Investigation of critical mathematical thinking ability, visual thinking and self-efficacy students’ in trigonometry

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Abstract. This study aims to investigate how students’ the ability of critical mathematical thinking ability, visual thinking and self-efficacy in the current trigonometry. Data were collected through the students’ answers, self-efficacy questionnaire and interview. The results obtained are then analyzed to see the students’ ability level in trigonometry. The result showed the students do not yet have the critical thinking ability are good but have good visual thinking ability in trigonometry. This is because students do not have the concept of underlying problem solving well. Furthermore, students feel less confident in their mathematical ability. This is evidenced of the analysis self-efficacy questionnaire. The results of this study encourage us as prospective teachers to find teaching methods that can help students understand the concepts of trigonometry well so that students love and eager to solve the problem of trigonometry.

1. Introduction

Trigonometry is a branch of mathematics that studies relationships involving the length and angle of a triangle. Trigonometry is a subject matter that students believe is very difficult and abstract compared to other materials so very few students who like this subject matter [1]. Orhun in his research investigating mistakes and misconceptions of trigonometry on student’s. He found that students did not develop the concept of trigonometry and they made some mistakes. When the basic concepts are not learned in the early stages, the learning process of the next stage becomes more difficult because mastery of the basic mathematical concepts is an important step in the development of concepts [2]. This is in line with the opinion Hudoyo argued that in mathematics if the concept of A and concept B under the concept of C then the concept of C cannot be studied before the concepts A and B studied first. Similarly, the new D concept can be learned when the concept of C has been understood, etc [3].

Students’ level of thinking consists of two parts: lower-order thinking and higher-order thinking. Lower-order thinking characterized as the recall of information or the application of concepts to familiar situations or applying algorithms familiar and higher-order thinking characterized as solving a non-routine problem and thinking skills of interpreting, analyzing or be able to manipulate previous information so that it is not monotonous [4]. Critical thinking is one of the higher-order thinking ability. Pagano and Roselle stated that critical thinking is a systematic, directed and efficient process in developing problem-solving skills through the process of evaluating relevant information and opinions collected in the reflection phase [5]. The ability to think critically is also an effective way to improve students’ understanding of mathematical concepts [6]. Mathematical critical thinking indicator in this
research derived from Ennis are: (1) Examining the truth of the argument, statement and solution process with reason and (2) Compile an answer or solve mathematical problem with reason [7].

The ability to visualize what cannot be seen directly is considered a critical skill in mathematics and science. Student understanding can be enhanced by using visualization simulation of a varied student thinking rather than just focus on the symbols and the formal approach [8]. Rosken and Rolka states that visual thinking in mathematics learning can be used to develop problem-solving and to give meaning to mathematical concepts and the relationship between them [9].

In addition to the ability to think critically mathematical and visual thinking, other research focus is one aspect of affective self-efficacy. Bandura states that self-efficacy is a reflection of an individual's belief about his ability to complete the task of achieving the result set [10]. Furthermore, Bandura states that the degree of self-efficacy refers to three dimensions: magnitude, strength and generality. Bong explains the three dimensions: (1) Dimensions magnitude indicates that the individual can complete the task, (2) Dimension strength reflects individual beliefs in completing tasks with varying degrees of difficulty and (3) Dimensions generality shows the breadth and level of accomplishment of accomplishing tasks [10]. Based on the above explanation, the purpose of this research is to investigate how students’ the ability of critical mathematical thinking ability, visual thinking and self-efficacy in the current trigonometry.

2. Method
This research is descriptive research using qualitative method. Subjects in this study involved 3 senior high school grade 11th students selected by accidental sampling. The research instruments are test instrument and non test instrument.

The test instrument is a question based on the ability to think critically and involves the ability of visual thinking that is to arrange mathematical models in the form of drawings and solve mathematical problems with the reasons of trigonometric material. Non test instruments are self-efficacy questionnaires and interviews. From the test results, researchers investigate the ability to think critically mathematically and students’ visual thinking. Questionnaire self-efficacy to see students’ confidence in their ability to solve the problem given. The results of the interviews were used to obtain information on students’ opinions about trigonometry.

3. Results and discussion
This section contains a discussion of the results of the study in the form of description of critical thinking ability of mathematics and visual thinking of students in trigonometry, the result of students’ answers is related to student self-efficacy, and the result of student opinion interview about trigonometry.

3.1. Analysis of the first question
A square garden $ABCD$ arbitrary with $AB = 20 \text{ m}$, $BC = 32 \text{ m}$ and $CD = 60 \text{ m}$. If the area of the triangle $DBC = 480 \sqrt{3} \text{ m}^2$ and $\angle DBA = 60^\circ$ calculate the area of the garden. Describe the situation above and then write down the steps to solve the problem with an explanation of the concept that underlies your answer.

![Figure 1. Results of students answers to the first question.](image)
Figure 1 shows that three students were able to identify the information given and then communicate it to the image. Images made by the three students correspond to the information provided. This means for the first question, the three students have good visual thinking skills.

Furthermore, the three students identified the calculation steps and solved them but were not accompanied by the concepts used in solving the problem. The answer of the first and second students is correct but the third student is not correct because the student assumes that the square is rectangular. This means the three students are only able to solve the problem but do not yet have good critical thinking skills because it does not provide the underlying explanation of the answer.

3.2. Analysis of the second question
Given a right triangle of $PQR$ with a right elbow in $Q$ and $\angle PRQ = 75^\circ$. $T$ is an extension point of $QR$ with $\angle PTR = 30^\circ$, $PT \perp RS$ and length $ST = \sqrt{6}$ cm. Describe the situation above then compute the length of the line $PQ$ and $\sin 75^\circ$ using ratio of trigonometry. Compare the value of $\sin 75^\circ$ obtained using the ratio of trigonometry with the sum and difference formulas of two angles. Explain the concept that underlies your answer.

![Figure 2](image.jpg)

Figure 2. Results of student answers to the second question.

Figure 2 shows that three students were able to identify the information given and then communicate it to the image. Images made by the three students correspond to the information provided. This means that for the second question, the three students have a good visual thinking skills.

The first student did not continue the problem-solving process. This means that students are only able to arrange math problems into image form but have not been able to solve the problem given. This means that the first student does not have the ability to think critically well. The second and third students identify the calculation steps and solve them but are not accompanied by the concepts used in solving the problem. The two students’ answers are not finished yet. Both students have been able to apply the concept of sinus rules in solving the problem. This means the two students have not had good critical thinking skills because they do not provide the underlying explanation for the answer.

3.3. Analysis of the third question
In the $PQR$ triangle, it is known that angle bisector $R$ intersect $PQ$ at M point. Jika $PR = RM = a$ and $PM = \frac{1}{2} a$ implication length $MQ = \frac{2}{3} a$. Describe the above situation then check whether the above statement is true? Explan of the concept that underlies your answer.
Figure 3 shows that three students were able to identify the information given and then communicate it to the image. This means that for the third question, they have good visual thinking skills.

The three students propose the same opinion that if the angular bisector of the triangle then the divided side has the same side length. This is contradicted by the definition of angle bisector, the line drawn from the vertex and dividing the angle into two equal parts [11]. If the line drawn from the vertex to the mid side in front of it then the side divided into two sides is equal, the line is called perpendicular bisector [11]. But the perpendicular bisector does not necessarily divide the angle equally. The perpendicular bisector will divide the angle equally when the triangle is an equilateral triangle of foot and equilateral. This means that for the third question, the student has the ability to think critically is good enough for them.
3.4. Analysis self efficacy questionnaire students

Here is the result of the student self-efficacy description:

| No. | Pernyataan                                                                 | SS  | S   | TS  | STS |
|-----|----------------------------------------------------------------------------|-----|-----|-----|-----|
| 1   | Saya mampu menyelesaikan masalah trigonometri yang diberikan dengan baik   |     |     |     | ✔  |
| 2   | Saya bersemangat ketika menyelesaikan masalah trigonometri yang sulit       | ✔   |     |     |     |
| 3   | Saya memikirkan dengan matang sebelum menyelesaikan soal trigonometri      | ✔   |     |     |     |
| 4   | Saya mencoba berkomunikasi dengan teman saat mengalami kesulitan dalam memahami konsep trigonometri | ✔   |     |     |     |
| 5   | Saya kesulitan menyelesaikan soal yang berbeda dari yang didengarkan oleh guru | ✔   | ✔  |     |     |
| 6   | Saya percaya diri atas kemampuan matematika yang saya miliki               | ✔   | ✔  |     |     |
| 7   | Saya bersedia menyelesaikan soal matematika di depan kelas                 | ✔   | ✔  | ✔  | ✔  |
| 8   | Saya menyerah menyelesaikan masalah matematika yang berhubungan dengan trigonometri | ✔   |     |     |     |
| 9   | Saya mengasah kemampuan matematika secara rutin                           | ✔   | ✔  | ✔  | ✔  |
| 10  | Saya menghindar mencoba cara yang berbeda dengan contoh dari guru         | ✔   | ✔  | ✔  | ✔  |

Figure 4. Results of students answers to the self-efficacy questionnaire.

Figure 4 shows that student self-efficacy analysis refers to three dimensions according to Bandura [10]. Based on the answers of the first and second statements, the first student does not have an optimistic and less enthusiastic view of the task of trigonometry. She acts selectively in achieving his goals. It is seen from the answers to the third statement. She did not despair when solving trigonometric problems. It is seen from the answers to the eighth statement. She made an effort to improve his performance well. It is seen from the answer to the fourth statement. She lacks confidence in his abilities but is willing to try new challenges and deal with things differently. It is seen from the statement answers the sixth, tenth and fifth. She has not had a good self-motivation for his own development. It is seen from the answers of the ninth statement. She has the courage to deliver the results obtained. It is seen from the seventh statement.

Based on the answers of the first, second and eighth statements, the second student has not had an optimistic view and is less enthusiastic about the task of trigonometry. She acts selectively in achieving his goals. It is seen from the answers to the third statement. She made an effort to improve his performance well. It is seen from the answer to the fourth statement. She lacks confidence in his ability, has not responded to anything different from the good and does not want to try new challenges. It is seen from the statement answers the sixth, fifth and tenth. She has not had a good self-motivation for his own development. It is seen from the answers of the ninth statement. She has the courage to deliver the results obtained. It is seen from the seventh statement.

Based on the answers of the first, second, sixth and eighth statements, the second student has an optimistic and confident view of his or her own ability but is less enthusiastic about the task of trigonometry. She acts selectively in achieving his goals. It is seen from the answers to the third statement. She made an effort to improve his performance well. It is seen from the answer to the fourth statement. She has not responded to anything different but is willing to try new challenges. It is seen from the statement answers the sixth, fifth and tenth. She has a good self-motivation for his development. It is seen from the answers of the ninth statement. She had the courage to present results. It is seen from the statement of the seventh.
3.5. Student opinions about trigonometry
The results of the interviews are used to obtain information about trigonometry student opinion. The first student does not like trigonometry because the way the teacher teaches at school is sometimes like just teaching because the students are too much so the atmosphere is less conducive. According to the first student, solving the problem by putting the mathematical model into the drawing form helps understand the problem easily.

The second student did not like trigonometry for complex calculations. According to the second student, to solve the problem by compiling a mathematical model in the form of pictures to help understand the problem easily as something abstract visible concrete.

The third student likes trigonometry when the given problem is easy but she does not like trigonometry when the given problem is difficult. According to the third student, can understand the problem easily by making the image in solving the problem because the image can be more clear than just illustrated.

4. Conclusions
From the results described in the previous section, we draw the following conclusions. The result showed the students do not yet have the critical thinking skills are good but have good visual thinking skills in trigonometry. At present there are still students who do not like trigonometry. Some students lack confidence in their mathematical skills and less honed their math skills regularly but dare to convey the results of answers obtained. This encourages us as prospective teachers to find learning methods that can help students understand the concept of trigonometry well so that students love and are excited about solving trigonometric problems.

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References
[1] Gur H 2009 Trigonometry Learning New Horizons in Education 57 67-80
[2] Usman M H and Hussaini M M 2017 Analysis of Students’ Error in Learning of Trigonometry among Senior Secondary School Students in Zaria Metropolis, Nigeria IOSR Journal of Mathematics 13 01-04
[3] Miksalmina M 2013 Penguasaan Siswa pada Materi Trigonometri di MAN Darussalam Aceh Besar Visipena 4 101-110
[4] Thompson T 2008 Mathematics Teachers’ Interpretation of Higher-Order Thinking in Bloom Taxonomy International Electronic Journal of Mathematics Education 3 96-109
[5] Prayitno A 2018 Characteristics of Students’ Critical Thinking in Solving Mathematics Problem The Online Journal of New Horizons in Education 8 46-55
[6] Chukwuyenum A N 2013 Impact of Critical Thinking on Performance in Mathematics among Senior Secondary School Student in Lagos State IOSR Journal of Research and Methode in Education 3 18-25
[7] Hendriana H and Soemarmo U 2017 Penilaian Pembelajaran Matematika (Bandung: Refika Aditama) pp 82
[8] Kashefi H, Alias N, Kahar M F, Buhari O and Othman S Z 2015 Visualisasion in Mathematics Problem Solving Meta-Analysis Research E-Proceeding of the International Conference on Social Science Research 8-9 June 2015 (Malaysia: Melia Hotel) p 803-812
[9] Rosken B and Rolka K 2006 A Picture is Worth A 1000 Words - The Role of Visualization in Mathematics Learning In Novotna J, Moraova H, Kratka M and Stehlikova N (Eds.) Proceedings 30th Conference of the International Group for the Psychology of Mathematics Education (Prague: PME) 4 457-464
[10] DeNoyelles A, Hornik S R and Johnson R D 2014 Exploring the Dimensions of Self-Efficacy in
Virtual World Learning: Environment, Task and Content *MERLOT Journal of Online Learning and Teaching* **10** 255-271

[11] Amarasinghe IS 2012 On the Standard Length of angle Bisector and the Angle Bisector Theorem *Global Journal of Advanced Research on Classical and Modern Geometries* **1** 15-27