Cosmology with Galaxy Clusters in the XMM Large-Scale Structure Survey

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Abstract. The upcoming XMM Large Scale Structure Survey (XMM-LSS) will provide a unique mapping of the distribution of X-ray sources in a contiguous 64 deg\textsuperscript{2} region. In particular, it will provide the 3-dimensional location of about 800 galaxy clusters out to a redshift of about 1. We study the prospects that this cluster catalogue offers for measuring cosmological parameters. We use the Press-Schechter formalism to predict the counts of clusters and their X-ray properties in several CDM models. We compute the detection efficiency of clusters, using realistic simulations of XMM X-ray images, and study how it differs from a conventional flux limit. We compute the expected correlation function of clusters using the extended halo model, and show that it is expected to evolve very little out to $z \approx 2$, once the selection function of the survey is taken into account. The shape and the amplitude of the correlation function can be used to brake degeneracies present when cluster counts alone are considered. The combination of cluster counts evolution and of the correlation function yields measurements of $\Omega_m$, $\sigma_8$ and $\Gamma$ with a precision of about 15%, 10% and 35%, respectively, in a $\Lambda$CDM model. This combination will also provide a consistency check for the $\Lambda$CDM model, and a discrimination between this model and the OCDM model. The XMM-LSS will therefore provide important constraints on cosmological parameters, complementing that from other methods such as the Cosmic Microwave Background.

Key words. X-rays: galaxies: clusters; Galaxies: clusters: general; Cosmology: cosmological parameters; Cosmology: large-scale structure of Universe; Surveys

1. Introduction

Clusters of galaxies are the most massive bound objects in the Universe and provide a powerful cosmological probe (see e.g. Borgani & Guzzo 2001 for a review). In particular, the number counts of clusters and its evolution yield a robust measure of both the amplitude of the matter power spectrum and of the geometry of the universe (e.g. Oukbir & Blanchard 1997; Eke et al. 1998; Viana & Liddle 1999; Kiyiyma & Suto 1997). The spatial correlation function of clusters quantifies the clustering of these objects and yields complementary constraints on cosmology (e.g. Mo, Jing & White 1996; Suto et al. 2000; Robinson 2000; Moscardini et al. 2000; Collins et al. 2000).

In this paper, we explore the prospects of measuring cosmological parameters with the upcoming XMM Large Scale Structure Survey (XMM-LSS; Pierre 2000). This survey consists of 10 ksec exposures of an $8 \times 8$ deg\textsuperscript{2} region with the XMM-Newton observatory, along with an extensive follow-up programme in the optical, IR and radio bands. In particular, it will provide the 3-dimensional location of about 800 clusters out to a redshift of about 1. Thanks to its uniform sensitivity across a contiguous region, this survey thus provides a unique database to measure the evolution of both the number counts and the correlation function of clusters.

To study how the clusters found in XMM-LSS can constrain cosmological models, we use the Press & Schechter (1974) formalism to predict the expected cluster counts in the survey. This is done using the selection function of the survey derived from detailed simulations of cluster detections in XMM-Newton images (see Valtchanov, Pierre & Gastaud 2001, VPG). Using the Mo & White (1996) formalism, we compute the expected spatial correlation function for the detected clusters. We then study how the cluster counts and correlation function, taken together, constrain cosmological parameters. Our results complement that of Moscardini et al. (2000) who considered the expected cluster counts and correlation function for XMM-LSS, but who assumed a simple flux limit, and did not compute the resulting joint constraints on cosmological parameters.

The paper is organized as follows. In §2 we summarize the characteristics of the XMM-LSS. In §3, we describe the simulations for cluster detection and derive the
cluster selection function. In \textsuperscript{1} we compute the expected cluster counts using the selection function combined with the Press-Schechter formalism. In \textsuperscript{2} we compute the correlation function for this cluster sample and show how it constrains cosmological parameters. Our conclusions are summarized in \textsuperscript{3}.

\subsection*{2. The XMM Large Scale Structure Survey}

The XMM-LSS Survey is a unique medium-deep cluster survey combining X-ray observations with an extensive optical, IR, and radio follow-up programme (Pierre \textsuperscript{2000}). The survey geometry - coverage and depth - was chosen to allow the measurement of the cluster two-point correlation function, with better than 15\% error on the correlation length, in two redshift intervals between \( z = 0 \) and 1.

The position of the \( 8 \times 8 \) deg\(^2 \) surveyed area on the sky (\( \alpha = 2h0 \text{m}, \delta = -5^\circ \)) is at a sufficiently high galactic latitude (\( \approx -60^\circ \)) in a region of moderate galactic absorption and without known bright X-ray sources. It will be covered by 24 \( \times \) 24 partially overlapping XMM pointings with individual exposure times of 10 ks, reaching a sensitivity of \( 3 \times 10^{-15} \) erg s\(^{-1}\) cm\(^{-2}\) in the [0.5 - 2] keV band for point sources, or of about \( 5 \times 10^{-15} \) erg s\(^{-1}\) cm\(^{-2}\) for cluster-like extended sources. Down to this limit, some 300 objects (mainly QSOs) per deg\(^2\) are expected according to the latest deep surveys (e.g. Hasinger et al. \textsuperscript{2001} Giacconi et al. \textsuperscript{2001}), and a total of about 10-15 clusters per deg\(^2\) out to \( z \approx 1 \). The survey is also well suited to probe the existence of massive clusters within the important 1 < \( z < 2 \) redshift range.

An extensive multi-wavelength follow-up programme has been undertaken by the XMM-LSS consortium\textsuperscript{1}. Special care is given to the optical identification of the X-ray sources: deep multi-color imaging of the entire region will be performed by the Canada-France-Hawaii Legacy Survey\textsuperscript{3} and subsequent redshift measurements by the VIRMOS/VLT instrument and other large telescopes to which the consortium has access. The main priorities are: (1) identification and redshift measurement of all X-ray clusters between 0 < \( z < 1 \), (2) NIR observations of distant (\( z > 1 \)) cluster candidates and, subsequently, determination of their redshift, (3) serendipitous spectroscopic observations of the X-ray QSOs, in order to study their clustering properties within the deep potential-well network traced by the clusters.

\section*{3. Simulations}

In order to estimate the detection probability of clusters in the survey, we performed a series of X-ray image simulations. While a detailed description can be found in VPG, we first review here the main features of the simulations.

\begin{itemize}
  \item We then show how they can be used to derive the selection function for the survey.
  \item Clusters of galaxies were modeled as spherically symmetric objects assuming a \( \beta \)-profile (e.g. Cavaliere & Fusco-Femiano \textsuperscript{1978}) with fixed core radius \( r_c = 250 h^{-1} \) kpc and slope \( \beta = 0.75 \). A thermal plasma spectrum (Raymond & Smith \textsuperscript{1977}) was assumed. The spectrum was normalized using the non-evolving luminosity-temperature (\( L - T \)) relation of Arnaud \& Evrard \textsuperscript{1999}. When generalized to arbitrary cosmological model, it is given by
  \begin{equation}
  L = 2.87 \times 10^{44} \left( \frac{T}{6 \text{keV}} \right)^{2.88} \left( \frac{D_L}{D_{L,\text{EdS}}} \right)^2 h^{-2} \text{erg s}^{-1},
  \end{equation}
  \end{itemize}

where \( L \) is the bolometric luminosity, \( T \) is the X-ray temperature, and \( D_L \) and \( D_{L,\text{EdS}} \) are the luminosity distances in the desired and Einstein-de Sitter cosmological models, respectively.

We took the neutral hydrogen column density to be \( N_H = 5 \times 10^{20} \) cm\(^{-2}\) and element abundances \( Z = 0.3Z_\odot \). Using XSPEC (Arnaud \textsuperscript{1996}), we calculated the total expected count rates for the extended and point-like sources for the three XMM EPIC instruments\textsuperscript{4} with thin filters in [0.5-2] keV energy band, for an integration time of 10 ks.

For each temperature and redshifts, we placed 13 clusters on a grid inside the inner 20\’ of the field-of-view. The detection was performed in the same way as in VPG, i.e. using multi-scale (wavelet) filtering assuming Poisson noise statistics (Starck & Pierre \textsuperscript{1998}), followed by \texttt{SExtractor} (Bertin & Arnouts \textsuperscript{1996}) detection and classification. As was pointed out in VPG, this is currently the most suitable method to detect, characterize and classify extended sources in XMM images. The raw photon and wavelet filtered images for clusters with \( T = 3 \) keV at a redshifts of \( z = 1 \) and 1.5 are shown in Figs. \textsuperscript{3} and \textsuperscript{4}.

To cross-identify the detected objects with the input clusters we have used a searching radius of 12\’. If a correspondence is found, we perform a classification based

\footnotesize
\begin{itemize}
  \item \textsuperscript{1} Official web page of the consortium:
    \url{http://vela.astro.ulg.ac.be/themes/spatial/xmm/LSS/}
    \url{http://cdsweb.u-strasbg.fr:2001/Instruments/Imaging/}
    \url{http://xmm.vilspa.esa.es/user/uhb/xmm_uhb.html}
  \item \textsuperscript{3} See e.g. the XMM-Newton User’s Handbook:
    \url{http://xmm.vilspa.esa.es/user/uhb/xmm_uhb.html}
\end{itemize}

\normalsize
on the half-light radius and the stellarity index, to determine whether the object is extended. (For the choice of the searching radius and the classification criteria, see VPG). A cluster is finally considered to be detected if the positional and the classification criteria are obeyed. This procedure is close to the planned analysis of the incoming XMM data, which will make use of the multi-color optical data to derive cluster positions.

\[ \phi(T, z) = N_{\text{det}}(T, z)/N_{\text{in}}(T, z) \]  

(2)

Fig. 3 shows the resulting selection function, which gives the probability for a cluster with temperature \( T \) and redshift \( z \) to be detected and classified as an extended object in the survey catalogue. Approximately, 90% of all clusters with \( T > 3 \) keV are detectable out to \( z \sim 0.6 \). The selection function is close to about 1 for \( T > 2 \) keV and \( z < 0.5 \) (not shown). Since we are only interested in clusters (and groups), we set the selection function to 0 for \( T < 2 \) keV at all redshifts. This corresponds to a minimum luminosity of about \( 10^{44} \text{ erg s}^{-1} \) (see Eq. [1]) and can thus be easily implemented in practice. As can be seen on the figure, low temperature clusters become progressively harder to detect as the redshift increases (compare Figs. 1 and 2). For example, at \( z = 2 \) only clusters with \( T > 6 \) keV yield a completeness better than 90%.

It is instructive to compare our selection function to that corresponding to a constant flux limit, as assumed in many previous studies. The flux \( S_{[0.5-2]} \) of a cluster in the \([0.5-2]\) keV band at redshift \( z \) can be derived from its temperature using the \( L - T \) relation (Eq. [1]), the Raymond-Smith spectrum and the luminosity-distance relation. Using this correspondence, we can express the selection function in terms of the flux rather than temperature. The resulting selection function \( \phi(S_{[0.5-2]}, z) \) is shown in Fig. 4. Our selection function clearly does not exactly correspond to a sharp flux limit. Instead, the completeness is a smooth function of flux and depends on the redshift. As is conspicuous on Fig. 4 an exact and constant flux limit of \( S_{[0.5-2]} = 5 \times 10^{-15} \text{ erg s}^{-1} \text{ cm}^{-2} \) would clearly overestimate the detection efficiency, especially for brighter clusters at large redshifts.

In the following, we will use our selection function which includes most of the relevant instrumental and observational limitations. For comparison, we will also consider the selection function corresponding to the above flux limit, along with that corresponding to temperature limit \( T > 2 \) keV. The latter selection is useful to study the effect of the removal of small clusters on our predictions.

4. Cluster Counts

We first compute the expected cluster number counts in the survey. This is done using the cluster selection function derived in the previous section combined with the

\[ \phi(T, z) = N_{\text{det}}(T, z)/N_{\text{in}}(T, z) \]  

(2)

Fig. 3. Selection function \( \phi(T, z) \) for \( \Lambda \)CDM model derived from the simulations. This function is the probability that a cluster with temperature \( T \) and redshift \( z \) is detected and classified as an extended object in the survey catalogue.

Fig. 4. Selection function \( \phi(S, z) \) expressed in terms of the flux \( S \) in the \([0.5-2]\) keV band for \( \Lambda \)CDM model. The vertical line shows the flux limit of \( 5 \times 10^{-15} \text{ erg s}^{-1} \text{ cm}^{-2} \) usually assumed for the extended objects detection in the survey; it illustrates deviations from a strict flux limit when realistic observing conditions are taken into account (esp. source confusion).
Schechter formalism. We first briefly review the main assumptions involved in our calculation of the mass function and of the temperature function of clusters. We then compute the expected projected number of clusters on the sky, as function of redshift. Finally, we show how the resulting redshift distribution constrains cosmological parameters within CDM models.

4.1. Mass Function

The Press-Schechter formalism provides an analytic expression for the abundance of dark matter halos (Press & Schechter 1974). At a given redshift $z$, the differential number of dark matter halos of mass $M$ per unit comoving volume is

$$\frac{dn}{dM} = \sqrt{\frac{2}{\pi}} \frac{\rho}{M} \frac{d\nu}{dM} e^{-\frac{\nu^2}{2}}$$

(3)
where $\bar{\rho}$ is the present mean matter density. The peak height is defined as $n(M) = \delta_c/\sigma(M)$, where $\sigma(M)$ is the linear rms fluctuation in a sphere containing a mean mass $M$. We compute $\sigma(M)$ for an arbitrary cosmological model by integrating the linear power spectrum $P_{lin}(k)$ derived from the BBKS transfer function (Bardeen et al. 1986; with the conventions of Peacock & Dodds 1997), evolved with the linear growth factor $D(z)$. The density threshold $\delta_c$ depends weakly on cosmology (i.e. on $\Omega_m$ and $\Omega_\Lambda$) and was computed using the fitting formulae of Kitiyama & Suto (1996).

### 4.2. Temperature Function

The X-ray temperature of a cluster at redshift $z$ is taken to be the virial temperature which is given by (see e.g. Eke, Cole & Frenk 1996)

$$kT \simeq \frac{7.75}{\beta_e} (1 + z) \Omega_m^{\frac{7}{4}} \left( \frac{M}{M_{15}} \right)^{\frac{2}{3}} \left( \frac{\mu}{0.59} \right) \left( \frac{\Delta_c}{178} \right)^{\frac{1}{2}} \text{keV},$$

where the average virial overdensity $\Delta_c(z, \Omega_m, \Omega_\Lambda)$ can be evaluated using the fitting formulae of Kitiyama & Suto (1996), $M_{15} = 10^{15} h^{-1} M_\odot$ and the value $\mu = 0.59$ for the number of particles per proton mass corresponds to a hydrogen mass fraction of 76%. The factor $\beta_e$ is equal to about 1 for a truncated singular isothermal sphere. We adopt this value as it provides a good fit to numerical simulations (Eke, Cole & Frenk 1996; Bryan & Norman 1997).

Combining Eqs. (3) and (4) we can derive the differential temperature function

$$\frac{dn}{dT} = \frac{dn}{dM} \frac{dM}{dT}. \quad (5)$$

It is often more convenient to consider the number density of clusters with temperatures above a given minimum, $n(T) = \int_T^{\infty} \frac{dT}{dN} dn(T)$. To illustrate the dependence of our prediction on cosmological parameters, we consider the three cosmological models listed in Tab. 1, i.e. a tilted ($\tau$) CDM, $\Lambda$CDM and OCDM. The normalization of these models is determined by $\sigma_8$, the amplitude of mass fluctuations on $8h^{-1}$ Mpc scale. Our chosen numerical values correspond to the constraints derived from current cluster surveys, namely $\sigma_8 \simeq 0.52^{0.052+0.13}_0$ for the flat case and $\sigma_8 \simeq 0.529^{0.046+0.101}_0$ for the open case (Eke et al. 1996). The shape of the matter power spectrum is controlled by the shape parameter $\Gamma$ which, unless otherwise specified, we fix at 0.23, as indicated by galaxy clustering surveys (see Viana & Liddle 1996 and reference therein).

### 4.3. Projected Cluster Counts

From the temperature function $dT/dT$ (Eq. 3), we can compute the projected surface density of clusters on the sky. Noting that the comoving volume element is $dV = R^2 d\chi d\Omega$, where $d\Omega$ is the infinitesimal solid angle, $\chi$ is

| Table 1. Cosmological Models |
|-----------------------------|
| Model | $h$ | $\Omega_m$ | $\Omega_\Lambda$ | $\sigma_8$ | $\Gamma$ |
| $\tau$CDM | 0.5 | 1 | 0 | 0.52 | 0.23 |
| $\Lambda$CDM | 0.7 | 0.3 | 0.7 | 0.93 | 0.23 |
| OCDM | 0.7 | 0.3 | 0 | 0.87 | 0.23 |

the comoving volume radius and $R(\chi)$ is the comoving angular-diameter radius, we find that the number of clusters per unit solid angle, temperature and redshift interval is

$$\frac{dN}{dT \, dz} = R^2 \frac{d\chi}{dz} \frac{dn}{dT}, \quad (6)$$

where $\frac{d\chi}{dz} = -n_0 \left( \frac{1}{a} \right)^2 \left[ (1 - \Omega) a^{-2} + \Omega_m a^{-3} + \Omega_\Lambda \right]^{-\frac{1}{2}}$ as derived from the Friedmann equations. As a result the observed surface density of clusters per unit redshift interval

$$\frac{dN_{\text{obs}}}{dz} = \int dT dN \phi(T, z), \quad (7)$$

where $\phi(T, z)$ is the survey selection function (Eq. B).

### 4.4. Predictions

The predicted projected counts as a function of redshift are shown on Fig. 4 for the three cosmological models whose parameters are listed in Tab. 1. The counts correspond to the full 64 deg$^2$ of the completed XMM-LSS survey. The counts from the three models agree at low redshifts ($z < 0.2$), as expected since the three models were normalized with the number of clusters in the local universe. On the other hand, the number counts differ greatly at larger redshifts. The $\tau$CDM model predicts much smaller number of clusters at $z > 0.2$, while the $\Lambda$CDM and OCDM models differ for $z > 0.6$. The predicted counts at $z > 1$ are larger in the OCDM model compared to that in the $\Lambda$CDM case, due to the somewhat slower evolution of the growth factor in the open model.

These differences illustrate the well known fact that the evolution of cluster counts is a powerful probe of $\Omega_m$ and $\Omega_\Lambda$.

In the 64 deg$^2$ of the survey, the expected number of detected clusters with $0 < z < 1$ is about 900, 1000 and 175 for the $\Lambda$CDM, OCDM and $\tau$CDM model, respectively. For $1 < z < 2$, the expected counts are about 400, 900 and 10, for each model respectively. Fig. 2 shows how these predictions for the $\Lambda$CDM model depend on $\sigma_8$ and $\Omega_m$. Clearly, the number counts are very sensitive on these two parameters. Taking $\sigma_8 = 0.93 \pm 0.07$ for $\Omega_m = 0.3$ (Eke et al. 1996), the expected number of clusters in this model is between 600-1200 for $0 < z < 1$ and 200-700 for $1 < z < 2$.

The effect of the selection function on these predictions are shown in Fig. 3. The number counts for the $\Lambda$CDM model are shown as a function of redshift, for our selection function $\phi(T, z)$ (as in Fig. 3), for the flux limited case and for the temperature limited case. The flux limited counts overpredict the counts for $0.4 < z < 1.4$, as expected from
Fig. 5. Projected number counts of clusters as a function of redshift in the three cosmological models. The selection function $\phi(T, z)$ for the XMM-LSS derived from image simulations was used for each model.

Fig. 6. Projected number counts of clusters for the $\Lambda$CDM model with different selection criteria: with the selection function $\phi(T, z)$ (as in figure 5), with a temperature limit only ($kT > 2$ keV), and with a flux limit ($S_{0.5-2} > 5 \times 10^{-15}$ erg s$^{-1}$ cm$^{-2}$).

Fig. 7. Cluster counts expected for the XMM-LSS as a function of $\sigma_8$ and $\Omega_m$ in the $\Lambda$CDM model. The XMM-LSS simulated selection function was used in all cases. Counts for the $0 < z < 1$ and $1 < z < 2$ redshift intervals are shown as the solid and dashed lines, respectively. In each case, models with $\Omega_m = 0.4$, 0.3 and 0.2, are shown from top to bottom, respectively.

4.5. Cosmological Constraints from Cluster Counts

We now wish to study how the cluster counts can be used to constrain cosmological parameters. For this purpose, we generated cluster counts from the predicted counts from Eq. (6) in $\Lambda$CDM model for several redshift bins (as shown in Fig. 6 for different models). We used the XLSS selection function $\phi(T, z)$ and survey area, and considered redshift bins of width $\Delta z = 0.1$ from $z = 0$ to $z = 1$ assuming Poissonian errors for the number counts in each bin. Note that the Poisson errors neglect the uncertainty arising from cosmic variance; this is a good approximation since this latter uncertainty is small for a survey area as large as that of the XLSS (compare the correlation length $\sim 20 h^{-1}$ Mpc to the survey size, $\sim 200 h^{-1}$ Mpc at $z \approx 0.5$). We then calculated the $\chi^2$-difference between the simulated counts and that expected in a $\Lambda$CDM model for a range of values for $\sigma_8$ and $\Omega_m$. This approach is rather computer intensive, but it is more accurate than the faster Fisher matrix method, which approximates the
confidence regions as ellipses and can be incorrect when the parameter space is non-Gaussian (Holder et al. 2001).

The resulting constraints on these cosmological parameters are shown as the solid lines in Fig. 8. The input ΛCDM model is that of Tab. 1 and is shown with a cross. The 90% and 95% confidence regions are shown as solid lines. The cluster counts alone will provide tight constraints on both parameters, with 95% uncertainties of about 0.06 and 0.05 for $\sigma_8$ and $\Omega_m$, respectively. As can be seen from the elongation of the contours, the two parameters are however somewhat degenerate. Any additional information on either parameters, can thus be used to reduce their respective uncertainty.

It is also interesting to investigate how cluster counts alone can constrain $\Gamma$, the shape parameter of the matter power spectrum. According to the Mo & White (1996) halo formalism, the correlation function of two sets of clusters with masses $M$ and $M'$ and with comoving separation $r$ is given by

\[
\xi(r, M, M', z) \simeq b(M, z)b(M', z)\xi_{\text{lin}}(r, z),
\]

where $\xi_{\text{lin}}(r, z) = \xi_{\text{lin}}(r, 0)D^2(z)$ is the linear correlation function which is the Fourier Transform of the linear power spectrum $P_{\text{lin}}(k, z)$. The mass dependent bias parameter of the halos is given $b(M, z) = 1 + (\nu^2 - 1)/\delta_c$, with the conventions of $\Omega$. Note that the separation of the clusters is assumed to be small compared to the scale in which any evolution takes place.

It is easy to show that the resulting observed correlation function in a narrow redshift interval is given by (Suto et al. 2000; Moscardini et al. 2000 and reference therein)

\[
\xi_{\text{obs}}(r, z) = b_{\text{eff}}^2(z)\xi_{\text{lin}}(r, z),
\]

5. Cluster Correlation function

We now turn to the cluster correlation function which quantifies the spatial clustering of clusters in the survey. We first use the extended halo model of Mo & White (1996) to predict the observed cluster correlation function at a given redshift. We then consider the average correlation function in a finite redshift range, compute its uncertainties, and study the constraints its measurement can place on cosmological parameters.

5.1. Observed Correlation Function : at a single redshift

According to the Mo & White (1996) halo formalism, the correlation function of two sets of clusters with masses $M$ and $M'$ and with comoving separation $r$ is given by

\[
\xi(r, M, M', z) \simeq b(M, z)b(M', z)\xi_{\text{lin}}(r, z),
\]

where $\xi_{\text{lin}}(r, z) = \xi_{\text{lin}}(r, 0)D^2(z)$ is the linear correlation function which is the Fourier Transform of the linear power spectrum $P_{\text{lin}}(k, z)$. The mass dependent bias parameter of the halos is given $b(M, z) = 1 + (\nu^2 - 1)/\delta_c$, with the conventions of $\Omega$. Note that the separation of the clusters is assumed to be small compared to the scale in which any evolution takes place.

It is easy to show that the resulting observed correlation function in a narrow redshift interval is given by (Suto et al. 2000; Moscardini et al. 2000 and reference therein)

\[
\xi_{\text{obs}}(r, z) = b_{\text{eff}}^2(z)\xi_{\text{lin}}(r, z),
\]
where the effective bias is

$$b_{\text{eff}}(z) = \int dM \frac{dn_{\text{obs}}}{dM} b(M) \Big/ \int dM \frac{dn_{\text{obs}}}{dM},$$

(10)
and where the observed differential number counts are given by $\frac{dn_{\text{obs}}}{dM} = \frac{d\xi_{\text{obs}}}{dM} \phi(T, z)$. Note that, in our analysis, we neglect redshift-space distortions which were shown to yield only about 10% corrections on the amplitude of the correlation function (Suto et al. 2000; Moscardini et al. 2000).

In general, the evolution of $\xi_{\text{obs}}(r, z)$ is determined by two competing effects. First, the growth of structures induces the linear correlation function $\xi_{\text{lin}}(r, z)$ to decrease as the redshift increases. On the other hand, the clusters which are detectable at large redshifts are more massive and therefore more strongly biased. The effective bias $b_{\text{eff}}(z)$ thus tends to be larger at high redshift. To study the interplay between these two effects it is convenient to define

$$b_{\text{eff}, 0}(z) \equiv \left( \frac{\xi_{\text{obs}}(r, z)}{\xi_{\text{lin}}(r, 0)} \right)^{\frac{1}{2}} = b_{\text{eff}}(z) D(z).$$

(11)

This quantity provides the bias of the observed cluster correlation with respect to the linear correlation function at $z = 0$, and therefore quantifies the evolution of the correlation function.

The behaviour of $b_{\text{eff}, 0}(z)$ is shown on Fig. 10 for the three cosmological models and for the XMM-LSS selection function. The curves are remarkably flat for all models, showing that the cluster correlation function evolves only very weakly from $z = 0$ to $z = 2$. It is interesting to study whether this lack of evolution depends on the selection function. Fig. 11 shows $b_{\text{eff}, 0}(z)$ for each selection scheme for the $\Lambda$CDM model. The evolution is also very weak for the temperature limited case. For the flux-limited sample, the evolution is somewhat stronger for $z > 1$. This evolution is lost when the full selection function $\phi(T, z)$ is used. In all cases, there is effectively no evolution for $0 < z < 1$. We shall thus, in the following, derive the constraints on cosmology integrating the correlation function over the $0 < z < 1$ range.

5.2. Observed Correlation Function: Redshift Average

To maximize the sensitivity, it is useful to measure the correlation function over a wide redshift range. In this case, we must take into account the pair-weighted evolution of the correlation function $\overline{\xi}(r, z)$ within the light-cone section of interest. The average correlation function in the redshift interval $z_{\text{min}} < z < z_{\text{max}}$ is thus given by (Suto et al. 2000; Moscardini et al. 2000 and reference therein)

$$\overline{\xi}_{\text{obs}}(r) = \int_{z_{\text{min}}}^{z_{\text{max}}} dz \frac{dz}{dz} \frac{R^2}{R_{\text{d}}^2} n_{\text{obs}}(z) \xi_{\text{obs}}(r, z)$$

$$= \int_{z_{\text{min}}}^{z_{\text{max}}} dz \frac{dz}{dz} \frac{R^2}{R_{\text{d}}^2} n_{\text{obs}}(z),$$

(12)

where the observed number density of clusters is $n_{\text{obs}}(z) = \int dT \frac{dT}{dT} \phi(T, z)$.

The resulting correlation functions for the three cosmological models are shown on Fig. 12. The XMM-LSS selection function was used in all cases, along with a redshift range of $0 < z < 1$. The correlation functions for the 3 models have very similar shape, as expected since the same value for $\Gamma$ was assumed in all cases. Notice that the $\tau$CDM correlation function has a lower amplitude than that for the other two models. This is expected since the former model has a lower value for $\sigma_8$ (see Tab. 1). The $\Lambda$CDM and OCDM models have very similar values of $\sigma_8$ and thus yield correlation functions with very similar amplitudes.
5.3. Uncertainties

We now wish to estimate the uncertainty involved in measuring the cluster correlation function. The simplest way to measure the correlation function is to count the number of pairs in excess of random in the survey volume. (This is sometimes taken as the definition of the correlation function). An estimator for \( \xi_{\text{obs}}(r) \) in a bin of radii between \( r \) and \( r + \Delta r \) is thus

\[
\hat{\xi}_{\text{obs}}(r, \Delta r) = \frac{N_{\text{obs}}^{\text{pairs}}(r, \Delta r)}{N_{\text{ran}}^{\text{pairs}}(r, \Delta r)} - 1, \tag{13}
\]

where \( N_{\text{obs}}^{\text{pairs}} \) and \( N_{\text{ran}}^{\text{pairs}} \) are the number of pairs (i.e. with clustering) and for a random distribution (i.e. without clustering), respectively, in the observed radius interval. Ignoring boundary effects, the random number of pairs is given by

\[
N_{\text{ran}}^{\text{pairs}}(r, \Delta r) \approx 2\pi Ar^2 \Delta r \int_{z_{\text{min}}}^{z_{\text{max}}} dz \frac{d\chi}{dz} R^2 n_{\text{ran}}^2(z), \tag{14}
\]

where \( A \) is the solid angle of the survey.

For weak signals, the error in measuring \( \hat{\xi}_{\text{obs}}(r, \Delta r) \) will be dominated by Poisson statistics, and will thus be given by (see e.g. Peebles 1980)

\[
\sigma[\hat{\xi}_{\text{obs}}(r, \Delta r)] \approx \frac{1 + \xi(r)}{N_{\text{obs}}^{\text{pairs}}(r, \Delta r)} = \sqrt{\frac{1 + \xi(r)}{N_{\text{ran}}^{\text{pairs}}(r, \Delta r)}}. \tag{15}
\]

This provides us with an expression for the error in the correlation function for a finite survey. The resulting errors for the \( \Lambda \)CDM model and for the full 64 deg\(^2\) of the XMM-LSS are shown in Fig. 12.

5.4. Combined Cosmological Constraints

We now study how the measurement of the cluster correlation function constrains cosmological parameters. As for the cluster counts (§5.3), we use a \( \chi^2 \)-fit to simulated measurements of the correlation function to derive confidence regions in parameter space. We considered a redshift interval of \( 0 < z < 1 \) and computed the errors using Eq. (13).

The resulting confidence contours for a joint fit of \( \Omega_m \) and \( \sigma_8 \) are shown as the dashed lines in Fig. 8 for the \( \Lambda \)CDM model. The constraints from the correlation function are rather weak on this plane alone, with little dependence on \( \sigma_8 \). These constraints are however somewhat orthogonal to that from cluster counts and are thus complementary.

The constraints from the correlation function on \( \Gamma \) and \( \Omega_m \) are shown in Figure 13. The confidence contours show little dependence on \( \Omega_m \) and are therefore nearly orthogonal to that from cluster counts alone. With the combined counts and correlation function constraints, \( \Omega_m, \sigma_8 \) and \( \Gamma \) can be measured with a precision of about 15%, 10%, and 35%, respectively at the 95% confidence level. The cluster correlation function thus provides the required information to break the degeneracy present when cluster counts alone are considered.

Until now, we have focused on the \( \Lambda \)CDM model, thus implicitly assuming prior knowledge that the universe is flat. While flatness is strongly indicated by measurements of the Cosmic Microwave Background anisotropies (e.g. Jaffe et al. 2001), it is interesting to establish whether our predictions depend on this assumption. To study this, we fitted an OCDM model to the \( \Lambda \)CDM predictions for both the cluster counts and the correlation function. The resulting constraints on \( \sigma_8 \) and \( \Omega_m \) are shown in Fig. 13. For the cluster counts, the best fit values for both of these parameters are now biased (compare the best fit, cross, to the input values, triangle). This bias again hampers the determination of \( \Omega_m \) using cluster counts alone. Thanksfully, the correlation function can again help overcome this limitation. Indeed, the contours for \( \xi(r) \) are now somewhat incompatible with that from the counts (compare to Fig. 8). The consistency between the counts and correlation function constraints can thus be used as a diagnostic and as a discriminant between the \( \Lambda \)CDM and the OCDM model.

6. Conclusions

Following the REFLEX survey (Guzzo et al. 1999, Böhringer et al. 2001) based on the ROSAT All-Sky-Survey, the XMM-LSS survey will be some 1000 times deeper and thus opens wide prospects for cosmology. Indeed, it will provide an independent measurement of cosmological parameter and thus complement Cosmic Microwave Background and Supernova experiments. It will also provide a test of important ingredients of the standard cosmological model, such as the gravitational instability paradigm and the gaussianity of initial fluctuations.
Within a ΛCDM model, we found that the redshift dependence of the XMM-LSS cluster counts will allow us to measure σ8 and Ωm, provided that the shape parameter Γ is known. In the absence of prior knowledge on Γ, the precision on these parameters degrades considerably, if only cluster counts are considered.

Clusters counts beyond z > 0.5 depend on cosmological parameters and can thus be used to constrain models. Within a ΛCDM model, we found that the redshift dependence of the XMM-LSS cluster counts will allow us to measure σ8 and Ωm with a precision of about 6% and 18% (95% CL), respectively, if the shape parameter Γ is known. In the absence of prior knowledge on Γ, the precision on these parameters degrades considerably, if only cluster counts are considered.

This limitation can be circumvented by considering the cluster correlation function. One of the strength of XMM-LSS is indeed to offer a uniform coverage over a wide contiguous area (64 deg²), with an extensive spectroscopic follow-up. This will allow us to measure the correlation function in several redshift bins out to z = 1. Using the extended halo formalism of Mo & White (1996), we computed the correlation function of clusters detectable in XMM-LSS. We find that, for the selection function of XMM-LSS, the correlation function is not expected to evolve significantly from z = 0 to 2. This results from the competing effects of the growth of mass perturbations and the stronger bias of the detectable massive clusters at large redshifts. It will thus be important to compare the correlation function measured in two redshift bins between z = 0 and z = 1: the verification of the lack of evolution provides a test of the bias model for haloes and of the gravitational instability paradigm.

The amplitude and shape of the cluster correlation function can be used to lift the degeneracies present when cluster counts alone are considered. Within a ΛCDM model, the correlation function function measured in XMM-LSS at 0 < z < 1, combined with the cluster counts, will constrain Ωm, σ8 and Γ with a precision of about 15%, 10% and 35%, respectively (95% CL). Moreover, the combination of the counts and the correlation function will provide a consistency check for the ΛCDM model, and a discrimination between this model and the OCDM model.

In our analysis, we have assumed that the L − T and M − T relations were known. By the time the XMM-LSS is completed, better insights will be gained in this respect, thanks to deep pointed cluster observations. It would nevertheless be useful to study how our results are affected by the uncertainties in these relations (see the analysis of Diego et al. 2001). In particular, it would be important to study how physical processes, such as feedback from early star formation and reionization, can modify these relationships and therefore yield detectable changes in the cluster counts and correlation function (see e.g. Muanwong et al. 2001 and reference therein). Another important issue is the existence of high redshift cooling flows which may affect the detectability of distant clusters and their L − T relationship (e.g. Henry 2000). Finally, the behaviour of the correlation function as a function of both cluster mass and redshift deserves further investigation. The study of these pending questions is left for future work.

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