REDUCED-RANK DOA ESTIMATION BASED ON JOINT ITERATIVE SUBSPACE OPTIMIZATION AND GRID SEARCH

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ABSTRACT

In this paper, we propose a novel reduced-rank algorithm for direction of arrival (DOA) estimation based on the minimum variance (MV) power spectral evaluation. It is suitable to DOA estimation with large arrays and can be applied to arbitrary array geometries. The proposed DOA estimation algorithm is formulated as a joint optimization of a subspace projection matrix and an auxiliary reduced-rank parameter vector with respect to the MV and grid search. A constrained least squares method is employed to solve this joint optimization problem for the output power over the grid. The proposed algorithm is described for problems of large number of users' direction finding with or without exact information of the number of sources, and does not require the singular value decomposition (SVD). The spatial smoothing (SS) technique is also employed in the proposed algorithm for dealing with correlated sources problem. Simulations are conducted with comparisons against existent algorithms to show the improved performance of the proposed algorithm in different scenarios.

Index Terms— Direction of arrival (DOA) estimation, array processing, joint iterative methods, reduced-rank methods.

1. INTRODUCTION

Direction of arrival (DOA) estimation techniques have been widely employed in many fields related to array processing [1]. Numerous DOA estimation approaches have been considered to date. Among them are the Capon [2], the conventional subspace-based methods that require the singular value decomposition (SVD), such as MUSIC [3] and ESPRIT [4], and more recent subspace techniques that do not require the SVD, such as the auxiliary vector (AV) estimation algorithm [5] and the conjugate gradient (CG) algorithm [6].

The Capon DOA estimation method minimizes the output power of the undesired interferences while maintaining a constant gain along the look direction. By computing and plotting Capon’s spectrum over the possible scanning directions, the DOAs can be estimated by locating the peaks in the spectrum. The estimation accuracy of the Capon method strongly depends on the number of snapshots and the array size. The subspace-based MUSIC and ESPRIT algorithms exploit the eigen-structure of the input covariance matrix to decompose the observation space into a signal subspace and a corresponding orthogonal noise subspace. ESPRIT has better performance by employing a displacement invariance in some specific array structures. The developed eigen-decomposition algorithms were proposed recently. The AV method is developed based on the orthogonality of an extended non-eigenvector signal subspace with the true signal subspace and the scanning vector itself. As the scanning vector drops in the signal subspace, the DOAs are determined by finding the collapse in the extended signal subspace. The CG method can be considered as an extended version of the AV method since it applies the residual vectors in place of the AV basis. Both algorithms show dominate in severe conditions with a small number of snapshots and at low SNR for both correlated and uncorrelated sources. However, they work inefficiently with a large number of sources or without exact information about the number of sources beforehand.

In this paper, we propose a DOA estimation algorithm by employing a novel reduced-rank signal processing strategy. The proposed algorithm is based on a joint iterative subspace optimization (JISO) and grid search with respect to the MV power spectrum evaluation. The implementation of the proposed DOA estimation algorithm amounts to designing a subspace projection matrix and an auxiliary reduced-rank parameter vector with respect to the MV criterion. We present a constrained least squares algorithm for jointly estimating the subspace projection matrix and the auxiliary reduced-rank parameter vector that calculate the output power over the possible scanning directions. The proposed algorithm is more practical, in comparison with the existing algorithms, since it is not limited by the array structure, does not require the SVD procedure, and works without information of the source number, which will be shown in simulations. The estimation accuracy is also satisfied with a large number of sources’ direction finding. In addition, the spatial smoothing (SS) technique, which was devised by Evans in [9] and further developed by Shan in [10], is employed in the proposed DOA estimation algorithm for dealing with the problem caused by correlated sources.

The rest of this paper is organized as follows: we outline a system model for DOA estimation and present the problem statement in Section 2. Section 3 derives the proposed DOA estimation algorithm and analyzes the complexity. The application of the SS technique in the proposed algorithm is also introduced briefly in this part. Simulation results are provided and discussed in Section 4, and conclusions are drawn in Section 5.

2. SYSTEM MODEL AND PROBLEM STATEMENT

2.1. System Model

Let us suppose that $q$ narrowband signals impinge on a uniform linear array (ULA) of $m$ ($m \geq q$) sensor elements. Note that the proposed DOA estimation algorithm can be applied to arbitrary array
structures. An extension to arbitrary arrays will be sought in a future work. The ULA here is adopted for using the SS technique and reaching a fair comparison with ESPRIT, which is applied to some specific array structures. The $i$th snapshot’s vector of sensor array outputs $x(i) \in \mathbb{C}^{m \times 1}$ can be modeled as

$$x(i) = A(\theta)s(i) + n(i), \quad i = 1, \ldots, N$$  \hspace{1cm} (1)

where $\theta = [\theta_0, \ldots, \theta_q]^{T} \in \mathbb{C}^{q \times 1}$ is the signal DOAs, $A(\theta) = [a(\theta_0), \ldots, a(\theta_q-1)] \in \mathbb{C}^{m \times q}$ is the matrix that contains the signal direction vectors $a(\theta_k)$, where $a(\theta_k) = [1, e^{-2\pi j \frac{d}{\lambda} \cos \theta_k}, \ldots, e^{-2\pi j (m-1) \frac{d}{\lambda} \cos \theta_k}]^{T} \in \mathbb{C}^{m \times 1}$, $(k = 0, \ldots, q - 1)$, $\lambda_c$ is the wavelength, and $d = \lambda_c/2$ in general) is the inter-element distance of the ULA. To avoid mathematical ambiguities, the direction vectors $a(\theta_k)$ are considered to be linearly independent [8]. $s(i) \in \mathbb{R}^{q \times 1}$ is the source data. $n(i) \in \mathbb{C}^{m \times 1}$ is the white sensor noise, which is assumed to be a zero-mean spatially and Gaussian process, $N$ is the number of snapshots, and $(\cdot)^{T}$ denotes transpose.

2.2. Problem statement

Based on the MV output power spectrum (or Capon output power spectrum), [2, 11], the output power to each scanning direction for DOA estimation is expressed by

$$\hat{\theta} = \arg \min_{\theta} w_{\theta}^{H} R w_{\theta}$$

subject to $w_{\theta}^{H} a(\theta) = 1$ \hspace{1cm} (2)

where $\hat{\theta}$ is the estimated direction and $w_{\theta} = [w_{\theta,1}, \ldots, w_{\theta,m}]^{T} \in \mathbb{C}^{m \times 1}$ is the weight vector corresponding to the current scanning direction $\theta$. $(\cdot)^{H}$ denotes Hermitian transpose. $R$ is the data covariance matrix

$$R = E[x(i)x^{H}(i)] = A(\theta)R_{s}A^{H}(\theta) + \sigma_{s}^{2}I$$  \hspace{1cm} (3)

where $R_{s} = E[s(i)s^{H}(i)]$ denotes the signal covariance matrix, which is diagonal if the sources are uncorrelated and is nondiagonal and nonsingular for partially correlated sources, and $E[n(i)n^{H}(i)] = \sigma_{n}^{2}I$ with $I$ being the corresponding identity matrix.

The MV (Capon) power spectrum estimation algorithm attempts to minimize the contribution of the total output power while maintaining an unity gain along a look direction $\theta$. By optimizing the weight vector $w_{\theta}$ and obtaining the output for all possible directions $\theta \in (0^\circ, 180^\circ)$, the DOAs can be determined by finding the peaks in the output power spectrum. The weight solution is [2, 11]

$$w_{\theta} = \frac{R^{-1}a(\theta)}{a^{H}(\theta)R^{-1}a(\theta)}$$ \hspace{1cm} (4)

Substituting (4) into (2), DOA estimation based on the MV (Capon) power spectrum is given by

$$\hat{\theta}_{\text{MV}} = \arg \max_{\theta} [a^{H}(\theta)R^{-1}a(\theta)]^{-1}$$  \hspace{1cm} (5)

Note that complete knowledge of $R$ cannot be obtained in practice. We may use a sample-average recursion to estimate this input covariance matrix, which is given by

$$\hat{R} = \frac{1}{N} \sum_{i=1}^{N} x(i)x^{H}(i)$$  \hspace{1cm} (6)

Where $\hat{R}$ is not invertible if the number of available snapshots is less than the number of sensors ($N \leq m$). It can be implemented by employing the diagonal loading technique [8].

The above MV based DOA estimation method suffers from a heavy computational load for large $m$ due to the matrix inversion and works inefficiently in the presence of correlated sources. Furthermore, the performance is inferior when large number of sources appear in the system.

3. PROPOSED DOA ESTIMATION ALGORITHM

In this section, we employ a reduced-rank strategy to perform DOA estimation. This is carried out via the proposed joint iterative subspace optimization (JISO) according to the MV criterion for estimating the subspace projection matrix and the auxiliary reduced-rank parameter vector followed by a grid search.

3.1. Proposed Reduced-Rank DOA Estimation Scheme

We introduce a subspace projection matrix $T_{r} = [t_{1}, t_{2}, \ldots, t_{r}] \in \mathbb{C}^{m \times r}$, which is responsible for the dimensionality reduction, to project the $m \times 1$ received vector $x(i)$ onto a lower dimension, yielding

$$\bar{x}(i) = T_{r}^{H}x(i)$$  \hspace{1cm} (7)

where $t_{l} = [t_{l,1}, t_{l,2}, \ldots, t_{l,r}]^{T} \in \mathbb{C}^{m \times 1}$, $(l = 1, \ldots, r)$ makes up the subspace projection matrix $T_{r}$, $\bar{x}(i) \in \mathbb{C}^{r \times 1}$ is the projected received vector, and in what follows, all $r$ dimensional quantities are denoted with a “bar”. $r < m$ is the rank. An auxiliary filter with the reduced-rank weight vector $f_{\theta} = [f_{\theta,1}, f_{\theta,2}, \ldots, f_{\theta,r}]^{T} \in \mathbb{C}^{r \times 1}$ is applied after the projection procedure. The aim of $T_{r}$ is to extract the key features of the original input vector $x(i)$ and form the reduced-rank input vector $\bar{x}(i)$. The auxiliary reduced-rank weight vector $f_{\theta}$ works on $\bar{x}(i)$ for obtaining the output power with respect to the current scanning direction $\theta$. Since the procedure is operated with a lower dimension $r$, the computational complexity will be reduced if $r << m$. Since DOA estimation depends on the number of sensor elements $m$ and on the eigenvalue spread of the input covariance matrix, the proposed reduced-rank estimation scheme will exhibit improved performance under conditions where $m$ is large [12]. Following the MV DOA estimation in (3), the proposed optimization problem can be expressed by

$$\hat{\theta}_{\text{JISO}} = \arg \min_{\theta} f_{\theta}^{H} T_{r}^{H} R T_{r} f_{\theta}$$

subject to $f_{\theta}^{H} T_{r}^{H} a(\theta) = 1$ \hspace{1cm} (8)

We find that the minimization with respect to (8) is equivalent to the joint optimization of the subspace projection matrix $T_{r}$, and the auxiliary reduced-rank weight vector $f_{\theta}$. After obtaining $T_{r}$ and $f_{\theta}$, DOA estimation can be determined by plotting the output power spectrum for the possible directions and searching for peaks that correspond to the DOAs of the sources. It is worth noting that, for $r > 1$, the novel scheme becomes a conventional full-rank MV scheme with an additional weight parameter $f_{\theta}$ that provides an amplitude gain. For $r > 1$, the signal processing tasks are changed and $T_{r}$ and $f_{\theta}$ are optimized for obtaining the proposed output power spectrum for the possible directions.

3.2. Proposed Joint Iterative Subspace Optimization Algorithm

The challenge left to us is how to efficiently compute the subspace projection matrix $T_{r}$ and the auxiliary reduced-rank weight vector
\( \bar{f}_\theta \) for solving the optimization problem [8]. We propose a constrained least squares (LS) algorithm to solve this joint optimization problem. The constraint in [8] can be incorporated by the method of Lagrange multipliers [13] in the form

\[
J = \sum_{i=1}^{i} \alpha^{-1} \left| \bar{f}_\theta (i) \right|^2 + \lambda \left| \bar{f}_\theta (i) \right|^2 - 1
\]

where \( \alpha \) is a forgetting factor, which is a positive constant close to but less than 1, and \( \lambda \) is a scalar Lagrange multiplier. Fixing \( \bar{f}_\theta (i) \), computing the gradient of [8] with respect to \( T_r (i) \), yields

\[
\nabla J_{T_r} = \sum_{i=1}^{i} \alpha^{-1} \left| x^H (i) x (i) \right|^2 + \lambda \left| \bar{f}_\theta (i) \right|^2 - 1
\]

where \( \bar{R} (i) = \sum_{i=1}^{i} \bar{x}^H (i) \bar{x} (i) \in C^{m \times m} \) is the estimated covariance matrix. According to [13], \( \bar{R} (i) \) can be written in a recursive form as

\[
\bar{R} (i) = \alpha \bar{R} (i-1) + 2 \bar{x} (i) \bar{x}^H (i)
\]

Making the above gradient terms equal to zero, multiplying \( \bar{f}_\theta (i) \) from the right of both sides, and rearranging the expression, it becomes

\[
T_r (i) \bar{f}_\theta = \lambda \bar{R} (i) \bar{f}_\theta
\]

where \( \bar{R} (i) \) is invertible by employing the diagonal loading technique.

If we define \( \bar{p} (i) = - \lambda \bar{R} (i) \bar{f}_\theta (i) \), the solution of \( T_r (i) \) can be regarded to find the solution to the linear equation

\[
T_r (i) \bar{f}_\theta = \bar{p} (i)
\]

In order to find an unique solution for \( T_r (i) \), we express the quantities involved in [13] by

\[
T_r (i) = \begin{bmatrix} \rho_1 (i) \\ \vdots \\ \rho_m (i) \end{bmatrix} \quad \rho_i (i) = \begin{bmatrix} f_{\theta,1} (i) \\ f_{\theta,2} (i) \\ \vdots \\ f_{\theta,m} (i) \end{bmatrix} \quad \bar{p}_r (i) = \begin{bmatrix} \bar{p}_1 (i) \\ \bar{p}_2 (i) \\ \vdots \\ \bar{p}_m (i) \end{bmatrix}
\]

The problem in [13] is equivalent to find \( \rho_j (i) \) (j = 1, . . . , m) for satisfying

\[
\min \| \rho_j (i) \|^2, \quad \text{subject to} \quad \rho_j (i) f_\theta = \bar{p}_j (i)
\]

which is obtained by using the Lagrange multiplier method

\[
\rho_j (i) = \bar{p}_j (i) f_\theta ^H (i) / \| f_\theta (i) \|^2
\]

and thus the projection matrix is

\[
T_r (i) = \bar{p} (i) f_\theta ^H (i) / \| f_\theta (i) \|^2
\]

Substituting the definition of \( \bar{p} (i) \) into [17], we have

\[
T_r (i) = - \lambda \bar{R} (i) \bar{f}_\theta (i) / \| f_\theta (i) \|^2
\]

The multiplier \( \lambda \) can be solved by incorporating [12] with the constraint in [8], which is

\[
\lambda_r = - \frac{1}{\bar{a}^H (i) \bar{R} (i) \bar{a} (i)}
\]

Substituting [19] into [18], we get the projection matrix

\[
T_r (i) = - \frac{\bar{R} (i) \bar{f}_\theta (i)}{\bar{a}^H (i) \bar{R} (i) \bar{a} (i)}
\]

At the same time, fixing \( T_r (i) \), taking the gradient of [8] with respect to \( \bar{f}_\theta (i) \), and making it equal to a null vector, we obtain

\[
\nabla J_{f_\theta} = \sum_{i=1}^{i} \alpha^{-1} \left| x^H (i) x (i) \right|^2 + \lambda \left| \bar{f}_\theta (i) \right|^2
\]

where \( \bar{R} (i) = \sum_{i=1}^{i} \bar{x}^H (i) \bar{x} (i) \in C^{r \times r} \) is the estimate of the reduced-rank covariance matrix. \( \bar{R} = E [\bar{x} (i) \bar{x}^H (i)] = T_r E [x (i) x^H (i)] T_r \)

Following the same procedures for calculating \( T_r (i) \), we obtain the result for the auxiliary reduced-rank weight vector \( \bar{f}_\theta (i) \)

\[
\bar{f}_\theta (i) = - \lambda \bar{R} (i) \bar{f}_\theta (i) / \| f_\theta (i) \|^2
\]

where \( \bar{a} (i) = T_r a (i) \) is the projected steering vector with respect to the current scanning direction. Note that [22] is similar in form to [2] if we do not consider the time instant \( i \). The proposed reduced-rank weight vector \( f_\theta (i) \) is more general when dealing with DOA estimation, namely, for \( r = m \), it is equivalent to the MV weight vector, and, for \( 1 < r < m \), it operates under lower dimensions for reducing the complexity and improving the performance.
3.3. DOA Estimation

After $N$ snapshots, substituting the weight solution $\hat{f}_{j}$ expressed in (24) with respect to the possible scanning directions $\theta \in (0^\circ, 180^\circ)$, and the subspace projection matrix $T_{ss}$ in (20) into (3), we obtain the corresponding output power spectrum for DOA estimation

$$P_{\text{ISO}}(\theta_n) = (\bar{a}^H(\theta_n)\bar{R}^{-1}\bar{a}(\theta_n))^{-1}$$

(25)

where the scanning direction $\theta_n = \theta_k$, $k \in \{0, \ldots, q - 1\}$, which corresponds to the transmitted sources, compared with other scanning angles that correspond to the noise level. Therefore, the output power spectrum shows peaks with respect to the sources when we plot it into the whole search range.

Considering correlated sources, we can use the SS technique [9] in our proposed algorithm. It is based on averaging the covariance matrix of identical overlapping arrays and so requires an array of identical elements equipped with some form of periodic structure. We divide the ULA into overlapping subarrays of size $n$, where $n = 1, 2, \ldots, 180^\circ/\Delta^\circ$. For a simple and convenient search, we make $180^\circ/\Delta^\circ$ an integer. We use a similar form to that of (11) for estimating $\hat{R}$. The proposed JISO algorithm for each scanning direction $\theta_n$ is summarized in Table 2, where $T_{ss}$ and $\hat{f}_{ss}$ are initialized to ensure the constraint. The proposed algorithm provides an iterative exchange of information between the projection matrix and the reduced-rank weight vector, which leads to the improved performance.

The output power in (25) is much higher if the scanning direction $\theta_n = \theta_k$, $k \in \{0, \ldots, q - 1\}$, which corresponds to the transmitted sources, compared with other scanning angles that correspond to the noise level. Therefore, the output power spectrum shows peaks with respect to the sources when we plot it into the whole search range.

| Table 2. The JISO-SS algorithm for each scanning direction |
|---------------------------------|
| **Initialization:** |
| $T_{ss,0}(0) = [I_{d}^r 0_{r \times (n-1)}^T]$ |
| $\hat{f}_{ss,0}(0) = (T_{ss,0}(0)\bar{a}_s(\theta_n))/(\|T_{ss,0}(0)\bar{a}_s(\theta_n)\|)^2$ |
| **Update for each time instant** |
| $j = 1, \ldots, J$ |
| $x_{j,ss}(i) = A_{ss} D^{-1} s(i) + n_{j,ss}(i)$ |
| $\hat{x}_{j,ss}(i) = T_{ss,0}^{H}(i-1)x_{j,ss}(i)$ |
| $\bar{P}_{j,ss}(i) = x_{j,ss}(i)\bar{x}_{j,ss}^{H}(i)$ |
| $\bar{a}_{ss}(\theta_n) = T_{ss,0}^{H}(i-1)\bar{a}_{ss}(\theta_n)$ |
| $\bar{R}_{ss}(i) = \alpha\bar{R}_{ss}(i-1) + \sum_{j=1}^{J} \bar{P}_{j,ss}(i)$ |
| $\hat{f}_{ss,ss}(i) = \hat{R}_{ss,ss}^{-1}(i)\bar{a}_{ss}(\theta_n)/(\bar{a}_{ss}^H(\theta_n)\hat{R}_{ss,ss}^{-1}(i)\bar{a}_{ss}(\theta_n))$ |
| $T_{ss,ss}(i) = \alpha\bar{R}_{ss}(i)\hat{f}_{ss,ss}(i)$ |
| **Output power** |
| $P_{\text{ISO-SS}}(\theta_n) = 1/(\bar{a}_{ss}^H(\theta_n)\hat{R}_{ss,ss}^{-1}\bar{a}_{ss}(\theta_n))$ |

than the MUSIC and ESPRIT methods and lower than the AV and CG algorithms. The JISO-SS algorithm is marginally more complex than the JISO one due to the SS preprocessing. Actually, we can employ other methods to solve the joint optimization problem (e.g., stochastic gradient, recursive least squares [13]) to avoid the matrix inversion for complexity reduction, which will be analyzed in the near future.

4. SIMULATIONS

Simulations are performed for an ULA with half wavelength inter-element spacing. We compare the proposed algorithm with the Capon. MUSIC, ESPRIT, AV, and CG methods, and run $K = 1000$ iterations to get each curve. The SS technique is employed for each algorithm to improve the performance. In all experiments, the BPSK signals’ power is $\sigma_s^2 = 1$ and the noise is spatially and temporally white Gaussian. The search step is $\Delta^\circ = 1^\circ$. The DOAs are considered to be resolved if $|\hat{\theta}_{\text{ISO}} - \theta_n| < 1^\circ$.

In Fig. 1 we consider the presence of $q = 2$ highly correlated sources separated by $3^\circ$ with correlation value $c = 0.9$, which are generated as follows:

$$s_1 \sim \mathcal{N}(0, \sigma_1^2) \quad \text{and} \quad s_2 = cs_1 + \sqrt{1 - c^2}s_3$$

(26)

where $s_3 \sim \mathcal{N}(0, \sigma_3^2)$. The sensor elements number is $m = 30$ and input SNR $= -2$dB. We set the forgetting factor $\alpha = 0.998$, the reduced dimension $r = 6$, and the diagonal loading $\delta = 5 \times 10^{-4}$ for the covariance matrix inverse in (18) and (19). The probability of resolution [5]. [6] is plotted against the number of snapshots. The proposed algorithm outperforms other existing methods with small number of snapshots. The curves between the proposed and the MUSIC algorithms are shown to intersect when the number of snapshots increases. The performance of the AV and CG methods can be seen to be inferior when compared to the proposed algorithm for all observation periods. Regarding the SS-based algorithms, we set the subarray size to $n = 26$, which accords with [10] and reaches a high probability of resolution. The performance of the algorithms with the SS technique is improved and the proposed algorithm still has better performance than the existing ones.
In the last experiment, we assess the performance of the proposed and analyzed algorithms with an uncorrect number of sources $q_w \neq q$ known by the receiver. This is more practical since the exact sources number has to be determined by procedures with extra computation cost and time. We keep the scenario as that in Fig. 2 but assume an uncorrect number of sources $q_w = 9$ instead of $q = 10$, and increase the number of snapshots for the operation. The fixed input SNR = 0dB. In Fig. 3 the MUSIC and its SS-based algorithms start to work with large number of snapshots, and the ESPRIT and its SS-based algorithms fail to resolve DOA estimation with the increase of the snapshots since $q$ is critical to the eigendecomposition for the partition of the signal subspace and the noise subspace in the input covariance matrix. Also, the design of the AV basis and CG residual vectors depends strongly on $q$. The Capon and its SS-based algorithms work well under this condition since they are insensitive to the number of sources. The same holds for the proposed and its SS-based algorithms, but both exhibit better performance and lower complexity. We consider $q_w > q$ condition and get the same result.

![Fig. 1. Probability of resolution versus number of snapshots (separation $3\Delta$, SNR = $-2$dB, $q = 2$, $c = 0.9$, $m = 30$, $r = 6$, $\delta = 5 \times 10^{-4}$, $\alpha = 0.998$, $n = 26$)](image1)

Next, we consider the sources to be uncorrelated but increase the number of sources by setting $q = 10$. The input SNR = $-5$dB and the number of sensor elements is set to $m = 50$. As can be seen in Fig. 2 the AV and CG methods are unable to obtain a DOA estimate with a large number of sources. The proposed algorithm demonstrates an improved performance and is the first to reach the highest resolution, as compared with the conventional Capon and the subspace-based MUSIC and ESPRIT methods, following the increase of number of snapshots. The subarray size is $n = 41$ in this scenario.

![Fig. 2. Probability of resolution versus number of snapshots (separation $3\Delta$, SNR = $-5$dB, $q = 10$, $m = 50$, $r = 6$, $\delta = 5 \times 10^{-4}$, $\alpha = 0.998$, $n = 41$)](image2)

![Fig. 3. Probability of resolution versus number of snapshots (separation $3\Delta$, SNR = 0dB, $q_w = 9$, $m = 50$, $r = 6$, $\delta = 5 \times 10^{-4}$, $\alpha = 0.998$, $n = 41$)](image3)

5. CONCLUDING REMARKS

We proposed a novel reduced-rank strategy to implement joint iterative subspace optimization and grid search for DOA estimation. The DOA estimation problem is formulated as a reduced-rank MV optimization problem. A subspace projection matrix is introduced to obtain the covariance matrix processed in the lower dimension so that computation cost is reduced and performance improved. An auxiliary reduced-rank parameter vector is combined to realize the joint iterative optimization with respect to the MV output power for each scanning direction. By searching the possible directions, the DOAs can be determined by finding the peaks in the output power spectrum. The proposed DOA estimation algorithm demonstrates advantages under large array condition with uncorrelated or correlated sources. Its performance is not significantly influenced by some parameters (e.g., the number of sources). In future work, we will pro-
vide the analysis including the Cramer-Rao Bound (CRB) and compare it with the estimation accuracy of the proposed algorithm. We will also consider unitary versions of the proposed reduced-rank algorithm for ULA that do not require grid search.

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