Constructive Graph Tasks in Distant Contests

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Abstract. The paper discusses a certain type of competitions based on distance interaction of a participant with simulation models of concepts from discrete mathematics and computer science. One of them is the “Construct, Test, Explore” (CTE) competition, developed by the authors, the other is the Olympiad in Discrete Mathematics and Theoretical Informatics (DM&TI). The tasks presented in this paper are generally devoted to the concept of a graph isomorphism. Most of the tasks are verified automatically.

Keywords: olympiad, computer science, discrete mathematics, electronic manipulator, CS competition, mathematical thinking, graphs, constructive tasks.

1. Background and Theory

One of the important issues in organizing mental activity in online (distance) competitions is to automate an analysis of mental actions which are performed by participants while solving tasks. Some information about the characteristics of mental actions may be obtained by analyzing competitions based on multiple choice questions (Yagunova, 2016). But this information is not enough to draw conclusions about the degree of understanding, because the correct answer could be selected accidentally, or by means of indirect reasoning that have nothing to do with an idea presented in the task. Moreover, the very form of a task with a multiple choice question is very limiting. Authors think that much more information about mental processes may be obtained by analyzing solutions of constructive tasks. Constructive tasks are convenient because they may be automated in a straightforward way. They may be stated inside an informational environment, built on computer tools and simulation models. Computer tools and simulation models allow us to analyze constructive solutions of participants. These solutions are very diverse and provide interesting information about how strongly have participants formed understanding of concepts, on which the task is based.
Also, while a participant works with a tool, rich information about his or her actions may be collected. However, the problem of actions logs analysis aimed to get mental acts is quite complicated and does not have full solution by now (Gibson, et al., 2016). So we do not consider logs analysis in this work. At the same time, the experience of working with imitation models has been gained (Honey, et al., 2010; Potkonjaka, 2016), their important role in tasks solving has been demonstrated (Baker, et al., 2008). It has been proved that the work with black-box models aids in better conceptualization of concepts: the work (Hosein, et al., 2008) demonstrates that “students using the black-box did better on the constructive tasks because of their increased explorations”. This work also shows that “students with low maths confidence resorted to using real-life explanations when answering tasks that were application related”. For this reason, this paper investigates subjects that have clear real-life sense and allow for transition into serious theoretical problems starting from understandable tasks that can be solved by participants only by common sense.

The work (Mislevy, 2011) demonstrates that it is important for concepts interpretations chosen for tasks statements to be obvious and natural. This approach corresponds to the idea of engineering design developed in the project WISEngineering (Chiu, 2013).

The type of feedback is important. The article (Attali, 2015) explores several feedback types: no feedback (NF), immediate knowledge of the correct response (KCR), multiple-try feedback with knowledge of the correct response (MTC), or multiple-try feedback with hints after an initial incorrect response (MTH), and determines that the latter is the most efficient. That is why this paper attaches great importance to the analysis of partial solutions and the automation of responding to them.

It is also interesting to study and develop such types of educational activities, that combine learning of new scientific and technical ideas with an ability to assess results of learners’ mental activities without specially set up additional assessments of obtained knowledge and skills. Such steps were taken in works (Pozdniakov, 2012, 2013; Posov, 2013; Yagunova et al., 2016; Akimushkin et al., 2015). This paper proceeds with a study of these means and abilities.

2. Methodology

We use a method of software supported subject tasks (Pozdniakov et al., 2013). By subject task we mean a task with some understandable real-world statement that does not need any specific knowledge to understand it and to make at least first steps in the solution. By software supported we mean that a task is accompanied with a computer tool, that demonstrates the statement and allows for searching for a solution (Honey et al., 2010, Hosein et al., 2008). Thus, such tasks have to be constructive, and a tool exposes their constructive nature.

The approach presented in this paper is connected with the study of possible feedback for partial solutions of subject tasks. To provide a feedback, a system should have access to information about students actions. Mental actions of a student are accompanied with
actions performed with the tool, and a potential ability to analyze and to control the solution search activity of a student arises.

To use the information about a work of a student within a computer tool while solving CTE constellation task we propose a method of a criteria system. These criteria assess partial solutions. The hierarchy of criteria is built in the way that every progress of a student is somehow evaluated with a usually numerical value. It allows for comparison of partial solution based on how close are these solutions to the full solution of the task. Criteria of higher levels are intended to reveal harder aspects of a subject. Thus, subject tasks may be considered both as quite simple (over the lower level criteria) and as olympiad (over higher level criteria). Subject tasks usually either implicitly or explicitly require to find the optimal solution.

The idea of constellations comes from the need to orient oneself in the space quickly. This can be achieved using stars in the sky. One should split the set of bright points-stars into groups, each of which is considered to be a single object. For that purpose people invented an idea of figures called constellations. Each constellation is obtained by connecting several points with segments. After that, the laws of human perception make a human quickly restore segments which are absent on the sky. He or she perceives a sky as a graph, consisting of several connectivity components. Thus, the task to introduce new constellations can be described in mathematical concepts as a construction of a non-connected undirected graph.

If the first stage is to come up with a metaphor, the second stage is to highlight a goal, that may be effectively supported by a computer tool. Such goal is to split stars into constellations. This task is natural from the context point of view, and thus the explanation of the “split” concept is not needed (see Fig. 1).

To measure the progress of students in achieving of the goal one needs to define criteria, that will assess the solution.

Criteria should not contradict the context of the problem, they should be natural inside this context, while leading a student to master a new idea, a concept or an algorithm. These requirements are satisfied by a splitting of stars, firstly, into different constellations, secondly, into constellations consisting of a fixed or upper-bounded number of stars. The requirement for the constellations to be different is described on the graph theory as a requirement for connectivity components not to be isomorphic. Thus, the important mathematical concept has a natural interpretation inside the chosen metaphor (context). The upper bound on the number of start is also natural, because a human can perceive a group of 5–7 objects as a unit, and he subconsciously splits the greater number of objects into subgroups with less number of elements.

In DM&TI task we also use an electronic manipulator. The manipulator generally allows the participant to construct a graph by adding vertices and edges. It has a number of customizable options: for some tasks we can allow the participant to color some vertices red and to highlight the neighbors of colored vertices.

However, in this paper we only consider the graph tasks which require to build a graph, satisfying certain conditions, without different manipulations with edges and vertices. The task considered in this paper are related to the same graph theory concepts, as the constellation task: planarity and isomorphism.
For assessing most of DM&TI tasks the automatical isomorphism verification was used. It means that, unlike the constellation task, partial points for the construction cannot be awarded. For some tasks text solutions are also required if we need the participant to prove that the construction is unique.

Let us look into a technique of designing software supported subject tasks on the example of the graph theory with the visual metaphor based on the idea of constellations. Such interpretation allows to decrease an introductory part of a statement because participants must have already been acquainted with the concept of constellations.

3. Constellations. Formalized Task Statement and Conditions of the Experiment

The task was presented to the participants of the competition. It was accompanied with a tool that allows to build configurations of constellations and evaluated numerical values of criteria for each configuration. Here is the problem statement:

This problem asks you to define constellations on a starry sky, connecting stars with segments using your mouse. Segments must not intersect. A constellation should contain at least 2 (4, 5 for other levels) stars and not more than one cycle (a closed poly-line).

The more different constellations you build, the better. How to tell whether two constellations are equal? The stars of equal constellations can be numbered in the way that if
a pair of stars is connected in one constellation, then in the other constellation stars with the same numbers are also connected (see Fig. 2).

If you do not succeed to create a new form of a constellation, then the more constellations you have (including same), the better is the result.

Two solutions with the same number of different constellations and the same total number of constellations are compared over the total length of all the segments. A better solution has a smaller sum.

The proposed subject is an example of subjects proposed annually in the “Construct, Test, Explore” competition. Each year schoolchildren get access to three software supported subject tasks. Participants use their browser to work with tasks. Solutions, that are best for a participant according to the criteria of the task, are saved automatically. Participants may also save any other partial solutions they want, and they can return to saved solutions any time with any browser on any computer.

The work on tasks usually starts in a class and continues at home. The competition lasts for one week, and the time to solve tasks is not bounded during this week.

All participants solutions have a log of actions made during the search for the solution. After the competition week, the best solutions of each participant are processed and compared between each other according to the same criteria, that were used for a single participant to obtain his or her best solution. So, each participant gets a rank for each task. The rank is a number demonstrating how many better solutions were found by other participants. The lower a rank is, the better is a solution. The participants are split into three groups according to their age, statements for each task for higher ages is more complicated. The ages split is: 0th level: 1–4 grades, 1st level: 5–8 grades, 2nd level: 9–11 grades.

4. Constellations. Results and Discussion

The qualitative analysis was done by viewing the best solutions of all participants in the order from the first rank to the last. Solutions sometimes contain mistakes that indicate that the participant has misunderstood some relevant mathematical concept. The bounds between solutions with and without such mistakes were found. Typical and bounding solutions are demonstrated on Fig. 3 and Fig. 4. This is the qualitative analysis of so-
solutions for the “constellations” problem for the 1st level (5–8 grades). There were 666 solutions in total.

Fig. 3(f) is the last solution with the maximal number of non-isomorphic graphs. Fig. 4(a) has 8 different graphs, and no isomorphic. Fig. 4(b) has 6 different graphs, and no isomorphic.

Fig. 3. Best solutions of the 1st level according to the 1st criterion: the number of non-isomorphic planar connected components.
Fig. 4(c) has 5 different graphs, and no isomorphic.
Fig. 4(d) has 5 different graphs, with some isomorphic.
Fig. 4(e) has 2 different graphs, with many isomorphic.
Fig. 4(f) has all graphs isomorphic.

Fig. 4. Examples of solutions of the first level with non-optimal solutions according to the main criterion.
Fig. 3 and Fig. 4 present examples of solutions, that demonstrate different levels of diving into the essence of the problem. 227 participants of 666 (34%) managed to find the optimal number of constellations, that is, they managed to build a graph from a maximal number of non-isomorphic planar components, with at least 4 vertexes each (Fig. 3a). What is the difference between the beginning of the list and its end? The answer came after the analysis: the difference is in the degree of taking into account the criterion of graph minimality (the minimality of total edges length). This is especially obvious from the last 227th solution, that surely does not take into account the minimality at all. Note, that well known greedy algorithms (for example, the Kruskal’s algorithm) are not applicable here because they do not consider the requirement of non-isomorphism of components being built.

The qualitative analysis of solutions demonstrates that during the solution almost all participants (more than 90%) gradually mastered the concept of isomorphism. Only the lowest rated solutions demonstrate participants that were trying to optimize the second criterion (not the main one) about the number of graph components. Note that best solutions do not use all the stars. There were no requirement to build a graph on all vertexes, however only a small fraction of participants were looking for a solution on a subset of stars. The two best found solutions have this form.

5. Constellations. Statistical Analysis of Solutions for the Task

In the “Constellations” task the best solutions over the main criterion (the number of non-isomorphic graphs) had also the best value of the second criterion (the total number of components) (Fig. 5, Fig. 6). In this case the good dispersion of results was achieved by the third criterion: the total length of planar graph edges.

Fig. 5. The number of different (non-isomorphic) graphs in the solution of the “Constellations” task on the 1st level. The first decrease from 12 to 11 corresponds to 227 optimal results according to the first criterion.
At the same time one can see, that the second criterion played the important role for solutions with a few non-isomorphic graphs (see Fig. 7). If only the dispersion of results is important, then the first and the third criteria are enough.

Fig. 6. A histogram of the number of different (non-isomorphic) graphs in the solutions of the “Constellations” problem on the 1st level.

Fig. 7. The number of all graphs in the solutions of the “Constellations” problem of the 1st level, results are ordered according to the number of non-isomorphic graphs (the main criterion). Splashes on the right of the plot shows differences by the second criterion for solutions that are neighbors by the first criterion.
However, the second criterion is important for the support of the individual work, because it is much more visually obvious for a participant: the number of built graphs is the first thing obvious for a participant, and the total length of edges is hard to be estimated visually and thus it plays smaller role in the feedback (Fig. 8, 9).

5.1. Constellations. Computer Form vs Paper Form

A comparative experiment was conducted to evaluate the effectiveness of using the developed means of informatization. The experiment involved 35 people of different ages, both from classes of the mathematical profile and from classes in the humanities. The “Constellations” task were offered to the students in the paper form.

After some time, the same participants were asked to do the same job in the computer form using the tool described above. None of them, using only the possibilities of working with a drawing on paper, could achieve an optimal solution according to the main criterion. At the same time, using the computer implementation of the task 32 people
improved their solution, and the optimal solution (in terms of the first criterion) was obtained by 29 participants, 2 people showed the same results, 1 participant worsened their decision.

It shows that using a computer helps participants to explore the problem more deeply and achieve optimal results experimentally.

Of course the improvement of the results is always expected when we give students an extra time to solve the task, but the huge number of optimal solution shows the benefit of using computer models for the graph tasks.

6. DM&TI Graph Tasks

6.1. Task 1. (Qualifying Round, 2018/19)

Build eight railroad stations and the largest number of direct railway lines between them so that the lines do not intersect. Points will be awarded in proportion to the number of lines built.

This task involves the concept of planarity.

One of the optimal solutions is shown at the Fig. 10. It could be proven using Euler formula, that maximal possible number of edges in this graph is 18.

Graphs drawn on a plane satisfy Euler’s formula: \( v - e + f = 2 \), where \( v \) is the number of vertices, \( e \) – the number of edges and \( f \) is the number of faces, i.e. regions bounded by edges. Also we have the inequality \( 2e \geq 3f \) as each edge is incident to exactly 2 faces, and each face is incident to at least 2 edges. Therefore we obtain \( e \leq 3(v - 2) \) which is 18 for \( v = 8 \).

In this task isomorphism verification is not enough. We also need to assess the target function which is the number of edges.

Fig. 10. Task 1 solution.
6.2. Task 2. (Final Round, 2018/19)

Build a graph that satisfies the following conditions:

1) Exactly 4 edges come out of each vertex.
2) There are no four vertices in the graph, every two of which would be interconnected.
3) A graph cannot be colored in three colors “properly”, that is, so that any two vertices of the same color are not adjacent.

3 points are awarded for an example, another 1 point for proving that the graph satisfies the second and third conditions. You will get 2 more points if your example has the minimal number of vertices and you prove it.

The required graph is the complement of a cycle at seven vertices.

In a cycle at seven vertices, among any four vertices, there are two neighboring ones. Therefore, its complement satisfies the condition of the task.

When coloring this graph in three colors, some three vertices will surely turn out to be the same color. The cycle at seven vertices does not contain triangles, which means that in this cycle some two of these three vertices are not adjacent. Consequently, they will be adjacent in the complement, which means that no coloring is “proper”.

The graph is minimal, because a graph on 5 vertices that all have degree 4 is a complete graph and it does not satisfy condition 2).

A graph on 6 vertices that have degree 4 is unique. For each vertex there is exactly one which is not adjacent to it. If we color each pair of such vertices in one color, we obtain the “proper” coloring in three colors, therefore, this graph does not satisfy condition 3.

Rules of assessment are included in the task formulation.

Fig. 11. Task 2 solution.

You will get 2 more points if your example has the minimal number of vertices and you prove it.
6.3. Task 3.1 (Qualifying Round, 2019/20)

The teacher drew a graph on the whiteboard. Three students redrew it into their notebooks (possibly not preserving the locations of the vertices), while each of them lost one vertex and all the edges incident to it (see figures). Help them to restore the graph that the teacher drew.

Three points are given for the correct graph; three more can be obtained by explaining why it is unique.

It is easy to see that the vertex, which is of degree 4 in the third graph, is absent in the second graph. From the first graph we see, that in initial graph this vertex must have degree 4, not 5. Then we understand, that, when added to the second graph, this vertex should be adjacent to four vertices, except the middle one. So the initial graph, that had to be reconstructed, is depicted on the Fig. 13.

Both task 2 and task 3 require the proof of uniqueness of the answer (up to isomorphism). Task 3 is related to the concept of isomorphism even stronger, as the constructive part itself requires the comprehension of the concept.

This task is a stepstone to graph reconstruction theory. Reconstruction conjecture states that every graph is determined uniquely (up to isomorphism) by its subgraphs. This hypothesis is still unproven, meanwhile its formulation is understandable even

![Fig. 12. Task 3.]

![Fig. 13. Task 3 solution.]
for some schoolchildren. Partial cases of this theory is an unexaustible source of interesting task.

One of our general ideas is that constructive tasks could be efficient on a way to theoretical ones. According to this approach, this task was followed by the next one, devoted to the same concepts.

6.4. Task 3.1 (Qualifying Round, 2019/20)

The teacher drew a graph with 10 vertices on the whiteboard. Ten students redrawn it into their notebooks (possibly not preserving the locations of the vertices), while each of them lost one vertex (each their own) and all the edges incident to it

a) (3 points) Six guys got graphs with 14 edges, and four got 13. How many edges were in the original graph if the children made no other mistakes?
b) (3 points) Six guys got graphs with 13 edges, and four got 14. Prove that one of them not only lost the top, but also messed up with the edges.

The key idea for this task is that every edge of the initial graph is missed exactly in two subgraphs. It means that every edge can be found at 8 students pictures, so to find the initial number of edges we should summarize the numbers of edges on the students pictures and divide it by 8. In the a) task we obtain 17, in the b) one the total sum cannot be divided by 8.

7. DM&TI Graph Tasks. Results and Statistics

Task 3.2 was successfully completed by 18 students. Another 2 student successfully completed only part a).

| Table 1 | Task 1 results |
|---------|----------------|
| Participants | 339 |
| Solutions | 226 |
| Optimal solutions | 107 |
| Suboptimal solutions | 79 |

| Table 2 | Task 2 results |
|---------|----------------|
| Participants | 60 |
| Solutions | 53 |
| Constructive tasks solved | 20 |
| Proofs of uniqueness | 4 |
Unfortunately, it was impossible to maintain a clear experiment with a control group within the framework of the Olympiad, but at least we can see that only one of the students, who have completed the theoretical tasks, failed with a constructive one. Thus we can conclude that some correlation exists.

**Concluding Remarks**

The qualitative and statistical analysis of results of 6 subjects (the paper contains only one subject) leads to the following conclusion:

1. Practically all participants (99%) got partial solutions of the task, that proves that subjects are understandable for participants.
2. Participants’ solutions demonstrate than more than 90% of participants not only mastered the concept of graph (vertex, edge) and learned to build graphs with different properties (planar, connected), but also formed an understanding of the isomorphism concept.
3. More than 30% participants managed to optimize a solution over the main criterion, they built a graph with the maximal number of non-isomorphic components. And the majority of them formed an understanding of building minimal spanning trees.
4. The stated task contained elements of an open research problem. The two best found solutions (different) lay in an unobvious region of a solutions set, they do not use all the stars as graph vertexes.
5. The proposed approach to construct tasks and their automated support provides an ability for a vast majority of participants to achieve success in solving proposed task and not to reject searching for solutions despite that problems in the formal statement are quite hard and belong to the class of olympiad problems.
6. The implementation of problems in the form of constructive-research subjects provides wide abilities for building own solutions and individual routes of searching for these solutions. The analysis of results demonstrates that this potential of problems really shows up in the works of students. More of that, it is interesting that participants usually add their own implicit criteria to their solutions, that are not stated in the problem and that may be considered “aesthetic”, that shows that it is possible to automatically support elements of creative activity by the proposed types of problems.
7. The proposed approach supposes an ability to state and assess success in solving problems with the optimal solution unknown to problems authors. The results of works sometimes really reveal effects of “discoveries” by participants and the arise of original solutions, that do not lay in the area of common tendencies of searching for the optimal solution. Thus, it confirms an ability to automate a support of creative activity by the proposed means.

8. The proposed approach of organization of online competitions, the usage of several subordinate criteria both for the feedback for the participants, and for the ranking of participants, allows for wide dispersion of results and an ability to objectively compare results of a big number of participants (thousands of different results).

9. The fact that subjects are based on hard and even unsolved problems connected with important ideas of mathematics and informatics, together with the huge amount of sensible solutions (more than 90%), show that it is possible to use proposed automated subjects for popularization of important and hard for understanding theoretical ideas, that do are not included in the school curriculum.

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