Generalized Sub-Array-Connected Hybrid Precoding Improves the Energy-Efficiency of Millimeter-Wave Massive MIMO Systems

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Abstract—In this paper, we propose a generalized sub-array-connected architecture for arbitrary radio frequency (RF) chain and antenna configurations, where the number of RF chains connected to a sub-array and the number of antennas in each sub-array can be arbitrary. Our design objective is to improve the energy efficiency of the hybrid precoder of millimeter-wave massive multiple input multiple output (MIMO) systems. We also propose a successive interference cancellation based hybrid precoding scheme for any given RF chain and antenna configuration. This scheme firstly decomposes the total achievable rate optimization problem into multiple sub-rate optimization problems, each of which is only related to a single sub-array and then it successively maximizes these sub-rates. Our simulation results demonstrate that the proposed scheme achieves a similar rate as the corresponding optimal unconstrained precoding scheme. Furthermore, we show that the energy-efficiency of the proposed scheme is better than that of the existing schemes in the fully-connected and sub-array-connected architectures.

Index Terms—MIMO, millimeter-wave communications, hybrid precoding, energy-efficiency.

I. INTRODUCTION

Millimeter-wave massive multiple input multiple output (MIMO) systems constitute promising candidate technologies for next-generation communication systems as a benefit of their substantial bandwidth and high spectral efficiency [1]. For example, at 30 GHz carrier frequency, the wave-length is 10 millimeters, which makes it possible to pack a large number of antennas in a compact area. As a benefit, a large antenna array is capable of providing significant precoding gains to compensate for the high path loss of millimeter-wave signals [2]. However, in traditional MIMO systems, the Transmit Precoding (TPC) is usually realized in the digital domain and requires the same number of radio frequency (RF) chains as the number of antennas [3], [4]. Hence digital TPC potentially imposes prohibitive energy consumption in millimeter-wave massive MIMO systems relying on large antenna arrays. To circumvent this problem, the hybrid TPC concept has been proposed, where the signals are firstly precoded by a low-dimensional digital TPC to cancel the interference and to allocate the transmit power. Then they are also precoded by a high-dimensional analog TPC to attain high beamforming gains [5].

Most hybrid TPC schemes consider the fully-connected (FC) and the sub-array-connected (SAC) architectures [6]. In the FC architecture, each RF chain is connected to all antennas by a large number of analogue phase shifters to achieve the maximum attainable TPC gains, which however leads to a high energy consumption [7]. By contrast, the SAC architecture requires a lower number of phase shifters, but has to tolerate some performance loss [8]. The best hybrid TPC architecture having the highest energy efficiency (EE) is still unknown. Recently, a more general SAC architecture, termed as hybrid-connection based architecture was proposed, where each sub-array may be connected to multiple RF chains [9]. However, the hybrid-connection based architecture assumes that the number of RF chains for all sub-arrays is the same, which limits the degree of freedom in improving the EE.

To circumvent this problem, we propose a generalized sub-array-connected (GSAC) architecture, where the number of RF chains connected to a sub-array and the number of antennas in each sub-array can be arbitrarily adjusted for improving the EE of hybrid TPC in millimeter-wave massive MIMO systems. For any given RF and antenna configuration, a successive interference cancellation (SIC) based hybrid TPC scheme is proposed. This scheme firstly decomposes the total achievable rate optimization problem into multiple sub-rate optimization problems, each of which is only related to a single sub-array. Then, it successively maximizes these sub-rates. Our simulation results demonstrate that the proposed scheme achieves a similar rate as the corresponding optimal unconstrained TPC scheme and the EE of the proposed scheme is the best in the family of the FC and SAC architectures.

II. SYSTEM MODEL AND CHANNEL MODEL

We consider a single-user millimeter-wave massive MIMO system, where the transmitter is equipped with \( N_t \) antennas and the receiver has \( N_r \) antennas. The \( N_t \times 1 \) received signal vector \( y \) can be presented as

\[
y = \sqrt{\rho} H F_{RF} F_{BB} s + n, \tag{1}
\]

where \( \rho \) is the average received power, \( F_{RF} \) of size \( N_t \times N_{RF} \) is the analog TPC matrix, \( F_{BB} \) of size \( N_{RF} \times N_s \) is the baseband TPC matrix, \( s \) is the \( N_s \times 1 \) signal vector and \( n \) is the vector of independent and identically distributed (i.i.d.) \( \mathcal{CN}(0, \sigma_n^2) \) noise. Furthermore, \( H \) is the \( N_t \times N_t \) millimeter-wave channel matrix expressed as

\[
H = \sqrt{N_t N_r} \frac{N_{cl} N_{ray}}{N_{cl} N_{ray}} \sum_{m=1}^{N_{cl}} \sum_{n=1}^{N_{ray}} \alpha_{m,n} a_m(\theta_{m,n}^t) a_n^*(\theta_{m,n}^r), \tag{2}
\]

where \( N_{cl} \) is the number of scattering clusters and each cluster contributes \( N_{ray} \) propagation paths, \( \alpha_{m,n} \) denotes the complex gain of the \( n^{th} \) path in the \( m^{th} \) cluster, while \( \theta_{m,n}^t \) and \( \theta_{m,n}^r \in \)
With the non-convex constraints imposed on the analog TPC matrix $F_{RF}$, it is an open challenge to obtain the globally optimal solution to (5). However, by exploiting that the hybrid TPC matrix $F$ is of block-diagonal structure, which implies that the TPC of each sub-array is mutually independent, the total achievable rate $R$ may be decomposed into multiple sub-rates, each of which is only related to a single sub-array. Then, we can solve (5) by maximizing each of the sub-rates.

The hybrid TPC matrix $F$ (when $N_{sub} > 1$) may be partitioned as

$$F = \begin{cases} \hat{f}_{N_{sub}-1} f_{N_{sub}}^\ast, & \text{if } N_{RF} = N_{sub} = 2, \\ \hat{F}_{N_{sub}-1} f_{N_{sub}}^\ast, & \text{if } N_{RF}, N_{sub} = 1, N_{RF} > 2, \\ \hat{f}_{N_{sub}-1} F_{N_{sub}}^\ast, & \text{if } N_{RF}, N_{sub} = N_{RF} - 1 > 1, \\ \hat{F}_{N_{sub}-1} f_{N_{sub}}^\ast, & \text{others}, \end{cases}$$

where $f_{N_{sub}}$ or $F_{N_{sub}}$ are the last $N_{RF}, N_{sub}$ columns of $F$, and $\hat{f}_{N_{sub}-1}$ or $\hat{F}_{N_{sub}-1}$ represent the first $N_{sub}-1$ hybrid TPCs. We then consider the case presented in (9) as an example to introduce the rate decomposition processes, while the corresponding operations for other cases can be carried out similarly. The total achievable rate $R$ can be rewritten as

$$R = \log_2 \left( \mathbf{I}_{N_{t}} + \frac{\rho}{N_{t} \sigma^2} \mathbf{H} F_{RF}^\ast \mathbf{H}^\dagger \right)$$

$$= \log_2 \left( \mathbf{I}_{N_{t}} + \frac{\rho}{N_{t} \sigma^2} \mathbf{H} \hat{F}_{N_{sub}-1} F_{N_{sub}}^\ast \hat{F}_{N_{sub}-1} \mathbf{H}^\dagger \right)$$

$$\overset{(a)}{=} \log_2 \left( \mathbf{I}_{N_{t}} + \frac{\rho}{N_{t} \sigma^2} \mathbf{H} F_{N_{sub}}^\ast \hat{F}_{N_{sub}-1} \mathbf{H}^\dagger \right)$$

$$+ \frac{\rho}{N_{t} \sigma^2} \mathbf{H} F_{N_{sub}}^\ast \hat{F}_{N_{sub}-1} \mathbf{H}^\dagger \right)$$

$$= \log_2 \left( \left( \mathbf{C}_{N_{sub}-1} \right) \right)$$

$$+ \log_2 \left( \left( \mathbf{I}_{N_{t}} + \frac{\rho}{N_{t} \sigma^2} \mathbf{H} F_{N_{sub}}^\ast \hat{F}_{N_{sub}-1} \mathbf{H}^\dagger \right) \right)$$

$$+ \frac{\rho}{N_{t} \sigma^2} \mathbf{H} F_{N_{sub}}^\ast \hat{F}_{N_{sub}-1} \mathbf{H}^\dagger \right)$$

$$\overset{(c)}{=} \log_2 \left( \left( \mathbf{C}_{N_{sub}-1} \right) \right)$$

$$+ \log_2 \left( \left( \mathbf{I}_{N_{t}} + \frac{\rho}{N_{t} \sigma^2} \mathbf{H} F_{N_{sub}}^\ast \hat{F}_{N_{sub}-1} \mathbf{H}^\dagger \right) \right)$$

$$= \log_2 \left( \left( \mathbf{C}_{N_{sub}-1} \right) \right)$$

$$+ \log_2 \left( \left( \mathbf{I}_{N_{t}} + \frac{\rho}{N_{t} \sigma^2} \mathbf{H} F_{N_{sub}}^\ast \hat{F}_{N_{sub}-1} \mathbf{H}^\dagger \right) \right)$$

$$+ \frac{\rho}{N_{t} \sigma^2} \mathbf{H} F_{N_{sub}}^\ast \hat{F}_{N_{sub}-1} \mathbf{H}^\dagger \right)$$

$$\overset{(d)}{=} \log_2 \left( \left( \mathbf{C}_{N_{sub}-1} \right) \right)$$

$$+ \log_2 \left( \left( \mathbf{I}_{N_{t}} + \frac{\rho}{N_{t} \sigma^2} \mathbf{H} F_{N_{sub}}^\ast \hat{F}_{N_{sub}-1} \mathbf{H}^\dagger \right) \right)$$

$$+ \frac{\rho}{N_{t} \sigma^2} \mathbf{H} F_{N_{sub}}^\ast \hat{F}_{N_{sub}-1} \mathbf{H}^\dagger \right)$$

where $C_{N_{sub}-1} = I_{N_{t}} + \frac{\rho}{N_{t} \sigma^2} \mathbf{H} F_{N_{sub}}^\ast \hat{F}_{N_{sub}-1} \mathbf{H}^\dagger$, $C_0 = I_{N_{RF}, i}$, and $\mathbf{A} = \log_2 \left( \left( \mathbf{C}_{N_{sub}-1} \right) \right)$. Equations (5) and (6) are valid since the hybrid TPC matrix $F$ is of block-diagonal structure and the TPCs of different sub-arrays are diagonal. Step (b) is true due to the fact that we have $|AB| = |A||B|$ and $\mathbf{A} = \log_2 \left( \left( \mathbf{C}_{N_{sub}-1} \right) \right)$, $\mathbf{B} = \log_2 \left( \left( \mathbf{C}_{N_{sub}-1} \right) \right)$, $\mathbf{C} = \log_2 \left( \left( \mathbf{C}_{N_{sub}-1} \right) \right)$. Furthermore, (c) is obtained due to the fact that $|I + AB| = |I + BA|$ by defining $\mathbf{A} = \mathbf{C}_{N_{sub}-1}^{-1} \mathbf{H} F_{N_{sub}}^\ast \mathbf{H}^\dagger)$, and $\mathbf{B} = \mathbf{F}_{N_{sub}}^\ast \mathbf{H}^\dagger$. Note that the second term $\log_2 \left( \left( \mathbf{C}_{N_{sub}-1} \right) \right)$ is similar to $R$. This observation implies that we can further decompose $\log_2 \left( \left( \mathbf{C}_{N_{sub}-1} \right) \right)$ utilizing a similar method to that in (10). Step (d) represents the result after $N_{sub}$ decompositions.
Similar to [8], we adopt the idea of SIC to optimize all the sub-rates. The sub-rate optimization problem of the \(i^{th}\) sub-array can be formulated as

\[
F_i^{opt} = \arg \max_{F_i} \log_2 \left( 1 + \rho \frac{N_t \sigma^2}{N_t^2} \right),
\]

where \(P_{i-1} = H^* C_{i-1}^{-1} H\) is an \((N_t \times N_t)\) Hermitian matrix. Note that only the elements spanning from the \(A^{th}\) row to the \(B^{th}\) row within \(F_i\) are non-zero (\(N_{i,0}\) is set to be 0). Therefore, the sub-rate optimization problem (11) can be written as

\[
\tilde{F}_i^{opt} = \arg \max_{\tilde{F}_i} \log_2 \left( 1 + \rho \frac{N_t \sigma^2}{N_t^2} \right),
\]

where \(\tilde{F}_i\) of size \(N_t \times N_{RF,i}\) is the sub-matrix of \(F_i\) spanning from the \(A^{th}\) row to the \(B^{th}\) row, \(P_{i-1}\) of size \(N_t \times N_{RF,i}\) is the sub-matrix of \(P_{i-1}\) from the \(A^{th}\) row and column to the \(B^{th}\) row and column. Let us define the singular value decomposition (SVD) of the Hermitian matrix \(P_{i-1}\) as

\[
\tilde{P}_{i-1} = V_{i-1} \Sigma_{i-1} V_{i-1}^*.
\]

The total optimal unconstrained TPC matrix \(F^{opt}\) is the block diagonal concatenation of \(\tilde{F}_i^{opt}\), which can be obtained through \(N_{sub}\) iterations formulated as

\[
F^{opt} = \begin{bmatrix}
\tilde{F}_1^{opt} & & \\
& \ddots & \\
& & \tilde{F}_{N_{sub}}^{opt}
\end{bmatrix}.
\]

Since there are constant modulus constraints placed on the elements of the analog TPC matrix \(F_{RF,i}\), we cannot directly set \(F^{opt}\) as the solution of the optimization problem (5). To obtain a practical solution, we try to further convert (12).

**Lemma 1:** When \(N_{RF,i} = 1\), the optimization problem (12) can be rewritten as

\[
\tilde{F}_i^{opt} = \arg \max_{\tilde{F}_i} \log_2 \left( 1 + \rho \frac{N_t \sigma^2}{N_t^2} \right),
\]

which is equivalent to

\[
\tilde{F}_i^{opt} = \frac{1}{\sqrt{\tilde{N}_{t,i}}} \| v_{i-1} \|_2 \exp[j \angle(v_{i-1})],
\]

where \(v_{i-1}\) is the first right singular vector of \(\tilde{P}_{i-1}\).

The solution to (17) can be readily expressed as

\[
a_i = \frac{1}{\sqrt{\tilde{N}_{t,i}}} \exp[j \angle(v_{i-1})],
\]

\[
d_i = \frac{\| v_{i-1} \|_2}{\tilde{N}_{t,i}},
\]

**Algorithm 1** Hybrid TPC For the GSAC Architecture

**Initialization:** \(H, N_{RF,i}, N_{RF,j}, N_{t,i}, i = 1, 2, ..., N_{sub}\)

**Output:** The total hybrid TPC \(F\)

1. \(P = H^* H\)
2. for \(i \leq N_{sub}\) do
   3. \(\tilde{P} = V \Sigma V^*\)
   4. if \(N_{RF,i} \leq 1\) then
      5. \(a_i = \frac{1}{\sqrt{\tilde{N}_{t,i}}} \exp[j \angle(v_{i-1})], d_i = \frac{\| v_{i-1} \|_2}{\tilde{N}_{t,i}}\)
   6. end if
   7. \(\tilde{F}_i = \frac{1}{\tilde{N}_{t,i}} \| v_{i-1} \|_2 \exp[j \angle(v_{i-1})] H^*\)
   8. else
      9. \(F_{RF,i} = \frac{1}{\tilde{N}_{t,i}} \exp[j \angle(v_{i-1})] H^*\)
   10. end if
11. end for
12. \(\tilde{F} = F_{RF,i} F_{BB,i}\)
13. \(F = F_{RF,i} F_{BB,i}\)

All the above details are summarized in **Algorithm 1**.
C. Energy efficiency

The energy efficiency can be defined as the ratio of the achievable rate and of the total power consumption \( \eta \), i.e.,

\[
\eta = \frac{R}{P_{\text{total}}} = \frac{R}{P_{\text{CO}} + N_{RF}P_{RF} + N_{t}P_{PA} + N_{PS}P_{PS}},
\]

where \( P_{\text{CO}} \) is the common power of the transmitter including site-cooling, baseband processing and synchronization. \( P_{RF}, P_{PA}, \) and \( P_{PS} \) represent the power consumption of each RF chain, phase shifter and power amplifier, respectively.

IV. SIMULATION RESULTS

In this section, the performance of the proposed scheme (marked as Generalized-SIC) is evaluated. We adopt the orthogonal matching pursuit (OMP) scheme in the FC architecture [8] as the benchmarks. Moreover, the scenario when the number of RF chains in the different sub-arrays is the same is labelled as “Generalized-SIC-equal-RF”. Moreover, the channel parameters are set as \( N_{cl} = 10 \) and \( N_{ray} = 5 \). The azimuth AOAs and AODs obey the Laplacian distribution with uniformly distributed mean angles within \( (0, 2\pi) \) and angular spread of \( 7.5^\circ \) [5]. The power consumptions of the different components are set as follows: \( P_{\text{CO}} = 10\text{W}, P_{RF} = 100\text{mW}, P_{PA} = 100\text{mW}, P_{PS} = 10\text{mW} \) [7]. Finally, the signal-to-noise ratio (SNR) is defined as \( \frac{P_{RF}}{\sigma^2} \).

Fig. 2 shows the achievable rate for different numbers of antennas at the transmitter, when \( N_{t} = 36, N_{a} = N_{RF} = 8 \). We compare different RF and antenna configurations, which are (7, 1), (6, 1, 1), (5, 2, 1), (4, 4), (3, 3, 2) and (2, 2, 2, 2). For example, the mapping (5, 2, 1) means that there are 3 sub-arrays and the numbers of RF chains connected to the sub-arrays are 5, 2 and 1, respectively. It can be observed that the proposed Generalized-SIC scheme achieves a similar rate to that of the corresponding optimal unconstrained TPC scheme (15) and we have \( R_{(7,1)} > R_{(6,1,1)} > R_{(5,2,1)} > R_{(4,4)} > R_{(3,3,2)} > R_{(2,2,2,2)} > R_{\text{SAC-SIC}} \).

Fig. 3 compares the EE of different schemes for different numbers of antennas at the transmitter. It may be observed that the Generalized-SIC scheme outperforms both the FC and the SAC architectures. As for the peak values of EEs, the configuration (5, 2, 1) is the best. Additionally, the configurations (6, 1, 1) and (5, 2, 1) are better than (4, 4) and (2, 2, 2, 2).

V. CONCLUSIONS

In this paper, a GSAC architecture was proposed for improving the EE of hybrid TPC in millimeter-wave massive MIMO systems relying on arbitrary RF and antenna configurations. A variety of results were provided by varying the RF and antenna configurations in the context of our energy-efficient architecture. Our simulation results verified that the proposed scheme achieves a similar rate to the corresponding optimal unconstrained TPC scheme and attains the best energy-efficiency among the schemes investigated.

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