Zonal flow generation in collisionless trapped electron mode turbulence

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Abstract

In the present work the generation of zonal flows in collisionless trapped electron mode (TEM) turbulence is studied analytically. A reduced model for TEM turbulence is utilized based on an advanced fluid model for reactive drift waves. An analytical expression for the zonal flow growth rate is derived and compared with the linear TEM growth, and its scaling with plasma parameters is examined for typical tokamak parameter values.

1. Introduction

The study of generation and suppression of turbulence and transport in tokamak plasmas is still a high priority topic in theoretical and experimental magnetic fusion research. In recent studies, the important role played by nonlinearly self-generated zonal flows for the regulation of turbulent transport has been emphasized [1–3]. These are radially localized flows (wave-vector \(q = (q_x, 0, 0)\)), propagating mainly in the poloidal direction, which can reduce the radial transport by shearing the eddies of the driving background turbulence.

The turbulence and anomalous transport observed in tokamak plasmas are generally attributed to short-wavelength drift-type instabilities, driven by gradients in the plasma density, temperature, magnetic field, etc. For the hot core region of a tokamak plasma, the two main drift-wave candidates are the toroidal ion temperature gradient (ITG) mode and the collisionless trapped electron mode (TEM).

Accordingly, the generation of zonal flows by nonlinear interactions among drift waves has recently been studied both analytically [4–11] and numerically [12–23]. While substantial effort has been devoted to the study of zonal flow generation by ITG modes (see, e.g. [5, 7, 9] for an analytical treatment), very little work has been published on zonal flows driven by pure TEM [16, 17]. In tokamak plasma experiments, TEMs are expected to play a dominant role in the hot electron regime \((T_e > T_i)\), relevant for experiments with dominant central electron heating [24, 25] and in advanced confinement regimes with electron transport barriers.
652 J Anderson et al. The study of zonal flows and turbulence driven by TEM is therefore crucial for the assessment of these regimes. In addition, an estimate of the zonal flow generation close to marginal stability is essential in order to discriminate between the TEM and the potentially important electron temperature gradient (ETG) mode [26,27] in comparison with experimental profiles against linear thresholds. The experimental temperature gradients (or inverse scale lengths) are typically found to be about a factor of 2 above the linear thresholds, both for ITG and TEM. In the Cyclone study [23], this factor was 1.7 for the ITG mode. However, the nonlinear upshift, due to zonal flows, increased the effective ITG threshold by a factor 1.5, thus bringing it much closer to the experimental gradient. Although a comprehensive and quantitative investigation of zonal flow generation would require a nonlinear gyrokinetic treatment, a qualitative analytical study, based on a reduced set of fluid equations, is feasible and also more transparent in terms of physics interpretation.

In the present paper, the generation of zonal flows by pure TEM is studied analytically in the limit \( \eta_i = 0 \) where the ITG mode is suppressed, using a reduced fluid model for the trapped electron dynamics. A system of equations is derived which describes the coupling between the background TEM turbulence, described by a wave-kinetic equation, and the zonal flow modes generated by Reynolds stress forces. The qualitative analytical technique used here follows closely the WKB analysis employed in [7,9] for zonal flow generation by ITG turbulence. The purpose of the study is to obtain a qualitative estimate of the zonal flow growth rate driven by TEM and to compare it with an ITG-driven case and, in addition, examine its scaling with plasma parameters for typical tokamak parameter values.

The paper is organized as follows. In section 2 the model equations for the ITG/TEM system and a reduced model for TEM turbulence are presented. The equations describing the coupling between the background TEM turbulence and the zonal flows are presented in section 3 and the most explicit derivations are shown in appendix A. In section 4 the results are discussed and finally a summary is given in section 5.

2. Reduced model for trapped electron modes

The description used for the coupled toroidal ITG and collisionless TEM system is based on the continuity and temperature equations for the ions and the trapped electrons [28]:

\[
\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j \vec{v}_E + n_j \vec{v}_{sj}) + \nabla \cdot (n_j \vec{v}_P + n_j \vec{v}_{\pi j}) = 0,
\]

(1)

\[
\frac{3}{2} \frac{n_j}{T_j} \frac{dT_j}{dt} + n_j T_j \nabla \cdot \vec{v}_j + \nabla \cdot \vec{q}_j = 0,
\]

(2)

\[
q_j = \frac{5}{2} \frac{p_j}{m_j \Omega_{ej}} (e \parallel \times \nabla T_j),
\]

(3)

where \( n_j, T_j \) are the density and temperature perturbations (\( j = i \) and \( j = et \) represents ions and trapped electrons) and \( \vec{v}_j = \vec{v}_E + \vec{v}_s + \vec{v}_P + \vec{v}_{\pi j}, \vec{v}_E \) is the \( E \times B \) velocity, \( \vec{v}_s \) is the diamagnetic drift velocity, \( \vec{v}_P \) is the polarization drift velocity, \( \vec{v}_{\pi j} \) is the stress tensor drift velocity and \( \vec{q}_j \) is the heat flux. The derivative is defined as \( d/dt = \partial/\partial t + \rho_s c_s \hat{z} \times \nabla \phi \cdot \nabla, \) and \( \phi \) is the electrostatic potential. In the forthcoming equations \( \tau = T_e/T_i, \) \( \vec{v}_s = \rho_s c_s \hat{y}/L_n, \) \( \rho_s = c_s / \Omega_{cs}, \) where \( c_s = \sqrt{T_e/m_i}, \) \( \Omega_{cs} = eB/m_ic. \) We also define \( L_f = -(d \ln f/dr)^{-1}, \)

\( \eta_j = L_n/L_{T_j}, \) \( \omega_{Dj}/\omega_{ej} = \epsilon_jg_j = (2L_n/R)g_j, \) where \( R \) is the major radius, \( \alpha_i = (1 + \eta_i/\tau \) and \( g_j \) represents the variation of \( \omega_{Dj} \) along the field line. The geometrical quantities are calculated in the strong ballooning limit (\( \theta = 0, g_j = 1). \) The perturbed field variables are normalized as \( \phi = (L_n/\rho_i)e\delta \phi/T_e, \) \( n = (L_n/\rho_i)\delta n/n_0, \) \( T_j = (L_n/\rho_i)\delta T_j/T_{e0}. \) The perpendicular length
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scale and time are normalized to $\rho_s$ and $L_n/c_s$, respectively. Equations (1) and (2) can now be simplified to

\[ \frac{\partial \tilde{n}_{et}}{\partial t} + f_i \frac{\partial \tilde{\phi}}{\partial y} + \epsilon_n g_e \left( -f_i \tilde{\phi} + \tilde{n}_{et} + f_i \tilde{T}_{et} \right) = -[\phi, \tilde{n}_{et}], \]

(4)

\[ \frac{\partial \tilde{T}_{et}}{\partial t} + \frac{5}{3} \epsilon_n g_e \frac{\partial \tilde{T}_{et}}{\partial y} + \left( \eta_e - \frac{2}{3} \right) \frac{\partial \tilde{\phi}}{\partial y} - \frac{2}{3} \frac{\partial \tilde{n}_{et}}{\partial t} = -[\phi, \tilde{T}_{et}] + \frac{2}{3} f_i [\phi, \tilde{n}_{et}], \]

(5)

\[ \frac{\partial \tilde{n}_i}{\partial t} - \left( \frac{\partial}{\partial y} - \alpha_i \frac{\partial}{\partial y} \right) \nabla^2 \tilde{\phi} + \frac{\partial \tilde{\phi}}{\partial y} - \epsilon_n g_i \frac{\partial}{\partial y} \left( \tilde{\phi} + \frac{1}{\tau} \left( \tilde{n}_i + \tilde{T}_i \right) \right) = -[\phi, \tilde{n}_i] + [\phi, \nabla_\perp^2 \tilde{\phi}] + \frac{1}{\tau} [\phi, \nabla_\perp^2 (\tilde{n}_i + \tilde{T}_i)], \]

(6)

\[ \frac{\partial \tilde{T}_i}{\partial t} - \frac{5}{3} \epsilon_n g_i \frac{\partial \tilde{T}_i}{\partial y} + \left( \eta_i - \frac{2}{3} \right) \frac{\partial \tilde{\phi}}{\partial y} - \frac{2}{3} \frac{\partial \tilde{n}_i}{\partial t} = -[\phi, \tilde{T}_i] + \frac{2}{3} f_i [\phi, \tilde{n}_i]. \]

(7)

Here $f_i = n_{et}/n_0$ is the fraction of trapped electrons. The Poisson bracket is $[A, B] = \partial A/\partial x \partial B/\partial y - \partial A/\partial y \partial B/\partial x$. The system is closed using the quasineutrality condition

\[ \delta n_i = \delta n_e = \delta n_{et} + \delta n_{ef}, \]

(8)

where a Boltzmann distribution has been assumed for the free electrons. After linearizing equations (4)–(7), the dispersion relation for the coupled ITG/TEM system is obtained as

\[ 0 = \frac{\omega}{N_i} \left[ \omega \left( 1 - \epsilon_n g_i \right) - \left( \frac{7}{3} - \eta_i - \frac{5}{3} \epsilon_n g_i \right) \omega D_i - k_y^2 \rho_i^2 (\omega - \omega_{ce} (1 + \eta_i)) \left( \frac{\omega}{\omega_{ce}} + \frac{5}{3} \epsilon_n g_i \right) \right] - f_i \frac{\omega}{N_e} \left[ \omega \left( 1 - \epsilon_n g_e \right) - \left( \frac{7}{3} - \eta_e - \frac{5}{3} \epsilon_n g_e \right) \omega D_e \right] - 1 + f_i, \]

(9)

where

\[ N_j = \omega^2 - \frac{10}{3} \omega \omega_{Dj} + \frac{5}{3} \omega_{Dj}^2. \]

(10)

Depending on the plasma parameters, the dispersion relation (9) may contain 0, 1 or 2 unstable modes. For modes propagating in the ion drift direction (usually the ITG mode), $N_i < N_e$, while for modes propagating in the electron drift direction (TEM), $N_e < N_i$. The modes become de-coupled when the inequalities are strong. Thus, for $N_e \gg N_i$ we obtain a pure ITG mode. For pure ITG mode physics, the fluid model used here has been found to be in good qualitative agreement with a number of gyrokinetic treatments. For example, both the $\eta_i$-scaling of the ion heat transport [23] and the nonlinear upshift of the linear ITG threshold due to zonal flows [23] have been recovered by the fluid model [29]. A more comprehensive version of the model, based on the full ITG and TE system (equations (4)–(7)), has been heavily used in predictive transport code simulations [30, 32] of tokamak discharges. The simulation results indicate that the model is able to reproduce experimental profiles of temperatures and density, inside the edge region, with good accuracy over a wide range of plasma parameters.

In the limit $N_i \gg N_e$, we obtain a pure TEM. In tokamak plasmas the TEM is expected to dominate in the hot electron regime ($T_e \gg T_i$) and/or in regimes with $\eta_e \gg \eta_i$. However, for peaked density profiles (small $\epsilon_n$), the ion and trapped electron responses are strongly coupled ($N_i \approx N_e$) and a density gradient driven TEM appears for weak temperature gradients. In the
following we will neglect the effects of ion perturbations on the TEM and hence only consider ETG-driven TEM (not including the ETG mode). The dispersion relation then takes the form:

\[ 0 = \omega^2 + \omega k_y \left( \xi (1 - \epsilon_n g) - \frac{10}{3} \epsilon_n g \right) + k_y^2 \epsilon_n g \left( \xi \eta_e - \frac{5}{3} \epsilon_n g - \frac{2}{3} (1 - \epsilon_n g) \right) + \frac{5}{3} \epsilon_n g, \quad (11) \]

\[ \xi = \frac{f_t}{1 - f_t}. \quad (12) \]

Equation (11) describes TEMs driven by \( R/L_{Te} \) and suppressed by \( R/L_n \) leading to a linear TEM stability threshold in the parameter \( \eta_e = L_n/L_{Te} \). In the considered limit the TEM is fairly symmetrical to the toroidal ITG mode, except that effects of finite-Larmor-radius and parallel electron dynamics do not appear in the TEM dispersion relation. The solution to equation (11) is given by

\[ \omega_r = -k_y \left( \xi (1 - \epsilon_n g) - \frac{10}{3} \epsilon_n g \right), \quad (13) \]

\[ \gamma = k_y \sqrt{\xi \epsilon_n g (\eta_e - \eta_{eth})}, \]

where \( \omega = \omega_r + i \gamma \) and the linear stability threshold is given by

\[ \eta_{eth} = \frac{2}{3} - \frac{\xi}{2} + \frac{10}{9 \xi} \epsilon_n g + \frac{5}{4} \xi \epsilon_n g. \quad (14) \]

In this regime we can define a reduced model for ETG-driven TEM turbulence by retaining equations (4) and (5) for the trapped electron fluid while neglecting ion dynamics (6) and (7) (see appendix A). The effect of the neglected ion dynamics on the linear physics will in the following be quantified by comparing the results of the reduced (equation (11)) with the complete (equation (9)) dispersion relation.

### 3. Zonal flow generation

In describing the large scale plasma flow dynamics it is assumed that there is a sufficient spectral gap between the small scale TEM fluctuations and the large scale flow. The electrostatic potential is represented by

\[ \phi(X, x, y, T, t) = \Phi(X, T) + \tilde{\phi}(x, y, t), \quad (15) \]

where \( \tilde{\phi}(x, y, t) \) is the fluctuating potential varying on the turbulent scales \( x, y, t \) and \( \Phi(X, T) \) is the zonal flow potential varying on the slow scale \( X, T \) (the zonal flow potential is independent of \( Y \)).

The evolution of the TEM turbulence in the background of the slowly varying zonal flow \( \Phi(X, T) \) can be described by the wave-kinetic equation \([5, 33, 34]\) for the adiabatic invariant \( N_k = C_k |\tilde{\phi}_k|^2 \) (see appendix A for a derivation of \( N_k \) and \( C_k \)).

\[ \frac{\partial}{\partial t} N_k(x, y, t) + \frac{\partial}{\partial k_x} \left( \omega_k + \tilde{v}_0 \right) \frac{\partial N_k(x, y, t)}{\partial x} - \frac{\partial}{\partial x} \left( \tilde{k} \cdot \tilde{v}_0 \right) \frac{\partial N_k(x, y, t)}{\partial k_x} = \gamma_k N_k(x, y, t) - \Delta \omega N_k(x, y, t)^2. \quad (16) \]

Here, \( \tilde{v}_0 \) is the zonal flow part of the \( E \times B \) drift. In this analysis it is assumed that the RHS is approximately zero (stationary turbulence). The role of nonlinear interactions among the TEM fluctuations (here represented by a nonlinear frequency shift \( \Delta \omega \)) is to balance linear growth rate, i.e. \( \gamma_k N_k(x, y, t) - \Delta \omega N_k(x, y, t)^2 \approx 0 \). The TEM turbulence is assumed to be adiabatically modulated by the slowly growing potential \( \Phi(X, T) \). Equation (16) is then
expanded under the assumption of small deviations from the equilibrium spectrum function; $N_k = N_k^0 + \tilde{N}_k$, where $\tilde{N}_k$ evolves at the zonal flow time and space scale $(\Omega_1, q_x, q_y = 0)$, as

$$
- i(\Omega - q_x v_{gs} + i\gamma k) \tilde{N}_k = k_y \frac{\partial^2}{\partial x^2} \Phi \frac{\partial N_k^0}{\partial k_x},
$$

(17)

$$
\tilde{N}_k = -q_x^2 k_y \frac{\partial N_k^0}{\partial k_x} \frac{i}{\Omega - q_x v_{gs} + i\gamma k}. \Phi_1.
$$

(18)

Here $v_{gs} = \partial \omega / \partial k_x \approx 0$, since the effects of electron FLR are neglected.

The evolution equations for the zonal flow are obtained after averaging the ion-continuity equation over the magnetic flux surface and over fast scales and employing quasineutrality (equation (8)):

$$
\frac{\partial}{\partial t} \gamma^2 x / \Phi_1 = \gamma^2 x \langle \frac{\partial}{\partial x} \tilde{\phi}_k \frac{\partial}{\partial y} \tilde{\phi}_k \rangle.
$$

(19)

Here we have assumed that turbulence is dominated by the TEM $(\tilde{\eta} < \tilde{n}_{ce})$ and hence only the small-scale self-interactions among the TEM contribute to the Reynolds stress on the RHS of (19) [35]. Expressing the Reynolds stress terms in equation (19) in $N_k$ we obtain

$$
- i \Omega_1 / \Phi_1 = \int d^2 k x k_y C_{k, -1} N_0. \Phi.
$$

(20)

The factor $C_{k, -1}$ defines the relationship between small-scale turbulence and the wave action density; see appendix equation (A.9) for details. Integrating by parts in $k_x$ and assuming a monochromatic wave packet $N_k^0 = N_0 \delta(k - k_0)$ and using equations (19) and (20) gives

$$
\Omega^2 = -q_x^2 C_{k, -1} k_y N_0.
$$

(21)

The dispersion relation for zonal flow $\Omega$ reduces to

$$
\Omega = iq_x k_y \sqrt{C_{k, -1} N_0}. \Phi.
$$

(22)

Hence, the zonal flow growth rate scales as $\Omega \propto |\phi_k|$. In expressing the zonal flow growth in dimensional form using equation (22), it is assumed that the background turbulence (in the absence of zonal flows) reaches the mixing length level for temperature gradient driven modes corresponding to $\bar{T}_c = 1 / k_x L_T$. We then obtain (see appendix A)

$$
\Omega = iq_x k_y F \frac{\eta_e}{k_y L_m},
$$

(23)

$$
F = \frac{\sqrt{(\Delta_k^2 + \gamma_k^2)}}{|\eta_e - (2/3\xi)e_{gs}(1 + \xi)|},
$$

(24)

where $q_x$ is the zonal flow wave number, $k_y$ is the TEM wave number, $\Delta_k = -(k_y/2)(\xi - e_{gs} \phi + \frac{4}{3} e_{gs})$ and $\gamma_k$ is the linear TEM growth rate. The function $F$ is usually large in regions close to marginal stability due to the denominator in equation (24). This is a result of the quasilinear treatment of the TEM perturbations $\phi_k$ (appearing in equation (23)) and $\bar{T}_c = 1 / (k_x L_{Te})$.

4. Results and discussion

An algebraic equation (23) describing the zonal flow growth rate driven by short-wavelength TEM turbulence has been derived. The zonal flow growth rates will in the following be calculated and compared with the linear TEM growth rates. First, the linear TEM descriptions are compared. In figure 1, the solutions to the full system of 4 equations describing the coupled ITG/TE system (squares, equation (9)) are compared with the reduced model with 2
Figure 1. The solutions for the TEM eigenvalues using the full system of 4 equations (□) are compared with the reduced model with 2 equations (∗). The growth rate and the real frequency with reversed sign (normalized the electron diamagnetic drift frequency) versus $\eta_e$ are displayed. The results are shown for $\tau = 1$, $\epsilon_n = 1.0$, $\eta_i = 0$ and $k \rho = 0.3$.

Figure 4 compares the zonal flow growth rate generated by pure TEM turbulence for $\eta_e = 3$, $\eta_i = 0$ with that generated by ITG mode turbulence for $\eta_e = 0$, $\eta_i = 3$; see
Figure 2. The zonal flow growth rate (normalized to the linear TEM growth rate) versus $\eta_e$ with $\epsilon_n$ as parameter is displayed. The results are shown for $\epsilon_n = 0.5$ (□), $\epsilon_n = 0.7$ (○). The other parameters are $\eta_i = 0$, $\tau = 1$ and $k_x\rho = k_y\rho = q_x\rho = 0.3$ and $f_i = 0.5$.

equation (33) in [9] (note the difference in the definition of $\tau$) as a function of $\tau$ with $\epsilon_n$ as parameter. The comparison is done assuming equal saturation levels for the ITG mode and TEM cases. The results are shown for $\epsilon_n = 0.5$ (diamonds), $\epsilon_n = 1.0$ (plus) and $\epsilon_n = 1.5$ (squares).

In the case of cold ions (small $1/\tau$), the ITG and TEM turbulence generates comparable levels of ZF growth, whereas for equal electron and ion temperatures, the ITG mode generates significantly larger levels of zonal flows (typically around 2 times larger ZF growth rates are obtained for the ITG case). This is due to the nonlinear diamagnetic effects which significantly contribute to the ITG-driven ZF growth [5]. The relatively weak generation of zonal flows obtained here by TEM is consistent with recent results from nonlinear gyrokinetic simulations of TEM turbulence [17].

5. Summary

The present paper investigates the generation of zonal flows by collisionless TEMs. An algebraic equation which describes the zonal flow growth rate in the presence of collisionless TEM turbulence is derived and solved numerically in the strong ballooning limit. A reduced model for the ETG-driven TEM is utilized based on an advanced fluid model including the trapped electron continuity and the electron temperature equations while neglecting the influence of ion perturbations. The generation of zonal flows is described by the vorticity equation and the time evolution of the TEM turbulence in the presence of the slowly growing zonal flow is described by a wave kinetic equation.

It is found that the reduced TE model (2-equations) is qualitatively able to reproduce the linear physics of the full model (4-equations) in the TEM dominated regimes where $T_e \gg T_i$. 
Figure 3. The zonal flow growth rate (normalized to the linear TEM growth rate) versus $\epsilon_n$ ($=2L_{eB}/L_B$) with the fraction of trapped electrons ($f_t$) as parameter. The other parameters are as in figure 2 with $\eta_e = 3$. The results are shown for $f_t = 0.5$ (•) and $f_t = 0.7$ (□).

and/or in regimes where $\eta_e \gg \eta_i$. For reasonable flat density profiles, the linear TE growth rates are typically within 20% except close to the linear stability threshold.

There is a significant increment in the zonal flow growth rate (normalized to the linear TEM growth) close to the linear TEM threshold where a resonance in the ZF generation is obtained. This may result in a larger level of zonal flow, and consequently a lower level of TEM turbulence, in this region.

A qualitative comparison of the zonal flow growth rate generated by pure TE and ITG mode turbulence shows that the ITG mode generates significantly larger levels of ZF growth except in the case of cold ions (small $1/\tau$) where the TEM-driven ZF growth is comparable to the ITG case.

In the present paper the generation of zonal flows has been studied analytically by analysing the linear zonal flow growth rates. A complete analysis should also involve an assessment of the stability properties of the generated flows. This is more suitable for numerical analysis and is outside the scope of the present paper. In addition, a more comprehensive comparison of TEM and ITG-driven zonal flows should include cases where $\eta_e \approx \eta_i$ and/or cases where the modes are strongly coupled (for peaked density profiles), using the full ITG/TEM system. This is left for future work.

Appendix A. Adiabatic invariant in TEM turbulence

In this appendix the derivation of the adiabatic invariant in TEM-driven turbulence is presented. The method has been described in detail in ([5, 9] and references therein) and only a brief
Summary is given here. From equations (4) and (5) we get

$$\frac{\partial \tilde{n}_{et}}{\partial t} - \xi \frac{\partial \tilde{n}_{et}}{\partial y} + \epsilon_n g \frac{\partial}{\partial y} (\xi \tilde{n}_{et} + \tilde{n}_{et} + f_t T_{et}) = -[\phi, \tilde{n}_{et}], \quad (A.1)$$

$$\frac{\partial \tilde{T}_{et}}{\partial t} + \frac{7}{3} \epsilon_n g \frac{\partial \tilde{T}_{et}}{\partial y} - (\eta_e - \frac{2}{3} \epsilon_n g) \xi \frac{\partial \tilde{n}_{et}}{\partial y} + \frac{2}{3} f_t \epsilon_n g \frac{\partial \tilde{n}_{et}}{\partial y} = -[\phi, \tilde{T}_{et}], \quad (A.2)$$

Here, the interaction between the TEM perturbations has been omitted (see discussion after equation (16) for this). It should be noted that the relationship between the electrostatic potential $\tilde{\phi}$ and the trapped electron density $\tilde{n}_{et}$ is given by $\tilde{\phi} = -1/(1 - f_t) \tilde{n}_{et}$ (see equation (8)). To determine the generalized wave action density $N_k = |\Psi_k|^2$, we introduce the normal coordinates $\Psi_k = \tilde{n}_{etk} + \alpha_k \tilde{T}_{etk}$, where $\alpha_k$ is to be calculated. Multiplying equation (A.2) by $\alpha_k$ and adding it to equation (A.1) gives

$$\frac{\partial}{\partial t} (\tilde{n}_{etk} + \alpha_k \tilde{T}_{etk}) + \left(-\xi + (1 + \xi) \epsilon_n g - \alpha_k \frac{\xi}{f_t} \left(\eta_e - \frac{2}{3} \epsilon_n g\right) + \frac{2}{3} f_t \epsilon_n g \alpha_k\right) \frac{\partial \tilde{n}_{etk}}{\partial y}$$

$$+ \left(\frac{7}{3} \epsilon_n g \alpha_k + f_t \epsilon_n g\right) \frac{\partial \tilde{T}_{etk}}{\partial y} = -[\Phi, \tilde{n}_{etk} + \alpha_k \tilde{T}_{etk}], \quad (A.3)$$

The normal coordinates are found if the equation is rewritten as in [9]:

$$\frac{\partial \Psi_k}{\partial t} + V_k \frac{\partial \Psi_k}{\partial y} = -[\Phi, \Psi_k], \quad (A.4)$$
where

\[ V_k = -\xi + (1 + \xi)e_n g - \alpha_k \frac{\xi}{f_i} \left( \eta_e - \frac{2}{3} e_n g \right) + \frac{2}{3} \frac{e_n g}{f_i} \alpha_k, \]  
(A.5)

\[ \alpha_k = \frac{(7/3)e_n g \alpha_k + f_i e_n g}{V_k}, \]  
(A.6)

which gives

\[ \alpha_k = -\frac{1}{2}(\xi - e_n g \xi + (4/3)e_n g) + \frac{i}{2} \frac{\xi e_n g (\eta_e - \eta_{\text{eth}})}{\xi f_i (\eta_e - (2/3 \xi)(1 + \xi)e_n g)}. \]  
(A.7)

The linear relations between \( \tilde{\phi}_k, \tilde{n}_{ck} \) and

\[ \tilde{T}_k = k_y (\eta_e - (2/3 \xi)e_n g (1 + \xi)) \frac{2i \gamma_k}{\omega - (7/3)e_n g k_y} \tilde{\phi}_k, \]  
(A.8)

enable one to express \( \Psi_k \) and \( N_k \) as

\[ \Psi_k = \tilde{n}_{ck} + \alpha_k \tilde{T}_k = \frac{2i \gamma_k}{\Delta_k + i \gamma_k} \tilde{\phi}_k, \]  
(A.9)

\[ N_k = |\Psi_k|^2 = \left( \frac{4(1 - f_i) \gamma_k^2}{\Delta_k + \gamma_k^2} + f_i^2 \right) |\tilde{\phi}_k|^2 = C_k |\tilde{\phi}_k|^2, \]  
(A.10)

\[ C_k = \left( \frac{4(1 - f_i) \gamma_k^2}{\Delta_k + \gamma_k^2} + f_i^2 \right), \]  
(A.11)

\[ \Delta_k = -\frac{k_y}{2} \left( \xi - e_n g \xi + \frac{4}{3} e_n g \right), \]  
(A.12)

\[ \gamma_k = k_y \sqrt{\xi e_n g (\eta_e - \eta_{\text{eth}})}. \]  
(A.13)

The equations (A.8)–(A.12) describe the normal variables \( \Psi_k \), the adiabatic invariant \( N_k \) and the linear TEM growth rate, respectively.

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