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Stability analysis and nonlinear current-limiting control design for DC micro-grids with CPLs

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Abstract: In this paper, a DC micro-grid architecture consisting of multiple paralleled energy resources interfaced by both bidirectional AC/DC and DC/DC boost converters and loaded by a constant power load (CPL) is investigated. By considering the generic dq transformation of the AC/DC converters’ dynamics and the accurate nonlinear model of the DC/DC converters, two novel control schemes are presented for each converter-interfaced unit to guarantee load voltage regulation, power sharing and closed-loop system stability. This novel control framework incorporates the widely adopted droop control expressions and using input-to-state stability theory, it is proven that each converter unit guarantees a desired current limitation during transients without the need for cascaded control and saturation blocks. Since CPLs are known to result in DC micro-grid instability due to their negative impedance behaviour, sufficient conditions to ensure closed-loop system stability with the proposed current-limiting control framework are analytically obtained and tested for different operation scenarios. The system stability is further analyzed from a graphical perspective, providing valuable insights of the CPLs influence onto the system performance and stability. The proposed control performance and the theoretical stability analysis are first validated by simulating a three-phase AC/DC converter in parallel with a bidirectional DC/DC boost converter feeding a common CPL in comparison with the traditional cascaded PI control technique. Finally, experimental results are also provided to further demonstrate the effectiveness of the proposed control approach on a real testbed.

1 Introduction

Driven by the energy crisis, environmental pollution and greenhouse gas emissions [1–3], the seamless integration of renewable energy sources (RES) has been actively pursued worldwide, over the past decades. With the uninterrupted growth of RES, the smart grid and micro-grid concepts have been proposed as a benchmark of the future grid to enable efficient utilization of renewable resources and distributed generations. The centrepiece of these frameworks is represented by the power converters [4] which are the interface devices of RES to the micro-grid system.

In DC micro-grids, distributed generation (DG) units are connected to a common DC bus through AC/DC and/or DC/DC converters, often operating in parallel leading to a series of non-trivial issues such as voltage regulation and accurate nonlinear model of the load power. A widely used technique to accomplish these tasks, implemented in a fully decentralized way, that does not require communication between each DG, is to introduce a virtual resistance at the output of each converter, a method also referred to as ‘droop control’ [5–9]. The main disadvantages of the conventional droop control consist of significant load voltage drop and inaccurate power sharing due to mismatches at the line impedances. Therefore, several methods have been proposed to tackle and improve its existing performance, such as the robust droop control [10, 11] where the line impedances are not considered, the nonlinear droop control [12] where each DG unit is optimized against hypothetical DGs, or the quadratic droop control [13], implemented as a special case of the general feedback controller. However, in the majority of these works, the stability of the parallel operated power converters has been insufficiently addressed mainly due to the complexity of the dynamics that increases with the nonlinear characteristics of the AC/DC and DC/DC converters and their nonlinear loads. Power converters fed by the main bus create unique dynamic characteristics and have been a research subject for years. As shown in [14, 15], under tight-speed regulation, the motor drive exhibits constant power behaviour at the DC bus, similar to tight regulated downstream converters [16–18].

The dynamic behaviour of constant power loads is equivalent to a dynamic negative impedance which can produce instability at the DC bus and, consequently, in the system [16]. Limitations of practical CPLs in real-world applications have been assessed in [17], and there is an increased interest in designing droop controllers that guarantee closed-loop system stability for DC micro-grids loaded by CPLs [19, 20].

The existing stability methods for investigating DC micro-grids are based on the small-signal model of the power devices and linear approximation approaches, mostly employing the Middlebrook and Cuk criterion [21]. Whilst small-signal modeling is useful to obtain the system’s open-loop gain by considering only the input impedance of the loads and output impedance of the sources [22, 23], the nonlinear dynamics of the power converters are not taken into account. Stability of reduced-order models has been investigated in [24, 25] and stable operating regions have been obtained, but they ignore the dynamic performances of the DC-DC converters. Global stability results can be obtained using nonlinear control techniques, such as passivity-based control (PBC) methods, which have been successfully applied to power converter systems applications [26, 27]. However, these control schemes require the knowledge of the system and load parameters, which may not be available in practice. To overcome this issue, advanced control techniques such as adaptive PBC [28] or the interconnection and damping assignment PBC (IDA-PBC) [29] have been designed. Particularly, the IDA-PBC guarantees closed-loop stability with enhanced system robustness as it is parameter free. However, its main shortcoming is that it needs the solution of a partial differential equation (PDE) system of order equal to the system order. Thus, in a DC micro-grid application with multiple DC/DC and AC/DC converters, the PDE solution cannot be analytically obtained.

Apart from achieving stability in the micro-grid, other control issues that relate to the technical requirements of each DG unit should be taken into account in the control design such as the capability of the power converters to be protected at all times, particularly
during transients, faults and unrealistic power demands. The overcurrent protection as presented in [30, 31], guarantees the converter operation and protection of the equipment without violating its technical limitations. Existing strategies are based on protection units such as using additional fuses, circuit breakers or relays [32–34], however it still represents a challenge to design control methods that ensure an inherent current-limiting property [35–37]. Although current-limiting control methods based on saturated PI controllers are often used to guarantee a given upper limit for the current, the shortcomings of these methods have not been completely overcome, e.g.: i) only the reference value of the converter’s input current is limited, i.e. overcurrent protection is not achieved during transients as shown in [31] and ii) closed loop stability cannot be analytically guaranteed since the controller can suffer from integrator windup problems that could potentially yield instability in the system [38].

For this reason, in this paper, two novel nonlinear droop control strategies are proposed for parallel operated bidirectional three-phase AC/DC and DC/DC boost converters feeding a CPL in a DC micro-grid architecture in order to ensure accurate distribution of the load power among the paralleled units in proportion to their power ratings and inherent overcurrent protection. Based on the nonlinear dynamics of the converters and using input-to-state stability (ISS) theory, it is proven that the proposed controllers guarantee an inherent current-limiting property for each converter independently from each other or the load. In addition, accurate power sharing and load voltage regulation close to the rated value are accomplished and the stability of the closed-loop system is proven when connected to a CPL using singular perturbation theory. The effectiveness of the proposed controllers and the stability conditions are verified through simulation testing and they are compared to the cascaded PI technique to highlight its superiority.

One distinctive fact is that compared to the cascaded PI approach or when a linear resistive load has been used [31, 39, 40], in this paper a new control structure is proposed that does not require the measurement of the converter output currents and additionally guarantees closed-loop system stability with a CPL. Moreover, in contrast to the control methods and stability analysis of the DC micro-grid presented in [20], the proposed approach not only guarantees stability but also has a better performance in achieving its control tasks whilst ensuring overcurrent protection at all times. The novel contributions of the proposed work are highlighted by the following aspects:

i) the parallel operation of both bidirectional three-phase AC/DC and DC/DC boost converters is investigated here, which are inherently nonlinear systems, opposed to only unidirectional boost converter [31], or only buck converters, as studied in [20] which have linear dynamics;

ii) compared to [20], a new droop control structure that achieves improved power sharing and output voltage regulation closer to the rated value is proposed and analyzed;

iii) an inherent current limitation is introduced via the proposed control design for all power converters;

iv) in contrast to [40] where a linear resistive load was considered, in this paper closed-loop stability is analytically guaranteed for the CPL case.

Therefore, proving closed-loop system stability of a DC micro-grid with a CPL using the nonlinear model of the bidirectional three-phase AC/DC and DC/DC boost converters investigated here, which are inherently nonlinear systems, opposed to only unidirectional boost converter [31], or only buck converters, as studied in [20] which have linear dynamics; is the subject of this paper.

The structure of this paper is divided as follows. In Section 2 the nonlinear model of a DC micro-grid consisting of multiple paralleled bidirectional three-phase AC/DC and DC/DC boost converters is presented. The control framework of the current-limiting droop controller is explained and analyzed in Section 3. In Section 4, the closed-loop system stability analysis is presented and then analyzed from a graphical perspective in Section 5. In Section 6, simulation results are displayed to test the controller performance, which is further validated in Section 7 on a real experimental testbed. Finally, in Section 8 some conclusions are drawn.

2 Nonlinear model of the DC micro-grid

2.1 Notation

Let \( x \in \mathbb{R}^{n \times n} \) be defined as the diagonal matrix whose diagonal entries are the elements of the n-dimensional vector \( x = [x_1, \ldots, x_n] \). Let \( O_n \in \mathbb{R}^{n} \) and \( O_{n \times n} \in \mathbb{R}^{n \times n} \) be the n-dimensional vector and \( n \times n \) square matrix, respectively, with all elements zero. \( I_n \) be the identity matrix and let \( I_n \in \mathbb{R}^{n} \) and \( I_{n \times n} \in \mathbb{R}^{n \times n} \) be the n-dimensional vector and \( n \times n \) square matrix, respectively, with all elements equal to one.

2.2 Dynamic model

A typical topology of a DC micro-grid is shown in Fig.1 consisting of several types of energy sources, power converters and loads connected to a common bus. The configuration of the DC micro-grid under investigation is shown in Fig.2, containing \( n \) bidirectional three-phase rectifiers and \( m \) bidirectional DC/DC boost converters feeding a constant power load, where \( L_{si} \) is the inductor at the input, a DC output capacitor \( C_i \) with a line resistance \( R_i \) and six controllable switching elements that operate using PWM and capable of conducting current and power in both directions. The input voltages and currents of the rectifier are expressed as \( u_{q1}, u_{q2}, u_{q3} \) and \( i_{s1}, i_{s2}, i_{s3} \), while output dc voltage is denoted as \( V_i \) with \( i \in \{1, 2, \ldots, n\} \). The bidirectional DC/DC converters have two switching elements, an inductor \( L_j \) at the input and a capacitor \( C_j \) with a line resistance \( R_j \) at the output, while \( V_j \) is the output voltage, where \( j \in \{n + 1, n + 2, \ldots, n + m\} \). At the input, the voltage and the current of the converter are represented as \( U_{j1} \) and \( i_{j1} \), respectively, with the latter being either positive or negative to allow a bidirectional power-flow.

To obtain the dynamic model of the rectifier, the average system analysis and the \( dq \) transformation can be used for three-phase voltages and currents, using Clarke and Park transformations [39]. Following [41], the mathematical model of the rectifiers in the \( dq \) coordinates is set up, in matrix form as

\[
L_s I_d = -\omega L_s I_q - \frac{1}{2} m_d V_r + U_d
\]

\[
L_s I_q = \omega L_s I_d - \frac{1}{2} m_q V_r
\]

\[
C V_r = \frac{3}{4} m_d I_d + \frac{3}{4} m_q I_q - i_r
\]

where \( \omega = [i_1, \ldots, i_n]^T \), \( V_r = [V_{1}, \ldots, V_{n}]^T \), \( L_s = \text{diag}(L_{si}) \), \( C_r = \text{diag}(C_i) \), \( \omega = \text{diag}(\omega_i) \) is the rotating speed, \( U_d = [U_{d1} \ldots U_{dn}]^T \) is the amplitude of the three-phase AC voltage source when voltage orientation on the \( d \) axis is considered and \( I_d = [i_{d1} \ldots i_{dn}]^T \), \( I_q = [i_{q1} \ldots i_{qn}]^T \) are the \( d \) and \( q \) components of the AC source currents, respectively, and \( m_d = \text{diag}(m_{di}) \), \( m_q = \text{diag}(m_{qi}) \) are the duty-ratio control inputs of the rectifier with
Fig. 2: Parallel operated three-phase AC/DC and bidirectional DC/DC boost converters feeding a common constant power load

\[ V_d \text{ and } V_b \text{ being the } d \text{ and } q \text{ components of the rectifier voltage } v = [v_d, v_q] \text{, respectively.} \]

Using Kirchhoff laws and average analysis \[ [42] \text{, the dynamic model, in matrix form, of the bidirectional DC/DC boost converter becomes} \]

\[ L_i \frac{dv}{dt} = U_{in} - (I_m - u) V_b \]

\[ C \frac{dV_b}{dt} = (I_m - u) V_L - i_b \]

where \[ i_L = [i_{L(n+1)} \ldots i_{L(n+m)}]^T \text{, } V_b = [V_{n+1} \ldots V_{n+m}]^T \text{, } i_b = [i_{n+1} \ldots i_{n+m}]^T \text{, and } U_{in} = [U_{n+1} \ldots U_{n+m}]^T \text{, } L = diag \{L_j\} \text{, } C = diag \{C_j\} \text{, u = diag \{u_j\}.} \]

One can observe that system (1)-(3), (4)-(5) is nonlinear, since the control inputs \( m_{di}, m_{qi} \) and \( u_j \) are multiplied with the system states, \( (I_d, I_q, V_s) \), and \( (i_L, V_b) \) respectively.

As the AC/DC and DC/DC converters supply a CPL, the power balance equation becomes

\[ P = V_k \sum_{k=1}^{n+m} i_k \]

\[ i_k = \frac{V_k - V_L}{R_k} \]

where \( V_k, i_k \) represent the output voltages and currents, respectively, with \( k \in [1, 2, \ldots, n+m] \), \( V_L \text{ is the load voltage, and } P \text{ is constant and represents the power of the CPL.} \text{ Consider now the following assumptions:} \]

**Assumption 1** It holds that

\[ \left( \sum_{k=1}^{n+m} \frac{V_k}{R_k} \right)^2 > 4P \sum_{k=1}^{n+m} \frac{1}{R_k} . \]

Thus, substituting the output current \( i_k \) from (7) into (6), one can obtain the following expression for the load voltage given by the real solutions of the second order polynomial

\[ V_L = \sqrt{\left( \sum_{k=1}^{n+m} \frac{V_k}{R_k} \right)^2 - 4P \sum_{k=1}^{n+m} \frac{1}{R_k} } \]

**Assumption 2** Let \( I_{k}^{\text{max}} = \{I_{1}^{\text{max}}, \ldots, I_{n+m}^{\text{max}}\} \) be the maximum current of each converter (maximum RMS current for AC/DC converters and maximum inductor current for DC/DC converters). Since for three-phase rectifiers \( V_i \geq 2U_{di} \) and for boost converters \( V_j \geq U_{ij} \), let

\[ \min(2U_{di}, U_{ij}) - I_{k}^{\text{max}} R_k \geq 0 \]

\[ \sum_{k=1}^{n+m} \frac{V_k}{R_k^2} \geq \sqrt{\sum_{k=1}^{n+m} \frac{V_k}{R_k^2}^2 - 4P \sum_{k=1}^{n+m} \frac{1}{R_k} } \]

\[ \frac{1}{2} \sum_{k=1}^{n+m} \frac{V_k}{R_k^2} \]

\[ \min(2U_{di}, U_{ij}) - I_{k}^{\text{max}} R_k \geq 0 \]

\[ \sum_{k=1}^{n+m} \frac{V_k}{R_k^2} \geq \sqrt{\sum_{k=1}^{n+m} \frac{V_k}{R_k^2}^2 - 4P \sum_{k=1}^{n+m} \frac{1}{R_k} } \]

\[ \frac{1}{2} \sum_{k=1}^{n+m} \frac{V_k}{R_k^2} \]

\[ \lambda_{Dk} = \frac{1}{2} \sum_{k=1}^{n+m} \frac{V_k}{R_k^2} \geq \sqrt{\sum_{k=1}^{n+m} \frac{V_k}{R_k^2}^2 - 4P \sum_{k=1}^{n+m} \frac{1}{R_k} } \]

\[ \forall k = 1, \ldots, n+m. \]
3 Nonlinear control design and analysis

3.1 The proposed controller

The purpose of the designed controller is to achieve accurate distribution of the load power and tight load voltage regulation close to the rated value, ensuring that the current of each converter does not violate certain bounds. The proposed concept is based on the idea of partially decoupling the inductor current dynamics, introducing a constant virtual resistance with a bounded controllable voltage for both the bidirectional three-phase AC/DC and the DC/DC boost converters. In both cases, the dynamics of the controllable virtual voltage will guarantee the desired upper bound for the converters’ currents regardless of the direction of the power flow.

3.1.1 Three-phase rectifier: Although a current-limiting controller was recently proposed in [39], it only allows unidirectional power flow, which is a significant limitation when storage units are introduced or the AC/DC converter represents an interface between a DC and an AC micro-grid. To overcome this problem, here the control inputs \( m_{di} \) and \( m_{qi} \), with \( i \in \{1, 2, \ldots, n\} \) are proposed to take the following form

\[
m_{di} = \frac{2}{V_i} (U_{di} - E_{di} - \omega_i L_s i_{di} + r_{vi} i_{di})
\]

\[
m_{qi} = \frac{2}{V_i} (\omega_i L_s i_{di} + r_{vi} i_{qi})
\]

where \( r_{vi} > 0 \) is a constant virtual resistance and \( E_{di} \) a virtual voltage change that charge according to the following nonlinear dynamics:

\[
\dot{E}_{di} = c_{di} \left( V^*-V_L - d_i \left( \frac{3}{2} U_{di} E_{di} - P_{seti} \right) \right) E_{dqi}^2
\]

\[
\dot{E}_{dqi} = -c_{dqi} \left( V^*-V_L - d_i \left( \frac{3}{2} U_{di} E_{di} - P_{seti} \right) \right) E_{dqi} E_{maxi}^2
\]

\[
- k_{dqi} c_{dqi} \left( \frac{E_{dqi}^2}{E_{maxi}^2} + E_{dqi}^2 - 1 \right) E_{dqi}
\]

with \( E_{dqi} \) representing an additional control state, \( V^* \) the load voltage reference, \( P_{seti} \) the set output power, \( d_i \) the droop coefficient, and \( c_{di}, E_{maxi}, k_i \) being positive constants. The proposed controller introduces the desired droop expression via the input \( m_{di} \), while it forces the current \( i_{di} \) to zero through \( m_{qi} \) in order to guarantee unity power factor operation, since \( Q_i = -\frac{3}{2} U_i i_{di} \).

3.1.2 Bidirectional DC/DC boost converter: Following a similar control framework with the AC/DC converter, for the DC/DC boost converter the control input \( u_j \), with \( j \in \{n+1, \ldots, n+m\} \), becomes

\[
u_j = 1 - \frac{r_{vbj} i_{Lj}}{V_j} + U_j - E_j
\]

where \( r_{vbj} > 0 \) represents a constant virtual resistance and \( E_j \) a virtual controllable voltage

\[
\dot{E}_j = c_j \left( V^*-V_L - d_j \left( \frac{U_j E_j}{r_{vbj}} - P_{setj} \right) \right) E_{bqj}^2
\]

\[
\dot{E}_{bqj} = -c_j \left( V^*-V_L - d_j \left( \frac{U_j E_j}{r_{vbj}} - P_{setj} \right) \right) E_{bqj} E_{maxj}^2
\]

\[
- k_{j} c_j \left( \frac{E_{bqj}^2}{E_{maxj}^2} + E_{bqj}^2 - 1 \right) E_{bqj}
\]

where \( E_{bqj} \) being an additional control state, \( P_{setj} \) the set output power, \( d_j \) the droop coefficient, and \( c_j, k_j, E_{maxj} \) positive constants. Compared to the robust droop controller [11], the proposed strategy does not require the measurement of the output current \( i_{si}, i_{sT} \) of each converter, thus leading to a simpler implementation. It is highlighted that a second controller state \( E_{dqi}, E_{bqj} \) is based on the bounded integral controller concept [43]. For more details on the bounded dynamics of the control states the reader is referred to [43] where it is shown that the control states are guaranteed to stay within their imposed bounds \( E_{di} \in [-E_{maxi}, E_{maxi}], E_{dqi} \in [-E_{maxi}, E_{maxi}] \) and \( E_{bqj}, E_{bqj} \in [0, 1] \) for all \( t \geq 0 \), given the set initial conditions \( E_{di} = E_{dqi} = 0 \) and \( E_{bqj} = E_{bqj} = 1 \). The block diagram depicting the controller implementation, measurement and actuation parts is presented in Figure 3. Having introduced the proposed control schemes, consider the additional assumptions for the system:

Assumption 3 For every constant \( E_{dqi} \in (-E_{maxi}, E_{maxi}) \) and \( E_{bqj} \in (-E_{maxi}, E_{maxi}) \), satisfying

\[
d_{di} = \frac{3}{2} \left( \frac{U_{di} E_{dqi}}{r_{dqi}} - P_{seti} \right) = \ldots = d_{bqj} \left( \frac{U_{bqj} E_{bqj}}{r_{bqj}} - P_{set(j+1)} \right)
\]

there exists a unique steady-state equilibrium point \( \{i_{di}, E_{di}^*, i_{deci}, V_{1e}, E_{dqi}, E_{maxi}, E_{bqj}^*, E_{bqj} \} \) corresponding to a load voltage regulation, \( V_{1e} \), where the load current \( \sum_{k=1}^{n} V_{ke} \in [0, 1], \forall k = 1, \ldots, n+m \).

Assumption 4 For \( \forall k = 1, \ldots, n+m \) it holds that \( \frac{U_{kan} E_{kan}}{r_{kan}} > \alpha_k \) when \( k = 1, \ldots, n \) and \( \alpha_k = 1 \) when \( k = n+1, \ldots, n+m \).

For the selection of \( E_{maxi} \) and \( E_{maxj} \) the following condition should hold

\[
|E_{maxi}| < U_k, \forall k = 1, \ldots, n+m.
\]

The desired current-limiting property for each converter can be now investigated in the next subsection.

---

**Fig. 3**: Block diagram with the control implementation of the proposed controllers
3.2 Current limitation

3.2.1 Three-phase rectifier: For system (1)-(2), consider the following continuously differentiable function

\[ V_i = \frac{1}{2} L_{qi} I_{di}^2 + \frac{1}{2} L_{di} I_{qi}^2. \]  

(18)

Substituting \( m_d, m_q \) from (11)-(12) into (1)-(2), and taking into account that \( E_{di} \in [-E_{maxi}, E_{maxi}] \) and \( E_{dq} \in [0, 1] \), the time derivative of \( V_i \) becomes

\[ \dot{V}_i = L_{si} I_{di} \dot{I}_{di} + L_{si} I_{qi} \dot{I}_{qi} = -v_r I_{di}^2 + E_{di} \dot{I}_{di} - v_r I_{qi}^2 \]

\[ \leq -\varepsilon v_r \left( I_{di}^2 + I_{qi}^2 \right) + |E_{di}| |I_{di}| \]

\[ \leq -\varepsilon v_r \|I\|^2_{2} + |E_{di}| \|I\|_{2} \]

where \( I = [I_{di} I_{qi}]^{T} \). Consider now that \( v_{ri} = \dot{v}_{ri} + \varepsilon > 0 \) for an arbitrarily small \( \varepsilon \). Then

\[ V_i \leq -\varepsilon \|I\|^2_{2}, \|I\|^2_{2} \geq \frac{|E_{di}|}{\varepsilon v_r} \]  

(19)

which means that system (1)-(2) is input-to-state stable (ISS) [44] with respect to the virtual voltage \( E_{di} \). Since \( E_{di} \) is bounded below the chosen maximum virtual voltage value \( E_{maxi} \), then both the \( d \) and \( q \) currents, \( I_d \) and \( I_q \) will remain bounded at all times.

Since \( I = [I_{di} I_{qi}]^{T} \) then taking into account the \( dq \) transformation, it results in

\[ \|I\|^2_{2} = \sqrt{I_{di}^2 + I_{qi}^2} = \sqrt{(\sqrt{2}I_{rms})^2} = \sqrt{2}I_{rms} \]  

(20)

For

\[ I_{rms} = \frac{E_{maxi}}{v_r} \]  

(21)

it is proven from the ISS property (19) that if initially the RMS AC/DC converter current is below the maximum allowed value \( I_{rms} \), i.e., \( I_{rms}(0) < I_{rms} \), then

\[ I_{rms}(t) \leq \frac{E_{maxi}}{v_r} = I_{rms}^\max, \forall t \geq 0. \]

Hence, the input current of each rectifier separately is always limited below \( I_{rms}^\max \) with the appropriate choice of \( E_{maxi} \) and \( v_r \) given in (21), ensuring power protection at all times. It is shown that the current-limiting property of each converter is guaranteed independently from the power sharing expression \( \hat{v}^* - \hat{v}_L = \frac{1}{2} \left( \frac{L_{qi}}{v_r} - \frac{L_{di}}{v_r} \right) \) that has to be regulated to zero. This means that each converter has as first priority to protect itself from high currents that can damage the device. When the current is below the maximum value, the converter contributes to the desired power sharing within the DC micro-grid.

3.2.2 Bidirectional boost converter: By applying the proposed controller expression (15) into the bidirectional converters dynamics (4), the closed-loop system equation for the inductor current \( i_{L} \) takes the following form

\[ L_{i} \dot{i}_L = -v_{ob} i_{L} + E_b. \]  

(22)

where \( v_{ob} = \text{diag}[v_{ob_{d}}, v_{ob_{q}}] \) and \( E_b = [E_{b1} \ldots E_{br+m}]^{T} \). One can clearly see that \( v_{ob} \) represents a constant virtual resistance in series with the converter inductor \( L \).

To investigate how the selection of the virtual resistance and the bounded controller dynamics of \( E \) are related to the desired overcurrent protection, let the following continuously differentiable function

\[ V_j = \frac{1}{2} L_{j} i_{L}^2 \]  

for closed-loop current dynamics (22). The time derivative of \( V_j \) yields

\[ V_j = L_{j} \dot{i}_{L} i_{L} = -r_{ob} \frac{i_{L}^2}{L_{j}} + E_{j} i_{L} \]

\[ \leq -r_{ob} \frac{i_{L}^2}{L_{j}} + |E_{j}| \|i_{L}\| \]

(23)

Considering \( r_{ob} = \dot{r}_{ob} + \varepsilon > 0 \) for arbitrarily small \( \varepsilon \), then

\[ V_j \leq -\varepsilon \frac{r_{ob} |i_{L}|^2}{v_{ob}}, \forall i_{L} \geq \frac{|E_{j}|}{r_{ob}}, \]  

(24)

which means that system (22) is input-to-state stable (ISS) with respect to the bounded virtual voltage, \( E_{j} \). Similar to the rectifier case, since \( |E_{j}| \in [-E_{maxj}, E_{maxj}] \) then

\[ i_{L} \leq \frac{E_{maxj}}{v_{ob}}, \forall t > 0, \]  

(25)

and guarantee that

\[ |i_{L}(t)| \leq \frac{E_{maxj}}{v_{ob}}, \forall t > 0. \]  

(26)

Any selection of the constant and positive parameters \( E_{maxj} \) and \( v_{ob} \) that satisfy (25) results in the desired overcurrent protection (26) of the converter’s inductor current regardless the load magnitude or system parameters.

It is underlined that compared to existing conventional overcurrent protection control strategies, here it has been mathematically proven according to the nonlinear ISS theory that the proposed controller maintains the current limited during transients and does not require limiters or saturation units which are prone to yield instability in the system. At the same time it maintains the continuous time structure of the closed-loop system that facilitates the stability analysis that follows.

4 Stability Analysis

By applying the proposed controller (11)-(14),(15)-(17) into the DC micro-grid dynamics (1)-(3),(4)-(5) the closed-loop system can be written in the following matrix form

\[
\begin{bmatrix}
\dot{i}_d \\
\dot{i}_q \\
\dot{i}_{ib} \\
\dot{V}_c \\
\dot{V}_b \\
\dot{E}_{dq} \\
\dot{E}_{duq} \\
\dot{E}_b \\
\dot{E}_{eq} \\
\dot{E}_{eqq}
\end{bmatrix} =
\begin{bmatrix}
\frac{L_{i}^{-1}}{L_{r}^{-1}}(-r_{ad} I_{d} + E_{ad}) \\
\frac{L_{i}^{-1}}{L_{r}^{-1}}(-r_{aq} I_{q} + E_{ad}) \\
\frac{L_{i}^{-1}}{L_{r}^{-1}}(-r_{ib} I_{b} + E_{ad}) \\
\frac{C_{i}^{-1}}{C_{r}^{-1}}[V_{r}^{-1} - (([U_{d}] - [E_{ad}] + r_{ad} I_{d}) I_{d} - r_{ad} I_{d}^2)] - C_{r}^{-1} \dot{v} \dot{r} \\
C_{i}^{-1} \dot{V}_b - C_{r}^{-1} \dot{v} \dot{r} - r_{ob} I_{b} [U_{b}] - [E_{b}] \dot{L}_C \\
-k_{d} [E_{eq}] - C_{r}^{-1} \dot{E}_{dq} - k_{b} [E_{eq}] - C_{r}^{-1} \dot{E}_{duq} \\
-k_{ad} [E_{eq}^2] - C_{r}^{-1} \dot{E}_{dq^2} - k_{b} [E_{eq}] - C_{r}^{-1} \dot{E}_{duq} \\
-c_{b} [E_{eq}] - C_{r}^{-1} \dot{E}_{eq} - (k_{d} [E_{eq}] + k_{b} [E_{eq}] - C_{r}^{-1} \dot{E}_{duq}) \\
-k_{ad} [E_{eq}^2] - C_{r}^{-1} \dot{E}_{dq^2} - k_{b} [E_{eq}] - C_{r}^{-1} \dot{E}_{duq}
\end{bmatrix}
\]  

(27)

where \( d_{d} = \text{diag} [d_{i}], \quad d_{b} = \text{diag} [d_{j}], \quad k_{d} = \text{diag} [k_{i}], \quad k_{b} = \text{diag} [k_{j}], \quad E_{d} = [E_{d1} \ldots E_{dn}]^{T}, \quad E_{eq} = [E_{eq1} \ldots E_{eqm}]^{T}. \)
\( \dot{E}_{eq} = [E_{eq}(i+1) \ldots E_{eq}(i+m)]^T, \quad \dot{v}_{eq} = diag \{ V_{eq} \}, \quad c_d = diag \{ c_d \}, \quad \phi = diag \{ c_i \}, \quad E_{eq} = diag \{ E_{eqmax} \}, \quad P_{eq} = \{ P_{set1}, \ldots, P_{setn} \}, \quad \phi_{eq} = \{ P_{set1}, \ldots, P_{setm} \}. \)

Consider an equilibrium point
\[
\begin{bmatrix}
\dot{E}_{eq}
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{n} \lambda_i \phi_{eq} \\phi_{eqmax} \\sum_{i=1}^{m} \lambda_i \phi_{eq} \\phi
\end{bmatrix} \begin{bmatrix}
\begin{bmatrix}
\dot{E}_{eq}
\end{bmatrix}
\end{bmatrix}.
\]

This equation can be simplified to
\[
\dot{E}_{eq} = \Phi \cdot \Phi_{eq} \cdot \Phi_{eqmax} \cdot \Phi_{eq},
\]
where \( \Phi \) is a diagonal matrix of eigenvalues and \( \Phi_{eq} \) is the equilibrium matrix. The characteristic polynomial can be calculated from
\[
\prod_{i=1}^{n} \left( \lambda_i - \frac{1}{\lambda_i} \right) E_{eq} = \left( \frac{1}{\lambda_i} - \frac{1}{\lambda_i} \right) E_{eq},
\]
where \( \lambda_i \) are the eigenvalues of \( \Phi \). The characteristic polynomial is given by
\[
\text{det}(\Phi - \lambda \Phi_{eq} \Phi_{eqmax} \Phi_{eq}) = 0.
\]

The roots of the above system can be computed as shown below
\[
E_{eq} = \begin{bmatrix}
\begin{bmatrix}
\dot{E}_{eq}
\end{bmatrix}
\end{bmatrix} = \begin{bmatrix}
\begin{bmatrix}
\dot{E}_{eq}
\end{bmatrix}
\end{bmatrix}.
\]

These roots can also be written as \( z = h(x) \) with \( \dot{E}_{eq} \in (E_{eqmax} \times E_{eq}), \quad \dot{F}_{eq} \in (E_{eqmax}), \quad \dot{v}_{eq} \in \Phi_{eq} \), and \( \dot{f}_{eq}, \quad \dot{b}_{eq} \in [0, 1] \), such that \( h(0) = 0 \). Thus, the roots also represent the equilibrium points of the nonlinear system. Exponential stability at the origin can be investigated via system's (31) corresponding Jacobian matrix:
\[
J_2 = \begin{bmatrix}
\begin{bmatrix}
\dot{E}_{eq}
\end{bmatrix}
\end{bmatrix} = \begin{bmatrix}
\begin{bmatrix}
\dot{E}_{eq}
\end{bmatrix}
\end{bmatrix}.
\]

where it is obvious that \( J_2 \) is negative definite as it is a linear system with a negative \( E_{eq} \).

The characteristic polynomial can be calculated as
\[
\prod_{i=1}^{n} \left( \lambda_i - \frac{1}{\lambda_i} \right) E_{eq} = \left( \frac{1}{\lambda_i} - \frac{1}{\lambda_i} \right) E_{eq},
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\prod_{i=1}^{n} \left( \lambda_i - \frac{1}{\lambda_i} \right) E_{eq} = \left( \frac{1}{\lambda_i} - \frac{1}{\lambda_i} \right) E_{eq},
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with \( \lambda_i \) as the eigenvalues of \( \Phi \). The characteristic polynomial is given by
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\begin{bmatrix}
\dot{E}_{eq}
\end{bmatrix}
\end{bmatrix} = \begin{bmatrix}
\begin{bmatrix}
\dot{E}_{eq}
\end{bmatrix}
\end{bmatrix}.
\]
Let the characteristic polynomial be
\[ |x^2D^{-1} + \lambda C + K||D| = 0, \]
Considering \( G = L^{-1}r_0Q_2 \), the characteristic polynomial becomes
\[ |G||x^2G^{-1}D^{-1} + \lambda C + K||D| = 0, \]
with \( \tilde{C} = G^{-1}C \) and \( \tilde{K} = G^{-1}K \). As the determinants \( |G| \) and \( |D| \) are positive, the polynomial reduces to
\[ |x^2G^{-1}D^{-1} + \lambda \tilde{C} + \tilde{K}| = 0, \]
which is a quadratic eigenvalue problem \((QE\tilde{P})\) with \( \tilde{K} \) symmetrical, and \( \tilde{C} \), according to Lemma 2 in [20], diagonalizable whose eigenvalues are all real, since it is a product of a positive-definite diagonal and a symmetrical matrix.

The characteristic equation then becomes
\[ |x^2G^{-1}D^{-1} + \lambda P\Lambda P^{-1} + \tilde{K}| = 0, \]
\[ |x^2P^{-1}G^{-1}D^{-1}P + \lambda \Lambda + P^{-1}K\Lambda P| = 0. \]

Note that \( \Lambda \) is a diagonal matrix with the eigenvalues of \( \tilde{C} \) as main entries and the similarity transformations \( P^{-1}G^{-1}D^{-1}P \) and \( P^{-1}\Lambda P \) are symmetrical, as \( P \) is orthogonal \((P^{-1} = P^T)\), and they share the same spectrum as \( G^{-1}D^{-1} \) and \( \tilde{K} \), respectively. If \( G^{-1}D^{-1} \), \( \Lambda \) and \( \tilde{K} \) are positive definite, then \( Re(\lambda) < 0 \) which means that \( J_2 \) is Hurwitz. Hence, since \( G^{-1}D^{-1} \) is already positive-definite, it is sufficient to show that \( \lambda > 0 \), or equivalently that \( \tilde{C} \) has positive eigenvalues, and \( \tilde{K} \) > 0. Since matrix \( \tilde{C} \) is represented by a multiplication where one term is the diagonal matrix \( G^{-1}Q_1 \) > 0, according to the same Lemma 2 in [20], the remaining symmetrical term, denoted \( \tilde{C}^* \), will have the same index of inertia as \( \tilde{C} \).

The condition \( \tilde{C}^* > 0 \) becomes
\[ \tilde{C}^* = Q_1^T(L^{-1}r_0D^{-1}B[i_{\text{vref}}]|V_e|^{-2}|U|D^{-1} + C^{-1}R^{-1}D^{-1}) - \tilde{I}_{\text{fr}}(\xi_{\text{fr}}) \]
which represents a sum between a diagonal positive-definite real matrix and the real symmetric matrix \(-\tilde{I}_{\text{fr}}(\xi_{\text{fr}})\). According to Lemma 1 in [20], if
\[
\left(\frac{r_0C_k}{L_k} + \frac{\alpha_k u_k}{V^2_k} + \frac{1}{R_k} \right) \lambda_{Dk} - \frac{\beta_k u_k}{V_k u_k d_k} + \frac{1}{R_k} (n+m) > 0, \quad (34)
\]
\( \forall k = 1 \ldots n+m \) holds, then \( \tilde{C}^* > 0 \) is satisfied. When \( k = 1 \ldots n \), then \( \alpha_k = 3 \) and \( \beta_k = 2 \), whereas when \( k = n+1 \ldots n+m \), then \( \alpha_k = \beta_k = 1 \). Regarding condition \( \tilde{K} > 0 \), taking into account Assumption 4, and according to the same Lemma 1 if
\[
\left(\frac{\alpha_k u_k}{V^2_k} + \frac{1}{R_k} \right) \lambda_{Dk} - \frac{\beta_k u_k}{V_k u_k d_k} (n+m) > 0, \quad (35)
\]
\( \forall k = 1 \ldots n+m \) holds, then \( \tilde{K} > 0 \) is satisfied. Hence, if the two conditions (34)-(35) are satisfied for each converter then there exist \( \rho_2 > 0 \) and a domain \( D_x \subseteq \{x \in \mathbb{R}^{2n}, \|x\| < \rho_2 \} \) where \( D_x \subseteq D_x \) such that the reduced model is exponentially stable at the origin.

According to Theorem 11.4 in [44], there exists \( \varepsilon^* > 0 \) such that for all \( \varepsilon < \varepsilon^* \) (or equivalently \( c_\text{g} > \frac{1}{2} \omega_1 + \delta_1 \) and \( c > \frac{1}{2} \omega_m + \delta \) ), the equilibrium point \( \{\bar{x}_d, \bar{V}_{\text{dc}}, \bar{V}_{\text{ac}}, \bar{V}_{\text{e}}, \bar{E}_{\text{dc}}, \bar{E}_{\text{e}}, \bar{E}_{\text{ac}}\} \) of (30)-(31) with \( E_{\text{dc}} \in (-E_{\text{max}}^d, E_{\text{max}}^d), E_{\text{e}} \in (-E_{\text{max}}^e, E_{\text{max}}^e) \) and \( E_{\text{ac}} \in (0, 1) \) is exponentially stable; thus completing the stability analysis of the entire DC micro-grid.

**5 Validation of closed-loop system stability**

In order to validate the theoretical stability analysis presented in Section 4 and demonstrate how conditions (34)-(35) can be tested, let us consider the system in Section 6 with parameters given in Table 1. Although (34)-(35) might seem difficult to verify, by taking into account that \( E_d \in [-E_{\text{max}}^d, E_{\text{max}}^d], E_e \in [-E_{\text{max}}^e, E_{\text{max}}^e] \) and \( E_{\text{ac}} \in [0, 1] \), which is guaranteed by the proposed control design, the procedure to verify whether the system is stable is the following: One can start by selecting a virtual voltage \( E_{\text{dc}} \) inside its defined range, for the rectifier. Then the values of the equilibrium points of the inductor current and load voltage are computed. Based on these obtained values, the remaining virtual voltages \( E_{\text{e}} \) of the DC/DC converter can be calculated. Thereafter, critical points of the
output voltages are calculated, followed by the eigenvalues of matrix \( D \). Finally, the two conditions can be tested for each converter.

Hence, following this procedure for different values of the set power, \( P_{set\,BAT} \), corresponding to the battery operation, charging and discharging, respectively, one can observe in Fig. 4 that for any \( E_d \) in the bounded range \((-E_d^{max}, E_d^{max}) = (-21, 21)\), the expressions (34)-(35) for each converter are positive, thus guaranteeing closed-loop stability.

To further validate the stability analysis, in Figure 5, a graphical interpretation of the stability conditions is provided for the entire range of the set power, \( P_{set\,BAT} \), to visually confirm that the two stability conditions always take positive values in the entire operating range of the particular DC micro-grid.

6 Simulation results

To test the proposed controller and compare it to the cascaded PI approach, a DC micro-grid consisting of a bidirectional boost converter and a three-phase rectifier feeding a CPL is considered having the parameters specified in Table 1. The aim is to achieve tight voltage regulation around the reference value \( V^* = 400\,V \), accurate power sharing in a 2 : 1 ratio among the paralleled AC/DC and DC/DC converters at the load bus while also assuring protection against overcurrents. But first the conditions for stability must hold.

The model has been implemented in Matlab Simulink and simulated for 45s considering a full testing scenario.

During the first 5s, the power requested by the load is 200W and it can be observed in Fig. 7b that the load voltage \( V_L \) is kept close to the reference value of 400V, at approximately 398V in both cases. But the power sharing is only accurately guaranteed (Fig. 7c) in a 2 : 1 manner with the proposed controller having \( i_{BAT} \approx 0.17\,A \) and \( i_{REC} \approx 0.34\,A \), unlike the case with cascaded PIs where \( i_{BAT} \approx 0.16\,A \) and \( i_{REC} \approx 0.35\,A \). The input currents haven’t reached their imposed limits yet as shown in Fig. 7a.

For the next 20s the operation principle of the battery is simulated. The direction of the power flow is reversed to allow the battery to charge and discharge. At \( t = 5s \) the power set by the battery controller becomes negative \( P_{set\,BAT} = -150W \), thus leaving the battery to be supplied by the three-phase rectifier. The input current of the battery becomes negative, while the rectifier’s input current increases to satisfy the new amount of power requested in the network (Fig. 7a). The power sharing ratio between the battery and the rectifier disappears since the current of the battery changes its direction, and becomes negative as shown in Fig. 7a. The load voltage remains closely regulated to the desired 400V value, at around 396.5V in both cases. After 10s the set value of the power returns to its initial 0 value, allowing the battery to return to its former discharging state. The power sharing ratio comes back to 2 : 1 as displayed in Fig. 7c.

At \( t = 25s \) the power requested by the load increases \( P = 400W \) and, thus, more power is needed from the battery and the three-phase rectifier to be injected in the micro-grid. The load voltage drops down to 396V according to Fig. 7b when using the proposed controller and \( V_{bat} = 395.5V \) when having cascaded PIs. At the same time, the input currents increase and, therefore, the power injected increases at the common bus (Fig. 7a). One can see that the sharing is kept between the two sources, the battery and rectifier, to the desired proportion of 2 : 1 having \( i_{BAT} \approx 0.34\,A \) and \( i_{REC} \approx 0.68\,A \) with the proposed controller, and \( i_{BAT} \approx 0.32\,A \) and \( i_{REC} \approx 0.68\,A \) with the cascaded PI approach.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| \( U_{MSS} \) | 110 V  | \( U_{bat} \) | 200 V  |
| \( R_{rec} \) | 0.7 Ω  | \( R_{bat} \) | 1.2 Ω  |
| \( L_{phases} \) | 2.2 mH | \( L_{bat} \) | 2.3 mH |
| \( C_{rec} \) | 1200 μF | \( C_{bat} \) | 2000 μF |
| \( \delta_{rec} \) | 0.015  | \( \delta_{bat} \) | 0.030  |
| \( P \) | 200 W  | \( k \) | 1000   |
| \( v_d \) | 2.1    | \( v_{bat} \) | 180    |
| \( r_v \) | 7 Ω    | \( r_{rec} \) | 5 Ω    |
| \( E_{ppm} \) | 21 V   | \( E_{ppm} \) | 5 V    |

Fig. 5: Graphical representation of the stability conditions (34)-(35)

Fig. 6: DC micro-grid considered for testing, containing a three-phase AC/DC converter connected to the grid, a bidirectional DC/DC boost converter interfacing a battery, and a CPL connected to the main bus and fed by the two converters.

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Fig. 7: Simulation results of the DC micro-grid system with PI cascaded control (left) and the proposed controller (right)

and $i_{REC} \approx 0.7A$ with the cascaded PI technique, as presented in Fig. 7c, given the fact that none of the inductor currents have reached their maximum allowed current. To test the input current protection capability, the power demanded by the load is further increased. Thus, at $t = 35s$ the power requested by the load reaches a higher value than before, $P = 640W$, forcing the battery and the three-phase rectifier to increase their power injection at the load bus. As noticed in Fig. 7a, the input current of the battery reaches its limit $i_{LBAT} = i_{LBAT_{max}} = 1.1A$ without violating it when using the proposed controller, but in the case of the cascaded PIs the transient current exceeds the upper limit prior reaching to steady-state. The power sharing is sacrificed (Fig. 7c) to ensure uninterruptible power supply to the load. The load voltage remains within the desired range, $V_L = 393.5V$ with a voltage drop of $6.5V$, which is about 1.5% when having the proposed controller and about $V_L = 392.5V$ with the cascaded PI approach.

Consequently, to further verify the theory presented, the controller states $E$, $E_d$ and $E_{dq}$, $E_{bq}$ are presented in Fig. 8a-8b. When the input current of the battery reaches its maximum, the virtual voltage of battery also arrives at its imposed limit $E_b = E_{b_{max}} = 5V$. One can notice in Fig. 8b that the corresponding control state $E_{bq}$ goes to zero when $E_b$ reaches maximum. It is noted that for the particular DC micro-grid scenario and the parameters used, the closed-loop performance with the cascaded PI control remains stable. However, this might not be true for a different system since there is no rigorous proof of stability. On the
other hand, the proposed control approach provides a strong theoretic framework, as proven in Section 4, that can be easily tested for different systems as well.

### 7 Experimental results

A DC micro-grid, with the parameters given in Table 2, consisting of two parallel Texas Instruments DC/DC boost converters connected to a common DC bus and feeding an ETPS ELP-3362F electronic load, operated in CPL mode, is experimentally tested. A switching frequency of $60\,kHz$ was used for the pulse-width-modulation of both converters. The aim is to experimentally validate the proposed nonlinear current-limiting control scheme. The main tasks are to regulate the output voltage to $V^* = 48\,V$ and regulate the power in a $2:1$ ratio, whilst ensuring overcurrent protection.

As one can see in Figure 9a, when the power changes from $40\,W$ to $60\,W$, the voltage is kept close to the reference value of $48\,V$, while the output currents are accurately shared proportionally to the sources rating, in a $2:1$ manner, having $i_2 \approx 0.45\,A$ and $i_1 \approx 0.9\,A$, provided the input currents, $i_{L1}$ and $i_{L2}$, have not reached their upper limit.

![Fig. 8: Dynamic response of the control states](image)

**Fig. 8:** Dynamic response of the control states

![Fig. 9: Experimental results under the proposed controller](image)

**Fig. 9:** Experimental results under the proposed controller

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| $U_1$      | 36 V   | $U_2$      | 24 V   |
| $R_1$      | 2.4 $\Omega$ | $R_2$      | 3 $\Omega$ |
| $L_{1,2}$  | 0.3 mH | $C_{1,2}$  | 300 $\mu F$ |
| $d_1$      | 0.2    | $d_2$      | 0.4    |
| $r_v,1,2$  | 20 V   | $k$        | 1000   |
| $c_1$      | 873    | $c_2$      | 655    |
| $E_{1,2}^{\max}$ | 30 V | $E_{2,2}^{\max}$ | 50 V |

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In Figure 9b, the load power demand decreases from 60W to 40W. The output current are accurately shared, having $i_{L1} \approx 0.6A$ and $i_{L2} \approx 0.3A$, and the load voltage is kept fixed at 48V.

To test the current-limiting capability, the power increases from 40W to 80W, as displayed in Figure 9c. One converter reaches to its imposed limit ($i_{L1} \approx 1.5A$), the power sharing is sacrificed to ensure the uninterrupted power supply of the load. The load voltage is still fairly close to the rated value of 48V. As it can be seen, the current limitation is not exactly at the 1.5A limit. This is due to the fact that the parasitic resistance, $r_{in}$, of the controller’s inductance is ignored, in the experimental and theoretical analysis, which in turn causes a slightly lower bound of the input current. If the parasitic resistance is considered, then based on the ISS analysis in Section 3, one can easily obtain that the controller parameters $E_{max}$ and $v_{th}$ should satisfy $i_{L1}^{max} = \frac{E_{max}}{v_{th}}$ in order to reach the upper limit of the converter. Nevertheless, it is clear that by ignoring this resistance, the current still remains below $i_{L1}^{max}$ as desired.

8 Conclusions

In this paper, a detailed control design was presented for multiple parallel operated three-phase AC/DC and bidirectional DC/DC boost converters in a DC micro-grid framework, loaded by a CPL. The nonlinear dynamic control scheme was developed to ensure load power sharing and output voltage regulation, with an inherent input current limitation. The stability of the entire DC micro-grid was analytically proven when the system supplies a CPL using singular perturbation theory. Introducing a constant virtual resistance with a bounded dynamic virtual voltage for the three-phase AC/DC and for the bidirectional DC/DC boost converter, it has been shown that the input currents of each converter will never violate a maximum given value. This feature is guaranteed without any knowledge of the system parameters and without any extra measures such as limiters or saturators, thus, addressing the issue of integrator wind-up and instability problems that can occur with the traditional overcurrent controllers’ design. The effectiveness of the proposed scheme and its overcurrent capability are verified by simulating a DC micro-grid considering different load power variations and battery operations (charging, discharging), and by experimentally testing a parallel converter micro-grid configuration feeding an electronic load, acting as a CPL.

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