Non-Relativistic Non-BPS Dp-brane

J. Klusoň *

Dipartimento di Fisica & Sezione I.N.F.N.
Università di Roma "Tor Vergata"
Via della Ricerca Scientifica, 1 00133 Roma ITALY
E-mail: Josef.Kluson@roma2.infn.it

ABSTRACT: We construct non-relativistic non-BPS Dp-brane action. Then we will study the properties of the tachyon kink solution on its world-volume. We will argue that this tachyon kink describes non-relativistic D(p-1)-brane.

KEYWORDS: D-branes, tachyon condensation.

*On leave from Masaryk University, Brno
1. Introduction and Summary

Non-relativistic string theory was determined some years ago in the context of the study of the string theories in the background with the electric field [1, 2, 3, 4, 5, 6]. Then it was soon recognised in [7, 8, 9] that non-relativistic string theory is consistent sector of the string theory with the manifest world-sheet conformal field theory description that has appropriate Galilean symmetry. For the case of strings, this can be accomplished if we consider wound strings in the presence of a background $B$-field and tuning the $B$-field so that the energy coming from the $B$-field cancels the tension of the string. Then it was shown in [13] that the similar procedure can be performed in case of Dp-branes $^2$. Since Dp-branes are charged with respect to Ramond-Ramond form $C_{p+1}$ it is possible to find the limit where the tension of the wound Dp-brane is cancelled by the coupling to the $C_{p+1}$ field. As a result we obtain a world-volume kappa symmetric action of a non-relativistic Dp-brane [13].

We can also ask the question whether some symmetries of the relativistic string theories remain symmetries of the non-relativistic string theories as well. It was shown in [13] that in the non-relativistic string theories one can find T-duality relation between effective actions for non-relativistic Dp-branes and also one can argue that non-relativistic D1 and D2-brane actions arise in the dimensional reduction from non-relativistic M2-brane action.

$^2$For related works, see [10, 11, 12, 14, 15]
On the other hand it is well known that all Dp-branes can be interpreted as the result of the tachyon condensation on the world-volume of unstable D-branes or on the world-volume of D-brane anti-D-brane pair \cite{16, 17}. One can then ask the question whether non-relativistic BPS Dp-branes can be considered as the result of the tachyon condensation on the world-volume of non-relativistic non-BPS Dp-brane. The goal of this paper is to study this problem.

Our approach closely follows \cite{13} with some mild differences. First of all we would like to find non-relativistic non-BPS Dp-brane action where the tachyon contribution has the schematic form $V(T)F(\eta^{ij}\partial_i T \partial_j T)$ with $F(u) \sim u^{1/2}$ for large $u$. The reason for this requirement is that for such a form of the non-BPS Dp-brane action one can find the tachyon kink solution that represents codimension one D(p-1)-brane \cite{34}. This condition however implies that in the scaling limit that defines non-relativistic Dp-brane from the relativistic one the tachyon has to scale in the same way as the world-volume scalar modes that define the embedding of Dp-brane along the directions spanned by the background Ramond-Ramond $C_p$ form. On the other hand if we try to apply procedure given in \cite{13} where these modes scale with the factor that goes to infinity we obtain that the argument of the tachyon potential goes to infinity that however implies that the tachyon potential goes to zero. As a result the non-BPS Dp-brane disappears in this definition of the non-relativistic limit. To resolve this paradox we use the fact that the non-relativistic limit can be interpreted as the limit where the world-volume scalar modes that parametrize directions transverse to the Ramond-Ramond background are small. More precisely, since the correct definition of the non-relativistic limit of BPS Dp-brane needs background Ramond-Ramond form $C_{p+1}$ it is natural to split the world-volume scalar modes to two sets, one that corresponds to the modes that parametrize directions along the Ramond-Ramond form $C_{p+1}$ and the second one that parametrizes directions transverse to it. In the case of the ordinary BPS Dp-brane its world-volume can span the directions specified by the background Ramond-Ramond $C_{p+1}$ form. However in case of a non-BPS Dp-brane the situation is different since there does not exist such a $C_{p+1}$ form. On the other hand it is well known that the non-BPS Dp-brane couples to the Ramond-Ramond forms through the Wess-Zumino (WZ) term that contains gradient of the tachyon and the tachyon potential \cite{29, 30, 31, 32, 33}. We will argue that thanks to the existence of this WZ term it is still possible to define the correct non-relativistic non-BPS Dp-brane as well.

As the further support of our result we will study the tachyon kink on the world-volume of a non-relativistic non-BPS Dp-brane. Using the approach given in \cite{34} we will construct the singular tachyon kink on the world-volume of the non-relativistic non-BPS Dp-brane that can be interpreted as a lower dimensional non-relativistic D(p-1)-brane. This result then suggests that we can define non-relativistic non-BPS

\footnote{For review, see \cite{18, 19}.}
Dp-brane that has the same properties as its relativistic version.

This paper is organised as follows. In the next section (2) we introduce the bosonic part of the non-BPS Dp-brane action in type IIA or type IIB theories. Then in section (3) we will consider its non-relativistic limit. In section (4) we will study the tachyon kink on its world-volume. In section (5) we review the basic properties of the supersymmetric form of the non-BPS Dp-brane in type IIA theory. Finally in section (6) we consider its non-relativistic limit.

2. Review of non-BPS Dp-brane

In this section we review the basic facts considering the bosonic part of the non-BPS Dp-brane action \[20, 21, 22, 23\]. The bosonic part of a non-BPS Dp-brane action takes the form

\[
S = S_{DBI} + S_{WZ},
\]

\[
S_{DBI} = -\tau_p \int d^{p+1}\xi e^{-\Phi}V(T)\sqrt{-\det A},
\]

\[
S_{WZ} = \tau_p \int \Sigma V(T)dT \wedge C \wedge e^{F+B},
\]

(2.1)

where

\[
A_{ij} = \partial_i X^M \partial_j X^N G_{MN} + \partial_i X^M \partial_j X^N B_{MN} + F_{ij} + \partial_i T \partial_j T,
\]

\[
F_{ij} = \partial_i A_j - \partial_j A_i,
\]

(2.2)

where \(A_i, i, j = 0, \ldots, p\) and \(X^{M,N}, M, N = 0, \ldots, 9\) are gauge and the transverse scalar fields on the world-volume of the non-BPS Dp-brane and \(T\) is the tachyon field. \(V(T)\) is the tachyon potential that is symmetric under \(T \rightarrow -T\), has maximum at \(T = 0\) equal to \(V(0) = 1\) and has its minimum at \(T = \pm \infty\) where it vanishes. \(\tau_p\) is a tension of the non-BPS Dp-brane that is related to the tension of the BPS Dp-brane \(T_p\) as \(\tau_p = \sqrt{2}T_p\). Finally \(G_{MN}, B_{MN}\) and \(\Phi\) are background metric, \(B\)-field and dilaton respectively. We restrict to the background with flat Minkowski metric \(G_{MN} = \eta_{MN} = \text{diag}(-1, 1, \ldots, 1)\), with vanishing \(B\)-field and constant dilaton \(\Phi_0\). In what follows we also include \(e^{-\Phi_0}\) into the definition of \(\tau_p\).

Note also that \(S_{WZ}\) given in (2.1) is WZ term for a non-BPS Dp-brane that expresses the coupling of the non-BPS Dp-brane to the Ramond-Ramond forms. In

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4General properties of the tachyon effective actions were also discussed in \[24, 25, 26, 27, 28\].

5We work in units \((2\pi\alpha') = 1\). Note also that \(p\) is odd in type IIA theory while \(p\) is even in type IIB theory.
\( \Sigma \) denotes the world-volume of the non-BPS Dp-brane and \( C \) collects all RR n-form gauge potentials (pulled back to the world-volume) as

\[ C = \oplus_n C_{(n)} . \] (2.3)

The form of the WZ term was determined from the requirement that the Ramond-Ramond charge of the tachyon kink is equal to the charge of D(p-1)-brane \([29, 31, 32, 33]\). In fact it was argued in \([29, 34]\) that the consistency requires:

\[ \tau_p \int_{-\infty}^{\infty} V(T) dT = T_{p-1} , \] (2.4)

where \( T_{p-1} \) is a tension of a BPS D(p-1)-brane.

### 3. Non-relativistic limit of the non-BPS Dp-brane

In this section we define non-relativistic limit of a non-BPS Dp-brane. As was argued in \([13]\) the correct definition of the non-relativistic limit of BPS Dp-brane is based on the existence of the constant background Ramond-Ramond form \( C_{p+1} \) and the BPS Dp-brane is extended in the directions determined by the orientation of \( C_{p+1} \) in order to cancel Dp-brane tension by the coupling to the \( C_{p+1} \) field. For non-BPS Dp-brane there is not suitable \( C_{p+1} \) form with the rank equal to the dimension of the world-volume of a non-BPS Dp-brane. Moreover the form of the WZ term given in (2.1) contains tachyon so that we cannot proceed exactly in the same way as in \([13]\). As we argued in the introduction we define the non-relativistic limit as the limit where the world-volume scalar modes that parametrise directions transverse to the directions spanned by the background Ramond-Ramond form \( C_p \) are small. Then we also demand that the tachyon and tachyon potential does not scale in the non-relativistic limit. More precisely let us presume that the Ramond-Ramond field \( C_p \) takes following form:

\[ C_{\mu_0...\mu_{p-1}} = (-1)^p \epsilon_{\mu_0...\mu_{p-1}} , \quad \mu, \nu = 0, \ldots, p - 1 . \] (3.1)

Then (3.1) suggests following scaling of the world-volume variables:

\[
\begin{align*}
X^\mu &= x^\mu , \\
X^a &= \lambda X^a , \\
\tau_p &= \lambda^{-2} \tau_{NRp} , \\
F_{ij} &= \lambda f_{ij} , \\
A_i &= \lambda W_i , \\
T &= T ,
\end{align*}
\] (3.2)
where $a, b = p, \ldots, 9$. Note that we have also presumed that the world-sheet gauge field $A_i$ scales in the same way as the transverse modes. The non-relativistic limit is defined as limit $\lambda \to 0$.

Now for (3.2) the matrix $A$ takes the form

$$A_{ij} = g_{ij} + \lambda f_{ij} + \partial_i T \partial_j T + \lambda^2 x^a x^b g_{ab} \equiv B_{ij} + \lambda C_{ij} + \lambda^2 D_{ij},$$

where

$$B_{ij} = g_{ij} + \partial_i T \partial_j T,$$

$$C_{ij} = f_{ij},$$

$$D_{ij} = X^a X^b g_{ab}. (3.3)$$

Inserting (3.3) into the DBI part of the tachyon effective action and considering the terms up to orders $\lambda^2$ we obtain the non-relativistic Dp-brane in the form

$$S_{NR} = -\frac{\tau_{NR} p}{\lambda^2} \int d^{p+1} \xi V(T) \sqrt{-\det B} - \frac{\tau_{NR} p}{2} \int d^{p+1} \xi V(T) \sqrt{-\det B} \left[ (B^{-1})^{ij} \partial_j X^a \partial_i X^b g_{ab} - \frac{1}{2} (B^{-1})^{ik} f_{kj} (B^{-1})^{jl} f_{li} + O(\lambda^2) \right]. (3.4)$$

Before we turn to the non-relativistic limit in the WZ term let us study in more details the matrix $B$. Its form suggests that it is natural to temporarily introduce the notation

$$Y^I \equiv (x^\mu, T), \quad Y^p = T, \quad I, J = 0, \ldots, p (3.5)$$

so that we can rewrite $B$ into the suggestive form

$$B_{ij} = \partial_i Y^I \partial_j Y^J g_{IJ} \equiv E^f_I E^f_J \eta_{IJ}, \quad \eta_{pp} = 1. (3.6)$$

Using this notation it is easy to see that the divergent contribution in (3.4) can be written as

$$S_{div} = \frac{\tau_{NR} p}{\lambda^2} \int \frac{1}{(p + 1)!} V(T) E^f_0 \wedge \ldots \wedge E^f_p, (3.7)$$

where $E^f_I \equiv E^f_I d\xi^J$.

Let us study the non-relativistic limit in the WZ term. We insert the pull-back of Ramond-Ramond background form (3.1) into it and we obtain

$$S_{WZ} = -\frac{\tau_{NR} p}{\lambda^2} \int V(T) dT \frac{1}{p!} (-1)^p \epsilon_{\mu_0 \ldots \mu_{p-1}} dX^\mu_0 \wedge \ldots \wedge dX^\mu_{p-1} =$$

$$-\frac{\tau_{NR} p}{\lambda^2} \int V(T) \frac{1}{p!} \epsilon_{\mu_0 \ldots \mu_{p-1}} dX^\mu_0 \wedge \ldots \wedge dX^\mu_{p-1} \wedge dT =$$

$$-\frac{\tau_{NR} p}{\lambda^2 (p + 1)!} \int V(T) \epsilon_{I_0 \ldots I_p} E^I_0 \wedge \ldots \wedge E^I_p. (3.8)$$
If we add (3.8) to (3.7) we obtain that all divergent contributions in the full non-relativistic non-BPS Dp-brane cancel. As the result we obtain finite non-relativistic non-BPS Dp-brane action

\[ S_{NRp}^{fin} = -\tau_{NRp} \frac{1}{2} \int d^{p+1} \xi V(T) \sqrt{-\text{det} B} \left( (B^{-1})^{ij} \partial_j X^a \partial_i X^b g_{ab} - \frac{1}{2} (B^{-1})^{ik} f_{kj} (B^{-1})^{il} f_{li} \right). \] (3.9)

In the next section we will study the tachyon kink on the world-volume of the action (3.9).

4. Tachyon kink on the world-volume of non-relativistic non-BPS Dp-brane

In this section we will study the tachyon kink solution on the world-volume of the non-relativistic non-BPS Dp-brane following [34] \(^6\).

As in [34] we start with the following ansatz

\[
\begin{align*}
T & = f(a(\xi^p - t(\xi^\alpha))) , \\
W_\alpha & = w_\alpha(\xi^\alpha) , \quad W_p = 0 , \\
x^\mu & = x^\mu(\xi^\alpha) , \quad \mu = 0, \ldots, p-1 , \\
X^a & = x^a(\xi^\alpha) , \quad a = p, \ldots, 9 , \\
\end{align*}
\] (4.1)

where \(\xi^\alpha, \alpha, \beta = 0, 1, \ldots, p-1\) are coordinates tangential to the the world-volume of the kink. The function \(f\) introduced in (4.1) satisfies

\[ f(-u) = -f(u) , \quad f'(u) > 0 \quad \forall u , \quad f(\pm) = \pm\infty \] (4.2)

but is otherwise an arbitrary function of the argument \(u\). \(a\) is a constant that we shall take to \(\infty\) in the end. In this limit we have \(T = \infty\) for \(\xi^p > 0\) and \(T = -\infty\) for \(\xi^p < 0\).

The first goal of our analysis is to shown that the action (3.9) evaluated on the ansatz (4.1) reproduces in the limit \(a \to \infty\) the action for non-relativistic D(p-1)-brane

\[ S_{NR(p-1)} = -\frac{T_{NR(p-1)}}{2} \int d^p \xi \sqrt{-\text{det} g(\tilde{g}^{\alpha\beta} \partial_\alpha x^a \partial_\beta x^b g_{ab} - \frac{1}{2} \tilde{g}^{\alpha\beta} f_{\beta\gamma} \tilde{g}^{\gamma\delta} f_{\delta\alpha})} , \] (4.3)

\(^6\)For generalisation of this approach to the curved background see [35, 36].
where
\[
\tilde{g} = \partial_\alpha x^\mu \partial_\beta x^\nu \eta_{\mu\nu} , \quad \mu, \nu = 0, \ldots, p - 1 ,
\]
\[
f_{\alpha\beta} = \partial_\alpha w_\beta - \partial_\beta w_\alpha , \quad \alpha, \beta = 0, \ldots, p - 1
\]
and \(x^a, a = p, \ldots, 9\) parametrise positions of non-relativistic Dp-brane transverse to its world-volume. For letter purposes we also determine the equations of motion that arise from (4.3)

\[
-\frac{1}{2} \partial_\alpha \left[ \eta_{\mu\nu} \partial_\beta x^\nu \tilde{g}^{\beta\alpha} \sqrt{-\det \tilde{g} \left( \tilde{g}^{\gamma\delta} \partial_\delta x^a \partial_\gamma x^b g_{ab} - \frac{1}{2} \tilde{g}^{\gamma\delta} f_{\delta\gamma} \tilde{g}^{\omega\sigma} f_{\sigma\omega} \right) } \right] - \partial_\alpha \left[ \sqrt{-\det \tilde{g} \tilde{g}^{\alpha\beta} \eta_{\mu\nu} \partial_\gamma x^\nu \tilde{g}^{\gamma\delta} \partial_\delta x^a \partial_\beta g_{ab} } \right] + \partial_\alpha \left[ \sqrt{-\det \tilde{g} \tilde{g}^{\alpha\beta} f_{\delta\gamma} \tilde{g}^{\gamma\delta} f_{\omega\sigma} \tilde{g}^{\omega\alpha} f_{\sigma\omega} g_{\alpha\beta} } \right] = 0
\]

(4.5)

that is the equation of motion for \(x^\mu\). On the other hand the equation of motion for \(x^a\) takes much simpler form

\[
\partial_\alpha \left[ \sqrt{-\det \tilde{g} \tilde{g}^{\alpha\beta} \eta_{\mu\nu} \partial_\gamma x^\nu \tilde{g}^{\gamma\delta} \partial_\delta x^a \partial_\beta g_{ab} } \right] = 0
\]

(4.6)

Finally, we determine the equation of motion for \(w_\alpha\)

\[
\partial_\alpha \left[ \sqrt{-\det \tilde{g} \tilde{g}^{\alpha\beta} \eta_{\mu\nu} \partial_\gamma x^\nu \tilde{g}^{\gamma\delta} \partial_\delta f_{\omega\sigma} \tilde{g}^{\omega\gamma} \partial_\gamma x^\nu \eta_{\mu\nu} } \right] = 0
\]

(4.7)

Let us now insert the ansatz (4.1) into the matrix \(B\) defined in (3.3)

\[
B_{ij} = \left( \begin{array}{cc}
\tilde{g}_{\alpha\beta} + a^2 f^2 \partial_\alpha t \partial_\beta t - a^2 f^2 \partial_\alpha t \\
-a^2 f^2 \partial_\beta t
\end{array} \right)
\]

(4.8)

and evaluate its determinant

\[
\det B = \left| \begin{array}{ccc}
B_{\alpha\beta} - B_{\alpha\gamma} B_{\gamma\rho} B_{\rho\beta} & 0 \\
B_{\gamma\rho} & B_{\rho\rho}
\end{array} \right| = a^2 f^2 \det \tilde{g}_{\alpha\beta}
\]

(4.9)

and its inverse matrix \(B^{-1}\)

\[
(B^{-1})^{\rho\rho} = \tilde{g}^{\alpha\beta} \partial_\alpha t \partial_\beta t , \quad (B^{-1})^{\rho\alpha} = \partial_\beta t \tilde{g}^{\beta\alpha} ,
\]

\[
(B^{-1})^{\alpha\rho} = \tilde{g}^{\alpha\beta} \partial_\beta t , \quad (B^{-1})^{\alpha\beta} = \tilde{g}^{\alpha\beta} .
\]

(4.10)

Note that the form of the matrix \(B^{-1}\) is exact for all \(a\). If we now insert (4.1), (4.9) and (4.10) into (3.9) and consider the limit of large \(a\) we obtain

\[
S_{NRp}^{fin} = \frac{T_{NR(p-1)}}{2} \int d^p \xi \sqrt{-\det \tilde{g} \left[ \tilde{g}^{\alpha\beta} \partial_\alpha x^a \partial_\beta x^b g_{ab} - \frac{1}{2} \tilde{g}^{\alpha\beta} f_{\beta\gamma} \tilde{g}^{\gamma\delta} f_{\delta\alpha} \right] } ,
\]

(4.11)
where we have used the fact that 
\[
\frac{\tau_{NP}}{2} \int d\xi^p V(f(a\xi^p))af'(\xi^p) = \frac{T_{NP}}{2} \int dmV(m) = \frac{T_{NP(p-1)}}{2} .
\] (4.12)

Comparing (4.3) with (4.11) we obtain the result that the tachyon kink reproduces the non-relativistic D(p-1)-brane action. We also see that \(t(\xi)\) does not appear in (4.11) and hence its interpretation is unclear at present.

Note however that this result does not prove that the dynamics of the kink is governed by the action (4.3). To do this we have to show, following [34], that any solution of the equations of motion derived from (4.3) will produce a solution of the equations of motions derived from (3.9) under the identification (4.1).

In order to establish this correspondence we firstly determine the equations of motion for \(x^\mu\)
\[
- \frac{1}{2} \partial_i \left[ V(T) \eta_{\mu\nu} \partial_j x^\nu (B^{-1})^{ij} \sqrt{-\det B} (B^{-1})^{ij} \partial_j X^b g_{ab} - \frac{1}{2} (B^{-1})^{ik} f_{kj} (B^{-1})^{jl} f_{li} \right] - 
- \frac{1}{2} (B^{-1})^{ik} f_{kj} (B^{-1})^{jl} f_{li} \right] - 
- \partial_i [V(T) \eta_{\mu\nu} \partial_j x^\nu (B^{-1})^{ij} \sqrt{-\det B} (B^{-1})^{ij} \partial_j X^a \partial_a x^b] + 
+ \partial_m [V(T) \eta_{\mu\nu} \partial_j x^\nu (B^{-1})^{ij} \sqrt{-\det B} (B^{-1})^{ij} \partial_j X^a \partial_a x^b] = 0 .
\] (4.13)

In the same way we determine the equations of motion for \(x^\mu\)
\[
- \frac{1}{2} \partial_i \left[ V(T) \eta_{\mu\nu} \partial_j x^\nu (B^{-1})^{ij} \sqrt{-\det B} (B^{-1})^{ij} \partial_j X^b g_{ab} - 
- \frac{1}{2} (B^{-1})^{ik} f_{kj} (B^{-1})^{jl} f_{li} \right] - 
- \partial_i [V(T) \eta_{\mu\nu} \partial_j x^\nu (B^{-1})^{ij} \sqrt{-\det B} (B^{-1})^{ij} \partial_j X^a \partial_a x^b] + 
+ \partial_m [V(T) \eta_{\mu\nu} \partial_j x^\nu (B^{-1})^{ij} \sqrt{-\det B} (B^{-1})^{ij} \partial_j X^a \partial_a x^b] = 0 .
\] (4.14)

and the equations of motion for \(X^a\)
\[
\partial_i [V(T) \sqrt{-\det B} (B^{-1})^{ij} \partial_j X^b g_{ba}] = 0 .
\] (4.15)

Finally, we determine the equation of motion for \(W_i\)
\[
\partial_j [V(T) \sqrt{-\det A} (B^{-1})^{jk} f_{kl} (B^{-1})^{li}] = 0 .
\] (4.16)

Now we will solve the equations of motions (4.13), (4.15), (4.16) with the ansatz (4.1). We start our discussion with the equation (4.13). Inserting (4.9), (4.10) and (4.1).
We proceed with (4.16). For \( i \) obeys (4.1) on condition that \( x^i \) is obeyed for the ansatz (4.1) on condition that \( \partial_x \) obeys (4.16). In the second step we have again used the antisymmetry of \( f \) so that \( \nabla^\alpha f_{\gamma\delta} \). Then the final form of the equation (4.19) implies that the ansatz (4.1) obeys the equation of motion (4.16). We can proceed in the same way in case when \( i = p \) is equal to \( \beta \) and we get

\[
\partial_j[V(T)\sqrt{-\det B(B^{-1})_j^{\alpha\beta}}] = a\partial_j[Vf(t)]\sqrt{-\det \tilde{g}\partial_x^\alpha\partial_x^\beta} + a\partial_j[Vf(t)]\sqrt{-\det \tilde{g}\partial_x^\alpha\partial_x^\beta} +
\]

\[
\partial_j[V(T)\partial_\alpha[Vf(t)]] = a\partial_j[Vf(t)]\sqrt{-\det \tilde{g}\partial_x^\alpha\partial_x^\beta} + a\partial_j[Vf(t)]\sqrt{-\det \tilde{g}\partial_x^\alpha\partial_x^\beta} ,
\]

\[(4.17)\]

where we have used the fact that

\[
\partial_x[Vf(t)] = -\partial_x[Vf(t)].
\]

(4.18)

Now the final form of the equation (4.17) implies that the equation of motion (4.16) is obeyed for the ansatz (4.11) on condition that \( x^\alpha \) obeys (4.6). In the same way we proceed with (4.16). For \( i = p \) we obtain

\[
\partial_j[V(T)\sqrt{-\det B(B^{-1})_j^{\alpha\beta}}] = a\partial_j[Vf(t)]\sqrt{-\det \tilde{g}\partial_x^\alpha\partial_x^\beta} + a\partial_j[Vf(t)]\sqrt{-\det \tilde{g}\partial_x^\alpha\partial_x^\beta} +
\]

\[
\partial_j[V(T)\partial_\alpha[Vf(t)]\sqrt{-\det \tilde{g}\partial_x^\alpha\partial_x^\beta} + a\partial_j[Vf(t)]\sqrt{-\det \tilde{g}\partial_x^\alpha\partial_x^\beta} .\]

(4.19)

where in the first step we have used the fact that \( \tilde{g}^\alpha = \tilde{g}^\beta = \tilde{g}^\alpha \) is symmetric while \( f_{\alpha\beta} \) is antisymmetric. In the second step we have again used the antisymmetry of \( f \) so that

\[
\tilde{g}^\alpha f_{\gamma\delta} \tilde{g}^\beta = 0 .
\]

(4.20)

Then the final form of the equation (4.19) implies that the ansatz (4.11) solves the equation of motion (4.16) on condition that the modes \( w_\alpha, x^\alpha, x^\mu \) obey (4.7).

We can proceed in the same way in case when \( i \) in (4.16) is equal to \( \beta \) and we get

\[
\partial_j[V(T)\sqrt{-\det B(B^{-1})_j^{\alpha\beta}}] = a\partial_j[Vf(t)]\sqrt{-\det \tilde{g}\partial_x^\alpha\partial_x^\beta} + a\partial_j[Vf(t)]\sqrt{-\det \tilde{g}\partial_x^\alpha\partial_x^\beta} .
\]

(4.21)

This result again implies that the ansatz (4.11) obeys the equation of motion (4.16) on condition that \( w_\alpha, x^\alpha, x^\mu \) solve (4.7).

Finally we will analyse the tachyon equation of motion (4.13). To simplify the calculation we use the fact that

\[
(B^{-1})^{ij} B_{jp} = \delta^i_p , \quad B_{pj} (B^{-1})^{ji} = \delta^i_p
\]

(4.22)
and then using (4.8) we obtain

\[(B^{-1})^{ip} - (B^{-1})^{i\alpha} \partial_{\alpha} t = \frac{1}{a^2 f'^2} \delta^i_p.\]

(4.23)

Let us start with the expression on the first and the second line in (4.13)

\[\frac{1}{2} V'(T) \sqrt{-\det B} \left( (B^{-1})^{ij} \partial_j X^a \partial_i X^b g_{ab} - \frac{1}{2} (B^{-1})^{ik} f_{kj} (B^{-1})^{jl} f_{li} \right) - \frac{1}{2} \partial_i \left[ \partial_j T (B^{-1})^{ji} \sqrt{-\det B} \left( (B^{-1})^{ij} \partial_j X^a \partial_i X^b g_{ab} - \frac{1}{2} (B^{-1})^{ik} f_{kj} (B^{-1})^{jl} f_{li} \right) \right] = \]

\[= \frac{1}{2} V'(T) \sqrt{-\det B} \left( (B^{-1})^{ij} \partial_j X^a \partial_i X^b g_{ab} - \frac{1}{2} (B^{-1})^{ik} f_{kj} (B^{-1})^{jl} f_{li} \right) - \frac{1}{2} V(f) \frac{1}{a^2 f'^2} \delta^i_p \sqrt{-\det B} \left( (B^{-1})^{ij} \partial_j X^a \partial_i X^j g_{ij} - \frac{1}{2} (B^{-1})^{ik} f_{kj} (B^{-1})^{jl} f_{li} \right) = 0 ,\]

(4.24)

where we have used (4.23). Moreover, using (4.23) in remaining terms in (4.13) we get that all these terms vanish. We see that the ansatz (4.1) solves the equation of motion (4.13) for arbitrary \(t(\xi)\). In fact this is a satisfactory result since we work with the action where the world-volume diffeomorphism invariance is not fixed and hence \(\xi^p\) is equivalent to \(\xi^p + t(\xi)\). This fact also implies that it is natural to consider \(t(\xi)\) as a parameter of the gauge diffeomorphism transformations and hence it has no physical significance.

Let us outline the results derived above. We have shown that the dynamics of the massless modes given in (4.1) is governed by the action (4.3). This result supports the interpretation of the tachyon kink as the D(p-1)-brane.

5. Supersymmetric Relativistic Non-BPS Dp-brane

In this section we define supersymmetric form of a non-relativistic non-BPS Dp-brane. We firstly review the basic facts about non-BPS D-branes in Type IIA theory, following [20] \(^7\). Let \(\xi^i, i, j = 0, ..., p\) are world-volume coordinates on a D-brane. On a non-BPS Dp-brane world-volume we have a 32 component anti-commuting field \(\theta\) that transforms as a Majorana spinor of the 10 dimensional Lorentz group. We also denote \(\Gamma^M\) the ten dimensional gamma-matrices that can be chosen real by taking charge conjugation matrix \(C = \Gamma_0\). These matrices obey the anti-commutation relation

\[\{\Gamma^M, \Gamma^N\} = 2\eta^{MN}\]

(5.1)

\(^7\)For non-BPS D-brane in Type IIB theory the situation is basically the same with difference in chirality of the Majorana-Weyl fermions.
with $\eta^{MN} = (-1, 1, ..., 1)$. We also introduce $\Gamma_{11} = \Gamma_0...\Gamma_9$, $(\Gamma_{11})^2 = 1$.

The supersymmetric generalisation of the the DBI part of the effective action for a non-BPS Dp-brane has a form [20, 21, 22, 23]:

$$S_{DBI} = -\tau_p \int d^{p+1}\xi V(T)\sqrt{-\det(G_{ij} + F_{ij} + \partial_i T \partial_j T)} = -\int L_{DBI} , \quad (5.2)$$

where

$$\Pi_i^M = \partial_i X^M + i \bar{\theta} \Gamma^M \partial_i \theta , \quad G_{ij} = \eta_{MN} \Pi_i^M \Pi_j^N \quad (5.3)$$

and

$$F_{ij} = F_{ij} - b_{ij} , \quad F_{ij} = \partial_i A_j - \partial_j A_i . \quad (5.4)$$

There also exists the supersymmetric generalisation of the WZ term

$$S_{WZ} = \tau_p \int \Sigma V(T) dT \wedge \Omega_p = \int \Sigma L_{WZ} . \quad (5.5)$$

The form $\Omega_p$ expresses the coupling of the non-BPS Dp-brane to the Ramond-Ramond fields [21]. In particular, its bosonic part denotes the pull back of the Ramond-Ramond fields and the combination of the field strength $F$. This part vanishes for vanishing Ramond-Ramond background but there is a part of $\Omega_p$ that survives even in the absence of any Ramond-Ramond background [37, 38, 39, 40]. Since the explicit form of $\Omega_p$ is complicated we introduce, following [13] $(p + 1)$ form $h_{p+1}$ such that

$$h_{p+1} = d\Omega_p . \quad (5.6)$$

For type IIA non-BPS Dp-branes (p-odd) the forms $b, h_{p+1}$ are equal to

$$b = -i \bar{\theta} \Gamma_{11} \Gamma_M d\theta \wedge (\Pi^M - \frac{i}{2} \bar{\theta} \Gamma^M d\theta) ,$$

$$h_{p+1} = -(-1)^n i d\bar{\theta} \wedge T_{p-1} \wedge d\theta , n = \frac{p - 1}{2} , \quad (5.7)$$

where $T_{p-1}$ is $p-1$ form. To define it, we introduce the formal sum of differential forms

$$T_A = \sum_{p' = even} T_{p'} = e^F C_A , \quad (5.8)$$

where

$$C_A = \Gamma_{11} + \frac{1}{2!} \psi^2 + \frac{1}{4!} \Gamma_{11} \psi^4 + \frac{1}{6!} \psi^6 + ... \quad (5.9)$$

and

$$\psi = \Pi^M \Gamma_M . \quad (5.10)$$

One can shown that the non-BPS Dp-brane action is invariant under rigid supersymmetry variations but it is not invariant under local $\kappa$ symmetry transformations [20]. The absence of this local symmetry implies that we cannot gauge away one half
of the fermions and hence the world-volume theory contains the correct number of massless degrees of freedom \[20\].

We can generalise the analysis presented in the previous section and switch on one more coupling in the world-volume of non-BPS Dp-brane consistent with all symmetries of the action \[13\]. From the space-time point of view, this is equivalent to turning on a closed Ramond-Ramond p-form \(C_p\) which does not modify the supergravity equations of motion. Then the total non-BPS Dp-brane action takes the form

\[
S = - \int d^{p+1} \xi L_{DBI} + \int L_{WZ} + \int L_{C_p} ,
\]

(5.11)

where

\[
L_{C_p} = \tau_p V(T) dT \wedge f^* C_p ,
\]

(5.12)

where \(f^*\) is the pullback of \(C_p\) on the world-volume of non-BPS Dp-brane.

6. Non-relativistic non-BPS Dp-brane with presence of fermions

We derive the action for non-relativistic non-BPS Dp-brane from the supersymmetric form of the relativistic non-BPS Dp-brane action. We again closely follow \[13\].

The non-relativistic limit is obtained by decoupling some charged light degrees of freedom that obey non-relativistic dispersion relation from the full relativistic theory. This decoupling is achieved in the similar way as in section \(2\). In other words we demand that the scalar modes that parametrise directions transverse to the background Ramond-Ramond p-form \(C_p\) are small. Considering fermionic degrees of freedom we will scale them in the way that is equivalent to the scaling introduced in \[11, 13\]

\[
X^\mu = x^\mu ,
X^a = \lambda X^a ,
\tau_p = \lambda^{-2} \tau_{NRp} ,
F_{ij} = \lambda f_{ij} ,
A_i = \lambda W_i ,
T = T ,
\theta = \theta_- + \lambda \theta_+ ,
C_{\mu_0...\mu_{p-1}} = -\epsilon_{\mu_0...\mu_{p-1}} ,
\]

(6.1)

where \(X^M\) has been split in \(X^\mu , \mu = 0, \ldots, p-1\) and \(X^a, a = p, \ldots, 9\). The \(X^\mu\) are coordinates of target space parallel to the orientation of \(C_p\) and \(X^a\) are transverse to it. Finally \(\lambda\) is scaling parameter that is sent to zero in the end.
The scaling of fermions depends on the splitting of the fermions under the matrix $\Gamma$:

$$\Gamma_\pm = \pm \theta_\pm .$$  \hfill (6.2)

The matrix $\Gamma_\pm$ is defined as

$$\Gamma_\pm = (-1)^{n+1} \Gamma_0 \ldots \Gamma_1^{n+1} , \, 2n = p - 1$$  \hfill (6.3)

and it obeys following relations

$$\overline{\theta}_\pm = \pm (-1)^{n+1} \theta_\pm \Gamma_\pm , \quad \Gamma_\pm \Gamma_\pm = -\Gamma_1 \Gamma_\pm , \quad \Gamma_\pm^\mu = (-1)^{n+1} \Gamma^\mu_\pm \Gamma_\pm .$$  \hfill (6.4)

Note also that $\Gamma_2 = I$. Now we insert (6.1) to the supertranslation 1-form given in (5.3) and we obtain

$$\Pi^\mu = \hat{e}^\mu + i \lambda \overline{\theta}_+ \Gamma^\mu \mu_+ d\theta_+ , \quad \hat{e}^\mu = e^\mu + i \overline{\theta}_- \Gamma^\mu \mu_- d\theta_- , \quad e^\mu = dx^\mu ,$$

$$\Pi^a = \lambda u^a , \quad u^a = dx^a + 2i \overline{\theta}_+ \Gamma^a \mu_+ d\theta_+ , \quad x^a = X^a + i \overline{\theta}_- \Gamma^a \theta_+ .$$  \hfill (6.5)

In the same way we determine the scaling of the form $F$

$$F = \lambda^3 F^{(1)} + \lambda^3 F^{(2)} ,$$

$$F^{(1)} = f + i f \overline{\theta}_+ \Gamma^\mu \mu_+ d\theta_+ + i \overline{\theta}_- \Gamma^\mu \mu_- d\theta_- (\hat{e}^\mu - i \overline{\theta}_- \Gamma^\mu \mu_- d\theta_-) +$$

$$+ i \overline{\theta}_- \Gamma^\mu \mu_- d\theta_-(u^a - i \overline{\theta}_- \Gamma^a \mu_+ d\theta_+ - i \overline{\theta}_+ \Gamma^a \mu_+ d\theta_+) ,$$

$$F^{(2)} = \frac{1}{2} (\overline{\theta}_+ \Gamma^\mu \mu_+ \mu_+ d\theta_+ + \overline{\theta}_- \Gamma^\mu \mu_- \mu_- d\theta_-) \overline{\theta}_+ \Gamma^\mu \mu_+ d\theta_+$$

$$+ i \overline{\theta}_- \Gamma^\mu \mu_- d\theta_-(u^a - i \overline{\theta}_- \Gamma^a \mu_+ d\theta_+ - i \overline{\theta}_+ \Gamma^a \mu_+ d\theta_+) .$$  \hfill (6.6)

Following [13] we will keep $\lambda$ small but finite in the intermediate computations and only send $\lambda$ to zero in the end. For that reason we keep explicitly terms in the action that scale as $\lambda^3$ (that are divergent) and terms that are independent on $\lambda$ (that are finite). We also drop terms that scale as $\lambda$ since they cannot contribute when we take the limit $\lambda \to 0$ at the end of the analysis.

More precisely, if we insert (5.3) into $G$ we obtain

$$G_{ij} = \Pi^M_i \Pi^N_j \eta_{MN} = G_{ij}^{(0)} + \lambda^2 G_{ij}^{(2)} + O(\lambda^3) ,$$

$$G_{ij}^{(0)} = \hat{e}^\mu \hat{e}_j \eta_{\mu \nu} , \quad G_{ij}^{(2)} = 2i \hat{e}^\mu \overline{\theta}_+ \Gamma^\nu \mu_+ d\theta_+ \eta_{\mu \nu} + u_i^a u_j^b \delta_{ab} .$$  \hfill (6.7)
where we have restricted to the terms up to order $\lambda^2$. Then if we insert (6.6) and (6.7) into the DBI part of the tachyon effective action (5.2) we obtain supersymmetric form of the non-relativistic non-BPS Dp-brane action in Type IIA theory

$$S_{NR,DBI} = -\frac{\tau_{NRp}}{\lambda^2} \int d^{p+1} \xi V(T) \sqrt{-\det B} -$$

$$-\frac{\tau_{NRp}}{2} \int d^{p+1} \xi V(T) \sqrt{-\det B} \left( (B^{-1})^{ij} G_{ji}^{(2)} - \frac{1}{2} (B^{-1})^{ij} F_{jk}^{(1)} (B^{-1})^{kl} F_{li}^{(1)} \right) + O(\lambda),$$

(6.8)

where

$$B_{ij} = G_{ij}^{(0)} + \partial_i T \partial_j T = \hat{e}_i^\mu \hat{e}_j^\nu \eta_{\mu\nu} + \partial_i T \partial_j T.$$  

(6.9)

In order to carefully treat with the divergent term in (6.8) we perform the same simplification as in section (2) and temporarily write

$$B_{ij} = \hat{e}_i^I \hat{e}_j^J \eta_{IJ}, \quad I, J = 0, \ldots, p,$$

$$\hat{e}_i^I = \hat{e}_i^\mu, \quad \text{for } \mu = I = 0, \ldots, p-1, \quad \hat{e}_i^p = \partial_i T.$$  

(6.10)

With the help of (6.10) we can rewrite the divergent term in (6.8) as

$$-\frac{\tau_{NRp}}{\lambda^2} \int d^{p+1} \xi V(T) \sqrt{-\det B} =$$

$$= \frac{\tau_{NRp}}{\lambda^2 (p+1)!} \int V(T) \epsilon_{I_0 \ldots I_p} \hat{e}_{I_0}^1 \wedge \ldots \wedge \hat{e}_{I_p}^p \equiv \frac{1}{\lambda^2} \int d^{p+1} \xi L^{div}_{DBI}.$$  

(6.11)

Now we insert (6.1) into the WZ term. As in [13] we easily obtain

$$\tau_p h_{p+1} = \tau_{NRp} h_{p+1}^{(2)} + \tau_{NRp} h_{p+1}^{(0)} + O(\lambda^2).$$  

(6.12)

The superficially divergent term can be determined as in [13] and takes the form

$$h_{p+1}^{(2)} = -id\theta_\perp \wedge \frac{1}{(p-1)!} \hat{e}^\mu_1 \wedge \ldots \wedge \hat{e}^{\mu_{p-1}}_1 \epsilon_{\mu_1 \ldots \mu_{p-1} \nu} \Gamma^\nu \wedge d\theta_\perp =$$

$$= -\epsilon_{I_0 \ldots I_p} \frac{1}{(p-1)!} d\hat{e}_I^\nu \wedge \hat{e}^I_1 \wedge \ldots \wedge \hat{e}^I_p.$$  

(6.13)

Now we consider following expression

$$d(d^{p+1} \xi L^{div}) = -d(d^{p+1} \xi L^{div}_{DBI}) + d(V(T) dT \wedge \Omega_p)^{div} =$$

$$= \tau_{NRp} V(T) \left[ d \left( \frac{1}{(p+1)!} \epsilon_{I_0 \ldots I_p} \hat{e}_{I_0}^1 \wedge \ldots \wedge \hat{e}_{I_p}^p \right) + dT \wedge h_{p+1} \right] =$$

$$= \tau_{NRp} V(T) \left[ \frac{1}{p!} \epsilon_{I_1 \ldots I_p} \hat{d}^J \wedge \hat{e}^J_1 \wedge \ldots \wedge \hat{e}^J_p + dT \wedge h_{p+1} \right] = 0$$  

(6.14)
using the fact that
\[ dV \wedge \hat{e}^p = \frac{dV}{dT} \hat{e}^p \wedge \hat{e}^p = 0, \quad d(dT) = 0 \quad (6.15) \]
and also
\[ -dT \wedge \epsilon_{\mu_1...\mu_{p-1}} \frac{1}{(p-1)!} d\hat{e}^\nu \wedge \hat{e}^{\mu_1} \wedge ... \wedge \hat{e}^{\mu_{p-1}} = \\
-(-1)^{(p+1)} \epsilon_{\nu \mu_1...\mu_{p-1}} \frac{1}{(p-1)!} d\hat{e}^\nu \wedge \hat{e}^{\mu_1} \wedge ... \wedge \hat{e}^{\mu_{p-1}} \wedge dT = \\
-\frac{1}{p!} \epsilon_{JI_1...I_p} d\hat{e}^J \wedge \hat{e}^{I_1} \wedge ... \wedge \hat{e}^{I_p}, \quad (6.16) \]
where we have used the definition of the exterior derivative \( d \) and also the fact that for type IIA non-BPS Dp-brane \( p \) is odd.

As the last term in (6.14) involves only the terms with fermions \( L_{DBI} \), this cancellation removes the terms with fermions in \( L_{DBI}^{div} \). There remains the pure bosonic term in \( L_{DBI}^{div} \) that is
\[ -d^{p+1} \xi L_{DBI, bos} = \tau_{NRp} V(T) \frac{1}{(p+1)!} \epsilon_{I_0...I_p} e^{I_0} \wedge ... \wedge e^{I_p}. \quad (6.17) \]
This term can be cancelled by turning on a closed Ramond-Ramond form \( C_p \) that gives an contribution to the WZ term
\[ \int L_{C_p}^{div} = -\tau_{NRp} \int V(T) dT \wedge \frac{1}{p!} \epsilon_{\mu_0...\mu_{p-1}} dx^{\mu_0} \wedge ... \wedge dx^{\mu_{p-1}} = \\
-\tau_{NRp} \int V(T) \frac{1}{(p+1)!} \epsilon_{I_0...I_p} e^{I_0} \wedge ... \wedge e^{I_p}. \quad (6.18) \]
Then we obtain the finite part of the supersymmetric generalisation of the non-relativistic non-BPS Dp-brane action in the form
\[ S_{NR} = -\tau_{NRp} \int d^{p+1} \xi V(T) \sqrt{-\text{det} \mathbf{B}} \left[ \left( \mathbf{B}^{-1} \right)^{ij} \bar{\theta}_+ \hat{e}^a_\nu \Gamma^\nu \eta_{\mu \nu} \partial_j \theta_+ + \\
\frac{1}{2} \left( \mathbf{B}^{-1} \right)^{ij} u_i^a u_j^b g_{ab} - \frac{1}{4} \left( \mathbf{B}^{-1} \right)^{ij} \bar{F}^{(1)}_{jk} \left( \mathbf{B}^{-1} \right)^{kl} \bar{F}^{(1)}_{li} \right] + \\
+ \tau_{NRp} \int V(T) dT \wedge \Omega^{(0)}_p, \quad (6.19) \]
where \( \Omega^{(0)}_p \) is the non-relativistic WZ term that has the same form as in case of non-relativistic BPS D(p-1)-brane.
Our goal is to show that the world-volume action for non-relativistic D(p-1)-brane in Type IIA theory arises from the action (6.19) as the tachyon kink. Recall that the action for non-relativistic D(p-1)-brane in Type IIA theory has the form

\[ S_{BP} = -T_{NR(p-1)} \int d^p \xi \sqrt{\det \tilde{g}} \left( i \tilde{g}^{\alpha \beta} \tilde{\partial}_+ \tilde{e}_+^{\mu} \eta_{\mu \nu} \partial_\beta \tilde{\theta}_+ + \frac{1}{2} \tilde{g}^{\alpha \beta} \tilde{u}_a \tilde{u}_b \tilde{g}_{ab} - \right. \\
- \left. \frac{1}{4} \tilde{g}^{\alpha \beta} \tilde{F}_{\beta \gamma}^{(1)} \tilde{g}^{\gamma \delta} \tilde{F}_{\delta \alpha}^{(1)} \right) + T_{NR(p-1)} \int \Omega_p^{(0)}, \]

(6.20)

where

\[ \tilde{g}_{\alpha \beta} = \tilde{e}_\alpha \tilde{e}_\beta \eta_{\mu \nu}, \quad \tilde{e}_\beta = \tilde{e}_\mu + i \tilde{\theta}_- \Gamma^\mu d \tilde{\theta}_-, \quad \tilde{e}_\mu = d \tilde{x}_\mu, \]
\[ \tilde{u}^a = d \tilde{x}^a + 2 i \tilde{\theta}_+ \Gamma^a d \tilde{\theta}_-, \quad \tilde{x}^a = \tilde{X}^a + i \tilde{\theta}_- \Gamma^a \tilde{\theta}_+, \]

(6.21)

and where \( \tilde{F}_{(1)} \) has the same form as in (6.6) with the gauge field \( \tilde{w}_\alpha \).

In order to show that the tachyon kink on the world-volume of the supersymmetric non-relativistic non-BPS Dp-brane describes the non-relativistic BPS D(p-1)-brane we should perform the same analysis as in section (2). As the first step we consider the ansatz

\[ T = f(a(t_x - t(x^a))), \]
\[ W_\alpha(x^p, \xi^\alpha) = \tilde{w}_\alpha(\xi^\alpha), \quad W_p = 0, \]
\[ x^\mu(x^p, \xi^\alpha) = \tilde{x}_\mu(\xi^\alpha), \quad X^\alpha(x^p, \xi^\alpha) = \tilde{x}_\alpha(\xi^\alpha), \]
\[ \theta_+(x^p, \xi^\alpha) = \tilde{\theta}_+(\xi^\alpha), \quad \theta_-(x^p, \xi^\alpha) = \tilde{\theta}_-(\xi^\alpha) \]

(6.22)

and insert it to the matrix \( B \) so that we obtain

\[ B_{ij} = \begin{pmatrix} \tilde{g}_{\alpha \beta} + a^2 f^2 \partial_\alpha t \partial_\beta t & -a^2 f^2 \partial_\alpha t \\ -a^2 f^2 \partial_\beta t & a^2 f^2 \end{pmatrix}. \]

(6.23)

Then we also get

\[ \det B = a^2 f^2 \det \tilde{g}_{\alpha \beta} \]

(6.24)

and

\[ \left( B^{-1} \right)^{p p} = \tilde{g}^{\alpha \beta} \partial_\alpha t \partial_\beta t, \quad \left( B^{-1} \right)^{p \alpha} = \partial_\beta t \tilde{g}^{\beta \alpha}, \]
\[ \left( B^{-1} \right)^{a p} = \tilde{g}^{a \beta} \partial_\beta t, \quad \left( B^{-1} \right)^{a \beta} = \tilde{g}^{a \beta}. \]

(6.25)
Then it is easy to see that when we insert (6.22), (6.23), (6.24) and (6.25) into (6.19), perform the integration over $\xi^p$ and use the relation (4.12) we obtain the non-relativistic D(p-1)-brane action (6.20).

To show the complete equivalence we should check that the ansatz (6.22) solves the equations of motion that arise from (6.19). However from the form of the ansatz (6.22) and corresponding matrix $B$ it is clear that the ansatz solves these equations on conditions that the massless modes given there solve the equations of motion that follow from (6.20).

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