TESTING UNIVERSAL RELATIONS OF NEUTRON STARS WITH A NONLINEAR MATTER–GRAVITY COUPLING THEORY

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ABSTRACT

Due to our ignorance of the equation of state (EOS) beyond nuclear density, there is still no unique theoretical model for neutron stars (NSs). It is therefore surprising that universal EOS-independent relations connecting different physical quantities of NSs can exist. Lau et al. found that the frequency of the f-mode oscillation, the mass, and the moment of inertia are connected by universal relations. More recently, Yagi and Yunes discovered the I–Love–Q universal relations among the mass, the moment of inertia, the Love number, and the quadrupole moment. In this paper, we study these universal relations in the Eddington-inspired Born-Infeld (EiBI) gravity. This theory differs from general relativity (GR) significantly only at high densities due to the nonlinear coupling between matter and gravity. It thus provides us an ideal case to test how robust the universal relations of NSs are with respect to the change of the gravity theory. Due to the apparent EOS formulation of EiBI gravity developed recently by Delsate and Steinhoff, we are able to study the universal relations in EiBI gravity using the same techniques as those in GR. We find that the universal relations in EiBI gravity are essentially the same as those in GR. Our work shows that, within the currently viable coupling constant, there exists at least one modified gravity theory that is indistinguishable from GR in view of the unexpected universal relations.

Key words: dense matter – equation of state – stars: neutron

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1. INTRODUCTION

Neutron stars (NSs) are compact objects that have densities several times the normal nuclear density in their cores. Due to their internal high-density environment, NSs have long been regarded as the most natural cosmic laboratories for studying dense nuclear matter, which is not well understood and poorly constrained by experiments performed on Earth. It is hoped that inferring global properties of NSs from observations may allow one to select the correct equation of state (EOS) among many theoretical possibilities. However, achieving this goal in practice is non-trivial. The reason is that it is in general not possible to extract all global properties of a NS simultaneously from observations. It is thus of great interest to search for empirical relationships, which are in general EOS dependent (e.g., the mass–radius relation of NSs), connecting only a few physical quantities of a NS so that one might hope to put constraints on the EOS by measuring these quantities. On the other hand, from a fundamental physics point of view, it would even be more interesting if the relationships are insensitive to the NS EOS models, since one could then use the relations to test the underlying gravitational theory despite our ignorance of dense nuclear matter (Yagi & Yunes 2013a, 2013b).

In the past decade, several empirical relations connecting different physical parameters of NSs have been proposed. Bejger & Haensel (2002) and Lattimer & Schutz (2005) discovered relationships relating the scaled moment of inertia I/\(MR^2\) and compactness \(M/R\), where \(I\) is the moment of inertia, \(M\) is the mass, and \(R\) is the radius. Making use of the discovered relation, Lattimer & Schutz (2005) suggested that the moment of inertia of star A in the double pulsar system J0737–3039 (Burgay et al. 2003; Lyne et al. 2004) could be determined to about 10% accuracy.

On the other hand, pulsating NSs are expected to be promising sources of gravitational waves. It is expected that studying the gravitational wave signals emitted by oscillating NSs can yield useful information about the internal structure of the stars. Several universal behaviors of the quadrupolar f-mode have been established (Andersson & Kokkotas 1996, 1998; Benhar et al. 1999, 2004; Tsui & Leung 2005; Lau et al. 2010). In particular, Lau et al. (2010) found a pair of nearly EOS-independent relations to connect the frequency and damping rate of the f-mode to the global properties \(M\) and \(R\) of the stars (see Section 4). It has furthermore been shown (Lau et al. 2010) that the values of \(M\), \(R\), and \(f\) of a NS can be inferred accurately from the f-mode gravitational wave signals.

More recently, Urbanec et al. (2013) discovered a universal relation between \(QM/J^2\) and \(M/R\), where \(Q\) is the spin-induced quadrupole moment and \(J\) is the angular momentum. Yagi & Yunes (2013a, 2013b) discovered universal relations relating \(I, Q\), the tidal Love number \(\lambda_{\text{tid}}\), and the rotational Love number \(\lambda_{\text{rot}}\). These newly discovered I–Love–Q relations will be directly relevant to the understanding of the gravitational wave signals emitted during the late stages of NS–NS binary mergers (see Section 5). Finally, Bauböck et al. (2013) found universal relations among \(I, J, Q, M/R\), and the ellipticity of the stellar surface.

While Yagi & Yunes (2013a, 2013b) have also studied the I–Love relation in an alternative theory of gravity (see Section 5), most of the universal relations of NSs discussed above are based on the assumption that gravity is described by the theory of general relativity (GR). But how well do we understand gravity? So far, the most successful theory of gravity is GR and it has been well tested in weak-field situations (for a review, see Will 2006). However, whether gravity behaves as GR predicts in strong-field situations, such as NS–NS binary mergers, is still an open question. It is hoped that testing GR in the strong-field limit will soon become possible in the coming decade through gravitational wave observations by ground based detectors such as Advanced LIGO, Advanced VIRGO,
and KAGRA (Will 1993, 2006; Gair et al. 2013; Yagi 2013; Yunes & Siemens 2013; Mirshekari 2013). On the other hand, we also know that GR is not complete because of its prediction of singularities in the Big Bang and those inside black holes. While it is generally believed that quantum gravity is needed to resolve these problems, it is still interesting to search for alternative theories of gravity that could avoid the singularity problems within the classical level.

In recent years, a new theory of gravity called Eddington-inspired Born-Infeld (EiBI) gravity proposed by Bañados & Ferreira (2010) has been gaining attention (see also Deser & Gibbons 1998; Vollick 2004). EiBI gravity is appealing because it reduces to GR in vacuum and can avoid the Big Bang singularity (Bañados & Ferreira 2010). The deviation between EiBI gravity and GR becomes significant only at high densities. The implications of EiBI gravity in cosmological (Bañados & Ferreira 2010; Scargill et al. 2012; Avelino & Ferreira 2012; Escamilla-Rivera et al. 2012; Liu et al. 2012; Cho et al. 2012, 2013; Harko et al. 2013a; Bouhmadi-Lopez et al. 2013) and astrophysical (Pani et al. 2011, 2012; Pani & Sotiriou 2012; Sham et al. 2012, 2013; Harko et al. 2013b) contexts have been widely investigated. Unlike GR, where gravity couples to matter linearly in the sense that the Einstein tensor $G_{\mu\nu}$ is proportional to the stress-energy tensor $T_{\mu\nu}$ in the Einstein field equations, EiBI gravity introduces nonlinear coupling between matter and gravity (see Section 2). It is in fact the nonlinear matter–gravity coupling in this theory that is responsible for avoiding some of the singularities that plague GR (Delsate & Steinhoff 2012). However, it has also been shown recently that the same nonlinear coupling leads to some pathologies, such as surface singularities (Pani & Sotiriou 2012; Harko et al. 2013; Harko et al. 2013b) contexts have been widely investigated. Unlike GR, where gravity couples to matter linearly in the sense that the Einstein tensor $G_{\mu\nu}$ is proportional to the stress-energy tensor $T_{\mu\nu}$ in the Einstein field equations, EiBI gravity introduces nonlinear coupling between matter and gravity (see Section 2). It is in fact the nonlinear matter–gravity coupling in this theory that is responsible for avoiding some of the singularities that plague GR (Delsate & Steinhoff 2012).

While the works of Pani & Sotiriou (2012) and Sham et al. (2013) cast doubt on its viability, EiBI gravity certainly stands as an interesting example of a more general class of nonlinear matter–gravity coupling theories, due to its equivalence to GR (see Section 2). It is in fact the nonlinear matter–gravity coupling that is responsible for avoiding some of the singularities that plague GR (Delsate & Steinhoff 2012). However, it has also been shown recently that the same nonlinear coupling leads to some pathologies, such as surface singularities (Pani & Sotiriou 2012) and anomalies associated with phase transitions (Sham et al. 2013), for compact stars in EiBI gravity.

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**2. EDDINGTON-INSPIRED BORN-INFELD GRAVITY**

The EiBI theory is based on a Palatini formulation of the action (Bañados & Ferreira 2010)

$$S = \frac{1}{16\pi} \int d^4 x \left( \sqrt{|g_{\mu\nu} + \kappa R_{\mu\nu}| - \lambda \sqrt{-g}} \right) + S_M[g, \Psi_M],$$

(1)

where $R_{\mu\nu}$ is the symmetric part of the Ricci tensor constructed solely by the connection $\Gamma^\alpha_{\beta\gamma}$, $S_M[g, \Psi_M]$ is the matter action, and $|f_{\mu\nu}| \equiv f$ denotes the determinant of a tensor field $f_{\mu\nu}$. The parameters $\kappa$ and $\lambda$ are related to the cosmological constant $\Lambda$ by $\Lambda = (\lambda - 1)/\kappa$. In the limit $\kappa \to 0$, it can be shown that the action (Equation (1)) reduces to the Einstein–Hilbert action for GR. We shall set $\lambda = 1$ hereafter and consider $\kappa$ as the only parameter of the theory. The current tightest constraint on $\kappa$ (Avelino 2012) is set by the existence of NSs and is given by $8\pi \kappa \epsilon_0 < 0.1$, where $\epsilon_0 = 10^{15}$ g cm$^{-3}$.

Varying the action (Equation (1)) with respect to the metric $g_{\mu\nu}$ and $\Gamma^\alpha_{\beta\gamma}$ separately yields

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu},$$

(2)

$$\sqrt{-q} q^{\mu\nu} = \sqrt{-g} g^{\mu\nu} - 8\pi \kappa \sqrt{-g} T^{\mu\nu},$$

(3)

where $q_{\mu\nu}$ is an auxiliary metric compatible with the connection

$$\Gamma^\alpha_{\beta\gamma} = \frac{1}{2} q^\alpha_{\delta\sigma} (\partial_\mu q_{\delta\beta} + \partial_\sigma q_{\mu\gamma} - \partial_\gamma q_{\delta\beta}).$$

(4)

Since the matter action $S_M$ is assumed to depend only on the metric $g_{\mu\nu}$ and the matter fields, but not on the connection $\Gamma^\alpha_{\beta\gamma}$, the stress-energy tensor $T^{\mu\nu}$ still satisfies the same conservation equations as in GR:

$$\nabla_\mu T^{\mu\nu} = 0,$$

(5)

where the covariant derivative refers to the physical metric $g_{\mu\nu}$. It should be noted that $q_{\mu\nu}$ is identical to the physical metric $g_{\mu\nu}$ when $T^{\mu\nu} = 0$. In fact, it can be shown that the action (Equation (1)) reduces to the Einstein–Hilbert action when the matter action $S_M$ vanishes (Bañados & Ferreira 2010). Hence, EiBI gravity is identical to GR in vacuum.

**3. APPARENT EOS FORMULATION OF EIBI GRAVITY**

Solving the field equations (Equations (2) and (3)) can be more difficult than in GR. The reason is that the auxiliary metric $q_{\mu\nu}$ involves both $g_{\mu\nu}$ and $T^{\mu\nu}$ and thus the Ricci tensor $R_{\mu\nu}$ contains second derivatives of the matter field. However, Delsate & Steinhoff (2012) have recently shown that the field equations (Equations (2) and (3)) can be written in a form that resembles the Einstein equations in GR:

$$G_\nu^\mu \equiv R_\nu^\mu - \frac{1}{2} \delta_\nu^\mu R = 8\pi \tau T^{\nu}_{\mu} - \frac{1 - \tau}{\kappa} \delta_\nu^\mu - 4\pi \tau T \delta_\nu^\mu \equiv 8\pi \tilde{T}_\nu^\mu,$$

(6)

1 Indeed, the dependence of $R_{\mu\nu}$ on derivatives of the matter field is the cause of the pathologies associated with compact stars in EiBI gravity found by Pani & Sotiriou (2012) and Sham et al. (2013).
where $R = R^\mu_{\nu}$ and $T = T^\mu_{\nu}$. The Einstein tensor $G^\mu_{\nu}$ is defined by the auxiliary metric $q_{\mu\nu}$ and $\tilde{T}^\mu_{\nu}$ is defined as the apparent stress-energy tensor. The scalar $\tau$ is defined by $\tau = \sqrt{g/\bar{q}}$ and can be expressed as

$$\tau = \left[ \det(\delta^\mu_{\nu} - 8\pi \kappa T^\mu_{\nu}) \right]^{-1/2}. \quad (7)$$

Note that Equation (6) in general still depends on the physical metric $g_{\mu\nu}$ through the standard energy tensor $T^\mu_{\nu}$. However, for the case of a perfect fluid,

$$T^\mu_{\nu} = (\epsilon + P) u^\mu u^\nu + P g^\mu_{\nu}, \quad (8)$$

where $\epsilon$ is the energy density, $P$ is the pressure, and $u^\mu$ is the four velocity of the fluid, Equation (6) can be made to decouple from $g_{\mu\nu}$ completely. In this special case, $\tau$ can be computed in a frame comoving with the fluid and is given by

$$\tau = \left[ (1 + 8\pi \kappa\epsilon)(1 - 8\pi \kappa P) \right]^{-1/2}. \quad (9)$$

The apparent stress-energy tensor can be written as

$$\tilde{T}^\mu_{\nu} = (\tilde{\epsilon} + \tilde{P}) u^\mu v^\nu + \tilde{P} \delta^\mu_{\nu}, \quad (10)$$

with the apparent energy density $\tilde{\epsilon}$ and pressure $\tilde{P}$ defined by

$$\tilde{\epsilon} = \tau \epsilon - \tilde{P}, \quad \tilde{P} = \tau P + \tilde{P}, \quad (11)$$

where $\tilde{P} \equiv (\tau - 1)/8\pi \kappa \tau - \tau (3P - \epsilon)/2$. The apparent four velocity $v^\mu$ satisfies the conditions

$$v^\mu v^\nu q_{\mu\nu} = -1; \quad v^\mu u^\nu = u^\mu u^\nu. \quad (13)$$

Note that the indices of $v^\mu$ and $u^\mu$ are lowered by $q_{\mu\nu}$ and $g_{\mu\nu}$, respectively.

The advantage of reformulating EiBI gravity in the form of Equation (6) has now become clear. For a given coupling parameter $\kappa$ and a given physical EOS $P(\epsilon)$, one can solve a problem in EiBI gravity by solving the same Einstein equations (Equation (6)) as in GR, but with $\tilde{q}_{\mu\nu}$ as the fundamental metric and an apparent EOS $\tilde{P}(\tilde{\epsilon})$ given by Equations (11) and (12). This implies that many theoretical ideas and numerical codes developed in GR can readily be transferred to EiBI gravity. For instance, instead of solving the field equations (Equations (2) and (3)) for the structure of compact stars, as was originally done (Pani et al. 2011, 2012; Sham et al. 2012, 2013; Haro et al. 2013b), one can simply solve the well-known Tolman–Oppenheimer–Volkov (TOV) equations in GR with an apparent EOS.

The advantage of this reformulation of EiBI gravity becomes even more transparent when one considers dynamical problems in EiBI gravity, where the algebraic complexity in the analysis grows quite rapidly with the problem size. For example, it is non-trivial to calculate non-radial oscillation modes of NSs even in GR. Formulating the same calculation in EiBI gravity starting with the field equations (Equations (2) and (3)) would be a more tedious task than in GR. However, due to the apparent EOS approach, we are able to make use of the numerical codes we previously developed in GR (Lau et al. 2010) to study the $f$-mode of NSs in EiBI gravity. As a side remark, we have indeed verified that the frequencies of the radial oscillation modes of NSs in EiBI gravity we obtained previously (Sham et al. 2012), by solving Equations (2) and (3), agree with the results (within numerical accuracy) obtained by solving the corresponding eigenvalue equation in GR (see, e.g., Kokkotas &Ruoff 2001) with apparent EOSs.

4. $F$-MODE UNIVERSALITY

The oscillation modes of NSs are damped by the emission of gravitational waves. They are in general called quasi-normal modes and each mode has a complex eigenfrequency $\omega = \omega_0 + i\omega_i$. The imaginary part $\omega_i$ corresponds to the damping rate of the oscillation mode. As discussed in Section 1, various attempts have been made to find relationships relating $\omega_0$ and $\omega_i$ of the $f$-mode to the global parameters of the star. Most of these works (Andersson & Kokkotas 1996, 1998; Benhar et al. 1999, 2004; Tsui & Leung 2005) used $M$ and $R$ as the parameters in the analysis. However, Lau et al. (2010) have recently established two EOS-independent relations using the parameters $M$ and $I$. The physical motivation for replacing $R$ by $I$ in the study of Lau et al. (2010) is that $R$ is sensitive to the low-density part of the EOS, while $I$ measures the mass distribution of the star globally. As the dynamics of $f$-mode oscillations are significantly affected by the mass distribution, it is thus expected that $I$ should relate more directly to the $f$-mode.

The universal relations found by Lau et al. (2010) are given by

$$M\omega_i = -0.0047 + 0.133\eta + 0.575\eta^2, \quad (14)$$

$$\tilde{P}^2 \omega_i / M^3 = 0.00694 - 0.0256\eta^2, \quad (15)$$

where the dimensionless factor $\eta \equiv \sqrt{M^3/I}$. Equations (14) and (15) are much improved universal relations in the sense that these relations are less sensitive to the EOS, compared with previous universal relations that use $R$ as a parameter. We refer the reader to Table 1 in Lau et al. (2010) for the accuracy of Equations (14) and (15).

Our aim in this section is to study whether the universal relations (Equations (14) and (15)) found for $f$-mode oscillations in GR remain valid in EiBI gravity. To this end, we calculate the $f$-mode frequency for NSs in EiBI gravity using the apparent EOS approach, as discussed in Section 3. The procedure is to (1) construct an apparent EOS $\tilde{P}(\tilde{\epsilon})$ for a given coupling parameter $\kappa$ and physical EOS $P(\epsilon)$, (2) construct an equilibrium background stellar model using the TOV equations in GR with the apparent EOS, and (3) calculate the $f$-mode frequency for the perturbed background model using the numerical codes that we previously developed in GR (Lau et al. 2010), but with $q_{\mu\nu}$ as the fundamental metric. The method for calculating oscillation modes of NSs in GR is well established and documented (for a review, see Kokkotas & Schmidt 1999). We refer the reader in particular to Lindblom & Detweiler (1983) and Detweiler & Lindblom (1985) for the formulation we used in the calculations.

In Lau et al. (2010), nine ordinary nuclear-matter and two quark-matter EOS models were considered in establishing the universal relations (Equations (14) and (15)). In this work, we consider four nuclear-matter EOS models: APR (Akmal et al. 1998), model FPS (Lorenz et al. 1993), model SLY4 (Douchin & Haensel 2000), and model WS (Lorenz et al. 1993; Wiringa et al. 1988). Among these four models, the model APR was also used in Lau et al. (2010). For the coupling parameter $\kappa$ in EiBI gravity, we consider three different values defined by $8\pi \kappa \epsilon_0 = -0.1, 0, 0.1$, where $\epsilon_0 = 10^{15}$ g cm$^{-3}$. These values are consistent with the constraint set by the existence of NSs (Avelino 2012). In particular, $\kappa = 0$ corresponds to the GR limit.

Note that we have corrected a typographical error in Equation (6) of Lau et al. (2010).
when enough that the resulting NSs in EiBI gravity differ significantly.

In order to show that the range of the coupling parameter \( \eta \) defined in the text. Similar to Figure 1, the values of \( 8\pi \kappa \epsilon_0 \) are shown in parentheses. The solid line is the fitting curve from Equation (14). The lower panel shows the relative fractional difference between the numerical results and the fitting curve.

(A color version of this figure is available in the online journal.)

Figure 3 plots the real part of the scaled \( f \)-mode frequency \( M_{0\delta} \) for our chosen EOS models (APR, FPS, SLy4, and WS) plotted against the effective compactness \( \eta \) defined in the text. Similar to Figure 1, the values of \( 8\pi \kappa \epsilon_0 \) are shown in parentheses. The solid line is the fitting curve from Equation (14). The lower panel shows the relative fractional difference between the numerical results and the fitting curve.

(A color version of this figure is available in the online journal.)

In order to show that the range of \( \kappa \) we consider is already large enough that the resulting NSs in EiBI gravity differ significantly from those in GR, we plot the gravitational mass \( M \) as a function of the central density \( \epsilon_c \) for the APR EOS in Figure 1. It can be seen from the figure that \( M \) can change by as much as \( \sim 30\% \) when \( 8\pi \kappa \epsilon_0 \) increases from \(-0.1\) to \(0.1\).

Figure 2 plots the real part of the scaled \( f \)-mode frequency \( M_{0\delta} \) against \( \eta \) for our chosen EOS models with different values of \( 8\pi \kappa \epsilon_0 \). It can be seen clearly that the data display universal relations that are essentially independent of the EOS models and the value of \( \kappa \), as long as \( \kappa \) is within the range constrained by the existence of NSs, as discussed above. We see that the data can be fit well by Equation (14). The relative fractional difference between the numerical results and Equation (14) is shown in the lower panel of the figure. Similarly, we plot \( \omega_i I^2 / M^5 \) against \( \eta^2 \) in Figure 3 and see that the data can be modeled well by

\[ \omega_i I^2 / M^5 = g_0 \eta^2 / \eta_{\text{I}^2} \]

Equation (15). We refer the reader to Lau et al. (2010) for the motivation of this way of plotting \( \omega_i \). In summary, we find that the universal relations (Equations (14) and (15)) for the \( f \)-mode of NSs in GR still hold for EiBI gravity as long as the coupling parameter \( \kappa \) is in the range \( |8\pi \kappa \epsilon_0| \lesssim 0.1 \).

5. I–LOVE–Q RELATIONS

After discussing the universality of \( f \)-mode oscillations, we now turn to the universal I–Love–Q relations discovered more recently by Yagi & Yunes (2013a, 2013b). The moment of inertia of a star is defined by \( I \equiv J / \Omega \), where \( J \) and \( \Omega \) are the angular momentum and angular velocity of the star, respectively. Physically, \( I \) determines how fast a star can spin for a given angular momentum. It thus seems quite natural that \( I \) should somehow relate to the spin-induced quadrupole moment \( Q \) of the star, since \( Q \) characterizes the deformation of the star due to self rotation. However, it is surprising that the relation between \( I \) and \( Q \) found by Yagi & Yunes (2013a, 2013b) is EOS-independent. On the other hand, the tidal Love number \( \lambda_{\text{tid}} \) measures the deformation of a NS due to the presence of a companion and is defined by \( Q_{ij} \equiv -\lambda_{\text{tid}} \tilde{\epsilon}_{ij} \), where \( Q_{ij} \) is the traceless quadrupole moment tensor of the star and \( \tilde{\epsilon}_{ij} \) is the tidal tensor that induces the deformation (see, e.g., Flanagan & Hinderer 2008). In general, there is no reason why there should exist EOS-independent universal relations relating the three quantities \( I \), \( Q \), and \( \lambda_{\text{tid}} \). More specifically, the universal relations concern the dimensionless quantities \( \tilde{I} \equiv I / M^3 \), \( \tilde{Q} \equiv -Q / (M^3 \chi^2) \) (with \( \chi \equiv J / M^2 \) being the dimensionless spin parameter), and \( \tilde{\lambda}_{\text{tid}} \equiv \lambda_{\text{tid}} / M^3 \).

The relevance of the I–Love–Q relations to astrophysics, gravitational wave, and fundamental physics has been discussed (Yagi & Yunes 2013a, 2013b). For instance, it was proposed that, for a detected gravitational wave signal emitted by an inspiralling NS binary, the relations could break the degeneracy between the NS quadrupole moment and the NS’s individual spins. It has also been shown by Maselli et al. (2013) that while the I–Love relation connecting \( I \) and \( \lambda_{\text{tid}} \) depends on the inspiral frequency during the inspiral, it nevertheless remains EOS independent. More recently, Haskell et al. (2013) showed that the I–Q universal relation fails for magnetized NSs with...
As in Section 4, we consider three different values \( \psi \) respectively, for our chosen EOS models and values of \( \psi \). Figure 4 shows the \( \ln \eta \) against \( \ln \lambda_{\text{tid}} \) for different EOS models and coupling constant \( \kappa \). The solid line is the fitting curve (Equation (16)) proposed by Yagi & Yunes (2013a, 2013b). Furthermore, as in the case of the \( f \)-mode universal relations, we find that the I–Love–Q relations are insensitive to the coupling parameter \( \kappa \) as long as it is in the range \([8\pi\kappa\epsilon_0] \lesssim 0.1\). We also note that our numerical results can be fit well by the following relation (the solid line in each figure), as suggested by Yagi & Yunes (2013a, 2013b):

\[
\ln y_i = a_i + b_i \ln x_i + c_i (\ln x_i)^2 + d_i (\ln x_i)^3 + e_i (\ln x_i)^4, \tag{16}
\]

where \( a_i, b_i, c_i, d_i, \) and \( e_i \) are some fitting coefficients. The relative fractional difference between our numerical results and Equation (16) is shown in the lower panel of each figure.

Yagi & Yunes (2013a, 2013b) also studied the universal relations in a modified theory called dynamical Chern–Simons (CS) gravity. They found that there also exists a universal I–Love relation connecting \( \eta \) and \( \lambda_{\text{tid}} \), although the relation is different from the corresponding I–Love relation in GR. They suggested that if one can measure \( \eta \) of the double-binary pulsar J0737–3039 to 10% accuracy and \( \lambda_{\text{tid}} \) to 60% accuracy with future gravitational wave observations, the CS theory can then be constrained much better than current tests by six orders of magnitude. For comparison, the universal I–Love–Q relations in EiBI theory are the same as the GR ones for the range of the coupling parameter \( \kappa \), which has already been constrained astrophysically. Hence, contrary to CS theory, EiBI gravity is an example of a modified theory where the I–Love–Q relations are degenerate with the corresponding GR relations and cannot be used to put a stronger constraint on the theory than that obtained by the current astrophysical one.

### 6. CONCLUSIONS

In this paper, we have studied the EOS-independent universal \( f \)-mode (Lau et al. 2010) and I–Love–Q (Yagi & Yunes 2013a, 2013b) relations for NSs in EiBI gravity. With the coupling parameter of the theory in the range \([8\pi\kappa\epsilon_0] \lesssim 0.1\), which is constrained by the existence of NSs (Avelino 2012), we find that the universal relations discovered in GR remain valid in EiBI gravity.

Naively, since EiBI gravity reduces to GR when the coupling constant \( \kappa \) vanishes, one might worry that the agreement...
between the universal relations in EiBI gravity and those in GR is simply because the values of $\kappa$ we consider are so small that the effect of nonlinear matter–gravity coupling in EiBI gravity is not apparent. However, this is not the case, as we have seen in the effect of nonlinear matter–gravity coupling in EiBI gravity (Yunes 2013a, 2013b), although the relation is different from the independent I–Love relation in dynamical CS gravity (Yagi & Yunes 2013a, 2013b), the theory would have one from some pathologies associated with compact stars (Pani & Sotiriou 2012; Sham et al. 2013), the theory would have one for our chosen EOS models. It will also be interesting to study whether the two apparently different sets of $f$-mode and I–Love–Q universal relations have a common origin or not. We hope to return to these issues in the future.

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