I. INTRODUCTION

Random lasing is a young and rapidly growing area of research. In fact, the phenomena “random lasing” comprise two distinctly different subfields of thorough theoretical and experimental studies. We dub them here “incoherent” random lasing and “coherent” random lasing, respectively. The essence of both subfields is the propagation of light in random media with gain.

Incoherent random lasing corresponds to a situation where, in the absence of a resonator, multiple scattering prevents light from leaving the optically pumped region. Such a disorder-induced feedback is sufficient for a drastic narrowing of the emission spectrum above a certain threshold pump intensity. Research on incoherent random lasing addresses various aspects such as steady state spectral characteristics and dynamics of this narrowing. Coherent random lasing, on the contrary pursues a scenario where disorder in a random medium leads to complete (or nearly complete) localization of the photonic modes. In other words, the disorder assumes the role of a Fabry-Perot resonator in a conventional laser. Therefore, above a threshold excitation intensity the emission spectrum from such a disordered medium comprises of a number of very sharp, laser-like lines.

The message of the present paper is that within the subfield of coherent random lasing there exist two different regimes, which we dub here “quantum” and “classical”. These two regimes are analogous to the quantum (Anderson) and classical localization of electrons in a random potential. If the potential is short-range, then the only length scale associated with it is the elastic mean free path, \( l^* \). Increasing disorder decreases the value of \( l^* \). Anderson localization is expected when \( l^* \) becomes smaller than \( \lambda \), where \( \lambda \) is the electronic wavelength. In contrast to the short-range potential, the long-range potential can be envisioned as a smooth landscape consisting of “lakes” and “mountains”. If a typical spatial scale of this potential is much bigger than \( \lambda \), then it is obvious that electrons with energies near the bottom of each lake are localized. The origin of this localization is purely classical. Interference effects, which are crucial for Anderson localization play a secondary role in the classical localization scenario; they are responsible only for the exact positions of the energy levels inside the lakes. Within this picture, the localization length of the electronic states is of the order of the lake diameter, and thus much bigger than \( \lambda \).

Since there is no one-to-one correspondence between the Schrödinger equation and the wave equation, the above picture of the classical localization cannot be directly applied to light. Nevertheless, as we argue below, the long-range fluctuations of the refraction index, \( n(\rho) \) are able to trap the light. The analog of a lake in optics is a region with higher \( n(\rho) \) than the average \( n \) of the surrounding space. Classical localization is then due to the total internal reflections from the boundaries of this region, as follows from geometrical optics.

In this paper we present evidence that coherent random lasing from \( \pi \)-conjugated polymer films is of the classical type. The basic argument in favor of this conclusion is that a long-range fluctuation of \( n(\rho) \) is similar to a Fabry-Perot resonator, in the sense that it gives rise to a number of localized modes having close frequencies and close quality factors. These modes are revealed in the emission spectrum. Since the frequencies of such modes are correlated, we could estimate the size, \( L \) of the underlying resonators from the power Fourier transform (PFT) of the spectrum. We indeed find that \( L \gg \lambda \).
a thickness variation

of the polymer film; the refractive indices of the polymer films is inhomogeneity of the film thickness variation $\delta h$; $L$ is the round trip resonator path-length.

The most probable reason for the fluctuations of $n(\rho)$ in the polymer films is inhomogeneity of the film thickness, $h$. Since light is confined within the film due to waveguiding, (see Fig. 1a) larger $h$ leads to higher in-plane wave vector of the light modes, i.e. to higher effective refraction index. By virtue of this mechanism, the long-range fluctuation of $h$ can result in the formation of a microdisk-type resonator (Fig. 1b). Such resonators have recently attracted a lot of attention, since they constitute the key element of semiconductor microlasers.

Certainly, alongside with long-range $n(\rho)$ fluctuations, short-range disorder is also present in the polymer films. This leads to $l^* \approx 10\lambda$, which we determined from coherent backscattering measurements. Such a weak short-range disorder is unable to localize light by itself. However, it radically affects lasing from the polymer film. This is because, whereas $l^* \gg \lambda$, it is still much shorter than the resonator size, i.e. $l^* \ll L$; this means that the ability of most random resonators to trap light is suppressed by the short-range disorder. There is a dramatic consequence of such a suppression: those resonators that “survive” the short-range disorder are sparse, and consequently almost identical. This scenario manifests itself in a spectacular way. At each position of the excitation spot the PFT of the random emission spectrum exhibits certain features. We found that averaging the PFTs of individual random spectra over the positions of the excitation spot on the polymer film not only does not smear these features, but, on the contrary, yields a series of distinct transform peaks. Moreover, we found that the shape of the averaged PFT is universal, i.e. increasing the disorder and correspondingly reducing $l^*$ does not change this shape: the average of the PFT spectra at different $l^*$ scales with $l^*$ to a universal curve. We have also developed a simple theory for the the PFT. Without specifying the shape of the resonators, the theory is based on a single assumption that the random resonators are exponentially sparse. This assumption is sufficient to reproduce the universal shape of the average PFT.

The paper is organized as follows. In Sect. II we review the current state of the theory and the experiment in the subfields of coherent and incoherent random lasing. In Sect. III we report on our measurements of random lasing spectra, PFTs and spatial intensity distribution of the emission from DOO – PPV polymer films. In Sect. IV a simple theory of random resonators formed due to long-range fluctuations of $n(\rho)$ is presented. Sect. V concludes the paper.

II. REVIEW OF RANDOM LASING

A. Incoherent Random Lasing; Theory

The first theoretical analysis of random lasing dates back to 1967 [3]. In the seminal paper of Letokhov [3] a random laser was modeled by an homogeneously pumped medium with scatterers that is contained within a certain volume. Due to the excitation pump, light diffusion inside the medium is accompanied by optical amplification. Lasing threshold is then determined from the condition that, in course of diffusion through the volume, light is amplified by a factor of $\sim 2$. Such an amplification compensates for the losses from the surface. The key idea of the theory [3] is that multiple scattering leads to incoherent feedback by forcing the light to spend a relatively long time inside the amplification region. Since light amplification can be viewed as multiplication of the number of photons, the condition for lasing is analogous to the criticality condition for chain reaction.

Interest revival in diffusive propagation of light in disordered gain media [4] has triggered further advancement of the incoherent random lasing theory. In the original work [3] it was assumed that the optical active centers are located inside the same particles that scatter light. The immediate consequence of this assumption is that a critical gain is independent of the scatterers concentration. To model realistic systems, the common situation that was considered in later papers was a homogeneous laser dye solution with suspended passive scattering microparticles. In this case, the description of random lasing requires solving the system of coupled equations for light diffusion with amplification, and also for population inversion. Various versions of such a system were invoked [5–9] to study the steady state [5,9] as well as the dynam-
ical properties of laser action in the scattering gain media.

In Ref. [6] a generic model of a dye with singlet and triplet electronic excitations was considered. Taking into account the possibility that the emitted light from the singlet band can serve as a pump for the triplet band yielded a bichromatic emission above a certain excitation threshold with a ratio of emission peaks depending on the pump intensity and dye concentration. In Refs. [6–8] the temporal response of a disordered laser material to the incident pump (or pump and probe) pulses was studied using a Monte Carlo simulation of random walks of the pump and emitted photons, which are interrelated through the population rate equation. These studies revealed a rich emission dynamics at excitation intensity close to the threshold. Finally, in Ref. [6] the Monte Carlo simulation was employed to study the emission spectral narrowing above the excitation threshold, as well as the input-output dependencies.

Summarizing, the original model of Ref. [6] yields a transparent and very appealing quantitative description of the incoherent random lasing process. However it leaves out two important physical effects: namely interference of diffusively moving waves, which is a precursor of photon localization [3, 4] and strong interference-induced fluctuations of the local light intensity, i.e., the mesoscopic phenomena [7, 8]. The fundamental role of the two effects was not appreciated back in 1967.

B. Incoherent Random Lasing; Experiment

The experimental results reported in the literature [9, 11, 12] essentially confirm the theoretical predictions. Except for studies on powder grains of laser crystal materials [39, 40] and \(\pi\)-conjugated polymer films, [41–43], the majority of experiments [20–35] have used dye solutions as amplifying media. Colloidal particles suspended in a solution served as random scatterers. In a typical experiment a short pulse is used for excitation. For below threshold pump intensities the emitted pulse is long. It shortens drastically to \(< 50\) ps as the laser threshold is exceeded. The emission spectrum narrows to \(< 20\) nm. Numerical solution of the the radiative transfer equations [3, 11, 12] yield the understanding of the dependence of the threshold energy on the concentration of scatterers, concentration of dye, cavity size, etc. However, the most convincing proof that laser action is due to “diffusive feedback” stems from the fact that below threshold the presence of scatterers has practically no effect on the emission.

C. Coherent Random Lasing; Theory

During the last decade the interplay of interference-induced localization effects and amplification of light in random media was the subject of intensive theoretical [44–46] studies. The theoretical prediction [44] that longer diffusive trajectories due to gain result in sharpening of coherent backscattering cone has been confirmed experimentally [47].

The next step was incorporation of disorder and interference into the conventional system of Maxwell equations coupled with the rate equations for the electronic population of a four-level-system involved in lasing. This program was realized in Ref. [48] for a model system representing a sequence of one dimensional layers of random thickness. One dimensional nature of the model and large (four times) contrast in dielectric functions between the neighboring layers allowed the authors to trace the crossover between the weak disorder i.e., long localization length (compared to the system size) and strong disorder regimes. The simulations of Ref. [48] have confirmed the basic idea of random lasing that increasing the disorder results in decreasing of the lasing threshold. These simulations have also verified that lasing lines in the emission spectrum have their origin in the localized modes of the disordered one dimensional system. Further studies reported in Ref. [49] have indicated that the distribution of electromagnetic field in the lasing mode is practically the same as in the localized mode in the absence of gain. The dependence of the emission spectrum on the gain frequency was also studied in the simulations of Ref. [50]. It was demonstrated that, upon varying the gain frequency, the order in which different localized modes overcome the lasing threshold can change.

D. Coherent Random Lasing; Experiment

Recently a group of experiments was reported [53–56], which revealed features of random lasing qualitatively different from those reported previously in Refs. [20–35]. The disordered gain media in which these measurements were done were very different, ranging from ZnO polycrystalline powders [53–54] to \(\pi\)-conjugated polymer films [55], organic dyes-doped gel films [55] and dye-infiltrated opal photonic crystals [56]. However, a unifying feature of the results reported using these media was the evolution of the emission spectra with increasing excitation intensity. Namely, the broad photoluminescence band at low intensities first drastically narrows; as the excitation intensity was increased even further, the emission spectrum transforms into a fine structure that consists of a number of sharp (\(\delta\lambda < 1\) nm) laser-like emission lines. Using the photon counting statistics [57], the coherent nature of the emitted light has been proved for powders [58] as well as for polymer films [59].
FIG. 2. (a) The measured emission spectrum of a DOO-PPV film above the threshold intensity for lasing. (b) Power Fourier transform of the emission spectrum in (a). The inset shows the coherent backscattering cone from a bulk sample of DOOPPV.

The most comprehensive experimental study of random lasing thus far has been performed by Cao et al. on ZnO powders that are extremely efficient scattering media; the light mean free path, \( l^* \), extracted from coherent backscattering measurements was \( l^* = 0.8\lambda \) in [53] and \( l^* = 0.5\lambda \) in [54]. On the basis of measurement of spatial distribution of the emitted light complemented with numerical simulations, it was demonstrated that the underlying mechanism of lasing was formation of closed loop paths of light with a characteristic size comparable to \( \lambda \). These paths are formed due to multiple scattering. It was argued in Ref. [54] that, due to interference or, in other words, localization effects, such closed loop paths serve as resonant cavities for light. Thus, the formation of these tiny cavities provides the feedback necessary for lasing. Correspondingly, the positions of the emission lines are determined by particular configurations of scatterers constituting each cavity [54].

The evolution of the emission spectrum with pump power observed in Ref. [53] is in qualitative agreement with numerical results of Ref. [51] obtained for the domain of small localization lengths.

III. EXPERIMENTAL RESULTS

A. Preliminary Discussion

In this paper we demonstrate that despite the similarity in the evolution of the emission spectra, the mechanism of random lasing in \( \pi \)-conjugated polymer films is fundamentally different from that reported for powders in Refs. [53, 54]. The difference can be briefly formulated as follows: In the case of Refs. [53, 54] it was reported that one emission line per random cavity was detected, whereas in polymer films one resonator gives rise to a number of correlated emission lines. This is because in our experiments with polymer films the relevant random resonators are much bigger in size; this is not surprising since the mean free path \( l^* \) in the polymer film was measured to be \( l^* \sim 10\lambda \). Our principal finding is that all random resonators are approximately identical. We come to this conclusion on the basis of the regularity in the experimentally observed emission spectra. Similarly to the case of electron transport in which the underlying period has been experimentally discerned from magneto-resistance fluctuations [11], the regularity in the random emission spectra is revealed here upon Fourier analysis of the data.

B. Random Spectra and Fourier Transforms

Our strategy was as follows. Fig. 2(a) is a typical emission spectrum above the laser threshold which was measured from a film excited with a stripe illumination of \( 2\text{mm} \times 100\mu\text{m} \) that is formed using a cylindrical lens. We note that in contrast, the excitation spot in Refs. [53, 54] was much smaller in size \( \sim 20\mu\text{m} \). Note also, that, despite the fact that in our case the two sizes of the excitation stripes differ by a factor of twenty. Nevertheless, they both are much bigger than \( l^* \). Clearly, therefore, the excited area cannot be considered as one dimensional [51, 52]. The spectrum was measured at an excitation intensity of 0.2\( \mu\text{j/pulse/mm} \) using 100ps pulses from the second harmonic of a Nd : YAG regenerative amplifier that operated at 100Hz repetition rate. The specific polymer was poly(dioclyoxy) phenylene vinylene, DOOPPV, which was synthesized by modifying a published procedure [62]. Fig. 2(b) shows the PFT of Fig. 2(a) and contains many apparent features. In general, the presence of such features in the PFT does not prove regularity in the emission spectrum. To illustrate this, we have generated a spectrum very similar to Fig. 2(a) by adding 12 Lorentzians centered at random wavelengths with random linewidths. Fig. 3(a) is a numerically simulated spectrum and Fig. 3(b) is the corresponding PFT, which also shows many apparent features. Next we generated 125 random spectra similar to Fig. 3(a), and averaged their individual PFTs. As would be expected, the sharp
PFT features vanished upon averaging. However when a similar procedure was performed on 125 real laser emission spectra obtained by shifting the excitation stripe on the surface of the polymer film, some features persisted in PFT; moreover, they became much more pronounced, as seen in Fig. 5 solid line. This supports the above statement about universality made in the Introduction.

Similarly to the features in magnetoresistance, which serve as fingerprints of actual disorder realization [61], the peaks in a single PFT reveal the properties of an individual random resonator. In Fig. 2(b) the peaks allow us to determine the size of a lasing resonator; more precisely the length $L$ of the light path is calculated via the relation $L = \pi d/n$, where $d$ is the peak position in the PFT and $n$ is the polymer index of refraction. Using $n = 1.8$ for the DOO-PPV polymer we get from Fig. 2(b) a round trip length $L = 56\mu m$ for the peak at $d = 18\mu m$, and $L = 216\mu m$ for the peak at $d = 60\mu m$, respectively. The smaller cavity corresponds to a length $L = 90\lambda$, which greatly exceeds the length scale $\sim \lambda$ as observed in Refs. [53, 54]. It is also important to note that the length $L$ also largely exceeds the mean free path, $l^*$, which was determined in DOO-PPV sample to be $l^* = 5.2\mu m$ or $8.3\lambda$ from the width of the coherent backscattering cone [63] as shown in Fig. 2(b), inset.

![FIG. 3. (a) A generated emission spectrum that consists of 12 Lorentzians. (b) Power Fourier transform of (a).](image-url)

C. Spatial Intensity Distribution

Figure 4 is a magnified image of approximately half of the excited area on the polymer film and measures the spatial distribution of the emission intensity at an excitation intensity above the laser threshold. The magnification is approximately 60 times. The emission intensity was measured through a long pass filter to remove the pump excitation light and avoid saturating the image array. The $z$-axis was computed by scaling the emission intensity of the red component of the color image. A crude estimate of the distance between the bright emitting spots is $40\mu m$, which is roughly comparable to that measured in Ref. [54]. However there is a stark difference between Fig. 4 here and the corresponding spatial distribution in Ref. [53], namely that in our case there are many more bright spots than spectral lines in the emission spectrum, as shown in Fig. 2(a). On the other hand, bright spots must correspond to certain lines in the emission spectrum. Recall, that from the PFT a characteristic size of the resonator $L = 56\mu m$ was inferred above. In the scale of Fig. 4 this corresponds to a single bright spot. The total number of bright spots in Fig. 4 is approximately three times the number of emission lines in Fig. 2(a). Thus, the emission spectrum which contains a limited number of lines reflects the emission from many resonators. The only resolution to this seemingly paradoxical situation is that all the resonators that are responsible for lasing in the polymer film are approximately the same. This fact, and the shape of the averaged PFT spectra seen in Fig. 2 require a theoretical explanation, which is given below.
FIG. 5. Average of 125 power Fourier transforms scaled by \( I^* \). The solid line is a theoretical model based on Eq. (6) superimposed on a smooth background; dotted lines are emission of pure polymer, and polymer doped with TiO\(_2\) scatterers, respectfully.

IV. THEORY

We do not really know the actual shape of the random resonators in the polymer films. However, it is apparent that our case is very different from the case in Refs. [3,34] since the round trip length, \( L \) of a typical resonator is much bigger than the mean free path \( I^* \), which, in turn, is much bigger than the wavelength \( \lambda \).

Large values of \( L \) suggest that the resonators are formed due to the long-range inhomogeneities of the refractive index, \( n(\rho) \). A plausible microscopic origin of these inhomogeneities is the fluctuation \( \delta h \) of the polymer film thickness, \( h \), as illustrated in Fig. 1(a). Due to the waveguiding in the polymer film, the effective refraction index, \( n_{\text{eff}} \), describing the in-plane propagation of light, is related to \( n \) as \( n_{\text{eff}} = \sqrt{n^2 - (p + 1)^2 \left( \frac{\lambda}{2h} \right)^2} \), where \( p \) is the number of the transverse waveguide mode. Therefore, a local increase of the film thickness by \( \delta h \) results in the enhancement of \( n_{\text{eff}} \) (see Fig. 1(a)) by \( \delta n_{\text{eff}} = (p + 1)^2 \lambda^2 \delta h / 4nh^3 \).

In fact, long-range variations of the film thickness in DOO-PPV polymer films were reported in Ref. [33]. Since \( \delta h \ll h \), and thus \( \delta n_{\text{eff}} \ll n_{\text{eff}} \), the angle of total internal reflection from the boundary (Fig. 1(b)) is close to 90°.

Thus, the microdisk-type resonator that is formed due to fluctuations of the film thickness (Fig. 1(b)) can only support the “whispering-gallery”, but not the “bow-tie” modes [2].

Below we demonstrate that the only assumption necessary to predict the shape of the average PFT is that the formation of resonators with \( L \gg I^* \) is highly unlikely. In other words, we assume that the the areal density, \( g(L) \), of resonators with a given length \( L \) falls off exponentially with \( L \). Quantitatively this fact can be expressed as follows

\[
g(L) = \frac{1}{L^2} \exp(-\varphi(L)),
\]

where the function \( \varphi(L) \) increases with \( L \), so that for \( L \gg I^* \) we have \( \varphi(L) \gg 1 \). The actual form of the function \( \varphi(L) \) is determined by the type of disorder in the film. For a given area of the excitation spot, \( S \), the maximal size, \( L_S \) of the resonator present within the area is determined by the condition \( Sg(L_S) \sim 1 \). Since \( g(L) \) is exponentially steep, this condition defines \( L_S \) with high accuracy. For values of \( L \) close to \( L_S \), i.e. \( |L - L_S| \ll L_S \), we can expand \( \varphi(L) \) as follows

\[
\varphi(L) = \varphi(L_S) + \frac{L - L_S}{L_0},
\]

where \( L_0 = (\partial \varphi / \partial L)_{L = L_S} \). Using Eq. (2) the average number \( N \) of resonators with the path length \( L \) within the illuminated area can be presented as

\[
\langle N(L) \rangle = \exp \left( \frac{L - L_S}{L_0} \right).
\]
The above arguments allow us to express the shape of the average PFT. The PFT for a single resonator with a given length, \( L \), can be presented as

\[
I(d) = \sum_m A_m \delta \left( mL - \frac{\pi d}{n} \right),
\]

where each \( m \) is an harmonics index that represents a round trip in the resonator. For all resonators \( A_m \) fall off with \( m \), but as gain starts to better compensate for losses, the harmonics amplitudes fall off more slowly \([67, 68]\).

The better gain compensates for losses, the slower is the decay of \( A_m \) with \( m \). It is usually approximated as \( r^m \) \([68]\), where \( r \) is a factor close unity; \((1 - r)\) characterizes the proximity to the lasing threshold. Below laser threshold Eq. \((4)\) is exactly valid; however above laser threshold the expected behavior of \( A_m \) may change. We have studied previously \([68]\) the change in the behavior of a single cavity crossing the lasing threshold. The results were that upon passing laser threshold the spacing between the components of the PFT stays the same, whereas the factors \( A_m \) decrease in amplitude. In Eq. \((4)\) we used the fact that the spectrum of the resonator is close to a delta function in the vicinity of the threshold \([68]\).

We note that the analysis of the coefficients \( A_m \) was recently suggested as a method to quantify resonator losses below lasing threshold \([68]\). Using Eq. \((4)\) the average (over excitation spots with area \( S \)) PFT takes the form

\[
< I(d) > \propto \sum_m A_m \int_{L_\gamma}^{\infty} dL \delta \left( mL - \frac{\pi d}{n} \right) < N(L) > \propto \sum_m \frac{A_m}{m} \exp \left[ -m^{-1} \left( \frac{\pi d}{n l_0} \right) \right] \Theta \left( \frac{\pi d}{n} - mL_\gamma \right),
\]

where \( \Theta(x) \) is the step-function. Eq. \((5)\) indicates that the average PFT contains structures that are dictated by the length \( L_\gamma \). Therefore, the presence of structures in Fig. \((6)\) has a simple physical interpretation. Namely, that only resonators with \( L \) above a certain, well defined value contribute to the laser emission spectrum. It is also seen from Eq. \((6)\) that the dependence of \( < I(d) > \) scales with \( l_0 \).

**A. Test of Universality**

The phenomenological parameter \( l_0 \) defined by Eq. \((4)\) is the characteristics of particular type of the disorder in the film. Within a simple model that is illustrated in Fig. 1 with long-range fluctuations of the film thickness playing the role of “mirrors,” this parameter should coincide with the light mean free path, \( l^* \), which is determined by the short-range disorder. Indeed, as follows from Eq. \((4)\), upon increasing the path length by \( \delta L \), the number of resonators diminishes by a factor

\[
\frac{< N(L + \delta L) >}{< N(L) >} = \exp \left( -\frac{\delta L}{l_0} \right).
\]

On the other hand, this reduction results from the fact that the light is scattered away from the mirrors within the segment \( \delta L \). The probability to survive this scattering is equal to \( \exp(-\delta L/l^*) \). Comparing this probability with the r.h.s. of Eq. \((6)\), we conclude that \( l_0 = l^* \).

In order to test the prediction that the averaged PFT Eq. \((4)\) scales with \( l^* \), we have intentionally introduced \( TiO_2 \) particles into the solution prior to the spin-casting process. From the coherent backscattering measurements on bulk samples, we have established that an additional short-range disorder caused a reduction of \( l^* \) from \( l^* = 5.2 \mu m \) in the pure film to \( l^* = 4.1 \mu m \) in the \( TiO_2 \) doped polymer film. In Fig. \((6)\) we plot the PFT of the random laser spectra for two polymer films, namely undoped and doped with \( TiO_2 \) particles, versus \( d/l^* \). Our first observation is that, whereas the peaks in the PFTs of individual spectra appear to be uncorrelated, the averaging over 125 spots yields a remarkable periodicity in the peak positions. This is in accordance with the theoretical dependence that is implied in Eq. \((6)\). The first harmonic near \( d/l^* = 3 \) differs by 20 percent from the data; this discrepancy is likely from the discreteness of the PFT. However, the discrepancy for the next five harmonics is maximum 6 percent. We thus conclude that the scaling is compelling.

In the same figure we also show a theoretical dependence, \( < I(d) > \), plotted using Eq. \((6)\) that is superimposed on a smooth background obtained from the structureless component of the experimental PFTs in Fig. \((6)\). The only fitting parameter in the theoretical \( < I(d) > \) is the dimensionless ratio \( d/l^* \), which was set to be 2.9. It is seen that the theoretical \( < I(d) > \) curve exhibits the same saw-tooth structure as the experimental curves.

![FIG. 6.](image-url) The average number of resonators \( < N > \) that exist within a sample with area \( S \) plotted versus the dimensionless pathlength \( L/l^* \) (Eq. \((6)\)). \( L_\gamma \) is the minimum length that is required to overcome losses. The shaded area represents resonators that are able to lase.
The fact that the peaks in the theoretical curve diminish much slower with $d$ than in the experimental PFTs can be accounted for incomplete averaging, since the higher is the peak index, the higher is the statistical error.

V. CONCLUSIONS

Until now, two different regimes of random lasing have been addressed in the literature, namely, incoherent and coherent random lasing, respectively. They correspond to different parameters domains of scattering media with optical gain. Incoherent random lasing occurs when, in the absence of gain, the propagation of light is diffusive, i.e., $l^* \gg \lambda$. Coherent random lasing should take place below or near the threshold of the Anderson localization transition, when $l^* \ll \lambda$. In this paper we demonstrate, however, that in the presence of long-range inhomogeneities of the refraction index with characteristic scale $L \gg \lambda$ the coherent random lasing is also possible even when $l^* \gg \lambda$, so that on average the propagation of light is diffusive. This is because inhomogeneities may trap light within the scale $\sim L$ in a “classical” fashion (due to total internal reflections). If $L \gg l^*$, which is the case for the polymer films that we have studied here, the areal density of resonators that are able to trap light decreases rapidly with $L$ [roughly as $\exp(-L/l^*)$]. On the other hand, at the lasing threshold, resonators with the largest $L$ lase first. These two opposite trends effectively “fix” the value of $L$. As a result of $L$ being approximately fixed, the positions of the emission lines are correlated. This correlation manifests itself in the shape of the average PFT of the emission spectra, which exhibits pronounced regular peaks.

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