Bianchi Type IX M-Branes

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Abstract

We present new M2 and M5 brane solutions in M-theory based on transverse self-dual Bianchi type IX space. All the other recently M2 and M5 branes constructed on transverse self-dual Taub-NUT, Eguchi-Hanson and Atiyah-Hitchin spaces are special cases of this solution. The solution provides a smooth transition from Eguchi-Hanson type I based M branes to corresponding branes based on Eguchi-Hanson type II space. All the solutions can be reduced down to ten dimensional fully localized intersecting brane configurations.

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1 Introduction

Fundamental M-theory in the low-energy limit is generally believed to be effectively described by $D = 11$ supergravity \[1, 2, 3\]. This suggests that brane solutions in the latter theory furnish classical soliton states of M-theory, motivating considerable interest in this subject. There is particular interest in supersymmetric $p$-brane solutions that saturate the Bogomol’nyi-Prasad-Sommerfield (BPS) bound upon reduction to 10 dimensions. Some supersymmetric solutions of two or three orthogonally intersecting 2-branes and 5-branes in $D = 11$ supergravity were obtained some years ago \[4\], and more such solutions have since been found \[5\].

Recently interesting new supergravity solutions for localized D2/D6, D2/D4, NS5/D6 and NS5/D5 intersecting brane systems were obtained \[6, 7, 8\]. By lifting a D6 (D5 or D4)-brane to four-dimensional Taub-NUT/Bolt, Eguchi-Hanson and Atiyah-Hitchin geometries embedded in M-theory, these solutions were constructed by placing M2- and M5-branes in the Taub-NUT/Bolt, Eguchi-Hanson and Atiyah-Hitchin background geometries. The special feature of these constructions is that the solution is not restricted to be in the near core region of the D6 (D5 or D4)-brane.

Taub-NUT, Eguchi-Hanson (the latter space will be referred to as Eguchi-Hanson type II in this paper) and Atiyah-Hitchin spaces are each special cases of the Bianchi type IX space. The Bianchi type IX spaces were used recently for construction cohomogeneity two metrics of $G_2$ holonomy which are foliated by twistor spaces \[9\]. The twistor spaces are two-sphere bundles over Bianchi type IX Einstein metrics with self-dual Weyl tensor.

Since the building blocks of M-theory are M2- and M5-branes, it is natural to investigate the possibility of placing M2- and M5-branes in the Bianchi type IX background space. This is the subject of the present paper, in which we consider the embedding of Bianchi type IX geometry in M-theory with an M2- or M5-brane. For all of the different solutions we obtain, 1/4 of the supersymmetry is preserved as a result of the self-duality of the Bianchi type IX metric. We then compactify these solutions on a circle, obtaining the different fields of type IIA string theory. Explicit calculation shows that in all cases the metric is asymptotically (locally) flat, though for some of the compactified solutions the type IIA dilaton field diverges at infinity.

The outline of this paper is as follows. In section 2 we discuss briefly the field equations of supergravity, the M2- and M5-brane metrics and the Killing spinor equations. In section 3, we present the different M2-brane solutions that preserve 1/4 of the supersymmetry. We find type IIA D2⊥D6(2) intersecting brane solutions upon dimensional reduction. In section 4 the alternative M2-brane solutions are presented. These solutions are obtained by continuation of the real separation constant into a pure imaginary separation constant. In section 5 we present different M5 solutions that preserve 1/4 of the supersymmetry. In section 6 we consider the decoupling limit of these solutions, especially M2 brane solutions.
2 M2- and M5- Branes and Kaluza-Klein Reduction

The equations of motion for eleven dimensional supergravity when we have maximal symmetry (i.e. for which the expectation values of the fermion fields is zero), are

\[ R_{mn} - \frac{1}{2} g_{mn} R = \frac{1}{3} \left[ F_{mpqr} F_n^{pq} - \frac{1}{8} g_{mn} F_{pqrs} F^{pqrs} \right] \]  

(2.1)

\[ \nabla_m F^{mnpq} = -\frac{1}{576} \varepsilon_{m_1 \ldots m_8 n p q} F_{m_1 \ldots m_4} F_{m_5 \ldots m_8} \]  

(2.2)

where the indices \( m, n, \ldots \) are 11-dimensional world space indices. For an M2-brane, we use the metric and four-form field strength

\[ ds^2_{11} = H(y, r)^{-2/3} \left( -dt^2 + dx_1^2 + dx_2^2 \right) + H(y, r)^{1/3} \left( ds_4^2(y) + ds_2^2(r) \right) \]  

(2.3)

and non-vanishing four-form field components

\[ F_{tx_1 x_2 y} = -\frac{1}{2H^2} \frac{\partial H}{\partial y} , \quad F_{tx_1 x_2 r} = -\frac{1}{2H^2} \frac{\partial H}{\partial r} . \]  

(2.4)

and for an M5-brane, the metric and four-form field strength are

\[ ds^2 = H(y, r)^{-1/3} \left( -dt^2 + dx_1^2 + \ldots + dx_5^2 \right) + H(y, r)^{2/3} \left( dy^2 + ds_4^2(r) \right) \]  

(2.5)

\[ F_{m_1 \ldots m_4} = \frac{\alpha}{2} \varepsilon_{m_1 \ldots m_5} \partial^{m_5} H , \quad \alpha = \pm 1 \]  

(2.6)

where \( ds_4^2(y) \) and \( ds_2^2(r) \) are two four-dimensional (Euclideanized) metrics, depending on the non-compact coordinates \( y \) and \( r \), respectively and the quantity \( \alpha = \pm 1 \), which corresponds to an M5 brane and an anti-M5 brane respectively. The general solution, where the transverse coordinates are given by a flat metric, admits a solution with 16 Killing spinors \[11\].

The 11D metric and four-form field strength can be easily reduced down to ten dimensions using the following equations

\[ g_{mn} = \begin{bmatrix} e^{-2\Phi/3} \left( g_{\alpha\beta} + e^{2\Phi} C_\alpha C_\beta \right) & \nu e^{4\Phi/3} C_\alpha \\
\nu e^{4\Phi/3} C_\beta & \nu^2 e^{4\Phi/3} \end{bmatrix} \]  

(2.7)

\[ F_{(4)} = \mathcal{F}_{(4)} + \mathcal{H}_{(3)} \wedge dx_{10} \]  

(2.8)

Here \( \nu \) is the winding number (the number of times the M5 brane wraps around the compactified dimensions) and \( x_{10} \) is the eleventh dimension, on which we compactify. We use hats in the above to differentiate the eleven-dimensional fields from the ten-dimensional ones that arise from compactification. \( \mathcal{F}_{(4)} \) and \( \mathcal{H}_{(3)} \) are the Ramond-Ramond (RR) four-form and the Neveu-Schwarz-Neveu-Schwarz (NSNS) three-form field strengths corresponding to \( A_{\alpha\beta\gamma} \) and \( B_{\alpha\beta} \).
Supersymmetric solutions to the equations of motion (2.1) and (2.2) can be constructed by looking for bosonic backgrounds that admit Killing spinors [12, 13]. For all bosonic backgrounds the supersymmetry variation of the gravitino field must vanish which yields,

$$\partial_m \epsilon + \frac{1}{4} \omega_{abm} \Gamma^{ab} \epsilon + \frac{1}{144} \Gamma_m^{npqr} F_{npqr} \epsilon - \frac{1}{18} \Gamma^{npqr} F_{mpqr} \epsilon = 0$$  \hspace{1cm} (2.9)$$

where $\epsilon$ is 32-component Majorana Killing spinor. The number of non-trivial solutions to the Killing spinor equation (2.9) determines the amount of supersymmetry of the solution [14].

In equation (2.9) the $\omega$’s are the spin connection coefficients and $\Gamma^{a_1 \ldots a_n} = \Gamma_{[a_1 \ldots a_n]}$. The indices $a, b, \ldots$ are 11 dimensional tangent space indices and the $\Gamma^a$ matrices are the eleven dimensional equivalents of the four dimensional Dirac gamma matrices, and must satisfy the Clifford algebra

$$\{ \Gamma^a, \Gamma^b \} = -2 \eta^{ab}$$  \hspace{1cm} (2.10)$$

In ten dimensional type IIA string theory, we can have D-branes or NS-branes. Dp-branes can carry either electric or magnetic charge with respect to the RR fields; the metric takes the form [11]

$$ds^2_{10} = f^{-1/2} (-dt^2 + dx_1^2 + \ldots + dx_p^2) + f^{1/2} (dx_{p+1}^2 + \ldots + dx_9^2)$$  \hspace{1cm} (2.11)$$

where the harmonic function $f$ generally depends on the transverse coordinates.

An NS5-brane carries a magnetic two-form charge; the corresponding metric has the form

$$ds^2_{10} = -dt^2 + dx_1^2 + \ldots + dx_5^2 + f (dx_6^2 + \ldots + dx_9^2)$$  \hspace{1cm} (2.12)$$

In what follows we will obtain a mixture of D-branes and NS-branes.

3 Embedding triaxial Bianchi type IX space in an M2-brane metric

The eleven dimensional M2-brane with an embedded transverse triaxial Bianchi type IX space is given by the following metric

$$ds^2_{11} = H(y, r)^{-2/3} (-dt^2 + dx_1^2 + dx_2^2) + H(y, r)^{1/3} (dy^2 + y^2 d\Omega_3^2 + ds^2_{\text{Bianchi IX}})$$  \hspace{1cm} (3.1)$$

and non-vanishing four-form field components

$$F_{tx1x2y} = -\frac{1}{2H^2} \frac{\partial H}{\partial y}, \quad F_{tx1x2r} = -\frac{1}{2H^2} \frac{\partial H}{\partial r}.$$  \hspace{1cm} (3.2)$$

The triaxial Bianchi type IX metric $ds^2_{\text{Bianchi IX}}$ is locally given by the following metric with an $SU(2)$ or $SO(3)$ isometry group [15]

$$ds^2_{\text{Bianchi IX}} = e^{2(A(\zeta)+B(\zeta)+C(\zeta))} ds^2 + e^{2A(\zeta)} a_1^2 + e^{2B(\zeta)} a_2^2 + e^{2C(\zeta)} a_3^2$$  \hspace{1cm} (3.3)$$
with
\[ \sigma_1 = d\psi + \cos \theta d\phi \]
\[ \sigma_2 = -\sin \psi d\theta + \cos \psi \sin \theta d\phi \]
\[ \sigma_3 = \cos \psi d\theta + \sin \psi \sin \theta d\phi \]
where \( \sigma_i \) are Maurer-Cartan one-forms with the property
\[ d\sigma_i = \frac{1}{2} \varepsilon_{ijk} \sigma_j \wedge \sigma_k. \] (3.5)

We note that the metric on the \( \mathbb{R}^4 \) (with a radial coordinate \( R \) and Euler angles \( (\theta, \phi, \psi) \) on an \( S^3 \)) could be written in terms of Maurer-Cartan one-forms via
\[ ds^2 = dR^2 + \frac{R^2}{4} (\sigma_1^2 + \sigma_2^2 + \sigma_3^2). \] (3.6)

with \( \sigma_1^2 + \sigma_2^2 \) is the standard metric of \( S^2 \) with unit radius; \( 4(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) \) gives the same for \( S^3 \). The metric (3.3) satisfies Einstein’s equations provided
\[
\frac{dA}{d\zeta} = \frac{1}{2} \left\{ e^{2A} - (e^{2B} - e^{2C})^2 \right\} - 4 \frac{e^{4A} + e^{4B} + e^{4C}}{4}
\]
and
\[
\frac{dB}{d\zeta} = \frac{1}{2} \left\{ e^{2B} - (e^{2C} - e^{2A})^2 \right\} - 4 \frac{e^{4B} + e^{4C} + e^{4A}}{4}
\]
Moreover self-duality of the curvature implies
\[
\frac{dA}{d\zeta} = \frac{1}{2} \left\{ e^{2B} + e^{2C} - e^{2A} \right\} - \alpha_1 e^{B+C}
\]
\[
\frac{dB}{d\zeta} = \frac{1}{2} \left\{ e^{2B} + e^{2C} - e^{2A} \right\} - \alpha_2 e^{A+C}
\]
\[
\frac{dC}{d\zeta} = \frac{1}{2} \left\{ e^{2A} + e^{2B} - e^{2C} \right\} - \alpha_3 e^{A+B}
\] (3.9)

where three constant numbers \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) must satisfy \( \alpha_1 \alpha_2 = \alpha_3, \alpha_2 \alpha_3 = \alpha_1 \) and \( \alpha_3 \alpha_1 = \alpha_2 \).

Choosing \( (\alpha_1, \alpha_2, \alpha_3) = (1, 1, 1) \), yields the Atiyah-Hitchin metric \[16\] in the form of (3.3) with
\[
A = \frac{1}{2} \ln(\frac{2 \vartheta_2 \vartheta_4}{\vartheta_3})
\]
\[
B = \frac{1}{2} \ln(\frac{2 \vartheta_2 \vartheta_4}{\vartheta_3})
\]
\[
C = \frac{1}{2} \ln(\frac{2 \vartheta_2 \vartheta_4}{\vartheta_3})
\] (3.10)

where the \( \vartheta \)'s are Jacobi Theta functions with complex modulus \( i\zeta \). Since Atiyah-Hitchin space embedded into the transverse geometry has been previously used to construct different M2 and M5 brane solutions \[8\], we don’t consider this case here.
By choosing \((\alpha_1, \alpha_2, \alpha_3) = (0, 0, 0)\) the differential equations (3.7), (3.8) and (3.9) can be solved exactly to give a metric of the form

\[
ds^2 = \frac{dr^2}{\sqrt{F(r)}} + \frac{r^2}{4} \sqrt{F(r)} \left( \frac{\sigma_1^2}{1 - a_1^4 r^4} + \frac{\sigma_2^2}{1 - a_2^4 r^4} + \frac{\sigma_3^2}{1 - a_3^4 r^4} \right) \quad (3.11)
\]

where

\[
F(r) = \prod_{i=1}^{3} \left(1 - \frac{a_i^4}{r^4}\right) \quad (3.12)
\]

and \(a_1, a_2\) and \(a_3\) are constants.

The metric (3.1) with \(ds_{\text{Bianchi IX}}^2\) given by (3.11) is a solution to the eleven dimensional supergravity equations provided \(H(y, r)\) is a solution to the differential equation

\[
y \sqrt{A_1 A_2 A_3} \frac{\partial^2 H}{\partial r^2} + \{6 + r \left( \frac{1}{A_1} \frac{dA_1}{dr} + \frac{1}{A_2} \frac{dA_2}{dr} + \frac{1}{A_3} \frac{dA_3}{dr} \right) \} \frac{\partial H}{\partial r} + \{2y r \frac{\partial^2 H}{\partial y^2} + 6r \frac{\partial H}{\partial y} \} = 0.
\]

(3.13)

where \(A_i = 1 - \frac{a_i^4}{r^4}\). This equation is straightforwardly separable. Substituting

\[
H(y, r) = 1 + Q_{M2} Y(y) R(r) \quad (3.14)
\]

where \(Q_{M2}\) is the charge on the M2 brane, we arrive at two differential equations for \(Y(y)\) and \(R(r)\). One solution of the differential equation for \(Y(y)\) is

\[
Y(y) = \frac{J_1 \left( \frac{c}{\sqrt{2}} y \right)}{y} \quad (3.15)
\]

which has the preferred damped oscillating behavior at infinity. The other solution is the Bessel function of the second kind, which diverges at infinity. The differential equation for \(R(r)\) is

\[
2r A_1 A_2 A_3 \frac{d^2 R_c(r)}{dr^2} + \{6 A_1 A_2 A_3 + r (A_2 A_3 \frac{dA_1}{dr} + A_3 A_1 \frac{dA_2}{dr} + A_1 A_2 \frac{dA_3}{dr}) \} \frac{dR_c(r)}{dr} - c^2 r \sqrt{A_1 A_2 A_3} R_c(r) = 0
\]

(3.16)

where \(c\) is the separation constant.

Although equation (3.16) does not have any analytic closed solution, we can solve it numerically. Several typical numerical solutions of (3.16) are given in figures 3.1, 3.2 and 3.3 for different values of \(a_1, a_2\) and \(a_3\) where we set \(a_1 \leq a_2 \leq a_3\) and so \(r > a_3\) in the metric (3.11).

We note that by increasing \(a_3\), the M2 brane metric function for constant \(r\) increases. The most general solution for the metric function will be a superposition of all possible solutions and takes the form

\[
H_{\text{Bianchi IX}}(y, r) = 1 + Q_{M2} \int_0^\infty \frac{dc}{y} J_1 \left( \frac{c}{\sqrt{2}} y \right) R_c(r) \quad (3.17)
\]
Figure 3.1: Numerical solution of the radial equation (3.16) for $R_k/10^8$ as a function of $\frac{1}{r-a_3}$, where we set $a_1 = 1, a_2 = 2$ and $a_3 = 3$ for simplicity. So for $r \approx a_3$ $R$ diverges. Note that the plot is only reliable for large $\frac{1}{r-a_3}$.

Figure 3.2: Numerical solution of the radial equation (3.16) for $R_k/10^8$ as a function of $\frac{1}{r-a_3}$, where we set $a_1 = 1, a_2 = 2$ and $a_3 = 500$ for simplicity. So for $r \approx a_3$ $R$ diverges. Note that the plot is only reliable for large $\frac{1}{r-a_3}$.

Figure 3.3: Numerical solution of the radial equation (3.16) for $R_k/10^8$ as a function of $\frac{1}{r-a_3}$, where we set $a_1 = 1, a_2 = 2$ and $a_3 = 1000$ for simplicity. So for $r \approx a_3$ $R$ diverges. Note that the plot is only reliable for large $\frac{1}{r-a_3}$.
as the metric function of the M2-brane solution (3.11) with transverse Bianchi type IX space, where the measure function is chosen via dimensional analysis.

An interesting result is obtained by taking the special case

\[ a_1 = 0 \]
\[ a_2 = 2kc \]
\[ a_3 = 2c \]

(3.18)

where we choose \( 0 \leq k \leq 1 \) and \( c > 0 \). For the special value of \( k = 0 \), where the smaller two \( a \)'s coincide, we obtain the following metric

\[
d s^2_{EHl} = \frac{d r^2}{f(r)} + \frac{r^2}{4} f(r) \left( d \theta^2 + \sin^2 \theta d \phi^2 \right) + \frac{r^2}{4 f(r)} (d \psi + \cos \theta d \phi)^2 \]

(3.19)

which is the Eguchi-Hanson type I metric with \( f(r) = (1 - (2c)^4 r^4)^{1/2} \). In the other extreme case where \( k = 1 \), the larger two \( a \)'s coincide and we obtain the Eguchi-Hanson type II metric

\[
d s^2_{EHII} = \frac{d r^2}{f^2(r)} + \frac{r^2}{4} f^2(r) \sigma^2_1 + \frac{r^2}{4} (\sigma^2_2 + \sigma^2_3) \]

(3.20)

This metric can be changed to the standard form given in [17]

\[
d s^2_{EH} = \frac{r^2}{4 g(r)} [d \psi + \cos(\theta) d \phi]^2 + g(r) d r^2 + \frac{r^2}{4} \left( d \theta^2 + \sin^2(\theta) d \phi^2 \right) \]

(3.21)

\[
g(r) = \left( 1 - \frac{a^4}{r^4} \right)^{-1}. \]

(3.22)

by making the substitutions \( \sigma_1 \leftrightarrow \sigma_3 \) and \( 2c = a \) in (3.20). Another form of the Eguchi-Hanson type I metric can be written as [18]

\[
d s^2_{EHl} = \tilde{f}^2(r) d r^2 + \frac{r^2}{4} \tilde{g}^2(r) \left( d \theta^2 + \sin^2 \theta d \phi^2 \right) + \frac{r^2}{4} (d \psi + \cos \theta d \phi)^2 \]

(3.23)

where

\[
\tilde{f}(r) = \frac{1}{2} (1 + \frac{1}{\sqrt{1 - \frac{a^4}{r^4}}})
\]

(3.24)

\[
\tilde{g}(r) = \frac{1}{2} (1 + \sqrt{1 - \frac{a^4}{r^4}})
\]

By increasing the parameter \( k \) from 0 to 1 we obtain series of M2 brane solutions that provide a smooth transition from Eguchi-Hanson type I M branes to corresponding branes based on Eguchi-Hanson type II space.

The second M2-brane solution containing Bianchi type IX in its transverse directions is

\[
d s^2_{11} = H(y, r)^{-2/3} \left( - d t^2 + d x_1^2 + d x_2^2 \right) + H(y, r)^{1/3} \left( d s^2_{TN} + d s^2_{Bianchi IX} \right) \]

(3.25)
where we substitute a Taub-NUT space

\[ ds^2_{TN} = f(y) \left( dy^2 + y^2 (d\alpha^2 + \sin^2(\alpha) d\beta^2) \right) + \left( \frac{4n}{f(y)} \right)^2 \left( d\sigma + \frac{1}{2} \cos(\alpha) d\beta \right)^2 \]  \hspace{1cm} (3.26)

\[ f(y) = \left( 1 + \frac{2n}{y} \right). \]  \hspace{1cm} (3.27)

for the other half of the transverse space. In this case, after separation of variables by the relation (3.14), we find exactly the same differential equation for \( R(r) \) as given in equation (3.16). The solution of the differential equation for \( Y(y) \) that has a damped oscillating behavior at infinity up to a constant is

\[ Y(y) = \frac{(i)^{\frac{1}{2}} W_M \left( \frac{-icn}{\sqrt{2}}, 1/2, \sqrt{2}icy \right)}{y} \]  \hspace{1cm} (3.28)

where \( W_M \) is the Whittaker function. The final general solution will be a superposition of all possible solutions in the form

\[ H_{TN} \otimes \text{Bianchi IX}(y, r) = 1 - i \frac{Q M_2}{y} \int_0^\infty d\sigma^4 W_M \left( \frac{-icn}{\sqrt{2}}, 1/2, \sqrt{2}icy \right) R_c(r) \]  \hspace{1cm} (3.29)

as the metric function of M2-brane solution (3.25). Note that the Whittaker function in the integrand is pure imaginary, yielding a real-valued \( H_{TN} \otimes \text{Bianchi IX}(y, r) \).

Compactifying over the circle parametrized by \( \sigma \) (and noting that \( \partial/\partial \sigma \) is a Killing vector) we find the NSNS fields

\[ \Phi = \frac{3}{4} \ln \left( \frac{H_{1/3}^{1/3} \otimes \text{Bianchi IX}}{f} \right) \]  \hspace{1cm} (3.30)

and RR fields

\[ C_\beta = 2n \cos(\alpha) \]

\[ A_{\alpha x_1 x_2} = H_{1/3}^{1/3} \otimes \text{Bianchi IX}. \]  \hspace{1cm} (3.31)

The metric in ten dimensions will be given by

\[ ds^2_{10} = H_{TN} \otimes \text{Bianchi IX}(y, r)^{-1/2} f^{-1/2} \left( -dt^2 + dx_1^2 + dx_2^2 \right) + H_{TN} \otimes \text{Bianchi IX}(y, r)^{1/2} f^{-1/2} ds^2_{\text{Bianchi IX}} + H_{TN} \otimes \text{Bianchi IX}(y, r)^{1/2} f^{1/2} \left( dy^2 + y^2 (d\alpha^2 + \sin^2(\alpha) d\beta^2) \right). \]  \hspace{1cm} (3.32)

This represents a D2\( \perp \)D6(2) system. To calculate how much supersymmetry is preserved by this solution in eleven dimensions, we use the Killing spinor equation (2.9). The Killing spinor equation is the supersymmetry variation of the gravitino field in \( D = 11 \) supergravity.

As we mentioned in section (2), we consider bosonic sector of eleven dimensional supergravity. Hence to make the vanishing of the gravitino consist with the presence of supersymmetry,
Figure 3.4: Numerical solution of equation (3.36) for $Y_k/10^5$ as a function of $1/y$ for non-zero separation constant $k$. The Eguchi-Hanson parameter $a$ is set to one and so for $y \approx a$, the function $Y_k$ diverges and for $y \approx \infty$, it vanishes.

one has to impose the constraint that the supersymmetry variations acting on such a bosonic background cannot restore a non vanishing gravitino. This means that the supersymmetry variation of the gravitino evaluated in the bosonic sector must vanish which yields equation (2.9). After a straightforward but lengthy calculation we have checked that for our brane solution, the supersymmetry variation vanishes with an arbitrary choice of one quarter of the components of the spinor $\epsilon$. More specifically, half of the components of the spinor $\epsilon$ is removed due to the presence of the brane and another half is removed due to the self-duality of Bianchi type IX space, so the solution preserves $1/4$ of the supersymmetry. We also have explicitly checked that the above 10-dimensional metric, with the given dilaton, one and three forms, is a solution to the 10-dimensional supergravity equations of motion.

A third possible M2-brane solution is given by

$$ds_{11}^2 = H(y, r)^{-2/3}(-dt^2 + dx_1^2 + dx_2^2) + H(y, r)^{1/3}(ds_{EH}^2 + ds_{Bianchi\ IX}^2) \quad (3.33)$$

where

$$ds_{EH}^2 = \frac{y^2}{4g(y)}[d\sigma + \cos(\alpha)d\beta]^2 + g(y)dy^2 + \frac{y^2}{4}(d\alpha^2 + \sin^2(\alpha)d\beta^2) \quad (3.34)$$

$$g(y) = \left(1 - \frac{a^4}{y^4}\right)^{-1}. \quad (3.35)$$

is the Eguchi-Hanson type II metric. In this case, after separation of variables by the relation (3.14), we find the same differential equation for $R(r)$ as given in equation (3.16). The differential equation for $Y(y)$ is

$$2y(y^4 - a^4)Y''(y) + 2(3y^4 + a^4)Y'(y) + c^2y^5Y(y) = 0. \quad (3.36)$$

While an analytic closed solution for the differential equation (3.36) is not available, the numerical solution shows that it has a damped oscillating behavior at infinity and that it diverges at $y \simeq a$. A typical numerical solution of $Y_k(y)$ is given in figure 3.4. The general
solution will be a superposition of all possible solutions in the form

\[ H_{EH} \otimes Bianchi \ IX(y, r) = 1 + Q_{M2} \int_{0}^{\infty} d\sigma^{5} Y_{c}(y) R_{c}(r) \]  

(3.37)

for the metric function of the M2-brane solution \( (3.33) \), where we have absorbed a constant into the M2-brane charge.

In this case, by compactification along the \( \sigma \) direction of Eguchi-Hanson metric, we find the NSNS fields

\[ \Phi = \frac{3}{4} \ln \left\{ \frac{H_{EH}^{1/3} \otimes Bianchi \ IX w^{2}}{4g} \right\} \]  

(3.38)

and RR fields

\[ C_{\beta} = a \cos(\alpha) \]  

(3.39)

and metric

\[ ds_{10}^{2} = \frac{w}{2} \left\{ H_{EH}^{-1/2} \otimes Bianchi \ IX g^{-1/2} \left( -dt^{2} + dx_{1}^{2} + dx_{2}^{2} \right) + H_{EH}^{1/2} \otimes Bianchi \ IX g^{-1/2} ds_{Bianchi \ IX}^{2} \right\} \]

\[ + H_{EH}^{1/2} \otimes Bianchi \ IX g^{1/2} a^{2} \left\{ dw^{2} + \frac{w^{2}}{4g} \left( d\alpha^{2} + \sin^{2} \alpha d\beta^{2} \right) \right\} \]  

(3.40)

where \( w = \frac{\alpha}{a} \). The metric describes an intersecting D2/D6 system where D2 is localized along the world-volume of the D6-brane and the world-volume of the D6 brane transverse to D2 is just Bianchi type IX space. We note that in the large \( w \) limit, the metric \( (3.40) \), reduces to the metric

\[ ds_{10}^{2} = \frac{w}{2} \left\{ -dt^{2} + dx_{1}^{2} + dx_{2}^{2} + ds_{Bianchi \ IX}^{2} + a^{2} \left( dw^{2} + \frac{w^{2}}{4g} \left( d\alpha^{2} + \sin^{2} \alpha d\beta^{2} \right) \right) \right\} \]  

(3.41)

which is again a 10D locally asymptotically flat metric with Kretchmann invariant

\[ R_{\mu \nu \rho \sigma} R^{\mu \nu \rho \sigma} = \frac{A(a, r)}{w^{2}} + \frac{224}{w^{6} a^{4}} \]  

(3.42)

which vanishes at large \( w \). \( A(a, r) \) is a complicated function of the Eguchi-Hanson parameter \( a \) and Bianchi type IX metric function \( F(r) \). All the components of the Riemann tensor in the orthonormal basis approach zero at \( w \to \infty \).

A fourth possible M2-brane solution is given by

\[ ds_{11}^{2} = H(y, r)^{-2/3} \left( -dt^{2} + dx_{1}^{2} + dx_{2}^{2} \right) + H(y, r)^{1/3} \left( ds_{AH}^{2} + ds_{Bianchi \ IX}^{2} \right) \]  

(3.43)

where

\[ ds_{AH}^{2} = \frac{c^{2}(y)}{y^{2}} dy^{2} + a^{2}(y) \sigma_{1}^{2} + b^{2}(y) \sigma_{2}^{2} + c^{2}(y) \sigma_{3}^{2} \]  

(3.44)
is the Atiyah-Hitchin metric and \( \sigma \) are Maurer-Cartan one-forms on the space with coordinates \( \sigma, \alpha \) and \( \beta \). The metric functions \( a(y), b(y) \) and \( c(y) \) can be expressed explicitly in terms of elliptic integrals \( K \) and \( E \) \cite{8} and the coordinate \( r \) ranges over the interval \([n\pi, \infty)\) where the positive number \( n \) is a constant number with unit of length that is related to NUT charge of metric at infinity obtained from Atiyah-Hitchin metric. In fact as \( y \to \infty \), the metric (3.44) reduces to

\[
\begin{align*}
\text{ds}_{AH}^2 & \to (1 - \frac{2n}{y})(dy^2 + y^2 d\alpha^2 + y^2 \sin^2 \alpha d\beta^2) + 4n^2 (1 - \frac{2n}{y})^{-1}(d\sigma + \cos \alpha d\beta)^2
\end{align*}
\] (3.45)

which is the well known Euclidean Taub-NUT metric with a negative NUT charge \( N = -n \).

In this case, after separation of variables by the relation (3.14), we find the same differential equation for \( R(r) \) as given in equation (3.16). The differential equation for \( Y(y) \) turns out to be

\[
2a(y)b(y)y^2Y''_c(y) + \{2a(y)b(y)y + 2a(y)y^2b(y)y + 2b(y)y^2d(y)\}\{Y''_c(y) + c^2a(y)b(y)c^2(y)Y_c(y) = 0. \}
\] (3.46)

While an analytic closed solution for the differential equation (3.36) is not available, we have found numerical solutions of this equation in paper \cite{8}. The numerical solutions show for \( y \approx n\pi \), function \( Y_c(y) \) diverges logarithmically by \( \ln(\frac{1}{y-n\pi}) \) and for large \( y \), it has oscillating damped behavior. A typical numerical solution of \( Y_k(y) \) is given in figure 3.4. So, the general solution for M2 metric function will be a superposition of all possible solutions in the form

\[
H_{AH @ Bianchi IX}(y, r) = 1 + Q_{M2} \int_0^\infty d\sigma c^5 R_c(r)Y_c(y)
\] (3.47)

Finally, the fifth possible M2-brane solution is given by

\[
\begin{align*}
ds_{11}^2 & = H(r_1, r_2) \frac{-2n}{y}\left( -dr^2 + dx_1^2 + dx_2^2 \right) + H(r_1, r_2)^{1/3}\left( ds_{Bianchi IX_1}^2 + ds_{Bianchi IX_2}^2 \right)
\end{align*}
\] (3.48)

where \( ds_{Bianchi IX_1}^2 \) and \( ds_{Bianchi IX_2}^2 \) are given by two copies of (3.11) with coordinate systems \( (r_1, \theta_1, \phi_1, \psi_1) \) and \( (r_2, \theta_2, \phi_2, \psi_2) \) and two sets of metric functions \( (A_1, A_2, A_3) \) and \( (\hat{A}_1, \hat{A}_2, \hat{A}_3) \), respectively. In this case, after separation of variables by the relation

\[
H(r_1, r_2) = 1 + Q_{M2}R_c(r_1)\hat{R}_c(r_2)
\] (3.49)

we find two differential equations

\[
2r_1A_1A_2A_3\frac{d^2 R_c(r_1)}{dr_1^2} + \{6A_1A_2A_3 + r_1(A_2^2A_3 + A_2A_3^2 + A_1A_2^2 + A_1A_3^2)\} \frac{dR_c(r_1)}{dr_1} - c^2r_1\sqrt{A_1A_2A_3R_c(r_1)} = 0
\] (3.50)

\[
2r_2\hat{A}_1\hat{A}_2\hat{A}_3\frac{d^2 \hat{R}_c(r_2)}{dr_2^2} + \{6\hat{A}_1\hat{A}_2\hat{A}_3 + r_2(\hat{A}_2^2\hat{A}_3 + \hat{A}_2\hat{A}_3^2 + \hat{A}_1\hat{A}_2^2 + \hat{A}_1\hat{A}_3^2)\} \frac{d\hat{R}_c(r_2)}{dr_2} + c^2r_2\sqrt{\hat{A}_1\hat{A}_2\hat{A}_3\hat{R}_c(r_2)} = 0.
\] (3.51)
Figure 4.1: Numerical solution of the radial equation \(3.51\) for \(\hat{R}_c/10^6\) as a function of \(\frac{1}{r-a_3}\), where we set \(a_1 = 1, a_2 = 2\) and \(a_3 = 3\) for simplicity. So for \(r \approx a_3\) \(R\) diverges. Note that the plot is only reliable for large \(\frac{1}{r-a_3}\).

The differential equation for \(R_c(r_1)\) is the same as equation \(3.16\) and typical solutions are presented in figures 3.1 - 3.3. As before, an analytic closed-form solution for the first differential equation \(3.51\) is not available. However the numerical solution presented in figure 4.1 shows that it has diverging behavior at \(r \approx a_3\) and damped oscillating behavior at infinity.

The general solution will be a superposition of all possible solutions in the form

\[ H_{\text{Bianchi IX}_1 \otimes \text{Bianchi IX}_2} (r_1, r_2) = 1 + Q_{M2} \int_0^\infty d\hat{c} c^5 \hat{R}_c(r_1) \hat{R}_c(r_2) \]

where we have absorbed additional constants into the M2-brane charge.

To summarize, all M2-brane solutions with a Bianchi type IX space in the transverse geometry preserve 1/4 of the supersymmetry. In all of these M2-brane solutions half of the supersymmetry is removed due to the presence of the M2-brane and another half is removed due to the self-duality of Bianchi type IX space.

4 A Second set of M2-brane solutions

A different set of M2-brane solutions can be obtained by reversing the sign of the separation constant \(c^2\) in the separated differential equations for \(Y(y)\) and \(R(r)\). As an example, by taking \(c \rightarrow i\hat{c}\) in the separable equations of embedded Bianchi type IX case, we find the solution of the differential equation for \(\tilde{Y}(y)\) as

\[ \tilde{Y}(y) = \frac{K_1(\sqrt{2}y)}{y} \]

where \(K_1\) is the modified Bessel function, diverging at \(y = 0\) and vanishing at infinity. The differential equation for \(\tilde{R}(r)\) is given by
Although the above equation does not have any analytic closed solution, we can solve it numerically. Typical solutions are presented in figure 4.1. The final general solution is the second M2-brane solution in (3.1), and is a superposition of all possible solutions

\[ \tilde{H}_{\text{Bianchi IX}}(y,r) = 1 + Q_{M2} \int_0^\infty d\tilde{c}c \frac{K_1(\tilde{c}^1 y)}{\tilde{c}^1} \tilde{R}(r) \]  

(4.3)

absorbing a possible constant into the charge \( Q_{M2} \).

The other two alternative solutions for the Taub-NUT \( \otimes \) Bianchi type IX and Eguchi-Hanson \( \otimes \) Bianchi type IX could be derived easily similar to the above case and so we do not present them here. In the Bianchi type IX \( \otimes \) Bianchi type IX case with metric function (3.52), the transformation \( c \to i \tilde{c} \) in (3.50) and (3.51), merely interchanges \( R_c(r_1) \to R_{\tilde{c}}(r_1) \) and \( \tilde{R}_{\tilde{c}}(r_2) \to R_c(r_2) \) and so yields no new solution.

5 Embedding Bianchi type IX space in an M5-brane metric

The eleven dimensional M5-brane metric with an embedded Bianchi type IX metric has the following form

\[ ds^2_{11} = H(y,r)^{-1/3} \left( -dt^2 + dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2 + dx_5^2 \right) + H(y,r)^{2/3} \left( dy^2 + ds^2_{\text{Bianchi IX}} \right) \]  

(5.1)

with field strength components

\[ F_{\psi \theta \phi y} = \frac{1}{16} \alpha r^3 \sin \theta \sqrt{A_1(r)A_2(r)A_3(r)} \frac{\partial H}{\partial r} \]
\[ F_{r \theta \psi} = \frac{1}{16} \alpha r^3 \sin \theta \frac{\partial H}{\partial y} \]  

(5.2)

We consider the M5-brane which corresponds to \( \alpha = +1 \); the \( \alpha = -1 \) case corresponds to an anti-M5 brane.

The metric (5.1) is a solution to the eleven dimensional supergravity equations provided \( H(y,r) \) is a solution to the differential equation

\[ 2A_1A_2A_3 \frac{\partial^2 H}{\partial r^2} + \left\{ A_2A_3 \frac{dA_1}{dr} + A_1A_2 \frac{dA_3}{dr} + A_1A_3 \frac{dA_2}{dr} + \frac{6}{r} A_1A_2A_3 \right\} \frac{\partial H}{\partial r} + 2\sqrt{A_1A_2A_3} \frac{\partial^2 H}{\partial y^2} = 0. \]  

(5.3)

This equation is straightforwardly separable. Substituting

\[ H(y,r) = 1 + Q_{M5} Y(y) R(r) \]  

(5.4)
where $Q_{M5}$ is the charge on the M5-brane. The solution of the differential equation for $Y(y)$ is
\[ Y(y) = \cos\left(\frac{c}{\sqrt{2}} y + \varsigma\right) \]  
(5.5)
and the differential equation for $R(r)$ is given by the equation (3.16). The numerical solution of this equation is presented in figures 3.1,3.2 and 3.3 for different values of $a_1, a_2$ and $a_3$. The final solution is a superposition of all possible solutions in the form of
\[ H(y, r) = 1 + Q_{M5} \int_0^\infty dc c^2 \cos\left(\frac{c}{\sqrt{2}} y\right) R_c(r) \]  
(5.6)
as the metric function of M5-brane solution (5.1), where we absorb the constant into the M5-brane charge.

A different M5-brane solution can be obtained by reversing the sign of the separation constant $c^2$ in the separated differential equations obtained from (5.3). In this case, by taking $c \to i\tilde{c}$, we find another solution in the form of
\[ \tilde{H}(y, r) = 1 + Q_{M5} \int_0^\infty d\tilde{c} c \tilde{c}^2 e^{-\tilde{c} y} \tilde{R}_c(r) \]  
(5.7)
where numerical plot of the function $\tilde{R}_c$ is given in figure 4.1. Although this is formally a solution, the integral in (5.7) is not convergent for all values of $y$. To make the integral convergent for $y < 0$, one can replace $e^{-\tilde{c} y}$ by $e^{-\tilde{c} |y|}$, but only at the price of introducing a source term at $y = 0$ in the corresponding Laplace equation for $\tilde{H}(y, r)$.

6 Decoupling limits

At low energies, the dynamics of the D2 brane decouple from the bulk, with the region close to the D6 brane corresponding to a range of energy scales governed by the IR fixed point [19]. For D2 branes localized on D6 branes, this corresponds in the field theory to a vanishing mass for the fundamental hyper-multiplets. Near the D2 brane horizon ($H \gg 1$), the field theory limit is given by
\[ g_{YM}^2 = g_s \ell_s^{-1} = \text{fixed}. \]  
(6.1)
In this limit the gauge couplings in the bulk go to zero, so the dynamics there decouple. In each of our cases above, the radial coordinates are also scaled such that
\[ Y = \frac{y}{\ell_s^2}, \quad U = \frac{r}{\ell_s^2} \]  
(6.2)
are fixed. As an example we note that this will change the harmonic function of the D6 brane in the TN $\otimes$ Bianchi IX case to the following (recall that the asymptotic radius of the 11th dimension is $R_{\infty} = 4n = g_s \ell_s$)
\[ f(y) = \left(1 + \frac{2n}{y}\right) = \left(1 + \frac{g_s \ell_s}{2y}\right) = \left(1 + \frac{g_s}{2\ell_s^2}\right) = \left(1 + \frac{g_{YM2} N_6}{2Y}\right) = f(Y) \]  
(6.3)
where we generalize to the case of $N_6$ D6 branes, giving the factor of $N_6$ in the final line above. A similar analysis shows for the EH metric, we get

$$g(y) \rightarrow \left(1 - \frac{A^4}{Y^4}\right) = g(Y)$$

(6.4)

where $a$ has been rescaled to $a = A\ell_s^{-2}$.

All of the D2 harmonic functions from the above solutions can be shown to scale as $H(Y,U) = \ell_s^{-4} h(Y,U)$. This form causes the D2-brane to warp the ALE region and the asymptotically flat region of the D6-brane geometry. The $h(Y,U)$’s are easily calculated; as an example, the TN $\otimes$ Bianchi IX function is given by

$$h_{TN \otimes \text{Bianchi IX}}(Y,U) = \int_0^\infty dPP^4 W_m \left(\frac{-i}{\sqrt{2}P g_{YM} Y} R_P(U)\right) (6.5)$$

where we used $\ell_p = g_s^{1/3} \ell_s$ to rewrite

$$Q_{M2} = 32\pi^2 N_2 \ell_s^6 = 32\pi^2 N_2 g_{YM}^2 \ell_s^8.$$  

(6.6)

In equation (6.5), $R_P(U)$ is the solution of

$$2U A_1 A_2 A_3 \frac{d^2 R_P(U)}{dU^2} + \{6A_1 A_2 A_3 + U (A_2 A_3 \frac{dA_1}{dU} + A_3 A_1 \frac{dA_2}{dU} + A_1 A_2 \frac{dA_3}{dU})\} \frac{dR_P(U)}{dU} - P^2 U \sqrt{A_1 A_2 A_3} R_P(U) = 0$$

(6.7)

where $A_i(U) = 1 - \gamma_i^4 U^4$ and we rescaled the integration variable $c$ and Bianchi IX parameters $a_i$ by $P/\ell_s$ and $\ell_s^2 \gamma_i$, respectively. Even when full analytic forms of $H(y,r)$ are not available, we can show that $H(Y,U) = \ell^{-4}_s h(Y,U)$ in the decoupling limit, due to the general forms of $H(y,r)$ we obtained above. As an example, we can see the ten-dimensional metric (3.32) scales as

$$\frac{ds_{10}^2}{\ell_s^2} = h_{TN \otimes \text{Bianchi IX}}(Y,U)^{-1/2} f(U)^{-1/2} \left(-dt^2 + dx_1^2 + dx_2^2\right) + h_{TN \otimes \text{Bianchi IX}}(Y,U)^{1/2} f(U)^{1/2} \left(\frac{du^2}{F(U)} + \frac{U^2}{4} F(U) \left(\frac{\sigma_1^2}{1 - \gamma_1^4} + \frac{\sigma_2^2}{1 - \gamma_2^4} + \frac{\sigma_3^2}{1 - \gamma_3^4}\right)\right) + h_{TN \otimes \text{Bianchi IX}}(y,r)^{1/2} f^{1/2} \left(dy^2 + Y^2 (d\alpha^2 + \sin^2(\alpha) d\beta^2)\right).$$

(6.8)

and there is only an overall normalization factor of $\ell_s^2$ in the above metric which is the expected result for a solution that is a supergravity dual of a quantum field theory.
We now want to perform an analysis of the decoupling limits of the solutions presented above. At low energies, the dynamics of IIA NS5-branes will decouple from the bulk. Near the NS5-brane horizon ($H >> 1$), we are interested in the behavior of the NS5-branes in the limit where string coupling vanishes

$$g_s \rightarrow 0$$

and

$$\ell_s = \text{fixed}. \quad (6.9)$$

In these limits, we rescale the radial coordinates such that they can be kept fixed

$$Y = \frac{y}{g_s \ell_s^2}, \quad U = \frac{r}{g_s \ell_s^2}. \quad (6.10)$$

We can show the harmonic functions (5.6) and (5.7) for the NS5-branes to rescale according to $H(Y, U) = g_s^{-2} h(Y, U)$. For example, the harmonic function (5.6) becomes

$$H(Y, U) = \pi N_5 g_s \int_0^\infty \frac{1}{g_s} dP \left( \frac{P^2}{g_s^2} \right) \cos \left( \frac{PY}{\sqrt{2}} \right) R_P(U) = \frac{h(Y, U)}{g_s^2} \quad (6.12)$$

where we have rescaled $c = P g_s^{-1} \ell_s^{-2}$ so that $h(Y, U)$ has no $g_s$ dependence and we have used $\ell_p = g_s^{1/3} \ell_s$ to rewrite $Q_{M5} = \pi N_5 \ell_p^3 = \pi N_5 g_s \ell_s^3$.

In the limit of vanishing $g_s$ with fixed $\ell_s$ (as we did in (6.9) and (6.10)), the decoupled free theory on NS5-branes should be a little string theory [20] (i.e. a 6-dimensional non-gravitational theory in which modes on the 5-brane interact amongst themselves, decoupled from the bulk).

### 7 Conclusion

By embedding Bianchi type IX space into M-theory, we have found new classes of 2-brane and 5-brane solutions to $D = 11$ supergravity. These exact solutions are new M2- and M5-brane metrics with metric functions (3.17), (3.29), (3.37), (3.52), (4.3), (5.6) and (5.7) – these are the main results of this paper. The important point is that all previously found M2 and M5 solutions based on embedded Taub-NUT, Eguchi Hanson type II and Atiyah-Hitchin spaces are special cases of our new solutions based on triaxial Bianchi type IX space. Moreover we found series of M2 (and similarly for M5) brane solutions by increasing the parameter $k$ from 0 to 1 in (3.18) that provide a smooth transition from Eguchi-Hanson type I M branes to corresponding branes based on Eguchi-Hanson type II space. The common feature of both solutions is that the brane function is a convolution of an exponentially decaying ‘radial’ function (for both branes) with a damped oscillating one. The ‘radial’ function vanishes far from the branes and diverges logarithmically near the brane core. The same logarithmic divergence near the brane happens in embedding of Eguchi-Hanson II metric in M-theory.
where the divergence is milder than \( \frac{1}{r} \), as in the case of embedding Taub-NUT space. Indeed, all of these properties of our solutions are similar to those previously obtained \[7, 8\] for the embedding of Taub-NUT, Eguchi-Hanson II and Atiyah-Hitchin spaces.

Our solutions preserve 1/4 of the supersymmetry due to the self-dual character of the Bianchi type IX metric. This is in contrast to earlier brane solutions of this type \[7\], for which M-supersymmetry could only be preserved for NUT-like transverse metrics; their bolt counterparts did not preserve any supersymmetry. Dimensional reduction of the M2 solutions to ten dimensions gives us intersecting IIA D2/D6 configurations that preserve 1/4 of the supersymmetry.

Finally we considered the decoupling limit of our solutions. In the case of embedded TN4 in M2 solution \[6\], when the D2 brane decouples from the bulk, the theory on the brane is 3 dimensional \( \mathcal{N} = 4 \) SU(2) super Yang-Mills (with eight supersymmetries) coupled to \( N_6 \) massless hypermultiplets \[21\]. This point is obtained from dual field theory and since some of our solutions preserve the same amount of supersymmetry, a similar dual field description should be attainable.

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