The generalized Uhlenbeck–Goudsmit hypothesis: ‘magnetic’ $S^\alpha$ and ‘electric’ $Z^\alpha$ spins

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Abstract
In this paper, the connection between the dipole moment tensor $D^{ab}$ and the spin four-tensor $S^{ab}$ is formulated in the form of the generalized Uhlenbeck–Goudsmit hypothesis,

$$D^{ab} = g_S S^{ab}.$$  

It is also found that the spin four-tensor $S^{ab}$ can be decomposed into two 4-vectors, the usual ‘space–space’ intrinsic angular momentum $S^{a}$, which will be called ‘magnetic’ spin ($mspin$), and a new one, the ‘time–space’ intrinsic angular momentum $Z^{a}$, which will be called ‘electric’ spin ($espin$). Both spins are equally good physical quantities. Taking into account the generalized Uhlenbeck–Goudsmit hypothesis, the decomposition of $S^{ab}$ and the decomposition of $D^{ab}$ into the dipole moments $m^a$ and $d^a$, we find that an electric dipole moment (EDM) of a fundamental particle, as a four-dimensional (4D) geometric quantity, is determined by $Z^a$ and not, as generally accepted, by the spin $S^a$. (The usual 3-vectors will be designated in boldface.) The relation (10) also shows that the MDM of a fundamental particle is determined by the $mspin$ $S^a$. In section 4, it is proved that neither the $T$ inversion nor the $P$ inversion are good symmetries in the 4D spacetime. In this geometric approach, only the world parity $W$, $W x^a = -x^a$, is well defined in the 4D spacetime. Some consequences for elementary particle theories and experiments that search for EDM are briefly discussed.

Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows and only a kind of union of the two will preserve an independent reality.
H Minkowski

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1. Introduction
In this geometric approach, it is considered that an independent physical reality, as in Minkowski’s statement above, is attributed to the geometric quantities that are defined on the four-dimensional (4D) spacetime and not, as usually accepted, by the 3-vectors. Such geometric quantities are introduced in section 2. There, using a general rule for the decomposition of a second rank antisymmetric tensor, the dipole moment tensor $D^{ab}$ is decomposed into the electric dipole moment (EDM) $d^a$ and the magnetic dipole moment (MDM) $m^a$ (2). The main results are obtained in section 3. Using the same rule, it is shown that the spin four-tensor $S^{ab}$ can be decomposed into two 4-vectors, the usual ‘space–space’ intrinsic angular momentum $S^a$, which will be called ‘magnetic’ spin ($mspin$), and a new one, the ‘time–space’ intrinsic angular momentum $Z^a$, (8), which will be called ‘electric’ spin ($espin$). Then, the connection between $D^{ab}$ and $S^{ab}$ is formulated in the form of the generalized Uhlenbeck–Goudsmit hypothesis,

$$D^{ab} = g_S S^{ab}.$$  

It is shown in (10) that an EDM of a fundamental particle, such as a 4-vector, is determined by the espin $Z^a$ and not by the spin $S^a$. (The usual 3-vectors will be designated in boldface.) The relation (10) also shows that the MDM of a fundamental particle is determined by the $mspin$ $S^a$. In section 4, it is proved that neither the $T$ inversion nor the $P$ inversion are good symmetries in the 4D spacetime. In this geometric approach, only the world parity $W$, $W x^a = -x^a$, is well defined in the 4D spacetime. Hence, in this approach, the existence of an EDM is not connected...
in any way with $T$ violation or, under the assumption of CPT invariance, with CP violation. In section 5, the results obtained are used to discuss recent experimental searches for a permanent EDM of particles, and different shortcomings in the interpretations of the results of measurements are considered. The results obtained in this paper significantly differ from the usual formulation. However, this is a consistent theory with intrinsically covariant objects. It could be considered as an ‘alternative’ but viable (in my opinion) view of the intrinsic angular momentums—spins and the associated dipole moments of the elementary particles. The treatment of the Trouton–Noble experiment [1] with the angular momentum four-tensor $M^{ab}$ and the torque four-tensor $N^{ab}$ provides important supporting evidence for the formulation presented here. More supporting evidence comes from the resolution of Jackson’s paradox, which is obtained in [2] using the same geometric quantities $M^{ab}$ and $N^{ab}$ as those in [1].

2. The 4D geometric approach

We shall deal with 4D geometric quantities that are defined without reference frames, e.g. the 4-vectors of the electric and magnetic fields $E^a$ and $B^a$, the electromagnetic field tensor $F^{ab}$, the dipole moment tensor $D^{ab}$, the 4-vectors of the EDM $d^a$ and the MDM $m^a$, etc. In the following, we shall rely on the results and the explanations from [3]; see also references therein. As stated in [3], according to [4], $F^{ab}$ can be taken as the primary quantity for the whole of electromagnetism. $E^a$ and $B^a$ are then derived from $F^{ab}$ and the 4-velocity of the observers $v^a$:

$$F^{ab} = (1/c)(E^av^b - E^b v^a) + e^{abcd}v_c B_d,$$
$$E^a = (1/c)F^{ab}v_b,$$  
$$B^a = (1/2c^2)e^{abcd}F_{bc}v_d,$$  

(1)

$$E^a v_a = B^a v_a = 0.$$  

The frame of ‘fiducial’ observers is the frame in which the observers who measure $E^a$ and $B^a$ are at rest. That frame with the standard basis $\{e_\mu\}_4$ in it is called the $e_0$-frame. (The standard basis $\{e_\mu; 0, 1, 2, 3\}$ consists of orthonormal 4-vectors with $e_0$ in the forward light cone. It corresponds to the specific system of coordinates with Einstein’s synchronization [5] of distant clocks and Cartesian space coordinates $x^\mu$. In the $e_0$-frame, $v^a = ce_0$, which, with (1), yields $E^a = B^a = 0$ and $E^0 = F^{00} = B^0 = (1/2c)e^{ijk}F_{ijk}$. Therefore $E^a$ and $B^a$ can be called the ‘time–space’ part and the ‘space–time’ part, respectively, of $F^{ab}$. The reason for the quotation marks in ‘time–space’ and ‘space–time’ will be explained in section 4.

As proved in, e.g., [6], any second rank antisymmetric tensor can be decomposed into two 4-vectors and a unit time-like 4-vector (the 4-velocity/c). This rule can be applied to $D^{ab}$. As shown in [3], $D^{ab}$ is the primary quantity for dipole moments. Then $d^a$ and $m^a$ are derived from $D^{ab}$ and the 4-velocity of the particle $u^a$:

$$D^{ab} = (1/c)(d^av^b - d^b v^a) + (1/c^2)e^{abcd}m_cu_d,$$
$$m^a = (1/2)e^{abcd}D_{bc}u_d,$$  

(2)

$$u^a = (1/c)D^{ab}v_b,$$

with $d^au_a = m^au_a = 0$. In the particle’s rest frame (the $K'$ frame) and the $\{e'_\mu\}$ basis, $u^a = ce'_0$, which, with (2), yields that $d^a = m^a = 0$, $u^a = D^{a0} = (c/2)e_0^0 K'_{ij}$. Therefore $d^a$ and $m^a$ can be called the ‘time–space’ part and the ‘space–space’ part, respectively, of $D^{ab}$.

In this geometric approach, the interaction term in the Lagrangian for the interaction between $F^{ab}$ and $D^{ab}$ can be written as a sum of two terms [3]:

$$(1/2)F_{ab}D^{bd} = (1/c^2)[-(E_a u^b + B_a m^b)(v_b u^a)$$

$$+ (E_a u^b)(v_b u^a) + (B_a u^b)(v_b m^a)]$$

$$- (1/c^2)(e^{abcd}(E_b m_d - c^2 B_d u_d)v_a u_c).$$  

(3)

Observe that every term on the rhs of (3) contains both velocities $u^a$ and $v^a$. This fact differs [3] from all previous expressions for the interaction between dipole moments and the electric and magnetic fields. As seen from the last two terms they naturally contain the interaction of $E^a$ with $m^a$, and $B^a$ with $d^a$, which are required for the explanations of the Aharonov–Casher effect and the Röntgen phase shift [3, 7], and also of different methods of measuring EDMs, e.g. such methods as in [8]. Moreover, there is no need for any transformation. We only need to choose the laboratory frame as our $e_0$-frame and then to represent $E^a$, $m^a$ and $B^a$, $d^a$ in that frame.

Furthermore, it is shown in [2] and [1] that the angular momentum four-tensor $M^{ab}$, given as $M^{ab} = x_0^a x_0^b - x^a x^b$ (i.e. in [2] and [1], the bivector $M = x \times p$), can be decomposed into the ‘space–space’ angular momentum of the particle $M^a_\gamma$ and the ‘time–space’ angular momentum $M^a_\gamma$ (both with respect to the observer with velocity $v^a$):

$$M^{ab} = (1/c)[(M^a_\gamma v^b - M^b_\gamma v^a) + e^{abcd}M^a_\gamma v_d v_c],$$  

$$M^a_\gamma = (1/c)\epsilon^{abcd}M^d_\gamma v_d M^a_\gamma = (1/c)M^{ab}v_b,$$  

(4)

with $M^a_\gamma v_a = M^a_\gamma v_a = 0$. $M^a_\gamma$ and $M^a_\gamma$ depend not only on $M^{ab}$ but also on $v^a$. Only in the $e_0$-frame $M^a_\gamma = M^a_\gamma = 0$ and $M^a_\gamma = (1/2c)\epsilon^{ijk}M_{ijk}$, $M^a_\gamma = M^a_\gamma$, $M^a_\gamma$ and $M^a_\gamma$ correspond to the components of $L$ and $K$ that are introduced, e.g., in [9]. However, Jackson [9], as all others, considers that only $L$ is a physical quantity whose components transform according to equation (11) in [9], which we write as

$$L_i^\gamma = L_i, \quad L'_i = \gamma(L_i + \beta K_i), \quad L'_i = \gamma(L_i - \beta K_i)$$  

(5)

(for the boost in the +x-direction). The components of $B$ (and of $E$) are transformed in the same way (see equation (11.148) in [10]), for example,

$$B'_i = B_i, \quad B'_i = \gamma(B_i + \beta E_i), \quad B'_i = \gamma(B_i - \beta E_i).$$  

(6)

The essential point is that in both equations, (5) and (6), the transformed components, $L'_i$ and $B'_i$, are expressed by the mixture of components, $L_i$, $K_i$ and $B_i$, $E_i$, respectively.

Recently [11] a fundamental result was achieved that the usual transformations (UT) of $E(r, t)$, $B(r, t)$, equations (11.148) and (11.149) in [10], differ from the Lorentz transformations (LT) (boosts) of the 4D geometric quantities that represent the electric and magnetic fields.

Also, it is worth mentioning an important result regarding the formulation of electromagnetism (as in [10]), which is presented in [12] and discussed in [13]. It is explained
in [12] that the usual \( \mathbf{E}(r, t) \) and \( \mathbf{B}(r, t) \) are not correctly defined as the quantities which have, in some basis of the 3D space, only three components, since they are space- and time-dependent quantities. This means that they are defined on the spacetime and that fact determines that such vector fields, when represented in some basis, have to have four components (some of them can be zero). It is argued in [12] that an individual vector has no dimension; the dimension is associated with the vector space and with the manifold where this vector is tangent. Hence, what is essential for the number of components of a vector field is the number of variables on which that vector field depends, i.e. the dimension of its domain. Thus, strictly speaking, the time-dependent \( \mathbf{E}(r, t) \) and \( \mathbf{B}(r, t) \) cannot be the 3-vectors, since they are defined on the spacetime. Therefore, from now on, we shall use the term ‘vector’ for a geometric quantity, which is defined on the spacetime and which always has in some basis of that spacetime, e.g. the standard basis \( \{e_i\} \), four components (some of them can be zero). This refers to \( m^i, d^i, M^a, M^b, \ldots \) as well. (In the preceding text, they are called the 4-vectors.) However, an incorrect expression, the 3-vector, will still remain for the usual \( \mathbf{E}(r, t) \) and \( \mathbf{B}(r, t) \), \( \mathbf{L, K} \), the spin \( S \), etc.

For the ‘fiducial’ observers, \( u^\mu = ce_\mu^0 \) and \( E^\mu = F^{\mu\nu}e_\nu^0 \), as shown in [11], and also in [13], both the field \( F^{ab} \) and the velocity \( (\ell c) \) of the ‘fiducial’ observer have to be transformed by the LT. This correct mathematical procedure yields that the components (in the standard basis) \( E^\mu \) transform by the LT as

\[
E^0 = \gamma (E^0 - \beta E^1), \quad E^1 = \gamma (E^1 - \beta E^0), \quad E^{2,3} = E^{2,3},
\]

(7)

for the boost in the \( +x^1 \)-direction. Of course, this is the way in which the components (in the standard basis) of any vector transform under the LT. Hence, the same transformations as (7) hold for the components \( B^a, M^a, d^a, m^a, S^a, Z^a \), etc. As noted in [12], and discussed in [13], Minkowski, in section 11.6 in [14], was the first who correctly transformed the electric and magnetic vectors.

The fundamental difference between the correct LT (7) of the components (in the standard basis) and the LT (5), (6) is that the components \( E^\mu \), i.e. \( B^a, M^a, d^a, m^a, S^a, Z^a \), transform by the LT again to the components \( E^\mu' \), i.e. \( B^a, M^a, d^a, m^a, S^a, Z^a \), respectively; there is no mixing of components.

As said in section 1, it is proved in [1] that the treatment with \( M^{ab} \) (or \( M^a, M^b \)) and the torque four-tensor \( N^{ab} \) (or vectors \( N^a \) and \( N^b \)) is in true agreement (independent of the chosen inertial reference frame and of the chosen system of coordinates in it) with the Trouton–Noble experiment. Similarly, in [2] it is shown that in such an approach with \( M^{ab} \) and \( N^{ab} \) the principle of relativity is naturally satisfied and there is no Jackson’s paradox. The true agreement with experiments, when using 4D geometric quantities, is also obtained in the second paper in [11] (the motional electromotive force), in the third paper in [11] (the Faraday disc) and also in [15] (the well-known experiments: the ‘muon’ experiment, the Michelson–Morley-type experiments, the Kennedy–Thorndike-type experiments and the Ives–Stilwell-type experiments). This true agreement with experiments directly proves the physical reality of the 4D geometric quantities. It is also shown in the mentioned papers ([1], [11] and [15]) that the agreement between the experiments that test special relativity and Einstein’s formulation of special relativity [5], which deals with the synchronously defined spatial length, i.e. the Lorentz contraction, with the conventional dilatation of time and also with the UT of the components of the 3-vectors \( E \) and \( B \), is not a true agreement since it depends on the chosen synchronization, e.g. Einstein’s synchronization or a drastically different, nonstandard, radio (‘\( r' \)’) synchronization; see also [16] and section 4 here.

### 3. The generalized Uhlenbeck–Goudsmith hypothesis: ‘time–space’ intrinsic angular momentum and the intrinsic EDM

The above consideration can be directly applied to the intrinsic angular momentum, the spin of an elementary particle. In the usual approaches, e.g. section 11.1.1A in [10], the relativistic generalization of the spin \( S \) from a 3-vector in the particle’s rest frame is obtained in the following way: ‘The spin 4-vector \( S^a \) is the dual of the tensor \( S^{\alpha\beta} \) in the sense that \( S^a = (1/2c)e^{ab}_{\gamma} u_\gamma S_{\beta\gamma} \), where \( u^\mu \) is the particle’s 4-velocity.’ The whole discussion above about \( F^{ab} \), \( D^{ab} \) (2) and particularly about \( M^{ab} \) (4) (spin is also an angular momentum) implies a more general geometric formulation of the spin of an elementary particle. In analogy with [1] and [2], we conclude that the primary quantity with definite physical reality for the intrinsic angular momenta is the spin four-tensor \( S^{ab} \), which can be decomposed into two vectors, namely the usual ‘space–space’ intrinsic angular momentum \( S^a \) and the ‘time–space’ intrinsic angular momentum \( Z^a \):

\[
S^{ab} = (1/c)[(Z^a u^b - Z^b u^a) + \epsilon^{abcd} S_{ab} d_{cd}],
\]

\[
S^a = (1/2c)\epsilon^{abcd} S_{ah} u_d, \quad Z^a = (1/c) S_i^{ah} u_h,
\]

(8)

where \( u^\mu = dx^\mu/dr \) is the velocity of the particle and it holds that \( S^a u_a = Z^a u_a = 0 \). \( S^a \) and \( Z^a \) depend not only on \( S^{ab} \) but on \( u^\mu \) as well. Only in the particle’s rest frame, the \( K' \) frame, and the \( \{e_i\} \) basis, \( u^\mu = e_0^\mu \) and \( S^0 = Z^0 = 0 \), \( S^i = (1/c) e^{0ij} S_{ij} \), \( Z^i = S^{i0} \). The definition (8) essentially changes the usual understanding of the spin of an elementary particle. It introduces a new ‘time–space’ spin \( Z^a \), which is a physical quantity in the same measure as is the usual ‘space–space’ spin \( S^a \).

In [17] it is asserted: ‘For an elementary particle, the only intrinsic direction is provided by the spin \( S \). Then its intrinsic \( \mu = \gamma_0 S \) and its intrinsic \( d = \delta_0 S \), where \( \delta_0 \) is a constant.’ (In [17] the unprimed quantities are in the particle’s rest frame.) Thus, both the MDM \( m^a \) and the EDM \( d^a \) (our notation) of an elementary particle are determined by the usual spin \( S^a \). In the usual approaches such a result is expected because only the ‘space–space’ intrinsic angular momentum is considered to be a well-defined physical quantity. In contrast to [17] and other usual approaches, we consider that the intrinsic direction in the 3D space is not important in the 4D spacetime, since it does not correctly transform under the LT. As already explained, in this geometric approach a definite physical reality is attributed to \( S^{ab} \) or to \( S^a \) and \( Z^a \) taken together (see (8)) in the same way as holds for the angular
momentum four-tensor $M^{ab}$ and the angular momenta $M^a_e$ and $M^a_i$ (4) (see [1] and [2]).

Furthermore, in the usual approaches, there is a connection between the magnetic moment $\mathbf{m}$ and the spin $\mathbf{S}$, $\mathbf{m} = \gamma_S \mathbf{S}$. This is the well-known Uhlenbeck–Goudsmit hypothesis [18]. The whole of the above consideration suggests that instead of the above connection between the 3-vectors $\mathbf{m}$ and $\mathbf{S}$ we need to have the connection between the dipole moment tensor $D^{ab}$ and the spin four-tensor $S^{ab}$. Obviously, it has to be formulated in the form of the generalized Uhlenbeck–Goudsmit hypothesis as

$$D^{ab} = g_S S^{ab}.$$ (9)

Taking into account the decompositions of $D^{ab}$ (2) and $S^{ab}$ (8), we find the connections between the dipole moments $m^a$ and $d^a$ and the corresponding intrinsic angular momenta $S^a$ and $\omega^a$, respectively, in a form that essentially differs from all usual approaches, e.g. [17]:

$$m^a = cg_S S^a, \quad d^a = g_S \omega^a.$$ (10)

In the particle’s rest frame and the $\{e_i\}$ basis, $d^0 = m^0 = 0$, $d^i = g_S \omega^i$, $m^i = cg_S S^i$. Comparing this last relation with $\mathbf{m} = \gamma_S \mathbf{S}$, we see that $g_S = \gamma_S/c$. Thus, the intrinsic MDM $m^a$ of an elementary particle is determined by the mspspin $S^a$, whereas the intrinsic EDM $d^a$ is determined by the spin Z$^a$; the names mspspin and espin come from the theories, e.g. in the standard model and in supersymmetric (SUSY) theories. However, in the geometric approach presented here, the relation $\mathbf{m} = \gamma_S \mathbf{S}$ is generalized, equation (9), replacing the 3-vectors $\mathbf{m}$ and $\mathbf{S}$ by the dipole moment tensor $D^{ab}$ and the spin four-tensor $S^{ab}$, respectively. In the same way we can generalize (11). Using (9), the generalized equation of motion for the spin four-tensor $S^{ab}$ becomes

$$dS^{ab}/dt = \mathbf{m}' \times \mathbf{B}' = \gamma_S \mathbf{S} \times \mathbf{B}',$$ (11)

where all the quantities are in the particle’s rest frame, the $K'$ frame, and the Uhlenbeck–Goudsmit hypothesis [18], $\mathbf{m}' = \gamma_S \mathbf{S}'$, is used. In section 11.11A in [10], a covariant generalization of (11) is presented and it refers to the spin 4-vector $\mathbf{S}^u$ (components in the $\{e_i\}$ basis). However, in the geometric approach presented here, the relation $\mathbf{m} = \gamma_S \mathbf{S}$ is generalized, equation (9), replacing the 3-vectors $\mathbf{m}$ and $\mathbf{S}$ by the dipole moment tensor $D^{ab}$ and the spin four-tensor $S^{ab}$, respectively. In the same way we can generalize (11). Using (9), the generalized equation of motion for the spin four-tensor $S^{ab}$ becomes

$$dS^{ab}/dt = F^{ac}_{gcd} D^{cd} - F^{bc}_{gcd} D^{da} = g_S [F^{a0}_{gcd} S^{cd} - F^{b0}_{gcd} S^{da}],$$ (12)

where $g_{ab}$ is the metric tensor. Equation (12) is written with primary 4D geometric quantities, for the electromagnetic field $F^{ab}$ and for dipole moments $D^{ab}$, i.e. for spins $S^{ab}$. In [22], a very similar equation of motion for the spin four-tensor $S^{uv}$ is derived (see equation (14) there); however, Peletminskii and Peletminskii [22] deal exclusively with components in the $\{e_i\}$ basis.) Of course one can use the decompositions (1) and (2), or (8), to obtain the generalized equation of motion (12) expressed in terms of the fields $E^a$, $B^a$ and the dipole moments $d^a$, $m^a$ or the spin $Z^a$ and the mspspin $S^a$. The consequences of (12) will not be investigated here, e.g. the generalization of the BMT equation ([11.164] in [10]), etc. We remark only that (12) reduces to the equation

$$dS^a/d\tau = e^{0ijk} m^j B_k = \gamma_S e^{0ijk} S^j B_k'$$ (13)

where $S^a$ is the mspspin in the $K'$ frame and the $\{e_i\}$ basis. The $K'$ frame is also chosen to be the $e_4$-frame, i.e. the observers who measure fields $E^a$ and $B^a$ move together with the dipole, $v^a = u^a = c e'_i$, and consequently $E^0 = B^0 = d^0 = m^0 = 0$. Furthermore, it is taken that in the $K'$ frame
$d^0 = 0$, i.e. $Z^a = 0$, and that $E^0 = F^0 = 0$. The relation (13) corresponds to the equation with the 3-vectors (11). However, (13) is correctly expressed by the components of the 4D geometric quantities, whereas (11) is written with the 3-vectors whose transformations are not the LT and also it contains the coordinate time and not the proper time.

Relations (8)–(10) with 4D geometric quantities $S^a$, $S^a$ and $Z^a$, $D^{ab}$, $m^a$ and $d^a$ are fundamentally new results that have not been mentioned previously in such a form in the literature.

In addition, it is worthwhile to mention the classical references, [23], on the relativistic theory of spin in classical electrodynamics. However, both Frenkel and Thomas [23], as almost all others later, finally expressed their covariantly generalized relations in terms of the usual 3-vectors considering that the 3-vectors are physical quantities and that the UT of $E$ and $B$, $d$ and $m$ are the relativistically correct LT. Particularly interesting is that Frenkel (first and second papers in [23]) considered that $d$ and $m$ ‘are connected with each other by the invariant relation $D^{i\nu}u_\nu = 0$, that is $d = (1/c)u \times m$,’ (our notation) (equations (3) and (4) in the first paper in [23]), expressing the fact—or rather the assumption—that in a coordinate system, in which the electron’s translational velocity $u$ is zero, the ‘electrical moment’ $d$ must vanish. In the geometric approach presented here, this invariant relation would be written as $D^{ab}u_b = 0$, which in the standard basis becomes $D^{i\nu}u_\nu e_\mu = c^2 d^\mu e_\mu = 0$. In the particle’s rest frame, the $x'$ frame $(u^\mu = (c, 0, 0, 0))$, one finds that $d^0 = 0$ and $D^{i\nu} = d^0 = 0$, which is Frenkel’s assertion, i.e. the assumption. However, Frenkel’s invariant relation $D^{i\nu}u_\nu = 0$ is, as Frenkel says, nothing else than an assumption, which is not founded in any way. This means that there is no physical or mathematical reason for the assumption that $d^0 = 0$. Moreover, as already argued several times, the 3-vectors and the relations with them, like $d = (1/c)u \times m$, are not meaningful in the 4D spacetime. Besides, in order to get from $D^{i\nu}u_\nu e_\mu = 0$ the relation with spatial components $(d^i = (1/c^2)\epsilon_{ijk}m_ju_k)$, which corresponds to Frenkel’s relation between the 3-vectors $d$ and $m$, some additional, not justified, assumptions (like $d^0 = 0$) are required.

4. $T$ and $P$ inversions and the world parity $W$

In elementary particle theories the existence of an EDM implies the violation of the time reversal $T$ invariance. Under the assumption of CPT invariance, a nonzero EDM would also signal CP violation. As stated in [24], ‘it is the $T$ violation associated with EDMs that makes the experimental hunt interesting.’ Let us briefly consider the connection between the EDM and the $T$ invariance, as it is explained in the usual formulation, e.g. [24]. Reversing time would reverse the spin direction but leave the EDM direction unchanged since the charge distribution does not change. In the elementary particle theories, e.g. the standard model and SUSY, the EDM direction is connected with a net displacement of charge along the spin axis, i.e. with an asymmetry in the charge distribution inside a particle; see, for example, [24]. Thus, with $t \rightarrow -t$, $S \rightarrow -S$ but $d \rightarrow d$. However, as in [17], $d$ is determined as $d = dS/S$. Hence $d$ has to be parallel to the spin $S$; it is considered that $S$ is the only available 3-vector in the rest frame of the particle. This yields that $d \rightarrow -d$, i.e. $d \rightarrow 0$. As stated in [24], ‘the alignment of spin and EDM is what leads to violations of $T$ and $P$.’

From the viewpoint of the geometric approach presented here, neither $T$ inversion nor $P$ inversion is well defined in the 4D spacetime; they are not good symmetries. For the position vector $x^a$, only the world parity $W$ (for the term see, e.g., [25]), according to which $Wx^a = -x^a$, is well defined in the 4D spacetime. In general, the $W$ inversion cannot be written as the product of the usual $T$ and $P$ inversions. But this will be possible for the representations of $W$, $T$ and $P$ in the standard basis $\{e_\mu\}$. It is easy to see that, e.g., $T$ inversion is not well defined and that it depends, for example, on the chosen synchronization.

As explained, e.g. in [16], different systems of coordinates (including different synchronizations) are allowed in an inertial frame and they are all equivalent in the description of physical phenomena. Thus, in [16], two very different but completely equivalent synchronizations, Einstein’s synchronization [5] and the ‘$r'$ synchronization, are exposed and exploited throughout the paper. The ‘$r'$ synchronization is commonly used in everyday life and not Einstein’s synchronization. In the ‘$r'$ synchronization, there is an absolute simultaneity. As explained in [26], ‘For if we turn on the radio and set our clock by the standard announcement’…’at the sound of the last tone, it will be 12 o’clock,’ then we have synchronized our clock with the studio clock according to the ‘$r'$ synchronization. In order to treat different systems of coordinates on an equal footing we have presented [16] the transformation matrix that connects Einstein’s system of coordinates with another system of coordinates in the same reference frame. Furthermore, in [16] we have derived a form of the LT that is independent of the chosen system of coordinates, including different synchronizations. The unit vectors in the $\{e_\mu\}$ basis and the $\{r_\mu\}$ basis, i.e. with the ‘$r'$’ synchronization, [16], are connected as

$$r_0 = e_0, \quad r_1 = e_0 + e_1.$$  

(14)

Hence, the components $g_{\mu r}$ of the metric tensor $g_{\mu \nu}$ are $g_{0 r} = 0$, and all other components are equal to 1. Remember that in the $\{e_\mu\}$ basis, $g_{\mu \nu} = \text{diag}(1, -1, -1, -1)$. (Note that in [16] and [15] the Minkowski metric is $g_{\mu \nu} = \text{diag}(-1, 1, 1, 1)$.)

Then, according to (4) from [16], one can use $g_{\mu r}$ to find the transformation matrix $R^0_\mu$ that connects the components from the $\{e_\mu\}$ basis with the components from the $\{r_\mu\}$ basis; $R^0_\mu = -R_\mu = 1$, and all other elements of $R^0_\mu$ are equal to 0. The inverse matrix $(R^0_\mu)^{-1}$ connects the ‘old’ basis, $\{e_\mu\}$, with the ‘new’ one, $\{r_\mu\}$. With such an $R^0_\mu$ one finds that the components of $x^a$ are connected as

$$x^0_r = x^0 - x^1 - x^2 - x^3, \quad x^i_r = x^i.$$  

(15)

Observe that $x^a = x^\mu e_\mu = x^0_r r_\mu$. (Obviously, the components of any vector transform in the same way as in (15), e.g. for the components of $E^a$ it holds that $E^a_r = E^0_r - E^1 - E^2 - E^3$, $E^i_r = E^i$.) It is clear from (15) that $T$ inversion, $t \rightarrow -t$, i.e. $x^0 \rightarrow -x^0$, does not give that $x^a_r \rightarrow -x^a_r$. This can be shown explicitly.
In the standard basis \([e_\mu]\) the matrix elements \(T^\mu_\nu\) of the time reversal operator \(T\) are \(T^0_0 = -1,\ T^i_i = 1\) and all other elements of \(T^\mu_\nu\) are equal to 0. Then, one can write \(x^\mu_T = T^\mu_\nu x^\nu\), where \(x^\mu_T\) are the components of the time-reversed position vector \(x^\mu_T = x^\mu T_r\), which are \(x^0_T = -x^0,\ x^i_T = x^i\). In the \([r_\mu]\) basis, the matrix elements \(T^\mu_\nu\) of the time reversal operator \(T\) which are different from zero are

\[
T^0_{0,r} = -1,\quad T^i_{i,r} = 1,\quad T^i_{0,r} = -2. \tag{16}
\]

Clearly, this is not a time reversal operation in the usual sense. In the \([r_\mu]\) basis, the components \(x^\mu_{T,r}\) of the ‘time-reversed’ position vector \(x^\mu_{T,r} = x^\mu r_r\) are

\[
x^0_{T,r} = -x^0 - x^1 - x^2 - x^3,\quad x^i_{T,r} = x^i
\]

and it holds that \(x^\mu_T = x^\mu r_r e_\mu = x^\mu_{T,r} r_r\). (Of course, \(x^\mu_{T,r} = R^\mu_\nu x^\nu_{T,r}\)). This means that the \(T\) inversion does not have a definite physical significance, since it depends on the chosen synchronization. Only when Einstein’s synchronization is used does the time reversal have the usual meaning. However, different synchronizations are nothing else than different conventions and physics must not depend on conventions.

In general, the same holds for the \(P\) inversion. In the \([e_\mu]\) basis \(P^\mu_\nu = 1,\ P^i_i = -1\) and all other elements of \(P^\mu_\nu\) are equal to 0. However, in the \([r_\mu]\) basis the matrix elements \(P^\mu_\nu\) of the parity operator \(P\) which are different from zero are

\[
P^0^0_0 = 1,\quad P^i_i = -1,\quad P^0_{i,r} = 2. \tag{18}
\]

Obviously, in the \([r_\mu]\) basis, \(P^\mu_\nu\) is not a spatial inversion. In that basis the components \(x^\mu_P,\ P^i_{\mu,i}\) of the ‘spatially reversed’ position vector \(x^\mu_P = x^\mu_P r_r\) are

\[
x^0_P = x^0 + x^1 + x^2 + x^3,\quad x^i_P = -x^i
\]

and it holds that \(x^\mu_P = x^\mu_P e_\mu = x^\mu_P r_r\). Thus, the parity operator \(P\) also depends on the chosen synchronization and therefore it is not a properly defined operation in the 4D spacetime. \(P\) has its usual meaning only when the standard basis \([e_\mu]\) is chosen in some inertial frame of reference.

On the other hand, the \(W\) inversion is properly defined because if \(x^\mu \rightarrow -x^\mu\) then \(x^\mu_T \rightarrow -x^\mu,\ x^\mu_P \rightarrow -x^\mu\), .... Thus

\[
W x^\mu = -I x^\mu,\quad W^\mu_\nu x^\mu e_\mu = W^\mu_\nu x^\mu r_r = \cdots = W^\mu_\nu x^\mu e_\mu = -W^\mu_\nu x^\mu r_r = \cdots = -I^\mu_\nu x^\mu r_r = -W^\mu_\nu x^\mu r_r = \cdots, \tag{20}
\]

where \(W^\mu_\nu,\ W^\mu_\nu,\ W^\mu_\nu,\ W^\mu_\nu\) are the matrix elements of the proper parity operator \(W\) in the bases \([e_\mu]\), \([r_\mu]\), \([e_\mu]\) and \([r_\mu]\) and all primed quantities in (20) are Lorentz transforms of the unprimed ones; see equation (1) in [16] for the general form of the LT. The LT in the \([r_\mu]\) basis are given in the same paper by equation (2). The elements that are different from zero are

\[
x^\mu_P = L^\mu_\nu x^\nu,\quad L^0_0 = K,\quad L^0_i = 0,\quad L^i_i = K - 1,\quad L^i_0 = L^0_i = -\beta/K,\quad L^i_j = \frac{1}{K},\quad L^2_2 = L^3_3 = \frac{1}{1},
\]

where \(K = (1 + 2\beta) / \sqrt{3}\) and \(\beta = dx^0 / dx^0 = \) the velocity of the frame \(S\) as measured by the frame \(S,\ \beta = (\beta - 1) / (\beta - 1)\) and it ranges as \(-1/2 < \beta < \infty\). In (20) it is the identity transformation. It can be easily checked that \(W^\mu_\nu = T^\mu_\nu P^\nu_\nu = L^\mu_\nu P^\nu_\nu = \cdots = I^\mu_\nu\). But the matrix elements \(T^\mu_\nu\) and \(P^\mu_\nu\), which are given by (16) and (18), respectively, are quite different from the usual ones from the \([e_\mu]\) basis, i.e., different from the matrix elements of the usual \(T\) and \(P\) inversions. They do not describe the time and space inversions and the notations for all, except \(TP\) in the \([e_\mu]\) basis, are not adequate.

It is worth noting that the same relations as in (20) hold also for \(d^A,\ Z^2,\ m^A,\ S^A,\ E^A,\ B^A,\) etc., i.e., \(W^A_\nu = -d^A_\nu,\ldots\). One law for the proper inversion \(W\) for all vectors! Hence, \(L^\text{int}_\text{in} (3)\) is unchanged under the proper inversion \(W\). The \(W\) inversion is well-defined symmetry in the 4D spacetime. This is drastically different from the usual \(T\) and \(P\) inversions for the 3-vectors. For the \(T\) inversion of \(d\) and \(S\), see, e.g., the beginning of this section.

This fact that \(T\) and \(P\) inversions are not well-defined symmetries in the 4D spacetime is one of the reasons why, contrary to the existing elementary particle theories, the \(T\) violation, i.e., the \(CP\) violation, cannot be connected in this approach with the existence of an intrinsic EDM.

Another reason is that, as already stated, neither the direction of \(d\) nor the direction of the spin \(S\) has a well-defined meaning in the 4D spacetime. The only Lorentz-invariant condition on the directions of \(d^A\) and \(S^A\) in the 4D spacetime is \(d^A u_a = S^A u_a = 0\). This condition does not say that \(d\) has to be parallel to the spin \(S\). The above discussion additionally proves that the relations (8), (10) and (9) are properly defined.

If an antisymmetric tensor (the components) \(A^{\mu\nu}\) (that tensor \(A^{ab}\) can be, e.g., \(F^{ab},\ M^{ab},\ S^{ab},\ D^{ab},\ldots\)) is transformed by \(R^\mu_\nu\) to the \([r_\mu]\) basis, then it is obtained that

\[
A_{10}^{ab} = A^{10} - A^{12} - A^{13}, \tag{22}
\]

which shows that the ‘time–space’ components in the \([r_\mu]\) basis are expressed by the mixture of the ‘time–space’ components and the ‘space–space’ components from the \([e_\mu]\) basis. Thus, for example,

\[
D_{10}^{ab} = -d^1 + (1/c)m^3 - (1/c)m^2. \tag{23}
\]

Similarly, it follows from (22) that \(F^{10} = E^1 + cB^3 - cB^2\).

The relation (23) and the one for \(F_{10}\) show, once again, that the components have no definite physical meaning. Only in the \([e_\mu]\), \([e_\mu]\) bases does it hold that \(E = F^{00},\ d^i = D^{0i},\ Z^a = S^{0a},\) etc. That is the reason why we always put the quotation marks in the expressions ‘time–space’ and ‘space–space’. One important consequence of (22) and (23) is that the usual EDMs and MDMs, \(\textbf{d}\) and \(\textbf{m}\), respectively, where, e.g., \(\textbf{d} = D^{01} + D^{02} + D^{03}\), have no definite physical meaning, since the components \(D^{0i}\) are dependent on the chosen synchronization. Of course, the same holds for the fields \(\textbf{E}\) and \(\textbf{B}\). In contrast with the usual covariant approach with coordinate-dependent quantities, all relations (1)–(10) are written in terms of 4D geometric quantities, i.e. they are defined without reference frames. This means that dipole moments \(d^a\) and \(m^a\) are well-defined quantities in the 4D spacetime but, according to (2), they depend not only on \(D^{ab}\) but also on the 4-velocity of the particle \(u^a\) as well. Hence,
as already stated, $D^{ab}$ is the primary quantity; it does not depend on $\nu^a$. The same assertion can be stated for the relation between $F^{ab}$ and $E^a$, $B^b$, $v^d$, as seen from (1).

All this proves that in the ‘r’ synchronization it is not possible to speak about time and space as separate quantities. So, in the 4D spacetime, $W$ inversion has an independent reality in Minkowski’s sense but not $T$ and $P$ inversions. Similarly, $D^{ab}$ has an independent reality but not the dipole moments $d$ and $m$. The same applies to $F^{ab}$ and the fields $E$ and $B$. By the explicit use of the ‘r’ synchronization I have mathematically formalized Minkowski’s words. Note that only in Einstein’s synchronization are the spatial and temporal parts of the interval between the two spacetime points separated. The usual covariant approaches implicitly use only Einstein’s synchronization and therefore the majority of physicists believe that $T$ and $P$ inversions taken separately are well-defined symmetries. A similar conclusion applies to $d$ and $m$ and the fields $E$ and $B$.

5. Shortcomings in the EDM searches

The results obtained offer some new interpretation of measurements of an EDM of a fundamental particle, e.g. [8, 24, 27]. In all experimental searches for a permanent EDM of particles, the UT of $E$ and $B$ are frequently used and considered to be relativistically correct; that is, that they are the LT of $E$ and $B$. Thus, in a recent new method of measuring EDMs in storage rings [8], the so-called motional electric field, our $E'$, is considered to arise ‘according to a Lorentz transformation’ from a vertical magnetic field $B$ that exists in the laboratory frame; $E' = \gamma c \beta \times B$. That field $E'$ plays a decisive role in the new method of measuring EDMs. It is stated in [8] that $E'$ ‘can be much larger than any practical applied electric field.’ and ‘Its action on the particle supplies the radial centrifugal force.’ Then, after introducing ‘$g - 2$’ frequency $\omega_d = a(eB/m)$, $a = (g - 2)/2$ is the magnetic anomaly) due to the action of the magnetic field on the muon magnetic moment, they say, ‘If there is an EDM of magnitude $d = \gamma eB/4m c \approx 4.7 \times 10^{-14} \text{ecm}$, there will be an additional precession angular frequency $\omega_a = (\gamma e/2m)\beta \times B$ about the direction of $E'$, ….’ The new technique of measuring EDM in [8] is to cancel $\omega_d$ so that $\omega_a$ can operate by itself. An important remark on such a treatment is that the field $E'$ is in the rest frame of the particle $K'$ but the measurement of EDM is in the laboratory frame $K$. A similar thing happens in [27] and many others in which ‘motional magnetic field’ $B' = (\gamma/c)E \times \beta$ appears in the particle’s rest frame as a result of the UT of the $E$ field from the laboratory. It is usually considered that the $(\gamma/c)E \times \beta$ field causes important systematic errors. Thus, it is stated already in the abstract in the first paper in [27]: ‘In order to achieve the target sensitivities it will be necessary to deal with the systematic error resulting from the interaction of the well-known $v \times E$ field with magnetic field gradients … . This interaction produces a frequency shift linear in the electric field, mimicking an EDM.’ The same interpretation with the UT of $E$ and $B$ appears when the quantum phase of a moving dipole is considered, e.g. [28]. For example, when the Röntgen phase shift is considered, it is asserted in the second paper in [28] that in ‘the particle rest frame the magnetic flux density $B$ due to the magnetic line is perceived as an electric field’ $E' = v \times B$. Then that $E'$ can interact with $d'$ in $K'$. This is objected to in [7]. In the usual approaches with the 3-vectors it is also possible to get the interaction between $B$ and $d$ by another method, which conforms more to a description in $K$. According to the second method, the magnetic field $B$ in $K$ interacts with the MDM $m$ that is obtained from the EDM $d'$ by the UT for $m$ and $d$; $m = \gamma v \times d'$. For the Aharonov–Casher effect, this method is mentioned in, e.g., [29]. However, as already said, the transformations of $E$ and $B$, (6), and of $d$ and $m$, are not the LT but the UT [11]. They have to be replaced by the LT of the corresponding 4D geometric quantities. Then, the LT transform $B'$ from $K$ again to $B''$ in $K'$ and, similarly, $E''$ from $K$ is transformed again to $E''$ in $K'$ (7); there is no mixing of components. The same holds for the LT of $d''$ and $m''$. Thus, in this approach, there is no induced $E'$ as in [8] and [28], and there is no ‘motional magnetic field’ $B'$ as in [27] and [29], and there is no induced $d$ in $K$ as in [29].

As already mentioned, in all EDM experiments the interaction between the electromagnetic field and the dipole moments is described in terms of the 3-vectors as $E \cdot d$ and $B \cdot m$. Moreover, the 3-vectors $d$ and $m$ (and also $E$ and $B$) are in the rest frame of the particle, whereas the measurements of EDM are in the laboratory frame. However, the last two terms in (3) show that in the geometric approach presented here there are direct interactions between the magnetic field $B^a$ and an EDM $d^a$ and also between $E^a$ and $m^a$, which are required for the explanation of measurements in [8] and [27]. In order to describe the interactions in $K$ one only needs to choose the laboratory frame $K$ as the $e_0$-frame and then to represent $E^a$, $B^a$ and $d^a$, $m^a$ in that frame. This can be explained in more detail comparing the expression (3) with the interaction term in the Lagrangian in, e.g., equation (17) in [17], which is

$$L_{int} = d' \cdot E' + B' \cdot m' + (1/c^2)u \cdot m' \times E' - u \cdot d' \times B'.$$  \hspace{1cm} (24)

This is written in our notation in which the particle’s rest frame is the $K'$ frame and the quantities from that frame are the primed quantities. (There is an ambiguity with the notation. Namely, $L_{int}$ (24) is in $K'$, but at the same time that expression contains the particle’s velocity $u$.)

Since the measurements are in the laboratory frame (the $K$ frame) we shall first choose the laboratory frame as the $e_0$-frame and represent $E^a$, $m^a$, $B^a$, $d^a$, … from (3) in that frame. Hence, in the laboratory frame, which is taken to be the $e_0$-frame, $\nu = c, 0, 0, 0$ and consequently $E^0 = B^0 = 0$; see (1). Then (3) becomes

$$L_{int} = -((E, d') + (B, m')) - (1/c^2)\eta^{ijk}(E, m_k - c^2 B, d_i) u_j + (1/c)((E, u')d' + (B, u')m') \hspace{1cm} (25)

Observe that in the laboratory frame there are contributions from the terms with $d^0$ and $m^0$. The contribution of the terms with the interaction of $E^a$ with $m^a$ ($B^a$ with $d^a$) is $u/c$ of the usual terms with the direct interaction of $E^a$ with $d^a$ ($B^a$ with $m^a$). The constraints of $d^0 u_0 = m^0 u_0 = 0$ can also be written in the $e_0$-frame, which enables us to express $d^0$ and $m^0$ by means of $d'_0$ and $m'_0$, respectively. Then, it can be seen that terms with $d^a$ and $m^a$ in (25) are $u^2/c^2$ of the usual terms $E_j d'_j$ or $B_i m'_i$ and therefore they can be neglected.
In the usual approach with the 3-vectors (neglecting terms of the order of \( u^2/c^2 \)), \( L_{\text{int}} (25) \) would correspond to

\[
L_{\text{int}} = d \cdot E + m \cdot B + (1/c^2) u \cdot m \times E - u \cdot d \times B, \tag{26}
\]

where, in contrast with (24), all quantities are in the laboratory frame. It is assumed that the spatial components with upper indices from (25) correspond to the spatial components of the 3-vectors in (26). Namely, the metric is diag(1, –1, –1, –1) and \( e^{0123} = 1 \).

In an unrealistic case when the rest frame of the dipole is chosen to be the \( e_0 \)-frame, i.e. when the observers who measure fields \( E^0 \) and \( B^0 \) move together with the dipole, \( v^a = u^a = c e_0^a \) and consequently \( E^0 \cdot B^0 = d^0 = m^0 = 0 \), then \( L_{\text{int}} \) from (3) becomes

\[
L_{\text{int}} = -(E' d') - (B' m'). \tag{27}
\]

In the usual picture with 3-vectors, it would correspond to

\[
L_{\text{int}} = d' \cdot E' + m' \cdot B'. \tag{28}
\]

The corresponding Hamiltonian is

\[
H_{\text{int}} = -d' \cdot E' - m' \cdot B'. \tag{29}
\]

\( H_{\text{int}} \) (29) is the form of the Hamiltonian of the interaction that is frequently used in the comparison with the standard theory and in the interpretation of the results of EDM experiments. \( E' \) and \( B' \) in (29) are both in the rest frame of the dipole and in all the usual approaches they are expressed in terms of the laboratory fields using the UT of \( E \) and \( B \). (6). Observe, once again, that \( L_{\text{int}} \) (28) and \( H_{\text{int}} \) (29) refer to the case when the observer who measures fields \( E' \) and \( B' \) 'sits' on the moving dipole.

In the 4D geometric approach presented in this paper, expressions like (26), (28), and (29) are meaningless because, as explained particularly in [12], there are not the usual time-dependent 3-vectors in the 4D spacetime. The relativistically correct 4D expressions are (25) in the laboratory frame (or, neglecting terms of the order of \( u^2/c^2 \)), \( L_{\text{int}} \) (25) without last two terms and (27) in the rest frame of the dipole when that frame is at the same time the \( e_0 \)-frame. They are derived from (3), while the Lagrangian \( L_{\text{int}} \) (3) is obtained using mathematically and physically correct decompositions (1) and (2).

For the phase shifts these questions are discussed in [3] and [7]. Accordingly, the experimentalists who search for an EDM, e.g. [8] and [27], and, for example, those who observe the Aharonov–Casher phase shift [30], will need to reexamine the results of their measurements taking into account the relations (3) and (8)–(10).

6. Conclusion

In conclusion, we believe that the new results (8)–(10) that are obtained in this paper, together with the expression (3) for the interaction term, [3], provide an alternative but viable formulation of spins and dipole moments. It will be of interest in different branches of physics, particularly elementary particle theories and experiments, and also theories and experiments that treat different quantum phase shifts with dipoles. It is worth noting that the relations (4), (8) and (10) are generalized to the quantum case and the new commutation relations for the orbital and intrinsic angular momentums and for the dipole moments are introduced in [31].

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