Flavored exotic multibaryons and hypernuclei in topological soliton models

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Abstract

The energies of baryon states with positive strangeness, or anti-charm (-beauty) are estimated in chiral soliton approach, in the "rigid oscillator" version of the bound state soliton model proposed by Klebanov and Westerberg. Positive strangeness states can appear as relatively narrow nuclear levels (Θ-hypernuclei), the states with heavy anti-flavors can be bound with respect to strong interactions in the original Skyrme variant of the model (SK4 variant). The binding energies of anti-flavored states are estimated also in the variant of the model with 6-th order term in chiral derivatives in the lagrangian as solitons stabilizer (SK6 variant). The latter variant is less attractive, and nuclear states with anti-charm and anti-beauty can be unstable relative to strong interactions. The chances to get bound hypernuclei with heavy anti-flavors increase within "nuclear variant" of the model with rescaled model parameter (Skyrme constant \( e \) or \( e' \) decreased by \( \sim 30\% \)) which is expected to be valid for baryon numbers greater than \( B \sim 10 \). The rational map approximation is used to describe multiskyrmions with baryon number up to \( \sim 30 \) and to calculate the quantities necessary for their quantization (moments of inertia, sigma-term, etc.).

1 Introduction

The remarkable recent discovery of the positive strangeness pentaquark state [1], its confirmation by several experiments [2] provided strong impact for the searches of other exotic states and revision of existing ideas on the structure of hadrons and the role of the valence quarks picture in their description [3, 4, 5, 6, 7, 8]. Subsequently, the discovery of the strangeness \( S = -2 \) state with charge \( -2 \), also manifestly exotic [9] (see [10] reviewing previously existing data), and evidence for a narrow anti-charmed baryon state [11] have been reported. Some experiments, however, did not confirm these results, see e.g. [12] and [13] where some negative results were summarized and pessimistic point of view was formulated. The HEP community is waiting now for results of high statistics experiments; some plans for future pentaquarks searches are presented, e.g. in [14].

The possible existence of such states has been foreseen theoretically within the quark models [15, 16, 17] \(^1\), as well as in chiral soliton models. The prediction of exotic states in chiral soliton models has not simple and instructive history, from the papers where the exotic antidecuplet and \{27\}-plet of baryons were mentioned [18], a resonant behaviour of the kaon-nucleon phase shift in the \( \Theta \) channel was obtained in some version of the Skyrme model[19], first estimates of the antidecuplet mass were made [20, 21], the masses of exotic baryon states were roughly estimated for arbitrary \( B \)-numbers [22], to papers where more detailed calculations of antidecuplet spectrum were performed [23, 24, 25], for recent discussion see also [26]. The mass of the dibaryon with \( S = +1 \), \( I = 1/2 \) was determined to be only \( 590 \text{ Mev} \) above nucleon-nucleon threshold within soft rotator quantization scheme [27]. It should be noted that the paper [24] which predicted narrow width and low mass of positive

\(^1\)The parity of lowest exotic states considered here is negative, see however [7], in difference from the chiral soliton model predictions, where it is positive. Spin and parity of exotic baryons are not measured yet.
strangeness state called $\Theta^+$ stimulated experimental searches for such states, in particular, experiments [1] have been arranged specially to check the prediction of [24].

Theoretical ideas and methods which led to the prediction of such states within the chiral soliton models [23, 24, 25] have been criticized with quite sound reasoning in [4] and, in the large $N_c$ limit, in [29, 30]. In the absence of the complete theory of strong interactions it was in principle not possible to provide the firm predictions for the masses of the states with accuracy better than about several tens of Mev, and similarly for the width of such states. One can agree with [29]: the fact that in some cases predictions coincided with the observed mass of $\Theta^+$ hyperon can be considered as ”accidental”, see also [28].

On the other hand, from practical point of view the chiral soliton approach is useful and has remarkable predictive power, when at least one of the exotic baryon masses is fitted. The masses of exotic baryons with strangeness $S = -2$ and isospin $I = 3/2$ predicted in this way [31], 1.79 Gev for the antidecuplet component and 1.85 Gev for the 27-plet component, are close to the value 1.86 Gev measured later [9]. Calculations of baryons spectra within chiral soliton approach were made more recently in papers [32, 33, 35, 36], not in contradiction with [31]; recent paper [37] where the interplay of rotational and vibrational modes has been investigated, should be mentioned specially. Some reviews and comparison of the chiral soliton approach with other models can be found, e.g. in [38].

This particular case of strangeness is in certain respect more complicated in comparison with the case of other flavors: the rigid rotator quantization scheme is not quite valid in this case [29], whereas the bound state approach also is not quite good [30]. In the case of heavy flavors the rotator quantization is not valid at all, but the bound state approach becomes more adequate compared to strangeness [30].

Baryons with heavy anti-flavors certainly is not a new issue: they have been discussed in literature long ago, with various results obtained for the energies of such states. The strange-anticharmed pentaquark was obtained bound [39] in a quark model with $(u, d, s) SU(3)$ flavor symmetry and in the limit of very heavy $c$-quark. There were already long ago statements and suggestions in the literature that anti-charm or anti-beauty can be bound by chiral solitons for the case of baryon number $B = 1$ [40, 41] (so called P-baryons).

In [42] the mass differences of exotic baryons ($\Theta^+$ and its analogs for anti-charm and anti-beauty) and nucleons were estimated in the flavor-symmetric limit for decay constants, $F_D = F_\pi$ of the chiral quark meson model. In [43] the anti-flavor excitation energies were calculated in the rigid oscillator version [44] of the bound state soliton model [45], for baryon numbers between 1 and 8. The rational map ansatz for multiskyrmions [46] has been used as starting configuration in the 3-dimensional minimization $SU(3)$ program [47]. These energies were found to be close to 0.59 Gev for anti-strangeness, 1.75 Gev for anti-charm and 4.95 Gev for anti-beauty, in the latter two cases these energies are smaller than masses of $D$ and $B$-mesons which enter the lagrangian [43]. The flavor symmetry breaking in flavor decay constants ($F_D/F_\pi > 1$) plays important role for these estimates. So, it was clear hint that such baryonic systems can be bound relative to strong interactions.

Similar results, in principle, follow from recent analysis within bound state soliton model [30], and also within the diquark model [4]. The spectra of exotic states with heavy flavors have been estimated in different models, already after discovery of positive strangeness pentaquark [48] (any baryon number), [49, 50, 51, 52, 53] and others. The possibility of existence of nuclear matter fragments with positive strangeness was discussed recently in [54].

In present paper we estimate the energies of ground states of multibaryons with baryon

\[ M_{\Theta} \simeq 1530 \text{MeV} \]

\[ \text{was to some extent a luck}. \]
numbers up to $\sim 30$ with different (anti)flavors using a very transparent "rigid oscillator" model [44]. In the next section the properties of multiskyrmions are considered which are necessary to calculate the energies of flavor excitations, using the rational map approximation for $B > 1$[46]. It is shown that $\Theta^+$ baryon is bound by nuclear systems, providing positive strangeness multibaryons ($\Theta$-hypernuclei), their binding energy can reach several tens of Mev. The multiskyrmion configurations possess some remarkable scaling properties, as a result, the flavor and antiflavor excitation energies are close to those for $B = 1$. The quantization scheme (slightly modified rigid oscillator version [44]) is described in section 3 where the flavor and antiflavor excitation energies are calculated as well. The masses (binding energies) of ground states of positive strangeness states - $\Theta$-hypernuclei - are presented in section 4, followed by those for anti-charmed or -beautiful states. The last section contains some conclusions and prospects.

2 Properties of multiskyrmions

Here we calculate the properties of multiskyrmion configurations necessary for calculation of flavor excitation energies and hyperfine splitting constants which govern the $1/N_c$ corrections to the energies of the quantized states. As it was already noted previously, the details of baryon-baryon interactions do not enter the calculations explicitly, although they influence implicitly via the integral characteristics of the bound states of skyrmions shown in Tables 1, 2.

The lagrangian of the Skyrme model in its well known form depends on parameters $F_\pi$, $F_D$ and $e$ and can be written in the following way [55, 56]:

$$
\mathcal{L} = -\frac{F_\pi^2}{16} Tr l_\mu l^\mu + \frac{1}{32e^2} Tr [l_\mu, l_\nu]^2 + \frac{F_\pi^2 m_\pi^2}{16} Tr (U + U^\dagger - 2) + \\
+ \frac{F_D^2 m_D^2 - F_\pi^2 m_\pi^2}{24} Tr (1 - \sqrt{3}\lambda_8)(U + U^\dagger - 2) + \\
+ \frac{F_D^2 - F_\pi^2}{48} Tr (1 - \sqrt{3}\lambda_8)(U l_\mu l^\mu + l_\mu l^\mu U^\dagger). 
$$

Here $U \in SU(3)$ is a unitary matrix incorporating chiral (meson) fields, and $l_\mu = \partial_\mu U U^\dagger$. In this model $F_\pi$ is fixed at the physical value: $F_\pi = 186 Mev$ and $m_D$ is the mass of $K$, $D$ or $B$ meson. The ratios $F_D/F_\pi$ are known to be 1.22 and 2.28$^{+1.4}_{-1.1}$ for, respectively, kaons and $D$-mesons. The Skyrme parameter $e$ is close to 4 in numerical fits of the hyperons spectra (see discussion at the end of this section). In the variant of the model with 6-th order term as solitons stabilizer the contribution is added to the lagrangian density [57, 58, 59]

$$
L_6 = -\frac{c_6}{48} Tr [l_\mu, l^\nu][l_\nu, l^\alpha][l_\alpha, l^\mu], 
$$

where we introduced the coefficient $1/48$ in the definition of the constant $c_6$ for further convenience. It is known that this term can be considered as approximation to the exchange of $\omega$-meson in the limit $m_\omega \to \infty$ [57]. The flavor symmetry breaking ($FSB$) in the lagrangian is of the usual form, and is sufficient to describe the mass splittings of the octet and decuplet of baryons within the collective coordinate quantization approach [60]. A nice

\footnote{We are using in (2) one of several possible forms of 6-th order term, but all give the same contribution to the static mass of the $SU(2)$ solitons, see also discussion in [57]. General consideration of high order terms and their role for skyrmion properties can be found in [58].}
and useful feature of the lagrangian (1,2) is that it contains only second power of the time derivative, and this allows to perform quantization without problems (next section).

The Wess-Zumino term, to be added to the action, which can be written as a 5-dimensional differential form \[ S^{WZ} = \frac{-iN_c}{240\pi^2} \int_{\Omega} d^5x \epsilon^{\mu\nu\lambda\sigma\tau} Tr(l_{\mu}l_{\nu}l_{\lambda}l_{\sigma}l_{\tau}), \] where \( \Omega \) is a 5-dimensional region with the 4-dimensional space-time Action (3) defines important topological properties of skyrmions, but it does not contribute to the static masses of classical configurations \[ 61, 21 \]. Variation of this action can be presented as a well defined contribution to the lagrangian (integrand over the 4-dimensional space-time).

We begin our calculations with \( U \in SU(2) \). The classical mass of \( SU(2) \) solitons, in the most general case, depends on 3 profile functions: \( f, \alpha, \beta \), and is given by

\[ M_{cl} = \int \left\{ \frac{F_f^2}{8} \left[ l_1^2 + l_2^2 + l_3^2 \right] + \frac{1}{2e^2} \left[ \left( l_1 l_2 \right)^2 + \left( l_2 l_3 \right)^2 + \left( l_3 l_1 \right)^2 \right] + \frac{1}{4} F_{\pi}^2 m_i^2 \left( 1 - c_f \right) + 2 c_6 \left( l_1 l_2 l_3 \right)^2 \right\} d^3r. \] (4)

Here \( l_k \) are the \( SU(2) \) chiral derivatives defined by \( \delta U U^\dagger = il_k \tau_k \), where \( k = 1, 2, 3 \). The general parametrization of \( U_0 \) for an \( SU(2) \) soliton we use here is given by \( U_0 = c_f + s_f \vec{n} \) with \( n_z = c_\alpha, n_x = s_\alpha c_\beta, n_y = s_\alpha s_\beta, n_f = \sin f, c_f = \cos f \), etc.

For the rational map (RM) ansatz we are using here as starting configurations \[ 46 \],

\[ n_x = \frac{2 Re R(\xi)}{1 + |R(\xi)|^2}, \quad n_y = \frac{2 Im R(\xi)}{1 + |R(\xi)|^2}, \quad n_z = \frac{1 - |R(\xi)|^2}{1 + |R(\xi)|^2}, \] (5)

where \( R(\xi) \) is a ratio of polynomials of the maximal power \( B \) of the variable \( \xi = t g(\theta/2) e^{i \phi} \), \( \theta \) and \( \phi \) being polar and azimuthal angles defining the direction of the radius-vector \( \vec{r} \). It is important assumption that vector \( \vec{n} \) depends on angular variables, but does not depend on \( r \), whereas the profile \( f(r) \) depends on distance from the soliton center, only. The explicit form of \( R(\xi) \) is given in \[ 46, 62 \] for different values of \( B \). Within RM approximation all characteristics of multiskyrmions necessary for us (including mass and moments of inertia) depend on two quantities, integrals over angular variables,

\[ N = \frac{1}{8\pi} \int r^2 (\partial_r n_k)^2 d\Omega, \quad I = \frac{1}{8\pi} \int r^4 (\partial_{\vec{n}} \vec{\partial} n_k)^2 d\Omega, \] (6)

satisfying inequality \( I \geq N^2 \) \[ 46 \]. For configuration of lowest energy \( N = B, f(0) - f(\infty) = \pi \) and the value \( I \) should be found by minimization of the map \( S^2 \rightarrow S^2 \) \[ 46 \]. The classical mass of multiskyrmion then simplifies to:

\[ M_{cl} = 4\pi \int \left[ \frac{F_f^2}{8} \left( f^2 + 2B \frac{s_f^2}{r^2} \right) + \frac{s_f^2}{2e^2r^2} \left( 2 f^2 B + s_f^2 \frac{I}{r^2} \right) + 4 c_6 I f^2 s_f^2 \frac{s_f^4}{r^4} + \rho_{M.L.} \right] r^2 dr, \] (7)

which should and can be minimized easily for definite \( B \) and \( I \). The mass term density is simple for starting \( SU(2) \) skyrmion: \( \rho_{M.L.} = F_{\pi}^2 m_s^2 (1 - c_f) / 4 \). The quantity \( \lambda \) can be introduced \[ 59 \] which characterizes the relative weight of the 6-th order term, according to \( \lambda / (1 - \lambda)^2 = c_6 F_{\pi}^2 e^4 \), or \( c_6 = \lambda / (F_{\pi}^2 e^4) \). For pure SK6 variant \( (\lambda = 1, e \rightarrow \infty \) and \( e' = e(1 - \lambda) \)-fixed), there is relation \( c_6 = 1 / (F_{\pi}^2 e^{4}) \).

The “flavor” moment of inertia plays a very important role in the procedure of \( SU(3) \) quantization \[ 61, 23 \], see formulas (16,17,23) etc. below. It defines the \( SU(3) \) rotational
energy $E_{rot}(SU_3) = \Theta_F(\Omega_4^2 + \Omega_5^2 + \Omega_6^2 + \Omega_7^2)/2$ with $\Omega_a$, $a = 4, \ldots, 7$ being the angular velocities of rotation in $SU(3)$ configuration space. For $SU(2)$ skyrmions as starting configurations and the $RM$ ansatz describing classical field configurations $\Theta_F$ is given by [63, 64]:

$$\Theta_F = \frac{1}{8} \int (1 - c_f) \left[ F_D^2 + \frac{1}{c^2} \left( f'^2 + 2B \frac{s_f^2}{r^2} \right) + 2c_0 \frac{s_f^2}{r^2} \left( 2B f'^2 + \frac{3}{r^2} \right) \right] r^2 dr.$$

(8)

It is simply connected with $\Theta_F^{(0)}$ of the flavor symmetric case ($F_D = F_\pi$):

$$\Theta_F = \Theta_F^{(0)} + \left( \frac{F_D^2}{F_\pi^2} - 1 \right) \Gamma/4,$$

(9)

with $\Gamma$ defined in (11) below.

| $B$ | $\Theta_I^{SK4}$ | $\Theta_F^{(0)SK4}$ | $\Gamma^{SK4}$ | $\Gamma^{SK6}$ | $\Theta_I^{SK6}$ | $\Theta_F^{(0)SK6}$ | $\Gamma^{SK6}$ | $\Gamma^{SK6}$ |
|-----|------------------|----------------------|----------------|-----------------|-----------------|----------------------|----------------|----------------|
| 1   | 5.56             | 2.05                 | 4.80           | 14.9            | 5.13            | 2.28                 | 6.08           | 15.8           |
| 2   | 11.5             | 4.18                 | 9.35           | 22.0            | 9.26            | 4.94                 | 14.0           | 24.7           |
| 3   | 14.4             | 6.34                 | 14.0           | 27.0            | 12.7            | 7.35                 | 20.7           | 30.4           |
| 4   | 16.8             | 8.27                 | 18.0           | 31.0            | 15.2            | 8.93                 | 24.5           | 33.7           |
| 5   | 23.5             | 10.8                 | 23.8           | 35.0            | 18.7            | 11.8                 | 32.8           | 38.3           |
| 6   | 25.4             | 13.1                 | 29.0           | 38.0            | 21.7            | 14.1                 | 39.3           | 41.6           |
| 7   | 28.9             | 14.7                 | 32.3           | 44.0            | 23.9            | 15.4                 | 42.5           | 43.4           |
| 8   | 33.4             | 17.4                 | 38.9           | 47.0            | 27.2            | 18.5                 | 51.6           | 46.9           |
| 9   | 37.8             | 20.6                 | 46.3           | 47.5            | 30.2            | 21.1                 | 59.1           | 49.7           |
| 10  | 41.4             | 23.0                 | 52.0           | 50.0            | 32.9            | 23.5                 | 65.8           | 51.9           |
| 11  | 45.2             | 25.6                 | 58.5           | 52.4            | 35.8            | 26.1                 | 73.6           | 54.3           |
| 12  | 48.5             | 28.0                 | 64.1           | 54.6            | 38.4            | 28.3                 | 79.9           | 56.2           |
| 13  | 52.1             | 30.5                 | 70.2           | 56.8            | 41.2            | 30.8                 | 87.1           | 58.1           |
| 14  | 56.1             | 33.6                 | 78.2           | 58.9            | 44.3            | 34.0                 | 96.9           | 60.5           |
| 15  | 59.8             | 36.3                 | 85.1           | 60.9            | 47.1            | 36.7                 | 105            | 62.4           |
| 16  | 63.2             | 38.9                 | 91.5           | 62.8            | 49.7            | 39.3                 | 112            | 64.1           |
| 17  | 66.2             | 41.2                 | 96.8           | 64.6            | 52.1            | 41.3                 | 118            | 65.4           |
| 18  | 70.3             | 44.5                 | 106            | 66.4            | 55.2            | 44.8                 | 129            | 67.5           |
| 19  | 73.9             | 47.4                 | 113            | 68.2            | 58.0            | 47.8                 | 138            | 69.2           |
| 20  | 77.5             | 50.4                 | 121            | 69.9            | 60.8            | 50.8                 | 147            | 70.8           |
| 21  | 80.9             | 53.2                 | 128            | 71.5            | 63.5            | 53.6                 | 156            | 72.4           |
| 22  | 84.3             | 56.0                 | 136            | 73.1            | 66.1            | 56.4                 | 164            | 73.8           |
| 23  | 88.0             | 59.2                 | 144            | 74.7            | 69.0            | 59.7                 | 174            | 75.4           |
| 24  | 91.3             | 62.0                 | 151            | 76.2            | 71.6            | 62.5                 | 183            | 76.7           |
| 25  | 94.7             | 64.9                 | 159            | 77.6            | 74.2            | 65.4                 | 192            | 78.0           |
| 26  | 98.2             | 68.1                 | 168            | 79.1            | 77.0            | 68.7                 | 202            | 79.4           |
| 27  | 102              | 71.1                 | 176            | 80.5            | 79.7            | 71.7                 | 211            | 80.8           |
| 28  | 105              | 74.3                 | 185            | 81.9            | 82.5            | 75.1                 | 222            | 82.2           |
| 29  | 108              | 77.6                 | 194            | 83.2            | 84.0            | 78.4                 | 233            | 83.8           |
| 30  | 111              | 80.9                 | 203            | 85.3            | 86.2            | 81.6                 | 244            | 85.7           |
| 31  | 114              | 84.2                 | 212            | 87.4            | 88.2            | 84.9                 | 255            | 87.7           |
| 32  | 118              | 86.4                 | 221            | 89.7            | 90.2            | 88.3                 | 266            | 89.7           |

**Table 1.** Static characteristics of multiskyrmions - moments of inertia and $\sigma$-term $\Gamma$, $\tilde{\Gamma}$ in the $SK4$ variant of the model with $e = 4.12$, and for the $SK6$ variant of the model, $e' = 4.11$, in $Gev^{-1}$. 


The isotopic momenta of inertia are the components of the corresponding tensor of inertia presented and discussed in many papers, see e.g. [61, 23, 63]. For majority of multiskyrmions we discuss, this tensor of inertia is close to the unit matrix multiplied by the isotopic moment of inertia: $\Theta_{ab} \simeq \Theta_{Iab}$, $\Theta_I = \Theta_{Iaa}/3$. This is exactly the case for $B = 1$ and, to within a good accuracy, for $B = 3, 7$. Considerable deviations take place for the torus with $B = 2$, smaller ones for $B = 4, 5, 6$, and generally, they decrease with increasing $B$-number. We shall use for our estimates very simple expression obtained within rational map approximation [63, 64]

$$\Theta_I = \frac{4\pi}{3} \int s_f^2 \left( \frac{F_2^2}{2} + \frac{2}{e^2} f^2 + 8c_6Bs_f^2 f^2 \right) r^2 dr$$

(10)

The isotopic inertia (10) at large enough baryon numbers receives the main contribution from the spherical envelope of multiskyrmion where its mass is concentrated. The dimensions of this spherical bubble grow like $R_B \sim \sqrt{B}$ [63], and moments of inertia are proportional roughly to the baryon number.

The quantity $\Gamma$ (or $\Sigma$-term) defines the contribution of the mass term to the classical mass of the soliton, $\Gamma$ enters due to the presence of $F_{SB}$ term proportional to the difference $F_{SB}^0 - F_{SB}^2$ in (1), the last term in (1). They define the potential where the rigid oscillator moves, and are given by:

$$\Gamma = \frac{F_2^2}{2} \int (1 - cf)d^3r, \quad \tilde{\Gamma} = \frac{1}{4} \int c_f[(\partial f)^2 + s_f^2(\partial n)^2].$$

(11)

The following relation can also be established: $\tilde{\Gamma} = 2(M_{cl}^{(2)}/F_2^2 - e^2\Theta_F^{SK4})$, where $M_{cl}^{(2)}$ is the second-order term contribution to the classical mass of the soliton, and $\Theta_F^{SK4}$ is the Skyrme term contribution to the flavor moment of inertia. The calculated momenta of inertia $\Theta_F$, $\Theta_I$, $\Gamma$ or $\Sigma$-term and $\tilde{\Gamma}$ for solitons with baryon numbers up to 32 are presented in Tables 1, 2.

Sigma-term $\Gamma$ gets contribution from the whole volume of multiskyrmion, where $c_f \sim -1$, and by this reason it grows faster than moment of inertia $\Theta_I$. Flavor inertia $\Theta_F$ gets contribution from the surface and the volume of multiskyrmion, and its behaviour is intermediate between that of $\Gamma$ and $\Theta_I$.

For both variants of the model, $SK4$ and $SK6$, we calculated static characteristics of multiskyrmions for two values of the only parameter of the model, constant $e$, or $e'$ for the $SK6$ variant (the connection between coefficient $c_6$ and $e'$ is $e' = 1/(F_2^2c_6)^{1/4}$). For the $SK4$ variant of the model and $e = 4.12$ the numbers given in Table 1 for $B = 1 - 8$ are obtained as a result of direct numerical energy minimization in 3 dimensions performed using the calculation algorithm developed in [47]. By this reason they differ slightly from those obtained in pure rational map approximation. This difference is the largest for the case of $B = 2$ and decreases with increasing $B$. In all other cases we used the $RM$ approximation with values of the Morse function $I$ given in [46, 62].

The second value of the constants, $e = 3.00$ and $e' = 2.84$, leads to the “nuclear variant” of the model which allowed to describe quite successful the mass splittings of nuclear isotopes for atomic (baryon) numbers between $\sim 10$ and $\sim 30$ [65]. The static characteristics of multiskyrmions change considerably when the change of the constants $e$ or $e'$ is made by about 30%, see Table 2, since dimensions of solitons scale like $1/(F_2e)$, and the isotopic mass splittings scale like $F_2e^3$. However, the flavor excitation energies change not crucially, even slightly for charm and beauty, according to the scale invariance of these quantities [63], as described in the next section.
| $B$ | $\Theta_{i}^{SK4^*}$ | $\Theta_{F}^{(0)SK4^*}$ | $\Gamma_{SK4^*}$ | $\Theta_{J}^{SK6^*}$ | $\Theta_{F}^{(0)SK6^*}$ | $\Gamma_{SK6^*}$ | $\Gamma_{SK6^*}$ |
|-----|----------------------|----------------------|-------------------|-------------------|-------------------|----------------|----------------|
| 1   | 12.8                 | 4.66                 | 10.1              | 19.6              | 14.2              | 6.21           | 15.3           | 22.3           |
| 2   | 24.3                 | 9.87                 | 20.9              | 28.8              | 25.7              | 13.6           | 35.9           | 34.7           |
| 3   | 34.7                 | 15.1                 | 31.7              | 35.6              | 35.5              | 20.4           | 53.9           | 42.5           |
| 4   | 42.9                 | 19.4                 | 40.1              | 41.1              | 43.2              | 25.0           | 64.6           | 46.9           |
| 5   | 53.5                 | 25.4                 | 53.2              | 46.2              | 52.9              | 32.9           | 86.2           | 53.1           |
| 6   | 62.6                 | 30.7                 | 64.7              | 50.6              | 61.4              | 39.4           | 103            | 57.4           |
| 7   | 69.6                 | 34.9                 | 72.5              | 54.4              | 68.0              | 43.3           | 112            | 59.8           |
| 8   | 79.9                 | 41.3                 | 87.4              | 58.2              | 77.3              | 51.7           | 135            | 64.4           |
| 9   | 88.9                 | 47.1                 | 101               | 61.7              | 85.7              | 58.9           | 154            | 67.9           |
| 10  | 97.4                 | 52.6                 | 113               | 64.9              | 93.5              | 65.3           | 171            | 70.8           |
| 11  | 106                  | 58.5                 | 126               | 67.9              | 102               | 72.5           | 191            | 73.8           |
| 12  | 114                  | 63.8                 | 138               | 70.8              | 109               | 78.7           | 207            | 76.1           |
| 13  | 122                  | 69.5                 | 151               | 73.6              | 117               | 85.4           | 225            | 78.6           |
| 14  | 132                  | 76.3                 | 168               | 76.3              | 125               | 94.0           | 249            | 81.5           |
| 15  | 140                  | 82.3                 | 182               | 78.8              | 133               | 101            | 269            | 83.9           |
| 16  | 148                  | 88.1                 | 196               | 81.2              | 141               | 108            | 287            | 86.0           |
| 17  | 155                  | 93.2                 | 207               | 83.5              | 148               | 114            | 302            | 87.6           |
| 18  | 164                  | 100                  | 225               | 85.9              | 156               | 123            | 328            | 90.1           |
| 19  | 173                  | 107                  | 241               | 88.1              | 164               | 131            | 350            | 92.2           |
| 20  | 181                  | 113                  | 257               | 90.3              | 172               | 139            | 372            | 94.1           |
| 24  | 213                  | 138                  | 320               | 98.2              | 202               | 170            | 457            | 101            |
| 28  | 245                  | 165                  | 387               | 105               | 232               | 202            | 550            | 107            |
| 32  | 275                  | 191                  | 454               | 112               | 261               | 234            | 640            | 113            |

Table 2. Static characteristics of multiskyrmions - moments of inertia and $\Gamma$, $\dot{\Gamma}$ for rescaled, or nuclear variants of the model: $e = 3.00$ in the $SK4$ and $e' = 2.84$ for the $SK6$ variants, also in $Gev^{-1}$.

3 Flavor and antiflavor excitation energies

The $SU(3)$ effective action defined by (1,3) leads to the collective lagrangian obtained in [61]. To quantize the solitons in their $SU(3)$ configuration space, in the spirit of the bound state approach to the description of strangeness proposed in [45, 44] and used in [63, 43], we consider the collective coordinate motion of the meson fields incorporated into the matrix $U$:

$$U(r, t) = R(t)U_0(O(t)\vec{r})\dot{R}(t), \quad R(t) = A(t)S(t),$$

(12)

where $U_0$ is the $SU(2)$ soliton embedded into $SU(3)$ in the usual way (into the upper left hand corner), $A(t) \in SU(2)$ describes $SU(2)$ rotations and $S(t) \in SU(3)$ describes rotations in the “strange”, “charm” or “beauty” directions and $O(t)$ describes rigid rotations in real space. In the quantization procedure of the rotator with the help of $SU(3)$ collective coordinates the following definition of angular velocities in $SU(3)$ configuration space is accepted [61]:

$$R^\dagger(t)\dot{R}(t) = -\frac{i}{2}\Omega_\alpha\lambda_\alpha,$$

(13)
with $\alpha = 1, \ldots, 8$, $\lambda_\alpha$ being $SU(3)$ Gell-Mann matrices. For the quantization method proposed in [44] and used here the parametrization (12) is more convenient, the components $\Omega_\alpha$ can be expressed via collective coordinates introduced in (12).

For definiteness we consider the extension of the $(u,d)$ $SU(2)$ Skyrme model in the $(u,d,s)$ direction, when $D$ is the field of $K$-mesons, but it is clear that quite similar extensions can also be made in the directions of charm or bottom. So

$$S(t) = \exp(iD(t)), \quad D(t) = \sum_{\alpha=4,\ldots,7} D_\alpha(t) \lambda_\alpha,$$  \tag{14}

where $\lambda_\alpha$ are Gell-Mann matrices of the $(u,d,s)$, $(u,d,c)$ or $(u,d,b)$ $SU(3)$ groups. The $(u,d,c)$ and $(u,d,b)$ $SU(3)$ groups are quite analogous to the $(u,d,s)$ one. For the $(u,d,c)$ group a simple redefinition of hypercharge should be made. For the $(u,d,s)$ group, $D_4 = (K^+ + K^-)/\sqrt{2}$, $D_5 = i(K^+ - K^-)/\sqrt{2}$, etc. For the $(u,d,c)$ group $D_4 = (D^0 + \bar{D}^0)/\sqrt{2}$, etc.

The angular velocities of the isospin rotations $\vec{\omega}$ are defined in the standard way [61]: $\vec{D} \dot{\vec{A}} = -i\vec{\omega} \vec{r}/2$. We shall not consider here the usual space rotations in detail because the corresponding momenta of inertia for baryonic systems $(BS)$ are much greater than the isospin momenta of inertia, and for the lowest possible values of angular momentum $J$, the corresponding quantum correction is either exactly zero (for even $B$), or small.

The field $D$ is small in magnitude. In fact, it is, at least, of order $1/\sqrt{N_c}$, where $N_c$ is the number of colors in $QCD$, see Eq. (22). Therefore, the expansion of the matrix $S$ in $D$ can be made safely.

The mass term of the lagrangian (1) can be calculated exactly, without expansion in the powers of the field $D$, because the matrix $S$ is given by [44] $S = 1 + iD \sin d/d - D^2(1 - \cos d)/d^2$ with $TrD^2 = 2d^2$. We find that

$$\Delta \mathcal{L}_M = -\frac{F_0^2 m_D^2 - F_\pi^2 m_\pi^2}{4} (1 - c_f) s_D^2$$ \tag{15}

The expansion of this term can be done easily up to any order in $d$. The comparison of this expression with $\Delta \mathcal{L}_M$ within the collective coordinate approach of the quantization of $SU(2)$ solitons in the $SU(3)$ configuration space [61, 23], allows us to establish the relation $\sin^2 d = \sin^2 \nu$, where $\nu$ is the angle of the $\lambda_4$ rotation, or the rotation into the “strange” (“charm”, “beauty”) direction. After some calculations we find that the Lagrangian of the model, to the lowest order in the field $D$, can be written as

$$L = -M_{d,B} + 4\Theta_F B \dot{D} \dot{D} - \left[ \Gamma_B \left( \frac{F_0^2}{F_\pi^2} m_D^2 - m_\pi^2 \right) + \tilde{\Gamma}_B (F_0^2 - F_\pi^2) \right] \dot{D} \dot{D} - \frac{i N_c B}{2} (\dot{D} \dot{D} - \dot{D} \dot{D}).$$ \tag{16}

Here and below $D$ is the doublet $K^+, K^0$ ($D^0$, $D^-$, or $B^+, B^0$): $d^2 = TrD^2/2 = 2D^2$. We have kept the standard notation for the moment of inertia of the rotation into the “flavor” direction $\Theta_F$ for $\Theta_c$, $\Theta_b$ or $\Theta_s$ [61, 60]; different notations are used in [44] (the index $c$ denotes the charm quantum number, except in $N_c$). The contribution proportional to $\Gamma_B$ is suppressed in comparison with the term $\sim \Gamma$ by a small factor $\sim (F_0^2 - F_\pi^2)/m_D^2$, and is more important for strangeness.

The term proportional to $N_c B$ in (1) arises from the Wess-Zumino term in the action and is responsible for the difference of the excitation energies of strangeness and antistrangeness (flavor and antiflavor in the general case) [45, 44].
Following the canonical quantization procedure the Hamiltonian of the system, including the terms of the order of $N_c^0$, takes the form \[44\]:

$$H_B = M_{d,B} + \frac{1}{4\Theta_{F,B}} \sum_i \Pi^i \Pi^i + \left[ \Gamma_B \bar{m}_D^2 + \bar{\Gamma}_B (F_D^2 - F_\pi^2) + \frac{N_c^2 B^2}{16 \Theta_{F,B}} \right] D^i D^i + i \frac{N_c B}{8 \Theta_{F,B}} (D^i \Pi - \Pi^i D),$$  \tag{17}

where $\bar{m}_D^2 = (F_D^2/F_\pi^2)m_D^2 - m_\pi^2$. The momentum $\Pi$ is canonically conjugate to variable $D$ (see Eq.(18) below). Eq. (17) describes an oscillator-type motion of the field $D$ in the background formed by the $(u,d) SU(2)$ soliton. After the diagonalization, which can be done explicitly following [44], the normal-ordered Hamiltonian can be written as

$$H_B = M_{d,B} + \omega_{F,B} a^i a + \bar{\omega}_{F,B} b^i b + O(1/N_c) \tag{18}$$

with $a^i, b^i$ being the operators of creation of strangeness (i.e., antikaons) and antistrangeness (flavor and antiflavor) quantum number, $\omega_{F,B}$ and $\bar{\omega}_{F,B}$ being the frequencies of flavor (antiflavor) excitations. $D$ and $\Pi$ are connected with $a$ and $b$ in the following way [44]:

$$D^i = \frac{1}{\sqrt{N_c B \mu_{F,B}}} (b^i + a^i), \quad \Pi^i = \frac{\sqrt{N_c B \mu_{F,B}}}{2i} (b^i - a^i) \tag{19}$$

with

$$\mu_{F,B} = [1 + 16 (F_D^2/F_\pi^2) \bar{\Gamma}_B] \Theta_{F,B} / (N_c B)^2]^{1/2}. \tag{20}$$

$\mu_{F,B}$ is slowly varying quantity, it simplifies for large mass $m_D$:

$$\mu_{F,B} \rightarrow 4 \sqrt{\frac{\Gamma_B \Theta_{F,B}}{N_c B}}. \tag{21}$$

Obviously, at large $N_c$, $\mu \sim N_c^0 \sim 1$, and dependence on the $B$-number is also weak, since both $\Gamma_B, \Theta_{F,B} \sim N_c B$ \footnote{Strictly, at large $B$, $\Gamma_B \sim B^{3/2}$ as explained above. But numerically at $B < 30$, $\Gamma_B \sim B$, as can be seen from Tables 1 and 2.}. For the lowest states the values of $D$ are small:

$$|D| \sim [16 \Gamma_B \Theta_{F,B} \bar{m}_D^2 + N_c^2 B^2]^{-1/4}, \tag{22}$$

and increase, with increasing flavor number $|F|$, like $(2|F| + 1)^{1/2}$. As it follows from (22) [44, 43], deviations of the field $D$ from the vacuum decrease with increasing mass $m_D$, as well as with increasing number of colors $N_c$, and this explains why the method works for any $m_D$, including charm and beauty quantum numbers.

The excitation frequencies $\omega$ and $\bar{\omega}$ are:

$$\omega_{F,B} = \frac{N_c B}{8 \Theta_{F,B}} (\mu_{F,B} - 1), \quad \bar{\omega}_{F,B} = \frac{N_c B}{8 \Theta_{F,B}} (\mu_{F,B} + 1). \tag{23}$$

The oscillation time can be estimated as $\tau_{osc} \sim \pi/\omega_{F,B} \sim \pi(\Theta_B/\Gamma_B)^{1/2}/m_D$, so, it decreases with increasing $m_D$. As was observed in [63, 43], the difference $\bar{\omega}_{F,B} - \omega_{F,B} = N_c B/(4 \Theta_{F,B})$ coincides, to the leading order in $N_c$, with the expression obtained in the collective coordinate
approach [61, 60], see Appendix. At large $m_D$ using (21) for the difference $\omega_{F,1} - \omega_{F,B}$ we obtain ($N_c = 3$):

$$\bar{\omega}_{F,1} - \bar{\omega}_{F,B} \simeq \frac{m_D}{2} \left[ \left( \frac{\Gamma_1}{\Theta_{F,1}} \right)^{1/2} - \left( \frac{\Gamma_B}{\Theta_{F,B}} \right)^{1/2} \right] + \frac{3}{8} \left( \frac{B}{\Theta_{F,B}} - \frac{1}{\Theta_{F,1}} \right).$$

(24)

Obviously, at large $m_D$, the first term in (24) dominates and is positive if $\Gamma_1/\Theta_{F,1} \geq \Gamma_B/\Theta_{F,B}$. This is confirmed by looking at Table 1. Note also that the bracket in the first term in (24) does not depend on the parameters of the model if the background $SU(2)$ soliton is calculated in the chirally symmetrical limit: both $\Gamma$ and $\Theta$ scale like $\sim 1/(F_{\pi}e^3)$. In a realistic case when the physical pion mass is included in (1) there is some weak dependence on the parameters of the model.

The $FSB$ in the flavor decay constants, i.e. the fact that $F_K/F_\pi \simeq 1.22$ and $F_D/F_\pi = 2.28^{+14}_{-11}$, should be taken into account. In the Skyrme model this fact leads to the increase of the flavor excitation frequencies which changes the spectra of flavored $(c, b)$ baryons and puts them in a better agreement with the data [40]. It also leads to some changes of the total binding energies of $BS$ [43]. This is partly due to the large contribution of the Skyrme term to the flavor moment of inertia $\Theta_F$. Note, that in [44] the $FSB$ in strangeness decay constant was not taken into account, and this led to underestimation of the strangeness excitation energies. Heavy flavors $(c, b)$ have not been considered in these papers.

The addition of the term $L_6$ into starting lagrangian (1) leads to modification of the flavored moment of inertia, according to simple relation $\Theta_F = \Theta_F^{kin} + \Theta_F^{SK1} + \Theta_F^{SK_8}$. But in order to take into account the symmetry breaking terms adequately, we have to express (in some order of $N_c^{-1}$) first set of coordinates (13) in terms of collective coordinates $A(t)$ and $S(t)$ and substitute into $L_{rot}$.

The terms of the order of $N_c^{-1}$ in the hamiltonian, which depend also on the angular velocities of rotations in the isospin and the usual space, are not crucial but important for the numerical estimates of the spectra of baryonic systems. To calculate them one should first obtain the lagrangian of $BS$ including all the terms upto $O(1/N_c)$. It can be written in a compact form, slightly different from that given in [44], as [42]:

$$L \simeq -M_d + 2\Theta_{F,B} \left[ 2\dot{D}^\dagger \dot{D} \left( 1 - \frac{d^2}{3} \right) - \frac{4}{3} (D^\dagger \dot{D} \dot{D}^\dagger D - (D^\dagger D)^2 - (\dot{D}^\dagger D)^2) + (\bar{\omega} \bar{\beta}) \right] +$$

$$+ \frac{\Theta_{L,B}}{2} (\bar{\omega} - \bar{\beta})^2 - \left[ \Gamma_B m_D^2 + (F_B^2 - F_\pi^2) \tilde{\Gamma}_B \right] D^\dagger D \left( 1 - \frac{d^2}{3} \right) +$$

$$+ \frac{i}{2} \frac{N_c B}{2} \left( \dot{D}^\dagger D - D^\dagger \dot{D} \right) - \frac{N_c B}{2} \bar{\omega} D^\dagger \bar{\tau} D,$$

(25)

where $d^2 = 2D^\dagger D$ and $\bar{\beta} = -i(\dot{D}^\dagger \bar{\tau} D - D^\dagger \bar{\tau} \dot{D})$.

(26)

As we mentioned already, the role of the term $L_6$ reduces to the modification of the flavored inertia $\Theta_F$ in (25). It is remarkable property of the starting lagrangian including $L_6$, that only quadratic terms in $\Omega_a$ enter (25). To get this expression we used the connection between components $\Omega_a$ and $D, \dot{D}, \omega_i$: $\Omega_1^2 + ... + \Omega_7^2 = 8\dot{D}^\dagger \dot{D}(1 - d^2/3) - 16(D^\dagger \dot{D} \dot{D}^\dagger D - (D^\dagger D)^2 - (\dot{D}^\dagger D)^2)/3 + 4(\bar{\omega} \bar{\beta})$, and the component $\Omega_8$ which defines the WZW term contribution, $\Omega_8 = \sqrt{3} \left[ i(1 - d^2/3)(D^\dagger \dot{D} - \dot{D}^\dagger D) + \bar{\omega} D^\dagger \bar{\tau} D \right]$. 


Taking into account the terms $\sim 1/N_c$ we find that the canonical variable $\Pi$ conjugate to $D$ is

$$
\Pi = \frac{\partial L}{\partial \dot{D}} = 4\Theta_{F,B} \left[ \dot{D} \left( 1 - \frac{d^2}{3} \right) - \frac{2}{3} \dot{D} \tilde{D} - \frac{4}{3} \dot{D} \tilde{D} \right] + i(\Theta_{I,B} - 2\Theta_{F,B})\dot{\overline{\tau}}D - i\Theta_{I,B}\tilde{\beta}\overline{\tau}D + i\frac{N_c B}{2} \left( 1 - \frac{d^2}{3} \right) D. \tag{27}
$$

From (25) the body-fixed isospin operator is:

$$
\hat{I}^I_F = \partial L/\partial \overline{\omega} = \Theta_{I,B}\overline{\omega} + (2\Theta_{F,B} - \Theta_{I,B})\tilde{\beta} - \frac{N_c B}{2} D\tilde{\tau}D, \tag{28}
$$

which can be written also as:

$$
\hat{I}^I_F = \Theta_{I}\overline{\omega} + (1 - \Theta_{I}) \hat{I}^I_F - \frac{N_c B \Theta_{I}}{4\Theta_F} D\tilde{\tau}D, \tag{29}
$$

with the operator

$$
\hat{I}^I_F = \frac{i}{2} (D\tilde{\tau}\Pi - \Pi\tilde{\tau}D) = (b^T \overline{\tau} a + a^T \overline{\tau} b)/2. \tag{30}
$$

Using connection between $\Pi$, $\dot{D}$ and $D$ given by (27) in leading order we obtain for the quantity $\beta$ in (26)

$$
\tilde{\beta} \simeq \frac{1}{2\Theta_F} \left( \hat{I}^I_F + \frac{N_c B}{2} D\tilde{\tau}D \right) \tag{31}
$$

For the states with definite flavor quantum number we should make substitution $D\tilde{\tau}D \to -2\hat{I}^I_F/(N_c B \mu_F)$ (for flavor) or $D\tilde{\tau}D \to 2\hat{I}^I_F/(N_c B \mu_F)$ for antiflavor, and we can write for matrix elements of states with definite flavor:

$$
\tilde{I}^I_F = \Theta_{I,B}\overline{\omega} + c_{F,B}\hat{I}^I_F, \tag{32}
$$

with

$$
c_{F,B} = 1 - \frac{\Theta_{I,B}}{2\Theta_{F,B}\mu_{F,B}}(\mu_{F,B} - 1). \tag{33}
$$

We used also that within our approximation

$$
\Theta_{I,B}\tilde{\beta} \simeq (1 - c_{F,B})\hat{I}^I_F. \tag{34}
$$

Relation, similar to (32) holds also for antiflavor with

$$
c_{F,B} = 1 - \frac{\Theta_{I,B}}{2\Theta_{F,B}\mu_{F,B}}(\mu_{F,B} + 1), \tag{35}
$$

so, it differs from (33) by change $\mu \to -\mu$. Using the identities:

$$
-i\tilde{\beta}\overline{\tau}D = 2D\tilde{\tau}D - (D\tilde{\tau}D + D\tilde{\tau}D)D \tag{36}
$$

and

$$
\tilde{\beta}^2 = 4D\tilde{\tau}D\tilde{\tau}D - (D\tilde{\tau}D + D\tilde{\tau}D)^2, \tag{37}
$$
we find that the $\sim 1/N_c$ zero mode quantum corrections to the energies of skyrmions can be estimated \cite{44} as:

$$
\Delta E_{1/N_c} = \frac{1}{2\Theta_{I,B}} [c_{F,B} I_r (I_r + 1) + (1 - c_{F,B}) I (I + 1) + (\bar{c}_{F,B} - c_{F,B}) I_F (I_F + 1)],
$$

(38)

where $I = I^{bf}$ is the value of the isospin of the baryon or BS. $I_r$ is the quantity analogous to the “right” isospin $I_r$, in the collective coordinate approach \cite{61}, and $\bar{I}_r = \bar{I}_r^0 - I_F$. The hyperfine structure constants $c_{F,B}$ are given in (33), and $\bar{c}_{F,B}$ are defined by relations:

$$
1 - \bar{c}_{F,B} = \frac{\Theta_{I,B}}{\Theta_{F,B}(\mu_{F,B})^2} (\mu_{F,B} - 1), \quad 1 - c_{F,B} = -\frac{\Theta_{I,B}}{\Theta_{F,B}(\mu_{F,B})^2} (\mu_{F,B} + 1).
$$

(39)

For nucleons $I = I_r = 1/2$, $I_F = 0$ and $\Delta E_{1/N_c}(N) = 3/(8\Theta_{I,1})$, for $\Delta$-isobar $I = I_r = 3/2$, $I_F = 0$, and $\Delta E_{1/N_c}(\Delta) = 15/(8\Theta_{I,1})$, as in $SU(2)$ quantization scheme. The $\Delta - N$ mass splitting is described satisfactorily, according to values of $\Theta_I$ presented in Table 1.

| $B$ | $\omega^{SK_4}$ | $\omega_{c}^{SK_4}$ | $\omega_{b}^{SK_4}$ | $\omega_{s}^{SK_6}$ | $\omega_{c}^{SK_6}$ | $\omega_{b}^{SK_6}$ | $\omega_{s}^{SK_6}$ | $\omega_{c}^{SK_6}$ | $\omega_{b}^{SK_6}$ | $\omega_{c}^{SK_6}$ | $\omega_{b}^{SK_6}$ |
|-----|----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1   | 0.307          | 1.54            | 4.80           | 0.336          | 1.61           | 4.93           | 0.345          | 1.55           | 4.77           | 0.375          | 1.62           | 4.89           |
| 2   | 0.298          | 1.52            | 4.77           | 0.336          | 1.64           | 4.98           | 0.339          | 1.54           | 4.75           | 0.386          | 1.66           | 4.95           |
| 3   | 0.293          | 1.51            | 4.76           | 0.342          | 1.64           | 4.98           | 0.336          | 1.54           | 4.74           | 0.385          | 1.66           | 4.95           |
| 4   | 0.285          | 1.50            | 4.74           | 0.328          | 1.62           | 4.95           | 0.330          | 1.52           | 4.72           | 0.377          | 1.64           | 4.93           |
| 5   | 0.290          | 1.51            | 4.75           | 0.334          | 1.63           | 4.96           | 0.334          | 1.53           | 4.74           | 0.380          | 1.65           | 4.94           |
| 6   | 0.290          | 1.51            | 4.76           | 0.332          | 1.63           | 4.96           | 0.334          | 1.54           | 4.74           | 0.379          | 1.65           | 4.94           |
| 7   | 0.285          | 1.50            | 4.74           | 0.324          | 1.62           | 4.95           | 0.331          | 1.53           | 4.73           | 0.374          | 1.64           | 4.93           |
| 8   | 0.290          | 1.51            | 4.76           | 0.329          | 1.63           | 4.96           | 0.335          | 1.54           | 4.75           | 0.377          | 1.65           | 4.94           |
| 9   | 0.292          | 1.52            | 4.77           | 0.331          | 1.63           | 4.97           | 0.336          | 1.54           | 4.76           | 0.378          | 1.65           | 4.94           |
| 10  | 0.293          | 1.52            | 4.78           | 0.331          | 1.63           | 4.97           | 0.337          | 1.55           | 4.76           | 0.378          | 1.65           | 4.94           |
| 11  | 0.295          | 1.53            | 4.79           | 0.332          | 1.63           | 4.97           | 0.338          | 1.55           | 4.77           | 0.378          | 1.65           | 4.95           |
| 12  | 0.295          | 1.53            | 4.79           | 0.331          | 1.63           | 4.97           | 0.338          | 1.55           | 4.77           | 0.378          | 1.65           | 4.95           |
| 13  | 0.296          | 1.53            | 4.79           | 0.332          | 1.63           | 4.98           | 0.339          | 1.55           | 4.77           | 0.378          | 1.65           | 4.95           |
| 14  | 0.300          | 1.54            | 4.80           | 0.335          | 1.64           | 4.98           | 0.342          | 1.56           | 4.79           | 0.379          | 1.65           | 4.95           |
| 15  | 0.301          | 1.54            | 4.81           | 0.336          | 1.64           | 4.99           | 0.343          | 1.56           | 4.79           | 0.380          | 1.66           | 4.95           |
| 16  | 0.302          | 1.54            | 4.81           | 0.336          | 1.64           | 4.99           | 0.343          | 1.56           | 4.79           | 0.380          | 1.66           | 4.96           |
| 17  | 0.302          | 1.54            | 4.81           | 0.335          | 1.64           | 4.99           | 0.343          | 1.56           | 4.79           | 0.379          | 1.66           | 4.95           |
| 20  | 0.308          | 1.56            | 4.84           | 0.340          | 1.65           | 5.00           | 0.347          | 1.58           | 4.81           | 0.382          | 1.66           | 4.96           |
| 24  | 0.312          | 1.57            | 4.85           | 0.343          | 1.66           | 5.01           | 0.351          | 1.58           | 4.83           | 0.384          | 1.66           | 4.97           |
| 28  | 0.316          | 1.58            | 4.87           | 0.347          | 1.66           | 5.02           | 0.354          | 1.59           | 4.85           | 0.385          | 1.67           | 4.98           |
| 32  | 0.319          | 1.59            | 4.88           | 0.349          | 1.67           | 5.02           | 0.356          | 1.60           | 4.86           | 0.386          | 1.67           | 4.98           |

**Table 3.** Flavor excitation energies for strangeness, charm and beauty, in Gev. $e = 4.12$ for the $SK_4$ variant and $e' = 4.11$ for the $SK_6$ model. For rescaled variants (numbers marked with *) $e = 3.00$ and $e' = 2.84$ for $SK_4$ and $SK_6$ variants, correspondingly. The ratio $F_D/F_\pi = 1.5$ for charm, and $F_B/F_\pi = 2$ for beauty.

As can be seen from Table 3, for the $SK_4$ variant there is some decrease of flavor excitation energies when $B$-number increases from 1 to 7, but further these energies increase
beauty excitation energies are very close to those of original variant (scaling property), but derived in [63, 66] for the binding energies of strange $S = -1$ hypernuclei are based mainly on the differences of these energies. The qualitative and in some cases quantitative agreement takes place between data for binding energies of ground states of hypernuclei with atomic numbers between 5 and 20 and results of calculations within $SK4$ variant of the chiral soliton model taking into account collective motion of solitons in $SU(3)$ configuration space [66].

Another peculiarity of interest is that for rescaled variant of the model the charm and beauty excitation energies are very close to those of original variant (scaling property), but differ more substantially for strangeness - greater by $\sim 30 - 40$ Mev. Such somewhat unexpected behaviour is connected with the fact that flavor excitation energies appear as a result of subtraction of two quantities, which behave differently when rescaling is made, see (23).

\begin{table}[h]
\centering
\begin{tabular}{|c|cccccccccc|}
\hline
$B$ & $\bar{\omega}^{SK4}_b$ & $\bar{\omega}^{SK4}_c$ & $\bar{\omega}^{SK6}_b$ & $\bar{\omega}^{SK6}_c$ & $\bar{\omega}^{SK4}_s$ & $\bar{\omega}^{SK4}_c$ & $\bar{\omega}^{SK6}_b$ & $\bar{\omega}^{SK6}_c$ & $\bar{\omega}^{SK4}_b$ & $\bar{\omega}^{SK6}_b$ & $\bar{\omega}^{SK4}_c$ & $\bar{\omega}^{SK6}_c$ \\
\hline
1   & 0.591 & 1.75 & 4.94 & 0.584 & 1.79 & 5.04 & 0.472 & 1.65 & 4.83 & 0.468 & 1.69 & 4.93 \\
2   & 0.571 & 1.72 & 4.90 & 0.571 & 1.80 & 5.08 & 0.459 & 1.63 & 4.81 & 0.470 & 1.72 & 4.99 \\
3   & 0.564 & 1.71 & 4.89 & 0.569 & 1.80 & 5.07 & 0.455 & 1.63 & 4.80 & 0.468 & 1.72 & 4.99 \\
4   & 0.567 & 1.71 & 4.87 & 0.580 & 1.80 & 5.06 & 0.454 & 1.62 & 4.78 & 0.468 & 1.71 & 4.97 \\
5   & 0.558 & 1.71 & 4.88 & 0.571 & 1.80 & 5.07 & 0.452 & 1.62 & 4.80 & 0.466 & 1.71 & 4.98 \\
6   & 0.555 & 1.71 & 4.88 & 0.571 & 1.80 & 5.07 & 0.451 & 1.62 & 4.80 & 0.465 & 1.71 & 4.98 \\
7   & 0.559 & 1.71 & 4.88 & 0.578 & 1.80 & 5.06 & 0.451 & 1.62 & 4.79 & 0.466 & 1.71 & 4.97 \\
8   & 0.553 & 1.71 & 4.89 & 0.571 & 1.80 & 5.07 & 0.450 & 1.63 & 4.80 & 0.465 & 1.71 & 4.98 \\
9   & 0.550 & 1.71 & 4.90 & 0.569 & 1.80 & 5.07 & 0.450 & 1.63 & 4.81 & 0.465 & 1.71 & 4.98 \\
10  & 0.549 & 1.71 & 4.90 & 0.569 & 1.80 & 5.07 & 0.450 & 1.63 & 4.82 & 0.465 & 1.71 & 4.98 \\
11  & 0.547 & 1.71 & 4.90 & 0.567 & 1.80 & 5.08 & 0.450 & 1.63 & 4.82 & 0.464 & 1.71 & 4.98 \\
12  & 0.547 & 1.72 & 4.91 & 0.568 & 1.80 & 5.08 & 0.450 & 1.63 & 4.82 & 0.464 & 1.71 & 4.98 \\
13  & 0.546 & 1.72 & 4.91 & 0.567 & 1.80 & 5.08 & 0.450 & 1.64 & 4.83 & 0.464 & 1.71 & 4.99 \\
14  & 0.543 & 1.72 & 4.92 & 0.564 & 1.80 & 5.08 & 0.450 & 1.64 & 4.84 & 0.464 & 1.71 & 4.99 \\
15  & 0.542 & 1.72 & 4.92 & 0.563 & 1.80 & 5.08 & 0.450 & 1.64 & 4.84 & 0.464 & 1.72 & 4.99 \\
16  & 0.541 & 1.72 & 4.93 & 0.562 & 1.80 & 5.08 & 0.450 & 1.64 & 4.85 & 0.464 & 1.72 & 4.99 \\
17  & 0.542 & 1.72 & 4.93 & 0.564 & 1.80 & 5.09 & 0.450 & 1.64 & 4.85 & 0.464 & 1.72 & 4.99 \\
18  & 0.540 & 1.72 & 4.93 & 0.561 & 1.81 & 5.09 & 0.451 & 1.65 & 4.85 & 0.464 & 1.72 & 5.00 \\
19  & 0.539 & 1.73 & 4.94 & 0.559 & 1.81 & 5.09 & 0.451 & 1.65 & 4.86 & 0.464 & 1.72 & 5.00 \\
20  & 0.538 & 1.73 & 4.94 & 0.558 & 1.81 & 5.09 & 0.451 & 1.65 & 4.86 & 0.464 & 1.72 & 5.00 \\
21  & 0.536 & 1.73 & 4.96 & 0.555 & 1.81 & 5.10 & 0.452 & 1.66 & 4.88 & 0.463 & 1.72 & 5.00 \\
22  & 0.533 & 1.74 & 4.97 & 0.552 & 1.81 & 5.10 & 0.453 & 1.67 & 4.89 & 0.463 & 1.72 & 5.01 \\
23  & 0.532 & 1.74 & 4.98 & 0.550 & 1.81 & 5.11 & 0.453 & 1.67 & 4.90 & 0.463 & 1.73 & 5.01 \\
\hline
\end{tabular}
\caption{Antiflavor excitation energies for strangeness, charm and beauty, as in Table 3. In original variants of the model $e = 4.12$ for the $SK4$ variant and $e^* = 4.11$ for the $SK6$ variant.}
\end{table}
The numbers with ∗ are for the rescaled variants of the model, e = 3.0 for the SK4 variant and 

e′ = 2.84 for the SK6 variant. The ratio \( F_D/F_\pi = 1.5 \) for charm and \( F_B/F_\pi = 2 \) for beauty.

Similar to flavor energies, there is remarkable universality of antiflavor excitation energies 
for different baryon numbers, especially for anti-charm and anti-beauty: variations do not 
exceed few %. It follows from Table 4 that there is some decrease of antiflavor excitation 
energies when \( B \) increases from 1, this effect is striking for the SK4 variant and especially 
for strangeness. Within the SK6 variant the \( B = 1 \) energies for anti-charm and anti-beauty 
are slightly smaller than for \( B \geq 2 \).

For the case of strangeness, the \( \bar{\omega}_s \) decreases with increasing B-number in most cases, 
as it can be seen from Table 4 (except the rescaled SK6 variant where \( B = 1 \) energy is 
slightly smaller than \( B = 2 \) one), but it is always greater than kaon mass, therefore, the state 
with positive strangeness can decay strongly into kaon and some final nucleus, or nuclear 
fragments.

The heavy antiflavors excitation energies also reveal notable scale independence, i.e. the 
values obtained with constant \( e = 4.12 \) and 3.00 (SK4 variant) shown in Tables 3,4, are 
close to each other within several percents, as well as values for \( e′ = 4.11 \) and 2.84 for 
the SK6 variant. It was really expected from general arguments for large values of \( F_{SB} \) 
meson mass [43]. The change of numerical values of these energies is, however, important 
for conclusions concerning the binding energies of nuclear states with antiflavors.

All excitation energies of antiflavors are smaller for rescaled variants, i.e. when constants 
\( e \) or \( e′ \) are decreased by about 30%. It seems to be natural since dimensions of multiskyrmions 
which scale like \( 1/(F_\pi e) \) increase due to this change, and all energies become ”softer”. Such 
behaviour is due to the fact that antiflavor energies are the sum of two terms (see above 
(23)) which behave (roughly!) in similar manner when rescaling is made. Remarkably, the 
decrease of energies due to rescaling is of the order of \( \sim 100 \text{ Mev} \) in all cases (e.g., for 
anti-strangeness and \( B = 1 \) it is 119 Mev (SK4 variant) and 116 Mev in SK6 variant), 
and slightly smaller for \( \bar{c} \) (decrease due to rescaling about \( \sim 100 \text{ Mev} \)) and \( \bar{b} \) (decrease by 
110 Mev).

4 The binding energies of \( \Theta^+ \)-hyperfine and anti-charmed (anti-beautiful) hypernuclei

In view of the large enough values of anti-strangeness excitation energies one cannot speak 
about positive strangeness hypernuclei, which decay weakly, similar to ordinary \( S = -1 \) hy-

pernuclei. However, one can speak about \( \Theta \)-hyperfine, where \( \Theta \)-hyperon is bound by sev-

eral nucleons. One of puzzling properties of pentaquarks is their small width, \( \Gamma_\Theta << 10 \text{ Mev} \) 
according to experiments where \( \Theta^+ \) has been observed [1, 2], and probably even smaller, 
according to analyses of kaon-nucleon interaction data [67]. Possible explanations, from some 
numerical cancellation [24] and cancellation in large \( N_c \) expansion [68] to qualitative one in 
terms of the quark model wave function [3, 4], and calculation using operator product 
expansion [69] have been proposed. The width of nuclear bound states of \( \Theta \) should be of 
same order of magnitude as the width of \( \Theta^+ \) itself, or smaller: besides the smaller energy 
release some suppression due to the Pauli blocking for the final nucleon from \( \Theta \) decay can 
take place for heavier nuclei.

For anti-charm and anti-beauty the excitation energies are smaller than masses of \( D \)- 
or \( B^- \) meson, and it makes sense to consider the possibility of existence of anti-charmed or

\[5\text{In most of variants of explanation it is difficult to expect the width of } \Theta \text{-hyperon of the order of } \sim 1 \text{ Mev, as obtained in [67]. Therefore, the checking of data analyzed in [67] seems to be of first priority.}\]
anti-beauty hypernuclei which have the time of life characteristic for weak interactions.

In the bound state soliton model, and in its rigid oscillator version as well, the states predicted do not correspond apriory to the definite \(SU(3)\) or \(SU(4)\) representations. They can be ascribed to definite \(SU(3)\) irreps, as it was shown in [44, 43]. Due to configuration mixing caused by the flavor symmetry breaking, each state with definite value of flavor, \(s\), \(c\) or \(b\), is some mixture of the components of several \(SU(3)\) irreps with given value of \(F\) and isospin \(I\) which is strictly conserved in our approach (until explicitly isospin violating terms are included into lagrangian). In the case of strangeness, as calculations show (see e.g. [27]), this mixture is dominated usually by the lowest \(SU(3)\) irrep, and admixtures do not exceed several percents, usually. Situation is changed for charm or beauty quantum numbers, where admixtures can have weight comparable with the weight of the lowest configuration. However, we consider here the simplest possibility of one lowest irrep, for rough estimates.

Let \((p, q)\) characterize the \(SU(3)\) irrep to which \(BS\) belongs, then quantization condition \((p + 2q) = N_c B\) [61], for arbitrary \(N_c\), changes to \((p + 2q) = N_c B + 3m\), where \(m\) is related to the number of additional quark-antiquark pairs \(n_{q\bar{q}}\) present in the quantized states, \(n_{q\bar{q}} \geq m\) [22, 70]. The non-exotic states with \(m = 0\), or minimal states, have \(p + 2q = 3B\), \((N_c = 3\) further), and states with lowest ”right” isospin \(I_r = p/2\) have \((p, q) = (0, 3B/2)\) for even \(B\), and \((p, q) = (1, (3B - 1)/2)\) for odd \(B\) [22, 27]. E.g., the state with \(B = 1\), \(|F| = 1\), \(I = 0\) and \(n_{q\bar{q}} = 0\) should belong to the octet of \((u, d, s)\), or \((u, d, c)\), \(SU(3)\) group, if \(N_c = 3\); see also [44]. For the first exotic states the lowest possible \(SU(3)\) irreps \((p, q)\) for each value of the baryon number \(B\) are defined by relations: \(p + 2q = 3(B + 1); p = 1, q = (3B + 2)/2\) for even \(B\), and \(p = 0, q = (3B + 3)/2\) for odd \(B\). E.g., for \(B = 2, 4, 6\) and \(8\) we have \(35, 3\bar{0}, \bar{1}43\) and \(224\)-plets, and for \(B = 3, 5\) and \(7\) - multiplets \(28, 55\) and \(91\), etc.

Since we are interested in the lowest energy states, we discuss here the baryonic systems with the lowest allowed angular momentum, ie \(J = 0\), for \(B = 4, 6\) etc. and \(J = 1/2\) for odd values of the \(B\)-number. There are some deviations from this simple law for the ground states of real nuclei, but anyway, the correction to the energy of quantized states due to collective rotation of solitons is small and decreases with increasing \(B\) since the corresponding moment of inertia increases proportionally to \(\sim B^2\) [63, 64]. Moreover, the \(J\)-dependent correction to the energy may cancel in the differences of energies of flavored and flavorless states, so, we neglect these contributions for our rough estimates.

For the non-exotic states we considered previously [66] the energy difference between the state with flavor \(F\) belonging to the \((p, q)\) irrep, and the ground state with \(F = 0\) and the same angular momentum and \((p, q)\). Situation is different for exotic states, since exotic and non-exotic states have different values of \((p, q)\). The difference between \(\bar{\omega} = \omega - \omega = N_c/(4\Theta_F)\) takes into account this distinction in the values of \((p, q)\), as it is shown explicitly in Appendix.

For the case of \(B = 1, 3, 5,...\) etc. we have for the ground state \(I = I_r = 1/2, I_F = 0\), therefore, the correction \(\Delta E_{1/N_c} = I(I + 1)/(2\Theta_{I,B}) = 3/(8\Theta_{I,B})\). For exotic antiflavored state we have \(I = 0, I_r = I_F = 1/2\), and correction equals to \(\Delta E_{1/N_c} = 3\bar{c}_{F,B}/8\Theta_{I,B}\). For the difference of energies between exotic and non-exotic ground states we obtain:

\[
\Delta E_{B,F} = \bar{\omega}_{F,B} + \frac{3(\bar{c}_{F,B} - 1)}{8\Theta_{F,B}} = \bar{\omega}_{F,B} + \frac{3(\mu_{F,B} + 1)}{8\mu_{F,B}^2\Theta_{F,B}}, \tag{40}
\]

Note that the moment of inertia \(\Theta_I\) does not enter the difference of energies (40).

For the \(B = 1\) case, the difference of masses of \(\Theta_F\) and the nucleon is

\[
\Delta M_{\Theta_{F,N}} = \omega_{F,1} - \frac{3(1 - \bar{c}_{F,1})}{8\Theta_{I,1}} = \omega_{F,1} + \frac{3(\mu_{F,1} + 1)}{8\mu_{F,1}^2\Theta_{F,1}}, \tag{41}
\]
The difference of masses of $\Theta$ and $\Lambda$ hyperons also is of interest and can be presented in simple form:

$$
\Delta M_{\Theta \Lambda} = \omega_{F,1} - \omega_{I,1} + \frac{3(c_{F,1} - \bar{c}_{F,1})}{8\Theta_{I,1}} = \frac{3(\mu_{F,1} + 1)}{4\mu_{F,1}} \Theta_{I,1}.
$$

(42)

For the case of even $B = 4, 6, \ldots$ etc. the ground state has $I = I_r = I_F = 0$ (like for nucleus $^4He$), and $\Delta E_{1/N_c} = 0$. For the first exotic states $I = I_F = 1/2$, and we have a choice for $I_r, I_r = 0$ or 1. If $c_{F,B} = 1 - \Theta_{I,B}(\mu_{F,B} + 1)/(2\Theta_{F,B}\mu_{F,B}) > 0$, $I_r = 0$; if $c_{F,B} < 0$, we should take $I_r = 1$. In the first case, the correction to the energy of the state $\Delta E_{1/N_c} = 3(1 + c_{F,B} - 2c_{F,B})/8\Theta_{I,B} = 3(\mu_{F,B} + 1)^2/(8\Theta_{F,B}\mu_{F,B}^2)$.

| $B$ | $\Delta e^{SK_4}$ | $\epsilon^{SK_4}$ | $\Delta e^{SK_6}$ | $\epsilon^{SK_6}$ | $\Delta e^{SK_4}$ | $\epsilon^{SK_4}$ | $\Delta e^{SK_6}$ | $\epsilon^{SK_6}$ |
|-----|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 2   | 47              | 47              | 75              | 75              | 25              | 25              | 17              | 17              |
| 3   | 67              | 76              | 45              | 45              | 26              | 26              | 12              | 12              |
| 4   | 20              | 49              | -4              | 24              | 9               | 9               | -8              | 20              |
| 5   | 81              | 108             | 47              | 74              | 30              | 57              | 6               | 33              |
| 6   | 56              | 88              | 24              | 56              | 20              | 52              | -1              | 31              |
| 7   | 83              | 121             | 41              | 80              | 32              | 70              | 7               | 45              |
| 8   | 69              | 126             | 31              | 87              | 24              | 81              | 2               | 58              |
| 9   | 94              | 152             | 53              | 110             | 33              | 90              | 8               | 66              |
| 10  | 79              | 144             | 39              | 103             | 27              | 92              | 4               | 68              |
| 11  | 99              | 173             | 56              | 130             | 33              | 108             | 9               | 84              |
| 12  | 86              | 178             | 43              | 135             | 28              | 120             | 5               | 97              |
| 13  | 101             | 196             | 56              | 152             | 33              | 129             | 9               | 104             |
| 14  | 93              | 197             | 50              | 154             | 29              | 133             | 6               | 111             |
| 15  | 105             | 219             | 61              | 175             | 33              | 147             | 9               | 123             |
| 16  | 96              | 224             | 53              | 181             | 29              | 157             | 7               | 134             |
| 17  | 105             | 235             | 61              | 191             | 33              | 163             | 9               | 139             |
| 18  | 100             | 237             | 56              | 194             | 29              | 167             | 7               | 144             |
| 19  | 109             | 255             | 65              | 211             | 33              | 178             | 10              | 156             |
| 20  | 103             | 263             | 60              | 220             | 29              | 190             | 8               | 168             |
| 21  | 111             | 276             | 67              | 232             | 32              | 197             | 10              | 175             |
| 22  | 105             | 279             | 62              | 236             | 29              | 203             | 8               | 182             |
| 23  | 113             | 297             | 69              | 253             | 32              | 216             | 10              | 194             |
| 24  | 107             | 305             | 64              | 263             | 29              | 228             | 8               | 206             |
| 25  | 113             | 316             | 70              | 273             | 31              | 235             | 10              | 213             |
| 26  | 109             | 321             | 66              | 278             | 29              | 241             | 8               | 220             |
| 27  | 115             | 337             | 72              | 294             | 31              | 253             | 10              | 232             |
| 28  | 111             | 347             | 69              | 305             | 29              | 265             | 9               | 245             |
| 29  | 116             | 358             | 73              | 315             | 31              | 273             | 10              | 252             |
| 30  | 112             | 363             | 70              | 321             | 29              | 279             | 9               | 259             |
| 31  | 117             | 376             | 75              | 335             | 30              | 290             | 10              | 270             |
| 32  | 113             | 385             | 71              | 343             | 29              | 300             | 9               | 281             |

Table 5. The binding energy differences and total binding energies of positive strangeness $\Theta^+$- hypernuclei (in Mev) for the SK4 and SK6 variants of the model in rational map approximation.
The binding energy differences $\Delta \varepsilon_{s,c,b}$ are the changes of binding energies of lowest $BS$ with flavor $s$, $c$ or $b$ and isospin $I = 0$ (for odd $B$) and $I = 1/2$ (for even $B$) in comparison with usual $u,d$ nuclei (when one nucleon is replaced by $\Theta$-hyperon - in other words). The classical masses of skyrmions are cancelled in such differences:

$$\Delta \varepsilon_{B,F} = \Delta E_{\text{gr.st.}}(B) - \Delta E(B,F) + \Delta M_{\Theta_{F,N}}. \quad (43)$$

It follows from (40) that this change of the binding energy of the system is, for odd $B$-number

$$\Delta \varepsilon_{B,F} = \bar{\omega}_{F,1} - \bar{\omega}_{F,B} + \frac{3(\mu_{F,1} + 1)}{8\mu_{F,1}^2 \Theta_{F,1}} - \frac{3(\mu_{F,B} + 1)}{8\mu_{F,B}^2 \Theta_{F,B}}. \quad (44)$$

Evidently, in the limit of very heavy flavor, $\mu_F \to \infty$,

$$\Delta \varepsilon_{B,F} \to \bar{\omega}_{F,1} - \bar{\omega}_{F,B}. \quad (45)$$

For $B$-numbers 4, 6, ... etc. we obtain

$$\Delta \varepsilon_{B,F} = \bar{\omega}_{F,1} - \bar{\omega}_{F,B} + \frac{3(\mu_{F,1} + 1)}{8\mu_{F,1}^2 \Theta_{F,1}} - \frac{3(\mu_{F,B} + 1)^2}{8\mu_{F,B}^2 \Theta_{F,B}}. \quad (46)$$

In the limit of very heavy flavor we have from (46)

$$\Delta \varepsilon_{B,F} = \bar{\omega}_{F,1} - \bar{\omega}_{F,B} - \frac{3}{8\Theta_{F,B}}, \quad (47)$$

so, in comparison with the case of odd $B$-numbers there is additional contribution decreasing with increase of the $B$-number (because inertia increase with $B$) from $\sim 25 \text{ Mev}$ for $B = 3$. Numerical estimates for the total binding energies of anti-flavored states, presented in Tables 5, 6, 7, are obtained by adding the values of (44, 46) to the binding energies of ordinary nuclei we take from existing data.

The case of $B = 2$ is a special one because the equality $I = J = 0$ is forbidden for two-nucleon system in $S$-wave due to the Pauli principle. In Tables 5-7 we present the binding energy estimates relative to $NN$-scattering state (quasideuteron) with isospin $I = 1$. To get them, we added the values $1/\Theta_{I,B=2}$ to the number given by (46).

One should keep in mind that for the $SK4$ model the value of $\Theta^+$ mass equals to $1588 \text{ Mev}$, i.e. by $\sim 150 \text{ Mev}$ above kaon-nucleon threshold. By this reason the states with biggest binding energy shown in Table 5 are unstable relative to strong interactions. For the $SK6$ variant $M_\Theta = 1566 \text{ Mev}$, and the binding energies are considerably smaller, by $\sim 40 - 50 \text{ Mev}$ in some cases - this is the main feature of the $SK6$ variant. For the rescaled variants the difference between both variants is reduced considerably, but in this case the binding energies are underestimated.

From phenomenological point of view we should describe the $B = 1$ states with original variants of models, i.e. $e = 4.12$, $e' = 4.11$ and states with $\sim 10 < B = A <\sim 30$ - using rescaled variants, as it is suggested by results of [65]. This procedure gives most optimistic values of $\Delta \varepsilon_{s=+1}$, about $145 \text{ Mev}$ for the $SK4$ variant and $\sim 140 \text{ Mev}$ for the $SK6$ variant. However, uncertainty of this prediction is few tens of $\text{MeV}$, at least.
For anti-charm and anti-beauty there is considerable difference between $SK4$ and $SK6$ variants (Tables 6,7). The mass of $\Theta_c$ hyperon in the $SK4$ model equals to 2700 $MeV$ and of $\Theta_b$ - to 5880 $MeV$, both well below threshold for strong decay. For the $SK6$ variant these masses are by 40 and 100 $MeV$ greater, but also below threshold. The $SK6$ variant is less attractive than $SK4$ variant, mainly due to the fact that the antiflavor excitation energies for $B = 1$ in the $SK6$ variant are smaller than for $B \geq 2$, and this leads to repulsion for $B > 1$, in comparison with the more familiar $SK4$ model. Considerable decrease of binding energies for large $B$, greater than $B \sim 20$, may be connected with fact that the rational map approximation becomes to be unrealistic for such big baryon numbers. The beauty decay constant $F_b$ is not measured yet, and this introduces additional uncertainty in our predictions. Probably, the value $F_b/F_\pi = 1.8$ is the best one for description of the $\Lambda_b$ mass.

In Table 7 we present the binding energies of hypernuclei with anti-charm and anti-beauty quantum numbers for rescaled $SK4$ and $SK6$ variants of the model.

Several peculiarities should be emphasized. The binding energies for rescaled variants are in general smaller than those for original variants (Table 6), mainly due to the decrease of excitation energies for the $B = 1$ configuration (by $\sim 100$ $MeV$ for the anti-charm and 110 $MeV$ for anti-beauty). For greater $B$-numbers this decrease is smaller. Since, however, rescaled or nuclear variant is valid for large enough baryon numbers, the binding energies can be greater than the values given in both Tables 6, 7, at least for $B$-numbers greater than $\sim 10$. This is similar to the situation with strangeness quantum number (Table 5 and its discussion).

| $B$ | $\Delta \epsilon_{\pi}^{SK4}$ | $\epsilon_{\pi}$ | $\Delta \epsilon_{\pi}^{SK4}$ | $\epsilon_{\pi}$ | $\Delta \epsilon_{\pi}^{SK6}$ | $\epsilon_{\pi}$ | $\Delta \epsilon_{\pi}^{SK6}$ | $\epsilon_{\pi}$ |
|-----|-------------------------------|-----------------|-------------------------------|-----------------|-------------------------------|-----------------|-------------------------------|-----------------|
| 2   | 61                            | 61              | 91                            | 91              | 56                            | 56              | 44                            | 44              |
| 3   | 38                            | 46              | 49                            | 57              | -8                            | 0               | -36                           | -28             |
| 4   | 15                            | 44              | 48                            | 76              | -29                           | -1              | -36                           | -7              |
| 5   | 44                            | 71              | 55                            | 82              | -5                            | 22              | -30                           | -3              |
| 6   | 27                            | 59              | 43                            | 75              | -20                           | 12              | -39                           | -7              |
| 7   | 47                            | 85              | 62                            | 101             | -5                            | 34              | -23                           | 16              |
| 8   | 31                            | 87              | 41                            | 98              | -17                           | 40              | -37                           | 19              |
| 9   | 42                            | 100             | 43                            | 100             | -6                            | 51              | -33                           | 24              |
| 10  | 31                            | 96              | 33                            | 98              | -15                           | 50              | -40                           | 25              |
| 11  | 40                            | 114             | 34                            | 108             | -7                            | 68              | -37                           | 37              |
| 12  | 31                            | 123             | 27                            | 119             | -15                           | 78              | -42                           | 50              |
| 16  | 27                            | 154             | 8                             | 136             | -15                           | 113             | -50                           | 78              |
| 17  | 32                            | 162             | 11                            | 141             | -10                           | 120             | -47                           | 83              |
| 20  | 22                            | 183             | -7                            | 154             | -15                           | 145             | -57                           | 104             |
| 24  | 19                            | 217             | -19                           | 179             | -16                           | 182             | -62                           | 136             |
| 28  | 15                            | 251             | -31                           | 205             | -17                           | 220             | -68                           | 169             |
| 32  | 12                            | 283             | -40                           | 232             | -18                           | 254             | -72                           | 200             |

Table 6. The total binding energies differences and binding energies themselves (in $MeV$) for the anti-flavored states, $SK4$-variant (first 4 columns), and $SK6$ variant (last 4 columns). $F_D/F_\pi = 1.5$, $F_B/F_\pi = 2$. 
The excitation energies of antiflavors are estimated within the bound state version of the chiral soliton model in two different variants of the model, \( SK_4 \) and \( SK_6 \), and for two values of the model parameter (\( \epsilon \) or \( \epsilon' \), see Tables 3, 4). The bounds for expected binding energies of hypernuclei are obtained in this way. These bounds are wide - variations of the total binding energy for the \( SK_4 \) and \( SK_6 \) models can reach 40 – 50 Mev. The difference between original (baryon) variant and rescaled (nuclear) variant is greater for strangeness and smaller for anti-charm and anti-beauty, where it is not greater than \( \sim 20 \) – 30 Mev for baryon numbers between 3 and \( \sim 20 \). If the logic is correct, that rescaled or nuclear variant of the model should be applied for large enough \( B \)-numbers, beginning with \( B \sim 10 \), then we should expect the existence of weakly decaying hypernuclei with anti-charm and anti-beauty.

The properties of multiskyrmion configurations, necessary for these numerical estimates, have been calculated within the rational map approximation [46] which provides remarkable scaling laws for the excitation energies of heavy antiflavors. This scaling property of heavy flavors (antiflavors) excitation energies, noted previously [43, 63], and confirmed in present paper by numerical calculations, is fulfilled with good accuracy. The relative role of the quantum correction of the order \( \sim 1/N_c \) (hyperfine splitting) decreases with increasing baryon number like \( 1/B \), therefore, besides \( 1/N_C \) expansion widely used and discussed in the literature, the \( 1/B \) expansion and arguments can be used to justify the chiral soliton approach at large enough values of baryon number.

Positive strangeness nuclear states are mostly bound relative to the decay into \( \Theta^+ \) and nuclear fragments, so, one can speak about \( \Theta^+ \) hypernuclei [54, 71]. The particular value of binding energy depends on the variant of the model, and is greater for the original \( SK_4 \) variant (Table 5). The existence of deeply bound states is not excluded by our treatment.

| \( B \) | \( \Delta \epsilon_{SK_4} \) | \( \epsilon_{c} \) | \( \Delta \epsilon_{SK_6} \) | \( \epsilon_{b} \) | \( \Delta \epsilon_{SK_4} \) | \( \epsilon_{c} \) | \( \Delta \epsilon_{SK_6} \) | \( \epsilon_{b} \) |
|---|---|---|---|---|---|---|---|---|
| 2  | 36 | 36 | 54 | 54 | -5 | -5 | -30 | -30 |
| 3  | 24 | 32 | 43 | -27 | -19 | -59 | -51 |
| 4  | 19 | 48 | 44 | 72 | -26 | 2 | -45 | -16 |
| 5  | 27 | 54 | 39 | 66 | -22 | 5 | -50 | -23 |
| 6  | 18 | 50 | 31 | 63 | -27 | 5 | -52 | -20 |
| 7  | 30 | 69 | 46 | 84 | -17 | 22 | -38 | 1 |
| 8  | 19 | 75 | 27 | 84 | -24 | 32 | -49 | 7 |
| 9  | 21 | 78 | 23 | 80 | -21 | 36 | -49 | 8 |
| 10 | 15 | 80 | 17 | 82 | -25 | 40 | -52 | 13 |
| 11 | 17 | 91 | 13 | 88 | -22 | 52 | -52 | 23 |
| 12 | 12 | 104 | 9 | 101 | -25 | 67 | -53 | 39 |
| 16 | 3 | 131 | -12 | 115 | -28 | 100 | -61 | 66 |
| 17 | 6 | 136 | -10 | 120 | -26 | 104 | -60 | 70 |
| 20 | -4 | 156 | -30 | 131 | -31 | 130 | -68 | 93 |
| 24 | -10 | 188 | -43 | 155 | -33 | 166 | -73 | 125 |
| 28 | -17 | 220 | -57 | 179 | -35 | 202 | -78 | 158 |
| 32 | -21 | 251 | -67 | 205 | -37 | 235 | -82 | 190 |

Table 7. Same as in Table 6, for rescaled \( SK_4 \) and \( SK_6 \) variants of the model.

5 Conclusions

The excitation energies of antiflavors are estimated within the bound state version of the chiral soliton model in two different variants of the model, \( SK_4 \) and \( SK_6 \), and for two values of the model parameter (\( \epsilon \) or \( \epsilon' \), see Tables 3, 4). The bounds for expected binding energies of hypernuclei are obtained in this way. These bounds are wide - variations of the total binding energy for the \( SK_4 \) and \( SK_6 \) models can reach 40 – 50 Mev. The difference between original (baryon) variant and rescaled (nuclear) variant is greater for strangeness and smaller for anti-charm and anti-beauty, where it is not greater than \( \sim 20 \) – 30 Mev for baryon numbers between 3 and \( \sim 20 \). If the logic is correct, that rescaled or nuclear variant of the model should be applied for large enough \( B \)-numbers, beginning with \( B \sim 10 \), then we should expect the existence of weakly decaying hypernuclei with anti-charm and anti-beauty.

The properties of multiskyrmion configurations, necessary for these numerical estimates, have been calculated within the rational map approximation [46] which provides remarkable scaling laws for the excitation energies of heavy antiflavors. This scaling property of heavy flavors (antiflavors) excitation energies, noted previously [43, 63], and confirmed in present paper by numerical calculations, is fulfilled with good accuracy. The relative role of the quantum correction of the order \( \sim 1/N_c \) (hyperfine splitting) decreases with increasing baryon number like \( 1/B \), therefore, besides \( 1/N_C \) expansion widely used and discussed in the literature, the \( 1/B \) expansion and arguments can be used to justify the chiral soliton approach at large enough values of baryon number.

Positive strangeness nuclear states are mostly bound relative to the decay into \( \Theta^+ \) and nuclear fragments, so, one can speak about \( \Theta^+ \) hypernuclei [54, 71]. The particular value of binding energy depends on the variant of the model, and is greater for the original \( SK_4 \) variant (Table 5). The existence of deeply bound states is not excluded by our treatment.
although in most cases the energy of the state is sufficient for the strong decay into kaon and residual nucleus or nuclear fragments.

The binding energies of the ground states of hypernuclei with heavy antiflavors (\(\bar{c}\) or \(\bar{b}\)) shown in Tables 6,7 are more stable relative to variations of the model parameters (\(\varepsilon\) or \(\varepsilon'\)), but more sensitive to the model itself. Similar to the case of antistrangeness, the binding energies for the \(SK_6\) variant of the model are systematically smaller than for the \(SK_4\) variant.

Within our approach it is possible to obtain the spectra of excited states - with greater values of isospin and angular momentum. The energy scale in the first case is given by \(1/\Theta_I\), in the second - by \(1/\Theta_J\), which is much smaller for large baryon numbers. Since for \(B = 1\) \(1/\Theta_I = 1/\Theta_J \simeq 180\text{ Mev}\), (see Table 1) it seems difficult to obtain within chiral soliton approach such small spacing between ground state and excited levels as derived, e.g. in [53] within the quark models.

Although we performed considerable numerical work, we feel that further refinements, improvements as well as more precise calculations are necessary. For example, possible contributions of the order \(1/N_c\) to the flavor excitation energies mentioned e.g. in [44] might change our conclusions. When calculations for the present paper have been finished, we became aware of papers [71] and [72], where the possibility of existence of anti-strange \(\Theta\) hypernuclei is discussed within a framework of more conventional approaches. Results obtained in [71] and [72] qualitatively agree with ours.

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6 Appendix. Comparison of flavor and antiflavor excitation energy difference in rigid rotator and bound state models

Here we show that the difference between flavor and antiflavor excitation energies given by (23) coincides with the difference of \(SU(3)\) rotation energies between exotic and nonexotic multiplets within rigid rotator approach, in the leading in \(N_c\) approximation. The method used here is similar to that of [22] applied for arbitrary B-numbers and \(N_c = 3\). Generalization for arbitrary \(N_c\) and \(N_F\) was made recently in [70]. For nonexotic multiplets we have quantization condition [61] \(p + 2q = N_c B\), and for odd \(B\)-numbers we take \(p = 1\), for even \(B\), \(p = 0\). The contribution to the \(SU(3)\) rotation energy depending on ”flavor” moment of inertia which is of interest here equals to [61]

\[
E^{rot}(SU_3) = \frac{1}{2\Theta_F}[C_2(SU_3)(p, q) - I_r(I_r + 1) - N_c^2 B^2/12] \tag{48}
\]

with \(C_2(SU_3) = (p^2 + q^2 + pq)/3 + p + q = [(p + 2q)^2 + 3p^2]/12 + (p + 2q)/2 + p/2\). The ”right” isospin for the lowest nonexotic states equals to \(I_r = p/2 = 0\) for even \(B\) (as for nuclei \(^4He, ^{12}C, ^{16}O\), etc.), and to \(I_r = p/2 = 1/2\) for odd \(B\) (as for isodoublets \(^3H - ^3He, ^5He - ^5Li\), etc.).

The lowest possible exotic \(SU(3)\) irrep \((p, q)\) for each value of the baryon number \(B\) are defined by relations: \(p' + 2q' = N_c B + 3m; m\) coincides with the number of additional quark-antiquark pairs for several lowest values of \(p'\).
The difference of the $SU(3)$ rotation energies for exotic and nonexotic multiplets equals to

$$\Delta E^{rot} = \frac{1}{2\Theta_{F,B}} \left[ C_2(SU_3)' - C_2(SU_3) - I_r'(I_r' + 1) + I_r(I_r + 1) \right]$$  \hspace{1cm} (49)$$

After simple transformations it can be written in the form:

$$\Delta E^{rot} = \frac{1}{2\Theta_{F,B}} \left[ (m(2N_cB + 3m) + p^2 - p'^2) / 4 + 3m/2 + (p' - p)/2 + (I_r - I_r')(I_r + I_r' + 1) \right].$$  \hspace{1cm} (50)$$

If $m = 1$, for lowest $SU(3)$ irreps $p' = 1$, $q' = (N_cB + 2)/2$ for even $B$, and $p' = 0$, $q' = (N_cB + 3)/2$ for odd $B$. We should keep in mind that the right isospin equals to $I_r' = (p' + 1)/2 = I_r + 1$ for $B = 2, 4...$ and $I_r' = (p' + 1)/2 = I_r$ for $B = 1, 3, 5,...$. For charm or beauty, due to large configuration mixing caused by large values of $D$ or $B$-meson masses present in the lagrangian such lowest irreps often are not the main component of the mixed state (in this connection the papers [51] may be of interest), but for strangeness they are.

For even $B$ ($m = 1, p = 1, p' = 0$) we have

$$\Delta E^{rot} = \frac{1}{4\Theta_{F,B}} [N_cB + 2].$$  \hspace{1cm} (51)$$

For odd $B$ ($p = 0, p' = 1$) we obtain

$$\Delta E^{rot} = \frac{1}{4\Theta_{F,B}} [N_cB + 3].$$  \hspace{1cm} (52)$$

For $N_c = 3$ and $B = 1$ this coincides with well known expression for the mass difference between andidecuplet and octet of baryons.

The leading in $N_c$ contribution is the same as given by (23). For arbitrary $m$ the leading contribution is $\Delta E^{rot} = mN_cB/(4\Theta_{F,B})$, for any multiplets with the final values of $p'$ and $I_r$, also different from those we took here. It is worth noting that the correction to the leading contribution decreases not only with increasing $N_c$, but also with increasing $B$ (recall that $\Theta_{F,B} \sim N_cB$). Therefore, convergence of both methods is better for larger values of $B$.

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