An adaptive fuzzy sliding mode controller applied to a chaotic pendulum

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Abstract

In this work, an intelligent controller is employed to the chaos control problem in a nonlinear pendulum. The adopted approach is based on the sliding mode control strategy and enhanced by an adaptive fuzzy algorithm to cope with modeling inaccuracies. The convergence properties of the closed-loop system are analytically proven using Lyapunov’s direct method and Barbalat’s lemma. Numerical results are also presented in order to demonstrate the control system performance.

INTRODUCTION

Chaotic response is related to a dense set of unstable periodic orbits (UPOs) and the system often visits the neighborhood of each one of them. Moreover, chaos has sensitive dependence to initial condition, which implies that the system evolution may be altered by small perturbations. Chaos control is based on the richness of chaotic behavior and may be understood as the use of tiny perturbations for the stabilization of an UPO embedded in a chaotic attractor. It makes this kind of behavior to be desirable in a variety of applications, since one of these UPO can provide better performance than others in a particular situation. Due to these characteristics, chaos and many regulatory mechanisms control the dynamics of living systems. Inspired by nature, it is possible to imagine situations where chaos control is employed to stabilize desirable behaviors of mechanical systems. Under this condition, these systems would present a great flexibility when controlled, being able to quickly change from one kind of response to another. Literature presents some contributions related to the analysis of chaos control in mechanical systems. Andrievskii and Fradkov (2004) and Savi et al. (2006) present an overview of applications of chaos control in various scientific fields. Intelligent control, on the other hand, has proven to be a very attractive approach to cope with uncertain nonlinear systems (Bessa, 2005; Bessa et al., 2005, 2017, 2018, 2019, 2020, 2021; Lima et al., 2018, 2020, 2021; Tanaka et al., 2013). By combining nonlinear control techniques, such as feedback linearization or sliding modes, with adaptive intelligent algorithms, for example fuzzy logic or artificial neural networks, the resulting intelligent control strategies can deal with the nonlinear characteristics as well as with modeling imprecisions and external disturbances that can arise. This contribution proposes a robust intelligent controller that can be applied to stabilize UPOs of chaotic attractors. The adopted approach is based on the sliding mode control strategy and enhanced by a stable adaptive fuzzy inference system to cope with modeling inaccuracies and external disturbances that can arise. The boundedness of all closed-loop signals and the convergence properties of the tracking error are analytically proven using Lyapunov’s direct method and Barbalat’s lemma. The general procedure is applied to a nonlinear pendulum that presents chaotic response (De Paula et al., 2006). Numerical simulations are carried out showing the stabilization of some UPOs of the chaotic attractor showing an effective response, demonstrating the controller performance.

CHAOTIC PENDULUM

The nonlinear pendulum consists of an aluminum disc with a lumped mass that is connected to a rotary motion sensor. This assembly is driven by a string-spring device that is attached to an electric motor and also provides torsional stiffness to the system. A magnetic device provides an adjustable dissipation of energy. An actuator provides the necessary perturbations to stabilize this system by properly changing the string length. It provides torsional stiffness to the system. A magnetic device provides an adjustable dissipation of energy. An motion sensor. This assembly is driven by a string-spring device that is attached to an electric motor and also

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\[ u = \dot{h}^{-1}(-\dot{f} - \dot{p} + \dot{\phi}_d - \lambda \dot{e}) - K \text{sgn}(s) \]  

where \( \dot{h}, \dot{f}, \text{and} \dot{p} \) are estimates of \( h, f \) and \( p \), respectively, and \( K \) is a positive control gain.

Regarding the development of the control law, the following assumptions should be made:

**Assumption 1** The function \( f \) is unknown but bounded, i.e. \( |\dot{f} - f| \leq F \).

**Assumption 2** The input gain \( h \) is unknown but positive and bounded, i.e. \( 0 < h_{\text{min}} \leq h \leq h_{\text{max}} \).

**Assumption 3** The perturbation \( p(t) \) is time-varying and unknown but bounded, i.e. \( |p(t)| \leq P \).

Based on Assumption 2 and considering that the estimate \( \hat{h} \) could be chosen according to the geometric mean \( \hat{h} = \sqrt{h_{\text{max}}h_{\text{min}}} \), the bounds of \( h \) may be expressed as \( \mathcal{H}^{-1} \leq \hat{h} \leq \mathcal{H} \), where \( \mathcal{H} = \sqrt{h_{\text{max}}h_{\text{min}}} \).

Under this condition, the gain \( K \) should be chosen according to

\[ K \geq \mathcal{H}^{-1}(\eta + |\dot{p}| + \mathcal{P} + \mathcal{F}) + (\mathcal{H} - 1)|\dot{u}| \]  

where \( \eta \) is a strictly positive constant related to the reaching time.

At this point, it should be highlighted that the control law [3], together with [4], is sufficient to impose the sliding condition

\[ \frac{1}{2} \frac{d}{dt} s^2 \leq -\eta |s| \]  

and, consequently, the finite time convergence to the sliding surface \( S \).

In order to obtain a good approximation to the disturbance \( p(t) \), the estimate \( \hat{p} \) will be computed directly by an adaptive fuzzy algorithm. The adopted fuzzy inference system was the zero order TSK (Takagi–Sugeno–Kang), whose rules can be stated in an appropriate linguistic manner. Considering that each rule defines a perturbation \( p(t) \) to the disturbance \( p \), the vector of adjustable parameters can be automatically updated by \( \hat{P} = \varphi s \Psi(s) \), where \( \varphi \) is a strictly positive constant related to the adaptation rate.

In order to evaluate the stability of the closed-loop system, let a positive-definite function \( V \) be defined as

\[ V(t) = \frac{1}{2} s^2 + \frac{1}{2\varphi} \delta^T \delta \]  

where \( \delta = \hat{P} - \hat{P}^* \) and \( \hat{P}^* \) is the optimal parameter vector, associated with the optimal estimate \( \hat{p}^*(s) \). Thus, by considering the time derivative of \( V \) and defining a minimum approximation error as \( \varepsilon = \hat{p}^*(s) - p \), and recalling the definitions of \( s, u \) and \( \hat{P} \), it is possible to verify that \( V \) becomes

\[ V(t) = -\left[(\dot{f} - f) + \varepsilon + \dot{h}^{-1}u - \dot{h}u - hK\text{sgn}(s)\right]s \leq -\eta |s| \]  

Here assumptions [3][4][5] are evoked and \( K \) is defined according to [6]. This implies \( V(t) \leq V(0) \) and that \( s \) and \( \delta \) are bounded integrating both sides of [5] and evoking Barbala’s lemma it is established that \( s \to 0 \) as \( t \to \infty \), which ensures the convergence of the states to the sliding surface \( S \) and to the desired trajectory. At this point, it should be noted that discontinuous terms can produce undesirable high frequency oscillations of the controlled variable. Therefore, it is convenient to use saturation functions that smoothes system discontinuities (Slotine and Li 1991).

**Numerical simulations**

In order to analyze the controller performance, numerical simulations are carried out considering the fourth order Runge-Kutta method. The model parameters are chosen according to De Paula et al. (2006) and control parameters are \( \lambda = 10.0, \eta = 0.5, \gamma = 1.0 \) and \( \mathcal{H} = 1 \). Basically, two different situations are treated. In the first case, Figure 1 a generic orbit \( [\varphi_d, \varphi_d] = [4.67\cos(2\pi t), 2.3\sin(2\pi t)] \) are considered, while in the second case, Figure 2 a period-1 UPO are chosen. Although both orbits are similar, it should be highlighted that the controller needs less effort to stabilize an UPO.

**Concluding remarks**

The present contribution considers the stabilization of orbits employing an adaptive fuzzy sliding mode controller. The stability and convergence properties of the closed-loop systems is proven using Lyapunov stability theory and Barbala’s lemma. As an application of the general formulation, numerical simulations of a nonlinear pendulum with chaotic response is of concern. The control system performance is investigated showing for the tracking of a generic orbit as well as for UPO stabilization. It is shown that the controller needs less effort to stabilize an UPO.
Figure 1: Tracking of $[\dot{\phi}_d, \phi_d] = [4.6\pi \cos(2\pi t), 2.3 \sin(2\pi t)]$.

Figure 2: Tracking of a Period-1 UPO.

References

B. R. Andrievskii and A. L. Fradkov. Control of chaos: Methods and applications, II - applications. Automation And Remote Control, 65(4):505–533, 2004.

W. M. Bessa. Controle por Modos Deslizantes de Sistemas Dinâmicos com Zona Morta Aplicado ao Posicionamento de ROVs. Tese (D.Sc.), COPPE/UFRJ, Rio de Janeiro, Brasil, 2005.

W. M. Bessa, M. S. Dutra, and E. Kreuzer. Thruster dynamics compensation for the positioning of underwater robotic vehicles through a fuzzy sliding mode based approach. In COBEM 2005 – Proceedings of the 18th International Congress of Mechanical Engineering, Ouro Preto, Brasil, November 2005.

W. M. Bessa, E. Kreuzer, J. Lange, M. A. Pick, and E. Solowjow. Design and adaptive depth control of a micro diving agent. IEEE Robotics and Automation Letters, 2(4):1871–1877, 2017. doi: 10.1109/LRA.2017.2714142.

W. M. Bessa, G. Brinkmann, D. A. Duecker, E. Kreuzer, and E. Solowjow. A biologically inspired framework for the intelligent control of mechatronic systems and its application to a micro diving agent. Mathematical Problems in Engineering, 2018:1–16, 2018. doi: 10.1155/2018/9648126.

W. M. Bessa, S. Otto, E. Kreuzer, and R. Seifried. An adaptive fuzzy sliding mode controller for uncertain underactuated mechanical systems. Journal of Vibration and Control, 25(9):1521–1535, 2019. doi: 10.1177/1077546319827393.

A. S. De Paula, M. A. Savi, and F. H. I. Pereira-Pinto. Chaos and transient chaos in an experimental nonlinear pendulum. Journal of Sound and Vibration, 294:585–595, 2006.

J. D. B. Dos Santos and W. M. Bessa. Intelligent control for accurate position tracking of electrohydraulic actuators. Electronics Letters, 55(2):78–80, 2019. doi: 10.1049/el.2018.7218.

G. S. Lima, W. M. Bessa, and S. Trimpe. Depth control of underwater robots using sliding modes and gaussian process regression. In LARS 2018 – Proceedings of the Latin American Robotic Symposium, João Pessoa, Brazil, 2018. doi: 10.1109/LARS/SBR/WRE.2018.00012.

G. S. Lima, S. Trimpe, and W. M. Bessa. Sliding mode control with gaussian process regression for underwater robots. Journal of Intelligent & Robotic Systems, 99(3):487–498, 2020. doi: 10.1007/s10846-019-01128-5.

G. S. Lima, D. R. Porto, A. J. de Oliveira, and W. M. Bessa. Intelligent control of a single-link flexible manipulator using sliding modes and artificial neural networks. Electronics Letters, 57(23):869–872, 2021. doi: 10.1049/el.12.12300.

M. A. Savi, F. H. I. Pereira-Pinto, and A. M. Ferreira. Chaos control in mechanical systems. Shock and Vibration, 13(4/5):301–314, 2006.

J.-J. E. Slotine and W. Li. Applied Nonlinear Control. Prentice Hall, New Jersey, 1991.

M. C. Tanaka, J. M. de Macedo Fernandes, and W. M. Bessa. Feedback linearization with fuzzy compensation for uncertain nonlinear systems. International Journal of Computers, Communications & Control, 8(5):736–743, 2013.