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A Novel Riccati Equation Grey Model And Its Application In Forecasting Clean Energy

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Abstract

Objective: An accurate prediction of clean energy can supply an important reference for governments to formulate social and economic development policies. This paper begins with the logistic equation which is the whitening equation of the Verhulst model, introduces the Riccati equation with constant coefficients to optimize the whitening equation, and establishes a grey prediction model (CCRGM(1,1)) based on the Riccati equation. This model organically combines the characteristics of the grey model, and flexibly improves the modelling precision. Furthermore, the nonlinear term is optimized by the simulated annealing algorithm. To illustrate the validation of the new model, two kinds of clean energy consumption in the actual area are selected as the research objects. Compared with six other grey prediction models, CCRGM(1,1) model has the highest accuracy in simulation and prediction. Finally, this model is used to predict the nuclear and hydroelectricity energy consumption in North America from 2019 to 2028. The results predict that nuclear energy consumption will keep rising in the next decade, while hydroelectricity energy consumption will rise to a peak and subsequently fall back, which offers important information for the governments of North America to formulate energy measures.

1. Introduction

British Petroleum (BP) noted that in 2018, coal consumption increased by 1.4%, oil consumption increased by 1.5%, and natural gas consumption increased by 5.3%. Thus, the energy consumption structure dominated by fossil energy increased carbon emissions by 2.0%, the fastest growth rate in nearly seven years, which also complicates the global environment and climate governance situation. However, the good news is that the consumption of clean energy represented by nuclear energy (up 2.4%) and hydroelectricity (up 3.1%) continues to increase. Clean energy has played a huge role in global environmental governance by virtue of its health, safety and pollution-free characteristics [1], thus reducing the proportion of fossil energy in total energy consumption. Therefore, in recent years, with the global promotion of energy conservation and emission reduction, countries have increased their research into clean energy [2]. Clean energy has become an important reference factor for governments to formulate policies.

The International Energy Agency (IEA) states that with vigorous promotion of clean energy, the global energy structure is undergoing significant changes, and this topic has spurred researchers to focus on building accurate energy prediction models to supply effective information for global energy policy-making. The most common models include commonly used mathematical statistical models (such as the autoregressive distributed lag model, ARIMA, Markov model [3–5]) and intelligent computer technology model [6,7]. These models predict the energy data accurately. However, the main disadvantage of statistical models is that sufficient input data are generally needed for parameter estimation to achieve accurate prediction [8]. The main disadvantage of intelligent computer models is that they usually require a large amount of data for training, and too little data might lead to an inaccurate model. However, significant changes have taken place in the global energy structure, and historical data are no longer reliable for future energy prediction, which has led to a significant reduction in the amount of useful energy data. In addition, no matter how much data are used to build the model, the computational results are not the real data. If the model can be built with less data to obtain relatively effective results, then the model is considered attractive [9]. In fact, grey models can meet these requirements. Therefore, in recent years, researchers have turned to the grey prediction models with small samples (at least four independent data points are...
required [10]).

To address “small-sample and poor information” systems, Professor Deng proposed the grey system [11], which is based on information coverage, via sequence generation and the grey model to explore the real law of movement of things, with the feature of “less data modelling”. Now, the grey model is widely used in predictions related to energy, finance, transportation, environment, manufacturing and other industries [12–16]. In this process, the grey model has also been extended from the original GM(1, 1) to Verhulst model, DGM(1, 1), NGM(1, k, c), FDGM (1, n), NIPGM (1, t, s) [17–22], and combined models, such as GM-ARIMA, GM-MARKOV [23,24] and the combined models of grey model and intelligent computer method [25–27]. To improve the performance of the grey model, researchers have conducted much in-depth and systematic research on the grey prediction model from the perspective of the data accumulation mode, optimization of the background value, model property, modelling mechanism, etc. [28–32], which has advanced the development and refinement of the grey prediction models.

Many grey energy prediction models consider only a single variable and can be divided into three main types: the first is the grey basic model. Zeng et al. [33] proposed the UGM(1,1) model based on the unbiased grey model and a weakening buffer operator to predict the shale gas constant in China. Wang et al. [34] proposed the DGGM(1,1) model to predict China’s hydroelectricity based on a small sample and data grouping. Ding [35] proposed a new grey model to predict China’s total and industrial electricity consumption from 2015 to 2020. The second type is the grey combination model. Wang [36] proposed MNGM-ARIMA model by combining linear and nonlinear models to predict shale gas production in the United States every month. Li [37] proposed a grey model with the regression method, and proposed the GM-ARIMA model to predict annual new installed capacity of China’s coal-fired growth in 2017–2026, which will reach 740 GW. Wang et al. [38] used the MVO-MNGBM model to predict that natural gas consumption will reach 354.1 billion cubic meters by 2020 in different regions of China. The third type is the grey model optimized by an optimization algorithm. Ding et al. [39] proposed the adaptive grey system SIGM(1,1) to predict natural gas demand in China via the ant colony optimization algorithm. Wu et al. [10] proposed the FANGBM(1,1) model to predict the renewable energy in a short period by PSO algorithm. Ma et al. [40] proposed a fractional time-delay grey model to predict coal and natural gas consumption in Chongqing with the latest grey wolf optimizer.

The above models have achieved good results, but they ignore the characteristics of energy data. The trend of consumption for fossil energy such as coal and oil presents a disordered and unsaturated S-shaped, and clean energy is no exception. In grey theory, the grey Verhulst model is better for ordered S-shaped data or single-peak data, and the classical GM(1,1) or DGM (1,1) model is better for exponential growth data. For the disordered S-shaped trend of energy data, the performance of the Verhulst model is affected. Therefore, based on the data characteristics and the above literature analysis, the grey Verhulst model is optimized and extended by the Riccati equation, and the mathematical properties of the new model are analysed. The nonlinear term of the new model is optimized by the simulated annealing algorithm, and modelling is performed. The good performance of the new model is verified by the validation cases. Therefore, this model is applied to predict clean energy consumption in North America. The results of the validation and application cases show that the new model is superior to the GM(1,1), DGM(1,1), NGM(1,1), ENGM(1,1) [41], ARGM(1,1) [42] and Verhulst models. Finally, the clean energy consumption over the next 10 years is predicted, providing important information to the government for policy-making.

In the full text, the different abbreviations are for different grey prediction models. Abbreviations and their meanings are listed in Table 1.

The remainder of this paper is arranged as follows. The CCRGM(1,1) model is discussed in detail in Section 2. Section 3 studies the accuracy of the CCRGM(1,1) model in three cases. Application is presented in Section 4. Conclusions and future work are summarized in Section 5.

2. CCRGM(1,1) model

2.1. The basis of the verhulst model

Definition 2.1 Assume that \(X^{(0)}\) is a real data sequence, \(X^{(1)}\) is a 1-accumulating generation operator (AGO) sequence of \(X^{(0)}\), where \(X^{(1)}(k) = \sum_{j=0}^{k} X^{(0)}(j)\), and \(Z^{(1)}\) is the mean sequence generated by consecutive neighbors of \(X^{(1)}\), where

\[
Z^{(1)}(k) = \frac{1}{2} \left( X^{(1)}(k) + X^{(1)}(k-1) \right)
\]

The Verhulst model is

\[
x^{(0)}(k) + ax^{(1)}(k) = bx^{(2)}(k) \tag{1}
\]

Definition 2.2 The whitening equation of grey Verhulst model is

\[
dx^{(1)}(t) + ax^{(1)}(t) = b \left( x^{(1)}(t) \right)^2 \tag{2}
\]

Definition 2.3 (1) The solution of whitening equation is

\[
x^{(1)}(t) = \frac{1}{e^{at} \left[ ax^{(1)}(0) \right] - \frac{b}{a} \left( 1 - e^{-at} \right)} \left[ ax^{(1)}(0) \right]
\]

\[
= \frac{ax^{(1)}(0)}{bx^{(1)}(0) + (a - bx^{(1)}(0))e^{at}} \tag{3}
\]
The time response equation is

$$\dot{x}^{(1)}(k+1) = \frac{ax^{(1)}(0)}{bx^{(1)}(0)} + (a - bx^{(1)}(0))e^{\frac{a}{b}k}$$  \hspace{1cm} (4)$$

In definitions 2.1–2.3, the least square estimate of the series parameter $\tilde{P} = |a, b|$ is $\tilde{P} = (B^TB)^{-1}B^TY$, where

$$B = [-z^{(1)}(2)(z^{(1)}(2))^2 - z^{(1)}(3)(z^{(1)}(3))^2; \cdots; -z^{(1)}(n)(z^{(1)}(n))^2]. Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}$$

2.2. The basis of riccati equation

The general Riccati equation is

$$\frac{dy}{dx} = p(x)y^2 + q(x)y + r(x)$$  \hspace{1cm} (5)$$

where $p(x), q(x)$ and $r(x)$ are continuous on the interval $I$, and $p(x)$ is not always 0. Eq. [5] generally has no elementary product decomposition. When $p(x) = -b, q(x) = a$ and $r(x) = 0$, Eq. [5] is the classical logistic model, and it is expressed as follows:

$$\frac{dy}{dx} = (a - by)y$$  \hspace{1cm} (6)$$

Eq. (6) is the whitening equation of the Verhulst model.

The Verhulst model has certain limitations in dealing with unordered S-shaped or single-peak data. To make the model more applicable, optimization is necessary. Because the logistic model is a special case of the Riccati equation, $p(x) = b, q(x) = -a$ and $r(x) = c(k - 1)^r + d$ can be considered from Riccati equation, and the following equation can be achieved:

$$\frac{dy}{dx} + ay = by^2 + c(k - 1)^r + d$$  \hspace{1cm} (7)$$

2.3. Grey prediction model based on riccati equation

In this section, according to the whitening equation of the

Verhulst model and the relationship between the logistic equation and the Riccati equation, a new grey prediction model is established. The properties of this model are studied, and the time response function of this model is obtained.

Definition 2.4 Set $X^{(0)}, X^{(1)}$ and $Z^{(1)}$ as in definition 2.1. Thus,

$$x^{(0)}(k) + az^{(1)}(k) = bz^{(1)}(k)^2 + c(k - 1)^r + d$$  \hspace{1cm} (8)$$

is CCRGM(1,1). The following whitening equation can be obtained from Eq. (7):

$$\frac{dx^{(1)}}{dx} + ax^{(1)} = b(x^{(1)})^2 + c(k - 1)^r + d$$  \hspace{1cm} (9)$$

CCRGM(1,1) is the grey Verhulst model and its extended model, $c(k - 1)^r$ and $a$ are nonlinear correction and constant correction terms respectively. In Eq. (9), when $c = 0$ and $d = 0$, CCRGM(1,1) is the grey Verhulst model.

Definition 2.5 Set parameter list is $P = (a, b, c, d)^T$, and

$$B = \begin{pmatrix} -z^{(1)}(2)(z^{(1)}(2))^2 11 - z^{(1)}(3)(z^{(1)}(3))^2 41; \cdots; -z^{(1)}(n)(z^{(1)}(n))^2(n - 1)^r 1 \end{pmatrix}. Y = \begin{pmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{pmatrix}, P = \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

Then parameters $a, b, c$ and $d$ satisfy

$$\tilde{P} = (a, b, c, d)^T = (B^TB)^{-1}B^TY$$  \hspace{1cm} (10)$$

Proof. Substituting non-negative raw data $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n))$ into Eq. (8), so

$$\begin{cases}
  x^{(0)}(2) + az^{(1)}(2) = b(z^{(1)}(2))^2 + c + d \\
  x^{(0)}(3) + az^{(1)}(3) = b(z^{(1)}(3))^2 + c + 2^r + d \\
  \vdots \\
  x^{(0)}(n) + az^{(1)}(n) = bz^{(1)}(n)^2 + c(n - 1)^r + d
\end{cases}$$  \hspace{1cm} (11)$$

that is, $Y = BP$.

Then, replacing $x^{(0)}(k), k = 2, 3, \ldots, n$ with $-az^{(1)}(k) + bz^{(1)}(k)^2 + c(k - 1)^r + d, e = Y - BP$ is obtained.

Sets $\epsilon^{(e)} = (Y - BP)I(Y - BP) = \sum_{k=2}^{n} x^{(0)}(k) + az^{(1)}(k) - bk)(k - 1)(k - 1)^2 - d)^2$, and the $a, b, c, d$ that minimize $s$ should satisfy

$$\begin{cases}
  \frac{\partial s}{\partial a} = 2 \sum_{k=2}^{n} x^{(0)}(k) + az^{(1)}(k) - bz^{(1)}(k)^2 - c(k - 1)^r - d) \cdot z^{(1)}(k) = 0 \\
  \frac{\partial s}{\partial b} = 2 \sum_{k=2}^{n} x^{(0)}(k) + az^{(1)}(k) - bz^{(1)}(k)^2 - c(k - 1)^r - d \cdot (k - 1)^2 = 0 \\
  \frac{\partial s}{\partial c} = 2 \sum_{k=2}^{n} x^{(0)}(k) + az^{(1)}(k) - bz^{(1)}(k)^2 - c(k - 1)^r - d \cdot (k - 1)^2 = 0 \\
  \frac{\partial s}{\partial d} = 2 \sum_{k=2}^{n} x^{(0)}(k) + az^{(1)}(k) - bz^{(1)}(k)^2 - c(k - 1)^r - d = 0
\end{cases}$$  \hspace{1cm} (12)$$
According to the above equations, the following equation can be obtained:

\[ B^T \varepsilon = 0 \Rightarrow B^T (Y - B \bar{P}) = 0 \Rightarrow B^T Y - B^T B \bar{P} = 0 \Rightarrow \bar{P} = (B^T B)^{-1} B^T Y \]

Therefore, definition 2.5 is proven.

**Definition 2.6** Set \( B, Y \) and \( P \) as in definition 2.5, the time response sequence of CCRGM(1,1) is

\[
\bar{x}^{(1)}(k) = \begin{cases} 
\frac{e^{2\sqrt{w}(bk+C_1)} (-m + \sqrt{w}) + m + \sqrt{w}}{1 + 2e^{2\sqrt{w}(bk+C_1)} + m} & w > 0 \\
\frac{1}{bk+C_2} + m & w = 0 \\
\sqrt{-w} \tan(\sqrt{-w}(bk+C_3)) + m & w < 0
\end{cases}
\]

where

\[ u = c(k - 1)^r + d, m = \frac{b}{2a} w = \frac{a^2}{4b^2} - \frac{u}{b} \]

\[ C_1 = \frac{1}{2\sqrt{w}} \ln \frac{\bar{x}^{(1)}(1) - m - \sqrt{w}}{\bar{x}^{(1)}(1) - m + \sqrt{w}} - b \]

\[ C_2 = -\frac{1}{\bar{x}^{(0)}(1) + p} - b \]

\[ C_3 = \frac{1}{\sqrt{-w}} \arctan \frac{\bar{x}^{(0)}(1) - m}{\sqrt{-w}} - b \]

\[ A = \frac{\bar{x}^{(1)}(1) - m - \sqrt{w}}{\bar{x}^{(1)}(1) - m + \sqrt{w}} \]

When \( A > 0 \), the first equation takes on a minus sign; otherwise, it takes on a positive sign.

**Proof.** Set \( c(k - 1)^2 + d = u \), Eq. (9) is changed as

\[
\frac{dx^{(1)}}{dt} = b \left( x^{(1)} \right)^2 - ax^{(1)} + u
\]

Then, integrating the two sides of Eq. (14):

\[
\int b dt = \int \frac{1}{\left( x^{(1)} - b \right)^2} - \left( \frac{a^2}{4b^2} - \frac{u}{b} \right) dx^{(1)}
\]

Set \( m = \frac{b}{2a} \) and \( w = \frac{a^2}{4b^2} - \frac{u}{b} \) so that

\[
\int b dt = \int \frac{1}{\left( x^{(1)} - m \right)^2 - w} dx^{(1)}
\]

Then, integrating the two sides of Eq. (16) to get

\[
bt + C_1 = \int \frac{1}{\left( x^{(1)} - m \right)^2 - w} dx^{(1)}
\]

The right end of Eq. (17) can be discussed in three cases:

(1) When \( w > 0 \):

\[
\int \frac{1}{\left( x^{(1)} - m \right)^2 - w} dx^{(1)} = \frac{1}{2\sqrt{w}} \ln \left[ \frac{x^{(1)}(1) - m - \sqrt{w}}{x^{(1)}(1) - m + \sqrt{w}} \right] = bt + C_1
\]

Set the initial value condition as \( x^{(1)}(1) = x^{(0)}(1) \), and substitute these into Eq. (18). The constant can be calculated as follows:

\[
C_1 = \frac{1}{2\sqrt{w}} \ln \left[ \frac{x^{(1)}(1) - m - \sqrt{w}}{x^{(1)}(1) - m + \sqrt{w}} \right] - b
\]

Move the left and right sides of Eq. (18) and solve to yield

\[
\frac{x^{(1)} - m - \sqrt{w}}{x^{(1)} - m + \sqrt{w}} = e^{2\sqrt{w}(bt+C_1)}
\]

(2) When \( w = 0 \):

\[
\frac{1}{\left( x^{(1)} - m \right)^2 - w} dx^{(1)} = -\frac{1}{x^{(1)}(t) - m} = bt + C_2
\]

Set the initial value condition as \( t = 1, x^{(1)}(1) = x^{(0)}(1) \), and substitute these into Eq. (22). Then,

\[
C_2 = -\frac{1}{x^{(0)}(1) - m} - b
\]

The time response equation can be obtained from Eq. (23):

\[
x^{(1)}(t) = -\frac{1}{bt + C_2} + m
\]

(3) When \( w < 0 \):
When using the SA algorithm, first, set the convergence conditions according to the actual needs, such as the size of the error, number of iterations or termination temperature. The cooling law is as follows: $V \in (0, 1)$ is the annealing coefficient, and $V$ is the iteration. Because $T$ has to decrease slowly, $V$ should be close to 1. The SA algorithm is given, as shown in Table 2.

2.5. Modelling steps

Based on the definition of CCRGM(1,1) and SA algorithm, this paper proposes a prediction process using CCRGM(1,1):

- **Step 1**: Input the original series $X^{(0)}$.
- **Step 2**: Compute the 1-AGO series $X^{(1)}$ and the mean sequence $Z^{(1)}$ of the 1-AGO series $X^{(1)}$.
- **Step 3**: Substitute the data of step 1 into Eq. (10) and initial nonlinear order to obtain parameters $a, b, c$ and $d$.
- **Step 4**: According to the relationship between $w$ and 0, choose the appropriate time response equation. When $w > 0$, choose Eq. (21); When $w = 0$, choose Eq. (24); When $w < 0$, choose Eq. (27).

Then, compute the restored value $\tilde{X}^{(0)}_t$ by Eq. (28).

- **Step 5**: Substitute the data of above three steps into Eq. (8) to construct the CCRGM(1,1) and compute the MAPE.

- **Step 6**: Use the SA algorithm to optimize nonlinear term and compute the lowest MAPE value.

- **Step 7**: Substitute the optimal parameters to reconstruct CCRGM(1,1) model and compute the simulated data $\hat{X}^{(0)}_t$ and MAPE.

Combined with the above modelling steps of the model, the modelling flow chart is obtained, as shown in Fig. 1.

3. Validation of CCRGM(1,1) model

To illustrate the validity of the CCRGM(1,1) model, three numerical cases are selected. The first is the share of renewable energy consumption, and the last two cases are nuclear energy and hydropower. In three numerical experiments, the results of CCRGM(1,1) are compared with those of GM(1,1) [20], DGM(1,1) [22], NGM(1,1, K, c) (referred to as NMG(1,1)) [23], ARGM(1,1) [46], ENGM(1,1) [45], and Verhulst models [21]. To comprehensively evaluate the prediction performance of the selected models, this paper evaluates the models from two different aspects: the first is to measure the performance of the evaluation metrics generated by these competition models. In addition to the APE and MAPE commonly used in grey model, RMSPE, MAE, MSE, IA, U1, U2, and R are also introduced. The definitions of these metrics are listed in Table 3. MAPE value is calculated in two stages: MAPEFIT in model building and MAPEPRE in prediction stage. Other metrics are calculated by the value of the entire process. Second, the APE comparison chart and curve trend chart are used to show the model performance. The curve trend chart is used to evaluate the fitting and approximation degree of the model simulation trend line and the actual data trend line. The higher the degree of fitting and approximation, the better the data fitting ability of the model is. This standard is reflected mainly by the data trend chart. In the data trend chart of three cases, because the error of the NMG(1,1) is very large, it is omitted.

In the following experiments, the CCRGM(1,1) model is calculated according to the steps in Fig. 1, for which the steps of the SA algorithm are calculated using MATLAB according to the steps in Table 2.

\[
\int \frac{1}{(x^{(1)} - m)^2} \, dx^{(1)} = \frac{1}{\sqrt{-w}} \arctan \frac{x^{(1)} - m}{\sqrt{-w}} = bt + C_3
\]

(25)

Take the initial value condition into Eq. (25), the constant is

\[
C_3 = \frac{1}{\sqrt{-w}} \arctan \frac{x^{(0)}(1) - m}{\sqrt{-w}} - b
\]

(26)

and the time response equation is

\[
x^{(1)}(t) = \sqrt{-w} \tan(\sqrt{-w}(bt + C_3)) + m
\]

(27)

Definition 2.6 is thus proven, and the reduced value can be obtained:

\[
x^{(0)}(k) = x^{(1)}(k) - x^{(1)}(k - 1), \quad k = 2, 3, \ldots, n
\]

(28)

2.4. Optimization of nonlinear correction term

This section derives the optimal order of nonlinear terms. First, to evaluate the accuracy of the model and the effect of order selection, the absolute percentage error (APE) and the mean absolute percentage error (MAPE) are introduced as evaluation metrics. The APE and MAPE are defined as follows:

\[
APE = \left( \frac{x^{(0)}(i) - \hat{x}^{(0)}(i)}{x^{(0)}(i)} \times 100\% \right), i = 1, 2, 3 \ldots n
\]

(29)

\[
MAPE = \frac{1}{n} \left( \sum_{i=1}^{n} \left( \frac{x^{(0)}(i) - \hat{x}^{(0)}(i)}{x^{(0)}(i)} \right) \right) \times 100\%, i = 1, 2, 3 \ldots n
\]

(30)

The nonlinear term optimization is primarily aimed at the orders in Eq. (8). CCRGM(1,1) uses the simulated annealing (SA) algorithm proposed by Kirkpatrick [43] to search for the optimal order of the 1-AGO series $X^{(1)}$ and the mean sequence $Z^{(1)}$ of the 1-AGO series $X^{(1)}$.

Step 1: Set the objective function and the maximum iteration number $T = V \cdot T(t)$

Step 2: Compute the 1-AGO series $X^{(1)}$ and the mean sequence $Z^{(1)}$ of the 1-AGO series $X^{(1)}$.

Step 3: Substitute the data of step 1 into Eq. (10) and initial nonlinear order to obtain parameters $a, b, c$ and $d$.

Step 4: According to the relationship between $w$ and 0, choose the appropriate time response equation. When $w > 0$, choose Eq. (21); When $w = 0$, choose Eq. (24); When $w < 0$, choose Eq. (27).

Step 5: Compute the restored value $\tilde{X}^{(0)}_t$ by Eq. (28).

Step 6: Use the SA algorithm to optimize nonlinear term and compute the lowest MAPE value.

Step 7: Substitute the optimal parameters to reconstruct CCRGM(1,1) model and compute the simulated data $\hat{X}^{(0)}_t$ and MAPE.

Combined with the above modelling steps of the model, the modelling flow chart is obtained, as shown in Fig. 1.

### Table 2
The steps of the SA algorithm.

| Algorithm: The SA algorithm to find the optimal |
| --- |
| Set the objective function and the maximum iteration number $T = V \cdot T(t)$ |
| Input: The original series and the number of modelling data |
| Output: The best order |
| for $t \in [0, 1, n]$ do |
| Substitute into $P = (B^T B)^{-1} B^T Y$ and obtain parameters $P = (a, b, c, d)^T$ |
| Substitute parameters to discrete equation Eq. (13) and compute the simulation value to obtain $\tilde{X}^{(1)}(k)$ |
| Compute $\tilde{X}^{(0)}(k)$ in Eq. (28) |
| Compute MAPE in Eq. (29)-(30) |
| End |
| Update the minimum MAPE value |
| Return the best by the SA algorithm. |

### Table 3
The definitions of the evaluation metrics.

| Metric | Formula | Definition |
| --- | --- | --- |
| APE | $\left( \frac{x^{(0)}(i) - \hat{x}^{(0)}(i)}{x^{(0)}(i)} \times 100\% \right)$ | Absolute percentage error |
| MAPE | $\frac{1}{n} \left( \sum_{i=1}^{n} \left( \frac{x^{(0)}(i) - \hat{x}^{(0)}(i)}{x^{(0)}(i)} \right) \right) \times 100\%$ | Mean absolute percentage error |

In the following experiments, the CCRGM(1,1) model is calculated according to the steps in Fig. 1, for which the steps of the SA algorithm are calculated using MATLAB according to the steps in Table 2.
3.1. Numerical simulation experiments

Validation case 1 Simulation experiment on the percentage of renewable energy consumption:
In the first case, data of the S-shaped renewable energy consumption proportion are taken from CHINA ENERGY STATISTICAL YEARBOOK 2018. The data from 1999 to 2013 are used to establish seven models in Table 1, and the other data are used to test the models. This numerical experiment primarily shows the advantages of the models under disordered S-shaped data. Table 4 shows
Table 3
Metrics for evaluating the effectiveness of the models [40].

| Name                                      | Abbreviation | Formulation |
|-------------------------------------------|--------------|-------------|
| Mean absolute percentage error           | MAPE         | $\frac{1}{n} \sum_{i=1}^{n} \left| \frac{X^{(0)}(k) - \hat{X}^{(0)}(k)}{X^{(0)}(k)} \right| \times 100\%$ |
| Root mean squares percentage error       | RMSPE        | $\sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( \frac{X^{(0)}(k) - \hat{X}^{(0)}(k)}{X^{(0)}(k)} \right)^2} \times 100\%$ |
| Mean absolute percentage error           | MAE          | $\frac{1}{n} \sum_{i=1}^{n} \left| X^{(0)}(k) - \hat{X}^{(0)}(k) \right| \times 100\%$ |
| Mean squares error                       | MSE          | $\frac{1}{n} \sum_{i=1}^{n} \left( X^{(0)}(k) - \hat{X}^{(0)}(k) \right)^2$ |
| Index of agreement                       | IA           | $1 - \frac{\sum_{i=1}^{n} (X^{(0)}(k) - \hat{X}^{(0)}(k))^2}{\sum_{i=1}^{n} (X^{(0)}(k) - \bar{X}^{(0)})^2}$ |
| Theil U statistic 1                      | U1           | $\frac{\sqrt{\sum_{i=1}^{n} (X^{(0)}(k) - \hat{X}^{(0)}(k))^2}}{\sqrt{\sum_{i=1}^{n} (X^{(0)}(k) - \bar{X}^{(0)})^2}}$ |
| Theil U statistic 2                      | U2           | $\frac{\sum_{i=1}^{n} (X^{(0)}(k) - \hat{X}^{(0)}(k))^2}{\sum_{i=1}^{n} (X^{(0)}(k) - \bar{X}^{(0)})^2}$ |
| Correlation coefficient                  | R            | $\frac{\text{Cov}(X^{(0)}, \hat{X}^{(0)})}{\sqrt{\text{Var}(X^{(0)}) \text{Var}(\hat{X}^{(0)})}}$ |

Table 4
Forecasting results of grey models in Validation Case 1.

| Year | Raw data | GM (1,1) (%) | APE | DGM (1,1) (%) | APE | NGM (1,1) (%) | APE | ARGGM (1,1) (%) | APE |
|------|----------|--------------|-----|---------------|-----|---------------|-----|----------------|-----|
| 1999 | 5.9      | 5.9          | 0   | 5.9           | 0   | 5.9           | 0   | 5.9            | 0   |
| 2000 | 7.3      | 7.1894       | −1.5152 | 7.1983        | −1.3933 | 8.8069        | 20.6429 | 6.8761         | 5.8063 |
| 2001 | 8.4      | 7.3417       | −12.5992 | 7.3495        | −12.5055 | 9.2502        | 10.1216 | 7.5359         | 12.865 |
| 2002 | 8.2      | 7.4972       | −8.5711 | 7.504         | −8.4881 | 9.8463        | 20.0763 | 7.9819         | 2.6598 |
| 2003 | 7.4      | 7.656        | 3.459 | 7.6617        | 3.5358 | 10.6477       | 43.8878 | 8.2833         | 11.936 |
| 2004 | 7.6      | 7.8181       | 2.8701 | 7.8226        | 2.9295 | 11.7253       | 54.2804 | 8.4871         | 11.672 |
| 2005 | 7.4      | 7.9837       | 7.8881 | 7.987         | 7.9327 | 13.1743       | 78.0308 | 8.6248         | 16.531 |
| 2006 | 7.4      | 8.1528       | 10.1732 | 8.1548        | 10.2006 | 15.1226       | 104.3588 | 8.7179         | 17.092 |
| 2007 | 7.5      | 8.3255       | 11.0067 | 8.3262        | 11.016 | 17.7422       | 136.5628 | 8.7908         | 17.077 |
| 2008 | 8.4      | 8.5018       | 1.2124 | 8.5012        | 1.2042 | 21.2646       | 153.15 | 8.8233         | 5.0396 |
| 2009 | 8.5      | 8.6819       | 2.1402 | 8.6798        | 2.1151 | 26.0008       | 205.8919 | 8.8521         | 4.142  |
| 2010 | 9.4      | 8.8658       | −5.6829 | 8.8622        | −5.7216 | 32.3691       | 244.3524 | 8.8715         | −5.6224 |
| 2011 | 8.4      | 9.0536       | 7.7809 | 9.0484        | 7.7189 | 40.932        | 387.2854 | 8.8846         | 5.7964 |
| 2012 | 9.7      | 9.2454       | −4.6871 | 9.2185        | −4.7576 | 52.4456       | 440.6761 | 8.8935         | −8.3144 |
| 2013 | 10.2     | 9.4412       | −7.4394 | 9.4126        | −7.5231 | 67.9268       | 565.9489 | 8.8995         | −12.7499 |
| MAPE |       | 6.2161       |       | 6.2161        |       | 41.5694       | 176.0945 | 6.9674         |       |
| 2014 | 11.3     | 9.6412       | −14.6801 | 9.6308        | −14.7713 | 88.7428       | 685.3349 | 8.9036         | −21.2074 |
| 2015 | 12.1     | 9.8454       | −18.6334 | 9.8332        | −18.7338 | 116.7321      | 864.7283 | 8.9063         | −26.3942 |
| 2016 | 13.3     | 10.0539      | −24.4098 | 10.0398       | −24.5125 | 154.3665      | 1060.6507 | 8.9082         | −33.0214 |
| 2017 | 13.8     | 10.2668      | −25.6026 | 10.2508       | −25.7189 | 204.9698      | 1385.2886 | 8.9094         | −35.4391 |
| MAPE |       | 20.8307      |       | 20.9341       |       | 999.0006      | 29.0155 |        |       |

Table continued...
the original data and calculation results of seven models. Table 5 lists the evaluation metrics of the seven models. The optimal of the CCRGM(1,1) is $r = 0.953875$.

In comparison with other grey models, Table 4 shows that the MAPEFIT values of CCRGM(1,1), GM(1,1), DGM(1,1), and ARG(1,1) are all lower than 10%, indicating that the fitting effect of these models is good, but the lowest is that of CCRGM(1,1), which is 5.1623%. The fitting effect of NGM(1,1) is the worst, and the MAPEFIT is as high as 176.0954%. In the prediction stage, the MAPEPRE of CCRGM(1,1) is the lowest, only 1.5014%, and that of other models exceeds 20%. Table 5 shows that all evaluation metrics of the CCRGM(1,1) are better than other models. For RMSPE, MSE, IA, U1, and U2 evaluation indexes, CCRGM(1,1) values are far better than those of the other models. To intuitively show the fitting error of all models, Table 4 is transformed into APE comparison chart and curve trend chart, as shown in Figs. 2 and 3 respectively. Because the error of NGM(1,1) model is too large to affect Figs. 2 and 3, it is omitted in the charts.
rapidly. Thus, the data in recent years are not S-shaped data, and energy later than other countries, but its development proceeded the largest countries in the world, China began to develop clean research object, with data taken from the literature [46]. One of consumption

Fig. 3 shows that the actual trend line exhibits a disordered S-shaped data. The data from 2006 to 2012 are used to establish CCRGM(1,1), whose MAPEFIT value is only 2.2628%, which is far better than those of the other models. However, NGM(1,1), which ranks second of the seven models, and other metrics are slightly better than those of the other models. However, NGM(1,1), ENGM(1,1) and Verhulst models, but approximate only a percentage point lower than that of GM(1,1), DGM(1,1) and ARGM(1,1). However, the MAPEPRE of CCRGM(1,1) model is much lower than that of other models, only 4.7724%, and that of other models exceeds 16%. In the comparison of other model is much lower than that of other models, only 4.7724%, and that of other models exceeds 16%. In the comparison of other

Validation case 2

This case selects China’s nuclear energy consumption as the research object, with data taken from the literature [46]. One of the largest countries in the world, China began to develop clean energy later than other countries, but its development proceeded rapidly. Thus, the data in recent years are not S-shaped data, and are used to illustrate the ability of CCRGM(1,1) model for non-S-shaped data. The data from 2006 to 2012 are used to establish the seven models, while the data from 2013 to 2017 are used to test the models. The original data and calculation results are shown in Tables 6 and 7. The optimal of CCRGM(1,1) is 2.599135.

Table 6

| Year | Raw data | GM (1,1) APE (%) | DGM (1,1) APE (%) | NGM (1,1) APE (%) | ARG (1,1) APE (%) | VERHULST APE (%) | CCRGM (1,1) APE (%) |
|------|----------|-----------------|-----------------|-----------------|-----------------|----------------|----------------|
| 2006 | 12.4     | 12.4            | 0               | 12.4            | 0               | 12.4           | 0              |
| 2007 | 14.1     | 13.6523         | 3.1752          | 13.6726         | 3.0313          | 17.5093        | 24.1795        |
| 2008 | 15.5     | 14.9241         | 3.7156          | 14.9433         | 3.5914          | 19.5799        | 26.3217        |
| 2009 | 15.9     | 16.3143         | 2.6059          | 16.3322         | 2.7181          | 22.6345        | 42.3556        |
| 2010 | 16.7     | 17.8341         | 6.7911          | 17.8501         | 6.8868          | 27.1411        | 62.5216        |
| 2011 | 19.5     | 19.4955         | 0.2323          | 19.5091         | 0.4666          | 33.7896        | 73.2802        |
| 2012 | 22       | 21.3116         | 3.1293          | 21.3223         | 3.0806          | 43.5982        | 98.1736        |
| 2013 | 25.3     | 23.2969         | 7.9176          | 23.304          | 7.8894          | 58.0688        | 129.5208       |
| 2014 | 30       | 25.4671         | 15.1097         | 25.4969         | 15.1004         | 79.4171        | 164.7238       |
| 2015 | 38.6     | 27.8395         | 27.8777         | 27.8371         | 27.8833         | 110.9124       | 187.3377       |
| 2016 | 48.2     | 30.4129         | 36.8613         | 30.4242         | 36.8792         | 157.3772       | 226.5087       |
| 2017 | 56.2     | 33.2679         | 40.8045         | 33.2519         | 40.8329         | 225.9267       | 302.0048       |
| 2018 | 6.7911   | 17.8501         | 6.8868          | 27.1411         | 62.5216         | 17.8281        | 44.7281        |

Table 7

| Metrics | GM (1,1) | DGM (1,1) | NGM (1,1) | ARG (1,1) | VERHULST | CCRGM (1,1) | CCRGM rank |
|---------|----------|-----------|-----------|-----------|----------|-------------|------------|
| RMSPE   | 18.6602  | 18.6671   | 142.4507  | 13.0347   | 55.6844  | 21.4582     | 5.6917     |
| MAE     | 5.1051   | 5.1055    | 41.1629   | 3.5150    | 15.2066  | 3.8693      | 21.8177    |
| MSE     | 82.0302  | 82.1168   | 4193.7546 | 40.1883   | 786.4197 | 22.0579     | 9.1250     |
| IA      | 0.7970   | 0.7966    | 0.2960    | 0.9193    | 0.6897   | 0.9769      | 0.9809     |
| U1      | 0.1745   | 0.1746    | 0.5300    | 0.1717    | 0.3299   | 0.0768      | 0.0498     |
| U2      | 0.3062   | 0.3063    | 2.1892    | 0.2143    | 0.9480   | 0.1588      | 0.1021     |
| R       | 0.9641   | 0.9640    | 0.9903    | 0.9833    | 0.9784   | 0.9807      | 0.9889     |
| MAPEFIT | 3.2401   | 3.2258    | 54.472    | 3.2505    | 9.6959   | 18.8782     | 2.2628     |
| MAPEPRE | 25.714   | 25.717    | 16.8311   | 6.9928    | 66.505   | 16.1082     | 4.7724     |

Fig. 4 shows that the APE values of CCRGM(1,1) are not the lowest in 7 years, but the those of other 12 years are much lower than other models, and those of six years are close to the zero line. Fig. 3 shows that the actual trend line exhibits a disordered S-shaped, the fitting lines of GM(1,1), DGM(1,1) ARGM(1,1), ENGM(1,1) are increasing, and far from the actual data trend line, while Verhulst model presents a complete S-shaped, and underestimates the actual value. Only the fitting line of CCRGM(1,1) is closest to the actual data line. In conclusion, two aspects show that CCRGM(1,1) has the best effect in dealing with disordered S-shaped data and alleviates the dependence of Verhulst model on saturated S-shaped data.
8 out of 12 years, most of which are close to the zero line, while the APE values of other four years are also not the largest. In Fig. 5, the actual data show an upward trend, not an S-shaped trend. GM(1,1), DGM(1,1) and ARGM(1,1) underestimate the actual value; ENGM(1,1) overestimates the actual value. The fitting line of Verhulst model shows an S-shaped trend, only that of CCRGM(1,1) is the closest to the actual data line. In conclusion, CCRGM(1,1) can not only effectively predict nuclear energy consumption, but also eliminate the dependence of Verhulst model on the saturated S-shaped.

Validation case 3 Predicting the hydroelectricity clean energy consumption:

These data are collected from *BP Statistical Review of World Energy 2019*. Countries in Commonwealth of Independent States (CIS) region, which is influenced by European and North American

| Year | Raw data | GM(1,1) APE (%) | DGM(1,1) APE (%) | ENGM(1,1) APE (%) | Verhulst APE (%) | CCRGM(1,1) APE (%) | MAPE (%) |
|------|----------|----------------|----------------|------------------|-----------------|-------------------|---------|
| 2008 | 47.0     | 47.0000        | 0.0000         | 47.0000          | 0.0000          | 47.0000           | 0.0000   |
| 2009 | 49.4     | 47.9984        | 2.8372         | 48.0122          | 2.8093          | 64.6302           | 30.8304  |
| 2010 | 49.1     | 48.5632        | 1.0932         | 49.574           | 1.0713          | 74.9167           | 52.5798  |
| 2011 | 48.1     | 49.1347        | 2.1512         | 49.1423          | 2.167           | 91.8805           | 50.0197  |
| 2012 | 48.2     | 49.7129        | 3.1389         | 49.7174          | 3.148           | 119.8561          | 50.8851  |
| 2013 | 51.9     | 50.298         | 3.0686         | 50.2991          | 3.0846          | 165.9918          | 51.0238  |
| 2014 | 50       | 50.8899        | 1.7797         | 50.8876          | 1.7753          | 242.0759          | 51.0946  |
| 2015 | 48.8     | 51.4887        | 5.5097         | 51.4831          | 5.4981          | 367.5492          | 51.1308  |
| 2016 | 53.1     | 52.0946        | 1.8933         | 52.0855          | 1.9106          | 574.4719          | 51.1492  |
| 2017 | 54.3     | 52.7077        | 2.9324         | 52.6949          | 2.9559          | 915.7161          | 51.1587  |
| 2018 | 55.4     | 53.3279        | 3.7402         | 53.3115          | 3.7699          | 1478.4751         | 51.1635  |
| MAPE | 2.7136   | 2.7134         | 2.7134         | 246.9467         | 3.5152          | 653.1745          | 4.7762   |

| Year | Raw data | ENGM(1,1) APE (%) | Verhulst APE (%) | CCRGM(1,1) APE (%) | MAPE (%) |
|------|----------|-------------------|-----------------|-------------------|---------|
| 2008 | 47.0     | 47.0000           | 0.0000          | 47.0000           | 0.0000   |
| 2009 | 49.4     | 47.2021           | 4.4492          | 48.6457          | 4.1296   |
| 2010 | 49.1     | 47.9227           | 2.3979          | 25.6467          | 2.0167   |
| 2011 | 48.1     | 48.645            | 1.1331          | 33.1389          | 1.0357   |
| 2012 | 48.2     | 49.3692           | 2.4258          | 41.7728          | 2.8317   |
| 2013 | 51.9     | 50.0952           | 3.4774          | 49.7343          | 3.1729   |
| 2014 | 50       | 50.8231           | 1.6462          | 55.8355          | 1.1671   |
| 2015 | 48.8     | 51.5528           | 5.641           | 58.6988          | 5.2895   |
| 2016 | 53.1     | 52.2843           | 1.5361          | 57.5967          | 1.2684   |
| 2017 | 54.3     | 53.0177           | 2.3616          | 52.8153          | 2.2734   |
| 2018 | 55.4     | 53.7529           | 2.9731          | 45.5075          | 1.8476   |
countries, have vigorously developed clean energy. Therefore, the data in recent years show a disordered S-shaped. The data of the first ten years are used to build the models, and the data of the last years are used to test the model. The optimal rank of CCRGM(1,1) model is \( r = 0.835622 \). The original data and fitting results are shown in Table 8, and the results of evaluation metrics are shown in Table 9.

As seen in Table 8, the difference between the MAPEFIT of GM(1,1), DGM(1,1), NGM(1,1), ARGGM(1,1), ENGM(1,1) and CCRGM(1,1) models is not large. The MAPEFIT of CCRGM(1,1) is approximately 1% lower than the other values, only 1.8476%, while NGM(1,1) has the worst fitting effect, with MAPEFIT reaching 460.9467%. The MAPEFIT of Verhulst model is slightly better than that of NGM(1,1), at 22.2604%. The MAPEPRE of CCRGM(1,1) model is close to zero, only 0.0018%, far lower than that of the other models. Table 9 clearly shows that the results of evaluation metrics of CCRGM(1,1) are better than those of other models. In the same way as the first two cases, Table 8 is transformed into APE comparison chart and curve trend chart, as shown in Figs. 6 and 7 respectively.

In Fig. 6, except for 2016, the APE of CCRGM(1,1) is the lowest every year, and all of the values are close to zero. In Fig. 7, the original data show a continuous multiple S-shaped trend, Verhulst model shows single saturated S-shaped; and the fitting curves of GM(1,1), DGM(1,1), ARGGM(1,1) and ENGM(1,1) show a straight upward trend. Only the fitting trend of CCRGM(1,1) is close to the actual data trend. The above analysis shows that CCRGM(1,1) is effective and accurate for the prediction of hydroelectricity energy.

3.2. Analysis of results

In this section, the results of three cases are analysed in combination with figures, and the following conclusions can be obtained:

(1) The Riccati equation is introduced into the classical Verhulst model to achieve a new grey model, and the accuracy of Verhulst model is substantially improved after the expansion. In the validation cases, the regularity of clean energy data is not obvious, but the effect of CCRGM(1,1) model is far better than that of the Verhulst model, which shows that the CCRGM(1,1) model has no obvious data requirements and alleviates the dependence of Verhulst model on S-shaped data, which makes CCRGM(1,1) model more suitable for clean energy prediction.

(2) In the comparison of CCRGM(1,1) with the GM(1,1), DGM(1,1), NGM(1,1), ARGGM(1,1) and ENGM(1,1) models, the evaluation metrics of CCRGM(1,1) model are the best, and the fitting and approximation degree between the trend lines of CCRGM(1,1) model and original data is the highest, far superior to those of the other grey models. This result shows that the CCRGM(1,1) model is effective and accurate in short-term and metaphase prediction of clean energy consumption.

4. Applications

North America usually refers to the United States, Canada and other regions. It is the most developed continent in the world and the first region to vigorously develop clean energy. According to BP Statistical Review of World Energy 2018, the annual growth rate of new energy consumption in North America from 2007 to 2017 was 13.9%, which effectively reduced the consumption of coal, oil and other resources. In 2016, North America reached a consensus that clean energy should account for 50% of energy consumption, thus, its predicted clean energy consumption trend plays a crucial role in formulating energy consumption policies.

Due to the good performance of the CCRGM(1,1) model, this model is applied to predict nuclear and hydroelectricity energy consumption in North America. The data are taken from BP Statistical Review of World Energy 2019, and divided into two parts: one
part is used to build the model, and the other part is used to test and compare the prediction results of the models.

### 4.1. Case 1: prediction of nuclear clean energy consumption

The actual data are shown in Table 10, in which the data from 2002 to 2007 are used to build the model, and the other data are used to test the models. The results of the seven models are shown in Table 10. The optimal ultimately identified is $r = 1.0291686$.

In Table 10, the errors of CCRGM(1,1) are the smallest, the MAPEFIT is only 0.6092%, and the MAPEPRE is only 1.0808%. The model with the largest MAPE value is Verhulst model. To visually compare the differences among the models, Table 10 is transformed into charts of the MAPE comparison and curve trend, as shown in Fig. 8.

**Table 10**

Forecasting results of grey models in Case 1.

| Year  | Raw data | GM (1,1) APE (%) | DGM (1,1) APE (%) | NGM (1,1) APE (%) | ARGM (1,1) APE (%) |
|-------|----------|-----------------|-------------------|-------------------|-------------------|
| 2002  | 205      | 205.0000        | 0.0000            | 205.0000          | 0.0000            |
| 2003  | 201.1    | 203.6943        | 1.453             | 203.6174          | 1.2518            |
| 2004  | 210.2    | 206.5670        | –1.728             | 206.5757          | –1.724             |
| 2005  | 209.4    | 209.5728        | 0.0825             | 209.5769          | 0.0845             |
| 2006  | 212      | 212.6224        | 0.2696             | 212.6217          | 0.2653             |
| 2007  | 215.4    | 215.7164        | 0.4649             | 215.7108          | 0.4433             |
|       |          | MAPE (%)        | 0.6993             | 0.6996             | 13.522             |
| 2008  | 215.8    | 218.8553        | 1.4158             | 218.8448          | 1.4109             |
| 2009  | 212.9    | 222.0400        | 4.2931             | 222.0243          | 4.2857             |
| 2010  | 213.9    | 225.7171        | 5.3602             | 225.4909          | 5.3048             |
| 2011  | 211.5    | 228.5490        | 8.0610             | 228.5225          | 8.0485             |
| 2012  | 206.5    | 231.8747        | 12.8880            | 231.8426          | 12.2724            |
| 2013  | 213.8    | 235.2488        | 10.0322            | 235.2109          | 10.0145            |
| 2014  | 216.2    | 238.6720        | 10.3941            | 238.6282          | 10.3738            |
| 2015  | 215.4    | 242.1459        | 12.4164            | 242.0951          | 12.3933            |
| 2016  | 217      | 245.6686        | 13.2113            | 245.6123          | 13.1854            |
| 2017  | 216.9    | 249.2434        | 14.9117            | 249.1807          | 14.8828            |
| 2018  | 217.9    | 252.8702        | 16.0488            | 252.8010          | 16.0170            |

| Year  | Raw data | ENGM (1,1) APE (%) | Verhulst APE (%) | CCRGM (1,1) APE (%) |
|-------|----------|-------------------|------------------|-------------------|
| 2002  | 205      | 9.8355            | 0.6122           |
| 2003  | 201.1    | 9.8355            | 0.6092           |
| 2004  | 210.2    | 9.8355            | 0.6122           |
| 2005  | 209.4    | 9.8355            | 0.6122           |
| 2006  | 212      | 9.8355            | 0.6122           |
| 2007  | 215.4    | 9.8355            | 0.6122           |
| 2008  | 215.8    | 9.8355            | 0.6122           |
| 2009  | 212.9    | 9.8355            | 0.6122           |
| 2010  | 213.9    | 9.8355            | 0.6122           |
| 2011  | 211.5    | 9.8355            | 0.6122           |
| 2012  | 206.5    | 9.8355            | 0.6122           |
| 2013  | 213.8    | 9.8355            | 0.6122           |
| 2014  | 216.2    | 9.8355            | 0.6122           |
| 2015  | 215.4    | 9.8355            | 0.6122           |
| 2016  | 217      | 9.8355            | 0.6122           |
| 2017  | 216.9    | 9.8355            | 0.6122           |
| 2018  | 217.9    | 9.8355            | 0.6122           |

| MAPE (%) | Year  | Raw data | ENGM (1,1) APE (%) | Verhulst APE (%) | CCRGM (1,1) APE (%) |
|----------|-------|----------|-------------------|------------------|-------------------|
| 2002     | 205   | 0.0000   | 0.0000            | 0.0000           |
| 2003     | 201.1 | 0.0000   | 0.0000            | 0.0000           |
| 2004     | 210.2 | 0.0000   | 0.0000            | 0.0000           |
| 2005     | 209.4 | 0.0000   | 0.0000            | 0.0000           |
| 2006     | 212   | 0.0000   | 0.0000            | 0.0000           |
| 2007     | 215.4 | 0.0000   | 0.0000            | 0.0000           |
| 2008     | 215.8 | 0.0000   | 0.0000            | 0.0000           |
| 2009     | 212.9 | 0.0000   | 0.0000            | 0.0000           |
| 2010     | 213.9 | 0.0000   | 0.0000            | 0.0000           |
| 2011     | 211.5 | 0.0000   | 0.0000            | 0.0000           |
| 2012     | 206.5 | 0.0000   | 0.0000            | 0.0000           |
| 2013     | 213.8 | 0.0000   | 0.0000            | 0.0000           |
| 2014     | 216.2 | 0.0000   | 0.0000            | 0.0000           |
| 2015     | 215.4 | 0.0000   | 0.0000            | 0.0000           |
| 2016     | 217   | 0.0000   | 0.0000            | 0.0000           |
| 2017     | 216.9 | 0.0000   | 0.0000            | 0.0000           |
| 2018     | 217.9 | 0.0000   | 0.0000            | 0.0000           |

**Fig. 8.** The MAPE of the models in Case 1.
The Verhulst model is very large, so it is omitted in Figs. 8 and 9 respectively. Table 10 shows that the MAPEPRE of the Verhulst model is very large, so it is omitted in Figs. 8 and 9.

In Fig. 8, the MAPEFIT values of GM(1,1) and DGM(1,1) are slightly 0.1% higher than that of CCRGM(1,1), but the MAPEPRE values are far higher than that of CCRGM(1,1), as shown in Table 11.

Table 11
Forecasting results of grey models in Case 2.

| Year | Raw data | GM (1,1) | APE (%) | DGM (1,1) | APE (%) | NGM (1,1) | APE (%) | ARGM (1,1) | APE (%) | ENGM (1,1) | APE (%) | CCRGM (1,1) | APE (%) |
|------|----------|----------|---------|-----------|---------|-----------|---------|------------|---------|------------|---------|--------------|---------|
| 2000 | 149.9    | 149.9000 | 0.000   | 149.9000  | 0.000   | 149.9000  | 0.000   | 149.9000   | 0.000   | 149.9000   | 0.000   |
| 2001 | 129      | 136.4277 | 5.7579  | 136.4262  | 5.7846  | 103.7961  | –19.5379| 144.7347   | 12.1975| 144.6337   | 1.0743 |
| 2002 | 143.1    | 138.4104 | 3.2771  | 138.4177  | 3.2581  | 120.8235  | –15.5671| 144.6337   | 1.0743 |
| 2003 | 141.6    | 140.422  | 0.8319  | 140.4418  | 0.8179  | 131.6093  | –7.0584 | 144.6355   | 2.1437 |
| 2004 | 141.7    | 142.4628 | 0.5383  | 142.475   | 0.5469  | 138.4322  | –2.3061 | 144.6355   | 2.0716 |
| 2005 | 148.5    | 144.5333 | 2.6712  | 144.5375  | 2.6683  | 142.7551  | –3.8686 | 144.6355   | 2.6024 |
| 2006 | 151.4    | 146.6338 | 3.1481  | 146.63   | 3.1506  | 145.4923  | –3.902  | 144.6355   | 4.4686 |
| 2007 | 144.4    | 148.7649 | 3.0228  | 148.7527  | 3.0143  | 147.2255  | 1.9567  | 144.6355   | 0.1631 |
| 2008 | 151.1    | 150.927  | 0.1145  | 150.9062  | 0.1283  | 148.323   | –1.8379| 144.6355   | –4.2783|
| 2009 | 150.9    | 153.1205 | 1.4715  | 153.0908  | 1.4518  | 149.0179  | –1.2473| 144.6355   | –4.1514|
| 2010 | 146.1    | 155.3458 | 6.3284  | 155.3071  | 6.3019  | 149.4579  | 2.2983  | 144.6355   | 3.6834 |
| 2011 | 164.7    | 157.6035 | 4.3078  | 157.5554  | 4.3379  | 149.7365  | –9.0833| 144.6355   | –12.1825|
| 2012 | 155.3    | 159.894  | 2.9582  | 159.8363  | 2.921   | 149.9129  | –3.4688| 144.6355   | –6.8671|
| 2013 | 155.3    | 162.2178 | 4.4545  | 162.1502  | 4.4109  | 150.0246  | –3.3969| 144.6355   | –5.5904|
| 2014 | 153.2    | 164.5754 | 7.4252  | 164.4976  | 7.3744  | 150.0954  | –2.0625| 144.6355   | –5.9044|
| 2015 | 149.2    | 166.9673 | 11.9083 | 166.879   | 11.8492 | 150.1401  | 0.6301  | 144.6355   | 3.0593 |
| 2016 | 154.2    | 169.3939 | 9.8533  | 169.2949  | 9.7892  | 150.1685  | –2.6145| 144.6355   | –6.2027|
| 2017 | 164.1    | 171.8557 | 4.7262  | 171.7457  | 4.6592  | 150.1865  | –8.4787| 144.6355   | –11.8614|
| 2018 | 160.3    | 174.3354 | 8.7669  | 174.232   | 8.6912  | 150.1978  | –6.302  | 144.6355   | –9.772 |

MAPEFIT values of GM(1,1) and DGM(1,1) are slightly 0.1% higher than that of CCRGM(1,1), but the MAPEPRE values are far higher than that of CCRGM(1,1).
higher than that of CCRGM(1,1). The MAPE\textsubscript{PRE} values of ARGM(1,1) and NGM(1,1) are similar to that of CCRGM(1,1), but the MAPE\textsubscript{FIT} values are higher. In Fig. 9, except for 2012, the CCRGM(1,1) predictions generally coincide with the original data, ARGM(1,1) and NGM(1,1) underestimate the nuclear energy consumption, and the fitting lines of GM(1,1) and DGM(1,1) essentially coincide, but similar to ENGM(1,1), they overestimate the nuclear energy consumption. In addition, Verhulst model presents a saturated S-shaped, which is contrary to the actual situation. Therefore, the CCRGM(1,1) model is the best for prediction of nuclear energy.

4.2. Case 2: prediction of hydroelectricity clean energy consumption

In this case, the CCRGM(1,1) model is established based on the hydroelectricity energy consumption in 2000–2009, and the consumption in subsequent nine years is used for prediction. The optimal\( r = 1.032092 \). The actual data and the results of seven grey models are shown in Table 11.

According to Table 11, the errors of CCRGM(1,1) are the smallest: the MAPE\textsubscript{FIT} is only 2.0603%, and the MAPE\textsubscript{PRE} is only 3.2004%. Similar to the previous case, the model with the largest error is the Verhulst model, whose MAPE\textsubscript{FIT} is 19.1260% and MAPE\textsubscript{PRE} is 130.6168%. This result also shows that in the prediction of clean energy, CCRGM(1,1) improves the prediction accuracy of Verhulst model. To directly compare the differences between models, the MAPE comparison chart and curve trend chart are constructed according to Table 11, as detailed in Figs. 10 and 11, respectively. Because the error of Verhulst model is too large to affect these Figs. 10 and 11, it is omitted.

In Fig. 10, the MAPE\textsubscript{FIT} of six models does not exceed 6.5%. In the prediction stage, only the MAPE\textsubscript{PRE} of ENGM(1,1) model is more than 10%, and that of the other models is less than 7.1%. These models can be used to predict hydroelectricity energy consumption, but the CCRGM(1,1) model has the best fitting effect. In Fig. 11, the trend of original data shows a disordered S-shaped, whereas GM(1,1), DGM(1,1) and ENGM(1,1) overestimate the hydroelectricity consumption, and ARGM(1,1) and NGM(1,1) slightly underestimate it. The Verhulst model shows a saturated S-shaped. Each point of CCRGM(1,1) predictions generally coincides with the original data. The above results show that CCRGM(1,1) is the most accurate model for prediction of hydroelectricity energy consumption in North America.

4.3. Future discussions

According to the results of the five cases, regardless of the shape of the data presented, the CCRGM(1,1) model is the most effective and accurate in predicting the clean energy consumption represented by nuclear energy and hydroelectricity energy. Therefore, this section uses this new model to predict the future energy consumption and supply information for the region to formulate energy consumption strategies.

4.3.1. Prediction of nuclear consumption in North America in the next 10 years

According to the prediction method of nuclear energy consumption in the previous section, CCRGM(1,1) model is established by data from 2013 to 2018. At this time \( r = 1.007939 \), and the MAPE\textsubscript{FIT} is only 0.3900%. Therefore, this model is used to predict nuclear consumption in the next 10 years, as shown in Table 12.

To more intuitively show the future consumption trend of nuclear energy, the data in Table 12 are converted into Fig. 12, showing clearly that the consumption of nuclear energy in North America is predicted to continue to increase, reaching 219.5282 million tons of oil equivalent in 2028, 0.9222% higher than that in 2018, but it does not reach a peak, indicating that the consumption of nuclear energy in North America will maintain an increasing trend. Therefore, North America is expected to increase the use of nuclear power and reduce the use of fossil fuels.

4.3.2. Prediction of hydroelectricity consumption in North America in the next 10 years

According to the hydroelectricity energy consumption
prediction method in Section 4.2, CCRGM(1,1) is established by data from 2009 to 2018. In this case, \( r = 2.035590 \), and the MAPEFIT is only 3.0028%. Then, the hydroelectricity consumption in the next 10 years is predicted, as shown in Table 13.

By presenting the data in Table 13 as Fig. 13, it can be clearly observed that the hydroelectricity consumption in North America shows a trend of continuous rise and fall and is predicted to reach a peak in 2022, reaching 161.9502 million tons of oil equivalent, followed by a fall to 152.7455 million tons of oil equivalent in 2029, down 4.71232% compared with 2018.

### 4.3.3. Uncertainty analysis

Two application cases show that CCRGM(1,1) model can effectively predict the consumption trend of nuclear energy and hydroelectricity in North America, but the energy situation is complex and changeable, especially with respect to the occurrence of unforeseen events, which leads to the results of prediction model might seem counterintuitive. According to literature [47], many large dams were built in North America before 1975. In recent years, environmental problems involving geological disasters and river ecology have become increasingly severe, and social problems have arisen. These problems were not foreseen, resulting in more dams being demolished by the government than are being built.
The United States Energy Information Administration noted that due to the development of wind power technology, wind power in the United States is expected to surpass hydropower for the first time in 2019, accounting for a large proportion of domestic power structure. Therefore, hydropower might show a downward trend after 2022 and the prediction results in Section 4.3.2 conform to the above analysis. To avoid the impact of such unforeseen events, CCRGM(1,1) model can be used for short-term prediction to avoid uncertainty due to medium and long-term prediction.

5. Conclusion

In this paper, based on the properties of Riccati equation with constant coefficients, the whitening equation of Verhulst model is proposed, and CCRGM(1,1) model is established. In practical cases, the eight evolution metrics of CCRGM(1,1) model are much better than those of GM(1,1), DGM(1,1), NGM(1,1), ARGM(1,1), ENGM(1,1), and Verhulst models, that is, the accuracy of CCRGM(1,1) model is the highest. In short, this paper makes two major contributions:

1. CCRGM(1,1) model optimizes the whitening equation of Verhulst grey model by mathematical equation, and optimizes the nonlinear term by SA algorithm, which reduces or even eliminates the dependence of the traditional Verhulst model on saturated S-shaped data, thus improving the accuracy and applicability of the traditional grey model. This is a generalization of traditional grey Verhulst model.

2. CCRGM(1,1) model can effectively predict the consumption of nuclear and hydroelectricity energy consumption in North America in the next decade, which will help local governments make policy decisions.

The CCRGM(1,1) model uses the classical Riccati equation to increase the nonlinearity of the Verhulst model, while the Verhulst model has a better effect on saturated S-shaped data or single-peak data, which has its own limitations [48]. The CCRGM(1,1), as an extension model, cannot completely eliminate these limitations. For clean energy data, the CCRGM(1,1) model alleviates the dependence of the Verhulst model on saturated S-shaped data to a certain extent, but this dependence has not been completely eliminated. Therefore, for countries that develop clean energy later, such as China, India and other countries, the various clean energy data of these countries may show insignificant data characteristics, and the prediction results may show large deviations. In addition, in the process of energy prediction, unforeseen events may occur, such as the global COVID-19 pandemic, which may lead to significant changes in energy consumption in the short term, resulting in a large deviation between the prediction results and the actual situation.

As a single variable grey model, CCRGM(1,1) model can effectively predict energy consumption. However, with the vigorous development of clean energy in China, the United States and other major countries, the world energy system is expected to become increasingly complex, and the factors affecting clean energy consumption are predicted to gradually increase, such as economic, population, and environmental factors. Furthermore, the influencing factors of different energy sources are different, which results in uncertainty in the prediction process. Therefore, fully elucidating the characteristics of these factors affecting different energy consumption and introducing them into CCRGM(1,1) model is a topic of interest. Via subsequent expansion of the CCRGM(1,1) to a multivariate model, the prediction effect might be further improved. How to further weaken the dependence of the Verhulst model on the trend of data change and how to build a multivariable CCRGM model for a variety of clean energy sources is our anticipated next main research direction.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Xilin Luo: Software, Visualization, Writing - original draft, Writing - review & editing, Validation, Data curation. Huiming Duan: Conceptualization, Methodology, Funding acquisition, Project administration, Supervision. Leiyuhang He: Investigation, Formal analysis, Validation, Data curation.

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