Low energy Collective Modes of a Superfluid Trapped atomic Fermi Gas

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We consider the low energy collective mode spectrum of a superfluid Fermi gas in a spherical trap in the collisionless regime. Using a self-consistent random-phase approximation, the effects of superfluidity on modes of dipole and quadrupole symmetries are systematically examined. The spectrum is calculated for varying pairing strength and temperature and we identify several spectral features such as the emergence of Goldstone modes that can be used to detect the onset of superfluidity. Our analysis is relevant for present experiments aimed at observing a superfluid phase transition in trapped Fermi gases.

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Properties of trapped Fermi gases are attracting increasing attention in the field of ultracold atomic systems. Experimentally, the trapping and cooling of fermionic alkalis has been demonstrated reaching temperatures as low as $4\text{K}$ and $^6\text{Li}$ $^6\text{Li}$. Several proposals for the detection of this phase transition using the interaction with light $^6\text{Li}$, thermodynamic and various collective and quasi particle properties $^6\text{Li}$ have been presented.

The purpose of this paper is to examine the effects of superfluidity on the low energy collective mode spectrum. The modes can be measured with high precision as shape oscillations of the atomic cloud and such studies and quasi particle properties $^6\text{Li}$ have been presented. The modes can be measured with high precision as shape oscillations of the atomic cloud and such studies and quasi particle properties $^6\text{Li}$ have been presented.

We present an analysis of the low energy collective mode spectrum of a trapped superfluid Fermi gas for various temperatures and coupling strengths. The trapping potential has a qualitative effect on the low energy modes as opposed to the high energy regime, where one can use the well-known results for a homogeneous superconductor $^6\text{Li}$. We assume that the underlying normal gas is in the collisionless limit such that the lifetime $\tau$ of the quasi particles is much longer than the characteristic period of motion ($\omega T\tau \gg 1$) for atoms in a harmonic trap of frequency $\omega_T$). It is shown that superfluidity has qualitative effects on the modes of dipole and quadrupole symmetries and we thus propose several ways of detecting the superfluid transition. An essential difference between the density and spin-density fluctuation modes is demonstrated and we also analyze the effects of a non-zero temperature.

Consider a gas of fermionic atoms of mass $m$ in two hyperfine states $|\sigma = \uparrow, \downarrow\rangle$ trapped by a spherically symmetric harmonic potential $U_0(r) = \frac{1}{2} m \omega_r^2 r^2$. The numbers $N_\sigma$ of atoms trapped in each hyperfine state are assumed to be equal as this is the optimum situation for Cooper pairing. The quasi particle excitations $\eta$ of the system are calculated by the Hartree-Fock-Bogoliubov (HFB) method. Using a zero-range pseudo-potential method to describe the interaction between the atoms, the Bogoliubov-de Gennes (BdG) equations become $^6\text{Li}$:

$$E_\eta u_\eta(r) = [H_0 + W(r)]u_\eta(r) + \Delta(r)v_\eta(r)$$

$$E_\eta v_\eta(r) = [-H_0 + W(r)]v_\eta(r) + \Delta(r)u_\eta(r). \quad (1)$$

Here, $H_0 = -(\hbar^2/2m) \nabla^2 + U_0(r) - \mu_F$, $W(r) \equiv g(\psi_\sigma^\dagger(r)\psi_\sigma(r)) = g(\rho_\sigma(r))$ is the Hartree potential, and $\psi_\sigma(r)$ is the field operator that annihilates an atom in spin state $\sigma$ at position $r$. The coupling constant is $g = 4\pi a \hbar^2/m$ with $a$ being the s-wave scattering length between atoms in the two different hyperfine states ($a < 0$, i.e. attraction) and $\mu_F$ denotes the chemical potential. The pairing field is defined as $\Delta(R) = -g \lim_{r \to 0} \partial_r [\rho_\uparrow(R + r/2) \psi_\uparrow(R - r/2)]$ $^6\text{Li}$. The quasi particles with energies $E_\eta$ are described by the Bogoliubov wave functions $u_\eta(r)$ and $v_\eta(r)$.

We are in this paper interested in the response of the gas in the superfluid phase to various particle conserving time-dependent external perturbations of the kind $\tilde{F}(t) \propto \exp(i \omega t) \sum_\sigma \int d^3r F_\sigma(r) \hat{\rho}_\sigma(r)$. In the collisionless limit, the appropriate framework to calculate the collective mode spectrum is the self-consistent random phase approximation generalized to superfluid systems $^6\text{Li}$. The linear response of the superfluid is characterized by a matrix consisting of two-particle correlation functions:

$$\Pi(\omega) = \begin{pmatrix}
\langle \hat{\rho}_\uparrow \hat{\rho}_\uparrow \rangle & \langle \hat{\rho}_\uparrow \hat{\rho}_\downarrow \rangle & \langle \hat{\rho}_\uparrow \hat{\chi} \rangle & \langle \hat{\rho}_\uparrow \hat{\chi}^\dagger \rangle \\
\langle \hat{\rho}_\downarrow \hat{\rho}_\uparrow \rangle & \langle \hat{\rho}_\downarrow \hat{\rho}_\downarrow \rangle & \langle \hat{\rho}_\downarrow \hat{\chi} \rangle & \langle \hat{\rho}_\downarrow \hat{\chi}^\dagger \rangle \\
\langle \hat{\chi} \hat{\rho}_\uparrow \rangle & \langle \hat{\chi} \hat{\rho}_\downarrow \rangle & \langle \hat{\chi} \hat{\chi} \rangle & \langle \hat{\chi} \hat{\chi}^\dagger \rangle \\
\langle \hat{\chi}^\dagger \hat{\rho}_\uparrow \rangle & \langle \hat{\chi}^\dagger \hat{\rho}_\downarrow \rangle & \langle \hat{\chi}^\dagger \hat{\chi} \rangle & \langle \hat{\chi}^\dagger \hat{\chi}^\dagger \rangle
\end{pmatrix}. \quad (2)$$

Here, $\langle \hat{A}\hat{B} \rangle$ is the Fourier transform of the retarded function $-i\Theta(t-t')\langle [\hat{A}(r,t),\hat{B}(r',t')] \rangle$ and $\langle .. \rangle$ denotes the thermal average. The operator $\hat{\chi} = \psi_\uparrow(r)\psi_\downarrow(r)$ describes the local fluctuations in the pairing field. Note
that we have split the correlation functions into the different hyperfine components since we also wish to consider spin modes in which the two hyperfine species oscillate in anti-phase. For the normal phase, the density and spin-density fluctuation modes can be obtained by calculating the upper left quadrant of \( \Pi(\omega) \) in Eq. (2). For the superfluid phase however, fluctuations in the pairing field are coupled to density fluctuation modes through the non-particle-conserving correlation functions such as \( \langle \langle \rho \chi \rangle \rangle \) and we need to consider the full matrix in Eq. (3).

Within RPA, we have

\[
\Pi(\omega) = [1 - \Pi_0(\omega)G]^{-1}\Pi(\omega)
\]

with

\[
G = \frac{g}{\hbar} \delta(r - r') \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}
\]

(describing the interaction between quasi particles in opposite hyperfine states. Here the matrix products denote: \( AB \equiv \int d^3r' A(r, r') B(r'', r') \) and \( \Pi_0(\omega) \) is the matrix in Eq. (2) calculated for non-interacting excitations using the HFB approximation given by Eq. (4).

For a spherically symmetric trap, the correlation functions split into terms describing the various multipole modes [13]. For instance, we obtain for the correlation function \( \langle\langle \rho_{\sigma}(r) \rho_{\sigma}(r') \rangle\rangle \):

\[
\langle\langle \rho_{\sigma}(r) \rho_{\sigma}(r') \rangle\rangle = \sum_{LM} \Xi_{LM}(r, r', \omega) Y_{LM}(\theta, \phi) Y_{L'M}(\theta', \phi')
\]

with \( r \) denoting the distance from the center of the trap and the \( Y_{LM}(\theta, \phi) \) the usual spherical harmonics. The expressions for the other correlation functions appearing in Eq. (4) are completely equivalent. Using Wicks theorem, we express the independent particle correlation functions \( \langle\langle ... \rangle\rangle \) used to form \( \Pi_0(\omega) \) in terms the quasi particle energies/wave functions obtained from Eq. (1). For instance, for \( \langle\langle \rho_{\sigma}(r) \rho_{\sigma}(r') \rangle\rangle_0 \), we obtain

\[
\Xi_{0L}(r, r', \omega) = \sum_{nm} \frac{h(2l + 1)(2l' + 1)}{4\pi(2L + 1)r^2} \langle\langle \langle L00|L0 \rangle \rangle \rangle_0^2
\]

\[
\sum_{nm'} \left[ \frac{uuuv(f - f')}{E - E' - \hbar\omega - i\delta} + \frac{uuuv(f + f' - 1)}{E + E' - \hbar\omega - i\delta} - \frac{vvuv(1 - f - f')}{E + E' + \hbar\omega + i\delta} + \frac{vvuv(f' - f)}{-E - E' - \hbar\omega - i\delta} \right].
\]

Here, we write the Bogoliubov wave functions from Eq. (1) with angular momentum quantum numbers \( lm \) and energy \( E_{nl} \) in the form \( u_{qlm}(r) = r^{-1} u_{ql}(r) Y_{lm}(\theta, \phi) \) and \( v_{qlm}(r) = r^{-1} v_{ql}(r) Y_{lm}(\theta, \phi) \). We define \( uuuv \equiv uuuv \chi \equiv uuuv(r)\chi(r') \) etc., and \( f = [\exp(\beta E_{nl}) + 1]^{-1} \) is the Fermi function with \( \beta = 1/k_BT \). The Clebsch-Gordan coefficients \( \langle L00|L0 \rangle \) yield strong selection rules for \( l \) and \( l' \): for the monopole modes \( (L = 0) \) one has \( l = l' \), for the dipole mode \( (L = 1) l' = l \pm 1 \) etc.

There is one last technical issue to resolve: The correlation function \( \langle\langle \chi(r)\chi^*(r') \rangle\rangle \) is ultraviolet divergent when expressed as a sum over Bogoliubov wave functions as in Eq. (6) [14]. Using the pseudo-potential method in a way similar to that described in Ref. [11], this divergence is removed by the substitution

\[
\langle\langle \chi(r)\chi^*(r') \rangle\rangle \rightarrow \langle\langle \chi(r)\chi^*(r') \rangle\rangle + \frac{1}{2} \delta(r - r') G_{\mu\nu}^{\text{int}}(r).
\]

Here \( G_{\mu\nu}^{\text{int}}(r) \) is the part of the the single particle Greens function \( G_{\mu\nu}(r, x) \equiv \langle r + x/2 | H_0^{-1} | r + x/2 \rangle \) that diverges as \( 1/x \) for \( x \to 0 \) [11]. One can show that for the homogeneous case, this procedure is equivalent to subtracting the non-regularized gap equation from the calculation of \( \langle\langle \chi^2 \rangle\rangle \) and the diagonalization of \( \Pi(\omega) \) then yields the well-known Bogoliubov-Anderson mode [13].

The structure of the self-consistent RPA calculation is then the usual one: First we obtain a self-consistent solution to the BdG equations given by Eq. (1); then \( \Pi_0(\omega) \) is formed using the obtained Bogoliubov wave functions/energies; and finally the RPA response function is calculated from Eq. (3). The poles of \( \Pi(\omega) \) then give the collective modes of the gas.

The quantity of experimental interest is the strength function

\[
S(F, \omega) = \sum_{nm} \frac{e^{-\beta E_n} - e^{-\beta E_m}}{Z} \langle\langle \langle n|\hat{F}|m \rangle \rangle^2 \delta(\hbar\omega + E_n - E_m)
\]

which is directly related to the net transitions per unit time with energy \( \hbar\omega \) induced by the operator \( \hat{F} \). Here \( Z \) is the grand partition function and \( |n\rangle \) is an eigenstate of the Hamiltonian with energy \( E_n \). For operators of the form \( \hat{F}(t) \propto \exp(i\omega t) \sum_\sigma \int d^3r F_\sigma(r) \rho_\sigma(r) \), it is given by

\[
S(F, \omega) = -\frac{1}{\hbar \pi} \sum_{\sigma, \sigma'} \int d^3r d^3r' F_\sigma F_{\sigma'}^* I_m([\langle\langle \rho_\sigma \rho_{\sigma'}^* \rangle\rangle]),
\]

We shall calculate it for \( F_\sigma(r) = F_\sigma(r) Y_{LM}(\theta, \phi) \), for \( F_\sigma(r) = F_\sigma(r) Y_{LM}(\theta, \phi) \) the quadrupole \( (L = 2) \) density mode. For the dipole symmetry \( (L = 1) \), we take \( F_\sigma(r) = -F_\sigma(r) \) exciting the lowest spin-dipole mode. Taking \( F_\sigma(r) = F_\sigma(r) \) for \( L = 1 \) would simply excite the center-of-mass mode at \( \omega = \omega_T \). Taking \( L = 0 \) excites modes of monopole symmetry. The influence of superfluidity on the monopole modes is somewhat complicated and will be analyzed in a future publication.

The properties of the superfluid gas in general depends on quantities such as the size of the coherence length \( \xi \) of the Cooper pairs, the quasi particle spectrum obtained by solving Eq. (1), \( \hbar\omega_T \) and the mean field splitting of the trap levels. For a large number of particles trapped and a strong interaction one has \( \Delta(r) \gg \hbar\omega_T \) for small \( r \) and therefore \( \xi(r) \ll R \) with \( R \) denoting the spatial
extent of the cloud. As we shall demonstrate, the pairing has strong effects on the low energy spectrum in this regime and should thus be relatively easy to detect. We therefore believe this limit to be of most relevance for the present experimental effort to observe the superfluid transition. To illustrate some important physical concepts, we also present results for weaker coupling where the pairing mainly occurs in the levels of the oscillator shell that is closest to the chemical potential (valence shell). In this case, there are quasi particle excitations with \( E_n = 0 \ell < \hbar \omega_T \) since the pairing energy is smaller than \( \hbar \omega_T \). However, we do not consider the technologically interesting \( \Delta < \hbar \omega_T \) limit in detail with \( \Delta \) denoting the pairing energy. In this regime the interplay between the shell structure of the trap and the Cooper pairing leads to a more complex phase diagram.

We first present typical results for a system with relatively weak pairing. Figure 1 exhibits \( S(F, \omega) \) (in trap units) given by Eq. (1) for the spin-dipole, and quadrupole modes. There are \( \sim 1.4 \times 10^4 \) atoms trapped with a coupling strength such that the pairing field in the center of the trap is \( \Delta(r = 0) \approx 0.69 \hbar \omega_T \) for \( T = 0 \). For the specific parameters chosen, it is mainly the levels within the harmonic oscillator shell with principal quantum number \( n = 33 \) (with energy \( \xi_n = (n + 3/2) \hbar \omega_T \) in the non-interacting case) which Cooper pair. The Hartree field has lowered the energy of this shell so that it is situated around the chemical potential at \( \mu_F = 32 \hbar \omega_T \). We have added a small imaginary part \( \Gamma = 0.005 \hbar \omega_T \) to the frequency to model a smooth response typically observed experimentally. For comparison we also plot the response for \( k_B T = \hbar \omega_T \) where the gas is in the normal phase \( (k_B T_c \approx 0.9 \hbar \omega_T) \) and both the quasi particle and the collective (RPA) response is shown. As expected, there is a large response at \( \omega \approx \omega_T, 2 \omega_T \) for the spin-dipole and quadrupole response respectively for the gas in the normal phase. The shift of these peaks away from the ideal gas result is, of course, a consequence of the particle-particle interaction \( \{4\} \). We will not discuss these shifts further in this paper as we are concentrating on features specific to superfluidity.

From Fig. (1), we see that the presence of superfluidity introduces a qualitative new feature in the quadrupole response: a low frequency response for \( \omega \) of the order \( \sim 0.4 \omega_T \). The low energy response essentially comes from the breaking of Cooper pairs in the lowest quasi particle shell. As pointed out by Baranov \( \{17\} \), the low energy quasi particle excitations \( \{E_n = 0 \ell \ll \Delta(r = 0)\} \) are surface modes since the pair correlations imply that the interior of the paired system is an incompressible medium which expels the low energy quasi particles. Reflecting this effect, the response observed at \( \sim 0.4 \hbar \omega_T \) corresponds to a pair of quasi particles moving in the outer part of the cloud; the strength of this response in RPA is increased and the frequency decreased as a result of the (weak) coupling with the collective mode at \( \sim 2 \hbar \omega_T \). This is illustrated by the inset in Fig. (1) which compares in detail the low energy single particle response calculated from \( \Pi_0(\omega) \) with the collective response. The single particle response is generated by breaking of Cooper pairs due to excitations of the kind \( \Phi_{n'=0 \ell' \pm} \rangle \Phi_{n=0 \ell} \rangle \) with \( \ell' = \ell \pm 2 \) costing \( E_{n=0 \ell} + E_{n'=0 \ell'} \sim \Delta \). Thus, the low energy response consists of pair breaking excitations with \( \omega \sim 2 \Delta \) which is \( \sim 0.4 \hbar \omega_T \) for this set of parameters. As we shall see, this response is the precursor of a Goldstone mode present at stronger pairing. Furthermore, the peak at \( \sim 2 \omega_T \) is fragmented and partially damped due to the pairing. We will demonstrate below that this mode simply disappears with the emergence of the Goldstone modes for stronger pairing.

For the spin-dipole mode, we see that the main effect of superfluidity in the \( \Delta < \hbar \omega_T \) regime is that it fragments and partially damps the normal phase response at \( \sim \omega_T \). As for the quadrupole mode at \( \sim 2 \omega_T \), this effect is the precursor of the situation in the strong pairing regime, where we will show that there is no well-defined low frequency spin-dipole mode.

Detailed effects of superfluidity on the mode spectrum in the weak pairing regime, such as the way the normal phase quadrupole/spin-dipole modes at \( \sim 2 \omega_T / \omega_T \) are fragmented and damped with increasing pairing, depend sensitively on the specific parameters of the system such as the exact location of the chemical potential relative to the quasi particle bands. We will discuss these effects in detail in a future publication.

We now consider the stronger coupling limit when \( \xi < R \). Figure 2 exhibits \( S(F, \omega) \) for \( \sim 1.6 \times 10^4 \) atoms trapped with a coupling strength such that the pairing field in the center of the trap is \( \Delta(r = 0) = 6 \hbar \omega_T \) for \( T = 0 \). We have taken \( \Gamma = 0.005 \hbar \omega_T \) and the response is calculated for \( T = 0 \) and \( k_B T = 1.4 \hbar \omega_T \) where the gas is still superfluid and \( k_B T_c = 3.0 \hbar \omega_T \) (where the gas is in the normal phase \( (k_B T_c \approx 2.8 \hbar \omega_T) \)). Again, we plot both the quasi particle and the collective response. A semi classical analysis shows that the lowest energy quasi particles are localized near the minimum of the function \( \Delta(r)^2 + |U_0(r) + W(r) + \hbar^2(l + 1/2)^2 / (2mr^2) - \mu_F|^2 \{7\} \). With increasing pairing, the width of this minimum narrows and the lowest quasi particle energy can be shown to increase as \( \ln(\mu_F / \hbar \omega_T) \). For the specific parameters chosen above, the lowest quasi particle states have energies \( E_n = 0 \ell \gtrsim \hbar \omega_T \).

From Fig. (3), we see that for the quadrupole mode there is for \( T = 0 \) now only one low frequency mode situated at \( \omega \approx 1.4 \omega_T \). This lower mode comes from the pairing degree of freedom \( \{18\} \) of the lower right \( 2 \times 2 \) part of the matrix \( \Pi \) in Eq. (3); it is an example of the well-known Goldstone modes \( \{18\} \) with a finite frequency due to the trapping potential. The low energy quasi particle response observed for weaker pairing \( \{3\} \) has...
now become a collective (Goldstone) mode. For these modes, one can develop a hydrodynamic theory valid for $\xi \ll R$ [13] predicting a quadrupole Goldstone mode at $\omega = \sqrt{2}\omega_T$ [13] in agreement with our result. Of experimental relevance is the temperature dependence of the spectrum. From Fig. (2), we see that as $T$ is lowered through $T_c$ the mode at $\omega \simeq 2\omega_T$ is damped and a low frequency response emerges. At intermediate temperatures $0 < T < T_c$, the collective mode may suffer appreciably Landau damping [see the spectrum for $T = T_c/2$ in Fig. (3)]. The spectrum in Fig. (2) is consistent with an interpretation in which the collective mode emerges as an undamped excitation as soon as the temperature is low enough so that the lowest quasi particle excitation energy is $\geq \sqrt{2}\hbar \omega_T$. For the parameters in Fig. (2), this occurs for $T \sim T_c/4$.

We finally consider the low $\omega$ spin-dipole response. From Fig. (2), we see that the normal phase mode at $\sim \omega_T$ is totally damped in the superfluid phase. This is to be expected for $\Delta > \hbar \omega_T$: The only kind of motion possible for the superfluid is potential flow, i.e. density fluctuation modes [4]. All other modes such as spin-density modes requires the presence of a normal component which decreases with $T \to 0$. The superfluid cannot participate in a mode of relative motion of the two superfluid states; one needs to break pairs and the response lies above the pairing energy. The damping of the spin-dipole mode can also be understood by writing the spin-dipole operator in terms of the quasi particle operators: $[\hat{\rho}_R(r) - \rho_R(r)]|\Phi_0\rangle \propto \sum_{\eta \eta'} |u_\eta(r)v_{\eta'}(r) - v_\eta(r)u_{\eta'}(r)|\gamma_\eta \gamma_{\eta'}^{+} |\Phi_0\rangle$. Pairing tends to mix a particle state with its time reversed hole-state yielding $u_\eta(r) \sim v_\eta(r)$ for a low energy excitation and therefore $|u_\eta(r)v_{\eta'}(r) - v_\eta(r)u_{\eta'}(r)| \sim 0$. Thus, the low $\omega$ spin-dipole mode excited by an odd operator under time-reversal will gradually disappear for $T \to 0$ as can be seen from Fig. (2).

In conclusion, we have identified and discussed the physical origin of several clear-cut spectral features of superfluidity on the low frequency collective mode spectrum of dipole and quadrupole symmetry in the collisionless regime for weak and strong pairing. For a weak pairing $\Delta < \hbar \omega_T$, the main effect of superfluidity is the presence of a low $\omega$ response for the quadrupole symmetry. Also, the normal phase quadrupole and spin-dipole modes at $\sim 2\omega_T$ and $\sim \omega_T$ respectively are partially damped. For stronger pairing with $\xi \ll R$, the Goldstone modes dominate the low frequency spectrum and the quadrupole mode is shifted to $\sim \sqrt{2}\omega_T$ in agreement with hydrodynamic theory. In this regime there is no well-defined low energy spin-dipole mode. As illustrated above, the emergence of these effects should be straightforward to detect as one lowers $T$ through $T_c$ and we believe the results presented to be relevant for the present experimental effort related to observing the predicted BCS transition.

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FIG. 1. The collective (RPA) and quasi particle (Q.P.) response for the spin-dipole and quadrupole modes for $\Delta < \hbar \omega_T$.

FIG. 2. The collective (RPA) and quasi particle (Q.P.) response for the spin-dipole and quadrupole modes for $\Delta > \hbar \omega_T$. The insets show various frequency regions in detail.