Complex Masses of Mesons and Resonances
In Relativistic Quantum Mechanics

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Relativistic bound state problem in hadron physics is studied. Mesons and their resonance excitations in the framework of Relativistic Quantum Mechanics (RQM) are investigated. Two-particle wave equation for the Lorentz scalar QCD inspired funnel-type potential with the coordinate dependent strong coupling $\alpha_S(r)$ is derived. The concept of distance dependent particle mass is developed. Two exact asymptotic expressions for the system’s squared mass are obtained and used to derive the meson interpolating complex-mass formula. Free particle hypothesis for the bound state is developed: quark and antiquark move as free particles in of the bound system. Practical applications of the model are given.

PACS numbers: 11.10.St; 12.39.Pn; 12.40.Nn; 12.40.Yx
Keywords: bound state, meson, width, resonance, complex mass, Regge trajectory

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INTRODUCTION

Most particles listed in the Particle Data Group (PDG) tables are mesons and their excited states — resonances. A thorough understanding of the physics summarized by the PDG is related to the concept of a resonance. These hadron states are simplest quark-antiquark ($q\bar{q}$) bound systems. The number of known hadrons is constantly increasing with the growing energies of accelerators and proposed experiments on LHC. According to the PDG and theoretical predictions, many hadrons are still absent from the summary tables.

There is no fundamental dynamic theory of hadron resonances at the present time. The description of the $q\bar{q}$ systems and resonances in a way fully consistent with all requirements imposed by special relativity and within the framework of Quantum Field Theory (QFT) is one of the great challenges in theoretical elementary particle physics. Calculations of hadron properties are frequently carried out with the help of phenomenological and relativistic quark models. One of the promising among them is the Regge method in hadron physics.

All mesons, baryons and their resonances in this approach are associated with Regge poles which move in the complex angular momentum plane. Moving poles are described by the Regge trajectories, $\alpha(s)$, which are the functions of the invariant squared mass $s = W^2$ (Mandelstam’s variable), where $W = E^*$ is the c. m. rest energy (invariant mass of two particle system). Hadrons and resonances populate their Regge trajectories which contain all the dynamics of hadron interaction in bound state and scattering regions.

Heavy $Q\bar{Q}$ mesons (quarkonia) can be considered as nonrelativistic (NR) bound systems and well described by the two-particle Shrödinger’s wave equation. Light $q\bar{q}$ states are relativistic bound states and require another approach. Within the framework of QFT the covariant description of relativistic bound states is the Bethe-Salpeter (BS) formalism. The homogeneous BS equation governs all the bound states. However, numerous attempts to apply the BS formalism to relativistic bound-state problems give series of difficulties. Its inherent complexity usually prevents to find the exact solutions or results in the appearance of excitations in the relative time variable of the bound-state constituents (abnormal solutions), which are difficult to interpret in the framework of quantum physics.

For various practical reasons and applications to both QED and QCD some simplified equations, situated along a path of NR reduction, are used. More valuable are methods which provide either exact or approximate analytic solutions for various forms of differential equations. They may be remedied in three-dimensional reductions of the BS equation; the most well-known of the resulting bound-state equations is the one proposed by Salpeter.

In this work we study mesons and their excited states (meson resonances) as relativistic two body systems from unified point of view in the framework of the potential approach. The problem under investigation is related to 1) two-particle relativistic equation of motion and 2) absence of a strict definition of the potential in relativistic theory. We use the modified funnel type potential in the framework of Relativistic Quark Model and treat it as the Lorentz-scalar function of the spatial variable $r$. Using lagrangian relativistic mechanics we derive the two particle classic equation of motion; then, on this basis with the help of the correspondence principle we derive the two-body wave equation. We obtain the exact solutions of the equation for two asymptotic terms of the potential — the Coulomb term and linear one. The two asymptotic expressions for the squared mass we use to write the complex-mass formula for the bound system. We show that the eigenvalues (masses) for this potential are complex and give several practical applications of the model.
I. RESONANCE AND ITS DEFINITIONS

There are two well-known definitions of these resonance’s parameters, both widely used in hadron physics [11]. One definition, known as the conventional approach, is based on the behavior of the resonance’s phase shift $\delta(E)$ as a function of the energy, while the other, known as the pole approach, is based on the pole position of the resonance and includes several approaches [12–15]. Fundamentals of scattering theory and strict mathematical definition of resonances in QM were considered in [13–15].

In particle physics resonances arise as unstable intermediate states with complex masses [15]. Resonances in QFT are described by the complex-mass poles of the scattering matrix [14]. In non-relativistic scattering theory the complex $k^2 = 2\mu E$ variable plane is used. A Riemann surface $k = \pm \sqrt{2\mu E}$ is obtained by replacing the $k^2$-plane with a surface made up of two sheets $R_0$ and $R_1$, each cut along the positive real axis [16] (Fig. 1).

The rigorous QM definition of a resonance requires determining the pole position in the second Riemann sheet of the analytically continued partial-wave scattering amplitude in the complex $\pm k$ variable plane [17]. In relativistic theory, a Riemann surface $\mathcal{M} = \pm \sqrt{s}$ is obtained by replacing the $s$-plane with a surface made up of two sheets $R_0$ and $R_1$. Both definitions above are formulated in scattering experiment. On the other hand, there are two basic problems in quantum physics: the scattering problem and bound state problem. Resonances are quasi-stationary states in the $s$-channel at $s > 0$; this means that one can use another approach to consider resonances. Such an approach to bound-state problem is RQM [18].

The formulation of RQM differs from non-relativistic quantum mechanics by the replacement of invariance under Galilean transformations with invariance under Poincaré transformations. The RQM is also known in the literature as relativistic Hamiltonian dynamics or Poincaré-invariant quantum mechanics with direct interaction [19]. A definition of resonance in RQM was considered in [20] [21], where mass and width of a resonance are defined from solution of the eigenvalue problem for the Cornell potential [22], the short-range Coulomb $V_S(r)$ term and linear one $V_L(r)$, $V(r) = V_S(r) + V_L(r) \equiv -\frac{4}{3} \alpha_s / r + \sigma r$; its parameters are directly related to basic physical quantities, $\alpha_s$ and $\sigma$.

Operators in ordinary QM are Hermitian and the corresponding eigenvalues are real. It is possible to extend the QM Hamiltonian into the complex domain while still retaining...
the fundamental properties of a quantum theory. This means one can start with the bound state problem and make the analytic continuation to the scattering region. The problem of particle decay within the Hamiltonian formalism was considered in generalized QM [23]. One of such approaches is complex quantum mechanics [24]. The complex-scaled method is the extension of theorems and principles proved in QM for Hermitian operators to non-Hermitian operators.

The Cornell potential is a special in hadron physics in that sense it results in the complex energy and mass eigenvalues. Separate consideration of two asymptotic components of the Cornell potential — $V_S(r)$ and $V_L(r)$ — results in the complex-masses expression for resonances, which in the center-of-momentum frame (c.m.f.) is ($\hbar = c = 1$) [20] [21]:

$$\mathcal{M}_N^2 = 4 \left[ \left( \sqrt{2\sigma N} + \frac{i\alpha m}{N} \right)^2 + \left( m - i\sqrt{2\alpha} \right)^2 \right],$$

where $\alpha_S = \frac{\alpha_S}{2}$, $N = N + (k + \frac{1}{2})$, $N = k + l + 1$, $k$ is radial and $l$ is orbit quantum numbers: it has the form of the squared energy $\mathcal{M}_N^2 = 4 \left[ (\pi N)^2 + \mu^2 \right]$ of two free relativistic particles with the quarks’ complex momenta $\pi_N$ and masses $\mu$. This formula allows to calculate in a unified way the centered masses and total widths of resonances,

$$\mathcal{M}_N^2 = \text{Re} \mathcal{M}_N^2 + i \text{Im} \mathcal{M}_N^2,$$

where

$$\text{Re} \mathcal{M}_N^2 = 4 \left[ 2\sigma N - \left( \frac{\alpha m}{N} \right)^2 + m^2 - \mu_1^2 \right], \quad \text{Im} \mathcal{M}_N^2 = 8m\mu_1 \left( \frac{\sqrt{\alpha N}}{N} - 1 \right).$$

The real-part mass in (3) exactly coincides with the universal mass formula obtained independently by another method with the use of the two-point Padé approximant [25] and is very transparent physically, as well as the Coulomb potential. It describes equally well the mass spectra of all $q\bar{q}$ and $Q\bar{Q}$ mesons ranging from the $ud$ ($d\bar{d}$, $u\bar{u}$, $s\bar{s}$) states up to the heaviest known $b\bar{b}$ systems [25] and glueballs [26] [27] as well. Besides, it allows one to get the Regge trajectories as analytic functions in the whole region from solution of the cubic equation for the angular momentum $J(M^2)$ [25]: the Regge trajectories including the Pomeron [26] [27] are “saturating” and appears to be successful in many applications [28] [30].

A Riemann surface $\mathcal{M}$ on Fig. 1 can be obtained from (2) by taking the square root $\pm \sqrt{\mathcal{M}_N^2}$ replacing the $\mathcal{M}_N^2$-plane with a surface made up of two sheets $R_0$ and $R_1$, each cut along the positive real axis [31]. The square root of the complex expression (2) gives

$$\mathcal{M}_N = \pm \sqrt{\mathcal{M}_N^2} \equiv \pm \left\{ \text{Re} \mathcal{M}_N + i \xi \text{Im} \mathcal{M}_N \right\},$$

where

$$\text{Re} \mathcal{M}_N = \pm \sqrt{\frac{|\mathcal{M}_N^2| + \text{Re} \mathcal{M}_N^2}{2}} = \mathcal{M}_N, \quad \text{Im} \mathcal{M}_N = \pm \sqrt{\frac{|\mathcal{M}_N^2| - \text{Re} \mathcal{M}_N^2}{2}} = -i\Gamma_N^\text{TOT}.$$

Here $|\mathcal{M}_N^2| = ((\text{Re} \mathcal{M}_N^2)^2 + (\text{Im} \mathcal{M}_N^2)^2)^{1/2}$, $\xi = \text{sgn}(\text{Im} \mathcal{M}_N^2)$.

Expressions (5) give process independent parameters of resonances, their centered masses $\mathcal{M}_N$ and total widths $\Gamma_N^\text{TOT}$. The imaginary-part mass in (5) defines the total width $\Gamma_N^\text{TOT} = -2i\text{Im} \mathcal{M}_N$ of the resonance. The resonance positions are symmetrically located in the Riemann $\mathcal{M}$-surface: if $\mathcal{M}_p = \text{Re} \mathcal{M}_p - i\text{Im} \mathcal{M}_p$ is a pole in the fourth quadrant of the surface $\pm \sqrt{\mathcal{M}_N^2}$, then $\mathcal{M}_p = -\text{Re} \mathcal{M}_p - i\text{Im} \mathcal{M}_p$ is also a pole, but in the third quadrant (antiparticle) [15].
II. RELATIVISTIC TWO-BODY PROBLEM

Considered above quarkonia are simplest among mesons. A more complicated case represent mixed $Q\bar{q}$ bound states in which quark masses are different. Solution of the relativistic two-body (R2B) problem in a way fully consistent with all requirements imposed by special relativity and within the framework of QFT is one of the great challenges in theoretical elementary particle physics \[32\]. Comprehensive description of $Q\bar{q}$ bound states is reduced to relativistic bound state problem.

There have been proposed several wave equations for the description of bound states within relativistic quantum theory such as the Klein-Gordon, the Dirac, quasipotential, etc. Relativistic description of two-body systems is usually based on the four-dimensional covariant BS equation \[7\]. This equation governs all the bound states and is appropriate framework for the description of the R2B problem within QFT. However, attempts to apply the BS formalism to the R2B problem give series of difficulties, the interaction kernel entering in this equation is not derivable from the first principles.

All these difficulties are the sources of the numerous attempts to reformulate the R2B problem of QFT \[4, 6\]; there exist various reductions of the BS equation (see \[6\]). Many authors have developed noncovariant instantaneous truncations of the BS equation \[6, 8\], but a better known is the Salpeter work \[33\]. After applying simplifying assumptions and approximations to the BS equation, one comes to the spinless Salpeter equation, which in the c.m.f. is \[6, 33\]

$$ \sqrt{\hat{p}^2 + m_1^2} + \sqrt{\hat{p}^2 + m_2^2} + V(r) = M\psi; \quad (6) $$

it is the conceptually simplest bound-state wave equation incorporating to some extent relativistic effects, where the potential $V(r)$ arises as the Fourier transform of the BS kernel $K(q)$. The equation \[6\] is sometimes denoted semirelativistic since is not a covariant formulation, however, even this simplest two-particle wave equation leads to several difficulties \[6, 8\].

The square roots of the operator in \[6\] cannot be used as they stand; they would have to be expanded in a power series before the squared momentum operator, $\hat{p}^2$, raised to a power in each term, could act on $\psi(r)$. As a result of the power series, the space and time derivatives are completely asymmetric: infinite-order in space derivatives but only first order in the time derivative, which is inelegant and unwieldy. The next problem is the noninvariance of the energy operator in \[6\], equated to the square root which is also not invariant. A more severe problem is that it can be shown to be nonlocal and can even violate causality \[6\].

The spinless Salpeter equation \[6\] can be obtained by another way without any approximations in the framework of RQM from the following simple and obvious consideration. The underlying classic equation of \[6\] is

$$ \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} + V(r) = M. \quad (7) $$

This is the total energy of two interacting particles in the c.m.f. and, at the same time, the mass of the system which can also be found from the following simple consideration.

Equation \[7\] is the zeroth component of the four-vector $P^\mu = p_1^\mu + p_2^\mu + W^\mu(q_1, q_2)$. Two particles with the momenta $p_1$, $p_2$ and the interaction field $W^\mu(q_1, q_2)$ (potential) together represent a closed system. The four-vector $P^\mu$ describes free motion of the bound system and can be separated into the two equations, i.e., $E = [p_1^2 + m_1^2]^{1/2} + [p_2^2 + m_2^2]^{1/2} + W(q_1, q_2) = \text{const}$ and $P = p_1 + p_2 + W(q_1, q_2) = \text{const}$, describing the energy and
momentum conservation laws. Because the energy $E$ of the system is the constant of motion, the corresponding Hamiltonian can not depend on time explicitly, the potential $W(q_1, q_2)$ should not depend on time, but depends on the relative coordinate $r = r_1 - r_2$ of particles, i.e., $W^0 = S(r)$ [8]. This means that the system’s relative time is zero, $\tau = 0$, and the interaction is instantaneous, the total energy $E$ in the c.m.f. is the mass $M$ of the system.

The potential in $W$ is the zeroth component of the Lorentz-vector $W^\mu(q_1, q_2)$. In general, there are two different relativistic versions: the potential is considered either as the zero component of a four-vector, or as a Lorentz-scalar [34]; its nature is a serious problem of relativistic potential models [35]. The relativistic correction for the case of the Lorentz-vector potential is different from that for the case of the Lorentz-scalar potential, the radial $S$-wave function $R(r)$ for the hydrogen atom diverges as $r \to 0$. This problem is very important in hadron physics where, for the vector-like confining potential, there are no normalizable solutions [35, 36]. There are normalizable solutions for scalar-like potentials, but not for vector-like. This issue was investigated in [10, 25]; the effective interaction has to be Lorentz-scalar in order to confine quarks and gluons.

The relativistic total energy $\epsilon_i(p_i)$ of a particle in $W$ given by $\epsilon_i^2(p_i) = p_i^2 + m_i^2$ can be represented as sum of the kinetic energy $\tau_i(p)$ and the rest mass $m_i$, i.e., $\epsilon_i(p) = \tau_i(p) + m_i$. Then the energy-mass $M$ in $W$ can be written as $M = [\tau_1(p) + m_1] + [\tau_2(p) + m_2] + V(r)$, the potential $V(r)$ can be considered as scalar-like $S(r)$ and shared among the two masses $m_1$ and $m_2$ as $m_i(r) = m_i + \frac{1}{2}S(r)$ [37]. Therefore, the system’s total energy $E$ (invariant system mass $M$) can be written in the form

$$M = \sqrt{p^2 + m_1^2(r)} + \sqrt{p^2 + m_2^2(r)}.$$  \hspace{1cm} (8)

The functions $m_i(r)$ in (8) are treated as the distance dependent particle masses and can be transformed to the squared relative momentum, $p^2 = [(s - m^2)](s - (m_+ + S)^2)/4s$, where $s = M^2$. This expression with the help of the fundamental correspondence principle (according to which physical quantities are replaced by operators acting onto the wave function) gives the two-particle spinless wave equation ($\hbar = c = 1$),

$$\left\{\Delta^2 + K(s)\left[M^2 - (m_+ + S)^2\right]\right\}\psi(\vec{r}) = 0, \hspace{1cm} (9)$$

where $K(s) = (s - m^2)/4s$, $m_+ = m_1 + m_2$, $m_- = m_1 - m_2$.

The Cornell potential we consider here is fixed by the two free parameters, $\alpha_S$ and $\sigma$. However, the strong coupling $\alpha_S$ in QCD is a function $\alpha_S(Q^2)$ of virtuality $Q^2$ or $\alpha_S(r)$ in configuration space. The potential can be modified by introducing the $\alpha_S(r)$-dependence, which is unknown. A possible modification of $\alpha_S(r)$ was introduced in [20],

$$V_{QCD}(r) = -\frac{4}{3}\frac{\alpha_S(r)}{r} + \sigma r, \hspace{0.5cm} \alpha_S(r) = \frac{1}{b_0 \ln[1/(\Lambda r)^2 + (2\mu_g/\Lambda)^2]},$$  \hspace{1cm} (10)

where $b_0 = (33 - 2n_f)/12\pi$, $n_f$ is number of flavors, $\mu_g = \mu(Q^2)$ — gluon mass at $Q^2 = 0$, $\Lambda$ is the QCD scale parameter. The running coupling $\alpha_S(r)$ in (10) is frozen at $r \to \infty$, $\alpha_\infty = \frac{1}{2}[b_0 \ln(2\mu_g/\Lambda)]^{-1}$, and is in agreement with the asymptotic freedom properties $[\alpha_S(r \to 0) \to 0]$.

### III. SOLUTION OF THE WAVE EQUATION

It is a problem to find the analytic solution of known equations, as well as [9], for the potential [10]. Instead, we solve the quasiclassical (QC) wave equation [38, 39].
Derivation of the QC equation is reduced to replacement of the operator $\hat{V}^2$ by the canonical operator $\Delta^c$ \cite{20} without the first derivatives, acting onto the state function $\Psi(\vec{r})$. Therefore, instead of \cite{9} we solve the QC equation for the potential \cite{10},

$$\left\{ \frac{\partial^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2} + K(s) \left[ M_s^2 - (m_+ + V_{QCD})^2 \right] \right\} \Psi(\vec{r}) = 0, \quad (11)$$

which is separated that gives the radial,

$$\left\{ \frac{d^2}{dr^2} + \frac{s - m_+^2}{4s} \left[ s - \left( m_+ - \frac{4\alpha_S(r)}{3} + \sigma r \right)^2 \right] - \frac{M_f^2}{r^2} \right\} R(r) = 0, \quad (12)$$

and the angular equations. Solution of the last was obtained in \cite{20} by the QC method in the complex plane, that gives $M_f = (l + \frac{1}{2})\hbar$, for the angular momentum eigenvalues. These angular eigenmomenta are universal for all spherically symmetric potentials in relativistic and NR cases.

The problem \cite{12} has four turning points and cannot be solved analytically by standard methods. We consider the problem separately by the QC method for the short-range Coulomb term and the long-range linear term. The QC wave equation \cite{12} for the Coulomb term has two turning points and the phase-space integral is

$$I = \oint_C \sqrt{\frac{s - m_+^2}{4s}} \left[ s - \left( m_+ - \frac{4\alpha_S(r)}{3} + \sigma r \right)^2 \right] - \frac{(l + \frac{1}{2})^2}{r^2} \, dr = 2\pi \left( k + \frac{1}{2} \right). \quad (13)$$

The phase-space integral \cite{13} is found in the complex plane with the use of the method of stereographic projection \cite{20} that gives

$$M_N^2 = \left( \sqrt{\epsilon_N^2} \pm \sqrt{(\epsilon_N^2)^*} \right)^2 \equiv 4 \left[ \text{Re} \{\epsilon_N^2\} \pm i\text{Im} \{\epsilon_N^2\} \right], \quad (14)$$

where $\epsilon_N^2 = \frac{1}{4} m_+^2 (1 - v_N^2) + \frac{1}{4} m_+ m_- v_N$, $v_N = \frac{2}{3} \alpha_\infty / N$, $N = k + l + 1$.

Large distances in hadron physics are related to the problem of confinement. The problem has four turning points, i.e., two cuts between these points. The phase-space integral \cite{13} is found by the same method of stereographic projection as above that results in the cubic equation: $s^3 + a_1 s^2 + a_2 s + a_3 = 0$, where $a_1 = 16\alpha_\infty \sigma - m^2$, $a_2 = 64\sigma^2 \left( \alpha_\infty^2 - \tilde{N}^2 - \tilde{\alpha}_\infty m_+^2 / 4\sigma \right)$, $a_3 = -(8\alpha_\infty \sigma m_-)^2$, $\tilde{N} = N + k + \frac{1}{2}$, $\tilde{\alpha}_\infty = \frac{4}{3} \alpha_\infty$.

The first root $s_1(N)$ of this equation gives the physical solution (complex eigenmasses), $M_N^2(N) = s_1(N)$, for the squared invariant mass.

Two exact asymptotic solutions, i.e., \cite{14} and the first root of the cubic equation above, are used to derive the resonance’s mass formula. The interpolation procedure for these two solutions is used \cite{25} to derive the resonance’s mass formula:

$$M_N^2 = (m_1 + m_2)^2 \left( 1 - \epsilon_N^2 \right) \pm 2i(m_1^2 - m_2^2) v_N + \text{Re} \{M_N^2(N)\}. \quad (15)$$

The real part of the square root of \cite{15} defines the centered mass and its imaginary part defines the total widths, $\Gamma_N^{\text{TOT}} = -2 \text{Im} \{M_N^2\}$, of the resonance \cite{20, 21}.

To demonstrate its efficiency we calculate the leading-state masses of the $\rho$ and $D^*$ meson resonances (see tables, where masses are in MeV).

The free fit to the data show a good agreement for the light and heavy $Q\bar{q}$ meson resonances. Note, that the gluon mass in the independent fitting is the same, $m_g = 416$ MeV. Besides, it is the same for glueballs \cite{26}. The $d$ quark effective mass is also practically the same, i.e., $m_d \simeq 273$ MeV, for the light and heavy resonances.
TABLE I. The masses of the $\rho^\pm(ud)$-meson resonances

| Meson       | $J^{PC}$ | $E_{ex}$  | $E_{th}$ | Parameters in (15) |
|-------------|----------|-----------|----------|--------------------|
| $\rho(1S)$  | 1$^{--}$ | 776       | 776      | $\Lambda = 500$ MeV |
| $a_2(1P)$   | 2$^{++}$ | 1318      | 1314     | $\mu_g = 416$ MeV  |
| $\rho_3(1D)$| 3$^{--}$ | 1689      | 1689     | $\sigma = 0.139$ GeV$^2$ |
| $a_4(1F)$   | 4$^{++}$ | 1996      | 1993     | $m_d = 276$ MeV     |
| $\rho(1G)$  | 5$^{--}$ | 2255      |          | $m_u = 129$ MeV     |
| $\rho(2S)$  | 1$^{--}$ | 1717      | 1682     |                    |
| $\rho(2P)$  | 2$^{++}$ | 1990      |          |                    |
| $\rho(2D)$  | 3$^{--}$ | 2254      |          |                    |

TABLE II. The masses of the $D^{*\pm}(cd)$-meson resonances

| Meson       | $J^{PC}$ | $E_{ex}$  | $E_{th}$ | Parameters in (15) |
|-------------|----------|-----------|----------|--------------------|
| $D^*(1S)$   | 1$^{--}$ | 2010      | 2010     | $\Lambda = 446$ MeV |
| $D_2^*(1P)$ | 2$^{++}$ | 2460      | 2464     | $\mu_g = 416$ MeV  |
| $D_3^*(1D)$ | 3$^{--}$ | 2845      | 2845     | $\sigma = 0.249$ GeV$^2$ |
| $D_4^*(1F)$ | 4$^{++}$ | 3178      | 3178     | $m_c = 1163$ MeV    |
| $D_5^*(1G)$ | 5$^{--}$ | 3478      |          | $m_d = 271$ MeV     |
| $D^*(2S)$   | 1$^{--}$ | 1820      | 2821     |                    |
| $D^*(2P)$   | 2$^{++}$ | 2011      | 3166     |                    |
| $D^*(2D)$   | 3$^{--}$ | 3471      |          |                    |

CONCLUSION

The constituent quark picture could be questioned since potential models have serious difficulties because the potential is non-relativistic concept. However, in spite of non-relativistic phenomenological nature, the potential approach is used with success to describe mesons as bound states of quarks.

We have modeled meson resonances to be the quasi-stationary states of two quarks interacting by the QCD-inspired funnel-type potential with the coordinate dependent strong coupling, $\alpha_S(r)$. Using the complex analysis, we have derived the meson complex-mass formula (15), in which the real and imaginary parts are exact expressions. This approach allows to simultaneously describe in the unified way the centered masses and total widths of resonances. We have shown here the results only for unflavored and charmed meson resonances, however, we have obtained a good description for strange and beauty mesons as well [37].

[1] K. A. Olive and et al.,The Particle Data Group, Chin. Phys., 2014, C38,pages=090001.
[2] W. W. Armstrong, . . . and M. N. Sergeenko and et al., ATLAS Collaboration, ATLAS: Technical proposal for a general-purpose pp experiment at the Large Hadron Collider at CERN, CERN-LHCC-94-43, December, 1994, 1–289.
[3] G. Morpurgo, Field theory and the nonrelativistic quark model: a parametrization of meson
masses, Phys. Rev. D, 1990, 41, 2865–2873.

[4] D. Ebert, R. N. Faustov and V. O. Galkin, Spectroscopy and Regge trajectories of heavy quarkonia and $B_c$ mesons, Euro. Phys. J. C, 2011, 71, 1825–1837.

[5] P. D. B. Collins, An Introduction to Regge Theory and High-Energy Physics, 1977, England: Cambridge Univ. Press.

[6] W. Lucha and F. F. Schöberl, Instantaneous Bethe-Salpeter Kernel for the Lightest Pseudoscalar Mesons, Phys. Rev. D, 2016, 93, 096005–096014.

[7] H. A. Bethe and E. E. Salpeter, Quantum mechanics of one- and two-electron atoms, Dover Publications, Mineola, N.Y., 2008, ISBN 978-0486466675.

[8] W. Lucha and F. F. Schöberl, Int. J. Mod. Phys. A, 1999, 14, 2309.

[9] E. E. Salpeter, Mass Corrections to the Fine Structure of Hydrogen-Like Atoms, Phys. Rev., 1952, 87, 328.

[10] Y.-S. Huang, Schredinger-Like Relativistic Wave Equation for the Lorentz-scalar potential, Found. Phys., 2001, 31(9), 1287–1294.

[11] A. Bernicha, G. L. Castro and J. Pestieau, Nucl. Phys. A., 1996, 597, 623.

[12] D. Morgan and M. R. Pennington, Phys. Rev. D, 1993, 48, 1185.

[13] P. D. Hislop and C. Villegas-Blas, arXiv:math-ph/1104.4466v1, 2011.

[14] J. R. Taylor, The Quantum Theory of Nonrelativistic Collisions, 2006, Dover Publications.

[15] N. Moiseyev, Quantum theory of resonances: calculating energies, widths and cross-sections by complex scaling, Phys. Rep., 1998, 302, 211–293.

[16] J. W. Brown and R. V. Churchill, Complex Variables and Applications, 2003, 7th edition, New York NY, McGraw Hill.

[17] J. Nieves and E. R. Arriola, Phys. Lett. B, 2009, 679, 449.

[18] W. Greiner, Relativistic quantum mechanics: wave equations, Springer-Verlag, Berlin–Heidelberg–New York–Barcelona; Hong Kong; London, third, 2000, ISBN 3-540-67457-8.

[19] P. A. M. Dirac, Forms of Relativistic Dynamics, Rev. Mod. Phys., 1949, 21, 392–415.

[20] M. N. Sergeenko, Masses and widths of Resonances for the Cornell Potential, Adv. HEP, 2013, 2013, Article ID 325431, 1–7.

[21] M. N. Sergeenko, Complex Masses of Resonances in the Potential Approach, Nonlin. Phen. in Compl. Sys., 2014, 17, 433–438.

[22] E. Eichten, S. Godfrey, H. Mahlke and J. L. Rosner, Rev. Mod. Phys., 2008, 80, 1161.

[23] E. C. G. Sudarshan, C. B. Chiu and V. Gorini, Phys. Rev. D, 1978, 18, 2914.

[24] C. M. Bender, D. C. Brody and D. W. Hook, J. Phys. A, 2008, 41, 352003.

[25] M. N. Sergeenko, An Interpolating mass formula and Regge trajectories for light and heavy quarkonia, Z. Phys. C., 1994, 64, 315–322.

[26] M. N. Sergeenko, Glueball masses and Regge trajectories for the QCD-inspired potential, Eur. Phys. J. C, 2012, 72, 2128.

[27] M. N. Sergeenko, Glueballs and the Pomeron, Europhys. Lett., 2010, 89, 11001–11007.

[28] P. Rossi, Physics of the CLAS collaboration: Some selected results, JLAB-PHY-03-14, Feb 2003. 11 pp. Talk given at 41-st International Winter Meeting on Nuclear Physics, Bormio, Italy, 26 Jan – 2 Feb, 2003, JLAB-PHY-03-14, 11 p., [arxiv.org/pdf/hep-ex/0302032].

[29] M. Battaglieri and et al., CLAS Collaboration, Photoproduction of the omega meson on the proton at large momentum transfer, Phys. Rev. Lett., 2003, 90, 022002–022014, [arxiv.org/pdf/hep-ph/0306153], [arxiv.org/pdf/hep-ex/0210023].

[30] L. Morand and et al., Deeply virtual and exclusive electroproduction of omega mesons, Eur. Phys. J. A, 2005, 24, 445–458, [arxiv.org/pdf/hep-ex/0504057].

[31] J. W. Brown and R. V. Churchill, Complex Variables and Applications, New York NY: McGraw Hill, New York, N.Y., 7th edition, 2003.

[32] B. Cagnac, M. D. Plimmer, L. Julien and F. Biraben, The hydrogen atom, a tool for
metrology, Rep. Prog. Phys., 1994, 57(9), 853–871.

[33] E. E. Salpeter, Mass Corrections to the Fine Structure of Hydrogen-Like Atoms, Phys. Rev., 952, 87, 328.

[34] B. Sahu et al., Behavior of relativistic wave functions near the origin for a QCD potential, Amer. J. Phys., 1989, 57, 886.

[35] J. Sucher, Phys. Rev. D, 1995, 51, 5965.

[36] C. Semay and R. Ceuleneer, Phys. Rev. D, 1993, 48, 5965.

[37] M. N. Sergeenko, Mesons and Resonances in Relativistic Quantum Mechanics For the Lorentz-Scalar Potential, [hep-ph/arXiv:1703.07766v3], 2017.

[38] M. N. Sergeenko, Relativistic semiclassical wave equation and its solution, Mod. Phys. Lett. A, 1997, 12, 2859–2871, [arXiv:quant-ph/9911081].

[39] M. N. Sergeenko, Semiclassical wave equation and exactness of the WKB method, Phys. Rev. A, 1996, 53, 3798–3804.