Nonlinear Tracking Control of Rigid Spacecraft under Disturbance using PID-type $H_\infty$ Adaptive State Feedback

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Future space programs will require an agile relative position and attitude control technology for spacecraft. Rendezvous and docking, capture of inoperative spacecraft and formation flight in orbit are the typical scenarios. One of the key technologies is designing the tracking controllers that can control the six degrees-of-freedom (d.o.f.) of spacecraft under the influence of physical parameter uncertainties and external disturbances. To achieve agility, the controller design must be formulated as nonlinear control problems where translational and rotational motions are dynamically coupled with each other. This paper proposes a tracking controller, proportional-integral-derivative (PID)-type $H_\infty$ adaptive state feedback controller, that can attenuate disturbances. The proposed controller has positive definite gain matrices whose conditions to be satisfied are given by linear matrix inequalities. The properties of the proposed controller were evaluated through numerical studies and compared with those of existing controllers.

Key Words: Spacecraft, Tracking Control, PID-type $H_\infty$ Adaptive State Feedback Control

Nomenclature

- $[l]$: inertial frame
- $[c]$: chaser body fixed frame
- $[f]$: target body fixed frame
- $\mathbb{R}^n$: linear space of real vectors of dimension $n$
- $\mathbb{R}^{n \times m}$: ring of matrices with $n$ rows and $m$ columns and elements in $\mathbb{R}$
- $S^q$: hypersphere of dimension $q$
- $t \in \mathbb{R}$: time
- $m, m_t \in \mathbb{R}$: mass of chaser and target
- $J, J_t \in \mathbb{R}^{3\times 3}$: inertia matrix of chaser and target
- $f, \tau \in \mathbb{R}^3$: control force and torque
- $d_f, d_t \in \mathbb{R}^3$: disturbance force and torque
- $r_c, r_t \in \mathbb{R}^3$: position vector of frame $[c]$ and $[t]$
- $[e^T \eta]^T \in S^3$: attitude vector (quaternion) of frame $[c]$
- $[e^T t]^T \in S^3$: attitude vector (quaternion) of frame $[t]$
- $v, v_t \in \mathbb{R}^3$: linear velocity vector of frame $[c]$ and $[t]$
- $o, o_t \in \mathbb{R}^3$: angular velocity vector of frame $[c]$ and $[t]$
- $p_i \in \mathbb{R}^3$: constant vector fixed frame $[f]$
- $x^s \in \mathbb{R}^{3\times 3}$: skew symmetric matrix derived from vector $x \in \mathbb{R}^3$
- $\|x\|_2 = \sqrt{x^T x}$: vector 2-norm
- $X > 0$: positive definite matrix
- $\lambda_X = \|X\|_2$: induced matrix 2-norm
- $I_n$: a unit matrix of size $n \times n$
- $O_{n \times m}$: a zero matrix of size $n \times m$
- $a_1, a_2, b_1, b_2, \gamma, \sigma_0 \in \mathbb{R}$: design parameter
- $a_r, a_v, a_m \in \mathbb{R}^{3\times 3}$: design parameter
- $k_{p1}, k_{p2}, k_{d1}, k_{d2} \in \mathbb{R}^{3\times 3}$: positive definite matrix feedback gain
- $\Gamma_1 \in \mathbb{R}$: positive scalar adaptive gain
- $\Gamma_2 \in \mathbb{R}^{6\times 6}$: positive definite matrix adaptive gain
- $\text{diag}[a, b, c, \ldots]$: diagonal matrix

1. Introduction

Future space programs will require an agile relative position and attitude control technology for spacecraft. Rendezvous and docking, capture of inoperative spacecraft and formation flight in orbit are the typical scenarios. One of the key technologies is designing the tracking controllers that can control the six degrees-of-freedom (d.o.f.) of spacecraft under the influence of physical parameter uncertainties and external disturbances. As for physical parameter uncertainties, we suppose that the mass and the inertia of the spacecraft vary as the results of fuel consumption when using thrusters. To achieve agility, the controller design must be formulated as nonlinear control problems where translational and rotational motions are dynamically coupled with each other.

Many studies have been carried out on nonlinear attitude control of rigid spacecraft. Among them, studies on passivity-based control1–12 appear to be the most promising because this control method is simple to implement and robustly stable against physical parameter uncertainties. In the same framework, attitude tracking using a proportional-derivative (PD)-type state feedback controller having positive scalar gains has been proposed1,4–9 and extended to backstepping control13,14. However, these passivity-based control methods ensure only asymptotic stability of the relative attitude under a disturbance-free environment. To achieve tracking control under disturbances, most research-
ers have focused on a nonlinear $H_{\infty}$ controller\textsuperscript{15–22} which is a PD-type state feedback controller whose scalar gains are tuned so as to make the $L_2$ gain of the closed-loop system from the disturbance to the controlled output less than a positive constant $\gamma$. However, although it generally requires high feedback gains to achieve high disturbance attenuation ability, it is not realistic because the maximum level of the control input is physically limited. Therefore, in order to suppress the disturbances, we consider it is not necessarily the best approach to employ the $H_{\infty}$ PD-type controller.

In fact, an integral action must be added to the controller to eliminate the steady-state error caused by constant disturbance as the proportional-integral-derivative (PID) control for linear systems.\textsuperscript{23} From this viewpoint, we propose a PID-type $H_{\infty}$ state feedback controller for nonlinear tracking control problems assuming that the change rate in space environment disturbance is smaller compared to the control system dynamics.\textsuperscript{24} We derived the conditions of the closed-loop asymptotic stability and $L_2$ gain less than or equal to $\gamma$ described by linear matrix inequalities (LMIs).\textsuperscript{24} Although it has achieved enough disturbance attenuation ability with moderate feedback gains, one problem still remains unsolved: the lack of closed-loop robust stability against physical parameter uncertainties.

Considering the above facts, this paper extended the results of Ikeda et al.\textsuperscript{24} to a PID-type $H_{\infty}$ adaptive state feedback tracking controller that has a parameter update law to cope with parameter uncertainties as well as to effectively attenuate constant signals included in the disturbance. The properties of the proposed controller were evaluated through numerical studies and compared with those of existing controllers.

2. Modeling and Problem Description

We consider a control problem in which a chaser spacecraft tracks a target point moving in an inertial frame under the influence of disturbances. Frames and vectors are defined in Fig. 1. Our objective is to control the chaser so that its mass center $C$ tracks point $P$ and frame $\{c\}$ tracks frame $\{t\}$.

The translation and rotation dynamics of the chaser fixed frame $\{c\}$ are given by the following equations\textsuperscript{25}:

\begin{align}
    m \ddot{v} + m \omega^T \dot{v} &= f + d_r, \quad \text{(1)} \\
    J \ddot{\omega} + \omega^T \dot{\omega} &= \tau + d_\omega. \quad \text{(2)}
\end{align}

The position of mass center $C$ and the attitude of $\{c\}$ with respect to $\{t\}$ are given by the following kinematics if a quaternion is used as the attitude parameterization:

\begin{align}
    \dot{r} &= v - \omega^T r, \\
    \dot{q} &= E(q) \omega = \frac{1}{2} \left[ \eta I_3 + \varepsilon^T \right] \omega, \quad \text{(3)}
\end{align}

where $q = [\varepsilon^T \eta]^T$ satisfies the constraint $\|q\| = 1$, $\forall t \geq 0$.

On the other hand, the dynamics and kinematics of the target motion are described as follows:

\begin{align}
    m_i \ddot{v}_i + m_i \omega_i^T \dot{v}_i &= 0, \quad \text{(4)}
\end{align}

\begin{align}
    \dot{r}_i &= v_i - \omega_i^T r_i, \\
    \dot{q}_i &= E(q_i) \omega_i = \frac{1}{2} \left[ \eta I_3 + \varepsilon_i^T \right] \omega_i, \quad \text{(6)}
\end{align}

Then, the position and velocity of point $P$ fixed in frame $\{t\}$ are given by

\begin{align}
    r_p &= r_i + p_i, \\
    v_p &= v_i + \omega_i^T p_i. \quad \text{(7)}
\end{align}

The objective of our tracking control problem is to find control laws such that

\begin{align}
    r &= r_p, \quad q = q_i, \quad v = v_p, \quad \omega = \omega_i \quad \text{as } t \to \infty.
\end{align}

To this end, an error system in $\{c\}$ is described as follows. Let the direction cosine matrix from $\{t\}$ to $\{c\}$ be

\begin{align}
    C &= \left( \eta_c^2 - \varepsilon_c^T \varepsilon_c \right) I_3 + 2\varepsilon_c \varepsilon_c^T - 2\eta_c \varepsilon_c^T \varepsilon_c \quad \text{(8)}
\end{align}

using the quaternion of relative attitude $q_c = [\varepsilon_c^T \eta_c]^T$, where $\varepsilon_c$ and $\eta_c$ are defined as

\begin{align}
    \varepsilon_c &= \eta_c \frac{\varepsilon_c - \eta_c}{\varepsilon_c^2 - \eta_c^2}, \quad \varepsilon_c = \eta_c \frac{1}{\varepsilon_c^2 - \eta_c^2}, \quad \eta_c = \eta_c \frac{1}{\varepsilon_c^2 - \eta_c^2} \quad \text{(9)}
\end{align}

The relative position, linear velocity, and angular velocity are given in the same frame $\{c\}$ as

\begin{align}
    r_c &= r - Cr_p, \quad v_c = v - Cv_p, \quad \omega_c = \omega - C \omega_i. \quad \text{(10)}
\end{align}

Substitution of Eq. (10) into Eqs. (1)–(3) using the identity $C = -\omega_c^T C$ yields the following relative motion equations:

\begin{align}
    m \ddot{v}_c &= -m \left( (\omega_c + C \omega_i)^T v_c + C \dot{v}_p + (C \omega_i)^T C v_p \right) + f + d_f, \quad \text{(11)}
    J \ddot{\omega}_c &= -(\omega_c + C \omega_i)^T J(\omega_c + C \omega_i) - J(C \omega_i - \omega_c^T C \omega_i) + \tau + d_\tau, \quad \text{(12)}
    \dot{r}_c &= v_c - (\omega_c + C \omega_i)^T r_c, \quad \text{(13)}
    \dot{q}_c &= E(q_c) \omega_c = \frac{1}{2} \left[ \eta_c I_3 + \varepsilon_c^T \right] \omega_c. \quad \text{(14)}
\end{align}

By transformation, the tracking control problem is reduced to a regulation problem to design control inputs $f$ and $\tau$ such that

\begin{align}
    (r_c, \varepsilon_c, v_c, \omega_c) \to (0, 0, 0, 0) \quad \text{as } t \to \infty \quad \text{under disturbance } d_f \text{ and } d_\tau,
\end{align}

according to
Eqs. (11)–(14).

Hereafter, we assume that the following variables

chaser: \( r, \varepsilon, \eta, v, \omega \)

target: \( r_i, \varepsilon_i, \eta_i, v_i, \omega_i, \dot{v}_i, \dot{\omega}_i \)

are directly measurable,\(^1\) and \( m \) and \( J \) are unknown but nominal values \( m_0 \) and \( J_0 \) are known. In addition, regarding the target states, the following assumption is made.

**Assumption 1**

The target states \( r_i, \varepsilon_i, \eta_i, v_i, \omega_i, \dot{v}_i, \dot{\omega}_i \) and \( \dot{\omega}_i \) are uniformly continuous and bounded for all \( t \in [0, \infty) \).

**Remark 1**

Quaternions \( \eta_e \) when \( \varepsilon_e = 0 \) exists as \( \eta_e = \pm 1 \) from the constraint of quaternion \( \| q_e \| = 1 \). In this paper, \( \eta_e \), which should be stabilized, is set to \( \eta_e = 1 \).

3. **PID-type** \( H_\infty \) **Adaptive State Feedback Controller**

In this section, we derive a PID-type \( H_\infty \) adaptive state feedback controller. For this design, we further transform Eqs. (11)–(14) as

\[
\ddot{v}_e = v_c - (C\alpha)^x v_e, \quad (15)
\]

\[
\ddot{f} = J \dot{m} \delta_r, \quad (16)
\]

\[
\ddot{t} = \tau - \delta_m \dot{\omega}_m, \quad (17)
\]

where \( \ddot{f}, \ddot{t} \in \mathbb{R}^3 \) are the new inputs, and

\[
\delta_r = 2(C\alpha)^y \ddot{v}_e + (C\alpha)^x (C\alpha)^y v_e + (C\alpha)^y \dot{v}_e \\
+ \left( C\alpha \right)^y \ddot{v}_e + \ddot{v}_p, \quad (18)
\]

\[
\delta_q = \delta_q_1(q_e, C\alpha) + \delta_q_2(C\alpha, \omega_e + C\alpha) + \ddot{q}_2(C\alpha), \quad (19)
\]

\( \alpha_0 = [J_{0,11} J_{0,12} J_{0,13} J_{0,22} J_{0,23} J_{0,33}]^T, \)

\[
\delta_q_1(x, y) = x^2 J_0 y, \quad \delta_q_2(x, y) = J_0 x, \quad \forall x, y \in \mathbb{R}^3 \]

\[
\delta_q_2(x) = \begin{bmatrix}
    x_1 & x_2 & x_3 & 0 & 0 & 0 \\
    0 & x_1 & x_2 & x_3 & 0 & 0 \\
    0 & 0 & x_1 & x_2 & x_3 & 0
\end{bmatrix}, \quad (20)
\]

\[
J_{0,ij} (i \leq j, i = 1, 2, j = 1, 2, 3) \text{ is the (i, j) element of matrix } J_0. \]

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\[
\dot{r}_e = \dot{v}_e - \alpha_e^x \dot{r}_e, \quad (22)
\]

\[
\dot{q}_e = E(q_e) \omega_e, \quad (23)
\]

where \( \Delta m = m - m_0 \) and \( \Delta \alpha = \alpha - \alpha_0 \). For the above system Eqs. (20)–(23), the following Lemma holds.

**Lemma 1**

The system Eqs. (20)–(23) are passive w.r.t input \( u = [\dot{f}^T \dot{t}^T]^T \) and output \( y = [\dot{v}_e^T \alpha_e^y]^T \) when \( \Delta m = 0, \Delta \alpha = 0, d_f = 0 \) and \( d_t = 0 \).

**Proof:** Let us define the storage function as

\[
S = \frac{m}{2} \| \dot{v}_e \|^2 + \frac{1}{2} \alpha_e^y J \omega_e. \quad (24)
\]

The time derivative of Eq. (24) along the trajectories of system Eqs. (20)–(23) with \( \Delta m = 0, \Delta \alpha = 0, d_f = 0 \) and \( d_t = 0 \) become

\[
\dot{S} = \ddot{v}_e^T (-m \alpha_e^x \dot{v}_e + \dot{f}) + \alpha_e^y ( - \alpha_e^y \dot{\omega}_e + \ddot{t} ) \\
= \ddot{v}_e^T \dot{f} + \alpha_e^y \ddot{t} \\
= \ddot{y}^T u.
\]

Therefore, system Eqs. (20)–(23) are passive w.r.t input \( u \) and output \( y \) when \( \Delta m = 0, \Delta \alpha = 0, d_f = 0 \) and \( d_t = 0 \).

By transforming Eqs. (15)–(17), the control problem is now ready to regulate system Eqs. (20)–(23) in order to design control inputs \( \ddot{f} \) and \( \ddot{t} \) such that

\( (r_e, \varepsilon_e, \eta_e, \dot{v}_e, \omega_e) \rightarrow (0, 0, 1, 0, 0) \)

as \( t \rightarrow \infty \).

Now, let us consider the PID-type adaptive state feedback controller that has positive definite gain matrices as follows:

\[
\begin{align*}
\dot{f} &= -\frac{1}{a_2} \left( k_{p1} \dot{r}_e + K_{d1} \dot{\omega}_e \right) - k_{i1} \xi_1 + \Delta \dot{m} \delta_r, \\
\dot{\xi}_1 &= \int_0^t \left( r_e + \frac{a_2}{a_1} \alpha_e^x r_e \right) dt, \\
\Delta \dot{m} &= -\Gamma \delta_t^T (a_1 \dot{r}_e + a_2 \dot{\omega}_e), \\
\dot{\xi}_2 &= \int_0^t \left( \varepsilon_e + \frac{b_2}{2b_1} \left( 2 - \eta_e \right) I_3 - \epsilon_e^x \omega_e \right) dt, \\
\Delta \dot{\omega} &= -\Gamma \delta_t^T (b_1 \varepsilon_e + b_2 \omega_e), \\
K(q_e) &= \left( \eta_e I_3 - \epsilon_e^x \right) K_{p3} + k_{p3} \left( 1 - \eta_e \right) I_3,
\end{align*}
\]

where \( \Delta \dot{m} \) and \( \Delta \dot{\omega} \) are estimated values of \( \Delta m \) and \( \Delta \omega \), respectively, and the output to be controlled is defined as

\( \zeta = [\dot{r}_e \dot{\varepsilon}_e^T 2 \cos^{-1}(|\eta_e|) \alpha_e^y]^T \).

From the definition of quaternion, \( \cos^{-1}(|\eta_e|) \) of the element of \( \zeta \) represents the eigen-angle around the unit vector (eigen-axis) with respect to relative attitude.\(^{17}\) Then, the following theorem can be obtained.

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291
Theorem 1

Given $a_1, a_2, b_1, b_2$, and $\gamma > 0$, the closed-loop system of Eqs. (20)–(23) with Eqs. (25), (26) satisfies the $L_2$ gain less than or equal to $\gamma$ from disturbance input $d = [d_1^T \ d_2^T]^T \in L_1[0, T]$ to controlled outputs $z$ if feedback gains satisfy the following conditions:

$$ F > 0, \quad 2k_{p3}I_3 > k_{p2} > k_{p3}I_3, \quad Q > 0, \quad (27) $$

$$ Q - \bar{\Sigma} \bar{\Sigma} - \frac{1}{4\gamma^2} W^T W > 0, \quad (28) $$

$$ F = \text{diag}(F_1, F_2), \quad Q = \text{diag}(Q_1, Q_2), $$

$$ F_1 = \begin{bmatrix} k_{p1}I_3 & a_1mI_3 & a_{k1}I_3 \\ * & a_2mI_3 & 0 \\ * & * & a_{k1}I_3 \end{bmatrix}, $$

$$ F_2 = \begin{bmatrix} 2k_{p2} & b_1J & b_{k2}I_3 \\ * & b_2J & 0 \\ * & * & b_{k2}I_3 \end{bmatrix}, $$

$$ Q_1 = \begin{bmatrix} a_1k_{p1}I_3 - a_{k2}I_3 & a_1d_2 \frac{K_{d1}}{a_2} \\ * & K_{d1} - a_1mI_3 \end{bmatrix}, $$

$$ Q_2 = \begin{bmatrix} \frac{b_1}{b_2} (2k_{p3}I_3 - K_{p2}) - (b_2 + \frac{b_3^2}{2b_1})k_{k2}I_3 \\ * & \frac{b_1}{2b_2} K_{d2} - \frac{b_3^2}{2b_1}k_{k2}I_3 \end{bmatrix}, $$

$$ W = \begin{bmatrix} a_1I_3 & a_2I_3 & O_{3x3} & O_{3x3} \\ O_{3x3} & O_{3x3} & b_1I_3 & b_2I_3 \end{bmatrix}. $$

$$ \bar{\Sigma} = \text{diag}(\sigma_1, \sigma_2, \pi, \sigma_3I, \sigma_3), $$

where symbol $*$ denotes a symmetric element. Moreover, the state variable of the closed-loop system becomes

$$ (r_e, \epsilon_e, \eta_e, \bar{v}_e, \omega_e, \xi_1, \xi_2, \Delta \hat{m}, \Delta \hat{a}) \rightarrow (0, 0, 1, 0, 0, 0, 0, 0, 0) $$
as $t \rightarrow \infty$ for arbitrary initial state when $d = 0$, where $c_1 \in \mathbb{R}$ and $c_2 \in \mathbb{R}^n$ are a constant scalar and a constant vector, respectively, determined by an initial condition of parameter update law.

Proof: Let us define the candidate of the Lyapunov function as

$$ V = \frac{1}{2} \chi^T F \chi + k_{p3}(\eta_e - 1)^2 + \frac{1}{2} \Delta \hat{m}^2 + \frac{1}{2} \Delta \hat{a}^T \Gamma_2^{-1} \Delta \hat{a}, $$

$$ \chi = [r_e^T \ \epsilon_e^T \ \eta_e^T \ \bar{v}_e^T \ \omega_e^T \ \xi_1^T \ \xi_2^T]^T, $$

where $\Delta \hat{m} = \hat{m} - \bar{m}$ and $\Delta \hat{a} = \Delta a - \bar{a}$ are estimated errors. Therefore, $V > 0$ if $F > 0$. The time derivative of Eq. (29) along the trajectories of the closed-loop system becomes

$$ \dot{V} = -\left(\frac{a_1k_{p1} - a_{k1}d_1}{a_2} \right) \|r_e\|^2 - \frac{a_1}{a_2} T K_{d1} \bar{v}_e $$

$$ + \bar{v}_e^T (K_{d1} - a_1mI_3) \bar{v}_e - \frac{b_1}{b_2} T K_{d2} \omega_e $$

$$ + \frac{b_3}{b_1} T (K_{d2} - \frac{b_3^2}{2b_1}k_{k2}I_3) \omega_e $$

$$ - \frac{b_2}{2b_1} T K_{d2} \omega_e + \frac{b_1}{2} T J T(q_e) \omega_e - b_1 \epsilon_e^T \omega_e J \omega_e $$

$$ + \frac{b_2}{2b_1} T k_{k2} \eta_e^T \omega_e + \frac{b_1}{2} T \eta_e J \omega_e $$

$$ + (a_1 \epsilon_e^T + a_2 \bar{v}_e^T) \frac{d}{dt} J, $$

$$ + (b_1 \epsilon_e^T + b_2 \bar{v}_e^T) \frac{d}{dt} J, $$

$$ \text{where} \quad T(q_e) = \eta_e I_3 + \epsilon_e^T. $$

In Eq. (30),

$$ \|T(q_e)\| = 1, \quad \|\epsilon_e\| \leq 1, \quad \|\omega_e\| = \|\omega_e\|, $$

$$ \left| \frac{1}{2} \epsilon_e^T J T(q_e) \omega_e - b_1 \epsilon_e^T \omega_e J \omega_e \right| \leq \frac{3}{2} b_1 \epsilon_e T \omega_e \omega_e, $$

$$ - \frac{b_2}{2b_1} T k_{k2} \eta_e^T \omega_e \leq \frac{b_2}{2b_1} T k_{k2} \left( \|\epsilon_e\|^2 + \frac{1}{4} \|\omega_e\|^2 \right), $$

and the identity $\epsilon_e^T T(q_e) = \epsilon_e^T \eta_e$ yields

$$ - \epsilon_e^T K_{d2} \omega_e = - \epsilon_e^T \left( \eta_e (K_{d2} - k_{p3}I_3) + k_{p3}I_3 \right) \epsilon_e $$

$$ = - \epsilon_e^T G(\eta_e) \epsilon_e, $$

where $G(\eta_e)$ is described as follows according to $\eta_e$.

$$ G(\eta_e) = \begin{cases} K_{p2}, & \eta_e = 1 \\ \eta_e (K_{p2} - k_{p3}I_3) + k_{p3}I_3, & 1 > \eta_e > 0 \\ -\eta_e (K_{p2} - k_{p3}I_3) + k_{p3}I_3, & 0 > \eta_e > -1 \\ 2k_{p3}I_3 - K_{p2}, & \eta_e = -1. \end{cases} $$

From the above equation, $G(\eta_e) > 0$ for all $\eta_e$ if $2k_{p3}I_3 > K_{p2} > k_{p3}I_3$. Furthermore, the minimum value of $G(\eta_e)$ is

$$ \min G(\eta_e) = G(-1) = 2k_{p3}I_3 - K_{p2}. $$

Therefore, if $2k_{p3}I_3 > K_{p2} > k_{p3}I_3$, then Eq. (31) becomes

$$ - \epsilon_e^T K_{d2} \omega_e \leq - \epsilon_e^T (2k_{p3}I_3 - K_{p2}) \epsilon_e, $$

and Eq. (30) satisfies

$$ \dot{V} \leq -\left(\frac{a_1k_{p1} - a_{k1}d_1}{a_2} \right) \|r_e\|^2 - \frac{a_1}{a_2} T K_{d1} \bar{v}_e $$

$$ - \bar{v}_e^T (K_{d1} - a_1mI_3) \bar{v}_e $$

$$ - \epsilon_e^T \left( \frac{b_1}{b_2} (2k_{p3}I_3 - K_{p2}) - (b_2 + \frac{b_3^2}{2b_1})k_{k2}I_3 \right) \epsilon_e $$

$$ - \epsilon_e^T \left( \frac{b_1}{2b_2} K_{d2} - \frac{b_3^2}{2b_1}k_{k2}I_3 \right) \epsilon_e $$

$$ - \omega_e^T (K_{d2} - \frac{3}{2} b_1 \epsilon_e J \omega_e) $$

$$ + \frac{b_2}{8b_1} T k_{k2} \eta_e^T \omega_e $$

$$ + (a_1 \epsilon_e^T + a_2 \bar{v}_e^T) \frac{d}{dt} J + (b_1 \epsilon_e^T + b_2 \bar{v}_e^T) \frac{d}{dt} J $$

$$ = -\chi^T Q \chi + \chi^T W^T d. $$

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\[ \ddot{\chi} = [r_e^T \hat{v}_e^T \hat{e}_e^T \hat{\omega}_e^T]^T. \] (32)

By completing the square, we obtain
\[ \dot{V} + \|c\|^2 - \gamma^2 \|d\|^2 \leq -\chi^T Q \dot{\chi} + \frac{1}{4\gamma^2} \chi^T W^T W \dot{\chi} \]
\[ -\gamma^2 \|d - \frac{1}{2\gamma^2} W \dot{\chi}\|^2 + \chi^T \Sigma^T \Sigma \chi. \]

If we note that
\[ d^T = \frac{1}{4\gamma^2} W \dot{\chi} \]
is the worst-case disturbance and that
\[ \chi^T \Sigma^T \Sigma \chi \leq \chi^T \Sigma^T \Sigma \dot{\chi} \]
from \(2 \cos^{-1}(\eta_e) \leq \pi \|e_e\|\), the inequality
\[ \dot{V} + \|c\|^2 - \gamma^2 \|d\|^2 \leq -\chi^T \left( Q - \Sigma^T \Sigma - \frac{1}{4\gamma^2} W^T W \right) \dot{\chi} \]
holds for all \(d \in L_2[0, T]\). Therefore, the condition in Eq. (28) implies
\[ \dot{V} \leq \gamma^2 \|d\|^2 - \|c\|^2 \]
indicating that the \(L_2\) gain of the closed-loop is less than or equal to \(\gamma\).³⁰

With regard to the asymptotic stability, when \(d = 0\), Eq. (32) becomes
\[ \dot{V} \leq -\chi^T Q \dot{\chi} \]
and \(\dot{V} \leq 0\) if \(Q > 0\). Therefore,
\[ V(x(t)) \leq V(x(0)), \quad \forall t \geq 0 \quad (x = [\chi^T \eta_e \Delta \hat{m} \Delta \hat{\omega}_e^T]^T) \]
and \(\dot{V}\) is bounded because \(V\) is radially unbounded. Furthermore, control inputs Eqs. (25) and (26) are bounded by the conditions in Assumption 1. Since \(\dot{\chi}\) is also bounded,
\[ \dot{V} \leq -\frac{1}{2\gamma^2} \chi^T Q \dot{\chi} \]
is bounded and \(\dot{V}\) is uniformly continuous with respect to \(t\). Additionally, since \(V\) is lower bounded from \(V \geq 0\),
\[ \dot{V} \rightarrow 0 \Rightarrow \dot{V} \rightarrow 0 \]
as \(t \rightarrow \infty\) from the Lyapunov-like lemma.³¹ Hence, \((\xi_1, \xi_2) \rightarrow (0, 0)\) and \(\eta_e \rightarrow 1\) when \(V = 0\). In addition, since the closed-loop system becomes
\[ \Delta \hat{m} = 0, \quad \Delta \hat{\omega} = 0, \]
\(\Delta \hat{m} = 0\) and \(\Delta \hat{\omega}\) converge to a constant scalar \(c_1\) and a constant vector \(c_2\), respectively. Therefore, the state variable of the closed-loop system becomes
\[ (r_e, e_e, \eta_e, \hat{v}_e, \hat{\omega}_e, \xi_1, \xi_2, \Delta \hat{m}, \Delta \hat{\omega}) \]
\[ \rightarrow (0, 0, 1, 0, 0, 0, 0, 0, c_1, c_2) \]
as \(t \rightarrow \infty\) for arbitrary initial state when \(d = 0\). ■

Remark 2

1) Since physical parameters \(m\) and \(J\) are in non-diagonal elements of matrix \(F\) such as \(a_1 m I_3\) and \(b_1 J\) and in diagonal elements of matrix \(Q\) such as \(-a_1 m I_3\) and \(-(3/2)b_1 J I_3\), the influence of parameter uncertainties can be reduced by setting \(a_1\) and \(b_1\) to small values. In addition, by setting \(\gamma\) and \(\Sigma\) to appropriate values, the term of \(-\Sigma^T \Sigma - (1/4\gamma^2) W^T W\) in condition Eq. (28) does not become large. Therefore, by setting \(a_1, b_1, k_{11}\) and \(k_{12}\) to small values and \(a_2, b_2, k_{p1}, k_{p2}, k_{p3}, K_{d1}\) and \(K_{d2}\) to large values under the condition \(2k_{p1} > K_{p2} > k_{p3} I_3\) and \(\gamma, \Sigma\) to appropriate values, the feedback gains satisfy conditions Eqs. (27) and (28) for small parameter uncertainties.

2) The obtained conditions in Eqs. (27) and (28) are LMIs with respect to feedback gains that are effectively solved using convex optimization tools.³²

3) If we set \(K_{p2} = k_{p3} I_3\), matrix \(K(q_e)\) becomes a positive scalar constant \(K_{p3}\).

4) It can be seen that the position and attitude can track their targets without offset errors when the disturbance is constant. At the steady state, \(\delta_e = 0\) and \(\omega_e = 0\) hold, assuming that \(r_e\) and \(e_e\) are very small. Therefore, as
\[ K(q_e) \approx K_{p2}, \quad \xi_1 \approx \int_0^t r_e \mathrm{d}t, \quad \xi_2 \approx \int_0^t e_e \mathrm{d}t, \]
\[ \delta_e \approx 0, \quad \delta_q \approx 0, \]
the closed-loop system is
\[ \frac{1}{a_2} k_{p1} r_e + k_{i1} \int_0^t r_e \mathrm{d}t - d_f = 0, \quad \frac{1}{b_2} k_{p2} e_e + k_{i2} \int_0^t e_e \mathrm{d}t - d_e = 0. \]

If we define
\[ e_1 = \int_0^t r_e \mathrm{d}t - \frac{1}{k_{i1}} d_f, \quad e_2 = \int_0^t e_e \mathrm{d}t - \frac{1}{k_{i2}} d_e, \]
then Eqs. (33) and (34) become
\[ \dot{e}_1 = -a_2 k_{i1} e_1, \quad \dot{e}_2 = -b_2 k_{i2} K_{p2}^{-1} e_2. \]

Since \(a_2, b_2, k_{p1}, k_{i1}\) and \(k_{i2}\) are greater than 0 and \(K_{p2} = K_{p2}^T > 0, (e_1, e_2) \rightarrow (0, 0)\) as \(t \rightarrow \infty\). Therefore, from Eqs. (33) and (34), it can be considered that \(r_e \rightarrow 0\) and \(e_e \rightarrow 0\) as \(t \rightarrow \infty\).

4. Numerical Study

The properties of the proposed controller were compared and discussed in this numerical study. For this purpose, we set the physical parameters of the target and chaser spacecraft as
\[ m_t = 300 \text{ kg}, \quad J_t = \text{diag}[50, 275, 275] \text{ kgm}^2, \]
\[ m_0 = 200 \text{ kg}, \quad J_0 = \begin{bmatrix} 75.0 & -28.1 & -28.1 \\ \ast & 75.0 & -28.1 \\ \ast & \ast & 75.0 \end{bmatrix} \text{ kgm}^2, \]
\[ \Delta m = 60 \text{ kg}, \quad \Delta J = \begin{bmatrix} 22.5 & -8.44 & -8.44 \\ \ast & 22.5 & -8.44 \\ \ast & \ast & 22.5 \end{bmatrix} \text{ kgm}^2. \]
The target position in frame $t$ is given as $p_t = [0 \ 0 \ 0]^T$. The initial conditions for the chaser spacecraft are \( r(0) = [10 \ 10 \ 10]^T \) m, \( v(0) = [0 \ 0 \ 0]^T \) m/s, \( q(0) = [0.06 \ 0.69 \ 0.06 \ 0.72]^T \), \( \omega(0) = [0 \ 0 \ 0]^T \) rad/s, and those for the target are \( r_t(0) = [3 \ 3 \ 3]^T \) m, \( v_t(0) = [0 \ 0 \ 0]^T \) m/s, \( q_t(0) = [0 \ 0 \ 0 \ 1]^T \), \( \omega_t(0) = [0.2 \ 0.2 \ 0.2]^T \) rad/s.

### 4.1. Tracking performance under a disturbance-free environment

First, we show the six d.o.f. tracking ability of the conventional controller\(^{(24)}\)

\[
\begin{align*}
\dot{f} &= -\frac{1}{a_2}(k_1 r_e + K_{d1} \dot{v}_e) - k_1 \dot{\xi}_1 + m_0 \delta_e \\
\dot{\xi}_1 &= \int_0^t \left( r_e + \frac{a_2}{a_1} \omega_e \dot{r}_e \right) dt, \\
\ddot{\xi}_2 &= \int_0^t \left[ \dot{v}_e + \frac{b_2}{2b_1} \left( 2 - \eta_e \right) I_3 - \eta_e \omega_e \right] dt,
\end{align*}
\]

and proposed controller in Eqs. (25) and (26) when \( d = 0 \). The conventional and proposed controller gains are selected to satisfy only the conditions in Eqs. (27) and (28), respectively. The controller gains are set as

\[
\begin{align*}
a_1 &= 0.2, \quad b_1 = 0.1, \quad a_2 = b_2 = 1, \\
k_1 &= 18, \quad K_{d2} = 31I_3, \quad k_2 = 31, \\
K_{d1} &= 180I_3, \quad K_{d2} = 300I_3, \\
k_3 &= 1.0, \quad k_2 = 0.4, \\
\Gamma_1 &= 40, \quad \Gamma_2 = 600I_6.
\end{align*}
\]

Figure 2 shows the responses of relative positions and relative quaternions of the conventional and proposed controllers. In the conventional controller, the chaser cannot track the target under the influence of physical parameter uncertainties. On the other hand, in the proposed controller, the chaser can track the target because the influence of physical parameter uncertainties is compensated by the parameter update law.

### 4.2. Tracking under constant disturbance

Then, we examine the six d.o.f. tracking ability of the conventional and proposed controllers under constant disturbances

\[
d_f = [3 \ 3 \ 3]^T N, \quad d_e = [3 \ 3 \ 3]^T N m.
\]

To compare only the six d.o.f. tracking ability under constant disturbances, the physical parameter uncertainties \( \Delta m \) and \( \Delta J \) are set to zero. The simulation results are shown in Fig. 3. Both controllers can eliminate the steady-state error caused by a constant disturbance, and the chaser tracks the target. However, time responses of relative position and attitude vibrate. Additionally since the proposed control law uses \( \Delta \dot{\eta} \), the control inputs vibrate. As a result, the time responses of relative position and attitude vibrate.

The simulation results with the physical parameter uncertainties under disturbances are shown in Fig. 4. The tracking performance of the conventional controller deteriorates under the influence of physical parameter uncertainties. On the other hand, the tracking performance of the proposed controller is almost the same as in the case of Fig. 3 except that \( \dot{\epsilon}_3 \).

### 4.3. Disturbance attenuation ability

Finally, we examine the disturbance attenuation ability. We compare the conventional and proposed controllers, both of which exhibit the \( H_\infty \) property when \( \gamma = 0.8 \) and \( \gamma = 0.2 \). The design parameters are set as

\[
\begin{align*}
a_1 &= 8, \quad b_1 = 4, \quad a_2 = b_2 = 40, \\
\sigma_r &= 6I_3, \quad \sigma_v = 1I_3, \quad \sigma_{\dot{\eta}} = 3, \quad \sigma_\omega = 1I_3,
\end{align*}
\]

![Fig. 2. Simulation results under a disturbance-free environment (solid line: proposed controller, dashed line: conventional controller).](image-url)
The disturbance input $d \in L_2[0, T]$ is

$$d_f = 3 \sin \left( \frac{\pi}{40} \right) [1 1]^T \text{N},$$

$$d_r = 3 \sin \left( \frac{\pi}{40} \right) [1 1]^T \text{Nm},$$

where $T = 100$. For the same initial conditions, the responses of relative position and attitude norms described by Euler angles of 3-2-1 system are obtained.

First, as in previous subsection, in order to compare only the disturbance attenuation ability, physical parameter uncertainties $\Delta m$ and $\Delta J$ are set to zero. The simulation results are shown in Fig. 5. Figure 5 show that the disturbance attenuation ability is higher in the proposed controller than in the conventional controller. This is considered that the parameters $\Delta \hat{m}$ and $\Delta \hat{a}$ are estimated to reduce the relative errors because the parameter update law uses the relative error information. Similar results are obtained for other periodic disturbances.

The simulation results with the physical parameter uncertainties are shown in Fig. 6. Although the disturbance attenuation performance of the conventional controller deteriorates due to the influence of physical parameter uncertainties, that of the proposed controller is almost the same as in the case of Fig. 5.

5. Conclusion

We investigated six d.o.f. nonlinear tracking control technologies of spacecraft under external disturbance in order to prepare for future space missions, and a PID-type $H_{\infty}$ adaptive state feedback controller was proposed. Conditions of the asymptotic stability of error systems and the $L_2$ gain properties of a closed-loop system were obtained. The performances of the proposed controller were compared and discussed through numerical studies.
Fig. 5. Simulation results under disturbance without parameter uncertainties (top: norm of relative position, bottom: norm of relative attitude, solid line: proposed controller, dashed line: conventional controller).

Fig. 6. Simulation results under disturbance with parameter uncertainties (top: norm of relative position, bottom: norm of relative attitude, solid line: proposed controller, dashed line: conventional controller).

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