Hidden variables or hidden theories?

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Abstract

We show that a modified Relativity Principle could explain in a "classical" way the strange correlations of entangled photons. We propose a gedanken experiment with balls and boxes that predicts the same distribution of probability of the Quantum Mechanics in the case of the EPR experiment with a pair of entangled photons meeting a pair of polarizers. In the light of this gedanken experiment, we find an alternative description of the real EPR experiment postulating the existence of two observers (one for each polarizer) embedded in two locally anisotropic spacetimes. In our model there is no need to invoke quantum non separability or instantaneous action at distance.

1 INTRODUCTION

Seventy years ago Einstein, Podolsky and Rosen published a famous paper [1] in which they consider a particular two particle state \( \psi_{12} \) that cannot be written as a product like \( \psi_1 \psi_2 \), but only as a sum of such products [2]. These quantum states of two particles are today called "entangled" and can be described "in such a way that their global state is perfectly defined, whereas the states of the separate particles remain totally undefined" [3]. Considering pairs of entangled particles EPR showed that Quantum Mechanics (QM) is not a complete theory and they hoped that QM could be

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improved or substituted by a theory in which the state of a particle is better specified knowing some still hidden variables. The probabilistic character of the quantum description of reality would be due only to the ignorance of these hidden variables. In 1964 Bell \cite{4} showed an inequality that is always satisfied by local hidden variable (LHV) theories and, in some cases, is violated by QM. The paper written by Clauser, Horne, Shimony and Holt \cite{5} made possible a lot of experimental tests that led to violations of their generalized Bell inequalities showing that LHV cannot explain the experiments \cite{6}. The conclusion is that it is impossible to construct a LHV theory that leads to the same predictions of QM and that QM agrees with the experiments. The hidden variable theories equivalent to QM (such as the de Broglie - Bohm approach \cite{7}) must be non local. So we must suppose, according to QM, that the entangled particles form a "non separable" system or that, in a hidden variable theory, an instantaneous action at distance can occur.

In a previous paper, written with Rampone, \cite{8} we discussed how the noise can alter the results of the experiments and the difficulties to find a "fair sample" of particles to test the inequalities. In this paper we consider the ideal case of tests without any noise or "experimental loopholes" (locality, detection efficiency, selection effects, etc). Although there are still some discussions about the interpretation of the results of Aspect’s like experiments, we want to believe that quantum mechanics gives correct predictions and so an alternative theory must reproduce exactly the same results. We do not propose a new deterministic theory that can complete, in the spirit of Einstein approach, the standard Quantum Mechanics. Our aim is only to show that in a particular case a derivation of the same predictions of quantum mechanics is possible without any need of instantaneous action at distance or invoking the non separability of the system of entangled particles. We have only to suppose the existence of a still hidden classical theory. To this aim we propose a gedanken experiment (section 3) in which the role of the hidden theory is played by the well known special relativity. Then we will describe the new Relativity Principle (section 4) and the hidden theory necessary to explain the strange correlations of quantum entangled systems (section 5). We consider in particular two entangled photons emitted by a source in opposite directions whose polarizations are measured using two polarizers. The photons have a probability to pass (or not to pass) the test of a polarizer with some chosen polarization axis. Up to now this probability can be predicted in the right way by the standard quantum mechanics (section 2) and not by the local hidden variable theories.
2 THE STANDARD THEORY

We analyze the famous experiment of pairs of entangled photons meeting polarizers. According with the standard interpretation of quantum mechanics the results of the EPR-type experiments with photons are described following these steps:

1. In the region A the polarizer on the left of the source, with a polarization axis in the position $\theta_A$, measures the polarization of the photon N. 1. Following the standard approach, the photons are emitted by the source with a completely undefined polarization, so the probability to pass the filter is $P(\text{Yes}, \ast) = 1/2$ and of course also $P(\text{No}, \ast) = 1/2$. If the photon passes the test, its wave function collapses in a state with a well defined polarization angle $\theta_A$.

2. As the state of the second photon is entangled with the first one, also the photon N. 2 instantaneously acquires a polarization at the same angle $\theta_A$. According with QM and the Malus law, it has a probability $\cos^2(\theta_A - \theta_B)$ to pass the test of the second polarizer placed in the region B on the right of the source, with a polarization axis in the position $\theta_B$.

3. So, the probability that both photons pass the tests is just $P(\text{Yes, Yes}) = \cos^2(\theta_A - \theta_B)/2$.

Hence the predictions of quantum mechanics are:

\[ P(\ast, \text{Yes}) = P(\ast, \text{No}) = P(\text{Yes, \ast}) = P(\text{No, \ast}) = 1/2 \quad (1) \]

\[ P(\text{Yes, Yes}) = P(\text{No, No}) = \frac{1}{2}\cos^2(\theta_A - \theta_B) \quad (2) \]

\[ P(\text{Yes, No}) = P(\text{No, Yes}) = \frac{1}{2}\sin^2(\theta_A - \theta_B) \quad (3) \]

and they are confirmed by the experiments.

3 A GEDANKEN EXPERIMENT

We want to show a completely classical case where an observer computes the same set of probabilities obtained in the previous experiment with photons. In this macroscopic example we will use balls instead of photons and boxes instead of polarizers. The Observer A is at rest with respect to a Box with
a tube inside that is open with a large section $S_1 = \Delta x \Delta z$ on the $xz$ plane and with a small section $S_2 = \Delta y \Delta z$ in the $zy$ plane (Fig. 1). There is a special filter inside the tube able to distinguish if a ball has a negative or a positive hidden charge. On the $xz$ plane of the Box there is also a circular instrument with an index that can be fixed at a whatever angle $\theta_A$. The observer A sees a lot of other observers with similar boxes that run with different constant velocity along the $y$ axis. He chooses his angle $\theta_A$ and one among these other observers (that we will call B) that has the index of its circular instrument on the position $\theta_B$. After a brief time interval the two Boxes reach a stable state in which the relation between the relative angle $\theta = \theta_A - \theta_B$ and their relative velocity is such that

$$V = c \sin \theta$$  \hspace{1cm} (4)$$

where $c$ is the speed of light. Furthermore the observer A turn on a machine that emits simultaneously two small balls that will go into the boxes of the observer A and B respectively entering the tubes from the $xz$ plane.

The observer A must compute the probability that the balls go out from
the two tubes and, repeating many times the experiment, must check if his prediction is right.

The rules of the game are the following:

1. The observer A knows that the machine emits with the same frequency pairs of negatively and positively charged balls.

2. From the instructions written on each Box, the observer A knows the probability that a positively charged ball passes the filter and goes out the box. It is proportional to the aperture $\Delta y$ (Fig.1):

$$P_+(\text{Yes}) = \frac{(\Delta y)^2}{(\Delta y)^2_{MAX}} = P_-(\text{No})$$

and is also the probability that a negatively charged ball has to be absorbed by the filter. He knows also the value of the normalization constant $(\Delta y)_{MAX}$ (the maximal length of the tube along the $y$-axis), but he ignores that all the Boxes are constructed with this maximal length.

3. He ignores the theory of Special Relativity.

4. He can measure all the lengths, velocities and angles.

In order to compute the probabilities, he measures the aperture of his tube and he finds the value $\Delta y_A = (\Delta y)_{MAX}$ and the aperture $\Delta y_B$ of the tube of the polarizer B. Repeating many times the experiment with different observers that travel at different velocities with respect to him, he can argue a phenomenological rule:

$$\Delta y_B = \Delta y_{MAX} \cos \theta$$

in which the aperture of the Box B is surprisingly related to the relative angle $\theta$ between the indexes of the two circular instruments.

He concludes that:

- $P_+(\text{Yes},*) = (\text{Prob. to find a positively charged ball})(\text{Prob. that it passes the filter A}) = 1/2$ and $P_+(\text{No},*) = 0$
- $P_-(\text{Yes},*) = (\text{Prob. to find a negatively charged ball})(\text{Prob. that it passes the filter A}) = 0$ and $P_-(\text{No},*) = 1/2$
- $P_+(*,\text{Yes}) = (\text{Prob. to find a positively charged ball})(\text{Prob. that it passes the filter B}) = 1/2 \cos^2 \theta = P_-(*,\text{No})$
\[ P_\text{−}(\ast, \text{Yes}) = (\text{Prob. to find a negatively charged ball})(\text{Prob. that it passes the filter B}) = \frac{1}{2}\sin^2 \theta = P_\text{+}(\ast, \text{No}) \]

The probability that both balls of a pair go out from the boxes is:
\[ P_\text{+}(\text{Yes, Yes}) = (\text{Prob. to find a pair of positively charged balls})(\text{Prob. that one passes the filter A})(\text{Prob. that the other passes the filter B}). \]

Hence:
\[ P_\text{+}(\text{Yes, Yes}) = \frac{1}{2} \left( \frac{(\Delta y_A)^2}{(\Delta y)_{\text{MAX}}^2} \right) \left( \frac{(\Delta y_B)^2}{(\Delta y)_{\text{MAX}}^2} \right) = \frac{1}{2} \cos^2(\theta_A - \theta_B) \quad (7) \]

Furthermore the observer A obtains:
\[ P_\text{+}(\text{Yes, No}) = \frac{1}{2} \sin^2(\theta_A - \theta_B) \quad (8) \]

and
\[ P_\text{+}(\text{No, Yes}) = P_\text{+}(\text{No, No}) = 0. \quad (9) \]

It is very easy to compute the corresponding probability for negatively charged balls:
\[ P_\text{−}(\text{No, No}) = \frac{1}{2} \cos^2(\theta_A - \theta_B) \quad (10) \]
\[ P_\text{−}(\text{No, Yes}) = \frac{1}{2} \sin^2(\theta_A - \theta_B) \quad (11) \]

and
\[ P_\text{−}(\text{Yes, No}) = P_\text{−}(\text{Yes, Yes}) = 0 \quad (12) \]

The observer B computes the probabilities in the same cases and obtains specular results that are different from A if one considers only positive (or negative) balls but are the same if one considers always the sum \( P_\text{+} + P_\text{−} \). Summing the probability of negatively and positively charged balls for each similar case, the observer A obtains exactly the same predictions of the quantum mechanics in the example of entangled photons (eqs. 1-3).

\[ P_\text{+}(\ast, \text{Yes}) + P_\text{−}(\ast, \text{Yes}) = P_\text{+}(\ast, \text{No}) + P_\text{−}(\ast, \text{No}) = \frac{1}{2} \quad (13) \]

\[ P_\text{+}(\text{Yes, \ast}) + P_\text{−}(\text{Yes, \ast}) = P_\text{+}(\text{No, \ast}) + P_\text{−}(\text{No, \ast}) = \frac{1}{2} \quad (14) \]

\[ P_\text{+}(\text{Yes, Yes}) + P_\text{−}(\text{Yes, Yes}) = P_\text{+}(\text{No, No}) + P_\text{−}(\text{No, No}) = \frac{1}{2} \cos^2 \theta \quad (15) \]

\[ P_\text{+}(\text{Yes, No}) + P_\text{−}(\text{Yes, No}) = P_\text{+}(\text{No, Yes}) + P_\text{−}(\text{No, Yes}) = \frac{1}{2} \sin^2 \theta \quad (16) \]

Of course this final result will be the same predicted by the observer B from his point of view.
The prediction is based only on the measures without knowing the theory of Special Relativity. From the Lorentz contraction of lengths we know that the observer A obtains for the aperture of the tube in the Box B the value

$$\Delta y_B = (\Delta y)_A \sqrt{1 - V^2/c^2}$$  \hspace{1cm} (17)

that using the eq.(4) leads to the result (6). But this is only a relativistic effect. The proper length of the aperture in B is equal to the one in A. So the problems for the observers A and B are due not only to hidden variables but also to a hidden theory.

4 THE MODIFIED RELATIVITY PRINCIPLE

The predictions of the two observers of the previous gedanken experiment could be used to explain the results of the real experiment with photons as an alternative to the standard QM postulating a sort of relativity in the polarizations. So we must think that there is an observer for each instrument. This is not a new idea because, for example, it was applied to EPR experiments by Smerlak and Rovelli using the words "any physical system provides a potential observer"[9]. Our aim is different. In order to save the locality, Rovelli [10], using the formalism of QM, gives a new point of view (called Relational Quantum Mechanics) alternative to the standard Copenhagen interpretation. On the contrary, we want to show that it is possible to explain in a "classical" way the EPR experiment without using the QM formalism.

Admitting the existence of two observers A and B, an experimental evidence is that they always obtain the same results if the corresponding polarizers have their optical axes with the same orientation. Different orientations can lead to different results of the experiments for two inertial observers and hence for two frames of reference. So one can choose between two alternatives:

1. To preserve the standard Galilean Relativity Principle compelling the two observers to compare their results only when their experimental devices are in the same conditions (the same orientation for the two polarizers)

2. To conclude that all possible inertial frames of reference are no longer physically equivalent. For each one of our observers a preferred direction (fixed by the optical axis of the polarizer) exists and the Relativity
Principle could be changed this way (we apply to our case a principle introduced by Bogoslovsky [11] in a different context):

"All laws of Nature are exactly the same only in such inertial frames of reference which have the same orientation with respect to the preferred direction"

If we consider the second point of view, the observer A of our experiment belongs to one class of equivalent frames of reference with the preferred direction given by the unit vector $\mathbf{\nu}_A$ and the observer B to another class of equivalent frames that has in common a different preferred direction $\mathbf{\nu}_B$. The preferred direction fixes also a preferred frame: the one with an axis (for example the y-axis) parallel to this direction. The outcomes of the experiments can be simply described this way:

1. Two quantum particles are considered "identical from the classical point of view" (same initial conditions, etc.) only if they are entangled.
2. If the preferred directions of the anisotropic spacetimes of the two observers coincide ($\mathbf{\nu}_A = \mathbf{\nu}_B$), the two inertial reference frames are equivalent and the two observers always obtain identical experimental results. The modified Relativity Principle holds.
3. If $\mathbf{\nu}_A \neq \mathbf{\nu}_B$ the probability that the two observers obtain the same results depends on the angle between the two preferred directions and it is $P(\text{Yes,Yes}) + P(\text{No,No}) = (\mathbf{\nu}_A \cdot \mathbf{\nu}_B)^2$.

5 THE RELATIVE POINT OF VIEW

But in the light of the example of the previous section, we can also describe the experiment with photons from a point of view closer to the example of balls and boxes. A source emits a pair of entangled photons with a circular polarization. At the polarizer the end of the electric field vector $\mathbf{E}$ (with a magnitude $E$) travels around a circle and stops either when it arrives at the direction of the optical axis or at the perpendicular direction. As the starting direction at the source is completely random (hidden variable), the fifty per cent of photons will arrive at the optical axis and the fifty per cent at the perpendicular direction. So there are two kinds of photons that could be called positively and negatively charged photons as in the example of balls. The probability that the photon passes is proportional to the square of the component of its electric field as measured by an observer along the preferred direction $(E_y/E)^2$ of his reference frame. If the observer A is in
the preferred frame $xOy$, he has the $y$ - axis parallel to the optical axis of the polarizer A but placed at an angle $\theta$ with respect to the optical axis of the polarizer B. He predicts that:

$$P_A^+(Yes, Yes) = \frac{1}{2} \left( \frac{E_A^y}{E} \right)^2 \left( \frac{E_B^y}{E} \right)^2 = \frac{1}{2} \cdot 1 \cdot \cos^2 \theta$$

as in the equation (7) of the section 3. Also the remaining distribution of probability is the same as the example with balls and boxes. Of course an observer placed on the polarizer B with a reference frame $x'O' y'$ with the $y'$ - axis parallel to the optical axis of the polarizer B computes

$$P_B^+(Yes, Yes) = \frac{1}{2} \left( \frac{E_A^{y'}}{E} \right)^2 \left( \frac{E_B^{y'}}{E} \right)^2 = \frac{1}{2} \cos^2 \theta \cdot 1$$

and, in particular, he obtains exactly the same probability distribution as A for the sum $P_+ + P_-$ as shown in equations (13 -16) of the section 3.

So we must think that there is an observer and a preferred frame for each instrument and that each observer A can assume that his instrument allows the maximal (minimal) probability for the transmission of positively (negatively) charged photons. On the other side he sees that on the other polarizer B, with a polarization axis rotated of an angle $\theta$ (with respect to the polarizer of the first observer) a probability contraction from 1 to $P_A^+(*, Yes) = \cos^2 \theta$ occurs for ”positively charged” photons and a probability dilation from zero to $P_A^-(*, Yes) = \sin^2 \theta$ for ”negatively charged” photons. These relations must be explained by a still hidden theory that plays the role of special relativity of the section 3. If this theory exists, it leads to the same probability distributions of the previous example of balls. In the case of photons these predictions are confirmed by the experiments so if it were possible to distinguish between negative and positive particles we would agree with Smerlak and Rovelli [9] when they argue that in QM ”different observers can give different accounts of the same sequence of events”. But if the charge of the photons remains an hidden variable, both the observers give the same predictions because the sum $P_+ + P_-$ for each case is an invariant.

Probably there will be several theories that can be right to play the role of special relativity in our example. A simple proposal for the hidden theory could be to substitute locally the standard special relativity with a special - relativistic theory of the locally anisotropic spacetime where the new Relativity Principle holds. The theory was formulated by Bogoslovsky

9
In that framework the length of a vector $X$ is given by (eq. n. 10 of Ref. [12])

$$||X|| = \left( \frac{\langle \nu_i X^i \rangle^2}{X^i X_i} \right)^{r/2} \sqrt{X^i X_i}$$  \hspace{1cm} (20)

where $r$ (such that $|r| < 1$) is the parameter that characterizes the magnitude of anisotropy. The interesting property of the equation (20) is that the vector magnitude is determined not only by its pseudo-Euclidean length, but also by its orientation with respect to a preferred direction given by the zero vector $\nu^i = (1, \vec{\nu})$ such that $\nu^i \nu_i = 0$. In our case we must think that the photon N. 1 is embedded in the locally anisotropic spacetime of the polarizer A with the preferred direction $\vec{\nu}_A$ given by its optical axis. So the length of the electric field vector is

$$||E|| = \left( \frac{\vec{\nu}_A \cdot \vec{E}_A}{E} \right)^r \sqrt{E^2}$$  \hspace{1cm} (21)

where $E^2 = E_x^2 + E_y^2 + E_z^2$. If we choose $r = 1/2$ (but it is possible to choose another value of $r$ and then to change the definition of the probability without changing the final result. For example the limit case $r = 1$ is very interesting,) and we denote with $\vec{E}_A$ the electric field of the photon that travels towards the polarizer A and with $\vec{E}_B$ the polarization vector of the photon that travels towards the polarizer B, we obtain the probability computed by the observer A that both the positively charged photons pass:

$$P_A^+(Yes, Yes) = \frac{1}{2} \left( \frac{||E_A||^2}{E^2} \right)^2 \cdot \left( \frac{||E_B||^2}{E^2} \right)^2$$

$$= \frac{1}{2} \left( \frac{\vec{\nu}_A \cdot \vec{E}_A}{E} \right)^2 \left( \frac{\vec{\nu}_A \cdot \vec{E}_B}{E} \right)^2 = \frac{1}{2} \cos^2 \theta$$  \hspace{1cm} (22)

Of course the observer B is embedded in a locally anisotropic spacetime with a preferred direction $\vec{\nu}_B$ given by the optical axis of his polarizer B.

$$P_B^+(Yes, Yes) = \frac{1}{2} \left( \frac{||E_A||^2}{E^2} \right)^2 \cdot \left( \frac{||E_B||^2}{E^2} \right)^2$$

$$= \frac{1}{2} \left( \frac{\vec{\nu}_B \cdot \vec{E}_A}{E} \right)^2 \left( \frac{\vec{\nu}_B \cdot \vec{E}_B}{E} \right)^2 = \frac{1}{2} \cos^2 \theta$$  \hspace{1cm} (23)
and this way one can compute all the probability distribution that will be again the same as the example of balls and boxes. So we have shown that there is a possibility to explain the strange behavior of entangled photons in EPR experiments, in a "classical" way without using quantum formalism or invoking "non separability" or instantaneous action at distance. Of course the price to pay is to admit the existence of two observers, of a new Relativity Principle and of a hidden theory such as the one of Bogoslovsky.

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