Cylindrical optimized nonmagnetic concentrator with minimized scattering

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Abstract: By using optimized transformation function, we research on a minimized scattering nonmagnetic concentrator, which can realize impedance matching at the inner and the outer boundaries. It has been demonstrated that the optimized transformation function method can improve the concentrating performance remarkably. The cylindrical anisotropic shell can be mimicked by radial symmetrical sectors which alternate in composition between two profiles of isotropic dielectrics, and the permittivity in each sector can be properly determined by the effective medium theory. The nonmagnetic concentrator has been validated by full-wave finite element simulations. We can believe that this work will improve the flexibilities for the EM concentrator design.

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In the past few years, the technique of transformation optic [1, 2] is a powerful and systematic approach in manipulating electromagnetic waves and producing new functionalities with artificial engineered structures [3, 4]. A series of applications have been reported based on this theoretical framework. Among all these applications, free-space invisibility cloaks [5–10] have definitely received most scientific interests. To achieve invisibility cloak at optical frequencies, Cai et al. proposed a nonmagnetic cloak with simplified material parameters for transverse magnetic (TM) waves [11]. Compared to the perfect set, such a nonmagnetic cloak has a drawback that significant scattering outside the cloak exists due to a mismatched impedance at the outer boundary. In order to match the wave impedance at the outer boundary, various coordinate transformation functions [12–15], such as quadratic, power, and piece power functions, have been applied to design the nonmagnetic cloak. Huang et al. proposed the generalized transformation functions and a kind of transformation functions with two parameters for the nonmagnetic invisibility cloak [16, 17]. The purpose of using these optimized functions is to avoid the reflection at the outer boundary of the nonmagnetic cloak. Recently, the optical transformation theory has been applied to the electromagnetic (EM) concentrator, and some theoretical analyses, numerical simulations, and parameter designs have been devoted to EM concentrating devices [18–22]. It is expected that the electromagnetic concentrator design possesses potential applications in optical microscopes, solar cells, and devices in which high field intensities are needed [22]. However, nonmagnetic concentrator with simplified material parameters also has significant scattering outside the structure, which is similar to that of nonmagnetic cloak. Thus, how to minimize the scattering of the nonmagnetic concentrator and improve the concentrating performance is highly urgent. In the process of design of optimized nonmagnetic concentrator, in order to minimize the scattering outside, the impedance of the inner and the outer boundaries must be considered simultaneously. The increasing of limited conditions will induce more complicated optimization process. Although people have achieved success in the minimizing the scattering of invisibility cloak theory, how to reduce the scattering of the electromagnetic concentrator has not been revealed.

In this paper, we research on how to further improve the impedance-matched nonmagnetic concentrator by properly designing the coordinate transformation function. The appropriate transformation function is given with the matched impedance at the inner and the outer boundaries of the nonmagnetic concentrator. With the aid of far field scattering patterns, we show the full-wave solution of the scattering by nonmagnetic concentrator. We can find that
this kind of concentrator has the perfect performance. Moreover, we find an approach to realize the practical fabrication, in which radial symmetrical layered structures of isotropic materials instead of using the artificial metamaterials with subwavelength structured inclusions. Just by properly choosing the permittivity in each isotropic dielectric layer, excellent concentrating performance can be achieved, which will improve the concentrator design.

2. Principle of transformation

For simplification purposes, we consider a bi-dimensional case of the nonmagnetic concentrator in a cylindrical coordinate system as schematically illustrated in Fig. 1. As pointed out in Ref. [18], the optical transformation for the concentrator can be expressed in the region \( r' \in [0, R_1] \) is compressed into the region \( r \in [0, R_1] \), namely, the core region. The region \( r' \in [R_2, R_3] \) is focused into the region \( r \in [R_2, R_3] \), namely, the circular region. Here, \( r \) and \( r' \) represent the radii of the physical space and the virtual space, respectively.

![Fig. 1. Sketch of the cylindrical nonmagnetic concentrator.](image)

The coordinate transformation function between the original coordinate \((r', \theta', z')\) and the transformation coordinate \((r, \theta, z)\) can be expressed as follows:

\[
r' = f(r), \quad \theta' = \theta, \quad z' = z
\]

According to the standard procedure of transformation optics, the relative permittivity and the permeability after the transformation can be written in terms of \( f(r) \) as

\[
\varepsilon_r = \mu_r = \frac{f(r)}{f'(r)}, \quad \varepsilon_{\theta} = \mu_{\theta} = \frac{1}{\varepsilon_r}, \quad \varepsilon_z = \mu_z = \frac{f'(r) f(r)}{r}
\]

Here, in order to remove the magnetic response of the material, which is especially important for making devices at optical frequency, the constitutive parameters of the proposed nonmagnetic concentrator for TM illumination can be further simplified as

\[
\varepsilon_r = \left( \frac{f(r)}{r} \right)^2, \quad \varepsilon_{\theta} = (f'(r))^2, \quad \mu_z = 1
\]

Compared to the ideal properties as depicted in Eq. (2), the reduced parameters can provide the similar wave trajectory inside the cylindrical region. According to Eq. (3), the wave impedance at the boundary becomes \( Z_{r=R} = \sqrt{\mu_z / \varepsilon_{\theta}} = 1 / f'(R) \), which depends on the transformation function.

According to the theory in Ref. [18], the transformation function can be easily selected as the linear function in core region.
Through the transformation function of Eq. (4), we observe that the enhancing ratio can be expressed as the ratio of \( R_2 \) to \( R_1 \), and enhancement theoretically diverges to infinity as \( R_1 \) tends to zero.

In circular region, the linear transformation function can be selected as

\[
f_{cir}(r) = \frac{R_2 - R_1}{R_1 - R_1} \left( r + \frac{R_2 - R_1}{R_1 - R_1} R_1 \right)
\]  

Then the reduced parameters of the core and circular region can be derived as

\[
\varepsilon_{(cor)} = \varepsilon_{(cor)} \mu = \left( \frac{R_1}{R_1} \right)^2, \quad \mu_{(cor)} = 1
\]

\[
\varepsilon_{(cir)} = \varepsilon_{(cir)} \mu = \left( \frac{R_3 - R_1}{R_3 - R_1} \right)^2, \quad \mu_{(cir)} = 1
\]

Parameters presented in Eq. (6) minimize the most challenging part of experimental design. But this set of parameter specification leads to impedance mismatch at the inner and outer boundary. The reduced parameters in core region have the impedance of \( Z_{r=R_1} = R_1 / R_2 \) at the boundary \( r = R_1 \) and the impedance of \( Z_{r=R_1} = (R_3 - R_1) / (R_3 - R_1) \) in free space, while the impedance of \( Z_{r=R_3} = (R_3 - R_1) / (R_3 - R_1) \) in circular region at the two boundaries. So the design approach using linear transformation function will not only induce some unfavorable scattering at the inner and the outer boundaries due to the impedance mismatch, but also lead to energy scattering. Meanwhile, the concentrating performance is degenerated based on this design approach.

To improve the concentrating efficiency, we need to derive another set of simplified material tensors, which also have simple tensor components and make the inner and the outer boundaries perfectly matched. In order to achieve the nonmagnetic concentrator with minimized scattering, the transformation functions are required to satisfy following conditions:

\[
f_{cor}(0) = 0, \quad f_{cor}(R_2) = R_2, \quad f_{cir}(R_1) = R_2
\]

\[
f_{cor}(R_1) = R_3, \quad f'_{cir}(R_1) = 1, \quad f'_{cor}(R_1) = f'_{cir}(R_1)
\]

3. Simulation and discussion

We notice that the choice for the transformation function which satisfy these restrictions is not unique, which indicates that the optimization might be accomplished by using some numerical methods like genetic algorithm [15,23]. If the variations in the distribution of the material parameter values along the radius are gently, the scattering of the proposed concentrator is smaller. We find that the high polynomial functions which satisfy Eq. (7) and \( f'_{cor}(R_1) = f'_{cir}(R_1) = 1 \) can meet the above-mentioned restrict conditions.

\[
f_{cor}(r) = \left( \frac{R_2 - R_1}{R_1} \right) r^2 + \frac{2R_2 - R_1}{R_1} r
\]

\[
f_{cir}(r) = Ar^3 + Br^2 + Cr + D
\]
where, \[ A = \frac{2(R_i - R_c)}{(R_i - R_c)^3}, \quad B = \frac{3(R_c - R_i)(R_i + R_c)}{(R_i - R_c)^3}, \quad C = \frac{R_i^2 - R_c^2 + 3R_cR_i(R_i + R_c - 2R_c)}{(R_i - R_c)^3}, \]
and
\[ D = \frac{R_i^2 (3R_c - R_i)(R_c - R_i)}{(R_i - R_c)^3}. \]
Substituting Eqs. (8) and (9) into Eq. (3), we can obtain
\[
epsilon_{(cor)\rho} = \left( \frac{R_i - R_c}{R_i} \right) r + \frac{2R_c - R_i}{R_i} \), \quad \epsilon_{(cor)\theta} = \left( 2 \frac{R_i - R_c}{R_i} r + \frac{2R_c - R_i}{R_i} \right) r, \quad \mu_{(cor)z} = 1 \quad (10)
\]
\[
epsilon_{(cor)\rho} = (Ar^2 + Br + C + D / r)^2, \quad \epsilon_{(cor)\theta} = (3Ar^2 + 2Br + C)^2, \quad \mu_{(cor)z} = 1 \quad (11)
\]
Figure 2 compares the results of two transformation functions: the linear and the polynomial function. We can find that the polynomial function curve is even smoother than the linear function curve, especially, on the inner boundary \( r = R_i = 0.2m \), which can realize the impedance matched on inner boundary perfectly. Based on the above results, the constitutive parameters of the nonmagnetic concentrator can be calculated through Eqs. (10) and (11), as illustrated in Fig. 3. It can be seen that the permittivity values of different regions on the inner boundary (\( r = R_i \)) are identical. We can expect that the concentrator with the new set of parameters would induce smaller scattering. Moreover, all material parameters have finite values. It is indicated that the materials are relatively easily realized with the metamaterials.

![Figure 2. Comparison of different coordinate transformation functions in core and circular region.](image)

![Figure 3. The material parameter values for the concentrator constructed through the polynomial function. (a) The material parameter value in core region. (b) The material parameter value in circular region.](image)

To compare the performance of the nonmagnetic concentrators with different transformation functions, we calculate the EM field scattering using the finite-element-method (FEM) based full-wave EM simulation with the commercial COMSOL Mutiphysics...
package. We consider the TM incident wave with the magnetic field polarized along the z axis. The TM wave propagates along the x direction. The geometry parameters are also selected as \( R_1 = 2R_2 = 3R_3 = 0.6m \), and the incident wave frequency is set at 1.5GHz.

The results of the spatial distributions of the magnetic field and the corresponding power flow distribution are shown in Fig. 4. Figure 4(a) shows the magnetic field distribution of concentrator with linear transformation. It is obvious that part of magnetic field is disturbed by mismatched impedance when crossing the concentrator, so the scattering field obviously emerges in the outside region. Figure 4(c) shows the magnetic field distribution with polynomial transformation function, it can be seen that the magnetic field is smoothly concentrated into the inner core region, and the field outside is undisturbed.

Fig. 4. (a) Magnetic field distribution of the concentrator with linear transformation function, (b) Normalized power flow of the concentrator with linear transformation function, (c) Magnetic field distribution of the concentrator with polynomial transformation function, (d) Normalized power flow of the concentrator with polynomial transformation function.

Furthermore, the corresponding power flow distribution with different transformations are calculated and shown in Fig. 4(b) and Fig. 4(d). Comparing the case of linear function with the case of polynomial function, the difference between the energy distribution of the two cases are conspicuous. For the case of linear transformation, part of the energy is scattered by the concentrator, which results in the disturbance of the energy in the whole space. As for the case of polynomial transformation, although the distribution of the energy in the core region is somewhat inhomogeneous, most of energy is concentrated.

In order to quantify the perfect concentrating performance of the proposed concentrator, we calculate the intensity of power flow on the centre line of core region with linear and polynomial transformation function, which is shown in Fig. 5. Comparing the two cases with different transformation functions, we can find that the concentrating performance with polynomial transformation is much better than that of linear transformation. To be specific, the normalized power flow of optimized concentrator is close to 1, but the maximal normalized power flow value of linear transformed concentrator just reaches 0.8 and what is
more, the distribution of the energy is highly non-uniform. Comparing curves in Fig. 5, we can find that the method using polynomial transformation function can enhance the concentrating performance by 25%. 

![Fig. 5](image)

**Fig. 5.** Normalized power flow of the concentrator with linear transformation and polynomial transformation on the centre line of core region \((x=0, y \in [-0.2, 0.2])\).

In order to make a further comparison, we plot the normalized far field scattering patterns of the nonmagnetic concentrator with different transformation functions, as shown in Fig. 6. By comparing the two cases with different transformation functions, we find that the backscattering reduction property can be distinctly improved by choosing the optimized polynomial transformation function other than the conventional linear transformation function.

![Fig. 6](image)

**Fig. 6.** The normalized far field scattering patterns of the nonmagnetic concentrator with different transformation functions. The black dashed curve indicates the case of linear transformation, and the red real curve indicates the case of polynomial transformation.

Based on the effective medium theory [24], a series of cylindrical or spherical anisotropic invisible cloaks have been mimicked by concentric layered structures consisting of alternating layers of two different homogeneous isotropic dielectric materials in subwavelength scale. Next, we restrict the practical fabrication of the nonmagnetic concentrator device. It can be seen from Eqs. (10) and (11) that the constitutive materials are all inhomogeneous and anisotropic in core and circular regions, which should be approximated by the structure of homogeneous and isotropic materials based on effective media theory.

Furthermore, from the parameters derived in Eqs. (10) and (11), we can find that the radial component is always larger than the angular components \((e_r > e_\theta)\) which is different from all the previous cases of invisibility cloaks [12–15]. Such required cylindrical anisotropy shell
can be mimicked by utilizing radial symmetrical sectors alternate in composition between two profiles of isotropic dielectrics instead of using the concentric alternating layers.

Supposing the tangential width of the dielectric profile-A and profile-B is identical, we find a new mapping of parameters between the cylindrical anisotropic shell and isotropic layered shell in interlaced profile-A and profile-B as

$$\varepsilon_r = \frac{\varepsilon_a + \varepsilon_b}{2}, \quad \frac{1}{\varepsilon_\theta} = \frac{1}{2\varepsilon_a} + \frac{1}{2\varepsilon_b}$$

(12)

Where, $\varepsilon_a$ and $\varepsilon_b$ represent permittivity patterns for profile-A and profile-B of the constitutive layers. Taking Eq. (10) to Eq. (12), we can obtain the nonmagnetic permittivity patterns $\varepsilon_\lambda$ and $\varepsilon_\theta$ for the two dielectric profiles. Figure 7 illustrates the permittivity patterns of the two interlaced dielectric profiles obtained by radial discretizing the circular region into $2M = 160$ constituents in the core region and the circular region. It can be seen that the value of permittivity for both profiles is positive, indicating the desirable cylindrical anisotropy can be effectively demonstrated by nonmagnetic right-handed material.

For a cylindrical cloaking shell ($\varepsilon_r < \varepsilon_\theta$), a similar configuration has been considered [25]. However, one of the permittivity patterns $\varepsilon_a$ and $\varepsilon_b$ should be negative, which is very demanding for the material in practical fabrication.

In order to validate such an isotropic layered concentrator device, the simulation results illuminated with a plane wave are shown in Fig. 8 and Fig. 9. Figure 8 shows that the transverse magnetic field of $(2M = 80)$ layered nonmagnetic concentrator with the linear and the polynomial transformation. It is found that the scattering of the nonmagnetic concentrator using optimized function is much smaller than that of linear transformation.
Figure 9 shows the comparison of different coordinate transformation functions of \((2M = 160)\) layered nonmagnetic concentrator. Following the comparison between Fig. 8(b) and Fig. 9(b), we can see that the concentrating performance of the nonmagnetic layered shell is quite pronounced when the discretizing number is large enough. Furthermore, from Fig. 9(b), the performance of the sufficient discretizing \((2M = 160)\) layered nonmagnetic concentrator using optimized polynomial function is remarkably close to the performance of ideal optimized nonmagnetic concentrator which is shown in Fig. 4(c). Therefore, we can conclude that the method of using optimized transformation function can generate the concentrating performance remarkably.

4. Conclusion

In summary, we have proposed an optimized transformation function for nonmagnetic concentrator with minimized scattering. Such transformation function offers us a great deal of flexibility in designing concentrators of high performance. Based on the effective medium theory, we present a radial symmetrical layered system in composition of two profiles as nonmagnetic concentrator device. Finite element method has been carried out to demonstrate the performance of the nonmagnetic isotropic layered concentrator, and excellent performance has been observed, respectively.
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