The (super)conformal BMS\textsubscript{3} algebra

Oscar Fuentealba\textsuperscript{a}, Hernán A. González\textsuperscript{b}, Alfredo Pérez\textsuperscript{c}, David Tempo\textsuperscript{d} and Ricardo Troncoso\textsuperscript{c}

\textsuperscript{a}Université Libre de Bruxelles and International Solvay Institutes, ULB-Campus Plaine CP231, B-1050 Brussels, Belgium
\textsuperscript{b}Facultad de Artes Liberales, Universidad Adolfo Ibáñez, Santiago, Chile
\textsuperscript{c}Centro de Estudios Científicos (CECs), Av. Arturo Prat 514, Valdivia, Chile
\textsuperscript{d}Departamento de Ciencias Matemáticas y Físicas, Universidad Católica de Temuco, Chile

E-mail: ofuentea@ulb.ac.be, hernan.gonzalez@uai.cl, aperez@cecs.cl, jtempo@uct.cl, troncoso@cecs.cl

ABSTRACT: The conformal extension of the BMS\textsubscript{3} algebra is constructed. Apart from an infinite number of ‘superdilatations,’ in order to incorporate ‘superspecial conformal transformations,’ the commutator of the latter with supertranslations strictly requires the presence of nonlinear terms in the remaining generators. The algebra appears to be very rigid, in the sense that its central extensions as well as the nonlinear terms coefficients become determined by the central charge of the Virasoro subalgebra. The wedge algebra corresponds to the conformal group in three spacetime dimensions $SO(3, 2)$, so that the full algebra can also be interpreted as an infinite-dimensional nonlinear extension of the AdS\textsubscript{4} algebra with nontrivial central charges. Moreover, since the Lorentz subalgebra ($sl(2, \mathbb{R})$) is non-principally embedded within the conformal (wedge) algebra, according to the conformal weight of the generators, the conformal extension of BMS\textsubscript{3} can be further regarded as a $W_{(2,2,2,1)}$ algebra. An explicit canonical realization of the conformal extension of BMS\textsubscript{3} is then shown to emerge from the asymptotic structure of conformal gravity in 3D, endowed with a new set of boundary conditions. The supersymmetric extension is also briefly addressed.
# Contents

1. Introduction .................................................. 2
2. The conformal BMS$_3$ algebra .............................. 3
3. Explicit realization: asymptotic structure of conformal gravity in 3D ............................................ 4
4. The superconformal BMS$_3$ algebra ......................... 6
A. Remarks on $so(3,2)$ and the conformal BMS$_3$ algebra ................................................................. 8
1 Introduction

The symmetries of special relativity are embodied through the Poincaré algebra. Thus, extensions thereof turn out to play a relevant role in theoretical physics. Indeed, for relativistic systems with scale invariance, the algebra is generically enhanced to that of the conformal group, including special conformal transformations, see e.g. [1, 2]. Conformal field theories, formulated in terms of these enhanced symmetries, have spanned a wealth impressive results in a wide variety of contexts [3–7]. Besides, extensions of the Poincaré algebra that contain additional fermionic generators of spin 1/2, known as super-Poincaré algebras, provide the building blocks for most of supersymmetric field theories, enjoying a prominent and complementary source of exciting developments [8–14]. Another very interesting extension of the Poincaré algebra, known as the BMS algebra, emerged from the structure of asymptotically flat spacetimes at null infinity [15, 16], in which translations are enhanced to an infinite-dimensional ideal of “supertranslations”. The BMS algebra can be further extended to admit “superrotations” [17–21] and it has recently attracted a great deal of attention due to its fascinating connections with soft theorems [22–24], the memory effect [25], and the information paradox [26, 27]. More recently, the robustness of the BMS algebra shows itself through its canonical realization either at null [28] or spatial infinity [29–31], and also near generic horizons [32].

It is then natural to wonder about the possible compatibility of these three time-honored, but wildly different extensions of the Poincaré algebra.

Conformal and supersymmetric extensions of the Poincaré algebra turn out to be perfectly compatible through the well-known superconformal algebra [8, 33]. Nevertheless, the supersymmetric extension of the BMS algebra remains intriguing. Indeed, among the infinite number of supertranslations, only the subset of standard translations possesses a fermionic “square root” being spanned by four fermionic generators, at null [34] or spatial infinity [35]. Inequivalent extensions with an infinite number of fermionic generators have been proposed in [34, 36, 37], and it is still unclear whether they could be canonically realized even at the linearized level [38].

On the other hand, a conformal extension of BMS has been recently constructed in [39], which successfully accommodates ‘superdilatations’. However, its structure is very different from that of the conformal group since standard special conformal transformations are not included. Thus, the BMS algebra seems to resist compatibility with the full conformal extension.

In the case of three-dimensional spacetimes, the conformal, supersymmetric and BMS extensions of the Poincaré algebra are also well-known. The compatibility of conformal and minimal supersymmetric extensions is also firmly established by the superconformal algebra osp(1|4) [40]. Interestingly, in contradistinction to the four-dimensional case, the BMS$_3$ algebra [42, 43] is known to admit a fully-fledged supersymmetric extension, in the sense that supertranslations possess suitable fermionic square roots, spanned by an infinite number of fermionic canonical generators [44]. However, as in four dimensions, a full conformal extension of BMS$_3$ has not been hitherto reported. In fact, as it has been recently pointed out from entirely different approaches in [45], [46], [47] the BMS$_3$ algebra...
can be suitably enlarged by superdilatations, but nonetheless, some difficulties in the closure of the algebra seem to preclude the inclusion of special conformal transformations.

2 The conformal BMS\(_3\) algebra

Here we show that the conformal extension of the BMS\(_3\) algebra that incorporates ‘super-special conformal transformations’ is a nonlinear algebra. In particular, the commutator of supertranslations with special conformal transformations strictly requires the presence of nonlinear terms in the remaining generators, which acquire support provided that the ‘BMS\(_3\)-Weyl’ subalgebra is endowed with nontrivial central extensions. This can be seen as follows.

It is simple to verify that the BMS\(_3\) algebra, spanned by superrotations \(J_m\) and supertranslations \(P_m\), once enlarged by superdilatations \(D_m\), admits only two nontrivial central extensions. The centrally-extended BMS\(_3\)-Weyl algebra then reads

\[
\begin{align*}
\{J_m, J_n\} & = (m-n)J_{m+n} + c(m^2-1) m \delta_{m+n,0} , \\
\{J_m, P_n\} & = (m-n)P_{m+n} , \\
\{J_m, D_n\} & = -nD_{m+n} , \\
\{P_m, D_n\} & = -iP_{m+n} , \\
\{D_m, D_n\} & = \tilde{c}m \delta_{m+n,0} ,
\end{align*}
\]

where \(m, n \in \mathbb{Z}\). Vanishing commutators are omitted hereafter. Note that in presence of superdilatations, the Jacobi identity excludes the possibility of a nontrivial central charge in the commutator of \(J_m\) and \(P_n\).

The generators of superspecial conformal transformations \(K_m\) can then be incorporated provided that the superdilatations “level” \(\tilde{c}\) coincides with the central charge of the Virasoro subalgebra (\(\tilde{c} = c\)), so that the remaining commutators of the full conformal BMS\(_3\) algebra are given by

\[
\begin{align*}
\{J_m, K_n\} & = (m-n)K_{m+n} , \\
\{K_m, D_n\} & = iK_{m+n} , \\
\{P_m, K_n\} & = -2(m-n)J_{m+n} - 2i(m^2 - mn + n^2 - 1) D_{m+n} \\
& \quad + (m-n)\Lambda_{m+n}^{(2)} + \Lambda_{m+n}^{(3)} - 2c(m^2 - 1) m \delta_{m+n,0} ,
\end{align*}
\]

where \(\Lambda_{m}^{(s)}\) stands for nonlinear terms defined through

\[
\begin{align*}
\Lambda_{m}^{(2)} & = \frac{4}{c} \sum_{n} D_{m-n} D_n , \\
\Lambda_{m}^{(3)} & = -\frac{4i}{c} \sum_{n} J_{m-n} D_n + \frac{4i}{c^2} \sum_{n,l} D_{m-n-l} D_n D_l ,
\end{align*}
\]

with (anomalous) conformal weight \(s\). Indeed, the conformal weight of \(J_m\), \(P_m\) and \(K_m\) is given by \(s = 2\), while \(D_m\) has conformal weight \(s = 1\).
It is worth highlighting that the central extensions as well as the coefficients at front of the nonlinear terms of the conformal BMS\textsubscript{3} algebra turn out to be entirely determined by the central charge \( c \) of the Virasoro subalgebra, and in this sense, the algebra is very rigid.

The wedge algebra reduces to that of the conformal group \( SO(3,2) \). It is recovered by restricting the integers that label the generators according to their conformal weight \( s \) as \(|m| < s\), dropping nonlinear terms (see Eqs. \( \text{(A.10)} \) and \( \text{(A.11)} \)).

Noteworthy, the conformal BMS\textsubscript{3} algebra can then also be interpreted as an infinite-dimensional nonlinear extension of the AdS\textsubscript{4} algebra with nontrivial central charges. In this way, the classical theorem of algebraic cohomology that precludes nontrivial central extensions for semisimple algebras (see e.g. \[ \text{48} \]), clearly does not apply in this case due to the nonlinearity of the extended algebra.

Furthermore, as the Lorentz subalgebra \((sl(2,\mathbb{R}))\), spanned by \( J_m \) with \( m = -1,0,1 \), is non-principally embedded within the wedge algebra \((so(3,2))\), taking into account the conformal weight of the generators, the conformal extension of BMS\textsubscript{3} can also be regarded as a \( W_{(2,2,2,1)} \) algebra (see e.g. \[ \text{49, 50} \]).

It is also worth pointing out that, as it occurs for classical \( W \)-algebras, the conformal BMS\textsubscript{3} algebra is well defined provided that the Virasoro central charge does not vanish; since otherwise, the coefficients that give support to the nonlinear terms would blow up. Nevertheless, this is not necessarily the case for the quantum algebra because these coefficients as well as the central extensions generically acquire corrections.

An explicit canonical realization of the conformal BMS\textsubscript{3} algebra is performed in the next section, while the superconformal extension of BMS\textsubscript{3} is briefly addressed in section 4.2.

## 3 Explicit realization: asymptotic structure of conformal gravity in 3D

The aforementioned link between the conformal BMS\textsubscript{3} and \( W_{(2,2,2,1)} \) algebras, naturally suggests an explicit realization in terms of a WZW model for \( SO(3,2) \) \[ \text{50} \], so that the conformal BMS\textsubscript{3} algebra could be obtained from the Kac-Moody extension of \( so(3,2) \) by virtue of a Sugawara-like construction\textsuperscript{2}. The Kac-Moody currents could also be endowed with suitable constraints so that the conformal BMS\textsubscript{3} algebra emerges from the Dirac brackets. The latter option can be holographically realized along the lines of \[ \text{61} \], so that the constraints would be automatically implemented through an appropriate choice of boundary conditions for a Chern-Simons theory of \( SO(3,2) \).

As shown in \[ \text{62} \], a Chern-Simons theory for \( SO(3,2) \), described by

\[
I_{CS} [A] = \frac{k}{4\pi} \int \left< AdA + \frac{2}{3} A^3 \right>,
\]

turns out to be related to conformal gravity in 3D \[ \text{63, 64} \], which admits an interesting class of black hole solutions \[ \text{65} \] (see also \[ \text{66} \]). Some choices of asymptotic conditions

\textsuperscript{1}An isomorphism between the pure BMS\textsubscript{3} algebra and the so-called \( W(2,2) \) algebra has also been pointed out in \[ \text{51} \], and further elaborated in \[ \text{52–57} \].

\textsuperscript{2}Indeed, this sort of Sugawara-like construction can be successfully implemented in order to obtain the (super) BMS\textsubscript{3} algebra from the affine extension of (super) Poincaré in 3D \[ \text{58, 59} \] (see also \[ \text{60} \]).
for conformal gravity in 3D have already been explored in [67–69], being such that the asymptotic symmetry algebra is given by the direct sum of a $U(1)$ current with either BMS$_3$ or two copies of the Virasoro algebra. Nevertheless, these choices do not accommodate the black holes in [65]. Thus, in what follows we propose a new set of boundary conditions that allows to include them, and also provides a canonical realization of the conformal BMS$_3$ algebra that emerges from the asymptotic symmetries.

Following [61], the radial dependence of the asymptotic form of the gauge field can be completely gauged away by virtue of a gauge choice of the form $A = g^{-1}ag + g^{-1}dg$, with $g = g(r)$, so that the components of the auxiliary connection $a = a_t dt + a_\phi d\phi$ depend only on time and the angular coordinate.

It is useful to express the generators of $SO (3,2)$ in a basis that matches that of the wedge algebra described in Section 2, being precisely defined in (A.10). Thus, the asymptotic behavior that we propose for $a_\phi$ can be readily written in terms of deviations with respect to a reference configuration that go along highest weight generators, i.e.,

$$a_\phi = J_1 - \frac{\pi}{k} \left( \left( J - \frac{\pi}{k} D^2 \right) J_{-1} + \frac{1}{2} P P_{-1} + \frac{1}{2} K K_{-1} - 2 D D \right),$$

(3.2)

where the dynamical fields $J$, $P$, $K$, $D$ depend on $t$, $\varphi$. This fall-off is maintained under gauge transformations $\delta a = d\Omega + [a, \Omega]$, where $\Omega = \Omega [\epsilon_J, \epsilon_P, \epsilon_K, \epsilon_D]$ depends on four arbitrary functions of $t$, $\varphi$ ($\epsilon_X = \epsilon_X (t, \varphi)$). The explicit form of $\Omega$ as well as the transformation law of the dynamical fields are given in Eqs. (A.12) and (A.14), respectively. According to [70, 71], the asymptotic symmetries are preserved by the evolution in time by choosing the asymptotic form of $a_t$ to be generically given by

$$a_t = \Omega [\mu_J, \mu_P, \mu_K, \mu_D],$$

(3.3)

where the “chemical potentials” $\mu_X = \mu_X (t, \varphi)$ are assumed to be fixed at the boundary. The fall-off of $a_t$ is then maintained by the asymptotic symmetries provided that the field equations hold in the asymptotic region, and the parameters $\epsilon_X$ fulfill suitable differential equations of first order in time (see (A.15)).

The asymptotic symmetry generators can then be obtained from different approaches [72, 73], which read

$$Q [\epsilon_J, \epsilon_P, \epsilon_K, \epsilon_D] = - \int (\epsilon_J J + \epsilon_P P + \epsilon_K K + \epsilon_D D) \, d\varphi.$$  

(3.4)

The algebra of the conserved charges (3.4) can then be obtained from their Dirac brackets, or more directly from the transformation law of the fields in Eq. (A.14) by virtue of
\{\mathcal{Q}[\eta_1], \mathcal{Q}[\eta_2]\} = -\delta_{\eta_1} \mathcal{Q}[\eta_2].\) It is explicitly given by

\[
\begin{align*}
\{\mathcal{J}_\phi, \mathcal{J}_\phi\} &= -2\mathcal{J}_\phi \delta'(\phi - \varphi) - \delta(\phi - \varphi)\mathcal{J}'_\phi + \frac{k}{2\pi}\delta''(\phi - \varphi), \\
\{\mathcal{J}_\phi, \mathcal{P}_\phi\} &= -2\mathcal{P}_\phi \delta'(\phi - \varphi) - \delta(\phi - \varphi)\mathcal{P}'_\phi, \\
\{\mathcal{J}_\phi, \mathcal{K}_\phi\} &= -2\mathcal{K}_\phi \delta'(\phi - \varphi) - \delta(\phi - \varphi)\mathcal{K}'_\phi, \\
\{\mathcal{J}_\phi, \mathcal{D}_\phi\} &= -\mathcal{D}_\phi \delta'(\phi - \varphi), \\
\{\mathcal{P}_\phi, \mathcal{D}_\phi\} &= -\mathcal{P}_\phi \delta(\phi - \varphi), \\
\{\mathcal{K}_\phi, \mathcal{D}_\phi\} &= \mathcal{K}_\phi \delta(\phi - \varphi), \\
\{\mathcal{D}_\phi, \mathcal{D}_\phi\} &= -\frac{k}{2\pi}\delta'(\phi - \varphi),
\end{align*}
\]

so that once expanded in Fourier modes, \(X = \frac{1}{4\pi} \sum_m X_m e^{im\varphi}\), it reduces to that in Eqs. (2.1) and (2.2), with \(\tilde{c} = c = k\), provided that the zero mode of \(\mathcal{J}_n\) is shifted as \(\mathcal{J}_0 \to \mathcal{J}_0 - \frac{X}{4\pi}\).

It is worth highlighting that the central extensions of the conformal BMS\(_3\) algebra, in this context are determined by the Chern-Simons level \(k\). This goes by hand with the fact that the conformal group \(SO(3,2)\) is semisimple, and hence, it admits a unique invariant bilinear form being given by the Cartan-Killing metric (up to a normalization that can fixed as in (A.6)).

A remarkable fact of the asymptotic behavior described above is that, since it accommodates the black holes in [65], it includes asymptotically (A)dS or flat three-dimensional spacetimes. Indeed, the precise value of the “cosmological constant” can be seen to be fixed by a suitable quotient of the chemical potentials. The structure of the generic form of the black holes that fit within our asymptotic conditions turns out to be very rich and intricate, and it can be carefully analyzed in terms of the conserved charges that span the conformal BMS\(_3\) algebra. This is left for a forthcoming work.

4 The superconformal BMS\(_3\) algebra

The conformal, supersymmetric and BMS extensions of the Poincaré algebra in 3D can also be seen fully compatible. Indeed, the fermionic generators of the superconformal algebra \(osp(1|4)\), associated to the square roots of translations \((Q)\) and special conformal transformations \((S)\), admit infinite-dimensional extensions that we denote by \(\psi^{[+]}_m\) and \(\psi^{[-]}_m\), corresponding to the square roots of supertranslations and superspecial conformal transformations, respectively.

The superconformal BMS\(_3\) algebra is then spanned by the set \(\{\mathcal{J}_m, \mathcal{P}_m, \mathcal{D}_m, \mathcal{K}_m, \psi^{[+]}_m, \psi^{[-]}_m\}\), so that the commutators of the BMS\(_3\)-Weyl subalgebra \(\{\mathcal{J}_m, \mathcal{P}_m, \mathcal{D}_m\}\) are given
by (2.1) with $\tilde{c} = c$; while the commutators of the generators of superspecial conformal transformations ($K_m$) with the remaining bosonic generators read as in (2.2), where the nonlinear term of conformal weight 3 in (2.4) acquires a quadratic shift in the fermionic generators, according to

$$\Lambda_{m}^{(3)} \rightarrow \Lambda_{m}^{(3)} + \frac{2i}{c} \sum_{n} \psi_{m-n}^{[-]} \psi_{n}^{[+]} .$$

(4.1)

The (anti-)commutators that involve fermionic generators read as

$$i \{ J_m, \psi_{n}^{[\pm]} \} = \left( \frac{m}{2} - n \right) \psi_{n}^{[\pm]} ,$$

$$i \{ D_m, \psi_{n}^{[\pm]} \} = \pm \frac{i}{4} \psi_{n}^{[\pm]} ,$$

$$i \{ P_m, \psi_{n}^{[-]} \} = 2 \left( \frac{m}{2} - n \right) \psi_{n}^{[+]} + \Lambda_{m+n}^{[+](5/2)} ,$$

$$i \{ K_m, \psi_{n}^{[\pm]} \} = -2 \left( \frac{m}{2} - n \right) \psi_{n}^{[-]} - \Lambda_{m+n}^{[-](5/2)} ,$$

$$i \{ \psi_{m}^{[\pm]}, \psi_{n}^{[\pm]} \} = \pm \frac{i}{4} \psi_{n}^{[\pm]} ,$$

$$i \{ \psi_{m}^{[\pm]}, \psi_{n}^{[-]} \} = -K_{m+n} ,$$

$$i \{ \psi_{m}^{[\pm]}, \psi_{n}^{[-]} \} = J_{m+n} - i(m - n) D_{m+n} - \frac{1}{4} \Lambda_{m+n}^{(2)} + \frac{2}{c} \sum_{n} \psi_{m-n}^{[-]} \psi_{n}^{[+]} .$$

(4.2)

where $\Lambda_{m}^{[\pm](5/2)} = \frac{2i}{c} \sum_{n} D_{m-n} \psi_{n}^{[\pm]}$, and the brackets between fermionic generators are symmetric. The fermionic generators are labelled by integers or half-integers for fermionic parameters with periodic or antiperiodic boundary conditions, respectively.

Note that the conformal weight of the fermionic generators $\psi_{m}^{[\pm]}$ is given by $s = 3/2$. For antiperiodic boundary conditions, the wedge algebra reduces to $osp(1|4)$, being recovered once nonlinear terms are dropped and the labels of the generators are restricted according to $|m| < s$, where $s$ is their conformal weight. Therefore, the conformal weight of the generators of the superconformal BMS$_3$ algebra, naturally suggests that it could be regarded as a $W(2,2,2,\frac{3}{2},\frac{3}{2},1)$ algebra.

As in the bosonic case, the superalgebra also appears to be very rigid, in the sense that the coefficients that characterize the nonlinear terms and all of the central extensions become completely determined by the central charge of the Virasoro subalgebra.

It is also worth pointing out that the superconformal extension of the BMS$_3$ algebra can be interpreted as an infinite-dimensional centrally-extended nonlinear extension of the super AdS$_4$ algebra ($osp(1|4)$), suggesting the possibility of a different version of the AdS$_4$/CFT$_3$ correspondence [74], presumably topological and with enhanced symmetries.

A canonical realization of the superconformal BMS$_3$ algebra can also be seen to arise from the asymptotic structure of conformal supergravity in 3D [41, 75], by virtue of a suitable supersymmetric extension of the new boundary conditions presented in section 3 (work in progress).

As a final remark, it might be interesting to explore whether the super BMS$_3$ algebras with $N > 1$ in [76–81], as well as the bosonic and fermionic higher spin extensions of BMS$_3$.
in [82–85] and [86, 87], respectively, could also be compatible with the conformal extension developed here. The compatibility with other possible extensions of BMS\(_3\) as in e.g., [88, 89] also deserves attention.

**Acknowledgments**

We thank Marcela Cárdenas, Ricardo Caroca, Joaquim Gomis, Marc Henneaux, Javier Matulich, Fábio Novaes, Miguel Pino, and Pablo Rodríguez for useful discussions. O.F. holds a “Marina Solvay” fellowship. This work was partially supported by the ERC Advanced Grant “High-Spin-Grav”, by FNRS-Belgium (conventions FRFC PDRT.1025.14 and IISN 4.4503.15). This research has been partially supported by FONDECYT grants N° 1171162, 1181031, 1181496, 1190427. The Centro de Estudios Científicos (CECs) is funded by the Chilean Government through the Centers of Excellence Base Financing Program of Conicyt.

### A Remarks on \(so(3,2)\) and the conformal BMS\(_3\) algebra

The \(so(3,2)\) algebra, spanned by generators \(J_{AB}\), which reads

\[
[J_{AB}, J_{CD}] = \eta_{AC} J_{BD} - \eta_{BC} J_{AD} + \eta_{AD} J_{CB} - \eta_{BD} J_{CA},
\]

is well-known to be isomorphic to the conformal algebra in 3D. Indeed, choosing \(\eta_{AB} = \text{diag}(-1, 1, 1, 1, -1)\) and splitting the index \(A\) according to \(A = \{a, 3, 4\}\), the following change of basis

\[
J_a = \frac{1}{2} \epsilon_{abc} J^{bc}, \quad P_a = J_{a3} - J_{a4}, \quad K_a = J_{a3} + J_{a4}, \quad D = J_{34},
\]

makes the algebra in (A.1) to read as

\[
[J_a, J_b] = \epsilon_{abc} J^c, \quad [P_a, J_b] = \epsilon_{abc} P^c, \quad [K_a, J_b] = \epsilon_{abc} K^c, \quad
[P_a, D] = P_a, \quad [K_a, D] = -K_a, \quad [P_a, K_b] = -2\epsilon_{abc} J^c + 2\eta_{ab} D.
\]

Therefore, if the Cartan-Killing metric is normalized according to

\[
\langle J^{AB}, J_{CD} \rangle = -\delta^{AB}_{CD},
\]

in the “conformal basis” the nonvanishing components of the invariant bilinear metric are given by

\[
\langle J_a, J_b \rangle = \eta_{ab} ; \quad \langle P_a, K_b \rangle = -2\eta_{ab} ; \quad \langle D, D \rangle = 1.
\]

Besides, \(so(3,2)\) also corresponds to the wedge algebra of the conformal extension of BMS\(_3\). In order to see that explicitly, it is useful to choose the Minkowski metric \(\eta_{ab}\) in light-cone coordinates, so that its nonvanishing components read \(\eta_{01} = \eta_{10} = \eta_{22} = 1\), and an orientation for which the Levi-Civita symbol fulfills \(\epsilon_{012} = 1\). The suitable change of basis can then be defined as
\[ J_0 \rightarrow -\frac{1}{2}J_{-1} \ ; \ J_2 \rightarrow J_0 , \quad (A.7) \]
\[ P_0 \rightarrow -\frac{1}{2}P_{-1} \ ; \ P_2 \rightarrow P_0 , \quad (A.8) \]
\[ K_0 \rightarrow -\frac{1}{2}K_{-1} \ ; \ K_2 \rightarrow K_0 , \quad (A.9) \]

and hence, the \( so(3,2) \) algebra in the new basis spanned by \((J_m,P_n,K_m,D)\), with \( m,n = -1,0,1 \), reduces to

\[
\begin{align*}
[J_m, J_n] &= (m-n) J_{m+n} , \\
[J_m, P_n] &= (m-n) P_{m+n} , \\
[J_m, K_n] &= (m-n) K_{m+n} , \\
[P_m, D] &= P_m , \\
[K_m, D] &= -K_m , \\
[P_m, K_n] &= -2 (m-n) J_{m+n} - 2 \left( m^2 - mn + n^2 - 1 \right) D ,
\end{align*}
\]

which agrees with the wedge algebra of the conformal BMS\(_3\) algebra provided that \( i \{ , \} \rightarrow [ , ] \), and

\[ J_m \rightarrow J_m , \ P_m \rightarrow P_m , K_m \rightarrow K_m , iD_0 \rightarrow D \ . \quad (A.11) \]

In the basis (A.10), the \( so(3,2)\)-valued parameter \( \Omega \) that preserves the asymptotic form of the gauge field \( a_\varphi \) in (3.2) is given by

\[
\begin{align*}
\Omega [\epsilon_J, \epsilon_P, \epsilon_K, \epsilon_D] &= \epsilon_J J_1 - \epsilon_K P_1 - \epsilon_P K_1 + \left( \epsilon_P + \frac{2\pi}{k} D\epsilon_J \right) D + \eta [\epsilon_J, \epsilon_P, \epsilon_K, \epsilon_D] , \quad (A.12) \end{align*}
\]

with

\[
\begin{align*}
\eta [\epsilon_J, \epsilon_P, \epsilon_K, \epsilon_D] &= -\epsilon_J J_0 + \left( \epsilon_K - \frac{2\pi}{k} D\epsilon_K \right) P_0 + \left( \epsilon_P - \frac{2\pi}{k} D\epsilon_P \right) K_0 \\
&- \frac{\pi}{k} \left( \left( J - \frac{\pi}{k} D^2 \right) \epsilon_J + K\epsilon_K + P\epsilon_P - \frac{k}{2\pi} \epsilon_J \right) J_{-1} \\
&+ \frac{\pi}{k} \left( \left( J - \frac{3\pi}{k} D^2 + D' \right) \epsilon_K + 2D\epsilon_K - \frac{1}{2} P\epsilon_J - \frac{k}{2\pi} \epsilon_K \right) P_{-1} \quad (A.13) \\
&+ \frac{\pi}{k} \left( \left( J - \frac{3\pi}{k} D^2 - D' \right) \epsilon_P - 2D\epsilon_P - \frac{1}{2} K\epsilon_J - \frac{k}{2\pi} \epsilon_P \right) K_{-1} ,
\end{align*}
\]
so that the transformation law of the dynamical fields reads

\[
\begin{align*}
\delta J &= 2J' \epsilon_J' - J' \epsilon_J - \frac{k}{2\pi} \epsilon''_J + 2P' \epsilon_P' + \mathcal{P}' \epsilon_P + 2K' \epsilon_K' + \mathcal{K}' \epsilon_K + \mathcal{D}' \epsilon_D, \\
\delta P &= 2P' \epsilon_P' + \mathcal{P}' \epsilon_P - 4 \left( J - \frac{4\pi}{k} D^2 \right) \epsilon_K' - 2 \left( J - \frac{4\pi}{k} D^2 \right)' \epsilon_K + \frac{k}{\pi} \epsilon''_P, \\
\delta K &= 2K' \epsilon_K' + \mathcal{K}' \epsilon_K - 4 \left( J - \frac{4\pi}{k} D^2 \right) \epsilon_P' - 2 \left( J - \frac{4\pi}{k} D^2 \right)' \epsilon_P + \frac{k}{\pi} \epsilon''_P, \\
\delta D &= \mathcal{D}' \epsilon_D + \mathcal{D}' \epsilon_D - \mathcal{P} \epsilon_P + \mathcal{K} \epsilon_K + \frac{k}{2\pi} \epsilon'_D.
\end{align*}
\]

As pointed out in [70], the asymptotic form of the field equations can be obtained from the fact that the evolution in time corresponds to a gauge transformation spanned by \( \Omega = \Omega [\mu_J, \mu_P, \mu_K, \mu_D] \), where \( \mu_X \) stands for the chemical potentials.

Finally, in order to maintain the fall-off of \( a_t \), the parameters \( \epsilon_X \) have to fulfill the following differential equations

\[
\begin{align*}
\dot{\epsilon}_J &= \mu_J \epsilon'_J - \epsilon_J \mu'_J + 2 \left( \epsilon_P \mu'_K - \mu_K \epsilon'_P - \frac{4\pi}{k} D \mu_K \epsilon_P \right) \\
&\quad + 2 \left( \epsilon_K \mu'_P - \mu_P \epsilon'_K + \frac{4\pi}{k} D \mu_P \epsilon_K \right), \\
\dot{\epsilon}_P &= \mu_P \epsilon'_P - \epsilon_P \mu'_P - \left( (\mu_D + \mu'_J) \epsilon_P - \mu_J \epsilon'_P \right) + \mu_P \epsilon_D, \\
\dot{\epsilon}_K &= \mu_K \epsilon'_K - \epsilon_K \mu'_K + \left( (\mu_D - \mu'_J) \epsilon_K + \mu_J \epsilon'_K \right) - \mu_K \epsilon_D, \\
\dot{\epsilon}_D &= -\epsilon_J \mu'_D + 2 \left( \mu'_K - \frac{4\pi}{k} J \mu_K + 2D \mu'_K - \frac{6\pi}{k} D^2 \mu_K \right) \epsilon_P \\
&\quad - 2 \left( \mu'_{K} - \frac{8\pi}{k} D \mu_K \right) \epsilon_P + 2 \mu_K \epsilon''_P \\
&\quad - 2 \left( \mu'_P - \frac{4\pi}{k} J \mu_P - \frac{6\pi}{k} D^2 \mu_P \right) \epsilon_K \\
&\quad + 2 \left( \mu'_P + \frac{8\pi}{k} D \mu_P \right) \epsilon_K - 2 \mu_P \epsilon''_K + \mu_J \epsilon'_D.
\end{align*}
\]

References

[1] J. Polchinski, “Scale and Conformal Invariance in Quantum Field Theory,” Nucl. Phys. B 303, 226-236 (1988) doi:10.1016/0550-3213(88)90179-4

[2] Y. Nakayama, “Scale invariance vs conformal invariance,” Phys. Rept. 569, 1-93 (2015) doi:10.1016/j.physrep.2014.12.003 [arXiv:1302.0884 [hep-th]].

[3] P. Di Francesco, P. Mathieu and D. Senechal, “Conformal Field Theory,” doi:10.1007/978-1-4612-2256-9
[4] M. B. Green, J. H. Schwarz and E. Witten, “Superstring Theory Vol. 1: 25th Anniversary Edition,” doi:10.1017/CBO9781139248563

[5] J. L. Cardy, “Scaling and renormalization in statistical physics,” Cambridge university press.

[6] C. Itzykson, H. Saleur and J. B. Zuber, “CONFORMAL INVARIANCE AND APPLICATIONS TO STATISTICAL MECHANICS,” WORLD SCIENTIFIC (1988) 979p doi:10.1142/0608.

[7] R. Blumenhagen and E. Plauschinn, “Introduction to conformal field theory: with applications to String theory,” Lect. Notes Phys. 779, 1-256 (2009) doi:10.1007/978-3-642-00450-6

[8] S. J. Gates, M. T. Grisaru, M. Rocek and W. Siegel, “Superspace Or One Thousand and One Lessons in Supersymmetry,” Front. Phys. 58, 1-548 (1983) [arXiv:hep-th/0108200 [hep-th]].

[9] H. P. Nilles, “Supersymmetry, Supergravity and Particle Physics,” Phys. Rept. 110, 1-162 (1984) doi:10.1016/0370-1573(84)90008-5

[10] H. E. Haber and G. L. Kane, “The Search for Supersymmetry: Probing Physics Beyond the Standard Model,” Phys. Rept. 117, 75-263 (1985) doi:10.1016/0370-1573(85)90051-1

[11] P. C. West, “Introduction to supersymmetry and supergravity,” World Scientific (1990) 425 p.

[12] S. P. Martin, “A Supersymmetry primer,” Adv. Ser. Direct. High Energy Phys. 21, 1-153 (2010) doi:10.1142/9789812839657_0001 [arXiv:hep-ph/9709356 [hep-ph]].

[13] S. Weinberg, “The quantum theory of fields. Vol. 3: Supersymmetry,” Cambridge University Press.

[14] P. Binetruy, “Supersymmetry: Theory, experiment and cosmology,” Oxford Univ. Pr. (2006) 520 p.

[15] R. Sachs, “Asymptotic symmetries in gravitational theory,” Phys. Rev. 128, 2851-2864 (1962) doi:10.1103/PhysRev.128.2851

[16] H. Bondi, M. G. J. van der Burg and A. W. K. Metzner, “Gravitational waves in general relativity. 7. Waves from axisymmetric isolated systems,” Proc. Roy. Soc. Lond. A 269, 21-52 (1962) doi:10.1098/rspa.1962.0161

[17] G. Barnich and C. Troessaert, “Symmetries of asymptotically flat 4 dimensional spacetimes at null infinity revisited,” Phys. Rev. Lett. 105, 111103 (2010) doi:10.1103/PhysRevLett.105.111103 [arXiv:0909.2617 [gr-qc]].

[18] G. Barnich and C. Troessaert, “Aspects of the BMS/CFT correspondence,” JHEP 05, 062 (2010) doi:10.1007/JHEP05(2010)062 [arXiv:1001.1541 [hep-th]].

[19] G. Barnich and C. Troessaert, “Supertranslations call for superrotations,” PoS CNCFG2010, 010 (2010) doi:10.22323/1.127.0010 [arXiv:1102.4632 [gr-qc]].

[20] M. Campiglia and A. Laddha, “Asymptotic symmetries and subleading soft graviton theorem,” Phys. Rev. D 90, no.12, 124028 (2014) doi:10.1103/PhysRevD.90.124028 [arXiv:1408.2228 [hep-th]].

[21] M. Campiglia and A. Laddha, “New symmetries for the Gravitational S-matrix,” JHEP 04, 076 (2015) doi:10.1007/JHEP04(2015)076 [arXiv:1502.02318 [hep-th]].

[22] F. E. Low, “Scattering of light of very low frequency by systems of spin 1/2,” Phys. Rev. 96, 1428-1432 (1954) doi:10.1103/PhysRev96.1428
[23] S. Weinberg, “Infrared photons and gravitons,” Phys. Rev. 140, B516-B524 (1965) doi:10.1103/PhysRev.140.B516

[24] T. He, V. Lysov, P. Mitra and A. Strominger, “BMS supertranslations and Weinberg’s soft graviton theorem,” JHEP 05, 151 (2015) doi:10.1007/JHEP05(2015)151 [arXiv:1401.7026 [hep-th]].

[25] A. Strominger and A. Zhilboedov, “Gravitational Memory, BMS Supertranslations and Soft Theorems,” JHEP 01, 086 (2016) doi:10.1007/JHEP01(2016)086 [arXiv:1411.5745 [hep-th]].

[26] S. W. Hawking, M. J. Perry and A. Strominger, “Soft Hair on Black Holes,” Phys. Rev. Lett. 116, no.23, 231301 (2016) doi:10.1103/PhysRevLett.116.231301 [arXiv:1601.00921 [hep-th]].

[27] S. W. Hawking, M. J. Perry and A. Strominger, “Superrotation Charge and Supertranslation Hair on Black Holes,” JHEP 05, 161 (2017) doi:10.1007/JHEP05(2017)161 [arXiv:1611.09175 [hep-th]].

[28] C. Bunster, A. Gomberoff and A. Pérez, “Regge-Teitelboim analysis of the symmetries of electromagnetic and gravitational fields on asymptotically null spacelike surfaces,” [arXiv:1805.03728 [hep-th]].

[29] M. Henneaux and C. Troessaert, “BMS Group at Spatial Infinity: the Hamiltonian (ADM) approach,” JHEP 03, 147 (2018) doi:10.1007/JHEP03(2018)147 [arXiv:1801.03718 [gr-qc]].

[30] M. Henneaux and C. Troessaert, “The asymptotic structure of gravity at spatial infinity in four spacetime dimensions,” [arXiv:1904.04495 [hep-th]].

[31] O. Fuentealba, M. Henneaux, S. Majumdar, J. Matulich and C. Troessaert, “Asymptotic structure of the Pauli-Fierz theory in four spacetime dimensions,” Class. Quant. Grav. 37, no.23, 235011 (2020) doi:10.1088/1361-6382/abbe6e [arXiv:2007.12721 [hep-th]].

[32] D. Grumiller, A. Pérez, M. M. Sheikh-Jabbari, R. Troncoso and C. Zwikel, “Spacetime structure near generic horizons and soft hair,” Phys. Rev. Lett. 124, no.4, 041601 (2020) doi:10.1103/PhysRevLett.124.041601 [arXiv:1908.09833 [hep-th]].

[33] R. Haag, J. T. Lopuszanski and M. Sohnius, “All Possible Generators of Supersymmetries of the s Matrix,” Nucl. Phys. B 88, 257 (1975) doi:10.1016/0550-3213(75)90279-5.

[34] M. A. Awada, G. W. Gibbons and W. T. Shaw, “CONFORMAL SUPERGRAVITY, TWISTORS AND THE SUPER BMS GROUP,” Annals Phys. 171, 52 (1986) doi:10.1016/0003-4916(86)80023-9.

[35] M. Henneaux, J. Matulich and T. Neogi, “Asymptotic realization of the super-BMS algebra at spatial infinity,” Phys. Rev. D 101, no.12, 126016 (2020) doi:10.1103/PhysRevD.101.126016 [arXiv:2004.07299 [hep-th]].

[36] S. G. Avery and B. U. W. Schwab, “Residual Local Supersymmetry and the Soft Gravitino,” Phys. Rev. Lett. 116, no.17, 171601 (2016) doi:10.1103/PhysRevLett.116.171601 [arXiv:1512.02657 [hep-th]].

[37] A. Fotopoulos, S. Stieberger, T. R. Taylor and B. Zhu, “Extended Super BMS Algebra of Celestial CFT,” JHEP 09, 198 (2020) doi:10.1007/JHEP09(2020)198 [arXiv:2007.03785 [hep-th]].

[38] O. Fuentealba, M. Henneaux, S. Majumdar, J. Matulich and T. Neogi, “Asymptotic structure of the Rarita-Schwinger theory in four spacetime dimensions at spatial infinity,” [arXiv:2011.04669 [hep-th]].
[39] S. J. Haco, S. W. Hawking, M. J. Perry and J. L. Bourjaily, “The Conformal BMS Group,” JHEP 11, 012 (2017) doi:10.1007/JHEP11(2017)012 [arXiv:1701.08110 [hep-th]].

[40] W. Nahm, “Supersymmetries and their Representations,” Nucl. Phys. B 135, 149 (1978) doi:10.1016/0550-3213(78)90218-3

[41] P. van Nieuwenhuizen, “Three-dimensional conformal supergravity and Chern-Simons terms,” Phys. Rev. D 32, 872 (1985) doi:10.1103/PhysRevD.32.872

[42] A. Ashtekar, J. Bicak and B. G. Schmidt, “Asymptotic structure of symmetry reduced general relativity,” Phys. Rev. D 55, 669-686 (1997) doi:10.1103/PhysRevD.55.669 [arXiv:gr-qc/9608042 [gr-qc]].

[43] G. Barnich and G. Compere, “Classical central extension for asymptotic symmetries at null infinity in three spacetime dimensions,” Class. Quant. Grav. 24, F15-F23 (2007) doi:10.1088/0264-9381/24/5/F01 [arXiv:gr-qc/0610130 [gr-qc]].

[44] G. Barnich, L. Donnay, J. Matulich and R. Troncoso, “Asymptotic symmetries and dynamics of three-dimensional flat supergravity,” JHEP 08, 071 (2014) doi:10.1007/JHEP08(2014)071 [arXiv:1407.4275 [hep-th]].

[45] H. Adami, M. M. Sheikh-Jabbari, V. Taghiloo, H. Yavartanoo and C. Zwikel, “Symmetries at null boundaries: two and three dimensional gravity cases,” JHEP 10, 107 (2020) doi:10.1007/JHEP10(2020)107 [arXiv:2007.12759 [hep-th]].

[46] L. Donnay, G. Giribet and F. Rosso, “Quantum BMS transformations in conformally flat space-times and holography,” [arXiv:2008.05483 [hep-th]].

[47] C. Batlle, V. Campello and J. Gomis, “A canonical realization of the Weyl BMS symmetry,” [arXiv:2008.10290 [hep-th]].

[48] D. B. Fuks, “Cohomology of infinite-dimensional Lie algebras,” Springer Science and Business Media, 2012. doi:10.1007/978-1-4684-8765-7

[49] P. Bouwknegt and K. Schoutens, “W symmetry,” Adv. Ser. Math. Phys. 22, 1-875 (1995)

[50] L. Frappat, E. Ragoucy and P. Sorba, “W algebras and superalgebras from constrained WZW models: A Group theoretical classification,” Commun. Math. Phys. 157, 499-548 (1993) doi:10.1007/BF02096881 [arXiv:hep-th/9207102 [hep-th]].

[51] J. Rasmussen and C. Raymond, “Galilean contractions of W-algebras,” Nucl. Phys. B 922, 435-479 (2017) doi:10.1016/j.nuclphysb.2017.07.006 [arXiv:1701.04437 [hep-th]].

[52] N. Aizawa and Y. Kimura, “Galilean conformal algebras in two spatial dimension,” [arXiv:1112.0634 [math-ph]].

[53] D. Adamovic and G. Radobolja, “Self-dual and logarithmic representations of the twisted Heisenberg-Virasoro algebra at level zero,” [arXiv:1703.00531 [math.QA]].

[54] W. Zhang and C. Dong, “W-algebra W(2,2) and the vertex operator algebra L(1/2,0)⊗L(1/2,0),” [arXiv:0711.4624 [math.QA]].

[55] W. Jiang and W. Zhang, “Verma modules over the W(2,2) algebras,” Journal of Geometry and Physics 98 (2015) 118-127.

[56] D. Adamovic and G. Radobolja, “On Free Field Realizations of W(2,2)-Modules,” SIGMA 12 (2016) 113, [arXiv:1605.08608 [math.QA]].
[57] T. Araujo, “Remarks on BMS3 invariant field theories: Correlation functions and nonunitary CFTs,” Phys. Rev. D 98, no.2, 026014 (2018) doi:10.1103/PhysRevD.98.026014 [arXiv:1802.06559 [hep-th]].

[58] G. Barnich and H. A. Gonzalez, “Dual dynamics of three dimensional asymptotically flat Einstein gravity at null infinity,” JHEP 05, 016 (2013) doi:10.1007/JHEP05(2013)016 [arXiv:1303.1075 [hep-th]].

[59] G. Barnich, L. Donnay, J. Matulich and R. Troncoso, “Super-BMS3 invariant boundary theory from three-dimensional flat supergravity,” JHEP 01, 029 (2017) doi:10.1007/JHEP01(2017)029 [arXiv:1510.08824 [hep-th]].

[60] N. Banerjee, A. Bhattacharjee, Neetu and T. Neogi, “New $\mathcal{N}=2$ SuperBMS3 algebra and invariant dual theory for 3D supergravity,” JHEP 11, 122 (2019) doi:10.1007/JHEP11(2019)122 [arXiv:1905.10239 [hep-th]].

[61] O. Coussaert, M. Henneaux and P. van Driel, “The Asymptotic dynamics of three-dimensional Einstein gravity with a negative cosmological constant,” Class. Quant. Grav. 12, 2961-2966 (1995) doi:10.1088/0264-9381/12/12/012 [arXiv:gr-qc/9506019 [gr-qc]].

[62] J. H. Horne and E. Witten, “Conformal Gravity in Three-dimensions as a Gauge Theory,” Phys. Rev. Lett. 62, 501-504 (1989) doi:10.1103/PhysRevLett.62.501

[63] S. Deser, R. Jackiw and S. Templeton, “Topologically Massive Gauge Theories,” Annals Phys. 140, 372-411 (1982) [erratum: Annals Phys. 185, 406 (1988)]

doi:10.1016/0003-4916(82)90164-6

[64] S. Deser, R. Jackiw and S. Templeton, “Three-Dimensional Massive Gauge Theories,” Phys. Rev. Lett. 48, 975-978 (1982) doi:10.1103/PhysRevLett.48.975

[65] J. Oliva, D. Tempo and R. Troncoso, “Static spherically symmetric solutions for conformal gravity in three dimensions,” Int. J. Mod. Phys. A 24, 1588-1592 (2009) doi:10.1142/S0217751X09045054 [arXiv:0905.1510 [hep-th]].

[66] J. Oliva, D. Tempo and R. Troncoso, “Three-dimensional black holes, gravitational solitons, kinks and wormholes for BHT massive gravity,” JHEP 07, 011 (2009) doi:10.1088/1126-6708/2009/07/011 [arXiv:0905.1545 [hep-th]].

[67] H. Afshar, B. Cvetkovic, S. Ertl, D. Grumiller and N. Johansson, “Conformal Chern-Simons holography - lock, stock and barrel,” Phys. Rev. D 85, 064033 (2012) doi:10.1103/PhysRevD.85.064033 [arXiv:1110.5644 [hep-th]].

[68] M. Bertin, S. Ertl, H. Ghorbani, D. Grumiller, N. Johansson and D. Vassilevich, “Lobachevsky holography in conformal Chern-Simons gravity,” JHEP 06, 015 (2013) doi:10.1007/JHEP06(2013)015 [arXiv:1212.3335 [hep-th]].

[69] H. R. Afshar, “Flat/AdS boundary conditions in three dimensional conformal gravity,” JHEP 10, 027 (2013) doi:10.1007/JHEP10(2013)027 [arXiv:1307.4855 [hep-th]].

[70] M. Henneaux, A. Perez, D. Tempo and R. Troncoso, “Chemical potentials in three-dimensional higher spin anti-de Sitter gravity,” JHEP 12, 048 (2013) doi:10.1007/JHEP12(2013)048 [arXiv:1309.4362 [hep-th]].

[71] C. Bunster, M. Henneaux, A. Perez, D. Tempo and R. Troncoso, “Generalized Black Holes in Three-dimensional Spacetime,” JHEP 05, 031 (2014) doi:10.1007/JHEP05(2014)031 [arXiv:1404.3305 [hep-th]].
[72] T. Regge and C. Teitelboim, “Role of Surface Integrals in the Hamiltonian Formulation of General Relativity,” Annals Phys. 88, 286 (1974) doi:10.1016/0003-4916(74)90404-7

[73] G. Barnich and F. Brandt, “Covariant theory of asymptotic symmetries, conservation laws and central charges,” Nucl. Phys. B 633, 3-82 (2002) doi:10.1016/S0550-3213(02)00251-1 [arXiv:hep-th/0111246 [hep-th]].

[74] O. Aharony, O. Bergman, D. L. Jafferis and J. Maldacena, “N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals,” JHEP 10, 091 (2008) doi:10.1088/1126-6708/2008/10/091 [arXiv:0806.1218 [hep-th]].

[75] M. Rocek and P. van Nieuwenhuizen, “N >= 2 SUPERSYMMETRIC CHERN-SIMONS TERMS AS d = 3 EXTENDED CONFORMAL SUPERGRAVITY,” Class. Quant. Grav. 3, 43 (1986) doi:10.1088/0264-9381/3/1/007

[76] N. Banerjee, D. P. Jatkar, I. Lodato, S. Mukhi and T. Neogi, “Extended Supersymmetric BMS3 algebras and Their Free Field Realisations,” JHEP 11, 059 (2016) doi:10.1007/JHEP11(2016)059 [arXiv:1609.09210 [hep-th]].

[77] I. Lodato and W. Merbis, “Super-BMS3 algebras from $N = 2$ flat supergravities,” JHEP 11, 150 (2016) doi:10.1007/JHEP11(2016)150 [arXiv:1610.07506 [hep-th]].

[78] N. Banerjee, I. Lodato and T. Neogi, “N=4 Supersymmetric BMS3 algebras from asymptotic symmetry analysis,” Phys. Rev. D 96, no.6, 066029 (2017) doi:10.1103/PhysRevD.96.066029 [arXiv:1706.02922 [hep-th]].

[79] R. Basu, S. Detournay and M. Riegler, “Spectral Flow in 3D Flat Spacetimes,” JHEP 12, 134 (2017) doi:10.1007/JHEP12(2017)134 [arXiv:1706.07438 [hep-th]].

[80] O. Fuentealba, J. Matulich and R. Troncoso, “Asymptotic structure of $\mathcal{N} = 2$ supergravity in 3D: extended super-BMS3 and nonlinear energy bounds,” JHEP 09, 030 (2017) doi:10.1007/JHEP09(2017)030 [arXiv:1706.07542 [hep-th]].

[81] R. R. Poojary and N. V. Suryanarayana, “On Asymptotic Symmetries of 3d Extended Supergravities,” JHEP 02, 168 (2019) doi:10.1007/JHEP02(2019)168 [arXiv:1712.09221 [hep-th]].

[82] H. Afshar, A. Bagchi, R. Faroughbal, D. Grumiller and J. Rosseel, “Spin-3 Gravity in Three-Dimensional Flat Space,” Phys. Rev. Lett. 111, no.12, 121603 (2013) doi:10.1103/PhysRevLett.111.121603 [arXiv:1307.4768 [hep-th]].

[83] H. A. Gonzalez, J. Matulich, M. Pino and R. Troncoso, “Asymptotically flat spacetimes in three-dimensional higher spin gravity,” JHEP 09, 016 (2013) doi:10.1007/JHEP09(2013)016 [arXiv:1307.5651 [hep-th]].

[84] M. Gary, D. Grumiller, M. Riegler and J. Rosseel, “Flat space (higher spin) gravity with chemical potentials,” JHEP 01, 152 (2015) doi:10.1007/JHEP01(2015)152 [arXiv:1411.3728 [hep-th]].

[85] J. Matulich, A. Perez, D. Tempo and R. Troncoso, “Higher spin extension of cosmological spacetimes in 3D: asymptotically flat behaviour with chemical potentials and thermodynamics,” JHEP 05, 025 (2015) doi:10.1007/JHEP05(2015)025 [arXiv:1412.1464 [hep-th]].

[86] O. Fuentealba, J. Matulich and R. Troncoso, “Extension of the Poincaré group with half-integer spin generators: hypergravity and beyond,” JHEP 09, 003 (2015) doi:10.1007/JHEP09(2015)003 [arXiv:1505.06173 [hep-th]].
[87] O. Fuentes, J. Matulich and R. Troncoso, “Asymptotically flat structure of hypergravity in three spacetime dimensions,” JHEP 10, 009 (2015) doi:10.1007/JHEP10(2015)009 [arXiv:1508.04663 [hep-th]].

[88] R. Caroca, P. Concha, E. Rodríguez and P. Salgado-Rebolledo, “Generalizing the bms_3 and 2D-conformal algebras by expanding the Virasoro algebra,” Eur. Phys. J. C 78, no.3, 262 (2018) doi:10.1140/epjc/s10052-018-5739-7 [arXiv:1707.07209 [hep-th]].

[89] R. Caroca, P. Concha, O. Fierro and E. Rodríguez, “On the supersymmetric extension of asymptotic symmetries in three spacetime dimensions,” Eur. Phys. J. C 80, no.1, 29 (2020) doi:10.1140/epjc/s10052-019-7595-5 [arXiv:1908.09150 [hep-th]].