Computer Control of Von Neumann Model on Civil Aviation Industry

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Abstract

The problem of computer control algorithm for the singular Von Neumann input-output model on civil aviation industry is researched. A kind of new mathematic method is applied to study the singular systems without converting them into general systems. A kind of stability condition under which the singular input-output model is admissible is proved with the form of linear matrix inequality. Based on this, a new state feedback stability criterion is established. Then the formula of a desired state feedback controller is derived.

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1. Introduction

Now enterprise resource plan shows more and more importance to the run of civil aviation industry. It is hoped that the development of civil aviation industry can be controlled by strong software which can adjust every section of the whole run of aviation enterprises. An integrated running period of aviation enterprises includes stock, transportation, services, repair and flying. If a company can firstly get the quantity of raw material and afterwards purchase the required material, the production will decrease the storage and production cost. So the relation between the input and the output is necessary. Firstly, economists studied the static company production. However, the running of enterprises is generally dynamic. So the economists want to study the current state of a company and research how various running plans can be used to move the company's production from its present status to a future more desirable state.
when they deal with those dynamic manufacture processes. A number of fundamental notions and methods based on the theory of economical cybernetics have been extended to the area of economics [1]. Economists and engineers include Holt (1962), Fisher (1962), Zellner (1966), Prescott (1971, 1972), Shupp (1972), Athans (1972), Aoki (1973) have been applying control theory to economic phenomenon in the early period. Their work is mainly characterized by the study of increasingly larger deterministic models and by movements into control theory. These pioneers' work were followed by many new area of studies by Kendrick (1981), Yang (1985), John (1998), Tang (2000), Cars (2005), Lynda (2006) etc. Robust control is applied to economical cybernetics lately [2]. Many methods and results on robust economic cybernetics have been reported by Tang, Cheng and Zhong [3]. Reference [4], [5] investigated the problem of consumption and investment decision with a higher interest rate. A macroeconomic model of one province is established and the problem of finding robust H-infinite control strategy is studied in [6].

When a more detailed description of the production side of an economy is desired with the development of macroeconomics, this leads to a so-called input-output analysis (i.e., input-output economics). Professor John von Neumann put forward the Von Neumann Model. Now, in the region of input-output economics, many models were established to describe the real economics [7], [8]. Firstly, the general linear system models were well investigated. However, many models are singular linear systems, which are much harder to solve than general linear systems. Singular systems are also referred to as implicit systems, descriptor systems, generalized systems, generalized state-space systems, differential-algebraic systems or semi-state systems [9], [10]. In the area of cybernetics there was a rapid progress in the control of singular systems because such systems can describe many real systems such as economic systems [10]. Many classic results of regular systems were extended to descriptor systems [11-18]. On the other hand, singular systems in economics are generally converted into general linear systems by means of the selection of state vector, control vector and output vector. Little research is about the direct disposal method to singular linear economic models. In this paper the singular Von Neumann Model civil aviation industry will be dealt with instead of converting into the general linear system.

The purpose of this paper is the research of the stability problem of singular Von Neumann Model on civil aviation industry, which belongs to singular linear systems. This paper will directly treat with the singular system via the linear matrix inequality approach. We will consider a sufficient condition under which the singular Von Neumann Model is admissible. It is hoped that this condition can be easily tested.

2. Modeling

A. Notation

\( \mathbb{R}^n \) is the n-dimensional Euclidean space; \( \mathbb{R}^{m \times n} \) denotes the \( m \times n \) real matrices space; \( I \) is the \( n \times n \) identity matrix; \( A^T \) denotes the matrix transposition; "*" is used as the term that are introduced by symmetry; when matrices \( X, Y \) are symmetric, \( X \succ Y \) means that matrix \( Y - X \) is positive-definite; \( \Omega \) is a compact set.

B. Economic model

In this section, the dynamic Von Neumann input-output model on civil aviation industry will be described in detail. In general, the Von Neumann dynamic input-output model is described by

\[
x(k) + Bx(k) = A_1 x(k+1) + A_2 x(k) + d(k),
\]

The vector \( x(k) = [x_1(k) \ldots x_n(k)]^T \in \mathbb{R}^n \) is the total output vector and \( x_i(k) \) is the total output from sector \( i \). \( d(k) \) is the consumption vector. \( A_i \) is the input coefficient matrix of production process of slow state. \( A_2 \) is the input coefficient matrix of production process of fast state. \( B \) is the residual coefficient matrix of depreciation. The above model is a discrete-time system. Sometimes, the continuous system need be investigated. From the above model, we can know:
In fact, \( d(k) \) can be considered as the discrete singular Von Neumann model’s control vector because we can affect the quantity of consumption by controlling the quantity of investment. Then \( x(k) \) can be treated as state vector. Thus the dynamic input-output model can be turned into state space model. In economics, matrix \( A \) is not always invertible. So the dynamic Von Neumann model is a singular system and \( \text{rank} A = r < n \). Then model (1) is a discrete-time Von Neumann system. Sometimes, the continuous system need be investigated. The continuous Von Neumann model is as follows:

\[
A \dot{x}(t) = (I + B - A_2 - A_1)x(t) - d(t) \tag{2}
\]

This system is hard to deal with because of the existence of \( dt \). Fortunately, the above system’s stability is equal to the following system:

\[
A \dot{x}(t) = (I + B - A_2 - A_1)x(t) \tag{3}
\]

which is easy to investigate. To get the stable condition of Von Neumann Model, we need the following definition:

**Definition 1:** The continuous singular system:

\[
Bx(t) = Ax(t) \tag{4}
\]

(a) System (4) is called to be regular if \( \det(sE - A) \) is not identically zero [9, 23].

(b) System (4) is causal if \( \deg(\det(sE - A)) = \text{rank} E \) [9, 19].

(c) System (4) is stable if any root of \( \det(sE - A) = 0 \) lies in the interior of the unit disk with center at the origin [9, 19].

(d) System (4) is called to be admissible if it is regular, causal and stable [9, 19].

**Remark 1:** Definition 1 is widely used in the area of the control problem of singular systems [9], [19]. In this paper, we try to find conditions of the stability for the discrete-time singular Von Neumann Model. In other words, we need to prove conditions under which system (1) is admissible.

### 3. Main result

In this section, we will give a computer control algorithm to the computer control problem of dynamic Von Neumann input-output system. Our aim of designing this strategy is that the dynamical Von Neumann model can move to the desired state. To get a kind of condition for the stability of the continuous Von Neumann input-output system, it is firstly supposed that \( Y(k) = 0 \) which means that all products of each sector are moved to other sectors. Then dynamical Von Neumann model is as follows:

\[
A \dot{x}(t) = (I + B - A_2 - A_1)x(t) \tag{5}
\]

Then the following results can be derived.

**Theorem 1:** The continuous singular Von Neumann input-output system (5) is admissible if there exists a nonsingular symmetrical matrix \( P \) such that

\[
A_i^TP \geq 0 \tag{6}
\]

\[
P + PB - PA_2 - PA_1 < 0 \tag{7}
\]

In order to get this result, we need the following lemmas:

**Lemma 1:** Any singular matrix is congruent with the matrix \[
\begin{bmatrix}
I & 0 \\
0 & 0
\end{bmatrix}
\]

**Lemma 2:** Any nonsingular matrix is congruent with the unit matrix \( I \).
Proof of Theorem 1: For system (5), according to Lemma 1, there exist two nonsingular matrices $M$ and $N$ such that

$$ A_i = M \begin{bmatrix} I_i & 0 \\ 0 & 0 \end{bmatrix} N \quad (8) $$

where $I_i \in \mathbb{R}^{\text{rank} A_i \times \text{rank} A_i}$ is an identity matrix.

Then, premultiplying $(I + B - A_2 - A_1)$ by $M^{-1}$ and postmultiplying $(I + B - A_2 - A_1)$ by $N^{-1}$ one can get a new matrix:

$$ M^{-1}(I + B - A_2 - A_1)N^{-1} = \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix}. \quad (9) $$

Together with (9), (7) can be rewritten as:

$$ PM \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix} N < 0. $$

This means that $T_i$ and $T_4$ are invertible. Then we can get the following formula:

$$ sA_i - (I + B - A_2 - A_1) = M \begin{bmatrix} sI_1 & 0 \\ 0 & 0 \end{bmatrix} N - M \begin{bmatrix} T_1 & T_2 \\ T_3 & T_4 \end{bmatrix} N $$

$$ = M \begin{bmatrix} sI_1 - T_1 & -T_2 \\ -T_3 & -T_4 \end{bmatrix} N $$

So, for system (5) it is easy to see that

$$ \det(sA_i - I - B + A_2 + A_1) = \det(M \begin{bmatrix} sI_1 - T_1 & -T_2 \\ -T_3 & -T_4 \end{bmatrix} N) $$

$$ = \det(MN) \det(\begin{bmatrix} sI_1 - T_1 & -T_2 \\ -T_3 & -T_4 \end{bmatrix}). $$

By Lemma 2, there exist two nonsingular matrices $L$ and $R$ such that

$$ L \begin{bmatrix} sI_1 - T_1 & -T_2 \\ -T_3 & -T_4 \end{bmatrix} R = \begin{bmatrix} sI_1 - T_1 & -T_2 \\ -T_3 & -T_4 \end{bmatrix}. $$

So, it can be easily seen that $\det(\begin{bmatrix} sI_1 - T_1 & -T_2 \\ -T_3 & -T_4 \end{bmatrix})$ is not identically zero and the degree is $\text{rank} A_i$. For $M$, $N$, $L$, and $R$ are nonsingular, $\det(sA_i - I - B + A_2 + A_1)$ is not identically zero and $\text{deg}(\det(sA_i - I - B + A_2 + A_1)) = \text{rank} A_i$. So the system (5) is regular and impulse free. Next, we will prove the system (5) is stable. We consider the following Lyapunov function:

$$ V(x) = x^T(t) A_i^T P x(t). $$

From (6), it is easy to know that $V(x) \geq 0$. 
Differentiating $V(x_t)$ with respect to $t$, we know:

$$
\dot{V}(x_t) = 2x^T(t)PA_1\dot{x}(t) = 2x^T(t)P(I + B - A_2 - A_1)x(t)
$$

$$
= 2x^T(t)(P + PB - PA_2 - PA_1)x(t)
$$

From (7), we can get $\dot{V}(x_t) < 0$.

Thus the system (5) is stable. So the system (5) is regular, impulse free and stable. In other words, the system (5) is admissible. The proof is completed.

Theorem 2: The continuous Leontief dynamic input-output system (3) is admissible if there exist non-singular symmetrical matrix $P$ such that:

$$
A^T_2 P \geq 0
$$

$$
P + PB - PA_2 - PA_1 > 0.
$$

In this case, a state feedback controller can be

$$
Y(t) = \varepsilon(I + B - A_2 - A_1)x(t)
$$

for any $\varepsilon > 1$.

Proof: With (3) and (12), system (3) can turn into

$$
A_1\dot{x}(t) = (1 - \varepsilon)(I + B - A_2 - A_1)x(t).
$$

From (11), it is easy to show that:

$$(1 - \varepsilon)(P + PB - PA_2 - PA_1) < 0.$$

It can be rewritten as:

$$
P(1 - \varepsilon) - P(1 - \varepsilon)A_2 - P(1 - \varepsilon)A_1 + (1 - \varepsilon)PB < 0.
$$

Together with (10), (14) and theorem 1, we can know the closed-loop system (13) is admissible. This completes the proof.

Remark 2: Theorem 2 provides the condition of the stabilization of the continuous dynamic Von Neumann input-output system. And this condition is linear matrix inequality. It can be easily computed.

4. Conclusion

In this paper, the stabilization of the discrete singular dynamic Von Neumann input-output model on civil aviation industry has been studied. This singular model is investigated without being transformed into the general linear system. A sufficient condition has been proved and the design of state feedback controller has been completed. The analysis of the problem is solved via linear matrix inequality approach. Finally, the computer control algorithm is completed.

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