The effect of heat conduction on the piston-in-cylinder dynamic pressure generator

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Abstract. This paper deals with the effect of heat conduction on the piston-in-cylinder dynamic pressure generator in order to indicate the transition from isothermal to adiabatic behaviour when increasing the pulsation frequencies. A mathematical model for the perfect gas in the cylindrical chamber is used to study the physical background of the effect of heat conduction on frequency characteristics of the pressure generator. Frequency characteristics of the pressure generator were theoretically obtained for different gases and average pressures.

1. Introduction
The increasing use of the pressure sensors to monitor rapidly time-varying pressures in many industrial and scientific applications caused growing needs for dynamic calibration of such sensors. The lack of primary standard for dynamic calibration of pressure sensors, except for sound pressure, results in the development of various configurations of dynamic pressure generators [1]. Due to the fact that the dynamic characteristics of the pressure sensors also depend on the value of the average pressure during the pressure pulsations, this paper discusses a piston-in-cylinder primary standard for dynamic calibration of pressure sensors at different average pressures (the mechanical implementation of which was presented by the authors of this paper in [2]). Its principle of operation is based on the sinusoidal motion of the piston with constant area, which causes volume changes of the gas in the cylinder chamber and consequently the pressure changes. The time-varying pressure can be traceable to static pressure and length measurements at the lowest and the highest pulsation frequencies, where the process in the dynamic pressure generator can be considered as isothermal and adiabatic, respectively. In order to provide SI-traceable dynamic calibrations in the transition frequency range, where process depends on the degree to which the heat transfers to the surroundings during the generation of time-varying pressure [3], the effect of heat transfer has to be employed in the frequency characteristics of the pressure generator. The heat conduction theory has been investigated by several authors in the field of acoustics, where the standard method for primary reciprocity calibrations of microphones provides two models for the heat conduction correction: the low frequency model and the broadband model [4-8]. The low frequency model is derived from Gerber’s analysis of heat conduction in a closed fluid-filled cylindrical chamber. He set up two forms of solutions for the complex temperature frequency response function, the full frequency range solution and simplified short-time solution. The latter is used in IEC standard 61094-2, although it is less accurate. The broadband model, on the other hand, expands the low frequency model by also considering a viscosity effect.
The aim of this paper is to present the investigations of the effect of heat conduction on the piston-in-cylinder dynamic pressure generator for different gases and average pressures. The analytical solution for the frequency characteristics of the pressure generator, in which the full frequency range solution for the complex temperature frequency response function is used, is presented in section 2. In section 3 the theoretical and experimental results are discussed and compared.

2. Frequency characteristics of the pressure generator with employed effect of heat conduction

The discussed piston-in-cylinder dynamic pressure generator consists of a cylindrical chamber (height \( h \), diameter \( d \)), which has on one end a piston and on another end a tested pressure sensor. The mathematical model assumes a relatively small volume of the chamber and small volume changes generated by the piston movement, and therefore the effects of wave motion, heat convection and viscosity can be neglected [5]. By also neglecting the heat radiation between the gas and the chamber walls and by applying the first law of thermodynamics, the energy equation for the heat conduction of the perfect gas can be written as [9]:

\[
\lambda \nabla^2 T = \rho c_v \frac{dT}{dt} + \rho p \frac{dv}{dt},
\]

where \( \lambda \) is the thermal conductivity, \( T \) is the temperature, \( \rho = p/RT \) is the density, \( p \) is the pressure, \( R = c_p - c_v \) is the gas constant, \( c_p \) is the specific heat at constant pressure, \( c_v \) is the specific heat at constant volume and \( v \) is the specific volume. Due to the fact that we consider a closed chamber without leakage to the surroundings, in (1) the relation \( \rho dv/dt = (1/V)dV/dt \) is used, where \( V \) is the volume of the gas. The harmonic changes of the volume, pressure and temperature of the gas in the chamber can be expressed as \( V = V_0 - V' \exp(i \omega t) \), \( p = p_0 + p' \exp(i \omega t) \) and \( T = T_0 + T' \exp(i \omega t) \), respectively, where \( V_0 = \pi d^2 h/4 \), \( p_0 \) and \( T_0 \) represent their time-averaged components and \( V' \), \( p' \) and \( T' \) complex amplitudes of their time-varying components, respectively, where \( \omega = 2\pi f \) is the angular frequency and \( f \) is the frequency. Equation (1) is linearized and solved for the boundary conditions at the chamber walls, where the time-averaged temperature of the gas is assumed to be equal to the temperature of the chamber walls. Taking into account the relations for the adiabatic index \( \gamma = c_p/c_v \) and the thermal diffusivity \( \alpha_t = \lambda/\rho c_v \), the solution for complex frequency response function between the relative spatially averaged temperature amplitude \( T'/T_0 \) and the relative volume amplitude \( V'/V_0 \) can be written as [8]:

\[
\frac{T'}{T_0} = (\gamma - 1) E_v, \quad E_v = \frac{32}{\pi^2} \sum_{z=0}^{\infty} \sum_{r=0}^{\infty} i \omega \left[ \frac{\pi^2 (2z + 1)^2}{h^2} + \frac{4j^2}{d^2} + \frac{i \omega}{\alpha_t} \right]^{-1},
\]

where \( j_r \) are the roots of the zeroth order Bessel function of the first kind \((J_0(j_r) = 0)\). By considering equation of state for perfect gas \( pV = mRT \), where \( m \) is the mass of the gas, the complex frequency response function between the relative spatially averaged pressure amplitude \( p'/p_0 \) and the relative volume amplitude can be written as:

\[
\frac{p'/p_0}{V'/V_0} = 1 + (\gamma - 1) E_v.
\]

The absolute value and the argument of the complex frequency response function (3) gives the amplitude and phase-frequency characteristics of the piston-in-cylinder dynamic pressure generator with the employed heat conduction effect, respectively. In (3) the indices \( z \) and \( r \) up to 200 were taken into account in order to produce the calculation error of less than 0.04% in amplitude ratio and 0.01° in phase lag up to \( f = 1000 \) Hz.

3. Results

The theoretical and measured amplitude-frequency characteristics of the developed piston-in-cylinder dynamic pressure generator (\( h = 12.05 \) mm, \( d = 80 \) mm) presented in figure 1 were determined for air under ambient conditions. The experimental results were determined by measuring the average
pressure with the piezoresistive pressure transmitter (Kistler, 4260A) and the generated pressure pulsations with the piezoelectric pressure transducer (Kistler, 7261) for the amplitude of the piston displacement equal to about 0.1 mm, which was measured with a laser triangulation displacement sensor (Micro-Epsilon, ILD 2300-2). In comparison with the theoretical results obtained with equation (3), the measured amplitude ratio is in general lower for the relative difference up to 4%. The difference between the theoretically and experimentally obtained amplitude ratios is expected to result from the fact that the actual pressure generator has a through piston rod (d_{rod} = 12 mm), which is not considered in the theoretical model.

The frequency characteristics of the pressure generator depend on the dimensions of the chamber and on the properties of the gas. Figure 2 presents the dependence of the thermal diffusivity of different gases on the average pressures at T_0 = 20 °C. From the figure it is evident that the carbon dioxide (CO_2) has lower thermal diffusivity than air and argon (Ar) and that the thermal diffusivity of the gases decreases when increasing the average pressure of the gas.

Figure 1. Amplitude-frequency characteristics of the pressure generator for air at p_0 = 100.7 kPa and T_0 = 22.5 °C.

Figure 2. Thermal diffusivity for different average pressures at T_0 = 20 °C.

Figure 3 shows the frequency characteristics of the pressure generator for different gases at p_0 = 100 kPa and T_0 = 20 °C. It is evident that the amplitude ratio and the phase lag at lower pulsation frequencies, where the system can be considered as isothermal, approach 1 and 0°, respectively. In the transition frequency range, where process depends on the degree to which the heat conducts to the surroundings during the generation of time-varying pressure, the phase lag increases up to 5.9° for CO_2, 7.7° for air and 11.8° for Ar. At higher pulsation frequencies, where the system can be considered as adiabatic, the values of the amplitude ratio and the phase lag approach γ (1.30 for CO_2, 1.40 for air and 1.67 for Ar) and 0°, respectively. The adiabatic system can be defined with the adiabatic limit frequency, at which the amplitude ratio reaches the value of 0.99γ and phase lag achieves the value of 0.6°. The results show that the adiabatic limit frequency is lower for the gases with lower adiabatic indices and thermal diffusivities, and is therefore 26 Hz for CO_2, 91 Hz for air and 204 Hz for Ar.

Figure 4 show the frequency characteristics of the pressure generator for air at different average pressures and T_0 = 20 °C. The adiabatic limit frequency decreases from 91 Hz to 19 Hz by increasing the average pressure from 100 kPa to 500 kPa. Due to the fact that the adiabatic index of the gas changes only slightly with the pressure, the adiabatic limit frequency at different average pressures mainly depends on the thermal diffusivity of the gas. As the thermal diffusivity of the gas decreases with increasing pressure (see figure 2), the heat at higher pressures conducts to the surroundings more slowly with respect to the volume changes over time and therefore the adiabatic system is more quickly approached when increasing the pulsation frequency.
Figure 3. Frequency characteristics of the piston-in-cylinder pressure generator at $p_0 = 100$ kPa and $T_0 = 20$ °C: (a) amplitude ratio; (b) phase lag.

Figure 4. Frequency characteristics of the piston-in-cylinder pressure generator for air at $T_0 = 20$ °C: (a) amplitude ratio; (b) phase lag.

4. Conclusion
This paper discusses the effect of heat conduction on the piston-in-cylinder dynamic pressure generator. The theoretical results show that by using the gases with smaller adiabatic indices and thermal diffusivities, the adiabatic limit frequency of the pressure generator is decreased. In the future work, we plan to improve the configuration of pressure generator by replacing the piston with the diaphragm without the through piston rod. The mathematical modelling will be extended by taking into account the effects of heat convection, heat radiation, leakage and viscosity.

5. References
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