Neutrino Mixing and Leptonic CP Phase in Neutrino Oscillations

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Oscillations of the Dirac neutrinos of three generations in vacuum are considered with allowance made for the effect of the CP-violating leptonic phase (analogue of the quark CP phase) in the lepton mixing matrix. The general formulas for the probabilities of neutrino transition from one sort to another in oscillations are obtained as functions of three mixing angles and the CP phase. It is found that the leptonic CP phase can, in principle, be reconstructed by measuring the oscillation-averaged probabilities of neutrino transition from one sort to another. The manifestation of the CP phase as a deviation of the probabilities of direct processes from those of inverse processes is an effect that is practically unobservable as yet. © 2001 MAIK “Nauka/Interperiodica”.

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Rapidly growing interest in neutrino physics has recently been stimulated by new data from facilities at Kamiokande [1], Super-Kamiokande [2], LSND [3], CHOOZ [4], and some others [5, 6]. These data are indicative, directly or indirectly, of the Pontecorvo (vacuum) oscillations [7] of neutrinos of three types, \( \nu_e, \nu_\mu, \) and \( \nu_\tau \). The presence of vacuum oscillations means that CP violation is an effect that is practically unobservable as yet. The manifestation of the CP phase as a deviation of the probabilities of direct processes from those of inverse processes is an effect that is practically unobservable as yet.

The Maki–Nagava–Sakata (MNS) mixing matrix \( V_{\alpha\beta} \) of Dirac neutrinos [9] has the same form as the SKM mixing matrix of quarks [10], but with its own mixing angles \( \theta_{12}, \theta_{13}, \) and \( \theta_{23} \) and its own CP-violating phase \( \delta_l \):

\[
V^l = c_{12} c_{13} s_{12} c_{13} e^{-i \delta_l}
\]

\[
= s_{12} c_{13} - c_{12} s_{23} s_{13} e^{i \delta_l} - c_{12} c_{23} s_{13} e^{i \delta_l} - c_{12} s_{23} s_{13} e^{i \delta_l} - c_{12} s_{23} s_{13} e^{i \delta_l}
\]

where \( s_{dk} = \sin \theta_{dk} \) and \( c_{dk} = \cos \theta_{dk} \). Like \( V_{CKM} \), the matrix \( V^l \) is unitary; i.e., \( V^l V^l = 1 \).

Previous analysis of experimental data [1–6] gave the following mixing angles [8, 9, 11]

\[
\theta_{12} = (42.1 \pm 6.9)^\circ, \quad \theta_{13} = (2.3 \pm 0.6)^\circ, \quad \theta_{23} = (43.6 \pm 3.1)^\circ
\]
for $\delta_1 = 0$ and

$$m_3 = (0.058 \pm 0.025) \text{ eV},$$

$$m_2 = (0.0060 \pm 0.0035) \text{ eV}, \quad m_1 \ll m_2.$$  \hspace{1cm} (5)

Here, the mean error in the mixing angles and the masses is taken from figures and tables in [8, 9, 11], where the CHOOZ data [4] from ground-based $\nu_e$ sources were taken into account.\(^1\)

Below, we will consider the possibility of determining the leptonic $CP$ phase $\delta_1$ from the data obtained in the today and immediate-future experiments of the type [1–6], where oscillations were not observed directly but only the oscillation-averaged probabilities $P(\nu_\alpha\nu_\beta)$ of $\nu_\alpha \rightarrow \nu_\beta$ transitions were measured.

The action of matrix (3) on the column vector $\nu^i$ gives [see Eq. (1)]

$$\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix} = \hat{V} \begin{pmatrix}
\nu^0_e \\
\nu^0_\mu \\
\nu^0_\tau
\end{pmatrix},$$

i.e.,

$$\nu_e(t) = [c_{12}c_{13}\nu^0_3(0) + s_{12}c_{13}\nu^0_2(0)e^{-i\varphi_{21}} + s_{13}\nu^0_3(0)e^{-i\varphi_{12}} - \frac{m_1^2}{2p_v}t],$$

$$\nu_\mu(t) = [ - (s_{12}c_{11} + c_{12}s_{13}e^{i\varphi_1})\nu^0_3(0) + c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\varphi_2} + c_{13}s_{23}e^{-i\varphi_3}]\nu^0_2(0)e^{-i\varphi_{21}},$$

$$+ c_{13}s_{23}\nu^0_3(0)e^{-i\varphi_{21}}] \exp\left(-\frac{m_2^2}{2p_v}t\right),$$

$$\nu_\tau(t) = [(s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\varphi_2})\nu^0_3(0) - (c_{12}s_{23} + c_{13}s_{23}e^{i\varphi_3})\nu^0_2(0)e^{-i\varphi_{21}} + c_{13}s_{23}\nu^0_3(0)e^{-i\varphi_{21}}]\exp\left(-\frac{m_1^2}{2p_v}t\right),$$

where, taking into account the dependence (2) of neutrino states on time $t = L$, one has

$$\varphi_{ij} = \frac{(m_i^2 - m_j^2)}{2p_v}t = 2.54 \frac{(m_i^2 - m_j^2)(e^2)^2}{2E_v \text{(MeV)}}L(m),$$  \hspace{1cm} (7)

with $E_v = p_v$ being the neutrino energy in a beam, $E_v \gg m_3 > m_2 > m_1$. Because the neutrino states $\nu^i(0)$ are orthonormalized, i.e., $(\nu^i_\nu^j) = \delta_{ik} \delta^2$ one has for the amplitudes $A_{\alpha \rightarrow \beta} = (\nu^\beta_\nu^i(0))$ and probabilities

$$P(\nu_\alpha\nu_\beta) = \left| (\nu^\beta_\nu^i(0)) \right|^2$$

of $\nu_\alpha(0) \rightarrow \nu_\beta(t)$ transitions in vacuum

$$P(\nu_e\nu_\mu) = \left| c_{12}^2 + s_{12}^2 e^{i\varphi_{21}} + s_{13}^2 e^{i\varphi_{12}} \right|^2,$$

$$+ \left| s_{12}c_{13} - s_{12}s_{23}s_{13}e^{i\varphi_2} + c_{13}s_{23}e^{i\varphi_3} \right|^2,$$

$$P(\nu_\mu\nu_\tau) = \left| s_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\varphi_2} + c_{13}s_{23}e^{i\varphi_3} \right|^2,$$

$$P(\nu_\tau\nu_e) = \left| c_{12}s_{23} - s_{12}s_{23}c_{13}e^{i\varphi_2} + c_{13}s_{23}e^{i\varphi_3} \right|^2,$$

and

$$P(\nu_\mu\nu_\tau) = \left| c_{12}c_{13}s_{12} + c_{12}s_{23}e^{i\varphi_1} \right|,$$

$$+ \left| s_{13}s_{12}c_{13}s_{23}e^{i\varphi_2} - s_{13}s_{12}s_{23}s_{13}e^{i\varphi_3} \right|^2,$$

$$P(\nu_\tau\nu_\tau) = \left| s_{12}s_{23}c_{13} - c_{12}s_{23}s_{13}e^{i\varphi_2} + c_{13}s_{23}e^{i\varphi_3} \right|^2,$$

$$+ \left| s_{13}s_{12}c_{13}s_{23}e^{i\varphi_2} - s_{13}s_{12}s_{23}s_{13}e^{i\varphi_3} \right|^2.$$

\(^1\)Unfortunately, results (4) and (5) obtained in [8, 9, 11] using the data reported in [1–6] are insufficiently reliable, especially those based on the data for solar and, partly, atmospheric electron neutrinos $\nu_e$, whose interaction with matter within the Sun or Earth can invert their spin and transform ($\nu_e$)$_\nu$ to the sterile, i.e., noninteracting, state ($\nu_e$)$_R$. This so-called MSW effect [12] does not occur in an analysis of the data from ground-based $\nu_e$ sources, e.g., CHOOZ data [4], whose processing yields very small angles $\theta_{13} \sim 2^\circ - 3^\circ$ [see Eq. (4)].

\(^2\)For the Majorana neutrinos, whose fields $\nu_\nu^0(0)$ with definite mass are real, the requirement that fields $\nu_e$, $\nu_\mu$, and $\nu_\tau$ (6) produced in the weak interaction be real even at $t = 0$ would mean that real matrix (3) is orthogonal, i.e., $\delta_1 = \delta_2 = \pi$ or 0. This is also true for the phases $\delta_{12}$ and $\delta_{23}$, which are omitted in Eqs. (3) and (6) because they lead to vanishingly small probabilities of the $\nu_i \rightarrow \nu_k$ transitions with amplitudes $\sim m_i E \sim 10^{-6} – 10^{-9}$. However, besides simplicity and aesthetics, there are no other reasons for requiring that fields (6) be real and $CP$ phase be zero. We intend to consider the Majorana neutrino oscillations elsewhere [13].
Averaging these probabilities over oscillations, i.e.,
over phases \( \phi_i \) [by setting \( \langle \sin^2 \phi_i \rangle = \langle \cos^2 \phi_i \rangle = 1/2 \) and \( \langle \cos(\phi_i + \delta_i) \rangle = \langle \cos \phi_i \rangle = 0 \) in Eqs. (8) and (9)], we obtain the following energy-independent probabilities, which were only measured to date \([1–6] \):

\[
\begin{align*}
\langle 1 - P(\nu_\alpha \nu_\beta) \rangle &= A_{ee}, \\
(1 - P(\nu_\mu \nu_\mu)) &= A_{\mu\mu} + B_{\mu\mu} \cos \delta + C_{\mu\mu} \cos ^2 \delta, \\
\langle 1 - P(\nu_\tau \nu_\tau) \rangle &= A_{\tau\tau} + B_{\tau\tau} \cos \delta + C_{\tau\tau} \cos ^2 \delta, \\
\langle \nu_\tau \nu_\mu \rangle &= A_{\mu\tau} + B_{\mu\tau} \cos \delta + C_{\mu\tau} \cos ^2 \delta, \\
\langle \nu_\mu \nu_\tau \rangle &= A_{\tau\mu} + B_{\tau\mu} \cos \delta + C_{\tau\mu} \cos ^2 \delta,
\end{align*}
\] (10)

where

\[
\begin{align*}
A_{ee} &= \frac{1}{2} \{c_{13}^2 \sin^2 (2 \theta_{12}) + \sin^2 (2 \theta_{13})\}, \\
A_{\mu\mu} &= \frac{1}{2} \{c_{13}^2 + (c_{12}^4 + s_{12}^4) s_{13}^2 \} \sin^2 (2 \theta_{23}), \\
B_{\mu\mu} &= \frac{1}{2} \{c_{23}^2 - s_{23}^2 s_{13}^2 \} s_{13} \sin (2 \theta_{23}) \sin (4 \theta_{12}), \\
C_{\mu\mu} &= -\frac{1}{2} s_{13} \sin^2 (2 \theta_{23}) \sin^2 (2 \theta_{12}), \\
A_{\tau\tau} &= \frac{1}{2} \{c_{13}^2 + (c_{12}^4 + s_{12}^4) s_{13}^2 \} \sin^2 (2 \theta_{23}), \\
B_{\tau\tau} &= -\frac{1}{2} s_{13} \sin (2 \theta_{23}) \{s_{23}^2 - c_{23}^2 s_{13}^2 \} \sin (4 \theta_{12}), \\
C_{\tau\tau} &= -\frac{1}{2} s_{13} \sin^2 (2 \theta_{23}) \sin^2 (2 \theta_{12}), \\
A_{\mu\tau} &= \frac{1}{4} \{1 + c_{12}^4 + s_{12}^4 \} s_{23}^2 \sin^2 (2 \theta_{13}) \\
&\quad + 2 c_{12}^2 s_{23}^2 \sin^2 (2 \theta_{12}), \\
B_{\mu\tau} &= \frac{1}{8} c_{13} \sin (2 \theta_{13}) \sin (2 \theta_{23}) \sin (4 \theta_{12}), \\
A_{\tau\mu} &= \frac{1}{4} \{1 + c_{12}^4 + s_{12}^4 \} c_{23}^2 \sin^2 (2 \theta_{13}) \\
&\quad + 2 c_{12}^2 c_{23}^2 \sin^2 (2 \theta_{12}), \\
B_{\tau\mu} &= -\frac{1}{8} c_{13} \sin (2 \theta_{13}) \sin (2 \theta_{23}) \sin (4 \theta_{12}),
\end{align*}
\] (11)

Note that the obvious relationships

\[
\begin{align*}
1 - P(\nu_\alpha \nu_\alpha) &= P(\nu_\mu \nu_\mu) + P(\nu_\tau \nu_\tau), \\
A_{\alpha\beta} &= A_{\beta\alpha}, \\
\alpha, \beta, \gamma &= e, \mu, \tau.
\end{align*}
\]

are satisfied, and \( P(\nu_\mu \nu_\mu) = [P(\nu_\mu \nu_\mu)]_{\delta \rightarrow \delta} \). Using the general formulas for oscillation probabilities from the Appendix, we obtain the following expressions for the differences between the probabilities of forth-back neutrino transitions:

\[
\begin{align*}
P(\nu_\mu \nu_\tau) - P(\nu_\nu \nu_\mu) &= a_0 (\sin \phi_2 + \sin \phi_3 - \sin \phi_{13}) \sin \delta, \\
P(\nu_\nu \nu_\mu) - P(\nu_\nu \nu_\mu) &= -a_0 (\sin \phi_2 + \sin \phi_3 - \sin \phi_{13}) \sin \delta, \\
P(\nu_\nu \nu_\nu) - P(\nu_\nu \nu_\nu) &= a_0 \left( \sin \phi_2 - 2 \sin \phi_{13} \cos \frac{\phi_{13} + \phi_{13}}{2} \right) \sin \delta,
\end{align*}
\] (12)

where \( a_0 = \frac{1}{2} c_{13} \sin 2 \theta_{12} \sin 2 \theta_{13} \sin 2 \theta_{23} = 0.07 \). Unfortunately, the phases \( \phi_k \) appearing in these relationships depend on the neutrino energy in a beam; therefore, to determine the \( \sin \delta \) value from Eq. (12), neutrinos \( \nu_\alpha \) and \( \nu_\beta \) should have the same energy \( E \) in an experiment. Modern beams include only continuous-spectrum neutrinos, and the effect reflected in Eq. (12) vanishes after averaging over the phases \( \phi_k \).

However, the CP phase can be obtained in a different way by using Eqs. (10) and (11) and the experimental data similar to those obtained in [1–6] but having higher accuracy in order to compensate the smallness of angle \( \vartheta_{13} \). For clarity, let us introduce the coefficients \( b_{ik} = B_{ik}/A_{ik} \) and \( c_{ik} = C_{ik}/A_{ik} \) in Eqs. (10) and (11). Because of the smallness of \( s_{13} = \sin \vartheta_{13} = 0.07 \), almost all of these coefficients are very small and are on the order of a fraction of percent:

\[
\begin{align*}
A_{ee} &= 0.499; \\
A_{\mu\mu} &= 0.636, \quad b_{\mu\mu} = 0.0058, \quad c_{\mu\mu} = -0.0038; \\
A_{\tau\tau} &= 0.613, \quad b_{\tau\tau} = 0.0055, \quad c_{\tau\tau} = -0.0040; \\
A_{\mu\tau} &= 0.261, \quad b_{\mu\tau} = 0.014; \quad \text{(13)}
\end{align*}
\]
\[ A_{\mu} = 0.238, \quad b_{\mu} = -0.015; \]
\[ A_{e\mu} = 0.373, \quad b_{\mu} = 0.0005, \quad c_{\mu\tau} = -0.0032. \]

For this reason, the ratio of the number of produced \( N_v \) to that of \( v_\mu \) in the primary \( v_\mu \) beam at large distances \( L \) (about 300–500 km) will weakly decrease with increasing \( \delta_3 \) from 0 to \( \pi \):

\[
\frac{N_{\nu_\mu}}{N_{\nu_e}} = \frac{\langle P(v_\mu v_\mu) \rangle}{\langle P(v_\nu v_\nu) \rangle} = \frac{A_{\mu\tau}}{A_{e\tau}}(1 + (b_{\mu\tau} - b_{e\tau})\cos \delta_3), \tag{14}
\]

where \( b_{\mu\tau} - b_{e\tau} = 2b_{\mu\tau} = 2.8\% \) and \( A_{\mu\tau}/A_{e\tau} = 1.04 \). Therefore, if the experimentally measured ratio (14) differs from 1.04 + 0.03 = 1.07 by more than 1–3%, this would indicate that \( \cos \delta_3 < 1 \); i.e., \( \delta_3 \neq 0 \).

If \( s_{13} = \sin \theta_{13} > 0.07 \), i.e., if \( s_{13} \) is larger than the value used in this work, then the \( CP \) phase will be manifested more strongly. In particular, for \( \theta_{13} = 14^\circ \), we have \( N_{\nu_e}/N_{\nu_\mu} = 1.07(1 + 0.08\cos \delta_3) \), and the coefficient \( a_{\nu_\mu} \) in Eq. (12) is \( a_{\nu_\mu} = 0.23 \).

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**APPENDIX**

The probabilities of all neutrino transitions in vacuum in the Pontecorvo oscillations with allowance made for the \( CP \) phase \( \delta_3 \) are determined by the following general algebraic formulas:

\[
1 - P(v_\nu v_\nu) = c_{12}\sin^2(2\theta_{13})\sin^2(\phi_{31}/2) + c_{13}\sin^2(2\theta_{12})\sin^2(\phi_{32}/2) + s_{12}\sin^2(2\theta_{13})\sin^2(\phi_{32}/2), \tag{A1}
\]

\[
1 - P(v_\nu v_\mu) = \{s_{23}^4\sin^2(2\theta_{12}) + s_{13}^2\sin^2(2\theta_{13}) + s_{23}^4\sin^2(2\theta_{23})
+ s_{13}^2s_{12}\sin^2(2\theta_{12}) + c_{12}s_{13}\sin^2(2\theta_{23})
+ \cos \delta_3\sin(4\theta_{12})\sin(2\theta_{23})(s_{13}c_{23} - s_{13}s_{23})
- \cos^2\delta_3s_{13}\sin^2(2\theta_{23})\sin^2(2\theta_{12})\}\sin^2(\phi_{32}/2), \tag{A2}
\]

\[
+ \{s_{12}^2c_{13}\sin^2(2\theta_{23}) + s_{12}^2s_{12}\sin^2(2\theta_{13}) + \cos \delta_3s_{23}c_{13}
\times \sin(2\theta_{12})\sin(2\theta_{23})\}\sin^2(\phi_{32}/2),
\]

\[
1 - P(v_\nu v_\tau) = \{s_{23}^4\sin^2(2\theta_{12}) + s_{13}^2\sin^2(2\theta_{23})
+ s_{23}^4s_{12}\sin^2(2\theta_{23})
+ \cos \delta_3s_{13}\sin^2(2\theta_{23})
+ \{s_{12}^2c_{13}\sin^2(2\theta_{23}) + s_{12}^2s_{12}\sin^2(2\theta_{13}) + \cos \delta_3s_{23}c_{13}
\times \sin(2\theta_{12})\sin(2\theta_{23})\}\sin^2(\phi_{32}/2),
\]

\[
+ \{s_{12}^2s_{12}\sin^2(2\theta_{23}) + s_{12}^2s_{12}\sin^2(2\theta_{13}) + \cos \delta_3s_{23}c_{13}
\times \sin(2\theta_{12})\sin(2\theta_{23})\}\sin^2(\phi_{32}/2). \tag{A3}
\]

\[
P(v_\nu v_\mu) = \frac{1}{4}\{s_{23}^4\sin^2(2\theta_{12}) + c_{12}s_{13}\sin^2(2\theta_{23})(s_{13}c_{23} - s_{13}s_{23})
- \cos \delta_3\sin(4\theta_{12})\sin(2\theta_{23})(s_{13}^2c_{23}^2 - s_{13}^2s_{23}^2)
- \cos \delta_3\sin(4\theta_{12})\sin(2\theta_{23})(s_{13}^2s_{23}^2 - s_{13}s_{23}^2)
+ \{s_{12}^2c_{13}\sin^2(2\theta_{23}) + s_{12}^2s_{12}\sin^2(2\theta_{13}) + \cos \delta_3s_{23}c_{13}
\times \sin(2\theta_{12})\sin(2\theta_{23})\}\sin^2(\phi_{32}/2),
\]

\[
+ \{s_{12}^2s_{12}\sin^2(2\theta_{23}) + s_{12}^2s_{12}\sin^2(2\theta_{13}) + \cos \delta_3s_{23}c_{13}
\times \sin(2\theta_{12})\sin(2\theta_{23})\}\sin^2(\phi_{32}/2). \tag{A4}
\]

\[
P(v_\nu v_\tau) = \frac{1}{4}\{s_{23}^4\sin^2(2\theta_{12}) + c_{12}s_{13}\sin^2(2\theta_{23})(s_{13}c_{23}^2 - s_{13}s_{23}^2)
- \cos \delta_3\sin(4\theta_{12})\sin(2\theta_{23})(s_{13}^2c_{23}^2 - s_{13}^2s_{23}^2)
+ \{s_{12}^2c_{13}\sin^2(2\theta_{23}) + s_{12}^2s_{12}\sin^2(2\theta_{13}) + \cos \delta_3s_{23}c_{13}
\times \sin(2\theta_{12})\sin(2\theta_{23})\}\sin^2(\phi_{32}/2),
\]

\[
+ \{s_{12}^2s_{12}\sin^2(2\theta_{23}) + s_{12}^2s_{12}\sin^2(2\theta_{13}) + \cos \delta_3s_{23}c_{13}
\times \sin(2\theta_{12})\sin(2\theta_{23})\}\sin^2(\phi_{32}/2). \tag{A5}
\]

\[
P(v_\mu v_\tau) = \frac{1}{4}\{s_{23}^4\sin^2(2\theta_{12}) + c_{12}s_{13}\sin^2(2\theta_{23})(s_{13}c_{23}^2 - s_{13}s_{23}^2)
- \cos \delta_3\sin(4\theta_{12})\sin(2\theta_{23})(s_{13}^2c_{23}^2 - s_{13}^2s_{23}^2)
+ \{s_{12}^2c_{13}\sin^2(2\theta_{23}) + s_{12}^2s_{12}\sin^2(2\theta_{13}) + \cos \delta_3s_{23}c_{13}
\times \sin(2\theta_{12})\sin(2\theta_{23})\}\sin^2(\phi_{32}/2),
\]

\[
+ \{s_{12}^2s_{12}\sin^2(2\theta_{23}) + s_{12}^2s_{12}\sin^2(2\theta_{13}) + \cos \delta_3s_{23}c_{13}
\times \sin(2\theta_{12})\sin(2\theta_{23})\}\sin^2(\phi_{32}/2). \tag{A6}
\]
\[
+ \left[ 2c_{13}\sin^2(2\vartheta_{23})(s_{12}^2s_{13}^2 - c_{12}^2) \right]
\]
\[
- \frac{1}{2}c_{13}\sin(2\vartheta_{13})\sin(2\vartheta_{13})\sin(4\vartheta_{23})c_{12}\cos(\varphi_{12})
+ 2c_{13}\sin(2\vartheta_{13})\sin(2\vartheta_{13})\sin(2\vartheta_{23})
\times \sin(\delta)\sin(\varphi_{21}/2)\cos\left(\varphi_{11} + \varphi_{12}\right)
\]
\[
+ s_{13}\sin(4\vartheta_{12})\sin(4\vartheta_{23})\cos\delta[1 + s_{13}^2\sin^2(\varphi_{21})/2]
- c_{13}\sin(2\vartheta_{12})\sin(2\vartheta_{13})\sin(2\vartheta_{23})\sin(\delta)\sin(\varphi_{21})
- 2s_{13}^2\sin^2(2\vartheta_{12})\sin^2(2\vartheta_{23})\cos(2\vartheta_{11})\sin^2(\varphi_{21}/2)\right].
\]

REFERENCES

1. W. W. M. Allison et al., Phys. Lett. B 449, 137 (1999); T. Mann, in Proceedings of the 19th International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, 2000; http://nu2000.sno.laurentian.ca/T.Mann/index.html.

2. Y. Fukuda et al., Phys. Lett. B 467, 185 (1999); Phys. Rev. Lett. 82, 2644 (1999); H. Sobel, in Proceedings of the XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, June 2000; T. Toshito, in Proceedings of the XXX International Conference on High Energy Physics, 2000, ICHEP 2000, Osaka, Japan.

3. S. Athanassopoulos et al. (LSND Collab.), Phys. Rev. Lett. 81, 1774 (1998).

4. M. Apollonio et al. (CHOOZ Collab.), Phys. Lett. B 420, 397 (1998); F. Boehm et al., hep-ex/9912050.

5. Y. Suzuki, in Proceedings of the XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, 2000; T. Takeuchi, in Proceedings of the XXX International Conference on High Energy Physics, 2000, ICHEP 2000, Osaka, Japan; B. T. Cleveland et al., Astrophys. J. 496, 505 (1998); R. Davis, Prog. Part. Nucl. Phys. 32, 13 (1994); K. Lande, in Proceedings of the XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, 2000; http://nu2000.sno.laurentian.ca/.

6. J. N. Abdurashitov et al. (SAGE Collab.), Phys. Rev. C 60, 055801 (1999); V. Garvin, in Proceedings of the XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, 2000; http://nu2000.sno.laurentian.ca; W. Hampel et al. (GALLEX Collab.), Phys. Lett. B 447, 127 (1999); E. Bellotti, in Proceedings of the XIX International Conference on Neutrino Physics and Astrophysics, Sudbury, Canada, 2000; http://nu2000.sno.laurentian.ca; F. Ronga, hep-ex/0001058.

7. M. Pontecorvo, Zh. Éksp. Teor. Fiz. 33, 549 (1957) [Sov. Phys. JETP 6, 429 (1958)]; Zh. Éksp. Teor. Fiz. 34, 247 (1958) [Sov. Phys. JETP 7, 172 (1958)]; V. N. Gribov and B. M. Pontecorvo, Phys. Lett. 288, 483 (1969); S. M. Bilenky and B. Pontecorvo, Phys. Rep. 41, 225 (1978).

8. Z. Berezhiani and A. Rossi, Phys. Lett. B 367, 219 (1996); Z. Berezhiani and A. Rossi, hep-ph/9811447; R. Barbieri, L. Hall, and A. Strumia, hep-ph/9808333.

9. Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. 28, 870 (1962).

10. Particle Data Group E and C, Eur. Phys. J. C 3, 1 (1998); Particle Phys. Booklet, VII/2000, pp. 157–159.

11. M. C. Gonzalez-Garsia, M. Maltoni, C. Peña-Garay, and J. W. F. Valle, hep-ph/0009350.

12. J. Bahcall, P. Krastev, and A. Smirnov, hep-ph/9807216; S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. 42, 1441 (1985) [Sov. J. Nucl. Phys. 42, 913 (1985)]; L. Wolfenstein, Phys. Rev. D 17, 2369 (1985).

13. K. A. Ter-Martirosyan, Is there difference in the oscillations of Majorana and Dirac neutrinos?, Phys. Lett. (in press).

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