Dzyaloshinskii-Moriya interaction as an agent to free the bound entangled states

Kapil K. Sharma* and S. N. Pandey†
Department of Physics, Motilal Nehru National Institute of Technology, Allahabad 211004, India.
E-mail: *scienceglobal@gmail.com, †snp@mnnit.ac.in
(Dated: January 7, 2015)

In the present article we investigate the efficacy of Dzyaloshinskii-Moriya (DM) interaction to convert the bound entangled states into free entangled states. Here we consider the tripartite hybrid system as a pair of non-interacting two qutrits initially prepared in bound entangled states and one auxiliary qubit. The auxiliary qubit interacts with any one of the qutrit of the pair through DM interaction. It has been found that the DM interaction free the bound entangled states as time advances. In the present work we consider two types of bound entangled states investigated by Horodecki. Further we find that the frequency of free entanglement conversion is same in both the states. We also investigate the phenomenon of entanglement and distillability sudden death and their possibilities. Here the realignment criteria and negativity have been used for detection and quantification of entanglement.

I. INTRODUCTION

Quantum entanglement [1, 2] is a physical phenomena which takes place when particles interact in microscopic world in such a way that the quantum state of one particle can be described in terms of each other. The entanglement phenomenon is expected to be a useful resource for future quantum technologies. Many applications, based on entanglement, have been investigated, like quantum teleportation [3], quantum imaging [4], quantum game theory [5], secure key quantum transmission [6], etc. To develop quantum technologies based on entanglement, the quantum community needs decoherence free quantum systems. Quantum systems are too evasive as they may lose their entanglement by external environmental interactions and can go for entanglement sudden death (ESD) for finite time [7, 8]. So dynamics of entanglement and its control under various environmental interactions conceptually underpinning the quantum information processing. Dynamics in various quantum spin chains have been studied under Dzyaloshinskii-Moriya (DM) interaction [10–12]. DM interaction is a useful resource in quantum information processing to entangle and disentangle the quantum systems. Recently Zang et al. studied the entanglement dynamics of two qubit pair by taking an third qubit or qutrit, which interact with a qubit of the pair through DM interaction [13–15]. They studied the dynamics of entanglement by taking a third qubit as a controller qubit. Further they studied the same by taking a third qutrit as a controller qutrit. They proposed that by manipulating the probability amplitude of third qubit or qutrit and DM interaction strength one can control the entanglement between two qubits induced by DM interaction and hence in various quantum spin chains [16, 17]. At this point we mention here that the same method may not only be used to control the entanglement, but it may also be used to free the bound entanglement [18, 21] in various bound entangled states. Once the bound entangled states are free then these are easily distillable [22].

The quantum states beyond the dimension $3 \otimes 3$ have been classified in two categories like free entangled states and bound entangled states. Free entangled states are distillable states or in other words noise free states. On the other hand bound entangled states are noisy states and no pure entanglement can be obtained by local operations and classical communication. It is difficult to use bound entangled states directly for practical quantum information processing. However, by providing additional resource, bound entanglement can be activated to increase the fidelity of quantum teleportation [24–26]. Up to now we don’t have satisfactory tools to quantify and detecting the bound entanglement. Recently the free entanglement production from bound entangled states is proposed by using the ancillary system which is coupled to the initial bound-entangled state via appropriate weak measurements [27].

In the present work, we consider qutrit-qutrit bound entangled bipartite states proposed by Horodecki [18, 21]. The main goal of the present work is to show that DM interaction can be a useful agent to convert the bound entangled state into free entangled state in two qutrit system. The motivation of this study comes from our recent work [28, 30].

The plan of the paper is as follows. In Sect. 2 we present the Hamiltonian of the system. Sect. 3. is devoted to the description of Horodecki’s bound entangled states and reduced system dynamics. In Sect. 4 we discuss the time evolution of negativity and realignment criteria. Lastly, in Sect 5 we report our conclusion.
II. HAMILTONIAN OF THE SYSTEM

We consider a qutrit (A)-qutrit (B) pair and an auxiliary qubit (C) which interact with any one of the qutrit of the pair through DM interaction. Here we assume that the auxiliary qubit (C) interact with the qutrit (B) of the pair. Now the Hamiltonian of the system can be written as

\[ H = H_{AB} + H_{BC}^{\text{int}}, \]

where \( H_{AB} \) is the Hamiltonian of qutrit (A) and qutrit (B) and \( H_{BC}^{\text{int}} \) is the interaction Hamiltonian of qutrit (B) and qubit (C). Here we consider uncoupled qutrit (A) and qutrit (B), so \( H_{AB} \) is zero. Now the Hamiltonian becomes

\[ H = H_{BC}^{\text{int}} = \mathbf{D} \cdot (\mathbf{\sigma}_B \times \mathbf{\sigma}_C), \]

where \( \mathbf{D} \) is DM interaction between qutrit (B) and qubit (C) and \( \mathbf{\sigma}_B, \mathbf{\sigma}_C \) are associated vectors of qutrit (B) and qubit (C) respectively. We assume that DM interaction exist along the z-direction only. In this case the Hamiltonian can be simplified as

\[ H = D \cdot (\sigma^X_B \otimes \sigma^Y_C - \sigma^Y_B \otimes \sigma^X_C), \]

where \( \sigma^X_B \) and \( \sigma^Y_B \) are Gell-Mann matrices for qutrit (B) and \( \sigma^X_C, \sigma^Y_C \) are X and Y Pauli matrices of qubit (C) respectively. The above Hamiltonian is a matrix having 6 \( \times \) 6 dimension. Further it is multiplied by the identity matrix of dimension 3 and we obtained the dimension as 18 \( \times \) 18. The matrix of Hamiltonian is easy to diagonalize by using the method of eigendecomposition. The unitary time evolution operator is easily commutable as

\[ U(t) = e^{-iHt}, \]

which is also a 18 \( \times \) 18 matrix. This matrix has been used to obtain the time evolution of density matrix of the system.

III. HORODIECKI’S BOUND ENTANGLLED STATES AND REDUCED SYSTEM DYNAMICS

In this section we describe two known bipartite Horodecki’s bound entangled states [18–21] in 3 \( \otimes \) 3 dimension. In subsection 3.1 we present the state 1 and in subsection 3.2 the state 2 is presented.

A. State 1.

The Horodecki’s bound entangled state [18, 19] in 3 \( \otimes \) 3 dimension is given by

\[ \rho_\alpha(0) = \frac{2}{I} P + \frac{\alpha}{21} Q + \frac{(5-\alpha)}{21} R, \quad 2 \leq \alpha \leq 5, \]

FIG. 1: Plot of entanglement and realignment criteria with \( D = 0.0 \)
FIG. 2: Variation of entanglement and realignment criteria vs. time with $D = 0.2$

where

$$P = |\psi\rangle\langle\psi|, \quad |\psi\rangle = \frac{1}{\sqrt{3}}(|00\rangle + |11\rangle + |22\rangle),$$  \hspace{1cm} (6)

$$Q = (|01\rangle\langle01| + |12\rangle\langle12| + |20\rangle\langle20|),$$  \hspace{1cm} (7)

$$R = (|10\rangle\langle10| + |21\rangle\langle21| + |02\rangle\langle02|).$$  \hspace{1cm} (8)

Horodecki demonstrated that

$$\rho_\alpha(0) = \begin{cases} 
\text{Separable for } & 2 \leq \alpha \leq 3, \\
\text{Bound entangled for } & 3 < \alpha \leq 4, \\
\text{Free entangled for } & 4 < \alpha \leq 5. 
\end{cases}$$  \hspace{1cm} (9)

B. State 2.

Another well known bound entangled state investigated by Horodecki \cite{20, 21} in $3 \otimes 3$ dimension is given as
Here $0 < a < 1$.

C. Reduced system dynamics

In this subsection we obtain reduce density matrix of qutrit-qutrit system. To begin with let us consider the auxiliary qubit (C) in pure state as

$$|\phi\rangle = c_0 |0\rangle + c_1 |1\rangle$$  \hspace{1cm} (11)

with normalization condition

$$c_0^2 + c_1^2 = 1.$$  \hspace{1cm} (12)

Qutrit (A)-qutrit (B) is prepared initially in bound entangled state. The density matrix of bound entangled state before interaction with qubit (C) is given in (5). Now we can obtain the composite density matrix of the open system,
**FIG. 4:** Variation of entanglement and realignment criteria vs. time with $D = 0.2$

\[ \rho_{\text{comp.}}(0) \] as

\[ \rho_{\text{comp.}}(0) = \rho_{\alpha}(0) \otimes \rho_{c}, \quad (13) \]

where $\rho_{c}$ is the density matrix of the auxiliary qubit (C). The density matrix after the interaction at time $t$ is given by

\[ \rho_{\text{comp.}}(t) = U(t)\rho_{\text{comp.}}(0)U^\dagger(t), \quad (14) \]

where $U(t)$ is unitary time evolution operator given in (4). Now we make the order same of all the matrices involved in equation (3.14) as $18 \times 18$. This can be done by doing the tensor product of $\rho_{\text{comp.}}(0)$ with identity matrix of the order $2 \times 2$. Now the order of the matrix $\rho_{\text{com.}}(t)$ becomes as $18 \times 18$. Taking partial trace of $\rho_{\text{com.}}(t)$ over the basis of auxiliary qubit (C), we get the reduced density matrix $\rho^{AB}$ of $9 \times 9$ dimension as,

\[ \rho^{AB} = \text{Ptr}_{c}[\rho_{\text{comp.}}(t)]. \quad (15) \]

Now we obtain the reduce density matrix for bound entangled state proposed by the Horodecki given by (5). The reduced density matrix is given below

\[
\rho^{AB} =
\begin{bmatrix}
X_{11} & 0 & X_{13} & 0 & X_{15} & 0 & X_{17} & 0 & X_{19} \\
0 & X_{22} & 0 & X_{24} & 0 & 0 & 0 & 0 & 0 \\
X_{31} & 0 & X_{33} & 0 & X_{35} & 0 & 0 & 0 & X_{39} \\
0 & X_{42} & 0 & X_{44} & 0 & 0 & 0 & 0 & 0 \\
X_{51} & 0 & X_{53} & 0 & X_{55} & 0 & X_{57} & 0 & X_{59} \\
0 & 0 & 0 & 0 & 0 & X_{66} & 0 & X_{68} & 0 \\
X_{71} & 0 & 0 & 0 & X_{75} & 0 & X_{77} & 0 & X_{79} \\
0 & 0 & 0 & 0 & 0 & X_{86} & 0 & X_{88} & 0 \\
X_{91} & 0 & X_{93} & 0 & X_{95} & 0 & X_{97} & 0 & X_{99}
\end{bmatrix},
\]
FIG. 5: Plot of entanglement and realignment criteria vs. time with $D = 0.0$

where

\[
X_{11} = X_{19} = X_{91} = X_{99} = \frac{2}{21}, \quad X_{13} = -X_{17} = X_{31} = -X_{39} = X_{71} = X_{79}
\]

\[
X_{15} = X_{51} = X_{59} = X_{95} = \frac{2}{21} q, \quad X_{22} = \frac{1}{21} \left[ \alpha c_0^2 - (\alpha - 5) \frac{p^2}{c_1^2} + \alpha c_1^2 q^2 \right],
\]

\[
X_{24} = X_{42} = -\frac{1}{21} \left[ (\alpha + (-5 + \alpha)q) p \right], \quad X_{33} = \frac{(7 - \alpha)}{42} - (\alpha - 3) \frac{s}{c_0 c_1} - 2(\alpha - 5)c_1^2,
\]

\[
X_{35} = X_{53} = (-3 + \alpha) r, \quad X_{44} = -\frac{1}{21} \left[ (\alpha - 5)c_0^2 + \frac{(5 - 2\alpha)s c_1}{c_0} + \frac{5}{42} \right],
\]

\[
X_{55} = \frac{1}{42} \left[ (\alpha + 2)c_0^2 - (\alpha - 7)c_1^2 \right] - s, \quad X_{57} = X_{75} = (-2 + \alpha) r,
\]

\[
X_{56} = \frac{5 c_0^2}{42} + \frac{(2\alpha - 5)s c_0}{c_1} + \frac{1}{21} \alpha c_1^2, \quad X_{65} = \frac{1}{21} \left[ -5 + \alpha + \alpha q \right] p,
\]

\[
X_{66} = \frac{5 c_0^2}{42} + \frac{(2\alpha - 5)s c_0}{c_1} + \frac{1}{21} \alpha c_1^2, \quad X_{68} = \frac{1}{21} \left[ 5 c_1^2 - ((-5 + \alpha)c_0^2 + \alpha c_1^2 q^2) \right]
\]

(16)

and $p = \sin[\sqrt{2}Dt]c_0 c_1$, $q = \cos[\sqrt{2}Dt]$, $r = \frac{1}{2} \sin[2\sqrt{2}Dt]c_0 c_1$, $s = \frac{1}{12} \cos[2\sqrt{2}Dt]c_0 c_1$. Observing the reduce density matrix we conclude that $c_0$, $D$ and $t$ are the key parameters which influence the entanglement between two qutrit pair. Similarly we obtain the reduced density matrix corresponding to another bound entangled state given by (3.10) with the help of (3.15). Further we obtain the partial transpose and realigned matrix of the reduced density matrices obtained for both the bound entangled states by using the relations.

\[
(\rho^T_{ij,kl}) = \rho_{il,kj} \quad (\rho^R_{ij,kl}) = \rho_{ik,jl}.
\]

(17)

(18)

These matrices have been used to calculate the quantities defined as,

\[
N_1 = \frac{||\rho^T|| - 1}{2}, \quad N_2 = \frac{||\rho^R|| - 1}{2},
\]

(19)

where $||..||$ is the trace norm of the matrix. The first quantity corresponds to negativity [31,35] and it has been used to measure the free entanglement while the second one has been used to detect the bound entanglement in the qutrit-qutrit system. Either $N_1 > 0$ or $N_2 > 0$ implies that the state is entangled. $N_1 = 0$ and $N_2 > 0$ implies that the state is bound entangled, and $N_1 > 0$ corresponds to free entangled.
IV. TIME EVOLUTION OF NEGATIVITY AND REALIGNMENT CRITERIA

In this section we study the evolution of free entanglement and realignment criteria obtained by using (3.19) for the bound entangled states 1 and 2. In case 1 we consider the bound entangled state described under the subsection 3.1 and in case 2 we do the same for another bound entangled state given in subsection 3.2. We represent the green color graphs for free entanglement $N_1$ and red color dotted graphs for realignment criteria $N_2$ in all the figures.

A. Case 1

In this case we consider the state 1 given in subsection 3.1. Here we replace $c_1$ in (3.19) in terms of $c_0$ by using the normalization condition given in (3.12). Now the resulting equation consists of the parameters $c_0$, $\alpha$, $D$ and $t$, which are required to obtain the evolution of free entanglement and realignment criteria in two qutrit system. First we plot the free entanglement and realignment criteria evolution in the absence of DM interaction i.e. ($D = 0$) in Fig. 1. The plots of free and bound entanglement satisfy the conditions given in (3.9).

Now we consider all the cases given in (3.9) with the parameter range $2 < \alpha \leq 5$. First we consider $2 \leq \alpha \leq 3$, the state is separable in this case. Next by varying the probability amplitude $c_0$ and DM interaction strength $D$ for different values of parameter $\alpha$ the results have been obtained. These results have been shown in Fig. 2. From the figure we observe that for $D = 0.2, c_0 = 0$ the state is still separable. Further as the value of probability amplitude $c_0$ increases, the entanglement has been produced in the state with a gap. This gap is called entanglement sudden death (ESD). Further we increase the value of $D$ from 0.2 to 0.4 and studied the quantities $N_1$ and $N_2$. The result plot is shown in Fig. 3. It is observed that as DM interaction increases, the frequency of entanglement production increases with ESD. While increasing value of probability amplitude ($c_0$) and the parameter $\alpha$ increases the amplitude of the free entanglement. When the auxiliary qubit reaches to maximally entangled state $c_0 = 1/\sqrt{2} = 0.707$, the maximum amplitude achieved in qutrit-qutrit system is 0.05. Next in Fig. 4 we plot the results for different values of probability amplitude $c_0$ with the parameter range $3 < \alpha \leq 4$. Here we fix the value of $D$ as 0.2. We recall that
and $N$ and $N$ and $N$ and $N$

$1$ $2$

$1$ $2$

$0.10$ $0.05$ $0.10$ $0.05$

$0.00$ $0.05$ $0.00$ $0.05$

$0.00$ $0.02$ $0.04$ $0.06$

$0.02$ $0.04$ $0.06$ $0.08$

$0.15$ $0.10$ $0.05$

FIG. 7: Variation of entanglement and realignment criteria vs. time with $D = 0.2$

the state is bound entangled with the parameter range $3 < \alpha \leq 4$. Observing all plots of Fig. 4 at $t = 0$ the realignment criteria $N_2$ achieve the positive values, so initially the state is bound entangled hence it can be made free. As time advances the DM interaction strength and probability amplitude $c_0$ convert the bound entangled state into free entangled state and again regain the nature of bound entangled state for a certain time of interval and the state goes under distillability sudden death (DSD). However the realignment criteria becomes negative as time advances and fail to detect the bound entanglement in the state, so there is the possibility of DSD. Adjusting the values of $c_0$, parameter $\alpha$ and DM interaction strength $D$, the bound entangled states can be converted to free entangled states, which is easily distillable. We repeat our study for $D = 0.4$ and conclude that increasing value of DM interaction strength increases the frequency of oscillations but not the amplitude of free entanglement. While increasing values of probability amplitude $c_0$ and parameter $\alpha$ increases the amplitude of the free entanglement. When the auxiliary qubit achieve the maximally entangled states with $(D = 0.2, c_0 = 0.707)$ corresponding to the parameter value $\alpha = 4.0$, we found that the free entanglement achieve the maximum amplitude $0.06$ and becomes zero at $t = 11$.

In Fig. 6 we plot the results for DM interaction strength $D = 0.2$ and increasing values of probability amplitude $c_0$ with the parameter range $4 < \alpha \leq 5$. Observing the figures we conclude that initially the state is free entangled but as the time advances the states convert into bound entangled state and goes under DSD. However the realignment criteria achieve the negative values with advancement of time. In this case the realignment criteria fail to detect the bound entanglement nature of the state and gives the possibility of DSD corresponding to the negative portion.
\begin{align*}
\{D=0.2, c_0=0.707\}
\end{align*}

FIG. 8: Comparision plot of entanglement in state 1 and 2 vs. time with \((D = 0.2, c_0 = 0.707)\).

B. Case 2

Under this subsection we explore our study of the bound entangled states given in subsection 3.2. In Fig. 5 we plot the result for DM interaction interaction strength \(D = 0\). The figure reveals that the state is bound entangled and carry no free entanglement. In Fig. 7, we plot the results for \(D = 0.2\) with increasing values of both the probability amplitude \(c_0\) and the parameter \(a\). Observing the figures we conclude that DM interaction produce the free entanglement and changes the nature of bound entangled state to free entangled state as time advances. However as the realignment criteria becomes negative with time and fail to detect the bound entanglement in the state. So there may be possibility of ESD or DSD as time evolves. When the auxiliary qubit achieve the maximally entangled state with \(D = 0.2, c_0 = 1/\sqrt{2} = 0.707\), the free entanglement achieve the maximum amplitude as 0.7 and becomes zero at \(t = 11\). We also compare the plots of both the states 1 and 2, when the auxiliary qubit achieve maximally entangled state i.e. \((c_0 = 1/\sqrt{2} = 0.707)\). These plots are corresponding to the parameter values \(D = 0.2, c_0 = 0.707\) and found that the amplitude of free entanglement in state 2 is greater by an amount 0.01, but frequency of free entanglement conversion is same in both the states 1 and 2. This result has shown in Fig. 8.

V. CONCLUSION

In this paper, we have studied the entanglement dynamics of two bipartite Horodecki’s bound entangled states for qutrit-qutrit system. We consider qutrit-qutrit system as a closed system and one auxiliary qubit, which interact with any one of the qutrit through DM interaction. For the Horodecki state 1 we have investigated all the cases. When the state 1 is separable with the parameter range \(2 < \alpha \leq 3\), the DM interaction produce the entanglement in the state with ESD. For the case of bound entanglement with the parameter range \(3 < \alpha \leq 4\), it has been found that DM interaction periodically makes free the bound entangled states as time advances. The state also goes under periodic DSD as time evolves. However the realignment criteria evolves with negative values during certain time intervals. In this case the criteria fail to detect the bound entangled nature of the state and the possibility of ESD or DSD have been investigated in the qutrit-qutrit system. We also studied the entanglement dynamics of Horodecki
state 2 and found that this state has also been converted into free entangled state through DM interaction with the possibility of ESD or DSD. The free entanglement conversion in both the states can be controlled by adjusting the DM interaction strength and probability amplitude of auxiliary qubit. We also compare the amplitude and frequency of free entanglement conversion, when the auxiliary qubit is in maximally entangled state i.e. \( c_0 = 1/\sqrt{2} = 0.707 \). It has been found that both the states have been convert in free entangled states with the same frequency and state 2 carry the free entanglement greater than the state 1 by an amount 0.01. Finally we conclude that the DM interaction can be useful agent to convert the bound entangled states into free entangled states, which are easily distillable. So the DM interaction can play an important role in this context to quantum information processing.

[1] Einstein, A., Podolsky, B., Rosen, N., Phys. Rev. 47 (1935), 777.
[2] Nielsen, M. A., Chuang, I. L., Quantum Computation and Quantum Information, Cambridge University Press, (2000).
[3] Bennett, C. H., Brassard, G., Crépeau, C., Jozsa, A., Peres, Wootters, W. K., Phys. Rev. Lett. 70 (1993), 1895.
[4] Lugiato, L., J. Opt. B: Quantum Semiclass. 4 (2002), 3.
[5] Meyer, D., Phys. Rev. Lett. 82 (1999), 1052.
[6] Ekert, A. K., Phys. Rev. Lett. 67 (1991), 661.
[7] Yu, T., Eberly, J. H., Phys. Rev. Lett. 93 (2004), 140404.
[8] Yu, T., Eberly, J. H., Science 30 (2009), 598.
[9] Affleck, I., Haldane, F.D.M., Phys. Rev. 36 (1987), 5291.
[10] Dzyaloshinsky, I., J. Phys. Chem. Solids 4 (1958), 241.
[11] Moriya, T., Phys. Rev. Lett. 4 (1960), 228.
[12] Moriya, T., Phys. Rev. Lett. 120 (1960), 91.
[13] Qiang, Z., Xiao-Ping, Z., Qi-Jun, Z., Zhong-Zhou, R., Chin. Phys. B 18 (2009), 3210.
[14] Qiang, Z., Ping, S., Xiao-Ping, Z., Zhong-Zhou, R., Chin. Phys. C 34 (2010), 1583.
[15] Qiang, Z., Qi-Jun, Z., Xiao-Ping, Z., Zhong-Zhou, R. Chin. Phys. C 35 (2011), 135.
[16] Affleck, I., Haldane, F. D. M., Phys. Rev. B 36 (1987), 5291.
[17] Albanese, C., J. Stat. Phys. 55 (1989), 297.
[18] Horodecki, M., Horodecki, P, Horodecki, R., G. Albert et al. (Ed.), Springer Tracts in Modern Physics 173, Springer Verlag Berlin, 2001, 151. (Also available as arXiv:0109124).
[19] Krammer, P., Diploma Thesis., Universitat Wien, 2005.
[20] Horodecki, P., Phys. Lett. A. 232 (1997), 333.
[21] Horodecki, M., Horodecki, P., Horodecki, R., Phys. Rev. Lett. 80 (1998), 5239.
[22] Horodecki, M., Horodecki, P., Horodecki, R., Quant. Inf. Comp.1 (2001), 45.
[23] Qian, G., Guan, X., Commun. Theor. Phys. 49 (2008), 343.
[24] Horodecki, P., Phys. Rev. Lett. 82 (1999), 1056.
[25] Diir, W., Cirac, J. I., Phys. Rev. A. 62 (2000), 022302.
[26] Kaneda, F., Shimizu, R., Ishizaka, S., Mitsumori, Y., Kosaka, H., Edamatsu, K., Phys. Rev. Lett. 109 (2012), 040501.
[27] Baghbanzadeh S., Rezakhani A. T., Phys. Rev. A 88, (2013) 062320.
[28] Sharma, K.K., Awasthi, S. K., Pandey, S. N., Quant. Inf. Pro. 12 (2013), 3437.
[29] Sharma, K.K., Pandey, S. N., Quant. Inf. Pro. 13 (2014), 2017.
[30] Sharma, K.K., Pandey, S. N., arXiv:1412.4833 [arXiv:1412.4838k1].
[31] Plenio, M. B., Virmani, S., Quant. Inf. Comp. 7 (2007), 1.
[32] Vidal, G., Werner, R. F., Phys. Rev. A 65 (2002), 032314.
[33] Horodecki, M., Horodecki, P., Horodecki, R., Phys. Lett. A 223 (1996), 1.
[34] Peres, A., Quantum theory: concepts and methods, Kluwer Academic Publishers, Netherlands, (1995).
[35] Schmidt, E., Math. Ann. 63 (1907), 433.
[36] Ekert, A., Knight, P. L., Am. J. Phys. 63 (1995), 415.