On the evolution law of the universe

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Abstract

The model of the homogenous and isotropic universe is considered in which the coordinate system of reference is not defined by the matter but is a priori specified. The scale factor of the universe changes following the linear law. The scale of mass changes proportional to the scale factor of the universe. The model under consideration avoids the flatness and horizon problems. The predictions of the model is fitted to the observational constraints: Hubble parameter, age of the universe and CMB data.

1 Introduction

As known [1, 2], the Friedmann model of the universe has fundamental difficulties such as the flatness and horizon problems due to the slow growth of the scale factor of the universe. In the Friedmann universe, the coordinate system of reference is associated with the matter of the universe. The evolution of the scale factor of the universe is given by

\[ a \sim t^{1/2}, \quad a \sim t^{2/3} \]

(1)

where the first equation corresponds to the matter as a relativistic gas, and the second, to the dust-like matter. Growth of the scale factor of the universe governed by the power law with the exponent less than unity is slower than growth of the horizon of the universe \( h \sim t \) that causes the flatness and horizon problems.

To resolve these problems an inflationary episode in the early universe is introduced [1, 2]. However there is another way of resolving the problems. This is based on the premise that the coordinate system of reference is not defined by the matter but is a priori specified.

2 Theory

Let us consider the model of the homogenous and isotropic universe. Let us assume that the coordinate system of reference is not defined by the matter but is a priori specified. Let the coordinate system of reference be the Euclidean space with the spatial metric \( dl \) and absolute time \( t \)

\[ dl^2 = a(t)^2(dx^2 + dy^2 + dz^2), \quad t. \]

(2)

That is the coordinate system of reference is the space and time of the Newton mechanics, with the scale factor of the universe is a function of time. Since the metric (2) is not defined by the matter, we can a priori specify the evolution law of the scale factor of the universe.
Let us take the linear law when the scale factor of the universe grows with the velocity of light

\[ a = ct. \tag{3} \]

In the Friedmann universe, the law (3) corresponds to the Milne model [3] which is derived from the condition that the density of the matter tends to zero \( \rho \to 0 \). Here Eq. (3) describes the universe in which the evolution of the scale factor do not depend on the presence of the matter. Hence the density of the matter is not equal to zero \( \rho \neq 0 \). The total mass of the universe relative to the background space includes the mass of the matter and the energy of its gravity. Let us adopt that the total mass of the universe is equal to zero, that is the mass of the matter is equal to the energy of its gravity

\[ c^2 = \frac{G m}{a}. \tag{4} \]

Allowing for Eq. (3), from Eq. (4) it follows that the mass of the matter changes with time as

\[ m = \frac{c^2 a}{G} = \frac{c^3 t}{G}, \tag{5} \]

and the density of the matter, as

\[ \rho = \frac{3c^2}{4\pi G a^2} = \frac{3}{4\pi G t^2}. \tag{6} \]

Hence the model under consideration yields the change of the scale of mass proportional to the scale factor of the universe. At the Planck time \( t_{Pl} = (\hbar G/c^5)^{1/2} \), the mass of the matter is equal to the Planck mass \( m_{Pl} = (hc/G)^{1/2} \). At present, the mass of the matter is of order of the modern value \( m_0 = c^2 a_0/G \).

Let us consider the flatness and horizon problems within the framework of the model under consideration. Remind [1, 2] that the horizon problem in the Friedmann universe is caused by that the universe observable at present consisted of the causally unconnected regions in the past that is inconsistent with the high isotropy of the background radiation. In the universe under consideration, the size of the universe (the scale factor of the universe) coincides with the size of the horizon during all the evolution of the universe. Hence the presented model avoids the horizon problem.

Remind [1, 2] that the essence of the flatness problem in the Friedmann universe is connected with impossibility to gain the modern density of the matter at present starting from the Planck density of the matter at the Planck time. In the presented theory, the density of the matter of the universe changes from the Plankian value at the Planck time to the modern value at the modern time. Hence the flatness problem is absent in the presented theory.

### 3 Predictions

In view of Eq. (3), we write down some relations describing the universe. The relation between the Hubble parameter and the age of the universe is given by

\[ H = \frac{1}{a} \frac{da}{dt} = \frac{1}{t}. \tag{7} \]
The scale factor of the universe at time $z$ is given by

$$a(z) = \frac{a_0}{z + 1}. \quad (8)$$

The age of the universe at time $z$ is given by

$$t(z) = \frac{1}{H_0(z + 1)}. \quad (9)$$

The angular diameter distance at time $z$ is given by

$$d(z) = \frac{c}{H_0} \frac{z}{z + 1}. \quad (10)$$

Let us fit the predictions of the model to the observational constraints: Hubble parameter, age of the universe and CMB data. The Hubble parameter deduced from the HST observations of Cepheids are converging to a value $H_0 = 63 \pm 10$ km/c/Mpc.

The age of the universe is estimated by the observed age of the oldest globular clusters $t_{GC} = 14 \pm 2$ Gyr [5]. According to Eq. (9), for $H_0 = 63$ km/c/Mpc, the age of the universe is $t_0 = 15.5$ Gyr.

Consider classical angular diameter distance test with the standard ruler provided by the cosmic microwave background (CMB) temperature anisotropy power spectrum [6]. Any feature projects as an anisotropy onto an angular scale associated with multipole

$$\ell = kr,$$

where $k$ is the size of the feature in $k$ space, $r$ is the distance to the feature. Last scattering in the epoch of recombination $z_{ls} = 1400$ projects the observed multipole $\ell = 260$ [7]. Usually the standard ruler is taken as the sound horizon at last scattering, and the distance to the ruler is an angular diameter distance to last scattering. Let us assume that the standard ruler is the size of the universe at last scattering

$$k_{ls} = \frac{H_0(z_{ls} + 1)}{c}. \quad (12)$$

The distance to the ruler is the sound horizon at last scattering measured at present

$$r_{ls} = \frac{d_{ls}}{\pi \sqrt{3}}. \quad (13)$$

Substituting Eqs. (11), (12), (13) into Eq. (11) we obtain

$$\ell_{ls} = \frac{z_{ls}}{\pi \sqrt{3}} = 257. \quad (14)$$

Let us estimate the rate of growth of density fluctuations within the framework of the Newton gravity. Density fluctuations change with time as [3]

$$\delta = \exp(\int \sqrt{4\pi G \rho dt}). \quad (15)$$
Substituting Eq. (6), we obtain

\[ \delta = \exp(\sqrt{3} \ln t) = t^{\sqrt{3}}. \]  

(16)

Let us determine the CMB anisotropy via the density fluctuations at the moment of recombination \( z_{ls} = 1400 \)

\[ \frac{\Delta T}{T} = \frac{1}{3} \delta(z_{ls}). \]  

(17)

If \( \delta \sim 1 \) corresponds to the formation of galaxy clusters in the epoch of reionization \( z_i = 5 \), then according to Eq. (16), \( \delta(z_{ls}) = 7.8 \cdot 10^{-5} \). This gives the CMB anisotropy \( \Delta T/T = 2.6 \cdot 10^{-5} \). COBE satellite [8] detected fluctuations in the CMB on the large scale at the level of \( \Delta T/T \sim 10^{-5} \).

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