The potentially multiferroic Aurivillius phase Bi$_5$FeTi$_3$O$_{15}$: cation site preference, electric polarization, and magnetic coupling from first principles

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We study the structural, ferroelectric, and magnetic properties of the potentially multiferroic Aurivillius phase material Bi$_5$FeTi$_3$O$_{15}$ using first principles electronic structure calculations. Calculations are performed both with PBE and PBEsol exchange correlation functionals. We conclude that PBE systematically overestimates the lattice constants and the magnitude of the ferroelectric distortion, whereas PBEsol leads to good agreement with available experimental data. We then assess a potential site preference of the Fe$^{3+}$-cation by comparing 10 different distributions of the perovskite B-sites. We find a slight preference for the “inner” site, consistent with recent experimental observations. We obtain a large value of $\sim 55 \mu C/cm^2$ for the spontaneous electric polarization, which is rather independent of the specific Fe distribution. Finally, we calculate the strength of the magnetic coupling constants and find strong antiferromagnetic coupling between Fe$^{3+}$-cations in nearest neighbor positions, whereas the coupling between further neighbors is rather weak. This poses the question whether magnetic long range order can occur in this system in spite of the low concentration of magnetic ions.

I. INTRODUCTION

Multiferroic materials, which exhibit simultaneous ferroelectric and magnetic order, have been attracting considerable attention during the last decade$^{[1,2]}$. Much of the corresponding research is motivated by the potential of these materials for non-volatile data storage, sensor applications, and many other areas of technology. However, materials that exhibit multiferroic properties above room temperature, and are thus suitable candidates for future device applications, are still extremely scarce. The search for alternative materials with robust multiferroic properties above room temperature is therefore of high relevance.

One possible route for the design of novel multiferroic materials is to start from a series of well-established ferroelectrics and create additional functionality by incorporating magnetic ions into these systems. A very promising class of materials for this purpose are the so-called Aurivillius phases. The Aurivillius phases are a family of compounds that form in a naturally-layered perovskite-related crystal structure which consists of $m$ perovskite layers ($A_{m-1}B_mO_{3m+1}$)$^{2-}$, stacked along the [001] direction, and separated by fluorite-like ($Bi_2O_2$)$^{2+}$ layers (see Fig. 1 for an example with $m = 4$). The overall chemical composition is thus $Bi_2A_{m-1}B_mO_{3m+3}$, where many different compositions can be incorporated on the $A$ and $B$ sites within the perovskite-like layers.$^3$ The Aurivillius phases are well-known for their excellent ferroelectric properties with very low fatigue$^{[4,5]}$, and offer great potential for tailoring specific properties by varying both ionic composition and number of layers.

A number of Aurivillius compounds exhibiting multiferroic properties have been reported recently. For example, room temperature ferromagnetism together with ferroelectric polarization has been reported for materials based on the 4-layer compound Bi$_3$FeTi$_3$O$_{15}$, but with half of the Fe cations substituted by either Co or Ni.$^{[6,7,8]}$ Furthermore, magnetic field-induced ferroelectric domain switching has recently been observed in a 5-layer system with composition Bi$_5$Fe$_{2.8}$Mn$_{0.2}O_{15}$.$^{[9]}$ However, it should be noted that in many cases the observed polarization and magnetization depend strongly on the synthesis method and annealing conditions.$^{[10-12]}$ and it has been pointed out that the weak magnetic signals found in these samples can indeed be caused by tiny inclusions of other phases, which are extremely hard to detect using standard characterization methods.$^{[13]}$

Even in the case of Bi$_5$FeTi$_3$O$_{15}$, the “parent compound” for many of the compositions that have been explored as potential multiferroics, the ferroelectric and magnetic properties are not well established. An antiferromagnetic Néel temperature of 80 K has been reported.$^{[15]}$ However, several more recent studies show paramagnetic behavior with no magnetic long-range order even at very low temperatures.$^{[13,17-19]}$ Fits of magnetic susceptibility data have found conflicting signs of the Curie-Weiss temperature in different samples, indicating either ferromagnetic$^{[13]}$ or antiferromagnetic$^{[17-19]}$ interactions between the Fe cations. Similarly, no well established value exists for the spontaneous electric polarization in Bi$_5$FeTi$_3$O$_{15}$, due to difficulties in obtaining fully saturated ferroelectric hysteresis loops. Reported values for the remanent polarization vary from 3.5 $\mu C/cm^2$ to $\sim 30$ $\mu C/cm^2$.$^{[14,19]}$

It is therefore highly desirable to establish the intrinsic properties of Bi$_5$FeTi$_3$O$_{15}$ and other potentially multiferroic Aurivillius phases using modern first principles electronic structure calculations. Such calculations provide a useful reference for experimental observations and can also be used as guideline for future studies.

Here we present results of such first principles calculations for the 4-layer Aurivillius phase Bi$_3$FeTi$_3$O$_{15}$. We show that the structural properties of this material can be obtained in good agreement with available experimental data using the PBEsol functional for the
exchange-correlation energy. We also show that there is a preference for the Fe$^{3+}$ cation to occupy the “inner” sites within the perovskite-like layers, and we suggest the possibility to reverse this preference using epitaxial strain. Furthermore, we calculate a large spontaneous ferroelectric polarization of $\sim 55 \mu C/cm^2$, which is similar in magnitude to the related 3-layer system Bi$_2$Ti$_3$O$_{12}$. This shows that the presence of the magnetic Fe$^{3+}$ cations is not detrimental to the ferroelectric properties of Bi$_2$FeTi$_3$O$_{15}$. Finally, we calculate the strength of the magnetic coupling and find a very strong and antiferromagnetic coupling between Fe in nearest neighbor positions. However, the coupling becomes nearly negligible beyond second neighbors, consistent with the short-range character of the underlying superexchange mechanism. This leaves the question of whether long-range magnetic order can be expected in Bi$_2$FeTi$_3$O$_{15}$ open for future studies.

This paper is organized as follows. In the next section we present the method we use in our first principles calculations, and discuss in particular how we treat the quasi-random distribution of Fe$^{3+}$ and Ti$^{4+}$ cations over the available sites within the perovskite-like layers. In Sections [III A] and [III B] we then present our results for the structural properties and the energetics of the different cation distributions. Sec. [III C] focuses on the calculation of the electric polarization, and in Sec. [III D] we give a brief discussion of the electronic structure in Bi$_2$FeTi$_3$O$_{15}$. Finally, in Sec. [III E] we present our analysis of the calculated magnetic coupling constants and discuss the possibility of long-range magnetic order. A summary and some further conclusions are given in Sec. [IV].

II. METHODS

A. Aurivillius crystal structure and different distributions of Fe/Ti cations

Virtually all members of the homologous series of Aurivillius compounds exhibit a tetragonal structure with space group symmetry $I4/mmm$ at high temperatures (see Fig. 1 for the case of $m = 4$). On cooling, this structure transforms into a polar structure corresponding to a subgroup symmetry of $I4/mmm$. Even-layered Aurivillius phases such as Bi$_2$FeTi$_3$O$_{15}$ ($i.e., m = 2, 4, \ldots$) transform to orthorhombic symmetry $A21$am (space group no. 36), whereas odd-layered members ($m = 1, 3, \ldots$) transform into $B2cb$ symmetry (space group no. 41) or an even lower symmetry corresponding to a subgroup of $B2cb$.

The symmetry-lowering from $I4/mmm$ to $A21$am leads to a doubling of the primitive unit cell, with 2 formula units (48 atoms) per unit cell in the $A21$am structure.

There are two inequivalent $B$ sites in the 4-layer structure: the inner sites within the perovskite layers, and the outer sites adjacent to the Bi$_2$O$_2$ layers. In Bi$_2$FeTi$_3$O$_{15}$ these octahedrally coordinated $B$ sites are shared between the Fe$^{3+}$ and Ti$^{4+}$ cations, and a quasi-random cation distribution is observed in experiments. However, recent Mössbauer experiments have reported a slight preference of Fe$^{3+}$ to occupy the inner sites within the perovskite layers, whereas the outer sites are preferentially occupied by Ti$^{4+}$ cations.

In our calculations for Bi$_2$FeTi$_3$O$_{15}$ we use the 48 atom unit cell corresponding to the primitive unit cell of the low-symmetry $A21$am structure, which contains 8 octahedrally-coordinated $B$ sites (4 inner and 4 outer sites). There are 10 symmetrically inequivalent ways to distribute 2 Fe and 6 Ti atoms over these 8 $B$ sites, all of which are depicted in Table 1. In this work we explicitly consider each of the 10 individual configurations listed in Table 1. This allows to evaluate the impact of a specific cation distribution on the physical properties and also to address a potential site preference of the Fe$^{3+}$ and Ti$^{4+}$ cations in Bi$_2$FeTi$_3$O$_{15}$. We note that the specific distribution of cations breaks further symmetries compared to the experimental $A21$am structure. One could think of the latter as a weighted average over all the possible configurations.

To allow for all distortions observed in the experimental $A21$am symmetry, we use lattice vectors that resem-
TABLE I. (Color online). The 10 symmetrically inequivalent configurations considered in this work, corresponding to different distributions of Fe and Ti over the octahedral $B$ sites. Fe and Ti sites are indicated by dark (brown) and light (blue) octahedra, respectively. The configurations are grouped into “outer”, “mixed”, and “inner”, depending on whether two, one, or none of the two Fe cations in the unit cell are situated in the perovskite layers adjacent to the $\text{Bi}_2\text{O}_2$ layers. The notation defined in the second line is used throughout this article to denote the various configurations.

| “outer” | “mixed” | “inner” |
|---------|---------|---------|
| O1      | M1      | I1      |
| O2      | M2      | I2      |
| O3      | M3      | I3      |

| “outer” | “mixed” | “inner” |
|---------|---------|---------|
| O1      | M1      | I1      |
| O2      | M2      | I2      |
| O3      | M3      | I3      |

We perform full structural relaxations of all degrees of freedom (lattice parameters and internal positions) for all 10 configurations listed in Table 1, both with parallel and antiparallel orientation of the magnetic moments of the two Fe$^{3+}$ cations within the unit cell. Lattice parameters and ionic positions are relaxed until the residual forces are smaller than $10^{-3}$ eV/Å and the total energy changes less than $10^{-8}$ eV. Calculations are converged using a $\Gamma$-centered $k$-point mesh with $4 \times 4 \times 2$ divisions along the three reciprocal lattice vectors and a plane wave cutoff energy of $E_{\text{cut}} = 550$ eV. For the calculation of the electronic density of states a denser (and more uniform) $12 \times 12 \times 3$ k-point mesh is used.

III. RESULTS AND DISCUSSION

A. Lattice parameters and energetics

In this section we present the energies and lattice parameters obtained by full structural relaxation of all 10 symmetrically inequivalent configurations with different distributions of Fe$^{3+}$ and Ti$^{4+}$ cations listed in Table 1. All presented results are obtained for antiparallel alignment of the Fe magnetic moments. As will be shown in Sec. III E this corresponds to the preferred magnetic state in all cases. Fig. 2 shows the lengths of the three lattice vectors obtained after full structural relaxation for each configuration, in comparison with experimental data taken from Ref. [21]. One can see that for both PBE and PBESol there is a clear correlation between the lattice constants and the distribution of Fe$^{3+}$ and Ti$^{4+}$ cations over the available $B$ sites. The three inner configurations have longer in-plane lattice constants, $a$ and $b$, and shorter out-of-plane constant, $c$, compared to the three outer configurations. Mixed configurations are in between these two cases. Differences between configurations of the same “type” (i.e. inner, outer, or mixed) are rather small. The larger tetragonality (larger $c/a$) of the crystallographic unit cell for the outer configurations is related to a consistently large local tetragonality of the Fe sites in the outer perovskite layer. For the following we define the local tetragonality of the perovskite sites as the ra-
Fully relaxed PBE

PBE with exp. lattice constants

lattice constant (Å)

\begin{tabular}{c c c c}
0.2 & 0.3 & 0.3 & 0.3 \\
0.4 & 0.4 & 0.4 & 0.4 \\
\end{tabular}

0

0.1

0.2

0.3

0.4

Energy wrt Lowest Config. (eV/2 f.u.)

Fully relaxed PBEsol

Fully relaxed PBE

PBE with exp. lattice constants

\begin{tabular}{cccc}
\hline
O1 & O2 & O3 & M1 \\
\hline
\hline
0 & 0.1 & 0.2 & 0.3 \\
0.4 & 0.5 & 0.6 & 0.7 \\
\hline
\end{tabular}

FIG. 2. (Color online). Lengths of the three lattice vectors \( a = |\vec{a}| \) (bottom, circles), \( b = |\vec{b}| \) (bottom, squares), and \( c = |\vec{c}| \) (top, triangles) for the 10 different configurations listed in Table I, computed using the PBEsol (filled symbols) and the PBE (open symbols) functionals. The horizontal dashed lines correspond to experimental data from Ref. 21.

FIG. 3. (Color online). Relative energies of all configurations for the three cases discussed in the main text: full relaxation using PBEsol (top), full relaxation using PBE (middle), and only ionic positions relaxed using PBE with lattice parameters fixed to experimental values (bottom).

The slight under- respectively overestimation of the lattice parameters compared to the experimental values, we consider three distinct cases: i) full structural relaxation using the PBEsol functional, ii) full relaxation using PBE, and iii) lattice parameters fixed to the experimental values from Ref. 21 and internal atomic coordinates relaxed using PBE. We find that in all three cases, the energies of the different configurations form a hierarchy according to the classification of the Fe distribution as inner, outer, and mixed (see Fig. 3). This indicates that the main factor determining the energetics is the total amount of Fe sitting in the inner (or outer) perovskite layers and that the specific spatial distribution within each layer is less important. It can further be seen that the full PBEsol relaxations, which give the best agreement with the experimental lattice constants, indicate a preference of the Fe\(^{3+}\) cations to occupy the inner site, whereas the full PBE relaxations, which slightly overestimate the lattice constants, prefer Fe on the outer sites. Interestingly, the calculations where the lattice parameters have been fixed to experimental values and only the internal positions of the ions are relaxed using PBE gives the same site preference as PBEsol, i.e. lower energies for the inner configurations. Therefore, the slight overestimation of the lattice parameters within PBE seems to be responsible for the different site preference obtained within PBE and PBEsol. This sensitivity of the observed site preference on the lattice parameters as well as the more elongated unit cell (larger \( c/a \) ratio) obtained for the outer configurations (see Fig. 2) suggest a potential way of tailoring the cation distribution using epitaxial strain, i.e. by growing thin films of B\(_5\)FeTi\(_3\)O\(_{15}\) on different substrates with either positive or negative lattice mismatch.

As can be seen from Fig. 3, the energy differences be-

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between different cation configurations are of the order of \(~100\,\text{meV}\) per unit cell. To obtain a rough estimate whether these energy differences can lead to noticeable deviations from a random distribution of Fe\(^{3+}\) and Ti\(^{4+}\) cations over the octahedral sites, we calculate the Fe occupation of the outer site as thermal average,

\[
\langle c_{\text{Fe}}^{(\text{out})} \rangle (T) = \frac{1}{Z} \sum_i c_{\text{Fe},i}^{(\text{out})} e^{-E_i/k_B T},
\]

where \(E_i\) and \(c_{\text{Fe},i}^{(\text{out})}\) are the energy and outer site occupation of configuration \(i\), and \(Z = \sum_i e^{-E_i/k_B T}\).

We note that this is an extremely oversimplified model that considers only the 10 configurations that correspond to our choice of periodic unit cell and does not take into account any kinetic effects during growth. We therefore do not expect this model to accurately predict the outer site occupation in realistic samples. Nevertheless, this model gives a rough idea of the temperature range in which a preferential site occupation can be expected based on simple energetics.

Fig. 4 shows the resulting temperature dependence of \(\langle c_{\text{Fe}}^{(\text{out})} \rangle\), i.e. the average Fe occupation of the outer site. At zero temperature the value of \(\langle c_{\text{Fe}}^{(\text{out})} \rangle\) is determined by the lowest energy configuration in each case (compare with Fig. 3), i.e. zero Fe occupation of the outer site within PBEsol and within PBE using experimental lattice parameters, while \(\langle c_{\text{Fe}}^{(\text{out})} \rangle = 0.5\) for the fully relaxed PBE case, corresponding to all Fe cations sitting on an outer site. Furthermore, for \(T \rightarrow \infty\) all three cases converge to a value of 0.25, which corresponds to a completely random distribution of Fe\(^{3+}\) cations over outer and inner sites. However, it can be seen that within the given approximations a fully random site occupation is not reached even at temperatures significantly above 1000 K. Indeed, at a typical synthesis temperature of 1000 K (see e.g. Refs. 21, 13 or 35) less than 20% of outer sites are occupied by Fe\(^{3+}\) within PBEsol and when using experimental lattice parameters. This means that more than 60% of all Fe\(^{3+}\) cations can be found on the inner sites. Thus, our results suggest that it is possible to alter the occupation of outer and inner sites by varying growth or sintering temperatures, cooling rates, and annealing times during synthesis.

Experimentally, while no indications for a deviation from the random distribution of Fe\(^{3+}\) and Ti\(^{4+}\) cations have been found in Rietfeld refinements of neutron diffraction data, recent Mössbauer spectroscopy indicates a small preference of the Fe\(^{3+}\) cations for the inner perovskite sites, consistent with our PBEsol results. This preferential occupation of inner (outer) sites with Fe\(^{3+}\) (Ti\(^{4+}\)) is even stronger in systems with a larger number of perovskite-like layers (\(m > 5\)) within the series Bi\(_{n+1}\)Fe\(_{n-3}\)Ti\(_3\)O\(_{3n+3}\), consistent with an earlier extended x-ray absorption fine structure (EXAFS) analysis of the 5-layer system Bi\(_4\)Fe\(_2\)Ti\(_3\)O\(_{18}\). A preferential occupation of the outer site with Ti\(^{4+}\) cations, has also been reported for the related system Bi\(_5\)CrTi\(_3\)O\(_{15}\).

It has been argued that the Ti\(^{4+}\) cation prefers the more distorted outer site because of its \(d^0\) electron configuration, which exhibits a strong tendency for off-center displacement due to the second-order Jahn-Teller effect. Interestingly, our relaxations show that, while it is indeed energetically more favorable that Fe occupies the inner site (at least within PBEsol and at the experimental volume), the presence of Fe\(^{3+}\) on the outer site in fact creates a significantly more distorted coordination octahedra compared to the Ti\(^{4+}\) cation (see the above discussion on local tetragonality). Another possible explanation for the outer site preference of Ti\(^{4+}\), suggested in Ref. 37, is that it is electrostatically more favorable to have the higher charged Ti\(^{4+}\) cation located closer to the negatively charged oxygen in the fluorite layer. However, none of these considerations provide an obvious explanation for the observed volume/lattice constant dependence of the site preference. While it is not possible to analyze the reason for the observed site preference further based on the available data, future studies of similar systems and on the effects of synthesis conditions would be of interest to clarify this issue.

**B. Internal structural parameters and mode decomposition**

Next, we analyze the differences in the internal atomic coordinates of the various configurations listed in Table I. Due to the numerous internal degrees of freedom and different symmetries for the different configurations, we do not compare the individual atomic coordinates directly. Instead, we perform a symmetry mode decomposition of all relaxed structures, as well as of the experimen-
tally observed structure, using the same high symmetry $I4/mmm$ reference structure in all cases (with equivalent atoms on all octahedrally coordinated sites). This allows for a more systematic quantitative comparison of the most relevant degrees of freedom.39

The position of each atom $\vec{r}_i$ in the unit cell, can be written as the sum of its position in the high-symmetry reference structure $\vec{r}^0_i$ (here taken to be the paraelectric $I4/mmm$ structure) and a displacement vector $\vec{u}_i$:

$$\vec{r}_i = \vec{r}^0_i + \vec{u}_i .$$

(2)

The set of displacement vectors can then be expressed as linear combinations of mode vectors $\vec{\epsilon}_{i,m}$ with amplitudes $A_m$:

$$\vec{u}_i = \sum_m A_m \vec{\epsilon}_{i,m} .$$

(3)

Thereby, each mode vector can be chosen such that it transforms according to a single irreducible representation, characterized by the index $m$, of the symmetry group of the reference structure. In general, these mode vectors consist of several components — the symmetry adapted basis modes for that irreducible representation, which correspond for example to displacements of cations on different Wyckoff positions. Here, we only discuss the total mode amplitudes for each irreducible representation, which is calculated from the vectorial sum of all the individual components. For more details please refer to Ref. 39. In the present work, the mode decomposition is performed using the tool AMPLIMODES available from the Bilbao Crystallographic Server.19

In analogy to the case of the 2-layered Aurivillius system SrBi$_2$Ta$_2$O$_9$42 there are three distinct symmetry modes that are involved in the transition from the paraelectric $I4/mmm$ group to the experimentally observed ferroelectric $A2_{1}am$ group: the polar $\Gamma_5^-$ mode and two zone-boundary modes, $X_3^-$ and $X_2^+$ (see Fig. 5). However, as stated in Sec. 1A the configurations in this work exhibit symmetries lower than $A2_{1}am$, and therefore contain additional distortion modes. Nevertheless, as already mentioned at the end of Sec. 1A we find that the departure from $A2_{1}am$ symmetry is small. In particular, we find that for all configurations the three modes listed above are the most dominant ones, in most cases with significantly larger amplitude than the “minor” modes related to the further symmetry lowering from $A2_{1}am$.

Fig. 5b-d shows the character of the three modes. These graphs correspond to configuration M2, but the main features are similar for all configurations. It can be seen that two zone-boundary modes, $X_3^-$ and $X_2^+$, correspond to “tilts” of the oxygen octahedra within the perovskite-like layers around the in-plane [010] direction and “rotations” around the [001] direction, respectively. Furthermore, the polar $\Gamma_5^-$ mode consists of an overall displacement of the ions in the Bi$_2$O$_2$ layer relative to the ions in the perovskite layer along the in-plane [010] direction. This resembles the so-called “rigid layer mode”, which has been identified as unstable polar mode in the 2-layer Aurivillius system SrBi$_2$Ta$_2$O$_9$.42,43 Similar unstable (or very close to unstable) polar phonons have also been found in the 1-layer system Bi$_2$WO$_6$ (Refs. 44 and 45) and in the 3-layer system Bi$_4$Ti$_3$O$_{12}$.41,42 Additionally, polar instabilities, corresponding to A site or B site displacive modes within the perovskite layer, have been identified for Bi$_2$WO$_6$ and Bi$_4$Ti$_3$O$_{12}$, respectively.41,42 We note that the $\Gamma_5^-$ mode depicted in Fig. 5c, which corresponds to the distortion in the relaxed structure and not to a specific phonon instability, also exhibits a strong displacement of the B site cations in the inner perovskite layer relative to the center of their coordination octahedra. An analysis of phonon instabilities in Bi$_2$FeTi$_3$O$_{15}$ would therefore be instructive to find further analogies or differences within the Aurivillius series. However, such an analysis is complicated to perform for Bi$_2$FeTi$_3$O$_{15}$, due to the lack of a well-defined high-symmetry configu-
However, in the case of BiFeTi$_3$O$_{15}$, the paraelectric high temperature phase and a suitably defined centrosymmetric reference, the spontaneous polarization is defined as the difference in polarization between the ferroelectric ground state structure and a too strong influence on the relative mode amplitudes.

The amplitudes of the three main modes for all configurations are summarized in Fig. 6. It can be seen that even though there is some variation between the different configurations, the specific cation distribution does not have a too strong influence on the relative mode amplitudes. The most striking feature is the significant overestimation of the $\Gamma^−_5$ mode amplitude within PBE. This effect can be related to the larger unit cell volume within PBE, and is thus very similar to the behavior found in many other ferroelectrics, where it has been shown that the ferroelectric mode is often very sensitive to the volume and generally becomes more dominant for larger volumes (see e.g. Refs. 47–49). On the other hand, there are only small differences between PBE and PBEsol for the two X modes. Taking into account both lattice parameters and mode amplitudes, we can conclude that overall PBEsol leads to good agreement with the experimental structure reported in Ref. 21 and is therefore preferable to PBE for calculating the physical properties of Bi$_2$FeTi$_3$O$_{15}$ and related materials.

C. Electric polarization

We now calculate the spontaneous electric polarization of Bi$_2$FeTi$_3$O$_{15}$ using the so-called “Berry-phase” or “modern” theory of polarization, 50–52. Thereby, the spontaneous polarization is defined as the difference in polarization between the ferroelectric ground state structure and a suitably defined centrosymmetric reference. In most cases, the paraelectric high temperature phase provides a suitable reference structure for this purpose. However, in the case of Bi$_2$FeTi$_3$O$_{15}$ this corresponds to the tetragonal I4/mmm structure shown in Fig. 1 which requires a random distribution of Fe and Ti cations over the different B sites within each perovskite layer. As discussed in Sec. 1A, the specific distributions of Fe and Ti considered in our calculations lead to symmetries lower than I4/mmm, in some cases even to polar symmetries. In the following we therefore focus on two representative configurations, I1 and O1, corresponding to “inner” and “outer” cases, respectively. Both configurations correspond to distributions of Fe and Ti cations that do not break inversion symmetry and thus allow to define suitable centrosymmetric reference structures.

These centrosymmetric reference structures are constructed by removing the polar $\Gamma^−_5$, the $X^+_2$, and all minor distortion modes from the fully relaxed ferroelectric structures. For both configurations this leads to non-polar $P2_1/m$ symmetry, whereas the corresponding fully relaxed structures have polar $P2_1$ symmetry. We note that the spontaneous polarization does not depend on a specific choice for the reference structure as long as the different reference structures can be transformed into each other by a continuous deformation that conserves the insulating character of the system and does not break inversion symmetry.

Another important consequence of the modern theory of polarization follows from the arbitrariness in the choice of the unit cell. As a result, the polarization inherits the periodicity of the crystal and is only determined modulo a polarization quantum, $\Delta P_Q = eQ_\text{cell}/(4\pi)$, with $e$ the electronic charge, $Q$ the volume of the unit cell, and $Q_\text{cell}$ the shortest lattice vector along the polarization direction. Nevertheless, polarization differences between two structures that are related to each other by a small deformation are well defined as long as the corresponding change in polarization is small compared to the polarization quantum. If this is not the case, then intermediate deformations need to be considered until each polarization value can be unambiguously assigned to a specific “branch” of the “polarization lattice” (see e.g. Ref. 53). Note that in the present case the polarization quantum is rather small, $\Delta P_Q \sim 14 \mu C/cm^2$, since the unit cell volume is quite large (due to the large unit cell length along $\hat{c}$) whereas the polarization is pointing along the short in-plane direction $\hat{b}$.

Fig. 7 shows the calculated polarization using PBEsol for the two selected configurations as function of distortion amplitude connecting the centrosymmetric reference (0% distortion amplitude) with the fully relaxed ferroelectric structure (100% distortion amplitude). For each distortion amplitude we have included all values of the polarization lattice within a given range, the differences being exactly equal to the polarization quantum defined above. In order to identify the polarization values at different distortion amplitudes that correspond to the same branch of the polarization lattice (i.e., how to connect the individual points in Fig. 7 by lines), we evaluate a point charge estimate of the polarization using the formal charges of the ions (Bi$^{3+}$, Fe$^{3+}$, Ti$^{4+}$, O$^{2-}$). While
this might give only a rough estimate of the magnitude of the spontaneous polarization, it clearly shows whether a positive or negative change in polarization is expected with increasing distortion. In the present case this is sufficient to make an unambiguous assignment of branches without the need to consider further intermediate distortion amplitudes. The resulting branches are indicated by lines in Fig. 7. The spontaneous polarization $P_s$ corresponds to the difference between 0 and 100% distortion evaluated on the same branch. It can be seen that the polarization quantum (i.e., the distance between two neighboring branches) is indeed significantly smaller than $P_s$.

The obtained spontaneous polarizations, $P_s$, together with the point charge estimates, $P_{pc}$, and the $\Gamma_5$ mode amplitudes for both configurations are summarized in Table II. All cases we obtain large values for $P_s$ between 52 and 66 $\mu$C/cm$^2$. Due to the overestimation of the $\Gamma_5$ mode amplitude within PBE compared to experiment (1.34 Å), we expect the spontaneous polarization calculated using the PBEsol functional to be more accurate. We thus predict a spontaneous polarization of Bi$_3$FeTi$_3$O$_{15}$ around 55 $\mu$C/cm$^2$.

While this is larger than what has been observed so far experimentally (see e.g. Refs. 14 and 15), it is quite similar to the spontaneous polarization for the closely-related 3-layer Aurivillius compound Bi$_4$Ti$_3$O$_{12}$, as reported both from experiments ($P_s = 50$ $\mu$C/cm$^2$, Ref. 54) and first principles calculations ($P_s = 46$ $\mu$C/cm$^2$, Ref. 55). The difference between our calculated $P_s \sim 55$ $\mu$C/cm$^2$ and available experimental data is therefore likely due to the difficulty to measure fully saturated polarization loops in Bi$_3$FeTi$_3$O$_{15}$.

We also observe that the polarization of the I1 configuration is about 10% larger than the one of the O1 configuration, for both PBE and PBEsol. On the other hand, the amplitude of the polar $\Gamma_5$ mode is larger for the O1 configuration. This indicates that the $\Gamma_5$ mode is slightly more polar in the I1 configuration than in the O1 case, i.e. the mode effective charge (see e.g. Ref. 56) of the $\Gamma_5$ mode differs somewhat for the two configurations.

Overall, it is clear that while some subtle differences between the two different configurations can be observed, there is only a moderate influence of the specific Fe distribution on the ferroelectric properties. In particular, the presence of the magnetic cation is not detrimental to the ferroelectric polarization, which remains large compared to other nonmagnetic Aurivillius compounds.

Finally, we note that the large difference between the polarization calculated using the Berry phase approach and the point charge estimate based on formal charges indicates a highly anomalous value of the mode effective charge, which is enhanced by about 50-60% compared to its formal value. Such enhanced effective charges are indicative of ferroelectricity that is driven by hybridization effects.

### Table II. Spontaneous polarization $P_s$ (calculated using the Berry phase approach) and point charge estimate $P_{pc}$ (based on formal ionic charges) together with the amplitude of the polar $\Gamma_5$ mode for the two representative configurations, O1 and I1, calculated using PBE and PBEsol energy functionals.

| Configuration | PBE     | PBEsol  | $\Gamma_5$ |
|---------------|---------|---------|------------|
| O1            | 60.3    | 39.1    | 1.97       |
| I1            | 66.3    | 43.9    | 1.73       |

**D. Electronic structure**

Fig. 8 shows the electronic densities of states (DOS), total and projected on selected atomic and orbital characters, calculated using PBEsol for the two exemplary outer and inner configurations, O1 and I1. It can be seen that while there are some differences, the overall features...
of the densities of states are the same for the two cases. This indicates that the specific Fe/Ti distribution has only a marginal impact on the global electronic structure.

For both configurations we obtain a large bandgap of 1.7 eV (for $U_{Fe} = 3.0$ eV). The valence band is formed by states with predominant O-p character and the conduction band by states with predominant Ti/Fe-d character (and some Bi-p character at slightly higher energies). There is a strong splitting ($\sim 7.5$ eV) between the Fe-d states with different local spin projection, consistent with a high spin $d^5$ electron configuration of Fe$^{3+}$. On the other hand, the Ti-d states are essentially empty (apart from small admixtures in the valence band due to hybridization with the O-p states), consistent with a $d^0$ configuration of the Ti$^{4+}$ cation.

In order to test the impact of the Hubbard $U$ parameter on the electronic structure, we compare the DOS obtained with $U_{Fe} = 3.0$ eV (the value used throughout this paper) with the one obtained for a somewhat higher value of $U_{Fe} = 5.0$ eV. The larger $U_{Fe}$ increases the spin splitting of the Fe $d$ states and pushes the unoccupied local minority spin Fe $d$ states to higher energies, so that the lower conduction band edge becomes more dominated by the Ti $d$ states and the band gap increases to 2.1 eV. Note that an even larger value for $U_{Fe}$ will not lead to a further increase of the bandgap, which is then fixed by the energy difference between the occupied O $p$ and unoccupied Ti $d$ states.

E. Magnetic coupling constants

In this section we calculate the strength and character of the magnetic coupling between the Fe cations in different relative positions. For this purpose we fully relax all atomic positions and lattice parameters in each configuration for two different relative alignments of the magnetic moments of the two Fe$^{3+}$ cations within the unit cell: i) parallel, i.e., ferromagnetic (FM) alignment, and ii) antiparallel, i.e., antiferromagnetic (AFM) alignment. Fig. 9 shows the resulting total energies calculated using PBEsol (PBE leads to very similar results).

It can be seen that for all configurations where the two Fe$^{3+}$ cations are in nearest-neighbor (NN) positions within the perovskite layer (O3, M4, I2, and I3), there is a large energy difference between the FM and the AFM case. For the two configurations where the Fe$^{3+}$ cations are in next-nearest neighbor (NNN) positions (M3 and I1), there is only a small energy difference between FM and AFM, whereas in all other configurations the energy differences are negligible. In all cases (even the ones with essentially negligible energy difference between FM and AFM) the AFM orientation leads to lower energy than the FM orientation.

The strong NN coupling and the rather short range of the magnetic coupling is indicative of the superexchange mechanism, which is generally the dominant magnetic coupling mechanism in insulating oxides such as Bi$_2$FeTi$_3$O$_{15}$. Furthermore, the preferred AFM alignment between NN cations is consistent with the...
TABLE III. Calculated magnetic Heisenberg coupling constants for nearest and next-nearest neighbor coupling between the Fe$^{3+}$ cations. Also listed are the corresponding (average) Fe-O bond distances, $d_{\text{Fe-O}}$, and the deviation, $\Delta \phi$, of the Fe-O-Fe bond angle from the ideal value of 180° for the nearest neighbor configurations. Upper part (rows 2-4) corresponds to PBE calculations, lower part (rows 5-7) to PBEsol calculations.

| configuration | I2 | M4 | I3 | O3 | I1 | M3 |
|---------------|-----|----|----|----|----|----|
| $J$ (meV)     |     |    |    |    |    |    |
| $d_{\text{Fe-O}}$ (Å) |     |    |    |    |    |    |
| $\Delta \phi$ (°) |     |    |    |    |    |    |

For a more quantitative analysis of the strength of the magnetic coupling, we extract coupling constants $J_{ij}$ by mapping the calculated energy differences on a Heisenberg model, $E = \sum_{\langle i,j \rangle} J_{ij} \vec{e}_i \cdot \vec{e}_j$, where the sum is over all “bonds” and the normalized vector $\vec{e}_i$ indicates the direction of the magnetic moment of the magnetic ion with index $i$. For this purpose, both FM and AFM energies are calculated using the relaxed structures obtained with AFM alignment of magnetic moments, in order to remove the structural component from the determination of the magnetic coupling constants. Since we consider only coupling between Fe in NN and NNN positions, each coupling constant is associated to exactly one particular ionic configuration. The resulting coupling constants are summarized in Table III.

We obtain rather strong nearest neighbor couplings of the order of $J_{\text{NN}} \sim 40-50$ meV (except for configuration M4) and relatively weak next-nearest neighbor coupling of $J_{\text{NNNN}} \sim 1-2$ meV. The couplings are slightly stronger in PBEsol compared to PBE, due to the slightly smaller bond distances within PBEsol. However, the differences between the two functionals are not very large. The nearest neighbor coupling between inner and outer perovskite layers (configuration M4) is smaller than the other three nearest neighbor couplings due to the strong sensitivity of the superexchange interaction on the bond distances and bond angles connecting the two coupled magnetic cations. The Fe-O bond distances, averaged over the two Fe-O bonds connecting two nearest neighbor Fe cations ($d_{\text{Fe-O}}$), and the corresponding Fe-O-Fe bond angles ($\Delta \phi$) are also listed in Table III. It can be seen, that the average Fe-O bond-length between adjacent inner and outer Fe sites is significantly larger for configuration M4 than for the other nearest neighbor configurations. This is related to the strong elongation of the oxygen octahedra surrounding the outer Fe site.

The calculated nearest neighbor coupling between the Fe cations in Bi$_5$FeTi$_3$O$_{15}$ is comparable in both sign and magnitude to the coupling in BiFeO$_3$, $J = 50.3$ meV, which we calculate using PBEsol and $U_{\text{Fe}} = 3$ eV (compare also to Ref. 59). We also evaluate the impact of the Hubbard $U$ parameter on the strength of the magnetic coupling by recalculating the coupling constants for $U_{\text{Fe}} = 5.0$ eV using PBE. This has the effect of reducing the average $J$ by $\sim 30\%$. For example, the magnetic coupling corresponding to configuration I2 is $J = 43.0$ eV with $U_{\text{Fe}} = 5$ eV, and becomes $J = 31.2$ eV using $U_{\text{Fe}} = 5$ eV. Such a decrease of the magnetic coupling strength with $U_{\text{Fe}}$ is expected with superexchange as the dominant coupling mechanism. Within the simplest model of superexchange, the coupling strength is proportional to $t^2/U$ where $t$ is the effective hopping between Fe-$d$-like Wannier states and $U$ is the corresponding Hubbard interaction parameter. While there is no direct correspondence between this $U$ and $U_{\text{Fe}}$ used in the DFT+$U$ calculations, this simple picture is sufficient to explain the overall decrease of the magnetic coupling constants with $U_{\text{Fe}}$.

Due to the relatively low concentration of magnetic cations in Bi$_5$FeTi$_3$O$_{15}$ (25% on the octahedrally coordinated perovskite-like sites) and the short range of the magnetic superexchange interaction, it is unclear whether magnetic long range order can occur in this system. One requirement for long range order is that percolation can be achieved throughout the material, i.e. the majority of Fe cations need to be connected with each other through continuous links between coupled neighbors. In a three-dimensional simple cubic lattice, to which the 4-perovskite thick layers can be related, the threshold for percolation is 31% of magnetic sites when considering only NN couplings, and 14% with both NN and NNN couplings. Thus, with 25% of magnetic sites as in Bi$_5$FeTi$_3$O$_{15}$, percolation is dependent on the rather weak NNN coupling, which will probably give rise to magnetic order only at rather low temperatures. Furthermore, the perovskite-like layers are separated by Bi$_2$O$_2$ layers that do not contain any magnetically active ions. The shortest superexchange path connecting two Fe cations on opposite sides of the Bi$_2$O$_2$ layer is of the form Fe-O-O-Fe, containing two O-O links, which will only give rise to a very weak coupling. In this work we do not attempt to determine the strength of this coupling, but we point out that such interlayer coupling is of course necessary to achieve three-dimensional percolation, and thus magnetic long-range order, in Aurivillius compounds.

IV. SUMMARY AND CONCLUSIONS

In summary, we have calculated from first principles the basic structural, ferroelectric, and magnetic properties of the Aurivillius phase Bi$_5$FeTi$_3$O$_{15}$. We have also addressed a potential site preference of the Fe$^{3+}$ and Ti$^{4+}$ cations within the perovskite-like layers and discussed the
possibility for magnetic long-range order within this material.

After systematically comparing our results obtained within PBE and PBEsol with available experimental data, we conclude that the PBEsol exchange-correlation functional provides an excellent description of the structural degrees of freedom in Bi$_2$FeTi$_3$O$_{15}$, superior to PBE, and we suggest to use PBEsol for further DFT studies of related members of the Aurivillius family.

Our results show that there is a preference for the Fe$^{3+}$ cations to occupy the inner sites within the perovskite-like layers, consistent with recent Mössbauer data. We have also shown that this site preference depends strongly on the lattice constants and reverses to an outer site preference for the slightly more extended lattice parameters within PBE. In addition, the c/a ratio of the crystallographic unit cell is systematically larger for configurations where Fe$^{3+}$ occupies the outer sites. This suggests the possibility to control the site occupancies via epitaxial strain in thin films of Bi$_5$FeTi$_3$O$_{15}$.

Moreover, we calculate a large value of 55 $\mu$C/cm$^2$ for the spontaneous electric polarization, which shows that the presence of the nominally non-ferroelectric magnetic Fe$^{3+}$ cation does not impede the ferroelectricity in this Aurivillius system.

Finally, we show that there is a strong antiferromagnetic coupling between Fe$^{3+}$ cations in nearest neighbor positions, characteristic for the superexchange interaction between $d^5$ cations, but that the coupling between further neighbors is rather weak and essentially becomes negligible beyond second neighbors. With a concentration of 25% magnetic cations on the perovskite B site, the system is above the percolation threshold for a simple cubic lattice with both NN and NNN interactions, but it is unclear how the interleaving Bi$_2$O$_2$ layers will influence percolation in the Aurivillius structure and what ordering temperature can be expected based on the weak NNN coupling.

It seems that, in order to achieve robust magnetic order above room temperature, a higher concentration of magnetic ions would be desirable. One obstacle for achieving such higher concentrations is the relation between the average valence of the B site cations and the number of perovskite layers m throughout the Aurivillius series. With a formal charge of 3+ on the A site (corresponding to Bi$^{3+}$) the required average valence on the B site varies from 4+ for m = 3 to 3+ for m → ∞. The possibility to substitute Mn$^{4+}$ for Ti$^{4+}$ in the 3-layer structure has been studied by Tinte and Stachiotti. However, they found only incipient ferroelectricity in the fully substituted system Bi$_4$Mn$_3$O$_{12}$. Furthermore, the Mn$^{4+}$ cation has a rather small ionic radius, which falls outside the range that is considered suitable for incorporation on the B site of the Aurivillius structure. Magnetic cations that are suitable for incorporation on the B site within the Aurivillius structure generally have charge states of only 3+ or even 2+. One possible route to increase the concentration of magnetic cations is therefore to focus on compositions with a higher number of perovskite layers, i.e. m > 4. Indeed, magnetic long range order has been reported for several m = 5 systems. Another possibility to increase the Fe$^{3+}$ content (or more generally the content of magnetic 3+ cations) in the m = 4 layer system would be to simultaneously replace Ti$^{4+}$ by other B site cations with higher valence, such as e.g. Ta$^{5+}$, Nb$^{5+}$, or W$^{6+}$, leading to compositions such as e.g. Bi$_5$Fe$_{1+x}$Ti$_3$$_{2-x}$Nb$_2$O$_{15}$.

One can therefore conclude, that the Aurivillius family is indeed a promising class of compounds to search for new multiferroics with good ferroelectric and magnetic properties. The main challenge is to increase the content of magnetic cations, in order to obtain robust magnetic long range order at elevated temperatures. The Aurivillius family provides enough chemical flexibility to explore new compositions that are promising in that respect. Our results indicate that it is conceivable to have ferroelectricity coexist with long-range magnetic order, and we hope that our work will stimulate further research on Aurivillius compounds as potential room-temperature multiferroics.

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