Review on chaotic weak signal processing

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Abstract: Weak fault signal not only refers to the signal with very small amplitude, but also refers to the signal existing in the background of strong noise. Weak signal detection technology is widely used in communication, radar, magnetism, vibration measurement, medicine and other research fields. Based on the essence of chaos, this paper summarizes the research status of chaos at home and abroad, including the basic characteristics of chaos system, the basic characteristics of chaos, the chaotic dynamics model and the discriminant method of chaos. The different methods of chaotic weak fault feature extraction are analyzed, their advantages and disadvantages are pointed out, and the research trend is forecasted.

1. introduction

Since the 1980s, chaos theory has been preliminarily established across almost all natural science and technology subjects, such as chaotic time series analysis, prediction theory and breakthrough progress in chaos control in 1990. The vigorous development of theoretical, experimental and applied research has provided necessary conditions for various applications of chaos. Chaos theory discusses the complexity, the unity of disorder and order, the unity of certainty and randomness that exist in nature and human society. For nonlinear systems, people ever think uncertainty incentives can cause the response, only leads to the randomness of random response, and the discovery of chaos phenomenon make people surprised to see that uncertainty can also cause the random response, this is a shock to people's traditional ideas, greatly widened people's horizons, deepen the understanding of the objective world[1].

Chaos research shows that the dynamic uncertainty system, exist in nature though they have only periodic motion, but it just a rare case of a measure to zero, the vast majority of nonlinear dynamic system, both the periodic motion and chaotic motion, although not all of the nonlinear system is chaotic motion, but the facts show that chaos is common behavior of nonlinear systems[2]. Chaos contains both disorder and order. Chaos is not an ordered state with periodicity and other obvious symmetries, nor is it absolute disorder. Instead, it can be regarded as a complex order that must be described by strange attractors, and an order contained in disorder.

After 1970s, more and more scholars at home and abroad began to conduct in-depth research on chaos theory, providing a new method of weak signal detection. The word chaos was first put forward by Li Tianyan and his tutor in 1975. Chaos is one of the most important achievements of the 20th century,
as important as relativity and quantum mechanics, and is known by many as the "third important physical revolution". Chaos is often used to represent the state of disorder. There is still a unified explanation for the definition of chaos in today's academic circles, but it can be roughly divided into three schools[3]. Firstly, Qian Xuesen, a domestic scholar, holds that chaos seems to be random, but in fact it is inherently orderly. Secondly, Hauperlin's view is that chaos is an order without periodicity; Thirdly, foreign scholar Feigenbaum's viewpoint is chaos is the random movement caused by the internal non-linearity of the system. Combined with these understandings, a general understanding can be given that, under certain conditions, chaos is produced by nonlinear dynamical systems, and it is a phenomenon with both order and disorder. Through the above understanding, we can know that chaos is required to produce conditions, it is more accurate to say that not all nonlinear systems can appear chaos. But if there is chaos in a system, then the system is nonlinear[4].

Starting from the basic characteristics of chaos, the following chapters analyze the development status of weak signal detection methods, as well as the classical chaotic system model and chaotic discrimination methods established today, and strive to have a deeper understanding of chaotic weak signal processing.

2. Five basic characteristics of chaotic system

Since a chaotic system is a nonlinear system, and chaos is produced by a nonlinear system, its trajectory is finite but never repeated. There are five basic characteristics of chaotic motion, among which the sensitivity to initial conditions and unpredictability are the most important.

(1) It is extremely sensitive to changes in initial values

Chaotic system is extremely sensitive to initial values, which is similar to the butterfly effect. Even if the dynamic equation is interfered very weakly, the movement trajectory of the system will be completely changed, and even may quickly deviate from the original trajectory.

(2) Not long term predictability

We cannot correctly predict the trajectory of chaos motion through the existing data and experiments, let alone predict the dynamic characteristics that will appear in the future with the chaos theory.

(3) There are defined areas of movement

In a chaotic state, no matter how much the system is disturbed by the outside world, the movement will always take place in a certain region and never leave this region.

(4) Ergodic

The motion of a system in a chaotic state passes through every point in the region of motion.

(5) Interval dimensions are fractions

Chaos thinks about the endless entanglement, folding and kinking of space. The fractal dimension of the Koch snow curve is 1.26, and the fractal dimension of the Lorentz time series is 2.06. The fractal dimension indicates that the chaotic motion is different from the random motion, and it is a regular motion[5].

3. Current status of weak fault signal detection methods

In order to be able to detect from under the strong background noise cover useful signal, through a large number of experiments for a long time, for a variety of the causes of noise and distribution laws are summarized, the statistical properties of useful signal carried on the thorough analysis and research, its purpose is to hope that through appropriate series method increases the SNR of the system, which can be well to extract useful signal from the overall signal containing noise. At present, the time domain detection methods commonly used in engineering include low noise preamplifier, lock amplification, narrowband filtering.

At present, the weak signal detection circuit that has been reported in China is mainly based on low noise pre-stage amplification. Due to the limited design and technological level, the noise of low noise amplifier can't be reduced infinitely. And the low noise amplifier can only restrain its own noise as far as possible, for its own signal to noise ratio is very low weak signal amplification processing is often powerless.
Narrow-band filters[6] only allow narrow-band useful signals and a very small amount of narrow-band noise through, the narrower the passband, the stronger the ability to suppress noise, so as to achieve the purpose of signal detection. But the center frequency of the narrowband filter is difficult to keep highly stable; Moreover, if the useful signal is extremely weak and submerged in the noise, the weak signal of the narrowband filter is still submerged by the noise although the noise voltage mean square value of the output signal is reduced and the signal-to-noise ratio is improved[7].

When dealing with the mixed signal whose frequency band of the noise and the measured signal coincide, the traditional method will filter out the noise while the useful signal will also be lost. With the deep research and wide application of chaos theory in various fields of modern science, people use chaos theory to detect weak signal, and form the theory and method of weak signal detection with chaos according to different types of background noise in the research and application. This method can detect the signal to be detected from the mixed signal with overlapping frequency band. The equipment structure is simple and the instrument cost is low.

Facts have proved that for some nonlinear dynamical systems, changing parameters will lead to essential changes in the solution, and chaotic systems have great potential in signal detection because of their sensitivity to small signals and immunity to noise. Using the principle of chaotic Duffing oscillator detection, the phase trajectory of a Duffing system cannot be changed even if the noise is very strong.

A detection system sensitive to weak signals can be built by using the characteristics of chaotic oscillator which is immune to noise and sensitive to weak signals. This detection method is simple, intuitive, suitable for the condition of low signal-to-noise ratio, and is less disturbed by noise. There are two main methods to distinguish chaotic characteristics of time series: intuitive analysis method and quantitative analysis method. Intuitive analysis method can directly determine the existence of chaos by analyzing the characteristics of signal in time domain or frequency domain. Its advantage is its intuitiveness and visualization. But this method has low efficiency, large error and is seriously affected by noise and other factors. Quantitative analysis method can judge the existence of chaos by some characteristic quantity of quantitative extreme chaos.

4. Basic characteristics of chaos and chaotic dynamics model

4.1 Chaos implementation

Chaos behavior is complex, people often think that its dynamic characteristics must be very complex, in fact, a few parameters, very simple dynamic characteristics of the system can also produce chaos phenomenon.

The study found that by changing the system parameters, make the system into the chaos of the first kind of way is period-doubling bifurcation, namely by the fixed point → two cycle → four cycle → ...Infinite times cycle → entered into a state of chaos. The path to chaos does not stop there. The second path is to go from equilibrium to periodic motion, to quasi-periodic state, all the way to chaos. The third way is that the system is in the process of approximate periodic motion. By changing the relevant parameters of the system, the system will appear paroxysmal chaotic process. With the adjustment of the parameters, the paroxysmal chaos becomes more and more frequent, and the approximate periodic motion becomes less and less, and finally enters into chaos[8].

When the actual system passes over the chaos region, some return to the periodic solution, some diverges directly, and some reverse bifurcation, which is related to the value of system parameters, which also indicates the diversity of chaos and the complexity of structure.

4.2 Three typical chaotic system dynamics models

Chaotic system is a kind of nonlinear dynamical system which is determined by a special deterministic mathematical model. State refers to the physical parameters that describe the basic state of a system, while dynamic characteristics are rules that describe how the state of a system changes over time. If dynamical systems are continuously changing over time, the mathematical tools used to describe such systems are mathematical models. For the complex dynamic system, when the parameters change, the
simple motion state is transformed to the chaotic motion state, and the relationship between the simple motion and chaotic motion is the chaotic system model[9].

On the one hand, the study of chaotic model provides a theoretical research model for the development of chaotic system theory, on the other hand, it also lays a foundation for the analysis and processing of chaotic signals and the application research of chaotic signals. There have been many models for generating and modeling chaotic signals, Lorenz's model and Rossler model from physical mechanics, Logistic mapping and other models from biology. These models exhibit abundant chaotic behavior and have been studied extensively and deeply[10]. At present, a variety of chaotic mathematical models have been established, and there are mainly two ways to realize them: one is to build a circuit to generate chaotic signals based on the mathematical model, and the other is to generate chaotic signals based on the mathematical model by using software.

1) Mathematical Model and Analysis of Duffing System
Duffing equation has been proved to be a chaotic system. The nonlinear dynamical system described by Duffing equation shows rich nonlinear dynamical characteristics, including oscillation, bifurcation and chaotic complex dynamics. It has become one of the commonly used models to study chaos[11].

The specific form of Duffing equation is:

\[ x''(t) + kx'(t) - x(t) + x^3(t) = acos\omega t \]  

In the formula, k is the damping ratio; a is the amplitude of periodic power; \(\omega\) is the angular frequency of the engine.

The following conclusions are drawn from the analysis:

① The amplitude of the policy force has a great influence on the chaotic motion. Under the same dynamic frequency, the dynamic behavior is different with the different amplitude of the dynamic, which shows that the phase trajectories of the chaotic motion are different.

② The system damping ratio K has an effect on the threshold value of chaotic motion.

③ From the analysis of the change of phase trajectory of the system, it can be seen that the dynamic behavior of the chaotic system is very sensitive to the initial parameters, so the weak signal can be detected by using the periodic signal to change the dynamic behavior of the system from the chaotic critical state to the phase trajectory change process of the large-scale periodic state.

2) Lorenz mathematical model and analysis
Lorenz, a meteorologist, made the discovery in the early 1960s when he ran computer experiments on an enhanced climate model, a mathematical model of the following form:

\[ \begin{align*}
    \dot{x} &= -\sigma(x - y) \\
    \dot{y} &= -y - xz + rx \\
    \dot{z} &= -bz + xy
\end{align*} \]  

Where R and \(\sigma\) are constants, as can be seen from the system, there are always solutions for \(x=y=z=0\).

And then there are solutions for \(r\) greater than or equal to 1, \(x = y = \sqrt{b(r - 1)}\), \(z = r - 1\). When \(r=1\), its orbits are entangled with each other, and after excessive attenuation, the complex geometry of the time traversal does not intersect itself, and it has no analytical solution. In the equation, \(x, y\) and \(z\) are dimensionless quantities. With any given initial value, the system will eventually return to a specific region of the state space. Its attractor is characterized by the adjacent orbital line of exponential divergence, and its evolution is aperiodic with exquisite and peculiar structure.

3) Logistic mathematical model and analysis
Logistic equation is an ecological model used to study the change of the total amount of a certain species in a long time. The discrete system Logistic is defined as

\[ x_{n+1} = f(x_n) = 1 - \mu x_n^2 \]  

Type, \(\mu\) is parameters, \(x_n\) is state. With the difference of \(\mu\), the Logistic equation shows obvious periodic or chaotic behavior. When \(0 \leq \mu \leq \mu_0 = 3.5699455\), The sequence \(\{x_k\}\) in cycle state. When \(\mu_0 \leq \mu \leq 4\), The Logistic system runs in a chaotic state.

Chaos also exists in neural networks, electricity, celestial bodies, circuits, molecular motion and other systems. In view of the current research situation, it is a problem to make full use of chaos theory to find
the chaotic dynamics model closer to the actual engineering and design the chaotic system according to the requirements of the actual engineering application.

5. Summary and Outlook
Due to the introduction of chaos, strict mathematical analysis, a large number of computer simulations and increasing laboratory observations are explaining the opposition and unity between the two descriptive systems of certainty theory and probability theory from another brand-new Angle. The phenomenon of chaos in many areas that many researchers than agreed to the traditional understanding of "random" theory, nonlinear dynamics system presents the chaos phenomenon is not because the system is affected by environmental noise source, also is not an infinite number of degrees of freedom because of the interaction, more is not associated with the uncertainty of random process, the nonlinear dynamics rules is a necessary condition for the existence of chaos[12].

In recent years, the use of chaotic oscillator to the unknown signal parameter estimation to get a high accuracy, no matter from the estimation precision and signal recognition ability than ever spectrum estimation method based on statistical theory proposed, higher order statistics, such as better, more satisfying is chaotic systems for any zero-mean noise immunity, that is to say, the influence of noise to chaotic system is less than the signal itself periodic perturbation on the effects of system dynamic behavior.

The sensitivity to small signals and immunity to noise make the chaotic system have a good application prospect in signal detection. For a nonlinear dynamic system, the parameter perturbation will cause changes in the nature of state solutions, the periodic signal under test as continuous nonlinear dynamics system disturbance input, even in a strong zero-mean noise background, the state of the system will be as the change of the input signal parameters have obvious evolution process, by using chaos method for detection of weak signal is a profound revolution in research of signal processing, the chaotic system is the harmony of certainty and randomness, the study of it, especially the study of chaotic signal processing method has far-reaching significance.

Chaos theory is becoming more and more mature under the research of scholars, but its practical application is relatively rare. Future research mainly focuses on the application of time, including the application of chaos technology in the extraction of leakage fault features of bearings, gears and pipelines, so as to enrich the practical application. At present, the development trend of this field is towards practical application. Now the nonlinear theory is still a hot research topic in the future. New methods will continue to appear and further develop towards practical application.

Acknowledgments
This research is supported by the National Natural Science of China (No.51909267,51579242).

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