Students’ concept formation stages in solving geometry problem based on personality types

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Abstract. Concept formation stages play an important role in developing mathematics learning. This study examined students’ concept formation stages, namely heap, complex, and concept toward Keirsey’s personality types. Twenty one students from a class were given the personality test and mathematics ability test. Regarding the test’s result, four students with higher mathematics achievement and represented each type of personality was selected as the subject of research. The data were collected from the geometry problem and interviews. The students’ concept formation stage was described based on problem solving behaviour that revealed certain stage. The results underlined that rational, guardian, and artisan students were in the complex stage. In the other hand, idealist student was in the concept stage. Rational student solved the given problem using symbol and formula directly. Idealist student solved the problem not only using symbol and formula but also representation. Artisan student solved the problem using representation and make it concrete by comparing to the other familiar figure. Guardian student solved the problem using concrete and structured ways. Regarding the diverse ways in solving problem, it suggested teachers to train the students to get used to solve any geometry problem creatively by considering students’ concept formation and personality type.

1. Introduction
Mathematics learning is identical with developing problem solving ability [6] which actually influenced by some aspects included knowledge and decision-making mechanism [8]. Moreover, mathematics learning also related with concepts that connected each other, developed deductive system, logical, and analytical [4]. It points on the process of how the concept is acquired that known as concept formation [4]. Thus, evaluating students’ concept formation stages will describe students’ capability in accommodating the knowledge while solving the problem and students’ approach in solving problems based on their capacity [1]. Especially in solving geometry problem, lack of basic knowledge, misconception, and lack of reasoning ability lead the students’ into difficulties in solving problem since geometry is a network of concepts, ways of reasoning and representation systems used to explore and analyze shape and space [5]. Thus, it is important to recognize the stages of students’ concept formation. Based on Vygostky’s theory of concept formation and Berger’s appropriation theory, it was categorized that there were three stages, i.e. heap, complex and concept [1, 2, 3]. Those stages can be identified based on students’ problem solving behavior [1].

In heap stage, the learners linked the ideas using their impression and concrete object to give a syncretic image [1, 2, 3]. In complex stage, the learners linked the ideas based on experience and association. Based on [1, 2, 3], there were four types of complex stage which were association, chain, representation, and pseudo-concept. The first type, association is tended to link the ideas by finding
the similarities between the ideas included adapting the procedure of using a certain formula. It consisted of three sub-types which were surface (focused on signifier ideas), example-oriented (adapted examples), and artificial (use familiar ideas). The second type, chain, tended to create not only connection between ideas but also the meaning of the connections that produced understanding toward the ideas. The third type, representation, tended to make the ideas more concrete by visualizing the connection of the ideas. The fourth type, pseudo-concept, tended to understand the connection between the ideas and generalize the ideas based on the experience toward concrete things. In concept stage, the learners linked the ideas systematically and logically. It consisted of two types of concept stages which were potential concept and concept. Potential concept type was actually a stage between the stages of complex to concept which formed as habit in mastering the rules of connections. The ideas were linked because of the complete process of complex stage such as association, chain, representation, and pseudo-concept. In the other side, the concept type considered as the stage that was not only able to master the rules of connection but it was also able to generalize the ideas. Related with the concept of geometry about area, developmental stages are shown on table 1.

| Stages    | Types       | Behaviors                                                                 |
|-----------|-------------|---------------------------------------------------------------------------|
| Heap      |             | Find connection between figures based on students’ impression.            |
|           |             | Unreasonable estimation (trial and error).                               |
| Complex   | Association | Compare the figures randomly.                                             |
|           | Surface     | Use incorrect formula and incorrect procedure.                           |
|           | Example – orient | Use correct formula and procedure, but use incorrect result.         |
|           | Artificial  | Use familiar figures to find connection between figures.                 |
| Chain     |             | Use third figures to compare the whole area.                             |
| Representation | Generalize area of figures by visualizing the connection.            |
| Pseudo-concept | Compare figures non-rigorously on visible area. Use correct formula with error in computational and unable to justify it. |
| Concept   | Potential Concept | Use correct formula with correct procedure in estimating visible area. |
|           | Concept     | Generalize and use the correct operation in estimating invisible area.   |

Some previous studies [1, 2, 3, 4] only focused on describing the stage without exploring the reason that might be influenced their behavior in solving problem further more. However, in this research, the stages of concept formation were also described by personality type of Keirsey. Different personality type directed to different responses of thinking and decision-making mechanism [9, 10]. Personality type could be classified based on how the students obtain their energy (introvert or extrovert), manage the information (sensing or intuitive), make decisions (thinking or feeling), and organize their lives (judging or perceiving) [9, 10]. Based on those classifications, Keirsey categorized four types of personality namely rational (intuitive-thinking), idealist (intuitive-feeling), guardian (sensing-judging), and artisan (sensing-perceiving) [10]. Some researches about Keirsey personality types showed that different personality types have different needs in solving problems [7, 9]. Instead of only observing the students’ concept formation, it was also important to consider about their personality type. Some characteristics related with the learning process of each personality type were shown on table 2.
Table 2. Characteristics of Each Personality Type

| Personality Types | Thinking Styles | Habits                                      |
|--------------------|-----------------|---------------------------------------------|
| Rational           | Intuitive       | Contemplative, logical, analytical, systematic and efficient Use less words or representatives in explaining their ideas |
| Idealist           | Intuitive       | Imaginative and creative Use less words or representatives in explaining their ideas |
| Guardian           | Sensing         | Structured, consistent and routines Prefer to describe factual things in words |
| Artisan            | Sensing         | Practical and realistic Use descriptive words to express ideas |

Keirsey personality type focused on describing the learning process that included decision-making, information processing, preferences, and organization [9, 10]. Thus, this study tried to describe students’ concept formation based on personality type of Keirsey. Personality By solving a given geometry problem, students with different personality types might indicate different problem behaviors which refers to a certain concept formation stage.

2. Method

This study used qualitative approach to examine grade seven students’ concept formation stages in solving a geometry problem. The research was held in SMP Mawar Sharon Surabaya. A class out of four classes, 7D, was chosen purposively. Twenty one students from that chosen class were given the Keirsey’s personality test, mathematical test, and geometry problem. Using Keirsey temperament sorter, it was identified that there were 3 rational students, 2 idealist students, 6 guardian students, and 10 artisan students. In the other side, based on mathematical test and geometry problem, it was obtained that there were 12 students with high achievement, 5 students with medium achievement, and 4 students with low achievement. Regarding on these results, four students with the highest rank who represent each type of Keirsey’s personalities were selected as the participants. The process of choosing the participants was showing in the following flowchart.

![Figure 1. Flowchart of choosing the participants process.](image-url)
All participants were interviewed individually based on their answer to ensure and validate their behavior. Predicted stages and exemplar of behavior from the given problem was shown on Table 3.

Table 3. Predicted Stages and Exemplar of Behavior

| Predicted stages | Exemplar of Behaviour |
|------------------|-----------------------|
| Heap             | Estimate the area of figures by guessing or trial and error. |
| Complex          | Estimate the area using the formula of figures but having incorrect calculation. |
|                  | Estimate the area of figures using representation by fitting the figures into. |
| Concepts         | Estimate the area using correct formula and procedure of using formula. |

The obtained data were analyzed qualitatively based on the indicators of concept formation stages and each characteristics of personality type.

3. Result and discussion
Since each personality type has different way to solve problem, the results of this study would describe the students’ concept formation stages which were revealed by the used approach and revealed behavior that was shown in solving the problem through each Keirsey’s personality type. It can be analyzed through their workings of problem and interview results. Rational student was coded as S1, idealist student was coded as S2, guardian student was coded as S3, and artisan student was coded as S4.

3.1. Concept formation stage of rational student
It was observed that the S1 got incorrect answer and conclusion. S1 was able to identify the figures by writing the names of each figure correctly and put the symbols using “A” for parallelogram, “B” for trapezoid, “C” for rhombus. S1 substituted the value of $a$ with 2 and then computed and calculated it directly without writing the formula. By doing substitution, he got the area of parallelogram was 4, the area of trapezoid was more than 4, and the area of rhombus was 4. Those results made him answering that parallelogram had the same area with rhombus and the trapezoid had the biggest area as shown in the figure 1.
Some questions were asked in order to clarify S1’s answers. Based on the interview, S1 explained that substituting the value of $a$ with 2 was the easiest way to estimate the area of the figures. S1 also declared that he was not sure about the formula which was used because, actually, he forgot the formula especially the formula of trapezoid and rhombus. Those interview’s records were shown below:

R : “Why you chose to substitute the value $a$ with 2?”
S1 : “Because it was the easiest way to predict the area if I used numbers”.

R : “How did you get these results of area?”
S1 : “I used the formula of base x height for the area of parallelogram. Since $a$ was equal to 2, so I got 4 for area of parallelogram. For trapezoid, I saw that it was consisted of square and 2 right angle triangles, so the area might be more than 4. Then, for rhombus, I used the formula of diagonal 1 x diagonal 2.”

R : “Were you really sure with the formula that you had used?”
S1 : “Actually, I was not sure. Only the formula of parallelogram which I really felt sure. I forgot the formula of trapezoid and rhombus.”

Regarding on S1’s written and interview results, it showed that he used correct formula in estimating the area of parallelogram and used incorrect formula and procedure in estimating the area of trapezoid and rhombus. Those were indicated that S1 was in stage of complex – association – surface since S1 estimated area of figures using incorrect formula and procedure. These caused S1 making the incorrect conclusions. S1 realized that it was because S1 forgot the formula of trapezoid and rhombus. S1 tended to use the easiest way solve problem by reducing the use of words. S1 preferred to use symbol or variable in solving the problem. These behaviours were in a line with the characteristics of rational student which tended to think contemplatively, logically, analytical, systematic and efficient [10]. Rational students also tended to use less words or representatives in explaining their ideas [9, 10]. Thus, they prefer to straight to the point and use the symbol or variable to explain their ideas. In solving geometry problem, [7] also described that rational student applied relevant thinking of the situation toward the formula used and realized if there is any mistake in thinking process.

### 3.2. Concept formation stage of idealist student

The findings show that S2 answered and made conclusion correctly. S2 was able to recognize that trapezoid and parallelogram had the same area. S2 converted the parallelogram into square with length of side $a$ and did same way toward the trapezoid. Then, S2 estimated the area of those two squares as
In order to show that the trapezoid and parallelogram had the same area. Using that square, he fitted the rhombus with diagonals of $a$ on that square. S2 obtained that there were left over areas between the square and rhombus. S2 also concluded that rhombus was smaller than the square. Thus, S2 was able to order that the smallest figure was the rhombus and then trapezoid and parallelogram had the same area which bigger than the rhombus. Overall, in answering the problem, S2 visualized the explanation using picture and using symbols as shown in figure 2.

**Figure 3. Idealist Student’s Answer**

In order to clarify S2’s answers, some questions were given orally after S2 had solved the problem. Based on the interview, S2 explained that, by drawing the explanation, it helped S2 to create imagination. S2 also declared that by using square, the other familiar figure which not mentioned in problem, it helped S2 to estimate the area of parallelogram and trapezoid easily without memorized the formulas. Those interview’s records were shown below:

R : “How did you conclude that the parallelogram has the same area with the trapezoid?”

S2 : “I moved out a right angle triangle from a parallelogram to the right so that it became a square”.

R : “What about the trapezoid?”

S2 : “I did the same thing. I changed it to be a square. I just rotated a right angle triangle from a trapezoid and then moved it out to the right.

R : “Why did you do that?

S2 : “It made me easier to estimate the area without memorized the formula.”
R : “How could you sure that those figures formed a square with the same area?”
S2 : “I remembered that a square had the same length of sides. Then, I saw that both of them had the same length of side which was a. So, the area supposed to be the same too.”
R : “What about the rhombus?”
S2 : “It had the length of diagonals a.
R : “How could you know that the area of rhombus was smaller than the other figures?”
S2 : “I just fitted the square that I got from the trapezoid and parallelogram on the rhombus. Then, I could see that the areas of the square were not covered all by the rhombus. So, the rhombus supposed to be smaller than the square”
R : “Why did you draw your answers?”
S2 : “Because it helped me to move out or rotate the figures.

Based on S2’s written and interview results, it showed that S2 was able to use the other familiar figure, square, to manipulate and estimate the area of parallelogram and trapezoid without using the formula directly. S2 was also able to estimate the area of rhombus by comparing the left over areas. Those were indicated that S2 was in stage of concept – potential concept since S2 compared the area of figures systematically and was able to generalize the formula. These caused S2 making the correct conclusions. All idealist students tended to think intuitively that more interest toward ideas and concepts rather than concrete things [10]. They also tended to use fewer words to explain their ideas by using symbols or variables [9, 10]. Since they were creative ones, they also preferred to visualize their ideas using representatives even though they were able to think abstractly [8]. In solving geometry problem, [7] also described that idealist student had intuitive nature could process information by seeing the patterns and connections.

3.3. Concept formation stage of guardian student
It was indicated that S3 was not able to make a correct conclusion. Related with S3’s answer, S3 wrote that the parallelogram was the first order, the trapezoid was the second order, and then the rhombus was the last order of having smaller area. S3 also wrote that the parallelogram had the same area with the trapezoid. S3 described the answer by drawing the trapezoid and then showed that when one of the right angle triangles from the trapezoid was switched, it formed a parallelogram with the same measurement. In the other way, S3 estimated the area of rhombus using the area of right angle triangle. S3 substituted the value of a with 10 and then S3 obtained that the height and the base of the triangle were 5. Then, S3 directly operated those values into the formula $\frac{1}{2} \times base \times height$ and it was obtained 12.5 cm$^2$ as the area of a right angle triangle. Since a rhombus was consisted of four right angle triangles, then it was obtained 50 cm$^2$ as the area of rhombus. Those responses were shown in the figure 3.
Some questions were asked in order to clarify S3’s answers. Based on the interview, S3 explained that she just ordered the area of figures like S3 was usually order something; from the biggest to the smallest without paid attention to the instruction. These caused her making incorrect conclusion. S3 also explained that by switching one of the right angle triangles from the trapezoid, it helped her to form the same parallelogram. S3 could not able to find the relation between the results of rhombus area and the area of parallelogram and trapezoid. S3 just guessed that the rhombus having the smallest area. Those interview’s records were shown below.

R : “Why did you order the parallelogram as the 1st, the trapezoid as the 2nd, and then the rhombus as the last order?” What did it mean?
S3 : “Because the parallelogram and the trapezoid have bigger area than the rhombus.”
R : “Did you know that you have to order from the smallest to the biggest?”
S3 : “I just used to order something from the biggest to the smallest. I did not pay attention on the instruction carefully”
R : “How did you know that the trapezoid had the same area with the parallelogram?”
S3 : “I switched one of the right angle triangles from the trapezoid and then it formed a parallelogram. They had the same length of side of a.”
R : “Why did you do that?”
S3 : “It helped me to form the same parallelogram.”
R : “What about the area of rhombus?”
S3 : “I did have any idea about the formula. I just knew that it consisted of 4 right angle triangles. Then, I looked for the area of a triangle by substituting a with 10, so I could get the area of triangle 12.5 by using the formula. Since rhombus consisted of 4 triangles, so the area of rhombus were 4 times of the triangles which was 50.”
R : “By getting those results, how did you know that the rhombus having smaller area or bigger area than trapezoid and parallelogram?”
S3 : “Actually, I did not know. I just saw and guessed that it was smaller than the other.”

Regarding on S3’s written and interview results, it showed that S3 was able to find connection using familiar figures, parallelogram and right angle triangle, in estimating the area. But, S3 did not able to find connection between the results of rhombus’ area, trapezoid area and parallelogram area since S3
did not really familiar with rhombus. S3 just guessed that rhombus had the smallest area than the other. Those were indicated that S3 was in stage complex-association-artificial. All guardian students tended to be sensing ones which preferred to work with something that concrete, familiar, and presented clearly [10]. S3 declared that S3 did not pay attention to the instruction carefully and just thought as S3’s habit in ordering something which was from the biggest to the smallest. These behaviours were in a line with the characteristics of guardian student who tended to think and do something in routines [9, 10]. In solving geometry problem, [7] described that it would be difficult for guardian student to connect the situation with the formula or concept used without clear guidance.

3.4. Concept formation stage of artisan student

It was noted that S4 made correct answer and conclusion. S4 explained the answer as descriptive and informative sentences. S4 wrote that trapezoid could be converted into a square. S4 also wrote that when the rhombus was put inside that square there was remaining space between. S4 explained that when the trapezoid and the parallelogram were converted into squares, there was no remaining space when those squares were fitted on. S4’s descriptions were shown in figure 4.

![Figure 4. Artisan Student’s Answer](image)

Translation:

If the trapezoid was converted into a square so the rhombus could be fitted on it and still had the left over area to make another rhombus.

Trapezoid and parallelogram could be converted into a square and there was no left over area when it was fitted on.

**Figure 5. Artisan Student’s Answer**

In order to clarify S4’s answers, some questions were given orally after S4 had solved the problem. Based on the interview, S4 explained that S4 easily described something in words. S4 was able to compare the area by converting to familiar figures and then compare the remaining area which covered by fitting the figures. Those interview’s records were shown below:

R : “Why did you explain your answer in those complete sentences?”
S4: “Because it was helped me to share ideas easily.”

R: “How did you get the conclusion that the rhombus was the smallest area and both trapezoid and parallelogram had the same area?”

S4: “I compare the trapezoid with the rhombus first. If the trapezoid was converted into a square and the rhombus was put inside it, there would be remaining space.”

R: “What did you mean by putting the rhombus inside the square then you would be able to make one more square?”

S4: “I saw that the rhombus has the same diagonals. If it was rotated, it could be considered as a square.”

R: “Did you mean that that square was the same square with the previous one that you obtained from the trapezoid?”

S4: “No. It smaller than it.”

R: “What did you know about the areas of trapezoid and parallelogram?”

S4: “Those were the same.”

R: “How could you sure that those had the same area?”

S4: “If the trapezoid and the parallelogram were converted into squares and then both of them were fitted on, there would be no remaining space.”

Analyzing S4’s written and interview results, it was showed that S4 was able to find connections using familiar figures, square, in estimating area and also able to compare the remaining area by fitting on the figures. Those behaviours showed that S4 tried to make the representation of figures more concrete by using another familiar figure which is square. S4 also almost confused between the concept of rhombus and square, but she was able to make it clear by looking at the rhombus that had the same diagonals. It was in a line with the characteristics of the artisan student who tended to think by sensing which concerned about concrete thinks [9, 10]. Regarding on her answers, it showed that she explain the answers in words which gave clear descriptions. This behaviour was described the way artisan students expressed their ideas. They tended to be more theoretically and descriptively, but in the other side their descriptions were realistic and practical [9]. In solving geometry problem, [7] described that artisan student not really good in applying formula but very good in identified strategy using concrete and clear things like representation and description.

Based on results and discussions above, the behaviours revealed by each participant were summarized on table 4.

| Personality Types | Predicted Stages | Exemplar of Behaviour | Habits |
|-------------------|------------------|-----------------------|--------|
| Rational (S1)     | Complex          | Used incorrect formula and incorrect procedure in estimating area of trapezoid and rhombus. | Find the fastest and easiest way to solve problem. Think abstractly. Realized the mistake |
| Idealist (S2)     | Concept          | Compared the area of figures by finding the left over area and generalize the formula. | Find connection by ideas and representation. Use picture, symbol and variable rather than words |
Table 4. Revealed Stages by participants and Exemplar of Behavior

| Personality Types | Predicted Stages | Exemplar of Behaviour | Habits |
|-------------------|------------------|-----------------------|--------|
| Guardian (S3)     | Complex          | Guessed in relating the area of rhombus with the area of parallelogram and trapezoid. Converted trapezoid into parallelogram to find that both of them had the same area. | Misunderstood the instruction. Stick to the routines |
| Artisan (S4)      | Complex          | Converted trapezoid and parallelogram into square to prove that they had the same area and fitted that square on rhombus to find that the rhombus was smaller than the square. | Practical and realistic; use familiar figure. Use descriptively words |

Regarding on the results, it underlined that each personality type had their own approach in solving geometry problem that obviously represented the way of their thought.

4. Conclusion

Regarding on the behaviour in solving geometry problem, rational student was in stage of complex and idealist student was in stage of concept. Both of rational and idealist were able to think abstractly. Rational student tended to invent the most efficient and fastest way to solve problem which somehow leaded him into mistake. Idealist student tended to find the most interesting and meaningful way to solve problem which directed him think intuitively and creatively. For guardian and artisan students, both of them were in the stage of complex. They were able to think concretely. Guardian student had difficulties in understanding unfamiliar task or concept instead she already knew it as routines or familiar knowledge. She needed clear instructions or guidance in order to activate her factual knowledge to associate the ideas since she tended to be structural and procedural in solving problem. As like guardian student, artisan student easily working with concrete ideas, but in the other way she tended to be realistic and practical in solving problem. In this study, every student showed different ways to think and solving problem. It leads mathematics education not only focus on improving problem solving skills and concept formation only. Therefore, in developing mathematics learning, it is suggested that educators need to train students to get used to solve any geometry problem creatively by considering also personality types. It would be more useful for further study if examine and explore the tendency of concept formation stages regarding their problem solving behaviour in a certain type of personality with different mathematics achievement.

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References

[1] Zhang P, Manouchehri A and Tague J 2013 Capturing middle school students’ understanding of the concept of area using Vygotsky’s concept formation theory Proc. of the 35th annual conf. of the Psychology of Mathematics Education – North American, Chicago, IL
[2] Berger M 2004 Heap, complex and concept (part 1) For the Learning of Mathematics, 24(2), pp 2-6
[3] Berger M 2004 Heap, complex and concept (Part 2) For the Learning of Mathematics, 24(3), pp 11-17
[4] Michael KM and Marc S2013 ‘The Area of a Triangle is 180º’—An Analysis of Learners’ Idiosyncratic Geometry Responses through the Lenses of Vygotsky's Theory of Concept Formation A frican Journal of Research in Mathematics, Science and Technology Education, 17 (1-2) pp 83-93
[5] Özerem A 2012 Misconceptions in geometry and suggested solutions for seventh grade students Procedia - Social and Behavioral Sciences 55 pp. 720-729
[6] Van de Walle J A, Karp K S and Bay-William J M 2013 Elementary and Middle School Mathematics: Teaching Developmentally (New Jersey: Pearson Education Inc)
[7] Fitriana L D, Fuad Y and Ekawati R 2017 Student’s Critical Thinking in Solving Open-Ended Problems Based on Their Personality Type Journal of Physics: Conf. Series, 947(1), pp 1-7
[8] Schoenfeld A H 2013 Reflection on Problem Solving Theory and Practice The Mathematic Enthusiast 10 (1) pp 9-34
[9] Huitt W G 1992 Solving and Decision Making: Consideration of Individual Differences Using the Myers-Briggs Type Indicator Journal of Psychological Type 24 (1) pp 33-44
[10] Keirsey D 1998 Please Understand Me II. (Del Mar: Prometheus Nemesis Book Company)