1. Introduction

The effectiveness of the use of firearms with appropriate ammunition depends on both, the stability of its basic ballistic characteristics and compliance of the values of these characteristics with the established norms [1]. Ballistic characteristics largely depend on the technical condition of weapons and ammunition, which affects the course of the
shot process and thus determines the laws of change of ballistic elements of the shot.

The main ballistic elements of the shot include the projectile path \( l \) in the barrel channel, the velocity \( v \) of the projectile relative to the barrel channel, and the average pressure \( p \) of powder gases on the projectile and the walls of the barrel channel in the discharge space [2].

The most important part of a firearm is its barrel, where the process of a shot occurs. During the service life, the barrel bore of any type of weapon is undergone by the mechanical wear, and it can swell under certain conditions. Deviations of the internal geometric parameters of the worn or swollen bore leads to the loss of energy of the powder gases during the shot, which is reflected in the ballistic elements of the shot, as a result, the initial velocity of the projectile decreases. These factors can lead to the impossibility of providing the required ballistic characteristics of the weapon and given shooting efficiency.

During long-term storage of ammunition, especially due to violations of its regime, the energy characteristics of gunpowder and the speed of its combustion change. This is reflected in the ballistic elements of the shot and leads to a deviation of the initial velocity of the projectile, and in some cases – a dangerous increase in the maximum pressure of powder gases in the barrel.

The study of the dependence of the ballistic characteristics of firearms on the technical condition of weapons and ammunition is associated with the need to simulate the influence of the parameters of the barrel and powder charge on the ballistic elements of the shot.

The possibilities of constructing such models of influence by the empirical method are very limited due to the complexity of practical reproduction, instrumental identification and description of real defects in the barrel channels and parameters of ammunition degradation. At the same time, the equations of internal ballistics [3, 4], which establish a connection between the ballistic elements of the shot and the loading conditions, are well known and tested. The loading conditions denote the set of parameters characterizing the design features of the barrel and ammunition and the physical and chemical characteristics of the powder charge and which affect on the ballistic characteristics of the weapon and the elements of the shot. Therefore, the possibility of constructing models of the influence of the deviation of the geometric parameters of the barrel and the degradation of the powder charge on the ballistic elements of the shot by calculation, for example, by numerical solution of the equations of internal ballistics is more promising.

Thus, the relevance of the work is to overcome the contradiction between the needs of applied ballistics and the capabilities of the existing scientific and methodological apparatus for solving the equations of internal ballistics. In the framework of this scientific direction, the way to the modeling the influence of deviations of geometrical parameters of the bore and the degradation of the powder charge on the course of the shooting process can be opened.

### 2. Literature review and problem statement

The analytical way of solving the equations of internal ballistics involves performing complex transformations that lead to a significant limitation of the accuracy of the solutions obtained, so the analytical solutions of the equations of internal ballistics are approximate [5, 6]. In addition, it is not possible to vary any charging parameters, which are considered unchanged in the process of firing according to the classical approach [7]. For example, it is impossible to take into account the variables along the length of the barrel channel and deviations of its geometric dimensions.

There are known methods for solving partial problems of internal ballistics, in particular for the barrel of a conical shape, which take into account the breakthrough of powder gases during shooting from a mortar [4]. But in the first case, the solution is simplified due to a linear change in the barrel diameter along the length, and in the second – due to the fact that the gap between the barrel walls and the mine is constant, hence the flow of powder gases through the gap depends only on their pressure. In the case of swelling or wear of the barrel channel, its diameter can vary according to a complex nonlinear law, the establishment of which is a separate task. As a result, it is not possible to obtain an analytical solution to the above problem in the final form.

In work [7], the approach to the account of wear of the barrel channel is stated, but only in a zone of a charging chamber and a bullet input, thus wear of the barrel channel along its length is not considered.

The methods of measuring the pressure of powder gases inside the barrel are outlined in [8]. Individual questions for the experimental determination of the ballistic elements of the shot are reflected in [9–11]. However, none of the works [8–11] doesn’t reveal the relationship between the ballistic elements of the shot and typical defects in the barrel channels and ammunition.

Experimental determination of ballistic elements of the shot is realized by the authors of [8, 9] only for cases of application of technically serviceable barrels and ammunition, but the research of influence of defects of the barrel channel and degradation of powder charges were not carried out.

It seems reasonable to obtain solutions of the equations of internal ballistics numerically if we take in account the substantial limitations of analytical and empirical methods, which are given above in [4–7], for modeling the process of a shot, as well as the capabilities of modern computer technology. The numerical solution of the equations of internal ballistics allows us to obtain a solution with a given accuracy, which will be determined by the size of the integration step in time and the accuracy of the initial data preparation [12]. A preliminary assessment of the capabilities of numerical methods suggests that they can be preferred in such complex cases, when one or more parameters change during the shot process (for example, variables are the size of the barrel section as a result of its swelling or wear), thus it is extremely difficult to take into account when using analytical methods. However, the known numerical methods for solving the equations of internal ballistics [5, 7, 13] do not take into account the loss of the fraction of powder gases due to their breakthrough between the walls of the damaged section of the barrel channel of complex shape and the projectile. Thus, these methods cannot be considered suitable for weapon defect simulations.

The review of information [4–13] indicates that the methodological basis for modeling the relationship between ballistic elements of the shot and typical defects in the barrel channels and ammunition has not been adequately covered. Therefore, it is advisable to develop numerical methods for solving the equations of internal ballistics in this direction.
3. The aim and objectives of the study

The aim of the study is to determine the principles of constructing a difference scheme for the numerical solution of the equations of internal ballistics, taking into account the influence of typical defects in the barrel channels and ammunition on the ballistic elements of the shot.

To achieve this aim, the following objectives are to be accomplished:

– to generate a list of initial data for the numerical solution of internal ballistics equations;
– to create recurrent expressions for step-by-step calculation of the required ballistic elements of the shot;
– to determine the initial conditions for the first and second periods of the shot;
– to carry out practical approbation of the difference scheme by numerical solution of the equations of internal ballistics for characteristic combinations of initial data.

4. Numerical method for modeling the shot process based on solving the equations of internal ballistics

The nature of the firing process depends on the technical condition of the barrel channel and ammunition and is determined by the type of dependencies of the ballistic elements of the shot on the time of the projectile motion along the barrel channel: \( p(t) \) and \( v(t) \).

The value of the ballistic elements of the shot at a certain time is determined by the equations of internal ballistics [14], which include:

1. The basic equation of thermodynamics (energy conversion equation)

\[
ps\ell_{\psi} + l = f \omega \psi \frac{\theta}{2} \phi mc^2, \tag{1}
\]

where \( s \) – the cross-sectional area of the barrel bore (including rifling); \( \ell \) – the reduced length of the free volume of the chamber; \( f \) – the power of gunpowder; \( \omega \) – the mass of the powder charge; \( \psi \) – the relative volume of powder grain, which burned down; \( \theta \) – the powder gas expansion parameter; \( \phi \) – the secondary factor (the ratio of the fictitious mass); \( m \) – the mass of the projectile.

2. The equation expressing the law of gunpowder burning:

\[
\frac{de}{dt} = u = u, p, \tag{2}
\]

\[
\frac{d\psi}{dt} = \frac{S}{A_1} \sigma u, p = x \sigma u, p, \tag{3}
\]

where \( \chi, \lambda \) – the geometrical characteristics of the powder grain; \( z \) – the relative thickness of the burnt layer of powder grain; \( u \) – the rate of gunpowder combustion; \( u, p \) – the powder burning rate at atmospheric pressure; \( e \) – the thickness of the burnt layer of powder grain; \( e, l \) – half the initial thickness of the gunpowder vault; \( \sigma \) – the relative surface of the powder grain.

3. The equation of projectile motion

\[
v = \frac{dl}{dt} = l' \quad l'' = \frac{d^2l}{dt^2}, \tag{6}
\]

and \( v \) is related to the equation \( z \)

\[
v = \frac{sl_1}{\phi m} (z - z_0), \tag{7}
\]

where

\[
z = z_0 + \frac{\phi m}{sl_1} v, \tag{8}
\]

where \( z_0 \) – the relative thickness of the burnt layer of powder grain at the beginning of the projectile motion.

The study of internal ballistics equations shows that elements \( v, \psi \) and \( p \) can be expressed as functions of \( l \) or their derivatives \( l', l'', l''' \) in time \( t \) (up to the third order inclusive). If we take the time \( t \) of the projectile motion along the barrel channel as an independent variable, and the path \( l \) of the projectile in the barrel channel as a variable that decomposes into a series, we can apply the Taylor series [15] to find the path \( l_{n+1} \) and its derivatives in the adjacent section, which corresponds to the time \( t_{n+1} = t_n + \Delta t \), and the values \( l_n \) and its derivatives for the previous time \( t_n \) must be known. Thus, it is possible to find all the elements of gunpowder combustion and projectile motion: \( z, \psi, v, p, l \) and \( t \).

The approach proposed in [16] is used as a basis for the construction of a difference scheme for the numerical solution of the equations of internal ballistics.

4. 1. Formation of a list of initial data for numerical solution of internal ballistics equations

The solution to this task involves the implementation of such operations.

1. Determination of the list of the required elements of the shot: \( l, v \) and \( l', p, \psi \).

2. Determination of the list of variables included in the equations of internal ballistics: \( z, l' \) the second derivative of the projectile path in time, \( l''' \) – the third derivative of the projectile path in time.

3. Formation of a list of charging conditions, which are the initial data for the simulation of the shot process: \( s, m, \psi, u, e, f, \omega, \theta, \chi, \lambda \), as well as \( W_0 \) – the initial volume of the charging chamber, \( p_0 \) – boost pressure, \( \delta \) – powder density, \( a \) – powder gas covolume.

In order to avoid confusion in the calculations and in the output of the calculation results, all the parameters that characterize the charging conditions are submitted in the international system of units of physical quantities (despite the fact that in some sources of information they are submitted by the authors in non-system units).

4. Determination of a set of parameters and coefficients that are derived from the charging conditions and characterize the shot process and are calculated from the values of the given initial data.

These parameters include:

\[
l_1 = \frac{e_1}{u_1} \quad \text{ – full gas pressure pulse at the end of gunpowder combustion;}
\]

\[
\Delta = \frac{\omega}{W_0} \quad \text{ – charging density;}
\]
\[ \psi_b = \frac{1 - \frac{1}{\delta}}{\frac{\omega}{s} + \alpha - \frac{1}{\delta}} \]  

is the fraction of powder charge burned before the projectile started moving:

\[ \sigma_k = \sqrt{1 + 4 \frac{\lambda}{\psi_0}} \]  

is the relative surface of the powder grain at the start of projectile motion;

\[ z_0 = \frac{2 \psi_0 \lambda}{\alpha (\alpha + 1)} \]  

is the relative thickness of the powder grain layer burned at the beginning of projectile motion;

\[ l_0 = \frac{W_0}{s} \]  

is the reduced length of the charging chamber;

\[ l_1 = \frac{1}{2} \left( W_0 - \frac{\omega}{s} \right) - l_0 \left( 1 - \frac{\Delta}{\delta} \right) \]  

is the reduced length of the free space of the charging chamber;  

\[ l_{w_1} = l_1 - a \psi_0 \]  

is the reduced length of the free volume (space) of the charging chamber at the beginning of projectile motion.

In addition, the coefficients are introduced, the use of which will reduce the recording of individual expressions and simplify calculations:

\[ a = \frac{\omega}{s} \left( \alpha - \frac{1}{\delta} \right), \]

\[ k_1 = \frac{\phi m}{s I_1}, \quad k_2 = \frac{f \omega}{s}, \quad k_3 = \frac{\phi m}{2 f \omega}, \quad k_4 = \frac{s}{\phi m}. \]  

4.2. Compilation of recurrent expressions for step-by-step calculation of the required elements of the shot

For the first period of the shot, recurrent expressions have the following form:

\[ l'_{w_1} = l'_{w_1} + \frac{h l'_{w_1}^2 + \frac{k^2}{2} l''_{w_1}}{2}, \]  

\[ v'_{w_1} = v'_{w_1}, \]  

\[ z_{w_1} = z_0 + k_1 v'_{w_1}, \]  

\[ \psi_{w_1} = \psi_{w_1} + k_2 v'_{w_1}, \]  

\[ l_{w_1} = l_{w_1} + h l'_{w_1} + 0.5 h^2 l''_{w_1} + \frac{k^2}{6} l''_{w_1}, \]  

\[ p_{w_1} = k_3 l_{w_1} \psi_{w_1} - k_4 v'_{w_1} + I_{m_1}, \]  

\[ I''_{w_1} = I'_{w_1} - \frac{1}{a} v'_{w_1} + I_{m_1}, \]  

\[ l''_{w_1} = 2 l''_{w_1} / h - 2 l''_{w} / h - l''_{w_1}. \]  

Further calculation of the shot elements for the second period is based on the same differential expressions as for the first, except for the absence of the need for the expressions for the calculation of \( \psi \) and \( z \). In the expression for \( p \), we substitute the value \( \psi \), which for all steps in time is equal to 1. Thus, the set of difference expressions for the second period of the shot gets the form:

\[ l_{w_2} = I_{w_2}, \]  

\[ v_{w_2} = l'_{w_2} + h l'_{w_2} + \frac{k^2}{2} l''_{w_2}, \]  

\[ l_{w_2} = l_{w_2} + h l'_{w_2} + 0.5 h^2 l''_{w_2} + \frac{k^2}{6} l''_{w_2}, \]  

\[ p_{w_2} = k_3 l_{w_2} \psi_{w_2} - k_4 v'_{w_2} + I_{m_2}, \]  

\[ I''_{w_2} = I'_{w_2} - \frac{1}{a} v'_{w_2} + I_{m_2}, \]  

\[ l''_{w_2} = 2 l''_{w_2} / h - 2 l''_{w} / h - l''_{w_2}. \]  

4.3. Determination of initial conditions for the first and second periods of the shot

Since the time \( t \) is counted from the beginning of projectile motion, at \( t = 0 \) the initial conditions for the first period of the shot will have the form:

\[ I(0) = 0, \quad p(0) = p_0, \quad z(0) = z_0, \quad \psi(0) = \psi_0. \]  

\[ I'(0) = v(0) = 0, \quad I''(0) = k_1 p_0. \]  

\[ I'''(0) = k_2 k_3 \frac{\psi_0}{l_1} \frac{l_2}{l_{w_0}}, \quad p_0 = I''(0) k_4 \frac{\psi_0}{l_1} \frac{l_2}{l_{w_0}}. \]  

The moment \( t_{l_1} \) of the end of the first period of the shot is determined by the achievement of the value \( \psi - 1 \) by the variable \( \psi \). For the moment of time \( t_{l_2} \), the values of all elements of the shot are fixed, corresponding to the end of gunpowder combustion: \( I_{l_{1}}, l_{l_{1}}, p_{l_{1}} \). These values define the initial conditions for the second shot period:

\[ I(0) = l_{l_{1}}, \quad I'(0) = v(0) = v_{l_{1}}, \]  

\[ I''(0) = k_1 p_{l_{1}}, \quad I'''(0) = - k_1 (1 + \theta) \frac{v_{l_{1}} p_{l_{1}}}{2 l_{l_{1}}}. \]  

The end of the second period of the shot is considered to be the fulfillment of the condition of reaching the muzzle end by the projectile:

\[ I \geq I_{\text{m}}. \]  

5. Practical approbation of the difference scheme by numerical solution of the equations of internal ballistics and its results

A tabular Microsoft Excel processor was used to test the difference scheme and to obtain numerical solutions of the internal ballistics equations for several characteristic combinations of the initial data.
In particular, the shot processes were simulated for such weapon samples as 7.62 mm SVD sniper rifle with 7.62×54 rifle cartridge and 122 mm D-30 howitzer with 122 mm shot with high-explosive shell and full charge at the nominal values of all charging parameters.

Examples of simulation results are shown in Fig. 1–4.

According to the graphs 1 in Fig. 1–4, the first and second periods of the shot are well identified, as well as the time \( t_{a} \) and the path of the projectile \( l_{m} \), which correspond to the maximum pressure of the powder gases. In addition, according to the graphs 1, 2 in Fig. 1–4, the maximum \( p_{m} \) and muzzle pressure \( p_{n} \), muzzle velocity \( v_{n} \) of the projectile and its velocity \( v_{m} \) at maximum pressure are determined.

The analysis of the obtained results showed that they coincide with the data of the internal ballistics tables [17] at the control points, namely, at the point where the maximum pressure is recorded: \( (p_{m}, t_{a}), (v_{n}, l_{m}) \), \( (p_{m}, l_{m}) \), \( (v_{n}, l_{m}) \), and also at the end point of projectile motion along the barrel channel \( (p_{m}, l_{m}), (v_{n}, l_{m}), (p_{n} t_{a}), (v_{n} t_{a}) \).

In addition, the simulation of the shot process for the cases of deviation of the parameters of the powder charge from the nominal values was carried out. In particular, solutions are obtained for the cases of reducing the force of powder \( f \), which may occur due to degradation processes during the aging of powder charges, for example, during their long-term storage. The examples of the results of this simulation are shown in Fig. 5, 6.

As shown in Fig. 5, the decrease in the force of gunpowder \( f \) leads to a drop in the pressure of gunpowder gases in the area of its expected maximum, whereby the rate of gunpowder combustion slows down, the pressure reaches a maximum value with a certain delay, and the duration of the shot process, particularly its first period, increases. From the very beginning of projectile motion there is a “blockage” of curves relative to the curve for technically serviceable ammunition on graphs 2–4 Fig. 6. The effect of this phenomenon is uncritical only with a slight (up to 3...7 %) reduction of the gunpowder force (curves 3, 4), since the muzzle velocity drop does not exceed the maximum permissible value, which is usually 5 %. With a further reduction of the gunpowder force, the effect becomes critical (drop in muzzle velocity greatly exceeds 5 %). Both the curves \( p(t) \) and the curves \( v(t) \) are quite informative, since the reduction of the gunpowder force is well identified by them.

Thus, the results of solving the equations of internal ballistics by the numerical method with the help of the considered difference scheme can be considered quite adequate. The proposed difference scheme can be used to obtain solutions to the equations of internal ballistics, modeling and analysis of the effect of changes in the initial data on the nature of the internal ballistic processes during the shot.
6. Discussion of results of construction of the difference scheme and its approbation

The course of the curves $p(t)$, $v(t)$, is determined by the parameters of the shot process flow and, in particular, by the geometric characteristics of the barrel channel, as well as by the energy characteristics of the powder charge and its combustion rate. Thus, there is an objective connection between the geometric characteristics of the barrel channel and the characteristics of the powder charge and the type of curves for the ballistic elements of the shot.

The change in the technical condition of the barrel channels and ammunition leads to a deviation of the charging conditions and changes in the parameters of the shot process, which can be observed by changing the type of the corresponding curves $p(t)$, $v(t)$. Each of the defects that are identified (for example, swelling, wear of the bore, fall of the powder force, increase in combustion rate) is characterized by characteristic features, and the size of the deviation determines the degree of manifestation of the defect.

It is necessary to implement their definition by measurements [18, 19] in order to obtain objective data on the actual ballistic elements of the shot, which correspond to the current technical condition of the barrel and ammunition. For such measurements, both means of measuring instantaneous pressure values of powder gases, and means of measuring instantaneous values of the projectile velocity can be used, so the principles of construction and characteristics of which are discussed in detail in [16, 20].

The results of this work can be used to determine the reference curves $p(t)$, $v(t)$, which correspond to the technically sound condition of the bore and ammunition. The proposed difference scheme will simulate the effect of defects in the barrel channel and degradation of powder charges on the ballistic elements of the shot. In addition, the proposed approach allows us to solve other partial problems of internal ballistics that require taking into account the variability of charging conditions. These tasks include modeling a shot from a weapon with a relatively low forcing pressure, taking into account the influence of the charge temperature on the barrel channel and degradation of powder charges on the barrel channel walls. However, these tasks are limited to the use of the difference scheme for those weapons for which the loss of energy due to heat exchange between the powder gases and the barrel channel walls cannot be neglected. In addition, the proposed difference scheme does not allow obtaining correct solutions for systems for which, by definition, there is no forcing pressure, although such systems have not been considered in this work.

Further research in this direction should be directed to the modeling of typical defects in the barrel channels and powder charges and the creation of an appropriate database of ballistic elements of the shot for practical use in the diagnosis of weapons and ammunition. Certain possibilities for improvement of the difference scheme are opened when the real law of change of the parameter of expansion of powder gases instead of application of the averaged value of this size is taken into account. The loss of energy due to heat exchange between the powder gases and the barrel channel walls can be taken into account indirectly, for example, by a corresponding decrease in the gunpowder force, but this issue requires additional research.

7. Conclusions

1. The list of charging conditions, which are the initial data for the simulation of the shot process, is determined. These output data include, in particular, the geometrical parameters of the barrel channel and the charging chamber, the physical and chemical parameters of the powder charge, as well as its mass.

2. Based on the decomposition of the equations of internal ballistics in the Taylor series, recursive expressions for step-by-step calculation of the required ballistic elements of...
the shot are compiled. These expressions allow you to find all the elements of gunpowder burning and projectile motion in successive steps in time.

3. The initial conditions for solving the equations separately for the first and the second periods of the shot were determined, which made it possible to take into account the change in the dynamics of the acceleration of the projectile after the cessation of the inflow of powder gases.

4. The simulation of the shot process using the proposed difference scheme for the characteristic combinations of the initial data is carried out. The deviation of simulation results from the data of internal ballistics tables at the control points does not exceed 1–2%. This indicates the adequacy of the difference scheme and the possibility of its further use in modeling the effect of defects in the barrel channels and ammunition on the nature of the shot process.

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