Hopping Parameter Analysis of Leptonic and Semi-Leptonic Heavy-Light Decays

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We study leptonic and semi-leptonic decays of $D$ and $B$ mesons. Use of the Hopping Parameter Expansion (HPE) for two-point functions allows us continuously to vary the pseudoscalar mass from below $m_D$ up towards $m_B$. We compute the pseudoscalar decay constants $f_D$ and $f_B$, and observe consistency with the value calculated in the static limit. From the measurement of three-point functions we compute the matrix element relevant to the decay $\bar{B} \to D\nu\bar{l}$ and extract the Isgur-Wise function $\xi(v,v')$. The HPE enables us freely to vary the initial state pseudoscalar mass at constant $v,v'$, and we investigate the $1/m_Q$ corrections to the heavy-quark limit.

1. Simulation Details

We work on a $16^3 \times 48$ lattice at $\beta = 6.0$ in the quenched approximation. We have analysed a total of 36 configurations, each separated by 1200 Hybrid Over-Relaxed sweeps (OR and Cabibbo-Marinari pseudo-heatbath updates in the ratio 5:1) after allowing 10800 sweeps for thermalisation from a hot start. We use the $\bar{O}(a)$-improved Sheikholeslami-Wohlert (SW) fermion action with the coefficient of the clover term set to its tree-level value (i.e. $c = 1$). Perturbative $O(a)$-improvement of the operators is achieved by rotating the quark propagators. Light-quark propagators have been calculated at three hopping parameter values, $\kappa_l = 0.1432, 0.1440, 0.1445$, and the relevant results from a light hadron analysis are summarised in table 1. The chiral limit corresponds to $\kappa_{\text{crit}} = 0.14556(6)$, and the strange quark to $\kappa_s = 0.1437(5)$. These results are consistent with those recently obtained by the APE collaboration.

2. Pseudoscalar Decay Constants

Two-point functions are computed using the Hopping Parameter Expansion (HPE), evaluated to 200th order to ensure that all heavy-light meson correlators are converged for $\kappa_h < 0.133$ (significantly lighter than $\kappa_{\text{charm}} \simeq 0.125$). We use extended operators, produced by the gauge-covariant Jacobi smearing algorithm, with an $\text{rms}$ smearing radius of four lattice units. The heavy-light pseudoscalar decay constants are extracted by analysing the ratio of the local-smeared axial-pseudoscalar and smeared-smeared pseudoscalar-pseudoscalar correlators and then extrapolating (interpolating) the light quark to $\kappa_{\text{crit}}(\kappa_s)$. Guided by the Heavy Quark Effective Theory (HQET), we extrapolate (interpolate) the results to $m_B$ ($m_D$) by fitting

$$\Phi(m_P) \equiv Z_A^{-1} f_P \sqrt{m_P} (\alpha_s(m_P)/\alpha_s(m_B))^{2/\beta_0}$$

(1)

to a quadratic function of $1/m_P$. This fit is the dotted curve in fig. 1, where only the five lightest values of $m_P$ (circles) have been included.

We also compute two-point functions with a static (i.e. infinitely massive) heavy quark using the same smearing radius. A similar analysis to that performed for propagating heavy quarks yields $Z_L^{\text{stat}} = 0.211(7)$, consistent with APE. The corresponding value of $f_B^{\text{stat}}$ is shown as a cross at $1/m_P = 0$ in fig. 1. We use the perturbative expressions for $Z_A$, evaluated with a boosted coupling, yielding $Z_A = 0.96$ and $Z_A^{\text{stat}} = 0.78$.

Table 1

The light-light meson spectrum (lattice units).

| $\kappa$  | $m_\pi$ | $m_\rho$ | $f_\pi/Z_A$ |
|-----------|---------|----------|-------------|
| 0.1432    | 0.386   | 0.51     | 0.088       |
| 0.1440    | 0.311   | 0.47     | 0.080       |
| 0.1445    | 0.257   | 0.43     | 0.075       |

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The results are summarised in table 2, where we take the scale from $m_\rho$. The first error is statistical, the second the systematic error from the scale uncertainty. Having computed both propagating and static results on the same configurations, we can perform a fully correlated fit of eqn. (1) to a quadratic function of $1/m_P$, including the static point. This fit is the solid curve in fig. 1, and has a $\chi^2$/dof of 1.5 showing that the data are consistent. Extrapolating to $m_B$ in this fashion raises the value of $f_B$ by 25%.

3. The Isgur-Wise Function

The matrix element relevant to the semileptonic pseudoscalar to pseudoscalar decay $P \to P'$ has the general decomposition

$$\langle P'| V_\mu | P \rangle = \sqrt{m_P m_P'} (h_+ (v.v')(v + v')^\mu + h_-(v.v')(v - v')^\mu) \quad (2)$$

Table 2

| Pseudoscalar Decay Constants | $f_D$ | $f_B$ | $f_{D_s}$ | $f_{B_s}$ | $f_{stat}$ | $f_{stat}$ |
|-----------------------------|-------|-------|-----------|-----------|-----------|-----------|
| $199$ +14 -15 +27 MeV       | $176$ +25 -24 +33 MeV       | $225$ +16 -24 -15 MeV       | $206$ +29 -28 -18 MeV       | $286$ +8 -10 +67 MeV       |

Figure 1. $Z_A(\Phi(m_P))$ vs. $1/m_P$ in lattice units.

Figure 2. The Isgur-Wise function.

In the HQET, $h_+$ is the universal Isgur-Wise function, $\xi$, and $h_-$ is zero [6]. The perturbative corrections to these simple relations, up to order $\alpha_s^2 z^2$ (where $z = m_{P'}/m_P$), have been calculated by Neubert [7]. The HQET symmetry-breaking corrections have not been calculated, but $h_+$ is protected by Luke’s theorem [8] so the corrections will be of order $1/m_P^2$ for $h_+$ at zero recoil.

We calculate three-point functions at one value of the light-quark mass, $\kappa_l = 0.1440$, three values of the the final state heavy-quark (charm) mass, $\kappa_{h'} = 0.129, 0.125$ and 0.120, and use the HPE for the initial state heavy quark (bottom). The $B$ meson is placed at rest, and the $D$ meson can have momentum $(8/\pi)|p| = 0, 1$ or $\sqrt{2}$.

To extract $\xi(v.v')$ we set $\kappa_h = \kappa_{h'}$ in the HPE and compute the elastic matrix element $\langle D'| V_4 | D \rangle$ [9]. We obtain $h_+$ from eqn. (2), apply radiative corrections, then fit to the BSW ansatz $\xi_v(v.v')$ to obtain the slope parameter $\rho^2 = -\xi_\rho^2(1)$ [10] (see fig. 2). This yields $\rho^2 = 1.2 \pm 0.9$, consistent with other determinations.

We can exploit the fact that $v.v' = E_{P'}/m_{P'}$ is independent of $\kappa_h$ to explore the $1/m_P$ corrections to the HQET using the HPE. For $\kappa_h \neq \kappa_{h'}$, we simultaneously fit the matrix elements of the spatial and temporal components of the vector current to extract both $h_+$ and $h_-$. In fig. 3 we plot $h_+$ (radiatively corrected) against $z$ at fixed $v.v'$ and fixed $m_{P'}$ ($\kappa_{h'} = 0.120$). Since $h_+$
is protected by Luke’s theorem, we would expect the $1/m_P$ corrections to be small close to zero recoil. This is confirmed by our results, $h_+$ being roughly constant over a large mass range.

In fig. 4 we plot $h_-/h_+$ as a function of $z$. In contrast to $h_+$, we see that there is a very strong dependence of $h_-$ on the pseudoscalar mass ratio. The observed slope is by no means accounted for purely by the radiative corrections, so we interpret fig. 4 as evidence for the existence of large $1/m_P$ contributions. This is to be expected as Luke’s theorem does not apply to $h_-$. Due to current conservation, $h_-$ should vanish for elastic scattering, and it is reassuring that it is indeed consistent with zero when $m_P = m_{P'}$.

4. Conclusions

We have shown that the use of rotated operators with the SW action gives a sensible normalisation for propagating heavy-quark fields, yielding values for pseudoscalar decay constants that are consistent between propagating and static heavy quarks. For semileptonic decays we have computed the Isgur-Wise function assuming that the HQET is valid, and find a slope parameter $\rho^2 = 1.2 \pm 0.9$. We have shown that the order $1/m_P$ corrections to the HQET predictions for the form factors are small for $h_+$ but large for $h_-$. These results are consistent with Luke’s theorem.

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