Geometric modeling of multilayer structures with the application of curves and surfaces Bezier

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Abstract. In the article is reported a method of constructing the model of three-dimensional parametric solids free-form and their description using Bezier curves and surfaces. There are shown the equations of solid patch that bounded curves and surfaces cubic Bezier. The experiment was carried on an octagonal carcass of curved solid patch. Computer experiment showed that the method works well and correctly determines the position of the external and internal surfaces of both the initial condition, and under deformation of the solid shape's with the aid of control points. This method makes easy controlling the shape and completely describes the behavior of the intermediate surfaces inside of the simulated object's solid. The proposed method can be used for numerical simulation of multilayer structures.

1. Introduction

Methods of geometric modeling of three-dimensional parametric solids, including multi-layer structures, make it possible to optimizing the design and development of a wide variety of industrial products different that often functionally different. For example, the volumetric geometric modeling method was developed for the design of parts manufactured from composite material by winding [1]. Capabilities and features of this method are also outlined for the designing of light industry products, which have a complex anthropomorphic form and the multi-layer structure [2, 3].

There are different approaches for simulating these solids. There are superficial solid model representation bodies through flat faces, modeling bodies with the help of constructive geometry solid methods [4]. Each of these methods has its advantages and disadvantages. The general approach is described the solid by sum total boundary portions, faces and edges that are defined parametrically.

Each patch is constructed from a set of matching with each other freeform surfaces, each of which contains full information about its borders and relations with neigh bors. Such a description is called a representation of the solid with the help of borders. It allows performing a number of operations over the bodies while maintaining a consistent way of “interior structure”. Presentation of solids through boundaries allows modeling objects of arbitrary shape and complexity, including multilayer structures.

In the works [1, 5] were used a generalized linear interpolation for describe the three-dimensional solid patch. This method of a solid description enables constructing a composite solid out of the patches of this type. Though at the boundaries of patches of the solid will be only continuous but not
smooth. Thus, in result composite solid is formed with order zero smoothness. Essential for most practical applications a continuous gradient is achieved by a more complicated way. In this way, the solid patch is determined not only by calculating the boundary surfaces, but also through the settlement boundary of the slopes in the directions that are transversally to the boundary surfaces. For this purpose, in [6] was posed and solved the problem of constructing the equation of the solid patch, which takes into account the specified slopes across the boundary surfaces. In this paper, the equation of patches obtained by applying the generalized Hermite interpolation. For a description of the boundary curves used parametric cubic splines. The obtained equation makes it possible to form a compound solid of the first order of smoothness. That kind of method is useful if a shape of the modeling object is known. Although the conversion of an algebraic equation Hermite equation in the form of the curve allows you to work with curve on an intuitive level, Hermitian form does not fully meet the requirements of application systems developers. It is not easy to predict the curve shape in a magnitude of the tangent vectors. Therefore, in some applications, such as in the designing of light industry products, where the physical model is often absent or present starting imperfections of product shape, it is preferable the modeling of curves and surfaces in the form of Bezier.

In this article is considered the problem of constructing a three-dimensional equation of the solid patch by a cubic Bezier curves and surfaces. The proposed method allows describing the composite solid of the first order of smoothness.

2. Problem formulation
Suppose we know the shape of the solid surface of the modeled object. Mesh of curves that defining the surface of the solid, was built. Grid of curves divides the surface into rectangular patches that are bounded \( u, v, w \) curves, as shown in figure 1. Let the length of these parametric curves varies from 0 to 1. Then \( r(u, v, w), 0 < u, v, w < 1 \), represents the interior of the solid patch, and \( r(i, v, w), r(u, j, w), r(u, v, k), i, j, k = 0, 1 \) determine its known boundary surfaces. It is necessary to determine the function \( r(u, v, w) \), provided that \( u = i, v = j, w = k \) represents the desired boundary surface of the solid patch.

![Figure 1. A patch of the solid](image)

3. Theory
A patch of the solid is an octagonal curved carcass consisting of 8 anchor points, 12 boundary curves, and 6 boundary surfaces (figure 2).
For a description of the boundary curves we used a parametric cubic Bezier curve (figure 3). The cubic Bezier curve equation in matrix form is (7):

\[
\mathbf{r}(t) = \mathbf{F}(t)\mathbf{M}\mathbf{P},
\]

where \( \mathbf{F}(t) = \begin{pmatrix} t^3 & t^2 & t & 1 \end{pmatrix} \) – row matrix of parameter \( t \);

\( \mathbf{P} = \begin{pmatrix} \mathbf{P}_0 & \mathbf{P}_1 & \mathbf{P}_2 & \mathbf{P}_3 \end{pmatrix}^T \) – column matrix of guided curve points;

\[
\mathbf{M} = \begin{pmatrix} -1 & 3 & -3 & 1 \\
3 & -6 & 3 & 0 \\
-3 & 3 & 0 & 0 \\
1 & 0 & 0 & 0 \\
\end{pmatrix}
\]

– matrix of coefficients cubic curve Bezier.

Using equation (1), we write the equation of the boundary curves of the solid patch:

\[
\mathbf{r}(u,j,k) = \mathbf{F}(u)\mathbf{M}_{ijk} \mathbf{r}(i,v,k) = \mathbf{F}(v)\mathbf{M}_{ijk} \mathbf{r}(i,j,w) = \mathbf{F}(w)\mathbf{M}_{ijk} \quad i,j,k \in \{0,1\},
\]

where \( \mathbf{M}_{ijk} \) – control points along curves \( u, v, w \).

The equations of the boundary surfaces of the solid patches are of the form:

\[
\mathbf{r}(i,v,w) = \mathbf{F}(v)\mathbf{M}_{i,jw}^T \mathbf{F}(w)^T, \quad \mathbf{r}(u,j,w) = \mathbf{F}(u)\mathbf{M}_{i,jw}^T \mathbf{F}(w)^T.
\]
where \( P_{ew}, P_{ujw}, P_{uwk} \) – control points of boundary surfaces.

Equation solid patch in matrix form is

\[
\mathbf{r}(u, v, k) = \mathbf{F}(u)\mathbf{M}P_{ew}\mathbf{M}^T\mathbf{F}(v)^T, i, j, k \in \{0,1\},
\]

where \( P_{uw0}, P_{uw1}, P_{uw2}, P_{uw3} \) – control points of surfaces.

4. Illustrative example

As an experiment was described solid's patch on an octagonal curved carcass. Computer experiment showed that the method works well and correctly determines the position of the external and internal surfaces of both the initial condition, and when the deformation forms using control points.

Computational experiments were carried out using mathematical MathCAD application. Changes the position of the nodal points of the solid patch and defined by the intermediate patch of the surface of the solid with parameter values of \( v = 0.25; 0.5; 0.75; 1 \). The experimental results are shown in figure 4.

![Figure 4. The intermediate surface of the solid patch](image)

5. Results

Computational experiments demonstrated the correctness of the obtained formula (4) for describe the patch of solid. Sequential substitutions of specific numerical values in the above formula satisfy the required conditions of the task. Manage form of simulated object can use the control points of the boundary surfaces Bezier. The boundary surfaces are changed by changing the carcass points. Thus the resulting structure is completely describes the behavior of the intermediate solid surface, including the middle surface. This is a very important result for certain applications where it is necessary to determine the median surface of the thick-walled multi-layer shell with variable thickness with the change in the shape of the boundary surfaces exposed to different loads.

6. Conclusions

This model setting the shape and structure of the multilayer product design will optimize the tasks of modeling the volume of products in various branches of industrial production. This will shorten the
product design, without causing deterioration of the quality of the designed object. Results of the experiments showed that the proposed method does not require the overriding the solid carcass when determining the position and shape of the intermediate surfaces. The shape of the solid surface is easily controlled with the aid of the control points. Using a parametric cubic Bezier curves and surfaces provides a composite with first order of smoothness.

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