We propose a computationally efficient algorithm that combines compressed sensing with imitation learning to solve text-based games with combinatorial action spaces. Specifically, we introduce a new compressed sensing algorithm, named IK-OMP, which can be seen as an extension to the Orthogonal Matching Pursuit (OMP). We incorporate IK-OMP into a supervised imitation learning setting and show that the combined approach (Sparse Imitation Learning, Sparse-IL) solves the entire text-based game of Zork1 with an action space of approximately 10 million actions given both perfect and noisy demonstrations.

1 Introduction

Combinatorial action spaces pose a challenging problem for AI agents, both from a computational and from an exploratory point of view. The reason being that (i) finding the best action may require iterating over all actions, an exponentially hard task, and (ii) absent prior knowledge, finding the best action requires testing all actions multiple times at each state [Brafman and Tennenholtz, 2002]. While the exploratory task is of great importance, in this work we focus on the computational aspects of the problem. Our method can be seen as a natural application of the Action Assembly Theory (AAT) [Greene, 2008]. According to Greene, behavior is described by two essential processes: representation and processing. Representation refers to the way information is coded and stored in the mind, whereas processing refers to the mental operations performed to retrieve this information [Greene, 2008]. Having good representations of information and an efficient processing procedure allows us to quickly exploit highly rewarding nuances of an environment upon first discovery.

In this work we propose the first computationally efficient algorithm (see Figure 1), called Sparse Imitation Learning (Sparse-IL), which is inspired by AAT and combines imitation learning with a Compressed Sensing (CS) retrieval mechanism to solve text-based games with combinatorial action spaces. Our approach is composed of:

(1) Encoder - the encoder receives a state as input (Figure 1). The state is composed of individual words represented by word embeddings that were previously trained on a large corpus of text. We train the encoder, using imitation learning, to generate a continuous action $a_{SoE}$ (a dense representation of the action). The action $a_{SoE}$ corresponds to a sum of word embeddings of the action that the agent intends to take, e.g., the embedding of the action ‘take egg’ is the sum of the word embedding vectors of ‘take’ and ‘egg’. As the embeddings capture a prior, i.e., similarity, over language, it enables improved generalization and robustness to noise when compared to an end-to-end approach.

(2) Retrieval Mechanism - given a continuous vector $a_{SoE}$, we reconstruct the $K$ best Bag-of-Words (BoW) actions $a_{BoW}$, composed of up to $l = 4$ words, from the continuous output of the encoder. We do this using an algorithm that we term Integer K-Orthogonal Matching Pursuit (IK-OMP). We then use a fitness function to score the actions, after which, the best action is fed into a language model to yield an action sentence $a_{env}$ that can be parsed by the game.

Main contributions: We propose a computationally efficient algorithm called Sparse-IL that combines CS with imitation learning to solve natural language tasks with combinatorial action spaces. We show that IK-OMP, which we adapted from White et al. [2016] and Lin et al. [2013], can be used to recover a BoW vector from a sum of the individual word embeddings in a computationally efficient manner, even in the presence of significant noise. We demonstrate that Sparse-IL can solve the entire game of Zork1, for the first time, while considering a combinatorial action space of approximately 10 million actions, using noisy, imperfect demonstrations.

This paper is structured as follows: Section 2 details relevant related work. Section 3 provides an overview
of the problem setting; that is, the text-based game of Zork and the challenges it poses. Section 4 provides an overview of CS algorithms and, in particular, our variant called IK-OMP. Section 5 introduces our Sparse-IL algorithm. Finally, in Section 6 we present our empirical evaluations, which include experiments in the text-based game Zork1 highlighting the robustness of IK-OMP to noise and its computational efficiency and showcasing the ability of Sparse-IL solving both the ‘Troll Quest’ and the entire game of Zork.

2 Related work

Combinatorial action spaces in text-based games: Previous works have suggested approaches for solving text-based games [He et al. 2016; Yuan et al. 2018; Zahavy et al. 2018; Zelimak 2018; Tao et al. 2018]. However, these techniques do not scale to combinatorial action spaces. For example, [He et al. 2016] presented the DRRN, which requires each action to be evaluated by the network. This results in a total of $O(|A|)$ forward passes. [Zahavy et al. 2018] proposed the Action-Elimination DQN, resulting in a smaller action set $O(|A'|)$. However, this set may still be of exponential size.

CS and embeddings representation: CS was originally introduced in the Machine Learning (ML) world by Calderbank et al. [2009], who proposed the concept of compressed learning. That is, learning directly in the compressed domain, e.g. the embeddings domain in the Natural Language Processing (NLP) setting. The task of generating BoW from the sums of their word embeddings was first formulated by White et al. [2016]. A greedy approach, very similar to orthogonal matching pursuit (OMP), was proposed to iteratively find the words. However, this recovery task was only explicitly linked to the field of CS two years later in Arora et al. [2018].

![Figure 2: Zork1 example screen.](image)

3 Problem setting

Zork - A text-based game: Text-based games [Côté et al. 2018] are complex interactive games usually played through a command line terminal. An example of Zork1, a text-based game, is shown in Figure 2. In each turn, the player is presented with several lines of text which describe the state of the game, and the player acts by entering a text command. In order to cope with complex commands, the game is equipped with an interpreter which deciphers the input and maps it to in-game actions. For instance, in Figure 2 a command “climb the large tree” is issued, after which the player receives a response. In this example, the response explains that up in the tree is a collectible item - a jewel encrusted egg. The large, combinatorial action space is one of the main reasons Zork poses an interesting research problem. The actions are issued as free-text and thus the complexity of the problem grows exponentially with the size of the dictionary in use.

Our setup: In this work, we consider two tasks: the ‘Troll Quest’ [Zahavy et al. 2018] and ‘Open Zork’, i.e., solving the entire game. The ‘Troll Quest’ is a sub-task within ‘Open Zork’, in which the agent must enter the house, collect a lantern and sword, move a rug which reveals a trapdoor, open the trapdoor and enter the basement. Finally, in the basement, the agent encounters a troll which it must kill using the sword. An incorrect action at any stage may prevent the agent from reaching the goal, or even result in its death (termination).

In our setting, we consider a dictionary $D$ of 112 unique words, extracted from a walk-through of actions which solve the game, a demonstrated sequence of actions (sentences) used to solve the game. We limit the maximal sentence length to $l = 4$ words. Thus, the number of possible, unordered, word combinations are $d^l/l!$, i.e., the dictionary size to the power of the maximal sentence length, divided by the number of possible permutations. This results in approximately 10 million possible actions.

Markov Decision Process (MDP): Text-based games can be modeled as Markov Decision Processes. An MDP $M$ is defined by the tuple $(S, A, R, P)$ [Sutton and Barto 1998]. In the context of text-based games, the states $s$ are paragraphs representing the current observation. $a_{env} \in A$ are the available discrete actions, e.g., all combinations of words from the dictionary up to a maximal given sentence length $l$. $R : S \times A \times S \rightarrow \mathbb{R}$ is the bounded reward function, for instance collecting items provides a positive reward. $P : S \times A \times S \rightarrow [0, 1]$ is the transition matrix, where $P(s' \mid s, a_{env})$ is the probability of transitioning from $s$ to $s'$ given $a_{env}$ was taken.

Action Space: While the common approach may be to consider a discrete action space, such an approach may be infeasible to solve, as the complexity of solving the MDP is related to the effective action space size. Hence, in this work, we consider an alternative, continuous representation. As each action is a sentence composed of words, we represent each action using the sum of the embeddings of its tokens, or constitutive words, denoted by $a_{emb}$ (Sum of Embeddings). A simple form of embedding is the BoW, it represents the word using a one-hot vector the size of the dictionary in which the dictionary index of the word is set to 1. Aside from the BoW embedding, there exist additional forms of embedding vectors. For instance, Word2vec and GloVe, which encode the similarity between words (in terms of cosine distance). These embeddings are pre-trained using unsupervised learning techniques and similarly to how convolutional neural networks enable generalization across similar states, word embeddings enable generalization across similar sentences,
i.e., actions. In this work, we utilize GloVe embeddings, pre-trained on the Wikipedia corpus. We chose GloVe over Word2vec, as there exist pre-trained embeddings in low dimensional space. The embedding space dimensionality is $m = 50$, significantly smaller in dimension than the size $d$ of the dictionary $D$, 112 in our experiments. Given the continuous representation of an action, namely the sum of embeddings of the sentence tokens $a_{\mathrm{SoE}} \in \mathbb{R}^m$, the goal is to recover the corresponding discrete action $a_{\mathrm{env}}$, that is the tokens composing the sentence. These may be represented as a BoW vector $a_{\mathrm{BoW}} \in \mathbb{N}^\ell$. Recovering the sentence from $a_{\mathrm{BoW}}$ requires prior information on the language model.

Provided a set of words, the goal of a language model, the last element in Figure 1 a central piece in many important NLP tasks, is to output the most likely ordering which yields a grammatically correct sentence. In this paper, we use a rule based approach. Our rules are relatively simple. For example, given a verb and an object, the verb comes before the object - e.g., 'sword', 'take' $\mapsto \text{take sword}$.

To conclude, we train a neural network $E_p(s)$ to predict the sum of embeddings $a_{\mathrm{SoE}}$. Using CS (Section 4), we recover the BoW vector $R(a_{\mathrm{SoE}}) = a_{\mathrm{BoW}}$, i.e., the set of words which compose the sentence. Finally, a language model $M$ converts $a_{\mathrm{BoW}}$ into a valid discrete-action, namely $M(a_{\mathrm{SoE}}) = a_{\mathrm{env}}$. The combined approach is as follows: $a_{\mathrm{env}} = M(R(E_p(s)))$.

4 Compressed sensing

This section provides some background on CS and sparse recovery, including practical recovery algorithms and theoretical recovery guarantees. In particular, we describe our variant of one popular reconstruction algorithm, OMP, that we refer to as Integer K-OMP (IK-OMP). The first modification allows exploitation of the integer prior on the sparse vector $a_{\mathrm{SoE}}$ and is inspired by White et al. [2016] and Sparrer and Fischer [2015]. The second mitigates the greedy nature of OMP using beam search [Lin et al., 2013]. In Section 4.1, we experimentally compare different sparse recovery methods and demonstrate the superiority of introducing the integer prior and the beam search strategy.

4.1 Sparse Recovery

CS is concerned with recovering a high-dimensional $p$-sparse signal $x \in \mathbb{R}^d$ (the BoW vector $a_{\mathrm{BoW}}$ in our setting) from a low dimensional measurement vector $y \in \mathbb{R}^m$ (the sum of embeddings $a_{\mathrm{SoE}}$). That is, given a dictionary $D \in \mathbb{R}^{m \times d}$:

$$\min ||x||_0 \text{ subject to } Dx = y.$$ (1)

To ensure uniqueness of the solution of (1), the sensing matrix, or dictionary, $D$ must fulfill certain properties. These are key to provide practical recovery guarantees as well. Well known such properties are the spark, or Kruskal rank [Donoho and Elad, 2003], and the Restricted Isometry Property (RIP) [Candes and Tao 2005]. Unfortunately, these are typically as hard to compute as solving the original problem (1). While the mutual-coherence (see Definition 1) provides looser bounds, it is easily computable. Thus, we focus on mutual-coherence based results and note that Spark and RIP based guarantees may be found in Elad [2010].

Definition 1 (Elad [2010] Definition 2.3) The mutual coherence of a given matrix $D$ is the largest absolute normalized inner product between different columns from $D$. Denoting the $k$-th column in $D$ by $d_k$, it is given by $\mu(D) = \max_{i \neq j} |d_i^T d_j| / \|d_i\| \cdot \|d_j\|$.

The mutual-coherence characterizes the dependence between columns of the matrix $D$. For a unitary matrix, columns are pairwise orthogonal, and as a result, the mutual-coherence is zero. For general matrices with more columns than rows ($m < d$), as in our case, $\mu$ is necessarily strictly positive, and we desire the smallest possible value so as to get as close as possible to the behavior exhibited by unitary matrices [Elad, 2010]. This is illustrated in the following uniqueness theorem.

Theorem 1 (Elad [2010] Theorem 2.5) If a system of linear equations $Dx = y$ has a solution $x$ obeying $p < \frac{1}{2} \left( 1 + \frac{1}{\mu(D)} \right)$, where $p = ||x||_0$, this solution is the sparsest possible.

We now turn to discuss practical methods to solve (1).

4.2 Recovery Algorithms

The sparse recovery problem (1) is non-convex due to the $\ell_0$-norm. Although it may be solved via combinatorial search, the complexity is exponential in the dictionary dimension $d$, and it has been proven that (1) is, in general, NP-Hard [Elad, 2010].

One approach to solve (1), basis pursuit, relaxes the $\ell_0$-minimization to its $\ell_1$-norm convex surrogate,

$$\min ||x||_1 \text{ s.t. } Dx = y.$$ (2)

In the presence of noise, the condition $Dx = y$ is replaced by $||Dx - y||_2 \leq \epsilon$. The Lagrangian relaxation of this quadratic program is written, for some $\lambda > 0$, as $\min ||x||_1 + \lambda ||y - Dx||_2$, and is known as basis pursuit denoising (BPDN).

The above noiseless and noisy problems can be respectively cast as linear programming and second order cone programming problems [Chen et al., 2001]. They thus may be solved using techniques such as interior-point methods [Ben-Tal and Nemirovski, 2001, Boyd and Vandenberghe 2004]. Large scale problems involving dense sensing matrices often precludes the use of such methods. This motivated the search for simpler gradient-based algorithms for solving (2), such as fast iterative shrinkage-thresholding algorithm (FISTA) [Beck and Teboulle, 2009].

Alternatively, one may use greedy methods, broadly divided into matching pursuit based algorithms, such as OMP [Blumensath and Davies, 2008], and thresholding based methods, including iterative hard thresholding [Blumensath and Davies 2009]. The popular OMP algorithm, proceeds by iteratively finding the dictionary column with the highest correlation to the signal residual,
computed by subtracting the contribution of a partial estimate of \( \mathbf{x} \) from \( \mathbf{y} \). The coefficients over the selected support set are then chosen so as to minimize the residual error. A typical halting criterion compares the residual to a predefined threshold.

4.3 Recovery Guarantees

Performance guarantees for both \( \ell_1 \)-relaxation and greedy methods have been provided in the CS literature. In noiseless settings, under the conditions of Theorem 1, the unique solution of (1) is also the unique solution of (2) [Elad 2010, Theorem 4.5]. Under the same conditions, OMP with halting criterion threshold \( \epsilon = 0 \) is guaranteed to find the exact solution of (1) [Elad 2010, Theorem 4.3]. More practical results are given for the case where the measurements are contaminated by noise [Donoho et al. 2006, Elad 2010].

4.4 Integer K-OMP (IK-OMP)

Algorithm 1 IK-OMP

Input: Measurement vector \( \mathbf{y} \in \mathbb{R}^m \), dictionary \( \mathbf{D} \in \mathbb{R}^{m \times d} \), maximal number of characters \( L \) and beam width \( K \).

Initial solutions \( \mathbf{X}^0 = [0_d, \ldots, 0_d] \)

for \( l = 1, L \) do

for \( i \in [1, \ldots, K] \) do

Extend: Append \( \mathbf{X}^{i-1}+1_j, \forall j \in [1, \ldots, d] \) to \( \mathbf{X}^{i-1} \)

Trim: \( \mathbf{X}^i = K \)-arg \( \min \) \( \mathbf{x}_i \in \mathbf{X}^{i-1} \| \mathbf{y} - \mathbf{D}\mathbf{x}_i \|_2^2 \)

return \( \mathbf{X}^L \)

An Integer Prior: While CS is typically concerned with the reconstruction of a sparse real-valued signal, in our BoW linear representation, the signal fulfills a secondary structure constraint besides sparsity. Its nonzero entries stem from a finite, or discrete, alphabet. Such prior information on the original signal appears in many communication scenarios [Candes et al. 2005, Axell et al. 2012], where the transmitted data originates from a finite set.

Beam Search OMP: As OMP iteratively adds atoms to the recovered support, the choice of a new element in an iteration is blind to its effect on future iterations. Therefore, any mistakes, particularly in early iterations, may lead to large recovery errors. To mitigate this phenomenon, several methods have been proposed to amend the OMP algorithm.

To decrease the greediness of the greedy addition algorithm (which acts similarly to OMP), [White et al. 2016] use a substitution based method, also referred as swapping [Andrle and Rebollo-Neira 2006] in the CS literature. Unfortunately, the computational complexity of this substitution strategy makes it impractical. Elad and Yavneh [2009] combine several recovered sparse representations, to improving denoising, by randomizing the OMP algorithm. However, in our case, the sum of embeddings \( \mathbf{a}_{\text{BoW}} \) represents a true sparse BoW vector \( \mathbf{a}_{\text{BoW}} \), so that combining several recovered vectors should not lead to the correct solution.

IK-OMP: We combine the integer-prior with the beam search strategy, and propose the IK-OMP (Algorithm 1). In the algorithm description, \( 1_j \) is the vector with a single nonzero element at index \( j \) and \( K \)-arg \( \min \) denotes the \( K \) elements with smallest value for the following expression. In this work, the selected BoW is the candidate which minimizes the reconstruction score.

5 Imitation Learning

In this section, we present our Sparse-IL algorithm and provide in-depth details regarding the design and implementation of each of its underlying components. We also detail the experiments of executing Sparse-IL on the entire game of Zork.

Sparse Imitation Learning: Our Sparse-IL architecture is composed of two major components - Encoder \( E_{\theta}(s) \) and Retrieval Mechanism (as seen in Figure 1). Each component has a distinct role and combining them together enables for a computationally efficient approach.

The Encoder (\( E \)) is a neural network trained to output the optimal action representation at each state. As we consider the task of imitation learning, this is performed by minimizing the \( \ell_2 \) loss between the Encoder’s output \( E_{\theta}(s) \) and the embedding of the action provided by the expert \( a_{\text{SoE}} \).

In all of the learning experiments, the architecture we use is a convolutional neural network (CNN) that is suited to NLP tasks [Kim 2014]. Due to the structure of the game, there exist long term-dependencies. Frame-stacking, a common approach in games [Mnih et al. 2015], tackles this issue by providing the network with the \( N \) previous states. For the “Open Zork” task, we stack the previous 12 states, whereas for the “Troll Quest” we only provide it with the current frame.

Retrieval Mechanism (\( R \)): The output of the Encoder, \( E_{\theta}(s) \), is fed into a CS algorithm, such as IK-OMP. IK-OMP produces \( K \) candidate actions, \( a_{\text{BoW}1}, \ldots, a_{\text{BoW}K} \). These actions are fed into a fitness function which ranks them, based on the reconstruction score \( \| E_{\theta}(s) - Da_{\text{BoW}i} \|_2 \), \( i = 1, \ldots, k \) (see Section 4), and returns the optimal candidate. Other CS approaches, e.g., OMP and FISTA, return a single candidate action.

6 Experiments

In this section, we present our experimental results. We begin by analyzing our proposed CS method, namely IK-OMP, in Section 6.1, and its ability to reconstruct the action when provided the sum of word embeddings \( a_{\text{SoE}} \). After evaluating our proposed method in a clean and analyzable scenario, we evaluate the entire system ‘Sparse Imitation Learning’ on the full game of Zork (Section 6.2).

6.1 Compressed Sensing

In this section, we focus on comparing several CS approaches. To do so, we follow the set of commands, extracted from a walk-through of the game, required to solve Zork1, both in the ‘Troll Quest’ and ‘Open Zork’ domains. In each state \( s \), we take the ground-truth action \( a_{\text{exp}}(s) \), calculate the sum of word embeddings \( a_{\text{SoE}}(s) \), add noise and test the ability of various CS methods to
We compare 4 CS methods: the FISTA implementation of BP, OMP, IK-OMP (Algorithm 1) and a Deep Learning variant we deem DeepCS described below. The dictionary is composed of $d = 112$ possible words which can be used in the game. The dimension of the embedding is $m = 50$ (standard GloVe embedding available online) and the sentence length is limited to at most 4 words. This yields a total number of $\approx 10$ million actions, from which the agent must choose one at each step. It is important to note that while accuracy and reward might seem similar, an inaccurate reconstruction at an early stage results in an immediate failure, even when the accuracy over the entire trajectory seems high.

Clearly, as seen from Figure 3, OMP fails to reconstruct the true BoW vectors $a_{\text{BoW}}$, even in the noiseless scenario. Indeed, the mutual-coherence (Definition 1) is $\mu = 0.97$ and from Theorem 1 there is no guarantee that OMP can reconstruct a sparse vector for any sparsity $p > 0$. However, our suggested approach, IK-OMP, is capable of correctly reconstructing the original action $a_{\text{BoW}}$, even in the presence of relatively large noise. This gives evidence that the integer prior, in particular, and the beam search strategy significantly improve the sparse reconstruction $a_{\text{env}}(s)$. We compare the run-time (Table 1), and the reconstruction accuracy (number of actions reconstructed correctly) and reward gained in the presence of noise (Figure 5). Specifically, the measured action is $a_{\text{mes}}(s) = a_{\text{BoW}}(s) + \epsilon$, where $\epsilon \sim N(0, 1)$ is normalized based on the signal to noise ratio (SnR).

Table 1: Runtime comparison.

| Algorithm  | Runtime |
|------------|---------|
| OMP        | 0.008   |
| IOMP, K=1  | 0.008   |
| IK-OMP, K=3| 0.021   |
| IK-OMP, K=20| 0.166 |
| IK-OMP, K=112| 1.116 |
| FISTA      | 0.881   |
| DeepCS     | 0.347   |

We compare 4 CS methods: the FISTA implementation of BP, OMP, IK-OMP (Algorithm 1) and a Deep Learning variant we deem DeepCS described below. The dictionary is composed of $d = 112$ possible words which can be used in the game. The dimension of the embedding is $m = 50$ (standard GloVe embedding available online) and the sentence length is limited to at most 4 words. This yields a total number of $\approx 10$ million actions, from which the agent must choose one at each step. It is important to note that while accuracy and reward might seem similar, an inaccurate reconstruction at an early stage results in an immediate failure, even when the accuracy over the entire trajectory seems high.

Clearly, as seen from Figure 3, OMP fails to reconstruct the true BoW vectors $a_{\text{BoW}}$, even in the noiseless scenario. Indeed, the mutual-coherence (Definition 1) is $\mu = 0.97$ and from Theorem 1 there is no guarantee that OMP can reconstruct a sparse vector for any sparsity $p > 0$. However, our suggested approach, IK-OMP, is capable of correctly reconstructing the original action $a_{\text{BoW}}$, even in the presence of relatively large noise. This gives evidence that the integer prior, in particular, and the beam search strategy significantly improve the sparse reconstruction $a_{\text{env}}(s)$. We compare the run-time (Table 1), and the reconstruction accuracy (number of actions reconstructed correctly) and reward gained in the presence of noise (Figure 5). Specifically, the measured action is $a_{\text{mes}}(s) = a_{\text{BoW}}(s) + \epsilon$, where $\epsilon \sim N(0, 1)$ is normalized based on the signal to noise ratio (SnR).

Table 1: Runtime comparison.

| Algorithm  | Runtime |
|------------|---------|
| OMP        | 0.008   |
| IOMP, K=1  | 0.008   |
| IK-OMP, K=3| 0.021   |
| IK-OMP, K=20| 0.166 |
| IK-OMP, K=112| 1.116 |
| FISTA      | 0.881   |
| DeepCS     | 0.347   |
recovery performance.

**Deep Compressed Sensing:** Besides traditional CS methods, it is natural to test the ability of deep learning methods to perform such a task. Here, we train a neural network to predict the BoW vector $\mathbf{a}_{\text{BoW}}$ which composes the continuous embedding vector. Our network is a multi layer perceptron (MLP), composed of two hidden layers, 100 neurons each. We use a sigmoid activation function to bound the outputs to $[0,1]$ and train the network using a binary cross entropy loss. In these experiments, we denote by $T$ the threshold above which an output is selected, e.g., when $T = 0.9$ all words which receive a weight of above 0.9 are selected.

Our results (Figure 6) show that the DeepCS approach works when no noise is present, however, once noise is added to the setup, it is clear that DeepCS performs poorly compared to classic CS methods such as IK-OMP. We observed similar results in the Troll domain. Besides, as DeepCS requires training a new model for each domain, it is data-specific and does not transfer easily, which is not the case with traditional CS methods.

![Open Zork](image)

**Figure 6:** Compressed Sensing - DeepCS: Comparison of the accuracy, and accumulated reward, of the DeepCS baselines, compared to the IK-OMP approach.

**6.2 Imitation Learning**

In an imitation learning setup, we are given a data set of state-action pairs $(s, a_{\text{env}})$, provided by an expert; the goal is to learn a policy that achieves the best performance possible. We achieve this by training the embedding network $E_\theta(s)$ to imitate the demonstrated actions in the embedding space, namely $a_{\text{SoE}}$, at each state $s$, using the MSE between the predicted actions and those demonstrated. We consider three setups: (1) Perfect demonstrations, where we test errors due to architecture capacity and function approximation, (2) Gaussian noise, $a_{\text{mes}}(s) = a_{\text{SoE}}(s) + \epsilon$ (See Section 6.1), and (3) discrete-action noise, in which a random incorrect action is demonstrated with probability (w.p.) $p$. This experiment can be seen as learning from demonstrations provided by an ensemble of sub-optimal experts.

Our results (Figure 5) show that by combining CS with imitation learning techniques, we are capable of solving the entire game of Zork1, even in the presence of discrete-action noise. In all our experiments, IK-OMP outperforms the various baselines, including the end-to-end approach DeepCS-2 which is trained to predict the BoW embedding $\mathbf{a}_{\text{BoW}}$ directly from the state $s$.

**Training:** Analyzing the training graphs presents an interesting picture. It shows that during the training process, the output of the Encoder can be seen as a noisy estimation of $a_{\text{SoE}}$. As training progresses, the effective SNR of the noise decreases which is seen by the increase in the reconstruction performance.

**Generalization:** In Figure 6 we present the generalization capabilities which our method Sparse-IL enjoys, due to the use of pre-trained unsupervised word embeddings. The heatmap shows two forms of noise. The first, as before, is the probability of receiving a bad demonstration, an incorrect action. The second, synonym probability, is the probability of being presented with a correct action, yet composed of different words, e.g., drop, throw and discard result in an identical action in the environment and have a similar meaning. These results clearly show that Sparse-IL outperforms DeepCS-2 in nearly all scenarios, highlighting the generalization improvement inherent in the embeddings.

**The benefit of meaningful embeddings:** In our approach, the Encoder $E_\theta$ is trained to predict the sum-of-embeddings $a_{\text{SoE}}$. However, it can also be trained to directly predict the BoW vector $\mathbf{a}_{\text{BoW}}$. While this approach may work, it lacks the generalization ability which is apparent in embeddings such as GloVe, in which similar words receive similar embedding vectors.

Consider a scenario in which there are 4 optimal actions (e.g., ‘go north’, ‘walk north’, ‘run north’ and ‘move north’) and 1 sub-optimal action (e.g., ‘climb tree’). With probability 0.15 we are presented with one of the optimal actions and with probability 0.4 the sub-optimal action. In this example, the expected BoW representation would include ‘north’ w.p. 0.6, ‘climb’ and ‘tree’ w.p. 0.4, and the rest w.p. 0.15. On the other hand, since ‘go’, ‘walk’, ‘run’ and ‘move’ have similar meanings and in turn similar embeddings, the expected $a_{\text{SoE}}$ is much closer to the optimal actions than to the sub-optimal one and thus an imitation agent is less likely to make a mistake.

**7 Conclusion**

We have presented a computationally efficient algorithm called Sparse Imitation Learning (Sparse-IL) that combines CS with imitation learning to solve text-based games with combinatorial action spaces. We proposed a CS algorithm variant of OMP which we have called Integer K-OMP (IK-OMP) and demonstrated that it can deconstruct a sum of word embeddings into the individual BoW that make up the embedding, even in the presence of significant noise. In addition, IK-OMP is significantly more computationally efficient than the baseline CS techniques. When combining IK-OMP with imitation learning, our agent is able to solve Troll quest as well as the entire game of Zork1 for the first time. Zork1 contains a combinatorial action space of 10 million actions. Future work includes replacing the fitness function with a critic in order to further improve the learned policy as well as testing the capabilities of the critic agent in cross-domain tasks.
References

Miroslav Andrl and Laura Rebollo-Neira. A swapping-based refinement of orthogonal matching pursuit strategies. *Signal Processing*, 86(3):480–495, 2006.

Sanjeev Arora, Mikhail Khodak, Nikunj Saunshi, and Kiran Vodrahalli. A compressed sensing view of unsupervised text embeddings, bag-of-n-grams, and lstms. 2018.

Erik Axell, Geert Leus, Erik G Larsson, and H Vincent Poor. Spectrum sensing for cognitive radio: State-of-the-art and recent advances. *IEEE Signal Processing Magazine*, 29(3):101–116, 2012.

Amir Beck and Marc Teboulle. A fast iterative shrinkage-thresholding algorithm for linear inverse problems. *SIAM journal on imaging sciences*, 2(1):183–202, 2009.

Ahorn Ben-Tal and Arkadi Nemirovski. *Lectures on modern convex optimization: analysis, algorithms, and engineering applications*. 2nd edition, 2001.

Thomas Blumensath and Mike E Davies. Gradient pursuits. *IEEE Transactions on Signal Processing*, 56(6):2370–2382, 2008.

Thomas Blumensath and Mike E Davies. Iterative hard thresholding for compressed sensing. *Applied and computational harmonic analysis*, 27(3):265–274, 2009.

Stephen Boyd and Lieven Vandenberghe. *Convex optimization*. Cambridge university press, 2004.

Ronen I Brafman and Moshe Tennenholtz. R-max-a general polynomial time algorithm for near-optimal reinforcement learning. *Journal of Machine Learning Research*, 3(Oct):213–231, 2002.

Robert Calderbank, Sina Jafarpour, and Robert Schapire. Compressed learning: Universal sparse dimensionality reduction and learning in the measurement domain. *preprint*, 2009.

Emmanuel Candès, Mark Rudelson, Terence Tao, and Roman Vershynin. Error correction via linear programming. In *Foundations of Computer Science, 2005. FOCS 2005. 46th Annual IEEE Symposium on*, pages 668–681. IEEE, 2005.

Emmanuel J Candès and Terence Tao. Decoding by linear programming. *IEEE transactions on information theory*, 51(12):4203–4215, 2005.

Scott Shaobing Chen, David L Donoho, and Michael A Saunders. Atomic decomposition by basis pursuit. *SIAM review*, 43(1):129–159, 2001.

Marc-Alexandre Côté, Ákos Kádár, Xingdi Yuan, Ben Kybartas, Tavian Barnes, Emery Fine, James Moore, Matthew Hausknecht, Layla El Asri, Mahmoud Adada, et al. Textworld: A learning environment for text-based games. *arXiv preprint arXiv:1806.11532*, 2018.

David L Donoho and Michael Elad. Optimally sparse representation in general (nonorthogonal) dictionaries via 11 minimization. *Proceedings of the National Academy of Sciences*, 100(5):2197–2202, 2003.

David L Donoho, Michael Elad, and Vladimir N Temlyakov. Stable recovery of sparse overcomplete representations in the presence of noise. *IEEE Transactions on information theory*, 52(1):6–18, 2006.

Michael Elad. *Sparse and Redundant Representations: From Theory to Applications in Signal and Image Processing*. Springer Publishing Company, Incorporated, 1st edition, 2010. ISBN 144197010X, 9781441970107.

Michael Elad and Irad Yavneh. A plurality of sparse representations is better than the sparsest one alone. *IEEE Transactions on Information Theory*, 55(10):4701–4714, 2009.

John O Greene. Action assembly theory. *The International Encyclopedia of Communication*, 2008.

Ji He, Jianshu Chen, Xiaodong He, Jianfeng Gao, Lihong Li, Li Deng, and Mari Ostendorf. Deep reinforcement learning with a natural language action space. In *Proceedings of the 54th Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*, volume 1, pages 1621–1630, 2016.

Yoon Kim. Convolutional neural networks for sentence classification. *arXiv preprint arXiv:1408.5882*, 2014.

P. Lin, S. Tsai, and G. C. Chuang. A k-best orthogonal matching pursuit for compressive sensing. In *2013 IEEE International Conference on Acoustics, Speech and Signal Processing*, pages 5706–5709, 2013.

Volodymyr Mnih, Koray Kavukcuoglu, David Silver, Andrei A Rusu, Joel Veness, Marc G Bellemare, Alex Graves, Martin Riedmiller, Andreas K Fidjeland, Georg Ostrovski, et al. Human-level control through deep reinforcement learning. *Nature*, 2015.

Susanne Sparrer and Robert FH Fischer. Soft-feedback omp for the recovery of discrete-valued sparse signals. In *Signal Processing Conference (EUSIPCO), 2015 23rd European*, pages 1-461–1465. IEEE, 2015.

Richard S Sutton and Andrew G Barto. *Reinforcement learning: An introduction*, volume 1. MIT press Cambridge, 1998.

Ruo Yu Tao, Marc-Alexandre Côté, Xingdi Yuan, and Layla El Asri. Towards solving text-based games by producing adaptive action spaces. *arXiv preprint arXiv:1812.00855*, 2018.

Lyndon White, Roberto Togneri, Wei Liu, and Mohammed Bennamoun. Generating bags of words from the sums of their word embeddings. In *International Conference on Intelligent Text Processing and Computational Linguistics*, pages 91–102. Springer, 2016.

Xingdi Yuan, Marc-Alexandre Côté, Alessandro Sordoni, Romain Laroche, Remi Tachet des Combes, Matthew Hausknecht, and Adam Trischler. Counting to explore and generalize in text-based games. *arXiv preprint arXiv:1806.11525*, 2018.

Tom Zahavy, Matan Harosh, Nadav Merlis, Daniel J Mankowitz, and Shie Mannor. Learn what not to learn: Action elimination with deep reinforcement learning. *Advances in neural information processing systems (NeurIPS)*, 2018.

Mikuláš Zelinka. Using reinforcement learning to learn how to play text-based games. *arXiv preprint arXiv:1801.01999*, 2018.