Spontaneous Spin Polarization of Quark Matter due to Tensor Selfenergies in NJL model

Tomoyuki Maruyama
College of Bioresource Sciences, Nihon University, Fujisawa 252-8510, Japan
E-mail: maruyama.tomoyuki@nihon-u.ac.jp

Toshitaka Tatsumi
Department of Physics, Kyoto University, Kyoto 606-8502, Japan

Abstract. We explore the spin-polarization of quark matter in the NJL model including the scalar and tensor interactions. There are two kinds of spin-polarized phases: one appears in the chiral-broken phase where the quark mass is non-zero, and the other in the chiral-restored phase where the quark mass is zero.

1. Introduction
The discovery of the magnetars, which is the neutron stars with strong magnetic field of $O(10^{15})$ G, revives the important question about the origin of the strong magnetic field [1]. The spontaneous spin polarization is one of the most possible candidates to explain such strong magnetic field. As an earlier work, Tatsumi [2] suggested a possibility of a ferromagnetic transition in quark matter interacting via one-gluon-exchange (OGE) force and showed that the maximum magnetic field can reach $B \sim O(10^{15-17}G)$ when the magnetar is a quark star.

In the relativistic framework the “spin density” can take the two forms [3], $\bar{\psi}\gamma^i\Sigma^i\psi(\equiv -\bar{\psi}\gamma_5\gamma^i\psi)$ and $\bar{\psi}\gamma^0\Sigma^i\psi(\equiv -\bar{\psi}\sigma^{12}\psi)$, with $\psi$ being the quark field. The former is a space-component of the axial-vector (AV) mean-field, and the latter is that of the tensor (T) one. These two mean-fields become equivalent to each other in the non-relativistic limit, while they are quite different in the ultra-relativistic limit (massless limit) [3]. In the text we shall call the former and latter polarizations the AV-type and T-type spin polarizations (SP), respectively.

When we introduce the AV interaction into the Nambu-Jona-Lasinio (NJL) model [4], then, the spin-polarized phase can appear only in a low density region just below the chiral phase transition (CPT) density, $\rho_c$ [5]. In contrast, the tensor-type spin polarizations (SP) can appear even if the quark mass becomes zero [6]. Then, the tensor-type SP can appear in the wide density region, especially in the mass-zero region.
2. Formalism

In order to examine the tensor-type SP we begin with the following NJL-type Lagrangian density with $SU(2)$ chiral symmetry,

$$
\mathcal{L} = \bar{\psi} \left( i \partial_\mu - m \right) \psi - \frac{G_s}{2} \left[ (\bar{\psi} \psi)^2 + (i \bar{\psi} \gamma_5 \tau \psi)^2 \right],
\quad - \frac{G_T}{2} \left[ (\bar{\psi} \sigma_{\mu \nu} \psi)(\bar{\psi} \sigma_{\mu \nu} \psi) + (i \bar{\psi} \tau_\alpha \gamma_5 \sigma_{\mu \nu} \psi)(i \bar{\psi} \tau_\alpha \gamma_5 \sigma_{\mu \nu} \psi) \right],
$$

where $\psi$ is a field operator of quark, $G_s$ and $G_T$ are the coupling constants for the scalar and tensor channels, respectively.

![Figure 1. The energy constant surfaces for $e - U_0 = 3M_q$ and $s = -1$, when $U_T = 3M_q$ (a) and when $U_A = 3M_q$ (b).](image)

In the present work, we restrict calculations and discussions to the flavor symmetric matter ($\rho_u = \rho_d$) at zero temperature. Within the mean-field approximation the quark Dirac spinor $u(p, s)$ is obtained as the solution of the following equation,

$$
\begin{bmatrix}
\hat{p} - M_q - U_0 \gamma^0 - U_T \Sigma_z
\end{bmatrix} u(p, s) = 0
$$

with $\Sigma_z = \text{diag}(1, -1, 1, -1)$ and

$$
M_q = -G_s \rho_s = -G_s < \bar{\psi} \psi >, \quad U_T = G_T \rho_T = G_T < \bar{\psi} \Sigma_z \psi >.
$$

The single particle energy is given by

$$
e(p, s) = \sqrt{\left( \sqrt{M_q^2 + p_x^2 + p_y^2 + sU_T} \right)^2 + p_z^2 + U_0}.
$$

where $p = (p_x, p_y, p_z)$, and $s = \pm 1$ indicates the spin of a quark.

It should be interesting to compare it with a single particle energy in the AV mean-field, $U_A$:

$$
e(p, s) = \sqrt{\left( \sqrt{M_q^2 + p_x^2 + sU_A} \right)^2 + p_y^2 + p_z^2 + U_0} = \sqrt{E_p^2 + 2sU_A \sqrt{M_q^2 + p_x^2 + p_y^2 + U_A^2} + U_0}.
$$

Here, we make a comment on the difference in the SP between the tensor and axial-vector interactions. When $U_T = 0$ and $U_A \neq 0$, we can obtain the above expression of $e(p, s)$ in Eq. (5)
from that in Eq. (4) by exchanging \( p_z \) and \( p_T \). The surfaces in the momentum space at the fixed energy have the same relation between the two types of SP. However, \( p_z \) is one-dimensional while \( p_T \) is the absolute value of the two dimensional vector.

In this work, we discuss only the case of \( G_T < 0 \) and the spin-polarized phase is isoscalar. However, we can apply the same argument to the isovector spin-polarized system for \( G_T > 0 \) and get the same strength of the SP for each quark.

In order to extract the vacuum part we use the proper time regularization (PTR) [?]. However, the vacuum part of the tensor density, strongly depends on the cut-off parameter \( \Lambda \) in the present model. So, in the NJL model it is not easy to apply a consistent method even for qualitative discussions. The lattice QCD calculation has shown that the negative magnetic susceptibility at the zero temperature limit is zero [7], and the vacuum contribution should be small in SP. Then, we perform actual calculation without the vacuum contribution for the tensor density.

3. Results

In this work we consider the chiral limit and use the parameter set, \( G_s \Lambda^2 = 6, \Lambda = 850 \text{ MeV} \), which are determined to reproduce the vacuum properties such as the pion decay constant, constituent quark mass, scalar condensate and so on [8]. In addition, we take the normal density to be \( \rho_0 = 0.17 \text{fm}^{-3} \) in the following calculations.

![Figure 2](image_url)

**Figure 2.** Upper panels (a,c): the T densities normalized with the normal nuclear matter density. The solid and dotted lines represent the results in the chiral broken and restored phases, respectively. Lower panels (b,d): the dynamical quark mass normalized with nucleon mass in the spin-polarized (solid lines) and spin-saturated phase (dashed lines). The left and right panes show the results when \( G_T/G_s = -1.2 \) (a,b) and \( G_T/G_s = -1.5 \) (c,d), respectively.
In Fig. 2, we show the baryon density \( \rho_B \) dependence of the tensor density \( \rho_T / \rho_0 \), and that of the dynamical quark mass (lower panels) when \( G_T = -1.2G_s \) (a,b) and \( G_T = -1.5G_s \) (c,d). In the upper panels the solid lines represent \( \rho_T / (N_c \rho_0) \) in the spin-polarized phase when \( M_q > 0 \), and the dotted lines indicate that when \( M_q = 0 \). In the lower panels, the solid and dashed lines represent the dynamical quark mass in the spin-polarized and spin-saturated phases, respectively.

These results suggest that there are two kinds of the spin-polarized phases: one is the chiral-broken SP (SP-I) which appears in the chiral-broken phase, \( M_q > 0 \), and the other is the chiral-restored SP (SP-II) which appears in the chiral-restored phase, \( M_q = 0 \).

We can see that two kinds of spin-polarized phases, SP-I and SP-II phases, appear. In addition, there is a density region where the three solutions corresponding to the spin-polarized phases,

When \( G_T = -1.5G_s \) (c,d), the SP-I phase appears at first, and the SP-II phase appears in the density region, \( \rho_B < \rho_c = 3.41\rho_0 \). Both the two SP phases exist in a same density region up to a density larger than the CPT density, \( \rho_c \), and the SP-I phase disappears at a density larger than \( \rho_c \), where \( M_q = 0 \) and \( U_T \neq 0 \).

The state with the minimum energy realizes among the spin-saturated, SP-I and SP-II phases. In this work, however, we discard the vacuum contribution to the tensor density and cannot define the total energy, so that we do not here discuss it.

In Fig. 3 we finally show the critical density between the spin-saturated and spin-polarized phases as a function of \( G_T M_N^2 \), where \( M_N \) is the nucleon mass. The solid and dashed lines represent the critical density normalized by the normal nuclera matter density \( \rho_0 \) in the chiral-broken phase, \( (M_q \neq 0) \) and that in chiral-restored phase, \( (M_q = 0) \), respectively. The critical density when \( M_q = 0 \) is determined only by \( G_T \), independently of \( G_s \).

![Figure 3](image-url)  
*Figure 3.* Critical density between the spin-saturated and spin-polarized phases as functions of \( G_T M_N^2 \) when \( M_q \neq 0 \) (solid lines) and \( M_q = 0 \) (dashed line).
4. Summary

We have studied the spontaneous SP of quark matter in the NJL model with the tensor interaction. The spin-polarized phase given by the tensor interaction remains even when the quark mass is zero. There appear two kinds of the spin-polarized phase, the SP-I and SP-II phases, where the dynamical quark mass is non-zero and zero, respectively. The SP-I phase appears when the T coupling $G_T$ is negatively large, but the SP-II phase can always appear.

The SP-I phase can exist in the density region above the CPT density and shifts the chiral transition to higher density. On the other hand, the SP-II phase can appear below the CPT density. The SP-I and SP-II phases can exist at the same density when $-G_T$ is large. Although we do not discuss the stability of each phase, we can easily suppose that the phase transition between the SP-I and SP-II phases is of the first order.

In this work we have made the discussion only when $G_T < 0$: the spin-polarized phase is isoscalar. When $G_T > 0$, the spin-polarized phase becomes isovector where the directions of the SP for $u$ and $d$ quarks are opposite. The strength of the induced magnetic field is much larger in the isovector spin-polarized phase than in the isoscalar spin-polarized phase because the charge of $u$ and $d$ quarks have opposite signs.

Realistically the current quark mass is non-zero, and the SC-I and SC-II phase are not distinguished, but qualitative behaviors must be the same. In most cases the SP-I phase must continuously connect with the SP-II phase. However, the spin-polarized phase may appear in two different regions as in Fig. 2a, and three SP states may appear at the same density as in Fig. 2b: the behaviors must depend on a value of $G_T$.

In this work, furthermore, we have not considered the AV channel of quark-quark interaction, which can be derived by the Fierz transformation of the one gluon exchange. The calculation of the spin-polarized phase is very difficult when both the T and AV interactions are introduced because the momentum distribution is very complicated. If the quark mass is zero, however, the calculation become much easier. The first author(T.M.) and his collaborators have published the results in Ref.[9]

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