Robust incentive Stackelberg strategy for Markov jump linear stochastic systems via static output feedback

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Abstract: In this study, a robust static output feedback (SOF) incentive Stackelberg game for a Markov jump linear stochastic system governed by Itô differential equations with multiple leaders and multiple followers is investigated. The existence conditions for the SOF incentive Stackelberg strategies are derived in terms of the solvability of a set of higher-order cross-coupled stochastic algebraic Lyapunov type equations (CCSALTEs). A classical Lagrange multiplier technique is employed to solve the CCSALTEs; therefore, the solution of the bilinear matrix inequality, which is a common NP-hard problem when designing SOF strategies, is not required. A heuristic algorithm is developed based on the CCSALTEs. In particular, it is shown that a robust convergence is guaranteed by combining the Krasnoselskii–Mann iterative algorithm with a new convergence condition. The performance of the proposed algorithm is discussed and a simple practical example is provided to demonstrate the effectiveness of the proposed algorithm and the SOF incentive Stackelberg strategies.

1 Introduction

Over the past few decades, many studies on the various control problems for a class of stochastic systems governed by Itô differential equations with Markovian jumping parameters have been conducted. The control problems associated with Markov jump systems and their applications have gained research attention because of the existence of rapid failure processes and sudden changes in the operating points of many systems [1, 2]. Moreover, in recent years, to deal with various games such as the Pareto optimal control problem and the Nash game, there have been rapid advances in the game dynamic theory for Markov jump linear stochastic systems (MJLSSs) (see e.g. [3–8]).

When designing a practical controller and implementing the control inputs, complete state information is not always available. The availability of this information further reduces if the systems are hierarchical and/or distributed large-scale systems. In such systems, a designing method that requires complete state information is not feasible as obtaining this state information is challenging. Therefore, implementing a controller based on the local or partial state information is a feasible alternative. Therefore, researchers have particularly focused on static output feedback (SOF) controls in which the complete state information is not required. However, to design a SOF control strategy, a numerical algorithm capable of solving a bilinear matrix inequality (BMI), which is a common NP-hard problem, is required. Although a few numerical approaches [9, 10] have been proposed to address this problem, an analytical analysis of the performances of these algorithms from a practical viewpoint does not exist. Therefore, the convergence rates of the algorithms for solving BMIs are still unclear.

The SOF control problems for Markov jump linear deterministic systems have been analysed in previous studies [11–13]. Additionally, the finite-horizon SOF $H_\infty$ control problem for Markov jump systems has also been studied, and the appropriate conditions have been established in the framework of the relaxed linear matrix inequality (LMI) [14]. Although many types of incentive Stackelberg games for linear stochastic systems have been studied based on state feedback strategies [5, 6, 15, 16], very few studies have focused on SOF incentive Stackelberg strategies in dynamic games for MJLSS [4]. An incentive Stackelberg game is a game in which the leader’s strategy can influence the decisions or actions of the followers such that the leader’s desired solution (usually, a team-optimal solution) can be achieved. Recently, the SOF incentive Stackelberg game for discrete-time MJLSS has been studied in [7]. The SOF Stackelberg game for a class of continuous-time MJLSSs has also been studied without using the incentive concept [8].

This study investigates a robust SOF incentive Stackelberg game for continuous-time MJLSSs with multiple leaders and followers. Unlike the previous studies [5, 6, 8, 17], the SOF incentive Stackelberg strategies for such a game have been developed for the first time. Developing the SOF incentive strategies for Stackelberg games with multiple leaders and followers for continuous-time MJLSSs is practically significant because (i) a real-world system is usually managed by many players (or decision-makers) in a hierarchical structure and (ii) the information structure is complex; hence, the information can only be partially accessed in most cases. This game is composed of multiple leaders and multiple followers in a two-level hierarchy. The leaders on the upper level are assumed to be non-cooperative in the game and the followers are assumed to be cooperative. Even though the preliminary concepts under the cooperative pattern have been introduced in [18], the complete results and their proofs are provided in this study. The leaders construct SOFT incentive Stackelberg strategies to induce the necessary behaviours of the followers to arrive at their Nash equilibrium and attenuate external disturbances. The followers are assumed to determine their SOF strategies to achieve their Pareto optimality. The existence conditions of the SOF incentive Stackelberg strategies are provided in terms of the solvability of a set of cross-coupled stochastic algebraic Lyapunov type equations (CCSALTEs). In particular, we adopt a classical Lagrange multiplier technique, which does not require the solution of bilinear matrix inequalities (BMIs), which are a common NP-hard problem when designing SOF strategies. Furthermore, we propose a new heuristic algorithm to compute the solution set of the CCSALTEs related to the MJLSSs, in comparison with the existing algorithm in [4–8, 17, 18]. We improve the convergence robustness of the algorithm by combining the Krasnoselskii–Mann (KM) iterative algorithm [19] with a new convergence condition. The performance of the KM iteration is also analysed. Thus, it is possible to estimate the convergence rate of the proposed algorithm. A simple practical example is provided.
2 Preliminary results

Throughout this paper, \( w(t), t \geq 0 \) is a one-dimensional Wiener process that is defined on a given filtered probability space \( (\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathbb{P}) \). Without loss of generality, it is assumed that \( \{w(t)\}_{t \geq 0} \) and \( \{r(t)\}_{t \geq 0} \) are independent stochastic processes. Furthermore, the probability transition is given by

\[
P(r_{t+\Delta t} = \ell | r_t = k) = \begin{cases} n_{\ell k} \Delta t + o(\Delta t), & \text{if } k \neq \ell \\ 1 + n_{k k} \Delta t + o(\Delta t), & \text{else} \end{cases}
\]  

where \( \lim_{\Delta t \to 0} o(\Delta t)/\Delta t = 0 \), \( n_{\ell k} \geq 0 \) for \( k \neq \ell \) and \( n_{k k} = - \sum_{\ell \neq k} n_{\ell k} \).

Now, we introduce the definitions of stochastic stabilisability and stochastic detectability, which are essential assumptions in the study.

Definition 1: Consider the following linear stochastically controlled system with Markovian jumps

\[
\begin{align*}
\dot{x}(t) &= [A(r_t)x(t) + B(r_t)u(t)]dt + A_p(r_t)x(t)dw(t), \\
y(t) &= C(r_t)x(t)
\end{align*}
\]  

where \( x(t) \in \mathbb{R}^n \) represents the state vector, \( u(t) \in \mathbb{R}^m \) represents the control inputs, \( y(t) \in \mathbb{R}^r \) represents the output measurement vector, \( w(t) \in \mathbb{R} \) represents a one-dimensional standard Wiener process defined in the filtered probability space.

The coefficients \( A, A_p, B \in \mathcal{M}_{n \times n}^\mathbb{R} \), and \( C \in \mathcal{M}_{r \times n}^\mathbb{R} \) with \( A(k), A_p(k), \) and \( B(k) \), \( k \in \mathcal{D} \), being constant matrices of compatible dimensions. First, system (2) or (A, B, A_p) is called stochastically stabilisable (in mean-square sense) by SOF, if there exists a feedback control \( u(t) = F(r_t)C(r_t)x(t) \) with \( F(1), F(2), \ldots, F(s) \) being constant matrices, such that for any initial state \( x(0) = x_0 \), \( r_0 = k \), the closed-loop system

\[
\dot{x}(t) = [A(r_t) + B(r_t)F(r_t)C(r_t)]x(t)dt + A_p(r_t)x(t)dw(t), \quad x(0) = x_0
\]  

is asymptotically mean-square stable (AMSS), i.e.

\[
\lim_{t \to \infty} \mathbb{E}[\|x(t)\|^2] = 0.
\]

Second, under the condition that \( B(r_t) \equiv 0 \) means autonomous systems, \( (A, A_p) \) is called stable, if (4) holds.

The following definition has been introduced in [20].

Definition 2: The state-measurement system (2) with \( u \equiv 0 \) or \( (A, A_p|C) \) is called stochastically detectable, if there exists constant matrix \( X \) such that \( (A + XC, A_p) \) is AMSS.

Then, we introduce a few useful lemmas. The following lemma is used to evaluate the cost value in the infinite time horizon with a Markov jump parameter.

The following lemma is used to evaluate the cost value in the infinite time horizon with a Markov jump parameter [21, 22].

Lemma 1: If \( (A, A_p|C) \) is stochastic detectable, then \( (A, A_p) \) is stable iff the stochastic algebraic Lyapunov equation (5a) has a unique positive semi-definite solution \( P \). Moreover, under the assumption that the Markov jump stochastic system (2) is AMSS, we have (5b)

\[
P(k)A(k) + A_p^T(k)P(k) + A_p(k)P(k)A_p(k) + \sum_{\ell \neq k} n_{\ell k} P(\ell)C^T(k)C(k) = 0,
\]  

\[
\mathbb{E}\left[\int_0^{\infty} x^T(r_t)Q(r_t)x(r_t)dt + 2x^T(r_0)L(r_0)u(0)\right] = 0.
\]  

The next lemma is an extension of the results reported in [8] for an infinite-horizon linear quadratic control problem via the SOF for MJSSs.

Lemma 2: Consider the following stochastic linear quadratic control via the SOF strategy in the following form:

\[
\begin{align*}
\min_{u(t)} & \left\{ J[u(t), x(t), k] \right\}, \quad s. t. \ldots,
\end{align*}
\]  

\[
u(t) = F(r_t)y(t) + F(r_t)C(r_t)x(t),
\]

where

\[
J(u(t), x(t), k) = \mathbb{E}\left[\int_0^{\infty} x^T(r_t)Q(r_t)x(r_t) + 2x^T(r_0)L(r_0)u(0)\right] + u^T(r_t)R(r_t)u(t)\bigg| r_0 = k.
\]  

Suppose that the closed-loop system (3) is AMSS and \( x(0) \) is a zero mean random variable satisfying \( \mathbb{E}[x(0)x^T(0)] = I_n \). Assume that there exist \( P \) and \( G \) that satisfy the following cross-coupled Lyapunov type equations (CCSALTEs) and \( C(k)G(k)C^T(k) \) is non-singular:

\[
P(k)A(k) + B(k)F(k)C(k)] + [A(k) + B(k)F(k)C(k)]^TP(k)
\]

\[
+ A_p^T(k)P(k)A_p(k) + \sum_{\ell \neq k} n_{\ell k} P(\ell)
\]

\[
+ L(k)F(k)C(k) + C^T(k)F^T(k)L(k)^T \quad + Q(k) + C^T(k)F^T(k)R(k)F(k)C(k) = 0.
\]  

\[
G(k)A(k) + B(k)F(k)C(k) + [A(k) + B(k)F(k)C(k)]G(k)
\]

\[
+ A_p(k)G(k)A_p(k) + \sum_{\ell \neq k} n_{\ell k} G(\ell) + I_n = 0.
\]  

Then, SOF control is given below

\[
u(t) = u(t) = u'(r_t)y(t) = F'(r_t)C(r_t)x(t),
\]

where
\[ F'(k) = -R^{-1}(k)[B'(k)P(k) + L'(k)G(k)C'(k)] \times [C(k)G(k)C'(k)]^{-1}. \]

Moreover

\[ J(u(t), x^2, k) \geq J(u^*(t), x^2, k) = \text{Trace}[P(k)]. \]  \hfill (10)

**Proof:** Since the proof can be completed by using the Lagrange multiplier approach as shown in [8], it is omitted. □

Consider the following stochastic linear system with Markovian jumps:

\[ \text{dx}(t) = \left[A(r(t))x(t) + D(r(t))v(t)\right]dt + A_p(r(t))x(t)dw(t), \]  \hfill (11a)

\[ z(t) = E(r(t))x(t), \]  \hfill (11b)

where \( v(t) \in \mathbb{R}^n \) and \( z(t) \in \mathbb{R}^n \) represent the external disturbance and controlled output, respectively.

The following result is already known as the stochastic version of the bounded real lemma (see Section 8.2 in [20]) related to the \( H_{\infty} \) control for the MJLSS.

**Lemma 3:** Let us consider MJLSS (11). For a given constant \( \gamma > 0 \), suppose there exists a symmetric non-negative definite solution \( Z \) to the following cross-coupled stochastic algebraic Riccati equations (CCSAREs):

\[ Z(k)A(k) + A^T(k)Z(k) + A_p^T(k)Z(k)A_p(k) + \sum_{\ell=1}^{s} \rho_{\ell} Z(\ell) \]

\[ + \gamma^{-2}Z(k)D(k)D^T(k)Z(k) + E^T(k)E(k) = 0. \]  \hfill (12)

Then, we have

(i) The stochastic linear system with Markov jumps (11) is AMSS internally.

(ii) The following inequality holds:

\[ \| L \|_{\infty}^2 = \sup_{v \in \mathcal{L}_2(\mathbb{R}), v \neq 0} \frac{\tilde{J}_1}{\tilde{J}_2} < \gamma, \]  \hfill (13)

where

\[ \tilde{J}_1 := \sum_{i=1}^{s} E\left[ \int_0^{\infty} \| z(t) \|^2 dt \left| n_i = k \right. \right], \]

\[ \tilde{J}_2 := \sum_{i=1}^{s} E\left[ \int_0^{\infty} \| v(t) \|^2 dt \left| n_i = k \right. \right]. \]

### 3 Problem formulation

Consider the following modified MJLSS with \( m \) leaders and \( N \) followers:

\[ \text{dx}(t) = \left[A(r(t))x(t) + \sum_{j=1}^{m} \sum_{j=1}^{N} \left[ B_{lj}(r(t))u_{lj}(t) \right] + \sum_{j=1}^{N} B_{lj}(r(t))u_{lj}(t)\right]dt \]

\[ + D(r(t))v(t)\]  \hfill (14a)

\[ + A_p(r(t))x(t)dw(t), \]  \hfill (14b)

\[ z(t) = \text{col}[E(r(t))x(t) u_{1l}(t) u_{2l}(t) \ldots u_{ml}(t)], \]  \hfill (14c)

\[ \gamma_s(t) = C_s(r(t))x(t), \]  \hfill (14d)

with

\[ u_{lj}(t) = \text{col}[u_{lj}(t) \ldots u_{lN}(t) \ldots u_{mN}(t)] \]

\[ \gamma_s(t) = \text{col}[\gamma_{1l}(t) \ldots \gamma_{1N}(t) \ldots \gamma_{mN}(t)] \]

\[ C_s(r(t)) = \text{col}[C_{1l}(r(t)) \ldots C_{1N}(r(t)) \ldots C_{mN}(r(t))]. \]

where \( u_{lj}(t) \in \mathbb{R}^{m_j} \) represents the leader \( L_s \)'s input for the follower \( F_j \). \( u_{lj}(t) \in \mathbb{R}^{r_j} \) represents the follower \( F_j \)'s output measurement vector. \( \gamma_s(t) \in \mathbb{R}^{p_s} \) represents the follower \( F_j \)'s output measurement vector. The coefficients \( A, B_{lj}, B_{lj}, A_p, C_{lj}, C_{lj} \) are constant matrices of compatible dimensions, \( i = 1, \ldots, M, j = 1, \ldots, N \).

Cost functionals of the leader \( L_s \) and the follower \( F_j \) are defined as follows:

\[ J_{L_s}(u_{1s}, \ldots, u_{Ms}, v, x^2, k) \]

\[ = E\left[ \int_0^{\infty} \left\{ \sum_{j=1}^{m} \sum_{j=1}^{N} \left[ u_{lj}^T(t)R_{lj}(r(t))u_{lj}(t) \right] + \sum_{j=1}^{N} u_{lj}^T(t)R_{lj}(r(t))u_{lj}(t) \right\} dt \right| n_i = k \]  \hfill (15a)

\[ J_{F_j}(u_{1s}, \ldots, u_{Ms}, v, x^2, k) \]

\[ = E\left[ \int_0^{\infty} \left\{ \sum_{j=1}^{m} \sum_{j=1}^{N} \left[ u_{lj}^T(t)R_{lj}(r(t))u_{lj}(t) \right] + \sum_{j=1}^{N} u_{lj}^T(t)R_{lj}(r(t))u_{lj}(t) \right\} dt \right| n_i = k \]  \hfill (15b)

where \( k \in \mathcal{D}, i = 1, \ldots, M, j = 1, \ldots, N \).

\[ u_{lj}(t) = \text{col}[u_{lj}(t) \ldots u_{lj}(t) \ldots u_{lj}(t)], \]

\[ Q_{lj}(r(t)) = \sum_{j=1}^{N} \sum_{j=1}^{N} \left[ u_{lj}^T(t)R_{lj}(r(t))u_{lj}(t) \right] + \sum_{j=1}^{N} u_{lj}^T(t)R_{lj}(r(t))u_{lj}(t) \]

\[ R_{lj}(r(t)) > 0, R_{lj}(r(t)) > 0, \]

\[ R_{lj}(r(t)) = R_{lj}(r(t)) > 0, R_{lj}(r(t)) = R_{lj}(r(t)) > 0. \]

For the incentive Stackelberg game, the leader \( L_s \) announces the following incentive strategy to the follower \( F_j \) ahead of time:

\[ u_{lj}(t) = F_{lj}(r(t))C_{lj}(r(t))x(t) \]

\[ + \Xi_{lj}(r(t))u_{lj}(t) \]

\[ = \Lambda_{lj}(r(t))x(t) + \Xi_{lj}(r(t))u_{lj}(t), \]  \hfill (16)

where \( \Lambda_{lj}(r(t)) = F_{lj}(r(t))C_{lj}(r(t))x(t) \)

\[ \Xi_{lj}(r(t)) = \Xi_{lj}(r(t))F_{lj}(r(t))C_{lj}(r(t))x(t) \]

The parameters \( \Lambda_{lj}(r(t)) \) and \( \Xi_{lj}(r(t)) \) are determined following the Nash equilibrium or Pareto optimality \( u_{lj}(t) \) of the followers. In this game, leaders achieve Nash equilibrium attenuating external disturbance \( v(t) \) in the sense of a \( H_{\infty} \) constraint.

Finally, a robust SOF incentive Stackelberg game with multiple leader–follower for MJLSS can be formulated as follows.

### 3.1 Problem formulation

For a given disturbance attenuation level \( \gamma > 0 \), find, if possible, the SOF strategies

\[ \hat{u}_{lj}(t) = \hat{F}_{lj}(r(t))\hat{C}_{lj}(r(t))x(t), \]  \hfill (17a)

\[ \hat{u}_{lj}(t) = \hat{F}_{lj}(r(t))\hat{C}_{lj}(r(t))x(t), \]  \hfill (17b)

such that the following holds:

(i) The trajectory of MJLSS (14) satisfies the Nash equilibrium conditions (18a) of the leaders with an \( H_{\infty} \) constraint condition (18b):
where \( i = 1, \ldots, M \) and

\[
J_f(u_1, \ldots, u_M, v; x^i, k) = \mathbb{E} \left[ \int_0^\infty \left[ \gamma^j(t) \| v(t) \|^2 - \| z(t) \|^2 \right] \, dt \right| r_0 = k,
\]

\[
\| z(t) \|^2 = x^T(t)E^j(r_0)E(r_0)x(t) + \sum_{j=1}^M u^T_{ji}(t)u_{ji}(t).
\]

On the other hand, consider the leader's incentive strategy (16) and the worst-case disturbance \( v(t) \in \mathcal{L}_\infty(R_+; \mathbb{R}^n) \). The follower's decision \( \hat{u}_j(t) \in \mathcal{L}_\infty(R_+; \mathbb{R}^m) \), \( j = 1, \ldots, N \) can be selected as follows.

(ii) The cooperative Pareto optimal strategy is assumed, the following objective function should be minimised:

\[
J_f(u_1, \ldots, u_M, v; x^i, k) = \mathbb{E} \left[ \sum_{j=1}^N \rho_j J_f(u_{1j}, \ldots, u_{Nj}, v; x^j, k) \right],
\]

where

\[
u_{ij}(t) = \Lambda_j(r_0)x_j(t) + \Xi_j(r_0)u_{ij}(t) = \phi_j(u_{ij}),
\]

\[
\sum_{j=1}^N \rho_j = 1, \quad 0 < \rho_j < 1, \quad i = 1, \ldots, M, \quad j = 1, \ldots, N.
\]

Remark 1: Suppose that \( \mathcal{U} \) is a set of admissible controls. The controls \( u^*_j := (u^*_{1j}, \ldots, u^*_{Nj}) \in \mathcal{U} \) is Pareto optimal if the set of inequalities

\[
J_f(u_1, \ldots, u_M, v; x^i, k) \leq J_f(u^*_{1j}, \ldots, u^*_{Nj}, v; x^j, k), \quad j = 1, \ldots, N.
\]

(with at least one of the inequalities being strict) does not allow for any solution in \((u^*_{1j}, \ldots, u^*_{Nj}) \in \mathcal{U} \) for any cost functions.

Furthermore, let \( \rho_j \in (0, 1) \), with \( \sum_{j=1}^N \rho_j = 1, \quad j = 1, \ldots, N \). Assume that \( u^*_j \in \mathcal{U} \) is such that

\[
u^*_j \in \arg \min_{u_j \in \mathcal{U}} \sum_{j=1}^N \rho_j J_f(u_{1j}, \ldots, u_{Nj}, v; x^j, k).
\]

Then \( u^*_j \) is Pareto optimal.

It is unclear if all Pareto optimal controls can be obtained using this method. It should be noted that this approach may not yield any Pareto solutions even though an infinite number of Pareto solutions exist [23].

On the contrary, it is stated the Pareto optimal solutions of every player can be obtained as the solution of a constrained optimisation problem such that \( u^*_j \) minimises

\[
J_f(u_{1j}, \ldots, u_{Nj}, v; x^j, k) \leq \inf_{u_j \in \mathcal{U}} J_f(u^*_{1j}, \ldots, u^*_{Nj}, v; x^j, k), \quad j \neq \ell
\]

for \( j = 1, \ldots, N \) to use this weighted-sum method [23]. It is claimed that the above-mentioned condition is equivalent to the condition that \( \mathcal{U} \) is Pareto optimal.

### 4 Main results

In this section, the Nash equilibrium strategies of the leaders and the Pareto strategies of the followers are derived.

#### 4.1 Leader's Nash equilibrium strategy

It is assumed that leaders are non-cooperative within their group, and they will find the Nash equilibrium. Therefore, the Nash equilibrium solutions \((u^*_{1i}(t), \ldots, u^*_{Mi}(t), v^*_{t}(i))\) of the leaders are investigated in terms of how they attempt to attenuate the disturbance under an \( H_\infty \) constraint. For this purpose, let us configure the MJLSS (14) as the following centralised system:

\[
dx(t) = \left[ A(r_0)x(t) + \sum_{i=1}^M B_i(r_0)u_{i}(t) + D(r_0)v(t) \right] dt + A_r(r_0)x(t) dw(t), \quad x(0) = x^0, \]

\[
z(t) = \text{col} (z(t)) = \begin{bmatrix} E(r_0)x(t) & u_1(t) & u_2(t) & \cdots & u_M(t) \end{bmatrix},
\]

\[
u_{i}(t) = F_{i}(r_0)\gamma_{i}(t) = F_{i}(r_0)C_{i}(r_0)x(t),
\]

where

\[
B_i(k) = \begin{bmatrix} B_{i1}(k) & B_{i2}(k) & \cdots & B_{iN}(k) \end{bmatrix},
\]

\[
F_{i}(k) = \text{block diag} \left( F_{i1}(k), \ldots, F_{iN}(k) \right),
\]

\[
R_i(k) = \text{block diag} \left( R_{i1}(k), \ldots, R_{iN}(k) \right),
\]

\[
R_{FI}(k) = \text{block diag} \left( R_{F1}(k), \ldots, R_{FN}(k) \right).
\]

To obtain the Nash equilibrium strategy of the \( i \)-th leader under the \( H_\infty \) constraint, the following results are provided through Lemma 2 assuming \( L = 0 \).

**Corollary 1:** For a given disturbance attenuation level \( \gamma > 0 \), suppose that there exist \( P_{i}, G_{i}, \) and \( Z \) that satisfy the following cross-coupled Lyapunov type equations (CCSALTEs) and that \( C_{i}(k)G_{i}(k)C_{i}^T(k) \) is non-singular:

\[
P_{i}(k) = A_{i}(k)P_{i}(k) + A_{i}^T(k)P_{i}(k)A_{i}(k) + \sum_{j=1}^{1} \pi_{i} \Phi_{i}(\ell) + Q_{i}(k)
\]

\[
+ C_{i}^T(k)F_{i}^T(k)R_{i}(k)F_{i}(k)C_{i}(k),
\]

\[
G_{i}(k)A_{i}(k) + A_{i}^T(k)G_{i}(k) + A_{i}(k)G_{i}(k)A_{i}(k) + \sum_{j=1}^{1} \pi_{i} \Phi_{i}(\ell) + I_{N} = 0,
\]

\[
Z(k)A_{i}(k) + A_{i}^T(k)Z(k) + A_{i}^T(k)Z(k)A_{i}(k) + \sum_{j=1}^{1} \pi_{i} Z(\ell) + \gamma^{-2}Z(k)D(k)D^T(k)Z(k) + E_{i}^T(k)E_{i}(k) = 0,
\]

\[
A_{i}(k) = A_{i}(k) + \sum_{j=1}^{1} \pi_{i} \Phi_{i}(\ell) + Q_{i}(k) + C_{i}^T(k)F_{i}^T(k)R_{i}(k)F_{i}(k)C_{i}(k),
\]

\[
G_{i}(k)A_{i}(k) + A_{i}^T(k)G_{i}(k) + A_{i}(k)G_{i}(k)A_{i}(k) + \sum_{j=1}^{1} \pi_{i} \Phi_{i}(\ell) + I_{N} = 0,
\]

\[
Z(k)A_{i}(k) + A_{i}^T(k)Z(k) + A_{i}^T(k)Z(k)A_{i}(k) + \sum_{j=1}^{1} \pi_{i} Z(\ell) + \gamma^{-2}Z(k)D(k)D^T(k)Z(k) + E_{i}^T(k)E_{i}(k) = 0,
\]

\[
J_{L}(u_{1i}, \ldots, u_{Mi}, v; x^i, k) \leq J_{L}(u^*_{1j}, \ldots, u^*_{Nj}, v; x^j, k), \quad j \neq \ell
\]

for \( j = 1, \ldots, N \) to use this weighted-sum method [23]. It is claimed that the above-mentioned condition is equivalent to the condition that \( u^*_j \in \mathcal{U} \) is Pareto optimal.
4.2 Follower’s Pareto optimal strategy

The minimization problem can be obtained

\[
\begin{align*}
J_k(t) & = A_k(t) + \sum_{j=1}^{M} B_{ij}(k) F_{ij}(k) C_{ij}(k), \\
A_k(t) & = A_k + \sum_{j=1}^{N} B_{ij}(k) F_{ij}(k) C_{ij}(k), \\
F_{ij}(k) & = -R_{ij}(k) B_{ij}(k) P_{ij}(k) G_{ij}(k) C_{ij}(k)^T, \\
F_{ij}(k) & = -R_{ij}(k) B_{ij}(k) P_{ij}(k) G_{ij}(k) C_{ij}(k)^T, \\
E_{ij}(k) & = \begin{bmatrix}
E(k) \\
F_{1i}(k) C_{1i}(k) \\
\vdots \\
F_{Mi}(k) C_{Mi}(k)
\end{bmatrix},
\end{align*}
\]

Then, the leader’s robust SOF incentive Stackelberg strategy set of the game.

\[
\begin{align*}
u_i(t) & = u_i^*(t) = F_{i}(r_i)\xi_i(t) = F_{i}(r_i)C_{i}(r_i)x(t), \\
v(t) & = v^*(t) = F_{t}(r_t)\xi_t(t) = \gamma^2D^T(r_t)Z(t)x(t).
\end{align*}
\]

4.2 Follower’s Pareto optimal strategy

In this subsection, each follower’s strategy associated with the incentive strategy (16) and the incentive parameters \(\Xi_j(r_j)\) is thereby established. Substituting (16) together with Nash strategy set (23) into MJNSS (14a) and the cost function (15b), the following optimisation problem can be obtained

\[
\begin{align*}
\min_{\nu \in \mathcal{F}} & \quad J_{F}(u_1, \ldots, u_N; \nu, x^0, k) \\
= & \mathbb{E}\left[ \int_0^\infty [T(t) \varphi(t) + 2x^T(t) \gamma^2 D^T(r_t) Z(t)x(t)] dt \right] \\
s.t. & \quad dx(t) = [A_k(r_k) x(t) + B_k(r_k) \nu_k(t)] dt \\
& \quad \text{s.t.} \\
& \quad x(t) = [u_1(t) \cdots u_N(t) \nu(t)]
\end{align*}
\]

\[
\begin{align*}
& \quad u_i(t) = \begin{bmatrix} u_{i1} \\ \vdots \\ u_{iN} \end{bmatrix}, \\
& \quad y_{i0}(t) = \begin{bmatrix} y_{i1} \\ \vdots \\ y_{iN} \end{bmatrix}, \\
& \quad y_{ij}(t) = \begin{bmatrix} y_{ij1} \\ \vdots \\ y_{ijn} \end{bmatrix},
\end{align*}
\]

\[
\begin{align*}
& \quad y_{ij}(t) = C_{ij}(x(t)).
\end{align*}
\]

Using Lemma 2 in the above-mentioned optimisation problem, the Pareto optimal strategy set can be obtained

\[
\begin{align*}
P_k(k) & [A_{k}, B_{k}(k) F_k(k) C_k(k)] \\
+ [A_k, B_k(k) F_k(k) C_k(k)]^T P_k(k) \\
+ A_k^T P_k(k) A_k + \sum_{j=1}^{N} \pi_{ij} P_k(k) \end{align*}
\]

\[
\begin{align*}
& \quad + L_{ij}(k) G_k(k) + C_k(k) F_k(k) L_{ij}(k) + Q_{ij}(k) \end{align*}
\]

\[
\begin{align*}
& \quad + C_k^T(k) F_k^T(k) R_{ij}(k) F_k(k) C_k(k) = 0, \\
& \quad G_k(k)[A_k, B_k(k) F_k(k) C_k(k)]^T \\
& \quad + A_k G_k(k) A^T_k + \sum_{j=1}^{N} \pi_{ij} G_k(k) + I_n = 0.
\end{align*}
\]

Then, the follower’s SOF strategy based on incentive (16) can be designed by using the result in (9) of Lemma 2

\[
u_i(t) = u_i^*(t) = F_{i}(r_i)\xi_i(t) = F_{i}(r_i)C_{i}(r_i)x(t).
\]

where

\[
\begin{align*}
F_{ij}(k) & = \text{block diag} [F_{1i}(k) \cdots F_{Ni}(k)], \\
F_{ij}(k) & = \text{block diag} [F_{1i}(k) \cdots F_{Ni}(k)], \\
F_{ij}(k) & = [\rho_i R_{ij}(k) + C_k(k) F_k(k) L_{ij}(k)] + Q_{ij}(k) \end{align*}
\]

\[
\begin{align*}
& \quad \times [C_k(k) F_k(k) = 0, \\
& \quad + A_k G_k(k) A^T_k + \sum_{j=1}^{N} \pi_{ij} G_k(k) + I_n = 0].
\end{align*}
\]

Finally, we can derive the equation \(\Xi_j(k), i = 1, \ldots, M, j = 1, \ldots, N\) related to the incentive (16) by computing the equivalence \(F_{ij}(k) = 0\). Namely, \(\Xi_j(k), i = 1, \ldots, M, j = 1, \ldots, N\) satisfy the following linear algebraic matrix equations (LAMEs):

\[
\begin{align*}
\Xi_j(k) & = (B_{ij}(k) + L_{ij}(k) E_k(k)) F_k(k) P_k(k) \\
+ R_{ij}(k) C_k(k) F_k(k) + B_{ij}(k) P_k(k) = 0.
\end{align*}
\]
4.3 Heuristic algorithm

Computational methods based on iterative sequential solutions for designing strategy sets have been the research objective of many studies. In a recent study, a computational method to obtain all pure-strategy Nash equilibria of a finite game has been developed [24]. In order to obtain the robust SOF incentive Stackelberg strategy set (16), we need to solve CCSALTEs (22) and (25), and LAMEs (27), which are high-order complex equations. Although we do not need to solve the BMLIs, which are a common NP-hard problem, we still need to develop a reliable numerical method to solve these high-order complex equations. We propose a novel heuristic algorithm to solve these equations. We only discuss the solution of CCSALTE (22). The solutions of CCSALTE (25) with LAMEs (27) can be obtained by using a similar technique. The novel algorithm is based on the KM iterative algorithm [19], which allows for a robust convergence to be attained. The detailed procedure is as follows:

Step 1. Set the initial values: choose \( F_{i}^{(0)}(k), i = 1, \ldots, N \), and \( F_{i}^{(0)}(k), k = 1, \ldots, s \), such that the closed-loop MJLSS in (20) is AMSS; choose an appropriate value of \( \kappa \), and let \( F_{i}^{(0)}(k) = G_{i}^{(0)}(k) = Z^{(0)}(k) = \kappa I_{n} \).

Step 2-1. For \( k = 1, \ldots, s \), solve the following CCSALTEs for \( P_{i}^{(r+1)}(k), r \in \mathbb{N} \):

\[
P_{i}^{(r+1)}(k)A_{i}^{(0)}(k) + A_{i}^{(0)T}(k)P_{i}^{(r+1)}(k) + \sum_{j=1}^{M} B_{i,j}(k)F_{j}^{(r+1)}(k)C_{i,j}(k) + \gamma^{-2}D_{i}(k)D_{i}^{T}(k)Z^{(r+1)}(k) = 0,
\]

where

\[
A_{i}^{(0)}(k) = A(k) + \sum_{j=1}^{M} B_{i,j}(k)F_{j}^{(r)}(k)C_{i,j}(k).
\]

Step 2-2. Solve the following CCSALTEs for \( G_{i}^{(r+1)}(k) \):

\[
G_{i}^{(r+1)}(k)A_{i}^{(0)}(k) + A_{i}^{(0)T}(k)G_{i}^{(r+1)}(k) + \sum_{j=1}^{M} B_{i,j}(k)G_{j}^{(r+1)}(k)A_{i,j}(k) + \gamma^{-2}D_{i}(k)D_{i}^{T}(k)Z^{(r+1)}(k) = 0,
\]

where

\[
A_{i}^{(0)}(k) = A(k) + \sum_{j=1}^{M} B_{i,j}(k)F_{j}^{(r)}(k)C_{i,j}(k),
\]

\[
E_{i}^{(r)}(k) = \left[ \begin{array}{c} E(k) \\ F_{i}^{(r)}(k)C_{i,j}(k) \\ \vdots \\ F_{i}^{(r)}(k)C_{i,N}(k) \end{array} \right].
\]

Step 3. Set

\[
\psi^{(r+1)} = \theta^{(r)}\psi^{(r+1)} + (1 - \theta^{(r)})\psi^{(r)}.
\]

Furthermore, \( \theta^{(r)} \in (0, 1] \) is chosen at each iteration to ensure that

\[
\psi^{(r)} > \psi^{(r+1)}.
\]

Step 4. If the iterative algorithm consisting of steps 2 to 3 converges, the obtained iterative solutions are the solutions of CCSALTE (22); otherwise, when the number of iterations reaches a preset threshold, declare that there is no strategy set. Stop.

The proposed heuristic algorithm is expected to obtain a robust SOF incentive Stackelberg strategy set because the algorithm always generates a non-increasing sequence of the cost, i.e.

\[
J^{(r)} > J^{(r+1)} > \cdots > J^{(0)} > 0.
\]

The procedure of (32) in step 4 is based on the KM iteration [19]. The proposed algorithm achieves robust convergence by combining the non-increasing cost sequence and KM iteration.

Therefore, we arrive at the following theorem.

**Theorem 2**: The proposed algorithm achieves the convergence if there exists \( \theta^{(r)} \in (0, 1] \) such that for all \( r \in \mathbb{N} \), \( J^{(r)} > J^{(r+1)} \). Furthermore, a converged solution set satisfies CCSALTE (22).

**Proof**: The sequence \( \{ J^{(r)} \} \) is decreasing, and the lower bound exists from (34). It is well known that the bounded and monotonic sequences are convergent. □

It should be noted that the proposed algorithm requires an initial stabilising strategy set in Step 1, which is a non-trivial problem. Hence, to apply the proposed algorithm, it is necessary to first find these stable gains. The computational complexity of this problem is NP-complete because the required strategy set can be computed via the proposed algorithm, within the polynomial-time although it is related to the well-known SOF problem involving two LMLIs and a non-convex coupling condition [25, 26].

In the following section, we discuss the performance of the proposed algorithm and review the performances of a few other
algorithms. Let us define the following non-expansive mapping $T$ that is defined by a numerical algorithm:

$$
\psi^{t+1} = T(\psi^t).
$$  \hspace{1cm} (35)

Moreover, let us define the following variable:

$$
\sigma_t := \sum_{i=0}^{t} \theta^e (1 - \theta^e).
$$  \hspace{1cm} (36)

In this case, the following estimation has been proved in [27], proved \( \lim_{t \to \infty} \sigma_t = \infty \):

$$
\| (I_{n^2} - T)(\psi^t) \| = \| \psi^{t+1} - \psi^t \| = o\left(\frac{1}{\sqrt{\sigma_t}}\right). \hspace{1cm} (37)
$$

For example, \( \theta^e = 1/2 \) is selected, then \( \sigma_t = t/4 \), \( \| (I_{n^2} - T)(\psi^t) \| \) can be established. This condition is used in the following calculations.

It should be noted that the quantity \( \| (I_{n^2} - T)(\psi^t) \| \) is used as a measure of the convergence rate because \( \| (I_{n^2} - T)(\psi^t) \| = 0 \) if and only if \( T(\psi) = \psi \) and the property \( \lim_{t \to \infty} \| (I_{n^2} - T)(\psi^t) \| = 0 \) is always satisfied [27].

Other algorithms to solve SOF-related problems have been proposed previously. For example, the numerical algorithm based on iterative sequential solutions of two convex LMI problems has been proposed to determine SOF stabilising controllers [9]. However, this algorithm does not always converge globally. It has been proved that the algorithm is capable of solving a few practical control examples. However, it is not clear if the algorithm is capable of solving SOF related dynamic games with multiple inputs. Moreover, there exists an alternative approach for solving the SOF control problem [28]. Even though the algorithm is practically feasible, the convergence of the algorithm has not been studied.

Although these well-known algorithms for solving SOF control problems are useful and reliable [9, 10], it is unclear whether these algorithms can also be used to solve SOF-related dynamic games. Unlike these algorithms, the proposed algorithm is capable of directly and iteratively solving higher-order complicated coupled non-linear equations without the use of other numerical procedures, thereby proving the superiority of the proposed algorithm in comparison with previous algorithms. The proposed algorithm can be used to determine the solution set of CCSALTE and SOF strategies.

The performance and computational complexity of the proposed algorithm have been analysed based on the KM iteration. The analysis results provide the convergence rate. Therefore, the convergence speed can be estimated via mathematical analyses.

\section{A simple practical example}

To demonstrate the effectiveness and usefulness of the theoretical results presented in the previous section, a simple practical example is provided in this section. Consider an \( R-L-C \) electronic circuit as shown in Fig. 1. In this system, \( R_i, k = 1, 2, \) and \( R_d(r_i) \), \( r_i = 1, 2 \) denote the resistances; \( L \) denotes the inductance and \( \eta_0 \) denotes the internal resistance of the inductor; \( C_k \), \( k = 1, 2 \) denote the capacitances. Moreover, \([u_1(t), u_2(t)]\), \([u_{21}(t), u_{22}(t)]\), \([w_1(t), w_2(t)]\), and \([w_{11}(t), w_{12}(t)]\) represent the applied voltages that are the control inputs of the leaders and followers, respectively; \( i(t) = x_i(t) \) represents the electrical current flowing through the inductor; \( V_i(t) := x_i(t), k = 1, 2 \) represent the voltage drop across the capacitors; and \( v(t) := i(t)/L \) represents an external deterministic disturbance. In practical applications, thermal noise is always present in electronic devices at normal operating temperatures. It is assumed that the position of switch SW3 follows a continuous-time Markov process \( \{r(t), t \geq 0\} \) with two states, \( s = 2 \). Thus, the system can be represented as a stochastic system governed by an MJLSS. If the impact of thermal noise can be modelled as a real-valued state-dependent Wiener process \( w(t) \) with diffusion coefficients, the stochastic system can be represented using

$$
\begin{align*}
\frac{dx(t)}{dt} &= \left[ A(r_i)x(t) + \sum_{i=1}^{2} \sum_{j=1}^{2} [B_{ij}(r_i)u_{ij}(t)] + B_{ij}(r_i)w_{ij}(t) \right]dt \\
&+ A(r_i)x(t)dw(t), x(0) = x^0. \\
\end{align*}
$$  \hspace{1cm} (38a)

$$
\begin{align*}
&z(t) = \text{col}[E(r_i)x(t), u_{11}(t), u_{22}(t)], \\
&y_{ij}(t) = C_{ij}(r_i)x(t). \\
\end{align*}
$$  \hspace{1cm} (38b)

where

\begin{align*}
M &= 2, N = 2, s = 2, \\
[\pi_{11} & \pi_{12} ] & [\pi_{21} & \pi_{22} ] \\
[0.3 & 0.3] & [0.7 & -0.7] \\
A(r_i) &= \begin{bmatrix}
-\frac{R_i(r)}{L} & 0 \\
0 & -\frac{1}{RC_i} \\
0 & 0 & -\frac{1}{RC_i^2}
\end{bmatrix}, \\
B_{ij}(r_i) &= 0.05A(r_i), \\
C_{ij}(r_i) &= [1 0, i,j = 11, 12, 21, 22], \\
D(r_i) &= 0 , E(r_i) = [1 1], R_i = 1, 2.
\end{align*}

It should be noted that we consider that 5% of the state coefficient \( A_p \) can be represented by a Wiener process for the diffusion coefficient as Brownian motion, due to thermal noise, caused by the discreteness of electric charges [29].

When numerically solving this problem, we set the following parameters in the simulations:
\[ R_d(1) = 10 \Omega, \quad R_d(2) = 30 \Omega, \quad r_d = 0.2 \Omega, \]
\[ R = 200 \Omega, \quad R_r = 400 \Omega, \]
\[ C_i = 1500 \mu F, \quad C_2 = 2500 \mu F. \]

The weight matrices of cost functions are given by

\[
Q_{k,k}(k) = Q_{k,k}(k) = Q_{k,k}(k)
\]
\[ R_{k,k}(k) = R_{k,k}(k) = R_{k,k}(k) = R_{k,k}(k)
\]
\[ R_{k,k}(k) = R_{k,k}(k) = R_{k,k}(k) = R_{k,k}(k)
\]
\[ R_k(k) = R_k(k) = R_k(k) = R_k(k)
\]
\[ R_{k,k}(k) = R_{k,k}(k) = R_{k,k}(k) = R_{k,k}(k)
\]

\[ \kappa = 1, 2, \quad \rho_{1} = 0.25, \quad \rho_{2} = 0.75. \]

Next, we select \( \gamma = 7 \). Using the proposed recursive algorithms, the leader's Nash strategy set \( u_c(t) = F(L_{i,j}(r_i))C_{i,j}(r_i)C(t) \), \( i = 1, 2, \)
\( j = 1, 2, \) and the worst case disturbance is computed as follows:

\[ F_{L(1)}(1) = \begin{bmatrix} 5.0536 \times 10^{-2} \end{bmatrix}, \quad F_{L(1)}(1) = \begin{bmatrix} 5.0536 \times 10^{-2} \end{bmatrix}
\]
\[ F_{L(2)}(1) = \begin{bmatrix} 5.0536 \times 10^{-2} \end{bmatrix}, \quad F_{L(2)}(1) = \begin{bmatrix} 5.0536 \times 10^{-2} \end{bmatrix}
\]
\[ F_{F(1)}(1) = \begin{bmatrix} 3.7582 \times 10^{-2} \end{bmatrix}, \quad F_{F(1)}(1) = \begin{bmatrix} 3.7582 \times 10^{-2} \end{bmatrix}
\]
\[ F_{F(2)}(1) = \begin{bmatrix} 3.7582 \times 10^{-2} \end{bmatrix}, \quad F_{F(2)}(1) = \begin{bmatrix} 3.7582 \times 10^{-2} \end{bmatrix}
\]
\[ F_{L(1)}(2) = \begin{bmatrix} 4.9608 \times 10^{-2} \end{bmatrix}, \quad F_{L(2)}(2) = \begin{bmatrix} 4.9608 \times 10^{-2} \end{bmatrix}
\]
\[ F_{F(1)}(2) = \begin{bmatrix} 4.9608 \times 10^{-2} \end{bmatrix}, \quad F_{F(2)}(2) = \begin{bmatrix} 4.9608 \times 10^{-2} \end{bmatrix}
\]
\[ F_{L(1)}(2) = \begin{bmatrix} 3.6655 \times 10^{-2} \end{bmatrix}, \quad F_{L(2)}(2) = \begin{bmatrix} 3.6655 \times 10^{-2} \end{bmatrix}
\]
\[ F_{F(1)}(2) = \begin{bmatrix} 3.6655 \times 10^{-2} \end{bmatrix}, \quad F_{F(2)}(2) = \begin{bmatrix} 3.6655 \times 10^{-2} \end{bmatrix}
\]
\[ F_{L(2)}(2) = \begin{bmatrix} 2.1938 \times 10^{-2} \end{bmatrix}, \quad F_{L(2)}(2) = \begin{bmatrix} 2.1938 \times 10^{-2} \end{bmatrix}
\]
\[ F_{F(2)}(2) = \begin{bmatrix} 2.1938 \times 10^{-2} \end{bmatrix}, \quad F_{F(2)}(2) = \begin{bmatrix} 2.1938 \times 10^{-2} \end{bmatrix}
\]

Finally, it can be observed that the strategy of the followers based on the incentive \( \Xi_{ij}(r_i) \) in (16) is induced to the Nash equilibrium strategy. In particular, the relation \( F_{F(j)}(r) = F_{F(j)}(r) \) is satisfied.

We employ the proposed KM iterative algorithm to obtain converged solutions and strategies. The initial gains are set as

\[ F_{i}^{(0)}(k) = - \begin{bmatrix} 1 \end{bmatrix}, \quad k = 5, \]

for \( k = 1, 2, \) and \( i = 1, 2. \) The initial condition was selected by the trial and error method, such that the closed-loop system is stable. The algorithms converge after 46 iterations with an accuracy of \( 10^{-10} \).

In step 3 of the heuristic algorithms, the value of \( \theta_{0} \) is set to 0.5. The cost values in the iterations of the KM iterative algorithm are listed in Table 1, which shows that the proposed algorithm generates a non-increasing sequence for the cost. Therefore, the convergence property of (34) is observed.

### 6 Conclusion

In this study, a robust SOF incentive Stackelberg game with \( M \) leaders and \( N \) followers in the two-level decision hierarchy for continuous-time MILSSs with external disturbances has been analysed. The \( M \) leaders are assumed to be non-cooperative in the game and the \( N \) followers are assumed to be cooperative in the game. This is the first time SOF constraint incentive Stackelberg strategies for the leaders to arrive at their Nash.
equilibrium and the followers to achieve Pareto optimality have been developed. The existence conditions of the SOF incentive Stackelberg strategies are provided in terms of the solvability of a set of CCSALTEs. A classical Lagrange multiplier technique is used to solve the considered optimisation problem, thereby eliminating the need for solving BMIs, which are a common NP-hard problem when designing SOF strategies. Furthermore, a novel numerical algorithm based on the KM iteration is developed to guarantee converged solutions. A simple practical example is solved to demonstrate the effectiveness of the proposed algorithm and the SOF incentive Stackelberg strategies.

7 References

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