Hadronic tau decays as New Physics probes in the LHC era

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We analyze the sensitivity of hadronic τ decays to non-standard interactions within the model-independent framework of the Standard Model Effective Field Theory (SMEFT). Both exclusive and inclusive decays are studied, using the latest lattice data and QCD dispersion relations. We show that there are enough theoretically clean channels to disentangle all the effective couplings contributing to these decays, with the τ → πτν channel representing an unexpected powerful New Physics probe. We find that the ratios of non-standard couplings to the Fermi constant are bound at the sub-percent level. These bounds are complementary to the ones from electroweak precision observables and pp → τντ measurements at the LHC. The combination of τ decay and LHC data puts tighter constraints on lepton universality violation in the gauge boson-lepton vertex corrections.

The agreement between the above-mentioned determinations of SM and QCD parameters with determinations using other processes represent a non-trivial achievement, which is only possible thanks to the impressive effort carried out in several fronts: experimental, lattice and analytical QCD methods. Needless to say, this agreement is easily spoiled if non-standard effects are present. However, the use of hadronic tau decays as New Physics (NP) probes has been marginal so far (see e.g. Refs. [11, 12]), with the exception once again of the simple τ → πτν, Kντ channels. The goal of this letter is to amend this situation presenting an unprecedented comprehensive analysis of the NP reach of hadronic tau decays.

We focus for sake of definiteness on the non-strange decays, which are governed by the following low-energy effective Lagrangian [13]

\[
\mathcal{L}_{\text{eff}} = -\frac{G_F V_{ud}}{\sqrt{2}} \left( 1 + \epsilon_L^c \right) \bar{\tau} \gamma_{\mu} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 - \gamma_5) d + \epsilon_R \bar{\tau} \gamma_{\mu} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \gamma^\mu (1 + \gamma_5) d + \bar{\tau} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \left[ \epsilon_S - \epsilon_P \gamma_5 \right] d + \epsilon_T^\tau \sigma_{\mu\nu} (1 - \gamma_5) \nu_\tau \cdot \bar{u} \sigma_{\mu\nu} (1 - \gamma_5) d + \text{h.c.,} \quad (1)
\]

where we use \( \sigma_{\mu\nu} = i [\gamma_{\mu}, \gamma_{\nu}] / 2 \) and \( G_F \) is the Fermi constant.

Through a combination of inclusive and exclusive τ decays, we are able to constrain all the Wilson coefficients in Eq. (1) – this is the main result of this paper. In the \( \overline{MS} \) scheme at scale \( \mu = 2 \) GeV we find the following central values and 1σ uncertainties:

\[
\begin{pmatrix}
\epsilon_L^c - \epsilon_R^c + \epsilon_R^c - \epsilon_R^c \\
\epsilon_R^c \\
\epsilon_S^c \\
\epsilon_P^c \\
\epsilon_T^c
\end{pmatrix}
= 
\begin{pmatrix}
1.0 \pm 1.1 \\
0.2 \pm 1.3 \\
-0.6 \pm 1.5 \\
0.5 \pm 1.2 \\
-0.04 \pm 0.46
\end{pmatrix} \cdot 10^{-2}, \quad (2)
\]

1 We have not included right-handed (and wrong-flavor) neutrino fields [14], which in any case do not interfere with the SM amplitude and thus contribute at \( \mathcal{O}(\epsilon^2) \) to the observables.
where \( e^\tau_{L,R} \) parametrize electron couplings to the first generation quarks and are defined in analogy to their tau counterparts. They affect the \( G_F V_{ud} \) value obtained in nuclear \( \beta \) decays \cite{15}, which is needed in the analysis of hadronic tau decays. The correlation matrix associated to Eq. (2) is

\[
\rho = \begin{pmatrix}
1 & 0.88 & 0 & -0.57 & -0.94 \\
0 & -0.86 & -0.94 \\
0 & 0 & 1 & 0.66 \\
1 & 0 & 0 & 1
\end{pmatrix}, \quad (3)
\]

Below we summarize how Eqs. (2)-(3) were derived.

I. EXCLUSIVE DECAYS

The \( \tau \to \pi \nu_\tau \) channel \cite{10} gives the following 68\% CL constraint:

\[
e^\tau_L - e^\tau_R - e^\mu_R - \frac{B_0}{m_\tau} e^\mu_P = (-1.5 \pm 6.7) \cdot 10^{-3}, \quad (4)
\]

where \( B_0 = m_\pi^2/(m_u + m_d) \). We included the SM radiative corrections \cite{17,19} and the latest lattice average for the pion decay constant, \( f_\pi = 130.2(8) \text{ MeV} \) \((N_f = 2 + 1)\) \cite{20}, from Refs. \cite{21,23}. We stress that the lattice determinations of \( f_\pi \) are a crucial input to search for NP in this channel, and despite its impressive precision, it represents the dominant source of error in Eq. (4), followed by the experimental error (2.4 times smaller), and the radiative corrections uncertainty. Because of this, significant improvement in the bound above can be expected in the near future. Alternatively, as often seen in the literature \cite{1}, one can obtain tighter constraints on the effective theory parameters by considering “theoretically clean” ratios of observables where the \( f_\pi \) dependence cancels out. For example, from the ratio \( \Gamma(\tau \to \pi \nu)/\Gamma(\pi \to \mu \nu) \) one can deduce

\[
e^\tau_L - e^\mu_R - e^\tau_R + e^\mu_R - \frac{B_0}{m_\tau} e^\mu_P - \frac{B_0}{m_\tau} e^\mu_P = (-3.8 \pm 2.7) \cdot 10^{-3}. \quad (5)
\]

This and similar constraints are not included in Eq. (2), which only summarizes the input from hadronic tau decays without using any meson decay observables. Instead, we later combine Eq. (2) with the results of Ref. \cite{21}, which derived a likelihood for the effective theory parameters based on a global analysis of pion and kaon decays. The combination effectively includes constraints from \( \Gamma(\tau \to \pi \nu)/\Gamma(\pi \to \ell \nu) \), once correlations due to the common \( f_\pi \) uncertainty are properly taken into account.

The \( \tau \to \pi \pi \nu_\tau \) channel, which is sensitive to vector and tensor interactions, is much more complicated to predict within QCD in a model-independent way. However, a stringent constraint can be obtained through the comparison of the spectral functions extracted from \( \tau \to \pi \pi \nu_\tau \) and its isospin-rotated process \( e^+ e^- \to \pi^+ \pi^- \), after the proper inclusion of isospin-symmetry-breaking corrections. The crucial point here is that heavy NP effects (associated with the scale \( \Lambda \)) can be entirely neglected in \( e^+ e^- \to \pi^+ \pi^- \) at energy \( \sqrt{s} \ll \Lambda \) due to the electromagnetic nature of this process. We can benefit from past studies that exploited this isospin relation to extract from data the \( \pi \pi \) component of the lowest-order hadronic vacuum polarization contribution to the muon \( g-2 \), usually denoted by \( a^{\text{had,LO}}_{\mu}[\pi \pi] \), through a dispersion integral. Such approach assumes implicitly the absence of NP effects, which however may contribute to the extraction from tau data. In this way we find a sub-percent level sensitivity to NP effects:

\[
\frac{a^{\tau}-a^{\mu}_{\text{ee}}}{2 a^{\mu}_{\text{ee}}} = e^{\tau}_{L} - e^{\tau}_{R} + e^{\mu}_{R} - 1.7 e^{\tau}_{T} = (8.9 \pm 4.4) \cdot 10^{-3}, \quad (6)
\]

where \( a^{\tau} = (516.2 \pm 3.6) \times 10^{-10} \) \cite{24} and \( a^{\mu}_{\text{ee}} = (507.14 \pm 2.58) \times 10^{-10} \) \cite{26} are the values of \( a^{\text{had,LO}}_{\mu}[\pi \pi] \) extracted from \( \tau \) and \( e^+ e^- \) data. The \( \sim 2\sigma \) tension with the SM reflects the well-known disagreement between both datasets \cite{26,27}. In order to estimate the factor multiplying \( e^{\tau}_{T} \) in Eq. (6), we have (i) assumed that the proportionality of the tensor and vector form factors, which is exact in the elastic region \cite{28,29}, holds in the dominant \( \rho \) resonance region (as is the case within the resonance chiral theory framework \cite{30}); (ii) used the lattice QCD result of Ref. \cite{31} for the \( \pi \pi \) tensor form factor at zero momentum transfer (see also Refs. \cite{32,33}).

The constraint above can be strengthened by directly looking at the s-dependence of the spectral functions (instead of the \( a_\mu \) integral), which would also allow us to disentangle the vector and tensor interactions. Moreover, the \( a^{\tau}_{\text{ee}} \) uncertainties include a scaling factor due to internal inconsistencies of the various datasets \cite{26}, which hopefully will decrease in the future. In fact, new analyses of the \( \pi \pi \) channel are expected from CMD3, BABAR, and possibly Belle-2 \cite{26,27}. All in all, we can expect a significant improvement in precision with respect to the result in Eq. (6) in the near future.

As recently pointed out in Ref. \cite{12}, a third exclusive channel that can provide useful information is \( \tau \to \eta \pi \nu_\tau \), since the non-standard scalar contribution is enhanced with respect to the (very suppressed) SM one. Because of this, one can obtain a nontrivial constraint on \( e^{\eta}_{S} \) even though both SM and NP contributions are hard to predict with high accuracy. Using the latest experimental results for the branching ratio \cite{16,31} and a very conservative estimate for the theory errors \cite{12,33,36} we find

\[
e^{\eta}_{S} = (-6 \pm 15) \times 10^{-3}, \quad (7)
\]

Recently Ref. \cite{24} found \( a^{\mu}_{\text{ee}} = 503.7 \pm 1.96 \) using similar data but a different averaging method than Ref. \cite{25}. This increases to \( 3\sigma \) the tension between \( a^{\mu}_{\tau} \) and \( a^{\mu}_{\mu} \).
which will significantly improve if theory or experimental uncertainties can be reduced. The latter will certainly happen with the arrival of Belle-II, which is actually expected to provide the first measurement of the SM contribution to this channel \( \mathcal{O}(\varepsilon_3^2) \) (see also Ref. \[28\] for Belle results). This is the only probe in this work with a significant sensitivity (via \( \mathcal{O}(\varepsilon_3^2) \) effects) to the imaginary part of \( \varepsilon_i \) coefficients. Including the latter does not affect the bound in Eq. (7) though.

II. INCLUSIVE DECAYS

Summing over certain sets of decay channels one obtains the so-called inclusive vector (axial) spectral functions \( \rho_{V(A)} \), which are nothing but the sum of the hadronic invariant mass distributions up to some constants and kinematic factors \[1, 2\]. In the SM they are proportional to the imaginary parts of the associated \( VV \) (\( AA \)) two-point correlation functions, \( \Pi_{VV(AA)}(s) \), but these relations are modified by NP effects \[39, 40\]. Thus, one could directly use the latest measurements of these spectral functions to constrain such effects if we had a precise theoretical knowledge of their QCD prediction. However, perturbative QCD is known not to be valid at \( \sqrt{s} < 1 \text{ GeV} \), especially in the Minkowskian axis, where the spectral function lies. Nevertheless, one can make precise theoretical predictions for integrated quantities exploiting the well-known analyticity properties of QCD correlators \[3\]. Here we extend the traditional approach to include also NP effects, finding \[39, 40\]

\[
\int_{4m^2_x}^{s_0} \frac{ds}{s_0} \omega \frac{s}{s_0} \exp_{\rho_{V(A)}} \Pi_{VV(AA)}(s) \approx \left( 1 + 2\varepsilon_V \right) X_{VV}^{\omega} \\
\pm (1 + 2\varepsilon_A) \left( X_{AA} - f^2 \omega \left( \frac{m^2_x}{s_0} \right) \right) + \varepsilon_T X_{VT},
\]

where \( \omega(x) \) is a generic analytic function and \( \rho_{V(A)}^{\exp} \) is the sum/diffERENCE of the vector and axial spectral functions, extracted experimentally under SM assumptions \[21, 25\]. We also introduced the couplings \( \varepsilon_{V/A} \equiv \varepsilon_{L/R} \). Last, the contributions \( X_{VV(AA)} \) and \( X_{VT} \) can be calculated via the Operator Product Expansion, as discussed in Appendix \[5\]. Eq. (8) shows how the agreement between precise SM predictions (RHS) and experimental results (LHS) for inclusive decays can be translated into strong NP constraints.

In the \( V + A \) channel, we find two clean NP constraints using \( \omega(x) = (1 - x)^2(1 + 2x) \), which gives the total non-strange BR, and with \( \omega(x) = 1 \). They provide respectively

\[
\varepsilon_{L+R}^{V,A} - \varepsilon_{L+R}^{V,A} - 0.78c^R_T + 1.71c^T_L = (4.16) \cdot 10^{-3}, \tag{9}
\]

\[
\varepsilon_{L+R}^{V,A} - \varepsilon_{L+R}^{V,A} + 0.89c^T_R + 0.90c^T_L = (8.5 \pm 8.5) \cdot 10^{-5}. \tag{10}
\]

The uncertainty in Eq. (9) comes mainly from the non-perturbative corrections, whereas that of Eq. (10) is dominated by experimental and Duality Violations (DV) uncertainties (see Appendix \[A\]).

In the \( V - A \) channel, where the perturbative contribution is absent, two strong constraints can be obtained using \( \omega(x) = (1 - x) \) and \( \omega(x) = (1 - x)^2 \):

\[
\varepsilon_{L+R}^{V,A} - \varepsilon_{L+R}^{V,A} + 3.1c^T_R + 8.1c^T_T = (5.0 \pm 50) \cdot 10^{-3}, \tag{11}
\]

\[
\varepsilon_{L+R}^{V,A} - \varepsilon_{L+R}^{V,A} + 1.9c^T_R + 8.0c^T_T = (10 \pm 10) \cdot 10^{-3}. \tag{12}
\]

DV dominate uncertainties for the first constraint, while experimental and \( f_\pi \) uncertainties dominates the latter one. This constraint could be improved with more precise data and \( f_\pi \) calculations, but at some point DV, much more difficult to control, would become the leading uncertainty. The non-negligible correlations between the various NP constraints derived above (due to \( f_\pi \) and experimental correlations) have been taken into account in Eq. (7).

III. ELECTROWEAK PRECISION DATA

If NP is coming from dynamics at \( \Lambda \gg m_Z \) and electroweak symmetry breaking is linearly realized, then the relevant effective theory at \( E \gg m_Z \) is the so-called SMEFT, which has the same local symmetry and field content as the SM, however the Lagrangian contains higher-dimensional operators encoding NP effects \[13, 41\]. The SMEFT framework allows one to combine in a model-independent way constraints from low-energy measurements with those from Electroweak Precision Observables (EWPO) and LHC searches. Moreover, once the SMEFT is matched to concrete UV models at the scale \( \Lambda \), one can efficiently constrain masses and couplings of NP particles. The dictionary between low-energy parameters in Eq. (1) and Wilson coefficients in the Higgs basis \[42, 43\] is

\[
\varepsilon^T - \varepsilon^L = \delta g^W_{Wq} - \delta g^W_{Wq} - \left[ c^{(3)}_{\ell q} \right]_{\tau\tau 11} + \left[ c^{(3)}_{\ell q} \right]_{\tau\tau 11}, \tag{11}
\]

\[
\varepsilon^T - \varepsilon^L = \delta g^W_{Wq} - \delta g^W_{Wq},
\]

\[
\varepsilon^T_{S,P} = -\frac{1}{2} \left[ c_{\ell q u} \pm c_{\ell q d} \right]_{\tau\tau 11},
\]

\[
\varepsilon^T_{\tau} = -\frac{1}{2} \left[ c^{(3)}_{\ell q u} \right]_{\tau\tau 11}, \tag{13}
\]

where we approximated \( V_{CKM} \approx 1 \) in these \( \mathcal{O}(\Lambda^{-2}) \) terms. \( \delta g^W_{Wq} \) are corrections to the SM \( W f f' \) vertex and \( c_i \) are 4-fermion interactions with different helicity structures; see Appendix \[B\] for their precise definitions. Note that \( c^{(3)}_{\ell q} \) is lepton-universal in the SMEFT, up to dim-8 corrections \[13, 44\]. We perform this matching at \( \mu = m_Z \), after taking into account the QED and QCD running of the low-energy coefficients \( \varepsilon_i \) up to the electroweak (EW) scale \[45\]. Electroweak and QCD running to/from 1 TeV is also carried out in the comparison with LHC bounds below.

Our results are particularly relevant for constraining lepton universality (LU) violation. To illustrate this...
decays imply novel limits on these coefficients. We find
\begin{equation}
\begin{bmatrix}
c_{\ell q} \\
c_{\ell e \ell q} \\
c_{\ell q d \ell q} \\
c_{\ell e q u}
\end{bmatrix}_{\tau \tau 11} = \left(\begin{array}{c}
1.2 \pm 2.9 \\
-0.2 \pm 1.1 \\
0.9 \pm 1.1 \\
-0.36 \pm 0.93
\end{array}\right) \times 10^{-2}
\end{equation}
after marginalizing over the remaining SMEFT parameters. These are not only very strong but also unique low-energy bounds. On the other hand Ref. \[43\] did access the right-handed vertex correction: \(\delta g_R^{W q_1} = -(1.3 \pm 1.7) \times 10^{-2}\), from neutron beta decay \[23\] \[47\]. Including hadronic tau decays in the global fit improves this significantly: \(\delta g_R^{W q_1} = -(0.4 \pm 1.0) \times 10^{-2}\) \[4\].

| Coefficient | ATLAS \(\tau \nu\) | \(\tau\) decays | \(\tau\) and \(\pi\) decays |
|-------------|-----------------|-----------------|-----------------|
| \(c^{(3)}\ell q\) | \([0.0, 1.6]\) | \([-12.6, 0.2]\) | \([-7.6, 2.1]\) |
| \(c_{\ell e q u}\) | \([-5.6, 5.6]\) | \([-8.4, 4.1]\) | \([-5.6, 2.3]\) |
| \(c_{\ell e d q, \ell q u}\) | \([-5.6, 5.6]\) | \([-3.5, 9.0]\) | \([-2.1, 5.8]\) |
| \(c_{\ell q u, \ell e q u}\) | \([-3.3, 3.3]\) | \([-10.4, -0.2]\) | \([-8.6, 0.7]\) |

**TABLE I.** 95% CL intervals (in \(10^{-3}\) units) at \(\mu = 1\) TeV, assuming one Wilson coefficient is present at a time. The third column uses Eq. (2), whereas the fourth one includes also clean LU ratios such as \(\Gamma(\tau \to \pi \nu)/\Gamma(\tau \to \mu \nu)\).

### IV. LHC BOUNDS

It is instructive to compare the NP sensitivity of hadronic tau decays to that of the LHC. While the experimental precision is typically inferior for the LHC, it probes much higher energies and may offer a better reach for the Wilson coefficients whose contribution to observables is enhanced by \(E^2/\nu^2\). We focus on the high-energy tail of the \(\tau \nu\) production. This process is sensitive to the 4-fermion coefficients \([c^{(3)}\ell q, c_{\ell e q u}, c_{\ell e d q}, c_{\ell q u}]\tau \tau 11\), which also affect tau decays. Other Wilson coefficients in Eq. (13) do not introduce energy-enhanced corrections to the \(\tau \nu\) production, and can be safely neglected in this analysis.\(^4\)

In Table I we show our results based on a recast of the transverse mass \(m_T\) distribution of \(\tau \nu\) events in \(\sqrt{s} = 13\) TeV LHC collisions recently measured by ATLAS \[50\]. We estimated the impact of the Wilson coefficients on the \(d\sigma(pp \to \tau \nu)/dm_T\) cross section using the \texttt{Madgraph} \[51\]/\texttt{Pythia} 8 \[52\]/\texttt{Delphes} \[53\] simulation chain. We assign 30% systematic uncertainty to that estimate, which roughly corresponds to the size of

\(^4\) A 50% stronger (weaker) bound on \(\delta g_R^{W q_1}\) is obtained using the recent lattice determination of the axial charge in Ref. \[49\] (Ref. \[41\]).

\(^5\) Other energy-enhanced operators do not interfere with the SM. Thus, their inclusion would not change our analysis.
the NLO QCD corrections to the NP terms (not taken into account in our simulations) \cite{54}. The SM predictions are taken from \cite{50}, and their quoted uncertainties in each bin are treated as independent nuisance parameters. We find that for the chirality-violating operators the LHC bounds are comparable to those from hadronic tau decays. On the other hand, for the chirality-conserving coefficient \( [c^{(3)}_{\ell q}]_{\tau r 11} \) the LHC bounds are an order of magnitude stronger thanks to the fact that the corresponding operator interferes with the SM \( q\bar{q} \rightarrow \tau\nu \) amplitude. Let us stress that SMEFT analyses of high-\( p_T \) data require additional assumptions though, such as heavier NP scales and suppressed dimension-8 operator contributions. Last, we observe an \( \mathcal{O}(2)\sigma \) preference for a non-zero value of \( [c^{(3)}_{\ell q}]_{\tau r 11} \) due to a small excess over the SM prediction observed by ATLAS in several bins of the \( m_\tau \) distribution.

The LHC and \( \tau \) decay inputs together allow us to sharpen the constraints on LU of gauge interactions. From Table 1 \( [c^{(3)}_{\ell q}]_{\tau r 11} \) is constrained by the LHC at an \( \mathcal{O}(10^{-3}) \) level, and similar conclusions can be drawn with regard to \( [c^{(3)}_{\ell q}]_{\tau c 11} \) \cite{55}. Then hadronic tau decays effectively become a new probe of the vertex corrections: \( \delta_g^{W \tau} - \delta_g^{W e} \) and \( \delta_g^{R \tau} - \delta_g^{R e} \), complementing the information from previous low-energy EWPO \cite{43}. The interplay between the two is shown in Fig. 2. The input from hadronic tau decays leads to the model independent constraint on LU of W boson interactions: \( \delta g_L^{W \tau} - \delta g_L^{W e} = 0.0134(74) \). The error is more than factor of two larger than in the previously considered scenario where \( \delta g_L^{W \tau} - \delta g_L^{W e} \) was the only deformation of the SM, however the present bound holds for generic heavy NP scenarios where all dimension-6 SMEFT operators can be generated with arbitrary coefficients.

V. CONCLUSIONS

We have shown in this work that hadronic \( \tau \) decays represent competitive NP probes, thanks to the very precise measurements and SM calculations. This is a change of paradigm with respect to the traditional approach, which considers these decays as a QCD laboratory where one can learn about hadronic physics or extract fundamental parameters such as the strong coupling constant. From this new perspective, the agreement between such determinations \cite{3–5} and that of Ref. \cite{10} in the lattice is recast as a stringent NP bound. Our results are summarized in Eq. (2) and can be easily applied to constrain a large class of NP models with the new particles heavier than \( m_\tau \). Hadronic \( \tau \) decays probe new particles with up to \( \mathcal{O}(10) \) TeV masses (assuming order one coupling to the SM) or even \( \mathcal{O}(100) \) TeV masses, for strongly coupled scenarios. They can be readily combined with other EWPO within the SMEFT framework to constrain NP heavier than \( m_\tau \). Including this new input in the global fit leads to four novel constraints in Eq. (14), which are the first model-independent bounds on the corresponding \( \tau\tau qq \) operators. Moreover, it leads to tighter bounds on the W boson coupling to right-handed quarks. Hadronic \( \tau \) decays represent a novel sensitive probe of LU violation (\( \tau \) vs. \( e \)), which competes with and greatly complements EWPO and LHC data. This is illustrated in Fig. 2 and Table 1 for vertex corrections and contact interactions respectively. Our constraints can thus be useful in relation with the current hints of LU violation in certain B mesons decays, or the old tension in W decays. For instance, our model-independent \( \mathcal{O}(1)\% \) constraints in Eq. (2) imply that the hints for \( \mathcal{O}(10)\% \) LU violation observed in \( B \rightarrow D^*\tau\nu \) decays \cite{55,56} cannot be explained by NP effects in the hadronic decay of the \( \tau \) lepton, but must necessarily involve (as is the case in most models) non-standard LU-violating interactions involving the bottom quark.

The discovery potential of these processes in the future is very promising since the constraints derived in this work are expected to improve with the arrival of new data (e.g. from Belle-II) and new lattice calculations. The \( \tau \rightarrow \pi\pi\nu_\tau \) channel represents a particularly interesting example through the direct comparison of its spectrum and \( e^+e^- \rightarrow \pi^+\pi^- \) data. Last, the extension of our analysis to strange decays of the tau lepton represents another interesting research line for the future.
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Appendix A: Inclusive decays calculation

The SM contributions $X_{VV/AA}$ to the inclusive observable in Eq. (8) can be written in the standard form of a contour integral in the complex plane:

$$X_{VV/AA} = \frac{i}{2\pi} \int_{|s| = s_0} ds \omega \left( \frac{s}{s_0} \right) \Pi^{(1+0)}_{VV/AA}(s), \quad (A1)$$

which can be calculated using the Operator Product Expansion (OPE) of the corresponding 2-point correlation function $\Pi^{(1+0)}_{VV/AA}$ [60], that is

$$\Pi^{(1+0)}_{VV/AA}(z) = f^{(pert.)}(\alpha_s; z) + \sum O_{2d}/(-z)^d, \quad (A2)$$

The first term is the dominant perturbative contribution, which is equal for vector and axial correlators. We calculate it using $\alpha_s(M_Z) = 0.1182(12)$ from the lattice [10] and the latest calculation of the correlator including $O(\alpha_s^2)$ contributions [60]. The contour integration is carried out using both fixed-order [3] and contour-improved [61, 62] perturbation theory. The difference between them give a subleading error to our constraints.

The second term in Eq. (A2) are small nonperturbative power corrections. In the $V + A$ channel we take them into account in a conservative way through $O_{2d}^{+A} = 0 \pm \Lambda^{2d}(d - 1)!$. We use $\Lambda \approx 0.4$ GeV as the naive scale of the power corrections, based on the few known vacuum condensates, and the factorial factor accounts for the possible asymptotic behaviour. In the $V - A$ channel only the dimension-six contribution is needed for the weight functions we chose. Using the naive estimate above for this contribution would give a very significant error to Eq. (12). Fortunately it is possible to obtain a more precise estimate for this particular term. Combining dispersive methods [63, 65] with the lattice results on $K \to \pi \pi$ matrix elements [60] we obtain $O_{V-A}^d = -0.0042(13)$ GeV$^6$.

Using also an OPE for the vector-tensor correlator [67] we find

$$X_{VT} = -24\delta_{n,0} \frac{\langle \bar{q}q \rangle}{s_0} \approx 0.086(10) \frac{m_f^2}{s_0} \delta_{n,0} \quad (A3)$$

for $\omega(x) = x^n$, and using the latest $N_f = 2 + 1$ lattice average for the quark condensate [20], from Refs. [68, 72].

Using the OPE result in the entire contour in Eq. (A1) introduces a systematic error that is known as Quark-Hadron Duality Violations [3, 5, 73-76]. They are typically small for $s_0 \sim m_f^2$ and for the appropriate weight functions, but its precise calculation is a nontrivial task. We estimate their size in this work using a conservative $s_0$-stability criteria.

Appendix B: SMEFT Lagrangian

The vertex corrections in Eq. (13) parametrize $W$ boson interactions with quarks and leptons in the SMEFT Lagrangian after electroweak symmetry breaking:

$$\mathcal{L} \supset \frac{g_W}{\sqrt{2}} W_\mu^+ \left[ \delta g_{Wq} \bar{u} \gamma_\mu P_R d + (1 + \delta g_{Lq}^\tau) \bar{\ell} \gamma_\mu P_L \nu \ell \right] \quad (B1)$$

where $\ell = \tau, e$. In the Higgs basis the $\delta g'$s are treated as independent parameters spanning the space of dimension-6 operators; in the Warsaw basis they can be expressed as linear combinations of several Wilson coefficients [32, 13]. The vertex correction $\delta g_{Lq}^\tau$ (parametrizing the $W$ coupling to left-handed up and down quarks) does not appear in Eq. (13) because its effect cancels in the $\epsilon_L^\tau - \epsilon_L$ difference. The four-fermion coefficients $c_i$ are defined by

$$\mathcal{L} \supset \left( \left[c_{iq}^{(3)} \right]_{\tau \tau, 11} (\bar{\ell} \gamma_\mu \sigma^i P_L \ell) (\bar{q} \gamma_\mu \sigma^i P_L q) \right. $$

$$+ \left[ c_{leq}^{(3)} \right]_{\tau \tau, 11} (\bar{P}_R \ell \rho) (\bar{q} P_R u)$$

$$+ \left[ c_{ledq}^{(3)} \right]_{\tau \tau, 11} (\bar{P}_R \ell \rho) (\bar{d} P_L q)$$

$$+ \left[ c_{leq}^{(3)} \right]_{\tau \tau, 11} (\bar{P}_R \ell \rho) (\bar{q} \sigma_{\mu\nu} P_R u) \right) \frac{1}{v^2}, \quad (B2)$$

where $l = (\nu, \tau)^T$, $q = (u, d)^T$, and $v \approx 246$ GeV is the vacuum expectation value of the Higgs field. $c_{ij1}^{(3)}$ is defined analogously with $l = (\nu, e)^T$. Both in the Higgs and the Warsaw basis the $c_i$ coefficients are independent parameters.

[1] A. Pich Prog. Part. Nucl. Phys. 75 (2014) 41–85, arXiv:1310.7922.
[2] ALEPH Collaboration, S. Schael et al. Phys. Rept. 421 (2005) 191–284, hep-ex/0506072.
[3] E. Braaten, S. Narison, and A. Pich Nucl. Phys. B373 (1992) 581–612.
[4] D. Boito, M. Golterman, K. Maltman, J. Osborne, and S. Peris Phys. Rev. D91 (2015), no. 3 034003.
[63] J. F. Donoghue and E. Golowich Phys. Lett. **B478** (2000) 172–184, [hep-ph/9911309].

[64] V. Cirigliano, J. F. Donoghue, E. Golowich, and K. Maltman Phys. Lett. **B522** (2001) 245–256, [hep-ph/0109113].

[65] V. Cirigliano, J. F. Donoghue, E. Golowich, and K. Maltman Phys. Lett. **B555** (2003) 71–82, [hep-ph/0211420].

[66] T. Blum et al. Phys. Rev. **D86** (2012) 074513, [arXiv:1206.5142].

[67] N. S. Craigie and J. Stern Phys. Rev. **D26** (1982) 2430.

[68] G. Cossu, H. Fukaya, S. Hashimoto, T. Kaneko, and J.-I. Noaki *PTEP* **2016** (2016), no. 9 093B06, [arXiv:1607.01099].

[69] Budapest-Marseille-Wuppertal Collaboration, S. Durr et al. Phys. Rev. **D90** (2014), no. 11 114504, [arXiv:1310.3626].

[70] A. Bazavov et al. *PoS LATTICE2010* (2010) 083, [arXiv:1011.1792].

[71] S. Borsanyi, S. Durr, Z. Fodor, S. Krieg, A. Schafer, E. E. Scholz, and K. K. Szabo Phys. Rev. **D88** (2013) 014513, [arXiv:1205.0788].

[72] P. A. Boyle et al. Phys. Rev. **D93** (2016), no. 5 054502, [arXiv:1511.01950].

[73] B. Chibisov, R. D. Dikeman, M. A. Shifman, and N. Uraltsev *Int. J. Mod. Phys. A* **12** (1997) 2075–2133, [hep-ph/9605465].

[74] M. A. Shifman, *Quark hadron duality*, in *At the frontier of particle physics. Handbook of QCD. Vol. 1-3*, (Singapore), pp. 1447–1494, World Scientific, World Scientific, 2001. [hep-ph/0009131 [3.1447(2000)].

[75] O. Cata, M. Golterman, and S. Peris *JHEP* **08** (2005) 076, [hep-ph/0506004].

[76] M. Gonzalez-Alonso, A. Pich, and J. Prades Phys. Rev. **D82** (2010) 014019, [arXiv:1004.4987].