Recent Developments in the PQCD Approach

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We review recent developments in the perturbative QCD approach to exclusive hadronic $B$ meson decays. We discuss the important next-to-leading-order corrections to $B \to \pi K$, $\pi \pi$, and the penguin-dominated $B \to PV$ modes, where $P(V)$ is a pseudo-scalar (vector) meson.

I. INTRODUCTION

$B$ factory experiments have accumulated a lot of data on exclusive hadronic $B$ decays, and have reported many interesting results [1]. Some observables, mixing-induced CP asymmetries for $b \to s$ penguin modes, branching ratios and direct CP asymmetries for $B \to \pi K$ and $\pi \pi$, and other observables, have exhibited some deviations from naive expectations in the Standard Model (SM). It is necessary to go beyond naive estimations for understanding the observed deviations, by including subdominant contributions, such as spectator and annihilation diagrams, and higher-order corrections. Most of the calculations of $B$ decay amplitudes rely on the factorization of decay amplitudes into a product of short-distance and long-distance physics. QCD-improved factorization (QCDF) [2] and soft-collinear effective theory (SCET) [3] are based on collinear factorization theorem, but the perturbative QCD (PQCD) approach [4] is based on $k_T$ factorization theorem. Employing collinear factorization theorem, some decay amplitudes involve a singularity arising from the end-point region of parton momentum fractions. An end-point singularity implies that the decay amplitude is dominated by soft dynamics and cannot be further factorized. Such soft contributions are regarded as phenomenological parameters, which are fitted from experimental data.

In the PQCD approach with $k_T$ factorization theorem, there is no end-point singularity because of the Sudakov factor [5]. All amplitudes then can be factorized into parton distribution amplitudes $\Phi$, the Sudakov factors $e^{-S}$, the jet function $J$, and the hard kernel $H$ as [6]

$$A(B \to M_2 M_3) = \Phi_{M_2} \otimes \Phi_{M_3} \otimes H \otimes J \otimes e^{-S} \otimes \Phi_B,$$

where the symbols $\otimes$ stand for convolutions in both longitudinal and transverse momenta of partons. The distribution amplitudes, which are universal in the processes under consideration, are determined from experiments, the light-cone QCD sum rules, lattice calculations, or other non-perturbative methods, and are the main source of uncertainty. The hard kernel is characterized by the hard scale $t \sim O(\sqrt{\Lambda m_b})$, where $\Lambda$ is a hadronic scale and $m_b$ the $b$ quark mass, and can be evaluated as an expansion in powers of $\alpha_s(t)$ and $\Lambda/t$. The schematic picture of the factorization theorem is displayed in Fig. 1. The parton transverse momenta $k_T$ in the mesons are of the order of $\Lambda$. With infinitely many collinear gluon emissions, $k_T$ accumulate and reach $O(\sqrt{\Lambda m_b})$ in the hard kernel. It ensures the absence of the end-point singularities [3]. The hard kernels of the spectator and the annihilation contributions, as well as the emission contribution, are calculable and start from $O(\alpha_s)$.

PQCD has applied to various two-body $B$ decays at leading order (LO) in $\alpha_s$, and has made reasonable predictions for many modes. Note that PQCD predicts a large direct CP asymmetry in the $B^0 \to \pi^+ \pi^-$ mode as a result of a large strong phase arising from scalar-penguin annihilation diagrams [4]. Recently, a part of next-to-leading-order (NLO) contributions have been included in the PQCD approach [7,8,9]. Only the most important NLO contributions coming from the NLO Wilson coefficients, the vertex corrections, the quark loops, and the magnetic penguin, shown in Fig. 2 have been considered [7], because other NLO corrections, mainly to the $B$ meson transition form factors, can be eliminated by choosing an appropriate scale $t \sim O(\sqrt{\Lambda m_b})$ [10]. In this talk, we summarize NLO PQCD predictions for $B \to \pi K$, $\pi \pi$, and the penguin-dominated $B \to PV$ modes, $P(V)$ being a pseudo-scalar (vector) meson.

This talk is organized as follows: In Sec. II we discuss the branching ratios and the CP asymmetries of $B \to \pi K$ and $\pi \pi$ with the NLO corrections. The predictions of the penguin-dominated $B \to PV$ decays are presented in Sec. III. The mixing-induced CP asymmetries for $b \to s$ penguin modes are in Sec. IV. Section V is a summary.

FIG. 1: PQCD factorization theorem for $B \to M_2 M_3$. 

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TABLE I: Branching ratios in units of $10^{-6}$ and direct CP asymmetries in percentage for the $B \to \pi K$ and $\pi\pi$ decays.

| Mode         | Data $[1]$ LO NLO | Br$(10^{-6})$ | $A_{\text{CP}}(10^{-2})$ |
|--------------|-------------------|---------------|--------------------------|
| $\pi^+ K^0$  | 23.1 $\pm$ 1.0    | 17.0          | 23.6$^{+4.4}_{-2.8}$    |
| $\pi^0 K^+ \mp$ | 12.8 $\pm$ 0.6   | 10.2          | 13.6$^{+0.3}_{-0.8}$    |
| $\pi^\mp K^\mp$ | 19.7 $\pm$ 0.6   | 14.2          | 20.4$^{+16.1}_{-13.4}$ |
| $\pi^0 K^0$  | 10.0 $\pm$ 0.6    | 5.7           | 8.6$^{+2.0}_{-1.1}$     |

| $\pi^\pm \pi^\mp$ | $5.2 \pm 0.2$ | $7.0 \pm 1.0$ | $6.5^{+3.3}_{-2.6}$ |
| $\pi^0 \pi^0$     | $5.7 \pm 0.4$ | $3.5$         | $4.0^{+3.4}_{-1.9}$   |
| $\pi^0 \pi^0$     | $1.31 \pm 0.21$ | $0.12$     | $0.29^{+0.50}_{-0.20}$ |

II. $B \to \pi K$ PUZZLE

The current data of the direct CP asymmetries of $B \to \pi K$ and the branching ratios of $B \to \pi\pi$ listed in Table I are inconsistent with the naive expectations

$$A_{\text{CP}}(B^\pm \to \pi^0 K^\mp) \approx A_{\text{CP}}(B^0 \to \pi^\mp K^\pm),$$

$$\text{ Br}(B^0 \to \pi^\mp \pi^\mp) \gg \text{ Br}(B^0 \to \pi^0 \pi^0).$$

These relations can be understood in the topological-amplitude decompositions (see Ref. [1] and references therein):

$$\sqrt{2} A(B^+ \to \pi^0 K^+) = -P^r - P^r_{\text{ew}} - (T^r + C^r) e^{i\phi_3},$$

$$A(B^0 \to \pi^- K^+) = -P^r - T e^{i\phi_3},$$

$$A(B^0 \to \pi^+ K^-) = -T - P e^{i\phi_2},$$

$$\sqrt{2} A(B^0 \to \pi^0 \pi^0) = (P - P_{\text{ew}}) e^{i\phi_2} - C,$$  

where $T^r$, $C^r$, $P^r$, and $P^r_{\text{ew}}$ stand for the color-allowed tree, color-suppressed tree, penguin, and electroweak penguin amplitudes, respectively, and $\phi_2$ and $\phi_3$ are the Cabibbo-Kobayashi-Maskawa (CKM) phase defined by $V_{ub} = |V_{ub}| \exp(-i\phi_2)$, $V_{td} = |V_{td}| \exp(-i\phi_1)$, and $\phi_2 = 180^\circ - \phi_1 - \phi_3$. Assuming the hierarchies, $P^r > T^r$, $P^r_{\text{ew}} > C^r$ for $B \to \pi K$ and $T > C$, $P > P_{\text{ew}}$ for $B \to \pi\pi$, the relations in Eq. (2) can be derived, where the asymmetries are written as

$$A_{\text{CP}}(B^\pm \to \pi^0 K^\mp) \approx 2 \text{Im} \left[ \frac{T^r + C^r}{P^r + P^r_{\text{ew}}} \right] \sin \phi_3,$$

$$A_{\text{CP}}(B^0 \to \pi^+ K^-) \approx 2 \text{Im} \left[ \frac{T^r}{P^r} \right] \sin \phi_3.$$  

The mixing-induced CP asymmetry for $B^0 \to \pi^0 K_S$ has exhibited another puzzle. Hence, the current data seem to require a large $C^r$, a large $P^r_{\text{ew}}$ with a new CP violating phase, or both of them (see, e.g., Refs. [2, 11] and references therein).

The LO PQCD predictions follow the naive expectations as listed in the third and sixth columns of Table I. The effect of the NLO corrections was studied in Ref. [7]. The sum of the quark loops and the magnetic penguin reduces the penguin amplitudes by about 10% in the $B \to \pi K$ decays and affects the CP asymmetries little. The vertex corrections much affect $C^r$, associated with the effective Wilson coefficient

$$a_2(\mu) = \left( C_1(\mu) + \frac{C_2(\mu)}{N_c} \right) + \frac{\alpha_s(\mu)}{2\pi} V_2(\mu) C_2(\mu),$$

where $N_c = 3$ is the number of colors and $V_2$ the loop function from the vertex corrections. The vertex correction term has a large effective coefficient $C_2$, whereas the LO ones cancel between $C_1$ and $C_2/N_c$. Therefore the vertex corrections enhance $C^r$ significantly and rotate its phase, such that $T^r + C^r$ is anti-parallel to $P^r$ as shown in Fig. 3, where $C^r$ is still subdominant compared with $T^r$. Thus, $A_{\text{CP}}(B^\pm \to \pi^0 K^\mp) \approx 180^\circ$ vanishes and NLO PQCD predicts the pattern $[2]$

$$|A_{\text{CP}}(B^\pm \to \pi^0 K^\mp)| \ll |A_{\text{CP}}(B^0 \to \pi^\mp K^\pm)|,$$

as listed in the last column of Table I.

Similarly, $C$ for $B \to \pi\pi$ is enhanced by the vertex corrections, but it is insufficient to accommodate $\text{Br}(B^0 \to \pi^0 \pi^0)$ to the measured value $[2]$. NLO PQCD predicts $|C/T| \approx 0.2$ for $B \to \pi\pi$, though a much larger $|C/T| \approx 0.8$ is required to explain the observed $\text{Br}(B^0 \to \pi^0 \pi^0)$. Note that the larger $C$ also contributes
The NLO corrections dramatically enhance the $B \to \rho K$, $\omega K$ branching ratios. As a result, the NLO predictions for the $B \to \rho K$ decays are in agreement with the data, while the central values of the NLO $B \to \omega K$ predictions are higher than the observed ones. The $B \to \omega K$ modes, as well as the $B \to \phi K$ modes, have an additional $a_5$ amplitude shown in Fig. 4(c). The vertex corrections to the $a_5$ amplitude increase (decrease) the $B \to \omega K$ ($\phi K$) branching ratios significantly. If the hard scale is lifted slightly, the $a_5$ contribution is moderated. $B(B \to \omega K)$ and $B(B \to \phi K)$ then decrease and increase, respectively, approaching the central values of the data.

The observed patterns of the direct CP asymmetries differ among the $B \to \pi K$, $\pi K^*$, and $\rho K$ modes as shown in Tables I and II. As explained in the last section, the penguin amplitude $P'$ in the $B \to \pi K$ decays is in the second quadrant with a strong phase coming from the scalar-penguin annihilation diagram in Fig. 4(d). The color-suppressed tree amplitude $C'$ is enhanced by the vertex corrections, such that $T' + C'$ is almost antiparallel to $P'$. It leads to the pattern in Eq. (6) and a negative asymmetry in the $B^0 \to \pi^\mp K^\pm$ mode. In the $B \to \pi K^*$ decays, $P'$ is more inclined to the positive imaginary axis compared to that in the $B \to \pi K$ decays. $T'$ and $P'$ are then more orthogonal to each other, and $T' + C'$ and $P'$ do not line up. Therefore, PQCD predicts the larger asymmetries with the pattern 9.

$$|A_{\text{CP}}(B^+ \to \pi^0 K'^\mp)| < |A_{\text{CP}}(B^0 \to \pi^\mp K^\pm)| \ , \ (7)$$

as shown in Table II. In the case of the $B \to \rho K$ decays, the real part of $P'$ diminishes due to the destructive interference between Figs. 4(a) and (b). Because of the sign flip of the scalar-penguin annihilation amplitude, $P'$ is roughly aligned with the negative imaginary axis. Therefore, the predicted direct CP asymmetries are larger and positive with 9.

$$A_{\text{CP}}(B^0 \to \rho^0 K^-) \approx A_{\text{CP}}(B^0 \to \rho^+ K^-) \ . \ (8)$$

Thus, NLO PQCD predicts the different patterns of the direct CP asymmetries among the $B \to \pi K$, $\pi K^*$, and $\rho K$ modes as shown in Eqs. (6), (7), and (8).

### IV. MIXING-INDUCED CP ASYMMETRIES

The mixing-induced CP asymmetries for the penguin-dominated modes are useful to search for new physics beyond the SM. Their asymmetries are naively expected to be identical to those for the tree-dominated $b \to c\bar{c}s$ modes, $S_{\rho K} = \sin(2\phi_1)$. The possible SM deviation $\Delta S_f \equiv S_f - \sin(2\phi_1)$ for the final states $f = \pi^0 K_S, \rho^0 K_S, \omega K_S, \phi K_S$ can be written as

$$\Delta S_f \approx 2 \left( \lambda^2 \sqrt{\rho^2 + q^2} - \Re \frac{C'}{P'} \right) \cos(2\phi_1) \sin \phi_3 , \ (9)$$

where $\lambda$, $\rho$, and $\eta$ are the CKM parameters. For the small $C'$ case, the deviation is small and positive ($\Delta S_f \sim 0.02$).
FIG. 4: Penguin emission and annihilation amplitudes, where the emission ones (a), (b), and (c) are associated with $a_4$, $a_5$, and $a_3$, respectively.

The NLO PQCD predictions for $\Delta S_{\rho}^{\pi K_S}$ are summarized in Table III, where those in QCDF [13] and in QCDF plus long-distance (LD) effects [14] are also shown for comparison. PQCD predicts positive deviations, opposite to the observed ones, except for the $B_0 \rightarrow \rho^0 K_S$ case. In the $B \rightarrow \rho K$ decays, both $P'$ and $C'$ are almost imaginary and parallel to each other. Therefore, PQCD predicts the negative $\Delta S_{\rho K_S}$. Among the modes studied here, $B_0 \rightarrow \phi K_S$ is the cleanest one, since it does not involve $C'$. The predictions are basically in agreement with those in QCDF, but differ from those in QCDF+LD, which predicted $\Delta S_{\rho K_S} > 0$.

V. SUMMARY

In this talk, we have summarized the recent works on exclusive $B$ meson decays in the PQCD approach, with focus on $B \rightarrow \pi K$, $\pi \pi$, and the penguin-dominated $B \rightarrow PV$ modes.

In the NLO PQCD analysis, it has been found that the color-suppressed tree amplitude is enhanced by the vertex corrections; the direct CP asymmetries for the $B \rightarrow \pi K$ modes have approached the observed values. The $B_0 \rightarrow \pi^0 \pi^0$ branching ratio is, however, still too small, compared with the measured value. NLO PQCD has also predicted the different patterns of the direct CP asymmetries among the $B \rightarrow \pi K$, $\pi K^*$, and $\rho K$ modes as shown in Eqs. (6), (7), and (8). Those patterns, if confirmed by data, will support the source of strong phases from the scalar-penguin annihilation diagrams in the PQCD approach. The predicted mixing-induced CP asymmetries for the penguin-dominated modes basically show the positive deviations, $S_{\pi^0 K_S, \omega K_S, \phi K_S} \gtrsim \sin(2\phi_1)$, except for $S_{\rho K_S} \approx 0.5$. Hence, the central values of the current data, $\Delta S_{\pi^0 K_S, \omega K_S, \phi K_S} < 0$, seem to be still puzzling.

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\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Mode & Data [1] & PQCD & QCDF & QCDF + LD \\
\hline
$\pi^0 K_S$ & $-0.35 \pm 0.21$ & 0.05 & 0.07 & 0.04 \& 0.03 & 0.01 \\
$\rho^0 K_S$ & $-0.48 \pm 0.57$ & -0.09 & -0.12 & 0.04 & 0.04 & 0.02 & 0.05 & 0.02 & 0.01 \\
$\omega K_S$ & $-0.20 \pm 0.24$ & 0.15 & 0.13 & 0.01 & 0.04 & 0.04 & 0.02 & 0.04 & 0.01 \\
$\phi K_S$ & $-0.29 \pm 0.18$ & 0.23 & 0.02 & 0.01 & 0.03 & 0.04 & 0.01 \\
\hline
\end{tabular}
\caption{Possible SM deviations of the mixing-induced CP asymmetries in PQCD [7, 9], in QCDF [13], and in QCDF+LD [14].}
\end{table}