Plate tensegrity structures controlled with self-stress

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Abstract. The present paper focuses on the analysis of plate tensegrity structures based on the 3-strut simplex module. Two structural systems are considered: a single plate tensegrity module and a multi-module segment. It is proved that the implementation of plate elements in the basic tensegrity module not only opens new application perspectives, but also increases its stiffness. The analyses are performed with the use of the finite element method (FEM), using the second order theory and the geometrically nonlinear analysis.

1. Introduction
Nowadays, tensegrity structures are becoming more and more popular – not only because of their modern appearance in real structures, but mostly because of their undeniable advantages such as a high stiffness-to-mass ratio, economic use of materials, increased capacity under the increasing external load or possibility to construct modular systems. The development of tensegrity has brought a new type of structural system – plate tensegrity structures.

Generally, tensegrity is a term that characterizes a construction rule, in which a structure consists of compressed elements (struts) joined by tensile members (cables) [1, 2]. These structural elements stabilize each other. In tensegrity structures there are infinitesimal mechanisms [3] that are eliminated by self-stress states, which guarantee structural stiffness. Self-stress forces are independent of supports and loads. Tensegrities are classified as a specific type of trusses. To their important features belongs adaptiveness to the environment due to self-stress [4-7].

Plate tensegrity structures have all previously mentioned properties and additionally, an integrated plate element which is a working part of the system, not an additional part attached to it. Such structures are developed by replacing some of the cables with plate elements. It allows us to change the designing approach – not only the construction stiffness is increased but also it opens new application possibilities.

Through years of evolution of tensegrity systems, plates have been repeatedly implemented into the structure, but always as an additional element, which only increased its functionality. The first real integration of plate and bars was made by Andreas Falk [8, 9]. In his consideration, Falk created two plate tensegrity modules – one based on a triangular plate and the second based on a rectangular plate. Plates are perforated in the middle and a strut goes through the opening. Both ends of the strut are joined to the plate corners by cables. The module is prestressed to eliminate an infinitesimal mechanism. As a result of prestressing, struts and plates are compressed and cables are tensed. The buckling length of the strut is halved by a plate which increases its capacity.

Using those modules Falk and Kirkegaard analyzed various slab structures [9] – from flat to arched. Each of them is created by joining plate tensegrity modules on plate level and both strut ends.
Plates transmit each other’s parallel forces – compression and shear. Struts are joined by cables. As a result, a three-level structure is obtained with external cable nets and an internal plate surface. The structure is in equilibrium when every element is prestressed with the set of forces attuned to modules.

Falk stated two ways of creating slabs – using one large plate or joining smaller modules. The first one creates a stiffer structure and decreases material usage. The second one allows us to create more flexible system – arched or with specific shapes. By stabilizing the systems with cable nets some of the modules may be removed and non-structural elements such as transparent plates or plates from lighter material may be applied. What is more, openings could be left in the slabs.

During their analysis, Falk and Kirkegaard noted that cables in nets and modules have different directions and therefore, prestressing of these cables could cause some deformation. To avoid the possibility of deformation they proposed a special geometrical ratio based on prestressing forces in cables. In order to adapt the structure to various load cases, self stress needs to be adjusted a few times during use. In his analysis, Falk consciously resigned from compression in tension rule. On this account, plate capacity can be bigger and it also helps to reapply prestressed elements.

In his analysis, Falk also mentions that plate tensegrity structures are stronger than tensegrity systems in case of cable damage. It resulted from the fact that damaging one cable provides variations in the prestressing forces set. The plate has a greater capacity limit and by manipulating its thickness and level of integration with cables, the structure is prepared for different load cases.

This feature confirms that plate tensegrity structures, just as traditional tensegrity systems, can be regarded as smart structures [2, 10]. Their smartness results from infinitesimal mechanisms that occur in the modules. There are four major inherent properties of smart structures [2]: self-control, self-diagnosis, self-repair, and self-adjustment (active control). Self-control refers to self-stiffening structures with the increase of applied load. Self-diagnosis refers to identification of damages by noting changes in the internal forces set. Damage of active members changes the level of self-stress in the entire structure. Self-repair comes directly from infinitesimal mechanisms stabilized by self-stress. Proper change of prestressing compensates damages in the structure and restores the initial state of displacements. Self-adjustment refers to the ability of adjusting self stress in the whole structure or separate parts of it. Each of these features is also present in plate tensegrity structures.

In present paper an analysis of a plate tensegrity module based on the simplex module is presented. The analysis involves comparing calculations in two types of theories – a second-order analysis which considers the prestressing forces and a geometrically nonlinear analysis. Moreover, an analysis of a multi-module plate tensegrity structure is presented.

2. Plate tensegrity module
A plate tensegrity structure is based on simplex (figure 1) – a basic tensegrity unit, which is composed of three struts and nine cables. It has the shape of a regular triangular prism. Struts – compressed elements – are diagonals of the prism sides and cables – tensed elements – are edges of the block. The final form of the module is created by rotating the upper and lower base of the prism in opposite directions creating a 150° angle. In plate tensegrity modules the upper and lower cables are replaced with panels.

The analyzed module is composed of two triangular plates with 1 m sides for the lower element and with 0.5768 m sides for the upper element. Both panels are 40 mm thick. For struts and cables, catalog cross-sections are applied. Compressed elements are steel tubes with 60 mm diameter and 2.9 mm wall thickness. Tensed elements are steel rods with 10 mm diameter.

For plate elements cross laminated timber (CLT) is applied. The production process eliminates anisotropy by gluing wooden boards in alternating cross-wise layers to obtain two-dimensional form stability. There is always an uneven number of layers and an angle between two next sheets is suited to the specific construction. CLT has also profitable weight to stiffness ratio which is the desired feature for tensegrity structures. Moreover, the production process is environmentally friendly and suited to the case.
Structural steel S355JR is used as a material for cables and struts. Material is commonly used in cable structures, not only because of high ultimate tensile strength and compression strength but also due to its plasticity and ease of shaping.

![Simplex module: (a) tensegrity, (b) plate tensegrity.](image)

The module is fixed by pin support in three points of the lower plate (figure 2). In node 1 movement in $X$, $Y$, $Z$ direction is blocked, in node 2 $Z$ motion is blocked and in node 3 $X$ and $Z$ movement is blocked.

The module was prestressed before applying external load to remove the infinitesimal mechanism encountered in tensegrity structures. Struts and cables are compressed with different forces: $S = S_0 \cdot a$, where $S_0$ – initial force value, $a$ – force multiplier value (table 1).

**Table 1.** Force multiplier values for struts and cables.

| Element | Force multiplier value |
|---------|------------------------|
| Strut   | -0.421637              |
| Cable   | 0.3333333              |

Table 2 presents the values of eigenfrequencies obtained for the module before and after the application of self-stress. It can be notice the prestressing forces eliminate the infinitesimal mechanism.

**Table 2.** Eigenfrequencies in the module.

| Eigenfrequencies | Before prestressing (Hz) | After prestressing (Hz) |
|------------------|--------------------------|--------------------------|
| 1                | 0.0                      | 24.14                    |
| 2                | 104.86                   | 104.53                   |
| 3                | 155.78                   | 148.53                   |
| 4                | 213.71                   | 207.41                   |
| 5                | 319.29                   | 314.96                   |
| 6                | 325.37                   | 324.72                   |
| 7                | 513.74                   | 511.50                   |
| 8                | 623.59                   | 616.06                   |
| 9                | 805.90                   | 807.28                   |
| 10               | 865.54                   | 862.19                   |
For the analysis, SOFiSTiK software was used. The analysis was performed for two load cases (figure 2). In the first case, three vertical loads equal to 10 kN each were applied to upper plate nodes 4, 5, 6. In the second one, the horizontal force equal to 10 kN in the global X direction was applied to node 5.

![Figure 2. Load cases: (a) vertical loads applied to upper plate nodes, (b) horizontal load applied to node 5.](image)

The module was analyzed with the use of the finite element method (FEM) in two types of theories – a second-order analysis which considers the stability of the structure and geometrically nonlinear analysis.

In the compressed element, buckling could occur. Intending to reduce that adverse mechanism, the applied prestress load needs to be limited. The critical load was calculated using Euler buckling formula. Its value is 247.81 kN. Analysis was limited by the critical load, the prestress applied to the module was in a range from 0 to 600 kN.

During the analysis the displacement of node 5 in the global Z direction for vertical load case (figure 3a) and global X direction for horizontal load case (figure 3b) was noted. Additionally, the behavior of the analyzed plate tensegrity module was compared to the characteristics of the basic 3-strut simplex module (figure 4). Displacement values are given in mm and force values in kN.

![Figure 3. Influence of self-stress on the displacement of node 5: (a) vertical load case, b) horizontal load case.](image)
Finite element analysis according to the second order theory allows to consider prestressing forces through geometric stiffness matrix and to obtain structural response for external loading. Fully nonlinear analysis leads to big differences within the scope of small prestressing forces, which disappear along with an increase of force values (figure 3).

Moreover, it can be noticed that the implementation of the plate element in the basic tensegrity module increases its stiffness (figure 4). The greatest difference is noted for small values of prestressing forces and slowly disappears with an increase of self-stress.

3. Multi-module plate tensegrity structure
Natural applications for multi-module plate tensegrity structures are slabs. Plate elements, which are integral part of the structure, could be an overlay as well as a construction surface. What is more, slabs can be created with single repetitive modules, what could simplify the erection process. Two major slab segment shapes – hexagon and elongated hexagon could be created by using simplex plate tensegrity module. The first segment is used in round or skew slabs, the second in rectangle ones.

Hexagonal segment is assembled from six simplex modules joined in one node (figure 5). Modules need to be arranged clockwise and counter-clockwise in rotation with the aim of stiffening the structure. The segment is fixed in four nodes.

The elongated hexagonal segment could be assembled with any amount of modules arranged clockwise and counter-clockwise, alternately (figure 6). Analogically to the hexagonal segment, in the elongated hexagonal segment the infinitesimal mechanism is compensated with the same prestress as in a single module.
Figure 6. Scheme of creating a basic elongated hexagonal segment.

Slab dimensions are limited only by a segment dimensions. Hexagonal segments are arranged concentrically around the chosen point in the case of round slabs or next to each other in rows for skew slabs. Elongated hexagonal segment creates a basic slab with a 4 m width and unlimited length. Slab with endless dimensions could be formed by joining more of those basic slabs.

Figure 7. 3D views of plate tensegrity segments: (a) hexagonal segment, (b) elongated segment.

The analyzed slab segment was created with fourteen modules of the same type – seven clockwise and seven counter-clockwise oriented (figure 8). The module used here is the single module analyzed before. The module is 1 m high. To create a continuous walking surface and avoid increasing the slab weight by adding more plate elements, modules need to be turned upside down.

As was mentioned before, modules are joined in rotation clockwise and counterclockwise oriented to block mutual movements. Modules are assembled in rows, seven in every and then both rows are joined together. Some of the struts are removed to avoid duplication in nodes. Finally, the structure is 1.73 meter wide, 4 m long and 1 m high.

The segment is fixed in four nodes on the lower surface by pin supports (figure 8). In node 1 movement in X, Y, Z direction is blocked, in node 7 X and Z motion is blocked, in node 23 Y and Z movement is blocked and in node 25 Z motion is blocked. The structure is loaded with vertical forces of 10 kN applied to the upper nodes (figure 8).
Figure 8. The elongated hexagonal segment with nodes numbering and loading.

Figure 9. Influence of self-stress on the vertical displacement of node.

It can be noticed that the behavior of the elongated segment is very similar to the characteristics of the single module, just as in the case of traditional tensegrity structures [11].

4. Conclusions

In present paper analyses of plate tensegrity structures based on the 3-strut simplex module are presented. Two types of theories are compared – a second-order analysis which considers the prestressing forces and a geometrically nonlinear analysis. Finite element analysis according to the second order theory allows to consider prestressing forces through geometric stiffness matrix and to obtain structural response for external loading. Fully nonlinear analysis leads to big differences within the scope of small prestressing forces, which disappear along with an increase of force values.

Two structural systems are analyzed: a single plate tensegrity module and a plate segment that consists of 14 modules connected in common nodes.

The performed analyses show that the implementation of plate element in the basic tensegrity module increases its stiffness and, in the same time, opens new application perspectives. The greatest difference is noted for small values of prestressing forces and slowly disappears with an increase of self-stress.

In the future studies the authors plan to analyze other structures based on the module considered in this paper, focusing on various support schemes and real loads acting in different directions. Moreover, new solutions of tensegrity modules will be searched for, where the cable elements may be replaced by rigid plates.
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