Three Components Evolution in a Simple Big Bounce Cosmological Model

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We consider a five-dimensional Ricci flat Bouncing cosmology and assume that the four-dimensional universe is permeated smoothly by three minimally coupled matter components: CDM+baryons \( \rho_m \), radiation \( \rho_r \) and dark energy \( \rho_x \). Evolutions of these three components are studied and it is found that dark energy dominates before the bounce, and pulls the universe contracting. In this process, dark energy decreases while radiation and the matter increase. After the bounce, the radiation and matter dominates alternatively and then decrease with the expansion of the universe. At present, the dark energy dominates again and pushes the universe accelerating. In this model, we also obtain that the equation of state (EOS) of dark energy at present time is \( w_x \approx -1.05 \) and the redshift of the transition from decelerated expansion to accelerated expansion is \( z_T \approx 0.37 \), which are compatible with the current observations.

**Keywords**: accelerating universe; dark energy; Big Bounce

1. Introduction

In recent decades, the observations of high redshift Type Ia supernovae reveal that the expansion of our universe is speeding up rather than slowing down \(^{123}\). Meanwhile, the discovery of Cosmic Microwave Background (CMB) anisotropy on degree scales indicates \( \Omega_{total} \approx 1 \), and the galaxy redshift surveys indicates \( \Omega_m \approx 1/3 \). All these strongly suggest that the universe is permeated smoothly by 'dark energy', which violate the strong energy condition and has negative pressure. The dark energy and accelerating universe has been discussed extensively from different points of view. Usually, inspired by inflation, dark energy was treated as a scalar field which is minimally coupled with conventional matter, such as in quintessence \(^5\), phantom \(^6\) and k-essence \(^7\) model. The kinematic interpretation of the relationship between SN Ia luminosity distance and red-shift implies that the transition from decelerated expansion to accelerated expansion is around \( z_T = 0.46 \pm 0.13 \) \(^3\).

It has been drawn great attention to the idea that our conventional universe is embedded in a higher dimensional world as required in the Kaluza-Klein theories.

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and the brane world theories. In this paper, we consider a class of five-dimensional cosmological model which, as an alternative candidate to the standard 4D FRW model, has been discussed by many authors \(^8\). Instead of the Big Bang singularity of the standard model, this 5D cosmological model is characterized by a ‘Big Bounce’, which corresponds to a finite and minimal size of the universe. Before the bounce the universe contracts, and after the bounce it expands. This model is 5D Ricci-flat, as in Space-Time-Matter (STM) theory \(^9\), implying that the 5D space-time is empty, and the matter of the conventional 4D universe is induced from the fifth dimension. This approach is guaranteed by Campbell’s theorem \(^10\) that any solution of Einstein equation of \(N\) dimensions can be locally embedded in Ricci-flat manifold of \((N + 1)\) dimensions. So, the theory is consistent with the general theory of relativity (GR) locally. But, at large scale, it maybe different from GR. This is the motivation of this paper. In a previous work \(^8\), a time variable cosmological ‘constant’ is isolated out in a natural way from the induced 4D energy-momentum tensor. In this paper, instead of isolating a cosmological ‘constant’ from the energy momentum tensor, we assume that the universe is permeated smoothly by three components: CDM+baryons \(\rho_m\) with pressure \(p_m = 0\), radiation \(\rho_r\) with pressure \(p_r = \rho_r/3\), and dark energy \(\rho_x\) with pressure \(p_x = w_x \rho_x\) (\(w_x\) is in general a function of time, which is not put by prior.). By studying we will find that dark energy dominates before the bounce and pulls the universe contracting. In this process, dark energy decreases and the radiation and the matter increase. After the bounce, the radiation and the matter dominates alternatively and then decrease with the expansion of the universe. At present stage, dark energy dominates again and pushes the universe accelerating.

2. The dimensionless density parameters of the three components in the 5D model

Within the framework of STM theory, an exact 5D cosmological solution was given by Liu and Mashhoon in 1995 \(^11\). Then, in 2001, Liu and Wesson \(^8\) restudied the solution and showed that it describes a cosmological model with a big bounce as opposed to a big bang. The 5D metric of this solution reads

\[
dS^2 = B^2 dt^2 - A^2 \left( \frac{dv^2}{1 - kr^2} + r^2 d\Omega^2 \right) - dy^2,
\]

where \(d\Omega^2 \equiv (d\theta^2 + \sin^2 \theta d\phi^2)\) and

\[
A^2 = (\mu^2 + k) y^2 + 2\nu y + \frac{\nu^2 + K}{\mu^2 + k},
\]

\[
B = \frac{1}{\mu} \frac{\partial A}{\partial t} = \frac{\dot{A}}{\mu}.
\]

Here \(\mu = \mu(t)\) and \(\nu = \nu(t)\) are two arbitrary functions of \(t\), \(k\) is the 3D curvature index (\(k = \pm 1, 0\)), and \(K\) is a constant. This solution satisfies the 5D vacuum
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equations $R_{AB} = 0$. So we have three invariants

$$I_1 \equiv R = 0, I_2 \equiv R^{AB}R_{AB} = 0, I_3 = R_{ABCD}R^{ABCD} = \frac{72K^2}{A^8},$$

(3)

which show that $K$ determines the curvature of the 5D manifold. Using the 4D part of the 5D metric (1) to calculate the 4D Einstein tensor, one obtains

$$(4) \ G_0^0 = \frac{3(\mu^2 + k)}{A^2},$$

$$(4) \ G_1^1 = (4) \ G_2^2 = (4) \ G_3^3 = \frac{2\mu\dot{\mu}}{AA} + \frac{\mu^2 + k}{A^2},$$

(4)

It can be seen that there are two kinds of singularities corresponding to $A = 0$ and $B = 0$, respectively. $A = 0$ represents the usual "Big Bang" singularity. $B = 0$ (with $A \neq 0$) represents a new kind of singularity at which the three invariants in (3) are regular while $A$ reaches to its minimum ($B = \dot{A}/\mu$). So this new kind of singularity corresponds to a bouncing.

In the previous work, the induced matter was assumed that to be a conventional matter plus a cosmological 'constant' (which in fact is not constant but a function of time). In this paper, we assume the induced matter contains CDM+baryons $\rho_m$, radiation $\rho_r$ and dark energy $\rho_x$, which are minimally coupled with each other. So, we have

$$\frac{3(\mu^2 + k)}{A^2} = \rho_m + \rho_r + \rho_x,$$

$$\frac{2\mu\dot{\mu}}{AA} + \frac{\mu^2 + k}{A^2} = -\rho_m - \rho_r - \rho_x,$$

(5)

with

$$p_m = 0, p_r = \rho_r/3,$$

$$p_x = w_x\rho_x.$$  

(6)  

(7)

From Eqs. (5), (6) and (7), we obtain the equation of state (EOS) of the dark energy

$$w_x = \frac{p_x}{\rho_x} = -\frac{2\mu\dot{\mu}/AA + (\mu^2 + k)/A^2 + \rho_{r0}A^{-4}/3}{3(\mu^2 + k)/A^2 - \rho_{m0}A^{-3} - \rho_{r0}A^{-4}}$$

(8)

and the dimensionless density parameters

$$\Omega_m = \frac{\rho_m}{\rho_m + \rho_r + \rho_x} = \frac{\rho_{m0}}{3(\mu^2 + k)A},$$

$$\Omega_r = \frac{\rho_r}{\rho_m + \rho_r + \rho_x} = \frac{\rho_{r0}}{3(\mu^2 + k)A^2},$$

$$\Omega_x = 1 - \Omega_m - \Omega_r.$$  

(9)  

(10)  

(11)

where $\rho_{m0} = \bar{\rho}_{m0}A_0^3$, $\rho_{r0} = \bar{\rho}_{r0}A_0^4$ denote the present volume of the matter and radiation, $\bar{\rho}_{m0}$, $\bar{\rho}_{r0}$ is the current value of CDM+baryons and radiation densities, respectively.
3. The evolution of the three components in the 5D model

The Eqs. (8)-(11) contain only two arbitrary functions $\mu(t)$, $\nu(t)$ and another two parameters $K$ and $y$. So, by choosing the function $\mu(t)$, $\nu(t)$ and the parameters $K$ and $y$ properly, we can obtain properties of the dark energy which coincide with the present astronomical observing data. Under these choices, we discuss the evolution of the three components. Because the observations support a flat universe, we only consider the case $k = 0$. In this special case, the EOS of the dark energy and the dimensionless density parameters become

$$w_x = -\frac{2\mu \dot{A}}{A^2} + \frac{\mu^2}{A^2} + \frac{\rho_{r0} A^{-4}}{3},$$
$$\Omega_m = \frac{\rho_{m0}}{3\mu^2 A}, \quad \Omega_r = \frac{\rho_{r0}}{3\mu^2 A}, \quad \Omega_x = 1 - \Omega_m - \Omega_r.$$

(12)

For the complexity of the solutions, we analyze the properties of the solutions numerically.

Now, we analyze the dimensionless density parameters and compare them with the observed data. From Eq. (9)-Eq. (11), using $A_0/A = 1 + z$ and $\Omega_m/\Omega_x = \gamma_z$, we obtain the relation

$$3\mu_0^2 \frac{A^2}{A_0^2} z = \left(1 + \frac{1}{\gamma_z}\right) \bar{\rho}_{m0} (1 + z) + \bar{\rho}_{r0} (1 + z)^2.$$

(14)

So, at present time $z = 0$, and at the transition time from decelerated to accelerated expansion we let $z = z_T$. Then (14) the relation gives

$$3\mu_0^2 \frac{A^2}{A_0^2} = \left(1 + \frac{1}{\gamma_0}\right) \bar{\rho}_{m0} + \bar{\rho}_{r0},$$
$$3\mu_T^2 \frac{A^2}{A_0^2} = \left(1 + \frac{1}{\gamma_T}\right) \bar{\rho}_{m0} (1 + z_T) + \bar{\rho}_{r0} (1 + z_T)^2.$$

(15)

(16)

At the equilibrium point $z_E$, the matter density equals to the radiation density. So we have $\bar{\rho}_{r0} \sim \bar{\rho}_{m0}/(1 + z_E)$, where $z_E \sim 6000$. Thus we obtain

$$\frac{\mu_0^2}{\mu_T^2} = \frac{1 + 1/\gamma_0}{(1 + 1/\gamma_T)(1 + z_T)}.$$

(17)

The observed value $\gamma_0$ is $\gamma_0 \sim 3/7$ at present $z = 0$, and $\gamma_T \sim 1$ at the transition $z = z_T$. So, we obtain

$$\frac{\mu_0^2}{\mu_T^2} \sim \frac{5}{3(1 + z_T)}.$$

(18)

This implies that the function $\mu(t)$ determines the transition from decelerated expansion to accelerated expansion at the late epoch of the universe. In addition, the other function $\nu(t)$ would be constrained by $z_E$.

The Hubble and deceleration parameters should be given as

$$H(t, y) = \frac{1}{A} \frac{dA}{dt} = \frac{1}{B} \frac{\dot{A}}{A} = \frac{\mu}{A}.$$
\[ q(t, y) \equiv -A \frac{d^2A}{d\tau^2} / \left( \frac{dA}{d\tau} \right)^2 = -\frac{A\dot{\mu}}{\mu A}, \tag{19} \]

from which we see that \( \dot{\mu}/\mu > 0 \) represents an accelerating universe, \( \dot{\mu}/\mu < 0 \) represents a decelerating universe. So the function \( \mu(t) \) plays a crucial role of defining the properties of the universe in late time again. The deceleration parameter \( q_T = 0 \) \( (\dot{\mu} = 0) \) corresponds to the transition from deceleration to acceleration.

The scale factor \( A \) in Eq. (2) can also be written in the form

\[ A^2 = \frac{1}{\mu^2 + k} \left[ (\mu^2 + k) y + \nu \right]^2 + \frac{K}{\mu^2 + k}. \tag{20} \]

The three values of \( K \) \((K > 0, = 0, < 0)\) represents three types of the 5D manifold. By rescaling \( \mu^2 \) and \( k \), we can set these types as \( K = +1, 0, -1 \). In this paper we consider the type \( K = 1 \) which corresponds to a bouncing universe. Meanwhile, from (20) we can also see that the form \( A(t, y) \) is invariant under a translation along the \( y \)-direction \( y \rightarrow y + y_0 \) provided we redefine \( \nu(t) \rightarrow \nu(t) - (\mu^2 + k)y_0 \). Therefore, we can set \( y = 1 \) without lose of generality. So, in Eq. (20), we only have to determinate \( \mu(t) \) and \( \nu(t) \). Supposing \( \mu(t) = at + b/t, \nu(t) = ct \). The three constants \( a, b \) and \( c \) should be constrained and determined by the observations, such as the dimensionless density parameters \( \Omega_{m0} \), the EOS of dark energy \( w_{x0} \), the decelerated factor \( q_0 \) and the transition redshift \( z_T \).

By the above choice, we can obtain the transition time \( t_T = \sqrt{b/a} \) from Eq. (19). It is to say that the ratio of the parameters \( b \) and \( a \) constrain the transition from deceleration to acceleration. The model independent estimation of the cosmological parameters values at the present is \( \Omega_{m0} \sim 0.3, \Omega_{x0} \sim 0.7 \). To meet these data, we choose \( K = 1, y = 1, \rho_{m0} = 1.1, \rho_{x0} = 2.4, a = 0.000009, b = 3.5, c = 0.11 \), the evolution of the scale factor \( A \) is plotted in Fig. 1 and Fig. 2. The evolution of the three components is plotted in Fig. 3. The present observed values are \( \Omega_m \approx 0.3, \Omega_x \approx 0.7 \). We can see that they correspond to the time \( t_0 \approx 928 \) in Fig. 3. Meanwhile, \( \Omega_m = \Omega_x \) corresponds to the time \( t \approx 595 \). So from Fig. 2, we can conclude that the redshift of the transition from deceleration to acceleration is \( z_T \approx 0.37 \), which is close to the observation in \(^3\). In Fig. 1, the bounce time is \( t_b \approx 3 \). So in Fig. 3, we can see that: Before the bounce, the dark energy dominates and pull the universe to contract. After the bounce, the radiation and CDM+baryons dominate alternatively. The radiation dominates firstly, then the CDM+baryons dominated. After \( t \approx 624 \), the dark energy dominates again and push the universe accelerating at present. Also, the EOS of dark energy is plotted in Fig. 4. From above calculation, we can read \( w_{x0} = -1.05 \) at present. The evolution of the deceleration factor with \( t \) is plotted in Fig. 5. The universe begins accelerating from \( z_T \) on. From the Fig. 5., the transition time from deceleration to acceleration is \( t_T \sim 624 \), which corresponds to the redshift is \( z_T \sim 0.37 \).
The evolution of the scale factor $A(t, y) = \sqrt{(\mu y + \nu/\mu)^2 + K/\mu^2}$ with time $t \in (0, 14)$. Here, $K = 1$, $y = 1$, $\rho_{m0} = 1.1$, $\rho_{r0} = 2.4$, $a = 0.000009$, $b = 3.5$, $c = 0.11$. The bounce can be seen from the figure.

The evolution of the scale factor $A(t, y) = \sqrt{(\mu y + \nu/\mu)^2 + K/\mu^2}$ with time $t \in (0, 1000)$. Here, $K = 1$, $y = 1$, $\rho_{m0} = 1.1$, $\rho_{r0} = 2.4$, $a = 0.000009$, $b = 3.5$, $c = 0.11$.

4. Conclusions

The classes of five-dimensional cosmological solution (1) is characterized by a 'Big Bounce' which contrasted with the 'Big Bang' in standard cosmological models.
Fig. 3. The evolution of the three components $\Omega_m$ (the solid line), $\Omega_r$ (the dashed line), $\Omega_x$ (the dotted line). Where, $K = 1$, $y = 1$, $\rho_m0 = 1.1$, $\rho_r0 = 2.4$, $a = 0.000009$, $b = 3.5$, $c = 0.11$. The redshift of transition from decelerated expansion to accelerated expansion is $z_T \sim 0.37$.

Fig. 4. The EOS of dark energy evolution with time $t$. Here, $K = 1$, $y = 1$, $\rho_m0 = 1.1$, $\rho_r0 = 2.4$, $a = 0.000009$, $b = 3.5$, $c = 0.11$.

Mathematically, the solution contains two arbitrary functions $\mu(t)$, $\nu(t)$. Different choices of the functions may give different models to describe different stages of the universe evolution. In this paper, the induced matter contains three components $\rho_m$, $\rho_r$, $\rho_x$. By choosing the two arbitrary functions properly, we conclude that before
Fig. 5. The evolution of deceleration parameter with time $t$. Here, $K = 1$, $y = 1$, $\rho_m = 1.1$, $\rho_r = 2.4$, $a = 0.000009$, $b = 3.5$, $c = 0.11$. The time $t$ and redshift of transition from decelerated expansion to accelerated expansion are $t_T = \sqrt{b/a} \sim 624$ and $z_T \sim 0.37$ respectively.

the bounce, the dark energy dominates and pull the universe to contract. After the bounce, the radiation and CDM+baryons dominate alternatively. Firstly the radiation dominates, then the CDM+baryons dominates. At not a distance past, the dark energy dominates again and push the universe accelerating. The equation of state of dark energy is $w_x \approx -1.05$ at present. The redshift of the transition from decelerated expansion to accelerated expansion is $z_T \approx 0.37$. These results are compatible with the current observations.

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