Analysis of Aggregate Production Planning Problem with Goal Programming Model

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Abstract. Aggregate production planning is medium range planning (3-18 months) that determines production quantities, production time management, and resources management. In the real world, companies that faced this problem usually want to optimize some objective functions that conflict each other. Therefore, the purposes of this study are to reconstruct goal programming model and to solve multiobjective aggregate production planning problem. Also, to obtain numerical solutions of production planning scenarios and weights of goal programming objective function. The model is followed by the constraint of raw material capacity, workforce level, storage space, and production capacity. The proposed planning period is six months with four objective functions to maximize profit, minimize inventory cost, minimize subcontract cost, and minimize workforce changes cost. A set of hypothetical data is used to test the validity of the proposed model with several different scenarios. The result for each scenario has been fulfilled the company’s requirement and objectives. It described through monthly quantity and mechanism of the production planning. For in-depth analysis, the effect of different weighting on target’s achievement is evaluated with 24 combinations of case.

1. Introduction

Production planning is a basic need in manufacturing process that is arranged synergically with business and marketing planning. The purpose of production planning is to minimize resources cost to meet customer demands. Aggregate production planning (APP) is medium-range planning (3-18 months) that determines the optimum production volume, managing production time, and resource management. Theoretically, APP falls between long-term strategic planning decisions such as product development and short-term planning that concerns about product scheduling [3].

Takey and Mesquita [11] utilized linear programming to solve APP with high-seasonal demand. They also mentioned that Mixed Integer Linear Programming (MILP) and heuristic methods can be used to solve APP. Moreover, there are three basic production strategies can be considered in solving APP, namely chase strategy, level strategy, and mixed strategy [3]. Further research on APP by MILP was conducted by Yazici et al. [14] and was applied to jewelry companies. Due to the production material flows that difficult to be defined precisely with mathematical models, the proposed models was then transformed into fuzzy models. Mifdal et al. [7] studied heuristic methods for production planning and used an integrated optimization approach to overcome machine maintenance and production planning problems by using one machine to produce several products. Therefore an appropriate machine maintenance strategy was needed to minimize resource costs.
In aggregate production planning, the company usually wants to optimize more than one objective function. Therefore to deal with this problem, Masud and Hwang [6] presented three multiple objective decision making (three-MODM). This method consists of goal programming, step method (STEM), and Sequential Multiple Objective Problem Solving (SEMOPS). Another approach that can be used to solve APP is Chance-Constrained Goal Programming (CCGP) [9]. This method works based on sequential solution in linear programming and suitable for preemptive goal programming and nonpreemptive goal programming methods. Then Deckro and Hebert [1] reviewed goal programming in completing Linear Decision Rule based on aggregate production planning problems. The study of Leung and Chan [4] used preemptive goal programming with three objective functions to solve aggregate production planning problem. The flexibility and robustness of the proposed model illustrated by changing the goal priorities of the objective function.

In this research, a nonpreemptive goal programming model is formulated to solve the aggregate production planning problem. The product used in this problem is a family product, i.e., goods produced by the same company under the same brand but have slightly different needs and tastes. The objective functions are (1) maximizing profit, (2) minimizing inventory costs, (3) minimizing subcontract costs, and (4) minimizing workforce changes costs. The model is tested in four planning scenarios, with the same weights of deviation variables. The effect of weights changes to the objective function’s achievement is evaluated.

2. Goal programming model
The formulation of goal programming is different from that of linear programming. One of the differences is the constraint function. In goal programming, the goal constraints are used to represent each of the goals. This means multiple goals can be handled by one model. It makes the goal constraints can be violated with certain penalties set by modeler. This doesn't apply in linear programming. Therefore, the constraints on goal programming are also known as soft constraints, while constraints on linear programming known as hard constraints [10].

Sarker and Newton [10] also mention that other difference between linear programming and goal programming lies in its components. The following are components of goal programming:
1. Decision variables, same as linear programming.
2. Deviation variables, consist of over-achievement deviation variables ($d_i^+$) and under-achievement deviation variables ($d_i^-$).
3. System constraints, identical to linear programming constraints with no deviations allowed (known as hard constraints).
4. Goal constraints, the target value to be achieved with deviations are allowed (known as soft constraints).
5. Objective function, minimizing the total weight of the undesirable deviations.

Hillier and Lieberman [2] explained that based on the importance of the objective function there are two types of goal programming:
1. Preemptive goal programming
   In this method, there is a hierarchical priority to describe level of importance of the objective function. The basic principle is that the goals of primary importance receive first-priority attention, those of secondary importance receive second-priority attention, and so forth. Then, there are two methods based on linear programming for solving preemptive goal programming problems, that is sequential and streamlined method.

2. Nonpreemptive goal programming
   In this method, the objective function is given a weight to describe the level of importance. The completion of each goal was done simultaneously.

A generalized model for goal programming is as follows:
\[
\begin{align*}
\min (Z) &= \sum_{i=1}^{k} w_i p_i (d_i^- + d_i^+) \\
\text{subject to:} & \quad \sum_{j=1}^{n} a_{ij} X_j - d_i^- + d_i^+ = b_i \quad (\forall \ i = 1, 2, \ldots, k)
\end{align*}
\]
\[ X_j, d^-_i, d^+_i \geq 0 \]
\[ w_i > 0 \]

with \( d^-_i \) is \( i^{th} \) under-achievement deviation variable and \( d^+_i \) is \( i^{th} \) over-achievement deviation variable. \[ \sum_{j=1}^{n} a_{ij}X_j - d^+_i + d^-_i = b_i \] is \( i^{th} \) goal constraint. This constraint is a reconstruction of objective function \( i \) that added by the deviation variable \((d^+_i, d^-_i)\) on the left-hand sides and the target value on the right-hand side [8].

Although, the objective function in goal programming is to minimize the sum of undesirable deviation but different types of objective function will give different types of deviation variables to be minimized. According to Orumie and Ebong [8], this term can be explained through the formulation model in Table 1:

### Table 1. Model formulation of goal programming

| Types of objective function | Deviation variables to be minimized |
|-----------------------------|-------------------------------------|
| \( \sum_{j=1}^{n} a_{ij}X_j \geq b_i \) | \( d^-_i \) |
| \( \sum_{j=1}^{n} a_{ij}X_j \leq b_i \) | \( d^+_i \) |
| \( \sum_{j=1}^{n} a_{ij}X_j = b_i \) | \( d^-_i + d^+_i \) |

3. Mathematical model

A company manufactures \( n \) products with two production times, regular and overtime. This company may assign subcontract and backorder when the demand exceeds the company’s capacity. The company can also decide to hire new workers to replace the workers with low performance. But, the new workers need training first. The planning period proposed by the company is six months with four production planning scenarios.

In the following, the parameters and the variables for the model are defined. Mathematical formulation of the proposed model, including goal constraints, system constraints, and objective function are also described.

3.1. Notations

Sets
- \( \mathbb{N} \): set of the products.
- \( \mathbb{T} \): set of the planning periods.
- \( \mathbb{K} \): set of the market places.

Indices
- \( n \in \mathbb{N} \): index of the product, \( n \in N = \{1, 2, \ldots, N\} \).
- \( t \in \mathbb{T} \): index of the planning period, \( t \in T = \{1, 2, \ldots, T\} \).
- \( k \in \mathbb{K} \): index of the market, \( k \in K = \{1, 2, \ldots, K\} \).

Decision variables
- \( QR_{nt} \): quantity of product \( n \) in period \( t \) manufactured at regular-time (unit).
- \( QO_{nt} \): quantity of product \( n \) in period \( t \) manufactured at overtime (unit).
- \( QS_{nt} \): subcontract quantity of product \( n \) in period \( t \) (unit).
- \( QB_{nt} \): backorder quantity of product \( n \) in period \( t \) (unit).
- \( QI_{nt} \): inventory quantity of product \( n \) in period \( t \) (unit).
- \( W_t \): number of workers in period \( t \) (men).
- \( WH_t \): number of workers hired in period \( t \) (men).
- \( WL_t \): number of workers laid-off in period \( t \) (men).
- \( OT_t \): overtime hours in period \( t \) (hours).
Deviation variables

- \( dp^+ \): over-achievement deviation variable of the profit goal ($).
- \( dp^- \): under-achievement deviation variable of the profit goal ($).
- \( dt^+ \): over-achievement deviation variable of the inventory cost goal ($).
- \( dt^- \): under-achievement deviation variable of the inventory cost goal ($).
- \( ds^+ \): over-achievement deviation variable of the subcontract cost goal ($).
- \( ds^- \): under-achievement deviation variable of the subcontract cost goal ($).
- \( dw^+ \): over-achievement deviation variable of the workforce changes cost goal ($).
- \( dw^- \): under-achievement deviation variable of the workforce changes cost goal ($).

Parameters

- \( P_{ntk} \): revenue of product \( n \) to supply to market \( k \) in period \( t \) ($/unit).
- \( CPR_{nt} \): regular-time production cost of product \( n \) in period \( t \) ($/unit).
- \( COP_{nt} \): overtime production cost of product \( n \) in period \( t \) ($/unit).
- \( CS_{nt} \): subcontract cost of product \( n \) in period \( t \) ($/unit).
- \( CB_{nt} \): backorder cost of product \( n \) in period \( t \) ($/unit).
- \( CI_{nt} \): inventory cost of product \( n \) in period \( t \) ($/unit).
- \( CWR_t \): regular-time workforce cost in period \( t \) ($/period).
- \( CWO_t \): overtime workforce cost in period \( t \) ($/hours).
- \( CH_t \): hiring cost in period \( t \) ($/man).
- \( CL_t \): lay-off cost in period \( t \) ($/man).
- \( D_{ntk} \): demand of product \( n \) at market \( k \) in period \( t \) (unit).
- \( MCR_{nt} \): maximum capacity at regular-time of product \( n \) in period \( t \) (unit).
- \( MCO_{nt} \): maximum capacity at overtime of product \( n \) in period \( t \) (unit).
- \( MCB_{nt} \): maximum backorder capacity of product \( n \) in period \( t \) (unit).
- \( S_{maxnt} \): maximum subcontract of product \( n \) in period \( t \) (unit).
- \( W_max \): maximum workforce level available by the company (men).
- \( Q_{maxnt} \): maximum product inventory in period \( t \) (unit).
- \( \alpha_n \): workforce time for manufacturing product \( n \) (hours/unit).
- \( \gamma_t \): working hours for workers in period \( t \) (hours/period).
- \( \delta \): percentage of workforce available for overtime (%).
- \( MU_n \): machine utilization time for manufacturing product \( n \) (minutes/unit).
- \( M_{maxnt} \): machine production capacity of product \( n \) in period \( t \) for regular-time (unit).
- \( M_{max\_OT\_nt} \): machine production capacity of product \( n \) in period \( t \) for overtime (unit).
- \( OT_{max\_t} \): maximum overtime hours in period \( t \) (hours).
- \( M \): binary dummy variables related to \( OT_{nt} \) and \( OT_{max\_nt} \).
- \( P \): binary dummy variables related to \( QS_{nt} \) and \( S_{maxnt} \).
- \( w_1 \): under-achievement deviation weights for the profit goal (unit).
- \( w_2 \): over-achievement deviation weights for the profit goal (unit).
- \( w_3 \): over-achievement deviation weights for the inventory cost goal (unit).
- \( w_4 \): over-achievement deviation weights for the workforce changes cost goal (unit).
- \( GP \): the desired profit by the company ($).
- \( GI \): the acceptable level for inventory cost ($).
- \( GS \): the acceptable level for subcontract cost ($).
- \( GW \): the acceptable level for workforce changes cost ($).
3.2. Goal Constraint

3.2.1. Goal 1: The profit goal
The first component of this goal is total revenue, which is obtained by multiplying product price with product demand. The second component is production cost, which is obtained by multiplying production cost with production volume at both regular and overtime. The next three components of this goal are inventory cost, backorder cost, and subcontract cost. The last component is workforce cost at both regular and overtime. It is assumed that the profit target set by company is \( GP \). Therefore, the formulation for Goal 1 is given as follows:

\[
\sum_{n=1}^{N} \sum_{t=1}^{T} \sum_{k=1}^{K} P_{nkt} D_{nkt} - \sum_{n=1}^{N} \sum_{t=1}^{T} C_{PR_{nt}} Q_{R_{nt}} - \sum_{n=1}^{N} \sum_{t=1}^{T} C_{PO_{nt}} Q_{O_{nt}} - \sum_{n=1}^{N} \sum_{t=1}^{T} C_{B_{nt}} Q_{B_{nt}} - \sum_{n=1}^{N} \sum_{t=1}^{T} C_{S_{nt}} Q_{S_{nt}} - \sum_{t=1}^{T} C_{WR_{t}} W_{t} - \sum_{t=1}^{T} C_{WO_{t}} O_{t} - \sum_{t=1}^{T} c_{dp} d_{p} - \sum_{t=1}^{T} c_{dp}^{-} d_{p}^{-} = GP.
\]  

3.2.2. Goal 2: The inventory cost goal
Inventory management is an important thing that need to be managed by the company. A stored product can be used later, but this condition involved additional costs. Moreover, a product that is stored for a long time will become outdated. Hence, it is necessary to have an optimal value to minimize inventory costs. In this goal, the acceptable inventory cost is \( GI \). Therefore, the formulation for Goal 2 is provided as follows:

\[
\sum_{n=1}^{N} \sum_{t=1}^{T} C_{I_{nt}} Q_{I_{nt}} - \sum_{n=1}^{N} \sum_{t=1}^{T} d_{i}^{+} d_{i}^{-} = GI.
\]  

3.2.3. Goal 3: The subcontract cost goal
Subcontract cost also needs to be minimized because the costs required for the subcontract are quite large. Moreover, when a company has a high volume of subcontracts, it will give advantages to the competitors. The acceptable subcontract cost for this goal is \( GS \). Therefore, the formulation for Goal 3 is obtained as follows:

\[
\sum_{n=1}^{N} \sum_{t=1}^{T} C_{S_{nt}} Q_{S_{nt}} - \sum_{n=1}^{N} \sum_{t=1}^{T} d_{s}^{+} d_{s}^{-} = GS.
\]  

3.2.4. Goal 4: The workforce changes cost goal
As mentioned before, there will be a change of workers when the worker has unsatisfying performance. However, this mechanism will give an additional cost to provide workers training or workers severance cost. Hence, the optimal value is needed to find the right combination between hired/laid-off workers. The company also determines that the acceptable workforce changes cost for this goal is \( GW \). Therefore, the formulation for Goal 4 is:

\[
\sum_{t=1}^{T} \left( C_{WH_{t}} W_{t} + C_{L_{t}} L_{t} \right) - \sum_{n=1}^{N} \sum_{t=1}^{T} c_{dw} d_{w}^{+} d_{w}^{-} = GW.
\]  

3.3. System Constraint
Based on the above problem, the system constraint consist of capacity constraints, labor constraints, and scenario fulfillment constraints. The formulations are proposed as follows:

\[
\begin{align*}
QR_{nt} &\leq MCR_{nt} \quad \forall n, t \\
QO_{nt} &\leq MCO_{nt} \quad \forall n, t \\
QB_{nt} &\leq MCB_{nt} \quad \forall n, t \\
\sum_{k=1}^{K} D_{nkt} &= Q_{I_{nt-1}} - Q_{B_{nt-1}} + Q_{R_{nt}} + Q_{O_{nt}} + Q_{S_{nt}} - Q_{I_{nt}} + Q_{B_{nt}} \quad \forall n, t \\
QB_{nt} &= Q_{I_{nt}} = 0 \quad \forall t \\
\sum_{n=1}^{N} \alpha_{n} (QR_{nt} + QB_{nt}) &\leq \gamma_{t} W_{t} \quad \forall n, t
\end{align*}
\]
\[ \sum_{n=1}^{N} \alpha_n Q_{nt} \leq OT_t \delta W_t \quad \forall t \] (15)

\[ W_t \leq W_{\text{max}} \quad \forall t \] (16)

\[ W_t = W_{t-1} + WH_t - WL_t \quad \forall t \] (17)

\[ WH_t, WL_t = 0 \quad \forall t \] (18)

\[ \sum_{n=1}^{N} Q_{nt} \leq Q_{\text{max}}_t \quad \forall t \] (19)

\[ MU_n (QR_{nt} + QB_{nt}) \leq MC_{\text{max}}_{nt} \quad \forall n, \forall t \] (20)

\[ MU_n QO_{nt} \leq MC_{\text{max}} \cdot OT_{nt} \quad \forall n, \forall t \] (21)

\[ \sum_{k=1}^{K} D_{ntk} \leq MCR_{nt} \rightarrow M = 1 \quad \forall n, \forall t \] (22)

\[ \sum_{k=1}^{K} D_{ntk} > MCR_{nt} \rightarrow M = 0 \quad \forall n, \forall t \]

\[ OT_{nt} \leq OT_{\text{max}} (1 - M) \quad \forall n, \forall t \]

Constraints (9), (10), and (11) restrict the production volume for each production time and the backorder volume. Constraint (12) explains that the demand in all markets for each product and each period equal to the inventory (or backorder) from the previous period plus regular and overtime production quantity in the current period as well as subcontract volume in the current period then minus the inventory (or backorder) in the current period. Constraint (13) states that inventory and backorder can not be at one period because backorder occurs when the company does not have inventory. Constraints (14) and (15) ensure that workforce times for manufacturing the products during regular and overtime should be limited to the workforce availability. The limitation of workforce availability for each period given by constraint (16). Constraint (17) stipulates that the workforce availability in a period equals the workforce in the previous period plus the change of workforce in the current period. Constraint (18) indicates the company can not hired and fired the workers in the same period and constraint (19) prevents the inventory from exceeding the maximum storage capacity. Machine production times during regular and overtime should not exceed the machine capacity, which is guaranteed by constraints (20) and (21). The above constraints have not distinguish the production planning when demand exceeds capacity. Therefore, constraints (22) and (23) are defined to satisfy these conditions.

3.4. Objective Function

Generally the objective function in goal programming model is to minimize the over-achievement and under-achievement deviation variables from each goal. If the goal is maximizing profit then the under-achievement deviation variables need to be minimized \((dp^-)\) and if the goal is minimizing the inventory cost, subcontract costs, and workforce level costs then the over-achievement deviation variables need to be minimized \((di^+, ds^+, dw^+)\). Moreover, in this study the deviation weights used are the standardized weights \(\left(\frac{w_1}{GP}, \frac{w_2}{GI}, \frac{w_3}{GS}, \frac{w_4}{GW}\right)\). This resulted a comparison between deviation variables can be done, so the proposed objective function is:

\[ \min Z := \frac{w_1}{GP} dp^- + \frac{w_2}{GI} di^+ + \frac{w_3}{GS} ds^+ + \frac{w_4}{GW} dw^+. \] (24)
4. Computational results

The proposed model is implemented in a company that manufactures three types of products within six months planning period and five markets to supply. Four production planning scenarios used to test the model are defined as follows:

(1) Scenario 1: Regular-time production
Demand in this scenario is fulfilled in regular-time basis because the demand itself is less than the raw material capacity for regular-time.

(2) Scenario 2: Regular and overtime production
Demand in this scenario is met by regular and overtime because the demand is between raw material capacity for regular-time production and raw material capacity for overtime production.

(3) Scenario 3: Regular-time production, overtime production, and backorder
Demand in this scenario is satisfied by regular-time production, overtime production, and backorder. It is because the demand exceeds the company raw material capacity. In other words, the company experiences out of stock.

(4) Scenario 4: Regular-time production, overtime production, backorder, and subcontract
Demand in this scenario is fulfilled by regular-time production, overtime production, backorder, and subcontract. It is because the demand exceeds the production capacity. So to fulfill the demand, company need to work with other company to produced certain product.

So based on constraints (22) and (23), Scenario 1 fulfilled when \( M = 1 \) and \( P = 1 \), Scenario 2 and Scenario 3 accomplished when \( M = 0 \) and \( P = 1 \), and Scenario 4 achieved when \( M = 0 \) and \( P = 0 \).

The weight of deviation variables and target values for each objective function used in Scenario 1-4 are given as follows:

\[
\begin{align*}
w_1 & : 1 \quad GP & : 3,500,000 \\
w_2 & : 1 \quad GI & : 7,000 \\
w_3 & : 1 \quad GS & : 1,000,000 \\
w_4 & : 1 \quad GW & : 10,000 \\
\end{align*}
\]

In this study, we use formula (25) to calculate the machine production capacity for regular and overtime production. \( M_{nt} \) is the number of machine used each product in period \( t \), \( EF_n \) is the percentage machine efficiency for product \( n \), and \( TP \) is the working hours per day which is assumed 7 hours/day for regular-time and 5 hours/day for overtime. It is also known that the company will increase the number of machines when the demand exceed the production capacity.

\[
MCmax_{nt} = \frac{M_{nt} \times TP \times 60}{MU_n} \times EF_n.
\]

Table 2 presents machine utilization time for manufacturing one unit product, machine efficiency, and workforce time for manufacturing one product. Table 3 shows the number of machines used and the production capacity for each production time for each product in six months. Table 4 shows the product revenue data for each product in each market for six months. Table 5 shows production costs, backorder costs, subcontract costs, and inventory costs. Workforce costs for both regular and overtime, regular-time working hours, and maximum overtime hours are presented in Table 6. Table 7 presented workforce related data and Table 8 indicates the material capacity for regular-time production, overtime production, backorder, maximum subcontract as well as maximum inventory.

| Product, \( n \) | Machine utilization time, \( MU_n \) (minutes/unit) | Machine efficiency, \( EF_n \) (%) | Workforce production time, \( \alpha_n \) (hours/unit) |
|-----------------|-------------------------------------------------|-----------------------------|------------------|
| 1               | 0.2                                             | 60                          | 1                |
| 2               | 0.5                                             | 80                          | 1.5              |
| 3               | 0.6                                             | 80                          | 5                |
### Table 3. Number of machine and production capacity for each production time

| Product, n | Period, t | Number of machines, \( M_{nt} \) (unit) | Regular-time production capacity, \( MC_{max_{nt}} \) (unit) | Overtime production capacity, \( MC_{max_{OT_{nt}}} \) (unit) |
|------------|-----------|------------------------------------------|-------------------------------------------------------------|-------------------------------------------------------------|
|            | 1         | 2                                       | 2,520                                                       | 1,800                                                       |
| 1          | 2         | 2                                       | 2,520                                                       | 1,800                                                       |
| 2          | 3         | 6                                       | 4,032                                                       | 2,880                                                       |
| 3          | 5         | 5                                       | 4,032                                                       | 2,880                                                       |
|            | 4         | 6                                       | 4,032                                                       | 2,880                                                       |
| 5          | 5         | 5                                       | 4,704                                                       | 3,640                                                       |
|            | 6         | 7                                       | 4,704                                                       | 3,640                                                       |

### Table 4. Product price, \( PP_{nkt} \) ($/unit)

| Product, n | Market, k | Period, t | 1   | 2   | 3   | 4   | 5   | 6   |
|------------|-----------|-----------|-----|-----|-----|-----|-----|-----|
| 1          | 1         | 105       | 105 | 105 | 105 | 105 | 105 | 105 |
| 2          | 2         | 135       | 135 | 135 | 135 | 135 | 135 | 135 |
| 3          | 3         | 150       | 150 | 150 | 150 | 150 | 150 | 150 |
| 4          | 4         | 130       | 130 | 130 | 130 | 130 | 130 | 130 |
| 5          | 5         | 105       | 105 | 105 | 105 | 105 | 105 | 105 |
| 2          | 1         | 480       | 480 | 480 | 480 | 480 | 480 | 480 |
| 3          | 2         | 475       | 475 | 475 | 475 | 475 | 475 | 475 |
| 4          | 3         | 445       | 445 | 445 | 445 | 445 | 445 | 445 |
| 5          | 4         | 400       | 400 | 400 | 400 | 400 | 400 | 400 |
### Table 5. Production cost, backorder cost, subcontract cost, and inventory cost

| Period, t | Regular-time production cost, $CPR_{nt}$ ($/unit) | Overtime production cost, $CPO_{nt}$ ($/unit) | Backorder cost, $CB_{nt}$ ($/unit) | Subcontract cost, $CS_{nt}$ ($/unit) | Inventory cost, $CI_{nt}$ ($/unit) |
|-----------|-----------------------------------------------|-----------------------------------------------|-----------------------------------|-----------------------------------|----------------------------------|
| 1         |                                               |                                               |                                   |                                   |                                  |
| 1         | 12                                            | 20                                            | 25                                | 45                                | 15                               |
| 2         | 12                                            | 20                                            | 25                                | 45                                | 15                               |
| 3         | 12                                            | 20                                            | 25                                | 46                                | 15                               |
| 4         | 15                                            | 20                                            | 25                                | 46                                | 15                               |
| 5         | 15                                            | 20                                            | 25                                | 47                                | 15                               |
| 6         | 15                                            | 20                                            | 25                                | 50                                | 15                               |

### Table 6. Workforce cost, regular-time working hours, and maximum overtime hours

| Periode, t | Workforce cost for regular-time, $CW_{Rt}$ ($/period) | Workforce cost for overtime, $CW_{Ot}$ ($/hours) | Regular-time working hours, $\gamma_t$ (hours/period) | Maximum overtime hours, $OT_{max_t}$ (hours) |
|------------|--------------------------------------------------------|---------------------------------------------------|------------------------------------------------------|------------------------------------------|
| 1          | 1,232                                                  | 25                                                | 154                                                 | 110                                     |
| 2          | 1,120                                                  | 25                                                | 140                                                 | 100                                     |
| 3          | 1,232                                                  | 25                                                | 154                                                 | 110                                     |
| 4          | 1,276                                                  | 25                                                | 147                                                 | 105                                     |
| 5          | 1,276                                                  | 25                                                | 147                                                 | 105                                     |
| 6          | 1,120                                                  | 25                                                | 140                                                 | 100                                     |

### Table 7. Workforce related data

| Initial workforce level, $W_0$ (men) | $30$ |
| Maximum workforce level, $W_{max}$ (men) | $300$ |
| Hiring cost, $CW_{H}$ ($/man)$ | $220$ |
| Laid-off cost, $CW_{L}$ ($/man)$ | $200$ |
| Percentage of workforce available for overtime, $\delta$ (%) | $50$ |
The goal programming model is numerically solved by LINGO 18.0. The optimal values of the objective function and deviation variables for each scenario is presented in Table 9, workforce and production planning are present in Table 10 as well as Table 11.

Table 8. Material capacity for regular-time production, overtime production, backorder, and maximum subcontract capacity

| Produk, n | Periode, t | Material capacity for regular-time production, $MC_{R_{nt}}$ (unit) | Material capacity for overtime production, $MC_{O_{nt}}$ (unit) | Backorder capacity, $MC_{B_{nt}}$ (unit) | Maximum subcontract capacity, $SM_{ax_{nt}}$ (unit) | Maximum inventory, $Q_{I_{max_t}}$ (unit) |
|-----------|------------|------------------------------------------------|------------------------------------------------|--------------------------------|--------------------------------|--------------------------------|
| Scenario 1 | 1          | 834                                           | 442                                           | 294                                     | 331                                     | 350                                     |
|           | 2          | 895                                           | 472                                           | 315                                     | 354                                     | 250                                     |
|           | 3          | 856                                           | 453                                           | 302                                     | 339                                     | 400                                     |
|           | 4          | 914                                           | 482                                           | 321                                     | 361                                     | 250                                     |
|           | 5          | 946                                           | 498                                           | 332                                     | 373                                     | 300                                     |
|           | 6          | 956                                           | 503                                           | 335                                     | 377                                     | 250                                     |
| Scenario 2 | 1          | 684                                           | 367                                           | 244                                     | 275                                     | 350                                     |
|           | 2          | 956                                           | 503                                           | 335                                     | 377                                     | 250                                     |
|           | 3          | 874                                           | 462                                           | 308                                     | 346                                     | 400                                     |
|           | 4          | 1,042                                         | 546                                           | 364                                     | 409                                     | 250                                     |
|           | 5          | 1,146                                         | 598                                           | 398                                     | 448                                     | 300                                     |
|           | 6          | 1,186                                         | 618                                           | 412                                     | 463                                     | 250                                     |
| Scenario 3 | 1          | 930                                           | 490                                           | 326                                     | 367                                     | 350                                     |
|           | 2          | 1,042                                         | 546                                           | 364                                     | 409                                     | 250                                     |
|           | 3          | 946                                           | 498                                           | 332                                     | 373                                     | 400                                     |
|           | 4          | 856                                           | 453                                           | 302                                     | 339                                     | 250                                     |
|           | 5          | 794                                           | 422                                           | 281                                     | 316                                     | 300                                     |
|           | 6          | 1,100                                         | 575                                           | 383                                     | 431                                     | 250                                     |

The goal programming model is numerically solved by LINGO 18.0. The optimal values of the objective function and deviation variables for each scenario is presented in Table 9, workforce and production planning are present in Table 10 as well as Table 11.

Table 9. Optimal values of objective function and deviation variable values for each scenario

|                       | Scenario 1 | Scenario 2 | Scenario 3 | Scenario 4 |
|-----------------------|------------|------------|------------|------------|
| $Z$                   | 0.334      | 0.2302     | 0.085      | 0.161      |
| $dp^-(\$)$            | 1,189,823  | 805,73     | 101,177    | 0          |
| $dt^+(\$)$            | 0          | 0          | 0          | 0          |
| $ds^+(\$)$            | 0          | 0          | 0          | 104,794    |
| $dw^+(\$)$            | 0          | 0          | 560        | 560        |
Table 10. Workforce planning for each scenario

| Scenario 1 | Period, \( t \) | 1 | 2 | 3 | 4 | 5 | 6 |
|------------|-----------------|---|---|---|---|---|---|
| \( WH_t \) |                 | 10| 11| 0 | 0 | 0 | 13|
| \( WL_t \) |                 | 0 | 0 | 8 | 0 | 0 | 0 |
| \( W_t \)  |                 | 40| 51| 43| 43| 43| 56|
| Scenario 2 |                 | 11| 8 | 0 | 0 | 0 | 14|
| \( WH_t \) |                 | 0 | 0 | 8 | 0 | 0 | 0 |
| \( WL_t \) |                 | 41| 49| 41| 41| 40| 54|
| \( W_t \)  |                 | 46| 59| 65| 73| 75| 53|
| Scenario 3 |                 | 33| 15| 0 | 0 | 0 | 0 |
| \( W_t \)  |                 | 63| 78| 78| 78| 78| 78|
| \( OT_t \) |                 | 110|100| 94| 92| 90|100|
| Scenario 4 |                 | 48| 0 | 0 | 0 | 0 | 0 |
| \( W_t \)  |                 | 78| 78| 78| 78| 78| 78|
| \( OT_t \) |                 | 110|100|105|105|105|100|

Table 11. Production planning for each scenario

| Product, \( n \) | 1 | 2 | 3 |
|-----------------|---|---|---|
| Period, \( t \) | 1 | 2 | 3 | 4 | 5 | 6 | 1 | 2 | 3 | 4 | 5 | 6 |
| Scenario 1 | \( QR_{nt} \) | 784| 845| 807| 863| 896| 906| 634| 906| 824| 992| 1,096| 1,136| 885| 987| 908| 794| 748| 1,046|
| \( QL_{nt} \) | 101| 174| 139| 162| 104| 99| 228| 503| 461| 546| 597| 549| 99| 100| 100| 100| 100| 100|
| Scenario 2 | \( QR_{nt} \) | 833| 820| 816| 852| 941| 956| 554| 553| 512| 596| 646| 736| 930| 1,042| 946| 856| 794| 1,100|
| \( QL_{nt} \) | 101| 174| 139| 162| 104| 99| 228| 503| 461| 546| 597| 549| 99| 100| 100| 100| 100| 100|
| Scenario 3 | \( QR_{nt} \) | 834| 895| 856| 914| 946| 956| 684| 956| 874| 1,042| 1,146| 1,186| 930| 1,042| 946| 856| 794| 1,100|
| \( QL_{nt} \) | 442| 435| 453| 482| 498| 502| 367| 490| 462| 546| 598| 522| 490| 546| 498| 453| 422| 523|
| Scenario 4 | \( QR_{nt} \) | 33 | 122| 193| 236| 262| 335| 57 | 117| 155| 209| 236| 409| 40 | 118| 143| 185| 238| 315|
| \( QL_{nt} \) | 442| 471| 453| 482| 498| 500| 367| 466| 462| 546| 598| 350| 490| 546| 498| 453| 422| 575|
| \( QS_{nt} \) | 27 | 51 | 83 | 113| 141| 334| 33 | 90 | 107| 128| 132| 412| 23 | 43 | 72 | 105| 281| 383|
| \( Q_{nt} \)  | 331| 354| 339| 361| 373| 377| 275| 377| 346| 408| 448| 463| 367| 409| 373| 339| 175| 348|

Based on Table 9, it can be seen that the under-achieved deviation for Goal 1 \((dp^-)\) occurs in Scenario 1, Scenario 2, and Scenario 3. This means the profit gained for Scenario 1 is $2,310,177, for Scenario 2 is $3,419,427, and for Scenario 3 is $3,398,823. It is also noted the over-achieved deviation of $560 for Goal 4 \((dw^+)\) occur in Scenario 3 and Scenario 4. Undesired deviation in Scenario 4 is an over-achieved that appear in Goal 3 \((ds^-)\) and Goal 4 \((dw^+)\). Hence, the subcontract cost and the workforce changes cost in this scenario respectively are $1,104,794 and $10,560.

The following solution is a monthly planning that explained quantity and mechanism for production as well as worker level. Therefore in this paper we don’t discuss about which production is the most optimum but how the monthly planning is like for each scenario, and whether the proposed model fit with the company’s requirement. The following will explain solution in Table 10, Table 11, and the compatibility with company’s requirement.

Workforce planning represented in Table 10 consists of hired workers \((WH_t)\), laid-off workers \((WL_t)\), needed workers \((W_t)\), and overtime hours \((OT_t)\). As it can be seen for Scenario 1 there is no overtime hours \((OT_t)\), it shows that the workforce planning fit with the demand requirements for
Scenario 1. Workforce planning for Scenario 3 and Scenario 4 does not require workers fired. It is because in both scenarios the production process exceeds the raw material capacity and the production capacity. In short, if the company lays-off the workers, the company could not fulfilled the demand optimally and may experience losses. Production planning in Table 11 has fulfilled the required conditions for each planning scenario. It is also revealed that inventory needed just for Scenario 1 with a small quantity, so it infers that this planning fit with the objective of minimizing inventory cost.

The weights for Goal 1 until Goal 4 used in the above planning are the same of 1. That means, the goal have the same importance’s level. Therefore the effect of different importance’s level to the objective function’s achievement will be tested, by assigning different weight to each goal. This study will use the demand in Scenario 4 with 24 different weight combinations. The result will be presented in Table 12.

**Table 12. Effect of weight changes to the objective function’s achievement**

| Case | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $dp^-$ ($) | $di^+$ ($) | $ds^+$ ($) | $dw^+$ ($) | $Z$ |
|------|------|------|------|------|------------|------------|------------|------------|-----|
| 1    | 1    | 2    | 4    | 8    | 0          | 0          | 171,994    | 120        | 0.784 |
| 2    | 1    | 2    | 8    | 4    | 0          | 0          | 104,794    | 560        | 1.062 |
| 3    | 1    | 4    | 2    | 8    | 0          | 0          | 205,594    | 0          | 0.411 |
| 4    | 1    | 8    | 2    | 4    | 0          | 0          | 171,994    | 120        | 0.392 |
| 5    | 1    | 4    | 8    | 2    | 0          | 0          | 93,852     | 780        | 0.906 |
| 6    | 1    | 8    | 4    | 2    | 0          | 0          | 104,794    | 560        | 0.531 |
| 7    | 8    | 1    | 2    | 4    | 0          | 0          | 171,994    | 120        | 0.392 |
| 8    | 4    | 1    | 2    | 8    | 0          | 0          | 205,594    | 0          | 0.411 |
| 9    | 4    | 1    | 8    | 2    | 0          | 0          | 93,852     | 780        | 0.906 |
| 10   | 8    | 1    | 4    | 2    | 0          | 0          | 104,794    | 560        | 0.531 |
| 11   | 2    | 1    | 4    | 8    | 0          | 0          | 171,994    | 120        | 0.784 |
| 12   | 2    | 1    | 8    | 4    | 0          | 0          | 104,794    | 560        | 1.062 |
| 13   | 4    | 8    | 1    | 2    | 0          | 0          | 171,994    | 120        | 0.196 |
| 14   | 8    | 4    | 1    | 2    | 0          | 0          | 171,994    | 120        | 0.196 |
| 15   | 2    | 4    | 1    | 8    | 0          | 0          | 205,594    | 0          | 0.205 |
| 16   | 2    | 8    | 1    | 4    | 0          | 0          | 205,594    | 0          | 0.205 |
| 17   | 8    | 2    | 1    | 4    | 0          | 0          | 205,594    | 0          | 0.205 |
| 18   | 4    | 2    | 1    | 8    | 0          | 0          | 205,594    | 0          | 0.205 |
| 19   | 4    | 8    | 2    | 1    | 0          | 0          | 104,794    | 560        | 0.265 |
| 20   | 8    | 4    | 2    | 1    | 0          | 0          | 104,794    | 560        | 0.265 |
| 21   | 8    | 2    | 4    | 1    | 0          | 0          | 93,852     | 780        | 0.453 |
| 22   | 4    | 2    | 8    | 1    | 0          | 0          | 90,372     | 1,000      | 0.823 |
| 23   | 2    | 4    | 8    | 1    | 0          | 0          | 90,372     | 1,000      | 0.823 |
| 24   | 2    | 8    | 4    | 1    | 0          | 0          | 93,852     | 780        | 0.453 |

Based on Table 12, undesired deviation occurs in Goal 3 and Goal 4 with various value. In term of reducing the deviation the company needs to increase the goal’s weight. As it can be seen in Table 12, Goal 3 has maximum deviation value of $205,594 when the weight is getting low and has minimum deviation value of $90,372 when the weight is getting high. Then as the deviation for Goal 3 minimum, the deviation for Goal 4 is maximum, vice versa. However at the end, the best decision of weights is determined by the decision-maker by choosing a combination that fits the company’s conditions.
5. Conclusion
In this study, it has been shown that aggregate production planning problem can be modeled by nonpreemptive goal. The proposed model is applied in the APP of a company that has three products, five markets, and six months period planning. This company also has four objective functions with several constraints that need to be optimized. The four scenarios that proposed in this study have fulfilled the company’s requirement with several production mechanism. Then, the effect of weights changes is assessed by applying demand in Scenario 4 with 24 cases. The best decision is left to the decision-maker in the company by considering company's conditions.

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