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Abstract: The present work deals with robustness and comparative analysis of grey wolf optimization (GWO)/proportional-integral-derivative (PID) approach in control of a ball and hoop (BH) system with different objective functions. The BH system is a device consisting of a ball rolling on the rim of a hoop and can be used to analyse the dynamics of liquid slosh problems. The commonly used objective functions are integral absolute error (IAE), integral square error (ISE), integral of square time multiplied by absolute error (ISTAE) and integral of time multiplied absolute error (ITAE). These objective functions have been minimized with the GWO algorithm to obtain parameters of the PID controller for the control of BH system. The robustness analysis of the proposed GWO/PID approach has also been carried out with ±5% perturbation in locations of the poles. The poles of BH system are perturbed by ±5% of the nominal value and the same PID controller tuned by the GWO algorithm has been applied on the perturbed systems. The comparative analysis of GWO/PID approach is demonstrated with several other existing techniques in terms of the transient response's parameters. The results of the GWO/PID approach outperform certain of the existing techniques in the literature, and the performance of the controller hardly alters with perturbation and objective functions; after it is tuned by GWO.

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PUBLIC INTEREST STATEMENT
The control of Ball and Hoop (BH) system is a challenging bench mark problem for control engineers. The BH system is important because it can be used to study the control of the oscillations of a liquid called “slosh” in a container, it can be dangerous when the container is moving and undergoing changes in velocity and direction. Some critical practical problems of liquid slosh are: oscillations of liquid fuel of missile, movement of liquid cargo in a road tanker, etc. The present work deals with control of BH system with the help of a GWO/PID controller. It is expected that outcome will be useful to the researchers around the world working in the area of optimization techniques, control, etc., as the results coming out are quite encouraging.
1. Introduction

Because of their simplicity, effectiveness and robustness, proportional-integral-derivative (PID) controllers are the most widely used controllers in process industries. Variations in process parameters and perturbations make the system unstable. In recent years, because of the increase in the complexity of processes in plants, certain challenges arise for tuning the parameters of a PID controller to achieve stability and good transient performance. Primarily, in all type of industries, PID tuning is performed manually, such manual tuning is a time consuming process. Zeigler–Nichols tuning (Ziegler & Nichols, 1942) is a widely used method over the past several years; however, this method requires prior knowledge of the plant model. Auto tuning procedures (Cominos & Munro, 2002; Zhuang & Atherton, 1993) are important for making the system stable. According to the literature, classical methods of tuning PID controllers, such as: the Manual tuning, Ziegler Nichol, Z–N Step Response and Cohen-coon methods are not capable of tuning complex higher order processes for achieving optimal performance.

Currently, optimal tuning of PID parameters represents a challenging task for control engineers (Ang, Chong, & Li, 2005; Astrom & Hagglund, 2001). Numerous soft computing techniques (Nagraj, Subha, & Rampriya, 2008) are available for optimal tuning of PID controller. Genetic algorithm (GA) is also one of the methods for solving control engineering optimization problems (Kumar & Goldberg, 1992; Maha, 2016; Wang, Spronck, & Tracht, 2003); however, it has slow convergence to find the global optimum in complex problems and it often converges towards local optima, rather than the global optimum solution.

Over the past two decades, meta-heuristic techniques (Mirjalili, Mohammad, & Lewis, 2014; Pareek, Kishnani, & Gupta, 2014) for optimization have become highly popular among researchers. Most of these techniques are inspired by physical phenomena, as well as the, behaviours of birds and other animals'. The main advantages of these algorithms are simplicity, flexibility, random search and avoidance of local optima compared to conventional optimization techniques. Some popular meta-heuristic techniques are Particle Swarm Optimization (PSO), Ant Colony Optimization (ACO), Artificial Bee Colony Optimization (ABC), Differential Evolution (DE), Grey Wolf Optimization (GWO), Simulated Annealing (SA) and Tabu Search (TS) (Roeva & Slavov, 2014; Rout, Sahu, & Panda, 2013). Kumar, Kumar, and Anantharaman (2010) proposed the particle swarm optimization (PSO) algorithm for tuning a PID controller for real-time industrial processes. PSO is a population based stochastic optimization algorithm; inspired by birds, bees and fishes. The limitation of PSO is premature convergence when solving complex problems. The artificial bee colony (ABC) algorithm (Karaboga & Akgay, 2009) is based on the intelligent foraging behavior of honey bees. Similar to other intelligent algorithms, ABC also suffers from premature convergence. Jain, Parmar, and Gupta (2016) applied the PSO, ABC and Bacterial Foraging Optimization (BFO) algorithm to tune a PID controller for a BH system.

The ball and hoop (BH) system (Sreekanth & Hari, 2016) is a simple electro-mechanical device that consists of a ball rolling on the rim of a hoop. A BH system is easy to construct and because of its good dynamics, it is preferred by control engineers for investigations. The open loop problems of a BH system are large rise, settling and peak times. The BH system is analogous to the liquid “slosh” problem. Several examples of liquid slosh problems are:

- The movement of liquid cargo in a road tanker, as it changes direction can alter the handling and stability of the truck.
- The liquid cargo of a ship will slosh when the ship is in heavy seas; this may reduce the stability of ship.
- The liquid fuel of a missile can oscillate when it makes a rapid change.
All of these examples are complex problems and require small rise, settling and peak times that can be obtained with tuning of the PID controller using meta-heuristic algorithms.

In recent years, many researchers have applied different meta-heuristic algorithms for the control of BH system. Mojallali, Gholipour, Khosravi, and Babae (2012) applied Chaotic Particle Swarm Optimization (CPSO) to tune the parameters of PID controller for BH system. Morkos and Kamal (2012) presented PSO and Adaptive Hybrid PSO (AHPSO) algorithms for tuning of PID parameters using IAE, ISE and MSE objective functions. Pareek et al. (2014) applied PSO, Bacteria Foraging Optimization (BFO) and Artificial Bee Colony (ABC) algorithms for tuning of the PID controller. Davendra, Zelinka, and Senkerik (2010) embedded Chaotic Maps inside DE (DEchaos) and the Chaos-Driven Self-Organizing Migrating Algorithm (SOMAchaos) for the tuning of the PID controller.

Most of the swarm intelligent techniques used to solve the optimization problems do not have the leader to control over the entire period. This drawback is rectified in GWO, in which the grey wolves have natural leadership mechanism. Furthermore, this algorithm has fewer parameters and is easy to implement, making it superior to the other algorithms. Because of the versatile properties of GWO, attempts have been made to implement GWO in tuning the parameters of a PID controller to control the BH system.

The present work involves application of the GWO algorithm in control of the BH system with the help of a PID controller. The parameters of the PID controller are tuned by GWO algorithm with different objective functions. The robustness analysis of the proposed GWO/PID approach has also been conducted with ±5% perturbation in the locations of the poles. Further, the results of comparative analysis are presented with some other existing techniques in graphical and tabular forms.

2. System under investigation

2.1. Ball and hoop system

The ball and hoop (BH) system illustrates the dynamics of a steel ball that is free to roll on the inner surface of a rotating circular hoop. There is a groove on inside edge of the hoop that allows the steel ball to roll freely inside the hoop. The hoop is continuously rotated by a motor. When the hoop is rotated, the ball will tend to move in the direction of the hoop rotation. At some point, gravity will overcome the frictional forces and the ball will fall back. By repeating this process, the ball will exhibit oscillatory motion. Figure 1 shows a schematic of the BH system which is a 4th order system, in which the key system variables are: hoop radius: \( R \), ball radius: \( r \), ball mass: \( m \), hoop angle: \( \theta \), ball angles with vertical (slosh angle): \( \psi \), ball position on the hoop: \( y \), input torque to the hoop: \( T(t) \).
As seen in Figure 1, the hoop is driven by a torque \( T(t) \) and the coordinates that define the dynamical behaviour of the system are \( \theta \) (angular position of the hoop w.r.t. a datum point A) and \( y \) (the position of the ball on the inner periphery of the hoop, measured w.r.t. the datum point A). Therefore, \( y \) is the output and \( \theta \) is the input for the BH system.

The transfer function of BH system is given by (Morkos & Kamal, 2012; Pareek & Gupta, 2014):

\[
G_{BH}(s) = \frac{y(s)}{\theta(s)} = \frac{1}{s^4 + 6s^3 + 11s^2 + 6s}
\]  

(1)

The four poles of the BH system are located at \( s = 0, s = -1, s = -2 \) and \( s = -3 \); this condition, makes it a stable system. In this study, Equation (1) was obtained by linearizing the BH system equations given in (Wellstead, 1983).

### 2.2. Perturbed ball and hoop systems

To validate the robustness of proposed GWO/PID approach, ±5% change in the location of the poles of the BH system is considered; such changes will result in perturbed systems. The poles of the BH system are perturbed by ±5% of the nominal value. A small change in perturbation will not change the location of poles from the left hand side to right hand side; thus, the system remains stable. A large percentage change in perturbation can cause the BH system to become unstable. The perturbation given to the BH system is measured in percentage.

#### 2.2.1. Perturbed BH system with +5% change in the locations of the poles

With +5% changes in the locations of the poles, the four poles of the perturbed BH system will be located at \( s = 0, s = -1.0500, s = -2.100, \) and \( s = -3.1500 \), and the transfer function of perturbed BH system will be:

\[
G_{PBH}(s) = \frac{1}{s^4 + 6.3s^3 + 12.127s^2 + 6.9457s}
\]  

(2)

#### 2.2.2. Perturbed BH system with −5% change in the locations of the poles

With −5% changes in the locations of the poles, the four poles of the perturbed BH system will be located at \( s = 0, s = -0.9500, s = -1.900, \) and \( s = -2.8500 \), and the transfer function of perturbed BH system will be:

\[
G'_{PBH}(s) = \frac{1}{s^4 + 5.7s^3 + 9.927s^2 + 5.1442s}
\]  

(3)

A comparison of the transient response parameters of BH, PBH (+5%) and PBH (−5%) systems is given in Table 1. Moreover, the closed loop step responses of these systems with unity feedback are compared in Figure 2.

| System         | Rise time (sec.) | Peak overshoot (%) | Peak time (sec.) | Settling time (sec.) |
|----------------|------------------|--------------------|------------------|----------------------|
| BH             | 8.9              | 0                  | >35              | 16.1                 |
| PBH (+5%)      | 11.2             | 0                  | >20.7            | 20.7                 |
| PBH (−5%)      | 7.01             | 0.42               | >16              | 11.8                 |
3. Performance indices and problem formulation

A performance index is used to quantify a system’s performance. A performance index is widely used to represent the performance of a PID controller and is helpful to design a system effectively with the desired specifications. The generally used performance indices in control system engineering are as follows:

3.1. IAE

The integral absolute error (IAE) performance index is given by Equation (4):

\[ IAE = \int_{0}^{\infty} |e(t)| \, dt \]  

(4)

3.2. ISE

The integral square error (ISE) has a characteristic that it penalizes large errors heavily and small errors lightly. A system designed based on this criterion tends to show a rapid decrease in a large initial error. ISE is given by Equation (5):

\[ ISE = \int_{0}^{\infty} e^2(t) \, dt \]  

(5)

3.3. ISTAE

Integral of the product between the squared time and absolute error (ISTAE) is given by Equation (6):

\[ ISTAE = \int_{0}^{\infty} t^2 |e(t)| \, dt \]  

(6)

3.4. ITAE

ITAE integrates the absolute error multiplied by the time. Designing a system using this criterion has small overshoot and well-damped oscillations. This ITAE is given by Equation (7):

\[ ITAE = \int_{0}^{\infty} t |e(t)| \, dt \]  

(7)

The complete Simulink model of the BH and perturbed systems with different objective functions is shown in Figure 3. The GWO algorithm is used to minimize the objective functions, i.e. IAE, ISE, ISTAE and ITAE separately to obtain the unknown parameters of PID controller. The optimization is run 5 times, and the best final solution among the 5 runs is chosen as the parameters of the PID controller for a particular objective function.
4. Grey wolf optimization

The GWO is recently proposed bio inspired heuristic algorithm inspired by both the social hierarchy and hunting strategy of grey wolves. The advantages of the GWO over other well-known evolutionary techniques include: no requirement of input parameters for implementation, less computational complexity, simple in nature and easy programming, etc. The drawback of the GWO is that, if any global optimization algorithm is applied alone, it will give the wide search space and may not give best solution. A balance of exploitation and exploration throughout the search procedure should be maintained to get excellent performance.

The search in the GWO starts with a population of randomly generated wolves (solutions). During hunting or optimization strategy, these wolves estimate the prey’s or optimum location through an iterative procedure. There are four groups; Alpha ($\alpha$), Beta ($\beta$), Delta ($\delta$), and Omega ($\omega$). The $\alpha$ represents the fittest solution and it is followed by $\beta$ and $\delta$ as the second and third best solutions, respectively. The rest of the solutions are considered as $\omega$ which are least important. The process of the
GWO technique completes in four steps; encircling the prey, hunting, attacking the prey (exploration process) and searching the prey; exploration capability. The functions of each group have also been defined in Figure 4 (Mirjalili et al., 2014; Soni, Parmar, & Kumar, 2016). The social hierarchy of wolves is modeled first during the designing stage as follows:

The main phases of grey wolf hunting are as follows:

4.1. **Encircling prey**

The encircling process of the grey wolves around the prey is given by:

\[
\mathbf{D} = |\mathbf{C} \cdot \mathbf{X}_p(t) - \mathbf{X}(t)|
\]

\[
\mathbf{X}(t + 1) = \mathbf{X}_p(t) - \mathbf{A} \cdot \mathbf{D}
\]

where; \( t \) represents the current iteration; \( \mathbf{A} \) and \( \mathbf{C} \) are coefficient vectors; \( \mathbf{X}_p \) is the position vector of the prey; \( \mathbf{X} \) is the position vector of wolf; The coefficient vectors are evaluated as:

\[
\mathbf{A} = 2\bar{\mathbf{a}} \bar{\mathbf{r}}_1 - \bar{\mathbf{a}}
\]

\[
\mathbf{C} = 2\bar{\mathbf{r}}_2
\]

where, \( \bar{\mathbf{r}}_1, \bar{\mathbf{r}}_2 \) are random vectors in the range 1 to 0 and vector \( \bar{\mathbf{a}} \) is linearly decreased during iterations from 2 to 0.

4.2. **Hunting process (updating of wolf position)**

The Alphas guide the hunting process with the help of Betas and Deltas. Three best positions corresponding to alphas, betas and deltas are saved which lead the optimum position and remaining solutions including omega are competed. The updated wolf positions around the prey are calculated as follows:

\[
\mathbf{D}_a = |\mathbf{C}_1 \cdot \mathbf{X}_a - \mathbf{X}|, \mathbf{D}_\beta = |\mathbf{C}_2 \cdot \mathbf{X}_\beta - \mathbf{X}|, \mathbf{D}_\delta = |\mathbf{C}_3 \cdot \mathbf{X}_\delta - \mathbf{X}|
\]

\[
\mathbf{X}_1 = \mathbf{X}_a - \mathbf{A}_1 \cdot (\mathbf{D}_a), \mathbf{X}_2 = \mathbf{X}_\beta - \mathbf{A}_2 \cdot (\mathbf{D}_\beta), \mathbf{X}_3 = \mathbf{X}_\delta - \mathbf{A}_3 \cdot (\mathbf{D}_\delta)
\]

\[
\mathbf{X}(t + 1) = \frac{\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3}{3}
\]

4.3. **Attacking Prey (Exploitation of Search Process)**

This phase enables the exploitation of search process in the GWO algorithm. When the prey stops, grey wolves attack the prey and stop their hunting. Mathematically, this process is expressed by decreasing \( \mathbf{A} \) which in turn decreases the variations in \( \mathbf{A} \). Initially, \( \mathbf{A} \) is a random value in the interval \([-\alpha, \alpha]\) and “\( \alpha \)” is decreased from 2 to 0 over the course of iterations. When \(|\mathbf{A}| < 1\), the wolves move towards the prey for attacking.

4.4. **Searching the prey (exploration of search process)**

This phase enables the exploration of search process in algorithm. Location of the alpha, beta, and delta grey wolves search for prey. The wolves move away from each other to search for the prey and come together to attack the prey. The exploration capability is incorporated in the GWO algorithm when the values of \( \mathbf{A} \) lie outside the range \([-1, 1]\). Wolves move away from the prey to search for a better prey when \(|\mathbf{A}| > 1\). The component \( \mathbf{C} \) also participates in exploration process and lies in the range
0 to 2. It assigns random weights for prey to define the distance. The search agents are allowed to update their position based on the location of $\alpha$, $\beta$, $\delta$ and attack towards the prey, after each iteration. The flow chart representation of the GWO is shown in Figure 5. The flow chart describes each stage performing the whole process. The pseudo code of the GWO can be found in (Mirjalili et al., 2014).

Two main parameters are initialized before starting the GWO. The first parameter is the “maximum number of search agents (SA)” or “grey wolves”. The second important parameter is the “number of iterations (Iter)”. These two parameters may vary according to the application. In present research work, the parameters used for simulation of the GWO algorithm are given in Table 2. The stopping criterion for the GWO is the maximum number of iterations.

5. Simulation results of GWO/PID approach during control of BH system
The GWO/PID approach is applied to the BH system for the minimization of different objective functions. The obtained parameters of the PID controller for different objective functions are given in Table 3. In this instance, the calculations are given only for the IAE objective function. Similarly, the closed loop transfer functions for other objective functions (ISE, ITAE and ISTAE) can be calculated (Table 7) according to the parameters of PID controller in Table 3.
5.1. IAE as an objective function

As per the scheme shown in Figure 3, the obtained parameters of PID controller are given by:

\[ K_p = 4.9900; K_i = 0.0010; K_d = 5.7056 \]  \hspace{1cm} (15)

Therefore, the PID controller is given by:

\[ G_c = 4.9900 + \frac{0.0010}{s} + 5.7056s \]  \hspace{1cm} (16)

Next, by multiplying the transfer functions of both the PID controller and the Ball & Hoop System, the open loop forward path transfer function is given by:

\[ G_F = \text{PID Controller} \left( G_c \right) \times \text{Ball Hoop} \left( G_{BH} \right) \]  \hspace{1cm} (17)

The closed loop transfer function of the BH system with a PID controller and unity feedback can be obtained from:

\[ G_CL = \frac{G_F}{1 + H(s) G_F} \]  \hspace{1cm} (18)

where, \( H(s) = 1 \).

### Table 2. Parameters used for the GWO algorithm

| Parameter                  | Value          |
|----------------------------|----------------|
| Number of search agents (population) | 30             |
| Dimension                  | 3              |
| Maximum iterations         | 50             |
| Lower bounds               | [0.0001 0.0001 0.0001] |
| Upper bounds               | [20 20 20]     |

### Table 3. Parameters of the PID controller with different objective functions

| Algorithm                  | IAE  | ISE  | ISTAE | ITAE  |
|----------------------------|------|------|-------|-------|
|                            | \( K_p \) | \( K_i \) | \( K_d \) | \( K_p \) | \( K_i \) | \( K_d \) | \( K_p \) | \( K_i \) | \( K_d \) |
| DE_{chaos} (Davendra et al., 2010) | 5.856 | 0.0043 | 11.835 | 5.204 | 0.1568 | 20.804 | - | - | - |
| SOMA_{chaos} (Davendra et al., 2010) | 5.856 | 0.0043 | 11.835 | 5.204 | 0.1568 | 20.804 | - | - | - |
| Z-N (Mojallali et al., 2012) | 6 | 1.9078 | 4.7178 | - | - | - | - | - | - |
| CPSO (Mojallali et al., 2012) | 5.8653 | 0.0001 | 11.4188 | - | - | - | - | - | - |
| GWO (Proposed) | 4.9900 | 0.0010 | 5.7056 | 6.9860 | 0.0018 | 9.0671 | 5.9880 | 0.0014 | 6.8870 | 3.9920 | 0.0010 | 4.4359 |

Therefore,

\[ G_CL (IAE) = \frac{5.7056s^2 + 4.9900s + 0.0010}{s^5 + 6s^4 + 11s^3 + 11.7056s^2 + 4.9900s + 0.0010} \]  \hspace{1cm} (19)

In Figure 6, the responses of the proposed GWO/PID approach with different objective functions for the control of BH system have been compared.
6. Robustness analysis of GWO/PID approach
To validate the proposed GWO/PID approach, robustness analysis is also conducted. The PID controller used for the BH system is applied to the perturbed systems and the simulation results are given.

6.1. Simulation results of GWO/PID approach in the control of PBH (+5%) system

6.1.1. IAE as an objective function
In this case, the PID controller is given by Equation (16), and by multiplying the transfer function of the PID controller and PBH system given by Equation (2), the open loop forward path transfer function is given by:

\[ G_F = \text{PID Controller}(G_C) \times \text{Perturbed Ball Hoop}(G_{PBH}) \]  

(20)

The closed loop transfer function of the PBH system with the PID controller and unity feedback can be obtained from Equation (18); therefore,

\[ G_{CL}(IAE) = \frac{5.7056 s^2 + 4.9900 s + 0.0010}{s^5 + 6.3 s^4 + 12.127 s^3 + 12.6513 s^2 + 4.9900 s + 0.0010} \]

(21)

Similarly, the closed loop transfer functions for other objective functions (ISE, ITAE and ISTAE) can be calculated for the PBH (+5%) system according to the parameters of the PID controller obtained by the GWO algorithm in Table 3. The responses of the proposed GWO/PID approach with different objective functions for the PBH (+5%) system are compared in Figure 7.

6.2. Simulation results of GWO/PID approach in the control of PBH (−5%) system

6.2.1. IAE as an objective function
In this case, the PID controller is given by Equation (16), and by multiplying the transfer functions of the PID controller and PBH system given by Equation (3), therefore,

\[ G_{CL}(IAE) = \frac{5.7056 s^2 + 4.9900 s + 0.0010}{s^5 + 5.7 s^4 + 9.927 s^3 + 10.8498 s^2 + 4.9900 s + 0.0010} \]

(22)

Similarly, the closed loop transfer functions for other objective functions (ISE, ITAE and ISTAE) can be calculated for the PBH (−5%) system according to the parameters of the PID controller in Table 3. The responses of the proposed GWO/PID approach with different objective functions for the PBH (−5%) system are compared in Figure 8.
7. Comparative analysis

In this section, the performance evaluation of the proposed GWO/PID approach with perturbations and other existing techniques is presented.

7.1. Comparative analysis of GWO/PID approach with perturbation

Here, the performance of GWO tuned PID controller is compared on BH, PBH (+5%) and PBH (−5%) systems for a particular objective function. Figures 9–12 show, the following: (1) the GWO/PID approach operates in a satisfactory manner on these systems and (2) there is no effect on the performance of the controller with perturbation in the locations of the poles of the BH system.

Furthermore, the transient response parameters, i.e. overshoot, rise and settling times of closed loop BH, PBH (+5%) and PBH (−5%) systems were also calculated; the parameters are given here in tabular forms.
Figure 9. Step responses of GWO/PID approach with IAE objective function for BH, PBH (+5%) and PBH (−5%) systems.

Figure 10. Step responses of GWO/PID approach with ISE objective function for BH, PBH (+5%) and PBH (−5%) systems.

Figure 11. Step responses of GWO/PID approach with ISTAE objective function for BH, PBH (+5%) and PBH (−5%) systems.
Tables 4–6, show that the transient response parameters of closed loop BH, PBH (+5%) & PBH (−5%) systems with GWO tuned PID controller for different objective functions are comparable, i.e. the performance of the controller hardly alters with perturbations and the objective functions after, it is tuned by GWO.

**Figure 12. Step responses of GWO/PID approach with ITAE objective function for BH, PBH (+5%) and PBH (−5%) systems.**

**Table 4. Comparison of the Rise time for the GWO/PID approach**

| Algorithm-index | Rise time (sec.) |
|-----------------|------------------|
|                 | BH system | PBH system (+5%) | PBH system (−5%) |
| GWO-JAE         | 1.52      | 1.74              | 1.35              |
| GWO-JSE         | 1.07      | 1.18              | 0.986             |
| GWO-ISTAE       | 1.28      | 1.43              | 1.16              |
| GWO-ITAE        | 1.93      | 2.29              | 1.67              |

**Table 5. Comparison of the overshoot for the GWO/PID approach**

| Algorithm-index | Overshoot (%) |
|-----------------|---------------|
|                 | BH system | PBH system (+5%) | PBH system (−5%) |
| GWO-JAE         | 10.4      | 4.35              | 17.7              |
| GWO-JSE         | 20.6      | 13.1              | 29.1              |
| GWO-ISTAE       | 16.2      | 9.27              | 24.1              |
| GWO-ITAE        | 5         | 0.473             | 11.2              |

**Table 6. Comparison of the settling time for the GWO/PID approach**

| Algorithm-index | Settling time (sec.) |
|-----------------|----------------------|
|                 | BH system | PBH system (+5%) | PBH system (−5%) |
| GWO-JAE         | 4.8       | 4.6               | 6.85              |
| GWO-JSE         | 5.68      | 5.64              | 7.42              |
| GWO-ISTAE       | 6.32      | 4.37              | 6.42              |
| GWO-ITAE        | 5.41      | 3.74              | 5.46              |
7.2. Comparative analysis of GWO/PID approach with other existing techniques

In this section, the performance of the proposed GWO/PID approach is compared with certain other existing techniques for the same BH system. In Table 7, different closed loop transfer functions of the BH system for the proposed and other existing techniques have been calculated, as per the parameters of PID controller in Table 3. Based on these closed loop transfer functions, the responses of the GWO/PID approach for the BH system with other existing techniques have been compared in Figures 13–15.

Figures 13–15 show that the GWO/PID approach gives better results in comparison to existing techniques. Furthermore, the parameters, i.e. overshoot and settling time of closed loop BH systems have also been calculated and compared with several existing techniques in the literature.

| S.No. | Algorithm               | Closed loop transfer function (G_CL)                                                                 | Objective function used |
|-------|-------------------------|-----------------------------------------------------------------------------------------------------|------------------------|
| 1     | DEChaos (Davendra et al., 2010) | \(11.835s^2 + 5.856s + 0.0043 \) \(6s^4 + 11s^3 + 17.31s^2 + 5.856s + 0.0043\) | IAE                    |
| 2     | Z-N (Mojallali et al.,2012) | \(6.7278s^2 + 6s + 1.9078 \) \(3s^4 + 6s^3 + 10.7178s^2 + 6s + 1.9078\) | IAE                    |
| 3     | CPSO (Mojallali et al.,2012) | \(11.488s^2 + 5.865s + 0.0001 \) \(6s^4 + 11s^3 + 17.835s^2 + 5.8653s + 0.0001\) | IAE                    |
| 4     | GWO (Proposed)          | \(5.7056s^2 + 4.9900s + 0.0010 \) \(3s^4 + 6s^3 + 11.7056s^2 + 4.9900s + 0.0010\) | IAE                    |
| 5     | SOMAChaos (Davendra et al., 2010) | \(20.804s^2 + 5.204s + 0.1568 \) \(6s^4 + 11s^3 + 26.804s^2 + 5.204s + 0.1568\) | ISE                    |
| 6     | GWO (Proposed)          | \(9.0671s^2 + 6.9860s + 0.0018 \) \(6s^4 + 11s^3 + 15.0671s^2 + 6.9860s + 0.0018\) | ISE                    |
| 7     | DEChaos (Davendra et al., 2010) | \(7.0235s^2 + 4.8436s + 0.00025 \) \(6s^4 + 11s^3 + 13.0235s^2 + 4.8436s + 0.00025\) | ITAE                   |
| 8     | SOMAChaos (Davendra et al., 2010) | \(7.3089s^2 + 4.963s + 0.0003 \) \(6s^4 + 11s^3 + 15.3089s^2 + 4.963s + 0.0003\) | ITAE                   |
| 9     | GWO (Proposed)          | \(4.4359s^2 + 3.9920s + 0.0014 \) \(6s^4 + 11s^3 + 10.4359s^2 + 3.9920s + 0.0014\) | ITAE                   |
| 10    | GWO (Proposed)          | \(6.8870s^2 + 5.9880s + 0.0016 \) \(6s^4 + 11s^3 + 12.8870s^2 + 5.9880s + 0.0016\) | ISTAE                  |

Note: Bold terms in Table 7 represents result of proposed work.
Tables 8 and 9 reveal that the GWO/PID approach outperforms existing techniques in the literature by giving low values of overshoot and settling times for all of the objective functions.

Table 8. Settling time comparison with the existing techniques of the GWO/PID approach for the BH system

| Algorithm                        | IAE   | ISE   | ITAE  |
|----------------------------------|-------|-------|-------|
| DEchaos (Davendra et al., 2010)   | 5.19  | 9.22  | 6.095 |
| SOMAchaos (Davendra et al., 2010)| 5.19  | 9.22  | 6.095 |
| Z-N (Mojallali et al., 2012)     | 10.6  | -     | -     |
| CPSO (Mojallali et al., 2012)    | 4.88  | -     | -     |
| Standard PSO (Morkos & Kamal, 2012)| 7.2  | 9.54  | -     |
| AHPSO Global (Morkos & Kamal, 2012)| 7.58 | 9.29  | -     |
| AHPSO Local (Morkos & Kamal, 2012)| 5.1  | 9.14  | -     |
| GWO (Proposed)                   | 4.8   | 5.68  | 5.41  |
8. Conclusions

This work describes performance evaluation of the GWO/PID approach in the control of a ball and hoop (BH) system with different objective functions and perturbations in the locations of poles. It is difficult to control the BH system, as the parameters of the same change constantly. The commonly used objective functions; IAE, ISE, ISTAE and ITAE are minimized with the help of the grey wolf optimization (GWO) algorithm to obtain the optimal parameters of a proportional-integral-derivative (PID) controller for the control of the BH system. The PID controller tuned by GWO gives satisfactory performance on the BH system with different objective functions. The robustness analysis of the proposed GWO/PID approach is also carried out with ±5% perturbation in the locations of the poles. The poles of the BH system are perturbed by ±5% of the nominal value and the same PID controller tuned by GWO algorithm is applied on the perturbed systems. The performance of the PID controller is found to change only slightly with the perturbation and the objective functions, once the PID controller is tuned by GWO. A comparison of the GWO/PID approach with other existing techniques is also presented in graphical and tabular forms. The GWO/PID approach is observed to provide better results in comparison to the existing techniques in the literature for all of the objective functions.

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Table 9. Overshoot comparison with the existing techniques of the GWO/PID approach for the BH system

| Algorithm | IAE | ISE | ITAE |
|-----------|-----|-----|------|
| DEchaos (Davendra et al., 2010) | 14.5 | 24.52 | 6.715 |
| SOMAchaos (Davendra et al., 2010) | 14.5 | 24.52 | 6.715 |
| Z-N (Mojallali et al.,2012) | 58.4 | - | - |
| CPSO (Mojallali et al.,2012) | 14.6 | - | - |
| Standard PSO (Markos & Kamal, 2012) | 25 | 25.95 | - |
| AHPSO Global (Markos & Kamal, 2012) | 15.5 | 28 | - |
| AHPSO Local (Markos & Kamal, 2012) | 14 | 25.89 | - |
| GWO (Proposed) | 10.4 | 20.6 | 5 |

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