Non-oscillating neutrinos in vacuum

Georgios Choudalakis

University of Athens
Physics Department
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Abstract

It is well known that matter effects affect the way neutrinos oscillate. The amplitude of oscillation in matter can be either enhanced, compared to the amplitude in vacuum, or suppressed, depending on the density of matter at the vicinity of a neutrino. Enhancement is less probable to occur than suppression.

This article demonstrates how matter effects can result into non-oscillating neutrinos even in vacuum.

1 The evolution equation in matter

We will firstly consider the case of two generations of neutrinos in matter. In the basis of flavor, the evolution equation of a neutrino in the 2-Dimensional flavor space takes [1] the form:

\[ i \frac{\partial}{\partial t} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} -A + M \\ B \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \]

(1)

where \( A \equiv \frac{\Delta m^2}{2E} \cos 2\theta_0 \), \( B \equiv \frac{\Delta m^2}{2E} \sin 2\theta_0 \), and \( M \equiv \sqrt{2} G_F N_e \). \( N_e \) is the numerical density of electrons, \( G_F \) is Fermi’s constant, \( \Delta m^2 \) is the mixing angle in vacuum, the one and only parameter of the \( 2 \times 2 \) mixing matrix for \( 2 \) generations. At any moment, the probability to observe the neutrino as a \( \nu_e \) equals \( |\nu_e|^2 \) and to observe it as a \( \nu_\mu \) equals \( |\nu_\mu|^2 \).

For constant \( N_e \), equation (1) can be solved analytically, as we will see in the next paragraph.

2 Constant \( N_e \)

In order to solve eq. (1) we need to diagonalize the hamiltonian \( H = \begin{pmatrix} -A + M & B \\ B & A \end{pmatrix} \).

It is easy to show that, for constant \( N_E \), the oscillation probabilities are
\[ P_{\nu_e \rightarrow \nu_{\mu}} = P_{\nu_{\mu} \rightarrow \nu_e} = \sin^2 2\theta \sin^2 \frac{\pi L}{L_m} \]  

(2)

where

\[ \tan 2\theta = \frac{2B}{2A - M} \]  

(3)

and

\[ L_m = \frac{2\pi}{E_A - E_B} \]  

(4)

with \( E_{A,B} \) being the eigenvalues of matrix \( H \), which are:

\[ E_{A,B} = \frac{M \pm \sqrt{M^2 + 4(A^2 + B^2 - AM)}}{2} \]  

(5)

This solution means that neutrinos in matter of constant density oscillate in a sinusoidal way, like in vacuum, while the amplitude and wavelength of oscillation in matter are different from those in vacuum. The difference results from the non-zero \( M \) term in equations (3) and (4). When \( N_e \) tends to 0, then \( M \) also tends to 0 and \( \theta \to \theta_0 \) and \( L_m \to L_0 = 4\pi \frac{E}{\Delta m^2} \).

An important result of this solution is that oscillation amplitude can be either enhanced or suppressed with respect to oscillation in vacuum, due to matter effects. This may be demonstrated by ranging \( M \) in eq. (3) from 0 to \( \infty \). For \( M \to +\infty \) we have \( \tan (2\theta) \to 0 \Rightarrow \theta \to 90^\circ \), which means zero amplitude, suppression of oscillation.

There is only one value of \( M \) which maximizes amplitude by giving \( \theta = 45^\circ \). This phenomenon is called MSW-resonance \[3] \) and happens when:

\[ M = M_{MSW} = 2A \Rightarrow \sqrt{2} G_F N_e = \frac{\Delta m^2}{2E} \cos 2\theta_0 \]  

(6)

Regarding the wavelength of oscillation \( L_m \), for zero \( N_e \) it equals \( L_0 \) while for \( N_e \to +\infty \) it tends to zero.

3 Variant \( N_e \)

As neutrinos travel through celestial bodies and other objects, they cross regions of extremely high and extremely low electron densities. Thus, solving eq. (1) for constant \( N_e \) does not help much. But when \( N_e \) is a function of position \( x \), then eq. (1) gets extremely complicated and can only be solved numerically\[3].

1By saying “neutrinos oscillation”, what we actually mean is “oscillatory variation of the survival/transition probability of neutrinos as time goes by”. Thus, amplitude and wavelength refer to the characteristics of the sinusoidal function which expresses their survival/transition probability.

2\( N_e \) is a function of position, but for neutrinos traveling at (almost) the speed of light, it is convenient to replace \( x \) with the neutrino time of flight \( t \), using Natural Units where \( c = \hbar = 1 \).

3There are only a few exceptions, like the well known adiabatic approximation \(2,4\), where the evolution equation with variant electrons density can be given an analytical approximate solution.
Figure 1: (a): The density profile for which we solved the evolution equation. It is a gaussian function which most of the time exceeds $N_{MSW}$. (b): The resulting survival probability of a neutrino which initially was a pure $\nu_e$. We can see the region of suppression at the center of the density distribution. We have assumed $E=3$ GeV, $\theta_0=32^\circ$, $\Delta m^2=7.2 \cdot 10^{-5}$ eV$^2$.

Either numerically or analytically, solutions of eq. (1) give us the amplitudes $\nu_e(t)$ and $\nu_\mu(t)$ as functions of time. Those amplitudes are complex numbers, but we usually express the solution in terms of $|\nu_e(t)|^2 \equiv P(\nu_e; t)$ and $|\nu_\mu(t)|^2 \equiv P(\nu_\mu; t)$, as done in eq. (2), because we have a more intuitive understanding of those probabilities. However, as it will have been explained by the end of this article, the complex nature of these amplitudes is important and may lead to observable phenomena.

When $N_e$ varies, $P(\nu_e; t)$ and $P(\nu_\mu; t)$ may vary in a chaotic fashion, depending on the density profile $N_e(t)$, the initial conditions $\nu_{e, \mu}(t=0)$, the energy $E$ of the neutrino and the input parameters $\theta_0$ and $\Delta m^2$. An example of such an irregular oscillation can be seen in Fig. 1. Nevertheless, in space (time) intervals where $N_e$ does not change very rapidly, the approximation of constant $N_e$ holds well and oscillation patterns in those regions look like those of eq. (2).
Another thing that should be noticed is that MSW-resonance may occur instantly if the density profile has not been adjusted so as to satisfy eq. (6) for long intervals. On the other hand, suppression is observed for \( N_e \gg N_{MSW} \), which holds for a whole range of density values, not just for a specific \( N_e \). So, if things have not been intentionally set up to be different, suppression is a phenomenon which may last for longer and is more probable than MSW-resonance.

### 4 The complex nature of \( \nu_{e,\mu} \)

Equation (1) is a system of complex differential equations and its solutions are complex functions of time, as stated earlier. The real and imaginary parts of \( \nu_e(t) \) and \( \nu_\mu(t) \) affect the real and imaginary parts of \( \nu_e(t + dt) \) and \( \nu_\mu(t + dt) \).

As for every differential equation, the solution of eq. (1) depends on the initial condition \( \nu_{e,\mu}(t = 0) \), which involves \( \nu_e(0) \) and \( \nu_\mu(0) \) as complex numbers:

\[
\nu_\alpha = |\nu_\alpha| e^{i\phi_\alpha}, \quad \alpha = e, \mu
\]

Starting from the same \( |\nu_\alpha| \) and different \( \phi_\alpha \) leads to completely different solutions \( P(\nu_\alpha, t) \), even in vacuum\(^4\). However, people don’t pay much attention to the arguments \( \phi_\alpha \). The reason is that neutrinos are always produced in weak vertices. All experiments show that weak bosons (\( W^\pm, Z^0 \)) couple with neutrinos in eigenstates of flavor. Thus, all neutrinos start their journey from the initial condition:

\[
\begin{pmatrix}
P(\nu_e; 0) \\
P(\nu_\mu; 0)
\end{pmatrix} = \begin{pmatrix}
1 \\
0
\end{pmatrix} \text{ or } \begin{pmatrix}
0 \\
1
\end{pmatrix}
\]

equivalently:

\[
\begin{pmatrix}
|\nu_e| \\
|\nu_\mu|
\end{pmatrix} = \begin{pmatrix}
1 \\
0
\end{pmatrix} \text{ or } \begin{pmatrix}
0 \\
1
\end{pmatrix}
\]

Of course, initial condition (8) says nothing about the arguments \( \phi_\alpha \). They can take any value. But if they are arbitrary and also drastically affect \( P(\nu_\alpha; t) \), then experiments should have observed extremely messy oscillations, different for identical neutrinos with only different \( \phi_\alpha \). The reason that this is not what happens is that the impact of \( \phi_\alpha \) on \( P(\nu_\alpha; t) \) is canceled when eq. (8) holds, which means always. It can be proved analytically and also be tested numerically.

In brief, neutrinos are eigenstates of flavor, not of mass, and that is why they oscillate. That is also why \( \phi_\alpha \) have no observable effect.

### 5 The condition of non-oscillation in vacuum

There are at least two methods to find the condition of non-oscillation of neutrinos in vacuum. The brute one is to solve analytically eq. (1) with the most

\(^4\)As we will see soon, there is only one case where \( \phi_\alpha \) makes no difference, and this case is the one nature always prefers.
general expression of initial conditions, find the expression for \( P(\nu_\alpha; t) \) in terms of the initial conditions and then demand that \( \frac{\partial P(\nu_\alpha; t)}{\partial t} = 0 \) \( \forall t \). This demand imposes a condition to the initial conditions which, if satisfied, guarantees constancy\(^5\) of \( P(\nu_\alpha; t) \).

Constancy of \( P(\nu_\alpha; t) \) is also guaranteed if the neutrino initially is in an eigenstate of mass, because \( E = \sqrt{p^2 + m^2} \) so, assuming definite momentum \( p \), definite mass means definite energy \( E \), which means constant state. So, the second, equivalent but much more elegant method to find the condition of non-oscillation is to demand that initially the neutrino has definite mass.

For two generations, the mixing of flavor and mass eigenstates is given by the expression:

\[
\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_0 & \sin \theta_0 \\ -\sin \theta_0 & \cos \theta_0 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \Rightarrow \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos \theta_0 & -\sin \theta_0 \\ \sin \theta_0 & \cos \theta_0 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}
\]  

\(9\)

with

\[ |\nu_e|^2 + |\nu_\mu|^2 = 1 \]  

\(10\)

Demanding that initially \( \nu_1 = 0 \), \(10\) and \(9\) give the condition:

\[
\frac{\nu_e}{\nu_\mu} = \tan \theta_0 \text{ and } |\nu_e| = \frac{1}{\sqrt{1+\cot^2 \theta_0}}
\]  

\(11\)

Demanding that initially \( \nu_2 = 0 \), from \(10\) and \(9\) we find:

\[
\frac{\nu_e}{\nu_\mu} = -\cot \theta_0 \text{ and } |\nu_e| = \frac{1}{\sqrt{1+\tan^2 \theta_0}}
\]  

\(12\)

Conditions \(11\) and \(12\) do not only refer to the amplitudes of \( \nu_e,\mu \), but also to the relationship of their arguments \( \theta_0 \), so the complex nature of \( \nu_e \) and \( \nu_\mu \) can not be margated and consider only \( |\nu_\alpha| \). For \(11\) to be satisfied, \( \nu_e \) and \( \nu_\mu \) must be in the same direction on the complex plane (homoparallel), while \(12\) demands them to point in opposite directions (antiparallel). \( \theta_0 \) is the only parameter in both non-oscillation conditions. Fig. 2 shows the configurations of \( \nu_e \) and \( \nu_\mu \) on the complex plane, which satisfy the non-oscillation conditions.

Let’s examine how \( \nu_e \) and \( \nu_\mu \) behave as complex numbers, when in vacuum. Let me substitute \( \nu_e \) and \( \nu_\mu \) with the symbols \( x \) and \( y \) respectively. Analytical solving of eq. \(1\) for \( M = 0 \) gives \( x \) and \( y \) revolving on ellipses on the complex plane, like in Fig. 3. Those ellipses always have their centers at the point where axes intersect (point 0) and are inclined with respect to the axes. Their inclinations depend on both \( x_0 \) and \( y_0 \). The analytical expression of those ellipses has been calculated to be:

\[
\begin{align*}
Re [x(t)] &= \sqrt{\Pi_1^2 + \Pi_2^2} \cos \left( st - \arctan \left( \frac{\Pi_2}{\Pi_1} \right) \right) \\
Im [x(t)] &= \sqrt{\Pi_3^2 + \Pi_4^2} \cos \left( st + \arctan \left( \frac{\Pi_4}{\Pi_3} \right) \right)
\end{align*}
\]  

\(13\)

\(^5\)Constancy of \( P(\nu_\alpha; t) \) does not mean constancy of the solution \( \nu_\alpha(t) \), but only of \( |\nu_\alpha(t)| \). In the complex plane, \( \nu_e(t) \) and \( \nu_\mu(t) \) always rotate.
Figure 2: Complex pairs like \((x_0, y_0)\) and \((\bar{x}_0, \bar{y}_0)\), pointing in any direction, satisfy (11) and (12) respectively, assuming \(\theta_0 = 32^\circ\) which determines the radii of the circles. \(x_0\) and \(y_0\) stand for \(\nu_e\) and \(\nu_\mu\).

\[
Re \{y(t)\} = \sqrt{B_1^2 + B_2^2} \cos \left( st - \arctan \left( \frac{B_1}{B_2} \right) \right)
\]

\[
Im \{y(t)\} = \sqrt{B_3^2 + B_4^2} \cos \left( st + \arctan \left( \frac{B_3}{B_4} \right) \right)
\]

(14)

where

\[\Pi_1 = X_{11} \text{Re} \{C_1\} + X_{21} \text{Re} \{C_2\}, \quad B_1 = X_{12} \text{Re} \{C_1\} + X_{22} \text{Re} \{C_2\}\]
\[\Pi_2 = X_{21} \text{Im} \{C_2\} - X_{11} \text{Im} \{C_1\}, \quad B_2 = X_{22} \text{Im} \{C_2\} - X_{12} \text{Im} \{C_1\}\]
\[\Pi_3 = X_{11} \text{Im} \{C_1\} + X_{21} \text{Im} \{C_2\}, \quad B_3 = X_{12} \text{Im} \{C_1\} + X_{22} \text{Im} \{C_2\}\]
\[\Pi_4 = X_{11} \text{Re} \{C_1\} - X_{21} \text{Re} \{C_2\}, \quad B_4 = X_{12} \text{Re} \{C_1\} - X_{22} \text{Re} \{C_2\}\]

(15)

where

\[
\begin{align*}
C_1 &= \frac{y_0 - X_{22} x_0}{X_{12} - X_{21} x_0} \\
C_2 &= \frac{x_0 - X_{11} C_1}{X_{21}}
\end{align*}
\]

(16)

where

\[
\begin{pmatrix}
X_{11} \\
X_{12} \\
X_{21} \\
X_{22}
\end{pmatrix} = \begin{pmatrix}
1 + \left( \frac{A}{B} \right)^2 \end{pmatrix}^{-1/2} \begin{pmatrix}
\frac{1}{A} \\
\frac{1}{B}
\end{pmatrix}
\]

(17)

and

\[
s = \frac{\Delta m^2}{4E}
\]

(18)
From (13) and (14) it is obvious that \(x(t)\) and \(y(t)\) revolve with the same angular frequency \(s\), so, if they are not initially collinear in the complex plane, they will never be. On the other hand, if we initially select \(x_0\) and \(y_0\) to satisfy one of the non-oscillation conditions, then the ellipses take the shape of the black circles of Fig. 3 and \(x\) and \(y\) keep revolving coherently, being always collinear.

![Figure 3: Trajectories of \(x(t)\) and \(y(t)\) in the complex plane when in vacuum, with initial conditions \(x_0 = 0.5e^{i\pi/6}, y_0 = 0.866e^{i0}\) (see the dots on the ellipses) and \(\theta_0 = 32^\circ\). Red ellipsis is the trajectory of \(x(t)\) and blue is of \(y(t)\). Black circles are the same we had in Fig. 2.](image)

6 Matter effects in complex trajectories

Non-oscillation in vacuum would only be fiction, if matter effects were not capable of achieving the non-oscillation condition, because nature produces neutrinos obeying eq. 5, which contradicts (11) and (12).

Numerical solution of eq. (1) with variant \(N_e\) has revealed that the trajectories of \(x\) and \(y\) in the complex plane get extremely complicated (see Fig. 4) and can happen to satisfy one of the non-oscillation conditions, even instantaneously. In other words, a neutrino in matter may transiently get into a mass eigenstate.

From the left part of Fig. 4 we realize that, as density increases and then decreases, \(x(t)\) and \(y(t)\) drift away from the ellipsoidal trajectories they initially followed in vacuum. When density becomes zero again, they return to elliptical trajectories which are different from those they initially followed when in vacuum. The reason is that the neutrino returned to vacuum from different initial conditions, i.e. the \(x\) and \(y\) it had at the moment it exited the matter region.
The survival probability of a neutrino which initially was a $\nu_e$ with $x_0 = 1 \cdot e^{i \frac{\pi}{6}}$, $y_0 = 0$ and crosses a region of high matter density, such as a star. We have assumed a gaussian $N_e(t)$ of the form of eq. (19) with $\rho_0 = 10^{-17}$ GeV$^3$, $t_0 = 2 \cdot 10^{24}$ GeV$^{-1}$ and $\sigma = 10^{23}$ GeV$^{-1}$. Suppression is obvious around $t = t_0$, as $N_e$ highly exceeds $N_{MSW}$. The oscillation is drastically modified by the passing through the star. The complex trajectories of $x(t)$ (red) and $y(t)$ (blue). Before entering the star, $x(t)$ circulates along the red ellipsis (red and blue dots indicate $x_0$ and $y_0$ respectively) and $y(t)$ along the blue ellipsis which is degenerated into a line for those (naturally default) initial conditions.

The most critical interval is while the neutrino lies in the middle of the matter region, where its oscillation is highly suppressed. Then, $y(t)$ moves along the curly blue line, around $y(t) \simeq \text{something} + i \cdot 0.9$. At the same time, $x(t)$ rapidly spins along the red circles of radius $\sim 0.3$. In what trajectories they will end up depends on when the spinning of $x(t)$ and the curling of $y(t)$ will stop, i.e. when $N_e$ will decline.

If $N_e$ rises appropriately and diminishes at the right time, then it is possible to make $x(t)$ and $y(t)$ satisfy (or almost satisfy) one of the non-oscillation conditions. After methodically trying several initial conditions (always obeying eq. (14)) and several $N_e$ shapes, I found one example of such a ‘flavor lens’. The neutrino was assumed to initially have $x_0 = 1 \cdot e^{i \theta_0}$, $y_0 = 0$ and energy $E=3$ GeV. It has also been assumed that $\theta_0 = 32^\circ$ and $\Delta m^2 = 7.2 \times 10^{-5}$ eV$^2$.

The density profile found can be seen in Fig. 5, together with the oscillation of survival probability of a $\nu_e$ passing through it. $N_e(t)$ rises according to the

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6Actually, trial-and-error method would have very hardly lead to any findings. I had to implement a 'trick': At each time step of the numerical solution I would check if any non-oscillation condition were almost satisfied. If it were, then I would 'cut' the density distribution and set it equal to zero from there on. In this way, the neutrino would be dropped back to vacuum on the right time.
Figure 5: A density profile which acts as a flavor lens for $\nu_e$ of $E=3$ GeV, provided that $\theta_0 = 32^\circ$ and $\Delta m^2 = 7.2 \cdot 10^{-5} \text{ eV}^2$.

The function:

$$N_e(t) = \rho_0 \exp \left( -\frac{(t - t_0)^2}{2\sigma^2} \right)$$  \hspace{1cm} (19)

with

$$\begin{cases} 
  t_0 = 4 \cdot 10^{24} \text{ GeV}^{-1} \\
  \rho_0 = 10^{-16} \text{ GeV}^{-3} \\
  \sigma = 10^{24} \text{ GeV}^{-1} \\
  N_e(t) = 0 \text{ for } t > 3.03 \cdot 10^{24} \text{ GeV}^{-1}
\end{cases}$$

The fact that I found this specific density profile does not mean that it is the only ‘flavor lens’ for this kind on neutrinos. The same distribution, with its center $t_0$ translated by an integer number of vacuum wavelengths $L_0$, would have the same result. Whether a density profile is a flavor lens or not depends on its position with respect to neutrino’s production point, on neutrino’s energy, on its shape and on the initial conditions of the neutrino $(x_0, y_0)$.

It has been tested and confirmed that $\phi_\alpha$ do not affect the oscillation through the flavor lens, as long as (8) was initially true. In the same way that (8) ‘hides’ the impact of $\phi_\alpha$ in oscillations in vacuum, it also also prevents the role of $\phi_\alpha$ from being revealed when matter is present. So, the same oscillation would occur whether $(x_0 = 1 \cdot e^{i0}, y_0 = 0)$ or $(x_0 = 1 \cdot e^{i\pi/3}, y_0 = 0)$ etc. (see Fig. 8).

7 3 Generations

In 3 generations things do not differ much in principle. The equation of evolution is a system of 3 differential equations, where there is a matter term equivalent to $M$ we had in 2 generations [1].

Demanding that a neutrino is at an eigenstate of mass $\nu_i$, instead of an eigenstate of flavor $\nu_\alpha$, we derive the non-oscillation condition in 3 generations.
Figure 6: Survival probability for a $\nu_e$ of $E=3$ GeV, as it passes through the flavor lens of Fig. 5. When it has emerged from the matter region it is almost non-oscillating. The fact that it undergoes very slight oscillation is because the non-oscillation condition was not exactly satisfied when matter dropped to zero, but was almost satisfied.

Figure 7: The trajectories of $x(t)$ (red) and $y(t)$ (blue) as a 3 GeV neutrino with $x_0 = 1 \cdot e^{i\theta}, y_0 = 0$ passes through the flavor lens of fig. 5.
Figure 8: The same like in fig. 7, but with initial conditions $x_0 = 1 \cdot e^{i\pi/3}, y_0 = 0$. Both trajectories are rotated by $\pi/3$, but their behavior is the same as fig. 7, so the survival probability remains the one in fig. 6.

Starting with the notation:

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i} \nu_i \Rightarrow \nu_i = \sum_{\alpha=e,\mu,\tau} U_{\alpha i}^* \nu_\alpha$$

(20)

and respecting the completion condition:

$$|\nu_e|^2 + |\nu_\mu|^2 + |\nu_\tau|^2 = 1$$

(21)

we find that to have an eigenstate of mass, i.e. to have $\nu_i'' = 1, \nu_i' = 0, \nu_i = 0$, the condition is:

$$\begin{cases}
|\nu_\mu| = 1/\sqrt{1 + |A|^2 + |B|^2} \\
\nu_e = A \nu_\mu \\
\nu_\tau = B \nu_\mu
\end{cases}$$

(22)

where

$$A = \frac{U_{ei}^* U_{\mu i}^* - U_{\mu i}^* U_{ei}^*}{U_{ei}^* - U_{\mu i}^* U_{ei}^*}, \quad B = \left(- \frac{U_{ei}^* A - U_{\mu i}^*}{U_{ei}^*}, \frac{U_{\mu i}^* A - U_{ei}^*}{U_{ei}^*}\right)$$

(23)

In 3 generations, the non-oscillation condition is triple, one for each eigenstate of mass, and depends only on the tree mixing angles of the PMNS matrix $U$.

Non oscillation condition in 3 generations is much more demanding than in 2 generations. It demands three complex numbers to be collinear in the
complex plane, to point in the right directions and to be of the right sizes. However, achieving its satisfaction with an appropriate density profile must not be impossible, though it is not easy to find an example.

Conclusions

Matter effects in neutrinos oscillation have been discussed. The complex character of $\nu_\mu$ and $\nu_e$ has been emphasized, as density $N_e$ affects their trajectories in the complex plane, potentially leading neutrinos, transiently, to mass eigenstates.

It has been demonstrated how an appropriate $N_e(t)$ can transform a regular $\nu_e$ into one which does not oscillate in vacuum.

Finally, the non-oscillation condition in 3 generations has been presented.

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