INFORMATION FROM THE BEGINNING

Estimating observable effects of quantum-gravity discreteness on patterns of cosmic background radiation

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Abstract. Requiring black hole evaporation to be quantum-mechanically coherent imposes a universal, finite “holographic bound”, conjectured to be due to fundamental discreteness of quantized gravity, on the amount of information carried by any physical system. This bound is applied to the information budget in the standard slow-roll model of cosmic inflation. A simple estimate suggests that when quantum gravity is included, fluctuations during inflation have a discrete spectrum with a limited information content, only about $10^5$ bits per mode, fixed by the inverse scalar perturbation amplitude. This scarcity of information may allow direct observation of quantum-gravity eigenmodes in the anisotropy of cosmic background radiation.

“This is what we found out about Nature’s book keeping system: the data can be written onto a surface, and the pen with which the data are written has a finite size.”
-Gerard ’t Hooft
1. Primordial quanta

Observations of the cosmic background now penetrate to a time when the number of quanta in the universe was a relatively small number. It is natural to ask whether we can learn something from such extraordinary data about the fundamental quantum structure of space and time. Here I offer arguments that this may indeed be possible, based on an estimate of the information content of the universe near the beginning.

It is now thought that cosmological perturbations originate as quantum fluctuations of the inflaton and the graviton fields during inflation. Their properties are calculated using the theory of quantum fields in curved spacetime [1, 2, 3, 4, 5, 6, 7]. Zero point fluctuations in the quantum modes of the the inflaton field give rise to scalar perturbations, those in the graviton field to tensor perturbations.

The spatial structure of field quanta in the original fluctuations “freezes out” as they increase in size beyond the inflationary event horizon, since causality prevents them from fluctuating further. From a quantum point of view, their states squeeze into a definite spatial projection, and then grow coherently by an exponential factor during the many subsequent e-foldings of inflation, creating an enormous number of coherent quanta (or holes in the inflaton condensate), in phase with the original quantum wave. From the classical point of view, the quantum fluctuations create permanent perturbations in the classical gravitational gauge-invariant potential $\phi_m[8]$, leading to observable background anisotropy and large scale structure[9, 10, 11, 12].

The large-scale classical perturbations observed today are thus a direct result of the quantum field activity during inflation; indeed, the pattern of microwave anisotropy on the largest scales corresponds to a faithfully amplified image of microscopic field configurations as they froze out during inflation. Roughly speaking, each hot or cold patch on the sky derives originally from about one quantum, in the sense that the occupation number at freeze-out was of order unity; the energy of a fluctuating patch was typically $E \approx hH$, where $h$ is Planck’s constant and $H$ is the expansion rate (Hubble’s constant) during inflation.

The model leads to a remarkable phenomenon: primordial anisotropies that are simultaneously the smallest and largest imaged entities in nature. COBE gave us our first glimpse [9, 10] of what these structures look like. When we look at the largest structures in the cosmic background radiation—the largest and most distant things we can possibly see, stretching across the sky at the edge of the universe—we are looking at images of particle wavefunctions imprinted when they were single quanta far smaller than the smallest subatomic structure seen in the lab. (The nearest labo-
ratory analogs are images of wavefunctions of trapped Bose-Einstein condensates that trace the zero-point mode of their trap, but even these are not free vacuum fluctuations.) The universe acts like a giant natural microscope, providing us with a natural virtual-particle observatory.

The standard calculation of these processes[13] uses a semiclassical approximation: spacetime is assumed to be classical (not quantized), and the perturbed fields (the inflaton and graviton) are described using relativistic quantum field theory, essentially (in the limit of free massless fields) an infinite collection of quantized harmonic oscillators. The Hilbert space of this system is infinite, so although the fields are quantized, they are continuously variable functions that can assume any values. The theory generically predicts random-phase gaussian noise with a continuous spectrum determined by the parameters of the inflaton potential. Although sky maps contain images of “single quanta,” the amount of information they carry is in principle infinite.

It has always been acknowledged that this description is incomplete, and will be modified by including a proper account of spacetime quantization. Although the fundamental theory of quantum gravity is not known, a “holographic entropy bound” already constrains with remarkable precision the total number of fundamental quantum degrees of freedom. The complete Hilbert space of a bounded volume is finite and discrete rather than infinite and continuous, limiting the range of accessible configurations in any region to a definite, calculable number. In particular this limit applies to the coupled system of inflaton-field and spacetime configurations, together with all their possible fluctuations, during inflation. Like bound states of an atom, but unlike a free quantum field or harmonic oscillator, the spectrum of states is discrete; indeed the number of eigenstates, unlike atoms, is even finite.

A simple estimate[14] suggests that in standard inflation, the amount of information in the anisotropy is remarkably limited: it can be described with only about $10^5$ bits per sky-harmonic mode, implying that the perturbations should be “pixelated” in some way. In principle, this effect may be observable, and provide concrete data on the discrete elements or eigenstates of quantum gravity.

2. The Holographic Principle

Hawking radiation from black holes presents a conundrum in information accounting. Matter falling into the hole travels to the central singularity long before its energy is radiated. It appears that either the information of infalling matter is lost, in contradiction with quantum mechanics, or else the black hole itself has coherent quantum states that store the information
between the time of infall and the time of particle evaporation.

The latter point of view is the one consistent with quantum mechanics and seems to be correct, but it has radical consequences.[15, 16, 17, 18] Hawking’s calculation of black hole evaporation showed that vacuum states of particles incoming from the distant past of a black hole spacetime become populated with a thermal spectrum in the far future, with a precisely calculable entropy. If quantum mechanics holds, then all the processes of black hole formation and evaporation should be time reversible. The evaporation products of a black hole can be time and parity reversed, then run backwards in time, to form a black hole that will slowly grow and eventually fly apart by throwing out whatever macroscopic objects (TV sets, astronauts, whatever) were thrown in originally. This time-reversed situation is also physical, it is just extremely unlikely. The example shows how the laws of gravity can be essentially statistical in nature, even for a black hole.

Assuming this coherent behavior, the evaporating black hole system allows a complete accounting of the number of internal quantum states of the black hole: it must be sufficient to store all the information needed for the radiated particles. A black hole, including its quantum-gravitational and particle degrees of freedom, has a density of states given by $e^S$, where the entropy $S$ depends only on the area $A$ of the event horizon of the hole: $S = A/4$ in Planck units. Roughly speaking, each radiated particle carries about a bit of information; when it is radiated, it reduces the mass of the hole by about $M^{-1}$ (with a wavelength at infinity of order the Schwarzschild radius, $M$), and reduces the area of the event horizon (and the entropy) by about one Planck area. Since a black hole is the maximal entropy state for any spatial region of a given bounding-area size, this result generalizes to a bound on the information content of any physical system.

Therefore, the Hilbert space of everything is finite and discrete: for everything in a three-dimensional region$^1$ bounded by a two-dimensional surface of area $A$, the complete quantum state is specified by at most $n = A/4 \ln 2$ binary numbers, and there are at most $2^n$ distinguishable outcomes of any experiment. The information content is the same as $n$ binary spins, corresponding to one spin per $0.724 \times 10^{-65}$ cm$^2$ of bounding area. (Since the spins are in general entangled, this is most correctly described as $n$ quantum bits, or “qubits”, of information). Field theory, even a single harmonic oscillator, has an infinite Hilbert space, so the holographic entropy bound imposes radical constraints on physics. Discreteness and nonlocality apparently exist in nature that are not modeled in field theory.

$^1$The three-dimensional regions in the most general formulation are null sheets.
3. Holography and Cosmology

If the holographic bound is correct, the observable universe today has at most \( 3\pi/\Lambda \ln 2 \approx 10^{120} \) qubits of information, where \( \Lambda \) is the current value of the cosmological constant. Although this is a lot less than infinity, nobody has yet suggested a way that such a large number of degrees of freedom could be distinguished in practice from a continuous system (see e.g. [19]).

On the other hand, the maximum information within the event horizon during inflation was much smaller:

\[
S_{\text{max}} = \pi/H^2.
\]  

(1)

We know from the cosmic background maps that \( H \) was less than about \( 10^{-5} \) (because graviton fluctuations produce tensor perturbations today with \( \delta T/T \approx H \)), but even at this level the total information content is only \( 10^{10} \) qubits; all possible physical situations could be encoded on a laptop computer.

A more detailed calculation,[14] based on the specific context of “slow-roll inflation”, suggests that for observable fluctuations, the amount of information is even less. The basic assumption of this calculation is a “discreteness ansatz” that the inflationary universe system—inflaton plus gravity—makes discrete transitions between states separated by one bit of entropy. This would follow naturally if the reason for the existence of the holographic bound is that the fundamental theory contains discrete elements. It seems plausible that this is the case, given recent development of candidates for fundamental theory such as M theory and loop quantum gravity[20] that display discrete and holographic features. It also seems natural given the nature of particle-emission processes in the black hole system, where emission of each particle is associated with reduction by about one bit in \( S \).

The classical evolution of slow-roll inflation then corresponds, from the quantum point of view, to adding binary qubits one at a time. The key assumption is that the background spacetime and horizon-scale fluctuations make transitions between discrete states, each of which adds (or subtracts) one bit of information to the total maximum observable entropy. That is, \( H \) comes in discrete steps of \( n \):

\[
H_i = \sqrt{\frac{\pi}{n_i \ln 2}}
\]  

(2)

where \( n_i \) are integers. The values of \( H \) and the inflaton \( \phi \) on the horizon scale are connected by the usual classical equations of inflation, so \( \phi \) also takes on discrete values.

The classical expectation value \( \phi_c \) of the inflaton field and the inflationary expansion rate \( H \) obey the Friedmann equation controlling the
expansion,
\[ H^2 = \frac{8\pi}{3} \left[ V(\phi_c) + \phi_c^2/2 \right]. \quad (3) \]

and the dynamical equation for \( \phi_c \),
\[ \ddot{\phi}_c + 3H \dot{\phi}_c + V'(\phi_c) = 0, \quad (4) \]

where the effective potential is \( V(\phi_c) \) and \( V' \equiv dV/d\phi_c \). For definiteness, assume that the observed modes crossed the inflationary horizon during a standard so-called “slow roll” or Hubble-viscosity-limited phase of inflation, corresponding to \( V'/V \leq \sqrt{48\pi} \) and \( V''/V \leq 24\pi \), during which \( \dot{\phi}_c \approx -V'/3H \). The rate of the roll is much slower than the expansion rate \( H \), so the kinetic term in Eq. (3) can be ignored in the mean evolution. The slow-roll phase of inflation creates approximately scale-invariant curvature perturbations.

We define a combination of inflationary parameters by
\[ Q_s \equiv \frac{H^3}{|V'|} \approx \left( \frac{V^3}{V'^2} \right)^{1/2} \left( \frac{8\pi}{3} \right)^{3/2}. \quad (5) \]

This combination can be estimated fairly accurately, since it also controls the amplitude of the scalar perturbations observed in the microwave background anisotropy. The best fit to the four-year COBE/DMR data, assuming scale invariance \((n = 1)\) and zero amplitude tensor modes, yields \([10, 14]\)
\[ Q_s = 9.4 \pm 0.84 \times 10^{-5}. \quad (6) \]

Putting this together one finds that there is a steady increase in observable entropy at a rate
\[ \dot{S}_{\text{max}} = \frac{8\pi^2 V'^2}{9 H^5} = \frac{8\pi^2}{9} HQ_s^{-2}. \quad (7) \]

Every inflationary e-folding, \( S_{\text{max}} \) increases by an amount of order \( 10^{10} \) due to the classical evolution of the system as \( H \) slowly decreases.

However, the information attached to the observable quantity—the horizon-scale perturbation in the inflaton—is much smaller than this. The variation in total entropy associated with a horizon-scale perturbation \( \delta \phi \) is:
\[ \delta S_{\text{tot}} = \frac{8\pi^2}{3} \left[ \frac{\delta \phi}{H} \right] Q_s^{-1}. \quad (8) \]

The standard field theory analysis for the horizon-size perturbations tells us that the quantity \( [\delta \phi/H] \) is statistically determined, with a continuous
gaussian statistical distribution of order unit width. The corresponding increment \( \delta S \approx (8\pi^2/3)Q_{S-1}^{-1} \approx 10^5 \) then is roughly the jump in the total observable cosmological entropy associated with the horizon-scale inflaton perturbations. This is much less than the increase given by Eq. (7) in the total information during an expansion time, of order \( Q_{S-1}^{-1} \) rather than \( Q_{S-2}^{-2} \). Presumably this can be attributed to the fact that almost all of the entropy growth is associated with the classical slow roll of the inflaton condensate, which leaves no imprint in the anisotropy.

Thus for any value of the inflationary Hubble constant, the horizon-size inflaton perturbations, which are the degrees of freedom frozen into our sky, contain only about \( 10^5 \) bits per mode. For example, the fluctuations seen in the COBE map, with about 1000 pixels, intrinsically contain only about \( 10^8 \) bits or 10 Megabytes. This could in principle be encoded in a color laptop display screen with no loss of information. If we had a dataset with enough resolution and a high enough signal-to-noise ratio, and knew the “encoding” (the projection of the quantum-gravity eigenstates onto the sky), we would find that the data could be fit surprisingly well using these modes, and that the fit would not improve with the addition of additional parameters. If there is substantial redundancy in the encoding, the number of distinguishable eigenstates could be much less than this limit, so a statistical search might find quantum-gravity signatures even in current datasets.

To put the same point more poetically: when the letters of the writing on the sky are known, the pattern will no longer appear as a meaningless jumble of random noise, and the significance of the whole pattern will be interpreted completely and transparently in terms of those letters—the eigenmodes of the inflationary system in fundamental theory. When that is done, we will know everything it is possible to know about the beginning. All we have done here is estimate how many letters there are.

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