The Use of Direct Solver in Vector Finite Element Modeling for Calculating 3-D Magnetotelluric Responses

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Abstract. In this work, we seek numerical solution of 3-D Magnetotelluric (MT) using edge-based finite element method. This approach is a variant of standard finite element method and commonly referred as vector finite-element (VFE) method. Nonphysical solutions usually occurred when the solution is sought using standard finite element which is a node based element. Vector finite element attempt to overcome those nonphysical solutions by using the edges of the element as vector basis. The proposed approach on solving second order Maxwell differential equation of 3-D MT is using direct solver rather than iterative method. Therefore, divergence correction to accelerate the rate of convergence for its iterative solution is no longer needed. The utilization of direct solver has been verified previously for correctness by comparing the resulting solution to those given by analytical solution, as well as the solution come from the other numerical methods, for earth layered model, 2-D models and COMMEMI 3D-2 model. In this work, further verification resulted from recent comparison model of Dublin Test Model 1 (DTM1) is presented.

1. Introduction
In this work, we seek numerical solution of 3-D Magnetotelluric (MT) using edge-based finite element method. This approach is a variant of standard finite element method and commonly referred as vector finite-element (VFE) method (Jin, 2002). Nonphysical solutions usually occurred when the solution is sought using standard finite element which is a node based element. Vector finite element attempt to overcome those nonphysical solutions by using the edges of the element as vector basis.

The proposed approach on solving second order Maxwell differential equation of 3-D MT is basically similar to the one developed by Farquharson and Miensopust, 2011. However, unlike their approach that used iterative solver to solve the resulting electric-field system of equations, robust and efficient direct solver PARDISO (Schenk and Gartner, 2004) is used here. Therefore, divergence correction to accelerate the rate of convergence for its iterative solution is no longer needed.

The correctness of modeling solution need to be verified by comparing with solution from other numerical method (e.g., Zhdanov et al., 1997). Comparison on the resulting solution to those given by analytical solution, as well as the solution come from the other numerical methods, for earth layered model, 2-D models and COMMEMI 3D-2 model has been presented in Prihantoro et al., 2015. In this work, further verification resulted from recent comparison model of Dublin Test Model 1 (DTM1) is presented.
2. Numerical Method
A computer program in FORTRAN has been developed that is capable of solving second order Maxwell differential equation in 3-D MT problem that can be written as follows,

\[ \nabla \times (\nabla \times E) + i \omega \mu_0 \sigma E = 0 \quad \text{in} \quad \Omega \]  

\[ n \times E = 0 \quad \text{on} \quad \Gamma \]  

where \( E \) is electric field, \( i \) is imaginary number, \( \omega \) is angular frequency, \( \mu_0 \) is magnetic permeability of free space, \( \sigma \) is conductivity, \( \Omega \) is volume domain under consideration and \( \Gamma \) is a surface boundary enclosing the domain. Robust and efficient direct solver PARDISO (Schenk and Gartner, 2004) is used to solve large sparse complex symmetric matrix arising from model discretization.

After all electric field is defined in all space of the volume domain, the corresponding magnetic field is computed from Faraday’s law. Comparison of electric and magnetic field at the surface of earth yield surface impedance tensor. Apparent resistivity, which is derived from surface impedance tensor, can then be calculated at the surface of the earth. These derives quantity is useful for interpretation since its indirectly reflecting the true subsurface resistivity structure and it is the common way of presenting MT field measurement.

**Figure 1.** Apparent resistivity (in logarithmic scale) pseudo-section of DTM1 along the centre profile for XY (left) and YX (right). The upper part is from data set obtained using finite-difference (FD) method (Miensopust et al., 2013).
3. Test on Magnetotelluric Response Function

Figure 1 showing pseudo-section of DTM1 along the Centre profile for XY (left) and YX (right) apparent resistivity. The resulting pseudo-section show good agreement with another results obtained using finite-difference (FD) method (Miensopust et al., 2013). However, slightly over determined apparent resistivity is observed. This might be due to difference in discretization of the model calculated using FD and VFE.

Horizontal contour map of the coordinate invariant apparent resistivity at three different frequency (10, 1 and 0.0001 Hz) and iso-volume for low and high apparent resistivity are shown in figure 2. Similarity is observed between the invariant apparent resistivity with the true geometrical structure of the resistive and conductive bodies of DTM1 model.

![Coordinate invariant apparent resistivity](image)

**Figure 2.** Horizontal contour map of coordinate invariant apparent resistivity (in logarithmic scale) at three different frequency (10, 1 and 0.0001 Hz) and iso-volume of low and high apparent resistivity those are correspond to geometrical structure of the resistive and conductive bodies of DTM1 model.

4. Conclusion

We have developed computer program in FORTRAN to compute numerical solution of 3-D Magnetotelluric (MT) response based on vector finite element method utilizing direct solver. Comparison with DTM1 model has shown that the resulting numerical solution is reliable. We have also demonstrated that the coordinate invariant apparent resistivity is in highly similarity to geometrical structure of the resistive and conductive bodies of DTM1 model.

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