Buried treasure in the sand of the QCD vacuum

P.J. Moran, D.B. Leinweber

Special Research Centre for the Subatomic Structure of Matter (CSSM), Department of Physics,
University of Adelaide, Adelaide SA 5005, Australia

The short-range structure of the 2+1 flavour QCD vacuum is studied through visualisations of the topological charge density. Of particular interest is a new Gaussian weighted smearing algorithm which is applied to the rough topological charge density to disclose underlying long-range structure. The results provide support for the view of the QCD vacuum as a sandwich of sign-alternating sheets of charge, with a long-range structure hidden beneath.

1. Introduction

The topological charge density of the QCD vacuum is most often obtained through either of two possible approaches. The first is through the gluonic definition,

\[ q(x) = \frac{g^2}{32\pi^2} \varepsilon_{\mu\nu\rho\sigma} \text{Tr}[F_{\mu\nu}(x)F_{\rho\sigma}(x)], \]

in combination with some cooling\cite{12,13} or smearing\cite{14,15,16} algorithm. Significant work has been invested into refining the relevant cooling/smearing operators in order to reduce the effects of discretisation errors during the cooling/smearing process\cite{17,18,19}.

Another approach is through the fermionic definition\cite{20,21}

\[ q(x) = -\text{tr} \left[ \gamma_5 \left( 1 - \frac{a}{2} D(0; x, x) \right) \right], \]

defined with the overlap Dirac operator\cite{22,23,24}

\[ D(0) = \rho \left( 1 + D_W / \sqrt{D_W^* D_W} \right) = \rho \left( 1 + \gamma_5 \text{sgn}(H_W) \right), \]

where \( H_W = \gamma_5 D_W \) and \( D_W \) is a Wilson-type Dirac operator. This definition satisfies the Atiyah-Singer index theorem

\[ Q = n_- - n_+, \]

and hence, will always return an integer total topological charge \( Q \). The overlap Dirac operator is far more computationally expensive to compute than using either
cooling or smearing, however advances in techniques and computing power have enabled some interesting studies.

Recently, Ilgenfritz et al.\textsuperscript{14} have demonstrated how the use of the spectral representation for the Dirac operator,\textsuperscript{16,17}

\begin{equation}
q_{\lambda\text{cut}}(x) = -\sum_{|\lambda|<\lambda\text{cut}} \left(1 - \frac{\lambda}{2}\right) \psi_\lambda^\dagger(x) \gamma_5 \psi_\lambda(x),
\end{equation}

enables one to calibrate the number of over-improved stout-link smearing\textsuperscript{3} sweeps to a given spectral cut-off, $\lambda\text{cut}$. In particular, five sweeps of smearing was matched with the full overlap density, and it was found that a $\lambda\text{cut}$ of 634 MeV corresponds to approximately 45 sweeps of over-improved stout-link smearing, as seen in Fig. 1.

The strong correlation between the different approaches justifies the use of over-improved smearing for efficient, large scale studies of QCD vacuum structure. For example, using five sweeps of smearing to generate a negative $\langle qq\rangle \equiv \langle q(x)q(0)\rangle$ correlator\textsuperscript{7} the effects of dynamical fermions on the short-range structure of the vacuum have been studied.\textsuperscript{15} In the following, we use over-improved stout-link smearing to further probe the UV structure of the QCD vacuum.

### 2. Short distance structure

The negative behaviour of the $\langle qq\rangle$ correlator\textsuperscript{19} suggests a sign-alternating sheet-like topological structure exists in the QCD vacuum.\textsuperscript{20,21}

Using a $28\times96$, dynamical lattice with $am_{u,d} = 0.0062$, $am_s = 0.031$, generously provided by the MILC collaboration\textsuperscript{22,23} we use five sweeps of over-improved smearing in combination with a 3-loop improved topological charge operator\textsuperscript{9} to study the short-range vacuum structure.

In Fig. 2 we present two visualisations of the topological charge density. In the left graphic we have plotted the full topological charge density range includ-
Fig. 2. [left] The short-distance sheet structure of the vacuum is clearly apparent after five sweeps of over-improved stout-link smearing. Negative charge density is green to blue, and positive charge density is yellow to red. [right] The same data, this time with the positive charge removed and the magnitude of the negative charge shown through the strength of the blue colouring.

Raising the cutoff threshold for $q(x)$ so that only the most intense regions of charge are shown presents a different view of the vacuum, as seen in the left graphic of Fig. 3. Here the vacuum appears to have a granular, sand-like structure. Ilgenfritz has investigated this idea of clustering deeply. In the right plot of Fig. 3 shows the topological charge density after 45 sweeps of over-improved stout-link smearing, where we see the familiar instanton-like lava lamp structure. It is tempting to try and find similarities between the two plots, however the vast number of sandy objects prohibits us from doing so.

In order to draw some comparisons between the two different sheet and lava pictures of the QCD vacuum we need some method of averaging the sheet structure of Fig. 2. For this, we define a Gaussian smoothing operation to act on the topological charge density itself. Given $q(x)$ for some gauge field, simultaneously update each point $x_0$ on the lattice according to,

$$
q(x_0) = \frac{1}{2\pi\sigma^2} \sum_x e^{-r^2/2\sigma^2} q(x),
$$

where $r$ is the Euclidean distance between $x$ and $x_0$, and $\sigma$ is the standard deviation of the Gaussian distribution in lattice units.

The evolution of the topological charge density of Fig. 2 under Gaussian smooth-
Fig. 3. [left] Placing a high cutoff on $q(x)$ such that only the most intense regions of charge are seen presents a picture of the QCD vacuum which resembles the sand of one of Australia’s many fine beaches. The grains of sand will diminish in size as the continuum limit is approached. [right] The topological charge after 45 sweeps of over-improved stout-link smearing.

ing with increasing $\sigma$ is shown in Fig. 4. The resulting effect appears to be quite similar to that seen in previous cooling/smearing animations.

In Fig. 5 there is a side by side comparison of the Gaussian smoothed charge density with $\sigma = 1.75$ and the topological charge density obtained after 45 sweeps of smearing. Recall that the Gaussian smoothed charge density was generated from the topological charge density after only five smearing sweeps. Although the resulting densities are certainly not identical, they still share many common features.

This kind of calculation suggests that the UV sheet structure of the vacuum is embedded on top of an underlying long-range structure. With the long-range structure revealed through the application of a cooling/smearing operator or through a truncated overlap Dirac operator. This kind of idea has been discussed previously by P. de Forcrand. 24

3. Conclusion

Using a Gaussian smoothing procedure the rough topological charge density obtained from five sweeps of over-improved stout-link smearing was compared with that obtained from 45 sweeps. Modest agreement was found using a Gaussian standard deviation of $\sigma = 1.75$. These results suggest that the QCD vacuum consists of a sandwich of high-energy fluctuations, with a long-distance structure hidden beneath. This is represented graphically in Fig. 6.
Fig. 4. The evolution of $q(x)$ obtained from five sweeps of over-improved stout-link smearing under Gaussian smoothing. The values of $\sigma$ used were 0.25 [top-left], 0.75 [top-right], 1.25 [bottom-left], and 1.75 [bottom-right].

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Fig. 5. [left] The Gaussian smoothed topological charge density after five smearing sweeps, using \( \sigma = 1.75 \). [right] The topological charge density after 45 smearing sweeps. The two pictures are far from identical, however it is remarkable that a few common features are present.

Fig. 6. An example of how UV fluctuations could be superimposed on a deeper long-range structure, as suggested by P. de Forcrand [24].

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