Spin transport and spin relaxation in semiconductor heterostructures with Rashba spin–orbit interaction

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Spin diffusion equations, which also contain the coupling between spins and charges, are discussed for semiconductor heterostructures with Rashba spin–orbit interaction in the presence of an electric field. The diffusion equations are applied to an investigation of the decay of a homogeneous initial magnetization. A field induced precession, which is observable if the electric field exceeds a characteristic field, is obtained. Moreover, the impact of quantum corrections on the spin transport coefficients is discussed.

1 Introduction

Investigations on the impact of the Rashba spin–orbit interaction on the spin and charge transport properties of non-magnetic semiconductors are of much current interest. The Rashba interaction is the conventional spin–orbit interaction with a constant electric field in a heterostructure based two-dimensional electron gas. An attractive feature of this interaction is the fact that its strength can be changed electrically (see, e.g. Ref. [1]). This fact has given rise to the vision that the electron spin can also be exploited in electronic devices, which are based on non-magnetic semiconductors [2].

However, the same features, which render systems with Rashba spin–orbit interaction attractive, lead also to the decay of the magnetization. The decay is caused by the coupling between the spin and the momentum of the particles, which is provided by the spin–orbit interaction. Due to this coupling the spin of every particle precesses around a different axis. Therefore, the magnetization dephases in a short amount of time. In systems with strong Rashba spin–orbit interaction, in which the spins perform many revolutions before the particles are scattered, the decay proceeds on a time scale, which is at most of the order of the momentum relaxation time [1]. Really long relaxation times can only be observed in systems with weak spin–orbit interaction, in which the particles are scattered so rapidly that the axis of the precession is changed before the spin can appreciably rotate. In this case the relevant mechanism, which governs the spin relaxation, is the Dyakonov–Perel mechanism [1]. This mechanism is characterized by spin relaxation times, which are large compared to the momentum relaxation time. The spin transport proceeds diffusely in this limit.

Today both systems with strong and weak Rashba spin–orbit interaction can be fabricated. The first have the advantage that steady state magneto-electric effects, like the spin accumulation induced by a constant electric field and the spin-Hall effect, are particular large in them. Therefore, these systems are particular popular in the moment. However, successful applications in electronic devices require also electric spin manipulation. Since the non-equilibrium magnetization in such systems decays on a time...
scale of the order of the momentum relaxation time the spin manipulation requires pulse techniques with pulses considerable shorter than the momentum relaxation time. This does not seem to be realistic today. Therefore, we expect that systems with weak Rashba spin–orbit interaction, in which the characteristic relaxation times are large compared to the momentum relaxation time, are more attractive for potential applications.

The investigation of spin transport properties in systems with weak spin–orbit interaction requires the derivation of the spin diffusion equations for such systems in the presence of external fields. Below we derive the diffusion equations in the presence of a constant electric field for a system with impurity scattering, apply this equation to the investigation of spin relaxation phenomena and investigate the impact of quantum corrections on the spin transport coefficients.

2 The model

Our investigation is based on the Hamiltonian

$$H = \frac{p^2}{2m} + (N, \sigma \times p) + V(x) + (F, x),$$  \hspace{1cm} (1)

where $V(x)$ is a random potential with Gaussian distribution function and standard deviation

$$\langle V(x) V(x') \rangle = \frac{h}{2\nu \tau} \delta(x - x'),$$  \hspace{1cm} (2)

$v = m/2\pi\hbar^2$ is the single particle density of states per spin, $\tau$ is the momentum relaxation time, $N = N_e$ is a vector perpendicular to the two-dimensional plane, $p$ is the momentum operator and $\sigma = \sigma_x, \sigma_y, \sigma_z$ is the spin operator. $e_x$ and $e_y$ are the unit vectors in the 2d plane. $e_z$ is a vector perpendicular to the plane. The labels $x$, $y$ and $z$ and the numbers 1, 2 and 3 are used simultaneously to identify components of vectors and tensors, respectively. $F$ is related to the electric field $E$ by the relationship $F = -eE$, where $e$ is the electric charge.

3 The diffusion equation

A diffusion equation, which takes also into account the coupling between spins and charges, has first been derived in the Refs. [3] and [4] for systems without external fields. Here we present a generalization of this equation to the presence of an electric field. For the derivation of the spin diffusion equation we use the ladder approximation [5]. In this approximation the diffusion equation can be deduced from a gradient expansion of the product of a configuration averaged retarded Green’s function and a configuration averaged advanced Green’s function. The Green’s functions are calculated in Born approximation. Details of the calculation are given in Ref. [6]. The calculation yields the spin diffusion equation

$$\partial_S S + R \times (S - S_e) - D \Delta S + D(F, V) \partial_S S - \omega_d (N \times \nabla) \times (S - S_e) = -\Omega \cdot (S - S_e) + \partial_S j_e.$$  \hspace{1cm} (3)

Here $S = S(x | \epsilon, t)$ is the density of spins of particles with kinetic energy $\epsilon$ at position $x$ at time $t$, $D = \tau e/m$, $\omega_d = 4mD/\hbar$ and

$$R = \frac{1}{2} \frac{d\omega_d}{d\epsilon} F \times N.$$  \hspace{1cm} (4)

1 The quantity $\epsilon$ is identical to the quantity $\mu$ of Ref. [6]. The $x$-dependence in Eq. (3) characterizes the shape of the packet.

If we take into account this fact we can write Eq. (49) of Ref. [6] in the form (3).
\( \Omega \) is a symmetric second rank tensor with matrix elements \( \Omega_{ik} = \Omega_{ki} (1 + \delta_{ik}) \) and \( j_{\epsilon} \) is the energy current \( (\Omega = 4N^mD/D\epsilon^2) \). The spin accumulation \( S_\epsilon = S_\epsilon(x, \epsilon, t) \) is given by

\[
S_\epsilon = -\tau N \times \nabla n + \tau N \times F \delta_{\epsilon,n} .
\]  

(5)

Here \( n = n(x | \epsilon, t) \) is the density of particles with kinetic energy \( \epsilon \) at position \( x \) and time \( t \).

Equation (3) shows that the electric field has much impact on spin transport properties. Apart from the conventional drift term (fourth term on the left hand side (lhs) of Eq. (3)) the field leads to a number of new terms. First, it leads to a field induced rotation of the non-equilibrium magnetization (second term on the lhs of Eq. (3)). Equation (3) shows, that only the deviation of the non-equilibrium magnetization from the spin accumulation \( S_\epsilon \) is subjected to the latter. The spin accumulation itself can be induced either by an electric field [7] or by a density gradient [4]. Equation (5) generalizes the existing results on the spin accumulation in an electric field to non-homogeneous particle densities. Second, the spin accumulation appears in a shift in the fifth term on the lhs of Eq. (3). We would like to mention that this fact is responsible for that there is no spin-Hall effect in this system, in line with the results in the literature. In the absence of the electric field this term drops out since \( N \times \nabla \| S_\epsilon \) in this case. Therefore, this term is absent in the Refs. [3] and [4]. From the physical point of view, however, we expect that the results which are obtained from a density gradient and an electric field are identical in the ohmic approximation. This fact suggests that the shift should better be taken into account also in the absence of an electric field to assure that the spin current is extracted correctly from the spin diffusion equation.

**4 Field induced rotation in a homogeneous system**

Here we consider the relaxation of a homogeneous non-equilibrium magnetization, which results from particles at the Fermi surface. In this case \( S(x | \epsilon, t) = S(t) \delta(\epsilon - \epsilon_r) \) and \( n(x | \epsilon, t) = 2\epsilon \theta(\epsilon_r - \epsilon) \), where \( \epsilon_r \) is the Fermi energy. To obtain a closed equation for \( S(t) \) we insert our ansatz into the diffusion equation and integrate the diffusion equation with respect to energy. Doing so, we obtain the following simple equation

\[
\frac{d}{dt} \delta S + R \times \delta S + \Omega \cdot \delta S = 0 .
\]  

(6)

Here \( \delta S \) is the deviation of the non-equilibrium magnetization from the spin accumulation.

Equation (6) shows that the impact of the field induced rotation on the spin relaxation depends on the interplay between the second and the third term on its lhs. For weak electric fields the decay dominates, viz the magnetization decays simply exponentially. However, if the electric field exceeds a characteristic field \( E_c \) the character of the decay changes [8]. For \( |E_c| > |E| \) we obtain damped precessions, which are superimposed to the decay. For such fields the solution to Eq. (6) takes the form

\[
\delta S_r(t) = e^{-\Omega_{2\omega_r}} \left( \frac{\Omega}{2\omega_r} \sin (\omega_r t) + \cos (\omega_r t) \right) \delta S_r(0) - \frac{R_c}{\omega_r} \sin (\omega_r t) \delta S_r(0) ,
\]  

(7)

\[
\delta S_r(t) = e^{-\Omega_{2\omega_r}} \left( -\frac{\Omega}{2\omega_r} \sin (\omega_r t) + \cos (\omega_r t) \right) \delta S_r(0) + \frac{R_c}{\omega_r} \sin (\omega_r t) \delta S_r(0) ,
\]  

(8)

where \( R_c = -2\tau NF/\hbar \) and

\[
\omega_r = \frac{\Omega}{2} \sqrt{\left( \frac{E}{E_c} \right)^2 - 1} .
\]  

(9)
The characteristic field is given by

\[ E_c = \pi \frac{N \hbar}{e} n_p, \]  

(10)

where \( n_p \) is the particle density. We would like to note that a similar rotation can also be observed in the hopping regime [9].

The question, whether the field induced rotation can also be observed in experiments or not, depends on the magnitude of the characteristic field. In many heterostructures based on indium combinations, like in InAlAs/InGaAs, InP/InAs, InAlSb/InAs systems, the magnitude of \( N \) ranges between \( \hbar N = 10^{-11} \ldots 10^{-12} \text{ eV m} \) [1]. Typical particle densities are ranging between \( 10^{11} \ldots 10^{12} \text{ /cm}^2 \). Thus, the characteristic field ranges between 30 V/cm and 3000 V/cm. Accordingly, we expect that damped precessions can easily be observed at least in systems with \( E_c = 30 \text{ V/cm} \). However, we would like to mention that in many systems containing indium the Rashba interaction is actually strong, so not all of them are suitable for an experimental verification of the field induced precession. Much more favourable conditions are met in Si/SiGe quantum well structures. In such systems the Rashba interaction is 10000 times smaller than in the systems mentioned before (see, e.g. Ref. [10]). Accordingly, we expect that the field induced precession can be observed in such systems already for extremely weak fields.

## 5 Quantum corrections

An important tool in the investigation of the structure of the spin–orbit interaction is the investigation of the transition from weak localization to antilocalization in the conductivity of such systems. Such investigations have been used by a number of experimental groups to find out what the structure of the spin–orbit interaction is and how large the coupling constants are. The weak localization and antilocalization effects themselves result from the interplay between quantum interferences and inelastic scattering events. However, while much attention has been paid to investigations on the impact of quantum interferences on charge transport properties little is known about their impact on spin transport properties.

The consideration of quantum interferences requires to go beyond the ladder approximation, which was used in Section 3, viz to take into account also the so called maximally crossed diagrams in the derivation of the diffusion equation (see, e.g. Ref. [11]). In this case the non-linear \( \sigma \)-model is a convenient tool for the calculation [11]. To get some insight into the impact of quantum interferences on the spin transport coefficients we have generalized the non-linear \( \sigma \) model to the presence of the Rashba spin–orbit interaction and the presence of an electric field [8] and have calculated the transport coefficients in one-loop approximation. Our investigation shows that the impact of the quantum interferences on spin transport properties differs from that on charge transport coefficients. The \( zz \)-component of the relaxation tensor \( \Omega^{(ii)} \), e.g., satisfies the relationship

\[ \frac{\Omega^{(ii)}_{zz}}{\Omega_{xz}} = 1 - \frac{1}{8\pi^2 D v \hbar} \left( \ln \left( \frac{\tau_{\phi}(T)}{\tau} \right) + \ln \frac{1}{\sqrt{(\tau/\tau_{\phi}(T))^2 + 3\Omega^2\tau^2/\tau_{\phi}(T) + 2\Omega^2\tau^2}} \right) \]  

(11)

in one-loop approximation (\(^{(ii)}\) indicates that the quantity is calculated in one-loop approximation and \( \tau_{\phi}(T) \) is a phenomenological temperature dependent phase relaxation time). Similar expressions are also obtained for the other components of the tensor \( \Omega^{(ii)} \), for the corrections to the spin diffusion coefficients and for the corrections to the quantity \( \phi \).

Equation (11) shows that there is no transition to an antilocalizing regime in the spin transport coefficients. The spin transport coefficients decrease continuously with decreasing temperature (increasing \( \tau_{\phi}(T) \)) at low temperatures. Thus, the relaxation times extracted from spin transport measurements are different from those extracted from charge transport measurements at low temperatures. Due to this difference spin packets spread slower than particle packets. Moreover, the lifetime of the magnetization increases continuously with decreasing temperature at low temperatures due to quantum interference.
effects. This fact reflects itself also in the field induced precession. Its frequency decreases with decreasing temperature at low temperatures (see Eq. (9)). The characteristic field (Eq. (10)), however, is temperature independent.

In contrast to the spin transport coefficients the spin accumulation (5) results from the coupling between spin and charge. It is therefore not a priori clear, whether the relaxation time in Eq. (5) is the relaxation time for spin or charge. A detailed investigation shows, that $S$ decreases also continuously with decreasing temperature [8]. Thus, the relaxation time in Eq. (5) is the spin relaxation time.

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