Redducibility of Killing tensors

in $d > 4$ NHEK geometry

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Abstract

An extremal rotating black hole in arbitrary dimension, along with time translations and rotations, possesses a number of hidden symmetries characterized by the second rank Killing tensors. As is known, in the near horizon limit the isometry group of the metric is enhanced to include the conformal factor $SO(2,1)$. It is demonstrated that for the near horizon extremal Kerr (NHEK) geometry in arbitrary dimension one of the Killing tensors decomposes into a quadratic combination of the Killing vectors corresponding to the conformal group, while the remaining ones are functionally independent.

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1. Introduction

Killing tensors are conventionally attributed to hidden symmetries as there are no coordinate transformations in spacetime associated with them. Along with Killing vectors, they provide important geometric characterization of a spacetime. As far as physical applications are concerned, they entail extra integrals of motion for the geodesic equations, which ensure separation of variables and integration of the equations in quadratures. A well-known example is the geodesic motion of a particle on the Kerr background [1, 2]. In field theory, they are responsible for the separation of variables in the Klein–Gordon and Dirac equations on the curved background [3] (for a review see [4] and references therein).

A spacetime may also admit an antisymmetric analogue of the Killing tensor known in the literature as the Killing–Yano tensor. In general, the Killing–Yano tensors may be used to construct the Killing tensors, but not every Killing tensor is decomposable into a combination of the Killing–Yano tensors (see e.g. [5]). It is also worth mentioning that, when considering a superparticle model on a curved background which admits the Killing–Yano tensors, extra supersymmetry charges can be constructed [6] which yield Killing tensors under the Poisson bracket.

Hidden symmetries of the Kerr–NUT–AdS black hole in arbitrary dimension and separation of variables in the geodesic equations were studied in a series of recent works [7]–[12]. In particular, it was shown that in $d = 2n + \epsilon$, where $\epsilon = 1$ for an odd-dimensional spacetime and $\epsilon = 0$ for an even-dimensional spacetime, there are $n$ functionally independent second rank Killing tensors, which also include the metric as a trivial Killing tensor. Along with time translations and rotations these guarantee the complete integrability of a particle model on such a background.

A salient feature of an extremal rotating black hole in arbitrary dimension is that one can consider the near horizon limit which yields a new vacuum solution of the Einstein equations. The latter differs from the parent geometry in many respects. The near horizon geometry is not asymptotically flat. The time translation symmetry is enhanced to the conformal group $SO(2, 1)$ [13]. The latter paved the way to the Kerr/CFT–correspondence [14] (for a review see [15]) and to the extensive study of the conformal mechanics models associated with the near horizon black holes [16]–[34]. For the case of four dimensions it was also demonstrated [25, 26] that the near horizon Killing tensor is reducible and can be expressed via a quadratic combination of the Killing vectors. Similar reducibility occurs for the Kerr–NUT–AdS black hole [35] and for the weakly charged extremal Kerr throat geometry [36].

As was mentioned above, the number of second rank Killing tensors grows with dimension. It is then important to study how many of them are functionally independent in the near horizon limit. The purpose of this paper is to demonstrate that for the class of extremal rotating black holes described by the Myers-Perry solution [37] (vanishing cosmological constant and the NUT charge) only one near horizon Killing tensor decomposes into a quadratic combination of the Killing vectors, while the remaining ones are functionally independent.

The paper is organized as follows. In Sect. 2 we first briefly review the construction of the second rank Killing tensors for an extremal rotating black hole in $d = 2n$. The near horizon
limit is discussed next and the reducibility of one of the Killing tensors is demonstrated. Similar analysis of the odd-dimensional case is carried out in Sect. 3.

2. Near-horizon Killing tensors in $d = 2n$

Our starting point is the Myers-Perry black hole solution \[37\] written in the coordinates introduced in \[38\]:

$$
\begin{align*}
    ds^2 &= \frac{U}{X} dr^2 + \sum_{\alpha=1}^{n-1} \frac{U^\alpha}{X^\alpha} dx^2 - \frac{X}{U} \left[ dt - \sum_{i=1}^{n-1} \left( \frac{\gamma_i}{\varepsilon_i} d\phi_i \right)^2 + \sum_{\alpha=1}^{n-1} \frac{X^\alpha}{U^\alpha} \left[ dt - \sum_{i=1}^{n-1} \frac{(r^2 + a_i^2)\gamma_i}{(a_i^2 - x_\alpha^2)\varepsilon_i} d\phi_i \right]^2 \right], \\
    U &= \prod_{\alpha=1}^{n-1} (r^2 + x_\alpha^2), \quad U_\alpha = -(r^2 + x_\alpha^2) \prod_{\beta=1, \alpha \neq \beta}^{n-1} (x_\beta^2 - x_\alpha^2), \\
    X_\alpha &= -\prod_{k=1}^{n-1} (a_k^2 - x_\alpha^2), \quad X = \prod_{k=1}^{n-1} (r^2 + a_k^2) - 2Mr, \\
    \gamma_i &= \prod_{\alpha=1}^{n-1} (a_i^2 - x_\alpha^2), \quad \varepsilon_i = a_i \prod_{k=1, k \neq i}^{n-1} (a_i^2 - a_k^2),
\end{align*}
$$

(1)

where $t$ is the temporal coordinate, $r$ is the radial coordinate and $x_\alpha, \phi_i$ with $i, \alpha = 1, \ldots, n-1$ are related to the angular variables. In Eq. (1) $M$ is the mass and $a_i$ are the rotation parameters. Without loss of generality, it can be assumed that $a_1 \leq a_2 \cdots \leq a_{n-1}$ in which case the coordinates $x_\alpha$ take their values in the intervals $a_\alpha \leq x_\alpha \leq a_{\alpha+1}$.

In what follows, we will also need the metric (1) written in special coordinates introduced in Ref. \[38\]. It is obtained by the linear change of the variables

$$
\begin{align*}
    B_j^{(k)} &\equiv \sum_{l_1 < l_2 \cdots < l_k \atop j \neq l_1, \ldots, l_k} a_{l_1}^2 a_{l_2}^2 \cdots a_{l_k}^2, \quad \frac{\phi_j}{a_j} = \sum_{k=0}^{n-2} B_j^{(k)} \psi_{k+1}, \\
    B^{(k)} &\equiv \sum_{l_1 < l_2 \cdots < l_k} a_{l_1}^2 a_{l_2}^2 \cdots a_{l_k}^2, \quad t = \psi_0 + \sum_{k=1}^{n-1} B^{(k)} \psi_k,
\end{align*}
$$

(2)

which yields \[38\]

$$
\begin{align*}
    ds^2 &= \sum_{\mu=1}^{n} \left\{ \frac{dx_\mu}{Q_\mu} + Q_\mu \left( \sum_{k=0}^{n-1} A^{(k)}_\mu \psi_k \right)^2 \right\},
\end{align*}
$$

(3)

where

$$
Q_\mu = \frac{X_\mu}{U_\mu}, \quad A^{(k)}_\mu = \sum_{\nu_1 < \nu_2 \cdots < \nu_k \atop \mu \neq \nu_1, \ldots, \nu_k} x_{\nu_1}^2 x_{\nu_2}^2 \cdots x_{\nu_k}^2.
$$

2
\[ U_\mu = \prod_{\nu=1 \atop \mu \neq \nu}^{n} (x_\nu^2 - x_\mu^2), \quad X_\mu = -\prod_{k=1}^{n-1} (a_k^2 - x_\mu^2) - 2\widehat{M} x_\mu \delta_{\mu n}, \]
\[ x_n = i r, \quad \widehat{M} = \pm i M, \] (4)

and \( A^{(0)}_\mu \equiv 1 \). In this coordinate system, the full set of \( n \) functionally independent Killing tensors was constructed in Ref. [12]

\[ K^{(k)} = \sum_{\mu=1}^{n} \left[ \frac{A^{(k)}_\mu}{X_\mu U_\mu} \left( \sum_{j=0}^{n-1} (-x_\mu^2)^{n-1-j} \partial_\psi_j \right)^2 + A^{(k)}_\mu Q_\mu (\partial_{x_\mu})^2 \right], \] (5)

where \( k = 0, \ldots, n - 1 \) and \( K^{(0)} \) stands for the metric inverse to (3).

As the next step, let us discuss the near horizon limit of the metric (1). In order to describe it, one considers the extremal case, for which \( X \) has a double zero at the horizon radius \( r_0 \)

\[ X|_{r=r_0} = 0, \quad X'|_{r=r_0} = 0. \] (6)

The equations (6) relate the mass, the rotation parameters, and \( r_0 \) [39]. Then the coordinates are redefined

\[ r \to r_0 + \lambda r_0, \quad \phi_i \to \phi_i + \alpha_i \beta t, \quad \alpha_i = \frac{a_i}{r_0^2 + a_i^2}, \]
\[ t \to \beta t, \quad \beta = \frac{\prod_{i=1}^{n-1} (r_0^2 + a_i^2)}{\lambda r_0 V}, \] (7)

where \( V = \frac{1}{2} X''|_{r=r_0} \), and the limit \( \lambda \to 0 \) is taken which yields [39]

\[ ds^2 = \frac{\widetilde{U}}{V} \left( -r^2 dt^2 + \frac{dr^2}{r^2} \right) + \sum_{\alpha=1}^{n-1} \frac{U_\alpha}{X_\alpha} dx_\alpha^2 \]
\[ + \sum_{\alpha=1}^{n-1} X_\alpha \left[ 2r_0 r \prod_{\beta=1}^{n} \left( \frac{r_0^2 + x_\beta^2}{V(r_0^2 + x_\beta^2)} \right) \right] dt + \sum_{i=1}^{n-1} \left( \frac{(r_0^2 + a_i^2)}{2} \right) \frac{\gamma_i}{(a_i^2 - x_\alpha^2)} d\phi_i. \] (8)

Here we denoted \( \widetilde{U} = U|_{r=r_0}, \widetilde{U}_\alpha = U_\alpha|_{r=r_0} \). It is straightforward to verify that Eq. (8) is a vacuum solution of the Einstein equations.

Near the horizon the isometry group is enhanced. In addition to time translation and rotations

\[ t' = t + \epsilon, \quad \phi'_i = \phi_i + \epsilon_i, \] (9)

it includes the dilatation

\[ t' = t + t \delta, \quad r' = r - r \delta \] (10)

and the special conformal transformation

\[ t' = t + \left( \frac{1}{r^2 + t^2} \right) V \sigma, \quad r' = r - 2rt V \sigma, \quad \phi'_i = \phi_i - \frac{c_i}{r} \sigma, \] (11)
where \( i = 1, \ldots, n - 1 \) and the constants \( c_i \) read

\[
c_i = \frac{4r_0a_i \prod_{j=1}^{n-1}(r_0^2 + a_j^2)}{(r_0^2 + a_i^2)^2}.
\]

(12)

The Killing tensors, which characterize the near-horizon geometry \( (5) \), are derived from \( \text{(5)} \) by taking into account the relation between the metrics \( \text{(3)} \) and \( \text{(1)} \), by applying the coordinate redefinition \( \text{(7)} \), and by taking the limit \( \lambda \to 0 \)

\[
\tilde{K}_{(k)} = \sum_{\mu=1}^{n-1} \frac{\tilde{A}_\mu^{(k)}}{X_\mu \tilde{U}_\mu} \left[ \sum_{i=1}^{n-1} a_i \prod_{j=1}^{n-1} (x_\mu^2 - a_j^2) \partial_{\phi_i} \right] + \sum_{\mu=1}^{n-1} \frac{A_\mu^{(k)}}{\tilde{U}_\mu} \left( \partial_{x_\mu} \right)^2
\]

\[- \frac{\tilde{A}_n^{(k)}}{V \tilde{U} r^2} \left[ \partial_t - 2rr_0 \sum_{i=1}^{n-1} a_i \prod_{j=1}^{n-1} (r_0^2 + a_j^2) \partial_{\phi_i} \right] + \frac{A_n^{(k)}}{\tilde{U}} r^2 \partial_r \partial_r. \]

(13)

Here \( \tilde{A}_\mu^{(k)} \) are the functions \( A_\mu^{(k)} \) given in \( \text{(4)} \) with \( x_n = \imath r_0 \) and \( k = 0, \ldots, n - 1 \).

In order to demonstrate that one of the near-horizon Killing tensors is reducible, let us consider the following linear combination:

\[
Q^{AB} = \sum_{k=0}^{n-1} r_0^{2(n-1-k)} K_{(k)}^{AB},
\]

(14)

where \( A, B = 1, \ldots, 2n \). It is straightforward to verify that \( Q^{AB} \) can be decomposed into the Killing vectors

\[
Q^{AB} = -\frac{1}{2} (k^{A(1)}_1 k^{B(1)}_1 + k^{A(2)}_3 k^{B(2)}_1) + \frac{1}{V} k^{A(2)}_1 k^{B(2)}_1
\]

\[- \frac{1}{4V} \left( \sum_{l=4}^{n+2} c_{l-3} k^{A(1)}_i \right) \left( \sum_{l=4}^{n+2} c_{l-3} k^{B(1)}_i \right),
\]

(15)

where \( k^{A(1)}_1 \) denote components of the Killing vector corresponding to the time translations, \( k^{A(2)}_1 \) are linked to the dilatations, \( k^{A(3)}_3 \) are associated with the special conformal transformations and the remaining ones are related to the shifts of the azimuthal angular variables \( \phi_i \). The constants \( c_j \), which enter the right hand side of \( \text{(15)} \), are taken from \( \text{(12)} \). Thus one Killing tensor can be expressed in terms of the others and a specific quadratic combination of the Killing vectors\(^1\) Similar results for \( d = 4 \) have been obtained earlier in [25, 26, 35, 36].

\(^1\)That there exists only one combination like \( \text{(15)} \) can be seen as follows. First it is verified that the tensors \( \text{(13)} \) are linearly independent. So are the vectors \( \text{(9)} - \text{(11)} \). Because the \((x_\alpha)\)-components of the vectors with \( \alpha = 1, \ldots, n - 1 \) are equal to zero, the corresponding components of quadratic combinations formed out of them vanish as well. In order to reduce a combination of the Killing tensors to a quadratic combination of the Killing vectors, one has to ensure that its \((x_\alpha, x_\beta)\)-components with \( \alpha, \beta = 1, \ldots, n - 1 \) vanish. It is straightforward to verify that this condition is equivalent to solving a system of linear algebraic equations whose general solution is given above in \( \text{(4)} \).
The fact that one of the Killing tensors turns out to be reducible near the horizon does not imply that the particle model on such a background fails to be completely integrable. As was mentioned above, near the horizon the isometry group is enhanced to include the dilatations and the special conformal transformations. Thus, as far as the issue of the complete integrability is concerned, one of the Killing tensors is lost while two extra Killing vectors appear. So the model is in fact minimally superintegrable. Similar situation occurs for the case of $d = 2n + 1$ which we discuss in the next section.

3. Near-horizon Killing tensors in $d = 2n + 1$

Rewritten in the coordinates introduced in \[38\], the rotating black hole solution of the Einstein equations in $d = 2n + 1$ reads

$$ds^2 = \frac{U}{X} dr^2 + \sum_{\alpha=1}^{n-1} \frac{U_{\alpha}}{X_{\alpha}} dx_{\alpha}^2 - \frac{X}{U} \left[ dt - \sum_{i=1}^{n-1} a_i^2 \gamma_i \frac{d\phi_i}{\varepsilon_i} \right]^2$$

$$+ \sum_{\alpha=1}^{n-1} \frac{X_{\alpha}}{U_{\alpha}} \left[ dt - \sum_{i=1}^{n-1} \frac{a_i^2 \gamma_i (r^2 + x_{\alpha}^2)}{a_i^2 - x_{\alpha}^2} \frac{d\phi_i}{\varepsilon_i} \right]^2 + \prod_{k=1}^n a_k^2 \left[ dt - \sum_{i=1}^{n} (r^2 + a_i^2) \gamma_i \frac{d\phi_i}{\varepsilon_i} \right]^2,$$

$$U = \prod_{\alpha=1}^{n-1} (r^2 + x_{\alpha}^2), \quad U_{\alpha} = -(r^2 + x_{\alpha}^2) \prod_{\beta=1}^{n-1} (x_{\beta}^2 - x_{\alpha}^2),$$

$$\gamma_i = \prod_{\alpha=1}^{n-1} (a_i^2 - x_{\alpha}^2), \quad \varepsilon_i = a_i \prod_{k=1}^{n} (a_i^2 - a_k^2),$$

$$X_{\alpha} = \frac{1}{x_{\alpha}^2} \prod_{k=1}^{n} (a_k^2 - x_{\alpha}^2), \quad X = \frac{1}{r^2} \prod_{k=1}^{n} (r^2 + a_k^2) - 2M,$$  \[16\]

where $t$ is the temporal coordinate, $r$ is the radial coordinate and $x_{\alpha}$ with $\alpha = 1, \ldots, n - 1$ and $\phi_i$ with $i = 1, \ldots, n$ are related to the angular variables. $M$ stands for the mass and $a_i$ denotes the rotation parameters. As in the even-dimensional case, the coordinate system will be used which is obtained by the linear change of the coordinates

$$B_j^{(k)} \equiv \sum_{l_1 < l_2 \cdots < l_k} \frac{a_{l_1} a_{l_2} \cdots a_{l_k}}{j \neq l_1, \ldots, l_k}, \quad \frac{\phi_j}{a_j} = \sum_{k=0}^{n-1} B_j^{(k)} \psi_{k+1},$$

$$B^{(k)} \equiv \sum_{l_1 < l_2 \cdots < l_k} \frac{a_{l_1} a_{l_2} \cdots a_{l_k}}{l_1 < l_2 \cdots < l_k}, \quad t = \psi_0 + \sum_{k=1}^{n} B^{(k)} \psi_k,$$  \[17\]
where \( x_n = ir \). In these coordinates the metric (16) reads [38]

\[
    ds^2 = \sum_{\mu=1}^{n} \left\{ \frac{dx_{\mu}}{Q_\mu} + Q_\mu \left( \sum_{k=0}^{n-1} A^{(k)}_\mu d\psi_k \right)^2 \right\} - \frac{c}{A(n)} \left( \sum_{k=1}^{n} A^{(k)}_\mu \partial \psi_k \right)^2,
\]

\[
    Q_\mu = \frac{X_\mu}{U_\mu}, \quad A^{(k)}_\mu = \sum_{\nu_1 < \nu_2 < \cdots < \nu_k} x_{\nu_1}^2 x_{\nu_2}^2 \cdots x_{\nu_k}^2, \quad A^{(k)} = \sum_{\nu_1 < \nu_2 < \cdots < \nu_k} x_{\nu_1}^2 x_{\nu_2}^2 \cdots x_{\nu_k}^2.
\]

\[
    U_\mu = \prod_{\substack{\nu=1 \\ \mu \neq \nu}}^{n} (x_{\nu}^2 - x_{\mu}^2), \quad X_\mu = \frac{1}{x_{\mu}} \prod_{k=1}^{n} (a_k^2 - x_{\mu}^2) + 2M\delta_{\mu n}, \quad c = \prod_{k=1}^{n} a_k^2,
\]

with \( A^{(0)}_\mu = A^{(0)} \equiv 1 \).

As in the preceding case, there are \( n \) functionally independent second rank Killing tensors [12]

\[
    K^{(k)} = \sum_{\mu=1}^{n} \left[ A^{(k)}_\mu \left( \sum_{j=0}^{n} (-1)^{j} \partial_{\psi_j} \right)^2 + A^{(k)}_\mu Q_\mu \partial_{\psi_n} \right] - \frac{A^{(k)}}{cA(n) A^{(n)}(\partial \psi_n)^2},
\]

where \( k = 0, \ldots, n-1 \) and \( K^{(0)} \) denotes the metric inverse to (18).

In order to construct the near-horizon geometry, the extremal case is considered

\[
    X|_{r=r_0} = 0, \quad X'|_{r=r_0} = 0,
\]

where \( r_0 \) is the horizon radius. Then the coordinate redefinition is used

\[
    r \rightarrow r_0 + \lambda rr_0, \quad \phi_i \rightarrow \phi_i + \alpha_i \beta t, \quad \alpha_i = \frac{a_i}{r_0^2 + a_i^2},
\]

\[
    t \rightarrow \beta t, \quad \beta = \frac{\prod_{i=1}^{n} (r_0^2 + a_i^2)}{\lambda r_0^3 V}
\]

and the limit \( \lambda \rightarrow 0 \) is taken. These yield [39]

\[
    ds^2 = \frac{\bar{U}}{V} \left( -r^2 dt^2 + \frac{dr^2}{r^2} \right) + \sum_{\alpha=1}^{n-1} \frac{X_\alpha}{U_\alpha} dx_{\alpha}^2
\]

\[
    + \sum_{\alpha=1}^{n-1} \frac{X_\alpha}{U_\alpha} \left[ 2r_0 r \prod_{\beta} (r_0^2 + x_{\beta}^2) dt + \sum_{i=1}^{n} \frac{a_i^2 (r_0^2 + a_i^2)}{r^2 (a_i^2 - x_\alpha^2) \varepsilon_i} d\phi_i \right]^2
\]

\[
    + \prod_{k=1}^{n} \prod_{a=1}^{n-1} \frac{a_k^2}{r_0^2} \left[ 2r \prod_{\beta} (r_0^2 + x_{\beta}^2) dt + \sum_{i=1}^{n} \frac{r_0^2 + a_i^2}{r_0^2} \gamma_i d\phi_i \right]^2,
\]

where \( V = \frac{1}{2} X''|_{r=r_0} \). As in the preceding section, the tilde over a function implies that it is evaluated at \( r = r_0 \).
For $d = 2n + 1$, the near horizon conformal symmetry transformations are realized as in Eqs. (9)-(11) above with the only difference that now the coefficients $c_i$ have the form

$$c_i = \frac{4a_i \prod_{j=1}^{n-1}(r_0^2 + a_j^2)}{r_0(r_0^2 + a_i^2)^2},$$

with $i = 1, \ldots, n$.

The second rank Killing tensors which characterize the near-horizon geometry are derived from (19) in three steps. First, the redefinition (21) is used. Then the link between (16) and (18) is taken into account. Finally, the limit $\lambda \to 0$ is taken. The result reads

$$\tilde{K}_{(k)} = \sum_{\mu=1}^{n-1} \frac{\tilde{A}^{(k)}_\mu (x_\mu + r_0^2)}{x_\mu X_\mu \tilde{U}_\mu} \left[ \alpha_i \prod_{j=1}^{n} (x_\mu + a_j^2) \partial_{\mu}, \right]^2 + \sum_{\mu=1}^{n-1} \frac{\tilde{A}^{(k)}_\mu X_\mu (\partial_{\mu})^2}{\tilde{U}_\mu},$$

where $k = 0, \ldots, n - 1$ and the functions $\tilde{A}^{(k)}_\mu, \tilde{A}^{(k)}$ are obtained from $A^{(k)}_\mu, A^{(k)}$ in Eq. (18) by setting $x_n = 1r_0$. The constants $b_i$, which appear in the previous expression, read

$$b_i = \frac{\prod_{j=1}^{n} a_j}{a_i(r_0^2 + a_i^2)}.$$

The fact that one of the near horizon Killing tensors is reducible is demonstrated by first considering the specific linear combination

$$Q^{AB} = \sum_{k=0}^{n-1} 2^{n-1-k} \tilde{K}_{(k)},$$

where $A, B = 1, \ldots, 2n + 1$ and then checking that the latter can be decomposed into the Killing vectors

$$Q^{AB} = \frac{1}{2} (k^{A}_{(1)} k^{B}_{(3)} + k^{A}_{(3)} k^{B}_{(1)}) + \frac{1}{V} k^{A}_{(2)} k^{B}_{(2)} - \frac{1}{4 V} \left( \sum_{l=4}^{n+3} c_{l-3} k^{A}_{(l)} \right) \left( \sum_{l=4}^{n+3} c_{l-3} k^{B}_{(l)} \right) + r_0^2 \left( \sum_{l=4}^{n+3} b_{l-3} k^{A}_{(l)} \right) \left( \sum_{l=4}^{n+3} b_{l-3} k^{B}_{(l)} \right),$$

where $c_i, b_i$ are given in (23), (25) above and $k^{A}_{(1)}, k^{A}_{(2)}, k^{A}_{(3)}$ correspond to the time translations, dilatations and special conformal transformations, respectively, while the remaining Killing
vectors are related to the shifts of the azimuthal angles $\phi_i$. As in the preceding section, we thus conclude that among the near horizon second rank Killing tensors only one is reducible, while the remaining ones prove to be functionally independent.

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