Massive Primordial Black Holes in Hybrid Inflation

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It is well known that energy density fluctuations at the early Universe give rise primordial black holes (BH) formation \[1\]. Widespread opinion is that these BHs are small enough with masses somewhere in the range \(M_{BH} \sim 10^{-5} - 10^{20}\) gram depending on specific model. What could be said about BHs with masses in the range \(10^{20} - 10^{40}\) gram? There exist a few number of inflationary models that predict BH production at some conditions \[2\], \[3\].

Here I discuss necessary conditions for BH formation in the framework of hybrid inflation \[4\]. It will be shown that it is hard to avoid BH production at early stage of the Universe evolution described by this model. New fluctuating mechanism of BH formation elaborated in \[3\] proved to be significant one if a potential of the inflaton field possesses at least two minima as it is in the case of hybrid inflation.

Potential of hybrid inflation has the form

\[
V(\chi, \sigma) = \kappa^2 \left( M^2 - \chi^2/4 \right)^2 + \frac{\lambda^2}{4} \chi^2 \sigma^2 + \frac{1}{2} m^2 \sigma^2
\]

Inflation takes place during slow rolling along the valley \(\chi = 0, \sigma > \sigma_c\), see Figure.1. Black dot marks critical point

\[
\sigma_c = \sqrt{2\frac{\kappa}{\lambda}} M.
\]
When the field $\sigma$ passes this value, the motion along the line $\chi = 0, \sigma < \sigma_c$ becomes unstable and the field $\chi$ quickly moves to one of the minima $\chi_{\pm} = \pm 2M, \sigma = 0$.

The field motion is ruled by classical equations

$$\ddot{\sigma} + 3H\dot{\sigma} + \frac{1}{2}\lambda^2\chi^2 \sigma + m^2 \sigma = 0,$$

$$\ddot{\chi} + 3H\dot{\chi} - \kappa^2 \chi (M^2 - \chi^2/4) + \frac{1}{2}\lambda^2 \chi \sigma^2 = 0,$$

were $H$ is Hubble parameter at this period.

Slow rolling, which is necessary condition for inflation, means slow variation of the field $\sigma$ along the valley $\chi = 0$ what takes place while $\sigma > \sigma_c$. This condition looks like $m << H$ for simple estimation. Neglecting second derivative as usual, solution of Eq. (1) acquires the form

$$\sigma(t) = \sigma_{in} \exp \left( -\frac{m^2}{3Ht} \right), \quad m << H.$$

The value of $\sigma_{in}$ could be obtained if we admit that the period of inflation is about $N_UH^{-1}$ in terms of Hubble parameter $H$. The value $N_U \approx 60$ is chosen for estimations. Thus, from the condition $\sigma(N_UH^{-1}) = \sigma_c$ one obtains initial value of the field $\sigma$

$$\sigma_{in} = \sigma_c \exp \left( \frac{m^2}{3H^2 N_U} \right).$$

Serious problem arises when we take into account field fluctuations during inflation. Indeed, the classical motion along the line $\chi = 0$ describes an average motion. The fact is that the space is constantly divided into increasing number of causally disconnected domains. Each of them is characterized by a field value slightly different from neighbor ones. Evidently, vast majority of them contain field $\chi \neq 0$. Thus, at the end of inflation, i.e. when the field $\sigma$ reaches the critical value $\sigma_c$ huge amount (about $10^{78}$) domains has been produced. Half of them, those with $\chi < 0$,
are directed to the minimum $\chi^- = -2M$, while the others - to the minimum $\chi^+ = +2M$. After the inflation, we come to the Universe separated by chaotically distributed domains with field values $\chi^+$ or $\chi^-$ inside them. The neighboring domains are separated by a field wall because a motion from $\chi^+$ to $\chi^-$ is accompanied by crossing a space point with potential maximum at $(\chi = 0, \sigma = 0)$. Such a wall - dominated period is unacceptable [5], because it prevents proper evolution of the Universe. Consequently, motion along the value $\chi = 0$, as it is usually supposed, is excluded.

The only way for our Universe to evolve into modern state is to be created with initial field value $\chi_{in} \neq 0$ at the beginning of inflation. During inflation, the field $\chi$ must slowly approaches critical line $\chi = 0$ for not to run into the problem discussed above.
One of the conditions of slow motion for the field $\chi$ is

$$\eta \equiv \frac{V''_{\chi\chi}}{3H^2} << 1.$$ 

Estimation for this value can be easily performed for $\sigma \approx \sigma_{in}$, $\chi = 0$ and $H \approx \sqrt{8\pi/3}\kappa M/M_P$, $M_P$ is Plank mass. The result is

$$\eta \equiv \frac{V''_{\chi\chi}}{3H^2} \approx \frac{1}{2} \frac{\lambda^2 \sigma_{in}^2 - \chi^2 M^2}{3H^2} \approx \frac{\zeta^2 M^2}{3H^2} \left( \frac{2m^2 N_U}{e^{3m^2 N_U}} - 1 \right) \approx \frac{2N_U M_P^4 m^2}{(8\pi)^2 \chi^2 M^6},$$

what leads to inequality

$$\eta = \frac{6N_U M_P^4 m^2}{(8\pi)^2 \chi^2 M^6} << 1. \quad (2)$$

Combining it with the formula for temperature fluctuations [6], [7]

$$\left( \frac{16\pi}{45} \right)^{1/2} \frac{\lambda \chi^2 M^5}{M_P^3 m^2} \sim \frac{\delta T}{T} \approx 10^{-5}, \quad (3)$$

we obtain an estimation for the parameter $\lambda$

$$\lambda \approx \frac{(8\pi)^2}{6N_U} \sqrt{\frac{45}{16\pi} \frac{\delta T}{T} \eta M_P}. \quad (4)$$

Evidently its numerical value is rather small unless $M >> M_P$.

It is worth to note that the hybrid model [4] was invented just to avoid the problem with very small coupling constants. Meantime Eq. (4) indicates unambiguously that $\lambda << 1$ at reasonable values of parameters and hence this problem remains in the hybrid inflation model.

If average field value approaches too close to critical line $\chi = 0$, the fluctuations of the field in some space domains could cross this line. In future, these domains will be filled by vacuum, say, $\chi_-$ surrounded by a sea of another vacuum $\chi_+$. The two vacua are
separated by a closed wall as it was discussed above. A number of such a walls depends in initial conditions at the moment of our Universe creation.

Let us estimate energy and size of the closed walls. To proceed, suppose that the field in the volume in question crossed critical line at e-folds number $N$ before the end of inflation. Its size is about Hubble radius, $H^{-1}$ and it will be increased in $e^N$ times during inflation. Surface energy density of the domain wall after inflation is

$$\epsilon = \frac{8\sqrt{2}}{3} \kappa M^3.$$  \hspace{1cm} (5)

Thus the energy $E_{wall}$ contained in the wall after inflation is at least

$$E_{wall} \approx 4\pi \epsilon \left(H^{-1}e^N\right)^2 = 4\sqrt{2} \frac{M_P^2}{\kappa M} e^{2N}.$$  \hspace{1cm} (6)

Numerical value $N$ varies in the interval $(0 < N < N_U \approx 60)$. Gravitational radius of the wall could be easily calculated

$$r_g = 2E_{wall}/M_P^2 \approx \frac{8\sqrt{2}}{\kappa M} e^{2N},$$

what is much larger than the wall width $d = 2\sqrt{2}/(\kappa M)$ for any e-fold $N$. It means that this wall will collapse into BH with mass $M_{BH} \approx E_{wall}$ 3.

Let us estimate masses of such a BHs for ordinary values of the parameters $\kappa = 10^{-2}$ and $M = 10^{16}\text{GeV}$. If $N = 40$ we obtain the mass of BH

$$M_{BH} \approx 3 \cdot 10^{59}\text{GeV} \sim 100\text{ Solar mass}.$$  

The same estimation for a mass of smallest BHs which are created at the e-fold number $N = 1$ before the inflation is finished gives

$$M_{BH,small} \approx 10^6 M_P.$$

Thus, hybrid inflation leads to BH production in the wide range $10^{25} \div 10^{59}$ GeV. A number of the massive Bhs depends on how
close average field value approaches to critical line. It, in turn, depends on the initial conditions and specific values of parameters of the model. Average dispersion of the field $\chi$ is about

$$\langle \delta \chi \rangle \approx \frac{H}{2\pi} \sqrt{NU}$$

If the field $\chi$ approaches to this value during its classical motion, overproduction of black holes is inevitable. Initial value of the field $\chi$ must satisfy inequality

$$\chi_{in} \geq \frac{H}{2\pi} \sqrt{NU} = \sqrt{\frac{2NU}{3\pi}} \frac{M^2}{M_P}$$ \hspace{1cm} (7)

what is necessary condition to avoid too many black holes after the end of the inflation.

In conclusion, the mechanism of massive BH production revealed in $[3]$ works effectively in the hybrid model of inflation. It proves to be powerful tool for testing of inflationary models and determining a range of their parameters. Careful investigation of fluctuations in the framework of hybrid inflation indicates that coupling constant must be very small to fit observations.

The author is grateful to M.Yu. Khlopov and A.S. Sakharov for discussion.

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