A Nonlinear System Stable Control Design by Firefly Algorithm and Extreme Learning Machine

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Abstract. In this paper, we introduce a novel iterative method for finding a series of fixed point and design a steady controllability approach for nonlinear systems by using Firefly algorithm and ELM. First, we construct a fixed-point search model, but it is just only looking for a fixed point. Because of the need of non-linear dynamic systems, we need to find a series of fixed point under certain circumstances. Then, we apply the Firefly Algorithm and ELM to design a steady controllability approach. Finally, we have an example to verify the method we have proposed. Through the method, we get a series of fixed point, which make dynamic Nonlinear Systems or nonlinear circuit is under a relatively stable state.

1. Introduction

In the last decades, researchers from many fields of natural sciences have studied phenomena that involved nonlinear systems exhibiting stable behavior [1–6]. This is because nonlinear systems demonstrate rich dynamics and have relatively stable state. Controllability is one of the most important properties of dynamical systems or nonlinear circuit. The various approaches to the study of the controllability of nonlinear systems include:

1) Methods based on the stability theory of Lyapunov;
2) Methods for systems defined on a manifold;
3) Methods, which are geometrical in nature;
4) Fixed-point methods.

In the fixed-point method, the controllability problem is transformed to a fixed-point problem for an appropriate nonlinear operator in a function space. An essential part of this approach is to guarantee the existence of an invariant subset for this operator. Optimization is the process of selecting the optimum solution from the set of alternative ones. We have to either maximize or minimize the objective function by calculating the value of function using several input values from the given range.
of values. Evolutionary algorithms are being widely used in optimization problems. Reproduction, mutation, crossover, recombination, etc., mechanisms are used in such algorithms. Biological aspects inspire optimization algorithms. Since the introduction of Genetic Algorithm [7] in 1975, many optimization algorithms such as Ant Colony Optimization [8], Particle Swarm Optimization [9,10], Modified Particle Swarm Optimization [11]. Firefly algorithm (FA) [12-20], which is a new member of this family, is currently an active focus of research, where several modifications and improvements were recorded within the past few years.

1.1. Firefly Algorithm
The firefly’s flash is used as a signal to attract other fireflies. The algorithm has three rules:

(1) All fireflies are unisexual, all other fireflies will attract so one firefly.
(2) Attractiveness is proportional to their brightness, and for any two fireflies, the less bright one will be attracted by the brighter one and thus move towards it, and the brightness decreases as the distance between the fireflies increases.
(3) If there are no fireflies brighter than a given firefly, it will move randomly. The brightness should be associated with the objective function.

Definition 1.1: The relative fluorescence intensity of fireflies:

\[ I = I_o \times e^{-\gamma r_{ij}} \]  

(1.1)

Where \( I_o \) is the biggest fluorescence brightness of firefly. \( \gamma \) is light intensity absorbed coefficient and it is constant. It is characteristics of the weak that under the influence of the fireflies emit light in the distance and medium by set light intensity absorbed coefficient. For two fireflies \( i \) and \( j \), \( r_{ij} \) is calculated as

\[ r_{ij} = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \]  

(1.2)

Eq. (1.2) is the distance between any two fireflies \( i \) and \( j \) at \( x_i \) and \( x_j \) respectively.

Definition 1.2: The attractiveness of firefly is:

\[ \beta = \beta_o \times e^{-\gamma r_{ij}^2} \]  

(1.3)

where \( \beta_o \) is the attractiveness at \( r = 0 \)

Definition 1.3: In each generation the fireflies move to nearby fireflies having more brightness as determined by Eq. (1.4) as

\[ x_{i}^{\text{new}} = x_{i}^{\text{old}} + \beta (x_{i}^{\text{old}} - x_{j}) + \alpha (\text{rand} - \frac{1}{2}) \]  

(1.4)

Where \( x_i \) and \( x_j \) is the distance between two fireflies \( i \) and \( j \). And \( \alpha \) is Step factor that is constant in [0, 1]; Rand is uniformly distributed random factors belonging to [0, 1].

Algorithm as follows:

Step 1: Initialize the parameters of the algorithm. Define that the number of fireflies is \( m \), the biggest attraction is \( \beta_o \), the light intensity absorbed coefficient is \( \gamma \), the step length factor is \( \alpha \), the maximum number of generations is \( T_{\text{max}} \) and the search accuracy is \( \epsilon \), respectively.

Step 2: \( I_o \) is calculated as the respective maximum fluorescence brightness based on initial position of fireflies.

Step 3: \( I \) and \( \beta \) are calculated by \( I = I_o \times e^{-\gamma r_{ij}} \) and \( \beta = \beta_o \times e^{-\gamma r_{ij}^2} \), and determine the direction of the fireflies to move.
Step 4: Update the locations of fireflies according to (1.4) and random disturbance in the optimum location.
Step 5: Recalculate the fluorescence intensity after position is updated.
Step 6: If the results meet the requirements, break to step 7; else, the number of iterations plus one, go to step 4.
Step 7: Output the global extreme point and the optimal individual values. \( O(m^2) \) is time complexity of algorithm.

1.2. Extreme Learning Machine

Extreme learning machine (ELM) is a simple and easy to use SLFNs learning algorithm for single hidden layer feed forward neural networks. Traditional neural network learning algorithms (such as BP algorithm) need to manually set a large number of network training parameters, and it is easy to produce a local optimal solution. ELM only needs to set the number of hidden nodes in the network. It does not need to adjust the input weights of the network and the offset of the hidden elements during the execution of the algorithm, and generates the only optimal solution. Therefore, it has the advantages of fast learning and generalization good performance advantages. ELM is a very useful and efficient tool to classify and predict [21-24] etc. In this study, ELM is utilized to fit nonlinear system. Its expressions is

\[ HV = Y \]  

(1.5)

where \( x, i = 1, 2, 3, ..., N \) is input data, \( y, i = 1, 2, 3, ..., N \) is output data. \( g(x) \) is hidden layer activation function, \( \mu \) and \( \nu \) are the weight coefficient and bias between input layer and hidden layer. When \( \mu \) and \( \nu \) are chosen, \( y \) is determined accordingly.

In this paper, inspired by [19-20], we introduce a novel iterative method to finding the fixed point and steady controllability for nonlinear function by Firefly Algorithm and ELM. The structure of this paper is as follows: In Section 1, brief overview of the Firefly algorithm and ELM. In Section 2, fixed-point problem is defined and search method for the fixed-point. In Section 3, design a steady controllability approach for nonlinear systems. Section 4 shows the experimental results obtained through proposed algorithm and conclusions are given in section5.

2. Fixed-point Problem

2.1. Fixed-Point Model

A nonlinear system with periodic inputs can, in general, be described by the following system of ordinary differential equations:

\[ f : x \rightarrow f(x) \]  

(2.1)

Where \( x \in \mathbb{R}^n \) is a vector of input values,
We denote the actual input values by \( x \) and the nominal values by \( x^* \). The purpose of steady-state solution is to obtain deviations \( \Delta x = x^* - x \to 0 \).

Once the actual parameter values are known, the dynamical systems can be identified, and calibrated to the nominal values.

Generally, these design specifications define a region in the parameter space called the feasible region \( R_c \) and can be defined as:

\[
R_c = \{ x \in R^n \mid f(x) = x + \alpha \} 
\]

(2.2)

Where \( f = (f_1, f_2, \cdots, f_m) : R^n \to R^m \), \( \alpha \) is the randomization parameter.

Definition 2.1.

In the paper, similarity is defined as

\[
T(x_i, x_j) = \frac{x_i \cdot x_j}{\| x_i \|^2 + \| x_j \|^2 - x_i \cdot x_j} 
\]

(2.3)

Where \( x_i, x_j \in R^n \), it is usually used similarity calculations, so we can find that the more closer \( T(x_i, x_j) \) is 1, the more similar instructions.

Definition 2.2.

The distance between the two points \( x_i, x_j \in R^n \) will be given by:

\[
d(x_i, x_j) = \| x_i - x_j \| = \sqrt{(x_i - x_j)^T T(x_i - x_j)} 
\]

(2.4)

Where \( T \) is the Similarity Coefficient.

Theorem 2.1: Feasible Coefficient.

Let \( x_0 \) be a nominal point and a sequence of boundary point \( \{ x_i, i = 1, 2, \cdots, k \mid x_i \in R_c \} \) if the existence of a Maximum at the nominal values \( x^* \), then we have

\[
\| x^* - x_0 \|^2 = \frac{\left( (g - 1)^T (x^* - x_0) \right)^2}{(g - 1)^T T^{-1} (g - 1)} 
\]

(2.5)

Where

\[
g = \nabla f(x_i) 
\]

and

\[
l = \frac{\left| (g - 1)^T (x^* - x_0) \right|}{\sqrt{(g - 1)^T T^{-1} (g - 1)}} 
\]

(2.6)

In the paper, \( l \) is called feasible radius.

Proof: By using the method of Lagrange multipliers, the lagrangian function will be:

\[
H(x, \alpha) = \sqrt{(x-x_0)T(x-x_0)} + \alpha \left( f(x) - x - \alpha \right) 
\]

(2.7)

When
Multiplying (2.8) by $g^T T^{-1}$ and rearranging we get:

$$
\alpha = - \frac{(g-1)^T (x^* - x_0)}{(g-1)^T T^{-1} (g-1)}
$$

(2.9)

Substitute for $\alpha$ in (2.9) and rearranging, then:

$$
T(x^* - x_0) = \frac{(g-1)^T (x^* - x_0)(g-1)}{(g-1)^T T^{-1} (g-1)}
$$

(2.10)

premultiply both sides by $(x^* - x_0)^T$, then

$$
(x^* - x_0)^T T(x^* - x_0) = \frac{[(g-1)^T (x^* - x_0)]^2}{(g-1)^T T^{-1} (g-1)}
$$

(2.11)

So

$$
\|x^* - x_0\|^2 = \frac{[(g-1)^T (x^* - x_0)]^2}{(g-1)^T T^{-1} (g-1)}
$$

(2.12)

So

$$
l = \frac{[(g-1)^T (x^* - x_0)]}{\sqrt{(g-1)^T T^{-1} (g-1)}}
$$

(2.13)

2.2. A Search Method For the Fixed-Point

In this section, an iterative search method will be used to solve the optimization problem with similar references [20]. The method generate a sequence of boundary points $\{x_i\}$ such that $f(x) - x - \alpha = 0$.

The sequence $\{x_i\}$ converges to the point $x^*$ which solves problem (2.2).

The technique starts with a point $x_i \in \mathbb{R}^n$ such that $f(x) - x - \alpha = 0$. Starting from $x_i$, a point $x_c \in \mathbb{R}^n$ is found and is given by:

$$
x_c = x_i + x_0 - \frac{n}{\sqrt{x_0 (g-1)^T T^{-1} (g-1)}} T^{-1} (g-1)
$$

(2.14)

Where
In general \( f(x) \neq x + \alpha \). After \( x_c \) is obtained a line search starting from \( x_c \) in the \( (x_0 - x_i) \) direction is carried out to find a point \( x_2 \in \mathbb{R}^n \) such that \( f(x_2) = x_2 + \alpha \). This iteration is repeated until convergence occurs and a point \( x_f \) is found. An iteration of the technique can be given by the following steps:

\[
x_{i+1} = x_i + \frac{\eta}{\sqrt{\left((g-1)^T (x_i - x_0)\right)^2 + \mu_i (x_i - x_i)}}
\]

Equation (2.16) can be written as:

\[
x_{k+1} = F(x_k), \quad k = 1, 2, \ldots
\]

where \( F \) is a transformation from \( \mathbb{R}^n \) into itself. A fixed point \( x_f \in \mathbb{R}^n \) for this transformation

\[
x_f = F(x_f)
\]

This point is an attraction point[14], if given a norm on \( \mathbb{R}^n \), there exists an open ball \( B_r = B_r \left( x^* \right) = \left\{ x \in \mathbb{R}^n : \|x - x^*\| < 1 \right\} \subseteq D \) such that for any initial approximation \( x_0 \in B_r \), the successive approximations \( x_{k+1} = F(x_k), \quad k = 1, 2, \ldots \) remain in \( D \) and converge to \( x^* \). Note that a finite number of iterates are allowed to lie outside \( B_r \).

Theorem 2.2 (Ostrowski; see, e.g., [25-26]). If \( F(x) \) is differentiable at the fixed point \( x^* \) and the spectral radius satisfies

\[
\rho \left( F'(x^*) \right) = \sigma < 1
\]

then \( x^* \) is an attraction point.

In the second part, we are just only looking for a fixed point. Because of the need of steady controllability for dynamic systems, you need to find a series of fixed point under certain circumstances, so that the system will be in a steady state. Then, let’s apply FA and ELM to nonlinear system control, we believe that its output is a number of fixed points

3. Steady Controllability Approach

3.1. Control Strategy

A nonlinear system with single inputs can, it can be described by

\[
x(t + 1) = f(x(t), \kappa)
\]

is a vector of input values, \( \kappa \) is system parameter.

As is showed in Fig. 1, the modified system is
\[ x(t+1) = f(x(t), \kappa) + \mu(t) \]  
(3.2)

Where \( \mu(t) \) is Controller. So it can be defined by

\[ \mu(t) = - \frac{\lambda t e^{-\lambda}}{t!} F(x(t), \kappa) + x_r(t) + \text{rand} \times (x(t) - x_r(t)) \]  
(3.3)

Thus, in this paper, \(- \frac{\lambda t e^{-\lambda}}{t!} F(x(t), \kappa), x(t), \text{rand} \) and \( x_r \) are defined as fitting function which related to the Poisson distribution, state variable, state parameter, reference signal and convergence parameter of nonlinear system, respectively.

![Figure1. The Nonlinear system control model](image)

As the analytical mathematical model is unknown clearly, its data can be obtained easily and fitted by ELM, which is

\[ \tilde{x}(t+1) = F(x(t), \kappa) \]  
(3.4)

So the residual is

\[ e(t) = x(t) - x_r(t) \]  
(3.5)

When \( t \) is relatively large \(- \frac{\lambda t e^{-\lambda}}{t!} F(x(t), \kappa) \rightarrow F(x(t), \kappa) \), Then

\[ e(t+1) = x(t+1) - x_r(t+1) \]

\[ = f(x(t), \kappa) - \frac{\lambda t e^{-\lambda}}{t!} F(x(t), \kappa) + x_r(t) + \text{rand} \times (x(t) - x_r(t)) - x_r(t+1). \]  
(3.6)

Assuming that fitting function and nonlinear system match is ideal, so the above equation can be rewritten as

\[ e(t+1) = \text{rand} \times (x(t) - x_r(t)) = \text{rand} \times e(t). \]  
(3.7)

This control strategy can make the nonlinear system output gradually become stable and reference signal can be traced effectively.
3.2. **Control Algorithm**

In this section, the processes to control nonlinear system are proposed. The proposed method is composed of three major steps:

**Initialization:**
Step 1: Initialize reference signal \( x \) and number of sample data \( M \) in nonlinear system.

Step 2: After sampling, we get training data and testing data \( x_i, i = 1, 2, 3, \ldots M \).

Step 3: Initialize parameters of FA: the number of fireflies \( n \), the maximum iterations \( i \), the random factor \( \varepsilon \) and reference accuracy \( \zeta \).

Step 4: Define number of neurons \( N \) in hidden layer and construct ELM fitting model.

**Search process:**
Step 5: Define the dimension \( D \) of firefly using the optimization object “weight matrix \( \mu \) and bias \( \nu \)”

\[
D = N \times \delta_i + N, \quad (3.8)
\]

where \( \delta_i \) and \( N \) are the number of neurons in input layer and hidden layer, respectively.

Step 6: Update locations of fireflies, brightness and attraction using (1.1-1.4).

Step 7: Calculate output training samples \( \hat{y}_i \) by (1.5).

Step 8: Calculate mean squared error (MSE)

\[
RESM = \frac{1}{m} \sum_{i=1}^{m} (\hat{y}_i - y_i)^2, i = 1, 2, \ldots, m. \quad (3.9)
\]

**Convergence:**
Step 9: Update the global optimal individual.

Step 10: Judge whether the current iteration \( l \) is equal to maximum iteration \( i \) and the current accuracy \( r \) is less than \( \zeta \) or not. If not, \( l = l + 1 \) and back to Step 6.

Step 11: Judge whether \( N \) is less than the maximum searching range in the hidden layer neurons. If not, initialize iterations, \( N = N + 1 \) and back to Step 4.

Step 12: Output optimal weight \( \mu_{\text{opt}} \), optimal bias \( \nu_{\text{opt}} \) and fitting function \( F \) with optimal ELM.

Step 13: Test nonlinear system with controller.

### 4. The Simulations and Discussions

In this section, in order to give an understanding of the method and how to use it, we demonstrate the algorithm by proposed in Part 3. According to the above algorithm, we designed a nonlinear systems (4.1)

\[
y(t+1) = 0.5y(t) - 0.5y(t-1) + 0.5 \frac{y^2(t-1) + u(k)}{1 + y^2(t-1)} + u(k) \quad (4.1)
\]

\[
u(k) = \sin(\pi y(t-1)) + 0.4
\]

we assume that the parameters of the firefly algorithm are: the size of fireflies is 30, the maximum iterations is 1000.

Respectively, with positive rotation signal and constant value signal as the control target.

Case 1 positive rotation signal
Figure 2. Positive rotation signal output
Case 2 constant value signal

Figure 3. Positive rotation signal fitness value curve

Figure 4. Constant signal output

Figure 5. Constant signal fitness value curve

All fitness value curves and outputs of system with different reference signals are shown clearly in Fig.2-5. It is clear that nonlinear systems can be controlled. The output indicates that the system will stabilize in a short time, and we think that the output is made up of some fixed points.

5. Conclusion
In this paper, we introduce a novel iterative method for design a steady controllability approach by using Firefly algorithm and ELM. Through the above algorithm, we get a series of fixed point, which make nonlinear system is under a relatively stable state. Example simulation results show that this analysis method could quickly obtaining steady-state conditions within a certain range.

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