New probe of modified gravity

A. Boyarsky1,2 and O. Ruchayskiy1

1Ecole Polytechnique Fédérale de Lausanne, FSB/ITP/LPPC, BSP 720, CH-1015, Lausanne, Switzerland
2Bogolyubov Institute of Theoretical Physics, Kyiv, Ukraine

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We suggest a new efficient way to constrain a certain class of large scale modifications of gravity. We show that the scale-free relation between density and size of Dark Matter halos, predicted within the $\Lambda$CDM model with Newtonian gravity, gets modified in a wide class of theories of modified gravity.

Models with the large scale modification of gravity are actively discussed in the recent years in connection with the observed accelerated expansions of the Universe and because they can be related to the existence of extra dimensions [1,2]. However, the general principles of gauge invariance and unitarity strongly constrain possible theories of gravity, modifying the Newton’s law at large distances (see analysis of [3]). Thus, in addition to its phenomenological applications, this problem is related to the fundamental questions of particle physics, field theory and gravity. It is therefore important to search for large scale modifications of gravity experimentally.

A possible set of consistent (as a spin-2 field theory) large scale modifications of gravity is described by two parameters – scale $r_c$ and a number $0 \leq \alpha < 1$ [3–5]. $r_c$ marks the distances at which at the linearized level gravitational law changes from $1/r^2$ to some other power $1/r^n$, and the parameter $\alpha$ determines the value of $n$. Phenomenologically, deviations from Newton’s law we are looking for may be represented in this parameter space. A significant fraction of this space is excluded by precision measurements of the Moon orbit [6]. Other natural probes of such modifications are cosmological observables (see e.g. [7–12] and refs. therein).

In this work we identify a new observable sensitive to the large scale modifications of gravity. We demonstrate that universal properties of individual dark matter halos are also affected by the modifications of gravity, and provide novel way to probe them. Namely, we show that the scale-invariant relation between density and size of dark matter halos, predicted by Newtonian gravity within the $\Lambda$CDM model [13] and found to hold to a good precision in observed dark matter halos [14], may receive non-universal (size-dependent) corrections for a wide range of parameters $r_c$ and $\alpha$.

Formation of structures in the Universe is an interplay between gravitational (Jeans) instability and overall Friedmann expansion. The gravitational collapse does not start until the potential energy $U$ of a gravitating dark matter system overpowers the kinetic energy of the Hubble expansion $K \sim 1/2H^2R^2$. Once the gravitational collapse has began, at any moment of time $t$ a dark halo is confined within a sphere of zero velocity or a turn-around sphere. As Hubble expansion rate $H(t)$ decreases with time, the turn-around radius $R_{ta}(t)$ grows. In the Newtonian cosmology with potential $\phi_N(r) = -GM/r$ the turn-around radius $R_{ta}$ is

$$R_{ta} \propto \left(\frac{GM}{H^2}\right)^{1/3}$$

(today for masses $\sim 10^{12}M_\odot$ the turn-around radius is $\sim 1$Mpc). Notice that at any moment of time the average density within a turn-around radius [1] is proportional to the cosmological density and is the same for halos of all masses:

$$\rho_{ta} \propto \frac{H^2}{G} \propto \bar{\rho}_{tot}(t)$$

(2)

It was shown in [13] that the property [2] leads to a universal relation between characteristic scales and densities of dark matter halos. This relation holds in wide class of dark-matter dominated objects (from dwarf galaxies to galaxy clusters) [14] (see also [15–17]). The relation is in a very good agreement with pure dark matter simulations [18,19], suggesting that baryonic feedback can be neglected in this case. Therefore, this relation can serve as a new tool of probing properties of dark matter and gravity at large scales.

The relation (2) continues to hold in the Universe where gravity is modified by the cosmological constant $\Lambda$. The gravitational energy of a body of mass $M$ at distance $r$ becomes $U_\Lambda = -\frac{GM}{r} - \frac{\Lambda r^2}{3}$. Comparing it with the kinetic energy of the Hubble flow $K$ one arrives once again to the relation [2]

$$\rho_{ta}(t) \propto \frac{\Lambda}{G}$$

(3)

(c.f. [13]). The relation (1) still holds and is again independent on the mass of the halo.

What is the most general form of gravitational potential, for which the property (2) remains true? Clearly, it will hold for all the gravitational potentials of the form

$$\phi(r) = -\frac{GM}{r} F\left(\frac{\rho(r)}{\rho_*}\right)$$

(4)

where $\rho_*$ is some constant with dimension of density. In particular, $\Lambda$-term obeys this property (with $\rho_* \propto \Lambda/G$). All the theories of the form [4] obey the property that relative correction to the Newtonian potential $\phi_N$ depends only on the density $\rho(r)$ within a radius $r$ (and not on the mass or the size of objects).

Next, we consider the modifications of gravity [3,4]. The gravitational potential of a spherically symmetric system of mass $M$ there has the form

$$\phi_\alpha(r) = -\frac{GM}{r} \frac{r}{Vr}$$

(5)

where $0 \leq \alpha < 1$. Here the characteristic (Vainshtein) radius $r_\nu$ is defined as [3,4,20]

$$r_\nu = \left(2GMr_c^2\right)^{1/3} \nu$$

(6)

where $\beta = 4(1-\alpha)$.
If the scale $r_c$ is of the order of $\sim H_0^{-1}$, such modifications of gravity can provide an explanation for the late-time cosmological expansion of the Universe \cite{1,2}. The corrections to Newton’s law become negligible as $r \to 0$ ($\pi(0) = 1$) and the radius $\Omega$ characterizes the scale where the deviations from Newton’s potential become of order unity. Using the relation

$$r_t = \left( \frac{r_{\Omega}^{1+\beta}}{2GMr_{\Omega}^2} \right)^{\frac{1}{1+\beta}} \rho M^{\frac{\beta-2}{1+\beta}} \frac{1}{\rho^{1/3}(2Gr_{\Omega}^2)^{\frac{1}{1+\beta}}} \tag{7}$$

(where $\rho = M/r^3$) we find that among the theories of modified gravity \cite{5} only $\beta = 2 (\alpha = 1/2)$, the DGP model \cite{1} possess the property \cite{3} and consequently \cite{4}.

The property \cite{2} can be probed experimentally. Extensive catalog of DM-dominated objects of all scales, collected in \cite{14}, exhibits a simple scaling relation of the properties of the DM halos. Dark matter distribution in the majority of observed objects can be described by one of the universal DM profiles (such as e.g. NFW \cite{21} or Burkert \cite{22}). Such profiles may be parametrized by two numbers, directly related to observations – a characteristic radius $r_c$ (off-center distance where the rotation curve becomes approximately flat, equal e.g. to $r_s$ for NFW) and a DM central mass density $\rho_c$, averaged inside a ball with the size $r_c$. It was shown that DM column density $S \propto \rho_c r_c$, (see \cite{13,14} for a detailed definition) changes with the mass as $S \propto M^\kappa$, where $\kappa \approx 0.22 - 0.33$. It was demonstrated in \cite{13} that in the simplest self-similar model (i.e., assuming that $r_c/R_{\text{ta}}$ is the same for the DM halos of all masses) property \cite{12} implies a scaling $S \propto M^{1/3}$, compatible with observations. Observations (see e.g. \cite{23,27}) demonstrate that the ratio of $r_c$ to the virial radius depends weakly (as $M^{0.10}$) on the mass of DM halos. The Fig.4 shows ratio of the virial radius of a halo to its $r_c$ for DM density profiles from the catalog of \cite{14}. These results are in perfect agreement with the $\Lambda$CDM numerical simulations (see e.g. \cite{12,28}). Due to this slight deviation from the self-similarity, the best fit value of the scaling parameter $\kappa = 0.23$ (see \cite{13} for discussion). For the qualitative discussion of this work, it is important that $S(M)$ is a featureless power-law dependence, whose slope does not depend on mass and that the deviation from the slope $1/3$ is small (as follows from observations).

We can conclude that in the theories, satisfying the condition \cite{5} (e.g. in DGP model) the properties of DM halos theory will follow the same scaling relation $S \propto M^{1/3}$ and the difference with the $\Lambda$CDM case will only be a different normalization of this scaling relation. For other theories, described in \cite{3,4}, with $\alpha \neq 1/2$, we can see from the Eq. \cite{7}) that the potential $\phi_\alpha(r)$ is not of the form \cite{4} and we can expect deviation from the universal scaling law.

I. $S - M$ RELATION FOR GENERAL $\alpha$

Let us work out the $S - M$ for a general $\alpha$ in details. A general expression, relating the turn-around time, turn-around radius and mass within this radius follows from energy con-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Comparison of $c_{\text{vir}} = R_{\text{vir}}/r_c$ as a function of DM halo mass for observed DM density profiles from the catalog of [14].}
\end{figure}
If the Vainstein radius $r_V \ll R_{\text{ta}}$ for halos of all masses that are experimentally observed (roughly from $\sim 10^8 M_{\odot}$ to $\sim 10^{16} M_{\odot}$), then for distances $r \gg r_V$ the corrections to the Newtonian potential reduce either to the order one renormalization of the gravitational constant (on the “normal branch”) or become indistinguishable from the $\Lambda$-term (“self-accelerated branch”). In both cases $S(M) \propto M^{1/3}$ with the renormalization, different from the pure Newtonian case.

In the opposite case $r_V \gg R_{\text{ta}}$, one can utilize the perturbative expansion of the function $\pi(x)$. The gravitational potential well inside the Vainstein radius is given by\[\pi(x \ll 1) \approx 1 + c_1 x^a \quad ; \quad a = \frac{\beta + 1}{2} = \frac{5 - 4\alpha}{2} \tag{12}\]

where $c_1 \sim O(1)$ and is positive for the “normal branch” and negative for the “self-accelerating branch” [5] in full analogy with the DGP model. Notice that $a > 1$ for $\alpha < \frac{1}{4}$. Using expansion (12) one arrives to

$$I(x_{\text{ta}} \ll 1) \approx 1 + \frac{c_1}{2} x_{\text{ta}} \left[ \frac{2}{\pi} \int_0^1 dx \frac{1 - x^{-a-1}}{\left( \frac{1}{2} - 1 \right)^{3/2}} \right]$$

where the function $I_1(\alpha)$ is shown on the Fig. 2.

Substituting the expression (13) back into equation (9) and using (7), we obtain

$$\left( \frac{\rho_0}{\rho_{\text{ta}}} \right)^{1/2} \left( 1 + \frac{c_1}{2} x_{\text{ta}} I_1(\alpha) \right) = 1 \tag{14}$$

As $x_{\text{ta}} \ll 1$ and $a > 1$, one finds that

$$\rho_{\text{ta}} \simeq \rho_0 \left( 1 + c_1 I_1(\alpha) x_{\text{ta}}^a (\rho_0) \right) \tag{15}$$

where to compute $x_{\text{ta}}$ we use Eq. (7) with $\rho_0$ instead of $\rho_{\text{ta}}$. From Eqs. (7) and (15) we see once again that for all $\alpha \neq \frac{1}{2}$ (i.e. $\beta \neq 2$) the turn-around density $\rho_{\text{ta}}$ loses its universality and becomes the function of the halo mass $M$. The turn-around radius $R_{\text{ta}}(M)$ is related to $\rho_{\text{ta}}$ via $R_{\text{ta}}(M) = (M/\rho_{\text{ta}})^{1/3}$.

Under the assumption of exact self-similarity, discussed above (i.e. $r_c/R_{\text{ta}} = \text{const}$) one arrives to the following expression for $S$ (recall that $\beta = 4(1 - \alpha)$):

$$S(M) = \rho_c r_c \propto M^{1/3} \rho_{\text{ta}}^{2/3} \tag{16}$$

$$\propto M^{1/3} \left( \frac{1}{3} + \frac{2}{3} c_1 I_1(\alpha) \left( \frac{M}{M_{\text{lim}}} \right)^{\frac{1-2\alpha}{3}} \right) \tag{17}$$

where

$$M_{\text{lim}} \equiv \frac{1}{G} \left[ \left( \frac{r_c}{2} \right)^3 \left( \frac{\pi}{l_0} \right)^{2(1+\beta)} \right]^{\frac{1}{1-\beta}} \tag{18}$$

An example of the relation (17) for several $\alpha$’s and $r_c$ is shown in Fig. 3.

Clearly, the most interesting case is when $r_V \approx R_{\text{ta}}$ for some range of observed halo masses. In this regime the deviations from Newtonian gravity become the strongest. The range of values $r_c$ for which this happens is shown in Fig. 4 (the value of $l_0$ is chosen to be the lifetime of the Universe $l_0 \simeq 1.3 \times 10^{10}$ years). We expect that for $r_c$ in the region Fig. 4 the slope of the relation $S \propto M^\alpha$ will change. Analysis of this case requires however an exact solution of the non-linear analog of the Poisson equation in theories with non-linear analog of the Poisson equation in theories with $\pi(x)$.

As $\alpha \rightarrow \frac{1}{2}$, for $\alpha < \frac{1}{2}$ and bigger than $r_V$ for $\alpha > \frac{1}{2}$.
The main purpose of this work was to identify a new observable that can be used to constrain the large-scale modifications of gravity. We see that the scaling properties of dark matter halos are sensitive to such modifications.

We demonstrated that the models with \( \alpha \neq \frac{1}{2} \) predict the deviation from a simple power-law scaling in the \( S(M) \) relation. Comparison of predictions of such models with the data, collected in [14] potentially allows to restrict the values of \( r_c \) from below for a given \( \alpha \). The improved data-processing and new observational data on DM distributions will allow to strengthen these bounds and make them quantitative.

In this work we have analyzed only the case when the turn-around sphere is well inside the Vainstein radius, \( r_V \gg R_{ta} \). To analyze a general case, a better theoretical understanding of the function \( \pi(r) \) (defined via Eq. (5)) is needed. Together with better quality of data this will allow to extend our analysis to a wider range of parameters.

In the case \( \alpha = \frac{1}{2} \) (the DGP model) the \( S(M) \) dependence remains featureless. In this case one has \( r_V \approx R_{ta} \) for all masses (for \( r_c \sim H_0^{-1} \)) and the deviations from the Newtonian gravity at turn-around radius will be strong. Therefore this model (in general, all models that have \( r_V \sim R_{ta} \) for halos of \( M \sim 10^{12} M_\odot \)) can be probed by studying the infall trajectories around the turn-around radius in the local group and nearby galaxies [29] using the available data and the data from forthcoming surveys of the Milky Way as well as GAIA mission.

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