A new proposal for measuring the Lense–Thirring effect with a pair of supplementary satellites in the gravitational field of the Earth

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Abstract

In this letter we propose a new observable for measuring the general relativistic Lense–Thirring effect with artificial satellites in the gravitational field of the Earth. It consists of the difference of the perigee rates of two satellites placed in identical orbits with supplementary inclinations. As in the well known LAGEOS–LARES project in which, instead, the sum of the residuals of the nodal rates would be used, the proposed observable would be able to cancel out the aliasing effect of the classical even zonal perigee precessions induced by the oblateness of the Earth. The possibility of using the already existing LAGEOS II and a twin of its, to be launched, in a supplementary orbit is briefly examined. While with the originally proposed LAGEOS–LARES mission only the sum of the nodal rates could be used because the perigee of LAGEOS is not a good observable, the implementation of the proposed mission would allow to adopt both the sum of the nodes and the difference of the perigees.
1 Introduction

In its weak-field and slow-motion approximation General Relativity predicts that, among other things, the orbit of a test particle freely falling in the gravitational field of a central spherical rotating body is affected by the so called gravitomagnetic dragging of the inertial frames or Lense–Thirring effect. More precisely, the longitude of the ascending node $\Omega$ and the argument of the perigee $\omega$ of the orbit $\Omega$ undergo tiny precessions $\dot{\Omega}$ and $\dot{\omega}$ (The original papers by Lense and Thirring can be found in english translation in [4] where the longitude of the perigee $\varpi = \Omega + \omega$ is used instead of $\omega$ which is an angle counted in the osculating orbital plane from the line of the nodes to the direction of the perigee.)

\[
\dot{\Omega}_{LT} = \frac{2GJ}{c^2 a^3 (1-e^2)^{3/2}}, \tag{1}
\]
\[
\dot{\omega}_{LT} = -\frac{6GJ \cos i}{c^2 a^3 (1-e^2)^{3/2}}, \tag{2}
\]

in which $G$ is the Newtonian gravitational constant, $J$ is the proper angular momentum of the central body supposed spherically symmetric and rigidly rotating, $c$ is the speed of light in vacuum, $a$, $e$ and $i$ are the semimajor axis, the eccentricity and the inclination, respectively, of the orbit of the test particle.

The first measurement of this effect in the gravitational field of the Earth has been obtained by analyzing a suitable combination of the laser-ranged data to the existing passive geodetic satellites LAGEOS and LAGEOS II [5]. The observable $\dot{\omega}$ is a linear trend with a slope of 60.2 milliarcseconds per year (mas/y in the following) and includes the residuals of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II. The Lense–Thirring precessions for the LAGEOS satellites amount to

\[
\dot{\Omega}_{LT}^{\text{LAGEOS}} = 31 \text{ mas/y}, \tag{3}
\]
\[
\dot{\Omega}_{LT}^{\text{LAGEOS II}} = 31.5 \text{ mas/y}, \tag{4}
\]
\[
\dot{\omega}_{LT}^{\text{LAGEOS}} = 31.6 \text{ mas/y}, \tag{5}
\]
\[
\dot{\omega}_{LT}^{\text{LAGEOS II}} = -57 \text{ mas/y}. \tag{6}
\]

\footnote{The perigee of LAGEOS was not used because it introduces large observational errors due to the smallness of the LAGEOS eccentricity.}\[6\] which amounts to 0.0045.
The claimed total relative accuracy of the measurement is $2 \times 10^{-1}$.

In this kind of experiment the major source of systematic errors is represented by the aliasing trends due to the classical secular precessions of the node and the perigee induced by the mismodelled even zonal harmonics of the geopotential $\delta J_2$, $\delta J_4$, $\delta J_6$, ... Indeed, according to the present knowledge of the Earth’s gravity field based on EGM96 model, they amount to a large part of the gravitomagnetic precessions of interest, especially for the first two even zonal harmonics. In the performed LAGEOS experiment the adopted observable allowed for the cancellation of the static and dynamical effects of $\delta J_2$ and $\delta J_4$. The remaining higher degree even zonal harmonics affected the measurement at a 12.9% level, according to the covariance matrix of the EGM96 up to degree $l = 20$.

In order to achieve a few percent accuracy, in [9] it was proposed to launch a passive geodetic laser-ranged satellite—the former LAGEOS III—with the same orbital parameters of LAGEOS apart from its inclination which should be supplementary to that of LAGEOS.

This orbital configuration would be able to cancel out exactly the classical nodal precessions, which are proportional to $\cos i$, provided that the observable to be adopted is the sum of the residuals of the nodal precessions of LAGEOS III and LAGEOS

$$\delta \dot{\Omega}^{\text{III}} + \delta \dot{\Omega}^{\text{I}}.$$  \hspace{1cm} (7)

The relativistic signature would be a linear trend with a slope of 62 mas/y. Later on the concept of the mission slightly changed. The area-to-mass ratio of LAGEOS III was reduced in order to make less relevant the impact of the non-gravitational perturbations and the eccentricity was enhanced in order to be able to perform other general relativistic tests: the LARES was born [10].

Currently, the observable of the LAGEOS–LARES mission is under revision in order to improve the obtainable accuracy [11].

The orbital parameters of LAGEOS, LAGEOS II and LARES are in Tab. 1.

2 A new perigee–only observable

The concept of a couple of satellites placed in identical orbits with supplementary inclinations could be fruitfully exploited in the following new way.
Table 1: Orbital parameters of LAGEOS, LAGEOS II and LARES.

| Orbital parameter | LAGEOS | LAGEOS II | LARES |
|-------------------|--------|-----------|-------|
| $a$ (km)          | 12,270 | 12,163    | 12,270|
| $e$               | 0.0045 | 0.014     | 0.04  |
| $i$ (deg)         | 110    | 52.65     | 70    |

An inspection of eq. (2) and of the explicit expressions of the rates of the classical perigee precessions induced by the even zonal harmonics of the geopotential [12] suggests to adopt as observable the difference of the residuals of the perigee precessions of the two satellites

$$\delta \dot{\omega}^i - \delta \dot{\omega}^{180^\circ - i},$$

so to obtain a secular trend with a certain slope in mas/y. Indeed, on one hand, the Lense–Thirring perigee precessions depend on $\cos i$, contrary to the nodal rates which are independent of the inclination, so that, by considering the relativistic effect as an unmodelled force entirely adsorbed in the residuals, in eq. (8) they sum up. On the other, it turns out that the classical even zonal perigee precessions depend on even powers of $\sin i$ and on $\cos^2 i$, so that they cancel out exactly in eq. (8). It may be interesting to notice that the proposed observable of eq. (8) is insensitive to the other general relativistic feature which affect the pericenter of a test body, i.e. the gravitoelectric Einstein precession. Indeed, as it is well known [2], it does not depend on the inclination of the orbital plane.

In regard to a practical application of such idea, we note that the LAGEOS–LARES mission would be unsuitable because the perigee of LAGEOS is not a good observable due to the notable smallness of the eccentricity of its orbit. For the sake of concreteness, we could think about a LARES II which should be the supplementary companion of LAGEOS II. In this case we would have a gravitomagnetic trend with a slope of -115.2 mas/y (which is almost twice that of the LAGEOS–LARES node–only mission). Moreover, since the magnitude of the eccentricity of LAGEOS II is satisfactory in order to perform relativistic measurements with its perigee, the LARES II, contrary to the LAGEOS–LARES mission, could be inserted in an orbit with the same eccentricity of that of LAGEOS II. So, the cancellation of the classical secular precessions would occur at a higher level than in the LAGEOS–LARES node–only observable [11].
course, a careful analysis of the time–dependent gravitational and, especially, non–gravitational perturbations (see [14] for the radiative perturbations and [14] for the thermal, spin–dependent perturbations), to which the perigee is particularly sensitive, contrary to the node, would be needed in order to make clear if also for such perturbations some useful cancellations may occur. Among the gravitational tidal perturbations, fortunately, the most relevant components of the semi–secular 18.6–year and the 9.3–year tides are even \((l = 2)\) zonal \((m = 0)\) [13], so that they would be canceled out. The attention should be focused on some insidious non–gravitational thermal perturbations. Indeed, some of them can generate linear perturbations, like the terrestrial Yarkovsky–Rubincam effect, which could mimic the relativistic signal. From very preliminary investigations based on [13] it seems that such terms would cancel out because proportional to \(\cos^2 i\), apart from other characteristics of the satellites like the thermal lag angle \(\vartheta\) and the square of the component \(S_z^2\) of the spin vector along the z axis of an inertial frame which could be identically prepared for the two satellites. On the contrary, according to [14] many components of the radiative perturbations would not cancel out because they depend on \(\cos i\). However, in regard to such other harmonic perturbations, even if not cancelled out, their impact would be less dangerous because, over long enough time spans, they could be viewed as empirically fitted quantities and removed from the signal. An optimal choice would be the launch of a couple of entirely new, geodetic satellites in highly eccentric orbits with supplementary inclinations to be carefully selected so to reduce the periods of the time–varying perturbations which could affect the observable. By the way, it should also be noticed that the implementation of such a mission would provide us with, at least, two different and complementary gravitomagnetic observables: \(\delta \dot{\Omega}^i + \delta \dot{\Omega}^{180^\circ - i}\) and \(\delta \dot{\omega}^i - \delta \dot{\omega}^{180^\circ - i}\). Instead, the originally proposed LAGEOS–LARES mission would allow to use \(\delta \dot{\Omega}^i + \delta \dot{\Omega}^{180^\circ - i}\) only. Moreover, the presence in orbit of LARES with LARES II or, eventually, two entirely new satellites would strongly enhance the possibility of fruitfully following the strategy of the multisatellite combined residuals, as sketched in [11]. More extensive and quantitative investigations can be found in [10, 17]. Of course, also a pair of drag–free satellites, although more expensive, could be employed so to reduce dramatically the impact of the non–gravitational perturbations.
3 Conclusions

In this letter we have proposed a new observable for the measurement of the general relativistic Lense–Thirring effect on the orbital motion of artificial satellites in the gravitational field of the Earth. It consists of the difference of the perigee rates of two satellites placed in identical orbits, except for the inclinations which should be supplementary. Such proposal could be implemented by means of a LARES II satellite with the same orbital parameters of the already existing LAGEOS II, except for the inclination, or by launching an entirely new couple of geodetic satellites in highly eccentric orbits with carefully selected supplementary inclinations. The latter choice could reduce the periods of the non–gravitational perturbations, which have, in general, a non negligible impact on the perigees of geodetic satellites, affecting the proposed observable. Forthcoming investigations of the time–dependent gravitational and non–gravitational perturbations will make clear their role in the sketched LAGEOS II–LARES II perigee–only mission. Of course, in addition to the proposed perigee–only measurement, also the sum of the nodes could be investigated, so to enforce and enlarge the experimental basis for a detection of the elusive Lense–Thirring effect.

Acknowledgements

I’m grateful to L. Guerriero for his support while at Bari and to D.M. Lucchesi for his helpful and important informations on the non–gravitational perturbations on LAGEOS II.

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