Spin Hall effects for cold atoms in a light induced gauge potential

Shi-Liang Zhu, Hao Fu, C.-J. Wu, S.-C. Zhang, and L.-M. Duan

1 FOCUS center and MCTP, Department of Physics, University of Michigan, Ann Arbor, MI 48109
2 Institute for Condensed Matter Physics and SPTE, South China Normal University, Guangzhou, China
3 Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106
4 Department of Physics, Stanford University, Stanford, CA 94305-4045

We propose an experimental scheme to observe spin Hall effects with cold atoms in a light induced gauge potential. Under an appropriate configuration, the cold atoms moving in a spatially varying laser field experience an effective spin-dependent gauge potential. Through numerical simulation, we demonstrate that such a gauge field leads to observable spin Hall currents under realistic conditions. We also discuss the quantum spin Hall state in an optical lattice.

PACS numbers: 05.30.Fk, 03.65.Vf, 72.25.-b, 73.43.-f

The spin Hall effect has recently attracted strong interest in condensed matter physics because of its connection to quantum Hall physics \[ \frac{1}{2} \] and its potential applications in spintronics \[ \frac{3}{4} \]. In analogy to the conventional Hall effect related to charge currents, the spin Hall effect refers to the generation of a spin current transverse to an applied electric field. It has been proposed to occur in certain solid-state systems with some primitive experimental demonstration \[ 3 \]. An essential requirement for observation of the spin Hall effect is generation of an effective spin-dependent gauge potential either in momentum space \[ \frac{5}{6} \] or in real space \[ 7 \]. In both cases, spin-orbit coupling in semiconductors or graphene are employed to provide such mechanisms.

It has been widely acknowledged that ultracold atomic gases provide an ideal playground to experimentally investigate some fundamental phenomena originally connected with condensed matter systems \[ 10 \]. The remarkable controllability in these systems allows a clean study of many complicated physics in a controllable fashion. The generation of an effective gauge potential in atomic systems has raised significant interest, from the earlier implementation of rotating traps \[ 11 \] to the more recent work on light induced gauge fields \[ 12, 13, 14, 15, 16, 17, 18 \]. Although most previous work focuses on the study of scalar gauge fields, it is natural to ask whether it is possible to study spin Hall effect in an atomic system.

In this paper, we propose an experimental scheme for observation of the spin Hall effects in a cold atomic gas. We show that for atoms with a simple Λ-type three-level configuration moving in a spatially varying laser field, a spin-dependent gauge potential in the real space naturally arises in connection with the Berry phase associated with the atomic motion. Under an applied effective “electric” field, which can be generated from gravity for instance, the atoms will follow a spin-dependent trajectory, which leads to a net spin current in the direction perpendicular to the “electric” and the gauge field while the mass current is zero. Furthermore, we show it is easy to generate different forms of the gauge field in this system, with a strong periodic gauge field as an example. The diverse configurations of the gauge field, in combination with the tunable interaction and the controllable potentials for the atomic gas, may allow us to study various kinds of interesting Hall physics in this system. With this gauge field, we also discuss the associated quantum spin Hall effect for fermionic atoms in an optical lattice.

We consider an atomic gas with each atom having an Λ-type level configuration as shown in Fig. 1a. The ground states \(|1⟩\) and \(|2⟩\) are coupled to an excited state \(|3⟩\) through spatially varying laser fields, with the corresponding Rabi frequencies \(Ω_1\) and \(Ω_2\), respectively. Different from the previous work \[ 13, 18 \], we assume here off-resonant couplings for the single-photon transitions with the same large detuning \(Δ\), and we will use the bright state as well as the dark state to realize a spin dependent gauge field for the atoms.

The full quantum state of the atoms \(|Φ(r)⟩\) (including both the internal and the motional degrees of freedom) can be expanded as \(|Φ(r)⟩ = ∑_{j=1}^{3} \phi_j(r)⟨j|⟩\), where \(r\) denotes the atomic position. The Hamiltonian of the atom has the form \(H = \frac{p^2}{2m} + V(r) + H_{int}\), where \(m\) is the atomic mass, \(V(r)\) denotes the external trapping potential which we assume to be diagonal in the internal states \(|j⟩\) with the form \(V(r) = ∑_j V_j(r)⟨j|⟩⟨j|\), and \(H_{int}\) is the laser-atom interaction Hamiltonian, given by:

\[ H_{int} = \begin{pmatrix} 0 & 0 & Ω_1 \\ 0 & 0 & Ω_2 \\ Ω_1^* & Ω_2 & 2Δ \end{pmatrix} \]  

in the basis \{\(|1⟩, |2⟩, |3⟩\}\}. We parameterize the Rabi frequencies through \(Ω_1 = Ω \sin θ e^{iφ}\) and \(Ω_2 = Ω \cos θ\) with \(Ω = \sqrt{Ω_1^2 + Ω_2^2}\). (\(θ\) and \(φ\) are in general spatially varying). The eigenvectors (the dressed states) \(|χ⟩ = (|χ1⟩, |χ2⟩, |χ3⟩)^T\) of the Hamiltonian \(H_{int}\) are specified by \(|χ⟩ = U(\{|1⟩, |2⟩, |3⟩\})^T\) \(^T\) denotes the trans-
have negligible contribution from the initial excited state which requires the off-diagonal elements of the matrices to be small.

\[ \text{and } \gamma \text{ is given by } \tan \gamma = \frac{\sqrt{\Delta^2 + \Omega^2} - \Delta}{\Omega}. \]

With the corresponding eigenvalues \( \lambda = (0, \Delta - \sqrt{\Delta^2 + \Omega^2}, \Delta + \sqrt{\Delta^2 + \Omega^2})^T. \) In the new basis \( |\chi_i\rangle \), the full quantum state of the atom \( |\Phi(r)\rangle \) is written as \( |\Phi(r)\rangle = \sum_j |\chi_j(r)\rangle |\chi_j(r)\rangle \), where the wave functions \( \Psi = (\Psi_1, \Psi_2, \Psi_3)^T \) obey the Schrödinger equation \( ih\partial_t \Psi = \tilde{H}\Psi \), with the effective Hamiltonian \( \tilde{H} \) taking the form:

\[ \tilde{H} = \frac{1}{2m}(-ih\nabla - \tilde{A})^2 + \tilde{V}(r), \]

where \( \tilde{A} = ihU\nabla U^\dagger \) and \( \tilde{V}(r) = MJ + UV(r)U^\dagger \) (\( I \) is the \( 3 \times 3 \) unit matrix). From Eq. (3), one can see that in the new basis the atoms can be considered as moving in a gauge potential \( \tilde{A} \) and a scalar potential \( \tilde{V}(r) \).

\[ \begin{pmatrix} 1 \end{pmatrix} \] (a) Three-level A-type atoms interacting with laser beams characterized by the Rabi frequencies \( \Omega_1, \Omega_2 \) through the Raman-type coupling with a large single-photon detuning \( \Delta \).
\[ \begin{pmatrix} 2 \end{pmatrix} \] (b) The Configurations of the Raman laser beams.

\[ \begin{pmatrix} 3 \end{pmatrix} \] (c) Configuration II: A periodic gauge field can be created by four overlapping laser beams propagating along the shown directions. The upper two form the Raman beam \( \Omega_1 \) while the lower two form \( \Omega_2 \). (d) A Raman configuration to transfer the bright state to a different hyperfine level \( |F'\rangle \) for detection. The \( |I\rangle \) and \( |2\rangle \) are assumed to be different Zeeman states on the same hyperfine level \( |F\rangle \).

We are interested in the subspace spanned by the two lower internal eigenstates \( \{|\chi_1\rangle, |\chi_2\rangle\} \) (called respectively the dark and the bright state). This gives an effective spin-1/2 system, and in the spin language we also denote \( |\chi_0\rangle \equiv |\chi_1\rangle \) and \( |\chi_1\rangle \equiv |\chi_2\rangle \). In the case of a large detuning \( \Delta \gg \Omega \), both states \( |\chi_1\rangle \) and \( |\chi_2\rangle \) have negligible contribution from the initial excited-state \( |3\rangle \), so they are stable under atomic spontaneous emission. Furthermore, we assume the adiabatic condition, which requires the off-diagonal elements of the matrices \( \tilde{A} \) and \( \tilde{V} \) to be much smaller than the eigenenergy differences \( |\chi_i\rangle \rangle, i, j = 1, 2, 3 \rangle \rangle \) of the states \( |\chi_i\rangle \rangle, i, j = 1, 2, 3 \rangle \rangle \). This gives the quantitative condition \( F \ll \Omega^2/2\Delta \), where \( F = \cos^2 \theta |v| \nabla (\tan \theta e^{i\varphi}) \) (\( v \) is the typical velocity of the atom) represents the two-photon Doppler detuning. Under this adiabatic condition, the Schrödinger equation for the wave function \( \Psi \) becomes diagonal in the basis \( \{|\chi_i\rangle\} \), and in the lower subspace spanned by \( \{|\chi_1\rangle, |\chi_2\rangle\} \), the effective Hamiltonian takes the form

\[ H_{\text{eff}} = \begin{pmatrix} H_1 & 0 \\ 0 & H_2 \end{pmatrix}, \]

where \( H_1 = \frac{1}{8\pi^2}(-ih\nabla - A_\sigma)^2 + V_\sigma(r) \), \( (\sigma = \uparrow, \downarrow) \). The gauge and the scalar potentials \( A_\sigma \) and \( V_\sigma \) for the spin-\( \sigma \) component are given by \( A_\sigma = ih\langle \chi_\sigma | \nabla | \chi_\sigma \rangle \) and \( V_\sigma(r) = \lambda_\sigma + \langle \chi_\sigma | V | \chi_\sigma \rangle + \frac{e^2}{2m} \nabla \chi_\sigma \nabla \chi_\sigma \rangle + \langle \chi_\sigma | V \chi_\sigma \rangle^2 \rangle \), respectively. Through Eq. (2), one can find out that \( A_\uparrow = -A_\downarrow = -\hbar \sin \theta \nabla \varphi \).

\[ B_\sigma = \nabla \times A_\sigma = -\eta_\sigma \hbar \sin(2\theta) \nabla \theta \times \nabla \varphi, \]

where \( \eta_\uparrow = -\eta_\downarrow = 1 \). We get exactly a spin-dependent gauge field from the above configuration of the laser-atom coupling, which is critical for the spin Hall effect.

We consider two specific configurations of the laser beams, which generate different spatial variations of the gauge field. First, two counter-propagating Gaussian laser beams with shifted centers generate a spatially slowly varying gauge field [18]. The spatial profiles of the corresponding Rabi frequencies \( \Omega_j = \Omega_0 \exp[-(x-x_j)^2/\sigma_0^2] \exp(-ik_jy), (j = 1, 2) \), where the propagating wave vectors \( k_1 = -k_2 = k/2 \) and the center positions \( x_1 = -x_2 = \Delta x/2 \) (see Fig. 1b). Under these two laser beams, the gauge field is given by

\[ A_\sigma' = \frac{-\eta_\sigma \hbar k}{1 + e^{-x/d}} e_y, \quad B_\sigma' = \frac{-\eta_\sigma \hbar k}{4d \cosh^2(x/2d)} e_z, \]

where \( d = \sigma_0^2/(4\Delta x) \). Second, through overlapping of two standing-wave laser beams as shown in Fig. 1c with the corresponding Rabi frequencies \( \Omega_1 = \Omega_0 \cos(kz \sin \alpha) e^{iky \cos \alpha} \) and \( \Omega_2 = \Omega_0 \sin(kz \sin \alpha) e^{-iky \cos \alpha} \) (\( \alpha \) is the angle of the propagating laser beams to the \( y \) axis), we generate a spatially periodic gauge field, given by

\[ A_\sigma'' = 2\eta_\sigma \hbar k \sin^2(k'x) e_y, \quad B_\sigma'' = 2\eta_\sigma \hbar k \sin(2k'x) e_z \]

with \( k' = k \sin \alpha \). Under a slowly varying gauge field \( B_\sigma'' \), one can have local Landau levels, and with a spatially periodic gauge field \( B_\sigma'' \), we expect to have Bloch-type of wave functions, similar to the case of particles in a periodic potential.

The spin Hall effect can be demonstrated by observing a spin-Hall current. In the following, first we propose an experiment to detect the spin Hall current in an atomic
gas with the above configuration of the light-atom coupling, and then we discuss the quantum spin Hall effect in an optical lattice. For observation of the spin Hall effect, we need an effective “electric” field $E$ which drives atoms in one direction, and a spin current should be observed in a direction perpendicular both to the “electric” field $E$ and the gauge field $B_x$. The “electric” field can be conveniently provided through gravity on the neutral atoms. We assume the internal state of the atoms is in superposition of the spin $\uparrow$ and $\downarrow$ components under the laser beams shown in Fig. 1b or 1c. The external atomic trap is turned off at time $t = 0$, and the atoms fall off due to gravity with an acceleration $g = 9.8 \text{ m/s}^2$ (along the direction $e_y$). Under the effective gauge field, the equations of motion are

$$\dot{x}_\sigma = p^{x}_\sigma/m, \quad \dot{p}^{x}_\sigma = [(\partial_x A_\sigma)p^{y}_\sigma - A_\sigma \partial_x A_\sigma]/m - \partial_x V_\sigma, \quad (8)$$

$$\dot{y}_\sigma = [p^{y}_\sigma - A_\sigma]/m, \quad \dot{p}^{y}_\sigma = mg, \quad (9)$$

where the gauge potential $A_\sigma$ is either $A_{\uparrow}^I$ or $A_{\downarrow}^I$, and $V_\sigma$ is the corresponding scalar potential induced by the same laser beams. The coordinates and the momenta $x_\sigma, y_\sigma, p^{x}_\sigma, p^{y}_\sigma$ are understood as variables (operators) in the classical (quantum) cases, respectively.

To have some intuitive idea, in Fig. 2 (a) and (b) we show the typical classical trajectories of the atoms under the gauge fields $B_{\uparrow}^I$ or $B_{\downarrow}^I$. One can clearly see that the trajectory of the atom depends on its spin state $\sigma$, and such a dependence leads to the spin Hall current in the direction $e_y$. The evolution of the density profile of the atomic gas with the above configuration of the light-atom coupling, and then we discuss the quantum spin Hall effect. The gauge potentials in Figs. (a,c) and (b,d) are generated by the laser configurations I and II, respectively. The sinusoids in Fig. (b) denote the effective gauge fields $B_y$. The directions of the Lorentz forces $F_\sigma$ change periodically in this case and are shown by the arrows there. The dotted (solid) vertical lines in (b) and (d) denote the stable equilibrium positions for spin-up (spin-down) atoms. In Figs. (c) and (d), the density profiles of the atomic gas are shown at time $t = 0, 4, 6 \text{ ms}$. For calculations in Figs. (a-d), we take the following typical experimental parameters with $\sigma = 10 \text{ mm}$, $\Delta x = 2.5 \text{ mm}$, $k = 10^7 \text{ m}^{-1}$ for the laser configuration I, and $k \sin \alpha = 5 \times 10^5 \text{ m}^{-1}$ for the laser configuration II. In both configurations, $\Omega_{2,4}^I/\Delta = 10^6 \text{ Hz}$. In Figs. (a) and (b), the initial atomic velocity is assumed to be zero, and the initial positions $x = 0, y = 0$ for (a) and $(x = 2.5, 5\text{ mm}, y = 0)$ for (b). The parameters for the atomic ensemble in Figs. (c) and (d) are given by $\sigma = 2.0 \mu \text{ m}$ and $\sigma_e = 0.5 \text{ cm/s}$. The atomic mass is taken to be the one for $^{87}\text{Rb}$. With the above parameters, we have checked the adiabatic condition is well satisfied during the evolution.

To experimentally detect the spin current (or spin separation) as shown in Fig.2, right before the imaging one can transfer the dressed bright state $|\chi\rangle$ to a different hyperfine level $|F'\rangle$ by turning on a laser pulse (with a Rabi frequency $\Omega_F$) that couples the excited state $|3\rangle$ to $|F'\rangle$ (see Fig. 1d). This pulse, together with the original laser beams $\Omega_1$ and $\Omega_2$, make a Raman transition with an effective Hamiltonian $H_R = (\Omega_F^\ast \Omega/\Delta)|\chi\rangle \langle F'| + h.c.$ (note that the dark state $|\chi\rangle$ is still decoupled because of the phase relation between $\Omega_1$ and $\Omega_2$). Although the
form of the bright state $|\chi_1\rangle$ is spatially varying, the Rabi frequency $\Omega$ (and thus also $\Omega^2/\Omega/\Delta$) is spatially constant (for the laser configuration II) or almost constant (in the overlap region for the laser configuration I). We can thus choose the pulse duration so that it makes a complete Raman transition ($\pi$-pulse), and the atomic motion can be neglected during such a short duration. After this Raman $\pi$-pulse, the initial different dressed spin states are mapped to different hyperfine levels, and the populations in different atomic hyperfine levels can be separately imaged with the known experimental techniques.

We now consider quantum spin Hall effect with fermionic atoms in an optical lattice. In this case, the scalar potential $V_\sigma(r)$ is spatially periodic. We assume the optical lattice has a higher intensity along the vertical direction so that the tunneling rate along the $z$-axis is negligible. One then has an effective 2D system in the $x-y$ plane. The gauge field $B_\sigma(r)$ (along the $z$-axis) is assumed to nearly constant or spatially periodic in the lattice (which corresponds to the above laser configurations I and II, respectively). The wave function in this case can still be written as $\Psi(\sigma) = \sum_n c_n^\sigma(k, r) e^{i k r}$, where $k$ is the Bloch wave-vector and the $n$-th band wavefunction $u_n\sigma(k, r)$ satisfies the Schrodinger equation with the effective Hamiltonian $^{20}$

$$H^\sigma_k = \hbar^2 \frac{\partial^2}{2m} \left[ (-i \partial_x + k_x)^2 + (-i \partial_y + k_y - A_\sigma(x))^2 \right] + V_\sigma(x, y)$$

Under an effective “electric” field $E$ along the $y$-direction ($E_y = mg$ through acceleration $g$), the Hall current along the $x$ direction is given by $J^\sigma_x = \langle \rho^\sigma_{xy} \rangle = \sigma^\sigma_{xy} E_y$ for the spin-$\sigma$ component with the linear response theory. The Hall conductivity then has the expression

$$\sigma^\sigma_{xy} = \frac{1}{2\pi \hbar} \sum_n \langle \rho^\sigma_{xy} \rangle \left( \partial_{k_\sigma} a_{n\sigma}^\sigma - \partial_{k_y} a_{n\sigma}^\sigma \right),$$

where $u_{n\mu}(k) \equiv i \hbar (\partial_{k_\mu} u_{n\sigma}(k) - \partial_{k_y} u_{n\sigma}(k))^\prime$ (where $\mu = x, y$) and $\rho^\sigma a_{n\sigma}^\sigma(k)$ denote the density of states of the $n$-th band with the band energy $\epsilon_n^\sigma(k)$. The mass and the spin currents are defined by $J^m_x = J^x_1 + J^x_2$ and $J^s_x = J^x_1 - J^x_2$, respectively.

Note added: During publication of this work, we became aware that the spin Hall effect was also discussed by Liu et al. in Ref. $^{21}$ under a different atomic configuration.

---

[1] K. v. Klitzing, G. Dorda, and M. Pepper, Phys. Rev. Lett. 45, 494 (1980).
[2] D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. 48, 1559 (1982).
[3] Y. K. Kato et al., Science 306, 1910 (2004); J. Wunderlich et al., Phys. Rev. Lett. 94, 047204 (2005).
[4] J. E. Hirsch, Phys. Rev. Lett. 83, 1834 (1999).
[5] S. Murakami, N. Nagaosa, and S. Zhang, Science 301, 1348 (2003).
[6] J. Sinova et al., Phys. Rev. Lett. 92, 126603 (2004).
[7] C. L. Kane and E. J. Mele, Phys. Rev. Lett. 95, 226801 (2005); L. Sheng et al., ibid. 95, 136602 (2005).
[8] B. A. Bernevig and S. C. Zhang, Phys. Rev. Lett. 96, 106802 (2006).
[9] B. G. Wang et al., Phys. Rev. Lett. 95 086608 (2005); L. Hu et al, Phys. Rev. B 70, 235323 (2004).
[10] J.R. Anglin and W. Ketterle, Nature 416, 211 (2002).
[11] N.K. Wilkin and J.M.F. Gunn, Phys. Rev. Lett. 84, 6
(2000); T.-L. Ho, ibid. 87 060403 (2001); T.-L. Ho and C. V. Ciobanu, ibid, 85, 4648 (2000); B. Paredes et al., ibid. 87, 010402 (2001).
[12] R. Dum and M. Olshanii, Phys. Rev. Lett. 76, 1788 (1996).
[13] S. K. Dutta, B. K. Teo, and G. Raithel, Phys. Rev. Lett. 83, 1934 (1999).
[14] D. Jaksch, P. Zoller, New J. Phys. 5, Art. 56 (2003).
[15] G. Juzeliunas and P. Ohberg, Phys. Rev. Lett. 93, 033602 (2004); J. Ruseckas et al., ibid. 95, 010404 (2005); P. Zhang et al., Eur. Phys. J. D. 36, 229 (2005).
[16] K. Osterloh et al., Phys. Rev. Lett. 95, 010403 (2005).
[17] A. Sorensen, E. Demler, M. Lukin, Phys. Rev. Lett. 94, 086803 (2005).
[18] G. Juzeliunas et al, Phys. Rev. A 73, 025602 (2006).
[19] F. Wilczek and A. Zee, Phys. Rev. Lett. 52, 2111 (1984).
[20] D. J. Thouless et al., Phys. Rev. Lett. 49, 405 (1982).
[21] X. J. Liu, X. Liu, L. C. Kwek, and C. H. Oh, Phys. Rev. Lett. (in press).