Simulation Study to Evaluate Full Information Maximum Likelihood as Parameter Estimation Methods for Spatial Vector Autoregressive Model with Calendar Variation

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Abstract. Spatial Vector Autoregressive models with calendar variation can be used to analyze the interrelationships between variables, the relationship of variables with their past and the relationship of variables in a location with those variables in other locations. It also can accommodate the effects of calendar variations. The parameter estimation of this model can be done using Full Information Maximum Likelihood (FIML). The purpose of this study is to evaluate the performance of FIML and it is compared to Ordinary Least Square (OLS) through simulation. The simulation is done using generated data which is designed following Spatial Vector Autoregressive model with calendar variation. There are three aspects studied in this simulation, namely how the effect of error correlation between equations, the variance of error and length of period on the performance of FIML Method and OLS Method. The result of the simulation is the variance of FIML parameter estimator is smaller than OLS, especially when the error correlation between equation are high. While the variance of errors and length of periods have no effect on performance of the estimator. The simulations also show that the mean of parameter estimators both FIML and OLS are very close to the parameters specified.

1. Introduction

In some fields of science such as economics, meteorology and health, some variables have space-time relationship. The model in statistics that can be used for space-time data analysis is Space-Time Autoregressive (STAR) discovered by [1]. Further enhanced to Generalized Space-Time Autoregressive (GSTAR) by [2] and [3]. The GSTAR assumes that the coefficients at each location may be different. This is a development of the STAR which assumes that the coefficients at each location are the same. The GSTAR development is done by [4] by finding Generalized Space-Time Autoregressive - Seemingly Unrelated Regression (GSTAR-SUR) for the GSTAR model with error correlation between locations. In addition, for seasonal data with errors correlated between locations, the Seasonal-Generalized Space Time Autoregressive-Seemingly Unrelated Regression (S-GSTAR-SUR) model was developed by [5]. Other GSTAR development was done by [6] which accommodates the existence of exogenous variables and errors correlation between locations with Generalized Space Time Autoregressive Exogen-Generalized Least Square (GSTARX-GLS) model.

Another model for space-time data is Vector Autoregressive (VAR). This models was developed by [7] and [8]. The STAR, GSTAR and VAR models can only be used to analyze one observed variable in
Spatial Vector Autoregressive with calendar variation is a model which accommodates an existence of relationship between some variables, a space-time relationship and a calendar variation effect on the endogenous variables. In the model consisting of several equations, there is a possibility of error correlation between equations. Therefore it is necessary to use appropriate parameters estimation methods. The parameter estimation of this model can use Full Information Maximum Likelihood (FIML) or Ordinary Least Square (OLS). The main difference between the FIML and OLS lies in the presence of the variance-covariance matrix in the FIML estimator formula that does not present in the OLS. The purpose of this research is to evaluate FIML performance compared to OLS through simulation. There are three aspects studied in this simulation, namely how the influence of variance of error, length of data (T) and correlation of error between equations on the FIML and OLS performances. Large variance and the correlation of error between equations can result in inefficient estimators [5]. While too short a period of time can produce unstable estimator.

2. Spatial Vector Autoregressive with Calendar Variation

The Spatial Vector Autoregressive model was developed by [10] and was a development of the Vector Autoregressive model. The Vector Autoregressive model can be used to analyze several variables at one location or one variable observed in several locations. The Spatial Vector Autoregressive model is used to analyze several variables and several locations. Due to some data, especially economic data shows influence by the calendar variation. Some researches who work on the effects of calendar variation are [11], [12], [13] and [14]. Some researches who work on calendar variation in Indonesia among them are [15], [16], [17] and [18].

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2.1. The Model

The Spatial Vector Autoregressive model with calendar variation is formed by adding exogenous variables in the Spatial Vector Autoregressive model from [10]. The exogenous variable is a dummy variable. The Spatial Vector Autoregressive model of order (1, p) from [10] is defined as

\[ y_t = B_1 y_{t-1} + \ldots + B_p y_{t-p} + \eta_t, \quad t = 1, \ldots, T \]  

(1)

Spatial Vector Autoregressive model with spatial order one and temporal order p [SpVAR(1, p)] with Calendar Variation that we propose can be written as

\[ y_t = X_t \gamma + B_1 y_{t-1} + \ldots + B_p y_{t-p} + \xi_t \]  

(2)

where

- \( y_t = [y_{1t}, y_{2t}, \ldots, y_{N_{1t}}, y_{12t}, y_{22t}, \ldots, y_{N_{2t}}, \ldots, y_{1Kt}, y_{2Kt}, \ldots, y_{NKt}] \),
- \( y_{nk}^{\text{m}} : \) the value of k-th variable observed at n-th location at time t and
- \( \gamma = [\gamma_1, \gamma_2, \ldots, \gamma_{N_{1}}, \gamma_{12}, \gamma_{22}, \ldots, \gamma_{N_{2}}, \ldots, \gamma_{1K}, \gamma_{2K}, \ldots, \gamma_{NK}] \),
- \( \gamma_{nk}^{\text{m}} = [\gamma_{nk1}^{\text{m}}, \gamma_{nk2}^{\text{m}}, \ldots, \gamma_{nk_{m}}^{\text{m}}] \), m is the number of dummy variable for calendar variation,
The likelihood function. In the model consisting of two or more equations we used Full Information Maximum Likelihood Method. This method obtains parameter estimators by maximizing parameter estimators that minimize the sum of the squared error. Another parameter estimation method is the Maximum Likelihood. The formula of FIML estimator is

$$\hat{\theta} = (Z'Z)^{-1}Z'y$$

2.2. Parameter Estimation Methods

The OLS is one of the widely used parameter estimation methods. The basic concept of OLS is to obtain parameter estimators that minimize the sum of the squared error. Another parameter estimation method is the Maximum Likelihood Method. This method obtains parameter estimators by maximizing likelihood function. In the model consisting of two or more equations we used Full Information Maximum Likelihood. The OLS estimator formula is

$$x_{nlk} = [x_{nk1l}, x_{nk2l}, \ldots, x_{nkm}]^T.$$

Following [10], $B_h$ is defined as

$$B_h = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1K} \\ A_{21} & A_{22} & \cdots & A_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ A_{K1} & A_{K2} & \cdots & A_{KK} \end{bmatrix},$$

where $A_{kl} = \Phi_{kl}^{(h)} + \Phi_{kl}^{(0)}W_{kl}$, 

$$\Phi_{kl}^{(h)} = \text{diag} \left( \begin{bmatrix} \phi_{l1}^{(h)} & \phi_{l2}^{(h)} & \cdots & \phi_{lN}^{(h)} \end{bmatrix} \right), \quad k, r = 1, \ldots, K; h = 1, \ldots, p,$$

where $\phi_{l}^{(0)}$ is the parameter for $spVAR$ model and $W_{kl}$ is $N \times N$ spatial weight matrix, where the element $w_{ij}(i, j)$ which is already known and assumed to be non-negative and positive if location $i$ and $j$ are neighbors and zero if $i = j$. Spatial weights are assumed to be fixed all the time. The autoregressive coefficients are assumed to vary between locations.

$$\xi_{kl} = \begin{bmatrix} \xi_{k1l} & \xi_{k2l} & \cdots & \xi_{kNl} \end{bmatrix},$$

$$\xi_{nlkt} : \text{error from model for } k\text{-th variable on the } n\text{-th location at time } t \text{ and is assumed } \xi_{nlkt} \sim N(0, \Sigma),$$

$$\Sigma = \begin{bmatrix} E(\xi_{11l}^2) & E(\xi_{12l}^2) & \cdots & E(\xi_{1Nl}^2) \\ E(\xi_{21l}^2) & E(\xi_{22l}^2) & \cdots & E(\xi_{2Nl}^2) \\ \vdots & \vdots & \ddots & \vdots \\ E(\xi_{N1l}^2) & E(\xi_{N2l}^2) & \cdots & E(\xi_{Nkl}^2) \end{bmatrix}, \quad \text{where } \xi_{nlkt} = \begin{bmatrix} \xi_{11l} \\ \xi_{12l} \\ \vdots \\ \xi_{Nkl} \end{bmatrix}$$
\[ \hat{\theta} = \left( Z' (\mathbf{I}_r \otimes \Sigma^{-1}_r) Z \right)^{-1} Z' (\mathbf{I}_r \otimes \Sigma^{-1}_r) y \]  

(6)

where \( \hat{\theta} \) is a vector of parameter estimator that contains all coefficients and \( Z \) is a matrix which contains of dummy variables, lag \( y \) and spatial elements.

2.3. Simulation Design and Algorithm

This simulation data using two variables, for example, Inflation and Money Supply and three locations to get six equations. Further variables for the calendar variation used are three. These variables are dummy variables and are defined as:

\[
\begin{align*}
    x_{1t} &= \begin{cases} 
        1, & \text{if } t \text{ is the month of Eid al Fitr} \\
        0, & \text{if } t \text{ is not the month of Eid al Fitr}, 
    \end{cases} \\
    x_{2t} &= \begin{cases} 
        1, & \text{if } t \text{ is a month before Eid al Fitr} \\
        0, & \text{if } t \text{ is not a month before Eid al Fitr}, 
    \end{cases} \\
    x_{3t} &= \begin{cases} 
        1, & \text{if } t \text{ is two month before Eid al Fitr} \\
        0, & \text{if } t \text{ is not two month before Eid al Fitr}.
    \end{cases}
\end{align*}
\]

Uniform weights are used as spatial weights, namely

\[ \mathbf{W} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix} \]

The length of the period used are 60 months, 120 months and 144 months. The parameters used are set so that the eigen value of \( \Phi \) is less than 1. The algorithm of this simulation are:

1. Error \( \xi_{nkt} \) is generated from a multivariate normal distribution with

\[ \mu = \begin{bmatrix} 0 \\ \text{var}(i) \text{ cov}(i, j) \text{ cov}(i, j) \text{ cov}(i, j) \text{ cov}(i, j) \text{ cov}(i, j) \\ \text{cov}(i, j) \text{ var}(i) \text{ cov}(i, j) \text{ cov}(i, j) \text{ cov}(i, j) \text{ cov}(i, j) \\ \text{cov}(i, j) \text{ cov}(i, j) \text{ var}(i) \text{ cov}(i, j) \text{ cov}(i, j) \text{ cov}(i, j) \\ \text{cov}(i, j) \text{ cov}(i, j) \text{ cov}(i, j) \text{ var}(i) \text{ cov}(i, j) \text{ cov}(i, j) \\ \text{cov}(i, j) \text{ cov}(i, j) \text{ cov}(i, j) \text{ cov}(i, j) \text{ var}(i) \end{bmatrix}, \Sigma = \begin{bmatrix} \text{cov}(i, j) \end{bmatrix} \]

The value of \( \text{var}(i) \) and \( \text{cov}(i, j) \) are adjusted for the purpose of simulation. The value of \( \text{var}(i) \) tested in this simulation are 0.25, 1 and 2. While the value of \( \text{cov}(i, j) \) is adjusted to the value of the tested correlation of 0.2, 0.5 and 0.9.

2. The value of \( y_{nkt} \) is generated from

\[
\begin{align*}
y_{11t} &= 0.57^* x_{1t} + 0.39^* x_{2t} + 0.54^* x_{3t} + 0.2^* y_{11t-1} \\
        &\quad + 0.1^* (0.5^* y_{21t-1} + 0.5^* y_{31t-1}) + 0.1^* y_{12t-1} + 0.25^* (0.5^* y_{22t-1} + 0.5^* y_{32t-1}) + \xi_{11t} \\
y_{21t} &= 0.78^* x_{1t} + 0.23^* x_{2t} + 0.88^* x_{3t} + 0.15^* y_{21t-1} \\
        &\quad + 0.15^* (0.5^* y_{11t-1} + 0.5^* y_{31t-1}) + 0.25^* y_{22t-1} + 0.25^* (0.5^* y_{12t-1} + 0.5^* y_{32t-1}) + \xi_{21t} \\
y_{31t} &= 0.12^* x_{1t} + 0.39^* x_{2t} + 0.6^* x_{3t} + 0.2^* y_{31t-1} \\
        &\quad + 0.2^* (0.5^* y_{11t-1} + 0.5^* y_{21t-1}) + 0.3^* y_{32t-1} + 0.3^* (0.5^* y_{22t-1} + 0.5^* y_{32t-1}) + \xi_{31t} \\
y_{12t} &= 0.66^* x_{1t} + 0.34^* x_{2t} + 0.14^* x_{3t} + 0.1^* y_{11t-1} \\
        &\quad + 0.15^* (0.5^* y_{21t-1} + 0.5^* y_{31t-1}) + 0.15^* y_{12t-1} + 0.2^* (0.5^* y_{22t-1} + 0.5^* y_{32t-1}) + \xi_{12t} \\
y_{22t} &= 0.12^* x_{1t} + 0.46^* x_{2t} + 0.22^* x_{3t} + 0.2^* y_{21t-1} \\
        &\quad + 0.25^* (0.5^* y_{11t-1} + 0.5^* y_{31t-1}) + 0.25^* y_{22t-1} + 0.25^* (0.5^* y_{12t-1} + 0.5^* y_{32t-1}) + \xi_{22t} \\
\end{align*}
\]
For $t = 0$, it is assumed that $y_{nit} = 0$

3. Using FIML and OLS to estimate the parameter of SpVAR model with calendar variation.

This simulation is repeated 1000 times then the value of the parameter estimator is averaged and the standard deviation of the parameter estimator are calculated.

3. Result and Discussion

The model used in this simulation consists of two endogenous variables and three locations so that there are six equations. In each equation, seven parameters or coefficient are estimated, three of which are coefficients for dummy variables of calendar variation, the other two are coefficients for autoregressive and the rest are coefficients for spatial elements. So the number of parameters that are estimated is 56 parameters. There are two measures that are calculated from the simulation in this research, that is, the mean of parameter estimators and standard deviation of parameter estimators. The mean of parameter estimators from the simulation for FIML are very close to the predetermined parameters. This happens in all conditions, i.e. at all lengths of periode (60, 120 and 144), all variance of error (0.25, 1 and 2) and all correlation of error between locations being tested (0.2, 0.5 and 0.9). Similar results are also experienced in OLS. These results show that both FIML and OLS are good enough to estimate parameters of SpVAR model with calendar variation, not dependent on length of periode, variance of error and correlation of error between locations.

The second measure calculated in this study is standard deviation of the parameter estimators. The result of this simulation for length of the periods of 60 presented on Table 1. The table shows that standard deviation of parameter estimator for FIML is smaller than OLS especially on the high correlation of error between locations that is 0.9. The same results are also shown from simulation for the length of the period of 120. The results for the length of period of 144 are also equal to the results for the length of period of the 60 and 120. The results of this simulation is an indication that FIML is more efficient than OLS. This finding is inline with [6] and [19] which stated that FIML typically produces the most efficient estimates. This occurs because in FIML, the correlation of error between equations is accommodated on the variance covariance matrix. When the correlation of error between equations is low then the variance-covariance matrix is close to the identity matrix so that the parameter estimator of OLS and FIML will show similarities. So if the error correlation between equations is low then OLS and FIML are equally good to be used, but if correlation of error between equations is high then FIML should be used.

4. Conclusion

The method of parameter estimation that can be used is FIML and OLS. Based on the simulation, the mean of parameter estimator for FIML and OLS is very close to the predetermined parameters. While standard deviation of parameter for FIML is smaller than OLS especially when the correlation of error between location is high. This is an indication that FIML is more efficient than OLS. This also shows that the correlation of error between locations has an effect on the parameter estimation of SpVAR model with calendar variation while the length of the period and variance of error have no effect.

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| Parameter | Corr=0.2 | Corr=0.25 | Corr=0.5 | Corr=0.75 | Corr=0.9 | Corr=2 | Variance=0.25 | Variance=0.5 | Variance=0.9 | Variance=2 |
|-----------|----------|-----------|----------|-----------|----------|-------|-------------|-------------|-------------|-------------|
|           | OLS      | FIML      | OLS      | FIML      | OLS      | FIML   | OLS         | FIML         | OLS         | FIML         |
| r_{11}    | 0.238    | 0.239     | 0.233    | 0.230     | 0.233    | 0.229  | 0.462       | 0.463       | 0.469       | 0.466       |
| r_{12}    | 0.240    | 0.239     | 0.234    | 0.232     | 0.252    | 0.239  | 0.459       | 0.458       | 0.472       | 0.467       |
| r_{13}    | 0.229    | 0.228     | 0.236    | 0.235     | 0.238    | 0.235  | 0.448       | 0.449       | 0.442       | 0.442       |
| r_{21}    | 0.248    | 0.247     | 0.246    | 0.242     | 0.242    | 0.237  | 0.501       | 0.499       | 0.456       | 0.453       |
| r_{22}    | 0.240    | 0.240     | 0.256    | 0.251     | 0.268    | 0.249  | 0.485       | 0.485       | 0.462       | 0.457       |
| r_{14}    | 0.225    | 0.225     | 0.236    | 0.235     | 0.235    | 0.230  | 0.455       | 0.455       | 0.468       | 0.465       |
| r_{24}    | 0.242    | 0.243     | 0.236    | 0.234     | 0.237    | 0.231  | 0.471       | 0.471       | 0.444       | 0.441       |
| r_{31}    | 0.253    | 0.253     | 0.250    | 0.245     | 0.282    | 0.253  | 0.492       | 0.491       | 0.463       | 0.460       |
| r_{32}    | 0.238    | 0.238     | 0.239    | 0.241     | 0.237    | 0.241  | 0.472       | 0.472       | 0.481       | 0.479       |
| r_{23}    | 0.237    | 0.237     | 0.238    | 0.235     | 0.238    | 0.236  | 0.465       | 0.464       | 0.448       | 0.445       |
| r_{12}    | 0.245    | 0.244     | 0.241    | 0.240     | 0.250    | 0.238  | 0.478       | 0.477       | 0.480       | 0.477       |
| r_{33}    | 0.235    | 0.236     | 0.233    | 0.232     | 0.237    | 0.234  | 0.464       | 0.464       | 0.464       | 0.460       |
| r_{22}    | 0.248    | 0.248     | 0.237    | 0.236     | 0.241    | 0.236  | 0.461       | 0.461       | 0.452       | 0.449       |
| r_{32}    | 0.241    | 0.239     | 0.253    | 0.246     | 0.274    | 0.254  | 0.506       | 0.507       | 0.474       | 0.468       |
| r_{33}    | 0.232    | 0.233     | 0.237    | 0.236     | 0.236    | 0.246  | 0.464       | 0.464       | 0.462       | 0.462       |
| r_{31}    | 0.668    | 0.666     | 0.668    | 0.668     | 0.673    | 0.671  | 0.459       | 0.458       | 0.476       | 0.476       |
| r_{21}    | 0.718    | 0.720     | 0.719    | 0.715     | 0.810    | 0.809  | 0.462       | 0.462       | 0.467       | 0.463       |
| r_{32}    | 0.645    | 0.646     | 0.668    | 0.667     | 0.640    | 0.642  | 0.460       | 0.459       | 0.488       | 0.485       |
| r_{12}    | 0.139    | 0.135     | 0.170    | 0.141     | 0.263    | 0.146  | 0.141       | 0.139       | 0.177       | 0.147       |
| r_{23}    | 0.145    | 0.141     | 0.174    | 0.150     | 0.226    | 0.148  | 0.182       | 0.179       | 0.228       | 0.208       |
| r_{32}    | 0.145    | 0.139     | 0.170    | 0.143     | 0.255    | 0.145  | 0.145       | 0.159       | 0.154       | 0.265       |
| r_{13}    | 0.210    | 0.093     | 0.095    | 0.092     | 0.100    | 0.096  | 0.172       | 0.172       | 0.220       | 0.206       |
| r_{24}    | 0.130    | 0.127     | 0.157    | 0.132     | 0.226    | 0.126  | 0.139       | 0.135       | 0.168       | 0.145       |
| r_{34}    | 0.164    | 0.161     | 0.188    | 0.166     | 0.277    | 0.174  | 0.175       | 0.172       | 0.217       | 0.199       |
| r_{14}    | 0.131    | 0.129     | 0.159    | 0.135     | 0.219    | 0.132  | 0.139       | 0.136       | 0.171       | 0.144       |
| r_{33}    | 0.096    | 0.095     | 0.094    | 0.092     | 0.100    | 0.096  | 0.192       | 0.190       | 0.237       | 0.208       |
| r_{23}    | 0.116    | 0.113     | 0.136    | 0.116     | 0.187    | 0.112  | 0.133       | 0.129       | 0.168       | 0.147       |
| r_{12}    | 0.172    | 0.170     | 0.205    | 0.182     | 0.302    | 0.186  | 0.184       | 0.221       | 0.202       | 0.338       |

Table 1. Standard Deviation of Estimator for T=60
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