REGULARIZATION OF A LIGHT-FRONT $Qqq$ MODEL

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Abstract

We study the mass of the ground state of $Qqq$ systems using different regularization schemes of the relativistic integral equation obtained with a flavor independent contact interaction in a QCD-inspired light-front model. We calculate the masses of the spin 1/2 low-lying states of the $\Lambda^0$, $\Lambda^+_c$ and $\Lambda^+_b$ for different values of the regularization cut-off parameter with a fixed nucleon mass. Our results are in remarkable agreement with the experimental data.

1. INTRODUCTION

The QCD-inspired light-front constituent quark model [1, 2], with two components in the interaction, a contact term and a Coulomb-like potential, was used previously to investigate the properties of mesons [1, 3] with reasonable success. It was also applied to investigate the binding energy of the ground state of spin 1/2 $Qqq$ baryons [4], where the Coulomb-like interaction was left out. In that work, a special regularization scheme was used in which the masses of the virtual two-body subsystems were constrained to be real, as a result the quark binding in the spin 1/2 low-lying states of the $\Lambda^0$, $\Lambda^+_c$ and $\Lambda^+_b$ was qualitatively reproduced. Recently, the integral equation for a three-boson system interacting with pairwise contact interaction in the light-front [5] was regularized with a sharp cut-off [6] and applied in the study of the nucleon. In the infinite cut-off limit it was previously shown that the three-boson system is stable for values of the two-boson bound state mass above a critical value [7]. This motivate us to study the effect of different cut-offs in the $Qqq$ ground state masses, obtained by solving the light-front integral equations with a contact interaction. Our aim here is to check to which extend the qualitative properties of the quark binding is regularization independent.

We use only the flavor independent contact interaction between the constituent quarks, which brings the physical scale of the ground state of the nucleon and includes the minimal number of physical scales to describe $Qqq$ systems. The spin is averaged out. The model has as inputs the constituent quark masses and the nucleon mass is used to fix the interaction strength. In our previous work [4], where the masses of the virtual two-quark subsystems were constrained to be real with the nucleon mass fixed at the experimental value, there was no freedom left in the $Qqq$ calculation. Here, even with the nucleon mass kept fixed still the cut-off has some freedom, which will be varied in our calculations. To obtain the $Qqq$ masses, the mass of one of the constituent quark ($Q$) is varied while the strength of the effective contact interaction is supposed flavor independent.

This work is organized as follows. In Sec.2, we briefly discuss the coupled integral equations for the $Qqq$ bound-state in the light-front and the regularization schemes. In Sec. 3, we present the numerical results for the binding energies of the $\Lambda^0$, $\Lambda^+_c$ and $\Lambda^+_b$ and give a summary of the work with our conclusion.

2. INTEGRAL EQUATIONS FOR THE $Qqq$ BOUND STATE

With the assumption that the spin is averaged out, the $Qqq$ system interacting through a contact pairwise force is represented by two spectator functions or Faddeev components of the vertex, which satisfies the coupled Bethe-Salpeter type equations, derived elsewhere [4], and shown diagrammatically in Fig. 1.
The coupled integral equations for the Fadeev components of the vertex function of the $Qqq$ system, represented in Fig. 1, are given by \[^{[4]}\]:
Fig. 2: Binding energy $B_{3B}$ as a function of the baryon mass $M_{3B}$. Attributed experimental binding energies for the spin 1/2 low-lying states of the nucleon, $\Lambda^0$, $\Lambda^+_c$ and $\Lambda^+_b$ from [4] (full squares). Model results from ref. [4] (solid curve). Present calculations with transverse momentum cut-off with the values of $3m_q$ (dashed curve), $5m_q$ (dotted curve) and $7m_q$ (dot-dashed curve).

discussed constraint. The two-quark scattering amplitudes, $\tau_{\alpha q}(M_{\alpha q}^2)$, are the solutions of the Bethe-Salpeter equations in the ladder approximation for a contact interaction between the quarks.

Here, we use a different regularization scheme of Eqs. (1) and (2), where $k_{\perp}^{max} = \Lambda_{\perp}$ and the theta functions in $x$ are dropped. The nucleon mass is fixed to its experimental value, therefore we have to adjust the renormalized coupling constant for each value of the transverse momentum cut-off

$$i \tau_{\alpha q}(M_{\alpha q}^2) = \left[ \lambda^{-1} - \frac{1}{2(2\pi)^3} \int_0^1 \frac{dxd^2p_{\perp}}{x(1-x)} \frac{1}{M_{\alpha q}^2 - p^2 + (m^2_{q} - m^2_{\alpha})x + m^2_{q}x(1-x)} \right]^{-1},$$

where $\lambda$ is the bare interaction strength. As its stands, Eq. (4) is ill-defined. The renormalization condition is chosen at an arbitrary subtraction mass point $\mu$ where the value of $\tau_{qq}(\mu^2)$ is known and given by $\lambda^{-1}_{ren}(\mu^2)$, the renormalized interaction strength, which is enough to remove the logarithmic divergence in Eq. (4). Therefore, the bare strength is given as:

$$\lambda^{-1} = \lambda^{-1}_{ren}(\mu^2) + \frac{1}{2(2\pi)^3} \int_0^1 \frac{dxd^2p_{\perp}}{x(1-x)} \frac{1}{\mu^2 - \frac{p^2 + m^2_{q}}{x(1-x)}},$$

which is supposed to be flavor independent. The strength of the effective interaction is determined by fitting the nucleon mass.

3. RESULTS AND SUMMARY

The physical inputs of the model defined by Eqs. (1) and (2), are the renormalized interaction strength ($\lambda_{ren}(\mu^2)$), the constituent quark masses $m_q = m_{u,d} = 0.386$ GeV and $m_Q (Q = s, c, b)$ which were
found in [4], where as well, it was attributed a binding energy to the spin 1/2 baryons (nucleon, \(\Lambda^0\), \(\Lambda^+\) and \(\Lambda^0_b\)) defined as \(B_{3B} = 2m_q + m_Q - M_{3B}\). Considering that the flavor-off-diagonal vector mesons are weakly bound systems of constituent quarks in the QCD-inspired model of Ref. [3], we obtained values of the constituent quark masses. The attributed values to the binding energies were found [4] using constituent quark masses and the experimental baryon masses from [8]. The attributed values for the constituent quark binding energies in the spin 1/2 low-lying states of the nucleon, \(\Lambda^0\), \(\Lambda^+_c\) and \(\Lambda^0_b\) are shown in Fig. 2 by the full squares, from the left to the right, respectively.

In Fig. 2, we show results for the calculation of the binding energy of the baryon for different renormalization schemes and values of the transverse momentum cut-off compared to the attributed baryon binding energies. For the values of the cut-off, \(3m_q\), \(5m_q\) and \(7m_q\), we fit the renormalized strength of the interaction to reproduce the nucleon mass. By changing the mass of the heavy quark, while the strength of the interaction was kept unchanged, the coupled integral equations were solved to obtain the mass and binding energy of the baryons. Again, as observed in the work [4] for the baryon mass above 2.3 GeV, the bound \(Qqq\) system goes to the diquark threshold. Our results show that the general behavior of the experimental data of masses and binding energies of the spin 1/2 low-lying states of \(\Lambda^0\), \(\Lambda^+_c\) and \(\Lambda^0_b\) are reproduced with different momentum cut-off’s constrained by the value of the nucleon mass.

In summary, in the present contribution we have studied how different regularization schemes in the relativistic coupled \(Qqq\) homogeneous integral equation affects the qualitative properties of the quark binding in \(Qqq\) baryons. Our calculations indicate that the general features of the low-lying spin 1/2 baryon binding energy as a function of its mass, is model independent to a large extent (small cut-off sensitivity) as long as the relativistic \(Qqq\) model has a flavor independent effective short-range interaction. Therefore, the conclusions drawn in our previous work [4] does not qualitatively depend on the regularization scheme.

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