ARTICLE

Development of River Meander Model

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1. Introduction

A functional relationship was attempt for the lower flow regime discharge to which the sediment rate may be related as \[1]:

\[
\frac{U}{U^*} = F(\psi') \tag{1}
\]

where \(U\) is the mean velocity and \(U^*\) is the shear velocity and \(\psi'\) is the intensity of shear on representative particles and is given by \[1\]:

\[
\psi'_{35} = \frac{(\rho_s - \rho) d_{35}}{\rho R s}. \tag{2}
\]

where \(\psi'_{35}\) is the intensity of shear on the particle with a diameter 35 mm sieve probe,

\(\rho_s = \) Sediment density and \(\rho = \) Water density

\(R' = \) Hydraulic radius of meandering boundary layer

\(S = \) Slope of the meandering boundary layer which is given in the Form resistance relationship of the 2nd equation.

2. Method

Another research works with the divided slope approach by assuming that the skin friction is due primarily to expansion losses associated with flow separation downstream of dune crest in meandering boundary layer \[1\].

The magnitude of the expansion head loss \(H''\) may be estimated from the formula \[1\]:

\[
\Delta H'' = \alpha \frac{(U_1 - U_2)^2}{2g} \tag{3}
\]

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where \( \alpha \) is the loss coefficient, \( U_1 \) is the mean velocity above the crest, and \( U_2 \) is the mean velocity over the trough. If the bed form height is given as \( h \) and the mean depth is given as \( D \), then the 3\(^\text{rd} \) relationship becomes \([1]\):

\[
\Delta H' = \frac{\alpha}{2g} \left[ \frac{q}{D} \frac{h}{2} - \frac{q}{D} \frac{h}{2} \right] - \frac{q}{A} \frac{h}{2} \approx \frac{U_1^2}{2g} \frac{h}{D} \Delta \rho
\]

where \( q \) is the discharge per unit channel width and \( U = q/D \) is the mean velocity and \( A = \text{unit.} \)

The energy gradient \( S'' \) is the head loss \( H'' \) divided by \( D \), then the 3\(^\text{rd} \) relationship becomes \([1]\):

\[
S'' = \frac{\Delta H''}{\lambda} = \frac{\alpha}{2gD} h^2 F^2
\]

and substituting the above equation into Eq. 5 yields \([1]\),

\[
S = S' + S''
\]

where the depth \( D \) is replaced by the hydraulic radius \( R \). Multiplying both sides by \( \gamma R(\gamma s - \gamma n) \) gives,

\[
\frac{\gamma RS}{(\gamma s - \gamma n) d} = \frac{\gamma RS}{(\gamma s - \gamma n)} d + \frac{\alpha}{2} \frac{\gamma h^2}{\gamma s - \gamma n} \lambda d
\]

### 3. Mathematical Model

In research of this model the vertical component of velocity is negligible, and the pressure is assumed as hydrostatic. The two-dimensional equations for water depth and averaged velocities is given according to \([14]\).

\[
\frac{\delta(uh)}{\delta s} + \frac{1}{r} \frac{\delta(yvh)}{\delta n} = 0
\]

\[
\frac{\delta u}{\delta s} u + \frac{\delta u}{\delta n} v + \frac{\delta v}{\delta r} = -\frac{1}{\rho} \frac{\delta p}{\delta s} - \frac{\tau_1}{\rho h} + 2 \frac{\delta}{\delta s} \left[ \frac{\delta u}{\delta s} + \frac{\delta u}{\delta n} \right] + \frac{\delta}{\delta n} \left[ \frac{\delta u}{\delta s} \right]
\]

\[
\left( \frac{\delta u}{\delta s} u + \frac{\delta u}{\delta n} v - \frac{u_1^2}{r} \right) = -\frac{1}{\rho} \frac{\delta p}{\delta s} - \frac{\tau_1}{\rho h} + \frac{\delta}{\delta s} \left[ \frac{\delta v}{\delta s} + \frac{\delta v}{\delta n} \right] + \frac{\delta}{\delta n} \left[ \frac{\delta v}{\delta s} \right]
\]

where \( u, v \) is the \( s \)- and \( n \)- components of the depth-averaged flow velocity, respectively, \( h \) is the depth of water, \( r \) is the radius of curvature of meandering boundary layer at \((s,n)\); \( \rho \) is the specific density of fluid; \( p = \rho g (h+z) \); \( z \) = bed elevation; \( \tau_r, \tau_n \) = \( s \)- and \( n \)- components of bed shear stress; respectively; and \( \epsilon \) = Diffusion coefficient.

In the model prediction the continuity equation for two-dimensional bed-load sedimentation equation is predicted \([14]\),

\[
\frac{\delta z}{\delta t} + \frac{1}{1 - \lambda} \frac{\delta q_{ls}}{\delta s} + \frac{1}{r} \frac{\delta (q_{ls} \epsilon)}{\delta n} = 0
\]

where \( t \) is the duration; \( \lambda \) is the porosity of bed material; and \( q_{ls}, q_{ln} = s \)- and \( n \)- components of the volume rate of bed-load transport per unit width of the bed, respectively. For calculation of bed-load transport equation the Meyer-Peter-Müller Formula is used \([1]\).

### 4. Equations with Empirical Background

Meander expansion is a significant sediment transport problem in river navigation which is well-researched by \([3]\). Many investigations have been made for the so-called regime theory \([5-7]\), which are given for empirical relationships. They are used for primarily designing the stable, straight channels. All the theories generally are given for predicting the value of meander formation that the channel width must be less than six-to-ten times depth in order for the channel remaining unchanged. Most river meanders have a width-depth ratio larger than six to ten, and their planform is variable. It consists of meanders that usually expanded by both downstream translation and lateral expansion. Most empirical equations cannot give rate and direction of expansion. Some expansion equations are given in the form of measured correlations between rates of bank retreat and width or width-radius ratio \([8-12]\).

Another theory is obtained by perturbation theory of river meandering boundary layer \([15]\). Perturbations are permitted into the system of observing the discharge which shows variation on channel planform migration by calculating the rate of growth of the oscillation at the boundary layer. The gain for river planform protection is that the perturbation – stability analysis enables river discharge characteristics in future basic flow conditions at the boundary layer. One –dimensional, straight-channel resistance formulas are given by \([14,15]\) where the perturbation analyses generally give some new models including those of sedimentation, and the description of flow and boundary layer formation in the channel in two different conditions. Two different stability formulations are given as:

(1) Bend formation theory which gives expansion of meandering features.

(2) Bar formation theory which gives alternating bar formation in straight channel.

New bend formation theories are given by \([15]\). It gives theories about river erosion conditions with centrifugally
induced secondary flow rates which influenced the river boundary layer topography and primary flow distributions. It is assumed that the bank erosion rate is proportional to the secondary current phase \[^{[15]}\]. It is assumed that the transverse bed slope has negligible impact on river boundary layer stability \[^{[15]}\]. In other research the secondary circulation is influenced by an external stress conditions which is controlled by different basic flow conditions.

5. Meandering Boundary Layers

A meandering flow model is searched to predict the symmetric and asymmetric meander loops. Results overlapping with experiments are given. Observations of the prediction to the development of meso-scale bed configuration in straight channels are also shown. The model predictions are conducted for three types of bed formations: alternating bars, braided bars, and no bars, according to the regime criteria of meso-scale bed formations \[^{[14]}\]. The migration velocity of bars in expanding boundary layers is also predicted.

In river management, it is the first condition to predict water surface height, water path and erosion and deposition of alluvial channels with sedimentation under different regime theories. Mathematical and physical models can be used in designing or planning for navigation use of river channels and hydraulic structures. Several researches have been conducted to evaluate flow and bed variation in expanding channels in nature \[^{[1,4]}\]. In meandering boundary layers the erosion and the deposition also shortly bed variation is computed by the continuity equation for sedimentation and discharge. Typical boundary layer formations and discharge with alternating bars and braided bars are predicted firstly in straight channels, and thus the formation and migration of bars are enabling prediction the boundary layer formations quantitatively.

6. Conclusions

Through an iteration process by prediction new finite difference method on a computational grid a result for steady-state condition is determined \[^{[1]}\]. The relationsips for discharge are given implicitly, while the continuity equation is given explicitly. The ratio of mean discharge is given as the square root of the mean energy gradient which is given by Manning \[^{[1]}\].

\[
\frac{Q + \Delta Q}{Q} \approx \sqrt{\left[\frac{I_e + \Delta I_e}{I_e}\right]} \approx \frac{1}{2} \frac{\Delta I_e}{I_e}
\]  

(12)

in which Q = the discharge; and Ie=energy slope is equal to the cross-sectional average water surface gradient.

If v =0 and a slip velocity \(u_w\) is emphasized as,

\[
\frac{\tau_w}{\rho} = \varepsilon \frac{\partial u}{\partial n} = C_{1_d} u L_{1_d} \varepsilon\]

(13)

where \(\tau_w\) is the shear stress at the boundary layer and \(C_{1_d}\) is the friction coefficient at boundary. The coefficient \(\varepsilon\) of the diffusion gives permission to compute the slip velocity \(u\) where \(v=0\) and \(z\) is the location at the upstream end. If the flow is supercritical, the boundary conditions are given as \(p\) at the upstream end and \(u,v\) are given at the downstream end.

7. Results

The results of model observations were carried out fort he same conditions as the mathemactical model at the Technical University of Berlin, Institute of Water Constructions and Water Resources, continued with a boundary layer without any gradient at continuous discharge conditions. Then the boundary layers were solidified, and precise measurement of the boundary layer formations and the new velocity conditions were attempted. It is observed \[^{[14]}\]:

1. The depth-averaged computational precise investigation was used to show equilibrium with natural observations at the symmetrical and asymmetrical meander loops.

2. The simulation with experimentation models and prototypes show good overlapping with this model.

3. The objective was to investigate a simple numerical relationship for prediction the variation of bed topography in prototypes.

In alluvial planforms, the relationships between hydraulic behaviour and plan geometries result in various of boundary layer formations which are dependent on migration bars and meandering river dunes.

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