Design and application of high-speed and high-precision CV gate on IBM Q OpenPulse system

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Abstract—Faster and more precise physical processing of quantum gate, by suitably designing a pulse sequence to implement the target gate, will greatly improve the performance of quantum algorithms in the presence of noise. In this paper, we demonstrate that, by employing OpenPulse design kit for IBM Q devices, the controlled-V gate (CV gate) can be implemented in about 34.5% shorter gate time, with 0.66% improvement in the average gate fidelity, compared to the standard version provided there. Then, based on the theory of Cartan decomposition, we show that the performance of several two-qubit gates containing CV gates can also be improved. Moreover, the average gate fidelity of Toffoli gate can be improved to 96.16% from 90.23% achieved in the default IBM Q package. These results imply the importance of our CV gate implementation technique, which, as an additional option for the basis gate set design, may shorten the overall computation time and consequently improve the accuracy of several quantum algorithms.

Index Terms—Controlled-V gate, Toffoli gate, IBM Q, OpenPulse

I. INTRODUCTION

The quantum computer used now or in the near future is called the noisy intermediate-scale quantum device or simply NISQ [20]. In general, NISQ quickly accumulates errors, which mainly come from the following two noise sources [24], [27]. One is the calibration errors of quantum gates, i.e., the gate error. The other one is an error due to the decoherence of the qubit over time. In order to maximize the capability of NISQ, we need to reduce these two errors as much as possible.

There are several type of platforms for implementing quantum computer, such as superconducting, ion, and optical devices. In this paper, we study the error reduction problem in IBM Q, which is a superconducting NISQ device with which Qiskit is provided as a software development environment. Qiskit has two representation languages for designing quantum circuits: QASM and OpenPulse [15]. QASM is the language for the circuit design with several quantum gates. Physically, each gate is decomposed into a set of precisely calibrated gates chosen from the universal quantum gate set [2], [3]. The universal gate set used in IBM Q is composed of single-qubit gates and the Controlled-X (CX) gate [24]. The point of taking this fixed gate set is that, because it contains only one two-qubits interaction gate (i.e., CX gate), the calibration process is relatively easy. In particular, CX gate can be implemented with high fidelity, via the cross resonance (CR) Hamiltonian [11], [21], [14], [26], [12], with the help of the echo scheme and the cancellation pulse technique [25]. However, the error rate of CX gate is still much higher than that of one-qubit gates [8], due to the longer pulse length than that of one-qubit gates and the effects of crosstalk [17], [16], [22]. Hence, if only the above-mentioned fixed gate set composed of single-qubit gates and CX gate is given, some quantum algorithm is realized on a circuit with very redundant way, which as a result introduces excess noise and reduce the accuracy of circuit.

The above-mentioned issue can be resolved by adding some 2-qubit gate to the conventional universal gate set composed of single qubit gates and CX gate. In this work, we take the controlled-V-gate (CV gate) as an additional element of the gate set, for the following two reasons. The first reason is the fact that the Toffoli gate needs six CX gates to implement if CX is only given to us, but it can be implemented using 2 CX and 3 CV gates if CV gate is further available [2], [4], [6]; that is, we can save one 2-qubit gate to implement Toffoli gate. Let us now recall that, as mentioned in the second paragraph, Qiskit provides OpenPulse; this is also a gate design environment yet at the pulse level, unlike QASM. The second reason for choosing CV gate is actually related to this fact; that is, CV gate can be readily generated using OpenPulse, by just halving the pulse length of the CR pulse used for generating CX gate. This means that the gate time of CV gate is shorter than that of CX gate. Therefore, if some CX gates of a quantum circuit can be replaced with the same number of CV gates, the total gate time of this circuit is reduced. As a result, the CV gate will improve the accuracy of the circuit by reducing the number of 2-qubit gates in the circuit design as well as by reducing the incoherent error by shortening the pulse length.

This paper is organized as follows. Section II is a preliminary section, showing the CR pulse and Cartan decomposition theory. In Section III, we show how to experimentally generate CV gate, using OpenPulse. The result is that the gate time of the CV gate is shortened by 34% and the fidelity is improved by 0.66%. In Section IV, we demonstrate that we can implement some 2-qubit gates such as the Controlled-U gate, based on Cartan decomposition and geometric structures. In particular, we demonstrate to generate $\sqrt{SWAP}$ and $\sqrt{SWAP}$, using the CV gates and confirm that the accuracy is improved thanks to the shorter gate length. In Section V, we discuss an efficient method for implementing Toffoli gate using the CV gate; the experimental results show that the accuracy is improved about 5% and the gate time is reduced.
by 2310 ns.

II. PRELIMINARIES

A. Gate design with OpenPulse

In the IBM Q device, the cross resonance (CR) interaction is used to couple two qubits \([21]\), by irradiating the control qubit with a microwave pulse at the transition frequency of the target qubit. The time-independent CR Hamiltonian is identified as follows \([25], [6], [14], [26]\):

\[
H_{CR} = \sum_{P=I,X,Y,Z} \frac{\omega_{ZP}(A,\phi)}{2} Z \otimes P + \sum_{Q=X,Y,Z} \frac{\omega_{IQ}(A,\phi)}{2} I \otimes Q, \tag{1}
\]

where the qubit ordering is control \(\otimes\) target. \(\omega_{ZP}\) and \(\omega_{IQ}\) represent the interaction strength, which are functions of the amplitude \(A\) and the phase \(\phi\) of the microwave pulse. In the absence of noise, the two qubits are driven by the unitary operator

\[
U_{CR} = \exp(-itH_{CR}(A,\phi)). \tag{2}
\]

Note that this unitary time evolution is valid under the assumption that the frequency of the pulse irradiated to the control qubit is the transition frequency of the target bit. Hence, for example, if the pulse is of square form that contains a high-frequency component in its rising and falling edges, then Eq. (2) may not hold. To make the model (2) valid, the Gaussian-Square pulse is employed in QASM \([5]\), which is of the form of flat-top Gaussian of length (width) \(\tau_w\), with rising and falling edges each of length (rise/fall) \(\tau_r\). The overall pulse waveform \(f(t)\), as a function of time \(t\), is thus given by

\[
f(t) = \begin{cases} 
A \exp\left(-\frac{1}{2\sigma^2}(t - \frac{\tau_r}{2})^2\right), & (0 \leq t < \tau_r) \\
A \exp\left(-\frac{1}{2\sigma^2}(t - \tau_r + \tau_w)^2\right), & (\tau_r \leq t < \tau_r + \tau_w) \\
A \exp\left(-\frac{1}{2\sigma^2}(t - \tau_r - \frac{\tau_w}{2})^2\right), & (\tau_r + \tau_w \leq t < \tau_d)
\end{cases} \tag{3}
\]

where \(A\) and \(\sigma^2\) are amplitude and variance of the pulse, respectively. Also

\[
\tau_d = 2\tau_r + \tau_w \tag{4}
\]

is the overall pulse length (duration). These parameters must be determined carefully, because the rising and falling part of the pulse may severely affect the performance of resulting unitary operator. For instance, if \(\tau_r\) is made smaller, then the waveform of the rising and falling edges becomes steeper, which can increase the high-frequency components of the pulse and thereby violate the assumption for deriving Eq. (2).

We now define the general two-qubits unitary operator

\[
[AB]^\theta = \exp\left(-i\pi\frac{\theta}{2}A \otimes B\right), \tag{5}
\]

where \(A\) and \(B\) are arbitrary single-qubit operators. With this notation, the CX gate (i.e., CNOT gate) is represented as \([21]\):

\[
CX = [ZI]^{1/2}[ZX]^{-1/2}[IX]^{1/2}. \tag{6}
\]

That is, the two qubits operation required to form the CX gate can only be served by the ZX Hamiltonian. However, Eq. (1) contains terms other than the ZX term, which should be eliminated by some means when implementing the CX gate via the CR Hamiltonian. This goal can be achieved, by using the two-pulse echoed gate and the cancellation pulse as illustrated in Fig. 1 in other words, these methods are effectively used to generate the CR Hamiltonian composed of only the ZX term \([1], [20]\). As a consequence, the CR Hamiltonian and the induced unitary operator that realizes the CX gate are given by

\[
H_{ZX} = \frac{\omega_{ZX}(A,\phi)}{2} Z \otimes X, \quad U_{ZX} = \exp(-it\frac{\omega_{ZX}(A,\phi)}{2} Z \otimes X) = [ZX]^{\omega_{ZX}(A,\phi)t}. \tag{7}
\]

Fig. 1. The CX gate pulse sequences generated in QASM for IBM Q Toronto. We assigned qubit 1 and the qubit 4 of this system as the control and the target qubit, respectively. The schedule consists of 2 CR pulses (colored with yellow) on the Control Channel u3 with the echo pulses applied on Drive Channel d1 of the control qubit and the active cancellation pulse on Drive Channel d4 of the target qubit. The local gates \([ZI]\) and \([IX]\) in Eq. (6) are also applied to the Drive Channel d1 of the target qubit. The value of \(\theta\) in \([ZX]^{\theta}\) increases in proportion to the pulse length of the CR pulse.

Lastly, we remark that the qubits on a currently available real device (IBM Q Toronto in our case) are not fully connected, and thus some pairs of qubits cannot be directly entangled. In the experiment presented in this paper, we used three qubits illustrated in Fig. 2, hence, qubit 0 and qubit 4 cannot be directly entangled. This limits the circuit design for the Toffoli gate, which will be discussed in Section 5.

Fig. 2. Topology of IBM Q Toronto device. The three qubits used are colored with black.
E. Evaluation of the gate accuracy

To evaluate the closeness between the quantum channel $\mathcal{E}$ constructed using OpenPulse and the target gate $U$, we use the average gate fidelity defined as follows. First, we apply the quantum process tomography (QPT) [19] to compute the superoperators $S_\mathcal{E}$ and $S_U$, corresponding to $\mathcal{E}$ and $U$, respectively. Note that the measurement error mitigation technique [9] is employed to correct the readout error. Then the average gate fidelity is calculated as [10], [18], [28].

$$F_{\text{ave}}(\mathcal{E}, U) = \int d\psi \langle \psi | U^\dagger \mathcal{E}(\psi) \psi \rangle^d,$$

$$= \frac{Tr[S_\mathcal{E}^d + d]}{d(d+1)},$$

where $d$ is the dimension of $\mathcal{E}$.

C. Cartan decomposition

The Cartan decomposition theory for SU(4) will be used to show, in Section IV, that several gates can be designed based on CX and CV gates. The Cartan decomposition means that an arbitrary 2-qubit unitary operation $U \in SU(4)$ has the following decomposition formula:

$$U = k_1 U_{[a,b,c]} k_2,$$

$$= k_1 \exp\left\{ \frac{1}{2} (aX \otimes X + bY \otimes Y + cZ \otimes Z) \right\} k_2,$$

where $k_1, k_2 \in SU(2) \otimes SU(2)$ are local single-qubit operations. When $U_1 = k_1 U_2 k_2$, we call that $U_1$ and $U_2$ are locally equivalent. From [29], [30], the class of locally equivalent 2-qubit gates has a one-to-one correspondence with the point $[a, b, c]$ on the tetrahedron $OA_1A_2A_3$ shown in Fig. 3. For example, the CX and CZ gates are both $[\pi/2, \pi/2, 0]$ and can be converted by applying the local Hadamard gates.

A useful result provided in this theory is that, by using $n \geq 3$ Controlled-U $[\gamma, 0, 0]$ gates with $\gamma \in (0, \pi/2]$, we can create arbitrary non-local gate $[a, b, c]$ that satisfies the following condition:

$$0 \leq a + b + c \leq n\gamma, \quad a - b - c \geq \pi - n\gamma. \quad (10)$$

Eq. (10) tells us that all the gates generated by $n$ applications of Controlled-U with local gates can be represented by two tetrahedras $OB_1B_2B_3$ and $A_1C_1C_2C_3$ in Fig. 3. Similarly, by using $n = 2$ Controlled-U $[\gamma, 0, 0]$ gates, we can create arbitrary non-local gate $[a, b, 0]$ that satisfies the following condition:

$$0 \leq a + b \leq 2\gamma, \quad a - b \geq \pi - 2\gamma. \quad (11)$$

That is, all the gates generated by 2 applications of Controlled-U with local gates can be represented by two triangles $OB_1B_2$ and $A_1C_1C_2$ in Fig. 4, which are within the triangle $OA_1A_2$. Note that a single Controlled-U can only take a point $[\gamma, 0, 0]$. Another important result is that some gates can be generated using the same number of CV gates or CX gates, e.g., arbitrary Controlled-U gate. Details will be given in Section IV.

III. CONTROLLED-V GATE

A. Implementation with OpenPulse

Matrix representation of CV gate is given as

$$CV = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \frac{1+i}{\sqrt{2}} & \frac{1-i}{\sqrt{2}} \\
0 & 0 & \frac{1-i}{\sqrt{2}} & \frac{1+i}{\sqrt{2}}
\end{bmatrix}. \quad (12)$$

From Eq. (6), by considering IX, ZX and ZI are commutable for each other, it is readily derived that CV gate can be decomposed into

$$CV = [ZI]^{1/4}[ZX]^{-1/4}[IX]^{1/4}. \quad (13)$$

In the QASM definition of CV gate, two CX gates is used to implement 2-qubit interaction part, $[ZX]^{-1/4}$, as shown in Fig. 5. In the present work, we directly implement $[ZX]^{-1/4}$

$$[RX(\frac{\pi}{4})]^{-1} \rightarrow U_2(0, \frac{\pi}{4}) \rightarrow \quad \text{Fig. 5. In the present work, we directly implement CX gate in QASM implementation.}$$

part, by taking the cross-resonance pulse duration $t_{CV}$ as

$$t_{CV} = \pi/4\omega_{ZX}(A, \phi), \quad (14)$$

which is half the duration of cross-resonance pulse of calibrated CX gate,

$$t_{CX} = \pi/2\omega_{ZX}(A, \phi). \quad (15)$$

Here, we expect that $\alpha$ of 2-qubit interaction part $[ZX]^\alpha$ is proportional to the duration of cross-resonance pulses. Note that we keep the basic structure and parameters of the pulse sequence in Fig. 1 unchanged except the cross-resonance pulse duration.
B. Experimental results

We performed all the experiments in this section with 8192 shots (repeated times), together with readout measurement error mitigation technique, by changing the following parameters

\[
\begin{align*}
\tau_w &= 1.75m + 42 \quad (16) \\
\tau_r &= 1.75m/2 \quad (17) \\
\tau_d &= 2\tau_r + \tau_w = 3.5m + 42 \quad (18)
\end{align*}
\]

where integer value \( m \) is taken from 1~34. This corresponds to the range of pulse duration \( \tau_d \) from 45.5 ns to 161 ns. We also applied QPT to implemented gate to identify the corresponding quantum channel, \( \mathcal{E} \), and compared to the ideal CV by calculating fidelity. Note that the half the duration of cross-resonance pulse of CX, 98 ns (\( \tau_d \) with \( m = 16 \)) is included in this range. We ran the same experiment three times at different days. Correspondingly, we ran the same experiment for the gates derived from calibrated CX gate, by changing the duration from 143.5 to 259 ns for comparison.

Figure 5 shows the fidelity dependence with respect to pulse duration, which is obtained from experiments. As shown in the figure, error-mitigation works well and gives the better fidelity values compared to raw (unmitigated) results. Black dotted line corresponds to the ideal fidelity which is calculated for CV and CX as

\[
\begin{align*}
CV_{cal} &= [ZI]^{1/4}[ZX]^{-(m+12)/112}[IX]^{1/4}, \\
CX_{cal} &= [ZI]^{1/2}[ZX]^{-(m+40)/112}[IX]^{1/2}.
\end{align*}
\]

As expected, the fidelity line of CV gate takes maximum value, 99.23% (averaged value for three different days) at 101.5 ns (\( m = 17 \)), which corresponds to half the duration of cross-resonance pulse of the calibrated CX gate. This value is 0.66% higher than that of CV gate fidelity of QASM implementation. Throughout all three different experiments, the maximum value is observed at 101.5 ns, which indicates that the optimal pulse duration did not fluctuate. In Fig. 5(a), one can see irregular behaviour of observed fidelity lines and deviation from the calculated line in the range 40 ~ 60 ns. This might be due to unwanted excitation to higher excited state caused by the high frequency components brought by steep rising feature of pulse envelope.

Figure 6 shows a comparison of the pulse sequence of a CV gate with pulse duration 101.5 ns and that of CV gate in QASM implementation with two CX gates. It is shown that the gate time is shortened by 651 ns while fidelity is improved by 0.66%.

![Fig. 5. (a) Fidelity of implemented CV gate with respect to the duration of cross-resonance pulse. (b) Fidelity of implemented CX gate with respect to the duration of cross-resonance pulse. The fidelity shows its maximum value when duration is 196 ns.](image)

![Fig. 6. (a) Pulse sequence for CV gate in QASM implementation. Total gate time is 994 ns. (b) Pulse sequence for CV gate by OpenPulse with the CR pulse duration, 101.5 ns. Total gate time is 343 ns.](image)

In this section, we have shown that one can implement efficient custom CV gate with shorter gate time (better fidelity) than that of QASM implementation through quite simple
procedure, i.e., replacing the cross resonance pulses in the calibrated CX gate with that of half duration while leaving other parameters unchanged. In the following sections, we apply the OpenPulse based CV gate created in this section to creating other useful gates.

IV. GATE DESIGN WITH CV GATES

A. THEORY

As shown in Sec. II, an arbitrary two-qubit operator $U \in SU(4)$ can be decomposed as,

$$U = k_1 U_{[a,b,c]}k_2$$

$$= k_1 \exp\left(\frac{i}{2}(aX \otimes X + bY \otimes Y + cZ \otimes Z)\right)k_2.$$

Through the above decomposition, it is readily shown that CV gate corresponds to the point $[\pi/4, \pi/4, 0]$ in the Weyl chamber. According to Ref. (130), two CV gates can generate a 2-qubit unitary gate corresponding to the point $[a,b,0]$ satisfying

$$0 \leq a + b \leq \pi/2, a - b \geq \pi/2,$$

which are the areas of $OLB_1$ and $A_1LB_2$ denoted in Fig. 7. On the other hand, two CX gates can generate gates in triangle $OA_1A_2$, which covers the area where two CV cannot generate, such as the point $[\pi/2, \pi/2, 0]$ for DCX/iSWAP. Although the area that two CV gates can span is limited compared to that of two CX gates, one should note that the line $OL$ is included in this area. This means one can generate arbitrary controlled-U gate, corresponding to $[\gamma, 0, 0]$, using two CV and local gates before, in between and after CV gates (See Fig. 7(b)). Thus, as OpenPulse based custom CV gate is faster than CX operators in QASM, one can generate arbitrary controlled-U gate with shorter gate time compare to the default implementation with two CX. Note also that not only controlled-U gates but also the gates that belong to the area, $OLB_1$ and $A_1LB_2$, can be implemented with 2 custom CV.

In the case of using three CV gates, one can generate 2-qubit gate for the point $[a, b, c]$ satisfying

$$0 \leq a + b + c \leq 3\pi/4, a - b - c \geq 3\pi/4,$$

which corresponds to the tetrahedrons $OB_1B_2B_3$ and $A_1C_1C_2C_3$ shown in Fig. 8. Note that the tetrahedron $OB_1B_2B_3$ contains the important points, such as $[\pi/4, \pi/4, 0]$ corresponding to $\sqrt{iSWAP}$ and $[\pi/4, \pi/4, \pi/4]$ corresponding to $\sqrt{SWAP}$.

Three CX gates can span the area of tetrahedron $OA_1A_2A_3$, which denotes the fact that 3 CX can generate arbitrary 2-qubit unitary gate. Just like the case of Fig. 7, the gates corresponding to the area, which three CV can span, can be implemented with custom CV gates for reducing the gate time.

B. Applications

Next, we apply the gate design concept with CV gates to useful gates, such as $\sqrt{SWAP}$ and $\sqrt{iSWAP}$. Matrix representations of these gates are given as

$$\sqrt{SWAP} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \\
0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},$$

$$\sqrt{iSWAP} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\
0 & -\frac{i}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.$$
As described in the previous subsection, $\sqrt{iSWAP}$ and $\sqrt{SWAP}$ can be created with two and three CV (CX) gates, respectively. Circuit diagrams and pulse sequences for these gates are shown in Fig. 9. For designing OpenPulse based custom gates, we first create the XX terms by applying local gates to the CV gate $[\pi/4, 0, 0]$. Next, convert it to YY and ZZ terms by local gate operations as shown in Ref [29], and combine these terms to create $\sqrt{iSWAP}$ gate $[\pi/4, \pi/4, 0]$ and $\sqrt{SWAP}$ gate $[\pi/4, \pi/4, \pi/4]$.

Fig. 9. Circuit diagram for $\sqrt{iSWAP}$ gate with 2 CX (above) and 2 CV (below).

![Circuit diagram for \(\sqrt{iSWAP}\) gate with 2 CX and 2 CV](image)

We performed the experiments with 8192 shots together with read-out error-mitigation. The gate fidelity calculated from the results of QPT is shown in Table I. The result shows that $\sqrt{iSWAP}$ gate with custom CV gates gives better fidelity by 0.87% compared with that of $\sqrt{iSWAP}$ with CX gates. Likewise, fidelity value of $\sqrt{SWAP}$ gate with custom CV shows 2.14% better than that of $\sqrt{iSWAP}$ with CX gates. From those results, one can see that introducing custom CV gate with OpenPulse technique is quite useful for 2-qubit gate designing.

V. TOFFOLI GATE IMPROVEMENTS

The Toffoli gate, also called the Controlled-Controlled-X gate, is a three-qubit gate. Toffoli gates are used, for example, in the three-qubit Grover’s algorithm, and their accuracy has a significant impact on the performance of the algorithm than a two-qubit gate. Besides, Toffoli gates are also important for
large-scale quantum algorithm implementation because we can easily extend them to \( n \)-controlled Toffoli gates by combining them with auxiliary qubits.

Theoretically, we can construct a Toffoli gate using six CX gates (and one-qubit gate), and a configuration using three CV gates and two CX gates is also known. In this section, we designed the Toffoli gate using the CV created in the previous section. We have confirmed that the accuracy and gate time are improved compared to the configuration without using CV gates.

### A. The design of Toffoli gates

![Circuit diagram](circuit-diagram.png)

Fig. 13. (a) Circuit diagram of Toffoli + SWAP gate with CV gate. It has 3 CX + 3 CV (b) Circuit diagram of Toffoli + 2SW AP gate used in QASM. It has 10 CX.

The Toffoli gate was implemented with CV gate as shown in \[13\] Since Toffoli gate is a 3-qubit gate, we add qubit 0 as the control qubit, in addition to the qubit 1 and 4 as the control qubit and the target qubit we used in the previous section. Qubit 0 and qubit 1 are connected, qubit 1 and qubit 4 are connected, too. However, qubit 0 and qubit 4 are connected is not. Therefore, we created pseudo-connection using SWAP gates. In the system, Toffoli gate with CV 4 gate is made with 3 CX + 3 CV, and with only CX gate is made with 10 CX. We didn’t implement last swap gates because we can balance the calculation in the gate design after the Toffoli gate. The \( CV \) gate we make is as follows.

\[
CV^\dagger = [IX]^{-1/4}[XI][ZX]^{-1/4}[XI][ZI]^{-1/4}
\] (23)

Fig. [14](#) shows the pulse sequence of the Toffoli gate using the made CV gate and the Toffoli gate implemented with only CX gates. From [14], the Toffoli gate with CX gates and CV gates can shorten the gate time 2310 ns more than the Toffoli gate by QASM.

### B. Experimental results

To evaluate accuracy, we perform quantum state tomography on eight initial states of \(|000\rangle \sim |111\rangle\) and calculated state fidelity \( F(\rho_{exp}, \rho_{ide}) \) of density matrices \( \rho_{exp} \) and \( \rho_{ide} \) in the following equation [13].

\[
F(\rho_{exp}, \rho_{ide}) = Tr[\sqrt{\sqrt{\rho_{exp}}\rho_{ide}\sqrt{\rho_{exp}}}^2].
\] (24)

We performed 8192 shots per initial state and performed error mitigation [9].

From Table II, the Toffoli gate generated with CV gates has high fidelity in all initial states. The average fidelity of the eight initial states was also about 5.93% higher. This is probably due to the reduction in the number of two-qubit gates, which have a high error rate, and the reduction gate time that generates decoherence. Compared to the results in Section 4, the accuracy has improved significantly. We expect that the greater the reduction in gate time, the greater the improvement in accuracy. Although we have discussed the gate design in this paper, we believe that incorporating gates into the actual algorithm will greatly improve the accuracy.

### VI. Conclusion

The use of only CX gates as two-qubit gates for quantum computation is now a de facto standard, and IBM Q is no exception. While this approach is less burdensome for calibration, it has the disadvantage that some gate designs are redundant. Therefore, we experimentally created CV gates
using open pulses with shorter gate times than CX gates. The CV gate parameters we designed are the same as those of the CX gate except for the pulse length, so the calibration burden is not significant. Based on multiple experiments results, we believe that the optimal pulse length does not change with each calibration. We have improved the accuracy of the \( \sqrt{SWAP} \) and Toffoli gates by incorporating square-root gates. From the practicality and feasibility viewpoint, we propose to add CV gate to the conventional CX gate and one-qubit gates as a basic gate set. We believe that the new gate set will improve NISQ devices’ performance by reducing the number of gates and the circuit size.

**APPENDIX**

Figure [5] shows the fidelity of the CV and CX gates for each of the date and time performed in section 3. We list the version of Qiskit packages used in the experiments in Table [II]. We performed each experiment on the dates listed in section 3. We list the average fidelity information for CX gate and one-qubit gates as a basic gate set. We believe that the new gate set will improve NISQ devices’ performance by reducing the number of gates and the circuit size.

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Fig. 15. Figure 5 in section 3 shows the relationship between pulse length and fidelity for CV and CX gates averaged over three experiments. Figure 15 shows the dates and experimental results for each of the three experiments.

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