Mass Distribution in a Spatial Mechanism to Reduce Shaking Moment

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Abstract

Background/Objectives: The inertial effects exerted on the supporting frame of a spatial mechanism depend upon the distribution of mass of various links for a particular geometry. The main objective of the present work is to provide a suitable mass distribution to reduce shaking forces and shaking moments. Methods/Statistical Analysis: In the present paper a Generalized Slider Crank Mechanism is analyzed. A geometrical model of seven points is used to represent the mass and moment of inertia of each link. The masses at the seven points are chosen in such a way that the masses on positive and negative sides of each of the three co-ordinate axis are equal but different from the masses located on the other two axes. The seventh point mass is taken at the origin. Each of the point masses are taken to vary between 0.01 and 0.99 times the link mass. Findings and Improvements: The decrease in the shaking moment achieved is found to be due to the overall decrease in the position-vectors of the individual point masses from the origin of the fixed co-ordinate system. Slight reduction in the shaking force is also observed. All the seven point masses considered are positive which differs from the earlier model where negative masses are also allowed.

Keywords: Point-Mass, Position Vector, Shaking Moment, Spatial Mechanism, Vibration

1. Introduction

Balancing of shaking effects in spatial mechanisms is important in order to improve their dynamic performance and fatigue life. The major achievement in the field of full shaking force balancing is “The method of linearly independent vectors”, by¹, where the mass center is made stationary. In² extended this method to spatial linkages. The method of reducing the shaking forces without counterweights and the decrease in input torques by decreasing the acceleration of the mass center of links is shown by³. The equivalent system of point-masses is shown in⁴ for the dynamic modeling of a mechanism. Several authors attempted for minimizing the shaking effects⁵–¹⁰. The mass of the mechanism along with other dynamic characteristics increases in the process of balancing shaking force¹. In⁶ used a dynamically equivalent system of the actual links and analyzed a Generalized Slider Crank Mechanism. But the results did not appreciably decrease the shaking effects. In the present paper a modification is proposed to Rehman’s model and generalized slider crank with all offsets is analyzed.

2. Co-Ordinate System of Point Masses

In order to reduce the inertia-induced forces, along with other dynamic quantities in a high-speed generalized slider crank mechanism, the method of using four point mass model dynamically similar to the actual links proposed by⁶ is shown in Figure 1. The acceleration of
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half point masses and point mass \( m_{i4} \) at the origin of the co-ordinate system are given by Equations (1) to (3). The link number is indicated by ‘i’ and the corresponding point masses on the co-ordinate axes are indicated by ‘j’.

Positive and negative sides of axes are indicated by P and N respectively. \( i+1 \text{d}_{ij} \) represents the location of point mass, from the distal coordinate system \( \{i+1\} \).

\[
a_{ijP} = \omega_1 x (\omega_1 x i+1 \text{d}_{ij}) + \frac{d\omega_1}{dt} x i+1 \text{d}_{ij} + \frac{d^2(Di)}{dt^2} \] \hspace{1cm} (1)

\[
a_{ijN} = \omega_1 x (\omega_1 x i+1 \text{d}_{ij}) + \frac{d\omega_1}{dt} x i+1 \text{d}_{ij} + \frac{d^2(Di)}{dt^2} \] \hspace{1cm} (2)

\[
a_{i4} = \omega_1 x (\omega_1 x i+1 \text{d}_{i4}) + \frac{d\omega_1}{dt} x i+1 \text{d}_{i4} + \frac{d^2(Di)}{dt^2} \] \hspace{1cm} (3)

The first two terms indicate the centripetal and tangential acceleration components of link (i) respectively. The third term gives the double derivative of the position vector of the origin of co-ordinate system at the distal end of \( i^{th} \) link. In this model it was considered that all the point masses on the axes are each equal to \( m/8 \) and the center mass is \( m/4 \). But the shaking effects did not change appreciably.

3. Proposed Methodology

The mass distribution of each moving link has been replaced by a system of point masses for minimizing the adverse effect of the inertia forces in a high-speed spatial mechanism. The vector coordinates of different point masses of various links of Slider Crank Mechanism with all offsets are calculated.

The slider crank mechanism considered is shown in Figure 2. Both the joints between the crank and the connecting rod and between connecting rod and slider are ball (spherical) joints which have three rotational degrees of freedom about each one of three mutually perpendicular axes. The joint between slider and frame is prismatic i.e., joint having only one translational degree of freedom.

A system of seven point masses at the positive and negative sides of the three co-ordinate axes and at the origin of the co-ordinate system as shown in Figure 3 is used. The masses at these seven points are chosen in such a way that the masses on positive and negative sides of each of a particular co-ordinate axis are equal but different from the mass located on the other two axes. The sum of all the point masses is taken to be equal to the link mass. In the proposed method of the mass distribution of link (equal masses on both sides of axis) each of the point masses are taken to vary between 0.01 and 0.99 times the link mass.

The following conditions of dynamical equivalence are derived for this model:

\[
\sum_{i=1}^{7} m_i = M \hspace{1cm} (4)
\]

\[
m_1 x_1 + m_2 x_2 = M x = 0 \hspace{1cm} (5)
\]

\[
m_3 y_1 + m_4 y_2 = M y = 0 \hspace{1cm} (6)
\]

\[
m_5 z_1 + m_6 z_2 = M z = 0 \hspace{1cm} (7)
\]
\[ I_x = m_y y_1^2 + m_y y_2^2 + m_z z_1^2 + m_z z_2^2 \]  \( \ldots \ldots \)  \( \text{(8)} \)

\[ I_y = m_x x_1^2 + m_x x_2^2 + m_z z_1^2 + m_z z_2^2 \]  \( \ldots \ldots \)  \( \text{(9)} \)

\[ I_z = m_x x_1^2 + m_x x_2^2 + m_y y_1^2 + m_y y_2^2 \]  \( \ldots \ldots \)  \( \text{(10)} \)

Here, \( m_1 = m_2 = m_3 = m_4 \); \( m_5 = m_6 \) and \( m_1 \neq m_3 \neq m_5 \neq m_7 \)

\[ X_1 = -X_2; Y_1 = -Y_2; Z_1 = -Z_2 \]  \( \ldots \ldots \)  \( \text{(11)} \)

Solving the above equations, we get
\[ X_1 = (I_{yy} + I_{zz} - I_{xx})/4m_1 \]
\[ Y_1 = (I_{xx} + I_{zz} - I_{yy})/4m_3 \]
\[ Z_1 = (I_{xx} + I_{yy} - I_{zz})/4m_5 \]

4. Results and Discussions

The Shaking forces and shaking moments are calculated at each position of the input link from 0° through 360° and plotted as shown in Figures 4 and 5. Because of the effect of the mass distribution on the position vector of each point mass, the reduction in shaking moment is encouraging as shown in Figure 5. It can be noticed from the Figure 5 of shaking moment Vs crank angle that the shaking moment is the lowest near 50° crank angle and highest at 240° crank angle. The decrement in shaking moment without any appreciable change in shaking force is studied thoroughly, and is found to be due to overall decrease in the vectors representing the position of the individual point masses from the origin of the Fixed Coordinate system.

The variation in the position vectors of the point masses \( M_{21} \) and \( M_{24} \) of connecting rod on the three co-ordinate axes from the origin of fixed system for 10 different sets of masses as compared to their corresponding literature values at 240° of crank rotation are as shown in Figures 6 (a), (b) (c) and 7(a), (b) (c) respectively. The position vector of the point mass \( M_{27} \) at the center of mass of connecting rod (a), (b), (c) The position vector of \( M_{21} \) on X, Y and Z axis from the origin of fixed frame for 10 different sets at 240°.
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In the recent studies by Rehman negative point masses are also allowed in parallelepiped model. In the present work all positive point masses are considered in the Co-ordinate system to find the suitable mass model of a link to decrease the shaking moment.

In literature Rehman has taken equal masses on all axes and one at the center. In this study the masses at the seven points are chosen in such a way that the masses on positive and negative sides of each of the co-ordinate axis are equal but different from the mass located on the other two axes. The maximum shaking moment obtained at

Figure 7. (a), (b), (c) The position vector of M_{24} on X, Y and Z-axis from the origin of fixed frame for 10 different sets of masses at 240°.

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Figure 8. (a), (b) The position vector of M_{27} on X, Y-axis from the origin of fixed frame for different sets at 240°.

Figure 9. (a), (b) The variation of position vector of point mass M_{21} on connecting rod for one complete rotation of the crank.
240° crank angle, 6.084e+002 Nm is 3.5% less compared to 6.303e+002Nm obtained by the literature model\[6\]. The variation of position vector of point mass $M_{21}$ on Connecting rod for one complete rotation of the crank is shown in Figure 9.

6. Conclusions

The masses at the seven points are chosen in such a way that the masses on positive and negative sides of each of the three co-ordinate axis are equal to one other but different from the mass located on the other two axes. The resulting decrement in shaking moment without any appreciable change in shaking force is studied thoroughly, and is found to be due to overall decrease in the position vectors of the individual point masses from the Fixed Coordinate system.

7. References

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