Practical Tracking Control of a Class of Uncertain Surface Vessels Under Weak Assumptions on Uncertainties and Reference Trajectories

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Abstract: This paper is devoted to the tracking control of a class of uncertain surface vessels. The main contributions focus on the considerable relaxation of the severe restrictions on system uncertainties and reference trajectory in the related literature. Specifically, all the system parameters are unknown and the disturbance is not necessarily to be differentiable in the paper, but either the unknown parameters or the disturbance is considered but the other one is excluded in the related literature, or both of them are considered but the disturbance must be continuously differentiable. Moreover, the reference trajectories in the related literature must be at least twice continuously differentiable and their time derivatives must be known for feedback, which are generalized to a more broad class of ones in the paper that are only once continuously differentiable without the measurements of their time derivatives.

To solve the control problem, a novel practical tracking control scheme is presented by using backstepping scheme and adaptive technique, and in turn to derive an adaptive state-feedback controller which guarantees that all the states of the resulting closed-loop system are bounded while the tracking error arrives at and then stays within an arbitrary neighborhood of the origin. Finally, simulation is provided to validate the effectiveness of the proposed theoretical results.

Keywords: Adaptive, practical tracking, surface vessel, uncertainties.

1. INTRODUCTION

The surface vessels, as a class of important transportation devices, have been widely used in many marine production fields, such as marine transportation, marine environment monitoring and marine service support [1,2], etc. In practice, certain control inputs should be designed for the systems which describe the dynamics of surface vessels so as to ensure desired control performance (for example, a surface vessel requires to move along the given course). This promotes the fast and persistent development of the controls for surface vessels over the past two decades [3-30,31,32]. It is necessary to point out that, control design for such systems is usually rather challenging due to the presence of serious unknowns or uncertainties (such as unknown parameters, nonlinear dynamics inside the system or the disturbances coming from the external environment). Therefore, many control problems in this field which are of great scientific and practical significance remain unsolved and deserve to be investigated.

Trajectory tracking control that concerns designing controller such that the system output tracks a desired reference trajectory has been an active topic among the controls of surface vessels [3-30]. Consequently, several control design schemes are proposed on this topic, such as PID or PD control [25,26], robust control [12,14,27], adaptive (non-robust) control [8,15,20,24], reinforcement learning strategy [23] and some intelligent control methods based on fuzzy logic or neural network [5-7,9,10,13,18,22]. However, the existing results are restricted by the severe assumptions on the system uncertainties about system parameters and disturbances. Specifically, in [3], all the system parameters are supposed to be exactly known and the disturbance is excluded, and hence no uncertainties are considered. In [7,11,12,14,15,20,22-24,27,28], either unknown parameter or disturbance is considered but the other one is excluded. Although in [5,21], both unknown parameters and disturbances are considered, the disturbances require to be smooth and hence exclude a large class of non-smooth ones. It is necessary to point out that, unknown parameters and disturbances are usually coexisting and contain serious uncertainties in prac-
tice due to the complicated structure of the surface vessel and the harsh environment it works in. For example, the disturbance (e.g., the effect of the ocean current acting on the surface vessel) may be non-smooth (even non-continuous). The presence of the serious uncertainties results into the ineffectiveness of the existing control schemes. Then, a novel control method should be developed which is powerful enough for the serious uncertainties.

The other restrictions of the existing results in the trajectory tracking control of the surface vessels focus on the generality of the reference trajectories to be followed, which are the following twofold: 1) High order smoothness of the reference trajectories are needed. Specifically, the reference trajectories require to be twice or four times continuously differentiable in [3-5,8,10-13,15-19,21,22,27] and even sufficient smooth in [6,23,24]. Then, a large class of reference trajectories with low order smoothness (e.g., only first continuously differentiable) are excluded and hence restricts the applicability of the existing results. 2) The time differentials of the reference trajectory are needed to be available for feedback. In fact, the first (or even second) order time differentials of the reference trajectory are need in the controllers of [3-5,8,11,12,15,17-19,21,22,24]. This implies that more measuring devices are required to obtain so many signals, and hence restricts the implementation of the controllers in the literature. Note the above twofold restrictions, once the reference trajectories become more generality (such as not necessarily to be twice or four times continuously differentiable) or only less measurable information is available for feedback, the existing control schemes will be incapable.

Notice the restrictions of the existing results on system uncertainties and the reference trajectories mentioned above, one natural and nontrivial control problem arise, i.e., for a class of surface vessels with serious uncertainties, how to design a tracking controller such that the system output follows a broad class of reference trajectories with less measurable information and weak assumption on the smoothness? That is also the main control objective of this paper. Remarkably, the restrictions on the system uncertainties and the reference trajectories are considerably relaxed since all the system parameters and dynamic matrices are unknown, the disturbance does not necessarily to be smooth, and moreover, the reference trajectories are not necessarily highly smooth (i.e., have second or more higher order derivatives) for which the time derivatives are not necessarily available for feedback. Then, by using backstepping scheme combining with adaptive technique, an adaptive controller is explicitly constructed in which the updating law is skilfully chosen to compensate the serious uncertainties coming from system parameters and external disturbance while guarantees the desirable tracking performance. It is proven that, for arbitrary reference trajectory and tracking accuracy given beforehand, the proposed controller guarantees that all the closed-loop system output practically tracks the reference trajectory, i.e., the tracking error arrives at and then stays within an arbitrary neighborhood of the origin.

In summary, the main novelties of the paper are highlighted by the following threefold:

1) More serious unknowns are allowed. On one hand, the disturbance in the paper is not necessarily to be differentiable, but those in [5,11,12,14,17,21,27] must be. On the other hand, the time derivatives of the reference trajectory are not necessarily to be available but those in [3-5,8,11,12,15,17-19,21,22,24] must be. Thus, more serious unknowns on disturbance and reference trajectory are allowed in the paper than the related literature.

2) More broader class of reference trajectories can be followed. In fact, the reference trajectories must be at least twice continuously differentiable in [3-6,8,10-13,15-19,21,24,27]. Differently, this paper just requires the reference trajectory to be one time continuously differentiable, and hence allows more broader class of trajectories.

3) The designed controller is more implementable. In [4,9,11-14,22-24], the tracking errors can converge to an arbitrary neighborhood of the origin by carefully choosing of multiple controller parameters or/and system parameters which are difficult to implement. Differently, the designed controller guarantees the similar control objective once two controller parameters (i.e., $c_1$, $c_2$ below) are positive, and hence is more implementable than those in the literature.

The remainder parts of the paper are organized as follows: Section 2 formulates the control problem under investigation where the surface vessel dynamics and control objective are given. Section 3 presents the procedure of control design. Section 4 analyzes the closed-loop system performance. Section 5 provides a simulation example to validate the effectiveness of the theoretical results. Section 6 gives some concluding remarks. The paper ends with Appendix A that collects the proof of an important property of the system.
2. PROBLEM FORMULATION

In this paper, we consider a surface vessel system (see Fig. 1) described by the following \cite{4}

\[
\begin{align*}
\dot{\eta} &= J(\varphi)v, \\
M\dot{v} + C(v)v + D(v)v &= \tau + d,
\end{align*}
\]

where \(\eta = (x, y, \varphi)^T\) and \(v = (u, v, r)^T\) are the system states with \((x, y)\), \(\varphi\) being the position and the heading in the earth-fixed frame and \(u, v, r\) being the velocities of surge, sway and yaw in the body-fixed frame; \(\tau = (\tau_1, \tau_2, \tau_3)^T\) is the control input with \(\tau_1, \tau_2\) being the force and \(\tau_3\) being the moment which is provided by the vessel’s propulsion system; \(d = (d_1, d_2, d_3)^T\) is the disturbance which represents the unmodeled dynamics and the effects suffered from the external environment; \(J(\varphi), M, C(v), D(v)\) are respectively the rotation matrix, the inertia matrix containing additional mass, the Coriolis and centripetal terms matrix, and the damping matrix which are defined as follows:

\[
M = \begin{pmatrix}
m_{11} & 0 & 0 \\
0 & m_{22} & m_{23} \\
0 & m_{32} & m_{33}
\end{pmatrix},
\]

\[
J(\varphi) = \begin{pmatrix}
\cos \varphi & -\sin \varphi & 0 \\
\sin \varphi & \cos \varphi & 0 \\
0 & 0 & 1
\end{pmatrix},
\]

\[
C(v) = \begin{pmatrix}
0 & 0 & -m_{22}v - m_{23}r \\
m_{22}v + m_{23}r & -m_{11}u & 0 \\
0 & m_{11}u & 0
\end{pmatrix},
\]

\[
D(v) = \begin{pmatrix}
d_{11}(u) & 0 & 0 \\
0 & d_{22}(v, r) & d_{23}(v, r) \\
0 & d_{32}(v, r) & d_{33}(v, r)
\end{pmatrix},
\]

where

\[
m_{11} = m - X_{du},
\]

\[
m_{22} = m - Y_{dr},
\]

\[
m_{23} = m \cdot x_g - Y_{dr},
\]

\[
m_{33} = I_z - N_{dr},
\]

\[
d_{11}(u) = -(X_u + X_{uu}|u|),
\]

\[
d_{22}(v, r) = -(Y_v + Y_{rv}|r| + Y_{rv}|v|),
\]

\[
d_{23}(v, r) = -(Y_v + Y_{rv}|r| + Y_{rv}|v|),
\]

\[
d_{32}(v, r) = -(N_v + N_{rv}|r| + N_{rv}|v|),
\]

\[
d_{33}(v, r) = -(N_v + N_{rv}|r| + N_{rv}|v|),
\]

with \(m, I_z, x_g, X_*, Y_*, N_*\) being unknown parameters, \(m\) represents the mass of ship, \(I_z\) is the moment of inertia about the yaw rotation, \(x_g\) denotes the distance between the origin of body-fixed frame and the gravity center of the surface ship and \(X_*, Y_*, N_*\) express hydrodynamic.

The following gives two important properties of system (1) which will used in the later controller design. Property 1 provides the characteristic of \(M, J(\varphi)\) which can be directly obtained from (2). Property 2 gives the estimation of matrices \(C(v)\) and \(D(v)\) whose derivation is postponed to Appendix A.

**Property 1:** Matrix \(J(\varphi)\) is an orthogonal matrix, i.e., \(J^T(\varphi)J(\varphi) = I_{3 \times 3}\), and \(M\) is a positive definite symmetric matrix.

**Property 2:** For \(C(v), D(v)\), there exist positive constants \(l_1, l_2\) such that

\[\|C(v)\| \leq l_1\|v\|, \quad \|D(v)\| \leq l_2(\|v\| + 1).\]

The control objective of this paper is to design a state feedback controller \(\tau\) for the system (1) such that all the states of the resulting closed-loop system are bounded and the system output \(\eta\) practically tracks a given reference trajectory \(\eta_d = (x_d, y_d, \varphi_d)^T\) which satisfies the following Assumption 1, i.e., for any prescribed \(\varepsilon > 0\), there is a finite time \(T > 0\), such that \(\|\eta - \eta_d\| < \varepsilon, \forall t > T\).

**Assumption 1:** There exists an unknown constant \(l_3\) such that \(\|\eta_d\| + \|\eta_d\| \leq l_3\).

**Assumption 2:** There exists an unknown constant \(l_4\) such that \(\|d\| \leq l_4\).

**Remark 1:** Assumption 1 shows that the reference trajectory requires to be only one time continuously differentiable and hence includes those \(3-6,8,10-13,15-19,21-24,27\) which are at least twice continuously differentiable. Then, a more broad class of reference trajectories can be followed in the paper which lead to the incapability of the control schemes in the literature. Therefore, a novel control design method should be developed to design a powerful controller which can track the general reference trajectories.
**Remark 2:** Assumption 2 shows that the disturbance is bounded without any requirements on the smoothness and hence includes those of [5,11,12,14,17,21,27] where the disturbance must be continuously differentiable. Actually, the disturbance may be quite unexpected in practice, such as non-smooth or even non-continuous, which cannot be overcome by the compensation technique in the literature.

**3. CONTROL DESIGN**

For system (1), we introduce the following coordinate transformation

\[
\begin{align*}
\dot{s}_1 &= \eta - \eta_d, \\
\dot{s}_2 &= v - \alpha,
\end{align*}
\]

with \(\alpha\) being virtual control to be chosen later. The following error system is obtained directly from (1) and (3):

\[
\begin{align*}
\dot{s}_1 &= \dot{\eta} - \dot{\eta}_d = J(\varphi)v - \dot{\eta}_d, \\
\dot{s}_2 &= \dot{v} - \dot{\alpha} \\
&= M^{-1}(\tau + d - C(v)v - D(v)v) - \dot{\alpha}.
\end{align*}
\]

In the next, controllers will be designed by the following 2-step backstepping transformation.

**Step 1:** Define Lyapunov function \(V_1 = \frac{1}{2} s_1^T s_1\). By the first equation of (4), we obtain that

\[
V_1 = \dot{s}_1^T \dot{s}_1 = s_1^T \left( J(\varphi)s_2 + J(\varphi)\alpha - \dot{\eta}_d \right).
\]

Then, by Young’s inequality and Assumption 1, we have

\[
-\frac{k(t)}{4} s_1^T \dot{\eta}_d \leq \frac{k(t)}{4} s_1^T s_1 + \frac{l_2^2}{k(t)},
\]

where \(k(t)\) subjects to the following adaptive law:

\[
k(t) = \begin{cases} 
\frac{\|s_1\|^2}{2} & \|s_1\| \geq \frac{\varepsilon}{2} \\
0 & \|s_1\| < \frac{\varepsilon}{2}
\end{cases},
\]

with \(k(0) \geq 1\).

Substituting (6) into (5) leads to

\[
V_1 \leq s_1^T \left( J(\varphi)s_2 + J(\varphi)\alpha + \frac{k(t)}{4} s_1 \right) + \frac{l_2^2}{k(t)}.
\]

By choosing the following virtual controller

\[
\alpha = -J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) s_1,
\]

with \(c_1\) being an arbitrary positive constant, we get

\[
V_1 \leq s_1^T \left( J(\varphi)s_2 - c_1 s_1 \right) + \frac{l_2^2}{k(t)}.
\]

By Young’s inequality and noting that \(J^T J = I_{3\times3}\), we have

\[
s_1^T J(\varphi)s_2 \leq \frac{c_1}{2} s_1^T s_1 + \frac{\|s_2\|^2}{\|s_1\|^2},
\]

substituting the above inequality into (9) leads to

\[
\dot{V}_1 \leq -\frac{c_1}{2} s_1^T s_1 + \frac{1}{2c_1} s_2^T s_2 + \frac{l_2^2}{k(t)}.
\]

**Remark 3:** Seeing from (7), the adaptive law designed in the paper only depends on the tracking accuracy parameter \(\varepsilon\), and hence is essentially different from the dead-zone based one in [33] which depends on the system parameters. Since all the system parameters in this paper are unknown, the controller based on the dead-zone adaptive parameters. Since all the system parameters in this paper are unknown, the controller based on the dead-zone adaptive law given in the literature fails to the control problem investigated in the paper.

**Step 2:** First, by (8), we have

\[
\dot{\alpha} = -J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) s_1 - J^T(\varphi) \frac{k(t)}{4} s_1
\]

\[
- J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) \dot{s}_1
\]

\[
= -J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) s_1 - J^T(\varphi) \frac{k(t)}{4} s_1
\]

\[
+ J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right)^2 s_1
\]

\[
- J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) (J(\varphi)s_2 + \dot{\eta}_d).
\]

Then, defining Lyapunov function \(V_2 = V_1 + \frac{1}{2} s_2^T M s_2\), the above equation and the second one of (4) give that

\[
V_2 = V_1 + s_2^T M s_2
\]

\[
= V_1 + s_2^T \left( M^{-1}(\tau + d - C(v)v - D(v)v) + J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) s_1 + J^T(\varphi) \frac{k(t)}{4} s_1 \right.
\]

\[
+ J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) J(\varphi)s_2
\]

\[
- J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right)^2 s_2 - J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) \dot{\eta}_d).
\]

By (8) and the second equation of (3), we have \(v = s_2 + \alpha = s_2 - J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) s_1\). Substituting it into above equality leads to

\[
V_2 = V_1 + s_2^T \left( \tau + d - (C(v) + D(v)) s_2 \right).
\]
+ \left( C(v) + D(v) \right) J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) s_1
+ MJ^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) s_1 + MJ^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right)^2 s_1
+ M \left( c_1 + \frac{k(t)}{4} \right) s_2 - MJ^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right)^2 s_1
- MJ^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) \eta_d \right). \quad (12)

In the following, we give the estimations of all the terms but the first one in the bracket of the second term on the right-hand side of (12).

Firstly, Assumption 2 directly gives that
\[ s_1^T d \leq \frac{k(t)}{4} \| s_2 \|^2 + \frac{l_2^2}{k(t)}. \quad (13) \]

Secondly, by Properties 1 and 2, we have
\[ -s_2^T (C(v) + D(v)) s_2 \leq s_2 \left( (l_1 + l_2) \| v \| + l_2 \right) s_2 \leq \frac{k(t)}{4} \| s_2 \|^4 (\| v \|^2 + 1) + \frac{(l_1 + l_2)^2 + l_2^2}{k(t)}, \quad (14) \]

and
\[ s_2^T (C(v) + D(v)) J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) s_1 \leq \| s_2 \| \| s_1 \| \| J(\varphi) \| \left( c_1 + \frac{k(t)}{4} \right) \left( (l_1 + l_2) \| v \| + l_2 \right) \leq \frac{k(t)}{4} \| s_2 \|^2 \| s_1 \|^2 (\| v \|^2 + 1) \left( c_1 + \frac{k(t)}{4} \right)^2 \]
\[ + \frac{(l_1 + l_2)^2 + l_2^2}{k(t)}. \quad (15) \]

Thirdly, to facilitate the estimation, we can describe \( \dot{J}(\varphi) \) as follows:
\[
\dot{J}(\varphi) = \frac{\partial J(\varphi)}{\partial \varphi} \varphi = Q \left( \begin{array}{c}
0 \\
0 \\
1
\end{array} \right)^T J(\varphi) (s_2 + \alpha)
= Q \left( \begin{array}{c}
0 \\
0 \\
1
\end{array} \right)^T J(\varphi) \left( s_2 - J^T(\varphi) \left( c_1 + \frac{k(t)}{4} \right) s_1 \right). \quad (16)
\]

with
\[
Q = \left( \begin{array}{ccc}
-\sin \varphi & -\cos \varphi & 0 \\
\cos \varphi & -\sin \varphi & 0 \\
0 & 0 & 0
\end{array} \right).
\]

Noting that \( \| Q \| = 1 \) and \( \| J(\varphi) \| = 1 \), we obtain that
\[
\| J(\varphi) \| \leq \| s_2 \| + \left( c_1 + \frac{k(t)}{4} \right) \| s_1 \|. \]

Then, we have
\[
\frac{k(t)}{4} \| s_2 \|^4 \| s_1 \|^2 \left( c_1 + \frac{k(t)}{4} \right)^2
+ \frac{k(t)}{4} \| s_2 \|^2 \| s_1 \|^4 \left( c_1 + \frac{k(t)}{4} \right)^2 + \frac{2 \| M \|^2}{k(t)}. \quad (17)
\]

Lastly, by (7) and Assumption 1, we obtain that
\[
\left\{ \begin{array}{l}
\frac{k(t)}{4} \| s_2 \|^4 \left( c_1 + \frac{k(t)}{4} \right)^2 + \frac{\| M \|^2}{k(t)}, \\
- \frac{k(t)}{4} \| s_2 \|^2 \| s_1 \|^2 \left( c_1 + \frac{k(t)}{4} \right)^4 + \frac{\| M \|^2}{k(t)}
\end{array} \right\} \leq \frac{k(t)}{4} \| s_2 \|^4 \left( c_1 + \frac{k(t)}{4} \right)^2 + \frac{\| M \|^2}{k(t)}, \quad (18)
\]

Substituting (10), (13)-(18) into (12) directly leads to
\[
\dot{V}_2 \leq -\frac{c_1}{2} s_1^T s_1 + \frac{1}{2c_2} s_2^T s_2 + s_1^T \tau + \frac{k(t)}{4} \| s_2 \|^2
+ \frac{k(t)}{4} \| s_2 \|^4 (\| v \|^2 + 1) + \frac{k(t)}{64} \| s_2 \|^2 \| s_1 \|
+ \frac{k(t)}{4} \| s_2 \|^2 \| s_1 \|^2 (\| v \|^2 + 1) \left( c_1 + \frac{k(t)}{4} \right)^2
+ \frac{k(t)}{4} \| s_2 \|^4 \left( c_1 + \frac{k(t)}{4} \right)^2 (1 + \| s_1 \|^2)
+ \frac{k(t)}{4} \| s_2 \|^2 \left( c_1 + \frac{k(t)}{4} \right)^2 (1 + \| s_1 \|^2)
+ \frac{k(t)}{4} \| s_2 \|^2 \| s_1 \|^2 \left( c_1 + \frac{k(t)}{4} \right)^4 (1 + \| s_1 \|^2)
+ \frac{l_2}{k(t)}.
\]
where
\[ l = 2(l_1 + l_2)^2 + 2l_1^2 + l_1^4 + 5\|M\|^2 + l_2^2 \|M\|^2, \]
\[ \Lambda = s_1 + s_2\|s_2\|^2(\|v\|^2 + 1) + \frac{s_2\|s_1\|^4}{16} \]
\[ + s_2\|s_1\|^2(\|v\|^2 + 1) \left( c_1 + \frac{k(t)}{4} \right)^2 \]
\[ + s_2 \left( c_1 + \frac{k(t)}{4} \right)^2 \left( 1 + \|s_2\|^2\|s_1\|^2 + \|s_2\|^2 \right) \]
\[ + s_2\|s_1\|^2 \left( c_1 + \frac{k(t)}{4} \right)^4 \left( 1 + \|s_1\|^2 \right). \] (19)

Choosing the controller
\[ \tau = -\frac{c_2}{2}s_2 - \frac{k(t)}{4}\Lambda - \frac{1}{2c_1}s_2, \] (21)
with \( c_2 \) being an arbitrary positive constant, we obtain that
\[ V_2 \leq -\frac{c_1}{2}s_1^Ts_1 - \frac{c_2}{2s_2^TMs_2} + \frac{l}{k(t)}. \] (22)

Noting that \( M \) is positive definite, we obtain that \( \bar{\Lambda}_m s_1^Ts_1 \leq \bar{\Lambda}_m s_2^TM s_2 \leq \bar{\Lambda}_m \) with \( \bar{\Lambda}_m, \bar{\Lambda}_m \) being the minimum and maximum eigenvalues of \( M \), respectively. Then, there holds that
\[ V_2 \leq -\frac{c_1}{2}s_1^Ts_1 - \frac{c_2}{2s_2^TM_{\bar{\Lambda}_m}}s_2 + \frac{l}{k(t)}. \] (23)

where \( \mu = \min \{ c_1, \frac{c_2}{\bar{\Lambda}_m} \}. \)

**Remark 4:** The designed controller (21) with (7) and (8) shows that no measurement information about the time derivatives of the reference trajectory are needed but those of \([3,5,8,11,12,15,17-19,21,22,24]\) must be. In practice, for a case that a surface vessel tracks a moving target, the target position and the velocity of the moving target (i.e., the time derivative of the reference trajectory) are unknown in advance, but the target position can be obtained in real time by a measuring device. For such case, the controller designed in the paper is effective but those in the related literature are ineffective.

### 4. PERFORMANCE ANALYSIS

In this section, the performance of the resulting closed-loop system is analyzed. As preparation, a proposition is given first. Then, a theorem is given to summarize the main results.

**Proposition 1:** \( k(t) \) defined by (7) is bounded on \([0, +\infty)\).

**Proof:** This proposition is proved by contradiction. Suppose that \( k(t) \) is unbounded on \([0, +\infty)\). Then, for a certain positive constant \( T^* = \frac{16\mu}{\varepsilon^2} \), there exists \( t_1 \in [0, +\infty) \) such that \( |k(t_1)| > T^* \). Noting that \( k(t) \geq 0 \), we obtain the \( k(t) \) is increasing, and \( k(t) \geq k(t_1) > T^* = \frac{16\mu}{\varepsilon^2} \), \( t \in [t_1, +\infty) \). Thus, equality (23) gives that
\[ V_2 + \mu V_2 \leq \frac{\varepsilon^2\mu}{16}, t \in [t_1, +\infty). \]

Multiply both sides of above inequality by \( \varepsilon^2 \), and then integrating it over \([t_1, t] \) leads to
\[ V_2(t) \leq \varepsilon^2(1 - e^{\mu(t-t_1)}) \leq e^{\mu(t-t_1)} V_2(t_1) + \frac{\varepsilon^2}{16}. \] (24)

Since \( \lim_{t \to +\infty} e^{\mu(t-t_1)} V_2(t_1) = 0 \), there is a time \( t_2 \in [t_1, +\infty) \) such that \( e^{\mu(t-t_1)} V_2(t_1) < \frac{\varepsilon^2}{16}, t > t_2 \). Then, (24) gives that \( V_2(t) < \frac{\varepsilon^2}{16}, t \in [t_2, +\infty) \), and hence, the definition of \( V_2 \) leads to
\[ s_1^Ts_1 \leq 2V_2(t) < \frac{\varepsilon^2}{4}, t \in [t_2, +\infty), \]
which implies that \( \|s_1\| \leq \frac{\varepsilon}{2}, t \in [t_2, +\infty) \), and hence \( \hat{k}(t) = 0, t \in [t_2, +\infty) \). Then, \( k(t) \) remains constant, and hence is bounded on \([t_2, +\infty) \). Noting that \( k(t) \) is continuous and increasing with \( k(0) \geq 1 \), \( k(t) \) is bounded on \([0, t_2) \). Thus, \( k(t) \) is bounded on \([0, +\infty) \). This contradicts the induction hypothesis given above. \( \square \)

**Theorem 1:** For system (1) with Assumptions 1 and 2, the controller (21) guarantees the following performances of the resulting closed-loop system:

1) all signals of the closed-loop system are bounded on \([0, +\infty)\);

2) for any constant \( \varepsilon > 0 \), there exists a finite time \( T > 0 \), such that \( \|s_1\| = \|\eta - \sigma_d\| < \varepsilon, \forall t > T \).

**Proof:** 1) Noting that \( k(t) \geq k(0) = 1 \), (23) gives that
\[ \begin{cases} V_2(t) \leq -\mu V_2(t) + \frac{l}{k(t)} \leq -\mu V_2(t) + l, \\ V_2(t) \leq V_2(0) e^{-\mu t} + \frac{l}{\mu} \leq V_2(0) + \frac{l}{\mu}. \end{cases} \]

The second inequality of above inequalities implies that \( V_2(t) \) and hence \( s_1, s_2 \) are bounded on \([0, +\infty) \). Then, (8) gives that \( \alpha \) is bounded on \([0, +\infty) \). So are \( \eta, v \) by (3) and \( \tau \) by (21).

2) We prove this claim by showing that \( \lim_{t \to \infty} \hat{k}(t) = 0 \). Noting that \( \hat{s}_1 \) is bounded (shown by the first equation of
Then, by Barbalat’s lemma, we have \( \lim_{t \to \infty} \dot{k}(t) = 0 \). Thus, for any \( \varepsilon > 0 \), there exists a finite time \( T > 0 \), such that \( |k(t)| < \frac{\varepsilon}{2}, \forall t > T \). Therefore, (7) gives that \( ||s_1|| < \frac{\varepsilon}{2} < \frac{\varepsilon}{3} \), and hence \( ||s_1|| = |\eta - \eta_d| < \varepsilon \).

5. SIMULATIONS

In this section, simulation results are first presented by using the designed controller which validate the effectiveness of the theoretical results in the paper. Then, the comparisons between the simulation results by using designed controller and the traditional PID controller are given so as to show the advantages of the designed controller in the paper.

5.1. Simulations results

In order to validate the effectiveness of the theoretical results for system (1), CyberShip II [34], a 1:70 scale replica of a supply ship, is used for simulation. The parameters in the system matrices \( J(\phi), M, C(v), D(v) \) are chosen as Table 1 in [5,34] and the disturbances are assumed as

\[
\begin{align*}
    d_u &= 2 + 2\sin(0.2t) + 3\sin(0.1t), \\
    d_v &= -1 + 3\cos(0.01t) + 2\cos(0.3t), \\
    d_r &= \begin{cases} 0, & t < 5, \\ 10, & t \geq 5. \end{cases}
\end{align*}
\]

(25)

For system (1), we choose the following reference trajectory with \( x_d = 2\sin(0.03r), \ y_d = 0.9\cos(0.03r + \frac{\pi}{6}), \ \phi_d = \frac{\pi}{4}\sin(0.04r) \). Then, by using controller (21) with the following two sets of data

Table 1. Values of the parameters for system (1).

| Parameters | Value | Parameters | Value |
|------------|-------|------------|-------|
| \( m \)    | 23.8  | \( x_d \)  | 0.046 |
| \( I_c \)  | 1.76  | \( X_{dx} \) | -2    |
| \( Y_{dr} \)| -10   | \( Y_{dr} \) | 0     |
| \( N_{dr} \) | -1    | \( N_x \)  | -0.7225 |
| \( X_{uu} \) | -1.3274 | \( Y_r \) | -0.8612 |
| \( Y_{rr} \) | -36.2823 | \( Y_{rr} \) | 2     |
| \( Y_{er} \) | 0.1079 | \( Y_{er} \) | 1     |
| \( N_{e} \)    | 3     | \( N_{e} \)  | 0.1052 |
| \( N_{tr} \)  | 5.0437 | \( N_{tr} \) | 5     |
| \( N_{e} \)    | 4     | \( N_{e} \)  | 0.5   |
| \( N_{tr} \)   | 0.8   |             |       |

we obtain six simulation figures, i.e., Figs. 2-7. Specifically, Fig. 2 indicates that the tracking errors enter the \( \varepsilon \)-neighborhood of the origin after about 1 second, and then stay therein afterwards. Figs. 3 and 4 show that the system outputs \( x, y, \phi \) respectively enter and then stay at the \( \varepsilon \)-neighborhood of \( x_d, y_d \) and \( \phi_d \) after about 1 second. Then, the prescribed tracking accuracy is achieved. Figs. 5 and 6 indicate that the closed-loop system states and control inputs are bounded, respectively. Fig. 7 shows that the time-varying controller gain \( k \) is always bounded and converges to a specific constant ultimately.
5.2. Comparison with PID control

In fact, trajectory tracking of the vessel system can also be ensured by PID control method. For system (1), we choose the following PID controller

$$\tau = c_p s_1 + c_i \int_0^t s_1(t)dt + c_d s_1,$$

(26)

with $c_p, c_i, c_d$ being controller parameters to be chosen later, $s_1$ being defined by (3) and $s_1$ being given by (4). As well known, the control performance of the PID controller severely depends on the choosing of the three controller parameters (i.e., $c_p, c_i, c_d$). A set of controller parameters which are arbitrary but not carefully chosen may not guarantee the desirable control performance. For example, for system with the disturbances and reference trajectory being chosen as above, the system parameters being chosen as Table 1, the initial conditions and the tracking accuracy parameters being respectively chosen as data 1) and 2), Fig. 8 shows that a set of PID controller parameters, i.e., $c_p, c_i, c_d$. A set of controller parameters which are arbitrary but not carefully chosen may not guarantee the desirable tracking objective. For detail, the tracking errors do not enter and then stay at the given neighborhoods of the origin.

It is worth pointing out that, the control performance of the proposed theoretical results in the paper does not depend on the choosing of the controller parameters (i.e., $c_1$ and $c_2$). This implies that any choosing of controller parameters can guarantee the desirable control performance, just as observed by Figs. 2-7.

6. CONCLUSIONS

In this paper, tracking control has been solved for a class of uncertain surface vessels. Different from the related literature where severe assumptions are imposed on system uncertainties and the reference trajectory, this paper allows more serious uncertainties and more broader class of reference trajectories since all the system parameters are unknown and the disturbance is not necessarily to be smooth, and moreover, the reference trajectory
is not necessarily to be twice continuously differentiable or its time derivatives are not available for feedback. To solve the control problem, a novel control strategy based on backstepping scheme and adaptive compensation technique is proposed. Then, an adaptive state feedback controller is designed which guarantees the boundedness of all the states of the resulting closed-loop system and the system output practically tracks the given reference trajectory, that is, the tracking error arrives at and then stays within arbitrary neighborhood of the origin.

The future research directions are twofold. First, tracking control via output feedback. Noting that the controller designed in the paper requires all the states of the system to be available for feedback, which is somewhat restrictive in application. Then, how to design a tracking controller for the surface vessels with less measurement (for example, only the heading angle $\phi$ is measured) will be interesting. For this, an appropriate state observer should be designed to reconstruct the in-measurable states and hence challenge the control problem. Second, tracking control joint with obstacle-avoidance (as considered in [35]). In practice, a surface vessel should track a moving target under the premise of obstacle-avoidance for the pursuit of security. Then, such control problem is of great significance in engineering. For this, a novel obstacle-avoidance mechanism should be developed, combining which with the schemes for the trajectory tracking under weaker assumption to establish a novel control framework.

**APPENDIX A: PROOF OF PROPERTY 2**

By (2), matrices $C(v)$ and $D(v)$ are represented as follows:

\[
C(v) = A_1u + A_2v + A_3r,
\]

\[
D(v) = B_1 + B_2|u| + B_3|v| + B_4|r|,
\]

where

\[
A_1 = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & m - X_{du} \\
0 & -(m - X_{du}) & 0
\end{pmatrix},
\]

\[
A_2 = \begin{pmatrix}
0 & 0 & -(m - Y_{dv}) \\
0 & 0 & 0 \\
m - Y_{dv} & 0 & 0
\end{pmatrix},
\]

\[
A_3 = \begin{pmatrix}
0 & 0 & -(mX_\nu - Y_{dr}) \\
0 & 0 & 0 \\
mX_\nu - Y_{dr} & 0 & 0
\end{pmatrix},
\]

\[
B_1 = \begin{pmatrix}
-X_a & 0 & 0 \\
0 & -Y_r & -Y_r \\
0 & -N_r & -N_r
\end{pmatrix},
\]

\[
B_2 = \begin{pmatrix}
-X_{au} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},
\]

\[
B_3 = \begin{pmatrix}
0 & 0 & 0 \\
0 & -Y_{rv} - Y_{rr} & 0 \\
0 & -N_{rv} - N_{rr}
\end{pmatrix},
\]

\[
B_4 = \begin{pmatrix}
0 & -Y_{rv} - Y_{rr} \\
0 & -N_{rv} - N_{rr}
\end{pmatrix}.
\]

Then, there exist positive constants $l_1, l_2$ such that

\[
\|C(v)\| \leq \|A_1\| |u| + \|A_2\| |v| + \|A_3\| |r| = l_1 \|v\|,
\]

\[
\|D(v)\| \leq \|B_1\| + \|B_2\| |u| + \|B_3\| |v| + \|B_4\| |r| = l_2(1 + \|v\|),
\]

with

\[
l_1 = \max \left\{ \sqrt{3}|m - X_{du}|, \sqrt{3}|m - Y_{dv}|, \sqrt{3}|mX_\nu - Y_{dr}| \right\},
\]

\[
l_2 = \max \left\{ |X_a|, \sqrt{Y_r^2 + N_r^2 + Y_r^2 + N_r^2}, \sqrt{3}|X_{au}|, \sqrt{Y_r^2 + N_r^2 + Y_r^2 + N_r^2}, \sqrt{3}\left| \frac{X_{au}}{X_u} \right| \right\}.
\]

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