Decoherence and Multipartite Entanglement of Non-Inertial Observers

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The decoherence effect on multipartite entanglement in non-inertial frames is investigated. The GHZ state is considered to be shared between partners with one partner in the inertial frame whereas the other two are in accelerated frames. One-tangle and π-tangles are used to quantify the entanglement of the multipartite system influenced by phase damping and phase flip channels. It is seen that for the phase damping channel, entanglement sudden death (ESD) occurs for \( p > 0.5 \) in the infinite acceleration limit. On the other hand, in the case of the phase flip channel, ESD behavior occurs at \( p = 0.5 \). It is also seen that entanglement sudden birth (ESB) occurs in the case of phase flip channel just after ESD, i.e. \( p > 0.5 \). Furthermore, it is seen that the effect of the environment on multipartite entanglement is much stronger than that of the acceleration of non-inertial frames.

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Quantum entanglement has been recognized as a powerful tool for manipulating information. The emerging field of quantum information processing has opened a new way of using entanglement for performing tasks that are impossible to achieve as efficiently with classical technologies. Quantum information and quantum computation can process multiple tasks which are intractable with classical technologies. Quantum entanglement is no doubt a fundamental resource for a variety of quantum information processing tasks, such as super-dense coding, quantum cryptography and quantum error correction. In recent years, two important features, entanglement sudden death (ESD) and entanglement Sudden Birth (ESB), have been investigated. Yu and Eberly have investigated loss of entanglement in a finite time under the action of pure vacuum noise for a bipartite system. Multitangle entangled states can be used to construct a variety of information transmission protocols, such as, quantum key distribution and teleportation. For the purposes of quantum communication, multipartite entangled states can serve as quantum communication channels in most of the known teleportation protocols. Furthermore, multipartite entanglement displays one of the most fascinating features of quantum theory, called the nonlocality of the quantum world.

However, entangled states are very fragile when they are exposed to the environment. Decoherence is the major enemy of entanglement which is responsible for the emergence of classical behavior in quantum systems. Therefore, it could be of great importance to study the deteriorating effect of decoherence in entangled states. Recently, researchers have focused on relativistic quantum information in the field of quantum information science due to conceptual and experimental reasons. In the last few years, much attention has been paid to the study of entanglement shared between inertial and non-inertial observers by discussing how the Unruh or Hawking effect will influence the degree of entanglement. Implementation of decoherence in non-inertial frames has been investigated for a two-qubit system by Wang et al. Recently, multipartite entanglement in non-inertial frames has been investigated, where it is shown that entanglement is degraded by the acceleration of the inertial observers. Its extension to the case of a qubit-qutrit system in non-inertial frames under decoherence can be seen in Ref. It is shown that ESB occurs in the case of the depolarizing channel.

In this Letter, the effect of decoherence is investigated for a multipartite system in non-inertial frames by using phase damping and phase flip channels. Three observers, Alice, Bob and Charlie, share a GHZ type state in non-inertial frames. Alice is considered to be stationary whereas the other two observers move with a uniform acceleration. One-tangles and π-tangles are calculated and discussed for both channels. Two important features of entanglement, ESD and ESB are investigated.

Let the three observers, i.e. Alice, an inertial observer, Bob and Charlie, the accelerated observers moving with uniform acceleration, share the following maximally entangled GHZ state

\[
|\Psi\rangle_{ABC} = \frac{1}{\sqrt{2}}(|0_{\omega_i}\rangle_A|0_{\omega_i}\rangle_B|0_{\omega_i}\rangle_C
\]

where \( |0_{\omega_i}\rangle_A \) and \( |1_{\omega_i}\rangle_A \) are vacuum states and the first excited states from the perspective of an inertial observer respectively. Let the Dirac fields, as shown in Refs., from the perspective of the uniformly accelerated observers, be described as an entangled state of two modes monocromatic with frequency \( \omega_i \).

\[
|\psi_i\rangle_M = \cos r_i|0_{\omega_i}\rangle_I|0_{\omega_i}\rangle_{IV} + \sin r_i|1_{\omega_i}\rangle_I|1_{\omega_i}\rangle_{IV},
\]

and the only excited state is

\[
|1_{\omega_i}\rangle_M = |1_{\omega_i}\rangle_I|0_{\omega_i}\rangle_{IV},
\]

where \( \cos r_i = (e^{-2\pi \omega_i/a_i} + 1)^{-1/2} \), \( a_i \) is the acceleration of the \( i \)th observer. The subscripts I and IV of the kets represent the Rindler modes in region I.

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and II, respectively, in the Rindler spacetime diagram (see Fig. 1 in Ref. [28]). Considering that an accelerated observer in Rindler region I has no access to the field modes in the causally disconnected region II and by taking the trace over the inaccessible modes, one obtains the following tripartite state in Rindler spacetime as given by [29,31]

\[
|\Psi\rangle_{AB|C_1} = \frac{1}{\sqrt{2}} \left[ \cos r_b \cos r_c |0\rangle_A |0\rangle_B |0\rangle_{C_1} + \cos r_b \sin r_c |0\rangle_A |0\rangle_B |1\rangle_{C_1} + \sin r_b \cos r_c |0\rangle_A |1\rangle_B |0\rangle_{C_1} + \sin r_b \sin r_c |1\rangle_A |1\rangle_B |1\rangle_{C_1} \right].
\]

For the sake of simplicity, the frequency subscripts are dropped and in density matrix formalism, the above state can be written as

\[
\rho_{AB|C_1} = \frac{1}{2} \left[ \cos r_b^2 \cos r_c^2 |000\rangle \langle 000| + \cos r_b^2 \sin r_c^2 |001\rangle \langle 001| + \sin r_b^2 \cos r_c^2 |010\rangle \langle 010| + \sin r_b^2 \sin r_c^2 |111\rangle \langle 111| \right].
\]

In order to simplify the calculations, it is assumed that Bob and Charlie move with the same acceleration, i.e. \( r_b = r_c = r \). The entanglement of a bipartite can be readily quantified by negativity [36] defined as

\[
N_{AB} = \|\rho_{12}\| - 1,
\]

where \( T_0 \) is the partial transpose of \( \rho_{AB} \) and \( \|\| \) is the trace norm of a matrix. Whereas for a three-qubit GHZ state \( |\Psi\rangle_{ABC} \), \( N_{AB} \) defines the two-tangle which is the negativity of the mixed state \( \rho_{AB} = \text{Tr}_C(|\Psi\rangle_{ABC}\langle\Psi|) \) and its one-tangles can be defined by

\[
N_{A(BC)} = \|\rho_{A|B|C}\| - 1
\]

and the \( \pi \)-tangle is given by

\[
\pi_{ABC} = \frac{1}{3} (\pi_A + \pi_B + \pi_C).
\]

where \( \pi_{A(BC)} \) is the residual entanglement and is defined as

\[
\pi_A = N_{A(BC)}^2 - N_{AB}^2 - N_{AC}^2.
\]

The interaction between the system and its environment introduces decoherence to the system, which is a process of undesired correlation between the system and the environment. The evolution of a state of a quantum system in a noisy environment can be described by the super-operator \( \Phi \) in the Kraus operator representation as [36]

\[
\rho_f = \Phi(\rho_i) = \sum_k E_k \rho_i E_k^\dagger,
\]

where the Kraus operators \( E_k \) satisfy the completeness relation \( \sum_k E_k^\dagger E_k = I \). The Kraus operators for the evolution of the tripartite system have been constructed from the single qubit Kraus operators by taking their tensor product over all \( n^3 \) combinations of \( \pi(i) \) indices as \( E_k = \otimes_{i=1}^n e_{\pi(i)} \), where \( n \) corresponds to the number of Kraus operators for a single qubit channel. The single qubit Kraus operators for the phase damping channel are given by

\[
E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\rho} \end{bmatrix}, \quad E_1 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{\rho} \end{bmatrix}
\]

and for the phase flip channel

\[
E_0 = \sqrt{1-\rho} I, \quad E_1 = \sqrt{\rho} \sigma_z,
\]

where \( \sigma_z \) represents the usual Pauli matrix. Using Eqs. (7) – (12) along with the initial density matrix as given in Eq. (5), the one-tangles and \( \pi \)-tangles of the tripartite system under different environments can be found as given in the following subsections.

The three one-tangles, influenced by the phase damping noise can be calculated by using the definition as given in Eq. (7), are given by

\[
N_{A(BC)} = -\frac{1}{2} + \frac{1}{4} \cos^4 r + \frac{1}{2} ((1 - p_0)(1 - p_1) \\
\cdot (1 - p_2) \cos^4 r)^{1/2} + \frac{1}{2} ((1 - p_0)(1 - p_1) \\
\cdot (1 - p_2) \cos^4 r + \sin^8 r)^{1/2} + \frac{1}{4} \sin(2r)^2,
\]

\[
N_{B(AC)} = -N_{C(AB)} = -\frac{1}{16} + \frac{1}{2} ((1 - p_0)(1 - p_1) \\
\cdot (1 - p_2) \cos^4 r)^{1/2} + \frac{1}{16} \cos(4r) \\
+ \frac{1}{8} ((16 - 16p_0)(1 - p_1)(1 - p_2) \cos^4 r \\
\cdot + \sin^4(2r)^{1/2},
\]

where \( p_0, p_1 \) and \( p_2 \) are the decoherence parameters corresponding to local coupling of the channel with the qubits of Alice, Bob and Charlie, respectively. The collective coupling corresponds to a situation when \( p_0 = p_1 = p_2 = p \). The \( \pi \)-tangle can be calculated easily by using Eq. (8) and is given by

\[
\pi_{ABC} = \frac{1}{3} \left( -\frac{1}{2} + \frac{1}{4} \cos^4 r + \frac{1}{2} ((1 - p_0)(1 - p_1) \\
\cdot (1 - p_2) \cos^4 r)^{1/2} + \frac{1}{2} ((1 - p_0)(1 - p_1) \\
\cdot (1 - p_2) \cos^4 r + \sin^8 r)^{1/2} + \frac{1}{4} \sin(2r)^2 \right)^2 \\
+ \frac{2}{3} \left( -\frac{1}{16} + \frac{1}{2} ((1 - p_0)(1 - p_1) \\
\cdot (1 - p_2) \cos^4 r)^{1/2} + \frac{1}{16} \cos(4r) \\
\cdot (1 - p_2) \cos^4 r)^{1/2} + \frac{1}{8} ((16 - 16p_0)(1 - p_1)(1 - p_2) \cos^4 r \\
\cdot + \sin^4(2r)^{1/2} \right)^2.
\]
The three one-tangles, influenced by the phase flip noise can be calculated by using the definition as given in Eq. (7), are given by

\[ \mathcal{N}_{A(BC)} = -\frac{1}{2} + \frac{1}{2} \cos^2 r (\text{abs}[(1-2p_0)(1-2p_1)] \\
\quad \cdot (1-2p_2) + \cos^2 r) + \frac{1}{2} ((1-2p_0)^2 \\
\quad \cdot (1-2p_1)^2 \cdot (1-2p_2)^2 \cos^4 r + \sin^8 r)^{1/2} \\
\quad + \frac{1}{4} \sin^2 (2r), \] (16)

\[ \mathcal{N}_{B(AC)} = \mathcal{N}_{C(AB)} = -\frac{1}{2} + \frac{1}{2} \text{abs}[(1-2p_0)(1-2p_1)] \\
\quad \cdot (1-2p_2) \cos^2 r + 1/2 \cos^4 r + \frac{1}{2} \sin^4 r \\
\quad + \frac{1}{8} \sin^2 (2r) + \frac{1}{8} (16(1-2p_0)^2(1-2p_1)^2 \\
\quad \cdot (1-2p_2)^2 \cos^4 r + \sin^4 (2r))^{1/2}. \] (17)

The \( \pi \)-tangle can be calculated easily by using Eq. (8) and is given by

\[ \pi_{ABC} = \frac{1}{3} \left( -\frac{1}{2} + \frac{1}{2} \cos^2 r (\text{abs}[(1-2p_0)(1-2p_1)] \\
\quad \cdot (1-2p_2) + \cos^2 r) + \frac{1}{2} ((1-2p_0)^2(1-2p_1)^2 \\
\quad \cdot (1-2p_2)^2 \cos^4 r + \sin^8 r)^{1/2} + \frac{1}{4} \sin^2 (2r)^2 \right) \\
\quad + \frac{2}{3} \left( -\frac{1}{2} + \frac{1}{2} \text{abs}[(1-2p_0)(1-2p_1)] \\
\quad \cdot (1-2p_2) \cos^2 r + 1/2 \cos^4 r + \frac{1}{2} \sin^4 r \\
\quad + \frac{1}{8} \sin^2 (2r) + \frac{1}{8} (16(1-2p_0)^2(1-2p_1)^2 \\
\quad \cdot (1-2p_2)^2 \cos^4 r + \sin^4 (2r))^{1/2} \right). \] (18)

The two-tangles \( \mathcal{N}_{AB}, \mathcal{N}_{BC} \) and \( \mathcal{N}_{AC} \) can be easily calculated by taking the partial trace of the final density matrix after the environmental effects over qubits \( C, A \) and \( B \) respectively. All the two-tangles remain zero as expected since the reduced density matrix is not affected by the environment.

Analytical expressions for one-tangles and \( \pi \)-tangles are calculated for a multipartite system in non-inertial frames influenced by phase damping and phase flip environments. The results are consistent with Refs. [30,31] and can be easily verified from the expressions of one-tangles and \( \pi \)-tangles. It is seen that for \( r = \pi/4 \) all the one-tangles become equal, i.e. \( \mathcal{N}_{A(BC)} = \mathcal{N}_{B(AC)} = \mathcal{N}_{C(AB)} \) for both the environments under consideration. To investigate the effect of decoherence on the multipartite system, the one-tangles and \( \pi \)-tangles are plotted as a function of the decoherence parameter, \( p \) for different values of acceleration \( r \) for phase damping channel in Fig. 1. Figures 1(a)–1(c) show the behavior of one-tangles and \( \pi \)-tangles when only Alice’s qubit is coupled to the phase damping channel with \( p_0 = p_2 = 0 \). Figures 1(d)–1(e) show the behavior of one-tangles and \( \pi \)-tangles when all three qubits are coupled to the phase damping channel with \( p_0 = p_1 = p_2 = p \). It is seen that the \( \pi \)-tangles are heavily influenced by the environment in both cases (local and collective couplings), where the terms local and collective coupling correspond to \( p_0 = p_2 = 0 \) and \( p_0 = p_1 = p_2 = p \) situations respectively. Furthermore, as the value of acceleration \( r \) increases, entanglement degradation is enhanced and it is more prominent for higher values of decoherence.
parameter \( p \). Hence a similar behavior of one-tangles \( \mathcal{N}_{A(BC)} \) and \( \mathcal{N}_{B(AC)} \) is seen, only \( \mathcal{N}_{A(BC)} \) is plotted for discussion. It is shown that the one-tangles and \( \pi \)-tangles go to zero at \( p = 1 \) for the phase damping channel. On the other hand, they go to zero at \( p = 0.5 \) in the case of the phase flip channel indicating an interesting aspect of ESB for \( p > 0.5 \).

In Fig. 2, the one-tangles and \( \pi \)-tangles are plotted as a function of decoherence for different values of acceleration \( r \) influenced by the phase flip channel. Similar to Fig. 1, Figs. 2(a)–2(c) show the behavior of one-tangles and \( \pi \)-tangles as a function of \( p_0 \) when only Alice’s qubit is coupled to the phase flip channel. Figures 2(d)–2(e) show the behavior of one-tangles and \( \pi \)-tangles when all three qubits are coupled to the phase flip channel, i.e. the case of \( p_0 = p_1 = p_2 = p \). It is seen that the \( \pi \)-tangles are heavily influenced by the environment in both the cases (local and collective couplings). It is seen that the maximum entanglement degradation occurs at \( p = 0.5 \) irrespective of the value of acceleration \( r \). It is seen that a sudden entanglement rebound process takes place for \( p > 0.5 \) in the case of phase flip noise. It is also seen that the one-tangles and \( \pi \)-tangles go to zero at \( p = 1 \). Since the rebound process is much mre stronger than the resistance of acceleration, one cannot ignore it anyway as it is much more prominent for \( p > 0.75 \). In Fig. 3, the three-dimensional graphs for one-tangles and \( \pi \)-tangles are given as a function of decoherence parameter \( p \) and acceleration \( r \) for phase damping and phase flip channels. It is shown that different environments affect the entanglement of the tripartite system differently. It is also seen that in the case of the phase flip channel, ESD behavior becomes independent of the acceleration. However, the sudden death of entanglement cannot be avoided for a phase flip noise around 50\% decoherence. Furthermore, entanglement dies out more quickly compared to the phase damping channel for a lower level of decoherence in the case of phase flip noise. The sudden birth and non-vanishing behavior of the one-tangles \( \pi \)-tangle at infinite acceleration is an interesting result. Since Rindler spacetime is similar to Schwarzschild spacetime, it enables one to conjecture that multipartite entanglement does not vanish even if one party falls into the event horizon of the black hole. Hence, some quantum information processing, for example, teleportation,\(^{[18]}\) can be performed within and outside the black hole.

In summary, environmental effects on tripartite entanglement in non-inertial frames is investigated by considering a maximally entangled GHZ type state shared between three partners. It is assumed that the two partners are in accelerated frames moving with the same uniform acceleration. In order to investigate the environmental effects on entanglement, one-tangles and \( \pi \)-tangles are calculated for the multipartite system. It is seen that in the case of the phase damping channel, entanglement sudden death (ESD) occurs for higher values of decoherence. However, for the phase flip channel, ESD behavior happens at \( p = 0.5 \). In addition, prominent behavior of entanglement sudden birth (ESB) is seen for \( p > 0.75 \) in the case of the phase flip channel. Therefore, it is investigated that the effect of the environment is much stronger than that of acceleration for multipartite systems.

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