Particle Size-Segregation and Spontaneous Levee Formation in Geophysical Granular Flows

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Particle size-segregation can have an important feedback on the bulk flow of geophysical granular avalanches. As a polydisperse material travels downhill the larger particles rise to the surface, where they are preferentially sheared to the flow front. This coarse-rich region experiences a greater resistance to motion and the large particles are shouldered aside to form lateral levees. Wider flows may break down into a series of these lobate, ‘finger-like’ structures. In either case, the static leveed regions channelise the finer, more mobile interior, causing the resulting run-out distances to be significantly enhanced. Modelling segregation-mobility feedback effects is therefore crucial for hazard mitigation. A new class of depth-averaged continuum models is introduced that describes the transport of large particles as well as the granular rheology. The feedback arises from a basal friction law that is composition dependent, implying greater friction where there are more large particles. Numerical simulations are used to show the spontaneous formation of leveed fingers.

Key words: granular media, size-segregation, leveed channels, depth-averaged

1. INTRODUCTION

Geophysical granular avalanches, such as pyroclastic flows, rock avalanches and debris flows, consist of dense mixtures of different sized particles that have a tendency to segregate during movement. Smaller particles are more likely to fall through gaps that open up as the material is sheared, which in turn exerts an upward force on the large particles, processes known as kinetic sieving and squeeze expulsion [Middleton, 1970; Savage and Lun, 1988]. This size segregation can lead to feedback effects that significantly alter the overall flow characteristics [Iverson and Vallance, 2001; Branney and Kokelaar, 2002; Iverson, 2003]. One such effect is the spontaneous formation of lateral levees [Calder et al., 2000; Conway et al., 2010]. Large particles are segregated to the flow surface and sheared to the flow front. Here they experience greater frictional forces and are pushed aside into the flow margins to form coarse-grained static regions, channelizing the interior of the flow.

Laboratory experiments [Kokelaar et al., 2014] show the formation of leveed channels for a bidisperse mixture. The flow self-organises into coarse-grained levees and a large-rich front. The levees are lined with a layer of finer material, which further reduces the friction and means that the run-out distances are greater than that of either type of particle in pure phase.

The same effect has also been investigated on much larger scales resembling geophysical mass flows. Figure 1 shows experiments conducted at the USGS debris flow flume [Johnson et al., 2012]. A mass of polydisperse, wet granular material is released from rest and flows down a 82 m long chute before being gradually brought to rest on a runout pad. Overhead images show the shearing of large particles to the front of the flow and subsequent deposition into the static lateral levees.

In these examples a confined, narrow inflow leads to the formation of a single leveed channel. For wider, unconfined flows particle size-segregation can cause a slightly different effect. As a plane of material moves downslope a large frictional front develops as before. This may become unstable and split into a number of different channels, or ‘fingers’, with the internal structure of each finger resembling that of a single leveed channel. This was first noticed in small-scale experiments [Pouliquen et al., 1997; Pouliquen
Field observations have also confirmed the presence of irregular large particles was required to drive the instability.

Field observations have also confirmed the presence of leveed channels in geophysical mass flows [Pierson, 1986]. Figure 2 shows the deposits left from the pyroclastic flow that resulted from the Mount St Helens eruption on July 22nd, 1980. The flow has broken down into a series of lobate structures, which have a width of several meters across. When these were excavated the front and lateral boundaries of each channel were found to contain high proportions of large pumice clasts, providing strong evidence for particle size-segregation. These features are associated with increased run-out distances and speeds of the hazardous pyroclastic flow and are therefore crucial to understand.

2. SMALL-SCALE EXPERIMENTS OF FINGERING INSTABILITIES

Laboratory experiments are carried out to show the development of fingering instabilities as a mass of granular material flows down an inclined plane. A chute is made rough by attaching a single layer of turquoise glass ballotini (750-1000 μm) to the base and then inclined at an angle of 28°. The hopper is filled with an initially homogeneous bidisperse mixture, consisting of 80% spherical white glass ballotini (75-150 μm) and 20% angular brown carborundum (315-355 μm). Material is released from rest through a 3 mm gate and allowed to propagate onto the empty plane.

As the grains begin to move, kinetic sieving and squeeze expulsion cause the large particles to migrate towards the free surface, where they are preferentially sheared towards the front. They may be overrun, but rise to the surface again and are re-circulated by particle size-segregation [Pouliquen et al., 1997; Gray and Ancey, 2009]. This forms a coarse flow head which experiences greater frictional resistance and is unstable to lateral variations in the distribution of particles. A region with a higher concentration of large particles will be less mobile and therefore advance more slowly than other areas of the flow. This causes an irregular front to develop, with the non-uniformity becoming more exaggerated until separate fingers are formed. As material continues to be supplied, the large particles are advected towards the edges of the fingers into levees, allowing the finer, more mobile material to flow down the centre of the channel.

Figure 3 shows the deposits left behind from the laboratory experiments. There are a number of distinct fingers, with dark boundaries signifying the presence of coarse-rich levees. On the inner edge of each levee is a light coloured fine-lining similar to that observed by Kokelaar et al. [2014]. Whilst there is a certain amount of natural fluctuation, the fingers all have a characteristic width, or wavelength, which
is around 2–3 cm for the experiments shown here. There is a maximum mass flux that can be supported in each channel, with faster, thicker flows breaking down into wider channels.

It is important to note that the above effect only happens when there are different sized particles in the flow. Experiments carried out using strictly monodisperse material do not spontaneously break up into leved fingers, and instead the front propagates evenly down the slope. Particle-size segregation acts as a driving force in these fingering instabilities and therefore will be incorporated into the theoretical model described below.

3. GOVERNING EQUATIONS

Consider a mass of granular material flowing down a slope inclined at an angle $\zeta$ to the horizontal, and define a coordinate system $Oxyz$ with the x-axis aligned with the slope, the y-axis in the transverse direction and z-axis normal to the base (Fig. 4). The bulk flow is assumed to have thickness $z = h(x, y, t)$ and depth-averaged velocity $\bar{u} = (\bar{u}, \bar{v})$,

$$\bar{u} = \frac{1}{h} \int_{0}^{h} u \, dz, \quad \bar{v} = \frac{1}{h} \int_{0}^{h} v \, dz,$$  \hspace{1cm} (1)

in the downslope and transverse directions respectively. For a bidisperse mixture the volume fraction of small particles is taken to be $\phi$, so that $(1 - \phi)$ is the proportion of large. Stratification pattern experiments [Gray and Hutter, 1997; Gray and Ancey, 2009] show that the different constituents rapidly segregate into inversely graded layers, so the concentration profile is assumed to be,

$$\phi = \begin{cases} \frac{1}{2}, & \text{if } 0 < z < \eta, \\ 0, & \text{if } \eta < z < h, \end{cases}$$  \hspace{1cm} (2)

i.e. there is a layer of pure small particles of thickness $\eta$ lying below a layer of pure large particles. It then follows that the depth-averaged concentration of smalls is $\bar{\phi} = \eta/h$, with $\bar{\phi}$ being defined analogously to Eq. (1). This concentration, along with the flow thickness $h$ and velocity field $\bar{u}$, are modeled using the depth-averaged equations,

$$\frac{\partial h}{\partial t} + \nabla \cdot (h \bar{u}) = 0,$$  \hspace{1cm} (3)

$$\frac{\partial}{\partial t} \left( h \bar{u} \right) + \nabla \cdot \left( h \bar{u} \otimes \bar{u} \right) + \nabla \left( \frac{1}{2} gh^2 \cos \zeta \right) = ghS + \nabla \cdot (vh^{3/2} \bar{D}),$$  \hspace{1cm} (4)

$$\frac{\partial}{\partial t} (h\bar{\phi}) + \nabla \cdot (h\bar{\phi} \bar{u}) - \nabla \cdot (h\bar{\phi}(1 - \bar{\phi})) = 0.$$  \hspace{1cm} (5)

The first two equations represent conservation of mass and momentum, where the source terms $S = (S_x, S_y)$ in Eq. (4) are due to a combination of gravity and basal friction,

$$S_x = \cos \zeta \left( \tan \zeta - \mu_b \frac{\bar{u}}{|\bar{u}|} \right),$$  \hspace{1cm} (6)

$$S_y = -\cos \zeta \left( \mu_b \frac{\bar{v}}{|\bar{u}|} \right).$$  \hspace{1cm} (7)

The basal friction coefficient $\mu_b$ provides a mechanism to incorporate segregation-mobility feedback effects into the equations. The angular large particles will experience greater frictional forces than the spherical smooth particles, and this is accounted for by taking a concentration-weighted sum [Pouliquen and Vallance, 1999; Woodhouse et al., 2012],

$$\mu_b = \bar{\phi} \mu_b^S + (1 - \bar{\phi}) \mu_b^L,$$  \hspace{1cm} (8)

where $\mu_b^S < \mu_b^L$ are the empirical friction laws for the individual constituents and depend on the flow thickness and granular Froude number, as well as material and bed properties [Pouliquen, 1999].

The final terms on the right hand side of the momentum balance (Eq. (4)) are higher order viscous terms, with $\bar{D}$ representing the depth-averaged strain-rate tensor and $\nu$ a constant (for fixed slope angle) coefficient of effective viscosity [Gray and Edwards, 2014]. These are typically small in magnitude and can therefore be safely neglected in many granular flow configurations. However, they play an important role in determining the cut-off frequency for rollwave instabilities [Gray and Edwards, 2014], as well as in modeling erosion-deposition waves [Edwards and Gray, 2015] and are crucial for the bidisperse flows model considered here. In the absence of viscosity, linear stability analysis of the steady uniform base state produces unbounded growth rates, meaning the equations are mathematically ill-posed [Woodhouse et al., 2012]. The inclusion of higher order terms into Eq. (4) is sufficient to regularize the model.
The depth-averaged concentration $\bar{\phi}$, or equivalently thickness $\eta = h\bar{\phi}$, of small particles is governed by Eq. (3). It is referred to as the ‘large particle transport equation’ [Gray and Kokelaar, 2010a,b; Woodhouse et al., 2012] because it captures the preferential shearing of large particles to the flow front and lateral margins via the $\bar{\phi}(1 - \bar{\phi})$ term. Note that this expression is zero in the monodisperse limits, meaning that the concentration is simply advected with the bulk velocity when in pure phase.

4. NUMERICAL SIMULATIONS

Numerical computations are carried out to simulate the laboratory experiments described in section 2. A shock-capturing finite volume scheme, capable of handling hyperbolic-parabolic systems of conservation laws [Kurganov and Tadmor, 2000], is used to solve Eqs. (3), (4) and (5) on a chute that is 1 m long and 0.2 m wide. The hopper release mechanism is modeled using initial conditions of an empty plane and prescribing steady uniform flow at the inflow boundary ($x = 0$). Transverse instabilities are introduced into the flow by randomly perturbing the inflow thickness about its steady uniform value, simulating experimental irregularities in the gate or base. The lateral boundaries are taken to be periodic.

Figure 5 shows the depth-averaged concentration $\bar{\phi}$ at different times $t$ and confirms that the governing equations described in section 3 are able to capture the development of fingering instabilities. The plots are coloured to match the particles used in the experiments, with dark brown regions corresponding to low concentrations (of smalls), therefore rich in carborundum. A dark front quickly develops ($t = 1$ s) as the large particles are sheared to the flow front, which becomes unstable and breaks into a series of distinct fingers ($t = 5$ s). Dark margins can be seen at the tip of each structure, and neighbouring fingers are separated by large-particle-rich regions, both of which are consistent with the presence of coarse-grained levees observed in the lab (see Fig. 3). The fingers become stably established and elongate as the material continues to flow downslope ($t = 10$ s).

The depth-averaged concentration $\bar{\phi}$ is plotted alongside the flow thickness $h$ and speed $|\vec{u}|$ in Fig. 6 for time $t = 10$ s, chosen so that the front has already broken down into well-defined fingers. The height profiles show a thicker section separating steady uniform flow near the gate from the propagating front, where the thickness vanishes to zero. Material in the bounding levees travels very slowly, with the velocities in these regions approaching zero. By contrast, the speeds are much greater in the centre of each channel, with material travelling faster than the prescribed steady inflow values.

Previous inviscid depth-averaged models [Woodhouse et al., 2012] have produced numerical simulations showing the same features as in Figs. 5, 6. However, the underlying ill-posedness of the equations meant the results were grid dependent, with the finger wavelength continuously decreasing as the computational domain was refined. The inviscid equations were therefore unable to quantitatively model fingering instabilities. The numerical simulations presented above, on the other hand, are found to be grid convergent, with the finger wavelength being controlled by the coefficient of effective viscosity $\nu$. For the parameters chosen here, this wavelength is around 3 cm. This is consistent with that seen in small-scale
experiments (Fig. 3) and therefore the new viscous model represents a significant step forward in modeling spontaneous levee and finger formation.

5. DISCUSSION AND CONCLUSIONS

The process of particle size-segregation is important in geophysical mass flows because it can lead to feedback effects that significantly alter the overall flow characteristics. Self-organization into leveed channels, or a series of leveed fingers, results in increased velocities and run-out distances, which are crucial considerations for hazard mitigation.

The model presented in this paper provides a basic framework to incorporate segregation-mobility feedback effects into the equations governing avalanche motion. The simplifying assumptions mean that some of the physical mechanisms are not currently accounted for. For example, all internal dynamics are lost by using a depth-averaged model, and assuming a perfectly inversely graded layer does not permit a lining of fine material in the lateral levees. Nevertheless, the model produces numerical solutions that are able to quantitatively match small-scale experiments for the formation of leveed fingers and it therefore could be scaled up to aid in the understanding of natural debris flows.

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