The supermembrane with central charge
as a bundle of D2-D0 branes

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Abstract. We discuss the consistency of the $D = 11$ supermembranes with non zero central charge arising from a nontrivial winding CSNW. The spectrum of its regularized Hamiltonian is discrete ans its heat kernel in terms of a Feynman formula may be rigorously constructed. The $N \to \infty$ limit is discussed. Since CSNW is equivalent to a noncommutative supersymmetric gauge theory on a general Riemann surface, its consistency provides a proof that all of them are well defined quantum theories. We interpret the supermembrane with central charge $n$, in the type $IIA$ picture, as a bundle of D2 branes with $n$ units of D0 charge induced by a nonconstant magnetic flux.

1. Introduction

The nonperturbative contributions to the superstring theory arise from the quantization of M-theory in 11 dimensions. Related to it, is the problem of quantization of 11 dimensional supermembranes. Recent progress to this problem has been done in the context of the quantization of supermembranes with nontrivial central charge [1, 2, 3, 4, 5, 6, 7]. This is a well defined sector of the complete theory. This sector is also related, as we discuss in the paper, with noncommutative gauge theories over Riemann surfaces of genus $g \geq 1$. The quantum consistency of supermembranes with fixed central charges would provide an indirect proof of consistency of all these noncommutative gauge theories.

In order to realize the nontrivial central charge condition, there must be a minimal immersion from $\Sigma$ to the target space which implies that, for the case of flat target spaces, the worldvolume of the supermembrane is a calibrated submanifold of the target space. Calibrations, and its generalizations, have been extensively used to describe

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supersymmetric branes in supergravity backgrounds. Minimal imersions can also describe non-BPS minimal solutions.

Supermembranes are extended objects of 2+1 dimensions, which live in 11D. They were originally proposed over a $D = 11$ Minkowski target space as candidates to fundamental objects in the context of M-theory. The continuity of the spectrum lead to reinterpret them as a many body theory. This property relies on two basic facts: first, the presence of singular configurations with zero energy at classical level, and second, supersymmetry. Under compactification this behaviour seems to remains the same, although as far as we know a rigorous proof has not yet been done.

In the following we will summarize the main properties of the supermembrane with central charges. This theory exhibits a completely different behaviour from the previous one: it does not contain string-like spikes, its supersymmetric spectrum at quantum level is purely discrete, and its heat kernel is perfectly well defined in terms of a Feynman formula in contrast to the supermembrane on a Minkowski target space.

The supermembrane with central charge due to the winding is equivalent to a symplectic noncommutative Super Yang Mills theory over a Riemannian surface of genus $g \geq 1$. The theory is invariant under area preserving diffeomorphisms, which in 2 dimensions coincide with the symplectomorphisms. The symplectic 2-form arising from the central charges, for $g \geq 2$ cannot be globally expressed in terms of a constant 2-form (as in the case of $g = 1$). However, by using Darboux theorem, we can reduce the symplectic two-form to a constant antisymmetric tensor on each open set of a Darboux covering of $\Sigma$ and hence to have a conventional noncommutative gauge theory at each open set of the covering, all patched together by symplectomorphisms.

In the dual picture we will discuss its interpretation in terms of D2 branes with fluxes. Fluxes are induced by the nontrivial winding and not by turning on a constant 3-form which is reduced in ten dimensions to a constant B field. These fluxes charge the D2 branes with $n$ units of D0 monopole charge, appearing a system of D2-D0 branes over the manifold. This mechanism imposes constraints on the target dimension where this system of D2-D0 branes can appear. It is a well known fact that supersymmetry preserving a set of intersecting branes may merge to form a single brane on a smooth calibrated surface. It allows to interpret the supermembrane with central charge in the dual picture as intersecting branes which recombine over the target manifold in this bundle of D2-D0 branes. The scalars parametrizing the position of this bundle in the transverse space becomes massive through the recombination process as a kind of Higgs mechanism. The effect is a bundle of D2 brane charged over the compactification manifold.

In the case where there are several M2 branes CSNW minimally immersed in different 2-cycles contained in an enough rich target manifold (with singularities, cycles) we speculate that it can lead to phenomenological models. In this interpretation each of these objects is thought as a fundamental object over a particular nontrivial 2-cycle and propagating in a 4D Minkowski space-time. The interpretation of the wrapped supermembrane as a fundamental object was also anticipated by Kallosh, based on a semiclassical quantization of the supermembrane done in . This semiclassical analysis in the light of our results of the quantum spectrum recover sense as a limit.
2. Supermembrane with non-trivial central charge

We consider the $D = 11$ supermembrane in the light cone gauge [15]. In this gauge the potential is given by

$$V(X) = \{X^M, X^N\}^2 \quad M, N = 1, ..., 9$$

where

$$\{X^M, X^N\}^2 = \frac{\epsilon^{ab}}{\sqrt{W(\sigma)}} \partial_a X^M \partial_b X^N.$$  

(2)

The scalar density $\sqrt{W(\sigma)}$ is introduced in the formulation by the partial gauge fixing procedure [15] and will take it to be the volume of the minimal immersion which is going to be introduced shortly. $X^M, M = 1, ..., 9$ are maps from a Riemann surface $\Sigma$, a torus in the discussions of this setting, to the target space which we assume to be $M_7 \times S^1 \times S^1$, $M_7$ being a seven dimensional Minkowski space-time.

The generalization to more general target spaces and Riemannian surfaces has been considered in [6]. We take $X_r, r = 1, 2$ to be maps from $\Sigma$ to $S^1 \times S^1$ and $X^m, m = 1, ..., 7$ the maps from $\Sigma$ to $M_7$.

The maps $X_r$ satisfy the conditions

$$\oint_{c_i} dX_r = 2\pi m_{ri} \quad r = 1, 2$$

where $C_i$ is a basis of homology over $\Sigma$. These winding conditions ensure that each $X_r, r = 1, 2$ is a map from $\Sigma \to S^1$. In order that the image of $\Sigma$ by $X_r, r = 1, 2$ describe also a torus, one has to impose

$$Z = \int_{\Sigma} (dX_r \wedge dX_s)e^{rs} = 2\pi n \neq 0$$

(4)

where $n = det(m_{jr})$ and $r = 1, 2$ and also $j = 1, 2$ since $\Sigma$ is also a torus. We notice that the condition $Z = 2\pi n \neq 0$ corresponds to have a nontrivial central charge on the supersymmetric algebra of the supermembrane. Between all the maps from the torus $\Sigma$ to the target space satisfying (4) there is one which minimizes the hamiltonian of the supermembrane. It is realized in terms of the basis of harmonic one-forms over the torus $\Sigma$, which we denote $dX^r, r = 1, 2$.

Any one-form over $\Sigma$ may be rewritten as

$$dX^r = m^r_s d\hat{X}^s + \delta^r_s dA_s$$

(5)

where $A_s$ is a single-valued object over $\Sigma$ and $det m^r_s = 1$. Using the residual invariance, the area preserving diffeomorphisms not connected with the identity we may fix at $m^r_s = \delta^r_s$. The transverse coordinates to the supermembrane $X^m, m = 1, ..., 7$ we assumed to be single-valued over $\Sigma$ since they are valued on Minkowski $D=7$.

The hamiltonian of the supermembrane with nontrivial central charge may now be rewritten in terms of $X^m, m = 1, ..., 7$ and $A_r, r = 1, 2$. The resulting expression is:

$$H = \int_{\Sigma} \left[ \frac{1}{2} \sqrt{W} [P^2_m + \Pi^2_r + \frac{1}{2} W \{X^m, X^n\}^2 + W(D_rX^m)^2 + \frac{1}{2} W(F_{rs})^2] 
\right.
\left. + \int_{\Sigma} \frac{1}{8} \sqrt{W} n^2 - \Lambda(D_r \Pi + \{X^m, P_m\}) \right]
\left. + \int_{\Sigma} \sqrt{W} [-\Psi \Gamma_{-} \Gamma_r D_r \Psi + \overline{\Psi} \Gamma_{-} \Gamma_m \{X^m, \Psi\} + \Lambda \{\overline{\Psi} \Gamma_{-}, \Psi\} \right]$$

(6)
where $P_m$ and $\Pi_r$ are the conjugate momenta to $X^m$ and $A_r$ respectively. $\mathcal{D}_r$ and $\mathcal{F}_{rs}$ are the covariant derivative and curvature of a symplectic noncommutative theory \cite{1,4}, constructed from the symplectic structure $\epsilon^{ab}$ introduced by the central charge. The last term represents its supersymmetric extension in terms of Majorana spinors. $\sqrt{W}$ is the worldvolume constructed from the minimal immersion $X^r, r = 1, 2$ of $\Sigma \to S^1 \times S^1$. This is the requirement \cite{6}, in order to extend the construction to more general target spaces. The image of $\Sigma$ under the minimal immersion is a calibrated submanifold.

In the general situation where the genus of $\Sigma$ is $\geq 2$ the symplectic two-form may only be considered constant, via Darboux theorem, in an open neighbourhood $U$ over $\Sigma$ which cannot be extended globally to all $\Sigma$. Only in the case when $\Sigma$ is a torus the constant global extension is possible. The noncommutative theory we obtain may be thought as a set of noncommutative gauge theories generated by a constant $B$ field, on each open set $U$ of a Darboux covering of $\Sigma$, all patched together. There is no Seiberg-Witten \cite{16} limit involved. This is an interesting point since CSNW contain within its configuration space the superstrings with winding.

3. Discretness of the spectrum and the Heat Kernel

The hamiltonian \cite{6} may be regularized \cite{3} making use of the property, we have already commented, that the harmonic modes may be fixed by using the area preserving diffeomorphisms not connected to the identity \cite{6}. We are left with the area preserving diffeomorphisms connected to the identity which are generated by the Gauss constraint of the noncommutative theory.

Once the regularized hamiltonian has been obtained one may proceed to analyse its spectrum. The bosonic potential $V_B$ has the property that $V_B \to \infty$ as the point in the configuration space goes to $\infty$. Furthermore $V_B = 0$ implies $X = A = 0$. By $V_B$ we refer to the contribution of the noncommutative Yang-Mills, without including the constant arising from the contribution of the ground state. Those properties imply that the spectrum of the bosonic hamiltonian is discrete \cite{4}, moreover taking into account the structure of the fermionic contribution it is possible to show that the fermionic potential is a perturbation relatively bounded of $H_B$ \cite{7}, it is then compatible with it and the whole hamiltonian has also a discrete spectrum \cite{5}. The heat kernel for the supersymmetric regularized hamiltonian $H$ has been constructed in \cite{7}. A Feymann-Kac formula has been obtained and its convergence in terms of the strong operator topology has been shown.

An important result in \cite{7} is related to the constructions of the semigroup $\exp^{-tH}$ as an operator belonging to the $C_r$ Neumann-Schatten class, a Banach space, where $r \sim N$ is the integer associated to the $SU(N)$ truncation of the original theory.

Since the Banach spaces $C_r$ satisfy

$$C_1 \subset C_2 \subset ... \subset C_r \subset C_s \subset C_\infty \quad r < s$$

it is expected for the supersymmetric case that the $N \to \infty$ limit makes sense. The supersymmetry should play a fundamental role in this limit in order to balance the bosonic sector which manifestly $\exp^{-tH_B} \to 0$ when $N \to \infty$. This limit would extend rigourously our results of the regularized hamiltonian of the supermembrane with central charge to the original one. The limit of large $N$ would define the quantization of the supermembrane with central charge. We hope to report on this limit soon.
Figure 1. Description of a supermembrane in Minkowski space with an irreducible winding $n$ on a Riemann surface of the transverse space. In the dual picture it is equivalent to a bundle of D2 branes with $n$ units of D0 branes attached and patched together around the compact manifold.

4. The supermembrane with central charge as a bundle of D2-D0 branes

The supermembrane with nontrivial central charge exhibits the new properties that we have discussed in previous sections:

- No singular configurations at classical level. A noncommutative Super Yang-Mills theory defined at its worldvolume. Discrete supersymmetric spectrum at quantum level.
- A well defined heat kernel. Massive transverse scalar fields that lift this moduli. A BPS absolute minimum.

All of these properties differ substantially from the case of the $D = 11$ supermembrane. There are some characteristics of the model that could be emphasized to understand its properties. First, in the compactification process we do not restrict in any way the dynamics of the supermembrane, so for a given time, it remains being a bidimensional surface for the whole target space. Second, because of an appropriate handle of the multivalued forms associated to the compactification process, we are able to extend DWHN matrix model [13] to compactified spaces and obtain an exact regularization of the hamiltonian in terms of D0 branes [3]. All of the interaction terms have been taken into account and emerge naturally from the hamiltonian.

In the dual picture, the supermembrane with fixed central charge $n$ induced by the nontrivial winding on a torus, can be interpreted in terms of a D2 brane with $n$ units of nonconstant magnetic flux. This flux verifies the following properties,

$$F_2(\sigma) = \epsilon^{rs}dX^r(\sigma) \wedge dX^s(\sigma)$$
$$dF_2 = 0$$
$$\int_{\Sigma_2} d^2\sigma F_2 = 2\pi n$$

The nontrivial winding is the quantization condition for the flux induced by the dualized scalar fields $dX^r$ over the worldvolume. The global condition represents a $2\pi n$ RR Dirac monopoles. On a torus it can be interpreted as a system of D2-D0 branes with $2\pi n$ units of D0 charge. We would like to emphasize the physical origin of this flux in the context of M-theory as purely topological. It is important to remark that since this flux is not defined by any external background, this bundle of D2-D0 branes cannot exists in 10 dimensions but in 9 or less dimensions.

Formally the matrix model theory on a torus with a constant antisymmetric tensor field as in [17, 18] looks like the same as the one presented here. In spite of its formal analogies as we have previously discussed, there are important differences that affect not only to
the spectral properties of the theory but to the interpretation. Our bracket is not a
Moyal bracket but a non-constant symplectic one. At leading order in their background
expansion and for the case of a torus where the global analysis is possible, both models
coincide. In a more general compactification manifold, i.e. a Riemannian surface, this
is not possible anymore. There exists a bundle of D2-D0 branes over the manifold.
The Darboux theorem ensures the existence of a finite covering of the manifold such
that on each open set of the covering the supermembrane is equivalent to a D2
brane with constant $B$ field. These charts are glued by the gauge group of diffeomorphisms
preserving the area and can be seen as intersecting D2 branes with different constant
flux that have been recombined into an stable D2-D0 brane over the nontrivial topology.
The stability is assured by the stability of the nontrivially wrapped supermembrane.
In general, intersecting supersymmetric Dp-branes can decay under certain topologies
on a recombined metastable Dp brane by giving mass to scalar fields. In our case this
is exactly what happens in the sense that the presence of central charge or in its dual
version, the flux, gives mass to the transverse scalar fields that parametrize the position
of the compact D2 brane in the target space. Nevertheless in this case the condition of
stability is much stronger that in other ones, since it is assured by the stability of the
supermembrane with central charge. The recombination process has been used to break
symmetries of gran unified models in order to obtain more realistic ones, and they are
understood as a Higgs mechanism. Here the worldvolume gauge fields are the responsible
of this induced Higgs mechanism. As they are only defined on the worldvolume they do
not get in conflict with Poincare invariance.
There is another interesting interpretation that naturally emerge from our picture. The
supermembrane with central charge, can be understood because of its spectral properties
as a fundamental object. We speculate that it eventually could lead to reproduce part of
the spectrum that we observe. Phenomenological approach have been done in the context
of braneworlds. There, Dp-branes have $p \geq 3$ to reproduce Minkowski space where
strings propagate. The spectrum is generated by the strings whose ends are attached
in different Dp-branes as well as closed string sector carrying gravity. Here the point
of view is different, we are thinking about the possibility of taking, the supermembrane
with central charge, as a fundamental object propagating in the target space which is
restricted in 4D to be Minkowski.

In this direction [19, 20] studied wrapped $D2$ branes on $C^2/Z_n \times X$ spaces. It contains
N 2-cycles and leaves $N^2$ possible wrapping configurations of $D2$ branes. The wrapped
$D2$ branes around singularities become massless and are able to reproduce the gauge
sector. In other type of singularities it is also possible to obtain chiral matter. The
gravity sector is carried by closed strings.³. In the same spirit, we can consider several
$M2$ nontrivially wrapped over generic 2-cycles minimally immersed in the target space
that collapse around adequate singularities able to reproduce the matter spectrum. The
gravity sector is supossed to be carried by the supermembrane in 11D propagating in
the target space since the supermultiplet of supergravity is conjectured to be the ground
state of the supermembrane. This sector would contain instabilities so a further study
of the complete theory taking all of the sectors into account would be needed to have
good insight of the theory. We leave all of these questions for a future work.

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