Interferometry through a quantum dot coupled to Majorana fermions

Akiko Ueda\textsuperscript{a*} and Takehito Yokoyama\textsuperscript{b}

\textsuperscript{a}University of Tsukuba, 1-1-1 Tennodai, Ibaraki, 305-8573, Japan
\textsuperscript{b}Tokyo Institute of Technology, Meguro-ku, Tokyo,152-8551, Japan

Abstract

We investigate transport properties of an Aharonov-Bohm interferometer with an embedded quantum dot with Majorana bound states at the end of the topological superconductor. The differential conductance is calculated by the Keldysh Green function formalism. The Fano resonance of the symmetric shape with the maximum value of $2e^2/h$ emerges as a function of the bias voltage. We find that when the energy of the quantum dot is fixed to the energy of the Majorana bound state, the conductance shows $\pi$ periodicity as a function of the Aharonov-Bohm phase induced by the magnetic flux penetrating the interferometer which differs from the case of the superconducting lead.

1. Introduction

Majorana fermions \cite{1} attract much interest owning to the exotic features of the particles and applications to the fault-tolerant topological quantum computers\cite{2-4}. In recent years, Majorana fermions are predicted in the topological superconductor induced by s-wave superconductivities on condensed matter systems\cite{5-12}. Recently, the experiment has reported the emergence of the zero-bias peak in normal metal-topological superconducting junctions that can be the signature of the Majorana bound states (MBSs)\cite{13-14}. More evidence is required to prove the existence. There are some proposals to detect the phase information of MBSs\cite{15-16}.

In this paper, we consider the Aharonov-Bohm (AB) interferometer with an embedded quantum dot coupling to the topological superconductor where the MBSs are located at the both ends of the wire. For the case of the normal
lead, the Fano resonance emerges due to the interference between the discrete level of an quantum dot and the continuum of the energy level of the reference arm[17]. The resonance peak consists of the asymmetric shape with the peaks and dips as a function of the bias voltage while the period of the conductance as a function of the phase shift on the electron waves due to the magnetic flux penetrating the interferometer is $2\pi$. Here, our interest is how the interference pattern is modified when the normal lead is replaced to the topological superconducting lead.

2. Model and Calculation method

We consider the AB interferometer with an embedded quantum dot attached to the topological superconductor as depicted in Fig. 1. The Majorana bound states are existed at both edges of the topological superconducting wire. The simplest effective Hamiltonian of the normal lead, the quantum dot, and the topological superconducting lead are written as [18]

$$H_s = \sum_k (\epsilon_k - \mu_L) c_k^\dagger c_k + \epsilon_0 c_0^\dagger c_0 + i E_M \gamma_2 \gamma_1 - i E_M \gamma_1 \gamma_2. \tag{1}$$

Here, $c_k$ and $c_k^\dagger$ are the creation and annihilation operators of the electrons in the normal lead. We consider the spinless Hamiltonian since only spins of the same direction are existed in the topological superconductor realized by s-wave superconductor. A single level $\epsilon_0$ is considered in the quantum dot with the creation and annihilation operators of the electron in the quantum dot, $c_0$ and $c_0^\dagger$. The operator of MBS coupling to the interferometer is $\gamma_1$. The MBS of the other edge has the operator $\gamma_2$. The MBS has the properties that $\gamma_i = \gamma_i^\dagger$ and $\gamma_i^2 = 0$. The Hamiltonian of the tunneling between the three regions is

$$H_T = t_L c_k^\dagger c_0 + t_L c_0^\dagger c_k - (W e^{i\varphi} c_k - W e^{-i\varphi} c_k^\dagger) \gamma_1 - (t_R c_0 - t_R^\dagger c_0) \gamma_1, \tag{2}$$

where the phase $\varphi$ indicates the AB phase induced by the magnetic flux penetrating the interferometer. In this paper, we normalize the parameters by the level broadening of the energy level of the quantum dot due to the coupling to the normal lead $\Gamma = \frac{\pi \nu^2}{t_L^2}$ with the density of the state of the normal lead $\nu$.

The current is expressed using the Keldysh Green function,

$$I = -\frac{e^2}{h} \int d\omega [t_L G^{<}_{k0}(\omega) + W e^{i\varphi} G^{<}_{k1}(\omega)], \tag{3}$$

where $G^{<}_{k0}(t,t') = i\langle c_0^\dagger(t') c_k(t) \rangle$ and $G^{<}_{k1}(t,t') = i\langle c_0^\dagger(t') c_k(t) \rangle$.

$G^{<}_{k0}(\omega)$ and $G^{<}_{k1}(\omega)$ are determined by solving the equation of motion method.

Fig. 1. Schematic picture of the model.
3. Result

We consider the case that the coupling between MBSs is absent ($E_M=0$) or present ($E_M=\Gamma$). The energy level of the quantum dot is tuned to $\epsilon_0=0$ or $\epsilon_0=\Gamma$. In Fig. 2, the conductance is plotted as a function of $eV$ for the four different cases. Three lines in each figure indicate the different AB phases, $\varphi=0$ (solid line), $\varphi=\pi/2$ (dotted line) and $\varphi=\pi$ (dashed line). In Figs. 2(a) and 2(c), the dashed line is located on the line of the solid line. The conductance shows Fano resonance consisting of peaks and dips. When $E_M=0$, the conductance shows a peak at $eV=0$ for any $\epsilon_0$ [Figs. 2 (a) and 2 (b)]. The peak reaches $2e^2/h$ due to the Andreev reflection ($e^2/h$ for normal lead). By contrast, the zero bias conductance become zero when $E_M \neq 0$ [Figs. 2 (c) and 2 (d)]. The Fano resonances are always symmetric because of the particle-hole symmetry for any $\varphi$ and $\epsilon_0$. We find that $dI/dV(0)=dI/dV(\pi)$ for $\epsilon_0=0$ while $dI/dV(0)\neq dI/dV(\pi)$ for $\epsilon_0 \neq 0$.

Figs. 3(a) and 3(b) show the conductance as a function of $\varphi$ when $eV=\Gamma$. The period of the AB oscillation is $\pi$ when $\epsilon_0=0$ [the solid line in Figs 3(a) and 3(b)] while the period becomes $2\pi$ when $\epsilon_0 \neq 0$ [the dotted line in Figs 3(a) and 3(b)]. Since the period of the AB oscillation is $2\pi$ when the topological superconducting lead is replaced to the normal lead or superconducting lead, the result can be the specific feature of the Majorana fermions.

In Fig.4, we plot the interference pattern. The AB oscillation shows $\pi$ periodicity for any bias voltage ($eV$) when $\epsilon_0=0$ while the $2\pi$ periodicity emerges for $\epsilon_0 \neq 0$. Also, the zero bias peak of the value $2e^2/h$ ($E_M=0$) or zero bias dip of the value zero ($E_M \neq 0$) is robust to $\varphi$. The observation of the interference patterns of such a system may be useful for proving the existence of the Majorana fermions.
Fig. 3. The differential conductance as function of $\varphi$. The solid line is the case of $\varepsilon_0=0$ and the dotted line is the case of $\varepsilon_0=\Gamma$. (a) $E_{\text{sc}}=0$ and (b) $E_{\text{sc}}=\Gamma$, $p\nu W^2=0.5\Gamma$ and $t_\varphi=\Gamma$.

Fig. 4. 2D plot of the differential conductance as function of $\varphi$ and $eV/\Gamma$ when (a) $E_{\text{sc}}=0, \varepsilon_0=0$; (b) $E_{\text{sc}}=0, \varepsilon_0=\Gamma$, (c) $E_{\text{sc}}=\Gamma, \varepsilon_0=0$, (d) $E_{\text{sc}}=\Gamma, \varepsilon_0=\Gamma$, $p\nu W^2=0.5\Gamma$ and $t_\varphi=\Gamma$.

4. Conclusion

We have examined the transport properties of the Aharonov-Bohm ring with an embedded quantum dot attached to the topological superconducting lead. The conductance shows the symmetric Fano resonance with maximum value of $2e^2/h$ due to the Andreev reflection. As a function of the AB phase induced by the magnetic flux penetrating the interferometer, the conductance shows $\pi$ periodicity when the energy level of the quantum dot is equal to the Majorana zero energy, while the conventional $2\pi$ periodicity shows up on the conductance when the energy level of the quantum dot is different from the Majorana zero energy.

Acknowledgements

This work was supported by Grant-in-Aid for Young Scientists (B) (No. 23740236, 24710111) and the “Topological Quantum Phenomena” (No. 25103709) Grant-in Aid for Scientific Research on innovative Areas from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan.

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