Magnetic measurement of mechanical stress in iron-based materials using magnetoimpedance

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Abstract

We present here a non-invasive method to measure variations of internal stress in magnetic steel elements. The method consists in the measurement of the magnetic permeability on the specimen and relays on the dependence of its imaginary component with applied load. Our method does not measure absolute values and hence, results very suitable for measurements in situ, where small changes of positions or vibrations can change those absolute values.

1. Introduction

Determination of internal stress in structural materials is critical to anticipate and prevent failure in service. Therefore, non-invasive methods to monitor internal stress in these materials are required. Iron is present in many structural materials, being steel the most used one. For these iron-based materials, the ferromagnetic properties of iron can provide a path to measure externally the stress of the piece. Magnetoelastic effects induce a coupling between mechanical deformation and magnetic properties of the materials [1]. Hence, any change in the mechanical strain of the piece would induce a change of magnetic properties that can be measured to detect mechanical effects. There are different methods developed for the magnetic measurement of internal stress [2,3,4,5].

In a recent paper[6], we proposed a new method to measure mechanical strain using static magnetic fields based on the measurement of the magnetostatic field created by
the piece biased by an external magnet. The main limitation for the application of the method in field applications is that small displacements or vibrations changing the distances induce errors in the measurements. A path to overcome this limitation could be the use of AC fields and instead of measuring the modulus, to determine rates or phase differences between quantities that are not so affected by small displacements.

In this paper, we analyze the possibility to detect mechanical stress in iron based materials, measuring the delay in the magnetic induction of the material.

Here we propose a new method using AC fields that exhibit important advantages with respect to existing ones in terms of sensitivity, robustness and simplicity.

2. Theory

Consider a magnetic cylinder with Radius \( R \) and relative permeability, \( \mu_r \), with a primary and a secondary coils. Upon application of AC electrical current on the primary coil, a field \( H_0 \) will appear in the cylinder and the magnetization will fluctuate with the same frequency that the electrical current. For low frequencies, fluctuation of magnetization will be in phase with the current, but not for high frequencies for which a delay will appear.

The reason is that, in this last case, the effective field acting on each surface with radius \( r<R \) is not only the applied field \( H_0 \) but also that created by eddy currents that follows Ohms law, \( j = \sigma E \), where the conductivity for metals is \( \sigma \sim 10^7 \Omega m^{-1} \).

The electric field, \( E \), responsible of the currents is generated by the time variation of B fields according to:

\[
\nabla \times E = -\frac{\partial B}{\partial t} = -\mu_0 \mu_r \frac{\partial H}{\partial t}.
\]

The H field created by these currents is determined by \( \nabla \times H = j \). From these two equations, taking rotational in both terms of the first one, we obtain

\[
\nabla^2 H = \mu_0 \mu_r \sigma \frac{\partial H}{\partial t} \quad (1)
\]

Now if we assume a sinusoidal profile of \( H \) as \( H(r, t) = H(r)e^{i\omega t} \) and writing equation (1) in cylindrical coordinates we get:

\[
\frac{1}{r \partial r} \left( r \frac{\partial H}{\partial r} \right) = i \sigma \mu \omega H \quad (2)
\]

That can be rewritten as

\[
\frac{d^2 H}{dr^2} + \frac{1}{r} \frac{dH}{dr} - k^2 H = 0 \quad (3)
\]

where \( k^2 = i \sigma \mu \omega \), that is, \( k = (1 + i) \sqrt{\frac{\sigma \mu \omega}{2}} \) and \( \omega = 2\pi v \)

Eq. (3) is the modified Bessel equation of order zero and therefore the solution is as
\[ H(r) = A I_0(kr) + B K_0(kr) \]  

(5)

As \( K_0(kr) \) diverges for \( r=0 \), where \( H \) is obviously finite, only the term \( A \cdot I_0(kr) \) has physical meaning. The boundary condition \( H(R) = H_0 \) lead to determination of \( A = \frac{H_0}{I_0(kR)} \)

Hence, the field can be written as

\[ H = H_0 \frac{I_0(kr)}{I_0(kR)} \]  

(6)

The average magnetic induction inside the cylinder will be:

\[ <B> = \frac{\mu_0 \mu_r H_0}{I_0(kR) \pi R^2} \int_0^R I_0(kr) 2\pi r dr = \frac{2}{kR} \frac{\mu_0 \mu_r H_0}{I_0(kR)} I_0'(kR) \]  

(7)

As \( B \) fluctuates periodically in time with frequency \( \omega \), a voltage \( i\omega <B> \) will be induced in the secondary coil. Hence, the effective permeability \( \mu_{ef} = <B>/H_0 \), will be:

\[ \mu_{ef} = \mu_0 \mu_r \frac{2I_0'(kR)}{kRI_0(kR)} \]  

(8)

If we consider a frequency of 1 KHz, relative permeability \( \mu_r=500 \) and conductivity \( 10^7 \) \( \Omega^{-1}m^{-1} \) which are typical of steels, we get:

\[ k = (1 + i) \sqrt{\frac{\sigma \mu_0}{2}} = 4 \cdot 10^3 (1 + i) m^{-1} \]  

(9)

Hence, for 1 MHz we obtain

\[ k = (1 + i) \sqrt{\frac{\sigma \mu_0}{2}} = 1.2 \cdot 10^5 (1 + i) m^{-1}. \]  

(10)

If we consider permeability of 100 (instead of 50), \( k \) values becomes about half; getting conductivity \( \sim 10^6 \) \( \Omega^{-1}m^{-1} \), the modulus of \( k \) are divided by three. In any case, we can assume that we will deal will \( k \) values that are between \( 10^3 \) and \( 10^6 \) m\(^{-1}\).

For \( kR>>1 \), this condition is fulfilled for steel cylinders with radius larger tan 1cm, so we can approximate

\[ \frac{\mu_{ef}}{\mu_0 \mu_r} = \sqrt{\frac{2}{kr}} (1 + \frac{1}{8(kR)^2} + \cdots ) \]  

(11)
Now, the delay $\varepsilon$ in the average induction, $<B>e^{i(\omega t+\varepsilon)}$, respect to $H_0$, (that is, of $\mu_{\text{ef}}$ respect to $\mu_0\mu_r$) must be so that, the first term in the above equation become real.

Therefore, $\frac{\text{Im}\mu_{\text{ef}}}{\text{Re}\mu_{\text{ef}}} = \frac{\sqrt{2}}{|kR|} = \tan\varphi = \frac{1}{\tan\varepsilon}$, that according to (9), at first order becomes

$$\tan\varepsilon = \frac{|kR|}{\sqrt{2}} \quad (12)$$

We address now the dependence of the dephasing with applied mechanical stress. According to (9), if the frequency of $H_0$ is constant, the delay can only vary if magnetic permeability does. For macroscopic polycrystalline materials permeability can vary due to texturing that induce grain orientation or internal stress coupling via magnetoelastic effects. Both effects are not independent since texturing induce stress fields.

We can estimate then the sensibility of the delay to the changes of anisotropy from (12), considering that the relative permeability is inversely proportional to the anisotropy constant:

$$\frac{\partial \tan\varepsilon}{\partial K^*} = \frac{\partial R\sqrt{\sigma}\omega}{\partial \mu} \frac{\partial \mu}{\partial K^*} = -\frac{R}{2\sqrt{\sigma}\omega} \frac{\mu_0 M^2_s}{K^{*2}} \quad (13)$$

Where we assumed $\mu_r = \frac{M_s^2}{K^*}$, being $M_s$ the spontaneous magnetization of the cylinder and $\mu = \mu_0\mu_r$.

A source of local variations of $K^*$ in macroscopic samples is the distribution of internal stress, yielding an average perturbation of local anisotropy of $(3/2)\lambda\tau$, being $\lambda$ and $\tau$ the magnetostriction constant and the average local shear stress respectively. Getting this value for $\Delta K^*$, equation (10) becomes

$$\Delta\tan\varepsilon = \frac{3R}{4\sqrt{\sigma}\omega} \frac{\mu_0 M^2_s}{K^{*2}} \lambda\tau \quad (14)$$

So, the relative variation of delay is

$$\frac{\Delta\tan\varepsilon}{\tan\varepsilon} = \frac{3\mu_0 M^2_s}{4\sigma\omega K^{*2}} \lambda\tau \quad (15)$$

For typical values $M_s = 10^8$ Am$^{-1}$, $K^* = 10^4$ J m$^{-3}$ and $\lambda = 10^{-6}$, relative variation of $\tan\varepsilon$ is:

$$\frac{\Delta\tan\varepsilon}{\tan\varepsilon} = \frac{3M^2_s}{4\sigma\mu_r\omega K^{*2}} \lambda\tau \approx \frac{10^2}{\sigma\mu_r\omega} \tau \quad (16)$$

For the particular case of a steel cylinder with conductivity values $\sigma \sim 10^6$ $\Omega$ m, $\mu_r \sim 10^2$ and $\omega \sim 10^5$ Hz we get

$$\frac{\Delta\tan\varepsilon}{\tan\varepsilon} = \frac{3M^2_s}{4\sigma\mu_r\omega K^{*2}} \lambda\tau \approx \frac{4 \cdot 10^2}{\sigma\mu_r\omega} \tau \approx 4 \cdot 10^{-10} \tau (\text{MPa}) \quad (17)$$
It means that mechanical stress of the order of 200 MPA (about half of the elastic limit of steels) relative variations are of the order of 10%, providing a method to detect internal stress well below failure.

3. Experimental method

In order to experimentally verify this result, we designed and fabricated an induction device consisting of a primary coil with 200 turns and a wire diameter of 0.5 mm and a secondary coil of 100 turns with wire diameter of 0.17 mm over a plastic spool with 27 mm of internal radius and 80 mm length.

Two specimens were tested. They correspond to cylinders with 100 mm length and 23 mm radius made of SR235JR steel (general construction and machine steel produced in accordance with EN 10025-2 [7] and F-114 steel (equivalent to AISI 1045).

For the experiments, the samples were placed inside the induction device and the primary coil was connected to a Hewlett Packard 33120A wave generator to obtain a 4V (PP) amplitude sine wave and a frequency of 40kHz. The secondary coil was connected to the input of a SRS844 lock-in amplifier from Stanford Research Systems. The output of the wave generator was also used as reference signal in the lock-in amplifier. The specimens were placed into an electromechanical universal tester EM2/200/FR from Microtest to apply controlled compressive stress. Figure 1a shows a picture of the home-made magneto-mechanic device while figure 1b present an scheme of the system.
Figure 1. (a) Image and (b) scheme of the experimental setup.
4. Results

The phase ($\theta$) variations in the signal of the secondary coil were measured during a load-unload compressive cycle of 0-200-0 MPa when applying a 4 V (PP) signal at a frequency of 40kH in the primary coil.

Figure 2 shows the dependence of the dephasing during a load cycle for the frequency of 40kHz for both samples. Curves are very similar, showing an increment of the dephasing of about 1.5-2% for 100 MPa stress. In addition, both curves exhibit hysteresis, which has a magnetic, but not mechanical, origin, since we are well below the elastic limit of the steel samples. The hysteresis is associated with the magnetic domain rearrangement produced by the applied stress. Even though the stress strength is always within the elastic range it induces magnetic rearrangements (90 degrees domain wall motion) that exhibit hysteresis [8]. Such hysteresis could be eliminated by demagnetizing the sample before measuring the phase at each applied stress. Anyway, the measurement indicates that the phase is a good parameter to detect stress changes and that its theoretically predicted dependence is in rather good agreement with the experimental results.

![Figure 2. Change of dephasing as a function of applied load for (a) SR235JR and (b) F-114 steel specimens at a frequency of 40kHz.](image)

Note that according to (17) for a $\omega \approx 4 \times 10^4$ Hz dephasing should change at about $\sim 2.5\%$, which is similar to that observed experimentally, confirming the reliability of the method.

5. Conclusions

In summary, we presented a new non-invasive method to measure variations of internal stress in magnetic steels elements. The method relays on measurement of phase and hence is not affected by small changes in positions or vibrations, and therefore suitable
for in situ sensors. For typical structural steels, changes are of the order of few per cent. at a mechanical stress of the order of 200 MPA (half of the elastic limit). Consequently, the method provides an interesting possibility to measure changes in the internal stress of steel elements in service.

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