Heavy-quark state production in A-A collisions at \( \sqrt{s_{pp}} = 200 \text{ GeV} \)

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Abstract

We estimate differential rapidity cross sections for \( J/\Psi \) and \( \Upsilon(1S) \) production via Cu-Cu and Au-Au collisions at RHIC, and the relative probabilities of \( \Psi'(2S) \) to \( J/\Psi \) production via p-p collisions using our recent theory of mixed heavy quark hybrids, in which the \( \Psi'(2S) \) mesons have approximately equal normal \( q\bar{q} \) and hybrid \( q\bar{q}g \) components. We also estimate the relative probabilities of \( \Psi'(2S) \) to \( J/\Psi \) production via Cu-Cu and Au-Au collisions, which will be measured in future RHIC experiments. We also review production ratios of \( \Upsilon(2S) \) and \( \Upsilon(3S) \) to \( \Upsilon(1S) \) in comparison to recent experimental results. This is an extension of our recent work on p-p collisions for possible tests of the production of Quark-Gluon Plasma via A-A collisions at BNL-RHIC.

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1 Introduction

In our recent work on the production of heavy quark states in p-p collisions[1] we used the color octet model[2, 3, 4], which was shown to dominate the color singlet model in studies of \( J/\Psi \) production at \( E=200 \text{ GeV} \)[5, 6]. Among other results we found that our mixed heavy quark hybrid theory[7] correctly predicted the experimental result for the ratio of \( \Upsilon(3S) \) to \( \Upsilon(1S) \) cross sections[8], while the standard model of \( |b\bar{b}| \) for the \( \Upsilon(3S) \) state was about a factor of three too small. Also the recent measurement of the ratio \( \frac{\sigma(\Upsilon(2S)) + \sigma(\Upsilon(3S))}{\sigma(\Upsilon(1S))} \) via p-p collisions by LHC-CMS[9] was in agreement with the mixed hybrid theory for the \( \Upsilon(3S) \), while the standard model was more than a factor of two smaller. This will be discussed in detail below.

In our present work we study heavy quark state production at RHIC, with \( E=\sqrt{s_{pp}} = 200 \text{ GeV} \) for \( A-A \) (\( A = N+Z \)) collisions. Since the present BNL-RHIC cannot measure the \( \Upsilon(1S), \Upsilon(2S), \Upsilon(3S) \) separately, we shall mainly study the \( J/\Psi(1S) \) and \( \Psi'(2S) \) production. In section 2 we briefly review the mixed hybrid theory for charmonium and bottomonium.
states, the color octet model, and the relation between the standard and mixed hybrid theories of heavy quark meson states needed for our present work. In section 3 we discuss the production of \( J/\Psi \) and \( \Upsilon(1S) \) states in Cu-Cu and Au-Au collisions, with results based on the research of many preceding theorists and experimentalists. Note that due to uncertainty in normalization of the absolute cross sections one main prediction is the shapes of the rapidity dependence rather than the magnitudes of the cross sections. The other main prediction is the ratios of cross sections. In section 4 we discuss the ratio of \( \Psi'(2S) \) to \( J/\Psi(1S) \) production and compare the hybrid vs standard theory to recent experiments with p-p collisions; and predict this ratio for future A-A collision RHIC experiments using recent experimental ratios of \( Pb - Pb \) to \( p - p \) \( \Upsilon(mS) \) production.

2 Review of mixed hybrid heavy quark mesons and the color octet model

We give a very brief review of the hybrid heavy quark and color octet models, and their relationship for the present work. See Ref[1] for details.

Using the method of QCD sum rules it was shown[7] that the \( \Psi'(2S) \) and \( \Upsilon(3S) \) are approximately 50-50 mixtures of standard quarkonium and hybrid quarkonium states:

\[
\begin{align*}
|\Psi'(2S)\rangle &= -0.7|c\bar{c}(2S)\rangle + \sqrt{1 - 0.5}|c\bar{c}g(2S)\rangle \\
|\Upsilon(3S)\rangle &= -0.7|b\bar{b}(3S)\rangle + \sqrt{1 - 0.5}|b\bar{b}g(3S)\rangle,
\end{align*}
\]

with a 10% uncertainty in the QCD sum rule estimate of the mixing probability, while the \( J/\Psi, \Upsilon(1S), \Upsilon(2S) \) states are essentially standard \( q\bar{q} \) states. This solves many puzzles[7, 1].

The cross sections for charmonium and bottomonium production in the color octet model are based on the cross sections obtained from the matrix elements for quark-antiquark and gluon-gluon octet fusion to a hadron \( H \), illustrated in Fig. 1

![Figure 1: Gluon and quark-antiquark color octet fusion producing hadron H](image)

With \( q\bar{q} \) models pp cross section ratios are[1] \( \sigma(2S)/\sigma(1S) \approx 0.039, \sigma(3S)/\sigma(1S) \approx 0.0064 \). On the other hand, for gluonic interactions with quarks there is an enhancement factor of \( \pi^2 \), for purely hybrid states, as illustrated in Fig. 2. For states that are approximately 50% hybrid, this gives an enhancement factor of \( \pi^2/4 \), with a 10% uncertainty, which accounts for the enhanced cross section ratios discussed above, in Ref[1], and below.
3 \quad J/Ψ and Υ(1S) production in Cu-Cu and Au-Au collisions with $\sqrt{s_{pp}} = 200$ GeV

The differential rapidity cross section for the production of a heavy quark state with helicity $\lambda = 0$ in the color octet model in A-A collisions is given by

$$\frac{d\sigma_{AA \rightarrow \Phi(\lambda=0)}}{dy} = R_{AA}^{E} N_{bin}^{AA} \frac{d\sigma_{pp \rightarrow \Phi(\lambda=0)}}{dy},$$

(2)

where $R_{AA}^{E}$ is the product of the nuclear modification factor $R_{AA}$ and $S_{\Phi}$, the dissociation factor after the state $\Phi$ (a charmonium or bottomonium state) is formed (see Ref[10]). $N_{bin}^{AA}$ is the number of binary collisions in the AA collision, and $< \frac{d\sigma_{pp \rightarrow \Phi(\lambda=0)}}{dy} >$ is the differential rapidity cross section for $\Phi$ production via nucleon-nucleon collisions in the nuclear medium. Note that $R_{AA}^{E}$, which we take as a constant, can be functions of rapidity. See Refs[11, 12] for a review and references to many publications.

Experimental studies show that for $\sqrt{s_{pp}} = 200$ GeV $R_{AA}^{E} \simeq 0.5$ both for Cu-Cu[13, 14] and Au-Au[15, 16, 17]. The number of binary collisions are $N_{bin}^{AA} = 51.5$ for Cu-Cu[18] and 258 for Au-Au. The differential rapidity cross section for pp collisions in terms of $f_{g}[19, 1]$, the gluon distribution function ($-0.8 \leq y \leq 0.8$ for $\sqrt{s_{pp}} = 200$ GeV with $f_{g}$ from Ref[1]), is

$$< \frac{d\sigma_{pp \rightarrow \Phi(\lambda=0)}}{dy} > = A_{\Phi} \frac{1}{x(y)} f_{g}(\bar{x}(y), 2m) f_{g}(a/\bar{x}(y), 2m) \frac{dx}{dy},$$

(3)

where $a = 4m^2/s$; with $m = 1.5$ GeV for charmonium, and 5 GeV for bottomonium, and $A_{\Phi} = \frac{5\pi^{2}a^{2}}{288m^{2}s} < O_{g}^{\Phi}(1S_{0}) > [1]$. For $\sqrt{s_{pp}} = 200$ GeV $A_{\Phi} = 7.9 \times 10^{-4}$nb for $\Phi=J/Ψ$ and $2.13 \times 10^{-3}$nb for $Σ(1S)$; $a = 2.25 \times 10^{-4}$ for Charmonium and $2.5 \times 10^{-3}$ for Bottomium.

The function $\bar{x}$, the effective parton x in a nucleus (A), is given in Refs[20, 21]:

$$\bar{x}(y) = x(y)(1 + \frac{\xi_{g}^{2}(A^{1/3} - 1)}{Q^{2}})$$

$$x(y) = 0.5 \left[ \frac{m}{\sqrt{s_{pp}}}(\exp y - \exp (-y)) + \sqrt{\frac{m}{\sqrt{s_{pp}}}(\exp y - \exp (-y))^{2} + 4a} \right],$$

(4)

with[22] $\xi_{g}^{2} = .12GeV^{2}$. For $J/Ψ$ $Q^{2} = 10GeV^{2}$, so $\bar{x} = 1.058x$ for Au and $\bar{x} = 1.036x$ for Cu, while for $Σ(1S)$ $Q^{2} = 100GeV^{2}$, so $\bar{x} = 1.006x$ for Au and $\bar{x} = 1.004x$ for Cu.
From this we find the differential rapidity cross sections as shown in the following figures for $J/\Psi$, $\Psi(2S)$ and $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$ production via Cu-Cu and Au-Au collisions at RHIC (E=200 GeV), with $\Psi(2S)$, $\Upsilon(3S)$ enhanced by $\pi^2/4$ as discussed above. The absolute magnitudes are uncertain, and the shapes and relative magnitudes are our main prediction.

Figure 3: $d\sigma/dy$ for $2m=3$ GeV, E=200 GeV Cu-Cu collisions producing $J/\Psi$ with $\lambda = 0$

Figure 4: $d\sigma/dy$ for $2m=3$ GeV, E=200 GeV Au-Au collisions producing $J/\Psi$ with $\lambda = 0$
Figure 5: $d\sigma/dy$ for $2m=3$ GeV, $E=200$ GeV Cu-Cu collisions producing $\Psi(2S)$ with $\lambda = 0$. The dashed curve is for the standard $c\bar{c}$ model.

Figure 6: $d\sigma/dy$ for $2m=3$ GeV, $E=200$ GeV Au-Au collisions producing $\Psi(2S)$ with $\lambda = 0$. The dashed curve is for the standard $c\bar{c}$ model.
Figure 7: $d\sigma/dy$ for $2m=10$ GeV, $E=200$ GeV Cu-Cu collisions producing $\Upsilon(1S)$ with $\lambda = 0$

Figure 8: $d\sigma/dy$ for $2m=10$ GeV, $E=200$ GeV Au-Au collisions producing $\Upsilon(1S)$ with $\lambda = 0$
Figure 9: $d\sigma/dy$ for $2m=10$ GeV, $E=200$ GeV Cu-Cu collisions producing $\Upsilon(2S)$, $\Upsilon(3S)$ with $\lambda = 0$. For $\Upsilon(3S)$ the dashed curve is for the standard $b\bar{b}$ model.

Figure 10: $d\sigma/dy$ for $2m=10$ GeV, $E=200$ GeV Au-Au collisions producing $\Upsilon(2S)$, $\Upsilon(3S)$ with $\lambda = 0$. For $\Upsilon(3S)$ the dashed curve is for the standard $b\bar{b}$ model.
4 Ratio of $\Psi'(2S)$ to $J/\Psi$ cross sections

In this section we discuss the ratios of the charmonium cross sections for p-p and A-A collisions at RHIC. In order to estimate the $\Psi'(2S)$ to $J/\Psi$ ratios in A-A collisions we make use of recent experimental results on $\Upsilon(mS)$ state production at the LHC.

4.1 Ratios for p-p collisions

In Ref[1] we discussed the $\Upsilon(mS)$ cross section ratios, showing that the error in the ratios is small as it is given by the wave functions for the standard model and the enhancement factor of $(1 \pm .1) \times \pi^2/4$ for the mixed hybrid, as discussed in Section 2. Now there are accurate measurements of the $\Psi'(2S)$ to $J/\Psi$ ratio for p-p production at RHIC. From the standard (st), hybrid model(hy) one finds for p-p production of $\Psi'(2S)$ and $J/\Psi$

$$\frac{\sigma(\Psi'(2S))}{\sigma(J/\Psi(1S))}|_{st} \simeq 0.27$$
$$\frac{\sigma(\Psi'(2S))}{\sigma(J/\Psi(1S))}|_{hy} \simeq 0.67 \pm 0.07,$$ (5)

while the PHENIX experimental result for the ratio[23] $\simeq 0.59$. Therefore, as in our earlier work the hybrid model is consistent with experiment, while the standard model ratio is too small.

4.2 Ratios for Pb-Pb collisions

The recent CMS/LHC result comparing Pb-Pb to p-p Upsilon production[24] found

$$\frac{[\Upsilon(2S) + \Upsilon(3S)]_{Pb-Pb}}{[\Upsilon(2S) + \Upsilon(3S)]_{Pb-Pb}} \simeq 0.31^ {+19}_{-15} \pm .013(syst),$$ (6)

while in our previous work on $p-p$ collisions we found the ratio $\sigma(\Upsilon(3S))/\sigma(\Upsilon(1S))|_{p-p}$ of the standard $|\bar{b}b>$ model was $4/\pi^2 \simeq 0.4$ of the hybrid model. This suggests a suppression factor for $\sigma(\bar{b}b(3S))/\sigma(\bar{b}b(1S))$, or $\sigma(c\bar{c}(2S))/\sigma(c\bar{c}(1S))$ of 0.31/4 as these components travel through the QGP; or an additional factor of 0.78 for $\Psi'(2S)$ to $J/\Psi$ production for $A-A$ vs $p-p$ collisions. Therefore from Eq(5) one obtains our estimate using our mixed hybrid theory for this ratio

$$\frac{\sigma(\Psi'(2S))}{\sigma(J/\Psi(1S))}|_{A-A} \simeq 0.52 \pm 0.05$$ (7)

5 Ratio of $\Upsilon(2S)$ and $\Upsilon(3S)$ to $\Upsilon(1S)$ cross sections

In our previous work[1] we estimated the ratios of $\Upsilon(2S)$ and $\Upsilon(3S)$ to $\Upsilon(1S)$ cross sections in comparison with an experiment published in 1991[8]. Our result for p-p collisions, with uncertainty due to separating $\Upsilon(2S)$ from $\Upsilon(3S)$, was

$$\frac{\Upsilon(3S)}{\Upsilon(1S)}|_{p-p} \simeq 0.14 - 0.22,$$ (8)
for our mixed hybrid theory, while the standard model would give $\frac{\Upsilon(3S)}{\Upsilon(1S)} \simeq 0.06$. A recent CMS result[25], with a correction factor for acceptance and efficiency of the $\Upsilon(3S)$ to the $\Upsilon(1S)$ state, which was estimated to be approximately 0.29, was found to be

$$\Upsilon(3S)/\Upsilon(1S)|_{p-p} \simeq 0.12,$$

with the mixed hybrid theory in agreement within errors, while the standard model differs by a factor of two.

The new CMS experiment’s main objective[25] is to test for $\Upsilon$ suppression in PbPb collisions, with estimates of the following quantities:

$$\frac{[\Upsilon(2S)/\Upsilon(1S)]_{PbPb}}{[\Upsilon(2S)/\Upsilon(1S)]_{pp}}$$

$$\frac{[\Upsilon(3S)/\Upsilon(1S)]_{PbPb}}{[\Upsilon(3S)/\Upsilon(1S)]_{pp}}.$$

(10)

The studies of AA collisions for Bottomonium states, which cannot be carried out at RHIC but are an important part of the LHC CMS program, will be carried out in our future research.

6 Conclusions

We have studied the differential rapidity cross sections for $J/\Psi$, $\Psi(2S)$ and $\Upsilon(nS)$ ($n = 1, 2, 3$) production via Cu-Cu and Au-Au collisions at RHIC (E=200 GeV) using $R_{AA}$, the product of the nuclear modification factor $R_{AA}$ and the dissociation factor $S_{\Phi}$, $N_{bin}$ the binary collision number, and the gluon distribution functions from previous publications. This should give some guidance for future RHIC experiments, although at the present time the $\Upsilon(nS)$ states cannot be resolved.

The ratio of the production of $\sigma(\Psi'(2S))$, which in our mixed hybrid theory is 50% $c\bar{c}(2S)$ and 50% $c\bar{c}g(2S)$ with a 10% uncertainty, to $J/\Psi(1S)$, which is the standard $c\bar{c}(1S)$, will be an important test of the production of the quark-gluon plasma. Using the hybrid model and suppression factors from previous theoretical estimates and experiments on $\Upsilon(mS)$ state production at the LHC, we estimate that the ratio of $\Psi'(2S)$ to $J/\Psi(1S)$ production at RHIC via A-A collisions will be about $0.52 \pm 0.05$.

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