Vortices generation and detection in superfluid helium by using quartz tuning forks

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Abstract. In this paper we present the results of an experimental investigation of the quantum vortices generation in superfluid helium by using quartz tuning forks; the estimates of the rate of the quantum vortexes generation and its decay are given.

Keywords: quantum vortices, turbulence, superfluid helium, tuning forks.

1. Introduction

Turbulence, formation and decay a turbulent state is one of the most intriguing and unsolved physics problems at present. Turbulence is a phenomenon that is observed every day, from a water motion in pipes to a fluid movement in rivers and oceans and clouds formation. Turbulence occurs at large deviations from equilibrium state in weakly dissipative media with numerous degrees of freedom. A kinetic turbulence is associated with the liquid or gas motion, when a kinetic energy of its movement exceeds the dissipative processes determined by the medium. In the laboratory conditions, the turbulence state is produced by mutual motion medium (liquid or gas) and a foreign body with velocities above the critical value. The critical velocities for kinetic turbulence are determined by the dimensionless parameter — the Reynolds number \( Re = \frac{vD}{\mu} \), where \( v \) is the velocity of reciprocal motion, \( D \) is the system size, and \( \mu \) is the dynamic viscosity. When \( Re \) reaches a value of the order of several thousands\(^3\), the laminar (predictable motion) changes to the turbulent (chaotic) motion. It is clear that a decrease in the dynamic viscosity makes it possible to sharply reduce the speed of mutual motion \( v \) or the system size \( D \) for achievement the turbulent state. The dynamic viscosity of the liquid hydrogen and liquid helium is in two orders of magnitude lower than the water’s viscosity. This is one of the reason for using cryogenic liquids for study the turbulent state’s. This characteristic allows drastically reduce the required speeds or system dimensions to achieve turbulence. The another advantage to use superfluid helium as a working medium is the creation of turbulent vortices, whose properties are determined by the quantum properties of superfluidity. In superfluid helium the created vortices have the quantized circulation of the superfluid component \( v_S \)

\[
\int_L v_S dx = n \frac{h}{m} = \kappa,
\]

where \( n \) is an integer, \( h \) is Planck's constant, \( m \) is the helium atom’s mass.

In superfluid helium, as well as in ordinary liquids, the transition to the turbulent state occurs when the velocity gradient exceeds a critical value. This may be relative body and fluid motion, for example, the fluid flow through a pipe or the solid motion in the fluid, the motion of charged particles in the fluid, or a counterflow of normal and superfluid components in helium II. Oscillating wires, balls, grids, quartz tuning
forks are used in superfluid helium as the moving solid bodies for generate the turbulent state. A maximum amplitude of vibration of these oscillators is achieved under resonant conditions.

In this work, we used quartz tuning forks for generation quantum vortices in superfluid helium. As a development of our research, we intend to use the tuning forks for detecting of turbulent state in superfluid helium at low temperatures take into account high quality of the tuning fork oscillator and a small mass of its sensitive element (tuning fork teeth).

2. The quartz tuning fork’s characteristics

Quartz tuning forks are commercially produced oscillators that are widely used in many areas - from standard frequency in wrist watches to small-displacement drives in optical and force microscopy. Such tuning forks are cheap, reproducible, rigid and durable devices, which are not sensitive to magnetic fields. The standard quartz tuning fork’s frequency is $2^{15} = 32768$ Hz. This is the standard frequency at which wristwatches work. For other purposes, in particular for study the emission of sound waves in different environments, tuning forks are used with different resonant frequencies from several kHz to hundreds of MHz. The quartz crystals have a Rhombohedral symmetry and demonstrate piezoelectric properties.

Standard watch tuning forks have oscillation with bending mode, though some special choice of crystallographic direction of quartz crystal allows to get tuning fork oscillators with torsional vibrations.

The quality of the quartz tuning fork in vacuum at room temperature reaches a value of $10^5$. The resonant frequency of the tuning fork is characterized by length $L$, thickness $W$ and width of one of its teeth $T$. Also the D-2T value is very important parameter, which determines the density of vortices between the teeth of a quartz tuning fork [1] (Fig. 1.).

At small amplitudes, the quartz tuning fork can be represented as a harmonic oscillator:

$$\frac{d^2x}{dt^2} + \gamma \frac{dx}{dt} + \frac{k}{m} x = \frac{F}{m},$$

where $\gamma$ is the attenuation coefficient, $k$ - rigidity of the tuning fork material, $m$ - effective mass of one tooth (oscillator mass), $F$ - external driving force acting on the tooth of the tuning fork. The quality factor of the oscillator is calculated from the relation $Q = \frac{\omega_0}{\Delta \omega}$ where $\Delta \omega$ is the width of the resonance curve at $\frac{\sqrt{2}}{2}$ of the maximum amplitude.

![Figure 1. Schematic representation of the quartz tuning fork](image)

The effective mass of a quartz tuning fork in vacuum can be calculated by the formula [6]:

$$m_{\text{vac}} = 0,24267 \rho_q LWT,$$

where $\rho_q = 2659$ kg / m$^3$ is the density of quartz.
The mechanical vibrations of the tuning fork are caused by the applied voltage from the external electrical circuit, and the movement of the tuning fork teeth is accompanied by the process of charging-discharging the electrodes of the tuning fork. In this case, the current caused by bending of the teeth of the tuning fork is proportional to the speed of movement of the teeth:

$$I = a \frac{dx}{dt},$$

and the driving force will be determined as

$$F_0 = \frac{a}{2} U_0$$

where $a$ is the electromechanical constant of the quartz tuning fork.

The measurement of the electromechanical constant experimentally is difficult, however, you can replace the equation of mechanical oscillator by an equivalent electrical circuit - then the expression takes the form:

$$\frac{d^2 I}{dt^2} + \frac{R}{L} \frac{dI}{dt} + \frac{I}{LC} = \frac{1}{L} \frac{dU}{dt}$$

Comparing this equation with the differential equation of a harmonic oscillator, you can connect the electrical and mechanical parameters of the system through a constant $a$.

Using the formula

$$R = \frac{2m\gamma}{a^2}$$

it is possible from the experimental results at resonance to calculate the constant $a$ as

$$a = \sqrt{\frac{2m\Delta\omega}{R}}$$

where $R=U_0I_0$, $I_0$ - current amplitude at resonance, $\Delta\omega$ - width of the resonance curve.

From the experiment it is difficult to determine the mass $m$. But the theoretical value of the mass is sufficiently accurate, which is confirmed by direct measurements of the maximum value of velocity in ideal conditions at resonance [2].

In a fluid with laminar motion, the drag force on the quartz tuning fork increases due to the presence of an attached, or hydrodynamic, mass and viscosity of the fluid. This effect leads to a decrease in the quality factor of the resonator and a shift of the resonant frequency of the tuning fork to lower frequencies.

3. Experimental part

3.1. Experimental setup

For the experiment, the measurement system was assembled, shown in Fig. 2
In this work, we use a Stanford Research Systems DS345 sinusoidal signal generator, a quartz tuning fork with a resonant frequency of 32768 Hz, an I-V converter and a Stanford Research Systems SR830 lock-in-amplifier. The signal from the generator excited a vibration of the quartz tuning fork, then the I-V converter amplifies the current on the quartz tuning fork and transforms it into voltage. The signal is displayed on a two channel synchronous digital voltmeter.

The experimental setup is a glass helium cryostat consisting of two Dewar vessels. Outer Dewar is nitrogen and inner one is helium. An insert with a quartz tuning fork is placed in the helium volume. The insert view is shown in fig. 3. In addition to the Tuning fork, the figure shows the system for generation (Heater in a long quartz tube) and registration (superconducting film Bolometer on a surface of a glass plate) of second-sound waves. The last system is used, in particular, for generation of quantum vortices. The helium pumping system allowed to carry out of experiments at temperatures down to 1.3 K, i.e. to pass the range of existence of both helium phase – normal (4.2-2.17 K) and superfluid (at T below 2.17 K).

3.2. Measurement results

As a first step of the experiment we calibrated the quartz tuning fork was in a vacuum. Vacuum is an ideal medium in which the Q-factor of the resonator is determined only by its own internal losses in the material of the tuning fork.
The quartz tuning fork resistance is approximately 10 kΩ at resonance in vacuum. The electromechanical coefficient \( a \) was calculated by using formula (6), and it is equal \( a = 4.05 \cdot 10^{-6} \text{ Ks/m} \). Using this coefficient, it is possible to determine the maximum speed of movement of its teeth from the current through the tuning fork, and the driving force applied to the mechanical resonator by the voltage of the generator signal.

The results of measurements of resonance curves of the in different media are presented on Fig.4. It can be seen that the resonant frequency shifts to lower values due to the added mass. The loss of the energy of the oscillator in the media leads to reduce of the maximum amplitude and the quality of the resonator.

Figure 4. Comparison of resonance curves in different media, excited by the same harmonic signal \( U_G = 1.0 \text{ V} \)

Naturally, we were interested in the behavior of the tuning fork in superfluid helium, the transition of the laminar motion of the tuning fork teeth under resonant conditions to the process of formation of quantum vortices. In fig. 5 shows the results of the measuring response of the tuning fork at resonance to the magnitude of the exciting voltage. The measured charging current of the tuning fork plates is proportional (3) to the maximum movement speed of the tuning fork teeth. At small values of the generator signal (\( F \sim U_G \)), the speed is proportional to the driving force and we observed the laminar motion of the oscillator. However, at \( U_G \approx 2.7 \text{ V} \), a transition to a turbulent motion occurs. This voltage corresponds to the velocity \( v \approx 11 \text{ cm/s} \). The transition to the turbulent motion and the creation of the vortices is accompanied by a change in the force-speed ratio \( F(v) \) from \( F \sim v \) to \( F \sim v^2 \) [3, 4]. The figure clearly shows that this change in the behavior of the resonator (tuning fork) is determined by the properties of superfluid helium while the moving the tuning fork teeth with the same speeds in vacuum does no indicate any peculiarities. (green line in Fig. 5.).

From the experimental data, it is possible to estimate the number of vortices generated every second by a quartz tuning fork. We estimate the power pumping into the vortex system as the difference between the continuation of the laminar straight line (the red line on the graph) and the experimentally measured value (blue dots). The power pumped into the vortex system may be write as follows:

\[
P_V = U_e I - U_L I = I(U_e - U_L),
\]  

(7)
where $I$ is the measured current value (real speed of the tuning fork teeth), $U_r$ is the voltage supplied from the generator (external force required for such movement) and $U_L$ is the calculated force for laminar movement with the same speed.

![Graph](image)

**Figure 5.** Transition from a laminar flow to a turbulent flow in superfluid helium, $T = 1.6$ K

So the current in 8 mA corresponds to the speed of movement of the tuning fork teeth 19.7 cm/s, the actual applied force required for such movement is $12.2 \times 10^6$ N ($U_r \approx 6$ V), while for laminar movement is required $9.1 \times 10^6$ N ($U_L \approx 4.5$ V). The vortex creation power is $P_V \approx 12$ mkW. Vortex length unit energy [5]

$$E_k = \frac{1}{2} \rho_S \int_0^c v^2_0 2\pi rdr = \rho_S \frac{k^2}{4n} \ln \frac{c}{b},$$  

(8)

where $b$ is the radius of the vortex core, having the order of the interatomic distance, $c$ is some conditional radius of the vortex. The vortex radius is related to the number of vortices per unit area $N$ through an obvious ratio

$$\pi b^2 = \frac{1}{N}$$  

(9)

It is not difficult to notice that the energy of a unit vortex length weakly depends on the density of the vortices. Thus, an increase in the density of vortices by two orders of magnitude (from $10^6$ to $10^8$ cm$^{-2}$) changes the value of $E_k \approx 30\%$, which is not significant for estimates in order of magnitude. In assessments, we take $E_k \approx 2 \times 10^{13}$ J/m. Now, by dividing $P_V$ by $E_k$, we can estimate the generated number of vortices as $dL/dt \sim 10^8$ m/s.

Thus, every second the tuning fork gives rise to $L \sim 10^8$ m of whirlwinds. It is worth noting that the system is in a stationary state, which means that as much vortex length appears per second the same number disappear due to their mutual vortex annihilation and their diffusion from the creation region into the surrounding space.

4. Conclusion

1. The experimental technique has been developed for generating vortices in He II using quartz tuning forks and the method for processing experimental data;
2. The experiments were carried out for using tuning fork in different media; the results of resonance of this kind of oscillator were obtained in different systems: vacuum, air, and superfluid helium;

3. During the transition from vacuum to more dense media, the Q-factor of the system decreases, which is associated with an increase in the density of added mass.

4. The critical amplitude \( U_G = 2.7 \text{ V} \) was determined, at which the system goes from laminar to turbulent flow, which corresponds to the critical velocity \( v = 11.4 \text{ cm/s} \), the temperature in the cryostat is 1.6 K;

5. The energy pumped into the vortex system by a quartz tuning fork in superfluid helium is estimated.

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