Single production of the gauge boson $W$ via polarized $e^-\gamma$ collisions in the littlest Higgs model

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Abstract

In the framework of the littlest Higgs ($LH$) model, we study single production of the standard model ($SM$) gauge boson $W^-_s$ and the heavy gauge boson $W^-_H$ via polarized $e^-\gamma$ collisions. We find that the corrections of the $LH$ model to the cross section $\sigma(W^-_s)$ might be observed only for the scale parameter $f \leq 1.5 TeV$ and the mixing parameter $c' \geq 0.4$ in future high energy linear $e^+e^-$ collider ($LC$) experiment with the center-of-mass ($CM$) energy $\sqrt{S} = 500 GeV$ and a yearly integrated luminosity $\mathcal{L} = 100 fb^{-1}$. However, with a suitably chosen polarization of the initial electron and positron beams, the possible signals of the heavy gauge boson $W^-_H$ can be easily detected via $e^-\gamma$ collisions in future $LC$ experiment with $\sqrt{S} = 3 TeV$ and $\mathcal{L} = 500 fb^{-1}$.

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I. Introduction

The high energy linear $e^+e^-$ collider (LC) has a large potential for the discovery of new particles[1]. Due to its rather clean environment, the LC will be perfectly suited for precise analysis of physics beyond the standard model (SM) as well as for testing the SM with an unprecedented accuracy. A unique feature of the LC is that it can be transformed to $\gamma\gamma$ collisions and $e^-\gamma$ collisions with the photon beams generated by the backward Compton scattering of the initial electron and laser beams. Their effective luminosity and energy are expected to be comparable to those of the LC. In some scenarios, they are the best instrument for the discovery of signatures of new physics.

The $e^-\gamma$ collisions can produce particles which are kinematically not accessible in the $e^+e^-$ collisions at the same collider[2]. For example, for the process $e^-\gamma \to AB$ with light particle A and new heavy particle B, the discovery limits can be much higher than in other reactions. The $e^-\gamma$ collisions can uniquely be identified due to the net $(-1)$ charge in the final state, the process $e^-\gamma \to AB$ offers the possibility for both new physics discovery and precision measurements. Thus, the $e^-\gamma$ collisions is particularly suitable for studying heavy gauge boson production. In Ref.[3], we have studied single production of the heavy gauge bosons $B_H, Z_H$ and $W_H$ predicted by the littlest Higgs (LH) model[4] via the unpolarized $e^-\gamma$ collisions and discuss the possibility of detecting these new particles in future LC experiments. We find that, in wide range of the parameter space preferred by the electroweak precision data, the gauge bosons $B_H$ and $Z_H$ should be observed via detecting the $e^-l^+l^-$ signal. However, the gauge boson $W_H$ can not be detected via the process $e^-\gamma \to \nu_e W^-_H$.

An important tool of the LC is using of the polarized beams. One expects that a high polarization degree between 80% and 90% can be reached[1,5]. Beam polarization is not only useful for a possible reduction of the background, but might also serve as a possible tool to disentangle different contributions to the signal and to directly analyze the interaction structure of new physics. Beam polarization of the electron and position beams would lead to a substantial enhancement of the production cross section for some specific processes with a suitably chosen polarization configuration. Furthermore, it has
been shown that the polarization of the initial laser beam and the electron beam will significantly affect the photon spectrum in $e\gamma$ or $\gamma\gamma$ collisions. Thus, a more detailed study of single production of heavy gauge bosons in $e^-\gamma$ collisions with polarized beams is needed. In this paper, we first consider the corrections of the $LH$ model to single production of the $SM$ gauge boson $W^-_s$ via $e^-\gamma$ collisions and discuss the possibility of detecting the virtual correction effects in future $LC$ experiment with the center-of-mass ($CM$) energy $\sqrt{S} = 500 GeV$ and a yearly integrated luminosity $\mathcal{L} = 100 fb^{-1}$. We find that the corrections might be observed only for the scale parameter $f \leq 1.5 TeV$ and the mixing parameter $c' \geq 0.4$. Then, we study single production of the heavy gauge boson $W^-_H$ via polarized $e^-\gamma$ collisions at the $LC$ with the $CM$ energy $\sqrt{S} = 3 TeV$ and $\mathcal{L} = 500 fb^{-1}$. We find that, for a suitably chosen polarization configuration $(p_e, p_\gamma) = (-0.8, 0.6)$ and $(-0.8, -0.6)$, the production cross section of the process $e^-\gamma \rightarrow \nu_e W^-_H$ can be significantly enhanced and the possible signals of the heavy gauge boson $W^-_H$ can be easily observed in wide range of the parameter space preferred by the electroweak precision data.

Little Higgs theory[6] was recently proposed as a possible mechanism of electroweak symmetry breaking ($EW SB$) and is a compelling possibility for physics beyond the $SM$. The key feature of this kind of theory is that the Higgs boson is a pseudo-Goldstone boson of a spontaneously broken approximate global symmetry. So far, a number of specific models have been proposed. The $LH$ model[4] is one of the simplest and phenomenologically viable models, which has all essential features of the little Higgs theory. So, in the rest of this paper, we will give our results in detail in the context of the $LH$ model.

In sec.II, we generally give the formula of the helicity amplitudes and the cross section for the process $e^-\gamma \rightarrow \nu_e W^-$, in which $W^-$ is the $SM$ gauge boson $W^-_s$ or the heavy gauge boson $W^-_H$ predicted by the $LH$ model. The relative corrections of the $LH$ model to single production of the $SM$ gauge boson $W^-_s$ in the $LC$ with $\sqrt{S} = 500 GeV$ and $\mathcal{L} = 100 fb^{-1}$ are calculated in sec.III. The cross section of single production for the heavy gauge boson $W^-_H$ via polarized $e^-\gamma$ collisions at the $LC$ with $\sqrt{S} = 3 TeV$ and $\mathcal{L} = 500 fb^{-1}$ are calculated in sec. IV. The observability of $W^-_H$ are also studied in this section. Section V contains our conclusions.
II. The helicity amplitudes and the cross section for the process $e^-\gamma \rightarrow \nu_eW^-$ in the LH model

At the tree-level, there are two Feynman diagrams contributing to the process $e^-\gamma \rightarrow \nu_eW^-$ for single production of the gauge boson $W^-$, as shown in Fig.1. The s-channel diagram is induced by the gauge couplings of the gauge bosons $\gamma$ and $W^-$ to fermions, while the t-channel diagram involves a triple gauge boson coupling making this process suitable for testing the non-Abelian gauge structure of the theory.

![Feynman diagrams for the process $e^-\gamma \rightarrow \nu_eW^-$](image)

Figure 1: Feynman diagrams for the process $e^-\gamma \rightarrow \nu_eW^-$. 

The LH model[4] consists of a non-linear $\sigma$ model with a global $SU(5)$ symmetry and a locally gauged symmetry $SU(2)_1 \times U(1)_1 \times SU(2)_2 \times U(1)_2$. The global $SU(5)$ symmetry is broken down to its subgroup $SO(5)$ by a vacuum condensate $f \sim \Lambda_s/4\pi \sim TeV$, which results in fourteen massless Goldstone bosons. Four of these massless Goldstone bosons are eaten by the SM gauge bosons, so that the locally gauged symmetry $SU(2)_1 \times U(1)_1 \times SU(2)_2 \times U(1)_2$ is broken down to its diagonal subgroup $SU(2) \times U(1)$, identified as the SM electroweak gauge group. This breaking scenario gives rise to the new gauge bosons $W_H^\pm$. In the LH model, the couplings of the gauge boson $W$ to ordinary particles, which are related our calculation, can be written as[7]:

$$g_L^{W\nu_e} = \frac{ie}{\sqrt{2}S_W}A, \quad g_R^{W\nu_e} = 0, \quad g^{WW} = -eB, \quad (1)$$

where $S_W = \sin \theta_W$, $\theta_W$ is the Weinberg angle. The coupling constant $B$ is always equal to 1 for the SM gauge boson $W_s$ and the heavy gauge boson $W_H$, while the coupling constant $A$ is equal to $1 - \frac{\nu^2}{27\pi^2}(c^2 - s^2)$ and $-\frac{\nu}{s}$ for the gauge bosons $W_s$ and $W_H$, respectively. $\nu = 246GeV$ is the electroweak scale and $c (s = \sqrt{1-c^2})$ is the mixing parameter between

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SU(2)\textsubscript{1} and SU(2)\textsubscript{2} gauge bosons.

In our calculation, we will neglect the initial state electron mass and think that the initial state electron beams are longitudinally polarized beams. Thus, the negative and positive polarized electrons coincide with their left- and right- chirality states, respectively.

In the SM and the LH model, the helicity of the incoming $e^-$ is fixed by the massless neutrino $\nu_e$, which implies that the right-handed electron has no contributions to the cross section of the process $e^-\gamma \rightarrow \nu_eW^-$. In this case, the single production process of the charged gauge boson $W^-$ can be written as:

$$e^-_L(p_e) + \gamma(k) \rightarrow \nu_e L(p) + W^-(p_W).$$  \hfill (2)

The helicity amplitudes of this process can be written as[8, 9]:

$$M_{\lambda\lambda'} = \frac{ie^2A}{\sqrt{2}S_W}u_{\nu}(p)e_\nu(p)e\varepsilon^{\nu}(k, \lambda)\varepsilon^{\mu'}_W(p_W, \lambda'),$$  \hfill (3)

where $\varepsilon^{\nu}(k, \lambda)$ and $\varepsilon^{\mu'}_W(p_W, \lambda')$ are the polarization vectors of the initial state photon and final state $W^-$, respectively. The helicities $\lambda = \pm 1$ and $\lambda' = \pm 1, 0$. The tensor $T_{\mu\nu}$ is the sum of the two terms corresponding to the s- and t- channel diagrams:

$$T_{\mu\nu} = \frac{\gamma_{\mu}(\hat{p}_e + \hat{k})\gamma_{\nu}}{\hat{S}} + \frac{1}{t - M_W^2}[\gamma^\rho - \frac{(\hat{p} - \hat{k})(p_W + k)^\rho}{M_W^2}]\Gamma_{\nu\mu\rho}$$  \hfill (4)

with

$$\Gamma_{\nu\mu\rho} = 2p_{W\nu}g_{\mu\rho} + 2k_{\mu}g_{\nu\rho} - (p_W + k)_{\rho}g_{\mu\nu},$$  \hfill (5)

where $t = (p - p_e)^2$, $\hat{S}$ is the CM energy of polarized $e^-\gamma$ collisions, and $M_W$ is the mass of the gauge boson W. The first term of Eq.(4) comes from the s-channel diagram and the second term comes from the t-channel diagram. Since there is pure vector-axial current and the charged gauge bosons $W^\pm$ has only left-handed couplings to the fermions, the s-channel diagram has non-zero contributions to the helicity amplitudes only for $\lambda = -1$ and $\lambda' = -1, 0$.

Using Eqs.(3) – (5), we can write the helicity amplitudes as:

$$M_{11} = \frac{C_1C_2\sqrt{S}}{t - M_W^2}(\cos\theta - 1)\sin\frac{\theta}{2},$$  \hfill (6)
\[ M_{1-1} = \frac{C_1 C_2}{t - M_W^2} [2|\vec{p}| \sin \theta - \sqrt{\hat{S}} (1 + \cos \theta) \sin \frac{\theta}{2}], \quad (7) \]
\[ M_{10} = \frac{C_2 C_3}{t - M_W^2} [\vec{p} (E_W + |\vec{p}|) - \sqrt{\hat{S} E_W}] \sin \theta \sin \frac{\theta}{2}, \quad (8) \]
\[ M_{-11} = \frac{C_1 C_2}{t - M_W^2} [\sin \theta \cos \frac{\theta}{2} - (1 + \cos \theta) \sin \frac{\theta}{2}], \quad (9) \]
\[ M_{-1-1} = -C_1 C_2 \left\{ \frac{2}{\sqrt{\hat{S}}} \sin \frac{\theta}{2} + \frac{1}{t - M_W^2} [2|\vec{p}| \sin \theta + \sqrt{\hat{S}} \sin \theta \cos \frac{\theta}{2} \right. \]
\[ \left. -\sqrt{\hat{S}} \sin (\cos \theta - 1) \sin \frac{\theta}{2} \right\}, \quad (10) \]
\[ M_{-10} = -C_2 C_3 \left\{ \frac{E_W + |\vec{p}|}{\sqrt{\hat{S}}} \cos \frac{\theta}{2} + \frac{1}{t - M_W^2} [\sqrt{\hat{S}} (|\vec{p}| + E_W \cos \theta) \cos \frac{\theta}{2} \right. \]
\[ \left. -(|\vec{p}| (E_W + |\vec{p}|) \sin \theta - \sqrt{\hat{S} E_W} \sin \theta \sin \frac{\theta}{2}) \right\} \quad (11) \]

with
\[ C_1 = \frac{ie^2}{2Sw} \sqrt{\hat{S} E\nu A}, \quad C_2 = 1 + \frac{|\vec{p}|}{E\nu}, \quad C_3 = \frac{ie^2}{\sqrt{2}M_W S_W} \sqrt{\hat{S} E\nu A}. \quad (12) \]

In above equations, we have taken the electron momentum to be along the z-axis and the \( \theta \) represents the angle between the electron momentum and the W momentum.

From above discussions, we can see that the chirality cross sections \( \hat{\sigma}_{RL} \) and \( \hat{\sigma}_{RR} \) vanish identically. Then the polarized cross section of the process \( e^-_L (p_e) + \gamma (k) \rightarrow \nu e_L (p) + W^- (p_{W}) \) can be written as:
\[ \hat{\sigma} (p_e, \xi_2, \hat{S}) = \frac{1}{4} (1 - p_e) [(1 - \xi_2) \hat{\sigma}_{LL} + (1 + \xi_2) \hat{\sigma}_{LR}], \quad (13) \]
where the Stokes parameter \( \xi_2 \) is given by [10]:
\[ \xi_2 = \frac{1}{D} \{ p_e r \xi [1 + (1 - x)(2r - 1)^2] - p_L (2r - 1) [\frac{1}{1 - x} + (1 - x)] \} \quad (14) \]
\[ D = \frac{1}{1 - x} + 1 - x - 4r (1 - r) - p_e p_L r \xi (2r - 1) (2 - x), \quad (15) \]
where \( p_e \) and \( p_e \) are the degrees of the longitudinal electron and positron polarization, respectively. \( p_L \) is the laser photon circular polarization. \( r = \frac{x}{\xi (1-x)} \). In our calculation, we will take \( \xi = 4.8 \), \( p_L = 1 \), and \( x_{max} = 0.83 \) as in Ref.[11].
The effective cross section $\sigma(S)$ for the subprocess $e^-\gamma \rightarrow \nu_eW^-$ in a LC with the CM energy $\sqrt{S}$, where the positron beam with the degree of longitudinal polarization $p_e$ is converted into the backscattered photon beam, is given by[10]:

$$\sigma(p_e, p_{\bar{e}}, S) = \int_{\frac{M_W^2}{S}}^{0.83} dx F(x, p_{\bar{e}}) \hat{\sigma}(p_e, \xi_2, \hat{S}), \quad (16)$$

in which $\hat{S} = xS$, the backscattered photon distribution function $F(x, p_{\bar{e}})$ for $p_L = 1$ and $\xi = 4.8$ is:

$$F(x, p_{\bar{e}}) = \frac{1}{1.83 + 0.15p_{\bar{e}}} \left[ \frac{1}{1 - x} + 1 - x - 4r(1 - r) - 4.8p_{\bar{e}}r(2r - 1)(2 - x) \right]. \quad (17)$$

In the following sections, we will use these formula to calculate the effective cross section of the sub-process $e^-\gamma \rightarrow \nu_eW^-$ for $W^- = W_s^-$ or $W_H^-$. 

III. Single production of the SM gauge boson $W_s^-$ via polarized $e^-\gamma$ collisions in the LH model

The process $e^-\gamma \rightarrow \nu_eW^-$ is one of the interesting processes for $e^-\gamma$ collisions which can uniquely be identified due to the net(-1) charge in the final state. It offers the possibility for both new physics discovery and precision measurements. Thus, studying this process in some popular specific models beyond the SM is very interesting. Single production of the SM gauge boson $W_s^-$ via the process $e^-\gamma \rightarrow \nu_eW_s^-$ receives two kinds of additional contributions in the LH model. One comes from the correction terms to the tree-level $W\nu_e e$ coupling shown as Eq.(1) and the other comes from the modification of the relation between the SM parameters and the precision electroweak input parameters.

In the LH model, the relation between the Fermi coupling constant $G_F$, the SM gauge boson $W_s$ mass $M_W$ and the fine structure constant $\alpha$ can be written as[12]:

$$\frac{G_F}{\sqrt{2}} = \frac{\pi \alpha}{2M_W^2 S_W^2} \left[ 1 - c^2(c^2 - s^2) \frac{\nu^2}{f^2} + 2c^4 \frac{\nu^2}{f^2} - \frac{5}{4} (c'^2 - s'^2) \frac{\nu^2}{f^2} \right]. \quad (18)$$

So we have

$$\frac{e^2}{S_W^2} = \frac{4\sqrt{2}G_F M_W^2}{1 - c^2(c^2 - s^2) \frac{\nu^2}{f^2} + 2c^4 \frac{\nu^2}{f^2} - \frac{5}{4} (c'^2 - s'^2) \frac{\nu^2}{f^2}}. \quad (19)$$

In the following numerical estimation, we will take $G_F = 1.16137 \times 10^{-5} GeV^{-2}$ and $M_W = 80.45 GeV[13]$ as input parameters and use them to represent other SM input parameters.
Except for the $SM$ input parameters, there are three free parameters: the mixing parameters $c$, $c'$, and the scale parameter $f$ in the expression of the relative correction parameter $R = \frac{\delta\sigma(W_s)}{\sigma^{SM}(W_s)}$ with $\delta\sigma(W_s) = \sigma^{LH}(W_s) - \sigma^{SM}(W_s)$. $\sigma^{LH}(W_s)$ and $\sigma^{SM}(W_s)$ are the production cross sections of the process $e^-\gamma \rightarrow \nu_e W_s$ predicted by the $LH$ model and the $SM$, respectively. The value of the relative correction parameter $R$ is insensitive to the degrees of the electron and positron polarization and the $CM$ energy $\sqrt{S}$. Thus, in our numerical calculation of this section, we do not consider the polarization of the initial states and take $\sqrt{S} = 500 GeV$.

![Figure 2: The relative correction parameter $R$ as a function of the mixing parameter $c'$ for $f = 1 TeV$ and three values of the mixing parameter $c$.](image)

In the $LH$ model, the custodial $SU(2)$ global symmetry is explicitly broken, which can generate large contributions to some electroweak observables. If one assumes that the $SM$ fermions are charged only under $U(1)_1$, then global fit to the electroweak precision data produces rather severe constraints on the free parameters of the $LH$ model[12,14]. However, if the $SM$ fermions are charged under $U(1)_1 \times U(1)_2$, the constraints become relaxed. The scale parameter $f = 1 \sim 2 TeV$ is allowed for the mixing parameters $c$ and $c'$.  

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$c'$ in the ranges of $0 \sim 0.5$ and $0.62 \sim 0.73$, respectively [15]. Taking into account the electroweak precision constraints on the $LH$ model, our numerical results are shown in Fig.2, in which we plot the relative correction parameter $R$ as a function of the mixing parameter $c'$ for $f = 1 TeV$ and three values of the mixing parameter $c$. From Fig.2 one can see that the contributions of the $LH$ model to the process $e^-\gamma \rightarrow \nu_e W_s^-$ is very small. In most of the parameter space allowed by the electroweak precision data, the absolute value of $R$ is smaller than 5%.

![Figure 3](image)

**Figure 3:** The value of SS as a function of the mixing parameter $c'$ for $c = 0.3$ and and three values of the scale parameter $f$.

In order to see whether the correction effects of the $LH$ model on the process $e^-\gamma \rightarrow \nu_e W_s^-$ can be observed in the near future $LC$ experiment with $\sqrt{S} = 500 GeV$ and $\mathcal{L} = 100 fb^{-1}$, we define the statistical significance (SS) of the signal as:

$$SS = \frac{|\sigma^{LH}(W_s) - \sigma^{SM}(W_s)|}{\sqrt{\sigma^{SM}}} \sqrt{\mathcal{L}}. \quad (20)$$

In order to ensure that the events are well within the detector range, we demand that the angles of all detectable final particles with the beam pipe are smaller than $15^0$. With this requirement, we assumed that 60% of the produced $W_s$ can be properly reconstructed.
In Fig.3, we plot SS as a function of the mixing parameter $c'$ for $c = 0.4$ and three values of the scale parameter $f$. From Fig 3 one can see that, for $f = 1 TeV$ and $0.62 \leq c' \leq 0.71$, the value of SS is larger than 2, while, for $f \geq 1.5 TeV$, its value is smaller than 2 in most of the parameter space. Thus, the correction effects of the LH model on the process $e^- \gamma \rightarrow \nu_e W^-_s$ might be detected at the LC with $\sqrt{S} = 500 GeV$ and $\mathcal{L} = 100 fb^{-1}$ only for $f \leq 1.5 TeV$ and $c' \geq 0.4$.

IV. Single production of the heavy gauge boson $W_H^{-}$ via polarized $e^-\gamma$ collisions

From above discussions, we can see that the single production cross section $\sigma(W_H)$ of the heavy gauge boson $W_H^{-}$ via the process $e^-\gamma \rightarrow \nu_e W_H^{-}$ dependents on the mixing parameter $c'$ only through the relation between the SM parameters and the precision electroweak input parameters as shown in Eq.(18). Thus, the $c'$ dependence of $\sigma(W_H)$ is very weak and we can take the fixed value for the mixing parameter $c'$. Taking into account the electroweak precision constraints on the LH model, we assume $c' = 0.65$, $c = 0.1 \sim 0.5$, and $M_{W_H} = 1 \sim 3 TeV$ in our following calculation.

Figure 4 shows the dependence of the production cross section $\sigma(W_H)$ on the mixing parameter $c$ for $\sqrt{S} = 3 TeV$, $c' = 0.65$, and four values of the $W_H$ mass $M_{W_H}$. To see the effects of the electron and positron beam polarization on $\sigma(W_H)$, we have chosen the different polarization configuration $(p_e, p_\bar{e}) = (0.8, 0.6), (0.8, -0.6), (-0.8, 0.6), (-0.8, -0.6)$, and $(0,0)$ in Fig.4. One can see from Fig.4 that the production cross section $\sigma(W_H)$ is indeed sensitive to the polarization of electron and positron beams. For a suitably chosen polarization configuration, the value of $\sigma(W_H)$ can be significantly enhanced. For example, for $(p_e, p_\bar{e}) = (-0.8, 0.6)$, and $(-0.8, -0.6)$, the values of $\sigma(W_H)$ are larger than those for $(p_e, p_\bar{e}) = (0,0)$ in all of the parameter space. If we take $0.1 \leq c \leq 0.5$ and $1 TeV \leq M_{W_H} \leq 2.5 TeV$, which is allowed by the electroweak precision constraints, then the single production section $\sigma(W_H)$ of the heavy gauge boson $W_H$ via polarized $e^-\gamma$ collisions in the future LC with $\sqrt{S} = 3 TeV$ are in the ranges of $60 fb \sim 1.4 \times 10^{-2} fb$, $72.9 fb \sim 3.8 \times 10^{-3} fb$, and $36.3 fb \sim 3.9 \times 10^{-3} fb$ for $(p_e, p_\bar{e}) = (-0.8, 0.6), (-0.8, -0.6)$, and $(0,0)$, respectively. If we assume that the yearly integrated luminosity of the future LC experiment with $\sqrt{S} = 3 TeV$ is $\mathcal{L} = 500 fb^{-1}$, then there are several tens and up to
thousand $W_H \nu_e$ events to be generated per year.

Figure 4: The cross section $\sigma(W_H)$ as a function of the mixing parameter $c$ for

different values of the $W_H$ mass $M_{W_H}$ and different polarization configurations.

In Fig.4 we have taken that the value of the degree $P_L$ of the laser-beam polarization

equals to 1. Certainly, we can also take $P_L = -1$. In this case, the value of the production
cross section $\sigma(W_H)$ is different from that for the same polarization configuration with
$P_L = 1$. However, the conclusion that a suitably polarization configuration of the initial
electron and positron beams can enhance the single production cross section $\sigma(W_H)$ is

not changed.

In general, the heavy gauge bosons are likely to be discovered via their decay products.
The decay channels $W_H^\pm \rightarrow l^\pm \nu$ can manifest itself via events that contain an isolated charged lepton and missing energy. In this case, the signal of single production of the heavy gauge boson $W_H$ via $e^-\gamma$ collisions should be an isolated charged lepton associated with large missing energy. For the hadron decay channels $W_H^\pm \rightarrow qq'$, the signal is a two jet event associated with large missing energy. In the narrow width approximation, the number of the \(l^-\nu \nu_e[2j + \nu_e]\) events can be approximately written as $N_{W_H} = \mathcal{L} \sigma(W_H) \times B_r(W_H \rightarrow l^-\nu)[B_r(W_H \rightarrow qq')]$. The branching ratio $B_r(W_H \rightarrow l^-\nu[qq'])$ can be easily estimated using the formula given by Ref.[16]. The most serious backgrounds for the $l^-\nu\nu_e$ signal come from the SM processes $e^-\gamma \rightarrow e^-Z \rightarrow e^-\nu\nu$ and $e^-\gamma \rightarrow W^-\nu_e \rightarrow l^-\nu\nu_e$. The scattered electron in the process $e^-\gamma \rightarrow e^-Z$ has almost same energy $E_e \approx \sqrt{S}$ for $\sqrt{S} \gg M_Z$. Thus, the process $e^-\gamma \rightarrow e^-Z$ could be easily distinguished from the signal[8,9]. Furthermore, the cross section for this process decreases as $\sqrt{S}$ increasing, while the cross section for the process $e^-\gamma \rightarrow \nu_e W_s^- \rightarrow l^-\nu\nu_e$ is approaching a constant at high energies. So, the most serious background process is $e^-\gamma \rightarrow \nu_e W_s^- \rightarrow l^-\nu\nu_e$.

![Figure 5](image.png)

Figure 5: The ratio $N$ as a function of $M_{W_H}$ for three values of the mixing parameter $c$. The polarization of the initial state beams are taken as $(p_e, p_{\bar{e}}) = (-0.8, 0.6)$ and $(-0.8, -0.6)$ in Fig.5(a) and Fig.5(b), respectively.

To compare the signal with background and discuss the possibility of detecting the heavy gauge boson $W_H$, we calculate the ratio of signal over square root of the background
\( N = N_{W_H}/\sqrt{B} \) in the parameter space of the \( LH \) model preferred by the electroweak precision data, in which \( B = \mathcal{L}_\sigma(W_s^-) \times B_r(W_s^- \rightarrow l^-\nu) \) for the lepton channel \( W_{H^-} \rightarrow l^-\nu \). The dependence of \( N \) on the \( W_H \) mass \( M_{W_H} \) is shown in Fig.5 for \( c' = 0.65 \) and three values of the mixing parameter \( c \). We have taken the polarization of the electron and positron beams as \((p_e,p_{\bar{e}}) = (-0.8, 0.6), \) and \((-0.8, -0.6)\) in Fig.5(a) and Fig.5(b), respectively. One can see from Fig.5 that the values of \( N \) increase as the mixing parameter \( c \) increasing and the \( W_H \) mass \( M_{W_H} \) decreasing. With reasonable values of the free parameters, the value of \( N \) is larger than 2. This means that, at least, the \( W_H \) signal can be separated from its \( SM \) background at \( 2\sigma \) confidence level. Thus, the \( W_H \) signal might be observed via detecting the \( l^-\nu\nu_e \) event.

\[ \mathcal{L}_\sigma(W_s^-) \times B_r(W_s^- \rightarrow l^-\nu) \]

Figure 6: In the case of detected the gauge boson \( W_{H^-} \) via the \( l\nu\nu_e \) final state, the dependence of the mixing parameter \( c \) on the \( W_H \) mass \( M_{W_H} \).

Compared to the \( SM \) fermions (quarks and leptons), the \( W_H \) mass \( M_{W_H} \) is very large. For the \( W_{H^-} \) decay channels \( W_{H^-} \rightarrow ff' \), we can neglect the fermion masses and there is \( Br(W_{H^-} \rightarrow l^-\nu) \approx Br(W_{H^-} \rightarrow qq') \). For the \( SM \) gauge boson \( W_s \), there is \( Br(W_s^- \rightarrow qq') > Br(W_s^- \rightarrow l^-\nu) \). Thus, the values of the ratio \( N \) for the lepton channels
$W_H^- \rightarrow l^-\nu$ are larger than those for the hadron decay channels $W_H^- \rightarrow q q'$. The possible signals of the new gauge boson $W_H^- \rightarrow q q'$ should be more easy observed via detecting the $l^-\nu \nu_e$ event than via detecting the $qq'\nu_e$ event.

It is well known that a appropriate cut on the SM background can generally enhance the ratio of signal over square root of the background. It has been shown that, with the suitably cut on the final lepton transverse momentum and rapidity, the SM background $l^-\nu \nu_e$ can be reduced by more than one order of magnitude[8]. Thus, we expect that, as long as the mixing parameter $c > 0.3$ and the $W_H$ mass $M_{W_H} < 2 TeV$, the heavy gauge bosons $W_H^\pm$ should be detected via polarized $e\gamma$ collisions in future LC experiment with $\sqrt{s} = 3 TeV$ and $\mathcal{L} = 500 fb^{-1}$.

If we assume that the heavy gauge boson $W_H^ -$ has been observed in high-energy experiments, such as LHC, then we can study the constraints on the free parameters of the LH model via considering the contributions of $W_H^-$ to the processes $e^-\gamma \rightarrow l^-\nu \nu_e$ and $qq'\nu_e$, in which $q$ and $q'$ are the SM quarks $u$ and $d$ or $c$ and $s$. At 95% confidence level, the constraints can be derived from

$$X^2 = \left[ \frac{\sigma(SM + W_H)}{\delta \sigma} - \sigma(SM) \right]^2 = 3.84,$$

where $\delta \sigma$ is the expected experimental uncertainty about the corresponding cross section.

In our numerical estimation, we will assume $\delta \sigma = 2\%$, the yearly integrated luminosity $\mathcal{L} = 500 fb^{-1}$, and the polarization of the electron and positron beams as $(P_e, P_\gamma) = (-0.8, -0.6)$, and take the $l^-\nu \nu_e$ final state as an example. The constraints on the free parameters $c$ and $M_{W_H}$ for $c' = 0.65$ are showed in Fig.6. From this figure, we can see that the constraints are very weak. For example, as long as the heavy gauge boson $W_H$ mass $M_{W_H}$ is smaller than $2 TeV$, the $W_H^-$ signals can be detected via the $l^-\nu \nu_e$ final state for $c \geq 0.1$. Certainly, we can also obtain the constraints on these free parameters from the $qq'\nu_e$ final state. However, the constraints are stronger than those from the $l^-\nu \nu_e$ final state.

V. Conclusions

To solve the so-called hierarchy or fine-tuning problem of the SM, the little Higgs theory was proposed as a kind of models of EWSB accomplished by a naturally light Higgs
sector. For all of the little Higgs models, at least two interactions are needed to explicitly break all of the global symmetries to make the Higgs boson as a pseudo-Goldstone boson. In general, these models predict the existence of the new heavy gauge bosons, colored fermions, and triplet scalars to cancel the quadratically divergent contributions to the Higgs mass induced by the $SM$ gauge bosons, Higgs boson, and the top-quark. These new particles might produce characteristic signatures at present or future high energy collider experiments[7,17]. Studying the possible signatures of these new particles can help to test little Higgs theory and further to probe $EWSB$ mechanism.

The $LH$ model is one of the simplest and phenomenologically viable models, which realizes the little Higgs idea. The high energy $e^-\gamma$ collision is particularly suitable for studying single production of the heavy gauge bosons. Thus, in the context of the $LH$ model, we study single production of the $SM$ gauge boson $W^-_s$ and the heavy gauge boson $W^-_H$ via polarized $e^-\gamma$ collisions. We find that the correction of the $LH$ model to the production cross section of the process $e^-\gamma \rightarrow \nu_e W^-_s$ is very small in most of the parameter space, which is very difficult to be detected in future $LC$ experiment with $\sqrt{S} = 500GeV$ and $\mathcal{L} = 100fb^{-1}$.

Beam polarization of the electron and positron beams would lead to a substantial enhancement of the production cross sections for some specific processes with a suitably chosen polarization configuration. Our numerical results show that the cross sections of the process $e^-\gamma \rightarrow \nu_e W^-_H$ for $(p_e, p_{\bar{e}}) = (-0.8, 0.6)$ and $(-0.8, -0.6)$ are larger than those for $(p_e, p_{\bar{e}}) = (0, 0)$ in all of the parameter space. For $0.3 < c \leq 0.5$ and $1TeV \leq M_{W_H} \leq 2TeV$ allowed by the electroweak precision data, the values of the single production cross sections for the heavy gauge boson $W^-_H$ are in the ranges of $0.46fb \sim 60fb$ and $0.3fb \sim 73fb$ for $(p_e, p_{\bar{e}}) = (-0.8, 0.6)$ and $(-0.8, -0.6)$, respectively. With reasonable values of the free parameters and a appropriate cut on the $SM$ background $e^-\gamma \rightarrow W^-_s \nu_e \rightarrow l^-\nu\nu_e$, the value of $N_{W_H}/\sqrt{B}$ can be significantly large. Thus, the possible signals of the heavy gauge bosons $W^\pm_H$ might be detected via polarized $e\gamma$ collisions in future $LC$ experiment with $\sqrt{S} = 3TeV$ and $\mathcal{L} = 500fb^{-1}$. 

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