Light hadron spectrum and quark masses

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Recent developments in lattice QCD calculations of the light hadron spectrum and quark masses are reviewed.

1. Introduction

One of the main goals of lattice QCD is to confirm the validity of QCD as the theory of strong interactions. Calculation of the hadron spectrum is fundamental in this respect. In the quenched approximation, large effort over the years has culminated in the work by CP-PACS[1] which revealed a systematic deviation of the quenched spectrum from experiment. Since then, the focus has shifted to full QCD including dynamical quarks, and a number of large-scale simulations have been executed in \( N_f = 2 \) full QCD[2]. A crucial issue here is how sea quark effects manifest in the spectrum. Work to explore this question has continued this year.

A realistic QCD simulation has to treat the strange quark dynamically. The algorithmic development to treat odd number of flavors has accelerated in the last few years[3]. This has led to several attempts toward realistic calculations in \( N_f = 3 \) QCD very recently.

One of the major uncertainties in lattice calculations arises from the chiral extrapolation toward realistically light quark masses. One possibility to control this extrapolation is to use chiral perturbation theory (ChPT)[4] as a guide. Test of the validity of ChPT is an important step for this purpose, and full QCD data to examine this issue are becoming available.

Another recent trend is the improvement of formulations of lattice QCD such as the domain-wall fermion[5] and fermions defined on anisotropic lattices[6]. Quenched QCD presents a testing ground of these formulations, and some realistic simulations have been made.

Spectrum calculations allow a simultaneous determination of the strong coupling constant and quark masses which are the fundamental parameters of the Standard Model. Precise determination of these quantities is an important issue in lattice QCD[7]. New studies in this area have also been reported at this conference.

In this review, we present the status of lattice calculations of the light hadron spectrum and the fundamental parameters of QCD with focus on the points above. We concentrate on results obtained with the Wilson-type quark actions, as those with the Kogut-Susskind (KS) action are covered in a separate talk[9]. In Sec. 2, we discuss sea quark effects in \( N_f = 2 \) full QCD. Recent developments toward \( N_f = 3 \) simulations are reviewed in Sec. 3, and verification of ChPT is discussed in Sec. 4. Section 5 is devoted to recent developments of spectroscopic studies in quenched QCD. The status of the strong coupling constant and quark masses are updated in Secs. 6 and 7. A brief conclusion is given in Sec. 8.

2. Light hadron spectrum in \( N_f = 2 \) QCD

2.1. Recent simulations

Recent simulations in two-flavor QCD are listed in Table 1. Most of the simulations with the Wilson-type quark action were made with the plaquette gauge action at a single lattice spacing \( a \sim 0.1 \) fm. An exception is the CP-PACS study[21] which explored the range \( a \sim 0.1 - 0.2 \) fm with the use of an improved gauge action. This work reported clear sea quark effects in the meson spectrum after the continuum extrapolation.

New results were reported by UKQCD[14] and JLQCD[13] at this conference. The two simulations still use the plaquette gauge action and work at a single lattice spacing \( a \sim 0.1 \) fm, but the quark action is fully \( O(a) \) improved with \( c_{SW} \)
Table 1
Recent simulations in $N_f = 2$ QCD. Values of $c_{SW}$ for the clover quark action\cite{10} are shown in brackets, where NP and TP stand for non-perturbative and tadpole-improved values, respectively.

| group       | gauge     | quark      | $a_s$ [fm] | $L_s$ [fm] | $m_{PS,sea}/m_{V,sea}$ | ref. |
|-------------|-----------|------------|------------|------------|-------------------------|------|
| SESAM       | plaquette | Wilson     | 0.08       | 1.3        | 0.68–0.83               | 11   |
| TχL         | plaquette | Wilson     | 0.08       | 1.9        | 0.57, 0.70              | 12   |
| SESAM-TχL   | plaquette | Wilson     | 0.09       | 1.5        | 0.68–0.85               | 13   |
| UKQCD       | plaquette | clover (1.76) | 0.12         | 1.0–1.9   | 0.67–0.86               | 14   |
| UKQCD       | plaquette | clover (NP) | 0.10       | 1.7        | 0.70–0.84               | 15,10|
| UKQCD       | plaquette | clover (NP) | 0.10       | 1.6        | 0.58                    | 15,10|
| QCDSF       | plaquette | clover (NP) | 0.09       | 2.1,1.5    | 0.69, 0.76              | 17   |
| JLQCD       | plaquette | clover (NP) | 0.09       | 1.1–1.8    | 0.60–0.80               | 18,19|

CP-PACS RG-improved\cite{20} clover (TP) | 0.11–0.22 | 2.5–2.6 | 0.55–0.81 | 21 |

Columbia plaquette KS | 0.09 | 1.5 | 0.57–0.70 | 22 |
MILC plaquette KS | 0.10–0.32 | 2.4–3.8 | 0.3–0.8 | 23 |
Columbia plaquette KS ($ξ \sim 1.8–5.0$) | 0.23–0.34 | 3.7–5.4 | 0.35–0.80 | 24 |
MILC Symanzik\cite{25} improved KS | 0.13 | 2.6 | 0.50 | 27 |

The smallest sea quark mass in most of the simulations stopped at $m_{PS,sea}/m_{V,sea} \sim 0.7$, which explains why sea quark effects were not clearly seen in these simulations. The CP-PACS data did cover the range down to $m_{PS,sea}/m_{V,sea} \sim 0.6$. The trend of sea quark effects as observed by JLQCD is present at $a \sim 0.1$ fm, while it is less clear on coarser lattices due to larger error of $r_0$\cite{31}.

Figure 2 shows recent results of the $J$ parameter\cite{32} calculated with differentiation in the valence quark mass as a function of the sea quark mass. The JLQCD data show a trend that the value of $J$, which is consistent with that of quenched QCD for heavier sea quark masses, increases as the sea quark mass decreases. The latter feature can be observed also in the UKQCD data. As a result, the values extrapolated to the physical sea quark mass are closer to experiment than in quenched QCD.

The $J$ parameter in full QCD can also be obtained using the experimental definition $J \equiv m_K - (m_{K^*} - m_p)/(m_K^2 - m_{K^*}^2)$, or from the combined chiral extrapolation as a function of sea and valence quark masses. The results are also plotted at $(r_0 m_{PS,sea})^2 = 0$ in Fig. 3. They lead to the same conclusion that $J$ increases with dynamical quarks. An exception is the SESAM point presumably due to heavier sea quark masses $m_{PS,sea}/m_{V,sea} \sim 0.7$ and scaling violation in their simulations. Recent results with KS fermions also
determined non-perturbatively\cite{28}. 

The UKQCD simulation shifts $\beta$ with sea quark mass, keeping the Sommer scale $r_0/a$\cite{28} fixed. The JLQCD runs were performed at a fixed $\beta$. Another important difference is the range of quark mass covered in the two simulations: JLQCD explored light sea quark masses down to $m_{PS,sea}/m_{V,sea} \sim 0.6$, whilst the UKQCD’s lightest point is around $m_{PS,sea}/m_{V,sea} \sim 0.7$\cite{11}. We note that Orth et al. attempts to reduce quark masses to even lighter points, but actual simulations have not yet been started\cite{39}.

2.2. Sea quark effects in meson spectrum

Figure 1 shows how the valence quark mass dependence of the vector meson mass varies as a function of sea quark mass in the JLQCD and UKQCD simulations. In both UKQCD and JLQCD data, the set of points for the sea quark masses down to $m_{PS,sea}/m_{V,sea} \sim 0.7$ almost overlap. On the other hand, the JLQCD data for a lighter sea quark mass $m_{PS,sea}/m_{V,sea} \sim 0.6$ clearly show a larger slope, and are closer to the experimental points marked by asterisks.

The smallest sea quark mass in most of the previous simulations stopped at $m_{PS,sea}/m_{V,sea} \sim 0.7$. For $\xi \sim 1.8–5.0$, $m_{PS,sea}/m_{V,sea} \sim 0.7$, the term $\beta$ is around $<1$ fm, while it is less clear on coarser lattices due to larger error of $r_0$\cite{31}.

Although UKQCD made another simulation at a smaller sea quark mass ($m_{PS,sea}/m_{V,sea} \sim 0.58$, $L_s \sim 1.6$ fm), finite size effects seem to be significant there\cite{39}.

\[\text{Table 1} \]

Recent simulations in $N_f = 2$ QCD. Values of $c_{SW}$ for the clover quark action\cite{10} are shown in brackets, where NP and TP stand for non-perturbative and tadpole-improved values, respectively.

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Figure 1. Vector meson mass as a function of pseudo-scalar meson mass squared, normalized in units of $r_0 = 0.49$ fm from JLQCD and UKQCD.

Figure 2. Recent results of $J$ parameters in $N_f = 2$ full QCD and quenched QCD. Results presented this year are plotted in filled symbols.

Figure 3. Strange vector meson masses with $m_K$ as input in $N_f = 2$ (filled symbols) and quenched QCD (open symbols).

2.3. Baryons and finite size effects

Significant sea quark effects are also expected in the baryon spectrum, as it sizably deviates from experiment in quenched QCD[1]. However, no group has found clear evidence. A possible reason is finite size effects, which we expect to be more important for baryons than for mesons.

Suppression of systematic errors to a few percent level is required for a convincing study of sea quark effects. In order to estimate the lattice size needed to achieve this accuracy, we fit hadron mass data of UKQCD[14] and JLQCD[19] on several lattice volumes to an ansatz $m(L) = m(L = \infty) + c/L^3$. Using the fitted value of $m(L = \infty)$, we show in Fig. 4 the relative magnitude of finite size effects as a function of the spatial extent $L$ (divided by $r_0$) at sea quark masses corresponding to $m_{PS}/m_V = 0.7$ and 0.6.

We observe that a lattice size of $L \sim 2$ fm suffices for mesons to keep finite size effects under a few percent level for sea quark masses down to $m_{PS}/m_V \sim 0.6$. On the other hand, calculations of the baryon masses need $L \sim 3$ fm at the same sea quark masses and the required size increases rapidly as the sea quark mass decreases. Further studies on such large volumes will be necessary to identify sea quark effects on the baryon spectrum.

2.4. Glueball masses

An unambiguous experimental identification of the glueball is still missing, and lattice QCD can provide helpful theoretical estimates. Calculations in quenched QCD have been performed for
Figure 4. Mass shift at finite lattice size $L$ as a function of $L/r_0$. The dashed and dotted lines show fit curve for data at sea quark mass corresponding to $m_{PS}/m_V = 0.7$ and 0.6, respectively.

The focus has now shifted to calculations in full QCD and mixings with quarkonia. UKQCD and SESAM-T reported new results this year. Both groups find a decrease of $m_{b^{++}}$ by about 20% with dynamical quarks. The observed sea quark mass dependence, however, is quite different: SESAM-T finds a significant slope, while UKQCD data are almost flat. It is, thus, too early to draw a definite conclusion on the sea quark effect. Scaling violation should be also studied carefully, as it is known to be large in quenched QCD. A similar indication was already seen by SESAM-T in full QCD.

3. QCD with dynamical up, down and strange quarks

Realistic simulations of QCD require up, down and strange quarks to be treated dynamically. While such studies already exist for the KS quark action using an approximate R algorithm, exact algorithms for odd number of Wilson-type quarks were developed only recently.

This year JLQCD initiated a study of $N_f = 3$ full QCD using an exact algorithm for the $O(a)$-improved Wilson fermion. As a first step, they studied the phase structure on the $(\beta, \kappa)$ plane and found an unexpected first-order phase transition with the standard plaquette gauge and the $O(a)$-improved Wilson quark action, as shown in Fig. 5.

4. Test of chiral perturbation theory

Chiral perturbation theory (ChPT) predicts the presence of logarithm singularities in the quark mass dependence of physical quantities, e.g., for the pseudo-scalar meson mass $m_{PS}$,

$$\frac{m_{PS}^2}{2B_0m_q} = 1 + \frac{1}{N_f}y\ln[y] + O(y)$$

where $y = 2B_0m_q/(4\pi f)^2$. A test of this relation is shown in Fig. 6, where lattice data for two-flavor QCD from JLQCD are plotted together with the fit curves of Eq. (1) assuming $f$ to be a free parameter or fixed to $f = 93$ MeV. The lattice results are not consistent with the curvature characterized by the chiral logarithm.
Recently UKQCDattempted to extract the low energy constants of chiral lagrangian from a fit of lattice data to the partially quenched ChPT formula using ratios proposed by ALPHA. The curvature expected from the chiral logarithm is also not clear in their data.

A possible reason behind these disagreements is that sea quark masses in current simulations are too large. In fact, the coefficient of the chiral logarithm is modified when the effect of flavor singlet meson is included into ChPT; it can take much smaller values than 1/N_f, if the pseudo-scalar meson is not so light compared to the flavor singlet meson. A recent work by Nelson et al. using KS fermions suggests a similar conclusion. The curvature is visible in their data at small sea quark masses, which are possible with KS fermions.

Another interesting subject of ChPT is determination of the low energy constants α_i from lattice data. α_8 is an important quantity since it is related to the possibility of m_u = 0. UKQCD attempted to extract α_i’s in two-flavor QCD, and obtained α_8 = 0.79(±2)(21), which disfavors m_u = 0. Nelson et al. obtained a consistent conclusion in three flavor QCD. These results, however, should be confirmed with lighter sea quark masses, for which ChPT at the one-loop level becomes more reliable.

5. Developments in quenched QCD

5.1. Test of improved formulations

Quenched QCD is now used to develop and test improved formulations of lattice QCD. Domain-wall fermion formalism is an important case, as it is expected to realize exact chiral symmetry at finite lattice spacings. Anisotropic lattices may provide a way to achieve simulations with large lattice cut-offs on large spatial volumes.

Figure 7 shows results of m_K^*/m_ρ mass ratio calculated with the domain-wall fermions and on anisotropic lattices. Clearly these formulations reproduce, and confirm, the small hyperfine splitting in the quenched meson spectrum obtained by the precision calculation by CP-PACS using the conventional Wilson quark action.

Recently, the Wilson quark action with a chirally twisted mass term has drawn attention as an alternative formulation to suppress exceptional configurations. Some evidences of such suppression have been presented at this conference. It will be interesting to apply this action to the non-perturbative determination of improvement and renormalization constants at strong coupling, where exceptional configurations cause a serious problem.

5.2. Negative parity baryon

An interesting subject in recent quenched simulations is to explore the rich structure of the excited baryons. There already exist several studies for the negative parity baryon N^*(J^P = 1/2^-) at finite lattice spacings. The first systematic study of scaling violation and finite size effects has been performed this year by the QCDSF-UKQCD-LHPC collaboration using the O(a)-improved Wilson action. Their result
of a systematic study already last year\cite{65}. They
fermions. Calculations of the charm quark mass
lation is the use of the domain-wall and overlap
7. Quark masses

By last year results for the light quark mass
$m_{ud}$ in quenched QCD converged within an accu-
curacy of about 15\%\cite{8}. For $m_s$ there turned out
an additional uncertainty of about 20\% due to the
choice of the input meson mass (e.g. $m_K$ or
$m_{\phi}$)\cite{11}. A noticeable trend in this year’s calcu-
lation is the use of the domain-wall and overlap
fermions. Calculations of the charm quark mass
$m_c$ have also been pursued in quenched QCD.

In $N_f=2$ full QCD, CP-PACS reported results of a systematic study already last year\cite{58}. They
found that sea quark effects reduce $m_{ud}$ and $m_s$
by about 25\%, and the discrepancy in $m_s$ with
$m_K$ or $m_{\phi}$ as input is remarkably reduced. This
year there has been less progress in full QCD.

7.1. Strange quark mass in quenched QCD

A new estimate of $m_s$ was obtained by CP-
PACS using the domain-wall fermion\cite{66}. Their
calculations were performed at $a^{-1} \sim 2$
and 3 GeV using one-loop perturbative renormal-
ization constants. Giusti et al. calculated
$(m_s + m_{ud})$ at $a^{-1} \sim 2$ GeV using the over-
lap fermion\cite{67}. Their renomalization constants
are evaluated non-perturbatively in the RI/MOM
scheme.

Recent results with $m_K$ as input are plotted in
Fig. 8. A reasonable agreement among the new
and previous results is observed. Therefore, the
average $m_s^{MS}(2 \text{ GeV}) = 105 \pm 15 \text{ MeV}$
does not change significantly since last year\cite{8}.

7.2. Light quark mass in quenched QCD

We consider the ratio $m_s/m_{ud}$, rather than
$m_{ud}$ itself, since systematic uncertainties partially
cancel in this ratio. A new result $m_s/m_{ud} =
26.2(2.3)$ by CP-PACS\cite{68} as well as previous re-
results are in good agreement with the prediction
of one-loop ChPT : 24.4(1.5)\cite{57}. We therefore
apply the ratio from ChPT to the average of $m_s$
to obtain $m_{ud}^{MS}(2 \text{ GeV}) = 4.3 \pm 0.7 \text{ MeV}$.

Dong et al. calculated $m_{ud}$ with the over-
lap fermion\cite{69}. Their estimate $m_{ud}^{MS}(2 \text{ GeV}) =
5.3(0.6)(0.7) \text{ MeV}$ agrees with the above average.

7.3. Charm quark mass in quenched QCD

There have been few studies for calculating the charm quark mass $m_c$\cite{71}. The reason is that
$O((am)^3)$ scaling violation is expected to

| Group   | Quark | $\alpha_s^{(N_f=5)}$ |
|---------|-------|----------------------|
| SESAM   | Wilson| 0.1118(17)           |
| QCDSF-UKQCD | clover(NP) | 0.1076(20)(18) |
| Boucaud et al. | Wilson | 0.113(3)(4) |
| Davies et al. | KS | 0.1174(24) |

The reason is that $O((am)^3)$ scaling violation is expected to
be large in relativistic formulations, while the charm quark is too light to be treated in the non-relativistic approximation.

New estimates of \( m_c \) have been reported by two groups this year, both using the \( O(a) \)-improved action to reduce scaling violation. Becirevic et al. performed a simulation at a fixed lattice spacing \( a^{-1} \sim 3 \text{ GeV} \)\(^{-1} \). The results significantly differ for \( m_c \) obtained from vector (VWI) and axial vector Ward identities (AVI). They take an average to quote \( m_c^{\text{MS}}(m_c) = 1.26(3)(12) \). The difference between the two methods (AVI and VWI) and the average is treated as a systematic error.

ALPHA performed simulations in a wide range of lattice spacing \( a^{-1} \sim 2-4 \text{ GeV} \) and performed the continuum extrapolation\(^\text{[72]} \). They observed a nice convergence of \( m_c \) determined from AVI and VWI toward the continuum limit, and obtained \( m_c^{\text{MS}}(m_c) = 1.314(40)(20)(7) \) in the continuum limit.

These results are plotted in Fig. 8 together with previous results\(^\text{[70]} \) and an average by PDG\(^\text{[73]} \). The two new results are consistent with previous ones as well as the PDG’s average.

A new estimate using the heavy quark action in the FNAL interpretation\(^\text{[74]} \) was presented by Juge et al. at this conference\(^\text{[75]} \). While the estimation of the systematic error is still in progress,

Figure 8. Continuum extrapolation of strange quark mass with \( m_K \) as input in \( \overline{\text{MS}} \) scheme at scale \( \mu = 2 \text{ GeV} \). Filled and open symbols show new and previous results, respectively. The choice of the quark action is written in brackets, where “W” = Wilson, “C” = clover, “KS” = Kogut-Susskind and “DWF” = domain-wall fermions.

their preliminary result is also around 1.3 GeV and consistent with the above new results.

7.4. Results in \( N_f = 2 \) full QCD

SESAM-T\( \chi \)L attempted a continuum extrapolation of \( m_{ud} \) and \( m_s \) from previous\(^\text{[76]} \) and new simulations at \( \beta = 5.5 \)\(^\text{[13]} \). Their range of the lattice spacing is, however, narrow and far from the continuum limit. This leads to a large uncertainty in their results, \( m_{ud}^{\text{MS}}(2 \text{ GeV}) = 4.5(1.7) \) MeV and \( m_s^{\text{MS}}(2 \text{ GeV}) = 92(83) \) MeV. In near future, we expect results from the UKQCD and JLQCD simulations with the \( O(a) \)-improved Wilson fermion.

8. Conclusions

In quenched QCD, improved formulations, such as the domain-wall and overlap fermions and anisotropic lattice actions, have been tested by calculating the light hadron spectrum and quark masses, and good consistency with previous results has been observed. These formulations are ready for applications to precise determinations of other observables.

In \( N_f = 2 \) full QCD, sea quark effects on the meson spectrum, which were previously observed by CP-PACS, have been confirmed using the \( O(a) \)-improved Wilson and an improved KS fermions. However we still need further studies for sea quark effects on the baryon and glueball spectrum. Exploring small sea quark masses with Wilson-type fermions is also an important issue to resolve the poor consistency between lattice data and ChPT.

Pursuit of realistic simulations in three flavor
QCD is one of the most important issues in spectroscopic studies. This year significant progress to this end has been made by several groups using both of the KS and Wilson fermions. Further systematic studies will be expected in near future.

And finally, there has been few progress in non-perturbative calculation of the improvement coefficient \( c_A \) and renormalization factors in full QCD. These calculations are urgently required for precise determinations of the quark masses and decay constants.

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