TIDAL EVOLUTION OF CLOSE-IN EXTRASOLAR PLANETS: HIGH STELLAR \( Q \) FROM NEW THEORETICAL MODELS

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ABSTRACT

In recent years it has been shown that the tidal coupling between extrasolar planets and their stars could be an important mechanism leading to orbital evolution. Both the tides the planet raises on the star and vice versa are important and dissipation efficiencies ranging over four orders of magnitude are being used. In addition, the discovery of extrasolar planets extremely close to their stars has made it clear that the estimates of the tidal quality factor, \( Q \), of the stars based on Jupiter and its satellite system and on main-sequence binary star observations are too low, resulting in lifetimes for the closest planets orders of magnitude smaller than their age. We argue that those estimates of the tidal dissipation efficiency are not applicable for stars with spin periods much longer than the extrasolar planets’ orbital period. We address the problem by applying our own values for the dissipation efficiency of tides, based on our numerical simulations of externally perturbed volumes of stellar-like convection. The range of dissipation we find for main-sequence stars corresponds to stellar \( Q \), of \( 10^5 \) to \( 3 \times 10^9 \). The derived orbit lifetimes are comparable to or much longer than the ages of the observed extrasolar planetary systems. The predicted orbital decay transit timing variations due to the tidal coupling are below the rate of ms yr\(^{-1} \) for currently known systems, but within reach of an extended \textit{Kepler} mission provided such objects are found in its field.

Key words: convection – methods: numerical – planetary systems – planets and satellites: dynamical evolution and stability – planet–star interactions – turbulence

Online-only material: color figures

1. INTRODUCTION

With the discovery of several giant extrasolar planets that are extremely close to their stars, it has become clear that the dissipation of the tides that the planet raise on the star plays an important role in determining the orbital evolution (cf. Dobbs-Dixon et al. 2004; Ibgui & Burrows 2009; Pont 2009; Jackson et al. 2008a, 2008b, 2009; Matsumura et al. 2008, among others). The problem already has a history (e.g., Rasio et al. 1996; Terquem et al. 1998; Sasselov 2003; Ogilvie & Lin 2004), which led to the basic conclusion that a comprehensive theoretical understanding of turbulent dissipation in stellar convection zones is lacking. The recent discovery of the very close-in giant planet WASP-19b made that very clear (Hebb et al. 2010).

In some cases the dissipation of the tides on the planet is important as well, but (at least for single planet systems) this eventually leads to the planet being on a circular orbit with its spin synchronized with the orbit, at which point the tides on the planet become independent of time and hence no dissipation occurs. The tides raised on the star will also have the tendency to synchronize the rotation of the star with the orbit, but in the case of planets usually the angular momentum of the planetary orbit is not enough to achieve that.

Commonly, the rate at which energy is dissipated from the stellar tides is parameterized in terms of a quality factor (\( Q_* \)) defined as the fraction of the tidal energy dissipated in one tidal period. It is then assumed that this value depends only on the star and not on the frequency of the tides, and its value derived from observations of main-sequence solar mass binary stars (Meibom & Mathieu 2005) is used: \( 10^5 \lesssim Q_* \lesssim 10^7 \). This dissipation, however, is found to lead to unreasonably short lifetimes for at least two systems—WASP-18b (Hellier et al. 2009) and WASP-19b (Hebb et al. 2010). For these systems the time it would take for the planets to plunge into the star, if the above \( Q_* \) values are assumed, is about three orders of magnitude smaller than the estimated ages of the systems and four orders of magnitude smaller than the total lifetime of the star, making it extremely unlikely that such a system should ever have been observed (Hellier et al. 2009), unless one assumes a mechanism that continuously resupplies these regions that are extremely close to the star with fresh planets.

There is however a fundamental difference between the tides in a binary star and a star–planet system, namely, that the mass of the secondary in the case of stars is large enough to spin the primary to synchronous rotation with the orbit, and in the case of planets it is not. And indeed for all transiting planetary systems for which the spin period of the star (\( p_* \)) is known it is found to be much longer than the orbital period (\( P \)). This distinction is important, because when \( |P| > |p_*|/2 \) the time variable tidal perturbation can resonantly excite inertial waves in the star, thus resulting in several orders of magnitude larger shear, and hence dissipation, compared to the static tide (Savonije & Papaloizou 1997; Dintrans & Ouyed 2001; Ogilvie & Lin 2004; Wu 2005a, 2005b; Papaloizou & Ivanov 2005; Ogilvie & Lin 2007; Ivanov & Papaloizou 2007; Ogilvie 2009).

For stars with surface convective zones the turbulent convective motions are thought to be causing a cascade of energy to occur from the large scales of the tides to smaller and smaller scales until eventually the finite viscosity of the plasma becomes important and converts this energy into heat. The usual treatment of this complicated process is to assume that it behaves like an effective turbulent viscosity coefficient (cf. Zahn 1966; Goldreich & Nicholson 1977). However, the different prescriptions have been difficult to reconcile with each other and with different observations (Goodman & Oh 1997).

Penev et al. (2009a) and Penev et al. (2007, 2008, 2009b) used numerical simulations of stellar-like convection to show that the dissipative properties of the turbulent convective flow are indeed well approximated by an effective viscosity coefficient. Therefore, the simulations allow us to derive its theoretical
value. In this paper, we combine these new results to construct a complete prescription for the effective viscosity and show that the resulting dissipation, in the absence of resonantly excited tidal waves, produces much higher \( Q_* \) values than those found observationally for main-sequence stars, and that those values are consistent with even the strongest current constraints on the tidal dissipation efficiency. In addition, we derive the magnitude of the expected transit timing variations (TTVs) due to orbital decay and compare them to possible observational constraints (e.g., by the Kepler mission) that could provide a direct test of the proposed viscosity prescription.

The organization of the paper is as follows: in Section 2 we introduce what is usually understood by the \( Q_* \) parameter, in Section 3 we discuss how we arrive at the turbulent viscosity we use to calculate orbital decay, in Section 4 we follow Scharlemann (1981, 1982) to convert our turbulent viscosity to a tidal torque, in Section 5 we present the stellar structure and evolution models used in the calculation of the torque, in Section 6 we calculate the effective \( Q_* \) which corresponds to this torque, the TTVs that our estimate of the dissipation would result in, and the future lifetimes of close-in extrasolar planets. Finally, in Section 7 we summarize our results.

2. CLASSICAL APPROACH TO \( Q_* \)

The tide raised by a planet on its star is a quadrupole wave of amplitude \( h \); the tidal motions are of order \((\omega - \Omega)h\) which set up shear inside the star. The orbital angular frequency, \( \omega \), exceeds the stellar spin one, \( \Omega \), in the case we consider here, and the tidal forcing due to \( \omega \) occurs with timescales that are much shorter than the correlation time of the turbulence in the star’s convection zone.

Dissipation in the star causes the tidal bulge \( h \) to lag behind by an angle \( \delta \), which is determined by the amount of coupling between the tide and the source of the dissipation, presumably the turbulent eddies in the convection zone. One could compare the response of the star to that of a forced harmonic oscillator and relate \( \delta \) to a specific dissipation function \( Q = (\omega - \Omega)E_0/\dot{E} = 1/2\delta \) (Murray & Dermott 1999). This is how the tidal dissipation quality factor \( Q_* \) is defined, with \( E_0 \) being the energy stored in the tidal bulge and \( \dot{E} \) the rate of viscous dissipation of energy. This is another way to determine the lag angle \( \delta \).

The turbulent viscosity is introduced in the sense of Rayleigh–Benard incompressible convection, \( v_i = \frac{1}{2}vl \approx l^2/\tau \), for a low forcing frequency. Here, \( l \) is the convective mixing length, \( v \) is the convective velocity, and \( \tau \) is the convective timescale (eddy turnover time). Then basically \( Q_* \propto G\mu/Mv_0\), where \( M \) and \( v_0 \) are the stellar mass and radius (see Sasselov 2003 for more details).

3. EFFECTIVE TURBULENT VISCOSITY

We need to combine the perturbatively derived effective viscosity of Penev et al. (2009a) and the perturbative estimates based on realistic low-mass star simulations (Penev et al. 2009b) and on an idealized simulation (Penev et al. 2008) all produce a linear scaling of the effective viscosity with period, for the range of periods available to those simulations. As is discussed in these works the linear scaling is not expected to hold for arbitrarily small periods, because at such timescales Kolmogorov cascade should be a good approximation to the flow and in that case Goodman & Oh (1997) show that the loss of efficiency should be quadratic with period. In addition, the direct calculations (Penev et al. 2009a) show that the effective viscosity saturates at long periods.

To accommodate these three scalings, we will assume the following form for the viscosity coefficients:

\[
K_m = \min \left[ s_m^* \left( \frac{P}{\tau} \right)^{2}; s_m \frac{P}{\tau} + \Delta_m; s_m \Pi_{\text{max}} + \Delta_m \right] \frac{1}{3} \frac{\rho (v^2)^{1/2}}{s_m},
\]

with

\[
s_m^* = s_m \frac{\Pi_{\text{min}}}{\Pi_{\text{max}}} + \frac{\Delta_m}{\Pi_{\text{min}}},
\]

where \( l \) is the mixing length, and \( \Pi_{\text{min}} \) and \( \Pi_{\text{max}} \) are the dimensionless periods between which the linear scaling applies, \( s_m \) are the dimension of slopes with period of each effective viscosity component, and \( \Delta_m \) are the corresponding zero crossings.

This expression has been chosen to be continuous with period \( P \), reproduce the quadratic scaling we expect to occur at small periods, the linear scaling at periods comparable to the local convective turnover time \( \tau \) with a possible offset \( \Delta_m \), and a constant effective viscosity for much longer periods.
From Penev et al. (2009a)

\[ s_1 = 0.084, \quad s_2 = 0.055. \]  (8)

Assuming that the scaling between the slopes of these components and the rest from the Penev et al. (2009b) perturbative calculation holds, we arrive at the following values for the remaining three \( s \) values:

\[ s_0 = 0.23, \quad s_0 = 0.07, \quad s_{0\gamma} = 0.02. \]  (9)

The offsets, \( \Delta m \), are less well constrained from the Penev et al. (2009a) and Penev et al. (2009b) simulations, because on one hand their values from the direct calculation depend on the method used to extract the effective viscosity and on the other the perturbative calculation does not predict any offsets so it is not clear of how to find them for the remaining components.

For the components not corresponding to radial shear we will assume no offset. The \( \Delta_1 \) component is expected to be between the values derived by the two direct methods of Penev et al. (2009a) for a forcing strength of zero. Ignoring the difference between the weak forcing and a zero forcing case, which according to Figure 13 of Penev et al. (2009a) is likely to be small, we conclude that

\[ 0.023 < \Delta_1 < 0.061. \]  (10)

For \( \Delta_{0\gamma} \) we will show results with two different assumptions:

\[ \Delta_{0\gamma}^0 = 0, \quad \Delta_{0\gamma}^1 = 2.6 \Delta_1. \]  (11)

The last case corresponds to assuming that the offset scales the same way as the slope between the \( m = 1 \) and \( m = 0 \) components.

Finally we need to specify \( \Pi_{\text{max}} \) and \( \Pi_{\text{min}} \). From Penev et al. (2009a) we see that \( \Pi_{\text{max}} = 2.4 \) is reasonable. The value of \( \Pi_{\text{min}} \) is not seen in the simulations, but clearly \( \Pi_{\text{min}} < 0.3 \), so we will consider two cases \( \Pi_{\text{min}} = 0.1 \) and \( \Pi_{\text{min}} = 0.01 \).

4. TIDAL TORQUE

An analytical expression for the tidal velocity and the associated tidal torque was derived with the smallest number of assumptions by Scharlemann (1981, 1982) for a circular orbit and close to synchronous internal rotation. Here, we will include the relevant results and refer to those works for their derivation.

Scharlemann (1982, Equation (17)) gives the tidal torque as

\[ T = \frac{96 \pi}{5} r_s^3 n \omega_{2s} f_2^2 \Lambda_1 (a_0 - a_{0c}), \]  (12)

where \( r_s \) is the radius of the star, \( f_2 \) is the maximum deviation of the equipotential surfaces from spheres, \( n \) is the orbital angular frequency, \( \Lambda_1 \) can be expressed through integrals of the viscous force over the star, \( a_0 - a_{0c} \) is the departure from synchronous rotation, and \( \omega_{2s} \) and \( a_0 \) describe the differential rotation assumed to have the form

\[ \Omega(r, \theta) = n \omega_{2s} \left( a_0 + \sum_i a_i r^i \left( \frac{3}{2} x^2 \sin^2 \theta \right) \right). \]  (13)

The second term in the above expression does not enter in \( \Lambda_1 \) and so will not play any further role.

The value of \( \Delta_1 \) is given by Scharlemann (1982, Equation (16)):

\[ \Delta_1 = I_2 + \frac{1}{7} I_{02} + \frac{20}{7} I_{02} - \frac{25}{28} I_{04} - \frac{5}{14} I_{04}. \]  (14)

where the indices, 2 and 4, refer to the \( P_2^3(\cos \theta) \) and \( P_4^4(\cos \theta) \) associated Legendre polynomial components, respectively, of the following integrals of the viscous force \( (F) \) acting on the tidal velocity field (Scharlemann 1982, Equations (5c)–(5e)):

\[ I_2(\theta) = \int_{r_*}^{r_*} r^4 \frac{F_r}{g} dr, \]  (15)

\[ I_0(\theta) = \int_{r_*}^{r_*} r^4 \tan \theta F_\theta dr, \]  (16)

\[ I_0(\phi) = \int_{r_*}^{r_*} r^4 \sin \theta F_\phi dr. \]  (17)

5. STELLAR MODELS

The viscosity tensor enters in the calculation of \( \Lambda_1 \) only through its 8th order moments:

\[ E_m = \int_{r_*}^{r_*} r^8 K_m(P, r) dr. \]  (18)

The depth dependence of \( K_m \) arises from the fact that we expect \( K_m \) to scale as \( \rho v/3 \) (\( \rho \), \( l \), and \( v \) are the density, the mixing length, and the convective velocity, respectively), all of which are functions of depth, and it also depends of the convective turnover time, approximated as \( l/v \), which also varies with depth.

In order to evaluate the right hand side of Equation (18) we need \( \rho \), \( l \), and \( v \) in the interior of the star. We will take these quantities from two stellar models: a 1.4 \( M_\odot \) and a 0.8 \( M_\odot \) main-sequence solar metalicity model calculated using the code described in Cody & Sasselov (2002).

The reason for choosing these two models is that presently almost all the known transiting extrasolar planets are around stars that fall within this mass range, so our results will span the range of currently known systems.

6. ORBITAL EVOLUTION

From the tidal torque we can calculate the rate at which the orbit of a circularized and synchronized planet orbiting a slowly rotating star shrinks. For comparison purposes we converted this rate to an effective \( Q_* \) value, shown in Figure 1.

As we can see, for both stellar models and all assumptions of Section 3, the dissipation efficiency is much smaller than the usually assumed values of \( 10^3 < Q_* < 10^7 \), implied observationally for main-sequence binary stars. As discussed in the introduction we expect that this is due to the fact that in the case of planets the tides have a frequency that is too high to resonantly excite inertial waves in the star.

The reason for twice as many curves being visible for the 0.8 \( M_\odot \) case than for the 1.4 \( M_\odot \) is that the two different assumptions for \( \Pi_{\text{min}} \) are indistinguishable for the higher mass model due to the fact that the convection in the high-mass star is much more vigorous and the orbital period does not get below 0.1 convective turnover times for most of the convective zone for the range of separations considered. The dissipation being
more efficient for the low-mass star is mostly due to the fact that it is much less centrally concentrated and hence a lot more mass is subject to the tides than for the high mass case.

In Figure 2, we show the TTVs that will be produced by this turbulent viscosity. Each colored region corresponds to the range of viscosity assumptions listed in Section 3, blue for the 1.4 M⊙ model; red for the 0.8 M⊙ model. The black circles and the green symbols correspond to the currently known transiting extrasolar planets that fall within the plotted range. For each stellar model, the values of the TTVs for the regions going from left to right are 10, 1, 0.1, 0.01, and 0.001, respectively. (A color version of this figure is available in the online journal.)

Figure 2. Rate of change of the orbital period of the planet in ms yr⁻¹. The colored regions correspond to the range of assumptions discussed in Section 3, blue for the 1.4 M⊙ model; red for the 0.8 M⊙ model. The black circles and the green symbols correspond to the currently known transiting extrasolar planets that fall in the plotted range. For each stellar model, the values of the TTVs for the regions going from left to right are 10, 1, 0.1, 0.01, and 0.001, respectively. (A color version of this figure is available in the online journal.)

Figure 1. Tidal quality factor corresponding to the combined viscosity from Penev et al. (2009a) and Penev et al. (2008, 2009b) for the range of assumptions described in Section 3. The blue dashed curves correspond to the 1.4 M⊙ model, the remaining curves are for the 0.8 M⊙ model: Πₘₜₜₜ = 0.1, green; Πₘₜₜₜ = 0.01, red.

(A color version of this figure is available in the online journal.)

Future lifetimes of single planet systems on a circular orbit around an initially non-rotating star for the two stellar models considered in the text: 0.8 M⊙, red; 1.4 M⊙, blue. Each colored region corresponds to the range of assumptions from Section 3. The black points and green symbols correspond to the currently known transiting solar planets that fall within the plotted range of masses and separations. The remaining lifetimes for each model corresponding to the shown regions are 0.01, 0.1, 1, 10, and 100 Gyr going from left to right. (A color version of this figure is available in the online journal.)

Figure 3. Future lifetimes of single planet systems on a circular orbit around an initially non-rotating star for the two stellar models considered in the text: 0.8 M⊙, red; 1.4 M⊙, blue. Each colored region corresponds to the range of assumptions from Section 3. The black points and green symbols correspond to the currently known transiting solar planets that fall within the plotted range of masses and separations. The remaining lifetimes for each model corresponding to the shown regions are 0.01, 0.1, 1, 10, and 100 Gyr going from left to right. (A color version of this figure is available in the online journal.)

To address the question of tidal destruction of planets we used the expression for the tidal torque to calculate the coupled evolution of the planetary orbit and the stellar spin for systems that are assumed to start with a non-spinning star and a planet in a circular orbit. We assume that the planet is synchronized, so we do not consider the tides on the planet and evolved the system forward in time until the dissipation drives the planet close enough to its star to be tidally destroyed, thus getting an estimate of the lifetime of the planet during which it will be observable.

These lifetimes for a range of planet masses and separations are shown in Figure 3 for the two stellar models discussed above. Each blue (narrow) region shows the range between the most and least dissipative effective viscosity assumptions for the 1.4 M⊙ model and the red (wide) region show the range for the 0.8 M⊙ model. The lines to which the curves for a given model asymptote in the upper right corner correspond to the critical planet–star separation beyond which the planetary orbit has sufficient angular momentum to eventually spin up the star to synchronous rotation thus halting tidal evolution. Again for reference we have shown the known transiting planets.

As we can see most of the currently observed planets have tidal lifetimes of many billions of years and (after considering WASP-12b, WASP-18b, OGLE-TR-56b, and WASP-19b individually) we see that no planet has a lifetime of less than a few hundred million years, the shortest lived being WASP-18b with a lifetime ranging between 100 and 160 Myr for a 1.25 M⊙ stellar model (the mass quoted in Hellier et al. 2009).
7. DISCUSSION

In this paper, we have presented a direct theoretical calculation of the dissipation efficiency of tides raised on stars by a short period giant planet. The estimates of this efficiency are based on direct simulations of the dissipation of externally driven perturbations in a small, but still significantly stratified, piece of a convective zone.

While due to numerical limitations we are unable to explore the entire range of the ratio of tidal frequency to local convective turnover time that occurs in the surface convective zones of low-mass stars, we show that the rate of dissipation of tidal energy averaged over the entire convective zone is constrained to within a factor of 10. This is a significant improvement over the current range of three to five orders magnitude that many authors wishing to calculate the tidal evolution of exoplanet orbits are forced to consider (e.g., Hellier et al. 2009; Jackson et al. 2009; Barker & Ogilvie 2009; Levrard et al. 2009; Pätzold et al. 2004; Adams & Laughlin 2006).

Since the usual way to parameterize the dissipation of tides is using the stellar quality factor $Q_\star$, we calculate effective values for this parameter as a function of the orbital frequency. We find that our estimates lie toward the upper end (small dissipation) of the usually assumed range for this parameter—$10^8 < Q_\star < 10^9$—and are significantly larger than the value needed to explain the fact that the maximum period to which solar type binary stellar orbits are circularized increases noticeably over the main-sequence lifetime of these systems.

We argue, however, that this is not necessarily a contradiction, since in the case of binary stars synchronization of the stellar rotation to the orbit happens much faster than the circularization of the orbit. This means that the tidal frequency is close to the rotation rate of the star, for most of the time circularization occurs. This opens the possibility that inertial waves are resonantly excited in the stars, which results in a much enhanced dissipation (Savonije & Papaloizou 1997; Dintrans & Ouyed 2001; Ogilvie & Lin 2004; Wu 2005a, 2005b; Papaloizou & Ivanov 2005; Ogilvie & Lin 2007; Ivanov & Papaloizou 2007; Ogilvie 2009), over what is likely to occur in planet star systems, which typically have the star rotating much slower than the tides raised by the planet.

This idea seems to be confirmed by a number of transiting extrasolar planets which are found to lie very close to their host star. So close, in fact, that were the value of $Q_\star$ implied by main-sequence binary stars applicable to star–planet systems, we would conclude that we caught several of those planets in the last less than 0.1% of their lifetime (e.g., Hellier et al. 2009; Penev et al. 2010), which seems rather unlikely given the current sample of about a hundred known transiting planets.

We show that our much smaller dissipation results in future lifetimes for all the currently known transiting planets comparable to their age (Figure 3) and is hence consistent with those observations.

We also calculate the inferred TTVs due to tidal decay of the orbit (Figure 2) and find values of $\lesssim 1$ ms yr$^{-1}$ for all currently know transiting planets. This is much smaller than we can hope to observe from the ground in the near future, but perhaps within reach of an extended Kepler mission if a very short period massive planet happens to be among its targets, leaving the possibility of measuring $Q_\star$ directly.

REFERENCES

Adams, F. C., & Laughlin, G. 2006, ApJ, 649, 1004
Barker, A. J., & Ogilvie, G. I. 2009, MNRAS, 395, 2268
Cody, A. M., & Sasselov, D. D. 2002, ApJ, 569, 451
Dintrans, B., & Ouyed, R. 2001, A&A, 375, L47
Dobbs-Dixon, I., Lin, D. N. C., & Mandell, A. 2004, ApJ, 610, 464
Goldreich, P., & Nicholson, P. D. 1977, Icarus, 30, 301
Goodman, J., & Oh, S. P. 1997, ApJ, 486, 403
Hebb, L., et al. 2010, ApJ, 708, 224
Hellier, C., et al. 2009, Nature, 460, 1098
Ibgui, L., & Burrows, A. 2009, ApJ, 700, 1921
Ivanov, P. B., & Papaloizou, J. C. B. 2007, MNRAS, 376, 682
Jackson, B., Barnes, R., & Greenberg, R. 2009, ApJ, 698, 1357
Jackson, B., Greenberg, R., & Barnes, R. 2008a, ApJ, 678, 1396
Jackson, B., Greenberg, R., & Barnes, R. 2008b, ApJ, 681, 1631
Levrard, B., Winn, J. C., & Chabrier, G. 2009, ApJ, 692, L9
Matsumura, S., Takeda, G., & Rasio, F. A. 2008, ApJ, 686, L29
Meibom, S., & Mathieu, R. D. 2005, ApJ, 620, 970
Murray, C. D., & Dermott, S. F. (ed.) 1999, Solar System Dynamics (Cambridge: Cambridge Univ. Press)
Ogilvie, G. I. 2009, MNRAS, 396, 794
Ogilvie, G. I., & Lin, D. N. C. 2004, ApJ, 610, 477
Ogilvie, G. I., & Lin, D. N. C. 2007, ApJ, 661, 1180
Papaloizou, J. C. B., & Ivanov, P. B. 2005, MNRAS, 364, L66
Pätzold, M., Carone, L., & Rauer, H. 2004, A&A, 427, 1075
Penev, K., Barranco, J., & Sasselov, D. 2008, arXiv:0810.5151
Penev, K., Barranco, J., & Sasselov, D. 2009a, ApJ, 705, 285
Penev, K., Sasselov, D., Robinson, F., & Demarque, P. 2007, ApJ, 655, 1166
Penev, K., Sasselov, D., Robinson, F., & Demarque, P. 2009b, ApJ, 704, 930
Pont, F. 2009, MNRAS, 396, 1789
Rasio, F. A., Tout, C. A., Lubow, S. H., & Livio, M. 1996, ApJ, 470, 1187
Sasselov, D. D. 2003, ApJ, 596, 1327
Savonije, G. J., & Papaloizou, J. C. B. 1997, MNRAS, 291, 502, 788
Scharlemann, E. T. 1981, ApJ, 246, 403
Zahn, J. P. 1966, Ann. d’Astrophys., 29, 489