Odd-parity multipole fluctuation and unconventional superconductivity in locally noncentrosymmetric crystal

Jun Ishizuka\(^1\) and Youichi Yanase\(^1\)

\(^1\)Department of Physics, Graduate School of Science, Kyoto University, Kyoto 606-8502, Japan

(Dated: July 17, 2018)

A microscopic calculation and symmetry argument reveal superconductivity in the vicinity of parity-violating magnetic order. An augmented cluster magnetic multipole order in a crystal lacking local space inversion parity may break global inversion symmetry, and then, it is classified into an odd-parity multipole order. We investigate unconventional superconductivity induced by an odd-parity magnetic multipole fluctuation in a two-dimensional two-sublattice Hubbard model motivated by Sr\(_2\)IrO\(_4\). We find that even-parity superconductivity is more significantly suppressed by a spin-orbit coupling than that in a globally noncentrosymmetric system. Consequently, two odd-parity superconducting states are stabilized by magnetic multipole fluctuations in a large spin-orbit coupling region. Both of them are identified as \(Z\) topological superconducting states. The obtained gap function of inter-sublattice pairing shows a gapped/nodal structure protected by non-symmorphic symmetry. Our finding implies a new family of odd-parity topological superconductors. Candidate materials are discussed.

I. INTRODUCTION

Recent intensive researches have clarified intriguing effects of a spin-orbit coupling in locally noncentrosymmetric (NCS) crystals [1–8]. The locally NCS crystal preserves global inversion symmetry in the crystal structure, although inversion symmetry on a local atomic site is lacked. A sublattice-dependent antisymmetric spin-orbit coupling (ASOC) appears in locally NCS crystals, and it may induce exotic superconductivity distinct from well-studied globally NCS superconductivity [9–24]. For instance, singlet-triplet multipole order is ubiquitous in materials. For instance, BaMn\(_2\)As\(_2\) [28, 48], Sr\(_2\)IrO\(_4\) [39, 40, 49–62], Cd\(_2\)ReO\(_7\) [33, 63–71], and SrTiO\(_3\) [72–74] have been studied from the viewpoint of odd-parity fluctuation-induced superconductivity from a microscopic point of view.

Another topic of recent interest in locally NCS crystals is an odd-parity electromagnetic multipole order [25, 28, 32–43] which spontaneously breaks the global inversion symmetry by an anisotropic spin and charge distribution. Although previous studies provided profound understanding of even-parity multipole order in strongly correlated electron systems [44–47], it is recently recognized that odd-parity electromagnetic multipole order is ubiquitous in materials. For instance, BaMn\(_2\)As\(_2\) [28, 48], Sr\(_2\)IrO\(_4\) [39, 40, 49–62], Cd\(_2\)ReO\(_7\) [33, 63–71], and SrTiO\(_3\) [72–74] have been studied from the viewpoint of odd-parity multipole order. More recently, more than 110 AFM compounds are identified as odd-parity magnetic multipole states by group-theoretical analysis [75]. For those compounds, a multipole moment in the unit cell (augmented cluster multipole) has an odd-parity and leads to parity violation.

Superconductivity near the odd-parity electromagnetic multipole order invokes an unconventional pairing mechanism induced by an odd-parity multipole fluctuation. However, theoretical studies based on microscopic models have not been conducted except for a few works on electric multipole fluctuation [76–78]. Because the AFM order in locally NCS crystals with the sublattice-dependent ASOC realizes the odd-parity magnetic multipole order [75], our study of fluctuation-induced superconductivity naturally reveals the superconductivity due to the magnetic odd-parity multipole fluctuation. The pairing interaction and the resulting superconducting state may be different from those of conventional magnetic-fluctuation-induced superconductivity. Therefore, a new platform of topological superconductivity may be found in this study.

Previous theories based on the random phase approximation (RPA) have investigated the superconductivity induced by AFM fluctuation in globally NCS crystals [15–18, 79]. In this paper, we clarify a peculiar superconducting state and magnetic multipole fluctuation in a locally NCS crystal with the same approximation. To be specific, we analyze a two-sublattice Hubbard model with a sublattice-dependent ASOC. The crystallographic point group is centrosymmetric \(D_{4h}\) and the local site symmetry is \(D_{2d}\) lacking inversion symmetry. This is a minimal model taking account of the locally NCS structure, spin-orbit coupling, and odd-parity magnetic multipole fluctuation. For instance, BaMn\(_2\)As\(_2\) and Sr\(_2\)IrO\(_4\) are captured by this model from the viewpoint of symmetry.

The seemingly conventional G-type AFM order in our model shows unbroken translation symmetry because of the two-sublattice structure peculiar to locally NCS crystals. Magnetic propagation vector is indeed \(q = 0\). Instead of the translation symmetry, the space inversion symmetry is broken. Therefore, the
AFM order is regarded as an odd-parity magnetic order. From the group theoretical study [28], the magneto-electric multipole moment has been classified based on the point group $D_{4h}$. The magnetic multipole order in the AFM state with $m \parallel c$ belongs to $B_{2g}$ irreducible representation (IR). The candidates of order parameter are identified as magnetic quadrupole moment and hexadecapole moment. On the other hand, the G-type AFM order with $m \perp c$ corresponds to the magnetic quadrupole and toroidal order. Our calculation takes into account all these magnetic multipole fluctuations. This work will be the first microscopic study of unconventional superconductivity induced by the odd-parity magnetic fluctuation.

This paper is constructed as follows. In Secs. II and III, symmetry operations for pair amplitudes with sublattice degree of freedom are revealed. We clarify the symmetry properties in the present crystal structure. Since our model preserves nonsymmetric crystal symmetry, the pair amplitudes have peculiar structures. Sec. IV introduces a two-sublattice Hubbard model with spin-orbit coupling and provides the formula of the microscopic calculation based on the RPA and Eliashberg equation. Numerical results are shown in Secs. V and VI. In Sec. V, we show the odd-parity magnetic fluctuation and its anisotropy. Effects of the ASOC on the magnetic fluctuations are discussed. In Sec. VI, we identify four stable pairing states which are distinguished by symmetry. Effects of the ASOC in locally NCS crystals are compared with those in globally NCS crystals. It is demonstrated that the local parity violation prefers the odd-parity superconductivity. Therefore, the Z2-topological odd-parity superconductivity in DIII class is stabilized in a large ASOC region. A brief summary and discussion are given in Sec. VII.

II. SYMMETRY OF SUPERCONDUCTIVITY IN MULTI-SUBLATTICE SYSTEMS

For the classification of pair amplitudes in multicomponent superconductors, we need to take into account internal degrees of freedom of electrons which were neglected in classical theories summarized by Sigrist and Ueda [80]. For instance, multi-orbital systems have been analyzed in Ref. 81. We here classify the systems with sublattice degree of freedom.

To study the locally NCS superconductors, it is important to clarify the inter-sublattice and intra-sublattice pairing amplitudes. A complete classification is given by introducing the permutation of sites. We study a single-orbital model for simplicity. An extension to multi-orbital and multi-sublattice systems is straightforward by considering the permutation of local orbitals.

A creation operator of a Bloch state $c_{m,s}^{\dagger}$ with spin $s$ on sublattice $m$ is transformed by a space group operation:

$$g c_{m,s}^{\dagger} g^{-1} = \sum_{R} g c_{(R + r_m)g}^{\dagger} e^{-i k \cdot R}$$

$$= \sum_{R,s'} \epsilon_{s'}^{\dagger} (p(R + r_m) + a) D_{s's}^{(1/2)}(p) e^{-i k \cdot R}$$

$$= \epsilon^{ipka} \sum_{s'} \epsilon_{s'}^{\dagger} D_{s's}^{(1/2)}(p) e^{-i p(k(r_{gm} - pr_m))}.$$ 

(1)

Here, $R$ is a basic lattice vector and $r_m$ is a relative coordinate of $m$-th sublattice in a unit cell. The operation $g = \{p | a \}$ is defined by a conventional Seitz space group symbol with a point-group operation $p$ and a translation $a$. By choosing a representation matrix indicating the permutation of sites as

$$D_{m'm}^{(\text{perm})}(g; k) = e^{-i p(k(r_{gm} - pr_m))} \delta_{m',gm},$$

(2)

the transformation is simply represented as

$$g c_{m,s}^{\dagger} g^{-1} = \epsilon^{ipka} \sum_{m',s'} \epsilon_{s'}^{\dagger} D_{s's}^{(1/2)}(p) D_{m'm}^{(\text{perm})}(g; k).$$

(3)

In a superconducting state, a pair amplitude is defined as

$$F_{msm's'}(k) = \langle c_{m,s}^{\dagger} c_{m's'} \rangle,$$

(4)

where $\langle \ldots \rangle$ denotes the thermal average. The fermion antisymmetry gives

$$F_{msm's'}(k) = -F_{ms's'm}(-k).$$

(5)

By Eqs. (3) and (4), the pair amplitude is translated as

$$g F_{msm's'}^{\Gamma_i}(k) g^T = \sum_{m_1m_2s_1s_2} F_{m_1s_1m_2s_2}^{\Gamma_i}(pk)$$

$$\times D_{s_1s_2s's}(1/2)(p) D_{m_1m_2m'm}^{(\text{perm})}(g; k) D_{1}(g; \Gamma_i).$$

(6)

Here, the corresponding representation matrix is

$$D_{m'm_1m_2m'm'}^{(\text{perm})}(g; k) = e^{ip(k(r_{gm} - pr_m))} e^{-i p(k(r_{gm'} - pr_{m'}))}$$

$$\times \delta_{m_1,gm} \delta_{m_2,gm'},$$

(7)

and $D_{1}(g; \Gamma_i)$ is the representation matrix of the $\Gamma_i$ IR of the gap function, whose characters are explicitly given in Table I for the $D_{4h}$ point group. Note that $D_{1}(g; \Gamma_i)$ is unity for $m = m'$. When the total Hamiltonian commutes with the space inversion operator $I$, the pair amplitude possesses an even-parity (odd-parity), namely

$$I F_{msm's'}^{\Gamma_i}(k) = (\pm) F_{msm's'}^{\Gamma_i}(k).$$

When local symmetry on each sublattice has inversion symmetry, $Im = m$, the spin-singlet and spin-triplet pairing states are distinguished by the intra-sublattice pair amplitude $IF_{msm's'}^{\Gamma_i}(k) = (\pm) F_{msm's'}^{\Gamma_i}(k)$. In the
absence of global inversion symmetry, the space inversion parity is not a good quantum number, and the parity mixing between the singlet and triplet channels occurs. On the other hand, in locally NCS superconductors the parity mixing appears in a different way. Consequently, the superconducting gap function shows a non-trivial and symmetry-protected structure. We focus on this case in the next section. Hereafter, we assume even-frequency and even-orbital pairings, which are thermodynamically stable states.

III. PAIR AMPLITUDE IN LOCALLY NONCENTROSYMMETRIC CRYSTAL

For a demonstration, we introduce a typical crystal structure lacking local inversion symmetry and examine the symmetry properties of pair amplitudes. The example considered throughout this paper is a tetragonal crystal lattice with two sublattices each of which lacks the local inversion symmetry. In a crystal structure of Sr$_2$IrO$_4$, which is depicted in Fig. 1(a), oxygen ions out of the Ir-O layer violate inversion symmetry at an Ir ion. Similar local parity violation appears in iron-based superconductors [Fig. 1(b)], whose zigzag structure of the pnictogen or chalcogen ions also breaks the local inversion symmetry owing to the absence of $\sigma_v$ mirror symmetry. Hereafter, we study the crystal structure in Fig. 1(a). A coset decomposi-

\[
G = \{E|0\} T + \{I|0\} T + \{2_z|\tau_x + \tau_y\} T
\]

\[
+ \{2_z|\tau_x\} T + \{2_y|\tau_x\} T + \{4_1^+|\tau_x\} T + \{4_2^-|\tau_y\} T
\]

\[
+ \{\sigma_x|\tau_x\} T + \{\sigma_y|\tau_x + \tau_y\} T
\]

\[
+ \{\sigma_{110}|\tau_x + \tau_y\} T + \{\sigma_{1-10}|0\} T
\]

\[
+ \{2_{110}|0\} T + \{2_{1-10}|\tau_x + \tau_y\} T
\]

\[
+ \{I4_4^+|\tau_x\} T + \{I4_4^-|\tau_y\} T
\],

where the translation group $T$ defines a Bravais Lattice, and $\tau_x = \frac{2\pi}{a} e_a$, $\tau_y = \frac{2\pi}{a} e_b$ are non-primitive translation vectors.

Let us consider the glide reflection $G_g = \{\sigma_y|\tau_x\}$. A Bloch state $\psi_{kms}^i$ is transformed as $(k_x, k_y, k_z) \rightarrow (k_x, -k_y, k_z)$, $(\sigma_x, \sigma_y, \sigma_z) \rightarrow (-\sigma_x, \sigma_y, -\sigma_z)$, and sublattice indices $(a, b) \rightarrow (b, a)$, and then the representation matrices are given by $D_{s'x/2}^{s/2}(\sigma_y) = i\sigma_y \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and $D_{m'm}^{(\text{perm})}([\tau_x, \tau_y]; k) = \begin{pmatrix} e^{-ik_{y}/2-k_{y}} & 0 \\ 0 & e^{-ik_{x}/2-k_{x}} \end{pmatrix}$. Therefore, $G_g$ gives a relative phase factor $e^{-ik_{x} z}$ between the two sublattices. The inter-sublattice hopping is forbidden at the zone face $k_x = \pm \pi$ owing to this phase factor.

The space group $G$ in Eq. (8) can be reduced to a subgroup $G_{\text{intra}}$ by restricting to sublattice-conserving operations (see Table II)

\[
G_{\text{intra}} = \{E|0\} T + \{2_z|\tau_x + \tau_y\} T
\]

\[
+ \{2_z|\tau_y\} T + \{2_y|\tau_x\} T
\]

\[
+ \{\sigma_{110}|\tau_x + \tau_y\} T + \{\sigma_{1-10}|0\} T
\]

\[
+ \{2_{110}|0\} T + \{2_{1-10}|\tau_x + \tau_y\} T
\]

\[
+ \{I4_4^+|\tau_x\} T + \{I4_4^-|\tau_y\} T
\],

From $G_{\text{intra}}$, we notice that the local site symmetry is $D_{2d}$. Thus, the parity mixing in intra-sublattice pair amplitudes is allowed by the symmetry reduction $D_{2d} \rightarrow D_{2d}$, which is determined by the compatibility relation shown in Table III. Note that this simple rule is only applicable to the intra-sublattice components.

For instance, the $B_{2g}$ IR mixes with the $A_{2u}$ IR because they are reduced to the same $B_2$ IR in $D_{2d}$. The admixture is, however, different from that in the globally NCS superconductors; (1) one of the admixed components has a staggered form between sublattices for an intra-sublattice pairing, and (2) a parity mixing in inter-sublattice components is forbidden, to preserve the global inversion symmetry. These properties of the pair amplitude in locally NCS superconductors are derived from Eq. (6); the inversion symmetry represented by $D_{s'x/2}^{s/2}(\sigma_y) = I_{4} \times 4$ and $D_{m'm}^{(\text{perm})}([\tau_x, \tau_y]; k) = \begin{pmatrix} 0 & \tau_x \\ \tau_y & 0 \end{pmatrix}$ imposes a constraint.
for intra- and inter-sublattice pair amplitudes

$$I F^\Gamma_{aαβα'}(k)y)I = \pm I F^\Gamma_{aαβα'}(-k) = \mp I F^\Gamma_{aαβα'}(k),$$

$$I F^\Gamma_{aαβα'}(k)y)I = \pm I F^\Gamma_{aαβα'}(-k) = \mp I F^\Gamma_{aαβα'}(k).$$

The sign ± corresponds to the even-parity and odd-parity superconductivity, respectively. Equation (10)

shows the staggered form of admixed spin-triplet (spin-singlet) components between sublattices. The

formula (11) prohibits the parity mixing in the inter-sublattice pair amplitude.

Recent progresses on the group-theoretical analysis of superconductivity have shown unusual nodal/gapped structure ensured by nonsymmorphic symmetry [82–89]. Similarly, we show peculiar structures of the pair amplitude. The space group symmetry of our model (8) is nonsymmorphic since it contains the glide symmetry. Let us consider the glide symmetry $G_x = \{\sigma_x|\tau_y\}$ represented by

$$D_{1/2}^{(1)}(\sigma_x) = \begin{pmatrix} -\sigma_x & -e^{-ik_y} \\ \sigma_x & e^{ik_y} \end{pmatrix},$$

$$D_{m_1m_2mm'}^{(perm)}(\{\sigma_x|\tau_y\}) = \begin{pmatrix} \sigma_x & \mp e^{-ik_y} \\ \mp e^{ik_y} & 1 \end{pmatrix}. \quad (13)$$

We especially focus on inter-sublattice Cooper pairs on the zone face $k_y = \pm \pi$ at $k_z = 0$. On this high

symmetry line, $G_x$ imposes a constraint,

$$G_x F_{a\uparrow\downarrow}(k_x, \pi, 0)G_x^T = e^{-i\phi} D^\Gamma_{a\uparrow\downarrow}(k_x, \pi, 0). \quad (14)$$

Therefore, it is indicated that inter-sublattice gap functions must be zero for glide-even superconducting states. For instance, we find that the inter-sublattice pair amplitudes in the $A_{1g}$ IR ($s$-wave state) and $B_{1g}$ IR ($d_{zz}-y^2$-wave state) have nodal lines at $k_z = 0$ and $k_{x,y} = \pm \pi$. On the other hand, the pair amplitude is finite for glide-odd IRs such as $B_{2g}$ IR ($d_{xy}$-wave state). These features are opposite to those expected from group-theoretical analysis of symmorphic superconductors [80].

In the following sections, we study superconductivity in a model preserving the space group symmetry (8). The gap functions obtained by numerical calculations satisfy the symmetry constraints which have been revealed in this section.

## IV. MODEL AND METHOD

In this section, we introduce a two-dimensional two-sublattice Hubbard model which was adopted for Sr$_2$IrO$_4$ [39]. We consider a two-dimensional IrO$_2$ plane of quasi-two-dimensional Sr$_2$IrO$_4$. The crystal structure has been illustrated in Fig. 1(a). We do not restrict our discussions to Sr$_2$IrO$_4$ and later propose some other candidate materials. However, it is significant to study a well-studied model for Sr$_2$IrO$_4$ as a typical example and to illustrate the effects of spin-orbit coupling and multipole fluctuations.

Total Hamiltonian is written as $H = H_0 + H_{\text{int}} + H_{\text{ASOC}}$. The Hamiltonian of kinetic energy terms is

$$H_0 = \sum_{m \neq m'} \varepsilon_0(k)c^\dagger_{km}c_{km'} + \sum_{kk'} \varepsilon_2(k)c^\dagger_{km}c_{km'}, \quad (15)$$

where $c^\dagger_{km}$ is the annihilation (creation) operator of an Ir-5d electron with pseudo-spin $s$ on sublattice $m = (a, b)$. The pseudo-spin corresponds to the $j_{eff} = 1/2$ doublet states formed by a strong spin-orbit coupling [50]. The single electron kinetic energy is described by taking into account the nearest- and next-nearest-neighbor hoppings,

$$\varepsilon_1(k) = -t_1(1 + e^{iks})(1 + e^{-iks}),$$

$$\varepsilon_2(k) = -2t_2(cos k_x + cos k_y). \quad (16)$$

The on-site Coulomb interaction on an Ir site is given by

$$H_{\text{int}} = U \sum_{m} n_{m\uparrow}n_{m\downarrow}. \quad (18)$$

The ASOC term is written as

$$H_{\text{ASOC}} = \sum_{m} \mathbf{g}(k) \cdot \mathbf{\sigma}_{ss'}^{m'} c^{\dagger}_{km} c_{km'}^{s'}, \quad (19)$$

where $\mathbf{g}(k)$ is the ASOC parameter.
where we consider the staggered ASOC arising from the spin-dependent intra-sublattice hopping,
\[
g(k) = \sin k_x \cos k_y \hat{x} - \sin k_y \cos k_x \hat{y}.
\] (20)

Superconductivity in this model is investigated by solving the Eliashberg equation,
\[
\lambda \Delta_{\xi \xi}(k) = -\frac{T}{N} \sum_{k'} \sum_{\xi' \xi''} V_{\xi \xi'}(k-k') \times G_{\xi' \xi''}(k') \Delta_{\xi' \xi''}(k'),
\] (21)
where \( \hat{G}(k) \equiv [(i\varepsilon_{m} - \mu)\hat{1} - \hat{H}(k)]^{-1} \) and \( i\varepsilon_{m} = i(2m + 1)\pi T \) is fermionic Matsubara frequency. Here, we use abbreviated notations \( k = (k, \varepsilon_m) \) and \( \xi = (m, s) \). In the RPA, effective pairing interaction is described by the generalized susceptibility in the \( 8 \times 8 \) matrix,
\[
\hat{V}(q) = -\hat{\Gamma}^{\alpha}_{\xi}(q)\hat{\Gamma}^{\alpha}_{\xi} - \hat{\Gamma}^{\alpha}_{\xi}(q).
\] (22)

In the two-sublattice single-orbital model, the bare irreducible vertex is obtained as
\[
\Gamma_{m_{s}m_{s}',m_{m}m_{m}'}^{s_{m}m_{m}'} = \frac{1}{2} \Gamma_{m_{s}m_{s}'}^{s_{m}m_{m}'} \sigma_{s_{m}m_{m}'} \cdot \sigma_{s_{m}m_{m}'} - \frac{1}{2} \Gamma_{m_{m}m_{m}'} \delta_{s_{m}m_{m}'} \delta_{s_{m}m_{m}'}.
\] (23)
and \( \hat{\Gamma}_{m_{m}m_{m}'}^{s_{m}m_{m}'} = U\delta_{m_{m}m_{m}'} \). The RPA susceptibility is given by
\[
\hat{\chi}(q) = \chi^{(0)}(q) \left[ 1 - \hat{\Gamma}^{\alpha}_{\xi}(q)\hat{\Gamma}^{\alpha}_{\xi}(q) \right]^{-1},
\] (24)
where the irreducible susceptibility is defined as \( \chi^{(0)}(q) = -(T/N) \sum_{k} \hat{G}(k+q)\hat{G}(k) \). Now we introduce magnetic susceptibilities
\[
\chi_{m_{m}m_{m}'}^{\mu\nu}(q) = \sum_{s_{m}s_{m}'} \sigma_{s_{m}m_{m}'} \chi_{m_{s}m_{s}m_{m}m_{m}'}(q) \sigma_{s_{m}m_{m}'}^\nu,
\] (25)
where \( \mu, \nu = x, y, z \). The magnetic fluctuation parallel (perpendicular) to the \( c \)-axis \( \chi_{m_{m}m_{m}'}^{\parallel}(q) \) is characterized by \( \chi_{m_{m}m_{m}'}^{\parallel}(q) \equiv \chi_{m_{m}m_{m}'}^{\parallel}(q) \) \( \chi_{m_{m}m_{m}'}^{\parallel}(q) \). We define the band filling \( n \) as the number of electrons per unit cell (e.g. \( n = 4 \) for full filling). The doping level \( x \) is related to the band filling as \( n = 2+2x \). A variational Monte Carlo study [60] for \( \text{Sr}_2\text{IrO}_4 \) shows that the \( d \)-wave superconducting state is stable near the doping level \( x = 0.2 \). Thus, we study \( x = 0.2 \) unless mentioned otherwise. We also discuss the result in the undoped case \( x = 0 \). We set \( (t_1, t_2) = (1.0, 0.26) \), \( 64 \times 64 \) k-point meshes, and 1024 Matsubara frequencies in the numerical calculations.

V. MAGNETIC FLUCTUATION

First, we study the magnetic fluctuation. When the ASOC is absent, the magnetic anisotropy does not exist, and therefore we have \( \chi^{\parallel} = \chi^{\perp} \). Then, the two sublattices are equivalent, and the model is equivalent to the ordinary single-sublattice Hubbard model which has been studied for a long time [90]. Near half-filling, the AFM fluctuation with a wave vector around \( q = (\pi, \pi) \) is enhanced. On the other hand, when the ASOC is turned on, the two sublattices are nonequivalent, and the Brillouin zone is folded.

In the folded Brillouin zone, the wave vector is \( q = 0 \) indicating a ferroic multipole fluctuation. In our model, the nesting of Fermi surface gives rise to the magnetic correlation parallel to the \( c \)-axis at \( q = 0 \) for \( \alpha > 0.2 \) as shown in Fig. 2(a). The anisotropy of the Ising-like magnetic fluctuation is compatible with the magnetic structure in \( \text{BaMn}_2\text{As}_2 \) [28, 48], which possesses a weak spin-orbit coupling. For \( \alpha > 0.2 \), the Ising-like magnetic fluctuation is significantly suppressed by the ASOC, and the incommensurate magnetic correlation perpendicular to the \( c \)-axis becomes predominant. This in-plane magnetic anisotropy is consistent with the 5d transition material oxide \( \text{Sr}_2\text{IrO}_4 \) having a strong spin-orbit coupling [39, 40, 49–62]. Thus, the model captures qualitative properties of these materials although a significantly simplified model is adopted. The suppression of magnetic fluctuation by the ASOC is a generic feature [16], and it has been confirmed by a NMR experiment in \( \text{CeCoIn}_5 \) superlattices [91]. A qualitatively same conclusion is obtained from the non-interacting magnetic susceptibility [see Fig. 3].

The ASOC dependence of the magnetic anisotropy may be attributed to the Fermi surface around the M point. Figs. 2(c) and 2(d) show the band structure and Fermi surfaces with the spin texture for \( \alpha = 0.1 \) and 0.3, respectively. We find a Lifshitz transition at \( \alpha \approx 0.2 \), involving the change of the spin texture on the Fermi surface. Since the inter-sublattice hopping disappears on the X-M line as ensured by the nonsymmorphic crystal symmetry, we can define the spin texture on each sublattice. As we show in the left panels of Figs. 2(c) and 2(d), the spin texture is anti-parallel (parallel) between the two Fermi surfaces for \( \alpha = 0.1 \) (0.3). The change of magnetic anisotropy around \( q = 0 \) coincides with the Lifshitz transition. Note that for the doping level \( x = 0 \), the ASOC-induced Lifshitz transition does not occur up to \( \alpha = 1 \), and then, the anisotropy is always \( \chi^{\parallel} > \chi^{\perp} \). Thus, it is implied that the change of the magnetic anisotropy is related to the Lifshitz transition.

Next, we classify the magnetic fluctuations into augmented cluster multipole fluctuations on the basis of the group theory [28]. When the AFM transition of \( \textbf{m} \parallel \text{c} \) occurs, the crystal symmetry of \( D_{4h} \) is reduced to the subgroup \( D_{2d} \) (the two-fold rotational symmetry axes of \( D_{2d} \) are rotated by \( 45^\circ \)). The IRs of \( D_{4h} \) are also reduced to representations of \( D_{2d} \). Since only the \( B_{2u} \) IR contains the fully symmetric \( A_1 \) IR of \( D_{2d} \), the magnetic order belongs to the \( B_{2u} \) IR of \( D_{4h} \). A basis function of the \( B_{2u} \) IR is a linear combination of magnetic quadrupole and hexadecapole moments. On the other hand, the AFM structure of \( \textbf{m} \perp \text{c} \) reduces \( D_{1h} \) to \( C_{2v} \). The \( E_u \) IR is the candidate of the
order parameter. A basis function of the $E_u$ IR contains the magnetic quadrupole and toroidal moments. Both $B_{2u}$ and $E_u$ IRs represent odd-parity orders, which spontaneously break global inversion symmetry. Thus, odd-parity multipole fluctuations are enhanced in our model.

To clarify the multipole fluctuations, we calculate the odd-parity multipole susceptibility defined as

$$
\chi_{B_{2u}(E_u)} = \sum_{mm'} \chi_{B_{2u}}^{\mu}(\tau_{m}\tau_{m'}),
$$

where $\tau_\mu$ is a Pauli matrix for sublattice degree of freedom [92]. When $\chi_{B_{2u}(E_u)}$ diverges at $q = 0$, the odd-parity magnetic multipole order accompanied by inversion symmetry breaking occurs. Indeed, the sublattice off-diagonal components $\chi_{ab} (= \chi_{ba})$ are negatively enhanced, and therefore, $\chi_{B_{2u}(E_u)}$ diverges by increasing $U$. Figures 4(a) and 4(b) show $q$ dependence of $\chi_{B_{2u}(E_u)}$ for $x = 0.2$. Here we adopt a large ASOC since we discuss unconventional superconductivity in this region later. As we have shown in Fig. 2, for a large ASOC the wave vector $q$ of the magnetic order is finite at $x = 0.2$. We call such incommensurate order the quadrupole-density-wave in a broad sense. On the other hand, the incommensurate susceptibility is not enhanced in the undoped case ($x = 0$) [Figs. 4(c) and 4(d)]. Then, the ferroic multipole fluctuation of $\chi_{B_{2u}}$ is predominant because of the absence of a specific nesting in Fermi surfaces. These odd-parity fluctuations affect the superconductivity, as we demonstrate in the next section.

VI. SUPERCONDUCTIVITY

Here we examine the superconductivity. Although this work is based on a model motivated by BaMn$_2$As$_2$ and Sr$_2$IrO$_4$, the following results are qualitatively
inter-sublattice pairing is equivalent to that of globally NCS superconductors, the selection role for inter-sublattice pairing is peculiar to the locally NCS superconductors. The selection rule of locally NCS superconductors, in contrast to the globally NCS superconductors, the selection rule for inter-sublattice pairing does not appear in inter-sublattice components. Thus, Table V illustrates the intra-sublattice component, parity-mixed intra-sublattice component, and inter-sublattice component. For a convenience we describe the order parameter of superconductivity in a standard manner, \( \Delta(k, i\pi T) = \sum_{\mu \nu} \Delta_{\mu \nu}^{\alpha}(k) \sigma_{\mu}^{\nu} \tau^s \), where \( \sigma^{\nu} \) and \( \tau^{s} \) are Pauli matrix for spin and sublattice degrees of freedom, respectively. We introduced \( \tau^{s} = [\sigma^{\nu} i\sigma^{y}]_{ss} \) for \( \mu = 0, x, y, z \). This notation is used in Table V.

The \( B_{2g} \) state corresponds to the spin-singlet \( d_{x^2-y^2} \) wave pairing state in the well-studied single-sublattice Hubbard model. Since the \( x \) and \( y \)-axes in the two-sublattice model are rotated by 45°, the predominant component of the order parameter is an inter-sublattice spin-singlet component \( \Delta_{\mu \nu}^{\alpha}(k) \) of \( d_{xy} \) wave symmetry. Consistent with many theoretical works on the single-sublattice Hubbard model near half-filling \([90] \), the \( B_{2g} \) state is stable at \( \alpha = 0 \). However, when the staggered ASOC is turned on, the eigenvalue of Eliashberg equation \( \lambda \) for the \( B_{2g} \) state is steeply suppressed [Fig. 5(a)]. This is mainly because the intra-sublattice spin-singlet pairing is ruled out by the selection rule of locally NCS superconductors, in strike contrast to the globally NCS superconductors (see Table IV).

To examine the effect of the staggered ASOC, we solved the Eliashberg equation for a similar model containing a uniform ASOC instead of the staggered ASOC. The eigenvalues \( \lambda \) of the two models are compared in Fig. 5(d). Consistent with the selection rule, the local parity violation more significantly suppresses the \( d \)-wave superconductivity than the global parity violation does. In both cases the superconductivity is suppressed by the ASOC owing to the suppressed magnetic fluctuation. In addition, the staggered ASOC causes pair breaking valid in a broad range of odd-parity magnetic multipole materials which have been recently identified [75].

Before showing numerical results of the Eliashberg equation, we discuss effects of the ASOC on superconductivity in locally NCS systems. The ASOC has two effects: (1) modulation of the one-particle Green’s function, and (2) that of the pairing interaction. Considering the effect (1), we may recognize that the stable superconducting state depends on whether the leading pairing channel is the intra-sublattice pairing or inter-sublattice pairing [3]. This gives a selection rule summarized in Table IV. The spin-singlet pairing state or spin-triplet pairing state with \( d(k) || g(k) \) are stable for the intra-sublattice pairing, while only the spin-triplet pairing state with \( d(k) \perp g(k) \) is stable for the inter-sublattice pairing. The other superconducting states are destabilized by the sublattice-dependent ASOC. Although the selection rule for intra-sublattice pairing is equivalent to that of globally NCS superconductors, the selection role for inter-sublattice pairing is peculiar to the locally NCS superconductors. The effect (2) occurs through the modification of magnetic fluctuation, which has been investigated in Sec. V. Later we show that the modified magnetic fluctuation stabilizes the odd-parity spin-triplet superconductivity.

Now we show the numerical results. Within the RPA theory, we obtained four stable superconducting states: \( B_{2g}, B_{1u}, B_{1g} \), and \( A_{1u} \) IRs. The leading pair amplitude and other admixed components of superconducting states are summarized in Table V. As we have shown in Sec. III, the parity-mixing does not appear in inter-sublattice components.

![Figure 4](image_url)

**FIG. 4.** Momentum dependence of the multipole susceptibilities \( \chi_{E\alpha}(q) \) and \( \chi_{B2\mu}(q) \) for \( U = 2.2, \) (a) \( \alpha = 0.7 \) and \( x = 0.2, \) (b) \( \alpha = 1 \) and \( x = 0.2, \) (c) \( \alpha = 0.7 \) and \( x = 0, \) and (d) \( \alpha = 1 \) and \( x = 0 \). The right panels show the Fermi surfaces.

### Table IV. Selection rules of the superconductivity in locally and globally NCS crystals [3].

|                  | Globally NCS Crystal | Locally NCS Crystal |
|------------------|----------------------|---------------------|
|                  | Intra-sublattice     | Inter-sublattice    |
| Singlet, d(k) || g(k) | Singlet, d(k) || g(k) | d(k) \perp g(k) |

|                  | Singlet, d(k) || g(k) | d(k) \perp g(k) |

[Note: Table content is not provided as it is in a visual format.]
through the modulation of one-particle Green’s function. The dominant pairing component $d^{00}(k)$, which is incompatible with the selection rule in Table IV, decreases in the same manner as $\lambda^{B_{2g}}$ by increasing $\alpha$. Instead of that, an intra-sublattice singlet component compatible with the selection rule monotonically increases as $d^{00}(k) \simeq d^{00}(k)|_{\alpha=0} + A\alpha \sin k_x \sin k_y$. Owing to the parity-mixing by the ASOC, an admixed sublattice-odd spin-triplet component, $(d^{xz}(k),d^{yz}(k)) \simeq B\alpha (\sin k_x - \sin k_y)$, appears. The momentum dependence of these components is shown in Fig. 5(c).

Although the $d$-wave superconductivity is stable in a broad range near the AFM critical point, it is significantly suppressed in the locally NCS crystals with a large spin-orbit coupling. Thus, we have a chance to see another exotic superconducting state. Candidates are the $B_{1g}$ and $A_{1u}$ states which show large eigenvalues of the Eliashberg equation. The other odd-parity IRs are less stable than these states. Both $B_{1g}$ and $A_{1u}$ states satisfy the condition, $d(k) \parallel q(k)$, in a part of $k$-space, compatible with the selection rule for the intra-sublattice pairing. However, $\lambda$ of all the odd-parity I Rs moderately decrease as increasing $\alpha$ [inset of Fig. 5(a)] due to the suppression of magnetic fluctuations. Thus, by looking at the $\alpha$-dependence of $\lambda$ we can not determine which superconducting states are preferred. To examine the superconductivity in a large ASOC region, we calculated $U$-dependence of $\lambda$ and investigated which superconducting states are stabilized in the vicinity of the magnetic critical point. Figure 5(b) shows that the $B_{1u}$ state is predominant for $x = 0.2$ and the eigenvalue of the Eliashberg equation reaches $\lambda = 1$ at $U \approx 2.4$. The second most stable superconducting state is the $A_{1u}$ IR. This state is the most stable in the undoped system $x = 0$. Thus, odd-parity superconductivity may be realized in a large ASOC region by the magnetic multipole fluctuations.

In order to further elucidate an essential role of the sublattice-dependent ASOC, we again compare our model to the model containing a uniform ASOC. Figure 5(e) compares the $U$-dependence of eigenvalues $\lambda$ for the $B_{2g}$ state and $B_{1u}$ state. As shown in the lower panel, the $B_{1u}$ superconducting state is more stable in our model than in the globally NCS model. In other words, the staggered ASOC favors the $B_{1u}$ state more significantly than the uniform ASOC does. Because the magnetic fluctuation is identified as the odd-parity magnetic multipole fluctuation only in the locally NCS model, it is implied that the modification of magnetic fluctuation by the staggered ASOC leads to the odd-parity multipole fluctuation and favors the odd-parity superconductivity. Note that the $B_{1u}$ state is compatible with the selection rule in Table IV for both models. In contrast, the even-parity $B_{2g}$ state is suppressed by the staggered ASOC [upper panel of Fig. 5(e)].

As we show in Table V, the leading order parameter of the $B_{1u}$ state is an intra-sublattice spin-triplet pairing $d^{xz}(k)$ and $d^{yz}(k)$, namely, $(\sin k_x \sigma^x - \sin k_y \sigma^y)\theta^0$. An admixed sublattice-odd spin-singlet component is $d^{xz}(k) \simeq \delta + \cos k_x + \cos k_y$. On the other hand, the leading order parameter of the $A_{1u}$ state is $(\sin k_x \sigma^x + \sin k_y \sigma^y)\theta^1$, and the induced component is $d^{yz}(k) \simeq \cos k_x - \cos k_y$ [see Fig. 5(c)].

Spin-triplet superconductors are known to be a platform of topological superconductivity, which has been one of the main subjects of modern condensed matter physics. The spin-triplet superconductivity elucidated in this work is also identified as topological superconducting states. According to the criterion for time-reversal invariant topological superconductivity in two dimension [93], both $B_{1u}$ and $A_{1u}$ states are $Z_2$-topological superconducting states in class DIII, because the number of Fermi surfaces enclosing time-reversal-invariant momentum $(\Gamma, X, \text{and} M \text{points})$ is odd.

Figure 6 shows the phase diagram as a function of $\alpha$ and $U$. From Fig. 6(a) for $x = 0.2$, we identify the stable odd-parity $B_{1u}$ state for $\alpha > 0.7$, while the $B_{2g}$ state is stabilized for $\alpha < 0.3$. The magnetic instability for $\alpha < 0.2$ is the $B_{2u}$ magnetic quadrupole and hexadecapole order, which is monotonically suppressed by the ASOC. The magnetic instability for $\alpha = 0.7$ is the $B_{1u}$ magnetic quadrupole-density-wave with an incommensurate period. In an intermediate ASOC region, the $B_{1g}$ state represented by the predominant intra-sublattice spin-singlet pairing $d^{00}(k) \sim \cos k_x - \cos k_y$ is stable. This state is stabilized by the incommensurate magnetic fluctuation with a small wave vector $q \sim (\pm 1.1, \pm 1.14)$. As is usually done by magnetic fluctuation, the sign change of the gap function between the Fermi surface connected by the wave vector is favored. The $B_{1g}$ superconducting state is compatible with this condition and also with the selection rule for locally NCS superconductors (Table IV).

In the undoped case, $x = 0$, we obtain a similar
FIG. 5. (a) ASOC dependence of eigenvalues of the Eliashberg equation $\lambda$ for $U = 1.6$. The $B_{1g}(d_{x^2-y^2} + p$-wave state), $B_{2g}(d_{x^2-y^2} + p$-wave state), $A_{1u}(p + d_{x^2-y^2}$-wave state), $A_{2u}(p + d_{x^2-y^2}$-wave state), $B_{1u}(p + s$-wave state), $B_{2u}(p + g$-wave state), $E_u(p + d_{x^2-y^2}$-wave state) IRs are depicted. (b) Coulomb interaction dependence of $\lambda$ for $\alpha = 0.9$. (c) Intra-sublattice and parity-mixed components of gap functions for $U = 1.6$ and $\alpha = 0.1$. For the spin-triplet components, $d^{ux}(k)$ and $d^{xy}(k)$ are shown for the $A_{1u}$ and $B_{1u}$ IRs, while $d^{yx}(k)$ and $d^{yy}(k)$ are shown for the $B_{2g}$ and $B_{1g}$ IRs. (d) ASOC dependence of $\lambda^{B_{2g}}$ for the $B_{2g}$ IR at $U = 1.6$. Solid line shows the result of our model introduced in Sec. VI for the locally NCS crystal. For comparison, we show the dashed line obtained for the model containing a uniform ASOC instead of the staggered ASOC. The latter corresponds to the globally NCS crystal. (e) Stoner factor $S = [\hat{\Gamma}(q)]_{\max}$ dependence of $\lambda^{B_{2g}}$ (upper panel) and $\lambda^{B_{1u}}$ (lower panel) for $\alpha = 0.9$. We again compare the locally NCS model (solid lines) with the globally NCS model (dashed lines).

but simpler phase diagram [Fig. 6(b)]. In a large ASOC region, the odd-parity $A_{1u}$ superconducting state is realized near the ferroic odd-parity magnetic multipole state. The magnetic wave vector is always $q = 0$. Thus, the incommensurate magnetic fluctuation is not a necessary condition for the odd-parity superconductivity. Irrespective of the wave vector of multipole fluctuations, the odd-parity superconducting states are stabilized in a large ASOC region. In contrast, the $B_{1g}$ superconducting state requires the incommensurate fluctuation, and it disappears in the phase diagram for $x = 0$.

Finally, we comment on a peculiar momentum dependence of gap function protected by nonsymmorphic space group symmetry. By Eq. (14) and a similar equation for $G_y = \{\sigma_y|\tau_z\}$, inter-sublattice spin-singlet gap function shows a unusual nodal/gapped structure. As shown in Fig. 7, $d^{\alpha x}(k)$ for the $B_{1g}$ IR shows nodal lines at $k_{x,y} = \pm \pi$, while that is gapped for the $B_{2g}$ IR. These nodal/gapped structures at the Brillouin zone boundaries are opposite to those in symmorphic crystals. The gap functions numerically obtained in this work satisfy the symmetry conditions.
VII. SUMMARY AND DISCUSSION

In this paper, we have investigated the superconductivity induced by odd-parity magnetic multipole fluctuations in the locally NCS crystal. The obtained results are summarized below.

First, we have revealed symmetry properties of the superconductivity with sublattice degree of freedom. The general representation for pair amplitudes including nonsymmorphic operations was derived. After introducing a space group of a specific crystal structure, an unconventional gapped/nodal structure protected by the glide symmetry was shown. For glide-even superconducting states, inter-sublattice pair amplitudes possess a node on a Brillouin zone boundary, while those are gapped for glide-odd superconducting states. On the other hand, the intra-sublattice amplitude shows local parity mixing in a staggered form with respect to the sublattices. The admixture of spin-singlet and spin-triplet pairings is classified by

the point group of local atomic sites. These symmetry analyses are consistent with the following numerical results.

Next, we have studied a two-dimensional two-sublattice model with the on-site Coulomb interaction term and the $D_{2d}$-type staggered ASOC term. The magnetic fluctuation is suppressed by increasing the ASOC, consistent with a theoretical study of CePt$_3$Si [16] and an experiment for CeCoIn$_5$ superlattices [91]. The ASOC term also induces an anisotropy in magnetic fluctuation. From the classification of multipole order parameters, the antiferromagnetism of $m_\parallel c$ is classified into the odd-parity magnetic multipole order belonging to the $B_{2g}$ IR. In the same way the $m_\perp c$ AFM state is classified into the odd-parity magnetic multipole order belonging to the $B_{2u}$ IR. Both $B_{2g}$ and $E_u$ IRs represent odd-parity magnetic multipole orders accompanied by spontaneous global inversion symmetry breaking. In our model, these odd-parity multipole fluctuations are enhanced in the vicinity of the magnetic critical point.

Superconducting instability has been analyzed by solving the Eliashberg equation with the use of RPA. We have demonstrated selection rules of locally NCS superconductors [3]. Since the inter-sublattice spin-singlet pairing is ruled out by the selection rule, the $B_{2g}$ state, which corresponds to the well-studied $d_{x^2-y^2}$-wave superconducting state in the Hubbard model, is rapidly suppressed by turning on the ASOC. Intriguingly, this behavior is in sharp contrast with the globally NCS superconductors. When the $d$-wave superconductivity is suppressed in a large ASOC region, the odd-parity superconductivity is stabilized by the enhanced odd-parity multipole fluctuations. We found that the $B_{1u}$ or $A_{1u}$ state is stable. From the criterion for time-reversal invariant topological superconductivity, both the $B_{1u}$ and $A_{1u}$ states are identified as the nontrivial $Z_2$ topological superconductivity in DIII class. Thus, our results may open a new platform of the odd-parity topological superconductivity.

Here we note that the staggered ASOC arising from the local parity violation in the crystal structure plays an essential role on stabilizing the odd-parity superconductivity. From the comparison with the globally NCS model, we have shown that the modifica-
tion of magnetic fluctuation by the staggered ASOC significantly enhances the odd-parity superconductivity. Such enhancement is not caused by the sublattice-independent ASOC in the globally NCS system. The modified magnetic fluctuation in the locally NCS system is regarded as odd-parity magnetic multipole fluctuations. Therefore, we conclude that the odd-parity superconductivity is stabilized by the odd-parity multipole fluctuations.

In the longstanding studies of unconventional superconductivity, spin-triplet superconductivity has attracted interest. Furthermore, renewed interest has been stimulated because the odd-parity superconductivity may be topologically nontrivial. However, only limited examples, such as Sr$_2$RuO$_4$ [94], UPt$_3$ [95], UCoGe [96], and so on, are known as strong candidates of spin-triplet superconductors. This is because conditions for spin-triplet pairing are quite unfavorable in most materials. Our study uncovered a new pairing mechanism favorable for the spin-triplet pairing. The local parity violation in crystal structures, large spin-orbit coupling, and enhanced magnetic multipole fluctuation are conditions for the spin-triplet superconductivity proposed in this work.

Finally, we discuss candidate materials of odd-parity fluctuation and superconductivity. First, Sr$_2$IrO$_4$ is a layered perovskite 5$d$ transition metal oxide and possesses K$_2$NiF$_4$-type structure as La$_2$CuO$_4$ does. A lot of similarities to the high-temperature cuprate superconductors have been recognized, and thus it is expected to be a superconductor from both experimental [54] and theoretical [39, 60] sides. AFM moments align to the a-axis with a small canted moment along the b-axis and show stacking patterns: $-++-$, $-+-+$, and $+++-$ patterns. From the viewpoint of multipole order, the $-++-$ pattern is magnetic octupole order preserving the space inversion symmetry, while the $-+-+$ pattern is odd-parity magnetic quadrupole order [40]. Another candidate is BaMn$_2$As$_2$ crystallizing in the locally NCS ThCr$_2$Si$_2$-type structure, which is isostructural to the 122 systems of iron-based superconductors. Undoped BaMn$_2$As$_2$ shows the G-type AFM order at $T_N = 625$ K [48]. The magnetic structure is classified into the odd-parity magnetic quadrupole and hexadecapole orderers [28]. Many related materials show the same odd-parity magnetic order, and some of them may be superconducting. A further experimental search is desired. The hole-doped (Ba$_1-x$K$_x$)Mn$_2$As$_2$ realizes the metallic state. However, superconductivity has not been observed up to now. A fascinating material is CrAs [97–105]. The space group is No. 62, Pna$_2$ (D$_{4h}$ point group) lacking local inversion symmetry at Cr sites. CrAs shows a first-order helical magnetic transition at $T_N \sim 265$ K [97, 98]. When the helical magnetic order is suppressed by applied pressures, superconductivity occurs [99]. The phase diagram implies superconductivity induced by the magnetic fluctuation. The wave vector of the helical magnetism is incommensurate, $q = (0, 0, q_z)$ with $q_z \sim 0.354$. Thus, the local parity violation and odd-parity magnetic fluctuation may promote odd-parity superconductivity in CrAs. Indeed, a recent experiment suggests the spin-triplet superconductivity [105]. As for the odd-parity electric multipole fluctuation, Sr$_2$TiO$_3$ [74] and Cd$_2$Re$_2$O$_7$ [67] show superconductivity in the vicinity of the nonmagnetic order accompanied by global inversion symmetry breaking. Recently, Ref. 76 theoretically proposed that the odd-parity electric fluctuation may induce the odd-parity superconductivity. To the best of our knowledge, this work is a first microscopic calculation of the superconductivity induced by odd-parity magnetic fluctuation, which uncovers a new mechanism of the odd-parity superconductivity.

ACKNOWLEDGMENTS

The authors are grateful to S. Sumita, Y. Yanagi, A. Daido, and H. Watanabe for fruitful discussions and comments. This work was supported by a Grant-in-Aid for Scientific Research on Innovative Areas “J-Physics” (Grant No. JP15H05884) and “Topological Materials Science” (Grant No. JP16H00991 and JP18H04225) from Japan Society for Promotion of Science (JSPS) and by JSPS KAKENHI (Grants No. JP15K05164, No. JP15H05745, and JP18H01178).

[1] M. Sigrist, D. F. Agterberg, M. H. Fischer, J. Goryo, F. Loder, S.-H. Rhim, D. Maruyama, Y. Yanase, T. Yoshida, and S. J. Youn, J. Phys. Soc. Jpn. 83, 061014 (2014).
[2] D. Maruyama, M. Sigrist, and Y. Yanase, J. Phys. Soc. Jpn. 81, 034702 (2012).
[3] M. H. Fischer, F. Loder, and M. Sigrist, Phys. Rev. B 84, 184533 (2011).
[4] S. Nakosai, Y. Tanaka, and N. Nagaosa, Phys. Rev. Lett. 108, 147003 (2012).
[5] T. Yoshida, M. Sigrist, and Y. Yanase, Phys. Rev. B 86, 134514 (2012).
[6] T. Yoshida, M. Sigrist, and Y. Yanase, J. Phys. Soc. Jpn. 82, 074714 (2013).
[7] T. Yoshida, M. Sigrist, and Y. Yanase, Phys. Rev. Lett. 115, 027001 (2015).
[8] T. Yoshida, A. Daido, Y. Yanase, and N. Kawakami, Phys. Rev. Lett. 118, 147001 (2017).
[9] Edited by E. Bauer and M. Sigrist, Non-Centrosymmetric Superconductors: Introduction and Overview, Lecture Notes in Physics Vol. 847 (Springer, Berlin, 2012).
[10] R. Settai, T. Takeuchi, and Y. ¯Onuki, J. Phys. Soc. Jpn. 76, 051003 (2007).
[11] V. M. Edelstein, Phys. Rev. Lett. 75, 2004 (1995).
[66] H. Harima, Journal of Physics and Chemistry of Solids 63, 1027 (2002).
[67] J.-I. Yamaura, K. Takeda, Y. Ikeda, N. Hiroa, Y. Ohishi, T. C. Kobayashi, and Z. Hiroi, Phys. Rev. B 95, 020102 (2017).
[68] J.-I. Yamaura and Z. Hiroi, J. Phys. Soc. Jpn. 71, 2598 (2002).
[69] Z. Hiroi, J.-I. Yamaura, Y. Murakoa, and M. Hanawa, J. Phys. Soc. Jpn. 71, 1634 (2002).
[70] M. Hanawa, J. Yamaura, Y. Murakoa, F. Sakai, and Z. Hiroi, Journal of Physics and Chemistry of Solids 63, 1027 proceedings of the 8th ISSP International Symposium.
[71] H. Sakai, K. Yoshimura, H. Ohno, H. Kato, S. Kambe, R. E. Walstedt, T. D. Matsuda, Y. Haga, and Y. Onuki, J. Phys.: Condens. Matter 13, L785 (2001).
[72] J. G. Bednorz and K. A. Müller, Phys. Rev. Lett. 52, 2289 (1984).
[73] J. F. Schooley, W. R. Hosler, and M. L. Cohen, Phys. Rev. Lett. 12, 474 (1964).
[74] C. W. Rischau, X. Lin, C. F. Grams, D. Finck, S. Harns, J. Engelmayer, T. Lorenz, Y. Galais, B. Faquè, J. Hemberger, and K. Behnia, Nat. Phys. 13, 643 (2017).
[75] H. Watanabe and Y. Yanase, ArXiv e-prints , arXiv:1805.10828 (2018).
[76] V. Kozii and L. Fu, Phys. Rev. Lett. 115, 207002 (2015).
[77] Y. Wang, G. Y. Cho, T. L. Hughes, and E. Fradkin, Phys. Rev. B 93, 134512 (2016).
[78] J. M. Edge, Y. Kedem, U. Aschauer, N. A. Spaldin, and A. V. Balatsky, Phys. Rev. Lett. 115, 247002 (2015).
[79] T. Takimoto and P. Thalmeier, J. Phys. Soc. Jpn. 78, 103703 (2009).
[80] M. Sigrist and K. Ueda, Rev. Mod. Phys. 63, 239 (1991).
[81] T. Nomoto, K. Hattori, and H. Ikeda, Phys. Rev. B 94, 174513 (2016).
[82] Y. Yanase, Phys. Rev. B 94, 174502 (2016).
[83] T. Nomoto and H. Ikeda, Phys. Rev. Lett. 117, 217002 (2016).
[84] T. Nomoto and H. Ikeda, J. Phys. Soc. Jpn. 86, 023703 (2017).
[85] M. R. Norman, Phys. Rev. B 52, 150503 (1995).
[86] T. Micklitz and M. R. Norman, Phys. Rev. B 80, 100506 (2009).
[87] S. Kobayashi, Y. Yanase, and M. Sato, Phys. Rev. B 94, 134512 (2016).
[88] S. Sumita and Y. Yanase, Phys. Rev. B 97, 134512 (2018).
[89] S. Kobayashi, S. Sumita, Y. Yanase, and M. Sato, Phys. Rev. B 97, 180504 (2018).