3D steerable pyramid decomposition and its application in seismic data processing and interpretation

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Abstract. The application of 3D multi-scale geometric analysis in seismic data processing and interpretation is still in the preliminary stage. The principle of 3D Steerable Pyramid decomposition and reconstruction in frequency domain is analyzed in detail. Spherical low-pass filters and steerable filters are adopted to realize multi-scale and multi-directional decomposition of 3D Steerable Pyramid, and applied to the actual 3D seismic data for random noise attenuation, detection of faults, channels and sand bodies by selecting weighting function in the process of reconstruction. The application of actual 3D seismic data shows that: 1) When the weights are all equal to 1, perfect reconstruction is achieved, the random noise can be removed by threshold method in the decomposition domain of 3D Steerable Pyramid, the denoising results are clean, and the continuity of the event, the characteristics of the reflected wave group and the breakpoint are clearer; 2) Weighting reconstruction can enhance some features. Compared with amplitude and coherent attribute analysis, the data weighting reconstructed by 3D Steerable Pyramid decomposition not only shows faults and channels completely and meticulously, but also presents an even more impressive picture of sand body’s boundary, thus achieving the optimal application effect.

1. Introduction

As a sparse representation of images developed in recent years, multi-scale geometric analysis is adopted to represent and process high-dimensional data, and provide multi-scale and multi-directional representation for them, which has been successfully applied to image denoising, compression, feature extraction and other fields. In seismic data processing, the current multi-scale geometric analysis tools mainly include Curvelet transform, Contourlet transform, Shearlet transform and Steerable Pyramid transform, etc. Partial transforms and their improvements have been widely applied in the field of seismic data processing and have achieved good results. Wang et al. studied the application of an iterative Curvelet thresholding algorithm for seismic random noise attenuation[1]. Liu et al. reconstructed missing seismic data by using compression characteristics of Curvelet transform[2]. Zhang et al. accurately revealed the development characteristics of faults and fractures based on multi-scale characteristics of Curvelet transform[3]. Lin et al. carried out faults detection and random noise attenuation in 2D Steerable Pyramid decomposition domain[4,5]. Geng et al. proposed compression method based on multi-scale Dreamlet transform[6]. Yuan et al. proposed non-dyadic Curvelet transform for suppressing surface wave interference in seismic data[7]. Bai et al. restored the seismic data with high SNR based on the iterative algorithm of steepest descent in the Curvelet transform domain[8]. Xie et al. proposed a seismic multi-attribute fusion method based on Contourlet transform[9]. Liu et al. used Shearlet transform to suppress random noise, and achieved satisfactory
results[10]. Dong et al. constructed a complex Curvelet transform algorithm to effectively separate primary wave and multiples[11]. Chen et al. used multi-scale fracture prediction technology to solve the high-degree nonhomogeneity and multi-scale characteristics of fracture prediction[12]. Zhao et al. highlighted the distribution of small-scale faults and tectonic fracture zones based on Contourlet transform[13]. Li et al. achieved better surface wave separation and suppression than conventional methods using Curvelet threshold iteration method based on energy ratio[14].

The above multi-scale geometric analysis tools are mainly applied to 2D data processing. However, 3D seismic exploration, as for now, remains the main method of oil-gas exploration. In general, the original seismic data obtained are 3D data. Therefore, most of the multi-scale methods used for seismic data processing are still based on 2D data processing, without taking into consideration the 3D information in seismic data when seismic sections or slices are being processed. To apply multi-scale geometric analysis to 3D data processing, these transforms need to be extended in the third dimension. 3D multi-scale geometric analysis is one of the hotspots in the field of multi-scale geometric analysis in recent years. It has been deeply studied in the fields of 3D images, video and geometric models, but is still at the preliminary stage in seismic data processing and interpretation. To apply multi-scale geometric analysis to 3D data processing, these transforms need to be extended in the third dimension.

2. 3D Steerable Pyramid decomposition and reconstruction

2.1. 3D Steerable Pyramid decomposition

Steerable Pyramid decomposition is a multi-scale image decomposer with translation invariance. It can decompose the image into sub-bands with different scales and directions, thus ensuring both translation invariance and direction steering, and duly describing the spatial scale and direction information of the image[20]. The 3D Steerable Pyramid decomposition is accomplished by recursive product operations in the frequency domain[15]. The process for 3D Steerable Pyramid decomposition with three levels and six directions is showed in Figure 1(a). $L_0, L_1, L_2$ are the spherical low-pass filters used to construct the pyramid layers, and $B_0, B_1, B_2, B_3, B_4, B_5$ are the six steerable filters; $P_0, P_1, P_2$ are low-pass filtered data, $q_{00}, q_{01}, q_{02}, q_{03}, q_{04}, q_{05}, q_{10}, q_{11}, q_{12}, q_{13}, q_{14}, q_{15}$ are the directionally filtered data; The remaining data $l$ is the coarse-scale layer and can be decomposed in the direction as required. To obtain more scales, the steps in the dashed black box on the left in the
Figure 1(a) can be called circularly until the requirements are met. Due to the large amount of data decomposed by 3D Steerable Pyramid, second-order down-sampling can be used to reduce the amount of data stored.

\[ L(k_x,k_y,k_z) = \left\{ \begin{array}{ll}
1; & k \leq k_s \\
\frac{1}{2}[1 + \cos(\pi \frac{k - k}{k_s - k})]; & k_s < k < k_s \\
0; & k \geq k_s
\end{array} \right. \]

Where \( k_s \) are respectively the starting point and cut-off point of the tapered domain of the filter, \( k = \sqrt{k_x^2 + k_y^2 + k_z^2} \). In the 3D case, the low-pass filter is a sphere, and a sphere ring is obtained by subtracting two adjacent low-pass filters.

2.1.2. Steerable filter. According to the steering theorems, at least six steerable filters are required in 3D Steerable Pyramid decomposition to obtain filtering responses in any direction if a second-order filter is used. John et al used the diagonal line of cubic octahedron to define the direction of the six basis filters, and defined the coordinate system in the 3D wave-number domain. At the same time, \( \cos^2 \) function is used to define the six basis filters, and the direction cosines are expressed which can be used directly:

\[ B_j(k_x,k_y,k_z) = \frac{(\alpha k_x + \beta k_y + \gamma k_z)^2}{k_x^2 + k_y^2 + k_z^2}, j = 0,1,..,5 \]

Where,
\begin{align}
\alpha_0 &= \frac{\sqrt{2}}{2}, \beta_0 = \frac{\sqrt{2}}{2}, \gamma_0 = 0 \\
\alpha_1 &= \frac{\sqrt{2}}{2}, \beta_1 = -\frac{\sqrt{2}}{2}, \gamma_1 = 0 \\
\alpha_2 &= \frac{\sqrt{2}}{2}, \beta_2 = 0, \gamma_2 = \frac{\sqrt{2}}{2} \\
\alpha_3 &= -\frac{\sqrt{2}}{2}, \beta_3 = 0, \gamma_3 = \frac{\sqrt{2}}{2} \\
\alpha_4 &= 0, \beta_4 = \frac{\sqrt{2}}{2}, \gamma_4 = \frac{\sqrt{2}}{2} \\
\alpha_5 &= 0, \beta_5 = \frac{\sqrt{2}}{2}, \gamma_5 = -\frac{\sqrt{2}}{2}
\end{align}

In this case, the sum of all filters results in a constant:
\[
\sum_{j=0}^{5} B_j = 2
\]

A 3D data superimposed by an amplitude time slice is showed in Figure 2, which has been processed by frequency division and the faults are relatively clear. According to the size of data, it is decomposed into two layers and six directions which is conducted in Matlab software, and second-order down-sampling isn’t used during the decomposition, and the remaining high-pass part isn’t displayed. The starting and cut-off points of the spherical low-pass filter \(L_0\) used in the pyramid decomposition are \(k_0 = 0.6\pi, k_0 = \pi\), and the value of the next layer is half that of the previous layer. Through decomposition, the distribution characteristics of faults can be observed in different scales and directions. The decomposed property maps are very useful for detecting and highlighting geological bodies with a certain distribution direction, such as channels and faults.

![Figure 2. 3D Steerable Pyramid decomposition for synthetic 3D data.](image)

((a) Synthetic 3D data, (b),(c),(d),(e),(f),(g) correspond to directionally filtered images \(B_0, B_1, B_2, B_3, B_4, B_5\) of the first layer respectively, (h),(i),(j),(k),(l),(m) correspond to directionally filtered images \(B_0, B_1, B_2, B_3, B_4, B_5\) of the second layer respectively)
2.2. 3D Steerable Pyramid reconstruction

The reconstruction process is in reverse order to the decomposition process, as shown in Figure 1(b). The filtered images in all directions of each layer are multiplied by their respective weighting function. If second-order down-sampling is adopted during decomposition, the data of the next layer need to be added to the data of the previous layer through second-order up-sampling to obtain the reconstructed image. If the second-order down-sampling is not adopted during decomposition, the second-order up-sampling is not required in reconstruction. If the weighting functions \( w_j \) are all equal to 1, that is, without weighting reconstruction, the amplitude of the output image is twice that of the input image, which is determined by formula (4); if the weighting functions are used, the output image is different from the input image, and certain features of the image (such as faults, channels, sand bodies, etc.) are enhanced.

2.2.1. Calculation of weighting function. According to the steering theorems [20], the weighting functions of the 3D steerable filters can be calculated through formula (5):

\[
\begin{pmatrix}
\alpha^2 \\
\alpha \beta \\
\alpha \gamma \\
\beta^2 \\
\beta \gamma \\
\gamma^2 
\end{pmatrix} =
\begin{pmatrix}
\alpha_0^2 & \alpha_1^2 & \alpha_2^2 & \alpha_3^2 & \alpha_4^2 \\
\alpha_0 \beta_0 & \alpha_1 \beta_0 & \alpha_2 \beta_0 & \alpha_3 \beta_0 & \alpha_4 \beta_0 \\
\alpha_0 \gamma_0 & \alpha_1 \gamma_0 & \alpha_2 \gamma_0 & \alpha_3 \gamma_0 & \alpha_4 \gamma_0 \\
\beta_0^2 & \beta_1^2 & \beta_2^2 & \beta_3^2 & \beta_4^2 \\
\beta_0 \gamma_0 & \beta_1 \gamma_0 & \beta_2 \gamma_0 & \beta_3 \gamma_0 & \beta_4 \gamma_0 \\
\gamma_0^2 & \gamma_1^2 & \gamma_2^2 & \gamma_3^2 & \gamma_4^2 
\end{pmatrix} w_j
\]

The calculated weighting functions satisfy:

\[ \sum_{j=0}^{s} w_j = 1 \]  

Therefore, a single layer can be reconstructed by the product of directionally filtered images and weighting functions:

\[ q(x, y, z) = \sum_{j=0}^{s} w_j (x, y, z) q_j (x, y, z) \]

The reconstructed image as the weighting functions are all equal to 1 is showed in Figure 3(a), while the weighting reconstructed image is showed in Figure 3(b), in both the outermost high-frequency information is removed. Comparing the two images, it can be seen that the fault characteristics have been retained and enhanced to a great extent after weighting reconstruction, and the whole image appears relatively clean, almost no redundant information except the faults. The slice of the data in Matlab shows that the occurrence and extension range of the faults in 3D space are also clearer.

![Reconstructed image as the weights are all equal to 1](image1)

(a) Reconstructed image as the weights are all equal to 1

![Weighting reconstructed image](image2)

(b) Weighting reconstructed image

**Figure 3.** Fault detection in 3D Steerable Pyramid decomposition domain for synthetic data.
3. Application of 3D Steerable Pyramid

YA area is located in the south of an Oilfield. Triassic system reservoir is the main oil and gas bearing formation, which belongs to a set of braided delta-lacustrine facies deposits, with channel microfacies sand bodies developed and low amplitude anticline lithologic traps controlled by lithology and faults. Therefore, it is particularly important to accurately depict the boundaries of faults, channels and sand bodies and the inside of sand bodies. The random noise attenuation is firstly carried out on the 3D seismic data in YA area in 3D Steerable Pyramid decomposition domain, and then the weighting reconstruction is used to finely describe faults, channels and sand bodies, which has brought optimal application effect.

3.1. Application of 3D Steerable Pyramid decomposition in random noise attenuation

Due to the multi-scale and multi-directional characteristics of 3D Steerable Pyramid decomposition and its direction steering, the edge and texture information of the 3D data can be soundly represented. When applied to the 3D seismic data, it can effectively distinguish the texture and noise, and boasts advantages of 3D data denoising[18].

The intercepted typical original amplitude data of YA area is showed in Figure 4(a), with obvious random noise and relatively lower signal-to-noise ratio, which affects the subsequent interpretation. The data is decomposed into three layers and six directions, and the adaptive reduced hard threshold (the threshold is reduced to 1/2) plus mean filtering[18] are adopted on the sub-band of each layer and direction. The weighting functions are all equal to 1 during reconstruction to obtain the denoising data, as shown in Figure 4(b). Contrast Figure 4 (a), Figure 4 (b) and Figure 4 (c), it can be seen that in the 3D Steerable Pyramid decomposition domain, the adaptive reduced hard threshold plus mean filtering denoising method can effectively remove the random noise, with clear continuity of seismic events, clear wave group relationship, clear fault block imaging and clear-cut breakpoint, and there is basically no event information in the difference data except the random noise removed.

(a) The original data (b) The denoising result (c) The difference data

Figure 4. The results of adaptive reduced hard threshold(1/2)plus mean filtering denoising in 3D Steerable Pyramid domain.

3.2. Application of 3D Steerable Pyramid decomposition in sand body imaging and fault detecting

Weighting reconstruction adopted in 3D Steerable pyramid decomposition domain can effectively highlight the features and distribution of faults, channels, sand bodies, etc.

An example of actual 3D seismic data detection in YA area is showed in Figure 5, and each figure is a display on the target layer. The original amplitude image is showed in Figure 5(a). The data is decomposed into three layers and six directions based on 3D Steerable Pyramid, and the outermost high-frequency information is removed during weighting reconstruction. The weighting reconstructed image is showed in Figure 5(b). For comparison, SMT software is used for coherence attribute analysis, as shown in Figure 5(c). Contrast Figure 5(a), Figure 5(b) and Figure 5(c), the edge information of faults and channels is displayed in a complete and detailed way through the two processing methods. Moreover, the 3D Steerable Pyramid decomposition and reconstruction is obviously superior to the amplitude and coherent attribute analysis in depicting the sand body’s
boundary, and the magnitude shows that the result generated from 3D Steerable Pyramid processing is no other but the fault and the sand body themselves, which can be further processed on this basis.

Figure 5. An example of actual 3D seismic data detection in YA area.

For further comparison, block YA3 in Figure 5(a) is extracted. Similarly, the data is decomposed into three layers and six directions based on 3D Steerable Pyramid, and the outermost high-frequency information is removed during weighting reconstruction. The original amplitude image is showed in Figure 6(a), the weighting reconstructed image is showed in Figure 6(b), and the coherence attribute map is showed in Figure 6(c). It can be seen from the comparison of various figures that both 3D Steerable Pyramid decomposition reconstruction and coherence attribute analysis can ensure the display of such edge information as faults, but the former is better in describing the boundary and interior of sand body.

Figure 6. An example of actual 3D seismic data detection in YA3 area.

4. Conclusion
The principle of 3D Steerable Pyramid decomposition and reconstruction is analyzed. After 3D Steerable Pyramid decomposition, the input 3D data can be used to analyze the characteristics and distribution in different scales and directions. Reconstruction methods vary each other in terms of applications. When the weighting function are all equal to 1, perfect reconstruction can be used for noise attenuation, and such information of certain enhanced features as faults, channels, sand bodies, etc. can be obtained through weighting reconstruction. In this Paper, examples are given to illustrate the application of 3D Steerable Pyramid decomposition in 3D seismic data processing. The results
show that in the 3D Steerable Pyramid decomposition domain, the adaptive reduced hard threshold plus mean filtering denoising method can effectively remove random noise, with clean denoising section and slice, clear continuity of the event, clear wave group relationship, clear fault block imaging and clear-cut breakpoint; weighting reconstruction in 3D Steerable Pyramid decomposition domain is used to enhance faults, channels and sand bodies, and thereby clear fault and sand body information is obtained, which shows the characteristics of these geological types in a complete and meticulous way.

Since 3D Steerable Pyramid decomposition is characterized by multi-scale and multi-direction, it can detect the edge and texture information of data in a more effective way, and can sensitively pick up directional information in 3D seismic data. Therefore, it has excellent practicability in noise attenuation and detection of faults, channels and sand bodies, and can be used together. In the design of the whole algorithm, there is no selection of special parameters, which is convenient for the processing and interpretation personnel.

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