DETUNED RANDALL-SUNDRUM MODEL: RADION STABILIZATION AND SUPERSYMMETRY BREAKING *

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In this talk I describe the low energy effective theory of the (supersymmetric) Randall-Sundrum scenario with arbitrary brane tensions. The distance between the branes is stabilized, at the classical level, by a potential for the radion field. In the supersymmetric case, supersymmetry can be broken by a VEV for the fifth-component of the graviphoton.

1. Introduction

Warped compactifications offer a completely new perspective on the hierarchy problem. In the Randall-Sundrum (RS) scenario [1], five-dimensional AdS space is compactified on an orbifold $S^1/Z_2$ with two opposite tension branes located at the orbifold fixed points. The ratio between the Planck mass and the electro-weak scale can be explained by the gravitational redshift of the metric along the fifth-dimension. The hierarchy problem is rephrased in terms of the distance between the branes which is a modulus of the compactification. A stabilization mechanism is necessary.

In this talk I will consider a generalized version of the RS model with "detuned" brane tensions [2]. An intriguing new feature of this scenario is the fact that the brane tensions fix the distance between the branes in the vacuum. I will present the low energy effective action for the radion for arbitrary tensions. I derive the results in the supersymmetric version of the

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model, where the low energy dynamics is controlled by an \( N = 1 \) supersymmetric \( \sigma \)–model. The low energy effective action includes a potential for the radion which stabilizes the size of the extra-dimension. I will also argue that in the detuned scenario supersymmetry can be broken spontaneously by a non-trivial Wilson line for the graviphoton. The effective theory in the non-supersymmetric scenario can be easily obtained from the supersymmetric case. The material presented here is based on the papers [3],[4] in collaboration with J. Bagger.

2. The detuned Randall-Sundrum model

The supersymmetric version of the model was constructed by Bagger and Belyaev [5]. The action is given by minimal 5D supergravity supplemented by brane actions. Neglecting the Chern-Simons term, the bosonic part of the action is simply

\[
S_{\text{bulk}} = -\frac{T}{6k} \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-G} \left( \frac{1}{2} R - 6k^2 + \frac{1}{4} F_{MN} F^{MN} \right) \\
S_{\text{brane}} = -T_0 \int d^4x \sqrt{-g_0} - T_\pi \int d^4x \sqrt{-g_\pi},
\]

(1)

where \( F_{MN} \) is the field strength of a \( U(1) \) gauge field, \( B_M \), called the graviphoton. I work in the orbifold covering space with the branes located at \( \phi = 0 \) and \( \phi = \pi \). Supersymmetry requires the tensions to satisfy the bound \( T_0 \leq T \leq T_\pi \) [5]. The full bulk-plus-brane theory is invariant under five-dimensional \( N = 2 \) supersymmetry in the bulk, restricted to four-dimensional \( N = 1 \) supersymmetry on the branes. This guarantees that the effective theory is \( N = 1 \) supersymmetric.

For generic tensions satisfying the bound above, the ground state metric is four-dimensional AdS space warped along the fifth-dimension,

\[
ds^2 = F(\phi)^2 g_{mn} dx^m dx^n + r_0^2 d\phi^2.
\]

(2)

Here the metric \( g_{mn} \) is AdS\(_4\) with radius \( L \) and the warp factor is,

\[
F(\phi) = e^{-kr_0|\phi|} + \frac{1}{4k^2L^2} e^{kr_0|\phi|}.
\]

(3)

The radius \( L \) is related to the tensions,

\[
\frac{1}{4k^2L^2} = \frac{T - T_0}{T + T_0}.
\]

(4)
In contrast to the original RS scenario (which corresponds to the choice \( T_0 = -T_\pi = T \)), the radius \( r_0 \) of the extra-dimension is fixed,

\[
2\pi kr_0 = \log \left( \frac{T + T_0(T + T_\pi)}{(T - T_0)(T - T_\pi)} \right)
\]

(5)

However the VEV of \( B_5 \) is not determined; it is the only modulus of the compactification.

3. Supersymmetric effective action

The general form of the effective action is determined by the symmetries of the five-dimensional theory up to four free constants.

The bosonic low energy effective action includes the fluctuations of the four-dimensional metric \( g_{nn} \), together with the light modes of \( G_{55} \) and \( B_5 \). The scalar associated with \( G_{55} \) can be identified as the proper distance between the branes, the radion field. The other scalar is obviously the zero mode of \( B_5 \). In the supersymmetric effective theory the two scalars join with the fifth-component of the gravitino to form a chiral multiplet. The zero mode of \( B_5 \) must be massless. The Kaluza-Klein reduction fixes the mass of the radion to be \( 4/L^2 \). This is precisely the value of the mass required by the representations of supersymmetry in AdS\(_4\).

The effective action is \( N = 1 \) supersymmetric, so it is determined by a Kähler potential \( K \) and a superpotential \( P \). The bosonic part of the action (setting \( M_4 = 1 \)) takes the form

\[
S_{\text{eff}} = -\int d^4x \sqrt{-g} \frac{1}{2} R + K_{\tau} g^{mn} \partial_m \tau \partial_n \overline{\tau} + e^K (K^{\tau} D_\tau P D_{\overline{\tau}} - 3P \overline{P}),
\]

(6)

where \( \tau \) is the lowest component of the radion superfield, and \( D_\tau P = \partial_\tau P + K_\tau P \). It can be shown that \( \tau = r + ib \) where \( r \) is the radion field and \( b \) is the zero mode of \( B_5 \).

To determine \( K \) and \( P \), one can observe that the bosonic part of the action (1) is invariant under a shift of \( B_5 \). This implies that, up to a Kähler transformation, \( K \) is a function of \( \tau + \overline{\tau} \) (it does not contain \( b \)). By the same argument, the potential in (6) is also a function of \( \tau + \overline{\tau} \). Since the superpotential is a holomorphic function of \( \tau \), this condition imply an infinite number of constraints on \( K \) and \( P \). We found that, in an AdS\(_4\)
The ground state, the most general solution of these constraints is

\[ K(\tau, \bar{\tau}) = -3 \log \left[ 1 - e^{-a(\tau + \bar{\tau})} \right] \]

\[ P(\tau) = p_1 + p_2 e^{-3a\tau}, \]

where \( p_1, p_2 \in \mathbb{C} \) and \( c, a \in \mathbb{R} \) are undetermined constants.

With a simple change of variables one can recognize that this is the Kähler potential of no-scale supergravity. In fact the superpotential is the generalization to AdS\(_4\) of the constant superpotential of ordinary no-scale supergravity [4].

4. Results

The unknown constants in (7) can be determined performing the Kaluza-Klein reduction of the bosonic fields. This requires a careful treatment of the tadpoles of the light fields with the heavy fields [4]. One finds that the Kähler potential and the superpotential for the radion are given by

\[ K(\tau, \bar{\tau}) = -3 \log \left[ 1 - e^{-\pi k(\tau + \bar{\tau})} \right] \]

\[ P(\tau) = \frac{k}{L} \sqrt{\frac{6}{T}} \left( 1 - e^{\pi kr_0} e^{-3\pi k\tau} \right). \]

The bosonic part of the action is then

\[ S_{\text{eff}} = -\int d^4x \left[ \frac{M_4^2}{2} R + 3k^2\pi^2 M_4^2 \frac{e^{-k\pi(\tau + \bar{\tau})}}{(1 - e^{-k\pi(\tau + \bar{\tau})})^2} g^{mn} \partial_m \tau \partial_n \bar{\tau} + V(\tau, \bar{\tau}) \right], \]

where I have inserted the four-dimensional Planck mass \( M_4 \). The scalar potential is

\[ V(\tau, \bar{\tau}) = -\frac{3 M_4^2}{L^2} \left( 1 - e^{-2k\pi r_0} \right) \left[ 1 - e^{-2k\pi(\tau + \bar{\tau} - r_0)} \right] \left( 1 - e^{-k\pi(\tau + \bar{\tau})} \right)^2. \]

As required, the potential is independent of \( b \). The ground state is AdS\(_4\); the potential stabilizes the radius of the extra-dimension (at \( r = r_0 \)), while \( b \) remains, at the classical level, a modulus of the compactification.

The purely gravitational case can be obtained by setting \( r = 0 \) in (9). For the case \( T_0, \tau \geq T \) the 4D ground state is de-Sitter space and the 5D theory cannot be supersymmetrized. The effective action can be still

\(^a\)This holds for \( m^2 = 4/L^2 \)
obtained from the supersymmetric result (9) replacing $L \rightarrow iL$. The ground state is unstable in this case.

From the 5D point of view one can show that when $b$ has a VEV (corresponding to a non-trivial Wilson line of the graviphoton), the Killing spinor equations have no solutions; supersymmetry is spontaneously broken [3] (see also [6]. For a similar effect in flat space see [7] and references therein). This effect however vanishes when the tensions are tuned. In the effective theory, unbroken supersymmetry requires that $D_\tau P$ vanish, when evaluated at the minimum of the potential. It is easy to check using (8) that supersymmetry is broken when $b \neq 2n/(3k)$, for $n$ integer. This mechanism of supersymmetry breaking is the AdS$_4$ analog of the one in no-scale supergravity.

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