IIB Matrix Model with D1-D5 Backgrounds

YUSUKE KIMURA \(^{a)\}^{1}\) and YOSHIHISA KITAZAWA \(^{a)\}^{2}\)

\(^{a)}\)High Energy Accelerator Research Organization (KEK),
Tsukuba, Ibaraki 305-0801, Japan

\(^{b)}\)Department of Physics, Tokyo Institute of Technology,
Oh-okayama, Meguro-ku, Tokyo 152, Japan

Abstract

We consider IIB matrix model with D1-D5-brane backgrounds. Using the fact that noncommutative gauge theory on the D-branes can be obtained as twisted reduced model in IIB matrix model, we study two-dimensional gauge theory on D1-branes and D5-branes. Especially the spectrum of the zero modes in the off-diagonal parts is examined. We also consider the description of D1-branes as local excitations of gauge theory on D5-branes. Relation to supergravity solution is also discussed.

\(^{1}\) e-mail address : kimuray@ccthmail.kek.jp or kimura@th.phys.titech.ac.jp
\(^{2}\) e-mail address : kitazawa@post.kek.jp
1 Introduction

Several kinds of Matrix Model have been proposed\cite{1,2,3,4} to study the nonperturbative aspects of string theory or M theory. These proposals are based on the developments of D-branes. They have been shown to play a fundamental role in nonperturbative string theory\cite{5,6}. Notable point is that supersymmetric gauge theory can be obtained on their world-volume as their low energy effective theory. The idea of matrix models is that supersymmetric gauge theory can describe string or M theory.

IIB Matrix Model is one of these proposals\cite{2}. It is a large $N$ reduced model of ten-dimensional supersymmetric Yang-Mills theory. It is postulated that it gives the constructive definition of type IIB superstring theory. It is crucial that this model has $\mathcal{N} = 2$ supersymmetry because it guarantees the existence of gravitons.

Recently, noncommutative Yang-Mills theory has been studied in many situations. It first appeared within the framework of toroidal compactification of Matrix theory\cite{8}. The world volume theory on D-branes with NS-NS two-form background is described by noncommutative Yang-Mills theory\cite{9}. It was shown\cite{10} that in matrix model picture noncommutative Yang-Mills theory is equivalent to twisted reduced model\cite{11}. In IIB matrix model, twisted reduced model is obtained as expanding the model around noncommutative backgrounds. A noncommutative background is a D-brane-like background which is a solution of equation of motion and preserves a part of supersymmetry. It is well known that gauge theory is realized in the world-volume of D-branes as their low energy effective theory. In IIB matrix model, gauge theory is realized as twisted reduced model. The remarkable point is that there is a relation between the coordinate space and the momentum space. Further researches about noncommutative Yang-Mills were done in\cite{12} using the property of twisted reduced model. The behaviors of Wilson loop operators in the large momentum scale region and the small momentum scale region are studied. In small momentum scale region Wilson loops are the ones in ordinary gauge theory while in large momentum scale region Wilson loops become open string like objects. It was pointed out that there was a crossover at noncommutative scale.

In this paper, we study IIB matrix model with D1-D5 backgrounds. The D1-D5 background is an interesting configuration since it is used by Strominger and Vafa\cite{13} in order to study the microscopic interpretation of the black hole entropy. It is further studied by Maldacena\cite{14} as $AdS/CFT$ correspondence. The organization of this paper is as follows. In section 2, we review the relation between noncommutative Yang-Mills and IIB matrix model. Noncommutative Yang-Mills is equivalent to twisted reduced model. Twisted reduced model is defined by the expansion around a D-brane-like background in IIB matrix model. In section 3, we consider D1-D5 backgrounds. For their description, we divide matrices into four parts. The upper left part and the lower right part represent D1-branes and D5-branes respectively. The remaining parts represent the interactions of them. In string theory picture, off-diagonal parts represent the states appearing as open strings connecting D1-branes and D5-branes. We examine the spectrum of this matrix model. The spectrum of open strings connecting D1-branes and D5-branes in string theory is discussed in\cite{9}. Comparison with string theory results is mentioned. We also consider another description of D1-branes as...
local excitations of gauge theory on the world-volume of D5-branes. This description gives
the description of D1-branes bounded to D5-branes. The counting of the zero modes is also
discussed. In section 4, we mention the relation to supergravity solutions. The relation
between supergravity and gauge theory is formulated as AdS/CFT correspondence[14]. B-
field background effect changes the behaviors of gauge theory, while supergravity solutions
are also changed[15, 16]. Interactions in very far region from branes, flat spacetime, are
described by the cluster property in matrix model. In near horizon region (so called AdS
region) gauge theory description can be used. We consider AdS$_5 \times S^5$ background in
matrix model. AdS$_3 \times S^3 \times M^4$ background is investigated in [10, 12]. Section 5 is devoted to
conclusions and discussions.

2 Noncommutative Yang-Mills in IIB matrix model

In this section, we review IIB matrix model[2, 17] and its relation to noncommutative
Yang-Mills which is equal to twisted reduced model[10, 12].

IIB matrix model is defined by the following action

\[ S = -\frac{1}{g^2} Tr \left( \frac{1}{4} [A_\mu, A_\nu] [A^\mu, A^\nu] + \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right). \]  

Here $\psi$ is a ten dimensional Majorana-Weyl spinor field, and $A_\mu$ and $\psi$ are $N \times N$ hermitian
matrices. This model is the large $N$ reduced model of ten-dimensional $\mathcal{N}=1$ supersymmetric
Yang-Mills theory. This is based on the observation that a large $N$ gauge theory is equivalent
to its reduced model[7]. This model has the following symmetries:

\[ \delta^{(1)} \psi = \frac{i}{2} [A_\mu, A_\nu] \Gamma^{\mu\nu} \epsilon \]  
\[ \delta^{(1)} A_\mu = i \bar{\epsilon} \Gamma^\mu \psi, \]  
and

\[ \delta^{(2)} \psi = \xi \]  
\[ \delta^{(2)} A_\mu = 0, \]  
and translation symmetry:

\[ \delta A_\mu = c_\mu 1. \]  

If we interpret the eigenvalues of $A_\mu$ as the space-time coordinates, we can regard the above
symmetry as $\mathcal{N} = 2$ supersymmetry. We take a linear combination of $\delta^{(1)}$ and $\delta^{(2)}$ as

\[ \tilde{\delta}^{(1)} = \delta^{(1)} + \delta^{(2)} \]  
\[ \tilde{\delta}^{(2)} = i \left( \delta^{(1)} - \delta^{(2)} \right), \]  
we can obtain $\mathcal{N} = 2$ supersymmetry algebra:

\[ \left( \tilde{\delta}^{(i)} \tilde{\delta}^{(j)} \tilde{\delta}^{(k)} \psi = 0, \right. \]  

3
\[
\left( \bar{\delta}^{(i)} \bar{\delta}^{(j)} - \delta^{(i)} \delta^{(j)} \right) A_\mu = 2i \epsilon^\mu \xi \delta_{ij} \tag{8}
\]

The equation of motion of (1) with \( \psi = 0 \) is

\[ [A_\mu, [A_\mu, A_\nu]] = 0. \tag{9} \]

The solution in which \([A_\mu, A_\nu] = c\)-number is an interesting solution because it is a BPS background. That is, half of the supersymmetry is preserved in this background. This corresponds to a D-brane background.

We expand the theory around the following classical solution,

\[ [\hat{p}_\mu, \hat{p}_\nu] = iB^{\mu\nu}, \tag{10} \]

where \( B^{\mu\nu} \) is anti-symmetric tensor and \( c\)-number. This is a solution of (9) and is a BPS background. We assume the rank of \( B^{\mu\nu} \) to be \( \tilde{d} \) and define its inverse \( C^{\mu\nu} \) in \( \tilde{d} \) dimensional subspace. \( \hat{p}_\mu \) satisfy the canonical commutation relations and span the \( \tilde{d} \) dimensional phase space. The volume of the phase space is

\[ V_p = n(2\pi)^{\tilde{d}/2} \sqrt{\det B}. \]

We expand \( A_\mu = \hat{p}_\mu + \hat{a}_\mu \) and Fourier-decompose \( \hat{a}_\mu \) and \( \hat{\psi} \) as

\[ \hat{a}_\mu = \sum_k \tilde{a}_\mu(k) \exp(iC^{\mu\nu}k_\mu \hat{p}_\nu), \tag{11} \]

\[ \hat{\psi} = \sum_k \tilde{\psi}(k) \exp(iC^{\mu\nu}k_\mu \hat{p}_\nu). \tag{12} \]

\( \exp(iC^{\mu\nu}k_\mu \hat{p}_\nu) \) is the eigenstate of \( P_\mu = [\hat{p}_\mu, \cdot] \) with eigenvalue \( k_\mu \). The Hermiticity requires that \( \tilde{a}_\mu^*(k) = \tilde{a}_\mu(-k) \) and \( \tilde{\psi}_\mu^*(k) = \tilde{\psi}_\mu(-k) \). Let \( \Lambda \) be the extension of each \( \hat{p}_\mu \). The volume of one quantum in this phase space is \( \Lambda^{\tilde{d}}/n = \lambda^{\tilde{d}} \) where \( \lambda \) is the spacing of the quanta, say, noncommutative scale. \( B \), which is the component of \( B^{\mu\nu} \), is related to \( \lambda \) as \( B = \lambda^2/2\pi \).

\( k_\mu \) is quantized in the unit of \( k_\mu^{\text{min}} = \lambda/n^{2/\tilde{d}} = \lambda/n^{1/\tilde{d}} \). The range of \( k_\mu \) is restricted as

\[ -n^{1/\tilde{d}} \lambda/2 \leq k_\mu \leq n^{1/\tilde{d}} \lambda/2. \]

Consider the map from a matrix to a function as

\[ \hat{a}_\mu \rightarrow a_\mu(x) = \sum_k \tilde{a}_\mu(k) \exp(ik_\mu x^\mu). \tag{13} \]

Under this map, we obtain the following map,

\[ \hat{a} \hat{b} \rightarrow a(x) \star b(x), \tag{14} \]

where \( \star \) is the star product defined as follows,

\[ a(x) \star b(x) \equiv \exp \left( \frac{iC^{\mu\nu}}{2} \frac{\partial^2}{\partial \xi^\mu \partial \eta^\nu} \right) a(x + \xi)b(x + \eta) \big|_{\xi = \eta = 0}. \tag{15} \]

\( Tr \) over matrices can be mapped on the integration over functions as
\[ Tr[\hat{a}] = \sqrt{\det B} \left( \frac{1}{2\pi} \right)^{\frac{d}{2}} \int d^{d}x a(x). \] (16)

Using these rules, the adjoint operator of \( \hat{p}_\mu + \hat{a}_\mu \) is mapped to the covariant derivative:

\[ [\hat{p}_\mu + \hat{a}_\mu, \hat{o}] \rightarrow \frac{1}{i} \partial_\mu o(x) + a_\mu(x) \ast o(x) - o(x) \ast a_\mu(x) \equiv [D_\mu, o(x)]. \] (17)

Noncommutative Yang-Mills action can be obtained by applying the above rules. Although we have discussed the momentum space, the coordinate space is also embedded in the matrices of twisted reduced model through the relation \( \hat{x}^\mu = C^{\mu\nu} \hat{p}_\nu \). This relation says that the coordinate space is related to the momentum space. This relation reminds us T-duality.

### 3 The Gauge theory of D1-D5 system in IIB matrix model

Following the rules of the previous section, we consider IIB matrix model with D1-D5 backgrounds. We use the coordinate space description instead of the dual momentum space. D1-D5-brane solutions are constructed by preparing the following matrices\(^2\),\(^3\),\(^4\),

\[
\begin{align*}
\hat{x}^0 &= \frac{T}{\sqrt{2\pi n_1}} \hat{q} \\
\hat{x}^1 &= \frac{L_1}{\sqrt{2\pi n_1}} \hat{p} \\
\hat{x}^2 &= \frac{L_2}{\sqrt{2\pi n_2}} \hat{q}' \\
\hat{x}^3 &= \frac{L_3}{\sqrt{2\pi n_2}} \hat{p}' \\
\hat{x}^4 &= \frac{L_4}{\sqrt{2\pi n_3}} \hat{q}'' \\
\hat{x}^5 &= \frac{L_5}{\sqrt{2\pi n_3}} \hat{p}''.
\end{align*}
\] (18)

where \([\hat{q}, \hat{p}] = i, [\hat{q}', \hat{p}'] = i, [\hat{q}'', \hat{p}''] = i\). D1-brane and D5-brane are constituted of \( n_{D1} (= n_1) \) quanta of the volume \( l_{NC}^2 \) and \( n_{D5} (= n_1 n_2 n_3) \) quanta of the volume \( l_{NC}^6 \) respectively, where \( TL_1 = n_{D1} l_{NC}^2 \) and \( TL_1 L_2 L_3 L_4 L_5 = n_{D5} l_{NC}^6 \). \( l_{NC} = 2\pi/\lambda \) is the noncommutative length scale.

\( \hat{x}^\mu \) satisfy the commutation relation such as

\[ [\hat{x}^\mu, \hat{x}^\nu] = -i C^{\mu\nu}, \] (19)

where \( C^{01} = C^{23} = C^{45} = 2\pi/B = l_{NC}^2 = (2\pi/\lambda)^2 \). In IIB matrix model, the D-brane is interpreted as not pure one but one with non-vanishing gauge field strength \( B_{\mu\nu} \). They
are equivalent to the branes in the presence of a constant NS-NS two-form background $b_{\mu\nu}$.  

We expand the model around the classical background, $A_{\mu}^c$, and the fluctuation is denoted by $X_\mu$,

\[
A_{\mu} = A_{\mu}^c + X_\mu. 
\]

We align a D1-brane along the 0, 1 directions and a D5-brane along the 0, 1, 2, 3, 4, 5 directions:

\[
A_{\mu}^c = \begin{pmatrix} \hat{x}_\mu & 0 \\ 0 & \hat{x}_\mu \end{pmatrix} \quad (\mu = 0, 1) \\
\begin{pmatrix} 0 & 0 \\ 0 & \hat{x}_\mu \end{pmatrix} \quad (\mu = 2, 3, 4, 5) \\
\begin{pmatrix} u_\mu & 0 \\ 0 & 0 \end{pmatrix} \quad (\mu = 6, 7, 8, 9) 
\]

\[
X_\mu = \begin{pmatrix} \hat{a}_\mu \hat{c}_\mu \\ \hat{c}_\mu^\dagger b_\mu \end{pmatrix}. 
\]

These $A_{\mu}^c$ satisfy the equation of motion \[^3\]. The upper left part and the lower right part of matrices represent a D1-brane and a D5-brane respectively. The remaining parts of matrices represent the interactions of them. $u_\mu$ are the positions of D1-branes in the 6, 7, 8, 9 directions. We will use the indices $\alpha, \beta$ as the indices running over the direction parallel to the branes and $a, b$ as the indices over the direction transverse to the branes. We replace $\hat{a}_\alpha$ and $\hat{b}_\alpha$ with $C^{\alpha\beta}a_\beta(x)$ and $C^{\alpha\beta}b_\beta(x)$ in our correspondence \[^4\], where $a_\alpha(x)$ and $b_\alpha(x)$ can be interpreted as gauge fields. We also replace $\hat{a}_\alpha$, $\hat{b}_\alpha$ and $\hat{c}_\mu$ with $(1/\sqrt{B})a_\alpha(x)$, $(1/\sqrt{B})b_\alpha(x)$ and $(1/\sqrt{B})c_\mu(x)$.

We can consider multiple branes by replacing $\hat{x} \rightarrow \hat{x} \otimes 1_{Q_1}$ or $\hat{x} \otimes 1_{Q_5}$. $Q_1$ and $Q_5$ are the number of D1-branes and D5-branes respectively. We call this system D1-D5 system. This system is described by two-dimensional supersymmetric $U(Q_1) \times U(Q_5)$ gauge theory in low energy limit. $a_\mu$ are $Q_1 \times Q_1$ hermitian matrices and transform as adjoints of $U(Q_1)$. $b_\mu$ are $Q_5 \times Q_5$ hermitian matrices and transform as adjoints of $U(Q_5)$. $c_\mu$ are $Q_1 \times Q_5$ matrices and transform as the product of the fundamental representation of $U(Q_1)$ and the anti-fundamental representation of $U(Q_5)$. $c_\mu^\dagger$ are two-dimensional fields while $b_\mu$ are six-dimensional fields.

We examine the condition whether this background is supersymmetric:

\[
\delta^{(1)} \psi = \frac{i}{2} [A_{\mu}, A_{\nu}] \Gamma^{\mu\nu} \epsilon
\]

\[^3\]This is the configuration considered in \[^5\].

\[^4\] To describe the compact space, we impose the following condition for the matrices,

\[
A_{\mu} + 2\pi R_{\mu}\delta_{\mu\nu} = \Omega_{\nu}A_{\mu}\Omega_{\nu}^\dagger \quad (\mu, \nu = 2, 3, 4, 5)
\]

where $\Omega_{\mu}$ are unitary matrices and $R_{\mu}$ are compactification radii. $\Omega_{\mu}$ don’t commute for the noncommutative torus \[^6\].
\[ \delta^{(2)} \psi = \xi. \] \hspace{1cm} (23)

We find that the following conditions are needed for this configuration to be supersymmetric:

\[ C^{01} \Gamma^{01} \epsilon = \xi, \]

\[ \Gamma^{2} \Gamma^{3} \Gamma^{4} \Gamma^{5} \epsilon = \epsilon. \] \hspace{1cm} (24)

Therefore one fourth of the supersymmetry are preserved in this configuration. This theory has \( \mathcal{N} = (4, 4) \) supersymmetry in two dimensions.

We next consider twisted reduced model action with the D1-D5 backgrounds. The bosonic part action with \( \hat{c}_\mu = 0 \) can be easily calculated and become

\[ S = \frac{TLQ_1}{4\pi g^2 B^2} - \frac{1}{2\pi B^3 g^2} \int d^2 x \text{tr}(\{D_\alpha, D_\beta\}[D_\alpha, D_\beta]) \]

\[ + 2B(D_\alpha a_\alpha)(D_\beta a_\beta) + B^2[a_\alpha, a_\beta][a_\alpha, a_\beta]_{\ast} \]

\[ + \frac{3BTL^5 Q_5}{16\pi^3 g^2 B} - \frac{1}{2\pi^3 g^2 B} \int d^6 x \text{tr}((\tilde{D}_\alpha, \tilde{D}_\beta)[\tilde{D}_\alpha, \tilde{D}_\beta]) \]

\[ + 2B(\tilde{D}_\alpha b_\alpha)(\tilde{D}_\beta b_\beta) + B^2[b_\alpha, b_\beta][b_\alpha, b_\beta]_{\ast}, \] \hspace{1cm} (25)

where \( D_\alpha \) is the covariant derivative constructed by the gauge fields \( a_\alpha(\alpha = 0, 1) \) while \( \tilde{D}_\alpha \) is the covariant derivative constructed by the gauge fields \( b_\alpha(\alpha = 0, 1, 2, 3, 4, 5) \). \( \text{tr} \) is taken over the color indices. \( \ast \) means that products are not usual ones but the ones defined by \(^5\). We have just considered the situation that D1-branes are overlapping and also that D5-branes are overlapping. By replacing left upper part of \( A_2^i \) with \( w_1^{(0)} \otimes 1_Q \) (\( i = \) color index) which is the positions of D1-branes in the \( x_2 \) direction, we can obtain the case that D1-branes are separated in \( x_2 \) direction. This makes \( a_\mu \) field massive. Other cases can be done in the same way.

We next consider \( c_\mu \) and \( c_\mu^\dagger \) fields. It corresponds to the states appearing in (1, 5) and (5, 1) string\(^5\) in string theory picture. We ignore the interaction terms. After the gauge fixing\(^4\), the kinetic and mass terms are given by

\[ S = - \frac{1}{8\pi B^2 g^2} \int d^2 x \text{tr} \sum_{\alpha=0}^{1} \sum_{\mu=0}^{9} \{ \partial_\alpha c_\mu \partial_\alpha c_\mu^\dagger + \partial_\alpha c_\mu^\dagger \partial_\alpha c_\mu \}, \]

\[ - \frac{1}{8\pi g^2} \int d^2 x \text{tr} \sum_{\alpha=2}^{5} \sum_{\mu=0}^{9} \{ c_\mu(\hat{x}_\alpha^2)_{c_\mu}^\dagger + U^2 c_\mu c_\mu^\dagger + c_\mu(\hat{x}_\alpha^2)_{c_\mu}^\dagger + U^2 c_\mu^\dagger c_\mu \}, \]

\[ + i \frac{1}{8\pi B g^2} \int d^2 x \{ c_2 c_3^\dagger - c_2^\dagger c_3 - c_3 c_2^\dagger + c_3^\dagger c_2 + c_4 c_5^\dagger - c_4^\dagger c_5 - c_5 c_4^\dagger + c_5^\dagger c_4 \}, \] \hspace{1cm} (26)

\(^5\) (1, 5) string is the open string with one end on a D1-brane and the other end on a D5-brane.
where \( U^2 \equiv \sum_{\mu=0}^{9} u_{\mu}^2 \). Mass terms are given in second and third lines in (20). \( \hat{x}_2^2 + \hat{x}_3^2 \) is the Hamiltonian of the charged particle moving on the two-plane in a uniform magnetic field (remember (19)). Therefore the energy eigenvalue of \( \hat{x}_2^2 + \hat{x}_3^2 \) is given by the Landau level, \( 2(N + \frac{1}{2})/B \) ( \( N = 0, 1, 2, \ldots \) ) and similarly for \( \hat{x}_4^2 + \hat{x}_5^2 \). Now we can rewrite mass terms in (20) as

\[
- \frac{1}{8\pi g^2} \int d^2xtr \left[ \left( (2M + 2)/B + U^2 \right) \left( \phi_1 \phi_1^\dagger + \phi_3 \phi_3^\dagger \right) + \left( 2M/B + U^2 \right) \left( \phi_2 \phi_2^\dagger + \phi_4 \phi_4^\dagger \right) \right]_*
\]

\[
+ \frac{1}{8\pi g^2 B^2} \int d^2xtr \left[ \left( (2N + 2)/B + U^2 \right) \left( \phi_2 \phi_2^\dagger + \phi_4 \phi_4^\dagger \right) + \left( 2N/B + U^2 \right) \left( \phi_1 \phi_1^\dagger + \phi_3 \phi_3^\dagger \right) \right]_*
\]

\[
- \frac{1}{8\pi g^2} \int d^2xtr \sum_{\mu \neq 2,3,4,5} \left[ \left( (2M + 1)/B + (2N + 1)/B + U^2 \right) \left( c_\mu c_\mu^\dagger + c_\mu^\dagger c_\mu \right) \right]_*
\]

where \( \phi_1 = \frac{1}{\sqrt{2}} (c_2 + ic_4) \), \( \phi_2 = \frac{1}{\sqrt{2}} (c_2 - ic_4) \), \( \phi_3 = \frac{1}{\sqrt{2}} (c_4 + ic_5) \) and \( \phi_4 = \frac{1}{\sqrt{2}} (c_4 - ic_5) \). \( U^2 \) is the distance between D1-branes and D5-branes. These mass terms mean that a field appearing in the string connecting two branes has a mass proportional to the distance between two branes in string picture. We can say that only \( \phi_2 \) and \( \phi_4 \) can be massless scalar at the ground state (\( M = 0 \) or \( N = 0 \)) and \( U^2 = 0 \) while \( \phi_1, \phi_3 \) and \( c_\mu (\mu \neq 2, 3, 4, 5) \) are always massive because of the zero point energy. Only four real degrees of freedom are massless degrees of freedom.

We next examine the fermionic part. We divide matrices into four parts in the same way as the bosonic part. Fermionic background is set to zero.

\[
\hat{\psi} = \begin{pmatrix}
\hat{\psi}_1 \\
\hat{\psi}_3 \\
\hat{\psi}_3^\dagger \\
\hat{\psi}_2
\end{pmatrix}
\]

Kinetic and mass terms are calculated as follows,

\[
S = - \frac{1}{4\pi g^2} \int d^2xtr \left( \bar{\hat{\psi}} \hat{\Gamma}_a \partial_a \hat{\psi} \right)_*
\]

\[
- \frac{1}{4\pi g^2} \int d^2xtr \left( \bar{\hat{\psi}}_3 \hat{\Gamma}_a \partial_a \hat{\psi}_3 + \bar{\hat{\psi}}_3^\dagger \hat{\Gamma}_a \partial_a \hat{\psi}_3 \right.
\]

\[
+ B \bar{\hat{\psi}}_3 (\Gamma_\gamma \hat{x}^\gamma - u_\tau \Gamma_\tau) \hat{\psi}_3^\dagger + B \bar{\hat{\psi}}_3^\dagger (\Gamma_\gamma \hat{x}^\gamma + u_\tau \Gamma_\tau) \hat{\psi}_3)_*
\]

\[
- \frac{B^2}{16\pi^3 g^2} \int d^6xtr \left( \bar{\hat{\psi}}_2 \hat{\Gamma}_\beta \partial_\beta \hat{\psi}_2 \right)_*
\]

where \( \hat{\Gamma}_\beta = \Gamma_\delta \epsilon^{\delta\beta} \), and \( \alpha = 0, 1, \beta = 0, 1, 2, 3, 4, 5, \gamma = 2, 3, 4, 5, \) and \( \tau = 6, 7, 8, 9 \). We consider the massless conditions for \( \hat{\psi}_3 \) and \( \hat{\psi}_3^\dagger \). Mass terms are given by the fourth and fifth terms of (29). Therefore the massless conditions for \( \hat{\psi}_3 \) and \( \hat{\psi}_3^\dagger \) are \( \Gamma_\gamma \hat{x}^\gamma \hat{\psi}_3 = 0 \) and \( \Gamma_\gamma \hat{x}^\gamma \hat{\psi}_3^\dagger = 0 \) and \( u_\tau = 0 \). We rewrite \( \Gamma_\gamma \hat{x}^\gamma = 0 \) as

\[
(\Gamma_2 + i\Gamma_3) \left( \hat{x}^2 - i\hat{x}^3 \right) + (\Gamma_2 - i\Gamma_3) \left( \hat{x}^2 + i\hat{x}^3 \right) + (\Gamma_4 + i\Gamma_5) \left( \hat{x}^4 - i\hat{x}^5 \right) + (\Gamma_4 - i\Gamma_5) \left( \hat{x}^4 + i\hat{x}^5 \right) = 0.
\]
From the commutation relation (19), we can make annihilation-creation operators:

\[
\begin{align*}
  a_{23} & \equiv \sqrt{\frac{B_2}{2}} (\hat{x}^3 + i\hat{x}^2) \\
  a_{23}^\dagger & \equiv \sqrt{\frac{B_2}{2}} (\hat{x}^3 - i\hat{x}^2)
\end{align*}
\]

(31)

\[
[a_{23}, a_{23}^\dagger] = 1.
\]

(32)

Annihilation-creation operators for \(\hat{x}^4\) and \(\hat{x}^5\) also can be made in the same way. So the following conditions are needed for \(\psi_3\) and \(\psi_3^\dagger\) to be massless:

\[
\begin{align*}
  a_{23}|\psi_{30}\rangle &= 0 \\
  a_{45}|\psi_{30}\rangle &= 0 \\
  (\Gamma_2 + i\Gamma_3)|a_{23}^\dagger\psi_{30}\rangle &= 0 \\
  (\Gamma_4 + i\Gamma_5)|a_{45}^\dagger\psi_{30}\rangle &= 0.
\end{align*}
\]

(33)

Last two conditions say that one fourth of \(\psi_3\) components survive. In other words, the massless mode of \(\psi_3\) fermion has eight components. We have seen that massless mode of \(c_\mu\) has real four degrees of freedom. Both \(c_\mu\) and \(\psi_3\) transform as the product of the fundamental of \(U(Q_1)\) and anti-fundamental of \(U(Q_5)\), i.e. these are super-partners. Since the massless physical degrees of freedom of \(\psi_3\) are four, they are same with the massless degrees of freedom of \(c_\mu\).

We compare these results with string theory results. Off-diagonal elements of matrices correspond to (1, 5) or (5, 1) string. Superstring theory is usually formulated in the Ramond-Neveu-Schwarz formalism, while matrix model is formulated in the Green-Schwarz formalism. We examine the spectrum of (1, 5)-string in the presence of \(b_{\mu\nu}\) field by the Ramond-Neveu-Schwarz formalism. In the R sector, the zero point energy is zero, because world-sheet bosons and world-sheet fermions have the same moding. The massless spectrum is not affected by a \(b_{\mu\nu}\) field. There are four periodic world-sheet fermions in the NN and DD directions (0,1 directions are not counted ). So, there are four zero modes. These zero modes generate \(2^{4/2} = 4\) ground states. After imposing the GSO projection, two of the four ground states survive. There are also the contribution from (5, 1) string. Therefore there appear four massless modes, which correspond to the massless modes of \(\phi_2\) and \(\phi_4\) from the R sector. In the NS sector, the massless spectrum is affected by the \(b_{\mu\nu}\) field. The zero point energy has the dependence of the value of the \(b_{\mu\nu}\) field. The spectrum is discussed in [9] about the case of the D0-D4 system. Same discussions are applied to the D1-D5 system. For large \(b_{23}\) and \(b_{45}\), a D5-brane is regarded as to be constituted of infinitely D1-branes or anti-D-branes. According to [9], the spectrums are different in these cases. (From the open string stretching between D-string and anti-D-string, tachyon state appears. So this system is unstable and not supersymmetric.) The case which is considered in this paper is the former case in which this system is supersymmetric. In this case, total four massless modes, which correspond to the massless modes of \(\psi_3\), appear from the NS sector after imposing
the GSO projection. From these facts, we conclude that the number of massless modes for the off-diagonal elements in matrix model matches the number of massless modes for (1,5) and (5,1) string.

We now consider the description of D1-branes embedded in D5-branes. We are interested in D1-D5 bound states because these states are associated with the dynamics of the black hole[13]. It can be shown that the instantons of $U(Q_5)$ gauge theory on the D5-branes carry R-R two-form charge and D1-branes considered in the previous part of this section correspond to the zero size limit of the instantons. Therefore $Q_1$ D1-branes embedded in D5-brane are constructed as $Q_1$ instantons in $U(Q_5)$ gauge theory[21, 22]. We now look at the Chern-Simon coupling in D5-brane action:

$$tr \int_{\Sigma_6} C_2 \wedge F_2 \wedge F_2$$

where $C_2$ is the R-R two form field and $F_2$ is the gauge field strength. In this coupling, $F \wedge F$ plays the role of D1-brane charge. In general, lower-dimensional D-branes embedded in higher-dimensional D-branes are expressed as the magnetic flux on higher-dimensional D-branes[23].

We consider $U(Q_5)$ gauge theory on D5-branes with a self-dual gauge field strength on $T^4$ in matrix model.

$$A_{\mu}^{cl} = \begin{pmatrix} \hat{x}_{\mu} & 0 \\ 0 & \hat{x}_{\mu} \end{pmatrix} \quad (\mu = 0, 1, 2, 3, 4, 5)$$

$$= \begin{pmatrix} 0 \\ \mu = 6, 7, 8, 9 \end{pmatrix}$$

$$X_{\mu} = \begin{pmatrix} \tilde{b}_{\mu}^{(1)} & \tilde{b}_{\mu}^{(3)} \\ \tilde{b}_{\mu}^{(3\dagger)} & \tilde{b}_{\mu}^{(2)} \end{pmatrix}. \quad (34)$$

The size of the right lower part of matrices are $2 \times 2$. The size of the left upper part of matrices are $(Q_5 - 2) \times (Q_5 - 2)$. The equation of motion in IIB matrix model is given by (4). After the replacements from matrices to functions, the equation of motion of this gauge theory become

$$[\tilde{D}_\mu, [\tilde{D}_\mu, \tilde{D}_\nu]] = 0 \quad (\mu, \nu = 0, 1, 2, 3, 4, 5, 6). \quad (36)$$

We analyze the situation that only gauge fields $b_{\mu}^{(2)}$ have nontrivial classical solutions. In other words, we embed an (anti-)instanton into $SU(2)$ part (right lower part). The instanton solution \footnote{Instantons on non-commutative $R^4$ space are constructed in [24].} can be constructed as follows,

$$\tilde{F}_{\alpha\beta}^{(2)} = \pm * \tilde{F}_{\alpha\beta}^{(2)}$$

$$\tilde{F}_{\rho\sigma}^{(2)} = 0$$

$$\tilde{F}_{\rho\sigma}^{(2)} = 0,$$
where $\alpha, \beta = 2, 3, 4, 5$ and $\rho, \sigma = 0, 1$. $\ast$ means the dual tensor on $T^4$. The star product which is defined in (14) is used in these equations. $\tilde{F}^{(2)}$ is the gauge field strength constructed by the gauge field $b^{(2)}$. Self-dual and anti-self-dual configurations are referred to instantons and anti-instantons respectively. In D-brane picture, the winding number of instanton represents the D1-brane charge. Therefore to express $Q_1$ D1-branes on D5-branes, we set the following condition:

$$Q_1 = \frac{1}{8\pi^2} tr \int_{T^4} \tilde{F}^{(2)} \wedge \tilde{F}^{(2)}. \quad (38)$$

Also, D5-branes carry D3-branes charge. Therefore we also need the following condition to obtain pure D1-D5 bound states, i.e. no D3-branes:

$$\frac{1}{2\pi} tr \int_{T^4} \tilde{F}^{(2)} = 0. \quad (39)$$

In these configurations, supersymmetry transformations (2) and (4) are given as follows:

$$\delta^{(1)} \psi = \frac{i}{2} [A_\mu, A_\nu] \Gamma^{\mu\nu} \epsilon$$

$$= \frac{1}{2} \left( \begin{array}{cc} C^{\mu\nu} \Gamma^{\mu\nu} & 0 \\ 0 & C^{\mu\nu} + iC^{\mu\tau}C^{\nu\rho} \left[ D_\tau^{(2)}, D_\rho^{(2)} \right] \end{array} \right) \Gamma^{\mu\nu} \epsilon$$

$$= \frac{1}{2} C^{\mu\nu} \Gamma^{\mu\nu} \epsilon$$

$$+ \left( \begin{array}{cc} 0 & 0 \\ 0 & iC^{\alpha\gamma}C^{\beta\delta} \left[ D_\gamma, D_\delta \right] \left( \frac{1+\Gamma^2\Gamma^3\Gamma^4\Gamma^5}{2} \right) \end{array} \right) \Gamma^{\alpha\beta} \epsilon$$

$$\delta^{(2)} \psi = \xi, \quad (40)$$

where repeated indices $\alpha, \beta, \gamma, \tau$ run over 2, 3, 4, 5. $\mp$ correspond to self-dual and anti-self-dual configurations respectively. Following constraints are needed for this configuration to be supersymmetric:

$$\frac{1}{2} C^{\mu\nu} \Gamma^{\mu\nu} \epsilon = \xi,$$

$$\Gamma^2\Gamma^3\Gamma^4\Gamma^5 \epsilon = \pm \epsilon. \quad (41)$$

We study the zero mode. It is known that the second Chern number is equal to the difference in the number of the chiral fermion zero modes whose chiralities are positive and negative (the index theorem). The difference of the chiral fermions is given by $\alpha_V Q_1 [24]$, where $\alpha_V$ is the Dynkin index depending on the representation and $Q_1$ is defined in (38). We expect that chiral fermions appear from the off-diagonal parts and the right lower part. These parts transform as the fundamental and adjoints of $SU(2)$ respectively. In these cases, $\alpha_V$ is 1 and 4 respectively. Therefore $Q_1 Q_5$ chiral fermion zero modes appear from the index theorem. From the condition (33), the zero mode of $\psi_3$ satisfies $\Gamma_2 \Gamma_3 \Gamma_4 \Gamma_5 \psi_{30} = -\psi_{30}$. This implies that $\psi_{30}$ has negative chirality as a $SO(4)$ (the rotation of $x_2, x_3, x_4, x_5$) Weyl spinor. We notice that this chirality is opposite to the spinor which satisfy the condition of unbroken supersymmetry (see (11)). The reason is that these zero modes are Goldstone modes. The degrees of freedom of these zero modes is associated with the dynamics of the black hole entropy [13].
4 Relation to supergravity solution

In this section, we would like to study the supergravity solution in IIB matrix model. The relevant supergravity solution to matrix model is supergravity solution with \( b_{\mu\nu} \) field since the D-brane in matrix model is not pure one but one with \( b_{\mu\nu} \) background. The alignment of D1-branes and D5-branes is same with the previous section. \( x_2, x_3, x_4, x_5 \), directions are compactified on a torus \( T^4 \). This gives black string solution if \( Q_1 \) and \( Q_5 \) are large. We will work out in Euclidean space. The supergravity solution of D1-D5 system with the \( b_{\mu\nu} \) fields which have non-zero components along all the directions parallel to the branes can be obtained likewise as in [26, 16]. The procedure is as follows. We apply T-duality transformation in the \( x_1 \) direction in a usual D1-D5 system. This gives D0-D4 bound states. Then we rotate the \((x_0, x_1)\)-plane with a rotation angle \( \theta \). Now we consider making the T-duality transformation in the \( x_1 \) (this is a rotated coordinate) direction. This gives the mixed boundary condition in the \( x_0, x_1 \) directions. In this T-duality transformation, \( b_{01} = \tan \theta f_{1}^{-1} f_{5}^{-1} h_{1} \), \( b_{23} = \tan \theta f_{1} f_{5}^{-1} h_{2} \), \( b_{45} = \tan \theta f_{1} f_{5}^{-1} h_{3} \), we obtain the following solution:

\[
\begin{align*}
    ds^2 &= f_{1}^{1/2} f_{5}^{-1/2} h_{1} \left( dx_0^2 + dx_1^2 \right) + f_{1}^{1/2} f_{5}^{1/2} \left( dr^2 + r^2 d\Omega_3^2 \right) \\
    &\quad + f_{1}^{1/2} f_{5}^{-1/2} \left[ h_2 (dx_2^2 + dx_3^2) + h_3 (dx_4^2 + dx_5^2) \right],
\end{align*}
\]

\[\begin{align*}
    e^{2\phi} &= g f_{1}^{2} h_{1} h_{2} h_{3}, \\
    f_{1,5} &= 1 + \frac{\alpha' R_{1,5}^2}{r},
\end{align*}\]

\[\begin{align*}
    b_{01} &= \tan \theta f_{1}^{-1} f_{5}^{-1} h_{1}, \\
    b_{23} &= \tan \theta f_{1} f_{5}^{-1} h_{2}, \\
    b_{45} &= \tan \theta f_{1} f_{5}^{-1} h_{3},
\end{align*}\]

where

\[\begin{align*}
    h_{1}^{-1} &= f_{1}^{-1} f_{5}^{-1} \sin^2 \theta + \cos^2 \theta, \\
    h_{2}^{-1} &= h_{3}^{-1} = f_{1} f_{5}^{-1} \sin^2 \theta + \cos^2 \theta.
\end{align*}\]

This solution is considered in string frame. \( \theta \) is a parameter which represents noncommutativity.

We consider the region which is very far from branes, that is, \( r \) is very large. The spacetime become the flat space-time. We can ignore the \( b_{\mu\nu} \) effect (i.e. noncommutativity) in this region. In such a region, the interactions are described as the interactions between diagonal blocks using the cluster property [2, 10] in matrix model. We consider the instanton and (anti-)instanton pair, each is in D5-brane, and the situation that their distance is very
far. Instanton configuration in D5-brane is discussed at the last part in the previous section. We embed D1-D5 bound states in the left upper part of matrices and the other D1-D5 bound states in the left upper part of matrices. The exchange of gravitons, dilatons and axions can be seen similar to D3-D(−1) case[10]. The interaction potential between instantons vanish because this system is BPS saturated. On the other hand, graviton exchange can be seen in the instanton and anti-instanton case[10].

It is conjectured by Maldacena[14] that two-dimensional CFT describing the Higgs branch of D1-D5 system on $M^4$ is dual to IIB string on $AdS_3 \times S^3 \times M^4$. In the spirit of this relation, supergravity solution dual to noncommutative Yang-Mills is studied by Maldacena and Russo[16], Hashimoto and Itzhaki[15]. Solution dual to two-dimensional gauge theory on D1-D5-branes with $b_{\mu\nu}$ field is obtained by taking the near horizon limit in the solution (42),

$$\alpha' \to 0, \quad \tan \theta = \frac{\tilde{b}}{\alpha'},$$

$$x_{0,1} = \frac{\alpha'}{\tilde{b}} \tilde{x}_{0,1}, \quad x_{2,3,4,5} = \left(\alpha'\right)^{\frac{1}{2}} \tilde{x}_{2,3,4,5},$$

$$r = \alpha' R^2 u, \quad g = \alpha' \tilde{g},$$

where $\tilde{b}$, $\tilde{x}_{0,1}$, $\tilde{x}_{2,3,4,5}$, $u$ and $\tilde{g}$ stay fixed and $R^2 = R_1 R_5$.

$$\frac{ds^2}{\alpha'} = R^2 \left[ u^2 \tilde{h} \left( dz_0^2 + dz_1^2 \right) + \frac{dU^2}{U^2} + d\Omega_3^2 + \frac{R_1}{R_5} d\tilde{x}_n d\tilde{x}_n \right],$$

$$\tilde{h} = \frac{1}{1 + a^2 u^2}, \quad a^2 = \tilde{b} R^2,$$

$$e^{2\phi} = \tilde{g}^2 \tilde{h}.$$

where $\tilde{g} = \tilde{g} \tilde{b} (R_1 / R_5)^2$ is the string coupling in the IR region. From the perspective of gauge theory, radial coordinate $u$ is interpreted as energy scale. In the IR region, this metric represents $AdS_3 \times S^3 \times M^4$. The string coupling constant depends on the energy scale. The string coupling decrease as the energy scale grows. This is the same behavior with one in $AdS_5 \times S^5$ which behaves like the twisted reduced model [10]. The behavior of the dilaton is

$$e^{\phi} \sim \tilde{g} \frac{1}{a^2 u^2} = \tilde{g} \frac{1}{\tilde{b} R^2 u^2}$$

at large $u$. This is in accordance with the fact that the relevant noncommutative Yang-Mills theory is two-dimensional [27].

In IIB matrix model, curved spacetime is thought to be given by assuming the following eigenvalue distribution [28],

$$\langle \rho(x) \rangle \sim \sqrt{g} e^{-\phi(x)},$$

where

$$\rho(x) = \sum_i \delta^{(10)}(x - x').$$
In this case, we can calculate as follows,
\[
\sqrt{g} e^{-\phi(x)} = (\alpha'R_1 R_5)^{\frac{1}{2}} \hat{g}^{-1} u \hat{h}^{\frac{1}{2}} \\
= (\alpha'R_1 R_5)^{\frac{1}{2}} \hat{g}^{-1} \left( \frac{u^2}{1 + a^4 u^4} \right)^{\frac{1}{2}},
\]
(48)
in only $AdS_3$ part. This is invariant under $u \to 1/a^2 u$ as expected. This eigenvalue distribution has a peak at $u = 1/a$ while ordinary $AdS$ has a peak at $u = \infty$ (i.e. boundary).

5 Discussions

In this paper, we have considered D1-D5 system in IIB matrix model. Using the equivalence between twisted reduced model and noncommutative Yang-Mills, we studied the two-dimensional gauge theory on D1-D5 system. Twisted reduced model is obtained by the expansion around the noncommutative background. D1-D5 background is supersymmetric and one fourth of the supersymmetry survive. We discussed the spectrum of the fields, especially the fields which appear in the off-diagonal parts of matrices. They correspond to the state which appear in $1, 5$ or $5, 1$ open string. We showed that only four degrees of freedom can be massless modes. This fact coincides with string theory result. D1-branes embedded in D5-branes are expressed as nontrivial solutions, instantons, in six-dimensional gauge theory on D5-branes. We also considered the zero modes of the instanton configuration. The topological charge of the instanton is related to the existence of the chiral fermion zero modes by the index theorem. It was shown that there were $Q_1 Q_5$ $SO(4)$ Weyl fermions as the zero modes. These degrees of freedom play an important role in the black hole physics. We also considered the anti-instanton solution which is interpreted as an anti-D1-brane embedded in a D5-brane. The anti-D1-brane background is constructed by replacing $\hat{x}_1$ with $-\hat{x}_1$ in (21). We can understand that anti-D1-brane and D5-brane system is unstable because this system is not supersymmetric. We can check it as follows. We obtain $-C^{01} \sigma_3 \Gamma^{01} \epsilon = 1_{2 \times 2} \xi$ instead of the first condition in (24). It is impossible to find the spinors which satisfy this condition. However, an Anti-D1-brane can dissolve in a D5-brane and it is described as anti-instanton in the gauge theory on D5-brane. We have shown that this anti-instanton solution is supersymmetric, so this system can be stable as the bound state.

In section 4, we discussed the supergravity solution which is dual to the gauge theory of the D1-D5 system in matrix model. In [10], D-instantons were constructed as local excitations of gauge theory on D3-branes. It was also shown that D-instantons couple to gravity by considering the interactions between diagonal blocks. In this paper, we considered D1-brane solutions as local excitations of gauge theory on D5-branes. In matrix model, D1-brane is expressed by infinitely many D-instantons. Therefore the interactions between D1-branes and (anti-)D1-branes can be viewed as the interactions between infinitely many D-instantons and (anti-)D-instantons.
Acknowledgments

We would like to thank N.Ishibashi and the members of the KEK theory group for useful discussions.

References

[1] T.Banks, W.Fischler, S.Shenker and L.Susskind,
   *M theory as a matrix model: a conjecture*, hep-th/9610043.

[2] N.Ishibashi, H.Kawai, Y.Kitazawa and A.Tsuchiya,
   *A Large N reduced model as superstring*, hep-th/9612113.

[3] R.Dijkgraaf, E.Verlinde and H.Verlinde, *Matrix string theory*, hep-th/9703030.

[4] H.Itoyama and A.Tokura, *USp(2k) Matrix Model: F theory Connection*, hep-th/9708123.

[5] J.Polchinski, *Dirichlet-Branes and Ramond-Ramond Charges*, hep-th/9510017.

[6] J.Polchinski, *TASI Lectures on D-Branes*, hep-th/9611050.
   J.Polchinski, *String Theory Volume II*, Cambridge 1998.

[7] T.Eguchi and H.Kawai, Phys.Rev.Lett.48 (1982) 1063.
   G.Parisi, Phys.Lett.112B (1982) 463.
   D.Gross and Y.Kitazawa, Nucl.Phys. B206 (1982) 440.
   G.Bhanot, U.Heller and H.Neuberger, Phys.Lett 113B (1982) 47.
   S.Das and S.Wadia, Phys.Lett. 117B (1982) 228.

[8] A Connes, M. R. Douglas and A. Schwarz,
   *Noncommutative Geometry and Matrix Theory : Compactification on Tori*, hep-th/9711162.

[9] N.Seiberg and E.Witten, *String theory and Noncommutative Geometry*, hep-th/9908142.

[10] H.Aoki, N.Ishibashi, S.Iso, H.Kawai, Y.Kitazawa and T.Tada,
    *Noncommutative Yang-Mills in IIB Matrix Model*, hep-th/9908141.

[11] A.Gonzales-Arroya and M.Okawa, Phys. Rev D27 (1983)2397.
[12] N.Ishibashi, S.Iso, H.Kawai and Y.Kitazawa,  
Wilson loops in Noncommutative Yang-Mills, hep-th/9910004.

[13] A.Strominger and C.Vafa,  
Microscopic Origin of the Bekenstein-Hawking Entropy, hep-th/9601029.

[14] J.Maldacena, The Large $N$ Limit of Superconformal Field Theories and Supergravity, hep-th/9911200.

[15] A.Hashimoto and N.Itzhaki,  
Noncommutative Yang-Mills and the AdS/CFT correspondence, hep-th/9907166.

[16] J.Maldacena and J.G.Russo,  
Large $N$ Limit of Non-Commutative Gauge Theories, hep-th/9908134.

[17] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, T. Tada and A. Tsuchiya,  
IIB Matrix Model, hep-th/9908038.

[18] I.Chepelev, Y.Makeenko and K.Zarembo,  
Properties of D-Branes in Matrix Model of IIB Superstring, hep-th/9701151.

[19] A.Fayyazuddin and D.J.Smith, P-brane solutions in IKKT IIB matrix theory, hep-th/9701168.

[20] I.Chepelev and A.A.Tseytlin,  
Interactions of type IIB D-branes from D-instanton matrix model, hep-th/9705120.

[21] M.R.Douglas, Branes within Branes, hep-th/9512077.

[22] M.S.Costa, Black hole dynamics from instanton strings, hep-th/9807183.

[23] W.Taylor, Lectures on D-branes, Gauge Theory and M(trices), hep-th/9801182.

[24] N.Nekrasov and A.Schwarz,  
Instantons on noncommutative $R^4$, and (2,0) superconformal six dimensional theory, hep-th/9802068.

[25] A.S.Schwarz,  
On Regular Solutions of Euclidean Yang-Mills Equations,  
Phys. Lett. B67, 172.  
Instantons and Fermions in the Field of Instanton, Comm. Math. Phys. 64, 223.
[26] J.G.Russo and A.A.Tseytlin,  
\textit{Waves, boosted branes and BPS states in M-theory}, \texttt{hep-th/9611047}.  
J.C.Breckenridge, G.Michaud and R.C.Myers,  
\textit{More D-brane bound states}, \texttt{hep-th/9611174}.  

[27] S.Minwalla, M.V.Raamsdonk and N.Seiberg,  
\textit{Noncommutative Perturbative Dynamics}, \texttt{hep-th/9912072}.  

[28] S.Iso and H.Kawai, \textit{Space-time and matter in IIB matrix model}, \texttt{hep-th/9903217}.