A Dynamical Classification of the Cosmic Web

J.E. Forero-Romero 1, Y. Hoffman 2, S. Gottlöber 1, A. Klypin 3 & G. Yepes 4

1 Astrophysikalisches Institut Potsdam, An der Sternwarte 16, D-14482 Potsdam, Germany
2 Racah Institute of Physics, Hebrew University, Jerusalem 91904, Israel
3 Department of Astronomy, New Mexico State University, Box 30001, Department 4500, Las Cruces, NM 880003, USA
4 Grupo de Astrofísica, Universidad Autónoma de Madrid, Madrid E-28049, Spain

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ABSTRACT

A dynamical classification of the cosmic web is proposed. The large scale environment is classified into four web types: voids, sheets, filaments and knots. The classification is based on the evaluation of the deformation tensor, i.e. the Hessian of the gravitational potential, on a grid. The classification is based on counting the number of eigenvalues above a certain threshold, $\lambda_{th}$, at each grid point, where the case of zero, one, two or three such eigenvalues corresponds to void, sheet, filament or a knot grid point. The collection of neighboring grid points, friends-of-friends, of the same web attribute constitutes voids, sheets, filaments and knots as web objects.

A simple dynamical consideration suggests that $\lambda_{th}$ should be approximately unity, upon an appropriate scaling of the deformation tensor. The algorithm has been applied and tested against a suite of (dark matter only) cosmological N-body simulations. In particular, the dependence of the volume and mass filling fractions on $\lambda_{th}$ and on the resolution has been calculated for the four web types. Also, the percolation properties of voids and filaments have been studied.

Our main findings are: (a) Already at $\lambda_{th} = 0.1$ the resulting web classification reproduces the visual impression of the cosmic web. (b) Between $0.2 \lesssim \lambda_{th} \lesssim 0.4$, a system of percolated voids coexists with a net of interconnected filaments. This suggests a reasonable choice for $\lambda_{th}$ as the parameter that defines the cosmic web. (c) The dynamical nature of the suggested classification provides a robust framework for incorporating environmental information into galaxy formation models, and in particular the semi-analytical ones.

1 INTRODUCTION

The large scale structure of the Universe, as depicted from galaxy surveys, weak lensing mapping and numerical simulations, shows a web like three dimensional structure. There are three features that can be generally observed. First, most of the volume resides in underdense regions; second, most of the volume is permeated by filaments; third, the densest clumps are located at the intersection of filaments (Bond et al. 1996). This motivates a classification of the cosmic web into at least three categories: voids (underdense regions), filaments and knots (densest clumps).

There are clear evidences for the correlations of the observed properties of galaxies with the environment. We have for instance the morphology density correlation, where elliptical galaxies are found preferentially in crowded environments, and spiral galaxies are found in the field (Dressler 1980). The same kind of correlation can be found in terms of the colors of the galaxies (Blanton et al. 2005).

According to the current paradigm of structure formation galaxies form and evolve in dark matter (DM) halos (White & Rees 1978). It follows that the study of such environmental dependence should commence with the effort to understand the formation of DM halos in the context of the cosmic web (Gao et al. 2005, Avila-Reese et al. 2005, Maulbetsch et al. 2007). This motivates us to search for a robust and meaningful method to classify the different environments in numerical simulations. Such a classification should provide the framework for studying the environmental dependence of galaxy formation.

Translating the visual impression into an algorithm that classifies the local geometry into different environments is not a trivial task. A somewhat less challenging, yet very closely related, task is that of identifying just the voids out of the cosmic web. A thorough review and comparison of different algorithms of void finders has been recently presented in Colberg et al. (2008). The void finders can be classified according to the method employed. Most are based on the point distribution of galaxies or dark matter (DM) halos and some on the smoothed density or potential fields. Some of the finders are based on spherical filters while others assume no inherent symmetry (Brunino et al. 2007, Colberg et al. 2008).
It should be emphasized that an environment finder should be evaluated by its merits and it cannot be labeled as correct or wrong. A good algorithm should provide a quantitative classification which agrees with the visual impression and it should be based on a robust and well defined numerical scheme. Yet, it is desirable for an algorithm to be based on an analytical prescription so that its outcome can be calculated analytically. Also, simplicity is always very highly desired.

A variety of approaches have been employed in the classification of the cosmic environment into its basic elements. The simplest way is based on the association of the environment with the local density, evaluated with a top-hat filter, say, of some width (Lemson & Kauffmann 1999). The density field can be analyzed in a much more sophisticated and elaborated way. This is the case of the web classification based on the multi-scale analysis of the Hessian matrix of the density field (Aragón-Calvo et al. 2007) or the skeleton analysis of the density field (Novikov et al. 2006; Sousbie et al. 2008). Both methods classify the cosmic web by pure geometrical tools applied to the density field. A very different approach is done within a dynamical framework in which the analysis of the gravitational potential is used to classify the web. This has been inspired by the seminal work of Zeldovich (1970) that led to the "Russian school of structure formation" (e.g. Arnold et al. 1982; Klypin & Shandarin 1983). The quasi-linear theory of the Zeldovich approximation predicts the existence of an infinitely connected web of pancakes (i.e. sheets), filaments and knots. This morphological classification is based on the study of the eigenvalues of the deformation tensor, namely the Hessian matrix of the linear gravitational field.

A recent application of the Zeldovich-based classification has been provided by Lee & Lee (2008) who used a Wiener filter linear reconstruction of the local density field and evaluated the linear deformation, and hence also the shear, tensor on a grid. The cosmic web has been classified according the structure of the shear tensor. A different approach has been followed by Hahn et al. (2007) who suggested that the full non-linear gravitational potential should be used for the geometrical classification. Apart from the difference between the linear and the non-linear potential, both Hahn et al. (2007) and Lee & Lee (2008) are using the same classification. Namely, the Hessian of the gravitational potential is evaluated on a grid and its eigenvalues are examined locally. A grid point is classified as a void, sheet, filament or knot point if the number of eigenvalues greater than a null threshold is zero, one, two or three.

The algorithm presented here is an extension and improvement of the one suggested by Hahn et al. (2007). The extension is represented in the selection of a new free parameter with a dynamical interpretation. As such it provides a classification of local environment. Namely each spatial point is flagged as belonging to a either a void, sheet, filament or a knot point. It is the collective classification of all points in space which gives rise to the geometrical construction we call the cosmic web. This opens the door for defining voids, sheets, filaments and knots as individual objects.

Each object is defined as a collection of connected points having the same environmental attribute. We use a friends-of-friends (FoF) algorithm to find connected sets of points in simulations. Having defined the web objects the statistical and dynamical properties of these can be readily studied. Here we shall focus on analyzing the statistical properties of the voids and filaments, aiming to better constrain the value for the free parameter we introduce. We perform as well a statistical study to assess effect of the cosmic variance on our conclusions.

This paper is organized as follows. The web classification scheme is described in §2. The N-body simulation used in the paper and the numerical implementation of the web classification are presented in §3. §4 describes the main properties of the cosmic web and in particular its dependence on the smoothing scale and the free parameter of our classification scheme. §5 concentrates on the properties of the voids sector of the cosmic web. §6 studies the fragmentation of filaments in order to give a confidence interval to the free parameter that was introduced. In §7 we revisit the sections §4 and §5 to study the effect of cosmic variance. The paper concludes with a general discussion and a summary of the main results of the paper (§8).

### 2 WEB CLASSIFICATION

Hahn et al. (2007) have recently suggested a new dynamical classification of the cosmic web. The basic idea of their approach is that the eigenvalues of the deformation tensor determine the geometrical nature of each point in space.

The deformation tensor, $T_{\alpha\beta}$, is defined by the Hessian of the gravitational potential $\phi$:

$$T_{\alpha\beta} = \frac{\partial^2 \phi}{\partial r_{\alpha} r_{\beta}}.$$  \hspace{1cm} (1)

The definition of the deformation tensor explicitly assumes that the matter density field is known and that it is smoothed with a finite kernel, or otherwise the derivatives are not defined. For simplicity the smoothed density field is defined over a (Cartesian) grid.

Hahn et al. (2007) considered the three eigenvalues of the deformation tensor, $\lambda_1 \geq \lambda_2 \geq \lambda_3$, and classified a grid point according the number of positive eigenvalues at that point. Namely, a void point corresponds to no positive eigenvalues, a sheet to one, a filament to two and a knot point to three positive eigenvalues. The sign of a given eigenvalue at a given grid node determines whether the gravitational force at the direction of the principal direction of the corresponding eigenvector is contracting (positive eigenvalue) or expanding (negative).

Hahn et al. (2007) provided a very attractive approach to the web classification problem. It is based on the dynamical nature of the web, and so it easily lends itself to a theoretical analysis. The ease of its application to cosmological simulations opens the door for a new framework for associating the properties of galaxies and dark matter with environment, as defined by the web classification.

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1. Hahn et al. (2007) call the $T_{\alpha\beta}$ tensor the ‘tidal tensor’. Usually the tidal tensor is defined as the traceless part of $T_{\alpha\beta}$.
Close inspection of the [Hahn et al. (2007)] classification scheme and its results reveals its shortcomings. The volume filling factor of [Hahn et al. (2007)] voids is very small. For their minimal smoothing scale, namely the highest resolution, the voids occupy only 17% of the simulated volume. This stands in contrast to the visual impression of voids in the actual universe and in simulation, where voids seem to occupy most of the volume but contain only a small fraction of the galaxies (in observations) or matter (in simulations). Furthermore, the [Hahn et al. (2007)] classification does not reproduce the visual perception of the cosmic web.

It is easy to understand the inability of the [Hahn et al. (2007)] approach to reproduce the visual impression. The web classification is based on the algebraic sign of the eigenvalues of the deformation tensor, namely the number of eigenvalues larger than a threshold value of zero. It follows that if an eigenvalue is only infinitesimally positive, the scheme assumes that the local neighborhood of the given grid point collapses along the corresponding eigenvector. Yet, the collapse proceeds over the dynamical time scale and if the value of the eigenvector is small enough the collapse will occur, if at all, only in the distant future. Visual inspection would not classify the region as collapsing at the present time. This leads us to consider an alternative approach, namely asking the eigenvalues to be larger than a positive threshold.

Seeing that the dimensionality of the deformation tensor is \([\text{time}]^{-2}\), its eigenvalues can be associated with the collapse time. It follows that the threshold value should be roughly determined by equating the collapse time with the age of the universe. In Appendix \(\Delta\) we rewrite the eigenvectors of the deformation tensor in a dimensionless way. In such a presentation one expects \(\lambda_{\text{th}} \approx 1\), where \(\lambda_{\text{th}}\) is the threshold value. For an isotropic collapse \(\lambda_{\text{th}}\) can be calculated explicitly (see Eqs. \([\Delta 3] \pm \sqrt{\Delta 7}\)). However, this provides a very rough guide as the collapse on the web is clearly not isotropic. Here, an empirical approach is to be used and \(\lambda_{\text{th}}\) is to be roughly determined from the geometrical nature of the cosmic web it defines.

### 3 N-BODY SIMULATION, NUMERICAL IMPLEMENTATION AND OBJECT DETECTION

We use two numerical simulation. The first assumes a WMAP3 cosmology ([Spergel et al. 2007]) with a matter density \(\Omega_{m} = 0.24\), a cosmological constant \(\Omega_{\Lambda} = 0.76\), a dimensionless Hubble parameter \(h = 0.73\), a spectral index of primordial density perturbations \(n = 0.96\) and a normalization of \(\sigma_{8} = 0.76\). A simulation of box size \(160 h^{-1}\text{Mpc}\) and 1024\(^3\) particles has been assumed, corresponding to a particle mass of \(3.5 \times 10^{8} h^{-1}\text{M}_\odot\). Starting at redshift \(z = 30\) the evolution is followed using the MPI version of the Adaptive Refinement Tree (ART) code described in [Gottlöber & Klypin (2008)]. The simulation used here is actually a constrained simulation of the local universe which is to be described at length at the forthcoming Yepes et al paper (in preparation). This is an updated and higher resolution version of the constrained simulation presented in [Klypin et al. (2003)]. Here the simulation is treated as just a random one and its constrained nature is completely ignored.

In order to estimate cosmic variance effects we have used a numerical simulation of box size \(1h^{-1}\text{Gpc}\). The assumed cosmology for this simulation is WMAP3-like with a matter density \(\Omega = 0.27\), a cosmological constant \(\Omega_{\Lambda} = 0.73\), a dimensionless Hubble parameter \(h = 0.70\), a spectral index \(n = 0.95\), a normalization \(\sigma_{8} = 0.79\) and 1024\(^3\) particles corresponding to a particle mass of \(9.8 \times 10^{8} h^{-1}\text{M}_\odot\). The simulation was performed using the ART code as well.

The analysis of the simulations proceeds as follows. The density field of the \(160h^{-1}\text{Mpc}\) simulation is calculated from the particle distribution on a \(256^3\) grid using the Cloud-In-Cell (CIC) scheme, then smoothed with a Gaussian kernel of width \(R_{s}\) and from which the deformation tensor is calculated directly, using an FFT solver. The deformation tensor is then diagonalized on the grid. The web characteristic of each grid point is determined by the number of eigenvectors, at that grid node, above the threshold. It should be realized that the classification is local by its nature, but the combined effect of all grid points results in the geometrical construction defined as the cosmic web.

In the \(1h^{-1}\text{Gpc}\) simulation we followed the same procedure, interpolating first the density on a \(512^3\) grid. Once the environment detection procedure is done we select \(6^3\) non-intersecting sub-boxes of \(160 h^{-1}\text{Mpc}\) on a side to test the effect of cosmic variance. Figure 1 presents the CIC density field and the cosmic web evaluated at the threshold values of \(\lambda_{\text{th}} = 0.00, 0.20, 0.40, 1.00\) and 2.00. The web is presented by a grey scale corresponding to the four web types, it is evaluated at a Gaussian smoothing of \(R_{s} = 0.625 h^{-1}\text{Mpc}\) in the Figure 1. The density field and the cosmic web are evaluated on a plane of the CIC grid. Visual inspection of the density field reveals a network of voids, filaments and dense knots. The density field is evaluated on a thin plane and therefore no clean distinction can be made between the 3-dimensional filaments and sheets.

The web defined by \(\lambda_{\text{th}} = 0.0\) consists of many small isolated voids that occupy only a small fraction of the total area of the plane. Only as \(\lambda_{\text{th}}\) increases the voids get bigger and connected and they become the dominant geometrical component of the web.

The non-zero threshold classification provides a better visual match to the density field than the null case. A qualitative analysis and comparison is presented in Figure 1. The analysis is based on two quantities: the volume occupied by each web type (volume filling fraction - VFF) and the fraction of mass contained in such a volume (mass filling fraction - MFF).

A characterization of the cosmic web is obtained by grouping, by a Friends-of-Friends (FoF) algorithm, neighboring grid points of a given web type into individual objects. The resulting objects are defined as voids, sheets, filaments and knots. The FoF association proceeds in the following way: the centers of the cells in the grid are used as the position of four different kinds of particles according to its web type, then a standard FoF is run over particles of the same kind with a linking length \(b = 1.1\) times the grid length. It means that only the six closest neighbors of a given cell are taken into account. The FoF void detection is done for different simulations: the \(160 h^{-1}\text{Mpc}\) simulation, the \(1h^{-1}\text{Gpc}\) simulation and 216 sub-boxes of \(160 h^{-1}\text{Mpc}\) on a side extracted from the largest simulation. To be consistent in the analysis of these three kinds of simulations,
Figure 1. The density field and five different environmental classifications. Upper left panel: slice of width 0.625\(h^{-1}\)Mpc depicting the density field in the simulation smoothed over a scale of \(R_\text{s} = 0.625\,h^{-1}\)Mpc, the color coding is logarithmic in the density — high density peaks are dark. The other panels show the environment classification using different values in for the threshold, \(\lambda_{\text{th}} = 0.0, 0.20, 0.40, 1.0,\) and 2.0. White corresponds to voids, clear gray to sheets, dark gray to filaments and black to peaks. The general impression is that the non-zero values of \(\lambda_{\text{th}}\) below 1.0 capture better the environment seen by eye in the density plot.
one should make the FoF detection without considering the periodic boundary conditions, as it is the right boundary condition for the sub-volumes extracted from the $1h^{-1}$Gpc simulation.

The FoF algorithm is used to detect voids for different threshold values in the different simulations which are smoothed with the same physical scale $R_s = 1.95h^{-1}\text{Mpc}$ in order to allow a fair comparison between them. In a detailed analysis on these voids is presented, paying special attention to its percolation properties as the threshold rises. The same kind of analysis is performed for the filaments in but only on the $1h^{-1}$Gpc simulation.

## 4 VOLUME AND MASS FILLING FRACTIONS

The web classification depends on two parameters that determine the environment. The first is the smoothing scale $R_s$, and the second is the threshold for the eigenvalues $\lambda_{th}$. The dependence of the volume and mass filling fractions (VFF and MFF, respectively) on these two parameters is studied here. This is done for the four web types.

The VFF and MFF are measured for every kind of environment. First by fixing $\lambda_{th} = 0$ and varying the smoothing scale $R_s$ between $0.625h^{-1}\text{Mpc}$ and $12.4h^{-1}\text{Mpc}$. Later, fixing the smoothing scale to $1.95h^{-1}\text{Mpc}$ and varying $\lambda_{th}$ between 0 and 1 with steps of 0.1. The results of these two kinds of cuts are shown in Figure 2. The VFF and MFF of the case $\lambda_{th} = 0.0$ and 1.0, with two different smoothing scales, $R_s = 0.625h^{-1}\text{Mpc}$ and $R_s = 1.95h^{-1}\text{Mpc}$ are presented in Table 1. The evolution of the filling fraction with $\lambda_{th} = 0$ reproduces the asymptotic results expected for large smoothing scales, which is 0.42 for sheets and filaments and 0.08 for voids and knots (Doroshkevich 1970).

The most striking feature that emerges from Figure 2 is that of the strong dependence of VFF and MFF of the voids on $\lambda_{th}$. This is to be contrasted with the other web types which show quite a similar behavior with the change of the smoothing and the change of the threshold. It follows that the voids can serve as a sensitive monitor and indicator of the cosmic web. For the case of a null threshold the VFF of these web elements are almost independent of $R_s$. The MFF, on the other hand, distinguishes very easily different values of the smoothing scale $R_s$.

## 5 PERCOLATION OF VOIDS

Given that each grid node is flagged as belonging to a void, sheet, filament or a knot one can define new objects made by connecting grid nodes of the same type. In particular we proceed here to define a void as an object made of the collection of neighboring void type grid nodes. Neighboring nodes are associated by the FoF algorithm to form individual objects as described at the end of 6.

Here we focus on the analysis of the statistical distribution of the sizes of the voids and their percolation. The emphasis on voids does not stem only from the extensive work done on their properties (Colberg et al. 2008) but also because they constitute the most sensitive gauge of the cosmic web and its dependence on the threshold of the eigenvalues. Filaments have received attention as well in the literature (Novikov et al. 2000), in the next section we present a brief analysis of their properties.

A simplified characterization was already performed in 4 by measuring the volume and mass occupied in the void environment. Here, the dependence of the number of voids and their percolation properties on $\lambda_{th}$ is examined. The percolation is quantified now by the fraction of volume of the most voluminous void to the total volume occupied by all voids.

The void identification is performed here at a fixed smoothing scale $R_s = 1.95h^{-1}\text{Mpc}$ and by varying the eigenvalues threshold in the range of $0 < \lambda_{th} < 0.3$. In the Figure 4 (left panel) one can see that the number of voids in the simulation roughly decays exponentially as a function of the threshold parameter $\lambda_{th}$, $N_{\text{void}} = N_0 \exp(-\lambda_{th}/\lambda_D)$, where $N_0$ is the number of voids for the null threshold and $\lambda_D$.

Figure 4 (right panel) presents the fraction of the volume of the largest (in volume) void to the total volume of all the voids. A transition occurs between $0.1 \lesssim \lambda_{th} \lesssim 0.2$ where it jumps from a ratio $\leq 1$ to $\geq 0.9$. Note that the percolation starts at the stage in which the VFF of voids is only $\sim 25\%$. It follows that in spite of the small VFF obtained for $\lambda_{th} \sim 0.1$ the voids start to coalesce and form one super-void which encompasses $90\%$ of the total volume of voids when the void VFF reaches $60\%$. As we will show in this transitional scale is dependent on the particular simulation under consideration.

## 6 FRAGMENTATION OF FILAMENTS

Along with voids, filaments have received some attention in the literature as a natural way to probe large scale structure (Sousbie et al. 2008). A study of the filaments is performed here, focusing on their percolation dynamics.

In the opposite sense to voids, filaments start to fragment as the threshold $\lambda_{th}$ is raised. This is clear in Figure 1 which shows that the filaments disappear as $\lambda_{th}$ increases, making room for the growing voids.

Consequently, one could expect that for some range of
Figure 2. Upper panel: the volume filling fraction of peaks, filaments, sheets and voids as a function of the smoothing scale $R_s$ for $\lambda_{th} = 0.0$ (left), and as a function of $\lambda_{th}$ for $R_s = 1.95 h^{-1}\text{Mpc}$ (right). Continuous line: voids, dashed: sheets, dotted: filaments, dotted-dashed: knots. Lower panel: same as the upper panel but for the mass filling fraction.

| Web type | Volume $R_s = 0.625 h^{-1}\text{Mpc}$ | Mass $R_s = 0.625 h^{-1}\text{Mpc}$ | Volume $R_s = 1.95 h^{-1}\text{Mpc}$ | Mass $R_s = 1.95 h^{-1}\text{Mpc}$ |
|----------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| void     | 0.16                             | 0.02                             | 0.76                             | 0.28                             |
| sheet    | 0.60                             | 0.27                             | 0.18                             | 0.25                             |
| filament | 0.24                             | 0.54                             | 0.05                             | 0.35                             |
| knot     | 0.01                             | 0.16                             | 5.0e-3                           | 0.12                             |

Table 1. Volume and mass filling fractions for the four web types for two different smoothing scales, $R_s = 0.625 h^{-1}\text{Mpc}$ and $R_s = 1.95 h^{-1}\text{Mpc}$. For each smoothing two extreme values of the threshold are used, $\lambda_{th} = 0.0$ and $\lambda_{th} = 1.0$. The volume filling fractions are similar for the same values of the threshold $\lambda_{th}$ regardless of which smoothing scale is used.
Figure 3. Upper row. Isocontours for the volume filling fractions for the different kinds of environment, in the plane $R_s - \lambda_{th}$. Lower row. Isocontours for the mass filling fractions for the different kinds of environment, in the plane $R_s - \lambda_{th}$. In both rows, from left to right: filaments, sheets and voids. The label on each isocontour is the percentage of the total volume filled by a given kind of environment.

Figure 4. Percolation results. Left panel: number of voids as a function of the threshold in eigenvalues normalized to the number of voids at $\lambda_{th} = 0.0$. A fixed smoothing of $R_s = 1.95h^{-1}\text{Mpc}$ is used here for all the simulations. The thick line shows the results for the $1h^{-1}\text{Gpc}$ simulation, the thin line shows the results for the $160h^{-1}\text{Mpc}$ simulation. The gray lines show the results for the sub-volumes extracted from the $1h^{-1}\text{Gpc}$ simulation. Right panel: fraction of the total void volume occupied by the most voluminous void. The line coding is the same as in the left panel. Results from the large $1h^{-1}\text{Gpc}$ differ greatly from the results in the simulation $160h^{-1}\text{Mpc}$, nevertheless the later is consistent within the scatter deduced from the sub-volumes. Even when it is clear that the detailed shape of the curve depends on the simulation size, it seems to be a robust feature that the largest change in the super-void size is presented around $0.1 \lesssim \lambda_{th} \lesssim 0.2$. 
\( \lambda_{th} \) there are two coexisting environments: a complex of percolating voids and a network of interconnected filaments, something close to the visual impression of the cosmic web. The percolation analysis of the filament and void networks can help to define an interval of \( \lambda_{th} \) values where the environment studies would be feasible.

We use the \( 1h^{-1}\text{Gpc} \) simulation following the approach of the previous section. Figure 3 shows the ratio of the volume of largest filament to the total volume occupied by filaments, overplotted is the same fraction for voids (presented in the right panel of Figure 3). The filament fraction evolves from a value close to \( \sim 1 \) for \( \lambda_{th} = 0.0 \) down to values close to zero for \( \lambda_{th} \sim 1.5 \). It confirms the visual intuition we had of a fully interconnected network of filaments that fragments as the eigenvalue threshold increases.

The percolation/fragmentation curves intersect at \( \lambda_{th} = 0.25 \), when the two fractional volumes are \( \sim 97\% \). Heuristically, one can assume a given network to exist, namely percolate, when its fractional volume exceeds 95%. The web is then defined at the threshold level at which the voids and filaments networks coexist. This implies a threshold value of \( \lambda_{th} \lesssim 0.40 \). Such a heuristic approach is in a good agreement and matches the visual impression of the LSS (Figure 4). This approximate interval should hold for lower smoothing scales, as the volume filling fraction (our gauge for the percolation dynamics) is almost independent of the smoothing scale for the range of thresholds considered, as seen in Figure 3 and suggested as well in the values of Table 1.

7 SIMULATION BOX SIZE AND COSMIC VARIANCE

In order to quantify the influence of the simulation box size and the impact of cosmic variance we measure for three different kind of simulations the MFF and VFF for voids and filaments, and the percolation of voids. The first is the \( 1h^{-1}\text{Gpc} \) simulation with the density field interpolated over a \( 512^3 \) grid and smoothed with \( R_s = 1.95h^{-1}\text{Mpc} \), used to quantify the effect of simulation size. The second is the \( 160h^{-1}\text{Mpc} \) simulation interpolated over a \( 256^3 \) grid smoothed over the same physical scale. The third kind is the \( 1h^{-1}\text{Gpc} \) simulation split in smaller non-intersecting cubes of \( 160h^{-1}\text{Mpc} \), used to quantify the effect of cosmic variance.

Figure 4 shows the result for the VFF and MFF. The results for the \( 160h^{-1}\text{Mpc} \) simulation are well within the variance calculated from the sub-volumes in the \( 1h^{-1}\text{Gpc} \) simulation. In the case of the MFF, the result for the \( 160h^{-1}\text{Mpc} \) are located far from the mean value in the \( 1h^{-1}\text{Gpc} \), nevertheless it is located with in the dispersion defined by the sub-volumes. In general, the VFF and MFF are consistent in all the three kinds of simulations. The agreement is less impressive than with the VFF, perhaps due as well to the different values of \( \sigma_s \) used in the simulations, the large simulation has \( \sigma_s = 0.79 \), while the small simulation has \( \sigma_s = 0.75 \).

The growth of the largest void in the simulation, Figure 3, is very different between the three kinds of simulations. The fraction of void volume occupied by the super-void is very dependent on the simulation size. For the \( 1h^{-1}\text{Gpc} \) simulation, the initial values of \( V_{\text{max}}/V_{\text{tot}} \) for \( \lambda_{th} = 0.0 \) are the lowest possible, this can be readily understood by the fact that \( V_{\text{tot}} \) grows with the simulation box size, while \( V_{\text{max}} \) should be on the same order of magnitude regardless of the simulation size. For values larger than \( \lambda_{th} = 0.2 \) the large simulation has almost percolated into a single super-void, while the sub-volumes still show a large dispersion in their percolation behavior. The results for the small \( 160h^{-1}\text{Mpc} \) simulation are consistent with such dispersion. In spite of that, in all the three cases there is a clear transitional behavior starting at \( \lambda_{th} = 0.1 \) and finishing around \( \lambda_{th} = 0.2 \), even when the detailed behavior with \( \lambda_{th} \) is far from being the same.

8 CONCLUDING REMARKS

This paper presents an improved method to identify large scale environment in dark matter simulations. Our scheme is based on the analysis of the Hessian of the gravitational potential generated by the dark matter distribution. The algorithm presented here constitutes an improvement on the scheme of Hahn et al. (2007), involving a pertinent reinterpretation of the dynamics in the problem.

The change is done through the addition of a free parameter \( \lambda_{th} \) related to the dynamical time associated with the collapse. This important improvement allows a more realistic treatment of the cosmic web.

Inspection of the different plots of Figure 2 reveals the striking difference in the way the cosmic web, and in particular the voids, respond to the changes in the Gaussian smoothing and the \( \lambda_{th} \)'s threshold. Keeping a null \( \lambda_{th} \) and changing \( R_s \) we see that the non-linear evolution does not change the ranking of the VFF and MFF found in the deep linear regime (i.e., \( R_s \approx 12.5h^{-1}\text{Mpc} \)). Namely, for the null threshold the sheets have the highest VFF and the filaments the highest MFF, independent of \( R_s \). Considering the case of a fixed \( R_s = 1.95h^{-1}\text{Mpc} \), the void VFF grows strongly
with $\lambda_{\text{th}}$ and above $\lambda_{\text{th}} \sim 0.5$ the voids have the highest VFF. The results on the VFF are extremely robust respect to the simulation size and cosmic variance effects. In the case of void MFF, it changes from the lowest one at $\lambda_{\text{th}} = 0$ to the second highest at $\lambda_{\text{th}} = 1.0$.

The nature of the web changes dramatically with the threshold and it becomes volume dominated by the voids as $\lambda_{\text{th}}$ increases. The MFF shows a larger cosmic variance, compared with the VFF. Also, it is equally sensitive to changes in the smoothing scale and the threshold value.

The web classification provides a set of flagged points on a grid and the collection of neighboring grid points of a given environmental type, connected by a FoF algorithm, forms objects we call voids, sheets, etc. The statistical properties of the system of voids and their dependence on $\lambda_{\text{th}}$ have been explored here. In particular the number of isolated voids and their sizes have been analyzed, finding that the number of isolated voids roughly decreases exponentially with $\lambda_{\text{th}}$.

In the $1h^{-1}\text{Gpc}$ simulation the system of voids percolates between $0.1 \lesssim \lambda_{\text{th}} \lesssim 0.2$, at which the largest void jumps from having less than 10% to 90% of the volume occupied by voids. The percolation dynamics is also seen on average in the different smaller simulations, with a wide spread on the way the percolation is observed, but keeping the same threshold interval for the transition.

The association of the eigenvectors of the deformation tensor with the collapse time, and hence with the age of the universe, enables in principle a theoretical determination of $\lambda_{\text{th}}$. Using the spherical collapse model, calculated within the WMAP3 cosmology, the threshold value is $\lambda_{\text{th}} = 3.21$. However, at that high value the web looks very fragmented, in particular the network of filaments. The application of the ellipsoidal collapse model (Sheth & Tormen 2002) might provide a better theoretical estimate.

Short of a “first principle” determination of $\lambda_{\text{th}}$ we resort to a heuristic approach. We look for the range of $\lambda_{\text{th}}$ over

Figure 6. Cosmic variance effects on volume and mass filling fractions for voids and filaments. Thick black line: simulation $1h^{-1}\text{Gpc}$, thin black line: simulation $160h^{-1}\text{Mpc}$, gray lines: sub-volumes extracted from the $1h^{-1}\text{Gpc}$ simulation. Upper panels: volume filling fraction of voids (left) and filaments (right) as a function of the eigenvalue threshold $\lambda_{\text{th}}$. Lower panels: same as the upper panels for the mass filling fractions.
which the percolated networks of the voids and filaments coexist for a fixed smoothing scale of $R_s = 1.95h^{-1}$Mpc in the $1h^{-1}$Gpc simulation. The voids and filaments behave in an opposite way in terms of the dependence of the percolation on the threshold value. At low $\lambda$, the voids are isolated and the filaments percolated and at high $\lambda$, the voids percolate and the filaments are fragmented. Adopting a 95% in the fractional volume as defining the percolation transition we find the web to be defined by a threshold in the range of $0.20 \lesssim \lambda_{th} \lesssim 0.40$. This range stands in good agreement with the visual impression obtained from the simulations.

The notion of the cosmic web is not new. The filamentary structure has been extensively studied, mostly within the context of the Zeldovich pancakes (Zeldovich 1970). The role of voids has also been heavily studied and many algorithms for voids finding have been suggested (see Colberg et al. 2008). We have been motivated by the computational simplicity and the elegance of the Hahn et al. 2007 approach and have modified it in a way that reproduces the web as it emerges from observations and simulations. Our main drive is to provide a simple, fast and precise tool for classifying the environmental properties of each point in space. Using the non-zero thresholding of the eigenvalues of the Hessian of the potential indeed provides a very efficient tool that can be easily applied to simulations.

The same analysis can be performed at different redshifts in the simulation, allowing the classification of environment as a function of time.

The dynamical nature of the web classification implies that the web type might affects the dynamical evolution of DM halos and of galaxy formation. This might have profound consequences on the star formation timescale.... (Gayler et al. 2008). It follows that the web classification can be introduced into semi-analytical modeling, as a dynamical tag that together with the mass of the DM halos dictate the the mode gas accretion onto galaxies.

A major challenge that is still to be addressed is the application of the method to the distribution of galaxies. Short of that, the algorithm remains in the theoretical realm of simulations and semi-analytical modeling of galaxy formation.

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APPENDIX A: SETTING THE THRESHOLD FOR WEB CLASSIFICATION

In this paper we use the eigenvalues of the deformation tensor, $\lambda_i$ normalized in a specific way. Here we provide details of the normalization and give motivation for selecting the threshold.

We write the Poisson equation in the following form:

$$\nabla^2 \tilde{\phi} = 4\pi G \tilde{\rho} = \tilde{\lambda}_1 + \tilde{\lambda}_2 + \tilde{\lambda}_3, \quad (A1)$$

where $\tilde{\rho}$ is the mean matter density of the universe and $\delta$ is the matter overdensity. One can re-scale the gravitational potential and the eigenvalues of the deformation tensor by dividing them by $4\pi G \tilde{\rho}$:

$$\nabla^2 \phi = \delta = \tilde{\lambda}_1 + \tilde{\lambda}_2 + \tilde{\lambda}_3. \quad (A2)$$

Note that $4\pi G \tilde{\rho}$ provides a natural scale to introduce dimensionless parameters $\lambda_i$. We solve this equation numerically.

The spherical collapse model is invoked here so as to get a rough estimate of $\lambda_{th}$. The (spherical) free-fall time is related to the local density by

$$\tau_{ff} = \sqrt{\frac{3\pi}{32G\tilde{\rho}}}. \quad (A3)$$

Recalling that

$$4\pi G \tilde{\rho} = \frac{3}{2} \Omega_m H_0^2, \quad (A4)$$

(where $\Omega_m$ is the value of the cosmological matter density and $H_0$ is the Hubble constant), Eq. (A1) is rewritten as:

$$\nabla^2 \tilde{\phi} = \tilde{\lambda}_1 + \tilde{\lambda}_2 + \tilde{\lambda}_3 = 4\pi G \tilde{\rho} - 4\pi G \tilde{\rho}. \quad (A5)$$

The threshold can be estimated by demanding that the free-fall time equals the age of the universe ($\tau_0$). Namely, the threshold separates between the principal axes that have collapsed by $\tau_0$ and the ones that have not. Substituting the free-fall time by the age of the universe the threshold is given by

$$3\beta(\lambda_i) \tilde{\lambda}_{th} = \frac{3\pi^2}{8\tau_0^2} - \frac{3}{2} \Omega_m H_0^2, \quad (A6)$$

where $\beta$ is a fiducial factor introduced to account for the deviation from local isotropy.

In terms of the dimensionless eigenvalues the threshold is given by:

$$\lambda_{th} = \frac{1}{3\beta(\lambda_i)} \left[ \frac{2}{4} \frac{1}{\Omega_m} (\tau_0 H_0)^{-2} - 1 \right] \quad (A7)$$

For the WMAP3 parameters used in the $160h^{-1}$Mpc simulation we have $\Omega_m = 0.24$, $h = 0.73$ and $\tau_0 H_0 = 0.983$. It follows that $\lambda_{th} = 9.63/(3.0\beta)$.

Assuming the spherical collapse model as a proxy to the full non-isotropic case, namely $\beta(\lambda_i) = 1$, then the threshold is $\lambda_{th} = 3.21$. 

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