Erosion potential mapping using analytical hierarchy process (AHP) and fractal dimension

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1. Introduction

China is severely affected by soil erosion. The total land area affected by wind and water erosion is estimated at 3 million km² and this approximates to about 32% of the landmass of China (Wang et al., 2016). It is expected that precipitation including extreme events such as droughts and heavy rainfall are bound to further significantly impact China in the distant future (Jiang et al., 2016). These predicted scenarios will further accelerate the erosion processes. The Loess Plateau of Northern Shaanxi Province of China is an important research area and has attracted considerable scholars from within China and abroad conducting various researches. Just as the rest of China, The Loess Plateau receives considerable losses from soil erosion through water and wind. Erosion risk mapping has been conducted in parts of China (Guo et al., 2015; Zhao et al., 2017). However, the consideration of the fractal geometry as a critical input in erosion potential mapping has been limited in The Loess Plateau in the Northern Shaanxi Province of China.

In view of this, the fractal geometry was explored in this study. The fractal geometry of nature was introduced as a way of describing the complex erratic shape of nature and the self-similarity concept. The popular use of fractal dimension for landscape mapping has been noted across the world (Chen et al., 2016; Feng and Liu, 2015; He et al., 2016; Julianni et al., 2016; Plexida et al., 2014). It has been used for image segmentation (Sarkar and Chaudhuri, 1994; Xi and Zhao, 2011), image data compression (Barnsley and Hurd, 1993; Fisher, 1994, 2012; Saupe, 1995; Walach and Karnin, 1986), computer graphics (Hughes et al., 2015; Kim et al., 2016; Peng et al., 2015; Shen et al., 2016), sedimentology (Dufresne et al., 2016; García-Hidalgo et al., 2016; Laurita et al., 2016; Liu et al., 2016) and particle morphology (Gonzalez-Jordan et al., 2016; Loh et al., 2012; Sarkar and Chaudhuri, 1994; Wu et al., 2016; Yang et al., 2016a, b; Zimmerman et al., 2014). It has also been applied to characterize urban growth (Chen, 2012, 2016; Reis et al., 2014) and river networks (Cámara et al., 2016; Donadio et al., 2013; Kim and Jung, 2015; Liang et al., 2016; Mandelbrot, 1983; Vieux, 2016; Yang et al., 2016a, b).
The main reason broad applicability of fractals is that they have the capacity to identify the slightest change in physical structures, their interconnections and couplings (Luc et al., 2005).

There are several methods available on Multi-Criteria Decision Analysis (MCDA) for suitability mapping purposes. One of the most widely method is the Analytical Hierarchy Process that is well adapted to solve complex decisions with multiple criteria. For example, AHP has been used for suitability mapping for flooding (Ghosh and Kar, 2018; Mungai et al., 2016; Sinha et al., 2008), soil erosion (Pradeep et al., 2015; Saha et al., 2019; Vijith and Dodge-Wan, 2019) and landslide mapping (Hung et al., 2016; Pourghasemi et al., 2012; Tazik et al., n.d.). The AHP has proved to be the most acceptable and adaptable.

Therefore, this study develops the erosion potential mapping (EPM) using the analytical hierarchical process (AHP). AHP was adopted as a result of its wide use and easy integration with Geographic Information System (GIS) (Desmet and Govers, 1995; Lu et al., 2004; Zabihi et al., 2019). The various criteria considered included Land Use Land Cover (LULC), Elevation, Slope, Fractal Dimension (FD) and. Each of the criteria was weighted using standard protocols. The findings from the study is expected to provide a baseline for erosion potential mapping and new insights about the integration of FD for EPM.

2. Materials and methods

2.1. Study area

The study area is located between 107°28‘E–111°15‘E and 35°20‘N–39°34‘N showing the middle belt of the Loess Plateau (Figure 1). According to report from Wang et al. (2017), the average annual surface runoff is 42.58 billion m³ for many years, the quantity of water resources 44.5 billion m³ (i.e. ranking the 19th in China). The same report also indicates that the average per capita water resources stand at 1,280m³, groundwater resources about 33.6 billion m³ and average evaporation losses estimated between 1000mm–2000mm annually.

2.2. Analytical hierarchical process (AHP)

The erosion potential mapping for the study area was conducted using GIS-multi-criteria analysis structure. The method made use of the existing capabilities in the support for geospatial data and the flexibility of the Multi-Criteria Decision Analysis (MCDA) to integrate several datasets such as Fractal dimension, slope and flow accumulation derived from DEM (Figure 2). The objective of the MCDA was to define the erosion potential based on the existing FD, slope and flow accumulation maps. The MCDA is a popular technique that has been used exhaustively. However, in this particular study the AHP method was adopted as a result of its wide spread use (Akinci, 2016).

The criteria selection for assessing the erosion potential is an important step for the overall analysis. Therefore, in the consideration of the derivation of the Erosion Potential Map (EPM), expert knowledge in the area resulted in the selection of six key factors such as: (1) the fractal dimension, (2) slope, (3) flow accumulation map, (4) elevation, (5) geomorphological and (6) land use land cover (LULC) maps. These factors were integrated together based on procedure clearly indicated in Figure 2.

In the determination of experts weight coefficients based on the AHP method, for each pair-wise comparison, a matrix Zk is determined by the consistency of the expert’s evaluation. Saaty suggested that the Consistency Ration (CR) was necessary for consistency check and the accepted limit was for CR to be less than 10% (Alonso and Lamata, 2006; Saaty, 1987). The calculation of the degree of consistency is conducted by two main steps. The first step is the consistency index (CI) and the second step is the estimation of the CR as shown mathematically in Eqs. (1) and (2) respectively. The relations used for computing the degree of consistency are shown below:

\[
CI = \frac{\left(\frac{1}{n} \sum_{i=1}^{n} x_i \right) - n}{n - 1}
\]

Figure 1. The Northern Shaanxi Province.
where $\beta_{\text{max}}$ is the maximum Eigen value of the comparison matrix

$N$ is the matrix rank

The second step calculates the CR as:

$$CR = \frac{CI}{RI}$$  \hspace{1cm} (2)

where RI is the random index.

The final erosion potential map was obtained using the Weighted Linear Combination (WLC). The WLC is a used for the process of criteria map integration and aggregation. This method allows for low scores to be compensated by high scores in another, and hence desired for eliminating bias. The method multiplies each standardised criteria map with accepted criteria weights obtained from the AHP method and then sum the results. After several iterations, the final results for the AHP are shown in Table 1.

Using Eq. (3), the final aggregation map was generated. The mathematical expression is shown below:

$$E_s = \sum W_i Y_i$$  \hspace{1cm} (3)

where $E_s$ is the erosion potential, $W_i$ is the normalised value of the factor weight and $Y_i$ is the criterion score of the factor $i$. On the basis of the above and the determined weight as shown in Figure 4, the WLC is estimated. The EPM generated is then re-classed into moderately high, high, and moderately low. It was then validated with geomorphological maps and historical records of erosion in the area.

### 2.3. Box counting method (BCM)

The BCM has been widely used to determine area of irregular cartographic features (Equation 4). The general mathematical form of the box counting method is:

$$N(r) = Cr^{-D}$$  \hspace{1cm} (4)

where $D$ is the fractal dimension, $N(r)$ is the number of boxes that cover the linear object measured, $r$ is the side length of the square box and $C$ is the constant. This equation can be represented in a log-log form as illustrated in Eq. (5).

$$\log[N(r)] = -D \log(r) + \log(C)$$  \hspace{1cm} (5)

To obtain the truthful value of $D$, one needs to count the $N(r)$ for different side lengths of $r$, then obtain the $D$ from the data pairs $N(r)$ and $r$ using least square regressions. BCM works best for self-similar linear objects. The primary data form used with this method, is a grid-based Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER) Global Digital Elevation Model (GDEM) Version 3. The GDEM has a spatial resolution of approximately 1 arc-second (30m) horizontally at the equator with an elevation error of 12.1m (Abrams and Crippen, 2018). This data was obtained from https://earthexplorer.usgs.gov/.

In this study, an automated Matlab function was developed to allow for the automatic unsupervised box counting (Angelo et al., 2004). The BCM was applied on the ASTER 30m DEM to produce a spatial FD map. Twelve (12) locations were selected as a result of their representatives for the

| Parameters             | Normalised Principal Eigen Vectors (%) |
|------------------------|----------------------------------------|
|                        | With Fractal Dimension | Without Fractal Dimension |
| Elevation              | 17.64                  | 20.02                     |
| Geomorphology          | 15.14                  | 8.19                      |
| LULC                   | 13.78                  | 4.54                      |
| Slope                  | 22.80                  | 65.21                     |
| Flow Accumulation      | 12.17                  | 2.04                      |
| Fractal Dimension      | 18.47                  | -                         |

Figure 2. Multi-Criteria Decision Analysis for erosion potential mapping.
study area as points for conducting validation with the historical records (Figure 3). This was done in order to ease the complexity of validating the qualitative records. In this same, each location record in the derived map was compared with the historical record and an inference was made. Each of the x-th location was denoted as SNet(x) location. Therefore, serially renamed as SNet(1), SNet(2),...SNet(12).

3. Results and discussion

3.1. Erosion potential map (EPM)

By having weighted spatial-accumulation of all the criteria and summing them up, two Erosion Potential Maps (EPMs) were produced for the study. The first one was EPM produced without FD inclusion (Figure 4h) and the second one, was EPM produced with FD integration (Figure 4g). For ease of presentation, EpmFD is denoted for EPM produced with FD while EpmNFD is used to denote EPM produced without FD. Therefore, the next Section discusses each of the cases, and concludes with the comparison of the two.

Three classes were created for each of the EPMs. These included Low Erosion Potential (LEP), Medium Erosion Potential (MEP) and High Erosion Potential (HEP). The EpmFD distribution shows that the representations of LEP as (29.4%, 29,986.9km²), MEP as (40.3%, 41,098.6km²) and HEP as (30.3%, 30,899.5km²) respectively. On the other hand, EpmNFD distribution shows the LEP as (46.7%, 47,619km²), MEP as (36.4%, 41,098.6km²) and HEP as (16.9%, 30,899.5km²). Both EpmFD and EpmNFD show closer proportions of the MEP in the study area. However, large differences in the representatives in terms of the LEP and HEP.

Findings from Zhou et al. (2010) show that erosion intensities for the North Shaanxi shows a variation of 2000-28000 t/km²/yr with a broad variation of extreme erosion towards the extreme North part of Shaanxi, and lower values in the range of 2000t/km²/yr at the extreme southern part. The extreme parts of the study area show erosion intensities in the range of 2000–10000 t/km²/yr while the extreme part to the West shows a variation of 5300–8000 t/km²/yr. The middle belt shows between 5000-10000 t/km²/yr. To the extreme North – evenly mixed case of LEP, MEP and HEP with more preference for LEP. To the East is mostly dominated by MP and HP. The southern part of the study area is dominated by LEP while to the West is LEP.
In this paper, it has been considered that, erosion intensities higher than 20000 t/km²/yr are considered extreme and should fall into the HEP zones. Areas within 10000–20000 t/km²/yr would be considered as MEP while any other areas less than 10000 t/km²/yr would be considered as LEP. In order to further conduct in-depth analysis of these differences, comparison is made with previous Chapters and works from Zhou et al. (2010). Table 2 shows the erosion intensities from works of Zhou et al. (2010), SNet locations and classification for both the EpmFD and EpmNFD.

Considering SNet (5) that represents the Desert Loess Transitional Area (DLTA) with erosion intensities of all less than 10000 t/km²/yr, it is expected that the EPM for both EpmFD and EpmNFD should show only LEP. However, this is not the case as EpmNFD is showing majority classes for MEP and some representation for LEP. For EpmFD, LEP is the majority representation with some scattered MEP. This implies that both EpmFD and EpmNFD are not able to explicitly represent LEP for the classification of the EPM. The possible reason could be attributed to the different sources of data and their inherent accuracies. This case is very similar to S Nets (1, 6, 8, 10 and 11) where EpmFD and EpmNFD could not explicitly represent only LEP.

At SNet(7), both EpmFD and EpmNFD show close representation of LEP as in the case of categorisation with the erosion intensities of less than 10000 t/km²/yr. For the case of S Nets (9, 12), erosion intensities are in the range of 5000–28000 t/km²/yr representing all 3 classes of LEP, MEP and HEP. For both S Nets (9 and 12), EpmFD shows close representation for all the classes. However, EpmNFD is only showing LEP and MEH with predominance for LEP.

The spatial distribution of the EPM of the EpmNFD is skewed towards extremes of LEP, MEP and HEP and hence does not take into the consideration the different complexities and the dynamics of the landforms. This is clearly represented in the results in the map presented in Figure 4h. For example, it groups large proportions as one class such as LEP, MEH or HEP. However from Zhou et al. (2010), it is clear that the Shaanxi North is characterised by several mixed variations as in the different landforms and the variability of erosion intensities even within a particular landform. In view of this, the classification of the EpmNFD is viewed as idealistic and does not take into consideration the natural phenomena of landform complexities and dynamics.

This gap has been clearly addressed by the EpmFD that shows the overall variation of the landform classification in terms of its complexity, characteristics and dynamics. The spatial distribution of the EPM is distributed fairly and showing the complex variation even within a particular landform. For example, at S Nets 9 and 12, EpmFD shows clearly all the classes of LEP, MEH and HEP. The findings from EpmFD shows the important role of FD in characterising landform complexity and more particularly providing insights into the EPM. Therefore, the integration of FD to the indicators for the EPM purpose becomes necessary. The purpose of the EPM is to qualitatively indicate the potential risks of erosion in the North Shaanxi and hence the EpmFD demonstrate this capacity.

The developed fractal contour map shown in Figure 5 was compared with the EpmFD show a close match spatial distribution with the erosion potential map obtained. This closeness of spatial distribution between the two maps also indicate the validity of the maps, and further confirms the important role of FD for EPM.

The integration of the FD into the analysis provides EPM results that are much closer to the reality, hence, making EpmFD maps more reliable and recommended compared to EpmNFD. In view of that, the results from the EpmFD shows that about 70% of the Northern Shaanxi Province which is part of the Loess Plateau is at a higher risk of erosion (Figure 4g). This is similar to the other records indicating that, the Loess Plateau is the most eroded area in the world with more than 60% of the land eroded with soil erosion modulus exceeding 8000tkm² per annum (Li et al., 2017). This is as a result of severe gully erosion influenced by the drainage basin and gully-head developments in the Loess Plateau (Guan et al., 2021). It is therefore recommended to consider sustainable measures that will protect the drainage basins. The study recommends efficient and effective conservation of soil and water for agricultural purposes and to reduce the effective drainage area of the gully-heads. For instance, promotion of ecological construction and the increment in vegetation cover will contribute to the decline in soil erosion within the

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**Table 2. Comparison of landforms, their erosion intensities and classes of EPM.**

| Landform Type                   | Erosion Intensities (t/km²/yr) | Class of EpmFD | Class of EpmNFD | SNet(s) |
|--------------------------------|--------------------------------|----------------|----------------|---------|
| Desert loess transitional area  | 4000–6700                      | LEP(m), MEP    | MEP(m), L      | 5       |
| Loess Ridge                    | 5000–10000                     | HEP(m), MEP    | HEP(m),        | 6,11    |
| Loess Incision Gorge Hill (Northern part) | 2000–25000                  | LEP(m), M      | LEP            | 8       |
| Loess Incision Gorge Hill (Eastern part) | 2000–10000                  | LEP(m), H      | LEP            | 10      |
| Loess tableland                | 2000–5000                      | LEP            | LEP            | 7       |
| Loess middle low mountain      | 2000–5000                      | LEP, MEP, HEP  | LEP, MEH, HEP  | 1       |
| Loess Hill Ridge               | 5000–28000                     | LEP, MEH, HEP  | LEP(m), MEH    | 9, 12   |

m implies majority distribution.
Northern Shaanxi Province. Stakeholders involvement and addressing all mutual benefits should be promoted in such management process in order to yield sustainable outputs (Wen and Zhen, 2020). The outcome of this research is vital in providing policy and decision makers and institutions with the framework for developing policies, strategies, solutions and investment-based infrastructure to support the remediation of the highly risk zones.

4. Conclusion

Erosion is a severe threat to the sustainable land use management in China. One of the regions affected is The Loess Plateau of the Shaanxi Province of China. Though some studies have been conducted on erosion potential mapping in the region; there is rare research on the integration of Fractal Dimension for erosion potential mapping. In this paper, the authors assess the development of EPM based on fractal dimension. The AHP method was used and to ensure that the method was consistent and accurate, eight (8) scales of test were used for the weighting purposes. The Inverse Linear Scale yielded the lowest CR and MER, and thus was adopted. Two maps were produced – EpmFD and EpmNFD – this was necessary to consolidate the role of the FD in EPM. The EpmNFD findings were in most cases unrealistic and not comparable to the landform types and distribution. However, the findings show that the EpmFD produced more realistic results in terms of the erosion intensities mapping and the overall characterising of the landform terrain complexity and dynamics. The EpmFD was further validated with the contour FD. These two maps also showed close proximity further confirming the major role of FD in characterising the terrain complexity in terms of EPM. The results from this study are important towards the sustainable land use management efforts towards renewed knowledge for tackling issues related to soil erosion in China and related regions across the World. These efforts are necessary in ensuring a sustainable soil erosion management initiative to ensure that all stakeholders contributing to potential risks of erosion are involved, and in the end, erosion risks in the study area are minimised for ecological restoration.

Declarations

Author contribution statement

Kamila J. Kabo-bah: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Tang Guoan, Xin Yang: Contributed reagents, materials, analysis tools or data; Wrote the paper.

Jiaming Na, Liyang Xiong: Contributed reagents, materials, analysis tools or data.

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