Application of structural equations of hemodynamics in evaluation of efficiency of physiotherapy of arterial hypertension

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Abstract. Evaluation of parameters of the expert structural model of normal hemodynamics in patients with arterial hypertension of the initial stage before and after special physiotherapy was carried out on the foundation of numerical methods of nonlinear optimization with conditions. A slight change in the consistency of the experimental data that were obtained after physiological treatment with the structural model of normal hemodynamics shows the lack of effectiveness of this physiotherapy for normalizing the regulation of blood pressure.

1. Introduction

The term structural equation modeling refers to a statistical model that allows the estimation and testing of correlative relationships between dependent variables and independent variables as well as the hidden structures in between. It can be checked whether the hypotheses adopted for the model agree with the given variables. It is assigned to the structure-testing multivariate method and has a confirmatory (confirmatory) character. Fundamentals of structural equation modeling can be fundamentally differentiated into covariance-based (eg Amos and LISREL) and variance-based (eg partial least squares, PLS) methods, which have similarities and differences [1].

Structural equations are a form of describing the dependencies between the measured and latent (not measurable) variables of the object under study. The method of modeling the relationship between several measured and latent variables was formed in the 1970s in the works of statisticians K. Joreskog and D. Serb [2], sociologists G. Blalock, O. Duncan [3, 4], econometricians A. Goldberger [5] and psychometrists P. Bentler [6]. In the general case, such dependencies can have a non-linear character of the functions.

The aim of this science work is to construct a structural model of normal hemodynamics and to assess the quality of the effect of physiotherapy procedures on the normalization of the functional state of hemodynamics on the foundation of an analysis of the change in the consistency of the structural model of normal hemodynamics with the experimental data of patients with arterial hypertension before and after physiotherapy procedures.

The objectives of this study are as follows:

- to form a structural model of normal hemodynamics on the basis of expert medical data;
- to assess the parameters of the expert structural model of normal hemodynamics and the consistency of the model with experimental data for individuals with arterial hypertension before and after special physiotherapy.

Various parameters characterizing the condition of patients with arterial hypertension served as experimental data. Since the experimental data are represented by a sample of the values of the measured variables for various objects under study, the minimum residuals can be used to estimate the parameters and values of the latent model variables specified by the structural equations as the sum of the residuals of the model computed for the entire sample of various objects. Additionally, restrictive conditions can be imposed on the parameters and values of the latent variables. It is proposed to use methods of nonlinear optimization with conditions: the method of configurations for solving the problem of minimizing the residuals of the model. Limitations imposed on the values of parameters and latent variables of the model are taken into account for using the penalty function method.
Evaluation of parameters of the expert structural model of normal hemodynamics was carried out for patients with arterial hypertension of the initial stage before and after special physiotherapy.

The structural model of normal hemodynamics can be interpreted as a dynamic model of the change in time of some indicators from others. Since the statistical data characterize the same physiological system, thereby different states and relationships of individual indicators of people can correspond to individual states in the temporal change of indicators.

It is possible to determine the residuals of the equations of the structural model for each object after evaluating the parameters and values of the latent variables of the structural equations of the contour of regulation of normal hemodynamics and maintaining normal blood pressure. The ratio of the number of large and small residuals corresponds to the consistency of the structural model of normal hemodynamics with the clinical state of the objects. The medical condition of individuals is all the better, the more consistent the physiological parameters of the states of people with arterial hypertension with the structural model of normal hemodynamics.

2. Theory of Structural Equation Modeling.

The following types of matrices are used in the theory of structural equations:
matrix \( Z \leftrightarrow z_{ij} \) - the matrix of values of the measured variables for the objects under study or the states of the dimension object \( m \times n \), where \( m \) is the number of measured parameters, \( n \) is the number of objects or states of the object (sample size),
matrix \( P \leftrightarrow p_{ij} \) - the matrix of values of latent variables of objects of dimension \( g \times n \), where \( g \) is the number of latent parameters,
matrix \( A \leftrightarrow a_{ij} \) - the matrix of parameters of structural equations of dimension \( k \times s \), where \( k \) is the number of structural equations, \( s \) is the number of parameters in structural equations.

The system of structural equations is given in the form:

\[
\begin{align*}
f_1 \left( a_{11}, a_{12}, \ldots, a_{1s}; t_1, t_2, \ldots, t_m; p_{11}, p_{12}, \ldots, p_{1s}; z_{11}, z_{12}, \ldots, z_{1n} \right) + \varepsilon_{1t} &= 0, \\
f_2 \left( a_{21}, a_{22}, \ldots, a_{2s}; t_1, t_2, \ldots, t_m; p_{21}, p_{22}, \ldots, p_{2s}; z_{21}, z_{22}, \ldots, z_{2n} \right) + \varepsilon_{2t} &= 0, \\
& \vdots \\
f_k \left( a_{k1}, a_{k2}, \ldots, a_{ks}; t_1, t_2, \ldots, t_m; p_{k1}, p_{k2}, \ldots, p_{ks}; z_{k1}, z_{k2}, \ldots, z_{kn} \right) + \varepsilon_{kt} &= 0,
\end{align*}
\]

where \( f_1, f_2, \ldots, f_k \) are nonlinear functions of their variables in the general case, \( \varepsilon_{1t}, \varepsilon_{2t}, \ldots, \varepsilon_{kt} \) are residuals of the model for the \( t \)-th object or the state of the object.

Additional conditions in the form of equalities and inequalities can be superimposed on the values of parameters and values of latent variables.

The optimal values of parameters and latent variables are those values that minimize the absolute values of the model residuals and satisfy all additional conditions. Optimization of the rotation criterion as functions of independent variables of a rotation matrix with constraints is proposed to be carried out by the method of penalty functions [7]. The configuration method was chosen as a method of unconditional optimization of the method of penalty functions [8].

3. Structural model of normal hemodynamics

In a scientific study, the type of structural model was used to describe the regulation of blood pressure was normal [9]:

\[
\begin{align*}
f_1 \left( a_{11}, a_{12}, \ldots, a_{1s}; t_1, t_2, \ldots, t_m; p_{11}, p_{12}, \ldots, p_{1s}; z_{11}, z_{12}, \ldots, z_{1n} \right) + \varepsilon_{1t} &= 0, \\
f_2 \left( a_{21}, a_{22}, \ldots, a_{2s}; t_1, t_2, \ldots, t_m; p_{21}, p_{22}, \ldots, p_{2s}; z_{21}, z_{22}, \ldots, z_{2n} \right) + \varepsilon_{2t} &= 0, \\
& \vdots \\
f_k \left( a_{k1}, a_{k2}, \ldots, a_{ks}; t_1, t_2, \ldots, t_m; p_{k1}, p_{k2}, \ldots, p_{ks}; z_{k1}, z_{k2}, \ldots, z_{kn} \right) + \varepsilon_{kt} &= 0,
\end{align*}
\]
Where the measured variables are:
systolic blood pressure (SBP) – $x_1$, diastolic blood pressure (DBP) – $x_2$, minute volume of the heart (MVH) – $x_3$, total peripheral vascular resistance (TPVR) – $x_4$, the Kerdo index – $x_5$, impact volume (IV) – $x_6$, heart rate – $x_7$, the Hildebrandt index – $x_8$, end-systolic size of the left ventricle (ESS) – $x_9$, end-systolic volume of the left ventricle (ESV) – $x_{10}$, end-diastolic size of the left ventricle (EDS) – $x_{11}$, end-diastolic volume of the left ventricle (EDV) – $x_{12}$, left ventricular ejection fraction (EF) – $x_{13}$, left ventricular shortening fraction (SF) – $x_{14}$, and the latent variables are:
venous return $p_1$, tone of veins $p_2$, pace of depolarization of the pacemaker $p_3$, baroreceptors $p_4$, contractility of the heart muscle $p_5$, adrenalin $p_6$, structural and geometric state of the heart $p_7$. The diagram of regulation of arterial pressure, which corresponds to this structural model, is presented in figure 1.

This scheme describes the effect of changing some variables from monotonous changes in others with normal regulation of blood pressure. For example, the minute volume of the heart increases with an increase in the heart rate and an increase in stroke volume in a stressful situation, which in case of increased total vascular resistance, for example, in excess weight, leads to increased arterial pressure.

In the case of a normal state of hemodynamics, the significant interdependence of the indices should lead to compensatory processes for the normalization of blood pressure. Therefore, it is important to confirm or refute the effectiveness of certain medical procedures to identify a positive effect on a significant increase in the interdependence of various hemodynamic parameters, which in turn characterizes the increase of dynamic compensation of the body's physiological systems for stabilizing blood pressure.
4. Result of numerical experiment

The numerical experiment of this scientific research consisted in testing the method of estimating the parameters of structural equations within the framework of the quadratic factor analysis model based on 15 biophysical indicators measured in 131 patients with arterial hypertension of the initial stage: weight, body mass index (BMI), breathing rate (BR), segmented neutrophils (S), lymphocytes (L), end-systolic size of the left ventricle (ESS), end-systolic volume of the left ventricle (ESV), end-diastolic size of the left ventricle (EDS), end-diastolic volume of the left ventricle (EDV), impact volume (IV), minute volume of the heart (MVH), total peripheral vascular resistance (TPVR), the Hildebrandt index (HI), left ventricular ejection fraction (EF), left ventricular shortening fraction (SF). All indicators were checked for normal distribution.

For 131 persons with arterial hypertension of the initial stage before and after special physiotherapy, the parameters of the structural equations were evaluated. The distribution of the residuals of the model of normal hemodynamics for individuals with arterial hypertension before and after physiotherapy is shown in figure 2 and figure 3 (V is the residual frequency, and A is the residual value).
Figure 2. Distribution of residuals of the model of normal hemodynamics before physiotherapy.

Figure 3. Distribution of residuals in the model of normal hemodynamics after physiotherapy.

On the distributions it is noticeable that the frequency of zero residuals after treatment has increased slightly. The number of residuals close to zero has decreased and medium residuals has increased. Good harmonized structural equations for various objects have become generally smaller, poor harmonized structural equations have become larger. Apparently, this indicates the lack of normalization of the regulation of arterial pressure after special physiotherapy.

5. Conclusion

The structural model of normal hemodynamics was formed on the basis of expert data. For individuals with arterial hypertension of the initial stage, the parameters and residuals of the model before and after special physiotherapy were evaluated. This model allows to evaluate the effect of physiotherapy on the normalization of blood pressure regulation. A slight change in the distribution of the residuals of the model after physiotherapy suggests a lack of normalization of the regulation of arterial pressure in arterial hypertension without improving the functional interrelation of the body systems.
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