Vector-based approaches for computing approximations in multigranulation rough set

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Abstract: Approximation computation is a significant issue when the rough set model is applied. However, few authors focus on how to calculate approximations of multigranulation rough set (MGRS). Herein, the authors clarify a fact that only a part of elements in the universe need to be judged whether they belong to approximations of MGRS. If \(X\) is a target concept which is approximated by approximations in MGRS, then the element whose equivalence class does not intersect with \(X\) is of no need to be judged. Based on the fact, the authors clarify that they proposed a vector-based algorithm to compute approximations in MGRS. Time complexity of the proposed algorithm is \(O(1|X|1|U|)\).

1 Introduction

Pawlak originally proposed a Rough set theory in 1980s and it is a powerful mathematical tool for characterising the uncertainty by the difference between lower and upper approximations. The rough set theory has been widely used in image processing [1, 2], machine learning [3–10], pattern recognition [11–18], data mining, and other relevant areas. However, multiple different types of attribute values appear in information systems in many real-world situations, e.g. missing ones, numerical ones, set-valued ones, and interval-valued ones. Classical rough set theory cannot be applied in analysing these data, means that classical rough set theory has some theoretical limitations. To overcome these limitations, a lot of extensions have been proposed such as covering-based rough set [19], which generalises rough set from the equivalence relation to the general binary relation, fuzzy rough set [20], which generalises rough set from the equivalence relation to fuzzy relation, and multigranulation rough set (MGRS) [21].

As we all know, Pawlak’s rough set is constructed by a single equivalence relation and that is too restrictive in many real-life applications. Multiple viewpoint has been used for many real application areas. In order to extend the application areas of rough set theory, Qian et al. [21] improved the theory and proposed a theory of MGRS, which includes optimistic and pessimistic lower and upper approximations. MGRSs are constructed by a family of attribute sets, which characterise different viewpoints. Approximation computation plays a significant role in applications of MGRS. However, since MGRSs have been proposed, few authors focus on designing a fast algorithm to compute approximations of MGRS. Hu et al. [22] proposed a matrix-based algorithm for computing approximation of MGRS which is much more efficient than naive algorithm. However, there is a defect in their algorithm which slows down the speed of the algorithm: all the elements in the universe need to be judged whether they belong to approximations of MGRS. If \(X\) is a target concept whose approximation in MGRS, then the element whose equivalence class does not intersect with \(X\) is of no need to be judged. Based on the fact, Hu’s authors clarify that they proposed a vector-based algorithm to compute approximations in MGRS. Time complexity of the proposed algorithm is \(O(1|X|1|U|)\).

2 Preliminaries

In this section, we review mainly the concepts in MGRSs.

2.1 Multigranulation rough sets

In the past decade, many extensions of MGRS have been proposed and since MGRS is our another basic model, we review its main results in this section.

Definition 1: Let \(IS = (U, AT, \chi_{AT}, f)\) be an information system, where \(U = \{x_1, x_2, \ldots, x_m\}\) is a non-empty finite set of the objects, the called the universe [1]. \(A = \{a_1, a_2, \ldots, a_r\}\) is a non-empty finite set of attributes. The element \(A \in AT\) is called an attribute set. \(\chi_{AT} = \bigcup_{A \in AT} V_A\) is a domain of attribute values, where \(V_A\) is the domain of attribute set \(A: f: U \times AT \to V\) is a decision function such that \(f:(x, A) \in V_A, \forall A \in AT, x \in U\).

Definition 2: Let \(IS = (U, AT, f)f\) be an information system, where \(A_k \in AT\) for any \(k \in \{1, 2, \ldots, m\}\), and \(\forall x \in U\) [21]. The optimistic multigranulation lower and upper approximation of \(X\) are denoted by \(\bigcup_{\ell=1}^m A_{\ell}^L(X)\) and \(\bigcup_{\ell=1}^m A_{\ell}^U(X)\), respectively

\[
\sum_{\ell=1}^m A_{\ell}^L(X) = \{x \in U|\{x_{\ell} \subseteq X : \forall \ell \in \{1, \ldots, m\}, \chi_{\ell} \subseteq X\}\}
\]

(1)

\[
\sum_{\ell=1}^m A_{\ell}^U(X) = \{x \in U|\{x_{\ell} \subseteq X : \forall \ell \in \{1, \ldots, m\}, \chi_{\ell} \subseteq X\}\}
\]

(2)
where \([x]_A_k\) is the equivalence class of \(x\) in terms of the attribute set \(A_k\). \(X\) is the complement of the set \(X\).

**Theorem 1**: Let \(IS = (U, AT, V_{AT, f})\) be an information system, where \(A_k \in AT\) for any \(k \in \{1, 2, ..., m\}\), and \(\forall X \subseteq U\). Since \([x]_{\sim A_k} \subseteq X \Leftrightarrow x \in X\), we have that

\[
\sum_{k=1}^{m} A^O_k(X) = \{x \in X | [x]_{\sim A_k} \subseteq X \lor \cdots \lor [x]_{\sim A_k} \subseteq X\},
\]

where \([x]_{\sim A_k}\) is the equivalence class of \(x\) in terms of the attribute set \(A_k\).

**Proof.** \(\forall x \in \sum_{k=1}^{m} A^O_k(X) \Rightarrow [x]_{\sim A_k} \in X(A \in AT)_k\), we have that \(\forall y \in [x]_{\sim A_k} \Rightarrow y \in X\), thus we have

\[
\sum_{k=1}^{m} A^O_k \subseteq \{x \in X | [x]_{\sim A_k} \subseteq X \land \cdots \land [x]_{\sim A_k} \subseteq X\};
\]

\(\Leftarrow \forall x \in \{x \in X | [x]_{\sim A_k} \subseteq X \land \cdots \land [x]_{\sim A_k} \subseteq X\} \Rightarrow x \in U\), since \([x]_{\sim A_k} \subseteq X\), we have that \(x \in \sum_{k=1}^{m} A^O_k(X)\), thus we have

\[
\{x \in X | [x]_{\sim A_k} \subseteq X \land \cdots \land [x]_{\sim A_k} \subseteq X\} \subseteq \sum_{k=1}^{m} A^O_k(X),
\]

this completes the proof. \(\Box\)

**Theorem 2**: Let \(IS = (U, AT, V_{AT, f})\) be an information system, where \(A_k \in AT\) for any \(k \in \{1, 2, ..., m\}\), and \(\forall X \subseteq U\).\([21]\) For the optimistic multigranulation upper approximation of \(X\), we have

\[
\sum_{k=1}^{m} A^U_k(X) = [x \in U | [x]_{\sim A_k} \cap X \neq \emptyset \land \cdots \land x \in U | [x]_{\sim A_k} \cap X \neq \emptyset]\}

\((4)\)

**Proof.** \(\forall x \in \sum_{k=1}^{m} A^U_k(X) \Rightarrow \forall k \leq m, [x]_{\sim A_k} \cap X \neq \emptyset \Rightarrow \forall y \in [x]_{\sim A_k} \cap X, \forall k \leq m\),

\[
[y]_{\sim A_k} \cap X \neq \emptyset \Rightarrow \exists \gamma \in X \cap ([x]_{\sim A_k}),\text{ s.t. } \forall k \leq m
\]

\[
\{[x]_{\sim A_k} \cap X \neq \emptyset \Rightarrow \sum_{k=1}^{m} A^U_k(X) = \{x \in U | [x]_{\sim A_k} \cap X \neq \emptyset \}.\}
\]

**Theorem 3**: Let \(IS = (U, AT, V_{AT, f})\) be an information system, where \(A_k \in AT\) for any \(k \in \{1, 2, ..., m\}\), and \(\forall X \subseteq U\). For the optimistic multigranulation upper approximation of \(X\), we have

\[
\sum_{k=1}^{m} A^U_k(X) = \cap \{[x]_{\sim A_k} | \\forall x \in X\}.
\]

**Proof.** \(\forall x \in \sum_{k=1}^{m} A^U_k(X) \Rightarrow \forall k \leq m, [x]_{\sim A_k} \cap X \neq \emptyset \Rightarrow \forall y \in [x]_{\sim A_k} \cap X, \forall k \leq m\),

\[
[y]_{\sim A_k} \cap X \neq \emptyset \Rightarrow \exists \gamma \in X \cap ([x]_{\sim A_k}),\text{ s.t. } \forall k \leq m
\]

\[
\{[x]_{\sim A_k} \cap X \neq \emptyset \Rightarrow \sum_{k=1}^{m} A^U_k(X) = \{x \in U | [x]_{\sim A_k} \cap X \neq \emptyset \}.\}
\]

3 **Vector-based algorithm for computing approximations in MGRS**

According to Hu's approach, all samples must participate in the computation process. However, Theorems 3 and 6 demonstrate that only part of the elements in the universe needs to be determined whether they belong to approximations or not. This inspired us to improve the algorithm to be more efficient.

The essential step of computing the approximations of MGRS is to judge an equivalence class, \([x]_{\sim A_k}\), for example, is contained in the target concept \(X\) or not. In addition, to set operation, there is a more efficient way which is introduced in [23].

**Definition 4**: [24] Let \(IS = (U, AT, V_{AT, f})\) be an information system, where \(A_k \in AT\) for any \(k \in \{1, 2, ..., m\}\), and \(\forall X \subseteq U\), the vector represent \(X\) is denoted as \(V(X) = [v_1(X), ..., v_m(X)]^T\) (\(T\) denotes the transpose operation, the same to \(\sim\)), where

\[
v_i(X) = \begin{cases} 1, & x_i \in X \\ 0, & x_i \notin X \end{cases} \quad i \in \{1, 2, ..., m\}
\]

**Lemma 1**: [23] Let \(IS = (U, AT, V_{AT, f})\) be an information system, where \(A_k \in AT\) for any \(k \in \{1, 2, ..., m\}\). \(U = \{x_1, x_2, ..., x_n\}\) \(\forall X, Y \subseteq U, \text{ if } Y \subseteq X\), then

\[
\sim (V^T(Y) \cdot \sim (V(X))) = 0.
\]
Corollary 1: Let IS = (U, AT, V_{AT}, f) be an information system, where A_k ∈ AT for any k ∈ {1, 2, ..., m}. U = \{x_1, x_2, ..., x_n\}. ∀X, Y ⊆ U, if Y ⊆ X, then
\[ V^f(\sim X) ⋅ V(Y) = 0. \]

Proof. This corollary can be easily obtained from Lemma 1. □

Example 1: Let IS = (U, AT, V_{AT}, f) be an information system, as shown in Table 1, where U = \{x_1, x_2, x_3, x_4, x_5\}, B = A ∪ d, and A = \{a_1, a_2, a_3\}. Let X = \{x_1, x_2, x_3\}. According to Definition 4, we have \[ V(X) = [0, 1, 1, 1, 0, 0]^T. \] Suppose Y = \{x_2, x_3\}, then \[ V(Y) = [0, 1, 1, 0, 0, 0]^T, \]
\[ Y \subseteq X. \]
\[ V(\sim X) = [1, 0, 0, 0, 1, 1]^T. \]
\[ V(Y) \cdot V(\sim X) = [0, 1, 1, 0, 0, 0] \cdot [1, 0, 0, 0, 1, 1]^T = 0. \]
\[ \text{Theorem 7: Let IS = (U, AT, V_{AT}, f) be an information system, where } A_k \in AT \text{ for any } k \in \{1, 2, ..., m\}, \text{ and } \forall X \subseteq U. \text{ For the optimistic multigranulation upper approximations of } X, \text{ we have} \]
\[ \sum_{k=1}^{m} A_k^U(X) = \cap_{i=1}^{m} \{ \cup \{x \in X \mid x \in A_k \} \}. \]

Proof. From Theorem 3
\[ \sum_{k=1}^{m} A_k^U(X) = \cap \{ \cup_{i=1}^{m} \{x \in X \mid x \in A_k \} \}, \]
and then ∀x, y ∈ X
\[ (\{x \in A_1 \} ∪ \{x \in A_2 \} ∪ \cdots ∪ \{x \in A_m \}) \cap (\{y \in A_1 \} ∪ \{y \in A_2 \} ∪ \cdots ∪ \{y \in A_m \}) \]
\[ = (\{x \in A_1 \} ∪ \{y \in A_1 \}) \cap (\{x \in A_2 \} ∪ \{y \in A_2 \}) \cap \cdots \cap (\{x \in A_m \} ∪ \{y \in A_m \}) \]
\[ = \cap_{i=1}^{m} \{ \cup \{x \in X \mid x \in A_i \} \}, \]
for the way we choose x and y, we can easily infer that \[ \sum_{k=1}^{m} A_k^U(X) = \cap_{i=1}^{m} \{ \cup \{x \in X \mid x \in A_k \} \}. \]

Lemma 2: Let IS = (U, AT, V_{AT}, f) be an information system, where A_k ∈ AT for any k ∈ {1, 2, ..., m}, and \forall X \subseteq U. \text{ For the optimistic multigranulation upper approximations of } X, \text{ we have}
\[ \sum_{k=1}^{m} A_k^U(X) = \cup_{i=1}^{m} \{ \cap \{x \in X \mid x \in A_k \} \}. \]

Proof. This lemma can be easily obtained from Theorem 6 □

By Theorem 7 and Lemma 2, we can propose a new approach to compute upper approximations of PMGRS and OMGRS:

Definition 5: Let IS = (U, AT, V_{AT}, f) be an information system, where A_k ∈ AT for any k ∈ {1, 2, ..., m}, and \forall X \subseteq U. The upper approximation character set of X can be calculated as
\[ I_k^U(X) = \{ \{x \in X \mid x \in A_k \} \}, \forall k = 1, 2, ..., m \] (11)

Table 1  Decision information system
| U | a_1 | a_2 | a_3 | d |
|---|-----|-----|-----|---|
| x_1 | 2   | 2   | 1   | 1 |
| x_2 | 2   | 3   | 1   | 1 |
| x_3 | 1   | 1   | 1   | 1 |
| x_4 | 1   | 1   | 2   | 2 |
| x_5 | 2   | 2   | 1   | 1 |
| x_6 | 1   | 1   | 0   | 3 |

Corollary 2: Let IS = (U, AT, V_{AT}, f) be an information system, where A_k ∈ AT for any k ∈ {1, 2, ..., m}, and \forall X \subseteq U. The optimistic and pessimistic upper approximations can be calculated by
\[ \sum_{k=1}^{m} A_k^U(X) = \cup_{i=1}^{m} I_k^U(X), \]
\[ \sum_{k=1}^{m} A_k^U(X) = \cap_{i=1}^{m} I_k^U(X). \]

Proof. This corollary can be easily obtained by Theorem 7 and Lemma 2 □

Example 2: Continuation of Example 1. From Table 1, we have that
\[ V(x_1) = V(x_2) = V(x_3) = [0, 1, 1, 1, 0, 1]^T. \]
\[ V(x_4) = V(x_5) = [0, 0, 0, 0, 1, 0]^T. \]
\[ V(x_6) = [1, 0, 0, 0, 0, 0]^T. \]
\[ V(\sim x_1) = [1, 0, 0, 0, 1, 1]^T. \]
\[ V(\sim x_2) = [1, 0, 0, 0, 1, 1]^T. \]
\[ V(\sim x_3) = [1, 0, 0, 0, 1, 1]^T. \]
\[ V(\sim x_4) = [1, 0, 0, 0, 1, 1]^T. \]
\[ V(\sim x_5) = [1, 0, 0, 0, 1, 1]^T. \]
\[ V(\sim x_6) = [1, 0, 0, 0, 1, 1]^T. \]

By Definition 5
\[ V(I_k^U(X)) = V(\{x \in X \mid x \in A_k \}) \]
\[ \vee V(x_1, x_2, ..., x_6) \]
\[ = V(x_1) \vee V(x_2) \vee V(x_3) \vee V(x_4) \vee V(x_5) \vee V(x_6) \]
\[ = [1, 1, 1, 1, 1, 1]^T. \]
\[ V(I_k^L(X)) = V(\{x \in X \mid x \in A_k \}) \]
\[ \vee V(x_1, x_2, ..., x_6) \]
\[ = V(x_1) \vee V(x_2) \vee V(x_3) \vee V(x_4) \vee V(x_5) \vee V(x_6) \]
\[ = [1, 1, 1, 1, 1, 1]^T. \]

By Corollary 2
\[ \sum_{k=1}^{m} A_k^U(X) = \cup_{i=1}^{m} I_k^U(X) \]
\[ V(\cup_{k=1}^{m} I_k^U(X)) = V(I_k^U(X)) \vee V(I_k^L(X)) \]
\[ = [1, 1, 1, 1, 1, 1] \vee [1, 1, 1, 1, 1, 1] \]
\[ = [1, 1, 1, 1, 1, 1]. \]
For \( V^T(\sim X) \cdot V([x]\cap A_k) \neq 0 \),
\[ V^T(\sim X) \cdot V([x]\cap A_k) \neq 0 \]
\[ V(\cap_{k=1}^{m} I^L_k(X)) = [0, 0, 0, 0, 0, 0] \]
\[ V(\cap_{k=1}^{m} I^U_k(X)) = [0, 1, 0, 0, 0, 0] \]
\[ V(\cap_{k=1}^{m} I^T_k(X)) = [0, 1, 1, 0, 0, 0] \]
\[ V(\cap_{k=1}^{m} I^R_k(X)) = [0, 0, 0, 0, 0, 0] \]

By Corollary 3
\[ \sum_{k=1}^{m} A^L_k(X) = \cap_{k=1}^{m} I^L_k(X) \]
\[ V(\cap_{k=1}^{m} I^L_k(X)) = V(\cap_{k=1}^{m} I^U_k(X)) \]
\[ \cap V(I^U_k(X)) \]
\[ = [0, 0, 0, 0, 0, 0] \]
\[ \cap [0, 1, 0, 0, 0, 0] \]
\[ = [0, 0, 0, 0, 0, 0] \]

By Definition 4
\[ \sum_{k=1}^{m} A^U_k(X) = \cup_{k=1}^{m} I^U_k(X) \]
\[ V(\cup_{k=1}^{m} I^U_k(X)) = V(\cup_{k=1}^{m} I^L_k(X)) \]
\[ \cup V(I^L_k(X)) \]
\[ = [0, 1, 0, 0, 0, 0] \]
\[ \cup [0, 0, 1, 1, 0, 0] \]
\[ = [0, 1, 1, 0, 0, 0] \]

By Definition 4
\[ \sum_{k=1}^{m} A^P_k(X) = \cap_{k=1}^{m} I^P_k(X) \]
\[ V(\cap_{k=1}^{m} I^P_k(X)) = V(\cap_{k=1}^{m} I^R_k(X)) \]
\[ \cap V(I^R_k(X)) \]
\[ = [0, 1, 0, 0, 0, 0] \]
\[ \cap [0, 0, 1, 1, 0, 0] \]
\[ = [0, 1, 1, 0, 0, 0] \]

Algorithm 1 (see Fig. 1) is a vector-based algorithm for computing the lower and upper approximations of optimistic and pessimistic MGRS whose time complexity is \( O(1|X|U) \). Steps 3–6 are to calculate \( I^L_k \) and \( I^U_k \) (\( k \in \{1, 2, \ldots, m\} \)) whose time complexity is \( O(1|X|U) \), steps 17–22 are to compute the approximations of MGRS whose time complexity is \( O(U^3) \), and in general, we have \( |X| \ll |U| \). Algorithm 1 (Fig. 1) is more efficient than matrix-based algorithm.

4 Experimental evaluations

In this section, several experiments have been conducted to verify the validity of the proposed vector-based algorithms. We have selected six data sets, which are described in Table 2. All the experiments have been carried out on a personal computer with Windows 10, Intel(R) Core(TM)I7-6700HQ @2.6 GHz and 8 GB memory. The programming language is Matlab R2015b.
First, since time complexity of Algorithm 2 is $O(|X| |U|)$ and time complexity of Algorithm 1 (Fig. 1) is $O(|U|^2)$, six group of experiments have been conducted to compare Algorithm 1 (Fig. 1) and matrix-based algorithm when the size of target concept $X$ was gradually increased by a 10% step in size of $U$, the strategy for selecting elements of $X$ is completely random.

Fig. 2 shows that the computation time of Algorithm 1 (Fig. 1) is less than matrix-based algorithm even though the cardinal number of target concept $X$ is the same. When the size of $X$ is increasing gradually, the computation time of Algorithm 1 (Fig. 1) has a positive growth while Hu's algorithm has almost no change at all. In Fig. 2, Algorithm 1 (Fig. 1) is more efficient than matrix-based algorithm on computing approximations of MGRS when the cardinal number of target concept $X$ is gradually increasing.

Fig. 3 shows that the computation time of Algorithm 1 (Fig. 1) and matrix-based algorithm when the size of universe is gradually increased by a 10% step in size of $U$, the strategy for selecting samples of $U$ and $X$ is completely random. The size of $X$ is 1/3 elements of the temporary universe.

Second, six group of experiments have been conducted to compare Algorithm 1 (Fig. 1) and matrix-based algorithm when the size of universe was gradually increased by a 10% step in size of $U$, the strategy for selecting samples of $U$ and $X$ is completely random. The size of $X$ is 1/3 elements of the temporary universe.

Fig. 3 shows that the computation time of Algorithm 1 (Fig. 1) and matrix-based algorithm when the size of universe is gradually increased by a 10% step in size of $U$, the strategy for selecting samples of $U$ and $X$ is completely random. The size of $X$ is 1/3 elements of the temporary universe.
and then a vector-based algorithm for computing approximations of whether they belong to approximations in MGRS has been clarified (No. 61379021, No. 11871259), National Youth Science

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7 References

5 Conclusion

In this paper, a fact that only the part of elements need to be judged whether they belong to approximations in MGRS has been clarified and then a vector-based algorithm for computing approximations of MGRS algorithm has been proposed. In the future, we will focus on updating approximation of MGRS while adding or deleting a granular structure, adding or deleting a sample by approaches which we verified.

Fig. 3 Computation time of Algorithm 1 (Fig. 1) and Algorithm 2 when the size of universe increasing gradually

5 Conclusion

In this paper, a fact that only the part of elements need to be judged whether they belong to approximations in MGRS has been clarified and then a vector-based algorithm for computing approximations of MGRS algorithm has been proposed. In the future, we will focus on updating approximation of MGRS while adding or deleting a granular structure, adding or deleting a sample by approaches which we verified.

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