Effects of collisions on the stability properties of plasma diodes

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Abstract. Stability features of the Pierce diode steady states are investigated in the presence of electron-background-particle collisions. The regime without electron reflection from the potential barriers is studied. Steady states are obtained analytically by means of the Lagrangian fluid description. Employing the first order perturbation theory, the relevant dispersion equation is derived. Both aperiodic and oscillatory solutions of this equation are found and their properties are studied.

1. Introduction

At high emitter temperatures, maximum specific power of the thermionic converter (TIC) is achieved and at cesium pressure corresponding to the transition regime between the collisionless and the electron collision one [1]. The electron-atom collision effect on the TIC regimes has to be studied in order to perform optimization of TIC’s characteristics.

On the other hand, it was suggested to use a TIC as a generator of alternate current in Ref. [2]. Its operation is based on the phenomenon of Bursian-Pierce instability. However, occurrence of electron–atom collisions in the inter-electrode gap may prevent this instability. Hence, switching on electron scattering can restrict from AC generation process in a TIC.

A lot of results for a TIC can be obtained analytically by means of the Pierce-like diode in which injected electrons with the beam-like velocity distribution move through the background of uniformly distributed immobile ions. This is due to the fact that the Pierce diode allows to approximate with high accuracy steady states of a collisionless TIC [3].

We studied the electron–atom collision effect on steady states of the Pierce diode in our recent paper [4]. We have revealed two types of solutions: the Bursian (which has a single minimum potential distribution) and non-Bursian (which has oscillation-type potential distribution). In this paper, we study effect of collisions on stability properties of both solution families.

2. Statement of the problem

We consider the planar diode with an electrode distance \(d\) and a potential difference \(U\) being applied across electrodes. The mono-energetic electron flow enters from the emitter \(z = 0\) with the density \(n_0\) and velocity \(v_0\) perpendicularly to the emitter surface. The inter-electrode space is uniformly occupied by infinitely massive immobile ions of constant density \(n_i\). Ion background
is taken into account by the dimensionless neutralization parameter

\[ \gamma = n_i/n_0. \] (1)

Electrons move with no collisions in the self-consistent electric field \( E \). It can be calculated from the scalar potential \( \varphi \) which depends on the coordinate \( z \) only.

In the 1D time-independent case, the basic governing equations are the continuity, momentum, and the Poisson’s ones. According to Refs. [4]–[6] in this case the presence of collisions can be incorporated in the momentum equation by a dissipative term which is proportional to the electron velocity with the coefficient \( \nu \) being ratio of an average collision frequency to the plasma frequency. Then the basic governing equations can be expressed in dimensionless form as:

\[
\begin{align*}
\frac{\partial n}{\partial t} + \frac{\partial}{\partial \zeta} (nu) &= 0, \\
\frac{\partial}{\partial t} + u \frac{\partial}{\partial \zeta} u &= -\varepsilon - \nu u, \\
\frac{\partial \varepsilon}{\partial \zeta} &= -n + \gamma, \\
\frac{\partial \eta}{\partial \zeta} &= \varepsilon/n.
\end{align*}
\] (2)

with the following boundary conditions: \( n(0, t) = 1, u(0, t) = 1, \eta(0, t) = 0 \) and \( \eta(\delta, t) = V \). All relevant variables are expressed in terms of dimensionless quantities. We introduced energy and length units which are the kinetic energy of electrons at the emitter \( W_0 \) and the beam Debye length \( \lambda_D \), respectively

\[ \lambda_D = \left[ \frac{2e_0 W_0}{e^2 n_0} \right]^{1/2} \approx 0.3238 \cdot 10^{-2} \frac{V_0^{3/4}}{j_0^{1/2}} [\text{cm}], \quad W_0 = \frac{mv_0^2}{2}. \] (3)

Here, the current density \( j_0 = en_0 v_0 \) and accelerating voltage \( V_0 = W_0/e = \frac{mv_0^2}{(2e)} \) of the injected beam are taken in Amperes per square cm and Volts, respectively; the symbols \( e \) and \( m \) represent the electron charge and its mass; the free-space permittivity \( \varepsilon_0 = 8.854 \cdot 10^{-12} \text{C}^2/\text{Nm}^2 \).

The dimensionless coordinate, time, velocity, potential and electric field strength are introduced as: \( \zeta = z/\lambda_D, \ t = t\omega_0, \ u = v/\sqrt{2W_0/m}, \ \eta = e\varphi/(2W_0), \ \varepsilon = eE\lambda_D/(2W_0); \) here \( \omega_0 = [e^2 n_0/(me_0)]^{1/2} \) is the characteristic frequency. Dimensionless inter-electrode gap and voltage between collector and emitter are denoted via \( \delta \) and \( V \) respectively.

After proceeding from Euler variables \((\zeta, t)\) to the Lagrangian ones \((t_0, t)\) [7]–[9] and carrying out the relevant transformations the set of equations (2) is transformed into

\[
\begin{align*}
\frac{\partial n}{\partial t} - n^2 \frac{\partial u}{\partial t_0} &= 0, \\
\frac{\partial u}{\partial t} &= -\varepsilon - \nu u, \\
\frac{\partial \varepsilon}{\partial t_0} &= 1 - \gamma, \quad \frac{\partial \eta}{\partial t_0} = \varepsilon/n.
\end{align*}
\] (4)

This system can be reduced to the single equation for an electron dynamics:

\[ \frac{\partial^2 \zeta}{\partial t^2} + \nu \frac{\partial \zeta}{\partial t} + \gamma \zeta = (t - t_0) - \varepsilon_0(t). \] (5)

3. Results

Parametric formulas for electron position and potential distribution could be obtained by the use of Eqs. (5). For time-independent case they are the following:

\[ \zeta(\tau) = \frac{1}{\gamma^2} - \frac{1}{\gamma} (\gamma \varepsilon_0 + \nu) + \frac{1}{\gamma^2} \exp \left( \frac{-\nu \tau}{2} \right) \times \]
\[
\eta(\tau) = -\frac{1}{2} \gamma \zeta^2(\tau) + (\tau - \varepsilon_0) \zeta(\tau) - \int_0^\tau \zeta(t) dt.
\]

Here “the effective frequency” is \( \beta = \sqrt{\gamma - \nu^2/4} \).

In Figure 1, steady states are visualized through the \( \varepsilon_0(\delta) \) parametric curves for the Pierce diode for several \( \nu \) values. We can see two types of solutions: the Bursian (with single potential minimum) and non-Bursian (oscillatory-type) families. Bursian branches lie in the left part of Figure 1 (for \( \delta < \pi \) and \( \varepsilon_0 > 0 \)). They have right bifurcation point – the \( SCL \)-point. The non-Bursian branches appear in the region \( 1 < \delta/\pi < 3 \). However, the non-Bursian branches only exist for sufficiently small values of collisional-frequency \( \nu \) and disappear when \( \nu \) increases. For the diode with \( \gamma = 1 \) it happens at \( \nu \approx 0.06 \).

![Figure 1](image-url)

**Figure 1.** Time-independent states presented in \( \varepsilon_0 \) vs \( \delta \) parametric curves drawn for \( \gamma = 1 \), \( V = 0 \) and various values of \( \nu \): \( \nu = 0 \) (curve 1), 0.02 (2), and 0.05 (3).

By the use of a perturbation approach we derived a relevant dispersion relation:

\[
F(\sigma; \delta, T) = -\exp(-\kappa T) \left[ 2\kappa \cos \beta T + \frac{\kappa^2 - \beta^2}{\beta} \sin \beta T \right] + (\gamma + \nu \sigma + \sigma^2)^2 \delta - (\gamma + \nu \sigma + \sigma^2) T + 2\kappa = 0.
\]

Here \( \sigma = \Gamma + i\Omega \) with growth rate \( \Gamma \) and frequency \( \Omega \), and \( \kappa = \sigma + \nu/2 \).

Our study shows that dispersion curves \( \Gamma(\delta) \) and \( \Omega(\delta) \) for the Bursian family are similar to that of the Bursian diode: i.e. for normal C branch \( (0 < \varepsilon_0 < \varepsilon_{0,SCL}) \) solutions are stable (negative growth rate) and for C-overlap branch \( (\varepsilon_0 > \varepsilon_{0,SCL}) \) they are unstable with respect to aperiodic perturbation. The oscillatory modes are found to be stable in similar way. Examples of dispersion curves \( \Gamma(\delta) \) for the non-Bursian branches are shown in two subgroups: Figure 2(a) \( (\varepsilon_0 < 0) \) and Figure 2(b) \( (\varepsilon_0 > 0) \). In all these curves, the sections which belong to positive parts of \( \Gamma \) are unstable and the sections which correspond to negative values of \( \Gamma \) are stable. Oscillatory modes of non-Bursian branches are always stable. However, a part of the aperiodic curve of non-Bursian solutions is found to be unstable.
4. Conclusion

In this work we have studied the stability features of the time independent solutions of the Pierce diode in the presence of electron-atom collisions. These solutions are obtained using Lagrangian formalism. To explore the stability features of the obtained solutions, we have derived a dispersion equation from a perturbative approach. For Bursian branches, the solutions which belong to the Normal C branch ($\varepsilon_0 < \varepsilon_0,_{SCL}$) are always stable with respect to small aperiodic perturbations whereas C-overlap branches ($\varepsilon_0,_{0} > \varepsilon_0 > \varepsilon_0,_{SCL}$) are found to be unstable. The stable and unstable regions of aperiodic modes are also explored for non-Bursian steady states. On the other hand, oscillatory branches of all solutions turn out to be always stable.

Our investigations open up opportunity to carry out more precise optimization of TIC characteristics taking into account electron-atom collisions. Our study provides also a possibility to predict the threshold magnitude of the collision parameter over which switching on the instability turns out to be impossible so that AC generator based on the TIC fails to perform.

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