Dissipation-based entanglement via quantum Zeno dynamics and Rydberg antiblockade

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A novel scheme is proposed for dissipative generation of maximally entanglement between two Rydberg atoms in the context of cavity QED. The spontaneous emission of atoms combined with quantum Zeno dynamics and Rydberg antiblockade guarantees a unique steady solution of the master equation of system, which just corresponds to the antisymmetric Bell state $|S\rangle$. The convergence rate is accelerated by the ground-state blockade mechanism of Rydberg atoms. Meanwhile the effect of cavity decay is suppressed by the Zeno requirement, leading to a steady-state fidelity about 90% as the single-atom cooperativity parameter $C \equiv g^2/(\kappa\gamma) = 10$, and this restriction is further relaxed to $C = 5.2$ once the quantum-jump-based feedback control is exploited.

The quantum dissipation arising from weak coupling between a quantum system and its surrounding reservoirs, is usually indicated by a Lindblad generator in Markovian quantum master equations for open quantum systems. Compared with unitary dynamics, the dissipative dynamics does provide a more accurate image for characterizing the evolution of quantum states in a realistic situation. It has always been regarded that the quantum dissipation plays a negative role in quantum information processing (QIP) tasks, since it causes decoherent effect on investigated quantum system. Nevertheless, the study in recent decades has changed people’s view of quantum states in a realistic situation. It has always been regarded that the quantum dissipation plays a negative role in quantum information processing (QIP) tasks, since it causes decoherent effect on investigated quantum system. Nevertheless, the study in recent decades has changed people’s view of quantum dissipation, and the environment can be used as a resource in QIP experimentally [1–7].

Quantum entanglement, as one of the most striking features in quantum theory, is defined to describe a strongly correlated system constituted by pairs or groups of particles, and a measurement made on either of the particles collapses the state of the system instantaneously. Thus it is an interesting question how can one prepare this kind of strongly ‘coherent’ system using the ‘decoherent’ factors. Currently, there are several representative schemes creating steady bipartite entanglement of high quality by dissipation [8–17]. For instance, in the context of cavity QED, the cavity decay was exploited to drive the system into a maximally entangled stationary state, making the spontaneous emission of atom be the solely detrimental element [9]. In neutral atom systems, two groups independently prepared high-fidelity steady-state entanglement between a pair of Rydberg atoms with dissipative Rydberg pumping [12, 13]. More recently, a highly entangled state with fidelity above 99% was achieved in ion traps, and the fidelity was further enhanced by detection of photons spontaneous emission emitted in to the environment [17]. However, it is challenging to detect atomic decay event in experiment.

In this work, we suggest an alternative scheme in the context of cavity QED for dissipatively preparing a maximally entangled state by capitalizing the advantages of above protocols. We combine the spontaneous emission of atom with quantum Zeno dynamics and Rydberg antiblockade effect to drive the system into a purely entangled steady state irrespective of initial state. The concept of quantum Zeno dynamics, formally proposed by Facchi et al but also adopted by Beige et al is a generalization of quantum Zeno effect [18–23]. The frequent measurements do not necessarily hinder the evolution of the quantum system but the system can evolve away from the initial state via the measurements as long as the projection is multi-dimensional.

For our concerning atom-cavity systems, the quantum Zeno subspaces via a strong continuous coupling is able to effectively suppress the occupation of photon in cavity and has been actively exploited to execute various of QIP based on unitary dynamics [24–27]. The function of Rydberg antiblockade interaction, primarily introduced to break the symmetry by shifting the energy level of triplet states, can also result in an effect of ‘ground-state blockade’ and then speed up the convergence rate of stationary entanglement for certain initial states, as this kind of interaction strength is much larger than the Rabi frequency of microwave field. Generally speaking, regarding two typical noise sources in cavity QED system, we take advantage of atomic decay and avoid the effect of cavity decay at the same time. Thence a fidelity $\sim 90\%$ of steady entanglement is available at a low value of single-atom cooperativity parameter $C \equiv g^2/(\kappa\gamma) = 10$. Moreover, by virtue of quantum-jump-based feedback control technology, the same fidelity can be achieved even as $C = 5.2$. This feature expands the feasible scope of our scheme from the perspective of experimental realization.

Our system consists of two $N$-type four level Rydberg atoms and an optical cavity, as shown in Fig. 1(a). A similar configuration of Rydberg atom has been employed to efficiently produce multi-particle entangled states using Rydberg blockade interactions by Saffman and Mølmer [28]. Here we replace the original Rydberg state $|p\rangle$ with an optical state, since our scheme is completely based on the spontaneous emission of atoms. The atoms in ground states $|g\rangle$ and $|e\rangle$ can execute an upward transition to the excited state $|p\rangle$ through resonant interaction with a quantized cavity field of coupling

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FIG. 1: (Color online) (a) Schematic view of atomic-level configurations. The transition from the ground state $|g\rangle$ to the optical state $|p\rangle$ is resonantly coupled to the quantized cavity field with coupling strength $g$, and the transition between states $|p\rangle$ and $|e\rangle$ is driven by an optical pumping laser acting on the two atoms with Rabi frequency $\pm \Omega_0$. In addition, there is a microwave field $\omega$ causing transitions from the ground state $|g\rangle$ to the ground state $|e\rangle$ which is then pumped to the excited Rydberg state $|r\rangle$ by a classical field $\Omega_c$, detuning $-\Delta$. (b) The effective transitions for two atoms. The whole system works well in a subspace of zero occupation for cavity mode due to the quantum Zeno dynamics. Under the condition of spontaneous emission of state $|D\rangle$, the optical pumping laser $\Omega_0$, the microwave driving field $\omega$, and the Rydberg blockade interaction $\lambda$, the system will be finally stabilized into the singlet state $|S\rangle$ because of $\Gamma \ll \gamma$.

strength $g$, and an optical pumping laser of Rabi frequency $\pm \Omega_0$, respectively. Apart from this, a microwave field of Rabi frequency $\omega$ is introduced to cause transitions between ground states $|g\rangle$ and $|e\rangle$, and an extra pumping laser field with Rabi frequency $\Omega_c$ drives the atom to the high-lying excited Rydberg state $|r\rangle$ from state $|e\rangle$ detuned by $-\Delta$. For the sake of simplicity, we have assumed the atom decays from the optical state $|p\rangle$ to the ground states $|g\rangle$ and $|e\rangle$ with the same spontaneous emission rate $\gamma/2$, and the life time of the Rydberg state $|r\rangle$ is supposed to be $1/\Gamma$.

The Hamiltonian of the system, in the interaction picture after performing a rotating with respect to $U = \exp(-i \Delta t \sum_i^2 |r\rangle\langle r|)$, can be written as ($\hbar = 1$)

$$H_I = H_z + H_r,$$

(1)

$$H_z = \sum_{i=1}^2 \left[ \Omega_0 (-1)^{i-1} |p\rangle\langle e| + g |p\rangle\langle g| + H.c. \right],$$

(2)

$$H_r = \sum_{i=1}^2 \left[ \omega |g\rangle\langle e| + \Omega_0 |e\rangle\langle i| + H.c. \right]$$

$$+ (U_{rr} - 2\Delta) |rr\rangle\langle rr|,$$

(3)

where the Rydberg-mediated interaction $U_{rr}$ originates from the dipole-dipole potential of the scale $C_6/r^6$ or the long-range van der Waals interaction proportional to $C_6/r^6$, with $r$ being the distance between two Rydberg atoms and $C_6(6)$ depending on the quantum numbers of the Rydberg state $[29–35]$.

For convenience, we reformulate the Hamiltonian $H_z$ as $H_z = \Omega_0 (H_c + K H_0)$, where $K = g/\Omega_0$, $H_c$ stands for the dimensionless interaction Hamiltonian between atoms and the classical field, and $H_0$ denotes the counterpart between atoms and the quantum cavity field. For a strong coupling limit $K \rightarrow \infty$, the Zeno requirement is satisfied and the above Hamiltonian takes the form $[19]$ $H_z = \Omega_0 (\sum_n P_n H_c P_n + K \varepsilon_n P_n)$, where $P_n$ is the eigenprojection of the $H_0$ belonging to the eigenvalue $\varepsilon_n$: $H_0 = \sum_n \varepsilon_n P_n$. When we confine our system into the subspace corresponding to $P_0$, the Zeno Hamiltonian reduces to $H_z = \Omega_0 |T\rangle \langle D| \otimes |0\rangle \langle 0| + H.c.$, with $|T\rangle = (|eg\rangle + |ge\rangle)/\sqrt{2}$, and $|D\rangle = (|pg\rangle - |gp\rangle)/\sqrt{2}$, $|0\rangle$ represents the vacuum state of cavity field. As for the Hamiltonian $H_c$, we can obtain its effective form in the regime of Rydberg antiblockade: $U_{rr} \sim 2\Delta \gg \Omega_0$, which is approximated to $H_r = \omega (|g\rangle\langle e| + |g\rangle_2\langle e|) + \lambda (|e\rangle\langle e| + H.c.) + (U_{rr} - 2\Delta + \lambda) |rr\rangle\langle rr|$, where $\lambda = 2\Omega_0^2/\Delta$ comes from the second-order perturbation theory, and we also omit the Stark-shift term of level $|e\rangle$. If we further assume that $(U_{rr} - 2\Delta + \lambda) = 0$, the Hamiltonian of Eq. (1) takes the following concise form

$$H_I = \Omega_0 |T\rangle \langle D| + \omega \sum_{i=1}^2 |g\rangle_{ii}\langle e| + \lambda |e\rangle\langle e| + H.c..$$

(4)

Note that the effective model is decoupled to the cavity field due to the restriction of quantum Zeno dynamics.

The Markovian master equation, describing the current system-environment interaction, is modeled in Lindblad form

$$\dot{\rho} = -i[H_I, \rho] + \sum_{j=1}^3 L_j \rho L_j^\dagger - \frac{1}{2} (L_j^\dagger L_j \rho + \rho L_j^\dagger L_j), $$

(5)

and the dominant operators for spontaneous emission read

$$L_{1(2)} = \sqrt{\frac{\gamma}{4}} |S\rangle \langle (T)| \langle D|, \quad L_3 = \sqrt{\frac{\gamma}{2}} |gg\rangle \langle D|,$$

(6)

where $|S\rangle = (|eg\rangle - |ge\rangle)/\sqrt{2}$. The effective transitions of current reduced system is depicted in Fig. 1(b). It can be seen clearly that atoms in the excited state $|D\rangle$ can spontaneously and independently decay into the ground states $|S\rangle$, $|gg\rangle$, and $|T\rangle$, respectively. For the atoms in state $|gg\rangle$ or $|T\rangle$, they will be pumped into other states by the microwave field or the Rydberg blockade interaction. Whereas for the atoms staying in state $|S\rangle$, they are invariant on account of $H_I (L_j) |S\rangle = 0$. Therefore, the process of pumping and decaying repeats again and again until the system is finally stabilized into the state $|S\rangle$. Although the spontaneous emission of Rydberg state may cause a leakage of quantum information out of related subspace, it contributes little effect on our scheme because $\Gamma \ll \gamma$ generally works for a realistic situation. In what follows, our
from Eq. (2) we find a specific value of $\lambda/\omega$ for relaxation. Interestingly, as the increase of the ratio $g/\omega$, simulations are all based on the full Hamiltonian of Eq. (1) without any specification.

In order to fully characterize the dependence of convergence rate on relevant parameters, we use the definition of purity $P(t) = \text{Tr}[\rho^2(t)]$ and plot its time evolution in Fig. 2 from different initial states. In Fig. 2(a), we suppose $\Omega_a = 0.1g$, which meets the condition of $\Omega_a \ll g$, and other parameters for the calculation are chosen as (a) $\Omega_a = 0.1g$, $\omega = 0.05g$, $\Omega_e = 0.05g$, $\Delta = 10g$; (b) $\Omega_a = 0.05g$, $\omega = 0.05g$, $\Omega_e = 0.5g$, $\Delta = 20g$; (c) $\Omega_a = 0.05g$, $\omega = 0.025g$, $\Omega_e = 5g$, $\Delta = 100g$; (d) $\Omega_a = 0.05g$, $\omega = 0.025g$, $\Omega_e = 10g$, $\Delta = 200g$.

![Purity vs. time for different initial states](image)

**FIG. 2:** (Color online) The purity $P(t) = \text{Tr}[\rho^2(t)]$ is plotted as a function of time with initial states $|gg\rangle$ (black solid line), $|T\rangle$ (red dashed line) and $|ee\rangle$ (green dotted line), respectively. The spontaneous emission rate of atom is assumed as $\gamma = 0.1g$, and other parameters for the calculation are chosen as (a) $\Omega_a = 0.1g$, $\omega = 0.05g$, $\Omega_e = 0.05g$, $\Delta = 10g$; (b) $\Omega_a = 0.05g$, $\omega = 0.025g$, $\Omega_e = 0.5g$, $\Delta = 20g$; (c) $\Omega_a = 0.05g$, $\omega = 0.025g$, $\Omega_e = 5g$, $\Delta = 100g$; (d) $\Omega_a = 0.05g$, $\omega = 0.025g$, $\Omega_e = 10g$, $\Delta = 200g$.

The purity of entangled states tends to be the target state $|ee\rangle$ which is activated for a quantum state initialized in $|gg\rangle$, resulting in a shortcut to the steady state. In contrast, by decreasing $g/\omega$, the fidelity is improved for a weaker driving field $\Omega_a = 0.05g$ at the cost of increasing the convergence time. The empty circles simulate the dynamics from the effective master equation corresponding to the dash-dotted line.

![Fidelity vs. time for different initial states](image)

**FIG. 3:** (Color online) The dependence of the fidelity $F(t) = \langle S|\rho(t)|S\rangle$ on time is illustrated from the initial state $|gg\rangle$ for a perfect cavity and a leaky cavity, respectively. The other fixed parameters are set as $\omega = 0.05g$, $\Omega_e = 0.5g$, $\Delta = 10g$ for the solid line and dashed line, and $\omega = 0.025g$, $\Omega_e = 0.5g$, $\Delta = 20g$ for the dash-dotted line and dotted line. Compared with the case of $\Omega_a = 0.1g$, the fidelity is improved for a weaker driving field $\Omega_a = 0.05g$ at the cost of increasing the convergence time. The empty circles simulate the dynamics from the effective master equation corresponding to the dash-dotted line.

initial states are stabilized into the target state with less time but higher purity than ones in Fig. 2(a). To reveal the physical principle behind this phenomenon, we must take the single excitation from state $|ee\rangle$ into account on the basis of Eq. (4) as a correction, i.e. $H_I = \Omega_a|T\rangle\langle D| + \omega\sum_{i=1}^{2}|g\rangle_{ii}\langle i| + \lambda|ee\rangle\langle rr| + \sqrt{2}\Omega_a|ee\rangle\langle B| - g|B\rangle\langle S|a + H.c.$, where $|B\rangle = (|pe\rangle - |ep\rangle)/\sqrt{2}$, and the interaction part related to state $|ee\rangle$ could be rewritten as

$$H_{ee} = \Omega_a(e^{i\delta t}\Phi_+ + e^{-i\delta t}\Phi_-) + e^{-igt}(\Phi_+ + e^{igt}(\Phi_-))$$

with dressed states $|\Phi_+\rangle = (|ee\rangle \pm |rr\rangle)/\sqrt{2}$, and $|\Phi_-\rangle = (|B\rangle|0_e\rangle \mp |S\rangle|1_e\rangle)/\sqrt{2}$. Therefore, a fast resonant transition occurs between states $|ee\rangle$ and $|B\rangle$ for $\lambda = g$, which then decays into the stationary entangled state $|S\rangle$ via spontaneous emission.

To show the robustness of our scheme against the leakage of cavity, we introduce the fidelity defined as $F(t) = \langle S|\rho(t)|S\rangle$, and simulate its dependence on time from the initial state $|gg\rangle$ with a full master equation including cavity decay $\kappa$ in Fig. 3.

The solid line and the dash-dotted line correspond to the solid lines of Fig. 1(a) and 1(b), respectively. It can be seen that in the presence of $\kappa = 0.1g$, a choice of small $\Omega_a$ favors a high fidelity 93.88% though the convergence time is prolonged, and the effectiveness of quantum Zeno dynamics is again verified.

The above analysis implies that our scheme can actively make use of atomic decay and prevent the malevolent effect...
of cavity decay simultaneously. Thence it is possible to produce a high-fidelity steady entangled state for a wide range of decoherence parameters. In Fig. 4(a), the contour of the fidelity for entanglement is plotted after solving the steady-state master equation numerically. The dash-dotted line represent a $\sim 90\%$ fidelity corresponding to the single-atom cooperativity parameter $C = 10$, which is much smaller than the values required by previous schemes [9, 10]. In addition, in virtue of quantum-jump-based feedback control, the threshold of $C$ can be further lowered [37–39]. In this situation, the feedback master equation is $\dot{\rho} = -i[H_1, \rho] + L_{\text{sp}}\rho + \kappa D[U_{\text{fb}, a}\rho, \rho]$, where $U_{\text{fb}} = \exp[-i\gamma/2\pi |g\rangle \langle g|] \otimes I_2$ is the feedback operator acting on the first atom. In Fig. 4(b), the contours of the enhanced fidelity for feedback-based steady state is simulated and a small value of $C = 5.2$ is big enough for the system to keep $90\%$ fidelity, as illustrated by the dash-dotted line.

In experiment, the Rydberg antiblockade condition $U_{rr} = (2\Delta - \lambda)$ is not readily satisfied, since it is difficult to accurately adjust the distance between two Rydberg atoms. Consequently, a deviation of $U_{rr}$ will deteriorate the quality of final entanglement. In Fig. 5, we suppose the Rydberg interaction term $U_{rr} = (2\Delta - \lambda + \delta)$ and simulate the steady-state fidelity as a function of the deviation $\delta/\Delta$. Fortunately, the current scheme permits $\delta/\Delta \in (-0.16, 0.2)$ so as to preserve the fidelity more than $90\%$. For the current available parameters in the cavity QED with Rydberg-blocked atoms [40–43], the strength coupling the transition between atomic ground level $5S_{1/2}$ and the optical level $5P_{3/2}$ of $^{87}\text{Rb}$ atom to the quantized cavity mode is $g/2\pi = 14.4$ MHz, the decay rates of the intermediate state $|p\rangle$ is $\gamma_p/2\pi = 3$ MHz, the decay rate of the cavity mode is $\kappa/2\pi = 0.66$ MHz, and the spontaneous emission rate for the Rydberg state is $\Gamma/2\pi = 1$ KHz. By modulating the Rabi frequencies, detuning and Rydberg interaction strength to be values of Fig. 4, we obtain the fidelity of steady state $98.6\%$ ($99.0\%$ when the feedback is off on).

In summary, we have proposed a new mechanism for generation of bipartite entanglement in cavity QED system by dissipation. The convergence time of the stationary entangled state is speeded up via the effect of ground-state blockade and a $90\%$ fidelity can be achieved without resorting to a large value of $C$. Thus our scheme is capable of adapting a wide range of experimental parameters from the perspective of steady state. We hope that this work may open a new venue for the experimental realization of entanglement in the near future.

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