High-Order Numerical Methods for the Thermal Activation of SMA Fibers

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The present paper establishes an axisymmetric benchmark model of a conducting loop, which implies an electromagnetic induction. Therein, the MAXWELL equations are demonstrated in a direct formulation (without a potential formulation) with its solution strategy. Standard high-order LAGRANGE shape functions and a harmonic ansatz are used to solve the electromagnetic behavior. Studies regarding the influence of a steel fiber in a block of concrete are analyzed.

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1 Motivation

An important concept in the improvement and reinforcement of concrete is to adapt its composition for enhanced properties. Ultra-High-Performance-Concrete can be produced with special additives and an improved packaging density. In addition, the strength in the pressure as well as in the tensile region can be brought about by introducing steel fibers. These steel fibers are intended to have certain shapes for improved strength, such as wave shape and or end-hook fibers. The advantage of the mechanical behaviors due to a modified design of the steel fibers is a disadvantage in the production process since the fresh still liquid concrete can no longer be properly incorporated in many components. Therefore, the idea is to use shape memory alloys (SMA), because these can be given an advantageous shape during processing and another advantageous shape for the mechanical properties by a change in the final state. SMA’s are characterized by the ability to adopt a previously imprinted shape due to thermal activation. This activation temperature would be applied in this case by a thermal aftertreatment, as used for the production of precast concrete products. In addition to experimental investigations, it is crucial to be able to simulate this new generation of concrete with the help of suitable numerical models. In order to be able to correctly predict these highly nonlinear processes during manufacturing as well as in the final stage, high-order methods in space and time are inalienable. The thermal activation in the aftertreatment can be generated either by contact heat or by electromagnetic induction. The main focus is on the investigation of high-order numerical methods in space and a harmonic ansatz for the thermal activation by the electromagnetic fields in a concrete material.

2 Weak Form of the Electromagnetic Field Equations

The basic relations of the electric and magnetic effects and their interactions are based on assumptions and experiments by FARADAY, AMPÈRE and GAUSS, cf. [4]. The fundamentals of classical electromagnetic field theory are embodied by MAXWELL’s equations in differential form, cf. [1]. In order to enable the application of the finite element method, the partial differential equations of MAXWELL and the NEUMANN boundary conditions, cf. [2], have to be formulated weakly through a multiplication with arbitrary test functions δE and δB and an integration over the volumes ΩE and ΩB. In addition, a strategy of DEMKOWICZ is applied in which the electromagnetic equations are extracted, by applying a further time derivative to the third and fourth Maxwell equations, cf. [1–3]. Hence, the weak forms δWE and δWB of the general initial-boundary value problem of electrodynamics can be derived:

\[ \delta W^E = \int_{\Omega_E} \delta E \cdot \epsilon \cdot \dot{E} \, dV + \int_{\Omega_E} \delta E \cdot \sigma \cdot \dot{E} \, dV + \int \nabla \times \left[ \kappa \cdot \delta E \right] \cdot \left[ \nabla \times E \right] \, dV \]

(1)

\[ + \int_{\Omega_E} \delta E \cdot J_{ext} \, dV + \int_{\Gamma_E} \nabla \cdot \delta E \kappa \cdot \nabla \cdot E \, dA + \int_{\Gamma_{ext}} \delta E \cdot \left[n \times \nabla \times E \right] \, dA = 0, \]

and

\[ \delta W^B = \int_{\Omega_B} \delta B \cdot \epsilon \cdot \dot{B} \, dV + \left[ \nabla \times \delta B \right] \cdot \left[ \nabla \times \kappa \cdot B \right] \, dV - \int_{\Omega_B} \left[ \nabla \times \delta B \right] \cdot \sigma \cdot E \, dV \]

(2)

\[ + \int_{\Gamma_B} \nabla \cdot \delta B \kappa \cdot \nabla \cdot B \, dA - \int_{\Gamma_{ext}} \left[ \nabla \times \delta B \right] J_{ext} \, dA + \int_{\Gamma_{ext}} \delta B \cdot \left[n \times \nabla \times B \right] \, dA = 0. \]

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3 Solution Procedure

To solve this multifield problem (1) - (2) a spatial discretization is realized using the finite element method. Therein, the primary variables \( E \) and \( B \), its time derivatives \( \dot{E}, \dot{\dot{E}} \) and \( \dot{B} \) and the test functions are approximated by \text{LAGRANGE} \ shape functions. For arbitrary test functions, the linear semidiscrete balance equation of electromagnetics is determined by evaluating the integrals of the spatially discretized terms with the Gauss integration and leads to:

\[
\begin{bmatrix}
M^{EE} & 0 \\
0 & M^{BB}
\end{bmatrix}
\begin{bmatrix}
\dot{E} \\
\dot{B}
\end{bmatrix}
+ \begin{bmatrix}
D^{EE} & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{E} \\
\dot{B}
\end{bmatrix}
+ \begin{bmatrix}
K^{EE} & 0 \\
0 & K^{BB}
\end{bmatrix}
\begin{bmatrix}
E \\
B
\end{bmatrix}
= \begin{bmatrix}
r^{EE} \\
r^{BB}
\end{bmatrix}.
\]

(3)

The ordinary differential equations of the electromagnetic fields are solved by means of a harmonic approach, cf. [2].

4 Axisymmetric Model of a Conducting Loop

To solve an axially symmetric model, the equations (3) must be converted from Cartesian coordinates into polar coordinates. In the \((r, \phi)\)-plane the electric current flows in the direction normal to this plane, thus only an electric field \( E_{\phi} \) and the magnetic components \( B_r \) and \( B_\phi \) occur. Fig. 1 left shows the dimensions (in millimeters), the boundary conditions, the material parameters and the load. In the present example, a concrete sample is surrounded by an induction coil. The dark gray marked concrete has in the inside a steel fiber (thick line), which represents a ring in an axial symmetrical case. The contour plot of the electric, magnetic and the Joule heating for the complete domain is depicted in the middle of Fig. 1. The electric field demonstrates clearly the penetration through the concrete (white dashed box) and the influence on the steel ring. Nothing can be recognized in the respective entire area of the magnetic field as well as of the Joule heating since the major influence is in the steel ring. For this purpose, the electric and magnetic field components and the Joule heat heating are plotted over the radial component in the center of the concrete on the right-hand side in Fig. 1 (marking of the plot is shown as a black solid line in the contour plots). While the electric field experiences a change in the steel ring, the magnetic field, as well as the Joule heating have both a strong peak in the steel ring.

5 Summary and Outlook

In the present paper an electromagnetic approach was depicted. Its numerical implementation using the finite element method together with a harmonic ansatz has been demonstrated and analyzed. It has been demonstrated that heating of iron-based steel fibers in a concrete body can be obtained by electromagnetic waves. In further investigations, the influence on many steel fibers in a block of concrete has to be investigated. At the same time, in order to be able to capture the influence on several fibers realistically, it is necessary to switch from an axially symmetric to a three-dimensional model. The electric field in Fig. 1 already shows that steel fibers lying one behind the other do not receive the same electromagnetic fields.

References

[1] L. Demkowicz, J. Kurts, D. Pardo, M. Paszyński, W. Rachowicz and A. Zdunek, Chapman & Hall/CRC Applied Mathematics and Nonlinear Science Series , (2007).
[2] T. Gleim, kassel university press GmbH, DOI 10.19211/KUP9783737602518, (2017).
[3] T. Gleim, B. Schröder and D. Kuhl, Archive of Applied Mechanics, 85(8), p. 1055-1073, (2017).
[4] F. Steinle, Franz Steiner Verlag, Boeithius, 50, (2000).

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