Adaptive channel selection for DOA estimation in MIMO radar

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Abstract—We present adaptive strategies for antenna selection for Direction of Arrival (DoA) estimation of a far-field source using TDM MIMO radar with linear arrays. Our treatment is formulated within a general adaptive sensing framework that uses one-step ahead predictions of the Bayesian MSE using a parametric family of Weiss-Weinstein bounds that depend on previous measurements. We compare in simulations our strategy with adaptive policies that optimize the Bobrovsky-Zakai bound and the Expected Cramér-Rao bound, and show the performance for different levels of measurement noise.

Index Terms—Adaptive Sensing, Antenna Selection, Array Processing, Weiss-Weinstein Bound, Bayesian Filtering, Direction of Arrival (DOA), MIMO, Cognitive Radar.

I. INTRODUCTION

Recent advances in millimeter-wave radar circuits make possible low-cost and compact multi-channel radar systems that can be controlled by software. This motivates the design of signal generation and processing algorithms that attempt to maximize the information extracted from the scene, in what is considered the basis of the perception-action cycle of a cognitive radar architecture [1], [2].

Algorithms for adaptive transmission typically employ a prediction of the conditional Bayesian mean-square error (BMSE) given previous observations. In the category of adaptive strategies that attempt to optimize one-step ahead predictions, recent works optimize parameters such as the pulse repetition frequency (PRF) in Pulse-Doppler radar in a joint framework for detection and tracking [3], [4], or the transmitted signal autocorrelation matrix in MIMO radar for DoA estimation [5], [6], using, respectively, the conditional Bayesian Cramér-Rao bound (BCRB) and the Reuven-Messer bound (RMB) [7]. In the category of algorithms that consider the consequences of actions based on some long-term reward, the work [8] schedules measurements in a tracking scenario where the target is temporarily occluded, and [9] optimizes waveform parameters using planning and reinforcement learning.

Few works have considered these approaches for adaptive antenna selection for Direction of Arrival (DoA) estimation. Accuracy of angular estimation improves with the length of the antenna array, and thus with the number of antenna elements that need to be adequately spaced to avoid ambiguity due to aliasing. Bigger apertures demand more Tx and Rx modules (and hence a higher system cost) and more data to be processed in real time. To overcome these constraints, the works [10], [11] study adaptive receiver selection algorithms for far-field DoA and SNR estimation with SIMO linear arrays based on optimization of the Bobrovsky-Zakai Bound (BZB), which provides better one-step ahead predictions than the Expected CRB (ECRB). The latter is not sensitive to sidelobe level but is related to the average mainlobe width of the array [12], and selects the receivers that yield biggest aperture regardless of previous measurements [10]. Alternatively, the Weiss-Weinstein bound [13] is computationally more expensive but predicts more accurately the contribution to the Mean-Square-Error (MSE) of sidelobe ambiguity at low SNR [14], [15].

We extend the work of [10], [11] to transmitter and receiver selection for DoA estimation in Time Domain Multiplexing (TDM) MIMO radar with linear arrays and propose a general algorithm for adaptive sensing using the Weiss-Weinstein bound. Using a particle filter [16] to incorporate sequentially the information from measurements into the belief distribution of the unknown parameter, we construct the conditional WWB (along with the BZB and the ECRB, for reference), that lower bounds the achievable MSE. This requires a double optimization procedure, first over the so-called test-points, to evaluate the tightest bound, and then over candidate sensing parameters. The resulting strategies are illustrated in simulations where we compare the performance of channel selection based on the WWB, the BZB, and the ECRB.

The rest of the paper is organized as follows: Section II proposes a general framework for adaptive sensing based on one-step ahead predictions of the MSE. The general strategy is then particularized in Section III to MIMO channel selection for DoA estimation. Finally, Section IV presents our conclusions and ideas for future work.

II. ADAPTIVE SENSING VIA WEISS-WEINSTEIN BOUND

In this section we present the general strategy to optimize sensing parameters based on a prediction of the MSE. First we introduce the WWB, then we connect it to the conditional BMSE, and finally we describe the computation of the conditional WWB involved in our algorithm.

A. Preliminaries on the Weiss-Weinstein bound

The WWB provides a lower bound on the BMSE of any estimator and thus gives an indication of the achievable estimation performance. Namely, the expected error of any estimator \( \hat{\theta}(x) \), over possible pairs of observations \( x \) and one-dimensional parameter values \( \theta \) modeled with probability distribution \( p(x, \theta) \), is bounded as follows,

\[
\mathbb{E}_{p(x, \theta)} [(\hat{\theta}(x) - \theta)^2] \geq \text{WWB}(s, h),
\]

where \( \text{WWB}(s, h) \) is the Weiss-Weinstein bound.
where \( \text{WWB}(s,h) \) is a member of the parametric family of bounds \([13, 15, \text{eq. 76}]\) given by

\[
\text{WWB}(s,h) := \frac{h^2 \eta(s,h)^2}{\eta(2s, h) + \eta(2 - 2s, -h) - 2\eta(s,2h)},
\]

where \( \eta \) is the moment generating function \([17, \text{pp. 337, pp. 65}]\) defined as

\[
\eta(\alpha, \beta) := \int_{\Omega} \frac{p(x|\theta + \beta)}{p^\alpha - 1(x|\theta)} dx d\theta = \int_{\Omega} \left( \int_{\Omega} \frac{p(x|\theta + \beta)}{p^\alpha - 1(x|\theta)} dx \right) p^\alpha(\theta + \beta) d\theta,
\]

where \( p(x|\theta) \) is the probability, or likelihood, of the observation \( x \in \Omega \subseteq \mathbb{R}^n \) given the parameter value \( \theta \in \Theta := \{ \theta \in \mathbb{R} : p(\theta) > 0 \} \), and \( p(\theta) \) is the prior probability distribution of \( \theta \), which is considered a modeling choice.

The value of the so-called test-point \( h \in (0, \infty) \), and the additional degree of freedom \( s \in (0,1) \), determine the bound on the BMSE, the tightest bound being obtained as \( \text{WWB} := \sup_{s,h} \text{WWB}(s,h) \). (For further generalizations we refer the reader to [17].) The BZB can be obtained from (2) in the limit cases \( s = 1 = 0 \).

\[
\text{BZB}(h) = \text{WWB}(s = 1, h) = \frac{h^2}{\eta(2, h) - 1}.
\]

The BCRB \([17, \text{pp. 72}]\) is in turn a particular case of (4), in the limit as \( h \to 0 \). In the next section we present the connection between the WWB described here and the conditional BMSE, relevant for our adaptive strategies.

### B. Conditional BMSE and adaptive sensing

Consider an estimation task where a sequence of observations \( X^{(k-1)} := (X^1, \ldots, X^{k-1}) \) of an unknown parameter \( \theta \) is obtained using a sequence of sensing parameters \( G^{(k)} := (G_1, \ldots, G_k) \) in a suitable domain, according to an observation model with joint probability distribution \( p(X^{(k)}, \theta|G^{(k)}) \). An adaptive sensing strategy or policy can be defined in general by a probability distribution over sensing parameters given previous measurements, \( p(G_k|X^{(k-1)}, G^{(k-1)}) \). In this work, the proposed strategies are evaluated with respect to the BMSE, which is defined, for any estimator \( \hat{\theta} \equiv \hat{\theta}(X^{(k)}, G^{(k)}) \), as the following integration over observations and realizations of the parameter,

\[
\text{BMSE}(\hat{\theta}, G^{(k)}) := \mathbb{E}_{p(X^{(k)}, \theta|G^{(k)})} \left[ (\hat{\theta} - \theta)^2 \right] = \mathbb{E}_{p(X^{(k-1)}|G^{(k)})} \left[ \mathbb{E}_{p(X^{(k)}, \theta|X^{(k-1)}, G^{(k)})} \left[ (\hat{\theta} - \theta)^2 \right] \right].
\]

Following [5] and [11], we consider the inner expectation above, called conditional BMSE (CBNSE),

\[
\text{CBMSE}(\hat{\theta}, X^{(k-1)}, G^{(k)}) := \mathbb{E}_{p(X^{(k)}, \theta|X^{(k-1)}, G^{(k)})} \left[ (\hat{\theta} - \theta)^2 \right],
\]

as an optimization metric for adaptive algorithms that attempt to find, at each step \( k \geq 1 \), a policy \( G_k \) that minimizes the BMSE given any sequences of previous sensing policies \( G^{(k-1)} \) and historical observations \( X^{(k-1)} \).

Such metric is usually impossible to compute explicitly, but can be lower-bounded in a similar fashion as the BMSE in relation (1). Motivated by this observation, we define the parametric family of conditional WWBS, denoted by \( \text{WWB}(s,h; X^{(k-1)}, G^{(k)}) \), as in (2), where in the definition (3) we use the likelihood function \( p(X|\theta, G^{(k)}) \) and replace the prior distribution by the posterior \( p_{k-1}(\theta) := p(\theta|X^{(k-1)}, G^{(k)}) \). The moment generating function in (3) becomes then

\[
\eta_k(\alpha, \beta) := \int_{\Theta} \int_{\Omega} \frac{p(x|\theta + \beta, G^{(k)})^\alpha}{p^\alpha - 1(x|\theta, G^{(k)})} dx \frac{p_{k-1}(\theta + \beta)^\alpha}{p_{k-1}(\theta)^{\alpha-1}} d\theta,
\]

where \( p_0(\theta) := p(\theta) \) is the prior probability. (Note that the domain of integration in (3) is such that \( p_{k-1}(\theta) > 0 \).)

**Proposition 2.1.** (Conditional WWB and CBMSE): Consider the observation model \( p(X^{(k)}, \theta|G^{(k)}) \) under the following two assumptions, i) \( X^k \) and \( X^{(k-1)} \) are conditionally independent given \( \theta \) and \( G^{(k)} \), i.e., \( p(X^k, X^{(k-1)}|\theta, G^{(k)}) = p(X^k|\theta, G^{(k)})p(X^{(k-1)}|\theta, G^{(k)}) \), and ii) \( p(X^k|\theta, G^{(k)}) = p(X^k|\theta, G_k) \). Then

\[
\text{CBMSE}(\hat{\theta}, X^{(k-1)}, G^{(k)}) \geq \text{WWB}(s,h; X^{(k-1)}, G^{(k)}).
\]

(6)

(Prop. 2.1 is standard and is omitted for lack of space.)

Motivated by the above result, we define adaptive strategies that select at step \( k \) the sensing policy \( G_k \) based on knowledge from previous measurements \( X^{(k-1)} \) and previous sensing policies \( G^{(k-1)} \), as the solution of

\[
G^* \in \arg\min_{G_k} \sup_{s \in (0,1), h \in (0, \infty)} \text{WWB}(s,h; X^{(k-1)}, G^{(k)}).
\]

In general, the inner optimization problem in (6) over test-points is nonconvex, requiring methods for global optimization such as simulated annealing [18], and the outer optimization over sensing policies can be discrete. In the next section we explain how to construct the parametric family of conditional bounds \( \text{WWB}(s,h; X^{(k-1)}, G^{(k)}) \) using the likelihood function, the sequence of measurements \( X^{(k-1)} \), and previous sensing policies \( G^{(k-1)} \).

### C. Computation of the conditional WWB

To evaluate \( \text{WWB}(s,h; X^{(k-1)}, G^{(k)}) \) we first re-write the moment generating function (5), consistently with the notation in [11], as

\[
\eta_k(\alpha, \beta) = \int_{\Theta} D_k(\theta, \alpha, \beta) \phi_{k-1}(\theta, \alpha, \beta) p_{k-1}(\theta) d\theta,
\]

\[
= \mathbb{E}_{p_{k-1}(\theta)} \left[ D_k(\theta, \alpha, \beta) \phi_{k-1}(\theta, \alpha, \beta) \right],
\]

(7)

where \( D_k(\theta, \alpha, \beta) \) contains the observation model,

\[
D_k(\theta, \alpha, \beta) := \int_{\Omega} \frac{p(x|\theta + \beta, G_k)^\alpha}{p(x|\theta, G_k)^{\alpha-1}} dx,
\]

and \( \phi_{k-1}(\theta, \alpha, \beta) \) contains the posterior distribution,

\[
\phi_{k-1}(\theta, \alpha, \beta) := \left( \frac{p_{k-1}(\theta + \beta) p_{k-1}(\theta)}{p_{k-1}(\theta)} \right)^\alpha.
\]

(8)
The computation of $\phi_{k-1}(\theta, \alpha, \beta)$ requires special attention. Again by Bayes Law and using the assumptions of Proposition 2.1, we can express the posterior probability as follows,

$$p_{k-1}(\theta) := p(\theta | X(k-1), G(k-1)) = \frac{p(\theta | G(k-1)) \prod_{m=1}^{k-1} p(X^m | \theta, G_m)}{p(X(k-1) | G(k-1))}.$$  

Next we make an observation that connects the iterative computation of the posterior in (9) with the approximation of the expectation in (7).

Remark 2.2: (Computation of the expectation) The random sampling approach in adaptive sensing schemes is absent in the literature. This is due to greedy adaptation of the sensing policies which is absent in the literature. For this, we use a linear array of $I$ omnidirectional antennas, with observation model

$$x_{j,k} = m_k(\theta)s_{j,k} + n_{j,k},$$  

where $x_{j,k} \in \mathbb{C}^I$ is the observation at snapshot $j \in \{1, \ldots, J\}$ in step $k \in \{1, 2, \ldots\}$, $m_k(\theta) \in \mathbb{C}^I$ is the steering vector for the unknown electronic azimuth $\theta := \sin(\phi)$, where $\phi \in (-\pi, \pi)$ is the azimuth or direction of arrival, $s_{j,k} \in \mathbb{C}$ is the target signal (which here we assume is known), and $n_{j,k}$ is the noise, modeled by independent and identically distributed zero-mean complex Gaussians with real and imaginary parts also independent with covariance $\sigma_I$ (i.e., a multiple of the identity matrix). In SIMO radar (i.e., a single transmitter and multiple receivers), $m_k(\theta)$ corresponds to the receive steering vector $a_{Rx}(\theta) := e^{j\kappa d_{Rx} \theta}$, where $\kappa_0 = 2\pi/\lambda$ is the wavenumber and $\lambda$ is the received wavelength. To incorporate into model the selection of Rx elements, we define the receive switching matrix $G_{Rx} \in \{0, 1\}^{I^x \times N}$, for a total of $I^x$ active receivers, such that the $i$th row contains a nonzero element only in column $n_i$, and each column has at most a nonzero element.

$$m_k^{MMO}(\theta) := G_{Rx}^* m_k(\theta) = e^{j\kappa_0 d_{Rx} \theta} = e^{j\kappa_0 G_{Rx}^* 2\pi \theta}.$$  

Similarly, for DoA estimation using MIMO arrays, we define the switched TDM MIMO steering vector as

$$m_k^{MIMO}(\theta) := (\gamma(f_D) \ast (G_{Tx}^T a_{Rx}(\theta))) \ast (G_{Rx}^* a_{Rx}(\theta)).$$  

In summary, measurements depend on the sensing policies, as prescribed by the likelihood. The likelihood serves two purposes, as shown in Fig. 1: i) filtering in the processor, where particles, i.e., guesses of the parameter, are re-sampled according to which ones make the measurements more likely, and ii) prediction in the controller, where the WWB is computed integrating the joint distribution that combines the likelihood of possible observations and the current posterior.

III. ADAPTIVE CHANNEL SELECTION

In this section, we particularize the strategy for adaptive sensing in DoA estimation with MIMO linear arrays.

2To our knowledge, the study of filtering performance in scenarios of dependent measurements due to greedy adaptation of the sensing policies is absent in the literature.

3For 1-dimensional cases, integration using the empirical PDF of the particles might be more efficient than Monte Carlo integration. We use this approach for DoA estimation with and without the approximation given by fitting the posterior by a Gaussian, which lowers the computational cost.

A. Problem statement

Here we consider the problem of angle of arrival estimation of a single far-field point target. For this, we use a linear array of $I$ omnidirectional antennas, with observation model

$$x_{j,k} = m_k(\theta) s_{j,k} + n_{j,k},$$  

where $x_{j,k} \in \mathbb{C}^I$ is the observation at snapshot $j \in \{1, \ldots, J\}$ in step $k \in \{1, 2, \ldots\}$, $m_k(\theta) \in \mathbb{C}^I$ is the steering vector for the unknown electronic azimuth $\theta := \sin(\phi)$, where $\phi \in (-\pi, \pi)$ is the azimuth or direction of arrival, $s_{j,k} \in \mathbb{C}$ is the target signal (which here we assume is known), and $n_{j,k}$ is the noise, modeled by independent and identically distributed zero-mean complex Gaussians with real and imaginary parts also independent with covariance $\sigma_I$ (i.e., a multiple of the identity matrix). In SIMO radar (i.e., a single transmitter and multiple receivers), $m_k(\theta)$ corresponds to the receive steering vector $a_{Rx}(\theta) := e^{j\kappa d_{Rx} \theta}$, where $\kappa_0 = 2\pi/\lambda$ is the wavenumber and $\lambda$ is the received wavelength. To incorporate into model the selection of Rx elements, we define the receive switching matrix $G_{Rx} \in \{0, 1\}^{I^x \times N}$, for a total of $I^x$ active receivers, such that the $i$th row contains a nonzero element only in column $n_i$, and each column has at most a nonzero element.

The switched receive steering vector is then defined as

$$m_k^{MMO}(\theta) := G_{Rx}^* m_k(\theta) = e^{j\kappa \theta} G_{Rx}^* 2\pi \theta = e^{j\kappa_0 \frac{2\pi}{\lambda} \theta} G_{Rx}^* 2\pi \theta.$$  

Similarly, for DoA estimation using MIMO arrays, we define the switched TDM MIMO steering vector as

$$m_k^{MIMO}(\theta) := (\gamma(f_D) \ast (G_{Tx}^T a_{Rx}(\theta))) \ast (G_{Rx}^* a_{Rx}(\theta)),$$  

where $a_{Rx}(\theta) := e^{j\kappa_0 d_{Rx} \theta} \in \mathbb{C}^M$ is the transmit steering vector for $M$ transmitters located at positions $d_{Rx} := [d_{Rx,1}, \ldots, d_{Rx,M}]$, the transmit switching matrix $G_{Tx} \in \{0, 1\}^{I^x \times M}$, for $I^x$ active transmitters, is such that the $i$th row contains only a nonzero element in column $m_i$ (and each column has at most a nonzero element); and $\gamma(f_D) := e^{j2\pi T [1, \ldots, I^x]} f_D \in \mathbb{C}^{I^x}$ contains the Doppler frequency shift $f_D \in \mathbb{R}$ (that we assume is known here), typical in a TDM scheme. The latter term results from the sequence of pulses from the active transmitters with inter-pulse duration $T > 0$.

With this notation, (12) can be written as

$$m_k^{MMO}(\theta) := [e^{j2\pi f_D T_1} e^{j\kappa_0 d_{Rx,1} \theta}, \ldots, e^{j2\pi f_D T_{I^x}} e^{j\kappa_0 d_{Rx,M} \theta}]^T \ast [e^{j\kappa_0 d_{Rx,1} \theta}, \ldots, e^{j\kappa_0 d_{Rx,M} \theta}]^T = e^{j\kappa_0 \frac{2\pi}{\lambda} \theta} G_{Rx}^* 2\pi \theta,$$  

where $d_{Tx} := 2\pi T [1, \ldots, I^x] \ast 1_{I^x}$, and

$$d_{Rx} := 1_{I^x} \ast (G_{Rx}^* d_{Rx}) + (G_{Rx}^* d_{Rx}) \ast 1_{I^x}.$$  

The goal is to choose a total of $I = I^x + I^T$ active transmitters and receivers, specified by $G_k = (G_{Tx}^T G_{Rx})$ at each step $k \in \{1, 2, \ldots\}$, that help extract the maximum
amount of information about the angle of arrival according to (6). The only part of the adaptive sensing strategy of Section III that needs to be particularized is the likelihood function, which naturally depends on the observation model above, cf. Fig. 4. Using the corresponding likelihood function for DoA estimation in SIMO and MIMO radar, in the next section we construct the WWB associated to these problems.

B. Conditional WWB for DoA estimation

To apply the general strategy of Section III to the problem of antenna selection, we need to use the likelihood function associated to the observation model (10), see Fig. 1. The likelihood function of \( J \) snapshots \( X^k = [x_{1,k}, \ldots, x_{J,k}] \), given \( \theta \) and sensing parameters \( G_k = \{G^x_k, G^r_k\} \), is distributed as a product of complex Gaussian distributions because snapshots are assumed independent, i.e.,

\[
p(X^k|\theta, G_k) = \prod_{j=1}^{J} \left( \frac{1}{\pi \sigma^2} \right) e^{-\frac{1}{\sigma^2} ||x_{j,k} - m_k(\theta)s_{j,k}||^2_2}.
\]

From the computation in [15, eq. (137)], one has

\[
D_k(\theta, \alpha, \beta) = \prod_{j=1}^{J} \int_{\Re^2} p^\alpha(x_{j,k}|\theta + \beta) \, p_{\theta, \beta}^{-1}(x_{j,k}|\theta) \, dx_{j,k}\]

\[
= e^{s_k^2 \alpha - \sum_{j=1}^{J} ||m_k(\theta + \beta) - m_k(\theta)||^2_2} - 2 \text{Re}\{ \sum_{i=1}^{R} m_k^{\text{SIMO}}(\beta^*)) \}
\]

where \( s_k^2 := \sum_{j=1}^{J} ||s_{j,k}||^2_2 \). (Note that the model with unknown stochastic target signals, called unconditional, requires a different calculation, cf. [11], [19].) In the SIMO case, using the definition (11), we get

\[
||m_k^{\text{SIMO}}(\theta + \beta) - m_k^{\text{SIMO}}(\theta)||^2_2 = \|m_k^{\text{SIMO}}(\theta + \beta)\|^2_2 + \|m_k^{\text{SIMO}}(\theta)\|^2_2
\]

which is related to the ambiguity surface (cf. [17, pp. 269, eq. 4.229]) for the selected receivers. Therefore,

\[
D_k^{\text{SIMO}}(\alpha, \beta) := e^{\frac{\alpha - 1}{\sigma^2} 2 (\text{Re}\{ \sum_{i=1}^{R} \cos(k_0 G^r_k \theta^i) \} \}.
\]

Similarly, for the MIMO case, using (13), we obtain

\[
D_k^{\text{MIMO}}(\alpha, \beta) := e^{\frac{\alpha - 1}{\sigma^2} (2 \text{Re}\{ \sum_{i=1}^{R} \cos(k_0 G^r_k \theta^i) \} \}.
\]

Equipped with the functions \( D_k^{\text{SIMO}}(\alpha, \beta) \) and \( D_k^{\text{MIMO}}(\alpha, \beta) \) (which incidentally do not depend on \( \theta \)), the parametric family of conditional bounds WWB \( s, h; X^{(k-1)}, G^i(\theta) \) can be expressed in terms of (7) according to (2). Note that the posterior can be approximated following Remark 2.2. We can then evaluate candidate sets of channels specified by \( G_k = \{G^x_k, G^r_k\} \), and select the optimal ones according to (6).

Next we present simulations with synthetic measurements.

C. Simulations

Here we compare in simulations the performance of channel selection policies that optimize the WWB, the BZB, and the ECRB for SIMO and MIMO arrays. The separation between adjacent transmitters and receivers is \( 0.9\lambda/2 \), the number of snapshots is \( J = 2 \), the target signal \( s_{j,k} \) is assumed known and equal to 1, and we assume an initial prior distribution for the electronic azimuth uniform in \([-1, 1]\). The target is static, \( f_D = 0 \), and therefore the order of transmitter activations is irrelevant for any given subset of them. We perform the inner optimization in (6) using simulated annealing [18] with a cooling speed of 100 intermediate temperatures when the SNR is less than 0, and 50 otherwise, and the posterior is sequentially updated using a particle filter with residual resampling [20] and 500 particles.

The channel choices for the SIMO and MIMO cases are shown in Figs. 2 and 3 for a single execution of our algorithm with SNR = −5. These choices depend on the posterior distribution updated by each strategy and thus on the unique history of previous measurements and channel selections. In the SIMO case, we observe a qualitative behavior for the policies that optimize the WWB and BZB analogous to the simulations in [10], [11], where during the first measurements receivers tend to be chosen closer together to avoid ambiguity in the estimation, and in subsequent measurements are selected farther apart to increase resolution. A similar behavior can be seen in the MIMO case.

We analyze the performance using the MSE of the conditional mean estimator \( \hat{\theta} \) that results from each sensing policy.

Fig. 2: Optimal channel choices in a typical execution in the SIMO case, with SNR = −5, where Rx 1 is always fixed.

Fig. 3: Optimal channel choices in the MIMO case for each policy, under a Gaussian approximation of the posterior at each step, where Tx 1 and Rx 1 are always fixed. (Overlapping virtual elements are represented with concentric circles.)

\[\text{Matlab code, by Héctor Corte, available in MathWorks File Exchange.}\]

\[\text{Matlab code “Resampling methods for particle filtering,” by J.-L. Blanco Claraco, available in MathWorks File Exchange.}\]
This is computed at each measurement step with respect to the true parameter value $\theta = \sin(\phi) = 0.3$ using 300 Monte Carlo realizations of each snapshot. In the SIMO case, Fig. 1 (top) shows the same single execution as in Fig. 2. In the MIMO case, we have simulated a computationally faster version of the adaptive policies where the expectation in (7) is approximated replacing the posterior given by the particles by a Gaussian distribution with the same mean and variance. Using the result in [15, eqs. (138), (152)], this allows us to obtain a close form for (7). With this approximation, Fig. 4 (bottom) compares the average MSE, over 20 algorithm trajectories for each SNR, of the conditional mean estimator at measurement step 8. We observe that optimizing the WWB yields slightly better performance than using the BZB for low SNR values. In addition, these adaptive policies outperform \textit{ad hoc} strategies with the same number of active antennas, including the SIMO “stair” switch and the fixed MIMO uniform virtual array. The evaluation of the WWB with $s = 0.5$ yields comparable performance to (but not always the same channel choices as) the WWB even though it uses a single test-point as the BZB. The discussion of the computational complexity depends on the cooling speed that is required for the optimization of each Bayesian bound and will be included in future work.

![Fig. 4: MSE of the conditional mean for each policy. In the SIMO case (top) we depict a single execution over time with SNR $= -5$. In the MIMO case (bottom) we plot the average MSE at step 8, over 20 executions, for each SNR.](image)

IV. CONCLUSIONS AND FUTURE WORK

Adaptive strategies based on the Weiss-Weinstein bound outperform some common channel selections for DoA estimation. The biggest concern is the computational time of policy evaluation at the controller, which for DoA estimation of a single target can be greatly reduced by fitting the output of the particle filter by a Gaussian, and also the number of candidate subsets of channels. Future work also includes target dynamics and estimation of model parameters such as the SNR or the Doppler frequency, and employing multi-step ahead predictions.

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