This Special Issue presents a comprehensive picture of the Casimir effect as a multidisciplinary subject that plays an important role in diversified areas of physics ranging from quantum field theory, atomic physics and condensed matter physics to elementary particle physics, gravitation and cosmology.

Originally this effect was predicted in 1948 by the Dutch physicist H. B. G. Casimir [1] as an attractive force acting between two parallel uncharged ideal metal planes spaced in vacuum at some separation at zero temperature. The unique feature of the Casimir force is that it depends only on the fundamental constants $\hbar$ and $c$, and on the separation distance between the planes. In 1955, E. M. Lifshitz [2] developed a unified theory of the van der Waals and Casimir forces between two thick plates described by the frequency-dependent dielectric permittivities kept at any temperature in thermal equilibrium with the environment. The Lifshitz theory is applicable to any material plates—metallic, dielectric or semiconductor.

According to the Lifshitz theory, the force arising between two uncharged plates is an entirely quantum effect caused by the zero-point and thermal fluctuations of the electromagnetic field. At short separations, where the relativistic effects are negligible, this force depends only on $\hbar$ and coincides with the van der Waals force, which is well-known in condensed matter physics. At zero temperature and sufficiently large separations, where the relativistic retardation comes into play, it coincides with the force predicted by Casimir and depends on both $\hbar$ and $c$. At arbitrary separations and temperature, besides $\hbar$ and $c$, the Casimir force depends on the temperature and material properties of the plates. Later, it was understood that the Casimir effect arises for all quantum fields in restricted quantization volumes and in spaces with nontrivial topology. In the latter case, the identification conditions imposed on the wave functions due to a nontrivial topology of space play the same role as the boundary conditions imposed on material surfaces restricting the quantization volume. This gave rise to numerous applications of the Casimir effect in both fundamental and applied science.

The broad role of this effect results from the fact that the zero-point and thermal fluctuations of quantized fields are inherent to any physical system. Due to this, the Casimir force has attracted much experimental and theoretical attention in condensed matter physics, atomic physics, nanotechnology, and in elementary particle physics, gravitation and cosmology. It has become the subject of thousands of articles and reviews, provoked hot discussions at numerous scientific conferences, and was reflected in many publications intended for the general public. Several monographs have also been devoted to this important effect [3–10].

During the last twenty years, many precise experiments measuring the Casimir force were performed using the new possibilities provided by modern laboratory techniques. During the same period of time, the Lifshitz theory was generalized to the arbitrary geometries, various material properties of boundary bodies, and to out of thermal equilibrium situations. On the one hand, Casimir-operated micromechanical chips have been created...
and, on the other hand, the Casimir effect was invoked in investigations of cosmological scenarios, dark matter and dark energy. When investigating the thermal Casimir force acting between metallic surfaces, it was discovered that the Lifshitz theory comes into conflict with the experimental data on measuring the Casimir force and one of basic principles of thermodynamics, the Nernst heat theorem, when the Drude response function is used in computations, accounting for the relaxation properties of conduction electrons. Surprisingly, an agreement with both the measurement data and thermodynamics is restored if, in place of the Drude model, the dissipationless plasma model is employed, which should be inapplicable at low frequencies. A similar problem arises for dielectric test bodies if one includes in computations the dc conductivity of dielectric materials which is not equal to zero at any nonzero temperature. These problems, which have not been resolved to date, are often referred to as the Casimir puzzle and Casimir conundrum.

This Special Issue, which contains both research and review articles, reflects all the above aspects of the Casimir effect, as well as a discussion of the complicated unresolved problems and plans for the future. Below we present a brief summary of the contributions devoted to different areas of the Casimir research grouped into seven groups.

Several modern approaches to the theoretical description of the Casimir effect are presented in [11–14]. These include a unification of bulk and surface formulations of the theory [11], a review of the Green function scattering approach [12], and computations of the Casimir interaction between spheres immersed in electrolytes and between a sphere and a plane at intermediate separations [13,14].

New experiments regarding the Casimir force are considered in [15–17]. It should be especially emphasized that in the experiment by R. S. Decca [15] the Casimir force between a sphere and a plate was measured precisely up to 8 µm separation and compared with exact theory, without using the proximity force approximation. The proposed experiments [16,17] provide new opportunities for measuring the Casimir interaction at very short and very large separations, respectively.

Problems arising in the Casimir physics mentioned above, including the Casimir puzzle and conundrum, are discussed in the review [18] and articles [19,20]. All these contributions contain some new ideas on how these problems could be solved. Specifically, new spatially nonlocal response functions are considered, which include dissipation of conduction electrons and could bring theoretical predictions of the Lifshitz theory in agreement with the measurement data [18,19].

The Casimir effect in condensed matter systems is investigated, including differences in nonlinear actuation of Casimir oscillators when using either the topological insulators or metals [21], thermal behavior in the Casimir–Polder interaction of atoms with graphene [22], and characteristic features of the Casimir force from topological matter [23]. The obtained results are useful for future applications of the Casimir effect in nanotechnology.

The dynamical Casimir effect and radiation-matter interaction are considered in [24–28]. These cover the Green functions’ scattering approach to the dynamical Casimir force [24], motion-induced radiation due to an atom situated near a graphene sheet [25], particle production from vacuum by an ultrarelativistic moving mirror [26], interactions between moving particles and the surfaces of cylindrical geometry [27], and optical forces acting on an oscillating dipole near the phase-change material [28].

Several articles have been devoted to the Casimir effect in quantum field theory [29–34]. Here, the main attention is devoted to the axionic Casimir effect [29], problems of vacuum polarization in different configurations [30,31,33,34], and to the Casimir effect for fermion condensate in conical rings [32].

Much attention has been paid to the relationship between the Casimir effect and the cosmos. The zeta function, as a powerful regularization method for obtaining finite Casimir energy in gravity theories and cosmology, is considered in review [35]. Constraints on theoretical predictions beyond the Standard Model from the Casimir effect, including those on non-Newtonian gravity and axionlike particles as the constituents of dark matter, are summatised in review [36]. Review [37] presents results related to the thermal Casimir
effect in Einstein and closed Friedmann universes, including ones with a cosmic string. Different configurations with cosmic strings are also considered in [38,39].

All together, the research articles and reviews included in this Special Issue present a comprehensive picture of the rapidly developing multidisciplinary field of physics which contains still-unresolved fundamental problems of great importance and simultaneously inspire with expectations of promising technological applications in the very near future.

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