Quelling the concerns
of
EPR and Bell

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Abstract
We begin with a review of the famous thought experiment that was proposed by Einstein, Podolsky and Rosen (EPR) and mathematically formulated by Bell; the outcomes of which challenge the completeness of quantum mechanics and the locality of Nature. We then suggest a reinterpretation of the EPR experiment that utilizes observer complementarity; a concept from quantum gravity which allows spatially separated observers to have their own, independent reference frames. The resulting picture provides a self-consistent resolution of the situation that does not jeopardize causality nor unitarity, nor does it resort to “spooky” (non-local) interactions. Our conclusion is that EPR and Bell rely on an overly strong definition of locality that is in conflict with fundamental physics.
1 Introduction

1.1 Prologue

Quantum mechanics is a field of study that is infested with counter-intuitive concepts, and many of our classical preconceptions are brought into question when we deal with situations in the quantum realm.

At the forefront of quantum paradoxes is the thought experiment that was put forward in the famous and influential paper by Einstein, Podolsky and Rosen (EPR). The EPR paper sparked a debate that is still in progress and requires a resolution if quantum mechanics is to be validated as a theory which gives a complete description of reality in its domain of applicability.

This paper examines the EPR argument as well as Bell’s formulation thereof and then, using a concept brought in from quantum gravity, suggests a different tact for addressing the unease of the situation. In brief, by reworking the EPR and Bell scenario within the framework of observer complementarity, any
“spooky action at a distance” is vanquished and the concerns of EPR and Bell are resolved.

1.2 Where it all started

The EPR thought experiment presented a challenge for quantum mechanics. It highlighted a fundamental paradox within the theory which suggested that, while the theory worked as a calculation device, it may not represent reality.

In the EPR paper [1], the argument begins with the consideration of a standard quantum-mechanical concept; namely, non-commuting operators. Let us consider an experimenter, called Alice, who has access to a particle in a state given by $|\Psi\rangle$. The mathematical description of Alice physically measuring a property of the particle, say momentum, is given by acting on the initial state with the relevant operator. The value that the operator retrieves is associated with the physical value that would be measured by Alice. EPR does not present a comprehensive definition of reality but, rather, remains satisfied with a “criterion of reality” by which they identify a physical reality with the corresponding physical quantity. By this criterion, the value produced by an operator acting on a state represents a real physical quantity; an element of reality.

However, quantum operators are not always so compliant and the issue of non-commuting operators arises. The momentum and position of a particle form just such a pair of incompatible operators. The EPR argument then states that, if Alice had measured the position of the particle rather than its momentum, the position has physical reality but the momentum does not. The original state is changed by the measurement interaction and the momentum value of the original state is lost as a consequence of measuring the position. And so there are pairs of operators for which the corresponding physical values may be determined but having a definite value for one implies that no definite value exists for the other.\footnote{This statement is essentially an iteration of the Heisenberg uncertainty principle.} By this reasoning, the EPR argument concludes that we must either accept that these physical quantities relating to incompatible observables do exist but quantum mechanics does not fully describe them or that the properties associated with two non-commuting operators cannot have physical reality at the same time. Only one can be considered an element of reality.

The next step of the EPR argument involves combining the EPR reality criterion with the assumption that quantum mechanics does offer a complete description. Their logic is perhaps more easily explained in terms of the spins
of two particles, rather than the position and momentum of a single particle as originally used in the EPR paper. Bohm introduced this newer formulation of the EPR setup and showed that the argument’s structure remained the same, regardless of which pair of non-commuting observables are being considered \[2\]. Bohm’s scenario uses a pair of particles in a singlet state and the role of non-commuting operators is now adopted by the different components of either particle’s spin vector. This state can be described, in standard Dirac notation, as

\[ |\Psi\rangle = \frac{1}{\sqrt{2}} \left[ |a+\rangle|b-\rangle - |a-\rangle|b+\rangle \right], \tag{1} \]

with the two particles, \( \alpha \) and \( \beta \), having anti-correlated spins of \( a\pm \) and \( b\mp \), respectively. Here, + or − refers to spin up or down with respect to some chosen reference axis (typically but not exclusively, the \( z \)-axis).

We now introduce two spatially separated observers, Alice and Bob. They are each sent one particle, \( \alpha \) and \( \beta \) respectively, from an initial starting point. The particles, prior to their being sent off, are prepared in a singlet state as described above. All the information about the particles is provided by the relevant theory; presumably, orthodox quantum mechanics. (Later on, \( \lambda \) is used to denote, schematically, the encapsulation of all this information.) When the particles arrive at Alice and Bob, each observer has a choice as to which direction they choose to measure. In general, they choose directions that differ from the initial reference axis and differ from one another.

Let us now focus on Alice’s measurement of \( \alpha \). Alice can determine \( \alpha \)’s spin in one direction only, say \( \hat{n}_a \). However, the perfect anti-correlation of the particles means that, by measuring the spin in a given direction, Alice will be able to predict \( \beta \)’s spin in that same direction. By the reality criterion of EPR, this means that \( \beta \) has a physical property relating to Alice’s measurement value. This physical property, as EPR points out, cannot have “sprung” into existence at Bob’s location as a result of Alice’s measurement as this violates the condition of locality \[1\]. A non-local action by Alice cannot instantaneously effect Bob. It should be noted, though, that this is not a violation of causality but of the definition of locality as assumed by EPR. Although Bob’s outcome can depend on the choice made by Alice (assuming that he, by chance, chooses \( \hat{n}_b = \hat{n}_a \)), she cannot communicate with or signal Bob using only the anti-correlation of the particles (\( e.g., [4] \)). In order to signal Bob, Alice would be required to pick up a phone, write an email or physically move to Bob’s location to relay any information regarding the chosen direction. The puzzle does not lie, then, in the

\[2\] The conventions of this paper are aligned with [3].
realm of faster-than-light signaling but in the world of “spooky” dependence of the state on non-local variables [5].

EPR’s contention, therefore, is that the physical quantity corresponding to the spin of $\beta$ in the direction $\hat{n}_a$ must have been determined before the spatial separation of the two spins. Then, remembering that Alice could have chosen any direction to measure in, EPR are lead to the conclusion that all the physical properties of $\beta$’s spin must have been similarly predetermined. This, by their reality criterion, implies a physical reality for the spin of $\beta$ in each and every direction. And so we must face the contradiction of having a set of non-commuting operators (the different components of spin) corresponding to simultaneously real properties.

As stated above, the EPR argument comes down to the following choice: The inability of quantum mechanics to completely describe physical reality versus the properties of non-commuting observables not simultaneously existing. But, since the above argument has shown that assuming the completeness of quantum mechanics leads to the simultaneous existence of non-commuting spin directions, the conclusion apparently must be that the quantum mechanics provides an incomplete description of all elements of reality. Although the EPR paper is very clear about its conclusion, there are those who disagree with this outcome. At the forefront of the dissension is Bohr’s interpretation of the EPR experiment [6]. One main concern for Bohr was EPR’s assumption that hypothetical experiments could be used in conjunction with experiments that were actually done; for example, considering the value of momentum after measuring the position. Bohr’s argument led him to conclude that the inability to describe certain situations is a part of reality.

This assumption of EPR — that physical meaning can be ascribed to hypothetical measurements — is a large part of the disagreement between Bell’s formulation of the EPR experiment and the argument that will be put forth in the current paper. The reliance of EPR and Bell’s theorem on counter-factual statements about such hypothetical events make their outcomes incompatible with observer complementarity, an important consequence of quantum gravity that may (and, as later argued, does) permeate into more “conventional” physics. This will be discussed in more detail below but, first, Bell’s position on the EPR experiment requires an explanation.
1.3 Bell’s theorem

Certainly, the most significant development arising out of the prolonged debate on the EPR experiment is Bell’s theorem [7, 8]. However, even though its importance is never disputed, what the theorem actually proves remains a matter of controversy (see, e.g., [9]). The discord can be attributed, in part, to an “assumption” that Bell makes in his theorem; namely, the infamous “hidden variables”. These being the variables that Bell includes in addition to those used in the standard description of quantum mechanics.

We will, for current purposes, turn to a modern interpretation of Bell’s work, as delineated in a series of articles by Norsen [10, 11, 12, 13]. This account of the theorem utilizes Bell’s description of locality as the primary starting point. (Bell’s locality definition is clarified in Section 2.) One of Norsen’s main points of emphasis is that Bell did not intend for his analysis to serve as a stand-alone argument. Instead, the EPR argument must be considered as the first of a two-step procedure, with the second step being the formulation of Bell’s celebrated inequality. This is a nuance that has been missed by some, leading to a misunderstanding about the significance of Bell’s “hidden variables”. All the relevant variables, whether hidden or otherwise, are meant to be contained in the initial-state parametrization, $\lambda$. Therefore, the inclusion or exclusion of hidden variables is really besides the point when Bell’s inequality is implemented. Consequently, what Norsen’s treatment shows is how any theory, with or without hidden variables, that is able to correctly describe the behavior of quantum particles must (in some cases) violate Bell’s inequality and, therefore, disobey his criteria for locality.

This argument will be reviewed, in due course, with particular emphasis on the role that is played by counter-factual statements; these being ubiquitous in Bell’s formulation. Such statements are, in this context, referring to hypothetical measurements that could have been performed by Alice and Bob but are never actually carried out. Following this review, we will present a different approach, from the perspective of observer complementarity, that resolves any issues regarding “spooky action at a distance”.

1.4 Nature’s censorship of paradoxes

The principle of observer complementarity arose out of horizon complementarity, which applies to Nature’s seemingly exotic behavior around the horizon of a black hole [14, 15, 16, 17]. In either case, the principle is that, although different observers can disagree on the occurrence of certain events, Nature will not allow
any observer to ever experience a paradoxical situation. In order to understand the basic argument, the notion of information flowing out of a black hole must first be understood.

It was Hawking who first showed that black holes slowly evaporate by emitting radiation with a nearly thermal (black-body) spectrum [18]. Importantly, this effect can be attributed to the influence of the black hole’s gravitational field on quantum matter fields; that is, black holes do not radiate unless quantum effects are accounted for. Therefore, black hole radiation is realized in a situation for which both gravity and quantum theory are important. This process, however, posed problems for the quantum side as a pure state entering the black hole will later be observed in the radiation as a mixed state; information is apparently lost [19]. Hawking later revised this position [20], as the viability of information loss was challenged by ideas from string theory, the tentative theory of quantum gravity [21]. These ideas suggest that any process which results in information loss is strictly forbidden in what is a manifestly unitary theory (namely, the quantum-field-theory dual to Einstein’s gravity [22]).

Hawking’s original calculation used only an approximation of quantum gravity because it described the black hole as a classical body. In recent work, a more rigorous version of Hawking’s calculation, using a description of the black hole in quantum terms, reveals that information is not lost [23] — much in the same way that information from a burning encyclopedia can, in principle, be recovered. These arguments are beyond the scope of the present paper, but what is essential is that information can indeed be accessed from the radiation.

The availability of this information to an observer outside of the black hole is the starting point of Susskind’s standard argument for horizon complementarity — see [24] for a simplified account.

To illustrate this argument, two observers are considered by Susskind; one falling freely into the black hole and one watching the black hole from afar. When the first observer, Alice, falls past the horizon of the black hole, she is seen by the distant observer, Bob, as being vaporized before ever reaching the horizon. This is because, from Bob’s perspective, the near-horizon region is extremely hot. Then, from the above description, it can be deduced that the “Hawking radiation” will eventually contain information about Alice that can be accessed by Bob.

However, Alice, from her point of view, experiences no change in scenery as

3 The horizon represents the “surface of no return”; classical matter can never escape from the black hole interior after passing through this causal boundary.
the space near the horizon of a large enough black hole is almost flat. And so we are left with a situation in which one or the other experiences a violation of the laws of nature; either Alice thermalizes instead of experiencing flat space or Bob observes Alice falling in without thermalizing. Susskind’s next step is to consider whether either observer must see a violation. An apparent resolution is to assume that the information (or Alice) is “cloned” at the horizon, so that one copy of Alice is radiated out and the second copy falls in. However, this requires violating a basic principle of quantum mechanics: Quantum information can not be duplicated in this way since it would be in conflict with the principle of linear superposition [27].

Considering the two observers, Susskind clarifies that the key to resolving this problem lies with the movement of information. Alice is safe from seeing any violation since she is trapped within the black hole horizon but Bob has the opportunity to gather Hawking radiation (and hence information) from the black hole and then follow Alice in. If, after entering the black hole, Bob could receive a signal from Alice, there would be a violation of Nature due to Bob having observed two cloned copies of the same information. Susskind, however, presents an argument which proves that this is not a problem.

After passing through the horizon, Alice has a limited time to send a signal to Bob before she hits the singularity of the black hole (where all matter would be destroyed by immensely large tidal effects). What Susskind shows — by applying the Heisenberg uncertainty principle and the knowledge that Bob has to wait a certain time before he can retrieve a copy of the signal from the Hawking radiation [28, 29] — is that Alice’s signaling device would then have to be more energetic than the black hole itself. Consequently, Alice’s device would no longer be able to fit inside the black hole [30], rendering the whole experiment as moot. And so we see that each observer has an individual account of events, which differ, but that neither observer can possibly compare these results and create a paradox; Nature simply does not allow it.

This idea is very relevant to the EPR problem. Horizon complementarity has since been molded into observer complementarity, which promotes the former principle to one having more general applicability [17]. Since horizon complementarity arises as a consequence of the synthesis of gravity and quantum theory, it appears to be a principle of quantum gravity. As the (presumed)
fundamental theory of physics, quantum gravity should be considered to be where all other theories emerge from, and so certain aspects of quantum gravity theory will apply to these emergent theories [31]. Certainly, not all aspects of quantum gravity will apply to all emergent theories, but there is no good reason not to consider the application of observer complementarity to, say, standard quantum mechanics. If an otherwise paradoxical situation can be resolved by observer complementarity, its usage can then be justified \textit{a posteriori}.

The above argument demonstrates that each observer of the black hole will have his or her own distinct account of the events having transpired. The expansion of this idea is that a theory describing the experiences of two or more observers must account for only one observer’s results at a time. A collective description of two (or more) experiments that are not causally connected can lead to paradoxical results and, if so, each experiment must then be described individually. We will apply this very idea to the EPR problem, allowing each observer to have his or her own description of the situation.

1.5 Similar stances

There are some approaches in the literature which are similar to that of the current paper but with different motivations.

One approach is that applied by Mermin in his so-called Ithaca interpretation of quantum mechanics [32, 33]. This viewpoint places its conceptual emphasis on the correlations between the constituent subsystems of the total quantum system. What Mermin shows is that these correlations are entirely captured by the system’s density matrix and can be revealed by suitable tracing procedures. He then argues that this is the correct framework for describing reality in the quantum world. Our stance is similar because, as seen later, applying observer complementarity is tantamount to tracing over the inaccessible variables of the density matrix.

Another such approach is that of “relational” quantum mechanics, as first presented by Rovelli [35]. This interpretation is founded on the idea of describing reality strictly in terms of relations between (quantum) observers. This is philosophically similar to but operationally distinct from observer complementarity. Indeed, Rovelli and Smerlak’s resolution of the EPR paradox [35] resembles the current presentation; nonetheless, our motivation will be focused on adhering to the requirements of observer complementarity without resorting to tracing the inaccessible variables.

Mermin originally asserted that correlations between subsystems provided a complete description of quantum reality but has since retracted this claim [34].
ing to additional assumptions and inputs from outside the realm of standard quantum mechanics.

Another common link between our treatment and Rovelli's is with regard to the concept of a “super-observer”. By assigning an element of reality to Alice's prediction of what Bob measures (or vice versa), EPR requires a hypothetical observer that can “see” the outcome of the prediction even if the implicated measurement never actually happens. Essentially, the predicted value must exist for some hypothetical observer who has access to all information that is held in the Universe. This element of the argument is elaborated on later in Section 4.

2 Bell (ala Norsen)

In order to better appreciate our observer-complementarity approach to the EPR paradox, we will first outline the results of Bell's mathematical treatment. The crux of Bell's argument lies in the formulation of his locality condition. Understanding this definition correctly is crucial to realizing the significance of Bell's inequality, and a diversity of locality definitions amongst authors has led to a divergence of opinions regarding the theorem and its results. Norsen, in particular, presents Bell's theorem by using a definition of locality that he refers to as “Bell locality” [10].

The condition relies mostly on Bell's original statements regarding a pair of space-like separated observables. The requirement of locality is that the probabilities associated with one of the observables, when worked out from a complete description of this one's past interactions, does not rely on the other, space-like separated observable. In other words, the information at Bob’s location must be irrelevant to the probabilities being calculated at Alice’s location (and vice versa).

And so, in terms of the EPR experiment, this locality condition translates into

\[ P(A|\hat{n}_a, \hat{n}_b, B, \lambda) = P(A|\hat{n}_a, \lambda), \]

where \( P \) is the probability of Alice's outcome being \( A \), conditioned by the specified variables on the right-hand side of the vertical divider. Also, \( B \) refers to the value measured by Bob, \( \hat{n}_a \) and \( \hat{n}_b \) refer to Alice and Bob's respective choices of measurement direction, and \( \lambda \) represents a complete description of the original singlet state as prescribed by the theory under scrutiny [9].
Equation (2), the mathematical statement of Bell’s criteria for locality, does not permit any non-causal action to influence the separated observations. It says that the probability of result \( A \) must be the same whether values from Bob’s location are taken into account or not, as any dependence of \( A \) on Bob’s outcome must be excised due to their acausal (space-like) separation. Put differently, Bob’s outcomes are non-local with respect to Alice’s.

Adjoint to this locality definition is the requirement of separability: The joint probability for Alice and Bob must factorize into the product of two separate probabilities, one for each observer individually [10]. This factorization should, as Bell argued [8], be considered as a consequence of the locality condition rather than a new input. The conceptual motivation for this being that the locality condition (2), as well as its \( A \leftrightarrow B \) converse, requires each observer to have independently calculated probabilities at their respective locations. But the joint probability for the singlet state must still be preserved, and so this expression should be separable into a pair of independent probabilities. The mathematical description of this factorization, in terms of the Alice–Bob setup, is then

\[
P(A, B|\hat{n}_a, \hat{n}_b, \lambda) = P(A|\hat{n}_a, \lambda) \cdot P(B|\hat{n}_b, \lambda).
\] (3)

Notice that, in addition to separating the joint probability on the left, we have applied Bell locality to each observer’s probability statement. Hence, each respective probability depends only on the locally chosen direction of measurement, \( \hat{n}_a \) or \( \hat{n}_b \), and information pertaining to the initial state of the measured particle as encoded in \( \lambda \). It is this relation that leads one to the inequality which Bell devised to test theories for adherence to locality.

The real problems emerge when this formalism is confronted with the results from actual quantum experiments. Rather than reiterate Bell’s derivation, we consider a simple example. As known from both experiment and standard quantum mechanics, the probability of measuring spin up or spin down for a member of a singlet pair, in any given direction, is 50%. Mathematically, this means for Alice that

\[
P(A = \uparrow |\hat{n}_a, \lambda) = \frac{1}{2},
\] (4)

where \( \lambda \) should now be regarded as the quantum-mechanical wave function describing the singlet state.

On the other hand, experimental results also tell us that Alice’s measurement
must adhere to the anti-correlation of the singlet, whereby

\[ P(A = \uparrow | \hat{n}_a, \lambda, \hat{n}_b = \hat{n}_a, B = \downarrow) = 1, \] (5)

\[ P(A = \uparrow | \hat{n}_a, \lambda, \hat{n}_b = \hat{n}_a, B = \uparrow) = 0. \] (6)

That is, when Bob’s chosen measurement direction corresponds with Alice’s choice, \( \hat{n}_b = \hat{n}_a \), the spin that Alice measures must be the opposite of Bob’s.

When these two outcomes are compared with the condition of Bell locality, one can see a clear violation. The locality condition demands that \( P(A|\hat{n}_a, \hat{n}_b, B, \lambda) = P(A|\hat{n}_a, \lambda) \). When \( \hat{n}_a \neq \hat{n}_b \), there is no such violation; for instance, the substitution of (4) and (5) into the locality condition (2) gives

\[ P(A = \uparrow | \hat{n}_a, \lambda) = \frac{1}{2} \neq 1 = P(A = \uparrow | \hat{n}_a, \lambda, \hat{n}_b \neq \hat{n}_a, B = \downarrow). \] (7)

However, when \( \hat{n}_a = \hat{n}_b \), the same substitutions rather yield

\[ P(A = \uparrow | \hat{n}_a, \lambda) = \frac{1}{2} \neq 1 = P(A = \uparrow | \hat{n}_a, \lambda, \hat{n}_b = \hat{n}_a, B = \downarrow). \] (8)

This inequality shows that, in order to respect Bell locality, quantum mechanics must include more than what is currently held in the wave function. It should be noted that the above outcome represents only one particular case of Bell’s more general mathematical statement, his celebrated inequality [7].

The inequality within (8) would seem to imply that quantum mechanics is simply incomplete as a theory and, in order to fully describe reality, extra or “hidden” variables are required. However, Norsen’s treatment shows that any theory which adheres to Bell locality, with or without these hidden variables, cannot explain the perfect anti-correlation which is verified by experiment [9]. This is because any such variables can be included \textit{a priori} in \( \lambda \) and, thus, lead to the very same conclusions. And so, in view of this argument, one is forced either to reject any adherence to Bell locality within quantum mechanics or to accept quantum mechanics only as a calculational device that does not fully describe reality.

The crucial part of Bell’s argument, as far as this paper is concerned, is the emergence of \textit{counter-factual definiteness} (CFD), as demonstrated and elucidated by Norsen [9]. CFD is the claim that a statement about a measurement which was \textit{not} performed can be discussed, in a meaningful way, alongside statements about actually performed experiments. This can be seen in (8), where the left-hand side of the expression assumes no knowledge of Bob’s measurement. This is equivalent to Bob not yet having performed any measurement, \footnote{Bohmian quantum mechanics is one such example of a hidden-variable theory [2].}.
as the left-hand side is not conditioned by any action taken by Bob. However, the right-hand side of (8) assumes Bob did perform a measurement and found a particular outcome which influences Alice’s result.

In order to illustrate this idea, let us consider the situation of placing a bet at a roulette wheel. A pessimistic gambler might suggest a “theory” that, if a bet is placed on red, the wheel will stop on black, otherwise it will stop on red. Perhaps, there is a particular spin of the wheel that substantiates the gambler’s (flawed) assertion. However, it cannot be claimed, after the fact, that a change in the bet would have reversed the outcome on the wheel. Such a claim constitutes a discussion of what could have happened, but did not, in lieu of what actually did.

This requirement of CFD within Bell’s theorem is an inherent part of the overall argument and cannot be arbitrarily eliminated — it arises as a direct consequence of Bell locality. The fact that assuming Bell locality necessarily implies the use of CFD is clearly demonstrated by Norsen [9]. Moreover, the same basic claim can be made for the stochastic or probabilistic reformulation of Bell’s inequality, which is known as the Clauser–Horne–Shimony–Holt inequality [37]. While probabilistic theories do remove CFD, they retain a slightly weaker form, counter-factual meaningfulness (CFM), which results in the same dependence on discussing events which could have happened but did not [11]. These CFD and CFM statements are, not only inevitable in the construction of Bell’s inequality, but will be a central element in our proposed resolution for alleviating the concerns of EPR and Bell.

3 Reworking EPR and Bell

We begin here with the same setup as previously considered: There is a prepared singlet state, as in (1), consisting of two particles, α and β. Each of the particles is sent to one of a pair of spatially separated observers, Alice and Bob, who are then free to measure the spin of their particles in the direction of their choosing. Alice will be measuring the spin of α, which is given by \( \vec{S}_\alpha \cdot \hat{n}_a = \pm \hbar / 2 \), and likewise for Bob, \( \vec{S}_\beta \cdot \hat{n}_b = \pm \hbar / 2 \).

Due to the properties of the singlet state, if Alice and Bob choose to measure the spin in different directions, then their results will have no correlation. For the sake of this argument, we are concerned with the case in which Alice and Bob

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\[ \int \rho(\lambda) |\lambda\rangle d\lambda \rightarrow \int \rho(\lambda) |\lambda\rangle d\lambda \] where \( \rho(\lambda) \) is a weight factor such that \( \int \rho(\lambda) d\lambda = 1 \). So that, for example, \( P(A = \uparrow | \hat{n}_a, \lambda) \rightarrow \int \rho(\lambda) P(A = \uparrow | \hat{n}_a, \lambda) d\lambda \).
measure along the same spin direction, as this is where the "problems" arise. We are assuming that they never plan to measure the same direction but end up doing so in the trial of immediate interest. Hence, this is not a description of a conspiracy between the observers to measure in the same direction.

Let us recall the singlet state \( \| \Psi_{\text{singlet}} \rangle = \frac{1}{\sqrt{2}} \left( |a+\rangle|b-\rangle - |a-\rangle|b+\rangle \right) \).

Given the coincidence of measurement directions, \( \hat{n}_a = \hat{n}_b \), the notation can now be reinterpreted as meaning \( a \pm = \hat{S}_a \cdot \hat{n}_a = \pm \hbar / 2 \) and \( b \mp = \hat{S}_b \cdot \hat{n}_b = \mp \hbar / 2 \). This is only a change of basis; the state has not been altered.

So far, we have been describing the particle system. To talk about observer complementarity, what we really need to look at is the particle–observer system. In this regard, a subtle point is that Alice and Bob cannot directly measure the spin of their respective particle. Each of their measurements involves some (other) observable, the measurement device, with its corresponding value indicating the result of the experiment. Let us denote this observable by \( |X_{\text{ready}} \rangle \) and conceptualize it as a pointer that begins in the horizontal position, facing zero. Then \( |X \uparrow \rangle \) and \( |X \downarrow \rangle \) will denote a measurement of spin up or spin down, respectively.

We now let \( |A \rangle \) and \( |B \rangle \) respectively represent the result of Alice and Bob’s measurements. The system for the particles \( \text{and} \) observers \( (P-O) \) can then be described as

\[
|\Psi_{P-O} \rangle = \frac{1}{\sqrt{2}} \left[ (|a+\rangle|A_{\text{ready}} \rangle|b-\rangle|B_{\text{ready}} \rangle - |a-\rangle|A_{\text{ready}} \rangle|b+\rangle|B_{\text{ready}} \rangle) \right],
\]

where the measurements have not yet been performed.

From here, we can calculate the density matrix of the combined system, \( \rho_{P-O} = \langle \Psi_{P-O} | \Psi_{P-O} \rangle \), giving

\[
\rho_{P-O} = \frac{1}{2} \left[ (|a+\rangle|A_{\text{ready}} \rangle|b-\rangle|B_{\text{ready}} \rangle \langle a + |A_{\text{ready}} \rangle|b - |B_{\text{ready}} \rangle \right. \\
\left. - |a-\rangle|A_{\text{ready}} \rangle|b+\rangle|B_{\text{ready}} \rangle \langle a + |A_{\text{ready}} \rangle|b - |B_{\text{ready}} \rangle \right. \\
\left. - |a+\rangle|A_{\text{ready}} \rangle|b-\rangle|B_{\text{ready}} \rangle \langle a - |A_{\text{ready}} \rangle|b + |B_{\text{ready}} \rangle \right. \\
\left. + |a-\rangle|A_{\text{ready}} \rangle|b+\rangle|B_{\text{ready}} \rangle \langle a - |A_{\text{ready}} \rangle|b + |B_{\text{ready}} \rangle \right].
\]

The density matrix provides us with a description of all the possible outcomes once the measurements occur. It should be stressed, though, that this density matrix is not attributed to any of our observers. It contains information
regarding two spatially separated locations; namely Alice and Bob’s measuring stations. In order to adhere to causality, we consider only local sources of information. For instance, since Alice is separated from Bob, her description must not access any information that is localized at Bob’s station. So that, to find Alice’s measurement outcomes, we must first determine the reduced density matrix relating to her experiment.

Given a quantum density matrix \( \rho \), it is standard operating procedure to remove what a given observer does not know by tracing out the hidden systems [38]. Mathematically, this entails calculating \( \rho_{\text{reduced}} = \sum_{a'} \langle a' | \rho | a' \rangle \), where \( |a'\rangle \) collectively represents the state kets for all the concealed systems. For Alice, this translates into

\[
\rho_{\text{Alice}} = \text{Tr}_{\text{Bob}}[\rho_{P-O}] = \sum_b \sum_B \langle b | \langle B | \rho_{P-O} | B \rangle | b \rangle,
\]

Therefore, Alice’s results are described by

\[
\rho_{\text{Alice}} = \frac{1}{2} \left[ |a+\rangle |A_{\text{ready}}\rangle (a + \langle A_{\text{ready}} | a - \langle A_{\text{ready}} |) , \right. \tag{12}
\]

and, for Bob, we find that

\[
\rho_{\text{Bob}} = \frac{1}{2} \left[ |b-\rangle |B_{\text{ready}}\rangle (b - \langle B_{\text{ready}} | b + \langle B_{\text{ready}} |) . \right. \tag{13}
\]

Let us briefly pause to consider the measurement process. For the sake of explanation, let us (following [39]) consider a device which measures the position of a particle. Given an initial state \( |x\rangle \) for the particle and \( |A\rangle \) for the measurement device, we can construct a description of the combined system as

\[
|\Psi\rangle = |x\rangle |A_{\text{ready}}\rangle.
\]

We can also define a unitary operator for the combined system — the interaction Hamiltonian, \( \hat{H}_{\text{int}} = \hat{X}\hat{\Pi}_A \). Here, \( \hat{X} \) is the position operator for the particle and \( \hat{\Pi}_A \) depicts a “momentum” operator that is associated with the measuring device. Formally, \( \hat{\Pi}_A \) is the canonical conjugate to \( \hat{A} \) and, conceptually, it is the operator which enables \( |A\rangle \) to record the measured result. Assuming a completely efficient measuring device, we can, without describing it explicitly, denote a measurement as

\[
|\Psi\rangle \to e^{i\hat{H}_{\text{int}}} |\Psi\rangle \tag{14}
\]

where \( \hat{X} \) and \( \hat{\Pi}_A \) are assumed to commute. And so the device has successfully registered the position of the particle. This is a very simple example. The
general case for quantum mechanics involves $|x\rangle \rightarrow |x'\rangle$. This is due to the
quantum state of the particle being changed by the interaction; the particle does
not necessarily end up in its original state.

Applying this basic idea of measurement to Alice's reduced density matrix,
we obtain
\begin{equation}
\rho_{\text{Alice}} = \frac{1}{2} \left[ \langle a^+ | A \uparrow \rangle \langle a + | A \uparrow \rangle + \langle a^- | A \downarrow \rangle \langle a - | A \downarrow \rangle \right]
\end{equation}

and analogously for Bob. Importantly, there is perfect correlation between the
pointer direction and the spin of the particle. Notice that the information from
Alice's measurement coupled with knowledge of the initial state would enable
her to predict the spin of $\beta$ in this same direction. But this can only be a prediction because of Alice being spatially separated from $\beta$. There would need to be some local interaction between Alice and $\beta$ (or Bob) to provide an actual observable that she could measure.

In order to ensure that quantum-mechanical consistency holds, we have to
verify that the correlation of the singlet state is preserved when Alice and Bob’s
notes are compared. To do this, we follow the example of [36] and consider a
third observer who checks up on Alice and Bob’s results after their measurements.

First, let us formulate the density matrix for the combined systems after
the measurements,
\begin{equation}
\tilde{\rho}_{P-O} = \frac{1}{2} \left[ \langle a^+ | A \uparrow \rangle \langle b^- | B \downarrow \rangle \langle a + | A \uparrow \rangle \langle b - | B \downarrow \rangle - \langle a^- | A \downarrow \rangle \langle b^+ | B \uparrow \rangle \langle a + | A \uparrow \rangle \langle b - | B \downarrow \rangle - \langle a^+ | A \uparrow \rangle \langle b^- | B \downarrow \rangle \langle a - | A \downarrow \rangle \langle b + | B \uparrow \rangle \right].
\end{equation}

Again, it should be pointed out that this density matrix is not attributed to any
observer in our system. Rather, it is a description that could only be utilized
by a super-observer having access to all the spatially separated situations. This
is in direct conflict with observer complementarity and so it must be made clear
that we do not consider this as a density matrix that can viably be compared
with those of the different observers.

We will rather use this matrix to determine the reduced density matrix for
the third observer, Carol, by tracing out what she has no access to; namely, the
initial states of $\alpha$ and $\beta$. This entails computing
\begin{equation}
\rho_{\text{Carol}} = \sum_{a'} \sum_{b'} \langle a'| \langle b' | \tilde{\rho}_{P-O} | b' \rangle | a' \rangle ,
\end{equation}

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where \( a' = \{a\pm\} \), \( b' = \{b\pm\} \) and, by tracing over the variables separately, we are not assuming any (anti-) correlations. The outcome is

\[
\rho_{\text{Carol}} = \frac{1}{2} \left[ |A\uparrow\rangle\langle B\downarrow| A\uparrow \langle B\downarrow | + |A\downarrow\rangle\langle B\uparrow| A\downarrow \langle B\uparrow | \right],
\]

which shows that the anti-correlation is preserved from Carol’s perspective.

And so we see that this series of local interactions, in compliance with observer complementarity and causality, holds no paradoxes. Any “spooky action at a distance” disappears when the view of each observer is restricted to his or her own reduced density matrix.

4 Discussion

4.1 Against counter-factual definiteness

The above results show that observer complementarity can account for the view of each respective observer, while still maintaining the anti-correlations between the particles as evident from Carol’s perspective.

On the other hand, Norsen’s formulation of Bell’s theorem proves that CFD is a necessary consequence of assuming Bell locality [9]. This can be weakened to CFM for a probabilistic theory but it still amounts to the same basic result. The theorem inevitably depends on discussing events that have no associated observer because, even though they could have happened, they did not actually happen. The idea of discussing, in a meaningful way, a statement about a hypothetical event is in opposition to the principle of observer complementarity. From the vantage point of this principle, an observer has no need to account for the results of an experiment that cannot be compared to his or her own findings. In other words, Alice need not account for Bob’s results (and vice versa) until they can be locally compared. Similarly, results from an experiment that was not actually performed can have no meaning in the observer-complementarity framework. It follows that CFD is an assumption that cannot be made in conjunction with observer complementarity; these being antithetical constructs. To show that one is true in the domain of a given theory would be enough to rule out the other within the same realm.

To this end, observer complementarity falls back on its origin. As already stated, quantum gravity can be considered as the fundamental theory from which all others are emergent. Therefore, as observer complementarity survives quantum gravity, it is the CFD assumption which ultimately must fall away. Then, in the spirit of Occam’s razor, the simplest way to relieve the tension
between CFD and observer complementarity is to discard CFD in any paradoxical situation. This would imply that, due to the inevitable appearance of CFD, Bell’s theorem is doomed from the start. With the assumption of Bell locality and, in the case of EPR, their criterion for reality, the respective arguments set themselves up to produce results that will imply either that Nature is non-local or that quantum mechanics is incomplete. However, considering the same situation without assuming Bell locality and, instead, using observer complementarity as the motivating principle, we see that the concerns fall away without forcing a rejection of either option. No contradiction is experienced, as shown by Carol’s results, and all action happens locally. This, along with the fact that observer complementarity is a consequence of quantum gravity, suggest that Bell locality and, by implication, CFD are too strong to define reality.

4.2 Against conspiracy

One might still be concerned as to how the particles “know” in advance which directions will be chosen by Alice and Bob. It becomes problematic if this information is not provided by the theory because the only other alternative is conspiratorial settings by Nature. That is, Nature would, somehow, be anticipating the choices of the observers and adjusting the particles accordingly but without allowing access to the determined values.

To address this question, let us consider the evolution of the system through time. It should be clear that the initial conditions of a system and its subsequent evolution produce the final conditions. But is the converse true? Can we not evolve the system back in time if given the final conditions? For a deterministic or classical environment, this is obviously acceptable. The quantum realm is not so deterministic, and so the answer is less clear. But, perhaps surprisingly, the proposition of evolving a quantum system back in time is valid as well. Indeed, the choice of final-state boundary conditions does not violate causality nor unitarity, and so we are free to apply reversed time evolution to our previously described experiment.

Let us then consider the backwards evolution from a time $t_1$, when Alice has measured a + spin on her particle in the direction $\hat{n}_a$. We can evolve this state backwards to a time $t_0$, just when the particles were separated. As particle $\alpha$ experiences no interactions during the intervening period, its time evolution is...
trivial,

\[ |\hat{n}_a, +\rangle_{t_0} = e^{i\hat{H}_{\text{free}}(t_0 - t_1)}|\hat{n}_a, +\rangle \]
\[ = (\text{phase})|\hat{n}_a, +\rangle, \]

where we have used that the free-particle Hamiltonian commutes with all spin operators. Since, at time \(t_0\), the particles are known to be anti-correlated, it can be deduced that

\[ (\text{phase})|\hat{n}_a, +\rangle|\hat{n}_b = \hat{n}_a, -\rangle \]

(20)

describes the state of the two particles.

Hence, as far as Alice is concerned, there has been no conspiracy, and the same is true for Bob. But what about the initial holder of the singlet state? After all, the state (20) differs from the original singlet state (9), and we cannot appeal to observer complementarity in this case since the initial location is causally linked to both Alice nor Bob. Nevertheless, what can be appealed to is that the wave function depends on an observer’s choice of “gauge”, much in the same way that the electromagnetic potentials are gauge-dependent fields. That is, even causally connected observers need only agree on gauge-invariant, physically measurable quantities. Here, all observers agree that the spins are anti-correlated, which is all that can ever be known with certainty.

There may also be concerns that the quantum particles do not “perceive” the arrow of time and, consequently, have the capability of time travel into the past. It is, in fact, already known that final-state selection does allow for such time travel via the process of post-state teleportation; an idea that is discussed at length [43]. While this is an area of study with many open questions, two important points will be made to assuage any discomfort at the suggestion of this type of particle behavior. (See [43] for further details and references.)

Firstly, this form of time travel can be formulated in a way that does not jeopardize the standard tenets of quantum mechanics, nor does it lead to any stereotypical “grandfather paradoxes”. And so this concept can safely be applied to quantum particles without worrying about any disorder permeating to the classical realm.

Secondly, given a consistent theory of quantum gravity, it is natural if not necessary that time travel be incorporated into the quantum side of the theory at some level. This can be understood as follows: In the classical (Einstein) theory

\footnote{This refers to the prototypical paradox of time travel: That a person could go back in time and shoot his or her own grandfather before ever being conceived.}
of gravity, closed time-loops or wormholes cannot be ruled out by any formal proof. For instance, the conditions around black holes produce effects which suggest that there are such time-ambiguous regions. Therefore, a consistent theory of quantum gravity requires that the quantum realm also allows for some notion of going backward in time. Post-state teleportation can be viewed as a tangible realization of this requirement.

4.3 Against super-observers

As alluded to earlier, the underlying premise of EPR’s criterion for reality, as well as the application of Bell locality, utilizes an assumption regarding a “special” observer with the capacity to view the Universe in its totality: the super-observer. This is a concept that has been used throughout science to build theories for explaining the world [44].

A useful approach to understanding this idea is through “Laplace’s demon”. This is a conceptual device that is used to represent the Universe as a deterministically working model [45]. It entails a hypothetical super-intelligence having access to all laws governing the Universe as well as a precise description of the Universe for a particular moment of time [46]. So that Laplace’s demon, this hyper-intelligent super-observer, would have accurate knowledge of all values for all things in the Universe. This, combined with its knowledge of the physical laws, would allow the prediction of all future occurrences as well as a retroactive prediction of all past events [45]: that is, the ability to describe the Universe, in its entirety, for all time given only one set of initial values. Such an entity is then representative of a deterministically functioning universe, where it is only our own ignorance of accurate values that prevents us from perfectly forecasting the future.

And so, for every moment of time, there would be a single and complete description of the Universe and all of its constituents. This is obviously problematic for observer complementarity, which does not allow a single observer to have access to all information since he or she cannot have access to any spatially separated values.

The necessity for a hypothetical super-observer has been debated over history. It was Heisenberg’s uncertainty principle which first provided a strong opposition. Insofar as some information is always inaccessible, there is a fundamental limitation to the predictive capabilities of Laplace’s hypothetical demon. In this way, the workings of quantum mechanics places a firm restriction on what can be known as a matter of principle. Even if given all available information...
at a particular time, the demon would still not be able to consistently make predictions about quantum events with certainty.

The final nail in the coffin for the super-observer came from a calculation that described Laplace’s demon as a computational device \[47\]. This paper showed that, for a device to accurately predict everything in a system, it cannot be part of the same system. Essentially, Laplace’s demon cannot be situated within the Universe if it is to provide a total description of the Universal state. And so the conclusion is that, for any set of natural laws, a computational device would not be able to predict everything within a world that it is also contained in. The concept of a Laplacian demon could still hold for one living “outside” the Universe. But such a demon has no utility because, in this case, its predictive power is then literally out of reach \[48\]. Hence, any theory could only describe “almost everything”; there will always be certain values that are forever out of its reach. (For a recent discussion, see \[49\].)

There is, therefore, no compelling reason to suggest that the contradiction between observer complementarity and the super-observer implies that former should be discarded. Rather, the evidence points to the dismissal of the super-observer as a \textit{part of} our Universe.

5 Conclusions

In closing, the approach of this paper reveals an oversight regarding the assumptions that are used in the EPR thought experiment and Bell’s formulation thereof. As made clear by Norsen and recapitulated above, the arguments rest squarely on the validity of CFD. However, as we have pointed out, the application of CFD is at odds with the principle of observer complementarity. Then, since the latter attains its pedigree from the fundamental theory of quantum gravity, we have argued that Bell locality is too strong a definition for reality and that this is what leads to the troubling outcomes. By dismissing Bell locality and utilizing observer complementarity, we have shown that all concerns regarding locality and the completeness of quantum mechanics fall by the wayside, as the EPR scenario naturally resolves itself in a self-consistent manner.

The acceptance of observer complementarity also requires one to dismiss the notion that a hypothetical super-observer is present in the Universe. This does not present a problem because, not only is there no argument for the necessity of a super-observer, there is considerable evidence against it. A super-observer may still fulfill the criteria of Laplace’s demon from outside the Universe. However, any interaction of the super-observer with the Universe requires it to be
part of the same; in which case, Laplace’s criteria can no longer be fulfilled. Therefore, any theory explaining measurement within the Universe cannot provide a deterministically complete description.

As locality and causality have been maintained by our reworked calculation, what must change is how reality is viewed; not as a single description of all subsystems but, rather, as a collection of descriptions from many different observer’s points of view. We would, if it were possible, be inclined to remind Einstein that such a notion is not much different than his theory of relativity.

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