Anomalous $WWZ$ couplings and $K_L \to \mu^+ \mu^-$

Xiao-Gang He

Research Center for High Energy Physics
School of Physics
University of Melbourne
Parkville, Vic. 3052 Australia

(June, 1993 (Revised July, 1993))

Abstract

We study contributions to $K_L \to \mu^+ \mu^-$ from anomalous $WWZ$ interactions. There are, in general, seven anomalous couplings. Among the seven anomalous couplings, only two of them contribute significantly. The others are suppressed by factors like $m_s^2/M_W^2$, $m_d^2/M_W^2$, or $m_K^2/M_W^2$. Using the experimental data on $K_L \to \mu^+ \mu^-$, we obtain strong bounds on the two anomalous couplings.
In this paper we study contributions to $K_L \rightarrow \mu^+\mu^-$ from the anomalous $WWZ$ interactions. The Minimal Standard Model of electroweak interactions is in very good agreement with present experimental data. However its structure should be tested in detail in order to finally establish the model. One of the important aspects is to test the structure of self-interactions of electroweak bosons. Such test will provide information about whether the weak bosons are gauge particles with interactions predicted by the MSM, or gauge particles of some extensions of the MSM which predict different interactions at loop levels, or even non-gauge particles whose self-interactions at low energies are described by effective interactions. In general there will be more self-interaction terms than the tree level MSM terms (the anomalous couplings) \[1\]. It is important to find out experimentally what are the allowed regions for these anomalous couplings. The process $K_L \rightarrow \mu^+\mu^-$ has been studied in the MSM extensively \[2\]. It has been used to study the allowed range for the top quark mass and the allowed ranges for some of the KM matrix elements. In this paper we show that $K_L \rightarrow \mu^+\mu^-$ also puts very strong constraints on some of the $WWZ$ anomalous couplings.

The most general form for the anomalous $WWZ$ interactions can be parametrized as

$$L = -g \cos \theta_W \left[ i \frac{g}{2} Z W_{\mu} W_{\nu} Z^{\mu \nu} - \frac{\kappa Z}{2 M_W^2} W_{\mu} W^{\mu} Z_{\nu} + \frac{\lambda Z}{2 M_W^2} W_{\mu} W^{\mu} Z_{\nu} + \frac{i \tilde{\kappa} Z}{2 M_W^2} W_{\mu} W^{\mu} Z_{\nu} + \frac{i \tilde{\lambda} Z}{2 M_W^2} W_{\mu} W^{\mu} Z_{\nu} + \frac{g Z}{2} W_{\mu} W_{\nu} - \partial^\mu Z^{\mu} + \partial^\nu Z^{\nu} \right],$$

where $W_{\mu}^{\pm}$ and $Z_{\mu}$ are the $W$-boson and $Z$-boson fields, $W_{\mu \nu}$ and $Z_{\mu \nu}$ are the $W$-boson and $Z$-boson field strengths, respectively; and $\tilde{Z}_{\mu \nu} = \frac{i}{2} \epsilon_{\mu \nu \alpha \beta} Z^{\alpha \beta}$. The terms proportional to $g Z_{1, 2, 3}$, $\kappa Z$, $\lambda Z$ and $g Z_{4}$ are CP conserving and $\tilde{\kappa} Z$, $\tilde{\lambda}$ and $g Z_{5}$ are CP violating.

To obtain amplitude for the process $K_L \rightarrow \mu^+\mu^-$, we first evaluate the effective coupling for $dsZ$ with the $Z$-boson off-shell. This coupling is induced at the one loop level. The effective Hamiltonian is given by
\[
H_{\text{eff}} = -ig\cos\theta_W \frac{g^2}{2} \epsilon^{\mu\nu} V_{tb} V_{ts}^* \partial_\gamma \epsilon_{\nu} \gamma_\beta \frac{1 - \gamma_5}{2} s \\
\times \int \frac{d^4k}{(2\pi)^4} \frac{k^\nu (g^{\alpha\alpha'} - k^{\alpha k_{\alpha'}} + k^{\beta k_{\beta'}}) (g^{\beta\beta'} - k^{\beta k_{\beta'}})}{(k^2 - m_t^2)(p - k)^2 - M_W^2)((p' - k)^2 - M_W^2) + H.C. .
\]

where \( l \) is summed over \( u, s, \) and \( t, \) and

\[
\Gamma_{\mu\alpha\beta}(q,k^+,k^-) = g_1^Z (g_{\alpha\beta}(k^- - k_\mu^+) + g_{\beta\mu} k^+ - g_{\alpha\mu} k^-) \\
- \kappa^Z (g_{\alpha\mu} q_\beta - g_{\beta\mu} q_\alpha) - \kappa^Z \epsilon_{\mu\alpha\beta\rho} q^\rho \\
+ \frac{\lambda^Z}{M_W^2} (g^\alpha_{\alpha'} k^{\delta} - g^\delta_{\alpha'} k^\alpha)(g_{\beta\delta} k_{\sigma} - g_{\beta\sigma} k\delta)(g_{\mu\rho} q^\sigma - g_{\rho\mu} q^\sigma) \\
+ \frac{\tilde{\lambda}^Z}{M_W^2} (g^\alpha_{\alpha'} k^{\delta} - g^\delta_{\alpha'} k^\alpha)(g_{\beta\delta} k_{\sigma} - g_{\beta\sigma} k\delta) \epsilon_{\rho\sigma\mu\tau} q^\tau \\
+ ig_5^Z (g_{\beta\alpha} q_\lambda + g_{\lambda\alpha} q_\beta) + ig_5^Z \epsilon_{\mu\alpha\beta\sigma}(k^{\sigma} - k^{-\sigma});
\]

where \( k, p, \) and \( p' \) are the internal, s-quark and d-quark momenta respectively, \( q = p' - p, \) \( k^+ = p - k \) and \( k^- = k - p', \) and \( \epsilon^{Z\mu} \) is the Z-boson polarization vector. Performing the standard Feynman parametrization, we have

\[
H_{\text{eff}} = -ig^3\cos\theta_W \epsilon^{\mu\nu} V_{tb} V_{ts}^* \partial_\gamma \gamma_\mu \frac{1 - \gamma_5}{2} s \\
\times \int_0^1 dx \int_0^{1-x} dy \int \frac{d^4k}{(2\pi)^4} \left( k^\nu (g^{\alpha\alpha'} - k^{\alpha k_{\alpha'}}) (g^{\beta\beta'} - k^{\beta k_{\beta'}}) \Gamma_{\mu\alpha\beta}(q,k^+,k^-) \right) \\
+ H.C. .
\]

Due to the anomalous nature of the couplings, the loop integrals are in general cut-off \( \Lambda \) dependent. To calculate such dependence we use dimensional regularization with a (modified) minimal subtraction renormalization scheme following the prescription in Ref. [3]. Substituting \( k' = k - (xp + yp') \) into eq.(3), the terms in odd powers of \( k' \) vanish. We find that among all the even power terms in \( k' \), only terms proportional to \( g_1^Z \) and \( g_5^Z \) will produce terms with no powers in external momenta. All other terms will be at least with two powers in external momenta. Therefore their contributions to \( K_L \rightarrow \mu^+\mu^- \) are suppressed by \( m_d^2/M_W^2, m_s^2/M_W^2 \) or \( m_q^2/M_W^2 \) compared with the contributions from the \( g_1^Z \) and \( g_5^Z \) terms. It is, then, obvious that the process \( K_L \rightarrow \mu^+\mu^- \) can only put useful constraints
on \( g_1^Z \) and \( g_5^Z \) but not the others. The \( g_1^Z \) and \( g_5^Z \) contributions to the effective \( dsZ \) coupling is given by

\[
H_{\text{eff}}(dsZ) = -\frac{1}{32\pi^2} g^3 \cos \theta_W V_{ts} V_{td}^* F_A(x_l) Z^\mu \bar{d} \gamma^\mu \frac{1 - \gamma_5}{2} s + H.C.,
\]  

where \( x_l = m_l^2 / M_W^2 \) and the function \( F_A(x) \) is given by

\[
F_A(x) = -g_1^Z \left( \frac{3x}{2} (x \ln \frac{\Lambda^2}{M_W^2} + x^2 (2 - x) \ln x + \frac{11x - 5x^2}{6(1 - x)} - \frac{x}{6}) \right. \\
+ \left. g_5^Z \left( \frac{3x}{1 - x} + \frac{3x^2 \ln x}{(1 - x)^2} \right) \right).
\]  

The amplitude for \( K_L \to \mu^+ \mu^- \) is obtained by exchanging a virtual \( Z \)-boson between \( ds \) and \( \mu^+ \mu^- \). At the quark level, we obtain

\[
H_{\text{eff}} = \frac{G_F^2 M_W^2}{2\pi^2} \cos^2 \theta_W V_{ts} V_{td}^* F_A(x_l) \bar{d} \gamma^\mu \frac{1 - \gamma_5}{2} s \bar{\mu} \gamma_\mu \left( \frac{1 - \gamma_5}{2} - 2 \sin^2 \theta_W \right) \mu + H.C. .
\]  

From this quark level effective Hamiltonian, we obtain the decay amplitude

\[
M(K_L \to \mu^+ \mu^-) = i \frac{G_F^2 M_W f_K m_\mu}{2\sqrt{2} \pi^2} \text{Re}(V_{ts} V_{td}^* \cos^2 \theta_W F_A(x_l) \bar{\mu} \gamma_\mu) + H.C.
\]  

where \( f_K \) is the decay constant of \( K \) and \( m_\mu \) is the muon mass. The function \( F(x) \) is given by

\[
F(x) = F_S(x) + \cos^2 \theta_W F_A(x).
\]  

Combining the contribution from the MSM, we obtain the total amplitude

\[
M'(K_L \to \mu^+ \mu^-) = i \frac{G_F^2 M_W f_K m_\mu}{2\sqrt{2} \pi^2} \text{Re}(V_{ts} V_{td}^* \eta_l F(x_l) \bar{\mu} \gamma_\mu)
\]  

where \( \eta_l \) are the QCD correction factors which are of order one [4]. The function \( F(x) \) is given by

\[
F(x) = F_S(x) + \cos^2 \theta_W F_A(x).
\]  

with the MSM contribution \( F_S(x) \) given by [5]

\[
F_S(x) = -\frac{2x}{1 - x} + \frac{x^2}{2(1 - x)^2} - \frac{3x^2 \ln x}{2(1 - x)^2}.
\]
We are now ready to use experimental data to put constraint on $g_5^Z$. The total branching ratio $Br^t$ for $K_L \rightarrow \mu^+\mu^-$ is $(7.3 \pm 0.4) \times 10^{-9}$ \([6]\). There are several different contributions to this decay which can be parametrized as $Br^t = R_{2\gamma} + R_{\text{dis}}$. Here $R_{2\gamma}$ is the absorptive contribution due to two real photons in the intermediate state and $R_{\text{dis}}$ is the dispersive contribution which contains the weak contribution $R_W$ from eq.(9) and long distance contribution $R_{\text{LD}}$. The absorptive part of the amplitude coming from real photons in the intermediate state has been unambiguously determined from the measured ratio $Br(K_L \rightarrow \gamma\gamma) = (5.7 \pm 0.27) \times 10^{-4}$ \([3]\). This gives $R_{2\gamma} = (6.83 \pm 0.29) \times 10^{-9}$. The dispersive contribution is then, $R_{\text{dis}} = (0.47 \pm 0.56) \times 10^{-9}$. When extracting the weak contribution from $R_{\text{dis}}$, one faces the problem of subtracting the long distance contribution. It has been argued that this contribution is small compared with the absorptive contribution by using data from $K_L \rightarrow e^+e^-\gamma$ \([7]\). The dispersive contribution may be solely due to weak contribution. At the present the long distance contribution is not well determined \([8]\). In our numerical analysis we will assume that $R_{\text{dis}}$ is saturated by the weak contribution $R_W$.

To minimize uncertainties in $f_K$ we scale the rate $\Gamma(K_L \rightarrow \mu^+\nu_\mu)$ due to the weak contribution by $\Gamma(K^+ \rightarrow \mu^+\nu_\mu)$. We have

$$Br(K_L \rightarrow \mu^+\mu^-) = \frac{\tau^0}{\tau^+} Br(K^+ \rightarrow \mu^+\nu_\mu) \frac{\Gamma(K_L \rightarrow \mu^+\mu^-)_W}{\Gamma(K^+ \rightarrow \mu^+\nu_\mu)} = \frac{\tau^0}{\tau^+} Br(K^+ \rightarrow \mu^+\nu_\mu) \frac{G_F^2 M_W^4}{8\pi^4} \frac{(1 - 4m_\mu^2/m_K^2)^{1/2}}{(1 - m_\mu^2/m_K^2)^2} \frac{|Re(V_{td}V_{td}^* \eta F(x_t))|^2}{|V_{us}|^2}.$$ \(11\)

The branching ratio $Br(K^+ \rightarrow \mu^+\nu_\mu)$ is 63.5%, and the lifetimes $\tau^0$ of $K_L$ and $\tau^+$ of $K^+$ are $5.17 \times 10^{-8}$ and $1.237 \times 10^{-8}$ s, respectively \([3]\). We will use $|V_{us}| = 0.22$, and $\eta = 0.9$. The dominant contribution is from the top quark in the loop. We must know the value for $Re(V_{ts}V_{td}^*)$. Unfortunately this quantity is not well determined at present. We will use the most recent estimate for $|V_{td}|$ in Ref. \([4]\) and take $Re(V_{ts}V_{td}^*)$ to be in the range $3.2 \times 10^{-4}$ to $6.7 \times 10^{-4}$. In our analysis we will let the top quark mass and the anomalous couplings $g_1^Z$ and $g_5^Z$ vary.

If $g_1^Z$ and $g_5^Z$ is set to zero, we obtain the MSM result. Using the experimental data and allowing the relevant KM matrix to span the allowed region, we find that the top quark
mass must be less than 240 GeV. This bound is weaker than the bound from LEP data [10]. In the following analysis, we consider the cases where one of $g_1^Z$ and $g_5^Z$ is not zero. In Tables 1, 2 and 3, we show the effects of non-zero $g_1^Z$. Table 1 shows how $R_{W}/R_{2\gamma}$ varies with $g_1^Z$ for different cutoffs $\Lambda$. We see that depending on the sign of $g_1^Z$, the anomalous coupling $g_1^Z$ can either increase or decrease $R_W$. Our results for the constraints on $g_1^Z$ at 2$\sigma$ level for two different cutoffs, $\Lambda = 1$ TeV and $\Lambda = 10$ TeV are shown in Table 2 and 3. The constraints on $g_1^Z$ in Table 2 and 3 are for $Re(V_{ts}V_{td}^{*})$ equal to $3.2 \times 10^{-4}$ and $6.7 \times 10^{-4}$, respectively. If $g_1^Z$ is positive the contribution from the anomalous interaction has the same sign as the MSM contribution. $g_1^Z$ is constrained to be in the range $-0.96$ to $0.57$ for $\Lambda = 1$ TeV. The constraints on $g_1^Z$ become tighter when the top quark mass is increased. In Tables 4, 5, and 6, we show the effects of non-zero $g_5^Z$. This contribution is cutoff independent. If $g_5^Z$ is positive, the contribution has the opposite sign as that of the MSM. $g_5^Z$ is constrained to be between $-3.36$ to $5.67$. Analysis with both $g_1^Z$ and $g_5^Z$ being non-zero can also be carried out. In this case cancellations between the anomalous contributions may happen. No significant additional constraints on $g_1^Z$ and $g_5^Z$ can be obtained using data only from $K_L \to \mu^+\mu^-$. The same analysis can be carried out for $B \to \mu^+\mu^-$. In this case the long distance contribution is expected to be small. When experimental data for this decay will become available, one may obtain better constraints on $g_1^Z$ and $g_5^Z$.

ACKNOWLEDGMENTS

I would like to thank G. Valencia for useful discussions. This work was supported in part by the Australian Research Council.
REFERENCES

[1] J.F. Gaemers and G. Gounaris, Z. Phys. C1 259(1979); K. Hagiwara, R. D. Peccei, D. Zeppenfeld and K. Hikasa, Nucl. Phys. B282, 253(1987).

[2] M.K. Gaillard and B.W. Lee, Phys. Rev. D9, 897(1974); V.A. Novikov, M.A. Shifman, A. I. Vainshtein and V.Z. Zakharov, Phy. Rev. D16, 223(1977); R.E. Schrock and M.B. Voloshin, Phys. Lett. B87, 375(1979); A.J. Buras, Phys. Rev. Lett. 46, 1354(1981); V. Barger, W.F. Long, E. Ma and A. Pramudita, Phys. Rev. D25, 1860 (1982); A. Paschos, B. Stech and U. Turke, Phys. Lett. B128, 240(1983); C.G. Geng and J.N. Ng, Phys. Rev. D41, 2351(1990).

[3] C. Burgess and D. London, Preprint, McGill-92/05, UdeM-LPN-TH-84.

[4] V.A. Novikov, M.A. Shifman, A. I. Vainshtein and V.Z. Zakharov, Phy. Rev. D16, 223(1977); A.J. Buras, Phys. Rev. Lett. 46, 1354(1981); C.O. Dib, I. Dunietz and F.J. Gilman, Phys. Rev. D39, 2639(1989).

[5] T. Inami and C.S. Lim, Prog. Theor. Phys. 65, 297(1981).

[6] Review of Particle Properties, Phys. Rev. D45, 1992.

[7] L. Bergstrom, E. Masso and P. Singer, Phys. Lett. B249, 141(1990).

[8] L. Littenberg and G Valencia, Preprint, FERMILAB-PUB-93/004-T.

[9] A. J. Buras, M. James and P.H. Weisz, Nucl. Phys. B347, 491(1990).

[10] S.C.C. Ting, Plenary talk given at the Annual Meeting of the Division of Particles and Fileds of the APS. Fermilab, 1992.
TABLE I. $R_W/R_{2\gamma}$ vs. $g_1^Z$ for $m_t = 150$GeV and $|Re(V_{ts}V_{td}^*)| = 5 \times 10^{-4}$.

| $g_1^Z$ | -1.0 | -0.8 | -0.6 | -0.4 | -0.2 | -0.1 | 0.0 | 0.2 | 0.4 | 0.6 |
|---------|------|------|------|------|------|------|-----|-----|-----|-----|
| $\Lambda = 1$ TeV | | | | | | | | | | |
| $R_W/R_{2\gamma}$ | 2.25 | 1.27 | 0.57 | 0.15 | $1.45 \times 10^{-4}$ | 0.03 | 0.13 | 0.54 | 1.22 | 2.18 |
| $\Lambda = 10$ TeV | | | | | | | | | | |
| $R_W/R_{2\gamma}$ | 12.35 | 7.51 | 3.86 | 1.41 | 0.17 | $7.3 \times 10^{-4}$ | 0.13 | 1.28 | 3.65 | 7.21 |

TABLE II. The constraints for $g_1^Z$ with $|Re(V_{ts}V_{td}^*)| = 3.2 \times 10^{-4}$.

| $m_t$(GeV) | 100 | 125 | 150 | 175 | 200 |
|------------|-----|-----|-----|-----|-----|
| $\Lambda = 1$ TeV | | | | | |
| $g_1^Z$ | -0.99~0.59 | -0.74~0.35 | -0.59~0.21 | -0.51~0.12 | -0.45~0.06 |
| $\Lambda = 10$ TeV | | | | | |
| $g_1^Z$ | -0.51~0.30 | -0.37~0.17 | -0.29~0.10 | -0.24~0.06 | -0.20~0.03 |

TABLE III. The constraints for $g_1^Z$ with $|Re(V_{ts}V_{td}^*)| = 6.7 \times 10^{-4}$.

| $m_t$(GeV) | 100 | 125 | 150 | 175 | 200 |
|------------|-----|-----|-----|-----|-----|
| $\Lambda = 1$ TeV | | | | | |
| $g_1^Z$ | -0.58~0.18 | -0.45~0.06 | -0.39~$-9.6 \times 10^{-4}$ | -0.34~0.04 | -0.31~0.07 |
| $\Lambda = 10$ TeV | | | | | |
| $g_1^Z$ | -0.30~0.09 | -0.23~0.03 | -0.19~$-5.2 \times 10^{-4}$ | -0.16~0.02 | -0.14~0.03 |
TABLE IV. $R_W/R_{2\gamma}$ vs. $g_5^Z$ for $m_t = 150\text{GeV}$ and $|Re(V_{ts}V_{td}^*)| = 5 \times 10^{-4}$.

| $g_5^Z$ | 6.0 | 5.0 | 4.0 | 3.0 | 2.0 | 1.37 | 1.0 | 0.0 | -1.0 | -2.0 | -3.0 |
|---------|-----|-----|-----|-----|-----|------|-----|-----|------|------|------|
| $R_W/R_{2\gamma}$ | 1.49 | 0.92 | 0.48 | 0.19 | $2.8 \times 10^{-2}$ | 0.0 | $9.4^{-3}$ | 0.13 | 0.39 | 0.79 | 1.33 |

TABLE V. The constraints for $g_5^Z$ with $|Re(V_{ts}V_{td}^*)| = 3.2 \times 10^{-4}$.

| $m_t$(GeV) | 100 | 125 | 150 | 175 | 200 |
|------------|-----|-----|-----|-----|-----|
| $g_5^Z$   | 5.67 ~ -3.36 | 4.73 ~ -2.21 | 4.2 ~ -1.47 | 3.91 ~ -0.94 | 3.74 ~ -0.53 |

TABLE VI. The constraints for $g_5^Z$ with $|Re(V_{ts}V_{td}^*)| = 6.7 \times 10^{-4}$.

| $m_t$(GeV) | 100 | 125 | 150 | 175 | 200 |
|------------|-----|-----|-----|-----|-----|
| $g_5^Z$   | 3.32 ~ -1.00 | 2.92 ~ -0.40 | 2.73 ~ 0.008 | 2.64 ~ 0.32 | 2.63 ~ 0.59 |