We show that the Standard Model electroweak interaction of ultrarelativistic electrons with nucleons ($eN$ interaction) in a neutron star (NS) permeated by a seed large-scale helical magnetic field provides its growth up to $\gtrsim 10^{15}$ G during a time comparable with the ages of young magnetars $\sim 10^{4}$ yr. The magnetic field instability originates from the parity violation in the $eN$ interaction entering the generalized Dirac equation for right and left massless electrons in an external uniform magnetic field. The averaged electric current given by the solution of the modified Dirac equation contains an extra current for right and left electrons (positrons). Such current includes both a changing chiral imbalance of electrons and the $eN$ potential given by a constant neutron density in NS. Then we derive the system of the kinetic equations for the chiral imbalance and the magnetic helicity which accounts for the $eN$ interaction. By solving this system, we show that a sizable chiral imbalance arising in a neutron protostar due to the Urca-process $e_{\text{L}}^{-} + p \rightarrow N + e_{\text{L}}$, diminishes very rapidly because of a huge chirality flip rate. Thus the $eN$ term prevails the chiral effect providing a huge growth of the magnetic helicity and the helical magnetic field.

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Some neutron stars (NSs), called magnetars, having magnetic fields $B \sim 10^{15} - 10^{16}$ G, can be considered as strongest magnets in our universe [1]. Despite the existence of various models for the generation of such strong fields, based, e.g., on the turbulent dynamo [2], the origin of magnetic fields in magnetars is still an open problem. Recently, in Ref. [3] the authors tried to apply the chiral magnetic effect [4, 5], adapted successfully for the QED plasma [6], to tackle the problem of magnetic fields in magnetars. The approach of Ref. [3] implies the chiral kinetic theory, where Vlasov equation is modified when adding the Berry curvature term to the Lorentz force [7].

The fate of such a chiral plasma instability is based on the Adler anomaly in QED with the nonconservation of the pseudovector current for massless fermions $\tilde{\gamma}_{\mu} \gamma_{5} \gamma^{\nu}$ in external electromagnetic fields. Since this current is the difference of right $j_{\mu}^{\text{R}}$ and left $j_{\mu}^{\text{L}}$ currents, the assumption of a seed imbalance between their densities given by the difference of chemical potentials, $(n_{R} - n_{L}) \sim \mu_{5} = (\mu_{R} - \mu_{L})/2 \neq 0$, where $n_{R,L}$ are the densities of right and left fermions (electrons) and $\mu_{R,L}$ are their chemical potentials, could lead to the magnetic field instability we study here adding electroweak interactions in the Standard Model (SM).

The same effect (while without weak interactions) was used in Ref. [8] to study the self-consistent evolution of the magnetic helicity in the hot plasma of the early Universe driven by the change of the lepton asymmetry $\sim \mu_{5}$. In Ref. [8] it was showed that such an asymmetry diminishes, $\mu_{5} \rightarrow 0$, due to the growth of the chirality flip rate in the cooling universe through electron-electron ($ee$) collisions, $\Gamma_{f} \sim \alpha_{\text{em}}^{2} (\frac{m_{e}}{T})^{2}$, where $\alpha_{\text{em}} = \frac{e^{2}}{\pi \alpha} \approx \frac{1}{137}$ is the fine structure constant, $m_{e}$ is the electron mass, and $T$ is the plasma temperature.

This negative result encouraged the appearance of Ref. [9], where another mechanism for the generation of magnetic fields was proposed. It is based on the parity violation in electroweak plasma resulting in the nonzero Chern-Simons (CS) term $\Pi_{2}$ that enters the antisymmetric part of the photon polarization operator in plasma of massless particles. Here we adopt the notation for the CS term from Ref. [9]. In Ref. [10], a similar CS term $\Pi_{2}^{(\nu)}$, based on the neutrino interactions with charged leptons, was calculated. Basing on this calculation, the magnetic field instability driven by neutrino asymmetries was revealed. This instability is implemented in different media such as the hot plasma of the early universe and a supernova (SN) with a seed magnetic field.

The amplification of a seed magnetic field during the SN burst driven by a non-zero electron neutrino asymmetry $\Delta n_{\nu_{e}} \neq 0$ which enters the CS term $\Pi_{2}^{(\nu)}$ was suggested in Ref. [10] to explain the generation of strongest magnetic fields in magnetars. Note that after the SN burst a cooling NS as the corresponding SN remnant emits equally neutrinos and antineutrinos. Thus, the neutrino asymmetry vanishes. The inclusion of the electroweak $ee$-interaction with a stable fraction of degenerate electrons $n_{e} \approx \text{const}$ instead of the $\nu e$ interaction with vanishing neutrino asymmetry $\Delta n_{\nu_{e}} \rightarrow 0$ has no sense since the corresponding parity violating CS term $\Pi_{2}^{(\nu)}$ tends to zero in the static limit $\omega \rightarrow 0$ for an elec-
electron gas, $\Pi^{(ee)}_2 \to 0$, as found in Ref. [11].

In the present Letter we suggest to take into account the electroweak electron-nucleon ($eN$) interaction providing a long time acting source of the $B$ field instability that plays a role of a CS term in the pseudovector electron current $J_5 \sim \Pi^{(eN)}_2 B$. Instead of the Matsubara technics used in Refs. [10, 11], here we calculate the total electric current in SM (additive to the standard ohmic current) solving the Dirac equation for the massless right and left electrons (positrons) in a magnetic field.

We start the derivation of the aforementioned CS term with solving the Dirac equation for a massless electron in the magnetic field $B = (0, 0, B)$ accounting for the electroweak $eN$ interaction in NS. This equation reads as

$$[\gamma^\mu (i\partial_\mu + eA_\mu) - \gamma^0 (V_{L,R} P_L + V_{R,L} P_R)] \psi_e = 0, \quad (1)$$

where $\gamma^\mu = (\gamma^0, \gamma^i)$ are the Dirac matrices, $A^\mu = (0, 0, Bx, 0)$ is the vector potential, $P_{L,R} = (1 \mp \gamma^5)/2$ are the chiral projection operators, $\gamma^5 = i\gamma^0 \gamma^1 \gamma^2 \gamma^3$, and $e > 0$ is the absolute value of the electron charge.

In Eq. (1) we assume that there are no macroscopic fluid (nucleon) currents in NS. The effective Lagrangians $V_{L,R}$ in Eq. (1) are given by the SM Lagrangians of the $eN$ interaction via neutral currents in the Fermi approximation (see, e.g., Ref. [12]),

$$\mathcal{L} = \sqrt{2} G_F \bar{\psi}_e \gamma^\mu (g_{l}^{(e)} P_L + g_{r}^{(e)} P_R) \gamma_\mu \psi_e \times [\bar{\psi}_n \gamma^\mu \psi_n - (1 - 4\xi) \bar{\psi}_p \gamma^\mu \psi_p], \quad (2)$$

where $G_F \approx 1.17 \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant $g_{l}^{(e)} = -\frac{1}{2} + \xi$ and $g_{r}^{(e)} = \xi$ are the standard coupling constants in SM with the Weinberg parameter $\xi = \sin^2 \theta_W \approx 0.23$, and $\psi_{n,p}$ are the neutron and proton wave functions. We reduced the total $eN$ Lagrangian in Ref. [12] to Eq. (2) omitting the axial nucleon currents $\sim \bar{\psi}_{n,p} \gamma^5 \gamma^\mu \gamma^\nu \psi_{n,p}$ irrelevant to our problem.

Taking the statistical averaging $\langle \ldots \rangle$ in Eq. (2) over the equilibrium (Fermi) distributions of nucleons in a neutron star and recalling that macroscopic nucleon currents are absent, i.e. $\langle \bar{\psi}_{n,p} \gamma^\mu \psi_{n,p} \rangle = 0$, we get the following definition of $V_{L,R}$ to be used in Eq. (3):

$$V_L = -\frac{G_F}{\sqrt{2}} \left[n_n - n_p (1 - 4\xi)\right] (2\xi - 1),$$

$$V_R = -\frac{G_F}{\sqrt{2}} \left[n_n - n_p (1 - 4\xi)\right] 2\xi, \quad (3)$$

where $n_{n,p} = \langle \bar{\psi}_{n,p}^0 \psi_{n,p} \rangle$ are the number densities of neutrons and protons.

Let us decompose $\psi_e$ in the chiral projections as $\psi_e = \psi_L + \psi_R$, where $\psi_{L,R} = P_{L,R} \psi_e$. Then, using Eq. (1) we get that $\psi_{L,R} = e^{-iE_{L,R} \tau + ip_y y + ip_z z} \psi_{L,R}(x)$, where

$$\psi_{L,R}^{(n)}(x) = \frac{1}{4\pi \sqrt{E_{L,R}}} \left(\begin{array}{c} \sqrt{E_{L,R} + p_z u_{n-1}(\eta)} \\ \mp i \sqrt{E_{L,R} + p_z u_n(\eta)} \\ \mp i \sqrt{E_{L,R} + p_z u_{n-1}(\eta)} \\ i \sqrt{E_{L,R} + p_z u_n(\eta)} \end{array}\right),$$

$$\psi_{L,R}^{(0)}(x) = \frac{1}{2\pi \sqrt{2}} \left(\begin{array}{c} 0 \\ u_0(\eta) \\ 0 \\ \mp u_0(\eta) \end{array}\right). \quad (4)$$

Here $\psi_{L,R}^{(n)}$ corresponds to $n = 1, 2, \ldots$, $\psi_{L,R}^{(0)}$ to $n = 0$, $\eta = \frac{\sqrt{\epsilon B x} + p_y}{\sqrt{\epsilon B}} u_0(\eta) = (\epsilon B)^{1/4} \exp(-\eta^2/2) H_n(\eta)$, and $H_n(\eta)$ is the Hermite polynomial. The upper signs in Eq. (4) stay for $\psi_L$ and the lower ones for $\psi_R$. To derive Eq. (4) we use the $\gamma$ matrices in the Dirac representation.

The energy levels $E_{L,R}$ in Eq. (4) can be obtained from the following expression: $(E_{L,R} - V_{L,R})^2 = p_z^2 + 2eBn$. It is worth mentioning that a more general solution of Eq. (4), which accounts for $m_e$, was found in Ref. [13].

Using the formalism developed in Ref. [4] and Eq. (4) one gets the averaged induced current given by the main Landau level $n = 0$ only,

$$J = -\frac{2\alpha_{em}}{\pi} (\mu_5 + V_5) B, \quad (5)$$

which is additive to the ohmic current $J_{O\text{hm}}$ in a standard QED plasma. This current is proportional to $\alpha_{em}$ and consists of the two parts: the vector term given in QED by the pseudoscalar coefficient $\mu_5 = (\mu_R - \mu_L)/2$ (\$\mu_L \to -\mu_5$ under spatial inversion) and the pseudovector current $J_5 = \frac{2\alpha_{em}}{\pi} V_5 B = \Pi^{(eN)}_2 B$ given in SM by the scalar factor $V_5 = (V_L - V_R)/2$, as seen from the parity violation term in the SM Lagrangian in Eq. (2).

The weak interaction coefficient

$$V_5 = \frac{G_F}{2\sqrt{2}} [n_n - (1 - 4\xi)n_p] \quad (6)$$

is of the order $V_5 \approx \frac{G_F}{2\sqrt{2}} n_n = 6 eV$ in NS with $n_n = 1.8 \times 10^{38}$ cm$^{-3}$, which corresponds to $\rho_n = 3 \times 10^{14}$ g cm$^{-3}$, since $n_p \ll n_n$. On the first sight, the electromagnetic QED term in the current in Eq. (5), $\sim \mu_5$, seems to be much bigger than the weak one in Eq. (6) [14]. However, we show below that the latter term remains as a stable source of the magnetic field instability in a supernova while the former one vanishes, $\mu_5 \to 0$, e.g., for helical magnetic fields with the maximum helicity contrary to the statement in Ref. [7] that an imbalance $\mu_5 \neq 0$ could lead to the generation of strong magnetic fields in magnetars.

To describe the magnetic field evolution driven by the $eN$ interaction we use fact that the Adler anomaly for the pseudovector current in electromagnetic fields,
\[ \partial_t (j^\mu - j^\mu_B) = \partial_\mu (\bar{\psi} \gamma^\mu \gamma^5 \psi) = (2\alpha_{em}/\pi)E \cdot B, \]

being integrated over the volume \( V \), is proportional to the magnetic helicity density change,

\[ \frac{dh(t)}{dt} = -\frac{2}{V} \int d^3x E \cdot B. \tag{7} \]

Thus the helicity conservation law in QED plasma reads

\[ \frac{d}{dt} \left[ n_R - n_L + \frac{\alpha_{em}}{\pi} h(t) \right] = 0. \tag{8} \]

Using Eq. (6) in Ref. [8] for the maximum magnetic helicity density spectrum \( h(k, t) = 2\rho_B(k, t)/k \) and adding the contribution of the electron-electron interaction in the NS core \( 2V_5 = 12 \) eV according to the current in Eq. (5), we get that

\[ \frac{\partial h(k, t)}{\partial t} = -\frac{2}{\sigma_{cond}} \left( k(\Delta\mu + 2V_5) \right) h(k, t), \tag{9} \]

where \( \Delta\mu = \mu_R - \mu_L = 2\mu_5 \). It should be noted that the helicity density \( h(t) \) in Eq. (7) is related to the helicity density spectrum in Eq. (8) by \( h(t) = \int d^3k h(k, t) \).

Taking into account that \( n_L = \mu_L^3/3\pi^2 \) and assuming that \( \mu_L \sim 2\mu_L \sim \mu \), where \( \mu \) is the chemical potential of the degenerate electron gas in NS, that is true at least at the beginning of the imbalance in NS, we get \( n_R - n_L \approx 2\mu_5^2/\pi^2 \). Eventually we obtain from Eq. (9), using the expression for the diffusion equation for \( \mu_5 \),

\[ \frac{d\mu_5}{dt} = \frac{\pi\alpha_{em}}{\mu^2\sigma_{cond}} \int dk k^2 h^2(k, t) \]

\[ -\left( \frac{2\alpha_{em}^2 \rho_B(t)}{\mu^2\sigma_{cond}} \right) (\mu_5 + V_5) - \Gamma_f \mu_5, \tag{10} \]

where the magnetic energy density \( \rho_B(t) = \int dk \rho_B(k, t) = \frac{1}{2} \int d^3k h(k, t) \).

In Eq. (10) we added the rate of chirality-flip processes, \( \Gamma_f \approx (m_e/\mu)^2/\nu_{coll} \), given by the Rutherford electron-proton (ep) collision frequency \( \nu_{coll} = \omega_p^2/\sigma_{cond} \) without flip. Here \( \omega_p = \mu \sqrt{4\alpha_{em}/3\pi} \) is the plasma frequency in a degenerate ultrarelativistic electron gas and \( \sigma_{cond} \) is the electric conductivity in a degenerate electron-proton plasma consisting of ultrarelativistic degenerate electrons and non-relativistic degenerate protons. Note that in a degenerate electron gas \( \nu_{coll} \) depends on the temperature \( T \); cf. Ref. [17]. This is due to the Pauli principle when all electron states with the momenta \( 0 \leq p \leq \mu \) are busy, i.e. ep scattering is impossible at \( T = 0 \).

One can see that Eq. (10) is different from the simplified kinetic approach \( \frac{d\mu_5}{dt} = \Gamma_{inst}\mu_5 - \Gamma_f \mu_5 \) used in Refs. [3, 4]. The first term in the rhs of Eq. (10) can be really estimated as \( \sim \alpha_{em}^2 \mu_5^2 \) for all “equal” parameters \( \mu \sim \mu_5 \sim \sigma_{cond} \) that is not the case we rely on. The more important difference is the appearance of the second term \( \sim \rho_B(t) \) that is the back reaction from magnetic field that diminishes an imbalance \( \mu_5 \).

Let us choose the simplest case of the monochromatic helicity density spectrum \( h(k, t) = h(t)\delta(k - k_0) \) where we can vary the wave number \( k_0 \) and the magnetic field scale \( \Lambda_B = k_0^{-1} \) to find later some critical regimes for the imbalance evolution \( \mu_5(t) \) through Eq. (10). Using the dimensionless functions \( M(\tau) = \frac{\alpha_{em}}{\pi k_0} \mu_5(t) \) and \( H(\tau) = \frac{\alpha_{em}^2}{\kappa_{em} k_0} h(t) \) which depend on the dimensionless diffusion time \( \tau = \frac{2\alpha_{em}^2}{\kappa_{em} k_0} t \) we can recast the self-consistent system of Eqs. (9) and (10) as

\[ \frac{dM}{d\tau} = (1 - M - V)H - GM, \]

\[ \frac{dH}{d\tau} = - (1 - M - V)H. \tag{11} \]

Here for fixed \( V_5 = 6 \) eV the dimensionless parameters \( V = \frac{2\alpha_{em}}{\pi k_0} V_5 \) and \( G = \frac{\alpha_{em}^2}{2k_0} \Gamma_f = \frac{2\alpha_{em}}{3\pi} \left( \frac{m_e}{k_0} \right)^2 \) are the function of the parameter \( k_0 \) only. Note that \( G \) does not depend on the conductivity \( \sigma_{cond} \) since the rate of the chirality flip can be estimated as \( \Gamma_f \approx \left( \frac{m_e}{\mu} \right)^2 \nu_{coll}^{(no flip)} \) where in the magento-hydrodynamic plasma \( \nu_{coll}^{(no flip)} = \omega_p^2/\sigma_{cond} \) is the ep collision frequency without flip. The dimensionless diffusion time \( \tau = \frac{2\alpha_{em}^2}{\kappa_{em} k_0} t \) depends on the conductivity found in Ref. [17].

\[ \sigma_{cond} = 1.6 \times 10^{28} \frac{m_e}{10^{36} \text{cm}^{-3}}^{3/2} \text{s}^{-1}, \tag{12} \]

that is valid for cooling NS matter consisting of degenerate non-relativistic nucleons and ultrarelativistic degenerate electrons.

For the magnetic field scale \( \Lambda_B \) comparable with the NS radius \( R_{NS} = 10 \) km, or for the small wave number \( k_0 = 1/R_{NS} = 2 \times 10^{-11} \) eV, one gets the electroweak interaction contribution in Eq. (11) as \( V = 7 \times 10^8 \) coming from the current in Eq. (5), where we substitute the small \( V_5 = 6 \) eV. We choose the initial chiral imbalance as \( \mu_5(0) = 1 \) MeV \( \ll \mu \), where for \( n_e = \mu^3/3\pi^2 = 10^{36} \text{cm}^{-3} \) in Eq. (12) the electron chemical potential equals to \( \mu = 60 \) MeV. Hence from the beginning the dimensionless chiral imbalance \( M(0) = \frac{\alpha_{em}^2}{\pi k_0} \mu_5(0) \sim 10^{14} \) occurs much bigger than the electroweak term \( V \). On the first glance, such inequality could be expected comparing electromagnetic and weak interaction effects, \( M(0) \gg V = \text{const} \). We assume also the constant temperature in a cooling neutron star \( T = 10^8 \) K [18]. Therefore the electric conductivity in Eq. (12) is also constant, \( \sigma_{cond} = 10^7 \) MeV.

The chirality flip rate

\[ G = \frac{2\alpha_{em}}{3\pi} \left( \frac{m_e}{k_0} \right)^2 = 9.8 \times 10^{30}, \tag{13} \]
is huge for the given small $k_0 = 2 \times 10^{-14}$ keV. If we change $m_e = 511$ keV → $m_{\text{eff}} = \mu \sqrt{2 \mu_{\text{eff}}}$, the rate in Eq. (13) would be even bigger diminishing $\mu_5$ faster in the first line in Eq. (11). Finally, for the acceptable initial magnetic field $B_0 = 10^{17}$ G, the initial helicity density $h(0) = B_0^2/k_0 = 2 \times 10^{13}$ MeV$^3$ gives $H(0) = \frac{1}{8 \pi} h(0) = 6 \times 10^{21}$.

We solved the system of the self-consistent kinetic equations in Eq. (11) numerically for the adopted $\nu$ and $G$ as well as the initial conditions $M(0) = 10^{14}$ and $H(0) = 6 \times 10^{21}$ chosen above. In Fig. 1 we plot the evolution of the chiral imbalance $M(\tau)$. In the inset, one can see how a large chirality imbalance $\mu_5 \sim O(\text{MeV})$ vanishes owing to the huge chirality flip rate in Eq. (13), $\mu_5 \rightarrow 0$, during a very short time $\tau \sim 10^{-30}$ corresponding to $t \sim 10^{-12}$ s. In the main plot one finds a sharp slope for $M$ somewhere at $\tau \approx 3 \times 10^{-8}$ that corresponds to the time $t \sim 8000$ yr. The obtained critical time is the order of young magnetar ages [1]. In Fig. 2 we see that, at the same time $\tau \approx 3 \times 10^{-8}$, the magnetic helicity density $H$ grows on about ten orders of magnitude, that corresponds to the growth of $B = \sqrt{K_0 h}$ on the five orders of magnitude, just getting $B \approx 10^{17}$ G if we started from the seed field $B_0 = 10^{12}$ G.

It is interesting to mention that, in Fig. 1 a positive primeval chirality imbalance, $\mu_5 = \left( \mu_R - \mu_L \right) / 2 > 0$, which appears, e.g. due to the direct Urca process, $\epsilon_L + p \rightarrow n + \nu_L$, becomes negative, $\mu_R - \mu_L < 0$. This happens due to the simultaneous growth of the helicity density $h$ (see in Fig. 2) that amplifies the negative derivative $\frac{d}{d\tau} M < 0$ much more intensively than the chirality flip $\sim G$. Vice versa, the attenuation of $M$ owing to the chirality flip is more important at the first stage illustrated in the inset of Fig. 1. Since $M \rightarrow -V = 7 \times 10^8$ (see in Fig. 2), while the decreasing sum $V + M$ remains positive, the value of the positive derivative $\frac{d}{d\tau} H > 0$ diminishes, or the helicity evolution simulates a saturation, see in Eq. (11) and in Fig. 2.

Finally we notice that rather helical magnetic fields determine the evolution of the chiral imbalance $\mu_5(t)$ than a non-zero seed $\mu_5 \neq 0$ leads to the growth of the magnetic helicity density $h = B^2/k_0$ or the magnetic field itself. This imbalance starting from a sizable value $\mu_5 \sim O(\text{MeV})$ decreases down to the $eN$ interaction term $|\mu_5| \sim V_5 \approx 6$ eV. We stress that namely the electroweak interaction term $V_5 > |\mu_5|$ drives the amplification of the seed magnetic field in NS, see in the second line in Eq. (11). Of course, taking into account the cooling of a neutron star, $\frac{d}{d\tau} T < 0$, a more realistic model to generate strong magnetic fields in magnetars can be developed. We plan to do that in our future work.

Of course, we considered here only the largest scale $k_0^{-1} = R_{\text{NS}} = 10$ km as the most interesting case for magnetic fields in NS. Our model is simplified both due to the choice of the maximum helicity density $kh(k, t)$, of $2\rho_{\text{NS}}(k, t)$ instead of the more general inequality $kh(k, t) \leq 2\rho_{\text{NS}}(k, t)$ [19], and due to the choice of the monochromatic helicity density spectrum $h(k, t) = h(t) \delta(k - k_0)$.

We would like to mention that recently in Ref. [20] the application of the chiral plasma instability in SN is also criticized due to a huge chirality flip underestimated in Ref. [3] in the approximation $\left( \frac{m^2}{\mu} \right)^2 \ll 1$. This is because in Ref. [3] the authors relied on the flip rate $\Gamma_f \sim \alpha^2 \left( \frac{m}{\mu} \right)^2 \mu_5$ meaning rather the collision frequency without flip as $\nu_{\text{coll}} \approx \left( \frac{\alpha_{\text{em}}^2}{\mu^2} \right) \mu^3 \mu \sim \mu_5$ in analogy with the common formula $\nu_{\text{coll}} = \sigma \nu_e$ where $\sigma \propto \alpha_{\text{em}}^2 / \langle E \rangle^2$ is the Rutherford cross-section for $ep$ collisions. This assumption is incorrect for a degenerate electron gas because the Pauli principle was not taken into account.

To resume we suggested here a novel mechanism for the magnetic field amplification in a neutron star that is absolutely different from the well-known model put for-
ward in Ref. [2] based on a strong turbulent convection in the core of SN and fast dynamo operating only for a few seconds, being driven by the high neutrino luminosity $L_\nu > 10^{52} \text{erg} \cdot \text{s}^{-1}$ at that time. We refute also arguments in Ref. [3] suggested to explain the generation of strong magnetic fields in magnetars based on the chiral magnetic instability. It should be noted that, in Ref. [21], it was found that protostars, which were progenitors to some magnetars, did not seem to reveal a fast rotation as required in Ref. [2]. This fact is in favor to our model where a magnetic field grows owing to the $eN$ interaction.

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