Why anthropic reasoning cannot predict $\Lambda$

Glenn D. Starkman$^{1,2}$ and Roberto Trotta$^1$

$^1$ Astrophysics Department, Oxford University, Denys Wilkinson Building, Keble Road, Oxford OX1 3RH, UK and
$^2$ Department of Physics, Case Western Reserve University, Cleveland, OH 44106-7079, USA

We revisit anthropic arguments purporting to explain the measured value of the cosmological constant. We argue that different ways of assigning probabilities to candidate universes lead to totally different anthropic predictions. As an explicit example, we show that weighting different universes by the total number of possible observations leads to an extremely small probability for observing a value of $\Lambda$ equal to or greater than what we now measure. We conclude that anthropic reasoning within the framework of probability as frequency is ill-defined and that in the absence of a fundamental motivation for selecting one weighting scheme over another the anthropic principle cannot be used to explain the value of $\Lambda$, nor, likely, any other physical parameters.

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One of the main goals of physics has been to explain the laws of nature from fundamental principles, such as the existence and breaking of symmetries. This program has so far been carried out with great success and attempts are being made to expand it to explain the values of the dimensionless constants that arise in our theories. Several of the most outstanding “problems” in fundamental physics of the last decades have been how to explain particularly deviant dimensionless numbers: the tiny ratio of the weak energy scale to the Planck mass (“the gauge hierarchy problem”), the large entropy of the universe (“the entropy problem”), the tiny value of the geometric curvature of space in Planck units when evolved back to the Planck time (“the flatness problem”) and the incredibly low energy density of the vacuum compared to the characteristic Planck energy density (the cosmological constant problem).

As an alternative to explanation from fundamental principles, a form of probabilistic reasoning known as the “anthropic principle” has become popular, especially as applied to the cosmological constant problem [1]. The argument can be summarized as follows. One assumes that the values taken on in our Universe by the constants of Nature are just one realization of a statistical ensemble. This ensemble may be thought of as causally disconnected patches of this Universe, as separate sub-universes (a.k.a. the multiverse) or as a superposition of states (in quantum cosmology). Given some a priori distribution of the values of the fundamental constants across the ensemble, the probability for a “typical” observer to measure a certain value of one or more of these constants is usually taken to be proportional to the number of observers (or the number of observers per baryon) that develop in a universe with that value. Values of the fundamental constants that are incompatible with the development of intelligent life will never be observed. In the case of the cosmological constant $\Lambda$, or equivalently the vacuum energy density $\rho_\Lambda$, this selection effect is claimed to successfully predict $\rho_\Lambda$ comparable to what is actually observed, i.e $\rho_\Lambda / M_{Pl}^4 \approx 10^{-123}$ [1].

As we cannot determine the actual number (density) of observers in our own Universe, not to mention in some hypothetical universe with different constants of nature, a more readily calculated surrogate is used – the physical number density of galaxies. We may question whether this is indeed the appropriate weighting factor. For one thing, since it is a function of time, we must choose when to evaluate it. For another, we might be concerned that it fails to differentiate between universes in which a typical galaxy lasts for a very short time (say one which recollapses after a billion years) and universes in which a typical galaxy persists for a very long time (perhaps trillions of years, or longer).

We shall therefore argue that there are other plausible weighting factors for universes, and that the answers to questions such as the expected value of $\Lambda$ depends enormously on the weighting. As an example, we introduce a weighting scheme based on the maximal number of allowed observations (MANO) in a universe. This quantity is clearly relevant to the expected value of a constant, say $\Lambda$, since a value that allows more observations to be carried out will be measured more often. It also has the advantage of being time-independent. In this Letter, we show that in this approach the anthropically predicted probability of measuring $\rho_\Lambda$ to be greater than or equal to the currently inferred value, in a universe otherwise similar to our own, is $\sim 10^{-5}$, in marked contrast to the usual result. The result also depends on other quantities that are effectively unknowable because they describe complex emergent phenomena. However, even in the very optimistic case of an early emergence of intelligent life, the probability of measuring a large $\Lambda$ is still $\sim 5 \cdot 10^{-4}$.

While we do not argue that our probabilistic weighting scheme is better than the traditional one, it is certainly no worse. Since the conclusions one derives depend enormously on which weighting scheme one uses, we conclude that anthropic reasoning based on such frequency arguments is ill-defined. It cannot be used to explain the value of $\Lambda$, nor, likely, any other physical parameters.
We focus on the case where only the cosmological constant varies from one realization to another. We keep fixed all other fundamental constants, as well as all remaining cosmological parameters. This approach has been widely used in the literature. (See [2] for a discussion of how the situation changes when more parameters are varied.)

As described above, the value that \( \Lambda \) takes on in our Universe is seen as the outcome of a sampling from a fundamental probability distribution \( f(\Lambda) \). The probability of observing a specific value \( \Lambda \) is then

\[
f_{\text{obs}}(\Lambda) = f(\Lambda)f_{\text{sel}}(\Lambda).
\]

\( f_{\text{sel}}(\Lambda) \) is the probability of the observation. It encapsulates the selection effects, giving different weights to different universes. There are at least two shortcomings to this approach (see also [3]).

The first point is that the very concept of probability as a limiting frequency of outcomes, though natural when applied to repeatable experiments, is not obviously applicable to the whole Universe. One solution is to use ergodic arguments to derive \( f(\Lambda) \). The validity of this approach remains unproven. More radical is the multiverse scenario, according to which there is an infinite collection of, by definition inaccessible, universes. This approach hardly seems economical. It is difficult to see how vastly increasing the number of universes could help determine the properties of the one Universe we actually observe. We will not address issues relating to \( f \) any further, but will make the usual assumption that \( f(\Lambda) \) is flat within the region where \( f_{\text{sel}} \) is non-vanishing.

The second aspect is that the selection effect probability \( f_{\text{sel}} \) is strongly dependent on the way one chooses to give weight to different universes. We introduce below a new possibility, namely the maximum possible number of observations (MANO) over the entire life of the universe. This concept has the advantage of being time-invariant, as opposed to e.g. the number of observers at any given time. All the different choices for \( f_{\text{sel}} \) (including MANO) suffer from an acute dependence on poorly understood micro-physical processes involved in the evolution of life, especially of conscious beings interested in making observations of the fundamental constants. Our approach does not claim to significantly improve over other treatments in this respect, but it stands conceptually at least on equal footing. The fact that our calculation gives an exceedingly small probability for \( \Lambda \) to be at least as large as the measured value, while others weightings give larger probabilities, is to be seen as an inherent failure of generic anthropic arguments.

We wish to evaluate the probability that an observer will measure his or her universe to have a vacuum energy density no smaller than what we measure in our Universe. For illustrative purposes and computability, we hold fixed all parameters of the universe other than the vacuum energy density, considering flat Lemaitre-Friedmann-Robertson-Walker universes with exactly the same matter and radiation contents and the same fluctuations as our own at the time of matter-radiation equality. This is a common setup in the literature, together with the assumption that the number of observers is proportional to the baryon fraction in halos (even though [4] showed that this result is critically dependent on fixing all other parameters). Below we show that even within this very restricted class of models, the result is completely dependent on the selection function one chooses. We consider only the case \( \Lambda > 0 \). This restriction can only increase the probability of observing \( \Lambda \) equal to or greater than the observed one, so we should interpret the probability we calculate as an upper limit.

As the selection function for observing \( \Lambda \) in the different realizations we put forward the total number of observations that observers can potentially carry out over the entire life of that universe (called MANO for brevity, for “Maximum Allowed Number of Observations”). This maximum number is the product of two factors – the number of observers and the maximum number of observations that each observer can make.

There is a fundamental difficulty in determining the total number of observers, since we can neither compute nor measure it. We might argue, as is usually done, that it is proportional to the number of galaxies, but the proportionality factor could be very large, incredibly small or anywhere in between. Indeed, if we require, as it seems we must, that our observers be sufficiently intelligent to make an observation of \( \Lambda \), then it is not clear that we even know how to define our criteria, never mind compute the probability per galaxy per unit time of meeting those criteria. However, in the limit where observers are rare (in a way we quantify below) the anthropic prediction for the probability of observing \( \Lambda \) will be independent of the density of observers. We therefore choose instead to focus on the second factor – the maximum number of observations that each observer can make.

In a \( \Lambda > 0 \) universe, the minimum temperature at which a system (e.g. an observer) can operate is the de Sitter temperature \( T_{\text{dS}} = \rho_{\Lambda}^{1/2}/(2\pi M_p) \). (Refrigerated subsystems can run cooler, but the energy consumption of the refrigeration more than compensates.)

As discussed in detail in [6], the maximum energy an observer can ever collect is

\[
E_{\text{max}} \propto \min \left( \frac{4\pi}{3} \left[ (n_{\text{obs}} - \eta_{\Lambda}) a_{\Lambda}^3 \right], 1 \right) \rho_m(a_{\Lambda}).
\]

Here \( \eta_{\Lambda} \) is the conformal time when the observer starts collecting, while at conformal time \( \eta_{\infty}, a(\eta_{\infty}) = \infty \). There are \( O(1) \) geometric prefactors that we ignore to focus on the functional dependence of Eq. [2]. The \( n_{\text{obs}}^{-1} \) term in [2] represents the cutoff in \( E_{\text{max}} \) due to competition with other observers. We consider the case of ”rare observers” – when there is at most one observer
within the comoving volume accessible to each from the time that they first become capable of making observations onward. We ignore this cutoff and focus on the first term.

The number of thermodynamic processes (such as observations of Λ) an observer can carry out is maximized if the observer saves up $E_{\text{max}}$ until the universe has reached the de Sitter temperature. Thus

$$N_{\text{max}} \leq E_{\text{max}} / k_B T_{\text{dS}}. \quad (3)$$

Following the arguments given above, we adopt $N_{\text{max}}$ as a probabilistic weight in the selection function.

We start from the Friedman equation for a flat universe containing cosmological constant, radiation and pressureless matter (both baryonic and cold dark matter)

$$\left(\frac{a'}{a}\right)^2 = \frac{8 \pi G}{3} (\rho_\Lambda + \rho_m + \rho_r) \quad (4)$$

(where a prime denotes derivative with respect to conformal time $\eta$). We define $RL^1$ to be the ratio of the vacuum energy density in the sample universe we are considering to the total energy density at matter–radiation equality, where $L^1 = (\Omega^1_\Lambda)_{eq}$ is the value of this ratio in our own Universe. (Throughout, quantities with dagger superscripts are the values measured in our Universe). We assume that the cosmological constant is always negligible at matter–radiation equality, i.e. we limit our analysis to the regime $R \ll 10^9$. Finally, we normalize the scale factor at matter–radiation equality, $a_{eq} \equiv 1$, and set $\alpha = a / a_{eq}$. It will prove useful to note that

$$L^1 = \frac{1}{2} \left(1 - \Omega^1_\Lambda \right) \frac{1}{a_{eq}^3} \approx \frac{3}{2} \alpha_{eq}^3, \quad (5)$$

where quantities subscripted with a 0 are as measured in our Universe today and $a_0 \sim 3000$. In the last equality we have used that $(\Omega^1_\Lambda)_{eq} \approx 3/4$, which we take henceforth as an equality.

With the above definitions, we readily compute

$$a_{eq} \mathcal{H}_{eq} (\eta_{\infty} - \eta) = \int_{\alpha_*}^{\infty} \frac{\alpha^{-2} d\alpha}{\sqrt{RL^1 + \frac{1}{2}(\alpha^{-3} + \alpha^{-4})}}. \quad (6)$$

The matter and radiation energy densities are $\alpha^{-3}/2$ and $\alpha^{-4}/2$ respectively because we assume we can ignore $\Lambda$ at matter–radiation equality. $\mathcal{H}_{eq}$ is the value of the Hubble parameter at equality. (So $\mathcal{H}_{eq} = \mathcal{H}_0^1$. Notice that having neglected $\Lambda$ at equality does not affect our results, since the time at which observers evolve (denoted by a subscript *$)$ is presumably much later than equality, $\alpha_* \gg 1$. We can also neglect the term proportional to $\alpha^{-1}$ in (6), and we can then solve analytically the Friedman equation for $a$ as a function of physical time $t$ (as opposed to conformal time $\eta$), and so obtain

$$\alpha_* = \alpha_0 \left(3R\right)^{-1/3} \sinh \left(\ln(\sqrt{3} + 2) \sqrt{R \tau}\right)^{2/3}. \quad (7)$$

We have introduced $\tau \equiv t_0 / t^1_0$, the time until observers smart enough to begin collecting energy arise, in units of 13.7 Gyrs, the age at which such observers (us, or our descendants) are known to have arisen in our Universe.

Before proceeding to a numerical evaluation of (6), it is instructive to look at its asymptotic limits. For the maximum number of observations, Eq. (3), we find

$$N_{\text{max}} \propto \begin{cases} \frac{2}{5} \alpha_0^{-3} R^{-2}, & R \gg 1, \\ 54 R^{-1}, & R \ll 1. \end{cases} \quad (8)$$

Following Eq. (1), we identify $f_{\text{sel}}(\Lambda) \propto N_{\text{max}}(R)$. Since we have assumed that $f(\Lambda)$ is flat in $\Lambda$ (and hence in $R$), we have $f_{\text{obs}} \propto N_{\text{max}}$. (This is the essence of our MANO weighting.) We see from (3) that $f_{\text{obs}}$ is a steeply decreasing function of $R$. Consequently, there is only a small anthropically conditioned probability that $\Lambda$ is larger than we observe, i.e.

$$p(R \geq 1 | \tau) \approx \int_1^\infty f_{\text{obs}}(R; \tau) \propto 1. \quad (9)$$

The probability $p(R \geq 1 | \tau)$ can be estimated by normalizing $f_{\text{obs}}$ approximately by integrating (8) up to $R = 1$. However, $f_{\text{obs}} \propto R^{-1}$ as $R \to 0$, so the normalization integral diverges logarithmically, and is dominated by the minimum cut–off value, $R_{\text{min}}$, if such exists. In the landscape scenario (see e.g. and references therein), for instance, the number of vacua is estimated to be of order $10^{500}$, and therefore the corresponding minimum value of $\Lambda$ can perhaps be taken to be $\Lambda_{\text{min}} \sim 10^{-500} M_{\text{Pl}}^4$. This gives $R_{\text{min}} \sim 10^{-377}$ or

$$p(R \geq 1 | \tau = 1) \sim 8 \cdot 10^{-6}. \quad (10)$$

This means that a low–$\Lambda$ universe is more probable than one with the value of $\Lambda$ that we observe.

In Figure 1 we plot the value of $f_{\text{obs}}(R|\tau)$ for a few values of $\tau$ around 1. The probability of measuring a value of $R \geq 1$ is very small, being $9\cdot 10^{-6}$ for $\tau = 1$ (this value is computed numerically) and falling to $4 \cdot 10^{-12}$ for $\tau = 10$. The situation is only marginally better if intelligent observers evolve before one–tenth of the current age of the universe, since for $\tau = 0.1$ $f_{\text{obs}}(R|\tau) = 5 \cdot 10^{-4}$.

So far, we have worked exclusively in the rare observer limit – where each intelligent observer is free to collect all of the energy within their apparent horizon without competition from other observers. One might imagine that as the density of observers rose, one would mitigate the preference for low $\Lambda$, but the case is by no means so clear. If the observer density is high, then the observers will come into competition for the universe’s (or at least their Hubble volume’s) same scarce resources. Our own historical experience is that such competition never leads to negotiated agreement to use those resources as conservatively as possible. More likely is that the competition for resources will lead to some substantial fraction
FIG. 1: Anthropically predicted probability density distribution as a function of $R$, the ratio of the cosmological constant in another part of the multiverse to the value it takes in our Universe, in our MANO scheme and for different values of the parameter $\tau$ controlling the cosmic time when intelligent life emerges (in units of 13.7 Gyrs, with $\tau = 1$ being the case of our Universe). The probability of observing a value of $\Lambda$ equal to or greater than what we measure (dashed vertical line) is very small as all the weight lies close the the minimum value.

of those resources being squandered in warfare until only one of the observers remains. Moreover, unless they eliminate all possible competitors, observers will continue to spend their finite supply of energy at a rate exceeding that which would otherwise be necessary. What is clear is that given our inability to predict or measure either the density of intelligent observers or the way in which they would behave when they meet, our ability to use anthropic reasoning can only be further compromised.

It is interesting to note that the rare observer limit would be better described as a rare civilization limit, since the individual observers need not be rare, but only the groups of them that act collectively. In this case, we could attempt to pass from weighting by the number of possible observations to weighting by the number of observers (although throughout all time). In the simplest case, where the number of observations that a civilization makes during its existence is a constant, the translation is trivial – the two weighting schemes are identical. However, one can easily imagine that this number grows or decreases with time in a way that we cannot possibly predict. Moreover, one would still have to solve the formidable problem of calculating the number density of civilizations.

We have argued that anthropic reasoning suffers from the problem that the peak of the selection function depends on the details of what exactly one chooses to condition upon – be it the number of observers, the fraction of baryons in halos or the total number of observations observers can carry out. Whereas a weighting proportional to the number density of galaxies implies that the expected value of $\Lambda$ is close to what we observe, the weighting scheme we propose – according to the maximum number of possible observations – implies that the expected value of $\Lambda$ is logarithmically close to its minimum allowed non-negative value (or is zero or negative). In its usual formulation, the anthropic principle does not offer any motivation – from either fundamental particle physics or probability theory – to prefer one weighting scheme over another. Ours is only one specific example out of many possible weighting schemes one might imagine (see e.g. [8] for an example involving holographic arguments). Since neither weighting scheme (nor any of the many others one can imagine) is clearly the correct one from a probability theory point of view (meaning one that does not lead to paradoxical or self-contradictory conclusions of the type described in [9]), we must conclude that anthropic reasoning cannot be used to explain the value of the cosmological constant. We expect that similar statements apply to any conclusions that one would like to draw from anthropic reasoning.

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