Mechanical integrity of tubing string in high-temperature and high-pressure ultra-deep gas wells based on uncertainty theory

Zhi Zhang¹ | Jian Ding¹ | Yushan Zheng² | Pengfei Sang³ | Lili Chen³ | Yu Fan³ | Yufei Li³

¹State Key Lab of Oil and Gas Reservoir Geology and Exploitation, Southwest Petroleum University, Chengdu, China
²CNPC Research Institute of Safety and Environment Technology, Beijing, China
³PetroChina Southwest Oil & Gasfield Company, Chengdu, China

Abstract
During the manufacturing process of tubing and casing, manufacturing deviations can occur in quality parameters, such as yield strength, outer diameter, and wall thickness, which can lead to uncertainty in the actual strength of the tubing string. Specifically, tubing string failures periodically occur in high-temperature, high-pressure, and ultra-deep gas wells owing to the uncertainty in string strength, attenuation of high-temperature material properties, and influence of complex load fluctuations. Therefore, in this study, Monte Carlo method is adopted to simulate the local fluctuation of downhole actual load, and the load variation rule of tubing string in high-temperature and high-pressure ultra-deep well is obtained. By considering the effect of manufacturing tolerance and corrosion, variation law of string strength is obtained. Additionally, the influence of load, temperature, manufacturing tolerance, and other uncertainties on the strength and stress of the tubing string is analyzed, and a quantitative evaluation method for the mechanical integrity of the tubing string is established. The calculation results show that the uncertainty of complex loads and influence of high temperature increase the failure probability of the mechanical integrity of tubing string. Furthermore, a failure risk can potentially exist under conditions wherein conventional verification results denote no failure. It is determined via sensitivity analysis that temperature, corrosion rate, and steel grade are the main factors for the failure of the tubing string mechanical integrity. By considering the influence of high temperature, the failure probability of the tubing string is increased by 25.33% when compared to the original basis. However, considering the manufacturing tolerance of tubing string, the actual yield strength of tubing string is higher than the nominal yield strength, which reduces the failure probability of the tubing string. This implies that reduction of the subjective influence in the risk assessment process can lead to precise and quantitative analysis of the failure...
risk of the tubing string mechanical integrity and optimization of the structure of the tubing string in high-temperature, high-pressure, and ultra-deep gas wells.

KEYWORDS
high temperature and high pressure, integrity, manufacturing tolerance, risk assessment, tubing string, ultra deep, uncertainty

1 | INTRODUCTION

With the development of the petroleum industry, oil and gas exploitation has been extended to high-temperature, high-pressure, and ultra-deep fields. In recent years, nearly 400 ultra-deep wells (above 7000 m) have been drilled in Tarim, and 1/3 of the major oil and gas well discoveries globally involve high-temperature and high-pressure fields. Among these fields, the Gulf of Mexico (204°C, 2.1 MPa/100 m), North Sea of England (199°C, 2.2 MPa/100 m), and South China Sea (249°C, 2.4 MPa/100 m) are three areas of high-temperature and high-pressure offshore. They also include China's Keshen, Shunnan, and Sichuan blocks. Failures of tubing strings occur in complex downhole conditions owing to the influence of high-temperature, high-pressure, and ultra-deep environments, and it was determined in a market survey conducted by Archer in 2010 wherein 41% of well integrity failure cases were due to tubing string failures.1-2 On a certain platform in the North Sea of Norway, there are 407 wells in total. Among these wells, 75 wells failed in well integrity and 30 wells failed in tubing string integrity, accounting for 40% of the total number of failed wells. Moreover, in a certain block of Tarim, there are currently a total of 110 wells, and a total of 17 wells failed in tubing string, accounting for 15.5% of the wells in this block.3 Once the failure of tubing string occurs, it destroys the integrity of the tube, resulting in production stoppage, workover, leakage, blowout, and other accidents in serious cases.4 Specifically, under harsh environments, such as high temperature and high pressure, the failure of tubing strings seriously threatens the safety of on-site production and personnel.5

Many studies have been conducted on the mechanical integrity and risk assessment of tubing strings in high-temperature, high-pressure, and ultra-deep gas wells. In 2019, Mustaffa et al. proposed an erosion model for integration into the existing burst strength models and assessments for using probabilistic approaches.6 In 2021, based on the Bayesian decision-making framework of risk, Akinsanya et al. evaluated the failure risk of tubing by jointly considering two failure forms: tubing clogging and leaking due to scale and pitting corrosion.7 In 2018, Zhang et al proposed an evaluation method based on finite life design theory, which utilizes a high-pressure high-temperature (HPHT) flow corrosion test.8 In 2020, Hassani et al. developed a methodological approach for assessing the vulnerability of major industrial risks in the oil and gas industry to humans, materials, and the environment using a semiquantitative method.9 In 2019, Elosta et al. used deterministic fatigue results as the starting point in conjunction with analytical reliability and developed practical reliability procedures and techniques applicable to offshore risers for evaluating probabilistic fatigue damage.10 In 2011, Brossia et al. proposed corrective and preventive measures for the integrity of aging oil and gas wells in the Gulf of Mexico to alleviate the aging process and proposed a method to determine the reliability of submarine oil wells.11

Many advances have been made in the study of the mechanical integrity of tubing strings. However, the manufacturing tolerances of tubing strings, influence of high temperature, and change in stress under complex downhole conditions have not been fully considered. Under the conditions of high temperature and pressure, there is a deviation between the actual and nominal strengths of the tubing string, and the load of the tubing string is large, leading to occasional failure of the tubing string.

In this study, Monte Carlo method is adopted to simulate the local fluctuation of downhole actual load. Furthermore, the load variation rule of tubing string in high-temperature, high-pressure, and ultra-deep well is obtained. By considering the influence of manufacturing tolerance and corrosion, the variation law of string strength is obtained. Additionally, the influence of load, temperature, manufacturing tolerance, and other uncertainties on the strength and stress of tubing strings is analyzed, and a quantitative evaluation method for the mechanical integrity of tubing strings is established to optimize the structure of tubing strings in high-temperature, high-pressure, and ultra-deep wells.

2 | QUANTITATIVE EVALUATION METHOD FOR MECHANICAL INTEGRITY OF TUBING STRING

Under the downhole high-temperature and high-pressure environment, the actual strength of the tubing string complies with a certain distribution rule owing to the influence
of the manufacturing tolerance. Meanwhile, the load also complies with a certain distribution rule owing to the production fluctuations. As shown in Figure 1, \( f(x) \) denotes the probability density distribution curve of the tubing string strength and \( f_s(x) \) denotes the probability density distribution curve of the tubing string stress. With an increase in the service life, the strength of the tubing string deteriorates, resulting in an interference area of the probability distribution curve, which is considered an unreliable case because the stress in the interference area is higher than the strength, and it is deemed to be a structural failure at this time.\(^{12}\)

It is generally believed that when the stress of a material is higher than its ultimate strength, the material is destroyed. According to OREDA, that is, offshore and onshore reliability databases, the failure modes of tubing strings generally include fracture, collapse, and tensile fracture.\(^{13}\) This implies that when the strength of the tubing string is less than the stress of the tubing string, the tubing string can be destroyed. Therefore, based on the stress–strength interference theory, \( Z_1 \), \( Z_2 \), and \( Z_3 \) are adopted to indicate the corresponding ultimate state of the tubing string when fractured, collapsed, and tensile fractured, respectively. Furthermore, the failure of the tubing string under a complex stress state is considered, and \( Z_4 \) is used to indicate the ultimate state of the tubing string under complex stress. The ultimate state equation of the tubing string is established as follows:\(^{14,15}\):

\[
\begin{align*}
Z_1 &= \sigma_1 - F_1, \\
Z_2 &= \sigma_2 - F_2, \\
Z_3 &= \sigma_3 - F_3, \\
Z_4 &= \sigma_4 - F_4,
\end{align*}
\]

(1)

where \( Z_1 \) denotes the dimensionless ultimate state of the fracture failure mode. Furthermore, \( Z_2 \) denotes the dimensionless ultimate state of the collapse failure mode, \( Z_3 \) denotes the dimensionless ultimate state of the tensile fracture failure mode, \( Z_4 \) denotes the dimensionless ultimate state of the complex stress condition. Additionally, \( \sigma_1 \) denotes the residual burst strength of tubing string (MPa), \( F_1 \) denotes the internal pressure borne by tubing string (MPa), \( \sigma_2 \) denotes the residual collapse strength of tubing string in MPa, \( F_2 \) denotes the external pressure borne by the tubing string (MPa), \( \sigma_3 \) denotes residual axial strength of tubing string (kN); \( F_3 \) denotes the axial stress borne by tubing string (kN), \( \sigma_4 \) denotes yield strength of tubing string (MPa), and \( F_4 \) denotes triaxial stress borne by tubing string (MPa).

When \( Z \) is equal to 0, the tubing string is in the ultimate state of the corresponding failure mode; when \( Z \) is lower than 0, the tubing string fails in the corresponding mode; and when \( Z \) is greater than 0, the tubing string is free of failure of the corresponding modes. We assumed that the probability density function of different ultimate state functions \( Z_i \) is \( f_{Z_i}(x) \) and its failure probability \( P_i \) can be calculated using Equation (2).\(^{16,17}\)

\[
P_i = \int_{-\infty}^{0} f_{Z_i}(x)dx,
\]

(2)

where \( f_{Z_i}(x) \) denotes the probability density function of different dimensionless limit state functions \( Z_i \), and \( P_i \) denotes the failure probability of different dimensionless limit state functions \( Z_i \).

According to the basic principle of probability statistics, when the ultimate state equation is a linear function and the input unknowns comply with the normal distribution, the corresponding ultimate state equation should also comply with the normal distribution. However, the probability distribution of stress \( F_i \) and strength \( \sigma_i \) is typically not normally distributed. Hence, it should be equivalently transformed into a normal distribution. Given the ultimate state function \( Z_i \), the following conditions must be satisfied to convert it into the normal variable \( Z_i' \) (\( \mu_{Z_i}', \sigma_{Z_i}' \)):

\[
F_{Z_i}(\mu_{Z_i}) = \phi\left(\frac{\mu_{Z_i} - \mu_{Z_i}'}{\sigma_{Z_i}'}\right),
\]

(3)

\[
f_{Z_i}(\mu_{Z_i}) = \phi\left(\frac{\mu_{Z_i} - \mu_{Z_i}'}{\sigma_{Z_i}'}\right) = \frac{1}{\sigma_{Z_i}'}\varphi\left(\frac{\mu_{Z_i} - \mu_{Z_i}'}{\sigma_{Z_i}'}\right),
\]

(4)

where \( F_{Z_i}(\mu) \) denotes the dimensionless distribution function of different limit state functions \( Z_i; \varphi(x) \) denotes a dimensionless distribution function of the standard normal distribution; \( \varphi(x) \) denotes dimensionless density function of the standard normal distribution; \( \Phi(x) \) denotes dimensionless derived function of standard normal distribution function; \( Z_i' \) denotes dimensionless equivalent normal variables of different limit state functions \( Z_i; \mu_{Z_i}' \) denotes dimensionless mean value of equivalent normal variables \( Z_i' \); \( \sigma_{Z_i}' \) denotes dimensionless standard deviation of

**FIGURE 1** Schematic diagram of stress–strength interference
equivalent normal variables $Z'_i$; and $\mu_{Z_i}$ denotes dimensionless mean value of different limit state functions $Z_i$.

We deduce inverse function of Equation (3) as follows:

$$\frac{\mu_{Z_i} - \mu'_Z}{\sigma'_Z} = \phi^{-1}(F_Z(\mu_{Z_i})). \quad (5)$$

The relation between mean value $\mu'_{Z_i}$ and standard deviation $\sigma'_{Z_i}$ of equivalent normal ultimate state function $Z'_i$ and mean value $\mu_{Z_i}$ and standard deviation $\sigma_{Z_i}$ of nonnormal ultimate state function $Z_i$ can be obtained as follows:

$$\sigma'_{Z_i} = \frac{\varphi(\phi^{-1}(F_Z(\mu_{Z_i})))}{f_Z(\mu_{Z_i})}, \quad (6)$$

$$\mu'_{Z_i} = \mu_{Z_i} - \sigma_{Z_i} \phi^{-1}(F_Z(\mu_{Z_i})). \quad (7)$$

The equivalent probability density function of the nonnormal ultimate state function $Z_i$ is obtained as follows:

$$f_{Z_i}(x) = \frac{1}{\sqrt{2\pi} \sigma'^2_{Z_i}} \exp \left( -\frac{(x-\mu')^2}{2\sigma'^2_{Z_i}} \right). \quad (8)$$

By substituting Equation (8) into Equation (2), the failure probability $P_i$ of the corresponding ultimate state $Z_i$ can be obtained. It is known that the occurrence of any failure mode can lead to the failure of tubing string. Hence, the failure probability of tubing string $P^18$ is as follows:

$$P = 1 - \prod_{i=1}^{n} (1 - R_i), \quad (9)$$

where $P$ denotes dimensionless failure probability of tubing string.

However, given the severity of tubing string failure, the risk matrix method was adopted and severity of tubing string failure was evaluated by field workers or experts to determine the mechanical integrity risk value, $L$, of the tubing string. Based on extant research and API 581 standards, the severity of each tubing string failure form is divided into five levels19 and the value range is shown in Table 1.

After obtaining the severity degree of different failure forms of the tubing string, the following equation was applied to calculate the severity degree $C$ of the tubing string as follows:

$$C = \frac{1}{4} \sum_{i=1}^{4} C_i, \quad (10)$$

where $C$ denotes the dimensionless severity degree of tubing string failure, and $C_i$ denotes the dimensionless severity degree of the corresponding tubing string failure form.

According to the definition of risk in risk assessment, the degree of risk is determined by the failure probability and severity degree of the factors. The equation for the mechanical integrity risk of the tubing string is as follows:

$$L = P \cdot C, \quad (11)$$

where $L$ denotes the dimensionless mechanical integrity risk value of tubing string.

After the mechanical integrity risk value of the tubing string was obtained, the mechanical integrity risk level of the tubing string was divided into four levels, as shown in Table 2, according to the requirements of the API 581 standard and safety assessment form for oil and gas production enterprises.

### 3 | EFFECT OF HIGH TEMPERATURE

Under the condition of high temperature at the shaft bottom, the mechanical properties of the material change owing to the influence of temperature. This reduces the strength of the tubing string. Therefore, the change in

| TABLE 1 | Severity of tubing string failure |
|----------|-------------------------------|
| Quantitative indicators of severity | Severity |
| (80, 100] | Catastrophic |
| (60, 80] | Serious |
| (40, 60] | Secondary |
| (20, 40] | Slight |
| (0, 20] | Negligible |

| TABLE 2 | Risk level of tubing string mechanical integrity |
|----------|-----------------------------------|
| Risk level | Range of risk | Risk state |
| IV | (84, 100] | Very high |
| III | (52, 84] | High |
| II | (24, 52] | Secondary |
| I | (0, 24] | Low |
yield strength of the tubing materials at different temperatures was obtained via an experimental test as shown in Figure 2.

The linear interpolation method was applied to fit the calculation equation of the yield strength reduction factor $\alpha_T$ of the tubing string according to the test data of tubing yield strength attenuation with temperature. Furthermore, the goodness of fit of the yield strength reduction coefficient is shown in Table 3.

$$\alpha_T = \begin{cases} 
-2.14 \times 10^{-4}T^2 - 0.007042 + 100.4 & \text{13Cr-80} \\
-7.07 \times 10^{-7}T^3 + 0.002636T^2 - 0.3273T + 106.7 & \text{13Cr-95} \\
8.244 \times 10^{-12}T^2 - 0.0640T + 101.7 & \text{13Cr-100} \\
2.067 \times 10^{-4}T^2 - 0.1216T + 102.8 & \text{13CrS-110} \\
2.767 \times 10^{-4}T^2 - 0.1526T + 103.6 & \text{13CrM-110} \\
3.034 \times 10^{-3}T^2 - 0.1618T + 103.7 & \text{22Cr-110} \\
2.307 \times 10^{-3}T^2 - 0.1356T + 103.7 & \text{25Cr-110} \\
6.749 \times 10^{-3}T^2 - 0.0726T + 102.0 & \text{25CrW-125} 
\end{cases} \quad (12)$$

where $\alpha_T$ denotes the dimensionless yield strength reduction factor of the tubing string, and $T$ denotes the service temperature of the tubing string in °C.

The actual yield strength of the tubing string under high temperature at the shaft bottom can be expressed as follows:

$$\sigma_t = \alpha_T \sigma_T'$$

where $\sigma_T'$-nominal denotes yield strength of tubing string (MPa).

Simultaneously, in a high-temperature corrosive environment, the wall thickness of the tubing string is reduced owing to corrosion, which causes the strength of the tubing string to decrease continuously. Based on the API 5C3 standard and principle of equivalent wall thickness and thick-walled cylinder, the calculation method of uniaxial residual strength after uniform corrosion of tubing string is obtained as follows:

$$\begin{align*}
\sigma_1 &= 0.875 \frac{2a_t(\delta - \nu t)}{R_o}, \\
\sigma_2 &= 2a_t \left\{ \left( \frac{R_o}{(\delta - \nu t)} \right) - 1 \right\}, \\
\sigma_3 &= \frac{\pi}{4} a_t (R_o^2 - (R_i + \nu t)^2),
\end{align*} \quad (14)$$

where $t$ denotes service time of the tubing string (a), $R_i$ denotes original inner diameter of tubing string (mm), $R_o$ denotes original outer diameter of tubing string (mm), $\delta$ denotes nominal wall thickness of tubing string (mm), and $\nu$ denotes corrosion rate of tubing string (mm).

### 4 | EFFECT OF MANUFACTURING TOLERANCE

In the manufacturing process of tubing strings, there are manufacturing tolerances, such as uneven wall thickness and ellipses, which lead to uncertainties in the actual strength of the tubing string. In ISO 10400, detailed statistics have been provided for the probability distribution of the tubing string yield strength and geometric parameters. The variation coefficient of the tubing string wall thickness is 2.59%, whereas, according to the variation coefficient table of metal material properties and engineering experience, the variation coefficient of the yield strength is 5.29%. All the variation coefficients of the parameters are listed in Table 4.

Accordingly, according to the law of the probability distribution function, equation $y = f(x_1, x_2, x_3, ..., x_n)$, and the probability distribution of each parameter is considered as a normal distribution. Hence, the corresponding random variable $Z$ follows a normal distribution. Then, the strength of the tubing string...
TABLE 4 Distribution function and variation coefficient of string parameters

| Thickness | Yield strength | Outer diameter |
|-----------|----------------|----------------|
| Normal    | Normal         | Normal         |

| Variation coefficient | 0.0259 | 0.0529 | 0.00181 |

complies with the normal distribution, and the probability distribution function is as follows:

\[
f_{\sigma}(x) = \frac{1}{\sqrt{2\pi} \sigma_x} e^{-\frac{(x-\mu_x)^2}{2\sigma_x^2}}, \tag{15}
\]

\[
\sigma^2_{\sigma} = \sum_{i=1}^{n} \left( \frac{\partial y_i}{\partial x_i} \right)^2 \sigma^2_{x_i}, \tag{17}
\]

where \(f_{\sigma}(x)\) denotes the dimensionless probability distribution function of the tubing string strength; \(\sigma_x\) denotes the dimensionless standard deviation of tubing string strength; \(\mu_x\) denotes the dimensionless mean value of tubing string strength; \(\sigma_{x_i}\) denotes the dimensionless standard deviation of tubing string performance parameters; and \(\mu_{x_i}\) denotes the dimensionless mean value of tubing string performance parameters.

5 | INFLUENCE OF LOAD UNCERTAINTY

In the production process of high-temperature and high-pressure ultra-deep gas wells, the output changes in the local range owing to the change in the production system. This leads to an inevitable fluctuation of the tubing string load, resulting in the uncertainty of the tubing string load. When in service, the tubing string load can be classified as follows: (1) external pressure \(F_1\), (2) internal pressure \(F_2\), and (3) axial force \(F_3\).

(1) External pressure

The external pressure of the tubing string is related to the annulus pressure, density of the annulus protection fluid, and depth of the packer \(^{25}\):

\[
F_1 = p_t + 0.00981 \rho_o h_s, \tag{18}
\]

where \(p_t\) denotes the annulus pressure (MPa), \(\rho_o\) denotes the density of the annulus protection fluid (kg/m\(^3\)), and \(h_s\) denotes the packer depth (m).

(2) Internal pressure

During the production process, the tubing string bears wellbore fluid pressure. Based on the acceleration pressure gradient, gravity pressure gradient, and friction pressure gradient of the wellbore fluid, the fluid pressure balance equation in the tubing string is obtained \(^{26}\) as follows:

\[
dp_t = -\frac{d(\rho_i c^2)}{2} - \rho_i g dz \cos \theta - f \frac{c^2}{4R_i} dh, \tag{19}
\]

where \(p_t\) denotes the fluid pressure in the tubing string (MPa), \(\rho_i\) denotes the density of the fluid in the tubing string (kg/m\(^3\)), \(c\) denotes the velocity of the fluid in the tubing string (m/s), \(g\) denotes the gravitational acceleration (m/s\(^2\)), \(h\) denotes the length of tubing (m), \(\theta\) denotes the well deviation angle (°), and \(f\) denotes the dimensionless friction coefficient.

The gas density and fluid velocity in a tubing string can be calculated using the following formula:

\[
F_i = \frac{pM}{Z_R RT}, \tag{20}
\]

\[
c = \frac{Q}{4\pi R_i^2}, \tag{21}
\]

where \(M\) denotes the molar mass of gas (kg), \(Z_R\) denotes the gas compressibility factor (dimensionless), \(R\) denotes the general gas constant \((R = 0.00831 \text{ MPa}\cdot\text{m}/\text{kmol}\cdot\text{K})\), \(\gamma\) denotes the relative density of gas (dimensionless), \(Q\) denotes gas production (m\(^3\)/d).

F_2 = p_{i}. \tag{22}

(3) Axial force

When there is a packer in the tubing, the axial force is calculated from the deformation of the tubing due to the Hooke’s law, balloon, thermal, and buckling \(^{27}\):

\[
F_3 = 4\pi R_i^2 E \frac{(\Delta h_1 + \Delta h_2 + \Delta h_3 + \Delta h_4)}{h}, \tag{23}
\]

where \(E\) denotes the elastic modulus of the tubing (MPa), \(\Delta h_1\) denotes the deformation of the tubing string due to Hooke’s law (m), \(\Delta h_2\) denotes the deformation of the tubing string due to the balloon (m), \(\Delta h_3\) denotes the deformation of the tubing string due to thermal (m), and \(\Delta h_4\) denotes the deformation of the tubing string due to buckling (m).
The deformation of tubing string due to the Hooke’s law is as follows:

$$\Delta h_1 = -\frac{h}{EA_s}((A_p - A_i)\Delta p_i - (A_p - A_o)\Delta p_o + WL),$$  \hspace{1cm} (24)$$

where $A_s$ denotes the cross-sectional area of the tubing wall ($m^2$), $A_p$ denotes the cross-sectional area of the packer sealing cavity ($m^2$), $A_i$ denotes the cross-sectional area of the tubing string inner diameter ($m^2$), $A_o$ denotes the cross-sectional area of the tubing string outside diameter ($m^2$), $\Delta p_i$ denotes the change in pressure in the tubing (MPa), $\Delta p_o$ denotes the change in annulus pressure (MPa), and $W$ denotes the weight per unit length of tubing (N/m).

The deformation of tubing string due to the thermal load is as follows:

$$\Delta h_2 = -\frac{2\mu}{E} \int_0^L \frac{p_x R_o^2 - p_x R_i^2}{R_i^2 - R_o^2} dh,$$  \hspace{1cm} (25)$$

where $\mu$ denotes the Poisson’s ratio of tubing (dimensionless).

The deformation of tubing string due to the thermal load is as follows:

$$\Delta h_3 = \alpha h \Delta T,$$  \hspace{1cm} (26)$$

where $\alpha$ denotes the thermal expansion coefficient of the tubing ($^\circ$C/m) and $\Delta T$ denotes the average temperature change of the wellbore ($^\circ$C).

The deformation of tubing string due to buckling is as follows:

$$\Delta h_4 = \left\{ \begin{array}{ll}
\frac{r^2 I_4 (p_x - p_y) h^2}{8EI} & h_Z \geq h, \\
-\frac{r^2 I_4 (p_x - p_y) h}{8EI} \left( 2 - \frac{hW}{A_p(p_x - p_y)} \right) & h_Z < h,
\end{array} \right.$$  \hspace{1cm} (27)$$

where $I$ denotes the moment of inertia of the tubing string cross section ($m^4$), and $r$ denotes the clearance between the tubing and casing, m.

Simultaneously, the triaxial equivalent stress is adopted to analyze the stress of the tubing string under the complex stress state. According to von Mises equivalent stress calculation method, the triaxial equivalent stress is calculated from radial stress $\sigma_r$, axial stress $\sigma_z$, and circumferential stress $\sigma_\theta$ according to the equation below as follows:

$$F_3 = \frac{1}{\sqrt{2}} \sqrt{\left( \sigma_z - \sigma_\theta \right)^2 + \left( \sigma_\theta - \sigma_r \right)^2 + \left( \sigma_r - \sigma_z \right)^2},$$  \hspace{1cm} (28)$$

where $\sigma_z$ denotes the axial stress of tubing string (MPa), $\sigma_r$ denotes the radial stress of tubing string (MPa); and $\sigma_\theta$ denotes the circumferential stress of tubing string (MPa).

Axial stress $\sigma_z$:

$$\sigma_z = \frac{F_3}{\pi (R_o^2 - R_i^2)}.\hspace{1cm} (29)$$

Radial stress $\sigma_r$:

$$\sigma_r = \frac{R_o^2 - R_i^2}{R_o^2 - R_i^2} F_2 - \frac{R_o^2 - R_i^2}{R_o^2 - R_i^2} F_1.\hspace{1cm} (30)$$

Circumferential stress $\sigma_\theta$:

$$\sigma_\theta = \frac{R_o^2 + R_i^2}{R_o^2 - R_i^2} F_2 - \frac{R_o^2 + R_i^2}{R_o^2 - R_i^2} F_1.\hspace{1cm} (31)$$

Furthermore, during the service of the tubing string, the stress of tubing string cannot be directly obtained, and the constant change in stress is mainly due to fluctuations in the production parameters. However, the probability distribution function $f_{x_i}(x)$ of production $Q$ can be obtained using statistical data. Simultaneously, the functional relationship between stress $F_i$ and production $Q$ is known. Therefore, the Monte Carlo simulation method is adopted to determine the stress distribution of the tubing string. The specific steps are as follows:

Determine the stress function $F_i = f(x_1, x_2, ..., x_n)$ and random variables $x_1, x_2, ..., x_n$.

Determine probability density function $f_{x_i}(x)$ of random variable $x_i$ in stress function.

We assume sampling times $j$ and general $N$ sample(s) of random variables according to the probability density function of random variable $x_i$ as follows:

$$\begin{bmatrix}
  x_{i1} & \cdots & x_{ij} \\
  \vdots & \ddots & \vdots \\
  x_{n1} & \cdots & x_{nj}
\end{bmatrix}\hspace{1cm} (32)$$

We substitute the randomly generated sample data $x_{ij}$ into stress function to calculate corresponding stress as follows:

$$[F_1 \cdots F_j].\hspace{1cm} (33)$$

We rank $F_j$ in ascending order and draw a stress histogram and fit the stress distribution to obtain the
stress probability distribution function $f_{xi}(x)$ and simultaneously perform fitness tests.

where $f_{xi}(x)$ denotes the dimensionless probability distribution function of random variable $xi$ in the stress function; $j$ denotes simulation time (times); $x_{ij}$ denotes a random number generated by random variable $xi$ simulating its distribution laws and is dimensionless; $F_j$ denotes corresponding stress obtained by substituting random simulation number $x_{ij}$ in the stress function (MPa); and $f_{Fi}(x)$ denotes the dimensionless stress probability distribution function.

**FIGURE 3** Well structure of X well

6 | APPLICATION CASES

Well X is a high-temperature, high-pressure, and ultra-deep gas well in a certain place, the relative density of the produced gas is 0.72, and the average CO$_2$ content is in the range of 0.108%–1.190%. The drilled well is 7045 m deep with a bottom temperature of 164°C and a pressure coefficient of 1.88, and its formation pressure is predicted as 131.79 MPa. Furthermore, the density of the annulus protection fluid is 1.4 g/cm$^3$, and the well has been in service for 5 years. The well structure is shown in Figure 3, production monitoring data of the well is shown in Figure 4, and specification parameters of the tubing string are listed in Table 5.

The production monitoring data are known, and the probability distribution density curve is obtained based on statistical analysis of the data (Figure 5). Hence, the results indicate that the production fluctuation complies with a bimodal Gaussian distribution. The Gaussian approximation method is adopted to fit the probability density distribution curve to obtain the probability density function, and the goodness of fit $R^2$ is 0.997.

**TABLE 5** Specification parameters of the tubing

| Outer diameter (mm) | Thickness (mm) | Thickness (mm) | Segment weight (kg/m) | Yield strength (Mpa) | Material | Corrosion rate (mm/a) |
|---------------------|----------------|----------------|-----------------------|----------------------|----------|-----------------------|
| 114.3               | 12.7           | 1200           | 32.144                | 758.42               | 13Cr     | 0.076                 |
| 114.3               | 9.65           | 1700           | 25.300                |                      |          |                       |
| 88.9                | 8.56           | 2900           | 18.900                |                      |          |                       |
| 88.9                | 9.53           | 6650           | 13.690                |                      |          |                       |
| 88.9                | 7.34           | 6900           | 19.272                |                      |          |                       |
The wellbore temperature distribution is obtained based on the production probability density function \( f_Q(x) \) as shown in Figure 6. Furthermore, Figure 6 shows that the production fluctuation slightly affects the change in wellbore temperature, and the wellhead temperature is maintained at approximately 120°C.

Based on the temperature distribution, the yield strength attenuation coefficient of the tubing string in a high-temperature environment is calculated using Equation (12) as shown in Figure 7. As shown in Figure 7, as the wellbore temperature ranges from 120 to 165°C, the yield strength reduction coefficient of the tubing string is in the range of 0.921–0.933. Hence, the actual yield strength of the downhole tubing string is reduced by 59.92 MPa, and the yield strength of the wellhead tubing string is also reduced by 50.81 MPa.

(2) Effect of the manufacturing tolerance

Based on the known performance parameters, parameter distribution function, and variation coefficient of the tubing string, the relationship between the variation coefficient, mean value, and standard deviation, the probability distribution function of tubing string parameters for different well sections can be obtained. Simultaneously, based on the probability distribution function of the tubing string parameter, Equations (15)–(17) were adopted to calculate the probability distribution function and probability distribution curve of the string strength (Figure 8). Furthermore, Figure 8 shows that the tubing string yield strength, axial strength, burst strength, and collapse strength conform to the normal distribution. However, given that the performance parameters of tubing strings in different sections are inconsistent, there is a difference between the mean value and standard deviation of the axial strength, burst strength, and collapse strength. However, the yield strength parameters of the tubing string in different well sections are consistent. Therefore, the probability distribution curves remained consistent.
FIGURE 7 Yield strength reduction factor of tubing string

FIGURE 8 Probability density curves of tubing strength
(3) Effect of the load uncertainty

When the probability density function of production $f(Q)$ is obtained, the variation in internal pressure, external pressure, axial force, and triaxial stress of the tubing string (Figure 9), and the probability density distribution curves of the internal pressure, external pressure, axial force, and triaxial stress of the tubing string at different depths (Figure 10) are obtained based on the determination method of the stress probability density function of the tubing string. As shown in Figure 9, when production fluctuates, the load of the tubing string also shows a corresponding change in trend although differences exist in the changes in the tubing string load at different depths.

As shown in Figure 10, the load distribution of tubing string tends to normal distribution although the mean value and standard deviation of internal pressure load increase with depth, and the mean value of external pressure load increases with depth. Nevertheless, the standard deviation of external pressure in the upper tubing string of packer is equal, whereas the standard deviation of external pressure in the lower tubing string of packer is higher. Furthermore, changes in the mean value and standard deviation of axial force and triaxial stress are complex, and they are in agreement with the mechanical change law of high-temperature and high-pressure tubing string.

The probability distribution curves of the stress and strength of the tubing string are obtained based on the strength and stress probability density functions (Figure 11). As shown in Figure 11, after 5 years of service, the distribution of the axial strength and yield strength of the tubing string significantly exceeds that of the axial stress and triaxial stress, whereas the distribution of the burst strength and collapse strength is lower than that of the internal pressure and external pressure, and the string begins to fail at this time.

In addition, the tubing string failure probability is obtained via calculations based on Equation (9). Furthermore,
the failure severity of the tubing string is evaluated by experts (Table 6) according to the quantitative index of the failure severity of the tubing string in Table 1.

The risk value of tubing string mechanical integrity $L$ is calculated based on formulas (10) and (11), where $L = 63.50 \times 0.9969 = 63.30$, which is classified as high risk. When compared to the scenario without parameter uncertainty, the scenario with parameter uncertainty increases failure risk of the tubing string. A high risk of failure exists in actual production even if the safety factor of the tubing string satisfies standard requirements.

6.1 Analysis on effect of well depth and service time

Figure 12 shows the failure probability of the tubing string under different ultimate stress states. As shown in the figure, failure probability of the tubing string increases with well depth under internal pressure and external pressure, and it decreases along the well depth under axial stress and triaxial stress. Furthermore, its change trend is consistent with the change trend of the tubing string load along the well depth.

Figure 13 shows the change trend of failure probability of tubing string with well depth and service time. As shown in the figure, given the effect of axial stress and triaxial stress, the failure probability of tubing string in the section of 0–500 m decreases slightly with an increase in well depth. However, it increases with increases in well depth and reaches the maximum at the bottom of the well. Additionally, the failure probability of the tubing string increases significantly with increases in service time. After 10 years of service, the failure probability of the tubing string at the bottom reaches 78.97% and the curve is consistent with the failure trend of bathtub curve.
6.2 | Analysis of effect of temperature and corrosion rate

Figure 14 shows the failure probability of the tubing string at different temperatures and corrosion rates. As shown in the figure, an increase in the corrosion rate of the tubing string and wellbore temperature decreases the strength of the tubing string, and the failure probability of the tubing string increases accordingly. When the temperature and corrosion rate increase from 25°C and 0 mm/a to 250°C and 0.25 mm/a, the failure probability increases from 12.39% to 61.27%. This in turn indicates that the risk of failure increases by 395%. Therefore, the reliability of the tubing string can be significantly improved, and the risk of failure can be reduced by controlling wellbore temperature and adopting anticorrosion measures to decrease the wellbore corrosion rate.

6.3 | Analysis on effect of manufacturing tolerances and steel grade

Figure 15 shows the failure probability of tubing strings with different manufacturing tolerances and steel grades. As shown in the figure, the failure probability of the tubing string decreases significantly with increases in the original strength of tubing string. The steel grade increases from 80 to 110 ksi, failure probability decreases from 69.22% to 2.48%, and the risk of failure decreases by 216%. Simultaneously, manufacturing tolerance of tubing string decreases, failure probability decreases from 14.4%
to 2.48%, and risk of failure decreases by 13.93%. Therefore, the failure probability and risk of tubing string can be significantly reduced by optimizing the steel grade design of the tubing string and controlling manufacturing tolerance of the tubing string.

7 | CONCLUSION

The aim of the study involves examining the failure of tubing strings in the production process of high-temperature, high-pressure, and ultra-deep gas wells by investigating the change law of tubing string load and strength and analyzing the failure probability of tubing strings under the effect of uncertainties such as temperature, manufacturing tolerance, and complex load. The following conclusions were obtained:

1. Monte Carlo method is adopted to simulate local fluctuation of downhole actual load, and the load variation rule of tubing string in high-temperature, high-pressure, and ultra-deep well is obtained. Furthermore, the effect of manufacturing tolerance and corrosion is considered and the variation law of string strength is obtained. Additionally, the effect of load, temperature, manufacturing tolerance, and other uncertainties on the strength and stress of the tubing string is analyzed, and a quantitative evaluation method for the mechanical integrity of the tubing string is established.

2. Under the action of high temperature, corrosion, and complex load, the reliability of the tubing string
gradually decreases with increases in well depth, and
the reduction rate of the tubing string reliability evi-
dently accelerated with increases in service time,
which conforms to the failure trend of the bathtub
curve.
3. The effect of the complex load and high temperature
decreases string strength and increases uncertainty of
the load, resulting in an increase in the failure prob-
ability of the tubing string. Furthermore, a high fail-
ure risk exists when the safety factor of the tubing
string satisfies standard requirements. However,
under the effect of manufacturing tolerance, the ac-
tual yield strength of the tubing string exceeds nom-
inal yield strength, which reduces the risk of tubing
string failure.
4. The main factors of tubing string failure include
temperature, corrosion rate, and steel grade. The
failure probability of the tubing string increased by
25.33% under the effect of high temperatures. Si-
multaneously, an increase in the corrosion rate and
manufacturing tolerance also increased failure prob-
ability. However, the failure probability of tubing
strings can be significantly reduced, and the mechanical integrity of the tubing string can be guaranteed via rational design of the steel grade, control of manufacturing tolerance, and reduction of the corrosion rate of the tubing string.

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