We consider the dynamics of a single electron in a chain of tunnel coupled quantum dots, exploring the formal analogies of this system with some of the laser-driven multilevel atomic or molecular systems studied by Bruce W. Shore and collaborators over the last 30 years. In particular, we describe two regimes for achieving complete coherent transfer of population in such a multistate system. In the first regime, by carefully arranging the coupling strengths, the flow of population between the states of the system can be made periodic in time. In the second regime, by employing a “counterintuitive” sequence of couplings, the coherent population trapping eigenstate of the system can be rotated from the initial to the final desired state, which is an equivalent of the STIRAP technique for atoms or molecules. Our results may be useful in future quantum computation schemes.

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I. INTRODUCTION

Population transfer in multistate quantum systems has been an active topic of research over the last half a century. In the context of atomic and molecular physics, coherent population transfer in optically-driven multilevel systems has been studied since the invention of lasers (1). Usually, the objective is to transfer the population from the initial to a well defined final state of the atom or molecule, via one or more intermediate states, while minimizing the loss of population through or its accumulation on the intermediate states. In early theoretical work, Shore and collaborators have studied population transfer in multilevel systems driven by resonant laser fields (2, 3, 4). In particular, they have found that it is possible to arrange the coupling strengths between the adjacent states in such a way that the system becomes analogous to a spin-J in a magnetic field, whose dynamic evolution is known to be periodic for any J \( \geq 3 \). This coupling scheme was therefore named spin-coupling.

Later, Hioe, Eberly, Bergmann and collaborators discovered the technique of stimulated Raman adiabatic passage (STIRAP) for three-level atomic/molecular systems (5). They have identified a specific eigenstate of the system, the so-called coherent population trapping (CPT) state, which contains a superposition of the initial and final states, and dates back to Alzetta et al. and Arimondo and Orrisio (6). The STIRAP technique is then based on first preparing the system in its initial bare state, which coincides with the CPT state, and then adiabatically rotating the CPT state towards the desired final bare state of the system. This technique has been subsequently polished (7) and extended to multilevel systems (8, 9, 10) with the active participation of Bruce W. Shore.

While the above studies were conducted in the context of multilevel atoms or molecules, here we show that similar effects can be found in the context of quantum transport in arrays of tunnel-coupled quantum dots (11, 12, 13, 14). Often referred to as artificial atoms, semiconductor quantum dots offer an unprecedented possibility of constructing at will and exploring situations ranging from practically single atom to a fully solid state many-body systems (15). The nanofabrication possibilities of tailoring structures to desired geometries and specifications, and controlling the number and mobility of electrons confined within a region of space, are some of the features that make these structures unique tools for the study of a variety of preselected set of phenomena, including the coherent population transfer in multistate systems.

Given the controllable quantum properties of the electrons in such structures, the possibility of their application to schemes of quantum computers (QCs) (16) has not escaped attention (17, 18, 19). The qubits of the QD-array based QC would be represented by the spin-states of single electrons confined in individual QDs, with the two-qubit nearest-neighbor coupling mediated by the controlled spin-exchange interaction (17, 18). One of the main difficulties with the existing proposals for integrated solid-state based QCs is that there is no efficient way of transferring the information between distant qubits. We consider here a single-electron tunneling in a one-dimensional array of QDs and establish the conditions under which the complete transfer of the electron wavepacket between two distant locations can be achieved. Our findings could therefore be relevant to the reliable information exchange between distant parts of an integrated quantum computer (20).

In Section II we outline the mathematical formalism describing a chain of QDs, in terms of which, in Section III we present the theory of coherent propagation and periodic oscillations of the electron wavepacket between the two ends of the chain. The single-electron transfer via an equivalent of multistate STIRAP is discussed in Section IV. In Section V we describe an envisioned implementation of a scalable quantum computer, followed by the concluding remarks.

*This paper is dedicated to Bruce W. Shore on the occasion of his 70th birthday.
2.

II. MATHEMATICAL FORMALISM

We consider electron transport in a linear array of $N$ nearly identical QDs which are electrostatically defined in a two-dimensional electron gas by means of metallic gates on top of a semiconductor heterostructure (GaAs/AlGaAs)\,\cite{11,12}. This system is described by the extended Mott-Hubbard Hamiltonian\,\cite{12,13}, which in its most general form is given by

$$ H = \sum_{j,\alpha} \varepsilon_{j\alpha} a_{j\alpha}^\dagger a_{j\alpha} + \frac{1}{2} \sum_j n_j (n_j - 1) $$

$$ + \sum_{i<j,\alpha} t_{ij,\alpha} (a_{i\alpha}^\dagger a_{j\alpha} + a_{i\alpha} a_{j\alpha}^\dagger) + \sum_{i<j} V_{ij} n_i n_j, $$

where $a_{j\alpha}^\dagger$ and $a_{j\alpha}$ are the creation and annihilation operators for an electron in state $\alpha$ with the single-particle energy $\varepsilon_{j\alpha}$, $U$ is the on-site Coulomb repulsion, $n_j = \sum_\alpha a_{j\alpha}^\dagger a_{j\alpha}$ the total electron number operator of the $j$th dot, $t_{ij,\alpha}$ are the coherent tunnel matrix elements between dots $i$ and $j$, and $V_{ij}$ is the interdot electrostatic interaction. In general, the index $\alpha$ refers to both orbital and spin states of an electron. In the tight-binding regime, when the on-site Coulomb repulsion and single-particle level-spacing $\Delta \varepsilon$ are much larger than the tunneling rates, $U > \Delta \varepsilon \gg t_{ij,\alpha}$, only the equivalent states of the neighboring dots are tunnel-coupled to each other\,\cite{21}. In the absence of a magnetic field, we can thus limit our consideration only to a single doubly- (spin-) degenerate level per dot ($\alpha \in \{\uparrow, \downarrow\}$), assuming further that the tunneling rates do not depend on the electron spin.

In this paper we are concerned with single-electron dynamics, considering a situation in which the environment is initially doped with one mobile electron, while all of the other dots of the chain are empty, as indicated in Fig. 1. Our aim is to determine the conditions under which the complete coherent transfer of the electron between the two ends of the chain can be achieved. The population transfer in this system is mediated by the tunneling between the neighboring QDs. The individual tunneling rates $t_j \equiv t_{j+1}$ are determined by the voltages applied to the gates defining the corresponding interdot tunneling barriers. A chain of $N$ tunnel-coupled QDs doped with a single electron is described by the following Hamiltonian,

$$ H_{1e} = \sum_{j,\alpha} \varepsilon_{j\alpha} a_{j\alpha}^\dagger a_{j\alpha} + \sum_{j,\alpha} t_j (a_{j\alpha}^\dagger a_{j+1,\alpha} + a_{j\alpha} a_{j+1,\alpha}^\dagger), $$

which obviously does not contain terms responsible for electrostatic interactions. Since this Hamiltonian preserves the electron number and its spin, the total state-vector of the system reads

$$ |\psi(\tau)\rangle = \sum_{j,\alpha} A_{j\alpha}(\tau) |j\alpha\rangle, $$

where $|j\alpha\rangle \equiv a_{j\alpha}^\dagger |01,\ldots,0_N\rangle$ denotes the state with one electron having spin $\alpha$ at the $j$th dot. The time-evolution of the system is governed by the Schrödinger equation

$$ i \frac{dA_{j\alpha}}{d\tau} = \varepsilon_j A_{j\alpha} + t_{j-1} A_{j-1\alpha} + t_j A_{j+1\alpha}, $$

where $t_0 = t_N = 0$. Obviously, the two sets of these amplitude equations with $\alpha = \uparrow$ and $\alpha = \downarrow$ are equivalent and decoupled from each other. As a result, if the electron is prepared in an arbitrary superposition of spin up and spin down states, $|\psi\rangle = A_j^\uparrow |j\uparrow\rangle + A_j^\downarrow |j\downarrow\rangle$, the two parts of the wavefunction evolve symmetrically and independently of each other. This assertion is valid as long as all the uncontrollable spin-flip processes are vanishingly small on the time scale of $t^{-1}$. In semiconductor QDs, the spin decoherence originates mainly from the spin-phonon coupling, as well as the coupling of the electron spin with the nuclear spins of the surrounding crystal (hyperfine interaction) or stray magnetic fields. The first decoherence mechanism is suppressed at low temperatures\,\cite{21}, at which the density of crystal phonons is negligible\,\cite{22}. For the uncontrollable hyperfine interactions, experimental measurements indicate spin-relaxation times in excess of 100 $\mu$s, which can be further improved by applying moderate magnetic fields or polarizing the nuclear spins\,\cite{23}. Another mechanism for decoherence in the process of electron (charge) transfer in our system originates from the structure imperfections and gate voltage fluctuations, which cause uncertainty in the intradot energy levels and interdot couplings. These fluctuations, however, are typically slow on the time scale of $t^{-1}$, and the resulting disorder in the system may be considered frozen during its dynamic evolution, as we have discussed in a previous publication\,\cite{21}.

Let us write the Hamiltonian for the electron with spin $\alpha$ in the matrix form

$$ H_{1e}^\alpha = \begin{pmatrix} \varepsilon_1 & 0 & \cdots & \cdots & 0 \\
1 & \varepsilon_2 & t_2 & \cdots & t_2 \\
0 & t_2 & \varepsilon_3 & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots & \cdots \\
0 & \cdots & \cdots & \cdots & \varepsilon_{N-1} \\
0 & \cdots & \cdots & \cdots & t_{N-1} \\
0 & \cdots & \cdots & \cdots & \varepsilon_N \end{pmatrix}, $$

which is obviously tridiagonal. Inspection of the amplitude equations\,\cite{4} or the Hamiltonian\,\cite{4} indeed verifies that our system is formally analogous to the laser-driven multilevel atomic or molecular systems studied by Shore and coworkers\,\cite{2,3,4} and Bergmann, Shore and others\,\cite{5,6,7,8}. Here, the tunneling rates $t_j$ between states...
(|j⟩ and |j + 1⟩ play the same role as the Rabi frequencies of the laser fields acting on the atomic transitions |j⟩ ← |j + 1⟩, while the energies εj of states |j⟩ correspond to the cumulative detunings of the atomic levels. In the following Sections, we describe two methods for achieving complete population transfer from the initial |1⟩ to the final |N⟩ state of the system, which turn out to be the counterpart of those in Refs. [3] and [10].

III. PERIODIC OSCILLATIONS OF POPULATION BETWEEN THE TWO END STATES

In this Section we consider the electron wavepacket dynamics in the chain with static couplings between the dots. Assume that at time τ = 0 the electron is localized on the first dot, |ψ(0)⟩ = |1α⟩, and the tunnel couplings are switched on. This switching should be fast enough on the time scale of t−1, so that no appreciable change in the initial state of the system occurs during the switching time τsw, but slow on the time scale of ε−1, so that no nonresonant coupling between the dots is induced: ε−1 < τsw < t−1. The aim is to determine the set of couplings between the states of the systems which will achieve a complete transfer of the electron population from the initial to the final dot.

To determine the time-evolution of the state vector we need to solve the eigenvalue problem  with which will yield the eigenvalues λk and corresponding eigenvectors |ψk⟩ of the Hamiltonian [3]. The state vector |ψ(τ)⟩ at any time τ ≥ 0 is given by

\[ |ψ(τ)⟩ = \sum_k^N e^{-iλ_kτ} |ψ_k⟩⟨ψ_k|ψ(0)⟩ = \sum_j^N A^α_j(τ) |jα⟩. \]  

(6)

Note that the matrix in Eq. 6 has the form of the tridiagonal Jacobi matrix. It is natural to first consider the case of equal tunneling rates between the dots: t_j = t. Assuming equal energies ε_j = ε and making the transformation \( A^α_j \rightarrow A^α_j e^{iετ} \), which is equivalent to the interaction picture, we find that the determinant \( D_N(λ) \equiv \text{det}(H^α_0 - λI) \) is identical to the Chebyshev polynomial of the second kind, which can be expressed as

\[ D_N(λ) = \prod_{k=1}^N (λ - λ_k). \]

The eigenenergies of the system are then given by the roots of this polynomial, namely

\[ λ_k = 2t \cos \left( \frac{kπ}{N+1} \right), \]

while the corresponding eigenvectors are

\[ |ψ^α_k⟩ = \sqrt{\frac{2}{N+1}} \sum_j^N \sin \left( \frac{jkπ}{N+1} \right) |jα⟩. \]

Using Eq. 6 and the initial conditions \( A_1^α = 1 \) and \( A_j^α = 0 \) for \( j = 2, 3, \ldots, N \), we obtain the solutions for the amplitudes as,

\[ A^α_j = \frac{2}{N+1} \sum_{k=1}^N \exp \left[ -i2tτ \cos \left( \frac{kπ}{N+1} \right) \right] \times \sin \left( \frac{jkπ}{N+1} \right) \left( \frac{kπ}{N+1} \right). \]  

(7)

It is thus evident that the eigenstates of the coupled system oscillate with incommensurate frequencies corresponding to the roots \( λ_k \) of \( D_N \), which in fact become increasingly densely spaced with increasing \( N \). As a consequence, the system never revives fully to its initial state, as is illustrated in Fig. 2a.

Clearly, it is highly desirable to tailor the parameters of the system so as to achieve a non-dispersive transfer of the single-electron wavepacket between the two ends of the chain. Recall from the theory of angular momentum that a spin-J particle subject to a constant magnetic field exhibits Larmor precession about the field direction. In particular, if one chooses the quantization direction along an axis perpendicular to the magnetic field direction and prepares the particle in its lowest spin eigenstate \(|J, M = −J⟩\), it will oscillate between this initial and the final state \(|J, M = J⟩\) in a perfectly periodic way. The matrix elements for the transitions \(|J, M⟩ \rightarrow |J, M + 1⟩\) between the neighboring states are proportional to \( \sqrt{(J-M)(J+M+1)} \). It is therefore clear that with the appropriate choice of the interdot tunneling matrix elements, the dynamics of the single-electron in a chain of QDs can mimic that of a spin-J in a magnetic field. Indeed, if we formally set \( N = 2J + 1 \) and \( j = J + M + 1 \), the tunneling rates \( t_j \) should be arranged according to \( t_j = t\sqrt{(N-j)} \) for \( j = 1, \ldots, N-1 \). Then again, by exploring the properties of the Jacobi polynomials, we find equally spaced eigenenergies of the system,

\[ λ_k = t(2k - N - 1), \]

while the corresponding eigenvectors can be expressed through the rotation matrices commonly used in the representation theory of angular momentum. With the initial conditions \( A_1^α = 1 \) and \( A_j^α = 0 \) for \( j = 2, 3, \ldots, N \), for the amplitudes of the state-vector [3], we then obtain simple analytic expressions given by the binomial form

\[ A^α_j = \left( \frac{N-1}{j-1} \right)^{1/2} \left[ -i\sin (tτ) \right]^{(j-1)} \cos (tτ)^{(N-j)}. \]  

(8)

Since the eigenstates of the system have incommensurate energies \( λ_k \), the electron wavepacket oscillates in a perfectly periodic way between the first and the last dots, whose occupation probabilities are given, respectively, by \( |A^α_i|^2 = \cos (tτ)^{2(N-i)} \) and \( |A^α_N|^2 = \sin (tτ)^{2(N-1)} \), which is illustrated in Fig. 2b. In particular, if at time \( τ = π/(2t) \) the tunneling rates are suddenly switched off, we obtain \( |A^α_1|^2 = 0 \) and \( |A^α_N|^2 = 1 \), i.e. complete population transfer from the initial to the final state of the system.
the eigenstate of Hamiltonian (5) with zero eigenvalue, \( \pi \) coupled quantum dots, assuming equal energies laser fields [7]. Equivalently, for a chain of three tunnel-photon (Raman) resonance, possesses a coherent population transfer which is acquire both, careful control of the individual tunneling in the chain with equal interdot tunneling rates

\[
|\psi_0^0\rangle = \frac{1}{\sqrt{N_0}}[t_2 |\lambda_0\rangle - \lambda_0 |\lambda_0\rangle + t_2 |\lambda_0\rangle],
\]

with corresponding eigenvalues \( \lambda_\pm = \pm \sqrt{t_1^2 + t_2^2} \), contain all three states \( |\lambda_\pm\rangle \). If for a given coupling strengths \( t_1 \) and \( t_2 \) the system is prepared in the CPT state \( |\lambda_0\rangle \), it will remain in this state as long as the couplings are constant in time. But even for time-dependent couplings, the system initially prepared in the CPT state can adiabatically follow this state, provided the tunneling rates change slowly enough. More quantitatively, the nonadiabatic coupling between the eigenstates of Hamiltonian \( \lambda_0 \) is small, if during the evolution the transition amplitude \( \langle \psi_0^0 | \psi_0^0 | \rangle \) remains much smaller than the energy separation between the corresponding eigenstates \( \lambda_0 \),

\[
|\psi_0^0 | \psi_0^0 | \rangle \ll |\lambda_\pm - \lambda_0|.
\]

Our objective is to transfer the electron from the first to the last QD using the time-dependent (pulsed) tunnel-couplings. From Eq. (9) one can see that if at an early time the tunnel coupling \( t_2 \) is switched on while \( t_1 \ll t_2 \), the CPT state coincides with the initial state \( |\lambda_0\rangle \). One then slowly (adiabatically) decreases \( t_2 \) while increasing \( t_1 \), so that at a later time \( t_1 \gg t_2 \) and the CPT state coincides with the final state \( |\lambda_\pm\rangle \). Assuming that \( t_2 \) and \( t_1 \) are represented by partially overlapping pulses, each having temporal width \( \tau_\omega \), the adiabaticity condition (10) requires \( \tau_\omega \gg 1 \).

In the field of atomic/molecular physics, this technique, involving the so-called counterintuitive sequence of pulses, is known as the stimulated Raman adiabatic passage (STIRAP) that is commonly used for coherent population transfer in three-state systems [2]. We note that the solid-state implementations of the CPT and STIRAP in a pair of coupled quantum dots driven by two electromagnetic fields has been proposed in [24].

The single electron transfer in a chain of three QDs via counterintuitive

**FIG. 2:** Time-evolution of a single-electron wavepacket in a chain of \( N = 9 \) QDs with static tunneling rates. (a) Population flow in the chain with equal interdot tunneling rates \( t_j = t \) (shown in the inset). (b) Population flow in the chain with spin-model tunneling rates \( t_j = t\sqrt{(N - j)} \) (shown in the inset). The time \( \tau \) is in units of \( t^{-1} \).
pulsing of tunnel-couplings as discussed above has been studied by Greentree et al. in [14], where it was termed coherent tunneling by adiabatic passage (CTAP). These authors also considered the extension of CTAP to multidot systems employing the so-called straddling scheme of [9]. Other schemes for adiabatic electron transport in tunnel-coupled QDs have been discussed in [25].

Another extension of the STIRAP technique to systems containing more than just three states has been given in [10]. This scheme can easily be adapted to our system, as described below. We thus consider a chain of $N$ sequentially coupled QDs and assume that the individual tunnel couplings can selectively and independently be manipulated. When $N$ is odd, i.e. $N = 3, 5, 7, \ldots$, the Hamiltonian (5) has a CPT eigenstate $|\psi_0^\alpha\rangle = \frac{1}{\sqrt{N_0}} \sum_{n=0}^{J} (-1)^n t_{\text{even}}^{-n} |(2n + 1)\alpha\rangle$, (12) which makes the above discussion more transparent. In particular, complete population transfer from the initial state $|1\alpha\rangle$ to the final state $|N_\alpha\rangle$ can be achieved by applying first the $t_{\text{even}}$ pulses and then the $t_{\text{odd}}$ pulses, the two sets of pulses partially overlapping in time, as shown in Fig. 3.

In order to minimize the nonadiabatic coupling of the CPT state to other eigenstates of the system, the rate of change of $t_{\text{even}}$ and $t_{\text{odd}}$, given approximately by the inverse pulse-width $\tau_w^{-1}$, should be small compared to corresponding eigenenergies $|\lambda| \sim \max|t_{\text{even}} + t_{\text{odd}}|$, which yields the same condition as above, $\tau_w^{-1} \gg 1$. One can see from the results in Fig. 3(a), which were obtained precisely for this reason, that when this condition is not very well satisfied, the population transfer is incomplete. As expected, when the tunneling rates are pulsed for longer times, or, equivalently, have larger amplitudes, the adiabaticity condition is satisfied better, resulting in the complete population transfer from the initial to the final dot of the chain, as seen in Fig. 3(b). The remarkable advantage of this method over the one described in the previous Section is that as long as the two sets of partially overlapping pulses are strong enough, the adiabatic transfer of population is expected to be robust with respect to small uncertainties and fluctuations of tunneling rates, just like its atomic/molecular counterpart in Refs. [5, 10]. On the other hand, the electron transfer via effective collective $\pi$ pulse can be achieved with smaller tunneling rates and/or reduced interaction times, provided a precise control of the tunneling amplitudes and timings is
possible. Depending on the characteristics of the particular system, one or the other method may prove to be more practical.

V. CONCLUSIONS

In the above Sections, we have studied the dynamics of a single-electron transport in a linear array of tunnel coupled quantum dots. We have identified two regimes under which a complete coherent transfer of electron wavepacket between the two ends of the array can be achieved. Our results could be used for reliable information exchange between distant parts of an integrated quantum computer. As already noted in the Introduction, one of the difficulties with the existing proposals for integrated QD based QCs is that the qubits (electron spins) interact with the nearest neighbors only, and there is no efficient way of transferring the information between distant qubits. As a way around such difficulties, one can envision an integrated quantum register composed of a large number of sub-registers, each containing two or more adjacent qubits, represented by spins of single electrons in individual QDs. The sub-registers are embedded in a two-dimensional array of empty QDs. As we have shown in an earlier publication, through the mechanism of transient Heisenberg coupling, combined with the control of tunnel-coupling between the dots studied in this paper, this two-dimensional grid could realize a flexible quantum channel, capable of connecting any pair of qubits within the register. Thus, to transfer the information, one connects distant sub-registers by a chain of empty QDs and applies one of the protocols described in the previous Sections to achieve a non-dispersive transfer of the qubit, followed by its controlled entanglement with a target qubit. Note that this scheme is analogous to a proposal for an integrated ion trap based QC, where, in order to circumvent the difficulties associated with a single large ion trap quantum register, it has been proposed to use many small sub-registers, each containing only a few ions, and connect these sub-registers to each other via controlled qubit (ion) transfer to the interaction region (entangler) represented by yet another ion trap.

We should note that the coherent electron dynamics in arrays of tunnel-coupled QDs bears many analogies with spin-wave dynamics in spin chains or electromagnetic field dynamics in periodic photonic crystals, where some of the effects described above should be observable. With an unprecedented control over system parameters, arrays of QDs doped with more than one electron allow for studies of numerous coherence and correlation effects in many-body physics.

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[1] B.W. Shore, The Theory of Coherent Atomic Excitation (Wiley, New York, 1990).
[2] J. H. Eberly, B.W. Shore, Z. Bialynicka-Birula and I. Bialynicki-Birula, Phys. Rev. A 16 (1977) 2038; Z. Bialynicka-Birula, I. Bialynicki-Birula, J.H. Eberly and B.W. Shore, Phys. Rev. A 16 (1977) 2048.
[3] R. Cook and B.W. Shore, Phys. Rev. A 20 (1979) 539.
[4] B.W. Shore and R. Cook, Phys. Rev. A 20 (1979) 1958; B.W. Shore, Phys. Rev. A 29 (1984) 1578.
[5] J. Oreg, F.T. Hioe and J.H. Eberly, Phys. Rev. A 29 (1984) 690; J.R. Kuklinski, U. Gaubatz, F.T. Hioe and K. Bergmann, Phys. Rev. A 40 (1989) 6741; U. Gaubatz, P. Rudecki, S. Schiemann and K. Bergmann, J. Chem. Phys. 92 (1990) 5363.
[6] G. Alzetta, A. Gozzini, L. Moi and G. Orriols, Nuovo Cimento 36B (1976) 5; E. Arimondo and G. Orriols, Lett. Nuovo Cimento 17 (1976) 333.
[7] K. Bergmann, H. Theuer and B.W. Shore, Coherent population transfer among quantum states of atoms and molecules, Rev. Mod. Phys. 70 (1998) 1003.
[8] Y.B. Band and P.S. Julienne, J. Chem. Phys. 95 (1991) 5681; J. Oreg, K. Bergmann, B.W. Shore and S. Rosenwaks, Phys. Rev. A 45 (1992) 4888.
[9] V.S. Malinovsky and D.J. Tannor, Phys. Rev. A 56 (1997) 4929; N.V. Vitanov, B.W. Shore and K. Bergmann, Eur. Phys. J. D 4 (1998) 15.
[10] B.W. Shore, K. Bergmann, J. Oreg and S. Rosenwaks, Phys. Rev. A 44 (1991) 7442.
[11] L.P. Kouwenhoven, F.W.J. Hekking, B.J. van Wees, C.J.P.M. Harmans, C.E. Timmering and C.T. Foxon, Phys. Rev. Lett. 65 (1990) 361; F.R. Waugh, M.J. Berry, D.J. Mar, R.M. Westervelt, K.L. Campman and A.C. Gossard, Phys. Rev. Lett. 75 (1995) 705; D.S. Duncan, M.A. Topinka, R.M. Westervelt, K.D. Maranowski and A.C. Gossard, Phys. Rev. B 63 (2001) 045311.
[12] C.A. Stafford and S. Das Sarma, Phys. Rev. Lett. 72 (1994) 3590; R. Kotlyar, C.A. Stafford and S. Das Sarma, Phys. Rev. B 58 (1998) R1746; C.A. Stafford, R. Kotlyar and S. Das Sarma, Phys. Rev. B 58 (1998) 7901.
[13] M.R. Wegewijs and Y.V. Nazarov, Phys. Rev. B 60 (1999) 14318.
[14] A.D. Greentree, J.H. Cole, A.R. Hamilton and L.C.L. Hollenberg, Phys. Rev. B 70 (2004) 235317.
[15] L.P. Kouwenhoven, C.M. Marcus, P.L. McEuen, S. Tarucha, R.M. Westervelt and N.S. Wingreen, Electron transport in quantum dots, in: L.L. Sohn, L.P. Kouwenhoven and G. Schönen, eds., Mesoscopic Electron Transport: Series E: Applied Sciences vol. 345 (Kluwer Academic, Dordrecht, 1997) 105-214; S.M. Reimann and M. Manninen, Rev. Mod. Phys. 74 (2002) 1283; W.G. van der Wiel, S.D. Franceschi, J.M. Elzerman, T. Fujisawa, S. Tarucha and L.P. Kouwenhoven, Rev. Mod. Phys. 75 (2003) 1.
[16] M.A. Nielsen and I.L. Chuang, Quantum Computation
and Quantum Information (Cambridge University Press, Cambridge, 2000).

[17] D. Loss and D.P. DiVincenzo, Phys. Rev. A 57 (1998) 120.

[18] L.M.K. Vandersypen, R. Hanson, L.H.Willems van Beveren, J.M. Elzerman, J.S. Greidanus, S. De Franceschi and L.P. Kouwenhoven, Quantum computing with electron spins in quantum dots, in Quantum Computing and Quantum Bits in Mesoscopic Systems (Kluwer Academic, Dordrecht, 2002); M. Friesen, P. Rugheimer, D.E. Savage, M.G. Lagally, D.W. van der Weide, R. Joynt and M.A. Eriksson, Phys. Rev. B 67 (2003) 121301(R).

[19] P. Zanardi and F. Rossi, Phys. Rev. Lett. 81 (1998) 4752.

[20] G.M. Nikolopoulos, D. Petrosyan and P. Lambropoulos Europhys. Lett. 65 (2004) 297; J. Phys.: Condens. Matter 16 (2004) 4991.

[21] Typically, in $\sim 50$ nm size GaAs/AlGaAs QDs, separated from each other by $\sim 100$ nm, one has $t_j \sim 0.05$ meV, $\Delta \varepsilon \sim 0.4$ meV, $U \sim 10$ meV, and at dilution-refrigerator temperatures $T \sim 2-10$ mK the thermal energy is $k_B T \sim 0.2-1 \mu$eV [15].

[22] T. Takagahara, J. Lumin. 70 (1996), 129.

[23] A.C. Johnson, J.R. Petta, J.M. Taylor, A. Yacoby, M.D. Lukin, C.M. Marcus, M.P. Hanson and A.C. Gossard, Nature 435 (2005) 925; S. Sasaki, T. Fujisawa, T. Hayashi, and Y. Hirayama, Phys. Rev. Lett. 95 (2005) 056803.

[24] T. Brandes and F. Renzoni, Phys. Rev. Lett. 85 (2000) 4148; U. Hohenester, F. Troiani, E. Molinari, G. Panzarini and C. Macchiavello, Appl. Phys. Lett. 77 (2000) 1864.

[25] F. Renzoni and T. Brandes, Phys. Rev. B 64 (2001) 245301; A.D. Greentree, A.R. Hamilton and F. Green, Phys. Rev. B 70 (2004) 041305(R).

[26] D. Kielpinski, C. Monroe and D.J. Wineland, Nature 417 (2002) 709.

[27] M. Christandl, N. Datta, A. Ekert and A.J. Landahl, Phys. Rev. Lett. 92 (2004) 187902; M. Christandl, N. Datta, T.C. Dorlas, A. Ekert, A. Kay and A.J. Landahl, Phys. Rev. A 71 (2005) 032312; S. Bose, Phys. Rev. Lett. 91 (2003) 207901.

[28] R. Sapienza, P. Costantino, D. Wiersma, M. Ghulinyan, C.J. Oton and L. Pavesi, Phys. Rev. Lett. 91 (2003) 263902; M. Ghulinyan, C.J. Oton, Z. Gaburro, L. Pavesi, C. Toninelli and D.S. Wiersma, Phys. Rev. Lett. 94 (2005) 127401.

[29] D.N. Christodoulides, F. Lederer and Y. Silberberg, Nature 424 (2003) 817.