Quantum Zeno Isolation of Quantum Dots

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We investigate whether and how the quantum Zeno effect, i.e., the inhibition of quantum evolution by frequent measurements, can be employed to isolate a quantum dot from its surrounding electron reservoir. In contrast to the often studied case of tunneling between discrete levels, we consider the tunnelling of an electron from a continuum reservoir to a discrete level in the dot. Realizing the quantum Zeno effect in this scenario can be much harder because the measurements should be repeated before the wave packet of the hole left behind in the reservoir moves away from the vicinity of the dot. Thus, the required repetition rate could be lowered by having a flat band (with a slow group velocity) in resonance with the dot or a sufficiently small Fermi velocity or a strong external magnetic field.

Introduction One of the major distinctions between classical and quantum physics is the role of measurements. As a consequence, it is impossible to directly observe quantum evolution taking place without actually affecting it. A striking example is the quantum Zeno effect which describes slowing down or even stopping quantum evolution by frequent measurements, see, e.g., [1, 2].

Let us discuss the basic picture by means of a simple example, two discrete states or levels [1] and [2] of equal energy which are tunnel-coupled. Preparing the initial state in one level |Ψ(t = 0)⟩ = |1⟩, the time-dependent probability \( P_2(t) = |\langle 2|Ψ(t)⟩|^2 \) for the other level reads \( P_2(t) = \sin^2(\gamma t) \), where the frequency \( \gamma > 0 \) is given by the tunneling strength. For short times \( t \ll 1/\gamma \), we obtain a quadratic growth \( P_2(t) \approx \gamma^2 t^2 \), because in quantum physics amplitudes instead of probabilities are added. However, if we measure the level occupation after such a short time, i.e., within the quadratic-growth regime, we project the state |Ψ(t)⟩ back to the initial state |1⟩ (assuming a strong, i.e., projective measurement) with high probability \( P_1(t) \approx 1 - \gamma^2 t^2 \) and thus the quantum evolution has to start again afterwards. In other words, the amplitudes before and after the measurement do no longer interfere constructively because we have obtained which-way information via the measurement. Now, repeating this measurement with a fast rate (much larger than \( \gamma \)) would effectively keep setting back the evolution such that the quantum state stays in the initial level |1⟩. This inhibition (or slowing down) of quantum evolution is usually referred to as the quantum Zeno effect [1]. Note that this line of argument crucially relies on the quadratic growth \( P_2(t) \approx \gamma^2 t^2 \) discussed above. In case of a linear behavior \( P_2(t) \sim t \), for example, setting back the evolution by measurements would not have this retarding effect.

The quantum Zeno effect is a striking example for quantum control [3, 4] and closely related to passive quantum error correction schemes (similar to the spin-echo method). Thus, this fascinating phenomenon is of fundamental interest and has already been observed experimentally in different systems, e.g., for ions in Penning traps [5], ultra-cold atoms in optical lattices [6, 7], Bose-Einstein condensates in magnetic traps [8, 9], Rydberg atoms in cavities [10] or superconducting qubits [11].

On much shorter time scales, electrons in quantum dots are also very suitable systems for studying the quantum Zeno effect: Their energy levels and tunneling rates can be manipulated (e.g., by the variation of voltages that are applied to suitably placed gate electrodes) or the size of the dot and they have quite long coherence times while their states can be read out, i.e., measured, quite fast by capacitive charge detectors [12, 13] or optical transitions [14, 24], for example.

In most previous works, the quantum Zeno effect has been studied in a regime where the aforementioned picture based on transitions between discrete levels can be applied [2, 5, 27, 24]. In the following, we shall study the more involved case of transitions between a discrete level and a continuum (a Fermi gas or liquid), see also e.g., [25, 26]. As a concrete experimental realization, we consider a quantum dot coupled to a reservoir in the form of an effectively two-dimensional electron gas (2DEG) [27, 31].

The Model We consider the quantum dot-reservoir system in good approximation [32] being described by the following many-body Hamiltonian (\( \hbar = 1 \))

\[
\hat{H} = \int d^2r \left( \frac{\nabla \hat{\psi}_1^\dagger \cdot (\nabla \hat{\psi}_1)}{m} + V_1(x, y)\hat{\psi}_1^\dagger \hat{\psi}_1 + \frac{\nabla \hat{\psi}_2^\dagger \cdot (\nabla \hat{\psi}_2)}{m} + (\gamma \hat{\psi}_1^\dagger \hat{\psi}_2 + \text{h.c.}) \right).
\]  

The first line represents the quantum dot for which we employ the standard [33] harmonic potential approximation \( V_1(x, y) = V_0 + m\alpha^2(x^2 + y^2)/2 \) where the offset \( V_0 \) can be tuned by a gate voltage. The second line describes the reservoir and its tunnel coupling to the dot with the coupling strength \( \gamma \). The field operators \( \hat{\psi}_1(x, y) \) and \( \hat{\psi}_2(x, y) \) describe the dot and reservoir states, respectively.

\[ \hat{\psi}_1^\dagger \hat{\psi}_1 = 1 \] for simplicity.
and \( \hat{\psi}_2(x, y) \) could be envisaged as lowest quantum well states in \( z \)-direction, for example.

If we focus on the ground state of the quantum dot \( \hat{\psi}_1(x, y) \to \hat{\Phi}_0(x, y)\hat{a}_d \) and Fourier transform the reservoir modes \( \hat{\psi}_2(x, y) \to \hat{\psi}_k \), the Hamiltonian (1) becomes

\[
\hat{H} = \epsilon_d \hat{a}_d + \int d^2 k \left[ \epsilon_k \hat{\psi}_k \hat{\psi}_k^{\dagger} + \left( \gamma_k \hat{a}_d \hat{\psi}_k^{\dagger} + \text{h.c.} \right) \right],
\]

where \( \epsilon_d \) denotes the energy of the quantum dot level and \( \hat{a}_d^{\dagger} \hat{a}_d \) the corresponding number operator, while \( \epsilon_k = k^2/(2m) \) are the single-particle energies of the reservoir modes \( k = (k_x, k_y) \). Finally, \( \gamma_k \) is determined by the Fourier transform of the ground-state wave-function inside the quantum dot (see e.g. [35]), i.e., Gaussian

\[
\gamma_k = \gamma \int \frac{d^2 r}{2\pi} \Phi_0^* (x, y) e^{i k \cdot r} \approx 0 \exp \left\{ -\frac{1}{2} k^2 \ell^2 \right\},
\]

where \( \ell = \sqrt{\hbar/(m \omega)} \) is the characteristic dot length scale. Note that the bi-linear Hamiltonian (1) does not explicitly include non-linear (e.g., Coulomb) interactions between the electrons nor their coupling to other degrees of freedom (e.g., phonons) whose potential impact will be discussed below. Therefore, we restrict ourselves to studying the transition between an empty and a singly-occupied dot, for which a charging energy of the quantum dot is irrelevant.

**Quantum Zeno Dynamics** Now we are in the position to study the tunneling process of an electron from the reservoir into the quantum dot. We start with the initial state \( |\text{in}\rangle \) where the dot is empty \( \hat{a}_d |\text{in}\rangle = 0 \) while the reservoir is filled up to the Fermi energy.

Although the above model can be solved exactly using e.g. Greens function techniques [35], we may already calculate the time-dependent probability \( P(t) \) for the occupation of the quantum dot using standard time-dependent perturbation theory with respect to \( \gamma \),

\[
P(t) = \langle \hat{a}_d(t) \rangle = 4 \int \frac{d^2 k}{\pi} |\gamma_k|^2 \sin^2 \left( \left| \epsilon_d - \epsilon_k \right| t/2 \right) \frac{\left( \epsilon_d - \epsilon_k \right)^2}{\left( \epsilon_d - \epsilon_k \right)^2},
\]

where the subscript \( F \) on the integral indicates that the integration runs up to Fermi energy \( \epsilon_F \).

**Quantum-Zeno Regime** Even within the region of applicability of perturbation theory (i.e., for small probabilities \( P(t) \ll 1 \)), we obtain two temporal regimes. For small times \( t \), the probability \( P(t) \) grows quadratically

\[
P(t) = \ell^2 \int \frac{d^2 k}{\pi} |\gamma_k|^2 + O(t^3).
\]

For the Gaussian case [36], this simplifies to

\[
P(t) = \pi \gamma^2 \frac{\ell^2}{\ell_2} \left( 1 - \exp \left\{ -k_F^2 \ell^2 \right\} \right) + O(t^3),
\]

where \( k_F = \sqrt{2m\epsilon_F} \) denotes the Fermi momentum. If the Fermi energy \( \epsilon_F \) is large enough \( 2m\epsilon_F \ell^2 \gg 1 \), i.e., \( k_F^2 \ell^2 \gg 1 \), we obtain \( P(t) = \pi \gamma^2 \ell^2/\ell_2^2 \). In the opposite limit \( k_F^2 \ell^2 \ll 1 \), the integration is cut off by \( k_F^2 \) and we find \( P(t) = \pi \gamma^2 \ell^2 k_F^2 \).

**Regime of Fermi’s Golden Rule** For later times (but still in the perturbative regime \( P(t) \ll 1 \)), we can use the standard approximation of the sinc function in Eq. (6) by a Dirac delta function and obtain a linear growth in time, consistent with Fermi’s Golden Rule

\[
P(t) \approx 2\pi \int \frac{d^2 k}{\pi} |\gamma_k|^2 \delta(\epsilon_d - \epsilon_k). \tag{7}
\]

Obviously, the above integral vanishes for \( \epsilon_d > \epsilon_F \) and for \( \epsilon_d < 0 \), i.e., if there are no filled reservoir modes in resonance with the dot. Assuming \( \epsilon_F > \epsilon_d > 0 \), we find for the Gaussian case (8) with \( \epsilon_k = k^2/(2m) \)

\[
P(t) \approx (2\pi)^2 \gamma \ell^2 \exp \left\{ \epsilon_d - \epsilon_k \right\} \tag{8}
\]

The exponential suppression \( \exp \left\{ -2m\epsilon_d \ell^2 \right\} \) for large dot energies \( 2m\epsilon_d \ell^2 \gg 1 \) stems from the wave-function mismatch between the Gaussian ground-state wave-function of the dot and the reservoir wave-functions of the corresponding energy \( \epsilon_d \) which are rapidly oscillating for large \( \epsilon_d \). In order to avoid this suppression, we assume small dot energies \( 2m\epsilon_d \ell^2 \ll 1 \) in the following.

As a peculiarity of the case of two spatial dimensions, the remaining pre-factor \( P(t) \approx (2\pi)^2 \gamma \ell^2 \) is actually independent of \( \epsilon_d \) because the energy \( \epsilon_k = k^2/(2m) \) and the volume factor \( d^2 k \) both display the same quadratic scaling in \( k \), such that the density of states per energy is constant.

**Crossover Time** Having found an initial period of quadratic growth (9) for early times (quantum Zeno regime) followed by a period of linear growth (7) for later times (regime of Fermi’s Golden Rule), we may estimate the crossover time \( T_c \) marking the transition between the two regimes (6) by comparing Eqs. (6) and (7). For the Gaussian limit we obtain with (6) and (8)

\[
T_c \approx 4\pi m \ell^2 \exp \left\{ -2m\epsilon_d \ell^2 \right\} \tag{9}
\]

Assuming small dot excitation energies \( 2m\epsilon_d \ell^2 \ll 1 \) and considering the limiting cases as discussed after Eq. (6), this simplifies to \( T_c \approx 4\pi \ell^2 k_F^2 \gg 1 \) and to \( T_c \approx 4\pi m/k_F^2 \) for \( \ell^2 k_F^2 \ll 1 \), respectively.

As explained in the Introduction, only measurements with a repetition time faster than \( T_c \) can induce the quantum Zeno effect. This time scale is typically quite short and can be visualized by the following intuitive picture.

Let us first consider the case \( \ell^2 k_F^2 \gg 1 \). If an electron tunnels with a small amplitude from the reservoir to the quantum dot, it leaves behind a hole in the reservoir. The typical size of the initial wave packet of this hole is determined by the characteristic length \( \ell \) of the ground-state wave function of the dot. Afterwards, this wave packet is spreading out or moving away with the typical
group velocity at that length scale $v_{\text{group}} \propto 1/(\ell m)$. Once the wave packet has spread out too much or moved away too far, further tunneling amplitudes would not be be added coherently and probabilities would add up instead. This is precisely the transition from the quantum Zeno regime to the regime of Fermi’s Golden Rule occurring at the crossover time $T_* \sim \ell/v_{\text{group}}$.

In the opposite limit ($k_F^2 \ell^2 \ll 1$), one should just replace the length scale $\ell$ of the dot by the Fermi length $\ell_F \propto 1/k_F$ and the associated group velocity by the Fermi velocity $v_F = k_F/m$. Then, the crossover time $T_*$ is basically the Fermi time $\tau_F$ which is related to the Fermi energy $E_F$ by Heisenberg’s uncertainty relation $[T_*, \mathcal{O} \{E_F^{-1}\}]$ and determines the minimum response time of hole states in the reservoir.

**General Band Structure**  Let us now discuss possible generalizations of the above results. For reservoir electrons propagating in a lattice, the single-particle energies $\epsilon_k$ of the reservoir modes $k$ should be replaced by the lattice band structure, which might also modify the (experimentally accessible) $\gamma_k$ couplings accordingly. Of course, the same would happen for non-harmonic quantum dot potentials $V_l(x,y) = V_0 + m\omega^2(x^2 + y^2)/2$. Nevertheless, one would still expect the effective Hamiltonian (4) to provide a good approximation. Thus, the results (6) and thus (9) and (7) remain valid, but with modified $\epsilon_k$ and $\gamma_k$. This may entail interesting consequences. For example, if we have a flat band (with $\epsilon_k = \text{const}$) below the Fermi energy in resonance with the quantum dot (i.e., $\epsilon_k = \text{const} = \epsilon_d$), the probability (9) per time would become very large while the initial growth (5) would remain almost unaffected since it is independent of $\epsilon_k$. This means that the crossover time $T_*$ would also become very large, which might help observing the quantum Zeno effect in this scenario. In terms of the intuitive picture sketched above, the group velocity becomes very small for such a flat band such that the wave packet stays quite long in the vicinity of the dot.

**Magnetic Field**  The above finding regarding flat or nearly flat bands motivates the study of a strong external magnetic field perpendicular to the reservoir, because this does also turn the reservoir modes into flat bands in the form the well-known Landau levels.

As usual, we represent the constant perpendicular magnetic field $B = B_0 e_z$ in the symmetric gauge by the vector potential $A = B \times r/2$. Then, after minimal coupling $\nabla \rightarrow \nabla - qA$ to the electron charge $q$, the Hamiltonian (1) acquires an additional angular-momentum contribution $-2A \cdot \nabla = B \cdot \mathbf{L} = B\mathbf{L}_z$ as well as a harmonic confinement potential $V_B = \Omega_B^2 (x^2 + y^2)/2$ from the quadratic term $A^2 = B^2 (x^2 + y^2)/4$ where $\Omega_B$ is the cyclotron frequency $\Omega_B = qB/m$.

Thus, the reservoir eigen-functions are no longer plane waves as in Eq. (2), but discrete Landau levels which can be represented by the modes of a two-dimensional harmonic oscillator in polar coordinates $r$ and $\phi$.

$$\psi_{n,l}(r, \phi) = f_{n,l}(r) e^{\pm i \phi}. \quad (10)$$

Here $n \in \mathbb{N}$ and $l \in \mathbb{Z}$ are the quantum numbers corresponding to energy $\epsilon_n = \Omega_B (n + 1/2)$ and angular momentum $L_z$, respectively. The radial mode functions $f_{n,l}(r)$ are given by polynomials multiplied by a Gaussian $\exp(-r^2/(2\ell_B)^2)$ with the magnetic length $\ell_B = 1/\sqrt{qB}$. For example, the lowest Landau levels with $n = 0$ have $f_{n=0,l}(r) \propto r^l \exp(-r^2/(2\ell_B)^2)$ with $l \geq 0$, which can be experimentally mapped.

The ground-state wave function of the quantum dot will also be modified, but merely by the additional harmonic confinement potential $V_B = \Omega_B^2 (x^2 + y^2)/8$ which narrows the Gaussian wave-function (i.e., decreases $\ell$) and increases the energy $\epsilon_d$. Since this wave function is rotationally symmetric (i.e., has zero angular momentum), it only tunnel couples to reservoir modes with $l = 0$.

If we now assume a strong magnetic field $B$ and/or a small Fermi energy $E_F < \Omega_B/2$ such that only the lowest Landau levels are occupied, the effective Hamiltonian (2) can be restricted to the mode $n = l = 0$ and reads

$$H = \epsilon_d \hat{a}_d + \frac{\Omega_B}{2} \hat{n}_{0,0} + \left( \gamma_{0,0} \hat{a}_d^\dagger \hat{a}_{0,0} + \text{h.c.} \right), \quad (11)$$

where the effective tunnel coupling $\gamma_{0,0}$ is determined by the overlap of the two Gaussian wave functions of the dot and $\gamma_{0,0} \propto \exp(- r^2/(2\ell_B)^2)$.

Assuming resonance $\epsilon_d = \Omega_B/2$, this Hamiltonian (11) is then formally equivalent to tunneling between two discrete modes (described by $\hat{a}_d$ and $\hat{a}_{0,0}$) such that we are back to the original picture of the quantum Zeno effect as described in the Introduction.

**Measurement Model**  So far, the Hamiltonian (11) does only describe the internal dynamics of the electrons, but not the actual measurement process – which is causing the quantum Zeno effect. To incorporate this, let us construct a simple toy model for the read out. We assume that the dot is strongly illuminated by laser light (saturation regime) which quickly transfers the electron in the quantum dot from the ground state level to an excited level $\hat{a}_d \hat{a}_d^\dagger$ as soon as the dot is occupied. This excited level then rapidly decays by emitting a photon as described by the bosonic creation and annihilation operators $\hat{b}^\dagger$ and $\hat{b}$, where we focus on one mode for simplicity. Assuming that these excitation–de-excitation cycles occur much faster than all the other time scales considered above (i.e., $T_*$), we may use an effective description where the dot emits photons as soon as it is occupied. Thus, we model the measurement by the additional Hamiltonian

$$\hat{H}_\text{measure} = \hat{n}_d \left( i\zeta \hat{b}^\dagger + \text{h.c.} \right) \quad (12)$$

with the detector coupling strength $\zeta > 0$, which is basically a pointer Hamiltonian for measuring the observable $\hat{n}_d$. 

Since $\hat{H}_{\text{measure}}$ commutes with the undisturbed Hamiltonian $H_0 = \epsilon_d \hat{a}_d + \int d^2 k \epsilon_k \hat{\psi}_k^\dagger \hat{\psi}_k$, we may incorporate it easily into the time-dependent perturbation theory used above for calculating $P(t)$. In the interaction picture, the dot annihilation operator $\hat{a}_d$ just acquires an additional operator-valued factor

$$\hat{a}_d(t) = e^{-i\epsilon_d t} \exp \left\{ (\zeta \hat{b}^\dagger - \text{h.c.}) t \right\} \hat{a}_d(0),$$

(13)

which represents the dynamics induced by $H_0 + H_{\text{measure}}$. When acting on the initial photonic vacuum state $|0\rangle$ with $\hat{b}|0\rangle = 0$, this additional factor $\exp\{ (\zeta \hat{b}^\dagger - \text{h.c.}) t \}$ just generates a coherent state $|\alpha(t)\rangle$ of the photon field whose amplitude $\alpha(t) = \zeta t$ grows linearly with time $t$.

The overlap of these coherent states at different times

$$\langle \alpha(t_1) | \alpha(t_2) \rangle = e^{-\zeta^2(t_1-t_2)^2/2}$$

modifies Eq. (11) via

$$P(t) = \int_0^t dt_1 \int_0^t dt_2 \int d^2 k \ |\gamma_k|^2 \exp \left\{ -\frac{\zeta^2}{2} (t_1 - t_2)^2 \right\} \times \exp\{ -i(\epsilon_d - \epsilon_k)(t_1 - t_2) \}.$$ 

(14)

For very short times $t \ll T_*$ and $t \ll 1/\zeta$, we recover the initial quadratic growth in Eq. (12). What happens after that initial period depends on the quantity $\zeta T_*$. For small $\zeta T_* \ll 1$, we recover Eq. (7). For large $\zeta T_* \gg 1$, on the other hand, the probability is strongly suppressed

$$P(t) \sim t \frac{\sqrt{2}}{\zeta} \int d^2 k \ |\gamma_k|^2,$$

(15)

which is a manifestation of the quantum Zeno effect by frequent measurements with the effective rate $\zeta$.

The detector signal can be very explicitly evaluated with methods of Full Counting Statistics or quantum polsyptra [24, 40]. However, already the analysis of average values reveals (see supplement) that – although delay effects occur – a transition between the timescales can be directly resolved.

**Conclusions** For the example of a quantum dot tunnel coupled to an effectively two-dimensional electron reservoir, we study whether and how the quantum Zeno effect can be employed to suppress tunneling between a discrete level and a continuum. In contrast to tunneling between two discrete levels, we find that the required measurement repetition rate is set by the time $T_*$, it takes the wave-packet in the reservoir to move away from the vicinity of the quantum dot. Hence this time scale can be increased (and thus the required repetition rate decreased) by lowering the Fermi energy or having a flat band (thus reducing the Fermi velocity) or by applying a strong perpendicular magnetic field, which effectively localizes the reservoir modes.

In a bigger picture, suppressing tunneling via the quantum Zeno effect and thereby effectively isolating the quantum dot from its environment is quite analogous to passive quantum error correcting schemes, such as the spin echo method. In contrast to active quantum error correcting schemes (typically involving redundancies), such passive schemes are based on interrupting the coherence between the quantum dot or qubit and its environment such that the constructive interference of the error amplitudes is turned into a (at least partially) destructive interference. As an intuitive picture, these methods work well as long as the wave-packet associated with the quantum error is still “lurking” in the vicinity of the quantum dot or qubit. Once they moved too far away, the impact of measurements or echo sequences is strongly reduced.

In order to discuss the experimental relevance of our findings, let us insert typical experimental parameters. For an effective mass of around 7% of the electron mass, a strong perpendicular magnetic field, which represents the dynamics induced by $H_0 + H_{\text{measure}}$. Assuming a typical level spacing in the quantum dot of around 50 meV, the characteristic length scale of the ground state wave function is $\ell \approx 5$ nm. With a Fermi momentum of order $10^5$ m$^{-1}$, we have $k_F \ell$ of order unity. Thus, we may estimate the crossover time $T_*$ by comparing the Fermi velocity with the length scale $\ell \approx 5$ nm which yields rather short times of order $T_* = O(10 \text{ fs})$. Unfortunately, although measurements can be performed quite fast (on the nano-second scale [20]), this $T_* = O(10 \text{ fs})$ is probably too short to observe the quantum Zeno effect in this set-up. Increasing this time scale $T_*$ by decreasing the Fermi energy (i.e., reducing the electron density in the reservoir) or by using materials with a flat band is possible in principle, but experimentally quite challenging.

Thus, probably the most viable option is to apply a strong perpendicular magnetic field $B$, say, of around 10 T, such that the 2DEG is in the Quantum Hall regime. The resulting magnetic length of $\ell_B \approx 8$ nm is roughly of the same order as the length scale $\ell \approx 5$ nm associated with the quantum dot and similarly the Landau level spacing of about 15 meV is not too far away from the other energy scales. With aligning the energy levels accordingly, one can arrange resonant tunnel transitions between the discrete level of the quantum dot and one of the discrete Landau levels in the reservoir. In this case, the line of arguments sketched in the Introduction can be applied and thus the measurement repetition rate is set by the tunneling rate $\gamma$ between these two levels. Since the time associated scale $1/\gamma$ can be quite long (in the micro- to milli-second regime or even longer, which would also enable an electric readout [17, 18]), a measurement time in the nano-second regime should be sufficient to observe the quantum Zeno effect.

In order to enable such an observation, one has to make sure that the environmental decoherence (which can have basically the same effect as a measurement; for a review of tunneling in a dissipative environment see e.g. [43]) is
weak enough, i.e., the associated coherence time is longer than the time between measurements. A frequently discussed decoherence mechanism is the scattering of the electrons or holes in the reservoir at local impurities. As long as these impurities (which can be characterized experimentally) are not too dense and thus far away from the dot, this mechanism is not relevant here because, as explained above, if the reservoir wave packet moved away from the dot far enough to see the impurity, it is already outside the region of applicability of the quantum Zeno effect. Another potentially important mechanism stems from the Coulomb interaction between the electrons or holes or their interaction with phonons. In order to suppress this decoherence mechanism, the temperature should be low enough such that the available phase space and number of excitations is reduced.

Also in this respect, working in a strong magnetic field can be helpful: While electron-phonon scattering in a field-free 2DEG happens in two-dimensional $k$-space, in the Quantum Hall regime electron-phonon scattering is limited to the one-dimensional (both in real-space and $k$-space) lines along which the electrons drifts. This reduced dimensionality strongly suppresses electron-phonon scattering and renders it negligible at milli-Kelvin temperatures. At these temperatures $T$, electron-electron scattering is the only relevant mechanism and follows a $T^2$ law [4] in a 2DEG in a strong magnetic field. Experimental studies of the universality in the Quantum Hall regime [40] allow one to estimate the associated time scale.

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Detector dynamics

To evaluate the detector occupation, we first note that in the interaction picture, the photon annihilation operator grows linearly in time

$$\hat{b}(t) = \hat{b}(0) + \zeta t \hat{n}_d(0).$$

(16)

The actually emitted photon number of our toy model then becomes to second order perturbation theory

$$\langle \hat{b}^\dagger \hat{b} \rangle_t = \zeta^2 \int_0^t dt_1 \int_0^{t_1} dt_2 \int_0^F d^2k |\gamma_k|^2 \exp\left\{-\frac{\zeta^2}{2} (t_1 - t_2)^2\right\}$$

$$\times (t - t_1)(t - t_2) \exp\{-i(\epsilon_d - \epsilon_k)(t_1 - t_2)\},$$

(17)

which also reflects the crossover from a $t^4$ scaling for $t \ll T_*$ and $t \zeta \ll 1$ to a reduced – at most $t^2$ – scaling for $t \gg T_*$. The initial delayed response can be understood as the detector has to react to the dynamics of the system first.

The expectation value of a photon annihilation operator (relevant e.g. for homodyne measurements) would become instead

$$\langle \hat{b}^\dagger \rangle_t = \zeta \int_0^t dt_1 \int_0^{t_1} dt_2 \int_0^F d^2k |\gamma_k|^2 \exp\left\{-\frac{\zeta^2}{2} (t_1 - t_2)^2\right\}$$

$$\times (t - t_2) \exp\{-i(\epsilon_d - \epsilon_k)(t_1 - t_2)\}$$

(18)

and exhibits a crossover from a cubic to (at most) quadratic scaling.

Considering the effect of a finite detector frequency $\Omega_{\text{measure}} \rightarrow \Omega_{\text{measure}} + \omega_{\text{det}} \hat{b}^\dagger \hat{b}$ implies an oscillatory behaviour of $\hat{b}(t)$ in the interaction picture and would eventually again lead to a linear long-term scaling in the Fermi golden rule regime.
Measuring empty dots

If the detector would be sensitive to the empty state, i.e.,

\[ \hat{H}_{\text{measure}} = \hat{a}_d \hat{a}_d^\dagger (\imath \zeta \hat{b}^\dagger + \text{h.c.}) \]  

(19)

instead of (12) in the main text, we would get in the interaction picture

\[ \hat{a}_d(t) = e^{-\imath \epsilon_d t} \exp \left\{ - (\imath \zeta \hat{b}^\dagger - \text{h.c.}) t \right\} \hat{a}_d(0), \]

\[ \hat{b}(t) = \hat{b}(0) + \zeta t \hat{a}_d(0) \hat{a}_d^\dagger(0). \]  

(20)

Since in the first line, we have only a flip in sign of \( \zeta \) in comparison to (13) in the main text, the arguments raised there for the dot occupation prevail in this case and we retain (14) in the main text (assuming of course the same initial conditions).

This is different however for the detector occupation. In this case, due to \( \text{Tr} \{ \hat{b}^\dagger(t) \hat{b}(t) \rho_0 \} = \zeta^2 t^2 \), a detector signal is seen already for \( \gamma_k = 0 \) for an initially empty dot. Up to second order, we get

\[ \langle \hat{b}^\dagger \hat{b} \rangle_t = \zeta^2 t^2 \]

(21)

\[ + \zeta^2 \int_0^t dt_1 \int_0^t dt_2 \int d^2k |\gamma_k|^2 \exp \left\{ - \frac{\zeta^2}{2} (t_1 - t_2)^2 \right\} \]

\[ \times [t_1 t_2 + t|t_1 - t_2| - t^2] \exp \{ - i (\epsilon_d - \epsilon_k)(t_1 - t_2) \}, \]

where the temporal crossover of the second order contribution is much harder to observe as the first order contribution is dominant. Thus, to see Zeno signatures in such a detector it would be more suitable to consider the reverse process of an electron tunnelling out of the dot.