Large-Scale Clustering in Bubble Models

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ABSTRACT

We analyze the statistical properties of bubble models for the large-scale distribution of galaxies. To this aim, we realize static simulations, in which galaxies are mostly randomly arranged in the regions surrounding bubbles. As a first test, we realize simulations of the Lick map, by suitably projecting the three-dimensional simulations. In this way, we are able to safely compare the angular correlation function implied by a bubbly geometry to that of the APM sample. Quite remarkably, we find that several bubble models provide an adequate amount of large-scale correlation, which nicely fits that of APM galaxies. Further, we apply the statistics of the count-in-cell moments to the three-dimensional distribution and compare them with available observational data on variance, skewness and kurtosis. Based on our purely geometrical constructions, we find that a well defined hierarchical scaling of higher order moments up to scales $\sim 70\, h^{-1}\text{Mpc}$. We show that this must be expected for any non-Gaussian distribution in the weak-coherence regime. The overall emerging picture is that the bubbly geometry is well suited to reproduce several aspects of large-scale clustering. Furthermore, the statistical test we apply are able to discriminate between different models. We find that models with fixed bubble radius have troubles to account for the angular correlation of APM galaxies, while a shallow spectrum fails to reproduce the observed three-dimensional skewness. A model with a rather steep spectrum of bubble of radii is rather adequate to account for the observational tests that we considered.
1 Introduction

The most striking feature of the large-scale structure of the Universe is the remarkable variety of coherent structures and the extension of the involved scales. Its interpretation really represents a difficult task for any theoretical model about the origin of primordial fluctuations. The most eminent victim of such a largely structured Universe is probably the standard cold dark matter (CDM) scenario. In its minimal version, this model assumes initial adiabatic and Gaussian fluctuations, dominated by highly non-relativistic non-baryonic particles in a $\Omega = 1$ Universe, with galaxies substantially more clustered than the underlying dark matter (see, e.g., Blumenthal et al. 1984; see also Davis et al. 1992 and Liddle & Lyth 1993, for recent reviews about CDM). Although successful at small and intermediate scales, this model has been shown to have serious troubles in accounting for clustering data at scales $> 20h^{-1}$Mpc, such as clustering of rich clusters (e.g., White et al. 1987), large-scale galaxy angular correlation (Maddox et al. 1990; Collins et al. 1991), large-scale galaxy peculiar motions (Vittorio, Juszkiewicz & Davis 1986; Bertschinger et al. 1990), count-in-cell statistics of IRAS-QDOT (Saunders et al. 1991) and Stromlo-APM (Loveday et al. 1992) galaxies, large-scale power spectra (Peacock 1991; Feldman, Kaiser & Peacock 1993), abundance of rich clusters (White, Efstathiou & Frenk 1993).

Several adjustments of the standard CDM model have been proposed in order to overcome its deficit of large-scale power, which can be divided into two main categories; those which modify the scheme for the production of primordial inhomogeneities and those which try to suitably modify the prescription for galaxy identification out of dark matter fluctuations. The first category includes models with primordial power-spectrum tilted away from the Zel’dovich one, predicted by standard inflation (e.g., Lucchin & Matarrese 1985; Cen et al. 1992), low-density Universe with non-vanishing cosmological constant ($\Omega_{\text{matter}} \simeq 0.2$, $\Omega_{\text{matter}} + \Omega_{\Lambda} = 1$; e.g., Efstathiou, Sutherland & Maddox 1990), non-Gaussian initial fluctuations (Moscardini et al. 1991; Weinberg & Cole 1992), inclusion of a hot particle component supplying large-scale power (Valdarnini & Bonometto 1985; Achilli, Occhionero & Scaramella 1985; Klypin et al. 1992). The second class of non-standard CDM models includes low-bias mechanisms of galaxy formation, in which the present time corresponds to a dynamically more evolved stage (Couchman & Carlberg 1992; see, however, Moscardini et al. 1993), and “cooperative” non-local biassing (Bower et al. 1993), in which galaxy formation is triggered by astrophysical mechanisms, able to introduce large-scale coherence (e.g., inhibition of galaxy formation due to high-redshift QSO reionization; Babul & White 1991).

In this paper we follow a purely geometrical approach by modelling the large-scale clustering by means of a distribution of spherical bubbles, with galaxies mostly arranged in the volume not occupied by such bubbles, while leaving them almost devoid. Therefore, we are not concerned with the dynamics underlying the above models for large-scale structure formation. Instead, we attempt to parametrize their dynamical content through a suitable geometrical construction.

Bubble models have been already considered in several contexts. Generation of bubbles can be provided by invoking primordial phase-transitions, occurred along with the inflationary expansion (e.g., La 1991). The only theory to date for primeval bubble generation, which also predicts a definite spectrum of bubble sizes, is the extended inflation (La & Steinhardt 1989;
for a review see Kolb 1991). In this scenario, bubbles are generated as true-vacuum regions, tumelling out from the initial false-vacuum state and subsequently stretched to cosmological sizes by the accelerated expansion. In the original model, the number of bubbles $N_B(R)$ with radius larger than $R$ is roughly a power-law,

$$N_B(R) = \left( \frac{R_M}{R} \right)^p,$$

where $p$ is related to the microphysical parameters of the phase transition and $R_M$ is the maximum bubble radius inside the horizon, also fixed by the model. If the inflation is driven by the Brans-Dicke field coupled through the constant $b$, like in La & Steinhardt (1989), then $p = 3 + 4/(b + 1/2)$. The determination of the slope $p$ of the present bubble spectrum may then be related to the fundamental constant $b$. A potential problem in this mechanism of bubble production lies in the predicted $R_M$ values, which, in all the models considered so far, are either in conflict with the microwave background isotropy or too small to be of cosmological interest (Liddle & Wands 1991). A primordial generation of bubbles clearly fixes non-Gaussian initial conditions, since the expectation of finding underdense regions is larger than that of finding overdense regions, and the resulting probability density function of the density field is negatively skewed.

A bubbly galaxy distribution naturally arises also in the explosion scenario (Ostriker & Cowie 1981; Ikeuchi 1981). Differently from the gravitational instability picture, in this scenario energy perturbations of non-gravitational origin drive material away from the seeds of the explosions, sweeping primordial gas into dense, expanding shells. As these shells cool, their fragmentation could give a further generation of objects, which again explode, thus amplifying the process and giving rise to large-scale structure formation. At the end, galaxies are arranged on spherical shells and rich clusters are expected to be placed at the intersection of three expanding bubbles. A variety of physical mechanisms may generate such explosions, such as supermassive stars or supernovae from the earliest galaxies. However, it is at present not clear whether such energy sources are sufficient to create the large voids that are observed. Moreover, suitable initial conditions are anyway needed to generate primordial objects, which act as seeds of explosions. It is also to be remarked that the recent analysis of the COBE FIRAS spectrum (Wright et al. 1993) seems to rule out the explosions scenario, since the energy release would have distorted the cosmic blackbody spectrum above the observational bounds.

A more conventional mechanism for nearly spherical bubble generation is provided by gravitational instability in the mildly non-linear regime. In the pancake scenario, overdense regions tend to increase their asphericity during the gravitational collapse, while underdense regions expand and become more and more spherical. As a result, the most part of space turns out to be occupied by almost devoid spherical regions, with galaxies arranged in sheet-like structures around them. It is however clear that in order for gravitational dynamics to produce as large voids as observed, a primordial fluctuation spectrum is required, which produces an adequate amount of large-scale fluctuation power, probably more than supplied by standard CDM.

Purely geometrical random bubble distributions have been already shown to be able to account for some features of large-scale clustering. In the framework of the explosion model, Weinberg, Ostriker & Dekel (1989) used a random distribution of expanding shells with both a power-law and a uniform distribution of radii. After identifying clusters at the intersection
of three of such shells, the statistics of their observed distribution is quite well reproduced. A detailed description of the galaxy distribution in terms of Voronoi tessellation (van de Weygaert & Icke 1989; Yoshioka & Ikeuchi 1989; van de Weygaert 1993) also reveals that a topology dominated by big voids surrounded by a sheet-like galaxy distribution gives a fair representation of several clustering features.

In this paper, we analyze statistical properties for simulations of bubbly galaxy distributions and compare them with similar tests, already presented in the literature, about the observed galaxy clustering. This allows us to check in detail the ability of this purely geometrical model to account for the large-scale texture of the Universe. In the simulations, the centers of spherical bubbles are randomly distributed and the radii are chosen either fixed or taken from a power law spectrum, like that of Eq.(1), with a lower cut-off radius \( R_m \) of the order of a few megaparsecs. A list of the parameters involved in each of the models that we will consider is presented in Table I. A letter F marks the models with fixed radius size. The model parameters are always tuned in such a way that the resulting bubble volume is 90% of the total volume. Galaxies are randomly arranged in the space not occupied by bubbles, while an unclustered component, a fraction \( f_u \) of the total number of particles, is also added to account for a residual galaxy population inside the underdense regions. The simulation box boundary is taken to be periodic. By varying the model parameters, we generate simulated galaxy distributions having different amounts of large-scale clustering. It is clear that using such simple simulations we completely neglect the presence of gravitational clustering at the scales of non-linearity. For this reason, we restrict our analysis to larger scales \( (> 5h^{-1}\text{Mpc}) \), where non-linear gravitational dynamics does not work and the clustering should be accounted for just by the large-scale geometry. The remarkable advantage of adopting such non-dynamical simulations is that we can reach very large scales (we take \( 450h^{-1}\text{Mpc} \) for the simulation box side), with a negligible computational cost, even when we average the results over several realizations. In a previous work (Amendola & Occhionero 1993), the bubble model has been compared to some observational constraints, such as the large-scale shape of \( 1 + \xi(r) \) (here, \( \xi(r) \) is the galaxy spatial 2-point correlation function) and the upper limits on anisotropy and spectral distortion of the cosmic microwave background. In particular, it was found that power-law bubble models can fit the observed spatial correlation function if

\[
R_M \approx (p/130)^{-1.3}
\]  

(tested in the range \( p = 4 - 11 \)) and if about 90% of the total volume is occupied by the bubbles. The latter requirement fixes the cut-off size \( R_m \), i.e. the radius of the smallest bubbles. All the power-law models we test in this paper roughly obey the prescriptions above. If bubbles were generated before decoupling, Amendola & Occhionero (1993) also show that power law spectra pass the cosmic microwave background tests only for \( 6 < p < 11 \); roughly speaking, shallower spectra have too many large bubbles, while steeper spectra too many small bubbles.

In Figure 1 we plot the resulting galaxy distribution for different bubble models, having both fixed and variable radii, according to the power-law spectrum of Eq.(1). Each panel reports the projection in the \( x - y \) plane of galaxies in a \( 15h^{-1}\text{Mpc} \) slice in the \( z \) direction. It is apparent the remarkable variety of structures generated by the bubble geometry, with rich clumps, filaments and underdense regions having sizes of some tens of Mpcs. Also the effect
of taking different model parameters is visible in this plot. Since each realization is generated with the same phase assignment, the persistence of several structures can be directly compared along the sequence of the model parameters. Fixed radius and steep spectra produces more isolated and concentrated galaxy aggregates, while shallower spectra turns into the presence of larger bubbles and, consequently, of more extended coherent structures. These differences will be quantified in more details in the remainder of this paper.

The plan of the paper is as follows.

After generating artificial Lick maps by projection of the spatial simulations, we analyze in Section 2 the resulting angular correlation functions and compare them with that observed for the APM sample (Maddox et al. 1990). Section 3 is devoted to the analysis of moments of cells counts (variance, skewness and kurtosis). The comparison with analogous results for observational data sets allows us to further restrict the parameter space for the permitted bubble models and to assess the reliability of this geometrical interpretation of large-scale clustering. In Section 4 we discuss our results and draw the main conclusions.

2 Angular correlation

As a first test of large-scale clustering, we investigate the projected (angular) correlation function implied by the bubble geometry. In order to implement this analysis, we generate artificial Lick galaxy catalogues, following the same procedure adopted by Coles et al. (1993, hereafter CMPLMM) and Moscardini et al. (1993). The observer is placed at the center of the simulation box. Then, the original box is reflected, so to generate a three dimensional observational cone having width and depth adequate to reproduce the Lick map. Although the characteristic depth of the Lick map is ∼210 h⁻¹ Mpc, also very luminous galaxies at a much larger distance are included. Taking for the luminosity function the Schechter-like expression

\[ \Phi(L) \propto \left( \frac{L}{L^*} \right)^{-\alpha} \exp \left( -\frac{L}{L^*} \right) \]  

with the parameters given by Efstathiou, Ellis & Peterson (1988; \( \alpha = -1.07 \) and \( M^* = -19.68 \)) for the absolute magnitude associated to the characteristic luminosity \( L^* \), few galaxies up to ∼700 h⁻¹ Mpc are included. Taking a simulation cube of 450 h⁻¹ Mpc aside, we select galaxies up to the distance of 675 h⁻¹ Mpc. With this kind of construction the observational cone encompasses the range of galactic latitude \( |b| \geq 45^\circ \). The procedure to assign luminosities to the galaxies is the same as described by CMPLMM. We suitably truncate \( \Phi(L) \) at the faint and bright tails, assign the apparent magnitude to a galaxy at a given distance by including \( K \)-correction and curvature effects. Therefore, we project only those galaxies brighter than the limiting apparent magnitude of the Lick map \( (m \leq 18.8) \). We tune the mean galaxy separation in the simulation box so to end up with a total number of projected galaxies of the same order of that included in the Lick map for \( |b| \geq 45^\circ \) (∼316,000). As observed by CMPLMM, although the observational cone includes galaxies belonging to different replica of the same simulation box, no substantial spatial periodicity should affect the projected clustering pattern, since each galaxy has a very small probability to be selected more than once. This problem is even less important in our case, being our simulation cube much larger.
than that used by CMPLMM, which used three levels of replicated boxes. After generating
the projected galaxy distribution, we collect them in counts with $10 \times 10$ arcmin cells, so to
reproduce the observational setup of the Lick map.

Simulating angular galaxy samples, like the Lick map, has been proved to be a reliable
approach to compare data and simulations. In fact, the lack of redshift information in angular
analysis is largely compensated by the extension of the available data sets. CMPLMM gen-
erated artificial Lick maps from N-body simulations, based on assuming both Gaussian and
skewed CDM initial conditions, in order to investigate the topology of the two-dimensional
galaxy distribution. Moscardini et al. (1993) used the same approach to investigate the angular
correlation functions of such simulated samples. Borgani et al. (1993) extracted synthetic
samples of galaxy clusters from the same Lick map simulations and applied several statistical
tests to compare them with clusters selected from the real Lick map (Plionis, Barrow & Frenk
1991).

Here, we analyze the angular correlation function, $w(\theta)$, of our Lick simulations and com-
pare them with the APM data, as provided by Maddox et al. (1990). The adopted estimator
of the angular two-point correlation function is

$$w(\theta) = \frac{\langle n_i n_j \rangle_\theta}{\langle \frac{1}{2}(n_i + n_j) \rangle_\theta^2} - 1.$$  (4)

Here, $n_i$ and $n_j$ are the galaxy counts in the $i$–th and $j$–th cell, respectively, placed at sepa-
ration $\theta$. The average is taken over all cell pairs with separation $\theta$. Since our simulations of a
bubbly Universe contain only geometry, we are not interested in the small-scale behaviour of
$w(\theta)$, which is expected to be determined by non-linear gravitational dynamics. Considering
only separation above $1^\circ.5$, which corresponds to $\lesssim 5 h^{-1}$ Mpc at the Lick depth (Maddox et
al.1990), allows us to considerably reduce the computational time. We group the small cells
to form counts in $1^\circ \times 1^\circ$ cells, and $w(\theta)$ is evaluated according to eq.(4), being $n_i$ and $n_j$ the
counts in such larger cells.

The results of our analysis are shown in Figure 2, where we plot the angular correlation
function for a list of bubble models, as compared to the APM data, suitably rescaled to the
Lick depth. In Table I we report the relevant parameters for the models we are consider-
ing. The upper left panel is for the fixed-radius model. It is apparent that the model with
$R_f = 12 h^{-1}$ Mpc displays a deficit of correlation strength at separations $\geq 3^\circ$, while providing
the correct small scale amplitude. Increasing the radius at $R_f = 14 h^{-1}$ Mpc increases $w(\theta)$,
thus providing an excess of small-scale clustering. The upper right panel is for the variable
radius models, with power index $p = 8$ and for two different values of the maximum bubble
radius. It is apparent that allowing for a variation of the bubble radius sensibly improves
the agreement. The shape of $w(\theta)$ is now quite well reproduced at all the relevant angular
scales. In the lower left panel we check the effect of taking an even wider spectrum of bubble
radii, by lowering the $p$ value. Also in this case, the overall shape of $w(\theta)$ is remarkably well
reproduced. Finally, in the lower right panel we verify the effect of taking different amounts of
unclustered galaxies within a fixed model. The plot shows that the $w(\theta)$ shape is left substi-
tionally unchanged apart from a variation of the small scale-amplitude. Therefore, increasing
the percentage of unclustered galaxies amounts essentially to shift $w(\theta)$ downwards. This sug-
gests that, once a model is shown to produce the correct correlation shape, its normalization
can be fixed a posteriori by choosing a suitable amount of random galaxies. It is however clear that this procedure is not completely arbitrary, since observational constraints on the percentage of weakly clustered galaxies in voids suggests that they should be $\gtrsim 10\%$ of the whole population. Note that several models produces a $w(\vartheta)$ slope at the smallest considered scales, which remarkably matches that for the APM sample. Since our simulations do not include gravitational dynamics, which is expected to be responsible for the small-scale $w(\vartheta)$ profile, this may suggest that the bubbly geometry can be rather adequate also to account for mildly non-linear clustering.

The above results about angular correlation analysis indicate that the geometrical pattern provided by a random bubble distribution is able to provide a rather adequate amount of the large-scale power traced by the largest available projected galaxy samples. However, although several models are clearly ruled out, and others, such as that with fixed radii, are only marginally acceptable, the $w(\vartheta)$ analysis seems not to be able to pick out one preferred model. For instance, looking at the plots of Figure 2, it is not clear whether a steep spectrum of bubble radii (with $p = 8$) with rather small $R_M$ is better than a having $p \simeq 5$ and a larger $R_M$ value. In order to further discriminate between different models, we apply in the following the higher-order statistics of count-in-cell moments to the three-dimensional distributions.

3 Moments of cell-counts

3.1 The method

The analysis of moments of count in cells has been recently widely employed to analyze the observed galaxy distribution and it has been found to be a powerful statistical test of large-scale clustering (Efstathiou et al.1990; Saunders et al.1991; Bouchet, Davis & Strauss 1992; Coles & Frenk 1991; Gatzañaga 1992; Loveday et al.1992). The main advantage of this kind of analysis lies in the fact that it is relatively easy to implement and turns out to be particularly suited to investigate the statistics at large scales, where statistical noise severely limits the usage of correlation functions.

In order to characterize the statistics of the moments of counts, let us introduce $\delta_R(x) = \rho_R(x)/\bar{\rho} - 1$ as the relative fluctuations of the density field $\rho_R(x)$, smoothed over a suitable angular scale $R$ by a window function $W_R(x)$. Therefore,

$$\delta_R(x) = \frac{1}{V_R} \int d^3 y \delta(y) W_R(x - y),$$

being $V_R = \int d^3 x W_R(x)$ the volume encompassed by the chosen window. At the scale $R$, the statistics of the fluctuation field can be described by using the moments $\mu_n(R) \equiv \langle \delta_R^n \rangle$. Let us consider $\delta_R$ as a random variable with zero mean and let $P(\delta_R)$ be its probability density function. Accordingly, its moment of order $n$ is

$$\mu_n(R) = \int d\delta_R P(\delta_R) \delta_R^n.$$

The corresponding generating function

$$M(\phi) = \langle e^{\phi \delta_R} \rangle = \int d\delta_R P(\delta_R) e^{\phi \delta_R}$$

being $M(\phi)$ the generating function of $\delta_R$. The first moment of $\delta_R$ is

$$\mu_1(R) = \langle \delta_R \rangle = 0.$$
is defined in such a way that the $\mu_n$ moments represent the coefficients of its McLaurin expansion,

$$M(\phi) = \sum_{n=0}^{\infty} \frac{\mu_n}{n!} \phi^n ; \quad \mu_n = \frac{d^n M(\phi)}{d\phi^n} \bigg|_{\phi=0}.$$  

(8)

An equivalent statistical description can also be given in terms of the **cumulant** generating function

$$K(\phi) = \log M(\phi) = \sum_{n=0}^{\infty} \frac{\kappa_n}{n!} \phi^n ; \quad \kappa_n = \frac{d^n K(\phi)}{d\phi^n} \bigg|_{\phi=0}.$$  

(9)

The cumulants $\kappa_n$ can be related to the moments $\mu_n$ by subsequently differentiating the corresponding generating functions. At the lowest orders, it is

$$\kappa_2 = \mu_2 ; \quad \kappa_3 = \mu_3 ; \quad \kappa_4 = \mu_4 - 3\mu_2^2.$$  

(10)

with more cumbersome relations holding at higher orders. In turn, the cumulant of order $n$ represents the average value of the connected $n$-point correlation function $\xi_n$ inside the window volume $V_R$:

$$\kappa_n(R) = \frac{1}{(V_R)^n} \int_{V_R} \left\{ \prod_{i=1}^{n} d^3 x_i W_R(x_i) \right\} \xi_n(x_1, ..., x_n).$$  

(11)

Under the assumption that the 2-point function behaves like a pure power-law, $\xi(r) = (r_o/r)^\gamma$, at the second order the variance is $\sigma^2 = \kappa_2 = J_2 \xi_2$, where

$$J_2 = \frac{1}{(V_1)^2} \int_{V_1} d^3 x_1 \int_{V_1} d^3 x_2 |x_1 - x_2|^{-\gamma} W_1(x_1) W_1(x_2)$$  

(12)

arises after changing the variables in the integrals of eq.(11), according to $x_i \rightarrow x_i/R$. In this way, $V_1$ represents the volume encompassed by the window of unit radius, $W_1(x)$, so that $J_2$ is a dimensionless coefficient, which only depends on the window profile and on the slope $\gamma$ of the 2-point function.

At higher orders, a popular model to describe the scaling of the moments is represented by the hierarchical expression

$$\kappa_n = S_n \kappa_2^{\eta_n}.$$  

(13)

where $\eta_n = n - 1$ and $S_n$ are suitable coefficients, which depends on the moment order, other than on the shape of the window function. For instance, the hierarchical 3-point function reads $\xi_3 = 3Q\xi_2^2$, so that eq.(14) gives $\kappa_3 = 3Q (J_3/J_2^2) \xi_2^2$, where

$$J_3 = \frac{1}{(V_1)^3} \int_{V_1} \left\{ \prod_{i=1}^{3} d^3 x_i W_1(x_i) \right\} |x_1 - x_2|^{-\gamma} |x_1 - x_3|^{-\gamma} |x_2 - x_3|^{-\gamma}$$  

(14)

plays the same role as $J_2$ for $n = 3$. On a theoretical ground the hierarchical recurrence relation is expected to be dynamically generated by strongly non-linear gravitational clustering (e.g., Peebles 1980). In this regime, the closure of the BBGKY equations provides hierarchical
correlations with specified $S_n$ coefficients (Fry 1984a; Hamilton 1988). Also in the mildly non-linear regime, second-order perturbative approaches to the evolution of gravitational clustering generates a hierarchical sequence of correlations (e.g., Fry 1984b; Juskiewicz & Bouchet 1992), but with values of the coefficients $S_n$ which are in general different from those arising from non-linear gravitational clustering (Lahav et al.1993).

Suppose now to represent the continuous density field with a discrete point distribution, as it occurs for the galaxy distribution. In this case, precise relations exist to connect the moments of the continuous field, $\mu_n$, to those, $m_n$, for the counts of the discrete realization. Under the minimal assumptions of Poissonian sampling of the continuous field, the discrete nature of the point distribution is accounted for by the usual change of variable $\phi \rightarrow e^\phi - 1$ in the functional dependence of the generators $M(\phi)$ and $K(\phi)$ (e.g., Layzer 1956; White 1979). As a consequence, new terms appear when differentiating them, and eqs. (10) become

$$\kappa_2 = m_2 - 1/\hat{N} \quad \text{(variance)};$$
$$\kappa_3 = m_3 - 3m_2/\hat{N} - 1/\hat{N}^2 \quad \text{(skewness)};$$
$$\kappa_4 = m_4 - 6m_3/\hat{N} - 3m_2^2 + 11m_2/\hat{N} - 6/\hat{N}^3 \quad \text{(kurtosis)}$$

(15) (Peebles 1980). Note, however, that the Poissonian representation to account for discreteness effects does not always provide a good prescription (Coles & Frenk 1991; Borgani et al.1993). This is the case, for instance, when the average count within $V_R$ is much less than unity, so that no informations about the underlying continuous statistics can be recovered. A further example is represented by the distribution of density peaks, which is far from being a Poissonian sampling of the background continuous field.

In our analysis we take the window to be a cubic cell of size $R$. In this case, if we make a partition of the simulation box into $N_b$ cubic cells, the corresponding moments of counts reads

$$m_n = \frac{1}{N_b} \sum_{i=1}^{N_b} \frac{(N_i - \hat{N})^n}{\hat{N}^n},$$

(16)

where $\hat{N}$ is the average cell count. Other authors considered different windows. Davis & Peebles (1983) analyzed the variance for the count of CfA galaxies inside “sharp” spheres, while Saunders et al. (1991) evaluated the variance and the skewness for the QDOT IRAS sample for counts within Gaussian spheres. In the following we will compare our analysis to analogous results from QDOT galaxies. Therefore, we need a suitable prescription to identify the size of the cubic cell to be associated to that of the Gaussian sphere. As for the variance, its value evaluated inside a Gaussian sphere of radius $R_{gs}$ coincide with that evaluated in a cubic cell of side

$$R_{cc} = (J_{2,cc}/J_{2,gs})^{1/\gamma} R_{gs} \quad ; \quad \gamma = 1.8,$$

(17)

with obvious meaning of the indices, quite independently of $\gamma$. In general one expect a different relation for the scales connecting the skewness estimate within the two different windows, but we find numerically a relation very close to (17) within the MonteCarlo quadrature accuracy, again with a negligible dependence on $\gamma$,

$$R_{cc} = (J_{3,cc}/J_{3,gs})^{1/2\gamma} R_{gs} \quad ; \quad \gamma = 1.8,$$

(18)
Note that, although the two prescriptions to identify the window sizes to be associated to a given moment value do depend on the moment order itself, in the case of interest we find only a rather weak dependence.

We focus now on the results for a representative subset of the models, which are in fair agreement with the angular APM correlation function. The three fiducial models are F.14.3, 8.38.1, and 5.66.1 (see Table I for the parameters relevant for these models).

3.2 Results

In Figure 3 we plot the scale-dependence of the variance for the three bubble models considered, as compared with observational data. The error bars for our models are not reported here for clarity; they have been calculated as the ensemble scatter over ten different realizations, and are plotted on the following figures. Open circles are for our models, while filled circles are for the Stromlo-APM galaxies (Loveday et al.1992) and filled triangles for QDOT galaxies as analyzed by Efstathiou et al.(1990; hereinafter QDOT90). The agreement on large scales is fairly good, while a residual discrepancy is always detected at small scales. The reason for this could be that the variance for the APM galaxies has been evaluated in redshift space and then corrected to real space according to the relation

\[ \sigma^2_{\text{redsh. sp.}} = f(\Omega, b) \sigma^2_{\text{real sp.}} \]

holds at the scales of linear clustering and gives an equal amount of clustering amplification at all the scales (Kaiser 1987). In eq. (19), \( b \) is the linear biasing factor, which describe the difference between relative fluctuations of galaxies and underlying dark matter, \( \delta_{\text{gal}} = b \delta_{DM} \). Loveday et al.(1992) corrected their variance estimate by taking \( f(\Omega, b) = 1.4 \) at all the scales. However, this could underestimate the clustering at small scales, where, instead, virial motions are expected to suppress the redshift-space correlation (e.g., Fisher et al.1993). It is however not clear how large this effect should be in our case, since the bubble simulations does not provide galaxy peculiar velocities. In any case, we do not believe that this small-scale discrepancy represents a serious problem. A better agreement could be reached by allowing for a further 10% of unclustered component, which should decrease the \( \sigma^2 \) values, without significantly changing the angular correlation (see Figure 2). However, here we are not interested in finding the model which best fits the observational data, rather we are analysing the ability of bubble geometries to reproduce the overall features of the galaxy distribution.

In Figure 4 we report the relations of skewness \( \mu_3 \) and kurtosis \( \mu_4 \) versus the variance \( \sigma^2 \). The models are compared with the corresponding relations found in the analysis of Gaztañaga (1992) for CfA and SSRS galaxies (dashed lines) and of Bouchet, Davis & Strauss (1992) for the Strauss et al.(1990) IRAS 1.2Jy sample (dot-dashed lines, hereinafter IRAS 1.2). The scales analyzed in the two observed samples are limited to about 30 h\(^{-1}\)Mpc for Gaztañaga (1992) and to about 70 h\(^{-1}\)Mpc for IRAS 1.2. We also plot the skewness-variance relation found by Saunders et al.(1991) in the QDOT survey with Gaussian windows (hereinafter QDOT91). The solid lines are the weighted best fit to our model’s values, considering only the points for which \( \sigma^2 < 1 \). The fitting parameters are reported in Table I; considering also
the points with $\sigma^2 > 1$ would result in values still inside the errors. We immediately note that, although the shallower power-law model 5.66.1 matches the APM angular correlation function, it is unable to follow the hierarchical relation \( [13] \) in the fully linear regime.

Despite the apparently fair fit provided by the hierarchical relations, Figure 5 shows that we cannot exclude deviation from hierarchicity, due to the large error bars in both our model’s and observational data. It appears indeed that the plots of skewness and kurtosis as functions of the variance may conceal even substantial departures from hierarchical scaling, departures which, on the contrary, are best evidenced by plotting the coefficients $S_3 = \kappa_3/\sigma^4$ and $S_4 = \kappa_4/\sigma^6$ against the box scale. Again open circles are for our simulations while filled triangles are now for the QDOT91 analysis by Saunders et al. (1991), with errors propagated from those originally quoted. Dashed lines and dot-dashed lines denote the surprisingly small $2\sigma$ uncertainty level for the analysis of Gatzañaga (1992) and the $1\sigma$ level for IRAS 1.2, respectively. We confirm with greater evidence that a shallow spectrum of bubble radii does not provide hierarchical correlations at large scales, and is totally at variance with the result at largest scale probed in QDOT91. On the contrary, steep power-law models and fixed radius models show a relative independence of $S_3$ with the scale, at least within the statistical uncertainties, while the coefficient $S_4$ remains completely undetermined at scales above $30 h^{-1}$Mpc. This does not allow a further discrimination among the models.

Finally, in Figure 6 we display the results for ten simulations of the models 8.38.1 and F.14.3 in the skewness-variance plane. These are compared to the result from the QDOT91 analysis of Saunders et al. (1991) with the largest Gaussian sphere, $R_{gs} = 20 h^{-1}$Mpc. Adopting the relation $R_{cc} = 3.3 R_{gs}$ one gets $R_{cc} = 66 h^{-1}$Mpc, but since this scale identification depends on a model for $\xi(r)$ which, at these scales, is quite questionable, we plot the results for the ten simulations taking box sizes of 60 $h^{-1}$Mpc (the rightward points) and 70 $h^{-1}$Mpc (the leftward points). The satisfactory agreement once again stresses the ability of certain bubble models to account for the large scale clustering of galaxies. Notice, however, that the results in QDOT91 have not been corrected for the redshift distortion; applying a correction factor $f(\Omega, b) > 1$ would push down the variance by the same factor and the skewness by $f(\Omega, b)^2$. Then the best agreement with our models would require somewhat less power on large scales, to be obtained, for instance, by increasing the unclustered component.

4 Discussion and conclusions

In this paper we realized a detailed clustering analysis of simulations of bubbly galaxy distributions. Due to the purely geometrical character of our simulations, we do not include small-scale gravitational clustering. Rather, we concentrate on large ($> 5 h^{-1}$Mpc) scales, where the clustering should be in the linear regime or weakly non-linear regime, so that it can be modelled by the geometry of the bubble distribution. In turn, the bubbly geometry can be thought to be either the result of a primordial phase transition or of quasi-linear gravitational dynamics, acting at intermediate and large scales. The advantage of this kind of approach is that we are able to generate very large simulation boxes of $450 h^{-1}$Mpc aside, which allow for a careful clustering analysis at large scales.

As a first test, we generated artificial Lick maps, by projecting galaxies contained in two levels of replicated boxes (see CMPLMM and Moscardini et al.1993). The resulting Lick
simulations include galaxies up to a distance of $675h^{-1}{\text{Mpc}}$ and is limited to the range of galactic latitude $|b| \geq 45^\circ$. The possibility of reproducing the observational setup of this angular sample enables us to realize a comparison between the bubbly large-scale clustering and that of observational data sets. As a remarkable result, by comparing the resulting angular two-point correlation function, $w(\vartheta)$, with that of the APM sample (Maddox et al.1990), we find that its large-scale behaviour can be fairly reproduced by several models. Models with fixed radius, which are in any case less motivated, are found to have troubles. In fact, choosing $R_M = 12h^{-1}{\text{Mpc}}$, the resulting $w(\vartheta)$ amplitude agrees at $\vartheta \simeq 2^\circ$ with APM data, but fails at larger scales. Vice versa, for larger radii the agreement at $\vartheta \gtrsim 5^\circ$ is sensibly improved, but it turns into an excess of clustering at smaller angles. Taking variable bubble radii introduces a modulation of the correlation amplitude, which succeeds at reproducing the observed $w(\vartheta)$ at all the relevant scales. However, we are not able to discriminate at this level between steep and shallow spectrum of radii.

In order to further discriminate between different models, we also applied the statistics of moments of count in cells and compared the results with similar analyses on observational data sets (Saunders et al.1991, Bouchet, Davis & Strauss 1992, Gatzañaga 1992, Loveday et al.1992). As for the scale-dependence of the variance, we find that all the models, which satisfy the angular correlation test, are also able to account for the Stromlo-APM redshift data (Loveday et al.1992). This is not surprising, since both angular and spatial analyses of second order correlations are based on the same parent sample. More interesting results appear as we consider at higher correlation orders the scaling of skewness and kurtosis. In fact, the bubble geometry turns out to develop a rather well defined hierarchical scaling of higher order cumulants up to scales $\lesssim 70h^{-1}{\text{Mpc}}$, with the exception represented by the large-scale behaviour of the shallower spectra 4.78.1 and 5.66.1 (see Figures 4 and 5 and Table I). This result may appear rather surprising, since hierarchical scaling is expected to have a dynamical origin, both in the mildly and in the strongly non-linear regime of gravitational clustering, while no dynamics is present in our simulations.

It is however possible to show that hierarchical scaling is a necessary consequence for any non-Gaussian distribution (like that implied by a bubbly geometry) in the weak coherence regime. To show this, let us assume to be at the scale $r$ in a regime of weak clustering for the fluctuation field $\delta_r(x)$. Let us also consider the coarse-grained field $\delta_R(x)$, smoothed at the scale $R \gg r$, that can be considered as the linear superposition of many independent processes $\delta_r(x)$:

$$
\delta_R(x) = \frac{1}{p} \sum_{i=1}^{p} \delta_r(x_i) .
$$

The corresponding moment of order $n$ is

$$
\mu_n(R) = \langle \delta_R^n(x) \rangle = \frac{\langle (\sum_{i=1}^{p} \delta_r(x_i))^n \rangle}{p^n} .
$$

Since $\delta_r$ values are assumed to be nearly independent, the average of all the cross products in the multinomial expansion of eq.(21) becomes negligible, so that $\mu_n(R) = p^{-(n-1)} \mu_n(r)$. 

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Therefore the same relation also holds between the cumulants and, since \( \sigma^2(R) \equiv \mu_2(R) = \mu_2(r)/p \), it implies the hierarchical scaling

\[
\kappa_n(R) = S_n [\sigma(R)]^{2(n-1)}.
\]

(22)

with coefficients \( S_n = \kappa_n(r)/[\sigma(r)]^{2(n-1)} \) fixed by the statistics at the reference scale \( r \) and independent of \( R \).

It is clear that this argument to explain the presence of hierarchical scaling crucially depends on the assumption of weak coherence. In most cases, this simply means that the clustering is well in the linear regime, \( \sigma^2 \ll 1 \). This remarkably agrees with the results presented in Table I for models having different percentages of unclustered galaxies. In fact, as the unclustered component is increased, so to decrease the correlation amplitude, the reliability of the hierarchical scaling improves (see the values of \( \eta_3 \) and \( \eta_4 \) for the three models with \( p = 4.5 \)). However, if the field \( \delta(x) \) keeps coherent over large scales, e.g. due to the presence of intrinsic phase correlations, coherent structures can still be present even in the small variance regime and the hierarchical scaling does not show up. This is the case, for instance, when large-scale phase correlation is imprinted in the initial conditions or when dealing with primordial spectra providing a lot of large-scale power, which rapidly develop extended structures in the clustering pattern. This may explains, for instance, why in the analysis of N-body simulations by Bouchet & Hernquist (1992) it is found that, while the hierarchical relation is closely followed by the CDM model in the mildly non-linear regime, the evidence of hierarchical behaviour on large scales for the HDM model is much less clear. This is also the case for our 4.78.1 and 5.66.1 models, which do not develop hierarchical behaviours at large scales. In fact, taking a shallower spectrum of bubble radii allows the presence of very large bubbles, which generates a high degree of large scale coherence.

In the introduction of this paper we addressed two questions concerning whether clustering models based on bubble geometry are adequate to account for the observed large-scale texture of the Universe and whether the resulting clustering measures are sensitive to the choice of the model parameters. Based on the results of our analysis we can give an affirmative answer to both questions. It is however clear that our findings demand for a dynamical interpretation about the origin of a bubbly galaxy distribution. In particular one may ask whether they are imprinted as initial conditions in the primordial fluctuations or develop through gravitational evolution. In this respect, one could be tempted to suggest that, while bubble generation through primordial phase transitions should give rise to a narrow spectrum of radii (which is also more likely to pass the cosmic microwave background constraints), gravitational evolution of a rather flat fluctuation spectrum should produce nearly spherical voids of sizes comprised in a large range. It is however clear that, before rush to definite conclusions on this point one should wait, from the observational side, for more extended data sets, which were able to encompass the largest scale involved by coherent structures, and, from the theoretical side, for more rigorous dynamical descriptions like those provided by running large N-body simulations.

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References

Achilli, S., Occhionero, F., Scaramella, R., 1985, ApJ, 299, 577
Amendola, L., Occhionero, F., 1993, ApJ, in press
Babul, A., White, S.D.M., 1991, MNRAS, 253, 311
Bertschinger, E., Dekel, A., Faber, S.M., Dressler, A., Burstein, D., 1990, ApJ, 364, 370
Blumenthal, G.R., Faber, S.M., Primack, J.R., Rees, M.J., 1984, Nature, 311, 517
Borgani, S., Coles, P., Moscardini, L., Plionis, M. 1993, MNRAS, submitted
Bouchet, F., Davis, M., Strauss, M., 1992, in Proc. of the 2nd DAEC Meeting on the Distribution of Matter in the Universe, eds. G.A. Mamon, & D. Gerbal, p.287 (IRAS 1.2)
Bouchet, F. R. Hernquist, L., 1992, ApJ 25, 400
Bower, R.G., Coles P., Frenk C.S., White, S.D.M., 1993, ApJ, 405, 403
Cen, R., Gnedin, N.Y., Kofman, L.A., Ostriker, J.P., 1992, ApJ, 399, L11
Coles, P., Frenk, C.S., 1991, MNRAS, 253, 727
Coles, P., Moscardini, L., Plionis, M., Lucchin, F., Matarrese, S., Messina, A., 1993, MNRAS, 260, 572 (CMPLMM)
Collins, C.A., Heydon-Dumbleton, N.H., MacGillivary, H.T., 1989, MNRAS, 236, 711
Couchman, H., Carlberg, R.G., 1992, ApJ, 389, 453
Davis, M., Efstathiou, G., Frenk, C.S., White, S.D.M., 1992, Nature, 356, 489
Davis, M., Meiksin, A., Strauss, M., Nicolaia da Costa, L., Yahil, A., 1988, ApJ, 333, L9
Davis, M., Peebles, P.J.E., 1983, ApJ, 267, 465
Efstathiou, G., Ellis, R.S., Peterson, B.A., 1988, MNRAS, 232, 431
Efstathiou, G., Kaiser, N., Saunders, W., Lawrence, A., Rowan-Robinson, M., Ellis R. S., Frenk C. S., 1990, MNRAS, 247, 10p (QDOT90)
Efstathiou, G., Sutherland, W.J., Maddox, S.J., 1990, Nature, 348, 705
Feldman, H., Kaiser, N., Peacock, J., 1993, preprint
Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A., Huchra, J.P., 1993, ApJ, 402, 42
Fry, J.N., 1984a, ApJ, 277, L5
Fry, J.N., 1984b, ApJ, 279, 499
Gaztañaga, E., 1992, ApJ, 398, L17
Hamilton, A.J.S., 1988, ApJ, 332, 67
Ikeuchi, S., 1981, Publ. Astr. Soc. Japan, 33, 211
Juszkiewicz, R., Bouchet, F.R., 1992, in Proc. of the 2nd DAEC Meeting on the Distribution of Matter in the Universe, eds. G.A. Mamon, & D. Gerbal, p.301
Kaiser, N., 1987, MNRAS, 227, 1
Klypin, A., Holtzman, J., Primack, J., Regös, E., 1992, preprint SCIPP 92/52
Kolb, E.W., 1991, Phys. Scr., T36, 199
La, D., 1991, Phys. Lett., B265, 232
La, D. Steinhardt, P.J., 1989, Phys. Rev. Lett., 62, 376
Lahav, O., Itoh, M., Inagaki, S., Suto, Y., 1993, ApJ, 402, 387
Layzer, D., 1956, ApJ, 61, 383
Liddle, A., Lyth, D.H., 1993, Phys. Rep., in press
Liddle, A., Wands D., 1991, MNRAS, 253, 637
— 1992, preprint SUSSEX-AST-92/2-2
Loveday, J., Efstathiou, G., Peterson, B.A., Maddox, S.J., 1992, ApJ, 400, L43
Lucchin, F., Matarrese, S., 1985, Phys. Rev., D32, 1316
Maddox, S.J., Efstathiou, G., Sutherland, W.J., Loveday, J. 1990, MNRAS, 242, 43
Moscardini, L., Borgani, S., Coles, P., Lucchin, F., Matarrese, S., Messina, A., 1993, ApJ, submitted
Moscardini, L., Matarrese, S., Lucchin, F., Messina, A., 1991, MNRAS, 248, 424
Ostriker, J.P., Cowie, L.L., 1981, ApJ, 243, L127
Peacock, J.A., 1991, MNRAS, 253, 11
Peebles, P.J.E., 1980, The Large Scale Structure of the Universe. Princeton University Press, Princeton
Plionis, M., Barrow, J.D., Frenk, C., 1991, MNRAS, 249, 662
Saunders, W., Frenk, C., Rowan-Robinson, M., Efstathiou, G., Lawrence, A., Kaiser, N., Ellis, R., Crawford, J., Xia, X.-Y., Parry, I., 1991, Nature, 349, 32 (QDOT91)
Strauss, M.A., Davis, M., Yahil, A., Huchra, J.P., 1990, ApJ, 361, 49
Valdarnini, R., Bonometto, S.A., 1985, A&A, 146, 235
Van de Weygaert, R., Icke, V., 1989, A&A, 213, 1
Van de Weygaert, R., 1993, preprint
Vittorio, N., Juszkiewicz, R., Davis, M., 1986, Nature, 323, 132
Weinberg, D.H., Cole, S., 1991, MNRAS, 259, 652
Weinberg, D.H., Ostriker, J.P., Dekel, A., 1989, ApJ, 336, 9
White, S.D.M., 1979, MNRAS, 186, 145
White, S.D.M., Efstathiou, G., Frenk, C.S., 1993, preprint
White, S.D.M., Frenk, C.S., Davis, M., Efstathiou, G., 1987, ApJ, 313, 505
Wright, E.L., et al., 1993, preprint
Yoshioka, S., Ikeuchi, S., 1989, ApJ, 341, 16
Figure captions

Figure 1. Slices of the bubbly galaxy distribution for different models (see Table I for the parameters of each model). Each slice is $15h^{-1}\text{Mpc}$ thick and all the realizations correspond to the same choice of initial phase-assignment.

Figure 2. The angular correlation function, $w(\vartheta)$, for simulations of Lick map, obtained by projecting the three-dimensional bubble simulations. Open circles refer to the APM correlation, rescaled to the depth of the Lick map, as provided by Maddox et al. (1990). Filled symbols are for our simulations. They are plotted only for $\vartheta \geq 1.5^\circ$, which correspond to the physical scales of linear clustering at the depth of the Lick map.

Figure 3. The scale dependence of the variance for counts in cubic cells. Open circles are for the bubble simulations, which are obtained after averaging over ten realizations. Filled triangles are for the IRAS QDOT analysis by Efstathiou et al. (1990) and filled circles are for Stromlo-APM galaxies (Loveday et al. 1992). For reason of clarity, we do not plot errorbars for our data.

Figure 4. The variance-skewness (left panels) and the kurtosis-skewness (right panels) relations. Open circles refer to our simulations, with errorbars denoting the r.m.s. scatter over an ensemble of ten realizations. Heavy bars for the skewness are the QDOT data by Saunders et al. (1991). The dot-dashed lines are the best fits to the analysis of the IRAS 1.2Jy survey by Bouchet, Davis & Strauss (1992) and dashed lines refer to the analysis of CfA and SSRS samples by Gatzañaga (1992). The solid lines are the best fits to our data, according to the parameters reported in Table I.

Figure 5. The scale dependence of the $S_3$ (left panels) and $S_4$ (right panels) hierarchical coefficients of eq. (13). Open circles are for our models, while dot-dashed and dashed lines delimit the uncertainty bands in the IRAS 1.2Jy analysis by Bouchet, Davis & Strauss (1992) and Gatzañaga (1992), respectively. Filled triangles for $S_3$ refer to the Saunders et al. (1991) analysis of QDOT data.

Figure 6. The variance-skewness relation for the QDOT data at the largest considered scale, corresponding to the size of $66h^{-1}\text{Mpc}$ for a cubic box. Each open circle refers to the result for one realization of the corresponding bubble model. Leftmost circles are for a box-size of $70h^{-1}\text{Mpc}$, while rightmost ones are for the $60h^{-1}\text{Mpc}$ scale.
| model | p  | $R_M$ | $R_m$ | $f_u$ | $S_3$ ($\Delta S_3$) | $\eta_3$ ($\Delta \eta_3$) | $S_4$ ($\Delta S_4$) | $\eta_4$ ($\Delta \eta_4$) |
|-------|----|-------|-------|-------|----------------------|-------------------------|----------------------|-------------------------|
| 4.78.1 | 4.5 | 78    | 5.5   | 0.1   | 1.44(40)             | 2.47(46)                | 1.41(1.03)           | 3.26(2.62)              |
| 4.78.2 | 4.5 | 78    | 5.5   | 0.2   | 1.64(22)             | 2.32(32)                | 1.89(65)             | 3.05(1.21)              |
| 4.78.3 | 4.5 | 78    | 5.5   | 0.3   | 1.85(27)             | 2.22(25)                | 2.37(87)             | 3.00(1.00)              |
| 5.66.1 | 5   | 66    | 6.5   | 0.1   | 1.84(1.10)           | 2.42(70)                | 2.25(4.04)           | 3.21(2.72)              |
| 8.36.1 | 8   | 36    | 9.    | 0.1   | 1.77(55)             | 1.97(33)                | 3.43(3.29)           | 2.81(1.15)              |
| 8.38.1 | 8   | 38    | 9.5   | 0.1   | 1.77(42)             | 1.97(31)                | 3.41(2.14)           | 2.81(1.09)              |
| F.12.3 | –   | 12    | –     | 0.3   | 2.11(30)             | 1.85(19)                | 4.70(1.10)           | 2.53(51)                |
| F.14.3 | –   | 14    | –     | 0.3   | 2.20(60)             | 1.85(33)                | 5.23(3.56)           | 2.54(88)                |