Glided acoustic higher-order topological insulators based on spoof surface acoustic waves

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Abstract
Higher-order topological insulator hosts both gapped edge states and in-gap corner states, which has garnered considerable attentions in the field of condensed matter physics, and most recently is further extended to the classical wave systems. Conventional acoustic metamaterials have intrinsic material and design limitations that prevent them from being used to create such states in subwavelength scale with function reconfigurability. Recently, the acoustic second-order topological insulators (SOTIs) composed of locally resonant metamaterials were reported to solve the problem, where the topological phase transition is induced by shrinking/expanding metamolecules. Here, we propose an acoustic SOTI in subwavelength scale by another protocol, i.e. gliding metamolecules, and the SOTI is pinned in the nontrivial region without the regular topological phase transition. Soda cans metamaterials in free space are utilized to support the spoof surface acoustic waves. With varying the introduced glided angle, the switching from the bulk to edge and corner states can be achieved accordingly. Furthermore, we not only experimentally observe this state switching process, but also illustrate the robustness of the topological corner states against various defects. Our results provide versatile ways to launch acoustic lower-dimensional topological states that might lead to interesting sound concentration applications.

1. Introduction
Higher-order topological insulators (HOTIs) [1–5] hosting the lower-dimensional boundary states have been reported as a new kind of topological materials with special bulk-boundary correspondence. For example, in two-dimensional (2D) systems, a second-order topological insulator (SOTI) supports the zero-dimensional (0D) localized corner states. Moreover, these lower-dimensional boundary states provide versatile routes to extraordinarily manipulate wave fields, which have led to extensive researches in the classical wave systems, such as elastics [6–8], microwaves [9], electric circuits [10], photonics [11–15], and acoustics [16–26]. Therein, the acoustic SOTIs have been experimentally proposed based on the enclosed resonators connected by the coupled tubes [16, 17, 22–24] and the rigid scatterers in the 2D waveguides [19]. The above reported acoustic implementations, for the most part, were designed in an acoustically rigid enclosure with space limitations and were built on a wavelength scale, which may hinder the application potentials. Most recently, the obstacles were removed by introducing the SOTIs supporting spoof surface acoustic waves (SAWs), which were composed of the perforated holey crystals [27] and soda cans metamaterials [28]. The topological phase transition between trivial and nontrivial ones was obtained by shrinking/expanding the metamolecules to modulate the coupling strength. Then it is worth investigating
whether the sonic SOTIs based on spoof SAWs can be realized by other protocols besides shrinking/expanding and if there exist SOTIs which are pinned to the nontrivial phase region without such topological phase transition process.

In this work, we marry the SOTI with the gliding operation together and propose a tunable subwavelength acoustic SOTI hosting the topological corner states in free space. Beginning with the square sonic lattice, the four meta-atoms in the metamolecule can be glided along the dotted line into arbitrary angles as illustrated in figure 1(a). In the gliding process, the bands degeneracies are lifted and the topological nontrivial phase appears whether it is glided clockwise or counter-clockwise. The switching among the bulk, edge and corner states at the same frequency can be achieved accordingly. In experiments, the soda cans metamaterials [29–34] acting as acoustic subwavelength resonators are utilized to build the sonic lattice [35–39], whose dispersions under the sound line guarantee the transmission of the spoof SAWs. We emphasize that the external trivial region, which is usually needed to prevent the energy leaking, is not prerequisite here and the resulted SOTI size will be much smaller. To be specific, the excited spoof SAW, forbidden by the free space, makes the outer enclosed structures unnecessary in our proposed soda cans array. Thanks to the cancellation of needless outer metamaterials and the intrinsic subwavelength resonance of soda cans (the lattice constant \( a = 0.25\lambda \) in the example), the total side length of the sample is greatly reduced to just two times the corresponding wavelength, which is much smaller than previous acoustic SOTI composed of rigid scatterers with the side length up to dozens of times of the wavelength. Then we experimentally observe the existences of the topological corner states, which confines sound energy at corners beyond the classical diffraction limit. Furthermore, the robustness of the proposed SOTI against typical kinds of defects is studied in detail. We demonstrate that our tunable SOTIs may realize the real-time transitions among the bulk, edge and corner states, which provides another way for the synchronized manipulations of sound waves.

2. Second-order topological insulators

Let us start with the regular square lattice shown in figure 1(a), the metamolecule is composed of four meta-atoms with the basis vectors \( \mathbf{a}_1 = (a, 0) \) and \( \mathbf{a}_2 = (0, -a) \). The identical adjacent interval of 0.5\( a \) between the meta-atoms in the metamolecule indicates that the intra-cell coupling strength is same as the inter-cell one. For the pristine structure, the metamolecule consisting of four meta-atoms can be regarded as four times of the primitive cells. As a result, the band diagrams based on the four-times enlarged metamolecule host a four-fold degenerate point at \( M \) point of the 1st Brillouin zone (BZ) due to the band folding mechanism. The specific band folding mechanism is shown in supplementary materials (https://stacks.iop.org/NJP/24/053009/mmedia) section I [40]. On this basis, the meta-atoms are gliding gradually around the center of the metamolecule with an angle \( \theta \) to break \( C_{4v} \) symmetry, as shown in figure 1(a). Note that the \( p4g \) space group symmetry is maintained during the gliding process, which contains the glide symmetry, \( G_x := (x, y) \rightarrow (\frac{a}{2} + x, \frac{a}{2} - y) \) and \( G_y := (x, y) \rightarrow (\frac{a}{2} - x, \frac{a}{2} + y) \). On this regard, the double degenerated dispersion curves appear along the BZ boundaries \( MX \) and \( MY \) [19, 41]. Meanwhile, a band gap arises and is further enlarged with increasing the angle \( \theta \). The nontrivial higher-order topological phase is induced by the symmetry reduction from \( p4mm \) to \( p4gm \) during the gliding process and increasing angle only modulates the energetic properties of the lattice but not affects the topological properties. As a result, the topological edge states and lower-dimensional corner states can be achieved in the complete band gap. Taking a specific working frequency as an example, the finite sonic crystal (SC) will host the bulk states, topological edge states and the topological corner states respectively under the increased gliding angle, as shown in figure 1(b).

To observe the above topological switching process in acoustics, the soda cans are utilized to take the role of meta-atoms, acting as acoustic resonators in sub-wavelength scale. Figure 2(a) illustrates the photograph of the subwavelength SC composed of soda cans before gliding, which are arranged in square lattice in free space. The metamolecule consists of four soda cans and the lattice constant is \( a = 21.3 \) cm. Figure 2(b) shows the detailed schematic of the metamolecule in full-wave simulations, which are performed in a finite-element software COMSOL multiphysics. During the simulations, the shell of the soda can is modeled as the acoustic rigid boundary conditions due to the huge impedance mismatch between the aluminum alloy and air. The mass density and sound velocity of air are set as \( \rho = 1.21 \) kg m\(^{-3}\) and \( c = 343 \) m s\(^{-1}\). The center-to-center distance between the soda cans in the metamolecule is defined as \( D \), which is chosen to be \( D/a = 0.5 \) corresponding to a regular square lattice. Top and side views of the single soda can are shown in the right inset with the height \( H = 13.02 \) cm and the radius \( R = 2.84 \) cm. The neck with the height of \( h_n = 0.2 \) cm can be considered as an ellipse with \( d = r_2 = 1.25 \) cm and \( r_1 = 0.75r_2 \). The calculated band diagram with the gliding angle \( \theta = 0^\circ \) is depicted in figure 2(c). The first four bands
are located near the frequency of the first-order resonance. It should be noted that the soda cans can be regarded as Helmholtz resonators where the opening plays the role of the neck of resonator, and the spoof SAWs are coupled through the opening of soda cans. Thus, the dispersions of the array will be affected by the opening of the soda cans, whose orientations should be kept consistent (supplementary materials section II [40]). As predicted previously, the four-fold degenerated point is found at \( M \) point and the double degenerated bands along \( MX \) direction are also clearly observed. The blue dashed curve represents the sound line, which is the acoustic dispersion relationship in air background defined as \( f = |\mathbf{K}| \cdot \omega_0 / 2\pi \). The propagation of spoof SAWs can be guaranteed by the bands located under the sound line. Then, the soda cans in the metamolecule are glided to \( \theta = 24^\circ \) under the glide symmetry \( G_x := (x, y) \rightarrow (\frac{x}{2} + x, \frac{y}{2} - y) \) and \( G_y := (x, y) \rightarrow (\frac{x}{2} - x, \frac{y}{2} + y) \) as shown in the inset of figure 2(d). From the resulted band diagram in figure 2(d), a complete band gap between the second and third bands is clearly observed. We demonstrate that the 1D topological edge states and 0D corner states coexist in this band gap, which are protected by a nontrivial 2D Zak phase.

The Zak phase can be calculated by the parities at high symmetric points as [42–44]

\[
P_i = \pi \left( \sum_n q^n_i \text{mod 2} \right), \quad (-1)^n = \frac{\eta_n(X_i)}{\eta_n(\Gamma)}.
\]

where \( n \) represents the number of bands under the target gap, \( \eta_n(k) \) is the parity (±) of the \( n \)th band at high symmetric point and \( X_i / \Gamma \) denotes the \( X/\Gamma \) point of the first BZ. The pressure fields distributions of the eigenmodes at high symmetric points \( \Gamma \) and \( X \) are illustrated in figure 2(e). We find that the profiles of the eigenmodes \( \Gamma_1, \Gamma_2 \) and \( X_1 \) possess the even parity (+). But the odd parity (−) is observed for the eigenmode \( X_2 \). According to equation (1), the nontrivial Zak phase of \( P_x = \pi \) is achieved. Due to the \( C_4 \) symmetry of the proposed structure, we have \( P_x = P_y \). As a result, the derived 2D Zak phase of \( P = (\pi, \pi) \) verifies the nontrivial topological phase of the proposed soda cans metamaterials. We demonstrate that the dipole moments of the occupied bulk states \( P_x / P_y \) guarantee the presence of the topological edge state along \( x/y \) direction according to the bulk-edge correspondence. Then the nontrivial dipole moments in both directions form a quadruple tensor at the 90° terminated corner, which is a convergence of the interface polarization along both directions and supports the existence of topological corner states [16–18, 45]. The emergence of the corner states is protected in a hierarchy of the bulk-edge and edge-corner correspondence which can be characterized by a topological corner charge [12, 18]

\[
Q_{\text{op}} = \frac{P_x P_y}{\pi^2}.
\]

Consequently, the topological corner charge of the proposed structure with \( \theta = 24^\circ \) is \( Q_{\text{corner}} = 1 \), which ensures the existence of the topological corner states at the corners of SOTI.

3. Topological corner states

Based on the above analysis, we design a SOTI based on the soda cans metamaterials as shown in figure 3(a). The SOTI is composed of \( 8a \times 8a \) soda can arrays with the gliding angle \( \theta = 24^\circ \) and \( D/a = 0.5 \). The topological boundary states in previously proposed acoustic topological insulators and
Figure 2. (a) Photograph of the subwavelength SC composed of soda cans arranged in square lattice. (b) Schematic of the metamolecule consisting of four soda cans with the adjacent interval of $D$. The top and front views of the single can are shown in the right two panels. Band diagrams of SC constructed by (c) the non-gliding metamolecule with $D/a = 0.5$ and (d) gliding metamolecule with $\theta = 24^\circ$. The blue dashed curve represents the sound line and the inset shows the first BZ. (e) Pressure field distributions in wave colors represent the eigenmodes profiles at high-symmetric points $\Gamma$ and $X$, as labeled in (d).

SOTIs exist at the boundaries between two structures with opposite topological invariants [19, 21, 46, 47]. Here, however, the metamolecules with nontrivial topological index are arranged in the free space without the constraint from the outer trivial structures as usual, which is attributed to the propagation of spoof SAWs. More details about the spoof SAW and the resonance mode of the soda cans are further discussed in supplementary materials section III [40]. This advantage makes the acoustic SOTI smaller and much easier to be built. To verify the existence of the lower-dimensional boundary state, we calculate the eigenfrequencies of the SOTI near the first-order resonant frequency. The dispersion relations of the ribbon-shaped SC are illustrated in supplementary materials section IV [40]. The eigenfrequency spectra in figure 3(b) unequivocally show the existence of bulk, edge and corner states as predicted. Besides the bulk states, multiple edge states and four nearly degenerated corner states can be found in the band gap. We demonstrate that the bulk, edge and corner states are distinguished based on the pressure fields distributions of the eigenmodes (supplementary materials section V) [40]. In order to clearly observe their distinctions, we show the simulated pressure field distributions of randomly selected eigenmodes in the following. The randomly distributed sound waves spread across the entire bulk of the SOTI as shown in figure 3(c), when the eigenfrequency is in the bulk region. The distributions of the edge states in figure 3(d) verify that the sound waves propagate along the SOTI boundary but are forbidden by the bulk. From the acoustic profiles of the eigenmodes presented in figure 3(e), topological corner states can be clearly discerned, showing how the sound energy confines within the cans that are located at the corners. We emphasize that the eigenfrequencies of the corner states are around 402 Hz. As a result, the ratio between the lattice constant and the corresponding wavelength is $a/\lambda = 0.25$, which unequivocally shows that the proposed SOTI can sustain subwavelength corner states in the audible range. Furthermore, we demonstrate that the nontrivial topological index and the corresponding corner states can be also obtained with gliding the metamolecule toward the other direction (supplementary materials section VI [40]).
Figure 3. (a) Photograph of the acoustic SOTI consisting of 24°-glided soda cans array. (b) Calculated eigenfrequencies spectrum of the SOTI near the resonant frequency. Bulk, edge and corner states are represented by gray, blue and red dots, respectively. The simulated absolute acoustic pressure distributions of the eigenmodes corresponding to the (c) bulk states, (d) edge states and (e) corner states as labeled in (b). Bottom panels represent the pressure distributions inside the cans and top panels represent the x–y–plane pressure distributions with a distance of 8 mm above the cans array.

4. Experimental measurements

The corresponding experimental setup of the proposed SOTI with the gliding angle $\theta = 24^\circ$ is shown in figure 4(a) (detailed experimental setup are referred in the supplementary materials section VII [40]). The loudspeaker is placed near the corner to excite spoof SAW, which is 41 cm away from the SOTI and of the same height with the opening of soda cans. The acoustic pressure amplitude is detected by the microphones located at the center of the openings. As shown in figure 4(a), two sites at the corners labeled by red region are chosen to measure the intensity response at corners. To obtain the spectral response along the edge, sound intensity is measured at 12 sites in the blue colored regions. For the bulk modes, the intensity response is experimentally measured at 64 sites out of $4a \times 4a$ metamolecules which are framed by gray region. At the same time, we also measured the intensity response of the whole sample across a similar frequency range. The resulted intensity spectra of these four cases are plotted in figure 4(b), where the red, blue, black and yellow curves show the response at corner, along the edge, in the bulk and of the whole sample, respectively. To neatly compare with the simulated frequency spectrum in figure 3(b), the corresponding simulated frequency ranges of the bulk, edge and corner states are marked by gray, blue and red regions in/above the sub-figures. One can clearly observe that the corner peak appears at $f = 402$ Hz.
Figure 4. (a) Experimental setup. Gray, blue and red colored regions represent the measuring positions of the soda cans used to determine the spectra of the bulk, edge and corner states, respectively. (b) Experimentally measured pressure spectra for corner, edge, bulk states and the whole sample. (c) Simulated and experimentally measured pressure amplitudes along $x$ and $y$ directions at the frequency of $f = 402$ Hz. (d) Simulated and experimentally measured pressure amplitudes along $z$ direction, which are probed above the cornered can. (e) Measured acoustic pressure distribution of (e) the corner state at $f = 402$ Hz and (f) the bulk state at $f = 389$ Hz, respectively. Permission for the inclusion of the Coca Cola logo has not been sought. The inclusion of the Coca Cola logo is not intended to indicate any endorsement by or collaboration of Coca Cola with this publication or any other IOP Publishing material. (top panel) and the edge peak appears at $f = 401$ Hz (second panel), which are both consistent with the simulated spectra and located in the band gap. The complete band gap can be clearly found in the bulk response (third panel), where the bulk peak is located at the lower-frequency bulk region ranging from 380 Hz to 400 Hz. Note that there should be also bulk states around 404 Hz according to the simulated results. But the experimental observation of this bulk peak is highly restricted by the nearly zero group velocity caused by the super-flat bulk band in dispersion relations of figure 2(d). The bottom panel of figure 4(b) illustrates the local response of the whole sample from which two points should be noted. (i) The entirely different peak location compared with the corner one confirms that the response peak of corner state is not resulted from the nearby placed loudspeakers and all the measured intensity responses are induced by the topological properties of the proposed metamaterials. (ii) Compared with the bulk peak in the third panel, there appears another yellow peak in the bulk band gap around 402 Hz, which is exactly the frequency range of the topological edge and corner states. It further confirms that the observed edge and corner states are not localized in the bulk but along the boundary or at the corners. In order to grasp the physical properties of the subwavelength corner state, we measure the pressure field distributions above the soda cans located along the $x$- and $y$-direction boundaries (yellow dashed lines in figure 4(a)). One of the corner cans is set as the original point. The experimentally measured results (red dots) in figure 4(c) agree very well with the simulated ones (black curves) at the frequency $f = 402$ Hz, where the sound energy is highly confined at the original point (corner can) and decreases rapidly away from the corner. At the same
time, through the pressure distributions along z direction above the corner can in figure 4(d), we find that the sound waves are tightly confined at the surface of the soda can arrays and decay exponentially into the free space, which proves that the topological corner state is constrained by the spoof SAW propagation. Furthermore, we scan every soda can and measure the acoustic pressure amplitude at the frequency $f = 402$ Hz as shown in figure 4(e), where the corner state is clearly observed. However for the bulk state at the frequency $f = 389$ Hz, the sound waves spread disorderly as shown in figure 4(f). It should be noted that since the soda cans can be regarded as Helmholtz resonators, the dissipation mainly localized in the opening of cans may decrease the transmission ratio of the topological edge states, while the topological corner states with high confinement properties suffer less from the impact of dissipation (supplementary materials section VIII [40]).

5. Topological switching process

In the following, we study the topological switching process from the bulk to the edge and corner states with varying the gliding angle $\theta$. Figure 5(a) illustrates the calculated eigenfrequency spectra based on the finite $8a \times 8a$ SC with the gradually varied angle from $-26^\circ$ to $26^\circ$. Note that the analyzed angle range is restricted by the finite size limitation of the soda cans. The gray and blue colored regions represent the bulk and edge states, respectively, and the red curves mark the eigenfrequencies of the topological corner states. The results are highly symmetric due to the intrinsic symmetry and we can only focus on the region from $0^\circ$ to $26^\circ$ instead. From figure 5(a), we can see that the band gap is enlarged with the increase of the gliding angle. The topological corner states are separated from the bulk states when the glide angle $\theta$ is larger than $18^\circ$, although the non-zero polarizations appear once gliding the structure. The spectral separation among bulk, edge and corner modes is an energetic result that depends on the value of the glide angle.
Interestingly, if the working frequency is fixed, $f = 402$ Hz for example as labeled by the dashed line, the topological switching process from the bulk to the edge and corner states can be realized with gliding the SC. For the method of obtaining spectral eigenfrequency regions with varying the gliding angles, see supplementary materials section IX [40]. Figure 5(b) illustrates the simulated pressure amplitude distributions of the SC at the frequency $f = 402$ Hz with different gliding angles. The point source is placed at the identical position as the situation in figure 4(a). When the gliding angle is $\theta = 6^\circ$, only the bulk states can be excited. Once the angle is increased to $\theta = 12^\circ$ which is the transition point from the bulk to edge states, the edge states can be excited although it is not obvious yet. When the angle is further glided to $\theta = 18^\circ$, the edge states can be observed much more clearly. The topological corner state appears with the gliding angle of $\theta = 24^\circ$ as displayed in the last plot, where the sound pressure is highly confined in the two soda cans at the corner. The experimentally measured results in figure 5(c) show great consistency with the simulations in figure 5(b). As a result, we realize the transition from the bulk to edge and corner states by changing the gliding angle at the same frequency in experiments. In this way, the system will support different topological states by changing the gliding angle in real time as needed, which provides a new method for real-time acoustic wave manipulation. We also demonstrate that there does not exist trivial gap during the glide operation. The topologically nontrivial polarizations appear without the regular phase transition between trivial and nontrivial ones [27, 28] once gliding the structure. Therefore, the spectral separation among bulk, edge and corner modes is an energetic result that depends on the value of the glide angle.

6. Robustness of SOTI against defects

As the smoking gun of the topological systems, the robustness of the proposed subwavelength SOTI against the defects is discussed here. For the soda can arrays, the placements may be slightly malposed during the arrangement. Also, some of the soda cans may also be deformed during the fabrications, which thus affects the resonance frequency. Therefore, it is necessary to study the robustness against the displacements and the deformation of the cans. In experiments, we introduce two kinds of defects into eight metamolecules located near the four corners of the SOTI as framed in figure 6(a). We demonstrate that the disorders located just close to the corners may lead to the stronger influences on the topological corner states and more detailed discussions on the robustness against other defects are studied in supplementary materials section X [40]. One is to move a can away from the pristine position as shown in figure 6(b) where the displacement shift is 5 cm. The other defect is to change the volume of the can to the greatest extent by placing the can upside down as shown in figure 6(c). Then the corresponding pressure spectra of the topological corner states without/with defects are experimentally measured as plotted in figure 6(d), where the red solid, blue dashed and green dashed curves represent the pressure amplitude measured in the SOTI without defects, with the displacements and with the defects by placing the can upside down, respectively. We demonstrate that the curves of topological corner state keep stable after introducing defects and the frequency shifts are negligible. It proves that the confined topological corner state has good robustness against both displacements and resonance shift.
7. Conclusion

In conclusion, we have introduced the gliding operation into the acoustic HOTI. The lattice symmetry is modulated through gliding the meta-atoms leading to the nontrivial topological indices. Then we theoretically proposed and experimentally realized an acoustic SOTI supporting the 0D topological corner state by utilizing the subwavelength-scale soda can arrays. The corner states are excited by spoof SAWs in the free space without the regular bulky outer structural restrictions. Furthermore, the topological switching process from the bulk to the edge and corner states at the same frequency through simply gliding the cans are experimentally observed. It provides a new idea for real-time manipulation of sound waves. At last, good robustness against both displacements and resonance shift are demonstrated. We state that our findings may provide possibilities to advance the design of reconfigurable acoustic functional devices based on the robust HOTIs in subwavelength scale.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Conflict of interest

The authors declare that there are no conflicts of interest related to this article.

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