Gaugino Condensation in M–theory on $S^1/Z_2$

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Abstract

In the low energy limit of for M–theory on $S^1/Z_2$, we calculate the gaugino condensate potential in four dimensions using the background solutions due to Hořava. We show that this potential is free of delta–function singularities and has the same form as the potential in the weakly coupled heterotic string. A general flux quantization rule for the three–form field of M–theory on $S^1/Z_2$ is given and checked in certain limiting cases. This rule is used to fix the free parameter in the potential originating from a zero mode of the form field. Finally, we calculate soft supersymmetry breaking terms. We find that corrections to the Kähler potential and the gauge kinetic function, which can be large in the strongly coupled region, contribute significantly to certain soft terms. In particular, for supersymmetry breaking in the $T$–modulus direction, the small values of gaugino masses and trilinear couplings that occur in the weakly coupled, large radius regime are enhanced to order $m_{3/2}$ in M–theory. The scalar soft masses remain small even, in the strong coupling M–theory limit.

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1 Introduction

New knowledge about the strongly coupled limits of heterotic string theory has opened new possibilities for building realistic low-energy models. Hořava and Witten [1, 2] have suggested that the strongly coupled $E_8 \times E_8$ heterotic string is described by M-theory on a $S^1/Z_2$ orbifold, where the low-energy effective theory is eleven-dimensional supergravity together with one set of $E_8$ gauge fields on each of the two orbifold fixed hyperplanes [1, 2]. Further, compactifying on a Calabi-Yau threefold, Witten has shown that it is possible to produce an $N = 1$ theory in four dimensions, where the tree level gravitational and gauge couplings match their known values [3]. One finds that the Newton constant and grand-unified coupling constant are given by

$$G_N = \frac{\kappa^2}{16\pi^2 V \rho}, \quad \alpha_{\text{GUT}} = \frac{(4\pi\kappa^2)^{2/3}}{2V},$$

where $\kappa$ is the eleven-dimensional gravitational coupling constant, $V$ is the volume of the Calabi-Yau space and $\pi \rho$ is the length of the orbifold interval. The GUT scale is set by the size of the Calabi-Yau three-fold, so that one takes $V^{-1/6}$ to be about $10^{16}$ GeV while the eleven-dimensional Planck scale is about a factor of two larger. The orbifold scale, $(\pi \rho)^{-1}$, is approximately a factor of ten smaller [4]. Thus, with increasing energy, the universe appears first four-, then five- and finally eleven-dimensional.

As always, one must address how the supersymmetry of the four-dimensional effective theory is eventually broken. A standard mechanism in weakly coupled string theory has been supersymmetry breaking via gaugino condensation in a hidden sector [5]. That such a mechanism could also work in the strongly coupled model was, first demonstrated in the full eleven-dimensional theory by Hořava [6]. The purpose of the present paper is to discuss this approach in the context of deriving the effective four-dimensional theory. In particular, we obtain the form of the four-dimensional gaugino potential directly from the M–theory effective action. We address the rôle of flux quantization in restricting the form of the gaugino potential. Finally, we discuss the structure of the soft supersymmetry breaking terms. These depend crucially on the form of the low-energy Kähler potential and superpotential, which were calculated directly from M–theory in a previous paper [7].

The question of gaugino condensation in the low-energy effective action of M–theory on $S^1/Z_2$ has already been addressed in various places in the literature [8, 9, 10, 11, 12, 13, 14]. However, we believe a number of points require clarification in these discussions, and in particular and most importantly, a clear derivation of the gaugino potential from the M theory effective theory has thus far been missing.

It is important in this discussion to be clear about the scale at which the condensate forms. There are two distinct cases, with the scale either above or below the scale set by the size of the orbifold interval. In the former case, the theory is still effectively five-dimensional when the condensate forms. The corresponding gaugino potentials, and the question of moduli stabilization,
should then be discussed in the context of a full five-dimensional theory \[13\]. A very interesting
property of this case is that, as pointed out by Hořava \[6\], locally the condensate does not break
supersymmetry. However, there is a global obstruction to preserving supersymmetry everywhere
across the orbifold interval. Consequently, only once one drops below the orbifold threshold to a
truly four-dimensional theory is the supersymmetry breaking seen locally. In this case, in contrast
to the conventional picture, the scale of low-energy supersymmetry breaking may then not set by
the condensate scale, but instead by the orbifold threshold. Since this scenario requires a five-
dimensional description, we will not pursue it further here.

In the other case, the theory becomes four-dimensional before the gauginos condense. In this
case, the supersymmetry breaking scale is set, as usual, by the condensate scale, and is independent
of the orbifold size, other than through the fact that the four-dimensional Planck scale depends on \(\rho\).
This is the scenario the majority of discussions have considered, since it is the phenomenologically
reasonable one. (One exception is Antoniadis and Quiros \[10\], who argue that the orbifold threshold
is actually down at \(10^{12}\) GeV, and take the former scenario. They claim the effects of the condensate
in the detailed five-dimensional theory are reproduced in four dimensions by a Scherk-Schwarz
compactification.) It is also worth noting that, in this second case, there is no way of resolving the
separation of the two orbifold planes since all momenta are below the orbifold threshold. Thus,
although the standard model fields may come from one plane and the hidden sector fields live on
the other, there is no sense in the four-dimensional theory that we are “living on one plane”.

In deriving the gaugino potential in the second scenario, it is crucial to observe, as was stressed
in \[7\] and has been missed in other discussions, that one cannot make a conventional dimensional
reduction. In general, it is not possible to simply excite fields which are independent of the orbifold
coordinate. In the Hořava-Witten description, the fields on the orbifold planes act as sources for
the eleven-dimensional supergravity fields. Only when the sources on the two planes are matched
can the bulk fields be taken to be independent of \(x^{11}\). As a result, one is always led to consider
truncations with non-trivial \(x^{11}\) dependence. It is the orbifold averages of these bulk fields which
lead to non-trivial couplings in the low-energy effective action. For instance, they are the source
of the Chern-Simons terms in four-dimensions \[7\], and here they are the source of the gaugino
condensate potential. It might appear from the form of the eleven-dimensional action that the
potential arises from delta-function terms localized at the orbifold planes. However, as we will show,
in fact, all terms in the eleven-dimensional action are smooth and the potential arises through the
average of an \(x^{11}\) dependent background.

Our essential result will be that the resulting gaugino potentials are the same as in the weakly
coupled limit. In a previous paper \[7\], we calculated the form of the rest of the effective action. One
finds that it again agrees in form with the large Calabi-Yau limit of the one-loop weak calculation,
barring one term which, while probably present in the weakly coupled limit, appears previously to
have been ignored. One finds the Kähler potential, superpotential and gauge kinetic functions are
given in terms of the usual $S$, $T$ and charged matter moduli $C$, by

$$
K = -\ln(S + \bar{S} - \beta|C|^2/2) - 3\ln(T + \bar{T} - |C|^2)
$$

$$
W = k d_{pq} C^p C^q C^r
$$

$$
f^{(1)} = S + \beta T
$$

$$
f^{(2)} = S - \beta T,
$$

where $k^2 = 4\alpha_{\text{GUT}}/G_N$. In the strongly coupled limit, one finds $\beta = \sqrt{2} \pi \rho^{16} \alpha_0 = O(\kappa^{2/3} \rho/\sqrt{3})$ where $\alpha_0$ is defined as in integral over the Calabi–Yau space [3, 4]. In the weak limit, $\beta$ characterizes the loop correction and is small, while in the strongly coupled limit it can be of order one. The previously neglected term is the correction $\beta|C|^2$ in the Kähler potential. Thus, while the form of the gaugino potentials are the same as in the weakly coupled limit, since $\beta$ need not be small we will find that this term can considerably alter the soft terms resulting from supersymmetry breaking. That the corresponding terms in the gauge kinetic functions can significantly raise the gaugino masses compared with the weak limit was previously noted in [11].

2 Review of M–theory on $S^1/Z_2$

To set the stage for the discussion of gaugino condensation, we would like to briefly review the low energy theory for M–theory on $S^1/Z_2$ [1, 2], and its solutions appropriate for a reduction to four dimensions [3, 6].

Our conventions are as follows. We use indices $I, J, K, \ldots = 0, \ldots, 9, 11$ for the full 11–dimensional space $M_{11}$. The orbifold $S^1/Z_2$ with radius $\rho$ is parameterized by $x^{11} \in [-\pi \rho, \pi \rho]$ so that the two 10–dimensional fixed hyperplanes $M_{10}^{(1)}, M_{10}^{(2)}$ are located at $x^{11} = 0$ and $x^{11} = \pi \rho$. Occasionally, we will find it useful to work in the boundary picture where only one half of the orbifold $S^1/Z_2$ is considered and the hyperplanes $M_{10}^{(i)}$ are viewed as boundaries of $M_{11}$. The coordinate $x^{11}$ is then restricted to $x^{11} \in [0, \pi \rho]$. The 10–dimensional space transverse to the orbifold is labeled by barred indices $\bar{I}, \bar{J}, \bar{K}, \ldots = 0, \ldots, 9$. For the reduction to four dimensions, we introduce indices $A, B, C, \ldots = 4, \ldots, 9$ for the Calabi–Yau space and four–dimensional indices $\mu, \nu, \ldots = 0, \ldots, 3$. Holomorphic and antiholomorphic indices on the Calabi–Yau space are denoted by $a, b, c, \ldots$ and $\bar{a}, \bar{b}, \bar{c}, \ldots$, respectively.

The effective action for M–theory on $S^1/Z_2$, up the order $\kappa^{2/3}$ in the 11–dimensional Newton constant $\kappa$, is given by [2]

$$
S = \frac{1}{2\kappa^2} \int_{M_{11}} \sqrt{-g} \left[ -R - \frac{1}{24} G_{IJKL} G^{IJKL} - \bar{\Psi}_I \Gamma^{IJK} D_J \left( \frac{\Omega + \hat{\Omega}}{2} \right) \Psi_K 
- \frac{\sqrt{2}}{192} (\bar{\Psi}_I \Gamma^{IJKLMN} \Psi_N + 12 \bar{\Psi}_J \Gamma^{KL} \Psi^M) (G_{JKLM} + \hat{G}_{JKLM}) \right]
$$

3
\[-\frac{\sqrt{2}}{1728}e^{I_1\ldots I_{11}}C_1^1I_2I_3G_{I_4\ldots I_7}G_{I_8\ldots I_{11}}\]
\[-\frac{1}{8\pi^2} \left(\frac{\kappa}{4\pi}\right)^{3/2} \sum_{i=1,2} \int_{M_{10}^{(i)}} \sqrt{-g} \text{tr} \left[ F_{ij}^{(i)} F_{ij}^{(i)} + 2\tilde{\chi}^{(i)} \Gamma^i D_i(\hat{\Omega}) \chi^{(i)} \right]
+ \frac{1}{2} \tilde{\Psi}_I \hat{G}_{IJKL} (F_{jk}^{(i)} + \hat{F}_{jk}^{(i)}) \chi^{(i)} - \frac{\sqrt{3}}{12} \tilde{\chi}^{(i)} \Gamma^{IJK} \chi^{(i)} \hat{G}_{IJKL} \right] \quad (3)

The bulk fields in this action are the 11–dimensional metric $g_{ij}$, the three–form $C_{IJK}$ with bulk field strength $G_{IJKL} = 24\partial [I_{C_{IJKL}}]$ and the gravitino $\Psi_I$. The two $E_8$ gauge fields $A_{ij}^{(i)}$, $i = 1, 2$ with field strengths $F_{ij}^{(i)}$ and their gaugino superpartners $\chi^{(i)}$ live on the 10–dimensional hyperplanes $M_{10}^{(i)}$. The supercovariant objects $\hat{\Omega}$, $\hat{F}_{ij}$ and $\hat{G}_{IJKL}$ are defined in ref. [2], but will not be needed explicitly in this paper. The above action has to be supplemented with the Bianchi identity

\[(dG)_{11IJKL} = -\frac{1}{2\sqrt{2\pi}} \left(\frac{\kappa}{4\pi}\right)^{3/2} \left[ J^{(1)} \delta(x^{11}) + J^{(2)} \delta(x^{11} - \pi\rho) \right]_{IJKL} \quad (4)\]

for $G$, where the sources are defined by

\[J^{(i)} = \left( \text{tr} F^{(i)} \wedge F^{(i)} - \frac{1}{2} \text{tr} R \wedge R \right) = d\omega^{(i)}. \quad (5)\]

The three–forms $\omega^{(i)}$ can be expressed in terms of the Yang–Mills and Lorentz Chern–Simons forms $\omega^{\text{YM}(i)}$ and $\omega^L$ as

\[\omega^{(i)} = \omega^{\text{YM}(i)} - \frac{1}{2} \omega^L, \quad (6)\]

The bulk fields should respect the $Z_2$ orbifold symmetry, which implies for the bosonic fields that $g_{ij}$, $g_{11,11}$, $G_{IJK}$ are even and $g_{111}$, $C_{IJK}$ are odd.

For this theory, the existence of background solutions suitable for compactification to four dimensions with preserved $N = 1$ supersymmetry has been demonstrated in ref. [3]. These solutions, based on the standard embedding of the spin connection into one of the $E_8$ gauge groups, are of the form $M_{11} = S^1/Z_2 \times X \times M_4$ where $X$ is a deformed Calabi–Yau space and $M_4$ is four–dimensional Minkowski space. The deformation of the Calabi–Yau space, as well as the existence of a nonzero form $G = G^{(W)}$ in those solutions, is due to the fact that the sources in the Bianchi identity (4) do not vanish, unlike in the weakly coupled case where this is guaranteed by the standard embedding. The modification of the solutions in the presence of gaugino condensation has been worked out by Hořava [3]. Since it is for those backgrounds that we will later compute the gaugino condensate potential, we will briefly review the main results of this work.

A crucial observation is that, similar to the weakly coupled case, the terms in the action which contain gaugino bilinears can, together with the $G^2$ term, be grouped into a perfect square

\[S^{(\kappa)} = -\frac{1}{48\kappa^2} \int_{M_{11}} \sqrt{-g} \hat{G}_{IJKL} \hat{G}^{IJKL} \quad (7)\]
with
\[ \tilde{G}_{IJKL} = G_{IJKL} \]
\[ \tilde{G}_{IJK11} = G_{IJK11} - \frac{\sqrt{2}}{16\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left( \delta(x^{11})\omega^{(x,1)} + \delta(x^{11} - \pi\rho)\omega^{(x,2)} \right)_{IJK} \]  
(8)
and the gaugino bilinears
\[ \omega^{(x,i)}_{IJK} = \text{tr} \tilde{\chi}^{(i)} \Gamma_{IJK} \chi^{(i)} . \]  
(9)
The terms proportional to \((\omega^{(x,i)})^2\) arising from this square are of order \(\kappa^4/3\) and have, therefore, been omitted from the action \((3)\). In ref. [2] they were shown to follow from the cancellation of the supersymmetry variation at order \(\kappa^4/3\), with exactly the right coefficient to fit into the perfect square. Note, however, that these terms are proportional to \((\delta(x^{11}))^2\) and are, therefore, not quite well-defined. In ref. [2], it was argued that this problem is due to the finite thickness of the boundary, which has not been taken into account in constructing the effective theory.

In terms of the redefined form field \(\tilde{G}\), the supersymmetry transformation of the gravitino reads
\[ \delta \Psi_i = D_i \eta + \frac{\sqrt{2}}{288} \tilde{G}_{JKLM}(\Gamma_i^{JKLM} - 8\delta_i^J \Gamma^{KLM}) \eta \]  
(10)
\[ \delta \Psi_{11} = D_{11} \eta + \frac{\sqrt{2}}{288} \tilde{G}_{JKLM}(\Gamma_{11}^{JKLM} - 8\delta_{11}^J \Gamma^{KLM}) \eta + \frac{1}{192\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left[ \delta(x^{11})\omega^{(x,1)} + \delta(x^{11} - \pi\rho)\omega^{(x,2)} \right]_{IJK} \Gamma_{IJK} \eta \]  
(11)
where the spinor \(\eta\) is subject to the \(Z_2\) restriction \(\eta(-x^{11}) = \Gamma^{11} \eta(x^{11})\). The equation of motion for \(\tilde{G}\) to be derived from eq. (7) \[\text{and the modified Bianchi identity (7)},\] are given by
\[ D_i \tilde{G}^{IJKL} = 0 \]  
(12)
\[ (d \tilde{G})_{11IJKL} = - \frac{1}{2\sqrt{2}\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left[ J^{(1)} \delta(x^{11}) + J^{(2)} \delta(x^{11} - \pi\rho) \right]_{IJKL} \]
\[ - \frac{\sqrt{2}}{16\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left[ J^{(x,1)} \delta(x^{11}) + J^{(x,2)} \delta(x^{11} - \pi\rho) \right]_{IJKL} \]  
(13)
where the sources \(J^{(x,i)}\) induced by the gaugino condensates are defined as
\[ J^{(x,i)} = d\omega^{(x,i)} . \]  
(14)
If these condensates are covariantly constant, that is \(J^{(x,i)} = 0\), the additional source terms in the above Bianchi identity disappear and \(\tilde{G}\) is governed by the same equations as \(G\) satisfied with vanishing gaugino condensates. In this case, which we consider from now on, one can find a solution of the theory if one uses Witten’s result for the metric \([3]\) and if one sets \(\tilde{G}\) equal to the form field \(G^{(W)}\) of Witten’s solution. Clearly, if the condensates are switched off in the solution constructed

\[ \text{We do not write the contribution from the Chern–Simons term in (6) to the equation of motion since it vanishes for the solutions we will consider.} \]
in this way, one goes back to a supersymmetry preserving configuration which satisfies the Killing spinor equations $\delta \Psi_I = 0$ for a certain spinor $\tilde{\eta}$. For nonzero condensates, of course, one cannot, a priori, expect to be able to fulfill those equations since supersymmetry might be broken. However, since the 10-dimensional part of the gravitino variation \((\mathcal{G})\) is unchanged by the condensate, the condition $\delta \Psi_I = 0$ can be fulfilled by the original spinor $\tilde{\eta}$. One can try to satisfy the remaining equation $\delta \Psi_{11} = 0$ by modifying the spinor $\tilde{\eta}$ to $\eta'$ (such that $\delta \Psi_{\bar{I}} = 0$ remains true). From eq. (11), this leads to the condition

$$\partial_{11}(\eta' - \tilde{\eta}) = - \frac{1}{192\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left[ \omega^{(x.1)}(x^{11}) + \omega^{(x.2)}(x^{11} - \pi \rho) \right] \Gamma^{\bar{I}JK} \eta_0$$

(15)

where $\eta_0$ is the Killing spinor of the original Calabi–Yau space. As pointed out by Hořava, this equation has a local solution for $\eta'$ everywhere but, in general, there exists no global solution. To see this more explicitly, we find it useful to rewrite eq. (15) in the boundary picture. In this picture, the delta–function sources on the fixed hyperplanes turn into boundary conditions and we have

$$\partial(\eta' - \tilde{\eta}) = 0$$

$$(\eta' - \tilde{\eta})_{x^{11}=0} = - \frac{1}{384\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \omega^{(x.1)} \Gamma^{\bar{I}JK} \eta_0$$

$$(\eta' - \tilde{\eta})_{x^{11}=\pi \rho} = - \frac{1}{384\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \omega^{(x.2)} \Gamma^{\bar{I}JK} \eta_0$$

(16)

Obviously, $\eta' - \tilde{\eta} = \text{const}$ is a solution everywhere locally but a problem arises if one tries to match it to both boundary values. Clearly, this requires

$$\left( \omega^{(x.1)} + \omega^{(x.1)} \right) \Gamma^{\bar{I}JK} \eta_0 = 0$$

(17)

which is fulfilled for equal but opposite condensates. In general, eq. (17) is not satisfied and supersymmetry is broken. This breaking mechanism can be called global in the sense that information from both hyperplanes is required in order to realize the breaking.

Several questions arise from the above discussion. Clearly, one would like to know the low energy potential that describes the dynamics of this type of supersymmetry breaking, and one would like to understand how the global nature of this breaking is encoded in the potential. From what was said so far, however, the computation of this potential leads to a number of problems. As already mentioned, the $\left( \omega^{(x.i)} \right)^2$ term from the perfect square (\(\mathcal{G}\)), which gives rise to a potential in the analogous weakly coupled setting, appears with a factor $\left( \delta(x^{11}) \right)^2$. If the low energy potential would indeed directly arise from this term, it would be proportional to $\delta(0)$ and therefore be poorly defined. Indeed, this has been found in ref. \([12]\), where the perfect square (\(\mathcal{G}\)) has simple been multiplied out to compute the low energy potential. In essence, such a result implies that the limitations of the effective 11-dimensional theory do not allow for a proper calculation of the low energy gaugino potential. On the other hand, following Hořava, we have set the modified form $\tilde{G}$
equal to Witten’s background $G^{(W)}$, which is smooth. In this approach, the reduction of the perfect square ($\Box$) does not produce any $\delta(0)$ singularities in the low energy action. Curiously, however, the gaugino condensate has also disappeared from the action and it would seem that no potential is generated at all.

In the following, we are going to resolve these and other problems and we will show that a well–defined gaugino condensate potential, which encodes the characteristics of the global breaking mechanism, can be computed by reduction of the 11–dimensional action.

3 Computation of the potential

The gaugino condensate potential which we are going to compute in this section will be part of a low energy theory otherwise specified by the Kähler potential, the superpotential and the gauge kinetic functions given in eq. (2). These quantities were systematically derived in ref. [7] by reducing the effective theory (3) on Witten’s deformed Calabi–Yau background to four dimensions. Let us first review the essential steps of this derivation so that we can, later on, precisely specify the modifications needed to incorporate gaugino condensation.

The bulk field configurations used for the reduction can be schematically written as

\[
g_{IJ} = g_{IJ}^{(0)} + g_{IJ}^{(1)} + g_{IJ}^{(B)}
\]

\[
G_{IJKL} = G_{IJKL}^{(0)} + G_{IJKL}^{(1)} + G_{IJKL}^{(B)} + G_{IJKL}^{(W)}
\]

where the contributions to zeroth order in $\kappa$ are explicitly given by

\[
g_{\mu\nu}^{(0)} = \bar{g}_{\mu\nu}, \quad g_{AB}^{(0)} = e^{2a} \Omega_{AB}, \quad g_{11,11}^{(0)} = e^{2c}
\]

\[
G_{\mu\nu\rho\lambda}^{(0)} = 3 \partial_{[\mu} B_{\nu\rho]} \quad G_{\mu AB}^{(0)} = \partial_{\mu} \phi \omega_{AB}
\]

In these expressions, $\Omega_{AB}$ and $\omega_{AB}$ are the metric and the Kähler form of the (undeformed) Calabi–Yau space with volume $V$. The generic moduli $a$ and $c$ measure the radii of the Calabi–Yau space and the orbifold, respectively, and the two–form $B_{\mu\nu}$ and the scalar $\phi$ are their bosonic superpartners. The Einstein frame metric $g_{\mu\nu}$ is related to $\bar{g}_{\mu\nu}$ by the Weyl rotation

\[
\bar{g}_{\mu\nu} = e^{-6a-c} g_{\mu\nu}.
\]

With the generic gauge matter field $C^p$ in the fundamental representation 27 of $E_6$ defined by

\[
A_b^{(1)} = \bar{A}_b + w_b c_T c_p C^p.
\]

(here $\bar{A}_b$ is the embedded spin connection), we can express the low energy moduli fields $S$ and $T$ as

\[
S = e^{6a} + i\sqrt{2} \sigma + \frac{1}{2} \beta |C|^2, \quad T = e^{c+2a} + i\sqrt{2} \phi + \frac{1}{2} |C|^2.
\]
where the dual $\sigma$ of the field strength $H_{\mu
u\rho} = 3\partial_{[\mu}B_{\nu\rho]}$ is defined by $H_{\mu
u\rho} = e^{-\frac{1}{12}\alpha}\epsilon_{\mu
u\rho}\partial_\sigma\sigma$. The constant $\beta$ is the same one that appears in the Kähler potential and the gauge kinetic functions [2], and its value can be explicitly computed for a given Calabi–Yau manifold [3, 4]. The bosonic field content of the low energy theory is completed by the observable gauge fields $A_{\mu}^{(1)}$ which, due to the standard embedding, belong to the gauge group $E_6$, and the hidden sector gauge fields $A_{\mu}^{(2)}$ with gauge group $E_8$.

The other contributions to the metric and the four–form in the eqs. (18), (19) deserve an explanation. The metric correction $g_{IJKL}^{(1)}$ contains the moduli dependent order $\kappa^{2/3}$ correction to the Calabi–Yau background. Correspondingly, $G_{IJKL}^{(1)}$ represents the correction of the form zero modes due to this background distortion. As already mentioned, $G^{(W)}$ is the form part of Witten’s solution needed to account for the sources in the Bianchi identity which are generated by internal gauge fields. Corresponding terms $G_{IJKL}^{(B)}$ arise for source terms switched on by the external gauge fields. These gauge fields also generate a nonvanishing stress energy on the boundaries, which makes the compensating part $g_{IJKL}^{(B)}$ of the metric necessary. All these quantities, and their dependence on the generic moduli, have been explicitly determined in ref. [7]. They have been proven to be essential for a correct derivation of the low energy action.

Let us now incorporate gaugino condensates into the above scheme. In analogy with the procedure for the background solution explained in the previous section, one might try to set the full solution (13) for the form equal to $\tilde{G}$. This is, however, not quite correct since a condensate which is covariantly constant as part of the pure background (and we will assume our condensates to be of that type) does not remain covariantly constant once the moduli are promoted to fields with explicit dependence on the four–dimensional coordinates. As a result, though the equation of motion for $\tilde{G}$, eq. (12), still formally coincides with the equation of motion for $G$ in the absence of a condensate, the Bianchi identity (13) for $\tilde{G}$ receives new sources from the condensate. We can account for these additional sources by adding yet another piece, $G_{IJKL}^{(\chi)}$, to the right hand side of eq. (19), so that $\tilde{G}$ takes the form

$$\tilde{G}_{IJKL} = G_{IJKL}^{(0)} + G_{IJKL}^{(1)} + G_{IJKL}^{(B)} + G_{IJKL}^{(W)} + G_{IJKL}^{(\chi)}$$

Since the first four terms in this expression take care of all sources in the Bianchi identity except the ones arising from condensates, $G_{IJKL}^{(\chi)}$ is subject to the following equations

$$D_I G^{(\chi)}_{IJKL} = 0$$
$$\left(\frac{\sqrt{2}}{16\pi}\left(\frac{\kappa}{4\pi}\right)^{2/3}\left[J^{(\chi,1)}\delta(x^{11}) + J^{(\chi,2)}\delta(x^{11} - \pi\rho)\right]\right)_{IJKL} = 0$$

As has been pointed out in ref. [2]: a solution to these equations should be free of delta–function singularities. Correspondingly, the first four parts of $\tilde{G}$ do not contain any delta–function contributions either, so that $\tilde{G}$ is a smooth object (in the sense that it contains at most step functions)
as already pointed out in [6]. Clearly, the solution can be expressed in terms of the original form field \(G\) via eq. (8) as well, but \(G\) now contains a compensating delta–function contribution so as to make \(\tilde{G}\) smooth. The crucial point is that the action (7) can be entirely written in terms of the smooth object \(\tilde{G}\) and, therefore, leads to a perfectly well defined finite low energy effective action without any \((\delta(x^{11}))^2\) terms, no matter what specific combination of \(G\) and the condensates is used to describe the solution. What we have really done is to define a new independent form field \(\tilde{G}\) such that the \((\delta(x^{11}))^2\) terms are eliminated from the action. The form field–gaugino coupling is then encoded in the new Bianchi identity (13), in a way analogous to the Chern–Simons terms. Since, as we have just argued, the solution for \(\tilde{G}\) should be smooth, the \((\delta(x^{11}))^2\) singularity at order \(\kappa^{4/3}\) is completely removed.

To find an explicit solution for \(G^{(\chi)}\), we rewrite the eqs. (26), (27) in the boundary picture as

\[
D_I G^{(\chi)IJKL} = 0
\]

\[
dG = 0
\]

\[
G_{IJKL}^{\chi,11}|_{x^{11}=0} = -\frac{\sqrt{2}}{32\pi} \left(\frac{\kappa}{4\pi}\right)^{2/3} J_{IJKL}^{(\chi,1)}
\]

\[
G_{IJKL}^{\chi,11}|_{x^{11}=\pi\rho} = \frac{\sqrt{2}}{32\pi} \left(\frac{\kappa}{4\pi}\right)^{2/3} J_{IJKL}^{(\chi,2)}.
\]

These equations can be solved approximately to lowest order term in a momentum expansion [7], where term of the form \(\rho \partial J^{(i)}\) are neglected. This is justified in the present case, since the sources are provided by low energy fields with momenta far smaller than the inverse orbifold radius \(\rho^{-1}\). The solution then reads

\[
G_{IJK11}^{(\chi)} = -\frac{\sqrt{2}}{32\pi^{2}\rho} \left(\frac{\kappa}{4\pi}\right)^{2/3} \left[\omega^{(\chi,1)} + \omega^{(\chi,2)}\right]_{IJK} + G'_{IJK11}
\]

\[
G_{IJKL}^{(\chi)} = -\frac{\sqrt{2}}{32\pi} \left(\frac{\kappa}{4\pi}\right)^{2/3} \left[J^{(\chi,1)} - \frac{2^{11}}{\pi\rho} (J^{(\chi,1)} + J^{(\chi,2)})\right]_{IJKL} + G'_{IJKL}
\]

where \(G'\) is an arbitrary zero mode; that is, a solution of the homogeneous equations

\[
D_I G'^{IJKL} = 0, \quad dG' = 0.
\]

In the terminology of ref. [16], a topologically nontrivial \(G'\) constitutes a “nonzero” zero mode which breaks supersymmetry explicitly. Such a piece should be generally admitted in a background with broken supersymmetry. It will, however, introduce some arbitrariness into the gaugino condensate potential, which we are going to fix later on by a flux quantization argument.

Let us now be more specific about the form of the condensate we are going to consider. For the time being, we will allow for condensates on both hyperplanes, though later on we will restrict the

\[\text{This argument does not apply to terms with derivatives in the Calabi–Yau direction since the Calabi–Yau radius can be smaller than the orbifold radius. For covariantly constant condensates, however, those terms vanish.}\]
discussion to the “physical” case of hidden $E_8$ condensates only. We use the standard expression for covariantly constant condensates

$$\omega_{abc}^{(\chi,i)} = \text{tr} \bar{\chi} \Gamma_{abc} \chi = \Lambda^{(i)} \epsilon_{abc}$$ (33)

where the $\Lambda^{(i)}$ represent the third powers of the condensation scales (to be explicitly determined later) and are, therefore, viewed as functions of the moduli $S$ and $T$. The nonvanishing components of the currents $J^{(\chi,i)}$ then read

$$J_{\mu abc}^{(\chi,i)} = \partial_\mu \Lambda^{(i)} \epsilon_{abc}$$ (34)

By means of eq. (32), $G'$ is a harmonic four–form on $S^1/Z_2 \times X$. There exist $h^{2,1} + 1$ independent such forms and a basis is provided by $\{\epsilon_{abc}, \omega_{abc}^{(i)}, i = 1, \ldots, h^{2,1}\}$ where $\omega_{abc}^{(i)}$ are the $h^{2,1}$ harmonic $(2,1)$–forms of the Calabi–Yau space $X$. Given the index structure of the condensate, only the first of those forms can mix with the gaugino condensate in the low energy potential and is, therefore, of interest to us. Consequently, we set

$$G'_{abc11} = -\frac{\sqrt{2}}{32 \pi^2 \rho} \left(\frac{\kappa}{4 \pi}\right)^{2/3} \Lambda^{(1)} + \Lambda^{(2)} + \lambda \epsilon_{abc}$$ (35)

(with all other components vanishing) where $\lambda$ is a free $x^{11}$–independent parameter. As a result, we find for $G^{(\chi)}$

$$G_{abc11}^{(\chi)} = -\frac{\sqrt{2}}{32 \pi^2 \rho} \left(\frac{\kappa}{4 \pi}\right)^{2/3} \left[\Lambda^{(1)} + \Lambda^{(2)} + \lambda \epsilon_{abc}\right]$$ (36)

$$G_{\mu abc}^{(\chi)} = -\frac{\sqrt{2}}{32 \pi} \left(\frac{\kappa}{4 \pi}\right)^{2/3} \partial_\mu \left[\Lambda^{(1)} - \frac{x^{11}}{\pi \rho}(\Lambda^{(1)} + \Lambda^{(2)})\right] \epsilon_{abc}$$ (37)

With these expressions, the field $\tilde{G}$ in eq. (23) is completely determined in the presence of the condensates and can be inserted into eq. (7) to obtain the low energy potential. Clearly, one contribution to the potential arises from the square of $G_{abc11}^{(\chi)}$. In addition, however, there can be mixing terms between $G_{abc11}^{(\chi)}$ and the other parts of $\tilde{G}$ in eq. (23) which produce potential terms. Inspection of those other parts as given in ref. [9], shows that there is only one component with the correct index structure; namely

$$G_{abc11}^{(B)} = -\frac{i}{\sqrt{2} \pi^2 \rho} \left(\frac{\kappa}{4 \pi}\right)^{2/3} d_{pqr} C^p C^q C^r \epsilon_{abc}$$ (38)

The corresponding mixing term is responsible for a cross term between the matter fields and the gaugino condensate that should appear in the scalar potential. For the discussion of gaugino condensation, this term is usually neglected since $C$, as a matter field, is thought of as fluctuating on small scales. It is, however, possible that $C$ acquires a large vacuum expectation value $\langle C \rangle$ which breaks the $E_6$ GUT symmetry. Though such an effect would be sizeable we will, in the following, stick to the case $\langle C \rangle = 0$, for simplicity. The generalization to include a nonzero
< C > is straightforward and does not change any of the essential conclusions. With these remarks in mind, we use the eqs. (36) to compute the potential

\[ S(\chi) = -\frac{V\pi\rho}{\kappa^2} \frac{1}{512\pi^2\rho^2} \left(\frac{\kappa}{4\pi}\right)^{4/3} \int_{M_4} \sqrt{-g} e^{-12\alpha - 3\epsilon} |\Lambda^{(1)} + \Lambda^{(2)} + \lambda|^2. \]  

We see that the potential can be expressed in terms of the condensates \( \Lambda^{(i)} \) (whose functional dependence on the moduli will be inserted later) and the arbitrary constant \( \lambda \) which originates from the explicitly supersymmetry breaking zero mode field \( G' \). The condensate terms in this potential do not arise directly from the corresponding terms in 11 dimensions, but rather as a part of the form field which is necessary to interpolate between condensate source terms on the boundaries, as explained in the introduction. Therefore, the potential does not contain any delta-function singularities. In the next section, we will argue that the free parameter \( \lambda \) can be fixed by a flux quantization argument in M–theory on \( S^1/Z_2 \), in analogy with the weakly coupled case \[17\].

Before we come to this, let us discuss the relation of the above potential to Hořava’s global breaking mechanism. Clearly, since we are working in four dimensions, the global nature of the breaking (which is global in the now invisible orbifold direction) cannot be really reflected in the potential. To see the global form one would need to formulate a five-dimensional effective theory, where the dependence on the orbifold coordinate is not integrated out. Despite this, there is one characteristic of the global breaking which should persist in four dimensions, namely the vanishing of supersymmetry breaking for equal but opposite condensates. In view of the eqs. (17) and (33) such a situation, namely the existence of a global Killing spinor, occurs for

\[ \Lambda^{(1)} + \Lambda^{(2)} = 0. \]  

On the other hand, this condition is identical with the vanishing of the gaugino part of the potential (39) which proves the required consistency. It is interesting to note that we could not arrive at such a consistency by simply multiplying out the perfect square (7) as has been done in ref. [12]. Since the cross term, being proportional to \( \delta(x^{11})\delta(x^{11} - \pi\rho) \), would (at least naively) vanish, such a procedure leads to a potential proportional to \( (\Lambda^{(1)})^2 + (\Lambda^{(2)})^2 \). This is not necessarily zero if the condition (40) is fulfilled.

### 4 Flux quantization in M–theory on \( S^1/Z_2 \)

To proceed, it is necessary to establish the flux quantization condition for Hořava-Witten theory. The appropriate quantization condition in M-theory was presented in ref. [18]. It states that, for any closed four-cycle \( C_4 \)

\[ \zeta \int_{C_4} \frac{G}{2\pi} + \int_{C_4} \frac{\lambda}{2} = n \]  

where \( n \) is any integer, \( G = dC \) locally, \( \lambda = \frac{1}{16\pi^2} R \wedge R \) and \( \zeta = \frac{1}{\sqrt{2}} \left(\frac{4\pi}{\kappa}\right)^{\frac{2}{3}}. \) As discussed in that paper, this result applies to any closed four-cycle in Hořava–Witten theory that is homologous to a
four-cycle on a single boundary plane. Thus, closed four-cycles purely in the 11-dimensional bulk, or lying partially or wholly in one boundary plane, satisfy (11). However, it is important to note that there is another class of cycles in Hořava–Witten theory; namely, those that wind around the orbifold or, in the boundary picture, stretch between both boundary planes. These new cycles are not closed in the boundary picture. Unlike closed cycles that, by definition, possess no boundary, these new cycles have a non-empty boundary three-cycle which is shared between both planes, \( \partial C_4 = \partial C_4^{(1)} + \partial C_4^{(2)} \). Such cycles are not homologous to closed four-cycles in the bulk or on one boundary and, hence, do not necessarily satisfy equation (11). It is precisely the flux quantization condition over these new cycles that is required in this paper to discuss the low energy effective action.

When one allows the new open four-cycles touching both boundary planes, in addition to the closed four-cycles in the bulk or touching a single boundary, we find that Witten’s flux quantization condition generalizes to

\[
\zeta \int_{C_4} \frac{G}{2\pi} + \frac{\lambda}{2} + \frac{1}{16\pi^2} \sum_{i=1}^{2} \int_{\partial C_4^{(i)}} \omega^{YM(i)} = n
\]

where \( \omega^{YM(i)} \) is the Yang-Mills Chern-Simons three-form on the \( i \)-th boundary. In this paper, we will justify this flux quantization formula by demonstrating that it is consistent with two limiting cases. First note, that for a closed four-cycle either in the bulk or touching one of the boundary planes, the Yang-Mills Chern-Simons term vanishes and we recover Witten’s expression (11). A second, far more non-trivial check is to consider a Hořava–Witten cycle that touches both boundaries. Furthermore, let us choose the radius of the boundary cycles, \( \partial C_4^{(i)} \), to be much larger than the radius of the \( S^1/Z_2 \) orbifold. In this limit, it was shown in ref. [7] that, at least locally, in the ten-dimensional space,

\[
G_{IJK11} = 3\partial_I B_{JK} - \frac{1}{4\sqrt{2}\pi^2 \rho} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left\{ \omega^{YM(1)} + \omega^{YM(2)} - \omega^L \right\}_{IJK} + \Delta H_{IJK}
\]

where \( \omega^L \) is the Lorentz Chern-Simons three-form. The \( B_{JK} \) and Chern-Simons forms are independent of \( x^{11} \). The \( \Delta H_{IJK} \) term is due to massive Kaluza-Klein modes on the orbifold and is explicitly \( x^{11} \) dependent. It is not necessary to know its exact form, but only to recall that it must satisfy the relation

\[
\langle \Delta H_{IJK} \rangle_{11} = 0
\]

where \( \langle \ldots \rangle_{11} \) indicates the average over the 11-direction. In this limit, we can evaluate the flux quantization condition. First, using the fact that

\[
\langle tr R \wedge R \rangle_{11IJK} = \left( d\omega^L \right)_{11IJK}
\]
equation (42) becomes
\[ \zeta \int_{C_4} G + \frac{1}{16\pi^2} \sum_{i=1}^{2} \int_{\partial C_4^{(i)}} (\omega^{YM(i)} - \frac{1}{2} \omega^{L(i)}) = n \]  
(46)

Now, inserting expression (43) into this equation and using relation (44), we find that the Chern-Simons terms exactly cancel and we are left with the relation
\[ \zeta \int_{C_4} H^0 = 2\pi n \]  
(47)

where locally \( H^0 = dB \). Now this limit, where the radius of the boundary cycles is much larger than the orbifold radius, corresponds to the strong coupling limit of the heterotic string. However, as we will show \cite{19}, the form of the ten-dimensional effective theory of the strongly coupled heterotic string is actually identical to that of the one-loop weakly coupled limit. Thus, since we expect quantization conditions to be independent of coupling, equation (47) should correspond to the flux quantization condition of the weakly coupled heterotic string. Comparing against the flux quantization condition derived in ref. \cite{17}, we see that this is indeed the case. We conclude, therefore, that expression (42) is the correct flux quantization law for any four-cycles in Hořava–Witten theory. This argument could be made stronger by considering the membrane action. For open membranes which end on the orbifold planes, one expects additional fields in the world-volume theory which live only on the boundary of the membrane. It is the coupling of these fields to the gauge fields on the orbifold planes that one expects to lead to the additional boundary terms in the quantization condition.

We are now ready to apply the quantization rule which we have just presented to gaugino condensation. To do that, we consider a cycle \( C_4 = S^1 \times C_3 \) where \( C_3 \) is the three–cycle in the Calabi–Yau space which is nonzero upon integration over \( \epsilon_{abc} \) and zero upon integration over the harmonic \( (2,1) \)–forms \( \omega_{ab}^{(i)} \). Then, the components of the four–form \( G \) which contribute to the integral are \( G_{abc11} \). Recalling the definition of \( \tilde{G} \) in terms of \( G \), eq. \( \text{[8]} \), and the form of \( \tilde{G} \), eqs. \( \text{[25]}, \text{[36]} \) we have for these components \footnote{Note that the only other part of \( \tilde{G} \) with a \((abc11)\)–component is \( \tilde{G}_{abc11}^{(2)} \) given in eq. \( \text{[38]} \). Since we have assumed that the matter fields \( C^\rho \) are small fluctuations it does not contribute to the quantization. In fact, even if it was included, we would find it was cancelled by a corresponding piece in the third term in the quantization condition, since both arise as the dimensional reduction of Chern-Simons terms.}

\[ G_{abc11} = -\frac{\sqrt{2}}{16\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \left[ \frac{1}{2\pi^2} (\Lambda^{(1)} + \Lambda^{(2)} + \lambda) - (\Lambda^{(1)} \delta(x^{11}) + \Lambda^{(2)} \delta(x^{11} - \pi \rho)) \right] \epsilon_{abc} . \]  
(48)

Furthermore, we note that for our background and the specific cycle we have chosen, the second and third integrals in the quantization rule do not contribute. We therefore have
\[ \zeta \int_{C_4} G = 2\pi n . \]  
(49)
Inserting the above expression for $G_{abc11}$, the condensate–dependent pieces cancel and we find a condition on $\lambda$ which reads

$$\lambda = \frac{32\pi^2}{c\sqrt{V}} n$$

(50)

where $c$ is an order one quantity defined by the relation $\int_{G_5} \epsilon_{abc}dx^a \wedge dx^b \wedge dx^c = c\sqrt{V}$ and $n$ is an integer as before.

Let us now discuss the implications of this result for the potential (39). From now on we will concentrate on the “physical” case of gaugino condensation in the hidden sector; that is, we will assume that $\Lambda^{(1)} = 0$ and $\Lambda^{(2)} \neq 0$. For a nonzero integer $n$ we have a minimum of the potential (39) at $\Lambda^{(2)} = \lambda$, which leads to soft supersymmetry breaking terms of order $m_{\text{soft}} \sim G_N \lambda \sim G_N n/\sqrt{V}$. For a realistic value $V^{-1/6} \sim 10^{16}\text{GeV}$, this implies soft breaking terms far too large to be compatible with low energy supersymmetry. Therefore, the only possible choice for the integer $n$ is $n = 0$, which implies

$$\lambda = 0 \ .$$

(51)

We have, therefore, seen that the arbitrary parameter in the low energy potential can be fixed by the M–theory quantization rule in a way which is very similar to the weakly coupled case [17].

To write the potential (39) in an explicitly moduli dependent form, we note that the condensates $\Lambda^{(i)}$ correspond to a nonstandard normalization of the four–dimensional gaugino kinetic terms. Taking this into account, we have [5]

$$\Lambda^{(2)} \sim \frac{1}{\sqrt{V}} \exp \left[ -\frac{6\pi}{b_0\alpha_{\text{GUT}}}(S - \beta T) \right]$$

(52)

and we get for the final potential

$$S(\chi) \sim \frac{\kappa^2}{\rho V^2} \int_{M_4} e^{-12a - 3c} \left| \exp \left[ -\frac{6\pi}{b_0\alpha_{\text{GUT}}}(S - \beta T) \right] \right|^2 .$$

(53)

Up to power law corrections, this leads to the superpotential

$$W(\chi) = h \exp \left[ -\frac{6\pi}{b_0\alpha_{\text{GUT}}}(S - \beta T) \right]$$

(54)

where $h$ is a constant of order $\kappa/\rho^{1/2}V$. This result exactly coincides with the one in the weakly coupled case [4] if one takes into account that the expression (1) for the gauge kinetic function also holds in the weakly coupled region. This contradicts claims [12] that the potential is significantly more complicated on the M–theory side. The crucial difference to the weakly coupled case is not the form of the potential, but the size of the parameter $\beta$ which is small for weak coupling but potentially large for strong coupling. It is also interesting to note that, for fixed $S$ (fixed Calabi–Yau radius), $T$ is driven toward small values; that is, toward the weak coupling region [4].
5 Soft terms

As a final application, we would like to calculate the generic pattern of soft terms that arises in the strongly coupled heterotic string. A similar analysis has been carried out in ref. [11].

Let us first discuss the possible goals and limitations of such a computation. As we have seen in the previous section, our simple gaugino condensate potential shows a runaway behaviour, as in the weakly coupled case, and does not provide us with a minimum for the moduli fields. Therefore, unless we go to more complicated models, like, for example, multi–gaugino condensation [20], the specific structure of supersymmetry breaking is not determined. In this paper, we will not attempt to explicitly construct such realistic models, but simple assume that supersymmetry breaking can be achieved in more complicated cases. Though we do not gain any information about the specific breaking pattern, we can still parameterize the effect of supersymmetry breaking by the auxiliary components $F_S$, $F_T$ of the moduli superfields $S$ and $T$. The interesting new feature which motivates an analysis of soft terms, even without detailed knowledge of the supersymmetry breaking mechanism, is the appearance of the correction terms proportional to $\beta$ in the Kähler potential and the gauge kinetic functions (2). We stress again that these terms are present in the weakly coupled case as well. The important difference is the magnitude of these terms in the weakly and strongly coupled case. Let us adopt a normalization where the moduli field $S$, $T$ take values of order one, so that the parameter $\beta$ describes the order of magnitude of the correction. Then $\beta = O(\kappa^{2/3} \rho / V^{2/3})$ is small in the weakly coupled regime, but it can be sizeable and even of the order one or larger in the strongly coupled regime. In fact, for the “phenomenological” values of $\rho$ and $V$ determined from the Newton constant and the grand unification coupling constant via eq. (1), it is of order one. The precise value depends somewhat on the specific Calabi–Yau space chosen and can be computed from the equations presented in ref. [7]. It should be kept in mind that the low energy theory specified by eq. (2) is constructed as an expansion in $\beta$ (up to the first order) and, consequently, breaks down if $\beta$ is too large. We will, therefore, assume that $\beta$ is still small enough so that the expressions in (2) are sensible. Clearly, given these remarks, a computation of soft terms only makes sense up to terms linear in $\beta$.

Having said this, the main goal of this section is to determine the $O(\beta)$ corrections to the soft terms which result from the $\beta$ dependent corrections to the Kähler potential and the gauge kinetic function. In particular, it is interesting to see to what extent the structure of soft terms, which is known to be somewhat special for weakly coupled, large radius Calabi–Yau compactifications, is enriched by those corrections.

The general structure of soft terms for hidden sector supersymmetry breaking has been computed in ref. [21, 22]. Here, we will use the convenient Kähler–covariant approach and the notation of ref. [23]. We should point out that the transmission of supersymmetry breaking from the hidden
to the observable sector is completely governed by four–dimensional supergravity as long as the theory is probed with momenta far below the orbifold and the Calabi–Yau scale. In particular, at such low momenta, the orbifold hyperplanes cannot be resolved and there is no notion of being on the “observable hyperplane” receiving information about the breaking from the “hidden plane”. Consequently, the suppression of the breaking scale from the hidden to the observable sector is not governed by the orbifold scale, but by the low energy Planck scale. The auxiliary fields $F^S$ and $F^T$ are, therefore, of the order $G_N \Lambda^{(2)}$. For a realistic scenario we should therefore have $(\Lambda^{(2)})^{1/3} \sim 10^{13}$ to $10^{14} \text{GeV}$ resulting in a gravitino mass $m_{3/2} \sim F^T \sim 10^3 \text{GeV}$.

Let us now rewrite the model in a form adequate as a starting point for the computation of soft masses. We introduce a four–dimensional $\kappa$–parameter defined by

$$\kappa^2 = \frac{8\pi G_N}{\lambda} = \frac{\kappa^2}{V \pi \rho}.$$  

In order to have matter fields $C$ of mass dimension one, we apply the rescaling $C \rightarrow \kappa \hat{P} C$. Then the Kähler potential (2), expanded up to second order in $C$, can be written as

$$K = \kappa^{-2} \hat{K}(S, T, \bar{S}, \bar{T}) + Z_{pq}(S, T, \bar{S}, \bar{T}) \bar{C} \hat{P} C^q$$  

with

$$\hat{K} = -\ln(S + \bar{S}) - 3 \ln(T + \bar{T}) , \quad Z_{pq} = \left( \frac{3}{T + \bar{T}} + \frac{\beta}{S + \bar{S}} \right) \delta_{pq} .$$  

The rescaled superpotential is given by

$$W = \frac{1}{3} \hat{Y} d_{pqr} C^p C^q C^r$$  

with a coupling $\hat{Y}$ of order one. Finally we need the gauge kinetic function of the observable $E_6$ gauge group

$$f^{(1)} = S + \beta T .$$  

Inserting these quantities into the formulae presented in ref. [23], we arrive at the following tree level soft terms:

$$m_{3/2}^2 = \frac{|F^S|^2}{3(S + \bar{S})^2} + \frac{|F^T|^2}{(T + \bar{T})^2} + \frac{F^S}{2(S + \bar{S})} + \beta \left( \frac{F^T}{2(S + \bar{S})} - \frac{T + \bar{T}}{(S + \bar{S})^2} F^S \right)$$  

$$m_{1/2}^2 = \frac{|F^S|^2}{(S + \bar{S})^2(T + \bar{T})} + \beta \left( \frac{5|F^S|^2}{3(S + \bar{S})^2} + \frac{2 \text{Re}(F^S \bar{F}^T)}{(S + \bar{S})^2(T + \bar{T})} \right)$$  

$$m_0^2 = \frac{\text{Re}(F^S \bar{F}^T)}{(S + \bar{S})^2} + \beta \left( \frac{F^T}{S + \bar{S}} - \frac{T + \bar{T}}{(S + \bar{S})^2} F^S \right) Y$$  

$$A = - \left( \frac{F^S}{S + \bar{S}} + \beta \left( \frac{F^T}{S + \bar{S}} - \frac{T + \bar{T}}{(S + \bar{S})^2} F^S \right) Y \right)$$

where $Y = e^{\hat{K}/2 \hat{Y}}$. Here, $m_{3/2}$, $m_{1/2}$, $m_0$ and $A$ are the gravitino mass, the gaugino mass, the scalar mass and the trilinear coupling. (Note that the scalar masses are not quite correctly normalized in these expressions, since we have not rescaled the Kähler metric out of the corresponding

\footnote{It is assumed that the cosmological constant in the hidden sector vanishes.}
kinetic terms.) Let us first consider the above expressions in the limit \( \beta \to 0 \). In this case, we recover the expression for the soft masses one obtains in a weakly coupled, large radius Calabi–Yau compactification \[24\]. In this limit, \( m_{1/2}, m_0 \) and \( A \) depend on the auxiliary component \( F^S \) only. This specific structure results from the form of the Kähler potential and (as far as the trilinear coupling is concerned) from the fact that the Yukawa couplings are constant in the large radius limit. Experience with gaugino condensation in the weakly coupled case shows that supersymmetry is broken frequently in the \( F^T \) direction only, in which case all soft couplings except \( m_{3/2} \) would be small.

Let us now discuss the effect of the additional terms proportional to \( \beta \). A priori, one could expect the \( F^T \) degeneracy of all couplings to be lifted by those corrections. While this is true for the gaugino masses \( m_{1/2} \), as has been pointed out in ref. \[11\], and the trilinear coupling \( A \), as we emphasize in this paper, the scalar soft masses still receive no contribution proportional to \( |F^T|^2 \). Consequently, for supersymmetry breaking in the \( F^T \)-direction, the scalar soft masses remain light in the M–theory regime while \( m_{1/2} \) and \( A \) are of the same order as \( m_{3/2} \). We note that the soft couplings also receive various new contributions proportional to \( F^S \) which will be relevant for the precise value of the couplings. They will, however, not cause a change in the order of magnitude in going from the weakly to the strongly coupled regime.

As this manuscript was prepared, ref. \[25\] appeared. This paper discusses some of the issues presented in this work, particularly the phenomenological results of section 5.

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