Various ground state properties are explored for full isotonic(isotopic) chain of neutron number N(proton number Z)=40 using different families of Relativistic Mean-Field theory. Several properties such as nucleon separation energies, pairing energies, deformation, radii and nucleon density distributions are evaluated and compared with the experimental data as well as those from other microscopic and macroscopic models. N=40 isotonic chain presents ample of support for the neutron magicity and articulates double magicity in recently discovered $^{60}_{20}$Ca and $^{68}_{28}$Ni. Our results are in close conformity with recently measured value of charge radius of $^{68}_{28}$Ni [S. Kaufmann et al., Phys. Rev. Lett. 124, 132502 (2020)] which supports the N=40 magicity. Contrarily, Zr isotopes (Z=40) display variety of shapes leading to the phenomenon of shape transitions and shape co-existence. The role of $3s_{1/2}$ state, which leads to central depletion if unoccupied, is also investigated. $^{56}_{26}$S and $^{122}_{40}$Zr are found to be doubly bubble nuclei.

1. Introduction
The recent discovery of $^{60}_{20}$Ca at the radioactive ion-beam factory operated by RIKEN Nishina Center and CNS, University of Tokyo and recent lifetime mea-
measurements demonstrating shape-coexistence in $^{98}_{40}\text{Zr}_{58}$ have certainly provided an extra impulse to ascent more theoretical and experimental studies for the nuclei consisting neutron(proton) number N(Z)=40. Latterly, the first spectroscopy of $^{62}\text{Ti}$ has been reported for the detail investigation of shell evolution of N=40 isotones towards $^{60}\text{Ca}$. Another nucleus $^{68}_{28}\text{Ni}_{40}$ has shown tremendous possibilities to look into the magicity, shapes and shape-coexisting configurations in excited states, giant resonances and pygmy dipole resonance (PDR). Evolution of structure in the Zr isotopes (Z=40) has been described to show the interplay between shape-phase transitions and shape coexistence.

Full systematic study using covariant density functional analysis of N=40 isotones has been done by Wang et al. which provides (a) reasonable description not only for the systematics of the low-lying states along the isotonic chain but also for the detailed structure of the spectroscopy in a single nucleus (b) spherical-oblate-prolate shape transition along the isotonic chain of N=40 and, (c) coexistence of low-lying excited $0^+$ states in neutron-deficient N=40 isotones. On the other hand, shape coexistence in Zr isotopes has been investigated using interacting boson model with configuration mixing which has confirmed spherical nature of the ground state of $^{94–98}\text{Zr}$ and deformed nature for $^{100–110}\text{Zr}$. Moreover, role of tensor force is inquired for the shape evolution of Zr nuclei using axially deformed Hartree-Fock (HF) calculations with the semi-realistic interaction M3Y-P6 and as a result a strong shape transition is found in Zr isotopes such as $^{80}\text{Zr}$ is found deformed whereas $^{86–96}\text{Zr}$ are found spherical. Prolate shape of $^{98–112}\text{Zr}$ switches to oblate for $^{114}\text{Zr}$ and then sphericity returns at $^{120}\text{Zr}$ and $^{122}\text{Zr}$. Therefore, Zr isotopes exhibit remarkable N-dependence in their structure leading to a demand of full systematic and deeper study which is one of the prime reason for this investigation. The existence of a new island of inversion around N=40 along with recent experimental studies waiting point nuclei along N=Z line for rapid proton capture reactions (rp-process) along with new doubly magic nuclei $^{60}\text{Ca}$ and $^{64}\text{Ni}$ are the driving forces which have prompted this systematic study of nuclei with (semi) magic number N(Z)=40.

We perform a systematic study of full chain of N=40 isotones and Z=40 (Zr) isotopes using relativistic mean-field (RMF) models. Majority of results are presented using the RMF model with density dependent meson coupling strength taking DD-ME2 parameter. At various places these results are compared with RMF models with density-dependent point coupling interactions using DD-PCX parameter and the nonlinear self- and mixed-interactions of the mesons viz. NL3* and FSU-Gold and experimental data as well. This systematic study consist of ground state properties which include deformation, separation energies, single particle energies, radii and density distribution etc.
2. Relativistic Mean-Field Theory

Three classes of mean-field models are employed for this investigation: the density-dependent meson-exchange (DD-ME) model, the nonlinear meson-nucleon coupling model (NL), and the density-dependent point-coupling (DD-PC) model. The main differences between these models consist in the treatment of the range of the interaction and in the density dependence. The density dependence is introduced either through an explicit dependence of the coupling constants (DD-ME & DD-PC) or via non-linear meson couplings (NL). Meson exchange model and point coupling model have an interaction of finite and of zero range, respectively. As a consequence, at present, these major classes of covariant energy density functionals exist dependent on the combination of above mentioned features (see Ref. for detail).

2.1. Variants of RMF

We perform the RMF calculations using effective nuclear interactions with density-dependent meson-nucleon vertex functions which represent a significant improvement in the relativistic self-consistent mean-field description of the nuclear many-body problem. This kind of effective interaction has been shown to provide a more realistic description of asymmetric nuclear matter, neutron matter, and finite nuclei, which includes a softer equation of state of nuclear matter (i.e., lower incompressibility) and a lower value of the symmetry energy at saturation in comparison to other standard nonlinear meson-exchange models. The employed effective Lagrangian density is similar to the Ref. and has the following form:

\[
\mathcal{L} = \bar{\psi} (i\gamma.\partial - M) \psi + \frac{1}{2} (\partial \sigma)^2 - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} \\
+ \frac{1}{2} m_\omega^2 \omega^2 - \frac{1}{4} \overrightarrow{R}_{\mu\nu} \overrightarrow{R}^{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
- g_\sigma \bar{\psi} \sigma \psi - g_\omega \bar{\psi} \gamma.\omega \psi - g_\rho \bar{\psi} \gamma.\rho \tau \psi \\
- \epsilon \bar{\psi} \gamma. A \left(1 - \tau_3\right) \psi
\] (1)

Here, vectors in isospin space are denoted by arrows, and the bold-faced symbols indicate vectors in ordinary three-dimensional space. The Dirac spinor \(\psi\) denotes the nucleon with mass \(M\). The masses of \(\sigma\), \(\omega\), and the \(\rho\) mesons are denoted by \(m_\sigma\), \(m_\omega\), and \(m_\rho\), respectively, with \(g_\sigma\), \(g_\omega\), and \(g_\rho\) being the corresponding coupling constants for the mesons to the nucleon. Here, \(\Omega^{\mu\nu}\), \(\overrightarrow{R}^{\mu\nu}\), and \(F^{\mu\nu}\) are the field tensors of the vector fields \(\omega\), \(\rho\), and of the photon:

\[
\Omega^{\mu\nu} = \partial^\mu \omega^\nu - \partial^\nu \omega^\mu \\
\overrightarrow{R}^{\mu\nu} = \partial^\mu \overrightarrow{\rho} ^\nu - \partial^\nu \overrightarrow{\rho} ^\mu \\
F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu
\] (2)

As per Ref. the density-dependent meson-exchange model (DD-ME) is used
in this work, in which the meson-nucleon strengths \( g_\sigma \), \( g_\omega \) and \( g_\rho \) have an explicit density dependence in the following form:

\[
g_i(\rho) = g_i(\rho_{\text{sat}}) f_i(x), \quad \text{for } i = \sigma, \omega
\]

where the density dependence is given by:

\[
f_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2}
\]

in which \( x \) is given by \( x = \rho / \rho_{\text{sat}} \), and \( \rho_{\text{sat}} \) denotes the baryon density at saturation in symmetric nuclear matter. For the \( \rho \) meson, density dependence is of exponential form and given by

\[
f_\rho(x) = \exp(-a_\rho(x - 1))
\]

For a comparison, we have used an improved parameterization of widely used and successful non-linear parameter (NL3*) for the RMF model, which contains only six phenomenological parameters. This variant has delivered accurate description of the ground state properties of many nuclei and simultaneously has provided an excellent description of excited states with collective character in spherical as well as in deformed nuclei.\(^{26}\) This variant includes linear terms for the \( \sigma \), \( \omega \) and \( \rho \) mesons, together with the non-linear term only for self interaction of the \( \sigma \) meson.

The effective Lagrangian density can be expressed by adding the nonlinear term of \( \sigma \) meson \((\frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4)\). For more details see Refs. \(^{25–28}\)

Our calculations are also performed with the nonlinear parameter FSU-Gold\(^{29}\) containing non-linear self interaction of \( \omega \) meson and the mixed interaction terms for \( \omega \) and \( \rho \) mesons. These two additional parameters are used by virtue of softening of both the EOS of symmetric matter and the symmetry energy. This variant FSU-Gold has been fitted to the binding energies and charge radii of a variety of magic nuclei. The interaction part of the Lagrangian, which describes the coupling of mesons to the nucleons and the non-linear self and mixed interactions of mesons, can be expressed as\(^{29}\)

\[
\mathcal{L}_{\text{int}} = -\bar{\psi} \left[ g_\sigma \sigma - \gamma^\mu \left( g_\omega \omega_\mu + \frac{1}{2} g_\rho \tau_\rho \rho_\mu + \frac{e}{2} (1 + \tau_3) A_\mu \right) \right] \psi
\]  
\[
- \frac{\kappa_3}{6M} g_\sigma m_\sigma^2 \sigma^3 - \frac{\kappa_4}{24M^2} g_\omega^2 m_\sigma^2 \sigma^4 + \frac{\zeta_0}{24} g_\omega^2 (\omega_\mu \omega_\mu)^2
\]  
\[
+ \frac{\eta_2}{4M^2} g_\omega^2 m_\rho^2 \omega_\mu \omega_\rho \rho_\nu.
\]

It includes an isoscalar-scalar \( \sigma \) meson field and three vector fields: an isoscalar \( \omega_\mu \), an isovector \( \rho_\mu \), and the photon \( A_\mu \). In addition to the Yukawa couplings, the Lagrangian is supplemented by four nonlinear meson interactions. The inclusion of isoscalar meson self-interactions (via \( \kappa_3, \kappa_4, \) and \( \zeta_0 \)) are used to soften the equation of state of symmetric nuclear matter, while the mixed isoscalar-iso-vector coupling \( (\eta_2) \) modifies the density dependence of the symmetry energy. Other symbols have the usual meaning and details can be found in Refs. \(^{25–27,29}\)
In analogy with meson-exchange model (DD-ME) described above, we have used density-dependent point coupling interaction (DD-PC) in the RMF calculations. The effective interaction DD-PCX used in this work represents the first effective interaction that is constrained using the binding energies, charge radii, and pairing gaps, together with a direct implementation of the ISGMR energy and dipole polarizability. This variant accurately describes the nuclear ground state properties including the neutron-skin thickness, as well as the isoscalar giant monopole resonance excitation energies and dipole polarizabilities. The effective lagrangian for this model in terms of nucleonic field can be expressed as:

$$\mathcal{L} = \bar{\psi}(i\gamma_\mu \partial - M)\psi - \frac{1}{2}\alpha_S(\rho)(\bar{\psi}\psi)(\bar{\psi}\psi) - \frac{1}{2}\alpha_V(\rho)(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma_\mu\psi) - \frac{1}{2}\alpha_{TV}(\rho)(\bar{\psi}\gamma_\mu\gamma_\mu\psi)(\bar{\psi}\gamma_\mu\gamma_\mu\psi) - \frac{1}{2}\delta_S(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - \frac{1}{2}\delta_{TV}(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi) - \epsilon\bar{\psi}\gamma_\mu A(1 - \tau_3)\psi$$

Here, the free nucleonic term contains isoscalar-scalar (S), isoscalar-vector (V) and isovector-vector (TV) interactions. The coupling constants $\alpha_i(\rho)$ are density dependent and employed as:

$$\alpha_i(\rho) = a_i + (b_i + c_i x)e^{-d_i x} \quad \text{for} \quad i = S, V, TV$$

where $x = \rho/\rho_o$, and $\rho_o$ denotes the nucleon density in symmetric nuclear matter at saturation point. It should be pointed out here that success of the DD-PCX interaction in the predictions of the dipole polarizabilities and neutron skin thicknesses in other nuclei not used in optimizing the model parameters validates the isovector channel of the functional and the respective symmetry energy properties.

### 2.2. Pairing Treatment

Pairing correlations play an important role in all open-shell nuclei and turn into particulary significant for the nuclei near the drip line. For the mean-field level pairing correlations are taken into account by Bardeen-Cooper-Schrieffer (BCS) or HartreeFock-Bogoliubov (HFB) theory and in the relativistic case by relativistic Hartree-Bogoliubov (RHB) theory. For the non-linear version of our calculations (NL3*), we have used the BCS scheme wherein the single particle continuum corresponding to the RMF is replaced by a set of discrete positive energy states. The results are found to be in close agreement with the experimental data and with those of recent continuum relativistic Hartree-Bogoliubov (RCHB) and other similar mean-field calculations. In the calculations, we use a delta force for the pairing interaction, i.e., $V = -V_0\delta(r)$ with the strength $V_0 = 350 \text{ MeV fm}^3$ which has been used in Refs. for the successful description of drip-line nuclei. Apart from its simplicity, the applicability and justification of using such a $\delta$-function form of interaction has been discussed in Ref. whereby it has been shown in the
context of HFB calculations that the use of a delta force in a finite space simulates
the effect of finite range interaction in a phenomenological manner (see also\textsuperscript{39} for
more details).

Whenever the zero-range $\delta$ force is used either in the BCS or the Bogoliubov
framework, a cutoff procedure must be applied, i.e. the space of the single-particle
states where the pairing interaction is active must be truncated. This is not only
to simplify the numerical calculation but also to simulate the finite-range (more
precisely, long-range) nature of the pairing interaction in a phenomenological way\textsuperscript{39}
In the present work, the single-particle states subject to the pairing interaction are
confined to the region satisfying

$$\epsilon_i - \lambda \leq E_{\text{cut}},$$

where $\epsilon_i$ is the single-particle energy, $\lambda$ the Fermi energy, and $E_{\text{cut}}=8.0$ MeV. For
further details of these formulations we refer the reader to Refs.\textsuperscript{41–43}

The pairing correlations for FSU-Gold lagrangian are treated in the constant
gap approximation\textsuperscript{44, 45} with gap parameters derived from the odd-even mass dif-
ferences. This approach works properly when the main effect of the pairing corre-
lations is a smearing of the Fermi surface. The constant pairing gaps are taken as
the following forms

$$\Delta_n = \Delta_p = \frac{11.2}{\sqrt{A}}$$

For the density dependent models, we use the TMR separable pairing force of
Ref.\textsuperscript{46} for the short range correlations. This kind of separable pairing force has
been adjusted to reproduce the pairing gap of the Gogny force D1S in symmetric
nuclear matter. Both forces are of finite range and therefore they show no ultraviolet
divergence and do not depend on a pairing cut-off. They provide a very reasonable
description of pairing correlations all over the periodic table with a fix ed set of
parameters. In the $^{1}S_0$ channel the gap equation is given by

$$\Delta(k) = \int_0^\infty \frac{k^2dk'}{2\pi^2} \langle k|V^{^{1}\!S_0}|k'\rangle \frac{\Delta k'}{2E(k')}$$
and the pairing force separable in momentum space is

$$\langle k|V^{^{1}\!S_0}|k'\rangle = -Gp(k)p(k')$$

The two parameters determining the force are, the pairing strength $G$ and $\alpha$ that
goes in the Gaussian ansatz $p(k) = e^{-\alpha^2k^2}$. Their value has been adjusted to $G =
728$ MeV fm\textsuperscript{3} and $\alpha = 0.644$ fm in order to reproduce the density dependence of the
gap at the Fermi surface, calculated with the D1S parametrization of the Gogny
force\textsuperscript{47}

3. Results and discussions

Study of $N=40$ isotones covers a chain of proton number from $Z=16$ to $Z=42$
whereas Zr isotopes ($Z=40$) contain a wide range of neutrons from $N=38$ to $N=112$
which in turn address to a broad area of periodic chart. We first describe shapes and structural properties of the nuclei of these chains which are followed by a detailed investigation of observed magicity. Thereafter, central depletion or bubble phenomenon in the nucleonic density is examined.

3.1. Shapes and Structural Properties

We have plotted two proton separation energy ($S_{2p}$) of N=40 isotones and two neutron separation energy ($S_{2n}$) of Zr isotopes in Fig. 1(a) and (b), respectively to validate our results. These energies are calculated using DD-ME2 parameter\(^{23}\) and compared also with the RMF calculations done with NL3*\(^{28}\), FSU-Gold\(^{29}\) and DD-PCX\(^{28}\) along with experimental data\(^{30}\). A reasonable match among all these parameters along with experimental results can be seen from Fig. 1(a) for N=40 isotones. Two proton drip-line for this chain is found \(^{82}\)Mo from DD-ME2 and DD-PCX parameter whereas NL3* and FSU-Gold parameters deliver to \(^{80}\)Zr. A sharp fall just after conventional magic numbers Z=20 and 28 is clearly observed affirming the magicity in \(^{60}\)Ca and \(^{68}\)Ni: the nuclei which are of current interest\(^{1, 4}\) as mentioned above.

![Figure 1. (Colour online) Two proton separation energies ($S_{2p}$) and two neutron separation energies ($S_{2n}$) calculated with DD-ME2, NL3*, FSU-Gold, and DD-PCX, parameters along with experimental separation energies\(^{30}\) for N=40 isotones and Z=40 isotopes are shown in panels (a) and (b), respectively. Difference between theoretical results with experimental data are also shown in panels (c) and (d). A reasonable agreement of theoretical results with experimental data is clearly visible in panels (a) and (b). A sharp fall in the values of $S_{2p}$ and $S_{2n}$ just beyond the magic number can also be looked at. Panels (c) and (d) results a closer agreement of DD-ME2 parameter as a whole.](image-url)
From Fig. 1(b), for Zr isotopes, a clear-cut dependency on the chosen parameter can be seen. To visualize this variance, we have plotted the differences of our theoretical estimates with available experimental values of $S_{2p}$ and $S_{2n}$ for $N=40$ isotones and Zr isotopes in Fig. 1(c) and (d), respectively. By and large, the results of DD-ME2 parameter are found in closest match with experimental data as compared to the other parameters. From density dependent parameters two-neutron drip line for Zr isotopes is reported as $^{122}$Zr, however, from the non-linear parameters i.e NL3* and FSU-Gold, it is found to extend up to a larger distance at $^{138}$Zr and $^{130}$Zr, respectively. It is tempting to associate the difference in the position of two-neutron drip lines with different symmetry energies $J$ ($J = 32.30, 38.6, 32.6$ and $31.12$ MeV for DD-ME2, NL3*, FSU-Gold and DD-PCX, respectively) and the slope parameter $L$ of the symmetry energy at saturation density ($L = 51.26, 122.6, 60.5, 46.32$ MeV for DD-ME2, NL3*, FSU-Gold and DD-PCX, respectively). A comparison of these nuclear matter properties does not clearly reveal any correlations with the locations of two-neutron drip lines as also has been described in Ref. [32]. In fact, the precise position of the drip line depends very much on the behavior of the tail of the neutron density which is not really relevant in comparison to the considered values of $J$ and $L$ of nuclear matter at saturation. However, the difference in the position of drip-line may be attributed to the pairing treatment adopted for the calculations. For an instance, the extension in neutron drip-line using state dependent BCS pairing has been explained by Yadav et al. [37] and Meng et al. [48] with the comment that for the drip-line nuclei the role of continuum states and their coupling to the bound states become exceedingly important, especially for the pairing energy contribution to the total binding energy of the system. The scattering of particles from bound to continuum states near the Fermi level and vice versa due to the pairing interaction pushes the drip-line towards more neutron side with a small increase in binding energy. Contrary to the variance in drip-line, sharp fall just after neutron number $N=50$ and $82$ from all the considered parameters demonstrate strong magicity for $^{90}$Zr and $^{122}$Zr, respectively in accord with recent studies [9, 12].

To look into the shapes of these nuclei, in Figs. 2 and 3, potential energy surfaces calculated by DD-ME2 parameter are shown as a function of deformation $\beta$ for $N=40$ isotones and Zr isotopes, respectively. Many of the potential energy surfaces of $N=40$ isotones as can be seen from Fig. 2 result to dominant ground state configurations with spherical shapes ensuing magic character of $N=40$, out of which $^{60}$Ca and $^{68}$Ni appear once again with eminent sphericity. However, a close examination of Fig. 2 shows that after $^{68}$Ni, for the nuclei between $^{70}$Zn and $^{84}$Ru the ground state configurations with only spherical shape are not so dominant as found for the cases before $^{68}$Ni up to $^{56}$S. The minima are either found very flat or with a few secondary minima which consequently lead to chances of shape co-existence. These nuclei are mentioned in Table 1 with the excitation energy: the difference of energy between two minima along with possible shape due to secondary minima.
(O) for oblate secondary minima and (P) for prolate secondary minima. It is gratifying to note that nuclei \(^{72}\text{Ge}, 74\text{Se}, 76\text{Kr}, 78\text{Sr}, 80\text{Zr}, 82\text{Mo}, 84\text{Ru}\) are indeed found as the possible candidates of shape co-existence in accord with various Refs.\(^{49–51}\) It is important to point out here that for the case of \(^{80}\text{Zr}\), spherical minima is found more dominant similar to what has been found in various other mean-field calculations\(^{12,22}\) whereas \(^{80}\text{Zr}\) is indeed deformed as has been described in various Refs.\(^{40,43,45}\) Nevertheless, the FSU-Gold parameter renders fairly large deformation for the case of \(^{78,80}\text{Zr}\) which may be attributed to the different choice of pairing in this variant of RMF. For other parameters, we have recalculated potential energy surface for \(^{80}\text{Zr}\) by increasing pairing strength and it has been found that the spherical minima goes flatter and the deformed minima becomes more dominant, which indicates need of deeper study along this direction. With this example of \(^{80}\text{Zr}\) near drip-line together with the variance in exact position of drip-line, it is concluded that for drip-line nuclei the treatment of pairing becomes very crucial which in turn leaves at somewhat distinct results from different variants of theoretical approaches.

In a similar manner, from Fig. 3 one can explore the shapes for full chain of Zr isotopes. As expected, \(^{90}\text{Zr}\) and \(^{122}\text{Zr}\) are found with a clean and single minima at zero deformation registering their spherical shapes similar to what observed by Hartree-Fock (HF) calculations with the semi-realistic interaction M3Y-P6.\(^{12}\) On the other hand, \(^{86,88,92}\text{Zr}\) and \(^{112–120}\text{Zr}\) are found with either flatter minima or
mainly with one shape. In connection to the recent measurements, isotopes are found with deformed shapes or with the coexisting shapes.

Figure 3. The potential energy surfaces of Zr (Z=40) isotopes as a function of the deformation parameter $\beta$ calculated by DD-ME2 parameter. $^{90}$Zr and $^{122}$Zr are found with a clean and single minima at zero deformation which establishes their spherical shapes and magicity. Many other isotopes are found with deformed shapes or with the coexisting shapes.

From our theory also many of Zr isotopes are found to show shape co-existence. These nuclei $^{82,84,94,96,98,100,102,106,108}$Zr are mentioned in Table 1 with their excitation energy and are found in agreement with Refs. 22, 19, 18 to register their candidature as nuclei showing shape co-existence. As mentioned above, in Table 1 (O) is used for oblate secondary minima and (P) for prolate secondary minima if dominant minima is spherical, whereas (O') and (P') refer to the oblate secondary and for prolate secondary minima while dominant minima is prolate and oblate, respectively.

After the multiple cases of shape-coexistence as mentioned in Table 1 it is important to compare ground state deformation with other considered parameters and other theories. In Table 2 we have tabulated all these ground state deformation to demonstrate the shape transition especially in Zr isotopes. The results of deformations are found in good match with all the considered parameters.

### 3.2. Magicity

The above detail investigation of shapes reinforce us to examine magicity in these chains of nuclei through the analysis of pairing energy which leads to magic char-
Table 1. Results of excitation energy (energy difference between two minima) as obtained in the RMF calculations using DD-ME2 force parameter for N(Z)=40 Isotones(Isotopes). The table clearly demonstrates many number of nuclei which are potential candidates showing shape-coexistence.

| Nucleus | Excitation Energy (MeV) | Other references |
|---------|--------------------------|------------------|
| $^{72}$Ge | 0.55 (O) | [50, 51] |
| $^{74}$Se | 0.17 (O) | [51] |
| $^{76}$Kr | 0.65 (O), 2.51 (P) | [51] |
| $^{78}$Sr | 0.45 (O), 0.54 (P) | [49] |
| $^{80}$Zr | 2.67 (O) | [49] |
| $^{82}$Mo | 1.64 (O) | [49] |
| $^{84}$Zr | 1.60 (O) | [49] |
| $^{82}$Zr | 1.38 (O) | [49] |
| $^{84}$Zr | 1.34 (O) | [49] |
| $^{90}$Zr | 0.95 (O’) | [51] |
| $^{92}$Zr | 1.49 (O’) | [51] |
| $^{94}$Zr | 0.80 (O’) | [51] |
| $^{100}$Zr | 0.21 (O’) | [51] |
| $^{102}$Zr | 0.85 (P’) | [51] |
| $^{104}$Zr | 1.28 (P’) | [51] |
| $^{106}$Zr | 0.45 (O) | [51] |

Figure 4. (Colour online) Proton and neutron pairing energy (MeV) for N(Z)=40 are shown in panels (a) and (b) respectively. Zero value of pairing energy exhibits magicity that results $^{60}$Ca, $^{68}$Ni and $^{80}$Zr as doubly magic nuclei in panel (a). Panel (b) indicates neutron magicity in $^{90}$, $^{110}$, $^{122}$Zr as only neutron pairing energy attains zero value for N=50, 70 and 82.

character, if it vanishes [50, 51] and manifests magicity. In Fig. 3 we have shown the proton and neutron pairing energy contribution for nuclei with N(Z)=40 calculated by DD-ME2 parameter. For N=40 isotones, neutron pairing energy is found zero for all the nuclei affirming their spherical shapes as found above. However, proton pairing energy reaches to zero along with neutron pairing energy if there is a doubly magic nucleus. This situation indeed arrives for the case of $^{60}$Ca, $^{68}$Ni and
Table 2. Ground state quadrupole deformation $\beta$ calculated with DD-ME2 parameter for the nuclei with $N(Z)=40$ are mentioned and compared with other considered parameters NL3*, FSU-Gold, and DD-PCX. These deformations are also compared with other theoretical model/parameter viz. RMF(TMA)\textsuperscript{20} FRDM\textsuperscript{21} Skyrme-Hartree-Fock-Bogoliubov mass formulas (HFB) using HFB-21\textsuperscript{22} Hartree-Fock-Bogoliubov approach using the Gogny D1S effective interaction (D1S for $N=40$ isotones)\textsuperscript{23} and Skyrme-Hartree-Fock-Bogoliubov formalism with SLy4 parameter (SLy4 for Zr isotopes)\textsuperscript{24}.

### $N=40$ Isotones

| Nucleus | DD-ME2 | NL3* | FSU | DD-PCX | TMA | FRDM | D1S | HFB |
|---------|--------|------|-----|--------|-----|------|-----|-----|
| $^{56}$S | 0.00   | -0.02 | 0.05 | 0.00   | 0.00 | -0.22 |     |     |
| $^{58}$Ar | 0.00  | 0.00  | 0.00 | 0.00   | 0.00 | -0.22 | 0.00 | 0.09 |
| $^{60}$Ca | 0.00  | 0.00  | 0.00 | 0.00   | 0.00 | 0.00  | 0.00 | 0.00 |
| $^{62}$Ti | 0.00  | 0.00  | 0.04 | 0.00   | 0.00 | 0.00  | 0.00 | 0.11 |
| $^{64}$Cr | 0.00  | 0.00  | 0.06 | 0.00   | 0.00 | 0.00  | 0.00 | 0.19 |
| $^{66}$Fe | 0.00  | 0.00  | 0.05 | 0.00   | 0.00 | 0.00  | 0.00 | 0.10 |
| $^{68}$Ni | 0.00  | 0.00  | 0.00 | 0.00   | 0.00 | 0.00  | 0.00 | 0.09 |
| $^{70}$Zn | 0.00  | 0.00  | 0.05 | 0.00   | 0.00 | 0.00  | 0.00 | -0.16|
| $^{72}$Ge | 0.00  | -0.20 | 0.06 | 0.00   | -0.21 | -0.22 | -0.2 | -0.24|
| $^{74}$Se | 0.00  | -0.24 | 0.00 | 0.00   | -0.23 | -0.24 | -0.2 | -0.23|
| $^{76}$Kr | 0.00  | 0.00  | 0.04 | 0.00   | -0.32 | 0.40  | 0.00 | -0.24|
| $^{78}$Sr | 0.00  | 0.49  | 0.00 | 0.00   | -0.49 | 0.40  | 0.00 | -0.22|
| $^{80}$Zr | 0.00  | 0.46  | 0.00 | 0.49   | 0.43  | 0.43  | 0.00 | -0.23|
| $^{82}$Mo | 0.00  | 0.03  | 0.00 | 0.59   | 0.47  | 0.00  | 0.00 | -0.22|
| $^{84}$Ru | 0.00  | 0.06  | 0.00 | -0.22  | -0.23 | 0.00  | 0.06 |     |
| $^{86}$Pd | 0.00  | 0.07  | 0.00 | -0.17  | -0.24 | -0.04 |     |     |

### $Z=40$(Zr) Isotones

| Nucleus | DD-ME2 | NL3* | FSU | DD-PCX | TMA | FRDM | SLy4 | HFB |
|---------|--------|------|-----|--------|-----|------|------|-----|
| $^{78}$Zr | 0.00  | 0.00  | 0.45 | 0.00   | 0.49 | 0.42  | 1.7  |     |
| $^{80}$Zr | 0.00  | 0.00  | 0.46 | 0.00   | 0.49 | 0.43  | 0.00 | -0.23|
| $^{82}$Zr | 0.00  | 0.00  | 0.01 | 0.00   | 0.59 | 0.44  | 0.00 | -0.23|
| $^{84}$Zr | 0.00  | 0.00  | 0.00 | 0.00   | -0.21 | -0.24 | 0.13 |     |
| $^{86}$Zr | 0.00  | 0.01  | 0.00 | 0.00   | -0.17 | 0.01  | -0.10|     |
| $^{88}$Zr | 0.00  | 0.00  | 0.00 | 0.00   | -0.01 | 0.00  | -0.09|     |
| $^{90}$Zr | 0.00  | 0.00  | 0.00 | 0.00   | -0.02 | 0.00  | 0.09 |     |
| $^{92}$Zr | 0.02  | 0.00  | 0.00 | 0.00   | -0.14 | 0.00  | -0.11|     |
| $^{94}$Zr | 0.20  | 0.20  | -0.01 | 0.10  | -0.22 | -0.16 | 0.00 | -0.15|
| $^{96}$Zr | 0.30  | 0.26  | 0.17  | 0.20  | 0.27  | 0.24  | -0.15|     |
| $^{98}$Zr | 0.24  | 0.51  | 0.21  | 0.40  | 0.34  | 0.34  | -0.20| -0.22|
| $^{100}$Zr | 0.30 | 0.49  | 0.21 | 0.50  | 0.41  | 0.36  | 0.42 | -0.23|
| $^{102}$Zr | -0.20 | 0.44  | 0.21 | 0.40  | 0.42  | 0.38  | 0.43 | -0.24|
| $^{104}$Zr | -0.20 | 0.42  | 0.33 | 0.40  | 0.41  | 0.38  | 0.43 | -0.24|
| $^{106}$Zr | -0.22 | 0.43  | 0.35 | 0.40  | 0.41  | 0.37  | 0.42 | -0.25|
| $^{108}$Zr | 0.02  | 0.42  | 0.35 | 0.40  | 0.41  | 0.36  | 0.41 | -0.24|
| $^{110}$Zr | 0.00  | 0.00  | 0.33 | 0.40  | 0.43  | 0.36  | 0.44 | -0.23|
| $^{112}$Zr | -0.02 | 0.00  | 0.00 | 0.40  | -0.19 | 0.36  | 0.00 | -0.20|
| $^{114}$Zr | 0.00  | 0.00  | 0.00 | 0.10  | -0.17 | -0.19 | -0.20|     |
| $^{116}$Zr | 0.10  | 0.00  | 0.00 | 0.10  | -0.15 | -0.16 | -0.11|     |
| $^{118}$Zr | 0.00  | 0.00  | 0.00 | 0.10  | 0.00  | -0.15 | -0.05|     |
| $^{120}$Zr | 0.02  | 0.00  | 0.00 | 0.00  | 0.00  | 0.00  | 0.05 |     |
| $^{122}$Zr | 0.00  | 0.00  | 0.01 | -0.10 | 0.00  | 0.00  | 0.00 |     |
Structural properties of nuclei with semi-magic number $N(Z)=40$

Figure 5. (Colour online) Neutron and proton single particle states for $N=40$ and $Z=40$ are shown in panels (a) and (b), respectively. The significant gap between neutron pf-shell and $\nu g_{9/2}$ confirms magicity at $^{60}\text{Ca}$ and $^{68}\text{Ni}$. In contrast, insignificant gap between proton pf-shell and $\pi g_{9/2}$ manifests the non-magic character of Zr isotopes.

$^{80}\text{Zr}$ which is another confirmation of double magicity. In Fig. 4(b), except $^{80}\text{Zr}$, none of the isotopes of Zr is found doubly magic as proton pairing energy does not vanish for any of the isotopes of Zr. However, neutron pairing energy attains zero value for $N=40$, 50, 70 and 82 confirming sphericity of $^{80,90,110,122}\text{Zr}$. Importantly, these all nuclei are found with dominant spherical shapes as discussed above in the description of Figs. 2 and 3.

To get more insight about magicity, we choose the above mentioned magic nuclei to describe magic nature on the basis of single particle energies. For the case of $N=40$ isotones, we have shown neutron single particle levels of $^{60}\text{Ca}$ and $^{68}\text{Ni}$ in Fig. 5(a). A large gap between $\nu f_{7/2}$ and $\nu f_{5/2}$ can be seen for all these considered nuclei indicating a strong shell closure at $N=28$. The gap which is responsible for shell closure at $N=40$ is the gap between neutron pf-shell and $\nu g_{9/2}$ state which is indeed evident from the figure for all these considered nuclei. This gap varies form 4.4 MeV to 5.7 MeV which is quite sizable as that of the gap found for conventional shell closure at $N=28$, therefore, $\nu g_{9/2}$ state remains vacant as can be seen from the occupancy mentioned (in blue number). This description of single particle levels serves as an accompaniment to strong magicity of $^{60}\text{Ca}$ and $^{68}\text{Ni}$. In a similar manner, proton single particle levels are plotted for the case of Zr isotopes by selecting nuclei $^{90,122}\text{Zr}$ in Fig. 5(b). As expected, the gap between $\pi f_{7/2}$ and $\pi f_{5/2}$ leads to shell closure at $Z=28$, however, the gap within proton pf-shell and $\pi g_{9/2}$ state is not as large as found for the case of nuclei with $N=40$ and resulting a
non-zero occupancy of $\pi g_{9/2}$. Hence, Zr isotopes are most likely to show non-magic character for proton magicity favouring the outcome obtained from proton pairing energy contribution in Fig. 4(b). Since the gap between single particle states is closely related to pairing correlations therefore the above findings of shell closures should be tested for various other considered models considering the fact that their pairing treatments are different as mentioned above. The gap between pf-shell and $1g_{9/2}$ state is calculated for $^{60}\text{Ca}$, $^{68}\text{Ni}$, $^{90}\text{Zr}$, and $^{\text{122}}\text{Zr}$ nuclei using DD-ME2, NL3*, FSU-Gold, and DD-PCX parameters and has been listed in Table 3. The values of gaps from all the considered parameters are found almost similar which establishes the magic character of $^{60}\text{Ca}$ and $^{68}\text{Ni}$ along with non-magicity of $^{90}\text{Zr}$ and $^{\text{122}}\text{Zr}$.

Table 3. Energy gap between pf-shell and $1g_{9/2}$ state obtained in the RMF calculations using various force parameters.

| Parameters   | Energy gap between single particle states (MeV) |
|--------------|-----------------------------------------------|
|              | Neutron S. P. States | Proton S. P. States |
|              | $^{60}\text{Ca}$ | $^{68}\text{Ni}$ | $^{90}\text{Zr}$ | $^{\text{122}}\text{Zr}$ |
| DD-ME2       | 5.7            | 4.4            | 2.0            | 1.4            |
| NL3*         | 5.4            | 4.1            | 2.0            | 1.8            |
| FSU-Gold     | 5.9            | 4.2            | 1.9            | 1.3            |
| DD-PCX       | 5.9            | 4.6            | 2.2            | 1.4            |

Figure 6. (Colour online) Normalized radii ($R_{p}^{\text{nor}} = R_{p}/R_{p}^{\text{Co}}$) for $N=40$ isotones calculated by DD-ME2, NL3*, FSU-Gold, and DD-PCX, parameters. Experimental data are taken from\textsuperscript{31} For $^{68}\text{Ni}$ the experimental data is taken from very recent measurement\textsuperscript{95} The kink observed for $^{60}\text{Ca}$ and $^{68}\text{Ni}$ is another demonstration of double magicity in these nuclei from all the considered parameters.
Further, we have also plotted proton rms radii for N=40 isotones calculated by DD-ME2, NL3*, FSU-Gold, and DD-PCX parameters. To eliminate the smooth mass number dependence of the proton rms radii \( R_p \), radii are normalized using the formula reported by Collard et al.\(^{66}\)

\[
R_p^{Co} = \sqrt{\frac{3}{5}} (1.15 + 1.80A^{-2/3} - 1.20A^{-4/3}) A^{1/3}
\]

The proton normalized radii (\( R_p^{nor} = R_p/R_p^{Co} \)) is plotted in the Fig. 6. This plot show a kink (change in the slope) particularly for shell closure 31, 67 in accordance with the work of Angeli et al. and indeed evident in Fig. 6 for the case of \(^{60}\)Ca and \(^{68}\)Ni. This kind of kink observed for \(^{60}\)Ca and \(^{68}\)Ni is another demonstration of double magicity. Moreover, it is gratifying to note that our theoretical results of radius of \(^{68}\)Ni calculated by DD-ME2 and DD-PCX parameters are in excellent agreement with the very recent measurement by Kaufmann et al.\(^{65}\) Therefore, from here one can reach to the conclusion that N=40 isotones own plenty of grounds for magicity. Various ground state properties obtained with DD-ME2, NL3*, FSU-Gold, and DD-PCX parameters are mentioned in Tables 4 and 5 with available experimental data 30, 31, 65 for N=40 isotones and Z=40 isotopes, respectively.

### 3.3. Bubble Structure

In addition to the ground state properties and related phenomenon of shape-coexistence and magicity discussed above, there is one more phenomenon which has raised substantial attention recently after the first experimental proof of central depletion in \(^{34}\)Si.\(^{69}\) It is the "bubble structure" named to the depletion of central density of a nuclei. Bubble structure has been discussed in many of the theoretical works 69–79 along with our recent work 80–83 from which it has been established that \(^{34}\)Si is a best candidate showing bubble or central depletion in its charge density distribution due to unoccupancy in \(\pi 2s_{1/2}\). Encouraging with these recent studies, we have analyzed density distribution at center for all the isotones of N=40 and isotopes of Z=40, which may be analogously influenced by \(3s_{1/2}\) state. From these chains of nuclei, it is found that many of the nuclei indeed possess the central depletion in their proton or neutron or both the densities. To visualize this phenomenon, in Fig. 7 we have shown proton and neutron densities together of some selected nuclei which are found with significant central depletion in their concern chain of isotones/isotopes. The diminution in the density at center can be express as a number defined as depletion fraction (DF) \( ((\rho_{max} - \rho_c)/\rho_{max}) \), where \(\rho_{max}\) is maximum density and \(\rho_c\) is the density at center\(^{71}\) which is also mentioned in the Fig. 7. From N=40 isotones, we have selected \(^{56}\)S and from Zr isotopes the isotopes considered are \(^{96}\)Zr and \(^{122}\)Zr. Bubble phenomenon is clearly observed in both neutron and proton densities with a reasonable value of DF as mentioned in the plots. However, for the case of \(^{122}\)Zr, neutron DF is found zero which is oblivious as no depletion in neutron density is observed.

To perform a systematic study of depletion in proton and neutron densities, in Fig. 8 we have displayed proton and neutron depletion fraction (DF) for full chain of
Table 4. Binding energy, charge radius ($R_c$), proton radius ($R_p$), neutron radius ($R_n$), and matter radius ($R_m$) for nuclei with $N=40$ are mentioned and compared with available experimental data which are taken from Refs. 30, 31, 65.

| Nuclei | Binding Energy (MeV) | $R_c$ (fm) | $R_p$ (fm) | $R_n$ (fm) | $R_m$ (fm) |
|--------|---------------------|------------|------------|------------|------------|
|        | DDME2 | NL3 | FSU | DDPCX | Exp. | DDME2 | NL3 | FSU | DDPCX | Exp. | DDME2 | NL3 | FSU | DDPCX | Exp. | DDME2 | NL3 | FSU | DDPCX | Exp. |
| $^{36}$S | 362.16 | 371.00 | 359.10 | 360.04 | 3.54 | 3.52 | 3.52 | 3.56 | 3.45 | 3.42 | 3.42 | 3.46 | 4.15 | 4.24 | 4.20 | 4.08 | 3.96 | 4.03 | 3.99 | 3.91 |
| $^{38}$Ar | 413.99 | 419.91 | 412.13 | 412.56 | 3.61 | 3.59 | 3.58 | 3.62 | 3.52 | 3.50 | 3.49 | 3.53 | 4.10 | 4.20 | 4.13 | 4.04 | 3.93 | 4.00 | 3.99 | 3.94 |
| $^{60}$Ca | 461.71 | 465.75 | 459.91 | 460.83 | 3.66 | 3.64 | 3.63 | 3.67 | 3.57 | 3.55 | 3.54 | 3.58 | 4.06 | 4.15 | 4.08 | 4.02 | 3.90 | 4.06 | 3.96 | 3.88 |
| $^{62}$Ti | 498.42 | 500.51 | 494.33 | 495.69 | 3.73 | 3.71 | 3.70 | 3.74 | 3.64 | 3.62 | 3.62 | 3.65 | 4.03 | 4.12 | 4.05 | 4.00 | 3.90 | 4.05 | 3.95 | 3.90 |
| $^{64}$Cr | 532.77 | 532.86 | 532.79 | 532.67 | 3.79 | 3.77 | 3.77 | 3.80 | 3.70 | 3.68 | 3.68 | 3.71 | 4.01 | 4.09 | 4.03 | 3.99 | 3.90 | 4.04 | 3.94 | 3.88 |
| $^{66}$Fe | 563.40 | 562.83 | 562.43 | 562.13 | 3.84 | 3.82 | 3.82 | 3.85 | 3.75 | 3.73 | 3.73 | 3.76 | 4.00 | 4.06 | 4.01 | 3.98 | 3.90 | 4.06 | 3.94 | 3.89 |
| $^{68}$Ni | 591.87 | 590.53 | 589.13 | 590.41 | 3.88 | 3.86 | 3.86 | 3.89 | 3.80 | 3.78 | 3.78 | 3.81 | 3.90 | 4.04 | 4.00 | 3.97 | 3.90 | 4.06 | 3.94 | 3.89 |
| $^{70}$Zn | 609.64 | 608.57 | 605.95 | 610.44 | 3.96 | 3.94 | 3.93 | 3.96 | 3.87 | 3.86 | 3.85 | 3.88 | 3.96 | 4.06 | 4.00 | 3.97 | 3.94 | 3.97 | 3.94 | 3.91 |
| $^{72}$Ge | 625.25 | 624.35 | 621.67 | 626.69 | 4.02 | 4.01 | 4.01 | 4.04 | 3.94 | 3.95 | 3.92 | 3.94 | 3.98 | 4.02 | 4.06 | 4.02 | 3.99 | 4.02 | 3.97 | 3.97 |
| $^{74}$Se | 638.74 | 638.56 | 635.89 | 638.89 | 4.08 | 4.06 | 4.05 | 4.08 | 4.00 | 3.97 | 3.97 | 4.00 | 3.99 | 4.04 | 4.00 | 3.97 | 3.90 | 4.06 | 3.94 | 3.90 |
| $^{76}$Kr | 650.13 | 649.00 | 647.53 | 650.25 | 4.14 | 4.12 | 4.11 | 4.14 | 4.06 | 4.04 | 4.04 | 4.05 | 4.12 | 4.06 | 4.04 | 3.99 | 4.06 | 3.93 | 4.01 | 3.97 |
| $^{78}$Sr | 659.28 | 658.99 | 658.07 | 659.01 | 4.19 | 4.17 | 4.16 | 4.19 | 4.12 | 4.10 | 4.10 | 4.12 | 4.17 | 4.12 | 4.10 | 4.05 | 4.09 | 4.06 | 4.07 | 4.11 | 4.08 |
| $^{80}$Zr | 665.72 | 665.33 | 664.75 | 664.40 | 4.25 | 4.23 | 4.22 | 4.25 | 4.18 | 4.15 | 4.15 | 4.17 | 4.18 | 4.10 | 4.07 | 4.07 | 4.14 | 4.11 | 4.11 | 4.11 |
| $^{82}$Mo | 667.77 | 667.01 | 666.38 | 666.09 | 4.30 | 4.28 | 4.27 | 4.30 | 4.23 | 4.21 | 4.21 | 4.20 | 4.10 | 4.11 | 4.08 | 4.09 | 4.17 | 4.18 | 4.15 | 4.14 |
| $^{84}$Ru | 665.75 | 667.25 | 666.20 | 665.41 | 4.35 | 4.34 | 4.32 | 4.35 | 4.28 | 4.26 | 4.26 | 4.25 | 4.12 | 4.12 | 4.10 | 4.10 | 4.20 | 4.18 | 4.18 | 4.18 |
| $^{86}$Pd | 662.93 | 665.02 | 659.44 | 662.82 | 4.40 | 4.38 | 4.37 | 4.40 | 4.32 | 4.31 | 4.31 | 4.29 | 4.33 | 4.23 | 4.22 | 4.21 | 4.23 | 4.22 | 4.21 |
Table 5. Binding energy, charge radius ($R_c$), proton radius ($R_p$), neutron radius ($R_n$), and matter radius ($R_m$) for Zr isotopes are mentioned and compared with available experimental data which are taken from Refs. [30,31,65]

| Nucleus | Binding Energy (MeV) | $R_c$ (fm) | $R_p$ (fm) | $R_n$ (fm) | $R_m$ (fm) |
|---------|----------------------|----------|----------|----------|----------|
| Zr$^{187}$ | 635.35 | 635.63 | 635.55 | 634.56 | 639.14 |
| Zr$^{80}$ | 665.72 | 665.33 | 664.75 | 664.40 | 668.80 |
| Zr$^{84}$ | 690.80 | 691.89 | 693.12 | 690.66 | 694.19 |
| Zr$^{74}$ | 714.98 | 715.67 | 717.32 | 715.31 | 718.12 |
| Zr$^{78}$ | 738.39 | 739.29 | 740.64 | 739.21 | 740.81 |
| Zr$^{82}$ | 761.79 | 761.85 | 763.14 | 762.25 | 762.61 |
| Zr$^{86}$ | 784.35 | 783.50 | 785.80 | 784.13 | 785.90 |
| Zr$^{88}$ | 796.39 | 796.63 | 799.67 | 797.78 | 799.73 |
| Zr$^{90}$ | 810.31 | 810.93 | 812.44 | 810.73 | 814.68 |
| Zr$^{92}$ | 824.01 | 823.51 | 826.03 | 823.24 | 828.99 |
| Zr$^{94}$ | 835.46 | 836.25 | 838.85 | 836.12 | 840.98 |
| Zr$^{96}$ | 847.02 | 848.29 | 850.15 | 848.81 | 852.21 |
| Zr$^{98}$ | 857.71 | 859.28 | 860.41 | 859.73 | 863.57 |
| Zr$^{100}$ | 869.06 | 869.39 | 871.30 | 869.84 | 873.85 |
| Zr$^{102}$ | 876.35 | 878.82 | 880.62 | 879.25 | 882.87 |
| Zr$^{104}$ | 886.13 | 887.55 | 888.94 | 887.86 | 891.76 |
| Zr$^{106}$ | 895.22 | 896.64 | 896.07 | 894.78 | 899.47 |
| Zr$^{108}$ | 901.41 | 901.45 | 905.46 | 900.45 | 906.53 |
| Zr$^{110}$ | 907.06 | 911.03 | 911.59 | 906.01 | 913.67 |
| Zr$^{112}$ | 912.45 | 916.95 | 917.04 | 912.37 | 919.94 |
| Zr$^{114}$ | 917.67 | 922.48 | 922.02 | 918.04 | 924.47 |
| Zr$^{116}$ | 922.35 | 927.39 | 926.50 | 920.07 | 929.15 |
| Zr$^{118}$ | 927.02 | 931.79 | 930.22 | 925.87 | 934.43 |
| Zr$^{120}$ | 926.32 | 932.02 | 932.82 | 927.49 | 936.95 |
| Zr$^{122}$ | 926.15 | 933.85 | 934.23 | 925.60 | 937.43 |
| Zr$^{124}$ | 925.60 | 934.68 | 934.77 | 924.92 | 936.58 |
| Zr$^{126}$ | 924.76 | 935.92 | 934.82 | 922.92 | 937.47 |
| Zr$^{128}$ | 923.68 | 936.98 | 934.54 | 922.67 | 936.40 |
| Zr$^{130}$ | 922.38 | 937.83 | 934.06 | 921.81 | 937.45 |
| Zr$^{132}$ | 929.94 | 937.83 | 933.35 | 929.33 | 935.74 |
| Zr$^{134}$ | 919.29 | 938.64 | 932.44 | 918.41 | 937.47 |
| Zr$^{136}$ | 917.48 | 938.64 | 931.34 | 916.68 | 937.63 |
| Zr$^{138}$ | 915.51 | 938.33 | 930.04 | 914.15 | 936.40 |
| Zr$^{140}$ | 913.43 | 938.43 | 928.60 | 911.22 | 935.77 |
| Zr$^{142}$ | 911.23 | 938.47 | 927.05 | 907.92 | 934.55 |
| Zr$^{144}$ | 906.91 | 938.39 | 925.18 | 904.26 | 933.71 |
| Zr$^{146}$ | 906.16 | 938.16 | 923.20 | 900.29 | 932.73 |
| Zr$^{148}$ | 903.43 | 927.65 | 920.98 | 896.01 | 930.76 |
isotones of N=40 and isotopes of Zr. From Fig. 8(a), neutron DF is found consistent in N=40 isotones, though with a low value, whereas proton DF is ascertained with the dependency on proton number Z. Proton depletion is found maximum for $^{56}_{16}$S which is due to vacant $\pi 2s_{1/2}$ state and reaches to zero for $Z \geq 20$ as $\pi 2s_{1/2}$ state occupies completely. The proton DF remains zero upto $Z < 40$ and then increases after $Z \geq 40$ which may be anticipated with the inclusion of vacant $\pi 3s_{1/2}$ state while filling of $\pi 1g_{9/2}$ state. For the case of Zr isotopes, this bubble phenomenon of proton density is found in a more general manner as can be seen from Fig. 8(b), which fortifies the role of unoccupied $\pi 3s_{1/2}$ state in this region of periodic chart. Towards neutron side this role of $3s_{1/2}$ state determining central depletion can be easily visualized for which we have plotted occupancies of $\nu 1g_{9/2}$, $\nu 3s_{1/2}$ and $\nu 1h_{11/2}$ single particle states for all the isotopes of Zr.

From Fig. 8 one can see that as neutron number increases for $N > 40$, $\nu 1g_{9/2}$ state starts to accommodate with the neutrons and fills completely at $N = 50$, however, upto this neutron number the $\nu 3s_{1/2}$ state remains unoccupied. After $N > 50$, $\nu 3s_{1/2}$ state starts to fill gradually along with few other single particle states viz. $\nu 1g_{7/2}$, $\nu 2d_{5/2}$ and $\nu 2d_{3/2}$. As one reaches to $N \geq 64$, $\nu 3s_{1/2}$ state has its occupancy > 0.5 leading to zero DF as can be seen in Fig. 8(b). This outcome where DF reaches to zero after 50% of occupancy of s-state is in agreement with our earlier work for $2s_{1/2}$ state. Therefore, after $N \geq 64$, DF reaches to zero and subsequently $\nu 3s_{1/2}$ and $\nu 1h_{11/2}$ single particle states get occupied upto $N = 82$, hence, no depletion has been found in the neutron density of $^{122}_{40}$Zr as seen in Fig. 7. Dependency of DF
Structural properties of nuclei with semi-magic number $N(Z)=40$:

Figure 8. (Colour online) Proton and neutron depletion fraction (DF) $\left(\frac{\rho_{\text{max}}-\rho_c}{\rho_{\text{max}}}\right)$ for $N(Z)=40$ isotones(isotopes). From panel (a) neutron DF is found consistent in $N=40$ isotones, though with a low value, whereas proton DF is dependent on proton number $Z$. Panel (b) depicts substantial bubble structure in the proton density of $Zr$ isotopes.

on various parameters of RMF has been discussed in detail already\cite{ref2} however, a deeper and separate study of bubble phenomenon in this region of periodic chart is required.

Figure 9. (Colour online) Occupation probability of $\nu 1g_{9/2}$, $\nu 3s_{1/2}$ and $\nu 1h_{11/2}$ of Zr isotopes. The figures evidently describes the role of $\nu 3s_{1/2}$ state in the bubble structure of Zr isotopes.

4. Conclusions

We describe ground state properties of $N=40$ isotones and $Z=40$(Zr) isotopes through investigating separation energies, deformation, single particle energies, pairing energies, radii, proton and neutron density profiles of even-even nuclei.
For this systematic study, we employ various relativistic mean-field (RMF) models comprising density dependent meson exchange model (DD-ME2), density dependent point coupling interaction (DD-PCX) and the model with nonlinear self- and mixed-interactions of the mesons viz. NL3* and FSU-Gold. The obtained results are compared within the RMF models, available experimental data and various other theories. N=40 isotones own strong candidature of magicity which declares $^{60}$Ca and $^{68}$Ni as doubly magic nuclei, whereas, most of the Zr isotopes are found deformed in which shape transition and shape co-existence are clearly observed. Among the nuclei from both chains, few nuclei are identified which show both proton and neutron central depletion in density distribution referred as doubly bubble nuclei. Evidently, $^{56}$S and $^{122}$Zr are reported as doubly bubble nuclei.

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