Dark energy induced by neutrino mixing

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The energy content of the vacuum condensate induced by the neutrino mixing is interpreted as dynamically evolving dark energy.

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I. INTRODUCTION

The neutrino-antineutrino pair condensate in the vacuum can provide a contribution to the cosmological dark energy without invoking ad hoc scalar field or further exotic mechanisms [1]. This result is obtained by analyzing the neutrino mixing [2–4] in the context of Quantum Field Theory (QFT) [5–14]. In fact, the unitary inequivalence has been recognized between the flavored vacuum and the massive neutrino vacuum [5, 6]. The non–perturbative vacuum structure associated with the field mixing [5–14] leads to new oscillation formulas [6–10] (similar results hold in the boson mixing case [12, 16]) and to a contribution to the vacuum energy [11–17].

The vacuum condensate can give rise to a dynamical (unclustered) source of energy contributing to the today observed acceleration of Hubble flow. Indeed, it provides a contribution \( \rho_{\text{vac}}^{\text{mix}} + 3p_{\text{vac}}^{\text{mix}} \geq 0 \), and behaves approximatively as a cosmological constant at present epoch. Here \( \rho_{\text{vac}}^{\text{mix}} \) is the vacuum pressure induced by the neutrino mixing. A value of \( \rho_{\text{vac}}^{\text{mix}} \), which is in agreement with the estimated value of the dark energy, may be obtained by using a cut-off on the momenta phenomenologically relevant for the neutrino mixing phenomenon.

In Sect.II, we outline the neutrino mixing formalism in Quantum Field Theory. In Sect.III we compute the mixing contribution to the dark energy and the conclusions are drawn in Sect.IV.

II. NEUTRINO MIXING IN QUANTUM FIELD THEORY

The main features of the QFT formalism for the neutrino mixing are summarized as follows. For simplicity we restrict ourselves to the two flavor case [12]. Extension to three flavors can be found in ref. [10], and similar results hold in the boson mixing case [12, 16] (for a detailed review see [11]).

We consider two Dirac neutrino fields. The Pontecorvo mixing transformations are [2]

\[
\begin{align*}
\nu_e(x) &= \nu_1(x) \cos \theta + \nu_2(x) \sin \theta \\
\nu_\mu(x) &= -\nu_1(x) \sin \theta + \nu_2(x) \cos \theta,
\end{align*}
\]

where \( \nu_e(x) \) and \( \nu_\mu(x) \) are the fields with definite flavors, \( \theta \) is the mixing angle and \( \nu_1 \) and \( \nu_2 \) are the fields with definite masses \( m_1 \neq m_2 \). \( \nu_1(x) \) and \( \nu_2(x) \) are

\[
\nu_i(x) = \frac{1}{\sqrt{2}} \sum_{k,r} \left[ u_{k,i}^e \alpha^c_{k,i}(t) + v_{k,i}^c \beta^\dagger_{k,i}(t) \right] e^{ikx},
\]

with \( i = 1, 2 \), \( \alpha^c_{k,i}(t) = \alpha^c_{k,i} e^{-i\omega_{k,i}t} \), \( \beta^\dagger_{k,i}(t) = \beta^\dagger_{k,i} e^{i\omega_{k,i}t} \), and \( \omega_{k,i} = \sqrt{k^2 + m_i^2} \).

The operators \( \alpha^c_{k,i} \) and \( \beta^\dagger_{k,i} \), \( i = 1, 2 \), \( r = 1, 2 \) are the annihilators for the vacuum state \( |0\rangle_{1,2} \equiv |0\rangle_1 \otimes |0\rangle_2 \):

\[
\alpha^c_{k,i} |0\rangle_{12} = \beta^\dagger_{k,i} |0\rangle_{12} = 0.
\]

The mixing transformation Eqs.[11] can be written as [5]:

\[
\begin{align*}
\nu_e^c(x) &= G^{-1}_\theta(t) \nu_1^c(x) G_\theta(t) \\
\nu_\mu^c(x) &= G^{-1}_\theta(t) \nu_2^c(x) G_\theta(t)
\end{align*}
\]

where \( G_\theta(t) \) is the mixing generator.

At finite volume \( G_\theta(t) \) is an unitary operator: \( G_\theta^{-1}(t) = G_{-\theta}(t) = G_\theta^0(t) \), preserving the canonical anticommutation relations, moreover \( G_\theta^{-1}(t) \) maps the Hilbert spaces for free fields \( \mathcal{H}_{1,2} \) to the Hilbert spaces for interacting fields \( \mathcal{H}_{e,\mu} \):
The today observed acceleration of the universe.

The flavor annihilators, relative to the fields $\nu_e(x)$ and $\nu_\mu(x)$ are defined as (we use $(\sigma, i) = (e, 1), (\mu, 2)$):

$$
\begin{align*}
\alpha_{k,\sigma}^r(t) &\equiv G_{\theta}^{-1}(t) \alpha_{k,\sigma}^l(t) G_{\theta}(t), \\
\beta_{k,\sigma}^r(t) &\equiv G_{\theta}^{-1}(t) \beta_{k,\sigma}^l(t) G_{\theta}(t).
\end{align*}
$$

The flavor fields can be then expanded in the same bases as $\nu_i$:

$$
\nu_\sigma(x, t) = \frac{1}{\sqrt{V}} \sum_{k,r} e^{i k \cdot x} \left[ u_{k,i}^r \alpha_{k,\sigma}^r(t) + v_{k,i}^r \beta_{k,\sigma}^r(t) \right].
$$

The flavor annihilation operators in the reference frame such that $k = (0, 0, |k|)$ are explicitly

$$
\begin{align*}
\alpha_{k,e}^r(t) = \cos \theta \alpha_{k,1}^r(t) + \sin \theta \left( |U_k| \alpha_{k,2}^r(t) + e^r |V_k| \beta_{k,2}^r(t) \right)
\end{align*}
$$

(6)

(7)

with $i, j = 1, 2$ and $i \neq j$. We have:

$$
|U_k| = \left( \frac{\omega_{k,1} + m_1}{2\omega_{k,1}} \right)^{1/2} \left( \frac{\omega_{k,2} + m_2}{2\omega_{k,2}} \right)^{1/2} \left[ 1 + \frac{k^2}{(\omega_{k,1} + m_1)(\omega_{k,2} + m_2)} \right]
$$

and

$$
|V_k| = \left( \frac{\omega_{k,1} + m_1}{2\omega_{k,1}} \right)^{1/2} \left( \frac{\omega_{k,2} + m_2}{2\omega_{k,2}} \right)^{1/2} \frac{k}{(\omega_{k,2} + m_2)} - \frac{k}{(\omega_{k,1} + m_1)}
$$

The condensation density is given by

$$
e,\mu \langle 0 | \alpha_{k,i}^r \alpha_{k,i}^r | 0 \rangle_{e,\mu} = e,\mu \langle 0 | \beta_{k,i}^r \beta_{k,i}^r | 0 \rangle_{e,\mu} = \sin^2 \theta |V_k|^2, \quad i = 1, 2.
$$

III. NEUTRINO MIXING AND DARK ENERGY

Experimental data \cite{18,20} support the picture that some form of dark energy, evolving from early epochs, induces the today observed acceleration of the universe.

There are many proposals to achieve cosmological models justifying such a dark component \cite{21-23}. In this Section, we present the contribution $\rho^{m\text{ix}}_{\nu\nu\text{c}}$ of the neutrino mixing to the vacuum energy density. We consider the Minkowski metric.

The Lorentz invariance of the vacuum implies that the vacuum state is the zero eigenvalue eigenstate of the normal ordered energy, momentum and angular momentum operators \cite{24}. Therefore $\mathcal{T}^{\nu\nu\text{c}}_{\mu\nu} = \langle 0 | \mathcal{T}_{\mu\nu} | 0 \rangle = 0$. The $(0,0)$ component of the energy-momentum tensor density $\mathcal{T}_{00}(x)$ for the fields $\nu_1$ and $\nu_2$ is then

$$
\mathcal{T}_{00}(x) := \frac{i}{2} : \left( \bar{\Psi}_m(x) \gamma_0 \frac{\partial}{\partial x_0} \Psi_m(x) \right) :
$$

where $\gamma_0$ denotes the customary normal ordering with respect to the mass vacuum in the flat space-time and $\Psi_m = (\nu_1, \nu_2)$.

In terms of the annihilation and creation operators of fields $\nu_1$ and $\nu_2$, the $(0,0)$ component of the energy-momentum tensor $\mathcal{T}_{00} = \int d^3 x \mathcal{T}_{00}(x)$ is given by

$$
\mathcal{T}_{(i)}^{00} := \sum_r \int d^3 k \omega_{k,1} \left( \alpha_{k,i}^r \alpha_{k,i}^r + \beta_{k,i}^r \beta_{k,i}^r \right),
$$

with $i = 1, 2$. Note that $\mathcal{T}_{(i)}^{00}$ is time independent.
The contribution $\rho_{\text{vac}}^{\text{mix}}$ of the neutrino mixing to the vacuum energy density is given by computing the expectation value of $T_{\nu}^{00}_{(i)}$ in the flavor vacuum $|0\rangle_{\nu,\mu}$:

$$\rho_{\text{vac}} = \frac{1}{V} \eta_{00} e,\mu (0(t)) \sum_{i} \langle T_{\nu}^{00}_{(i)} (0) : |0(t)\rangle_{\nu,\mu}.$$ \hspace{1cm} (13)

Within the QFT formalism for neutrino mixing, we have

$$e,\mu (0) : T_{\nu}^{00}_{(i)} : |0\rangle_{\nu,\mu} = e,\mu (0(t)) : T_{\nu}^{00}_{(i)} : |0\rangle_{\nu,\mu}$$ \hspace{1cm} (14)

for any $t$. We then obtain

$$\rho_{\text{vac}}^{\text{mix}} = \sum_{i,j} \int \frac{d^3k}{(2\pi)^3} \omega_{k,i} \left( e,\mu \langle 0 | \alpha_{k,i}^\dagger \alpha_{k,i} | 0 \rangle e,\mu + e,\mu \langle 0 | \beta_{k,i}^\dagger \beta_{k,i} | 0 \rangle e,\mu \right).$$

By using Eq. (13), we get

$$\rho_{\text{vac}}^{\text{mix}} = \frac{2}{\pi} \sin^2 \theta \int_0^K dk k^2 (\omega_{k,1} + \omega_{k,2}) |V_k|^2,$$ \hspace{1cm} (15)

where the cut-off $K$ has been introduced.

In a similar way, the contribution $\rho_{\text{vac}}^{\text{mix}}$ of the neutrino mixing to the vacuum pressure is given by the expectation value of $T_{\nu}^{jj}_{(i)}$ (where no summation on the index $j$ is intended) on the flavor vacuum $|0\rangle_{\nu,\mu}$:

$$\rho_{\text{vac}}^{\text{mix}} = \frac{1}{V} \eta_{jj} e,\mu (0) \sum_{i} \langle T_{\nu}^{jj}_{(i)} : |0\rangle_{\nu,\mu}.$$ \hspace{1cm} (16)

where no summation on the index $j$ is intended. Being

$$\langle T_{\nu}^{jj}_{(i)} \rangle := \sum_{r} \int d^3k k^j k^j \omega_{k,i} \left( \alpha_{k,i}^\dagger \alpha_{k,i}^\dagger \beta_{k,i} \beta_{k,i}^\dagger \right),$$ \hspace{1cm} (17)

in the case of the isotropy of the momenta we have $T^{11} = T^{22} = T^{33}$, then

$$\rho_{\text{vac}}^{\text{mix}} = \frac{2}{3} \frac{\sin^2 \theta}{\pi} \int_0^K dk k^4 \left( \frac{1}{\omega_{k,1}} + \frac{1}{\omega_{k,2}} \right) |V_k|^2.$$ \hspace{1cm} (18)

By Eqs. (15) and (18) we have that Lorentz invariance of vacuum neutrino-antineutrino condensate is broken, indeed $\rho_{\text{vac}}^{\text{mix}} \neq -\rho_{\text{vac}}^{\text{mix}}$ for any value of $m_1$ and $m_2$ and independently of the choice of the cut-off. We note that the adiabatic index $w = \rho_{\text{vac}}^{\text{mix}} / \rho_{\text{vac}}^{\text{mix}} \approx 1/3$ when the cut-off is chosen to be $K \gg m_1, m_2$.

The values of $\rho_{\text{vac}}^{\text{mix}}$ and $\rho_{\text{vac}}^{\text{mix}}$ we obtain are time-independent since, for the sake of simplicity, we are taking into account the Minkowski metric. When the curved background metric is considered, $|V_k|^2$ is time-dependent. Such a result holds in the early universe epochs, when the curvature radius is comparable with the oscillation length.

At the present epoch, the breaking of the Lorentz invariance is negligible and then $\rho_{\text{vac}}^{\text{mix}}$ comes from space-time independent condensate contributions (i.e. the contributions carrying a non-vanishing $\partial_\mu \sim k_\mu = (\omega_k, k_j)$ are missing). This means that the stress energy tensor of the vacuum condensate is approximatively equal to

$$e,\mu (0) : T_{\mu\nu} : |0\rangle_{\nu,\mu} = \eta_{\mu\nu} \sum_{i} m_i \int \frac{d^4x}{(2\pi)^4} e,\mu \langle x | \tilde{v}_i (x) \nu_i (x) : |0\rangle_{\nu,\mu} = \eta_{\mu\nu} \rho_{\text{mix}}^{\text{mix}}.$$ \hspace{1cm} (19)

Since $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and, in a homogeneous and isotropic universe, the energy momentum tensor is $T_{\mu\nu} = \text{diag}(\rho, p, p, p)$, then, consistently with Lorentz invariance, the state equation is $\rho_{\text{mix}}^{\text{mix}} = -p_{\text{mix}}$. That is, today the vacuum condensate coming from the neutrino mixing, contributes to the dynamics of the universe, with a behavior similar to that of the cosmological constant $\Lambda$. Explicitly, we have

$$\rho_{\text{mix}}^{\text{mix}} = \frac{2}{\pi} \sin^2 \theta \int_0^K dk k^2 \left( \frac{m_1^2}{\omega_{k,1}} + \frac{m_2^2}{\omega_{k,2}} \right) |V_k|^2.$$ \hspace{1cm} (20)
It is shown that during the Big Bang Nucleosynthesis, flavor particle-antiparticle pairs are produced by mixing and oscillations with typical momentum $k \sim \frac{m_1+m_2}{2}$. This introduces us to comment on the problem of the choice of the cut-off $K$ in the integrations in the equations above.

We do not have the solution for such a problem. However, in our approach there is an indication of possible choices suggested by the natural energy scale of the neutrino mixing in the QFT formalism. The ultraviolet cut-off at Planck scale, as well as the QCD one, give huge unacceptable values for the today observed vacuum energy density. It is therefore imperative to explore alternative routes. One of the merits of the present approach is indeed to point out that, although the arbitrariness problem is not solved, other possible choices exist which not only are consistent with the intrinsic energy scale of the mixing phenomenon, but also lead to quite acceptable values for the vacuum energy density. We thus arrive at the cut-off choice which is suggested by the natural scale appearing in the QFT formalism of the mixing phenomenon, i.e. we may set $K \simeq \sqrt{m_1 m_2}$. Another possibility, as suggested in Ref. [20] on similar grounds, is the sum of the two neutrino masses, $K = m_1 + m_2$. Both choices lead to values of $\rho^\text{mix}_\Lambda$ compatible with the observed value of $\rho_\Lambda$. The latter choice is also quite near to another possibility, $K \sim \frac{m_1+m_2}{2}$, which could be related to the discussion of Ref. [23], although this is referred to background neutrinos. It is indeed an interesting open question the relation between the (hot) dark matter and the dark energy, namely, from the perspective of the present paper, of the relation between dark matter and the vacuum structure. Such a question will be object of our future study.

By using $\sin^2 \theta \simeq 0.3$, $m_i$ of the order of $10^{-3} eV$, so that $\delta m^2 = m_2^2 - m_1^2 \simeq 8 \times 10^{-5} eV^2$, and one of the above choices for $K$, for example $K \sim \frac{m_1+m_2}{2}$, we obtain $\rho^\text{mix}_\Lambda \sim 1.1 \times 10^{-47}GeV^4$, which is in agreement with the estimated value of the dark energy. The other two choices lead to values of $\rho^\text{mix}_\Lambda$ also compatible with the estimated value of $\rho_\Lambda$, i.e. $\rho^\text{mix}_\Lambda \sim 0.7 \times 10^{-47}GeV^4$ and $\rho^\text{mix}_\Lambda \sim 5.5 \times 10^{-47}GeV^4$, respectively.

We remark that, unless one works in the present approach, such incredibly small values for the cut-off would be ruled out, since one would think that regularization of quantum effects from the physics beyond the standard model should come at a very high scale, e.g. the Planck scale. However, in the present case such a belief is actually unfounded: it is indeed in conflict with simple facts such as the disagreement of 123 orders of magnitude with the observed dark energy value. On the contrary, being bounded to a flat computational basis, as observed in Refs. [17, 20], the presence of $|V_k|^2$ (with its behavior as a function of the momentum) in the integrations naturally leads to one of the above small cut-off choices. The non-perturbative physics of the neutrino mixing thus points to the relevance of soft momentum (long-wave-length) modes.

In this connection, we also remark that Eqs. (15) and (20) show that the contribution to the dark energy induced from the neutrino mixing depends on the specific QFT nature of the mixing: indeed, it is absent in the quantum mechanical treatment of the mixing.

IV. CONCLUSIONS

We have reported on recent results showing that the vacuum condensate due to neutrino mixing contributes to the dark energy of the universe. The expectation value of the energy-momentum tensor has been computed in the vacuum state where neutrino oscillations are observed and the energy content of the vacuum condensate induced by the neutrino mixing is interpreted as dynamically evolving dark energy. By careful choice of the momentum cut-off we have obtained acceptable values for vacuum energy density.

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