Enhancing the Student’s Reasoning Ability in Solving Real Number System Problems Using the Concept Map

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Abstract. The reasoning is an important aspect of learning mathematics. In studying real analysis, students must have the reasoning ability to understand the concepts and the relation of each other. The aim of this research is to examine the effect of using a concept map for learning on the student reasoning ability in solving a real number system problem. The reasoning can be seen from the indicators: analysis, synthesis, verification, and generalization. This research is quantitative research with pre-test post-test group design. The subject of this research is eighteen students who enrolled in a real analysis course in the Department of Mathematics Education, Universitas Widya Dharma Klaten academic year 2017/2018. The data was collected by test and documentation. The reasoning test was given twice, first is the pre-test that was given before the implementation of the concept map model in the learning process, and the second is post-test that given after the implementation of the model. The data were analyzed by descriptive statistics. The results showed that there was an enhancement of student reasoning ability in solving real number system problems by using a concept map.

1. Introduction
Real analysis is one of the courses in the analytical group taught in almost all universities. In this course, real number systems, real number sequence, limit functions, and continuity are studied. Objects in real analysis such as concepts, definitions, theorems and the consequences of theorems can be described as a network that is interrelated and related. The ability to connect mathematical objects is one aspect of ability in mathematical reasoning. Brodie [1] states that reasoning is something very essential in mathematics because it is needed as a tool to understand, associate, and build the structure of mathematical knowledge.

NCTM [2] states that “Reasoning is drawing conclusions from evidence, grounds or assumptions. It involves developing logical arguments to deduce or infer conclusions”. In other words, reasoning is an attempt to make a conclusion by using logic rules based on assumptions, principles, traits, and pre-existing evidence. This shows that in order to be able to reason well it requires an understanding of logical rules and associates knowledge about concepts, rules, principles, traits, or evidence that have been learned to formulate or make a new conclusion or proof. The reasoning ability according to Mullis et al. [3] includes the following aspects:
1. **Analysis**, that is the ability to determine the relationships between variables or objects in mathematical situations and make conclusions that are appropriate based on the information provided.

2. **Synthesis**, that is the ability to make connections between different elements and connect related mathematical ideas. It also combines mathematical facts, concepts, and procedures to determine results and combine results to obtain further results.

3. **Verification**, that is the ability to prove by referring to the results or properties of mathematics that have been known

4. **Generalization**, that is the ability to expand the domain so that the results of mathematical thinking or problem-solving can be applied more generally or more broadly.

Based on the analysis of the results of student work both in the middle and final test, it shows that the main problems of students in studying Real Analysis are the lack of understanding of the concepts have studied, the difficulty of linking between concepts, and the inability to use concepts have learned to build new concepts. In other words, the students' reasoning abilities are still weak.

The concept map developed by Novak [4] is a strategy for building knowledge structures, to show the relationship of concepts in a topic that are graphically presented. Important characteristics of concept maps are cross-links [5]. The cross-link is the relationship between concepts in different segments or domains of concept maps. By cross-links, it can be seen how concepts in one domain of knowledge are related to a concept in another domain that is displayed on a concept map. Cross-links often represent a creative leap as part of generating knowledge in the creation of new knowledge.

There are two features of concept maps that are important in facilitating creative thinking, namely the hierarchical structure represented in the concept map and the ability to search for and characterize new cross-links [5].

Serhan et al. [6] had found that after learning process through concept maps, students had an understanding of concepts richer on the Euler Circuit concept and able to construct more representations of this concept. This result is also in line with the research of Chen et al. [7]. The aim of this research is to examine the effect of using a concept map for learning on student’s reasoning ability in solving real number system problems. The reasoning ability will be investigated based on Mullis aspects, namely, analysis, synthesis, verification, and generalization.

2. **Methods**

The research is conducted by quantitative approach. The population is sixth semester students of Department of Mathematics Education, Universitas Widya Dharma, Klaten academic year 2017/2018 who took Real Analysis courses. The data was collected using test and documentation methods. The reasoning ability test is given twice, before and after the implementation of the model. Because all members of the population are a sample, the data are analyzed using descriptive statistics that is by comparing the average value of the pretest and posttest.

3. **Results**

The learning process by using a model based on the concept map can be seen in Figure 1. The implementation of learning model and lesson plan based on concept maps is carried out in two sessions with learning indicators: explaining the order properties of \( \mathbb{R} \), proving the properties derived from the order properties and applying the order properties to the inequalities in \( \mathbb{R} \). Before beginning the lesson with concept maps, students are given a pre-test to find out their initial reasoning abilities. The learning process begins with the lecturer giving greetings, apperception, conveying the goals and plans of learning using concept maps. At the main learning activities, the group discussions were held where each group consisted of 3-4 students and the lecturer facilitated the students to discuss the material. Then, each group discussed the student worksheets to determine the main and additions ideas of the topic learned. After that, the student starts to construct the concept map by placing the main ideas in the top of the map and the first addition ideas below of the main ideas and then connecting them by a linking label. The student placed the second addition ideas below of the first addition ideas,
then connecting them by a linking label, and so on. Furthermore, one of the group member presented the concept map that had been constructed to another in the front of the class. At the end of learning activities with the concept map, students are given a post-test to examine the improvement of their reasoning abilities.

![Searching material](image1)

![Identifying the main and addition idea](image2)

![Verification the concept map](image3)

![Construct the concept map](image4)

**Figure 1.** Implementation of Learning Model Based on Concept Map

Table 1 and Figure 2 shows the pre-test results of students' reasoning abilities before the implementation of the model.

| Interval | $x_i$ | $F_i$ | $F_k$ | Relative Frequency |
|----------|-------|-------|-------|-------------------|
| 25-34    | 29.5  | 4     | 4     | 22%               |
| 35-44    | 39.5  | 3     | 7     | 17%               |
| 45-54    | 49.5  | 1     | 8     | 6%                |
| 55-64    | 59.5  | 5     | 13    | 27%               |
| 65-74    | 69.5  | 3     | 16    | 17%               |
| 75-84    | 79.5  | 2     | 18    | 11%               |
| Total    | 18    |       |       | 100%              |
Figure 2. Histogram of pre-test results

Based on the results of students’ pre-test, the highest score 81.25 and the lowest 25. The average score of 18 students was 52.08 with standard deviation (SD) 17.94, median 56.5, and mode equal to 61.17. Table 2 and Figure 3 describe the results of post-test the student reasoning ability after implemented the concept map model in learning process.

Table 2. Post-Test Results

| Interval | $x_i$ | $F_i$ | $F_k$ | Relative Frequency |
|----------|------|------|------|--------------------|
| 24-36    | 30   | 3    | 3    | 17%                |
| 37-49    | 43   | 1    | 4    | 6%                 |
| 50-62    | 56   | 4    | 8    | 22%                |
| 63-75    | 69   | 2    | 10   | 11%                |
| 76-88    | 82   | 5    | 15   | 27%                |
| 89-101   | 95   | 3    | 18   | 17%                |
| Total    | 18   |      |      | 100%               |

Figure 3. Histogram of post–test results
Based on the post-test results, the students obtain the highest score 100 and the lowest score 25. The average score of 18 participants was 66.32 with a standard deviation (SD) 25.10, median 69, and the mode 83.3.

In this research, all members of the population was used as a sample. This is because students who took the real analysis courses are only 18 students. Therefore, to examine the effectiveness of the learning model based on concept maps are can be done with compare the average score of student pre-test and post-test. The results of pre-test and post-test demonstrated that the average score of the student pre-test is 51.64 and the post-test is 66.32. Therefore, it can be concluded that the concept map learning model is effective in improving student’s reasoning ability.

4. Discussions
In this research, the student's reasoning ability was investigated from four aspects, namely, analysis, synthesis, verification, and generalization. Analysis is the ability to determine the relationships between variables or objects in mathematics situations and draw the right conclusions based on the information provided. Synthesis is the ability to make connections between different elements and connect related mathematical ideas. Furthermore, verification is the ability to prove by referring to the results or known of the mathematical properties. While a generalization is the ability to expand the domain so that the results of mathematical thinking or problem-solving can be applied more generally or more broadly.

The following examples of the student results show that they have a good reasoning ability after the implementation of the model.

![Figure 4. Analysis and Synthesis Abilities](image)

Figure 4 show that the students have been able to make connections between concepts and conclusions based on existing mathematical statements. They are able to explain that

\[
\text{if } a > b \text{ and } b > c \text{ then } a - b \in P \text{ and } b - c \in P. 
\]

In other words, students have the accurate analysis. In addition, students are also able to make connections between different elements and connect mathematical ideas, namely being able to determine the relationship

\[
\text{if } a - b \in P \text{ and } b - c \in P \text{ then } (a - b) + (b - c) \in P. 
\]
Hence, students are able to synthesize correctly.

![Figure 5](image)

**Figure 5. Verification Ability**

Figure 5 shows that students are able to prove by referring the results or known mathematical properties. In this case, by using the property that every natural number is always positive, that is

\[ 2 \in \mathbb{N} \text{ then } 2 > 0, \]

and the inequality property that states the sign of inequality will not change if each side is multiplied by the same positive real number:

\[ 2a < a + b \text{ and } a + b < 2b, \frac{1}{2} > 0 \quad \text{then} \quad \frac{1}{2}2a < \frac{1}{2}(a + b) \text{ and } \frac{1}{2}(a + b) < \frac{1}{2}2b. \]

Students are also able to use the order properties, that is

\[ a < b \text{ and } b < c \Rightarrow a < b < c, \]

to prove

\[ a < \frac{1}{2}(a + b) < b. \]

Thus students have proper verification ability.

![Figure 6](image)

**Figure 6. Generalization Ability**
Figure 6, students are able to generalize because they can expand the results of mathematical thinking or problem-solving points, that is (a) applied more generally or more broadly for points (b) and points (b) applied more broadly to point (c). Thus students have appropriate generalization ability.

The results of the study are inseparable from the use of models and learning tools of concept maps that are implemented during learning process. With this model, students involve actively in discussing, determining the main and additions ideas, and compiling the relationship between ideas or between existing concepts. This study is in accordance with the results of the study of Passmore et al. [8] which stated that metacognitive strategies such as concept maps allow students to learn actively. In addition, Karakuyu [9] states that concept maps can be used as a tool to improve the meaningfulness of learning and improve student understanding. Nirmala and Shakuntala [10] compare the scores of pre-test and post-test in implementing the teaching and learning strategy using concept maps. Their study showed that the results of concept maps strategy significantly different in all aspects of concept maps than hierarchy. Whereas Moahmed [11] emphasizes that concept maps are learning strategies that are suitable for all fields of learning. His study showed that the concept map strategy able to improve students' mathematical connection skills, especially in trigonometry.

The learning model of the concept maps has several benefits. According to Brinkman [12], concept maps is helping the students to organize relevant topic information, organize knowledge in easy categories and subcategories to spin and return. Concept maps is also an effective learning tools, that is helping students to organize and understanding new material, a tool for knowing the structure of student knowledge, helping memory because of the structure of concept maps and images in a single unit. At the end of the lesson, concept maps able to arranged as repetitions and to gain knowledge about the topic being studied.

Learning model based on concept maps can improve student's reasoning ability. The reasoning is a process or cognitive activity to make conclusions based on principles, rules, properties, or existing evidence [13]. In the reasoning process, to make a conclusion or assess a conclusion must be based on information or data that has been known. The reasoning process requires the ability to think logically and systematically to be able to produce concise and correct conclusions. Ministry of Education [14] states that "Mathematical reasoning refers to the ability to analyze mathematical situations and construct logical arguments". Through concept maps, students are able to demonstrate the interrelationships between existing concepts, able to use principles, rules, properties, and evidence to make a conclusion. In other words, the results of this study confirmed the previous studies.

5. Conclusion
Based on the results of the study, it can be concluded that the concept map learning model is effective in improving student's reasoning ability in a real analysis course, especially in the topic of the real numbers system. In this case, reasoning ability is investigated based on four aspects, namely analysis, synthesis, verification, and generalization.

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