Photo- and electro-production of medium mass Λ-hypernuclei

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Abstract

The characteristic and selective nature of the electro-magnetic production of Λ-hypernuclei in exciting states is demonstrated assuming the medium-mass targets \textsuperscript{28}Si, \textsuperscript{40}Ca, and \textsuperscript{52}Cr. In the analysis, formalism of DWIA is used adopting the Saclay-Lyon, Kaon-MAID, Adelseck-Saghai, and Williams-Ji-Cotanch models for the elementary production process and various nuclear and hypernuclear wave functions. The elementary amplitudes are discussed in detail presenting their basic properties and comparison with data. The unique features of the electro-magnetic production of Λ-hypernuclei shown on the examples are the selective excitation of unnatural parity highest-spin states (natural parity ones for the \textit{LS}-closed targets) and a possibility to investigate the Λ single-particle energies including a spin-orbit splitting using variety of medium-mass targets.

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I. INTRODUCTION

The hypernucleus is a strongly-interacting multibaryonic system with hyperon(s) and its lifetime is as long as the typical order of the Λ weak-interaction decay (τ ≈ 10^{-10} sec). Thus a variety of hypernuclear phenomena provide us with new knowledge of baryon-baryon interactions and novel behaviors of many-body structures. Among these systems the Λ-hypernuclei have been relatively well produced mainly through the (K^-, π^-) and (π^+, K^+) reactions. However, as these experiments have more or less limitation due to their own kinematics, more variety of hypernuclear production processes should be challenged in order to get information on detailed properties of hyperon-nucleon interactions. In this paper we like to demonstrate a new and wide possibility of the photo- and electro-production of hypernuclei, by taking the actual experimental facilities into account, and will show a novel aspect of the reaction spectroscopy.

Historically, the stopped K^- absorption reaction was utilized to produce Λ-hypernuclei of which decays were identified in bubble chamber and nuclear emulsion. Next, the in-flight reaction K^- + n → π^- + Λ with the kaon beam momenta p_K = 400–800 MeV/c had a unique role of getting particular hypernuclear excited states taking an advantage of the recoilless condition (ΔL = 0) with very weak spin-flip transition. Then a new stage of hypernuclear production was realized by employing the π^+ + n → K^+ + Λ reaction mostly at p_π ≈ 1.05 GeV/c. This (π^+, K^+) reaction on nuclear targets played a great role of producing a series of all Λ single-particle states inside the nucleus, since the process with recoil momentum q ≈ 350 MeV/c preferentially excites hypernuclear high-spin stretched configurations with natural parity mainly due to the spin-non-flip component of the elementary amplitudes. More details can be found in the review papers [1, 2, 3].

In general, hypernuclear states consist of multiplets which are based on the nuclear core angular momentum J_c coupled with that of a hyperon j_Λ, so that low-lying doublets with a s-state Λ have spin partners: J_H = J_c ± 1/2. The (K^-, π^-) reaction with spin-non-flip nature leads to excitation of one of the doublet member with natural parity. Although the (π^+, K^+) spin-flip component is sizable enough to produce appreciable polarization in the final hypernuclear states [4], actually the favored states with large cross section are solely restricted to the natural parity states and their unnatural parity partners get very small cross section [4, 3]. In addition the typical energy resolution achieved in the (π^+, K^+) re-
action spectroscopy is 1.45 MeV at best [6], which is much larger than the characteristic
spin-doublet splittings of the order of a few tens keV. Therefore, only the energy of natural
parity member of the doublets can be accessible within the restriction of such energy reso-
lation. The coincidence $\gamma$-ray measurement proved to be a nice tool to know the detailed
hypermnuclear level energies with a remarkably good resolution of several keV [2].

In contrast to the production processes mentioned above, which convert a neutron into $\Lambda$, here we focus our attention on $K^+$ photo- and electro-production process. These reactions
convert a proton in nuclear targets into $\Lambda$, so that new variety of hypernuclei can be pro-
duced. Indeed much larger amount of spectroscopic data for other hypernuclei with various
excited states are quite necessary to draw some clear conclusion about the nature of hyperon-
nucleon interaction for which the elementary scattering experiments are not available. Over
10 years ago, three of the present authors predicted the photo-production cross sections for
$^7$Li, $^{10}$B, $^{12}$C, and $^{16}$O targets [7, 8]. Among others the $^{12}$C case has been remarkably well
confirmed by the first experiment on the nuclear target at the Thomas Jefferson National
Laboratory (JLab) [9].

These factors explain the reasons why recently the photo- and electro-production of both
hyperons and hypernuclei has attracted much attention in strangeness nuclear physics. In
addition the following facts encourage us to extend the theoretical estimates to wider pos-
sibilities of precise reaction spectroscopy:

- The quality of CEBAF beam (high intensity and energy resolution) at JLab makes it
  possible to identify various hypernuclear energy levels with appreciable cross sections
together with a resolution of about 400 keV [2, 10].

- The photo- and electro-production reactions, $(\gamma, K^+)$ and $(e,e'K^+)$, are characterized
  by the large momentum transfer ($q \geq 350$ MeV$/c$) and the strong spin-flip terms.
  This means especially that photons (real or virtual) have unique characteristics of
  exciting unnatural parity high-spin hypernuclear states preferentially including states
  with deeply-bound $\Lambda$ hyperon [11].

- In contrast to $(K^-, \pi^-)$ or $(\pi^+, K^+)$ reactions, the electro-magnetic production of $K^+\Lambda$
  pair on the proton makes it possible to study some “proton-deficient” hypernuclear
  species, such as $^7_\Lambda$He, $^9_\Lambda$Li and $^{12}_\Lambda$B, which are not available otherwise. Such ability
  opens a new possibility of producing neutron-rich hypernuclei with large excess of
neutral baryons. The same hypernuclei may be produced in \((K^-, \pi^0)\) reaction \cite{12}, but due to the weak spin-flip interaction in the latter the two reactions will afford complementary information on hypernuclear spectroscopy.

The possibility to use the electro-production of strange particles as a tool for study of hypernuclei was first mentioned in a pioneering work by Fetisov et al. \cite{13}. In 1980’s, early works on photo- and electro-production of hypernuclei have been done based on simplified models \cite{14, 15, 16, 17}. The production cross section and polarization of produced hypernuclei in photo-production process was examined in Refs. \cite{8, 18, 19} for schematic as well as for very realistic shell-model wave functions and with full inclusion of distortion effects. Electro-production of \(0\)\textit{p}-shell hypernuclei was investigated carefully in Refs. \cite{7, 20} with close link to the JLab experimental program \cite{21, 22, 23}.

The aim of this paper is, first, to demonstrate the characteristic and selective nature of the photo-production reaction in exciting hypernuclear states. For this purpose we adopt a typical medium-mass target of \(^{28}\text{Si}\) and discuss the general and novel rules based on the properties of the elementary amplitudes. We also make clear the effect of kaon distortion and the different hypernuclear wave functions. Second, we examine the properties of the recent models of elementary kaon electro-magnetic production. Third, by choosing medium-heavy nuclear targets such as \(^{28}\text{Si}\), \(^{40}\text{Ca}\) and \(^{52}\text{Cr}\), we present theoretical predictions of excitation spectra corresponding to the hypernuclear programs at the JLab facility \cite{23}.

This paper is organized as follows: Section 2 briefly describes the minimum framework of calculation of the hypernuclear production cross section. In order to demonstrate the successful prediction done 10 years ago, the updated excitation spectrum for \(^{12}\text{C}(e,e'K^+)\Lambda^1\text{B}\) is shown. In section 3 the basic properties of modern elementary amplitudes of kaon electromagnetic production are discussed. Section 4 summarizes the excitation spectra predicted for the \((\gamma, K^+)\) reaction on \(^{28}\text{Si}\). In order to show the novel and general selectivity found for the \((\gamma, K^+)\) reaction, the simple configuration estimates are presented in subsection 4.1, while sophisticated wave functions are employed in subsection 4.2 for the realistic predictions. In section 5, the calculation is further applied to heavier hypernuclear production by choosing the \(^{40}\text{Ca}\) and \(^{52}\text{Cr}\) targets. Conclusions and summary are given in section 6.
II. FORMALISM FOR HYPERNUCLEAR PRODUCTION

The cross section for the electro-magnetic production of $^{12}_ΛB$ hypernucleus was predicted already many years ago [7, 8] but until recently a good quality data [9] allows for a comparison with experiment which stimulates further development of the models. In Fig. 1 we demonstrate that a very good agreement was achieved for the $^{12}C(e,e'K^+)^{12}_ΛB$ reaction [9].

The calculation presented in Fig. 1 was performed for the real photon whereas the measurement done at JLab is for the virtual photons. However, kinematics of this experiment is very close to the photo-production case because the photon momentum squared ($q^2 = -Q^2$) is $-2.6 \times 10^{-6}$ (GeV/c)$^2$ at the electron scattering angle $0.1^\circ$. Thus the photo-production cross section is practically identical to the one for the electro-production process. The successful first experiment has opened a variety of hypernuclear production by electron beams [21, 22, 23], and accordingly it is challenging to make theoretical predictions for wide range of hypernuclear productions together with improved treatment.

The production cross section and polarization of hypernuclear states are evaluated by choosing the coordinate frame $\{S_2\}$ defined as follows (see Fig. 2)

$$\hat{z} = \hat{n} = \frac{p_\gamma \times p_K}{|p_\gamma \times p_K|}, \quad \hat{y} = \hat{p}_\gamma, \quad \hat{x} = \hat{y} \times \hat{z}. \quad (1)$$

For the real photon, the hypernuclear production cross section is expressed in DWIA as

$$\frac{d\sigma}{d\Omega}(\theta_K^f) = \frac{s p_K^2 E_K E_H}{p_K(E_H + E_K) - E_\gamma E_K \cos\theta_K^f} \sum_{M_f} R(f_i; M_f), \quad (2)$$

where $p$'s and $E$'s are the momenta and energies in the $A$-body laboratory frame, and $s$ is the square of the sum of the energies of $\gamma$ and proton in their center of mass system. $R(f_i; M_f)$ is a transition strength defined as

$$R(f_i; M_f) = \frac{1}{[J_i]} \sum_{M_i} |\langle J_f M_f T_f \tau_f |O| J_i M_i T_i \tau_i \rangle|^2, \quad (3)$$

where the magnetic subspace quantum number $M_f$ is kept explicitly and the convention $[a] = 2a + 1$ is used.

The transition operator $O$ for the $(\gamma, K^+)$ reaction is written as

$$O = \int d^3r \chi_K^{(-)^*(p, \xi r)} \chi_\gamma^{(+)}(k, r) \sum_{\nu=1}^A V^{(\nu)}(r - \eta r_\nu) \delta(r - p,t) |k, p, t, 0\rangle_{lab}, \quad (4)$$
where \( \xi = M_A/M_H \) and \( \eta = (M_H - M_A)/M_A \) are introduced to take the recoil effect into account. \( V^{(\nu)} \) represents a V-spin lowering operator which converts a proton into a \( \Lambda \) hyperon and is zero for neutron. The \( t \)-matrix in the 2-body lab system can be given by using the baryon spin operators as
\[
\mathcal{M} \equiv \langle k - p, p | t | k, 0 \rangle_{\text{lab}} = \epsilon_0 (f_0 + g_0 \sigma_0) + \epsilon_x (g_1 \sigma_1 + g_{-1} \sigma_{-1}) .
\] (5)

Here \( \epsilon_0 \) and \( \epsilon_x \) denote the photon polarization in the \( z \) and \( x \) directions, respectively, and the coefficients of \( g_0, g_1, \) and \( g_{-1} \) are defined in Ref. [8].

The actual evaluation of \( R(f_i; M_f) \) is carried out in the coordinate frame \( \{S_1\} \) defined by
\[
\hat{z} = \hat{p}_\gamma, \quad \hat{y} = \hat{n}, \quad \hat{x} = \hat{n} \times \hat{z} ,
\] (6)
which is more suitable for calculations of incident and outgoing distorted waves. The coordinate frame \( \{S_2\} \) is obtained from \( \{S_1\} \) by the rotation \( \mathcal{R}(\tau, \tau, \frac{\pi}{2}) \). The \( R(f_i; M_f) \) is, then, expressed in terms of the “reduced effective number” \( \rho(f_i; M_f) \) as
\[
R(f_i; M_f) = |f_0|^2 \rho_{ff}(f_i; M_f) + |g_0|^2 \rho_{gg}(f_i; M_f) + 2 \text{Re}[f_0 g_0^* \rho_{fg}(f_i; M_f)]
+ |g_1|^2 \rho_{gg+}(f_i; M_f) + |g_{-1}|^2 \rho_{gg-}(f_i; M_f) + 2 \text{Re}[g_1 g_{-1} \rho_{gg++}(f_i; M_f)] .
\] (7)

The expressions of \( \rho_{ff}, \rho_{gg}, \) and \( \rho_{fg} \) are formally the same as those in the \((\pi^+, K^+)\) reaction, so that one may refer to the Appendix of Ref. [4]. The other quantities \( \rho_{gg+}, \rho_{gg-}, \) and \( \rho_{gg++} \) are essentially new ones which appear in the \((\gamma, K^+)\) reaction for the first time. Their expressions are listed in the Appendix.

III. BASIC PROPERTIES OF THE ELEMENTARY KAON PHOTO-PRODUCTION AMPLITUDES

In hypernuclear production a magnitude of the momentum transfer to the \( \Lambda \) hyperon plays an important role in exciting a series of hypernuclear highest-spin states. Before discussing properties of typical elementary amplitudes for kaon photo-production, we compare in Fig. 3 the momentum transfer \( q_\Lambda \) for \( p(\gamma, K^+)\Lambda, n(\pi^+, K^+)\Lambda, \) and \( n(K^-, \pi^-)\Lambda \) reactions, the last two having been often performed until present. In the figure the two curves, which correspond to the kaon (pion) angle \( \theta^L = 0^\circ \) (lower one) and \( \theta^L = 10^\circ \) (upper one), are shown for each reaction drawn as a function of the incident momentum \( P_{in} \). In the following
hypernuclear calculations we choose $P_\gamma = 1.3\text{ GeV/c}$, which is close to the energy selected in several proposals of experiments at JLab. The momentum transfer $q_\Lambda$, for the photo-production, then amounts to 353 MeV/c at $\theta^b=0^\circ$ and 425 MeV/c at $\theta^b=10^\circ$. These values are well comparable to those for the $n(\pi^+, K^+)\Lambda$ reaction at $P_\pi = 1.05\text{ GeV/c}$: 406 ($\theta^b=0^\circ$) and 447 ($\theta^b=10^\circ$) MeV/c. On the other hand, one may refer to smaller momentum transfer involved in the $n(K^-, \pi^-)\Lambda$ reaction, $q_\Lambda = 50$ and 147 MeV/c at $P_K = 0.8\text{ GeV/c}$ and $q_\Lambda = 109$ and 285 MeV/c at $P_K = 1.5\text{ GeV/c}$.

Numerous theoretical attempts have been made to describe the elementary $\gamma p \rightarrow \Lambda K^+$ process. In the kinematical region assumed here, $E^L_\gamma = 0.91-2\text{ GeV}$, the isobaric models [24, 25, 26, 27, 28, 29, 30, 31] based upon the Feynman diagram technique are of particular interest. In these models the amplitude is derived from an effective hadronic Lagrangian in the tree level approximation. It gains contributions from the extended Born diagrams, where the proton, lambda, $\Sigma^0$, and kaon are exchanged in the intermediate state, and diagrams which account for exchanges of moderate mass (less than 2 GeV) nucleon, hyperon, and kaon resonances. Unfortunately, due to absence of a dominant baryon resonance in the process [32], on the contrary to the pion and eta production, many of exchanged resonances have to be a priori assumed [28] introducing a rather large number of free parameters in calculations, the appropriate coupling constants. The free parameters are determined by fitting the cross sections and polarizations to the experimental data which, however, provides a copious number of possible sets of parameters [26, 28]. This large number of models which describe the data equally well can be reduced implementing duality hypothesis, crossing symmetry, and SU(3) symmetry constraints.

According to duality principle most of the nucleon resonances exchanged in the s-channel, especially those with a high spin, can be mimic by the lowest-mass kaon poles K$^*$ and K$_1$ in the t-channel [27, 28]. The crossing symmetry constraint requires in order that a realistic model for the $\gamma p \rightarrow \Lambda K^+$ process yields simultaneously a reasonable description of the radiative capture of $K^-$ on the proton with $\Lambda$ in the final state [27, 28]. The flavor SU(3) symmetry allows to relate the main coupling constants $g_{K\Lambda N}$ and $g_{K\Sigma N}$ to the better established one, $g_{\pi NN}$. For the 20% breaking of the symmetry the following limits can be obtained for them: $-4.4 \leq g_{K\Lambda N}/\sqrt{4\pi} \leq -3.0$ and $0.8 \leq g_{K\Sigma N}/\sqrt{4\pi} \leq 1.3$ [26, 28]. Analysis of data performed under different assumptions [24, 25, 26, 27, 28, 29] showed that a moderate number of resonances is sufficient to get a reasonable agreement with the experimental data.
In the isobaric models discussed above the baryons are assumed as a point-like particles in the strong interaction vertices which is forced by the gauge invariance principle. Recently, however, baryon form factors were successfully introduced in the model in a gauge-invariant way \[33, 34\]. Accounting for the composite structure of the baryons proves to be important in suppression of the contribution of the Born terms \[31\].

More elementary approach to study of the reaction mechanism of $\gamma p \rightarrow \Lambda K^+$ was performed in terms of quark degrees of freedom in Refs. \[35, 36\]. These models being in a closer connection with QCD than those based on the baryon degrees of freedom, need a smaller number of parameters to describe the data. Moreover, the quark models assume explicitly an extended structure of the baryons which was found to be important for a reasonable description of the photo-production data \[36\]. Other approaches based on the Regge trajectory formalism \[37\] and chiral perturbation theory \[38\] are not suited for using here because they are applicable to photon energies larger than 3-4 GeV and to the threshold region, respectively.

In hypernuclear calculations performed here we adopt four isobaric models denoted hereafter as Kaon-MAID (KMAID) \[30\], Adelseck-Saghai (AS1) \[26\], Williams-Ji-Cotanch (C4) \[27\], and Saclay-Lyon A (SLA) \[29\] which characterize nowadays understanding of the $\gamma p \rightarrow \Lambda K^+$ process for $E_{\gamma}^{\text{Lab}} \leq 1.5$ GeV. The models KMAID, C4, and SLA are extended to the higher energies, $E_{\gamma}^{\text{Lab}} \leq 2.2$ GeV, and to the electro-production process. They also aimed at description of the channels with $\Sigma^0$ and $\Lambda(1405)$ (C4) or $\Sigma^+$ (KMAID, SLA) in the final state. The models C4 and SLA describe also data on the radiative capture of $K^-$ on the nucleon.

A common feature of the four models, KMAID, AS1, C4, and SLA, is that besides the extended Born diagrams they include also the kaon resonant ones, $K^*(890)$ and $K_1(1270)$. As shown in Ref. \[27\] these t-channel resonant terms in combination with s- and u-channel exchanges improve an agreement with the data in the intermediate energy region. The models differ in a particular choice of nucleon and hyperon resonances. The models AS1, and C4 assume only the spin 1/2 baryon exchanges with different masses, whereas in the more elaborate models, KMAID and SLA, the spin 3/2 nucleon resonance $N(1720)$ was added. Moreover, to account for the resonant structure seen in the SAPHIR data \[39\] and confirmed in the latest measurements \[40, 41\] the KMAID model includes a resonance $D_{13}(1895)$ which was predicted by the quark model \[42\] but which was not observed yet.
The higher-spin resonances were omitted in the C4 assuming the duality hypothesis whereas in the Saclay-Lyon model they were introduced to improve an agreement with the data at higher energies [28]. The model SLA is a simplified version of the full Saclay-Lyon model [28] in which the nucleon resonance with spin 5/2 appears in addition. Since predictions of the both models are very similar for the cross sections and polarizations in the kaon photo-production [29] we have chosen the simpler version SLA here. The only KMAID model includes the baryon form factors realized in the prescription by Haberzettel et al. [33] which improves its predictions for higher energies.

The electro-magnetic and strong coupling constants were fixed by fitting to various sets of the experimental data which defines a range of validity of the models. The model AS1 confines itself to the kaon photo-production data for \( E_{\gamma}^{L} < 1.4 \text{ GeV} \), violates the crossing principle by over-predicting the branching ratio of the radiative capture [26] but it fulfills the SU(3) symmetry limits for the two main coupling constants. Parameters of the model C4 were fitted to the data on the kaon photo- and electro-production and the radiative capture of \( K^- \). The energy range was extended to \( E_{\gamma}^{L} < 2.1 \text{ GeV} \). The C4, however, violates the SU(3) symmetry constraint for both the main coupling constants: \( g_{K\Lambda N}/\sqrt{4\pi} = -2.38 \) and \( g_{K\Sigma N}/\sqrt{4\pi} = 0.27 \). The model SLA focused to description of the data in the energy range up to 2.5 GeV. The SU(3) limits for the coupling constants are fulfilled since they were included in the fitting procedure. More details and comparison of the models can be found in Ref. [43].

In Figure 4 we compare differential cross sections as they are predicted by the four models with experimental data at fixed kaon angle as a function of the photon laboratory energy. In the bottom part (b), the c.m. cross section is shown at kaon c.m. angle of 90°. Predictions of all models are in a very good agreement with the data for energies up to 1.4 GeV but for the higher energies the models AS1, and C4 overestimate significantly the SAPHIR data [39] which were not included in the fitting procedure of neither of them. Only the KMAID and SLA models are consistent with the data up to 2 GeV. All the four models provide acceptable description of the process at \( E_{\gamma}^{L} \approx 1.3 \text{ GeV} \) concerned in the present hypernuclear calculations. The amount of the cross section difference coming from these different models constitutes a part of theoretical uncertainty in predicting the hypernuclear production rate.

However, in the hypernuclear calculations only the forward angle laboratory amplitude
for the elementary process enters effectively into the calculations. In Figure 4(a) we show behavior of theoretical laboratory cross sections at $\theta^L_K = 3^\circ$. Predictions of the models AS1, C4 and SLA reveal a constant discrepancy in most of the energy range whereas the KMAID model predicts considerably different values with respect to the SLA near the threshold and especially at energies larger than 1.6 GeV. At 1.3 GeV and $\theta^L_K = 3^\circ$, SLA gives by 40% larger cross section than KMAID.

In Figure 5 we plot angular distribution of the cross sections at $E^L_{\gamma} = 1.3$ GeV. Predictions of the models are in a good agreement with the data (the bottom part (a)) for all angles except for $\theta^{c.m.}_K < 30^\circ$ where C4 over-predicts the data. Note here, probably a systematic, discrepancy of SAPHIR data [39, 40] and those by Bleckmann et al. [44] for $\theta^{c.m.}_K < 45^\circ$. The latest CLAS measurements [41] confirm the discrepancy between the data sets which makes fixing the theoretical models at small angles problematic. In Figure 5(a) large differences between results of the models at forward angles are demonstrated showing that an uncertainty can amount up to 80% (KMAID and C4) at very forward angles ($\theta^L_K = 3^\circ$). Figure 4(a) shows that the C4 predicts too large cross sections at small angles in the whole energy range whereas KMAID drops down at $E^L_{\gamma} > 1.6$ GeV making the difference still larger.

The most remarkable difference between predictions of the models is revealed when the hyperon polarization is plotted. In Figure 6 we compare the $\Lambda$ polarization at $\theta^{c.m.}_K = 90^\circ$ as a function of photon laboratory energy. The quality of the data, however, does not allow to prefer some of the models. This is not the case of the angular dependence of the polarization shown in Fig. 7 where the results of the models exhibit significant deviations at the backward angles. Prediction of the AS1 model is out of the data at large angles having opposite sign to that of the data points. However, all the four models seem to have a problem with description of the data points around 60$^\circ$. Of the four models the KMAID and SLA provide large values of the polarizations at backward angles with the proper sign. These models deviate also from the others for $E^L_{\gamma} \approx 2$ GeV at $90^\circ$ (Fig. 6).

In order to give an idea of relative importance of the spin-non-flip ($f_0$) and spin-flip ($g_0$, $g_1$, $g_{-1}$) amplitudes, in Table I we list values of the amplitudes in the laboratory frame calculated for $\theta^L_K = 3^\circ$ and 10$^\circ$ at $E^L_{\gamma} = 1.3$ GeV. The amplitudes are normalized as follows

$$
\left( \frac{d\sigma}{d\Omega} \right)_L = \frac{s \, p^2_K E_K E_\Lambda}{p_K(E_\Lambda + E_K) - E_\gamma E_K \cos \theta^L_K} \, \frac{1}{2} \left( |f_0|^2 + |g_0|^2 + |g_1|^2 + |g_{-1}|^2 \right), \quad (8)
$$
where the energies, momenta, and angle are in the laboratory frame. It is evident that at small scattering angles which are relevant in hypernuclear calculations the three spin-flip amplitudes are much larger in magnitude than the spin-non-flip one. This observation together with the high momentum transfer to Λ (Fig. 3) suggest that a large number of possible hypernucleus final states with high spins and unique features can be selectively populated.

An extension of the isobaric models to the electro-production process is carried out by introduction of a $Q^2$-dependence into the electro-magnetic vertices by means of form factors [27, 28]. However, in the calculations of the electro-production of hypernuclei, performed here (small kaon and electron angles), very small values of $Q^2 \left[ Q^2 < 0.1 \, (\text{GeV}/c)^2 \right]$ are reached which, due to a smooth $Q^2$-dependence of the form factors, results in a very small effect in the amplitudes. It means that the elementary photo-production amplitude $A_{S1}$ can be also utilized for the calculations of the electro-production of hypernuclei at small angles. This is also why we did not discuss results of the models for the electro-production here but it can be found in Ref. [43].

IV. EXCITATION SPECTRA PREDICTED FOR $^{28}\text{Si}(\gamma, K^+)^{28}\Lambda\text{Al}$

In order to demonstrate characteristics of photo-production of hypernuclei, here we choose $^{28}\text{Si}$ as a typical nuclear target. The excitation spectra have been evaluated at $E_{\gamma}^L = 1.3$ GeV and $\theta_{K}^L = 3^\circ$ which are close to the kinematical condition in the experimental proposal. In the first subsection, for demonstration the $^{28}\text{Si}$ target ground state is assumed to have the lowest proton-closed shells $[s^4p^{12}(0d_{5/2})_{pn}^{12}]$. In the second subsection, we extend the calculation to employ realistic wave functions solved in the full $[s^4p^{12} (sd)_{pn}^{12}]$ shell-model space.

A. A demonstrative model with $(0d_{5/2})_p^6$

As the proton shells are closed up to $(0d_{5/2})_p^6$, the final hypernuclear states are described, respectively, with 1p-1h configurations $[(nlj)^{−1}_p (nlj)^{\Lambda}]_J$. For the single-particle wave functions, we employ the DDHF solutions so as to be as realistic as possible. The calculated cross sections are summarized in Table II.
The characteristic result to be emphasized first is the selective excitation of the highest-spin state within each $1h-1p$ multiplet. In fact one sees in Table III that the $[d^\pm_{5/2} s^\Lambda_{1/2}]_{3^+}$, $[d^\pm_{5/2} p^\Lambda_{3/2}]_{4^-}$, $[d^\pm_{5/2} p^\Lambda_{1/2}]_{3^-}$, $[d^\pm_{5/2} d^\Lambda_{3/2}]_{5^+}$, and $[d^\pm_{5/2} d^\Lambda_{1/2}]_{4^+}$ states are very strongly excited and that the cross sections to the lower-spin states are much smaller. The preferential excitation of the high-spin states are attributed to the large momentum transfer (about 350 MeV/c) as similarly as in the case of the $(\pi^+, K^+)$ reaction.

Secondly, such a novel fact is revealed that the selectively excited state in each combination $[j^-_1 j^\Lambda_>_J]$ has an unnatural parity with the maximum spin value of $J = J_{\text{max}} = j_> + j^\Lambda_> = l_p + l_\Lambda + 1 = L_{\text{max}} + 1$. In Table III, one may refer to the $[d^-_{5/2} s^\Lambda_{1/2}]_{3^+}$, $[d^-_{5/2} p^\Lambda_{3/2}]_{4^-}$, and $[d^-_{5/2} d^\Lambda_{3/2}]_{5^+}$ cases for reconfirmation. The $[p^\Lambda_{3/2} j^\Lambda_>]_{J=2^-,3^+,4^-}$ states in the lower block of Table III have the similar nature. This kind of selectivity is not seen in the other hypernuclear production processes such as $(\pi^+, K^+)$ and $(K^-, \pi^-)$ reactions. This is attributed to the spin-flip transition dominance in the elementary hyperon photo-production reaction (see Tab. II). It is also noted that, in the other combinations such as $[j^-_1 j^\Lambda_>_J]$ or $[j^-_1 j^\Lambda_<_J]$, the highest spin is limited to $J'_{\text{max}} = l_p + l_\Lambda$ and accordingly the natural parity to $(-1)^{l_0 + l_\Lambda}$.

The numerical results of Table III are schematically shown in Fig. 8 where relative strengths in each J-multiplet are easily understood.

Figure 9 shows the calculated angular distributions for the pronounced peaks. All these differential cross sections decrease quickly as the kaon lab scattering angle increases. It is interesting to note that the relative strength for the $[d^-_{5/2} d^\Lambda_{3/2}]_{J=4^+}$ and $[d^-_{5/2} p^\Lambda_{3/2}]_{J=4^-}$ states changes at $\theta_{K^\ell} \simeq 7^\circ$.

### B. Use of the $(sd)^n$ full space wave function

Here we use sophisticated wave functions solved in the full $(0d_{5/2} 0d_{3/2} 1s_{1/2})_{pn}^{11,12}$ space for $^{27,28}\text{Si}$. It is remarked that the use of such detailed wave functions should predict several new but minor states in addition to the pronounced peaks which are expected in the simplified configuration adopted in the preceding subsection. The present estimates are based on the spectroscopic amplitudes for proton pick-up from $^{28}\text{Si}$ leading to the excited states of $^{27}\text{Al}$ which are calculated with the $(sd)^n$ model space.

In order to predict a realistic excitation spectrum for the $^{28}\text{Si}(\gamma, K^+)^{28}\Lambda\text{Al}$ reaction, one has to take the empirical proton-hole widths into account, although they are not always available.
Figure 10 shows the result where the following proton widths are employed tentatively: \( \Gamma_p(0s_{1/2}) = 10 \text{ MeV}, \Gamma_p(0p_{3/2}) = 6 \text{ MeV}, \Gamma_p(0p_{1/2}) = 3 \text{ MeV}, \) and \( \Gamma_p(0d_{5/2}) = 0 \text{ MeV}. \) At the same time, for the \( \Lambda \) bound states the width \( \Gamma_\Lambda(j) = 0.3 \text{ MeV} \) is used which is about half of the energy resolution expected at the Jefferson Lab, while \( \Gamma_\Lambda(j) = 1.0 \text{ MeV} \) for \( 0 < E_\Lambda < 2 \text{ MeV} \) and \( \Gamma_\Lambda(j) = 3 \text{ MeV} \) for \( E_\Lambda > 2 \text{ MeV} \) are assumed rather arbitrarily. Furthermore the energy splittings between members of the \( [j^{-1}j^\Lambda]_J \) multiplet are taken from the YNG(\( \Lambda N \)) h-p interaction [47] derived from the Nijmegen model-D, and it is notable that the splittings are mostly of the order of 0.1 MeV.

Major 3 doublets (6 peaks) structure obtained with the simplified wave functions \( [d^{-1}_{5/2}J^\Lambda] \) (see Table 2) well persist also in the new estimates. It is quite interesting to note that, for the major peaks, the use of the full space wave functions results in the reduction of the cross sections by a factor of about 0.65 in comparison with the single-\( j \) estimate with \( (0d_{5/2})^6_p \). It should be also remarked that two pronounced peaks obtained at \( E_\Lambda \approx -8.5 \text{ MeV} \) correspond to the \( [d^{-1}_{5/2}p^\Lambda_{3/2}]_{4^-} \) and \( [d^{-1}_{5/2}p^\Lambda_{1/2}]_{3^-} \) structure, respectively. As the hole-particle interactions for high-spin states are generally very small, the energy difference between these two peaks, if separated experimentally, provides us the spin-orbit splitting of the \( \Lambda p \)-state.

The third major doublet obtained at \( E_\Lambda \approx 0 \text{ MeV} \) includes the unnatural parity highest-spin state \( [d^{-1}_{5/2}d^\Lambda_{5/2}]_{5^+} \) which should get the biggest cross section. If the \( d^\Lambda_{5/2} \) state is bound or it is not bound but the energy position is not so high above the threshold, this peak width might be sharp enough to be identified in the experiment with the good energy resolution expected at JLab. As the partner has the dominant structure of \( [d^{-1}_{5/2}d^\Lambda_{3/2}]_{4^+} \) and the \( \Lambda N \) hole-particle interactions in high-spin states are very small, the energy difference between these two peaks is almost equal to the spin-orbit splitting of the \( d \)-state \( \Lambda \). Thus the photo- and electro-production reactions will provide a nice opportunity of looking at such splittings in heavy systems if the energy resolution is good enough.

It is interesting to see how different the theoretical hypernuclear production cross sections are when one uses different models of hyperon photo-production amplitudes. In section 3 we have already discussed remarkable difference among the AS1, KMAID, C4, and SLA models. In Table III the cross sections of strongly excited states in \( ^{28}\text{Si}(\gamma, K^+)^{28}\Lambda\text{Al} \) are compared as calculated with the SLA and KMAID amplitudes. They are the cross sections expected within the \( (sd)^n \) shell model framework, noting that use of the \( (sd)^n \) wave functions give rise to a reduction factor of 0.61 in comparison with the single-\( j \) estimate of \( (d_{5/2})^n \). One notes
from Table III that the KMAID amplitude gives approximately 30% smaller cross sections of the SLA values when compared at $\theta_K^L = 3^\circ$. Such difference has been suggested already in section 3.

The present treatment is a direct extension of that for the $p$-shell hypernuclear photo-production calculation \[7, 8\] where always the full $p$-shell wave functions have been easily employed. In the case of $p$-shell hypernuclear production, the particles involved in the low-lying state are in the $s$- and $p$-orbits, so that the “high-spin” selectivity mentioned above is realized as the transitions with $\Delta J = \Delta L + \Delta S = 2^-, 2^+, \text{ and } 3^+$. It is worthwhile to remark that in Fig. 10 there appear side peaks at $E_{\Lambda} \simeq -16$ and $-14$ MeV in $^{28}_{\Lambda}\text{Al}$ as similarly as confirmed in $^{12}_{\Lambda}\text{C}$ ($1^-_{2}$ and $1^-_{3}$). They are based on the $s$-state $\Lambda$ particle coupled to the excited states ($3/2^+$ and $7/2^+$) in $^{27}\text{Al}$.

V. PHOTO-PRODUCTION WITH THE $^{40}\text{Ca}$ AND $^{52}\text{Cr}$ TARGETS

The next sample target with heavier mass is $^{40}\text{Ca}$ and the calculated excitation function for the $^{40}\text{Ca}(\gamma, K^+)^{40}_{\Lambda}\text{K}$ is shown in Fig. 11. This target is doubly $LS$-closed up to the $0d_{3/2}$ shell, so that the situation is different from the $^{28}\text{Si}$ case because $^{40}\text{Ca}$ has the uppermost proton orbit of the $j<$-type. Therefore the highest spin in a $[d_{3/2}^{-1}j_{>}]_L$ multiplet is $J'_{\max} = j_\Lambda + j^A_\Lambda = l_p + l_\Lambda = L_{\max}$ with a natural parity. On the other hand, in a $[d_{3/2}^{-1}j_{<}]_L$ multiplet the highest spin is $J''_{\max} = j_\Lambda + j^A_\Lambda = l_p + l_\Lambda - 1 = L_{\max} - 1$, so that this restriction on the smaller angular momentum transfer makes the latter cross sections much smaller than the former case with $J'_{\max} = L_{\max}$. This situation of the $j_<$-closed shell explains why the dominant peak series shown in Fig. 11 consists of the natural parity $2^+, 3^-$, and $4^+$ states accompanied with only minor side peaks of the $1^+, 2^-$, and $3^+$ states, respectively. In other word, if these pronounced peak positions are measured with good energy resolution in a future experiment, they tells us the exact energies of the $\Lambda$ particle in the $s^A_{1/2}$, $p^A_{3/2}$, and $d^A_{5/2}$, respectively. The broad background shown in Fig. 11 reflects the strengths originating from the deeper proton shell, $0d_{5/2}$, which has the spreading width of several MeV. In reality we also expect extra small peaks between the pronounced peaks, as they might be based on the fragmentation of the $0d_{3/2}$ proton hole strength. In spite of such factors, the photo-production reaction with this kind of doubly $LS$-closed target might give a nice example of showing up the $\Lambda$ single-particle energies in medium-heavy mass region.
Finally we add the case of the $^{52}\text{Cr}$ target as a typical candidate from the $fp$-shell region of nuclei. The calculated results for the $^{52}\text{Cr}(\gamma,K^+)^{52}\Lambda\text{V}$ reaction spectrum is displayed in Fig. 12 where the vertical axis shows simply the cross section value in nb/sr. Therefore, differently from the former figures, here we do not take the smearing width into account when we show the major series of the pronounced peaks by solid lines. The C4 elementary amplitude is employed here, although it leads to overestimates of the cross section by about 25%. It is also noted that in Fig. 12 we show positions of broad peaks by gray blocks originating from deeper proton hole orbits such as $(0d_{5/2})_p$ and $(0d_{5/2})_p$. The height×width of each gray block correspond to the calculated cross section estimates.

The $^{52}\text{Cr}$ target has four protons in the uppermost $j > = 0f_{7/2}$ shell and the neutron is $jj$-closed. Therefore it is easy to understand that the major peak series are based on the conversion of the $0f_{7/2}$ protons into the $\Lambda$-particle sitting in the $s, p, d, \text{and} f$ orbits. In fact we confirmed by the calculation that the dominant peak series consists of the unnatural parity $[f^{-1}_{7/2}j^\Lambda_>]_{J = J_{\max}}$ states with $J_{\max} = j_1 + j^\Lambda_2 = l_p + l_\Lambda + 1 = L_{\max} + 1$ where $j^\Lambda = s_{1/2}(J = 4^-)$, $p_{3/2}(J = 5^+)$, $d_{5/2}(J = 6^-)$, and $f_{7/2}(J = 7^+)$, respectively. On the other hand, the $\Lambda$ spin-orbit partner states $[f^{-1}_{7/2}j^\Lambda_<]_{J = L_{\max}}$ (with $J = 3^-, 4^+, 5^-, 6^+$) gain about 60% production rate of the corresponding biggest peak within each multiplet. Therefore, if the energy resolution is good enough, there will be a chance to get information on the $ls$-splitting in this medium-mass region. In the present case, the other angular momentum states also have certain (non negligible) contributions to each peak. It should be mentioned that, if we improve the description of the $^{51}\text{V}$ nuclear excited states, we will also get very weak excited side peaks among the strong peaks shown here. See also the report [48].

VI. CONCLUSION

The first $(e,e'K^+)$ experiment on the nuclear target ($^{12}\text{C}$) was reported recently from the Jefferson Lab, proving it to be a nice tool for spectroscopic study of $\Lambda$ hypernuclei. As our theoretical prediction has been confirmed remarkably by this experiment, we applied its framework to typical heavier targets with careful consideration of modern elementary amplitudes for hyperon photo-production. In this paper we presented the $(\gamma,K^+)$ excitation spectra for producing medium-mass $\Lambda$-hypernuclei, where we focused our attention to the novel and characteristic features of the reaction process.
We started with discussions on the basic properties of modern elementary amplitudes of kaon photo-production process. Among many theoretical attempts, the isobaric models for the $\gamma p \rightarrow \Lambda K^+$ process based on the Feynman diagram technique are of interest and successful in the kinematical region assumed here, $E_{\gamma}^L = 0.91 - 2.0$ GeV. Four elementary amplitudes of the models are adopted in this paper, i.e., those denoted as Kaon-MAID (KMAID), Adelseck-Saghai (AS1), Williams-Ji-Cotanch (C4) and Saclay-Lyon A (SLA) models. The models differ in their own choices of nucleon and hyperon resonances. Detailed comparison of the models has been done for the differential cross sections and polarizations for the elementary $\gamma p \rightarrow \Lambda K^+$ and their results are compared with the available experimental data. Some variances of agreement between the models exist at higher energies $E_{\gamma}^L \geq 1.4$ GeV. However, all the four models are acceptable at $E_{\gamma}^L \approx 1.3$ GeV which is concerned in the present hypernuclear calculations. The unique and interesting feature of the $\gamma p \rightarrow \Lambda K^+$ amplitudes lies in that the spin-flip amplitudes are much larger in magnitude than the spin-non-flip one even at the forward angles at $\theta_{KL}^L = 3^\circ$ and $\theta_{KL}^L = 10^\circ$ and at energy $E_{\gamma}^L = 1.3$ GeV. This feature of the amplitude together with the large momentum transfer ($q_{\Lambda} \approx 350 - 400$ MeV/$c$) yields the novel and selective features of the excitation spectra in the medium-heavy $\Lambda$-hypernuclei.

Here we choose $^{28}\text{Si}$, $^{40}\text{Ca}$ and $^{52}\text{Cr}$ as the typical medium-mass target nuclei for the spectroscopic study of the $\Lambda$-hypernuclei in the $(\gamma, K^+)$ reactions. First the excitation spectra of $^{28}\text{Si}(\gamma, K^+)_{\Lambda}^{28}\text{Al}$ are discussed for two model calculations of $^{28}\text{Si}$. When the simple $(0\,d_{5/2})_{p}$-closed configuration is assumed for $^{28}\text{Si}$ as a demonstrative model, the characteristic and unique excitation function is clearly seen. The result emphasized first is the selective excitation of the highest-spin state within each 1h-1p multiplet such as $[d_{5/2}^{-1}p_{3/2}^{1}\Lambda]_{4^{-}}, [d_{5/2}^{-1}p_{1/2}^{1}\Lambda]_{3^{-}}, [d_{5/2}^{-1}d_{3/2}^{1}\Lambda]_{5^{+}}$ and $[d_{5/2}^{-1}d_{3/2}^{1}\Lambda]_{4^{+}}$. This is due to the large momentum transfer for the reaction process. Second, such a novel fact is revealed that the selectively excited states with $[j_{A}^{-1}j_{p}^{1}]_{J}$ are strongly populated and have unnatural parities with maximum spins of $J = J_{\text{max}} = j_{p} + j_{A} = \ell_{p} + \ell_{\Lambda} + 1 = L_{\text{max}} + 1$. This is attributed to the spin-flip transition dominance in the elementary hyperon photo-production reaction. This kind of selectivity is not seen in other hypernuclear production processes such as $(K^{-}, \pi^{-})$ and $(\pi^{+}, K^{+})$ reactions.

The calculation has been extended to the $(sd)^n$ full space model to describe $^{28}\text{Si}$. Excitation function for the $^{28}\text{Si}(\gamma, K^+)_{\Lambda}^{28}\text{Al}$ is presented, taking the empirical particle-hole width into account tentatively so as to make the comparison with the future experiment possible.
The major doublets and peaks obtained with the simplified model well persist also in the extended estimates. The energy splitting of the two peaks with structures, such as \([d_{5/2}^{-1} p_{3/2}^\Lambda]_4^-\) and \([d_{5/2}^{-1} p_{1/2}^\Lambda]_3^-\), and \([d_{5/2}^{-1} d_{5/2}^\Lambda]_5^+\) and \([d_{5/2}^{-1} d_{3/2}^\Lambda]_4^+\) corresponding to the proton hole in \(d_{5/2}\) and \(\Lambda\) in \(j_>\) and \(j_<\), if observed experimentally, might give us information on the spin-orbit splitting of the \(\Lambda\) \(p\)- and \(d\)-states, respectively.

The photo-production reactions with \(^{40}\text{Ca}\) and \(^{52}\text{Cr}\) targets are discussed as other interesting cases. The former nucleus is \(LS\)-closed up to \(j_<=0d_{3/2}\) shell orbit, so that the highest spin in the \([d_{3/2}^{-1} j_>]^\Lambda J\) multiplet is \(J'_\text{max} = j_>+j_\Lambda^\Lambda = \ell_p + \ell_\Lambda = L_{\text{max}}\) with a natural parity and such a state is strongly excited. However, the states of \([d_{3/2}^{-1} j_<^\Lambda]_J\)-type with \(J''_{\text{max}} = \ell_p + \ell_\Lambda - 1 = L_{\text{max}} - 1\) have weak strengths in the excitation function. The \(j_<\)-closed shell nuclear target presents the dominant peaks of natural parity, i.e., \(2^+, 3^-\) and \(4^+\) as seen in \(^{40}\Lambda\)K. The present case seems to be a nice example of getting the \(\Lambda\) single-particle energies in this mass region which has not been explored with good energy resolution.

The \(^{52}\text{Cr}\) target contains four protons in an active \(0f_{7/2}\) shell orbit. The dominant peak series in the \((\gamma, K^+)\) reaction consists of unnatural parity states of \([f_{7/2}^{-1} j_<^\Lambda]_J\)-type with \(J_{\text{max}} = \ell_p + \ell_\Lambda + 1 = L_{\text{max}} + 1\) and \(\ell_p = 3, \ell_\Lambda = 0, 1\) and \(2\), respectively, while the \(\Lambda\) spin-orbit partner states \([f_{7/2}^{-1} j_<^\Lambda]_J=L_{\text{max}}\) have reduced strength as about 60% of the biggest peak within each multiplet.

Conclusively, the \((\gamma, K^+)\) photo-production reactions on the medium-mass nuclear targets will provide us with novel and unique features in the \(\Lambda\)-hypernuclear spectroscopy through the selective excitation of unnatural parity highest-spin states (natural parity high-spin states for the \(LS\)-closed targets). A possibility is also pointed out that we might have a chance to investigate \(\Lambda\) single-particle energies over whole periodic table including the spin-orbit splitting for \(p\)-, \(d\)-, and \(f\)-orbits. It is also notable that the reaction on various nuclear targets produces proton-deficient hypernuclear species otherwise unaccessible. Thus the photo- and electro-production of \(\Lambda\)-hypernuclei will offer a nice opportunity to get knowledge on the dynamical behavior of hyperon-nucleus coupling and on the baryon behavior deeply inside the nucleus.
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APPENDIX: EXPRESSIONS OF THE REDUCED EFFECTIVE NUMBER $\rho(f_i; M_f)$

The product of incident $\gamma$ and outgoing K meson distorted waves is expanded in partial waves

$$\chi^{(-)}(p_k, r) \chi^{(+)}(p, r) = \sum_{km} \sqrt{4\pi[k]} \tilde{j}_{km}(p, p_k, \theta, r) Y_{km}(\hat{r}) . \quad (A.1)$$

Then the $\rho^{gg+}(f_i; M_f)$, $\rho^{gg-}(f_i; M_f)$ and $\rho^{gg^{+}}(f_i; M_f)$ are expressed as follows

$$\rho^{gg+}(f_i; M_f) \equiv \rho^{ggx}(f_i; M_f) \quad \text{with} \quad x = 1 , \quad (A.2)$$

$$\rho^{gg-}(f_i; M_f) \equiv \rho^{ggx}(f_i; M_f) \quad \text{with} \quad x = -1 , \quad (A.3)$$

and

$$\rho^{ggx}(f_i; M_f) = \frac{4\pi}{2|J_f|} \sum_{k_1k_2} \sum_{K_1K_2} \sqrt{|k_1||k_2||\hat{K}_1||\hat{K}_2|} (-1)^{k_1+k_2-K_1-K_2} (-1)^{J_f-M_f} \langle \hat{j}_{k_1-k_2} \rangle$$

$$\times \sum_{\kappa p} W(K_1K_2J_fJ_f; \kappa J_i) \left( J_f M_f J_f - M_f |\kappa 0 \right) W(K_1K_2k_1k_2; p'1)$$

$$\times \sum_m (K_1 - (m + x) K_2 m + x |\kappa 0) (K_1m + x K_2 - (m + x) | p'0)$$

$$\times \sum_{m'm''} (-1)^{m''} (k_1m'k_2 - m''|pq) \hat{\sigma}^{[q]} d(p, q)$$

$$\langle J_f || \hat{j}_{k_1m'}(p, p_k, \theta, r) [Y_{k_1} \times \sigma]_{k_1} || J_i \rangle_{S_1}$$

$$\times \langle J_f || \hat{j}_{k_2m''}(p, p_k, \theta, r) [Y_{k_2} \times \sigma]_{k_2} || J_i \rangle_{S_1} , \quad (A.4)$$
where \( d(p, q) \) is a function defined in Ref. [4].

\[
\rho^{gg+}(f; M_f) = \frac{4\pi}{2|J_i|} \sum_{k_1,k_2,K_1,K_2} \sqrt{|k_1||k_2||K_1||K_2|} (-1)^{k_1-K_1} (-1)^{J_i-M_f} \hat{d}^{k_1-k_2}
\]

\[
\times \sum_{np'p} \left\{ W(K_1 K_2 J_f J_f; \kappa J_i) (J_f M_f J_f - M_f |\kappa 0) W(K_1 K_2 k_1 k_2; p'1) \right. \\
\times \sum_m (K_1 - (m+1) K_2 m + 1 |\kappa 0) (K_1 m + 1 K_2 m + 1) |p'2m + 2) \\
\times \left. (k_1 - m k_2 - m - 2 |p' - 2m - 2) (k_1 m k_2 - m - 2 |p - 2) \right. \\
\times \sum_{m'm''} (-1)^{m''} (k_1 m' k_2 - m'' |pq) \hat{i}^j d^p_{-q,2} \pi \left(\frac{\pi}{2}\right) \\
\cdot \langle J_f || \tilde{J}_{k_1 m'}(p_\gamma, p_K, \theta, r) [Y_{k_1} \times \sigma]_{K_1} || J_i \rangle_{S_1} \\
\cdot \langle J_f || \tilde{J}_{k_2 m''}(p_\gamma, p_K, \theta, r) [Y_{k_2} \times \sigma]_{K_2} || J_i \rangle_{S_1}^* \right), \tag{A.5}
\]

where \( \hat{d}^{J_{mm'}}_{mm'}(\theta) \) is a rotation matrix [49].
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TABLE I: Comparison of the spin-independent \( \langle f_0 \rangle \) and spin-dependent parts \( \langle g_0, g_1, g_{-1} \rangle \) of the elementary amplitude is shown for the four models adopted in this paper. The values are evaluated for two kaon laboratory angles at \( E_L^\gamma = 1.3 \) GeV. Units of \( |f_0|^2 \), \( |g'|^2 \), and \( \text{Re}(f_0g_0^*) \) are \( \mu b/sr/\text{GeV}^4 \). The laboratory cross sections are in \( \mu b/sr \).

| \( \theta_L^K \) | Model | \( |f_0|^2 \) | \( |g_0|^2 \) | \( |g_1|^2 \) | \( |g_{-1}|^2 \) | \( \text{Re}(f_0g_0^*) \) | \( d\sigma/d\Omega \) | Pol(\( A \)) |
|---|---|---|---|---|---|---|---|---|
| 3° | AS1 | 0.0067 | 0.374 | 0.179 | 0.193 | −0.0141 | 1.91 | −0.055 |
| | KMAID | 0.0149 | 0.296 | 0.148 | 0.148 | −0.0642 | 1.54 | −0.212 |
| | C4 | 0.0002 | 0.572 | 0.272 | 0.281 | −0.0063 | 2.85 | −0.019 |
| | SLA | 0.0023 | 0.424 | 0.211 | 0.208 | −0.0079 | 2.14 | −0.016 |
| 10° | AS1 | 0.0515 | 0.352 | 0.164 | 0.192 | −0.0397 | 1.84 | −0.142 |
| | KMAID | 0.1205 | 0.267 | 0.121 | 0.129 | −0.1793 | 1.55 | −0.575 |
| | C4 | 0.0010 | 0.534 | 0.179 | 0.204 | −0.0179 | 2.23 | −0.066 |
| | SLA | 0.0239 | 0.368 | 0.175 | 0.167 | −0.0235 | 1.78 | −0.053 |
TABLE II: Differential cross sections (in nb/sr) for the $^{28}\text{Si}(\gamma, K^+)^{28}\text{Al}$ reaction calculated in DWIA (a) at $E_L^{\gamma} = 1.3$ GeV and $\theta_K = 3^\circ$ with the Saclay-Lyon A amplitude \footnote{29}. The final hypernuclear states are expressed by $[(nlj)^{-1}(nlj)\Lambda]_J$. The $\Lambda$ DDHF single-particle energies ($E_\Lambda$) in MeV are listed in the parentheses where the $\Lambda N$ spin-orbit interaction is neglected for simplicity. In the lower section (b), the estimates without Kaon distortion are listed for comparison in the case of $[(0d_{5/2}^{-1})_p(nlj)\Lambda]_J$.

|        | (a) DWIA | 0s_{1/2}^1 | 0p_{3/2}^1 | 0p_{1/2}^1 | 0d_{5/2}^1 | 0d_{3/2}^1 | 1s_{1/2}^1 | ($E_\Lambda$) | (E) | (E) | (E) | (E) |
|--------|----------|------------|------------|------------|------------|------------|------------|-------------|-----|-----|-----|-----|
|        |          | (−16.92)   | (−8.40)    | (−8.40)    | (0.69)     | (0.69)     | (0.32)     |              |    |    |    |    |
| (p-hole) | J=0     | 0.0        | 0.0        | 0.0        | 0.0        |           |            |              |    |    |    |    |
|         | J=1     | 17.8       | 11.2       | 0.5        | 2.7        | 4.8        | 9.7        |              |    |    |    |    |
|         | J=2     | 52.8       | 0.2        | 68.6       | 1.8        | 15.7       | 29.0       |              |    |    |    |    |
|         | J=3     | −          | 110.6      | −          | 4.8        | 108.8      | −          |              |    |    |    |    |
|         | J=4     | −          | −          | 145.4      | −          |            |            |              |    |    |    |    |
| (0p_{1/2}^{-1}) | J=1 | 18.0       | 14.8       | 29.4       | −          | 14.7       | 56.8       |              |    |    |    |    |
|         | J=2     | −          | 44.0       | −          | 29.6       | 44.1       | −          |              |    |    |    |    |
|         | J=3     | −          | −          | 58.7       | −          |            |            |              |    |    |    |    |
| (b) PWIA | 0s_{1/2}^1 | 0p_{3/2}^1 | 0p_{1/2}^1 | 0d_{5/2}^1 | 0d_{3/2}^1 | 1s_{1/2}^1 | ($E_\Lambda$) | (E) | (E) | (E) | (E) | (E) | (E) | (E) | (E) | (E) | (E) |
|        |          | (−16.92)   | (−8.40)    | (−8.40)    | (0.69)     | (0.69)     | (0.32)     |              |    |    |    |    |
| (p-hole) | J=0     | 0.0        | 0.0        | 0.0        | 0.0        |           |            |              |    |    |    |    |
|         | J=1     | 10.9       | 59.6       | 15.3       | −          |            |            |              |    |    |    |    |
|         | J=2     | 68.3       | 4.03       | 33.4       | 0.4        | 71.9       | 1.4        |              |    |    |    |    |
|         | J=3     | 135.6      | 8.3        | 155.3      | 46.5       | 58.9       | 2.7        |              |    |    |    |    |
|         | J=4     | −          | 250.5      | −          | 0.4        | 186.9      | −          |              |    |    |    |    |
|         | J=5     | −          | −          | 248.1      | −          |            |            |              |    |    |    |    |
TABLE III: Comparison of excitation cross sections of $^{28}\text{Si}(\gamma, K^+)^{28}\text{Al}$ calculated using SLA and KMAID amplitudes at $E_{\gamma}^L = 1.3$ GeV and $\theta_K^L = 3^\circ$ and $10^\circ$. Cross sections are in units of nb/sr.

| $J^\pi$ | $E_x$ | $\langle d\sigma/d\Omega \rangle$ | $\langle d\sigma/d\Omega \rangle$ |
|---------|------|-------------------------------|-------------------------------|
|         | [MeV] | ($\theta_K = 3^\circ$) | ($\theta_K = 10^\circ$) |
| SLA     | KMAID|
| 2$^+$   | 0.0  | 19.8                         | 14.3                         | 8.2  | 7.6  |
| 3$^+$   | 0.3  | 39.4                         | 28.1                         | 15.1 | 12.0 |
| 4$^-$   | 8.6  | 84.9                         | 60.5                         | 44.9 | 36.3 |
| 3$^-$   | 8.9  | 52.4                         | 36.7                         | 28.9 | 21.2 |
| 5$^+$   | 17.6 | 99.6                         | 70.9                         | 69.2 | 56.0 |
| 4$^+$   | 18.1 | 74.7                         | 52.3                         | 53.4 | 39.5 |

FIG. 1: Comparison of the experimental excitation spectrum for the $^{12}\text{C}(e,e'K^+)^{12}\text{B}$ reaction [9] with the theoretical calculation done for real photons (see text) at $E_{\gamma} = 1.42$ GeV and $\theta_K^L = 2^\circ$ using the Saclay-Lyon A model. The comparison illustrates a predictive power of calculations performed in the DWIA framework.
FIG. 2: Hyperon electro-magnetic-production kinematics in the laboratory frame. Real or virtual photons are assumed.

FIG. 3: Momentum transfer to the Λ hyperon is plotted as a function of projectile laboratory momentum $P_{\text{in}}$. The two curves for each reaction correspond to the two values of the kaon (pion) angle $\theta^L = 0^\circ$ (lower curves) and $\theta^L = 10^\circ$ (upper curves).
FIG. 4: Laboratory (a) and center of mass (b) cross sections are plotted as a function of the photon laboratory energy at fixed kaon angles. Predictions of the models KMAID, AS1, C4, and SLA (see the text) are compared with experimental data. In the laboratory frame (a) only the theoretical curves are shown. The data are from Refs. [44] (solid circles), [45] (circles), [39] (triangles), and [40] (squares).
FIG. 5: The same as in Fig. 4 but for the angular dependence at $E_\gamma = 1.3$ GeV.

FIG. 6: Polarization of $\Lambda$ is plotted as a function of the photon laboratory energy at $90^\circ$ in the $\gamma p \rightarrow A K^+$ reaction. Results of the four theoretical models (see the text) are compared with experimental data. The data are from Refs. [44] and [46] (solid circles), [39] (triangles), and [40] (squares).
FIG. 7: The same as in Fig. 6 but for the angular distribution at $E_L^\gamma = 1.3$ GeV. The data are from Refs. [44] (solid circles), [45] (empty circles), [39] (triangles), and [40] (squares).

FIG. 8: Divided contributions to the particle-hole $J$-multiplet state $[j_p^{-1}j_A^\Lambda]_J$ as calculated for the $^{28}$Si$(\gamma,K^+)^{28}$Al reaction at $E_\gamma = 1.3$ GeV and $\theta_K^c = 3^\circ$. The DDHF wave functions are used with the Saclay-Lyon A amplitude [29]. The height of each pillar corresponds to the differential cross section, while the width has no special meaning.
FIG. 9: Calculated angular distributions of the states excited strongly in $^{28}\text{Si}(\gamma, K^+)^{28}\text{Al}$ reaction at $E_\gamma = 1.3$ GeV. The excited states denoted with $J$ correspond to the $[j_p^{-1} j^\Lambda]_J$ multiplets shown in Fig. 8.

FIG. 10: Theoretical excitation function calculated with the full $(sd)^n$ wave functions for the $^{28}\text{Si}(\gamma, K^+)^{28}\text{Al}$ reaction at $E_\gamma = 1.3$ GeV and $\theta_K = 3^\circ$ using the Saclay-Lyon A model. For simplicity to draw pronounced doublet peaks, the artificial $ls$-splitting is introduced as 0.17($2l+1$) in MeV, so that in actual case such multiplet may be seen as a degenerate one. The hypernuclear energy is measured from the $^{27}\text{Al}(\text{g.s.})+\Lambda$ threshold, so it is expressed in terms of the hyperon energy $E_\Lambda$.  

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FIG. 11: Excitation function for the $^{40}\text{Ca}(\gamma,K^+)_{\Lambda}^{40}\text{K}$ reaction calculated in DWIA at $E_\gamma = 1.3$ GeV and $\theta_K^L = 3^\circ$ using the Saclay-Lyon A model. For simplicity to draw pronounced doublet peaks, the artificial $ls$-splitting is introduced as $0.17(2l+1)$ in MeV. In actual case such multiplet may be seen as a degenerate one. The hypernuclear energy is measured from the $^{39}\text{K(g.s.)}+\Lambda$ threshold, so it is expressed in terms of the hyperon energy $E_\Lambda$.

FIG. 12: Excitation function for the $^{52}\text{Cr}(\gamma,K^+)_{\Lambda}^{52}\text{V}$ reaction calculated in DWIA at $E_\gamma = 1.3$ GeV and $\theta_K^L = 3 \text{ deg}$ using the C4 model. For simplicity to draw pronounced doublet peaks, the artificial $ls$-splitting is introduced as $0.17(2l+1)$ in MeV. In actual case such multiplet may be seen as a degenerate one. See text for gray blocks. The hypernuclear energy is measured from the $^{51}\text{V(g.s.)}+\Lambda$ threshold, so it is expressed in terms of the hyperon energy $E_\Lambda$. 