Quark Mass Ratios and Mixing Angles from $SU(3)$ Family Gauge Symmetry

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We explore a framework for the computation of quark mass ratios and CKM mixing angles based on an $SU(3)$ family gauge symmetry. The four ratios $m_d/m_b$, $m_s/m_b$, $m_u/m_t$, and $m_c/m_t$ can be fit at one-loop in the family gauge interaction. The same is true of the quark mixing angles $\theta_{12}$ and $\theta_{23}$, although the result for $\theta_{13}$ is too small. The CP violating phase is naturally $O(1)$.

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| $SU(3)_1$, $SU(3)_2$, $Z_3$, $SU(2)_L$, $U(1)_Y$ |
|---|---|---|---|---|
| $q$ | 3 | 1 | $1''$ | 3 | 2 | $\frac{1}{3}$ |
| $\bar{q}'$ | 3 | 1 | $1'$ | 3 | 1 | $\frac{1}{3}$ |
| $d$ | 3 | 1 | $1'$ | 3 | 1 | $\frac{1}{3}$ |
| $c$ | 3 | 1 | $1'$ | 3 | 1 | $\frac{4}{3}$ |
| $h$ | 1 | 1 | 1 | 1 | 2 | $\frac{1}{2}$ |
| $S$ | 6 | 1 | $1'$ | 1 | 1 | 0 |
| $\Sigma$ | 6 | 1 | $1''$ | 1 | 1 | 0 |
| $H$ | 1 | 6 | 1 | 1 | 1 | 0 |

Table I: Field content and symmetries of the model. The $Z_3$ labels refer to the three cube roots of unity. All fermions are LH chiral fields.

**Introduction**—We examine the radiative generation of quark mass ratios and mixing angles when the standard model (SM) with three families of quarks and leptons is enlarged to include an $SU(3)$ family gauge interaction. We take the family symmetry to be broken at some scale $F$, large enough to suppress flavor-changing neutral currents. With electroweak breaking described by a Higgs-doublet field, some mechanism, such as the inclusion of supersymmetry, must be invoked to stabilize the Higgs mass. We do not address this problem here. We assume that any additional new physics, such as that proposed by a top partner, can be included perturbatively.

**The Model**—To compute the ratios $m_d/m_b$, $m_s/m_b$, $m_u/m_t$, and $m_c/m_t$, and the CKM mixing angles radiatively in the family gauge interaction, these quantities must vanish in its absence. To this end, we introduce two global symmetries, $SU(3)_1 \times SU(3)_2$, with the standard model fermions and a set of partners (the "visible" sector) transforming according to $SU(3)_1$, and additional fields of a "hidden" sector transforming according to $SU(3)_2$. The $SU(3)$ family gauge interaction arises from gauging the diagonal subgroup of $SU(3)_1 \times SU(3)_2$. We also introduce an additional, discrete $Z_3$ symmetry.

The breaking pattern in the visible sector preserves two $Z_2$ subgroups of $SU(3)_1$, and can be shown to arise naturally from a broad class of potentials. This pattern leads to the vanishing of the mixing angles and quark-mass ratios in the absence of the gauge interaction. The breaking pattern in the hidden sector also preserves two $Z_2$ subgroups of $SU(3)_2$. The unbroken discrete symmetries are misaligned, such that no $Z_3$ symmetry remains when the sectors are gauge coupled. The gauge coupling therefore leads to non-vanishing values for the mass ratios and mixing angles. Phases of order unity also arise naturally in the breaking in each sector.

The fields of our model, together with their transformation properties under the $SU(3)_1 \times SU(3)_2 \times Z_3$ symmetries and the SM symmetries, are shown in Table I. In addition to the SM fermion fields, there are two fermionic fields, $\chi$ and $\chi'$, with the SM quantum numbers of the up-type quarks. They differentiate the up- and down-sectors, and play an important role in up-type mass generation. Each fermion transforms as a $3$ under $SU(3)_1$, meaning that this symmetry must be broken to generate fermion mass. In addition to the usual Higgs scalar $h$, two scalar (SM-singlet) multiplets, $S$ and $\Sigma$, both 6's (symmetric tensors) under $SU(3)_1$, couple to the fermions. Finally, we describe the hidden sector by one (SM-singlet) scalar multiplet, $H$, a 6 under $SU(3)_2$.

The EFT for physics below the family breaking scale is comprised of the fermions, the Higgs boson, the SM gauge bosons, the family gauge bosons, and the components of the $S$ and $\Sigma$ fields that survive as pseudo-Goldstone bosons (PGB’s). The $SU(3)$ family gauge interaction is universal with respect to the up-type and down-type fermions. It is, so far, anomalous, requiring the existence of additional, heavy fermions to remove the anomalies. When integrated out, they generate an appropriate Wess-Zumino-Witten (WZW) term in the EFT which must be included in the analysis. It will not affect the fermion mass ratios and mixing angles to leading order, and we will not discuss it further here.

Since we are interested in the generation of the Yukawa couplings of the Standard Model, we focus on operators that are bilinear in the fermion fields. The operators...
allowed by the symmetries (including the $Z_3$), and involving only a single power of $S$ or $\Sigma$, are given by

$$L_Y = y_1 \frac{q h S d}{F} + y_2 \frac{q h S}{F} + y_3 \frac{\chi S \Sigma}{F} + h.c.$$  \hspace{1cm} (1)

In the EFT below the cutoff $M_F \equiv 4 \pi F$, nonlinear constraints on $S$ and $\Sigma$ insure that they describe only GB and PGB degrees of freedom. Each of the $y_i$ couplings is a dimensionless parameter determined by physics above $M_F$. Each except for $y_1$ will be small compared to the family gauge coupling $g$, which will be $O(1)$, that is, $\alpha/\pi \equiv g^2/4\pi^2 = O(1/40)$. This will justify using the $y_i$ couplings at lowest order, with quantum corrections arising from the family gauge interactions. Operators bilinear in the fermions fields but with higher powers of $S$ and $\Sigma$ are also allowed by the symmetries. We argue that they naturally produce small effects due to the smallness of the $y_i$ couplings, after describing the role of the above operators.

**Symmetry Breaking**—We assume that the visible-sector physics above the family breaking scale is such as to give the following VEV's for $S$ and $\Sigma$:

$$\langle S \rangle = F \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 \\ s \end{pmatrix} \hspace{1cm} \langle \Sigma \rangle = F \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 \\ 0 \end{pmatrix}.$$  \hspace{1cm} (2)

Here $s$ and $\sigma$ are complex numbers of roughly unit magnitude. They preserve a $Z_2$ subgroup of $SU(3)_1$, generated, for example, by

$$P_1^{(1)} = \text{diag}\{1, -1, -1\} \hspace{1cm} P_1^{(2)} = \text{diag}\{-1, 1, -1\},$$  \hspace{1cm} (3)

and also break the discrete $Z_3$ symmetry.

To justify this pattern of symmetry breaking, it is helpful to analyze potential terms with the high energy theory described by a linearized theory of the scalars $S$ and $\Sigma$. The potential couples the $S$ and $\Sigma$ fields, containing terms even in these fields as well as terms such as $\text{Tr}[S \times S], \text{Tr}[\Sigma \times \Sigma]$, where $S \times S$ denotes the 6 in the product of the two 6's. These terms preserve $SU(3)_1 \times Z_3$, but explicitly break $U(1)_S$ and $U(1)_\Sigma$ associated with $S$ and $\Sigma$.

The potential Eq. (2), a special case of the general class of $Z_2^3$-preserving vacua, which can have arbitrary diagonal entries, emerges for a wide class of potentials. For $S$, an example is the potential $V = (\text{Tr} SS^* - F^2)^2 + \lambda \text{Tr}[S \times S]S + \kappa F \text{Tr}[S \times S] + h.c. + \ldots$, where $\lambda$ and $\kappa$ are dimensionless parameters. Taking $\langle S \rangle$ diagonal by convention, this potential leads to the above form providing only that $\lambda > 0$ and that the cubic term is not too large. Similar terms for the $\Sigma$ field, together with dimension-4 coupling terms such as $\text{Tr}[(SS^*)^2(S^*\Sigma)]$ (with positive definite coefficient), lead to a diagonal form for $\langle \Sigma \rangle$, also with a single entry, and prefer to anti-align it with $\langle S \rangle$. Taking the non-zero entries to be 33 and 22, we have the form Eq. (2). This pattern can be thought of as preserving two, distinct $SU(2)$ symmetries for the $S$ and $\Sigma$ sectors separately, with the coupling of the sectors preferring to mis-align them, leaving no unbroken continuous subgroup of the common $SU(3)_1$. Eight Goldstone bosons (GB's) are formed. For a similar construction see [2].

The hidden-sector potential is also taken to include terms which preserve $SU(3)_2$ but explicitly break $U(1)_H$. In the absence of the gauge coupling, the sextet $H$ is assumed to develop a VEV $\langle H \rangle$ of $O(F)$, but leaving no continuous subgroup of the $SU(3)_2$. This, too, is natural depending on the parameters of the potential. With only one sextet in this sector, since its VEV can be diagonalized, a $Z_2^2$ subgroup of the $SU(3)_2$ necessarily remains unbroken.

When the hidden and visible sectors are coupled through the family gauge interaction, with $\langle S \rangle$ and $\langle \Sigma \rangle$ diagonal as above, $\langle H \rangle$ takes a general form:

$$\langle H \rangle = F \begin{pmatrix} b_1^2 & b_2 & b_3 \\ b_2 & a_1 & a_3 \\ b_3 & a_3 & b_2 \end{pmatrix},$$  \hspace{1cm} (4)

where $a_i$ and $b_i$ are dimensionless complex numbers. We assume here that the potential terms generated by the family gauge interactions prefer this pattern with its nonvanishing off-diagonal elements. It means that no discrete ($Z_3^2$) subgroup of the $SU(3)$ family gauge group remains.

We also assume the existence of a moderate hierarchy in $\langle H \rangle$, where each $|a_1| = O(a) = O(1)$ and each $|b_i| = O(b) < O(1)$. This form, with one diagonal element quadratic in $b$, emerges naturally from certain potential terms, for example those that constrain $|\text{det}(H)|^2$ to be $O(b^4)$. (These terms by themselves respect a global $U(1)_H$, an approximate symmetry of the full potential if the explicit breaking terms are somewhat smaller.) The orientation of $\langle H \rangle$ relative to $\langle S \rangle$ and $\langle \Sigma \rangle$, with the suppression factor $b$ appearing in $H_{ij}$ for either index equal to 1, also emerges naturally from a broad class of such potential terms. In the limit $b \to 0$, $\langle H \rangle$ preserves an (approximate) $U(1)$, a product of $U(1)_H$ and a $U(1)$ subgroup of $SU(3)_2$. This limit also preserves a single, exact $Z_2$ subgroup of the $SU(3)$ family gauge group. Thus the breaking is sequential, first preserving this $Z_2$ symmetry, and then breaking it.

The hidden sector produces 8 GB’s. They combine with the 8 GB’s of the visible sector to produce 8 exact GB’s which are eaten by the family gauge bosons. The other 8 combinations are PGB’s, which acquire a (small) mass through the explicit symmetry breaking of the family gauge interaction. We discuss the effect of the PGB’s later.

**Quark Mass Matrices at Tree Level**—After elec-
Perturbation theory is valid providing that additional suppression factors expressed by powers of matrix can be read off from the diagonal matrix \( \tilde{t} \). This is due to weak breaking, the down-type quark mass matrix is symmetric, forbidding the direct, dimension-5 coupling of \( \Sigma \) to the down sector. Hence, for the down quark mass matrix, we have

\[
M_d = y_d \nu \frac{(S)}{F} = y_d \nu \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & s \end{pmatrix},
\]

(5)

where \( v \approx 250 \text{GeV} \) is the VEV of Higgs doublet \( h \). At this level, only the \( b \) quark develops a mass, of the right order for \( y_d \approx 10^{-2} \). This pattern relies on the \( Z_3 \) symmetry, forbidding the direct, dimension-5 coupling of \( \Sigma \) to the down sector.

The up-type quark mass matrix is \( 6 \times 6 \):

\[
(u \chi) \bar{M}_u \begin{pmatrix} \nu^c \\ \chi^c \end{pmatrix} = (u \chi) \begin{pmatrix} 0 & y_1 \nu \frac{(S)}{y_3 \langle \Sigma \rangle} \\ y_2 \langle S \rangle & y_3 \langle \Sigma \rangle \end{pmatrix} \begin{pmatrix} \nu^c \\ \chi^c \end{pmatrix}.
\]

(6)

The squares of the eigenvalues of this (non-symmetric) matrix can be read off from the diagonal matrix \( M_u \bar{M}_u^\dagger \). There are three non-vanishing eigenvalues of order \( g^2 y^2 \), \( y_2^2 F^2 \), and \( y_3^2 F^2 \). The first corresponds to the \( t \)-quark. That is, its left-handed component is the \( SU(2)_L \)-doublet t-field. The latter two are very large providing only that \( v/F \ll y_2, y_3 \), and correspond completely to \( SU(2)_L \)-singlet fermion.

Thus, in the absence of the family gauge interaction, there are no family mixings, no masses for the \( u, d, c \), and \( s \) quarks, and no mass for one additional up-type, \( SU(2)_L \)-singlet fermion.

**SU(3) Family Gauge Interactions**—The corrections to the quark mass matrices depend directly on the mass matrix for the family gauge bosons. It arises from the kinetic terms of the three scalar sextets:

\[
L_K = \frac{1}{2} \text{Tr}[(D_\mu S)(D^\mu S)^*] + \frac{1}{2} \text{Tr}[(D_\mu \Sigma)(D^\mu \Sigma)^*]
+ \frac{1}{2} \text{Tr}[(D_\mu H)(D^\mu H)^*],
\]

(7)

with \( D_\mu G = \partial_\mu G + ig A_{\mu a} t_a G + ig A_{\mu a} G t_a^\dagger \), where \( t_a, a = 1, \ldots, 8 \) are the generators of the \( SU(3) \) family gauge symmetry and \( G \) represents the three scalars \( S, \Sigma \) and \( H \). In terms of the scalar VEV’s, the gauge-boson mass operator is then

\[
L_M = \frac{1}{2} A_a (M_{S\bar{a}b}^2 + M_{\Sigma \bar{a}b}^2 + M_{H \bar{a}b}^2) A_b,
\]

(8)

where, for example,

\[
M_{S\bar{a}b}^2 = g^2 \text{Tr}[t_a \langle S \rangle s^* \langle S \rangle^* + t_a \bar{b} \langle S \rangle \langle S \rangle^*] + (a \leftrightarrow b).
\]

(9)

The entries of \( M_{S\bar{a}b}^2 + M_{\Sigma \bar{a}b}^2 + M_{H \bar{a}b}^2 \) are of order \( g^2 F^2 \), but, due to the form of \( \langle H \rangle \), some off-diagonal terms have additional suppression factors expressed by powers of \( b \).

**Radiative Corrections to the Quark Mass Matrices**—Perturbation theory is valid providing that \( g^2/4\pi^2 \ll 1 \). We will require \( g \approx 1 \) phenomenologically.

The loop corrections may be viewed as corrections to \( \langle S \rangle \) and \( \langle \Sigma \rangle \) in Eq. (2). At one loop, we find

\[
\delta(S)_{ij} = -\frac{\alpha}{\pi} s F \log\left( \frac{M_F^2}{M_B^2} \right) (t_a)_{i1}^3 (t_b)_{j2}^3 O_{ab},
\]

(10)

where \( i, j = 1, 2, 3 \) are the family indices and \( a, b, c = 1, \ldots, 8 \) label the 8 gauge bosons. \( M_F \) is the cutoff scale and the \( M_B^2 \) are the mass eigenvalues of the family gauge bosons. The matrix \( O \) is the orthogonal transformation diagonalizing the gauge boson mass matrix. The small parameter \( b \) appears in this matrix. The \( M_F \) dependence survives in only the 33 element, giving a cut-off dependent renormalization of \( s \). A similar expression obtains for \( \delta(\Sigma)_{ij} \), with the index 3 replaced by 2. To derive these expressions, it is easiest to work in a renormalizable gauge such as Landau gauge.

In addition to the above contributions to \( \delta(S)_{ij} \) and \( \delta(\Sigma)_{ij} \), there are contributions from fermion wave function renormalization (kinetic-energy mixing). They lead to corrections of the same general form, and we don’t exhibit them explicitly.

The corrected forms of the \( \langle S \rangle \) and \( \langle \Sigma \rangle \) matrices thus include \( O(\alpha/\pi) \) entries replacing the 0’s in Eq. (2). The presence of factors \( b \) and \( b^2 \) in the first row and column of \( \langle H \rangle \) leads to a similar presence in the corrected \( \langle S \rangle \):

\[
\langle S \rangle' = \langle S \rangle + \delta\langle S \rangle = F \begin{pmatrix} 0 & (\frac{\alpha}{\pi} b) & O(\frac{\alpha}{\pi}) \\ O(\frac{\alpha}{\pi}) & (\frac{\alpha}{\pi} b) & O(\frac{\alpha}{\pi}) \\ O(\frac{\alpha}{\pi}) & O(\frac{\alpha}{\pi}) & 0 \end{pmatrix}
\]

(11)

The complex coefficient in each entry depends on the values of the parameters in \( \langle H \rangle \). Having explicitly exhibited \( b \), all these coefficients are \( O(1) \), and can be expressed as functions of the parameters \( b_i \) and \( a_j \).

Similarly, the form of the corrected \( \langle \Sigma \rangle \) matrix is

\[
\langle \Sigma \rangle' = \langle \Sigma \rangle + \delta\langle \Sigma \rangle = F \begin{pmatrix} 0 & O(\frac{\alpha}{\pi} b^2) & O(\frac{\alpha}{\pi} b) \\ O(\frac{\alpha}{\pi}) & O(\frac{\alpha}{\pi}) & (\frac{\alpha}{\pi} b) \\ O(\frac{\alpha}{\pi}) & O(\frac{\alpha}{\pi}) & O(\frac{\alpha}{\pi}) \end{pmatrix}
\]

(12)

Here again, each entry in the first row and column carries suppression factor of \( O(b) \).

The mass matrix for the down-type quarks is

\[
M_d = y_d \nu \frac{(S)}{F}.
\]

Diagonalizing this matrix, we obtain the mass ratios for down-type quarks. To leading order in \( \alpha/\pi \), we find

\[
\frac{m_d}{m_b} \approx \frac{\alpha}{\pi} b^2
\]

(13)

\[
\frac{m_s}{m_b} \approx \frac{\alpha}{\pi}
\]

(14)

with the \( b \)-quark mass given by \( m_b \approx y_d \nu \). We have dropped corrections of \( O(b^2) \) in each expression. Each includes a coefficient of \( O(1) \) arising from physics above \( M_F \).
The up-type masses are obtained from Eq. (6) using the corrected forms $\langle S \rangle'$ and $(\Sigma)'$. We determine the eigenvalues and mixing angles from the $6 \times 6$ symmetric matrix

$$
\tilde{M}_u \tilde{M}_u^\dagger = \left( \begin{array}{ccc}
y_1^2 & y_1 y_3 & y_1 y_3^\dagger \\
y_1 y_3 & y_3^2 + y_3^\dagger & y_3 y_3^\dagger \\
y_1 y_3^\dagger & y_3 y_3^\dagger & y_3 + y_3^\dagger \end{array} \right).
$$

The three heavy eigenvalues are $\mathcal{O}(y_2^2 F^2), \mathcal{O}((y_3^2 F^2)$, and $\mathcal{O}((\alpha/\pi)^2 F^2 b^2 (y_3^2 + y_3^\dagger))$. Each is well beyond experimental reach for the range of parameters considered here. We diagonalize this matrix and get the up-type quark masses and mixing angles from the $6 \times 6$ matrix

$$
M_u M_u^\dagger = y_1^2 v^2 \frac{\langle S \rangle'}{F} \left( I + \frac{\langle \Sigma \rangle'^\dagger \langle \langle S \rangle' \rangle'^\dagger}{z^2} - \frac{\langle \Sigma \rangle'}{F} \right),
$$

valid for $v/F \ll y_2 \alpha/\pi$, $y_3 \alpha/\pi$. Here, $z \equiv y_2/y_3$.

Diagonalizing this matrix gives the up-type quark masses and mixing angles. To lowest non-vanishing order in $\alpha/\pi$, they take simple algebraic forms. In the limit $(\alpha^2/\pi^2) b^2 \ll z^2 \ll b^2 \ll 1$, appropriate for our numerical fits, we find

$$
m_u \approx \frac{\alpha^2}{\pi^2} b \quad \text{(17)}
$$

$$
m_t \approx \frac{\alpha}{\pi} \quad \text{(18)}
$$

Each expression includes a coefficient of $O(1)$ arising from physics above $M_F$. The $t$-quark mass is given by $m_t \approx y_1 v$. Recall that for $\alpha = 0$, the mass-eigenvalue of order $y_1 v$ corresponds to the $SU(2)_L$-doublet $t$-field. As $\alpha$ is increased to $(\alpha/\pi) b \gg v/F$, this eigenvalue grows to be $O((\alpha/\pi)^2 F^2 b^2 (y_3^2 + y_3^\dagger))$, but its $t$-component decreases to nearly zero. Meanwhile, one of the zero mass-eigenvalues at $\alpha = 0$ grows to $O(y_1 v)$, with its $t$-component growing from zero to nearly $100\%$.

The (small) CKM mixing angles emerge as differences between the diagonalization angles for $M_d M_d^\dagger$ and $M_u M_u^\dagger$. Using conventional definitions [3], we find, again to lowest non-vanishing order in $\alpha/\pi$ and in the limit $(\alpha^2/\pi^2) b^2 \ll z^2 \ll b^2 \ll 1$,

$$
\theta_{23} \approx \frac{\alpha}{\pi} \quad \text{(19)}
$$

$$
\theta_{13} \approx \frac{\alpha^2}{\pi^2} b \quad \text{(20)}
$$

$$
\theta_{12} \approx \frac{\alpha}{\pi} \quad \text{(21)}
$$

The $O(1)$ phases appearing throughout the mass matrices naturally generate an $O(1)$ CKM phase.

Before turning to the phenomenology, it is worth noting that although the expressions for $m_u/m_t$ and $\theta_{13}$ are $O(\alpha^2/\pi^2)$, they are one-loop results. The form of $m_u/m_t$ is due simply to the product of $\alpha$ factors appearing in the "seesaw" expression Eq. (16) for $M_u M_u^\dagger$. This can be seen directly by computing $\det M_u M_u^\dagger$, or by computing $\det M_u M_u^\dagger$ Eq. (15). The latter quantity can be shown to be proportional to $(\alpha/\pi)^6 (z^2 v^F F^6)$ in the limit $(\alpha^2/\pi^2) b^2 \ll z^2 \ll b^2 \ll 1$. The values of the three large eigenvalues of this matrix are as reported above, with one proportional to $\alpha^2$. But only three of its six eigenvalues vanish as $\alpha \to 0$, and one $(m_t^2)$ is proportional to $\alpha^2$. If higher-order corrections are added to the expressions for $(\Sigma)'$ and $(\Sigma)'$, this will not change the leading-order value of $\det M_u M_u^\dagger$. Thus the expression Eq. (17) for $m_u/m_t$ will not be affected by two-loop contributions.

We also find $\theta_{13} = O(\alpha^2/\pi^2)$ Eq. (20), again a one-loop result. We compute $\theta_{13}$ by first noting that the up-sector mixing angles are determined by the second term in the large brackets in Eq. (16), that is, from the diagonalization angles for

$$
\alpha \approx \pi \frac{\langle \langle S \rangle' \rangle'^\dagger}{F} \left( I + \frac{\langle \Sigma \rangle'^\dagger \langle \langle S \rangle' \rangle'^\dagger}{z^2} - \frac{\langle \Sigma \rangle'}{F} \right),
$$

using $\det \tilde{M}_u = 0$. Each of the 7 approximate expressions (13,14,17,18,19,21) depends on one or more of the 3 small parameters $\alpha/\pi, b$, and $z$. Each except $\theta_{12}$ vanishes as $\alpha \to 0$. The coefficients of order unity in each expression depend on the $O(1)$ parameters in $\langle H \rangle$, $\langle S \rangle$, and $\langle \Sigma \rangle$.

We compare the above expressions to the measured values of the mixing angles and the quark mass ratios [4]. These depend of course on the scale at which they are defined. The Yukawa interactions derived here should be regarded as defined at the scale $M_F \gg v$, to be evolved to lower scales through SM interactions. We disregard these renormalization group effects here, and simply compare our expressions with the quark masses and CKM angles defined at $M_Z$. The quark masses in GeV units are $m_t(M_Z) = 176 \pm 5$, $m_b(M_Z) = 2.95 \pm 0.15$, $m_u(M_Z) = 0.65 \pm 0.12$, $m_s(M_Z) = 0.062 \pm 0.015$, $m_d(M_Z) = 0.0017 \pm 0.0005$, $m_d(M_Z) = 0.0032 \pm 0.0009$. The mixing angles measured in tree-level processes, and as defined in Ref. [4], are: $\sin \theta_{12} = 0.2243 \pm 0.0016$, $\sin \theta_{13} = 0.0413 \pm 0.0015$, $\sin \theta_{13} = 0.0037 \pm 0.0005$.

The 6 approximate expressions (18,19,20,21) (excluding the expression for $\theta_{13}$), can be fit to the data, up to coefficients of order unity, by the choices

$$
\frac{\alpha}{\pi} \approx 0.04 \quad \text{(22)}
$$

$$
b \approx 0.2 \quad \text{(23)}
$$

$$
z \approx 0.04. \quad \text{(24)}
$$
While the 6 expressions used in the fit are accurate for these values, we have also performed a numerical study of the model, using the complete expressions for each of the mass ratios and mixing angles, and found that a good agreement with the experimental values can be obtained with the above choices. Finally, the $t$ and $b$ masses are fit with $y_t \approx 4$ and $y_b \approx 10^{-2}$.

The mixing angle $\theta_{13}$, however, is not well fit by the expression Eq. [20]. It is too small by nearly an order of magnitude assuming that the coefficient is $O(1)$. Since this is a one-loop result in the gauge interaction, and since none of the couplings of our effective theory is larger than the (weak) gauge coupling, there appears to be no ingredient in the theory that could produce a large enough value for $\theta_{13}$. Additional ingredients may be necessary to generate a $\theta_{13}$ at the measured ($O(4 \times 10^{-3})$) level. This problem is currently under study.

**Discussion**—There are additional corrections to the quark mass matrices from the emission and re-absorption of the 8 PGB’s of the EFT, whose masses can be estimated to be $O(\sqrt{g^2/4\pi} F)$. Their coupling to the light fermions is suppressed by $v/F$, but they couple to the up-type heavy fermions with strength $y_2$ or $y_3$. Their corrections to the up-type mass matrices can thus be as large as $O(y_2^2/4\pi^2)$. These will be smaller than the family gauge corrections providing only that $y_3 \ll g \approx 1$, as we have already assumed.

In addition to the operators bilinear in the fermion fields that we have analyzed so far (Eq. [1]), there are more such operators, with higher powers of $S$ and $\Sigma$. The nonlinear constraints of the EFT mean that they are not dimensionally suppressed, and they can contribute directly to the quark mass matrices. Among the operators with two powers of the scalar fields, because of the structure of $\langle S \rangle$ and $\langle \Sigma \rangle$, only one contributes to the quark masses to zeroth order in $\alpha$. It is $g q (S \times \Sigma)^* u^*/F^2$. Since $\langle S \times \Sigma \rangle = diag\{seF^2, 0, 0\}$ to zeroth order, it gives a direct contribution to the mass of up quark. Its presence reflects the fact that the diagonal zero’s in $\langle S \rangle$ and $\langle \Sigma \rangle$ Eq. [2] are not stable against quantum corrections.

The natural size of this operator can be argued, however, to be small. It can be estimated from quantum loops based on the interactions of Eq. [1]. We first note that such quantum loops do not destabilize the assumed smallness of the $y_i$ themselves. They are protected by chiral symmetries in the limit $y_i \to 0$. For the operator $g q (S \times \Sigma)^* u^*/F^2$, its coefficient can be estimated to be $O(y_1 y_2 y_3/4\pi^2)$. For the range of $y_i$ values employed here, this gives a contribution to $m_u$ comfortably smaller than the family-gauge contribution. Similar analyses can be applied to the full tower of operators bilinear in the fermion fields.

It is important to extend the framework suggested here to the charged leptons and neutrinos. A neutrino seesaw mechanism could emerge using the large scale $F$, providing it is large enough. It is also possible that the present framework can be combined with grand unification at still higher scales [4].

**Conclusion**—We conclude that the general pattern of up-type quark masses, down-type quark masses, and CKM mixing angles, with the exception of $\theta_{13}$, can be understood as arising radiatively from a relatively weak $SU(3)$ family gauge interaction, with a sequential breaking of this symmetry – first preserving a discrete $Z_2$ subgroup of the gauged $SU(3)$ and then breaking it. The up-type quark mass ratios are generated via mixing with heavy fermions after the radiative corrections are included. The detailed predictions depend on the two additional small parameters $b$ (in the scalar potential of the hidden sector) and $z$ (a ratio of two Yukawa couplings of the up-type fermions), as well as various $O(1)$ parameters, all of which are determined by physics above the family breaking scale. The values of $m_u$ and $m_d$ are determined by the parameters $y_d$ and $y_t$, which also arise from physics above the family breaking scale. The framework leads, however, to a value for $\theta_{13}$ that is smaller than the measured value by nearly an order of magnitude, indicating the need for additional ingredients.

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