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Combinatorics in the exterior algebra and the Bollobás Two Families Theorem. (English) J. Lond. Math. Soc., II. Ser. 104, No. 4, 1812-1839 (2021)

Summary: We investigate the combinatorial structure of subspaces of the exterior algebra of a finite-dimensional real vector space, working in parallel with the extremal combinatorics of hypergraphs. Using initial monomials, projections of the underlying vector space onto subspaces, and the interior product, we find analogs of local and global LYM inequalities, the Erdős-Ko-Rado theorem, and the Ahlswede-Khachatrian bound for $t$-intersecting hypergraphs. Using these tools, we prove a new extension of the Two Families Theorem of Bollobás, giving a weighted bound for subspace configurations satisfying a skew cross-intersection condition. We also verify a recent conjecture of D. Gerbner et al. [“Set systems related to a house allocation problem”, Preprint, arXiv:1910.04666] on pairs of set systems satisfying both an intersection and a cross-intersection condition.

MSC:

05D05 Extremal set theory
15A75 Exterior algebra, Grassmann algebras
14N20 Configurations and arrangements of linear subspaces

Keywords: Erdős-Ko-Rado theorem; Ahlswede-Khachatrian bound

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