Symmetry breaking indication for supergravity inflation in light of the Planck 2015

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Abstract. Supergravity (SUGRA) theories with exact global U(1) symmetry or shift symmetry in Kähler potential provide natural frameworks for inflation. However, quadratic inflation is disfavoured by the new results on primordial tensor fluctuations from the Planck Collaboration. To be consistent with the new Planck data, we point out that the explicit symmetry breaking is needed, and study these two SUGRA inflation in detail. For SUGRA inflation with global U(1) symmetry, the symmetry breaking term leads to a trigonometric modulation on inflaton potential. Coefficient of the U(1) symmetry breaking term is of order $10^{-2}$, which is sufficient large to improve the inflationary predictions while its higher order corrections are negligible. Such models predict sizeable tensor fluctuations and highly agree with the Planck results. In particular, the model with a linear U(1) symmetry breaking term predicts the tensor-to-scalar ratio around $r \sim 0.01$ and running spectral index $\alpha_s \sim -0.004$, which comfortably fit with the Planck observations. For SUGRA inflation with breaking shift symmetry, the inflaton potential is modulated by an exponential factor. The modulated linear and quadratic models are consistent with the Planck observations. In both types of models the tensor-to-scalar ratio can be of order $10^{-2}$, which will be tested by the near future observations.

Keywords: inflation, supersymmetry and cosmology, particle physics - cosmology connection

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1 Introduction

To realize inflation [1–3] in supergravity (SUGRA) theory, the flat conditions give strong constraints on the F-term scalar potential with an exponential factor $e^{K(\Phi, \bar{\Phi})}$ which is too steep to generate inflation by the field close or above the reduced Planck scale. This is the well-known $\eta$ problem for SUGRA inflation. The $\eta$ problem can be solved if the Kähler potential admits certain symmetry, such as in no-scale SUGRA with global SU($N, 1$)/SU($N$) × U(1) symmetry [4–8]. The classical quadratic inflation [3] is simply realized in supergravity theory with global U(1) symmetry [9, 10] or shift symmetry [11] in the Kähler potential. The quadratic inflation predicts large primordial tensor fluctuations with the tensor-to-scalar ratio $r \simeq 0.15$. However, the recent observations from the Planck and BICEP2/Keck Array Collaborations have provided strong constraints on the primordial tensor fluctuations [12–14], $r < 0.11$ ($r < 0.12$ from BICEP2/Keck Array) at 95% Confidence Level (C.L.). So the simplest quadratic inflation is disfavoured. In light of the new Planck results, another proposal for supergravity inflation with slightly explicit symmetry breaking becomes important. This is based on the fact that the $\eta$ problem can also be solved by an approximate symmetry in the Kähler potential while the inflationary observables are sensitive to such potential variation. Interestingly, the inflationary observables can be significantly modified while the models are still free from the $\eta$ problem.

A natural solution to the $\eta$ problem is from the global U(1) symmetry in the Kähler potential of the minimal supergravity (mSUGRA) $K = \Phi \bar{\Phi}$, which is invariant under the U(1) rotation: $\Phi \rightarrow \Phi e^{i\theta}$. To employ the phase $\theta$ as the inflaton, it requires strong field stabilization in the radial direction and phase monodromy in the superpotential, which are simply fulfilled in helical phase inflation driven by the potential with helicoid structure [9]. The global U(1) symmetry is of specially importance, because it not only provides a new solution for the $\eta$ problem, but also protects the models away from quantum loop corrections, which can appear only in the Kähler potential but not in the superpotential, and they depend on the radial component instead of the phase so have little effect on phase inflation. Moreover, according to the Lyth bound [15] on the inflationary models with tensor-to-scalar ratio larger than 0.01, the super-Planckian field excursion is needed, which potentially makes the models...
unreliable due to the quantum gravity effects. In helical phase inflation, the super-Planckian field excursion is fulfilled along a helix trajectory, and then the problem is solved. Moreover, helical phase inflation can realize a super-Planckian phase decay constant through phase monodromy, which corresponds to the explicit U(1) symmetry breaking in the superpotential and provides a simple phase-axion alignment.

Helical phase inflation remarkably relates to many important and interesting developments on large field inflation. The idea to employ the phase, a pseudo-Nambu-Goldstone boson (PNGB) as inflaton was first proposed in ref. [16, 17] to protect the flat potential away from the Ultra Violet (UV) corrections. The inflation driven by the PNGB potential has been studied extensively in refs. [24–32]. The axion alignment mechanism for super-Planckian axion decay constant was first proposed in ref. [33]. The monodromy inflation as an attractive proposal to realize the super-Planckian field excursion in string theory was provided in refs. [34–39]. In refs. [40–42] the axion alignment mechanism is explained as a special type of monodromy inflation realized by axions. In fact, a similar name “helical inflation” was first used in ref. [41], in which the helical structure refers to the alignment of axions, while in our models the “helical” path is from the single phase component of a complex field with stabilized radial component.

Considering the crucial role of the global U(1) symmetry for inflation, it is questionable if the merits of helical phase inflation maintain after the U(1) symmetry breaking. Since the global U(1) symmetry is broken at tree level, the extra terms may be generated from quantum loop corrections or after integrating out heavy fields and then break the U(1) symmetry further. To fit the experimental observations, the inflationary observables are of order $10^{-2}$. So the U(1) symmetry breaking is at the same order, which is large enough to improve the inflation predictions while its higher order corrections are too small to induce notable effect. In this work we will show that by introducing a small U(1) symmetry breaking term, the phase potential will be slightly modulated by a trigonometric factor, and their predictions on inflation are highly consistent with the new Planck results. Specifically, it predicts interesting running of spectral index with magnitudes depending on the U(1) symmetry breaking term.

Another concise solution of the \( \eta \) problem has been proposed for years [11]. Its Kähler potential is constrained by an extra shift symmetry: \( \Phi \rightarrow \Phi + iC \) so that \( K = K(\Phi + \bar{\Phi}) \) is independent of the \( \text{Im}(\Phi) \) and then the \( \eta \) problem is evaded for inflation driven by \( \text{Im}(\Phi) \). Interestingly, this kind of models was shown to be closely related to the helical phase inflation models with U(1) symmetry, as studied in [10]. Through a field redefinition \( \Phi = e^\Psi \) and ignoring the higher order terms which vanish after field stabilization, the models with U(1) symmetry reduce to the models with shift symmetry \( \Psi \rightarrow \Psi + iC \).

In ref. [19] we for the first time proposed the shift symmetry breaking in the Kähler potential. The quadratic potential, as well as other power-law potentials are modulated by an exponential factor, which generates inflation with a scalar spectral index \( n_s \approx 0.96 \sim 0.97 \) and especially a broad range of tensor-to-scalar ratio \( r \). The predictions are well consistent with the Planck results published in 2013 [20]. The potential role of the symmetry breaking term was discussed in ref. [21] and the tensor-to-scalar ratio can be as large as \( r \approx 0.2 \), as shown in ref. [22] in light of the BICEP2 results on large tensor fluctuations evaluated from B-mode polarizations that are significantly affected by the dust contributions [23]. Because the new Planck results provided stronger constraint on the tensor-to-scalar ratio, it is very important to compare these models with the new observations further.

In this work, we study SUGRA inflation with breaking global U(1) symmetry or shift symmetry in the Kähler potential, and compare their predictions with the new Planck re-
results. In the SUGRA inflation with global U(1) symmetry, a trigonometric modulation on inflaton potential is generated from the symmetry breaking term. The coefficient of the U(1) symmetry breaking term at order $10^{-2}$ is large enough to improve the inflationary predictions while the higher order corrections are negligible. The predicted sizeable tensor fluctuations are highly consistent with the Planck results. Especially, the model with a linear U(1) symmetry breaking term has the tensor-to-scalar ratio around $r \sim 0.01$ and running spectral index $\alpha_s \sim -0.004$, which comfortably fit with the Planck observations. In the SUGRA inflation with breaking shift symmetry, the modulated linear and quadratic models agree with the Planck observations due to the additional exponential factor in the inflaton potential. In these two kinds of models, the tensor-to-scalar ratio can be of order $10^{-2}$, which will be tested by the near future experiments. Therefore, the new Planck data strongly suggest the global U(1) or shift symmetry breaking in Kähler potential for SUGRA inflation.

This paper is organized as follows. In section 2 we review helical phase inflation based on the minimal supergravity with global U(1) symmetry. In sections 3 and 4, we study the inflationary models associated with U(1) symmetry breaking and shift symmetry breaking, respectively. Our conclusion is given in section 5.

2 Brief review of helical phase inflation

Initially helical phase inflation was introduced to solve the $\eta$ problem for inflation in supergravity theory [9, 10]. It starts from the trivial fact that in the minimal supergravity, the Kähler potential for a chiral superfield $\Phi$: $K(\Phi, \bar{\Phi}) = \Phi \bar{\Phi}$ is invariant under a global U(1) transformation: $\Phi \to \Phi e^{i\theta}$, and so is the factor $e^K$ in the F-term scalar potential. In consequence, by using the phase component of $\Phi$ as inflaton the $\eta$ problem is solved automatically. Nevertheless, we need to solve two problems related to this proposal: stabilization of the radial component of $\Phi$ and realization of the phase monodromy in superpotential.

Firstly, the radial component of $\Phi$ should be stabilized otherwise it would generate notable iso-curvature perturbations that contradict with the experimental observations. However, it is non-trivial to stabilize the norm of $\Phi$ while keep its phase light as they couple with each other. Because the superpotential $W(\Phi)$ is an holomorphic function of $\Phi$, without extra U(1) charged field $W$ is not invariant under general U(1) transormation. If, for a whole circular U(1) rotation $\Phi \to \Phi e^{i2\pi}$, the superpotential $W$ is invariant, then the F-term scalar potential of $\Phi$ is exactly periodic under $\theta \to \theta + 2\pi$. With a sub-Planckian field norm $|\Phi| \leq M_P$, such potential cannot provide sufficient trans-Planckian field excursion that is needed for large field inflation with tensor-to-scalar ratio $r > 0.01$. Therefore, the superpotential $W$ should break the global U(1) symmetry in the way without the discrete symmetry $\Phi \to \Phi e^{2\pi i}$. In the other words, there is a phase monodromy in $W$. For different phase monodromies in $W$, we can get different types of large field inflationary models, for example, the quadratic inflation or natural inflation with super-Planckian decay constant.

2.1 Quadratic inflation

Helical phase inflation with quadratic potential was proposed in ref. [10]. The Kähler potential and superpotential in supergravity theory are

$$K = \Phi \bar{\Phi} + X \bar{X} - g(X \bar{X})^2, \quad W = a \frac{X}{\Phi} \ln(\Phi). \quad (2.1)$$
The global U(1) symmetry is broken by the superpotential while restored when $a = 0$, so the global U(1) symmetry is technically natural [18]. The phase monodromy in the superpotential $W$ is

$$\Phi \rightarrow \Phi e^{2\pi i}, \quad W \rightarrow W + 2\pi ai \frac{X}{\Phi}. \tag{2.2}$$

As shown in refs. [9] and [10], the above superpotential $W$ in eq. (2.1) can be obtained from the following superpotential

$$W_0 = \sigma X \Psi(T - \delta) + Y(e^{-\alpha T} - \beta \Psi) + Z(\Psi \Phi - \lambda), \tag{2.3}$$

by integrating out the heavy fields, where the coupling $Ye^{-\alpha T}$ can be generated through non-perturbative effect. The phase monodromy in eq. (2.2) originates from the approximate global U(1) symmetry in $W_0$

$$\Psi \rightarrow \Psi e^{-i\alpha \theta}, \quad \Phi \rightarrow \Phi e^{i\alpha \theta}, \quad Y \rightarrow Ye^{-i\alpha \theta}, \quad T \rightarrow T + i\alpha \theta. \tag{2.4}$$

This global U(1) symmetry is exact in the last two terms of $W_0$ while is broken explicitly by its first term, which is hierarchically smaller but dominates the inflation process. The phase monodromy of $W_0$ under the circular U(1) rotation is

$$\Psi \rightarrow \Psi e^{-i2\pi}, \quad W_0 \rightarrow W_0 + i2\pi \frac{1}{\alpha} X \Psi. \tag{2.5}$$

The field $X$ during inflation is strongly stabilized at $X = 0$ due to the large mass obtained from the factor $e^{X \bar{X}}$ in the $F$-term scalar potential. With this field stabilization the $F$-term scalar potential is simplified as

$$V = e^{\Phi \bar{\Phi}} W X \bar{W} X = a^2 e^{\frac{1}{r^2} \left( (\ln r)^2 + \theta^2 \right)}, \tag{2.6}$$

in which $\Phi = re^{i\theta}$. The potential has interesting helicoid structure, and provides strong stabilization on the radial component $\langle r \rangle = 1$ in Planck unit $M_P = 1$, where $M_P$ is the reduced Planck scale (the Planck unit is adopted throughout the paper). The phase component of $\Phi$ decouples with the radial component, and its Lagrangian after field stabilization becomes

$$L = \partial_\mu r \partial^\mu r + r^2 \partial_\mu \theta \partial^\mu \theta - a^2 e^{\frac{1}{r^2} \left( (\ln r)^2 + \theta^2 \right)} \approx \partial_\mu \theta \partial^\mu \theta - e a^2 \theta^2, \tag{2.7}$$

which gives the quadratic inflation. The inflaton evolves along a helix trajectory — the local valley of helicoid potential.

### 2.2 Natural inflation

According to the new Planck results [13], natural inflation locates in the region with 95% confidence level for the effective axion decay constant $f_a \geq 6.9 M_P$. The effective large axion decay constant can be obtained from axion alignment mechanism [33]. Nevertheless, its supergravity or string realization is rather difficult [43, 44], since generically the axions (phase) couple with other components in the $F$-term scalar potential, and it is highly non-trivial to stabilize all the extra components while keep the axions light. The axion alignment with consistent moduli stabilization was fulfilled in ref. [45], where the anomalous U(1) gauge symmetry plays a crucial role as its D-term potential automatically separates the axions from
extra components. Inflation based on the anomalous U(1) gauge symmetry has been studied extensively [46–51].

We showed that the supergravity setup given by eq. (2.1) can be slightly modified to accommodate natural inflation [10]. With the same Kähler potential, we considered the following superpotential

\[ W_1 = \sigma X \Psi (e^{-\alpha T} - \delta) + Y (e^{-\beta T} - \mu \Psi) + Z (\Psi \Phi - \lambda), \tag{2.8} \]

in which \( 1 \ll \alpha \ll \beta \) since for each single phase, its decay constant is much lower than the Planck mass, and a small hierarchy between \( \alpha \) and \( \beta \) is needed to get super-Planckian phase decay constant. The last two terms in eq. (2.8) are the same as these in eq. (2.3), while the first term, which is perturbative in eq. (2.3), now is replaced by the non-perturbative coupling in eq. (2.8). And the phase monodromy becomes

\[ \Psi \rightarrow \Psi e^{-2 \pi i}, \quad W_1 \rightarrow W_1 + \sigma X \Psi (e^{-2 \pi i \frac{\alpha}{\beta}} - 1). \tag{2.9} \]

By integrating out the heavy fields the superpotential (2.8) reduces into

\[ W' = a X \Phi (\Phi - b - c), \tag{2.10} \]

in which \( b = \frac{\alpha}{\beta} \ll 1 \) and \( c \approx 1 \). Here, the small \( b \) arising from the small hierarchy between \( \alpha \) and \( \beta \) is crucial to realize large phase decay constant. The potential with fractional power was introduced in ref. [26] to get super-Planckian field excursion and large axion decay constant, it also plays a key role in ref. [47] to obtain the super-Planckian axion decay constant together with anomalous U(1) gauge symmetry. And then the scalar potential is

\[ V = e^{e^2 a^2 r^2 (r^{-2 b} + c^2 - 2 c r^{-b} \cos(b \theta))} + e^{e^2 a^2 c r^2 (\sin(b \theta)^2)}, \tag{2.11} \]

where the first term has a minimum at \( \langle r \rangle = r_0 = c^{-\frac{1}{2}} \approx 1 \) and the minimum of the second term locates at \( \langle r \rangle = r_1 = \sqrt{1 + \frac{b}{2}} \). Giving \( r_0 \approx r_1 \), the scalar potential has a global minimum around \( r_0 \approx 1 \), where the radial component is well stabilized. The Lagrangian of the phase becomes

\[ L = \partial_\mu \theta \partial^\mu \theta - \Lambda^4 [1 - \cos(b \theta)], \tag{2.12} \]

which generates the natural inflation with super-Planckian phase decay constant.

3 The helical phase inflation with U(1) symmetry breaking

The global U(1) symmetry of Kähler potential plays a fundamental role to realize phase inflation in supergravity. This symmetry provides a flat direction for inflation and resolves the \( \eta \) problem. As we pointed out before, technically the \( \eta \) problem can also be solved by an approximate symmetry, i.e., the symmetry which protects the flatness of potential can be explicitly broken, as firstly shown in ref. [19] for the SUGRA inflation with shift symmetry.

Besides providing a flat direction for inflation, the global U(1) symmetry can protect the phase potential away from the quantum corrections [10]. The superpotential is free from
quantum loop corrections because of the non-renormalized theorem, but the correction terms do appear in the Kähler potential. In particular, to generate the effective superpotential like eqs. (2.1) and (2.10), we need to integrate out the heavy fields above the inflation energy scale. Such process may introduce higher order corrections to the Kähler potential as well. The global U(1) symmetry guarantees that as long as these corrections do not break U(1) symmetry, they can only slightly affect the field stabilization along the radial direction instead of modify the phase potential. If the global U(1) symmetry is explicitly broken at tree level, then the quantum loop effect and heavy fields are likely to generate extra terms that break the U(1) symmetry further, and then the inflation process may be seriously affected by these corrections depending on the magnitude of the symmetry breaking term. Fortunately, the slow-roll parameters of inflation

\[ \epsilon = \frac{M_P^2}{2} \left( \frac{V_\phi}{V} \right)^2, \quad \eta = \frac{M_P^2 V_{\phi\phi}}{V}, \]

(3.1)

at the stage when the current universe scale crossed the horizon, are of order $10^{-2}$. So the symmetry breaking term at the order $10^{-2}$ is sufficient to affect the slow-roll parameters. While the higher order corrections from the symmetry breaking term are of order $10^{-4}$ or even smaller, which are far beyond the scope of current observations.

The global U(1) symmetry can be broken by the real terms like $c(\Phi^\dagger + \bar{\Phi})$ or $i c(\Phi^\dagger - \bar{\Phi})$. Taking $c(\Phi^2 + \bar{\Phi}^2)$ as an example, we have

\[ K = \Phi \bar{\Phi} + c(\Phi^2 + \bar{\Phi}^2) + X \bar{X} - g(X \bar{X})^2, \]

\[ W = a \frac{X}{\Phi} \ln \Phi, \]

(3.2)

where the coefficient $c$ is of order $10^{-2}$. During inflation $X$ is stabilized at $\langle X \rangle = 0$, and then the $F$-term scalar potential is

\[ V = e^{\Phi + c(\Phi^2 + \bar{\Phi}^2)} X \bar{X} \frac{1}{r^2} \left( (\ln r)^2 + \theta^2 \right). \]

(3.3)

So the global vacuum is $\langle r \rangle = 1$, $\theta = 0$. For large $\theta$ during inflation, the coefficient of above potential has a minimum at $r_1 = \frac{1}{\sqrt{1 + 2c\cos(2\theta)}}$. Thus, the potential possesses a helicoid structure, as shown in figure 1.

During inflation $r \approx r_1$, the first term $(\ln r)^2 \sim c^2 \sim 10^{-4} \ll \theta^2 \sim O(10)$ is negligible. Applying the radial component stabilization to eq. (3.3), we obtain the phase Lagrangian

\[ L = \frac{1}{1 + 2c\cos(2\theta)} \partial_\mu \theta \partial^\mu \theta - a^2 \epsilon \left( 1 + 2c\cos(2\theta) \right) \theta^2. \]

(3.4)

The radial component depends on $\theta$ and then is slowly changing during inflation, nevertheless its kinetic energy is of the order $c^2$ and dropped in above formuluar. The quadratic phase potential is modulated by a cosine factor because of the U(1) symmetry breaking term. The modulated potential is presented in figure 2. The effects of the potential modulation on inflation have been widely studied before [52–62]. The potential modulation leads to an effective inflaton mass slowly varying with time, or equally the flatness condition of inflaton potential oscillates during inflation. Both the spectral index and tensor-to-scalar ratio are
Figure 1. Left: the helicoid structure of the potential in eq. (3.3) with $c = 0.01$, the red dashed line indicates the local valley with $r_1 = \frac{1}{\sqrt{1+2c \cos(2\theta)}}$, along which the inflaton evolves. Right: the helix trajectories with broken U(1) symmetry (red dashed) and exact U(1) symmetry (blue).

Figure 2. The phase potentials for exact U(1) symmetry (dotted) and explicitly U(1) symmetry breaking (blue and red). The blue and red curves are respectively the potentials modulated by cosine and sine factors, which originate from different U(1) symmetry breaking terms. Here, the coefficient of U(1) symmetry breaking term is $c = 0.04$.

determined by the flatness condition of inflaton potential. The tensor-to-scalar ratio could be increased or suppressed depending on the phase of modulation when the mode $k_i = a_i H_i$ crosses the horizon for the first time. Furthermore, as a result of modulation, the flatness condition of inflaton potential, and so is the spectral index, changes notably during field excursion, which introduces significant running of spectral index. Without modulation the potential is smooth and generically the running of spectral index is insignificant, such as in the quadratic inflation $V = \frac{1}{2} m^2 \phi^2$. In our model, the U(1) symmetry breaking term also slightly modulates the phase kinetic term. The predicted inflationary observables can
Figure 3. $r$ versus $n_s$ for the cosine modulated quadratic inflation with quadratic symmetry breaking term (right) and the sine modulated quadratic inflation with linear symmetry breaking term (left). The two purple regions represent the 95% and 68% C.L. according to the Planck results [13]. In each graph, from the left-top to right-bottom, the curves present the $n_s - r$ relations with different e-folding numbers $N = \{50, 52, 54, 56, 58, 60\}$. The curves start from the predictions of exact quadratic inflation corresponding to $c = 0$ and extend with increasing parameter $c$. The results in the left graph, which relate to the quadratic U(1) symmetry breaking term, strongly depend on the e-folding number $N$, indicating the notable scalar spectral index running. In contrast, for the results from linear U(1) breaking term, as shown in the right graph, they just slightly shift for different $N$ and correspondingly, the running of spectral index is rather small.

be significantly improved by the modulation, as presented in figure 3. In particular, the tensor-to-scalar ratio spreads in a broad range. The $n_s - r$ relations strongly depend on the e-folding number, indicating a notable running of spectral index $\alpha_s$. Moreover, we can get the small running of spectral index as well.

Next, we shall consider the U(1) symmetry breaking by the linear term $ic(\Phi - \bar{\Phi})$. The Kähler potential in eq. (3.2) is replaced by

$$K = \Phi \bar{\Phi} - ic(\Phi - \bar{\Phi}) + X \bar{X} - g(X \bar{X})^2. \quad (3.5)$$

The scalar potential after field stabilization $X = 0$ is

$$V = a^2 e^{r^2 + 2cr \sin \theta} \frac{1}{r^2} ((\ln r)^2 + \theta^2). \quad (3.6)$$

The radial component is stabilized at $r \approx 1 - \frac{1}{2} \sin \theta$ up to the order $c$ ($O(c)$), and then the phase Lagrangian becomes

$$L = \frac{1}{1 + c \sin \theta} \partial_{\mu} \theta \partial^{\mu} \theta - a^2 c(1 + 2c \sin \theta) \theta^2. \quad (3.7)$$

The corrections from higher order terms $O(c^n), n \geq 2$ on the inflationary observables are far beyond the current observations so can be ignored. Different from the potential in eq. (3.4), the above potential is modulated by a sine factor instead of cosine factor as well as a factor 2 difference on modulation.

The predictions of the inflationary observables from potentials in eqs. (3.4) and (3.7) are given in figure 3. Both models can nicely agree with the new Planck observations with U(1) symmetry breaking parameter $c$ in certain range. For the potential in eq. (3.4), the results are altered significantly from $N \sim 50$ to $N \sim 60$. The potential modulation from
quadratic U(1) symmetry breaking term has introduced notable running of scalar spectral index. In the regions at 68% C.L., for example with \( c \approx 0.025 \), \( n_s \in [0.96, 0.97] \) and \( r \approx 0.04 \), the model predicts the running of spectral index \( \alpha_s \approx -0.019 \) for \( N = 56 \). While for the model with linear U(1) symmetry breaking term, the results are much different. It predicts a rather smaller running of spectral index, which is about \( \alpha_s \approx -0.005 \) or even closer to zero depending on the ranges of \( n_s \) and \( r \). Such difference in running spectral index is caused by the modulation term, which is proportional to \( \cos 2\theta \) in eq. (3.4) while proportional to \( \sin \theta \) in eq. (3.7). The modulation term with higher frequency increases the magnitude of the running of spectral index. The running of spectral index also oscillates according to the phase of the sinusoidal modulation. The running changes its magnitude and even reverse its sign at different pivot scale, or equivalently, with different e-folding number \( N \). Specifically, given \( c = 0.025 \), the model (3.4) predicts that the running changes its sign around \( N = 69 \), and similarly, in the model (3.7), with the same parameter \( c = 0.025 \) the running becomes positive with e-folding number \( N \geq 64 \).

The two U(1) symmetry breaking terms \( c(\Phi^n + \bar{\Phi}^n) \) and \( ic(\Phi^n - \bar{\Phi}^n) \) are related by a discrete phase shift \( \theta \to \theta + \frac{\pi}{2n} \). Instead of adopting different U(1) symmetry breaking forms, such phase shift can also be fulfilled by introducing an extra phase parameter in the superpotential

\[
W = a \frac{X}{\Phi} \ln \frac{\Phi}{\Lambda},
\]

where \( \Lambda = e^{i\phi_0} \). It gives a continuous phase shift and the quadratic potential is modulated by a trigonometric factor. Starting from the same Kähler potentials in eqs. (3.2) or (3.5), with above superpotential we can finally get the phase Lagrangians up to the order \( c (O(c)) \)

\[
L = \frac{1}{1 + 2c \cos(2\theta - \phi_0)} \partial_{\mu} \theta \partial^{\mu} \theta - a^2 e(1 + 2c \cos(2\theta - \phi_0)) \theta^2,
\]

or

\[
L = \frac{1}{1 + c \sin(\theta - \phi_0)} \partial_{\mu} \theta \partial^{\mu} \theta - a^2 e(1 + 2c \sin(\theta - \phi_0)) \theta^2,
\]

where \( \phi_0 \) is a constant and the discrete phase shift \( \theta \to \theta + \frac{\pi}{2n} \) is included as a special choice of the constant phase \( \phi_0 \). The results of the potentials in eqs. (3.9) and (3.10) are given in figure 4, where the \( n_s - r \) curves are estimated with a fixed e-folding number \( N = 56 \). For general \( N \in [50, 60] \), the curves will be slightly modified with the shifted intersection point.

The major difference between the potentials in eqs. (3.9) and (3.10) appears in the running spectral index \( \alpha_s \). As shown in figure 5, for \( r < 0.10 \) and fixed \( N = 56 \), the running spectral index \( \alpha_s \) generated by the potential in eq. (3.9) is about \(-0.02 \sim -0.01 \), while the running spectral index is around \(-0.006 \sim -0.002 \) for that in eq. (3.10). So the different U(1) symmetry breaking terms can be clearly distinguished from the observations of running spectral index. Such a small running spectral index is preferred according to the Planck observations [12, 13], although a conclusive result is still absent. Future observations on the running spectral index will determine the U(1) symmetry breaking term of such kind of models.

In addition, one can study the U(1) symmetry breaking effect for natural inflation. The original inflaton potential \( V = \Lambda^4(1 - \cos b\theta) \) is also modulated by a trigonometric factor as below

\[
V \propto (1 + 2c \cos 2\theta)^{1 + \frac{b}{2}}(1 - \cos b\theta),
\]

(3.11)
Figure 4. $r$ versus $n_s$. Left: the inflaton potential in eq. (3.9) with $\phi_0 = \{0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}\}$ for the curves with colors of blue, red, yellow, black, green, respectively. The e-folding number is fixed at $N = 56$. Right: the inflaton potential in eq. (3.10). All the curves intersect at a point of exact quadratic inflation with the same e-folding number. Apart from this point, the curves are parameterized by positive or negative $c$.

Figure 5. The running of spectral index $\alpha_s$ versus $r$ for the inflaton potentials in eqs. (3.9) (left) and (3.10) (right) with the fixed e-folding number $N = 56$. The color coding is the same as in figure 4 and the points with the same color are parameterized by $c$.

or a similar form depending on the U(1) symmetry breaking term. The inflationary predictions are similar to the above modulated quadratic inflations, except a slight shift of limitation related to $c \to 0$. Because in such kind of model there are two free parameters (phase decay constant and symmetry breaking coefficient) while gives the similar results as the modulated quadratic inflation presented in figure 3 which only has one free parameter, it seems to be less attractive at current stage.

4 The shift symmetry breaking in light of Planck 2015

The shift symmetry as a solution to $\eta$ problem for supergravity inflation was proposed in [11]. We first suggested that by breaking the shift symmetry in the Kähler potential, one can get a broad range of tensor-to-scalar ratio $r$ without changing the scalar spectral index $n_s$ [19].
The potential role of the symmetry breaking term was discussed in ref. [21], and such a symmetry breaking model could generate r as large as r \sim 0.20 [22].

In ref. [10], it was shown that helical phase inflation can reduce to the supergravity realization of quadratic inflation with shift symmetry [11]. From the inflationary model given by eq. (2.1), taking the field redefinition Φ = e^Ψ, the Kähler potential and superpotential become

\[ K = e^{Ψ + ¯Ψ} + \cdots = 1 + Ψ + ¯Ψ + \frac{1}{2}(Ψ + ¯Ψ)^2 + \cdots, \]
\[ W = aXΨe^{-Ψ}. \]

The higher order terms have no contribution to inflation after field stabilization: |Φ| = e^{Re(Ψ)} = 1, Re(Ψ) = 0. Through a Kähler transformation

\[ K(Ψ, ¯Ψ) \rightarrow K(Ψ, ¯Ψ) + F(Ψ) + ¯F( ¯Ψ), \quad W \rightarrow e^{-F(Ψ)}W, \]

the Kähler potential and superpotential in eq. (4.1) reproduce the well-known model proposed in ref. [11]

\[ K = \frac{1}{2}(Ψ + ¯Ψ)^2 + X ¯X + \cdots, \quad W = aXΨ. \]

The simple connection between helical phase inflation and supergravity model with shift symmetry remains in the symmetry breaking scenario. One simple choice of the Kähler potential with explicitly broken shift symmetry is

\[ K = ic(Ψ − ¯Ψ) + \frac{1}{2}(Ψ + ¯Ψ)^2 + X ¯X + \cdots. \]

According to the field redefinition Φ = e^Ψ, one can easily figure out the corresponding Kähler potential in the supergravity inflation with U(1) symmetry

\[ K = ic(ln Φ − ln ¯Φ) + Φ ¯Φ + X ¯X + \cdots. \]

The U(1) symmetry breaking term ic(ln Φ − ln ¯Φ) linearly depends on the phase and introduces the expected factor e^{-2cθ} on the quadratic phase potential. Through a Kähler transformation the U(1) symmetry can be restored, the equivalent model is

\[ K = Φ ¯Φ + X ¯X + \cdots, \quad W = aXΦ^b ln Φ, \]

where the power b = −1 + ic is complex. However, the physical origin of the complex power is rather obscure and it is more natural to break the global U(1) symmetry in the way discussed above.

The linear potential in the shift symmetry breaking scenario. We shall study the linear potential modulated by an exponential factor, which can be easily realized in the supergravity inflation with breaking shift symmetry, and comfortably fits with the new Planck results. Let us start from the following Kähler potential and superpotential [19]

\[ K = i \frac{c}{√2}(Φ − ¯Φ) + \frac{1}{2}(Φ + ¯Φ)^2 + X ¯X − g(X ¯X)^2, \]
\[ W = aX √Φ. \]
Following the usual procedure, the field $X$ is stabilized at $X \rightarrow \langle X \rangle = 0$, and the $F$-term scalar potential becomes

$$V(\sigma, \chi) = e^K |W_X|^2 = e^{-c\sigma} \sqrt{\sigma^2 + \chi^2},$$

in which the complex field $\Phi$ is replaced by $\Phi = \frac{1}{\sqrt{2}} (\chi + i\sigma)$. The imaginary component obtains a mass above the Hubble scale and runs into the global minimum $\langle \chi \rangle = 0$ rapidly. So we get the exponentially modulated linear potential

$$V(\sigma) = e^K |W_X|^2 = e^{-c\sigma} |\sigma|.$$  

By taking a different superpotential $W = a X \Phi$ in eq. (4.7), we obtain the inflaton potential $V(\sigma) = e^{-c\sigma} \sigma^2$, which is the quadratic potential with exponential modulation.

The predictions of such kind of inflation models are presented in figure 6. Although the quadratic inflation is disfavoured, especially comparing with the results with B-mode polarizations, its exponential modulated form remains to be consistent with the new Planck results. The modulated linear potential perfectly agrees with the observations with or without the B-mode polarizations. With scalar spectral index $n_s \approx 0.966$, it predicts strong tensor fluctuations with tensor-to-scalar ratio $r$ around 0.03, which can be strictly tested at the future observations.

There are different choices to break the shift symmetry following the idea of eq. (4.7). For example, one may introduce a shift symmetry breaking term $c(\Phi - \bar{\Phi})^2$, together with the same superpotential the final scalar potential for inflation is $\sigma e^{-c\sigma^2}$ or $\sigma^2 e^{-c\sigma^2}$, which gives similar $n_s - r$ relations as those in figure 6.

5 Discussions and conclusion

We have studied the effects of explicitly symmetry breaking for the supergravity inflation. To solve the $\eta$ problem, the Kähler potential admits an exact global symmetry, either the global U(1) symmetry or shift symmetry, which can realize the quadratic or natural inflation.
However, these simplest setups are disfavoured according to the new Planck results. We found that by introducing small symmetry breaking term the inflationary predictions are significantly improved, in the meanwhile the $\eta$ problem remains absent even though the symmetry is approximate.

For the supergravity inflation with global U(1) symmetry, the inflaton is the phase component of a complex field which evolves along helix trajectory during inflation. The global U(1) symmetry plays a crucial role to protect the inflation dynamics away from the quantum loop corrections. It also realizes the super-Planckian field excursion without involving in the physics above the Planck scale. Because the U(1) symmetry is broken explicitly at tree level, the quantum loop corrections will introduce higher order corrections. Fortunately, to explain the Planck new results, the needed U(1) symmetry breaking term is of order $10^{-2}$, and then the higher order corrections are much smaller and ignorable. The U(1) symmetry breaking term modulates the simple quadratic potential with a cosine or sine factor, consequently, the $n_s - r$ relation is deformed. For the scalar spectral index range $n_s \in [0.96, 0.97]$, the tensor-to-scalar ratio can continuously reduce down to $0.015$ in the model with quadratic U(1) breaking term. Similarly in the model with linear U(1) symmetry breaking term, the tensor-to-scalar ratio falls into the range $r \in [0.006, 0.03]$. The major difference between these two different U(1) symmetry breaking models is about the running spectral index. The model with quadratic U(1) breaking term predicts a notable running $\alpha_s \in [-0.02, -0.01]$ for small $r < 0.08$, in contrast, the model with linear U(1) symmetry breaking term gives a small running $\alpha_s \in [-0.006, -0.003]$, which comfortably agrees with the new Planck results. Moreover, we studied the U(1) symmetry breaking model with continuous phase shift. The modulation factor changes under the phase shift and so are the inflationary observables, the results distribute in the $n_s - r$ plane with a beautiful pattern, and generically they admit deformations with tensor-to-scalar ratio around $r \sim 0.01$.

In the supergravity inflation with shift symmetry breaking, the inflaton potential is modulated by an exponential factor, which significantly improves the inflationary predictions as well. Both the quadratic and linear inflation with shift symmetry breaking can nicely fit with the new Planck results. These models predict the large tensor fluctuations $r \simeq 0.04$ with $n_s \simeq 0.966$, which can be tested at the near future experiments.

In short, we have shown that the supergravity inflation with broken global symmetry, as a deformation of the classical quadratic or linear inflation, stays in the region preferred by the new Planck results. For all these models there is a remarkable threshold on the tensor fluctuations $r \simeq 0.01$, which is well-known for large field inflation [15]. Here it is of special importance since for the inflationary models obtained from global symmetry breaking, under the limitation of scalar spectral index $n_s \in [0.96, 0.97]$, the tensor-to-scalar ratio is expected to be of order $0.01$. The future experiments may finally tell us whether the Nature adopted the explicitly symmetry breaking or not.

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