A Model of Superoutbursts in Binaries of SU UMa Type

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Abstract

A new mechanism explaining superoutbursts in binaries of SU UMa type is proposed. In the framework of this mechanism the accretion rate increase leading to the superoutburst is associated with formation of a spiral wave of a new “precessional” type in inner gas-dynamically unperturbed parts of the accretion disc. The possibility of existence of this type of waves was suggested in [1]. The features of the “precessional” spiral wave allow explaining both the energy release during the outburst and all its observational manifestations. The distinctive characteristic of a superoutburst in a SU UMa type star is the appearance of the superhump on the light curve. The proposed model reproduces well the formation of the superhump as well as its observational features, such as the period that is 3–7% longer than the orbital one and the detectability of superhumps regardless of the binary inclination.

1 Introduction

Binaries of SU UMa type are dwarf novae with orbital period shorter than 3 hr that display superoutbursts. At present, this traditional definition is expanded to include the requirement of presence of the superhump during the superoutburst [2]. Normal outbursts in binaries of this type are rather short and irregular. These ones are explained well by standard models of
dwarf novae (see, e.g., [2]). Superoutbursts are essentially more prolonged, more rare but more regular. For instance, binary OY Car displays normal outbursts each 25–50 days with amplitude \( \sim 3^m \) and duration about 3 days. Superoutbursts are repeated with period \( \sim 300 \) days, their amplitude reaches \( 4^m \) and the duration is as large as 2 weeks. The analysis of observational data shows that the majority of superoutbursts has practically identical profile: the brightness rises sharply for a time of \( \sim 0.1 \) of superoutburst duration, then the brightness diminishes slowly during a time of \( \sim 0.8 \) of superoutburst duration (an extend sloping “plateau”), after that the system rapidly comes back to the quiescent state. Superhumps with period \( P_s \) of a few per cents longer than the orbital period \( P_{orb} \) have been observed in every SU UMa star for which high speed photometry during a superoutburst has been obtained. At their full development the superhumps have a range of \( \sim 0.3 - 0.4^m \) and are equally prominent in all SU UMa stars independent of inclination.

The presence of periodic superoutbursts accompanied by superhumps place dwarf novae of SU UMa type among the most enigmatic phenomenon in astronomy. Despite plenty of both observational data and theoretical models, the understanding of its nature is far from completion. In present work we suggest a new mechanism for explanation of superoutbursts in SU UMa stars. The essence of this mechanism is as follows: (i) during the time between the superoutbursts the accretion disc is formed, it accumulates matter and becomes more dense as compared to the matter of the stream, a gas-dynamically non-perturbed zone is formed in the inner part of the disc; (ii) in accordance with [1], a spiral wave of “precessional” type is generated in the disc’s inner part were gasdynamical perturbations are negligible; (iii) the formation of the wave is accompanied by substantial (up to the order of magnitude) increase of the accretion rate; (iv) retrograde precession of the spiral wave as well as the compactness of inner zone with increasing energy release explain the superhump period which is longer than orbital one as well as the superhump being observable independently of the inclination of the binary. Note, the assumption on the generation of gasdynamically non-perturbed region as far as the matter accumulates in the disc up to some limit permit us to stay in a paradigm of constant mass transfer rate in the binary system. This substantially simplifies the model, since one doesn’t need any more to appeal to variations of conditions in mass-losing star for the explanation of periodic superoutbursts anymore.

The presentation of the new mechanism explaining the superoutbursts in SU UMa stars is given in the paper as follows: Section 2 contains a brief review of observational data and theoretical models for superoutbursts; Section 3 contains the results of 3D gasdynamical simulation of the morphology of gaseous flows in semidetached binaries as well as the description of basic
features of the “precessional” spiral wave forming in the inner part of the
disc; the comparison of observational manifestations of superoutbursts and
computational results confirming the consistency of the new model is given
in Section 4.

2 Observations and theoretical models
for superoutbursts

Following to Warner (see monograph [2]), let us summarize the basic features
of superoutburst and superhump on SU UMa variables.

The superoutburst peculiarities can be stated as:

1. it is $\sim 0.5 - 1^m$ brighter than a normal outburst; total energy released
   in a superoutburst is $E \simeq 10^{40}$ erg, so it is $\sim 10$ times larger than that
   for a normal outburst;

2. its duration is few times larger than that for normal outburst and can
   be several weeks long;

3. the intervals between superoutburst are rather long and, as a rule, are
   hundreds of days, but for some systems can reach few thousands of
   days; the intervals are approximately constant;

4. all superoutbursts have the same profile: rapid rise, extended sloping
   plateau, rapid decline;

5. the slope of the plateau is almost invariant and equal to $\sim 9\pm 1$ day/mag,
   i.e. the brightness diminishes one stellar magnitude during approx-
   imately 9 days; since the duration of superoutbursts is ranged from
   $\sim 10$ to $\sim 30$ days, the changing of brightness in plateau is $\sim 1 - 3^m$;

6. the brightness and color changes in the rise to a superoutburst are in
   general indistinguishable from those for a normal outburst in the same
   system, so a superoutburst begins to develop as a normal outburst;

7. there are no recorded instance of a normal outburst occurring during
   a superoutburst, or immediately at the end of it; basing on this fact it
   is usually concluded that normal outbursts and superoutbursts are not
   independent phenomena;
8. the superhumps appear some time after maximum of superoutburst (the so called interregnum, this time is, as a rule, from \( \frac{1}{20} \) to \( \frac{1}{2} \) of the plateau length);

9. in the eclipsing SU UMa systems (e.g., OY Car and Z Cha) light curves have recurrent dips at phases \( \sim 0.25 \) and 0.75, these ones are usually ascribed to increases in vertical thickness of the disc at the corresponding phases; UV flux distributions also show minima at phases \( \sim 0.2 \) and 0.8;

10. asymmetrical line profiles show that during superoutbursts a substantial fraction of the disc is in non-circular motion; the non-circular component rotates with period \( P_s \) not \( P_{orb} \), so it is intimately linked with the superhump process;

11. for some SU UMa stars (e.g., VW Hyi) after the rapid brightness decrease at the end of plateau there are both normal orbital humps and also a modulation at the superhump period shifted in phase by \( \sim 180^\circ \); these are known as the “late superhumps”;

The superhump peculiarities can be stated as:

12. superhumps have been observed for every SU UMa star for which high speed photometry during a superoutburst has been obtained;

13. as mentioned above, the asymmetrical line profiles show that the superhump phenomenon is linked with non-circular motion of the disc components;

14. the superhump period is 3–7\% longer than the orbital period;

15. for some cases the substantial decrease (up to 1.25\%) of the superhump period during the superoutburst is observed;

16. superhumps are equally prominent in all SU UMa stars independent of inclination (even in stars like V436 Cen, WX Hyi and SU UMa, which have no detectable orbital humps during quiescence);

17. at their full development the superhumps have a range of \( \sim 0.3 - 0.4^m \);

18. usually the amplitude of superhumps decreases faster than the system brightness, causing them to disappear before the end of the extended slope plateau of the superoutburst;
19. Multicolor photometry of superhumps shows that the superhump light is bluest at minimum and reddest at superhump maximum, thus there appear to be an inverse correlation between color temperature and brightness of the superhump;

20. The eclipse mapping technique permits to reveal the superhump light sources, it was found that the main sources are located in three regions in the disc.

A number of models were suggested to explain the peculiarities of superoutbursts and superhumps enumerated above. The first models of superoutbursts and superhumps were based on possible non-synchronously rotating mass-losing star [3], intermediate polar model [4], slight eccentricity of binary orbit [3], mass transfer variations [17], disc instability [8], as well as combination of two latter [9]. The review of early models as well as its criticism can be found in the monograph by Warner [2].

In 1982 Vogt [10] suggested that the accretion disc takes up an elliptical shape during superoutburst, later Osaki [7,8] and Mineshigi [11] proposed that a slowly precessing elliptical disc develops during superoutburst. Whitehurst [12,13] confirmed numerically the possibility of disc’s precession for sufficiently small values of $q$. This model currently is thought to be the essence of the process producing superoutbursts in SU UMa stars. The precession appearance in this model is due to the disc instability caused by Lindblad eccentric resonance 3:1 [14].

In accordance with [8,14,16], the superoutburst onset can be explained as follows: in the course of accretion the matter is accumulated in the disc so the disc radius increases; the short normal outbursts occurs during this time, the latter are caused by the same thermal instability (see, e.g., [17,18]) as that of ordinary dwarf novae of U Gem type; a tidal instability occurs only when a normal outburst pushes the outer edge of the disc beyond a critical radius for the eccentric 3:1 Lindblad resonance, that is, a normal outburst triggers a superoutburst. During the superoutburst the shape of disc becomes elliptical, the disc begins to precess, and the superhump is formed.

1Nonaxisymmetric modes in the disc are expressed as $\exp[i(k\varphi - l\Omega t)]$ and $(k, l)$ are an integer pair specifying a particular mode. Lubow [14] have shown that the density perturbations produced by tidal perturbation with (3,3) couples with an imposed eccentric perturbations with (1,0) to excite two-armed spiral density waves with (2,3) at the eccentric inner Lindblad resonance which is given by $\omega = m\Omega/(m - 2)$ for $m = 3$, that is the 3:1 resonance.
Yet this model can not explain all observational manifestations of SU UMa stars. Firstly, it can’t explain the appearance of superhump for binaries that don’t display the normal hump in quiescence, i.e. for near face-on SU UMa stars (e.g., V436 Cen, WX Hyi, SU UMa [2]).

The model of precessing accretion disc also can’t explain the appearance of superoutbursts and superhumps in SU UMa binaries with relatively large mass ratio \((q > 0.22, \text{e.g., VY Scl [19]})\).

The “standard” model also fails in explanation of late superhumps. Osaki [7] and Whitehurst [12] proposed that the eccentric disc survives for several days after the end of superoutburst, and the modulation of “bright spot” brightness produces late superhumps (see also [20, 21]). But an independent determination of disc eccentricity for OY Car doesn’t meet this model [22, 23].

Taking into account all these inconsistencies there are some doubts in adequacy of existing models explaining superoutbursts and superhumps in SU UMa binaries.

3 Results of 3D gasdynamical simulation of cool accretion discs. The peculiarities of “precessional” spiral wave.

Results of both qualitative analysis and 3D gasdynamical simulations of the morphology of gaseous flows in semidetached binaries when the gas temperature is low \((\sim 10^4 \text{ K})\) permit us to reveal the basic features of the structure of cool accretion discs [1, 24]. In general, the flow structure is qualitatively the same as for the case of high gas temperature [25, 27], namely: the gasdynamical structure of gaseous flows is governed by the stream of matter from \(L_1\), accretion disc, circumdisc halo and circumbinary envelope; the interaction between the stream and the disc is shock-free; the interaction of matter of circumdisc halo and circumbinary envelope with the stream results in the formation of the shock – “hot line” – located along the edge of the stream. At the same time the gas temperature decrease leads to a number of differences. Thus the cool accretion disc becomes sufficiently more dense as compared to the matter of the stream, the disc is thinner and has more circular form. The second arm of tidal spiral shock (discovered in [28, 30]) is formed, the both arms don’t reach the accretor but are located in the outer part of the disc. Taking into account that the stream acts on the dense inner part of the disc weakly as well as that all shocks (“hot line” and two arms of tidal wave) are located in the outer part of the disc we can identify a new
element of flow structure for low-temperature case: the inner region of the accretion disc where the impact of gasdynamical perturbations is negligible.

Formation of gasdynamically non-perturbed region in the inner part of the disc allows to consider the latter as an slightly elliptical disc with typical size of $\sim 0.2-0.3A$ ($A$ is the binary separation) embedded in the gravitational field of binary. It is known (see, e.g., [2,31]), that the influence of companion star results in precession of orbits of particles rotating around of the binary’s component. The precession is retrograde and its period increases with approaching the accretor. We have shown in [1] that the retrograde precession with specified law of precession rate results in formation of the density spiral wave of “precessional” type in the inner part of the disc. This wave is formed by apastrons of flowlines and its appearance leads to growth of radial component of matter flux $F_{rad} \propto \rho v_r$ due to increasing of both density $\rho$ and radial velocity $v_r$. The increasing of radial component of matter flux after passing the wave results in increasing of accretion rate in the region where “precessional” wave approaches the accretor.

Right panels of Fig. 1 depict the density distribution and velocity vectors in the equatorial plane for four instants of time (beginning from some chosen one and then more with the interval of one orbital period), namely for $t = t_0$, $t_0 + P_{orb}$, $t_0 + 2P_{orb}$, $t_0 + 3P_{orb}$. Left panels show the distribution of radial flux of matter in the inner parts of the disc for the same instants of time. The analysis of these results confirm that: the dense accretion disc as well as the compact circumdisc halo are formed; the interaction of matter of circumdisc halo and circumbinary envelope with the stream results in the formation of the shock – “hot line” – located along the edge of the stream; the two-armed tidal spiral shock is formed, both its arms don’t reach the accretor but are located in the outer part of the disc. We also can see one more spiral wave in the inner part of the disc.

We have conducted our simulations for the binary with characteristics of the dwarf novae IP Peg: $M_1 = 1.02M_\odot$, $M_2 = 0.5M_\odot$, $A = 1.42R_\odot$. Considering the computational results for four instants of time we can see that the inner spiral wave precesses retrogradely and the velocity of its revolution in inertial frame (i.e. in the observer’s frame) equals $\approx -0.13$ revolution per one orbital period of binary. Analyzing the distribution of radial flux of matter in the disc we have found that it increases with approaching to the accretor and in the maximum it is more than 10 times greater than in other parts of the disc. Thus we can expect more than tenfold increase of the accretion rate.

Of course, the outburst of IP Peg has characteristics that differ from those of superoutbursts in SU UMa binaries due to relative smallness of the disc and, correspondingly, the smallness of the region which isn’t subjected to
Figure 1: Left panel: density isolines and velocity vectors in the equatorial plane of binary; right panel: the same for the central part of the disc.
Figure 1 (continued): The same as in Fig. 1 but after one orbital period.
Figure 1 (continued): The same as in Fig. 1 but after two orbital periods.
Figure 1 (continued): The same as in Fig. 1 but after three orbital periods.
gasodynamical perturbations. Indeed, for a typical binary of SU UMa type the components’ mass ratio is \( q = M_2/M_1 \approx 0.1 \) and this value is substantially smaller than that for IP Peg. Decreasing of the components’ mass ratio implies that the size of Roche lobe increases \( (x_{L_1} \to A \text{ as } q \to 0) \), here \( x_{L_1} \) is the distance between the accretor an the inner Lagrangian point \( L_1 \)), hence forming accretion disc should have a larger size. In accordance to Paczyński, the size of accretion disc depends on \( q \) as

\[
\frac{R_d}{A} = \frac{0.6}{1+q},
\]

i.e. it can reach the value of 0.54\( A \) for SU UMa stars. Consequently, the region without gasdynamical perturbations can be rather extended, so the mechanism resulting in the formation “precessional” could be effective.

To be convinced in this, we have conducted gasdynamical simulation for a binary with parameters of OY Car \( (q=0.147) \): \( M_1 = 0.95M_\odot \), \( M_2 = 0.14M_\odot \), \( A = 0.69R_\odot \). Analysis of the results of simulation shows that “precessional” wave is initiated at the distance \( \approx 0.25A \) from the accretor, the velocity of its revolution turns out to be equal to \( \approx -0.03 \div -0.04 \) revolution per one orbital period of binary, and the accretion rate after passing the wave increases more than 10 times.

Kinematical properties of the “precessional” wave as well as an substantial growth of the accretion rate due to this wave permit us to engage it for the explanation of superoutbursts and superhump in SU UMa stars.

4 Foundations of the new mechanism for superoutburst.

Let us review the basic statements of the suggested mechanism explaining superoutbursts and superhumps in binaries of SU UMa type. In the framework of this mechanism the formation of accretion disc is suggested, the accumulation of matter in the disc makes it more dense as compared to matter of the stream from \( L_1 \), hence the inner part of the disc turns out to be not subjected by gasdynamical perturbations. A spiral density wave of “precessional” type is formed in the gasdynamically non-perturbed parts of the disc; the formation of the wave is accompanied by substantial (up to the order of magnitude) increase of the accretion rate. The growth of accretion rate results in brightening of the binary star, i.e. developing of superoutburst. Retrograde precession of the density wave with the rate of \( \sim \) few hundredth of revolution per one orbital period of binary as well as the compactness of
energy release zone permits to explain the formation of superhump as well as its observational peculiarities.

These observational peculiarities of superoutbursts and superhumps in SU UMa stars were enumerated in Section 2. To be sure that the “precessional” spiral wave can explain these peculiarities let us provide a consequential, item by item comparison between computed and observational peculiarities of both superoutbursts and superhumps.

Energy release, recurrence time and duration of superoutburst (peculiarities #1–#3 from Section 2). Our mechanism suggests that energy release and period of superhumps are determined by the mass and accumulation time of the disc. During the superoutburst approximately a half of the disc mass is accreted (see, e.g., [2]), this results in releasing of energy $E \simeq 10^{40}$ erg. Assuming that a half of binding energy is radiated we can evaluate the mass of accreted part of the disc $(1/2m_d)$ using the formula $E \simeq \frac{1}{2} \frac{GM_1}{R_1} \cdot \frac{1}{2}m_d$. Adopting characteristic parameters for SU UMa stars $M_1 \simeq 1M_\odot$ and $R_1 \simeq 10^9$, we get $m_d \simeq 1.5 \times 10^{-10}M_\odot$. Since the outburst recurrence time (i.e. the time of accumulation of mass $1/2m_d$) is approximately equal to one year we can evaluate the value of mass transfer rate as $10^{-10}M_\odot$/year, the latter is in a good agreement with the standard evaluation of mass transfer rate for cataclysmic variables. Variation of recurrence time for different binaries can be easily explained by variations of value of mass transfer rate, range of the latter being reasonable – for recurrence time $\sim 100$ days mass transfer rate is $\dot{M} \simeq 3 \times 10^{-10}M_\odot$/year and for recurrence time $\sim 1000$ days it is $\dot{M} \simeq 3 \times 10^{-11}M_\odot$/year. A strong regularity of superoutbursts is determined by the constant rate of mass transfer (we can neglect the evolutionary changes on these timescales). Duration of superoutburst is determined by the ratio of the mass of accreted matter to the accretion rate, the latter is approximately equal to the mass transfer rate during quiescence so increasing of accretion rate up to the order of magnitude (or even more) during superoutburst gives the ratio of recurrence time to the superoutburst duration as 10–20, that is in a good agreement with observations as well.

The superoutburst profile (peculiarities #4 and #5 from Section 2). The rapid brightening at the superoutburst onset is stipulated by the appearance of “precessional” spiral wave in the disc and induced increase of accretion rate, the latter is the result of the more effective outward transport of angular momentum in the disc. In the quasi-steady mode the angular momentum transfer rate is apparently constant. In the framework of proposed mechanism the inner regions of the disc are accreted first, and after that
the more outer regions. Due to the law of angular momentum distribution \( \propto r^{1/2} \) for Keplerian disc it takes more time to accrete matter from remote orbits, consequently, the accretion rate will decrease slightly with time and this fall will be displayed as the extended slope plateau. Interestingly, since the angular momentum transfer is determined not by the parameters of binary but by the characteristics of the wave, this is can serve as the explanation of approximately constant value of the plateau slope for different systems. The value of the slope is equal to \( \approx 9 \text{ days/mag} \), i.e. the brightness decreases by one stellar magnitude (or, the same, the accretion rate decreases 2.5 times) during 9 days. It means that in 9 days the more distant \((2.5^2 \rho_{\text{accr}} \approx 0.1A)\) portion of the disc than previous one begins to accrete. Based on these considerations and having the measurements of superoutburst duration and the plateau slope we can evaluate the wave size (i.e. the distance between the initialization and termination points) and deduce some physical characteristics of the disc. Indeed when knowing both the wave size (i.e the radius of accreted part of the disc) and energy release during the superoutburst we can evaluate the mass of the disc and its density as well. The variations in superoutburst duration is apparently due to variations of the wave size, so for given binary parameters and, consequently, for known maximum disc radius we can deduce the maximum superoutburst duration for this system. Adopting \( q \approx 0.1 \) for a typical SU UMa star we can get the maximum radius for the disc (and maximum possible wave size as well) is \( 0.54A \) \cite{15}, hence the brightness can change up to \( \sim 4.25^m \) at most, and maximum superoutburst duration is \( \sim 40 \) days. Adopting again that only a half of disc mass accretes and accepting the density of disc matter to be constant we can find that the wave size is \( \sqrt{2} \) less than disc radius. So new estimation for maximum brightening is \( \sim 3.9^m \), and for maximum duration of superoutburst is \( \sim 35 \) days. The analysis of observational data shows that there are no SU UMa type stars not satisfying these estimations.

Connection between superoutbursts and normal outbursts (peculiarities #6 and #7 from Section 2). Our mechanism suggests that superoutbursts are caused by abrupt increasing of the accretion rate. Normal outbursts are caused by increase of the accretion rate too. Hence, though the increase of accretion rate in these two cases have different nature (formation of the “precessional” wave for superoutburst and disc instability for normal outburst) their observational manifestations can coincide for early stages. It explains the superoutburst beginning as a normal outburst (peculiarity #6). As about the peculiarity #7 the absence of observations of normal outbursts occurring on, or immediately at the end of, a superoutburst doesn’t prove the connection between these types of outbursts. In our mechanism superoutbursts and normal outbursts have different nature but all the same we
can explain the peculiarity #7 as follows: superoutburst destroys the entire inner part of the disc so no outbursts are possible until it will be filled again. The filling rate is determined by efficiency of the outward transport of angular momentum (for instance, due to the turbulent viscosity) but even if the efficiency is rather high it takes a considerable refilling time which is comparable to the outburst duration.

**Appearance of superhump (peculiarity #8 from Section 2).** The superhump appearance is also a consequence of the “precessional” wave formation in the disc. Increasing of accretion rate after gas particle passing the wave is spatially localized in azimuth direction, hence matter arrives to the accretor surface over rather compact zone. During the outburst developing both gas heating and difference in rotational velocities of the accretor and the wave will increase this zone up to the forming of a belt. Nevertheless there will be a “kernel” of energy release characterized by the increased values of accretion rate. This “kernel” is rather compact and located on the accretor surface so it can’t be detectable at some orbital phases. An observer registers the formation of superhump in the moment of egress of “kernel” from eclipse, when the “kernel” is oriented toward the observer. The “kernel” is connected to the “precessional” wave so its rotational velocity is determined by the velocity of the wave. The “kernel” amplitude is determined relatively to the energy release on the entire surface so the amplitude depends on the properties of the accretor and the disc but not on those of the wave. The fact that the superhump is usually registered some time after the beginning of superoutburst is a consequence of the “kernel” compactness. If we suppose that at the moment of superoutburst beginning the wave location (and, therefore, the position of the superhump) is azimuthally distributed with uniform probability then, in average, it takes about a half of precessional period for “kernel” to egress from eclipse (i.e. to reach the direction toward the observer) and will be registered as superhump.

**The details of the disc structure from observations of superhumps (peculiarities #9 and #10 from Section 2).** Observations of light curves of eclipsing SU UMa systems display appearance of dips at phases $\sim 0.2 - 0.25$ and $0.75 - 0.8$ both in optical and UV bands. In our model we can find two arms of the tidal shock wave on these phases. The heating of gas on shocks results in increasing of the disc thickness and can explain the observed dips. The peculiarity #10 dealing with non-circular motion of the disc fraction with period $P_s$ also can be explained in the framework of our model since “precessional” spiral wave is formed in the region of non-circular motion and rotates with the superhump period. These regions being observed can serve as the direct confirmation of our model.
The “late superhump” appearance (peculiarity #11 from Section 2). Observations show that for some SU UMa stars after completion of superoutburst there are both normal orbital humps and also a modulation of brightness at the superhump period but shifted in phase at \( \sim 180^\circ \); these are known as the “late superhumps”. In the framework of our model the appearance of “late superhump” can be explained as follows: (i) the “precessional” spiral wave is formed in the region of apastrons of flowlines; (ii) during the superoutburst the accretion of matter results in the formation of an empty zone (or, more exactly, the zone of a decreased density) in the inner part of the disc. The shape of this zone being non-circular – it is gaunt in the area where the wave was and is located closer to the accretor at the opposite side (in the regions ofperiastrons of flowlines); (iii) after the end of superoutburst and the wave disappearing we will have an elliptical ring of matter instead of the disc, the periastron of this ring is shifted in phase at \( \sim 180^\circ \) as compared to the former location of the wave (or, the same, the superhump phase); (iv) after the superoutburst the transport of angular momentum and, consequently, the accretion are due to viscosity which is the process with uniform azimuthal distribution. Hence, gas particles will lose angular momentum in an axially-symmetrical way, so they will reach the accretor surface faster in the region of closer starting positions, i.e. in the region of periastron of the elliptical ring. This results in the formation of the “late superhump”, that is the modulation of brightness at the superhump period but shifted in phase at \( \sim 180^\circ \). The “late superhump” lifetime is determined by the time of circulization of flowlines in the disc.

Superhump characteristics (peculiarities #12–#16 from Section 2). The superhump observed in every SU UMa star for which high speed photometry during a superoutburst has been obtained is the evidence of the presence of a common mechanism of formation of both superhump and superoutburst. Our model suggests that both superhump and superoutburst are generated by appearance of “precessional” spiral wave in the disc. This wave is formed in the region of non-circular flowlines and the velocity of its rotation is determined by retrograde precession of flowlines (\( \sim \) few hundredth of revolution per one orbital period), hence the superhump peculiarities #13 (connection of superhump and non-circular rotation of the disc) and #14 (3–7% excess of superhump over the orbital period) are natural consequences of the formation mechanism. The superhump period is determined by the period of the wave \( P_{\text{wav}} \) and orbital period \( P_{\text{orb}} \) in accordance to

\[
P_s = \frac{P_{\text{wav}}P_{\text{orb}}}{P_{\text{wav}} - P_{\text{orb}}}.
\]

The decrease of the superhump period during the superoutburst (peculiar-
ity #15) is also quite natural if considering the appearance of superhump to be due to the formation of the “precessional” spiral wave in the disc. Indeed, the value of the precessional period is the average between the period of “fast” outer flowlines and “slow” inner ones (see [1] for details). As the superoutburst develops the wave size will decrease and “slow” inner flowlines will exert more influence. The wave rotation velocity will decrease (i.e., $P_{\text{wav}}$ will increase), and it will result in diminishing of superhump period. As mentioned above, the appearance of superhumps being independent on the binary inclination (peculiarity #16) is explained by the existence of the compact “kernel” of energy release which is spatially localized in azimuthal direction, so the brightness variation is due to the “kernel” being located on the visible/invisible side of accretor.

**The superhump amplitude** (peculiarities #17 and #18 from Section 2). The typical superhump amplitude is $\sim 0.3 - 0.4\%$, this means the energy release in the “kernel” is $\sim 10\%$ of the energy release on the rest of accretor surface. It is quite natural since up to the moment of first registration of superhump by observer the accretor revolves few times, so the accretion zone will be a belt with a small “kernel” (see also comment on the peculiarity #8). For the systems where the interregnum is absent and superhump is registered right away after superoutburst onset the “kernel” is seen just in the beginning and the belt is formed later, as far as the matter accretes. In particular, this effect is displayed in changing of the superhump shape during the outburst: the initial shape is asymmetric (since the rate of energy release as well as the size of region of energy release are different for phases prior and after the superhump), but when the belt is formed the superhump shape becomes symmetrical. The amplitude of superhumps decreasing faster than the system brightness (peculiarity #18) can be explained as follows: as the superoutburst develops the accretion occurs from more and more high orbits, so the “kernel” localization in azimuthal direction becomes weaker. This results in decreasing of ratio of energy release in the compact “kernel” to the total energy release, i.e. to the amplitude of superhumps decreasing faster than the superoutburst amplitude.

**Inverse correlation between the superhump color temperature and brightness** (peculiarity #19 from Section 2). The superhump is observed in the moment of time when the “kernel” of energy release zone is oriented towards the observer. In this time there is the density spiral wave of “precessional” type between the accretor and the observer, this results in reddening due to enlarge absorption in the superhump maximum in comparison with its minimum when the wave doesn’t prevent the observations.

**Superhump light sources** (peculiarity #20 from Section 2). When applying the eclipse mapping technique to SU UMa stars it can be found
three light sources in the outer regions of the disc located in areas coinciding with constituents of presented model: two arms of tidal spiral shock and “hot line” shock. The fact that the eclipse mapping technique doesn’t display the compact energy release zone on the accretor surface is thought to be due to the “excess of azimuthal symmetry” of the method [2], so it smears the fine azimuthal details. This doesn’t prevent the investigations of the outer parts of the disc, but the method fails in revealing of bright spot on the accretor surface.

Resuming the comparison of computed and observed peculiarities for superoutbursts and superhumps we can state that for the first time all, with no exceptions, peculiarities (including even “late superhumps” that are very hard to interpret) are naturally explained in the framework of unified model.

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