Smooth double critical state theory for

type-II superconductors

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Abstract

Several aspects of the general theory for the critical states of a vortex lattice and the magnetic flux dynamics in type-II superconductors are examined by a direct variational optimization method and widespread physical principles. Our method allows us to unify a number of conventional models describing the complex vortex configurations in the critical state regime. Special attention is given to the discussion of the relation between the flux line cutting mechanism and the depinning threshold limitation. This is done by using a smooth double critical state concept which incorporates the so-called isotropic, elliptical, T and CT models as well-defined limits of our general treatment. Starting from different initial configurations for a superconducting slab in a 3D magnetic field, we show that the predictions of the theory range from the collapse to zero of transverse magnetic moments in the isotropic model, to nearly force-free configurations in which paramagnetic values can arbitrarily increase with the applied field for magnetically anisotropic current–voltage laws. Noteworthily, the differences between the several model predictions are minimal for the low applied field regime.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The study of the critical state theory of a vortex lattice in type-II superconductors is a stimulating problem. It relates to a wide list of physical phenomena and also affects a number of practical applications. The original concept of a critical state dates back to the work by Bean [1–3] who assumed that external magnetic field variations are opposed by the maximum (critical) current density in the material, i.e. when non-vanishing $|J| = J_c$. Physically, the driving force due to the currents circulating in the superconducting sample is balanced by the limiting pinning force acting on the vortex lattice so as to prevent destabilization and the consequent propagation of dissipative states. It occurs that Bean’s simplifying ansatz straightforwardly leads to predicting the proper response of the sample provided the electrical current density vector $J$ is perpendicular to the local magnetic field vector $B$, i.e. $J(r) = J_{\perp}(r)$. We just recall that magnetostatic forces are given by $J \times B$. However, unless for highly symmetric situations, $J$ does not necessarily satisfy the condition $J = J_{\perp}$. Therefore, the stronger limitation of Bean’s model is that one can just apply it to vortex lattices composed by parallel flux lines perpendicular to the current flow. On the other hand, rotations of $B$ can lead to entanglement and recombination of neighboring flux lines which brings a component of the current density along the local magnetic field, $J_{\parallel}$. This component generates distortions which also become unstable when a threshold value $J_{c\parallel}$ is exceeded, giving way to the so-called flux cutting phenomenon. Thus, when the conditions $J_{\parallel} = J_{c\parallel}$ and $J_{\perp} = J_{c\perp}$ become active, the so-called double critical state appears.

From the mathematical point of view, the critical state problem consists of finding the equilibrium distribution for the circulating current density $J(r)$ defined by the conditions $J_{\parallel} \leq J_{c\parallel}$ and $J_{\perp} \leq J_{c\perp}$, both consistent with the Maxwell equations in quasistationary form, i.e. displacement currents are neglected [4]. Customarily, one also considers situations where the local components of the magnetic field $H(r)$ along the superconductor are much higher than the lower critical field $H_{c1}$ and well below $H_{c2}$ to allow the use of the linear relation $B = \mu_0 H$.

The general statement of the critical state, in the above-described terms, was done by Clem and Pérez-González [5–12]. In particular, these authors have provided the physical background for successfully understanding an
important number of experiments with rotating and oscillating magnetic field components. On the other hand, the theoretical scenario has been successively enlarged by a number of alternative approaches that focus on different aspects of the vast number of experimental activities in this field, e.g. one can identify the so-called:

(i) isotropic models [13–17] in which the critical state law reads \( J_\parallel^2 + J_\perp^2 \leq J_c^2 \) in the spirit of Bean’s original hypothesis [3].

(ii) elliptical model [18–20], posed through the condition \( J_\parallel^2 / J_\parallel^2 + J_\perp^2 / J_\perp^2 \leq 1 \).

(iii) \( T \) states characterized by \( J_\perp \leq J_c \) and \( J_\parallel \) unbounded [21].

In this work, we will show that all the above-mentioned models may be unified within a continuous two-parameter theory that poses the problem in terms of geometrical concepts within the \( J_\parallel - J_\perp \) plane. Our analysis is performed by using the methodology introduced in [4]. In that paper, a general critical state theory for investigating the flux dynamics in type-II superconductors was introduced. In brief, it was shown that the quasistatic magnetic response of the superconductor may be obtained by minimizing variations of a current density functional over the sample. Minimization must be performed for constrained values of such a quantity, i.e. \( \mathbf{J}(\mathbf{r}) \in \Delta r \), with this condition standing for the material law describing the superconductor. Taking advantage of that framework, in this paper we show that by the application of our variational statement, one is able to specify almost any critical state law by means of a simple mathematical model that includes an index \( n \), accounting for the smoothness of the \( J_\parallel / J_\perp \) relation and a certain bandwidth characterizing the magnetic anisotropy ratio \( \chi \equiv J_\parallel / J_\perp \). The systematic consideration of the influence of these parameters will allow a straightforward elucidation of the relation between diverse physical processes and the actual material law.

This paper is organized as follows. In section 2 we put forward some details about the most remarkable features of the critical state theory. The physical interpretation of the underlying approximations is focused on within our variational formulation. Then, section 3 is devoted to observe some properties of the electrodynamical behavior of the superconductor for different choices of the above-defined parameters \( n \) and \( \chi \). Specifically, we consider 3D magnetic field structures in the infinite slab geometry for different initial states and processes (see figure 1). These configurations have been already studied before [4] and are proposed as basic arrangements for the visualization of intriguing phenomena such as the magnetization collapse and the appearance of paramagnetic responses. A global discussion of our results and some concluding remarks are finally presented in section 4.
2. Magnetic anisotropy of the critical state

As stated above, the most complete description of irreversible phenomena in type-II superconductors at a macroscopic level is done through the commonly called double critical state model (DCSM) introduced by Clem and Pérez-González [5–12]. Let us recall some basic ideas that will be essential for our further treatment. The material law introduced by the above authors in the form of the threshold conditions $J_1 \leq J_{c1}$ and $J_2 \leq J_{c2}$ for the current density flowing either parallel or perpendicular to the local magnetic field has been expressed in a geometrical language in previous articles [4, 22]. Essentially, our concept is to define a region $\Delta_r(J)$ within the $J_1$–$J_2$ plane such that nondissipative current flow occurs when the condition $J = J_1 + J_2 \in \Delta_r$ is verified. In contrast, a very high dissipation is to be assumed when $J$ is driven outside $\Delta_r$. Recall that this scheme has allowed us to translate the DCSM physics onto a region of currents defined in 3D by a cylinder [4] with its axis parallel to the local magnetic field $\mathbf{B}$ and a rectangular longitudinal section in the plane defined by the vectors $\mathbf{J}_1 = J_1 \mathbf{\hat{E}}_1$ and $\mathbf{J}_2 = J_2 \mathbf{\hat{E}}_2$. Such a section is shown in figure 1. We recall that, in 2D problems with in-plane currents and magnetic field, the current density region straightforwardly coincides with the above-mentioned longitudinal section. In this scheme, the parts of the sample where only the flux depinning threshold has been reached are denoted as T zones or flux transport zones ($J_1 = J_{c2} < J_2$). They are represented by points in a horizontal band. Physically, the flux lines are migrating while basically retaining their orientation. On the other hand, regions where only the cutting threshold is active are denoted as C zones or flux cutting zones ($J_1 = J_{c1} > J_2$). They are represented by points in a vertical band. Those regions where both mechanisms have reached their critical values are defined as CT zones ($J_1 = J_{c1}$ and $J_2 = J_{c2}$). The current density vector belongs to the corners of a rectangle. Finally, the regions without energy dissipation are called O zones and the current density vector belongs to the interior of the rectangle.

In this section, and corresponding to the regions depicted in the lower part of figure 1, we investigate the magnetic response of type-II superconductors, whose material law is obtained by smoothly modifying the standard DCSM rectangular region until the elliptic cases are reached. As we will focus on the role of the smoothing index $n$, only the extreme cases $\chi = 1$ and $\chi \rightarrow \infty$ will be considered. Notice that the smooth limiting cases (elliptic or isotropic) can be interpreted as the manifestation of averaged values of the critical current restrictions due to the inhomogeneity of the material, as well as a consequence of flux line interactions at a mesoscopic level that introduce coupling between the thresholds $J_{c\parallel}$ and $J_{c\perp}$. In any case, smooth models have to be considered as related to a number of experiments that one could not explain within piecemeal smooth statements [23–28]. Hereafter, we will use the notation $\text{Sm-DCSM}$ for such models.

2.1. Variational statement

As indicated above, the selection of appropriate restrictions for the macroscopic current density is a significant step forward to reveal the intrinsic structure of the mechanisms involved. In conventional approaches, related to the material law $\mathbf{J}(|\mathbf{E}|)$, and starting from the Maxwell equations, one may obtain the penetration profiles for the magnetic field from a differential equation statement of diffusive type (i.e. $\partial_t \mathbf{H} = f(\nabla^2 \mathbf{H})$). In our case, the basis of the Sm-DCSM relies on a parallel (and equivalent) formulation that uses a discretization scheme of the magnetic field in terms of time steps connected by the finite-difference expression $\mu_0 (\mathbf{H}_{t+1} - \mathbf{H}_t)$. The evolution from one magnetostatic state to another is obtained variationally. Thus, we minimize the functional

$$F[\mathbf{H}_{t+1}(\mathbf{r})] = \frac{\mu_0}{2} \int_V [\mathbf{H}_{t+1} - \mathbf{H}_t]^2 + \rho \cdot (\nabla \times \mathbf{H}_{t+1} - \mathbf{J})$$  \hspace{1cm} (1)

where the Lagrange multiplier enforces Ampère’s law [4, 29].

In addition, the minimization is performed with the local distribution of currents constrained by the law $\mathbf{J} \in \Delta_r$. Notice that either material or extrinsic anisotropy can be easily incorporated by prescribing $\Delta_r$ to be the appropriate region: for instance, by modeling $\Delta_r$ as an elliptical [4, 18–20, 22] or a rectangular [4–12, 30] region oriented over selected axes. Mathematically, such kinds of regions are hosted as limiting cases of a smooth expression defined by the two-parameter family of superelliptic functions:

$$(\frac{J_1}{J_{c1}})^{2n} + (\frac{J_2}{J_{c2}})^{2n} \leq 1.$$  \hspace{1cm} (2)

The reader can immediately verify that an index $n = 1$ and a bandwidth defined by $\chi = J_{c1}/J_{c2} = 1$ correspond to the standard isotropic model [13]. On the other hand, when one assumes enlarged bandwidth (i.e. $\chi > 1$), the Sm-DCSM becomes the standard elliptical model introduced by Romero-Salazar and Pérez-Rodríguez [18, 19]. When the bandwidth $\chi$ is extremely large, i.e. $J_{c1} \gg J_{c2}$, one recovers the so-called $T$ states treated by Brandt and Mikitik [21]. Rectangular regions strictly corresponding to the DCSM [5–12] are obtained for the limit $n \rightarrow \infty$ and arbitrary $\chi$. Finally, allowing $n$ to take values over the positive integers, a wide scenario describing anisotropy effects is envisioned (figure 1(b)). Such regions will be named superelliptical and their properties can be understood in terms of the rounding (or smoothing) of the corners for the DCSM. We note in passing that, in general, $n$ could be chosen as a positive real. However, integer values allow us to model the smoothing process very accurately and are more tractable from the numerical point of view.

2.2. Numerical treatment

In order to illustrate the effect of the material law, below we will show the behavior of the field and current density profiles for the system illustrated in figure 1 with different selections of the region $\Delta_r$. To be specific, we have considered an infinite slab made up by $2 \times N_y$ longitudinal sheets arranged along the $x$–$y$ plane and filling the space $|z| \leq a$. Depending on the external excitation, either symmetry or antisymmetry conditions for the different electrodynamical quantities can be applied along the $N_y$ sheets arranged in $0 \leq z \leq a$. On the other hand, one can assume in-plane position...
independence of \( \mathbf{J} \), i.e. one has \([I_x(z_i), J_y(z_i)]\). Note that this ensures a divergenceless current density as required by charge conservation in steady states. Incidentally, we have to mention that in this work we have used \( N_s = 300 \). From the technical point of view, we also mention that the physical parameters that define the problem have been used to renormalize the physical quantities. Thus, we use \( h \equiv \mathbf{H}/I_n a \) and \( \mathbf{J} \equiv \mathbf{J}/I_n a \). Recall that one may assume the numerical value \( I_n \) as known a priori or obtained from experiment. Finally, the position within the slab will be expressed in terms of \( z \equiv z/a \).

Now, following [4] the variational statement for the Sm-DCSM, in numerical form, leads to minimizing the function

\[
F([I_{i,j+1}]) = \frac{1}{2} \sum_{i,j} I_{i,j+1} M_{i,j} I_{i,j+1} - \sum_{i,j} I_{i,j} M_{i,j} I_{i,j+1} + \sum_i I_{i,j+1} \Delta h(z_i),
\]

where the set of unknown current values for the time layer \( l+1 \), i.e. \([I_{i+,j}]\), are defined within a collection of circuits (indexed by \( i, j \)) whose mutual inductance coefficients are represented by \( M_{i,j} \). In the slab symmetry the circuits are just sheets made up of straight lines along the \( x \) and \( y \) axes and \([I_i]\) is a compact notation for the whole set. Finally, \( \Delta h(z_i) \) defines the time discretization of the corresponding applied magnetic field components \( (\Delta h(z_i) \equiv h_{i+1}(z_i) - h(z_i)) \). As was shown in [4], the geometrical coefficients \( M_{i,j} \) are given by

\[
M_{i,j} \equiv 1 + 2[\text{min}(i,j)], \quad \text{or} \quad 2(\frac{1}{2} + i - 1),
\]

\( i \neq j \), or

\[
M_{i,j} \equiv 2(\frac{1}{2} + i - 1), i = j.
\]

Recall that inductive coupling only occurs for \( x \) and \( y \) layers separately, and that the corresponding coefficients are identical.

Eventually, the response of the superconductor is obtained as a pair of surface current functions \( \{j_x(z_i), j_y(z_i)\} \) for each one of the \( N_s \) sheets. The magnetic field profiles and magnetic moments may be obtained by numerical integration. Thus, the magnetic moment components per unit area are obtained from

\[
M = \int_{-a}^{a} \mathbf{z} \times \mathbf{j} \, dz.
\]
Figure 2. Profiles of the magnetic field component $h_z, h_x(\alpha)$ and their corresponding current density profiles $j_y(z, h\alpha)$ starting from the first-time step defined by $h_x(\alpha) = 1.1$ and $h_z = 1.5$ in the diamagnetic configuration (figure 1(c)). The current component $j_x(z, h\alpha)$ and the cutting component $j_\perp$ are also shown. The curves are labeled according to the longitudinal magnetic field component at the surface of the slab corresponding to the values $h_x(\alpha) = 0.040, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.6, 2.0, 4.0$ and $8.0$, and $h_z = 1.5$. First row: profiles for the isotropic model $\chi^2 = 1, n = 1$. Second: profiles for the Sm-DCSM with $\chi^2 = 1, n = 4$. Third: profiles for the DCSM with $\chi^2 = 1, n = \infty$. Finally, in the final row, the profiles for the $T$-state model, i.e. $\chi^2 \to \infty, j_\perp \neq 0$ are shown.

Figure 3. Same as figure 2, but the initial paramagnetic configuration illustrated in figure 1(d) with $h_x(\alpha) = 1.1$ and $h_z = 1.5$. Here, the curves have been labeled according to the values $h_x(\alpha) = 0.040, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 2.0, 3.0, 4.0, 5.0, 6.0, 7.0$ and $8.0$. 

5
The magnetic moment components $M_x$ (top) and $M_y$ (bottom) per unit area as a function of the applied magnetic field component $h_y(a)$ in the diamagnetic (left) and paramagnetic (right) initial configurations defined by $h_x(a) = 1.1$ and $h_z = 1$. Results for several models are shown, e.g. the $T$-state model ($\chi^2 \to \infty$, $j_{c\parallel} \neq 0$), the conventional DCSM ($\chi^2 = 1$, $n \to \infty$), the $Sm$-DCSM with $\chi^2 = 1$ and $n = 1$, and the $Sm$-DCSMs with $\chi^2 = 1$ and $n = 2, 3, 4, 6$ and 10. The insets shows a zoom of the behavior around the minima.

Figure 4. The magnetic moment components $M_x$ (top) and $M_y$ (bottom) per unit area as a function of the applied magnetic field component $h_y(a)$ in the diamagnetic (left) and paramagnetic (right) initial configurations defined by $h_x(a) = 1.1$ and $h_z = 1.5$. Results for several models are shown, e.g. the $T$-state model ($\chi^2 \to \infty$, $j_{c\parallel} \neq 0$), the conventional DCSM ($\chi^2 = 1$, $n \to \infty$), the $Sm$-DCSM with $\chi^2 = 1$ and $n = 1$, and the $Sm$-DCSMs with $\chi^2 = 1$ and $n = 2, 3, 4, 6$ and 10. The insets shows a zoom of the behavior around the minima.

Figure 4. The magnetic moment components $M_x$ (top) and $M_y$ (bottom) per unit area as a function of the applied magnetic field component $h_y(a)$ in the diamagnetic (left) and paramagnetic (right) initial configurations defined by $h_x(a) = 1.1$ and $h_z = 1$. Results for several models are shown, e.g. the $T$-state model ($\chi^2 \to \infty$, $j_{c\parallel} \neq 0$), the conventional DCSM ($\chi^2 = 1$, $n \to \infty$), the $Sm$-DCSM with $\chi^2 = 1$ and $n = 1$, and the $Sm$-DCSMs with $\chi^2 = 1$ and $n = 2, 3, 4, 6$ and 10. The insets shows a zoom of the behavior around the minima.

We emphasize that whatever region is considered (excepting the limiting cases $\chi^2 = 1$, $n = 1$ (isotropic model) and $\chi^2 \to \infty$ (T or infinite bandwidth model)), the peak effect in the paramagnetic case is predicted for both components of the magnetization. In this sense, we argue that the peak effect cannot be interpreted as direct evidence of an elliptical material law. Instead of this, it is a universal signal of the anisotropy effects involved in a general description of the material law. The evolution of the peak effect as a function of $\chi^2$ has been shown in figures 17 and 18 of [4]. There, we note that an increase of the bandwidth $\chi^2$ produces a stretched magnetic peak. Consequently, paramagnetic effects are visible over a wider range as the cutting threshold value $j_{c\parallel}$ increases. We emphasize that the overall effect of increasing the value $\chi^2 = (j_{c\parallel}/j_{c\perp})^2$ is that the components of $M$ get closer to the master curves defined by $\chi \to \infty$. Several further distinctive signals for the different models are highlighted below.

On the one hand, for the isotropic model, the collapse of the magnetization is achieved while $j_{c\parallel}$ is monotonically reduced (upper rows in figures 2 and 3). When the material law is the infinite bandwidth model or $T$-state model ($\chi^2 = \infty$), the magnetization collapse does not take place and there is no restriction on the longitudinal component of the current that increases arbitrarily towards the center of the sample. This corresponds to the absence of flux cutting, i.e. $j_{c\parallel}$ does not saturate by reaching a threshold value $j_{c\parallel}$. For rectangular or smooth rectangular regions (intermediate rows in figures 2 and 3), together with the absence of collapse, one also observes that $j_{c\parallel}$ basically saturates to a value that depends on the smoothing parameter $n$ (exactly to $j_{c\parallel}$ for the strictly rectangular case $n \to \infty$).

Remarkably, when a rectangular section is assumed, the sample globally reaches the CT state (corner of the rectangle). As a consequence of the sharp limitation for $j_{c\parallel}$, a well-defined corner in the magnetic moment components $M_x$ and $M_y$ appears, both for the diamagnetic and paramagnetic cases (see figure 4). This clear trace of the DCSM establishes the departure from the master curves defined by the $T$ state, and has been assigned to the instant at which the sample reaches the CT state. We call the readers’ attention to the noticeable gap in figure 4, separating the isotropic model ($\chi^2 = 1$, $n = 1$) and the square model ($\chi^2 = 1$, $n \to \infty$).

Thus, the question arises about the possibility of inverting an experimental response within this interval so as to uniquely determine a given smooth region $\Delta_y$ for the components of $J$. As will be argued below, complimentary information about the limitations $J_{c\parallel}$ and $J_{c\perp}$ is due. Thus, if one compares figure 4, and figures 17 and 18 in [4] one can realize that smooth models...
for a given ratio $\chi \equiv J_{c\parallel}/J_{c\perp}$ will fill the gap between the master limiting curves defined by the rectangular ($\chi, n \to \infty$) and elliptic ($\chi, n = 1$) models, and their corresponding curves for different values of $\chi$ will intersect in a complicated fashion. In other words, the magnetization curves by themselves do not provide exhaustive information on the material law which defines the critical state dynamics in type-II superconductors. In fact, notice that in the regime of low fields $h_{\perp} \sim h_{\perp}(a)$ (or a weak oscillating magnetic field in the presence of a strong constant field [13, 16]) the material law is indistinguishable and the magnetic moment may be reproduced even by the isotropic model.

Nevertheless, a deeper insight in figures 2 and 3 reveals that the local behavior of the current density profiles, if available, should give clear indications. Thus, we notice that, although the dynamics of the profiles $h_{\perp}, j_{\perp}, j_{\parallel}$ is almost indistinguishable between the smooth and rectangular models (third and fourth rows of figures 2 and 3), a clear distinction arises by analyzing $J_{c\parallel}$. On the one hand, when the rectangular model is assumed $j_{\parallel}$ reaches the threshold value $j_{c\parallel}$ and the entire specimen verifies a CT state as the applied magnetic field increases. On the other hand, when the rectangular region is smoothed by the index $n$, the parallel component of the current density eventually decreases to a value that depends on the values of $n$ and $\chi$.

4. Conclusions

In type-II superconductors, an incomplete isotropy for the limitations of the current density relative to the orientation of the local magnetic field arises from the different physical conditions of current flow either along or across the Abrikosov vortices. One can thus talk about magnetically induced anisotropy. In this work, we have explored the application of the so-called smooth double critical state model to anisotropic material laws in such a case. This model relies on our general critical state theory [4] that allows us to incorporate the above-mentioned physical structure in the form of mathematical restrictions for the circulating current density. In this paper, taking advantage of such a potential, a systematic study has been performed and theoretical predictions have been made that allow us to establish a relation between a number of experimental observations and anisotropies of the material law. Two fundamental material-dependent quantities play key roles in this theory ($J_{c\parallel}, J_{c\perp}$) related to the flux cutting and pinning thresholds. We have applied the theory to the case of a type-II superconducting slab assuming translational symmetry along its surface and a 3D magnetic field.

Motivated by the possibility of modeling the influence and mutual interaction between the critical current thresholds, we have investigated situations with current density vectors belonging to some smooth region $\Delta_r$ in the $J_{c\parallel}-J_{c\perp}$ plane. This has been done by describing the boundary of $\Delta_r$ by means of a superelliptic relation $((J_{c\parallel}/J_{c\perp})^{2n} + (J_{c\perp}/J_{c\parallel})^{2n} = 1)$. Notoriously, the material law $\Delta_r$ is determined by the index $n$ and a proper bandwidth $\chi \equiv J_{c\parallel}/J_{c\perp}$. Thus, our predictions cover a wide range of laws: (i) the isotropic model ($\chi^2 = 1, n = 1 \Rightarrow \Delta_r$ is an ellipse), (ii) the elliptical model ($\chi^2 > 1, n = 1 \Rightarrow \Delta_r$ is a rectangle) and (iv) the infinite band model ($\chi^2 \to \infty$). After a detailed analysis that entails the local electrodynamics for material laws that cover a wide range of values for $n$ and $\chi$ we conclude that a considerable amount of experimental observations may be explained in this framework. Thus, we have shown that (i) the magnetic moment collapse by a perpendicular field is clearly assigned to the isotropic behavior of $\Delta_r$, (ii) paramagnetic magnetization induced by the application of a perpendicular field is always predicted if anisotropy in the region $\Delta_r$ is allowed, (iii) paramagnetic peak effects induced by a perpendicular magnetic field are expected in a wide range of conditions, (iv) differences between the several models studied are smeared out for low magnetic fields and (v) unless for the extreme cases (isotropic $\chi = 1, n = 1$ and T states $\chi \to \infty$) the inversion of magnetic data $(M_{c\parallel}, M_{c\perp})$ so as to elucidate the specific critical state region for a given sample is not straightforward. Complementary information about the maximal values of $J_{c\perp}$ and $J_{c\parallel}$ is required so as to extract the complete material law $J_{c\parallel}(J_{c\perp})$. Further research along this line is suggested, i.e. the design of some experimental routine that defines a well-posed inverse problem for the determination of $\Delta_r$.

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