ManyClaw: Slicing and dicing Riemann solvers for next generation highly parallel architectures

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Abstract

Next generation computer architectures will include order of magnitude more intra-node parallelism; however, many application programmers have a difficult time keeping their codes current with the state-of-the-art machines. In this context, we analyze Hyperbolic PDE solvers, which are used in the solution of many important applications in science and engineering. We present ManyClaw, a project intended to explore the exploitation of intra-node parallelism in hyperbolic PDE solvers via the Clawpack software package for solving hyperbolic PDEs. Our goal is to separate the low level parallelism and the physical equations thus providing users the capability to leverage intra-node parallelism without explicitly writing code to take advantage of newer architectures.

1 Introduction

As the scientific computing community approaches the hurdles necessary to reach exascale computation, many are looking at how current PDE solver packages can be easily transformed for the coming changes. Current practices require heavy involvement from application scientists and engineers in the algorithm and design process. Lowering such barriers will facilitate the adoption of next-generation computer architectures for a wider number of applications. Many of the current solvers require the use of methods such as adaptive mesh refinement, that make the task of identifying data exchanges for not only MPI tasks but now in the context of intra-node parallelism which requires the modeling of shared data and execution on coprocessors.

In this paper we propose and analyze a number of different strategies for intra-node parallelism in the context of hyperbolic PDE solvers via ManyClaw[5]. ManyClaw is a many-core implementation of Clawpack [2], a popular and representative finite volume package that has a large number of applications ranging from tsunami propagation across the Pacific to elasticity problems in complex material including the human body. We hope to provide a number of different approaches to leveraging intra-node parallelism analyzing each for their scalability and suitability for easy adaptation by application developers.

1.1 Hyperbolic PDE Solvers and Finite Volume Methods

Hyperbolic PDEs form the basis of many important physical phenomena that are of interest to a wide variety of scientists and engineers. The most basic hyperbolic PDEs include the linear wave equation and itself capture many important processes. These linear hyperbolic PDEs are often approximations to fully nonlinear systems of PDEs which can exhibit complex behavior such as the development of discontinuities from smooth initial conditions in finite time (commonly called shocks). Solvers that attempt to solve these...
nonlinear systems are often costly and require the use of sophisticated nonlinear computational kernels such as Riemann solvers and limiters. Clawpack is one such solver employing a finite volume wave-propagation algorithm that includes these computational kernels and provides a proven approach to solving hyperbolic PDEs in general.

Finite volume methods discretize a domain by partitioning it into cells and then evolving the average value of each quantity of interest. At the edges of these cells a flux must be calculated which often involves the computation of the solution to the Riemann problem which itself is composed of the original PDE on an infinite domain with a discontinuity in the initial conditions at $x = 0$. The simplification of the general problem into a series of simpler problems is one of the core advantages of finite volume types of methods and allow for the physics of the problem to be largely decoupled from the algorithm and computational method being used. For this reason, ManyClaw currently concentrates on implementing the solution of a variety of Riemann solvers using multiple different approaches to intra-node parallelism with the requirement that the underlying Riemann solver does not have to be aware of the parallelism and is left unchanged from a serial execution of the Riemann solvers.

### 1.2 Computational Decomposition

The kernels used to evolve a solution forward in time can be mapped by arithmetic intensity to various levels of a heterogeneous compute architecture. In the case of Clawpack, the most intense kernels including the Riemann solver and limiter routines can be mapped to coprocessor technologies, mid-level kernels such as local time step management and flux accumulation to the multicore node, and low intensity kernels such as grid management to the distributed set of nodes. For this paper we concentrate on only the high intensity Riemann solver routines as this approach has the potential to significantly accelerate many application developer codes with minimal intervention on their part.

Some threads will handle larger portions of the data and dynamically schedule pieces to finer-grained threads. Such a dynamic execution model, matching exascale machine designs, allows for local precision with minimal effects to global time stepping.

Grid management must marshal data between processes running different portions of the grid. Additionally, it must manage the refinement between grids and occasionally adjust the load balance between processes. Finally, it serves as the dynamic dispatcher for the time stepping kernels. As the intra-node parallelism is increased, this operation will be managed even between devices and shared memory pools. For the most part, this problem has been solved \cite{3,1} and scales to leadership-class machines. Thus we leave its analysis for other work.

To mitigate the CFL condition limits, each refined grid can manage its own time step. This requires an update procedure on refinement boundaries which varies by order of spacial or temporal interpolation. These operations will have to grab data from the values produced by fine grained threads, such as those on a coprocessor, and work in its own node level thread.

The Riemann problem implements the updates of the cell fluxes from field values based upon the specific modeling equations. This is the most arithmetic intense portion of the code and is dependent on the equations being solved. The minimal overlap between cells lends it to be highly amenable to coprocessors.

The final kernel to discuss is the limiter which updates the field values based upon the fluxes computed by the Riemann problem. These simple operations require more cell data in the computation but is still

| Kernel                  | Algorithmic Intensity | Data in                          | Data out                          |
|-------------------------|-----------------------|----------------------------------|-----------------------------------|
| Grid Management         | low                   | grid and refinement details      | refined grids                     |
| Dynamic Time stepping   | low to mid            | time step of adjacent grid       | updated values at grid boundaries |
| Riemann Problem         | high                  | grid cell centers                | values at grid boundaries         |
| Limiters                | high                  | values at grid boundaries        | grid cell centers                 |

Table 1: Summary of kernel for explicit methods for hyperbolic PDEs
local to the Riemann problem. Limiters also must match up between different sizes of grids requiring extra overhead associated with the mesh refinement.

2 Threading the Riemann solver

The rest of this preliminary report will focus on the scalability of threading different Riemann problems. Two recent debates in the Clawpack community guided this study. The first issue is the type of code that must be written for speed. While the numerical scientist thinks about the Riemann problem most naturally in a pointwise manner, i.e. considering only a single interface at a time, the Clawpack code used vectorized Riemann problems presumably for speed. The second issue regarded the CUDA implementation of Clawpack that has not been adopted by the community. While CUDACLAW [4] showed speedups of up to 40X over the Clawpack implementation, it did so at considerable code complexity. To keep the decomposition easy to reuse by the community, we tried to keep the work in standard work arrays that in practice will cause bandwidth bottlenecks. The resolution of this second issue has been left as further work.

To address these issues, we implemented the four standard Riemann problems with varying arithmetic intensity via vectorized Fortran, OpenMP, and TBB. The code was factored to allow any threading model to call a Riemann problem, provided for by the Clawpack community, over a structured grid. Our hope is to add a more threading models, such as Intel SPMD Program Compiler and Thrust CUDA, to further improve the study.

The main function evaluated is the Riemann problem, which takes the states and auxiliary variables of two or more adjacent cells and computes the flux between the cells. The simplest loop is to compute this at each cell interface for each direction one at a time. We also implemented versions that compute each cell direction at the same time and then one that tiles the data for better cache locality. The simplest and most efficient turned out to be calculating each direction at the same time. This was the basis for the threading models.

2.1 Fortran Code Comparison

The Fortran results are meant as a representative sample of how the original Clawpack code performs. A number of different comparisons were produced based on the Fortran code that is currently a part of Clawpack. The Clawpack algorithms are currently implemented as a vector evaluation of a strip of the discretized domain and sweeps are done in each direction. Three different methods of access to the entire grid were implemented in an effort to understand the access overhead that is imposed by the artificial access patterns required for the tests. The first is the nearest implementation to what is currently done in Clawpack, a direct access slice through the strided arrays. The second uses Fortran pointers in order to window into the appropriate data. The last is an artificial test that removes all overhead due to accessing strips of the array. The last implementation reported is a point-wise evaluation of the Riemann solver similar to what has been implemented in the other tests.

2.2 OpenMP parallel for

The OpenMP code used C++ and the parallel for construct over the cell centers. For our tests, this code simply adds a pragma statement to the outermost cell iterator. In practice, when there are many threads managing the time stepping and other grids we expect memory affinity and thread management to degrade performance. Our numbers in this since give us a best-effort performance when there are no other factors of a highly parallel code running.

2.3 Thread Building Blocks

The Thread Building Blocks (TBB) implementation used C++, as required by the tool, and the parallel for execution over a blocked_range2d iterator. TBB uses various iterators to smartly schedule the running ker-
In our results we see that this strategy which often requires some warm-up time to schedule effectively works well for larger thread pools and problem sizes.

### 3 Numerical Tests and Results

Our numerical tests focused on four common Riemann problems and scalability on both a 12 core Westmere and a 30 core Knight’s Ferry supporting 120 threads. Both these architectures were used at the Texas Advanced Computing Center. The Westmere is a commodity architecture similar to what many in the Clawpack community use. While Clawpack does not take advantage of intra-node parallelism, it is understood to scale on coming highly parallel architectures the Knight’s Ferry is a good test ground.

The four kernels implemented are advection, constant coefficient acoustics, variable coefficient acoustics and Euler. We have a rough calculation of the arithmetic intensity of each kernel in Table 2; this number is based on hand counting the stores and flops in source and is a rough order of magnitude number.

In general, the C++ kernels outperformed the Clawpack Fortran kernels but not the rewritten point-wise Fortran, see Table 3. These numbers were surprising to the authors, but the Fortran array indexing seems to be causing the difference. The artificial test shows that calling the Riemann problem was not the major overhead.

The threading tests are not quite conclusive and show several interesting characteristics. We show the timing and scaling of the runs with the Westmere to give the reader a feel of how long these steps take. Each millisecond counts as it is quite often that the time stepper will require millions of timesteps. We show the advection (Figure 1) and Euler (Figure 2) problems as they do the least and most amount of work, respectively. While the timing data is pretty smooth for both, the scalability jumps around a bit. Perhaps the most notable trend is the different scalability trends the Euler problem takes based on the grid size. Additionally TBB does worse on the advection kernel presumably due to the lack of warm up time for scheduling.

Figure 3 shows the scaling of the code running on the Knight’s Ferry. It continues to get about 50% scaling for Euler and 40% for advection, despite the large number of cores. This bodes well for the programmer who can scale the code on a current core such as the Westmere.

### 4 Conclusion

In the paper we demonstrated an easy-to-use system that allows Clawpack users to utilize intra-node parallelism with reasonable scaling results. The work is far from done as there still needs several parts of the algorithm to complete. Notably, limiters, dynamic time stepping, and automatic mesh refinement. Additionally, we expect to explore several other libraries for parallelism, especially Thrust, Intel SMPD Program.
Figure 1: Timing and Scaling of different grids on advection problems using Westmere.

Compiler, OpenCL, and CUDA.

References

[1] David I Ketcheson, Kyle T Mandli, Aron Ahmadia, Amal Alghamdi, Manuel Quezada, Matteo Parsani, Matthew G Knepley, and Matthew Emmett. Accessible, Extensible, Scalable Tools for Wave Propagation Problems. arXiv.org, math.NA, November 2011.

[2] Randall J. LeVeque, Marsha J Berger, and et al. Clawpack, 2012. http://www.clawpack.org

[3] Kyle T Mandli, David I Ketcheson, and et al. PyClaw, 2012. http://www.pyclaw.org

[4] Jeff A. Stuart, H. Gorune Ohannessian, Kyle Mandli, George Turkiyyah, John D. Owens, and David Ketcheson. CUDACLAW: An interactive data-parallel hyperbolic pde solver. poster, SC11, 2011.

[5] Andy R. Terrel, Kyle Mandli, and W. Nathan Bell. Manyclaw, 2012. http://github.com/aterrel/Manyclaw all tests from this paper use commit 6228c3067a.
Figure 2: Timing and Scaling of different grids on Euler problems using Westmere.

Figure 3: Scaling of various problems using Knight’s Ferry on a 1024x1024 grid.