Magnetoelastic metamaterials

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The study of advanced artificial electromagnetic materials, known as metamaterials, provides a link from material science to theoretical and applied electrodynamics, as well as to electrical engineering. Being initially intended mainly to achieve negative refraction, the concept of metamaterials quickly covered a much broader range of applications, from microwaves to optics and even acoustics. In particular, nonlinear metamaterials established a new research direction giving rise to fruitful ideas for tunable and active artificial materials. Here we introduce the concept of magnetoelastic metamaterials, where a new type of nonlinear response emerges from mutual interaction. This is achieved by providing a mechanical degree of freedom so that the electromagnetic interaction in the metamaterial lattice is coupled to elastic interaction. This enables the electromagnetically induced forces to change the metamaterial structure, dynamically tuning its effective properties. This concept leads to a new generation of metamaterials, and can be compared to such fundamental concepts of modern physics as optomechanics of photonic structures or magnetoelasticity in magnetic materials.

The original approach to achieve a nonlinear response in metamaterials was realized by either engineering the elements of a metamaterial with a nonlinear component, or employing a nonlinear host medium. In those approaches the nonlinear response is obtained on the level of individual elements. On the other hand, by varying the mutual interaction between elements, one can efficiently control bulk metamaterial properties mechanically or thermally. It is therefore quite promising to explore the possibilities of nonlinear mutual interaction in metamaterials.

To illustrate the idea of magnetoelastic metamaterials, we consider a metamaterial composed as an anisotropic lattice of resonant elements, such as split-ring resonators or capacitively loaded rings, as shown in Fig. 1. In response to electromagnetic waves with a magnetic field $H_0$ along the axial direction, such a metamaterial shows resonant magnetic behaviour. The currents induced in the resonators not only affect each other through mutual inductance, but also result in an attractive Ampère force between the resonators (it is attractive provided that the neighbouring currents are in phase). Therefore, if the resonators are allowed to move along the axial direction, they will displace from their original positions, thus changing their mutual impedance, which in turn affects the current amplitudes, interaction forces and so on. The balance is maintained by a restoring Hooke force, which originates from the elastic properties of the host medium.

The forces that occur in such a metamaterial are entirely attractive, which is an important advantage when compared with the quasi-optical forces recently discussed in metamaterials in a photonic crystal regime, where the effects analogous to optomechanical phenomena can be foreseen. Thus, the magnetoelastic phenomena can be also observed in the terahertz and even radiofrequency range, where optical forces are not practical. As we show below, with moderate power the required elasticity coefficient is rather small with respect to the bulk properties of conventional materials. However, the required response can be engineered in practice with the help of appropriately bent thin filaments or small plastic springs with realistic geometrical parameters, which fit to the suggested geometry and therefore require a fabrication effort on the same scale as for resonators themselves. Furthermore, our theoretical estimates imply that with an increase of frequency, although the size of resonators should decrease, the elasticity should not, so we do not expect that scaling the system down would raise further fabrication challenges.

Ideally, the electromagnetic resonance frequency in metamaterials is such that the wavelength is much larger than the element size and lattice constants (this can be easily achieved for example by using elements with an appropriate capacitance, or broadside-loaded rings, as shown in Fig. 1). In response to the electromagnetic field, the metamaterial is compressed by the electromagnetic forces acting between the elements. Dimensionless lattice parameters $a$ and $b$ are normalized to the resonator radius $r_0$.

**Figure 1** | An anisotropic magnetic metamaterial combined with an elastic medium. Two layers of the bulk sample are shown. **a**, The metamaterial before the electromagnetic field is applied. **b**, The metamaterial is compressed by the electromagnetic forces acting between the elements. Dimensionless lattice parameters $a$ and $b$ are normalized to the resonator radius $r_0$.
coupled resonators \(^{26}\) with sufficiently small gaps). At the same time, characteristic response times for mechanical movements are much larger than the period of electromagnetic oscillations. Therefore, currents induced in neighbouring elements are all in phase and also uniform along the resonator circumference, so that the attraction force can be evaluated in the same way as the static force for d.c. currents, weighted with a time-averaged current amplitude (see Methods). Note that electric interaction is either irrelevant (if the gaps are sufficiently small) or can be minimized by choosing an appropriate mutual orientation of the rings\(^{26}\).

Self-action, resulting in nonlinear behaviour, occurs through the mutual inductance between the rings, which depends on the lattice constant \(b\). In metamaterials, the effect of mutual interaction between all of the rings is accounted for by the so-called lattice sum \(\Sigma\), which depends on the lattice type and parameters\(^{20}\). Then, the balance equation between the Ampère \((F_1)\) and Hooke \((F_2)\) forces is coupled with the complete impedance equation (see Methods), and the system can be numerically solved to find the lattice constant \(b\) at equilibrium.

To outline the expected phenomena, we depict some examples of the interplay of the involved forces in Fig. 2. The solutions to the balance of forces are graphically seen as the crossing points, and the stable equilibrium positions are such that \(|F_1| < |F_2|\) for an attempted decrease in \(b\). Note that the resonant nature of the currents, induced in the rings depending on \(b\), defines the resonant character of the force \(F_2\). Thus, when three mathematical solutions are available, only two are actually stable; or otherwise there is a single stable state. The phenomenology is qualitatively clear: when the current amplitude exceeds a certain threshold (indicated with cross ‘2’ in Fig. 2b), the initial ‘right-side’ equilibrium (such as at circle ‘1’) cannot be achieved, so the lattice distance \(b\) attempts to collapse. However, this also changes the mutual interaction markedly, leading to a significant shift of the resonance frequency, so the current magnitude drops, permitting the other (‘left-side’) equilibrium state (Fig. 2, circle ‘4’), corresponding to the same force curve. On the other hand, with a decreasing amplitude, the ‘left-side’ balance remains stable as long as the peak attraction force is sufficient to counter the elastic force (down to a threshold point, cross ‘3’), from where the system jumps back to the corresponding ‘right-side’ solution (circle ‘1’).

We can then characterize the entire metamaterial with a nonlinear and resonant magnetization \(M(H_0, \omega)\) dependence (see Methods). We illustrate typical patterns of the arising nonlinearity (Fig. 3) with elements of radius \(r_0 = 5\, \text{mm}\), resonating individually at 1 GHz (the frequency values below are normalized with respect to the corresponding angular frequency \(\omega_0\), with a quality factor of 100. These are arranged in a metamaterial with \(a = 4\), \(b_0 = 0.3\) and \(b_{\min} = 0.1\), with a stiffness coefficient \(k = 0.44\, \text{mN m}^{-1}\).

At frequencies lower than the eigenfrequency of the initial state, we observe a slightly nonlinear \(M(H_0)\) dependence as the amplitude grows, until the metamaterial abruptly switches to a stronger compression. However, when the amplitude is decreased, the metamaterial remains in the compressed state until much lower magnitudes, exhibiting a hysteresis-like behaviour (Fig. 3a). Close to the original resonance, the hysteresis disappears but the nonlinearity is quite strong (Fig. 3b).

More spectacular phenomena can be observed with the frequency dependence, which reveals a rich array of complex bistable behaviour (Fig. 4). With moderate to high amplitudes, the lattice distance \(b\) declines slowly with growing frequency, until at some stage the initial balance of forces is lost and the metamaterial jumps to a more compressed state, from where it gradually returns back to the original state with further frequency increase; the magnetization pattern reflects these changes (see the blue circles in Fig. 4a,c). However, when the frequency is decreased from the high values, the structure remains in the compressed state across that threshold, and continues to compress until the mechanical limit at \(b_{\min}\) is reached. It remains fully compressed until the currents induced at decreasing frequency become low enough to release the entire jump back to the ground state (Fig. 4a, red circles). This is followed by the magnetization, but note that in the frequency range of full compression the response of the metamaterial is linear and hence we can observe a purely linear resonance (see the red circles in Fig. 4c).

However, at low amplitudes, we can observe highly unusual behaviour (Fig. 4b,d). The frequency hysteresis described above still applies, but in addition there appears a frequency range where the metamaterial is stable in a compressed state, but which, however, cannot be directly accessed in this hysteresis loop (see the green squares in Fig. 4b,d). The only ways to reach this range of compressions are to push the structure once with an external mechanical force, or to temporarily increase the intensity. Once there, the metamaterial remains stable in the corresponding frequency window, jumping to the ground state when decreasing or increasing the frequency past the window limits. This exotic effect exists because below certain amplitudes, the currents induced in the intermediate range of \(b\) are not sufficient to hold the force balance even at resonance; however, they are still able to do it for smaller \(b\) because of the effectively \(1/b^2\) law in \(F_1\) (see Methods). The range of amplitudes where this can be observed is relatively narrow (in our example,
between 0.38 and 0.41 A m\(^{-1}\)). The corresponding frequency range becomes increasingly narrow with decreasing \(H_0\), until finally disappearing, although the main hysteresis is still in place up to very low amplitudes.

To assess the plausibility of the predicted effect, we carry out a transmission experiment on elastically coupled resonators. For simplicity, the interaction here is observed in pairs of elements, so that the attraction and repulsion occurs separately in each pair. In such a system, the corresponding governing equations differ from those in the bulk system only by a lack of volumetric enhancement factor \((\sim \frac{1}{r^3}, \text{see Methods})\), which does not bring any qualitative difference for small compressions; but makes us confident that an effect observed for a small number of elements is a reliable proof of practical feasibility of large systems. Indeed, Fig. 5 demonstrates a remarkable effect of increasing electromagnetc wave power, with a resonance frequency shift of 13 MHz. The experimental power dependence of the resonance matches very well with the theory (after the straightforward amendments required to account for two resonators instead of a bulk material), with the corresponding parameters and assuming a stiffness coefficient of 0.13 ± 0.01 N m\(^{-1}\).

The experiments indicate that although the performance of the volumetric magnetoelastic systems is expected to be better, the metamaterials without a total compression, with the effect confined within a subgroup of elements, are still efficient and can be employed when fabrication of volumetric structures faces significant technological difficulties.

We have suggested a new type of magnetoelastic metamaterials introducing mechanical degrees of freedom, and demonstrated that this provides a clearly measurable effect with the parameters close to our assumptions. We believe that the magnetoelastic coupling will lead to many interesting but not yet explored effects in metamaterials that can be useful for further design of artificial media as well as for their future applications in microwave and terahertz ranges.
axis of a WR 229 rectangular waveguide on a dielectric rod with grooves at a distance of 1 mm (θ₀ = 0.3r₀), so that the rings are able to swing towards each other when an attractive force is induced. To provide an elastic force, we attached a keratin filament between the resonators. In this set-up, an additional contribution to the restoring force is provided by gravity, which was taken into account for the theoretical curve shown in Fig. 5. (It amounts to about 20% for the largest inclination observed in experiments).

The waveguide is excited by a vector network analyser (Rohde and Schwarz ZVB-20), which is amplified by a signal amplifier (Agilent 83020A), having a maximum output power of approximately 32 dBm. A broadband directional coupler (Agilent 11692D) and power meter (Rohde and Schwarz NRP-Z23) are used to monitor the output of the amplifier. The output power of the network analyser is then adjusted to compensate for the nonlinearity of the amplifier. This ensures that the power incident on the metamaterial sample does not vary with frequency.

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Author contributions

Theoretical analysis was carried out by M.L. and experiments were carried out by I.V.S. and D.A.P. and the figures were prepared by M. Gorkunov and A. Sukhorukov. Additional information is available online at http://www.nature.com/reprints. Correspondence and requests for materials should be addressed to M.L.