Path tracking control optimization algorithm for mobile robot based on backstepping control algorithm

Li Peifeng, Bao Hong, Xu Cheng
Beijing Union University, Key Laboratory of Beijing Information Service Engineering, Beijing 100101
Email: baohong@buu.edu.cn

Abstract. Path tracking control of wheeled mobile robot is always an important research content in intelligent control research. Aiming at the control problem that the nonholonomic nature of wheeled mobile robot and external interference factors affect the tracking control effect, an improved backstepping control algorithm is proposed based on backstepping algorithm and speed compensation control method. Firstly, the kinematics model of wheeled mobile robot is constructed, and the mathematical relations of kinematics and path tracking state error are deduced. Secondly, the kinematics speed control law of mobile robot is designed based on the backstepping control algorithm and speed compensation control method, which improves the robustness of path tracking control. Finally, the design is verified by Lyapunov stability theorem. The tracking convergence of the tracking control algorithm is discussed. Compared with the traditional backstepping control algorithm, the improved algorithm improves the robustness of path tracking control. The tracking control algorithm proposed in this paper can be applied in the related fields of wheeled mobile robot, and the practical benefit is remarkable.

1. Introduction
Wheeled mobile robots are widely used in rescue, warehouse management, transportation and other scenarios due to their high flexibility and small size compared to cars. Wheeled mobile robot is a nonholonomic constraint system, which also brings certain challenges to the research of wheeled mobile robot path tracking control, and promotes the research of many control algorithms in mobile robot path tracking control.

The traditional PID control algorithm is widely used in the design of path tracking controller for mobile robot [1,2]. The sliding mode controller of mobile robot is designed based on the design idea of sliding mode control algorithm [3]; [4] proposes a novel adaptive indirect controller that uses integrated improved integrated sliding mode control (ISMC) and adaptive neural networks (NNs) to solve the path tracking control problem. The design idea of model predictive control algorithm is also applied to the design of mobile robot path tracking controller [5,6]. Based on the design idea of backstepping control algorithm, combined with other control technologies, the path tracking controller of mobile robot is optimized and designed, such as sliding mode control algorithm, fuzzy logic reasoning idea, nonlinear disturbance observer and other technologies [7-9].

The research is mainly aimed at the problem of poor tracking stability of the four-wheel mobile robot during the path tracking process due to its own and external factors. A control method based on the speed compensation method to improve the backstepping control algorithm is proposed.
The content of this article is mainly expanded from the following parts. The first part is mainly about the construction and analysis of the wheeled mobile robot control model; the second part is the improvement of the backstepping control algorithm design based on the speed compensation method and the Lyapunov stability verification; the third part is mainly through MATLAB Simulink simulation experiment to verify the improved design algorithm and related data analysis; and the final conclusion.

2. Construction of wheeled robot control model

2.1 Control model based on kinematics

The research object of this paper is an indoor mobile robot driven by four independent wheels. In this paper, the rotation speed of the same wheel of the mobile robot is set to be the same, and the steering motion of the robot is realized by the rotation speed difference between the two driving wheels. This paper studies and constructs two coordinate systems, namely global coordinate system X0Y and local coordinate system x0y, to describe the motion state and pose state of mobile robot. Therefore, the kinematic control model of mobile robot is shown in Figure 1.

![Figure 1. Kinematic tracking control model](image)

The research uses a pose vector $q = \begin{bmatrix} x & y & \theta \end{bmatrix}^T$ and a speed vector $u = \begin{bmatrix} v & \omega \end{bmatrix}^T$ to describe the pose and speed state of the mobile robot. In the figure, $L$ represents the length of the vehicle body, $D$ represents the distance between the wheels on both sides, and $r$ represents the radius of the wheels.

Taking the position in Figure 1 as the reference point, the mobile robot is regarded as a mass point in the research, and the kinematic relation of the mobile robot is obtained as the following formula (1):

$$\dot{q} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{\theta} \end{bmatrix}^T = \Gamma(q) \cdot u = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}$$

(1)

2.2 Error model based on tracking state

Before studying the path tracking control of the mobile robot, the path tracking error model of the mobile robot is established. The schematic diagram of the tracking error model is shown in Figure 1. Combined with the kinematics mathematical model analysed in the previous section, the mathematical relationship of the tracking state error is obtained.

The point M in Figure 2 represents the current pose of the mobile robot, and the point T represents the desired pose of the mobile robot to track.

According to the orthogonal rotation transformation matrix, the global tracking error is transformed into the local tracking error. The orthogonal rotation transformation matrix is $T_u$, The path tracking error model of the mobile robot is obtained from the path tracking error analysis in Figure 1 and the orthogonal rotation transformation matrix:
The research of path tracking control of mobile robot is to make the tracking error of the mobile robot, tend to zero through the control input, so that the mobile robot can move from the current position to the target tracking path point to complete the tracking control task.

3. Improved algorithm based on the backstepping control algorithm

3.1 Backstepping control model

The design of the path tracking control law of the mobile robot is based on the tracking error of the mobile robot. When the heading angle error reaches the set value, the input control value derived from the control law design based on the backstepping algorithm will be compensated. According to the derivation of the path tracking error model of formula (2) and combining with formula (1), the tracking error state model is as follows:

\[
e_{p} = T_{M} \cdot (q_{p} - q_{M}) = \begin{bmatrix}
\cos \theta_{M} & \sin \theta_{M} & 0 \\
-\sin \theta_{M} & \cos \theta_{M} & 0 \\
0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
x_{p} - x_{M} \\
y_{p} - y_{M} \\
\theta_{p} - \theta_{M}
\end{bmatrix}
\]  

(2)

The path tracking controller of the mobile robot is designed based on the idea of the path tracking error model and the backstepping algorithm of the mobile robot. The Lyapunov function is designed through the idea of the backstepping control algorithm, as shown in the following formula (4):

\[
V = \frac{(k_{i} + k_{j})}{2} (e_{i}^{2} + e_{i}^{2}) + \frac{1}{k_{i}} (1 - \cos e_{o})
\]  

(4)

Differential derivation of equation (4) and substituting equation (3) into the following equation:

\[
\dot{V} = (k_{i} + k_{j}) e_{i} (v_{r} \cos e_{o} + \omega_{t} e_{r} - v_{M}) + (k_{i} + k_{j}) e_{i} (v_{r} \sin e_{o} - \omega_{t} e_{i}) + \frac{1}{k_{i}} \sin e_{o} (\omega_{t} - \omega_{o})
\]  

(5)

The tracking speed control law is designed based on the backstepping control model as formula (6):

\[
\begin{align*}
v_{M} & = v_{r} \cos e_{o} + (k_{i} + k_{j}) v_{r} e_{i} \\
\omega_{t} & = \omega_{o} + (k_{i} + k_{j}) k_{r} v_{r} e_{r} + k_{r} v_{r} \sin e_{o}
\end{align*}
\]  

(6)

Substituting formula (6) into formula (5) to simplify:

\[
\dot{V} = - (k_{i} + k_{j}) v_{r} e_{r}^{2} - v_{r} \sin^{2} e_{o}
\]  

(7)

The setting of formula (7) mainly includes $k_{i}, k_{j} > 0$ and $v_{r} > 0$, It can be concluded that as long as the input control speed is bounded, the path tracking error of the mobile robot is also bounded.

3.2 Improved backstepping algorithm control model

Considering that in the process of path tracking control, due to the influence of external interference, mobile robot's own characteristics and path's own attitude factors, the stability of the inversion control model cannot meet the path tracking effect, and there will be large tracking error in the tracking process. Aiming at the problem of poor tracking robustness caused by various disturbance factors, the design of backstepping control model is improved based on speed compensation method and tracking state error.

When the heading angle error is $2/3 \pi > e_{o} \geq 1/2 \pi$ and $-2/3 \pi < e_{o} \leq -1/2 \pi$, the compensation design value of formula (8) is input as follows:

\[
u_{\text{comp}} = \begin{bmatrix}
v_{\text{comp}} \\
\omega_{\text{comp}}
\end{bmatrix}^T = \begin{bmatrix}
e_{r} & e_{r} \\
pi & \pi
\end{bmatrix}^T
\]  

(8)
When tracking the heading angle error is $e_\theta \geq 2/3 \pi$ and $e_\theta \leq -2/3 \pi$, the compensation design input is as follows:

$$u_{\text{comp}} = \begin{bmatrix} v_{\text{comp}} \\ \omega_{\text{comp}} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\pi}{e_\theta} \omega_\theta \end{bmatrix}$$ (9)

4. Experimental results and analysis

4.1 Experimental process

In this paper, based on the kinematics model of mobile robot provided by MATLAB, the path tracking control model is designed, and the proposed path tracking control algorithm is simulated and verified. For the improved control algorithm proposed by the research, the parameters need to be initially set during the simulation process. The path tracking simulation experiment parameters are shown in Table 1 below:

| Project                  | Parameters                        |
|-------------------------|-----------------------------------|
| Initial pose            | $q_0 = [0 \ 0 \ 0]^T$             |
| Initial expected pose   | $q_i = [1 \ 1 \ \pi / 4]^T$       |
| Desired tracking speed  | $v = \text{lm/s}$, $\omega = 0.5 \text{rad/s}$ |
| Control law parameters  | $k_1 = 0.5, k_2 = 0.8, k_3 = 3.5$ |

Tracking path points are a series of self-defined random path points. The desired pose angle for tracking path points is the angle between the path desired point and the previous path desired point and the abscissa.

4.2 Analysis of experimental results

The schematic diagram of the path tracking simulation experiment is shown in Figure 2 below. From the tracking diagram in Figure 2, the mobile robot has good tracking effect, small tracking error and strong robustness in the face of sudden external interference.

Under the condition that the simulation experiment environment is consistent, this paper compares the traditional single inversion control algorithm with respect to the lateral control error, and draws the two parameter values into curves, as shown in Figure 3 below.

Figure 2. Schematic diagram of path tracking

Figure 3. Tracking position error

Figure 4 and Figure 5 are the comparison of the linear velocity and angular velocity curves in the path tracking control.
It can be seen from the curve in Figure 3 that the tracking error of the improved inversion control model based on the velocity compensation method is controlled within 0.2m under the interference factors, and the error fluctuation is small, while the peak value of the transverse error of the traditional inversion control algorithm is 0.4m, and the transverse error fluctuation is large. It can be concluded that the robustness of the tracking control algorithm designed in this paper is far better than that of the traditional inversion control algorithm.

From the curve analysis in Figure 4 and Figure 5, the speed control robustness of the optimization algorithm is obviously improved. The results show that the speed control robustness and stability of the optimal control algorithm are better than the traditional inversion control algorithm, and it is more suitable for the safety design of mobile robot controller.

5. Conclusion
In this paper, aiming at the mobile robot's path tracking stability is easily affected by external interference factors, an improved backstepping tracking control algorithm based on speed compensation method is designed, and the effectiveness of the design scheme is verified by simulation experiments, which improves the robustness of path tracking control under the influence of external interference, and the control effect is good. There are still many deficiencies in this paper. In the future research, we will further consider the detailed influence of the dynamic model and the influence of the four-wheel drive mathematical model of the four-wheel mobile robot on the tracking control.

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Acknowledgments
This work was supported, the National Natural Science Foundation of China (Grant No. 61932012), the Beijing Municipal Commission of Education Project (No.KM202111417001), the Academic Research Projects of Beijing Union University (No. ZB10202003, ZK80202001, XP202015).