Perturbation on the perfect lens: the near-perfect lens

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New Journal of Physics 8 (2006) 271
Received 8 October 2006
Published 13 November 2006
Online at http://www.njp.org/
doi:10.1088/1367-2630/8/11/271

Abstract. We present the full wave solution for a dipole source above a slab of left-handed medium of refractive index $n = -1$, but with $\epsilon$ and $\mu$ differing from those ideal values that create the perfect lens through taking $\epsilon = -(1 + \delta)^{-1}$ and $\mu = -(1 + \delta)$, where $\delta$, and hence $\epsilon$ and $\mu$, are real. Any modifications in resolution are therefore not due to loss effects within the lens. Finite solutions for the form of the fields throughout all space are obtained using the method of Hertz potentials, thereby regularizing the singular perfect lens solution. This regularization will facilitate subsequent perturbation analyses. Using an appropriately defined criterion, we examine the sensitivity of the lens resolution to the material imperfection $\delta$. It is found that the resolution converges logarithmically with $\delta$ upon that of the perfect lens. Comparison of theoretical results with experimental data suggests that even this slow convergence is the best that can be achieved.

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1. Introduction

In [1], it is proposed that a material with a refractive index of $-1$ could be used to make an optical lens. The effect of such a lens would be two-fold. Firstly, the negative angle of refraction at the interface enables propagating waves, which represent the far-field, to be brought to a focus by a planar slab rather than a conventional curved lens. Secondly, such a slab has an amplifying effect upon the near-fields of a source, which are absent from the image produced by a conventional lens, in such a way that the source is reproduced exactly at the focus. Since the image produced by a left-handed lens contains all the near-and far-field information of the source the resolution is perfect leading to such devices being dubbed ‘super-lenses’.

Objections are raised to this concept [2] on the grounds that, for evanescent waves in an ideal, lossless, non-dispersive medium, the behaviour claimed in [1] is limited to thin slabs of left-handed medium (LHM) in order that the fields do not diverge within the material. Moreover, the maximum allowable thickness does not permit the super-lensing properties. Further, it is shown that the presence of absorption (characterized in this instance by an imaginary part to the electric permittivity alone) gives rise to evanescently decaying waves within the LHM, rather than the evanescently growing waves that are key to the behaviour described in [1], indicating the sensitivity of the phenomenon to dissipative effects. However, such theoretical concerns are restricted solely to the interpretation of the physics since left-handed metamaterials have been constructed at microwave frequencies and have been used to achieve sub-diffraction limited imaging [3]. More recently, materials that display left-handed characteristics at optical frequencies have been fabricated [4].

Chew [5] considers the Sommerfeld integrals for a slab of lossless LHM and shows that super-resolution is possible in the case where the source is a distance from the slab precisely equal to the slab thickness. It is also claimed that the limits on super-resolution occur only as a result of the dissipation in an LHM slab, a claim that will be disputed here.

The problem of deviations of the material parameters away from those of the perfect lens was first addressed in [6]. In [6] a small perturbation away from the perfect lens value ($\mu = -1$) is made to the real part of the magnetic permeability. The real part of the permittivity remained at its ideal value ($\epsilon = -1$) and a small lossy part was introduced to both parameters. The sensitivity of the resolution to the perturbation was investigated and the relevance of the surface modes stressed, with the conclusion that superlenses were feasible only over a fairly restrictive range of perturbation parameters. The resolution deteriorated rapidly with the perturbation, and further
with increasing slab thickness. In making the perturbation only to the magnetic permeability however the refractive index differs from $-1$ and so the perfect lensing effect is lost for propagating radiation. The small imaginary part of the material parameters was introduced to facilitate the numerical calculations.

An elegant approach to addressing deviations from the perfect lens scenario whilst maintaining a refractive index of $-1$ is introduced in [7, 8]. Electromagnetic parameters $\epsilon$ and $\mu$ are given a form such that their product remains unity whilst they each deviate separately from $-1$, being the perfect lens values. This is achieved by taking

$$
\epsilon = -\frac{1}{1+\delta}, \quad \mu = -(1+\delta),
$$

where $\delta$ is real and need not necessarily be small. In [7], the parameterization (1) is used to establish the relevance of plasmon poles in achieving super-resolution of a line source by a left-handed slab. Despite the presence of infinitely many poles in the complex plane of the wavevector, all physical quantities remain finite with an appropriately chosen contour used in the evaluation of integrals. In [7] the contour is applied to the field integrals in the two-dimensional (2D) problem as a means of obtaining a solution. Furthermore, although evidence of subwavelength resolution is given for a particular choice of the parameter $\delta$, no systematic study of the dependence of the resolution upon this parameter is undertaken.

Another intriguing application of LHMs is given in [8], in which it is proposed that two perfectly out of phase line sources situated either side of a left-handed slab such that one source is located at the focus of the other (and vice versa) will localize the fields entirely between them. It is noted that in the perfect lens case of $\epsilon = \mu = -1$, however, the field integrals become divergent, as noted previously, and the above parameterization is used to resolve this issue. The effect of the perturbation is to blur slightly the boundary of the area of localization due to reflection arising from the slight impedance mismatch introduced at the interfaces of the LHM. Again, this approach appears to have been used solely to facilitate the calculation, rather than to study the impact of the perturbation upon the system. The localization effect relies on the interference between both propagating and evanescent modes and so requires a true LHM, rather than a metallic sheet for which only $\epsilon$ is negative.

It is not necessary for a material to be truly left-handed in order for sub-wavelength resolution to be achievable [1, 9, 10]. A slab of silver, which at certain frequencies supports surface plasmons due to its negative electric permittivity, can be used to resolve objects separated by less than a wavelength [1]. Although such a situation does indeed result in sub-wavelength imaging, the images produced cannot be said to be perfect since propagating modes emanating from a given source do not contribute to the image since they attenuate within the slab. In [9] retardation effects are included, an extension upon the electrostatic limit work undertaken in [1]. It is found that retardation effects mean that a ‘lens’ with $\epsilon = -1$ but with arbitrary $\mu$ is not perfect but that significant resolution improvements are possible. Jiang and Pike [10] simulate the near-field image produced by a silver slab and compare the results with those obtained in the electrostatic limit. It is again found that enhanced resolution may be achieved despite the fact that the permeability is positive. It is also observed that the resolution is highly sensitive to the slab thickness and attempts to overcome this using several thinner slabs and gain media are considered. Whilst metallic sheets have been used as a means of obtaining sub-wavelength resolution they are not appropriate for a localization device such as that in [8] because their imaginary refractive index prohibits the transport of propagating modes.
Following this introduction, in section 2 of the paper, the full 3D solution to the problem of a dipole source located above a slab of LHM for general negative $\epsilon$ and $\mu$ is derived via the method of Hertz potentials. The form of the solution is found to be in such a form so as to transparently illustrate the physics of the problem. In section 3, the perfect lens case ($\epsilon = \mu = -1$) is considered and shown to possess the problem of divergent integrals seen in [2] in such a way that the form of the solution is found to illustrate the origin of the divergence in a physically intuitive manner. Section 4 sets out the details of the calculation performed when the material parameters take the form (1). Using a contour analogous to that found in [7] to perform the integrals, it is possible to arrive at a finite solution despite the presence of the plasmon resonance and this has two main implications. Firstly, the existence of a regularized solution provides a basis for further perturbation analyses. Secondly, the solution obtained here allows detailed analysis of the sensitivity of the resolution to the perturbation. It is the sensitivity of the resolution that is considered in section 5. It is found that the resolution deviates rapidly away from the arbitrary resolution of the perfect lens as the perturbation increases, although resolution is still possible below the diffraction limit. The link between the resolution and the plasmon resonance is also clearly visible in the results obtained. Furthermore, in the presence of a perturbation it is found that there is a deterioration in resolution that occurs as the slab thickness, $d$, increases. For thick lenses (greater than the order of wavelengths across), the value of $\delta$ that is required to obtain a given resolution is found to decay exponentially with $d$ and there therefore exists a trade-off between the slab thickness and perturbation size. The theoretical results are discussed in relation to experimental data [3] and it transpires that even the rapid deterioration in resolution seen in the regularized solution is a best case scenario. Finally, conclusions are given in section 6.

2. Hertz potential solution for a vertical electric dipole above a left-handed slab

Considered here is the problem of a dipole situated on the $z$-axis a height $h$ above a planar slab of LHM as shown in figure 1. The upper and lower surfaces of the slab are the planes $z = 0$ and $z = -d$ respectively ($d > h$ in order that a real image is formed on the far side of the lens). Regions 1 and 3 ($z > 0$ and $z < -d$ respectively) are vacuum while region 2 ($0 > z > -d$) has material parameters $\epsilon < 0$ and $\mu < 0$ such that the refractive index $n = -1$.

Following the approaches of [11, 12], the problem is solved using the Hertz vector potentials, of which there are two: one electric and one magnetic. However, since it is an electric dipole source that is considered here, only the electric Hertz potential, $\Pi$, is required and this is introduced by means of the relations

$$\mathbf{A} = \epsilon \mu \frac{1}{c} \frac{\partial \Pi}{\partial t}, \quad (2)$$

$$\phi = -\nabla \cdot \Pi, \quad (3)$$

where $\mathbf{A}$ and $\phi$ are the familiar vector and scalar potentials. It is possible to express the electric Hertz vector in terms of the source polarization, $\mathbf{P}$, in a simpler way than with the vector and scalar potentials, namely

$$\nabla^2 \Pi - \frac{\epsilon \mu}{c^2} \frac{\partial^2 \Pi}{\partial t^2} = -\frac{4\pi}{\epsilon} \mathbf{P}. \quad (4)$$
The electric and magnetic fields are readily obtained from the Hertz potential via the relations

\begin{align}
    \mathbf{E} &= \nabla (\nabla \cdot \Pi) + \epsilon \mu k_0^2 \Pi, \\
    \mathbf{H} &= -ik_0 \epsilon \nabla \times \Pi,
\end{align}

where \( k_0 = \omega/c \) is the wavenumber in vacuo and all quantities are assumed to vary as \( e^{-i\omega t} \). For a vertical electric dipole of dipole moment \( p_0 \mathbf{\hat{z}} \), the source polarization is given by

\begin{equation}
    \mathbf{P} = p_0 \delta(z - h)\delta(R) \mathbf{\hat{z}},
\end{equation}

where \( \delta(x) \) is the Dirac delta function. The Hertz potential in this case has only a \( z \)-component, \( \Pi_z \), and the boundary conditions are that the quantities \( \epsilon \Pi_z \) and \( \partial_z \Pi_z \) are continuous across each interface. The full solution expressed as a function of the cylindrical coordinates \((R, \phi, z)\) is given by

\begin{align}
    \Pi_1(R, z) &= \int_0^\infty J_0(\alpha R) \left( f_{1z}(\alpha) e^{-\gamma_1(z+h)} + \frac{p_0 \alpha}{\gamma_1} e^{-\gamma_1|z-h|} \right) d\alpha, \\
    \Pi_2(R, z) &= \int_0^\infty J_0(\alpha R) \left( f_{2z}(\alpha) e^{\gamma_2 z - \gamma_1 h} + g_{2z}(\alpha) e^{-\gamma_2 z - \gamma_1 h} \right) d\alpha, \\
    \Pi_3(R, z) &= \int_0^\infty J_0(\alpha R) f_{3z}(\alpha) e^{\gamma_1(z-h)} d\alpha,
\end{align}

Figure 1. Geometrical configuration for a dipole situated above a slab. Also shown are the cylindrical coordinates \((R, \phi, z)\).
where

\[
f_{1z}(\alpha) = \frac{p_0 \alpha}{\gamma_1} \left( \frac{\gamma_2 - \epsilon \gamma_1}{\gamma_2 + \epsilon \gamma_1} \right) \left( \frac{e^{-2\gamma_2d}}{D_\epsilon(\alpha)} - 1 \right),
\]

\[
f_{2z}(\alpha) = \frac{p_0 \alpha}{\gamma_1} \left( \frac{2\gamma_1}{\gamma_2 + \epsilon \gamma_1} \right) \frac{1}{D_\epsilon(\alpha)},
\]

\[
g_{2z}(\alpha) = \frac{p_0 \alpha}{\gamma_1} \left( \frac{2\gamma_1}{\gamma_2 + \epsilon \gamma_1} \right) \left( \frac{\gamma_2 - \epsilon \gamma_1}{\gamma_2 + \epsilon \gamma_1} \right) \frac{e^{-2\gamma_2d}}{D_\epsilon(\alpha)},
\]

\[
f_{3z}(\alpha) = \frac{p_0 \alpha}{\gamma_1} \left( \frac{2\gamma_1}{\gamma_2 + \epsilon \gamma_1} \right) \left( \frac{-2\epsilon \gamma_2}{\gamma_2 + \epsilon \gamma_1} \right) \frac{e^{(\gamma_1 - \gamma_2)d}}{D_\epsilon(\alpha)},
\]

and

\[
D_\epsilon(\alpha) = 1 - \left( \frac{\gamma_2 - \epsilon \gamma_1}{\gamma_2 + \epsilon \gamma_1} \right)^2 e^{-2\gamma_2d}.
\]

The integration variable \(\alpha\) corresponds to the component of the wavevector parallel to the plane of the interfaces, and \(\gamma_i\) the component normal to the interfaces (the \(z\)-component) in region \(i\). The dispersion relation dictates that \(\gamma_i = \sqrt{n_i^2 k_0^2 - \alpha^2}\), where \(n_i = \sqrt{\varepsilon_i \mu_i}\) is the refractive index of the medium occupying region \(i\). The integral solution given above can therefore be interpreted as a superposition of outgoing cylindrical waves in the radial direction that propagate in the \(z\)-direction for \(\alpha < n_i k_0\) but that are evanescent for \(\alpha > n_i k_0\).

In regions 1 and 3 the boundary conditions at infinity \([5]\) mean that the following signs of the square root must be taken:

\[
\gamma_1 = \begin{cases} 
-i \sqrt{k^2 - \alpha^2} & \text{for } \alpha < k_0, \\
\sqrt{\alpha^2 - k^2} & \text{for } \alpha > k_0.
\end{cases}
\]

(16)

In region 2 the left-handedness of the medium means that the sign of the square root is reversed for the propagating components whilst being unchanged for the evanescent components:

\[
\gamma_2 = \begin{cases} 
i \sqrt{\varepsilon \mu k^2 - \alpha^2} & \text{for } \alpha < \sqrt{\varepsilon \mu} k_0, \\
\sqrt{\alpha^2 - \varepsilon \mu k^2} & \text{for } \alpha > \sqrt{\varepsilon \mu} k_0.
\end{cases}
\]

(17)

3. Divergence of the perfect lens solution

For the perfect lens, \(\varepsilon = \mu = -1\) and therefore

\[
\gamma_2 = \begin{cases} 
-\gamma_1 & \text{for } \alpha < k_0, \\
\gamma_1 & \text{for } \alpha > k_0.
\end{cases}
\]

(18)

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Figure 2. Diagram illustrating the geometry and location of the foci of a perfect lens. Also labelled is the region between the two foci where the integrals for the Hertz vector diverge.

The full solution in this case is simply

\[ \Pi_1(z, R, z) = \int_0^\infty \frac{p_0 \alpha}{\gamma_1} J_0(\alpha R) e^{-\gamma_1|z-h|} \, d\alpha, \]  

\[ \Pi_2(z, R, z) = \int_0^\infty \frac{p_0 \alpha}{\gamma_1} J_0(\alpha R) e^{-\gamma_1(z+h)} \, d\alpha, \]  

\[ \Pi_3(z, R, z) = \int_0^\infty \frac{p_0 \alpha}{\gamma_1} J_0(\alpha R) e^{\gamma_1(z-h+2d)} \, d\alpha. \]

The solution in region 1 is readily identified as the Hertz potential solution for a dipole situated on the \( z \)-axis at \( z = h \) and this is to be expected since for \( \varepsilon = \mu = -1 \) all radiation incident upon the slab is transmitted through it.

The solution in regions 2 and 3 reveal the lensing nature of the slab, see figure 2. For \( -h < z < 0 \) the potential in region 2 is identical to that of a dipole situated at \( (R, z) = (0, -h) \). The effect of the slab is therefore to generate a convergent spherical wave where the propagating modes are phase reversed and the evanescent modes grow with decreasing \( z \). This is exactly equivalent to a divergent spherical wave emergent from \( (R, z) = (0, -h) \), whose oscillatory modes propagate away from the ‘source’, and whose evanescent modes decay with increasing \( z \). Thus a source appears to be located at \( (R, z) = (0, -h) \), which is the first focus of the lens.

However in the region \( -d < z < -h \) the integral diverges due to the fact that within the integral the factor \( z + h \) in the argument of the exponential is now negative and for large \( \alpha \), \( -\gamma_1 \) is similarly large and negative, their product being large and positive. This divergence arises purely from the evanescent part of the integral since for \( 0 < \alpha < k \) the integral is well defined. The physical origin of the integral divergence is therefore connected to the effect of the slab upon the evanescent modes, which is to amplify them, as noted in [1].
emanating from the source at \((R, z) = (0, h)\) decays by a factor of \(e^{-\gamma_1 h}\) before reaching the interface, whereupon it begins to grow. At \(z = -h\) it has grown by a factor of \(e^{\gamma_1 h}\) compared with its value at the interface and has therefore returned to the magnitude with which it left the source. The integral over all these evanescent modes, together with the propagating modes, gives rise to the Hertz potential for the dipole, which is infinite at the origin. Amplifying the evanescent modes beyond this point gives rise to the divergence in the integrals.

For \(h - 2d < z < -d\) exactly the same thing occurs because \(\gamma_1\) is again large and positive for large \(\alpha\) whilst the factor \(z - h + 2d\) is positive also. This region again corresponds to that where the evanescent modes are amplified to a value beyond that with which they left the source.

For \(z < h - 2d\) the integral again becomes convergent as the factor \(z - h + 2d\) changes sign and the Hertz potential is identical to that attributable to a dipole situated at \((R, z) = (0, h - 2d)\), this being the second focal point of the lens.

The divergence in the Hertz potential manifests itself in the fields via the relations (5) and (6), and hence also in physically measurable quantities such as intensity. This divergence renders the perfect lens solution unphysical, as noted in [2]. The physical origin of the divergence is that in order to reach the steady state, implied by the harmonic ansatz used, an infinite amount of energy has been fed into the slab resonances. Any attempt to expand the Hertz potentials about the perfect lens solution is doomed to failure because of this divergence. If an alternative, non-divergent solution can be found about which to expand, the resulting expansion would be of considerable use in investigating the behaviour of the solution for material parameters that are close to those of the perfect lens case. Such a solution is found in the following section.

4. Near-perfect material parameters

It is to avoid the divergence inherent in the perfect lens solution that the following model for the material parameters is introduced:

\[
\epsilon = -\frac{1}{1 + \delta}, \quad \mu = -(1 + \delta),
\]

(22)

where \(\delta\) is a real and positive parameter. Clearly for \(\delta = 0\) the perfect lens scenario \(\epsilon = \mu = -1\) is recovered and so a small value of \(\delta\) corresponds to a small deviation from the ideal case. In this case the slab shall be referred to as a near-perfect lens (NPL). Since \(n^2 = \epsilon \mu = 1\), it is still the case that the angle of refraction is equal and opposite to the angle of incidence, as for the perfect lens. However, the impedance of each interface is now given by

\[
Z = \sqrt{\frac{\epsilon}{\mu}} = \frac{1}{1 + \delta},
\]

(24)

which is no longer equal to unity, although for small \(\delta\), \(Z \approx 1 - \delta\) and the impedance is close to that of the vacuum. Whereas for the perfect lens all radiation incident upon the lens is transmitted across the interface, some energy is reflected from the slab surface upon the introduction of a nonzero \(\delta\).
The full solution for such a choice of material parameters is

\[ \Pi_1(R, z) = \int_0^\infty J_0(\alpha R) \frac{p_0\alpha}{\gamma_1} \left( e^{-\gamma_1(z-h)} + \frac{\delta(2 + \delta)(e^{-2\gamma_1 d} - 1)}{-\delta^2 + (2 + \delta)^2 e^{-2\gamma_1 d}} e^{-\gamma_1(z-h)} \right) d\alpha, \]  

(25)

\[ \Pi_2(R, z) = \int_0^\infty J_0(\alpha R) \frac{p_0\alpha}{\gamma_1} \left( \frac{-2(1 + \delta)(2 + \delta)e^{-2\gamma_1 d}}{-\delta^2 + (2 + \delta)^2 e^{-2\gamma_1 d}} e^{-\gamma_1(z+h)} \right. 
\]  

\[ + \frac{-2\delta(1 + \delta)}{-\delta^2 + (2 + \delta)^2 e^{-2\gamma_1 d}} e^{\gamma_1(z-h)} \right) d\alpha, \]  

(26)

\[ \Pi_3(R, z) = \int_0^\infty J_0(\alpha R) \frac{p_0\alpha}{\gamma_1} \left[ -\frac{4(1 + \delta)}{\gamma_1} e^{\gamma_1(z-h)} \right] d\alpha. \]  

(27)

What is immediately clear is that none of the above integrals diverge in the same way as for the perfect lens: the exponential terms always decay for large \( \alpha \) for all \( z \). However, the integrand now contains a pole due to the factor \([- \delta^2 + (2 + \delta)^2 e^{-2\gamma_1 d}]^{-1}\) that occurs in the integrand for all three regions.

This first order pole occurs at a critical wavenumber, \( \alpha_c \), given by

\[ \alpha_c = k_0 \left[ 1 + \frac{1}{(k_0 d)^2} \left( \frac{2 + \delta}{\delta} \right)^2 \right]^{1/2} > k_0, \]  

(28)

and this corresponds to the wavenumber of the plasmon resonance in the plane of the interfaces. The integrals may be performed by taking a small semi-circular detour above the pole into the region \( \text{Im}[\alpha] > 0 \). Inserting the explicit form of \( \gamma_1 \), and performing the integration around the pole it is found that in region 3, which is of particular interest since this is where the image is formed,

\[ e^{i \Pi_3} = \int_0^k \frac{i p_0\alpha J_0(\alpha R)}{(k_0^2 - \alpha^2)^{1/2}} \frac{4(1 + \delta)e^{-\delta(z-h)}}{-\delta^2 + (2 + \delta)^2 e^{2\delta k_0^2 - \alpha^2 - \alpha^2 \gamma_1 d}} d\alpha 
\]  

\[ + \text{CPV} \int_k^\infty \frac{p_0\alpha J_0(\alpha R)}{(\alpha^2 - k_0^2)^{1/2}} \frac{4(1 + \delta)e^{\delta(z-h)}}{-\delta^2 + (2 + \delta)^2 e^{2\delta k_0^2 - \alpha^2 - \alpha^2 \gamma_1 d}} d\alpha 
\]  

\[ + \frac{2\pi i p_0}{d} \frac{(1 + \delta)}{(2 + \delta)^2} J_0(\alpha_c R) e^{\frac{\alpha_c^2 - k_0^2}{2}(z-h+2d)}, \]  

(29)

where CPV denotes the Cauchy principal value of the integral. The integrals may now be evaluated numerically, as may the integrals in regions 1 and 2, which are calculated in a similar way.

**5. Effect of perturbation upon resolution**

To investigate the resolution in the second focal plane of the lens, given by the plane \( z = h - 2d \), the normalized intensity is defined as

\[ I(R) = \frac{|E(R, z = h - 2d)|^2}{|E(0, z = h - 2d)|^2}, \]  

(30)
and the measure of resolution, $R_{1/2}$, is taken to be the radial distance over which the normalized intensity falls to half its maximum value, which is attained at $R = 0$, i.e. $I(R_{1/2}) = I(0)/2$. For the perfect lens case, divergences aside, the lens creates an image at the second focus that is identical to the source. The normalized intensity for a dipole placed at the second focus, denoted $I_{pl}(R)$, can be shown to be

$$I_{pl}(R) = \begin{cases} 1 & \text{for } R = 0, \\ 0 & \text{otherwise}. \end{cases}$$ (31)

The width of this ‘spike’ is clearly zero and therefore so too is the measure of resolution: the perfect lens, were it physically realizable, would have arbitrarily fine resolution as predicted by Pendry [1]. The value of $R_{1/2}$ as a function of $\delta$ is plotted as the solid line on a log-linear scale in figure 3. Also shown as the dashed line is the normalized length scale of the plasmon resonance, given by $C/\alpha_c$. The normalization factor, $C$, is chosen so that as $\delta \to \infty$, and hence $\alpha_c \to k_0$, the plasmon length scale approaches the resolution achieved by the diffraction limited propagating mode for which $\alpha = k_0$.

It is clear from figure 3 that there is a very strong correspondence between the radial resolution of the full wave solution and the length scale of the resonant wave vector. This indicates that the dominant physical process for the resolution is that of the resonance: the small length scale of the surface waves in planes parallel to the lens surface enables resolution to occur on a finer scale than is possible in their absence.

The agreement between the solid and dashed curves appears to be almost exact for the smallest value of $\delta$ and although there is an increase in the difference between the two curves around $\delta = 0.1$ this difference remains slight. Beyond $\log_{10}(\delta) = -1$ the difference again becomes less pronounced. The slight deviations can be explained due to one main factor: the solution contains contributions from all modes, be they propagating or evanescent. Interference between these modes will necessarily lead to a broadening of the intensity peak. The reason that this effect does not completely destroy the super-resolution phenomena is that there is a heavier weighting towards those modes close to the resonance due to the large value of the integrand there.

An important feature of figure 3 is the rate at which the resolution approaches that of the perfect lens as $\delta \to 0$. The convergence is very slow, with the plasmon length varying as $[\ln(\delta)]^{-1}$ for small $\delta$. When viewing the parameter $\delta$ as an imperfection in the material parameters arising in the manufacture of a LHM, it can be seen that below a certain point, substantial reductions in $\delta$ result in negligible improvements in resolution. The key point being made here is not that sub-diffraction imaging is impossible (in [3], for example, experimental results are presented in which resolution is achieved below the diffraction limit); rather, it is stressed that there comes a point beyond which improvements in resolution become prohibitively difficult to obtain. This creates a lower bound that any practical attempts to construct improved lenses will encounter, precluding the achievement of perfect resolution.

The situation described above becomes more restrictive as the thickness of the slab increases. The results in figure 3 were calculated for a slab thickness equal to $\lambda/2\pi$, substantially less than a wavelength. As the slab thickness is increased the resolution deteriorates rapidly as shown in figure 4, in which $R_{1/2}$ is plotted against $d/\lambda$ for three values of $\delta$. It can be clearly seen for all curves that once the slab thickness is of the order of 10 wavelengths there is only a marginal improvement in resolution despite the slight nature of the perturbations. To achieve significant improvements in resolution only very thin slabs are suitable, regardless of the value of $\delta$. 

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Figure 3. Radial resolution in the second focal plane as a function of $\delta$ for $d = k = 1$ and $h = 0.5$ (solid line). Also shown is the normalized plasmon length scale (dashed line).

Figure 4. Effect of slab thickness upon resolution of the NPL. Upper, middle and lower curves are for values of $\delta = 10^{-3}$, $\delta = 10^{-6}$ and $\delta = 10^{-9}$ respectively.

In [3], a metallic grid loaded with capacitors and inductors is used to construct a planar left-handed lens with relative refractive index very close to $-1$. The experiments were performed for a slab thickness, $d$, of 50.4 mm and with radiation of wavelength, $\lambda = 15.59$ cm ($k_0 = 40.31$ rad m$^{-1}$). Using the half-power beamwidth as the measure of resolution, a resolution of $0.21\lambda$ was achieved, an improvement on the diffraction limited value of $0.36\lambda$.

If it is assumed that the slab material parameters have the form (1) then the above improvement in resolution would correspond in our model to a perturbation $\delta = 0.125$. The relative material parameters used in [3] are $\epsilon = -0.9988 + 0.0491i$ and $\mu = -1.0000 + 0.0177i$ and these are not accountable for with our model. Aside from the fact that the $'\delta'$ would need to be imaginary, the requirement that the imaginary part of both parameters be positive breaks the symmetry. However it is interesting to note that despite the experimental values being much closer to the ideal values of $-1$ ($|\epsilon + 1| = 0.0492$ and $|\mu + 1| = 0.0177$) the resolution achieved is equivalent in our model to a far greater perturbation of $\delta = 0.125$. A perturbation similar in magnitude to the experimental values, $\delta = 0.05$ say, would give rise in our model to a resolution
of 0.13λ, well below that achieved in the experiment. This suggests that a perturbation that is asymmetric and complex causes a more severe deterioration in resolution than one that is real and symmetric, such as (1). Consequently, the resolution achievable by the regularized solution is a best case scenario. Any deviation from the symmetric and lossless parameters will result in yet further impairments to resolution, as seen in [3], making it even harder to achieve finer resolution.

6. Conclusions

In this paper a 3D, exact and analytical approach to the superlens problem has been presented. This approach provides a physically transparent form of the solution which clearly displays the divergent integrals noted in [2] that correspond to the plasmon resonances. By taking the material parameters to be of the form (1), with the refractive index remaining equal to −1, the problems with divergent integrals do not arise and a regularized solution is obtained. This regularized solution could find use as the basis of an expansion approach to treating materials whose parameters have addition real or imaginary perturbations to their values.

For the specific form of the material parameters given by (1), the resolution is found to be highly sensitive to the perturbation from the perfect lens limit as well as to the thickness of the lens. The deterioration in resolution upon the introduction of the perturbation is not due to dissipation within the lens since the material parameters are real, and is therefore a fundamental property of the system. The limits imposed upon the perturbation size and slab thickness indicate that the manufacture of left-handed media for use in lensing devices needs to be extraordinarily stringent, particularly for imaging at short wavelengths.

Acknowledgments

This study was funded by the Engineering and Physical Sciences Research Council, and by a case award from BAE Systems. Additional funding was provided by a Royal Society International Joint Project with the Universidad de Cantabria in Santander, Spain.

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