Experimental verification of trade-off relation for coherence and disturbance

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Keywords: trade-off relation, coherence, disturbance

Abstract
When a quantum system is sent through a noisy channel, it is usually disturbed. At the same time, the system undergoes decoherence and tends to lose some delicate quantum features. For a particular basis, the coherence of the state changes. Noisy channels lead to both disturbance and decoherence, it is natural to ask about the relation between disturbance and decoherence. Recently, a trade-off relation for coherence and disturbance has been presented by using the relative entropy of coherence for measurement channels as well as for general channels (completely positive trace-preserving maps) in reference [20]. In this paper, with entangled photon pairs and linear optics, we experimentally verify this trade-off relation for a single-qubit system undergoing various noisy channels. Our experimental results agree with the theoretical predictions and provide a quantitative understanding of the relation between quantum channels and resources.

1. Introduction
Quantum features hold the promise of revolutionizing information processing in ways that surpass current approaches, including quantum computations and quantum communications. However, quantum resources are very fragile. When a quantum system is sent through a noisy channel, its state is usually disturbed [1–12]. At the same time, the system undergoes decoherence [13–19] and tends to lose some delicate quantum features. For a particular basis, the coherence of the state changes. Noisy channels lead to both disturbance and decoherence. Then, it is natural to ask about the relation between disturbance and decoherence. Recently, a coherence disturbance trade-off relation has been presented by using the relative entropy of coherence for measurement channels as well as for general channels (completely positive trace-preserving maps) in reference [20].

Coherence is a property of the physical system in the quantum world that can be used to drive various nonclassical phenomena. Coherence originates from the superposition principle of quantum mechanics and reflects one of the fundamental essences in quantum phenomena [21–25]. The resource theory of coherence has been developed [26, 27], in which coherence is viewed as one kind of fundamental quantum resources. Especially, coherence exhibits quantum features and quantum advantages without the requirement of two- or multi-partite systems. All these motivate us to study the relation between coherence and disturbance of the single quantum system and to explore the idea that the initial coherence should respect a trade-off relation with the disturbance caused to the system state, whenever some information is extracted from the system or a measurement is performed on a quantum system.

Further, we compare the coherence-disturbance trade-off relation and the information-disturbance trade-off relation. In quantum theory, any measurement that provides information about a physical system also inevitably disturbs the system’s state, which drives the information-disturbance trade-off relation [2, 3, 10, 28–32]. Whereas, being sent through a noisy channel, though no information is extracted, the state of the system is disturbed and its coherence is changed for a particular given basis. These two kinds of
trade-off relations originate from quantum measurements and quantum noisy channels, respectively. As the trade-off between information and disturbance is of great interest in establishing the foundations of quantum mechanics [33, 34], the trade-off between coherence and disturbance is also expected to play an important role in quantum information processing.

In this paper, we report the experimental verification of this trade-off relation between the quantum coherence and disturbance for a single-qubit state with the qubit undergoing various noisy quantum channels. Our experimental results agree with the theoretical predictions in [20] and provide a quantitative understanding of the relation between quantum resources and quantum channels.

2. Experimental verification of trade-off relation for coherence and disturbance

A trade-off relation between the coherence and the disturbance is proven by Sharma and Pati [20]

\[ 2 \mathcal{C}(\rho) + D(\rho, \epsilon) \leq 2 \log_2 d. \]  

(1)

Here \( \mathcal{C}(\rho) \equiv S(\rho^\rho) - S(\rho) \) is a measure of coherence proposed by Baumgratz et al [22], \( S(\rho) = -\text{Tr}[\rho \log_2 \rho] \) is the von Neumann entropy, and \( \rho^\rho \) denotes the state which is obtained by deleting the off-diagonal elements of the state \( \rho \) in the computational basis \( \{|i\}\} \). Whereas, \( D(\rho, \epsilon) \equiv S(\rho) - I_{\text{coh}}(\rho, \epsilon) \) [35] is a measure of disturbance, which causes difference between the initial and final states of the system being measured. Here \( I_{\text{coh}} \equiv S(\epsilon(\rho)) - S(\epsilon \otimes I) \mathcal{P}(\mathcal{P}) \) is the coherent information [36, 37] of the system passing through a noisy channel \( \epsilon \). The pure state \( |\Psi\rangle \) is obtained by purification of the state \( \rho \). The map \( \epsilon \otimes I \) acts on the pure state \( |\Psi\rangle \) with \( \epsilon \) acting on the Hilbert space of the system and \( I \) on the Hilbert space of an ancilla. For a \( d \)-dimensional state \( \rho \), \( 0 \leq \mathcal{C}(\rho) \leq \log_2 d \) and \( 0 \leq D(\rho, \epsilon) \leq 2 \log_2 d \) are satisfied. This trade-off relation in (1) holds for all quantum channels.

We experimentally verify the coherence-disturbance trade-off relation for a single-qubit system undergoing various single-qubit noisy channels with entangled photons and linear optics. Let us consider a two-dimensional system with a density matrix \( \rho \). The system share a pure bipartite state \( |\Psi\rangle_{12} \) with an ancilla qubit and then the state of the system is \( \rho = \text{Tr}_2(|\Psi\rangle_{12} \langle \Psi|) \). In our experiment, the pure bipartite state is prepared in a computational basis \( \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\} \)

\begin{align*}
|\Psi\rangle_{12} = & \begin{pmatrix}
\sqrt{\lambda_0} \cos 2\theta \cos 2\varphi + \sqrt{\lambda_1} \sin 2\theta \sin 2\varphi \\
\sqrt{\lambda_0} \cos 2\theta \sin 2\varphi - \sqrt{\lambda_1} \sin 2\theta \cos 2\varphi \\
\sqrt{\lambda_0} \sin 2\theta \cos 2\varphi - \sqrt{\lambda_1} \cos 2\theta \sin 2\varphi \\
\sqrt{\lambda_0} \sin 2\theta \sin 2\varphi + \sqrt{\lambda_1} \cos 2\theta \cos 2\varphi
\end{pmatrix},
\end{align*}

(2)

where \( \lambda_{0(1)} \in \mathbb{R} \) and \( \lambda_0 + \lambda_1 = 1 \). With various parameters \( \lambda_{0(1)}, \theta \) and \( \varphi \), we have a family of initial states.

In our experiment, qubits are encoded in the polarizations of single photons \( \{|H\rangle \equiv |0\rangle, |V\rangle \equiv |1\rangle\} \).

Our experimental setup illustrated in figure 1, consists of an entangled photon source, entangled state preparation, single-qubit noisy channel and polarization measurement. We use two adjacent nonlinear crystals (\( \beta \)-barium borate, BBO) which are pumped by a 405 nm laser diode to produce spontaneous parametric down-conversion (SPDC) followed by three half-wave plates (HWP) with setting angles \( \theta_0, \varphi_0 \) and \( \varphi_1 \), and then we can produce the polarizations of photon pairs in the family of entangled states in (2) with \( \lambda_0 = \sin^2 2\theta_0 \) [38–42]. We choose the parameters for the initial states as \( \theta = \varphi = \frac{\pi}{4}, \lambda_0 = 0.50, 0.89, 1 \), \( \theta = \frac{\pi}{12}, \varphi = \frac{\pi}{3}, \lambda_0 = 0.02, 0.08, 0.23 \) and \( \theta = \frac{\pi}{3}, \varphi = \frac{\pi}{3}, \lambda_0 = 0.01, 0.07, 0.18 \). For the initial state of the system and ancilla, we reconstruct the density matrices with quantum state tomography. For a two-qubit state, making 16 measurements of the polarization correlations in various bases

\( \{|HH\rangle, |HV\rangle, |HR\rangle, |HD\rangle, |VH\rangle, |VV\rangle, |VR\rangle, |VD\rangle, |RH\rangle, |RV\rangle, |RR\rangle, |RD\rangle, |DH\rangle, |DV\rangle, |DR\rangle, |DD\rangle\} \)

allows tomographic reconstruction of the density matrices of the two-qubit states [43–48], where \( |R\rangle = (|H\rangle - i|V\rangle)/\sqrt{2} \) and \( |D\rangle = (|H\rangle + |V\rangle)/\sqrt{2} \). With the reconstructed density matrices of the initial states of the system and ancilla, we can obtain the density matrices of the state of the system \( \rho \) by tracing out the ancilla qubit and then calculate \( S(\rho) \) and \( \mathcal{C}(\rho) \).

After the initial state preparation, one photon of the entangle pair serves as a qubit state of the system and passes through various single-qubit noisy channels. The other servers as an ancilla and does not pass the noisy channel. We then verify the trade-off between quantum coherence and disturbance for three different noisy channels, including weak measurement channel, depolarizing channel, and amplitude damping channel.
In the polarization basis which is used in our experiment, these noisy channels are realized by the dual interferometer setup implemented by splitting the two polarization controls inside a Sagnac interferometer [43, 49, 50]. The Sagnac interferometer consists of a polarizing beam splitter (PBS) and two HWPs with the setting angles $\alpha$ and $\beta$, respectively. The PBS splits photons with two polarization components by directly transmitting the horizontally polarized photons and reflecting the vertically polarized photons. The following HWP with the setting angle $\alpha$ rotates the polarizations of photons (in $|H\rangle$) transmitted by the PBS to a superposition of horizontal and vertical polarizations. Then the horizontally (vertically) polarized photons are transmitted (reflected) by the PBS again and then enter the spatial modes $A$ ($B$). Similarly, another HWP with the setting angle $\beta$ rotates the polarizations of photons (in $|V\rangle$) reflected by the PBS to a superposition of horizontal and vertical polarizations. After that the horizontally (vertically) polarized photons are transmitted (reflected) by the PBS again and then enter the spatial modes $B$ ($A$).

To realize noisy channels, different module (a HWP at 45° for a weak measurement channel, a quartz for a depolarizing channel, and null for an amplitude damping channel) is inserted in the mode BS. The Sagnac interferometer acts as a continuously variable BS in which the spatial modes ratio $A$ can be linearly varied by the setting angles $\alpha$ and $\beta$ of the two HWPs.

2.1. Weak measurement channel

First, we consider a single-qubit weak measurement channel. The Kraus operators for the noisy channel [51, 52] are given by

$$K(x) = \sqrt{\frac{1-x}{2}} \Pi_0 + \sqrt{\frac{1+x}{2}} \Pi_1, \quad K(-x) = \sqrt{\frac{1+x}{2}} \Pi_0 + \sqrt{\frac{1-x}{2}} \Pi_1,$$

where $\Pi_0 = |0\rangle\langle 0|$ and $\Pi_1 = |1\rangle\langle 1|$ are projectors in the computational basis and $x \in [0, 1]$ denotes the measurement strength. The weak measurement Kraus operators satisfy $K(x)K(x) + K(-x)K(-x) = I$.

Under the weak measurement channel, a single-qubit state $\rho$ changes as

$$\rho \rightarrow \varepsilon(\rho) = K(-x)\rho K(-x) + K(x)\rho K(x) = \begin{pmatrix} \rho_{00} & \sqrt{(1-x^2)}\rho_{01} \\ \sqrt{(1-x^2)}\rho_{10} & \rho_{11} \end{pmatrix},$$

where $\rho_{ij}$ is the element of the density matrix of $\rho$. In our experiment, a module for realizing a weak measurement channel is a HWP at 45° placed in the spatial mode $B$ of the Sagnac interferometer. As
horizontal polarization (vertical polarization) is transmitted (reflected) partially, a single-photon polarization qubit undergoes a weak measurement channel. The strength of the weak measurement is tighter than that in (1). Note that in our experiment, the solid line representing $C$ bars represented by three red triangles are on the solid line.

Next, we consider a single-qubit depolarizing channel. The Kraus operators are given by

$$K_1 = \sqrt{1 - \frac{3p}{4}} I, \quad K_2 = \sqrt{\frac{p}{4}} \sigma_x, \quad K_3 = \sqrt{\frac{p}{4}} \sigma_y, \quad K_4 = \sqrt{\frac{p}{4}} \sigma_z,$$

where $d_E$ is the dimension of the Hilbert space of the environment, and $C(\rho)$ and $D(\rho, \varepsilon)$ are basis dependent. If the dimension of the system is as same as that of the environment, the trade-off relation in (5) is tighter than that in (1). Note that in our experiment, $d_E = d = 2$.

As illustrated in figure 2, the trade-off relation in (5) has a tighter bound compared to that in (1), i.e., the solid line representing $C(\rho) + D(\rho, \varepsilon) = 1$ is below the dashed line $2C(\rho) + D(\rho, \varepsilon) = 2$, and it also holds for all the states we choose. With the parameters of the initial states $\{\theta = \varphi = \frac{\pi}{2}, \lambda_0 = 0.50, 0.89, 1\}$ and the parameter of the weak measurement channel $x = 1$, the trade-off relation in (5) is saturated as predicted theoretically and is confirmed by our experimental results, i.e., the experimental data with error bars represented by three red triangles are on the solid line.

2.2. Depolarizing channel

Next, we consider a single-qubit depolarizing channel. The Kraus operators are given by

$$K_1 = \sqrt{1 - \frac{3p}{4}} I, \quad K_2 = \sqrt{\frac{p}{4}} \sigma_x, \quad K_3 = \sqrt{\frac{p}{4}} \sigma_y, \quad K_4 = \sqrt{\frac{p}{4}} \sigma_z,$$
Finally, we consider a single-qubit amplitude damping channel. The Kraus operators are given by

\[ \rho \rightarrow \varepsilon(\rho) = \sum_{i=1,2} K_i \rho K_i^\dagger = \begin{pmatrix} 1 - \frac{p}{2} & \frac{p}{2} \rho_{10} + \frac{p}{2} \rho_{01} \\ \frac{p}{2} \rho_{01} & (1 - p) \rho_{11} + \frac{p}{2} \rho_{00} \end{pmatrix}. \]  

(7)

In our experiment, a module for realizing a single-qubit depolarizing channel is a quartz crystal. The single-qubit state found at the spatial modes A and B are identical to the input state \( \rho \) but with the probability amplitudes \( 1 - p \) and \( p \), respectively. The angles of the HWPs are set as \( \alpha = \frac{1}{2} \arccos(\sqrt{1 - p}) \) and \( \beta = \frac{1}{2} \arccos(\sqrt{1 - p}) \). The single-qubit state found at the two outputs of the BS is described precisely as \( \frac{1}{2}(1 - p) \rho ) \), indicating that the input state has gone through the depolarizing quantum operation \( \varepsilon(\rho) \). Finally, we perform quantum state tomography to reconstruct the final states and calculate the left- and right-hand sides of the trade-off relation in (1).

The experimental data are shown in figure 3. It is clear that the trade-off relation (1) between the disturbance and coherence holds for all the states. Note that with the parameters of the initial states \( \{ \theta = \varphi = \frac{\pi}{4}, \lambda_0 = 0.50, 0.89, 1 \} \) and the parameter of the depolarizing channel \( p = 1 \), the trade-off relation in (1) is saturated as predicted theoretically and is confirmed by our experimental results, i.e., the experimental data with error bars represented by three red triangles are on the dash line indicating \( 2C(\rho) + D(\rho, \varepsilon) = 2 \).

### 2.3. Amplitude damping channel

Finally, we consider a single-qubit amplitude damping channel. The Kraus operators are given by

\[ K_1 = \sqrt{q} |0\rangle \langle 1|, \quad K_2 = |0\rangle \langle 0| + \sqrt{1 - q} |1\rangle \langle 1|. \]

(8)

Under the amplitude channel, the single-qubit state changes as

\[ \rho \rightarrow \varepsilon(\rho) = \sum_{i=1,2} K_i \rho K_i^\dagger = \begin{pmatrix} \rho_{00} + \frac{q}{2} \rho_{11} & \sqrt{1 - q} \rho_{01} \\ \sqrt{1 - q} \rho_{10} & (1 - q) \rho_{11} \end{pmatrix}. \]

(9)

In our experiment, we remove the module in the spatial mode \( B \). Amplitude-damping decoherence is realized by using a set of a PBS, a BS and two HWPs, such that the path length difference between the two spatial paths is much bigger than the coherence length of the photons. The angles of the HWPs are set as \( \alpha = 0 \) and \( \beta = \frac{1}{2} \arcsin(\sqrt{q}) \). After the PBS, the horizontal polarization does not change, while the vertical
polarization is converted into the horizontal one with the probability \( q \) according to the setting angle \( \beta \) of the HWP. Then, we perform quantum state tomography to reconstruct the final states and calculate the left- and right-hand sides of the trade-off relation in (1).

The experimental data are shown in figure 4. For the amplitude damping channel \( D(\rho, \varepsilon) \) and \( C(\rho) \) also follow the trade-off relation in (1). Note that with the parameters of the initial states \( \{\theta = \varphi = \frac{\pi}{2}, \lambda_0 = 0.50, 0.89, 1\} \) and the parameter of the depolarizing channel \( q = 1 \), the trade-off relation in (1) is saturated as predicted theoretically and is confirmed by our experimental results, i.e., the experimental data with error bars represented by three red triangles are on the dash line indicating \( 2C(\rho) + D(\rho, \varepsilon) = 2 \).

3. Analysis of experimental imperfections

There are three sources of imperfections in our experiment: fluctuations in the counts of photon, inaccurate wave plates and decoherence (dephasing) caused by the imperfect interferometers. First, the imperfection caused by fluctuations in the counts of photon increases as the counts of photon decrease. The counts of photon in our experiments is about 3000. Second, for each wave plate, the uncertainty of the setting angle is about \( \theta + \delta \theta \), where \( \delta \theta \) is randomly chosen from an interval. The range of the interval is determined through Monte Carlo simulations to fit the deviations of experimental data from their theoretical predictions. Third, the dephasing caused by the imperfection of the Sagnac interferometer is described by the noise channel \( \chi(\rho) = \tau \rho + (1 - \tau)\sigma_z \rho \sigma_z \), where \( \rho \) and \( \chi(\rho) \) are the density matrices of the input and output states, \( \sigma_z \) is the Pauli matrix, and \( \tau \) is the dephasing rate. Due to the instability of the Sagnac ring, the uncertainty of the dephasing rate is \( \delta \tau \). We estimate \( \tau \) to be \( \tau = 0.976 \pm 0.014 \) from the experimental results. Among the three imperfections, the reconstructed density matrices are most sensitive to the inaccuracy of the wave plates.

In order to elaborate on the influence on the disturbance from the imperfections in our experiment, we perform numerical simulations of the disturbance by considering all the experimental imperfections. We choose an initial state with the parameters \( \theta = \frac{\pi}{2}, \phi = \frac{\pi}{4}, \lambda_0 = 0.01 \). We choose three different uncertainties of the setting angles of the wave plates \( \delta \theta = 0.02, 0.08, 0.14 \) as examples. As illustrated in figure 5, the data (represented by symbols) are obtained by numerical simulations. The setting angle of each wave plate is sampled randomly from the interval \( [\theta - \delta \theta, \theta + \delta \theta] \). Error bars are obtained by Monte Carlo simulation by randomly sampling the setting angles of the wave plates for 100 times.

As it is shown in figure 5, error bars \( \delta D \) increase with \( \delta \theta \) as the disturbance is very sensitive to the inaccuracy of the wave plates. Thus, in our experiment, the experimental result of disturbance could possibly be any data within the interval \( [D - \delta D, D + \delta D] \) with the influence of the experimental imperfections.
imperfections. That is, the experimental results of the disturbance do not increase with the increasing of the measurement strength $x$ for some parameters due to the imperfections of our experiments. For different setting angles, the inaccuracy is different. For example, the inaccuracy of an integer angle is much less than that of a non-integer angle. That is the reason why the experimental results of the disturbance with some parameters agree with their theoretical predictions better than those with other parameters.

Most importantly, we emphasize that in this work even with all these imperfections of the experiment, the trade off relation for coherence and disturbance can be tested experimentally. In our experiments, this conclusion holds for all initial states and channel parameters even with all these experimental imperfections.

4. Discussion and conclusions

With entangled photons and linear optics, we experimentally verify the coherence-disturbance trade-off relation for a single-qubit system undergoing various single-qubit noisy channels. Our experimental results agree with the theoretical predictions. From the experimental results, it can be seen that the trade-off between coherence and disturbance is channel dependent. The trade-off relation obeyed for a single-qubit state in the case of a weak measurement channel is tighter, while the depolarizing and amplitude damping channels follow the original relation for an arbitrary single-qubit state. Our achievement relies on replacing time-sharing noisy channels by space-multiplexed noisy channels using a practical, linear-optical interferometric network. Our demonstration serves as a foundation for future experimental simulations employing networks of single-qubit channel simulations. Our work provides a quantitative understanding of the relation between quantum resources and quantum channels.

Acknowledgments

National Natural Science Foundation of China (Grant Nos. 12025401, 12104036, 12104009, U1930402, 12088101).

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.
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**References**

[1] Bennett C H, Brassard G and Mermin N D 1992 Phys. Rev. Lett. 68 557
[2] Fuchs C A and Peres A 1996 Phys. Rev. A 53 2038
[3] Banaszek K 2001 Phys. Rev. Lett. 86 1366
[4] D’Ariano G M 2003 Fortschr. Phys. 51 318
[5] Maccone L 2006 Phys. Rev. A 73 042307
[6] Sciarrino F, Ricci M, De Martini F, Filip R and Mišta L 2006 Phys. Rev. Lett. 96 020408
[7] Andersen U L, Sabuncu M, Filip R and Leuchs G 2006 Phys. Rev. Lett. 96 020409
[8] Sacchi M F 2006 Phys. Rev. Lett. 96 220502
[9] Buscemi F and Sacchi M F 2006 Phys. Rev. A 74 052320
[10] Buscemi F, Hayashi M and Horodecki M 2008 Phys. Rev. Lett. 100 210504
[11] Cheong Y W and Lee S-W 2012 Phys. Rev. Lett. 109 190402
[12] Terashima H 2016 Phys. Rev. A 93 022104
[13] Zurek W H 2003 Rev. Mod. Phys. 75 715
[14] Mani A and Karimipour V 2015 Phys. Rev. A 92 032331
[15] Schlosshauer M 2016 Phys. Rev. A 93 012115
[16] Wootters W K and Zurek W H 1982 Nature 299 802
[17] D’Ariano G M and Yu H P 1996 Phys. Rev. Lett. 76 2832
[18] Xue P and Xiao Y-F 2006 Phys. Rev. Lett. 97 140501

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[20] Sharma G and Pati A K 2018 Phys. Rev. A 97 062308
[21] Streltsov A, Adesso G and Plenio M B 2017 Rev. Mod. Phys. 89 041003
[22] Baumgratz T, Cramer M and Plenio M B 2014 Phys. Rev. Lett. 113 140401
[23] Napoli C, Bromley T R, Cianciaruso M, Piani M, Johnston N and Adesso G 2016 Phys. Rev. Lett. 116 150502
[24] Piani M, Cianciaruso M, Bromley T R, Napoli C, Johnston N and Adesso G 2016 Phys. Rev. A 93 042107

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[20] Sharma G and Pati A K 2018 Phys. Rev. A 97 062308
[21] Streltsov A, Adesso G and Plenio M B 2017 Rev. Mod. Phys. 89 041003
[22] Baumgratz T, Cramer M and Plenio M B 2014 Phys. Rev. Lett. 113 140401
[23] Napoli C, Bromley T R, Cianciaruso M, Piani M, Johnston N and Adesso G 2016 Phys. Rev. Lett. 116 150502
[24] Piani M, Cianciaruso M, Bromley T R, Napoli C, Johnston N and Adesso G 2016 Phys. Rev. A 93 042107
[25] Xue P, Sanders B C and Leibfried D 2009 Phys. Rev. Lett. 103 183602
[26] Chitambar E and Hsieh M-H 2016 Phys. Rev. Lett. 110 190402
[27] Singh U, Bera M N, Dhar H S and Pati A K 2015 Phys. Rev. A 91 052115
[28] Fuchs C A and Jacobs K 2001 Phys. Rev. A 63 062305
[29] Banaszek K and Devetak I 2001 Phys. Rev. A 64 052307
[30] Ozawa M 2004 Ann. Phys., NY 311 350
[31] Ren X-J and Fan H 2014 J. Phys. A: Math. Theor. 47 305302
[32] Chen G et al 2014 Phys. Rev. X 4 021043
[33] Gill N, Ribordy G, Tittel W and Zbinden H 2002 Rev. Mod. Phys. 74 145
[34] Nielsen M A and Chuang I 2000 Quantum Computation and Quantum Information (Cambridge: Cambridge University Press)
[35] Maccone L 2007 Europhys. Lett. 77 40002
[36] Schumacher B and Nielsen M A 1996 Phys. Rev. A 54 2629
[37] Schumacher B and Westmoreland M D 1997 Phys. Rev. A 56 131
[38] Zhan X, Zhang X, Li J, Zhang Y S, Sanders B C and Xue P 2016 Phys. Rev. Lett. 116 090401
[39] Xiao L et al 2017 Nat. Phys. 13 1117–23
[40] Wang K K, Qiu X, Xiao L, Zhan X, Bian Z H, Yi W and Xue P 2019 Phys. Rev. Lett. 122 020501
[41] Xiao L, Deng T, Wang K, Zhu G, Wang Z, Yi W and Xue P 2020 Nat. Phys. 16 761–6
[42] Bian Z H, Majumdar A S, Jебarathinam C, Wang K K, Xiao L, Zhan X, Zhang Y S and Xue P 2020 Phys. Rev. A 101 020301 (R)
[43] Wang K K, Wang X P, Zhan X, Bian Z H, Li J, Sanders B C and Xue P 2018 Phys. Rev. A 97 042112
[44] Xiao L, Wang K, Zhan X, Bian Z, Li J, Zhang Y, Xue P and Pati A K 2017 Opt. Express 25 17904–10
[45] Xiao L, Wang K, Zhan X, Bian Z, Kavabata K, Ueda M, Yi W and Xue P 2019 Phys. Rev. Lett. 123 230401
[46] Xue P, Zhang R, Qin H, Zhan X, Bian Z H, Li J and Sanders B C 2015 Phys. Rev. Lett. 114 140502
[47] Bian Z, Li J, Qin H, Zhan X, Zhang R, Sanders B C and Xue P 2015 Phys. Rev. Lett. 114 203602
[48] Wang K, Qiu X, Xiao L, Zhan X, Bian Z, Sanders B C, Yi W and Xue P 2019 Nat. Commun. 10 2293
[49] Kim Y-S, Lee J-C, Kwon O and Kim Y-H 2012 Nat. Phys. 8 117–20
[50] Jeong Y-C, Lee J-C and Kim Y-H 2013 Phys. Rev. A 87 014301
[51] Oreshkov O and Brun T A 2005 Phys. Rev. Lett. 95 110409
[52] Singh U and Pati A K 2014 Ann. Phys., NY 343 141