Measurement of neutrino masses from relative velocities

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We present a new technique to measure neutrino masses using their flow field relative to dark matter. Present day streaming motions of neutrinos relative to dark matter and baryons are several hundred km/s, comparable with their thermal velocity dispersion. This results in a unique dipole anisotropic distortion of the matter-neutrino cross power spectrum, which is observable through the dipole distortion in the cross correlation of different galaxy populations. Such a dipole vanishes if not for this relative velocity and so it is a clean signature for neutrino mass. We estimate the size of this effect and find that current and future galaxy surveys may be sensitive to these signature distortions.

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Introduction. Neutrinos are now established to be mass-

ive, and the mass differences have been measured, but

the mass hierarchy and absolute mass values remain un-

known 1. Precision large scale structure data can be

used to measure or constrain the sum of neutrino masses,

as cosmic neutrinos with finite masses slightly suppress

the growth of structure on scales below the neutrino ther-

mal free-streaming scale2–5. But the challenge of this

method is to conclusively disentangle the complex and

poorly understood baryonic effects as many processes can

lead to power suppression on small scales. In this letter,

we present an astrophysical effect which provides a new

way to measure the neutrino masses by using a distinct

signature in current or future galaxy surveys.

We consider the relative velocity between the cold dark

matter (CDM) and neutrinos. Neutrinos decouple early in

the history of the universe when they are still relativ-

istic, but their energy gradually decreases as the uni-

verse expands until they behave as non-relativistic par-

ticles. At this point they can cluster under the action of

gravity. Nevertheless, due to their low masses the neu-

trinos can travel relatively large distances (even at low

redshifts), and be perturbed by the underlying gravita-

tional potential along their trajectories. The large scale

structures can induce a significant bulk relative velocity

field between the CDM and neutrinos, with typical

velocities comparable to the neutrino thermal velocity

dispersion. As we shall show below, such a bulk relative

velocity field will cause a local dipole asymmetry in the

CDM-neutrino cross-correlation function. The concept of

dipole asymmetry in correlation functions was discussed

in 6 recently. The CDM-neutrino cross correlation may

be inferred from the cross-correlation of different galaxy

populations, and such a dipole asymmetry provide a dis-

tinctive and robust signature of neutrino mass, since such

dipole anisotropy would be absent if not for this effect.

In this letter, we delineate the principle of this method, make an analytical estimate of the size of this effect, and then forecast the detectability of this effect in a simplified galaxy bias model.

The Relative Velocity. We treat the CDM and neutrinos as two fluids 7–8 interacting with each other through gravity. The CDM particles and neutrinos are collision-

less, nevertheless much of their behaviour in gravitational

fields can still be modelled with the introduction of an

“effective pressure”, which takes into account the velocity
dispersion or thermal motion of the particles 7. In the

fluid approximation, the (thermal) motion of individual

particles are then ignored, while the bulk motions of the
two fluids are considered. The two fluids have different
effective pressure, so they will acquire different distribu-
tions and velocities even though they are under the action
of the same gravitational field. We use the moving back-
ground perturbation theory (MBPT) 8 to calculate an-
alytically the evolution of the density perturbations and

velocities of the two fluids, the details of this calculation

are given in the Supplement. The basic idea is to assume

that within a certain volume of radius R, each fluid has

a coherent bulk velocity, which can be expanded around

a background velocity as \( \mathbf{v}_i(x, t) = \mathbf{v}_{i(b)}(t) + \mathbf{u}_i(x, t) \),

where \( i \) refers to neutrino (\( \nu \)) or cold dark matter (c).

The background velocity \( \mathbf{v}_{i(b)} \) is a slowly varying velocity

long mode. Linear perturbative calculation then can be

applied within the region to obtain the cross-correlation of

the two fluids.

Starting at a high redshift (we use \( z = 15 \) in our cal-

culation) when the relative bulk mach number is small,

we evolve the MBPT equations down to lower redshifts,

and obtain the relative velocity field \( \mathbf{v}_{i(c)}(x, z) \). We

estimate the variance of this relative velocity analytically

by taking the ensemble average for the given distribu-
tion of primordial fluctuations. We plot the evolution of the neutrino thermal velocity dispersion $\sigma_{\nu}$ and the standard deviation of the gravity-induced bulk relative velocity $\sqrt{\langle v_{\nu c}^2 \rangle}$ in Fig. 1. The thermal velocity dispersion of the neutrinos decreases as the Universe expands. On the other hand, the bulk relative velocity as represented by $\sqrt{\langle v_{\nu c}^2 \rangle}$ grows to its maximum at $0 < z < 1$, then begins to decay. At low redshifts it is comparable with the thermal velocity dispersion.

We plot the relative velocity correlation function $\xi_{\nu c}$ (defined in the Supplement) for four redshifts in Fig. 2. The bulk velocity correlation function for different neutrino masses are almost identical at very high redshifts, but become increasingly differentiated at low redshifts, as the correlation functions of the lighter neutrinos have larger amplitudes and longer correlation lengths. The coherent scale $R_\xi$ which is defined as the scale at which the correlation function $\xi_{\nu c}$ drops to half of its maximum value, are 14.5, 10.3, 7.0 and 4.6 Mpc$/h$ respectively for the four neutrino masses at $z = 0$. However, the neutrinos are not visible, so we can not use this correlation function to measure neutrino mass directly.

Power spectra and correlation functions. Due to the bulk relative velocity between the CDM and neutrinos, the reflection symmetry along the direction of the flow is broken locally, and within a velocity coherent region the cross-correlation contains a dipole term,

$$\xi_{\nu c}(r, v_{\nu c}^{(bg)}) = \xi_{\nu c0}(r, v_{\nu c}^{(bg)}) + \mu \xi_{\nu c1}(r, v_{\nu c}^{(bg)}),$$

(1)

where $\mu = r \cdot v_{\nu c}^{(bg)}$. This also appears as an imaginary part in the CDM-neutrino cross power spectrum$^1$:

$^1$ We can see this by noting that when taking the Hermite conjugate of $P_{\nu c}$, the imaginary part changes sign and so the angular dependent part is anti-symmetric in "c", i.e. $\xi_{\nu c}(r, v_{\nu c}^{(bg)}) = \xi_{\nu c0}(r, v_{\nu c}^{(bg)}) - \mu \xi_{\nu c1}(r, v_{\nu c}^{(bg)})$.

FIG. 1: Redshift evolution of neutrino velocity dispersion and neutrino-CDM relative velocity for different neutrino masses.

FIG. 2: The relative flow correlation function $\xi_{\nu c}(r)$ at different redshifts. The amplitude and scale of the relative flow depends on neutrino mass. The tick marked correlation length.

FIG. 3: The power spectra of CDM and neutrinos. The CDM auto-power $P_{\nu c}$, neutrino monopole $P_{\nu c0}$ and dipole $P_{\nu c1}$, and smoothed dipole term $P_{\nu c, s}$ are plotted.

$$P_{\nu c}(k, v_{\nu c}^{(bg)}, \mu) = P_{\nu c0}(k, v_{\nu c}^{(bg)}) + i\mu P_{\nu c1}(k, v_{\nu c}^{(bg)})$$

This imaginary term would otherwise be zero if not for the relative flow between neutrinos and CDM. This effect is similar to gravitational redshift$^2$, which breaks the reflection symmetry along the line of sight, and causes an imaginary part in the cross power spectrum between two types of galaxies.

Taking $\sqrt{\langle v_{\nu c}^2 \rangle}$ as the representative value for the background velocity, we calculate the induced density correlations using MBPT. Fig. 8 shows the monopole and the absolute value of the dipole (most parts of it are negative) terms of the CDM-neutrino cross power spectrum as well as the CDM autopower spectrum for four different neutrino masses. The oscillations in $P_{\nu c1}$ (dotted line) are due to the sharp sound horizon which is an artifact of the
fluid approximation of neutrinos in our calculation. We have thus smoothed the dipole power spectrum and obtained an average $P_{\nu}(k)$, which is shown as the solid line, for the different neutrino masses the power spectra are different and distinguishable. Fig. 3 shows respectively the CDM autocorrelation function, the neutrino autocorrelation function, and the monopole and dipole part of CDM-neutrino cross correlation functions. We find that the neutrino autocorrelation grows as the neutrino mass increases, since the more massive neutrinos tend to form more structures. The dipole term of the cross power spectrum have a broad peak or hump, its amplitude also grows with the neutrino mass. The scales of the peaks in the correlation function decrease with neutrino mass, and are located at 16, 11, 7, and 5 Mpc/h respectively for the four neutrino masses.

We have taken a single value of $v^{(b)} = \sqrt{\langle v_{\nu}^{2} \rangle}$ for each neutrino mass. For a given background velocity value, the dipole correlation depends on the neutrino mass value, as shown in the equations in the Supplement. But in fact the bulk relative velocity varies from point to point in space. A more rigorous treatment would require a consideration of the distribution of the bulk velocity. The fact that both the typical value of bulk velocity and the dipole correlation for a given background velocity depend on the neutrino mass enhances the sensitivity for this technique. Below for simplicity we will consider only the typical values.

Observability. Neither the neutrinos nor the dark matter can be observed directly, but as their densities affect galaxy densities, their cross power can be inferred from the cross power of galaxies of different populations, provided that the biases of the two population have different dependence on neutrinos and dark matter. Galaxies are known to be biased relative to each other relative to the dark matter, whereas the bias for luminous red galaxies is typically greater than 1. For a galaxy population, we assume its density contrast is related to the dark matter and neutrino density constraints $\delta_\alpha$, $\delta_\beta$ as $\delta_\alpha = b_\alpha f_\alpha \delta_\alpha + b_\nu f_\nu \delta_\nu$ as in [12]. The neutrino bias $b_\nu$ must be close to 1, since the halo mass scale $10^{12} \sim 10^{13} M_\odot$ (or $1 h^{-1} \sim 2 h^{-1} Mpc$ in unperturbed universe) is smaller than the neutrino free streaming scale and coherent scale, so the effect of neutrinos on the halos are linear. If we consider the cross-correlation of two galaxy populations denoted by $\alpha, \beta$, and use $b_\alpha, b_\beta$ to denote $b_\alpha$ for $\alpha, \beta$, then

$$\xi_{\alpha \beta} = \langle \delta_\alpha \delta_\beta \rangle = b_\alpha b_\beta \nu c k^2 \xi_\nu + (b_\alpha + b_\beta) f_\nu \xi_{\nu\nu} + f_\nu^2 \xi_\nu.$$ 

Now consider the $\mu$-dependence of $\xi_{\alpha \beta}$: because the cross correlation function is anti-symmetric in “$v\nu$”, a dipole $\mu(b_\alpha - b_\beta) f_\nu \xi_{\nu\nu}$ appears. The observability of this dipole depends on the relative bias, $\Delta b \equiv b_\alpha - b_\beta$. The known spread in formation bias provides a lower bound on $\Delta b \gtrsim 0.5$. For sensitivity estimation, we will adopt $\Delta b = 1$. The actual error bar of the inferred neutrino mass will depend on the product of $\Delta b$ and galaxy number density $n_g$.

For this measurement, the bulk velocity field can be reconstructed from the observed density field,

$$v_{\nu c}(k) = \frac{\delta_\nu(k)}{T_{\theta,\nu}(k)} \cdot \frac{T_{\theta, c}(k)}{k^2}.$$ (2)

Here $T_{\theta,\nu}(k), T_{\theta, c}(k)$ are the velocity-divergence transfer functions for neutrino and dark matter respectively, and $T_{\theta, g}(k)$ is the density transfer function, which depends on the unknown neutrino mass. In practice, one can iterate the reconstruction with different trial masses $m_\nu$, until a self-consistent relative velocity field $v_{\nu c}$ and dipole value is found. At the high sampling densities considered here, the fractional error in $v_{\nu c}$ is comparable to the error in the CDM density field $\delta$. The shot noise is much smaller than the sample variance, making the error on the velocity field negligible at the scales of interest.

The correlation function provides a local operational procedure to measure the dipole,

$$\xi_{\alpha \beta}(r, \mu) = \frac{1}{N} \sum_{x} \sum_{x+\Delta x} \delta_{\alpha}(x) \delta_{\beta}(x + \Delta x),$$ (3)

where $N$ is appropriate normalization. The dipole term can be extracted from this anisotropic correlation as in

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2 We have verified that the oscillation period is inversely proportional to the effective sound speed, so it is due to the (false) acoustic oscillation in the fluid. Real neutrinos are not a collisional fluid, and the effective sound speed is actually a superposition of different sound speeds, so we do not expect the true cross power spectrum to exhibit these oscillations.
Table I: The forecasted error on neutrino mass with a survey of \( V_\gamma = 1.0 h^{-1}\text{Gpc}^3 \), \( n_g = 2.4 \times 10^{-2} h^3\text{Mpc}^{-3} \) and with current survey data, modeled with SDSS and 2dF as \( V_\gamma = 0.2 h^{-1}\text{Gpc}^3 \), \( n_g V_\gamma = 1 \times 10^6 \). Note that substantial uncertainties exist due to unknown galaxy neutrino bias, which is a nuisance parameter that we marginalize over.

| \( m_\nu \) (eV) | \( \sigma_{m_\nu} \) (current (SDSS) relative error) | \( \sigma_{m_\nu} \) (future relative error) |
|---|---|---|
| 0.05 | 0.045 | 0.90 | 0.0042 | 0.084 |
| 0.10 | 0.044 | 0.44 | 0.0041 | 0.041 |
| 0.20 | 0.079 | 0.40 | 0.0074 | 0.037 |
| 0.40 | 0.097 | 0.24 | 0.0091 | 0.023 |

Taking the Fourier transform then yields the power spectrum dipole. The error bar is easier to specify for the power spectrum than the correlation function, since \( k \)-bins are statistically independent. The transformation from real space to redshift space does not change our error estimate because the dipole is orthogonal to the effect of redshift distortion, which is a quadrupole distortion.

In Fig. 5 we plot the expected error bars of the angular-dependent CDM-neutrino cross power spectrum for a survey with volume \( V_\gamma = 1.0 h^{-1}\text{Gpc}^3 \) and \( n_g \Delta b = 2.4 \times 10^{-2} h^3\text{Mpc}^{-3} \). This corresponds to an all-sky survey out to redshift \( z < 0.2 \), comparable to the SDSS main sample volume, but with a tenfold higher galaxy sampling density, about the density of HIPASS galaxies. The two populations of galaxies could be, for example, a deep optical survey and an HI survey at low redshifts. Alternatively, the second tracer might be obtained by a non-linear weighting of the same density field such as the cosmic tide field.

We proceed to calculate the error on the neutrino mass measurement using a Fisher matrix estimate. We use five \( k \)-bins (\( k = 0.059, 0.12, 0.24, 0.47, 0.94 h/\text{Mpc} \)) in Fig. 5. Modes with smaller \( k \) are not used because MBPT is not a very good approximation unless the background velocity comes from scales larger than the \( k \)-mode. We fit for two parameters: a multiplicative (relative) galaxy bias \( \Delta b \), treated as a nuisance parameter, and a neutrino mass, and marginalize the result over the relative bias. The result is given in Table 1 for the four different neutrino masses. Existing galaxy redshift data may result in a detection for optimistic neutrino mass and bias parameters. Future surveys can measure the neutrino masses precisely.

**Discussions.** The neutrino mass measurement method proposed here differs from the one based on small scale power spectrum suppression, and it is more robust to scale-dependent galaxy biasing. In the approach based on power suppression, if for some reason there is a weak scale-dependent variation of bias at the level of \( \sim 1\% \), it can completely swamp the neutrino signal. In our dipole cross correlation approach, the measured signal arises only from the relative velocity effect. If galaxy bias were to depend on scale, the impact on the inferred neutrino mass would only be proportionate to any such changes, unlike for total power measurements where any uncertainty in bias is amplified by two orders of magnitude or more.

For the cases we considered, the correlation function peaks occur at scales (16, 11, 7.5 Mpc/h) comparable to the relative velocity field coherency scales (14.5, 10.3, 7, 0.46 Mpc/h); this is not unexpected as it is the coherence of the bulk velocity which induces such correlation. However, for the analytical MBPT calculation we used here, it does pose a problem, because strictly speaking the MBPT approximation is valid only for scales below the coherence scale. The non-linear effects become significant for \( k \gtrsim 0.1h/\text{Mpc} \). Nevertheless, the essence of large scale velocity modulation and the expected physical effect (the dipole structure) is still captured in the calculation, though quantitatively it may not be very accurate at the largest scales. This can be remedied with numerical simulations. We will study this in a future paper; preliminary results, however, show that the result is generally consistent with the analytical one.

In our Fisher analysis, we have treated the galaxy relative bias as a nuisance parameter. As described above, the sensitivity to this effect depends on \( n_g \Delta b \) and the galaxy density needed to detect this dipole depends on the bias. In any given detection of the dipole, \( \Delta b \) is immediately known, and thus the error on the neutrino mass would also be known. The uncertainty in the bias, and thus the error, is proportionate to the significance of the detection, i.e. for a 10\( \sigma \) detection, there is an additional 10% uncertainty in the error itself.

In the above we have considered a single neutrino mass. In fact, unlike the power spectrum suppression effect, which is sensitive only to the sum of the neutrino masses, the dipole effect discussed here can in principle be used to measure the mass of a single neutrino. For multiple neutrinos, the different mass eigenstates will have different bulk velocity directions for each of them, which at least in theory can be solved independently by repeating this procedure once for each mass. In practice this may be difficult, but if one or two neutrino masses are dominant and degenerate, then the procedure discussed in this paper is already sufficient. For an inverted neutrino mass hierarchy, the effect would be twice as large and enhance the possibility of detection.

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SUPPLEMENTAL MATERIAL

In this supplement, we give the details of our MBPT calculation. We treat the CDM and neutrinos as two fluids. At low redshifts, when the perturbations are well inside the horizon, we can write the evolution equations as the pressureless fluid equations for the CDM, the fluid inside the horizon, we can write the evolution equations for the CDM and neutrinos are different. The fluid fluid approximation for neutrinos is found to be accurate to $\sim 25\%$ for density contrast in the mass range of $0.05 - 0.5$ eV, quite sufficient to produce qualitative results.

The above equations can be solved and we will obtain the large scale bulk velocity for the neutrino fluid with respect to the CDM. Then for smaller scale modes, we can treat the long wavelength modes as background velocity, $v_c(x, t) = v_c^{(bg)}(t)$, and similarly for $v_\nu(x, t)$, where $v_c$ and $v_\nu$ are slow varying (constant) with position and their evolution are governed by large scale linear velocity field. We can then estimate the induced CDM-neutrino cross correlation by using the MBPT.

Perturbing around the background solution, $v_c(x, t) = v_c^{(bg)}(t) + u_c(x, t)$ and similarly for $v_\nu(x, t)$, we have

$$\frac{\partial \delta_c}{\partial t} + \frac{1}{a} v_c \cdot \nabla \delta_c = - \frac{1}{a} \frac{\nabla \Phi}{a} - H u_c,$$

and

$$\nabla^2 \Phi = 4\pi G a^2 \bar{\rho}_m \delta_m. \quad (4)$$

where the effective sound speed for neutrinis is $c_s = \sqrt{\frac{\rho_\nu}{\rho_c}}$. We see that the equation of motion for the CDM and neutrinos are different. The fluid fluid approximation for neutrinos are found to be accurate to $\sim 25\%$ for density contrast in the mass range of $0.05 - 0.5$ eV, quite sufficient to produce qualitative results.

We work in the CDM rest frame, i.e. set $v_c^{(bg)} = 0$ and $v_\nu^{(bg)} = v_\nu^{(bg)}$. Transforming these equations into Fourier space, and using $\theta \equiv \nabla \cdot \nu$ as our velocity variables yields:

$$\frac{\partial \delta_c}{\partial t} = - \frac{1}{a} \theta_c,$$

$$\frac{\partial \theta_c}{\partial t} = - \frac{3}{2} a H^2 \Omega_m (f_c \delta_c + f_\nu \delta_\nu) + H \theta_c,$$

$$\frac{\partial \delta_\nu}{\partial t} = - \frac{i}{a} v_\nu \cdot \kappa \delta_\nu - \frac{1}{a} \theta_\nu,$$

$$\frac{\partial \theta_\nu}{\partial t} = - \frac{i}{a} v_\nu \cdot \kappa \delta_\nu - \frac{3}{2} a H^2 \Omega_m (f_c \delta_c + f_\nu \delta_\nu)$$

$$- H \theta_\nu + \frac{1}{a} c_s^2 k^2 \delta_\nu,$$

where $f_c = (\Omega_m + \Omega_\nu)/\Omega_m$, $f_\nu = \Omega_\nu/\Omega_m$, with $\Omega_m$ evaluated at the corresponding redshifts. Compaed with the ordinary linear perturbative equations with vanishing bulk velocity, these equations also include the coupling terms $i v_\nu \cdot \kappa \delta_\nu$, which represents the interaction of the short mode with the long velocity mode.

We then make a multipole expansion. Taking the z-axis along the direction of $v_\nu$, $v_\nu \cdot \kappa = v_\nu k_\mu$, where $\mu = \cos \theta$, and expanding in Legendre polynomials $P_l(\mu)$, $l = 0, 1, ..., \delta_c(k, v_\nu^{(bg)}, \mu) = \delta_{c0}(k, v_\nu^{(bg)}) + \mu \delta_{c1}(k, v_\nu^{(bg)}) + ... \quad (7)$
and a set of multipole evolution equations is obtained. Truncating at \( l = 1 \), we find

\[
\begin{align*}
\frac{\partial \delta_{cl}}{\partial t} &= -\frac{1}{a} \theta_{cl}, \\
\frac{\partial \theta_{cl}}{\partial t} &= -\frac{3}{2} a H^2 \Omega_m (f_c \delta_{cl} + f_\nu \delta_{\nu l}) - H \theta_{cl}, \\
\frac{\partial \delta_{\nu l}}{\partial t} &= -\frac{1}{3} \frac{i}{a} v_{bg}^{(bg)} k \delta_{\nu l(1-l)} - \frac{1}{a} \theta_{\nu l}, \\
\frac{\partial \theta_{\nu l}}{\partial t} &= -\frac{1}{3} \frac{i}{a} v_{bg}^{(bg)} k \theta_{\nu l(1-l)} - \frac{3}{2} a H^2 \Omega_m (f_c \delta_{cl} + f_\nu \delta_{\nu l}) \\
&\quad - H \theta_{\nu l} + \frac{c_s^2 k^2}{a} \delta_{\nu l},
\end{align*}
\]

where \( l = 0, 1 \). In the above we have omitted \( l > 1 \) terms as they will be small, since the initial distribution is isotropic.

Writing the real and imaginary parts of the variables explicitly, i.e. \( \delta_{c0} = \delta_{c0}^R + i \delta_{c0}^I \), we find the evolution equations for these 8 variables in the form of \( \delta_R \), \( \theta_R \), \( \delta_I \) and \( \theta_I \) form a subset of total 16 equations. These variables remain zero as the initial conditions are null. Our initial conditions are purely real numbers and isotropic, i.e. without dependence on angle \( \mu \), when they are generated from linear Boltzmann code. So we are actually evolving 8 fluid equations. We use the CLASS code\(^[13]\) to generate CDM and neutrino initial conditions at \( z = 15 \) when the bulk relative velocity is still very small, then evolve the above set of equations down to \( z = 0 \).

**Bulk Velocity Variance**

The neutrino thermal velocity dispersion (shown as the red dash straight line in Fig1) is given by

\[
\sigma_v^2 = \frac{1}{m^2} \int \frac{dp}{f} \frac{p^2 dp}{f(p)^2 dp}
\]

(9)

where \( f(p) = 1/\text{exp}[p/T] + 1 \) is the relativistic Fermi-Dirac distribution function. By contrast the bulk velocity of the neutrinos is induced by the inhomogeneous matter distribution. We can calculate the variance of the bulk velocity in real space using velocity power spectrum in Fourier space: \( \Delta^2 v_{\nu c}(k, z) = \langle v_{\nu c}^2(k, z) \rangle \Delta^2 k/2\pi^2 \), and it can be evaluated from the primordial curvature perturbation spectrum \( \Delta^2 \zeta \):

\[
\langle v_{\nu c}^2(z) \rangle = \int \frac{dk}{k} \Delta^2 \xi(k) \left[ \frac{\theta_{\nu}(k, z) - \theta_{c}(k, z)}{k} \right]^2
\]

(10)

\[
\Delta^2 v_{\nu c}(k, z) = \int \frac{dk}{k} \Delta^2 \xi(k, z)
\]

(11)

where \( \theta \equiv \nabla \cdot v \) is the velocity divergence. The relative contributions to the velocity variance \( \langle v_{\nu c}^2(z) \rangle \) from different scales are shown in Fig.5. The correlation function of the bulk relative velocity is

\[
\xi_{\nu c}(r) \equiv \langle v_{\nu c}(x) v_{\nu c}(x + r) \rangle = \int \frac{dk}{k} \Delta^2 v_{\nu c} \delta_0(k r). \quad (12)
\]

We can take the coherent scales \( R \) to be determined by \( \xi_{\nu c}(R)/\xi_{\nu c}(0) = 1/2 \).

**Power Spectra and Correlation functions**

The CDM and neutrino autopower spectra are

\[
\begin{align*}
P_c(k, v_{\nu c}^{(bg)}) &= \langle \delta_c^* \delta_c \rangle = \langle \delta_c^R \delta_c^R \rangle, \\
P_\nu(k, v_{\nu c}^{(bg)}) &= \langle \delta_\nu^* \delta_\nu \rangle = \langle \delta_\nu^R \delta_\nu^R \rangle
\end{align*}
\]

(13)

and the cross power spectrum between CDM and neutrinos:

\[
P_{cv}(k, v_{\nu c}^{(bg)}, \mu) = P_{cv0}(k, v_{\nu c}^{(bg)}) + \mu i P_{cv1}(k, v_{\nu c}^{(bg)}),
\]

where

\[
\begin{align*}
P_{cv0}(k, v_{\nu c}^{(bg)}) &= \delta_{c0}^R \delta_{\nu0}^R, \\
P_{cv1}(k, v_{\nu c}^{(bg)}) &= \delta_{c0}^R \delta_{\nu1}^R - \delta_{c1}^R \delta_{\nu0}^R.
\end{align*}
\]

(14)

and \( P_{cv}(k, v_{\nu c}^{(bg)}, \mu) = P_{cv}(k, v_{\nu c}^{(bg)}), \mu \). Here an imaginary part appears in the cross power spectrum but not the auto power spectrum; this is very similar the gravitational redshift effect in \( \xi_{\nu c} \).

With the CDM-Neutrino cross power spectrum, we can calculate CDM-Neutrino cross correlation function:

\[
\xi_{cv}(r, v_{\nu c}^{(bg)}) = \int \frac{d^3 k}{(2\pi)^3} \left[ P_{cv0} + \mu i P_{cv1} \right] e^{i kr} \quad (15)
\]
For special configurations, e.g. \( r \) is parallel to \( \mathbf{v}_{\nu c} \), this integral can be done analytically:

\[
\xi_{\nu c}(r, v_{\nu c}^{(bg)}) = \int \frac{k^2 dk}{2\pi^2} \left[ P_{\nu c0} j_0(kr) - P_{\nu c1} j_1(kr) \right] \tag{16}
\]

When \( r \) is perpendicular to \( \mathbf{v}_{\nu c} \), we have:

\[
\xi_{\nu c}(r, v_{\nu c}^{(bg)}) = \int \frac{k^2 dk}{2\pi^2} \left[ P_{\nu c0} j_0(kr) \right] \tag{17}
\]

When \( r \) is anti-parallel to \( \mathbf{v}_{\nu c} \), we have:

\[
\xi_{\nu c}(r, v_{\nu c}^{(bg)}) = \int \frac{k^2 dk}{2\pi^2} \left[ P_{\nu c0} j_0(kr) + P_{\nu c1} j_1(kr) \right]
\]

Finally, we have

\[
\xi_{\nu c}(r, v_{\nu c}^{(bg)}) = \xi_{\nu c0}(r, v_{\nu c}^{(bg)}) + \mu \xi_{\nu c1}(r, v_{\nu c}^{(bg)}).
\]

because the imaginary part in the cross power spectrum changes sign when we take the Hermite conjugate of power spectrum.