Shadowing in multiparton proton – deuteron collisions

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We study the screening effect for the multiparton interactions (MPI) for proton–deuteron collisions in the kinematics where one parton belonging to the deuteron has small $x_1$ so the leading twist shadowing is present while the second parton ($x_2$) is involved in the interaction where shadowing effects are small. We find that the ratio of the shadowing and the impulse approximation terms is approximately factor of two larger for MPI than for the single parton distributions. Overall shadowing leads to a strong reduction of the double parton antishadowing effect due to the independent interactions of the partons of the protons with two nucleons of the deuteron. For example, for the resolution scale $Q^2_1 \sim 4 \text{ GeV}^2$ of the interaction with parton $x_1$ we find that shadowing reduces the DPA effect by $\sim 30\%$.

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I. INTRODUCTION

Recently there was a renewed interest to the theoretical studies of the multiparton interactions (MPI) in which at least two partons of one of the colliding particles are involved in the proton - nucleus collisions\textsuperscript{[1–7]}. To large extent this is due to the first experimental studies of such collisions at the LHC \textsuperscript{[8–11]}. It was suggested in \textsuperscript{[1–4]} that MPI would be easier to observe experimentally in $pA$ collisions than in $pp$ collisions since they are parametrically enhanced in the pA case by a factor $A^{1/3}$ \textsuperscript{[1]}. General formulae for this cross section were derived in \textsuperscript{[2]} within perturbative QCD (pQCD) in the impulse approximation (that is neglecting deviations of the nuclear parton distribution functions (pdf) from the additive sum of the nucleon pdfs). The analysis demonstrated connection of the pQCD treatment to the parton model calculation of \textsuperscript{[1]} for the large A limit and uncorrelated nucleon distribution in the nucleus.

The calculation of \textsuperscript{[2]} employed the formalism developed in Refs. \textsuperscript{[12–15]}, which is based on the use of the generalized double parton distributions in momentum space introduced in Ref. \textsuperscript{[12]}. 

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The calculation was done explicitly in the impulse approximation.

It was argued in Refs. \[5–7\], that the impulse approximation is not a complete answer and one must include also the so called interference diagrams, although no explicit estimates of their relative strength was performed. In Ref. \[2\] the arguments were presented that interference diagrams become important for small \(x\) due to the leading twist (LT) nuclear shadowing phenomenon.

The purpose of the current paper is to calculate explicitly the interference corrections to the impulse approximation due to the nuclear shadowing for the case of proton - deuteron scattering (for the recent review of the theory of the leading twist shadowing phenomena see \[16\]). We will focus on the limit when one of partons in the deuteron has small enough \(x\), so that nuclear shadowing is present for the deuteron pdf while the second parton is probed in the kinematics where shadowing effects are absent. We will demonstrate that in this limit nuclear shadowing induced interference is present already on the level of diagrams where one of the nucleons is active in the \(|\text{in}\rangle\) - state and two in the \(|\text{out}\rangle\) - state (or vise versa), and that it has the same magnitude as the enhancement of MPI due to the interaction with two nucleons in the impulse approximation.

While the actual experiments are done with the heavy nuclei, we believe that the deuteron case provides a simple "laboratory" for the studying possible mechanisms of shadowing in four jet production processes. In the case of heavy nuclei combinatorics is much more complicated \[\textit{[6]}\]. It will be considered elsewhere.

The shadowing in the multijet production differs significantly from the LT shadowing for nuclear pdfs since the two partons belonging to the projectile proton are typically located in a very small transverse area of the radius \(\sim 0.5\) fm. As a result they scatter off two different but very close in the impact parameter space nucleons that may be rather strongly correlated. This is especially true for the case of scattering off the deuteron which is a highly correlated system. Hence the analysis presented here can serve as the stepping stone to a discussion of similar effects for MPI with heavy nuclei.

In the current experimental studies one usually starts with a trigger on a hard process of large virtuality - say dijet with \(p_t\)’s larger than 50 ÷ 100 GeV and one next looks for a second hard subprocess in the underlying event. Since the LT nuclear shadowing for \(p_t \geq 100\) GeV/c is very small we will focus here on consideration of the MPI in which one of the subprocesses has large enough \(x\) or large virtuality so that nuclear shadowing can be neglected in this case.

The paper is organized as following. In section 2 we apply the general expressions relating double hard four jet cross section for the collision of hadrons \(A\) and \(B\) in terms of \(2\) GPDs (Eq. \[3\]) to obtain a compact expression for the double parton antishadowing contribution (DPA) taking into
account the finite transverse size of the gluon GPD in the nucleon. In section 3 we summarize first the theory of the LT shadowing for the deuteron pdfs and next use it to calculate the shadowing correction to the MPI rate for the case when $x$ of one of the partons of the deuteron participating in collision is large and another is small. We demonstrate that the shadowing in the case of MPI is a factor of two stronger than in the case of the deuteron pdfs. At the same time an additional contribution to MPI due to the pQCD evolution induced correlations in the proton wave function reduces this enhancement. In section 4 we present the numerical results. We find that shadowing effect is smaller but of the same magnitude as DPA for modest virtualities ($Q^2 \sim 4\text{GeV}^2$). Also we show explicitly that the double parton shadowing is negligible when both of the partons have large $x$, confirming the results of Ref. [2].

II. IMPULSE APPROXIMATION FOR THE PROTON - DEUTERON SCATTERING

A. Leading term

Let us first consider the case when both partons of the nucleus involved in the interaction belong to the same nucleon – the impulse approximation. (Fig. 1).

![Impulse approximation](image)

FIG. 1: Impulse approximation.

This is the dominant contribution in the deuteron case, though it becomes subleading for heavy nuclei [1, 2]. The corresponding cross section is, obviously, twice the cross section of the MPI $pp$ scattering (we neglect here difference of the quark distributions in proton and neutron). It is given by

$$\sigma_{\text{imp$_4$}}(pD) = 2\sigma_{\text{imp$_4$}}(pN).$$

(1)
So introducing so called $\sigma_{eff}(pD)$ we can write

$$\frac{1}{\sigma_{eff pA}} = \frac{\sigma_{imp}}{\sigma_1\sigma_2} = 2 \int \frac{d^2\Delta_t}{(2\pi)^2} F_{g2}(\Delta^2, x_1) F_{g2}(\Delta^2, x_2) F_{g2}(\Delta^2, x_{1p}) F_{g2}(\Delta^2, x_{2p})(1 + N), \quad (2)$$

where $\sigma_1, \sigma_2$ are the elementary cross sections of production of jets in the parton - parton interaction; the factor $F_{g2}$ is the two gluon form factor of nucleon [17]. The factor $1 + N$ parameterizes the enhancement of the observed cross section as compared to the calculation in the mean field approximation.

A significant positive contribution to $N$ originates from the pQCD evolution induced parton - parton correlations - the $1 \otimes 2$ processes [12][15] which enhance the cross section as compared to the one calculated assuming dominance of the collisions of two independent pairs of partons - the $2 \otimes 2$ processes. Our numerical studies found $1 + N \sim 2.2$ for $pp$ scattering in quasi symmetric kinematics which is consistent with the LHC data for $x \sim 0.001 - 0.01$. In the kinematics we consider here - one large $p_t$ pair of $p_t \sim 30 \text{ GeV/c}$ jets and another pair with moderate $p_t$'s of the order $2,3,10 \text{ GeV/c}$ the mechanism of [12][14] leads to an expectation of $N \sim 0.3,0.6,1.0$ respectively. Note that these values of $N$ are slightly larger than the corresponding values in $pp$ collisions at the LHC for the same hard transverse scales, since the c.m. energy in $pA$ collisions is smaller ($\sqrt{s} = 5 \text{ TeV}$) and corresponding $x$ are larger by a factor 1.3 than in $pp$ collisions for $\sqrt{s} = 8 \text{ TeV}$.

B. Antishadowing contribution.

The second contribution, which becomes dominant in the case of scattering off heavy nuclei, results from the process in which two partons from a incoming proton interact with two different nucleons of the deuteron. Corresponding diagram is depicted in Fig. 2.

![FIG. 2: Double parton antishadowing correction.](image)
It can be calculated using the general expression relating double hard four jet cross section for the collision of hadrons $A$ and $B$ in terms of 2GPDs

\[
\frac{d\sigma_{4\text{jet}}^{AB}}{dt_1 dt_2} = \int \frac{d^2 \vec{\Delta}}{(2\pi)^2} \frac{d\vec{\sigma}_1(x'_1, x_1)}{dt_1} \frac{d\vec{\sigma}_2(x'_2, x_2)}{dt_2} 2G_A(x'_1, x'_2, \vec{\Delta}) 2G_B(x_1, x_2, \vec{\Delta}),
\]

(3)

where in our case $G_A, G_B$ are the 2 parton GPDs of the nucleon and the deuteron[12]. Here $x'_1 = x_{1p}, x'_2 = x_{2p}$ are the light-cone fractions for the partons of the projectile nucleon, and $x_1, x_2$ are the light-cone fractions for the target nucleon/nucleons. It was demonstrated in [2] that this contribution can be written through the two-body nuclear form factor. In the case of scattering off the deuteron (diagram of Fig. 2 this form factor is easily calculated and expressed through the deuteron form factor (since in this case there is a simple relation between two-body and single-body form factors) Indeed, the contribution of the corresponding diagram is given by (cf. Fig. 2 and Eqs. 19 - 21 in [2]).

\[
\frac{\sigma_{\text{DPA}}}{\sigma_1 \sigma_2} = 2 \times \int \frac{d^4 \Delta}{(2\pi)^4} F_{2g}(\Delta_t, x_1) F_{2g}(\Delta_t, x_2) F_{2g}(\Delta_t, x_{1p}) F_{2g}(\Delta_t, x_{2p})
\]

\[
\times \int \frac{d^4 k}{(2\pi)^4} \frac{\Gamma(p/2 + k, p/2 - k)}{((p/2 + k)^2 - m^2)((p/2 + k - \Delta)^2 - m^2)((p/2 - k - \Delta)^2 - m^2)((p/2 - k + \Delta)^2 - m^2)}
\]

(4)

The factors $\Gamma$ are the two vertex functions depicted in Fig. 2. We can now integrate in a standard way over $k^0, \Delta^0$, and use the fact that the corresponding denominators are dominated by nonrelativistic kinematics: $k^0, \Delta^0 \sim \vec{k}^2/M$, and the longitudinal transfer $\Delta_z = 0$. After performing the integration we immediately obtain:

\[
\frac{\sigma_{\text{DPA}}}{\sigma_1 \sigma_2} = 2 \times \int \frac{d^2 \Delta_t}{(2\pi)^2} F_{2g}(\Delta_t, x_1) F_{2g}(\Delta_t, x_2) F_{2g}(\Delta_t, x_{1p}) F_{2g}(\Delta_t, x_{2p}) S(\vec{\Delta}^2).
\]

(5)

We define here the deuteron form factor as (see e.g. [16]):

\[
S(\vec{\Delta}^2) = \int \frac{d^3 k}{(2\pi)^3 8M} \frac{\Gamma(\vec{k}^2)}{(A^2 + \vec{k}^2)(A^2 + (\vec{k} - \vec{\Delta})^2)},
\]

(6)

where the $\Gamma$ is the deuteron to two nucleons vertex, and

\[
A^2 = m^2 - M^2/4.
\]

(7)

Here $M$ is the deuteron mass, $m$ is the nucleon mass, and the momenta of nucleons in the deuteron are $\vec{p}/2 + \vec{k}$, and $\vec{p}/2 - \vec{k}$. Here we used the fact than the deuteron is a nonrelativistic system, so the form factors $\Gamma$ depend only on the differences of the spacial components of the nucleon momenta.
Using the relation between the vertex functions and wave functions of the deuteron we can rewrite the latter expression in terms of the deuteron nonrelativistic wave functions as

\[ S(\Delta^2) = \int d^3\vec{p} \left[ u(\vec{p})u(\vec{p} + \Delta) 
+ w(\vec{p})w(\vec{p} + \Delta) \left( \frac{3}{2} \frac{(\vec{p} \cdot (\vec{p} + \Delta))^2}{p^2(p + \Delta)^2} - \frac{1}{2} \right) \right], \]

where \( u \) and \( w \) are the \( S \)-wave and \( D \)-wave components of the deuteron wave function respectively (here in difference from Eq. 6 we give the expression for the spin-1 deuteron).

Note that Eqs. 5, 6 accurately take into account the finite transverse size of the nucleon GPDs which is numerically rather important (see section 4).

At the same time we neglected in this calculation the nucleon Fermi motion effect which shifts the x-argument of the bound nucleon pdfs. The reason is that this effects is a very small correction which enters only on the level of the terms \( \propto \vec{k}^2/m^2 \) which are very small for the deuteron, cf. discussion in [2].

Finally, let us mention that we must multiply this expression by \( 1 + N_L \), where \( N_L \) is the enhancement of 4 jet cross section relative to mean field approximation in the given kinematics due to parton correlations. In our kinematics this number is very small. Indeed, in difference from the case of \( pp \) collisions the \( \Delta \) dependence of the nucleus and nucleon factors in the corresponding equation is very different. As a result one does not have in this case an enhancement factor of \( \sim 2 \) from \( 1 \otimes 2 \) which is present in the \( pp \) case. In addition, the transverse integral is dominated by the same deuteron form factor both in \( 1 \otimes 2 \) and \( 2 \otimes 2 \) contributions, leading to \( N_L \leq 0.1 \) (see section 4).

III. SINGLE SHADOWING: ONE TO TWO PROCESSES.

A. Leading twist shadowing for the deuteron pdfs

Before discussing the shadowing for MPI in the deuteron it is worth reminding the picture of the LT shadowing for the case of the deuteron pdfs. It was demonstrated in [18] that the shadowing correction to the deuteron pdf can be expressed in the model independent way through the diffractive nucleon pdfs. In the reference frame where deuteron is fast, the process can be pictured as the hard interaction in \( |in\rangle \)-state with a small \( x \) parton in which the nucleon in the final state carries most of its initial momentum fraction \( - (1 - x_F) \), while in the final state the
diffractive system which carries the light-cone fraction $x_F$ combines with the second nucleon into a nucleon with momentum fraction $1 + x_F$, see Fig. 3.

As a result one finds for the shadowing correction (see Eq. 98 and Fig. 28 in Ref. [16])

$$\Delta f_D(x, Q^2) = 2f_N(x, Q^2) - f_D(x, Q^2),$$

where $\beta = x/x_F$ and $F^{D(4)}(\beta, Q^2, x_F, q_t)$ is the diffractive pdf. It is easy to see that the shadowing originates from configurations where two nucleons are roughly behind each other. For these configurations shadowing is large as long as the effective cross section of the rescattering:

$$\sigma_2 \approx 16\pi \frac{\int x^0 dF_j^{D(4)}(\beta, Q^2, x_F, t_{min})}{xf_j/N(x, Q^2)},$$

is comparable to the pion-nucleon cross section which is the case for the gluon channel for $x \leq 10^{-3}, Q^2 \leq 10 \text{ GeV}^2$.

**B. Single shadowing for MPI**

The DPA contribution which we considered above corresponds to collisions where two nucleons of the deuteron are located at small relative transverse distance of the order of the nucleon transverse gluon size - $\sim 0.5 \text{ fm}$. For such two nucleon configuration LT nuclear shadowing is large since the effective cross section of the rescattering interaction is large. Hence it may strongly reduce the DPA effect. The shadowing term corresponds to the diagrams which are an analog of the LT
shadowing diagrams for the deuteron pdf with an extra blob corresponding to the non screened second interaction (Fig. 4).

The screening contribution requires that the first nucleon experiences the diffractive interaction, while the second hard blob is a generic hard nucleon nucleon interaction. Similar to the DIS case this diagram gives negative contribution to the cross section.

As usual only the diagrams with elastic $IP - nucleon - IP$ vertex contribute, since we work in conventional two nucleon approximation for the deuteron when all other components of the deuteron wave function are neglected.

Hence the shadowing is described by four diagrams one of which is depicted in Fig. 4. The combinatorial factor of two arises since the parton ”1” can belong to either of two nucleons. Another factor of two is due to the possibility to attach the Pomeron line to the first nucleon either in the initial or in the final state. The shadowing contribution can be written as:

\[
\frac{\sigma_{SS}}{\sigma_1 \sigma_2} = -4 \int \frac{d^4q d^4k}{(2\pi)^{12}} \frac{F_{D}^{(4)}(\beta, Q_1^2, q_1^2, x_{F}, \Delta_t)}{G_N(x_1, Q_1^2) \frac{1}{((p/2 + k)^2 - m^2)((p/2 + k - q + \Delta)^2 - m^2)}} \\
\times \frac{F_{2g}(\Delta_t, x_{1p}) F_{2g}(\Delta_t, x_{2p}) F_{2g}(\Delta_t, x_{2})}{((p/2 - k)^2 - m^2)((p/2 - k - \Delta)^2 - m^2)((p/2 - k - \Delta + q)^2 - m^2)} + (1 \leftrightarrow 2),
\]

with the factor of four reflecting presence of four diagrams. The Pomeron exchanges carry three-momenta $\vec{q} = (\vec{q}_t, q_z)$ and $\vec{q} + \Delta$. 

FIG. 4: Shadowing correction to 4 jet production in pD scattering.
We carry the integration over $q_0, k_0, \Delta_0$ in exactly the same way as in the previous section, where we calculated the diagram of Fig. 2, taking into account that the vector $\vec{\Delta}$ is transverse. Using Eq. 6 for the deuteron form factor we can rewrite Eq. 12 as

$$
\frac{\sigma_{SS}}{\sigma_1 \sigma_2} = -4 \int \frac{d^2q_t d^2\Delta_t dx_{\mathbf{p}'} F^{D(4)}(\beta, Q^2_{12}, q^2_t, x_{\mathbf{p}'}, \vec{\Delta}_t)}{G_N(x_1, Q^2_1)} S((q + \vec{\Delta})^2)
\times F_{2g}(\Delta_t, x_{1p})F_{2g}(\Delta_t, x_{2p})F_{2g}(\Delta_t, x_2) + (1 \leftrightarrow 2).
$$

(13)

Overall, we can see from the comparison of Eqs. 10 and 13 that in the limit when radius of the deuteron is very large so one could neglect the $q_t$ dependence of all other factors, the ratio of shadowing and impulse approximation terms in the case of the MPI is a factor of two larger than for case of DIS. This reflects the enhancement of the central collisions in the MPI which we mentioned above.

IV. NUMERICAL ESTIMATES.

A. Antishadowing.

For numerical estimates it is convenient to approximate the deuteron form factor calculated with a realistic deuteron wave functions by a sum of two exponentials [19]

$$
S(\vec{\Delta}^2) = 0.6 \exp(-K^2_{1D} \vec{\Delta}^2) + 0.4 \exp(-K^2_{2D} \vec{\Delta}^2),
$$

(14)

where

$$
K^2_{1D} = 22.7 \text{GeV}^{-2}, K^2_{2D} = 127 \text{GeV}^{-2}.
$$

(15)

The momentum dependence of the two gluon form factor can be extracted [17] from the $J/\psi$ photoproduction data. The exponential fit gives

$$
F_{2g}(\vec{\Delta}^2, x) = \exp(-\vec{\Delta}^2 B_N(x)),
$$

(16)

where

$$
B_N \approx 1.43 + 0.14 \log[x_0/x] \text{ GeV}^{-2}.
$$

(17)

and $x_0 = 0.1$. For understand better qualitative features of the interplay between the distance scales related to the deuteron and to the nucleon GPDs we shall use below a simplified form of the deuteron form factor

$$
S(\vec{\Delta}^2) = \exp(-K^2_D \vec{\Delta}^2),
$$

(18)
while in the numerical calculations we will use Eq. 14 (the radius $K_D^2$ is related to the electric radius of the deuteron as $K_D^2 = (2/3)R_{D,e.m.}^2$). Performing integration in Eq. 2 we obtain for the leading term:

$$\frac{\sigma_{imp}}{(\sigma_1\sigma_2)} = \frac{1}{2\pi} \frac{(1+N)}{K(x_1,x_2,x_{1p},x_{2p})}.$$  (19)

where

$$K(x_1,x_2,x_{1p},x_{2p}) = B_N(x_1) + B_N(x_2) + B_N(x_{1p}) + B_N(x_{2p}).$$  (20)

The function $K$ is determined by the two gluon form factors of the nucleon. It is independent of the deuteron wave function. The answer for the DPA correction to the cross section is obtained by taking integral over $\Delta$ in Eq. 5 using parametrization 18:

$$\frac{\sigma_{DPA}}{\sigma_1\sigma_2} = \frac{1}{2\pi} \frac{1}{K_D^2 + K(x_1,x_2,x_{1p},x_{2p})}.$$  (21)

Using parametrization 14 for the deuteron form factor, we obtain the DPA correction of the order 8% when all $x$’s are $\sim 0.01$, (neglecting $N_L$) and slowly decreasing with a further decrease of $x$’s. This is in very good agreement with a more explicit calculation using a expression 8 for the form factor and the Paris deuteron wave functions, which gives 7.3%. Note here that neglecting the nucleon finite size as compared to the deuteron size (putting $B_N$ to zero in Eq. 21) would result in an overestimate of the discussed contribution to the cross section by $25\div 30\%$.

**B. Single shadowing.**

We now use the simple parametrization for the nucleon diffractive pdf $F_D^{4D}$:

$$F^{4(D)}(\beta,Q^2,x_{\beta},q_t) = B_D \exp(-B_D q_t^2) F^{3(D)}(\beta,Q^2,x_{\beta}),$$  (22)

where $\beta = x_1/x_\beta$. In the limit of small $x$ when we can neglect $t_{\text{min}} = -m_N^2 x_{\beta}^2/(1 - x_\beta)$, integral over longitudinal and transverse degrees of freedom in Eq. 10 decouple. In this limit Eq. 10 for the shadowing correction can be rewritten as (we can neglect $x_{\beta}$ in the argument of the deuteron form factor)

$$\Delta G(x,Q^2) = -I(x,Q^2)B_D \left( \frac{0.6}{K_{1D}^2 + B_D} + \frac{0.4}{K_{2D}^2 + B_D} \right) = -S \cdot I(x,Q^2) = -0.166 I(x,Q^2),$$  (23)
where we defined
\[ I(x, Q^2) = \int_x^{0.1} dx_F \beta F_3(\beta, Q^2, x_F)/8\pi^2. \] (24)
and \( S \) is the integral over transverse momenta:
\[ S = B_D \left( \frac{0.6}{K_{1D}^2 + B_D} + \frac{0.4}{K_{2D}^2 + B_D} \right) \] (25)

Here \( B_D = 7 \text{ GeV}^{-2} \) is the slope of diffractive structure function of the nucleon based on the HERA experimental data which indicates that \( B_D \) practically does not depend on \( x_F \) \[20\]. In this approximation the function \( I(x, Q^2) \) can be easily determined from numerical results for \( \Delta G(x, Q^2) \) \[16\].

Now we can use expression \[13\] for the single parton shadowing in four jet production to calculate the value of the shadowing effect. For the exponential parametrization we can write
\[ F^{4(D)}(\beta, Q^2, x_F, q_t, \Delta_t) = B_D \exp(-q_t^2 B_D/2 - (q_t + \Delta_t)^2 B_D/2) F_3(\beta, Q^2, x_F). \] (26)

Hence the shadowing correction is
\[ \frac{\sigma_{SS}}{\sigma_1 \sigma_2} = \frac{4(I(x_1, Q_1^2)U(x_1, x_2, x_{1p}, x_{2p}) + I(x_2, Q_2^2)U(x_2, x_1, x_{1p}, x_{2p}))}{4\pi}. \] (27)

Here the longitudinal function \( I \) is given by Eq. \[24\] and the transverse integrals \( U \) are obtained by using Eq. \[13\] and explicit Gaussian parametrization for the form factor.

The ratio \( K = \sigma_{SS}/\sigma_{DPA} \) is presented in Fig. 5 as a function of \( x_1 \) and \( Q_1^2 \) for the LHC kinematics of production of two jets with \( p_t = Q \) and \( 4Q^2 = x_1 x_{1p} s \), \( s = 2.5 \times 10^7 \text{ GeV}^2 \). The second \( x_2 = 0.1, Q_2^2 = 1000 \text{ GeV}^2 \) being fixed to stick to the kinematics under discussion. In Fig. 6 we also present the ratio of the shadowing correction for this kinematics and the full impulse approximation result.

For typical \( x_1 \sim 0.001, x_2 \sim 0.05 \) in LHC kinematics we find shadowing of order 30% relative to DPA for low \( Q_1^2 \sim 4 \text{ GeV}^2 \). We also see from Fig. 5 that the shadowing contribution to the cross section decreases with the increase of the transverse scale.

Note also that the account for the finite size of the nucleon reduces the the absolute value of the correction by \( \sim 10\% \). The same reduction occurs also for the DPA, so the ratio of shadowing and DPA contributions is practically not sensitive to the finite nucleon radius.
FIG. 5: The ratio $K$ of shadowing and DPA corrections to the four jet production cross section as a function of $x_1 \equiv x$ for hard scales $Q_1^2 = 4, 10, 100$ GeV$^2$, $Q_2^2 = 1000$ GeV$^2$. We put $x_2 = 0.1$ and $x_{1p} = 4Q_1^2/(x_1 s), x_{2p} = 4Q_2^2/(x_2 s) \sim 0.0016$.

FIG. 6: The ratio of shadowing correction to DPA and full impulse cross section as a function of $x$ for hard scales $Q_1^2 = 4, 10, 100$ GeV$^2$. $Q_2^2 = 1000$ GeV$^2$. We put $x_2 = 0.1$ and $x_{1p} = 4Q_1^2/(x_1 s), x_{2p} = 4Q_2^2/(x_2 s) \sim 0.0016$.

In the limit of very small $x_1 \leq 10^{-3}$ and $x_2$ large one maybe close to the black disk regime and the LT approximation would break down. Still our calculation indicate that in this limit suppression effect should be large -- $\sim 0.5$. relative to DPA.

It is instructive to compare the shadowing correction to the total differential cross section of the four jet production in $pD$ collision in the impulse approximation to the shadowing correction to deuteron structure functions. The internal over the longitudinal momenta is the same for both corrections and hence their ratio is given then by the ratio of transverse integrals which is of the
order one. Indeed, the ratio of shadowing and impulse contributions can be rewritten as

\[
\frac{\sigma_{SS}}{\sigma_{imp4}} = \frac{\Delta G_N(x_1, Q_1^2)}{G_N(x_1, Q_1^2)} \frac{2}{1 + N} \frac{U \cdot K}{S},
\]

(28)

where we used Eqs. 23, 25. Thus we see that the shadowing correction for DPI is proportional to the shadowing correction to the deuteron gluon PDF, the proportionality coefficient being the the product of the factor \(2/(1 + N)\) and the ratio of transverse integrals. The latter one is always close to one. For logarithmic parametrization of \(B_N\) the transverse factor \(U \cdot K/S\) does not depend on \(x_1\) (only on the hard scales). The factor \(2/(1+N)\) also depends on \(x_1\) only weakly, at least for \(x_1 \geq 0.001\) and is close to one for large \(Q_1^2\) while it is of the order 1.5 at \(Q \sim\) few GeV in the chosen kinematics\[14\].

Altogether we see that the \(x\)-dependence of the ratio (28) is the same as for the shadowing correction for the corresponding deuteron pdf, but the absolute value depends on the ratio of the transverse integrals (which is of the order of one) and the value of \(N\). As a result the ratio is of the order of \(2/(1 + N)\). The factor 2 shows that there is a different combinatorics in MPI in pD scattering and in the DIS scattering of the deuteron, i.e. one does not obtain the screening correction simply by substituting the nuclear pdf (that includes shadowing) instead of nucleon pdf in the impulse approximation equations.

Finally, let as note that the ratio \(\sigma_{DPA}/\sigma_{imp4}\) of DPA and impulse approximation is \(x\)-independent and depends only on hard scales. It is equal to

\[
\sigma_{DPA}/\sigma_{imp4} \sim (0.16 \div 0.18)/(1 + N),
\]

(29)

where 0.18 corresponds to the hard scale 4 GeV\(^2\) and 0.16 to the 100 GeV\(^2\) scale. So the ratio slowly decreases with the change of the hard scale, mostly due to the change of \(N\), decreasing from \(\sim 1\) at the hard scale 10 GeV to \(\sim 0.3\) at 2 GeV, due to the dynamical dependence of \(N\) on the scale, found in \[13\] \[14\].

The \(1 \otimes 2\) contributions to DPA is small. Indeed, as it was already mentioned above, there is no factor 2 that is present in the \(pp\) collisions due to asymmetric kinematics. Also, the integral over \(\vec{\Delta}\), for the \(1 \otimes 2\) term in the \(pp\) collisions is proportional to \(4B_N/2B_N\), enhancing \(1 \otimes 2\) contributions by a factor of two relative to the \(2 \otimes 2\) contribution. This enhancement however is absent in DPA, where the corresponding ratio is \((K_D + 4B_N)/(K_D + 2B_N) \sim 1.1\). Altogether this results is a strong suppression of the \(1 \otimes 2\) contribution in DPA so it can be safely neglected. Similar effect for heavy nuclei was discussed in Ref. \[2\].
V. CONCLUSION.

We calculated the contribution of the nucleon shadowing to the four jet MPI cross section in the proton - deuteron collisions in the limit when one of the probes has small $x$ and another has $x, Q^2$ in the range where shadowing is small. We have seen that shadowing increases with the decrease of $x$, and decreases rapidly with the increase of hard scale. For large $p_t$ of one of the probes corresponding to a typical jet trigger in $pA$ collisions at the LHC and small $p_t$ of the other probe we obtain correction of the order of 30%. This contribution is not reduced to the substitution of the deuteron pdf instead of nucleon pdf in the impulse approximation formula – it is twice as large as a such naive guess. There is a reduction by the factor $1/(1 + N)$ that may be of order 1/2, depending on kinematics, due to a completely different mechanism of $1 \otimes 2$ enhancement of the four jet cross section.

Our analysis will serve as a starting point to a more complicated calculation of shadowing in the case of heavy nuclei for the similar kinematics. Further studies will be necessary for calculations of the shadowing in the kinematics when both $x'$s of the partons from the nucleus are small and hence more complicated diagrams contribute to the the nuclear shadowing.

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Appendix A: Correspondence with the Glauber model of $pA$ scattering

It is easy to see that the structure of the double scattering term is very close to that for the double scattering term for the total cross section of $pA$ scattering in the Glauber model. This similarity holds for any nuclear wave functions as the two-body form factor which enters in both cases is the same. Since the relevant expressions for the heavy nucleus case were derived before in $[1]$ it is convenient to check the correspondence taking the limit of large $A$, and neglecting nucleon - nucleon correlations.

The ratio of the double and single scattering terms in the Glauber series for the total cross section of $hA$ scattering:

$$
\frac{\sigma_{tot}^{hA}}{\sigma_{tot}} = \int d^2b2(1 - \exp(-\sigma_{tot}T(b))/2) = \sigma_1 - \sigma_2 + \sigma_3 - ..., \quad (A1)
$$
is given by

\[ \sigma_2 / \sigma_1 = \frac{1}{4} \sigma_{\text{tot}} \int T^2(b) d^2 b / A. \]  

(A2)

This expression differs from the ratio of the cross section of production of four jets in the interaction with two and one nucleons (Eqs. 2, 3) by the factor of \(\frac{1}{4}\) and substitution \(\sigma_{\text{tot}} \rightarrow \pi R_{\text{int}}^2\). The factor of four could be understood on the basis of the Abramovsky, Gribov, Kancheli cutting rules [21] which state that the double cut diagram enters with the extra factor of two as compared to the shadowing correction to the total cross section. An another factor of two reflects combinatorics of emission of "pair one" from either first or second nucleon.

Using this observation it is straightforward to find the expressions for the double interaction contribution if the expression for the shadowing for the total cross section is known (including the effects of nucleon - nucleon correlations)

For example, in the case of the scattering off the deuteron contribution of the diagram 2 to \(G_2(x_1, x_2, \Delta)\) is given by (For the discussion of proton - deuteron four jet production in the coordinate space representation see [5][7].)

\[ 2 G_D(x_1, x_2, \Delta) = 2 G_N(x_1, \Delta) G_N(x_2, \Delta) \cdot S_D(\Delta), \]  

(A3)

Here \(S_D(\Delta)\) is the standard deuteron form factor defined above (Eq. 6), which enters in the Glauber double scattering term. Factor of two in Eq. A3 is due to combinatorics (the factor of \(A(A-1)\)). This is just the result obtained in section II - Eq. 5.

Similarly, one can obtain the expressions for the triple MPIs matching the corresponding expressions of ref. [1].

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