Research on Dynamics of Vehicle Height Adjustment for Automobile with Double-Wishbone ECAS

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Abstract: This paper addresses the problem of dynamics of vehicle height adjustment for automobile with Double-wishbone electronically-controlled air suspension (ECAS) under considering the effects of suspension geometry and kinematics. The model of vehicle height adjustment consists of air spring model and suspension dynamics model. The air spring model was derived by thermodynamic theory for variable-mass gas charge/discharge system. The suspension dynamics model was formulated by kinematics analysis and dynamics modeling of double-wishbone suspension. Then, the influence of suspension geometry and kinematics on the model of vehicle height adjustment was quantitatively analyzed through simulation. Finally, Simulink-Adams co-simulation results show the accuracy of the model.

1. Introduction
ECAS are widely installed in luxury car to improve the performance of the vehicles, such as trafficability, ride comfort and handling stability[1]. Vehicle height adjustment is one of the main features, which can adjust the vehicle height by inflating and deflating the air spring[2, 3].

However, it is still a challenge to build the dynamic model of vehicle height adjustment system for automobile with double-wishbone ECAS, especially if we consider the influence of geometry and kinematics of double-wishbone suspension.

In previous work, the vehicle height adjustment model is obtained based on the classical two-degree-of-freedom model of suspension and the air spring model derived from thermodynamic theory for variable-mass gas charging/discharging system [4-6]. Nonetheless, the influence of suspension kinematics and geometry during motion is not considered in their models.

Inspired by the aforesaid research, the main contributions of this paper include: 1) The vehicle height adjustment model is developed based on air spring model and suspension dynamics model derived from kinematics analysis and dynamics modeling; 2) Through the comparison of different models, the specific influence of suspension geometry and kinematics are discussed; 3) The model validation are conducted through the Simulink-Adams co-simulation.

The remainder of this paper is organized as follows. Section 2 presents the principle of vehicle height adjustment system of ECAS and problems in modeling. Section 3 develops a new dynamic model for automobile with double-wishbone ECAS. Section 4 quantitatively analyzes the influence of suspension geometry and kinematics on the model. The Simulink-Adams co-simulation results are presented in Section 6 to show the accuracy of the new model.
2. System Description
The target vehicle height adjustment system for automobile with double-wishbone ECAS is represented in Fig 1, which includes double-wishbone air spring, charging/discharging system, sensors and ECU. The double-wishbone air spring consists of vehicle body, control arms, an air spring, a damper and wheel. The charging/discharging system that regulates air flowing into or out of the air spring mainly contains a compressor, a dryer, an air reservoir, a solenoid valve and pipes. Sensors that send the signals to the ECU includes height sensor and pressure sensor.

![Fig 1 Vehicle height adjustment system](image)

According to the ECAS structure, the adjustment of vehicle height can be summarized into the following three situations:

1. Lifting procedure: When the vehicle body needs to be lifted, the solenoid valve is opened to allow high-pressure air in the air reservoir to flow into the air spring. By increasing the air mass in the air spring, the vehicle body can be lifted. When the vehicle height meets the requirements, the solenoid valve is closed.

2. Lowering procedure: When the vehicle body needs to be lowered, the solenoid valve is opened to allow the air in the air spring to flow into the environment. By decreasing the air mass in the air spring, the vehicle body can be lowered. Similarly, when the vehicle height meets the requirements, the solenoid valve is closed.

3. Maintaining procedure: When the vehicle body need to be maintained, the solenoid valve is closed. The air mass in the air spring remains unchanged.

3. Mechanism Model

3.1. Air Spring Model
To reduce the complexity of the model, some assumptions are made as follows[7]:

1. The air in the system is view as a perfect gas. Its kinetic and potential energy can be ignored.
2. The temperature and pressure of the air are homogeneous in the ECAS.
3. The cross-sectional area of the air spring is constant within the range of target vehicle height.
4. The air reservoir is simplified as a high-pressure air source with constant pressure. Meanwhile, the environment is simplified as a low-pressure air source with constant pressure.
5. The damping coefficient of the damper is constant.

As shown in Fig 2, according to first law of thermodynamics, the model of air spring is express as:

\[
Q_{\text{heat}} + W_{\text{as}} + h_{\text{in}} q_{\text{in}} - h_{\text{out}} q_{\text{out}} = U_{\text{as}}
\]

where \(Q_{\text{heat}}\) is the heat transfer between the air spring and the environment; \(W_{\text{as}}\) is the power caused by the volume change of the air spring; \(U_{\text{as}}\) is the internal energy of the air spring; \(q_{\text{in}}\) and \(q_{\text{out}}\) are the increment and decrement of the air mass in the air spring. \(h_{\text{in}}\) and \(h_{\text{out}}\) are the specific enthalpy of the air flowing into and outside the air spring.
Hence, the model of the air spring can be sorted out as follows:

\[
p_{as} V_{as} = \kappa R T_{as} (q_{in} - q_{out}) - \kappa p_{as} \dot{V}_{as}
\]

where \( p_{as} \) is the air pressure of the air spring; \( T_{as} \) is the air temperature of the air spring; \( \kappa \) refers to the polytropic index; \( V_{as} \) is the air spring volume.

According to assumption (3), the air spring volume can be rewritten as:

\[
V_{as} = A_{as} H_{as} / \cos \beta = A_{as} (H_{as0} + z_s - z_u) / \cos \beta
\]

where \( H_{as0} \) is the initial vehicle height; \( z_s \) and \( z_u \) are the displacement of the vehicle body and wheel; \( \beta \) is the angle of air spring.

![Fig 2](image)

**Fig 2** Energy change of the air spring

3.2. Solenoid Valve

According to the theory of orifice, the air mass flow rate through the solenoid valve can be derived as:

\[
q(p_u, p_d) = \begin{cases} 
\frac{2}{\kappa+1} \left( \frac{\kappa-1}{2(\kappa-1)} \right)^{\kappa \over \kappa-1} \frac{\kappa}{RT} p_u S \sqrt{1 - \left( \frac{p_d}{p_u} b \right)^2} & (0 < p_d/p_u \leq b) \\
\frac{2}{\kappa+1} \left( \frac{\kappa-1}{2(\kappa-1)} \right)^{\kappa \over \kappa-1} \frac{\kappa}{RT} p_u S \sqrt{1 - \left( \frac{p_d}{p_u} b \right)^2} \left( b < p_d/p_u \leq 1 \right) & (0 < p_d/p_u \leq 1)
\end{cases}
\]

where \( q(p_u, p_d) \) refers to the air mass flow rate between the high pressure source and the low pressure source; \( p_u \) and \( p_d \) are the upstream air pressure and downstream air pressure respectively; \( R \) is the perfect gas constant; \( T \) is the air temperature; \( S \) is the effective cross-sectional area of the solenoid valve; \( b \) is the critical pressure ratio.

3.3. Double-Wishbone Suspension Dynamics

The following assumptions are made to develop the model of the suspension.

1. The vehicle body undergoes a vertical motion.
2. The control arms are considered as rigid bodies and its mass are negligible.
3. All joints are considered as ideal.
4. The displacement of vehicle body \( z_s \) and the displacement of wheel \( z_u \) are considered as generalised coordinates. As shown in Fig 3, The Z-Y absolute coordinate system is established with the origin point O under static equilibrium.

3.3.1. Kinematics Analysis

As shown in Fig 3, in Z-Y plane, the finite displacement of the wheel assembly can be viewed as a rotation and a translation.
\[
\begin{bmatrix}
N_x & P_x \\
N_z & P_z \\
1 & 1
\end{bmatrix}
= \begin{bmatrix}
a_{11} & a_{12} & C_x - (a_{12}C_y0 + a_{12}C_z0) \\
a_{21} & a_{22} & C_z - (a_{22}C_y0 + a_{22}C_z0) \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
N_{y0} & P_{y0} \\
N_{z0} & P_{z0} \\
1 & 1
\end{bmatrix}
\]

where \((N_{y0}, N_{z0}), (P_{y0}, P_{z0}), (C_{y0}, C_{z0})\) are the initial coordinates of \(N, P, C\) respectively; \((N_{y}, N_{z}), (P_{y}, P_{z}), (C_{y}, C_{z})\) are the instantaneous coordinates of \(N, P, C\) respectively, \(C_{z}\) can be written as \(C_{z} = C_{z0} + z_{u}\); \(a_{11} = a_{22} = \cos \phi \approx 1, a_{12} = a_{21} = \sin \phi \approx \phi\), \(\phi\) is the wheel camber.

The above four formulation with six unknown parameters cannot be solved. The length of control arms are introduced as the constraint equations. To facilitate the solution, the constraint equation is linearized as follows:

\[
(N_{y0} - M_{y0})(N_{y} - N_{y0}) + (N_{z0} - M_{z0})(N_{z} - N_{z0} - z_{r}) = 0
\]

\[
(P_{y0} - O_{y0})(P_{y} - P_{y0}) + (P_{z0} - O_{z0})(P_{z} - P_{z0} - z_{s}) = 0
\]

The equation obtained by combining equation (5) and equation (6) can solve the value of \(N_{y}, N_{z}, P_{y}, P_{z}, C_{y}, \phi\) in the generalised coordinates \(z_{r}\) and \(z_{s}\).

Hence, the angles of the control arms and the air spring can be expressed as follows:

\[
\theta_{1} = \arccos \left[ \frac{(N_{y} - M_{y0})}{l_{MN}} \right]
\]

\[
\theta_{2} = \arccos \left[ \frac{(P_{y} - O_{y0})}{l_{OP}} \right]
\]

\[
\beta = \arctan \left[ \frac{(A_{y} - B_{y})}{(B_{z} - A_{z})} \right]
\]

where, \(\theta_{1}, \theta_{2}, \beta\) are the angles of upper control arm, lower control arm and air spring respectively; \((A_{y}, A_{z}), (B_{y}, B_{z})\) are the instantaneous coordinates of \(A, B, A_{y} = O_{y} + l_{OP}(P_{y} - O_{y})/l_{OP}, A_{z} = O_{z} + l_{OP}(P_{z} - O_{z})/l_{OP}, B_{y} = B_{y0}, B_{z} = B_{z0} + z_{s}; l_{OP}\) and \(l_{MN}\) are the length of lower and upper control arm.

3.3.2. Dynamics Modeling

First, according to the force analysis of wheel assembly in Fig 4, the equation is established as follows. Among them, wheel camber \(\phi\) is small enough to be approximate as 0.

\[
F_{z} + F_{i} \sin \theta_{1} - F_{Lz} = m_{u} \ddot{z}_{u}
\]

\[
F_{U} \cos \theta_{1} - F_{Ly} = 0
\]

\[
F_{o}d_{CU} + F_{Ly}d_{CLy} - F_{Lz}d_{CLz} = 0
\]

where \(m_{u}\) is the mass of wheel; \(F_{z}\) is the force of the road on the wheel assembly, \(F_{z} = k_{t}(z_{r} - z_{u})\), \(z_{r}\) is the road disturbance, \(k_{t}\) is the stiffness of the tire; \(F_{U}\) is the force of the upper control arm on the wheel assembly at point \(N\); \(F_{Lz}\) and \(F_{Ly}\) are the force of lower control arm on the wheel assembly at point \(N\) in
the Z axis direction and Y axis direction; $d_{CLz}$, $d_{CLy}$, and $d_{CU}$ are the force arm from point C to force $F_{Lz}$, $F_{Ly}$, and $F_{U}$.

Hence, the force $F_{Lz}$, $F_{Ly}$, and $F_{U}$ can be solved. Then, as shown in Fig 5, the force analysis of lower control arm is performed as follows:

$$F_{Oz} = -F_{Ly} - F_{as} \sin \beta$$
$$F_{Ox} = F_{as} \cos \beta - F_{Lz}$$

(9)

where $F_{Oz}$ and $F_{Ox}$ are the force of vehicle body on the lower control arm at point O in the Z axis direction and Y axis direction; $F_{as}$ is the force of air spring and damper on the lower control arm at point A, $F_{as} = A_{as}P_{as} - c_{as}(\dot{z}_{a} - \dot{z}_{u}) / \cos \beta$.

Afterwards, the wheel dynamics is derived from the torque balance of lower control arm.

$$F_{Lz}d_{OLz} - F_{Ly}d_{OLy} - F_{as}d_{\beta} = 0$$

(10)

Finally, the vehicle body dynamics is derived from the force balance as shown in Fig 6.

$$F_{Lz} - F_{as} \cos \beta + F_{as} \cos \beta - F_{Lz} \sin \theta = m_{\beta} \ddot{z}_{i}$$

(11)

The double-wishbone suspension dynamics can be achieved by combining equation (10) and equation (11).

$$m_{\beta} \ddot{z}_{i} = \lambda \left[ -A_{as}P_{as} + c_{as}(\dot{z}_{a} - \dot{z}_{u}) / \cos \beta \right] + k_{i}(z_{r} - z_{u})$$
$$m_{\beta} \ddot{z}_{i} = \lambda \left[ A_{as}P_{as} - c_{as}(\dot{z}_{a} - \dot{z}_{u}) / \cos \beta \right]$$

(12)

where $\lambda = (d_{CU} + d_{CL} \cos \theta - d_{CLz} \sin \theta_{0}) \int d\beta / [(d_{CU} + d_{CL} \cos \theta_{0})d_{OLz} - d_{OLy}d_{CL} \cos \theta_{0}]$

3.4. Factor Analysis

The vehicle height adjustment model can be obtained by combining equation (2) and equation (12). Among them, the variable factor $\lambda$ and $\beta$ reflect the influence of the double-wishbone suspension geometry and kinematics. It is not difficult to find that when $\lambda = \lambda_{0}$, $\beta = \beta_{0}$, the model only considers the suspension geometry and ignores the kinematics. The constant factor $\lambda_{0}$ and $\beta_{0}$ are defined as:

$$\lambda_{0} = (d_{CU} \cos \theta - d_{CLz} \sin \theta_{0}) \int d\beta / [(d_{CU} + d_{CL} \cos \theta_{0})d_{OLz} - d_{OLy}d_{CL} \cos \theta_{0}]$$

The solenoid valve command and B-class road disturbance are applied to the vehicle height adjustment system. The variables including vehicle height $H_{as}$ and air pressure of air spring $P_{as}$ are selected for comparison in Fig 7. They reflect the dynamic characteristics of the vehicle height adjustment system.

The comparison results are shown in Fig 7 and Table 1. The maximum error of vehicle height between model 1 and model 2 is 0.414mm. The maximum error of vehicle height between model 1 and
model 3 is 4.321 mm. Therefore, the response error of vehicle height is reduced by 90.42% through considering the influence of suspension geometry. The response error of pressure is reduced by 9.58% through considering the influence of suspension geometry. The same is true for air pressure of air spring.

In conclusion, it is found that the suspension geometry has the greatest influence on the model of vehicle height adjustment for double-wishbone ECAS, while the suspension kinematics has little influence.

### Table 1 Parameters comparison of vehicle height adjustment models

| Parameters          | model 1-model 2 | model 1-model 3 | unit   |
|---------------------|-----------------|-----------------|--------|
| Max error of Vehicle height | 0.414           | 4.321           | mm     |
| Max error of Pressure   | 425.1           | 8795.2          | Pa     |

### 4. Co-simulation verification

In order to ensure the accuracy of the model, the Simulink-Adams co-simulation is used to verify the model of vehicle height adjustment for double-wishbone ECAS. Because the air spring model has been verified in the previous literature\[4\], this paper focuses on the influence of geometry and kinematics of the suspension. Therefore, the air spring model established by Simulink and the double-wishbone suspension model established by Adams are used as the co-simulation model. The simulation workflow is shown in Fig 8. Using the same solenoid valve command and road disturbance as Section 3.4, the simulation results of vehicle height $H_{av}$ and air pressure of air spring $P_{as}$ are shown in Fig 9.

The vehicle height error RMS (Root Mean Square) and air pressure of air spring error RMS between the model 1 and co-simulation are 3078 Pa and 1.902 mm. The error RMS between the model 1 and co-simulation are 3954 Pa and 3.110 mm. This illustrates that the model of vehicle height adjustment considering the influence of suspension geometry and kinematics is more accurate than the models in previous literature.
5. Conclusions
In this paper, a new dynamic model of vehicle height adjustment for double-wishbone ECAS by considering the influence of suspension geometry and kinematics is established and verified by Simulink-Adams co-simulation. Based on the new model, the influence of suspension geometry and kinematics are quantitatively analyzed. According to the modeling and simulation, the following conclusions can be drawn:

(1) The model of vehicle height adjustment that considers the influence of suspension geometry and kinematics is more accurate than the models without considering.

(2) The influence of suspension geometry on the model of vehicle height adjustment is greater than suspension kinematics.

The derived model of vehicle height adjustment for double-wishbone ECAS provides a better reference for the design of ride height controller.

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