Update analysis of two-body charmed $B$ meson decays

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Abstract

The charmed $B$ decays, $B \rightarrow DP$, $D^*P$ and $DV$, are re-analyzed using the latest experimental data, where $P$ and $V$ denote the pseudoscalar meson and vector meson, respectively. We perform global fits under the assumption of flavor SU(3) symmetry. The size of the decay amplitudes and the strong phases between the topologically distinct amplitudes are studied. Predictions of the related $B_s$ decay rates are made based upon the fitted results. We also note a serious SU(3) symmetry breaking or inconsistency in the $DV$ sector.

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I. INTRODUCTION

The hadronic decays of $B$ mesons have provided us with a good place to study CP violation in particle physics. In particular, the detection of direct CP violation in a decay process requires that there exist at least two contributing amplitudes with different weak and strong phases. The direct CP-violating effect in the $B$ system has finally been observed in the $B^0 \rightarrow K^+\pi^-$ decay at the $B$-factories [1, 2], proving the existence of nontrivial strong phases in $B$ decays. It is therefore of consequence to find out the patterns of final-state strong phases for a wider set of decay modes.

Since the CKM factors involved in charmed $B$ meson decays are purely real to a good approximation, the phases associated with the decay amplitudes thus have the origin of strong interactions. Such final-state rescattering effects have been noticed from data in these decays [3, 4], and estimated to be at 15-20% level [5]. Unfortunately, no satisfactory first-principle calculations can yield such strong phases [6]. In Ref. [7], we performed an analysis based upon the experimental data available at that time. A few theoretical and experimental questions are left unanswered. As more decay modes have been observed and others are measured at higher precisions, it becomes possible for us to look at and answer those questions. In this paper, flavor SU(3) symmetry is employed to relate different amplitudes and strong phases of the same topological type. Moreover, we will take a different approach by fitting theoretical parameters to all available branching ratios simultaneously. An advantage of this analysis is that the parameters thus obtained are insensitive to statistical fluctuations of individual modes.

This paper is organized as follows. In Section II we give the amplitude decomposition of modes under flavor SU(3) symmetry and the current branching ratio data. Theoretical parameters involved in our analysis are defined. In Section III we consider three sets of charmed decay modes: $DP$, $D^*P$, and $DV$, where $P$ and $V$ denote charmless pseudoscalar and vector mesons, respectively. A summary of our findings is given in Section IV.

II. FLAVOR AMPLITUDE DECOMPOSITION AND DATA

In the decomposition of decay amplitudes, relevant meson wave functions are assumed to have the following quark contents, with phases chosen so that isospin multiplets contain no
relative signs:

- **Beauty mesons:** $B^0 = b\bar{d}$, $B^- = -b\bar{u}$, $\overline{B}_s = b\bar{s}$.
- **Charmed mesons:** $D^0 = -c\bar{u}$, $D^+ = c\bar{d}$, $D_s^+ = c\bar{s}$, with corresponding phases for vector mesons.
- **Pseudoscalar mesons $P$:** $\pi^+ = u\bar{d}$, $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\pi^- = -d\bar{u}$, $K^+ = u\bar{s}$, $K^0 = d\bar{s}$, $\bar{K}^0 = s\bar{d}$, $K^- = -s\bar{u}$, $\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$, $\eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}$, assuming a specific octet-singlet mixing [8, 9] in the $\eta$ and $\eta'$ wave functions.
- **Vector mesons $V$:** $\rho^+ = u\bar{d}$, $\rho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\rho^- = -d\bar{u}$, $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$, $K^{*+} = u\bar{s}$, $K^{*0} = d\bar{s}$, $K^{*-} = -s\bar{u}$, $\phi = s\bar{s}$.

The amplitudes contributing to the decays discussed here involve only three different topologies [8, 9, 10, 11]:

1. **Tree amplitude $T$:** This is associated with the transition $b \to cd\bar{u}$ (Cabibbo-favored) or $b \to cs\bar{u}$ (Cabibbo-suppressed) in which the light (color-singlet) quark-antiquark pair is incorporated into one meson, while the charmed quark combines with the spectator antiquark to form the other meson.

2. **Color-suppressed amplitude $C$:** The transition is the same as in the tree amplitudes, namely $b \to cd\bar{u}$ or $b \to cs\bar{u}$, except that the charmed quark and the $\bar{u}$ combine into one meson while the $d$ or $s$ quark and the spectator antiquark combine into the other meson.

3. **Exchange amplitude $E$:** The $b$ quark and spectator antiquark exchange a $W$ boson to become a $c\bar{u}$ pair, which then hadronizes into two mesons by picking up a light quark-antiquark pair out of the vacuum.

After factoring out the CKM factors explicitly, we obtain the flavor amplitude decomposition of the charmed $B$ decay modes in Tables I, II, and III. In these tables, we introduce positive $\xi$’s to parameterize the flavor SU(3) breaking effects. This symmetry is respected between strangeness-conserving and strangeness-changing amplitudes when $\xi$’s are taken to
be unity. As we will discuss in the next section, $\xi$’s will be allowed to change in order to test the assumption. Using the Wolfenstein parameters $^{[12]}$, the relevant CKM factors are:

\[ V_{cb} = A \lambda^2, \quad V_{ud} = 1 - \frac{\lambda^2}{2}, \quad \text{and} \quad V_{us} = \lambda, \quad (1) \]

none of which contain a weak phase to the order we are concerned with. In the following analysis, we take the central values $\lambda = 0.2272$ and $A = 0.809$ quoted by the CKMfitter group $^{[13]}$.

Since only the relative strong phases are physically measurable, we fix the tree $(T, T_P, \text{and} T_V)$ amplitudes to be real and pointing in the positive direction. We then associate the color-suppressed and exchange amplitudes with the corresponding strong phases explicitly as follows:

\[ C = |C| e^{i \delta_C}, \quad E = |E| e^{i \delta_E}, \quad (2) \]

\[ C_P = |C_P| e^{i \delta_{CP}}, \quad E_P = |E_P| e^{i \delta_{EP}}, \quad (3) \]

\[ C_V = |C_V| e^{i \delta_{CV}}, \quad E_V = |E_V| e^{i \delta_{EV}}. \quad (4) \]

The magnitude of invariant decay amplitude $\mathcal{A}$ for a decay process $B \rightarrow M_1 M_2$ is related to its partial width via the following relation:

\[ \Gamma(B \rightarrow M_1 M_2) = \frac{p^*}{8\pi m_B^2} |\mathcal{A}|^2, \quad (5) \]

with

\[ p^* = \frac{1}{2m_B} \sqrt{m_B^2 - (m_1 + m_2)^2} \sqrt{m_B^2 - (m_1 - m_2)^2}, \quad (6) \]

where $m_{1,2}$ are the masses of $M_{1,2}$, respectively. To relate partial widths to branching ratios, we use the world-average lifetimes $\tau^+ = (1.638 \pm 0.011)$ ps, $\tau^0 = (1.530 \pm 0.009)$ ps, and $\tau_s = (1.466 \pm 0.059)$ ps computed by the Heavy Flavor Averaging Group (HFAG) $^{[14]}$.

III. STRONG PHASES

In our analysis, we take the amplitude sizes and the strong phases as theoretical parameters, and perform $\chi^2$ fits to all the branching ratios in each category ($B_{u,d} \rightarrow DP, D^*P, \text{and} DV$). We consider three schemes to test the flavor SU(3) assumption:

1. $\xi_T = \xi_C = \xi_T_V = \xi_C_P = \xi_T_P = \xi_C_V = 1$. This is the exact flavor SU(3)-symmetric case.
TABLE I: Branching ratios and flavor amplitude decomposition for $B \to DP$ decays. Data are quoted from Refs. [15, 16, 17, 18, 19, 20, 21, 22, 23, 24]

| Decay          | $m_B$ (GeV) (in units of $10^{-4}$) | $|f|$ (GeV) $|f|$ | Representation                                                                 |
|----------------|----------------------------------|----------------|--------------------------------------------------------------------------------|
| $B^- \to D^0\pi^-$ 5.2791 | 47.5 ± 2.1                       | 2.308 7.61 ± 0.17 | $-V_{cb}V_{ud}^*(T+C)$                                                          |
| $\to D^0K^-$      | 4.08 ± 0.24                      | 2.281 2.24 ± 0.07 | $-V_{cb}V_{us}^*(\xi_T T + \xi_C C)$                                           |
| $\bar{B}^0 \to D^+\pi^-$ 5.2793 | 29 ± 2                         | 2.306 6.11 ± 0.21 | $-V_{cb}V_{ud}^*(T+E)$                                                          |
| $\to D^+K^-$      | 2.0 ± 0.6                        | 2.279 1.63 ± 0.24 | $-V_{cb}V_{us}^* T$                                                             |
| $\to D^0\eta^0$  | 2.61 ± 0.24                      | 2.308 1.85 ± 0.09 | $V_{cb}V_{ud}^*(E - C)/\sqrt{2}$                                                |
| $\to D^0\eta$    | 2.0 ± 0.2                        | 2.274 1.62 ± 0.08 | $V_{cb}V_{ud}^*(C + E)/\sqrt{3}$                                               |
| $\to D^0\eta'$   | 1.25 ± 0.23                      | 2.198 1.31 ± 0.12 | $-V_{cb}V_{ud}^*(C + E)/\sqrt{6}$                                              |
| $\to D^0\bar{K}^0$ | 0.52 ± 0.07                    | 2.280 0.83 ± 0.05 | $-V_{cb}V_{us}^* C$                                                             |
| $\to D^+_s K^-$  | 0.27 ± 0.05                      | 2.242 0.61 ± 0.06 | $-V_{cb}V_{ud}^* E$                                                             |
| $\bar{B}^0_s \to D^+\pi^-$ 5.3696 | 2.357                         | 2.359 $V_{cb}V_{us}^* E E/\sqrt{2}$                                           |
| $\to D^0\pi^0$   | 2.332                           | 2.326 $(\xi_E E - \xi_C C)/\sqrt{3}$                                          |
| $\to D^0K^0$     | 2.251                           | 2.251 $V_{cb}V_{ud}^*(2\xi_C C + \xi_E E)/\sqrt{6}$                          |
| $\to D^+\pi^-$   | 38 ± 3 ± 13 $^a$                | 2.321 7.30 ± 1.28 | $-V_{cb}V_{ud}^* T$                                                             |
| $\to D^+_s K^-$  | 2.294                           | 2.294 $-V_{cb}V_{us}^*(\xi_T T + \xi_E E)$                                    |

$^a$ Ref. [16].

2. $\xi_T = \xi_C = \xi_{T_V} = \xi_{C_P} = f_K/f_\pi \simeq 1.22$, and $\xi_{T_P} = \xi_{C_V} = f_{K^*}/f_\rho \simeq 1.00$. This takes into account the difference in the decay constants for the charmless meson in the final states.

3. All $\xi_T$’s and $\xi_C$’s are taken as free parameters and determined by the $\chi^2$ fit in each individual category.

Here we have taken the decay constants $f_\pi = 130.7$ MeV, $f_K = 159.8$ MeV $^b$, $f_{K^*} = 210.4$ MeV and $f_\rho = 210.4$ MeV $^{24}$. For Scheme 3 in the $DV$ sector, it turns out that this scheme
TABLE II: Branching ratios and flavor amplitude decomposition for $B \to D^* P$ decays. Data are quoted from Refs. [15, 16, 17, 18, 19, 20, 21, 22, 23, 24].

| Decay | $m_B$ (GeV) | Branching ratio | $p^*$ (GeV) | $|A|$ (10$^{-7}$ GeV) | Representation |
|-------|-------------|-----------------|-------------|------------------------|-----------------|
| $B^- \to D^{*0} \pi^-$ 5.2791 | 50 ± 4 | 2.256 | 7.87 ± 0.32 | $-V_{cb}V^*_{ud}(T_V + C_P)$ |
| $\to D^{*0} K^-$ | 3.7 ± 0.4 | 2.227 | 2.16 ± 0.12 | $-V_{cb}V^*_{us}(\xi_T T_V + \xi_C P C_P)$ |
| $\bar{B}^0 \to D^{+} \pi^-$ 5.2793 | 28.5 ± 1.7 | 2.255 | 6.17 ± 0.19 | $-V_{cb}V^*_{us}(T_V + E_P)$ |
| $\to D^{*+} K^-$ | 2.14 ± 0.20 | 2.226 | 1.70 ± 0.08 | $-V_{cb}V^*_{us}\xi_T T_V$ |
| $\to D^{*0} \pi^0$ | 1.7 ± 0.3 | 2.256 | 1.52 ± 0.12 | $V_{cb}V^*_{ud}(E_P - C_P)/\sqrt{2}$ |
| $\to D^{*0} \eta$ | 1.8 ± 0.6 | 2.220 | 1.55 ± 0.24 | $V_{cb}V^*_{ud}(C_P + E_P)/\sqrt{3}$ |
| $\to D^{*0} \eta'$ | 1.23 ± 0.35 | 2.141 | 1.32 ± 0.19 | $-V_{cb}V^*_{ud}(C_P + E_P)/\sqrt{6}$ |
| $\to D^{*0} K^0$ | 0.36 ± 0.12 $^{b}$ | 2.227 | 0.70 ± 0.12 | $-V_{cb}V^*_{us}\xi_C P C_P$ |
| $\to D^{*+}_s K^-$ | 0.20 ± 0.05 ± 0.04 $^{c}$ | 2.185 | 0.53 ± 0.08 | $-V_{cb}V^*_{us}E_P$ |
| $\bar{B}^0_s \to D^{+} \pi^-$ 5.3696 | | 2.306 | | $-V_{cb}V^*_{us}\xi_E P E_P$ |
| $\to D^{*0} \pi^0$ | | 2.308 | | $V_{cb}V^*_{us}\xi_E P E_P/\sqrt{2}$ |
| $\to D^{*0} K^0$ | | 2.279 | | $-V_{cb}V^*_{ud}C_P$ |
| $\to D^{*0} \eta$ | | 2.273 | | $V_{cb}V^*_{us}(\xi_E C_P E_P - \xi_C P C_P)/\sqrt{3}$ |
| $\to D^{*0} \eta'$ | | 2.195 | | $-V_{cb}V^*_{us}(2C_P + \xi_E E_P)/\sqrt{6}$ |
| $\to D^{*+}_s \pi^-$ | | 2.267 | | $-V_{cb}V^*_{ud}T_V$ |
| $\to D^{*+}_s K^-$ | | 2.238 | | $-V_{cb}V^*_{us}(\xi_T T_V + \xi_E E_P)$ |

$b$ Ref. [17], $^c$ Ref. [18].

does not work well with the present available experimental data. We will discuss this issue in Subsection III C.

Among all $B_{u,d}$ decays considered in this work, no Cabibbo-suppressed decay involves the exchange diagram. The only place to test this is the $B_s$ decays, of which we know very little at the moment. We thus assume $\xi_E$’s = 1 when we predict the branching ratios of those decays.

The strong phases given in the following results are subject to a two-fold ambiguity. This is because only the cosines of the relative strong phases are involved in the branching ratios.
TABLE III: Branching ratios and flavor amplitude decomposition for $B \to DV$ decays. Data are quoted from Refs. [15, 16, 17, 18, 19, 20, 21, 22, 23, 24]

| Decay | $m_B$ (GeV) | Branching ratio | $p^*$ (in units of $10^{-4}$) | $|A|$ (GeV) | Representation |
|-------|-------------|----------------|-------------------------------|-------------|----------------|
| $B^- \to D^0 \rho^-$ 5.2791 | 134 ± 18 | 2.237 | 13.0 ± 0.9 | $-V_{cb}V_{ud}^*(T_P + C_V)$ |
| $\to D^0 K^{*-}$ | 5.3 ± 0.4 | 2.213 | 2.60 ± 0.11 | $-V_{cb}V_{us}^*(\xi_{TP} + \xi_{C}C_V)$ |
| $\overline{B}^0 \to D^+ \rho^-$ 5.2793 | 75 ± 12 | 2.235 | 10.1 ± 0.8 | $-V_{cb}V_{ud}^*(T_P + E_V)$ |
| $\to D^+ K^{*-}$ | 4.5 ± 0.7 | 2.211 | 2.48 ± 0.19 | $-V_{cb}V_{us}^*\xi_{TP}T_P$ |
| $\to D^0 \rho^0$ | 3.2 ± 0.5 | 2.237 | 2.07 ± 0.16 | $V_{cb}V_{ud}^*(E_V - C_V)/\sqrt{2}$ |
| $\to D^0 \omega$ | 2.6 ± 0.3 | 2.235 | 1.87 ± 0.11 | $-V_{cb}V_{us}^*(C_V + E_V)/\sqrt{2}$ |
| $\to D^0 K^{*0}$ | 0.42 ± 0.06 | 2.212 | 0.76 ± 0.06 | $-V_{cb}V_{us}^*\xi_{C}C_V$ |
| $\to D^+_s K^{*-}$ | < 8 | 2.172 | < 3 | $-V_{cb}V_{us}^*E_V$ |
| $\overline{B}^+_s \to D^+ \rho^-$ 5.3696 | 2.288 | 2.264 | $-V_{cb}V_{us}^*\xi_{E}E_V$ |
| $\to D^+_s K^{*-}$ | 2.264 | $-V_{cb}V_{us}^*(\xi_{TP} + \xi_{E}E_V)$ |
| $\to D^0 \rho^0$ | 2.289 | $V_{cb}V_{ud}^*E_V/\sqrt{2}$ |
| $\to D^0 K^{*0}$ | 2.265 | $-V_{cb}V_{ud}^*C_V$ |
| $\to D^0 \omega$ | 2.288 | $-V_{cb}V_{us}^*\xi_{E}E_V/\sqrt{2}$ |
| $\to D^0 \phi$ | 2.237 | $-V_{cb}V_{us}^*E_V/\sqrt{2}$ |
| $\to D^+_s \rho^-$ | 2.250 | $-V_{cb}V_{us}^*\xi_{C}C_V$ |
| $\to D^+_s K^{*-}$ | 2.226 | $-V_{cb}V_{us}^*(\xi_{TP} + \xi_{E}E_V)$ |

Therefore, it is allowed to flip the signs of all the phases simultaneously without changing the fitting quality and our predictions. In view of this, we will restrict the strong phase associated with the color-suppressed amplitudes to the $[-180^\circ, 0^\circ]$ range in our analysis.

A. $B \to DP$ decays

In Table IV we see that $\chi^2_{\min}$ is greatly reduced by the introduction of the SU(3) breaking factors $\xi_T$ and $\xi_C$. The smallness of $\chi^2_{\min}$ in Schemes 2 and 3 also shows the consistency of input observables.
The values of $|T|$ and $|C|$ can be directly obtained from the $B^0 \to D^+K^-$ and $D^0\bar{K}^0$ decays via the U-spin symmetry, i.e., exchange between $d$ quark and $s$ quark. They are respectively $14.0 \pm 2.1$ and $7.17 \pm 0.46$ in units of $10^{-6}$ GeV. Here we take $\xi_T = \xi_C = f_K/f_\pi$. Likewise, $|E|$ is inferred from the $B^0 \to D_s^+K^-$ mode to be $(1.49 \pm 0.14) \times 10^{-6}$ GeV. These values directly extracted from individual modes are consistent with those given in Table IV and in general have larger errors except for $|E|$. The SU(3) breaking parameter $\xi_T$ can also be extracted from $B^0 \to D^+K^-$ and $B^0_s \to D_s^+\pi^-$. It leads to $\xi_T = 0.96 \pm 0.22$. This is smaller than the fitted value of $\xi_T$ in Table IV.

According to our wave functions for $\eta$ and $\eta'$, the ratio $B(D^0\eta)/B(D^0\eta')$ is predicted to be 2, in comparison with $1.58 \pm 0.33$ given by the current data. From these decays, we determine $|C + E| = (7.07 \pm 0.30) \times 10^{-6}$ GeV. On the other hand, $|C - E| = (6.42 \pm 0.30) \times 10^{-6}$ GeV is inferred from the $D^0\pi^0$ mode. Therefore, one can form the combination $|C|^2 + |E|^2 = (45.6 \pm 2.9) \times 10^{-12}$ GeV$^2$ from these three modes, consistent with $(53.6 \pm 6.6) \times 10^{-12}$ GeV$^2$ that is derived from the $D^0\bar{K}^0$ and $D_s^+K^-$ modes assuming $\xi_C = f_K/f_\pi$.

![Figure 1](image.png)

**FIG. 1:** $\Delta \chi^2=1$ (pink, solid) and 2.30 (blue, dotted) contours on the $\delta_C$-$|C/T|$, $\delta_E$-$|E/T|$, and $|E/T|$-$|C/T|$ planes in Scheme 3.

In Fig. 1, we show the $\Delta \chi^2 = 1$ and 2.30 contours on the $\delta_C$-$|C/T|$, $\delta_E$-$|E/T|$, and $|E/T|$-$|C/T|$ planes in Scheme 3, respectively, showing the correlations between each pair of parameters. The projections of the $\Delta \chi^2 = 1$ contours to individual axes give the 68.3% confidence level (CL) ranges of the corresponding quantities. In particular, we find that $|C/T| = 0.48 \pm 0.02$ and $|E/T| = 0.11 \pm 0.01$. Our result shows an enhancement in the color-suppressed amplitude. This can be explained by non-factorizable effects or final state interactions. The three flavor amplitude sizes fall into a hierarchy: $|T| > |C| > |E|$, with $|E|$ being about one order of magnitude smaller than $|T|$. This is the reason why the 1 σ
bounds on $\delta_E$ are relatively loose. Moreover, we observe non-trivial strong phases $\delta_C$ and $\delta_E$. These results are consistent with previous studies [7, 24, 28].

We note in passing that in our contour plots, the planes are scanned by minimizing $\chi^2$, keeping all the other parameters free to vary. Therefore, our results are different from those given in Ref. [28]. In addition, their formalism corresponds to our Scheme 1.

Based upon our fit results, we give predictions for all the $B_{u,d,s}$ meson decays in this category in the lower part of Table IV. For the $B_s$ decays involving the exchange diagram, we take $\xi_E = 1$. The predicted branching ratios of those modes could be changed if we take into account SU(3) breaking in $E$. Conversely, measurements of those modes can provide useful information about the magnitude of the SU(3) breaking effect in the exchange diagram.

$B^{0}_s \to D_s^{+}\pi^-$ is a Cabibbo-favored decay involving the tree amplitude. Therefore, it has the largest decay rate among the channels in this group. Our preferred value for its branching ratio is $(22 \pm 1) \times 10^{-4}$. On the other hand, a recent measurement of this mode by CDF gives $(38 \pm 3 \pm 13) \times 10^{-4}$ [16]. The discrepancy is $1.2 \sigma$. Further measurements of this and other $B_s$ decay modes with better precision will help settling the question whether flavor SU(3) symmetry can be reliably extended to the sector of $B_s$ meson decays or not.

From the naive factorization (NF) approximation, the SU(3) breaking parameters are given by

$$\xi_{T}^{\text{NF}} = \frac{f_KF_0^{BD}(m_K^2)}{f_\pi F_0^{BD}(m_\pi^2)} \simeq 1.23 \quad \text{and} \quad \xi_{C}^{\text{NF}} = \frac{(m_B^2 - m_K^2)F_0^{BK}(m_D^2)}{(m_B^2 - m_\pi^2)F_0^{B\pi}(m_D^2)} \simeq 1.37 \quad (7)$$

where the form factors are calculated using the covariant light-front model [29]: $F_0^{BD}(m_\pi^2) = 0.67$, $F_0^{BD}(m_K^2) = 0.67$, $F_0^{B\pi}(m_D^2) = 0.28$, $F_0^{BK}(m_D^2) = 0.38$. These theoretical predictions are very close to our fitted values: $\xi_T = 1.24 \pm 0.02$ and $\xi_C = 1.33 \pm 0.02$.

The ratio of the two effective Wilson coefficients $a_{1,2}^{\text{eff}}$ for these decay processes can be extracted as

$$\left| \frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} \right|_{DP} = \left| \frac{C}{T} \right| \frac{(m_B^2 - m_D^2)f_\pi F_0^{BD}(m_\pi^2)}{(m_B^2 - m_\pi^2)f_D F_0^{B\pi}(m_D^2)} = 0.59 \pm 0.03 \quad (8)$$

where $|C/T| = 0.48 \pm 0.02$ as obtained from the $\chi^2$ analysis in Scheme 3, and $f_D = 222.6$ MeV [15] is used. In Ref. [27], $|a_2^{\text{eff}}/a_1^{\text{eff}}|_{DP}$ is found to be $0.54 - 0.70$ at the $1 \sigma$ level using the data of $B^- \to D^0\pi^-$, $B^0 \to D^+\pi^-$ and $B^0 \to D^0\pi^0$ modes, which is consistent with our result. In the pQCD calculation [30], it is found that $|a_2^{\text{eff}}/a_1^{\text{eff}}|_{DP} = 0.42 - 0.51$, and the
relative phase between $a_1^{\text{eff}}$ and $a_2^{\text{eff}}$ is estimated to be $-65.3^\circ < \arg(a_2^{\text{eff}} / a_1^{\text{eff}})_{DP} < -61.5^\circ$ without the exchange diagram.

**B. $B \to D^*P$ decays**

We see again in Table IV that $\chi^2_{\text{min}}$ is significantly lowered by the introduction of the SU(3) breaking factors $\xi_{TV}$ and $\xi_{CP}$.

In this category, $|T_V| = (14.7 \pm 0.7) \times 10^{-6}$ GeV, $|C_P| = (6.0 \pm 1.0) \times 10^{-6}$ GeV and $|E_P| = (1.29 \pm 0.21) \times 10^{-6}$ GeV can be directly extracted from the $D^{*+}K^-$, $D^{*0}\bar{K}^0$ and $D_s^{*+}K^-$ modes respectively, taking $\xi_{TV} = \xi_{CP} = f_K/f_\pi$. Another way to constrain $|C_P|$ is to deduce from the $D^{*0}(\pi, \eta, \eta')$ and $D_s^{*+}K^-$ modes. Using this method, we find $|C_P| = (6.2 \pm 0.5) \times 10^{-6}$ GeV.

FIG. 2: $\Delta \chi^2=1$ (pink, solid) and 2.30 (blue, dotted) contours on the $\delta_{C_P}\cdot|C_P/T_V|$, $\delta_{E_P}\cdot|E_P/T_V|$, and $|E_P/T_V|\cdot|C_P/T_V|$ planes in Scheme 3.

In Fig. 2 we show the $\Delta \chi^2 = 1$ and 2.30 contours on the $\delta_{C_P}\cdot|C_P/T_V|$, $\delta_{E_P}\cdot|E_P/T_V|$, and $|E_P/T_V|\cdot|C_P/T_V|$ planes in Scheme 3, respectively. We find that $|C_P/T_V| = 0.40^{+0.07}_{-0.04}$ and $|E_P/T_V| = 0.08^{+0.02}_{-0.01}$. As in the $DP$ category, there can be sizable non-factorizable contributions to the color-suppressed amplitude or final state interactions. The hierarchy among $|T_V|$, $|C_P|$ and $|E_P|$ is also very similar to those in the $DP$ category in Section III A. However, the strong phase, $\delta_{E_P}$ can be zero within the 68.3% CL region.

Predictions for all the $B_{u,d,s}$ meson decays according to our fit results are listed in the lower part of Table IV. The most dominant mode $D_s^{*+}\pi^-$ of $B_s$ decays is predicted to have a branching ratio of $(27 \pm 4) \times 10^{-4}$, similar to that of the $D_s^{+}\pi^-$ mode.
From the naive factorization approximation, the SU(3) breaking parameters are given by

\[
\xi_{T_P}^{\text{NF}} = \frac{p_{DK^*}^* f_{K^*} A_0^{BD}(m_{K^*}^2)}{p_{D^*}^* f_{D^*} A_0^{BD}(m_{D^*}^2)} \simeq 1.22 , \quad \xi_{C_V}^{\text{NF}} = \frac{p_{DK^*}^* f_{K^*} A_0^{BK}(m_{K}^2)}{p_{D^*}^* f_{D^*} A_1^{BK}(m_{D^*}^2)} \simeq 1.36 , \tag{9}
\]

where \( A_0^{BD}(m_{K^*}^2) = 0.64, A_0^{BD}(m_{D^*}^2) = 0.65, F_1^{BK}(m_{D^*}^2) = 0.31 \) and \( F_1^{BK}(m_{D^*}^2) = 0.43 \) [29]. Unlike the \( DP \) sector, our fitted SU(3) breaking factors are somewhat smaller than the naive factorization expectations.

The ratio of the two effective Wilson coefficients can be extracted as

\[
\left| \frac{a_2^{\text{eff}}}{a_1^{\text{eff}}} \right|_{D^*P} = \left| \frac{C_P}{T_V} \right| \frac{f_{D^*} A_0^{BD}(m_{D^*}^2)}{f_{D^*} A_1^{BD}(m_{D^*}^2)} = 0.42 \pm 0.04 , \tag{10}
\]

where \( |C_P/T_V| = 0.40^{+0.07}_{-0.04} \) as obtained from the \( \chi^2 \) analysis in Scheme 3, and \( f_{D^*} = 256.0 \) MeV [15, 31] is used. In the pQCD approach [30], \( |a_2^{\text{eff}}/a_1^{\text{eff}}|_{D^*P} \) is found to be \( 0.47 - 0.55 \) and their relative phase is estimated to be \(-64.8^\circ < \arg(a_2^{\text{eff}}/a_1^{\text{eff}})_{D^*P} < -61.4^\circ \) without the exchange diagram. These are consistent with our results at the 1 \( \sigma \) level.

### C. \( B \to DV \) decays

The decays in this category render a very different pattern from the previous two in the \( \chi^2 \) fitting. First, Scheme 1 and Scheme 2 yield the same result. This is because \( f_{K^*}/f_\rho \simeq 1.00 \). Furthermore Scheme 3 does not work well, unlike the \( DP \) and \( D^*P \) sectors. It is found that \( \chi^2_{\text{min}} = 0.045, \xi_{T_P} = 0.83, \xi_{C_V} = 4.58, |T_P| = 31.49, |C_V| = 1.75 \) and \( |E_V| = 6.64 \) in units of \( 10^{-6} \) GeV, if we take \( \xi_{T_P} \) and \( \xi_{C_V} \) as free fitting parameters. Theoretically, we do not expect \( |C_V| < |E_V| \). This unreasonable result is partly caused by the fact that \( |E_V| \) is less constrained by the experiment, \( \bar{B}^0 \to D^+_s K^{*-} \). Therefore we here adopt another prescription in which \( \xi_{T_P} \) and \( \xi_{C_V} \) are fixed by the naive factorization calculation, i.e.,

\[
\xi_{T_P}^{\text{NF}} = \xi_{T_P} = \frac{p_{DK^*}^* f_{K^*} F_1^{BD}(m_{K^*}^2)}{p_{D^*}^* f_{D^*} F_1^{BD}(m_{D^*}^2)} \simeq 1.00 , \quad \xi_{C_V}^{\text{NF}} = \xi_{C_V} = \frac{p_{DK^*}^* A_0^{BK}(m_{D}^2)}{p_{D^*}^* A_0^{B}(m_{D}^2)} \simeq 1.09 , \tag{11}
\]

where \( F_1^{BD}(m_{D^*}^2) = 0.69, F_1^{BD}(m_{D^*}^2) = 0.69, A_0^{B}(m_{D}^2) = 0.35 \) and \( A_0^{BK}(m_{D}^2) = 0.38 \) [29].

\( |T_P| = (26.1 \pm 2.0) \times 10^{-6} \) GeV can be extracted from the \( D^+ K^{*-} \) mode using the U-spin symmetry and taking \( \xi_{T_P} = f_{K^*}/f_\rho \). This is slightly larger than our fit result in Scheme 2. Directly from the \( \bar{B}^0 \to D^+_s K^{*-} \) mode, we have only a poor upper bound of \( 8.2 \times 10^{-6} \) GeV on \( |E_V| \).
The observable $\mathcal{B}(D^0\rho^-)$ has the largest contribution to the total $\chi^2_{\text{min}}$. From Table III we observe that the area of the triangle formed from the $B^- \rightarrow D^0\rho^-$, $\bar{B}^0 \rightarrow D^+\rho^-$ and $\bar{B}^0 \rightarrow D^0\rho^0$ decays is very small, while that of the triangle formed from the $B^- \rightarrow D^0K^{*-}$, $\bar{B}^0 \rightarrow D^+K^{*-}$ and $\bar{B}^0 \rightarrow D^0K^{*0}$ modes is not. This is the reason why the global $\chi^2$ fits in the $DV$ sector are not as satisfactory as those in the $DP$ and $D^*P$ sectors.

In Ref. [7], we noted that $|C_V|$ extracted from $D^0K^{*0}$ was inconsistent with $\sqrt{|C_V|^2 + |E_V|^2}$ extracted from a combination of the $D^0\rho^0$ and $D^0\omega$ modes. Currently, the former is $(8.01 \pm 0.60) \times 10^{-6}$ GeV if we take $\xi_{C_V} = f_{K^*}/f_{\rho}$, and the latter is $(6.86 \pm 0.34) \times 10^{-6}$ GeV. There is still a discrepancy at the 1.7 $\sigma$ level, or this discrepancy implies that the SU(3) breaking factor $\xi_{C_V}$ should be greater than about 1.17. A determination of $\mathcal{B}(D^+_sK^{*-})$ and better measurements of related modes will be very useful in providing further insights into this problem.

![FIG. 3](image_url)

FIG. 3: $\Delta \chi^2 = 1$ (pink, solid) and 2.30 (blue, dotted) contours on the $\delta_{C_V}|C_V/T_P|$, $\delta_{E_V}|E_V/T_P|$, and $|E_V/T_P| - |C_V/T_P|$ planes in Scheme 3.

In Fig. 3 we show the $\Delta \chi^2 = 1$ and 2.30 contours on the $\delta_{C_V}|C_V/T_P|$, $\delta_{E_V}|E_V/T_P|$, and $|E_V/T_P| - |C_V/T_P|$ planes in Scheme 3, respectively. We find that $|C_V/T_P| = 0.27 \pm 0.02$ and $|E_V/T_P| = 0.03^{+0.06}_{-0.03}$. We see that the magnitude of $T_P$ is larger than $T$ and $T_V$, resulting in a more hierarchical structure among $|T_P|$, $|C_V|$ and $|E_V|$. Another result of the large $T_P$ is reflected in the bigger branching ratio prediction for the most dominant $B^0_s \rightarrow D^+_s\rho^-$ mode. As in the $D^*P$ sector, the central value of $\delta_{E_V}$ is non-zero, but is still consistent with zero within the 68.3% CL region.

The ratio of the two effective Wilson coefficients can be extracted as

$$\left|\frac{a_2^{\text{eff}}}{a_1^{\text{eff}}}\right|_{DV} = \frac{|C_V|}{|T_P|} \frac{f_{\rho} F^{BD}(m_{\rho}^2)}{f_D A_{B_p}^{BD}(m_D^2)} = 0.50 \pm 0.04,$$ (12)
where \(|C_V/T_P| = 0.27 \pm 0.02\) as obtained from the \(\chi^2\) analysis in Scheme 3. In Ref. [27], it is estimated that \(|a_{2,1}^{\text{eff}}/a_{1}^{\text{eff}}|_{DV} = 0.24 - 0.42\) at the 1 \(\sigma\) level using the data of the \(B^- \rightarrow D^0 \rho^-\), 
\(B^0 \rightarrow D^+ \rho^-\) and \(B^{0*} \rightarrow D^0 \rho^0\) modes.

Finally, we summarize our findings in Fig. 4. These diagrams are constructed by taking the central values of the fitted parameters in each category using Scheme 3. They illustrate the sizes and relative phases among the tree, color-suppressed, and exchange amplitudes.

**FIG. 4:** Amplitude diagrams of (a): \(DP\) decays; (b): \(D^*P\) decays; and (c): \(DV\) decays.

**IV. CONCLUSIONS**

We have used the \(\chi^2\) fit approach to re-analyze the two-body charmed \(B\) meson decays in the flavor SU(3)-symmetric formalism, taking into account different symmetry breaking
schemes as well. In the $DP$ and $D^*P$ decays, there are significant improvement in the $\chi^2$ minimum between Scheme 1 and Scheme 2, but not much between Scheme 2 and Scheme 3. This shows that the major SU(3) breaking effect can be accounted for by the decay constant ratio $f_K/f_\pi$, as demanded for example by naive factorization. The same feature, however, is not observed in the $DV$ sector, where the corresponding decay constant ratio is approximately one.

In our analysis, the fit results are generally consistent with those extracted from individual modes. We have found that the color-suppressed amplitudes are enhanced in the $DP$ and $D^*P$ sectors, but not in the $DV$ sector. This strongly suggests that non-factorizable effects or final-state rescattering effects cannot be neglected in the former two sectors.

In the $DV$ sector, it is observed that the Cabibbo-suppressed $D^0K^*0$ yields a $|C_V|$ that exceeds the bound $\sqrt{|C_V|^2 + |E_V|^2}$ given by a combination of the $D^0\rho^0$ and $D^0\omega$ branching ratios at 1.7 $\sigma$ level, or $\xi_{C_V}$ should be greater than about 1.17. We urge the measurement of $\mathcal{B}(\mathcal{B}^0 \to D_s^+K^{*-})$ for a direct determination of the exchange amplitude, which may provide a possible solution to this problem.

Finally we note that the exchange diagrams are at least an order of magnitude smaller than the dominant tree topologies in these decays. Consequently, it is difficult to determine their phases, particularly in the $D^*P$ and $DV$ sectors, unless data precision can be significantly improved in the future.

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The decay constants of $K^*$ and $\rho$ are extracted using the method given in the appendix of Ref. [26] and the updated branching ratios given in Ref. [15].
TABLE IV: $B \rightarrow DP$ decays. Theoretical parameters are extracted from global $\chi^2$ fits in different schemes explained in the text. The amplitude sizes are given in units of $10^{-6}$. Predictions of branching ratios are made with $\xi_E = 1$ and given in units of $10^{-4}$ unless otherwise noted.

| Scheme 1 | Scheme 2 | Scheme 3 |
|---------|---------|---------|
| $|T|$    | 16.26$^{+0.61}_{-0.68}$ | 13.74 ± 0.45 | 13.71 ± 0.46 |
| $|C|$    | 6.77$^{+0.20}_{-0.21}$  | 6.67 ± 0.20  | 6.57 ± 0.22  |
| $|E|$    | 1.47$^{+0.13}_{-0.15}$  | 1.48$^{+0.13}_{-0.15}$ | 1.49$^{+0.13}_{-0.15}$ |
| $\delta_C$ (degrees) | $-69.0^{+9.2}_{-7.5}$ | $-47.0^{+9.5}_{-8.2}$ | $-48.7^{+9.8}_{-8.5}$ |
| $\delta_E$ (degrees) | $-146.2^{+13.9}_{-12.0}$ | $30.4^{+11.6}_{-11.8}$ | $28.9^{+11.9}_{-12.1}$ |
| $\xi_T$ | 1 (fixed) | $f_K/f_\pi$ (fixed) | 1.24 ± 0.02 |
| $\xi_C$ | 1 (fixed) | $f_K/f_\pi$ (fixed) | 1.33 ± 0.02 |
| $\chi^2_{\text{min}}$ | 45.28 | 3.53 | 1.41 |
| $\chi^2_{\text{min}}$/dof | 11.32 | 0.88 | 0.71 |

$B^- \rightarrow D^0\pi^-$: 52.8 ± 5.3 48.6 ± 3.7 47.5 ± 3.8

$B^- \rightarrow D^0K^-$: 2.84 ± 0.28 3.91 ± 0.30 4.08 ± 0.34

$B^0 \rightarrow D^+\pi^-$: 29 ± 3 29 ± 2 29 ± 2

$B^0 \rightarrow D^+K^-$: 1.8 ± 0.1 1.9 ± 0.1 2.0 ± 0.1

$B^0 \rightarrow D^0\pi^0$: 2.76 ± 0.37 2.68 ± 0.35 2.61 ± 0.36

$B^0 \rightarrow D^0\eta$: 2.2 ± 0.3 2.1 ± 0.2 2.1 ± 0.2

$B^0 \rightarrow D^0\eta'$: 1.06 ± 0.12 1.03 ± 0.11 1.00 ± 0.12

$B^0 \rightarrow D^0\overline{K}^0$: 0.31 ± 0.02 0.45 ± 0.03 0.52 ± 0.04

$B^0 \rightarrow D^+_sK^-$: 0.27 ± 0.05 0.27 ± 0.05 0.27 ± 0.05

$B^0_s \rightarrow D^+\pi^-$ (in units of $10^{-6}$): 1.4 ± 0.3 1.4 ± 0.3 1.4 ± 0.3

$B^0_s \rightarrow D^0\pi^0$ (in units of $10^{-6}$): 0.7 ± 0.1 0.7 ± 0.1 0.7 ± 0.1

$B^0_s \rightarrow D^0\overline{K}^0$: 5.4 ± 0.3 5.3 ± 0.3 5.1 ± 0.3

$B^0_s \rightarrow D^0\eta$: 0.09 ± 0.01 0.14 ± 0.01 0.16 ± 0.02

$B^0_s \rightarrow D^0\eta'$: 0.20 ± 0.02 0.29 ± 0.02 0.33 ± 0.03

$B^0_s \rightarrow D^+_s\pi^-$: 31 ± 2 22 ± 1 22 ± 1

$B^0_s \rightarrow D^+_sK^-$: 1.8 ± 0.1 2.0 ± 0.1 2.0 ± 0.1
TABLE V: $B \to D^*P$ decays. Theoretical parameters are extracted from global $\chi^2$ fits in different schemes explained in the text. The amplitude sizes are given in units of $10^{-6}$. Predictions of branching ratios are made with $\xi_E = 1$ and given in units of $10^{-4}$ unless otherwise noted.

|                      | Scheme 1         | Scheme 2         | Scheme 3         |
|----------------------|------------------|------------------|------------------|
| $|T_V|$                | $16.45^{+0.55}_{-0.61}$ | $14.85^{+0.60}_{-1.00}$ | $15.34^{+0.84}_{-1.70}$ |
| $|C_P|$                | $6.03^{+0.43}_{-0.46}$      | $6.21^{+0.39}_{-0.43}$      | $6.14^{+0.46}_{-0.50}$      |
| $|E_P|$                | $1.37^{+0.18}_{-0.20}$      | $1.26^{+0.20}_{-0.23}$      | $1.29^{+0.19}_{-0.22}$      |
| $\delta_{C_P}$ (degrees) | $-63.4^{+13.2}_{-10.8}$    | $-54.5^{+24.9}_{-12.0}$    | $-57.3^{+30.0}_{-12.6}$    |
| $\delta_{E_P}$ (degrees) | $-126.8^{+21.4}_{-19.3}$  | $-84.7^{+84.7}_{-27.8}$  | $-100.9^{+100.9}_{-30.0}$ |
| $\xi_{TV}$           | 1 (fixed)         | $f_K/f_\pi$ (fixed) | $1.17^{+0.03}_{-0.02}$ |
| $\xi_{C_P}$          | 1 (fixed)         | $f_K/f_\pi$ (fixed) | $1.20 \pm 0.05$          |
| $\chi^2_{\text{min}}$ | 12.00             | 1.39              | 0.72              |
| $\chi^2_{\text{min}}$/dof | 3.00              | 0.35              | 0.36              |

| $B^- \to D^{*0}\pi^-$ | $52 \pm 6$ | $49 \pm 8$ | $50 \pm 10$ |
| $\to D^0K^-$          | $2.8 \pm 0.3$ | $3.9 \pm 0.6$ | $3.7 \pm 0.8$ |
| $B^0 \to D^{*+}\pi^-$ | $30.3 \pm 2.8$ | $27.9 \pm 5.4$ | $28.4 \pm 7.3$ |
| $\to D^{*+}K^-$       | $1.80 \pm 0.13$ | $2.19 \pm 0.24$ | $2.14 \pm 0.37$ |
| $\to D^{*0}\pi^0$     | $1.9 \pm 0.5$ | $1.6 \pm 0.6$ | $1.7 \pm 0.9$ |
| $\to D^{*0}\eta$      | $1.9 \pm 0.4$ | $2.2 \pm 0.4$ | $2.1 \pm 0.6$ |
| $\to D^{*0}\eta'$     | $0.89 \pm 0.17$ | $1.05 \pm 0.21$ | $1.00 \pm 0.29$ |
| $\to D^{*0}\bar{K}^0$ | $0.24 \pm 0.04$ | $0.38 \pm 0.05$ | $0.36 \pm 0.06$ |
| $\to D^{*+}_sK^-$     | $0.23 \pm 0.06$ | $0.19 \pm 0.06$ | $0.20 \pm 0.06$ |
| $B^0_s \to D^{*+}\pi^-$ (in units of $10^{-6}$) | $1.2 \pm 0.3$ | $1.0 \pm 0.3$ | $1.1 \pm 0.3$ |
| $\to D^{*0}\pi^0$ (in units of $10^{-7}$) | $6.0 \pm 1.6$ | $5.0 \pm 1.7$ | $5.3 \pm 1.7$ |
| $\to D^{*0}\bar{K}^0$ | $4.2 \pm 0.6$ | $4.5 \pm 0.6$ | $4.4 \pm 0.7$ |
| $\to D^{*0}\eta$      | $0.06 \pm 0.02$ | $0.09 \pm 0.02$ | $0.09 \pm 0.04$ |
| $\to D^{*0}\eta'$     | $0.16 \pm 0.03$ | $0.27 \pm 0.03$ | $0.25 \pm 0.05$ |
| $\to D^{*+}_s\pi^-$   | $31 \pm 2$ | $25 \pm 3$ | $27 \pm 4$ |
| $\to D^{*+}_sK^-$     | $1.8 \pm 0.1$ | $2.1 \pm 0.3$ | $2.2 \pm 0.5$ |
TABLE VI: $B \to DV$ decays. Theoretical parameters are extracted from global $\chi^2$ fits in different schemes explained in the text. The amplitude sizes are given in units of $10^{-6}$. Predictions of branching ratios are made with $\xi_E = 1$ and given in units of $10^{-4}$ unless otherwise noted.

| Scheme 1 | Scheme 2 | Scheme 3 |
|----------|----------|----------|
| $|T_P|$ | $25.60^{+1.56}_{-1.62}$ | $25.60^{+1.56}_{-1.62}$ | $25.87^{+1.61}_{-1.72}$ |
| $|C_V|$ | $7.07^{+0.29}_{-0.33}$ | $7.07^{+0.29}_{-0.33}$ | $6.95^{+0.29}_{-0.37}$ |
| $|E_V|$ | $0.57^{+1.32}_{-0.43}$ | $0.57^{+1.32}_{-0.43}$ | $0.77^{+1.53}_{-0.66}$ |
| $\delta_{C_V}$ (degrees) | $-75.1^{+19.1}_{-15.8}$ | $-75.1^{+19.1}_{-15.8}$ | $-79.2^{+18.0}_{-14.9}$ |
| $\delta_{E_V}$ (degrees) | $143.4^{+36.6}_{-108.8}$ | $143.4^{+36.6}_{-108.8}$ | $158.6^{+21.4}_{-128.5}$ |
| $\xi_{TP}$ | 1 (fixed) $f_{K^*}/f_{\rho}$ (fixed) $\xi_{NF}^{TP}$ (fixed) | 1 (fixed) $f_{K^*}/f_{\rho}$ (fixed) $\xi_{NF}^{TP}$ (fixed) |
| $\chi^2_{min}$ | 5.91 | 5.91 | 4.18 |
| $\chi^2_{min}/dof$ | 2.96 | 2.96 | 2.09 |

$B^- \to D^0\rho^-$ | $105 \pm 18$ | $105 \pm 18$ | $103 \pm 18$ |
$\to D^0 K^{*-}$ | $5.7 \pm 1.0$ | $5.7 \pm 1.0$ | $5.6 \pm 1.0$ |

$B^0 \to D^+\rho^-$ | $78 \pm 11$ | $78 \pm 11$ | $78 \pm 12$ |
$\to D^+ K^{*-}$ | $4.3 \pm 0.5$ | $4.3 \pm 0.5$ | $4.4 \pm 0.6$ |
$\to D^0 \rho^0$ | $3.5 \pm 0.8$ | $3.5 \pm 0.8$ | $3.4 \pm 1.0$ |
$\to D^0 \omega$ | $2.7 \pm 0.7$ | $2.7 \pm 0.7$ | $2.7 \pm 0.9$ |
$\to D^0 K^*$ | $0.33 \pm 0.03$ | $0.33 \pm 0.03$ | $0.38 \pm 0.04$ |
$\to D^+ K^{*-}$ | $0.04 \pm 0.12$ | $0.04 \pm 0.12$ | $0.07 \pm 0.20$ |

$B^+_s \to D^+\rho^-$ (in units of $10^{-7}$) | $2.1 \pm 6.3$ | $2.1 \pm 6.3$ | $3.8 \pm 10.7$ |
$\to D^+ K^{*-}$ | $4.2 \pm 0.6$ | $4.2 \pm 0.6$ | $4.2 \pm 0.6$ |
$\to D^0 \rho^0$ (in units of $10^{-6}$) | $1.9 \pm 5.8$ | $1.9 \pm 5.8$ | $3.5 \pm 9.8$ |
$\to D^0 K^*$ | $5.8 \pm 0.5$ | $5.8 \pm 0.5$ | $5.6 \pm 0.5$ |
$\to D^0 \omega$ (in units of $10^{-7}$) | $1.0 \pm 3.2$ | $1.0 \pm 3.2$ | $1.9 \pm 5.4$ |
$\to D^0 \phi$ | $0.31 \pm 0.03$ | $0.31 \pm 0.03$ | $0.35 \pm 0.03$ |
$\to D^+ \rho^-$ | $75 \pm 9$ | $75 \pm 9$ | $77 \pm 10$ |
$\to D^+ K^{*-}$ | $4.1 \pm 0.6$ | $4.1 \pm 0.6$ | $4.2 \pm 0.6$ |