A New Framework for Expressing, Parallelizing and Optimizing Big Data Applications

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Abstract—The Forelem framework was first introduced as a means to optimize database queries using optimization techniques developed for compilers. Since its introduction, Forelem has proven to be more versatile and applicable beyond database applications. In this paper we show that the original Forelem framework can be used to express and optimize Big Data applications, more specifically: k-Means clustering and PageRank, resulting in automatically generated implementations of these applications. These implementations are more efficient than state-of-the-art, hand-written MPI C/C++ implementations of k-Means and PageRank, as well as significantly outperform state-of-the-art Hadoop implementations.

1 INTRODUCTION

When the Forelem framework was first introduced, it addressed the optimization of (embedded) database queries together with the wrapping C/C++ code and its associated API layer. In order to do this, database queries were automatically transformed into explicit loop structures (forelem loops), which together with the wrapping C/C++ code allowed for integral optimization. This integral optimization led to significant performance improvements of database applications.

In order to allow for this integral optimization in Forelem, database tables were translated into reservoirs of tuples and database queries were translated into forelem loop structures, which iterate over a reservoir of tuples. To ensure a versatile and flexible optimization process, each iteration of the forelem loop body was assumed to be tuple-based and atomic, allowing the specification to be inherently parallel. For database applications this restriction proved to be very natural, but surprisingly this restriction has also proven to be very natural for other applications. In essence this restriction yields a program specification which is free from common artifacts, like explicit data and loop structures, and associated data dependencies. As a result, this tuple-based and inherently parallel Forelem specification forces the program design for many applications to be reconsidered, reimplemented and broken down to their essence. This results in a more flexible, versatile and parallel implementation of the original algorithm. On top of this, the Forelem framework generates data structures automatically at the end of the compile chain, so that specific characteristics of the applications can be taken into account.

Previous work has already shown that the Forelem framework was successful in optimizing (sparse) matrix computations and LU factorization. In this paper, we investigate the usefulness of the Forelem framework for Big Data Applications, more specifically, k-Means and PageRank. For both algorithms, we first describe how the Forelem specification is derived. Second, we demonstrate how through the application of simple transformations, different implementations can be automatically obtained.

The derived implementations are benchmarked with both Hadoop and C/C++ MPI implementations. For k-Means clustering, the Hadoop implementation is used that was developed for the scalable machine learning and datamining project Apache Mahout. As a C/C++ MPI implementation a code originally developed by W.-K. Liao from Northwestern University is selected. For PageRank, an implementation using Hadoop MapReduce from the Pegasus project is used. The evaluated C/C++ MPI implementation is taken from Indiana University’s Parallel BGL. We show that using the Forelem framework implementations of k-Means clustering and PageRank are automatically derived, that improve the C/C++ MPI implementation and can outperform the Hadoop implementations.

This paper is organized as follows. Related work is discussed in Section 2. In Section 3 the Forelem framework is described. Section 4 discusses how Forelem specifications are derived for the k-Means clustering and PageRank algorithms. Section 5 describes a number of transformations and explains how these are applied to the initial Forelem specifications of the k-Means and PageRank to derive different implementations. The performance of the different implementations is evaluated in Section 6. Section 7 concludes the paper.

2 RELATED WORK

As the Forelem framework is based on atomic, tuple-based operations, it may appear similar to Linda, the tuple space coordination model. Linda was introduced by David Gelernter in the eighties using tuples as a basic operation for coordination and communication of parallel processes. The basic enabler for these operations was the fact that all tuples were stored in a physical, shared, virtual, associative memory. All these operations retrieve tuples from this memory, operate on these tuples, and store the result tuple back in this memory. Unlike this approach, the Forelem
framework does not make any assumption on where these
tuples are stored, instead tuples are a conceptual notion and
act as a placeholder for the software optimization process
of Forelem. It is this difference which enables Forelem
to automatically generate data structures and their mapping
into physical memory. Whereby Linda suffered from
performance limitations thereby hindering the widespread
use for high performance parallel applications, the Forelem
framework is far more versatile allowing highly efficient
implementations to be derived for many applications.

As the Forelem framework is tuple-based, there is a
natural connection to the concept of Dataflow computing.
Dataflow has been a major topic in computer architecture
research in the seventies and early eighties [9]. Dataflow
computing is token based and at runtime these tokens are
matched and computed on. Several Dataflow computing
architectures were proposed, both for the storage of tokens
as well as the matching unit. For the token storage mostly a
content addressable memory was foreseen. As with Linda,
Dataflow computing suffered from performance issues for
general use and its application, although influential, was
limited to specific areas in computer hardware and software
design. As such the Forelem framework can be seen as a
generalization of Dataflow computing, enabling a full
optimization chain for general applications.

3 FORELEM
Within Forelem operations are tuple-based and atomic, and
are organized by the use of two different loop structures: the
forelem and whilelem loops. Both structures iterate over the
tuples in a tuple reservoir. These tuple reservoirs are neither
physical nor virtual, but are defined on a conceptual level
without specifying any order in which tuples are scheduled.

A Forelem specification captures the essence of an algo-

rithm, yielding a specification free from common artifacts
like explicit data and loop structure and associated data
dependencies. This section describes how such specifica-
tions are derived for the k-Means clustering and PageRank
algorithms. The derived specifications are used in the next
section to demonstrate how different implementations can
be generated through the application of basic transforma-
tions.

4 DERIVING A FORELEM SPECIFICATION
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tions.

4.1 k-Means Clustering
The k-Means clustering algorithm divides a given set of
data points of dimension \(d\) into \(k\) clusters. The number \(k\) is
specified beforehand by the user. To start, the algorithm first
initializes the $k$ cluster centers. This can be done in various ways. A standard distribution consists of randomly assigning data points to one of the $k$ clusters, then calculating the mean of the assigned data points to obtain the cluster center.

The algorithm consists of several iterations. During each iteration the algorithm loops over each data point, calculating the Euclidean distance to each cluster center. After each iteration, the data points are assigned to the cluster whose center was closest. After reassigning all data points, the cluster centers are set to the mean of all data points that were assigned to this cluster during this iteration:

```plaintext
color{change} = true {color}
while (change) { {color}
    // assume there is no change
    // reassign data points to clusters
    for (x = 1 to N) { {color}
        mindist = LARGE
        for (m = 1 to k) { {color}
            if (dist(COORDS[x], M_COORDS[m]) <= mindist) {
                a = m
                mindist = dist(COORDS[x], M_COORDS[m])
            }
            if (a != M[x]) {
                M[x] = a
                change = true
            }
        }
        // recalculate cluster centers
        if (change) { {color}
            for (m = 1 to k) { {color}
                mean, count = 0
                for (x = 1 to N) { {color}
                    if (M[x] == m) {
                        mean = mean + COORDS[x]
                        count = count + 1
                    }
                }
                M_COORDS[m] = mean/count
            }
        }
    }
}
```

Here $x$ is a data point with $N$ the total number of data points, $m$ is a cluster with $k$ the total number of clusters, $dist$ a function that calculates the Euclidean distance, $M[x]$ the cluster data point $x$ is currently assigned to, $COORDS[x]$ the coordinates of data point $x$ and $M_COORDS[m]$ the coordinates of the cluster center of cluster $m$. Note that $COORDS[x]$ and $M_COORDS[m]$ are $n$-dimensional, and any operations involving them, including the distance function, are in fact $n$-dimensional operations.

### 4.1.1 Forelem Specification of k-Means

Recall that the Forelem framework allows “random” execution of the (tuple) operations in any order and for an arbitrary amount of times, in contrast to the classic implementation of k-Means which explicitly determines the order of operations. The key difference between the Forelem implementation and other parallel implementations for k-Means clustering will be that the computation must be reduced to its core. To do so we first note that the classic algorithm consists of a main while loop which continues operations until no change is made. This naturally corresponds to using a while loop in the Forelem specification, which by definition terminates as soon as all tuples result in a no-op operation. In fact, the resulting Forelem specification will use only a single outer while loop.

Next we note that the loop body of the classic algorithm is split into two separate steps: reassigning the data points and recalculating the cluster centers. For both steps the classic algorithm contains a 2-dimensional for-loop, looping over each possible combination of a data point and a cluster. In the first step, the distance between the data point and the cluster is compared to the best recorded distance and if this distance is smaller, the best recorded distance is updated. In the second step, the data point is then taken into account when recalculating the cluster center, only if the distance to the given cluster is the best recorded distance.

For the specification of the while loop these two steps are merged into one, thereby removing the necessary bookkeeping such as the change variable seen in the classic algorithm. After all, these variables do not contribute to the essence of the computation in the classic algorithm. At the same time this merger results in the removal of the artificial barrier between the two steps, resulting in a single while loop in which all steps of the two separate inner for loops are combined into single point operations. In order to ensure that these point operations can be executed in a random fashion and independently of each other, the following observations are used:

1) The first step is to capture the while loop body as a simple if-statement: if the distance between a data point $x$ and a cluster $m$ is smaller than the best recorded distance, i.e., the distance to the cluster a data point is currently assigned to ($M[x]$), then we must reassign this data point. If not, then no operation is needed, as is also captured by the use of a boolean recording whether a change occurred in the classic implementation.

2) The first observation combined with the observation that - if $M[x] == m$ then clearly the distance will not be strictly smaller – gives us the condition of the if-statement in the while loop:

```plaintext
if (M[x] != m && dist(COORDS[x], M_COORDS[m]) < dist(COORDS[x], M_COORDS[M[x]])) { ...
```

where $dist$ calculates the Euclidean distance.

3) As a result, the reassigning of the data point $x$ can then be captured in the body of the if-statement by simply stating $M[x] = m$.

Therefore, the while loop in the Forelem specification will need to loop over each combination of a cluster $m$ and a data point $x$, and take the same steps in the loop body as the classic algorithm. As a consequence, our reservoir $T$ will contain tuples $\langle m, x \rangle$. The cluster centers are updated accordingly.

In essence, k-Means clustering can therefore be captured by the following Forelem while loop given in [Algorithm K.1](#)

[Algorithm K.1] Here $x$ is a data point and $M[x]$, respectively $m$, is the cluster $x$ is currently assigned to, respectively to be assigned to, $dist$ calculates the Euclidean distance, $COORDS[x]$ is the coordinates of data point $x$ and $M_COORDS[m]$ and $M_SIZE[m]$ are the cluster center and
size of a cluster $m$ respectively. Note again $\text{COORDS}[x]$ and $\text{M\_COORDS}[m]$ are n-dimensional and all operations that involve them are in fact n-dimensional operations, including the distance function $\text{dist}$. These operations have been abbreviated to improve readability.

### Algorithm K.1 The initial Forelem specification of k-Means clustering.

```
whilelem (\langle m, x \rangle \in T) 
  \text{if} (M[x] != m \&\& dist(\text{COORDS}[x], M\_COORDS[m]) < dist(\text{COORDS}[x], M\_COORDS[M[x]])) 
    \text{M\_COORDS}[M[x]] = 
    \text{(M\_COORDS}[M[x]]*M\_SIZE[M[x]] - \text{COORDS}[x]) /
    \text{(M\_SIZE}[M[x]] - 1)
    M\_SIZE[M[x]] -= 1
    \text{M\_COORDS}[m] = 
    \text{(M\_COORDS}[m]*M\_SIZE[m] + \text{COORDS}[x]) /
    \text{(M\_SIZE}[m] + 1)
    M\_SIZE[m] += 1
    M[x] = m
```

As for the classic algorithm for k-Means, the algorithm converges, albeit not necessarily to a global optimum. A proof of the correctness of this specification and, therefore, its convergence property and its termination, is given in Appendix A.1.

### 4.2 PageRank

PageRank [12] is an algorithm to rank a set of web pages based on an objective notion of importance. Intuitively the ranking models random surfers, who, after arriving on a web page, follow a random link until they stop following successive links. Once they stop, they pick a random website and continue from there. Equation 1 shows the definition of the PageRank of a vertex, where the likelihood of stopping is modeled using a constant $0 < d < 1$, also referred to as the damping factor.

$$PR(v) = \frac{1-d}{|V|} + d \sum_{u \in \text{nebhd}(v)} \frac{PR(u)}{\text{deg}^+(u)}$$  

(1)

Usually PageRank is calculated in an iterative fashion, where in every iteration a vertex “donates” all its PageRank to its successors, and receives it from its predecessors. As an example, a vertex with a PageRank of 0.2, and an outdegree of 5, yields a rank of $\frac{0.2}{5} = 0.04$ to all its successors. Additionally, the damping factor is applied to all incoming rank, and the (graph-) constant value $\frac{1-d}{|V|}$ is added.

In [13] a damping factor of approximately 0.85 is suggested to yield the best results. In the rest of this paper we will assume $d = 0.85$, unless stated otherwise.

The de facto standard iterative algorithm for PageRank works by taking the current guess for the PageRank values (\text{PR}) and calculating the next set of values based on those. Whenever an iteration results in the same values as the previous generation, the algorithm has converged to the right values. The pseudocode for such an implementation is shown here:

```
for (v = 1 to N) 
  PR[v] = 1/N
```

whilelem (PR != PRold) {
  PRold = PR
  for (v = 1 to N) {
    PR[v] = (1-d)/N
    for (u = 1 to N) 
      if (edge (u, v) exists) 
        PR[v] += d*PRold[u]/Dout[u]
  }
}
```

Here $PR$ represents the PageRank values of the current generation, $PRold$ the values of the previous generation, $N$ the number of vertices, and $Dout$ the outdegrees of the vertices.

#### 4.2.1 Forelem Specification of PageRank

As is the case for k-Means clustering, the PageRank algorithm explicitly determines an order of operation. When designing a Forelem specification of PageRank it is important to reduce the computation such that the basic operations can be executed independently.

The core computation in PageRank is the value of the sum, shown in Equation 1 and the imposed order is simply to prevent summing the wrong values. The iterations are intended to “forget” the previous calculation, so old (wrong) values can be removed. In order to reduce the algorithm to its essence and therefore can be executed independently, the Forelem implementation keeps track of differences of these updates for each tuple. Then these differences are used to adjust the PageRank values instead of each time recalculating the summation. Implementing this change leads us to the following Forelem specification:

```
whilelem (w \in V) 
  \text{if} (PR[w] != OLD[w]) 
    \text{forelem} ((u, v) \in E.u[w]) 
      PR[v] += d*(PR[w] - OLD[w])/Dout[w]
  OLD[w] = PR[w]
```

Here $OLD$ replaces $PRold$, but serves a very similar purpose. Instead of the values of $PR$ in the previous generation, it now contains the value of $PR[w]$ of the last time the loop body was executed for $w$.

Note that this specification quite naturally ended up being a “push-style” algorithm, pushing changes forward in the graph, rather than pulling them forward. Another noteworthy change is the addition of the if-statement. Rather than continuously executing this loop, it is only executed when a change is produced. To correct for the missing reinitialization, instead of initializing the values of $PR$ at $\frac{1}{|V|}$, they now need to be initialized at $\frac{1-d}{|V|}$.

Unfortunately, while this specification is much less strict about order, it still contains a double iteration: first over vertices, and then over edges. To enable the Forelem framework to fully optimize this specification, we want to impose as little structure as possible. Fortunately, this can be solved exactly like the first step: with $OLD$. By expanding $OLD$ to be per-edge, rather than per-vertex, it is no longer required to do all updates for a vertex at once. In turn, this enables iterating over edges, getting rid of the imposed push style, and optionally enabling a pull style as well. The resulting specification is shown in Algorithm K.2. A proof of the correctness of this specification as well as its convergence properties is given in Appendix A.2.
Algorithm P.1 The initial Forelem specification of PageRank.

\[
\text{whilelem } \langle (u, v) \in E \rangle \{ \\
\quad \text{if } (PR[u] != OLD[u,v]) \{ \\
\quad \quad PR[v] += \frac{d \times (PR[u] - OLD[u,v])}{Dout[u]} \\
\quad \quad OLD[u,v] = PR[u] \\
\quad \} \\
\}
\]

5 Transformations and Implementations

The Forelem specifications given in Algorithm K.1 and Algorithm P.1 capture the essence of the applications and are used as a starting point to derive several implementations, by an automated process. This automated process functions through the application of sequences of transformations. In this section, the transformations are described that are used to derive the final implementations of which the performance is evaluated in Section 6. Note that more transformations, and many more implementations are possible. For a detailed description we refer to [11], and for the automatic generation of the associated data structures and the overall automation process we refer to [1], [14].

5.1 Orthogonalization

The orthogonalization transformation can be used to optimize the order in which tuples are visited. It introduces an outer loop, which adds an order to the processing of the tuples. The outer loop selects one or more fields of the tuples, the inner loop then loops over those tuples in the original reservoir which contain the selected values for these fields. Thereupon the reservoir splitting transformation, that will be described next, can be applied to this outer loop. As a result, tuples are now processed in particular groups, and reservoir splitting can be performed based on the values for certain fields of the tuple.

For example, if orthogonalization is applied to k-Means, the outer loop will iterate over all data points and the inner loop over all clusters. This can be seen in Algorithm K.2. The orthogonalization transformation can be used to optimize the order in which tuples are visited. It introduces an outer loop, which adds an order to the processing of the tuples. The outer loop selects one or more fields of the tuples, the inner loop then loops over those tuples in the original reservoir which contain the selected values for these fields. Thereupon the reservoir splitting transformation, that will be described next, can be applied to this outer loop. As a result, tuples are now processed in particular groups, and reservoir splitting can be performed based on the values for certain fields of the tuple.

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5.2 Reservoir Splitting

The Forelem framework provides an inherently parallel specification. Similar to the automatic data partitioning and loop blocking optimization techniques in optimizing compilers, Forelem can automatically partition the tuple reservoir to parallelize the execution of forelem and whilelem loops. Contrary to traditional optimizations like data partitioning and loop blocking which have to take into account loop iteration data dependencies and array bounds, within the Forelem framework this partitioning transformation is very straightforward: as no structure is imposed on the shared spaces, all iterations are naturally parallel and all tuples have the same structure. Given a reservoir \( R \) to iterate over, we partition it in subreservoirs, then iterate over these separately, ideally in parallel. Any partitioning of \( R \) works, as long as \( \bigcup_i S(R) = R \). Note that because of the use of the union operator, the tuples are allowed to reside in multiple partitions. Usually a fair partitioning is used, where every \( S(R) \) has roughly the same size.

If we apply single-value reservoir splitting on the initial Forelem specification of PageRank (Algorithm P.1), this results in Algorithm P.3. Note that although this variant is simple, in practice it will be suboptimal, since it requires synchronization between different partitions on the various writes to \( PR \), see Section 5.5. If reservoir splitting is applied to Algorithm P.2 this would result in a program specification in which every \( PR \) value has exactly one writer (see Algorithm P.7).

If reservoir splitting is applied to the orthogonalized code of the k-Means algorithm (Algorithm K.2), both loops are replaced by:

\[
\text{whilelem } \langle y \in S(T) \_i.x \rangle \\
\text{forelem } \langle (m, x) \in S(T) \_i.x[y] \rangle \\
\]

in case the reservoir splitting is based on single values of the field \( x \). The reservoir splitting can also be based on a range of values of field \( x \) in this case the two loops are replaced by

\[
\text{whilelem } \langle y \in T.x[ \min(T.x)+i*\{(\max(T.x)-\min(T.x))/S\}, \min(T.x) + (i+1)*\{(\max(T.x)-\min(T.x))/S\}-1] \rangle \\
\text{forelem } \langle (m, x) \in T.x[y] \rangle \\
\]

in which \( i \) ranges over the numbers \([0, S-1]\). The resulting code of the single-value reservoir splitting on Algorithm K.2 is shown in Algorithm K.3. Note that for this resulting code, each partition only needs the \( \text{COORDS} \) and \( \text{M} \) values that apply to its own data points.

Algorithm K.2 The Forelem specification of k-Means clustering (Algorithm K.1) after using orthogonalization.

\[
\text{whilelem } \langle y \in T.x \rangle \\
\text{forelem } \langle (m, x) \in T.x[y] \rangle \\
\]

\[
\begin{align*}
\text{if } (M[x] != \text{OLD}[u,v]) & \\
\quad \text{PR}[v] = \frac{d \times (PR[u] - \text{OLD}[u,v])}{Dout[u]} & \\
\quad \text{OLD}[u,v] = PR[u] \\
\end{align*}
\]
Algorithm K.3 The Forelem specification of k-Means clustering after the application of reservoir splitting on Algontm K.2.
whilelem \((y \in S(T)_i . x)\)
  forelem \((m , x) \in S(T)_i . x [y]\)
  if \((M[x] != m \&\&\) dist \((\text{COORDS}[x] , M \_COORDS[m])< \text{dist} \((\text{COORDS}[x] , \text{M \_COORDS}[M[x]])\) \}\{ 
    M \_COORDS[M[x]] = \((\text{M \_COORDS}[M[x])] \times \text{M \_SIZE}[M[x]]\) 
    - \text{COORDS}[x]) / (\text{M \_SIZE}[M[x]] - 1) 
    \text{M \_SIZE}[M[x]] -= 1 
    \text{M \_COORDS}[m] = \((\text{M \_COORDS}[m] + \text{M \_SIZE}[m]) + \text{COORDS}[x]) 
    / (\text{M \_SIZE}[m] + 1) 
    \text{M \_SIZE}[m] += 1 
  \}
  \}  

Algorithm K.4 The Forelem specification of k-Means clustering after applying the localization transformation.
whilelem \((y \in S(T)_i . x)\)
  forelem \((m , x) \in S(T)_i . x [y]\)
  if \((c , x) != m \&\&\) dist \((x , M \_COORDS[c , x])< \text{dist} \((x , M \_COORDS[c , x])\) \}\{ 
    M \_COORDS[c , x] = \((\text{M \_COORDS}[c , x] + \text{M \_SIZE}[c , x] - x) / (\text{M \_SIZE}[c , x] - 1) 
    \text{M \_SIZE}[c , x] -= 1 
    \text{M \_COORDS}[m] = \((\text{M \_COORDS}[m] + \text{M \_SIZE}[m] + x) 
    / (\text{M \_SIZE}[m] + 1) 
    \text{M \_SIZE}[m] += 1 
  \}
  \}

Algorithm P.3 The Forelem specification of PageRank (Algorithm P.1) after the application of reservoir splitting.
whilelem \((\{u , v\} \in S(E))\)
  if \((PR[u] != \text{OLD}[u , v])\) \{ 
    PR[v] = d \times (PR[u] - \text{OLD}[u , v]) \times \text{Dout}[u]) 
    \text{OLD}[u , v] = PR[u] 
  \}

5.3 Localization
In order to take full advantage of memory hierarchies, the localization transformation brings the data in shared spaces directly to the tuples in the reservoir. So, instead of data stored in shared spaces, this data is included in the fields of the tuples. Where shared space data is initially stored separate from tuples, localization causes shared space data to be stored — or localized — in the tuples directly.

For k-Means, using the localization transformation (followed by orthogonalization and reservoir splitting) we obtain the specification shown in Algorithm K.4. A tuple \(\langle m, x, c-x \rangle\) in \(T\) now contains the value of a data point \(x\) and the associated cluster as \(c-x\). Note that the \(x\) now represents the actual data point, instead of the index at which this data can be found.

For PageRank, the localization transformation will include the \(\text{OLD}\) data with the edge data, yielding tuples \(\langle u, v, \text{old} \rangle\). See Algorithm P.4.

Algorithm K.5 The Forelem specification of PageRank (Algorithm P.1) after applying Tuple Reservoir Reduction.
whilelem \((\{u,v\} \in E)\)
  if \((PR[u] != \text{OLD}[u, v])\) \{ 
    if \((v == C)\) 
      forelem \((w \in V \setminus \{u\})\)
      \(PR[w] = d \times (PR[u] - \text{OLD}[u, v]) \times \text{Dout}[u]) \}
  else 
    \(PR[v] = d \times (PR[u] - \text{OLD}[u]) \times \text{Dout}[u] \}
    \text{OLD}[u , v] = PR[u] 

5.4 Tuple Reservoir Reduction
The tuple reservoir reduction transformation reduces the iterated tuple reservoir’s size by identifying common subsets \(C\) in the reservoir which can be compacted when initializing the reservoir and expanded on demand. In order to guarantee an efficient implementation, these subsets \(C\) of \(T\) are only identified if the tuples corresponding to these subsets \(C\) can be enumerated in linear (constant) time by a simple enumeration function \(\text{C}_C\).

Having identified subsets and their enumeration functions then the tuple reservoir can be reduced by deleting all tuples of a subset and replacing them by a simple stub to the corresponding enumeration function. Then at execution time this subset is being generated one at a time and the loop body is replicated for each of the tuples corresponding to this subset.

For k-Means, this transformation is not directly applicable. In the case of PageRank, these subsets \(C_u\) originate from these vertices \(u\) which originally had an outdegree of 0 and consist of all added tuples \(\langle u, v \rangle\) (where \(u \neq v\)). Assuming the vertices are numbered 1 to \(|V|\), then each subset \(C_u\) consists of \(\{(u, i) | 1 \leq i \leq |V|\}\) and the enumeration function ends up being a simple for-loop from 1 to \(|V|\). When this transformation is applied to Algorithm P.1 this results in Algorithm P.5.

Note that in contrast to the initial specification of the tuple reservoir \(E\), for which for every vertex with outdegree 0 additional tuples \(\langle u, v \rangle\) were created for every \(v \neq u\), by using tuple reservoir reduction all these tuples were identified as reducible subsets and therefore deleted. In fact, the initial expansion of the tuple reservoir was needed to obtain a clean and simple representation in the Forelem framework — thereby allowing a cleaner convergence proof and facilitating other transformations to be applied to this specification, see below. Instead of the generation of the Forelem construct enumerating all the elements of the subset also an arbitrary element of this subset could have been chosen. In this case it is important that the enumeration
function can produce an arbitrary element of the subset in constant time.

5.5 Shared Space Allocation

Because the operations in the forelem and whileelem loop structures are atomic and inherently parallel, the parallel execution of these loop structures is straightforward even in the presence of the fact that different tuples can still access the same address in shared space. So all these accesses through the address function \( F_A \) can be executed independently and in any order, whether they are writes or reads. So, when parallelizing these loop structures the only choice remains whether to distribute or not the shared spaces involved in these loop structures. Whenever the parallel execution is scheduled on a global shared-memory architecture, i.e., multi-core architecture, the shared space can just reside in this global memory. If the parallel execution is scheduled on a distributed parallel computer, i.e., MPI-based HPC cluster architectures, then a decision has to be made whether the shared space is replicated or distributed among the nodes of such a cluster. In both cases, any updates done by tuples on shared space locations in one node have to be communicated at some time to copies of this location residing on other nodes. For a general allocation scheme of the shared space in general this can be very inefficient, therefore the Forelem framework relies on allocation of the shared space, whereby the allocation is guided by an orthogonalization transformation followed by the reservoir splitting transformation (see Section 5.2). This is done in such a way that the reservoir splitting transformation is determined either directly by the outer loop or by a “loop blocking” transformation of the outer loop that was introduced by orthogonalization (see Section 5.1). As a result, we have the following parallel loop structure on a tuple reservoir \( T \) with orthogonalization performed on a field \( x \):

\[
\text{distrfor} \ (p \in [0, P-1]) \\
\text{whileelem} \ (y \in T.x) \\
\quad [ \min(T.x) + \frac{p \times ((\max(T.x) - \min(T.x)) / P)}{2^p} + \frac{((\max(T.x) - \min(T.x)) / P - 1]}} \]

The shared space allocation in this case is being done in such a way that all shared space addresses being referenced by the tuples of the whileelem loop are being allocated in the same node. Note that, because \( F_A \) is an affine mapping and the shared space allocation is determined by the values of a single field \( x \), determining which shared addresses are “shared” between different nodes can be computed efficiently by determining the subset of these spaces.

The only issue which remains is how the actual communication is performed. Note that because of the inherent parallel nature of the whileelem loop, the updates do not have to be executed instantaneously but can be done at any time during which the whileelem loop executes. So, the shared space location at different nodes do not necessarily have to be consistent and containing the same values at all times. So it can be the case that there are multiple (even more than two) values residing in the same shared space location in different nodes. Also, it does not matter whether one copy is used to update another copy or whether the other copy updates the first copy.

In order to make the data exchange more efficient the Forelem framework will try to accumulate multiple updates. This is done in several ways. The first way (buffered data exchange) buffers all updates on each node in such a way that multiple iterations of the whileelem loop structure are first executed before initiating this data exchange. The second way (master data exchange) is established by sending all of these buffered updates to one master node which reduces all the individual updates to one single update over all copies which is communicated to all participating nodes. This latter accumulation step can be optimized further in case we have update statements (of the form \( x = a + 3 \text{ or } b = b / 2 \)), in which case multiple updates of the same variable and be first combined instead of being performed repeatedly. A third way (indirect data exchange) of optimizing this data exchange can be achieved by relating the update statements with non-shared space data by program assertions in the original Forelem specification. As an illustration, if we look at the M-then the updates on \( M_{COORDS} \) and \( M_{SIZE} \) are directly related to the means reassignment statement, see Algorithm K.1. So, if by program assertion the coupling is made explicit by asserting that \( \text{assert}(M_{SIZE}[i] = \text{SUM}(i == M[x]) \text{ for } x \text{ in } T.x) \) then the accumulated update can be replaced by recomputing this information explicitly for any shared space location of \( M_{SIZE} \).

5.6 Materialization

So far we have been iterating over tuple reservoirs, without specifying the relevant data structure. Materialization is the first step in the process of deriving different data structures and associated shared space data exchanges. In the Materialization step, an initial choice will be made on how tuples will be retrieved from the tuple reservoir using an indexing structure. This index structure will identify a unique integer to every tuple in the reservoir, without fixating the order in which these tuples receive this index. So, the original iteration of an unordered reservoir \( (t \in T) \) will be replaced by a sequential iteration \( i \in [0, N-1] \). As a consequence, every tuple referral in the loop body is replaced with \( PT[i] \). Note that this is just a syntax change of the loop structure and as such has no consequences for the execution order of the original forelem or whileelem loop. As an illustration when materialization is applied to Algorithm P2 we get the loop structure of Algorithm P6.

**Algorithm P6** Materialization applied to the initial Forelem Specification of PageRank (Algorithm P7).

```plaintext
whileelem \ (i \in [0,|PE|-1])
\text{if} \ \{ \text{PR}[PE[i].u] != OLD[PE[i]] \} \ { \text{PR}[PE[i].v] = \text{d} * (\text{PR}[PE[i].u] - OLD[PE[i]]) \} * \text{i/DS}[PE[i].u] \} \ \) \ OLD[PE[i]] = \text{PR}[PE[i].u] \}
```

The next step towards actual data structure generation is a concretization step in which the order of the iterations is fixated and an initial choice of actual data structures is made. This data structure specification does not specifically specify whether to use pointer-structured lists, arrays, dictionaries, etc., but rather specifies the logical structure of the data structures. For instance, array of structures, arrays of arrays or structures of arrays. After the concretization phase and at the code generation phase these logical structures...
will be replaced by (target code) dependent data structure declarations, for instance plain arrays, arrays represented as linked lists, pointer-linked grid structures, STL containers, balanced trees, etc.

One might wonder why the initial materialization step is not combined with the concretization step. This choice was made so that the initial materialization step can be combined with the code transformations as described before, thereby creating opportunities for deriving unexpected data structure choices. For instance consider the Forelem specification of sparse matrix multiplication (see Section 3). Then, if we apply localization of shared space to the specification, followed by orthogonalization on $i$ and $j$ successively, materialization of tuple reservoir and storing the materialized tuple fields in separate arrays, we obtain the loop structure:

\[
\text{forelem } (j \in [0,N-1])
\]

\[
\text{forelem } (i \in [0,N-1])
\]

\[
C[i,j] += PA[i][kk]*B[PB[i][kk],j]
\]

Then after a loop interchange transformation of the two inner loops the resulting loop structure is:

\[
\text{forelem } (j \in [0,N-1])
\]

\[
\text{forelem } (kk \in [0,|PA[i]|-1])
\]

\[
\text{forelem } (i \in [0,N-1])
\]

\[
C[i,j] += PA[i][kk]*B[PB[i][kk],j]
\]

Now, if concretization is applied to this loop structure, then we obtain a jagged diagonal/ITPACK data structure implementation for sparse matrix multiplication [16]. This resulting data structure would not have happened if we would have combined the concretization with the materialization step, because after concretization loop interchange, and possible other transformations, will be obscured by the fixed logical structure of the data structures. This is the prime reason why optimizing compiler techniques thus far have not been able to transform any matrix multiplication into this jagged diagonal/ITPACK form. This implementation relying on jagged diagonal/ITPACK has been very successful in exploiting long pipelined executions (vector processing) architectures but always have been developed by hand [16], [17].

5.7 Putting It All Together

The transformations as described in the previous section can be composed so that their effect is multiplied. Note that, except for concretization, the transformations have a Forelem specification as input and produce Forelem specifications as output, so they are inherently composable. The composition of multiple transformations allowing different orders of application — including re-use of transformations — leads to many different implementation of the initial Forelem specification. These implementations can differ in many aspects: iteration order of the tuples, reservoir partitioning, generated data structure and the shared space data exchange that is used.

5.7.1 k-Means

As a running example in the previous section, we have already seen the composition of transformations applied to the initial k-Means specification that was given in Algorithm K.1. Orthogonalization (resulting in Algorithm K.2) caused the tuple reservoir to be iterated point-by-point by the outer loop, introducing an inner loop to evaluate the distance to each mean for this particular point. This was followed by reservoir splitting (resulting in Algorithm K.3), which partitioned the tuple reservoir enabling parallel execution whereby each process is executing the loop body for a subset of the tuple reservoir. The subsequent application of the localization transformation results in a code (Algorithm K.4) in which data from the COORDS and $M$ shared spaces is included in, or grouped with, the tuples.

We now demonstrate that the application of materialization at different steps in the transformation chain leads to different data structures to be generated as well as different data exchange schemes. First consider Algorithm K.3. Then, after applying orthogonalization and materialization we obtain Algorithm K.5 in which all shared spaces became separate array structures.

Algorithm K.5 The Forelem specification of k-Means clustering after using orthogonalization and reservoir splitting (Algorithm K.3) followed by materialization.

```
whilelem (i \in [0,|PM|-1])
  \text{forelem } (m \in [0,k-1])
  \text{if } (PM[i] != m \&\&
    \text{dist}(PCOORDS[i],PM_COORDS[m]) < \text{dist}(PCOORDS[i],PM_COORDS[PM[i]]))
    \text{PM_COORDS}[i] = (PM_COORDS[M[i]]*PM_SIZE[M[i]] - PCOORDS[i]) / (PM_SIZE[M[i]] - 1)
    PM_SIZE[M[i]] = (PM_COORDS[m]*PM_SIZE[m] + PCOORDS[i]) / (PM_SIZE[m] + 1)
    PM_SIZE[m] += 1
  \text{PM}[i] = m
```

When allocating the shared spaces among distributed memory nodes, the most natural choice for Algorithm K.5 can be either distributed via buffered and/or indirect data exchange. The straightforward implementation of buffered data exchange is to perform this data exchange after one or multiple sweeps of the inner forelem loop. With respect to the indirect data exchange the updates on $PM_SIZE$ can be computed indirectly from the updates on the $PMCOORDS$ and $PM$. See also Section 5.5.

As a second materialization possibility, we consider the application of materialization to Algorithm K.4 which is a localized version of Algorithm K.1. Also in this case, the outer loop is replaced with a loop over an integer interval, but contrary to Algorithm K.1 the values in the tuples are not residing in shared memory anymore but are part of the tuples themselves. So, the tuples in the reservoir also need to be materialized. Note that for the materialization of the tuples like the materialization of the shared space an explicit data structure is not chosen at this step, but rather an indexing structure is set up that is indexed with integers. As a result we obtain Algorithm K.6. Observe that DATA and $M$ are not stored as arrays in this case, due to the application of localization. In the concretization step, this materialized version of the tuple reservoir will be transformed to an
array of structures in which each structure contains a vector to store \( x \) and integer to store \( c_x \). The allocation of the shared spaces for this variant is similar to Algorithm K.5 and therefore not further discussed.

**Algorithm K.6** The Forelem specification of k-Means clustering after using orthogonalization, reservoir splitting and localization (Algorithm K.4) followed by materialization.

```java
whilelem (i ∈ \{0, |PT|-1\})
  forelem (m ∈ \{0, k-1\})
    if (PT[i].c_x != m &&
        dist(PT[i].x, PM_COORDS[m]) < dist(PT[i].x, PM_COORDS[PT[i].c_x])) {
      PM_COORDS[PT[i].c_x] =
        (PM_COORDS[PT[i].c_x] + PM_SIZE[PT[i].c_x])
        / (PM_SIZE[PT[i].c_x] - 1)
      PM_SIZE[PT[i].c_x] -= 1
      PM_SIZE[m] = (PMCOORDS[m] + PM_SIZE[m])
        / (PM_SIZE[m] + 1)
      PM_SIZE[m] += 1
      PT[i].c_x = m
    }
```

5.7.2 PageRank

Up till now, we have only seen the application of individual transformations to the initial PageRank specification. In this section, we will consider compositions of these transformations. For k-Means, we first applied orthogonalization, followed by reservoir splitting. If this is done for the initial PageRank specification (Algorithm P.1) this results in Algorithm P.7. Since \( \text{OLD} \) is always indexed using \( e \in E \), \( \text{OLD} \) can be made part of the tuple to avoid repeated separate shared spaces access. This can be achieved with the localization transformation, resulting in Algorithm P.8.

**Algorithm P.7** The Forelem specification of PageRank (Algorithm P.1) after the application of orthogonalization and reservoir splitting.

```java
forelem (w ∈ S(V)_i) {
  forelem ((u,v) ∈ E.v[w]) {
    if (PR[u] != OLD[u,v]) {
      PR[v] = d* (PR[u] - OLD[u,v]) * (1/Dout[u])
      OLD[u,v] = PR[u]
    }
  }
}
```

**Algorithm P.8** The Forelem specification of PageRank (Algorithm P.1) after the application of orthogonalization, localization and reservoir splitting, in order.

```java
whilelem (w ∈ S(V)_i) {
  forelem ((u,v,old) ∈ E.v[w]) {
    if (PR[u] != old) {
      PR[v] = d*(PR[u]-old) * (1/Dout[u])
      old = PR[u]
    }
  }
}
```

The shared space data exchange scheme that is generated depends on the distribution of the tuples and shared spaces that is implied by the transformations that have been performed on a specification. For the case of Algorithm P.9 only shared space PR is written, all other shared spaces are only read. From analysis follows that PR is only written by a subcript \( v \), which has only been distributed to a single process during reservoir splitting, so all writes are local. Because other processes will read this value, PR must be kept current. The buffered data exchange scheme that is generated for PageRank is the same as for k-Means: each process buffers the changed values and communicates these during a data exchanged. Initially, this data exchange will take place after each iteration of the inner loop, but again for efficiency reasons these changed values can be collated such that the data exchange is only performed after one (or multiple) sweeps of the inner forelem loop.

Finally, note that in addition to the compositions of transformations that have been described above, also tuple reservoir reduction as described in Section 5.4 can be applied to all resulting algorithms.

6 Experiments

To evaluate the performance of the derived k-Means clustering and PageRank implementations we ran several experiments, using two implementations from the BigDataBench benchmark as a baseline for each algorithm. We will first briefly discuss these implementations in Section 6.1 and Section 6.2 for k-Means clustering and PageRank respectively. The experimental setup is explained in Section 6.3 and the results are given in Section 6.4.

6.1 k-Means Baseline Implementations

For obtaining state-of-the-art codes we use as a repository the BigDataBench benchmark [18], which contains several parallel implementations of the k-Means clustering algorithm. For our first baseline we choose to use the Hadoop implementation, which uses the implementation included in the Apache Mahout project [4]. Hadoop provides a way to easily parallelize existing algorithms, which is something the Forelem framework also wishes to achieve. We will refer to this implementation as the Hadoop-Mahout implementation.

Since the final implementations generated using the Forelem framework will use C/C++ code and MPI, we also use the C/C++ MPI implementation from the BigDataBench
benchmark, which originated from Northwestern University and was written by W.-K. Liao [5]. It takes a more traditional approach to parallelizing k-Means clustering, first all processes reassign the data points in parallel, then they recalculate the cluster centers in parallel. We will refer to the C/C++ MPI implementation as Kmeans_MPI.

### 6.2 PageRank Baseline Implementations

The PageRank Hadoop implementation in the BigDataBench benchmark [18] is the one as implemented in the Pegasus [6] project. It is a fairly straightforward MapReduce version of the original algorithm. Each iteration evaluates the contributions of each edge, then all those contributions are summed up to produce the next PageRank values. The implementation stops when the change between iterations in smaller than a given error bound $\epsilon$. In [12] empirical evidence is given this algorithm tends to converge in a small number of iterations. Throughout this section, we will refer to this implementation as the PageRank_Hadoop implementation.

Like for k-Means, BigDataBench also comes with a C++/MPI implementation of PageRank, which is taken from Indiana University’s Parallel BGL [7]. We will refer to this implementation as the PageRank_MPI implementation.

### 6.3 Experimental Setup

For the k-Means experiments we have chosen to write a random data generator to exclude any bias towards initial distributions or other artifacts. The generator is given the total number of data points to generate, the dimension of the desired data points and the number of clusters to generate the data in. It first generates the intended cluster centers using a uniform distribution in the interval $[0, 10]$ and a standard deviation for each cluster, uniform random in the interval $[0.1, 0.2]$. To generate a data point, the generator first uniform randomly chooses a cluster to assign it to, then uses a normal distribution with the generated center as a mean and the generated standard deviation. Note that it is possible for coordinates of the generated data points to fall outside the interval $[0, 10]$. All data sets used in the experiment contained data points of dimension 4, generated in 4 clusters. For each implementation the data is stored in ASCII format, since the format for the Kmeans_MPI and Hadoop_Mahout implementation differ slightly. Kmeans_MPI and Hadoop_Mahout implementation can read both formats.

For the experiments with the PageRank implementations we use the data generator provided by the BigDataBench benchmark [18]. This generator uses parameters derived from a Google webgraph to create Kronecker graphs of the requested size. Due to the generation method a very small amount of vertices may not be connected, but this poses no problems for any of the used implementations.

Note that the Kmeans_Hadoop implementation uses a convergence delta to determine whether the process has converged. If the change in the cluster centers during an iteration is less than this convergence delta, the calculation terminates. To allow a fair comparison, this convergence delta was added to Implementation 1 to 4. Similarly, the Kmeans_MPI implementation uses a threshold to determine convergence. If the fraction of data points that are switched to a different cluster center during an iteration is less than the given threshold, calculation terminates. This was also added to the Forelem implementations of k-Means.

The experiments ran on (up to) 16 nodes of the DAS-4 cluster [19]. A node in this cluster consists of 2 CPU sockets, each containing a 4-core CPU with Hyper-Threading. This yields a total of 8 physical cores and 16 virtual cores per node. So, up to a total of 256 threads were run in parallel.

Throughout this section, we will refer to the implementation of k-Means clustering corresponding to Algorithm K.2 using the buffered data exchange scheme as Kmeans_MPI. Implementation Kmeans_2 also corresponds to Algorithm K.2, but uses the indirect data exchange scheme. Similarly, Implementation Kmeans_3 and Implementation Kmeans_4 correspond to Algorithm P.4 and use the indirect data exchange scheme and buffered data exchange scheme respectively. Similarly, for the PageRank implementations, we will refer to the implementation corresponding to Algorithm P.3 as PageRank_1, the implementation corresponding to Algorithm P.9 as PageRank_2, implementation PageRank_3 corresponds to Algorithm P.8 and implementation PageRank_4 corresponds to Algorithm P.7. All these implementations use buffered data exchange.

### 6.4 Results

For the first experiment, we ran the Forelem k-Means implementations using 64 threads for data sets containing $2^{20}$ to $2^{28}$ data points, the results are shown in Figure 1. These results show that applying localization, as is done for the Kmeans_3 and Kmeans_4 implementations, decreases the calculation time. Similarly, for larger data sets, the calculation time is decreased by using the derived communication scheme instead of the recalculation communication scheme, as shown by implementations Kmeans_2 and Kmeans_4 performing better than implementations Kmeans_1 and Kmeans_3 respectively. Both effects become more apparent on the larger data sets. Note that the calculation time shown excludes the time needed for input and output. We focus our experiments on the part of the code that was specified and optimized in the Forelem framework: time is measured from the start of the initialization of the cluster centers until the execution of the while loop terminates.

Similarly, we ran the Forelem PageRank implementations using 4 threads per node and 8 threads per node (leading to a total of 64 and 128 threads respectively) for data sets containing approximately $2^{20}$ to $2^{28}$ vertices, the results of which are shown in Figure 2 and Figure 3. These results also show that the applied transformations result in an optimization of the final implementations, since the PageRank_1 implementation is clearly outperformed by the other three implementations. In most cases the other three implementation perform roughly the same, with a deviation in the 128-thread case. Due to its smaller memory footprint, PageRank_2 scales better than the other implementations when the memory bus gets close to saturation.

In a second experiment, the Forelem implementations were run on configurations containing different numbers of threads and the data sets containing $2^{20}$ data points or vertices. The calculation times for the four k-Means implementations are shown in Figure 4. The results for the four
Fig. 1. The calculation time the Forelem k-Means implementations, using 64 threads and a convergence delta of 0.0001.

Fig. 2. The runtime of the Forelem PageRank implementations using 64 threads. Hadoop was left out to improve legibility.

Fig. 3. The runtime of the Forelem PageRank implementations using 128 threads.

Fig. 4. The calculation time of the Forelem k-Means implementations for varying numbers of threads, using a convergence delta of 0.0001.

Fig. 5. The runtime of the Forelem PageRank implementations for varying numbers of threads.

PageRank implementations are shown in Figure 5. Note that when different configurations would yield the same number of threads, the configuration using the lowest number of nodes was used. The results for the k-Means implementations show that when the number of threads double, the calculation time becomes roughly half of the calculation time, thus showing that the implementations scale very well. The PageRank implementations achieve similar results. Due to the range needed to show all results in Figure 4 and Figure 5, the graphs may appear to approach a limit for the higher number of threads, but in fact they continue to go down at a similar rate as before. From 32 to 64 threads, the calculation time of the k-Means implementations decreases with a factor 1.6 on average. For PageRank, this is a factor 1.3 on average.

Finally, to further investigate the behaviour of the Forelem k-Means implementations we also ran an experiment using data sets with different dimensions and numbers of clusters. Both experiments were run on data sets of size $2^{26}$. The results when running on data sets with $k=4$ and different dimensions are shown in Figure 6. These results show that the calculation time slightly increases
when the dimension does, which is due to the increase in operations needed to calculate the Euclidean distance and recalculate the cluster centers. However, the increase in calculation time is very small compared to the increase in dimension: an increase of a factor 8 in dimension only results in an increase of about a factor 2 in calculation time.

The results for data sets with dimension 4 and different numbers of clusters are shown in Figure 7. Similar to the experiment with different dimensions, the calculation time appears to increase slightly as the number of clusters increases. This is due to an increased amount of information needing to be communicated, when the processes communicate the cluster centers. However, since the frequency of this communication does not increase, only the length of the messages does, the increase in calculation time is small.

From these first experiments we can conclude that the Forelem framework implementations scale well. The additional optimizations applied in the Kmeans_4 and PageRank_2, PageRank_3 and PageRank_4 implementations indeed improve the performance, given that Kmeans_4 is the best performing implementation in almost all cases and PageRank_1 the worst. To get a better understanding of the performance of the Forelem implementations, we will use the Kmeans_MPI, Kmeans_Hadoop, PageRank_MPI and PageRank_Hadoop implementations as baselines.

For the comparison with the Kmeans_MPI implementation, we note that to allow a fair comparison we will measure only the time taken by the core calculation, similar to how we measure the time for the Forelem k-Means implementations as noted before. The time measurement is taken from the moment the Kmeans_MPI implementation calls the function that will execute the iterations, until the moment the process has converged.

The performance of the Forelem k-Means implementations and the Kmeans_MPI implementation on data sets of different sizes are shown in Figure 8. It should be noted that switching to a threshold of 0.0001 instead of a convergence delta caused outliers. For about 15% of the runs the number of iterations used became far greater than normally seen (up to 490 iterations in a single run, where runs with 3 to 10 iterations were normal). The Kmeans_MPI implementation also exhibited this behaviour. These outliers were excluded from the results shown.

The results show that the performance of Forelem k-Means implementations is close to the performance of the Kmeans_MPI implementation. The Kmeans_1 implementation, the slowest of the four Forelem implementations, proved to be slower than the Kmeans_MPI implementation. The fastest Forelem implementation, Kmeans_4, proved to be faster for seven out of nine input sizes. For the remaining two input sizes, the Kmeans_4 implementation was just slightly slower (in the order of 1%). Note that the comparison is done with a hand-coded MPI version whilst our implementations concern codes which were constructed by an automated process.

The performance of the Forelem PageRank implementations was compared to the performance of the PageRank_MPI implementation, the results of which are shown in Figure 9. On the smallest graphs, all of our implementations outperform PageRank_MPI. In the other cases, PageRank_MPI only outperforms the most naive of Forelem
implementations, and all transformed implementations (PageRank_1 and PageRank_4 are shown) significantly outperform PageRank_MPI. Of particular note is the memory consumption: on graphs larger than $2^{18}$ vertices, the naive Forelem implementation and PageRank_MPI consumes too much memory to calculate a result, hence their omission in other experiments.

Finally, a comparison with the two Hadoop baselines was made. Figure 10 shows the execution times of the Forelem k-Means implementations and Figure 11 shows the speedup of the Forelem implementations compared to the Kmeans_Hadoop implementation for various input sizes. The Kmeans_Hadoop implementation was given a maximum number of iterations of 10. Note that the Forelem k-Means implementations are between 20 to 70 times faster than the Kmeans_Hadoop implementation. While the Kmeans_Hadoop implementation first becomes more efficient compared to the Forelem implementations as the data size increases, it becomes less efficient for larger data sizes. However, since for most implementations this effect is only shown for the largest data set, this may be coincidental (due to the randomness of the initialization and its influence on performance). It was not possible to run the Kmeans_Hadoop implementation for a data set larger than $2^{25}$ data points, because it ran out of memory.

Similarly, we compare the performance of the Forelem PageRank implementations to the performance of the PageRank_Hadoop implementation, the results of which are shown in Figure 12. The figure clearly shows that all Forelem-based implementations outperform the original benchmark implementation. As expected from the previous results, the performance of Implementations 2, 3 and 4 are closely tied, whereas PageRank_1 performs the worst. The minimum speedup achieved by Implementations 2, 3 and 4, at $2^{27}$ vertices, is approximately a factor 60. Interestingly, for even larger datasets the speedup of these three implementations starts to increase again. It is likely that this is caused by the fact that for larger datasets, the I/O performed by Hadoop to write intermediate results to disk is becoming a larger and larger bottleneck.

7 Conclusions

The Forelem framework is based on inherently parallel specifications. This demands that a specification process reduces an existing algorithm to its core idea. By doing so, all computational steps are being desynchronized, allowing for an automatic translation process which takes into account all possible computation orders and communication structures, leading to highly efficient parallel implementations. In this paper we have seen that through the Forelem...
framework the application of a sequence of transformations to a simple specification of an algorithm leads to the automated derivation of highly efficient implementations. The performance of these implementations is shown to be superior to the Hadoop baseline implementations, being approximately 40 to 60 times faster. Also, these implementations are more efficient than state-of-the-art, handwritten MPI/C++ implementations. Future work will include the further automation of this process, and the demonstration of the effectiveness of this framework on various other examples and algorithms.

Fig. 12. Speedup of the Forelem PageRank implementations compared to Hadoop using 128 threads.

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