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ORIGINAL RESEARCH PAPER

Joint Power allocation and target detection in distributed MIMO radars

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Abstract
Here, a power allocation method jointly optimising the total transmit power and the probability of interception is proposed based on target detection applications in distributed multiple-input multiple-output (MIMO) radars. Power allocation is performed by solving a non-linear constrained optimisation problem formulated to minimise the total transmit power based on target detection applications. Then, the Neyman-Pearson detector is designed under the Rayleigh scatter model and the Lagrangian method is used to solve the optimisation problem. Moreover, a positioning algorithm optimising the placement of transmitters and receivers in distributed MIMO radars is applied to minimise the total transmit power satisfying the target detection criterion. The results show that uniform power allocation is not the optimal strategy and the proposed power allocation algorithm provides either better target detection performance for the same power budget, or requires less power to provide the same detection performance. Numerical simulations and the theoretic analysis confirm the effectiveness of the proposed algorithm.

1 | INTRODUCTION

In recent years, multiple-input multiple-output (MIMO) radar systems have attracted more attention because of their performance improvement compared with classical systems with single antenna. In MIMO radars, each transmit antenna is able to create an independent waveform [1]. It is shown that optimal performance of target detection in MIMO radars is achieved when orthogonal signals are transmitted. Hence, the MIMO radars employ multiple antennas for transmitting several orthogonal waveforms and multiple antennas for receiving signals reflected from the target [2]. Based on antenna configurations, MIMO radars can be classified into two principal types: (i) MIMO radars equipped with co-located antennas [3], and (ii) MIMO radars equipped with widely separated antennas [4].

The first type is known as the co-located MIMO radars, where the transmitter and receiver antennas are closely spaced to transmit a beam towards a certain direction on the space [3].

The second type is called as statistical MIMO radars or distributed MIMO radars, where its antennas are located sufficiently far from each other, such that a target can be viewed from different spatial directions simultaneously to achieve spatial diversity gain making diverse paths exhibit different amplitudes. In this type of MIMO radars, there is no correlation between the received signals in the receivers. Distributed MIMO radars are capable of significantly improving the target detection, localisation, parameters estimation and identification performances by exploiting the spatial diversity and multiplexing gain [4–7].

Some challenges need to be solved when multiple antennas are used in a radar. One of these problems is to choose the antennas’ positions properly [8, 9]. In [10], it has been shown that antennas’ positions can affect performance of the MIMO radar considerably.

Another problem is power allocation between transmit antennas. In previous studies, it has been shown that radar performance in detection, localisation and parameter estimation can be improved with an increase in either the number of transmit antennas or the amount of transmit power [11–16]. As the radar resources may practically be limited, optimising the resource allocation in multiple radar tasks to achieve desired
performance is necessary. In this case, the resource allocation problem in distributed MIMO radars becomes a hot topic, especially in systems with low probability of interception (LPI). Power allocation is a simple way to provide the LPI property, in which only the amount of power required to fulfil the desired performance is allocated to each radar. LPI radars usually use wideband-modulated signals, such as phase modulation, continuous-wave frequency modulation, frequency hopping, and antennas with low side-lobe level, high processing gain, and power management [17–20].

To suppress multiple source interference in radars, while achieving high detection performance or low detection error with minimum power consumption, an optimal resource allocation is required. Hence, resource allocation has become an important research field in MIMO radars that its goal is on achieving less error and better performance in different system tasks, different target characteristics and surroundings [21].

In [22–24], a game-theoretic power allocation scheme is presented in a multi-static MIMO radar network, that is organised into multiple clusters, where there is no communication between the distributed clusters. In these papers, the primary objective is to minimise transmission power, while satisfying a certain detection criterion.

In [25], power allocation in distributed MIMO radars is investigated to reduce the probability of interception. The orthogonal waveforms are commonly used in MIMO radars, and there are two orthogonal waveforms generally: frequency-diversity (FD) or phase-coded (PC) waveforms. In this study, the LPI optimisation problems are formulated based on the FD and PC radar waveforms, and a power allocation strategy is developed to support the LPI design in the distributed MIMO radar for two cases: FD and PC orthogonal waveforms. Both of the LPI optimisation problems with PC and FD orthogonal waveforms are formulated as a non-linear optimisation problem, and then a lower bound of the detection probability function is used to relax the problem as a linear optimisation problem.

In [26], the subarray selection integrated with a joint power and bandwidth allocation scheme is considered in the distributed MIMO radar, which makes joint multi-target tracking and new target detection. In this study, the bandwidth allocation is used to improve the target tracking accuracy and the detection efficiency becomes closer to the predicted posterior Cramer-Rao lower bound (PCRLB).

In [27], for better utilisation of limited system resources, a collaborative detection and power allocation (CDPA) scheme is developed for the application of target tracking in clutter, by using multiple radar system (MRS), with the aim of enhancing the target tracking accuracy. The CDPA scheme is based on optimisation techniques to control the false alarm rate (FAR) and transmit power of each radar, while achieving better target state estimation accuracy. This study proved that the CDPA scheme can expand the detection range, increase the resource utilisation efficiency of the MRS, and improve the target tracking accuracy. This scheme is formulated as a mathematical optimisation problem, and the Bayesian Cramer–Rao lower bound (BCRLB) is derived, relaxed, and subsequently utilised as its cost function.

In [28], a non-cooperative game-theoretic distributed power control scheme is proposed in a radar network system based on low probability of intercept (LPI) subject to the signal-to-interference-plus-noise ratio (SINR) constraint and the transmit power constraint of each radar. In this article, all the radars in the network share the same frequency band. The main goal in this article is to improve the LPI performance by reducing the transmit power caused by some radars’ SINRs over the specified threshold.

In [29], a joint resource allocation method is proposed to address the velocity estimation problem for multiple targets tracking in the distributed MIMO radar systems. This paper focuses on improving the tracking performance for key target by maximising the system resource utilisation of general targets. In [29], the optimisation problem is formulated as a three-step suboptimal method where each optimisation problem is transformed into a second-order cone programming (SOCP) form by convex relaxation, and finally, an approximate optimal solution is presented.

In [30], a joint resource allocation scheme is proposed by selecting an optimal subset of sensors with the predetermined size and implementing power allocation and bandwidth strategies among them for range-only target tracking in distributed MIMO radar systems. This algorithm is capable to achieve better performance with the same resource constraints. In [30], three main steps are presented to formulate the problem, which are listed as follows:

**Step 1** The Bayesian Cramer-Rao bound (BCRB) is derived.

**Step 2** A criterion for minimising the BCRB at the target location among all targets tracking in a certain range is derived.

**Step 3** The optimisation problem involved with three variable vectors is formulated.

The aforementioned optimisation problem is simplified and solved by the cyclic minimisation algorithm incorporated with the sequential parametric convex approximation (SPCA) algorithm.

In [31], a distributed MIMO dual-function radar-communication (DFRC) system is introduced. This configuration is composed of multiple distributed dual-function transmitters, multiple radar receivers and multiple communication receivers. Such a system is capable of performing target state estimation and information transferring tasks simultaneously. In this article, the authors developed a low probability of intercept (LPI)-based power resource allocation (PRA) scheme that is called LPI-PRA for the distributed MIMO-DFRC system with the mission of performing radar target state estimation and information transfer simultaneously. The main purpose in this case is to minimise the total power resource consumption of the overall system. In [31], the transmit power allocation is optimised subject to a predetermined target estimation CRLB threshold. This problem is
a constrained, non-linear, and non-convex optimisation problem, which is solved by a developed efficient three-stage solution technique.

In [32], a joint subarray selection and power allocation (JSSPA) strategy is developed to solve the joint subarray selection and power allocation problem for tracking multiple targets in clutter environments using large-scale distributed MIMO radar networks. The proposed optimisation model is a non-convex problem, and a two-stage local search-based algorithm is presented to solve it.

In [33], an LPI-based collaborative power and bandwidth allocation (CPBA) strategy is developed for multi-target tracking in distributed radar networks. In [33], the primary aim is on minimising the total power consumption of the distributed radar network by collaboratively optimising radar node selection, power and bandwidth allocation, resulting in an improved LPI performance for the distributed radar network system. The LPI-CPBA scheme is presented as a mathematical optimisation problem. This problem is the mixed-Boolean, non-linear and non-convex optimisation problem. Then, an efficient two-stage solution procedure is proposed to solve this optimisation problem.

In this paper, a power allocation method jointly optimising the total transmit power and the probability of interception is proposed based on target detection applications in distributed MIMO radars. Our main contributions are as follows:

(1) **Antenna placement**: To minimise the total transmit power and satisfying the target detection performance criterion simultaneously, a positioning algorithm optimising the placement of transmitters and receivers in distributed MIMO radars is used. Hence, an iterative method, introduced in [13], is used to optimise the placement of both TXs and RXs. To do this, first, the transmitter antennas placement is performed for a non-stationary target as the receiver antennas positions are fixed. Second, the receiver antennas positions are optimised for the same target.

(2) **Power allocation**: In this paper, two power allocation strategies have been developed to support power-aware design for target detection in distributed MIMO radars. The first power allocation algorithm minimises the total transmit power, and the second one minimises the maximum transmit power. Both optimisation problems are presented to satisfy the probability of detection constraint. The proposed power allocation methods can be applicable in LPI radars. Nowadays, in green communication systems, minimising total power consumption is of a crucial importance and the algorithms presented in this paper have some practical applications in green radar systems with low probability of interception.

The rest of the paper is organised as follows: The system model and detection formulation is defined in Section 2. In Section 3, the optimal power allocation strategies are introduced based on target detection performance and the algorithms are proposed to solve them. Simulation results are presented in Section 4. Finally, Section 5 concludes the paper’s results and achievements.

## 2 | SYSTEM MODEL AND DETECTION FORMULATION

### 2.1 | Signal model

Consider a distributed MIMO radar with M transmitters and N receivers, which are located in two-dimensional planes. The m-th transmitter is positioned at the coordinate \((x^t_m, y^t_m)\) for \(m = 1, \ldots, M\), the n-th receiver is positioned at the coordinate \((x^r_n, y^r_n)\) for \(n = 1, \ldots, N\) and the position of target is \((x^t, y^t)\). Figure 1 illustrates a distributed MIMO radar architecture with \(M = N = 2\) antennas.

A set of orthogonal waveforms are transmitted from the transmitter antennas and the corresponding baseband signals are denoted by \(s_m(t)\) for \(m = 1, \ldots, M\), where \(\int |s_m(t)|^2 dt = 1\) and \(\tau_m\) is the duration time for the m-th transmitted signal. Then, orthogonality of the transmitted waveforms can be expressed by:

\[
\int_{-\infty}^{\infty} s_m(t)s_{m'}^*(t-\tau) dt = \begin{cases} 
1 & \text{for } m = m', \tau = 0 \\
0 & \text{otherwise}
\end{cases}
\]

The transmit powers of different antennas are denoted by \(P_m\) for \(m = 1, \ldots, M\), therefore, the vector of transmit powers is characterised by \(P_m = [P_1 P_2 \ldots P_M]^T\). The upper and lower bound vectors of transmit powers are given by \(P_{m-max} = [P_{1-max} P_{2-max} \ldots P_{M-max}]^T\) and \(P_{m-min} = [P_{1-min} P_{2-min} \ldots P_{M-min}]^T\), respectively. The baseband signal transmitted from the m-th transmitter antenna and received by the n-th receiver antenna can be represented as [16]:

\[
r_{nm} = \sqrt{L_{nm}}P_m C_{nm} s_m(t-\tau_{nm})e^{j2\pi f_m t}e^{j2\pi f_r t} + \eta_n(t)
\]

as

\[
\sqrt{L_{nm}}P_m = \begin{bmatrix} \sqrt{L_{1,1} P_1} & \sqrt{L_{1,2} P_2} & \cdots & \sqrt{L_{1,M} P_M} \\
\vdots & \vdots & \ddots & \vdots \\
\sqrt{L_{N,1} P_1} & \sqrt{L_{N,2} P_2} & \cdots & \sqrt{L_{N,M} P_M} \end{bmatrix}_{N \times M}
\]

and

\[
C_{nm} = \begin{bmatrix} c_{1,1} & c_{1,2} & \cdots & c_{1,M} \\
\vdots & \vdots & \ddots & \vdots \\
c_{N,1} & c_{N,2} & \cdots & c_{N,M} \end{bmatrix}_{N \times M}^T
\]
where \( L_{nm} \) is the propagation loss that includes antenna gains, \( P_m \) is the transmit power, \( C_{nm} \) is a scattering amplitude that includes any unknown oscillator phase terms, \( s_m \) is the signal transmitted by the \( m \)-th transmitter antenna, \( \tau_{nm} \) is the bi-static time delay between \( m \)-th transmitter and \( n \)-th receiver antenna, \( f_{nm} \) is the Doppler shift, \( f_c \) is the carrier frequency and \( \eta_n(t) \) is noise and interference term.

The baseband-received signal at the \( n \)-th receiver can be expressed as [13, 16]:

\[
r_n(t) = \sum_{m=1}^{M} r_{nm}(t) = \sum_{m=1}^{M} D_{nm}.C_{nm}.S_n^T(t) + \eta_n(t)
\]  
\( (5) \)

where

\[
D_{nm} = \left[ \sqrt{L_{n,1}P_1}, \sqrt{L_{n,2}P_2}, \ldots, \sqrt{L_{n,M}P_M} \right]
\]  
\( (6) \)

\[
C_{nm} = [c_{n,1}, c_{n,2}, \ldots, c_{n,M}]^T
\]  
\( (7) \)

and

\[
S_n = \begin{bmatrix}
s_1(t - \tau_{n,1})e^{j2\pi(f_{c,1}t - f_c\tau_{n,1})} \\
\vdots \\
S_M(t - \tau_{n,M})e^{j2\pi(f_{c,M}t - f_c\tau_{n,M})}
\end{bmatrix}
\]  
\( (8) \)

The noise-free signal received by the \( n \)-th receiver is represented by \( R_{mn} \) and it can be defined as [13]:

\[
R_{mn} = \sum_{m=1}^{M} D_{nm}.C_{nm}.S_n^T(t)
\]  
\( (9) \)

Note that, according to the free space propagation loss equation, the amplitude of received signal is obtained by [8, 9, 13]:

\[
\alpha_{nm} = \sqrt{\frac{P_m.G_m.G_n.I_P.\sigma_{nm}^2.\lambda^2}{(4\pi)^2.r_{nm}^2.L_c.L_r}}
\]  
\( (10) \)

where, \( G_m \) and \( G_n \) are the transmitting and receiving antenna gains, respectively, \( I_P \) is the processing gain at the receiver, \( \sigma_{nm}^2 \) is the average RCS of the target, \( \lambda \) is the wavelength, \( L_c \) is the scattering loss, \( L_r \) is the receiver loss, \( r_{nm} \) and \( r_{n,m} \) are the distances between target-\( n \)-th transmitter and target-\( m \)-th receiver, respectively. In distributed MIMO radars, it can be assumed that the different \( \alpha_{nm} \)s are independent due to the existence of widely separated antennas.

Based on Equation (2) and (5)–(7), \( \alpha_{nm} \) can be defined as follows:

\[
\alpha_{nm} \triangleq D_{nm}.C_{nm}.d^{2\pi f_{c,n}t}.e^{j2\pi f_c\tau_{nm}}
\]  
\( (11) \)

Hence,

\[
R_{mn} \triangleq \sum_{m=1}^{M} \alpha_{nm}.s_m(t - \tau_{nm})
\]  
\( (12) \)

The vector of noise-free signals received is represented by \( R_0 \) and can be written as:

\[
R_0 \triangleq [R_{0,1}, R_{0,2}, \ldots, R_{0,N}]^T
\]  
\( (13) \)

The received signal vector that passed through the filter bank and sampler in the range gates is called \( r \) and it is a decision vector consisting of \( NM \) samples for each range gate can be expressed as:

\[
r \triangleq [r_1, r_2, \ldots, r_N]^T
\]  
\( (14) \)

Both the Swerling cases 1 and 2 are applied to a target that is made up of many independent scatterers of roughly equal areas such as aeroplanes [34]. In this paper, the Swerling 1 model is assumed for the RCS of the target and the Neyman-Pearson detector is considered under the Rayleigh scattering model (shown in Table 1). In this case, the target is assumed to consist of many independent scatterers with predominant ones that are free or none and the scattering coefficients are zero-mean complex circular Gaussian as well.

### 2.2 | Antenna placement

#### 2.2.1 | Transmitters’ positioning

In this section, for transmitters’ positioning, the proposed method in [13] is used to find the optimal positions of transmitter antennas. This positioning method is given as follows.
**Genetic algorithm (GA) as follows:** Hence, in [9], receivers’ positions are optimised by the receivers’ positions maximising the minimum detection probability. In this part, for receivers’ positioning, the proposed method in [9] is used to determine the optimal positions of receiver antennas with the detection probability criterion to improve detection performance. First, the transmitters’ positions are calculated in the previous section. It is necessary to find the optimal location of the first transmit antenna (which is the optimal place for this transmitter antenna. Hence, the optimal location of the first transmit antenna \(X_1\) is the position with the highest average power.

The aforementioned method is repeated to find the optimal position of the second transmitter antenna; however, it should be noted that the first transmitter’s power that is already positioned at \(X_1\) is used as a weighting coefficient before taking the average, at each \(Q\).

The next transmitters’ positions are found similar to the second one.

### 2.2.2 Receivers’ positioning

In this part, for receivers’ positioning, the proposed method in [9] is used to determine the optimal positions of receiver antennas with the detection probability criterion to improve detection performance. First, the transmitters’ positions are calculated in the previous section. It is necessary to find the receivers’ positions maximising the minimum detection probability. Hence, in [9], receivers’ positions are optimised by the Genetic algorithm (GA) as follows:

1. Initial value for the population of GA (initial value for receivers’ position) is set.
2. \(P_d\) for all possible target positions is calculated for each population.
3. The minimum value (worst value) of \(P_d\) is found among all possible target positions for each population.
4. All populations of GA will be updated.
5. If population fitness is lower than the defined value go back to step 2.
6. The population with maximum of the minimum values of \(P_d\) is selected.

### 2.3 Detector model

Since, the existence of the target in a bi-static range cell is a random process and also the priori probabilities are unknown, the Neyman-Pearson detector is a good choice [9]. Detection hypotheses are given by:

\[
\begin{align*}
H_0 : \quad & r_n = \eta_n, \quad n = 1, \ldots, N \\
H_1 : \quad & r_n = \eta_n + \sum_{m=1}^{M} \alpha_{nm} s_m(t - \tau_{nm}), \quad n = 1, \ldots, N
\end{align*}
\]

It is assumed that the system noise model is a Gaussian-distributed model, hence,

\[
\begin{align*}
\left\{ \begin{array}{l}
f_{R_x}(r_n|H_0) \sim N(0, \sigma^2 I) \\
f_{R_x}(r_n|H_1, \alpha_{nm}) \sim N(\mathbf{R}_{\alpha n}, \sigma^2 I) \\
\end{array} \right.
\]

where \(\sigma^2\) is the noise variance. Then, the likelihood ratio test for the Neyman-Pearson detector can be written as:

\[
L(\tau) = \frac{f_{R_x}(\tau|H_1)}{f_{R_x}(\tau|H_0)} \leq \frac{H_1}{T}
\]

Then, this likelihood ratio is compared with threshold \(T\) to perform target detection. That is, if \(L(\tau)\) is greater than the threshold, the hypothesis \(H_1\) is chosen; otherwise, the hypothesis \(H_0\) is selected. This threshold is determined according to the desired false alarm probability; therefore, for the null hypothesis, it can be written as

\[
f_{R_x}(\tau|H_0) = \frac{1}{(2\pi \sigma^2)^{N_2}} \prod_{n=1}^{N} e^{-\frac{|\mathbf{R}_{\alpha n}|^2}{2\sigma^2}}
\]

\[
= \frac{1}{(2\pi \sigma^2)^{N_2}} e^{-\sum_{n=1}^{N} \frac{|\mathbf{R}_{\alpha n}|^2}{2\sigma^2}}
\]

### TABLE 1 Scattering model [34]

| RCS Model | Slow Fluctuation 'Scan-To-scan' |
|-----------|-------------------------------|
| Exponential | \(f(x) = \frac{1}{\pi \sigma_x^2}\) | Swerling 1 |
| Rayleigh | \(f(x) = \frac{1}{\pi \sigma_x^2} e^{-\frac{x^2}{\sigma_x^2}}\) | Swerling 1 |

Abbreviation: RCS, radar cross section.
and for $H_1$ hypothesis,

$$ f_R(r|H_1) = \int_0^\infty \ldots \int_0^\infty \left\{ \sum_{n=1}^N \prod_{m=1}^M f_{\alpha_{nm}}(\alpha_{nm}) \right\} \, d\alpha_{nm} $$

(20)

On the other hand, according to (17),

$$ f_R(r|H_1, \alpha_{nm}) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp \left\{ -\frac{1}{2\sigma^2} \sum_{n=1}^N \| r_n \|^2 \right\} e^{-\frac{1}{2\sigma^2} \sum_{n=1}^N \frac{[r_n - \alpha_{nm}]^2}{\sigma^2}} $$

(21)

and

$$ f_R(r|H_1, \alpha_{nm}) = \frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2} \sum_{n=1}^N \| r_n \|^2} $$

(22)

where

$$ \sum_{n=1}^N \| r_n \|^2 = M \sum_{m=1}^M \sum_{n=1}^N \| r_n \|^2 $$

(23)

According to (1) and (12),

$$ \sum_{n=1}^N \| \mathbf{R}_0 \|^2 = \sum_{n=1}^N \| \mathbf{R}_0 \|^T \mathbf{R}_0 = \sum_{n=1}^N \sum_{m=1}^M \sum_{q=1}^Q \alpha_{nm} \cdot \mathbf{s}_m(t - \tau_{nm}) \cdot \mathbf{s}_q^T(t - \tau_{nq}) $$

(24)

and substituting (26)

$$ \sum_{n=1}^N \mathbf{r}_n^T \mathbf{R}_0 \mathbf{r}_n = \sum_{n=1}^N \sum_{m=1}^M \mathbf{r}_n^T \mathbf{\alpha}_{nm} \cdot \mathbf{s}_m(t - \tau_{nm}) $$

(25)

Substituting (23)-(25) in (22),

$$ f_R(r|H_1, \alpha_{nm}) = \frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2} \sum_{n=1}^N \frac{[r_n - \alpha_{nm}]^2}{\sigma^2}} $$

(26)

and substituting (26) in (20), $f_R(r|H_1)$ is obtained in (27).

In Appendix A, the likelihood ratio test for the Neyman-Pearson detector is calculated as follows:

$$ f_R(r|H_1) = \int_0^\infty \ldots \int_0^\infty \left\{ \frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2} \sum_{n=1}^N \frac{[r_n - \alpha_{nm}]^2}{\sigma^2}} \prod_{m=1}^M \prod_{n=1}^N f_{\alpha_{nm}}(\alpha_{nm}) \right\} \, d\alpha_{nm} $$

(27)
\[
L(\mathbf{r}) \approx \prod_{m=1}^{M} \prod_{n=1}^{N} \left[ \sqrt{2\pi \sigma} A_2 e^{\frac{\lambda^2}{2\sigma^2}} \right] \leq H_1 \Rightarrow T
\]

For simplicity, the logarithm of \( L(\mathbf{r}) \) is used for detection. In Appendix B, the log-likelihood ratio equation is obtained as follows:

\[
\ln\{L(\mathbf{r})\} = \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln \left[ \sqrt{\frac{2\pi}{\sigma}} \frac{\nu_{nm}}{\sigma^2} \sqrt{2P_m e^{\frac{\nu_{nm}^2 P_m}{\sigma^2}}} \right] - \ln \left[ 1 + \frac{\nu_{nm}^2 P_m}{\sigma^2} \right] \right\} \geq T_2
\]

\section{POWER ALLOCATION STRATEGIES}

As mentioned before, the antennas’ positions have a significant effect on the whole system’s performances including target detection performance. Hence, the method proposed in [13] is used to find the optimal positions of the transmitter antennas to achieve highest average power received by the target, and the method proposed in [9] is also used to calculate the optimal positions of the receiver antennas to maximise the minimum value of \( P_d \).

Next, the sum of the transmitters’ power is minimised by allocating resources among the transmitter antennas subject to acceptable target detection performance. This optimisation problem is written as:

\[
\begin{aligned}
\text{minimize} & \quad \sum_{m=1}^{M} P_m \\
\text{subject to} & \quad \ln\{L(\mathbf{r})\} > T_2 \\
& \quad P_m \leq P_{m-\text{max}}, \quad m = 1, \ldots, M \\
& \quad P_m \geq P_{m-\text{min}}, \quad m = 1, \ldots, M
\end{aligned}
\]

Substituting Equation (29) in Equation (30), the optimisation problem in Equation (30) is rewritten as:

\[
\begin{aligned}
\text{minimize} & \quad \sum_{m=1}^{M} P_m \\
\text{subject to} & \quad \ln\{L(\mathbf{r})\} > T_2 \\
& \quad P_m \leq P_{m-\text{max}}, \quad m = 1, \ldots, M \\
& \quad P_m \geq P_{m-\text{min}}, \quad m = 1, \ldots, M
\end{aligned}
\]

The designed constrained optimisation problem in Equation (31) is non-convex and non-linear, due to the first inequality constraint. To solve this problem, the Lagrange multipliers, Karush-Kuhn-Tucker (KKT) conditions, and decomposition techniques [35, 36] are used. First, this optimisation problem is rewritten in the following standard form.

\[
\begin{aligned}
\text{minimize} & \quad \sum_{m=1}^{M} P_m \\
\text{subject to} & \quad \ln\{L(\mathbf{r})\} > T_2 \\
& \quad P_m \leq P_{m-\text{max}}, \quad m = 1, \ldots, M \\
& \quad P_m \geq P_{m-\text{min}}, \quad m = 1, \ldots, M
\end{aligned}
\]

To solve the power allocation problem given in Equation (31), the Lagrangian function is given by:

\[
L = \lambda_1 \left\{ T_2 - \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln \left[ \sqrt{\frac{2\pi}{\sigma}} \frac{\nu_{nm}}{\sigma^2} \sqrt{2P_m e^{\frac{\nu_{nm}^2 P_m}{\sigma^2}}} \right] - \ln \left[ 1 + \frac{\nu_{nm}^2 P_m}{\sigma^2} \right] \right\} \right\} + \lambda_2 \{ P_m - P_{m-\text{max}} \} + \lambda_3 \{ P_{m-\text{min}} - P_m \} + \sum_{m=1}^{M} P_m
\]

Consequently, to obtain the KKT conditions:

\[
\frac{\partial L}{\partial P_m} = -\lambda_1 \sum_{n=1}^{N} \left( \frac{1}{2P_m} + \frac{\nu_{nm}^2}{2\sigma^2} \right) - \lambda_2 = 0
\]

Since \( \sigma^2 \) is too small and, \( \frac{\nu_{nm}^2}{\sigma^2} \approx \frac{1}{P_m} \)

\[
\frac{\partial L}{\partial P_m} \approx -\lambda_1 \sum_{n=1}^{N} \left( \frac{1}{2P_m} + \frac{\nu_{nm}^2}{2\sigma^2} \right) - \lambda_2 = \lambda_2 - \lambda_3 + 1 = 0
\]

Then,
The second and third inequality constraints in Equation (32) are chosen to be inactive, a solution to Equation (32) is obtained; therefore, \( \lambda_2 = \lambda_3 = 0, \quad \forall m = 1, \ldots, M \). Reducing the set of KKT conditions in Equation (40),

\[
\begin{aligned}
\lambda_1 \left( T_2 - \left[ \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln \sqrt{\pi \frac{\nu_{nm}}{\sigma}} \sqrt{2P_m \cdot e^{\frac{\nu_{nm}^2 \nu_{nm}^2}{2\sigma^2}}} \right\} \right] \right) = 0
\end{aligned}
\]

and Equation (42)

\[
\begin{aligned}
P_m = \frac{-N\lambda_1}{2 + \sum_{m=1}^{M} \frac{\nu_{nm}^2}{\sigma^2}}
\end{aligned}
\]

On the other hand, according to the complementary slackness conditions, the optimal solution must satisfy the states of Equation (44) as:

\[
\begin{aligned}
\lambda_1 & \left( T_2 - \left[ \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln \sqrt{\pi \frac{\nu_{nm}}{\sigma}} \sqrt{2P_m \cdot e^{\frac{\nu_{nm}^2 \nu_{nm}^2}{2\sigma^2}}} \right\} \right] \right) = 0

\begin{cases}
\text{if } \lambda_1 = 0 \Rightarrow T_2 < \left[ \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln \sqrt{\pi \frac{\nu_{nm}}{\sigma}} \sqrt{2P_m \cdot e^{\frac{\nu_{nm}^2 \nu_{nm}^2}{2\sigma^2}}} \right\} \right] - \ln \left[ 1 + \frac{\nu_{nm}^2 P_m}{\sigma^2} \right] \\
\text{if } \lambda_1 > 0 \Rightarrow T_2 = \left[ \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln \sqrt{\pi \frac{\nu_{nm}}{\sigma}} \sqrt{2P_m \cdot e^{\frac{\nu_{nm}^2 \nu_{nm}^2}{2\sigma^2}}} \right\} \right] - \ln \left[ 1 + \frac{\nu_{nm}^2 P_m}{\sigma^2} \right]
\end{cases}
\end{aligned}
\]
\[
\lambda_2 \{ P_m - P_{m-max} \} = 0, \quad m = 1, \ldots, M
\]

if \( \lambda_2 = 0 \) \( \Rightarrow P_m - P_{m-max} < 0 \) \( \Rightarrow P_m < P_{m-max} \)

if \( \lambda_2 > 0 \) \( \Rightarrow P_m - P_{m-max} = 0 \) \( \Rightarrow P_m = P_{m-max} \)

(44-b)

\[
\lambda_3 \{ P_{m-min} - P_m \} = 0, \quad m = 1, \ldots, M
\]

if \( \lambda_3 = 0 \) \( \Rightarrow P_{m-min} - P_m < 0 \) \( \Rightarrow P_{m-min} < P_m \)

if \( \lambda_3 > 0 \) \( \Rightarrow P_{m-min} - P_m = 0 \) \( \Rightarrow P_{m-min} = P_m \)

(44-c)

Completing transmitter antennas power allocation, the maximum transmit power of antennas is minimised. This strategy keeps a balance between power of different antennas in the radar system. Since the interception probability is a function of the transmit power, hence, the LPI optimisation is equivalent to minimise the maximum transmit power. As a result, the LPI optimisation problem is defined as:

\[
\min \max \limits_{P_m} P_m \quad \text{s.t.} \quad \begin{cases} 
\ln(L(r)) > T_2 \\
P_m \leq P_{m-max}, \quad m = 1, \ldots, M \\
P_m \geq P_{m-min}, \quad m = 1, \ldots, M
\end{cases} \quad (45)
\]

where \( P_m \) is the m-th transmitter power, \( P_{m-max} \) and \( P_{m-min} \) are the maximum and minimum powers of each transmit antenna, respectively. Hence, the above min–max optimisation problem can be converted to a minimisation problem with additional constraints. This problem can be rewritten as follows [36]:

\[
\min q \quad \text{s.t.} \quad \begin{cases} 
\ln(L(r)) > T_2 \\
P_m \leq P_{m-max}, \quad m = 1, \ldots, M \\
P_m \geq P_{m-min}, \quad m = 1, \ldots, M \\
P_m \leq q, \quad m = 1, \ldots, M
\end{cases} \quad (46)
\]

First, this problem is written in the following standard form as:

\[
\min_{P_m, q} \quad \text{s.t.} \quad \begin{cases} 
\ln(L(r)) > T_2 \\
P_m \leq P_{m-max}, \quad m = 1, \ldots, M \\
P_m \geq P_{m-min}, \quad m = 1, \ldots, M \\
P_m \leq q, \quad m = 1, \ldots, M
\end{cases} \quad (47)
\]

Then, the Lagrangian method is applied to solve this problem:

\[
L = q + \lambda_1 \left\{ T_2 - \left[ \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln\left( \sqrt{\pi} \frac{V_{nm}}{\sigma} \sqrt{2P_m} \right) - \ln\left( 1 + \frac{V_{nm}^2 P_m}{\sigma^2} \right) \right\} \right] \right\} + \lambda_2 \{ P_m - P_{m-max} \} \\
+ \lambda_3 \{ P_{m-min} - P_m \} + \lambda_4 \{ P_m - q \}
\]

(48)

Consequently, to obtain the KKT conditions:

\[
\frac{\partial L}{\partial q} = 1 - \lambda_4 = 0, \quad m = 1, \ldots, M
\]

\[
\frac{\partial L}{\partial \lambda_1} = T_2 - \left[ \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln\left( \sqrt{\pi} \frac{V_{nm}}{\sigma} \sqrt{2P_m} \right) - \ln\left( 1 + \frac{V_{nm}^2 P_m}{\sigma^2} \right) \right\} \right] = 0
\]

(49)

According to the complementary slackness conditions, the optimal solution must satisfy the states of Equation (50).

\[
\lambda_1 \left\{ T_2 - \left[ \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln\left( \sqrt{\pi} \frac{V_{nm}}{\sigma} \sqrt{2P_m} \right) - \ln\left( 1 + \frac{V_{nm}^2 P_m}{\sigma^2} \right) \right\} \right] \right\} = 0
\]

if \( \lambda_1 = 0 \) \( \Rightarrow T_2 < \left[ \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln\left( \sqrt{\pi} \frac{V_{nm}}{\sigma} \sqrt{2P_m} \right) - \ln\left( 1 + \frac{V_{nm}^2 P_m}{\sigma^2} \right) \right\} \right] \)

(50-a)

if \( \lambda_1 > 0 \) \( \Rightarrow T_2 = \left[ \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln\left( \sqrt{\pi} \frac{V_{nm}}{\sigma} \sqrt{2P_m} \right) - \ln\left( 1 + \frac{V_{nm}^2 P_m}{\sigma^2} \right) \right\} \right] \)
\begin{align*}
\lambda_2 \{ P_m - P_{m\text{-max}} \} = 0, \ m = 1, \ldots, M \\
\text{if } \lambda_2 = 0 \Rightarrow P_m - P_{m\text{-max}} < 0 \Rightarrow P_m < P_{m\text{-max}} \\
\text{if } \lambda_2 > 0 \Rightarrow P_m - P_{m\text{-max}} = 0 \Rightarrow P_m = P_{m\text{-max}} \\
\lambda_3 \{ P_{m\text{-min}} - P_m \} = 0, \ m = 1, \ldots, M \\
\text{if } \lambda_3 = 0 \Rightarrow P_{m\text{-min}} - P_m < 0 \Rightarrow P_{m\text{-min}} < P_m \\
\text{if } \lambda_3 > 0 \Rightarrow P_{m\text{-min}} - P_m = 0 \Rightarrow P_{m\text{-min}} = P_m \\
\lambda_4 \{ P_{m\text{-min}} - P_m \} = 0, \ m = 1, \ldots, M \\
\text{if } \lambda_4 = 0 \Rightarrow P_{m\text{-min}} - P_m < 0 \Rightarrow P_{m\text{-min}} < P_m \\
\text{if } \lambda_4 > 0 \Rightarrow P_{m\text{-min}} - P_m = 0 \Rightarrow P_{m\text{-min}} = P_m \\
\end{align*}

4 | SIMULATION RESULTS

The power allocation method proposed in Section 3 decreases total power consumption. In this section, numerical analysis is presented for the proposed antenna positioning and power allocation algorithms.

In this paper, a distributed MIMO radar system located in a 2-dimensional plane is considered with \( M = 3 \) transmitters and \( N = 4 \) receivers. Hence, a square-shaped region with sides equal to 30 km is considered for radar antennas placement. The target fluctuations are considered according to the Swerling 1 model and the radar parameters are given in Table 2. The first goal is the optimal placement of three transmitters and four receivers in a widely separated MIMO configuration based on the method presented in Section 2. Following the antennas positions optimisation, the results shown in Tables 3 and 4 are obtained for transmitters and receivers antennas.

**Table 2** System parameters

| Parameter | Description                      | Value   |
|-----------|----------------------------------|---------|
| \( G_m \) | Transmitting Antenna Gain        | 30 dB   |
| \( G_r \) | Receiving antenna gain           | 30 dB   |
| \( I_p \) | Processing gain at receiver      | 20 dB   |
| \( BW \) | Radar bandwidth                   | 5 MHz   |
| \( f_c \) | Carrier frequency                 | 10 GHz  |
| \( \lambda \) | Wavelength                        | 3 cm    |
| \( \sigma^2 \) | Average RCS of the object        | 2 m²    |
| \( L_s = L_c \) | Radar system losses              | 0 dB    |
| \( M \) | Number of transmitter antennas   | 3       |
| \( N \) | Number of receiver antennas      | 4       |
| \( P_{m\text{-max}} \) | Maximum transmit power            | 20 kW   |
| \( P_{m\text{-min}} \) | Minimum transmit power            | 0 kW    |

**Abbreviation**: RCS, radar cross section.

Figure 2 illustrates the resulting antennas geometry with the given assumptions. Then, the power allocation problem must be solved.

4.1 | Power allocation-case 1

In previous studies, it has been shown that the uniform or equal power allocation is not necessarily the optimal choice. In this paper, the transmit powers are calculated by the proposed power allocation scheme in Section 3. In this case, a constrained non-linear optimisation problem is defined to minimise the total transmit power by allocating resources among the transmitter antennas subject to acceptable target detection performance. Then, this problem is solved using Lagrange multipliers, the Karush-Kuhn-Tucker (KKT) conditions, and decomposition techniques. The resulting powers for three transmitters are presented in Table 5. It can be verified that the power assigned to transmitters is lower than the uniform power allocation, and the proposed power allocation algorithm provides either better target detection performance for the same power budget, or requires less power to establish the same detection performance.

**Table 3** Transmit antennas positions

| Number of Transmitter | x(km) | y(km) |
|-----------------------|-------|-------|
| #1                    | 5     | 0     |
| #2                    | -15   | 5     |
| #3                    | 5     | 10    |

**Table 4** Receive antennas positions

| Number of Receiver | x(km) | y(km) |
|--------------------|-------|-------|
| #1                 | 0     | 0     |
| #2                 | -10   | -8    |
| #3                 | 10    | -5    |
| #4                 | 3     | 8     |

![Antennas geometry](image-url)
The comparison of the uniform and proposed power allocation algorithms is shown in Figure 3.

4.2 Power allocation-case 2

As mentioned before in Section 3, in distributed MIMO radars, LPI optimisation means minimising the total antennas transmit power. Hence, the total allocated power is used as the LPI metric and a min-max optimisation problem is designed to minimise the maximum transmit power. Therefore, the min–max optimisation problem is converted to a minimisation problem with additional constraints, and then the simulation results are presented in Table 6. The results listed in Tables 5 and 6 confirm that the proposed min-max optimisation problem can greatly reduce the total power.

The comparison of the powers assigned to the transmitters is shown in Figure 4 for two power allocation algorithms (The proposed power allocation algorithm and the optimal power allocation algorithm).

The comparison of the probability of detection is shown as receiver-operating-characteristic (ROC) curves in Figure 5 for three power allocation algorithms (Uniform power allocation, proposed power allocation algorithm in case 1 and optimal power allocation algorithm in case 2).

In this case, the probability of the false alarm is on the interval $10^{-5}$ to 1. The simulation results show superiority of the proposed methods in comparison with uniform power allocation. Figure 5 indicates the probability of detection in both proposed power allocation strategies are almost equal, but power allocation in case 2 decreases the total transmit power more than that in case 1.

The powers allocated to the transmitters in a 120-s time interval for two proposed methods (Proposed power allocation in case 1 (PPA) and Optimal power allocation in case 2 (OPA)) are presented in Figure 6a and Figure 6b.

Figure 6c shows the total transmit power achieved by three methods (Uniform power allocation (UPA), PPA and OPA) in a 120 s time interval. The colours represent the amount of total transmit power allocated for distributed MIMO radar to detect a certain target. Specifically, the blue colour corresponds to the minimum of the power allocated to this radar.

As it can be seen in Figure 6, in comparison with uniform power allocation strategy, significant power savings can be obtained by the both the proposed power allocation methods; as a result, LPI performance is also improved significantly in the both proposed power allocation strategies.

**Table 5** Transmit Antennas power (in case1)

| Number of Transmitter | Uniform power allocation(kW) | Proposed power allocation(kW) |
|-----------------------|-----------------------------|-------------------------------|
| #1                   | 20                          | 5.7                           |
| #2                   | 20                          | 14.6                          |
| #3                   | 20                          | 11.2                          |

**Table 6** Transmit Antennas power (in case2)

| Number of Transmitter | Proposed power allocation(kW) | Optimal power allocation(kW) |
|-----------------------|-------------------------------|-----------------------------|
| #1                   | 5.7                           | 5.4                          |
| #2                   | 14.6                          | 11.5                         |
| #3                   | 11.2                          | 10.8                         |

**Figure 3** Powers assigned to the transmitters (in case 1)

**Figure 4** Powers assigned to the transmitters (in case 2)

**Figure 5** Comparison of receiver-operating-characteristic (ROC) curves for three power allocation methods
decomposition methods. The results have shown that uniform power allocation is not the optimal strategy, and accuracy improvements or significant power savings can be obtained through the proposed scheme. Simulation results further confirm the convergence and stability of the proposed method.

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**FIGURE 6** Power allocation results over a 120 s time interval. (a) proposed power allocation, (b) optimal power allocation, (c) total transmit power comparison for three methods

5 | CONCLUSION

Both target detection performance and the total transmit power depend on placement of TXs and RXs in distributed MIMO radars. In this paper, a reasonable approach is used to place transmitter antennas based on the maximum average power received by a moving target in the vision area of the radar, and another reasonable approach is also used to place receiver antennas based on maximising the minimum value of $P_d$.

In this paper, two power allocation strategies are developed to support resource-aware design for target detection in distributed MIMO radar systems. One of these strategies minimises the total transmit power, and the other one minimises the maximum transmit power. Both optimisation problems have been proposed to satisfy the probability of detection constraint.

The resulting power allocation non-linear constrained optimisation problem has been obtained through using the Lagrange multipliers and the KKT conditions and domain
By using (27), $f_{R}(r|H_1)$ is calculated as follows:

$$f_{R}(r|H_1) = \frac{1}{(2\pi\sigma^2)^{Nd^2}} \prod_{m=1}^{M} \prod_{n=1}^{N} \left( \int_{\Omega} - \frac{1}{M} \| r_n \|^2 - 2r_n^T \alpha_{nm} s_m (t - r_{nm}) + \alpha_{nm}^2 \right)^{-\frac{1}{2}} 2\sigma^2 \cdot f_{\alpha_{nm}}(\alpha_{nm}) d\alpha_{nm}$$

(51)
By using $Q(\mu) = \int_{\mu}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$ and substituting $U = \frac{\xi}{\sqrt{2}}$ and $d\xi = \sqrt{2}dU$ in $\int_{\sqrt{2}U}^{\infty} e^{-U^2} dU$:

$$\int_{\sqrt{2}U}^{\infty} e^{-U^2} dU = \sqrt{2}\pi \int_{\frac{\xi}{\sqrt{2}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2} d\xi = \sqrt{2}\pi \int_{\frac{\xi}{\sqrt{2}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2} dx = \sqrt{2}\pi. Q\left(\frac{-A_2}{\sqrt{2A_1}}\right)$$

(56)

Hence,

$$I_{nm} = \frac{A_2^2}{2A_1^2} \int_{x=A_1}^{\infty} e^{-U^2} dU + \frac{A_2}{A_1} \int_{x=A_1}^{\infty} e^{-U^2} dU$$

$$= \frac{A_2^2}{2A_1^2} \int_{x=A_1}^{\infty} e^{-U^2} dU + \frac{A_2}{A_1} \int_{x=A_1}^{\infty} e^{-U^2} dU$$

(57)

Finally, $f_B(r|H_0)$ and $f_B(r|H_1)$ are calculated as (58).

$$f_B(r|H_0) = \frac{1}{(2\pi\sigma^2)^{\frac{N+2}{2}}} \prod_{m=1}^{M} N_{m=1}^{N} \prod_{n=1}^{N} f_B(r|H_1)$$

$$= \frac{1}{(2\pi\sigma^2)^{\frac{N+2}{2}}} \prod_{m=1}^{M} N_{m=1}^{N} \prod_{n=1}^{N} \frac{1}{2A_1^2} \int_{x=A_1}^{\infty} e^{-U^2} dU$$

(58-a)

$$= \frac{A_2}{A_1} \int_{x=A_1}^{\infty} e^{-U^2} dU$$

(58-b)

According to (18) and (58):

$$I(r) = \frac{f_B(r|H_1)}{f_B(r|H_0)} = \prod_{m=1}^{M} N_{m=1}^{N} \frac{1}{1 + \frac{\sigma_m^2}{\sigma}}$$

(59)

$$+ \frac{A_2}{A_1} \int_{x=A_1}^{\infty} e^{-U^2} dU$$

Assuming $\sigma_m \gg 1$ in $\frac{1}{2\sigma} + \frac{1}{\sigma_m} = A_1^2$, it can be written

$$\frac{1}{\sigma} \approx A_1^2.$$ Using $Q(-x) \approx 1 - \frac{1}{\sqrt{2\pi x}} e^{-\frac{x^2}{2}}$, $x \gg 1$ in (59):
\[ L(r) \approx \prod_{m=1}^{M} \prod_{n=1}^{N} \left( \frac{1}{1 + \sigma_{nm}^2 / \sigma^2} \right) \left[ 1 + \sqrt{2 \pi} \sigma A_2 e^{\frac{x_n^2}{2 \sigma^2}} \right] \left[ 1 + \frac{1}{\sqrt{2 \pi} \sigma A_2} e^{\frac{x_n^2}{4 \sigma^2}} \right] \ln \left[ \frac{1}{1 + \sigma_{nm}^2 / \sigma^2} \right] \approx T_1 \]

\[ \text{APPENDIX B} \]

The log-likelihood ratio can be written by:

\[ \ln \{L(r)\} = \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln \left[ \sqrt{2 \pi} \sigma A_2 e^{\frac{x_n^2}{2 \sigma^2}} \right] - \ln \left[ 1 + \sigma_{nm}^2 / \sigma^2 \right] \right\} \geq T_2 \]

(61)

Where \( T_2 \) is the logarithm of \( T_1 \). By substituting \( \frac{1}{\sqrt{2 \pi}} \approx A_1 \) in (61):

\[ \ln \{L(r)\} = \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln \left[ \sqrt{2 \pi} \sigma A_2 e^{\frac{x_n^2}{2 \sigma^2}} \right] - \ln \left[ 1 + \sigma_{nm}^2 / \sigma^2 \right] \right\} \geq T_2 \]

(62)

Using \( x_{nm} = \frac{4 \gamma}{\nu} \) in (62):

\[ \ln \{L(r)\} = \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln \left[ \sqrt{2 \pi} x_{nm} e^{\frac{x_n^2}{2 \sigma^2}} \right] - \ln \left[ 1 + \sigma_{nm}^2 / \sigma^2 \right] \right\} \geq T_2 \]

(63)

Also, \( x_{nm} \) can be written as:

\[ x_{nm} = \sqrt{2} \sigma \frac{v_n^T s_m(t - \tau_{nm})}{\sigma^2} \]

\[ = \sqrt{2} \sigma \frac{v_n^T s_m(t - \tau_{nm})}{\sigma} \]

\[ = \sqrt{2} \sigma \frac{\sum_{m=1}^{M} D_{nm} C_{nm} s_n^T(t) + \nu_n(t)}{\sigma} . s_m(t - \tau_{nm}) \]

\[ = \frac{2 \sigma D_{nm} C_{nm}}{\sigma} = \sqrt{2} \frac{\sqrt{1/m} . P_m . C_{nm}}{\sigma} \]

(64)

By substituting \( v_{nm} = \sqrt{1/m} C_{nm} \), \( x_{nm} = \frac{v_{nm} \sqrt{2} P_m}{\sigma} \) and \( \sigma_{nm}^2 = \nu_{nm}^2 \sigma^2 P_m \) in (63), the log-likelihood ratio is rewritten as follows:

\[ \ln \{L(r)\} = \sum_{m=1}^{M} \sum_{n=1}^{N} \left\{ \ln \left[ \sqrt{2} \sigma \frac{v_{nm}^T s_m(t - \tau_{nm})}{\sigma^2} \right] - \ln \left[ 1 + \frac{\nu_{nm}^2 \sigma^2 P_m}{\sigma^2} \right] \right\} \geq T_2 \]

(65)