Kinematic Modeling for the Nutation Drive Based on Screw Theory

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Abstract

The paper describes a study of a relative screw motion of a pair of internal and external spiral bevel gears with double circular-arc tooth profile in a nutation drive by screw theory and focuses on the kinematic modeling for the nutation drive. According to the instantaneous screw motion theory, the meshing function of the nutation drive for the kinematic modeling is developed based on the gear meshing theory and the coordinate system transformation between a crown gear and an internal or an external spiral bevel gear. The equations for the tooth profile of the crown gear, internal and external spiral bevel gears are further obtained based on the standard double circular-arc tooth profile and equal strength principles. Finally, the controllable precision three dimension modeling method for the tooth profile of the external and internal spiral bevel gears is further proposed.

Keywords: Screw theory; Spiral bevel gear; Nutation drive; Kinematic modeling

1. Introduction

Nutation motion is a kind of transmission proposed on the basis of the motion principle of celestial planet or gyroscopic. The principle of this rotation is used to create a kind of mechanical drive called the nutation drive. The kinematic diagram of one-stage nutation drive system shown in Fig. 1 is composed of an input shaft 3, external spiral bevel gear 1, fixed internal spiral bevel gear 2, equiangular speed ratio mechanism 4 and an output shaft 5. The external gear 1 meshing with the internal gear 2 is free to rotate about its axis, which is inclined to the axis of the input shaft.

As a kind of transmission form, the nutation drive has a broad development spaces in the field of robotics wrist and aerospace craft with the advantages of low noise, higher carrying capacity, steady transmission ability, higher transmission ratio and small volume. Thus, the nutation drive mechanism has great prospect development and research significance, leading wide research of experts [1,2].

For a kind of transmission, it is imperative to analysis the kinematic modelling and consequently many researches had been completed in this fields. Litvin et al analyzed the meshing contact of gear tooth surfaces [3]. Wang et al [4] presented generating line method to formed exact spherical involutes tooth surfaces of manufacturing spiral bevel gears. Duan [5] defined Bertrand curve and proposed the basic theory of Bertrand conjugate surface, given the design of loxodrome normal circular arc bevel gear. Yao et al [6,7] presented the analysis and modelling for nutation drives with double circular-arc helical bevel gears.

However the analysis methods of those literatures are complicated, so this paper proposes to use the screw theory to analysis the kinematic modelling of nutation drive.

The screw theory with the advantages of clear geometric concepts, clear physical meaning, simple expression and convenient algebraic operation has been wildly used in a
spatial mechanism for kinematic analysis [8,9] and can be easily combined with the vector, the matrix and the kinematic influence coefficient analysis method for further discussion. Dai et al [10] proposed to use the screw system to explore the compliance modeling and analysis of a 3-RPS parallel kinematic mechanism. Zhao et al [11] presented a force analysis of a 3-RPS parallel mechanism by utilizing the screw theory. Zhang [12,13] gave the screw and the relative screw motion of gears. But, there is no literature associated with the analysis of the nutation drive by using the screw theory.

The purpose of this paper is to propose a clear, simple and convenient kinematic modeling method for the nutation drive by the screw theory. The meshing function of a pair of internal and external spiral bevel gears with double circular arc tooth profile in a nutation drive is given by the screw method. The equations for the tooth profile of the crown gear, internal and external spiral bevel gears are obtained based on the standard double circular-arc tooth profile and equal strength principles. Finally, the controllable precision 3D modeling method for the tooth profile of the spiral bevel gears is further proposed.

2. Kinematic Modeling on Screw Theory

2.1. Meshing function of the Spiral Bevel Gear

In the transmission process of nutation drive, the tooth profiles of two mating spiral bevel gears are common tangent contact, called meshing contact, at any instant contacting curves. The meshing function is a relation among the geometric and kinematic parameters of the two mating spiral bevel gears to ensure the common tangent mating. The meshing parameters are the geometric and kinematic parameters of the two mating spiral bevel gears. In order to obtain the meshing function, screws $\mathbf{S}_1$ and $\mathbf{S}_2$ are proposed to present the known screw of two mating spiral gears respectively, and the screw $\mathbf{S}_3$ is the sum screw. To calculate the linear combination of the two screws, coordinate system is established and shown in Fig. 2. By setting Z-axis along the direction of the base tangent of two screws, the coordinate axis $X$ and $Y$ and coordinate origin $O$ can freely choose. The axis of $\mathbf{S}_1$ and $\mathbf{S}_2$ are intersect with Z-axis at point $A$ and $B$. The angle $\varphi$ present the angle between two known screws.

Based on the screw theory, a screw $\mathbf{S}_i$ can be divided into real unit $\mathbf{S}$ and dual unit $\mathbf{S}^\dagger$ of dual vector. According to the algorithms of screw, the real unit and dual unit of sum screw are the sum of original vector and dual vector of two screws respectively. The linear combination of two Screws can be obtained from [9] as

$$\mathbf{S}_3 = \mathbf{S}_1 + \mathbf{S}_2 $$

(1)

For any screw, can be given from as

$$\mathbf{S}_3 = \mathbf{S}_1 + e \mathbf{S}_1^\dagger = \mathbf{S}_1 + e (\mathbf{S}_{10} + h_1 \mathbf{S}_1) (i=1,2,3) $$

(2)

and then the original vector and dual vector of sum Screw are

$$\mathbf{S}_3^\dagger = \mathbf{S}_{30} + h_3 \mathbf{S}_3 = \mathbf{S}_{30} + h_3 (\mathbf{S}_1 + h_1 \mathbf{S}_1) $$

(3)

$$\mathbf{S}_3^\dagger = \mathbf{S}_{30} + h_3 \mathbf{S}_3 = \mathbf{S}_{30} + h_1 \mathbf{S}_1 + h_3 \mathbf{S}_2 $$

(4)

Where $h_i$ $(i=1,2,3)$ is the pitch of these screws, $\mathbf{S}_{30}$ $(i=1,2,3)$ is the moment of line.

From Fig.2, the line moment can be obtained as

$$\mathbf{S}_{30} = \mathbf{r}_i \times \mathbf{S}_i = a_i a_{12} \times \mathbf{S}_j (i=1,2,3) $$

(5)

Where $a_i (i=1,2,3)$ is the distance of screws to origin of coordinate. The unit vector $\mathbf{a}_{12}$ is the common normal of $\mathbf{S}_1$ and $\mathbf{S}_2$ with a unit value. The vector $\mathbf{r}_i (i=1,2,3)$ present the the distance $a_i (i=1,2,3)$, respectively.

Because the unit vector is the common normal of the two screws, so the sum of two screws is vertical to the common normal. And

$$\mathbf{S}_3 \cdot \mathbf{a}_{12} = 0 $$

(6)

The pitch of the sum screw can be obtained by the dot product between the $\mathbf{S}_3$ and equation (4) and expressed as

$$h_3 = h_3 S_{10}^2 + h_3 S_{12} S_{20} \cos \varphi + h_3 S_{12}^2 + h_3 S_2 S_1 \cos \varphi - (a_2 - a_1) S_3 S_1 \sin \varphi$$

$$S_3^2 + S_{12}^2 + 2 S_1 S_2 \cos \varphi$$

(7)

where $\varphi$ represents the angle between $\mathbf{S}_1$ and $\mathbf{S}_2$.

The cross product of between the $\mathbf{S}_3$ and equation (4) can be given as below

$$a_3 = (h_3 - h_1) S_3 S_2 \sin \varphi + (a_2 - a_1) S_2 S_1 \cos \varphi + a_2 S_{12}^2 + a_2 S_{20}^2$$

$$S_3^2 + S_{12}^2 + 2 S_1 S_2 \cos \varphi$$

(8)

Then the magnitude, location, pitch and direction of the sum screw can be determined by equations (3), (7) and (8).

Particularly, in the nutation drive, the cone vertex of the bevel gear pair is coincidence, and consequently the point $A$, $B$ and $C$ is coincidence, i. e. $a_1 = 0$ and $a_2 - a_1 = 0$. Compare with Fig. 1 and Fig. 2, the screw system of nutation drive are further illustrated in Fig. 3.

In nutation gear drive system, bevel gear pair meshes only for pure rotation, and the pitch of the screw is $h_3 = h_2 = 0$. Thus, the pitch of the screw and the distance between sum screw $\mathbf{S}_3$ and coordinate system $S(X,Y,Z)$ is $h_3 = 0$ and $a_3 = 0$.

The meshing between the external and internal spiral bevel gears in the nutation drive can be considered as the external and internal spiral bevel gear meshing with crown gear (an imaginary gear), which has a pitch cone angle of $90^\circ$, and the pitch cone is at right angle to its axis. The pitch cone angle of the internal bevel gear is larger than $90^\circ$, and the pitch cone angle of the external bevel gear is less than $90^\circ$. The motion between the crown gear and the external or internal bevel gear can be considered as the pure rolling of two pitch cones.
To establish the meshing function of spiral bevel gears, five coordinate systems are established in Fig. 4 and Fig. 5 to describe the meshing between the crown and external bevel gear and the meshing between the crown and internal bevel gear. This paper takes the meshing between crown gear and external spiral bevel gear as an example.

The screw $S_1$ and $S_2$ represent instantaneous screw of spiral bevel gear and crown gear that relative to the frame, respectively, and screw $S_1$ presents relative spiral of $S_1$ to $S_2$.

The fixed coordinate system $S_0(i_0,j_0,k_0)$ represents the original location of the external and internal spiral bevel gears attached coordinate system $S_k(i_k,j_k,k_k)$ ($k=1,2$) in Fig. 4 and Fig. 5.

The coordinate system $S_0(i_0,j_0,k_0)$ fixed to the crown gear, represents the original location of the crown gear rotatable system.

The coordinate system $S_1(i_1,j_1,k_1)$ is attached to the external spiral bevel gear and rotates about the axis $k_1$ of the centre of the external spiral bevel gear by angular speed $\omega_1$ in the Fig. 5, with $\phi_1$ represents the rotational angle of the external spiral bevel gear.

The coordinate system $S_2(i_2,j_2,k_2)$ is attached to the internal spiral bevel gear and rotates about the axis $k_2$ of the centre of the external spiral bevel gear by angular speed $\omega_2$ in the Fig. 5, with $\phi_2$ represents the rotational angle of the external spiral bevel gear.

The direction of screw $S_1$ is along the axis $k_1$ of the centre of spiral bevel gear, and the direction of screw $S_2$ is along the axis $k_2$ of the centre of the crown gear. For a pair of spatial meshing gears, not only rotating around the gear axis, but also moving along the axis. Therefore, the instantaneous screw motion of screws is

$$\begin{align*}
\sigma_k &= (\sigma_{k_1} + \sigma_{k_2}) + \omega_1 \times n_k + \omega_2 \times n_k \\
\end{align*}$$

Where $\sigma_{k_1}$ and $\sigma_{k_2}$ represent the movement speed along the axis $k_1$ and $k_2$, respectively.

To ensure the common tangent of the mating profiles, the meshing function can be expressed by the vertical relation between the common normal $c_n$ and relative speed $c_v$. The meshing function is given as,

$$\Phi = n_c \cdot v_c = 0$$

Where $\Phi$ represents the meshing function, $n_c$ is the common normal vector of the crown gear, and $v_c(k=1,2)$ is the vector of relative velocity of two meshing gears.

To ensure the common tangent of the mating profiles, the meshing function can be expressed by the vertical relation between the common normal $n_c$ and relative speed $v_c$. The meshing function is given as,

$$\Phi = n_c \cdot v_c = 0$$
The velocity of crown gear with respect to external spiral bevel gear can be obtained as

$$v_{cG} = \omega_1 \times r_{cG}$$

where $\omega_1 = -\sin \phi \phi_2 i_x + \sin \phi \cos \phi_2 j_x - \cos \phi k_x$

The transformation matrix from coordinate system 0 to 1 is

$$M_0^1 = \begin{bmatrix}
\cos \phi & \sin \phi & 0 & 0 \\
-\sin \phi & \cos \phi & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

(19)

The transformation matrix from $S_c$ to $S_1$ is

$$M_{cG}^1M_0^G = \begin{bmatrix}
a_{11} & a_{12} & a_{13} & 0 \\
a_{21} & a_{22} & a_{23} & 0 \\
a_{31} & a_{32} & a_{33} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

(20)

Where

$$a_{11} = -\cos \phi \sin \phi_2 - \sin \phi_1 \sin \delta_1 \sin \phi_2$$
$$a_{12} = -\cos \phi \sin \phi_1 - \sin \phi_1 \sin \delta_1 \cos \phi_2$$
$$a_{13} = \sin \phi_1 \cos \delta_1$$
$$a_{21} = \sin \phi \cos \phi_2 - \cos \phi \sin \phi_2 \sin \delta_1 \sin \phi_2$$
$$a_{22} = \sin \phi \sin \phi_2 + \cos \phi \sin \phi_2 \sin \delta_1 \cos \phi_2$$
$$a_{23} = \cos \phi \cos \delta_1$$
$$a_{31} = -\cos \delta_1 \sin \phi_2$$
$$a_{32} = \cos \delta_1 \cos \phi_2$$
$$a_{33} = -\sin \delta_1$$

The parameter $\delta_1$ represents the pitch cone angle of external spiral bevel gear.

Thus, combining equations (13) and (14), the relative velocity of crown gear with respect to external spiral bevel gear is

$$v_{cG} = \omega_1 \times r_{cG}$$

(15)

Similarly, the relative velocity of crown gear with internal spiral bevel gear can be obtained as

$$v_{cG} = \omega_2 \times r_{cG}$$

where $\omega_2 = -\cos \delta_1 \sin \phi_2 i_x - \cos \delta_1 \cos \phi_2 j_x$

The transformation matrix from coordinate system 1 to 2 is

$$M_1^2 = \begin{bmatrix}
b_{11} & b_{12} & b_{13} & 0 \\
b_{21} & b_{22} & b_{23} & 0 \\
b_{31} & b_{32} & b_{33} & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

(22)

Where

$$b_{11} = -\cos \phi \sin \phi_2 - \sin \phi_1 \sin \delta_2 \sin \phi_2$$
$$b_{12} = -\cos \phi \sin \phi_1 - \sin \phi_1 \sin \delta_2 \cos \phi_2$$
$$b_{13} = \sin \phi_1 \cos \delta_2$$
$$b_{21} = \sin \phi \cos \phi_2 - \cos \phi \sin \phi_2 \sin \delta_2 \sin \phi_2$$
$$b_{22} = \sin \phi \sin \phi_2 + \cos \phi \sin \phi_2 \sin \delta_2 \cos \phi_2$$
$$b_{23} = -\cos \phi \cos \delta_2$$
$$b_{31} = \cos \delta_2 \sin \phi_2$$
$$b_{32} = -\cos \delta_2 \cos \phi_2$$
$$b_{33} = -\sin \delta_2$$

2.2. Tooth Equation of the Double Circular-Arc Spiral Bevel Gear

As shown in Fig. 6, the basic tooth profile of the spiral bevel gears in the normal section is a double circular-arc profile, and adopts the profile of the model GB 12759-91 as the basic tooth profile, which consists of eight sections.

For the purpose of establishing the equation of the crown gear’s tooth profile, the coordinate system $S_c(x_c, y_c, z_c)$ is attached to the tooth profile in Fig. 6, and the coordinates of a point on the basic tooth profile can be illustrated in the coordinate system $S_n$ as
\[ r_{ai} = \left[ x_{ai}, y_{ai}, r_{ai} \right]^T = \left[ r_{ai} \cos \alpha_i + F_i, \right. \]
\[ \left. r_{ai} \sin \alpha_i + F_i, 0 \right] \]  

(24)

Where \( n(i=1, \ldots, 8) \) represents the number of arcs, \( \{E_i, F_i\} \) represents the circle centre of arc \( i \), \( r_{ai} \) presents the arc radius and \( \alpha_i \) presents the angle of arc \( i \).

Fig. 6. Basic tooth profile of the double circular-arc profile

As shown in Fig. 7, the coordinate system \( S_c(i, j, k_c) \) is attached to the crown gear and rotates about the axis \( k_c \) of the centre of the gear. And the curve \( p \) presents the actual boundary curve of the tooth profile; the curve \( p' \) presents the actual curve centre of boundary curve. The transformation matrix from tooth profile coordinate system \( S_c \) to the crown gear coordinate system \( S \) is given as

\[
M_c^s = \begin{bmatrix}
0 & \sin \theta \Delta \theta_j + \beta & 0 \\
0 & -\cos \theta \Delta \theta_j + \beta & 0 \\
1 & 0 & 1
\end{bmatrix}
\]

(25)

Thus, the equation of tooth profile of the crown gear can be obtained by combining equations (24) and (25).

\[
[r_{j,l}]^T = M_c^s [r_{ai}]^T = [x_j, y_j, z_j]^T
\]

\[
\begin{bmatrix}
\{x_j - [\tau(p_j - l_j)] \sin \theta - \Delta \theta_j + \beta + e^{\rho \beta} \cos \theta - \Delta \theta_j \} \\
\{y_j - [\tau(p_j - l_j)] \sin \theta - \Delta \theta_j + \beta + e^{\rho \beta} \cos \theta - \Delta \theta_j \} \\
1
\end{bmatrix}
\]

(26)

Where \( j (j=l, r) \) is presented left and right tooth profile.

The parameter \( \tau(p_j - l_j) \) is the distance of half width of the tooth thickness, where \( -\rho(j - l_j) \) represents the left profile and \( +\rho(j - l_j) \) represents the right profile. The rotating angle \( \theta \) is between two limiting values \( \theta \in (\theta_l, \theta_r) \).

The angle \( \theta_l \) at the small end can be represented as

\[
\theta_l = \ln r_{ai} \tan \beta
\]

(27)

and the radius \( r_{ai} \) can be represented as

\[
r_{ai} = \frac{m_s z_j}{2 \cos \beta}
\]

(28)

Where \( m_s \) is the module in the normal section of the bevel gear, \( z_j \) is the crown gear’s tooth number, and \( \beta \) is the helical angle.

The angle \( \theta_r \) at the big end of spiral bevel gear can be obtained by the following equation:

\[
\theta_r = \ln r_{ai} \tan \beta
\]

(29)

Where the rotating angle \( \theta \) can be represented as

\[
r_{ai} = \frac{R}{\sin \delta_i}
\]

(30)

The angle \( \Delta \theta_j \) can be determined by

\[
\Delta \theta_j = \theta_j - \pi / 2 \pm \Delta \phi_j
\]

(31)

Where the rotating angle \( \Delta \phi_j \) of the loxodrome is obtained by the following equation:

\[
\tan \Delta \phi_j = \frac{\rho_j}{r_{ai} \cos \beta}
\]

(32)

From Eq. (25), the normal vector in \( S_c \) is

\[
N_c = r_{ai}^l \times r_{ai}^r = \begin{bmatrix}
\frac{\partial x_j}{\partial \alpha_c} & \frac{\partial y_j}{\partial \alpha_c} & \frac{\partial z_j}{\partial \alpha_c} \\
\frac{\partial x_j}{\partial \theta} & \frac{\partial y_j}{\partial \theta} & \frac{\partial z_j}{\partial \theta}
\end{bmatrix}
\]

(33)

and the unit normal vector is given as

\[
\vec{n}_c = \frac{N_c}{|N_c|} = \begin{bmatrix}
\cos \alpha_c \sin \theta - \Delta \theta_j + \beta \\
\cos \alpha_c \sin \theta - \Delta \theta_j + \beta \sin \alpha_i
\end{bmatrix}
\]

(34)

Thus, combining Equations (11), (15), (16) and (34), the meshing function is

\[
\Phi = (-1)^{j} [E_c \cos \alpha_i - F_i \sin \alpha_i \pm (\rho_j - l_j) \sin \alpha_i \sin \theta - \Delta \theta_j + \beta - \phi_2] + (-1)^{j} e^{\rho \beta} \cos \theta - \Delta \theta_j - \phi_i \sin \alpha_i
\]

(35)

The meshing parameter \( \alpha_i \) between the crown gear and spiral bevel gears is expressed as

\[
\tan \alpha_i = \frac{E_c \sin \theta - \Delta \theta_j + \beta - \phi_2}{[F_j - \tau(p_j - l_j)] \sin \theta - \Delta \theta_j + \beta - \phi_2 + e^{\rho \beta} \cos \theta - \Delta \theta_j - \phi_i}
\]

(36)

The tooth equations of the spiral bevel gear in their fixed rotatable coordinate system \( S_k \) are obtained by matrix transformation as

\[
r_k = (x_k, y_k, z_k) = M_c^s (x_i, y_i, z_i, l)^T (i, j, k_c)
\]

(37)

By combining equations (20), (22), (26) and (37), the detail is given as
2.3. The Controllable Precision Modeling for The Tooth Profile

The 3D models of tooth profile of spiral bevel gear are created using data processing software to verify the mathematical equations. By importing the equation (38) into MatLab software system, the profile of the spiral bevel gear can be plotted by selecting the major design parameters, and Fig.8 presents a three-dimensional model of the tooth profile of the spiral bevel gear. Moreover, the modelling precision of the 3D tooth profile model can be controlled by grid resolution.

![](image)

(a) The grid number is 100x25  
(b) The grid number is 100x100

Fig. 8. Mathematical model of tooth profile

By importing the 3D coordinates data of the tooth profile into the 3D entity modelling software such as Solidworks, the 3D models of tooth profile of spiral bevel gear had been obtained. The meshing surface and mathematical equations of the tooth profiles of double-circular-arc spiral bevel gears had been developed. These mathematical equations were then verified by three dimension modelling software and by creating controllable precision 3D tooth profile model and 3D entity model. It is easier to express the meshing relationship between the meshing gears and the meshing function is more concise by introducing the screw theory in the kinematic modelling process for the spiral bevel gears. The proposed tooth profile equations of double-circular-arc spiral bevel gears could be applied in the establishing of the controllable precision modelling in the nutation drive.

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