Vibration and harmonic analysis of moderately thick anti-symmetric cross-ply laminated composite plate using FEM

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Abstract. This work presents vibration and harmonic behaviour of cross-ply composite plates with anti-symmetric lamination scheme of layers. Convergence test and comparison studies for different parameters with respect to number of modes have been carried out using the finite element method. The finite element package ANSYS is used to determine the frequency parameter and resonance amplitude of the plate. The effects of different parameters on the vibration and harmonic behaviour, are discussed. The frequency-amplitude relationships are obtained and presented on frequency response function graph. The clamped, simply supported, free and the combinations of boundary conditions are considered. Effects of damping factor on harmonic response are also investigated.

Keywords: Composite plate, Free Vibration, Harmonic Response, Frequency Response Function, FEM.

1. Introduction
Efforts are being made to find light weight, stronger and tougher structural materials for making component used in aircraft, automobile sector, wind turbine etc. Laminated composite plates being part of these components are often subjected to dynamic force, resulting vibration. If vibration reached to its natural frequency, resonance occurred. System can fail due to improperly constructed design structures. Therefore, the study for these aspects is necessary for safe and accurate design.

Many researchers have shown more attention in studying the dynamic behaviour of different types of composite plates by employing different methods. Kant and Swaminathan (2001) employed the higher-order refined theory to study the symmetric/anti-symmetric composite as well as sandwich plate and found the solutions and formulations for vibration analysis. Karami and Malekzadeh (2002) analysed the static as well as stability of thin quadrilateral plates using Differential Quadrature Method (DQM) and the free vibration of thin sector plates was analysed by Wang and Wang (2004) by using a new version of DQM. Sharma A. et al. (2005) presented a analytical solution for various eigenvalue problems of buckling as well as free vibration behaviour of cylindrically orthotropic sector plates taking different combinations of boundary conditions. Ohya et al. (2006) observed free vibration behavior of rectangular Mindlin plates using superposition method and shown the accurate result for these plates having internal column supported on uniform elastic edge.
Khaldoon F. Brethee (2009) analyzed for free vibration of laminated symmetric/anti-symmetric composite plate with cut-out of different geometry in centre. Asadi and Fariborz (2011) obtained the governing equations as well as required boundary conditions for analyzing the vibration of anti-symmetric/symmetric composite plates using a global HSDT. Rao and Reddy (2012) analyzed the displacements and natural frequencies of a propeller of composite material with metal using ANSYS software. Useche et al. (2012) studied the vibration and harmonic characteristics of orthotropic plates having crack and having a shear deformable property. Author used a Method of Boundary Element. Sharma A. K. et al. (2014) presented the effect of materials, geometrical, and support parameters on the free vibration of anti-symmetric angle-ply laminated rectangular plates of composite materials for both translational and rotational edge boundary conditions. Maithry and Rao (2015) studied the dynamic behavior of orthotropic plates for different fiber direction using ANSYS 13.0 software. Viswanathan K.K. et al. (2016) analyzed the vibration behavior of anti-symmetric angle-ply orthotropic plates having clamped constrained using RBF and spline approximation numerical approaches. Veysel ALANKAYA (2017) investigated bending deformation of composite plates for varying boundary conditions and laminations. Nor Hafizah A. K. et al. (2018) studied the vibrational behaviour of angle-ply laminated composite plates with anti-symmetric lamination scheme of variable thicknesses using HSDT.

In this paper, free vibration and harmonic analysis of anti-symmetric cross ply laminated composite plates with different boundary conditions are investigated by finite element method. The accuracy, convergence and versatility of the algorithm are demonstrated via solving different cases of moderately thick laminated plates. This work, thus, aims to study the free vibration and harmonic study of anti-symmetric cross ply laminated composite plates with different boundary conditions which appear to have not been studied as yet.

2. Modelling

For modal analysis, an orthotropic square plate of 1m × 1m of various thicknesses is considered in the study. For providing the material properties of the plate, the default parameters like $E_1/E_2 = \text{open}$, $\nu = 0.25$, $G_{12}/E_2 = G_{13}/E_2 = 0.6$, $G_{23}/E_2 = 0.5$ and $\rho = 1 \text{ kg/m}^3$ are employed where the subscripts 1 denotes parallel and 2 denotes perpendicular directions with respect to the direction of fibres in a layer.

A 1 N force is applied at the node location (0.30769 m, 0.53846 m) of same plates, to know the harmonic behaviour of the plate. An orthotropic plate under clamped boundary condition subjected to a force is shown in fig. 1(a) and cross ply orientation (0°/90°) of this plate is shown in fig. 1(b).

![Figure 1. (a) cross ply plate with anti-symmetric lamination scheme under a force of 1 N (b) Orientation of same plate i.e. (0°/90°).](image)

The SHELL281 element is chosen to mesh the model.
3. Verification of present analysis

3.1. Convergence Study

For choosing proper mesh size, convergence test has been performed for cross-ply orthotropic plate as shown in table 1 and 2. The convergence has been found at the mesh size of 13 × 13.

Table 1. Converged non-dimensionalised frequency parameters for a square cross-ply laminated antisymmetric plate having two layers [0°/90°] under all edges clamped boundary conditions for thickness ratios (t/b) of 0.1 and 0.2.

| a/b | t/b | Mesh Size | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|-----|-----|-----------|----|----|----|----|----|----|----|----|
| 1   | 0.1 | 9x9       | 17.5028 | 30.0879 | 39.1025 | 46.4919 | 46.6349 | 53.0407 | 53.0407 |
|     |     | 11x11     | 17.5018 | 30.0779 | 39.0886 | 46.4343 | 46.5773 | 52.9851 | 52.9851 |
|     |     | 13x13     | 17.5012 | 30.0740 | 39.0827 | 46.4098 | 46.5555 | 52.9633 | 52.9633 |
|     |     | 15x15     | 17.5012 | 30.0740 | 39.0827 | 46.4085 | 46.5435 | 52.9533 | 52.9533 |
|     |     | 17x17     | 17.5012 | 30.0740 | 39.0787 | 46.3965 | 46.5376 | 52.9474 | 52.9474 |
|     |     | 19x19     | 17.5012 | 30.0720 | 30.0700 | 39.0787 | 46.3925 | 46.5356 | 52.9454 | 52.9454 |
| 1   | 0.2 | 9x9       | 11.2926 | 18.0084 | 23.0085 | 26.6485 | 26.6803 | 30.3253 | 30.3253 |
|     |     | 11x11     | 11.2926 | 18.0044 | 23.0035 | 26.6247 | 26.6565 | 30.3015 | 30.3015 |
|     |     | 13x13     | 11.2926 | 18.0035 | 23.0015 | 26.6148 | 26.6465 | 30.2925 | 30.2925 |
|     |     | 15x15     | 11.2926 | 18.0035 | 23.0015 | 26.6148 | 26.6426 | 30.2886 | 30.2886 |
|     |     | 17x17     | 11.2926 | 18.0025 | 23.0006 | 26.6078 | 26.6396 | 30.2856 | 30.2856 |
|     |     | 19x19     | 11.2926 | 18.0025 | 18.0025 | 22.9996 | 26.6068 | 26.6386 | 30.2846 | 30.2846 |

Table 2. Converged non-dimensionalised frequencies parameters for a square cross-ply laminated antisymmetric plate having four layers [0°/90°]2 with thickness ratio as 0.2 under different boundary conditions i.e. SSSS and CFCF.

| a/b | Boundary Condition | Mesh Size | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  |
|-----|--------------------|-----------|----|----|----|----|----|----|----|----|
| 1   | SSSS               | 9x9       | 10.7045 | 18.9969 | 24.6408 | 28.2579 | 28.2679 | 32.3281 | 32.3281 |
|     |                    | 11x11     | 10.7045 | 18.9939 | 24.6358 | 28.2341 | 28.2440 | 32.3053 | 32.3053 |
|     |                    | 13x13     | 10.7045 | 18.9929 | 24.6338 | 28.2251 | 28.2341 | 32.2963 | 32.2963 |
|     |                    | 15x15     | 10.7045 | 18.9929 | 24.6338 | 28.2251 | 28.2341 | 32.2924 | 32.2924 |
|     |                    | 17x17     | 10.7045 | 18.9919 | 24.6318 | 28.2182 | 28.2271 | 32.2894 | 32.2894 |
|     |                    | 19x19     | 10.7045 | 18.9919 | 18.9919 | 24.6318 | 28.2162 | 28.2261 | 32.2884 | 32.2884 |
| 1   | CFCF               | 9x9       | 8.8422  | 9.0427  | 11.8907 | 17.6130 | 17.6130 | 18.0025 | 23.5390 | 23.7844 |
|     |                    | 11x11     | 8.8420  | 9.0424  | 11.8897 | 17.6091 | 17.6806 | 17.9985 | 23.5340 | 23.7784 |
|     |                    | 13x13     | 8.8419  | 9.0422  | 11.8917 | 17.6701 | 17.6796 | 17.9965 | 23.5330 | 23.7774 |
|     |                    | 15x15     | 8.8419  | 9.0422  | 11.8917 | 17.6701 | 17.6796 | 17.9965 | 23.5331 | 23.7794 |
|     |                    | 17x17     | 8.8419  | 9.0422  | 11.8917 | 17.6701 | 17.6796 | 17.9955 | 23.5311 | 23.7814 |
|     |                    | 19x19     | 8.8419  | 9.0422  | 11.8917 | 17.6701 | 17.6796 | 17.9955 | 23.5311 | 23.7844 |

3.2. Comparison Study

In order to illustrate the accuracy and usefulness of the present adopted method, square plates having cross-ply laminate have been studied and compared with the published literature of Kant and Swaminathan (2001). Table 3 shows the comparison study of frequency parameter (ω) for antisymmetric square laminated plate having four and six layers under simply supported boundary condition.
for different modulus ratio. From the table, it is clear that the maximum variation is limited to 2% that is the results seems to close propinquity.

Table 3. Comparison for non-dimensional frequency parameters ($\bar{\sigma} = \omega (b^2/t) \sqrt{\rho/E_2}$) of anti-symmetric cross-ply square laminated plate under simply supported boundary condition for various modulus ratio and increasing no. of layers.

| No. of Layer | t/b = 0.2 | $\bar{\sigma}$ | %Error |
|--------------|-----------|----------------|--------|
| E1/E2        | 10        | 20             | 30     | 40     |
| Present      |           |                |        |        |
| (0/90)₂      | 8.0640    | 9.4393         | 10.2088| 10.7045|
| Kant and Swaminathan (2001) | 8.1482    | 9.4675         | 10.2733| 10.8221|
| %Error       | 1.033     | 0.297          | 0.628  | 1.086  |
| (0/90)₃      | 8.2246    | 9.7552         | 10.6389| 11.2191|
| Kant and Swaminathan (2001) | 8.3852    | 9.8346         | 10.7113| 11.3051|
| %Error       | 1.914     | 0.807          | 0.675  | 0.760  |

4. Results and discussion

4.1. Vibration Analysis

The natural frequencies and mode shapes of a structure or an object are determined during free vibration in the study of modal analysis. In this paper, the first mode of frequencies has been studied for anti-symmetric four ply ($0°/90°$)₂ composite plate and found the effects of boundary condition, number of layers, modulus ratio and various thickness ratios. The $13 \times 13$ mesh size is employed and the Block Lanczos solver is used.

4.1.1. Effect of number of layers. The laminated composite plate having anti-symmetric arrangement of layers with thickness ratio as 0.2 and modulus ratio as 40, are studied to examine the effect of number of layers on the frequency parameter ($\bar{\sigma}$). Cross-ply orientation is used with all edges fixed boundary condition i. e. CCCC. The results are shown in fig. 2. In this, 2, 4, 6 and 8 layered cross-ply plates with anti-symmetric lamination schemes are used to study the vibration behaviour of plates. It has been observed that there is an increment on non-dimensional frequency parameters ($\bar{\sigma}$) as the layers are increasing.

4.1.2. Effect of Boundary Condition. The variation of non-dimensional fundamental frequency of the plate under different boundary conditions is shown in figure. 3. Four-ply ($0°/90°$)₂ anti-symmetric laminated plate having thickness ratio 0.2 and modulus ratio 40, is analysed to show the effect of boundary conditions. The various boundary conditions such as all edges simply supported (SSSS), all
edges clamped (CCCC), one edge clamped and other free (CFFF) and two edges clamped and two edges free (CFCF) are used. From the figure, it is found that the values are much close for the plate with clamped edges and simply supported. It is also observed the values are low for cantilever plate.

![Figure 3](image.png)

**Figure 3.** Effect of boundary conditions on non-dimensionalised fundamental frequencies.

4.1.3. Effect of modulus ratio ($E_1/E_2$). The four-ply ($0^\circ/90^\circ$)$_2$ laminates under clamped boundary condition having thickness ratio of 0.2 are studied to examine the effect of modulus ratio on the non-dimensional frequency parameters. The results for cross-ply laminates are shown in figure. 4. It has been clearly shown from figure that the fundamental frequency increases with increase in modulus ratio.

![Figure 4](image.png)

**Figure 4.** Effect of Modulus Ratio on Non-dimensionalised frequency parameters.

4.1.4. Effect of thickness ratio. The four-ply ($0^\circ/90^\circ$)$_2$ laminates having modulus ratio as 40 under clamped boundary condition (CCCC) are analyzed to determine the effect of various thickness ratios (0.001, 0.05, 0.1 and 0.2) as shown in figure. 5. It has been found that the fundamental frequency decreases with increase in thickness ratio.
4.2. Harmonic Analysis

The analysis is done to compute the resonant amplitude at the natural frequencies. The plates are loaded by externally applied force and the responses of plates are drawn on the Frequency Response Function (FRF) graph. An FRF graph is a frequency-based measurement function and is used to identify the mode shapes, resonant frequencies and damping effect of a physical structure. We have come to know its resonance point from this graph.

In this analysis, the effect of boundary condition, number of layers, damping ratio and modulus ratio are obtained.

4.2.1. Effect of boundary condition. The four layered (0°/90°)\textsubscript{2} anti-symmetric cross-ply laminates with modulus ratio of 40 for various thickness ratios are analyzed to demonstrate the effect of boundary conditions on natural frequency and resonance amplitude. The results are obtained for various boundary conditions i.e. CCCC, SSSS and CFFF as shown in Table 4. It has been found that the natural frequency is higher for clamped boundary condition i.e. CCCC for same thickness ratio and the resonance amplitude is lower for the same case.

Table 4. Effect of boundary condition on the frequency response amplitude for anti-symmetric laminated composite plate with various thickness ratios.

| t/b | Natural frequency in Hz for different boundary condition | Resonant amplitude in mm for different boundary condition |
|-----|---------------------------------------------------------|---------------------------------------------------------|
|     | CCCC         | SSSS         | CFFF         | CCCC         | SSSS         | CFFF         |
| 0.001| 192.38  | 93.38      | 21.09        | 301.373      | 955.282      | 3263.88      |
| 0.05 | 7797.8   | 4386.1     | 1030.3       | 0.0033688    | 0.00861437   | 0.0285367    |
| 0.1  | 10958    | 7588.6     | 1932         | 0.000771155  | 0.0014277    | 0.00446159   |

Frequency Response Function (FRF) Plot

Harmonic responses on cross-ply anti-symmetric laminated plate for different boundary condition are shown on FRF plot from figure 6(a) to figure 6(c) at various thickness ratios. It has been found from the FRF graph that resonance occurs at natural frequency for every mode. Resonance is maximum for first mode of frequency.
Figure 6(a) shows the FRF plot for various thickness ratios (i.e. 0.001, 0.05 and 0.1) under clamped boundary condition. Resonance 301.373 mm occurs at 192.38 Hz natural frequency for thickness ratio 0.001, 0.0033688 mm occurs at 7797.8 Hz for 0.05 and 0.000771155 mm occurs at 10958 Hz for 0.1.

Figure 6(b) shows the FRF plot for various thickness ratios (i.e. 0.001, 0.05 and 0.1) under simply supported boundary condition. Resonance 955.282 mm occurs at 93.38 Hz natural frequency for thickness ratio 0.001, 0.00861437 mm occurs at 4386.1 Hz for 0.05 and 0.0014277 mm occurs at 7588.6 Hz for 0.1.

**Figure 6(a).** FRF for four layered (0°/90°)_2 plate for different thickness ratios under all edges fixed boundary condition (CCCC).

Figure 6(b) shows the FRF plot for various thickness ratios (i.e. 0.001, 0.05 and 0.1) under simply supported boundary condition. Resonance 955.282 mm occurs at 93.38 Hz natural frequency for thickness ratio 0.001, 0.00861437 mm occurs at 4386.1 Hz for 0.05 and 0.0014277 mm occurs at 7588.6 Hz for 0.1.
**Figure 6(b).** FRF for four layered ($0^\circ$/90$^\circ$)$_2$ plate for different thickness ratios under all edges simply supported condition (SSSS).

Figure 6(c) shows the FRF plot for various thickness ratios (i.e. 0.001, 0.05 and 0.1) with one edge fixed and other edges free boundary condition. Resonance 3263.88 mm occurs at 21.09 Hz natural frequency for thickness ratio 0.001, 0.0285367 mm occurs at 1030.3 Hz for 0.05 and 0.00446159 mm occurs at 1932 Hz for 0.1.

**Figure 6(c).** FRF for four layered ($0^\circ$/90$^\circ$)$_2$ plate for different thickness ratios under one edge clamped and other edges free boundary condition.
4.2.2. Effect of number of layers. The cross-ply laminated plate with anti-symmetric scheme of layers for various thickness ratios are examined to demonstrate the effect of number of layers on natural frequency and resonance amplitude. The results are found for two layered, four layered and six layered plate under clamped boundary conditions as shown in table 5. It has been found that the natural frequency increases with increasing layer but the resonance amplitude decreases for same cases.

Table 5: Effect of number of layers on the natural frequency and resonance amplitude for anti-symmetric laminated composite plate with various thickness ratios i.e. CCCC and $E_1/E_2 = 40$.

| t/b | Natural frequency in Hz for increasing layers | Resonant amplitude in mm for increasing layers |
|-----|---------------------------------------------|---------------------------------------------|
|     | $(0^\circ/90^\circ)_1$ | $(0^\circ/90^\circ)_2$ | $(0^\circ/90^\circ)_3$ | $(0^\circ/90^\circ)_1$ | $(0^\circ/90^\circ)_2$ | $(0^\circ/90^\circ)_3$ |
| 0.001 | 121.8 | 192.38 | 202.8 | 749.752 | 301.373 | 135.621 |
| 0.05 | 5480.4 | 7797.8 | 8262.6 | 0.00706337 | 0.0033688 | 0.00300689 |
| 0.1 | 8808.2 | 10958 | 11669 | 0.00126327 | 0.000771155 | 0.000681675 |

Frequency Response Function (FRF) Plot

Harmonic responses of this plate for different boundary condition with increasing layers are shown on FRF plot from fig. 7(a) to fig. 7(c) at various thickness ratios. Figure 7(a) shows the FRF plot for two ply lamination schemes with clamped boundary condition. Critical amplitude of 749.752 mm occurs at natural frequency of 121.8 Hz for thickness ratio 0.001, 0.00706337 mm occurs at 5480.4 Hz for 0.05 and 0.00126327 mm occurs at 8808.2 Hz for 0.1.

Figure 7(a). FRF for two layered $(0^\circ/90^\circ)$ plate for different thickness ratios under all edges fixed boundary condition.
Figure 7(b) shows the FRF plot for four ply lamination schemes with clamped boundary condition. Critical amplitudes of 301.373 mm, 0.0033688 mm and 0.000771155 mm occur at natural frequency of 192.38 Hz, 7797.8 Hz and 10958 Hz respectively for various thickness ratios.

![Figure 7(b)](image)

**Figure 7(b).** FRF for four layered (0°/90°)$^2$ plate for different thickness ratios under all edges fixed boundary condition.

Figure 7(c) shows the FRF plot for six ply lamination schemes with clamped boundary condition. Critical amplitudes of 135.621 mm, 0.00300689 mm and 0.000681675 mm occur at natural frequency of 202.8 Hz, 8262.6 Hz and 11669 Hz respectively for various thickness ratios.

![Figure 7(c)](image)
Figure 7(c). FRF for six layered (0°/90°)_3 plate for different thickness ratios under all edges fixed boundary condition.

4.2.3. Effect of Modulus Ratio. Four layered (0°/90°)_2 plate with anti-symmetric scheme of lamination is used for presenting the effect of modulus ratio on natural frequency and resonance amplitude for various thickness ratios as shown in Table 6. It has been observed that as the modulus ratio increases, natural frequency also increases but resonance amplitude decreases for all thickness ratios.

Table 6. Effect of Modulus Ratios on the frequency response amplitude for anti-symmetric laminated composite plate with clamped Boundary conditions.

| t/b   | MODULUS RATIO (E_1/E_2) | 10       | 20       | 30       |
|-------|--------------------------|----------|----------|----------|
|       | 0.001                    | 105.95   | 140.83   | 168.59   | 192.38   | 301.373 |
| 0.05  | 4913.8                   | 0.00888557| 6225.5   | 7124.7   | 7797.8   | 0.0033688 |
| 0.1   | 8342.1                   | 0.00144592| 9726.0   | 10480.0  | 10958.0  | 0.000771155 |

Frequency Response Function (FRF) Plot

Harmonic responses of the plate for different modulus and thickness ratios are shown on FRF plot 8(a) to 8(c).

Figure 8(a) shows the FRF plot for various modulus ratios with thickness ratio of 0.001. Critical amplitude of 989.367 mm occurs at 105.95 Hz natural frequency for modulus ratio 10, 561.064 mm occurs at 140.83 Hz for modulus ratio 20, 389.74 mm occurs at 168.59 Hz for modulus ratio 30 and 301.373 occurs at 192.38 Hz for modulus ratio 40.
Figure 8(a). FRF for four layered (0°/90°) plate for different modulus ratios at thickness ratio of 0.001 under all edges fixed boundary condition.

Figure 8(b) shows the FRF plot for thickness ratio of 0.05. Critical amplitude of 0.00888557 mm occurs at 4913.8 Hz natural frequency for modulus ratio 10, 0.0054791 mm occurs at 6225.5 Hz for modulus ratio 20, 0.00409423 mm occurs at 7124.7 Hz for modulus ratio 30 and 0.0033688 occurs at 7797.8 Hz for modulus ratio 40.
Figure 8(b). FRF for four layered $(0^\circ/90^\circ)_2$ plate for different modulus ratio at thickness ratio of 0.05 under all edges fixed boundary condition.

Figure 8(c) shows the FRF plot for thickness ratio of 0.1. Critical amplitude of 0.00144592 mm occurs at 8342.1 Hz natural frequency for modulus ratio 10, 0.00102515 mm occurs at 9726 Hz for modulus ratio 20, 0.000859724 mm occurs at 10480 Hz for modulus ratio 30 and 0.000771155 occurs at 10958 Hz for modulus ratio 40.

Figure 8(c). FRF for four layered $(0^\circ/90^\circ)_2$ plate for different modulus ratio at thickness ratio of 0.1 under all edges fixed boundary condition.
4.2.4. Effect of Damping Ratio. The effect of damping ratios on natural frequency and resonance amplitude are presented for four layered (0°/90°) cross ply plate with anti-symmetric lamination scheme for various thickness ratios. The results are obtained for all edges fixed boundary condition. The results are shown in Table 7. From the results, increments are shown in natural frequency for increasing damping ratio but decrement is shown in resonance amplitude for same cases.

Table 7. Effect of Damping Ratios on the frequency response amplitude for anti-symmetric laminated composite plate under clamped Boundary conditions.

| t/b  | DAMPING RATIO |       |       |       |       |       |
|-----|---------------|-------|-------|-------|-------|-------|
|     | 0.01          | 0.05  | 0.1   | 0.01  | 0.05  | 0.1   |
|     | Frequency in Hz | Amplitude in mm | Frequency in Hz | Amplitude in mm | Frequency in Hz | Amplitude in mm |
| 0.001 | 192.38  | 301.373 | 192.38  | 60.2505 | 192.38  | 30.0881 |
| 0.05  | 7797.8 | 0.00033688 | 7797.8 | 0.000673383 | 7797.8 | 0.000336114 |
| 0.1   | 10958 | 0.000771155 | 10958 | 0.000154249 | 10958 | 0.000077148 |

Frequency Response Function (FRF) Plot
Harmonic responses of this plate for different damping and thickness ratios are shown on FRF plot. It can be seen that a large damping ratio has a fairly flat resonance curve as in shown in figure 9(a) to 9(c).

Figure 9(a) shows the FRF plot for various damping ratios with thickness ratio 0.001. Critical amplitude 301.373 mm occurs for damping ratio 0.01, 60.2505 occurs for damping ratio 0.05 and 30.0881 mm occurs for damping ratio 0.1 at natural frequency 192.38 Hz.
Figure 9(a). FRF for four layered (0°/90°) 2 plate for different damping ratios at thickness ratio of 0.001.

Figure 9(b) shows the FRF plot for various damping ratios with thickness ratio 0.05. Critical amplitude 0.0033688 mm occurs for damping ratio 0.01, 0.000673383 occurs for damping ratio 0.05 and 0.000336114 mm occurs for damping ratio 0.1 at same natural frequency of 7797.8 Hz.

Figure 9(b). FRF for four layered (0°/90°) 2 plate for different damping ratios at thickness ratio of 0.05.
Figure 9(c) shows the FRF plot for various damping ratios with thickness ratio 0.1. Critical amplitude 0.000771155 mm occurs for damping ratio 0.01, 0.000154249 mm occurs for damping ratio 0.05 and 0.000077148 mm occurs for damping ratio 0.1 at same natural frequency of 10958 Hz.

![Figure 9(c)](image_url)

**Figure 9(c).** FRF for four layered (0°/90°) plate for different damping ratio at thickness ratio of 0.1.

5. Conclusions
In this paper, vibration and harmonic behaviour of various cross-ply plates with anti-symmetric lamination schemes are analyzed. Convergence tests and comparison studies have been carried out using FEM. The proper mesh size is converged at 13 x 13. The comparison results have shown reasonably good agreement with published literature for different modulus and thickness ratios as well as different support conditions.

It has been concluded from the results that the maximum variation in the non-dimensional fundamental frequency for anti-symmetric cross ply laminated plate limited to 0 to 2%. The non-dimensional fundamental frequency increases as the number of layers increases but the variation is negligible beyond four layers. It has been observed that as the thickness ratio increases the fundamental frequency increases but the value of resonance amplitude decreases for all boundary conditions and also found that as the damping ratio increases the fundamental frequency increases but the value of resonance amplitude decreases for all thickness ratios.
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