On the Time-Varying Efficiency of Cryptocurrency Markets

Akihiko Noda\textsuperscript{a,b}\footnote{Corresponding Author. E-mail: noda@cc.kyoto-su.ac.jp, Tel. +81-75-705-1510, Fax. +81-75-705-3227.}

\textsuperscript{a} Faculty of Economics, Kyoto Sangyo University, Motoyama, Kamigamo, Kita-ku, Kyoto 603-8555, Japan
\textsuperscript{b} Keio Economic Observatory, Keio University, 2-15-45 Mita, Minato-ku, Tokyo 108-8345, Japan

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Abstract: This study examines whether the market efficiencies of major cryptocurrencies (e.g., Bitcoin, Ethereum, and Ripple) change over time based on the adaptive market hypothesis (AMH) of Lo (2004). In particular, we measure the degree of market efficiency using Ito et al.'s (2014; 2016; 2017) generalized least squares-based time-varying model. The empirical results show that (1) the degree of market efficiency varies with time in cryptocurrency markets, (2) the market efficiency level of Bitcoin is higher than that of the other markets over most periods, and (3) the market efficiency of cryptocurrencies has evolved. We conclude that the results support the AMH for the established cryptocurrency market.

Keywords: Cryptocurrency Markets; Adaptive Market Hypothesis; Efficient Market Hypothesis; GLS-Based Time-Varying Model Approach; Degree of Market Efficiency.

JEL Classification Numbers: C32; G12; G14.
1 Introduction

Since the study of Nakamoto (2008), cryptocurrency markets have expanded, their total market capitalization reaching USD 800 billion by January 2018. However, these markets have subsequently experienced a crisis, their total market capitalization decreasing to USD 100 billion by the end of 2018. As such, we can conclude that the investors on financial markets have managed cryptocurrencies as an asset. Further, economists consider that investigating the efficiency of the cryptocurrency market in the sense of Fama’s (1970) is essential for evaluating the price mechanism of financial markets. Therefore, several recent studies on cryptocurrency markets aim to determine whether these markets are efficient.

There exists a large body of literature on the weak-form of Fama’s (1970) efficient market hypothesis (EMH) for cryptocurrency markets, especially the Bitcoin market. However, there exists controversy over the cryptocurrency market efficiency between the proponents and opponents of the EMH. For example, Urquhart (2016), Nadarajah and Chu (2017), Bariviera (2017), and Tiwari et al. (2018) conclude that the Bitcoin market is almost efficient. By contrast, Yonghong et al. (2018), Cheah et al. (2018), and Al-Yahyae et al. (2018) present skeptical empirical results that do not support the EMH for this market. One of the reasons for this controversy is that efficiency varies over time. As a result, Lo (2004) proposes the adaptive market hypothesis (AMH) as an evolutionary alternative to the EMH, reinforcing the view that the market evolves over time and that market efficiency also change over time. Specifically, he estimates the time-varying first-order autocorrelation of returns on the U.S. stock market using 60-month rolling windows. His empirical results show that market efficiency keeps evolving over time.

To examine the AMH, two approaches have been adopted in the literature. The first one is measuring the degree of market efficiency together with its statistical inference. Some studies employ Ito et al.’s (2014, 2016, 2017) generalized least squares (GLS)-based time-varying model to estimate the degree of market efficiency on international stock markets (see Ito et al. (2014, 2016) and Noda (2016) for details). Particularly, Noda (2016) tests the AMH using Japanese stock market data and concludes that the degree of market efficiency varies with time. Another approach is based on a conventional statistical test to examine the AMH under the moving-window method. In practice, Urquhart (2016) and Nadarajah and Chu (2017) employ Kim et al.’s (2011) automatic variance ratio test to examine the AMH for the Bitcoin market. However, the moving-window method poses the problem of choosing an optimal window width for the test statistic. Unlike the moving-window method, a GLS-based time-varying model has the superior property that it does not depend on sample size.

As such, this study examines Lo’s (2004) AMH on cryptocurrency markets from the viewpoint of market efficiency. Specifically, we focus on three major cryptocurrencies (i.e., Bitcoin, Ethereum, and Ripple) whose trading volumes and market capitalization are different. Particularly, we first estimate their market efficiency degrees using the GLS-based time-varying model approach with statistical inferences. Second, we analyze the changes in their market efficiency degrees over time and whether they show different efficiencies depending on trading volume and market capitalization. Finally, we explore what types of markets support the AMH.

1The historical data of total market capitalization is available at the web page of CoinMarketCap (https://coinmarketcap.com/charts/).
2 Method

2.1 Market Efficiency

Malkiel (1992) explicitly defines the EHM as the market fully and correctly reflecting all relevant information in determining security prices when the it is said to be efficient. This implies that market price reflects exogenous shocks immediately on financial markets. Generally, Malkiel's (1992) definition is mathematically represented as follows:

\[ E[x_t | I_{t-1}] = 0, \]  

where \( x_t \) denotes the return of a security at \( t \) period and \( I_{t-1} \) is the set of available information at \( t - 1 \) period, some \( \sigma \)-field to which \( x_{t-1}, x_{t-2}, \cdots \) is adapted. The EMH holds when the price of the security follows a random walk process. Further, the security price is randomly determined, it is namely "determined by chance." Therefore, it is natural to consider that an (excess) stock return follows a moving average process with infinite terms MA(\( \infty \)) when the hypothesis does not hold:

\[ x_t = u_t + \beta_1 u_{t-1} + \beta_2 u_{t-2} + \cdots , \]  

where \( \{u_t\} \) is an i.i.d. process. Since \( I_{t-1} \) is a \( \sigma \)-field to which \( x_{t-1} \) is adapted and \( \{I_{t-1}\} \) a system of sets of available information, the following equation holds:

\[ E[x_t | I_{t-1}] = \beta_1 u_{t-1} + \beta_2 u_{t-2} + \cdots . \]

Then, the EMH in the weak sense holds if and only if \( \beta_i = 0 \) for all \( i \).

As previously mentioned, Lo (2004) proposes the AMH which is alternative to the EMH. His hypothesis is based on evolutionary theory to economic interactions such as the changes in the market environment. In fact, several studies demonstrate that stock market efficiency has been gradually changing (see Ito et al. (2014, 2016), Kim et al. (2011), and Noda (2016)). However, there exists hardly studies that the time-varying structure of cryptocurrency market efficiency. The literature on the time-varying structure of cryptocurrency market efficiency is only Yonghong et al. (2018). They employ the rolling-window method using the generalized Hurst exponents to explore the time-varying structure. As mentioned above, the AMH implies that the degree of market efficiency fluctuates over time and reflects the evolving market environment. As such, we measure the time-varying degree of market efficiency and investigate whether the cryptocurrency market evolves over time from the viewpoint of the efficiency.

2.2 GLS-Based Time-Varying AR Model

We employ a GLS-based time-varying autoregressive (AR) model of Ito et al. (2016, 2017) to analyze financial data for which the data-generating process is time-varying. The conventional AR model,

\[ x_t = \alpha_0 + \alpha_1 x_{t-1} + \cdots + \alpha_q x_{t-q} + u_t , \]

has been frequently used to analyze the time series of the returns of assets, where \( \{u_t\} \) satisfies \( E[u_t] = 0, E[u_t^2] = 0, \) and \( E[u_t u_{t-m}] = 0 \) for all \( m \). While \( \alpha_i \)'s are assumed to
be constant in standard time series analysis, we assume that the coefficients of the AR model change over time. We thus apply a GLS-based time-varying AR (TV-AR) model to analyze cryptocurrency markets because financial markets have been facing structural changes by several reasons, such as economic crises (see Lim and Brooks (2011) for details).

A GLS-based TV-AR model is expressed as follows:

\[ x_t = \alpha_{0,t} + \alpha_{1,t}x_{t-1} + \cdots + \alpha_{q,t}x_{t-q} + u_t, \]  

(3)

where \( \{u_t\} \) satisfies \( E[u_t] = 0, E[u_t^2] = 0 \), and \( E[u_tu_{t-m}] = 0 \) for all \( m \). Furthermore, we assume that parameter dynamics restrict the parameters when we estimate a GLS-based TV-AR model using such data. Particularly,

\[ \alpha_{\ell,t} = \alpha_{\ell,t-1} + v_{\ell,t}, \quad (\ell = 1, 2, \cdots, q), \]  

(4)

where \( \{v_{\ell,t}\} \) satisfies \( E[v_{\ell,t}] = 0, E[v_{\ell,t}^2] = 0 \) and \( E[v_{\ell,t}v_{\ell,t-m}] = 0 \) for all \( m \) and \( \ell \). We solve a system of simultaneous equations using Equations (3) and (4).

According to Ito et al. (2017), a GLS-based TV-AR model has two major advantages over the conventional Bayesian method (e.g., Kalman filtering and smoothing). First, this method is quite simple and the calculation speed is fast. Unlike the conventional Bayesian method, no iteration by Markov chain Monte Carlo (MCMC) algorithms is required. Second, prior distributions of parameters are unnecessary when we employ a GLS-based TV-AR model. We can thus employ conventional statistical inferences (e.g., residual-based bootstrap method) on the time-varying estimates to conduct statistical inferences.

2.3 Time-Varying Degree of Market Efficiency

In this subsection, we first calculate the time-varying impulse responses from TV-AR coefficients over each period. Then, we calculate the confidence intervals for each coefficient based on the estimated covariance matrix. While the concept of a GLS-based TV-AR model is quite simple, two caveats exist: (1) a GLS-based TV-AR model is only an approximation of the real data-generating process, which is supposed to be a complex nonstationary process; and (2) we assume the estimated stationary AR(\( q \)) model index by period \( t \), which is stationary, as a local approximation of the underlying complex process.

We define the time-varying degree of market efficiency based on Ito et al.’s (2016, 2017) as follows:

\[ \zeta_t = \frac{\sum_{j=1}^{p} \hat{\alpha}_{j,t}}{1 - \left( \sum_{j=1}^{p} \hat{\alpha}_{j,t} \right)}. \]  

(5)

We measure the deviation from the zero coefficients on the corresponding time-varying moving-average model to the TV-AR model. Hence, this implies that large deviations of \( \zeta \) from zero are evidence of market inefficiency. We know that that degree \( \zeta_t \) crucially depends on sampling errors. Thus, we construct confidence intervals for \( \zeta_t \)’s on the condition that the market is efficient. We find the market at time \( t \) period is inefficient when \( \zeta_t \) exceed than the upper limit at \( t \) period of the intervals.

Specifically, the interval is constructed as follows. We first identify the returns with the residuals of a TV-AR(\( q \)) estimation under the above hypothesis that all coefficients
are zero. Second, we extract $N$ samples as an empirical distribution of the residuals. Third, we fit a TV-AR model to the $N$ bootstrap samples and derive $N$ sets of estimates. We then compute the $N$ bootstrap samples of $\zeta$ from the estimates. Finally, we construct confidence intervals from the $N$ bootstrap samples. Therefore, the bootstrap is conducted under the null hypothesis of zero autocorrelation. The estimates of the degree of efficiency exceed the 99% confidence intervals in Figure 2 imply a rejection of the null hypothesis of no return autocorrelation at the 1% significance level.

3 Data

We utilize the daily returns for the prices of the three major cryptocurrencies; Bitcoin (BTC), Ethereum (ETH), and Ripple (XRP). The start dates for the datasets are different for the three cryptocurrencies: July 7, 2010 for BTC, August 6, 2015 for ETH, and January 21, 2015 for XRP. On the other hand, the end dates are the same for all cryptocurrencies (December 31, 2018). We take the log first difference of the time series of prices to obtain the returns of the cryptocurrencies. Figure 1 presents time series plots of the returns for each cryptocurrency.

(Figure 1 around here)

Table 1 demonstrates the descriptive analysis for the returns. We confirm that the mean (standard deviation) of returns on the BTC is higher (lower) than those of ETH and XRP. This means that the BTC is a more established market than the others because a lower standard deviation of returns indicates better liquidity.

(Table 1 around here)

For estimations, all variables that appear in the moment conditions should be stationary. We apply the augmented Dickey–Fuller (ADF) test to confirm whether the variables satisfy the stationarity condition. The ADF test rejects the null hypothesis that the variables (all returns) contain a unit root at the 1% significance level.

4 Empirical Results

4.1 Preliminary Estimations

We assume a standard AR($q$) model with constant parameters and employ Schwarz's (1978) Bayesian information criterion (SBIC) to select the optimal lag order. Consequently, we choose the AR(7) model for BTC, AR(6) for ETH, and AR(2) for XRP. Table 2 entire summarizes the preliminary results for the standard AR($q$) models using the whole sample.

(Table 2 around here)

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2We employ daily closing price data to calculate the returns. The data are available from the website of Yahoo! Finance (https://finance.yahoo.com).
The AR estimates are statistically different from zero, except for some AR coefficients. However, this is because we use daily data, which fluctuate more widely than low-frequency ones.

Then, we utilize Hansen's (1992) test under the random parameters hypothesis to investigate whether the parameters are constant in the above AR models. As presented in Table 1, Hansen's (1992) $L_C$ statistics indicate that we reject the null of constant parameters against the parameter variation at the 1% significance level. This implies that the parameters follow the random walk process. Therefore, we estimate the time-varying AR models to investigate whether gradual changes occur on the three major cryptocurrency markets.

4.2 Time-Varying Degree of Market Efficiency

As previously mentioned, we employ the GLS-based time-varying AR model of Ito et al. (2016, 2017) to obtain the degree of market efficiency. We measure the cryptocurrency markets’ deviation from the efficient condition by using Equation (5) because the degree is based on the spectral norm. Then, the degree of market efficiency shows how the market is different from the efficient market. If $\zeta_t = 0$ for time $t$, the market is shown to be efficient at that time.

Figure 2 indicates the degree of market efficiency based on the above TV-AR models. We first find that the degrees of the BTC, ETH, and XRP change over time. Figure 2 also demonstrates the markets are inefficient during some bubble periods or economic crises. In practice, these correspond with the rapid price increases of cryptocurrencies and financial security breaches due to “Mt. Gox” in February 2014.

We confirm three significant differences among the major cryptocurrencies in terms of their degrees of market efficiency. First, since August 20, 2015, BTC shows the highest market efficient, being followed by XRP and ETH in this order. The averages of BTC, XRP, and ETH are 0.18, 0.23, and 0.32, respectively. Second, the market efficiencies of XRP and ETH fluctuate more widely than that of BTC. In fact, the standard deviations of the degrees of BTC, XRP, and ETH are 0.17, 0.18, and 0.27, respectively. Third, the market efficiency of the BTC has been less volatile since the financial security shock in February 2014, but those of XRP and ETH have not.

The differences among the BTC, ETH, and XRP in terms of trading volumes and market capitalization might explain these differences in market efficiency, as shown in Brauneis and Mestel (2018) and Wei (2018).

Particularly, Figure 3 demonstrates that trading volumes and market capitalizations are quite different among BTC, ETH, and XRP. Additionally, it is widely known that the market capitalization of BTC accounts for more than 50% of the total market capitalization on the entire cryptocurrency market. This means that the BTC is the market dominator.

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3We compare the market efficiencies over the same sample period because the periods are different among currencies. The average of BTC, XRP, and ETH are 0.66, 0.24, and 0.32, respectively using the entire sample for reference (see Table 1 for details).
on cryptocurrency markets and trade openness differs among cryptocurrencies. Figure 2 also shows that the market efficiency of BTC not only gradually changed over time but also has evolved since the financial security shock in early 2014. The empirical results are consistent with Urquhart’s (2016) and he shows that market efficiency tends to improve when using sub-sample estimation. The market efficiency of the BTC reflects the shock, whereas those of the ETH and XRP do not. Thus, the empirical results support Lo’s (2004) AMH on the more qualified cryptocurrency market, as shown in Noda (2016).

5 Concluding Remarks

In this study, we investigate whether the market efficiencies of cryptocurrencies change over time, based on the AMH of Lo (2004). Particularly, we estimate the degree of market efficiency based on Ito et al.‘s (2014; 2016; 2017) time-varying model approach. The empirical results show that (1) market efficiency varies with time on the cryptocurrency markets, (2) the market efficiency of the BTC is higher than that of the other cryptocurrencies in most periods, and (3) the market efficiency of the BTC has been less volatile since the financial security shock in February 2014, while those of XRP and ETH have not. Therefore, we conclude that the empirical results support Lo’s (2004) AMH for the more established market cryptocurrency market.

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References

Al-Yahyaee, K. H., Mensi, W., and Yoon, S. M. (2018), “Efficiency, Multifractality, and the Long-Memory Property of the Bitcoin Market: A Comparative Analysis with Stock, Currency, and Gold Markets,” *Finance Research Letters*, 27, 228–234.

Bariviera, A. F. (2017), “The Inefficiency of Bitcoin Revisited: A Dynamic Approach,” *Economics Letters*, 161, 1–4.

Brauneis, A. and Mestel, R. (2018), “Price Discovery of Cryptocurrencies: Bitcoin and Beyond,” *Economics Letters*, 165, 58–61.

Cheah, E.-T., Mishra, T., Parhi, M., and Zhang, Z. (2018), “Long Memory Interdependency and Inefficiency in Bitcoin Markets,” *Economics Letters*, 167, 18–25.

Lim and Kim (2011) and Noda (2016) conclude that trade openness is associated with market efficiency on stock markets.
Fama, E. F. (1970), “Efficient Capital Markets: A Review of Theory and Empirical Work,” *Journal of Finance*, 25, 383–417.

Hansen, B. E. (1992), “Testing for Parameter Instability in Linear Models,” *Journal of Policy Modeling*, 14, 517–533.

Ito, M., Noda, A., and Wada, T. (2014), “International Stock Market Efficiency: A Non-Bayesian Time-Varying Model Approach,” *Applied Economics*, 46, 2744–2754.

— (2016), “The Evolution of Stock Market Efficiency in the US: A Non-Bayesian Time-Varying Model Approach,” *Applied Economics*, 48, 621–635.

— (2017), “An Alternative Estimation Method of A Time-Varying Parameter Model,” [arXiv:1707.06837], Available at https://arxiv.org/pdf/1707.06837.pdf.

Kim, J. H., Shamsuddin, A., and Lim, K. P. (2011), “Stock Return Predictability and the Adaptive Markets Hypothesis: Evidence from Century-Long U.S. Data,” *Journal of Empirical Finance*, 18, 868–879.

Lim, K. P. and Brooks, R. (2011), “The Evolution of Stock Market Efficiency Over Time: A Survey of the Empirical Literature,” *Journal of Economic Surveys*, 25, 69–108.

Lim, K. P. and Kim, J. H. (2011), “Trade Openness and the Informational Efficiency of Emerging Stock Markets,” *Economic Modelling*, 28, 2228–2238.

Lo, A. W. (2004), “The Adaptive Markets Hypothesis: Market Efficiency from an Evolutionary Perspective,” *Journal of Portfolio Management*, 30, 15–29.

Malkiel, B. G. (1992), “Efficient Market Hypothesis,” in *New Palgrave Dictionary of Money & Finance*, eds. Newman, P., Milgate, M., and Eatwell, J., Macmillan, London, vol. 1, pp. 739–741.

Nadarajah, S. and Chu, J. (2017), “On the Inefficiency of Bitcoin,” *Economics Letters*, 150, 6–9.

Nakamoto, S. (2008), “Bitcoin: A Peer-to-Peer Electronic Cash System,” Online Available at https://bitcoin.org/bitcoin.pdf.

Newey, W. K. and West, K. D. (1987), “A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix,” *Econometrica*, 55, 703–708.

Noda, A. (2016), “A Test of the Adaptive Market Hypothesis using a Time-Varying AR Model in Japan,” *Finance Research Letters*, 17, 66–71.

Schwarz, G. (1978), “Estimating the Dimension of a Model,” *Annals of Statistics*, 6, 461–464.

Tiwari, A. K., Jana, R., Das, D., and Roubaud, D. (2018), “Informational Efficiency of Bitcoin–An Extension,” *Economics Letters*, 163, 106–109.

Urquhart, A. (2016), “The Inefficiency of Bitcoin,” *Economics Letters*, 148, 80–82.
Wei, W. C. (2018), “Liquidity and Market Efficiency in Cryptocurrencies,” *Economics Letters*, 168, 21–24.

Yonghong, J., He, H., and Weihua, R. (2018), “Time-Varying Long-Term Memory in Bitcoin Market,” *Finance Research Letters*, 25, 280–284.

Figure 1: The Returns on the BTC, ETH, and XRP

![Graph of Returns on BTC](image)

![Graph of Returns on ETH](image)

![Graph of Returns on XRP](image)
### Table 1: Descriptive Statistics and Unit Root Tests

|       | Mean  | SD    | Min   | Max    | ADF   | Lag | N   |
|-------|-------|-------|-------|--------|-------|-----|-----|
| $R_{BTC}$ | 0.0036 | 0.0684 | -0.8488 | 1.4744 | -22.0756 | 4   | 3089 |
| $R_{ETH}$ | 0.0031 | 0.0774 | -0.9163 | 0.3830 | -18.7072 | 2   | 1242 |
| $R_{XRP}$ | 0.0022 | 0.1114 | -0.9973 | 1.0280 | -27.9951 | 1   | 1440 |

Notes:
1. “ADF” denotes the ADF test statistics and “Lag” denotes the lag order selected by the BIC.
2. In computing the ADF test, a model with a time trend and a constant is assumed. The critical value at the 1% significance level for the ADF test is “−3.96”.
3. “N” denotes the number of observations.
4. R version 3.5.3 was used to compute the statistics.

### Table 2: Preliminary Estimations and Parameter Constancy Tests

|       | $R_{BTC}$ | $R_{ETH}$ | $R_{XRP}$ |
|-------|-----------|-----------|-----------|
| Constant | 0.0034 | 0.0033 | 0.0026 |
| $R_{t-1}$ | [0.0012] | [0.0023] | [0.0026] |
|          | 0.0409 | −0.0599 | −0.3095 |
| $R_{t-2}$ | [0.0345] | [0.0377] | [0.0830] |
|          | −0.1611 | 0.0239 | 0.0981 |
| $R_{t-3}$ | [0.0746] | [0.0300] | [0.0556] |
|          | 0.0228 | 0.0699 | − |
| $R_{t-4}$ | [0.0301] | [0.0341] | − |
|          | 0.1028 | −0.0185 | − |
| $R_{t-5}$ | [0.0679] | [0.0439] | − |
|          | 0.0796 | 0.0131 | − |
| $R_{t-6}$ | [0.0302] | [0.0331] | − |
|          | −0.0025 | 0.0141 | − |
| $R_{t-7}$ | [0.0474] | [0.0309] | − |
|          | −0.0254 | − | − |
| $R^2$    | 0.0511 | 0.0049 | 0.1244 |
| $L_C$    | 69.3123 | 84.5505 | 39.4939 |

Notes:
1. “$R_{t-p}$,” “$R^2$,” and “$L_C$” denote the AR(p) estimate, the adjusted $R^2$, and the Hansen’s 1992 joint $L$ statistic with variance, respectively.
2. Newey and West’s 1987 robust standard errors are between brackets.
3. R version 3.5.3 was used to compute the estimates.
Notes:

(1) The panels of the figure show the time-varying degree of market efficiency for the BTC (first panel), ETH (second panel), and XRP (third panel).

(2) The dashed red lines represent the 99% confidence intervals of the efficient market degrees.

(3) We run bootstrap sampling 20,000 times to calculate the confidence intervals.

(4) R version 3.5.3 was used to compute the estimates.
Figure 3: Trading Volumes and Market Capitalizations

Notes:

(1) The panels of the figure show trading volumes (left panel) and market capitalizations (right panel) for the BTC, ETH, and XRP.

(2) The dataset is obtained from the web page of CoinMarketCap [https://coinmarketcap.com/].

(3) R version 3.5.3 was used to compute the statistics.