Development of a semi-empirical method to determine the efficiency of a gamma radiation detector

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Abstract. In the present work we present the development of a new method to calibrate the Full energy peak (FEP) efficiency of a gamma radiation detector. The method consists of the experimentally determination of the detector efficiency for an energy using a monoenergetic gamma source and from this value to extrapolate constructing the complete detector efficiency curve using the physics of first principles of gamma detection theory. For this reason the proposal method corresponds to a semi-empirical method.

In this first work we show the application and the study for the validation of the method by means of simulations based on the Monte Carlo method using FLUKA code. The energies considered in the simulations correspond to those emitted by the radioisotopes $^{137}\text{Cs}$, for the determination of the efficiency for one energy we will call reference efficiency, and $^{152}\text{Eu}$, to perform the extrapolation of the intrinsic efficiency from the reference efficiency.

So far, we have calibrated the total instrinsic efficiency of a bare cylindrical detector by applying the proposed method. A systematic difference between the expected values of the intrinsic efficiency obtained by statistical count for the $^{152}\text{Eu}$ gamma energies, and those determined by the proposed method is observed. Despite this, the results show a good agreement with relative biases less than $5\%$ in most cases.

One of the principal advantages of this method is that it let us obtain a calibration curve with all phenomenology involved in the detection, such as the resonance peaks.

1. Introduction
The gamma spectrometry using a Hiper Pure Germanium (HPGe) detector is the most popular technique for the determination and quantification of the radioactive nuclei present in a radioactive source. When we want to know the quantities of the radioactive nuclei present in a like point-source, we need to have knowledge of the full energy peak (FEP) efficiency of the detector for a point source and for the different energies emitted by the radioactive source under study.

The more common way to determine the FEP efficiency curve is experimentally, using a set of radioactive sources with well known activity (named calibration sources) and covering the complete gamma energy range of interest. Following this, a curve is fitted to the experimental points to construct the FEP efficiency curve. Then, the FEP efficiency for the energy of interest can be determined by interpolation. If we count on a calibration source compounded by the radionuclide under study we can determine the activity of this source directly by the relative
method. Due to the uncertainty produced in the interpolation, the best way is using the relative method, but calibration point source of any radionuclide is not easy to obtain.

In this work we are developing a new semi-empirical method to calibrate the FEP efficiency by means of extrapolation from one experimental point of the detector efficiency. The experimental point is used to discover the average traveled path by the photons in the detector and then using the linear attenuation coefficient of the detector material for the interest energy, we determined the FEP efficiency for such energy.

The advantages of the method are: we do not need to use multigamma sources for the calibration, which require complicated corrections in the process as true coincidence correction; we do not need to apply a fit to determine the efficiency curve, and one of the principal advantages of this method is that we can obtain the efficiency curve with all phenomenology involved in the detection, such as the resonance peaks.

In this first aproximation to the validation we have used FLUKA code [1] to determine the efficiencies for the different energies used in this work and XCOM data base [2] to know the attenuation coefficients.

2. Semi-empirical Method to Determine the Efficiency of a Detector

The intrinsic efficiency of a detector for photons with energy \( E \) emitted by a gamma source positioned in \( \vec{r} \) is defined as,

\[
\epsilon_{\text{int}}(E, \vec{r}) = 1 - e^{-\lambda(E)d(\vec{r})},
\]

where \( \lambda \) is the linear attenuation coefficient and \( d(\vec{r}) \) the average traveled path by photons in the detector.

If we know the intrinsic efficiency and the linear attenuation coefficient for an energy \( E_1 \), \( \epsilon_{\text{int}}(E_1, \vec{r}) \) and \( \lambda(E_1) \), the average traveled path \( d(\vec{r}) \) by the photons in the detector can be determined by the following equation,

\[
d(\vec{r}) = \frac{\ln(1 - \epsilon_{\text{int}}(E_1))}{-\lambda(E_1)}.
\]

Now, if we know the linear attenuation coefficient for another energy \( E_2 \), \( \lambda(E_2) \), we can determine the intrinsic efficiency for that energy using the next equation,

\[
\epsilon_{\text{int}}(E_2, \vec{r}) = 1 - e^{-\lambda(E_2)d(\vec{r})}.
\]

On the other hand, the intrinsic efficiency is related to the absolute efficiency by the next equation,

\[
\epsilon_{\text{int}} = \epsilon_{\text{abs}} \frac{4\pi}{\Omega},
\]

where \( \Omega \) is the solid angle that subtends the point source over the detector.

2.1. Hypothesis I: Application of the method to calibrate the FEP efficiency (Direct method)

The linear attenuation coefficient has three important components in the gamma spectrometry, the partial linear attenuation coefficients associated to the photoelectric effect, Compton
scattering and pair production. Considering this, we can write the total linear attenuation coefficient as,

$$\lambda(E) = \lambda_{ph}(E) + \lambda_{cs}(E) + \lambda_{pp}(E), \quad (5)$$

where $\lambda_{ph}$, $\lambda_{cs}$ and $\lambda_{pp}$ represent the partial linear attenuation coefficients related to the photoelectric effect, Compton scattering and pair production respectively. Now, if we consider only the absorptions that deposit all the energy in the detector, i.e. the photons that are recorded in the FEP, we can define the complete absorption coefficient as,

$$\lambda = \lambda_{ph}(E) + \beta \lambda_{cs}(E) + \gamma \lambda_{pp}(E), \quad (6)$$

where $\beta$ and $\gamma$ are parameters that represent the probability of a photon that interacted by Compton scattering or pair production finally deposited all the energy in the detector.

The total linear attenuation coefficient, shown in Equation (5), evaluated in Equation (1) corresponds to the total intrinsic efficiency of the detector, while the evaluation of the Equation (6) in the Equation (1) corresponds to the FEP intrinsic efficiency.

Finally, if we can determine the FEP area removing the Compton scattering and Pair production contributions, i.e. if we achieve that $\beta$ and $\gamma$ are zero, then we could determine the FEP intrinsic efficiency only using the photoelectric linear attenuation coefficient and we do not need to know $\beta$ and $\gamma$ parameters. According to the Equation (3), the FEP intrinsic efficiency would be,

$$\epsilon_{int}(E, \vec{r}) = 1 - e^{-\lambda_{ph}(E) d(\vec{r})}. \quad (7)$$

2.2. Hypothesis II: Semi-empirical method to calibrate the FEP efficiency using the total efficiency (Indirect method)

In contrast to the previous method, which we wanted to determine the FEP efficiency curve by direct extrapolation from the reference FEP efficiency. Now, we want to determine the FEP efficiency curve by indirect method. This means, we will calibrate the total intrinsic efficiency and from this we will determine the FEP intrinsic efficiency. Theoretically, this corresponds to the evaluation of the Equation (5) in the Equation (1), as indicated in

$$\epsilon_{int}(E, \vec{r}) = 1 - e^{-(\lambda_{ph}(E) + \lambda_{cs}(E) + \lambda_{pp}(E)) d(\vec{r})}. \quad (8)$$

Then, we can determine the average traveled path by the photons in the detector using the total intrinsic efficiency and the total linear attenuation coefficient (both for the same energy) using Equation (2), and to extrapolate the total intrinsic efficiency of the detector for another energy using Equation (3).

Subsequently, from the calibration curve of the total intrinsic efficiency of the detector we can determine the FEP intrinsic efficiency by means of the following equation [3],

$$\epsilon_{int, peak} = \frac{P}{T} \epsilon_{int, total}, \quad (9)$$

where $\epsilon_{int, peak}$ represents the FEP intrinsic efficiency of the detector, $\epsilon_{int, total}$ is the total intrinsic efficiency of the detector and $P/T$ is the Peak-Total ratio. The Peak-Total ratio must be determined using simulations as Moens indicates in his work [3].
3. Simulation
For simplicity, we have simulated a bare cylindrical detector of Germanium, whose dimensions are 2.5 cm radius and 4.0 cm height.

The gamma sources have been simulated like monoenergetic isotropic sources emitting the interest energies. In Table 1 we show the interest energies and the radionuclides associated to these.

Table 1. Emitted energies by the simulated monoenergetic sources and the radionuclide associated to these.

| Energy (MeV) | Radionuclide |
|--------------|--------------|
| 0.1218       | $^{152}$Eu   |
| 0.2447       | $^{152}$Eu   |
| 0.3443       | $^{152}$Eu   |
| 0.4111       | $^{152}$Eu   |
| 0.6617       | $^{137}$Cs   |
| 0.7789       | $^{152}$Eu   |
| 0.8674       | $^{152}$Eu   |
| 1.086        | $^{152}$Eu   |
| 1.333        | $^{152}$Eu   |
| 1.408        | $^{152}$Eu   |

The simulated sources were positioned on the axial axis of the cylindrical photon detector. In all simulations the number of photons emmited by the source was $10^7$. In Figure 1 we can see the simulated setup.

Figure 1. This figure shows the simulated setup. In this we can see the cylindrical gamma detector and the isotropic gamma source positioned on the axial axis of the detector.
4. Validations

4.1. Hypothesis I: Application of the method to calibrate the FEP efficiency (Direct method)

First, we simulated a point isotropic source emitting photons of 661.65 keV to 10 cm from the detector. From the obtained spectrum we determined the FEP absolute efficiency of the detector and using Equation (4) we obtained the reference FEP intrinsic efficiency. Afterwards we determined the average traveled path for the photons in the detector using Equation (7). The value of the linear attenuation coefficient was determined using XCOM data base [2].

Then, using the obtained average traveled path and the linear attenuation coefficient for the energies associated to the $^{132}$Eu decay from XCOM data base [2], we calculated the efficiencies of the detector for these energies using Equation (7).

On the other hand, we simulated a set of mononenergetic isotropic sources, one by one, emitting gamma rays with the energies associated to the $^{132}$Eu decay and we determined the intrinsic efficiency by statistical count.

The obtained results are shown in Table 2.

### Table 2. Comparison between the efficiencies obtained by the statistical count and the direct method.

| Energy (MeV) | Photoelectric attenuation coefficient (1/cm) | $\epsilon_{\text{int}}$ Simulated | $\epsilon_{\text{int}}$ Method | $\Delta$% |
|--------------|--------------------------------------------|----------------------------------|-------------------------------|---------|
| 0.1218       | $1.125 \times 10^0$                        | $8.410 \times 10^{-1}$           | $1.000 \times 10^0$           | 15.9    |
| 0.2447       | $1.385 \times 10^{-1}$                     | $4.967 \times 10^{-1}$           | $9.362 \times 10^{-1}$        | 46.9    |
| 0.3443       | $5.174 \times 10^{-2}$                     | $3.396 \times 10^{-1}$           | $6.425 \times 10^{-1}$        | 47.1    |
| 0.4111       | $3.168 \times 10^{-2}$                     | $2.817 \times 10^{-1}$           | $4.672 \times 10^{-1}$        | 39.7    |
| 0.7789       | $6.446 \times 10^{-3}$                     | $1.451 \times 10^{-1}$           | $1.203 \times 10^{-1}$        | −20.7   |
| 0.8674       | $5.147 \times 10^{-3}$                     | $1.295 \times 10^{-1}$           | $9.726 \times 10^{-2}$        | −33.1   |
| 1.086        | $3.160 \times 10^{-3}$                     | $1.054 \times 10^{-1}$           | $6.088 \times 10^{-2}$        | −73.1   |
| 1.333        | $2.169 \times 10^{-3}$                     | $8.794 \times 10^{-2}$           | $4.219 \times 10^{-2}$        | −108.4  |
| 1.408        | $1.963 \times 10^{-3}$                     | $8.196 \times 10^{-2}$           | $3.826 \times 10^{-2}$        | −114.2  |

We can see that the results obtained with the direct method are not good. This is because we assume we can remove Compton and Pair production contribution to the peak is false. Although we can discount multiple Compton contribution to the peak we can not discount the interactions that start with a Compton scattering or with a Pair production and follow it by a photoelectric absorption. These contributions to the FEP area can not be separated from the photons that immediately interact by photoelectric effect. Therefore this method can not be applied.

4.2. Hypothesis II: Semi-empirical method to calibrate the FEP efficiency using the total efficiency (Indirect method)

First, we simulated a point isotropic source emitting photons of 661.65 keV to 10, 20 and 30 cm from the detector. From the obtained spectrum we determined the total absolute efficiency of the simulated detector and using Equation (4) we obtained the reference total intrinsic efficiency. Afterwards we determined the average traveled path for the photons in the detector using Equation (8).

Then, using the obtained average traveled path and the linear attenuation coefficient for the energies associated to the $^{132}$Eu decay from XCOM data base [2], we calculated the efficiencies of the detector for these energies using Equation (8).
On the other hand, we simulated a set of monenergetic isotropic sources, one by one, emitting gamma rays with the energies associated to the $^{132}$Eu decay and we determinate the intrinsic efficiency by statistical count. The obtained results for the relative bias between the total intrinsic efficiencies determined by statistical count and by extrapolation of the reference efficiency are shown in Table 3.

Table 3. Comparison between the total intrinsic efficiencies obtained by statistical count and by extrapolation. The values correspond to the relative bias between this quantities.

| Energy (MeV) | Source-detector distance (cm) |
|--------------|-----------------------------|
|              | 10  | 20  | 30  |
| 0.1218       | −9.44 | −6.39 | −5.61 |
| 0.2447       | −5.85 | −4.50 | −3.64 |
| 0.3443       | −3.68 | −2.89 | −2.78 |
| 0.4111       | −2.31 | −2.00 | −1.65 |
| 0.7789       | 0.71  | 0.42  | 0.29 |
| 0.8674       | 1.62  | 0.20  | 0.38 |
| 1.086        | 2.29  | 1.19  | 0.59 |
| 1.333        | 3.47  | 1.65  | 1.04 |
| 1.408        | 3.15  | 1.90  | 1.43 |

From the results shown in Table 3 we can see that the results are better while we move away from the detector. Another characteristic we can see is that the results are better for energies close to the reference energy (661.65 keV). And finally, we can identify a sistematic error in the method, because the results are greater than the expected values for energies less than the reference energy, and lesser than the expected values in opposite case.

In general the results are in a good agreement, except the result obtained for the 121.8 keV. The obtained results for energies between 344.3 keV and 1408 keV are in an excellent agreement with the expected values.

The results can be improved considering a set of total intrinsic efficiencies along the energy range of interest as references and to do the extrapolation in a short range around these points. Experimentally, such gamma sources can be a set of monoenergetic sources, because the use of a monoenergetic source represents the easier way to determine the efficiency.

The next step to complete the method is to determine the FEP intrinsic efficiency to the energies under study using Equation (9). Due to that this work has been performed using simulaton, the transfer of the total intrinsic efficiency to the FEP intrinsic efficiency can not be carried out, because this needs to be performed by simulation, as Moens indicates \[3\]. But, as the application of the Equation (9) has already been demostrated we expect it to be successful.

In order to apply the method as proposed, we need a well characterized detector, i.e. a detector with well known all geometric parameters and materials.

5. Conclusions
The semi-empiric method to determine the FEP intrinsic efficiency of the detector can not be directly applied using Equation (7), because when we determine the FEP area of the peak we can not remove the counts on the peak contributed by the Compton and Pair production interactions.

The method to determine the Total intrinsic efficiency by extrapolation, Equation (9) has been successfully applied, except for the energy 121.8 keV.
The next step to improve the method is to use a set of reference efficiency, and to realize a local extrapolation.

The final step to complete this work is to use a well known detector and to apply the complete method.

References
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