Running coupling constant and propagators in $SU(2)$ Landau gauge

Jacques C. R. Bloch$^a$, Attilio Cucchieri$^{b†}$, Kurt Langfeld$^a$ and Tereza Mendes$^{b†}$

$^a$Institut für Theoretische Physik, Universität Tübingen, D-72076 Tübingen, Germany

$^{b}$IFSC São Paulo University, C.P. 369 CEP 13560-970, São Carlos (SP), Brazil

We present a numerical study of the running coupling constant and of the gluon and ghost propagators in minimal Landau gauge. Simulations are done in pure $SU(2)$ lattice gauge theory for several values of $\beta$ and lattice sizes. We use two different lattice setups.

1. INTRODUCTION

We consider, on the lattice, a running coupling constant $g^2(p)$ defined by

$$g^2(p) \equiv g_0^2 \left[ p^2 D(p) \right] \left[ p^2 G(p) \right]^2$$

(1)

where $D(p)$ and $G(p)$ are, respectively, the gluon and ghost propagators evaluated in Landau gauge. Clearly $g^2(p)$ is a gauge-dependent quantity; however, notice that $g^2(p)$ is renormalization-group invariant in Landau gauge since, in this case, $Z_0 Z_\perp^{1/2} Z_\parallel = Z_1 = 1$. This running coupling strength enters the quark Dyson-Schwinger equation directly and can be interpreted as an effective interaction strength between quarks.

Studies of the coupled set of Dyson-Schwinger equations for the gluon and ghost propagators have shown that: (i) the gluon propagator behaves as $D(p) \sim p^{-2+4\kappa}$ in the infrared limit [and thus $D(0) = 0$ if $\kappa > 0.5$], (ii) the ghost propagator behaves as $G(p) \sim p^{-2-2\kappa}$ at small momenta and (iii) the running coupling strength $\alpha_s(p) = g^2(p)/4\pi$ defined in eq. (1) has a finite value $\alpha_c$ at zero momentum (infrared fixed point).

Using different approximations, in order to solve the Dyson-Schwinger equations, the following values have been obtained: $\kappa \approx 0.92$ and $\alpha_c \approx 9.5$ [8], $\kappa \approx 0.77$ and $\alpha_c \approx 11.5$ [9], $\kappa \approx 0.60$ and $\alpha_c \approx 8.9/N_c$ [10]. [Here, the first two results refer to $SU(3)$.] We stress that the large value for $\alpha_c$ obtained in [8] is related to the angular approximation used in the integration kernels. Let us notice that, using stochastic quantization [5], Zwanziger also obtained that the transverse gluon propagator in the infrared limit behaves as $D(p) \sim p^{-2+4\kappa}$ with $\kappa \approx 0.52$.

From the lattice point of view we know that lattice gauge-fixed Landau configurations belong to the region $\Omega$ delimited by the first Gribov horizon, and that $\Omega$ is not free of Gribov copies. One can also prove [11] that the restriction of the path integral to the region $\Omega$ implies a suppression of the (unrenormalized) transverse gluon propagator $D(p)$ in the infrared limit. At the same time, the Euclidean probability gets concentrated near the Gribov horizon and this implies enhancement of $G(p)$ at small momenta [12].

2. RESULTS

Simulations have been done in São Carlos for $\beta = 2.2, 2.3, \ldots, 2.8$ and $V = 14^4, 20^4, 26^4$, and in Tübingen for $\beta = 2.1, 2.15, \ldots, 2.5$ and $V = 12^3 \times 24, 16^3 \times 32$. The simulations carried out in Tübingen are based on a direct evaluation of the form factors $F(p) = D(p) p^2$ and $G(p) p^2$ appearing in eq. (1). Also, for the evaluation of $F(p)$, the gluon field has been defined in terms of the adjoint links [12] instead of the usual link variables. The gluon field obtained in this way is invariant under non-trivial $Z_2$ transformations.

Gribov-copy effects for the two propagators, if present, are smaller than the numerical accuracy [13]. Preliminary results have been presented in [14].
Figure 1. Fit for the running coupling using eq. (2) with $c_0 = 1.4(2)$, $a_0 = 5.5(3)$, $\delta = 1.77(9)$, $\Lambda = 0.83(4)$ and $\lambda$ set to 2.2.

In order to compare lattice data obtained for the two propagators at different $\beta$ values we used a standard scaling analysis \[11\] based on maximum overlap without considering any phenomenological fit functions. (Details will be presented in \[12\].) Also, for the data produced in São Carlos, we have discarded data points at small momenta that are affected by finite-size effects. (These finite-size effects are less pronounced when one evaluates the form factor directly.)

We have considered two different sets of fitting functions, namely

\[
\alpha(p) = \frac{1}{c_0 + t^\delta}\left[c_0 a_0 + \alpha_2(t + \lambda)t^\delta\right] \tag{2}
\]

\[
D(p) p^2 = A \frac{t}{c_1 + c_2 t^\gamma + t} \alpha^{13/22}(p) \tag{3}
\]

\[
G(p) p^2 = B \left(\frac{c_1 + c_2 t^\gamma + t}{t}\right) \alpha^{9/44}(p) \tag{4}
\]

where $t = p^2/\Lambda^2$ and $\alpha_2(p)$ is the 2-loop running coupling constant \[13\], and

\[
\alpha(p) = C p^{\psi}/\left[(p^4 + m)s(a)\right] \tag{5}
\]

\[
D(p) = A p^{\psi}/\left[(p^4 + m)s^{\gamma D}(a_D)\right] \tag{6}
\]

\[
G(p) = B / \left[p^2 s^{\gamma G}(a_G)\right] \tag{7}
\]

where $s(a) = (11/24\pi^2)\log[1 + (p^2/\Lambda^2)^a]$, $\gamma_D = 13/22$ and $\gamma_G = 9/44$. Note that, in the first case, the fitting functions correspond to $\kappa = 0.5$, while in the second case one has $\kappa_G = a_G\gamma_G$ and $\kappa_D = 1 - a_D\gamma_D/2$. Also, both sets of fitting functions satisfy the leading ultraviolet behavior of the two propagators.

Results of the fits are reported\[1\] in Figs. 1–5. From our data there is evidence for the suppression of the transverse gluon propagator $D(p)$ in the infrared limit and for the enhancement of the ghost propagator $G(p)$ in the same limit. Also, the running coupling strength $\alpha_s(p)$ defined in eq. (1) probably has a finite value at zero momentum. However, in order to probe the infrared region and give a final value for $\kappa$ and $\alpha_c$ one needs to simulate at larger lattice volumes.

REFERENCES

1. L. von Smekal, A. Hauck and R. Alkofer, Phys. Rev. Lett. 79 (1997) 3591; Ann. Phys. 267 (1998) 1, Erratum-ibid. 269 (1998) 182; Comput. Phys. Commun. 112 (1998) 166.

2. D. Atkinson and J. C. R. Bloch, Phys. Rev. D58 (1998) 094036; Mod. Phys. Lett. A13 (1998) 1055.

3. R. Alkofer and L. von Smekal, Phys. Rept.\footnote{Notice the logarithmic scale in the $y$ axis in Figs. 3, 4.}
Figure 3. Fit for the ghost propagator using eq. (7) with $B = 0.924(4)$, $a_G = 1.73(3)$ and $\Lambda = 1.322(8)$; this gives $\kappa_G = a_G\gamma_G = 0.354(6)$. If $\gamma_G$ is also a fitting parameter we get $\gamma_G = 0.202(5)$ to be compared with $9/44 \approx 0.2045$.

353 (2001) 281; J. C. R. Bloch, Phys. Rev. D66 034032, hep-ph/0202073.

4. C. S. Fischer, R. Alkofer and H. Reinhardt, Phys. Rev. D65 (2002) 094008; C. S. Fischer, R. Alkofer, Phys. Lett. B536 (2002) 177; C. Lerche and L. von Smekal, Phys. Rev. D65 (2002) 125006; D. Zwanziger, Phys. Rev. D65 (2002) 094039.

5. D. Zwanziger, hep-th/0206053.

6. D. Zwanziger, Phys. Lett. B257 (1991) 168; Nucl. Phys. B364 (1991) 127.

7. D. Zwanziger, Nucl. Phys. B412 (1994) 657.

8. K. Langfeld, H. Reinhardt and J. Gattnar, Nucl. Phys. B621 (2002) 131.

9. A. Cucchieri, Nucl. Phys. B508 (1997) 353.

10. A. Cucchieri, T. Mendes and D. Zwanziger, Nucl. Phys. B (Proc. Suppl.) 106 (2002) 697.

11. D. B. Leinweber et al., Phys. Rev. D60 (1999) 094507, Erratum-ibid. D61 (2000) 079901; D. Becirevic et al., Phys. Rev. D61 (2000) 114508.

12. J. C. R. Bloch, A. Cucchieri, K. Langfeld and T. Mendes, in preparation.

13. I. Hinchliffe, Eur. Phys. J. C15 (2000) 85.

Figure 4. Fit for the gluon propagator using eq. (6) with $A = 1.02(9)$, $a_D = 1.9(3)$ and $m = 0.8(3)$; this gives $\kappa_D = 1 - a_D\gamma_D/2 = 0.44(9)$. Here $\Lambda$ has been set to 1.322 (see Fig. 3). If $\gamma_D$ is also a fitting parameter we get $\gamma_D = 0.579(7)$ to be compared with $13/22 \approx 0.591$.

Figure 5. Fit for the running coupling $\alpha(p)$ using eq. (5) with $C = 0.072(8)$, $a = 1.9(3)$, $\Lambda = 1.31(1)$ and $m = 1.0(6)$. 