Supersymmetric Scalar Masses, $Z'$, and E(6)

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Abstract

Assuming the existence of a supersymmetric U(1) gauge factor at the TeV energy scale (motivated either by the superstring-inspired $E_6$ model or low-energy electroweak phenomenology), several important consequences are presented. The two-doublet Higgs structure at the 100 GeV energy scale is shown to be different from that of the Minimal Supersymmetric Standard Model (MSSM). A new neutral gauge boson $Z'$ corresponding to the extra U(1) mixes with the $Z$. The supersymmetric scalar quarks and leptons receive new contributions to their masses from the spontaneous breaking of this extra U(1). The assumption of universal soft supersymmetry breaking terms at the grand-unification energy scale implies a connection between the U(1) breaking scale and the ratio of the vacuum expectation values of the two electroweak Higgs doublets.

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Introduction

Consider the sequential reduction in rank of the symmetry group $E_6$:

\[
E_6 \rightarrow SO(10) [\times U(1)_\psi] \quad (1)
\]
\[
\rightarrow SU(5) [\times U(1)_\chi] \quad (2)
\]
\[
\rightarrow SU(3)_C \times SU(2)_L [\times U(1)_Y]. \quad (3)
\]

At each step, a $U(1)$ gauge factor may or may not appear, depending on the details of the symmetry breaking. If $E_6$ is indeed the grand-unification group, it is often assumed that a single $U(1)$ survives down to the TeV energy range, given by

\[
U(1)_\psi \times U(1)_\chi \rightarrow U(1)_\alpha. \quad (4)
\]

This talk is concerned mainly with the phenomenological consequences of extending the MSSM to include this $U(1)_\alpha$.

New $U(1)$ and New Particles

Under the maximal subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R$, the fundamental representation of $E_6$ is given by

\[
27 = (3, 3, 1) + (3^*, 1, 3^*) + (1, 3^*, 3). \quad (5)
\]

Under the subgroup $SU(5) \times U(1)_\psi \times U(1)_\chi$, we then have

\[
27 = (10; 1, -1) [(u, d), u^c, e^c] \\
+ (5^*; 1, 3) [d^c, (\nu_e, e)] \\
+ (1; 1, -5) [N] \\
+ (5; -2, 2) [h, (E^c, N^c_E)].
\]
\begin{align*}
+ (5^*; -2, -2) \ [h^c, (\nu_E, E)] \\
+ (1; 4, 0) \ [S],
\end{align*}

(6)

where the U(1) charges refer to \(2\sqrt{6}Q_\psi\) and \(2\sqrt{10}Q_\chi\). Note that the known quarks and leptons are contained in \(10; 1, -1\) and \((5^*; 1, 3)\), and the two Higgs scalar doublets are represented by \((\nu_E, E)\) and \((E^c, N_E^c)\). Let

\[Q_\alpha = Q_\psi \cos \alpha - Q_\chi \sin \alpha,\]

(7)

then the so-called \(\eta\)-model\([1, 2]\) is obtained with \(\tan \alpha = \sqrt{3/5}\) and we have

\[\begin{align*}
27 &= \ (10; 2) + (5^*; -1) + (1; 5) \\
&\quad + (5; -4) + (5^*; -1) + (1; 5),
\end{align*}\]

(8)

where \(2\sqrt{15}Q_\eta\) is denoted; and the \(N\)-model\([3]\) is obtained with \(\tan \alpha = -1/\sqrt{15}\) resulting in

\[\begin{align*}
27 &= \ (10; 1) + (5^*; 2) + (1; 0) \\
&\quad + (5; -2) + (5^*; -3) + (1; 5),
\end{align*}\]

(9)

where \(2\sqrt{10}Q_N\) is denoted. The \(\eta\)-model is theoretically attractive because it is obtained if the symmetry breaking of \(E_6\) occurs via only the adjoint \(78\) representation which is what the superstring flux mechanism may do\([4]\). It is also phenomenologically interesting because it allows for an explanation of the experimental \(R_b\) excess\([4]\). The \(N\)-model is so called because \(N\) has \(Q_N = 0\). It allows \(S\) to be a naturally light singlet neutrino and is ideally suited to explain the totality of all neutrino-oscillation experiments\([3]\). It is also a natural consequence of an alternative \(SO(10)\) decomposition\([4]\) of \(E_6\), \(i.e.

\[\begin{align*}
16 &= [(u, d), u^c, e^c; h^c, (\nu_E, E); S],
\end{align*}\]

(10)

\[\begin{align*}
10 &= [h, (E^c, N_{E}^c); d^c, (\nu_e, e)],
\end{align*}\]

(11)

\[1 = [N],
\]

(12)
which differs from the conventional assignment by how the $SU(5)$ multiplets are embedded.

**Higgs Sector**

The Higgs sector of the $U(1)_\alpha$-extended supersymmetric model consists of two doublets and a singlet. They transform under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\alpha$ as follows.

\[
\Phi_1 \equiv \begin{pmatrix} \phi_1^0 \\ -\phi_1^- \end{pmatrix} \equiv \begin{pmatrix} \tilde{\nu}_E \\ \tilde{E} \end{pmatrix} \sim \left(1, 2, -\frac{1}{2}; -\frac{1}{\sqrt{6}} \cos \alpha + \frac{1}{\sqrt{10}} \sin \alpha \right),
\]

\[
\Phi_2 \equiv \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix} \equiv \begin{pmatrix} \tilde{E}^c \\ \tilde{N}_E \end{pmatrix} \sim \left(1, 2, \frac{1}{2}; -\frac{1}{\sqrt{6}} \cos \alpha - \frac{1}{\sqrt{10}} \sin \alpha \right),
\]

\[
\chi \equiv \tilde{S} \sim \left(1, 1, 0; \frac{\sqrt{2}}{3} \cos \alpha \right).
\]

Hence the Higgs potential has the contribution

\[
V_F = f^2[(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)(\bar{\chi}\chi)],
\]

where $f$ is the Yukawa coupling of the $\Phi_1 \Phi_2 \chi$ term in the superpotential. From the gauge interactions, we have the additional contribution

\[
V_D = \frac{1}{8}g_2^2[(\Phi_1^\dagger \Phi_1)^2 + (\Phi_2^\dagger \Phi_2)^2 + 2(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) - 4(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)]
\]

\[
+ \frac{1}{2}g_1^2[-\frac{1}{2} \Phi_1^\dagger \Phi_1 + \frac{1}{2} \Phi_2^\dagger \Phi_2]^2 + \frac{1}{2}g_\alpha^2[-\frac{1}{\sqrt{6}} \cos \alpha + \frac{1}{\sqrt{10}} \sin \alpha] \Phi_1^\dagger \Phi_1
\]

\[
+ \left(-\frac{1}{\sqrt{6}} \cos \alpha - \frac{1}{\sqrt{10}} \sin \alpha \right) \Phi_2^\dagger \Phi_2 + \sqrt{\frac{2}{3}} \cos \alpha \bar{\chi}\chi]^2.
\]

Let $\langle \chi \rangle = u$, then $\sqrt{2}Re\chi$ is a physical scalar boson with

\[
M^2 = \frac{4}{3} \cos^2 \alpha \ g_\alpha^2 u^2,
\]

and the $(\Phi_1^\dagger \Phi_1)\sqrt{2}Re\chi$ coupling is

\[
F = \sqrt{2}u \left[f^2 + g_\alpha^2 \sqrt{\frac{2}{3}} \cos \alpha \left(-\frac{1}{\sqrt{6}} \cos \alpha + \frac{1}{\sqrt{10}} \sin \alpha \right) \right].
\]

\[4\]
The effective \((\Phi^\dagger_1 \Phi_1)^2\) coupling \(\lambda_1\) is thus given by\[5, 6, 7\]
\[
\lambda_1 = \frac{1}{4}(g_1^2 + g_2^2) + g_\alpha^2 \left( -\frac{1}{\sqrt{6}} \cos \alpha + \frac{1}{\sqrt{10}} \sin \alpha \right)^2 - \frac{F^2}{M^2} \\
= \frac{1}{4}(g_1^2 + g_2^2) + \left(1 - \sqrt{\frac{3}{5}} \tan \alpha \right) f^2 - \frac{3f^4}{2 \cos^2 \alpha \ g_\alpha^2}. \tag{20}
\]
Similarly,
\[
\lambda_2 = \frac{1}{4}(g_1^2 + g_2^2) + \left(1 + \sqrt{\frac{3}{5}} \tan \alpha \right) f^2 - \frac{3f^4}{2 \cos^2 \alpha \ g_\alpha^2}, \tag{21}
\]
\[
\lambda_3 = -\frac{1}{4}g_1^2 + \frac{1}{4}g_2^2 + f^2 - \frac{3f^4}{2 \cos^2 \alpha \ g_\alpha^2}, \tag{22}
\]
\[
\lambda_4 = -\frac{1}{2}g_2^2 + f^2, \tag{23}
\]
where the effective two-doublet Higgs potential has the generic form
\[
V = m_1^2 \Phi^\dagger_1 \Phi_1 + m_2^2 \Phi^\dagger_2 \Phi_2 + m_{12}^2 (\Phi^\dagger_1 \Phi_2 + \Phi^\dagger_2 \Phi_1) \\
+ \frac{1}{2} \lambda_1 (\Phi^\dagger_1 \Phi_1)^2 + \frac{1}{2} (\Phi^\dagger_2 \Phi_2)^2 + \lambda_3 (\Phi^\dagger_1 \Phi_1)(\Phi^\dagger_2 \Phi_2) + \lambda_4 (\Phi^\dagger_1 \Phi_2)(\Phi^\dagger_2 \Phi_1). \tag{24}
\]
From Eqs. (20) to (23), it is clear that the MSSM is recovered in the limit of \(f = 0\). Let \(\langle \phi_{1,2}^0 \rangle \equiv v_{1,2}, \tan \beta \equiv v_2/v_1, \) and \(v^2 \equiv v_1^2 + v_2^2, \) then this \(V\) has an upper bound on the lighter of the two neutral scalar bosons given by
\[
(m_h^2)_{\text{max}} = 2v^2[\lambda_1 \cos^4 \beta + \lambda_2 \sin^4 \beta + 2(\lambda_3 + \lambda_4) \sin^2 \beta \cos^2 \beta] + \epsilon, \tag{25}
\]
where we have added the radiative correction due to the \(t\) quark and its supersymmetric scalar partners, \textit{i.e.}
\[
\epsilon = \frac{3g_\top^2 m_t^4}{8\pi^2 M_W^2} \ln \left(1 + \frac{\tilde{m}_t^2}{m_t^2} \right). \tag{26}
\]
Using Eqs. (20) to (23), we obtain
\[
(m_h^2)_{\text{max}} = M_Z^2 \cos^2 2\beta + \epsilon \\
+ \frac{1}{\sqrt{2} G_F} \left[ f^2 \left(\frac{3}{2} - \sqrt{\frac{3}{5}} \tan \alpha \cos 2\beta - \frac{1}{2} \cos^2 2\beta \right) - \frac{3f^4}{2 \cos^2 \alpha \ g_\alpha^2} \right]. \tag{27}
\]
Hence the MSSM bound can be exceeded for a wide range of values of $\alpha$ and $\beta$. Normalizing $U(1)_Y$ and $U(1)_\alpha$ at the grand-unification energy scale, we find it to be a very good approximation\[^8\] to have $g^2_\alpha = (5/3) g^2_1$. We use this and vary $f^2$ in Eq. (27) subject to the condition that $V$ be bounded from below. We find the largest numerical value of $m_h$ to be about 142 GeV, as compared to 128 GeV in the MSSM, and this is achieved with
\[
\tan \alpha = -\frac{2\sqrt{3/5} \cos 2\beta}{3 - \cos^2 2\beta},
\] (28)
which is possible in the $\eta$-model.

**Z - Z’ Sector**

The new $Z'$ of this model mixes with the standard $Z$ so that the experimentally observed $Z$ is actually
\[
Z_1 = Z \cos \theta + Z' \sin \theta,
\] (29)
where
\[
\theta \simeq \frac{1}{2} \sqrt{3} \frac{1}{2 \cos \alpha} g_Z \left( \sin^2 \beta - \frac{1}{2} + \frac{1}{2} \sqrt{3} \tan \alpha \right) \frac{v^2}{u^2},
\] (30)
resulting in a slight shift of its mass from that predicted by the standard model, as well as a slight change in its couplings to the usual quarks and leptons. These deviations can be formulated in terms of the oblique parameters\[^1\]:
\[
\epsilon_1 = \frac{1}{4} \left( \sin^4 \beta - \frac{1}{4} \left( 1 - \sqrt{3} \tan \alpha \right)^2 \right) \frac{v^2}{u^2} \simeq \alpha T,
\] (31)
\[
\epsilon_2 = \frac{1}{4} \left( 3 - \sqrt{15} \tan \alpha \right) \left[ \sin^2 \beta - \frac{1}{2} \left( 1 - \sqrt{3} \tan \alpha \right) \right] \frac{v^2}{u^2} \simeq -\frac{\alpha U}{4 \sin^2 \theta_W},
\] (32)
\[
\epsilon_3 = \frac{1}{4} \left[ 1 - 3 \sqrt{3} \tan \alpha + \frac{1}{2 \sin^2 \theta_W} \left( 1 + \sqrt{3} \tan \alpha \right) \right] \left[ \sin^2 \beta - \frac{1}{2} \left( 1 - \sqrt{3} \tan \alpha \right) \right] \frac{v^2}{u^2} \simeq \frac{\alpha S}{4 \sin^2 \theta_W},
\] (33)
Note that for $\sin^2 \beta$ near $(1/2)(1 - \sqrt{3/5} \tan \alpha)$, $\epsilon_{1,2,3}$ are all suppressed. In any case, the experimental errors on these quantities are fractions of a percent, hence $u \sim \text{TeV}$ is allowed.

The mass of $Z'$ is approximately equal to that of $\sqrt{2} \text{Re} \chi$, i.e. $M$ of Eq. (18). Its interactions are of course determined by $U(1)_\alpha$. In particular, in the $N$-model, two $S$’s are light singlet neutrinos, hence the ratio of the decay rates of $Z'$ to $\nu\bar{\nu} + S\bar{S}$ over $Z'$ to $\ell^-\ell^+$ is 62/15, instead of 4/5 without the $S$’s. This would be a great experimental signature.

**Supersymmetric Scalar Masses**

Consider the masses of the supersymmetric scalar partners of the quarks and leptons:

$$m_B^2 = m_0^2 + m_R^2 + m_F^2 + m_D^2,$$

where $m_0$ is a universal soft supersymmetry breaking mass at the grand-unification scale, $m_R^2$ is a correction generated by the renormalization-group equations running from the grand-unification scale down to the TeV scale, $m_F$ is the explicit mass of the fermion partner, and $m_D^2$ is a term induced by gauge-boson masses. In the MSSM, $m_D^2$ is of order $M_Z^2$ and does not change $m_B$ significantly. In the $U(1)_{\alpha}$-extended model, $m_D^2$ is of order $M_{Z'}^2$ and will affect $m_B$ in a nontrivial way. For example, for the ordinary quarks and leptons,

$$\Delta m_D^2(10; 1, -1) = \frac{1}{8} M_{Z'}^2 \left(1 + \sqrt{\frac{3}{5}} \tan \alpha \right),$$

$$\Delta m_D^2(5^*; 1, 3) = \frac{1}{8} M_{Z'}^2 \left(1 - 3 \sqrt{\frac{3}{5}} \tan \alpha \right).$$

This would have important consequences on the experimental search of supersymmetric particles. In fact, depending on $m_F$, it is possible for exotic scalars to be lighter than the usual scalar quarks and leptons.

Another important outcome of Eq. (34) is that the $U(1)_\alpha$ and electroweak symmetry breakings are related. To see this, go back to the two-doublet Higgs potential $V$ of
Eq. (24). Using Eqs. (20) to (23), we can express the parameters $m^2_{12}$, $m^2_1$, and $m^2_2$ in terms of the mass of the pseudoscalar boson, $m_A$, and $\tan \beta$.

\begin{align*}
  m^2_{12} &= -m^2_A \sin \beta \cos \beta, \quad (37) \\
  m^2_1 &= m^2_A \sin^2 \beta - \frac{1}{2} M^2_Z \cos 2\beta \\
  &\quad - \frac{2f^2}{g^2_Z} M^2_Z \left[ 2 \sin^2 \beta + \left( 1 - \sqrt{\frac{3}{5}} \tan \alpha \right) \cos^2 \beta - \frac{3f^2}{2 \cos^2 \alpha g^2_\alpha} \right], \quad (38) \\
  m^2_2 &= m^2_A \cos^2 \beta + \frac{1}{2} M^2_Z \cos 2\beta \\
  &\quad - \frac{2f^2}{g^2_Z} M^2_Z \left[ 2 \cos^2 \beta + \left( 1 + \sqrt{\frac{3}{5}} \tan \alpha \right) \sin^2 \beta - \frac{3f^2}{2 \cos^2 \alpha g^2_\alpha} \right]. \quad (39)
\end{align*}

On the other hand, using Eq. (34), we have

\begin{align*}
  m^2_{12} &= f A_f u, \quad (40) \\
  m^2_1 &= m^2_0 + m^2_R(\tilde{g}, f) + f^2 u^2 - \frac{1}{4} \left( 1 - \sqrt{\frac{3}{5}} \tan \alpha \right) M^2_{Z'}, \quad (41) \\
  m^2_2 &= m^2_0 + m^2_R(\tilde{g}, f) + f^2 u^2 - \frac{1}{4} \left( 1 + \sqrt{\frac{3}{5}} \tan \alpha \right) M^2_{Z'} + m^2_R(\lambda_t), \quad (42)
\end{align*}

where $f A_f$ is the coupling of the soft supersymmetry breaking $\Phi_1 \Phi_2 \chi$ scalar term, $\tilde{g}$ is the gluino, and $\lambda_t$ is the Yukawa coupling of $\Phi_2$ to the $t$ quark. Matching Eqs. (37) to (39) with Eqs. (40) to (42) allows us to determine $u$ and $\tan \beta$ as a function of $f$ for a given set of parameters at the grand-unification scale.

In the MSSM assuming Eq. (34),

\begin{equation}
  m^2_1 - m^2_2 = -m^2_R(\lambda_t) = -(m^2_A + M^2_Z) \cos 2\beta. \quad (43)
\end{equation}

Since $m^2_R(\lambda_t) < 0$, we must have $\tan \beta > 1$. In the $U(1)_\alpha$-extended model, because of the extra D-term contribution, $\tan \beta < 1$ becomes possible. Another consequence is that because of Eq. (35), a light scalar $t$ quark is not possible unless $\tan \alpha < -\sqrt{5/3}$.
Conclusions

(1) Supersymmetric $U(1)_\alpha$ from $E_6$ is a good possibility at the TeV scale. (2) The two-doublet Higgs structure at around 100 GeV will be different from that of the MSSM. (3) Supersymmetric scalar masses depend crucially on $U(1)_\alpha$. (4) The $U(1)_\alpha$ breaking scale and $\tan \beta$ are closely related.

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