Excited states of $^4$He

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Abstract. The study of the $^4$He nucleus is important because it is the most basic sub-unit (cluster) in nuclei. We have investigated the structures and the reaction mechanisms in $^4$He by using the correlated Gaussian basis function with the global vector representation. In order to treat the boundary condition for the \textit{ab-initio} calculation of the four nucleons, we employ the Microscopic R-matrix Method (MRM) and the Complex Scaling Method (CSM). Elastic-scattering phase shifts for four-nucleon systems are studied in a \textit{ab-initio} type cluster model with MRM in order to clarify the role of the tensor force and to investigate cluster distortions in low energy $d+d$ and $t+p$ scattering. For $^1S_0$, the calculated phase shifts show that the $t+p$ and $h+n$ channels are strongly coupled to the $d+d$ channel for the case of the realistic interaction.

1. Introduction

"\textit{Ab-initio}"-type studies have been carried out for the $^4$He nucleus by many groups [1]. $^4$He is an interesting nucleus because it is a most basic sub-unit (cluster) in nuclei and has a double closed s-shell nature. However, for excited states of $^4$He, theoretical analyses have been limited mainly because the treatment of unbound states are very difficult. One of the useful method for unbound states is the Microscopic R-matrix Method (MRM) [2] which is combined with the microscopic cluster model. The microscopic cluster model is one of the successful models to study the structure and reactions of light nuclei [3]. Since the conventional cluster model does not directly treat the $D$-wave component, the strong attraction of the nucleon-nucleon interaction due to the tensor term is assumed to be renormalized into the central term of the effective interaction. However, it is well known that the ground states of the typical s-shell clusters $d$, $t$, $h$ and $^4$He have non-negligible admixtures of $D$-wave component due to the tensor interaction. Therefore, we need an extension of the cluster model to an "\textit{ab-initio}"-type one. Recently, we have studied the importance of the tensor force in the $d+d$ scattering [4] and the $d(d,\gamma)^4$He radiative capture [5] with the extended cluster model combined with the MRM. The calculated results for the transfer and capture reactions will be also shown by Arai in the proceedings. Another useful method to treat unbound states is the Complex Scaling Method (CSM) [6, 7]. In the MRM, the "\textit{ab-initio}" solution in the internal region is connected to the two body asymptotic solution (Coulomb and Whittaker functions) at a channel radius. Three and four body continuums are approximately and reasonably treated through the rearrangement in the MRM. The CSM can directly treat three-body and four-body continuums in the excited region of $^4$He. However, the computational time with the CSM is very slower than that with the MRM. Thus, the comparative analyses with both method are important. In the present paper, we present the latest studies for excited states of $^4$He with the MRM and the CSM.
2. Method

We explain the present method in brief. The wave function of the four nucleon system in the channel representation is described,

\[ \Psi_{JM\pi}^{\alpha} = A \left[ \left( \psi_{La}^{(\text{space})} \psi_{S_a}^{(\text{spin})} \right) I_a \left( \psi_{Lb}^{(\text{space})} \psi_{S_b}^{(\text{spin})} \right) I_b \right] \chi_{\alpha}(\rho_\alpha) \psi_{T_a M_a}^{(\text{isospin})} \psi_{T_b M_b}^{(\text{isospin})}, \]

(1)

where \( \psi_{La}^{(\text{space})} \), \( \psi_{S_a}^{(\text{spin})} \) and \( \psi_{T_a M_a}^{(\text{isospin})} \) denote the space, spin and isospin parts of the cluster wave function. \( \chi_{\alpha} \) is a relative wave function which is labeled by a channel \( \alpha \). The matrix elements can easily calculated by the so-called \( f \)-function in the global vector representation[8, 4]. In order to treat unbound states, we use with the MRM and the CSM. Both method have been applied many systems, and the detail of the methods was written in [2, 6].

The four nucleon Hamiltonian is described as

\[ H = \sum_{i=1}^{4} T_i - T_{\text{c.m.}} + \sum_{i<j}^{4} V_{ij} + \sum_{i<j<k}^{4} V_{ijk}, \]

(2)

where \( T_i \) is a kinetic energy of each nucleon, \( T_{\text{c.m.}} \) is a kinetic energy of the center of mass motion, \( V_{ij} \) is a two nucleon interaction and \( V_{ijk} \) is a three nucleon interaction. We want to compare the phase shifts obtained with two Hamiltonians that differ in the type of two nucleon interactions. One is a realistic interaction called the AV8′ potential [9] that includes central, tensor and spin-orbit components. We also add an effective three-nucleon force (TNF) in order to reproduce reasonably the binding energies of \( t, h \) and \( ^4\text{He} \) [10], which makes reasonable thresholds. Another is an effective central interaction called the Minnesota (MN) potential [11], which reproduce reasonably the binding energies of \( d, t, h \) and \( ^4\text{He} \), though it has central terms alone (with an exchange parameter \( u = 1 \)). The Coulomb potential is included for both potentials.

![Figure 1](image_url)

**Figure 1.** The calculated two-body and three-body threshold energies for AV8′+TNF, MN and the experimental threshold energy. The dashed line is the \( p+p+n+n \) four-body threshold.
3. Results

Figure 1 displays two-body and three-body decay thresholds above 20 MeV excited energy region. The lowest three thresholds are corresponding to two-body decay physical channels: $t+p$, $h+n$, $d+d$. The next lowest threshold are corresponding to a three-body decay physical channel: $d+p+n$. In the present calculation, we reproduce the experimental threshold position both for the AV8′+TNF and MN potentials as seen in Figure 1.

If we directly diagonalize the hamiltonian matrix with a bound state approximation, there are many continuum states whose origin are two-body, three-body and four-body continuum states. By using the complex scaling method, we can easily distinguish these continuum states because the complex eigenvalues corresponding continuum states are the corresponding so-called “$2\theta$-lines” [6]. Furthermore, we can get the bound state solutions on the negative real axis as usual, and also the resonant solution in the wedged region whose real part and imaginarily part are corresponding to the resonance energy and the decay width ($\Gamma/2$), respectively. In Figure 2, we can see continuum lines of $t+p$, $h+n$, $d+d$, $d+p+n$, $p+p+n+n$, whose starting points are threshold energies. Furthermore, we get the bound ground state solution of $^4$He just on the negative real axis.

Figure 3 displays the $^1S_0$ elastic-scattering phase shift ($d+d$) obtained with the AV8′+TNF potential. The dotted line is the phase shift calculated with $\{d(1^+)+d(1^+)\}$ channel (deuteron spin $I_d = 1^+$), and the dashed line is the phase shift with $\{d(1^+)+d(1^+)\}&\{d(0^+)+d(0^+)\}$ channels ($I_d = 1^+, 0^+$). The phase shifts calculated by including further excited deuterons are also plotted by the dash-dotted and solid lines that correspond to the channels ($I_d \leq 2^+$) and ($I_d \leq 3^+$), respectively. A naive picture that the $^1S_0$ elastic-scattering phase shift might ($d+d$) be well described in channel $d(1^+)+d(1^+)$ alone completely breaks down in the case of the AV8′+TNF potential.

Because the deuteron has a virtual state $\bar{d}$ with $0^+$, it is reasonable that the inclusion of the $\{d(0^+)+d(0^+)\}$ channel gives rise to a considerable attractive effect of several tens of degrees on the phase shift, as shown by the dashed line of Figure 3. However, the phase shift exhibits no converging behavior even when the higher spin states such as $d(2^+)$ and $d(3^+)$ are taken into account in the calculation. One may conclude that the deuteron is strongly distorted even in the low energy $^1S_0$ elastic scattering ($d+d$). But this should be a consequence that there exist two observed $0^+$ states below the $d+d$ threshold. Obviously the $d+d$ scattering wave function is

Figure 2. The complex scaled eigenvalue distribution for four nucleons system of $^4$He ($0^+$) with the Minnesota potential.
Figure 3. $^1S_0$ elastic-scattering phase shift calculated with the AV8$'$+TNF potential. The phase shifts are all obtained within the $d+d$ channels. The set of included channels is successively increased from $\{d(1^+)+d(1^+)\}$ (dotted) to $\{d(1^+)+d(1^+)&d(0^+)+d(0^+)\}$ (dashed), $\{d(1^+)+d(1^+)&d(0^+)+d(0^+)&d(2^+)+d(2^+)\}$ (dash-dotted), and $\{d(1^+)+d(1^+)&d(0^+)&d(2^+)+d(2^+)&d(3^+)+d(3^+)\}$ (solid).

subject to the structure of those states in the internal region and the deuteron can not remain a simple deuteron ground state of $1^+$. Especially, the second $0^+$ state of $^4$He lying about 4 MeV below the $d+d$ threshold, which is known to have a $3N+N$ cluster structure [12, 10]. This $3N+N$ state together with the ground state influence to the $d+d$ scattering state.

Figure 4 displays the $^1S_0$ elastic-scattering phase shifts ($d+d$) obtained with the AV8$'$+TNF potential (left) and the MN potential (right). The $\{2N+2N\} & \{3N+N\}$ calculation (solid line in Figure 4) means that major $2N+2N$ and $3N+N$ channels are coupled. The $R$-matrix

Figure 4. $^1S_0$ d+d elastic-scattering phase shift calculated with the AV8$'$+TNF (left) and MN (right) potentials. The solid line is a FULL calculation, while the dotted line is a $d+d$ channel calculation. Crosses correspond to the $R$-matrix analysis of Ref. [13].
analysis (crosses) [13] is reproduced well by both the AV8'+TNF and MN potentials with the \{2N+2N\} & \{3N+N\} calculation. Compared to the uncoupled phase shift (dotted line), one clearly sees that the 3N+N channel produces a very large effect on the d+d elastic phase shift, especially in the case of the AV8'+TNF potential. The slow convergence seen in Figure 3 is thus attributed to the neglect of the 3N+N channel, indicating that a proper account of the \(^1S_0\) d+d elastic phase shift at low energy can be possible only when the coupled channels \{d(1^+)+d(1^+)\} + \{d(0^+)+d(0^+)\} + \{t(1/2^+)+p(1/2^+)\} + \{h(1/2^+)+n(1/2^+)\} are considered. Thus, the slow convergence in Figure 3 suggests that the 2N+2N partition is not an economical way to include the effects of the 3N+N channel. In the case of the MN potential (right panel in Figure 4), the situation is very different from the AV8'+TNF case. The channel coupling effect is rather modest, and the size of the \(^1S_0\) elastic phase shift (d+d) is already accounted for mostly in the d+d channel calculation.

4. Summary
We have investigated the influence of clusters’ distortion in the low-energy \(^1S_0\) elastic scattering phase shift using a microscopic cluster model with the global vector method. We showed that the tensor interaction changes the phase shifts very much by comparing a realistic interaction (AV8’) and an effective interaction (MN). For the realistic interaction, the calculated \(^1S_0\) phase shift shows that the t+p and h+n channels strongly couple with the d+d channel because of the tensor interaction. On the contrary, the coupling of these 3N+N channels plays a relatively minor role for the case of the effective interaction because of the absence of tensor term. In the future work, we want to analyze the details of the excited structure of \(^4\)He applying the CSM.

Acknowledgments
The main part of the present work presents research results of Bilateral Joint Research Projects of the JSPS (Japan) and the FNRS (Belgium) collaborated with K. Arai, Y. Suzuki, P. Descouvemont and D. Baye. And also, this work is supported by a Grant-in-Aid for Scientific Research (No.24540262). The author thank A.T. Kruppa and R.G.lovaz for fruitful discussions for the application of the CSM to the four nucleons system, which is supported by the short visiting program and Bilateral Joint Research Projects between the JSPS and the HAS(Hungary). A part of computational calculations was carried out in T2K-Tsukuba and SR16000(Yukawa Institute).

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