The Large $N$ Limit of the $(2, 0)$ Superconformal Field Theory

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Abstract
We discuss the large $N$ limit of the $(2, 0)$ field theory in six dimensions. We do this by assuming the validity of Maldacena’s conjecture of the correspondence between large $N$ gauge theories and supergravity backgrounds, here $AdS_7 \times S^4$. We review the spectrum of the supergravity theory and compute the spectrum of primary operators of the conformal algebra of arbitrary spin.

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1 Introduction

An interesting development stemming from investigations of string duality is the existence of higher dimensional interacting superconformal fixed points. The simplest such fixed point is the $(2,0)$ field theory in six dimensions. The theory can be discovered on the world-volume of the M-theory 5-branes \cite{3}, and in the compactification of type IIB string theory on a singular $K3$ \cite{2}. This theory, compactified on a 5-torus, is also relevant to Matrix theory \cite{4} compactifications on $T^4$ \cite{3, 11}.

The realization of the theory as embedded in M-theory leaves many questions open. The theory of a single fivebrane is a free field theory containing one free tensor multiplet. For $N$ fivebranes one expects to get an interacting field theory at a fixed point of the renormalization group \cite{11}. One would like therefore to compute dimensions of primary operators and their correlation functions in order to extract information about the theory.

Some progress has been made in Ref. \cite{1}, in which the DLCQ of the $(2,0)$ theory was discussed, via the Matrix description of M-theory. Several quantities were computed, for low values of $N$.

Recently, Maldacena \cite{7, 8} has made the suggestion that large $N$ gauge theories are related to supergravity on anti-deSitter ($AdS$) space. In the case of maximal supersymmetry, the full space is $AdS_n \times C$, where $C$ is a sphere of appropriate dimension. This conjecture has been significantly clarified by the recent work of \cite{10}. In the present paper, we study the case of supergravity on $AdS_7 \times S^4$, which is related to the theory of the M5-brane, and thus should give information on the $(2,0)$ theory in the large $N$ limit. Similar studies have been done \cite{13, 14} in the $AdS_5 \times S^5$ case, where the corresponding gauge theory is the relatively well-understood $N = 4, d = 4$ $SU(N)$ Yang-Mills theory. In this case, however, we expect to learn something new about the $(2,0)$ theory, at least in the infinite $N$ limit.

Assuming the validity of the conjecture, there is a correspondence between the spectrum of masses on the AdS space and the spectrum of dimensions of operators in the conformal field theory \cite{10}. In section 2 we extend the analysis of \cite{10} to non-scalar operators. In particular we find a correlation between the sign of the mass of a fermionic state on $AdS_7$ and the chirality of the operator it induces on the boundary. Using this analysis we construct in section 3 the spectrum of the conformal primary operators of arbitrary spin.

\footnote{For examples of related work, see \cite{9}.}
in the (2,0) theory. According to the conjecture in [7] this is a complete list of the primary operators whose dimension is finite in the infinite $N$ limit.

After this work was completed, we received two papers which discuss similar issues [19, 20]. Our results, where they overlap, are in agreement.

2 KK Modes of $d = 11$ Supergravity on $S^4$

In [10] an interpretation of Maldacena’s conjecture was suggested. The gauge theory is described by the supergravity background via a holographic image on the boundary of the $AdS$ space. There is a one-to-one mapping of bulk supergravity modes and conformal operators in the boundary theory. The Kaluza-Klein modes on the sphere play a special role, since they describe the complete set of operators whose dimensions are independent of $N$ in the large $N$ limit. It is known that all such Kaluza-Klein modes appear as short multiplets of the supersymmetry algebra in an $AdS$ background [18].

The scaling dimension of a given boundary operator is related to a mass parameter of the corresponding bulk solution. In this section, we review this relation and extend it to operators of arbitrary spin. The Kaluza-Klein modes of $d = 11$ supergravity on $S^4$ are known, having been worked out in Refs. [16, 15]. It should thus be a simple task to identify the primary operators of the $d = 6$ superconformal theory. The operators, being elements of short multiplets, should saturate certain unitarity bounds which have recently been worked out in Ref. [12].

We will refer to the bulk as $M$, and the 6-dimensional boundary as $B$. We denote a mode in the bulk generically as $j\{\alpha\}$, where $\alpha$ is a collection of quantum numbers labeling the representation of the superconformal algebra $Sp(6,2|4)$; it is sufficient to give quantum numbers of the maximal compact subgroup, here $Spin(5,1) \times Spin(2) \times Sp(2)_R$. The bulk mode couples to an operator on the boundary $\mathcal{O}\{\bar{\alpha}\}$, where $\bar{\alpha}$ is the conjugate representation. For example, if $j$ is a chiral spinor, the corresponding $\mathcal{O}$ is an anti-chiral spinor; similarly if $j$ is a self-dual tensor, then $\mathcal{O}$ is anti-self-dual. The coupling is simply of the form

$$\int_B j\{\alpha\} \cdot \mathcal{O}\{\bar{\alpha}\}$$

(1)
2.1 Dimension formulas

The component fields of the supergravity solutions on $AdS_7 \times S_4$ appear in Table 1 of Ref. [16]. All the rows in that table are determined from the first by supersymmetry and group theory; however, it is interesting to see how this works in detail from the point of view of supergravity solutions for the various component fields. In what follows we analyze the behaviour of the 7-dimensional solutions at the boundary. Throughout, we work in the metric

$$ds^2 = dy^2 + \sinh^2 y \eta_{ij} dx^i dx^j$$ (2)

The boundary is at $y \to \infty$.

**Scalars:** As given in [10], scalar solutions behave as $e^{\lambda y}$ at infinity, and the Klein-Gordon equation for the scalar in this background implies $\lambda(\lambda + d) = m^2$. The dominant root is the larger of the two, and is generically positive.\(^3\) The corresponding operator in the SCFT then has dimension $\Delta = \lambda + d$. There are three series:

\begin{align*}
  m^2 &= 4k(k - 3) : \quad \Delta = 2k, \quad (k = 1, 2, \ldots) \quad (3) \\
  m^2 &= 4(k^2 + 7k + 10) : \quad \Delta = 2k + 10, \quad (k = 0, 1, \ldots) \quad (4) \\
  m^2 &= 4(k^2 + 9k + 18) : \quad \Delta = 2k + 12, \quad (k = 0, 1, \ldots) \quad (5)
\end{align*}

As we will see below, in a generic massive multiplet, there are scalar components which fit into each of these series. The first few multiplets have scalars only from the first series.

**Spinors:** Here, we should study solutions of the Dirac equation in the AdS background at infinity to find the dimensions of the corresponding operators. After a brief calculation, we find that there is an important contribution from the spin connection in this background:

$$\left(\partial_7 + m\right)\Psi = \left[\gamma^7 \left(\partial_y + d/2 \coth y\right) + m + \frac{1}{\sinh y} \partial_6\right] \Psi$$ (6)

and so at infinity

$$\Psi \sim e^{\lambda y} u_\infty$$ (7)

\(^2\)For the rest of the Lorentz representations, we also take the corresponding $\lambda$ to be positive.

\(^3\)In Table 1 of Ref. [16], masses should be scaled by a factor of $e = 2$. 

3
\[
(\lambda + d/2)\gamma^7 + m \mid u_\infty = 0
\]  
(8)

In the 5 + 1-dimensional sense, \(\gamma^7\) gives the chirality. Since \(\lambda\) is taken to be positive, this correlates the sign of the mass with chirality. Hence:

\[
m > 0 : \quad \Delta_- = +m + d/2
\]

\[
m < 0 : \quad \Delta_+ = -m + d/2
\]

This is an important result, and as we shall see in the next section, precisely what is needed for the interpretation in terms of operators in the (2, 0) theory.

From table 1 of [16] (shifting range of \(k\) appropriately in some cases), there are four series of spinors:

\[
m_1 = e(k - 1/4) \quad \Delta_- = 2k + 5/2 \quad (k = 0, 1, \ldots)
\]

\[
m_2 = -e(k + 1/4) \quad \Delta_+ = 2k + 15/2 \quad (k = 0, 1, \ldots)
\]

\[
m_3 = e(k + 3/4) \quad \Delta_- = 2k + 21/2 \quad (k = 0, 1, \ldots)
\]

\[
m_4 = -e(k + 5/4) \quad \Delta_+ = 2k + 23/2 \quad (k = 0, 1, \ldots)
\]

with 5 + 1-dimensional chirality indicated as before.

One should note that in the corresponding 4-dimensional \(N = 4\) case, obtained from Type IIB on \(AdS_5 \times S_5\), we should interpret the spinors with definite scaling as \(d = 5\) symplectic-Majorana spinors and the mass as a Majorana mass. In this case, for each mode in the bulk, we find an equation like \(\gamma^7(\lambda_i + d/2)u_{i,\infty} = me_{ij}u_{j,\infty}\). Choosing \(\lambda_i\) positive, we get two Weyl spinors of opposite chirality, with the same dimension. The spectrum is then non-chiral, as it must be to have \(N = 4\) supersymmetry on the boundary.

Vectors and \(p\)-forms: Here we are solving an equation of the form

\[
\partial_\mu \left( \sqrt{-g} g^{\mu\nu} \varrho^{\sigma} F_{\nu\sigma} \right) = m^2 \sqrt{-g} g^{\mu\nu} A_{\mu}
\]

(15)

To see in detail the behaviour at infinity, we choose a convenient gauge \( \nabla_{(6)} \cdot A = 0, A_7 = 0 \). The remaining components of eq. (15) are now

\[
\partial_\ell \left( \sqrt{-g} g^{\ell j} g^{ki} F_{ji} \right) + \partial_y \left( \sqrt{-g} g^{77} g^{ki} F_{7i} \right) = m^2 \sqrt{-g} g^{ki} A_i
\]

(16)

At infinity, if \(A_i \sim e^{\lambda y} f_i(x)\), the first factor in (16) is subleading, and we find

\[
\lambda(\lambda + d - 2) = m^2
\]

(17)
It is important to realize that there is a factor of $g^{77}$ in the second factor of (16) in doing the power counting.

For $p$-form gauge fields satisfying a similar Maxwell-like equation, the result generalizes to

$$\lambda(\lambda + d - 2p) = m^2$$

(18)

The dimension of the corresponding operator has

$$\Delta = -p + \lambda + d,$$

and so we get

$$(\Delta + p - d)(\Delta - p) = m^2$$

(19)

As an example, we note that for the first vector series, we have $m^2 = 4(k^2 - 1)$ and so $(\Delta - 5)(\Delta - 1) = 4k^2 - 4$, which gives $\Delta_{v1} = 3 + 2k, k = 1, 2, 3, \ldots$.

For the first 3-form series, we have $m^2 = 4k^2$, and so $(\Delta - 3)^2 = 4k^2$, i.e. $\Delta_{T1} = 3 + 2k, k = 0, 1, 2, \ldots$. All other $p$-form series may be found in the Table at the end of this paper.

**Graviton**: For spin 2 fields, the equations of motion can be shown to reduce to the scalar case, and thus $\Delta(\Delta - d) = m^2$. The correct value of $m^2$ is $4(k^2 + 3k)$ (a shift of $-1/2$ appeared incorrectly (for the notation used here) in Table 1 of eq. [16]). Thus $\Delta = 6 + 2k, k = 0, 1, 2, \ldots$.

**Gravitini**: Here, we repeat a similar analysis as for the spinors. The result is the same and again the results are chiral.

## 3 The Spectrum of Operators

An operator in the $(2,0)$ theory is characterized by Lorentz quantum numbers and by the $Sp(2)_R$ R-symmetry representation. Let the highest weights of the operators under $Spin(5,1)$ be $(h_1, h_2, h_3)$ with index $T(h)$, and the highest weights under $Sp(2)_R$ be $(\ell_1, \ell_2)$.

A representation of the superconformal algebra may be constructed by starting with a superconformal primary operator, which by definition is annihilated by the special conformal generators $K_i$ and by the special SUSY generators $S_{\alpha}$. Any superconformal descendant is constructed by acting with “raising” operators which are the SUSY generators $Q_{\alpha}$ and the momentum operators $P_i$.

A superconformal primary operator is called of level $r$ if acting with the raising operators $r$ times we encounter for the first time a null state. Such
primary operators generate short representations of the superconformal algebra, and saturate unitarity bounds derived in \[12\]. The dimension, $\Delta$, of such an operator is specified by its quantum numbers as follows:

For level 1 operators one has:

$$\Delta = T(h) + 2(\ell_1 + \ell_2)$$  \hspace{1cm} (20)

For level 2 operators there are 3 possibilities:

$$\Delta = T(h) + 2(\ell_1 + \ell_2) + a$$  \hspace{1cm} (21)

where $a$ can be 2, 4 or 6.

Given that the Kaluza Klein modes are in short multiplets, we expect them to saturate one of the bounds above. Indeed all operators are in fact level one.

It turns out that we can classify, at least in terms of $Spin(6) \times Sp(2)_R$ quantum numbers, the gauge invariant primary operators that appear in the superconformal theory in terms of place-holder fields

$$\phi, \psi, H$$  \hspace{1cm} (22)

all of which are taken to be in the adjoint representation of $U(N)$ and transform as a tensor multiplet of the $(2,0)$ supersymmetry. The $Spin(6)$ weights are respectively

$$(0,0,0), \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right), (1,1,1).$$  \hspace{1cm} (23)

Similarly, the $Sp(2)_R$ weights are

$$(1,0), \left(\frac{1}{2}, \frac{1}{2}\right), (0,0).$$  \hspace{1cm} (24)

The primary operators may then simply be identified with

$$O_{m,n,k} = \text{tr} \ H^m \psi^n \phi^k$$  \hspace{1cm} (25)

A hierarchy is built up based on the integer $p = m + n + k = 0, 1, 2, \ldots$. Clearly the $p = 0$ operator is just the identity operator, with dimension zero. It is a singlet of the supersymmetry algebra. The $p = 1$ operators $\text{tr} \ H$, $\text{tr} \ \psi$, $\text{tr} \ \phi$ are in $Sp(2)_R$ representations $1, 4, 5$ respectively. Clearly, they touch only the Abelian part of $U(N)$; they correspond to the doubleton and have free dimensions $3, 5/2, 2$ respectively.
In the bulk supergravity theory, the doubleton representation corresponds to decoupled, pure gauge modes. In the boundary theory, they represent a free Abelian tensor multiplet which decouples from the interacting fixed point theory. Note that we build all representations from the place-holder fields (25). These should not be confused with the doubleton representation.

We collect the results of Section 2 in a Table below. All of the Kaluza-Klein modes give rise to operators (25) whose dimensions can be read off by simply adding the dimensions of the place-holder fields. All the superconformal primary fields saturate the level 1 unitarity bounds. We note also for example that the additional series of scalars (1) and (5) appear as scalar ”composites” of the place-holder fields. Furthermore, the chirality of the various spinor series is determined by the $Spin(6)$ group theory, and agrees with the supergravity analysis of the last section.

The structure of the table, and in particular the ability to represent all operators as simple composites of a set of place-holder fields, is similar to the structure of a free field theory. Indeed, far along the flat directions of the (2,0) field theory, the infra-red physics is free. The eigenvalues of the place-holder fields are then interpreted as the $N$ free tensor multiplets, whose scalars parametrize the moduli space $\mathbb{R}^N / S_N$. As we move towards the singularities of the moduli space, we expect to reach an interacting (2,0) superconformal fixed point. A general operator will undergo significant renormalization, unless it is protected by SUSY non-renormalization theorems.

For this reason, not all possible combinations of the fields $\phi, \psi, H$ appear in the table. The only superconformal primary fields appearing in the table correspond to the completely symmetric combination of the $\phi$ fields. For the completely symmetric combination, the R-symmetry weights $\ell_1, \ell_2$ simply add up. Therefore the unitarity bounds given above completely determine the dimension of these operators (and therefore also their descendants). In other words the operators found in the table are precisely those which are protected from renormalization.

Other combinations of $\phi$’s, involving their commutators, are not constrained by this argument to saturate the unitarity bound. Such operators, since not protected by non-renormalization theorems, are expected to have dimensions which diverge with $N$, and indeed we find that they do. In the context of Maldacena’s conjecture, they correspond to M-theory modes which are not seen in the supergravity approximation.

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| Spin(6) | Sp(2)$_R$ | dimension | Operator |
|---------|-----------|-----------|----------|
| 1       | 5         | 0 + 2(1)  | tr $\phi$ |
| 4       | 4         | 5/2 + 2(0) | tr $\psi$ |
| 10      | 1         | 3 + 2(0)  | tr $H$   |
| 1       | 14        | 0 + 2(2)  | tr $\phi\phi$ |
| 4       | 16        | 5/2 + 2(1)| tr $\psi\phi$ |
| 10      | 5         | 3 + 2(1)  | tr $H\phi$ |
| 6       | 10        | 5 + 2(0)  | tr $\psi\psi$ |
| 20      | 4         | 11/2 + 2(0)| tr $\psi H$ |
| 20'     | 1         | 6 + 2(0)  | tr $HH$  |

| 1       | 30        | 0 + 2(3)  | tr $\phi\phi\phi$ |
| 4       | 40        | 5/2 + 2(2)| tr $\psi\phi\phi$ |
| 10      | 14        | 3 + 2(2)  | tr $H\phi\phi$ |
| 6       | 35        | 6 + 2(1)  | tr $\psi\psi\phi$ |
| 20      | 16        | 11/2 + 2(1)| tr $\psi H\phi$ |
| 20'     | 5         | 6 + 2(1)  | tr $HH\phi$ |
| 4       | 20        | 15/2 + 2(0)| tr $\psi\psi\psi$ |
| 15      | 10        | 8 + 2(0)  | tr $\psi\psi H$ |
| 20      | 4         | 17/2 + 2(0)| tr $\psi HH$ |
| 10      | 1         | 9 + 2(0)  | tr $HHH$  |

| 1       | 55        | 0 + 2(4)  | tr $\phi\phi\phi\phi$ |
| 4       | 80        | 5/2 + 2(3)| tr $\psi\phi\phi\phi$ |
| 10      | 30        | 3 + 2(3)  | tr $H\phi\phi\phi$ |
| 6       | 81        | 6 + 2(2)  | tr $\psi\psi\phi\phi$ |
| 20      | 40        | 11/2 + 2(2)| tr $\psi H\phi\phi$ |
| 20'     | 14        | 6 + 2(2)  | tr $HH\phi\phi$ |
| 4       | 64        | 15/2 + 2(1)| tr $\psi\psi\psi\phi$ |
| 15      | 35        | 8 + 2(1)  | tr $\psi\psi H\phi$ |
| 20      | 16        | 17/2 + 2(1)| tr $\psi HH\phi$ |
| 10      | 5         | 9 + 2(1)  | tr $HHH\phi$ |
| 1       | 35        | 10 + 2(0) | tr $\psi\psi\psi\psi$ |
| 4       | 20        | 21/2 + 2(0)| tr $\psi\psi\psi H$ |
| 6       | 10        | 11 + 2(0) | tr $\psi\psi HH$ |
| 4       | 4         | 23/2 + 2(0)| tr $\psi HHH$ |
| 1       | 1         | 12 + 2(0) | tr $HHHH$ |

**Table:** The first four superconformal multiplets (multiplets are separated by **
a double line). All other multiplets are obtained by appending extra φ's in the traces of the last multiplet. Throughout the table, dimensions are given in the form \( n + 2(k) \), where \( k \) counts the number of φ's in the given operator. The R-symmetry index structure of the operators is suppressed and can be recovered from the corresponding representation.

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