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Modal Phase Measuring Deflectometry

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Abstract: In this work, a model based method is applied to phase measuring deflectometry, which is named as modal phase measuring deflectometry. The height and slopes of the surface under test are represented by mathematical models and updated by optimizing the model coefficients to minimize the discrepancy between the reprojection in ray tracing and the actual measurement. The pose of the screen relative to the camera is pre-calibrated and further optimized together with the shape coefficients of the surface under test. Simulations and experiments are conducted to demonstrate the feasibility of the proposed approach.

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References and Links

1. P. de Groot, "Principles of interference microscopy for the measurement of surface topography," Advances in Optics and Photonics 7, 1-65 (2015).
2. F. Chen, G. M. Brown, and M. Song, "Overview of three-dimensional shape measurement using optical methods," Optical Engineering 39, 10-22 (2000).
3. M. C. Knauer, J. Kaminski, and G. Häusler, "Phase measuring deflectometry: a new approach to measure specular free-form surfaces," in Optical Metrology in Production Engineering, Proc. SPIE (SPIE, 2004), 366-376.
4. M. Petz and R. Tutsch, "Reflection grating photogrammetry: a technique for absolute shape measurement of specular free-form surfaces," in Optical Manufacturing and Testing VI, Proc. SPIE (SPIE 2005), 58691DS8691-58691D85612.
5. T. Bothe, W. Li, C. von Kopylow, and W. P. O. Jüptner, "High-resolution 3D shape measurement on specular surfaces by fringe reflection," in Optical Metrology in Production Engineering, Proc. SPIE (SPIE, 2004), 411-422.
6. C. Faber, E. Olesch, R. Krobot, and G. Häusler, "Deflectometry challenges interferometry: the competition gets tougher!," in Proc. SPIE, 84930R-84930R-84915.
7. Y. Tang, X. Su, Y. Liu, and H. Jing, "3D shape measurement of the aspheric mirror by advanced phase measuring deflectometry," Opt. Express 16, 15090-15096 (2008).
8. L. Huang, C. S. Ng, and A. K. Asundi, "Dynamic three-dimensional sensing for specular surface with monoscopic fringe reflectometry," Opt. Express 19, 12809-12814 (2011).
9. Y. Liu, E. Olesch, Z. Yang, and G. Häusler, "Fast and accurate deflectometry with crossed fringes," Advanced Optical Technologies 3, 441-445 (2014).
10. P. Su, R. E. Parks, L. Wang, R. P. Angel, and J. H. Burge, "Software configurable optical test system: a computerized reverse Hartmann test," Applied Optics 49, 4404-4412 (2010).
11. P. Su, Y. Wang, J. H. Burge, K. Kazmachev, and M. Idır, "Non-null full field X-ray mirror metrology using SCOTS: a reflection deflectometry approach," Optics Express 20, 12393-12406 (2012).
12. R. Huang, P. Su, J. H. Burge, and M. Idır, "X-ray mirror metrology using SCOTS/deflectometry," in Proc. 88480G-88480G-88486.
13. H. Yue, Y. Wu, B. Zhao, Z. Ou, Y. Liu, and Y. Liu, "A carrier removal method in phase measuring deflectometry based on the analytical carrier phase description," Optics Express 21, 21756-21765 (2013).
14. T. Su, A. Maldonado, P. Su, and J. H. Burge, "Instrument transfer function of slope measuring deflectometry systems," Applied Optics 54, 2981-2990 (2015).
15. D. Sprenger, C. Faber, M. Seraphim, and G. Häusler, "UV-Deflectometry: No parasitic reflections," in Proc. DGaO, 2010, A19.
1. Introduction

Three-dimensional (3D) shape information is always placed at the top of the wish list while
an object is under investigation. Extensive studies have been conducted to measure 3D shapes
of different surfaces in multiple scales [1, 2]. Similar to interferometry, phase measuring
deflectometry (PMD) was proposed primarily for specular surfaces [3-5], but different from
interferometry, PMD is an incoherent method with a capability of providing full-field
topography data for free-form surfaces [6]. During years after its proposal, progress in several
aspects has been made: Tang et al. demonstrated PMD is heading towards being a serious competitor
for interferometry [6]. Su et al. carried out series of work with their SCOTS on mirror measurement
including telescope mirrors and x-ray mirrors [10-12]. Yue et al. addressed the carrier
removal in PMD with analytical description [13]. Not limited in visible light, deflectometry
has been pushed into fields of infrared [14] and ultraviolet [15]. Via comparison with
interferometry, Faber et al. demonstrated PMD is heading towards being a serious competitor
for interferometry [6].

However, as a slope metrology tool, there is an inherent ambiguity in deflectometry about
the calculation for the height and slopes of the surface under test (SUT). Classically the
surface height is reconstructed from slopes through integration [16, 17], while the slopes are
determined with height knowledge in prior. Due to the uncertainty of the prior knowledge on
height, the introduced slope error will propagate into the resultant height via an integration process. The height-slope ambiguity issue was traditionally ignored or partially solved by roughly assuming a prior height distribution [18], a height-known seeding point [19], iterative reconstruction [20], or stereo vision [3]. Recently, Liu et al. proposed several ways to reconstruct the specular surface shape from a single image with no ambiguity [21, 22]. Another issue we will discuss is the system calibration, which is an everlasting topic in metrology. The classical PMD calibration method can be split into three separate steps for the calibration of screen, cameras, and the system geometry [3, 4]. A flexible way to calibrate the PMD system with a markerless flat mirror was proposed by Xiao et al. [23] and a similar idea has been applied to calibrate stereoscopic PMD system by Ren et al. [24]. Laser tracker was employed to measure the system geometry [12, 25]. In this work, we present a method to simultaneously estimate the height and slopes of the SUT in PMD by using mathematical models (e.g. Chebyshev polynomials, Zernike polynomials, or B-splines), which is called modal phase measuring deflectometry (MPMD). Moreover, a post-optimization for the screen geometry after the pre-calibration of the PMD system is also addressed in MPMD.

Section 2 introduces the principle of the proposed MPMD with its post-optimization. Simulations for both monoscopic PMD (mono-PMD) and stereoscopic PMD (stereo-PMD) configurations are carried out in section 3 to demonstrate the performance of MPMD. The feasibility of the proposed method is verified with experiments in section 4. Some features in using the proposed MPMD are discussed in section 5, and section 6 concludes the work.

2. Principle

The fundamental principle of deflectometry is the law of reflection. The angle of incidence equals the angle of reflection when light coming from the screen goes into the camera after the reflection on the specular surface as depicted in Figure 1(a). Conventionally, this scene is considered in a reverse way illustrated in Figure 1(b). The probe ray \( P = (x_n, y_n, 1) \) from the camera is reflected by the specular surface at point \( X = (x_c, y_c, z_c) \) = \( z_c P \) and hits the screen at its corresponding point \( m \), where \( (x_n, y_n) \) stands for the sampling position in camera coordinates, and \( (x_n, y_n) = (x_c, y_c) / z_c \) denotes the normalized camera coordinates in the plane \( z_c = 1 \).

Figure 1. Sketches of PMD can be drawn either in physical principle with light from the screen (a) or in ray tracing with probe from camera (b).

Unlike the classical PMD that calculates the slopes from the fringe phases with pre-known or roughly known height values and then integrates slopes to get the resultant height distribution, the proposed method reconstructs both height and slopes at the same time. The modal wavefront reconstruction method [26-28] is introduced into PMD with a deeper consideration on simultaneously reconstructing both height and slopes from the measured fringe phases, instead of just getting height from slopes. The essential idea of MPMD is to
represent both height and slopes of the SUT with well-established mathematical models \( w \)
and to optimize the coefficient vector \( c \) to best explain the captured fringe patterns in
measurement. In order to avoid recalculating basis function during iterations in optimization,
which could be time consuming, the shape models are defined in the normalized camera
coordinates \((x_n, y_n)\).

\[
\begin{align*}
  z_c &= w^T c, \\
  \frac{\partial z_c}{\partial x_n} &= w_x^T c, \\
  \frac{\partial z_c}{\partial y_n} &= w_y^T c,
\end{align*}
\]

where \( w \) is the vector of the known basis functions, which can be Chebyshev polynomials,
Zernike polynomials, or B-splines. Vectors \( w_x \) and \( w_y \) are the known derivatives derived from
the corresponding basis function \([29, 30]\). The unknown vector \( c \) contains the coefficients to
be optimized for each mode. The length of \( c \) is determined by the number of terms in
Chebyshev and Zernike polynomials, or the number of control points in B-splines.

Considering the following relations

\[
\begin{array}{l}
  \frac{\partial z_c}{\partial x_c} = \frac{\partial z_c}{\partial x_n} \frac{\partial x_n}{\partial x_c} + \frac{\partial z_c}{\partial y_n} \frac{\partial y_n}{\partial x_c} = \frac{\partial z_c}{\partial x_n} \left( z_c + x_n \frac{\partial z_c}{\partial x_n} \right) + y_n \frac{\partial z_c}{\partial y_n} \frac{\partial z_c}{\partial x_n}, \\
  \frac{\partial z_c}{\partial y_c} = \frac{\partial z_c}{\partial x_n} \frac{\partial x_n}{\partial y_c} + \frac{\partial z_c}{\partial y_n} \frac{\partial y_n}{\partial y_c} = x_n \frac{\partial z_c}{\partial x_n} + \frac{\partial z_c}{\partial y_n} \frac{\partial z_c}{\partial y_n} \left( z_c + y_n \frac{\partial z_c}{\partial y_n} \right),
\end{array}
\]

The \( x \)- and \( y \)-slopes in camera coordinates \((x_c, y_c)\) are represented as

\[
\begin{align*}
  \frac{\partial z_c}{\partial x_c} &= \frac{\partial z_c}{\partial x_n} \left( z_c + x_n \frac{\partial z_c}{\partial x_n} + y_n \frac{\partial z_c}{\partial y_n} \right), \\
  \frac{\partial z_c}{\partial y_c} &= \frac{\partial z_c}{\partial y_n} \left( z_c + x_n \frac{\partial z_c}{\partial x_n} + y_n \frac{\partial z_c}{\partial y_n} \right).
\end{align*}
\]

Therefore, the normal of the mirror surface facing towards the camera is

\[
N = \begin{pmatrix} \frac{\partial z_c}{\partial x_c}, & \frac{\partial z_c}{\partial y_c}, & -1 \end{pmatrix}^T.
\]

For a probe vector \( P \), its sampling point on the SUT is \( X \) and the vector of the reflected ray from \( X \) can be calculated as

\[
r = p - 2 \langle p, n \rangle n,
\]
where \( n = \frac{N}{\|N\|} \) is the normalized normal and \( p = \frac{P}{\|P\|} \) is the normalized probe vector.

The reflected ray can be determined once its direction \( (r) \) and one point on it \( (X) \) are known. If we want to calculate the corresponding intersection \( \hat{m} \) on the screen, it is required to have the pose of the screen relative to the camera \( (3 \times 1 \text{ rotation vector } \omega, 3 \times 1 \text{ translation vector } T) \), which can be obtained through a pre-calibration procedure. The same as the classical calibration [3, 4], the pre-calibration includes the camera calibration, screen calibration, and geometric calibration. Once the pose of the screen is initialized via pre-calibration, the intersection \( \hat{m} \) can be calculated by solving equations of the screen plane and reflected ray. Another purpose of pre-calibration is to initialize coefficient vector \( c \) by fitting a camera calibration plane close to where the SUT is going to place.

The coefficient vector \( c \) and the pose of the screen \( (\omega, T) \) in camera coordinates are optimized by minimizing the differences between the reprojection \( \hat{m} \) and the measurement \( m \) on the screen among the \( N \) pairs of correspondence in a least squares sense. This nonlinear least squares problem in Eq. (8) can be solved by the Levenberg-Marquardt algorithm.

\[
\left[ \omega, T, c \right] = \arg \min_{\omega, T, c} \sum_{n=1}^{N} \left\| \hat{m}_n (\omega, T, c) - m_n \right\|^2.
\]  

In addition, the proposed MPMD method can easily adapt to multi-camera PMD system as shown in Figure 2. The probe vector of the primary camera \( p \) determines the sampling point on SUT \( X_p = z_p p \). The model is established in the normalized camera coordinates of the primary camera.

![Figure 2. The MPMD approach can be applied to multi-camera PMD.](image)

The probe vector for the same point \( X_p \) in the \( k \)th camera can be determined with the camera calibration parameters. Interpolation on the measured correspondences is necessary to get \( m_k \) for point \( X_p \) in the \( k \)th camera. The calculation procedure of reprojection point \( \hat{m}_k \) is the same as that for mono-PMD. We minimize the discrepancy between the reprojection \( \hat{m}_{k,n} \) and the measurement \( m_{k,n} \) on screen for \( N \) sampling points in \( K \) cameras in total. The optimization problem finally yields

\[
\left[ \omega, T, c \right] = \arg \min_{\omega, T, c} \sum_{k=1}^{K} \sum_{n=1}^{N} \left\| \hat{m}_{k,n} (\omega, T, c) - m_{k,n} (\omega, T, c) \right\|^2.
\]  

\[ (9) \]
where \( \mathbf{m}_{k,n} \) is also a function of parameters \((\omega, T, c)\) except the primary camera which always keeps the same probe ray with no interpolation process. The problem in Eq. (9) can be solved by a nonlinear least squares solver.

3. Simulation

Series of simulations for both mono-PMD and stereo-PMD system are conducted to demonstrate the feasibility of the proposed MPMD approach. In this section, the system configuration in our simulation is described at first, and then the evaluation of the measured shape error is discussed. After the reconstruction under perfect system calibration is addressed, the reconstruction under imperfect pre-calibration will be demonstrated to show the effectiveness of the post-optimization.

System configurations

In simulation, the stereo-PMD system keeps the same configuration of the mono-PMD system shown in Figure 3(a-b) by just adding one more camera as the right eye shown in Figure 3(b). Both the SUT-to-camera distance and the SUT-to-screen distance are about 2.5 meters. The dimensions of the SUT are around 200 mm \( \times \) 200 mm \( \times \) 0.55 mm. The shape can be analytically expressed as

\[
z = \frac{1}{10} \left( \cos \left( \frac{2\pi x}{200 + x} \right) + \sin \left( \frac{2\pi y}{100} \right) \right) + 10^{-5}x^2 + 2 \times 10^{-5}y^2, \quad -100 \leq x \leq 100, -100 \leq y \leq 100.
\]

Figure 3. Simulated PMD systems: (a) mono-PMD, (b) stereo-PMD, and (c) the SUT.

The noise on fringe phase is set as \( 2\pi/100 \) rad rms which is not difficult to achieve in practical phase retrieval, especially in PMD. The fringe period is 16 screen pixels with the pixel pitch of the screen is 187.5 \( \mu \)m. Therefore, the measurement uncertainty of a pattern point in one direction on the screen is 30 \( \mu \)m rms.

Error evaluation

If we only concern on the shape of a SUT, instead of its absolute coordinates, the piston, tip and tilt terms in the result can be removed from the error evaluation as illustrated in Figure 4. In MPMD, the screen pose can be adjusted with the model coefficient during optimization. It will introduce additional piston, tip and tilt terms into the absolute height result, which are the dominating errors of the reconstrated absolute height result.
In most applications of the PMD, the shape of the specular surface is the goal, so we evaluate the shape error with piston, tip, and tilt removed in this work.

Reconstruction under perfect calibration

If the system calibration is perfect, there is no need to optimize the screen pose. The shape is reconstructed by estimating coefficients \( c \). The number of terms used in Chebyshev polynomials or Zernike polynomials is selected as 289, the same as the number of control points in B-splines, to have comparable results and avoid possible one-sided understandings. It could be interesting to compare different models in MPMD, but it is out of the scopes of this work.

Reconstruction under imperfect calibration

However, the practical system calibration is never perfect. In fact, the inaccurate system calibration is one of the major error sources in PMD, which limits its practical application. The pre-calibration error is simulated and added into the true pose of the screen as shown in Figure 6.
Figure 6. The calibration error of the screen pose is added in simulation.

If we trust the inaccurate system calibration and directly apply the MPMD approach with Chebyshev polynomials as the model for an instance, large shape error shown in Figure 7(e) will observed from the reconstruction result in Figure 7.

Figure 7. Large error occurs if inaccurate geometric calibration is trusted in the optimization with Chebyshev polynomials in the mono-PMD system. (a) Reconstructed shape, (b) reprojection on the screen, (c) reprojection residuals, (d) vectors of reprojection residuals, and (e) shape error.

The height and slopes represented together by coefficients are modified to “best” reflect the probe rays onto the screen as illustrated in Figure 7(b-d). However, the screen in ray tracing is “placed” at a wrong location owing to the incorrect geometry knowledge. The residuals in Figure 7(c) and (b) are typical illustrations implying the vector field of residuals is not curl-free.

If the pre-calibration result is only used as an initial value in the post-optimization, the screen pose can be optimized with the shape estimation as described in Eq. (8) for a mono-PMD system.
Figure 8. Shape error significantly reduces, when the inaccurate pose of screen is used as an initial guess in the optimization with Chebyshev polynomials in the mono-PMD system. (a) Reconstructed shape, (b) reprojection on the screen, (c) reprojection residuals, (d) vectors of reprojection residuals, and (e) shape error.

The model coefficients and the screen pose are adjusted to have the best match between the reprojected and measured correspondences on the screen (see Figure 8). Along with the reduction of reprojection residuals in iteration, the shape error decreases as well. Comparing to the shape error by trusting inaccurate calibration, the post-optimization effectively reduces the shape reconstruction error from 449 μm rms in Figure 7(e) to 3.15 μm rms in Figure 8(e).

A similar observation is made in Figure 9 when the post-optimization is implemented to a stereo-PMD system according to Eq. (9).

Figure 9. The shape of SUT is successfully reconstruction in a form of Chebyshev polynomials in the stereo-PMD system. (a) Reconstructed shape, (b) reprojection of the two camera rays on the screen, (c) reprojection residuals, (d) vectors of reprojection residuals, and (e) shape error.

Figure 9 (b-d) indicates the reprojections of both left and right camera rays are close to the measured correspondences on screen. The resultant shape error is 0.461 μm rms in the stereo-PMD system.

4. Experiment

Experiments are carried out to demonstrate the feasibility of MPMD in practical measurements. An LCD screen (Dell P2414H with 1920×1080 pixels and 0.2745 mm×0.2745 mm pixel pitch) and a CCD camera (Manta G-145 with 1388×1038 pixels and 12-bit pixel depth) compose a mono-PMD system. Another camera (Manta G-145 with 1388×1038 pixels and 12-bit pixel depth) is added for a stereoscopic configuration as shown Figure 10. The SUT is 200 mm long and 95.3 mm wide and the mirror is concave along one dimension.
Before we can take a measurement, the PMD system needs to be calibrated. As mentioned above, the purpose of the pre-calibration is to initialize the parameters for screen pose and model coefficients.

Pre-calibration for both mono-PMD and stereo-PMD systems

The pre-calibration can be the same as the classical PMD calibration. An LCD screen displaying phase shifting fringe patterns is employed as the calibration target for camera calibration. A flat mirror with markers is used in geometry calibration to determine the screen pose in camera coordinates. An optimization is finally operated to refine the screen pose as suggested by Xiao et al. [23]. The pre-calibrated system geometry is illustrated in Figure 11.

Measurement

Phase shifting method with multi-frequency phase unwrapping strategy [31-33] is utilized for absolute phase measurement. The period of the finest fringe is 16 screen pixels. The sinusoidal fringe patterns coded with $x$-phase and $y$-phase are sequentially displayed on the screen and captured by cameras. Figure 12(a) and (b) show two typical images captured by the camera. The corresponding screen coordinates shown in Figure 12 (c) and (d) are calculated from the absolute phases, fringe period and screen pixel pitch.
Figure 12. Two typical captured fringe patterns with x- (a) and y- (b) phases, and the corresponding coordinates on screen in x- (c) and y- (d) directions.

Once the correspondence \( m \) is determined according to fringe phases, the model coefficients of the SUT \( c \) and the screen pose \( (\omega, T) \) can be estimated together. The initial values of the model coefficients can be calculated from one of the camera calibration planes, which should be close to the SUT. The initial pose of the screen are set as the screen geometry from pre-calibration.

At first, by using the proposed MPMD method, we reconstruct the SUT with the mono-PMD data. Uniform cubic B-splines with 17 \( \times \) 17 control points are used as the shape model. Figure 13 illustrates the iterative results in the mono-PMD system.

Figure 13 indicates that the root mean square (RMS) of the reprojection residuals declines along iterations in the mono-PMD configuration. Large reprojection errors in Figure 13(a) are expected while the initial values of the model coefficients and screen pose are entered. It can be found in Figure 13(b) the reprojected correspondence \( \hat{m} \) is getting closer to the measured one \( m \) while the iteration updates. The resultant shape is finalized in Figure 13(c) when the shape change or the variation of the reprojection error is negligible.
Moreover, the stereo-PMD data are also utilized to reconstruct the same SUT with MPMD method based on Eq. (9). After several iterations as illustrated in Figure 14, the stereo-PMD delivers the shape result. Figure 14(a) shows both camera rays trace large errors at the start of the iteration. It is mainly because the shape coefficients are initialized from a plane pose used in camera calibration, which makes the shape guess is so different from the SUT.

Figure 14 reveals the reprojection errors of both left and right rays are getting smaller during optimization and finally reduce to 82.3 μm rms for the left ray and 67.3 μm rms for the right ray in Figure 14(c). These RMS values are close to that in mono-PMD (81.7 μm rms) in Figure 13(c). The two reconstruction results in Figure 13(c) and Figure 14(c) are compared in Figure 15(a-b) and their difference is displayed in Figure 15(c). Their height difference is about 8.20 μm rms. The mono-PMD offers the resultant radius of curvature as 153.69 mm and the stereo-PMD finds a result of 154.48 mm, while the radius of curvature of the SUT is about 154.88 mm.
In order to investigate the repeatability of MPMD used in both mono-PMD and stereo-PMD datasets, another set of measurement is carried out and the results of mono-PMD and stereo-PMD are shown in Figure 15(d-e). Figure 15(f) displays their height difference with 8.45 μm rms. The main interest is to compare the two sets of measurement with the same reconstruction method to see how repeatable the reconstructions are. Figure 15(g) denotes there is 0.725 μm rms height difference between the reconstructions from two different sets of mono-PMD data with the proposed MPMD. A similar observation is made on stereo-PMD side with 0.898 μm rms height difference in Figure 15(h). With the proposed MPMD method, sub-micron level repeatable results can be reconstructed from both mono-PMD and stereo-PMD data in our experiment.

5. Discussion

Several aspects of MPMD including the initial values, noise influence, and model mismatch are discussed and highlighted in this section for a better and comprehensive understanding of this method.

Initial values

The post-optimization in MPMD reduces the requirement on calibration by including the screen pose into the optimization with the model coefficients of the SUT. Usually certain affordable geometry error on screen pose will affect the shape result little. However, it does not mean the geometry calibration is not important anymore. In fact, the system pre-calibration provides initial values of the screen pose and the SUT model coefficients for the nonlinear optimization, which is vital to achieve good reconstruction with less shape error, especially in a form of the two-dimensional 2nd order polynomial. An example under different quality of pre-calibrations by using Zernike polynomials as the shape model is demonstrated in Figure 16.
Figure 16. Pre-calibration is still important although MPMD relaxes the calibration. A good calibration (a) offers better initial values for a better reconstruction with less shape error (b), comparing to a poor calibration (c) with its corresponding large reconstruction error (d).

Figure 16 reveals that MPMD method gets a relatively lower shape error under a good pre-calibration. The reconstruction can be much worse if the pre-calibration is poorly handled with providing an initial guess far from the reality as shown in Figure 16 (c-d).

**Noise influence**

In classical PMD, the phase noise directly links to the slope noise, so the noise influence occurs at the integration step and depends on the integration algorithm. However, in MPMD, the measurement noise affects the nonlinear optimization and usually it ends up with a global shape error as a result.

**Mismatch between the SUT and the model**

Another important error source is the mismatch between the SUT and the model with a limited number of polynomial orders or control points. For example, Figure 17(a-d) shows a reconstruction of the SUT by 289 orders of Zernike polynomials. In contrast, if we only use 36 orders of Zernike polynomials in reconstruction, the larger reprojection residuals and shape error can be observed in Figure 17(e-h) due to the insufficient Zernike modes to well represent the SUT.

Figure 17. Number of model coefficients may influence the reconstruction accuracy. Reprojection residuals (a-b) are smaller and the reconstructed shape (c) is more accurate with less shape error (d) with 289 orders of Zernike polynomials than those (e-h) with 36 orders of Zernike polynomials.
The proposed MPMD is not good at reconstructing local waviness on the specular SUT. In some applications, high frequency components are interested and significant. It requires a large number of model coefficients to represent these high frequency terms. However, this will highly increase the computing time and become impractical. How to better deal with the reprojection residuals and reconstruct the high frequency components in MPMD will be addressed in our future work.

6. Conclusion

Modal phase measuring deflectometry is proposed in this work for mono-PMD, stereo-PMD, and multi-camera PMD configurations. The surface shape is represented by mathematical models (e.g., Chebyshev, Zernike, or B-spline). The model coefficients are adjusted to minimize the reprojection error on the screen. The reconstruction problem becomes a nonlinear optimization problem to match the reprojected correspondences with measured ones. In addition, the screen geometry from calibration is not simply trusted, but used as initial values and optimized with the surface shape optimization. Both simulation and experiment demonstrate the proposed MPMD is effective in 3D shape reconstruction for specular surfaces.

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