Research on Error Correction Algorithm of Vehicle SINS and Odometer Integrated Navigation

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Abstract. In order to solve the problem of odometer small differential error and pulse calculation error and improve the precision of SINS and Odometer Integrated Navigation, this paper proposes an error correction algorithm of vehicle SINS and odometer integrated navigation, compares the calculated position of SINS with the calculated position of odometer as the measured value of integrated navigation within the fixed mileage range and presents a method to correct the truncation error of odometer counting. The simulation results show that these methods can quickly separate the sensor errors of the integrated navigation system, have the high positioning and orientation accuracy. Also, these methods can be applied in the practical vehicle SINS system.

1. Introduction

Odometer is an autonomous auxiliary measurement sensor, which is suitable for high autonomy requirements and the integrated navigation field limited by GPS. The odometer integrated navigation can achieve high positioning and orientation accuracy. The speed and position measurement schemes based on odometer are all commonly measurement schemes in SINS and odometer integrated navigation. In reference 7, the MEMS-IMU was installed in the center of the carrier. The velocity measurement could be obtained by using the moving feature of the MEMS-IMU rotated with the wheel, thus the three-dimensional velocity measurement, together with the non-holonomic constraint of the lateral and vertical velocity, could be utilized to improve the performance of the integrated navigation. In reference 8, odometer speed information and non-holonomic data were utilized to improve positioning precision, and the odometer could effectively restrain the error divergence of the navigation system. From the experiment, the position accuracy was improved more than 90%, when GNSS was interrupted for 60s or 10min. In reference 9, state equations and measurement equations of SINS/DR integrated navigation were established, the difference of position between SINS and DR was used as the input of kalman filter, and installation error and scale factor error were estimated to compensation for dead reckoning. In reference 10, position measurement method is adopted with the 18 dimension state space. The difference between the increment of SINS position and the increment of dead reckoned position in 1s was taken as measurement values. It overcame the shortcomings of the traditional position measurement scheme in 21 dimensional state space and slow convergence of attitude error angle. However, due to the different vehicle speed, the accumulated position increment of 1s is different, and the noise distribution is also different in each time period. Under the condition of low-speed driving, the output pulse of odometer has less numbers.
in 1s. If the accuracy of odometer is not high, the truncation error of counter will cause the mileage increment error \cite{11}.

This paper adopts the scheme of position measurement, which does not increase the dimension of state. The measurement value is provided according to the difference between the position of SINS calculation and dead reckon value of odometer in the fixed mileage range. At the same time, the truncation error of odometer is corrected to reduce the incremental error of odometer under different speed and noise distribution. The effectiveness of the scheme is verified by system simulation.

2. Integrated navigation model of SINS based on odometer

The geocentric coordinate system \( i \) is selected as the inertial coordinate system, the rotation rate of the earth is \( \omega_n \), and East-North-Up coordinate is selected as navigation coordinate system \( n \). The origin of vehicle coordinate system \( b \) is at the vehicle centroid. The \( x_b \) axis is right along the transverse axis of the vehicle, the \( y_b \) axis is forward along the longitudinal axis of the vehicle, and the \( z_b \) axis is up along the longitudinal axis of the vehicle. The origin of odometer coordinate system \( m \) is located at the intersection of wheel and ground. The \( x_m \) axis points to the right along the transverse axis of the car body, the \( y_m \) axis points to the front along the longitudinal axis of the car body, and the \( z_m \) axis is perpendicular to the \( x_m \) axis and \( y_m \) axis, forming the right-hand rectangular coordinate system.

The state error \( X \) of integrated navigation system is 18 dimensional vector, which is expressed as:

\[
X = \begin{bmatrix} \phi^T & (\delta V^n)^T & (\delta P^n)^T & (\epsilon^b)^T & \delta k_d & \delta \xi^b_m & \delta \zeta^b_m \end{bmatrix}^T
\]

(1)

\( \phi^n \) is the attitude error, \( \delta V^n \) is the velocity error, \( \delta P^n \) is the position error, \( \epsilon^b \) is the accelerometer bias, \( \delta k_d \) is the gyro drift, \( \delta k_d \) is the odometer scale factor error, \( \xi^b_m \) is pitch installation angle between \( b \) and \( m \) and \( \zeta^b_m \) is azimuth installation angle. The integrated navigation system adopts the Kalman filter, and the state equation is:

\[
\dot{X} = \begin{bmatrix} A_1 & A_2 & A_3 & C_n^e & 0_{3 \times 3} & 0_{3 \times 3} \\
A_4 & A_5 & A_6 & 0_{3 \times 3} & -C^n_b & 0_{3 \times 3} \\
0_{3 \times 3} & A_9 & A_8 & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\
0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\
0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \end{bmatrix} X + W
\]

(2)

\( A_1 \sim A_6 \) are shown below:

\[
A_1 = \begin{bmatrix} 0 & -a_y & a_z \\
a_y & 0 & -a_x \\
-a_z & a_x & 0 \end{bmatrix}, \quad A_2 = \frac{1}{h} \begin{bmatrix} v_y \tan L - v_x & 2h \omega_n \sin L + v_y \tan L & -2v_x \omega_n \cos L + v_x \sin L \\
-v_x \omega_n \sin L & v_y \cos L + v_x \tan L & 2v_x \omega_n \cos L + v_y \sin L \end{bmatrix},
\]

\[
A_3 = \frac{1}{h} \begin{bmatrix} 2h \omega_n (v_y \cos L + v_x \sin L) & 2h \omega_n (v_y \cos L) & 0 \\
-2v_x \omega_n \sin L & 2h \omega_n v_x \sin L & 0 \end{bmatrix},
\]

\[
A_4 = \frac{1}{h} \begin{bmatrix} 2h \omega_n (v_x \cos L + v_y \sin L) & 2h \omega_n (v_y \cos L) & 0 \\
-2v_x \omega_n \sin L & 2h \omega_n v_y \sin L & 0 \end{bmatrix},
\]

\[
A_5 = \frac{1}{h} \begin{bmatrix} 2h \omega_n (v_x \cos L + v_y \sin L) & 2h \omega_n (v_y \cos L) & 0 \\
-2v_x \omega_n \sin L & 2h \omega_n v_y \sin L & 0 \end{bmatrix},
\]

\[
A_6 = \frac{1}{h} \begin{bmatrix} 2h \omega_n (v_x \cos L + v_y \sin L) & 2h \omega_n (v_y \cos L) & 0 \\
-2v_x \omega_n \sin L & 2h \omega_n v_y \sin L & 0 \end{bmatrix},
\]

\[
A_7 = \begin{bmatrix} 2h \omega_n (v_x \cos L + v_y \sin L) & 2h \omega_n (v_y \cos L) & 0 \\
-2v_x \omega_n \sin L & 2h \omega_n v_y \sin L & 0 \end{bmatrix},
\]

\[
A_8 = \begin{bmatrix} 2h \omega_n (v_x \cos L + v_y \sin L) & 2h \omega_n (v_y \cos L) & 0 \\
-2v_x \omega_n \sin L & 2h \omega_n v_y \sin L & 0 \end{bmatrix},
\]

\[
A_9 = \begin{bmatrix} 2h \omega_n (v_x \cos L + v_y \sin L) & 2h \omega_n (v_y \cos L) & 0 \\
-2v_x \omega_n \sin L & 2h \omega_n v_y \sin L & 0 \end{bmatrix},
\]
\[ a^a = \begin{bmatrix} a_x & a_y & a_z \end{bmatrix}^T \]
\[ v^a = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}^T \]

\section{3. Error correction of odometer integrated navigation}

\subsection{3.1 Odometer error model}

The odometer uses pulse mode to output the mileage increment within the sampling time interval. The odometer scale factor is expressed as \( k_d \), the number of pulses output is expressed as \( N_A \) in the sampling interval \( i \). Then mileage increment of odometer output in system \( b \) is:

\[ \Delta S_i^b = \begin{bmatrix} \cos \varphi_m^b & \sin \varphi_m^b & \rho_m^b & \cos \varphi_m^b & \sin \varphi_m^b \end{bmatrix}^T k_d N_A \quad (3) \]

It can be seen that \( \Delta S_i^b \) is only related to \( \varphi_m^b \) and \( \varphi_m^b \), but not to roll installation angle \( \psi_m^b \). The SINS / odometer integrated navigation system is calibrated before use, and the error components are small after calibration. If the second order is ignored, the projection \( \Delta \hat{S}_i^b \) of the actual output increment of odometer in vehicle coordinate system \( b \) can be expressed as:

\[ \Delta \hat{S}_i^b = \Delta S_i^b + \begin{bmatrix} \Delta S_i^{bn} \delta k_d + \Delta S_i^{bn} \delta \varphi_m^b \\ \Delta S_i^{bn} \delta k_d + \Delta S_i^{bn} \delta \varphi_m^b - \Delta S_i^{bn} \delta \varphi_m^b \\ \Delta S_i^{bn} \delta k_d - \Delta S_i^{bn} \delta \varphi_m^b \end{bmatrix} + n \quad (4) \]

\subsection{3.2 Modification of odometer error}

Due to technology and cost, \( N_D \) is generally not too high, and \( \Delta t \) is short. Under the condition of low-speed driving, the calculation value of odometer pulse may have the truncation error, so that the odometer pulse count is not equal to the resolution \( N_D \) of odometer in a circle. Therefore, the truncation error of the odometer value will cause the mileage increment error, reduce dead reckoning accuracy of the odometer, and affect the accuracy of the integrated navigation system. In order to reduce the truncation error of the odometer counter, the odometer pulse value must be modified within one driving circle. Let \( \Delta N_D = N_D - \sum_{j=1}^{N_j} N_j \), based on the odometer coordinate system \( m \), the SINS position increment is \( \left( C_m^b \right)^T C_b^a \left( t_{j-1} \right)^T V^n \Delta t \) in \( \Delta t \) time. The forward position increment of odometer is corrected by formula (5).

\[ \Delta S_i^{wo} = K_p N_j + \frac{\Delta N_D}{\sum_{j=1}^{N_j} \left( C_m^b \right)^T C_b^a \left( t_{j-1} \right)^T} T \left( C_m^b \right)^T C_b^a \left( t_{j-1} \right)^T V^n \quad (5) \]
3.3 Measurement equation

Using the position measurement scheme, the measurement value $Z$ is the difference between the calculated position of SINS and the calculated position of odometer. Within the $i$-th SINS solution time $\Delta t$, the solution position $\Delta P^n_i$ of SINS and the calculated position $\Delta P^n_d$ of odometer are respectively:

$$\Delta P^n_i = \Delta t \left( V^n_i + \delta V^n_i \right)$$  \hspace{1cm} (6)

$$\Delta P^n_d = C^n_b(t_{i-1}) \Delta \tilde{S}^b = C^n_b(t_{i-1}) \Delta S^n_i + C^n_b(t_{i-1}) \Delta S^n_b \phi^n + \sum_{j=1}^{K} \left( C^n_b(t_{i-1}) M^n_j \right) \delta k^n_j + C^n_b(t_{i-1}) n$$  \hspace{1cm} (7)

$V^n_i$ is the true velocity of the vehicle at $t_i$, $\delta V^n_i$ is the velocity error of SINS, and $C^n_b(t_{i-1})$ is the attitude matrix of the vehicle at $t_{i-1}$.

In order to avoid different noise caused by different vehicle speed in the unit time and mileage increment error caused by counter truncation error under the condition of vehicle low speed, etc, the photoelectric coding odometer with positioning signal is used as auxiliary navigation equipment in this paper. When the vehicle wheels are driven for a circle, the odometer outputs high-level positioning pulse signal to represent a fixed mileage. The difference between the calculated value of inertial navigation and the dead reckon value of odometer is taken as the position measurement value in the fixed mileage range. Assuming that the number of calculation cycles of SINS is $N$ in the fixed mileage range, the measured value is expressed as:

$$Z = \sum_{i=1}^{N} \Delta P^n_i - \Delta P^n_d$$  \hspace{1cm} (8)

$\Delta P^n_d$ is dead reckoning position increment of odometer in the fixed mileage range. when the wheel is driven for a circle, the output of the odometer is fixed in the odometer coordinate system $m$. However, due to the change of vehicle attitude matrix, the projection of odometer is different in navigation coordinate system $n$. Therefore, $\Delta P^n_d$ is still calculated by formula (7).

The real position of SINS solution is the same as dead reckoning real position of odometer during driving, then $V^n_i \Delta t = \sum_{j=1}^{K} \left( C^n_b(t_{i-1}) \Delta S^n_j \right)$. The values of $\Delta P^n_i$ and $\Delta P^n_d$ can be put into formula (8). $\phi^n, \delta V^n, \delta k^n_j, \delta k^n_{mb}$ and $\delta m^n_{mb}$ can be approximately regarded as a constant value, when the wheel is driven for a circle. The measurement equation of SINS/odometer integrated navigation can be obtained as follows:

$$Z = \left[ K \Delta t \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \sum_{j=1}^{K} \left( C^n_b(t_{i-1}) \Delta S^n_j \right) \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} - \sum_{j=1}^{K} \left( C^n_b(t_{i-1}) M^n_j \right) \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right]^T X + V$$  \hspace{1cm} (9)

4. Experimental Verification

The fiber optic gyro SINS IMU is selected, gyro random constant drift is $0.02^\circ/h$, random noise variance is $0.01^\circ/h$, accelerometer random constant bias is $1 \times 10^{-4} g$, random noise variance is $1 \times 10^{-5} g$. IMU is installed on wheeled vehicle and calibrated in advance. The calibration value of odometer scale factor is $K_p = 0.0146 (m/p)$. The initial alignment of the stationary base is carried out for 5 minutes before the vehicle is running, then the vehicle runs in the circles repeatedly. After two hours of continuous movement, the data are analyzed offline, and the results are shown in Figure...
1-5. Figure 1 shows the positioning error, figure 2 shows RMSE curve of attitude error angle, Figure 3 shows RMSE curve of gyro drift, figure 4 shows RMSE curve of accelerometer bias, and Figure 5 shows RMSE curve of odometer scale factor error and installation error angle.

It can be seen that the horizontal positioning error of the vehicle is about 30 m. The height error is divergent, because there is no height measuring equipment such as altimeter. With the increase of driving time, the horizontal positioning error does not increase continuously, and it tends to be stable. From figure 2-5, it can be seen that the algorithm can effectively estimate the main error sources such as attitude error angle, gyro drift, acceleration zero deviation, odometer calibration error and system installation error, etc. The estimated errors can be fed back to the system to reduce the influence of the main error sources on the horizontal positioning error, and suppress the divergence trend of the horizontal positioning error effectively.
5. Conclusion
The position measurement scheme is adopted in the SINS/odometer integrated navigation system. The measured value of the system is the difference between the dead reckon value of the odometer and the calculated position value of the SINS within the fixed mileage range. The dead reckoned position value of odometer is corrected by the truncation error correction algorithm. With this scheme, the sensor errors of SINS can be calculated quickly without amplifying the measurement noise, and the accuracy of position and orientation can meet the requirements of the vehicle integrated navigation system.

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![Figure 5. RMSE curve of odometer scale factor error and installation error angle](image-url)
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