The numerical analysis on the buoyancy-driven conjugate heat transfer and fluid flow under the influence of magnetic field in the cubic cavity using OpenFOAM

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Abstract. The influence of Lorentz force on the heat propagation in the conjugate cavity with various Rayleigh number (Ra) and Hartmann number (Ha) is explored in this study. The conjugate thermal and electromagnetic coupling is considered for the fluid-wall interface in the present analysis and includes the wall thickness within the computational domain. The computation of electric potential field and conduction heat transfer is enabled in the wall domain. The numerical code for conjugate buoyancy-driven convective flow with magnetohydrodynamics principle is developed on the open source platform OpenFOAM. The developed solver is capable to simulate steady and unsteady flows on complex geometry. The influence of thermal Rayleigh number (Ra) on the fluid flow is also investigated in this study for different Ra range. The outer vertically opposite side walls are kept isothermal and remaining outer walls are maintained as thermally insulated. The inner vertically opposite walls are kept thermally conductive and other walls of the fluid domain are maintained as insulated. The magnetic field is actuated normal to the temperature gradient in the range of Ha = 0 - 100, with corresponding Rayleigh number of Ra = 10⁴ – 10⁶. The enforced magnetic field on the enclosure generates the Lorentz force, which is the consequence of the interaction between electric current and magnetic field. The generated Lorentz force is responsible for the retardation in the heat and fluid flow and hence the Nusselt number.

Keywords: OpenFOAM, Conjugate Heat Transfer, Lorentz Force, Magnetic field.

1. Introduction

The conjugate heat transfer method termed the communication between the conduction heat transfer and the convection heat transfer of fluid beside a solid surface. In various real-world applications, such as nuclear reactors, heaters, heat exchangers, pipe insulation arrangements, solar energy collector, etc., the influence of conduction within the solid surface is important and must be considered in the analysis. Therefore, the study of conjugate heat transfer phenomena keeps essential coupling of the convection in the fluid surrounding the conduction in the solid body. The convective heat transfer within the cavity filled with the non-magnetic and electrically conducting fluid under the impression of magnetic field has several industrial application such as to control the flow of fluid and heat transfer in the crystal growth in casting process, floating glass production, smelting, solidification or magnetic force is operated for separators in lubricants and dryers [1]. Several researchers focused on forced convection heat transfer in tubes with axially fluctuating thermal boundary situations at wall neglect the wall thickness and its conductance, i.e., the conjugate effect [2-3]. Serrano-Arellano et al. [4]
numerically examined the effect of buoyancy produced by the temperature gradient in the conjugate heat transfer between two air fluids circulating in the parallel pipes.

### Nomenclature

| Symbol | Description |
|--------|-------------|
| $U$    | velocity of fluid, m s$^{-1}$ |
| $p$    | Pressure, N m$^{-2}$ |
| $T$    | Temperature, K |
| $T_{ref}$ | Ambient Temperature, K |
| $j$    | Electric current density, A m$^{-2}$ |
| $B$    | Applied magnetic field, kg s$^{-2}$ A$^{-1}$ |
| $g$    | Acceleration due gravity, m s$^{-2}$ |
| $\phi$ | Electric potential, m$^2$ kg s$^{-3}$ A$^{-1}$ |
| $L$    | Side length of cube, m |
| $Gr$   | Grashof number |
| $Ha$   | Hartmann number |
| $Ra$   | Rayleigh number |
| $F$    | Lorentz force ($j \times B$), N m$^{-3}$ |

### Greek symbols

| Symbol | Description |
|--------|-------------|
| $\alpha$ | Thermal diffusivity, m$^2$ s$^{-1}$ |
| $\beta$ | Coefficient of thermal expansion, K$^{-1}$ |
| $\rho$ | Fluid density, kg m$^{-3}$ |
| $\sigma$ | Fluid electrical conductivity, s$^{-3}$ A$^2$ m$^{-3}$ kg$^{-1}$ |
| $\nu$ | Kinematic viscosity, m$^2$ s$^{-1}$ |

MHD flow with heat transfer method is nowadays a significant investigating area due to their possible implementations in industrial fields and engineering area. MHD power generators, actuators, accelerators, and chilling of nuclear reactors and casting industries are included in this area. Therefore, an extensive amount of exploration has been consummated on the consequence of electrically conducting fluids such as water, liquid metals or water mixed with an acid and others in the incidence of the orthogonal magnetic field on the heat transfer and fluid flow physiognomies over different domains. For example, heat dissipation in natural convection between the mercury and a horizontal cylinder or solid conducting body placed in the cavity under the stimulus of the magnetic field are the field of consideration [5-7].

It is observed from the above findings that the several researchers had discussed the effect of the wall conductance and magnetic field on the behavior of flow pattern and heat transfer, which is affected by the occurrence of the electric current and its flow. However, the scarce attention is given towards the study of the distribution of electric current in the domain for buoyancy driven flow in the cavity. In this study, the solver is developed for conjugate heat transfer in buoyancy-driven flow under the consequence of the magnetic field in the OpenFOAM using electric potential formulation. The Lorentz force is activated in the present code. Therefore, the directional influence of Lorentz force on the behavior of flow and heat dissipation can be easily traced. Hence, the magnitude wise distribution of electric current and its outcome on the flow pattern and heat transfer is conveyed in the present study.

### 2. Mathematical formulations, Problem definition, grid independence and validation test

The cubical enclosure with the thickness of the wall is taken into consideration ($t_w = 0.06 \times L$). The outer vertically opposite sides are maintained as isothermal and all other outer sides are as thermally insulated. The side walls of the solid block and fluid domain is kept as electrically insulating. The wall conductance ratio ($C_w = \sigma_{e} \times t_w / \sigma_{f} \times L_f$) are kept constant for all the simulation as $C_w = 0.01$. The numerical simulations are performed for three different Rayleigh number ($Ra = 10^4$, 10$^5$, and10$^6$) and corresponding Hartmann number varies in the range of $Ha = 0$, 50 and 100 for fixed Prandtl number of $Pr = 0.71$. The magnetic field is imposed in the direction parallel to the isothermal walls. The side length ($L_f$) of the inside block is kept as one. The schematic view of the test case and the mesh distribution in the cavity at the plane $z = 0.5$ is shown in Figure 1 (a and b).
Figure 1. (a) schematic view of conjugate cubic block, (b) Mesh distribution at the $z = 0.5$

Let us considered the developed laminar buoyancy-driven flow in the cubic cavity heated from the outer wall of the domain. The physical properties of the walls and fluid are assumed to be independent of temperature. The incompressible, viscous, and electrically conducting fluid is taken into consideration. The thermal and electrical conductivity of the wall is assumed to be consistent. So, the governing equation for the fluid domain can be written as follows:

Continuity equation:
$$\nabla \cdot \dot{U} = 0$$

Momentum equation with Lorentz force and Boussinesq approximation:
$$\frac{\partial \dot{U}}{\partial t} + (\dot{U} \cdot \nabla) \dot{U} = -\nabla p + \rho \nabla^2 \dot{U} + \frac{j \times B}{\rho} - \beta (T - T_n) \dot{g}$$

Ohm’s law Current density for fluid:
$$\dot{j} = \sigma (-\nabla \phi + \dot{U} \times B)$$

Conservation of charge:
$$\nabla \cdot \dot{j} = 0$$

Conservation of thermal energy of fluid:
$$\frac{\partial T}{\partial t} + (\dot{U} \cdot \nabla) T = \nabla \cdot (\alpha_f \nabla T)$$

The poisons equation for an electric potential for the fluid domain is obtained by comparing the Ohm’s law of current density (Eq. 3) and conservation of charge (Eq. 4) is written as follow:
$$\nabla^2 \phi = \nabla \cdot (\dot{U} \times \dot{B})$$

The thermal energy and Laplacian equation of an electric potential is solved in the solid domain to get the temperature distribution and electric flow within the wall respectively. The governing equation in the solid wall is written as follows:
$$\frac{\partial T}{\partial t} = \nabla \cdot (\alpha_s \nabla T)$$
$$\nabla^2 \phi = 0$$

Where $\alpha_f$ and $\alpha_s$ are the thermal diffusivity of fluid and solid respectively. The fluid-solid interface conditions are mandatory to solve the governing equations for conjugate heat transfer in the FVM.
$T_f = T_i$
Where \( T_f \) and \( T_s \) is the temperature of fluid and solid on the solid-fluid interface and \( n \) is the normal direction to the interface. The three non-dimensional numbers are incorporated in this study to analyse the fluid behaviour under the domination of the magnetic field are as follows:

\[
Ha = BL_f \sqrt{\frac{\sigma}{\rho \nu}} \\
Ra = \frac{g \beta (T_f - T_w) L_f^3}{\nu \alpha} = Gr \cdot Pr \\
P r = \frac{v}{\alpha}
\]

Here, the \( Ha \) is the Hartmann number, it measures the strength of the magnetic field intensity. The \( Ra \) is the Rayleigh number, the product of Prandtl number (\( Pr \)) and Grashof number (\( Gr \). Here \( L_f \) stands for the characteristic length is kept as the heated side length of the fluid domain. The Direct numerical simulation is executed to solve the entire set of the equations (Eqn. 1-10) by the present solver in FVM. To analyse the fluid flow in natural convection, the Boussinesq approximation is activated in the solver. The velocity is used in the electric potential equation to determine electric potential, which will be further used to obtain electric current and hence Lorentz force. The electromagnetic force acts as source term along with buoyancy force in the momentum equation. The second-order central difference scheme is used to discretize the convection and diffusion term. The temporal term is discretized by first-order accurate Euler scheme. Table 1 shows the boundary condition and non-dimensional number used in the present study to perform computation.

### Table 1. Boundary specifications and non-dimensional values

| Parameters | Outer walls (solid) | Fluid-Solid interface |
|------------|----------------------|-----------------------|
| \( U \) | Side 1 | Side 2 | Other sides | Side 1 | Side 2 | Other sides |
| \( p \) | - | - | No slip |
| \( T(K) \) | 350 | 300 | \( \partial T / \partial n = 0 \) | \( T_f = T_w \) | \( T_f = T_w \) | \( \partial T / \partial n = 0 \) |
| \( \phi \) | \( \partial \phi / \partial n = 0 \) | \( \partial \phi / \partial n = 0 \) |
| \( B \) | Fixed value | Fixed value |
| \( Ha \) | 0, 50, 100 |
| \( Ra \) | \( 10^3, 10^5 \) and \( 10^6 \) |
| \( Pr \) | 0.71 |

The results obtained from the present solver has been verified against the data shown in the published paper by Serrano-Arellano et al [4]. The boundary condition and geometrical information are maintained similar as mentioned in the literature [4]. Figure 2 (a) Show the comparison of the temperature profile obtained for parallel flow heat exchanger against the result shown by the reference [4]. The mesh size for present study is decided by grid independence analysis performed on three different grid size for the \( Ra = 10^6 \) and \( Ha = 100 \) as follows; grid 1 = 61 \( \times \) 61 \( \times \) 61, grid 2 = 71 \( \times \) 71 \( \times \) 71 and grid 3 = 81 \( \times \) 81 \( \times \) 81. The non-uniform mesh with near wall spacing is maintained as \( 10^{-3} \) and 12% increment in all the direction is chosen. The uniform spacing of grid with 10 grid in the solid domain is taken into consideration for computation in the solid domain. Figure 2 (b) shows the grid independence result and hence it can be concluded that the grid 2 has enough mesh quality to accommodate all the boundary layer physics and can be used for the further simulation in the present study.

### 3. Results and Discussions

In this section, the computational investigation of natural convection flow inside the cavity of the cube and the temperature variation in the solid wall as well as the fluid domain is shown for MHD and without MHD conditions. The non-dimensional Hartmann number (\( Ha \)) is used here to vary the
magnetic field intensity. The ranges of $Ha$ considered for the present study are $Ha = 0, 50$ and $100$ for corresponding Rayleigh number of $Ra = 10^4, 10^5$ and $10^6$ at fixed Prandtl number of $Pr = 0.71$. The boundary specifications and non-dimensional values are shown in the following Table 1. Figure 3 shows the distribution of normalized electric current ($j/j_{\text{max}}$) in the $y$-$z$ plane at $x = 0.5$. The solid-fluid interface is kept as insulated thus the electric current is not allowed to flow outside the fluid domain and hence the impact of the electromagnetic force is increased within the fluid domain. The temperature is allowed to flow in the $x$-direction, hence the velocity gradient is accessible in $x$ and $y$-direction only, the $z$-components of velocity has negligible value hence neglected in the Lorentz force calculation. The magnetic field is imposed only in the $y$-direction, therefore the electric current density ($j \approx U \times B$) is generated in the $z$-direction as shown in Figure 3. The Lorentz force ($j \times B$), which is proportionate to the square magnitude of the magnetic field ($B^2$) and electric current is obtained in the $x$ and $y$-direction only with increasing strength for the higher magnetic field.

The Rayleigh number plays a substantial role in the up-surge the heat transfer in the natural convection process. The rise in $Ra$ number, viscosity of the fluid be likely to reduce since all the constraints are set as constant, thus Rayleigh number and viscosity are associated as inversely proportional. Consequently, for higher $Ra$, the unsteadiness in the flow is higher, which results in the higher convection flow and heat dissipation. The following figure 4 shows the variation of the horizontal velocity in the $x$-direction at different Hartmann number as well as Rayleigh number. It is understood from the Figure 4 (a) that the velocity of fluid suppressed after application of magnetic field at higher Hartmann number ($Ha = 100$) and convectional flow, as well as heat transfer reduces. The buoyancy force at higher $Ra$ is more than the viscous force which decelerates the horizontal components of velocity and further amplification of magnetic field pauses the motion of fluid in the domain, which leads to suppression of heat flow in the domain.

![Figure 2](image1.png)

**Figure 2.** (a) Validation of present result with the reference [4]. (b) Streamwise velocity comparison in the $y$-direction at plane $x = 0$ for three different grids at $Ra = 10^6$ and $Ha = 100$.

![Figure 3](image2.png)

**Figure 3.** Normalized Electric current distribution in the $y$-$z$ plane at the $x = 0.5$ for $Ra = 10^6$ at (left) $Ha = 50$ and (right) $Ha = 100$. 
The fluid gets heated and the system becomes unsteady beyond the critical Rayleigh number ($Ra_c = 1000$) and fluid starts to appear in the regular sequence of convection rolls. If the fluid is electrically conducting and the magnetic field is imposed on the system, then the natural convection exhibit remarkably distinct flow pattern than the ordinary hydrodynamic buoyancy. Figure 5 shows the streamlines at different $Ra$ and corresponding maximum $Ha$. The heat distribution within the solid domain drops the temperature up to the solid-fluid interface and hence the heating effect of fluid is lesser compare to the cooling. Therefore, the convection rolls formation is not evenly distributed in the domain as shown in Figure 5. The fluid tends to cool in the half of the length as it approaches to cooling side and return back to the heated solid-fluid interface without rolling around the cold solid-fluid interface. After imposing the magnetic field the cooling effect of fluid is further reduced and hence the flow is shifted towards the heated solid-fluid interface. The shift in the flow of heat from convection to conduction occurs due to the generation of Lorentz force. The interaction between the electric current density and the intensity of magnetic field leads to produce the opposing Lorentz force. The generated Lorentz force is solely responsible for the flow suppression which retards the heat dissipation of fluid and hence the convection rolls of fluid are suppressed and the diffusion heat transfer within the fluid increases at the higher intensity of the magnetic field. The heat dissipation is faster at $Ha = 0$ and with increasing $Ra$, this is so because the fluid becomes lighter at higher $Ra$ and flows freely within the domain causing the faster convective heat dissipation. Therefore, it is seen from Figure 6 (a, c and e), that the temperature distribution for $Ha = 0$ has propagated throughout the core area and gravitate down near the cold surface. The convective heat transfer in the domain is represented by the horizontal isotherms in the core area as the $Ra$ increases the horizontal isotherms occupy mainly in the core volume and the vertical isotherms shrink into the fine layer towards the isothermal solid-fluid interface. The vertical isotherms are present only near to the isothermal walls, it means that the dominant conductive heat transfer changes to convectional heat transfer or fluid flow as the $Ra$ increases. The boundary layers for the temperature develops near to the isothermal walls, and it tends to thinner as $Ra$ increases. Although, thermal boundary layers do not seem to appear near to insulated walls. The further increase in the Rayleigh number causes the chaotic phenomenon for the velocity as well as the isotherms, which means heat transfer is then time-dependent. As soon as the magnetic field is applied the convection flow is retarded and thus, the temperature distribution reduces gradually for higher Hartmann number for all $Ra$ numbers. The variation of temperature distribution with the implementation of the different magnetic field at various Rayleigh number is shown in the following.

![Figure 4](imageURL)

**Figure 4.** $U_x$ in the $x$-direction for different magnetic field at (a) $Ra = 10^4$, (b) $Ra = 10^5$ and (c) $Ra = 10^6$.

Figure 6 (b, d and f). At higher Hartmann number, the Lorentz force flows in the reverse direction of motion of the fluid flow and damps the motion of fluid produced from buoyancy. Therefore, it can be understood from the Figure 6 (b, d and f) that the increase in the $Ha$ shows the change in the isotherms profile, the profile of isotherms changes from horizontal isotherms to almost vertical at low Rayleigh number ($Ra = 10^4$) and $Ha = 100$, and the intensity of the magnetic field needs to be higher for higher $Ra$ to achieve the diffusion heat transfer. Hence, for the higher $Ra$, the applied magnetic field is lesser
effective to suppress the flow and heat transfer. The isotherms are shown in Figure 6 (d and f) approaches to shift in the vertical direction for the given magnetic strength, hence diffusion heat transfer at higher Ra can only be possible at higher Hartmann number. This is so, because the buoyancy force for higher Ra is more than the viscous force, in order to drop the buoyancy force and Lorentz force has to be more. Therefore, the requirement of magnetic field intensity is higher to shift in the change of heat dissipation from convection to diffusion.

4. Conclusions
The present study deals with numerical simulation of the conjugate heat transfer and buoyancy-driven natural convection flow with MHD and non-MHD phenomena. The magnetic field is forced in the vertical y-direction and the temperature gradient is maintained in the x-direction. It is observed from the result that the heat dissipation from the walls is faster at the higher Ra, this is because of the faster movement of fluid near to the heated wall. The dissipation of heat from the wall tends to reduce as the magnetic field intensity rises. The increase in the magnetic field drops the flow of fluid and hence the convection heat transfer shifted towards the diffusion heat transfer. The temperature isotherms show the effect on the magnetic field on the heat dissipation mode. For the non-MHD case, the heat transfer near the wall is diffusion and convective heat transfer in the central area of the cube. When the magnetic intensity is imposed on the system the, the intermediate phase of heat dissipation starts within the domain, the diffusion heat transfer near the wall to central area is approaching as dominant at lower Ra and at higher Ra, the dominance of convective heat transfer is present though magnetic force is there. This indicates that the lower magnetic strength at higher Ra does not have severe influence on the flow of the fluid. The severity of Lorentz force can be achieved by increasing the magnetic field in the fluid domain and the buoyancy force in the fluid can be suppressed down as per the required level to achieve desired heat transfer mode or to control the heat flow within the domain.

**Figure 5.** Streamlines comparison at different Ha, (a, b) $Ra = 10^4$, (c, d) $Ra = 10^5$, and (e, f) $Ra = 10^6$. 
Figure 6. The isotherms variations at various $Ha$ at, (a, b) $Ra = 10^4$, (c, d) $Ra = 10^5$, and (e, f) $Ra = 10^6$.  

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