The description of multiplicity distributions in high energy $e^+e^-$ annihilation within the framework of Two Stage Model

E.S. Kokouлина
The Pavel Sukhoy Gomel State Technical University, 246746 Belarus

Abstract

The multiplicity distributions in high energy $e^+e^-$ annihilation are described within the Two Stage Model. It is shown that oscillations in sign of the ratio of factorial cumulant moments over factorial moments of increasing order are confirmed. Parameters of parton stage and hadronization of this model are analyzed.

\footnote{Talk given at the International School -Seminar ”The Actual Problems of Particle Physics”, Gomel, August 7 – 16, 2001}
1 Introduction

Multiparticle production (MP) is one of important regions in high energy physics \[1\]. Modern accelerators had made possible the intensive and detailed study of multiparticle processes. Quantum chromodynamics (QCD), developing theory of high energy physics \[2\] and a lot of phenomenologic models are tested by the process of MP. We should use phenomenologic models on account of absent total theory of QCD.

MP is begun at high energy. Among of all producing particles we can observe a lot of hadrons. So, number of producing particles can be 50 and more in $e^+ e^-$ annihilation \[3\].

On one hand we want to know high energy physics, but on the other hand the increase of the inelastic channels is making difficulty in the description of this process with customary methods by using of the diagram technics. We have similarly situation as we had at the time of developing the thermodynamics and statistical physics. The total analysis of MP process is very difficult. The consideration of such reactions begins from the analysis of behavior of charge multiplicity.

**Definition**: Multiplicity is the number of secondaries $n$ in process MP:

$$A + B \rightarrow a_1 + a_2 + \ldots + a_n$$

**Definition**: Multiplicity distribution (MD) $P_n$ is the ratio cross-sections $\sigma_n$ to $\sigma = \sum_n \sigma_n$

$$P_n = \sigma_n / \sigma.$$

This quantity has such sense: the probability of producing of $n$ charge particles in process (1).

The description MP with using statistical methods is based on a lot of particles in this process. We can construct common quantities like mean values, moments of MD, correlations and so on.

The history of study MP is very interesting. It connects on one hand with the developing theory and on the other hand with increase energy of accelerators. We have stopped on the studying MP in $e^+ e^-$ - annihilation at high energy. At accordance with theory of strong interactions QCD this MP can be realize through the production of $\gamma$– or $Z^0$–boson into two quarks:

$$e^+ e^- \rightarrow (Z^0 / \gamma) \rightarrow q \bar{q}$$

We can analyze the general features of hadronic decays up to the highest available center-of-mass (c.m.) energies. In the last years LEP produced $e^+ e^-$ -collisions of c.m. energies at 172, 183 and 189 GeV. The charge particle multiplicity can be more than 60 \[3\].

QCD can describe process of fission partons (quarks and gluons) at high energy, because the strong coupling $\alpha_s$ is small at that energy. This stage can be called as the stage of cascade. After fission, when partons have not high energy, they must change into hadrons, which we can observe. On this stage we shouldn’t apply QCD, because $\alpha_s$ is not small. Phenomenological models are used for description of hadronization (transformation partons (quarks and gluons) into hadrons).
It is expected on the stage quark-gluon cascade perturbative QCD will be applied \[4, 5\]. Certain features of the predictions at the parton level are expected to be insensitive to details of the hadronization mechanism. They can be tested directly by using hadron distributions \[6\].

The hadronization models are more phenomenological and are built on the experience gained from the study of low-$p_T$ hadron collisions. It is proposed to use Two Stage Model (TSM) \[8\] for description MD and other characteristics of $e^+e^-$ annihilation. It is usually considered that the producing of hadrons from partons is universal one. We can use these MD for calculation factorial and cumulant factorial moments and oscillations of their ratio and so on.

Investigation of MP led to discovery of jets. Jets phenomena can be studied in all processes, where energetic partons are produced. The most common ones are in $e^+e^-$ annihilation, deep inelastic scattering of $e, \mu$ or $\nu$ on nucleons and hadron- hadron scattering, involving high-$p_T$ particles in final state.

The $e^+e^-$-reaction is simple for analysis, as the produced state is pure $q\bar{q}$. It is usually difficult to determine the quark species on event-by-event basis. The experimental results are averaged over the quark type. Because of confinement the produced quark and gluons fragment into jets of observable hadrons.

### 2 Two Stage Model

Parton spectra in QCD quark and gluon jets have been studied by Konishi K., Ukawa A. and Veneciano G.\[4\]. Working at the leading logarithm approximation and avoiding IR divergences by considering finite $x$, the probabilistic nature of the problem has been established \[4\].

At the study of MP at high energy we decided to use idea A. Giovannini \[7\] for describing QCD jets as Markov branching process. Giovannini proposed to interpret the natural QCD evolution parameter as the thickness of the jets and their development through subnuclear matter as Markov process. The Markov nature of elementary process ("gluon fission", "quark bremsstrahlung" and "quark-pair creation") leads to unsuspected properties of QCD jets. The stochastic approach gives clean complete solutions for the parton MD. QCD evolution parameter is

\[
Y = \frac{1}{2\pi b} \ln[1 + ab \ln(Q^2/\mu^2)],
\]

where $2\pi b = \frac{1}{6}(11N_C - 2N_f)$ for a theory with $N_C$ colours and $N_f$ flavours.

Three elementary process contribute to the quark or gluon distributions into QCD jets:

1. gluon fission;
2. quark bremsstrahlung;
3. quark pair creation.

Let $A\Delta Y$ be the probability that gluon in the infinitesimal interval $\Delta Y$ will convert into two gluons, $\bar{A}\Delta Y$ be the probability that quark will radiate a gluon, and $B\Delta Y$ be the
probability that a quark-antiquark pair will be created from a gluon. \( A, \bar{A}, B \) are assumed to be \( Y \)-independent constants and each individual parton acts independently from others, always with the same infinitesimal probability.

Definite the probability that \( m_g \) gluons and \( m_q \) quarks will be transformed into \( n_g \) gluons and \( n_q \) quarks over a jet of thickness \( Y \) and call it \( P_{m_g,m_q;n_g,n_q}(Y) \). The probability generating function for a gluon jet and quark jet will be, respectively

\[
G(u_g, u_q; Y) = \sum_{n_g,n_q=0}^{\infty} P_{1,0;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q},
\]

\[
Q(u_g, u_q; Y) = \sum_{n_g,n_q=0}^{\infty} P_{0,1;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q}.
\]

Due to the independent action of the individual partons it can be shown through straightforward that

\[
\sum_{n_g,n_q=0}^{\infty} P_{m_g,m_q;n_g,n_q}(Y) u_g^{n_g} u_q^{n_q} = [G(u_g, u_q; Y)]^{m_g} [Q(u_g, u_q; Y)]^{m_q}.
\]

Eq. (6) says that from a probabilistic point of view the total parton population (\( m_g \) gluons and \( m_q \) quarks) evolving through thickness \( Y \) behaves as \( (m_g + m_q) \) independent parton population, each with one initial quark or gluon. Since, the process is homogeneous in \( Y \) the transition probabilities obey Chapman–Kolmogorov equations:

\[
P_{m_g,m_q;n_g,n_q}(Y + Y') = \sum_{l_g,l_q=0}^{\infty} P_{l_g,l_q;m_g,l_q,n_g,n_q}(Y) P_{l_g,l_q;0,m_q}(Y').
\]

For the gluon jet one obtains

\[
P_{0,1;n_g,n_q}(Y + Y') = \sum_{l_g,l_q=0}^{\infty} P_{1,0;l_g,l_q}(Y) P_{l_g,l_q;0,n_q}(Y').
\]

and for a quark jet

\[
P_{0,1;n_g,n_q}(Y + Y') = \sum_{l_g,l_q=0}^{\infty} P_{0,1;l_g,l_q}(Y) P_{l_g,l_q;n_g,0}(Y').
\]

Then (see eq. (10), (11))

\[
G(u_g, u_q; Y + Y') = G[G(u_g, u_q; Y'), Q(u_g, u_q; Y'); Y],
\]

\[
Q(u_g, u_q; Y + Y') = Q[G(u_g, u_q; Y'), Q(u_g, u_q; Y'); Y].
\]

Making the substitution \( Y' \rightarrow \Delta Y \), we obtain

\[
\frac{\partial G(u_g, u_q; Y)}{\partial Y} = \frac{\partial G}{\partial u_g} w^{(g)}(u_g, u_q) + \frac{\partial G}{\partial u_q} w^{(q)}(u_g, u_q),
\]

\[3\]
\[
\frac{\partial Q(u_g, u_q; Y)}{\partial Y} = \frac{\partial Q}{\partial u_g} w^{(g)}(u_g, u_q) + \frac{\partial Q}{\partial u_q} w^{(q)}(u_g, u_q),
\]

(13)

where the infinitesimal generating function for quark and gluon jets \(w^{(g)}(u_g, u_q)\) and \(w^{(q)}(u_g, u_q)\), respectively) are

\[
w^{(g)}(u_g, u_q) = \sum_{m_g, m_q=0}^\infty a^{(g)}_{m_g, m_q} u_g^{m_g} u_q^{m_q} = (-A - B) u_g + A u_g^2 + B u_q^2,
\]

(14)

\[
w^{(q)}(u_g, u_q) = \sum_{m_g, m_q=0}^\infty a^{(q)}_{m_g, m_q} u_g^{m_g} u_q^{m_q} = (-\tilde{A} u_q + \tilde{A} u_q u_g).
\]

(15)

Eqs. (12) and (13) recognize the forward Kolmogorov equations for the generation functions of the transition probabilities \(P_{m_g, m_q; n_g, n_q}(Y)\). The corresponding backward Kolmogorov equations follow from eqs. (10, 11)

\[
\frac{\partial G}{\partial Y} = w^{(g)}[G(u_g, u_q; Y), Q(u_g, u_q; Y)],
\]

(16)

\[
\frac{\partial Q}{\partial Y} = w^{(q)}[G(u_g, u_q; Y), Q(u_g, u_q; Y)].
\]

(17)

Recalling eqs. (14) and (15), eqs. (16) and (17) become

\[
\frac{\partial G}{\partial Y} = -AG + AG^2 - BG + BQ^2,
\]

(18)

\[
\frac{\partial Q}{\partial Y} = -\tilde{A}Q + \tilde{A}QG.
\]

(19)

These eqs. were obtained in [7]. We can find the probability for quark (or gluon) to produce in the interval \((Y + \Delta Y)\) \(n_g\) gluons and \(n_q\) quarks through process (1)-(3). It follows for gluon jet

\[
P_{1,0; n_g, n_q}(Y + \Delta Y) = [1 - \tilde{A} n_q \Delta Y - A n_q \Delta Y - B n_g \Delta Y] P_{1,0; n_g, n_q}(Y)
\]

\[+ \tilde{A} n_q \Delta Y P_{1,0; n_g-1, n_q} + A (n_g - 1) \Delta Y P_{1,0; n_g-1, n_q}(Y)
\]

\[+ B (n_g + 1) \Delta Y P_{1,0; n_g+1, n_q-2}(Y) + o(\Delta Y)
\]

(20)

Letting \(\Delta Y \to 0\) we get the system of differential equations:

\[
\frac{dP_{1,0; n_g, n_q}(Y)}{dY} = [\tilde{A} n_q - B n_g - A n_q] P_{1,0; n_g, n_q}
\]

\[+ \tilde{A} n_g P_{1,0; n_g-1, n_q}(Y) + A (n_g - 1) P_{1,0; n_g-1, n_q}(Y)
\]

\[+ B (n_g + 1) P_{1,0; n_g+1, n_q-2}(Y).
\]

(21)
Finding the explicit solutions both in terms of generating functions \(^{(18,19)}\) or of the exclusive cross-sections\(^{(20)}\) is not easy. Approximate solutions can be obtained for particular case \(B = 0, \tilde{A} \neq A \neq 0\).

The meaning of this approximation is next: we don’t allow gluons to split into quark-antiquark pair. Eqs. for gluon and quark jets in this case are

\[
\frac{dP_{1,0;ng,0}(Y)}{dY} = A(n_g - 1)P_{1,0;ng-1,0}(Y) - An_g P_{1,0;ng,0}(Y),
\]

\[
\frac{dP_{0,1;ng,1}(Y)}{dY} = -\tilde{A}P_{0,1;ng,1}(Y) - An_g P_{0,1;ng,1}(Y) + \tilde{A}P_{0,1;ng-1,1}(Y) + An_g P_{0,1;ng,1}(Y)
\]

with initial conditions \(P_{0,1;0,1}(0) = 1, P_{1,0;1,0}(0) = 1, P_{1,0;ng,0}(0) = 0\) and \(P_{0,1;ng,1}(0) = 0\) \(\forall n_g > 1\).

For gluon jet we have

\[
P_{1,0;ng,0}(Y) = \frac{1}{\langle n_g \rangle} \left( 1 - \frac{1}{\langle n_g \rangle} \right)^{ng-1}
\]

with a average gluon multiplicity \(\langle n_g \rangle = e^{\tilde{A}Y}\). For quark jet \(\eqref{25}\) one obtains MD

\[
P_{0,1;0,1}(Y) = e^{-\tilde{A}Y},
\]

\[
P_{0,1;ng,1}(Y) = \frac{\mu(\mu + 1)\ldots(\mu + ng - 1)}{ng!} e^{-\tilde{A}Y} (1 - e^{-AY})^{ng},
\]

where \(\mu = \frac{\tilde{A}}{A}\). The average gluon multiplicity will be \(\langle n_g \rangle = \mu(e^{\tilde{A}Y} - 1)\) and the normalized exclusive cross section for producing \(ng\) gluons

\[
\frac{\sigma_{n_g}^{(0,q)}}{\sigma_{tot}} \equiv P_{0,1;ng,1}(Y) = \frac{\mu(\mu + 1)\ldots(\mu + ng - 1)}{ng!} \left[ \frac{\langle n_g \rangle}{\langle n_g \rangle + \mu} \right]^{ng} \left[ \frac{\mu}{\langle n_g \rangle + \mu} \right]^\mu.
\]

The generating function will be given by

\[
Q = \sum_{ng=0}^{\infty} u_g^{ng} u_q P_{0,1;ng,1}(Y) = u_q \left[ \frac{e^{-AY}}{1 - u_g(1 - e^{-AY})} \right]^\mu.
\]

Eq.\(\eqref{27}\) is Polya-Egenberger distribution, where \(\mu\) is not integer.

In Two Stage Model \(\cite{8}\) we took \(\eqref{28}\) for description of parton stage and added for final stage hadronization supernarrow distribution of Bernulli (binomial distribution). We chose it based oneself on analysis experimental data in \(e^+e^-\) annihilation at 9 - 22 GeV. Second
correlation moments were negative at this energy and it was one, which could describe experiment.

We consider that hypothesis of soft bleachment is right. We added stage of hadronization to parton stage with aid of their factorization. MD in this process can be writing

\[ P_n(s) = \sum_m P^P_m P^H_n(m, s), \]  

(29)

where \(P^P_m\) - MD for partons (27), \(P^H_n(m, s)\) - MD for hadrons, are produced from \(m\) partons on the stage of hadronization.

In accordance with TSM the stage of hard fission of partons is described by negative binomial distribution (NBD) for quark jet

\[ P^P_m(s) = \frac{k_p(k_p + 1) \ldots (k_p + m - 1)}{m!} \left( \frac{\bar{m}}{m+k_p} \right)^k p \left( \frac{k_p}{k_p+m} \right)^m, \]  

(30)

where \(k_p = \tilde{A}/A\), \(\bar{m} = \sum_m m P^P_m\). We are neglecting process (3) quark pair production \((B = 0)\). Two quarks fragment to parton independently each other. Total MD of two quarks is equal to (30) too. MD \(P^P_m\) and generating function for MD \(Q^P(s, z)\) are

\[ P^P_m = \frac{1}{m!} \frac{\partial^m}{\partial z^m} Q^P(s, z) \bigg|_{z=0}, \]  

(31)

\[ Q^P_m(s, z) = \left[ 1 + \frac{\bar{m}}{k_p}(1 - z) \right]^{-k_p}. \]  

(32)

From TSM MD of hadron are formed from parton are described in form

\[ P^H_n = C^m_{N_p} \left( \frac{\tau^h_p}{N_p} \right)^n \left( 1 - \frac{\tau^h_p}{N_p} \right)^{N_p-n}, \]  

(33)

\(C^m_{N_p}\) - binomial coefficient) with generating function

\[ Q^h = \left[ 1 + \frac{\tau^h_p}{N_p}(z - 1) \right]^{N_p}, \]  

(34)

where \(\tau^h_p\) and \(N_p (p = q, g)\) have sense of average multiplicity and maximum secondaries of hadrons are formed from parton on the stage of hadronization. MD of hadrons in \(e^+e^-\) annihilation are determined by convolution of two stages

\[ P_n(s) = \sum_{m=0}^{\infty} P^P_m \frac{\partial^n}{\partial z^n}(Q^H)^{2+n} \bigg|_{z=0} \]  

(35)

Further we assume next simplification for second stage: \(\tau^h_p \approx \bar{\tau}^h_q\). Let \(\alpha = \frac{N_q}{N_g}, N = N_q, \bar{n}^h = \bar{n}^h_q\). Then

\[ Q^H_q = \left( 1 + \frac{\bar{n}^h}{N}(z - 1) \right)^N, \]
\[ Q_g^H = \left(1 + \frac{\bar{n}_q}{N}((z - 1)\right)^{\alpha N}. \]

From (35) we have

\[ P_n(s) = \sum_{m=0}^{M_g} P_m P^N_{C^{n}} \left(\frac{\bar{n}_q}{N}\right)^n \left(1 - \frac{\bar{n}_q}{N}\right)^{(2+\alpha m)N-n}. \] (36)

For comparing with experimental data the normalized factor Ω was introduced into (36) and a number of summarized gluons were restricted by \( M_g \) - maximal number of gluons are giving cascade

\[ P_n(s) = \Omega \sum_{m=0}^{M_g} P_m P^N_{C^{n}} \left(\frac{\bar{n}_q}{N}\right)^n \left(1 - \frac{\bar{n}_q}{N}\right)^{(2+\alpha m)N-n}. \] (37)

The results of comparison of model expression MD(37) with experimental data [3], [10] represented in the Table 1. Apparently that behavior of parameters of TSM is giving the reasonable picture of \( e^+e^- \) annihilation process. The average multiplicity of gluons \( \bar{m}_g \) formed on fission stage changes from 3 - 4 at 22 GeV to 15 - 16 at 189 Gev. This increase takes place with the growing energy, but at the same energy this value accepts a big one (at 43.6 GeV).

From QCD is following that the parameter \( k_p \) must decrease at more high energies. The parameter of parton spectra \( k_p \) changes: from 140 at low energy (22 GeV) up 5 - 6 at 172 - 189 GeV. QCD are giving limiting value \( k_p \) is equal to 1.

The most interesting dependencies are discovered for parameters of hadronization \( \bar{n}_q \), \( N_q \) and \( \alpha \). The parameter \( N_q \) determines maximum number of hadrons, which can be formed from quark on stage of hadronization. It changes unusually, it can be 5 - 6 or 8 - 15 at different energies, so it equal about 9 at 172 GeV, but equal 54.6 at 183 and again 11 at 189 GeV. After that analysis we can say about the existing some special energies when process hadronization changes (at those energies value of \( \chi^2 \) is changed unconsiderably).

At the same time the parameter of hadronization \( \bar{n}_q \) (mean number of hadrons formed from quark on stage of hadronization) discovers insignificant growth with increasing of energy. It is changed from 3 - 4 at 22 GeV up 4 - 5 at 172 - 189 GeV. The existing models give quantity \( n_q^h \) equal two (it is observed really at energies lower than 40 GeV). Such behavior of parameter may be connected with the growth of spectrum of mass hadron states (appearance of new mass states with increase of energy). It is interesting that the ratio \( \frac{n_q^h}{N_q} \) (the parameter of PBD) is approximately regularity and is equal to 1/2. It should mark breaking such behavior at some energies (43.6, 183 GeV) again.

Parameter \( \alpha \) is stayed approximately constant and equal to 0.2 in order case and become smaller than 0.1 at special cases (43.6 and 183 GeV). The number of hadrons are produced from quark jet on the stage of hadronization is more than from gluon jet. The normalized factor Ω is stayed constant and equal to 2.
3 Oscillation of moments in MD

At the recent years it was shown [9] that the ratio of factorial cumulative moments over factorial moments changes sign as a function of order. We can use MD formed in TSM for explanation of this phenomena.

The factorial moments can be obtained from MD $P_n$ through the relation

$$F_q = \sum_{n=q}^{\infty} n(n-1)\ldots(n-q+1)P_n,$$

and factorial cumulative moments are found from expression

$$K_q = F_q - \sum_{i=1}^{q-1} C_{q-i}^{i} K_{q-i} F_i.$$  (39)

The ratio of their quantities is

$$H_q = K_q/F_q.$$  (40)

We can use the generating function for MD of hadrons (36) in $e^+e^-$ annihilation $G(z)$

$$G(z) = \sum_{m=0}^{\infty} P^g_m [Q^H_g(z)]^m Q^2_q(z) =$$

$$= Q^g (Q^H_g(z))^m Q^2_q(z).$$  (41)
We are calculating $F_q$ and $K_q$ in TSM using (41)

$$F_q = \frac{1}{n^q(s)} \frac{\partial^n G}{\partial z^n} \bigg|_{z=0} \quad (42)$$

$$K_q = \frac{1}{n^q(s)} \frac{\partial^n \ln G}{\partial z^n} \bigg|_{z=0}. \quad (43)$$

The expression (41) for $G(z)$ after taking a logarithm

$$\ln G(s, z) = -k_p \ln[1 + \frac{m}{k_p}(1 - Q^H_g)] + 2 \ln Q^H_q$$

and the expansion to series in power on $Q^H_g$ will be

$$\ln G(s, z) = k_p \sum_{m=1} \left(\frac{m}{m+k_p}\right)^m \frac{Q^m_q}{m} + 2 \ln Q. \quad (44)$$

Inserting $Q_g$ into (44)

$$\ln G(s, z) = k_p \sum_{m=0} \left(\frac{m}{m+k_p}\right)^m \frac{1}{m} \left[1 + \frac{n^q(s)}{N} (z - 1)\right]^{amN} + 2N \ln[1 + \frac{n^q(s)}{N} (z - 1)],$$

and using (43) we obtain

$$K_q = \left(k_p \sum_{m=1} \alpha m (\alpha m - \frac{1}{N}) \ldots (\alpha m - \frac{q-1}{N}) \left(\frac{m}{m+k_p}\right)^m \frac{1}{m}ight.$$

$$\left.-2(-1)^q \frac{(q-1)!}{N^{q-1}} \right) \left(\frac{n^q(s)}{n^q(s)}\right)^q \quad (45)$$

where $\overline{n}(s)$ is the average multiplicity of hadrons in process (2). It is possible to find $F_q$ using (42)

$$F_q = \sum_{m=0} (2 + \alpha m)(2 + \alpha m - \frac{1}{N}) \ldots (2 + \alpha m - \frac{q-1}{N}) P_m \left(\frac{n^q(s)}{n^q(s)}\right)^q \quad (46)$$

with $P_m$ equal (30).

The sought-for expression for $H_q$ will be

$$H_q = \Omega_1 \sum_{m=1} \frac{k_p \alpha m (\alpha m - \frac{1}{N}) \ldots (\alpha m - \frac{q-1}{N}) \left(\frac{m}{m+k_p}\right)^m - 2(-1)^q \frac{(q-1)!}{N^{q-1}}}{(2 + \alpha m - \frac{1}{N}) \ldots (2 + \alpha m - \frac{q-1}{N}) P_m} \quad (47)$$

where $\Omega_1$ is the normalized factor. The comparison with experimental data [10] shows that (47) is describing the ratio of factorial moments. It is seen minimum at $q=5$.

The immediate calculations $H_q$ based on (38)-(40) with using MD [37] are giving very good description of the oscillation value of $H_q (\chi^2 \approx 2)$. Significant oscillations begin near region producing of $Z^0$ and can be explain by not integer values of parameters of hadronization $N_q$ and $N_g = \alpha N_q$ or by convolution of wide NBD and narrow PBD.
4 Conclusions

It is shown that TSM does not contradict to the experimental data on MD and the oscillations ratio of factorial moments (small $\chi^2$) (Figure 1).

Table 1. Parameters of TSM.

| $\sqrt{s}$ GeV | $\overline{m}$ | $k_\rho$ | $N$ | $\pi^0$ | $\alpha$ | $\Omega$ |
|----------------|-----------|----------|-----|--------|--------|--------|
| 14             | .08       | $2.4 \times 10^8$ | 27.7 | 2.87   | .97    | 2      |
| 22             | 3.01      | 4.91     | 20.2 | 4.34   | .2     | 2      |
| 34.8           | 6.58      | 6.96     | 12.5 | 4.1    | .195   | 2      |
| 43.6           | 10.9      | 39.4     | 6.1  | 2.43   | .386   | 2      |
| 91.4           | 10.9      | 7.86     | 11.2 | 4.8    | .226   | 2      |
| 172            | 20.1      | 9.11     | 9.17 | 4.34   | .195   | 1.98   |
| 183            | 13.2      | 1.48     | 54.6 | 8.9    | .086   | 2.06   |
| 189            | 15.1      | 6.9      | 11.6 | 5.15   | .215   | 2.01   |

References

[1] Manjavidze J., Sissakyan A.N. Phys.Rep. 346(2001)1.
[2] Politzer H.P. Phys.rep. 14(1974)129.
[3] OPAL coll. CERN-EP/99-178.
[4] Konishi K., Ukawa A., Veneciano G. Nucl.Phys. B157(1979)45.
[5] Dokshitzer Yu. et al. Proc.XIII Winter school.LINP(1978).
[6] Cvitanovic P., Hoyer P., Konishi K. Phys.Lett. 85B(1979)413.
[7] Giovannini A. Nucl.Phys. B161(1979)429.
[8] Kuvshinov V.I., Kokouлина E.S. Acta Phys.Pol.B13 (1982) 533; Kokouлина E.S., Kuvshinov V.I. Proc.6 Int.Conf. on HEP Problems; Dubna, D1, 2-81 (1981) JINR, 299.
[9] Kokouлина E.S. Proc. Sem. NPCS (1991,1995,1999).
[10] OPAL-coll. Z.Phys. C53 (1992) 539; DELPHY-coll. Z.Phys. C52 (1991) 271; TASSO-coll. Z.Phys. C45 (1989) 193; HRS-coll. Phys.Rev. D34 (1986) 3504.
[11] SLD-coll. Phys.Lett. B371 (1996) 149; Dremin I.M. Phys.Lett. B341 (1994) 95; Giovannini A. et al. World Scientific, Singapore (1995) 67.