Child universes UV regularization?

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Abstract

It is argued that high energy density excitations, responsible for UV divergences in quantum field theories, including quantum gravity, are likely to be the source of child universes which carry them out of the original space time. This decoupling prevents these high UV excitations from having any influence on physical amplitudes. Child universe production could therefore be responsible for UV regularization in quantum field theories which take into account gravitational effects. Finally child universe production in the last stages of black hole evaporation, the prediction of absence of tranplanckian primordial perturbations, connection to the minimum length hypothesis and in particular the connection to the maximal curvature hypothesis are discussed.

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I. INTRODUCTION

Quantum field theory and quantum gravity in particular suffer from UV divergences. While some quantum field theories are of the renormalizable type, quantum gravity is not and the UV divergences cannot be hidden into a finite number of ”counter-terms”. Perturbative renormalizability does not appear to be available for quantum gravity.

In an apparently unrelated development, the ”child universe” solutions have been studied [1], [2]. These child universes are regions of space that evolve in such a way that they disconnect from the ambient space time. Inflationary bubbles of false vacuum correspond to this definition [1], [2]. In this case an exponentially expanding inflationary bubble arises from an ambient space time with zero pressure which the false vacuum cannot displace. The inflationary bubbles disconnect from the ambient space generating a child universe.

Here we want to explore the possibility that high energy density excitations, associated to the UV dangerous sector of quantum field theory could be the source of child universes, which will carry the UV excitations out of the original ambient space time. Child universe production could be therefore responsible for UV softening in quantum field theory that takes into account gravitational effects. It implies also the existence of a maximum energy density and curvature.

We will now show now, using a simple model, that very high UV excitations have appreciable tendency to disconnect from the ambient space time.

II. THE SUPER HIGH UV BUBBLE

We describe now the model which we will use to describe a high UV excitation which will be associated with the production of a child universe. This model for high UV excitation will consist of a bubble with very high surface tension and very high value of bulk energy density inside the bubble.

The entire space-time region consists of two regions and a boundary: 1) Region I de Sitter space 2) Region II, Schwarzschild space and the domain wall boundary separating regions I and II.

In Region I: The de Sitter space. The line element is given by
\[ ds^2 = -(1 - \chi^2 r^2) dt^2 + (1 - \chi^2 r^2)^{-1} dr^2 + r^2 d\Omega^2 \]  

(1)

where \( \chi \) is the Hubble constant which is given by

\[ \chi^2 = \frac{8}{3} \pi G \rho_0 \]  

(2)

\( \rho_0 \) being the vacuum energy density of the child universe and \( G = \frac{1}{m_P} \), where \( m_P = 10^{19} \) GeV.

In **Region II**: The Schwarzschild line element is given by

\[ ds^2 = -(1 - \frac{2GM}{r}) dt^2 + (1 - \frac{2GM}{r})^{-1} dr^2 + r^2 d\Omega^2 \]  

(3)

The Einstein’s field equations,

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \]  

(4)

are satisfied in regions I and II and determine also the domain wall evolution [1], using the methods developed by Israel [3]. Using gaussian normal coordinates, which assigns to any point in space three coordinates on the bubble and considers then a geodesic normal to the bubble which reaches any given point after a distance \( \eta \) (the sign of \( \eta \) depends on which side of the bubble the point is found). Then energy momentum tensor \( T_{\mu\nu} \) is given by

\[ T_{\mu\nu}(x) = \begin{cases} -\rho_0 g_{\mu\nu}, & (\eta < 0) \text{ for the child universe, Region I, (negative pressure)} \\ 0, & (\eta > 0) \text{ for the Schwarzschild region, Region II} \\ -\sigma h_{\mu\nu} \delta(\eta) & \text{for the domain wall boundary.} \end{cases} \]  

(5)

where \( \sigma \) is the surface tension and \( h_{\mu\nu} \) is the metric tensor of the wall, that is \( h_{\mu\nu} = g_{\mu\nu} - n_\mu n_\nu \), \( n_\mu \) being the normal to the wall.

The eq. of motion of the wall give[1]

\[ M_{\text{cr}} = \frac{1}{2G\chi} \frac{\gamma^3 z^6_m (1 - \frac{1}{2} \gamma^2)^{\frac{3}{2}}}{3\sqrt{3(z^6_m - 1)^2}} \]  

(6)

where the \( M_{\text{cr}} \) is the mass at (or above) which there is classically a bubble that expands to infinity into a disconnected space, the child universe. In the above equation

\[ \gamma = \frac{8\pi G \sigma}{\sqrt{\chi^2 + 16\pi^2 G^2 \sigma^2}} \]

\[ z^3_m = \frac{1}{2} \sqrt{8 + (1 - \frac{1}{2} \gamma^2)^2 - \frac{1}{2}(1 - \frac{1}{2} \gamma^2)} \]  

(7)
where $z^3 = \frac{\chi^2 r^3}{2G M}$ and $\chi^2 = \chi^2 + \kappa^2$, $\kappa = 4\pi G \sigma$. $r_m$ is the location of the maximum of the potential barrier that prevents bubbles with mass less than $M_{cr}$ to turn into child universes.

We expect this representation of a high UV excitation to be relevant even for a purely gravitational excitation, which can be associated, after an appropriate averaging procedure, to an effective energy momentum, a procedure that gets more and more accurate in the UV limit.

Let us now focus our attention on the limit where $\sigma \to \infty$ (while $\rho_0$ is fixed), which we use as our first model of a super UV excitation. Then, we see that $\gamma \to 2$ and $M_{cr} \to 0$. Alternatively, we could obtain another model of super UV excitation, by considering the energy density inside the bubble, $\rho_0 \to \infty$, while keeping $\sigma$ fixed. This also leads to $M_{cr} \to 0$ as well. Finally, letting both $\sigma \to \infty$ and $\rho_0 \to \infty$ while keeping their ratio fixed, also leads to $M_{cr} \to 0$. In all these limits we also get the the radius of the critical bubble $r_m \to 0$ as well.

In [1] the above expression for $M_{cr}$ was explored for the case that energy densities scales (bulk and surface) were much smaller than the Planck scale, like the GUT scale. This gave a value for $M_{cr} = 56 kg >> m_p$. Here we take the alternative view that the scale of the excitations are much higher than the Planck scale, giving now an arbitrarily small critical mass. Defining the ”scale of the excitation” through by $\rho_0 \equiv M_{exc}^4$, then the pre-factor $\frac{1}{2G\chi}$ in eq (3), goes like $\left(\frac{m_p}{M_{exc}}\right)^2 m_p$. We see that for trans planckian excitations, i.e. if $M_{exc} >> m_p$, we obtain a very big reduction for $M_{cr}$. This is a kind of ”see saw mechanism”, since the higher the $M_{exc}$, the smaller $M_{cr}$.

This means that in these models for high UV excitations there is no barrier for the high UV excitation to be carried out to a disconnected space by the creation of a child universe. Notice also the interesting ”UV -IR mixing” that takes place here: although we go to very high UV limits in the sense that the energy density in the bulk or the surface energy density are very high, the overall critical mass goes to zero.

III. THE CONJECTURE

This allows us to formulate the conjecture that the dangerous UV excitations that are the source of the infinities and the non renormalizability of quantum gravity are taken out of the original space by child universe production, that is, the consideration of child universe
production in the ultrahigh (trans planckian) sector of the theory could result in a finite quantum gravity, since the super high UV modes, after separating from the original space will not be able to contribute anymore to physical processes.

The hope is that in this way, child universes could be of interest not only in cosmology but could become also an essential element necessary for the consistency of quantum gravity. One situation where all the elements required (high energy densities, since the temperature is very big) necessary for obtaining a child universe appears to be the late stages of Black Hole evaporation. If the ideas explained here are correct, we should not get contributions to primordial density perturbations from the trans planckian sector, since these perturbations would have disconnected from our space time. Also, any attempt to measure distances smaller than the Planck length will be according to this also impossible since such a measurement will involve exciting a high UV excitation that will disconnect. This means that there must be a minimum length that we could measure, of the order of the Planck scale.

It appears there is a maximal energy density according to this, since now bubbles with high energy density will be quickly disconnected, being replaced in the observable universe by regions of Schwarzschild space, which has zero energy density, i.e., a very big energy density must decay in the observable universe. The "maximal curvature" hypothesis (here we focus on scalar curvature) is justified by this maximal energy density result, if we use eq. An effective dynamics that takes into account the effect of child universe production (i.e., integrates out this effect) could resemble indeed that of. Notice that the maximal scalar curvature hypothesis gives rise to very interesting dynamics.

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