Azimuthal Dependence of DIS with Spin-1 Target

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Abstract

We study DIS with a spin-1 hadron or a nucleus polarized in arbitrary direction. The differential cross-section in this case will have an azimuthal dependence. We derive the dependence by including contributions from twist-2- and twist-3 QCD operators. The twist-3 contribution is computed at tree-level. A spin-1 hadron or nucleus can have nonzero tensor polarization. We find that all structure functions related to the tensor-polarization of the initial hadron can be extracted by study the azimuthal dependence. A non-zero result of the structure functions of a nucleus from experiment will indicate nontrivial inner structure of the nucleus.

DIS experiment has been done mostly with a proton target. It has provided a wealth of information about the inner structure of proton and has played important role in testing QCD as the theory of strong interaction. If one takes a nucleus target as a weakly bounded state, one may expect that DIS cross-section is a sum of DIS cross-sections of each nucleon in the nucleus. But, EMC experiment has shown that it is not the case[1]. In fact, the interaction between nucleons in a nucleus can not be neglected, the nucleons are correlated. This is clearly shown by the measurement in [2] of DIS cross-sections with Bjorken variable of a nucleon larger than one. It implies that in DIS scattering with a nucleus more than one nucleon is involved.

DIS with a spin-1 target like a spin-1 nucleus has been theoretically analyzed in [3]. There are more structure functions than those in DIS with a proton. Especially, there are structure functions related the tensor polarization of the target. These functions do not exist for a proton target. It has been shown in [3] that these structure functions are zero, if nucleons inside the nucleus are at rest and do not interact with each other. In a recent experiment, one has found the evidence that one of these structure functions of deuteron, called as $b_1$, is non-zero[4]. It is clear that progresses made in experimental study of DIS with a nucleus will help to explore interactions or correlations between nucleons in a nucleus, and partonic picture of a nucleus.

In this work we study DIS with a spin-1 target. We consider the case that the initial hadron is polarized in an arbitrary direction. In this case the differential cross-section will have an azimuthal dependence. We will derive this dependence. Measuring the dependence will help to disentangle various structure functions. Our analysis is made in the framework of QCD factorization. We include not only contributions of twist-2 operators but also twist-3 operators. We re-analyze contributions of twist-2 operators and confirm the existing results with twist-2 operators. At twist-3 there are two additional distributions contributing to structure functions at tree-level. Again, these distributions can be extracted from the azimuthal dependence.
We consider the process:
\[ \ell + H \to \ell + \gamma^* + H \to \ell + X, \]  
where \( H \) is a spin-1 hadron. We will consider the case in which \( H \) is polarized along an arbitrary direction and the polarization is described by a polarization vector \( \epsilon \) with \( \epsilon \cdot P = 0 \), where \( P \) is the momentum of \( H \). We take a frame in which \( H \) moves in the \( z \)-direction with the momentum \( P \). The initial hadron \( H \) is polarized. The initial lepton carries the momentum \( k \) and is polarized with the helicity \( \lambda \).

The hadronic tensor is defined as:
\[ W^{\mu\nu} = \frac{1}{4\pi} \sum_X \int d^4xe^{iq \cdot x} \langle H|J^{\mu}(x)|X\rangle\langle X|J^{\nu}(0)|H\rangle. \]  

The relevant standard variables are:
\[ x_B = -\frac{q^2}{2q \cdot P} = \frac{Q^2}{2q \cdot P}, \quad y = \frac{q \cdot P}{P \cdot k}. \]

The Bjorken variable \( x_B \) is in the range \( 0 < x_B < 1 \). In the case that the initial hadron is a nucleus with the atomic number \( A \), one can define the Bjorken variable \( x_b \) of a nucleon with the relation \( x_b = Ax_B \). As mentioned at the beginning, the DIS cross-section with a nucleus is not zero for \( x_b > 1 \). We assume that the initial hadron is polarized along the direction \( \vec{m} \) in the sense that the projection of the spin vector along \( \vec{m} \) is quantized, i.e., \( \vec{m} \cdot \vec{S} = \lambda_m = 0, \pm 1 \). We introduce a frame in which the initial lepton moves in the \(-z\)-direction and the initial hadron moves in the \(+z\)-direction. The \( x \)-axis of the frame is chosen so that \( \vec{m} \) is in the \( xz \)-plane with the angle \( \theta_m \) to the \( z \)-direction. In this frame the lepton in the final state moves in the direction described with the polar angle \( \theta \) and the azimuthal angle \( \phi \). We call this frame as laboratory frame. This is illustrated in Fig.1.

![Figure 1: The defined frame for the differential cross-section.](image)

In the framework of collinear factorization the hadronic tensor can be factorized as convolutions of perturbative coefficient functions with various matrix elements of QCD operators at different twists. In this work we only consider the contributions from twist-2- and twist-3 QCD operators. Therefore we write the hadronic tensor as:
\[ W^{\mu\nu} = W^{\mu\nu}_{\text{twist-2}} + W^{\mu\nu}_{\text{twist-3}} + \cdots. \]
The quantities introduced from the symmetric- and trace-less part of $Q$ is relatively suppressed by the leading term in the Bjorken limit, the second term receives contributions only from twist-3 operators and is of higher twists are suppressed by $Q^{-n}$ with $n \geq 2$ relative to the first term. In the separation of Eq. we neglect the effect of the hadron mass $M$ except the effect in the relation between $x_B$ and the momentum fraction $x$ of partons. The neglected effect are suppressed at least by $M^2/Q^2$ in our results.

The hadronic tensor is conveniently analyzed in the light-cone frame in which the virtual photon moves in the $-z$-direction and the hadron in the $z$-direction. In this frame a vector $a^\mu$ is expressed as $a^\mu = (a^+, a^-, a_\perp) = ((a^0 + a^3)/\sqrt{2}, (a^0 - a^3)/\sqrt{2}, a^1, a^2)$ and $a_\perp^2 = (a^1)^2 + (a^2)^2$. We introduce two light-cone vectors $l^\mu = (1, 0, 0, 0)$ and $n^\mu = (0, 1, 0, 0)$. Two tensors related to the two vectors are defined as: $g_{\perp}^{\mu \nu} = g^{\mu \nu} - n^\mu l^\nu - n^\nu l^\mu$ and $\epsilon_{\perp}^{\mu \nu} = \epsilon^{\alpha \beta \mu \nu} l_\alpha n_\beta$. In the frame the momentum $P$ is $P^\mu = (P^+, P^-, 0, 0)$ with $P^2 = M^2$ and the virtual photon carries the momentum $q^\mu = (q^+, q^-, 0, 0)$. The introduced frame is not equivalent to the laboratory frame in Fig.1. However, the obtained hadronic tensor will be given in a covariant form by noting that $g_{\perp}^{\mu \nu}$ and $\epsilon_{\perp}^{\mu \nu}$ can be defined in terms of $g^{\mu \nu}$, $P^\mu$ and $q^\mu$ in an arbitrary frame. From any vector $a^\mu$ one can project the vector $a^\mu_{\perp} = g_{\perp}^{\mu \alpha} a_\alpha$, which is transverse to $P$ and $q$.

With the polarization vector $\epsilon^\mu$ one can define the spin density matrix as $\rho^{\mu \nu} \propto \epsilon^\mu \epsilon^\nu$. Instead of using polarization vectors, it will be convenient to use the following quantities to describe the spin. Although we explicitly define them in the light-cone frame, these quantities can also be defined covariantly as discussed in the above. From the symmetric- and trace-less part we introduce:

$$T_{\perp}^{\mu \nu} = \frac{1}{2} \left( \epsilon_{\perp}^{\mu} \epsilon_{\perp}^{\nu} + \epsilon_{\perp}^{\nu} \epsilon_{\perp}^{\mu} - g_{\perp}^{\mu \nu} \epsilon_{\perp} \cdot \epsilon_{\perp} \right),$$

$$\tilde{S}_{\perp}^\mu = \frac{M}{P^+} \left( n \cdot \epsilon_{\perp}^\mu + n \cdot \epsilon_{\perp} \cdot \epsilon_{\perp} \right), \quad \tilde{S}_L = -\frac{3}{2} \left( n \cdot \epsilon_{\perp} \cdot \epsilon_{\perp} + l \cdot n \cdot \epsilon_{\perp} \right), \quad (5)$$

and from the anti-symmetric part of the spin-density matrix one can define a transverse vector and the helicity constant $S_L$:

$$S_{\perp}^\mu = -i \epsilon_{\perp}^\mu \frac{M}{P^+} \left( n \cdot \epsilon_{\perp} l_{\perp} - n \cdot \epsilon_{\perp} \epsilon_{\perp} \right), \quad S_L = i \left( \epsilon_{\perp} \epsilon_{\perp} - \epsilon_{\perp} \epsilon_{\perp} \right). \quad (6)$$

The quantities introduced from the symmetric- and trace-less part of $\rho^{\mu \nu}$ describe the tensor polarization of $H$. If there is no tensor polarization, all of these quantities are zero. Those from the antisymmetric part describe the vector polarization. It is easy to find that $S_L$ is the helicity. Comparing the spin description of a spin-1/2 hadron, where one only needs a helicity and a transverse-spin vector for helicity-flip, corresponding to $S_L$ and $S_{\perp}$, respectively. Interpretations of the quantities introduced in the above are discussed in detail in [3, 6], where production of a spin-1 hadron in semi-inclusive DIS is studied.

The twist-2 contributions have been derived in [3, 6] with the technique of operator product expansion (OPE). We re-derive these results and organize the results in the way that the one-loop corrections of $\alpha_s$ can be easily obtained from existing results of DIS with spin-1/2 target. The relevant matrix element of twist-2 quark operator can be parametrized as:

$$\int \frac{d\lambda}{2\pi} e^{-ix\lambda P^+} \langle H | \bar{\psi}_i(\lambda n) \gamma_5 \psi_j(0) | H \rangle = \frac{1}{2} \left[ (q(x) + \tilde{S}_L \bar{q}_L(x)) \gamma^- + S_L q_L(x) \gamma_5 \gamma^- \right]_{ji} + \cdots, \quad (7)$$

where $ij$ are Dirac indices. The color indices of quark fields are contracted. Gauge links along the $n$-direction are suppressed in the above expression, or one can eliminate them in the light-cone gauge $n \cdot G = 0$. In the above we have neglected higher-twist contributions and the contribution at leading
twist with the structure $\gamma_5\gamma_\perp^\mu\gamma^-$. The later is called as transversity distribution in the case of a spin-1/2 hadron\cite{7}. The distribution defined with the structure $\gamma_5\gamma_\perp^\mu\gamma^-$ will not contribute in our case because of its chirality. Comparing with quark distributions of a spin-1/2 hadron, there is an additional quark distribution related to the tensor polarization of the spin-1 hadron. It is also noted that the additional distribution $q_L(x)$ in Eq.(7) is multiplied with the same $\gamma^-$ as the unpolarized quark distribution $q(x)$. Therefore, the perturbative coefficient functions related to the two distributions will be the same.

Four gluon distributions can be defined with the twist-2 gluonic operator. They are:

$$\frac{1}{xP^+} \int \frac{d\lambda}{2\pi} e^{-ix\lambda P^+} \langle H|G_{\mu\nu}(\lambda n)G^{\mu\nu}(0)|H\rangle = -\frac{1}{2} \bar{g}_{\perp}(x) + \bar{g}_L(x) \tilde{S}_L - \frac{1}{2} T_{\perp} g_T(x) - i \frac{\bar{g}_{\perp}}{2} g_L(x) S_L,$$

where $G_{\mu\nu}$ is the gluon field strength tensor and the indices $\mu$ and $\nu$ are transverse. Again, gauge links should be supplemented to make the definition gauge-invariant. Comparing with gluon distributions of a spin-1/2 hadron, here we have two additional gluon distributions related to the tensor polarization. The distribution $g_T$ was first identified in \cite{8} with OPE.

The introduced parton distributions or their certain linear combinations have the interpretation of the probabilities to find the parton with a definite polarization in the hadron. E.g., such an interpretation of $g_T$ is given in \cite{6}. The parton distributions depend on the renormalisation scale $\mu$. The $\mu$-dependence of the most distributions is known. The $\mu$-dependence of $q(x)$ and $g(x)$, is governed by the standard DGLAP equation. $\bar{q}_L(x)$ and $\tilde{g}_L(x)$ satisfy the same DGLAP equation as $q(x)$ and $g(x)$ do. Similarly, the $\mu$-dependence of distributions related to $\hat{S}_L$ is the same as that of the correspond distributions of a spin-1/2 hadron. The $\mu$-dependence of $g_T(x)$ has been derived in \cite{9}. We have re-derived it. The result is:

$$\frac{\partial g_T(x, \mu)}{\partial \ln \mu} = \frac{\alpha_s}{\pi} \int_x^1 \frac{d\xi}{\xi} g_T(\xi, \mu) \left[ \frac{2N_c z}{1-z} + \delta(1-z) \left( \frac{11}{6} N_c - \frac{1}{3} N_f \right) + \mathcal{O}(\alpha_s) \right],$$

with $z = x/\xi$. This confirms the result in \cite{9}.

The hadronic tensor with the contributions from twist-2 operators or twist-2 distributions can be calculated in a standard way. We will skip the detail of the calculation and only give the results here. We introduce the following quantities:

$$p^\mu = (P^+, 0, 0, 0), \quad \hat{p}^\mu = p^\mu - \frac{q \cdot p}{q^2} p^\mu, \quad x = \frac{Q^2}{2p \cdot q} = \frac{2x_B}{1 + \sqrt{1 + 4M^2 x_B}}.$$  \hspace{1cm} (10)

Here $x$ is Nachtmann variable introduced in \cite{10} which takes the target mass correction from kinematics into account. For $Q^2$ large enough, one can neglect the hadron mass and has $x \approx x_B$. Our result for the twist-2 part of the hadronic tensor takes the form:

$$W^{\mu\nu} = \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) \left( F_1(x_B, Q^2) + \bar{S}_L \tilde{F}_1(x_B, Q^2) \right) + \frac{x}{p \cdot q} \hat{p}^\mu \hat{p}^\nu \left( F_2(x_B, Q^2) + \bar{S}_L \tilde{F}_2(x_B, Q^2) \right) + \frac{i}{p \cdot q} S_{L\mu\nu} \alpha_s p_\alpha q_\beta G_1(x_B, Q^2) + T_{\perp}^{\mu\nu} \hat{G}_T(x_B, Q^2),$$

and the structure functions at leading order of $\alpha_s$ are expressed of twist-2 parton distributions as:

$$F_1(x_B, Q^2) = \frac{1}{2} q(x), \quad F_2(x_B, Q^2) = q(x), \quad \hat{G}_1(x_B, Q^2) = q_L(x), \quad \hat{G}_T(x_B, Q^2) = \frac{\alpha_s}{4\pi} \int_x^1 \frac{d\xi}{\xi^2} \hat{g}_T(\xi, \mu^2).$$  \hspace{1cm} (12)
In the factorization the structure functions are expressed as convolutions of perturbative coefficient functions with twist-2 parton distributions. For the first 5 structure functions in Eq. (11) they are \( \delta \)-functions. We do not explicitly give the contributions with anti-quark distributions here and later in the case with twist-3 contributions. They can easily be obtained. The results of \( \tilde{F}_{1,2} \) can be obtained from the results of \( F_{1,2} \) by replacing \( q(\xi, \mu^2) \) with \( \bar{q}_L(\xi, \mu^2) \), and \( g(x, \mu^2) \) with \( g(x, \mu^2) \) beyond the leading order.

We have written our results in the form so that the perturbative coefficient functions are exactly the same appearing in the corresponding structure functions for DIS with a spin-1/2 hadron. Therefore, the one-loop corrections to the results in the first line in Eq. (12) and to \( \tilde{F}_{1,2} \) are known. The leading order of \( \hat{G}_T \) is of \( \alpha_s \). We have extracted the perturbative coefficient function of \( \hat{G}_T \) from the forward scattering \( \gamma^* g \to q\bar{q} \to \gamma^* g \). The result agrees with that in [6]. It should be noted that \( \hat{G}_T \) only receives contributions from gluons in the target. It does not receive any contributions from quarks at twist-2.

Now we turn to the twist-3 part of the hadronic tensor. At twist-3 there are many different operators, e.g., there are twist-3 operators defined with bilinear quark fields like \( \bar{\psi}(\lambda_1 n) G^{+\mu}(\lambda_2 n) \psi(0) \) and those defined with bilinear quark fields combined with one gluon field strength operators like \( \bar{\psi}(\lambda_1 n) G^{+\mu}(\lambda_2 n) \psi(0) \) or with the covariant derivative \( D^\mu(\lambda_2 n) = \partial^\mu + ig \lambda_2 G^{\mu}(\lambda_2 n) \) instead of \( G^{+\mu}(\lambda_2 n) \) given in [11, 12]. However, not all of their matrix elements are independent, as shown in [13, 14, 15]. This will also be shown later in this work. To consistently factorize the relevant structure functions, one should take those operators which are independent. One should use an adequate set of independent operators for a given process. In our case, the twist-3 effect comes from the transverse motion of a single incoming parton as given in Fig. 2a and from multi-parton scattering as given in Fig. 2b. Therefore, it is convenient to take the operators defined with the covariant derivative, in which the part with the derivative represents the effect of the transverse motion. We define here:

\[
P^+ \int \frac{dy_1 dy_2}{(2\pi)^2} e^{-iy_2(x_2-x_1)P^+} e^{-iy_1 x_1 P^+} \langle P, \epsilon | \bar{\psi}(y_1 n) D^\mu(y_2 n) \psi(0) | P, \epsilon \rangle = \frac{1}{4} \left[ \gamma^\perp \left( D_F(x_1, x_2) \epsilon_\perp^{\mu \nu} S_\perp^{\mu \nu} + i \tilde{D}_F(x_1, x_2) \tilde{S}_\perp^{\mu \nu} \right) + \gamma_5 \gamma^\perp \left( D_\Delta(x_1, x_2) S_\perp^{\mu \nu} + i \tilde{D}_\Delta(x_1, x_2) \epsilon_\perp^{\mu \nu} \tilde{S}_\perp^{\mu \nu} \right) \right] + \cdots \]  

where \( \cdots \) denote power-suppressed contributions. The definition is given in the light-cone gauge \( n \cdot G = 0 \). In other gauges, gauge links need to be supplemented. From symmetries of QCD one can show:

\[
D_F(x_1, x_2) = -D_F(x_2, x_1), \quad D_\Delta(x_1, x_2) = D_\Delta(x_2, x_1), \quad \tilde{D}_F(x_1, x_2) = \tilde{D}_F(x_2, x_1), \quad \tilde{D}_\Delta(x_1, x_2) = -\tilde{D}_\Delta(x_2, x_1). \]  

Similarly, there are four independent twist-3 gluon distributions defined with operators of gluon field strength tensor. Two of them are defined with the structure constant \( f^{abc} \):

\[
\frac{i f^{abc} g_{\alpha \beta}}{P^+} \int \frac{dy_1 dy_2}{4\pi} e^{i(-y_1 x_1 + y_2 x_2) P^+} \langle H | G^{a, +\alpha}(y_1 n) G^{b, +\mu}(0) G^{c, +\beta}(y_2 n) | H \rangle = T_G(x_1, x_2) \epsilon_\perp^{\mu \nu} S_\perp^{\mu \nu} + i \tilde{T}_G(x_1, x_2) \tilde{S}_\perp^{\mu \nu}. \]  

The contributions involving these matrix elements of the twist-3 gluonic operators appear at higher order of \( \alpha_s \). They will be not considered here. We only notice here that symmetries, like Bose symmetry of gluons, give constraints of the form of the two distributions. Different parameterizations of these twist-3 gluon distributions exist, e.g., in [16]. There are another two twist-3 gluon distributions defined by
replacing $i f^{abc}$ in Eq. (15) with $d^{abc}$. These two distributions will not give contributions to DIS, because that the operator with $d^{abc}$ is odd under charge conjugation and the product of currents in the hadronic tensor in Eq.(2) is $C$-even. The scale-dependence of twist-3 operators has been studied in [17, 18, 19, 20].

Figure 2: The diagrams for twist-3 contributions. The black dots denote the insertion of electromagnetic current operators. A cut is implied for all diagrams.

The contributions from these twist-3 matrix elements to the hadronic tensor at leading order of $\alpha_s$ are given by diagrams in Fig.2. In this letter, we work with the light-cone gauge. As discussed in detail in [21], the contributions from Fig.2a contains not only the contributions of twist-2 but also the contributions of twist-3 or higher twist. The twist-2 contributions from Fig.2a are those given in the first line of Eq. (12). The leading-power contribution of Fig.2b is of twist-3 in the gauge $n \cdot G = 0$. We will take this gauge to derive our results. The contribution from Fig.2a reads:

$$W_{\mu\nu}^{(a)} = \frac{1}{4\pi} \int \frac{d^4k d^4y}{(2\pi)^4} e^{-iyk} |\bar{\psi}(y)H^{\mu\nu}(k, q)\psi(0)|^2, \quad H^{\mu\nu}(k, q) = 2\pi \delta((k + q)^2)\gamma^{\mu}\gamma \cdot (k + q)\gamma^\nu, \quad (16)$$

where $k$ is the four momentum of the quark entering the scattering with the virtual photon. To separate the contributions of different twists, we first expand $H^{\mu\nu}(k, q)$ around the momentum $\hat{k}^{\mu} = (k^+, 0, 0, 0)$:

$$H(k, q) = H(\hat{k}, q) + k^\rho \frac{\partial H(k, q)}{\partial k_{\perp \rho}} \Big|_{k = \hat{k}} + \cdots. \quad (17)$$

Then we decompose the quark field into the large- and small component in the high energy limit:

$$\psi(x) = \psi_+(x) + \psi_-(x), \quad \psi_+(x) = \frac{1}{2} \gamma^- \gamma^+ \psi(x), \quad \psi_-(x) = \frac{1}{2} \gamma^+ \gamma^- \psi(x), \quad (18)$$

where $\psi_+(\psi_-)$ is the large(small) component. With equation of motion one can derive:

$$2\partial^+ \psi_-(x) = -\gamma^+ \gamma_\perp \cdot D_\perp \psi_+(x). \quad (19)$$

This indicates that the small component $\psi_-$ in comparison with the large component $\psi_+$ is power-suppressed in the matrix element in Eq.(16). One can use the expansion in Eq.(17) and the decomposition in Eq.(18) to obtain contributions of different twists.

The leading twist contribution from Fig.2a is given by taking the first term in the expansion and neglecting the small component of quark fields in Eq.(16). The twist-3 contribution from Fig.2a has two parts. The first one is from the second term in the expansion in Eq.(17), where all quark fields are the large components in Eq.(16). This gives a twist-3 contribution involving operators like $\bar{\psi}\partial_\perp \psi$. The second part is given by taking the first term in Eq.(17), where one small component in Eq.(16) is involved. This gives another twist-3 contribution involving operators like $\bar{\psi}D_\perp \psi$. 

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The twist-3 contribution from Fig.2b is obtained in a straightforward way, where one lets all partons have the momenta along the +−-direction and the gluon line is for a transversely polarized gluon. One then obtains the twist-3 contribution from Fig.2b involving the operator \( \bar{\psi} G_\perp \psi \). Summing this contribution with the first part of Fig.2a, we find that the sum can be written in a form involving the operator \( \bar{\psi}(y_1 n) D_\perp (y_2 n) \psi(0) \). At the end we have the total twist-3 contribution of the hadronic tensor:

\[
W^{\mu\nu} \bigg|_{\text{twist-3}} = -\frac{1}{2p \cdot q} \left[ \tilde{S}^{\mu}_\perp \tilde{p}^\nu + \tilde{S}^{\nu}_\perp \tilde{p}^\mu \right] \int dy \left( \tilde{D}_F(x, y) - \tilde{D}_\Delta(x, y) \right) \\
+ i \frac{1}{2g^2} \epsilon^{\mu\nu\alpha\beta} q_\alpha S_{\perp\beta} \int dy \left( D_\Delta(x, y) + D_F(x, y) \right), \tag{20}
\]

with the integration range \( 1 > y > -1 + x \). It should be noted that the case with \( y > 0 \) and that of \( y < 0 \) represent different partonic processes. For \( y < 0 \), the process is the forward scattering of \( \gamma^* q\bar{q} \rightarrow \gamma^* g \), while for \( y > 0 \) the partonic process is \( \gamma^* qg \rightarrow \gamma^* q \). This can be seen clearly by using multi-parton states as a target to calculate the hadronic tensor and twist-3 matrix elements, similarly to the study of single spin asymmetries in [22], where the factorization involves twist-3 operators.

Comparing DIS with a spin-1/2 hadron, the last term in Eq. (20) corresponds to the structure function related to the transverse polarization of the spin-1/2 hadron. This structure function has been studied in detail in [14,15], where one-loop correction has been obtained. Therefore, the one-loop correction to the last term in Eq. (20) is also known. It may be possible to extract one-loop correction to the first term in Eq. (20) from the results of [14,15].

It is interesting to realize that the special form of the tree-level result in Eq. (20) can be written in a more simple form by using equation of motion of QCD, as discussed in the case of a spin-1/2 hadron in [14]. For this we define the following two twist-3 functions with the mentioned bilinear quark operator:

\[
xP^+ \int \frac{d\lambda}{2\pi} e^{-ix\lambda P^+} \langle H| \bar{\psi}(\lambda n) \gamma_5 \gamma_\perp \psi(0) |H\rangle = q_T(x) S^{\mu}_\perp, \\
xP^+ \int \frac{d\lambda}{2\pi} e^{-ix\lambda P^+} \langle H| \bar{\psi}(\lambda n) \gamma_\perp \psi(0) |H\rangle = \tilde{q}_T(x) \tilde{S}^{\mu}_\perp. \tag{21}
\]

One can find the following operator-identities and the hadronic tensor:

\[
q_T(x) = \int dy \left( D_F(x, y) + D_\Delta(x, y) \right), \\
\tilde{q}_T(x) = -\int dy \left( \tilde{D}_F(x, y) - \tilde{D}_\Delta(x, y) \right), \\
W^{\mu\nu} \bigg|_{\text{twist-3}} = \frac{1}{2p \cdot q} \left( \tilde{S}^{\mu}_\perp \tilde{p}^\nu + \tilde{S}^{\nu}_\perp \tilde{p}^\mu \right) \tilde{q}_T(x) - i \frac{1}{2g^2} q_T(x) \epsilon^{\mu\nu\alpha\beta} q_\alpha S_{\perp\beta}. \tag{22}
\]

However, for factorization one should use the result in Eq. (20) for higher-order corrections and study of \( Q^2 \)-dependence as discussed in [14]. In this work we do not introduce new notations of structure functions for the twist-3 part. In the below, we will use the notation of our tree-level results in Eq. (22) for differential cross-sections. It should be kept in mind that beyond tree-level the structure functions of the twist-3 part should be written as convolutions with those twist-3 matrix elements in Eq. (13). Our results in Eq. (11) and Eq. (22) are covariant as discussed before about the light-cone frame, and are \( U_{em}(1) \)-gauge invariant.

With our results of structure functions we can obtain the differential cross-section for arbitrary \( \bar{m} \)

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with $\lambda_m = 0, \pm 1$:

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\frac{d\sigma(\lambda_m)}{dx_B dy d\phi} = \frac{2y\alpha^2}{Q^2} \left[ F_1(x_B, Q^2) + a_m \tilde{F}_1(x_B, Q^2) + \frac{1-y}{y^2} \left( F_2(x_B, Q^2) + a_m \tilde{F}_2(x_B, Q^2) \right) \right]
- \frac{1}{2y^2} b_m \sin^2 \theta_m \hat{G}_T(x_B, Q^2) \cos 2\phi - \lambda_m \frac{2-y}{y^2} \cos \theta_m \hat{G}_1(x_B, Q^2)
- \frac{1}{2y^2} b_m \sin 2\theta_m \hat{q}_T(x) \right] + \left( 2 - y \right) b_m \sin \theta_m \hat{q}_T(x)
\right]
\right],
$$

(23)

where $a_m$ and $b_m$ are parameters of $\lambda_m$. They are:

$$
b_m = 3|\lambda_m| - 2, \quad a_m = \frac{1}{4} b_m (1 - 3 \cos^2 \theta_m).
$$

(24)

Eq.(23) is our main result. The differential cross-section is given in the laboratory frame, the azimuthal angle $\phi$ is defined in Fig.1. The terms in the first two lines are of leading-twist part of $W^{\mu\nu}$. They are also at the leading power of $1/Q$ in the differential cross-section. The terms in the last line are from the twist-3 part of $W^{\mu\nu}$, they are suppressed by $1/Q$. The result is at the accuracy of the next-to-leading power of $1/Q$. In Eq.(23) we have neglected the effect of finite target mass in kinematics except the effect in the relation between $x$ and $x_B$. From the result one can see that the tensor-polarized gluon distribution $\hat{q}_T$ in Eq.(5) characterizes a $\cos 2\phi$-dependence at leading power of $1/Q$. The twist-3 matrix element $\tilde{q}_T$ involving tensor polarization of $H$ leads to a $\cos \phi$-dependence.

Some terms vanish if one takes $\theta_m = 90^\circ$ in Eq.(23), e.g., the contributions of $\hat{G}_1$ and $\tilde{q}_T$. In fact these contributions do not vanish, they are kinematically suppressed by an overall factor $M/Q$. To see this we give the differential cross-section in the case of $\theta_m = 90^\circ$:

$$
\frac{d\sigma(\lambda_m)}{dx_B dy d\phi} = \frac{2y\alpha^2}{Q^2} \left[ F_1(x_B, Q^2) + \frac{b_m}{4} \tilde{F}_1(x_B, Q^2) + \frac{1-y}{y^2} \left( F_2(x_B, Q^2) + \frac{b_m}{4} \tilde{F}_2(x_B, Q^2) \right) \right]
- \frac{1}{2y^2} b_m (1-y) \hat{G}_T(x_B, Q^2) \cos 2\phi + \lambda_m \frac{1-y}{y Q} q_T(x) \cos \phi
\right]
\right] + \left( 2 - y \right) \frac{2M x_B \sqrt{1-y}}{y Q} \left[ \lambda_m \hat{G}_1(x_B, Q^2) - b_m \frac{1-y}{y Q} \hat{q}_T(x) \right] \cos \phi
\right]
\right].
$$

(25)

From the above equation one can see that in the case of $\theta_m = 90^\circ$, the contribution from $\hat{G}_1$ becomes power-suppressed and is $\cos \phi$-dependent. The contribution from $\tilde{q}_T$ is now at order of $1/Q^4$. In comparison with Eq.(23), the contribution from $\hat{G}_1$ and $\tilde{q}_T$ are suppressed by the overall factor $M/Q$ in the case $\theta_m = 90^\circ$. This factor is purely from kinematics. The effect from higher twist in $\hat{G}_1$ or higher-twist correction to contribution of $\tilde{q}_T$ will also be suppressed by the same kinematic factor and are not at leading power of $Q$. From Eq.(25) the contributions from $\hat{G}_1$ and $q_T$ give a $\cos \phi$-dependence at the same order of $1/Q$, although they are of different twist.

Another special case is with $\theta_m = 0^\circ$. In this case we have:

$$
\frac{d\sigma(\lambda_m)}{dx_B dy d\phi} = \frac{2y\alpha^2}{Q^2} \left[ F_1(x_B, Q^2) - \frac{b_m}{2} \tilde{F}_1(x_B, Q^2) + \frac{1-y}{y^2} \left( F_2(x_B, Q^2) - \frac{b_m}{2} \tilde{F}_2(x_B, Q^2) \right) \right]
- \frac{2M^2 x_B^2 (1-y)^2}{y^2 Q^2} \hat{G}_T(x_B, Q^2) - \lambda_m \frac{2-y}{y} \hat{G}_1(x_B, Q^2)
\right] + \left( 2 - y \right) b_m \hat{q}_T(x) - \lambda_m y q_T(x) \right]
\right].
$$

(26)
There is no azimuthal dependence as expected. Here, the contributions from $\hat{G}_T$ and the twist-3 part of $W^{\mu\nu}$ are suppressed by the factor $M/Q$ from kinematics, as in the case discussed before.

Experimentally, the azimuthal dependence in DIS with a spin-1 nucleus can be studied with fixed target experiment of J-Lab and at the proposed EIC[23]. It will be interesting to see evidences of the existence of those structure functions related to the tensor polarization. They are $\tilde{F}_{1,2}$, $\hat{G}_T$ and $\hat{q}_T$. If there is no interaction between nucleons in a nucleus, they are zero. Hence, experimental study of the azimuthal dependence in DIS with a spin-1 polarized nucleus will provide more information about the inner structure of the nucleus. The experimental results in [4] indicates that the structure function called $b_1$ according to [3] can be nonzero. $b_1$ is related to the tensor polarization $\tilde{S}_L$ and is equivalent to the defined structure function $\tilde{F}_1$ or $\tilde{F}_2$ here. From our result, the structure function $\tilde{F}_{1,2}$ gives a constant contribution in the azimuthal distribution.

Among the structure functions related to tensor polarization, $\hat{G}_T$ is of the most interesting, as discussed in [6]. The existence of this structure function will clearly indicate that a nucleus has a nonzero gluon content which can not be interpreted with the gluon content of each individual nucleon in the nucleus. From Eq.(12), the contribution of $g_T$ to the differential cross section is suppressed by $\alpha_s$ relatively to other structure functions. It may be easier to extract $g_T$ from DIS with the restriction that the final state consists a heavy quark $Q\bar{Q}$-pair with other possible hadrons. Then, all parton distributions contribute at the same order of $\alpha_s$. The results for the first three structure functions in Eq.(12), hence also for $\tilde{F}_{1,2}$, can be found in literature about DIS with a spin-1/2 target. We denote $\hat{G}_T$ in this case as $\hat{G}_{T,QQ}$ and derive here the result for $\hat{G}_{T,QQ}$ by considering the forward scattering of $\gamma^* g \rightarrow Q \bar{Q} \rightarrow \gamma^* g$:

$$\hat{G}_{T,QQ}(x_B, Q^2) = \frac{\alpha_s}{4\pi} \int_{x/(1-4x_m)}^{1} \frac{d\xi}{\xi} g_T(\xi) \left[ \beta(z^2 + 2x_m(1-z)) + 4x_m(z + x_m) \ln \frac{1 + \beta}{1 - \beta} \right] + O(\alpha_s^2),$$

$$z = \frac{x}{\xi}, \quad x_m = \frac{m_Q^2}{2p \cdot q}, \quad \beta^2 = 1 - \frac{4x_m}{1 - z}. \quad (27)$$

In the above $m_Q$ is the mass of the heavy quark $Q$. In the limit of $m_Q \rightarrow 0$, we find from $\hat{G}_{T,QQ}$ the result of $\hat{G}_T$ in Eq.(12).

To summarize: We have studied the azimuthal dependence of DIS with a spin-1 hadron or nucleus, where the initial hadron is polarized in an arbitrary direction. In the first step we have re-derived the twist-2 contributions and the evolution of the tensor-polarized gluon distribution. The results agree with existing one. We then have derived the twist-3 contributions at tree-level. There are two structure functions from twist-3 operators. One is related to the tensor polarization, another is related to the vector polarization. The azimuthal distribution is given for the case that the initial hadron is polarized in an arbitrary direction. In general structure functions related to the tensor polarization of a spin-1 nucleus will be zero, if nucleons do not interact or are not correlated inside the nucleus. Therefore, experimental study of DIS with a spin-1 nucleus will provide information about the interaction or the correlation. Our result shows that these structure functions can be extracted from the studied azimuthal distribution.

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