Maximum capacity model of grid-connected multi-wind farms considering static security constraints in electrical grids

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Abstract. An increasing interest in wind energy and the advance of related technologies have increased the connection of wind power generation into electrical grids. This paper proposes an optimization model for determining the maximum capacity of wind farms in a power system. In this model, generator power output limits, voltage limits and thermal limits of branches in the grid system were considered in order to limit the steady-state security influence of wind generators on the power system. The optimization model was solved by a nonlinear primal-dual interior-point method. An IEEE-30 bus system with two wind farms was tested through simulation studies, plus an analysis conducted to verify the effectiveness of the proposed model. The results indicated that the model is efficient and reasonable.

1. Introduction
Since wind energy is an environment friendly and renewable resource, installed wind power generation capacity is continuously increasing. However, as wind energy is increasing integrated into power systems, it has significant influence on the steady-state security of electrical grids, such as voltage stability problems and overloading of some transmission lines. For the power users, over-voltage will lead to a reduction in equipment lifetime, increase in energy consumed with no improvement in performance and even a puncture of insulation. While under-voltage will lead to the failure of premature equipment, failing to start up, and then increase temperature in the case of motor windings and loss of service. For the power system itself, it will suffer from a collapse of voltage and ultimately cascading blackouts. Therefore, the accepted maximum capacity issue of wind farms in power systems should be investigated in order to avoid system stability problems[1-5].

Similar studies for a given power system have been performed in literature [6-8], which present methods to find the maximum capacity of one grid-connected wind farm. The determination of the capacity in a wind farm is formulated as a maximization issue of the number of wind turbines.

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However, there are generally more than one wind farm in an actual power system, and thus this paper investigates the maximum grid-connected capacity issue considering multi-wind farms.

The whole paper is structured as follows: firstly, the introduction is given; secondly, the power model in the wind farm is described, and then this paper introduces the proposed mathematics model of grid-connected wind capacity considering multi-wind farms in detail; finally, numerical simulation and analysis are elaborated to verify the feasibility of the proposed optimization model and method.

2. Power model in the wind farm

The power output of wind turbine is determined by the type of wind turbines. Usually, there are two kinds of wind turbine systems in current applications \cite{8,9}: the fixed-speed wind turbine system and the variable speed wind turbine system. The double-fed wind power generator regarded as a widely used variable speed wind turbine is chosen in this paper. So the active power for a variable speed wind turbine is as follows:

\[
P_{\text{wind}} = \begin{cases} 
0 & 0 \leq v < v_i \\
a + bv^3 & v_i \leq v < v_r \\
P_v & v_r \leq v \leq v_0 \\
0 & v > v_0
\end{cases}, \quad a = \frac{P_v v_i^3}{v_r^3 - v_i^3}, \quad b = \frac{P_v}{v_r^3 - v_i^3}
\]  

\(1\)

where \(v\) is wind speed, \(v_i\) is cut-in wind speed, \(v_r\) is rated wind speed, \(v_0\) is cut-out wind speed, \(P_v\) is rated power of wind turbine.

If all the wind turbines are identical in a wind farm, and then the total active power output of the wind farm is given by

\[
P_{\text{farm}} = NP_{\text{wind}}
\]

\(2\)

where \(N\) is the number of wind turbines in a wind farm.

The power factor or voltage of the bus which is connected to a wind farm will maintain constant by effective control, supposing the wind turbines operate at the constant power factor. Therefore, the reactive power output of the wind turbine can be expressed as

\[
Q_{\text{wind}} = \frac{P_{\text{wind}}}{\cos \phi} \left(1 - \cos^2 \phi \right)^{1/2}
\]

\(3\)

where \(\cos \phi\) is power factor. The total reactive power output of the wind farm is equal to

\[
Q_{\text{farm}} = NQ_{\text{wind}}
\]

\(4\)

3. Mathematics model of grid-connected wind capacity issue

As mentioned above, the determination of total capacity of all the grid-connected wind farms can be formulated as a maximization wind power accepted by systems, while satisfying the power flow equations and other several operation limits, including the restrictions of system stability, security and power quality, etc. In this paper, the objective function of optimization model is the sum of the power outputs of all the wind farms which can be expressed as

\[
\max \sum_{m=1}^{M} N_m P_{\text{wind},m}
\]

\(5\)

where \(N_m\) and \(P_{\text{wind},m}\) are the number of wind turbines and power output per wind turbine with wind farm of \(m\), respectively.
It can also be rewritten in the following form:

$$\min - \sum_{n=1}^{N} N_n P_{\text{wind},n}$$  \hspace{1cm} (6)

The constraints include equality and inequality constraints, and the power flow equations at each bus are treated as equality constraints, which must be satisfied. For the bus which is not connected to a wind farm, the active power and reactive power flow equations are

$$\Delta P = P(x) - P_G + P_L = 0$$
$$\Delta Q = Q(x) - Q_G + Q_L = 0$$  \hspace{1cm} (7)

where $x = [V \quad \theta \quad P_G \quad Q_G \quad N]^T$, $x$ is the optimization variable, $V$ is the magnitude vector of bus voltages, $\theta$ is the angle vector of bus voltages, here $N$ denotes the number vector of wind turbines, $P_G$ and $Q_G$ are active power and reactive power of conventional generators, $P_L$ and $Q_L$ are active power and reactive power demands. $P(x)$ and $Q(x)$ are nodal active and reactive power functions.

While the power flow equations at the bus connected to a wind farm are given by

$$\Delta P_{\text{wind}} = P_{\text{wind}}(x) - P_{\text{farm}} - P_{G,i_{\text{wind}}} + P_{L,i_{\text{wind}}} = 0$$
$$\Delta Q_{\text{wind}} = Q_{\text{wind}}(x) - Q_{\text{farm}} - Q_{G,i_{\text{wind}}} + Q_{L,i_{\text{wind}}} = 0$$  \hspace{1cm} (8)

where $i_{\text{wind}}$ indicates the bus number at which the wind farm is connected to the grid.

The main concerns of this paper are operational limits of conventional generator, including the voltage limits, active power and reactive power limits of conventional generators, and the thermal limits of transmission lines in the grid system. They are given by

$$V_{\text{min}} \leq V \leq V_{\text{max}}$$
$$P_{G,\text{min}} \leq P_G \leq P_{G,\text{max}}$$
$$Q_{G,\text{min}} \leq Q_G \leq Q_{G,\text{max}}$$
$$-P_{L,\text{max}} \leq P_L \leq P_{L,\text{max}}$$
$$-P_{L,\text{max}} \leq P_L \leq P_{L,\text{max}}$$  \hspace{1cm} (9)

4. Case study analysis

4.1. Setup of the test system

To examine the feasibility and merits of the proposed optimization model, the IEEE-30 test system consisting of two wind farms was studied. There are 2 generators and 4 synchronous condensers in the power system. Bus 1 is considered as a slack bus, and the synchronous condenser at bus 11 and 13 are replaced by wind farms. Double-fed wind turbine is selected maintaining a constant power factor, and the power factors in these two different wind farms are set to 0.95 and 0.98, respectively. All the maximum and minimum voltage limits are set to 1.06 (p.u.) and 0.94 (p.u.), respectively. In order to simplify the calculation, only active power constrains of main line are involved in the model. And the thermal limits of main line 7, 9 and 22 are equal to 150 MW, 130 MW and 110 MW, respectively. The rated power output of wind turbine in the wind farm connected at bus 11 is 2 MW, and that in the wind farm connected at bus 13 is 1.5 MW. The initial wind turbine numbers of these two wind farms are both set to 100.

4.2. Simulation results
In order to obtain the optimal solution of the proposed model, the nonlinear primal-dual interior point method is applied to solve the grid-connected wind capacity issue \cite{10}. The related program was coded using Matlab language, and then the result was achieved by running the optimization program for 2.9 seconds. The variation process of total capacity of grid-connected wind farms is given in Figure 1. Simulation results demonstrate that the final capacity of the grid-connected wind farms is 375.9 MW, and the optimal wind turbine number of the wind farm connected to bus 11 is 88, the other one is 132.

![Figure 1. The iteration curve of total capacity of grid-connected wind farms.](image)

| Generator bus number | Active power (MW) | Generator bus number | Reactive power (Mvar) |
|----------------------|-------------------|----------------------|-----------------------|
| 1                    | -200              | 1                    | 101.17                |
| 2                    | 0                 | 2                    | 50                    |
| 5                    | 100               | 5                    | 40                    |
| 8                    | 27.39             | 8                    | 40                    |
| 11                   | 177.11            | 11                   | 58.21                 |
| 13                   | 198.79            | 13                   | 40.37                 |

**Table 1.** Active and reactive power of conventional generator and wind farm in the optimal solution.

| Bus number | Magnitude of bus voltage (p.u.) | Bus number | Magnitude of bus voltage (p.u.) | Bus number | Magnitude of bus voltage (p.u.) |
|------------|---------------------------------|------------|---------------------------------|------------|---------------------------------|
| 1          | 1.0600                          | 11         | 1.0346                          | 21         | 0.9669                          |
| 2          | 1.0543                          | 12         | 0.9927                          | 22         | 0.9676                          |
| 3          | 1.0253                          | 13         | 1.0096                          | 23         | 0.9582                          |
| 4          | 1.0228                          | 14         | 0.9779                          | 24         | 0.9549                          |
| 5          | 1.0522                          | 15         | 0.9693                          | 25         | 0.9588                          |
| 6          | 1.0233                          | 16         | 0.9748                          | 26         | 0.9400                          |
| 7          | 1.0276                          | 17         | 0.9736                          | 27         | 0.9754                          |
| 8          | 1.0261                          | 18         | 0.9593                          | 28         | 1.0162                          |
| 9          | 0.9842                          | 19         | 0.9571                          | 29         | 0.9545                          |
| 10         | 0.9798                          | 20         | 0.9618                          | 30         | 0.9424                          |

**Table 2.** Magnitude of bus voltage in the optimal solution.

The specific optimal results are shown in Tables 1-3. It can be found that the active power output of the conventional generator connected to bus 5 reaches the upper boundary, the active power outputs
of the conventional generators connected to bus 1 and bus 2 reach the lower boundary, the reactive power outputs of the conventional generators connected to bus 2, bus 5 and bus 8 reach the upper limit, the voltage magnitude of bus 1 reaches the upper boundary, and that of bus 26 reaches the lower boundary. Other optimal variables are between the upper and lower limits. That means we can get the acceptable maximum capacity of grid-connected wind farms while satisfying the operation constraints of power systems.

Table 3. Calculation results for the active power flow of transmission line.

| Branch number | Active power flow of transmission line $P_{ij}$ (MW) | Active power flow of transmission line $P_{ji}$ (MW) |
|---------------|-------------------------------------------------|-------------------------------------------------|
| 7             | -34.74                                          | 34.88                                           |
| 9             | 44.74                                           | -44.14                                          |
| 22            | 12.98                                           | -12.79                                          |

5. Conclusions
A deterministic optimization model is used to obtain the maximum capacity of grid-connected multi-wind farms, in which several important limitations are considered including voltage stability limits, active power and reactive power limits of conventional generators, and the thermal limits of transmission lines in the power system. The proposed model is implemented on an IEEE-30 bus test system with two wind farms. The optimal solution is determined by the nonlinear primal-dual interior point method. The results indicate that the proposed model is efficient and the solution is reasonable. In other words, we can get the acceptable maximum capacity of grid-connected wind farms and the optimal allocation of system voltages and generator power outputs while satisfying the static security constraints in electrical grids. Moreover, in order to consider the variable and stochastic characteristics of wind power output, the uncertainty of wind speed will be involved using a probabilistic method in the future works.

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