Intersecting M-branes

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ABSTRACT

We present the magnetic duals of Güven’s electric-type solutions of D=11 supergravity preserving 1/4 or 1/8 of the D=11 supersymmetry. We interpret the electric solutions as $n$ orthogonal intersecting membranes and the magnetic solutions as $n$ orthogonal intersecting 5-branes, with $n = 2, 3$; these cases obey the general rule that $p$-branes can self-intersect on $(p - 2)$-branes. On reduction to $D = 4$ these solutions become electric or magnetic dilaton black holes with dilaton coupling constant $a = 1$ (for $n = 2$) or $a = 1/\sqrt{3}$ (for $n = 3$). We also discuss the reduction to D=10.
1. Introduction

There is now considerable evidence for the existence of a consistent supersymmetric quantum theory in 11 dimensions (D=11) for which the effective field theory is D=11 supergravity. This theory, which goes by the name of M-theory, is possibly a supermembrane theory [1]; in any case, the membrane solution of D=11 supergravity [2], and its magnetic-dual 5-brane solution [3], (which we refer to jointly as ‘M-branes’) play a central role in what we currently understand about M-theory and its implications for non-perturbative superstring theory (see, for example, [4,5,6,7,8,9,10]). It is therefore clearly of importance to gain a fuller understanding of all the p-brane-like solutions of D=11 supergravity.

For example, it was shown by Güven [3] that the membrane solution of [2] is actually just the first member of a set of three electric-type solutions parametrized, in the notation of this paper, by the integer $n = 1, 2, 3$. These solutions are

\[
\begin{align*}
    ds^2_{(11)} &= -H^{-2n}dt^2 + H^{\frac{n-3}{2}}ds^2(E^{2n}) + H^\frac{n}{2}ds^2(E^{10-2n}) \\
    F_{(11)} &= -3 dt \wedge dH^{-1} \wedge J ,
\end{align*}
\]

where $H$ is a harmonic function on $E^{10-2n}$ with point singularities, $J$ is a Kähler form on $E^{2n}$ and $F_{(11)}$ is the 4-form field strength of D=11 supergravity. The proportion of the D=11 supersymmetry preserved by these solutions is $2^{-n}$, i.e. $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{8}$, respectively. The $n = 1$ case is the membrane solution of [2]. We shall refer to the $n = 2$ and $n = 3$ cases, which were interpreted in [3] as, respectively, a 4-brane and 6-brane, as the ‘Güven solutions’. Their existence has always been something of a mystery since D=11 supergravity does not have the five-form or seven-form potentials that one would expect to couple to a 4-brane or a 6-brane. Moreover, unlike the membrane which has a magnetic dual 5-brane, there are no known magnetic duals of the Güven solutions.

In our opinion, the p-brane interpretation given by Güven to his electric $n = 2, 3$ solutions is questionable because of the lack of $(p+1)$-dimensional Poincaré
invariance expected of such objects. This is to be contrasted with the \( n = 1 \) case, for which the solution (1.1) acquires a 3-dimensional Poincaré invariance appropriate to its membrane interpretation. In this paper we shall demystify the Güven solutions by re-interpreting them as orthogonally intersecting membranes. We also present their magnetic duals which can be interpreted as orthogonally intersecting 5-branes. The latter are new magnetic-type solutions of D=11 supergravity preserving, respectively, \( \frac{1}{4} \) and \( \frac{1}{8} \) of the D=11 supersymmetry. A novel feature of these solutions is that they involve the intersection of D=11 fivebranes on 3-branes. We shall argue that this is an instance of a general rule: \( p \)-branes can self-intersect on \((p - 2)\)-branes.

Particle solutions in four dimensions (D=4) can be obtained from M-brane solutions in D=11 by wrapping them around 2-cycles or 5-cycles of the compactifying space. This is particularly simple in the case of toroidal reduction to D=4. In this case, wrapped membranes and 5-branes can be interpreted [4] as, respectively, electric and magnetic \( a = \sqrt{3} \) extreme black holes (in a now standard terminology which we elaborate below). Here we show that Güven’s solutions, and their magnetic duals, have a D=4 interpretation as either \( a = 1 \) (for \( n = 2 \)) or \( a = 1/\sqrt{3} \) (for \( n = 3 \)) extreme electric or magnetic black holes. This D=11 interpretation of the \( a = 1, 1/\sqrt{3} \) extreme black holes in D=4 is in striking accord with a recent interpretation [11] of them (following earlier suggestions [12], and using results of [13]) as bound states at threshold of two (for \( a = 1 \)) or three (for \( a = 1/\sqrt{3} \)) \( a = \sqrt{3} \) extreme black holes.

Rather than reduce to D=4 one can instead reduce to D=10 to find various solutions of IIA supergravity representing intersecting \( p \)-branes. We shall briefly mention these at the conclusion of this paper. There is presumably an overlap here with the discussion of intersections [14] and the ‘branes within branes’ [15,16,17,18,19] in the context of D-branes, but we have not made any direct comparison. The general problem of intersecting super \( p \)-branes was also discussed in [20] in the context of flat space extended solitons. We must also emphasize that the D=11 supergravity solutions we discuss here have the interpretation we give them only
after an integration over the position of the intersection in the ‘relative transverse space’; we argue that this is appropriate for the interpretation as extreme black holes in D=4.

2. Intersecting p-branes

We begin by motivating our re-interpretation of the D=11 supergravity solutions (1.1). The first point to appreciate is that infinite planar p-branes, or their parallel multi p-brane generalizations, are not the only type of field configuration for which one can hope to find static solutions. Orthogonally intersecting p-branes could also be static. The simplest case is that of pairs of orthogonal p-branes intersecting in a q-brane, q < p. The next simplest case is three p-branes having a common q-brane intersection. Here, however, there is already a complication: one must consider whether the intersection of any two of the three p-branes is also a q-brane or whether it is an r-brane with r > q (we shall encounter both cases below). There are clearly many other possibilities once one considers more than three intersecting p-branes, and even with only two or three there is the possibility of intersections of orthogonal p-branes for different values of p. A limiting case of orthogonal intersections of p-branes occurs when one p-brane lies entirely within the other. An example is the D=11 solution of [21] which can be interpreted as a membrane lying within a 5-brane. For the purposes of this paper, orthogonal intersections of two or three p-branes for the same value of p will suffice.

Consider the case of n intersecting p-branes in D-dimensions for which the common intersection is a q-brane, with worldvolume coordinates $\xi^\mu$, $\mu = 0, 1, \ldots, q$. The tangent vectors to the p-branes’ worldvolumes that are not tangent to the q-brane’s worldvolume span a space $V$, which we call the ‘relative transverse space’; we denote its coordinates by $x^a$, $a = 1, \ldots, \ell$, where $\ell = \dim V$. Let $y$ denote the coordinates of the remaining ‘overall transverse space’ of dimension $D - q - \ell$. The D-dimensional spacetime metric for a system of static and orthogonal p-branes
intersecting in a $q$-brane should take the form

\[ ds^2 = A(x,y)d\xi^\mu d\xi^\nu \eta_{\mu\nu} + B_{ab}(x,y)dx^a dx^b + C_{ij}(x,y)dy^i dy^j. \]  

Note the $(q+1)$-dimensional Poincaré invariance. We also require that $A \to 1$, and that $B, C$ tend to the identity matrices, as $|y| \to \infty$, so the metric is asymptotic to the D-dimensional Minkowski metric in this limit.

A metric of the form (2.1) will have a standard interpretation as $n$ intersecting $p$-branes only if the coefficients $A, B, C$ functions are such that the metric approaches that of a single $p$-brane as one goes to infinity in $V$ while remaining a finite distance from one of the $n$ $p$-branes. The Güven solutions (1.1) do not have this property because they are translation invariant along directions in $V$. Specifically, they are special cases of (2.1) of the form

\[ ds^2 = A(y)d\xi^\mu d\xi^\nu \eta_{\mu\nu} + B(y)dx^a dx^b \delta_{ab} + C(y)dy^i dy^j \delta_{ij}. \]  

Because of the translational invariance in $x$ directions, the energy density is the same at every point in $V$ for fixed $y$. However, the translational invariance allows us to periodically identify the $x$ coordinates, i.e. to take $V = T^\ell$. In this case, the metric (2.2) could be viewed as that of a $q$-brane formed from the intersection of $p$-branes after averaging over the intersection points in $V$. If we insist that the $p$-branes have zero momentum in $V$-directions orthogonal to their $q$-brane, then this averaging is an immediate consequence of quantum mechanics. This delocalization effect should certainly be taken into account when the size of $V$ is much smaller than the scale at which we view the dynamics in the $y$ directions, i.e. for scales at which the effective field theory is $(D-\ell)$-dimensional. The $q$-brane solution of this effective field theory can then be lifted to a solution of the original D-dimensional theory; this solution will be of the form (2.2).

Thus, metrics of the form (2.2) can be interpreted as those of $p$-branes intersecting in a common $q$-brane. However, the solution does not determine, by itself,
the combination of $p$-branes involved. That is, when interpreted as a $q$-brane intersection of $n_\alpha p_\alpha$-branes (for $\alpha = 1, 2, \ldots$) the numbers $(n_\alpha, p_\alpha)$ are not uniquely determined by the numbers $(D, q, \ell)$. For example, the $n = 2$ Güven spacetime could be interpreted as intersections at a point of (i) 4 strings, or (ii) 2 strings and one membrane or (iii) a 0-brane and a 4-brane or (iv) 2 membranes. Additional information is needed to decide between these possibilities. In the context of M-theory, most of this additional information resides in the hypothesis that the ‘basic’ $p$-branes are the M-branes (i.e. the membrane and 5-brane), where ‘basic’ means that all other $p$-branes-like objects are to be constructed from them via orthogonal intersections, as described above. There is also additional information coming from the form of the 4-form field strength, which allows us to distinguish between electric, magnetic and dyonic solutions. With this additional information, the intersecting $p$-brane interpretation of the $n = 2, 3$ Güven solutions is uniquely that of 2 or 3 intersecting membranes.

It is convenient to consider the Güven solutions cases as special cases of $n$ $p$-branes in D dimensions pairwise intersecting in a common $q$-brane, i.e. $\ell = n(p - q)$. To see what to expect of the magnetic duals of such solutions it is convenient to make a periodic identification of the $x$-coordinates in (2.2), leading to an interpretation of this configuration as a $q$-brane in $d \equiv D - n(p - q)$ dimensions. The magnetic dual of a $q$-brane in $d$ dimensions is a $\tilde{q}$-brane, where $\tilde{q} = d - q - 4$. We must now find an interpretation of this $\tilde{q}$-brane as an intersection of $n$ $\tilde{p}$-branes in D-dimensions, where $\tilde{p} = D - p - 4$. The consistency of this picture requires that the dimension of the space $\tilde{V}$ spanned by vectors tangent to the $\tilde{p}$-branes’ worldvolumes that are not tangent to the $\tilde{q}$-brane’s worldvolume be $D - d = D - n(p - q)$. This is automatic when $n = 2$ (but not when $n > 2$). As an example, consider the $n = 2$ Güven solution, interpreted as two orthogonal membranes with a 0-brane intersection. Periodic identification of the $x$-coordinates leads to a particle-like solution in an effective D=7 supergravity theory. A particle in D=7 is dual to a 3-brane. This 3-brane can now be interpreted as the intersection of two 5-branes. The vectors tangent to the 5-branes’ worldvolumes that are not tangent to the
3-brane’s worldvolume span a four-dimensional space, so the total dimension of the spacetime is \(7 + 4 = 11\), as required.

Consider now the \(n = 3\) Güven solution, interpreted as three orthogonal membranes intersecting at a common 0-brane. Periodic identification of the \(x\)-coordinates now leads to a particle-like solution in an effective D=5 supergravity theory. A particle is dual to a string in D=5, so we should look for a solution in D=11 representing three orthogonal 5-branes whose common intersection is a string. The dimension of the space \(\tilde{V}\) spanned by the vectors tangent to the 5-branes’ worldvolumes that are not tangent to the string’s worldsheet depends on whether the common intersection of all three 5-branes is also the intersection of any pair. If it were then \(\tilde{V}\) would take its maximal dimension, \(3(5 - 1) = 14\), leading to a total spacetime dimension of \(5 + 14 = 19\). Since this is inconsistent with an interpretation in D=11, we conclude that the pairwise intersection of the three 5-branes must be a \(q\)-brane with \(q \geq 2\). In fact, the consistent choice is \(q = 3\), i.e. each pair of 5-branes has a 3-brane intersection and the three 3-branes themselves intersect in a string \(^*\). In this case \(\tilde{V}\) has dimension six, leading to a total spacetime dimension of eleven.

Note that all the cases of intersecting \(p\)-branes which we have argued should occur in M-theory have the property that \(p\)-brane pairs (for the same value of \(p\)) intersect on \((p - 2)\)-branes. Specifically, we have argued that 2-branes can intersect on 0-branes, that 5-branes can intersect on 3-branes and that these 3-brane intersections can themselves intersect on 1-branes. We shall conclude this section by explaining why we believe that this is a general rule, i.e. \(p\)-branes can self-intersect on \((p - 2)\)-branes.

Recall that the possibility of a membrane having a boundary on a 5-brane [16,17] arises from the fact that the 5-brane worldvolume contains a 2-form potential which can couple to the membrane’s string boundary. The same argument

\(^*\) A useful analogy is that of three orthogonal planes in \(\mathbb{E}^3\) which intersect pairwise on a line. The three lines intersect at a point.
does not obviously apply to intersections but it is plausible that it does, at least for those cases in which it is possible to view the $q$-brane intersection within a given $p$-brane as a dynamical object in its own right. Thus, it is reasonable to suppose that a condition for a $p$-brane to support a $q$-brane intersection is that the $p$-brane worldvolume field theory includes a $(q + 1)$-form potential to which the $q$-brane can couple. We now observe that $p$-brane worldvolume actions always contain $(D - p - 1)$ scalar fields. If one of these scalars is dualized then the worldvolume acquires a $(p - 1)$-form potential, which can couple to a $(p - 2)$-brane. Hence the rule stated above; the freedom of choice of which scalar to dualize corresponds to the possibility of an energy flow into the $p$-brane, at the intersection, in any of the directions orthogonal to its worldvolume.

3. Magnetic duals of Güven solutions

We now have sufficient information to find the magnetic duals of the series of electric solutions (1.1) of D=11 supergravity. They should be of the form (2.2) with $q = 7 - 2n$ and they should preserve some fraction of the D=11 supersymmetry. Solutions that preserve some supersymmetry can most easily be found by seeking bosonic backgrounds admitting Killing spinors. The Killing spinor equation can be found directly from the supersymmetry transformation law for the gravitino field $\psi_M$ ($M = 0, 1, 2, \ldots, 10$), and is

$$\left[ D_M + \frac{1}{144} (\Gamma_M^{NPQR} - 8\delta_M^N \Gamma^{PQR}) F_{NPQR} \right] \zeta = 0 , \quad (3.1)$$

where $D_M$ is the standard covariant derivative. Solutions $\zeta$ of this equation (if any) are the Killing spinors of the bosonic background, i.e. the D=11 metric and 4-form field strength $F_{MNPQ}$. Backgrounds admitting Killing spinors for which the Bianchi identity for $F_{(11)}$ is also satisfied are automatically solutions of D=11 supergravity. The proportion of the D=11 supersymmetry preserved by such a solution equals the dimension of the space of Killing spinors divided by 32.
By substituting an appropriate ansatz for the metric and 4-form into (3.1) we have found a series of magnetic solutions parametrised by the integer $n = 1, 2, 3$. These are

$$ds^2_{(11)} = H^{-\frac{n}{3}}(d\xi \cdot d\xi) + H^{-\frac{n+3}{3}}ds^2(E^{2n}) + H^{\frac{2n}{3}}ds^2(E^3)$$

$$F_{(11)} = \pm 3 \star dH \wedge J,$$

where $\star$ is the Hodge star of $E^3$, $J$ is the Kähler form on $E^{2n}$ and $n = 1, 2, 3$. Our conventions for forms are such that

$$J = \frac{1}{2} J_{ab} dx^a \wedge dx^b$$

$$F_{(11)} = \frac{1}{4} F_{MNPQ} dx^M \wedge dx^N \wedge dx^P \wedge dx^R.$$ (3.3)

The function $H$ is harmonic on $E^3$ with point singularities. Asymptotic flatness at ‘overall transverse infinity’ requires that $H \to 1$ there, so that

$$H = 1 + \sum_i \frac{\mu_i}{|x - x_i|},$$

for some constants $\mu_i$. Note that these solutions have an $8 - 2n$ dimensional Poincaré invariance, as required.

In the $n = 1$ case the metric can be written as

$$ds^2_{(11)} = H^{-\frac{4}{3}} d\xi \cdot d\xi + H^{\frac{2}{3}}ds^2(E^5)$$

which is formally the same as the 5-brane solution of [3]. The difference is that the function $H$ in our solution is harmonic on an $E^3$ subspace of $E^5$, i.e. our solution is a special case of the general 5-brane solution, for which $H$ is harmonic on $E^5$.

The $n = 2, 3$ cases are new solutions of D=11 supergravity with the properties expected from their interpretation as intersecting 5-branes. The solutions of the
Killing spinor equation for the background given by (3.2) are

\[ \zeta = H^{-\frac{n}{12}} \zeta_0 \]
\[ \bar{\Gamma}_a \zeta_0 = \pm J_{ab} \gamma^* \bar{\Gamma}^b \zeta_0, \quad (3.6) \]

where \( \{ \bar{\Gamma}_a; a = 1, \ldots, 2n \} \) are the (frame) constant D=11 gamma matrices along the \( E^{2n} \) directions, \( \gamma^* \) is the product of the three constant gamma matrices along the \( E^3 \) directions and \( \zeta_0 \) is a constant D=11 spinor. It follows from (3.6) that the number of supersymmetries preserved by the magnetic intersecting 5-brane solutions is \( 2^{-n} \), exactly as in the electric case.

4. D=4 Interpretation

We now discuss the interpretation of the solutions (1.1) and (3.2) in D=4. The D=4 field theory obtained by compactifying D=11 supergravity on \( T^7 \) can be consistently truncated to the massless fields of N=8 supergravity. The latter can be truncated to

\[ I = \int d^4x \sqrt{-g} \left[ R - 2(\partial \phi)^2 - \frac{1}{2} e^{-2a\phi} F^2 \right], \quad (4.1) \]

where \( F \) is an abelian 2-form field strength, provided that the scalar/vector coupling constant \( a \) takes one of the values\(^*\)[4,22]

\[ a = \sqrt{3}, 1, \frac{1}{\sqrt{3}}, 0. \quad (4.2) \]

The truncation of N=8 supergravity to (4.1) is not actually a consistent one (in the standard Kaluza-Klein sense) since consistency requires that \( F \) satisfy \( F \wedge F = 0 \). However, this condition is satisfied for purely electric or purely magnetic field configurations, so purely electric or purely magnetic solutions of the field equations

\* We may assume that \( a \geq 0 \) without loss of generality.
of (4.1) are automatically solutions of N=8 supergravity, for the above values of $a$. In particular, the static extreme electric or magnetic black holes are solutions of N=8 supergravity that preserve some proportion of the N=8 supersymmetry. This proportion is $1/2, 1/4, 1/8, 1/8$ for $a = \sqrt{3}, 1, 1/\sqrt{3}, 0$, respectively.

It is known that the membrane and fivebrane solutions of D=11 supergravity have a D=4 interpretation as $a = \sqrt{3}$ extreme black holes. Here we shall extend this result to the $n = 2, 3$ cases by showing that the electric solutions (4.1) of D=11 supergravity, and their magnetic duals (3.2) have a D=4 interpretation as extreme black holes with scalar/vector coupling $a = \sqrt{(4/n) - 1}$. As we have seen, the D=11 solutions for $n = 2, 3$, electric or magnetic, have a natural interpretation as particles in D=7 and D=5, respectively. It is therefore convenient to consider a two-step reduction to D=4, passing by these intermediate dimensions. The $n = 3$ case is actually simpler, so we shall consider it first. We first note that for $a = 1/\sqrt{3}$ the action (4.1) can be obtained from that of simple supergravity in D=5, for which the bosonic fields are the metric $ds^2_{(5)}$ and an abelian vector potential $A$ with 2-form field strength $F_{(5)}$, by the ansatz

$$ds^2_{(5)} = e^{2\phi}ds^2 + e^{-4\phi}dx^2_5$$

$$F_{(5)} = F,$$  \hspace{1cm} (4.3)

where $ds^2$, $\phi$ and $F$ are the metric and fields appearing in the D=4 action (4.1). Note that this ansatz involves the truncation of the D=4 axion field $A_5$; it is the consistency of this truncation that requires $F \wedge F = 0$. As mentioned above, this does not present problems in the purely electric or purely magnetic cases, so these D=4 extreme black hole solutions can be lifted, for $a = 1/\sqrt{3}$, to solutions of D=5 supergravity. The magnetic black hole lifts to the D=5 extreme black multi string solution [23]

$$ds^2_{(5)} = H^{-1}(-dt^2 + dx^2_5) + H^2 ds^2(\mathbb{E}^3)$$

$$F_{(5)} = * dH,$$  \hspace{1cm} (4.4)

where $*$ is the Hodge star of $\mathbb{E}^3$ and $H$ is a harmonic function on $\mathbb{E}^3$ with some
number of point singularities, i.e. as in (3.4). We get a magnetic \( a = 1/\sqrt{3} \) extreme black hole by wrapping this string around the \( x_5 \) direction.

The electric \( a = 1/\sqrt{3} \) extreme multi black hole lifts to the following solution of D=5 supergravity:

\[
ds_{(5)}^2 = -H^{-2}dt^2 + Hds^2(\mathbb{E}^3 \times S^1) \\
F_{(5)} = dt \wedge dH^{-1},
\]

where \( H \) is a harmonic function on \( \mathbb{E}^3 \). This solution is the ‘direct’ dimensional reduction of the extreme electrically-charged black hole solution of D=5 supergravity [24]. The latter is formally the same as (4.5) but \( \mathbb{E}^3 \times S^1 \) is replaced by \( \mathbb{E}^4 \) and \( H \) becomes a harmonic function on \( \mathbb{E}^4 \).

To make the connection with D=11 we note that the Kaluza-Klein (KK) ansatz

\[
ds_{(11)}^2 = ds_{(5)}^2 + ds^2(\mathbb{E}^6) \\
F_{(11)} = F_{(5)} \wedge J,
\]

where \( J \) is a Kähler 2-form on \( \mathbb{E}^6 \), provides a consistent truncation of D=11 supergravity to the fields of D=5 simple supergravity. This allows us to lift solutions of D=5 supergravity directly to D=11. It is a simple matter to check that the D=5 extreme black hole solution lifts to the \( n = 3 \) Güven solution and that the D=5 extreme black string lifts to the magnetic \( n = 3 \) solution of (3.2).

The \( a = 1 \) case works similarly except that the intermediate dimension is D=7. The KK/truncation ansatz taking us to D=7 is

\[
ds_{(11)}^2 = e^{-4\phi} ds_{(7)}^2 + e^{2\phi} ds^2(\mathbb{T}^4) \\
F_{(11)} = F_{(7)} \wedge J.
\]

where \( ds_{(7)}^2 \) is the string-frame D=7 metric. Consistency of this truncation restricts \( F_{(7)} \) to satisfy \( F_{(7)} \wedge F_{(7)} = 0 \), but this will be satisfied by our solutions. The ansatz
then taking us to $D=4$ is

$$d\hat{s}^2_{(7)} = ds^2 + ds^2(T^3)$$
$$F_{(7)} = F,$$

where $ds^2 = e^{2\phi}ds^2$ is the string-frame $D=4$ metric. Combining the two KK ansätze, it is not difficult to check that the electric $a = 1$ extreme black hole lifts to the $n = 2$ Güven solution in $D=11$ and that the magnetic $a = 1$ extreme black hole lifts to the new $n = 2$ magnetic $D=11$ solution of this paper.

5. Comments

We have extended the $D=11$ interpretation of $D=4$ extreme black hole solutions of $N=8$ supergravity with scalar/vector coupling $a = \sqrt{3}$ to two of the other three possible values, namely $a = 1$ and $a = 1/\sqrt{3}$. While the $a = \sqrt{3}$ black holes have a $D=11$ interpretation as wrapped M-branes, the $a = 1$ and $a = 1/\sqrt{3}$ black holes have an interpretation as wrappings of, respectively, two or three intersecting M-branes. We have found no such interpretation for the $a = 0$ case, i.e. extreme Reissner-Nordström black holes; we suspect that their $D=11$ interpretation must involve the gauge fields of KK origin (whereas this is optional for the other values of $a$).

The solution of $D=11$ supergravity representing three intersecting 5-branes is essentially the same as the extreme black string solution of $D=5$ supergravity. For both this solution and the $D=11$ 5-brane itself the singularities of $H$ are actually coordinate singularities at event horizons. Moreover, these solutions were shown in [23] to be geodesically complete, despite the existence of horizons, so it is of interest to consider the global structure of the solution representing two intersecting 5-branes. For this solution the asymptotic form of $H$ near one of its singularities is $H \sim 1/r$, where $r$ is the radial coordinate of $\mathbb{E}^3$. Defining a new radial coordinate
\[ \rho \text{ by } r = \rho^3, \text{ we find that the asymptotic form of the metric near } \rho = 0 \text{ is} \]

\[ ds^2_{(11)} \sim \rho^2 d\xi \cdot d\xi + \frac{1}{\rho} ds^2(\mathbb{E}^4) + 9 d\rho^2 + \rho^2 d\Omega^2 \tag{5.1} \]

where \( d\Omega^2 \) is the metric of the unit 2-sphere. ‘Spatial’ sections of this metric, i.e. those with \( d\xi = 0 \), are topologically \( \mathbb{E}^4 \times S^2 \times \mathbb{R}^+ \), where \( \rho \) is the coordinate of \( \mathbb{R}^+ \). Such sections are singular at \( \rho = 0 \) although it is notable that the volume element of \( \mathbb{E}^4 \times S^2 \) remains finite as \( \rho \to 0 \).

We have concentrated in this paper on solutions representing intersecting \( p \)-branes in \( D=11 \), i.e. M-branes, but the main idea is of course applicable to supergravity theories in lower dimensions. In fact, the intersecting M-brane solutions in \( D=11 \) can be used to deduce solutions of \( D=10 \) IIA supergravity with a similar, or identical, interpretation by means of either direct or double dimensional reduction. Direct reduction yields solutions of \( D=10 \) IIA supergravity with exactly the same interpretation as in \( D=11 \), i.e. two (for \( n = 2 \)) or three (for \( n = 3 \)) membranes intersecting at a point, in the electric case, and, in the magnetic case, two 5-branes intersecting at a 3-brane (for \( n = 2 \)) or three 5-branes intersecting at a string (for \( n = 3 \)). On the other hand, double dimensional reduction of the electric \( D=11 \) \( n > 1 \) solutions, i.e. wrapping one membrane around the \( S^1 \), gives solutions of \( D=10 \) N=2A supergravity theory representing either a string and a membrane intersecting at a point (for \( n = 2 \)) or a string and two membranes intersecting at a point (for \( n = 3 \)). In the magnetic case, the wrapping can be done in two different ways. One way, which is equivalent to double-dimensional reduction, is to wrap along one of the relative transverse directions, in which case the \( D=10 \) solutions represent either a 5-brane and a 4-brane intersecting at a 3-brane (for \( n = 2 \)) or two 5-branes and a 4-brane intersecting at a string (for \( n = 3 \)). The other way, which might reasonably be called ‘triple dimensional’ reduction, is to wrap along one of the directions in the common \( q \)-brane intersection, in which case one gets \( D=10 \) solutions representing either two 4-branes intersecting at a membrane (for \( n = 2 \)) or three 4-branes intersecting at a point (for \( n = 3 \)). We expect that some of these IIA \( D=10 \) solutions will have a superstring description via Dirichlet-branes.
Finally, we point out that the solutions (1.1) and (3.2) can both be generalized to the case in which $ds^2(\mathbb{R}^{2n})$ is replaced by any Ricci-flat Kähler manifold $\mathcal{M}^n$ of complex dimension $n$. Examples of compact manifolds $\mathcal{M}^n$ for $n = 1, 2, 3$ are $\mathcal{M}^1 = \mathbb{T}^2$, $\mathcal{M}^2 = K_3$, $\mathcal{M}^3$ a Calabi-Yau space. The new solutions of D=11 supergravity obtained in this way generalize the corresponding KK vacuum solution of D=11 supergravity to one representing an M-brane, or intersecting M-branes, wrapped around cycles in the the compactifying space. In any case, it is clear that the results of this paper are far from complete. It seems possible that a recent classification [25] of $p$-brane solutions of maximal supergravities in dimensions $D < 11$ might form a basis of a systematic M-theory interpretation, along the lines presented here, of all $p$-brane like solutions of D=11 supergravity.

Acknowledgments: We would like to thank M.B. Green, J. Gauntlett and E. Witten for helpful discussions. G.P. is supported by a University Research Fellowship from the Royal Society.

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