Last minute only bidding is implausible in eBay sealed bid type-of-auctions

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Abstract
In recent years internet auctions have attracted much attention. This paper discusses the "last minute bidding" (sniping) phenomenon, first investigated by Roth and Ockenfels (American Economic Review 92, 1093-1103) in eBay, fixed-end, auctions. Unlike standard auctions where each offer is processed by the auctioneer due to system traffic, and connection time, in eBay auctions very late bids may be left unprocessed. In the paper we consider a simple two-players, two-periods sealed-bid auction model, inspired by eBay auctions, with private values and complete information. The main difference with respect to eBay auctions is given by the first period. Indeed, players can submit at most one bid per period. In the first stage bids are processed with certainty by the system while in the second stage, with positive probability, bids may not be processed due to system congestion. Unlike eBay we show how in this model last minute, that is second stage only, bidding may have low plausibility, as it can be a Nash equilibrium only under very specific circumstances and is never a unique best reply. Intuitively this is because the first stage is sealed-bid, with at most one offer per bidder, and players have no guarantee that they could outbid the opponent in the second period, once after the first round they realise that their bid is not the best.

Keywords eBay · Nash equilibrium · Last minute bidding · System congestion

1 Introduction
Over the last two decades internet auctions attracted much attention [1–12], both because of their large commercial success but also because of some interesting empirical evidence concerning bidders’ behavior, which has been considered as puzzling. This paper focuses on one such main intriguing features, the so called “late
“bidding” phenomenon also named *sniping*, first identified by Roth-Ockenfels [13] in sale auctions conducted by the eBay and Amazon sites [14, 15]. The two auction sites adopted a “second price” design, which can either be seen as an extension to a dynamic context of the Vickrey (1961) second-price sealed bid auction, or alternatively as a generalization of the standard oral English ascending auction.

Though they shared a number of similarities, a main difference between the two auctions sites is their end-rule, which can account for rather different bidding behavior by participants [16, 17]. In particular, the phenomenon of late bidding appears to be much more visible in eBay auctions, characterized by a fixed-end time length. For this reason, in the paper we concentrate on them. Unlike Amazon, where an auction could be extended beyond the initially predefined time length until no offer is received within the last ten minutes from the last valid bid, in e-Bay the auction time length cannot be changed, independently of when bidders submit their bids. As a consequence, because of system traffic in eBay a bid too close to the end of the auction may have positive probability of not being processed by the system.

Single-object eBay auctions work as follows. Participants submit their offers, which the system receives and conceals to the other bidders; the system interprets the offers as bidders’ maximum willingness to pay for the object. What instead becomes commonly known, publicly visible on the site screen, is the current price namely the price at which the object would be sold should the auction end at that time. The current price is computed by adding to the second highest offer, received by the system, a (small) monetary fee $\epsilon > 0$ fixed by eBay before the auction starts. This is the price at which the bidder who submitted the highest offer wins and pays the object. For this reason, this is a second-price type of auction; moreover, the auction is ascending because bidders, while competing, cannot submit bids lower than those they previously placed.

Given that the winner pays less than what she offers, and since the highest offer is kept secret by the system, the eBay site would encourage participants to place their maximum willingness to pay early in the auction, which would then be handled by the system in the way we described [18]. This bidding strategy is called by eBay *proxy bidding*.

In doing so the site seems to translate to this extended version of the Vickrey design a main feature of the standard second price sealed-bid format, namely that bidding the own object evaluation is a weakly dominant strategy. Clearly, if followed, the indication would not only induce participants to bid early in the auction but would also prevent multiple individual offers. Empirical evidence however is often, though not always [19, 20], inconsistent with this. Indeed, during the auction bidders may offer more than once, though early offers may not be competitive, while most of the (winning) bids concentrate around the very end of the competition. The *intensity* of late bidding has however different degrees, depending upon (i) the end-rule of the auction and (ii) the type of good being auctioned. In particular, as we remarked earlier, it is more likely to occur with a fixed-end, but also in common value auctions for example such as those in which antiques are sold.

Among the explanations offered for late bidding, summarized and extensively discussed by [2, 3, 13] identify the possibility to prevent incremental bidding, or naive bidding (when the eBay auction is misunderstood to be a first price English
Last minute only bidding is implausible in eBay sealed bid Auction. Bajari-Hortascu [1] emphasize the presence of late bidding as a way to prevent other bidders to learn from one’s offer, which instead may take place when bids are placed early in the competition. Indeed, bidding near the end would reduce, if not eliminate, the possibility of being outbid. This argument would be particularly cogent in common value auctions, where participants are uncertain about the object value, and learning from others is important. In fact, it is in a common value framework that they characterize late bidding as a symmetric Nash Equilibrium of the eBay auction game. Other contributions have instead emphasized the sequential nature of eBay auctions [21, 22], within which late bidding could be rationalized.

Furthermore, Ockenfels-Roth [23] proposed a game theoretic characterization of eBay auctions where they show that no weakly dominant strategy exists, and formalize late bidding as a Nash Equilibrium with both common and private values. The equilibrium discussed is based on the idea that late only bidding may take place as an outcome of tacit collusion among players, a strategy adopted to avoid starting a price war early in the auction. Such collusion is sustained by forms of trigger strategies, which could be credibly implemented because the early part of the auction takes place in continuous time, allowing for multiple bids to be placed before the end. Their results hinge on the probability $p > 0$, for a last-minute bid to be successfully placed. In their model however, as long as $p$ is less than one its value plays no specific role.

Within a simple single object private value, two-players and two-periods, model this paper studies late bidding focusing on the possible strategic role of internet traffic, system congestion, as formalized by the probability value for a successful late bid. The two stages are modelled as sealed-bid [24], where players bid simultaneously. At each stage players can submit at most one offer; after the first stage bids are publicly revealed before the second round of bids. In the first stage all bids are accepted with certainty while, as well as in eBay, in the second stage only probabilistically. For this reason, our model modifies the one adopted online by eBay, presenting a variation which to our knowledge has not been investigated yet in the literature. Conceptually the main difference with respect to the eBay auction is given by the first period which is framed as a single round sealed bid phase, and below we discuss how this could meaningfully affect bidders’ behavior.

Because of such modification and system congestion the model exhibits some novel results which, although our framework differs from the online eBay auction, interestingly appear compatible with some recent empirical evidence [20]. Indeed, such evidence discusses how squatting, namely early bidding, also seems to take place in some auctions’ categories.

First, it suggests that last minute only bidding equilibria may have low plausibility, since it depends on very specific circumstances on system congestion. In particular, a necessary and sufficient condition for such equilibria to exist is that when the system receives two late bids it will process at least one of them. We call this absence of joint congestion. Moreover, even when optimal against an opponent’s strategy, late only bidding is never a unique best reply. Intuitively this is because the first stage of the auction, being sealed-bid, does not allow players to punish the opponent, when deviating from a tacitly collusive behavior to prevent triggering a price war early in the auction. Yet despite the eBay site suggesting to bid early,
and in so doing perhaps revealing a preference for early bidding as compared to late only bidding, in our model such preference may be reversed since the auctioneer’s expected revenue may be higher in the latter case. The paper is structured as follows. In Sect. 2 we expose the model and the main results while Sect. 3 concludes the work.

2 The model and main results

We consider an independent private value auction with complete information, a single object on sale and two players (c and d) submitting price offers to buy it, whose object (reservation) values are given by \( v_c > v_d > 0 \). In the Appendix we shall also consider an extension of the model to three bidders. At each round players submit their bids simultaneously, though sequentially along the two dates (stages) \( t = 0, 1 \). Date \( t = 0 \) is the “early part” and \( t = 1 \) is the final part (“last minute”) of the auction. The two bidding stages are sealed bid, and at each date a player can submit at most one price. After having bid at \( t = 0 \), and before bidding at \( t = 1 \), price offers are revealed to players. The main difference between the two periods is that at \( t = 0 \) all bids are accepted with certainty by the system while, instead, at \( t = 1 \) their acceptance is probabilistic. Though the eBay site adopts a different criterion for tie breaking to simplify the analysis, with no major loss of generality, in case of ties we assume player c will obtain the object.

We begin with the following definition of a bidding (pure) strategy. This is the notion of strategy that we shall focus on in the paper where, with no meaningful loss of generality, we shall not consider mixed strategies.

**Definition 1 (Bidding Strategy)** A (pure) bidding strategy, for player \( i = c, d \) in the auction game, is a pair of numbers \( b_{i(0)} = (b_{i(0)}(0) = x; b_{i(1)} = y = y(m(0))) \), with \( x, y \geq 0 \), defined as follows. If \( b(0) = (b_c(0); b_d(0)) \in \mathbb{R}_+^2 \) specifies the bids submitted by the two players to the system at \( t = 0 \), then it is \( m(0) = \max(b_c(0); b_d(0)) \) and \( b(1) = (b_c(1); b_d(1)) : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+^2 \) is a function mapping any \( b(0) \) into a pair of price offers, one for each player, at date \( t = 1 \), such that the following holds. No bidding at \( t \) is indicated by \( b_{i(t)} = 0 \), which is not allowed as a bid. If \( b_i(1) > 0 \) then it must be \( m(0) < b_i(1) \), for both \( i = c, d \).

Few comments on the above definition, are in order. It specifies that either a player does not bid at \( t = 1 \) or otherwise his bid must be higher than \( m(0) = \max(b_c(0); b_d(0)) \) since the auction is ascending and bids in the second stage must be larger than the best bid in the first stage. More in general, \( b_c(1) = y(m(0)) \) is defined as a function of \( m(0) \), hence of \( b(0) \), that is conditional on bids at time \( t = 0 \). For example,

\[
b_c(1) = \begin{cases} 
    y > m(0) & \text{if } 0 < m(0) < v_c \\
    0 & \text{otherwise}
\end{cases}
\]
is a properly defined $b_c(1)$ which is not expressed by any number, rather by only those which satisfy the inequality $y > m(0)$, which makes $b_c(1)$ conditional on the bidding history $b(0)$.

For our purposes however, to simplify the notation and without much loss of generality, in what follows we consider $b_i(1)$ just as a number, bearing in mind that $(b_x(0) = 0; b_x(0) = y; b_x(0) = 0; b_x(0) = w), (b_x(0) = x; b_x(0) = x; b_x(0) = w)$, with $x, w > 0$ are the possible four types of bids profiles in the first stage of the game. The reason why such simplification does not represent a problem in the paper is because our analysis will focus on specific findings, where the general definition of $b_i(1)$, conditional on $b(0)$, is implicit in the reasoning.

Still as an example, define strategy $b_{c(eBay)}$ for player $c$ as $b_{c(v,0)}$; that is,

$$b_c(0) = v_c \text{ and } b_c(1) = 0$$

while any strategy $b_{c(xy)}$ for player $c$, is defined by the following second bid

$$b_c(1) = \begin{cases} 
y > 0 & \text{if } m(0) < y \\
0 & \text{otherwise}
\end{cases}$$

We now develop and formalize the idea of system congestion, inspired by internet traffic.

**Definition 2** (System congestion) Consider the joint events.

$$\{b_i(t) \geq 0 \text{ is (not) accepted}; b_{-i}(t) \geq 0 \text{ is (not) accepted}\},$$

with $t = 0, 1, i = c, d, -i$ being $i$'s opponent, and let

$$P(b_i(t) \geq 0 \text{ is (not) accepted}; b_{-i}(t) \geq 0 \text{ is (not) accepted})$$

be their probability. Then

(i) $P(b_i(t) = 0 \text{ is accepted}; b_{-i}(t) = 0 \text{ is accepted}) = 1$ for $t = 0, 1$

(ii) $P(b_i(0) \geq 0 \text{ is accepted}; b_{-i}(0) \geq 0 \text{ is accepted}) = 1$

(iii) $P(b_i(1) > 0 \text{ is accepted}; b_{-i}(1) = 0 \text{ is accepted}) = \alpha$

(iv) $P(b_i(1) > 0 \text{ is accepted}; b_{-i}(1) > 0 \text{ is accepted}) = \lambda - \delta$

(v) $P(b_i(1) > 0 \text{ is accepted}; b_{-i}(1) > 0 \text{ is accepted}) = \delta$

(vi) $P(b_i(1) > 0 \text{ is not accepted}; b_{-i}(1) > 0 \text{ is not accepted}) = \mu = 1 - 2\lambda + \delta$

with $0 \leq \delta \leq \lambda \leq \alpha \leq 1$.

(Single congestion) if $\lambda < \alpha$ then the system exhibits single congestion.
(Joint congestion) if $\mu > 0$ then the system exhibits joint congestion.

Points (i) and (ii) of the definition formalize the idea that at $t = 0$ all bids are accepted by the system; namely, there is no meaningful system traffic in the first stage of the auction.

At $t = 1$ we consider two possible cases, depending upon the number of submitted bids. Conditional on player $i$ only bidding, there is a positive probability $1 - \alpha$ that her offer would not get through just because of the underlying level of traffic. However, if both players submit a price traffic may increase and there are four possibilities: no bid is accepted by the system, only player $c$’s bid goes through, otherwise only player $d$’s bid or both bids are accepted. The probability that player $i$’s offer is accepted is $\lambda$. We assume $\lambda \leq \alpha$. To capture the idea that at $t = 1$ network traffic may increase due to the opponent’s bid. As above, we define $\lambda < \alpha$ as single congestion, to mean that for a single individual the opponent’s bid generates a negative externality, with the difference $\alpha - \lambda > 0$ quantifying the extent of such externality. If $\alpha = 1$ and $0 < \lambda < 1$, then the only reason for traffic congestion would be the opponent’s bid, while $\lambda = \alpha$ means that the opponent’s activity imposes no negative externality on player $i$’s probability for a successful late bid. This last case is what Ockenfels-Roth [23] consider. Finally, $\delta$ is the probability that both late offers would go through, which implies that the probability of at least one bid being accepted is $2\lambda - \delta$. Hence, $\mu = 1 - 2\lambda + \delta$ is the probability that no late bid would be accepted by the system; we define the case $\mu > 0$ to be of joint congestion since it is possible that no bid would go through. Note that because $\mu = 1 - 2\lambda + \delta$ then, given the constraint $\alpha \geq \lambda \geq \delta$, it follows that

$$\lambda = \min\left(\alpha, \max\left(\frac{1 - \mu + \delta}{2}, \delta\right)\right)$$

Since at $\delta = 0$ it is $\frac{1 - \mu}{2} \geq 0$ then

$$\max\left(\frac{1 - \mu + \delta}{2}, \delta\right) = \begin{cases} \frac{1 - \mu + \delta}{2} & \text{if } \delta \leq 1 - \mu \\ \delta & \text{otherwise} \end{cases}$$

where $\delta = 1 - \mu$ solves the equation $\frac{1 - \mu + \delta}{2} = \delta$.

Moreover, (conditional) independence would require $\delta = \lambda^2$, and so that $\mu = 1 - 2\lambda + \lambda^2 = (1 - \lambda)^2$ is uniquely solved by $\lambda = 1 - \sqrt{\mu}$. Hence, $\delta \neq \lambda^2$ implies correlation in the success (failure) of last-minute bids. In particular, $\delta = \lambda > 0$ entails

$$P(b_i(1) > 0 \text{ is accepted}|b_{-i}(1) > 0 \text{ is accepted}) = \frac{\delta}{\lambda} = 1$$

that is perfect positive correlation, while $\delta = 0$ and $\lambda > 0$ implies
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\[ P(b_i(1) > 0 \text{ is accepted} | b_{\neg i}(1) > 0 \text{ is accepted}) = \frac{\delta}{\lambda} = 0 \]

namely perfect negative correlation.

Therefore, with independent bids, for all \(0 \leq \lambda < 1\) there is joint congestion, that is strictly positive probability that no offer will be accepted. In what follows, presence or absence of joint congestion will play an important role for the possibility of last minute only (LMO) bidding equilibria.

Finally, it is worth pointing out that joint congestion does not necessarily entail single congestion and vice versa. Indeed, it could be that either \(\mu > 0\) and \(\lambda = \alpha\), or \(\mu = 0\) and \(\lambda < \alpha\) as the following numerical examples show. Suppose \(\lambda = \alpha = 0.5\) and \(\delta = 0.4\); then \(\mu = 0.4\) and there would be joint congestion only. Assume instead \(\alpha = 0.8\), \(\delta = 0.4\) and \(\lambda = 0.7\); then \(\mu = 0\) and there is single congestion only.

Finally, with no major loss of generality, we assume that a non-eligible bid, submitted in the second stage, will be rejected by the system and treated as a non-bid.

We now introduce the following notion.

**Definition 2 (Offer)** For each \(i = c, d\) we define

\[ O(b_{i(t)}(x,y)) = \max(x,y) \text{accepted by the system} \]

player \(i\)’s offer associated to strategy \(b_i\).

Again, let \(b_c = (b_c(0) = 10; b_c(1) = 20)\) and suppose that only \(b_c(0) = 10\) is accepted by the system. Then \(O(b_c) = b_c(0) = 10\), despite \(b_c(1) = 20\) being higher than \(b_c(0)\).

In general terms, the payoff function of player \(c\) (the one for player \(d\) is analogous) is then given by

\[ \Pi_c(b_c,b_d) = \begin{cases} v_c - O(b_d) - \epsilon & \text{if } O(b_c) \geq O(b_d) \\ 0 & \text{otherwise} \end{cases} \]

Therefore the auction is second price and the payoff function is defined in terms of the two players’ bidding strategies; its components are however specified by the players’ offers. The quantity \(\epsilon \geq 0\) is a (small) monetary fee fixed by the internet auction site which, for simplicity, we assume to be constant though, in general, could depend upon the amount offered. To avoid trivial conclusions, throughout the paper we assume \(v_i > \epsilon, i = c, d\).

We can now give the definition of pure strategy Nash equilibrium; since we shall not consider mixed strategies we’ll refer to it simply as Nash equilibrium (NE).

**Definition 3 (NE)** A NE in the auction game is a pair of strategies \((b^*_c, b^*_d)\) such that.

\[ E\Pi_i(b^*_i, b^*_d) \geq E\Pi_i(b_{\neg i}, b^*_i) \]

for \(i = c, d\) and all \(b_i\)

The above definition is given in terms of expected payoffs, since when players submit a price at \(t = 1\) bid acceptance is probabilistic.
The following result presents a first important characterization of the bidding strategy suggested by eBay

\[ b_{i(eBay)} = (b_i(0) = v_i; b_i(1) = 0) \]

which consists in submitting one’s reservation price only, early in the auction. The suggestion is inspired by the standard (one stage) sealed-bid second price Vickrey auction, where bidding the own value is a weakly dominant strategy. However both eBay and our auction are dynamic, with system congestion in the second period, and that would make a difference with respect to the static version.

Indeed, assuming for simplicity a minimum price equal to zero, the next finding may provide a first insight on why bidders seem to be reluctant to follow the recommendation of choosing strategy \( b_{i(eBay)} \). Though the two models are somewhat different, an analogous result is also obtained by Ockenfels-Roth [23].

**Proposition 1** For no \( \epsilon > 0 \) strategy \( b_{i(eBay)} \) is weakly dominant.

**Proof** Fix \( \epsilon > 0 \), take player \( c \) and consider strategy \( b_{d(w_0)} \) chosen by \( d \), such that \( w < v_c < w + \epsilon \). Then, considering \( b_{c(x_0)} \), with \( x < w \) it follows that.

\[
E\Pi_c(b_{c(x_0)}, b_{d(w_0)}) = 0 > (v_c - w - \epsilon) = E\Pi_c(b_{c(eBay)}, b_{d(w_0)})
\]

which implies that \( b_{c(eBay)} \) is not weakly dominant for player \( c \). Similar considerations hold for player \( d \) and the result follows QED. □

In the above Proposition system congestion plays no role, as when players only bid early in the auction strategy \( b_{i(eBay)} \) is not weakly dominant simply because of the presence of \( \epsilon > 0 \). However, as we shall see below, there could be other more effective, and interesting, reasons for \( b_{i(eBay)} \) failing to be weakly dominant, which hold even with \( \epsilon = 0 \).

The following proposition makes it explicit that system congestion can have a role when players consider late bidding. In particular, it suggests that LMO bidding may be a NE under specific conditions.

**Proposition 2** Suppose \( \alpha \geq \lambda \geq \delta \) and \( \mu > 0 \); then for no pair of numbers \( y > 0, z > 0 \) strategies \( b_{c(0y)}, b_{d(0z)} \) are a NE. Furthermore, for all \( \mu \geq 0 \) and \( \alpha > \lambda \) strategy \( b_{i(eBay)} \) is not best reply against LMO bidding.

**Proof** Suppose \( \mu > 0 \), which implies \( (1 - \lambda) > (\lambda - \delta) \), and assume \( d \) plays \( b_{d(0z)} \). Then if \( c \) chooses \( b_{c(0y)} \) his expected payoff will be.

\[
E\Pi_c(b_{c(0y)}, b_{d(0z)}) = \begin{cases} 
\delta (v_c - z - \epsilon) + (\lambda - \delta) (v_c - \epsilon) & \text{if } z \leq y \\
(\lambda - \delta) (v_c - \epsilon) & \text{if } y < z 
\end{cases}
\]

Assume instead \( c \) would choose strategy \( b_{c(xy)} \), with \( x > 0 \), then
Suppose \( v_c - z - \varepsilon \geq 0 \) then the maximum payoff in (1) is \( \delta(v_c - z - \varepsilon) + (\lambda - \delta)(v_c - \varepsilon) \) while in (2) is \( \delta(v_c - z - \varepsilon) + (1 - \lambda)(v_c - \varepsilon) \) which, because \( \mu > 0 \) and so \( (1 - \lambda) > (\lambda - \delta) \), is higher than the former. If \( (v_c - z - \varepsilon) < 0 \) then \( (\lambda - \delta)(v_c - \varepsilon) \) is maximum in (1) and \( (1 - \lambda)(v_c - \varepsilon) \) is in (2) which, still due to \( \mu > 0 \), is again higher than the former.

Though the above already proves the first part of the result, for completeness, assume now \( c \) plays \( b_{c(t_0)} \). Then if \( d \) selects \( b_{d(0, z)} \) his expected payoff will be

\[
E\Pi_c(b_{c(t_0)}, b_{d(0, z)}) = \begin{cases} 
\delta(v_c - z - \varepsilon) + (1 - \lambda)(v_c - \varepsilon) & \text{if } x < z \leq y \\
(1 - \lambda)(v_c - \varepsilon) & \text{if } y < z
\end{cases}
\] (2)

if instead \( d \) selects strategy \( b_{d(w, z)} \), with \( w > 0 \), then

\[
E\Pi_d(b_{c(t_0)}, b_{d(z)}) = \begin{cases} 
\delta(v_d - y - \varepsilon) + (\lambda - \delta)(v_d - \varepsilon) & \text{if } y < z \\
(\lambda - \delta)(v_d - \varepsilon) & \text{if } z \leq y
\end{cases}
\] (3)

As well as for (1) and (2), \( \mu > 0 \) implies that \( b_{d(0, z)} \) is not best reply for player \( d \), which proves the first part of the proposition.

To prove the second part suppose again player \( d \) chooses \( b_{d(0, z)} \); then

\[
E\Pi_c(b_{c(t_0)}, b_{d(0, z)}) = (1 - \alpha)(v_c - \varepsilon)
\] (5)

Therefore, if \( (v_c - z - \varepsilon) \geq 0 \) it’s easy to check that \( \delta(v_c - z - \varepsilon) + (1 - \lambda)(v_c - \varepsilon) \) is the maximum in (2) and, because \( (1 - \lambda) > (1 - \alpha) \), it is higher than (5). Likewise, if \( (v_c - z - \varepsilon) < 0 \) then the maximum in (2), \( (1 - \lambda)(v_c - \varepsilon) \), is again larger than (5). QED.

Therefore, according to Proposition 2 with joint congestion, \( \mu > 0 \), LMO bidding cannot be a NE as when the opponent bids late only, for a player is strictly best to play both early and late. Moreover, the second part shows that with single congestion, \( \alpha - \lambda > 0 \), the eBay strategy cannot be weakly dominant. \( \square \)

The above result is not in Ockenfels-Roth [23], since in their model LMO bidding equilibrium obtains with joint congestion. As we already mentioned, this is due to the different frameworks in the early part of the auction.

How the main findings of Proposition 2 could extend to more than 2 players is discussed in the Appendix.

The following Corollary shows that without single congestion the eBay strategy could be optimal against late bidding.

**Corollary 1** Suppose \( 1 > \alpha = \lambda \geq \delta \) and \( \mu \geq 0 \); then \( b_{LMO} \) could be best reply to LMO bidding.
**Proof** Consider \( (v_c - z - \varepsilon) \leq 0 \) and \( (v_d - y - \varepsilon) \leq 0 \). Since \( \alpha = \lambda \) then the maximum in both (2) and (4) coincides with (5), so that \( b_{(eBay)} \) can be best reply against LMO bidding. QED. 

The next result clarifies that LMO bidding can be a NE if and only if joint congestion is precluded.

**Proposition 3** Suppose \( \lambda < 1 \); then strategy profiles \( (b_c(0y), b_d(0z)) \) can be NE of the game if and only if \( \mu = 0 \).

**Proof** If \( \lambda < 1 \) and \( \mu = 0 \) it is \( (1 - \lambda) = (\lambda - \delta) \). Hence, expressions (1) and (2) take the same values and the same happens for expressions (3) and (4). As a consequence truthful bidding, for example, that is the pair \( (b_c(0y), b_d(0z)) \) with \( y = v_c - \varepsilon \) and \( z = v_d - \varepsilon \) is a NE of the game. Indeed, in this case \( (v_c - z - \varepsilon) > 0 \) and from (1) it follows that \( y \), such that \( y \geq z \), is best for player \( c \), hence also \( y = v_c - \varepsilon \). Moreover, \( (v_d - y - \varepsilon) < 0 \) and from (3) it follows that \( z \), such that \( y \geq z \), is best for player \( d \), hence included \( z = v_d - \varepsilon \). It is immediate to check that other LMO bidding pairs can be NE of the game such as, for instance, the pairs \( (b_c(0y), b_d(0z)) \), with \( y < v_d - \varepsilon \) and \( z > v_c - \varepsilon \). Finally, it can be verified that necessity follows immediately. QED. 

Notice that under the same conditions of the above proposition there is also a multiplicity of NE where players bid both early and late, and which provide the same expected payoff to players as \( (b_c(0y), b_d(0z)) \).

These results and considerations can be summarized as follows.

**Corollary 2** Take any \( \mu \geq 0 \). Then against LMO bidding, for a player submitting at both dates is always a best reply.

That is, against LMO bidding is best to bid at both stages. This point could be further extended, as the next proposition suggests. \( z > v_c - \varepsilon \)

**Proposition 4** LMO bidding is never a unique best reply.

**Proof** To prove the proposition we need to show that, against any possible strategy adopted by the opponent, either LMO bidding is not best reply or, if it is optimal, is not the unique best reply.

To see this start with player \( c \) and suppose \( d \) chooses \( b_d(wz) \). If \( c \) chooses \( b_c(0y) \) his expected payoff would be

\[
E\Pi_c(b_c(0y), b_d(wz)) = \begin{cases} 
(\lambda - \delta)(v_c - w - \varepsilon) + \delta(v_c - z - \varepsilon) & \text{if } z \leq y \\
(\lambda - \delta)(v_c - w - \varepsilon) & \text{if } w \leq y < z
\end{cases}
\]

while if \( c \) selects \( b_c(xy) \), with \( x > 0 \), his expected profit will be
for \( z \leq y \) and equal to

\[
E\Pi_c(b_{c(xy)}, b_{d(wz)}) = \begin{cases} 
(\lambda - \delta + \mu)(v_c - w - \varepsilon) + \delta(v_c - z - \varepsilon) & \text{if } w \leq x < y \\
(\lambda - \delta)(v_c - w - \varepsilon) & \text{if } x < w \leq z \\
(\lambda - \delta)(v_c - w - \varepsilon) & \text{if } x < w \leq z
\end{cases}
\]

for \( y < z \).

Suppose now \( v_c - \varepsilon > z \) then \((\lambda - \delta)(v_c - w - \varepsilon) + \delta(v_c - z - \varepsilon)\) is the maximum in (6) while \((\lambda - \delta + \mu)(v_c - w - \varepsilon) + \delta(v_c - z - \varepsilon)\), which is no lower than the former, is the maximum in (7). If \( w < (v_c - \varepsilon) \leq z \) then \((\lambda - \delta)(v_c - w - \varepsilon)\) is the maximum in (6) while \((\lambda - \delta + \mu)(v_c - w - \varepsilon)\), which again is no lower, is maximum in \((7')\). Finally, if \( v_c - \varepsilon \leq w \) it follows that no bidding for player \( c \) is best in (6), (7) and \((7')\).

Similarly, if player \( d \) bids early only, adopting strategy \( b_{d(w0)} \), player \( c \)'s expected payoff when choosing \( b_{c(0y)} \) is given by

\[
E\Pi_c(b_{c(0y)}, b_{d(w0)}) = a(v_c - w - \varepsilon)
\]

while if \( c \) chooses strategy \( b_{c(xy)} \) then

\[
E\Pi_c(b_{c(xy)}, b_{d(w0)}) = \begin{cases} 
v_c - w - \varepsilon & \text{if } w \leq x \\
a(v_c - w - \varepsilon) & \text{if } x < w \leq y
\end{cases}
\]

and it is easy to check that also in this case LMO is not a unique best reply.

Analogous considerations hold for player \( d \) and the result is proved. QED. □

Below we see that the profile \((b_{c(eBay)}, b_{d(eBay)})\) can be a NE under any condition on system traffic.

**Proposition 5** Suppose \((v_c - v_d - \varepsilon) \geq 0, 1 \geq \alpha \geq \lambda \geq \delta \) and \( \mu \geq 0 \); then the strategy profile \((b_{c(eBay)}, b_{d(eBay)})\) is a NE of the game.

**Proof** Suppose player \( d \) chooses \( b_{d(eBay)} \); if \( c \) adopts \( b_{c(xy)} \) his expected profit would be.

\[
E\Pi_c(b_{c(xy)}, b_{d(eBay)}) = \begin{cases} 
v_c - v_d - \varepsilon & \text{if } v_d \leq x \\
a(v_c - v_d - \varepsilon) & \text{if } x < v_d
\end{cases}
\]

while if \( c \) adopts \( b_{c(eBay)} \) then \( E\Pi_c(b_{c(eBay)}, b_{d(eBay)}) = (v_c - v_d - \varepsilon) \). Therefore, if \((v_c - v_d - \varepsilon) \geq 0\) strategy \( b_{c(eBay)} \) is best against \( b_{d(eBay)} \), since \((v_c - v_d - \varepsilon)\) is the largest possible payoff for \( c \).

Assume now player \( c \) chooses \( b_{c(eBay)} \); if \( d \) adopts \( b_{d(w2)} \) his expected profit would be.
\[ E\Pi_d(b_{c(eBay)}, b_{d(wc)}) = \begin{cases} (v_d - v_c - \epsilon) & \text{if } v_c \leq w \\ \alpha(v_d - v_c - \epsilon) & \text{if } w < v_c \end{cases} \]

while if \( d \) adopts \( b_{d(eBay)} \) then \( E\Pi_d(b_{c(eBay)}, b_{d(eBay)}) = 0 \). Therefore, since \((v_d - v_c - \epsilon) < 0 \) strategy \( b_{d(eBay)} \) is best against \( b_{c(eBay)} \) because 0 is the largest possible payoff for \( d \). □

3 Conclusions

In the paper we considered a two-bidders, two-stages sealed bid eBay type-of-model with one object on sale and complete information [25]. Inspired by eBay auctions, we assumed that in the second stage, because of system traffic congestion, bids may have positive probability of not being processed by the system. However, unlike eBay and related contributions in the literature, we modelled the first stage as a simultaneous, sealed bid, phase. We discussed how late only bidding equilibria have low plausibility, as they are based on a specific necessary and sufficient condition of system traffic, which we defined as absence of joint congestion. This is when the probability that no last-minute bid will be processed is zero.

We also argued how such seemingly low plausibility is due to the early part of the auction being structured as a single stage sealed bid. Indeed, this would prevent sustaining tacit collusion leading to late only bidding, that could hinge on trigger strategies. Finally, we also showed that, even when it is a best reply against a choice by the opponent, late only bidding is never the unique optimal strategy.

To summarise, we believe the paper findings can provide interesting insights on how to organize e-auctions to mitigate LMO bidding, when late bidding is undesirable for the auctioneer. Such two-stages auction should be sufficiently easy to implement on e-platforms, and the relevant congestion parameters easy to set. In the paper we considered that after the first stage both bids are publicly observable, but a variation of the model could only allow public observability of the lowest first stage bid, as in eBay where only the highest bid is kept hidden by the system until the end. Moreover, no first bid observability before the second bid could also be considered. In both cases we conjecture that the main findings of the model may not change much.

Appendix

In this section we discuss how the main findings of Proposition 1 could extend to more than 2 players. We do so by considering the simplest extension, to 3 players, to show how the logic underlying the proof may generalize. The generic model with \( n > 2 \) number of players would need much more cumbersome notation, however conveying a similar intuition.
Consider three players \((c, d, e)\) whose object (reservation) values are given by \(v_c > v_d > v_e > 0\). In this case, to simplify, below we only discuss how the relevant success probabilities modify, when the three players all bid at \(t = 1\). It remains of course true that all parameters values are such that probabilities fall within the closed unit interval.

**Definition 4 (Joint congestion with three players)** Consider the joint events when \(b_i(1) > 0\) for \(i = c, d, e\)

1. \(P(b_i(1) > 0 \text{ is accepted}) = \lambda\)
2. \(P(b_-(1) > 0 \text{ is accepted}; b_i(1) > 0 \text{ is not accepted}) = \delta\)
3. \(P(b_i(1) > 0 \text{ is accepted}; b_-(1) > 0 \text{ is accepted}) = \gamma\)

Hence

4. \(P(b_i(1) > 0 \text{ is accepted}; b_-(1) > 0 \text{ is not accepted}) = \lambda - (2\delta + \gamma)\)
5. \(P(b_i(1) > 0 \text{ is not accepted}; b_-(1) > 0 \text{ is not accepted}) = \mu = 1 - 3(\lambda - \delta) + 2\gamma\)

with \(0 \leq \gamma \leq \delta \leq \lambda \leq \alpha \leq 1\).

**(Absence of joint congestion)** The system has no joint congestion if \(\mu = 0\) hence if \(\lambda = \frac{1+2\gamma}{3} + \delta\).

Notice that \(\mu = 0\) implies \(\frac{1}{3} \leq \lambda\) that is, consistently with the intuition, as compared to the model with two players absence of joint congestion can obtain with individual bids having lower success probability. Moreover, \(\lambda = 1\) obtains according to the combinations of \(\delta\) and \(\gamma\) satisfying \(\delta = \frac{2(1-\gamma)\gamma}{3}\) with \(0 \leq \gamma \leq 1\) and so \(0 \leq \delta \leq \frac{2}{3}\).

It is now possible to extend Proposition 2 as follows.

**Proposition 6** Suppose \(1 > \lambda \geq \delta \geq \gamma\) and \(\mu > 0\); then for no triples \(y > 0, z > 0, u > 0\), strategies \(b_{c(0y)}, b_{d(0z)}, b_{e(0u)}\) are a NE.

**Proof** Suppose \(\mu > 0\), which implies \((1 - 2\lambda + \delta) > (\lambda - 2\delta - 2\gamma)\), and assume \(d\) and \(e\) play, respectively, \(b_{d(0z)}\) and \(b_{e(0u)}\), with \(z > u\) (a similar reasoning applies for \(u \geq z\)). Then if \(c\) chooses \(b_{c(0y)}\) his expected payoff will be.

\[
\begin{align*}
\mathbb{E} p_i(b_{c(0y)}, b_{d(0z)}, b_{e(0u)}) &= \begin{cases}
(\delta + \gamma)(v_c - z - \epsilon) + \delta(v_e - u - \epsilon) + (\lambda - 2\delta - \gamma)(v_c - \epsilon) & \text{if } z \leq y \\
\delta(v_e - u - \epsilon) + (\lambda - 2\delta - \gamma)(v_c - \epsilon) & \text{if } u \leq y < z \\
(\lambda - 2\delta - \gamma)(v_c - \epsilon) & \text{if } y < u
\end{cases}
\end{align*}
\]

Assume now \(c\) would choose strategy \(b_{c(xy)}\), with \(x > 0\), then
Following a similar argument as in Proposition 1, whatever the sign of \((v_c - z - \varepsilon)\) and \((v_c - u - \varepsilon)\), because \((1 - 2\lambda + \delta) > (\lambda - 2\delta + 2\gamma)\) payoffs in (11) can always be larger than payoffs in (10), which proves the result. □

The intuition behind the result refers mainly to the probability of the largest outcome \((v_c - \varepsilon)\). When joint congestion is the case, and players bid both early and late, the likelihood of \((v_c - \varepsilon)\) strictly increases with respect to late bidding only, since in case no late bid is accepted the early bid would guarantee the auction victory and object award.

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References

1. Bajari, P., & Hortacsu, A. (2003). The winner’s curse, reserve prices, and endogenous entry: empirical insights from eBay auctions. *Rand Journal of Economics*, 34, 329–355.
2. Bajari, P., & Hortacsu, A. (2004). Economic insights from internet auctions. *Journal of Economic Literature*, 42, 457–486.
3. Bajari, P., & Hortacsu, A. (2004). Cyberspace auctions and pricing issues: A survey of empirical findings. *The new economy handbook*. Academic Press.
4. Barrot, C., Albers, S., Skiera, B., & Schafers, B. (2010). Vickrey vs eBay: Why second price sealed-bid auctions lead to more realistic price-demand functions. *International Journal of Electronic Commerce*, 14, 7–38.
5. Elfenbein, D., & Mc, M. B. (2010). Last minute bidding in eBay charity auctions. *Economics Letters*, 107, 42–45.
6. Hasker, K., & Sickles, R. (2010). eBay in the economic literature: Analysis of an auction marketplace. *Review of Industrial Organization*, 37, 3–42.
7. Hayne, S., Bugbee, B., & Wang, H. (2010). Bidder behaviours on eBay: Collectibles and commodities. *Electronic Markets*, 20, 95–104.
8. Steiglitz, K. (2007). *Snipers, shills and sharks. EBay and human behaviour*. Princeton University Press.
9. Stryszowska, M. (2013). Multiple and last-minute bidding in competing internet auctions. *Review of Economic Design*, 17, 273–305.
10. Varian, H. (2000). Economic scene: Online users as laboratory rats. New York Times, 16 November
11. Ward, S., & Clark, J. (2002). Bidding behavior in on-line auctions: An examination of the eBay Pokemon card market. *International Journal of Electronic Commerce, 6*, 139–155.

12. Wintr, L. (2008). Some evidence on late bidding in eBay auctions. *Economic Inquiry, 3*, 369–379.

13. Roth, A., & Ockenfels, A. (2002). Last minute bidding and the rules for ending second price auctions: Evidence from eBay and amazon auctions on the internet. *American Economic Review, 92*, 1093–1103.

14. Barbaro, S., & Bracht, B. (2021). Shilling, squeezing, sniping: A further explanation for late bidding in online second-price auctions. *Journal of Behavioral and Experimental Finance, 31*, 100553.

15. Schildler, J. (2003). Late bidding on the internet, University of Vienna

16. Ariely, D., Loewenstein, G., & Prelec, D. (2005). An experimental analysis of ending rules in internet auctions. *The Rand Journal of Economics, 34–6*, 890–907.

17. Muthitacharoen, A., & Tams, S. (2016). The role of auction duration in bidder strategies and auction prices. *International Journal of Electronic Commerce, 21*, 67–98.

18. Hortacsu, A., & Nielsen, E. (2010). Do bids equal values on eBay? *Marketing Science, 29*, 994–997.

19. Ely, J., & Hossain, T. (2009). Sniping and squatting in auction markets. *American Economic Journal Microeconomics, 1*, 68–94.

20. Groenwegen, P. (2017). Squatting, sniping, and online strategy: Analyzing early and late bidding in eBay auctions, Mimeo University of Yale

21. Kolodziejczyk, M. (2003). Late and multiple bidding in competing second price internet auctions, Mimeo University of Yale

22. Wang, J. T. (2006). *Is last minute bidding bad?* Mimeo Caltech.

23. Ockenfels, A., & Roth, A. (2006). Late and multiple bidding in second price internet auctions: Theory and evidence concerning different rules for ending an auction. *Games and Economic Behaviour, 55*, 297–320.

24. Zeithammer, R., & Adams, C. (2010). The sealed bid abstraction in online auction. *Marketing Science, 29*, 964–987.

25. Ockenfels, A., & Roth, A. (2001). The timing of bids in internet auctions: market design, bidder behavior and artificial agents. *Artificial Intelligence Magazine, 23*, 79–87.

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