\[ \Lambda - \Sigma^0 \textit{mixing in finite nuclei} \]

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(February 9, 2008)

Abstract

Properties of hypernuclei are studied in the context of a chiral Lagrangian which successfully describes ordinary nuclei. \( \Lambda - \Sigma^0 \) mixing arises from non-diagonal vertices in flavor space induced by the vector mesons and by the electromagnetic field. The set of Dirac equations for the coupled hyperon system is discussed. Results are presented for energy spectra and electromagnetic properties of hyperons in the nuclear environment. Simple estimates suggest that flavor mixing can lead to sizable changes in the lifetimes of \( \Lambda \) hypernuclei.
I. INTRODUCTION

Hyperons embedded in the nuclear medium provide an unique tool for studying the elementary nucleon-hyperon and hyperon-hyperon interaction. On the experimental side, physics of hypernuclei has entered a phase in which not only ground state energies but also excitation spectra and decay properties are being measured \cite{1}. Theoretically, binding energies and single particle spectra of hypernuclei are studied in a variety of nonrelativistic \cite{2} and relativistic \cite{3,4} mean-field models. The relativistic models usually involve the baryon octet and several strange \cite{5–8} and nonstrange mesons.

The concepts and methods of effective field theory (EFT) have lead to new insights into relativistic mean field models for nuclear structure \cite{9,10}. For ordinary nuclei a chiral effective Lagrangian has been constructed which successfully reproduces low-energy nuclear phenomenology \cite{9}. One important implication is that all interaction terms that are consistent with the underlying symmetries of QCD should be included.

Recently, this approach was generalized and applied to strange nuclear matter \cite{8}. It was demonstrated that a framework which includes the most general types of interactions leads to new and interesting many-body effects. Most prominently, D-type couplings between baryons and vector mesons give rise to \(\Lambda - \Sigma^0\) flavor mixing. In the present work we extend the analysis to finite systems and include various tensor couplings and couplings to the electromagnetic field which are relevant in hypernuclei. Following Ref. \cite{9}, the low-energy electromagnetic structure of the hyperons is described within the effective Lagrangian approach so that no external form factors are needed.

The central point in the present discussion is the analysis of \(\Lambda - \Sigma^0\) flavor mixing. The primary result is that a hypernucleus is generally in a state of mixed flavor rather than in a state with distinct \(\Lambda\) and \(\Sigma^0\) particles. A particle interpretation is only possible in terms of the actual energy eigenstates which are a superposition of the flavor eigenstates. As a consequence, systems which contain \(\Lambda\) hyperons always have a small admixture of \(\Sigma^0\) hyperons, and vice versa. The physical nature of this effect is similar to the recently much discussed neutrino oscillations \cite{11}.

Flavor mixing is mainly driven by the mean field of the \(\rho\) meson and signatures of the effect are more likely to be observed in very heavy asymmetric nuclei. We therefore focus on hypernuclei that consist of a \(^{208}\text{Pb}\) core and examine single particle spectra, electromagnetic properties and lifetimes. The basic result of our analysis is that the flavor mixing is a rather small effect. The shifts in the single-particle spectra are well below typical hypernuclear spin-orbit splittings. A more direct signature of the mixing are deviations of hypernuclear magnetic moments from their single-particle values. Our results indicate changes at the 2\% level. Because of the hyperon mass differences, \(\Lambda\) and \(\Sigma^0\) hypernuclei are characterized by strikingly different lifetimes and widths which can be as much as twelve orders of magnitude apart. Although the admixture of a \(\Sigma^0\) in a \(\Lambda\) hypernucleus is very small, the dramatic difference in the decay width leads to sizable effects. Based on simple assumptions we show that the width of the \(\Lambda\) in excited states can be significantly enhanced by flavor mixing.

The outline of this paper is as follows: In Sec. \[\text{II}\] we present the relevant part of the effective Lagrangian. Section \[\text{III}\] contains a discussion of the Dirac equation which governs the coupled hyperon system. In Sec. \[\text{IV}\] we discuss the impact of the flavor mixing on specific properties of hypernuclei. Section \[\text{V}\] contains a short summary.
II. THE EFFECTIVE INTERACTION

The model is based on an effective Lagrangian that realizes chiral symmetry and vector meson dominance. In the nucleon sector this approach has been successful in describing the properties of ordinary nuclei [9,10]. More recently, these ideas have been generalized to describe strange nuclear matter [8]. In our contribution we will extend this analysis and develop new aspects which arise in strange finite systems. Most of the material can be found in Ref. [8] and we will quote only the new ingredients relevant for hypernuclei.

The effective degrees of freedom are the baryon octet and the vector meson nonet. The baryons and the vector meson octet are collected in $3 \times 3$ traceless hermitian matrices

$$B = \begin{pmatrix} \frac{1}{\sqrt{6}} \Lambda + \frac{1}{\sqrt{2}} \Sigma^0 & \Sigma^+ & p \\ \Sigma^- & \frac{1}{\sqrt{6}} \Lambda - \frac{1}{\sqrt{2}} \Sigma^0 & n \\ \Xi^- & \Xi^0 & \frac{2}{\sqrt{6}} \Lambda \end{pmatrix}, \tag{1}$$

$$V_\mu = \begin{pmatrix} \frac{1}{\sqrt{6}} V^8_\mu + \frac{1}{\sqrt{2}} \rho^0_\mu \\ \rho^-_\mu \end{pmatrix} \begin{pmatrix} \rho^+_\mu \\ \frac{1}{\sqrt{6}} V^8_\mu - \frac{1}{\sqrt{2}} \rho^0_\mu \\ K^{*+}_\mu \end{pmatrix}. \tag{2}$$

The physical $\omega$ and $\phi$ mesons arise from the mixing of the $V^8_\mu$ and the vector meson singlet $S_\mu$ via

$$\omega_\mu = \cos(\theta) S_\mu + \sin(\theta) V^8_\mu, \quad \phi_\mu = \sin(\theta) S_\mu - \cos(\theta) V^8_\mu. \tag{3}$$

We also include a light isoscalar scalar meson $\varphi$ which simulates the exchange of correlated pions and kaons.

The couplings of the vector mesons to the baryons are contained in

$$L_{BMB} = -g_F \text{Tr}(\overline{B}[\mathcal{V}, B]) - g_D \text{Tr}(\overline{B}\{\mathcal{V}, B\}) - g_S \text{Tr}(\overline{B}S B)$$

$$- \frac{f_F}{4M} \text{Tr}(\overline{B}[\sigma_{\mu\nu} \mathcal{V}^{\mu\nu}, B]) - \frac{f_D}{4M} \text{Tr}(\overline{B}\{\sigma_{\mu\nu} \mathcal{V}^{\mu\nu}, B\}) - \frac{f_S}{4M} \text{Tr}(\overline{B}\sigma_{\mu\nu} \mathcal{S}^{\mu\nu} B). \tag{4}$$

Couplings to the electromagnetic field are introduced by

$$L_{EM} = -e \text{Tr}(\overline{B}[QA, B]) - e \frac{\mu_D}{4M} \text{Tr}(\overline{B}\{QA, B\}) - e \frac{\mu_F}{4M} \text{Tr}(\overline{B}QA B)$$

$$+ e \frac{\beta_D}{2M^2} \text{Tr}(\overline{B}\gamma^\nu \partial^\mu F_{\mu\nu}[Q, B]) + e \frac{\beta_F}{2M^2} \text{Tr}(\overline{B}\gamma^\nu \partial^\mu F_{\mu\nu}[Q, B]), \tag{5}$$

where $Q = \text{diag}\{2/3, -1/3, -1/3\}$ is the quark charge matrix. Combined with vector meson dominance, the Lagrangian Eq. (3) describes the low energy electromagnetic structure of the baryons so that external form factors are not needed. For example, the tensor couplings generate the magnetic moments of the baryons. In the vacuum Eq. (3) implies the Coleman and Glashow [12] relations,
\[
\frac{1}{2} \mu_n = \mu_A = -\mu_{\Sigma^0} = -\frac{1}{\sqrt{3}} \mu_{\Lambda \Sigma^0} .
\] (6)

To constrain the couplings we follow closely Ref. [8]. For the meson-baryon couplings we assume \( SU(3) \) symmetry and that the OZI rule holds, \( i.e. \) the couplings between nucleons and the \( \phi \) meson vanish. Furthermore, relation Eq. (3) is implemented with the ideal mixing angle \( \sin(\theta) = 1/\sqrt{3} \). For given values of the corresponding \( \omega \) and \( \rho \) couplings to the nucleons, the set of parameters \( (g_F, g_D, g_S, f_F, f_D, f_S) \) is then fixed \( (M \) is taken to be the nucleon mass).

The Coleman and Glashow relations Eq. (3) hold only in the strict \( SU(3) \) limit. The physical values of the magnetic moments can be generated by adding appropriate symmetry breaking terms. We use the Particle Data Group \([13]\) values

\[
\mu_\Lambda = -0.613 \quad , \quad \mu_{\Lambda \Sigma^0} = 1.61 .
\] (7)

For the magnetic moment for the \( \Sigma^0 \), which is experimentally not accessible, we employ the result of the chiral perturbation theory calculation in Ref. [14]

\[
\mu_{\Sigma^0} = 0.65 .
\] (8)

The parameters \( \beta_F \) and \( \beta_D \) contribute to the charge radii of the baryons which are not known in the hyperon sector. For simplicity, we assume \( SU(3) \) symmetry which determines the parameters from the corresponding values in the nucleon sector. To specify the coupling of the \( \Lambda \) to the scalar field \( \varphi \) we fit our model to reproduce the lowest \( \Lambda \) level in \( ^{17}\Lambda O \) which can be extracted from the experimental results in Ref. [15]. For the \( \Sigma^0 \) we follow the phenomenological approach of Refs. [8,16] and require that the coupling reproduces the hyperon potential in nuclear matter which is taken to be

\[
U^\Sigma = g_\Sigma^\varphi \varphi - g_\Sigma^\omega \omega^0 = 25\text{MeV} .
\] (9)

In the nucleon sector we employ the parameter set G1 of Ref. [4]. The corresponding hyperon couplings are listed in Table [4].

**III. \( \Lambda - \Sigma^0 \) MIXING IN FINITE NUCLEI**

We consider \( \Lambda, \Sigma^0 \) hypernuclei in a mean field approximation based on the interaction terms introduced in the last section. We are primarily interested in studying the new features arising from the flavor mixing. As a simple approximation we neglect the response of the nuclear core to the hyperons so that the hyperons can be treated separately. As the first step in this valence-hyperon approximation we perform the calculation for the nuclear core and substitute the resulting meson mean fields in the Dirac equations that govern the hyperons. To maintain rotational invariance we consider doubly magic nuclei.

The nondiagonal terms in flavor space given in Eq. (4) and Eq. (5) mix the \( \Lambda \) and \( \Sigma^0 \). The wave function of the combined system can be written as

\[
\Psi(r, t) = e^{-iEt} \begin{pmatrix} \Psi_\Lambda(r) \\ \Psi_\Sigma(r) \end{pmatrix} ,
\] (10)
with
\[ \Psi_{\Lambda,\Sigma}(r) = \begin{pmatrix} \frac{iG_{n\kappa}^{\Lambda,\Sigma}(r)}{r} \mathcal{Y}_{\kappa m} \\ -\frac{F_{n\kappa}^{\Lambda,\Sigma}(r)}{r} \mathcal{Y}_{-\kappa m} \end{pmatrix}, \tag{11} \]

where \( n \) is the principal quantum number and \( \mathcal{Y}_{\kappa m} \) is a spin-\( \frac{1}{2} \) spherical harmonic. The nonzero integer \( \kappa \) determines \( j \) and \( l \) through \( \kappa = (2j + 1)(l - j) \). The separation leads to a coupled set of equations for the radial wave functions:

\[ \Delta^+_{\Lambda} G^\Lambda + \left[ \frac{d}{dr} - \frac{\kappa}{r} + T_{\Lambda} \right] F^\Lambda = V_{\Lambda\Sigma} G^\Sigma - T_{\Lambda\Sigma} F^\Sigma \tag{12} \]
\[ \Delta^+_{\Sigma} F^\Sigma - \left[ \frac{d}{dr} + \frac{\kappa}{r} - T_{\Sigma} \right] G^\Sigma = V_{\Lambda\Sigma} F^\Lambda - T_{\Lambda\Sigma} G^\Lambda \tag{13} \]
\[ \Delta^\pm_{\Sigma} F^\Sigma - \left[ \frac{d}{dr} + \frac{\kappa}{r} - T_{\Sigma} \right] G^\Sigma = V_{\Lambda\Sigma} F^\Lambda - T_{\Lambda\Sigma} G^\Lambda \tag{14} \]
\[ \Delta^\pm_{\Lambda} G^\Lambda + \left[ \frac{d}{dr} + \frac{\kappa}{r} - T_{\Lambda} \right] F^\Lambda = V_{\Lambda\Sigma} G^\Sigma - T_{\Lambda\Sigma} F^\Sigma \tag{15} \]

where we have introduced the notation
\[ \Delta^\pm_{\Lambda,\Sigma} = E - V_{\Lambda,\Sigma} \pm (M_{\Lambda,\Sigma} - g_{\Lambda,\Sigma} \varphi). \tag{16} \]

The individual vector and tensor potentials are listed in Table II. Bound state solutions are normalized according to
\[ \int_0^\infty dr (|G^\Lambda|^2 + |F^\Lambda|^2) + \int_0^\infty dr (|G^\Sigma|^2 + |F^\Sigma|^2) \equiv N_\Lambda + N_\Sigma = 1, \tag{17} \]
so that the system contains one strange baryon. For bound states we seek solutions that are regular at the origin and that fall off sufficiently fast at infinity. The behavior at the origin can be studied by assuming constant values of the vector and scalar potentials and zero values for the tensor terms. In this case, the set of equations \( \text{(12)} \) is easily solved by writing
\[ G^{\Lambda,\Sigma} = g^{\Lambda,\Sigma} r j_l(\alpha r), \quad F^{\Lambda,\Sigma} = f^{\Lambda,\Sigma} r j_{l\pm 1}(\alpha r), \tag{18} \]
with \( l + 1 \) for \( \kappa < 0 \) and \( l - 1 \) for \( \kappa > 0 \). Substituting Eq. \( \text{(18)} \) into Eq. \( \text{(12)} \) leads to a set of algebraic equations for the unknown coefficients \( g^{\Lambda,\Sigma} \) and \( f^{\Lambda,\Sigma} \). Solutions exist only if the condition
\[ \alpha^4 - \alpha^2 (\Delta^+_{\Lambda} \Delta^-_{\Lambda} + \Delta^+_{\Sigma} \Delta^-_{\Sigma} + 2V^2_{\Lambda\Sigma}) + (\Delta^-_{\Lambda} \Delta^-_{\Sigma} - V_{\Lambda\Sigma}^2) (\Delta^+_{\Lambda} \Delta^+_\Sigma - V^2_{\Lambda\Sigma}) = 0, \tag{19} \]
is fulfilled. The roots of this equation constitute two qualitatively different solutions. A type of solutions which reduces to a pure \( \Lambda \) with \( g^\Sigma = f^\Sigma = 0 \) if the mixing is turned off, and a second type of solutions which reduces to a pure \( \Sigma^0 \) with \( g^\Lambda = f^\Lambda = 0 \). A general solution to the radial equations consists of a super position of the two types; the relative weight has to be determined numerically.
The radial equations decouple in the asymptotic regime $r \to \infty$ and, because there is no direct electromagnetic coupling, can be replaced by the corresponding free equations. The solutions are

$$G^{\Lambda,\Sigma}_{r \to \infty} = c^{\Lambda,\Sigma} \sqrt[r]{K_{l+1/2}(\beta^{\Lambda,\Sigma} r)} , \quad F^{\Lambda,\Sigma}_{r \to \infty} = -c^{\Lambda,\Sigma} \left[ r \frac{M_{\Lambda,\Sigma} - E}{M_{\Lambda,\Sigma} + E} \right]^{1/2} K_{l \pm 1/2}(\beta^{\Lambda,\Sigma} r) ,$$

with $\beta^{\Lambda,\Sigma} = \sqrt{M_{\Lambda,\Sigma}^2 - E^2}$. The asymptotic coefficients $c^{\Lambda}$ and $c^{\Sigma}$ together with the relative weight at the origin and the energy variable $E$ constitute four unknowns which may be determined by matching the two large and two small radial wave functions at some matching radius.

The energy spectrum depends on the strength of the mixing potentials and on the mass difference of the two hyperons. When mixing is suitably weak as here, there are two qualitative different sets of solutions. Solutions dominated by the $\Lambda$ part of the wave function ($\Lambda$-like) that can be characterized by

$$N_\Lambda \gg N_\Sigma ,$$

and $\Sigma$-like solutions with

$$N_\Sigma \gg N_\Lambda .$$

Because the energy eigenstates contain both flavors, a hypernucleus is generally in a state of mixed flavor rather than in a state with distinct $\Lambda$ and $\Sigma^0$ particles. A (quasi) particle interpretation is only possible in terms of the actual energy eigenstates, $\Lambda$-like or $\Sigma$-like, which are not flavor eigenstates. A (time-dependent) flavor eigenstate can only arise as a superposition of energy eigenstates [8].

$\Lambda - \Sigma^0$ mixing is superficially similar to the phenomenon of neutrino flavor mixing which gives rise to neutrino oscillations. However, the origin of both effects is fundamentally different. Neutrino oscillations are assumed to occur in the vacuum arising from a nondiagonal mass matrix in flavor space which contains the vacuum mass parameters [11]. This effect can be appreciably enhanced when neutrinos pass through dense matter as predicted by the MSW effect [17]. In contrast, the $\Lambda - \Sigma^0$ mixing is a true many body effect arising from the nondiagonal self energies which are generated by the nuclear medium. As long as small isospin violations can be neglected, the vacuum self energies are diagonal in flavor space and the asymptotic states can be properly identified as the pure $\Lambda$ and $\Sigma^0$ flavor states.

**IV. RESULTS AND DISCUSSION**

**A. Ground state Properties**

Armed with an understanding of the general features of the solutions, we now turn to specific properties of hypernuclei. Flavor mixing is primarily driven by the mean field of the neutral $\rho$ meson and the effect is very small in light and symmetric nuclei. To generate the mean fields we therefore consider a $^{208}$Pb core nucleus.
Strict Σ-like bound states only exist if the mass difference between the two hyperons is small. In the chiral limit with equal Λ and Σ⁰ masses two series of bound states, Λ-like and Σ-like, arise. Increasing the mass of the Σ⁰ increases the energy of the states in the Σ-like part of the spectrum. At some point the energy of the most weakly bound Σ-like state exceeds the mass of the Λ, whereupon the parameter \( \beta^\Lambda = \sqrt{M^2_\Lambda - E^2} \) in Eq. (20) turns imaginary and the Λ part of the wave function has to be replaced by a continuum wave function. Using the physical values for the hyperon masses, we find that, for all Σ-like states, the Λ part of the wave function is unbound. As a consequence, only true bound states of Λ-like configurations exist.

The impact of the flavor mixing on the energy spectrum is rather small. The situation is illustrated in Fig. 1. To make it easier to study the effect we have scaled the mixing potentials \( T_{\Lambda \Sigma} \) and \( V_{\Lambda \Sigma} \) by a factor \( \zeta \). Part (a) indicates the energy of the lowest Λ-like states. The mixing is attractive and tends to lower the binding energies. However, for the realistic situation \( \zeta = 1 \) the effect is very small, the energy levels decrease by less than 0.01 MeV. To set the proper scale this number has to be compared to typical hypernuclear spin-orbit splittings (∆E \( \lesssim 0.5 \))MeV that can be resolved experimentally. The parabolic shape of the curves reflects the fact that the mixing is a second order effect in perturbation theory. For the Λ the tensor couplings significantly reduce the spin-orbit potential which leads to the very small splittings of the \( (1p_{3/2}, 1p_{1/2}) \) and \( (1d_{5/2}, 1d_{3/2}) \) for zero and weak mixing. Part (b) of Fig. 1 indicates the values of \( N_\Lambda \) and \( N_\Sigma \) as defined in Eq. (17) for the lowest \( s_{1/2} \) state. Other states show very similar behavior. For the realistic situation \( \zeta = 1 \) the system consists primarily of a Λ with a very small admixture (0.003%) of the Σ⁰ flavor.

A direct indication of the flavor mixing is a nonvanishing mixed baryon density

\[
\rho_{\Lambda \Sigma} = <\bar{\Psi}_\Lambda \gamma^0 \Psi_\Sigma> + <\bar{\Psi}_\Sigma \gamma^0 \Psi_\Lambda>
\]

which is shown in Fig. 2 together with the pure flavor densities

\[
\rho_{\Lambda, \Sigma} = <\bar{\Psi}_{\Lambda, \Sigma} \gamma^0 \Psi_{\Lambda, \Sigma}>
\]

Although only 1% of the size of the Λ density, the mixed density is much bigger than the Σ density. This is because \( \rho_{\Lambda \Sigma} \) arises from the amplitude for a Σ⁰ admixture whereas \( \rho_\Sigma \) arises from the Σ⁰ probability.

We now turn to electromagnetic properties. In the ΛΣ⁰ sector the electromagnetic current obtained from Eq. (5) is given by

\[
J_{EM}^\mu = -\frac{\mu_\Lambda}{2M} \partial_\nu \left( \bar{\Psi}_\Lambda \sigma^{\nu \mu} \Psi_\Lambda \right) - \frac{\mu_\Sigma}{2M} \partial_\nu \left( \bar{\Psi}_\Sigma \sigma^{\nu \mu} \Psi_\Sigma \right) - \frac{\mu_{\Lambda \Sigma}}{2M} \partial_\nu \left( \bar{\Psi}_\Lambda \sigma^{\nu \mu} \Psi_\Sigma + \bar{\Psi}_\Sigma \sigma^{\nu \mu} \Psi_\Lambda \right)
\]  

\[
+ \frac{\beta_D}{6M^2} \partial^2 \left( \bar{\Psi}_\Lambda \gamma^\mu \Psi_\Lambda - \bar{\Psi}_\Sigma \gamma^\mu \Psi_\Sigma \right) - \frac{\beta_D}{2\sqrt{3}M^2} \partial^2 \left( \bar{\Psi}_\Lambda \gamma^\mu \Psi_\Sigma + \bar{\Psi}_\Sigma \gamma^\mu \Psi_\Lambda \right),
\]

where we have disregarded terms that do not contribute in the mean field approximation. The corresponding charge density can be decomposed according to the flavor indices. This is illustrated in Fig. 3. Indicated are two states with total angular momentum \( j = 1/2 \) for which the charge density is radially symmetric. Part (a) shows the various contributions for a \( 1s_{1/2} \) state. The charge density is dominated by the Λ; the contribution of the Σ⁰ is again rather small. The mixed density leads to a sizable increase near the origin. Part (b) shows the same situation for the \( 1p_{1/2} \) state. Here the mixed density contribution is less important.
We can determine the magnetic moments of the hypernucleus from the current in Eq. (25). The Coleman-Glashow relations in Eq. (6) suggest that the mixing leads to deviations of the moments from the pure flavor values [18]. For the magnetic moments, the mixing induces changes which are of first order. The impact is therefore much greater than for the energy levels. This can be studied in Fig. 4 which indicates moments for different levels. Similar as in Fig. 1 we have scaled the mixing potentials by a factor \( \zeta \).

Changes with respect to the single particle value are mainly induced by the transition moment \( \mu_{\Lambda\Sigma} \) decreasing the size of the moments for the \( \Lambda \)-like states. For very large mixing the moments even change sign. However, in the relevant region (\( \zeta = 1 \)) the deviations of the moments from their pure flavor values are only \( \approx 2\% \). In our simple valence-hyperon approximation the magnetic moments are very close to the corresponding nonrelativistic Schmidt values. As demonstrated in [20,21] this remains true when the response of the nuclear core is taken into account. This is in contrast to the nucleon case where the core response has to be included properly to restore the magnetic moments to their Schmidt values [22].

Although there has been no measurement of hypernuclear magnetic moments, proposals have been made for future experiments [23]. Our results indicate that very precise measurements are necessary in order to find signatures of the flavor mixing. For comparison, the experimental uncertainties of \( \Lambda \) magnetic moments in free space are of the order \( \Delta \mu_{\Lambda} < \sim 1\% \) [13].

B. Continuum Solutions

As mentioned earlier, there are no bound \( \Sigma \)-like solutions for the physical values of the hyperon masses. However, signatures of bound pure \( \Sigma^0 \) states, which arise when the mixing is turned off, can be found as resonances. In the energy range \( M_\Lambda \leq E \leq M_\Sigma \), the asymptotic form for the \( \Lambda \) part of the wave function in Eq. (20) has to be replaced by

\[
\begin{pmatrix}
G^\Lambda \\ F^\Lambda
\end{pmatrix}_{r \to \infty} \propto \alpha_j^\Lambda r^{j_l(\beta_\Lambda r)} \left( \mp \frac{E-M_\Lambda}{E+M_\Lambda} \right)^{1/2} y_{l\pm 1}(\beta_\Lambda r) + \alpha_y^\Lambda r^{y_l(\beta_\Lambda r)} \left( \mp \frac{E-M_\Lambda}{E+M_\Lambda} \right)^{1/2} y_{l\pm 1}(\beta_\Lambda r),
\]

with \( \beta_\Lambda = \sqrt{E^2 - M_\Lambda^2} \). The numerical coefficients \( \alpha_j^\Lambda, \alpha_y^\Lambda \) determine the phase shifts \( \delta^\Lambda \) and are calculated by matching the wave functions similar as discussed in Sect. [11] for the bound states. The function \( \sin^2(\delta^\Lambda) \) for a 1p3/2 state is indicated in Fig. 5. Two bound states which arise for a pure \( \Sigma^0 \) flavor lead to very narrow resonances at \( E \approx 1174.16\text{MeV} \) and \( E \approx 1184.64\text{MeV} \). The complete hyperon spectrum for the \( ^{208}\text{Pb} \) core nucleus is indicated in Fig 6. On the left-hand side are the truly bound \( \Lambda \)-like states and on the right-hand side the resonance energies of the \( \Sigma \)-like configurations. The figure nicely displays the quite different character of the spin-orbit force [19] for the two flavors. For the \( \Lambda \)-like states it leads to a sizable reduction of the splittings and an increase for the \( \Sigma \)-like states.

For energies \( E \geq M_\Sigma \) two physical different situations can be realized. \( \Lambda \)-like scattering states characterized by the asymptotic form given in Eq. (26) and by

\[
\begin{pmatrix}
G^\Sigma \\ F^\Sigma
\end{pmatrix}_{r \to \infty} \propto \gamma_y^\Sigma r^{y_l(\beta_\Sigma r)} \left( \mp \frac{E-M_\Sigma}{E+M_\Sigma} \right)^{1/2} y_{l\pm 1}(\beta_\Sigma r),
\]
for the $\Sigma^0$ part of the wave function. Eq. (24) together with Eq. (27) describes an incoming pure $\Lambda$ scattered off the target nucleus. In addition to the $\Lambda$, the outgoing wave function also contains a (small) contribution of the $\Sigma^0$, indicating a possible transition between a $\Lambda$ and a $\Sigma^0$ during the collision. Similarly, $\Sigma$-like scattering states can be described by interchanging the flavor indices in Eq. (26) and in Eq. (27).

In our simple mean field picture the $\Lambda$ and $\Sigma^0$ are treated on an equal footing. In a more realistic approach various decay mechanisms which lead to significantly different properties of $\Lambda$ and $\Sigma$ hypernuclei have to be taken into account. Hypernuclei are typically produced through hadronic reactions such as ($K^\pm, \pi^\pm$). Eventually, they will decay through nonleptonic weak processes which involve the emission of pions or nucleons. For $\Lambda$-hypernuclei the dominant process is the nonmesonic decay mode $\Lambda N \rightarrow NN$ which leads to lifetimes comparable to the lifetime of a free $\Lambda$. Properties of $\Sigma$-hypernuclei are still controversial. Complications arise from the strong $\Sigma N \rightarrow \Lambda N$ conversion process, which leads to widths in the excitation spectrum of several MeV. In a realistic scenario this process completely dwarfs the resonances indicated in Fig 5 with $\Gamma \approx 1$keV. Because of the strikingly different scales set by the $\Lambda$ and $\Sigma^0$ decay widths it is interesting to examine the impact of the flavor mixing on lifetimes and width of hypernuclei. Let us assume that the decay of the $\Lambda$ and $\Sigma^0$ can be described by adding appropriate optical potentials [24] to our mean field model. The mixing potentials are very small and we can use perturbation theory. To first order a $\Lambda$ like solution can be written as

$$\Psi = \left( \begin{array}{c} \psi^0_\Lambda \\ C_{\Lambda \Sigma} \psi^0_\Sigma \end{array} \right),$$

(28)

where the wave functions are normalized pure flavor solutions with complex energies

$$E^0_\Lambda = E^0_\Lambda - \frac{i}{2} \Gamma^0_\Lambda, \quad E^0_\Sigma = E^0_\Sigma - \frac{i}{2} \Gamma^0_\Sigma.$$

(29)

To second order the width is then given by

$$\Gamma_\Lambda = (1 - |C_{\Lambda \Sigma}|^2) \Gamma^0_\Lambda + |C_{\Lambda \Sigma}|^2 \Gamma^0_\Sigma.$$

(30)

The coefficient $C_{\Lambda \Sigma}$ which characterizes the admixture of the $\Sigma^0$ can be estimated by

$$|C_{\Lambda \Sigma}|^2 \approx \frac{N_\Sigma}{N_\Lambda},$$

(31)

with the norms introduced in Eq. (17). Using typical numbers [11] for the in-medium width $\Gamma^0_\Sigma \approx 10^{12} \Gamma^0_\Lambda$ this leads to the rough estimate

$$\Gamma_\Lambda \approx 10^7 \Gamma^0_\Lambda.$$

(32)

Thus, flavor mixing significantly decreases the weak lifetimes of $\Lambda$-like hypernuclear systems. The resonances populated in hypernuclear reactions are often highly excited states which may decay by electromagnetic processes prior to their weak decays. As argued in Ref. [18] the corresponding lifetimes can also be modified by the flavor mixing. Electromagnetic decays are typically much slower than the $\Sigma N \rightarrow \Lambda N$ conversion. However, they are of the
same order of magnitude for Λ and Σ states and, according to Eq. (30), flavor mixing can only lead to very small modifications.

The fact that level mixing can change the lifetime of states is a well known phenomenon. For example, isospin mixing in $^{12}\text{C}$ has an appreciable effect on decay rates and form factors [25,26]. At this point, however, some caveats must be added. The optical potentials, i.e. the imaginary part of the self energies, are strongly energy dependent and one can expect that Eq. (32) is only applicable for excited and continuum Λ-like states. This conclusion is based on the requirement that if all the decay channels of the Λ are turned off, the system must have a stable Λ-like ground state.

At present, the understanding of hypernuclear decay is still at a primitive stage. Theoretical predictions for details of the dominant nonmesonic decay are not compatible with experimental data (see, e.g., Refs. [27,28]). Experimental studies of relatively light hypernuclei lead to weakly mass-dependent lifetimes, slightly smaller than the lifetime of a free Λ [29] in contrast to our simple estimate in Eq. (32). However, the experiments typically average over the lowest lying states in the spectrum where we expect the lifetimes not to be effected by the flavor mixing. Although our discussion is an oversimplification of the problem we believe it is useful for providing a first orientation. A more rigorous calculation of lifetimes has to include the various decay channels and the flavor mixing in a self-consistent manner. This will be an important topic for future investigations.

V. SUMMARY

In this paper we study $\Lambda - \Sigma^0$ flavor mixing in hypernuclei. Our analysis is based on a chiral effective Lagrangian containing the baryon octet, the vector meson nonet and a light scalar singlet. We extend the nuclear matter analysis of Ref. [8] by adding various tensor couplings and couplings to the electromagnetic field which are relevant in finite systems. The electromagnetic structure of the hyperons is described within the theory. As a consequence, electromagnetic properties of hypernuclei can be calculated without introducing external form factors [9].

The vector coupling constants are related to the corresponding couplings in the non-strange sector via $SU(3)$ symmetry. We employ a parameter set which has been obtained in a fit to properties of normal nuclei [9]. The scalar hyperon couplings are determined by using phenomenological information on energy levels in Λ hypernuclei and on the $\Sigma^0$ potential in nuclear matter.

The most important feature of the model is that D-type couplings between baryons and mesons lead to a nondiagonal self-energy in the $\Lambda - \Sigma^0$ sector of flavor space. As a consequence $\Lambda - \Sigma^0$ flavor mixing arises.

We discuss the coupled set of Dirac equations that govern the hyperons. The solutions characterize a hypernucleus as a state of mixed flavor, in contrast to the familiar description with distinct $\Lambda$ and $\Sigma^0$ particles. Since the mixing is relatively small, two qualitatively different types of solutions arise, each dominated by a particular flavor. However, this implies that systems which contain $\Lambda$ hyperons always have a small admixture of $\Sigma^0$ hyperons, and vice versa.

To search for observable signatures of the effect, we study a $^{208}\text{Pb}$ nucleus with one strange baryon added. Because of the large mass difference of the hyperons, only $\Lambda$-like
states are truly bound. The $\Lambda$ part of the wave function which describes $\Sigma$-like states is a
continuum wave function. These quasi-bound states can be found as very sharp resonances
in scattering phase shifts.

We study the energy spectrum and electromagnetic properties of the hyperons embed-
ded in the nuclear medium. The impact of the flavor mixing on the energy levels is very
small, much smaller than typical hypernuclear spin-orbit splittings. The effect is more pro-
nounced for electromagnetic properties. The $\Lambda\Sigma$ transition moment leads to deviations of
hypernuclear moments from their single particle values at the 2% level.

We also examine the impact of the flavor mixing on lifetimes of hypernuclei. Based on
simple assumptions we find that the lifetimes of excited $\Lambda$-like hypernuclei can be decreased
significantly. Although, the discussion is rudimentary it suggests that a more rigorous ex-
amination of lifetimes, experimentally and theoretically, could reveal signatures of $\Lambda - \Sigma^0$
mixing in strange nuclear systems.

ACKNOWLEDGMENTS

This work was supported in part by U.S.DOE under Grant No. DE-FG03-93ER-40774.
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TABLES

TABLE I. Hyperon coupling constants.

|       |       |       |
|-------|-------|-------|
| $g_\omega^\Lambda$ | 7.9125 |
| $g_\omega^\Sigma$ | 6.0381 |
| $f_\omega^\Lambda$ | -4.7597 |
| $g_\omega^\Xi$ | 8.2582 |
| $g_\omega^\Sigma$ | 6.0753 |
| $f_\omega^\Xi$ | 11.739 |
| $g_\rho^\Lambda\Sigma$ | 0.29942 |
| $f_\rho^\Lambda\Sigma$ | 14.288 |
| $\beta_D$ | -0.41754 |

TABLE II. Vector and tensor potentials. $V^0$ and $b^0$ are the time like component of the $\omega$ and the $\rho^0$ mean field respectively, and $A^0$ is the Coulomb potential.

|       |       |       |
|-------|-------|-------|
| $V_\Lambda$ | $g_\omega^\Lambda V^0 - \frac{\beta_D}{6M^2} \Delta A^0$ |
| $V_\Sigma$ | $g_\omega^\Sigma V^0 + \frac{\beta_D}{6M^2} \Delta A^0$ |
| $V_{\Lambda\Sigma}$ | $g_\omega^\rho_{\Lambda\Sigma} b^0 + \frac{\beta_D}{2\sqrt{3}M^2} \Delta A^0$ |
| $T_\Lambda$ | $\frac{1}{2M} (f_\omega^\Lambda V^0 + \mu_\Lambda A^0)$ |
| $T_\Sigma$ | $\frac{1}{2M} (f_\omega^\Sigma V^0 + \mu_\Sigma A^0)$ |
| $T_{\Lambda\Sigma}$ | $\frac{1}{2M} (f_\rho^{\Lambda\Sigma} b^0 + \mu_{\Lambda\Sigma} A^0)$ |
FIGURES

FIG. 1. (a) Binding energies of several Λ like states. The mixing potentials are scaled by $\zeta$. (b) Flavor fractions of the $1s_{1/2}$ state.

FIG. 2. Pure flavor and mixed flavor baryon densities. (a) shows a $1s_{1/2}$ state and (b) a $2s_{1/2}$ state.

FIG. 3. Λ, Σ and mixed contribution to the charge density. (a) shows a $1s_{1/2}$ state and (b) a $1p_{1/2}$ state.

FIG. 4. Magnetic moments for several Λ like states. The mixing potentials are scaled by $\zeta$.

FIG. 5. The function $\sin^2(\delta_\kappa)$ for a $1p_{3/2}$ state. The resonances at 1174.16 MeV and 1184.64 MeV are signatures of pure $\Sigma^0$ bound states which arise when the mixing is turned off.

FIG. 6. Λ-like and Σ-like spectrum for the $^{208}Pb$ core nucleus.
FIGURE 1
FIGURE 1
FIGURE 2
FIGURE 2
\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{A graph showing various density functions as functions of radial distance \(r\) in femtometers (fm). The labels denote different density contributions: 
- \(\rho^\Lambda_c\)
- \(\rho^\Sigma_c \times 500\)
- \(\rho^{\Lambda\Sigma}_c \times 2\)
- \(\rho^\Lambda_c + \rho^{\Sigma}_c + \rho^{\Lambda\Sigma}_c\)

At the right of the graph, \(1s_{1/2}\) is indicated.}
\end{figure}
FIGURE 3
FIGURE 4
FIGURE 5
FIGURE 6