The observability of the Hofstadter spectrum generated by a Wigner crystal using photoluminescence techniques is studied. Itinerant hole geometries are examined, in which a hole may combine directly with electrons in the lattice. It is found that when the effect of lattice distortions of the WC due to interactions with the hole are accounted for, only the largest Hofstadter gaps are observable. To overcome the problems of lattice distortion, a novel geometry is proposed, involving a two layer system with electrons in one layer forming a WC and in the other a full Landau level. It is found that recombination of electrons in the full Landau level with localized holes reflects the full Hofstadter spectrum of the lattice.

73.20 Dx, 73.40 Hm
An electron gas is expected to condense into a Wigner crystal (WC) below some critical density. In the absence of a magnetic field, this condensation occurs when the cost in kinetic energy due to crystallization is outweighed by the decrease in Coulomb energy. For a two-dimensional electron system (2DES), the kinetic energy scales like \( K = \frac{\hbar^2}{m^* a_0^2} \), while the Coulomb energy scales like \( V = \frac{e^2}{\epsilon a_0} \), where \( a_0 \) is the mean inter-electron distance and \( \epsilon \) is the dielectric constant of the host material, and \( m^* \) and \( e \) are the electron mass and charge respectively. The relevant parameter is the ratio \( r_s = \frac{V}{K} = \frac{a_o}{a_B} \), where \( a_B = \frac{\hbar^2 \epsilon}{m^* e^2} \) is the Bohr radius. Monte Carlo simulation predicts that a 2DES crystallizes for \( r_s \geq 37 \).

When a strong magnetic field is applied perpendicular to the 2DES, the situation is changed qualitatively, as the kinetic energy is quenched into discrete Landau levels, and the zero-point fluctuations in the lowest Landau level are confined within a magnetic length \( l_o = \left( \frac{\hbar c}{eB} \right)^{1/2} \), where \( c \) is the speed of light and \( B \) is the applied magnetic field. Once \( l_o \) is sufficiently small compared to the typical inter-particle distance \( a_o \), crystallization occurs. The ratio \( l_o/a_o \) can be characterized by the Landau level filling factor \( \nu = \frac{2\pi l_o^2 n}{\nu} \), where \( n \) is the density of the 2DES. Crystallization will occur for sufficiently small \( \nu \) for any given density. Early theoretical estimates put the critical filling factor of crystallization at about \( \nu_c \sim 1/6.5 \).

Recent numerical calculations suggest that larger values of the critical filling factor are possible by considering the Landau level mixing.

Crystallization in the absence of a magnetic field has been observed for electrons trapped on the surface of liquid helium. The magnetic-field-induced WC has not been unambiguously observed. Reentrant insulating phases around \( \nu = 1/5 \) for a 2DES and around \( \nu = 1/3 \) for a hole system have been found and studied by various techniques. While the assumption of a pinned WC as the ground state of the insulating phase is found to be qualitatively consistent with many of the experimental results, alternative interpretations based on disorder-induced states are not ruled out. Despite increased efforts in recent years, the nature of the insulating phase still remains unclear. One of the experiments which may help to resolve this puzzle is the photoluminescence (PL) from the recombination of the electrons in the insulating phase with nearby holes. It has been shown...
that PL spectra can yield information about the single particle density of states for both the holes and the recombining electrons. This means that PL potentially contains important information about the electronic state, because a charged particle subject both to a periodic potential due to an electron lattice and a magnetic field has a characteristic energy spectrum. This so-called Hofstadter spectrum consists of a series of bands and gaps that is very sensitive to the number of magnetic flux quanta \( \phi/\phi_0 \) passing through each unit cell of the lattice: for \( \phi/\phi_0 = p/q \), with \( p \) and \( q \) integers, there are \( p \) bands and \( p - 1 \) gaps. It has been proposed that itinerant hole PL experiments, in which some of the holes are thermally excited to higher Hofstadter bands, offers a possible avenue for observing the lowest few bands of this spectrum. Because of their great sensitivity to the magnetic field, successfully identifying these bands from the PL spectrum would provide convincing evidence for the existence of a WC.

In this work, we shall study the relevant densities of states (DOS) that enter into the PL spectrum. We begin with the case of itinerant holes interacting with a nearby electron crystal. We will argue that most of the gaps in the Hofstadter spectrum are greatly suppressed, due to the lattice distortion caused by the electron-hole interaction. Only the energy levels corresponding to the localized orbitals of a hole trapped at a single lattice electron are likely to be observed in PL experiments. The energies of these levels are computed, and their relationship with the Hofstadter spectrum of a system without lattice distortions is discussed. We shall then propose a novel geometry involving a double-layer structure which avoids all the difficulties introduced by the lattice distortions, and provides a great potential to observe the unperturbed Hofstadter spectrum associated with the electron Wigner crystal.

Our results for itinerant holes may be summarized as follows. Firstly, the hole plane must be at least a minimum distance \( d > d_{tr} \approx 0.45a_0 \) away from the WC to avoid inducing the formation of an interstitial defect in the lattice. For \( d \) larger than this critical value, the electron-hole potential still induces lattice distortions, although they are considerably weaker than for small \( d \). It is found that only the largest gaps in the Hofstadter spectrum actually survive these weak distortions, and that the splittings among the PL lines from holes
thermally excited to higher energy states are very close to, and are best understood in terms of, the spectrum of a hole bound to a single electron in the lattice. In this situation, one expects to see a single new line in the hole DOS as the flux per unit cell (or, equivalently, per electron) is increased by a single flux quantum. Thus, lattice distortions eliminate much of the sensitivity to the precise number of flux quanta per unit cell seen in the full Hofstadter spectrum. Nevertheless, if this spectrum were observed in actual PL experiments, this would provide strong evidence that the electrons have localized at individual sites, and would distinguish between a liquid or “Hall insulator” ground state [15] and one in which the individual electrons have localized [20].

To analyze the expected hole DOS, we employ the Hartree-Fock approximation. One must find a sensible method for dealing with the lattice deformations (i.e., polaron effects [21]) that necessarily accompany a hole moving near the WC. Within the Hartree-Fock approximation [22], two possible states exist for the system. One is the state where the hole has an equal possibility of residing on each lattice site and the crystal symmetry is preserved. The other is the state where the hole is localized at a single lattice site and the crystal symmetry is destroyed. We find that the state where the hole is trapped at a single lattice site is energetically favored over the other state. Therefore, we shall take this case as the relevant configuration for the system at \( d > d_{tr} \), and study the single-particle excitation spectrum of the hole in this configuration.

To understand the connection of this localized hole spectrum with the Hofstadter spectrum, it is most convenient to use a semiclassical approach [23]. We begin by considering filling factor \( \nu = 1/m \), ignore lattice distortions, and focus on the lowest few states. Semiclassically, there is a set of bound states of the hole for each lattice site, characterized by constant energy contours where the wavefunctions are maximized, which represent classical drift orbits for the hole in the potential of the electron lattice. The allowed quantum states associated with these orbits are determined by the requirement that each state’s orbit must contain approximately one more flux quantum than the state immediately below it [24]. Bound states in different unit cells enclosing a given number of flux quanta are coupled to-
gether via tunneling to form the Hofstadter bands. The tunneling matrix element between sites for the low-lying states scales \[ e^{-\alpha a_0^2/l_0^2} \] as \( e^{-\alpha a_0^2/l_0^2} \), where \( \alpha \) is a number of order unity that depends on the precise shape of the potential. It should be kept in mind that in the low filling factor limit, this is quite small, so that the bands will be very narrow.

When the filling factor is changed slightly away from \( \nu = 1/m \) to \( \nu = p/q \), the magnetic field introduces an extra phase that must be accumulated when the hole tunnels between nearest neighboring sites. This breaks up each band near the bottom of the spectrum into \( p \) separate bands \[23\]. However, because these gaps arise due to phases introduced in tunneling, the gaps themselves also scale as \( e^{-\alpha a_0^2/l_0^2} \), which is quite small. This is in accordance with more exact calculations of the Hofstadter spectrum, for small filling factors \[18\].

The result is that the Hofstadter gaps based on the bound state spectrum of the holes will be relatively easy to observe \[25\] (these gaps are typically of order of several degrees \( K \)), whereas the smaller gaps based on tunneling between sites are quite difficult to resolve. The effect of lattice distortions greatly compounds this. Even with lattice distortions, \textit{in principle} the hole can tunnel between lattice sites, although in doing so the lattice distortion around the initial site must relax and a new lattice distortion must develop around the final site. This significantly reduces the effective tunneling matrix element between sites for the hole. With such small effective tunneling amplitudes, for experimentally relevant quantities it may be neglected. Thus, only the largest gaps in the Hofstadter spectrum should be observable in these geometries, and for these low-lying levels they are to an excellent approximation given by the bound state spectrum of an individual lattice site. The Hartree-Fock state in which the lattice deforms around a localized hole should give an excellent approximation for the effects of lattice distortion on these lowest few states.

In order to apply the Hartree-Fock approximation \[22\] to the present case where the crystal symmetries no longer exist, we use a super-cell technique, \textit{i.e.}, we consider a periodic array of holes, each trapped at a single lattice site. The super-lattice has primary lattice vectors which are \( k \) times as large as the lattice vectors of the original electron crystal, where \( k \) is an integer. In this way we are dealing with a hole-electron
double-layer system. These two layers are coupled through an electrostatic potential
\[ H_{h-e} = -\left(\frac{1}{\Omega}\right) \sum_G (2\pi e^2/\epsilon G) e^{-Gd} \rho_h(G) \rho_e(-G), \]
where \( \Omega \) is the area of the system and \( \rho_h(G) \) and \( \rho_e(G) \) are respectively the hole and electron densities. For simplicity the hole-hole interaction may be ignored, because in practice the hole density is much smaller than the electron density. The equations from this Hartree-Fock approximation are then solved numerically at zero-temperature and under the condition that only the lowest Landau level is occupied.

The density of states of the holes is shown in Fig. (1). It was calculated with an electron Landau level filling factor \( \nu = 1/6 \), a hole setback distance of \( d = 0.7a_0 \), and with 9 electrons per super-cell. One can see that, due to the deformation of the electron crystal, the density of states differs greatly from the 6-band spectrum expected in the case of an unperturbed crystal. The result of Fig. (1) can be understood as the bound state spectrum for the hole in the potential well caused by the lattice deformation. We note that, while this density of states looks quite different than the Hofstadter spectrum, the energy differences between the low-lying energy levels are almost identical to the separations between the lowest few Hofstadter bands of the undistorted lattice. This just reflects the fact that the large gaps in the Hofstadter spectrum are essentially those of the spectrum for a hole bound to a single lattice site. The several lowest levels in the excitation spectrum of Fig. (1), which may be observable in experiment, are marked by the dotted lines. Figs. 2 and 3 illustrate the evolution of these energy levels as a function of \( d \) and \( B \).

As can be seen from the above discussion, there are several limitations associated with the itinerant hole experiments. Lattice distortions collapse all but the largest Hofstadter gaps, leaving a spectrum that is best understood as the bound states of a hole with a single lattice site. Even among these states, only the lowest few are expected to be observed in the actual PL spectrum. There are two reasons for this: higher states of the holes are only occupied due to thermal fluctuations, and those holes which do occupy such states tend to be physically distant from the electrons, so that the resulting PL power will be small [26]. Clearly, it is highly desirable to find a situation in which the Hofstadter spectrum may be
observed unperturbed, in its entirety.

A geometry that overcomes essentially all these problems and provides a great potential to observe the unperturbed Hofstadter spectrum associated with the electron Wigner crystal is illustrated in Fig. 4. It is a system of two layers, one with low enough density that the electrons would be expected to crystallize (we will call this the “crystal layer”), the other at much higher density, enough to fill an integral number of Landau levels (for concreteness, we will take $\nu = 1$, and call this the “probe layer.”) A layer of acceptors may then be grown close to the $\nu = 1$ layer. The idea is to observe PL from recombination processes between the $\nu = 1$ layer and localized holes in the cores of the acceptors. In this geometry, PL is a measure of the density of states for the $\nu = 1$ electron layer. Because there is a large gap to excited states of this layer, it is easily shown that its density must be uniform, so that the probe layer cannot affect the electronic structure of the crystal layer. Nevertheless, one may compute the density of single particle states for the probe layer, and it is easily shown that it is identical to that of a single (negatively) charged particle moving in the periodic potential due to the crystal layer.

That the probe layer density of states reflects the Hofstadter spectrum may be seen most easily in the Hartree-Fock approximation \(^\text{(22)}\). The retarded Green’s function for electrons in the probe layer satisfy the equation

$$ (\omega + i\delta + \mu)G_p(G, \omega) - U(0)G_p(G, \omega) - \sum_{G'} V(G - G') e^{iG \times G'/2} G_p(G', \omega) = \delta_{G,0}, $$

where $G$ are the reciprocal lattice vectors of the WC, $G_p(G, \omega)$ is the Green’s function for electrons in the probe layer, $U(G) \equiv W(G) < \rho_p(G) >$, $W$ is the difference of the direct and exchange interactions, and $V$ is the periodic potential due to the WC in which the electrons in the probe layer move. The key insight is that because the probe layer is a precisely filled Landau level, if we ignore mixing of higher Landau levels, the electron density in the probe layer must be uniform. This means that $U(G)$ is only different from zero for $G = 0$, so that the only effect of electron-electron interactions within the probe layer is to shift the chemical potential $\mu$ by an overall constant. Thus, $G_p$ obeys precisely the equation of motion for a
single electron in a periodic potential. Finally, observing that the density of states \( D(E) \) is just proportional to \( \text{Im}G(G = 0, E) \), one sees that this is identical to the result expected for a single electron in a periodic potential.

There are several advantages to this geometry over presently available ones: (1) The lattice is not distorted either by the core holes (which are screened in their initial state \( [13] \)) or by the probe layer, so that the Hofstadter spectrum is limited only by the intrinsic disorder of the sample, not by the probes themselves. None of the Hofstadter gaps are collapsed by lattice distortions. (2) Unlike the itinerant hole experiments, \( all \) the Hofstadter bands are filled in the probe layer (which is why it has a net uniform density), so that all the bands can contribute to the PL. (3) Since the density of the probe layer is uniform, and the holes are placed randomly in the plane, the electrons in \( all \) the Hofstadter states will have a significant overlap with the holes, not just the lowest few. (4) Since the geometry employs localized holes, there are no ambiguities as to the initial state of the holes, which has made presently available valence band hole data \( [14] \) difficult to interpret. (5) Unlike present localized hole (and valence band hole) experiments, one is not removing an electron from the crystal layer, so shakeup effects \( [27] \) should not be very pronounced; this will especially be true if the acceptors are close to the probe layer. We believe this geometry takes advantage of the best features of both itinerant carrier and localized hole experiments, and has an excellent potential to probe the Hofstadter spectrum.

In summary, we have studied the single-particle density of states for a hole near an electron WC with the Hartree-Fock approximation. The holes are not bounded to any impurity or disorder sites. We take into consideration the perturbation of the crystal by the holes. We have found that the low-lying excitations for the holes at \( d > d_{tr} = 0.45a_o \) consist of well-separated energy levels. We have evaluated the energy splittings for these levels at different distance \( d \) and for different magnetic fields. We suggest that these low-lying levels can produce additional lines in the photoluminescence spectrum of the hole-WC system under experimentally relevant conditions. A novel geometry using a full Landau level as a probe layer is proposed to overcome the problems intrinsic to itinerant hole geometries.
This geometry provides a great potential to observe the unperturbed Hofstadter spectrum associated with the electron Wigner crystal.

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FIGURES

FIG. 1. Density of states for the holes at a setback distance of \( d/a_0 = 0.7 \) and at a filling factor of \( \nu = 1/6 \). The lowest five levels, marked by \( \epsilon_i \) for \( i = 1 \sim 5 \), are well-separated in energy. Each of the five levels has a degeneracy equal to the number of the holes in the system.

FIG. 2. The energies of the four lowest excitation levels of the holes \( \epsilon_i \) (\( i = 2 \sim 5 \)) as functions of the hole setback distance \( d \) at a filling factor of \( \nu = 1/6 \). The energies are with respect to the energy of the lowest level \( \epsilon_1 \).

FIG. 3. The energies of the four lowest excitation levels of the holes \( \epsilon_i \) (\( i = 2 \sim 5 \)) as functions of magnetic field strength at a hole setback distance of \( d/a_0 = 0.7 \). The energies are with respect to the energy of the lowest level \( \epsilon_1 \).

FIG. 4. Proposed two layer geometry for observing the Hofstadter spectrum of a Wigner crystal.