SUPERSYMMETRY AND CHIRAL SYMMETRY

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Abstract

We dispute a recent claim for a nonperturbative nonrenormalisation theorem stating that mass cannot be spontaneously generated in supersymmetric QED. We also extend a long-standing perturbative result, namely that the effective potential is zero to all orders of perturbation theory, to the nonperturbative regime for arbitrary numbers of flavours.

I. INTRODUCTION

The supersymmetric (SUSY) nonrenormalisation theorem for mass to all orders in perturbation theory has been established for some time [1–3]. Several authors [4–6] have investigated the possibility of a nonperturbative nonrenormalisation theorem in SUSY quantum electrodynamics (SQED), which asserts that the chiral solution to the Dyson-Schwinger equation (DSE) in SQED is not merely favoured but unique, i.e. that there is no achiral solution. The first was Clark and Love [4] who, using the superfield formalism and after truncating diagrams containing seagull and higher order $n$-point vertices, found that the effective mass $M$ contains a prefactor $\xi^{-1}$ which vanishes in Feynman gauge. Reasoning that if the mass vanishes in one gauge then it must vanish in all gauges, they conclude that there can be no dynamic chiral symmetry breaking in SQED, even beyond the rainbow approximation.

This approach was criticized by Kaiser and Selipsky on two grounds [5]. Firstly they argue that the truncation of seagull diagrams is too severe as it ignores contributions even at the one-loop level. Secondly they point out that infinities arising from the infrared divergences which plague the superfield formalism can counter the vanishing prefactor and allow spontaneous mass generation. They also point out that the original nonrenormalisation theorem did not forbid mass corrections, only infinite mass counter-terms.

The issue was taken up by Campbell-Smith and Mavromatos [6] who investigated chiral symmetry breaking in $2 + 1$ dimensional SQED ($\text{SQED}_3$) using superfields with both two- and four-component spinors. In the four-component theory [6] they also find a nonrenormalisation theorem. Their analysis dimensionally reduces SQED$_4$ to SQED$_3$, introducing a compactification scale in the process. After truncating all two-particle irreducible diagrams

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from the DSE, taking the limit that all momenta are small compared to the momentum scale of the compactification, and making several other approximations, they find the same prefactor in front of the effective mass as Clark and Love, and claim that its cancellation by infrared divergences is subverted by the lack of a corresponding prefactor in the renormalisation factor $Z$. Since their argument depends on dimensional reduction of SQED$_{4}$, it cannot be applied in $3 + 1$ dimensions.

Our analysis in the component formalism finds no evidence for such a theorem. It is certainly the case that no vanishing gauge dependent prefactor emerges.

The CJT effective potential [7] expresses the potential of a theory as a functional of its propagators and can be used to compare the chiral and achiral propagators in ordinary field theories. It has been known for some time that in perturbation theory [8,9] the effective potential is exactly zero to all orders in a SUSY theory. (Pisarski has adapted these proofs to the many flavour limit in the nonperturbative theory [10].) However, a rigorous result in perturbation theory cannot be assumed to hold in the nonperturbative regime. So while the favoured solution must have a potential of exactly zero since SUSY is unbroken [11], it is still reasonable to ask if the unfavoured one does not.

There has been some confusion regarding the calculation of the CJT effective potential (for example see [12]) in SUSY theories. This arises from the fact that it is not clear how to treat Green’s functions with auxiliary fields. This problem was solved in a previous paper [13], and substituting the propagators from that paper into the CJT effective potential yields that it is uniformly zero. This renders the CJT effective potential ineffective for choosing between solutions to the DSE in the absence of SUSY breaking.

Sec. II gives our argument against the existence of a nonperturbative nonrenormalisation theorem. Substituting in the most general possible form for the three-point vertices [13], we find that it is impossible to find an acceptable ansatz in which the effective mass must be zero. However we do not succeed in finding an achiral solution so the (remote) possibility that the DSE cannot be solved with a dressed mass remains open.

Our analysis of the CJT effective potential, in which we show that it is uniformly zero, is presented in Sec. III. This result is not spoiled by vacuum polarisation.

II. CHIRAL SYMMETRY BREAKING IS PERMITTED IN SQED

The Lagrangian of SQED is

$$L = |f|^2 + |g|^2 + |\partial_\mu a|^2 + |\partial_\mu b|^2 - \bar{\psi} \not{\partial} \psi$$

$$- m(a^* f + a f^* + b^* g + b g^* + i \bar{\psi} \psi)$$

$$-ie A^\mu (a^* \not{\partial_\mu} a + b^* \not{\partial_\mu} b + \bar{\psi} \gamma_\mu \psi)$$

$$-e [\bar{\lambda}(a^* + i \gamma_5 b^*) \psi - \bar{\psi}(a + i \gamma_5 b) \lambda]$$

$$+ie D(a^* b - ab^*) + e^2 A_\mu A^\mu(|a|^2 + |b|^2)$$
The electron is represented by $\psi$ and its propagator is of the general form
\[
S(p) \equiv \frac{-i}{\gamma \cdot p A(p^2) + B(p^2)} = -i \frac{Z(p^2)}{\gamma \cdot p + \mathcal{M}(p^2)},
\]
where $Z(p^2), \mathcal{M}(p^2), A(p^2)$ and $B(p^2)$ are scalar functions. It makes up the chiral multiplet together with the selectrons $a, f, b, g$. The selectron propagators are restricted by SUSY Ward identities \cite{4,13} to be
\[
D_{aa}(p^2) \equiv \langle a^* a \rangle(p^2) = \frac{A(p^2)}{p^2 A(p^2) - B^2(p^2)},
\]
\[
D_{af}(p^2) \equiv \langle a^* f \rangle(p^2) = \frac{B(p^2)}{p^2 A(p^2) - B^2(p^2)},
\]
\[
D_{ff}(p^2) \equiv \langle f^* f \rangle(p^2) = \frac{p^2 A(p^2)}{p^2 A(p^2) - B^2(p^2)}.
\]

The photon and photino fields are $A_\mu$ and $\lambda$, respectively.

The DSE in SQED is given by \cite{13}
\[
S^{-1}(p) - S^{-1}_0(p) = -\int \frac{d^d q}{(2\pi)^d} \{ D_{\mu\nu}(p - q) \gamma^\mu S(q) \Gamma^\nu_{\psi A_\nu}(q, p) + S_\lambda(p - q) D_{aa}(q) \Gamma_{\lambda a^* \psi}(q, p) + S_\lambda(p - q) D_{af}(q) \Gamma_{\lambda f^* \psi}(q, p) \}.
\]

The three-point vertices in Eq. (2.6) are given by \cite{13}
\[
\Gamma_{\lambda a^* \psi}(p, q) = \frac{e}{p^2 - q^2} (p^2 A(p^2) - q^2 A(q^2)) + \frac{e}{p^2 - q^2} (B(p^2) - B(q^2)) \gamma \cdot q
\]
\[
+ \frac{1}{2} e(p^2 - \gamma \cdot q) T_{aa}(p^2, q^2, p \cdot q) + \frac{1}{2} e p^2 (q^2 - \gamma \cdot p) T_{ff}(p^2, q^2, p \cdot q) + \frac{1}{2} e [\gamma \cdot p (p^2 - q^2) - 2 \gamma \cdot q (p^2 - p \cdot q)] T_{af}(p^2, q^2, p \cdot q),
\]

\[
\Gamma_{\lambda f^* \psi}(p, q) = \frac{-e}{p^2 - q^2} (A(p^2) - A(q^2)) \gamma \cdot q - \frac{e}{p^2 - q^2} (B(p^2) - B(q^2))
\]
\[
+ \frac{1}{2} e (\gamma \cdot p - \gamma \cdot q) T_{aa}(p^2, q^2, p \cdot q) + \frac{1}{2} e (p - q)^2 T_{af}(p^2, q^2, p \cdot q)
\]
\[
- \frac{1}{2} e \gamma \cdot q (p^2 - \gamma \cdot p) T_{ff}(p^2, q^2, p \cdot q).
\]
where

\[ \Gamma_{\nu A \psi}^{\mu}(p, q) = \Gamma_{BC}^{\mu}(p, q) + \frac{ie}{p^2 - q^2} (A(p^2) - A(q^2)) \left[ \frac{1}{2} T_3^\mu - T_8^\mu \right] \]

\[ - \frac{ie}{p^2 - q^2} (B(p^2) - B(q^2)) T_5^\mu + \frac{1}{2} i e T_{aa}(p^2, q^2, p \cdot q) T_3^\mu \]

\[ + ie T_{af}(p^2, q^2, p \cdot q) \left( \frac{1}{2}(p - q)^2 T_1^\mu - T_4^\mu \right) \]

\[ + \frac{1}{2} i e T_{ff}(p^2, q^2, p \cdot q) \left[ T_2^\mu - p \cdot q T_3^\mu - (p - q)^2 T_8^\mu \right], \quad (2.9) \]

and

\[ \Gamma_{BC}^{\mu}(p, q) = \frac{ie}{2} \frac{1}{p^2 - q^2} (\gamma \cdot p + \gamma \cdot q) (A(p^2) - A(q^2))(p + q)^\mu \]

\[ + i e \frac{1}{2} (A(p^2) + A(q^2)) \gamma^\mu + \frac{ie}{p^2 - q^2} (B(p^2) - B(q^2))(p + q)^\mu, \quad (2.10) \]

\[ T_1^\mu = \gamma^\mu (q^2 - p \cdot q) + q^\mu (p^2 - p \cdot q), \quad (2.11) \]

\[ T_4^\mu = (\gamma \cdot p + \gamma \cdot q) T_1^\mu, \quad (2.12) \]

\[ T_3^\mu = \gamma^\mu (p - q)^2 - (\gamma \cdot p - \gamma \cdot q)(p - q)^\mu, \quad (2.13) \]

\[ T_5^\mu = \sigma^{\mu\nu} (p - q)_\nu, \quad (2.14) \]

\[ T_8^\mu = \frac{1}{2} (\gamma \cdot p \gamma \cdot q \gamma^\mu - \gamma^\mu \gamma \cdot q \gamma \cdot p), \quad (2.15) \]

and represent the electron-a-photino, electron-f-photino and electron-photon vertices respectively.

Performing the Wick rotation into Euclidean space and substituting the full vertices into the DSE gives us the following integral equations:

\[ B(p^2) = 2e^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{(p - q)^2} \frac{1}{p^2 - q^2} \left[ D_{af}(q^2)p^2 A(p^2) \right. \]

\[ + (p^2 - 2q^2) D_{aa}(q^2) B(p^2) \]

\[ + (\xi - 1)e^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{(p - q)^4} \left[ D_{af}(q^2)p^2 A(p^2) + D_{aa}(q^2)q^2 B(p^2) \right] \]

\[ - (\xi - 1)e^2 \int \frac{d^d q}{(2\pi)^d} \frac{p \cdot q}{(p - q)^4} \left[ D_{af}(q^2)A(p^2) + D_{aa}(q^2)B(p^2) \right] \]

\[ - \frac{1}{2} e^2 \int \frac{d^d q}{(2\pi)^d} D_{af}(q^2) T_{aa}(p^2, q^2, p \cdot q) \]

\[- e^2 \int \frac{d^d q}{(2\pi)^d} D_{aa}(q^2) T_{af}(p^2, q^2, p \cdot q) \left[ (p \cdot q)^2 - p^2 q^2 \right. \]

\[ \left. \frac{(p - q)^2}{(p - q)^2} + q^2 - p \cdot q \right] \]

\[ + \frac{1}{2} e^2 \int \frac{d^d q}{(2\pi)^d} \left. \right] D_{af}(q^2) T_{ff}(p^2, q^2, p \cdot q) \left[ p^2 q^2 - p \cdot q \right. \]

\[ \left. \frac{(p - q)^2}{(p - q)^2} + q^2 \frac{p^2 - p \cdot q}{(p - q)^2} \right], \quad (2.16) \]
\[ A(p^2) - 1 = 2e^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{(p-q)^2} \frac{1}{p^2 - q^2} D_{aa}(q^2) \left[ (p^2 - 2q^2)A(p^2) + q^2 A(q^2) \right] \]
\[ -2e^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{(p-q)^2} \frac{1}{p^2 - q^2} D_{af}(q^2) [B(p^2) - B(q^2)] \]
\[ + (\xi - 1)e^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{(p-q)^4} D_{aa}(q^2) q^2 [A(p^2) + A(q^2)] \]
\[ - (\xi - 1)e^2 \int \frac{d^d q}{(2\pi)^d} \frac{1}{(p-q)^4} D_{af}(q^2) [B(p^2) - B(q^2)] \]
\[ + (\xi - 1) \frac{e^2}{p^2} \int \frac{d^d q}{(2\pi)^d} \frac{p \cdot q}{(p-q)^2} D_{af}(q^2) [B(p^2) - B(q^2)] \]
\[ - (\xi - 1) \frac{e^2}{p^2} \int \frac{d^d q}{(2\pi)^d} \frac{p \cdot q}{(p-q)^2} D_{aa}(q^2) [p^2 A(p^2) + q^2 A(q^2)] \]
\[ + \frac{e^2}{p^2} \int \frac{d^d q}{(2\pi)^d} D_{aa}(q^2) T_{aa}(p^2, q^2, p \cdot q) \left[ \frac{3}{2} p \cdot q - p^2 q^2 - p \cdot q \left( \frac{p^2 - q^2}{(p-q)^2} \right) \right] \]
\[ + \frac{1}{2} \frac{e^2}{p^2} \int \frac{d^d q}{(2\pi)^d} D_{af}(q^2) T_{af}(p^2, q^2, p \cdot q) \left[ p \cdot q - 3p^2 + 2(p^2 - p \cdot q) \frac{q^2 - p \cdot q}{(p-q)^2} \right] \]
\[ + \frac{3}{2} \frac{e^2}{p^2} \int \frac{d^d q}{(2\pi)^d} D_{aa}(q^2) T_{ff}(p^2, q^2, p \cdot q) q^2 p^2, \tag{2.17} \]

where the scalar functions \( T_{aa}(p^2, q^2, p \cdot q), T_{af}(p^2, q^2, p \cdot q), T_{ff}(p^2, q^2, p \cdot q) \) are symmetric in \( p, q \) due to charge conjugation invariance [13] but are unconstrained by either the Ward-Takahashi or SUSY Ward identities. In a slight abuse of language we refer to these functions as ‘transverse’ functions since they contribute only to the transverse components of the vertices. The contribution to \( B(p^2) \) from the transverse components of the vertices is referred to as the transverse contribution. Eqs. (2.16), (2.17) were derived in the quenched approximation. The effects of vacuum polarisation will be considered shortly.

Considering first the minimal SUSY Ball-Chiu ansatz where the transverse functions are set to zero, we see immediately that there is no reason why \( B(p^2) \) must vanish. We can also eliminate the possibility of the transverse functions inducing a nonrenormalisation theorem by cancelling off the minimal contribution. To see this, recall that the transverse functions are symmetric in \( p \) and \( q \) and examine the coefficients of \( D_{af}(q^2) \). Its coefficients in the minimal contribution are asymmetric in \( p, q \) because of the \( A(p^2) \) factor whereas those in the transverse contribution are exactly symmetric. (A corresponding argument for terms in Eq. (2.16) proportional to \( D_{aa}(q^2) \) cannot be made because the transverse contribution has an asymmetric component.) It follows that the integrand of \( B(p^2) \) will not vanish, regardless of the choice of transverse functions. This result will still hold after angular integration as the transverse contribution will be a symmetric function multiplied by \( q^{d-1} \) whereas the minimal contribution will not. However we cannot eliminate the possibility that Eqs. (2.16), (2.17) may simply not be solvable unless \( B(p^2) \) is set to zero.
This apparent contradiction between ourselves and previous superfield analyses \[4,6\] requires explanation. If an achiral solution is forbidden in the superfield formalism then it must also be forbidden in the component formalism, and yet our analysis finds no evidence that an achiral solution cannot exist. We can eliminate the possibility that our choice of the quenched approximation has obscured a nonrenormalisation theorem. Indeed, the photon propagator is of the general form

\[
D_{\mu\nu}(p) \equiv \frac{1}{p^2} \left( g_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{p^2} \right) - \frac{1}{1 + \Pi(p^2)} + \frac{\xi p_{\mu}p_{\nu}}{p^4},
\]

where \(\Pi(p^2)\) is the vacuum polarisation. The photino and \(D\) propagators are then restricted by SUSY to be \[14\]

\[
S_\lambda(p) = \frac{-i}{\gamma \cdot p} \frac{1}{1 + \Pi(p^2)}, \quad (2.19)
\]

\[
D_D(p^2) = \frac{1}{p^2} \frac{1}{1 + \Pi(p^2)}, \quad (2.20)
\]

respectively. In the absence of approximations we can confidently choose to work in a specific gauge, and in Landau gauge \((\xi = 0)\) the right hand sides of Eqs. \((2.16,2.17)\) are simply multiplied by a factor of \(\frac{1}{1 + \Pi(p^2)}\), which preserves our result.

It would seem that the vanishing gauge dependent prefactor in superspace treatments \[4,6\] is an artifact of the extensive approximations used. The approximations in \[6\] were generally chosen so as to have minimal impact in the infrared region where chiral symmetry breaking is largely determined, but to combine such approximations with a gauge dependent argument is dangerous. Consider, for example, the equivalent of Eq. \((2.16)\) in non-SUSY QED\(_3\) in the quenched rainbow approximation,

\[
B(p^2) = (\xi + 2) \frac{e^2}{4\pi^2 p} \int_0^\infty dq \frac{qB(q^2)}{q^2 A^2(q^2) + B^2(q^2)} (\ln|p + q| - \ln|p - q|). \quad (2.21)
\]

In the special gauge of \(\xi = -2\) the right hand side of Eq. \((2.21)\) vanishes unless \(A(q^2)\) and \(B(q^2)\) conspire to cancel this prefactor. It does not follow though that chiral symmetry is unbroken. In fact non-SUSY QED\(_3\) is known to break chiral symmetry from lattice studies \[15,16\]. The vanishing prefactor in Eq. \((2.21)\) is an artifact of the rainbow approximation.

III. THE CJT EFFECTIVE POTENTIAL IN THE NONPERTURBATIVE REGIME

Finding the CJT effective potential for any theory obviously requires the correct form of the propagators. The correct form of the selectron propagators in SUSY theories is made a little obscure by the auxiliary fields \(f, g\) and was obtainable only with the recent realisation \[13\] that the free Lagrangian becomes quadratic when the selectrons \(a, f\) \((b, g)\) form a column matrix \(\begin{pmatrix} a \\ f \end{pmatrix}\) \(\begin{pmatrix} b \\ g \end{pmatrix}\). The selectron propagator is

\[
[D(p^2)] = \begin{bmatrix}
D_{aa}(p^2) & D_{af}(p^2) \\
D_{fa}(p^2) & D_{ff}(p^2)
\end{bmatrix} = \begin{bmatrix}
D_{bb}(p^2) & D_{bg}(p^2) \\
D_{gb}(p^2) & D_{gg}(p^2)
\end{bmatrix}. \quad (3.1)
\]
The CJT effective potential is

\[
V[S, [D]_a, [D]_b] = \int \frac{d^d p}{(2\pi)^d} \left( \text{Tr} \ln(S_0^{-1}(p)S(p)) + \frac{1}{2} \text{Tr}(1 - S_0^{-1}(p)S(p)) \right) - 2 \int \frac{d^d p}{(2\pi)^d} (\text{Tr} \ln([D(p^2)]_0^{-1}[D(p^2)]) \\
+ \frac{1}{2} \text{Tr}(1 - [D(p^2)]_0^{-1}[D(p^2)])) \\
+ \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} (\text{Tr} \ln(S_0^{-1}(p)S(p)) + \text{Tr}(1 - S_0^{-1}(p)S(p))) \\
- \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} (\text{Tr} \ln(D_{\mu\nu}^{-1}(p)D_{\nu\sigma}(p)) + 4 - D_{\mu\nu}^{-1}(p)D_{\nu\mu}(p)) \\
- \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} (\text{Tr} \ln(D_{\mu\sigma}^{-1}(p^2)D_{\nu\lambda}(p^2)) + \text{Tr}(1 - D_{\mu\sigma}^{-1}(p)D_{\nu\lambda}(p^2))).
\]

(3.2)

(The calculation in [12] used ordinary scalar propagators \(D_{aa}(p^2).\)) Substituting Eqs. (2.2) to (2.3) and Eqs. (2.18) to (2.20) into Eq. (3.2) reveals that the CJT effective potential is zero at its extrema, and therefore uniformly zero. Note that there is another term in Eq. (3.2), given by

\[
\int \frac{d^3 p}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} D_{\mu\nu}(q) \text{Tr}([\Gamma_{a^* A_a A_{a^*} A_a}^{\mu\nu}(p, p, q)[D(p^2)]],
\]

(3.3)

where \([\Gamma_{a^* A_a A_{a^*} A_a}^{\mu\nu}]\) is the four-point vertex for two \([a]\)s and two photons, corresponding to the vacuum graphs that give rise to the tadpole contributions in the boson DSE. As we have written it, Eq. (3.2) only gives half of these particular diagrams. However, Eq. (3.3) is proportional to the massless tadpole integral and is therefore zero by dimensional regularisation [17].

So Pisarski’s result is not restricted to the many flavour limit, ie. the nonrenormalisation theorem for the effective potential also applies nonperturbatively. It is trivial to extend our result to arbitrary numbers of flavours and it follows that the CJT effective potential cannot be used to select the dynamically favoured propagator in a SUSY theory without SUSY breaking.

IV. CONCLUSION

There is no reason to suppose the nonexistence of an achiral solution to the DSE in either SQED_3 or SQED_4, and no nonperturbative nonrenormalisation theorem in either of these theories. The apparent theorem found in superfield treatments is an artifact of the substantial approximations required by the form of the DSE in the superfield notation. It must be admitted, however, that an actual achiral solution has so far eluded us so we have yet to prove that such a solution actually exists, although there is no evidence that it doesn’t.

Nor have we addressed the issue of whether it is the chiral or the achiral solution that is dynamically favoured. We have demonstrated, however, that the CJT effective potential cannot be used to determine this in the absence of SUSY breaking since it is uniformly zero, even in the nonperturbative regime.
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