Asymmetries from the interference between Cabibbo-favored and doubly-Cabibbo-suppressed $D$ decays

Dao-Neng Gao†

Interdisciplinary Center for Theoretical Study, University of Science and Technology of China, Hefei, Anhui 230026 China

Abstract

A phenomenological analysis of $D \rightarrow K\pi$ and $D_s^+ \rightarrow KK$ decays including both Cabibbo favored and doubly Cabibbo suppressed modes have been presented by employing the present experimental data. SU(3) symmetry breaking effects from the decay constants and form factors have been taken into account in the analysis. Three asymmetries, $R(D^0)$, $R(D^+)$, and $R(D_s^{+})$, which are generated through interference between Cabibbo favored and doubly Cabibbo suppressed decays, are estimated. Theoretical results agree well with the current measurements.

† Email address: gaodn@ustc.edu.cn
1 Introduction

Two-body hadronic $D$ decays could provide useful information for the study of the weak and strong interactions. These processes contain three types: Cabibbo favored (CF), singly Cabibbo suppressed (SCS), and doubly Cabibbo suppressed (DCS) decays. The first two types have largely been observed experimentally, while suffering from the backgrounds of CF decays, only a few channels have been measured for the third one \[1\]. On the other hand, as pointed out by Bigi and Yamamoto \[2\] (and also in Ref. \[3\]), DCS modes involving neutral kaons may show their existence by studying some interesting asymmetries due to interference between CF transitions (producing an $s$ quark, and thus a $\bar{K}^0$) and DCS transitions (producing an $\bar{s}$ quark, and thus a $K^0$), which, for instance, can be defined as

$$R(D) \equiv \frac{B(D \to K_S\pi) - B(D \to K_L\pi)}{B(D \to K_S\pi) + B(D \to K_L\pi)} \quad (1)$$

for $D \to K\pi$ decays. By explicitly setting

$$\frac{A(D \to K^0\pi)}{A(D \to K^0\pi)} = r e^{i\phi} \quad (2)$$

which is the ratio of DCS and CF amplitudes and $\phi$ is the strong phase between them. One can further get

$$R(D) = -\frac{2r \cos \phi}{1 + r^2} \quad (3)$$

and for the small $r$, we have $R(D) \simeq -2r \cos \phi$. Thus, the measurement of these asymmetries may help to extract some information about the DCS processes. Experimentally, these measurements have been done by the CLEO Collaboration \[4\] as

$$R(D^0) = 0.108 \pm 0.025 \pm 0.024, \quad R(D^+) = 0.022 \pm 0.016 \pm 0.018. \quad (4)$$

Similar asymmetry for $D_s^+$ induced from the decays $D_s^+ \to K^+K^0$ and $D_s^+ \to K^+\bar{K}^0$, namely,

$$R(D_s^+) \equiv \frac{B(D_s^+ \to K_SK^+) - B(D_s^+ \to K_LK^+)}{B(D_s^+ \to K_SK^+) + B(D_s^+ \to K_LK^+)} \quad (5)$$

will be reported by the BES Collaboration soon \[5\].

The effective Hamiltonian relevant for CF and DCS decays can be given by

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ud} V_{cs}^* [C_1(\bar{s}i_c)i_{V-A}(\bar{u}jdj)_{V-A} + C_2(\bar{s}i_c)_{V-A}(\bar{u}jdj)_{V-A}] ight.$$

$$+ V_{us} V_{cd}^* [C_1(\bar{d}i_c)_{V-A}(\bar{u}jsj)_{V-A} + C_2(\bar{d}i_c)_{V-A}(\bar{u}jsj)_{V-A}] \} + \text{H.c.} \quad (6)$$

where $V - A$ denotes $\gamma_{\mu}(1 - \gamma_5)$. The first line in eq. (6) is for CF transitions and the second line for DCS transitions. Historically, the naive factorization approach has long been utilized in the analysis of the hadronic $D$ decays, although there is an obvious shortcoming.
that it cannot lead to the scale and scheme independence for the final physical amplitude. On the other hand, some interesting methods, such as the QCD factorization [6] and pQCD [7], which work very well for the non-leptonic $B$ decays, cannot lead to reliable predictions for $D$ decays [8] for the charm quark mass is not heavy enough.

In Ref. [8], we have performed a phenomenological analysis of $D \to K\pi$ decays including both CF and DCS modes based on the quark-diagrammatic approach [10]. In order to determine all decay amplitudes of these transitions using the present experimental data, some SU(3) symmetry breaking effects have been taken into account. $R(D^0)$ and $R(D^+)$ have been calculated, which are consistent with the results reported by the CLEO Collaboration [4]. The purpose of the present study is twofold. First, we will generalize the study in Ref. [8] to the $D^+_s \to K^0 K^+$ and $D^+_s \to \bar{K}^0 K^+$ decays, since $R(D^+_s)$ of eq. (5) will be measured by the BES Collaboration soon. Second, we would like to reanalyze these processes since some data have been updated after the publication of Ref. [8]. It is easy to see that, CF and DCS $D \to K\pi$ and $D^+_s \to K\bar{K}$ decays, which are guided by eq. (6), are free of penguin contributions. Studies of penguin contributions might be very interesting to understand SU(3) symmetry breaking effects and/or CP violation in SCS $D \to \pi\pi, K\bar{K}$ decays [9]. However, it has been pointed out in Ref. [8] that the present analysis cannot be directly extended to the case of SCS processes.

The remainder of the paper is organized as follows. In section 2, we shall discuss the amplitude decompositions of $D \to K\pi$ and $D^+_s \to K\bar{K}$ decays, and some useful constraints will be obtained. In Section 3, a phenomenological analysis is carried out and asymmetries $R(D)$’s will be estimated. Our main results are summarized in Section 4.

2 Amplitude decompositions

In terms of the quark-diagram topologies $T$ (color-allowed), $C$ (color-suppressed), $E$ ($W$-exchange), and $A$ ($W$-annihilation) [10], the decay amplitudes for $D \to K\pi$ and $D^+_s \to K^0(\bar{K}^0)K^+$ transitions can be written as

\begin{align*}
A(D^0 \to K^-\pi^+) &= i \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* (T + E), \\
\sqrt{2} A(D^0 \to \bar{K}^0\pi^0) &= i \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* (C - E), \\
A(D^+ \to \bar{K}^0\pi^+) &= i \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* (T + C), \\
A(D^+_s \to \bar{K}^0 K^+) &= i \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* (C_s + A_s), \\
A(D^0 \to K^+\pi^-) &= i \frac{G_F}{\sqrt{2}} V_{us} V_{cd}^* (T' + E'), \\
\sqrt{2} A(D^0 \to K^0\pi^0) &= i \frac{G_F}{\sqrt{2}} V_{us} V_{cd}^* (C' - E'), \\
A(D^+ \to K^0\pi^+) &= i \frac{G_F}{\sqrt{2}} V_{us} V_{cd}^* (C' + A'),
\end{align*}
Figure 1: $W$-exchange or $W$-annihilation diagrams via gluon emission. The solid square denotes the weak vertex.

\[
\sqrt{2} A(D^+ \to K^+ \pi^0) = i \frac{G_F}{\sqrt{2}} V_{us} V_{cd}^* (T' - A'), \tag{14}
\]

\[
A(D_s^+ \to K^0 K^+) = i \frac{G_F}{\sqrt{2}} V_{us} V_{cd}^* (T'_s + C'_s). \tag{15}
\]

For our notations, we have extracted the CKM matrix elements and factor $G_F/\sqrt{2}$ from the quark-diagram amplitudes, and the prime is added to DCS amplitudes. Using the factorization hypothesis, the quark-diagram amplitudes $T'$'s and $C'$'s appearing in above equations can be further expressed as

\[
T = f_\pi (m_D^2 - m_K^2) F_0^{D \to K} (m_\pi^2) a_{1\text{eff}},
\]

\[
C = f_K (m_D^2 - m_\pi^2) F_0^{D \to \pi} (m_K^2) a_{1\text{eff}},
\]

\[
T' = f_K (m_D^2 - m_\pi^2) F_0^{D \to \pi} (m_K^2) a_{1\text{eff}},
\]

\[
C' = f_K (m_D^2 - m_\pi^2) F_0^{D \to \pi} (m_K^2) a_{2\text{eff}},
\]

\[
C'_s = f_K (m_{D_s}^2 - m_\pi^2) F_0^{D_s \to K} (m_K^2) a_{2\text{eff}},
\]

\[
T'_s = f_K (m_{D_s}^2 - m_\pi^2) F_0^{D_s \to K} (m_K^2) a_{1\text{eff}},
\]

where $a_{i\text{eff}}$'s are regarded as the effective Wilson coefficients fixed from the data (in the naive factorization, $a_{1,2} = C_{1,2} + C_{2,1}/N_c$), and $F_0^{D(s) \to \pi(K)}(q^2)$'s are the form factors for $D(s) \to \pi(K)$ transitions.

For the $W$-exchange and $W$-annihilation amplitudes, it has been pointed out in [8, 11] that the diagrams induced by the topologies of gluon emission arising from the quarks of the weak vertex, as shown in Fig. 1, play important roles in the hadronic $D$ decays. This is also the case for $B$ decays [12]. This contribution has been given in Refs. [11, 12], which reads

\[
\mathcal{E} = f_D f_K f_\pi \frac{C_F}{N_C} \pi \alpha_s C_1 \left[ 18 \left( X_A - 4 + \frac{\pi^2}{3} \right) + 2 r^K_X r^K A_X \right], \tag{17}
\]

\[
\mathcal{E}' = \mathcal{E}, \tag{18}
\]

\[
\mathcal{A}' = f_D f_K f_\pi \frac{C_F}{N_C} \pi \alpha_s C_2 \left[ 18 \left( X_A - 4 + \frac{\pi^2}{3} \right) + 2 r^K_X r^K A_X \right], \tag{19}
\]

and

\[
\mathcal{A}_s = f_{D_s} f_K f_\pi \frac{C_F}{N_C} \pi \alpha_s C_2 \left[ 18 \left( X_A - 4 + \frac{\pi^2}{3} \right) + 2 r^K_X r^K A_X \right], \tag{20}
\]
where $X_A$ is introduced to parameterize the logarithmically divergent integrals due to the end-point singularity, $C_1, C_2$ are the Wilson coefficients in (6), and

$$r_\chi^p = \frac{2m_p^2}{m_e(m_1 + m_2)}$$

with $m_{1,2}$ are the current quark mass inside the $P$ meson. As shown in Ref. [13], in the isospin limit, there exists

$$m_\pi^2 = \frac{m_{K^+}^2}{m_u + m_s}. \tag{22}$$

This means $r_\chi^\pi = r_\chi^K$. Consequently, one can get some constraints for weak annihilation amplitudes

$$A' = \frac{C_2}{C_1} E' = \frac{C_2}{C_1} E, \tag{23}$$

and

$$A_s = \frac{f_D f_\pi}{f_D f_K} A'.$$ \tag{24}

Thus, from eq. (16) together with eqs. (23) and (24), one will find that only three complex amplitudes, chosen, for example, as $T, C, E$, are independent, which could be determined using the present experimental data. On the other hand, it is easy to see that eq. (23) seems to be not very physical since $C_1$ and $C_2$ are both scale and scheme dependent [14]. As shown in [8], the ratio $C_2/C_1$ is about $-0.5 \sim -0.3$ for the scale $\mu$ around $1.0 \sim 1.5$ GeV, which is the range of the scale relevant for $D$ decays. Eq. (23) supports that the relative phase between $A'$ and $E'$ is $180^\circ$. Since theoretical determination for the absolute value of $C_2/C_1$ cannot be done unambiguously, in this paper, we will adopt

$$A' = -\kappa \ E' = -\kappa \ E \tag{25}$$

instead of eq. (23), where $\kappa$ is the positive parameter fixed from the experimental data.

### 3 Asymmetries

In order to go into the analysis of the amplitudes from the data, first we need to know the information about the form factors $F_{D(s)\rightarrow \pi(K)}^D(q^2)$. Here we shall use the same way as in Ref. [8], by adopting the Bauer-Stech-Wirbel model [15], in which the form factors are assumed to behave as a monopole,

$$F_{D(s)\rightarrow P}^D(q^2) = \frac{F_{D(s)\rightarrow P}^D(0)}{1 - q^2/m_\pi^2}, \tag{26}$$

where $P$ denotes $\pi$ or $K$, and $m_\pi$ is the pole mass, which has been shown in [15] for $P = \pi$ or $K$. $F_{D\rightarrow P}^D(0) (P = \pi, K)$ can be obtained via $F_{D\rightarrow P}^D(0) = F_{D\rightarrow P}^D(0)$, since the latter can be measured in semi-leptonic $D^0 \rightarrow \pi^- \ell^+\nu$ and $D^0 \rightarrow K^- \ell^+\nu$ decays. The latest experimental values from the CLEO Collaboration [16] give

$$F_{+\rightarrow K}^D(0) = 0.739 \pm 0.007 \pm 0.005,$$

$$F_{+\rightarrow \pi}^D(0) = 0.666 \pm 0.004 \pm 0.003. \tag{27}$$
Let us move to the determination of the decay amplitudes from currently available data. As mentioned in the previous section, we have three independent complex amplitudes: \( T, C, \) and \( \mathcal{E} \). Without loss of generality, \( T \) is set to be real. \( \delta_C, (\delta_E) \) is the relative strong phase of \( C \) (\( \mathcal{E} \)) to \( T \). Recall that the positive parameter \( \kappa \) introduced in eq. (25), totally we have six real parameters: \( T, |C|, \delta_C, |\mathcal{E}|, \delta_E, \) and \( \kappa \), which could be calculated from six branching ratios: \( B(D^0 \to K^-\pi^+), B(D^0 \to \bar{K}^0\pi^0), B(D^+ \to \bar{K}^0\pi^+), B(D^0 \to K^+\pi^-), B(D^+ \to K^+\pi^0), \) and \( B(D^+ \to \bar{K}^0K^+) \), given by particle data group [1]. The results of \( T, C, \mathcal{E}, \) and \( \kappa \) are summarized in Table 1, and the error is due to the uncertainties of experimental branching ratios. Other amplitudes such as \( T', C', \mathcal{E}', A', C'_s \), \( T'_s \), and \( A_s \) can be easily derived using eqs. (16), (21) and (25). Note that we get \( \kappa = 0.33 \pm 0.19 \), which is consistent with the range of \( C_2/C_1 : -0.5 \sim -0.3 \) used in Ref. [8], and also the previous fits by the CLEO Collaboration [17] and Bhattacharya and Rosner [18]:

\[
A' = (-0.32 \pm 0.24)\mathcal{E}.
\]  

Now we start to estimate the asymmetries \( R(D) \). For the neutral \( D \) decays, it has been shown in [8] that

\[
A(D^0 \to K^0\pi^0) = -\tan^2 \theta_C A(D^0 \to \bar{K}^0\pi^0),
\]  

which implies that the relative strong phase between these two amplitudes vanishes. Here \( \theta_C \) is the Cabibbo angle. Consequently, one gets

\[
R(D^0) = \frac{2\tan^2 \theta_C}{1 + \tan^4 \theta_C} \approx 2\tan^2 \theta_C \approx 0.106,
\]  

which is in agreement with the measurement in eq. (4). The same result has been obtained in Refs. [2] [19]. As pointed out in Ref. [19], the decays \( D^0 \to K^0\pi^0 \) and \( D^0 \to \bar{K}^0\pi^0 \) are related to each other under the \( U \)-spin symmetry \( s \leftrightarrow d \), thus the SU(3) symmetry breaking is expected to be extremely small in the relation (29).

In the \( D^+ \) case, we have

\[
\frac{A(D^+ \to K^0\pi^+)}{A(D^+ \to \bar{K}^0\pi^+)} = -\tan^2 \theta_C \frac{C' + A'}{C + T} = -\tan^2 \theta_C \frac{C' - \kappa \mathcal{E}}{C + T}.
\]  

One cannot expect a similar analytic relation as eq. (29) for neutral modes. However, the direct numerical calculation leads to

\[
R(D^+) = -0.010 \pm 0.026,
\]  

Table 1: Numerical results of quark-diagram amplitudes \( T, C, \mathcal{E}, \) and parameter \( \kappa \) estimated by using the present data.
which is consistent with the observed value $R(D^+) = 0.022 \pm 0.016 \pm 0.018$ [4]. Here we have corrected a sign error in the calculation of $R(D^+)$ in Ref. [8], some updated experimental data for $D \rightarrow K\pi$ decays have been used, and $D^+_s \rightarrow K^0(\bar{K}^0)K^+$ decays have been included in the present analysis. $R(D^+)$ was also predicted to be $-0.006^{+0.033}_{-0.028}$ in [18], $-0.005 \pm 0.013$ in [20], and $-0.019 \pm 0.016$ in [21].

Similar work can be done for the decays $D^+_s \rightarrow K^0K^+$ and $D^+_s \rightarrow \bar{K}^0K^+$. Using

$$\frac{A(D^+_s \rightarrow K^0K^+)}{A(D^+_s \rightarrow \bar{K}^0K^+)} = -\tan^2 \theta_C \frac{T'_s + C'_s}{C_s + A_s}$$

and eqs. (16), (24) and (25), we obtain

$$R(D^+_s) = -0.008 \pm 0.007.$$  

At present, there is no experimental measurement available for this asymmetry. It may be reported by the BES Collaboration soon. Theoretically, the prediction of $R(D^+_s)$ has also been given by $-0.003^{+0.019}_{-0.017}$ in [18], $-0.0022 \pm 0.0087$ in [20], and $-0.008 \pm 0.007$ in [21].

4 Concluding remarks

We have presented a phenomenological analysis of $D \rightarrow K\pi$ and $D^+_s \rightarrow KK$ decays including both CF and DCS modes. In terms of quark-diagram approach and factorization hypothesis, all decay amplitudes for these processes have been determined using the present data. SU(3) symmetry breaking effects from the decay constants and form factors have been taken into account in the analysis. Asymmetries $R(D^+)$’s due to interference between CF and DCS transitions have been evaluated, and the predictions of $R(D^0)$ and $R(D^+)$ are in agreement with the experimental data.

Comparing with Ref. [8], we take the absolute value of the ratio $\mathcal{A}'/\mathcal{E}$, namely $\kappa$ in this paper, as a parameter fixed from data, instead of an input. Some updated experimental $D \rightarrow K\pi$ branching ratios and the latest measurements for $F_{D^+P(0)}$ from the CLEO Collaboration have been used in the calculation. We also include CF and DCS $D^+_s \rightarrow KK$ decays in the present work, $R(D^+_s)$ is thus estimated, which is consistent with other predictions. It is expected that experimental measurement for $R(D^+_s)$ may come soon.

It will be interesting to extend the present formalism to describe CF and DCS $D$ decays involving $\eta$ or $\eta'$ mesons. However, it is seen that the relation $r^{\pi}_\chi = r^K_\chi$ from eq. (22) is essential to get the constraint (24). This relation will be complicated or explicitly violated when one includes $\eta$ or $\eta'$. A further discussion of this issue is open for the future investigation.

Acknowledgements

The author is grateful to Hai-Bo Li for helpful communications and discussions. This work was supported in part by the NSF of China under Grants No. 11075149 and 11235010.
References

[1] K.A. Olive et al., Particle data group, Chin. Phys. C 38 (2014) 090001.
[2] I.I. Bigi and H. Yamamoto, Phys. Lett. B 349 (1995) 363.
[3] Z.-Z. Xing, Phys. Rev. D 55 (1997) 196.
[4] Q. He et al., CLEO Collaboration, Phys. Rev. Lett. 100 (2008) 091801.
[5] H.-B. Li, private communication.
[6] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. 83 (1999) 1914; Nucl. Phys. B591 (2000) 313.
[7] Y.Y. Keum, H.-n. Li, and A.I. Sanda, Phys. Lett. B 504 (2001) 6; Phys. Rev. D 63 (2001) 054008; Y.Y. Keum and H.-n. Li, Phys. Rev. D 63 (2001) 074006.
[8] D.-N. Gao, Phys. Lett. B 645 (2007) 59.
[9] Y. Grossman, A.L. Kagan, and Y. Nir, Phys. Rev. D 75 (2007) 036008; H.-Y. Cheng and C.-W. Chiang, Phys. Rev. D 85 (2012) 034036; B. Bhattacharya, M. Gronau, and J.L. Rosner, Phys. Rev. D 85 (2012) 054014; Y. Grossman, A.L. Kagan, and J. Zupan, Phys. Rev. D 85 (2012) 114036; H.-n. Li, C.-D. Lü, and F.-S. Yu, Phys. Rev. D 86 (2012) 036012; J. Brod, Y. Grossman, A.L. Kagan, and J. Zupan, J. High Energy Phys. 1210 (2012) 161; H.-Y. Cheng and C.-W. Chiang, Phys. Rev. D 86 (2012) 014014; F. Buccella, M. Lusignoli, A. Pugliese, and P. Santorelli, Phys. Rev D 88 (2013) 074011; B. Bhattacharya, M. Gronau, and J.L. Rosner, Phys. Rev. D 87 (2013) 074002; G. Isidori, arXiv: 1302.0661 [hep-ph]; A. Lenz, arXiv: 1311.6447 [hep-ph].
[10] L.-L. Chau and H.-Y. Cheng, Phys. Rev. Lett. 56 (1986) 1655; Phys. Rev. D 36 (1987) 137.
[11] J.-H. Lai and K.-C. Yang, Phys. Rev. D 72 (2005) 096001.
[12] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Nucl. Phys. B606 (2001) 245; M. Beneke and M. Neubert, Nucl. Phys. B675 (2003) 333.
[13] A. Pich, Effective field theory: Course, hep-ph/9806303.
[14] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125.
[15] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C 29 (1985) 637; M. Bauer, B. Stech, and M. Wirbel, Z. Phys. C 34 (1987) 103.
[16] D. Besson et al., CLEO Collaboration, Phys. Rev. D 80 (2009) 032005.
[17] M. Artuso et al., CLEO Collaboration, Phys. Rev. D 77 (2008) 092003.
[18] B. Bhattacharya and J.L. Rosner, Phys. Rev. D 77 (2008) 114020.
[19] J.L. Rosner, Phys. Rev. D 74 (2006) 057502.

[20] B. Bhattacharya and J.L. Rosner, Phys. Rev. D 81 (2010) 014026.

[21] H.-Y. Cheng and C.-W. Chiang, Phys. Rev. D 81 (2010) 074021.