$a_1$ meson-baryon coupling constants in the framework of soft-wall and hard-wall AdS/QCD models

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ABSTRACT

We calculate the $a_1$ meson-nucleon coupling constant in the framework of soft-wall and hard-wall AdS/QCD models. In the bulk of AdS space were written bulk interaction Lagrangians for a minimal gauge coupling, a magnetic gauge coupling and a triple coupling. To use AdS/CFT correspondence and these bulk interaction Lagrangians we calculate the $a_1$ axial-vector meson-nucleon coupling constant in the boundary of AdS space within both models. We observe that the numerical values for the $g_{a_1NN}$ coupling constant in the framework of both models are more close to the experimental value than results in other works. We also calculate the $g_{a_1\Delta\Delta}$ axial-vector meson-$\Delta$-baryon coupling constant in the framework of hard-wall AdS/QCD model.

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I. INTRODUCTION

The theoretical and experimental study of the coupling constants and form-factors are one of the most important problems of hadron physics. Some theoretical approaches such as chiral quark model, QCD sum rules and etc. are used for solving this problem.

The holographic principle for QCD also has important consequence to solve phenomenological problems of strong interaction, such as calculation of coupling and decay constants, form-factors, mass spectrum and etc. Since, at a low energy limit the ordinary perturbation theory does not work, for example in QCD the strong coupling constant get a high value in a small value of the transferred momentum. Thus, direct application of perturbative methods at a low energy limit to QCD is impossible. Holographic QCD does not run into such difficulties and is used to solve QCD problems without restraints to the transferred momentum and energy region. So, the holographic QCD is considered to be very effective method in QCD at a low energies.

There are two approaches in holographic QCD. First one is top-down and other one is bottom-up approaches. In the top-down approach of the holographic QCD, the QCD models based on the string and D-brane theories. The bottom-up approach is constructed according to direct application of the AdS/CFT principle to the theory of strongly interacting particles and named AdS/QCD models. The AdS/CFT principle is a correspondence between the fields in the 5-dimensional bulk of an anti-de Sitter (AdS) space with the field theory operators defined on the 4-dimensional ultraviolet (UV) boundary of AdS space \[1–9\]. There are two models of AdS/QCD: hard-wall and soft-wall models. These models include non-perturbative aspects such as, chiral symmetry breaking and confinement and were constructed under the finiteness condition of the 5D action at the infrared (IR) boundary of AdS space. In the hard-wall model this condition is provided by cut off the space at this boundary \[7, 10, 11\]. In the soft-wall model this condition is ensured by introducing an extra exponential factor to the integral expression over the extra dimension at infinite values of this dimension \[8, 9\].

After building the AdS/QCD models, the coupling constants and form-factors were calculated in the framework of these models \[7–20\].

In this paper, we calculate the \(g_{a_1 N N}\) \(a_1\)-axial-vector meson-nucleon coupling constant within the soft-wall and hard-wall AdS/QCD models and the \(g_{a_1 \Delta \Delta}\) \(a_1\)-axial-vector meson-spin 3/2 \(\Delta\)-baryon coupling constant in the framework of the hard-wall AdS/QCD model. Note, that this coupling constant was investigated in the framework of a hard-wall model by Maru and Tachibana in \[10\] and in the framework of a soft-wall model in \[19\] by Fang. In both cases, this constant was
calculated using only two-gauge interaction Lagrangian term contained in the covariant derivative of action and Pauli interaction Lagrangian term. So, we decided to take into account the triple interaction Lagrangian term that characterizes the interaction between the scalar, axial-vector and fermion fields in the bulk of AdS space. This kind of term is similar to the Yukawa interaction term and was included into calculation in [13], where was studied axial-vector form factor of nucleons. We have calculated the numerical values of this constant in the framework both-hard-wall and soft-wall models and within the different parameters of the chiral condensate, the quark mass and the infrared boundary parameters fixed on the AdS/QCD models, to determine the accuracy of the model and the effect of parameters.

We also calculated the \( g_{a_1,\Delta\Delta} \)-axial-vector meson-spin 3/2 \( \Delta \)-baryon coupling constant taking into account triple interaction Lagrangian in the framework of hard-wall AdS/QCD model and predict the numerical values for this coupling constant. Unfortunately, there is no experimental data for this constant to compare with them.

The present paper was arranged as follows: First, we describe basic features of AdS space, then we write profile functions for the axial-vector, nucleon and spin 3/2 \( \Delta \)-baryons in the framework of hard-wall AdS/QCD model. In third section we describe profile functions for the axial-vector and nucleon in the framework of soft-wall AdS/QCD model. In next section, we write a Lagrangian for the axial-vector meson-nucleon and the axial-vector meson-3/2 \( \Delta \)-baryon interactions in the bulk of AdS space and derive the boundary \( g_{a_1,NN} \) coupling constant within the soft-wall and hard-wall AdS/QCD models and \( g_{a_1,\Delta\Delta} \) coupling constant in the framework of hard-wall AdS/QCD model from the bulk Lagrangians. At last, we present a numerical results for these coupling constants in a Table form and make comparison of the obtained values with the other one.

II. THE HARD-WALL MODEL

As was noted above the finiteness condition of the 5D action at the infrared (IR) boundary of AdS space is provided by cut off the space at this boundary in the hard-wall model and so, action for the hard-wall model is written like this [3, 10, 11, 21]:

\[
S = \int_0^{z_m} d^4x dz \sqrt{g} \mathcal{L}(x,z),
\]

where \( g = |\det g_{MN}| \) \( (M,N = 0, 1, 2, 3, 5) \) and the fifth coordinate \( z \) varies in the range \( \epsilon \leq z \leq z_m (\epsilon \to 0) \). The metric of AdS space is chosen in Poincare coordinates:

\[
ds^2 = \frac{1}{z^2} (-dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu), \quad \mu, \nu = 0, 1, 2, 3,
\]
where $\eta_{\mu\nu}$ is a 4-dimensional Minkowski metric \[7, 10, 11, 21\]:

$$
\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1).
$$

(3)

A. $a_1$ meson in hard-wall model

According to the holographic QCD there is a correspondence between 4-dimensional axial-vector current and the 5-dimensional gauge fields. So, to obtain axial-vector-nucleon coupling constant in the boundary QCD, one need to introduce in the bulk of AdS space two gauge fields $A^M_L$ and $A^M_R$, which transform as a left and right chiral fields under $SU(2)_L \times SU(2)_R$ chiral symmetry group of the model \[10, 22\]. In the bulk of AdS space is also introduced the scalar $X$ field, which transform under the bifundamental representation of symmetry group. Due to the interaction of the bulk gauge fields with the scalar field $X$ the chiral symmetry group is broken to the isospin group $SU(2)_V$. An agreement to the AdS/CFT correspondence the bulk $SU(2)_V$ symmetry group is the symmetry group of the dual boundary theory and the $a_1$ meson is described by this representation of the $SU(2)_V$ group. From the $A^M_L$ and $A^M_R$ gauge fields the bulk vector and axial-vector fields are constructed as follow:

$$
V^M = \frac{1}{\sqrt{2}} (A^M_L + A^M_R),
$$

$$
A^M = \frac{1}{\sqrt{2}} (A^M_L - A^M_R).
$$

In common with vector mesons, according to the AdS/CFT correspondence the axial-vector mesons are the KK modes of the transverse part of the axial-vector gauge field.

Action for the gauge field sector have been written in terms of bulk vector and axial-vector fields as following in \[22\]:

$$
S = \int_0^{2\pi} d^5 x \sqrt{g} Tr \left\{ |DX|^2 + 3|X|^2 - \frac{1}{4g_5^2} Tr \left[ F^2_V + F^2_A \right] \right\}
$$

(4)

where $F_{MN} = \partial_M A_N - \partial_N A_M - i [A_M, A_N]$ is a field stress tensor, $A_M = A^a_M t^a$, $t^a = \sigma^a/2$ and $\sigma^a$ are Pauli matrices. The 5-dimensional coupling constant $g_5$ is related to the number of colors $N_c$ in the dual theory as $g_5^2 = 12\pi^2/N_c$ and for the dual boundary $SU(2)$ gauge group it has the value $g_5 = 2\pi$. The 4-dimensional transverse components of axial-vector field at UV boundary ($A_\mu(x, z = 0)$) correspond to the source for the axial-vector current and fluctuations of the transverse components bulk axial-vector field corresponds to the axial-vector mesons on the boundary. $X(x, z) = v(z) \frac{U(x, z)}{2}$ is a bulk scalar field, $U(x, z) = \exp(2it^a \pi^a (x, z))$ is a product of chiral field, $v(z) = \frac{1}{2} m_q z + \frac{1}{2} \sigma z^3$, where the coefficient $m_q$ is the mass of $u$ and $d$ quarks and the $\sigma$ is the value of chiral condensate \[22\]. The coefficients $m_q$ and $\sigma$ were established from the UV and IR boundary conditions on the solution for the $X$ field. Expanding $U(x, z)$ in powers of $\pi^a$ gives
the appropriate part of the action as

\[ S_{Axial}^{AdS} = \int_0^{z_m} d^5x \sqrt{g} Tr \left\{ -\frac{1}{4g_5^2 z} + \frac{v^2(z)}{2z^3} (A_M^a - \partial_M n^a)^2 \right\} \] (5)

Thus the \( S_{Axial}^{AdS} \) Lagrangian is bringing a \( z \)-depending mass to break the axial-vector gauge-symmetry like Higgs mechanism. It is useful to write the transversal part of bulk axial-vector gauge field \( A_{\perp \mu}^a(x, z) \) in momentum space by help of Fourier transformation. Equation of motion for Fourier components \( \tilde{A}_{\perp \mu}^a(p, z) \) is easily obtained from the action (5) \[10, 22\] and has the form

\[ z^3 \partial_z \left( \frac{1}{z} \partial_z \tilde{A}_{\mu}^a(p, z) \right) + p_2^2 z^2 \tilde{A}_{\mu}^a(p, z) - g_5^2 v^2 z^3 \tilde{A}_{\mu}^a(p, z) = 0. \] (6)

The \( \tilde{A}_{\perp \mu}^a(p, z) \) can be written as \( \tilde{A}_{\perp \mu}^a(p, z) = A_{\mu}^a(p) A(p, z) \) and at IR boundary \( A(p, z) \) satisfies the condition \( A(p, 0) = 1, A'(p, z_0) = 0 \) \[10, 22\]. For the \( n \)-th mode \( A_n(z) \) in the Kaluza-Klein decomposition \( A(p, z) = \sum_{n=0}^{\infty} A_n(z) f_n(p) \) with \( p^2 = m_n^2 \) the equation (6) has the form \[10\]:

\[ \left[ \frac{-m_n^2}{z} - \partial_z \left( \frac{1}{z} \right) \partial_z + \frac{2g_5^2 v^2}{z^2} \right] A_n(z) = 0, \] (7)

This equation of motion for the axial-vector field has a \( z \)-dependent mass term, so that it can not be analytically solvable. In the approximation the EOM is the same with vector meson, where the bulk mass is supposed to be the brane localized mass at QCD brane (\( z = z_m \)). The boundary condition at \( (z = z_m) \) is as follow \[10\]:

\[ 0 = \left( \partial_z + \frac{2g_5^2 v^2}{z^2} \right) A_n(z) \bigg|_{z=z_m}, \] (8)

by using from the boundary condition (8) normalized wave function for the first excited axial vector meson \( a_1 \) was found as below \[10\]:

\[ A_n(z) = \frac{z J_{1}(m_{a_1} z)}{\sqrt{\int_0^{z_m} dz [J_{1}(m_{a_1} z)]^2}} \] (9)

### B. Nucleons in hard-wall model

In the framework of soft-wall model nucleons were introduced in \[8, 9\] and within hard-wall model nucleons were introduced in \[11, 21\]. According to the AdS/CFT correspondence in sequence to represent nucleon operators in the boundary of the AdS space it is necessary to insert \( (\Psi_1(x, z)) \) and \( (\Psi_2(x, z)) \) two spinor fields for each nucleon operators in the bulk of AdS space, because at the boundary of AdS space each nucleon operators has the left- and right-handed components, which transform differently under the \( SU(2)_L \times SU(2)_R \) chiral symmetry group of QCD. Other two
chiral components are vanishing by the helping boundary conditions at IR boundary \cite{10, 11, 21}. Following \cite{10, 11, 21} we shall present here profile function for the spinor field in the framework of hard-wall model, which is constructed according AdS/CFT correspondence. The action for the bulk ($\Psi_1(x,z)$) spinor field without the interaction with the gauge fields is written as follows:

\[
S_{F_1} = \int_{0}^{m_0} d^4x dz \sqrt{g} \left[ \frac{i}{2} \bar{\Psi}_1 e^A_M D_M \Psi_1 - \frac{i}{2} (D_M \Psi_1) \Gamma^0 e^A_M \Gamma^A \Psi_1 - m_5 \bar{\Psi}_1 \Psi_1 \right],
\] (10)

where $D_B = \partial_B - \frac{i}{4} \omega_B^{MN} \Sigma_{MN} - i (A_L^5)_B T^a$ is the covariant derivative, $\Sigma_{MN} = \frac{1}{2} \Gamma_{MN}$ and $\Gamma_{MN} = \frac{1}{2} [\Gamma_M, \Gamma_N]$, $e^A_M$ is a vielbein and chosen as $e^A_M = \frac{1}{z} \eta^A_M$ and the non-zero components of spin connection $\omega_B^{MN}$ are $\omega_5 A^M = -\omega_5 A_5^M = \frac{1}{z} \delta^A_M (\mu = 0, 1, 2, 3)$. From the action (10) the equation of motion is obtained as a Dirac equations in the AdS space:

\[
(i \Gamma^A D_A \mp m_5) \Psi_{1,2} = 0,
\] (11)

The boundary term to obtain the equation of motion as Dirac equation is below:

\[
(\delta \bar{\Psi}_1 e^A_5 \Gamma^A \Psi_1) \bigg|_{z = 0} = 0.
\] (12)

$\Psi_z = 0$ is a condition to vanish an extra $\Psi_z$ degrees of freedom \cite{11, 21}. $\gamma^5 \Psi_L = \Psi_L$ and $\gamma^5 \Psi_R = -\Psi_R$ is a properties the left- and right-handed components of the spinor fields, Fourier transformation for them is written as follow:

\[
\Psi_{L,R}(x, z) = \int d^4p e^{-ip \cdot x} F_{L,R}(p, z) \psi_{L,R}(p)
\] (13)

where $\psi(p)$ is a 4D spinor field and $\psi_1(p)$ obeys the 4D Dirac equation

\[
\not{p} \psi_1(p) = |p| \psi_1(p)
\] (14)

where, $|p| = \sqrt{p^2}$ for a time-like four-momentum $p$ and thus the 5D Dirac equation (11) will lead to equations over the fifth coordinate $z$ for $f_{L,R}$ amplitudes:

\[
\left( \frac{\partial_z^2}{z} - \frac{4}{z} \frac{\partial_z}{z} + \frac{6 \pm m_5 - m_5^2}{z^2} \right) f_{L,R} = -p^2 f_{L,R}.
\] (15)

The equation (15) will give us relations between the profile functions $f_L(p, z)$ and $f_R(p, z)$ \cite{9}:

\[
\left( \frac{\partial_z}{z} + \frac{2 - m_5}{z} \right) f_{1L} = -pf_{1L},
\]

\[
\left( \frac{\partial_z}{z} - \frac{2 + m_5}{z} \right) f_{1R} = pf_{1R}.
\] (16)
Using the relation (16) we obtain the second order differential equations for the profile functions $f_{L,R}(p,z)$

\[
\left[ \frac{\partial^2}{\partial z^2} - \frac{4}{z} \frac{\partial}{\partial z} + \frac{6 + m_5 - m_5^2}{z^2} + p^2 \right] f_{1L}^n(z) = -m_n^2 f_{1L}^n(z),
\]

\[
\left[ \frac{\partial^2}{\partial z^2} - \frac{4}{z} \frac{\partial}{\partial z} + \frac{6 - m_5 - m_5^2}{z^2} + p^2 \right] f_{1R}^n(z) = -m_n^2 f_{1R}^n(z) = 0. \tag{17}
\]

The $n$-th normalized Kaluza-Klein mode $f_{L,R}^{(n)}(z)$ of the solutions $f_{L,R}$ with $p^2 = m_n^2$ can be expressed in terms of Bessel functions:

\[
f_{1L}^n(z) = c_1^n (z)^{\frac{5}{2}} J_2(m_n z),
\]

\[
f_{1R}^n(z) = c_1^n (z)^{\frac{5}{2}} J_3(m_n z) \tag{18}
\]

The constants $c_1^n$ are found from the normalization condition and are equal to

\[
|c_{1,2}| = \frac{\sqrt{2}}{z_m J_2(m_n z_m)}. \tag{19}
\]

There are following relations between the profile functions of first and second bulk fermion fields [10, 11, 21]

\[
f_{1L} = f_{2R}, \quad f_{1R} = -f_{2L}, \tag{20}
\]

For obtaining only a left-handed component of the nucleon from $\Psi_1$ the right-handed component of this spinor is eliminated by the boundary condition at $z = z_m$:

\[
\Psi_{1R}(x,z_m) = 0. \tag{21}
\]

This condition gives the Kaluza-Klein mass spectrum $M_n$ of excited states, which is expressed in terms of zeros $\alpha_n^{(3)}$ of the Bessel function $J_3$

\[
M_n = \frac{\alpha_n^{(3)}}{z_m}. \tag{22}
\]

The quantum number $n$ corresponds to the excitation number of a nucleon in the dual boundary theory.

III. THE SOFT-WALL MODEL

As was noted in introduction, in the soft-wall model the finiteness condition of the 5-dimensional action is ensured by introducing an extra exponential factor to the integral expression over the
extra dimension at infinite values of this dimension \([7–9]\). Thus, action for the soft-wall model in general is written in the form:

\[
S = \int_0^\infty d^4x dz \sqrt{g} e^{-\Phi(z)} \mathcal{L}(x, z),
\]

where \(g = |\det g_{MN}| (M, N = 0, 1, 2, 3, 5)\) and fifth dimension \(z\) varies in the range \(\epsilon \leq z < \infty\) \((\epsilon \to 0)\) \((z \to \infty)\) and \(\Phi (z) = k^2 z^2\) is the dilaton field and belong only to the soft-wall model \([7–9]\).

A. \(a_1\) meson in soft-wall model

In the framework of hard-wall model we have already introduce in the bulk of AdS space two gauge fields \(A^M_L\) and \(A^M_R\), which transform as a left and right chiral fields under \(SU(2)_L \times SU(2)_R\) chiral symmetry group, the scalar field \(X\) which transform under the bifundamental representation of this group and two bulk fermion fields having opposite signs of 5D mass \(M\) and gave common information about bulk and boundary fields.

To notice generalized common information for the model, here we introduce profile function for the \(a_1\)-meson within soft-wall model. In the framework of this model from the two gauge fields \(A^M_L\) and \(A^M_R\) bulk action was written in terms of bulk vector and axial-vector fields as below in \([7, 9]\):

\[
S_{\text{gauge}} = -\frac{1}{4 g_5^2} \int_0^\infty d^5x \sqrt{g} e^{-\Phi(z)} Tr \left[ F^2_L + F^2_R \right] = -\frac{1}{4 g_5^2} \int_0^\infty d^5x \sqrt{g} e^{-\Phi(z)} Tr \left[ F^2_V + F^2_A \right],
\]

To write the transverse part of the bulk axial-vector field \(A^{aT}_\mu(x, z)\) in momentum space by help of Fourier transformation, the equation of motion for Fourier components \(\tilde{A}^{aT}_\mu(p, z)\) is easily obtained from the action (24) \([19, 24]\):

\[
\partial_z \left( \frac{1}{z} e^{-k^2 z^2} \partial_z \tilde{A}^{aT}_\mu(p, z) \right) + p^2 \frac{1}{z} e^{-k^2 z^2} \tilde{A}^{aT}_\mu(p, z) = 0.
\]

The \(\tilde{A}^{aT}_\mu(p, z)\) can be written as \(\tilde{A}^{aT}_\mu(p, z) = A^{a}_\mu(p) A(p, z)\) and \(A(p, z)\) satisfies the condition \(A(p, \epsilon) = 1\) at UV boundary. In the Kaluza-Klein decomposition \(A(p, z) = \sum_{n=0}^\infty A_n(z) f_n(p)\) for the \(n\)-th mode \(A_n(z)\) with mass \(m_n^2 = p^2\) the equation (25) is written as follows:

\[
\partial_z \left( e^{-B(z)} \partial_z A_n \right) + m_n^2 e^{-B(z)} A_n = 0,
\]

where \(B(z) = \Phi(z) - A(z) = k^2 z^2 + \ln z\). After substitution

\[
A_n(z) = e^{B(z)/2} \psi_n(z)
\]
the equation (27) is convert to the Schroedinger equation form and has a solution with Laguerre polynomials $L^m_n$ as follows:

$$A_n^{\text{soft}}(z) = e^{-k^2 z^2/2} (kz)^{m+1/2} \sqrt{\frac{2n!}{(m+n)!}} L^m_n (k^2 z^2).$$

(28)

For the eigenvalues $m^2_n$ there is a linear dependence on the number $n$: $m^2_n = 4k^2(n + 2)$, which enables us to fix the free parameter $k$. In the AdS/CFT correspondence $m^2_n$ is identified with the mass spectrum of the vector mesons in the dual boundary QCD. For the $a_1$-meson we have $m = 1$ and the $A_n(z)$ becomes

$$A_n^{\text{soft}}(z) = k^2 z^2 \sqrt{\frac{2}{n+1}} L^1_n (k^2 z^2).$$

(29)

Notice that solution (29) was obtained for the free axial vector field case and does not take into account the back reaction of the bulk spinor field, which one was introduce in the next paragraph to describe nucleons.

B. Nucleons in soft-wall model

As was noted in previous section, in the framework of the soft-wall model nucleons were introduced in [8] and their excited states within this model were considered in [20]. According to [8] and [20] we present here some formulas of profile function for the spinor field in the soft-wall model. Within soft-wall model Lagrangian contains additional term $\Phi \Psi \Psi$, which describe coupling of a dilaton field with the bulk fermion fields [8] and the sign of this term for the second fermion field is chosen oppositely to the one for the first fermion [11]. The action for the first free spinor field is written as follows:

$$S_{F_1}^{\text{soft}} = \int_0^\infty d^4xdz\sqrt{g}e^{-\Phi(z)} \left[ \frac{i}{2} \bar{\Psi}_1 e^N_A \Gamma^A D_N \Psi_1 - \frac{i}{2} (D_N \Psi_1)^\dagger \Gamma^0 e^N_A \Gamma^A \Psi_1 - (M + \Phi(z)) \bar{\Psi}_1 \Psi_1 \right],$$

(30)

The equation of motion for the spinor field obtained from the action (30) is written as follows:

$$\left[ ie^N_A \Gamma^A D_N - \frac{i}{2} (\partial_N \Phi)e^N_A \Gamma^A - (M + \Phi(z)) \right] \Psi_1 = 0.$$  

(31)

To solve (31) for $\Psi$ in terms of left- and right-handed components we must notice $\bar{\Psi}_{L,R} = (1/2) (1 + \gamma^5) \Psi$ expression. In other side in momentum space $\bar{\Psi}_{L,R}$ are written like this: $\Psi_{L,R}(p, z) = z^\Delta \Psi^0_{L,R}(p) f_{L,R}(p, z)$. After notice a connection $p^\mu \Psi^0_R(p) = p^\mu \Psi^0_L(p)$ between boundary fields $\Psi^0_{L,R}(p)$ and consider that spinors in 5 dimension are the ones in 4 dimension, the equation
(31) is written as follows in [8]:
\[
\left( \partial_z - \frac{d/2 - \Delta + \Phi + (M + \Phi)}{z} \right) f_{1R} = -pf_{1L},
\]
\[
\left( \partial_z - \frac{d/2 - \Delta + \Phi - (M + \Phi)}{z} \right) f_{1L} = pf_{1R}.
\] (32)

To use the relation \( \Delta = \frac{d}{2} - M \) (\( d = 4 \) in our model) in (32) was obtained the differential equations for the profile functions \( f_{L,R}(p, z) \) [8]:
\[
\left[ \partial_z^2 - \frac{2}{z}(M + k^2 z^2) \partial_z + \frac{2}{z^2}(M - k^2 z^2) + p^2 \right] f_{1R} = 0,
\]
\[
\left[ \partial_z^2 - \frac{2}{z}(M + k^2 z^2) \partial_z + p^2 \right] f_{1L} = 0.
\] (33)

After solving the second order equation of motions (33) with \( p^2 = m_n^2 \), we get the expression for the \( f_{L,R}^{(n)}(z) \) \( n \)-th normalized Kaluza-Klein modes which correspond to the nucleons state:
\[
f_{1L}^{(n)}(z) = n_{1L}(kz)^{2\alpha} L_n^{(\alpha)}(kz),
\]
\[
f_{1R}^{(n)}(z) = n_{1R}(kz)^{2\alpha-1} L_n^{(\alpha-1)}(kz),
\] (34)

where the \( L_n^{(\alpha)} \) are the Laguerre polynomials, \( \alpha \) is related to the 5-dimensional mass \( M \) as \( \alpha = M + \frac{1}{2} \), the mass of the \( n \)-th mode \( m_n^2 = 4k^2(n + \alpha) \) and the constants \( n_{L,R} \) are found from the normalization condition \( \int dz \frac{e^{-k^2 z^2}}{z^2} f_{1L}^{(n)} f_{1L}^{(m)} = \delta_{nm} \) and are equal to
\[
n_{1L} = \frac{1}{k^{\alpha-1}} \sqrt{\frac{2\Gamma(n + 1)}{\Gamma(\alpha + n + 1)}},
\]
\[
n_{1R} = n_{1L} \sqrt{\alpha + n},
\] (35)

where \( \alpha = 2 \), because \( M = \frac{3}{2} \) and \( f_{1L} = f_{2R}, f_{1R} = -f_{2L} \), is a relations between the profile functions of the first and second bulk fermion fields [11].

IV. BULK INTERACTION LAGRANGIANS AND THE \( g_{a_1 NN} \) AND THE \( g_{a_1 \Delta \Delta} \) COUPLING CONSTANTS

In this section we calculate the \( g_{a_1 NN} \) constant in the framework hard-wall and soft-wall models and the \( g_{a_1 \Delta \Delta} \) coupling constant in the framework hard-wall model of AdS/QCD.

At first we calculate the \( g_{a_1 NN} \) constant. Then repeat all calculation steps for \( g_{a_1 \Delta \Delta} \) constant and give final expression and the numerical results for this constant.
A. The $g_{a_1NN}$ coupling constant in the framework of hard-wall model

Let us study $g_{a_1NN}$ axial-vector meson-nucleon coupling constant. For this aim we have used from the 5D action for the interaction between the all-axial-vector, fermion and scalar fields in the bulk of the AdS space, from the expression of generating function and from the expression of 4D axial-vector current of nucleons. Let us write interaction action in the bulk of AdS space:

$$ S_{int} = \int_0^{z_m} d^4xdz\sqrt{g}L_{int}. $$

(36)

According to the AdS/CFT correspondence there is connection between 5D bulk fields with 4D boundary particles. In our case axial-vector current of nucleons in 4D boundary correspond to the bulk axial-vector field in 5D bulk of AdS. According to holographic principle the 4-dimensional axial-vector current of nucleons is found by taking variation from the generating function $Z$ for the vacuum expectation value of the axial-vector field in dual theory boundary. In our case this principle will be written as

$$ <J_\mu> = -i\frac{\delta Z_{QCD}}{\delta \tilde{A}_\mu^0}|_{\tilde{A}_\mu^0=0}, $$

(37)

where $Z_{QCD} = e^{iS_{int}}$, $\tilde{A}_\mu^0 = \tilde{A}_\mu(q, z = 0) = A_\mu(q)$ is the boundary value of the axial-vector field ($A(z = 0) = 1$) and $J_\mu$ obtained from the variation of $exp(iS_{int})$ will be identified with the nucleon current and $\tilde{A}_\mu^0$ is the source for the current $J_\mu$. In other side in the boundary of AdS space 4D current of axial-vector field is written as follow:

$$ J_\mu(p', p) = g_{a_1NN}\bar{u}(p')\gamma^5\gamma_\mu u(p), $$

(38)

where, $q = p' - p$ is an energy-momentum conservation relation between 4D momenta $q$, $p'$ and $p$. From the equivalence of the right-hand side of (37) and (38) we get formula for the $g_{a_1NN}$ coupling constant as an integral expression over $z$ when two currents, the fermionic current on the boundary and the nucleon current are identified according to AdS/CFT correspondence.

Now, we need explicit form of $L_{int}$ interaction Lagrangian in the bulk of AdS space to put on the formula (36). The interaction Lagrangian is constructed based on the gauge invariance of using model and include different kinds of interaction terms between the bulk fields [10, 20, 21]. As we noted above this coupling constant was calculated using only two-gauge interaction Lagranian term contained in the covariant derivative of action and Pauli interaction lagranian terms, within a hard-wall model by Maru and Tachibana [10] and within a soft-wall model [19] by Fang. In addition this two interaction terms, using a new triple interaction term, which was introduced in [13] similar
to the Yukawa interaction term we have calculated $a_1$-axial-vector meson-nucleon coupling constant within both of soft-wall and hard-wall models. Since, the triple interaction Lagrangian term that characterize the interaction between the scalar, axial-vector and fermion fields in the bulk of AdS space is not taken into action to calculate the $a_1$-axial-vector meson-nucleon coupling constant within both of models. Thus we have used from the three interaction Lagrangian terms as we show below:

1) $\mathcal{L}_{\text{int}}$ contains a term of a minimal gauge interaction of the axial-vector field with the current of fermions $[10,21]$

\[
\mathcal{L}_{a1NN}^{(0)} = \frac{1}{2} \left[ \overline{\Psi}_1 \Gamma^M A_M \Psi_1 - \overline{\Psi}_2 \Gamma^M A_M \Psi_2 \right],
\]  \hspace{1cm} (39)

2) $\mathcal{L}_{\text{int}}$ include a magnetic gauge coupling of spinors with axial-vector field $[10,19,21]$

\[
\mathcal{L}_{a1NN}^{(1)} = \frac{i}{2} k_1 \left\{ \overline{\Psi}_1 \Gamma^{MN} F_{MN} \Psi_1 + \overline{\Psi}_2 \Gamma^{MN} F_{MN} \Psi_2 \right\},
\]  \hspace{1cm} (40)

where the field stress tensor of the axial-vector field is $F_{MN} = \partial_M A_N - \partial_N A_M$.

3) $\mathcal{L}_{\text{int}}$ is also contain from the triple interaction term, which was introduced in $[13]$

\[
\mathcal{L}_{a1NN}^{(2)} = g_Y \left( \overline{\Psi}_1 X \Gamma^M A_M \Psi_2 + \overline{\Psi}_2 X^\dagger \Gamma^M A_M \Psi_1 \right),
\]  \hspace{1cm} (41)

After substitution the interaction Lagrangian terms (39), (40), (41) in the action for interaction (36), then performing the integrals in momentum space and applying the holography principle this Lagrangian term gives the following contributions to $g_{a_1NN}^{\text{h.w.}}$ constant represented in terms of integral over the $z$. To know briefly contribution of all interaction term, to the coupling constant we carry out separately calculation for each interaction Lagrangian terms and get following formulas for $g_{a_1NN}^{\text{h.w.}}$ coupling constant within hard-wall model:

\[
g_{a1NN}^{(0)\text{h.w.}} = \frac{1}{2} \int_0^z \frac{dz}{z^4} A_0(z) \left( |f_{1R}^{(n)}(z)|^2 - |f_{1L}^{(m)}(z)|^2 \right),
\]  \hspace{1cm} (42)

where the $A_0$ is the profile function of the axial-vector field, $f_{1,2L,R}$ are the profile function of the nucleons and the superscript indices $n$ and $m$ indicate the number of excited states of the initial and final nucleons respectively.

\[
g_{a1NN}^{(1)\text{h.w.}} = \frac{k_1}{2} \int_0^z \frac{dz}{z^3} \partial_z A_0(z) \left( |f_{1L}^{(n)}(z)|^2 + |f_{1R}^{(m)}(z)|^2 \right),
\]  \hspace{1cm} (43)

\[
g_{a1NN}^{(2)\text{h.w.}} = 2 g_Y \int_0^z \frac{dz}{z^4} A_0(z) v(z) f_{1R}^{(n)}(z) f_{1L}^{(m)}(z),
\]  \hspace{1cm} (44)
Thus, the $g_{a_1NN_{h.w.}}$ constant will be sum of three terms:

$$g_{a_1NN_{h.w.}} = g_{a_1NN}^{(0)nmh.w.} + g_{a_1NN}^{(1)nmh.w.} + g_{a_1NN}^{(2)nmh.w.}. \quad (45)$$

We carry out a numerical analysis of these terms separately in the last section.

**B. The $g_{a_1NN}$ coupling constant within soft-wall model**

Let us study $g_{a_1NN}$ axial-vector meson-nucleon coupling constant in the framework of soft-wall model to get the effect of the model. For this aim we have wrote the 5D action for the interaction between the all-axial-vector, fermion and scalar fields in the bulk of the AdS space within soft-wall model as below:

$$S = \int_0^\infty d^4xdz \sqrt{g} e^{-\Phi(z)} \mathcal{L}(x,z) \quad (46)$$

and used from the expression of generating function and from the expression of 4D axial-vector current of nucleons. Since the 4D current of axial-vector field \( (37), (38) \) and explicit form of $\mathcal{L}_{int}$ interaction Lagrangian in the bulk of AdS space \( (39), (40), (41) \) are not dependent on the model, so we can use from the same expression for this quantity. After substitution the interaction Lagrangian terms \( (39), (40), (41) \) in a formula of interaction \( (46) \), then performing the integrals in momentum space and applying the holography principle this Lagrangian term gives the following contributions to $g_{a_1NN_{h.w.}}$ constant represented in terms of integral over the $z$. We also carry out separately calculation for each interaction Lagrangian terms and get following formulas for $g_{a_1NN_{s.w.}}$ coupling constant within soft-wall model:

$$g_{a_1NN}^{(0)nmss.w.} = \frac{1}{2} \int_0^\infty \frac{dz}{z^3} e^{-\Phi(z)} A_0(z) \left( |f_{1R}^{(n)ss}(z)|^2 - |f_{1L}^{(m)ss}(z)|^2 \right), \quad (47)$$

where the $A_0$ is the profile function of the axial-vector meson expressed as \( f_{1,2L,R}^s \) are the profile function of the nucleons within soft-wall model of AdS/QCD expressed as \( f_{1,2L,R}^s \) and the superscript indices $n$ and $m$ indicate the number of excited states of the initial and final nucleons respectively.

$$g_{a_1NN}^{(1)nmss.w.} = \frac{k_1}{2} \int_0^\infty \frac{dz}{z^3} e^{-\Phi(z)} \partial_z A_0(z) \left( |f_{1R}^{(n)ss}(z)|^2 + |f_{1L}^{(m)ss}(z)|^2 \right), \quad (48)$$

$$g_{a_1NN}^{(2)nmss.w.} = 2g_Y \int_0^\infty \frac{dz}{z^4} e^{-\Phi(z)} A_0(z) v(z) f_{1R}^{(n)ss}(z) f_{1L}^{(m)ss}(z), \quad (49)$$
Thus, we describe the $g_{a_1NN}^{s.w.}$ constant as the sum of three terms:

$$
g_{a_1NN}^{s.w.} = g_{a_1NN}^{(0)nms.w.} + g_{a_1NN}^{(1)nms.w.} + g_{a_1NN}^{(2)nms.w.}. \tag{50}
$$

We carry out a numerical analysis of these terms separately in the last section too.

Similarly the $g_{a_1NN}$ constant in the framework of hard-wall model, we study the $g_{a_1\Delta\Delta}$-axial-vector meson-spin $3/2$ $\Delta$-baryon coupling constant similar to ones for the $g_{a_1NN}$ constant. We have used interaction lagrangian terms for this coupling constant similar to ones for the $g_{a_1NN}$ constant:

$$
\mathcal{L}_{a_1\Delta\Delta}^{(0)} = \frac{1}{2} \left[ \nabla_1^N \Gamma^M A_M \Psi_{1N} - \nabla_2^N \Gamma^M A_M \Psi_{2N} \right],
\tag{51}
$$

$$
\mathcal{L}_{a_1\Delta\Delta}^{(1)} = \frac{i}{2} k_1 \left\{ \nabla_1^N \Gamma^{MN} F_{MN} \Psi_{1N} + \nabla_2^N \Gamma^{MN} F_{MN} \Psi_{2N} \right\},
\tag{52}
$$

$$
\mathcal{L}_{a_1\Delta\Delta}^{(2)} = g_Y \left( \nabla_1^N X \Gamma^M A_M \Psi_{2N} + \nabla_2^N X^\dagger \Gamma^M A_M \Psi_{1N} \right),
\tag{53}
$$

Boundary spin $3/2$ baryons is described by the $\Psi_{1,2}^N$ and $\Psi_{1,2N}$ Rarita-Schwinger fields in the bulk of AdS space. In much the same way by using $\text{[33, 34, 35]}$, the interaction Lagrangian terms and describing the integrals in momentum space, at last using AdS/CFT correspondence, we get the following contributions for $g_{a_1\Delta\Delta}$ constant. We carry out separately calculation for each interaction Lagrangian terms here too:

$$
g_{a_1\Delta\Delta}^{(0)nm} = \frac{1}{2} \int_0^{z_m} \frac{dz}{z^2} A_0(z) \left( |F_{1R}^{(n)}(z)|^2 - |F_{1L}^{(m)}(z)|^2 \right),
\tag{54}
$$

where $F_{1,2L,R}$ are the profile function of the $\Delta$ baryons and is given in $\text{[10, 11, 21]}$:

$$
F_{1L}^{(n)}(z) = C_1^n (z)^{\frac{5}{2}} J_2 (m_n z),
$$

$$
F_{1R}^{(n)}(z) = C_1^n (z)^{\frac{5}{2}} J_3 (m_n z)
\tag{55}
$$

and constants $C_1^n$ are found from the normalization condition:

$$
|C_{1,2}| = \frac{\sqrt{2}}{z_m J_2 (m_n z)}.
\tag{56}
$$

$$
g_{a_1\Delta\Delta}^{(1)nm} = \frac{k_1}{2} \int_0^{z_m} \frac{dz}{z^2} A_0(z) \left( |F_{1R}^{(n)}(z)|^2 + |F_{1L}^{(m)}(z)|^2 \right),
\tag{57}
$$

$$
g_{a_1\Delta\Delta}^{(2)nm} = 2g_Y \int_0^{z_m} \frac{dz}{z^2} A_0(z)v(z)|F_{1R}^{(n)}(z)|F_{1L}^{(m)}(z),
\tag{58}
$$

Thus, we describe the $g_{a_1\Delta\Delta}$ constant as the sum of three terms:

$$
\begin{align*}
g_{a_1\Delta\Delta}^{h.w.} &= g_{a_1\Delta\Delta}^{(0)nm} + g_{a_1\Delta\Delta}^{(1)nm} + g_{a_1\Delta\Delta}^{(2)nm}.
\tag{59}
\end{align*}
$$
V. NUMERICAL ANALYSIS

To carry out a numerical analysis for $g_{a_1NN}^{h.w.}$ coupling constant, we calculate the integrals for the expression $g_{a_1NN}^{(0)nmh.w.}$, $g_{a_1NN}^{(1)nmh.w.}$, and $g_{a_1NN}^{(2)nmh.w.}$, corresponding (42), (43), (44) and for $g_{a_1NN}^{s.w.}$ coupling constant, we calculate the integrals for the expression $g_{a_1NN}^{(0)nmw.s.}$, $g_{a_1NN}^{(1)nmw.s.}$ and $g_{a_1NN}^{(2)nmw.s.}$, corresponding (47), (48), (49) expressions. For this aim, to get numerical results we have used from the Matematica package and then was compare our results with in both model with the experimental and the other results adopted within the soft-wall and hard-wall models AdS/QCD. Also in our model the free parameters $k_1$, $g_Y$, $m_q$ and $\sigma$ should be fixed. The parameter $k_1 = -0.98$ was fixed from the fitting $g_{a_1NN}$ and $g_{\pi NN}$ coupling constants results within hard-wall model with the experimental data [21]. The constant $g_Y=9.182$ was fixed and taken from [23].

For the $\sigma$ and $m_q$ parameters we have used three different kind of values in the framework of AdS/QCD models in [10, 21, 24]: 1) the values $\sigma = (0.225)^3$ and $m_q = 0.00145$ were found from the fitting of the $\pi$ meson mass $m_\pi$ and its decay constant $f_\pi$ obtained in the soft-wall model framework with their experimental values in [24], 2) the values $\sigma = 0.00722$ and $m_q = 0.00234$ were found from the fitting the results for these constants obtained in the hard-wall model with their experimental values [10, 21, 24], 3) the values $\sigma = (0.311)^3$ and $m_q = 0.00234$ were found from the fitting the results for these constants obtained in the hard-wall model with their experimental values [10]. The all parameters of values and constants presented here are in GeV units.

$g_{a_1NN}^{exp} = 4.7 \pm 0.6$ value is given in [25] and it was measured in [26]. The hard-wall value $g_{a_1NN}^{h.w.} = 1.5 \sim 4.5$ was taken from the [10], the value $g_{a_1NN}^{h.w.} = -2.93$ also was taken from the [10] and the value $g_{a_1NN}^{h.w.} = 0.42$ was taken from the [10]. The soft-wall value $g_{a_1NN}^{s.w.} = 0.14$ was taken from the [19]. To have an idea of relative contributions of different terms of Lagrangian, we present results for $g_{a_1NN}^{(0)nm}$, $g_{a_1NN}^{(1)nm}$ and $g_{a_1NN}^{(2)nm}$ coupling constants in the framework of hard-wall model separately. The numerical results for the set 1) is given in Tables I and IV, for the set 2) it are shown in Tables II and V and for the set 3) it are shown in Tables III and VI.

Comparison of results shows that for all values of parameters results obtained here more close to the experimental data. We also notice that the our result for $g_{a_1NN}$ coupling constant is more sensitive to the value of parameter $\sigma$ and $m_q$ than parameter $k$. Unfortunately, there is no experimental data for the $g_{a_1NN}$ coupling constant in the case of excited states of nucleons, so we can not compare our results for this case.

For determination of $g_{a_1\Delta\Delta}$ coupling constant we need to calculate integrals for the $g_{a_1\Delta\Delta}^{(0)nm}$ and $g_{a_1\Delta\Delta}^{(1)nm}$ and $g_{a_1\Delta\Delta}^{(2)nm}$ appearing in equations (54), (57) and (58). We carry out this calculation and
give the results in TABLE III.

| n | $m_N$ | $m_{h.w.}^{N}$ | $g_{a_1NN}^{(0)nN}$ | $g_{a_1NN}^{(1)nN}$ | $g_{a_1NN}^{(2)nN}$ | $g_{a_1NN}^{exp}$ | $g_{a_1NN}^{s.w.}$ | $g_{a_1NN}^{h.w.}$ |
|---|-------|----------------|----------------------|------------------|------------------|----------------|---------------|-----------------|
| 0 | 0.94  | 1.089         | -0.652               | 7.958            | -17.46           | 4.7±0.6        | 0.14          | -2.93 (0.42)   |
| 1 | 1.44  | 1.323         | -0.017               | 3.995            | 0.36             | 4.338          | —             | —               |
| 2 | 1.535 | 1.556         | 0.137                | 3.221            | -2.431           | 0.927          | —             | —               |

TABLE I: Numerical results for $g_{a_1NN}$ coupling constant, for $k = 0.383$, $m_{a_1}^{h.w.} = 1.230$, $k_1 = -0.98$, $g_Y = 9.182$, $\sigma = (0.225)^3 \text{GeV}^3$ and $m_q = 0.00145 \text{GeV}$.

| n | $m_N$ | $m_{h.w.}^{N}$ | $g_{a_1NN}^{(0)nN}$ | $g_{a_1NN}^{(1)nN}$ | $g_{a_1NN}^{(2)nN}$ | $g_{a_1NN}^{exp}$ | $g_{a_1NN}^{s.w.}$ | $g_{a_1NN}^{h.w.}$ |
|---|-------|----------------|----------------------|------------------|------------------|----------------|---------------|-----------------|
| 0 | 0.94  | 1.089         | -0.652               | 7.958            | -11.184          | 4.7±0.6        | 0.14          | -2.93 (0.42)   |
| 1 | 1.44  | 1.323         | -0.017               | 3.995            | 0.249            | 4.227          | —             | —               |
| 2 | 1.535 | 1.556         | 0.137                | 3.221            | -1.536           | 1.822          | —             | —               |

TABLE II: Numerical results for $g_{a_1NN}$ coupling constant, for $k = 0.383$, $m_{a_1}^{h.w.} = 1.230$, $k_1 = -0.98$, $g_Y = 9.182$, $\sigma = 0.00722 \text{GeV}^3$ and $m_q = 0.00234 \text{GeV}$.

| n | $m_{\Delta}$ | $g_{a_1\Delta\Delta}^{(0)n\Delta}$ | $g_{a_1\Delta\Delta}^{(1)n\Delta}$ | $g_{a_1\Delta\Delta}^{(2)n\Delta}$ | $g_{a_1\Delta\Delta}^{s.w.}$ |
|---|---------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------|
| 0 | 1.232         | -0.983                             | 8.197                             | 10.109                            | 17.316          |
| 1 | 1.700         | 0.263                              | 4.231                             | 6.658                             | 11.153          |
| 2 | 1.920         | 0.473                              | 2.232                             | 2.684                             | 5.389           |

TABLE III: Numerical results for $g_{a_1\Delta\Delta}$ coupling constant, for $k = 0.383$, $m_{a_1}^{h.w.} = 1.230$, $k_1 = -0.98$, $g_Y = 9.182$, $\sigma = (0.311)^3 \text{GeV}^3$ and $m_q = 0.00234 \text{GeV}$.
### TABLE IV: Numerical results for $k = 0.383$, $m_{a_1}^{s.w.} = 1.230$, $k_1 = -0.98$, $g_Y = 9.182$, $\sigma = (0.225)^3 \text{ GeV}^3$ and $m_q = 0.00145$ GeV.

| $n$ | $m_N$ | $m_{a_1}^{s.w.}$ | $g_{a_1NN}^{(0)\text{nm}}$ | $g_{a_1NN}^{(1)\text{nm}}$ | $g_{a_1NN}^{(2)\text{nm}}$ | $g_{a_1NN}^{\text{exp.}}$ | $g_{a_1NN}^{s.w.}$ | $g_{a_1NN}^{h.w.}$ |
|-----|-------|------------------|-------------------------|-------------------------|-------------------------|-------------------------|----------------|----------------|
| 0   | 0.94  | 1.089            | -0.09                   | -1.588                  | 6.088                   | 4.41                    | 4.7±0.6       | 0.14           | -2.93 (0.42)   |
|     |       |                  |                         |                         |                         |                         |                 |                | 1.5 ~ 4.5      |
| 1   | 1.44  | 1.323            | -0.068                  | -2.073                  | 24.275                  | 22.135                  | —              | —              |
| 2   | 1.535 | 1.556            | -0.056                  | -2.468                  | 54.563                  | 52.039                  | —              | —              |

### TABLE V: Numerical results for $k = 0.383$, $m_{a_1}^{s.w.} = 1.230$, $k_1 = -0.98$, $g_Y = 9.182$, $\sigma = 0.00722$ GeV$^3$ and $m_q = 0.00234$ GeV.

| $n$ | $m_N$ | $m_{a_1}^{s.w.}$ | $g_{a_1NN}^{(0)\text{nm}}$ | $g_{a_1NN}^{(1)\text{nm}}$ | $g_{a_1NN}^{(2)\text{nm}}$ | $g_{a_1NN}^{\text{exp.}}$ | $g_{a_1NN}^{s.w.}$ | $g_{a_1NN}^{h.w.}$ |
|-----|-------|------------------|-------------------------|-------------------------|-------------------------|-------------------------|----------------|----------------|
| 0   | 0.94  | 1.089            | -0.09                   | -1.588                  | 3.896                   | 4.7±0.6                 | 0.14           | -2.93 (0.42)   |
|     |       |                  |                         |                         |                         |                         |                 |                | 1.5 ~ 4.5      |
| 1   | 1.44  | 1.323            | -0.068                  | -2.073                  | 15.461                  | 13.321                  | —              | —              |
| 2   | 1.535 | 1.556            | -0.056                  | -2.468                  | 34.696                  | 32.172                  | —              | —              |

### TABLE VI: Numerical results for $k = 0.383$, $m_{a_1}^{s.w.} = 1.230$, $k_1 = -0.98$, $g_Y = 9.182$, $\sigma = (0.311)^3 \text{ GeV}^3$ and $m_q = 0.00234$ GeV.

| $n$ | $m_N$ | $m_{a_1}^{s.w.}$ | $g_{a_1NN}^{(0)\text{nm}}$ | $g_{a_1NN}^{(1)\text{nm}}$ | $g_{a_1NN}^{(2)\text{nm}}$ | $g_{a_1NN}^{\text{exp.}}$ | $g_{a_1NN}^{s.w.}$ | $g_{a_1NN}^{h.w.}$ |
|-----|-------|------------------|-------------------------|-------------------------|-------------------------|-------------------------|----------------|----------------|
| 0   | 0.94  | 1.089            | -0.09                   | -1.588                  | 16.038                  | 14.36                   | 4.7±0.6       | 0.14           | -2.93 (0.42)   |
|     |       |                  |                         |                         |                         |                         |                 |                | 1.5 ~ 4.5      |
| 1   | 1.44  | 1.323            | -0.068                  | -2.073                  | 64.028                  | 61.88                   | —              | —              |
| 2   | 1.535 | 1.556            | -0.056                  | -2.468                  | 143                     | 141.4                   | —              | —              |

### VI. SUMMARY

In present letter we calculated the strong coupling constant of $a_1$ mesons with nucleons within the hard-wall and soft-wall models AdS/QCD. We found that the predictions of these models for $g_{a_1NN}$ coupling constant are close to experimental value. The $g_{a_1NN}$ coupling constant is more sensitive to the value of parameter $\sigma$ and $m_q$ than parameter $k$ within both of models. Unfortunately, there is not experimental data for the $g_{a_1NN}$ coupling constant in the excited states of nucleons, so we did not compare our results for this case.

We also calculated the $g_{a_1\Delta\Delta}$-axial-vector meson-spin 3/2 $\Delta$-baryon coupling constant in the
framework of hard-wall AdS/QCD model and predict the numerical values for this coupling constant.
[25] V.G.J. Stoks and Th.A. Rijken, Nucl.Phys. A613 (1997) 311, arXiv:nucl-th/9611002

[26] O. Andreev $1/q^2$ Corrections and Gauge/String Duality Phys.Rev. D73 (2006) 107901