Isocurvature forecast in the anthropic axion window

J. Hamann\textsuperscript{1}, S. Hannestad\textsuperscript{2}, G. G. Raffelt\textsuperscript{3} and Y. Y. Y. Wong\textsuperscript{4}

\textsuperscript{1} LAPTh, Universit\'e de Savoie, CNRS
BP 110, F-74941 Annecy-le-Vieux Cedex, France
\textsuperscript{2} Department of Physics and Astronomy
University of Aarhus, DK-8000 Aarhus C, Denmark
\textsuperscript{3} Max-Planck-Institut f"ur Physik (Werner-Heisenberg-Institut)
F"ohringer Ring 6, D-80805 M"unchen, Germany
\textsuperscript{4} Theory Division, Physics Department, CERN, CH-1211 Genève 23, Switzerland

E-mail: hamann@lapp.in2p3.fr, sth@phys.au.dk, raffelt@mppmu.mpg.de and yvonne.wong@cern.ch

Abstract. We explore the cosmological sensitivity to the amplitude of isocurvature fluctuations that would be caused by axions in the “anthropic window” where the axion decay constant $f_a \gg 10^{12}$ GeV and the initial misalignment angle $\Theta_i \ll 1$. In a minimal $\Lambda$CDM cosmology extended with subdominant scale-invariant isocurvature fluctuations, existing data constrain the isocurvature fraction to $\alpha < 0.09$ at 95\% C.L. If no signal shows up, Planck can improve this constraint to 0.042 while an ultimate CMB probe limited only by cosmic variance in both temperature and $E$-polarisation can reach 0.017, about a factor of five better than the current limit. In the parameter space of $f_a$ and $H_i$ (Hubble parameter during inflation) we identify a small region where axion detection remains within the reach of realistic cosmological probes.
1. Introduction

The Peccei-Quinn (PQ) mechanism and concomitant axion is arguably the most plausible explanation for the smallness of the Θ parameter of QCD [1, 2]. This case has become stronger in recent years because the experimental precision exploration of the quark mixing matrix reveals that the Kobayashi–Maskawa mechanism of explicit CP violation accounts perfectly well for all observations [3], so one can hardly argue that CP was a fundamental symmetry that is only spontaneously broken. Likewise, the possibility of a massless up-quark is now convincingly excluded [3], leaving few credible alternatives to the PQ mechanism. Perhaps the greatest advantage of the PQ mechanism is that it can be verified by the detection of axions.

The CP-violating term in the QCD Lagrangian is of the form $(\alpha_s/8\pi)\Theta G^\mu\nu \tilde{G}_{a\mu\nu}$, with $\alpha_s$ the strong fine structure constant and $G$ the colour field-strength tensor. With the identification $\Theta \rightarrow a/f_a$ the axion field $a$ inherits its defining interaction with the axial QCD anomaly. The axion decay constant $f_a$ is the primary parameter for axion physics. In particular, all couplings are inversely proportional to $f_a$ and the axion mass is determined by

$$m_a f_a = \frac{\sqrt{m_u m_d}}{m_u + m_d} m_\pi f_\pi \equiv \Lambda_a^2.$$  \hspace{1cm} (1.1)

Here $m_\pi = 135.5$ MeV is the pion mass, $f_\pi = 93$ MeV its decay constant, and $m_{u,d}$ the up- and down-quark masses with $Z = m_u/m_d = 0.35–0.6$ [3]. A series of overlapping experimental and astrophysical constraints suggests $f_a \gtrsim 10^9$ GeV [3, 4], so axions are very light and “invisible,” in any case far more elusive than neutrinos.

However, during the QCD epoch of the early universe, a non-thermal mechanism produces axions as nonrelativistic coherent field oscillations that can play the role of cold dark matter [5]. In terms of the initial “misalignment angle” $\Theta_i = a_i/f_a$ relative to the CP-conserving minimum of the axion potential, the cosmic axion density is [6]

$$\Omega_a h^2 \simeq 0.195 \Theta_i^2 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{1.184}.$$  \hspace{1cm} (1.2)

If $\Theta_i^2$ is of order unity, axions provide the dark matter of the universe if $f_a \sim 10^{12}$ GeV ($m_a \sim 10 \mu\text{eV}$). As such they can be detected by experiments that exploit the generic axion–two photon interaction, allowing for the coherent conversion into excitations of a high-quality microwave cavity placed in a strong $B$-field [2, 7]. If axions are in the mass range 1–100 $\mu\text{eV}$ and form the dark matter of our galaxy, the ADMX and New CARRACK experiments are poised to find them within a decade. Such a discovery would both reveal the nature of dark matter and verify the PQ mechanism for CP symmetry restoration.‡

Meanwhile one may study other “windows of opportunity” where axions could leave a detectable trace. Ongoing efforts include helioscope searches that are sensitive to solar

‡ In view of the fundamental importance of such a measurement it is perplexing how little worldwide effort is directed towards this goal compared with the vast range of WIMP dark matter searches.
Isocurvature forecast in the anthropic axion window

Axions if \( f_a \sim 10^7 \) GeV, corresponding to sub-eV masses \([8, 9]\). Of course, if solar axions were found something would have to be wrong with the supernova 1987A energy-loss limit that requires \( f_a \gtrsim 10^9 \) GeV. For \( f_a \lesssim 10^8 \) GeV axions thermalise in the early universe after the QCD epoch, so sub-eV mass axions would provide a hot dark matter component \([10–12]\). If future surveys were to reveal a hot dark matter fraction beyond the minimal neutrino component, an axion interpretation would be conceivable.

We consider here axions in another range beyond the classical cosmological window. In a scenario where the PQ symmetry is not restored during or after inflation, a single value \( -\pi < \Theta_i < +\pi \) determines the axion density. It is possible that \( \Theta_i \ll 1 \), allowing for \( f_a \gg 10^{12} \) GeV. This case is motivated because the PQ mechanism presumably is embedded in a greater framework. In particular, the PQ symmetry emerges naturally in many string scenarios, but then it is hard to obtain \( f_a \) much below \( 10^{16} \) GeV \([13]\). The axion is the Nambu–Goldstone boson of the global \( U(1)_{\text{PQ}} \) symmetry that is broken at a scale \( v_{\text{PQ}} = N f_a \) where \( N \) is an integer. (Both \( a/v_{\text{PQ}} \) and \( a/f_a \) must be \( 2\pi \)-periodic because \( a/v_{\text{PQ}} \) is the phase of a new Higgs field responsible for spontaneous symmetry breaking and the QCD vacuum is periodic in \( \Theta = a/f_a \).) Since \( -\pi < a/v_{\text{PQ}} < +\pi \) encounters \( N \) minima of the axion potential, there is a formidable cosmological domain-wall problem unless \( N = 1 \) or inflation occurs after PQ symmetry breaking, further motivating a “late-inflation scenario” \([5]\).

Postulating \( \Theta_i \ll 1 \) may look like shuffling the QCD problem of a small \( \Theta \) parameter into an unnatural initial condition for our universe. However, a GUT-scale value of \( f_a \sim 10^{16} \) GeV would only require a modest fine-tuning of \( \Theta_i \lesssim 3 \times 10^{-3} \) \([14]\). More importantly, anthropic selection suggests that the cosmic baryon/dark matter ratio can not be too small, so for any \( f_a \) the most probable value for \( \Theta_i \) is near the one required for the observed dark-matter density \([15, 16]\).

Following these papers we emphasise that in our late-inflation scenario \( \Theta_i \), and therefore the axion dark matter density, is an environmental parameter of our universe, not a fundamental one, and that its prior probability distribution is known to be flat on the interval \( -\pi < \Theta_i < +\pi \). This contrasts with typical other cases of anthropic reasoning where usually it is not known if a given cosmological or particle-physics parameter is fundamental or environmental. In the axion case anthropic reasoning is unavoidable to quantify if a given value of \( \Theta_i \) is natural or not.

Therefore, the “anthropic axion window” with \( f_a \gg 10^{12} \) GeV is well motivated. In this case \( m_a \ll 10 \text{ \mu eV} \) and axion dark matter is difficult to detect with the cavity technique because of the large required cavity size and magnetic field region.\(^\S\) An alternative signature is provided by primordial isocurvature fluctuations that can show up in future data. In our late-inflation scenario, the massless axion field is present during inflation and thus acquires the usual amplitude fluctuations imprinted by the de Sitter expansion. When axions acquire a mass during the QCD epoch, these fluctuations become dynamically relevant in the form of isocurvature fluctuations that

\(^\S\) Ideas for overcoming this limitation were discussed by S. Thomas in a talk given at the workshop “Axions at the Institute for Advanced Study” (20–22 Oct 2006, IAS, Princeton, New Jersey).
are uncorrelated with the adiabatic fluctuations inherited by all other matter and radiation from the inflaton field. The isocurvature amplitude depends on both $f_a$ and $H_I$, the Hubble parameter during inflation, so observational limits on isocurvature fluctuations exclude certain regions in this parameter space [17–22].

Since there is no trace of isocurvature fluctuations in existing data, perhaps a more interesting question is the remaining window for axions to show up in future. One forecast was recently provided by the CMBPol Study Team Collaboration [23]. The purpose of our paper is to provide a more detailed sensitivity forecast on the amplitude of isocurvature fluctuations. We explore possible degeneracies that may exist between the isocurvature fluctuations and other cosmological parameters in both present and future data, and determine the region in the underlying axion parameter space that is realistically within the reach of future cosmological probes.

To this end we review in Sec. 2 the axion dark matter and isocurvature contributions in terms of underlying physical parameters. In Sec. 3 we derive our constraints and sensitivity forecasts on the axion-type isocurvature fraction. We interpret these results in terms of underlying physical parameters in Sec. 4 and conclude in Sec. 5.

2. Cosmological imprint of axions

2.1. Energy density

To specify the cosmological imprint of axions we begin with their contribution to the matter density of the present-day universe. When the PQ symmetry breaks at some large temperature $T \sim v_{PQ}$, the relevant Higgs field will settle in a minimum corresponding to $\Theta_i = a_i/f_a$, where $-\pi \leq \Theta_i \leq +\pi$. We assume that this happens before cosmic inflation, so throughout our observable universe we have the same initial condition except for fluctuations imprinted by inflation itself. When the universe cools and evolves towards the QCD epoch, instanton effects become important and produce a potential for the axion field. The corresponding temperature-dependent axion mass is [6]

$$m_a(T) \simeq \begin{cases} \beta_{\text{inst}}^{1/2} \left( \frac{T}{\text{GeV}} \right)^{-n/2} / f_a & \text{for } T \gtrsim \Lambda_{QCD}, \\ \Lambda_a^2 / f_a & \text{for } T \ll \Lambda_{QCD}, \end{cases}$$

(2.1)

where $\beta_{\text{inst}} \simeq 3.964 \text{ MeV}^4$, $n \simeq 6.878$, $\Lambda_{QCD} \simeq 380 \text{ MeV}$, and $\Lambda_a \simeq 77 \text{ MeV}$, assuming $Z = 0.5$.

The axion field begins oscillating at $3H(T_{osc}) \simeq m_a(T_{osc})$. The Hubble parameter during radiation domination is

$$H^2(T) = \frac{8\pi}{3M_{Pl}^2} \frac{\pi^2}{30} g_*(T) T^4,$$

(2.2)

where $M_{Pl} = 1.221 \times 10^{19} \text{ GeV}$ is the Planck mass. For $T \gtrsim \Lambda_{QCD}$ the effective number of thermally excited degrees of freedom is $g_*(T_{osc}) \simeq 61.75$, whereas $g_*(T_{osc}) \simeq 10.75$ for
\[ T \ll \Lambda_{\text{QCD}}. \] Together equations (2.1) and (2.2) give

\[
T_{\text{osc}} \simeq \begin{cases} 
9.16 \times 10^2 \text{ MeV} \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{-0.184} & \text{for } T_{\text{osc}} \gtrsim \Lambda_{\text{QCD}}, \\
6.65 \times 10^4 \text{ MeV} \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{-0.5} & \text{for } T_{\text{osc}} \ll \Lambda_{\text{QCD}}.
\end{cases}
\] (2.3)

We see that the limit \( T_{\text{osc}} \ll \Lambda_{\text{QCD}} \) is only relevant for \( f_a \gg 3 \times 10^{16} \text{ GeV} \). We will usually focus on \( f_a \) values around or below \( 10^{16} \text{ GeV} \) and therefore concentrate on the case where the axion field begins oscillating during the QCD epoch.

The traditional calculation of the present-day axion density [5] was recently revisited in detail, taking into account modern values for the relevant QCD parameters and anharmonic corrections for the axion potential [6]. However, since we are interested in the anthropic axion window where the initial misalignment angle is small, anharmonic effects are not important. We also ignore the possibility of axion dilution by late entropy production, so

\[
\omega_a \equiv \Omega_a h^2 \simeq 0.195 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{1.184} (\Theta_i^2 + \sigma_{\Theta}^2).
\] (2.4)

Here, the initial misalignment angle \( \Theta_i \) is to be interpreted as the average in our Hubble volume, \( \Theta_i = \langle \Theta \rangle_i \), while

\[
\sigma_{\Theta}^2 = \frac{H_I^2}{4\pi^2 f_a^2}
\] (2.5)

is the inflation-induced variance, with \( H_I \) the Hubble parameter during inflation. In other words, the relevant initial parameter is \( \langle \Theta^2 \rangle_i = \Theta_i^2 + \sigma_{\Theta}^2 \).

We will usually assume that all of the cold dark matter consists of axions, so according to current cosmological data \( \omega_a = \omega_c \simeq 0.11 \) [20]. Assuming \( \sigma_{\Theta} \) is small, we then find

\[
\Theta_i \simeq 0.75 \left( \frac{10^{12} \text{ GeV}}{f_a} \right)^{0.592}
\] (2.6)

as a unique relationship between the initial misalignment angle and the axion decay constant.

### 2.2. Isocurvature fraction

Since the axion is essentially massless during inflation, quantum fluctuations are imprinted on the axion field in the same manner that the inflaton field acquires its perturbations. Because of the axion’s negligible contribution to the total energy budget of the universe during inflation, these fluctuations do not perturb the total energy density and manifest themselves only as perturbations in the ratio of axion number density to entropy density, i.e., \( \delta \rho = 0 \), and \( \delta (n_a/s) \neq 0 \). For this reason they are known as entropy or isocurvature perturbations. As the axion field provides some or all of the cold dark matter of the universe, these perturbations contribute to the temperature and polarisation fluctuations in the CMB.
Axion-induced isocurvature fluctuations are uncorrelated with the adiabatic fluctuations inherited by other matter and radiation components from the inflaton. The spectrum of entropy perturbations is
\[ S(k) = \theta^2 - \langle \theta^2 \rangle, \]
(2.7)
Assuming \( \delta \theta \equiv \theta - \langle \theta \rangle \) has a Gaussian distribution, the perturbation power spectrum can be written as
\[ \langle |S(k)|^2 \rangle = \frac{2 \sigma^2_\theta (2 \theta_1^2 + \sigma^2_\theta)}{(\theta_1^2 + \sigma^2_\theta)^2}, \]
(2.8)
where \( \theta_1 \) and \( \sigma^2_\theta \) are the mean and the variance of the initial \( \theta \) distribution. As we shall see later, for all of the viable parameter space, \( \theta_1^2 \gg \sigma^2_\theta \), thus
\[ \langle |S(k)|^2 \rangle \approx \frac{H^2_f}{\pi^2 f_a^2 \theta_1^2} \left( \frac{k}{k_0} \right)^{n_{\text{iso}}-1}, \]
(2.9)
In the second equality we have approximated the power spectrum by a power law with spectral index \( n_{\text{iso}} = 1 - 2 \epsilon \approx 1 \), where \( \epsilon \) is the first inflationary slow-roll parameter and \( k_0 = 0.002 \text{ Mpc}^{-1} \) the pivot scale. Likewise, adiabatic perturbations sourced by the inflaton field are encoded in the curvature power spectrum,
\[ \langle |R(k)|^2 \rangle = \frac{H^2_f}{\pi M^2_P \epsilon} \propto \left( \frac{k}{k_0} \right)^{n_{\text{ad}}-1}, \]
(2.10)
where \( n_{\text{ad}} \) is the adiabatic spectral index.

Because fluctuations of the axion field are uncorrelated with those of the inflaton field, the primordial scalar power spectrum is simply the incoherent sum of the adiabatic and isocurvature perturbation spectra,
\[ P(k) = \langle |R(k)|^2 \rangle + \langle |S(k)|^2 \rangle. \]
(2.11)
The resulting CMB temperature and polarisation anisotropy spectra are given by a similar sum, but with the appropriate transfer functions \( T_X(k) \) folded in \( (X = T, E) \),
\[ C_{XY}(\ell) = C_{XY}^{\text{ad}}(\ell) + C_{XY}^{\text{iso}}(\ell) \]
\[ \propto \int \frac{dk}{k} T^{\text{ad}}_X(k) T^{\text{ad}}_Y(k) \langle |R(k)|^2 \rangle + \int \frac{dk}{k} T^{\text{iso}}_X(k) T^{\text{iso}}_Y(k) \langle |S(k)|^2 \rangle. \]
(2.12)
Here it is useful to define the isocurvature fraction \( \alpha \),
\[ \alpha \equiv \frac{\langle |S(k)|^2 \rangle}{\langle |R(k)|^2 \rangle + \langle |S(k)|^2 \rangle} \bigg|_{k=k_0} \approx \begin{cases} \frac{H^2_f}{A_{\text{S}} \pi^2 f_a^2 \theta_1^2} & \text{for } \theta_1^2 \gg \sigma^2_\theta, \\ \frac{2}{A_{\text{S}}} & \text{for } \theta_1^2 \ll \sigma^2_\theta, \end{cases} \]
(2.13)
where \( A_{\text{S}} = P(k = k_0) \) is the amplitude of the total primordial scalar power spectrum at the pivot scale \( k_0 \), and we have assumed \( \alpha \ll 1 \). Given \( A_{\text{S}} \simeq 10^{-9} \), the solution \( \theta_1^2 \ll \sigma^2_\theta \) cannot be realised. In the opposite limit \( \theta_1^2 \gg \sigma^2_\theta \) we find
\[ \alpha \simeq 7.5 \times 10^{-3} \left( \frac{2.4 \times 10^{-9}}{A_{\text{S}}} \right) \left( \frac{0.109}{\omega_c} \right) \left( \frac{H_f}{10^7 \text{ GeV}} \right)^2 \left( \frac{10^{12} \text{ GeV}}{f_a} \right)^{0.816}, \]
(2.14)
where we have used equation (2.4).
2.3. Gravity waves

The misalignment mechanism per se does not produce gravity waves. However, all inflationary scenarios produce some amount of tensor perturbations, which is in principle manifest in the CMB temperature and polarisation anisotropies. The quantity of interest is the tensor-to-scalar ratio $r = A_T/A_S$, defined as the ratio of the primordial tensor to the scalar perturbation spectrum at $k = k_0$. The amplitude of tensor perturbations from inflation is $A_T = 16H_I^2/(\pi M_{Pl}^2)$. Thus, the Hubble expansion rate during inflation

$$H_I = \frac{1}{4}\sqrt{\frac{\pi A_S r}{M_{Pl}}}$$

$$\simeq 1.33 \times 10^{14} \text{ GeV} \left(\frac{A_S}{2.4 \times 10^{-9}}\right)^{1/2} \left(\frac{r}{0.25}\right)^{1/2}$$

is directly probed by a measurement of $r$.

3. Present constraints and future sensitivities

3.1. Observational signatures of axion-type isocurvature perturbations

To discuss the observational imprint in the CMB we recall that the axion field causes scale-invariant CDM isocurvature fluctuations that are uncorrelated with the dominant inflaton-induced adiabatic fluctuations. Compared to adiabatic initial conditions, the growth of isocurvature CDM density perturbations with wavenumber $k$ is suppressed by a factor of $k^2$ on sub-horizon scales [24]. Hence, if we are looking for traces of a subdominant isocurvature perturbation with a similar primordial scale dependence as the dominant adiabatic mode, the signal will be strongest at large scales. We illustrate this point in Fig. 1, where we plot the CMB angular power spectra for a model with purely adiabatic perturbations ($\alpha = 0$) and $n_{\text{ad}} = 0.96$, and for a corresponding model with purely CDM isocurvature perturbations ($\alpha = 1$) with $n_{\text{iso}} = 1$. For uncorrelated isocurvature perturbations, cases with $0 < \alpha < 1$ simply correspond to weighted sums of these two spectra, so even for largish values of $\alpha$, the spectrum at multipoles $\ell \gtrsim 200$ will be completely dominated by the adiabatic contribution.

It is also useful to contrast the isocurvature signal with the cosmic variance, which poses a fundamental limit to the amount of information to be extracted from CMB anisotropies. In Fig. 2 we compare the contribution $\Delta C_\ell$ of a subdominant isocurvature part to the total $C_\ell$ with the intrinsic uncertainty due to cosmic variance, $\Delta_{\text{CV}}^{XY} = \sqrt{C_\ell^{XX}C_\ell^{YY}} \sqrt{\frac{2}{2\ell+1}}$, with $X, Y \in [T, E]$. This shows that data of small-scale perturbations by themselves carry hardly any information about axion-type isocurvature.

Based on existing measurements of the CMB anisotropies, the main signature of scale-independent uncorrelated CDM isocurvature, i.e., an enhancement of the Sachs–Wolfe plateau, can be mimicked to a certain extent by changing the values of other cosmological parameters. In particular, one can expect $\alpha$ to be degenerate with the total matter density $\Omega_m$ and the adiabatic spectral index $n_{\text{ad}}$. Adding data of the large-scale matter power spectrum and of distance probes such as supernovae will however
**Figure 1.** CMB angular power spectra for purely adiabatic initial conditions ($\alpha = 0$, dark red lines) with $n_{\text{ad}} = 0.96$ and for purely CDM isocurvature initial conditions ($\alpha = 1$, light red lines) with a scale invariant spectrum $n_{\text{iso}} = 1$. All other cosmological parameters are kept fixed.

**Figure 2.** Scale dependence of the “signal to noise” for isocurvature fractions of $\alpha = 0.1$ (blue lines), and $\alpha = 0.001$ (pale blue lines). Plotted is the distortion $\Delta C_{\ell}$ of the CMB angular power spectra caused by adding a subdominant scale-invariant uncorrelated CDM isocurvature component, given in units of the cosmic variance $\Delta CV$. 
allow us to break these degeneracies and thus improve constraints on $\alpha$ indirectly.

3.2. Constraints and forecast

For inferring constraints from present data and predicting the discovery potential of future CMB experiments, we consider a cosmological model with seven free parameters: the isocurvature fraction $\alpha$ plus the usual six parameters of the “vanilla” $\Lambda$CDM model. These parameters and their benchmark values to generate mock data for our forecasts are the baryon density $\omega_b = 0.0226$, the dark matter density $\omega_c = 0.109$, the Hubble parameter $h = 0.715$, the redshift to reionisation $z_{\text{re}} = 11.36$, the scalar spectral index $n_{\text{sd}} = 0.962$, the normalisation $\ln(10^{10} A_s) = 3.195$, and the isocurvature fraction $\alpha = 0$. The spectral index of the primordial isocurvature power spectrum is fixed to $n_{\text{iso}} = 1$. For the other parameters we adopt flat priors in the data analysis.

We do not vary the tensor-to-scalar ratio $r$, since in the anthropic parameter region any possibly observable amount of tensor perturbations would imply a value of $H_I$ high enough to make the isocurvature fraction close to unity, which is clearly inconsistent with observations [30].

Parameters are estimated using standard Markov Chain Monte Carlo techniques, as implemented in the publicly available package CosmoMC [31]. As input we use four different data sets:

1. WMAP: the five-year WMAP data [25].
2. WMAP+SDSS-LRG+SN: WMAP plus the galaxy power spectrum from the luminous red galaxy subsample of the Sloan Digital Sky Survey [26] and the luminosity distances of supernovae from the Union compilation [27].
3. Planck: Simulated $TT$, $TE$ and $EE$ spectra up to $\ell = 2000$ from the Planck satellite [28], assuming isotropic white noise, a sky coverage of 70%, and 14 months of observation in the 70, 100, and 143 GHz channels.
4. CVL: Simulated, noiseless (i.e., cosmic variance limited) $TT$, $TE$ and $EE$ spectra up to $\ell = 2000$.

The mock CMB data sets 3 and 4 are generated using the method discussed in Ref. [29]. The likelihood function $L$ is defined as

$$\chi^2_{\text{eff}} \equiv -2 \ln L = \sum_{\ell=2}^{\ell_{\text{max}}} (2\ell + 1) f_{\text{sky}} \left[ \text{Tr} \left( \hat{C}_\ell^{-1} \tilde{C}_\ell \right) + \ln \left| \frac{\tilde{C}_\ell}{\hat{C}_\ell} \right| - n \right], \quad (3.1)$$

where $\hat{C}_\ell \doteq \hat{C}^{XY}_\ell$ with $X, Y \in [T, E]$ denotes the mock data covariance matrix, $\tilde{C}_\ell \doteq C^{XY}_\ell + N^{XY}_\ell$ is the total covariance matrix comprising theoretical predictions of the CMB anisotropy spectrum $C^{XY}_\ell$ and the noise power spectrum $N^{XY}_\ell$. The quantity $n$ counts the number of observable modes, where $n = 2$ for observations in temperature and $E$-type polarisation.

Using data set 2, we find 95% credible intervals for the observables discussed in Sec. 2,

$$0.1023 < \omega_c < 0.1165, \quad (3.2)$$
Isocurvature forecast in the anthropic axion window

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3}
\caption{Degeneracies of the isocurvature fraction $\alpha$ with the other parameters of the standard ΛCDM-model for different data sets. Plotted are the marginalised two-dimensional 95% credible contours for WMAP data only (pale red), WMAP+SDSS-LRG+SN (red), and for simulated data from Planck (thin pale blue dotted) and a cosmic variance limited experiment (thin blue dotted lines).}
\end{figure}

\begin{align}
3.105 &< \ln[10^{10}A_S] < 3.260, \\
\alpha &< 0.09,
\end{align}

consistent with the findings of Komatsu et al. [20]. As noted by these authors, adding large-scale structure and supernova data leads to a significantly tighter bound on $\alpha$ because these data sets contain additional information on $\Omega_m$, and thus help alleviate the degeneracy problem somewhat. This is illustrated in Fig. 3, where we show the two-dimensional joint constraints on $\alpha$ with the other parameters of the vanilla model for data sets 1–4.

For the future experiments and if no isocurvature signal shows up, we forecast 95%-credible upper limits of

\begin{align}
\alpha &< 0.042 \quad \text{(Planck)}, \\
\alpha &< 0.017 \quad \text{(CVL)},
\end{align}

in rough agreement with the CMBpol White Paper [23]. We emphasise that upcoming

\| The noise properties of the CMB probes considered in Ref. [23] are roughly equivalent to our CVL experiment. Some other subtle differences include a slightly larger sky coverage of 80% (which leads to a general $\sqrt{0.8/0.7 - 1 \sim 7\%}$ improvement in the parameter sensitivities compared with numbers derived from a 70% sky coverage) and the inclusion of $B$-polarisation (which has no bearing on isocurvature detection). A major difference between Ref. [23] and our analysis is the forecast method: we have analysed mock future data, while the forecasts of Ref. [23] are based on the Fisher matrix which assumes Gaussian posterior distributions.
CMB experiments will break essentially all remaining degeneracies without the need for additional data. The improvement one can expect from Planck is partly due to this effect, but the improved sensitivity to $E$-polarisation will also play a role (Planck $E$-polarisation data will essentially be cosmic variance limited for $\ell \lesssim \mathcal{O}(10)$). Going from Planck to CVL will increase the sensitivity to $\alpha$ by another factor of 2.5, mostly because CVL covers the “sweet spot” in the $E$-polarisation signal-to-noise, where the first acoustic peak of the isocurvature signal coincides with the first acoustic trough of the adiabatic signal, around $\ell = 200$ (see Fig. 2). Because the isocurvature signal affects predominantly the largest scales, any significant further increase in the sensitivity to $\alpha$ from CMB observations beyond the CVL benchmark value is most likely unrealistic.

4. Axion parameter space

It is now easy to translate our constraints and sensitivity forecasts on the isocurvature fraction $\alpha$ into axion parameters by virtue of equation (2.14). It can be written as

$$H_I = 3.5 \times 10^7 \text{ GeV} \left(\frac{\alpha}{0.09}\right)^{1/2} \left(\frac{\omega_c}{0.109}\right)^{1/2} \left(\frac{f_a}{10^{12} \text{ GeV}}\right)^0.408,$$

where we have used our present-day $\alpha$-constraint from equation (3.2) as a benchmark value. Assuming axions are the dark matter, we show this constraint in Fig. 4 with a line marked “Current data.”

In this plot, which is inspired by a similar figure in Ref. [19], we also show the relationship between $f_a$ and $\Theta_i$ as dashed lines marked with the relevant value of $\Theta_i$. We have also marked the region where the PQ scale is smaller than $H_I$ and where therefore our “late inflation scenario” would not apply. Also displayed is the region of $H_I$ values excluded by excessive tensor modes, corresponding to $r > 0.25$. Future sensitivities to $\alpha$ from Planck and CVL based on equation (3.5) are shown labelled “Planck” and “CVL” respectively.

5. Conclusions

We have studied current cosmological limits and future sensitivities for axion-type isocurvature fluctuations. Our present-day limit on the isocurvature fraction $\alpha < 0.09$ (95% C.L.) agrees with similar recent results by other authors. We have used a $\Lambda$CDM cosmological model with six standard parameters and in addition a scale-invariant isocurvature fluctuation spectrum that is uncorrelated with the dominant adiabatic fluctuations.

The main effect of axion isocurvature fluctuations is to modify the Sachs–Wolfe plateau at small CMB multipole orders. However, breaking degeneracies with other parameters, future CMB probes can significantly improve the sensitivity to $\alpha$ and thus to $f_a$. This phenomenological limitation can be circumvented in a class of extended axion models that lead to a significantly blue-tilted spectrum [32], affecting also CMB anisotropies at larger multipoles.
the presence of axions in the universe. A non-detection of isocurvature modes by Planck would improve the limit to $\alpha < 0.042$, while an ultimate CMB probe limited only by cosmic variance in both temperature and $E$-polarisation up to $\ell = 2000$ could reach $\alpha < 0.017$, about a factor of five more restrictive than current limits. These forecasts agree roughly with previous estimates, notably by the CMBPol Collaboration [23]. The small differences are probably due to their using Fisher-matrix techniques in contrast to our full-fledged analysis of mock data sets.

We emphasise that the CVL sensitivity to $\alpha$ is essentially “as good as it gets”, since all relevant degeneracies have been lifted and no other cosmological probe besides the CMB is directly sensitive to isocurvature fluctuations. In terms of the axion decay constant $f_a$, the cosmic-variance limited probe shifts the sensitivity region by approximately a factor of five for some fixed Hubble parameter during inflation $H_I$, and thus opens a sliver of parameter space where axions can still leave a detectable imprint in the microwave sky. If a tentative signal were found, an experimental search for axion dark matter in this parameter range would be crucial, yet technologically challenging.

The idea of dark-matter axions with $f_a$ at the GUT scale or larger has received relatively little attention, at least from the perspective of experimental searches, because
the initial misalignment angle would have to be much smaller than unity. However, in our late-inflation scenario the axion dark matter density is fixed by the random number $-\pi < \Theta_i < +\pi$ with a flat prior distribution. In this case anthropic selection is practically unavoidable and implies that the required small $\Theta_i$ value is not unnatural. Therefore, as stressed by previous authors, the anthropic axion window is a plausible parameter range. In the absence of direct experimental searches in this window, the appearance of isocurvature fluctuations is the only opportunity for axions to become visible in this scenario. However, in view of the fundamental limitations posed by cosmic variance on all cosmological probes, a large chunk of parameter space in $f_a$ and $H_I$ will likely remain unreachable by this method.

Acknowledgements

We acknowledge use of computing resources from the Danish Center for Scientific Computing (DCSC). In Munich, partial support by the Deutsche Forschungsgemeinschaft under the grant TR 27 “Neutrinos and beyond” and the Cluster of Excellence “Origin and Structure of the Universe” is acknowledged.

References

[1] R. D. Peccei, “The strong CP problem and axions,” Lect. Notes Phys. 741 (2008) 3 [hep-ph/0607268].
[2] J. E. Kim and G. Carosi, “Axions and the strong CP problem,” Rev. Mod. Phys., to be published [arXiv:0807.3125].
[3] C. Amsler et al. [Particle Data Group], “Review of particle physics,” Phys. Lett. B 667 (2008) 1.
[4] G. G. Raffelt, Lect. Notes Phys. 741 (2008) 51 [hep-ph/0611350].
[5] P. Sikivie, “Axion cosmology,” Lect. Notes Phys. 741 (2008) 19 [astro-ph/0610440].
[6] K. J. Bae, J. H. Huh and J. E. Kim, “Update of axion CDM energy,” JCAP 0809 (2008) 005 [arXiv:0806.0497].
[7] S. J. Asztalos, L. J. Rosenberg, K. van Bibber, P. Sikivie and K. Zioutas, “Searches for astrophysical and cosmological axions,” Ann. Rev. Nucl. Part. Sci. 56 (2006) 293.
[8] Y. Inoue et al., “Search for solar axions with mass around 1 eV using coherent conversion of axions into photons,” Phys. Lett. B 668 (2008) 93 [arXiv:0806.2230].
[9] E. Arik et al. [CAST Collaboration], “Probing eV-scale axions with CAST,” JCAP 0902 (2009) 008 [arXiv:0810.4482].
[10] S. Hannestad, A. Mirizzi and G. Raffelt, “New cosmological mass limit on thermal relic axions,” JCAP 0507 (2005) 002 [hep-ph/0504059].
[11] A. Melchiorri, O. Mena and A. Slosar, “An improved cosmological bound on the thermal axion mass,” Phys. Rev. D 76 (2007) 041303 [arXiv:0705.2695].
[12] S. Hannestad, A. Mirizzi, G. G. Raffelt and Y. Y. Y. Wong, “Cosmological constraints on neutrino plus axion hot dark matter: Update after WMAP-5,” JCAP 0804 (2008) 019 [arXiv:0803.1585].
[13] P. Svrcek and E. Witten, “Axions in string theory,” JHEP 0606 (2006) 051 [hep-th/0605206].
[14] So-Young Pi, “Inflation without tears,” Phys. Rev. Lett. 52 (1984) 1725.
[15] A. D. Linde, “Inflation and axion cosmology,” Phys. Lett. B 201 (1988) 437.
[16] M. Tegmark, A. Aguirre, M. Rees and F. Wilczek, “Dimensionless constants, cosmology and other dark matters,” Phys. Rev. D 73 (2006) 023505 [astro-ph/0511774].
[17] D. H. Lyth, “A limit on the inflationary energy density from axion isocurvature fluctuations,” Phys. Lett. B 236 (1990) 408.
[18] M. Beltrán, J. García-Bellido and J. Lesgourgues, “Isocurvature bounds on axions revisited,” Phys. Rev. D 75 (2007) 103507 [arXiv:hep-ph/0606107].
[19] M. P. Hertzberg, M. Tegmark and F. Wilczek, “Axion cosmology and the energy scale of inflation,” Phys. Rev. D 78 (2008) 083507 [arXiv:0807.1726].
[20] E. Komatsu et al. [WMAP Collaboration], “Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Cosmological interpretation,” Astrophys. J. Suppl. 180 (2009) 330 [arXiv:0803.0547].
[21] D. B. Kaplan and A. E. Nelson, arXiv:0809.1206 [astro-ph].
[22] L. Visinelli and P. Gondolo, “Dark matter axions revisited,” arXiv:0903.4377 [astro-ph.CO].
[23] D. Baumann et al. [CMBPol Study Team Collaboration], “CMBPol mission concept study: Probing inflation with CMB polarization,” arXiv:0811.3919 [astro-ph].
[24] M. Bucher, K. Moodley and N. Turok, Phys. Rev. D 62 (2000) 083508 [astro-ph/9904231].
[25] M. R. Nolta et al. [WMAP Collaboration], “Five-Year Wilkinson Microwave Anisotropy Probe (WMAP) observations: Angular power spectra,” Astrophys. J. Suppl. 180 (2009) 296 [arXiv:0803.0593].
[26] M. Tegmark et al. [SDSS Collaboration], “Cosmological constraints from the SDSS luminous red galaxies,” Phys. Rev. D 74 (2006) 123507 [astro-ph/0608632].
[27] M. Kowalski et al., “Improved cosmological constraints from new, old and combined supernova datasets,” Astrophys. J. 686 (2008) 749 [arXiv:0804.4142].
[28] M. Bersanelli et al., eds., [Planck Science Team], “Planck: The scientific programme,” arXiv:astroph/0604069.
[29] L. Perotto, J. Lesgourgues, S. Hannestad, H. Tu and Y. Y. Y. Wong, “Probing cosmological parameters with the CMB: Forecasts from full Monte Carlo simulations,” JCAP 0610 (2006) 013 [astro-ph/0606227].
[30] P. Fox, A. Pierce and S. D. Thomas, arXiv:hep-th/0409059.
[31] A. Lewis and S. Bridle, “Cosmological parameters from CMB and other data: A Monte-Carlo approach,” Phys. Rev. D 66 (2002) 103511 [astro-ph/0205436].
[32] S. Kasuya and M. Kawasaki, arXiv:0904.3800 [astro-ph.CO].