Meissner effect in holographic superconductors with Dirac-Born-Infeld electrodynamics

Debabrata Ghoraia, Sunandan Gangopadhyaya, Rabin Banerjeea

a Department of Theoretical Sciences, S.N. Bose National Centre for Basic Sciences, JD Block, Sector III, Salt Lake, Kolkata 700098, India

Abstract

In this paper, we have investigated the Meissner effect of holographic superconductors in the presence of Dirac-Born-Infeld electrodynamics. The matching method is applied to obtain the critical magnetic field and the critical temperature. The critical magnetic field obtained from this investigation shows the effects of the Dirac-Born-Infeld parameter $b$. However, the result differs from that obtained by using Born-Infeld electrodynamics because of the extra $\vec{E} \cdot \vec{B}$ term in the Dirac-Born-Infeld theory. It is observed that the critical magnetic field increases in Dirac-Born-Infeld theory compared to that in the Born-Infeld theory, thereby favouring the Meissner effect.

1 Introduction

The physics of strongly coupled systems poses difficulties when approached by conventional methods. The gauge/gravity duality [1]-[4] proved to be a powerful mathematical tool to study strongly coupled field theoretical systems by investigating weakly coupled gravitational systems. Constructing gravitational duals of strongly coupled physical phenomena one may explain some of its properties which may in turn give some insight in the intricacies of the duality itself. In the past decade, the dual gravitational theories played a very important role in theoretical physics to study quantum chromodynamics [5]-[10], fluid dynamics [11]-[16], entanglement entropy [17]-[20] and condensed matter physics [21]-[23]. Holographic superconductors [24]-[41] is a gravitational dual model which explains some basic properties of high $T_c$ superconductors. The model gives a mechanism for the formation of scalar hair outside a AdS black hole below a certain critical temperature via spontaneous breakdown of a local $U(1)$ symmetry near the black hole horizon [42],[43]. Using the gauge/gravity duality, an enormous amount of work has been done to understand various properties of holographic superconductor/metal phase transition in the framework of usual Maxwell electromagnetic theory [24]-[31] and Born-Infeld (BI) electrodynamics [32]-[41]. However, the study of non-linear effects on the critical magnetic field in the presence of Dirac-Born-Infeld (DBI) electrodynamics [44]-[46] has not been carried out so far in the literature. The difference between the DBI electrodynamics and BI electrodynamics is important only when the magnetic field is switched on. This is because of the extra $\vec{E} \cdot \vec{B}$ term in the DBI theory which is absent in the BI theory. The theory is important in its own right
as it removes the divergence in the self energy of point charged particles and also enjoy enjoys electromagnetic duality. These features motivate to study holographic superconductors in the presence of DBI electrodynamics. It should be noted that there is no difference between the DBI theory and BI theory in the absence of a magnetic field.

In this paper we investigate the effects of magnetic field on holographic superconductors by considering DBI electrodynamics. Our intention is to study how the presence of extra \( \vec{E} \cdot \vec{B} \) term in DBI theory affects the Meissner effect. In particular we would like to observe the non-linear effects coming from DBI electrodynamics on the critical magnetic field at which superconducting order gets destroyed. We calculate analytically the critical magnetic field at which the superconducting state becomes normal metallic state. In this work the matching method is used in which we match the asymptotic behaviour of fields with the horizon behaviour of the fields. The critical magnetic field obtained from the DBI electrodynamics incorporates the non-linear effects. Our analysis differs from the previous study \[36\] in the sense that we solve the scalar field equation in the electrodynamic sector (in the absence of the matter field) taking into account the magnetic field. It is through this equation that the non-linear parameter (in the DBI electrodynamics) coupled with the magnetic field once again makes an entry into the entire analysis.

This paper is organized as follows. In section 2, the basic formalism for the holographic superconductors coupled to DBI electrodynamics is presented. In section 3, we have considered the Meissner effect upto first order in DBI parameter \( b \). Section 4 contains the concluding remarks. Finally, we have an appendix.

## 2 Basic formalism

In \( 3 + 1 \)-dimensions, the action for the model of a holographic superconductor in the framework of DBI electrodynamics consists a complex scalar field coupled to a \( U(1) \) gauge field in AdS black hole spacetime

\[
S = \int d^4 x \sqrt{-g} \left[ \frac{1}{2\kappa^2} (\mathcal{R} - 2\Lambda) + \frac{1}{b} \left( 1 - \sqrt{1 + \frac{b}{2} F_{\mu\nu} F_{\mu\nu} - \frac{b^2}{16} (G_{\mu\nu} F_{\mu\nu})^2} \right) \right. - (D_\mu \psi)^* D^\mu \psi - m^2 \psi^* \psi \]  

(1)

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu ; (\mu, \nu = t, r, x, y) \), \( G_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \), \( D_\mu \psi = \partial_\mu \psi - iq A_\mu \psi \), \( \Lambda = -\frac{3}{L^2} \) is the cosmological constant, \( \kappa^2 = 8\pi G \), \( G \) being the Newton’s universal gravitational constant, \( b \) is the Born-Infeld parameter, \( A_\mu \) and \( \psi \) represent the gauge and scalar fields.

It should be noted that in the existing literature on holographic superconductors one considers the Born-Infeld theory \[44, 45\] instead of the Dirac-Born-Infeld theory \[46\]. In the DBI theory, the BI theory gets augmented by the third term under the square root in eq. (1). This is basically the anomaly term \( (\vec{E} \cdot \vec{B})^2 \), which turns out to be very important when we study the effects of the magnetic field on holographic superconductors since this term is proportional to \( \vec{E} \cdot \vec{B} \) and would give an additional contribution along with \( F_{\mu\nu} F^{\mu\nu} \) for a non-zero magnetic field. However, in the absence of the magnetic field, there is no difference between the BI and the DBI theories.

The plane-symmetric black hole geometry reads (setting the AdS radius \( L = 1 \))

\[
ds^2 = -f(r) dt^2 + \frac{1}{f(r)} dr^2 + r^2 (dx^2 + dy^2)  

(2)

where

\[
f(r) = r^2 \left( 1 - \frac{r^3}{L^2} \right) .

(3)\]
The Hawking temperature of this black hole spacetime reads
\[ T = \frac{f'(r_+)}{4\pi} = \frac{3r_+}{4\pi}. \] (4)

This is interpreted as the temperature of the dual field theory at the boundary.

The equation of motion for the gauge and matter fields read
\[
\partial_\alpha \left[ \sqrt{-g} F^{\alpha\beta} \right] - \frac{b}{4} \partial_\alpha \left[ \sqrt{-g} G^{\alpha\beta} G^{\mu\nu} F_{\mu\nu} \right] = 2\sqrt{-g} q^2 A^\beta |\psi|^2 + i q \sqrt{-g} \left[ \psi^* \partial^\beta \psi - \psi \partial^\beta \psi^* \right].
\] (5)

\[
\partial_\alpha \left[ \sqrt{-g} \partial^\alpha \psi \right] = \sqrt{-g} \left[ q^2 A_\mu A^\mu + m^2 \right] \psi + i q \left[ 2\sqrt{-g} A^\mu \partial_\mu \psi + \sqrt{-g} (\partial_\mu A^\mu) \psi + (\partial_\mu \sqrt{-g}) A^\mu \psi \right].
\] (6)

Making the ansatz for the gauge field and the scalar field as [21]
\[ A_\mu = (\phi(r), 0, 0, 0), \quad \psi = \psi(r) \] (7)

leads to the following equations of motion for the gauge and matter fields
\[
\phi''(r) + \frac{2}{r} \phi'(r) - \frac{2}{r} b \phi'(r)^3 - 2 q^2 \frac{\phi(r) \psi^2(r)}{f(r)} (1 - b \phi'(r)^2)^{3/2} = 0 \] (8)

\[
\psi''(r) + \left( \frac{2}{r} + \frac{f'(r)}{f(r)} \right) \psi'(r) + \left( \frac{q^2 \phi^2(r)}{f(r)^2} - \frac{m^2}{f(r)} \right) \psi(r) = 0 \] (9)

where prime denotes derivative with respect to \( r \). The conditions \( \phi(r_+) = 0 \) and \( \psi(r_+) \) to be finite imposes the regularity of the fields at the horizon.

Setting \( q = 1 \) and changing the coordinate from \( r \) to \( z = \frac{r_+}{r} \), the field eqs. (8) look like
\[
\phi''(z) + \frac{2b z^3}{r_+^2} \phi'(z)^3 - \frac{2r_+^2 \psi^2(z)}{z^4 f(z)} \left( 1 - \frac{b z^2}{r_+^2} \phi'(z)^2 \right)^{3/2} \phi(z) = 0 \] (10)

\[
\psi''(z) + \frac{f'(z)}{f(z)} \psi'(z) + \frac{r_+^2}{z^4} \left( \frac{\phi^2(z)}{f(z)^2} - \frac{m^2}{f(z)} \right) \psi(z) = 0 \] (11)

where prime denotes derivative with respect to \( z \). To solve these equations, we have to impose the boundary behaviour of the fields.

The fields near the boundary of the bulk obey [20]
\[
\phi(z) = \mu - \frac{\rho}{r_+} z \] (12)

\[
\psi(r) = J_- z^{-\Delta_-} + J_+ z^{\Delta_+} \] (13)

where
\[
\Delta_\pm = \frac{3 \pm \sqrt{9 + 4m^2}}{2} \] (14)
are the conformal weights of the conformal field theory living on the boundary. The interpretation of the parameters \(\mu\) and \(\rho\) are given by the gauge/gravity dictionary. They are interpreted as the chemical potential and charge density of the conformal field theory on the boundary. For the choice \(\psi_+ = 0\), \(\psi_-\) is interpreted as the dual of the expectation value of the condensation operator \(O_{\Delta}\) at the boundary.

For \(m^2 = -2\) we have \(\Delta_+ = 2\) and \(\Delta_- = 1\). Here, we consider the case \(J_+ = 0\), so the relevant conformal dimension is \(\Delta = \Delta_- = 1\) and hence the matter field near the AdS boundary is given by

\[
\psi(z) = \frac{J_-}{r_+} z .
\] (15)

In order to study the effect of the magnetic field, we first need to investigate the relation between the critical temperature and the charge density. This we do using the matching method in which we match the asymptotic behaviour of fields with the horizon behaviour of field at any arbitrary point \((z_m)\) between \([0, 1]\). The details of this study are presented in the Appendix.

The critical temperature \(T_c\) at zero magnetic field reads [36]

\[
T_c = \frac{3}{4\pi} \frac{\sqrt{\rho}}{\sqrt{\beta\{1 + 2b\beta^2(1 - z_m)\}}}
\] (16)

where

\[
\beta = 2\sqrt{\frac{1 + 2z_m^2}{1 - z_m^2}}.
\] (17)

These results will be used in the subsequent discussion to find the effect of the magnetic field on holographic superconductors.

3 Effect of magnetic field

In this section, we add a magnetic field in the bulk. The asymptotic value of this magnetic field represents a magnetic field in the boundary field theory. The following ansatz is taken to study the Meissner effect of holographic superconductors

\[
A_\mu = (\phi(r), 0, 0, Bx) , \quad \psi \equiv \psi(r, x) .
\] (18)

Using the above ansatz, we obtain from eqs.(5,6)

\[
\left(1 + bB^2\right) \frac{r^2}{f(r)} \phi''(r) + \frac{2}{r} \phi'(r) - m^2 \frac{\phi(r)}{f(r)} = \frac{2q^2r^2\phi^2(r, x)}{f(r)} \psi(r, x)
\] (19)

\[
\partial_r^2 \psi(r, x) + \left( f'(r) + \frac{2}{r} \right) \frac{\partial_r \psi(r, x)}{f(r)} - m^2 \frac{\phi(r)}{f(r)} \psi(r, x)
\]

\[
+ \frac{1}{r^2 f(r)} \frac{\partial_x^2 \psi(r, x)}{r^2 f(r)} - \frac{q^2B^2x^2}{r^2 f(r)} \psi(r, x) = \frac{-q^2\phi^2(r)}{f^2(r)} \psi(r, x) .
\] (20)

Now we proceed to solve the gauge field equation which reads up to first order in the DBI parameter

\[
\left(1 + bB^2 + \frac{bB^2}{r^4}\right) \phi''(r) + \frac{2}{r} \left(1 + bB^2 + \frac{2bB^2}{r^4} - b\phi^2(r)\right) \phi'(r) = \frac{2q^2\phi^2(r, x)}{f(r)}
\]

\[
\times \left[1 + \frac{3b}{2} \left(\frac{B^2}{r^4} - \phi^2(r)\right)\right] .
\] (21)
Changing variables to \( z = \frac{r}{r^+} \), we find the matter field and gauge field equations in \( z \) coordinate to be

\[
\frac{\partial^2 \psi(z, x)}{\partial z^2} + f'(z) \frac{\partial \psi(z, x)}{\partial z} - \frac{m^2 r^2}{z^4 f(z)} \psi(z, x) + \frac{q^2 r^2 \phi^2(r)}{z^4 f^2(z)} \psi(z, x) = -\frac{1}{z^2 f(z)} \left[ \frac{\partial^2 \psi(z, x)}{\partial x^2} - q^2 B^2 x^2 \psi(z, x) \right] \quad (22)
\]

\[
\left( 1 + bB^2 + \frac{bB^2 z^4}{r^+} \right) \frac{d^2 \phi(z)}{dz^2} - \frac{2b B^2 z^3 d\phi(z)}{dz} + \frac{2b z^3}{r^2} \left( \frac{d\phi(z)}{dz} \right)^3 = \frac{2q^2 \psi^2(z, x)}{f(z)} \left[ \frac{r^2}{z^4} + \frac{3b}{2} \left( \frac{B^2}{r^2} - \phi^2(r) \right) \right]. \quad (23)
\]

At \( T = T_c \), the matter field \( \psi(z) \) vanishes. Putting \( \psi(z) = 0 \) in eq.\((23)\), we obtain

\[
\frac{d^2 \phi(z)}{dz^2} - \frac{2b B^2 z^3}{1 + bB^2 + \frac{bB^2 z^4}{r^+}} \frac{d\phi(z)}{dz} + \frac{2b z^3}{1 + bB^2 + \frac{bB^2 z^4}{r^+}} \left( \frac{d\phi(z)}{dz} \right)^3 = 0. \quad (24)
\]

Inserting the integrating factor \( \frac{1}{\sqrt{1 + bB^2 + \frac{bB^2 z^4}{r^+}}} \) converts the above equation to the following form

\[
\frac{d\zeta(z)}{dz} = -\frac{2b}{r^+} z^3 \zeta^3(z) \quad (25)
\]

where \( \zeta(z) = \sqrt{\frac{\phi'(z)}{1 + bB^2 + \frac{bB^2 z^4}{r^+}}} \). To solve this equation, we need to impose the asymptotic behaviour of the gauge field which is

\[
\phi(z) = \mu - \frac{\rho}{r^+} z. \quad (26)
\]

Now we integrate eq.\((25)\) in the interval between boundary and the event horizon, that is \([0, 1]\)

\[
\int_0^1 \frac{d\zeta(z)}{dz} = -\frac{2b}{r^+} \int_0^1 z^3 dz \quad (27)
\]

\[
\Rightarrow \frac{1}{\zeta^2(1)} = \frac{b}{r^+} + \frac{1}{\zeta^2(0)}. \quad (28)
\]

We also integrate eq.\((25)\) in the interval \([1, z]\) and use the above relation to get

\[
\frac{1}{\zeta^2(z)} = \frac{b}{r^+} (z^4 - 1) + \frac{1}{\zeta^2(1)} \quad (29)
\]

\[
\Rightarrow \frac{1}{\zeta^2(z)} = \frac{b z^4}{r^+} + \frac{1}{\zeta^2(0)}. \quad (30)
\]

Using the asymptotic behaviour of \( \phi(z) \) \((26)\), we finally obtain

\[
\phi'(z) = -\sqrt{\frac{1 + bB^2 \left( 1 + \frac{z^4}{r^+} \right)}{1 + b \left( B^2 + \frac{\rho^2 z^4}{r^+} \right) r^+}} \frac{\rho}{r^+}. \quad (31)
\]
Note that this expression takes into account the effects of the magnetic field coming from both $F_{\mu\nu}F^{\mu\nu}$ and $F_{\mu\nu}G^{\mu\nu}$ terms. To be precise, the last term in the numerator and denominator arise from the BI part of the theory and the second term in the numerator and denominator arises from the $\vec{E}\cdot\vec{B}$ term in the DBI theory. This relation will be used in the subsequent discussion to calculate the critical magnetic field.

Now we turn our attention at the matter field equation near $T_c$. Employing the separation of variable technique $\psi(z,x) = X(x)R(z)$ and setting $q = 1$, eq. (22) takes the form

$$
\frac{R''(z)}{R} + \frac{f'(z)R'(z)}{f(z)R(z)} - \frac{m^2 r_+^2}{z^4 f(z)} + \frac{r_+^2 \phi^2(z)}{z^4 f(z)} = \frac{1}{z^2 f(z)} \left[ -\frac{X''}{X} + B^2 x^2 \right].
$$

This finally gives on separation the following equation for $X(x)$

$$
\left( -\frac{d^2}{dx^2} + B^2 x^2 \right) X = \kappa^2 X.
$$

The above equation for $X(x)$ is identified as the Schrödinger equation in one dimension with a $B$-dependent frequency which leads us to identify $\kappa^2 = (2n + 1)B$ where $n$ is an integer. For $n = 0$, we find that $\kappa^2 = B$ and this helps in finding the critical magnetic field.

The radial part of the matter field takes the form

$$
R''(z) + \frac{f'(z)}{f(z)} R'(z) + \left( \frac{r_+^2}{z^4} \frac{\phi^2(z)}{f^2(z)} - \frac{m^2 r_+^2}{z^4 f(z)} - \frac{\kappa^2}{z^2 f(z)} \right) R(z) = 0.
$$

From the above equation and using the fact that $f(1) = 0$, we find

$$
R'(1) = -\left( \frac{m^2}{3} + \frac{\kappa^2}{3r_+^2} \right) R(1) \quad (35)
$$

$$
R''(1) = \left[ \frac{m^2}{3} + \frac{m^4}{18} + \frac{\kappa^4}{18r_+^4} + \frac{m^2 \kappa^2}{9r_+^2} - \frac{\phi'^2(1)}{18r_+^2} \right] R(1). \quad (36)
$$

We now expand $R(z)$ around $z = 1$ which reads

$$
R(z) = R(1) - R'(1)(1-z) + \frac{R''(1)}{2}(1-z)^2 + \ldots
$$

$$
\approx R(1) - R'(1)(1-z) + \frac{R''(1)}{2}(1-z)^2. \quad (37)
$$

Substituting the value of $R''(1)$ and $R'(1)$ in eq. (37), we find

$$
R(z) = \left[ 1 + \left( \frac{m^2}{3} + \frac{\kappa^2}{3r_+^2} \right)(1-z) + \left( \frac{m^2}{3} + \frac{m^4}{18} + \frac{\kappa^4}{18r_+^4} + \frac{m^2 \kappa^2}{9r_+^2} - \frac{\phi'^2(1)}{18r_+^2} \right) \frac{(1-z)^2}{2!} \right] R(1). \quad (38)
$$

Setting $m^2 = -2$ and equating eq. (38) and eq. (15) and their derivatives at $z = z_m$, we obtain

$$
\frac{J_- z_m}{r_+} = R(1) \left[ 1 - \left( \frac{2}{3} - \frac{\kappa^2}{3r_+^2} \right)(1-z_m) + \left( -\frac{4}{9} + \frac{\kappa^4}{18r_+^4} - \frac{2\kappa^2}{9r_+^2} - \frac{\phi'^2(1)}{18r_+^2} \right) \frac{(1-z_m)^2}{2} \right]
$$

$$
\frac{J_-}{r_+} = R(1) \left[ \frac{2}{3} \frac{\kappa^2}{3r_+^2} - \left( -\frac{4}{9} + \frac{\kappa^4}{18r_+^4} - \frac{2\kappa^2}{9r_+^2} - \frac{\phi'^2(1)}{18r_+^2} \right)(1-z_m) \right]. \quad (39)
$$
From the above relations, we finally get
\[ \kappa^4 + 4 \left( \frac{2 + z_m^2}{1 - z_m^2} \right) r_+^2 \kappa^2 + 4 \left( \frac{1 + 2 z_m^2}{1 - z_m^2} \right) r_+^4 - \phi'^2(1) r_+^2 = 0 \quad (41) \]
which in turn implies, using \( \kappa^2 = B \)
\[ B^2 + 4 \left( \frac{2 + z_m^2}{1 - z_m^2} \right) r_+^2 B + 4 \left( \frac{1 + 2 z_m^2}{1 - z_m^2} \right) r_+^4 - \phi'^2(1) r_+^2 = 0. \quad (42) \]
The last term in the above equation upto \( \mathcal{O}(b) \) can be obtained from eq.(31) and reads
\[ \phi'^2(1) = \left[ 1 + \frac{b}{r_+^2} \left( B^2 - \rho^2 \right) \right] \frac{\rho^2}{r_+^2}. \quad (43) \]
Substituting eq.(17) and eq.(43) in eq.(42), we get upto order \( \mathcal{O}(b) \)
\[ \left( 1 - b \frac{\rho^2}{r_+^2} \right) B^2 + 4 a_2 r_+^4 B + \beta^2 r_+^4 - \rho^2 \left( 1 - b \frac{\rho^2}{r_+^2} \right) = 0 \quad (44) \]
where \( a_2 = \frac{2 + z_m^2}{1 - z_m^2} \). The solution of the above equation reads
\[ B_c = \frac{1}{\left( 1 - b \frac{\rho^2}{r_+^2} \right)} \left[ \sqrt{4 a_2^2 r_+^4 - \left( 1 - b \frac{\rho^2}{r_+^2} \right) \left\{ \beta^2 r_+^4 - \rho^2 \left( 1 - b \frac{\rho^2}{r_+^2} \right) \right\} - 2 a_2 r_+^2} \right]. \quad (45) \]
Now let us denote \( T_c \equiv T_c(B) \), then from eq.(16) and eq.(4) we find
\[ \frac{\rho^2}{r_+^2} = \beta^2 \left( 1 + 2 b \beta^2 (1 - z_m) \right)^2 \frac{T_c^4(0)}{T^4} \quad (46) \]
\[ = \beta^2 \left( 1 + 4 b \beta^2 (1 - z_m) + \mathcal{O}(b^2) \right) \frac{T_c^4(0)}{T^4}. \quad (47) \]
Substituting the above equation and \( r_+ = \frac{4 \pi}{3} T \) in eq.(45), we finally obtain
\[ B_c \approx B_0 + b \beta^2 \left[ B_0 \frac{T_c^4(0)}{T^4} + \frac{8 \pi^2 \beta}{9} \left( 5 - 4 z_m - 2 T_c^4(0) T^4 \right) \right] \quad (48) \]
where
\[ B_0 = B_c|_{b=0} = \frac{16 \pi^2}{9} \beta T_c^2(0) \left[ \sqrt{1 + \left( \frac{4 a_2^2}{\beta^2} - 1 \right) \frac{T^4}{T_c^4(0)} - \frac{2 a_2}{\beta} \frac{T^2}{T_c^2(0)}} \right]. \quad (49) \]
We observe that the critical magnetic field \( B_c \) incorporates the effects of the DBI parameter \( b \). Note that the critical magnetic field upto \( \mathcal{O}(b) \) differs from that obtained in the BI theory [36]. The first term in the square bracket is an extra contribution that arises due to the DBI theory together with the fact that the gauge field equation has been solved taking into account the effect of the magnetic field. Further, the critical magnetic field increases in the DBI theory compared to that in the BI theory. This clearly indicates that the extra \( \vec{E}.\vec{B} \) term present in the DBI theory is favourable for the Meissner effect as it increases the critical magnetic field at which the superconductivity order gets destroyed.
4 Conclusions

In this paper we have studied the effects of magnetic field on holographic superconductors by considering Dirac-Born-Infeld electrodynamics. The investigation is important in its own right as most of the studies carried out so far in the literature with non-linear electrodynamics have been with Born-Infeld electrodynamics [32]-[41]. However, the study involving Dirac-Born-Infeld electrodynamics has not been carried out. The importance of this study lies in the fact that the Dirac-Born-Infeld theory of electrodynamics has an extra $\vec{E}.\vec{B}$ which is non-zero in the presence of a magnetic field. We observe from our analysis that the critical magnetic field increases with increase in the Dirac-Born-Infeld parameters and its value is greater than the corresponding result obtained in Born-Infeld electrodynamics [36]. This indicates that DBI is favourable for the Meissner effect in superconductivity.

Appendix

Here we briefly sketch the matching method to obtain the relation between the critical temperature and the charge density. The technique is to match the asymptotic behaviour of fields with the horizon behaviour of fields at an arbitrary point $z_m$ between horizon and the AdS boundary. First, we expand the scalar and matter fields near the horizon ($z = 1$)

$$
\phi(z) = \phi(1) - \frac{\phi'(1)}{1!}(1-z) + \frac{\phi''(1)}{2!}(1-z)^2 + ... \quad (50)
$$

$$
\psi(z) = \psi(1) - \frac{\psi'(1)}{1!}(1-z) + \frac{\psi''(1)}{2!}(1-z)^2 + ... \quad (51)
$$

Using fact that $f(1) = 0$, $f'(1) = -3r_+^2$, $f''(1) = 6r_+^2$ together with regularity condition $\phi(1) = 0$, we can find from the gauge field equation (10)

$$
\phi''(1) = -\left[\frac{2b}{r_+^2}\phi^2(1) + \frac{2}{3}\psi^2(1)\left(1 - \frac{b}{r_+^2}\phi^2(1)\right)^{3/2}\right]\phi'(1). \quad (52)
$$

Similarly from equation for the matter field (11), we find

$$
\psi'(1) = -\frac{m^2}{3}\psi(1), \quad \psi''(1) = \left(\frac{2m^2}{9} - \frac{\phi^2(1)}{18r_+^2}\right)\psi(1). \quad (53)
$$

Substituting the above expressions in eq.(s)(50), (51), we get

$$
\phi(z) \approx -(1-z) + \left\{\frac{b}{r_+^2}\phi^2(1) + \frac{1}{3}\psi^2(1)\left(1 - \frac{b}{r_+^2}\phi^2(1)\right)^{3/2}\right\}(1-z)^2 \phi'(1) \quad (54)
$$

$$
\psi(z) \approx \left[1 + \frac{m^2}{3}(1-z) + \left(\frac{m^2}{9} - \frac{\phi^2(1)}{36r_+^2}\right)(1-z)^2\right]\psi(1). \quad (55)
$$

Setting $m^2 = -2$ in the above equations, we then match the above behaviour of the scalar and matter fields near horizon with those in the asymptotic region at $z = z_m$. The same thing is carried for their derivatives also. This yields

$$
\mu - \frac{\rho}{r_+}z_m = \beta(1-z_m) + \beta\left[b\beta^2 + \frac{\alpha^2}{3}\left(1 - \frac{b\beta^2}{r_+^2}\right)^{3/2}\right](1-z_m)^2 \quad (56)
$$

$$
\frac{\rho}{r_+} = \beta + 2\beta\left[b\beta^2 + \frac{\alpha^2}{3}\left(1 - \frac{b\beta^2}{r_+^2}\right)^{3/2}\right](1-z_m) \quad (57)
$$
where $\beta = -\phi'(1)$ and $\alpha = \psi(1)$. Using $T = \frac{3\rho}{4\pi}$ and using the above equations, we get

$$\alpha^2 = \frac{3}{2} \frac{1 + 2b\beta^2(1 - z_m)}{1 - z_m(1 - b\beta^2)^{3/2}} \left( \frac{T_c^2}{T^2} - 1 \right)$$

where

$$T_c = \frac{3}{4\pi} \sqrt{\rho} \sqrt{\beta \{1 + 2b\beta^2(1 - z_m)\}}$$

with $\tilde{\beta} = \frac{\beta}{r_+}$. Treating the matter field sector in a similar way yields

$$\frac{J_-}{r_+} z_m = \frac{\alpha}{3} + \frac{2\alpha}{3} z_m - \frac{\alpha}{9} \left( 2 + \frac{\beta^2}{4r_+^2} \right) (1 - z_m)^2$$

$$\frac{J_-}{r_+} = \frac{2\alpha}{3} + \frac{\alpha}{9} \left( 4 + \frac{\beta^2}{2r_+^2} \right) (1 - z_m).$$

The above relations give

$$\tilde{\beta} = \frac{\beta}{r_+} = 2 \sqrt{\frac{1 + 2z_m^2}{1 - z_m^2}}.$$

**Acknowledgments**

DG would like to thank DST-INSPIRE, Govt. of India for financial support. SG acknowledges the support by DST SERB under Start Up Research Grant (Young Scientist), File No.YSS/2014/000180. SG also acknowledges the Visiting Associateship at IUCAA, Pune.

**References**

[1] J. M. Maldacena, “The Large N Limit of Superconformal Field Theories and Supergravity”, Adv. Theor. Math. Phys. 2, 231 (1998).

[2] E. Witten, “Anti De Sitter Space And Holography”, Adv. Theor. Math. Phys. 2, 253 (1998).

[3] S.S. Gubser, I.R. Klebanov, A.M. Polyakov, “Gauge Theory Correlators from Non-Critical String Theory”, Phys. Lett. B 428, 105 (1998).

[4] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri, Y. Oz, “Large N Field Theories, String Theory and Gravity”, Phys. Rept. 323, 183 (2000).

[5] J. Babington, J. Erdmenger, N.J. Evans, Z. Guralnik, I. Kirsch, “Chiral Symmetry Breaking and Pions in Non-Supersymmetric Gauge/Gravity Duals”, Phys. Rev. D69 (2004) 066007.

[6] S.J. Brodsky, G.F. de Teramond, “Light-Front Hadron Dynamics and AdS/CFT Correspondence”, Phys. Lett. B582 (2004) 211.

[7] T. Sakai, S. Sugimoto, “Low energy hadron physics in holographic QCD”, Progr. Theoret. Phys. 113 (2005) 843.
[8] J. Erlich, E. Katz, D.T. Son, M.A. Stephanov, “QCD and a Holographic Model of Hadrons”, Phys. Rev. Lett. 95 (2005) 261602.

[9] Y. Kim, D. Yi, “Holography at Work for Nuclear and Hadron Physics”, Adv. High Energy Phys. 2011 (2011) 259025.

[10] Y. Kima, I.J. Shina, T.Tsukioka, “Holographic QCD: Past, present, and future”, Progress in Particle and Nuclear Physics 68 (2013) 55.

[11] G. Policastro, D. T. Son, and A. O. Starinets, The shear viscosity of strongly coupled N = 4 supersymmetric Yang-Mills plasma, Phys. Rev. Lett. 87 (2001).

[12] S. Bhattacharyya, V. E. Hubeny, S. Minwalla, and M. Rangamani, “Nonlinear Fluid Dynamics from Gravity, JHEP 0802 (2008) 045.

[13] M. Rangamani, “Gravity & Hydrodynamics: Lectures on the fluid-gravity correspondence”, Class. Quant. Grav. 26 (2009) 224003.

[14] C.P. Herzog, “Lectures on holographic superfluidity and superconductivity”, J. Phys. A 42, 343001 (2009).

[15] Jae-Hyuk Oh, “Gauge-gravity duality and its application to cosmology and fluid dynamics”, University of Kentucky Doctoral Dissertations (2011) 178.

[16] S. Das, S. Gangopadhyay, D. Ghorai, “Viscosity to entropy density ratio for non-extremal Gauss-Bonnet black holes coupled to Born-Infeld electrodynamics”, Eur.Phys.J. C77 (2017) 615.

[17] S. Ryu and T. Takayanagi, “Holographic derivation of entanglement entropy from AdS/CFT, Phys. Rev. Lett. 96 (2006) 181602.

[18] S. Ryu and T. Takayanagi, Aspects of holographic entanglement entropy, JHEP 08 (2006) 045.

[19] M. Rangamani, T. Takayanagi, “Holographic Entanglement Entropy”, arXiv:1609.01287 (2016).

[20] S. Karar, D. Ghorai, S. Gangopadhyay, “Holographic entanglement thermodynamics for higher dimensional charged black hole”, Nucl. Phys. B 938 (2019) 363.

[21] S.A. Hartnoll, “Lectures on holographic methods for condensed matter physics”, Class. Quantum Grav. 26, 224002 (2009).

[22] S.-S. Lee, “A Non-Fermi Liquid from a Charged Black Hole : A Critical Fermi Ball”, Phys. Rev. D 79, 086006 (2009).

[23] H. Liu, J. McGreevy, D. Vegh, “Non-Fermi liquids from holography”, Phys. Rev. D 83, 065029 (2011).
[24] S.A. Hartnoll, C.P. Herzog, G.T. Horowitz, “Building a Holographic Superconductor”, Phys. Rev. Lett. 101, 031601 (2008).

[25] S. A. Hartnoll, C. P. Herzog, G. T. Horowitz, “Holographic superconductors”, JHEP 12, 015 (2008).

[26] G. T. Horowitz, M. M. Roberts, “Holographic superconductors with various condensates”, Phys. Rev. D 78, 126008 (2008).

[27] G.T. Horowitz, “Introduction to Holographic Superconductors”, arXiv:1002.1722 [hep-th].

[28] G. T. Horowitz, M. M. Roberts, “Zero Temperature Limit of Holographic Superconductors”, JHEP 0911 (2009) 015.

[29] R. Gregory, S. Kanno, J. Soda, “Holographic Superconductors with Higher Curvature Corrections”, JHEP 0910 (2009) 010.

[30] G. Siopsis, J. Therrien, “Analytic calculation of properties of holographic superconductors”, JHEP 05 (2010) 013.

[31] T. Nishioka, S. Ryu, T. Takayanagi, “Holographic Superconductor/Insulator Transition at Zero Temperature”, JHEP 1003, 131 (2010).

[32] J. Jing, S. Chen, “Holographic superconductors in the Born-Infeld electrodynamics”, Phys. Lett. B 686 (2010) 68.

[33] S. Gangopadhyay, D. Roychowdhury, “Analytic study of properties of holographic superconductors in Born-Infeld electrodynamics”, JHEP 05 (2012) 002.

[34] Q. Pan, J. Jing, B. Wang, S. Chen, “Analytical study on holographic superconductors with backreactions”, JHEP 06 (2012) 087.

[35] R. Banerjee, S. Gangopadhyay, D. Roychowdhury, A. Lala, “Holographic s-wave condensate with nonlinear electrodynamics: A nontrivial boundary value problem”, Phys. Rev. D 87 (2013) 104001.

[36] S. Gangopadhyay, “Holographic superconductors in Born-Infeld electrodynamics and external magnetic field”, Mod. Phys. Lett. A 29 (2014) 1450088.

[37] Rong-Gen Cai, Li Li, Li-Fang Li, Run-Qiu Yang, “Introduction to Holographic Superconductor Models”, Sci.China Phys.Mech.Astron. 58 (2015) 060401.

[38] D. Ghorai, S. Gangopadhyay, “Higher dimensional holographic superconductors in Born-Infeld electrodynamics with back-reaction”, Eur.Phys.J. C76 (2016) 146.

[39] D. Ghorai, S. Gangopadhyay, “Noncommutative effects of spacetime on holographic superconductors”, Phys.Lett. B758 (2016) 106.
[40] D. Ghorai, S. Gangopadhyay, “Non-linear effects on the holographic free energy and thermodynamic geometry”, EPL 118 (2017) 31001.

[41] D. Ghorai, S. Gangopadhyay, “Conductivity of holographic superconductors in Born-Infeld electrodynamics”, Nucl. Phys. B 933 (2018) 1-13.

[42] S.S. Gubser, “Phase transitions near black hole horizons”, Class. Quant. Grav. 22, 5121 (2005).

[43] S.S. Gubser, “Breaking an Abelian gauge symmetry near a black hole horizon”, Phys. Rev. D 78, 065034 (2008).

[44] M. Born, “On the Quantum Theory of the Electromagnetic Field”, Proc. R. Soc. Lond. A 143 (1934) 410.

[45] M. Born and L. Infeld, “Foundations of the New Field Theory”, Proc. R. Soc. Lond. A 144 (1934) 425.

[46] P.A.M Dirac, “A Reformulation of the Born-Infeld Electrodynamics”, Proc. R. Soc. Lond. A 257 (1960) 32.