Thermal lensing-induced bifocusing of spatial solitons in Kerr-type optical media

A. M. Dikandé(a)

Laboratory of Research on Advanced Materials and Nonlinear Science (LaRAMaNS), Department of Physics, Faculty of Science, University of Buea - P.O. Box 63 Buea, Cameroon

received 12 February 2011; accepted 9 April 2011
published online 16 May 2011

PACS 42.65.Tg – Optical solitons; nonlinear guided waves
PACS 42.70.Gi – Light-sensitive materials
PACS 44.10.+i – Heat conduction

Abstract – Thermo-optical effects cause a bifocusing of incoming beams in optical media, due to the birefringence created by a thermal lens that can resolve the incoming beams into two-component signals of different polarizations. We propose a nonperturbative theoretical description of the process of formation of double-pulse solitons in Kerr optical media with a thermally induced birefringence, based on solving simultaneously the heat equation and the propagation equation for a beam in a one-dimensional medium with uniform heat flux load. By means of a nonisospectral Inverse Scattering Transform assuming an initial solution with a pulse shape, a one-soliton solution to the wave equation is obtained that represents a double-pulse beam whose characteristic properties depend strongly on the profile of heat spatial distribution.

Introduction. – In dielectric media with thermo-optical effects, the modulation of incoming beams can lead to a wavefront distortion [1–3] reflecting their instability. Generally this instability gives rise to a depolarization of a high-power field [4–9], due to a thermally induced birefringence which is attributed [10–12] to a change in the refractive index of the medium. It has been established [1] that this change in the refractive index originates from heat deposition in the propagation medium, resulting in a space-dependent temperature gradient (so-called thermal lensing) [1,4,13]. For media with linear indices, the thermal lens leads to a drastic change in the irradiance along the beam axis such that the resulting depolarization can strongly degrade the beam quality requiring thermal lensing compensation. However, in nonlinear media such as Kerr media, the nonlinearity can be a relevant self-compensation scheme [14] for stabilizing multi-wave modes generated by the thermal birefringence.

While several materials exhibiting thermo-optical effects are known in the literature, most investigated ones are semiconductor lasers [1] (including photonic materials) and the family of solid-state lasers YAG [1,4,13,15]. These last materials are generally represented as rod crystals in which the change in temperature of the materials induces thermal distortion of the heat-carrying laser beams [9]. For this particular family of materials, several theoretical attempts have been made to formulate the spatial profile of the temperature gradient along the rod exploiting available experimental data. In particular, in refs. [1,13,15] it was found that in the cylindrical-rod configuration where the heat is generated at a constant rate [9,13], a quadratic spatial distribution provides a very good description of the experimentally observed birefringence and the resulting beam bifocusing [9,16].

But thermo-optical processes are actually common to a broad class of materials, not just solid-state lasers. Namely some photonic crystals displaying nonlocal thermal and photothermal properties have been considered in the recent past from both experimental and theoretical points of view [17,18]. These materials share with solid-state lasers the common fact that heat resulting from the input pump source causes physical variation of the material, for instance the material expands with the heat load due to a stress gradient formed which produces space-dependent birefringence in the material. Quite remarkably, it is observed [4,9,19] that when the thermal gradient is strong enough the thermally-induced birefringence can resolve a polarized high-power input beam into two-component beams. Upon recombination after traversing the bulk, the two beams resulting from this thermally-induced...
bifocusing of the input signal are no longer in phase with one another such that the polarization state of the input beam cannot be recovered [3,19].

The thermally-induced depolarization phenomenon together with the resulting beam bifocusing have been investigated experimentally and theoretically by several authors [4–9]. In photonic crystals with a nonlinear Kerr-type dielectric susceptibility competing with diffraction, nonlocal thermal properties of the propagation medium have very recently been shown to induce twin-mode spatial solitons, so-called dipole solitons [20–23]. The robustness of these double-pulse structures provides strong indication that the competition between nonlinearity and thermal lensing might be a stabilizing factor for the double-polarized modes, potentially observable in a broad class of optical materials including solid-state lasers, photonic crystals as well as optical fibers with pronounced thermo-optical properties.

The present work aims at proposing a first nonperturbative description of the generation of a double-polarized pulse beam in optical materials with thermally-induced linear birefringence, paying particular attention to nonlinear optical media with a quadratic spatial profile of temperature distribution. Our objective is to provide a consistent theoretical framework for better understanding the effects of thermal lensing on beam propagation in nonlinear optical materials, of which the YAG fiber laser whose temperature profile was experimentally established [1,13,15]. We will first demonstrate, by solving the heat equation, that the topography of temperature distribution is quadratic in the case of a one-dimensional (1D) anisotropic material uniformly loaded at a constant heat flux rate. Next, using the relationship between the temperature gradient and the induced inhomogeneous optical index, we shall derive an effective refractive index for the material by combining the thermally induced optical index and a Kerr-type refractive index reflecting the intrinsic nonlinearity of the optical properties of the 1D anisotropic material. With the help of this effective refractive index we shall formulate the propagation equation for beams in a thermal nonlinear medium. Note that this equation is a nonlinear Schrödinger equation with a repulsive external quadratic potential, that has already been derived in ref. [24] but solved perturbatively. Here instead, a nonperturbative treatment will be proposed based on a nonisospectral Inverse Scattering Transform (IST) method [25,26] with emphasis on IST’s initial solution being single-pulse shaped.

**Double-pulsed one-soliton solution to the nonlinear wave equation.** – Thermal lensing can be described in simple words as a thermo-optical process associated with a weak absorption of an input beam that induces a nonzero temperature gradient across the material, leading to a spatial variation of its refractive index [1,13,21]. Recent experimental as well as theoretical developments on this process suggest that the underlying mechanism involves a local refractive index change $\Delta n(x)$ which increases linearly [15,21] with the temperature change $\Delta T(x)$, i.e. $\Delta n = \beta \Delta T$, where $\beta = dn/dT$ refers to the thermo-optic coefficient [21,27]. Thus when the optical beam of a uniform thermal load gets slightly absorbed and heats the material, this produces heat that is conveyed by the electromagnetic wave. If $\rho$ denotes the uniform heat flux density and $\kappa$ the heat conductivity coefficient, the heat diffusion in the material along a preferred direction (for 1D anisotropic materials of current interest), driven by the uniform heat flux load, is determined by the heat equation

$$\kappa \nabla^2 T(x) = -\rho. \quad (1)$$

Since eq. (1) is a key to the current analysis we consider its most general solution given by:

$$T(x) = -\frac{1}{2} a_2 x^2 + a_1 x + T_0, \quad (2)$$

where $a_2 = \rho/\kappa$, $a_1$ and $T_0$ being two arbitrary real constants. Formula (2) is consistent with the quadratic law of temperature variation found for most laser fibers in the presence of a uniform heat load. More specifically in YAG fiber lasers this law is common [1,13,15] and is consistent with the optical bifocality associated with a thermally induced birefringence, that promotes double-polarized laser beams from an input laser field.

For the sake of simplicity we require the temperature gradient to be zero and temperature to take a bare value $T_0$ at $x = 0$ (i.e. the ambient temperature). The change of temperature in the material along the $x$-axis, hereafter assumed to be the axis of beam propagation, then reads

$$\Delta T(x) = T_0 - T(x) = \frac{1}{2} a_2 x^2. \quad (3)$$

With the last formula we derive the following expression for the local refractive index change:

$$\Delta n(x) = \frac{1}{2} \alpha x^2, \quad \alpha = \rho \beta/\kappa. \quad (4)$$

Now if the intrinsic optical properties of the material are dominated by Kerr-type phenomena, the homogeneous part of the refractive index can be expressed as $(k^2 c/2\pi) n_2 I$ where $I$ is the beam intensity. With the help of (4) we can readily define an effective refractive index $n(x)$ for the thermal nonlinear material, viz:

$$n[x, I] = \Delta n(x) + (k^2 c/2\pi) n_2 I. \quad (5)$$

Assuming that the wave motion is fast along the axis of anisotropy (i.e. $x$) but very slow along $z$ [21], the paraxial approximation on the 2D wave equation for an electromagnetic field $q(x, z)$ leads to

$$\frac{\partial^2 q}{\partial x^2} + 2i k \frac{\partial q}{\partial z} + n[x, I] q = 0. \quad (6)$$
As already stressed eq. (6) is actually not new, indeed the same equation was obtained [24] for the same problem but solved following the collective-coordinate method. To this last point, eq. (6) is an inhomogeneous nonlinear Schrödinger equation and so can in principle be solved using the collective-coordinate method. However this is a perturbative method and consequently requires that the thermal lensing is sufficiently weak, so that the Kerr nonlinearity remains the main governing factor in the modulation and stability of signals in the thermal nonlinear medium. So to say any input beam sent in the medium must be modulated into a signal of permanent single-pulse shape, with eventually an acceleration or slowdown of the pulse due to the thermal lensing. In fact this consideration is very far from any acceptable consistency with the physics of the process under study, of which the double polarization of the incident beam is a most salient aspect.

Being interested in a solution to eq. (6) which is more consistent with experiments we try a nonperturbative approach. Let us start by remarking that this equation can be rewritten in the following form:

\[
\frac{\partial^2 q}{\partial x^2} + 2ik \frac{\partial q}{\partial x} + [(k^2 c/2\pi)\eta_2 |q|^2 - V(x)] q = 0, \tag{7}
\]

corresponding to a nonlinear Schrödinger equation with an external potential

\[V(x) = -\alpha x^2/2, \tag{8}\]

which is quadratic in x with a maximum at \(x = 0\). The physics behind this quadratic potential is contained both in its profile and the parameter \(\alpha\) defined in (4), which determines the strength of the thermal birefringence on the beam shape. One remarkable side of this physics emerges from the assumption of the heat flux density \(\rho\) and the heat conductivity \(\kappa\) as being fixed, such that \(\alpha\) appears to be increasing with an increase in the thermooptic coefficient \(\beta\). Thus when \(\beta\) is increased the curvature of the quadratic profile of the heat distribution in the material is more and more pronounced so that the effect of thermal lensing on beam modulation is stronger and stronger. In fact the external potential is expected to be much effective on the beam position and according to the profile of this potential given by (8), the centre of mass of the input beam should experience a trapping force from the manifestly explosive quadratic potential.

When \(\alpha\) is large enough such that the potential field erected by the thermal lensing process on the beam path is strongly localized, its contribution must be fully taken into consideration. In this last respect we follow the nonisospectral IST technique proper [25,26] to equations of this specific kind, considering an initial signal \(q(x, z = 0)\) of a permanent single-pulse shape along \(x\) at \(z = 0\). For eq. (6) this technique leads to the following one-soliton solution:

\[q(x, z) = Q_0(x, z) \text{sech} \left[\Psi(x, z)\right] e^{i\Phi(x, z)}, \tag{9}\]

with

\[Q_0(x, z) = \sqrt{\frac{2\pi}{\alpha c^2}} \frac{x f(z)}{k}, \tag{10}\]

\[\Psi(x, z) = \ln \left(2|f(z)|^2\right) - 2V(x)f(z) + \Psi_0, \tag{11}\]

\[\Phi(x, z) = 2V(x)g(z) + \Phi_0, \tag{12}\]

\[f(z) = \frac{\text{sech}^2(z/\zeta)}{1 + 8(\rho \beta/\kappa)^2 \tanh^2(z/\zeta)}, \tag{13}\]

\[g(z) = \frac{\kappa^2 + 8(\rho \beta)^2}{2\kappa^2 \rho \beta \kappa} \frac{\tanh(z/\zeta)}{1 + 8(\kappa \beta/\kappa)^2 \tanh^2(z/\zeta)}. \tag{14}\]

\[\zeta = (2/\alpha)^{1/2} k. \tag{15}\]

If the “sech” function in (9) reminds a pulse signal, the complicated dependence of its prefactor \(Q_0(x, z)\) in \(x\) and \(z\) clearly suggests an actually complex pulse structure for the beam on the \(xz\) plane. To gain insight about what this dependence implies for the signal profile, in fig. 1 we plotted the beam amplitude \(|q(x, z)|\) as a function of \(x\) at two different positions in the direction \(z\) transverse to the

![Fig. 1: (Colour on-line) x-axis profiles of \(|q(x, z)|\) for \(z = 0.5\) (left graphs) and \(z = 10\) (right graphs). Values of \(\alpha\) are (a) 0.1, (b) 0.05, (c) 0.01, (d) 0.005.](image-url)
Concluding remarks. — The double-pulse structure obtained, as well as its ring profile emerging in 2D, are reminiscent of dipole and ring-vortex solitons predicted recently in some nonlocal nonlinear media [21,23,28]. This double-pulse structure in our specific context is due to a uniform heat load which causes a thermal birefringence of the optical material [1,13,15], as we established by solving the corresponding heat equation.

As already noted a previous attempt [24] to model the same problem led to eq. (6), however in this previous work the equation was treated perturbatively and so results could not reflect the remarkable aspects underlying the physics of thermal lensing in Kerr media. To end, let us remark that the universality of the IST one-soliton solution (9) can be checked by applying any other exact spectral method to the generating equation, such as the Darboux method [29] combined with a Lax-pair formalism with nonconserved spectral parameters [30,31].

Part of this work was done at the Abdus Salam International Centre for Theoretical Physics (ICTP) Trieste, Italy. I thank M. Müller, M. Marsili, M. Kiselev and V. Kravtsov for their kind hospitality.

REFERENCES

[1] KOECHNER W., *Solid State Laser Engineering*, 6th edition (Springer, Berlin) 2006.
[2] FOSTER J. D. and OSTERINK L. M., *J. Appl. Phys.*, 41 (1970) 3656.
[3] KLEIN C. A., *Opt. Eng.*, 29 (1990) 343.
[4] EICHLER H. J., HASSE A., MENZEL R. and SHEMONETT A., *J. Phys. D*, 26 (1993) 1884.
[5] OSTERMeyer M., KLEMPZ G. and MENZEL R., *Proc. SPIE*, 4629 (2002) 67.
[6] MOSHE I. and JACKEL S., *J. Opt. Soc. Am. B*, 22 (2005) 1228.
[7] MACHAVARIAN G., LUMER Y., MOSHE I., MEIR A., JACKEL S. and DAVIDSON N., *Appl. Opt.*, 46 (2007) 3304.
[8] WANG Y., INOUKE K., KAN H. and WADA S., *J. Phys. D*, 42 (2009) 235108.
[9] WANG Y., KAN H., OGAWA T. and WADA S., *J. Opt.*, 13 (2011) 015703.
[10] KHRAZANOV E. A., KULAGIN O. V., YOSHIDA S., TANNER D. B. and REITZE H. D., *IEEE Quantum Electron.*, 35 (1999) 1116.

beam propagation, and for four distinct values of $\alpha$. More explicitly the four left graphs in fig. 1 represent $|q(x,z)|$ vs. $x$ at $z = 0.5$ for $\alpha = 0.1, 0.05, 0.01$ and 0.005, while the four right graphs represent $|q(x,z)|$ vs. $x$ for the same values of $\alpha$ but at $z = 10$. As one can see, the signal intensity is a two-component pulse whose intensities are strongly dependent on the strength of the thermal lens potential. It is quite noticeable on exploring the height figures that when $\alpha$ increases for a fixed transverse position $z$, the intensities of the two-component pulse increase while their widths at half-tails diminish. The last behaviour is consistent with the $\alpha$-dependence of parameter $\zeta$ defined in (15), which indeed represents the average spatial extension of the pulse along the $x$-axis. Another relevant feature emerging from the graphs of fig. 1 is the fact that when $\alpha$ is decreased for a fixed value of $z$, the two constituents pulses in the double-polarized beam preserve their shapes but their peak positions (i.e. centres) are gradually shifted. The last feature is more transparent in the right graphs corresponding to a relatively large value of $z$. In fact, the last behaviour can be interpreted in terms of a ring profile for the signal intensity in the $xz$ plane as reflected by the contour plots of fig. 2, where shadows of the double-pulse soliton in the $xz$ plane are represented for different values of $\alpha$. The figure clearly indicates an increase of the separation between pulses in the double-pulse signal, implying that the radius and curvature of the ring signal in the $xz$ plane are fixed by the magnitude of $\alpha$.

Fig. 2: (Colour on-line) Contour plots of $|q(x,z)|$ for different $\alpha$ listed as: (a) 0.01, (b) 0.005, (c) 0.001, (d) 0.0005, (e) 0.0001, (f) 0.00001.

44004-p4
Two-pulse solitons in thermal Kerr media

[11] Genereux F., Leonard S. W. and Driel H. M., Phys. Rev. B, 63 (2001) 161101(R).
[12] Shoji I., Sato Y., Kurimura S., Lupei V., Taira T., Ikesue A. and Yoshida K., Opt. Lett., 27 (2002) 234.
[13] Koechinger W., Appl. Opt., 9 (1970) 2548.
[14] Koujelev A. S. and Dudelzak A. E., Opt. Eng., 47 (2008) 085003.
[15] Eichler H. J., Haase A., Menzel R. and Siemoneit A., J. Phys. D, 26 (1993) 1884.
[16] Frede M., Wilhelm R., Brendel M., Fallnich C., Seifert F., Wilke B. and Danzmann K., Opt. Express, 12 (2004) 3581.
[17] Vasudevan S., Chen G. C. K. and Ahluwalia B. S., Opt. Lett., 33 (2008) 2779.
[18] Andika M., Chen G. C. K. and Vasudevan S., J. Opt. Soc. Am. B, 27 (2010) 796.
[19] Wielandy S. and Gaeta A. L., Phys. Rev. Lett., 81 (1998) 3359.
[20] Yakimenko A. I., Zaliznyak Y. A. and Kivshar Y. S., Phys. Rev. E, 71 (2005) 065603.
[21] Rotschild C., Cohen O., Manela O., Segev M. and Carmon T., Phys. Rev. Lett., 95 (2005) 213904.
[22] Skupin S., Saffman M. and Krölikowski W., Phys. Rev. Lett., 98 (2007) 263902.
[23] Ye F., Kartashov Y. V., Bambi Hu and Torner L., Opt. Lett., 34 (2009) 2658.
[24] Gharaati A., Elahi P. and Cari S., Acta. Phys. Pol. A, 112 (2007) 891.
[25] Balakrishnan R., Phys. Rev. B, 32 (1985) 1144.
[26] Dikandé A. M., J. Math. Phys., 49 (2008) 073520.
[27] Sowade R., Breunig I., Tulea C. and Buse K., Appl. Phys. B, 99 (2010) 63.
[28] Ye F., Malomed B. A., He Y. and Bambi Hu, Phys. Rev. A, 81 (2010) 043816.
[29] Xu Z., Li L., Li Z., Zhou G. and Nakkeeran K., Phys. Rev. E, 68 (2003) 046605.
[30] Rhada R., Kumar V. R. and Porsezian K., J. Phys. A, 41 (2008) 315209.
[31] al Khawaya U., J. Phys. A, 39 (2006) 9679; 42 (2009) 265206.