CALIBRATION OF THE MID-INFRARED TULLY–FISHER RELATION

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ABSTRACT

Distance measures on a coherent scale around the sky are required to address the outstanding cosmological problems of the Hubble constant and of departures from the mean cosmic flow. The correlation between galaxy luminosities and rotation rates can be used to determine the distances to many thousands of galaxies in a wide range of environments potentially out to 200 Mpc. Mid-infrared (3.6 μm) photometry with the Spitzer Space Telescope is particularly valuable as a source of luminosities because it provides products of uniform quality across the sky. From a perch above the atmosphere, essentially the total magnitude of targets can be registered in exposures of a few minutes. Extinction is minimal and the flux is dominated by the light from old stars, which is expected to correlate with the mass of the targets.

In spite of the superior photometry, the correlation between mid-infrared luminosities and rotation rates extracted from neutral hydrogen profiles is slightly degraded from the correlation found with I-band luminosities. A color correction recovers a correlation that provides comparable accuracy to that available at the I band (~20% in an individual distance) while retaining the advantages identified above. Without color correction, the relation between linewidth and [3.6] magnitudes is $M_{\rm C(3.6)} - 20.34 - 9.74(\log W_{\rm max} - 2.5)$. This description is found with a sample of 213 galaxies in 13 clusters that define the slope and 26 galaxies with Cepheid or tip of the red giant branch distances that define the zero point. A color-corrected parameter $M_{\rm C(3.6)}$ is constructed that has reduced scatter: $M_{\rm C(3.6)} - 20.34 - 9.13(\log W_{\rm max} - 2.5)$. Consideration of the seven calibration clusters beyond 50 Mpc, outside the domain of obvious peculiar velocities, provides a preliminary Hubble constant estimate of $H_0 = 74 \pm 4$ km s$^{-1}$ Mpc$^{-1}$.

Key words: cosmological parameters – distance scale – galaxies: clusters: general – galaxies: distances and redshifts – galaxies: photometry – radio lines: galaxies

Online-only material: color figures, machine-readable table

1. INTRODUCTION

Soon after the discovery of the power-law correlation between the rotation rates of galaxies and their luminosities (Tully & Fisher 1977), it was suggested (Aaronson et al. 1979) that the methodology might be improved by moving to near-infrared bands, particularly when it is used to measure distances. Obscuration corrections within the hosts and due to our Galaxy are minimized and light from old stars, which peaks in the infrared, should optimally represent the baryonic mass that presumably couples to the rotation rate. However, progress with infrared observations of galaxies has been difficult because of the high and variable sky foreground at near-infrared wavelengths and overwhelming thermal emission at mid-infrared wavelengths with ground-based observations. The most serious modern attempts to use an infrared form of the correlation have drawn on the ground-based observations. The most serious modern attempts to use an infrared form of the correlation have drawn on the ground-based observations. The most serious modern attempts to use an infrared form of the correlation have drawn on the ground-based observations. The most serious modern attempts to use an infrared form of the correlation have drawn on the ground-based observations. The most serious modern attempts to use an infrared form of the correlation have drawn on the ground-based observations. The most serious modern attempts to use an infrared form of the correlation have drawn on the ground-based observations. The most serious modern attempts to use an infrared form of the correlation have drawn on the ground-based observations.

The situation changed dramatically with the launch of the Spitzer Space Telescope (Werner et al. 2004). With observations using the Infrared Array Camera (IRAC; Fazio et al. 2004), the “sky” is far reduced from observations on the ground, and is now dominated by diffuse zodiacal light and the stochastic distribution of background high-redshift galaxies. Imaging with of the order of four minute integrations in the [3.6] band with this facility permits area photometry at levels that reach slightly fainter than ground-based optical imaging with comparable exposures, i.e., to levels that include all but a few percent of the total light of a galaxy (Sorce et al. 2012a). In addition, and as a very important point, the photometry has consistent properties in all of the directions on the sky.

Real progress on this program had to await the exhaustion of cryogenics on the Spitzer Space Telescope. During the subsequent “warm” mission, observations have only been possible with the two shortest wavelength passbands for the facility, at 3.6 μm and 4.5 μm, and there has been an emphasis on large programs that can usefully work in these bands. This article results from a commonality of interests between two of these programs. One of these, initiated in the Spitzer proposal cycle 6, is the Carnegie Hubble Program (CHP). The intent of this program is to reduce systematics arising in the determination of the Hubble constant. Part of the CHP focuses on a mid-infrared calibration of the Cepheid period–luminosity relation, and a second part addresses the properties of the rotation rate–luminosity correlation of galaxies, the Tully–Fisher Relation (TFR). The two parts are related since the TFR zero point is established by the Cepheid distance measurements. Freedman et al. (2011) describe the goals of the CHP and Freedman et al. (2012) report on the results of the Cepheid calibration that gives a distance modulus for the Large Magellanic Cloud (LMC) of 18.48 ± 0.03. The second program, initiated in...
cycle 8, is Cosmic Flows with Spitzer (CFS). The goals in this case are to acquire distances to several thousand galaxies using the mid-infrared TFR in order to map deviations from the Hubble flow. The two programs use overlapping data from Spitzer and require a similar calibration of the rotation rate–luminosity correlation. This paper presents the calibration that will be used in subsequent work with both the CHP and CFS.

The ensuing discussion borrows heavily from the recent re-calibration of the $I$-band correlation by Tully & Courtois (2012, hereafter TC12). That paper outlines a strategy of forming a template relation using samples from 13 galaxy clusters and establishing a zero point using nearby galaxies with independent Cepheid period-luminosity or tip of the red giant branch (TRGB) distances. It turns out that [3.6] magnitudes now exist for a substantial majority of the same galaxies. In this paper, we use the same H$\alpha$ profile and inclination information as in the $I$-band calibration paper. The only significant difference is the replacement of mid-infrared with optical luminosities. It turns out that although the new photometry has high fidelity and the photometry correction terms are small, there is an intrinsic color term in the [3.6]-band TFR. Scatter in the relation is reduced with the application of a color correction. We conclude with an estimate of the Hubble constant.

2. DATA

2.1. Calibrators

The slope and zero-point calibrator samples are described in detail in TC12. The correlation slope is established from a template built from galaxies in 13 clusters. The only departure in terms of an extension from the $I$-band calibration occurs in the case of A2634. The CHP included observations of a larger region including A2666. The two clusters are close in projection and, evidently, in distance. We find no discernable difference in distance between the galaxies closest on the sky to A2634 versus those closest to A2666. We propose averaging over the entire complex.

Each cluster sample is comprised of galaxies that are likely to be at similar distances. We attempted to include all of the galaxies with suitable properties down to a defined faint luminosity level to have an unbiased sampling of the cluster volume to a magnitude limit. Candidates are chosen out of a projection-velocity window. We care about minimizing relative distance effects in the TFR, so it is more important to minimize interlopers than maximize true members. Cluster members that are “window outsiders” would not be expected to lie in any preferred part of the TF diagram. The selection criteria are the following: (1) morphological types earlier than Sa are excluded, (2) H$\alpha$ profiles with adequate signal to noise are required (see the next section), (3) no evidence of confusion or tidal disruption, and (4) inclinations inferior to 45° are rejected. Tests with samples that satisfy this limit have not revealed any distance bias with inferred inclinations (TC12). Criteria for the inclusion of zero-point calibrators are similar, with the additional requirement that they have very well known distances from either Cepheid or TRGB measurements. In our earlier papers, the Cepheid scale had been set by a distance modulus of the LMC of 18.50 (Freedman et al. 2001). Here, we adopt the slightly modified modulus 18.48 ± 0.03 based on mid-infrared photometry of Cepheids in the LMC and in our Galaxy, the latter anchored with trigonometric parallaxes (Monson et al. 2012). Our TRGB distances are based on a Population II calibration but have been demonstrated to be on a consistent scale (Rizzi et al. 2007; Tully et al. 2008).

The CFS program was only beginning when the current analysis was carried out. However, because of the overlap in interests with the CHP, a large fraction of the $I$-band calibrators used by TC12 have already been observed in the earlier Spitzer cycle for the same purpose of a TFR calibration, and most others have been observed serendipitously in other Spitzer programs. At this time, 230 of the 314 galaxies (73%) used in the $I$-band calibration (plus nine other galaxies introduced with the extension of the A2634 sample to include A2666) have Spitzer [3.6] photometry, including 26 of the 36 (72%) that set the zero point.

The completion is greater than 60% for 12 of the 13 template clusters (the Pisces filament is an exception). It is deemed appropriate to present a preliminary calibration with the available material. In a later section, there will be a review of the impact of the current completeness level on the small Malmquist bias that we make.

2.2. H$\alpha$ linewidths

The Cosmic Flows project has now analyzed H$\alpha$ profiles for over 14,000 galaxies in a consistent way, deriving a linewidth parameter $W_{50}$ with suitable precision (error estimate $\leq 20$ km s$^{-1}$) for over 11,000 galaxies (Courtois et al. 2009, 2011b). This parameter is a measure of the H$\alpha$ profile width at 50% of the mean flux within the velocity range encompassing 90% of the total H$\alpha$ flux. The newly measured H$\alpha$ profiles of thousands of galaxies are available for public use at the Extragalactic Distance Database (EDD) Web site. This observed parameter $W_{50}$ is transformed into the more physically motivated parameter $W_{\text{max}}$ through steps that are justified in Courtois et al. (2009, 2011b) and reviewed by TC12. $W_{\text{max}}$ statistically approximates twice the maximum rotation velocity of a galaxy.

These transformations remove a slight relativistic broadening and a broadening due to finite spectral resolution, adjust to twice the projected maximum rotation velocity, and de-project to edge-on orientation. Linewidth error estimates are based on the level of the signal, $S$, at 50% of the mean flux divided by the noise, $N$, measured beyond the extremities of the signal. Profiles with error estimates smaller than 20 km s$^{-1}$ are retained. These profiles meet a minimum flux per channel requirement of $S/N \geq 2$ and are accepted after visual inspection.

Uncertainties in the rotation rate parameter are illustrated in the error bars of the figures presented in the next section. It will be seen that errors in the linewidth parameter dominate observational uncertainties. Errors in the logarithmic linewidth parameter tend to be larger for slow rotators since a typical measurement uncertainty of 10–20 km s$^{-1}$ causes a larger fractional uncertainty with a narrow profile. The largest uncertainties are associated with more face-on galaxies, those toward the 45° cut-off. At this limit, a 5° error in inclination results in an 8% error in linewidth.

2.3. [3.6] Photometry

The photometric data have all been obtained with IRAC ch. 1, passband center 3.55 μm. CHP (Freedman et al. 2011) provides 60% of the data. In addition, S$^4$G, the Spitzer Survey of Stellar Structure in Galaxies (Sheth et al. 2010), gives 17%, and SINGS, the Spitzer Infrared Nearby Galaxies Survey (Dale et al. 2005, 2009).

5 http://edd.ifa.hawaii.edu; catalog “All Digital HI.”
to reach a surface brightness 26.5 mag arcsec$^{-2}$ in 240 s, occasionally 120 s). The integrations are sufficiently deep but the integration times are the same within a factor of 2 (mostly for galaxies. The information comes from a multitude of programs.

The new CFS program has only contributed 3% of the current data. Smaller programs during the warm Spitzer cryogenic phase, while the first two were conducted during the cryogenic mission. The new CFS and CHP averaged-corrected magnitude (mag). The photometric reductions were carried out by two independent procedures. The method utilized by the CHP uses software developed for the Galaxy Evolution Explorer Large Galaxy Atlas (M. Seibert et al. 2013, in preparation). The method developed in anticipation of the arrival of CFS data is based on the Archangel photometry package (Schombert 2007) described by Sorce et al. (2012a, hereafter SCT12) and earlier by Courtois et al. (2011a) in the context of optical photometry. In a comparison of 171 galaxies (SCT12), the 2 procedures result in an agreement at the level of 0.01 mag with an rms scatter of 0.052. Partitioned equally, the internal uncertainty (reductions of the same data by different methods and individuals) is ±0.037 mag. There are marginal differences for galaxies brighter than [3.6] = 11 (CHP brighter at the level of 0.03), which is probably attributable to sky settings. We choose to average over the CHP and CFS photometric values.

Uncertainties for apparent magnitudes have been shown to be very small, cumulatively ±0.05 (SCT12). The photometry reaches isophotal levels that require only a few percent extrapolation to give total magnitudes, and the scale is stable to better than 0.01 mag across the sky (IRAC Instrument Handbook V2.0, 2011). Setting the sky remains a dominant uncertainty at a level of 0.04 mag. IRAC ch.1 [3.6] luminosities receive the following corrections:

### Table 1
Calibrator Parameters

| PGC | Name | [3.6]ave | [3.6]CHP | Δ [3.6] | b/a | Inc | W$_{3.6}$ | W$_{5.8}$ | log(W$_{3.6}$) | Sam |
|-----|------|---------|---------|---------|-----|-----|---------|---------|----------|-----|
| 9332 | NGC 0925 | 10.866 | 10.231 | 10.549 | −1.589 | 0.57 | 57 | 194 | 231 | 2.364 | Zerp |
| 13179 | NGC 1365 | 8.818 | 8.812 | 8.815 | −0.725 | 0.61 | 54 | 371 | 459 | 2.662 | Zero |
| 13602 | NGC 1425 | 10.693 | 10.700 | 10.697 | −1.197 | 0.46 | 65 | 354 | 391 | 2.592 | Zero |
| 17819 | NGC 2090 | 10.477 | 10.287 | 10.382 | −1.052 | 0.43 | 67 | 277 | 301 | 2.478 | Zero |
| 21396 | NGC 2403 | 8.558 | 8.370 | 8.464 | −1.354 | 0.53 | 60 | 226 | 261 | 2.417 | Zero |
| 23110 | NGC 2541 | 11.949 | 11.949 | 11.949 | −0.773 | 0.57 | 57 | 254 | 303 | 2.481 | Zero |
| 26512 | NGC 2841 | 8.558 | 8.644 | 8.644 | −1.114 | 0.45 | 66 | 592 | 650 | 2.813 | Zero |
| 28120 | NGC 2976 | 9.904 | 9.210 | 9.209 | −0.879 | 0.70 | 47 | 262 | 359 | 2.556 | Zero |
| 30197 | NGC 3198 | 11.739 | 11.739 | 11.739 | −0.889 | 0.56 | 58 | 264 | 312 | 2.494 | Zero |
| 32207 | NGC 3370 | 11.750 | 11.750 | 11.750 | −1.630 | 0.40 | 69 | 106 | 113 | 2.054 | Zero |
| 32007 | NGC 3351 | 9.208 | 9.208 | 9.208 | −0.879 | 0.70 | 47 | 262 | 359 | 2.556 | Zero |
| 32207 | NGC 3370 | 11.739 | 11.739 | 11.739 | −0.889 | 0.56 | 58 | 264 | 312 | 2.494 | Zero |
| 34554 | NGC 3621 | 8.989 | 9.035 | 9.012 | −1.002 | 0.45 | 66 | 266 | 292 | 2.465 | Zero |
| 34695 | NGC 3627 | 8.314 | 8.254 | 8.284 | −0.894 | 0.53 | 60 | 333 | 385 | 2.585 | Zero |
| 37422 | NGC 4244 | 10.333 | 10.333 | 10.333 | −1.413 | 0.20 | 90 | 192 | 192 | 2.283 | Zero |
| 40692 | NGC 4414 | 9.368 | 9.368 | 9.368 | −0.638 | 0.60 | 55 | 378 | 463 | 2.666 | Zero |
| 41812 | NGC 4535 | 9.783 | 9.751 | 9.767 | −0.817 | 0.72 | 45 | 265 | 374 | 2.573 | Zero |
| 41823 | NGC 4536 | 9.840 | 9.856 | 9.848 | −0.818 | 0.38 | 71 | 322 | 341 | 2.533 | Zero |
| 42408 | NGC 4605 | 10.161 | 10.161 | 10.161 | −0.971 | 0.41 | 69 | 154 | 165 | 2.219 | Zero |
| 42510 | NGC 4603 | 10.682 | 10.663 | 10.673 | −0.913 | 0.64 | 52 | 353 | 450 | 2.653 | Zero |
| 42741 | NGC 4639 | 11.250 | 11.255 | 11.253 | −1.073 | 0.60 | 55 | 274 | 336 | 2.526 | Zero |
| 43451 | NGC 4725 | 8.922 | 8.893 | 8.908 | −1.068 | 0.56 | 58 | 397 | 470 | 2.672 | Zero |
| 51344 | NGC 5584 | 11.763 | 11.819 | 11.791 | −1.171 | 0.73 | 44 | 186 | 267 | 2.426 | Zero |
| 69327 | NGC 7331 | 8.409 | 8.377 | 8.393 | −0.873 | 0.44 | 66 | 501 | 547 | 2.738 | Zero |
| 73049 | NGC 7793 | 9.298 | 9.298 | 9.298 | −1.048 | 0.62 | 53 | 162 | 202 | 2.306 | Zero |

Notes.

a Principal Galaxies Catalog (PGC) number.

b Common name.

c CFS-corrected magnitude (mag).

d CHP-corrected magnitude (mag).

e CFS and CHP averaged-corrected magnitude (mag).

f Optical to mid-infrared color, AB mag.

g Axial ratio.

h Inclination (degrees).

i Linewidth not corrected for inclination (km s$^{-1}$).

j Linewidth corrected for inclination (km s$^{-1}$).

k Logarithm of the inclination-corrected linewidth.

l Sample.

(This table is available in its entirety in a machine-readable form in the online journal. A portion is shown here for guidance regarding its form and content.)
Figure 1. Tully–Fisher relation in the [3.6] band for the Virgo Cluster. The solid line gives the inverse fit of the universal template correlation. The dotted line is the inverse fit of the correlation for the Virgo Cluster alone.

1. $A_{[3.6]}^b$: galactic extinction (Cardelli et al. 1989; Schlegel et al. 1998),
2. $A_{[3.6]}^i$: internal extinction (Giovanelli et al. 1995, 1997b; Tully et al. 1998),
3. $A_{[3.6]}^k$: shift in the flux due to the Doppler effect (Oke & Sandage 1968; Huang et al. 2007),
4. $A_{[3.6]}^a$: extended emission from the point-spread function outer wings and from scattered diffuse emission across the IRAC focal plane (Reach et al. 2005).

These corrections are all discussed in SCT12. The resulting apparent magnitude in the AB system is

$$[3.6]_{b,i,k,a} = [3.6] - A_{[3.6]}^b - A_{[3.6]}^i - A_{[3.6]}^k + A_{[3.6]}^a. \quad (1)$$

The data that are used in the following discussion are collected in Table 1. This table, with the complete version provided in the online publication, includes the CFS and CHP total magnitudes, each including the four adjustments just described, and averages of the two methods. The table also gives inclination and linewidth information drawn from TC12 and color correction terms described in Section 3.3. The galaxies in Table 1 are either part of the zero-point calibration (sample ZeroPt) or members of a cluster contributing to the slope template.

3. [3.6]-BAND CALIBRATION

The TFR calibration requires the definition of a slope and the establishment of an absolute scale. The slope is the trickiest item because there is a correlation between its value and a form of Malmquist bias. Given two galaxies at the same distance with the same linewidth, the brighter galaxy might be chosen but not the fainter one. The potential bias depends on the slope of the correlation because with a relatively flat slope most intrinsically luminous galaxies lie above the correlation, while with a very steep slope these same galaxies tend to lie below the correlation. Consider a target for a distance measurement in the field that intrinsically lies above the assumed mean relation, the trend for distant galaxies if the relation is flat. With the distance measurement, the target is assigned the mean luminosity of the correlation at the target's linewidth, and so is given a distance that is too small. This bias has repeatedly been discussed at length, most recently by TC12. The salient point is that the so-called inverse relation (ITFR), the least-squares regression where errors are taken to be in linewidth only, gives results that are close to bias free. Willick (1994) pointed out that while in his experiments the ITFR bias was reduced by a factor of six from that incurred using the direct relation, a small bias remained because the sample selection was not made in the band he considered. We have the same problem. Our strategy is to use the ITFR and then evaluate the bias with simulations anticipating that, like with the $I$-band calibration, the effects will be small. The bias tests are discussed in a later section.

The calibration process has been described in detail by Tully & Pierce (2000) and TC12. With the $I$-band relation, there is no clear evidence for scatter due to a third parameter, but the situation at [3.6] is different. A color term is found and that matter will be discussed.

3.1. Relative Distances and ITFR Slope

The measurement of distances requires the hypothesis of a universal correlation. To begin, we make inverse fits to each of the clusters separately. The dotted lines in Figures 1 and 2...
illustrate the inverse fits of the TFR for each cluster. Slopes are quite similar between clusters. Slopes and their uncertainties are given for each cluster in Table 2. The individual fits are consistent with the soon to be derived best fit, and hence with the universal correlation hypothesis. As cluster distances increase, the faint luminosity limits increase. However, no dependence of the slope with distance is seen, as would be a marker of the Malmquist bias (we still make a tiny correction for bias to cluster moduli as described in Section 3.4).

The next step is to combine the 13 individual cluster correlations by vertical translations. The Virgo Cluster is used for reference. Each preliminary zero point from the individual fits provides us with a first estimate of the relative distance between the Virgo Cluster and the cluster in question.

Apparent magnitude zero points confirm that Virgo, Fornax, and Ursa Major are the closest clusters. Then come Antlia–Centaurus–Pegasus, then Hydra–Pisces–Cancer, and finally Coma and the three Abell clusters A1367, A400, and A2634/66. To establish the best universal slope and the best relative distances between clusters, we follow an iterative procedure. We initially consider the nearest three clusters because they are observed to comparable depths in intrinsic magnitude. The Fornax and Ursa Major Major magnitude scales are shifted according to the difference in zero point with respect to Virgo. A least-squares fit of the ITFR is made to this ensemble. The new slope is assumed in a fit to the three individual clusters, with only the zero point as a free parameter in each case. Given the new zero-point offsets, the cycle is repeated, leading to rapid convergence. This procedure is repeated with the addition of each distance group in turn. Again, the convergence is rapid.

We must stress that this procedure works because, following expectations, the slope of the ITFR is not affected by the magnitude level of truncation. This procedure would manifestly not work with the direct or bivariate relations where the slopes vary with the level of truncation. In the end, we obtain a slope of $-9.74 \pm 0.22$ for the template ITFR. Zero-point offsets with this “universal” slope are shown in Figure 3 and give relative distance moduli of clusters referenced to the Virgo Cluster. The

![Figure 3. Template Tully–Fisher relation in the [3.6] band obtained with data from 213 galaxies in 13 clusters. Offsets given with respect to the Virgo Cluster represent distance modulus differences between each cluster and Virgo. The solid line is a least-squares fit to all of the galaxies with errors entirely in line - image not available here.](image-url)
universal slope of the ITFR is displayed in Figure 3, as well as by the solid lines in Figures 1 and 2.

3.2. Zero Point and Absolute Distances

Presently, [3.6] photometry is available for 26 nearby galaxies with suitable morphologies, inclinations, and linewidths that also have well-measured distances from either the Cepheid period-luminosity or TRGB methodologies. These 26 are a subset of the 36 absolute calibrator galaxies used in the I-band calibration (TC12). Their luminosity–linewidth correlation is seen in Figure 4, where now the ordinate is absolute magnitudes from the established distances. The line is a least-squares fit with a slope $-9.74$ prescribed by the template. The zero point is $-20.34 \pm 0.10$. The most deviant point is the fastest rotator, NGC 2841, with a deviation of 2.7σ with respect to the template dispersion. This galaxy was a 2.3σ deviant in the I-band calibration. There is nothing unusual about this galaxy other than its extreme rotation rate, so we see no reason to disregard it as a calibrator.

The distance to the Virgo Cluster is given by the zero point of the constrained slope shown in Figure 3, minus the zero point of the absolute calibration shown in Figure 4. Application of this shift allows both cluster template and zero-point calibrator galaxies to be plotted together, as seen in Figure 5. The ITFR expression in the [3.6] band is given by

$$M_{[3.6]}^{b,i,k,a} = -(20.34 \pm 0.10) - (9.74 \pm 0.22)(\log W_{\text{mx}}^i - 2.5).$$

(2)

The TFR scatter in magnitudes (relevant for distance measurements) is given by

$$\sigma_{\text{TFR}} = \sqrt{\frac{\chi^2}{N - 1}},$$

(3)

where $\chi^2$ is the minimum of

$$\sum \left( M_i - (a + b(\log W_{\text{mx}}^i - 2.5)) \right)^2,$$

with $a$ and $b$ being the zero point and slope of the ITFR, respectively, and $N - 1$ are the degrees of freedom. The scatter for the entire cluster template sample is $\pm 0.49$ mag from the universal ITFR, corresponding to a scatter in distance of 25%. The scatter for the 26 zero-point calibrators is a similar 0.44 mag. Dispersion increases toward fainter magnitudes, as was well documented at the I band by Giovanelli et al. (1997a).

The sample presented here is still limited, but the dispersion is consistent with a Gaussian distribution. With large samples (Tully et al. 2008), one finds that about 3% of candidates are more deviant than anticipated by Gaussian statistics. The causes are not always evident.

Scatter may arise from (1) measurement uncertainties affecting magnitudes, inclinations, and linewidths; (2) correction uncertainties applied to measured parameters; and (3) “cosmic” scatter, e.g., cluster depth effects or interlopers, deviations from disk planarity, other gravitational and photometric asymmetries, variations in the stellar population make-up, variations in disk-to-bulge ratios, etc. Whatever the sources, we have a standard to meet that was set by the I-band analysis. The samples used in the current analysis involve 80% of the samples used in the I-band calibration (TC12). Inclinations and linewidths are the same, the factors mentioned associated with cosmic scatter are the same, corrections to photometric parameters are reduced in the mid-infrared, and the integrity of the magnitude measurements must be at least as good as or better than the Spitzer observations since observations are made all-sky with the same instrumental configuration. Error bars on magnitudes are reduced in Figures 1–5 compared with those on equivalent plots in the I-band calibration paper (TC12) to the degree that observational errors in magnitudes are a minimal component of uncertainties.
Figure 6. TFR in \(B\), \(R\), \(I\), and [3.6] bands. \(B\)- and \(R\)-bands data are from Tully & Pierce (2000), \(I\)-band data are from TC12, and [3.6]-band data are from SCT12. Linewidths are the same as used by TC12. The slopes steepen from blue to red, with values \(-7.27\) at \(B\), \(-7.65\) at \(R\), \(-8.81\) at \(I\), and \(-9.74\) at [3.6].

(A color version of this figure is available in the online journal.)

3.3. A Color Term

It has long been known that the TFR steepens toward longer wavelengths (Tully et al. 1982). The effect is seen in Figure 6. (Note: in the discussions in this section, all of the optical photometry values have been transferred from Vega to AB zero points.) There is a strong color correlation with linewidth, more rapidly rotating galaxies tend to be redder, so at longer wavelengths the high rotation end of the TFR rises with respect to the low rotation end. Within a small linewidth interval, redder galaxies will rise more than bluer galaxies. It follows that red and blue galaxies cannot be well mixed in the TFR at all wavelengths. The trends that could be anticipated are shown in Figure 7 (only a portion of the sample has photometric measurements at the \(B\) band). The comparison of fluxes at four bands from \(B\) to [3.6] for the individual sources given in Figure 8 confirms the well-known linkage between galaxy type and color. Early-type galaxies have relatively more infrared flux relative to late-type galaxies. This point was also illustrated with the representative spectral energy distribution plots in Figure 1 of SCT12. Galaxies that are more luminous and earlier in type are dominated by older, more metal-enriched red giant stars emitting more in the infrared.

It has long been suggested that the dispersion in the TFR might be reduced by including additional parameters. In an early instance (Rubin et al. 1985), when only photographic or photoelectric magnitudes were available, the case was framed in terms of galaxy types that are strongly correlated with color. Masters et al. (2006) have maintained the use of a type separation with \(I\)-band work. Tully & Pierce (2000) acknowledged the hint of a type dependence in the \(I\)-band relation but concluded that the evidence remained too weak to warrant adding complexity to the TFR analysis.

The situation changes with the mid-infrared information. In spite of superior photometry, the scatter in the TFR is increased and there is a significant color signature. The variations in the cluster template sample, lower with a significance of \(2\sigma\), and 0.36 mag for the zero-point calibrators. As much as half of the increase in magnitude scatter will occur because the slope of the correlation is steeper in the mid-infrared. However, there could be an additional explanation for the increased scatter found at [3.6].

Figure 7. Top three panels: deviations from the mean ITFR relation as a function of \(I - [3.6]\) color. Solid and dotted lines are best fits and 95% probability limits. Top: at the \(B\) band red galaxies tend to lie below the mean relationship. Top middle: at the \(I\) band there is a hint that red galaxies lie low although the correlation fit is dominated by a few extreme cases. Bottom middle: at the [3.6] band the sense of the correlation has flipped and red galaxies tend to lie above the mean relation. Bottom: the correlation between linewidth and color.
spectral energy distribution implicit in the range of representative colors shown in Figure 8 provide a natural explanation given the extended lever arm from the optical to the [3.6] band.

There is also the possibility that some flux in the [3.6] band may come from other than old stars. Meidt et al. (2012) determined that 12% ± 5% of the [3.6] flux arises from hot dust, polycyclic aromatic hydrocarbon emission, or young-to-intermediate age stars in six representative spiral galaxies observed with the Spitzer Space Telescope. However, the variance of 0.05 mag is small compared with the ITFR scatter. Moreover, it can be anticipated that the galaxies most affected by manifestations of star formation are later, bluer types, where the augmented flux will tend to diminish a color term arising from old stars.

Whatever the cause, it can be anticipated that the scatter can be decreased with the introduction of a color correction. To address this issue, we consider the straight line fits included in the top three panels of Figure 7. The fits are least-squares minimizations on the ordinate parameter: the difference in magnitude of a target from the mean TFR. The bottom panel shows the concordant variation of color with linewidth. Faster rotators tend to be redder.

In the mid-infrared case, the offset for an individual galaxy from the mean fit in the figure is

$$\Delta M_{[3.6]}^\text{color} = M_{b,i,k,a}^{[3.6]} + 20.34 + 9.74(\log W_{mx}^i - 2.5).$$  (4)

An equivalent correction can be constructed with apparent magnitudes rather than absolute magnitudes, $\Delta [3.6]^{\text{color}} = \Delta M_{[3.6]}^\text{color}$, with an appropriate replacement of the zero-point constant in Equation (4). The correction term commensurate with the fit in the third panel of Figure 7 is

$$\Delta [3.6]^{\text{color}} = - (0.47 \pm 0.11)(I - [3.6]) + 0.77.$$  (5)

We introduce a new color-adjusted magnitude parameter $C_{[3.6]} = [3.6]^{b,i,k,a} - \Delta [3.6]^{\text{color}}$, where the distinct nomenclature emphasizes the composite nature of this pseudo-magnitude. Next, the analysis discussed in Section 3.2 leading to the construction of Figure 4 is repeated. Likewise, the adjustments are applied to the calibrators with independently established distances and the procedures are repeated that lead to Figure 5. The adjusted relations are shown in Figure 9. The new correlation is described by the formula

$$M_{C_{[3.6]}} = - (20.34 \pm 0.08) - (9.13 \pm 0.22)(\log W_{mx}^i - 2.5).$$  (6)

The flattening of the adjusted relation comes about because redder systems move downward and redder galaxies tend to have larger linewidths. The overall magnitude scatter in the new relation is ±0.44 mag (corresponding to a scatter in distance of 22%), down from 0.49 mag before adjustment, and comparable with 0.41 found at the I band with an otherwise
comparable analysis (TC12). The comparable numbers for the zero-point calibrators alone are a scatter of 0.37, with the adjusted parameter $C_{[3.6]}$, 0.44 before the adjustment, and 0.36 at the $I$ band. The comparisons between $[3.6]$ and $I$ are somewhat imprecise because the sample sizes for the latter are 25% greater. The TFR parameters derived from alternative samples and bandpasses are summarized in Table 3.

### 3.4. Bias

Willick (1994) showed that a small Malmquist bias exists in the use of the ITFR, although reduced from the direct TFR by a factor of six in the situation he explored (Willick et al. 1995), reducing the bias reflected in the Hubble constant from 17% to 3%. The residual bias arises from two effects. First, sample selection departs from an idealized case of a flat magnitude limit because samples have been selected in blue bands and color terms translate into a slope in the limiting magnitude in the infrared: slower rotators which tend to be bluer are favored for inclusion over faster rotators which tend to be redder (see Figure 7, bottom). Second, the shape of the galaxy luminosity function contributes to the bias because there are more intrinsically fainter galaxies that scatter brightward through errors than intrinsically brighter galaxies that scatter faintward (Eddington 1913). The bias increases with distance as the effect of the exponential cutoff of the luminosity function plays an increasing role.

The amplitude of the bias from the two effects was explored with the calibration at the $I$ band (TC12). The situation now with the $[3.6]$ band sample is slightly worse than at $I$ because the wavelength interval from selection at $B$ is larger. The bias analysis carried out in the case of the $B$-band calibration is repeated here, tailored to the current situation. We first combine the Virgo, Fornax, and Ursa Major samples to improve statistics and include contributions from a range of environments. This ensemble is described by a Schechter (1976) function with a faint end slope $\alpha = -0.9$ and a bright end cutoff at $M_{[3.6]}^* = -22$. Then, we randomly populate an artificial TFR to match the observed $[3.6]$-band relation, drawing from the Schechter luminosity function. The faint limit is determined empirically to roughly obey the relation $M_{[3.6]}^{\text{lim}} = C_\ell - 2.70(\log W_{mx}^1 - 1.8)$, where $C_\ell$ couples with distance. The artificial TFR and the cutoff for the nearest clusters are shown in the top panel of Figure 10. The dashed blue line indicates the cutoff experienced at a distance modulus of 31. The cutoff, characterized by $C_\ell$, slides to brighter (more negative) magnitudes linearly with an increasing distance modulus. The bias $\langle \Delta M \rangle_{\text{measured}}$ is determined at intervals of $C_\ell$ corresponding to increasing distance. Here, $\langle \Delta M \rangle_{\text{measured}}$ is the average deviation from the fiducial relation where $\langle \Delta M \rangle_{\text{true}} = 0$ by construction. The growth of the bias as a function of cutoff magnitude is seen in the bottom panel of Figure 10. The solid curve, normalized to unity at a distance modulus $\mu = 31$ where even the faintest of useful candidates are included, is described by the formula

$$b = -0.0065(\mu - 31)^2.$$  

By comparison, the coefficient in the case of the $I$-band analysis is $-0.005$. The letters at the bottom of the figure are codes for the 13 calibrating clusters (see Table 4 for the translation of codes). Their horizontal positions indicate sample limits and vertical intercepts with the solid curve give the corresponding biases.

### Table 3

| Sample       | Ngal | Slope    | rms | Zero Point |
|--------------|------|----------|-----|------------|
| $I$ template | 267  | $-8.81 \pm 0.16$ | 0.41 | $\ldots$ |
| $I$ zero point | 36   | $\ldots$ | 0.36 | $-21.39 \pm 0.07$ (Veg) |
| $[3.6]$ template | 213 | $-9.74 \pm 0.22$ | 0.49 | $\ldots$ |
| $[3.6]$ zero point | 26  | 0.44 | $-20.34 \pm 0.10$ (AB) | $\ldots$ |
| $M_C$ template | 213 | $-9.13 \pm 0.22$ | 0.44 | $\ldots$ |
| $M_C$ zero point | 26  | 0.37 | $-20.34 \pm 0.08$ (AB) | $\ldots$ |

(A color version of this figure is available in the online journal.)
are recorded in Table 2 and are reflected in the adjusted cluster moduli and distances. For a galaxy in the field, the corrected distance modulus $\mu^c$ can be expressed as

$$\mu^c = (C_{[3.6]} - M_{C_{[3.6]}}) + 0.0065[(C_{[3.6]} - M_{C_{[3.6]}}) - 31]^2. \quad (8)$$

4. THE HUBBLE CONSTANT

The last column in Table 2 records the “Hubble parameter” for each cluster: the velocity of the cluster in the cosmic microwave background (CMB) frame divided by the measured distance. These quantities are plotted against distance in Figure 11. A similar figure was presented as a summary of results from the $I$-band calibration with the same 13 clusters (TC12: distances compared in Table 4).

Here, as there, we see a large scatter in the Hubble parameter for the nearer clusters and a small scatter for the more distant clusters. It can be anticipated that the measures for the nearer clusters are strongly affected by peculiar motions. The five clusters within 40 Mpc are all part of our extended supercluster complex, either within the historic Local Supercluster or the so-called Great Attractor region. The low scatter among the 7 clusters more distant than 50 Mpc ($V_{\text{CMB}} > 4000 \text{ km s}^{-1}$) suggests that the relative contributions of peculiar velocities have a modest effect on redshifts at such large distances.

In the case of the $I$-band calibration, the mean value of the Hubble parameter for the 7 most distant clusters was $75.1 \pm 2.7 \text{ km s}^{-1} \text{ Mpc}^{-1}$, where the error is just the rms scatter of the seven contributions. That value would increase to 75.8 with the revised LMC distance from Monson et al. (2012). With the present calibration, including the new LMC distance, the fit shown in Figure 11 gives a value of $H_0 = 73.8$, with an rms scatter of 1.1 and a standard deviation of 0.4 $\text{ km s}^{-1} \text{ Mpc}^{-1}$ for the same seven clusters considered previously. If the fit is extended to include the Pegasus Cluster at 44.5 Mpc, then $H_0 = 74.4$ and the scatter is 2.0 $\text{ km s}^{-1} \text{ Mpc}^{-1}$. The effect of a deviant radial motion of 200 $\text{ km s}^{-1}$ is illustrated in the figure as a function of distance.

The uncertainty from the fit in Figure 11 is given by the statistics of the deviations of the seven contributions and is unrealistically low. This error is what is expected if there is perfect Hubble expansion. If peculiar motions of 200 $\text{ km s}^{-1}$ are the norm, and given the expected statistical errors in the distance of each cluster, then the anticipated scatter around the mean Hubble value is $\pm 2.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$. We consider this to be our 1$\sigma$ random error. We have several sources of systematic error. The dominant component, creating almost 4% uncertainty in $H_0$, comes from the uncertainty in the TFR zero point with just 26 calibrators. Combined with a small uncertainty from the finite population of the template, the uncertainty in $H_0$ associated with the TFR calibration (assuming the zero-point calibrator distances are perfect) is $\pm 2.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The zero-point calibrator distances are not perfect. Freedman et al. (2012) argue that with new Milky Way parallaxes for Cepheid stars and mid-infrared Spitzer photometry, the uncertainty in the Cepheid scale is $\pm 1.9 \text{ km s}^{-1} \text{ Mpc}^{-1}$. The cumulative systematic error in $H_0$ is $\pm 3.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Combining random and systematic components, we find $H_0 = 73.8 \pm 2.6 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

5. CONCLUSIONS

A great concern with studies of motions on large scales with the TFR has been the possibility that systematic errors in photometry could create spurious flows. Small offsets between different observers, instruments, conditions, hemispheres, or seasons could be sky-sector dependent. Probably the single most important advantage of the use of space-based photometry such as is offered by the Spitzer mission comes from the confidence that measurements are on the same scale at better than 1% in all parts of the sky. There are other advantages. Obscuration is minimal both within targets and from our Galaxy. This latter point is especially significant because studies of galaxy flow patterns can now reach high levels of completion across the sky. Then it is a considerable advantage that the great majority of flux at the [3.6] band arises from old stars, mainly those on the red giant branch. It can be surmised from the modest scatter in the TFR that there is a close coupling between the mass in stars and the dynamical mass. And there is an advantage, at least vis à vis ground infrared observations, with the sensitivity achieved because of very low sky noise. All but a few percent of the total flux is measured within isophots resolved from the noise.

A small disadvantage with the mid-IR TFR calibration has been revealed by the documentation of a color term. This color term is understood to be the natural consequence of the correlation between galaxy rotation rate or luminosity and color (Tully et al. 1982). At a given linewidth, red galaxies progressively get brighter relative to blue galaxies as one considers the TFR at longer wavelengths. Evidence is accumulating that intrinsic scatter in the simple two parameter TFR is minimal with photometry at about 1 $\mu$m. A consequence of the color dependence is a steepening of the TFR toward the infrared. If one is interested in the physical implications of the TFR rather than its use as a distance tool, then the bivariate fit is of interest. Our template

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**Table 4**

Comparison with Courtois & Tully (2012)

| Cluster      | This Paper* | TC12* | Cluster      | This Paper* | TC12* |
|--------------|-------------|-------|--------------|-------------|-------|
| V Virgo      | 14.7 ± 0.9  | Pi Pisces | 65 ± 3 | 64 ± 2  |
| F Fornax     | 17.4 ± 1.2  | Ca Cancer | 67 ± 4 | 65 ± 3  |
| U U Ma       | 18.0 ± 0.9  | Co Coma | 95 ± 6 | 90 ± 4  |
| An Antlia    | 37 ± 2      | A4 A400 | 97 ± 5 | 94 ± 5  |
| Ce Cen30     | 39 ± 4      | A1 A1367 | 96 ± 6 | 94 ± 5  |
| Pe Pegasus   | 45 ± 3      | A2 A2634/66 | 112 ± 7 |       |
| H Hydra      | 56 ± 4      | A2634 | ... | 121 ± 7 |

Note. * Distance (Mpc).
sample has the bivariate dependence $M_{c,[3.6]} \propto W^{3.8\pm0.1}$, which is 0.4 steeper than was found with almost the same sample at the $I$ band (TC12).

At the expense of the requirement of extra knowledge in the form of a color, the TFR in the [3.6] band can be reformulated into a form with scatter that matches the best optical formulations. The correction is small and not acutely dependent on the color measurement. The appropriate ITFR equation for the measurement of distances is

$$M_{c,[3.6]} = -(20.34 \pm 0.08) - (9.13 \pm 0.22)(\log W^{3.8\pm0.1} - 2.5),$$

where $M_{c,[3.6]}$ is derived from the corrected apparent magnitude [3.6]$^{k,i,k,a}$ of a source minus the color term

$$\Delta [3.6] \text{color} = -0.36 - 0.47(I - [3.6]).$$

The slope of this formulation has been derived from a sample of 213 galaxies distributed in 13 clusters, while the zero point is established from 26 calibrators with Cepheid or TRGB distances. The rms scatter in distances found with these galaxies (cluster template and zero-point calibrators combined) of 0.42 mag, 21% in distance, is insignificantly different from the accuracy found with the strongly overlapping I-band study.

Distance measures derived with this calibration are subject to a small Malmquist bias, requiring the distance modulus correction $\mu = \mu + 0.0065(\mu - 31)^2$. After the application of bias and color corrections, a preliminary estimate of the Hubble constant can be made from the velocities and distances to seven clusters at $V_{\text{CMB}} > 4000$ km s$^{-1}$. Accounting for all of the error sources, the determination is $H_0 = 74 \pm 4$ km s$^{-1}$ Mpc$^{-1}$. The difference between the value determined with this mid-IR analysis compared with the $I$-band value found with the same procedures and a strongly overlapping sample is $\Delta H_0 = -2$ km s$^{-1}$ Mpc$^{-1}$, not a formally significant difference. We reiterate that the great strength of the present calibration is the high confidence in uniformity over the entire sky. Nevertheless the present sample of only seven clusters beyond the domain of known extreme peculiar velocities is unsatisfactorily small. In a subsequent paper (Sorce et al. 2012b), the [3.6]-band calibration is extended to a calibration of the Type Ia supernova scale, analogous to what has been done at the I band (Courtis & Tully 2012), permitting a determination of $H_0$ at $z \sim 0.1$.

The data used in this paper are available at the Extragalactic Distance Database. The photometric data are found by selecting the catalog Spitzer [3.6]-Band Photometry and then a galaxy of choice, while the H1 profiles are found in the catalog All Digital HI. We thank Tom Jarrett, part of the Cosmic Flows with Spitzer collaboration, for advice on Spitzer photometry and James Shobert for his development of Archangel. Much of the data used here come from the Spitzer Space Telescope archive. We thank Kartik Sheth for the contribution of his program Spitzer Survey of Stellar Structure in Galaxies. NASA through the Spitzer Science Center provides support for CHP, the Carnegie Hubble Program, cycle 6 program 61009 and for CFS, Cosmic Flows with Spitzer, cycle 8 program 80072. R.B.T. receives support from the US National Science Foundation with award AST-0908846.

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