Effect of curvature on confinement-deconfinement phase transition.

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Abstract. First order confinement-deconfinement phase transition is studied through hadronic bubble nucleation in an expanding quark-gluon plasma in the context of heavy ion collisions. For this study we consider interacting quark and hadron gas and incorporate the effects of curvature energy. We find that the interactions have the effect of hastening the phase transition whereas the curvature energy has mixed behaviour. Lowering surface tension has the effect of increasing super cooling and slowing down the process of hadronisation in contrast to the case of early Universe. Higher values of bag pressure tend to speed up transition. An interesting feature is the beginning of hadronisation process as soon as the QGP is formed.

1. Introduction
The heavy ion collision experiments are aimed at studying the behavior of quantum chromodynamics (QCD) at high energies [1]. A hot baryon free plasma of quarks and gluons (QGP) is expected to be created whereas at lesser energies, it shows large stopping of the nuclei and hint at the baryon rich matter. The phase transition from QGP to hadron resonance gas (HRG) has been extensively studied for the early Universe where the time scale is of the order of $10^{-6} - 10^{-7}$ s [2] [3] [4]. What emerges is a QGP fireball of $\sim 150 \text{fm}^3$ created at the initial temperature of $\sim 300-350$ MeV, which is about twice the expected critical temperature $T_c$. The effect of curvature energy term has been studied earlier by Mardor and Svetitsky [6] and it has been pointed out that the curvature energy term $8\pi \gamma r$, in addition to the surface energy $4\pi r^2 \sigma$ (where $\sigma$ and $\gamma$ are the surface and curvature energy densities respectively) plays an important role [6] [7] [8] [9]. A first order thermodynamic phase transition would proceed through the nucleation of hadronic bubbles. Csernai and Kapusta [5] have computed a nucleation rate and applied it to a first order phase transition in a set of rate equations. We examine here the dynamics of the phase transition by treating the QGP as a gas of massless $u,d$ quarks, massive $s$ quarks and massless gluons. QCD interactions are treated perturbatively to order $g^3$ and the long range confinement effects are parameterized by the bag pressure $B$. For the hadron phase we use the known masses of the low lying 33 baryons and 45 mesons whose masses and degeneracy factors are taken from the Particle Data Group Summary [10]. We take baryons and antibaryons to have the same size as protons, given by $V_p = m_p/4B$. 
2. Bubble nucleation

The critical radius for bubble nucleation is obtained by extremizing the thermodynamic work expanded to create a bubble, i.e.

\[ W \equiv -\frac{4\pi}{3}r^3 (P_h - P_q) + 4\pi r^2 \sigma - 8\pi (\gamma_q - \gamma_h)r, \]

(1)

where \( P_h \) and \( P_q \) are the pressures in the hadronic and quark phase respectively, and \( \gamma = (\gamma_q - \gamma_h) \) is the curvature coefficient. Following Mardor and Svetitsky [6] the curvature energy in the MIT model for massless quarks is given by

\[ E_c = \frac{g^2}{3\pi} \int_0^\infty dk k \left\{ 1 + \exp\left(\frac{k - \mu_B}{T}\right) \right\}^{-1}. \]

(2)

We get \( \gamma \simeq \frac{\gamma_q^2}{2\beta} \), assuming \( \gamma_h << \gamma_q \) and \( \mu_B << T \). The critical radii for extremums (a maxima) and (a minima) below \( T_c \) are given as

\[ r_{c+} = \frac{\sigma}{\Delta P} (1 + \sqrt{1 - \beta}) \]

(3)

and

\[ r_{c-} = \frac{\sigma}{\Delta P} (1 - \sqrt{1 - \beta}) \]

(4)

respectively, where \( \Delta P = P_h - P_q \) and \( \beta = \frac{2\Delta P}{\pi^2} \gamma \). Since \( \beta > 1 \), a real solution exists only if \( \beta \leq 1 \). The equilibrium (minima) bubbles do not grow. There is a extremum (a minima) in the free energy even above the critical temperature,

\[ r_{c+>} = \frac{\sigma}{\Delta P} (-1 + \sqrt{1 + \beta}). \]

(5)

Significantly, the restriction of \( \beta \leq 1 \) is not present for these bubbles. This ensures that the phase transition actually begins well above the critical temperature having interesting consequences.

The bubble nucleation rate at temperature \( T \) is given by \( I = I_0 e^{-W_c/T} \) where \( I_0 \) is the prefactor having dimensions of \( T^4 \). The prefactor used traditionally in the early Universe studies is given by \( I_0 = \left(\frac{W_c}{2\pi T}\right)^{3/2} T^4 \). Csernai and Kapusta have calculated this in a coarse grain effective field theoretic approximation to QCD and give

\[ I_0 = \frac{16}{3\pi} \left(\frac{\sigma}{3T}\right)^{3/2} \frac{\sigma \eta_q r_c}{\xi_q^2 (\Delta w)^2}, \]

(6)

where \( \eta_q = 14.4T^3 \) is the shear viscosity in the plasma phase, \( \xi_q \) is a correlation length in the plasma phase, and \( \Delta w \) is the difference in the enthalpy densities of the two phases.

Taking contribution of all the critical bubbles, hadron fraction \( h(t) \) is now given by

\[ h(t) = \int_{t_0}^t dt' I_+ (T(t')) \{1 - h(t')\} V(t',t) + \int_{t_0}^t dt' I_- (T(t')) \{1 - h(t')\} \frac{4\pi}{3} r_{c-}^3 (T(t')) + \int_{t_0}^t dt' I_+ (T(t')) \{1 - h(t')\} \frac{4\pi}{3} r_{c+>}^3 (T(t')). \]

(7)

The dynamical equation using the longitudinal scaling hydrodynamics of Bjorken is given by

\[ \frac{de}{dt} = \frac{w}{t}. \]

(8)
Figure 1. Free energy $W$ in units of MeV as a function of bubble radius $r$ in fm. In fig.1a Solid (with $\gamma$) and dashed (without $\gamma$) curves are for $B^{1/4} = 235$MeV and $\sigma \sim 7$MeV fm$^{-2}$. Long dashed (with $\gamma$) and dotted (without $\gamma$) curves are for $B^{1/4} = 300$MeV and $\sigma \sim 7$MeV fm$^{-2}$. Curves in 1a are at a temperature of $1.01T_c$. Curves in 1b are labelled as in 1a except that they are at a temperature of $0.99T_c$.

Figure 2. Temperature $T/T_c$ as a function of time $t$ in fm. Solid, dashed, dotted and long dashed curves are labelled as in fig. 1a.

Figure 3. The hadron fraction $h$ as a function of time $t$ in fm. Curves are labelled as in fig. 1a.

where $e(T)$ and $w(T)$ are the energy and enthalpy densities respectively given by $e(T) = h(t)e_h(t) + (1 - h(t))e_q(t)$, likewise for enthalpy density. The growth velocity of bubbles is taken to be $v(T) = v_0(1 - T/T_c)^{3/2}$ where $v_0 = 3c$ for $T > (2/3)T_c$. We have assumed that for $T < (2/3)T_c$, $v = c/\sqrt{3}$. The volume of an expanding bubble at time $t$ nucleated at time $t'$ is
given by

\[ V(t', t) = \frac{4\pi}{3} \{ r_c + (T(t')) + \int_{t'}^{t} dt'' v(T(t'')) \}^3. \]  

(9)

We use the parameters \( \xi_q = 0.7 \text{fm} \) and \( \eta_q = 14.4T^3 \).

3. Results and discussion

We find that the inclusion of the interactions in both the phases lowers the critical temperature, so higher values of the bag pressure are required for having reasonable values of \( T_c \). In fig.1a it is clearly seen that equilibrium sized hadron bubbles appear even above the critical temperature. In fig.1b we find that below the critical temperature the curvature effect produces a minima as well as a maxima in the free energy. In fig.2 the degree of supercooling depends on the parameters and can be a significant fraction of \( T_c \). The important feature to see here is that the prefactor \( I_0 \) is the deciding factor for supercooling and the duration of the transition rather than the exponent in contrast to early Universe studies. The inclusion of \( \gamma \) term in this case actually reduces the time to complete the phase transition. This can be contrasted with the case corresponding to the other set of parameters in fig.1b. Comparing with the no curvature case, it is seen that increasing \( B \) speeds up the phase transition whereas decreasing it slows it down if the \( \gamma \) term is included. We also see that the decrease in the surface tension increases the supercooling of the plasma by significant amount (can be close to 50 percent). However, increasing \( B \) reduces supercooling when \( \gamma \) is included. Finally, fig.3 shows the hadron fraction \( h \) as a function of time \( t \). Figure shows that for low bag pressures, the effect of \( \gamma \) is to delay the phase transition (solid curve) but for larger values of \( B \) we see that the curvature correction actually speeds up the process of hadronization (long dashed curve).

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