Observing actions in global games

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Abstract
We study Bayesian coordination games where agents receive noisy private information over the game’s payoffs, and over each others’ actions. If private information over actions is of low quality, equilibrium uniqueness obtains in a manner similar to a global games setting. On the contrary, if private information over actions (and thus over the game’s payoff coefficient) is precise, agents can coordinate on multiple equilibria. We argue that our results apply to phenomena such as bank-runs, currency crises, recessions, or riots and revolutions, where agents monitor each other closely.

Keywords Coordination games · Global games · Conjectural equilibrium

JEL Classification D82 · D83

ERNIE: There is something funny going on over there at the bank, George, I’ve never really seen one, but that’s got all the earmarks of a run.

PASSEBY: Hey, Ernie, if you have any money in the bank, you better hurry.1

1 From the movie script “It’s a wonderful life.”

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Introduction

The global games approach assumes that players face asymmetric private information over the game’s payoff structure. In turn, each agent has to use his own signal over the game’s payoffs to infer the signals, and thus the actions, of the other agents. Such inference makes it difficult to coordinate on multiple equilibria, and whenever private information over the game’s payoffs is very precise, but not perfect, the global games structure selects unique equilibria.

One implicit assumption in the global games approach of Rubinstein (1989); Carlsson and van Damme (1993) and Morris and Shin (1998) is that agents cannot observe each others’ actions directly. At the same time, global games are used extensively to model phenomena such as bank-runs, riots and revolutions, or joint investment projects, where agents study each others’ actions. That is, in the context of a bank-run, depositors can observe the length of a queue, respectively the lack thereof, in front of their local bank branch. To understand the equilibria that agents play, we therefore argue that agents’ information over the game’s payoff coefficients and information over each others’ actions should be taken into account.

Information over actions plays a dual role in the current model. First, actions depend on the game’s payoff coefficient, and signals over actions therefore carry information over the game’s payoff coefficients. In this interpretation, the signal over actions is just another signal over the game’s fundamental. Taking this view, private information over actions should reinforce the global games mechanism, where private information over fundamentals selects unique equilibria. The second function of signals over actions is that they inform players of each others’ actions, which helps coordination. In equilibrium, we find that this second effect dominates, and multiple equilibria are ensured whenever private information over actions is sufficiently precise.

Multiple equilibria obtain in a manner similar to an epidemic. That is, in the context of a bank-run, increases in the mass of agents who line up to withdraw their deposits are observed, and induce additional agents to join the queue. This infectious process allows agents to coordinate on multiple equilibria whenever the precision, with which they observe each other, is sufficiently high. The equilibria of the game are then situations where no agent, given his information over the queue and the game’s fundamental, would like to join/leave the queue of agents, who are waiting for the bank to open. In turn, once the bank opens, the agents in the queue withdraw their money.

Technically, equilibria are steady states where the period \( n - 1 \) mass of waiting agents \( A^{n-1} \), which is partially revealed through agents’ private information, coincides with the mass of waiting agents \( A^n \), such that \( A^n = A^{n-1} = A \). In the main text we will suppress index \( n \), and solve for steady states only. We do use the time index in appendices B.2 and D (see Supplementary Information) to obtain these steady states via a sequential model, where agents join and leave the crowd of agents, who are waiting to withdraw their money, until a steady state is reached.
Alternative interpretations: Riots and revolutions, joint investment projects, or coordinated attacks are problems where actions are strategic complements. Agents will therefore monitor each other closely.

During riots and revolutions, people see whether the number of protesters in the street is large or small. Bystanders, who observe that turn-out is large, may decide to join the protest. Protesters, who see that turn-out is small, may withdraw. The present model studies the steady state crowd, which gathers before the protesters clash with the riot police.2

Political parties and parliaments often perform nonbinding test-votes.3 These test votes can be interpreted as coordination devices, which allow politicians to observe the "queue" of politicians who support a particular motion. After these test-votes, they proceed to the final, binding, vote.

In the context of large investment projects, investors sign nonbinding letters of intent. These letters serve as signals regarding the future action, i.e., the expected number of investors who will actually participate once the binding investment contract is signed.

Finally, we may refer to an example from nature: sardines rely on a very fine pressure sense in their bodies, which helps them to detect the movements/actions of their nearest neighbors. This fine sense allows sardines to match each others’ actions, respectively, to from swarms that protect them from large, fast moving, predators.

Related literature: Agents who observe each others’ actions learn about the game’s fundamental. This learning channel has been studied by Chamley (1999); Angeletos et al. (2007, 2007b), and Frankel (2012). These models assume that agents observe (i) the actions that they took when they played the game in the past, or (ii) the irreversible45 actions of a group of early movers. This type of information over actions is equivalent to information over the game’s fundamental. The global games mechanism thus applies in the models of Chamley (1999); Angeletos et al. (2007, 2007b); Frankel (2012), and ensures unique equilibria whenever private information over actions/fundamentals is sufficiently precise.

The current paper, on the contrary, studies steady states of crowds that form before the final, binding, action is taken. In the context of a bank-run, an equilibrium is a steady state queue of agents that forms before the bank opens. That is, an equilibrium is a situation where none of the agents in the queue, given their information over the queue’s length and their information over the bank’s reserves, would like to withdraw from the queue. At the same time, none of the agents outside of the queue,

2 The January 2021 storming of the US Capitol may serve as an example for this: protesters were gathering at various rallies throughout Washington before they moved to overwhelm the police.
3 One may view such votes as public signals. In an alternative interpretation, one may consider the relevant information as being private in the sense that the voting population of politicians discusses the composition of the aggregate ‘yes’ and ‘no’ votes in small circles. That is, politicians discuss in private who voted ‘yes’ and ‘no’, to understand the outcome of the final, binding, vote.
4 That is, agents are partitioned into early and late movers; late movers observing the irreversible actions taken by those who moved early.
5 See (Kovac and Steiner 2013) for a two period global games model with reversible actions. (Kovac and Steiner 2013) focus on environments where actions are unobservable, and thus equilibria are unique.
given their information over the queue’s length and their information over the bank’s reserves, would like to join the queue.⁶

Angeletos and Werning (2006) find that public signals over players’ steady state actions, just like public signals over fundamentals, help agents to coordinate on multiple equilibria. The present model shows that private information over actions, unlike private information over fundamentals, induce multiple equilibria.

Modern technology increasingly allows agents to draw on diverse sources of information. Taking this perspective, we contribute to the effort aimed at enriching the global games information structure. (Izmalkov and Yildiz 2010; Steiner and Steward 2011; Kuhle 2016; Grafenhofer and Kuhle 2016; Bergemann and Morris 1909; Binmore and Samuelson 2001) have recently emphasized the role of heterogeneous priors, strategic information revelation, learning, as well as environments where agents receive signals over each others’ information.

Finally, the current model connects the literature on conjectural equilibrium, (Battigalli and Guaitoli 1997; Minelli and Polemarchakis 2003; Rubinstein and Wolinsky 1994; Esponda 2013),⁷ emphasizing noisy information over steady state actions, with arguments from the literature on global games, where uncertainty over actions originates from parameter uncertainty.

Organization: Section 2 outlines the model. Section 2.1 recalls the Morris and Shin (2004) model. Section 2.2 provides a tractable example of global games featuring private signals over actions. In Section 3 we prove our results for more general signal structures. Section 4 concludes.

Model

There is a status quo and a unit measure of agents indexed by \( i \in [0, 1] \). Each of these agents \( i \) can choose between two actions \( a_i \in \{0, 1\} \). Choosing \( a_i = 1 \) means to attack the status quo. Choosing \( a_i = 0 \) means that the agent abstains from attacking the status quo. An attack on the status quo is associated with a cost \( c \in (0, 1) \). If the attack is successful, the status quo is abandoned, and attacking agents receive a net payoff \( 1 - c > 0 \). If the attack is not successful, an attacking agent’s net payoff is \( -c \). The payoff for an agent who does not attack is normalized to zero. The status quo is abandoned if the aggregate size of the attack \( A := \int_0^1 a_i di \) exceeds the strength of the status quo \( \theta \), i.e., if \( A > \theta \). Otherwise, if \( A < \theta \), the status quo is maintained, and the attack fails.

The timing of the game may be thought of as follows:

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⁶ In the alternative context of a riot, we study the crowd that gathers before the actual attack takes place. On the contrary, (Chamley 1999; Angeletos et al. 2007, 2007b; Frankel 2012), study agents who learn from the outcomes of past attempts to, e.g. storm the US capitol, or from actions of early movers who have already committed to the attack.

⁷ See also (Hahn 1977) for Walrasian economies, where agents hold conjectures over each others’ supply and demand functions, which need not be true.
• A group of agents, who observe each other, gathers in front of the bank. An equilibrium is a situation where no agent, given his information over the length of the queue of waiting agents and the game’s fundamental, would like to join/leave the queue.
• The bank opens, and the agents, who are waiting in the queue, withdraw their money.

In what follows, we study steady state equilibria, where the past queue of waiting agents $A^{n-1}$, which agents observe, coincides with the contemporaneous queue of waiting agents $A^n$ such that $A^n = A^{n-1} = A$. That is, in the main text we suppress the time index and solve for steady states only. We do use the time index $n$ in appendices B.2 and D (see Supplementary Information) to obtain these steady states via an iteration over the crowd of attacking agents. Finally, in order to focus on the role of private information, we assume throughout the paper that players hold a uniform uninformative prior over $\theta$. The game, with the exception of the exogenous fundamental $\theta$ and the endogenous size of the attack $A(\theta)$, is common knowledge.

The Morris and Shin (2004) benchmark: observing actions indirectly

Let us briefly recall the Morris and Shin (2004) benchmark model, where agents receive private information over the game’s fundamental only. That is, agents receive signals

$$x_i = \theta + \sigma_i\varepsilon_i, \quad \varepsilon_i \sim \mathcal{N}(0, 1),$$

which inform them of the game’s fundamental $\theta$ with precision $\alpha_x := \frac{1}{\sigma_i^2}$. Expected utility is thus:

$$E[U(a_i|x_i)] = a_i(P(\theta < A|x_i) - c).$$

We denote by $x^*$ the signal threshold that separates signal values $x_i < x^*$, for which agents attack $a_i = 1$, from signal values $x_i > x^*$, for which agents do not attack $a_i = 0$. Using (2), we have an indifference condition

$$P(\theta < A|x^*) = c,$$

which characterizes $x^*$. Given threshold $x^*$, and given a fundamental $\theta$, the mass of attacking agents is

$$A = P(x < x^*|\theta) = \Phi(\sqrt{\alpha_x(x^* - \theta)}),$$

where $\Phi()$ denotes the cumulative density function of the standard normal distribution. Equation (4), helps to compute threshold values $\theta^*$, which separate fundamental values $\theta < \theta^*$, for which the attack succeeds from fundamental values $\theta > \theta^*$, for which the attack fails:

$$A(\theta^*) = \theta^* \iff \Phi(\sqrt{\alpha_x(x^* - \theta^*)}) = \theta^*.$$
Combining (3) and (5) we rewrite the payoff indifference condition:

\[ P(\theta < \theta^* | x^*) = \Phi(\sqrt{\alpha_x}(\theta^* - x^*)) = c. \]  

(6)

Together (5) and (6) yield:

**Proposition 1** There exists a unique equilibrium \( x^*, \theta^* \).

**Proof** We solve equation (6) for \( x^* = -\Phi^{-1}(c) \frac{1}{\sqrt{\alpha_x}} + \theta^* \). Substituting into (5) yields \( \theta^* = \Phi(-\Phi^{-1}(c)) \). In turn, \( x^* = -\Phi^{-1}(c) \frac{1}{\sqrt{\alpha_x}} + \Phi(-\Phi^{-1}(c)) \).

\[
\Box
\]

That is, according to Proposition 1, equilibria are unique regardless of the precision with which agents observe the game’s fundamental.

How well do agents observe each others’ actions? Agents receiving signals \( x_i \) over the game’s fundamental \( \alpha \) know that the other agents’ equilibrium action is an invertible function of the fundamental \( A = \psi(\theta) \) such that \( \theta = \psi^{-1}(A) \). Hence, in the model of Morris and Shin (2004), the signal over the game’s fundamental informs agents of each others’ contemporaneous actions. Rewriting (4), we have \( \theta = -\frac{1}{\sqrt{\alpha_i}} \Phi^{-1}(A) + x^* \). Signals \( x_i = \theta + \sigma_x \epsilon_i \) thus carry information over actions \( x_i = -\frac{1}{\sqrt{\alpha_i}} \Phi^{-1}(A) + x^* + \sigma_x \epsilon_i \), and we have \( (x^* - x_i) \sqrt{\alpha_x} = \Phi^{-1}(A) - \epsilon_i \). That is, information is specified such that the precision with which agents observe actions is independent of the precision \( \alpha_x \) with which they observe the game’s fundamental.

**Observing actions directly**

We begin with a simple one-dimensional example, in which agents receive only one private signal over the attack’s net size \( A - \theta \). In Section 3, we extend the current result to a model where agents receive two separate signals over \( \theta \) and \( A \). Moreover, appendices B.2 and C (see Supplementary Information) derive the (steady state) equilibrium function \( A(\theta) \) via an iteration argument in which agents observe past actions.

The signal

\[ z_i = A - \theta + \sigma_z \epsilon_i, \quad \epsilon_i \sim \mathcal{N}(0,1) \]  

(7)

informs players \( i \) with precision \( \alpha_z := \frac{1}{\sigma_z} \) of the attack’s net size \( A - \theta \), and we have:

**Proposition 2** If private information is precise, \( \alpha_z > 2\pi \), agents can coordinate on multiple equilibria. If information is imprecise, \( \alpha_z < 2\pi \), there exists a unique equilibrium.

**Proof** We proceed in three steps. First, we compute the threshold signal \( z^* \), for which agents are indifferent between attacking and not attacking. Given this threshold,
we compute the mass of attacking agents. Third, we show that there exist multiple equilibria.

1) Payoff indifference condition: Given a signal \( z_i \), agents \( i \) choose an action \( a_i \in \{0, 1\} \), to maximize expected utility:

\[
E[U_i|z_i] = a_i(P(A - \theta > 0|z_i) - c).
\]

Agent \( i \) is therefore just indifferent between attacking, \( a_i = 1 \), and not attacking, \( a_i = 0 \), when he receives a signal \( z_i = z^* \) such that:

\[
P(A - \theta > 0|z^*) = c.
\]

It follows from (9) that agents attack if \( z > z^* \), and they do not attack whenever \( z \leq z^* \).

2) Given the critical signal \( z^* \), we can compute the mass of attacking agents:

\[
A = P(z > z^*|A, \theta)
\]

For normally distributed signal errors, (10) can be rewritten as:

\[
A = 1 - \Phi(\sqrt{\sigma^2_A(z^* - A + \theta)}),
\]

where \( \Phi() \) is the cumulative normal distribution. From (11), we have

**Lemma 1** If \( \alpha > 2\pi \) then, for every level \( z^* \), there exists an interval \([\hat{\theta}(z^*), \hat{\theta}(z^*)]\) such that (11) has three solutions \( A_j(\theta, z^*), j = 1, 2, 3 \) whenever \( \theta \in [\hat{\theta}(z^*), \hat{\theta}(z^*)] \).

To construct equilibrium functions \( A(\theta, z^*) \), we use the solutions \( A_j(\theta, z^*) \) from Lemma 1. More specifically, we focus on the (stable) solutions \( j = 1, 3 \). In turn, we obtain, indexed by \( t \), different functions \( A_t(\theta, z^*) \), which are downward sloping such that \( \frac{\partial A_t}{\partial \theta} < 0 \). Given these functions, it remains to show that there exist values \( z^* \) that satisfy the payoff indifference condition

\[
P(A_t(\theta, z^*) - \theta > 0|z^*) = c.
\]

In Appendix A (see Supplementary Information) we show that there exist small values \( \hat{z} \) such that \( P(A_t(\theta, \hat{z}) - \theta > 0|\hat{z}) < c \) and large values \( \tilde{z} \) such that \( P(A_t(\theta, \tilde{z}) - \theta > 0|\tilde{z}) > c \).

Equilibrium values \( z^* \) now either obtain as solutions to (12), or the critical \( z^* \) values are those values where the function \( P(A_t(\theta, z^*) - \theta > 0|z^*) \) is discontinuous in \( z^* \). In that case we have a \( z^* \), such that for small \( \delta > 0 \), \( P(A_t(\theta, z^* + \delta) - \theta > 0|z^* + \delta) > c \) and \( P(A_t(\theta, z^* - \delta) - \theta > 0|z^* - \delta) < c \). That is, the expected value of attacking/not attacking changes discontinuously at the agents’ equilibrium cutoff value \( z^* \).

Intuitively, multiple equilibria obtain since increases in the mass of attacking agents are observed. This, in turn, increases the number of attacking agents, which is again visible and induces even more agents to attack... . Hence, if the private signal’s precision is sufficiently high, runs feed on themselves, and agents can coordinate on multiple equilibria.
Diagrammatic discussion

Figure 1 plots Eq. (11) for the case where the precision of the private signal is high, such that there exist multiple equilibria. The red, blue, and green lines graph the attack for different (decreasing) levels of the fundamental $\theta$. Figure 1 in particular shows that there exists a range of $\theta$ values for which there are up to three intersections of the function $A(\theta, A)$ with the 45° line. Each such intersection represents a single equilibrium value $A(\theta)$. In turn, we can vary $\theta$, to plot one equilibrium curve $A_t(\theta)$ in Fig. 2. Such a curve $A_t(\theta)$, represents one solution of the game. If private information is of high quality, there exists a continuum of such curves.

Figures 3 and 4 illustrate the alternative case, where private information is of low quality, such that the game’s equilibrium is unique.

Separate signals over actions and fundamentals

The previous section’s signal structure, where agents receive private information over $A - \theta$, was tractable, and thus easy to interpret. In what follows, we prove our results for a more realistic, but much less tractable, signal structure. In particular, we assume

![Diagram of separate signals over actions and fundamentals](image-url)
that each player $i$ receives two pieces of private information: a noisy signal $x_i$ over the strength of the status quo $\theta$, and another signal $y_i$ over the other players’ (steady state) actions $A$:

- Signal over the fundamental: $x_i = \theta + \epsilon_i^x$
- Signal over the aggregate attack: $y_i = A + \epsilon_i^y$  (13)

Given signals (13), agents choose actions $a_i$ to maximize:

$$E[U(a_i)|x_i, y_i] = a_i(P(A > \theta|x_i, y_i) - c).$$  (14)

Action $a_i = 1$ is thus optimal whenever $P(A > \theta|x_i, y_i) \geq c$. Agents are just indifferent between attacking and not attacking when signals $x_i$ and $y_i$ are such that $P^* := c$. We denote the error terms’ joint density function by $f(\epsilon^x, \epsilon^y)$. Given agents’ information, and the critical probability $P^*$, we define:
Definition 1 (Equilibrium) An aggregate attack function $A(\theta)$ is an equilibrium of the game, if for all $\theta \in \mathbb{R}$ the following holds:\footnote{$\chi$ denotes the indicator function.}

$$A(\theta) = \int_{\mathbb{R}^2} \chi\{ (e^x, e^y) : p[A(\theta) \geq \theta | x_i = \theta + e^x, y_i = A(\theta) + e^y, A(c)] \geq p^* \} f(e^x, e^y) de^x de^y. \quad (15)$$

To characterize equilibria, we treat cases where error distributions have compact, respectively unbounded support, separately. We begin with the case where error terms have compact support:

Proposition 3 Suppose that $\epsilon^y_i \in [-\sigma, \sigma]$, i.e. $f(e^x, e^y) = 0$ for all $|e^y| > \sigma$ and all $e^x \in \mathbb{R}$. Further assume that the precision of the signal about the aggregate attack $y_i$ is precise enough, i.e. $0 < \sigma < \frac{1}{2}$. Then, there exists a continuum of equilibria.

Proof Pick an arbitrary $t \in [0, 1]$ and define

$$A_t(\theta) := \begin{cases} 1 & \theta < t \\ 0 & \text{otherwise.} \end{cases} \quad (16)$$

To establish that the $A_t$ functions in (16) are equilibria, we have to show that (15) holds. First, we determine the distribution of signal realizations $y_i$ for a given $\theta$. There are two cases:

Case $\theta < t$ : \quad $y_i = 1 + \epsilon^y_i > 1 - \sigma \geq \frac{1}{2}$

Case $\theta \geq t$ : \quad $y_i = 0 + \epsilon^y_i < \sigma \leq \frac{1}{2}$. \quad (17)

Notice, that signal realizations $y_i$ do not overlap: when $\theta < t$, all signal realizations are above $\frac{1}{2}$. On the contrary, whenever $\theta > t$, all signal realizations are below $\frac{1}{2}$.

Now suppose that an agents learns that his signal $y_i$ is larger than $\frac{1}{2}$: then he knows that this is only possible when $\theta < t$. In turn, $\theta < t$ means that a successful attack is underway, which he should join. Indeed, looking at the conjectured functions $A_t(\theta)$, we know that all agents will attack as in Fig. 5.

Fig. 5 Perfect coordination
If the agent’s signal $y_i$ is smaller than $\frac{1}{2}$, he knows $\theta < t$. That is, given the conjectured aggregate attack $A_t(\theta)$, the attack is not successful. Hence, it is optimal for the agent to abstain from attacking.

Equation (15) holds for all $\theta$: when $\theta < t$ all agents attack, and the aggregate attack is 1. In the other case, no agent attacks and the aggregate attack is 0. Thus, $A_t(\theta)$ satisfies the requirements of an equilibrium.

Proposition 3 generalizes our multiplicity result from Proposition 2 to a setting where agents receive separate signals over actions and fundamentals. Figures 5 and 6 graph the equilibria characterized by Proposition 3 for the case where private information over actions is very precise. Figures 5 and 6 also show that, due to the error terms’ compact support, agents can infer upper and lower bounds for the unknown fundamental $\theta$, which allows them to achieve perfect coordination if signals over actions are sufficiently precise.

It is difficult to identify the probability distributions that govern the information that agents have in the context of bank-runs, currency crises, recessions, or riots and revolutions. Accordingly, to cover a wider range of potential information structures, we will now prove an analog to Proposition 3 for the case where error terms have unbounded support:

**Proposition 4** Assume that $e_i^x$ and $e_i^y$ are distributed according to pdf $f = f_x f_y$ (cdfs $F_x, F_y$), where $f_x$ and $f_y$ are symmetric, and there exist $\delta > 0$, $\gamma > 0$, and $\xi > 0$ such that $1 - \delta \geq \gamma$, $1 > 3\delta + 2\gamma$, and the following conditions hold:

\[
\frac{F_x(\xi)}{1 - F_x(\xi)} \sup_{\eta \in [0, \delta]} f_y(\eta - a) \leq \frac{1 - c}{c} \quad \text{for all } \eta \geq 1 - \delta - \gamma
\]  \hspace{1cm} (18)

\[
F_x(\xi) F_y(\gamma) \geq 1 - \delta
\]  \hspace{1cm} (19)

Whenever these conditions hold, there exists a continuum of equilibria.

**Proof** The proof of Proposition 4 is similar to that of Proposition 3, and is given in Appendix B (see Supplementary Information). The main difference between both

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9 The symmetry assumption shortens the proof.
proofs lies in two regularity conditions, (18) and (19), which require that private signals over actions are sufficiently informative. Otherwise, the proof is organized in two steps. First, Section B.1 (see Supplementary Information), characterizes signal combinations for which agents do, respectively, do not attack. To perform this characterization, we impose condition (18). In turn, we impose condition (19), which requires that the mass of agents, who receive private signals with very large errors, is sufficiently small. Taken together, conditions (18) and (19) provide upper and lower boundaries, as shown in Fig. 7, for the mass of attacking agents.

Step two, which is given in Section B.2 (see Supplementary Information), starts with a guess of agents’ aggregate attack $A_0^t(\theta)$. In turn, we compute a best response $A_1^t(\theta)$, which yields another best response $A_2^t(\theta)$ and so on... Via this iteration we construct a converging sequence of aggregate attacks/best responses. In the limit, we obtain an equilibrium, or steady state, attack function for every $t$. Finally, since conditions (18) and (19) provide upper and lower boundaries, as shown in Fig. 7, for the mass of attacking agents.

Fig. 7 Construction of $A(\theta)$

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10 Roughly interpreted, condition (18) requires that agents put sufficient weight on the signal $y_i$, which informs them of the other agents’ actions. Put differently, condition (18) requires that high (low) signals over actions induce agents to believe that the attack is going to be a success (a failure). In terms of our model’s bank-run interpretation, condition (18) requires that a large, clearly visible, queue in front of a bank induces agents to join the queue. Likewise, if agents observe that there is no queue, they will not withdraw their deposits. Technically, condition (18) follows from the requirement that signal combinations where $y_i \geq 1 - \delta - \gamma$ and $x_i \leq t + \xi$, imply $P[\theta < t|A(\cdot), (x_i, y_i)] \geq c$, while signals $y_i \leq \delta + \gamma$ and $x \geq t - \xi$, imply $P[\theta \geq t|A(\cdot), (x_i, y_i)] \geq c$. In turn, condition (19) requires that the mass of agents, who receive pairs of private signals $x_i, y_i$ with very large errors, is sufficiently small. In terms of our model’s bank run interpretation, condition (19) requires that agents’ private information over the bank’s reserves, as well as over the queue of waiting agents, is sufficiently precise.

11 That is, due to the error terms’ unbounded support, agents will never be able to coordinate perfectly. Accordingly, the proof of Proposition 4 differs from that of Proposition 3, where our initial guess, i.e.
there exists a continuum of permissible \( t \) values, we have shown that a continuum of equilibria exists.

In the special case where error terms are normally distributed, we have:

**Example 1** Suppose error terms \( e^x, e^y \) are normally distributed, and denote the precisions of the respective signals by \( \alpha_x = \frac{1}{\sigma_x^2} \) and \( \alpha_y = \frac{1}{\sigma_y^2} \). For that case, we have
\[
f_x(\xi) = \phi(\sqrt{\alpha_x} \xi) \quad \text{and} \quad F_x(\xi) = \Phi(\sqrt{\alpha_x} \xi)
\]
\[
f_y(\gamma) = \phi(\sqrt{\alpha_y} \gamma) \quad \text{and} \quad F_y(\gamma) = \Phi(\sqrt{\alpha_y} \gamma),
\]
where \( \phi \) is the density and \( \Phi \) is the cumulative density function of the standard normal distribution. In turn, we can choose \( \delta \) and \( \gamma \) such that \( 1 > 3\delta + 2\gamma \), e.g., \( \delta = 0.2, \gamma = 0.1 \). Moreover, we can choose \( \alpha_x \xi > \Phi^{-1}(0.8) \). Finally we let \( \alpha_y \to \infty \), such that (18) and (19) are both satisfied.

**Discussion**

The basic global games approach to bank-runs assumes that agents cannot observe the queues that form before the bank opens. The only information that agents possess concerns the bank’s financial strength. In turn, each agent has to use his own signal over the bank’s strength to infer the signals, and thus the actions, of the other agents. This indirect reasoning makes coordination difficult.

The current model emphasizes that bank-runs, riots and revolutions, or joint investment projects, are phenomena where agents monitor each other closely. That is, queues in front of a bank are observable, and induce additional depositors to withdraw their money. This infectious process ensures multiple equilibria whenever the precision with which agents observe each other is sufficiently high. Equilibria in our model are then situations where no agent, given his information over the length of the queue of waiting agents and the game’s fundamental, would like to join/leave the queue, which forms before the bank opens.

Our model’s comparative statics accommodate a range of environments that differ regarding the information that agents have over each others’ actions and over fundamentals. Equilibrium uniqueness obtains in a manner similar to a global games setting whenever information over actions is of low quality. On the contrary, if private information over actions (and thus the game’s fundamental) is precise, agents can coordinate on multiple equilibria. These equilibria are quite different from what basic global games predict. Hence, we argue that the current model helps to better

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Footnote 11 (continued)
equation (16), turned out to be correct. We thus have to rely on an iteration argument to find the equilibrium function \( A_t(\theta) \).

Likewise, sardines have highly developed pressure senses, which allow them to monitor each others’ movements/actions. This helps them to coordinate quickly, respectively, to form swarms, which protect them from predators.
understand the equilibria that agents play in environments where it is reasonable to assume that they can observe each other.

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