Neutron Star Cooling with a Dynamic Stellar Structure

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Abstract

The observations combined with theory of neutron star (NS) cooling play a crucial role in achieving the intriguing information of the stellar interior, such as the equation of state, composition, and superfluidity of dense matter. The traditional NS cooling theory is based on the assumption that the stellar structure does not change with time. The validity of such a static description has not yet been confirmed. We generalize the theory to a dynamic treatment; that is, continuous change of the NS structure (rearrangement of the stellar density distribution with the total baryon number fixed) as the decrease of temperature during the thermal evolution, is taken into account. It is found that the practical thermal energy used for the cooling is slightly lower than that estimated in a static situation, and hence the cooling of NSs is accelerated correspondingly but the effect is rather weak. Therefore, the static treatment is a good approximation in the calculations of NS cooling.

Key words: stars: neutron

1. Introduction

Neutron stars (NSs) as a type of compact objects, contain matter of supranuclear density in their interiors, which are not accessible in modern laboratory experiments. They have a lot of extreme and intriguing features that are unique in the universe, including rapid rotation, extremely strong magnetic field, superstrong gravitation, superfluidity and superconductivity inside, and superprecise spin period (Haensel et al. 2006). Accordingly, as natural laboratories, they involve superrich physics that are related to various branches of current physics and astronomy, and hence they greatly promote the development of fundamental physics in extreme environments. At present, many properties can be measured with increasingly improved accuracies, such as the mass, radius, spin period (and its derivative with respect to time), glitch, giant flares, quasi-periodic oscillations, and surface magnetic field, which help one to grasp important information on NSs. However, very little knowledge of the NS interior can be achieved solidly through these observations. The NS thermal evolution provides a possibility to study its interior physics and some difficult issues in nuclear physics such as the equation of state (EOS) of supranuclear densities (Page et al. 2004).

The rapid cooling of the NS in Cassiopeia A was reported from an analysis of several Chandra observations (Heinke & Ho 2010). It has ignited a great interest in the exploration of NS thermal evolution, and thus numerous developments have been performed on the theoretical models that are used to interpret these observational data (Page et al. 2011; Shurnin et al. 2011; Yang et al. 2011; Blaschke et al. 2012, 2013; Newton et al. 2013; Noda et al. 2013; Sedrakian 2013; Bonanno et al. 2014). Owing to the well-known age (Fesen et al. 2006) and well-measured surface temperature for 10 years (Heinke & Ho 2010), this NS serves as a valuable opportunity to explore the knowledge of the neutron star matter at high densities. For instance, the observed rapid cooling was interpreted as the triggering of enhanced neutrino emission resulting from the neutron {3}P₂ pairing in the NS core, and it was claimed that such rapid cooling is the first direct evidence that superfluidity and superconductivity occur at supranuclear densities within NSs (Page et al. 2011). Quantitatively, the superfluidity gap of the neutron {3}P₂ channel is found to be around 0.1 MeV (Page et al. 2011). However, another group reported that a statistically significant temperature drop is not seen for the NS in Cassiopeia A (Posselt et al. 2013). And also a microscopic calculation gives a small {3}P₂ pairing, which differs from the result of D. Page et al. (Dong et al. 2013, 2016). Anyway, reliable observations of NS thermal evolution provide a powerful probe to grasp information on the NS interior.

As a consequence of improved measurements of thermal emission from cooling NSs, it has become clear that the observations cannot be explained on the basis of a single universal cooling curve (Yakovlev & Pethick 2004). Thus, a reliable theory for the NS cooling is indispensable to predict accurately the evolution of the NS surface temperature, and to gain important information about the stellar interior in combination with observations. At present, the structure of a given isolated NS is believed to not change with time in all the previous investigations of thermal evolution, referred to as the static treatment. In view of the great importance of the NS cooling in both astrophysics and nuclear physics, in this work, the previous static approach to describe the NS cooling is generalized to a dynamic one; that is, the temperature-dependent (and hence the time-dependent) change of the stellar structure during the cooling is included.

2. EOS of Dense Matter at Finite Temperature

It is necessary to establish an NS structure for the calculation of NS cooling. The EOS of NS matter as the input for the building of stellar structure is obtained from the relativistic mean-field (RMF) theory in which the temperature effect can be readily introduced. For the dense matter made of nucleons ($B = p$, $n$) and leptons ($l = e$, $\mu$), the total interacting
Lagrangian density is given by
\[
\mathcal{L} = \overline{\psi}_B (i\gamma^\mu \partial_\mu - M - g_s \sigma - \frac{g_w}{2} \gamma^\mu \tau \cdot \rho_\mu + g_\omega \gamma_\mu \omega_{\mu}) \psi_B \\
+ \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \left( \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{3} g_s \sigma^3 + \frac{1}{4} g_\omega \omega_{\mu}^\mu \right) \\
- \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu} + \frac{1}{2} m_{\omega}^2 \omega_{\mu}^\mu \omega_{\mu}^\mu - \frac{1}{4} R_{\mu\nu} \cdot R^{\mu\nu} \\
+ \frac{1}{2} m_{\rho}^2 \rho_\mu \cdot \rho_\mu + \frac{2}{3} g_\omega \overline{\rho}_\mu^\mu \omega_{\mu}^\mu + \overline{\psi}_B (i\gamma^\mu \partial_\mu - m_\rho) \psi_B, \\
\]
where \( M, m_\sigma, m_\omega, \) and \( m_\rho \) are the nucleon-, lepton-, \( \sigma \)-, \( \omega \)-, and \( \rho \)-meson masses, respectively. The field tensors for the vector meson are given as \( \Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \) and by similar expression for \( R_{\mu\nu} \) of \( \rho \) meson. The nucleon field \( \psi_B \) interacts with the \( \sigma, \omega, \rho \) meson fields \( \sigma, \omega_\mu, \rho_\mu \) with the coupling constants \( g_s, g_\omega, \) and \( g_\rho \), respectively, and the lepton field \( \psi_B \) is free field. The self-coupling term of the \( \sigma, \omega \) meson with coupling constants \( g_\sigma, g_\omega \) is responsible for reducing the compression modulus of the nuclear matter to an empirical value (Boguta & Bodmer 1977). The self-coupling of omega-meson described by the coupling constant \( \zeta \), is introduced to soften the EOS at high density. The nonlinear mixed isoscalar-isovector described by \( \Lambda_0 \) modifies the density-dependence of the symmetry energy (Fattoyev et al. 2010). In the mean-field approximation, the meson field operators are replaced by their expectation values. There is no current in uniform nuclear matter and thus the spatial vector components of \( \omega_\mu, \rho_\mu \) vanish, with only the timelike components \( \omega_\mu, \rho_\mu \) left. In addition, the charge conservation makes sure that only the third component of the isospin of \( \rho \) meson, i.e., \( \rho_3 \), is nonzero. In a word, the \( \sigma, \omega_0, \) and \( \rho_3 \) are the nonvanishing expectation values of mesons fields in nuclear matter. The employed effective interaction in the RMF approach is IU-FSU here, which gives a good description of ground state properties as well as excitations of free nuclei (Fattoyev et al. 2010). Accordingly, the energy density \( \varepsilon \) and pressure \( P \) for a zero-temperature NS matter are given as
\[
\varepsilon = \sum_{B=\sigma,\omega} \frac{1}{2 \pi^2} \int_0^{k_F} \frac{dk}{k^2} \sqrt{k^2 + (m_B - g_B \sigma)^2} \\
+ \sum_{l=e,\mu} \frac{1}{2 \pi^2} \int_0^{k_F} \frac{dk}{k^2} \sqrt{k^2 + m_{l}^2} + \frac{1}{2} m_{l}^2 \sigma^2 \\
+ \frac{1}{3} g_\omega \sigma^3 + \frac{1}{2} g_\sigma \omega_{\mu}^\mu \sigma^2 + \frac{1}{2} m_{\omega}^2 \omega_{\mu}^\mu + \frac{1}{2} m_{\rho}^2 \rho_{\mu}^\mu \\
+ 3 \Lambda_0 g_\omega m_{\rho_3}^2 \rho_{\mu}^\mu + \frac{2}{3} g_\omega \omega_{\mu}^\mu \omega_{\mu}^\mu. \\
\]
\[
P = \sum_{B=\sigma,\omega} \frac{1}{2 \pi^2} \int_0^{k_F} \frac{dk}{k^2} \frac{k^2}{\sqrt{k^2 + (m_B - g_B \sigma)^2}} \\
+ \sum_{l=e,\mu} \frac{1}{2 \pi^2} \int_0^{k_F} \frac{dk}{k^2} \frac{k^2}{\sqrt{k^2 + m_{l}^2}} - \frac{1}{2} m_{l}^2 \sigma^2 \\
- \frac{1}{3} g_\omega \sigma^3 - \frac{1}{4} g_\sigma \omega_{\mu}^\mu \sigma^2 + \frac{1}{2} m_{\omega}^2 \omega_{\mu}^\mu + \frac{1}{2} m_{\rho}^2 \rho_{\mu}^\mu \\
+ \Lambda_0 g_\omega m_{\rho_3}^2 \rho_{\mu}^\mu + \frac{2}{3} g_\omega \omega_{\mu}^\mu \omega_{\mu}^\mu. \\
\]
where \( k_{F,B} \) and \( k_{F,l} \) are the Fermi momenta of nucleons and leptons, respectively. The EOS based on the Haensel–Zdunik–Dobaczewski and Negele–Vautherin (Haensel & Zdunik 1990, Negele & Vautherin 1973) is applied for the NS crust.

The stellar interior is assumed to be isothermally described by a coordinate temperature (redshifted temperature), since NSs are excellent conductors with quite high thermal conductivities (Haensel et al. 2006). For the convenience of calculation, we use the constant proper temperature (local temperature) instead, which does not hinder us from discussing the physics we are concerned with. Such finite temperature is not expected to affect the stellar structure substantially. Accordingly, the influence of temperature on the EOS acts as a small perturbation in the present work. For the \( \beta \)-stable matter with a given total baryon number density \( n_B \), we derive the modifications of the energy density and pressure due to the presence of temperature \( T \) which are given by
\[
\Delta \varepsilon = m_B^2 \sigma \Delta \sigma + g_2 \sigma^2 \Delta \sigma + g_3 \sigma \omega \Delta \omega + m_{\omega}^2 \omega \Delta \omega_0 \\
+ m_{\rho}^2 \rho_3 \Delta \rho_3 + 6 \Lambda_0 g_\omega m_{\rho_3}^2 \omega_0 \Delta \omega_0 + \omega_0 \Delta \rho_3 \}
\]
\[
\frac{g_\omega}{2} (g_\omega^2 \omega^2 + \sum_{B=\sigma,\omega} \frac{1}{2 \pi^2} \left[ \frac{m_B^2 \omega_B^2}{\sqrt{m_B^2 - m_{\omega}^2}} \right] \Delta \mu_B \\
+ \frac{1}{6} (k_B T^2) \frac{3 \mu_B^2 - 2 \mu_B m_{\omega}^2}{\sqrt{m_B^2 - m_{\omega}^2}} \\
+ \sum_{B=\sigma,\omega} \frac{g_\omega}{8 \pi^2} \left[ m_{\omega}^2 - \frac{2}{m_B^2 \omega_B^2} m_{\omega}^2 m_B^2 - \frac{2}{m_B^2 \omega B^2} m_{\omega}^2 m_B^2 \right] \\
\times \frac{\mu_B^2 + \mu_B \sqrt{m_B^2 - m_{\omega}^2}}{m_B^2 - m_{\omega}^2} \\
+ \sum_{l=e,\mu} \left[ \frac{1}{2 \pi^2} \left( \sqrt{m_{\omega}^2 - m_{\omega}^2} \right) \right] \Delta \mu_l \\
+ \frac{1}{6} (k_B T^2) \frac{3 \mu_l^2 - 2 \mu_l m_{\omega}^2}{\sqrt{m_l^2 - m_{\omega}^2}}. \\
\]
\[
\Delta P = -m_B^2 \sigma \Delta \sigma - g_2 \sigma^2 \Delta \sigma - g_3 \sigma \Delta \sigma + m_{\omega}^2 \omega \Delta \omega_0 \\
+ m_{\rho}^2 \rho_3 \Delta \rho_3 + 2 \Lambda_0 g_\omega m_{\rho_3}^2 \omega_0 \Delta \rho_3 + \rho_3 \Delta \rho_3 \}
\]
\[
+ \frac{g_\omega}{2} (g_\omega^2 \omega^2 + \sum_{B=\sigma,\omega} \frac{1}{2 \pi^2} \left[ \frac{m_B^2 \omega_B^2}{\sqrt{m_B^2 - m_{\omega}^2}} \right] \Delta \mu_B \\
+ \frac{1}{6} (k_B T^2) \frac{3 \mu_B^2 - 2 \mu_B m_{\omega}^2}{\sqrt{m_B^2 - m_{\omega}^2}} \\
- 5 m_B^2 \omega_B^2 \mu_B \omega_B^2 + 12 m_{\omega}^2 \ln \frac{\mu_B + \sqrt{\mu_B^2 + m_{\omega}^2}}{m_B^2 - m_{\omega}^2} \\
+ \sum_{l=e,\mu} \left[ \frac{1}{2 \pi^2} \left( \sqrt{m_{\omega}^2 - m_{\omega}^2} \right) \right] \Delta \mu_l \\
+ \frac{1}{6} (k_B T^2) \frac{3 \mu_l^2 - 2 \mu_l m_{\omega}^2}{\sqrt{m_l^2 - m_{\omega}^2}}. \\
\]
Here \( k_B, \mu, m^* = m - g_\sigma \sigma \) are the Boltzmann constant, zero-temperature chemical potential, and nucleonic Dirac effective mass, respectively. It should be stressed that the \( \mu_B \)
for nucleons here is a translational chemical potential defined as 
$\mu_B = \sqrt{k_F^2 + m^*^2}$ for the sake of derivation. $\Delta \mu_B$ and $\Delta \mu_l$ are the changes in the chemical potentials of nucleons and leptons, respectively, due to thermal effects. The changes in the meson fields induced by thermal effects for a given baryon number density, namely the $\Delta \sigma$, $\Delta \omega_0$, and $\Delta \rho_{30}$, are obtained with the conditions of $\beta$-stable and electric neutrality of NS matter, which are, respectively, given as

$$
\Delta \sigma = g_\sigma m^* \left[ \sum_{\beta = n, p} \left( \frac{(kT)^2}{6} \frac{\mu_B}{\sqrt{\mu_B^2 - m^*^2}} \right) - \sigma \right] 
$$

$$
\Delta \omega_0 = \frac{4k^2 g_h^2 g^2 \rho_{30}}{m^2 + 3\sigma \omega_0 + 2\lambda g_h^2 g^2 \rho_{30}^2} \Delta \rho_{30},
$$

$$
\Delta \mu_e = \frac{(x_e + x_p)(x_e + x_p + x_p - x_p)}{x_p x_e} \Delta \rho_{30},
$$

$$
\Delta \mu_p = \frac{(x_e + x_p) \Delta \mu_e + y_p - y_p}{x_p},
$$

$$
\Delta \mu_n = \Delta \mu_p + \Delta \mu_e + \frac{g_\sigma}{2} \Delta \rho_{30},
$$

$$
\frac{g_\sigma}{2} \left( x_e + x_p \Delta \mu_e + y_p - y_p \right) = \frac{1}{\pi} \sqrt{\mu_B^2 - m^*^2} \mu_B, \quad B = n, p
$$

The above equations are solved in a self-consistent way. The static thermal energy density under the temperature $T$ is $\Delta \epsilon$. The electrons in the neutron star crust may be influenced considerably since the temperature in the crust is sufficiently high for the motion of the electrons. Considering that the pressure in the crust is primarily yielded by the electrons, the effects of temperature on electrons are simply taken into account via

$$
\Delta \epsilon = \frac{1}{6} (k_T^2)^2 \mu_e \sqrt{\mu_e^2 - m_e^2},
$$

$$
\Delta P = \frac{(k_T^2)^2}{18} \frac{\mu_e}{\mu_e} \sqrt{\mu_e^2 - m_e^2} \left( \frac{\mu_e + m_e^2}{\mu_e} \right),
$$

Due to the very small proportion of a star mass, the $\Delta \epsilon$ and $\Delta P$ from the crust contribute rather insignificantly to the rearrangement of stellar structure. Figure 1 displays the static thermal energy density $\Delta \epsilon$ as a function of density. Since the chemical potential of each component changes slightly due to the presence of temperature, the percentage of each component changes accordingly as the result of chemical balance under a given baryon number. The calculated $\Delta \epsilon$ without the effect of the change in the constitute concentrations, is also shown for comparison, and we do not give the tedious formula here. It shows that the two calculations are almost identical to each other, indicating that the change in the concentrations due to the temperature effect almost does not provide additional energy in the present investigation. On the other hand, the $\Delta \epsilon$ can be calculated by $\Delta \epsilon = \sum_i c_i dT$, where $c_i$ is the heat capacity per unit volume for the species $i$. The $c_i$ is given as $c_i = m_i^* k_F k_B^2 T / (3h^3)$ (Cumming et al. 2017), where $m_i^*$ is the Landau effective mass. The nucleonic (nonrelativistic) Landau mass is exactly equal to the Dirac mass $m^* = m - g_\sigma \sigma$ here because the self-energy in the RMF approach is momentum-independent. The obtained $\Delta \epsilon$ with this method is also almost identical to the above results, which indicates the correctness of our calculated thermal energy density.

For the NS matter with temperature $T$ and baryon density $\rho_B$, the energy density and pressure are written as $\varepsilon + \Delta \varepsilon$ and $P + \Delta P$, respectively. With the obtained EOS for zero- or finite-temperature, the mass-versus-radius relation, and other relevant quantities of a spherically symmetric nonrotating NS can be determined by solving the following TOV equation (Oppenheimer & Volkoff 1939) based on the general relativity

$$
dP(r) = \left( \frac{\varepsilon(r)m(r)}{r^2} \right) \left( 1 + \frac{P(r)}{\varepsilon(r)} \right)
$$

$$
\times \left( 1 + \frac{4\pi r^3 P(r)}{m(r)} \right) \left( 1 - \frac{2m(r)}{r} \right)^{-1},
$$

$$
m(r) = \int_0^r 4\pi r^2 \varepsilon(r) dr,
$$

where $P(r)$ is the pressure of the star at distance $r$, $m(r)$ is the mass inside a sphere of radius $r$, and $c = G = 1$. The radius $R$ and mass $m(R)$ of a neutron star are obtained from the boundary condition $P(R) = 0$. Our EOS marginally fulfills the recent observational maximum mass (Demorest et al. 2010; Antoniadiis et al. 2013).
constitute is presented for comparison. The Astrophysical Journal, 5

Our starting point is to calculate the static thermal energy $E_{\text{th}}^*$, but that which can be stored in stars. The energy balance equation offers an intuitive physical picture to describe the NS cooling. Current simulations of thermal evolution including the present work are displayed in Figure 2. The stored $E_{\text{th}}^*$ that will be lost during the cooling, is lower than the static thermal energy $E_{\text{th}} = \int C_v dT$. Importantly, both of them are proportional to $T^4$, which allows us to introduce an effective heat capacity $C_{\text{v,eff}}$ and hence the dynamic thermal energy is written as $E_{\text{th}}^* = \frac{1}{2} C_{\text{v,eff}} T$. It is necessary to look for the relationship between $C_{\text{v,eff}}$ and $C_v$. Fortunately, the ratio of $\eta = C_{\text{v,eff}}/C_v$, defined as the efficiency factor here, is independent of temperature $T$, as shown in the inset of Figure 2. For example, $\eta = 0.87$ for the canonical NS with the IU-PSU interaction here, indicating that only 87% of static thermal energy is available for the NS cooling. As a result, the traditional heat capacity $C_v$ should be replaced by the effective one $C_{\text{v,eff}}$ or $\eta C_v$, to give a dynamic description of NS cooling. Here we determine an average with regard to the whole star body for the efficiency factor $\eta$, i.e., $\eta$ is a constant everywhere.

For NSs with different masses, we calculate the $E_{\text{th}}$, $E_{\text{th}}^*$ and the corresponding efficiency factor $\eta$, and present the results in Figure 3. For low-mass NSs such as $M = 0.5 M_{\odot}$, the $E_{\text{th}}$ is not much different from the $E_{\text{th}}^*$, where the efficiency factor is as large as $\eta = 0.94$. However, with the increase of the star mass, the difference between these two energies becomes larger and larger, and hence the $\eta$ reduces gradually. For high-mass NSs such as $M = 1.9 M_{\odot}$, the difference is so substantial that $\eta$ is merely 0.78; that is, 22% of the static thermal energy is not available. Figure 1 has already shown that the static thermal energy density $\Delta \varepsilon$ (and hence the heat capacity per volume) grows with density $\rho_0$, which is consistent with the calculations by Cumming et al. (2017). However, the NS radius remains almost unchanged for the mass ranging from $0.4 M_{\odot}$ to $1.8 M_{\odot}$, as shown in the inset of Figure 3. Therefore, the $E_{\text{th}}$ grows as the stellar mass increases in this range. Beyond $1.8 M_{\odot}$, NSs tend to be more compact with increasing mass, leading to the $E_{\text{th}}$ decreases with stellar mass as the result of these two competing effects. It is well-known that the white dwarf is supported by the electron degeneracy pressure, where gravity cannot compress it any more because Pauli principle prevents it from complete collapse. However, for NSs, it is the strong nuclear force among nucleons instead of the nucleonic degeneracy pressure that contributes primarily to the pressure. The alternation in the pressure $\Delta P$ induced by the temperature stems primarily from the change of the $\sigma$ meson field, where the $\sigma$ meson describes the intermediate range attraction of nuclear force. Other meson fields change rather insignificantly because the source terms in their respective motion equations do not involve the temperature directly. The nonmonotonic

![Figure 1](image_url)
behavior of $E_{th}^*$ as a function of NS mass mainly results from the temperature-induced nonmonotonic variation of the $\sigma$ meson field versus density, yet, the $\eta$ decreases monotonically with increasing mass.

The interior temperature of an NS is $10^7 \sim 10^9 \text{K}$ usually, which is much lower than the corresponding Fermi temperature of dense matter inside the star. Therefore, in order to achieve a sufficient accuracy, we treat the effect of the temperature on the EOS of dense matter as a tiny perturbation, and then derive the analytical formulas (Equations (4), (5)) for the changes in the total energy density and in pressure induced by temperature, which is a pivotal step of the present study. By setting up an initial central density, the TOV equation can be integrated with the energy density $\varepsilon (\varepsilon + \Delta \varepsilon)$ and pressure $P (P + \Delta P)$ as inputs for a zero- (finite-) temperature NS. The loop runs over the central density of the finite-temperature NS until its total baryon mass $m_b$ converges exactly to that of the zero-temperature one. In fact, the interior structure of the zero-temperature NS, i.e., density and pressure distributions, is quite close to that of the finite-temperature one because of $\Delta \varepsilon \ll \varepsilon$ and $\Delta P \ll P$. Since the energy density $\varepsilon'$ of the finite-temperature NS is given by the zero-temperature energy density $\varepsilon'|_{\varepsilon=0}$ plus the thermal energy density $\Delta \varepsilon$, the dynamic thermal energy $E_{th}^*$ is evaluated by

$$E_{th}^* = \int_0^R 4\pi r^2 (\varepsilon'|_{\varepsilon=0} + \Delta \varepsilon) \, dr - \int_0^R 4\pi r^2 \varepsilon|_{\varepsilon=0} \, dr,$$

where the first and second integrals denote the change of the zero-temperature internal energy and of the gravitational potential energy induced by temperature, marked as $\Delta E_{th,U}$ and $\Delta E_{th,g}$ respectively. Figure 4 displays the calculated $\Delta E_{th,U}$ and $\Delta E_{th,g}$ versus stellar mass. The central density of the finite-temperature NS is slightly lower than that of zero-temperature one. That is, due to the presence of temperature, the dense matter deep inside the star moves outward. This leads to the decrease of zero-temperature internal energy but the increase of the gravitational potential energy, and such changes are mainly subject to the change of the compactness parameter $2m/r$. Because the thermal energy density $\Delta \varepsilon$ becomes larger and larger with increasing density as shown in Figure 1, the difference between the compactness parameter $2m/r$ of finite-temperature NS and $2m/r$ of the corresponding zero-temperature one, i.e., $\sim 2E_{th}(r)/r$ with $E_{th}(r) = \int_0^r 4\pi r^2 \Delta \varepsilon \, dr$, grows with NS mass. Therefore, the absolute values of both $\Delta E_{th,U}$ and
The core temperature is chosen to be $5 \times 10^8$ K.

| $\Lambda_V$ | $g_V^2$ | $L$(MeV) | $\Lambda_V$ | $g_V^2$ | $L$(MeV) |
|-----------|---------|---------|------------|---------|---------|
| 0.00      | 84.4175 | 119     | 0.01       | 95.7143 | 91      |
| 0.02      | 110.5015| 72      | 0.03       | 130.6928| 60      |
| 0.04      | 159.9126| 51      | 0.05       | 205.9605| 45      |

Note. $L = 3\rho \partial E_{\text{sym}}(\rho) / \partial \rho |_{\rho_0}$ is the slope parameter of the symmetry energy $E_{\text{sym}}$ at the saturation density $\rho_0$.

$\Delta E_{\text{th},g}$ grow with stellar mass. The net temperature effect is a result of these two competing trends.

Nowadays the properties of dense matter at supersaturation densities remain uncertain due to poor knowledge of the nuclear forces and the difficulty of solving the many-body problem. Therefore, it is necessary to test the uncertainty of the $E_{\text{th}}$ and $\eta$ caused by different interactions. The IU-FSU interaction used here describes well the EOS of symmetric matter (Fattoyev et al. 2010), but the symmetry energy that characterizes the isospin-dependent part of the EOS of asymmetric nuclear matter (Baran et al. 2005; Steiner et al. 2005; Lattimer & Prakash 2007; Li et al. 2008) is not yet well determined at high densities. To explore the sensitivity of $E_{\text{th}}^*\eta$ and $\eta$ to the symmetry energy, we build a family of IU-FSU interactions by adjusting the isovector parameters $\Lambda_V$ and $g_V^2$ in such a way that the value of the symmetry energy remains fixed at 25.70 MeV at a baryon density of $\rho_0 = 0.1$ fm$^{-3}$ within a procedure as in the work of Piekariewicz (2011). The parameters $\Lambda_V$ and $g_V^2$ are listed in Table 1 with the slope $L = 3\rho \partial E_{\text{sym}}(\rho) / \partial \rho |_{\rho_0}$ at the saturation density $\rho_0$, which could also be employed to investigate the effects of the density-dependent symmetry energy in some subjects in the future.

To illustrate the behavior of the mean-field interactions, we display in the upper panel of Figure 5 the symmetry energy predicted by all of these models, providing a stiff symmetry energy ($\Lambda_V = 0.00$) to a soft one ($\Lambda_V = 0.05$). The convergence of all interactions at a density of $\rho_0 = 0.1$ fm$^{-3}$ is clearly discernible, and is divergent visibly at high densities. For instance, the stiff symmetry energy with $\Lambda_V = 0.00$ gives $E_{\text{sym}}(3\rho_0) = 116$ MeV while the relatively soft one with $\Lambda_V = 0.05$ gives $E_{\text{sym}}(3\rho_0) = 62$ MeV, where $3\rho_0$ is close to the central density of a canonical NS. The lower panel of Figure 5 illustrates the $E_{\text{th}}$ and $E_{\text{th}}^*$ under the different behavior of the symmetry energy. A stiffer symmetry energy gives a larger $E_{\text{th}}$ and $E_{\text{th}}^*$. From $\Lambda_V = 0.00$ to $\Lambda_V = 0.05$, the $E_{\text{th}}$ and $E_{\text{th}}^*$ reduce by 21% and 25%, respectively. However, on the whole, the effect of the symmetry energy on the efficiency factor $\eta$ is not so intense because the effect of symmetry energy plays a similar role for the $E_{\text{th}}$ and $E_{\text{th}}^*$ as exhibited in the inset. The depressed model dependence enhances the reliability of the presently obtained $\eta$.

The central task of NS cooling theory is to calculate the cooling curve, i.e., the surface temperature as a function of age. To show the difference between the dynamic and static treatments of the NS cooling, we compute the cooling curves for canonical (1.4 $M_\odot$) and large mass (1.9 $M_\odot$) NSs based on the publicly available code NSCool written by D. Page. The minimal cooling paradigm is used, i.e., without charge-meson condensate and exotic degrees of freedom. However, for large mass NSs, the direct Urca process is open within the IU-FSU interaction since the matter density in the core could exceed the threshold. Although such a process is believed to enhance the NS cooling most efficiently, it is not included in the standard cooling scenario for canonical NSs. The results are shown in Figure 6. Because the dynamic thermal energy is less than the static one, the NS cooling is found to be faster in dynamic treatment than that in static treatment. For the canonical NS and the large mass NS, the two approaches do not exhibit considerable difference, indicating that the static description is a good approximation. A reliable NS cooling theory is indispensable to help one to explore the knowledge of stellar interior. However, owing to the complexity.
of neutron star physics (such as anisotropic magnetic field, and composition of stellar envelope), it still has a long way to go to establish an ultimate cooling theory. On the other hand, much more observational data are required to constrain in turn the effective mass is taken from the APR EOS (Akmal et al. 1998).

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Figure 6. Cooling curves of 1.4\(M_\odot\) and 1.9\(M_\odot\) NSs. The calculations are carried out with the static (traditional) and dynamic (present) treatments. The stellar structure is also built by employing the RMF approach with the IU-FSU interaction, and the effective mass is taken from the APR EOS (Akmal et al. 1998).

The weak rearrangement of stellar structure discussed above stems from the decrease of temperature where the total baryon number of a star is conserved. In fact, the rearrangement can be achieved through many other avenues, such as the spin-down, magnetic field decay, and possible phase transitions, which should improve our understanding of reheating mechanism in NS cooling and is perhaps responsible for some intriguing features of NSs.

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