A possible alternative to the Breit frame in DIS

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Abstract
A new Lorentz frame for DIS jet finding is suggested.

Talk presented at the 6th Int. Workshop
on Deep Inelastic Scattering and QCD: DIS 98
(Brussels, 4-8 April 1998)
1 Motivation

The traditional choice of a Lorentz frame to perform jet finding in $e^\pm p$ DIS final state is the Breit frame, since in such a frame $k_\perp$-type jet clustering algorithms would preserve factorization which is an important feature of QCD. Vectors, whether partons or calorimeter cells, used as the input to the jet algorithm, have to be boosted from the laboratory frame to the Breit frame and then clustered into jets. However, boosting from the laboratory frame to the Breit frame introduces systematic errors that may affect the jet-finding results. In particular, when boosting calorimeter cells, a problem arises near the outer edges of the forward and rear sections of a cylindrical calorimeter system. This is the region where the cells are least projective radially, and the longitudinal variation in the energy deposit in these cells results in large differences in the polar angle between cells after boosting them to the Breit frame. Many methods have been tried to reduce this problem, but in the end, none give a satisfactory result.

Our goal is to demonstrate the existence of a new Lorentz frame, more suitable for DIS jet finding.

2 The Breit Frame

In order to define fully a jet clustering algorithm one needs to introduce an auxiliary vector $\bar{p}$ of the form

$$\bar{p} = x f(Q^2)P + g(Q^2)q .$$

Here $P$ and $q$ are the incoming proton and virtual photon four-momenta and $f, g$ are any function of $Q^2$. The simplest example of a suitable auxiliary vector is

$$\bar{p} = 2xP + q$$

with $\bar{p}^2 \simeq Q^2$ and $\bar{p} \cdot q = 0$. The last equation can be used to specify a frame of reference in which the cluster resolution variables $d_{ij}$ are to be evaluated. For frames of reference where a virtual photon is purely space-like ($q^* = 0$) there are two solutions of the equation $\vec{p}^* \cdot q^* = 0$. The first solution ($\vec{p}^* = 0$) corresponds to the rest frame of $\bar{p}$ and known as the Breit frame (BF) of reference.

In terms of $\bar{p}$ the Lorentz parameters of the BF are as follows

$$\gamma = \bar{p}_o/\sqrt{\bar{p}^2}, \quad \bar{\eta} = \bar{p}/\sqrt{\bar{p}^2}.$$  \hspace{1cm} (1)

Fig. 1a shows the Lorentz factor $\gamma$ as a function of $x$ at different $y$. The arrow shows a unique point $x = x_o = k/P, y = 1$ where the HERA laboratory and Breit frames coincide ($\gamma = 1$). Here $k$ and $P$ are the incoming lepton and proton momenta. A large variation of $\gamma$ with $(x, Q^2)$ causes problems noted in Sec.1 and discussed in.

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1This may be one of the reasons why there are still very few published results with data analysis from HERA using the $k_\perp$ jet algorithm.
The second solution of the equation
\[ \vec{p}_o \vec{q}_o^* - \vec{p}^* \cdot \vec{q}^* = 0 \]
corresponds to \( \vec{p}^* \neq 0 \) and \( \vec{p}^* \perp \vec{q}^* \) at \( q_o^* = 0 \). In the general form the Lorentz parameters of a new frame, called the Photon frame of reference, are
\[ \gamma = \frac{\ell_o}{\sqrt{\ell^2}}, \quad \vec{\eta} = \frac{\vec{\ell}}{\sqrt{\ell^2}} \]  
with
\[ \ell = (\sqrt{q_o^2 + Q^2}, q_o \vec{q}/\sqrt{q_o^2}) \]
and \( \ell^2 = Q^2, \ \ell \cdot q = 0 \).

Here we would like to enumerate some properties of the new frame. From (2) one sees that the laboratory and Photon frames are connected by a boost along the direction of the momentum transfer vector \( \vec{q} \). Fig. 1b shows the Lorentz factor (2) as a function of \( x \) at different \( y \). At \( x > 10^{-3} \gamma_{ph} \) depends on \( (x, Q^2) \) values in a very different way compared with \( \gamma_{Br} \) in Fig. 1a. In the range \( 10^{-2} < x < 10^{-1} \) the Photon frame (PF) is very close to the HERA frame though \( Q^2 \) varies significantly. At \( x = x_o \) (the arrow in Fig. 1b) the PF coincides with the HERA frame along the line in the phase space independent of \( Q^2 \) and \( y \) values and a virtual photon is pure space-like in the laboratory frame of reference.

Deep inelastic lepton-nucleon scattering in the PF is described in the parton model (zeroth order QCD) by the space diagram in Fig. 2a. An auxiliary angle between the scattered lepton and quark is denoted as \( \alpha \). Angles \( \delta, \theta, \xi \) and \( \alpha \) relate to \( q_o, Q^2 \) and the incoming lepton and proton energies, \( \epsilon, E \), as follows.

\[ \begin{aligned} \gamma & = \ell_o/\sqrt{\ell^2}, & \vec{\eta} & = \ell/\sqrt{\ell^2} \\ \ell & = (\sqrt{q_o^2 + Q^2}, q_o \vec{q}/\sqrt{q_o^2}) \end{aligned} \]
The Photon frame in dynamics

\[ y = 0.5 \]

\[ \cos \xi \]

\[ \cos \theta \]

\[ \cos \alpha \]

\[ -1 \]

\[ 1 \]

\[ -1 \]

\[ 1 \]

\[ 0 \]

\[ 1 \]

\[ 10^{-3} \]

\[ 10^{-1} \]

\[ 10 \]

\[ 10^{-4} \]

\[ 10^{-2} \]

\[ 10^{-1} \]

\[ 1 \]

\[ x_0 \]

\[ x \]

\[ \cos \delta = \frac{\sqrt{Q^2 + q_0^2}}{2x - q_0}, \quad \cos \xi = \frac{\sqrt{Q^2 + q_0^2}}{2xE + q_0}, \quad (3) \]

\[ \cos \theta = 1 - 2\cos^2 \xi, \quad \cos \alpha = 1 - \frac{2}{y} \cos \delta \cdot \cos \xi \quad (4) \]

with

\[ q_0 = \frac{(k - xP)Q^2}{2x(kE + \epsilon P)} \approx (k - xP)y. \]

Due to the relations (4) in between \( \delta, \theta, \xi \) and \( \alpha \) there are only two independent angles. Fig. 2b shows variation of these angles with \( x \) at \( y = 0.5 \). At \( x < 10^{-3} \) the BF and the PF are very close to each other (\( \theta \approx \pi, \xi \approx 0 \)). Direct comparison of (1) and (2) also confirms the last conclusion, since at small \( x \gamma_{Ph} \sim \gamma_{Br} \approx q_0/Q, \quad \vec{\eta}_{Ph} \sim \vec{\eta}_{Br} \approx \vec{q}/Q \).

In the parton model the line \( x = x_o \) has a special significance, since both incoming and outgoing \( e^\pm \) and parton have the same energy and back-scatter off each other (\( \alpha = \pi \)).

We point out that jet finding in the PF preserves factorization. Careful analysis of examples given in [1] shows that in term of the vector \( \vec{p}^\ast = 2xP^\ast + q^\ast \) in the PF the \( k_{\perp} \)-type resolution variable \( b_{ij} \) has the same form as Eqs. (25)-(26) of Ref. [1]. As compared with the PF the BF current hemisphere due to the static geometry is dominated by the fragments of the struck quark. This makes comparisons of multiplicities in \( e^+e^- \) and the current region of \( e^\pm p \) easier in the BF. On the other hand, to perform the DIS jet finding in \( e^\pm p \) it is preferable to use the PF because it reduces the above-mentioned problems.

Figure 2
4 Conclusions

A new Lorentz frame, called the Photon frame, with a pure space-like virtual photon is found. Many features of the Photon frame make of it attractive for jet finding. In the kinematical region interesting for DIS jet study boosts are small, substantially reducing the systematic errors.

Acknowledgment. I would like to thank the DESY Directorate and the organisers of this meeting for financial support and to acknowledge friendly assistance and discussions with E. De Wolf, J. Hartmann, R. Klanner, G. Wolf. I am grateful to P. Bussey for reading the manuscript and comments. This work supported in part under the DFG grant #436 RUS 113/248/1.

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