Neutrino effects in two-body electron-capture measurements at GSI

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Abstract

I conjecture that the time modulated decay rates reported in single ion measurements of two-body electron capture decay of hydrogen-like heavy ions at GSI may be related to neutrino spin precession in the static magnetic field of the storage ring. These ‘GSI Oscillations’ arise from interference between amplitudes of decay within and without the magnetic field, a scenario that requires a Dirac neutrino magnetic moment six times lower than the Borexino solar neutrino upper limit of $0.54 \times 10^{-10}$ Bohr magneton. I also show in a way not discussed before that the time modulation associated with interference between massive neutrino amplitudes, if such interference could arise, is of a period at least four orders of magnitude shorter than reported and must average to zero given the time resolution of the GSI measurements.

Key words: neutrino interactions, mass, mixing and moments; electron capture

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1 Introduction

Measurements of weak interaction decay of multiply ionized heavy ions coasting in the ion storage-cooler ring ESR at the GSI laboratory, since the first report in 1992 [1], open up new vistas for dedicated studies of weak interactions. In particular, electron capture (EC) decay rates in hydrogen-like and helium-like $^{140}$Pr ions have been recently measured for the first time [2] by following the motion of the decay ions (D) and the recoil ions (R). The overall decay rates $\lambda_{\text{EC}}$ of these two-body $^{140}$Pr $\rightarrow$ $^{140}$Ce + $\nu$ EC decays, in which no neutrino $\nu$ is detected, are well understood within standard weak interaction calculations of the underlying $e^- p \rightarrow \nu_e n$ reaction [3,4]. However, a
time-resolved decay spectroscopy applied subsequently to the two-body EC decay of H-like $^{140}$Pr and $^{142}$Pm single ions revealed an oscillatory behavior, or more specifically a time modulation of the two-body EC decay rate $^5$:  

$$\lambda_{EC}(t) = \lambda_{EC}[1 + a_{EC} \cos(\omega_{EC}t + \phi_{EC})],$$  

(1)

with amplitude $a_{EC} \approx 0.2$, and angular frequency $\omega_{EC}^{\text{lab}} \approx 0.89 \text{ s}^{-1}$ (period $T_{EC}^{\text{lab}} \approx 7.1 \text{ s}$) in the laboratory system which is equivalent in the rest frame of the decay ion to a minute energy $\hbar \omega_{EC} \approx 0.84 \times 10^{-15} \text{ eV}$. Subsequent experiments on EC decays of neutral atoms in solid environment have found no evidence for oscillations with periodicities of this order of magnitude $^6,^7$. Thus, the oscillations observed in the GSI experiment could have their origin in some characteristics of the H-like ions, produced and isolated in the ESR, and in the electromagnetic fields specific to the ESR which are not operative in normal laboratory experiments. It is suggested here, in Sect. 3, that the ‘GSI Oscillations’ could indeed be due to the magnetic field which stabilizes and navigates the motion of the ions in the ESR.

Several works, by Kienle and collaborators, relegated the ‘GSI Oscillations’ to interference between neutrino mass eigenstates that evolve coherently from the electron neutrino $\nu_e$ $^8,^9,^10,^11,^12$. This idea apparently also motivated the GSI experiment $^5$. Such interferences, according to these works, lead to oscillatory behavior given by Eq. (1) with angular frequency $\omega_{\nu_e}$ where, again in the decay-ion rest frame,

$$\hbar \omega_{\nu_e} = \frac{\Delta (m_\nu c^2)^2}{2 M_D c^2} \approx 0.29 \times 10^{-15} \text{ eV},$$  

(2)

Here, $\Delta (m_\nu c^2)^2 = (0.76 \pm 0.02) \times 10^{-4} \text{ eV}^2$ from accumulated solar $\nu$ plus KamLAND reactor $\bar{\nu}$ data $^13$ for the two mass-eigenstate neutrinos that almost exhaust the coupling to $\nu_e$, and $M_D \approx 130 \text{ GeV}/c^2$ is the mass of the decay ion $^{140}$Pr$^{58+}$. Although the value of $\hbar \omega_{\nu_e}$ on the r.h.s. of Eq. (2) is about three times smaller than the value of $\hbar \omega_{EC}$ required to resolve the ‘GSI Oscillation’ puzzle, getting down to this order of magnitude nevertheless presents a remarkable achievement if correct. However, it is shown here in Sect. 2 by following the methodology of Ref. $^12$ that the correct energy scale under circumstances allowing oscillatory behavior is given by

$$\hbar \Omega_{\nu_e} = \frac{\Delta (m_\nu c^2)^2}{2 E_\nu} \approx 0.95 \times 10^{-11} \text{ eV},$$  

(3)

$^1$ Eq. (2) was also obtained by Lipkin $^14$ assuming interference between two unspecified components of the initial state with different momenta and energies that can both decay into the same final state, an electron neutrino and a recoil ion with definite energy and momentum. This scenario was criticized by Peshkin $^15$. 
where $E_{\nu} \approx 4$ MeV is a representative value for neutrino energy in the H-like $^{140}\text{Pr} \rightarrow ^{140}\text{Ce} + \nu_e$ and $^{142}\text{Pm} \rightarrow ^{142}\text{Nd} + \nu_e$ EC decays [3]. The energy $\hbar \Omega_{\nu_e}$ is larger by over four orders of magnitude than $\hbar \omega_{\text{EC}}$ or $\hbar \omega_{\nu_e}$ given by Eq. (2), and so it would lead to modulation period shorter by over four orders of magnitude than the 7 s period reported by the GSI experiment. Given a time measurement resolution of order 0.5 s [5], the effect of such oscillatory behavior would average out to zero.

Other authors [16,17,18,19,20,21] have rejected any link between neutrino mass eigenstates and the EC decay rate oscillatory behavior reported by the GSI experiment [5], the underlying argument being that since no neutrino is detected, the EC decay rate sums incoherently over neutrino mass eigenstates, whereas any oscillatory behavior requires interference between amplitudes summed upon coherently. It is instructive, however, to demonstrate this assertion also by adopting the methodology of Ref. [12], but with a caveat explained below. To this end the time-dependent EC transition amplitude $A_{\nu_e}(i \rightarrow f; t)$, from initial state $i$ (D injected at time $t = 0$) to a final state $f$ (R plus a coherent combination of neutrino mass eigenstates at time $t$), is written in terms of transition amplitudes $A_{\nu_j}(i \rightarrow f; t)$ that involve propagating mass-eigenstate neutrinos $\nu_j$ as

$$A_{\nu_e}(i \rightarrow f; t) = \sum_j U_{ej} A_{\nu_j}(i \rightarrow f; t),$$  \hspace{1cm} (4)

where $U_{ej}$ is a neutrino mixing matrix element of the $3 \times 3$ unitary matrix $U$

$$|\nu_\alpha\rangle = \sum_{j=1}^{3} U^{*}_{\alpha j} |\nu_j\rangle \quad (\alpha = e, \mu, \tau) \hspace{1cm} (5)$$

between the emitted electron-neutrino $\nu_e$ and a mass-eigenstate neutrino $\nu_j$ [22]. For times of order seconds, appropriate to the ‘GSI Oscillations’, the coherence implied by Eq. (4) is still in effect [20] and the flavor basis is of physical significance. If the GSI experiment were to detect neutrino $\nu_\beta$ by a flavor measurement, the corresponding amplitude would have been generated by projecting Eq. (4) onto flavor $\beta$:

$$A_{\nu_e \rightarrow \nu_\beta}(i \rightarrow f; t) = \sum_j U_{ej} A_{\nu_j}(i \rightarrow f; t) U^{*}_{\beta j},$$  \hspace{1cm} (6)

in close analogy with the discussion of neutrino oscillations in dedicated oscillation experiments (Eq. (13.4) in Ref. [22]). The probability associated with the amplitude (6) is then given by

$$P_{\nu_e \rightarrow \nu_\beta}(i \rightarrow f; t) = |A_{\nu_e \rightarrow \nu_\beta}(i \rightarrow f; t)|^2.$$  \hspace{1cm} (7)
Interference terms $A_{\nu_j}A_{\nu_j}^*$ will arise in $\mathcal{P}_{\nu_e \rightarrow \nu_\beta}(i \rightarrow f; t)$, leading to oscillations as shown in Sect. 2. Since the GSI experiment does not detect any neutrino, the overall probability is the sum of probabilities $\mathcal{P}_{\nu_e \rightarrow \nu_\beta}(i \rightarrow f; t)$ over all three flavors $\beta$. The probability of observing the transition $i \rightarrow f$, in which $D$ decays to $R$ and a neutrino is emitted but remains undetected, is thus given by

$$\mathcal{P}_{\nu_e}(i \rightarrow f; t) = \sum_{\beta} \left| \sum_j U_{ej}A_{\nu_j}(i \rightarrow f; t)U_{\beta j}^* \right|^2. \quad (8)$$

Note that the probability $\mathcal{P}_{\nu_e}(i \rightarrow f; t)$ cannot be written as a square of one amplitude, simply because it involves different-flavor final states which require measurement schemes differing from each other and therefore adding up incoherently. Using the unitarity of the mixing matrix $U$, the summation over $\beta$ in Eq. (8) gets rid of the interference terms, leading to the final expression:

$$\mathcal{P}_{\nu_e}(i \rightarrow f; t) = \sum_j \left| U_{ej} \right|^2 \left| A_{\nu_j}(i \rightarrow f; t) \right|^2 \approx \left| A_{\nu_e}(i \rightarrow f; t) \right|^2, \quad (9)$$

where the dependence of the absolute-squared terms $\left| A_{\nu_j}(i \rightarrow f; t) \right|^2$ on the species $\nu_j$ was neglected enabling repeated use of unitarity. The final result, Eq. (9), is that the probability $\mathcal{P}_{\nu_e}(i \rightarrow f; t)$ for the two-body EC decay to occur is what standard weak interaction theory yields for a massless electron neutrino, regardless of its coupling to the mass-eigenstate neutrinos. This holds true also for the total EC decay rate which is obtained by time differentiation of $\mathcal{P}_{\nu_e}(i \rightarrow f; t)$ and which is found identical with the time-independent decay rate $\lambda_{EC}$ derived ignoring neutrino mixing. Thus, although the mass-eigenstate components of the emitted neutrino oscillate against each other, the total EC decay rate does not exhibit any oscillatory behavior owing to the unitarity of the matrix $U$, Eq. (5), which transforms incoherence in one basis into incoherence in the other basis. If $U$ is nonunitary, the above argumentation breaks down, but this does not spoil the more straightforward argumentation that mass-eigenstate neutrinos, as distinct mass particles, have to be summed upon incoherently; one then goes directly from the amplitude $A_{\nu_e}$, Eq. (4), into the probability $\mathcal{P}_{\nu_e}$, Eq. (9), which indeed is an incoherent sum over the neutrino mass-eigenstates $\nu_j$.

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2 This neglect does not hold for interference terms $A_{\nu_j}A_{\nu_j'}^*$, $j \neq j'$, which give rise to oscillatory behavior, as discussed in Sect. 2.
2 Detection of a flavor neutrino, neutrino oscillations

Oscillatory behavior of EC decay rates is possible when a neutrino of a given flavor is detected. The relatively small energy of order few MeV released in EC limits the detected neutrino to $\nu_e$. Here I show within a straightforward ‘gedanken’ extension of the GSI experiment, in which an electron-neutrino $\nu_e$ is detected, that the corresponding angular frequency of the oscillations is given by $\hbar \Omega_{\nu_e}$, Eq. (3). To this end, the specific time-dependent first-order perturbation theory amplitudes $A_{\nu_j}(i \to f; t)$ introduced by Ivanov and Kienle [12] are followed as much as possible:

$$A_{\nu_j}(i \to f; t) = -i \int_0^t \langle f(\bar{q})\nu_j(\bar{k}_j)|H_{e\nu_j}(\tau)|i(\bar{0})\rangle d\tau,$$

with a weak-interaction Hamiltonian for the leptonic transition $e^- \to \nu_j$ given by

$$H_{e\nu_j}(\tau) = \frac{G_F}{\sqrt{2}} V_{ud} \int d^3x [\bar{\psi}_n \gamma^\lambda(1 - g_A \gamma^5)\psi_p][\bar{\psi}_{\nu_j} \gamma^\lambda(1 - \gamma^5)\psi_{e^-}].$$

Here, $x = (\tau, \vec{x})$, $G_F$ is the Fermi constant, $V_{ud}$ is the CKM matrix element, $g_A$ is the axial coupling constant, and with $\psi_n(x)$, $\psi_p(x)$, $\psi_{\nu_j}(x)$ and $\psi_{e^-}(x)$ denoting neutron, proton, mass-eigenstate neutrino $\nu_j$ and electron field operators, respectively. EC decays occur at any time $\tau$ within $[0, t]$, from time $t' = 0$ of injection of D into the ESR to time $t' = t$ of order seconds and longer at which the EC decay rate is evaluated. In the single-ion GSI experiment [3] the heavy ions revolve in the ESR with a period of order $10^{-6}$ s and their motion is monitored nondestructively once per revolution. The decay is defined experimentally by the correlated disappearance of D and appearance of R, but the appearance in the frequency spectrum is delayed by times of order 1 s needed to cool R. The order of magnitude of the experimental time resolution is similar, about 0.5 s, as reflected in the time intervals used to exhibit the experimental decay rates $R(t)$ in Figs. 3,4,5 of Ref. [3]. The decay rates determined in the ESR appear to agree with those measured elsewhere, e.g. for $^{142}$Pm [6], and this consistency suggests that details of kinematics and motion of the heavy ions in the storage ring affect little the overall decay rates which are evaluated here in conventional time-dependent perturbation theory. Therefore, it is plausible to assume that the evolution of the final state in these single-ion EC measurements at GSI proceeds over times of order 1 s which is used here as a working hypothesis.

To obtain the time dependence of the amplitude $A_{\nu_j}(i \to f; t)$ (similarly structured to Eq. (6) of Ref. [12]), recall that the time dependence of the
The integrand in Eq. (10) is given by \( \exp(i\Delta_j \tau) \) where

\[
\Delta_j(q) = E_R(-q) + E_j(q) - M_D
\] (12)

with

\[
E_R = \sqrt{M_R^2 + (-q)^2}, \quad E_j = \sqrt{m_j^2 + \vec{q}^2}
\] (13)

for the recoil ion and neutrino \( \nu_j \) energies, respectively, in the decay-ion rest frame. Integrating on this time dependence results in a standard time-dependent perturbation-theory energy-time dependence \[23\]

\[
A_{\nu_j}(i \rightarrow f; t) \sim \frac{1 - \exp(i\Delta_j t)}{\Delta_j}. \quad (14)
\]

Finally, the EC partial decay rate \( R_{\nu_e \rightarrow \nu_e}(i \rightarrow f; t) \) is obtained from the probability \( P_{\nu_e \rightarrow \nu_e}(i \rightarrow f; t) \), Eq. (7), by differentiating: \( R = \partial_t P \). The term ‘partial’ applied to the rate \( R_{\nu_e \rightarrow \nu_e} \) owes to its limitation to the detection of one particular kind of flavor neutrinos: depleted electron neutrinos. Using Eq. (14) for the time dependence of \( A_{\nu_j}(i \rightarrow f; t) \), one gets for the contribution of any \( j' = j \) non oscillatory term to \( R_{\nu_e \rightarrow \nu_e} \):

\[
R_{\nu_j} = \frac{d}{dt} |A_{\nu_j}(i \rightarrow f; t)|^2 \sim \frac{2 \sin(\Delta_j t)}{\Delta_j} \rightarrow 2 \pi \delta(\Delta_j), \quad (15)
\]

where the last step requires a sufficiently long time \( t \). The properly normalized contribution of these terms to \( R_{\nu_e \rightarrow \nu_e}(i \rightarrow f; t) \) is given by

\[
\sum_j R_{\nu_j} = \lambda_{EC} \sum_j |U_{ej}|^4 \delta(\Delta_j). \quad (16)
\]

Similarly, the contribution of the \( j' \neq j \) oscillatory terms to the EC partial decay rate \( R_{\nu_e \rightarrow \nu_e}(i \rightarrow f; t) \), again for sufficiently long times, is given by

\[
\lambda_{EC} \sum_{j \neq j'} |U_{ej}|^2 |U_{ej'}|^2 [\delta(\Delta_j) + \delta(\Delta_{j'})] \cos[(\Delta_j - \Delta_{j'})t]. \quad (17)
\]

The \( \delta \) symbols in Eqs. (16) and (17) differ from Dirac \( \delta \) functions in that no further integration on the implied c.m. momentum \( \vec{q} \) has to be done. Their meaning is straightforward for the non oscillatory terms, but more delicate for the oscillatory terms in which the sum of \( \delta \) symbols imply that \( \Delta_j - \Delta_{j'} \)

\[\text{From here on } \hbar = c = 1 \text{ units are almost exclusively used.}\]
be evaluated for momentum once derived from the constraint \( \Delta_j(q) = 0 \) and once derived from \( \Delta_j'(q') = 0 \). On each occasion, using a generic notation \( k \) for the momentum implied by each one of the \( \delta \) symbols, one obtains to an excellent approximation
\[
\Delta_j(k) - \Delta_j'(k) = E_j(k) - E_j'(k) = \hbar \Omega_{jj'},
\]
where \( \Omega_{jj'} \) is related to \( \Omega_{\nu e} \) of Eq. (3):
\[
\hbar \Omega_{jj'} = \frac{m_j^2 - m_j'^2}{2 E_{\nu}} \approx \hbar \Omega_{\nu e}.
\]

The requirement of sufficiently long times for Eq. (17) to hold translates in the present case to requiring \( t \gg \Omega_{\nu e}^{-1} \sim 7 \times 10^{-5} \) s, which is comfortably satisfied given the experimental time resolution scale of \( \sim 0.5 \) s [4].

The final expression for the depleted \( \nu_e \) rate is obtained by integrating over the \( \delta \) symbols in Eqs. (16) and (17), resulting in
\[
\mathcal{R}_{\nu_e \rightarrow \nu_e}(i \rightarrow f; t) = \lambda_{EC}\{\sum_j |U_{ej}|^4 + 2 \sum_{j>j'} |U_{ej}|^2 |U_{ej'}|^2 \cos(\Omega_{jj'} t)\}.\]  

(20)

Using the unitarity of \( U \), Eq. (20) may be simplified to the following form:
\[
\mathcal{R}_{\nu_e \rightarrow \nu_e}(i \rightarrow f; t) = \lambda_{EC}\{1 - 4 \sum_{j>j'} |U_{ej}|^2 |U_{ej'}|^2 \sin^2(\frac{\Omega_{jj'} t}{2})\}.
\]

(21)

This expression is identical with the probability for \( \nu_e \rightarrow \nu_e \) oscillation in neutrino spatial oscillation experiments (Eq. (13.9) in Ref. [22]) upon making the identification \( t = L/c \), where \( L \) is the distance traversed by the neutrino between its source and the detector. A more rigorous wave-packet treatment is required to justify this transition from \( t \) to \( L \) [24]. A further simplification of Eq. (21) occurs when only two of the mass-eigenstate neutrinos are coupled to \( \nu_e \):
\[
\mathcal{R}_{\nu_e \rightarrow \nu_e}(i \rightarrow f; t) = \lambda_{EC}\{1 - \sin^22\theta \sin^2(\frac{\Omega_{\nu e} t}{2})\},
\]

(22)

where \( \theta \) is the \( \nu_1 \leftrightarrow \nu_2 \) mixing angle (cf. Eq. (13.20) in Ref. [22]). Note that it is the neutrino energy \( E_{\nu} \) to which the period of oscillations is proportional, not to the mass \( M_D \) of the decaying heavy ion in the GSI experiments.\footnote{Ivanov and Kienle [12] overlooked this distinction by using in Eq. (18) simultaneously on energy shell momentum values \( k_j \) and \( k_{j'} \) implied by \( \delta(\Delta_j) \) and}
3 Magnetic field effects

The preceding discussion ignored a possible role of the electromagnetic fields surrounding the ESR for guidance and stabilization of the heavy-ion motion. The nuclei $^{140}$Pr and $^{142}$Pm in the GSI experiment [5] have spin-parity $I^\pi = 1^+$, and the electron-nucleus hyperfine interaction in the decay ion forms a doublet of levels $F^\pi_i = (\frac{1}{2}^+, \frac{3}{2}^+)$, the ‘sterile’ $\frac{3}{2}^+$ level lying about 1 eV above the ‘active’ $\frac{1}{2}^+$ g.s. from which EC occurs to a $F_f = \frac{1}{2}^+$ final state of a fully ionized recoil ion with spin-parity $I^\pi_f = 0^+$ plus a left-handed neutrino of spin $\frac{1}{2}$.

The lifetime of the $F^\pi_i = \frac{3}{2}^+$ excited level is of order $10^{-2}$ s, so that it de-excites sufficiently rapidly to the $F^\pi_i = \frac{1}{2}^+$ g.s. [2,4]. Periodic excitations of this ‘sterile’ state cannot explain the reported time dependence and intensity pattern [25]. The static magnetic field which is perpendicular to the ESR, $B = 1.19$ T for $^{140}$Pr [26], gives rise to precession of the $F^\pi_i = \frac{1}{2}^+$ initial-state spin with angular frequency $\omega_i$ of order $\hbar \omega_i \approx 0.7 \times 10^{-4}$ eV [27], where $\mu_B$ is the Bohr magneton. The corresponding time scale of order $10^{-11}$ s is substantially shorter than even the ESR revolution period $t_{\text{revol}} \approx 0.5 \times 10^{-6}$ s, so any oscillation arising from this initial-state precession would average out to zero over 1 cm of the approximately 100 m long circumference. A nonstatic magnetic field could lead through its high harmonics to oscillations with the desired frequency between the magnetic substates of the $F^\pi_i = \frac{1}{2}^+$ g.s. [28], but the modulation amplitude $\omega_{\text{EC}}$ expected for such harmonics is below a 1% level, and hence negligible. Furthermore, the associated mixing between the two hyperfine levels $F^\pi_i = (\frac{1}{2}^+, \frac{3}{2}^+)$ is negligible. In conclusion, no initial-state coherence effects are expected from internal or external electromagnetic fields in the GSI experiment.

In the final configuration, interferences may arise from the precession of the neutrino spin in the static magnetic field of the ESR. The corresponding angular frequency $\omega_{\mu\nu}$ is given by $\hbar \omega_{\mu\nu} = \mu_{\nu} \gamma B < 0.5 \times 10^{-14}$ eV in the decay ion rest frame, due to the neutrino anomalous magnetic moment $\mu_{\nu}$ interacting with the static magnetic field $B$. Here, $\gamma = 1.43$ is the Lorentz factor relating the rest frame to the laboratory frame, and $\mu_{\nu} < 0.54 \times 10^{-10}\mu_B$ from the Borexino solar neutrino data [29]. Below I show how the total EC rate gets time-modulated with angular frequency $\omega_{\mu\nu}$. To agree with the reported GSI measurements, $\omega_{\mu\nu} = \omega_{\text{EC}}$, a value of the electron-neutrino magnetic moment $\mu_{\nu} \approx 0.9 \times 10^{-11}\mu_B$ is required which is six times smaller than provided by the published Borexino solar neutrino upper limit [29].

The subscript $f$ in this section relates to both the recoil ion and the neutrino.
3.1 Interference due to a Dirac neutrino magnetic moment

For definiteness I first assume that neutrinos are Dirac fermions with only diagonal magnetic moments \( \mu_{jk} = \mu_j \delta_{jk} \), and that these diagonal moments are the same for all 3 species: \( \mu_j = \mu_\nu \). The emitted electron-neutrino is a left-handed lepton. The amplitude for producing it right-handed, namely with a positive helicity is negligible, of order \( m_\nu/E_\nu < 10^{-7} \) and thus may be safely ignored. A static magnetic field perpendicular to the ESR flips the neutrino spin. Each of the mass-eigenstate components of the emitted neutrino will then precess, with amplitude \( \cos(\omega_\mu \nu t) \) for the depleted left handed components and with amplitude \( \sin(\omega_\mu \nu t) \) for the spin-flip right handed components [30]. Both are legitimate neutrino final states which are summed upon incoherently. The summed probability is of course time independent: \( \cos^2(\omega_\mu \nu t) + \sin^2(\omega_\mu \nu t) = 1 \).

However, the magnetic field dipoles of the storage ring do not cover its full circumference, except for about 35\% of it [26]. This results in interference between the decay amplitude \( A^0_\nu j \) for events with no magnetic interaction and the decay amplitude \( A^m_\nu j \) for events undergoing magnetic interaction (superscript m) with depleted left handed components, i.e. with a superimposed amplitude of \( \cos(\omega_\mu \nu t) \):

\[
A^0_\nu j \sim -i \int_0^t \exp(i\Delta_j \tau) d\tau, \quad A^m_\nu j \sim -i \int_0^t \exp(i\Delta_j \tau) \cos[\omega_\mu \nu (t - \tau)] d\tau, \quad (23)
\]

using the same normalization as in Eq. (14) for any of the left-handed mass-eigenstate neutrinos. This expression for \( A^m_\nu j \) is a crude approximation, but has the merit of representing physically the sequential time dependence anticipated for magnetic interactions. For completeness, I also list the amplitude \( A^R_\nu j \) for events undergoing magnetic interaction which have resulted in a right-handed neutrino (superscript R), with a superimposed amplitude of \( \sin(\omega_\mu \nu t) \):

\[
A^R_\nu j \sim -i \int_0^t \exp(i\Delta_j \tau) \sin[\omega_\mu \nu (t - \tau)] d\tau. \quad (24)
\]

Repeating the same steps in going from amplitudes \( A_\nu j \), Eq. (14), to decay rates \( R_\nu j \), Eq. (15), and adopting the same normalization, the decay rates associated with each one of these three amplitudes are given by:

\[
R^0_\nu j \sim 2\pi \delta(\Delta_j), \quad (25)
\]

\[
R^m_\nu j \sim \frac{\pi}{2} [\delta(\Delta_j + \omega_\mu \nu) + \delta(\Delta_j - \omega_\mu \nu)](1 + \cos(2\omega_\mu \nu t)), \quad (26)
\]
\[ R_{\nu_j}^R \sim \frac{\pi}{2} [\delta(\Delta_j + \omega_{\mu\nu}) + \delta(\Delta_j - \omega_{\mu\nu})] (1 - \cos(2\omega_{\mu\nu}t)). \] (27)

Note that although the two latter expressions, for rates associated with the magnetic interaction, are time dependent, their sum is time independent as expected from summing incoherently over the two separate helicities. The only time dependence in this schematic model arises from interference of the two amplitudes \( A^0_{\nu_j} \) and \( A^m_{\nu_j} \) for a left-handed neutrino. The sum of these partial rates, all of which correspond to \( \nu_j \), and incorporating this interference, is given by

\[ R_{\nu_j} \sim |a_0|^2 2\pi \delta(\Delta_j) + |a_m|^2 2\pi [\delta(\Delta_j + \omega_{\mu\nu}) + \delta(\Delta_j - \omega_{\mu\nu})] \\
+ 2\text{Re}(a_0 a^*_m) \frac{\pi}{2} [\delta(\Delta_j + \omega_{\mu\nu}) + 2\delta(\Delta_j) + \delta(\Delta_j - \omega_{\mu\nu})] \cos(\omega_{\mu\nu} t) \\
- 2\text{Im}(a_0 a^*_m) \frac{\pi}{2} [\delta(\Delta_j + \omega_{\mu\nu}) - \delta(\Delta_j - \omega_{\mu\nu})] \sin(\omega_{\mu\nu} t), \] (28)

where \( |a_m|^2 \sim 0.35 \) and \( |a_0|^2 \sim 0.65 \), with unknown relative phase between the probability amplitudes \( a_m \) and \( a_0 \) for undergoing or not undergoing magnetic interaction, respectively. Working out the complete normalization of this expression, the final rate expression is given by

\[ R_{\nu_e} = \lambda_{EC} [1 + 2\text{Re}(a_0 a^*_m) \cos(\omega_{\mu\nu} t)], \] (29)

showing explicitly a time modulation of the kind Eq. (1) reported by the GSI experiment [5]. It is beyond the present schematic model to explain the magnitude of the modulation amplitude \( a_{EC} \) and the phase shift \( \phi_{EC} \), except that \( |a_{EC}| < 1 \). In particular, a more realistic calculation is required in order to study effects of departures from the idealized kinematics implicitly considered above by which both the recoil ion and the neutrino go forward with respect to the decay-ion instantaneous laboratory forward direction. Whereas this is an excellent approximation for the recoil-ion motion, it is less so for the neutrino\(^6\). Nevertheless, for a rest-frame isotropic distribution, it is estimated that neutrino forward angles in the laboratory dominate over backward angles by more than a factor five.

For distinct diagonal Dirac-neutrino magnetic moments, Eq. (29) gets generalized to

\[ R_{\nu_e} = \lambda_{EC} [1 + 2\text{Re}(a_0 a^*_m) \sum_j |U_{ej}|^2 \cos(\omega_{\mu_j} t)], \] (30)

resulting in a more involved pattern of modulation.

\(^6\) I owe this observation to Eli Friedman.
For vanishing diagonal magnetic moments, and nonzero values of transition magnetic moments, the discussion proceeds identically to that for Majorana neutrinos in the next subsection.

### 3.2 Majorana neutrino magnetic moments

Majorana neutrinos can have no diagonal electromagnetic moments, but are allowed to have nonzero transition moments connecting different mass-eigenstate neutrinos, or different flavor neutrinos. A static magnetic field perpendicular to the storage ring will induce spin-flavor precession \[31\]. However, the magnetic interaction effect is masked in this case by neutrino mass differences, such that the amplitudes \(\cos(\omega_{\mu\nu} t')\) and \(\sin(\omega_{\mu\nu} t')\) in Eqs. (23) and (24) are replaced, to leading order in \(\omega_{\mu\nu}/\Omega_{\nu_e} \ll 1\), by

\[
\begin{align*}
\cos(\omega_{\mu\nu} t') & \to \exp(-i\Omega_{jj'} t'), \\
\sin(\omega_{\mu\nu} t') & \to \frac{\omega_{\mu\nu} t'}{\Omega_{jj'}} \sin(\Omega_{jj'} t'),
\end{align*}
\]

where \(\hbar \omega_{jj'} = \mu_{jj'} \gamma B\), and \(\Omega_{jj'}\) is defined by Eq. (18). The period of any oscillation that might be induced by these amplitudes is of order \(\Omega_{\nu_e}^{-1} \sim 7 \times 10^{-5}\) s which is several orders of magnitude shorter than the time resolution scale of \(\sim 0.5\) s in the GSI experiment \[5\]. Therefore, such oscillations will completely average out to zero over realistic detection periods.

### 4 Discussion and summary

In conclusion, it was shown that interference terms between different propagating mass-eigenstate neutrino amplitudes in two-body EC reactions on nuclei cancel out to zero when no neutrinos are detected. Coherence between propagating mass-eigenstate neutrinos is evident within each one of the amplitudes for detecting a given flavor neutrino. It is only when all final flavor rates are summed upon incoherently, as motivated by the different flavor measurements required, that cancelations occur and the overall rate becomes independent of time and is reliably calculable from standard weak interaction theory for massless neutrinos. The underlying logic here is that summing on all possible phase space for flavor neutrinos is equivalent within quantum mechanics to not detecting any specific neutrino.

Interference terms from different propagating mass-eigenstate neutrinos would survive and give rise to oscillatory behavior of the EC decay rate, if and only if neutrinos are detected. It was shown that the period of oscillations in such a case is \(T \sim 4\pi E_\nu/\Delta(m^2_\nu)\) which for \(E_\nu \approx 4\) MeV as in the GSI
experiments\,[3], and for $\Delta(m^2_e) \approx 0.76 \times 10^{-4}\,\text{eV}^2$\,[13], assumes the value $T \sim 4.4 \times 10^{-4}\,\text{s}$, shorter by over four orders of magnitude than the period reported in these experiments. The oscillation period cited here is in full agreement with the oscillation length tested in dedicated neutrino oscillation experiments, provided the time $t$ is identified with $L/c$ where $L$ is the distance traversed by the neutrino. In particular, besides the $\Delta(m^2_e)$ neutrino input, it depends on the neutrino energy $E_\nu$, not on the mass $M_D$ of the decay ion.

On the positive side, I have proposed a possible explanation of the ‘GSI Oscillations’ puzzle connected with the magnetic field that guides the heavy-ion motion in the ESR, requiring a Dirac neutrino magnetic moment $\mu_\nu$ about six times smaller than the laboratory upper limit value from the Borexino Collaboration\,[29]. The underlying mechanism is the interference between decay amplitudes not affected by the static magnetic field of the ESR and decay amplitudes affected by this field which induces spin precession of the emitted neutrino. The motion of the recoil ion in the ESR is constrained by the interference long after the neutrino has fled away. This mechanism does not work for Majorana neutrinos that can have no diagonal magnetic moments. For nonzero values of transition magnetic moments, the resulting spin-flavor precession is suppressed by neutrino mass differences, and it becomes impossible to relate then the GSI Oscillations puzzle to magnetic effects. It is not yet resolved experimentally whether neutrinos are Dirac or Majorana fermions, although the theoretical bias rests with Majorana fermions, in which case the present paper accomplished nothing towards providing a credible explanation of this puzzle.

For experimental verification, note that the time-modulation period $T_{\text{EC}}^{\text{lab}}$ is inversely proportional to $B$, so the effect proposed here may be checked by varying $B$, for example by varying $\beta = v/c$ for the coasting decay ions. For a fixed value of $\beta$, $B$ depends on the charge-to-mass ratio of the decay ion which varies only to a few percent with the decay-ion mass $M_D$. Finally, the proposed effect is unique to two-body EC reactions, since three-body weak decays do not constrain the neutrino direction of motion with respect to the fixed direction of $\vec{B}$. Indeed, preliminary data on the three-body $\beta^+$ decay of $^{142}\text{Pm}$ indicate no time modulation of the $\beta^+$ decay rate, limiting its modulation amplitude to $a_{\beta^+} < 0.03(3)$\,[32].

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References

[1] H. Geissel, K. Beckert, F. Bosch, et al., Phys. Rev. Lett. 68 (1992) 3412.
[2] Yu.A. Litvinov, F. Bosch, H. Geissel, et al., Phys. Rev. Lett. 99 (2007) 262501.
[3] Z. Patyk, J. Kurcewicz, F. Bosch, H. Geissel, Yu.A. Litvinov, M. Pfützner, Phys. Rev. C 77 (2008) 014306.
[4] A.N. Ivanov, M. Faber, R. Reda, P. Kienle, Phys. Rev. C 78 (2008) 025503.
[5] Yu.A. Litvinov, F. Bosch, N. Winckler, et al., Phys. Rev. Lett. B 664 (2008) 162.
[6] P.A. Vetter, R.M. Clark, J. Dvorak, et al., Phys. Lett. B 670 (2008) 196.
[7] T. Faestermann, F. Bosch, R. Hertenberger, L. Maier, R. Krücken, G. Rugel, Phys. Lett. B 672 (2009) 227.
[8] A.N. Ivanov, R. Reda, P. Kienle, arXiv:0801.2121 [nucl-th].
[9] M. Faber, arXiv:0801.3262 [nucl-th].
[10] H. Kleinert, P. Kienle, EJTP 6, No. 22 (2009) 107 (arXiv:0803.2938 [nucl-th]).
[11] A.N. Ivanov, E.L. Kryshen, M. Pitschmann, P. Kienle, arXiv:0804.1311 [nucl-th]; A.N. Ivanov, P. Kienle, E.L. Kryshen, M. Pitschmann, EJTP 6, No. 22 (2009) 97.
[12] A.N. Ivanov, P. Kienle, Phys. Rev. Lett. 103 (2009) 062502.
[13] B. Aharmim, et al. (SNO Collaboration), Phys. Rev. Lett. 101 (2008) 111301.
[14] H.J. Lipkin, arXiv:0801.1465,0805.0435,0905.1216,0910.5049 [hep-ph].
[15] M. Peshkin, arXiv:0811.1765 [hep-ph].
[16] C. Giunti, Phys. Lett. B 665 (2008) 92.
[17] H. Burkhardt, J. Lowe, G.J. Stephenson Jr., T. Goldman, B.H.J. McKellar, arXiv:0804.1099 [hep-ph].
[18] H. Kienert, J. Kopp, M. Lindner, A. Merle, J. Phys. Conf. Ser. 136 (2008) 022049.
[19] A.G. Cohen, S.L. Glashow, Z. Ligeti, Phys. Lett. B 678 (2009) 191.
[20] A. Merle, Phys. Rev. C 80 (2009) 054616.
[21] V.V. Flambaum, arXiv:0908.2039 [nucl-th].
[22] C. Amsler, M. Doser, M. Antonelli, et al., Reviews of Particle Physics, Phys. Lett. B 667 (2008) 163-171; B. Kayser, arXiv:0804.1497 [hep-ph].
[23] Gordon Baym, Lectures on Quantum Mechanics (W.A. Benjamin, Inc. 1969), Eq. (12-14).

[24] M. Beuthe, Phys. Rep. 375 (2003) 105; for a recent review, see E.Kh. Akhmedov, A.Yu. Smirnov, Phys. At. Nucl. 72 (2009) 1363 [arXiv:0905.1903 [hep-ph]], and references to earlier work cited therein.

[25] N. Winckler, K. Siegień-Iwaniuk, F. Bosch, H. Geissel, Y. Litvinov, Z. Patyk, arXiv:0907.2277 [nucl-th].

[26] T. Faestermann, private communication. The magnetic field in the dipoles does not apply to the full circumference of the ESR, the value $B = 0.44$ T given by T. Faestermann, arXiv:0907.1557 [nucl-th] is an average over the circumference.

[27] M. Faber, A.N. Ivanov, P. Kienle, M. Pitschmann, N.I. Troitskaya, J. Phys. G 37 (2010) 015102.

[28] I.M. Pavlichenkov, arXiv:1002.0075 [physics.atom-ph].

[29] C. Arpesella, et al. (Borexino Collaboration), Phys. Rev. Lett. 101 (2008) 091302. An improved limit $0.32 \times 10^{-10} \mu_B$ for reactor antineutrinos is due to A.G Beda, et al. (GEMMA experiment), arXiv:0906.1926 [hep-ex].

[30] K. Fujikawa, R.E. Shrock, Phys. Rev. Lett. 45 (1980) 963.

[31] J. Schechter, J.W.F. Valle, Phys. Rev. D 24 (1981) 1883.

[32] P. Kienle, for the SMS Collaboration, Nucl. Phys. A 827 (2009) 510c.