Super-twisting sliding mode control of permanent magnet synchronous motor based on voltage compensation

Xu Jinli, Peng Zhen
(School of Mechanical and Electrical Engineering, Wuhan University of Technology, Wuhan 430070 China)

Abstract: In this paper, a compensation method for the expected voltage value was proposed to the problem of speed overshoot in traditional Super-twisting sliding mode control applied to permanent magnet synchronous motors (PMSM). This method combines the voltage change caused by the load angle change, overcomes the overshoot situation when the speed changes rapidly, and speeds up the time to reach the given speed. The proposed control theory is verified by MATLAB/Simulink software. Simulation results show that the proposed compensation method can successfully suppress speed overshoot and improve the dynamic response of speed.

1. Introduction
In the electric drive system of electric vehicles, permanent magnet synchronous motors are widely used in servo systems of various electric vehicles due to their high efficiency, high power density, and low maintenance costs. Direct Torque Control (DTC) has simple structure design, fast dynamic response speed, and good robustness. But on the other hand, when DTC is used as control strategy for electric vehicles, the ripple of the magnetic flux and torque become obvious, and it becomes inevitable especially at low speeds.

In order to improve the stability of DTC control, scholars have conducted in-depth research. [1] started from the voltage space vectors and optimized the number of voltage space vectors in order to reduce the torque ripple through better choices. [2, 3, 4] introduced Space Vector Pulse Width Modulation (SVPWM), which further provided the voltage vector required for the closest control process through vector composition of multiple basic voltage space vectors. [5] proposed to replace the PI control in the traditional torque and flux controller with fuzzy control and Sliding Mode Control (SMC) to improve the anti-interference and adaptive ability. However, in the application process of sliding mode control, there is a large chattering problem, which decreases its spreading.

In this paper, an improved STSMC method is introduced in the torque controller and flux controller to replace the traditional PI control based on the traditional DTC. The expected voltage input is analyzed and corrected in new method, which effectively reduces the the overshoot when the speed changes, and makes the control process smoother.

2. Mathematical model of PMSM
The mathematical model of PMSM in the two-phase rotating coordinate (in dq axis) can be described as the following equations:
In this formula, \( \psi_s \), \( i_s \), and \( u_s \) are the stator flux linkage, stator current, and stator voltage. \( \psi_d \) and \( \psi_q \), and \( i_d \) and \( i_q \), \( u_d \) and \( u_q \) are the components of the stator flux, current and voltage of \( d \) and \( q \)-axis. \( \psi_f \) is the rotor permanent magnet flux linkage, \( R_s \) is the stator resistance, \( \omega \) is the synchronous rotational angular velocity, \( J \) is the moment of inertia, \( B \) is the coefficient of friction, and \( p \) is the number of pole pairs of the motor. \( L_d \) and \( L_q \) are the direct and quadrature-axis inductance of the motor, respectively.

For the hidden-pole PMSM, \( L_d = L_q \).

According to the rotor flux vector relationship, \( \psi_q = 0 \), namely \( \psi_s = \psi_d \). From (1) we know:

\[
\frac{d}{dt} \psi_s = \frac{d}{dt} \psi_d = u_s - R_s i_s
\]

(2)

The derivative of the motor torque \( T_e \) with time can be transformed into the following equation:

\[
\frac{d T_e}{dt} = \frac{3}{2} p \psi_f \frac{di_q}{dt} + \frac{3}{2} p \psi_f \left( -p \omega i_d - \frac{R_s}{L_q} i_d - \frac{p \psi_f}{L_d} \omega + \frac{u_q}{L_d} \right)
\]

(3)

Among them, \( R_s \), \( \psi_f \), \( p \), \( L_d \), \( \omega \), and \( i_d \) are limited values. Therefore, it can be concluded from (2) and (3) that not \( u_d \) has a linear relationship with the change rate of the magnetic flux amplitude, but \( u_q \) and the change rate of torque has a linear relationship.

### 3. DTC algorithm based on improved STSMC

#### 3.1. DTC algorithm based on traditional STSMC

According to STSMC, the sliding mode switching functions \( s_1 \) and \( s_2 \) of the flux controller and torque controller are designed as follows:

\[
\begin{cases}
    s_1 = |\psi_s^*| - |\psi_s| \\
    s_2 = T_e^* - T_e
\end{cases}
\]

(4)

In (4), \( |\psi_s^*| \) and \( T_e^* \) are the given values of the magnetic flux and torque. \( |\psi_s| \) is a known quantity. \( \Delta n \) is the difference between the actual speed \( n \) and the given speed \( n^* \), and \( T_e^* \) can be calculated through PI transformation of \( \Delta n \). \( |\psi_s| \) and \( T_e \) are the results of actual estimation of flux and torque.

The magnetic flux controller and torque controller designed according to the switching function can be expressed as:

\[
\begin{align*}
    u_{sd} &= K_{s1} |s_1|^{0.5} \text{sign}(s_1) + u_{sd1} \\
    \frac{du_{sd1}}{dt} &= K_{d1} \text{sign}(s_1) \\
    u_{sq} &= K_{s2} |s_2|^{0.5} \text{sign}(s_2) + u_{sq1} \\
    \frac{du_{sq1}}{dt} &= K_{d2} \text{sign}(s_2)
\end{align*}
\]

(5)

(6)
In (6), \( u_{sd} \) and \( u_{sq} \) are the reference voltages generated by STSMC; \( K_{d1}, K_{d2}, K_{q1} \) and \( K_{q2} \) are the controller gains, and the stability of the system is determined by the Lyapunov function.

### 3.2. Analysis of reference voltage vector

In traditional DTC, the reference voltage is selected from the voltage space vectors using a limited number of basic voltage vectors with a specific amplitude and direction for each control cycle. In SVPWM-based DTC, the control system can be used to synthesize any voltage vector through the basic voltage vector at each sampling interval, so that the error between the desired voltage vector and the actual generated reference voltage vector can be reduced.

At time \( k+1 \), the stator voltage formula (1) can be discretized as:

\[
    u_s(k + 1) = \frac{\Psi'_s(k + 1) - \Psi'_s(k)}{T_s} + i_s(k)R_s
\]

(7)

\( T_s \) is the sampling interval in the SVPWM module. A new reference voltage vector signal is generated at the end of each sampling and transferred to the inverter after conversion. \( u_s(k+1) \) and \( \Psi'_s(k+1) \) are the desired voltage vector and the desired magnetic flux vector value, which are required to bring the magnetic flux closer to the reference magnetic flux value \( |\Psi'_s| \) at time \( k+1 \). The relationship is shown in Figure 1. \( \Delta \theta_s \) is the rotor angle increment, namely the magnetic flux angle increment, which can be measured by a position sensor; in the same time period, the angle between the expected magnetic flux and the stator flux is the load angle \( \phi \).

The actual magnetic flux components \( \Psi_{sa}, \Psi_{sb} \) in \( \alpha-\beta \) axis and the reference magnetic flux components \( \Psi'_{sa}, \Psi'_{sb} \) at time \( k \) can be calculated by the following equations:

\[
\begin{align*}
    \Psi_{sa} &= \Psi_s(k) \cos \theta_s \\
    \Psi'_{sa} &= \Psi'_s(k) \cos (\theta_s + \Delta \theta_s) \\
    \Psi_{sb} &= \Psi_s(k) \sin \theta_s \\
    \Psi'_{sb} &= \Psi'_s(k) \sin (\theta_s + \Delta \theta_s) \\
    \Delta \theta_s &= T_s \omega_p + \Delta \phi
\end{align*}
\]

(8)

Figure 1. Vector analysis of magnetic flux in \( \alpha-\beta \) axis.

In (8), \( \Delta \phi \) is the amount of change in the load angle. Each sampling period can be considered as a micro-division. When PMSM running at a constant speed, the speed only changes within a small range, so the amount of change in the flux angle is approximately constant; when the speed changes, the speed change of a sampling period cannot be ignored, at this time the change of the flux angle is no longer constant. According to (8), the component \( T_s \omega_p \) of the flux angle change amount will be different in each sampling interval. When the given speed changes, such as the speed increases rapidly, the calculation will be smaller than the actual value according to the traditional magnetic flux angle calculation method, in that way the traditional STSMC will have a certain degree of overshoot.
3.3. Improved sliding mode controller
In order to solve this overshoot, this paper come up whit a method to compensates the reference voltage vector generated by the traditional STSMC. For the two-phase rotating reference voltage calculated by the traditional STSMC, a compensation term $\Delta \delta$ is added to the process of caculating the two-phase static voltage by Park transformation, and the rotor angle $\theta_s$ used in the process is compensated according to the change in speed:

$$\Delta \delta = \begin{cases} 
\Delta \theta_{\text{max}} - T_s \omega p, & \Delta n > 0 \\
0, & \Delta n = 0 \\
-\Delta \theta_{\text{max}} - T_s \omega p, & \Delta n < 0 
\end{cases}$$

(10)

$\Delta \theta_{\text{max}}$ is the maximum magnetic flux angle increment and it can be expressed as:

$$\Delta \theta_{\text{max}} = 2 \arcsin \frac{2 |\Psi_s|}{u_d T_s}$$

(11)

$u_d$ is the power supply voltage and $\Psi_s$ is a specific value, in that way $\Delta \theta_{\text{max}}$ is a constant value. The block diagram of DTC based on this paper is shown in Figure 2.

4. Simulation
The proposed method is compared with the traditional STSMC. The PMSM stator resistance $R_s$ used in the simulation is 1.3Ω, the stator inductance $L$ is 0.835mH, the rotor flux $\psi_f$ is 0.175Wb, the moment of inertia $J$ is 0.0008kg·m², the number of pole pairs $p$ is 4, and the stator flux reference value $\Psi_s^*$ is 0.22Wb, SVPWM sampling interval $T_s$ is 0.0002s, load torque is provided by constant speed.

![Figure 2. Block diagram of improved control algorithm.](image)

The simulation lasts 1s, at time of 0s, the given speed is 800rpm start from 0s. At time of 0.3s, the speed is increased to 1000rpm, and at time of 0.5s, it is reduced to 500rpm until the end of the simulation. In the simulation, the motor parameters and control cycle are the same.

The speed and its partial view waveforms using traditional STSMC are shown in Figure 3, and waveforms using improved method in this paper are shown in Figure 4.

From Figure 3(a) and its partial enlargement Figure 3(b), it can be seen that when the speed is increased from 800rpm to 1000rpm, there is a large overshoot. The peak value of overshoot is about
37rpm. It can be seen from Figures 4(a) and 4(b) that the overshoot of the speed change is 5rpm, and the speed tends to stabilize after 0.007s. From Figure 4(a) and Figure 4(b) it can be known that the improved method can reduce the speed overshoot value effectively when the speed changes rapidly, and it has better dynamic performance.

5. Conclusion
In order to solve the problem of overshoot when the speed of the traditional STSMC changes rapidly, this paper analyzes the voltage control input change when the load angle changes in the traditional STSMC, and comes up a method to corrects and compensates the expected voltage value. The compensation term makes it more reasonable before it becomes a vector pulse-width modulated wave. The conclusion is verified by MATLAB/Simulink simulation. The simulation shows that the method proposed in this paper can effectively reduce the overshoot situation when the speed changes rapidly, so that the overshoot value decreases from 37rpm to 5rpm, a decrease of 86%, the time required to reach the rated speed decreases from 0.02s to 0.007s, an increase of 65%. The improved method reduces the overshoot situation effectively when the speed changes rapidly, and speeds up the arrival time for the given speed.

References
[1] Y. L, Wen-Ke W and Yen-Chang C 2004 Novel switching techniques for reducing the speed ripple of AC drives with direct torque control IEEE T IND ELECTRON 51 768-75
[2] S. S, G. F and M. F R 2008 SVM direct torque control of IPM synchronous machines at very low speeds. In: 2008 4th IET Conference on Power Electronics, Machines and Drives, pp 296-300

[3] Jun Z, Zhuang X, Lixin T and M. F R 2005 A Novel Direct Load Angle Control for Interior Permanent Magnet Synchronous Machine Drives with Space Vector Modulation. In: 2005 International Conference on Power Electronics and Drives Systems, pp 607-11

[4] Song Q, Li Y and Jia C 2018 A Novel Direct Torque Control Method Based on Asymmetric Boundary Layer Sliding Mode Control for PMSM ENERGIES 11 657

[5] H. F E S and M. E E 2008 Improving the Torque Ripple in DTC of PMSM Using Fuzzy Logic. In: 2008 IEEE Industry Applications Society Annual Meeting, pp 1-8