Abstract

Axioms can be used to model derived predicates in domain-independent planning models. Formulating models which use axioms can sometimes result in problems with much smaller search spaces and shorter plans than the original model. Previous work on axiom-aware planners focused solely on state-space search planners. We propose axiom-aware planners based on answer set programming and integer programming. We evaluate them on PDDL domains with axioms and show that they can exploit additional expressivity of axioms.

1 Introduction

Currently, in the most commonly studied classical planning models, all changes to the world are the direct effects of some operator. However, it is possible to model some effects as indirect effects which can be inferred from a set of basic state variables. Such derived predicates can be expressed in modeling languages such as PDDL and formalisms such as SAS+ as axioms, which encode logical rules defining how the derived predicates follow from basic variables. Planners have supported various forms of derived predicates since relatively early systems (Manna and Waldinger 1987; Barrett et al. 1995), and PDDL has supported axioms which specify derived predicates as a logic program with negation-as-failure semantics since version 2.2 (Edelkamp and Hoffmann 2004).

Consider, for example, the well-known single-agent puzzle game Sokoban, in which the player pushes stones around in a maze. The goal is to push all the stones to their destinations. The standard PDDL formulation of Sokoban used in the International Planning Competition (IPC) consists of two kinds of operators, push and move. push lets the player push a box in one direction, while move moves the player into an unoccupied location.

Ivankovic and Haslum (2015a) proposed a new formulation of Sokoban with axioms and showed that this leads to a problem with a smaller search space and shorter plan (Ivankovic and Haslum 2015b). They remove the move operators entirely, and introduce axioms to check whether the player can reach a box to push it. The reformulated operators now have a derived predicate reachable(loc) instead of at-player=loc as their precondition. The values of the derived predicates are determined by the following axioms:

1. reachable(loc) ← at-player=loc
2. reachable(loc) ← reachable(from), clear(loc), connected(from,loc)

Intuitively, the first axiom means that the current location of the player is reachable. The second axiom means a location next to a reachable location is also reachable. With axioms, the search space only has the transitions caused by push operators, resulting in smaller search space and shorter plan.

Previous work on derived predicates and axioms for planning has focused on the advantages of expressivity (compactness) of domain modeling using axioms, (Thiébaux, Hoffmann, and Nebel 2005), as well as forward state-space search algorithms which are aware of axioms (Coles and Smith 2007; Gerevini, Saetti, and Serina 2011; Ivankovic and Haslum 2015b).

While previous work focused solely on axiom-aware state search planners, to our knowledge, little to no work has been done to evaluate model-based planners on PDDL domains with axioms. A standard approach to model-based planning is to translate a planning problem instance into a k-step model of other formalisms such as SAT and Integer Programming (IP), where a feasible solution to the k-step model corresponds to a solution to the original planning problem with n < k “steps”.

We propose two axiom-aware model-based planners called ASPlan and IPlan, which are based on answer set programming (ASP) and integer programming (IP) respectively, and show that additional expressivity of axioms help model-based planners as well.

Answer set programming (ASP) is a form of declarative programming based on answer set semantics of logic programming, and thus is a natural candidate for integrating axioms. Early attempts to apply ASP to Planning (Subrahmanian and Zaniolo 1995) and (Dimopoulos, Nebel, and Koehler 1997a) predated PDDL, and were not evaluated on large sets of benchmarks. To our knowledge, the first ASP-based planner compatible with PDDL is plasp (Gebser, Kaufmann, and Schaub 2012). However, plasp does not handle axioms. We developed ASPlan based on plasp, but used PDDL to SAS+ translator of FastDownward (Helmert 2006) to make it compatible with larger sets of IPC domains including domains with axioms.

IPlan is an IP-based planner based on Optiplan (van den
Briel and Kambhampati (2005). We integrated axioms into IPlan by using the ASP to IP translation method by (Liu, Janhunen, and Niemelä (2012). To our knowledge, ASPlan and IPlan are the first model-based planners which handle axioms.

2 Preliminaries

2.1 Normal Logic Problem

We introduce a normal logic problem, adopting the notations used in (Liu, Janhunen, and Niemelä (2012).

Definition 1. A normal logic problem (NLP) \( P \) consists of rules of the form

\[ a \leftarrow b_1, ..., b_m, \neg c_1, ..., \neg c_n. \]

where each \( a, b_i, c_j \) is a ground atom.

Given a rule \( r \in P \), we denote the head of the rule \( a \) by \( H(r) \), the body \( \{ b_1, ..., b_m, \neg c_1, ..., \neg c_n \} \) as \( B(r) \), the positive body literals \( \{ b_1, ..., b_m \} \) by \( B^+(r) \) and the negative body literals \( \{ \neg c_1, ..., \neg c_n \} \) by \( B^-(r) \). We use \( At(P) \) for the set of atoms which appear in \( P \).

A set of atoms \( M \) satisfies an atom \( a \) if \( a \in M \) and a negative literal not \( a \) if \( a \notin M \), denoted \( M \models a \) and \( M \models \neg a \), respectively; \( M \) satisfies a set of literals \( L \), denoted \( M \models L \), if it satisfies each literal in \( L \); \( M \) satisfies a rule \( r \), denoted \( M \models r \), if \( M \models H(r) \) whenever \( M \models B^+(r) \). A set of atoms \( M \) is a model of \( P \), denoted \( M \models P \), if \( M \) satisfies each rule of \( P \).

An answer set of a program is defined through the concept of reduct.

Definition 2. For a normal logic program \( P \) and a set of atoms \( M \), the reduct \( P^M \) is defined by

\[ P^M = \{ H(r) \leftarrow B^+(r) \mid r \in P, B^-(r) \cap M = \phi \}. \]

Definition 3 (Gelfond and Lifschitz (1988). A model \( M \) of a normal logic program \( P \) is an answer set iff \( M \) is the minimal model of \( P^M \).

We are particularly interested in a class of logic programs called locally stratified programs, which disallow negation through recursions. Locally stratified programs were originally introduced in Przymusinski (1988).

Definition 4. A normal logic program \( P \) is locally stratified if and only if there is a mapping \( l \) from \( At(P) \) to \( \{ 1, ..., \mid At(p) \mid \} \) such that:

- for every rule \( r \) with \( H(r) = a \) and every \( b \in B^+(r) \), \( l(b) \leq l(a) \)
- for every rule \( r \) with \( H(r) = a \) and every \( c \in C^-(r) \), \( l(c) < l(a) \)

With stratifications the unique model, called a perfect model can be computed by a stratified fixpoint procedure (Apt, Blair, and Walker (1988). It is known that the perfect model for a locally stratified program coincides with the unique answer set of the program (Eiter, Ianni, and Krennwallner (2009).

2.2 SAS+ and Axioms

We adopt the definition of axiom-enhanced SAS+ used in Helmer (2009) and Ivanovic and Haslum (2015b).

Definition 5. An SAS+ problem with axioms \( \Pi \) is a tuple \( (V, O, I, G, U, A) \) where

- \( V \) is a set of primary variables. Each variable \( v_i \) has a finite domain of values \( D(v_i) \).
- \( O \) is a set of operators. Each operator \( o \) has a precondition \( (pre(o)) \), which consists of variable assignments of the form \( v_i = x \) where \( x \in D(v_i) \). We abbreviate \( v_i = 1 \) as \( v_i \) when we know the variable is binary. An operator \( o \) is applicable in a state \( s \) iff \( s \) satisfies \( pre(o) \).

Each operator \( o \) also has a set of effects \( (eff(o)) \). Each effect \( e \in eff(o) \) consists of a tuple \( (\text{cond}(e), \text{affected}(e)) \) where \( \text{cond}(e) \) is possibly empty variable assignments and \( \text{affected}(e) \) is a single variable and value pair. Applying an operator \( o \) with an effect \( e \) to a state \( s \) where \( \text{cond}(e) \) is satisfied results in a state \( (o(s)) \) where \( \text{affected}(e) \) is true.

\( \text{cost}(o) \) is the associated cost of the operator \( o \).

- \( I \) is an initial assignment over primary variables.
- \( G \) is a partial assignment over variables that specifies the goal conditions.
- \( U \) is a set of secondary variables. Secondary variables are binary and do not appear in operator effects. Their values are determined by axioms after each operator execution.

- \( A \) is a set of rules of the form (1). Axioms and primary variables form a locally stratified logic program. At each state, axioms are evaluated to derive the values of secondary variables. We denote the result of evaluating a set of axioms \( A \) in a state \( s \) as \( A(s) \). Note that \( A(s) \) is guaranteed to be unique due to the uniqueness of the model for locally stratified logic programs.

A solution (plan) to \( \Pi \) is an applicable sequence of operators \( o_0, ..., o_n \) that maps \( I \) into a state where \( G \) holds.

2.3 \( \forall \) vs. sequential (seq-) semantics

A standard, sequential search strategy for a mode-based planner first generates a 1-step model (e.g., SAT/IP model), and attempts to solve it. If it has a solution, then the system terminates. Otherwise, a 2-step model is attempted, and so on (Kautz and Selman (1992). Adding axioms changes the semantics of a “step” in the k-step model. As noted by Dimopoulos et al (1997b) and Rintanen et al (2006), most model-based planners, including the base IPlan/ASPlan planners we evaluate below, use \( \forall \) semantics, where each “step” in a k-step model consists of a set of actions which are independent of each other and can therefore be executed in parallel. In contrast, sequential semantics (seq-semantics) adds exactly 1 action at each step in the iterative, sequential search strategy. In general, \( \forall \) semantics is faster than seq-semantics, since \( \forall \) semantics significantly decreases the \# of iterations of the sequential search strategy. For the domains with axioms, however, we add a constraint which restricts the number of actions executed at each step to 1, imposing seq-semantics. This is because a single operator can have far-reaching effects on derived variables, and establishing independence with respect to all derived variables affected by multiple operators is nontrivial (future work).
3 ASPlan

We describe our answer set programming based planner ASPlan. ASPlan adapts the encoding of plasp (Gebser, Kaufmann, and Schaub 2012) to the multi-valued semantics of SAS+. While plasp directly encodes PDDL to ASP, ASPlan first obtains the grounded SAS+ model from PDDL using the FastDownward translator, and then encodes the SAS+ to ASP. Using the FastDownward translator makes it easier to handle more advanced features like axioms and conditional effects.

3.1 Baseline Implementation

We translate a planning instance to a normal logic program with $k$ steps. Having $k$-steps means that we have $k + 1$ states or "layers" to consider.

\[
\text{const } n = k. \text{ step } 1..n. \text{ layer } 0..n. \tag{3}
\]

For each $v_i \in I$, we introduce the following rule which specifies the initial state.

\[
\text{holds}(f(v_i,x),0) \tag{4}
\]

Likewise, for each $v_j \in G$, we have the following rule.

\[
\text{goal}(f(v_i,x)) \tag{5}
\]

The next rule (6) makes sure that all of the goals are satisfied at the last step.

\[
\leftarrow \text{goal}(F), \neg \text{holds}(F,n) \tag{6}
\]

For each operator $o \in O$ and each $v_i = x$ and $v_j = y$ in $o$’s preconditions and effects respectively, we introduce the following rules.

\[
\text{demands}(o,f(v_i,x)). \text{ add}(o,f(v_j,y)). \tag{7}
\]

Rules (8) and (9) require that applied operators’ preconditions and effects must be realized.

\[
\leftarrow \text{apply}(O,I), \text{demands}(O,F), \neg \text{holds}(F,I-1), \text{step}(I). \tag{8}
\]

\[
\text{hold}(O,I) \leftarrow \text{apply}(O,I), \text{add}(O,F), \text{step}(I) \tag{9}
\]

Inertial axioms (unchanged variables retain their values) are represented by rules (10)-(11). An assignment $v = x$ to a SAS+ variable is mapped to an atom $f(v,x)$.

\[
\text{changed}(X,I) \leftarrow \text{apply}(O,I), \text{add}(O,f(X,Y)), \text{step}(I) \tag{10}
\]

\[
\text{hold}(f(X,Y),I) \leftarrow \text{hold}(f(X,Y),I-1), \text{step}(I), \quad \text{inertial}(f(X,Y)), \neg \text{changed}(X,I) \tag{11}
\]

Note that every primary variable $v$ is marked as inertial with the following rule.

\[
\text{inertial}(f(v,\text{val})) \tag{12}
\]

With the sequential semantics, the rule (13) ensures that only one operator is applicable in each step.

\[
\text{apply}(A,I) : \text{actions}(A), \text{step}(I) \tag{13}
\]

On the other hand, with $\forall$-semantics, (14)-(16) requires that no conflicting operators are applicable at the same step.

\[
\leftarrow \text{apply}(A,I), \text{actions}(A), \text{step}(I) \tag{14}
\]

\[
\leftarrow \text{apply}(A,I), \text{apply}(B,I), \text{add}(A,f(X,Y)), \quad \text{demands}(B,f(X,Z)), \text{step}(I), A \neq B, Y \neq Z \tag{15}
\]

\[
\leftarrow \text{apply}(A,I), \text{apply}(B,I), \text{add}(A,f(X,Y)), \quad \text{add}(B,f(X,Z)), \text{step}(I), A \neq B, Y \neq Z \tag{16}
\]

We also need the following constraints to realize negation-as-failure semantics while being compatible with the formulations above.

\[
\text{hold}(f(u,0),I) \leftarrow \text{hold}(f(u,1),I), \text{layer}(I). \tag{19}
\]

\[
\neg \text{hold}(f(u,0),I), \neg \text{hold}(f(u,1),I), \text{layer}(I). \tag{20}
\]

3.2 Integrating Axioms

Integrating axioms to ASPlan is fairly straightforward. For an axiom $a \leftarrow b_1, \ldots, b_n, notc_1, \ldots, cm$, we introduce the following rule.

\[
\text{hold}(f(a,1),I) \leftarrow \text{hold}(f(b_1,1),I), \ldots, \text{hold}(f(b_n,1),I), \quad \text{not hold}(f(c_1,0),I), \ldots, \text{not hold}(f(c_m,0),I), \text{step}(I). \tag{18}
\]

We also need the following constraints to realize negation-as-failure semantics while being compatible with the formulations above.

\[
\text{hold}(f(u,0),I) \leftarrow \text{hold}(f(u,1),I), \text{layer}(I). \tag{19}
\]

\[
\neg \text{hold}(f(u,0),I), \neg \text{hold}(f(u,1),I), \text{layer}(I). \tag{20}
\]

3.3 Integrating Conditional Effects

We describe how to integrate conditional effects into ASPlan. For each operator $o$ and its effect $e \in \text{eff}(o)$, we introduce the following rule

\[
\text{fired}(E,I) \leftarrow \text{apply}(A,I), \quad \text{hold}(f(v_1,z),I), \ldots, \text{hold}(f(v_n,z),I), \text{step}(I) \tag{21}
\]

where $v_1 = x, \ldots, v_n = z$ are in cond$(e)$.

With conditional effects, applying operators does not necessarily mean their effects get triggered. We replace the rules (10) and (11) with the following rules.

\[
\text{hold}(F,I) \leftarrow \text{fired}(E,I), \text{add}(E,F), \text{effect}(E), \text{step}(I) \tag{22}
\]

\[
\text{changed}(X,I) \leftarrow \text{fired}(E,I), \text{add}(E,f(X,Y)), \text{step}(I) \tag{23}
\]

4 IPlan

4.1 Baseline Implementation

We describe our baseline integer-programming planner IPlan, which is based on Optiplan (van den Briel and Kambhampati 2005). Optiplan, in turn, extends the state-change variable model (Vossen et al. 1999), and the Optiplan model is defined for af propositional (STRIPS) framework. IPlan adapts the Optiplan model for the multi-valued SAS+ framework, exploiting the FastDownward translator (Helmert 2006).
An assignment \( v=x \) to a SAS+ variable is mapped to a fluent \( f \). For all fluents \( f \) and time step \( t \), Optiplan has the following state change variables. \( \text{pre}_f, \text{add}_f, \text{del}_f \) denote a set of operators that might require, add, or delete \( f \) respectively. Intuitively, state change variables represent all possible changes of fluents at time step \( t \).

\[
x_{f,t}^{\text{maintain}} = \begin{cases} 1 & \text{if fluent } f \text{ is propagated in period } t \\ 0 & \text{otherwise} \end{cases} \quad (24)
\]

\[
x_{f,t}^{\text{preadd}} = \begin{cases} 1 & \text{if operator } a \text{ is executed in period } t \\ & \text{such that } a \in \text{pre}_f \land a \not\in \text{del}_f \\ 0 & \text{otherwise} \end{cases} \quad (25)
\]

\[
x_{f,t}^{\text{predel}} = \begin{cases} 1 & \text{if operator } a \text{ is executed in period } t \\ & \text{such that } a \in \text{pre}_f \land a \not\in \text{add}_f \\ 0 & \text{otherwise} \end{cases} \quad (26)
\]

\[
x_{f,t}^{\text{add}} = \begin{cases} 1 & \text{if operator } a \text{ is executed in period } t \\ & \text{such that } a \not\in \text{pre}_f \land a \in \text{add}_f \\ 0 & \text{otherwise} \end{cases} \quad (27)
\]

\[
x_{f,t}^{\text{del}} = \begin{cases} 1 & \text{if operator } a \text{ is executed in period } t \\ & \text{such that } a \not\in \text{pre}_f \land a \in \text{del}_f \\ 0 & \text{otherwise} \end{cases} \quad (28)
\]

For all operators \( o \) and time step \( t \), Optiplan has the operator variables

\[
y_{o,t} = \begin{cases} 1 & \text{if operator } o \text{ is executed in period } t \\ 0 & \text{otherwise} \end{cases} \quad (29)
\]

The constraints (30) and (31) represent the initial states constraints.

\[
x_{f,0}^{\text{add}} = 1 \quad \forall f \in I \quad (30)
\]

\[
x_{f,0}^{\text{add}}, x_{f,0}^{\text{maintain}}, x_{f,0}^{\text{preadd}} = 0 \quad \forall f \notin I \quad (31)
\]

The constraint (32) ensures that the goals are satisfied at the last step \( T \).

\[
x_{f,t}^{\text{sat}} \geq 1 \quad \forall f \in G, t=T \quad (32)
\]

For all fluents \( f \) and time step \( t \), Optiplan has the following constraints to make sure the state change variables have the intended semantics.

\[
\sum_{o \in \text{add}_f \setminus \text{pre}_f} y_{o,t} \geq x_{f,t}^{\text{add}} \quad (33)
\]

\[
y_{o,t} \leq x_{f,t}^{\text{add}} \quad \forall o \in \text{add}_f \setminus \text{pre}_f \quad (34)
\]

\[
\sum_{o \in \text{del}_f \setminus \text{pre}_f} y_{o,t} \geq x_{f,t}^{\text{del}} \quad (35)
\]

\[
y_{o,t} \leq x_{f,t}^{\text{del}} \quad \forall o \in \text{del}_f \setminus \text{pre}_f \quad (36)
\]

\[
\sum_{o \in \text{pre}_f \setminus \text{del}_f} y_{o,t} \geq x_{f,t}^{\text{preadd}} \quad (37)
\]

\[
y_{o,t} \leq x_{f,t}^{\text{preadd}} \quad \forall o \in \text{pre}_f \setminus \text{del}_f \quad (38)
\]

\[
\sum_{o \in \text{pre}_f \setminus \text{del}_f} y_{o,t} = x_{f,t}^{\text{predel}} \quad (39)
\]

The constraints (40) and (41) restrict certain state changes from occurring in parallel.

\[
x_{f,t}^{\text{add}} + x_{f,t}^{\text{maintain}} + x_{f,t}^{\text{del}} + x_{f,t}^{\text{predel}} \leq 1 \quad (40)
\]

\[
x_{f,t}^{\text{add}} + x_{f,t}^{\text{maintain}} + x_{f,t}^{\text{del}} + x_{f,t}^{\text{predel}} \leq 1 \quad (41)
\]

The constraint (42) represents the backward chaining requirement.

\[
x_{f,t}^{\text{preadd}} + x_{f,t}^{\text{maintain}} + x_{f,t}^{\text{predel}} \leq
x_{f,t}^{\text{preadd}} + x_{f,t-1}^{\text{add}} + x_{f,t-1}^{\text{maintain}} \quad \forall f \in F, t \in 1, ..., T \quad (42)
\]

**New Mutex Constraints** IPlan augments the Optiplan model with a new set of mutex constraints. In addition to the above variables and constraints which are from Optiplan, IPlan introduces a set of auxiliary binary variables \( x_{f,t}^{\text{sat}} \) with the following constraints. \( x_{f,t}^{\text{sat}} \) corresponds to the value of the fluent \( f \) at the time step \( t \).

\[
x_{f,t}^{\text{sat}} \leq x_{f,t}^{\text{add}} + x_{f,t}^{\text{preadd}} + x_{f,t}^{\text{maintain}} \quad (43)
\]

\[
x_{f,t}^{\text{sat}} \geq x_{f,t}^{\text{add}} \quad (44)
\]

\[
x_{f,t}^{\text{sat}} \geq x_{f,t}^{\text{preadd}} \quad (45)
\]

\[
x_{f,t}^{\text{sat}} \geq x_{f,t}^{\text{maintain}} \quad (46)
\]

Using \( x_{f,t}^{\text{sat}} \), mutex constraints can be implemented as follows. For every mutex group \( g \) (at most one fluent in \( g \) can be true at the same time) found by the FastDownward (Helmert 2006) translator, IPlan adds the following constraint.

\[
\sum_{f \in g} x_{f,t}^{\text{sat}} \leq 1 \quad (47)
\]

### 4.2 Integrating Axioms

We describe how to integrate axioms into an IP-based model. For each time step \( t \), axioms form the corresponding normal logic program (NLP) \( P_t \). The models for \( P_t \) correspond to the truth values for the derived variables. We translate each NLP \( P_t \) to an integer program (IP) using the method by Liu, Janhunen, and Niemelä (2012), and add these linear constraints to the IPlan model.
**Level Rankings**  Translation from a NLP to an IP by (Liu, Janhunen, and Niemelä 2012) relies on characterizing answer sets using level rankings. Intuitively, a level ranking of a set of atoms gives an order in which the atoms are derived.

**Definition 6** (Niemelä 2008). Let $M$ be a set of atoms and $P$ a normal program. A function $l: M \rightarrow N$ is a level ranking of $M$ for $P$ iff for each $a \in M$, there is a rule $r \in P_M$ such that $H(r) = a$ and for every $bcB^+(r)$, $l(r(a)) - 1 \geq l(r(b))$.

Note that $P_M$ is the set of support rules, which are essentially the rules applicable under $M$.

**Definition 7** (Niemelä 2008). For a program $P$ and $I \subseteq At(P)$, $P_I = \{r \in P | \forall a \in I \exists b \in B^+(r)\}$ is the set of support rules.

The level ranking characterization gives the condition under which a supported model is an answer set.

**Definition 8** (Apt, Blair, and Walker 1988). A set of atoms $M$ is a supported model of a program $P$ iff $M = P$ and for every atom $a \in M$ there is a rule $r \in P$ such that $H(r) = a$ and $M = B^+(r)$.

**Theorem 1** (Niemelä 2008). Let $M$ be a supported model of a normal program $P$. Then $M$ is an answer set of $P$ iff there is a level ranking of $M$ for $P$.

**Components and Defining Rules** Liu, Janhunen, and Niemelä (2012) defines a dependency graph of a normal logic program to be used for the translation to IP.

**Definition 9** (Liu, Janhunen, and Niemelä 2012). The dependency graph of a program $P$ is a directed graph $G = \langle V, E \rangle$ where $V = At(P)$ and $E$ is a set of edges $\langle a, b \rangle$ for which there is a rule $r \in P$ such that $H(r) = a$ and $b \in B^+(r)$.

**Definition 10** (Liu, Janhunen, and Niemelä 2012). For a program $P$ and an atom $a \in At(P)$, respective sets of defining rules, externally defining rules, and internally defining rules are defined as follows:

\[
\text{Def}_P(a) = \{r \in P | H(r) = a\} \quad (48)
\]

\[
\text{Ext}_P(a) = \{r \in \text{Def}_P(a) | B^+(r) \cap SCC(a) = \emptyset\} \quad (49)
\]

\[
\text{Int}_P(a) = \{r \in \text{Def}_P(a) | B^+(r) \cap SCC(a) \neq \emptyset\} \quad (50)
\]

The set of internally supporting atoms is defined as

\[
\text{IS}(a, r) = \text{SCC}(a) \cap B^+(r). \quad (51)
\]

**Translation** We are now ready to describe how to translate a normal logic program $P_I$ formed by axioms to linear constraints based on the method by Liu, Janhunen, and Niemelä (2012). The translation consists of linear constraints developed below.

1. For each secondary variable $u \in U$, introduce a binary variable $x_{u,t}^{sat}$.

2. For each secondary variable $u \in U$, include the following constraint

\[
\sum_{r \in \text{Def}_P(u)} bd_r t - |\text{Def}_P(u)| \cdot x_{u,t}^{sat} \leq 0 \quad (52)
\]

where $bd_r t$ is a binary variable for each $r \in \text{Def}_P(u)$ and step $t$. Intuitively, $bd_r t$ represents whether the body of $r$ is satisfied at step $t$. The constraint ensures that when one of the rules defining $u$ ($\text{Def}_P(u)$) is satisfied, $x_{u,t}^{sat}$ must be true ($x_{u,t}^{sat} = 1$).

3. For each axiom $r \in A$, include the following constraints.

\[
\sum_{b \in B^+(r)} x_{b,t}^{sat} - \sum_{c \in B^-(r)} x_{c,t}^{sat} - |B(r)| \cdot bd_r t \geq -|B^-(r)| \quad (53)
\]

\[
\sum_{b \in B^+(r)} x_{b,t}^{sat} - \sum_{c \in B^-(r)} x_{c,t}^{sat} - bd_r t \leq |B^+(r)| - 1 \quad (54)
\]

Constraints (53) and (54) express the fact that the body of rule $r$ is satisfied iff each literal in $B(r)$ is satisfied.

4. For each secondary variable $u \in U$, include the constraint

\[
\sum_{r \in \text{Ext}_P(u)} bd_r t - \sum_{r \in \text{Int}_P(u)} s_r t - x_{u,t}^{sat} \geq 0 \quad (55)
\]

where $s_r t$ is a binary variable for each $r \in \text{Int}_P(u)$ and each step $t$. Intuitively, the binary variable $s_r t$ represents whether the respective ranking constraints for the rule $r$ are satisfied in addition to its body. The constraint requires $x_{u,t}^{sat}$ to be true when one of its externally defining rules is satisfied or one of its internally defining rules is satisfied while respecting the ranking constraints.

5. For each secondary variable $u \in U$ and each $r \in \text{Int}_P(u)$, include the constraints

\[
bd_r t - s_r t \geq 0 \quad (56)
\]

\[
\sum_{b \in \text{IS}(u, r)} gt_{u,b} t - \text{IS}(u, r) \cdot s_r t \geq 0 \quad (57)
\]

where $gt_{u,b} t$ is a binary variable for each $b \in \text{IS}(u, r)$, which represents whether the rank of $u$ is greater than that of $b$.

6. For each secondary variable $u \in U$ and each $r \in \text{Int}_P(u)$, and each $b \in \text{IS}(u, r)$, include the constraint

\[
z_{u,t} - z_{b,t} = |\text{SCC}(u)| \cdot gt_{u,b} t \geq 1 - |\text{SCC}(u)| \quad (58)
\]

where $z_{u,t}$ and $z_{b,t}$ are integer variables representing level rankings for $u$ and $b$ respectively. The constraint (58) guarantees that if $gt_{u,b} t = 1$ then $z_{u,t} \geq z_{b,t}$.

With the above constraints, $x_{u,t}^{sat}$ corresponds to the values of a secondary variable $u$ at time step $t$. Since secondary variables only appear in operators’ preconditions, we only need to make sure applied operators’ preconditions are satisfied.

\[
y_{u,t} \leq x_{u,t}^{sat} \quad \forall a \in \text{pre}_f \setminus \text{del}_f \quad (59)
\]

For the reasons described earlier, with domains with axioms, we need to add the following constraint to restrict the number of operators executed at each time step to 1.

\[
\sum_{a \in \text{pre}_f \setminus \text{del}_f} y_{u,t} \leq 1 \quad (60)
\]

**5 Experimental Results**

All experiments are performed on a Xeon E5-2670 v3, 2.3GHz with 2GB RAM and 5 minutes limit. In all experiments, the runtime limits include all steps of problem solving, including translation/parsing, and search. We use clingo4.5.4, a state-of-the-art ASP solver (Potassco 2016) to solve the ASP models produced by ASPlan. The IP models produced by IPare are solved using Gurobi Optimizer 6.5.0, single-threaded.

The rules and constraints used in each of our planner configurations are summarized in Table 2.
5.1 Baseline Evaluation on PDDL Domains without Axioms

We first evaluated ASPlan and IPlan on PDDL domains without axioms to compare them against existing planners. A thorough comparison of model-based planners is non-trivial because of the multitude of combinations possible of translation schemes, solvers, and search strategies. The purpose of this experiment is to show that the baseline ASPlan and IPlan planners perform reasonably well compared to similar, existing model-based planners which (1) use a simple search strategy which iteratively solves k-step models, and (2) use models which are solved by “off-the-shelf” solver algorithms, i.e., this excludes planners that such as Madagascar (Rintanen, Heljanko, and Niemelä 2006), which uses a more sophisticated search strategy and customized solver algorithm, as well as the flow-based IP approaches (van den Briel, Vossen, and Kambhampati 2008).

We compared the following: (1) APlanS (ASPlan with seq- semantics) (2) plasp\(^1\) (3) IPlanS (IPlan with seq- semantics) (4) IPlan (IPlan with \(\forall\)-semantics) (5) ASPlan (ASPlan with \(\forall\)-semantics) (6) The state of the art CSP-based planner TCPP (Ghooshchi et al. 2015).

Since plasp uses incremental grounding, we used iClingo\(^2\) (Gebser et al. 2008) as an underlying solver. As for TCPP, we could not obtain the source code from the authors as of this writing, so we use the results from the original paper. The results are shown in Table 1.

ASPlanS dominated plasp in every domain, indicating that ASPlan is a reasonable, baseline ASP-based solver. This is to be expected, since ASPlanS is based on the plasp model. Among \(\forall\)-semantics solvers ASPlan, IPlan and TCPP, ASPlan and IPlan are highly competitive with TCPP despite the fact that the results for TCPP were obtained with a significantly longer (30min. vs. 5min) runtime limit (on a different machine).

5.2 Evaluation on PDDL Domains with Axioms

We evaluated ASPlan and IPlan on PDDL domains with axioms.

The results are shown in Table 3.

About the domains used in experimental evaluations, Sokoban has been discussed already. The new domains are discussed below.\(^3\)

Verification Domains by Ghosh, Dasgupta, and Ramesh (2015) Ghosh, Dasgupta, and Ramesh (2015) proposed a modeling formalism for capturing high level functional specifications and requirements of reactive control systems. The formulation consists of two agents, namely the environment which disturbs the system, and the controller which tries to return the system to safe state. If there is a sequence of operators for the environment that leads to an unsafe state, the control system has vulnerability. We used two compiled versions of the formulation: compilation into STRIPS proposed by (Ghosh, Dasgupta, and Ramesh 2015): and compilation into STRIPS with axioms discussed in (Ivankovic and Haslum 2015b).

The Adaptive Cruise Control (ACC) is a well known driver assistance feature present in many high end automotive systems. The ACC is designed to take away the burden of adjusting the speed of the vehicle from the driver, mostly under light traffic conditions. ACC is a verification domain for the ACC.

The GRID domain is a synthetic planning domain loosely based on cellular automata and incorporates a parallel depth first search protocol for added variety. Note that this domain is completely different from grid used in IPC.

Multi-Agent Domains by Kominis and Geffner (2015) Kominis and Geffner (2015) proposed a framework for handling beliefs in multiagent settings, building on the methods for representing beliefs for a single agent. Computing linear multiagent plans for the framework can be mapped to a classical planning problem with axioms.

Muddy Children, originally from Fagin et al. (2004), is a puzzle in which \(n\) children play together. During their play, \(k\) of the children get mud on their foreheads. Each can see the mud on the others, but not their own. The goal for

| Domain   | ASPlan | ASPlanS | IPlan | IPlanS | TCPP |
|----------|--------|---------|-------|--------|------|
| grid     | 5      | 2       | 1     | 1      | 0    |
| gripper  | 20     | 2       | 2     | 4      | 3    |
| movie    | 30     | 30      | 30    | 30     | NaN  |
| logistics-98 | 35    | 2      | 2     | 1      | 16   |
| mprime   | 35     | 27      | 24    | 21     | 25   |
| mystery  | 30     | 16      | 16    | 13     | 16   |
| blocks   | 35     | 18      | 15    | 28     | 28   |
| freecell | 80     | 7       | 6     | 7      | 18   |
| logistics00 | 28   | 7      | 7     | 6      | 28   |
| depot    | 22     | 2       | 2     | 11     | 9    |
| driverlog| 20     | 7       | 4     | 4      | 11   |
| rovers   | 40     | 4       | 0     | 4      | 21   |
| zenithavel| 20   | 7      | 6     | 3      | 13   |
| satellite| 36     | 4       | 3     | 5      | 8    |

Table 1: Results on Domains without Axioms. Note that for TCPP, the results from the original paper (Ghooshchi et al. 2015) on a PC with Intel 3.5GHz CPU, 8GB memory limit and 30 minute timeout are shown. NaN indicates a lack of the results. # denotes the number of instances for each domain.

| Domain   | ASPlan | ASPlanS | IPlan | IPlanS |
|----------|--------|---------|-------|--------|
| grid     |        |         |       |        |
| gripper  |        |         |       |        |
| movie    |        |         |       |        |
| logistics-98 |       |         |       |        |
| mprime   |        |         |       |        |
| mystery  |        |         |       |        |
| blocks   |        |         |       |        |
| freecell |        |         |       |        |
| logistics00 |       |         |       |        |
| depot    |        |         |       |        |
| driverlog|        |         |       |        |
| rovers   |        |         |       |        |
| zenithavel|       |         |       |        |
| satellite|        |         |       |        |

Table 2: Summary of the rules or constraints used in each planner configuration

\(^1\)plasp was sourced from (https://sourceforge.net/p/potassco/code/7165/tree/trunk/plasp-2.0/releases).
\(^2\)iClingo was obtained from (https://sourceforge.net/projects/potassco/files/iclingo/3.0.5/iclingo-3.0.5-x86-linux.tar.gz/download)
\(^3\)The benchmarks are available with more detailed descriptions from (https://github.com/dosydon/axiom_benchmarks).
a child is to know if he or she has mud by sensing the beliefs of the others. Muddy Child is a reformulation of Muddy Children.

In Collaboration through Communication, the goal for an agent is to know a particular block’s location. Two agents volunteer information to each other to accomplish a task faster than that would be possible individually.

In Sum, there are three agents, each with a number on their forehead. It is known that one of the numbers equals the sum of the other two. The goal is for one or two selected agents to know their numbers.

Wordrooms is a variant of Collaboration through Communication where two agents must find out a hidden word from a list of n possible words.

PROMELA IPC-4 benchmarks contain another domain with axioms called PROMELA aside from PSR. The task is to validate properties in systems of communicating processes (often communication protocols), encoded in the Promela language. Edelkamp (2003) developed an automatic translation from Promela into PDDL, which was extended to generate the competition examples.

We used verification domains for Dining Philosophers problem, and the so-called Optical Telegraph protocol.

PSR The task of PSR (power supply restoration) is to reconfigure a faulty power distribution network so as to resupply customers affected by the faults. The network consists of electronic lines connected by switched and power sources with circuit breaker. PSR has been investigated by the AI community, including the works such as Bonet, Thiébaux, and others (2003). We used the version of PSR used in IPC-4, which is a simplified version of Bonet, Thiébaux, and others (2003) with full observability of the world and no numeric optimization.

It is interesting to note that in Sokoban, acc, psr-middle, and philosophers ASPPlan (and sometimes IPlan) solved more instances from the formulations with axioms than the compiled formulations without axioms. This is because compiled instances tend to have longer plans, which model-based planners such as ASPPlan and IPlan had difficulty in handling.

6 Conclusion

We proposed axiom-aware model-based planners ASPPlan and IPlan. ASPPlan is an ASP-based planner, which is able to handle axioms and conditional effects. IPlan is an IP-based planner based on Optiplan (van den Briel and Kambhampati 2005). We integrated axioms into IPlan using the ASP to IP translation method by (Liu, Janhunen, and Niemelä 2012). We evaluated ASPPlan and IPlan on PDDL domains with axioms and showed that axiom-aware model-based planners can exploit shorter plan with formulations with axioms.

| 2GB, 5min | # | ASPPlan | IPlan | Axioms |
|---|---|---|---|---|
| Domains from Ivankovic and Haslum (2015b) | | | | |
| sokoban-axioms | 30 | 5 | 4 | Y |
| sokoban-opt08-strips | 30 | 4 | 0 | N |
| trapping_game | 7 | 4 | NaN | Y |
| Domains from Ghosh, Dasgupta, and Ramesh (2015) | | | | |
| acc-com | 8 | 0 | 0 | N |
| acc | 8 | 7 | 1 | Y |
| door-fixed | 2 | 1 | 1 | Y |
| door-broken | 2 | 0 | 0 | Y |
| grid-axiom | 99 | 0 | 0 | Y |
| Domains from Kominis and Geffner (2015) | | | | |
| muddy-child-kg | 7 | 1 | NaN | Y |
| muddy-children-kg | 5 | 1 | NaN | Y |
| collab-and-comm-kg | 3 | 1 | NaN | Y |
| wordrooms-kg | 5 | 1 | NaN | Y |
| Domains from IPC-4 | | | | |
| psr-middle | 50 | 48 | NaN | Y |
| psr-middle-noce | 50 | 43 | 25 | Y |
| psr-middle-com | 50 | 1 | NaN | Y |
| psr-large | 50 | 21 | NaN | N |
| optical-telegraphs | 48 | 0 | NaN | Y |
| optical-telegraphs-com | 48 | 0 | NaN | N |
| philosophers | 48 | 2 | NaN | Y |
| philosophers-com | 48 | 0 | NaN | N |

Table 3: Results on domains with axioms. NaN indicates a lack of results since the current implementation of IPlan is not compatible with conditional effects. # denotes the number of instances for each domain. The columns "Axioms" indicates whether the corresponding domain has axioms.
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