Optimizing orbits for (e)LISA

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Abstract. Earth and other planets gravitational fields induce perturbations of the (e)LISA constellation geometry (mostly flexing and Doppler frequency shifts). We present here a numerical optimization method minimizing these disturbances, subject to constraints (maximum distance to Earth and launch mass).

1. Introduction
The three, free-falling, satellites of (e)LISA form an (quasi) equilateral triangle trailing the Earth by about 20°. While orbiting around the Sun, the distances between the S/C evolve, leading to breathing and flexing effects, which should be accommodated by the satellite design, e.g. telescope steering (or laser in-field pointing capability) and wide photodetector bandwidth (due to the Doppler frequency shifts). These fluctuations can be efficiently minimized analytically for pure keplerian orbits [1]. However, the constellation is also perturbed by the gravitational influence of the Earth and, to a lesser extent, Jupiter and Mars. Numerical optimization is required to find the optimal initial orbital parameters and, possibly, velocity increments for orbit control during the mission lifetime.

The optimization process is also subject to two other constraints. The first constraint is due to the data link budget which requires a maximum distance to Earth of ≈ 70 × 10^6 km[2]. The capability of the launcher sets a second constraint on the total wet mass (i.e. including propellant for orbital maneuvers) of the constellation.

2. Optimization Method
In the present work, the numerical optimization process combines an orbit propagator and the minimization of a cost function. The orbit propagator is based on a VEFRL (Velocity Extended Forest-Ruth Like) integrator [3], which is a constant step size, quasi 6th order symplectic integrator and optimized for a weak perturbation of a central force potential. The chosen minimization method is an Active Covariance Matrix Adaptation Evolution Strategy (aCMAES) [4] optimizer, developed for non-linear, non-convex, black-box optimization and especially used for continuous cost functions with sharp edges, local minima, outliers, etc.
The cost function is expressed as a linear combination of various penalties:

\[ C(\mathcal{P}) = \alpha_A \cdot C_A(\mathcal{P}) + \alpha_X \cdot C_X(\mathcal{P}) + \alpha_F \cdot C_F(\mathcal{P}) + \alpha_M \cdot C_M(\mathcal{P}) + \alpha_D \cdot C_D(\mathcal{P}) \]

\[ C_A(\mathcal{P}) = \frac{1}{3} \sum_{i=1}^{3} \left( \frac{L_i(t) - L_{\text{ref}}}{L_{\text{ref}}} \right)^2 \]

\[ C_X(\mathcal{P}) = \max_{i \in [1:3]} \frac{\max_x \theta_i(t) - \min_x \theta_i(t)}{60^\circ} \]

\[ C_M(\mathcal{P}) = P_{Th} \left( \sum_{i=1}^{3} \frac{M_i - M_{\text{max}}}{M_{\text{spread}}} \right) \]

\[ C_D(\mathcal{P}) = P_{Th} \left( \frac{\max_i D_i(t) - D_{\text{max}}}{D_{\text{spread}}} \right) \]

\[ P_{Th} = \frac{e^{4x}}{1 + e^{4x}} + 0.025 \log \left( 1 + e^{4x} \right) \]

Where:

- \( \mathcal{P} \) is the set of initial orbital parameters and maneuver velocity increments to be optimized
- \( C_A(\mathcal{P}) \) is the armlength penalization, minimizing the mean quadratic difference between the length of arm \( i \) at time \( t \) \( (L_i(t)) \) and the desired arm length \( L_{\text{ref}} \)
- \( C_X(\mathcal{P}) \) is the flexing penalization, minimizing the amplitude of the fluctuations of the inner angle at vertex \( i \) \( (\theta_i(t)) \)
- \( C_F(\mathcal{P}) \) is the Doppler shift penalization, minimizing the amplitude of frequency fluctuations on arm \( i \) \( (\Delta F_i(t)) \)
- \( C_M(\mathcal{P}) \) is the mass penalization, with \( M_{\text{max}} \) being the maximum possible launched mass, \( M_{\text{spread}} \) the allowed mass dispersion and \( M_i \) the computed wet mass of satellite \( i \). \( P_{Th}(x) \) is a non-linear (threshold) penalization function.
- \( C_D(\mathcal{P}) \) is the distance to Earth penalization, with \( D_{\text{max}} \) being the maximum possible distance to Earth, \( D_{\text{spread}} \) the allowed distance dispersion and \( D_i(t) \) the distance to Earth of satellite \( i \) at time \( t \)

\( P_{Th}(x) \) is used to define a threshold function: \( P_{Th}(x) \) exhibits a smooth transition from 0 to 1 with a slope of 1 at \( x = 0 \) and an additional small linear increase (slope of 0.1) for \( x > 0 \). The hyper-parameters \( \alpha_u \) are used to tune the relative importance of the different penalizations factors.

In the present work (based on the former NGO mission profile [2]), the estimation of the propellant mass is done using the following scenario:

- Launch on a Geostationary Transfer Orbit \((200 \text{ km} \times 36 \text{ 000 km})\)
- Escape manoeuvre on the ecliptic plane computed for a cruise time of 15 months
- 6 to 12 months Lambert transfer maneuver at the longitude of the ascending or descending node of the targeted orbital plane towards the initial position (and time) of the science mission.
- ‘Stopping’ maneuver to reach the desired initial orbital parameters at the beginning of the science mission.

Additional periodic velocity increments can also be included in the optimization process.

### 3. Optimization results

As an example, the above optimization scheme has been applied with the following constellation configuration: \( 3 \times 10^6 \text{ km armlength (L_{ref}) ; 10 years mission lifetime (2030 - 2040) ; 8 t maximum launch mass (compatible with Ariane 64 capability to geostationary transfer orbit) ; 70 \times 10^6 \text{ km maximum distance to Earth.} \) The combined gravitational perturbing effects of the Earth, Moon, Mars and Jupiter have been considered.
3.1. Optimization with no additional orbital maneuvers

In a first optimization run, maneuvers are only used to position the S/C on their initial orbits, with no further orbit control maneuver during the duration of the mission.

Figure 1 represents the resulting time evolution of flexing angle, arm length, frequency shift and distance to Earth. In this case, the optimized initial orbital parameters lead to a maximum vertex angle fluctuations amplitude of \( \approx 1.5 \) deg peak-to-peak (for each vertex), a maximum Doppler frequency shift of \( \approx 14 \) MHz peak-to-peak and a total wet mass of the constellation of about 7835 kg.

Noticeably, due to the relatively large wet mass allowed by an Arinae 6.4 launch, a full stopping maneuver can be performed on the 3 spacecraft, so that they are subsequently following a ballistic trajectory towards the Earth. Actually, the S/C are slightly faster than the Earth. Its pulling effect increases the semi-major axis of each S/C orbit, hence slowly decreasing the S/C orbital velocity, and finally inverting the relative movement between the Earth and the satellites (see figure 1d). This numerical solution is similar to the semi-analytic ‘projectile’ solution from [5].

Moreover, the geometrical center of the constellation is oscillating by \( \pm 8 \times 10^6 \) km above and below the ecliptic plane. This movement mitigates the pulling effect of the Earth on the S/C (and therefore reduces its perturbing influence).

3.2. Optimization with periodic orbital maneuvers

A second optimization run was performed including orbital maneuvers (i.e. velocity increments) every 18 months (i.e. 6 maneuvers for each S/C in the mission lifetime), see fig. 2.

Thanks to these additional maneuvers, the amplitude of the vertex angle fluctuations is reduced to \( \approx 0.6 \) deg peak-to-peak (for each vertex) and the maximum Doppler frequency shift is of \( \approx 4 \) MHz peak-to-peak. The total wet mass of the constellation is slightly reduced to about 7431 kg, taking into account the additional propellant required for the orbit control maneuvers.
Figure 2: Evolution of constellation geometry for optimized initial orbital parameters, with additional periodic orbit control maneuvers (every 18 months)

(using $\Delta V$ impulses ranging from 70 to 140 m/s). Moreover, these maneuvers result in a globally constant distance to Earth (see figure 2d), hence allowing a longer mission lifetime (regardless of propellant consumption).

4. Conclusion
The optimization method described above allows to find the orbital parameters minimizing the disturbances due to the gravitational field of the Earth and other planets. This method can also be used to optimize and compute the impact of periodic additional orbital maneuvers during the mission lifetime. Clearly, a few control maneuvers can significantly reduce the beam pointing constraint and Doppler effect, with no mass penalty. These maneuvers could also be useful to compensate for previous maneuvers uncertainties, unmodeled perturbations and velocity dispersion from the initial positioning maneuvers.

As drawbacks, orbit control maneuvers will require to suspend the science acquisition, grab the test masses and, probably, re-acquire the laser links after the maneuvers. They might also induce significant changes in the mass distribution (hence the local gravitational field) of the S/C.

References
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