High-density QCD: the effects of strangeness

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I discuss the zero temperature phase diagram of QCD, as a function of baryon density and strange quark mass. The noteworthy points are that at sufficiently high density chiral symmetry is always restored, and at low strange quark mass there need be no phase transition between nuclear matter and quark matter. I comment on the possibility that introducing a strange quark may make it easier to see finite-density physics on the lattice.

1. Introduction

Although lattice gauge theory has been successfully applied to QCD at zero baryon density and non-zero temperature, we know very little about QCD at high density and low temperature, a regime which is physically relevant to neutron star physics and low-energy heavy-ion collisions. Strong evidence has been marshalled [1–3] that the ground state in this regime spontaneously breaks the color gauge symmetry by a condensate of “Cooper pairs” of quarks. The pattern of symmetry breaking has been found to be very different for the cases of 2 and 3 flavors. In this paper I report on the results of recent investigation into the more realistic 2+1 flavor theory.

Let us start by reviewing the two- and three-flavor cases.

1. Two massless flavors. The ground state is two-flavor color superconducting (2SC): chirally symmetric $u$-$d$ pairing [2,3]. The pattern is $\langle q_\alpha^i C \gamma_5 q_j^\beta \rangle \sim \epsilon^{i\alpha\beta} \epsilon^{ij}$ (color indices $\alpha, \beta$, flavor indices $i, j$), which breaks $SU(3)_{\text{color}} \rightarrow SU(2)_{\text{color}}$, leaving $SU(2)_L \times SU(2)_R$ unbroken.

2. Three massless flavors. The ground state is 3-flavor color-flavor locked (CFL), with pairing between all flavors. Chiral symmetry is broken [4]. The pattern is $\langle q_\alpha^i C \gamma_5 q_j^\beta \rangle \sim \delta^\alpha_\beta \delta^i_j - c \delta^\alpha_j \delta^i_\beta$, breaking $SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R \rightarrow SU(2)_{\text{color}+L+R}$. The ansatz is symmetric only under equal and opposite color and flavor rotations. Since color is vectorial, this breaks the axial flavor symmetry. Even though it only pairs left-handed quarks with left-handed and right-handed with right-handed, color-flavor locking breaks chiral symmetry.

3. 2+1 flavors. Even if one quark is massive, there is still a CFL phase. The pattern is more complicated than the 3-flavor case (see [5] for details), but the essence is that $u$-$s$ and $u$-$d$ pairing breaks the $SU(3)_{\text{color}} \times SU(2)_L \times SU(2)_R$ to $SU(2)_{\text{color}+L+R}$, breaking chiral symmetry because the flavor symmetries are locked to color.

2. Phase diagram

![Figure 1. Conjectured phase diagram for 2+1 flavor QCD at zero temperature as a function of chemical potential $\mu$ and strange quark mass $m_s$. The global symmetries of each phase are labelled.](image-url)
Hamiltonian they leave unbroken. These are the $SU(2)_L \times SU(2)_R$ flavor rotations of the light quarks, and the $U(1)_S$ of strangeness.

We make the following assumptions about the quark phases: (1) $m_u = m_d = 0$. Including small $u, d$ masses would have little effect on pairing [2]. (2) Zero temperature. (3) The quark phase can be described by an NJL-type model, with a 4-fermion interaction normalized by the zero-density chiral condensate. (4) Electromagnetism is ignored. (5) Weak interactions are ignored.

In the baryonic phases, we assume that baryon Fermi surfaces are unstable to pairing in channels which preserve rotational invariance, breaking internal symmetries such as isospin if necessary.

To explain Fig. 1 imagine following two lines of increasing density ($\mu$), one at high $m_s$, then one at low $m_s$.

At high $m_s$, we start in the vacuum, where chiral symmetry is broken. At $\mu_0 \sim 300$ MeV, one finds nuclear matter, in which $p-p$ and $n-n$ pairing breaks isospin. At $\mu_V$, we find a first-order phase transition to the 2SC phase of color-superconducting quark matter, in which the red and green $u$ and $d$ quarks pair in isosinglet channels. Pairing of the blue quarks is weak [14], and we ignore it. When $\mu$ exceeds the constituent strange quark mass we enter “2SC+s” in which there is a strange quark Fermi surface, with weak $s-s$ pairing [15], but no $u-s$ or $d-s$ pairing. Finally, when the chemical potential is high enough that the Fermi momenta for the strange and light quarks become comparable, there is a first-order phase transition to the color-flavor locked (CFL) phase. Chiral symmetry is once again broken. Gapless superconductivity [16] occurs in the metastable region near the locking transition.

At low $m_s$, the story starts out the same way. As density rises we enter the nuclear matter phase with $pp$ and $nn$ pairing. Then we enter the strange baryonic matter phase, with Fermi surfaces for the $\Lambda$ and/or $\Sigma$ and $\Xi$. These pair with themselves in spin singlets, breaking $U(1)_S$ and isospin and chirality. We can imagine two possibilities for what happens next as $\mu$ increases further. (1) Deconfinement: the baryonic Fermi surface is replaced by $u, d, s$ quark Fermi surfaces, which are unstable against pairing, and we enter the CFL phase, described above. An “isospin” $SU(2)_{\text{color}+V}$ is restored, but chiral symmetry remains broken. (2) No deconfinement: the Fermi momenta of all of the octet baryons are now similar enough that baryons with differing strangeness can pair in isosinglets ($p\Xi^-, n\Xi^0, \Sigma^+\Sigma^-, \Sigma^0\Sigma^0, \Lambda\Lambda$), restoring isospin. The interesting point is that scenario (1) and scenario (2) are indistinguishable. They both have a global $SU(2)$ symmetry, and an unbroken $U(1)$ gauge symmetry. This is the “continuity of quark and hadron matter” described by Schäfer and Wilczek [9]. We conclude that for low enough strange quark mass, $m_s < m_s^{\text{cont}}$, there may be a region where sufficiently dense baryonic matter has the same symmetries as quark matter, and there need not be any phase transition between them. The mapping between the baryonic and quark gaps is given in Table 1 along with their transformation properties under the unbroken symmetries.

### Table 1

| Quark | $SU(2)_{c+V}$ $Q'$ | Hadron $SU(2)_V$ $Q$ |
|-------|-------------------|-------------------|
| $bu$  | 2 +1 $p$ 2 +1     |                   |
| $bd$  | 0 $n$ 0          |                   |
| $gs$  | 2 0 $\Xi^0$ 2 0  |                   |
| $rs$  | -1 $\Xi^-$ -1    |                   |
| $ru - gd$ | 0 $\Sigma^0$ 0 |                   |
| $gu$  | 3 +1 $\Sigma^+$ 3 +1 |          |
| $rd$  | -1 $\Sigma^-$ -1 |                   |
| $ru + gd$ +$\xi_-$ $bs$ | | $\Lambda$ 1 0 |
| $ru + gd$ -$\xi_+$ $bs$ | |                   |

The mapping between quark and hadronic states in the CFL phase.

### 3. Possible lattice calculations

It has been pointed out [17] that there is a tricritical point in the $\mu-T$ plane for two-flavor QCD, which may be experimentally detectable in heavy-ion collisions [18]. As $m_s$ is reduced below its
Figure 2. The order of the finite-temperature chiral phase transition of QCD for $m_{ud} = 0$. The wedge at top right corresponds to the 2SC phase, in which the ground state respects chiral symmetry, so there is no chiral-restoring transition as $T$ rises. If $m_{ud}$ is non-zero, the second-order region becomes a crossover, and the tricritical line becomes a critical line.

physical value $m_s^{phys}$, this tricritical point moves to lower chemical potential, and at strange quark mass $m_s^*$ it occurs at $\mu = 0$ [11] (Fig. 2). For lower $m_s$ the phase transition is first-order. Lattice calculations give $m_s^* \approx \frac{1}{3} m_s^{phys}$ [12].

For $m_s$ just above $m_s^*$, $\mu_{crit}$ is very low, so one should be able to estimate it by calculating derivatives of observables such as the chiral susceptibility with respect to $\mu$ at $\mu = 0$. (Previously, $\mu$-derivatives have only been calculated at $m_s = \infty$, where there is no nearby critical behavior, and no clear signal was seen [13]). This could be extrapolated to give some indication of $\mu_{crit}(m_s^{phys})$, the value of the critical chemical potential in real-world QCD—a prediction that would be of direct value to heavy-ion experimentalists, who need to know at what energy to run their accelerators in order to see the phenomena predicted to occur near the critical point [14].

It should also be noted that existing finite-$\mu$ techniques such as imaginary chemical potential [14] may well be useful in this crossover/critical region, since baryon masses become light as the baryons continuously melt into quarks, so there will be little suppression of the amplitudes of higher Fourier modes in imaginary $\mu$.

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