Progress report on the staggered $\epsilon'/\epsilon$ project

T. Bhattacharya$^a$, G.T. Fleming$^b$, G. Kilcup$^b$, R. Gupta$^a$, W. Lee$^c$ and S. Sharpe$^d$

$^a$MS–B285, T-8, Los Alamos National Lab, Los Alamos, New Mexico 87545, USA

$^b$Department of Physics, Ohio State University, Columbus, OH 43210, USA

$^c$School of Physics, Seoul National University, Seoul, 151-747, South Korea

$^d$Department of Physics, University of Washington, Seattle, WA 98195, USA

We report on progress and future plans for calculating kaon weak matrix elements for $\epsilon'/\epsilon$ using staggered fermions.

The first goal of this project is to provide a check of the results ($\epsilon'/\epsilon < 0$) obtained using quenched Domain Wall fermions (DWF) by the CP-PACS and RBC collaborations. The second goal is to extend the work to dynamical simulations, since quenching is likely one of the main systematic errors in present calculations. The third goal is either to find a window for new physics or to confirm the standard model, when our numerical results are compared with the observed values.

Staggered fermions are appropriate for this purpose, because they preserve enough chiral symmetry to prevent composite operators from mixing with wrong chirality operators and to protect quark mass from additive renormalization, which is essential to calculate $\epsilon'/\epsilon$. They have the advantages over DWF of requiring less CPU time and dynamical simulations are already possible with relatively light quark masses. By construction, staggered fermions carry four degenerate flavors (also called “tastes”). They allow large flavor changing quark-gluon interactions, which makes it impractical to non-perturbatively determine matching constants for operators of our interest. Another disadvantage is that the unimproved action and operators receive large perturbative corrections at one loop and have large scaling violations of order $a^2$. Both of these disadvantages can be alleviated by improving staggered fermions using smeared links. Flavor symmetry breaking in the pion spectrum is significantly reduced with such links.

Our project has progressed through the following steps.

1. We calculated the current-current diagrams for the gauge-invariant unimproved staggered fermions at one loop, which provided a complete set of perturbative matching formula for $\epsilon'/\epsilon$, combined with existing results for penguin diagrams.

2. We performed a numerical study on $\epsilon'/\epsilon$ using the Columbia QCDSP supercomputer. Using the matching formula given in [3], we constructed fully one-loop matched gauge-invariant operators. As expected, we found large perturbative corrections for $B_6$, so that we cannot quote quantitative results for this although the statistical uncertainty is under control. An unexpected result was that different quenched transcriptions of the continuum operators on the lattice (proposed by Golterman and Pallante) lead to substantially different values for $B_6$. This indicates a large quenching uncertainty and deserves further study.

3. The goal was to find an improvement scheme which can reduce perturbative correction down to 10% or smaller. To achieve this goal, we calculated, explicitly, one loop matching factors for a variety of improved staggered actions and opera-
tors: 1) Fat7, 2) Fat7+Lepage, 3) HYP, and 4) Asqtad-like (Fat7+Lepage+Naik) actions. We observed that all the above improvement schemes significantly reduce the size of the matching coefficients. After a higher level of mean-field improvement, the HYP and Fat7 links lead to the smallest one-loop corrections. Since the HYP action reduces the non-perturbative flavor symmetry breaking more efficiently in the pion spectrum, we adopted the HYP scheme in our numerical study.

4. We studied further on the HYP link. The HYP links possess some universal properties, which are summarized in the following 5 theorems.

**Theorem 1 (SU(3) Projection)**
Any fat link can be expanded in powers of gauge fields ($A_\mu$).

\[ B_\mu = B^{(1)}_\mu + B^{(2)}_\mu + B^{(3)}_\mu + \cdots \]
\[ B^{(n)}_\mu = O(A^n) \]

1. The linear term, $B^{(1)}_\mu$, is invariant under SU(3) projection.
2. The quadratic term, $B^{(2)}_\mu$, is antisymmetric in gauge fields.

**Theorem 2 (Triviality of renormalization)**
1. At one loop level, only the $B^{(1)}_\mu$ term contributes to the renormalization of the gauge-invariant staggered fermion operators.
2. At one loop level, the contribution from $B^{(n)}_\mu$ for any $n \geq 2$ vanishes.
3. At one loop level, the renormalization of the gauge-invariant staggered operators can be done by simply replacing the propagator of the $A_\mu$ field by that of the $B^{(1)}_\mu$ field.

This theorem is true, regardless of details of the smearing transformation.

**Theorem 3 (Multiple SU(3) projections)**
1. The linear gauge field term $B^{(1)}_\mu$ in the perturbative expansion is universal.
2. In general, the quadratic terms may be different from one another. But all of them are antisymmetric in gauge fields.
3. This theorem is true, regardless of the details of smearing.

**Theorem 4 (Uniqueness)**
If we impose the perturbative improvement condition of removing the flavor changing interactions on the HYP action, the HYP link satisfies the following:

1. The linear term $B^{(1)}_\mu$ in perturbative expansion is identical to that of the SU(3) projected Fat7 links.
2. The quadratic term $B^{(2)}_\mu$ is antisymmetric in gauge fields.

**Theorem 5 (Equivalence at one loop)**
If we impose the perturbative improvement condition to remove the flavor changing interactions, at one loop level,

1. The renormalization of the gauge invariant staggered operators is identical between the HYP staggered action and those improved staggered actions made of the SU(3) projected Fat7 links.
2. The contribution to the one-loop renormalization can be obtained by simply replacing the propagator of $A_\mu$ by that of $B^{(1)}_\mu$.

The first two theorems were used in [11] and [12], although they did not present their derivations. For derivations of all five theorems and further details, see [10]. As a result of these theorems, we can prove that for each Feynman diagram,

\[ \| C_{\text{fat}} \| < \| C_{\text{thin}} \| . \]

(1)

Here, $C_{\text{fat}}$ ($C_{\text{thin}}$) represents perturbative corrections to gauge-invariant staggered operators constructed using SU(3) projected fat links (thin links). This inequality is not valid for those fat links without SU(3) projection. Hence, this lead
to a conclusion that we may view the SU(3) projection of fat links as a tool of tadpole improvement for the staggered fermion doublers [10]. We also present alternative choices of constructing fat links to improve staggered fermions in [11]. The above five theorems make the perturbative calculation simpler for the HYP scheme, because one can perform the calculation merely by replacing the thin link propagator with that of the HYP links. This simplicity is extensively used in calculating the renormalization constants of the four-fermion operators in the next stage.

5. We calculate the current-current diagrams to obtain perturbative matching coefficients for the staggered four-fermion operators constructed using the HYP links. Especially, we are interested in the \( (O_3)_{II} \) operator (we use the same notation as in [8]), because this receives large perturbative corrections (≈ 1) in the case of unimproved staggered fermions.

\[
(O_3)_{II} = 2([P \times P][P \times P] - [S \times P][S \times P])_{II}
\]

In Tables 1 and 2, we present values of one-loop corrections to \( (O_3)_{II} \) constructed using both the HYP link and unimproved thin link (denoted as NAIVE). Here, \( C_N = 13.159 \) and \( C_H = 0.7709 \), which correspond to tadpole improvement contributions. The results shows that, by choosing the HYP scheme, the perturbative corrections are reduced to ≈ 10% level. In the case of the HYP scheme, note that the size of one-loop correction is already under control even without tadpole improvement. For further details, refer to [13].

6. At present, we are performing a numerical study using the Columbia QCDSF supercomputer. We calculate weak matrix elements for \( \epsilon' / \epsilon \) using the HYP links at quenched \( \beta = 6.0 \) on the \( 16^3 \times 64 \) lattice. So far, we have completed measurements on about 80 gauge configurations.

We plan to complete the one-loop calculations with HYP fermions, including the penguin dia-

| Operators | \( [P \times P][P \times P]_{II} \) | \( [S \times P][S \times P]_{II} \) |
|-----------|-------------------------------|-------------------------------|
| NAIVE     | \( 2 \times (111.3 - 2C_N) \) | \( 2 \times (14.6 - 6C_H) \) |
| HYP       | \( 2 \times (8.25 - 2C_H) \)   | \( 2 \times (95.6 - 6C_N) \)   |

Table 1
One-loop correction to \( (O_3)_{II} \).

Table 2
One-loop correction to \( (O_3)_{II} \).

This will allow us to match the complete set of \( \Delta I = 1/2 \) and \( 3/2 \) four-fermion operators constructed using HYP links to the corresponding continuum operators in, say, the NDR scheme.

We have also generalized the definition of the optimal matching scale, \( q^* \) [14], to the case of matching factors for operators with non-vanishing anomalous dimensions [13]. Ultimately, we wish to extend the calculation to partially quenched and unquenched QCD.

We thank N. Christ, C. Jung, C. Kim, G. Liu, R. Mawhinney and L. Wu for their support of this project and assistance with numerical simulations on the Columbia QCDSF supercomputer.

REFERENCES

1. CP-PACS Collaboration, hep-lat/0108013.
2. RBC Collaboration, hep-lat/0110075.
3. T. Bhattacharya et al., Nucl. Phys. B (Proc. Suppl.) 106 (2002) 311.
4. K. Orginos et al., Phys. Rev. D 60 (1999) 054503.
5. A. Hasenfratz and F. Knechtli, Phys. Rev. D 64 (2001) 034504.
6. W. Lee, Phys. Rev. D 64 (2001) 054505.
7. S. Sharpe and A. Patel, Nucl. Phys. B417 (1994) 307.
8. M. Golterman and E. Pallante, JHEP 0110 (2001) 037; these proceedings.
9. W. Lee and S. Sharpe, hep-lat/0208018, hep-lat/0208030.
10. W. Lee, hep-lat/0208032.
11. A. Patel and S. Sharpe, Nucl. Phys. B395 (1993) 701.
12. C. W. Bernard and T. DeGrand, Nucl. Phys. B (Proc. Suppl.) 83, (2000) 845.
13. W. Lee et al., in preparation.
14. P. Lepage and P. Mackenzie, Phys. Rev. D 48 (1993) 2250.