Parametric Study of Cantilever Plates Exposed to Supersonic and Hypersonic Flows

A Sri Harsha², M Rizwan², Kuldeep S², A Giridhara Prasad², J Akhil¹ and S R Nagaraja¹³
¹B.Tech student, Department of Mechanical Engineering, Amrita School of Engineering, Bengaluru, Amrita Vishwa Vidyapeetham, Amrita University, India
²Department of Mechanical Engineering, Amrita School of Engineering, Bengaluru, Amrita Vishwa Vidyapeetham, Amrita University, India
³E-mail: sr_nagaraja@blr.amrita.edu

Abstract. Analysis of hypersonic flows associated with re-entry vehicles has gained a lot of significance due to the advancements in Aerospace Engineering. An area that is studied extensively by researchers is the simultaneous reduction aerodynamic drag and aero heating in re-entry vehicles. Out of the many strategies being studied, the use of aerospikes at the stagnation point of the vehicle is found to give favourable results. The structural stability of the aerospike becomes important as it is exposed to very high pressures and temperatures. Keeping this in view, the deflection and vibration of an inclined cantilever plate in hypersonic flow is carried out using ANSYS. Steady state pressure distribution obtained from Fluent is applied as load to the transient structural module for analysis. After due validation of the methods, the effects of parameters like flow Mach number, plate inclination and plate thickness on the deflection and vibration are studied.

1. Introduction

A lot of emphasis has been placed on the design of hypersonic re-entry vehicles, especially with regards to drag reduction and heat flux reduction. The fact that these are conflicting constraints makes the design process difficult. On one hand, streamlining the vehicles would result in reduced drag forces, but very high values of heat flux are experienced owing to attached shock formation and a very thin shock layer. On the other hand, blunt bodies provide better heat dissipation at the expense of increased drag forces. Attaching slender bodies (aerospikes) to the stagnation point of re-entry vehicles has proved to be an effective way of countering both drag and heat flux problems.

Fluid-structural interaction problems are usually analysed by numerical simulation as they are too complex to solve analytically. It was observed that under the action of a hypersonic flow, various structural parameters vary [1-3]. P. Le Tallec et al[4], Ergin and Ugurlu[5] have discussed these effects on cantilever beams. Fluid-Structure Interaction (FSI) problems are widely studied by researchers all over the world. The conditions under which a one-way coupling solution is appropriate and the differences between one way and two way FSI methods and solutions was discussed in literature[6, 7]. The differences in various methods of formulation for the fluid flow was discussed by Piperno[8].
The use of aerospikes was first suggested by Alexander in 1947[9]. It was recorded that after a certain Mach number the surface pressures and aero-thermodynamic heating rates may collectively be severe enough to cause failure of the aerospike[10]. Gopala Krishnan et al. [11] and Sreekanth et al. [12] are among the researchers who studied various configurations of aerospikes numerically. Various experimental analyses were done in literature which is similar to the present study[13-15]. Two of them are explained in detail below, and one of them is validated in section 2.

Research by Gaetano M. D. Currao et al.[16] discussed the fluid structure interaction of a cantilevered plate exposed to Mach 5.8 air flow. Analytical and FEM numerical estimation of plate oscillation and plastic deformation was done. The plate displacement as well as flow features like shock location and boundary layer were measured using high-speed Schlieren video, while fast response pressure-sensitive paint was used to study the pressure distribution history on the surface of the plate. Experimental and Numerical results showed reasonable agreement in predicting the plastic deformation of the plate, and suggestions for improving the experiment were discussed.

Another paper by Ali Arman[17] is a wind-tunnel study of an unsteady flow of Mach number of 7 in Helium. Several sting-mounted wedges, double-wedge and flat-plate air-foil models with three different leading-edge radii were subjected to this flow. The data was obtained by taking high-speed schlieren motion pictures of the decaying motion of the model as it was released from an initial deflection. Nagaraja S R et al [18, 19] have studied shock structure interactions used for forming of thin metallic plates.

The literature survey has helped the authors in understanding the scope of structural analysis of cantilevers exposed to hypersonic flows, and has given a proper direction in which to proceed. A parametric study of hypersonic flow over inclined cantilever plates is done with an intention to extend the study to model Fluid-Structure Interaction (FSI) in hypersonic re-entry vehicles. Chapter 2 describes the details of numerical modelling and validation, and the results obtained using these models is described in Chapter 3. A conclusion and scope for further studies are given in Chapter 4.

2. Modelling and validation

The vibration and deflection of an inclined cantilever plate is carried out using the Fluent and Structural modules of ANSYS. A 2-D analysis is done in Fluent to get the flow field and pressure distribution around the plate. This pressure distribution is assumed to be constant in the perpendicular (z-) direction. For the Fluent analysis, the domain and the mesh is as shown in Figure 1.

![Figure 1: Meshed domain.](image)

The left (inlet), right (outlet) and top sides of the domain are modelled as pressure far-field and all other edges are treated as no-slip wall. A density based solver is used to simulate the flow of air, which is considered as an ideal gas ($C_p = 1006.43\ J/kg-K$, Thermal conductivity = 0.0242w/m-K and Sutherland’s viscosity model). A 3-D model of the plate with a mesh that is consistent with edge
sizing used in Fluent is generated using the Structural module. Structural steel (density = 7850 Kg/m3, yield strength = 2.5E+08 Pa, Young’s modulus = 2E+11 Pa, Poisson’s ratio = 0.3 and Ultimate strength = 4.6E+08 Pa) was taken as the material for the plate, and Bilinear Kinematic hardening condition (with Young Modulus (E) = 200GPa, Tangent Modulus (E_t) = 10GPa and Yield stress (σ_y) = 205MPa) was used to account plastic deformation. While a steady state analysis is carried out in Fluent, a transient analysis is done in the Structural module with end time of 0.1 seconds, and sub-step size of 1 ms. A major assumption used here is that the effect of vibration and deflection of the plate on the flow field is negligible. Thus, there is a one-way coupling between the Fluent and structural modules, wherein the flow field is causing the deflection, but the deflection is not affecting the flow field.

Validation of the numerical model was carried out with a paper by Gaetano M. D. Currao et al. [16] in which experimental and numerical study was carried out on a 1mm thick, 20° inclination cantilever exposed to Mach 5.8 air flow with free-stream conditions of P_∞ = 680 Pa, T_∞ = 73 K. The experimental model used was a 1 mm thick and 230 mm long steel plate that was mounted on a rigid 20° wedge, and the numerical analysis was carried out considering only the cantilevered part of the structure with transient fluid and structural analysis with two way coupling (Figure 2).

![3D model](image)

**Figure 2:** 3D model used for validation (from drawings).

The pressure acting on the sides of the plate obtained from 2-D FLUENT analysis was used as the input to the 3-D transient structural analysis, assuming uniform pressure distribution in the z-direction. The deflection of the first cycle compared with the results from the paper, as shown in Figure 3 shows good agreement, thus validating the modelling methodology undertaken.

![Deflection comparison](image)

**Figure 3:** Comparison of deflection for the first cycle.

The same approach is followed in the subsequent section and the structural behaviour of the inclined cantilever plate is studied under different Mach numbers, plate thicknesses and inclinations.
3. Results and analysis

Our study is aimed at analysing the variation in deflection with Mach number, plate thickness and plate inclination. Plates of widths 5 mm, 10 mm and 15 mm, inclined at 20° and 30°, for free stream Mach numbers from 3 to 8 are considered. For each case, the deflection is found to oscillate initially and then reach a steady value. The results will be presented in three sections; each section depicting the variation with respect to one of the mentioned parameters, while the others remain constant.

3.1. Mach number

To study the effect of Mach number, a 5 mm thick plate inclined at 20° is considered. It can be seen from Figure 4 that the amplitude of vibration and the final steady state value of deflection are increasing with Mach number, with Mach 8 having the largest amplitude and final deflection.

![Figure 4: Deflection vs time for 5mm 20deg plate.](image)

This behaviour can be understood from Figure 5, which shows the pressure distribution along the sides of the plate. As the Mach number increases, the strength of the shock wave formed increases, thus the pressure difference experienced between the sides (left and right) of the plate becomes higher. The location of peak pressure on the left side of the plate shifts towards the top (away from the fixed...
region) thus increasing the bending moment experienced. The increase in the value of pressure, and the shift in the location of peak pressure contribute to larger deflections as the Mach number increases. Similar variation is observed with Mach number, for other values of thickness and inclination.

3.2. Inclination

For the same thickness, same length of the plate (100mm) and same Mach number, it is also observed that the deflection of the plate with 30° inclination is more than that with 20° inclination (Figure 6). A 5 mm thick plate exposed to Mach 8 is chosen for this analysis. For the same Mach number, the shock strength (pressure ratio) increases with the inclination. Thus, a larger pressure is experienced by the plate inclined at 30° (Figure 7). For other values of Mach number and thickness, the magnitude of deflection and amplitude varies, but the trend remains the same, values being higher for the plate with the larger inclination.

![Figure 6: Deflection of the plate vs time.](image)

![Figure 7: Pressure vs distance.](image)

3.3. Thickness

For the same angle of inclination and same Mach number, it is observed that the deflection of the plate increases with decrease in thickness (Figure 8). Plates of varying thickness, inclined at 30° and exposed to Mach 8 air flow are considered. Same Mach number and inclination result in almost uniform pressure distribution on the left side of the plate (Figure 9). As the thickness increases, the stiffness and thus the resistance to deformation increases, thus resulting in lower amplitudes of vibration and magnitudes of deflection.

![Figure 8: Deflection for varying thickness.](image)

![Figure 9: Pressure acting for varying thickness.](image)
4. Conclusion

A one way FSI was carried on a cantilevered plate with varying thickness of 5mm, 10mm and 15mm and at angles of attack of 20 and 30 degrees. The pressure profile and deflection were analysed for Mach numbers varying from 3 to 8.

Constant free-stream conditions of $P_\infty = 680$ Pa, $T_\infty = 73$ K, were taken for the steady state Fluent analysis. The pressure acting on the sides of the plate was used as the input to the 3-D transient structural analysis, assuming uniform pressure distribution in the z-direction. The transient deflection was observed and reported for the cases mentioned. It was found that the magnitude of deflection oscillated during the initial stages and eventually reached a steady value.

The maximum steady state deflection and amplitude of the vibration of the plate is found to be increasing with increase in Mach number, owing to larger values of pressure on the left side of the plate, and the shift in location of the peak pressure away from the fixed point. A reduction in deflection as the plate thickness increases, because of increased stiffness, is also observed. It is also observed that the deflection of the plate with 30° inclination is more than that with 20° inclination which can be explained by the increased shock strength and pressure ratio at higher inclination. However, the angles chosen were less than the limiting value for the Mach numbers used, and thus, only attached oblique shocks are considered in the simulations. For higher angles, detached and curved shock waves will be formed, and the deformation of the plate under these conditions should be studied. A two-way coupling between the Fluent and Structural modules can be considered for analysis and the results should be compared with the current results in order to arrive at an effective analysis procedure, before applying it to actual re-entry applications.

5. References

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