Bimodal Accretion Disks: SSD-ADAF Transitions

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We show that, unlike the results presented previously in the literature, the transition from an outer Shakura-Sunyaev disk (SSD) to an advection-dominated accretion flow (ADAF) is possible for large values of the viscosity parameter $\alpha > 0.5$. The transition is triggered by thermal instability of a radiation-pressure-supported SSD. The transition radius is close to the central black hole. We confirm our qualitative prediction by actually constructing global bimodal SSD-ADAF solutions.

*Subject headings:* accretion, accretion disks - black hole physics - hydrodynamics
1. Introduction

The model of bimodal accretion disk, which consists of a geometrically thin, optically thick Shakura-Sunyaev disk (SSD) (Shakura & Sunyaev 1973) as the outer part and a quasi-spherical, optically thin advection-dominated accretion flow (ADAF) as the inner part, has been quite successfully applied to black hole X-ray binaries and galactic nuclei (see Narayan, Mahadevan, & Quataert 1998 for a review). In this model, the accretion flow switches from an SSD to an ADAF at a transition radius $R_{tr}$. However, $R_{tr}$ was inferred only by making plausible assumptions on how it depends on the accretion rate $\dot{M}$. The precise mechanism through which the SSD material is converted into an ADAF remains a matter of debate, and no self-consistent global solution for such a transition has been found so far. Indeed, Narayan, Kato, & Honma (1997) obtained a number of examples of global transonic ADAF solutions which connect outward to geometrically thin disks. Lu, Gu, & Yuan (1999) recovered the whole class of such global ADAF-thin disk solutions. But these authors did not take the local radiative cooling into account, thus their solutions could not show the variation of the optical depth and could not be regarded as bimodal SSD-ADAF solutions. On the other hand, Chen, Abramowicz, & Lasota (1997) considered the local radiative cooling as provided by thermal bremsstrahlung, but did not find SSD-ADAF solutions. Igumenshchev, Abramowicz, & Novikov (1998) also adopted bremsstrahlung cooling and found that only an outer Shapiro-Lightman-Eardley (SLE) disk (Shapiro, Lightman & Eardley 1976) could smoothly match an inner ADAF. Such SLE-ADAF solutions are optically thin everywhere. More impressive in this respect is the result of Dullemond & Turolla (1998, hereafter DT98). They concluded that the SSD-ADAF transition was not permitted, and only an outer SLE disk could match an inner ADAF or SSD. But, as they noticed, because of the thermal instability of the SLE disk, such SLE-ADAF or SLE-SSD models most probably do not exist in nature.
It would be a pity if such a promising bimodal SSD-ADAF model could not be actually constructed. In this Letter we also use an argument based on energetic considerations which is similar to DT98, but find that the SSD-ADAF transition is possible for large values of viscosity parameter. We confirm our qualitative prediction by presenting a numerical example of the global SSD-ADAF bimodal solution.

2. Equations

The system of equations adopted here, which is similar to DT98, makes the following standard assumptions:

(1): The vertical half-thickness of the disk is expressed as \( H = c_s/\Omega_K \), where \( c_s = (p/\rho)^{1/2} \) is the isothermal sound speed, with \( p \) and \( \rho \) being the total pressure and the mass density at the equatorial plane, respectively, and \( \Omega_K \) is the Keplerian angular velocity calculated by using the Newtonian potential, \( \Omega_K = \left(\frac{GM}{R^3}\right)^{1/2} \).

(2): The kinematic viscosity coefficient is expressed as \( \nu = \alpha c_s H \).

(3): \( p \) is the sum of gas and radiation pressure, \( p = p_g + p_r \). \( p_g = \rho R(T_i + T_e) \), where \( T_i \) and \( T_e \) are the ion temperature and the electron temperature, respectively, and \( T_e = \min(T_i, 6 \times 10^9 K) \). \( p_r = Q_{rad}(\tau + 2/\sqrt{3})/4c \), where \( Q_{rad} \) is the radiative cooling rate and \( \tau = \kappa \rho H \) is the total optical depth.

(4): The opacity \( \kappa \) is the sum of electron scattering and absorption opacity, \( \kappa = \kappa_{es} + \kappa_{abs} \), where \( \kappa_{es} = 0.34 cm^2 g^{-1} \) and \( \kappa_{abs} = 0.27 \times 10^{25} \rho T_e^{-3.5} cm^2 g^{-1} \).

Introducing the radial velocity \( \upsilon_R \), the angular velocity \( \Omega \), and the surface density \( \Sigma = 2H\rho \), the continuity, radial momentum, azimuthal momentum, and energy equations take the form (e.g., Narayan & Yi 1994; DT98)

\[
\dot{M} = -2\pi \Sigma R \upsilon_R ,
\] (1)
The viscous heating $Q^+$ has the usual expression

$$Q^+ = \nu \Sigma \left( R \frac{d\Omega}{dR} \right)^2 .$$

The advective cooling is expressed as

$$Q_{\text{adv}} = \Sigma v_R T_e \frac{ds}{dR} = \Sigma v_R \left( \frac{1}{\gamma - 1} \frac{d\epsilon_s^2}{dR} - \frac{c_s^2}{\rho} \frac{d\rho}{dR} \right) .$$

The radiative cooling rate $Q_{\text{rad}}$ is calculated using a bridging formula (DT98),

$$Q_{\text{rad}} = 8 \sigma T_e^4 \left( \frac{3 \tau}{2} + \sqrt{3} + \frac{8 \sigma T_e^4}{Q_{\text{brem}}} \right)^{-1} .$$

Eq.(7) is valid in both optically thin and optically thick regimes. The bremsstrahlung cooling is given by (Abramowicz et al. 1995)

$$Q_{\text{brem}} = 1.24 \times 10^{21} H \rho^2 T_e^{1/2} \text{ergs s}^{-1} \text{cm}^{-2} .$$

Abramowicz et al. (1995) and Chen et al. (1995, hereafter Chen95) obtained a unified $\Sigma - \dot{M}$ picture of accretion flows around black holes with an additional assumption that the disk always rotates at $\Omega = \Omega_K$. However, it is known that slim disks (Abramowicz et al. 1988) and ADAFs are quite sub-Keplerianly rotating. We therefore assume:

$$\Omega = \omega \Omega_K (0 \leq \omega \leq 1) ,$$

where the parameter $\omega$ is obtained from the self-similar solution (Narayan & Yi, 1994). We fix $\gamma = 1.5$ and adopt the self-similar method, then the equations (2), (3), (5) and (6) are reduced to the following algebraic form:

$$\frac{1}{2} v_R^2 + \frac{5}{2} c_s^2 + (\omega^2 - 1) \Omega_K^2 R^2 = 0 .$$
\[ v_R = -\frac{3 \nu}{2 R} = -\frac{3}{2} \alpha c_s \frac{H}{R}, \]  

(10)

\[ Q^+ = \frac{3}{4\pi} \dot{M} \Omega^2, \]  

(11)

\[ Q_{adv} = \frac{1}{4\pi} \frac{\dot{M} c_s^2}{R^2}. \]  

(12)

From Eq. (9) one easily obtains: \( h = H/R = c_s/\Omega_K R < \sqrt{2/5} \approx 0.63 \). We use a parameter \( f = Q_{adv}/Q^+ = h^2/3\omega^2 \) to measure the degree to which the flow is advective. If the radial velocity is neglected, Eq. (9) is simplified to: \( (\omega^2 - 1) + \frac{5}{2} h^2 = 0 \). For radiative cooling-dominated flows \( (f \approx 0) \) such as SSDs or SLE disks, \( \omega \approx 1 \), and \( h \approx 0 \); while for advection-dominated flows \( (f \approx 1) \) such as slim disks or ADAFs, \( \omega \approx \sqrt{2/17} \), and \( h \approx \sqrt{6/17} \). Thus the assumption \( \Omega = \omega \Omega_K \) is more reasonable as it is valid for both radiative cooling-dominated and advection-dominated accretion flows.

3. SSD-ADAF Solutions

Abramowicz et al. (1995) first obtained a unified \( \Sigma - \dot{M} \) picture for accretion flows at a fixed radius \( R \) in the case of low viscosity, which includes four classes of solutions, namely SSDs, SLE disks, slim disks and ADAFs. Chen95 found that two types of \( \Sigma - \dot{M} \) picture should exist, which are separated by a critical viscosity parameter \( \alpha_{crit} \). We recover here these two types of \( \Sigma - \dot{M} \) picture with our assumption \( \Omega = \omega \Omega_K \), which are given in Fig.1, (a) for \( \alpha < \alpha_{crit} \) and (b) for \( \alpha > \alpha_{crit} \). The solid lines in the figures represent thermal equilibrium solutions, i.e. with \( Q^+ = Q^- \). In Fig.1(a), the right S-shaped curve consists of three branches, of which the lower one is for gas-pressure-supported SSDs, the middle one for radiation-pressure-supported SSDs, and the upper one for slim disks; while the left curve consists of two branches, of which the lower one is for SLE disks, and the upper one
for ADAFs. In Fig.1(b), the straight line is for ADAFs and slim disks, while the \( n \)-shaped curve consists of three branches, of which the two branches on the right are the same as the middle and lower branches of the \( S \)-shaped curve in Fig.1(a), and the branch on the left is for SLE disks. The unstable branches are those which have \( Q^+ > Q^- \) above and \( Q^+ < Q^- \) below, while the stable branches are just opposite. Thus gas-pressure-supported SSDs, slim disks and ADAFs are thermally stable, whereas radiation-pressure-supported SSDs and SLE disks are thermally unstable.

DT98 investigated the thermal instability in Fig.1(a), and argued that the SSD-ADAF transition is not permitted. We agree with DT98 that such a transition is indeed prohibited in Fig.1(a). The arrows in Fig.1(a) show a limit-cycle behavior resulting from the thermal instability of a radiation-pressure-supported SSD.

However, the other type of picture as shown in Fig.1(b) was ignored by DT98. Because a bridging formula like Eq.(7) is used, the optically thick, high \( \dot{M} \) slim disk solution and the optically thin, low \( \dot{M} \) ADAF solution can be described by a single line in Fig.1(b), the line extends over the entire range of \( \dot{M} \) without break. It is this feature that makes the SSD-ADAF transition possible. A thermal disturbance on a radiation-pressure-supported SSD can trigger the flow to behavior in the following way: The flow first jumps to a slim disk solution and becomes thermally stable. But because the accretion rate \( \dot{M} \) does not match that of the outer SSD, then the slim disk must evolve into an ADAF, for which \( \dot{M} \) matches that of the outer SSD. The whole process is indicated by the two arrows in Fig.1(b). An SSD-ADAF transition is realized.

To confirm this qualitative prediction, we now go on to present our numerical models for bimodal SSD-ADAF disks. The general thermal instability condition is

\[
\left( \frac{\partial Q^+}{\partial T} \right)_{\Sigma} - \left( \frac{\partial Q^-}{\partial T} \right)_{\Sigma} > 0
\]  

(13)

We denote \( \beta = p_g/(p_g + p_r) \), and \( \lambda = \kappa_{\text{abs}}/(\kappa_{\text{es}} + \kappa_{\text{abs}}) \). In previous researches on SSDs the
disk was usually divided into three separate regions, for which (from the inner to the outer) the two parameters \((\beta, \lambda)\) are \((0,0)\), \((1,0)\) and \((1,1)\), respectively. Here we let both \(\beta\) and \(\lambda\) vary continuously from 0 to 1, thus the SSD solution obtained will smoothly extend over these three regions. In our formulation Eq.(13) takes an explicit form
\[
\delta = 4 - 10\beta - 7.5\lambda - 0.5\beta\lambda > 0 .
\] (14)

When the opacity is dominated by the electron scattering, i.e. \(\lambda = 0\), this condition is reduced to the usual expression: \(\beta < 0.4\).

Once the viscosity parameter \(\alpha\) and the relative accretion rate \(\dot{m}\) \((\dot{m} = \dot{M}/M_{Edd}\) with \(M_{Edd}\) being the Eddington accretion rate) are given, we can search for the global SSD-ADAF solution by the following steps:

(i) Provided that the given viscosity parameter \(\alpha > 0.5\), a critical radius \(R_c\) exists. The \(\Sigma - \dot{M}\) picture is of the type of Fig.1(a) for \(R > R_c\), and of the type of Fig.1(b) for \(R < R_c\). We first calculate the value of \(R_c\) which corresponds to the given \(\alpha\).

(ii) We solve the equations of SSD inward from an outer boundary \(R_{out} = 2000R_g\). The SSD solution breaks off when the thermal instability condition Eq.(14) is met at a certain radius \(R_b\). We calculate the value of \(R_b\) which corresponds to the given \(\alpha\) and \(\dot{m}\). If the condition Eq.(14) is never met for the given \(\alpha\) and \(\dot{m}\), then the SSD solution is thermally stable everywhere.

(iii) If \(R_b < R_c\), the SSD-ADAF transition will occur at \(R_{tr} = R_b\). We can obtain the self-similar ADAF solution inside \(R_{tr}\). Combining with the SSD solution outside \(R_{tr}\), we obtain the whole global SSD-ADAF solution. On the contrary, if \(R_b > R_c\), the limit-cycle behavior will occur at \(R_b\), and the SSD-ADAF transition will not occur.

In Fig.2 we show how the behavior of an original SSD flow depends on \(\alpha\) and \(\dot{m}\). It is seen that three possible cases, namely the stable SSD, the limit cycle behavior, and the SSD-ADAF transition, each occupy a certain region of the \(\alpha - \dot{m}\) parameter space,
and the SSD-ADAF transition occurs for $\alpha > 0.5$ and a definite range of $\dot{m}$. To construct numerically a global bimodal SSD-ADAF solution, we choose $\alpha = 0.8$. We then obtain the corresponding critical radius $R_c = 12.0R_g$, and find from Fig.2 that the SSD-ADAF transition occurs for $0.03 < \dot{m} < 0.17$. The SSD is stable everywhere for $\dot{m} < 0.03$, and the limit-cycle behavior appears for $\dot{m} > 0.17$. An example of the global SSD-ADAF solution for $\dot{m} = 0.1$ is presented in Fig.3. The SSD becomes unstable at $R_b = 8.0R_g$. The figure gives a $\log(\Sigma) - \log(\dot{m})$ picture at $R = 8.0R_g$, which belongs to the type of Fig.1(b). The arrows indicate the transition from the SSD solution (filled circle) to an ADAF solution (filled square).

4. Discussion

In this Letter we show that the thermal instability of a radiation-pressure-supported SSD can possibly trigger two different kinds of behavior of the flow, namely the limit-cycle and the SSD-ADAF transition. For low values of viscosity parameter $\alpha < 0.5$, only the limit-cycle behavior can occur; while for large values $\alpha > 0.5$, either of the two kinds of behavior can, and which one is actually realized is determined by $\dot{m}$. We use two parameter, $\beta$ and $\lambda$, to smoothly connect the three usually separated regions of an SSD, so that the exact position of the SSD-ADAF transition can be found, and the global bimodal SSD-ADAF solution can be obtained.

The range of $\alpha$ values necessary for the SSD-ADAF transition to occur should depend on, among other things, radiative cooling mechanisms adopted. Chen95 used two bridging formulae of $Q_{rad}$ to calculate $\alpha_{crit}$ for a fixed radius $R$, one of which is introduced by Narayan & Yi (1995), who considered bremsstrahlung and synchrotron cooling, and Comptonization; and the other of which is introduced by Wandel & Liang (1991), who considered bremsstrahlung cooling with Comptonization only. They found that the value
of $\alpha_{\text{crit}}$ calculated with the former formula (i.e. with more sources of radiative cooling) is higher than that calculated with the latter one. For example, for $R = 5R_g$, they had $\alpha_{\text{crit}} = 0.41$ with the former formula, and $\alpha_{\text{crit}} = 0.245$ with the latter one. Since in bridging formula Eq.(7) only bremsstrahlung cooling is taken into account, we guess that if other sources of radiative cooling such as synchrotron emission and Comptonization are included, the lowest required value of $\alpha$ for the SSD-ADAF transition would become larger than 0.5, i.e. the necessary condition on $\alpha$ for the existence of bimodal SSD-ADAF disk would seem even more restrictive. It is unclear yet if there are other factors which could help to relax this condition on $\alpha$. However, the good thing for the SSD-ADAF transition is that the larger the allowed value of $\alpha$ is, the wider the corresponding range of $\dot{m}$ is, as seen clearly from Fig.2.

The SSD-ADAF transition found in this Letter results from the instability of the radiation-pressure-supported region, i.e. the inner part of SSD. Thus the transition position is close to the central black hole, as we have calculated here. On the other hand, observations of some black hole X-ray binaries (Narayan et al. 1998 and references therein) seem to imply a SSD-ADAF transition radius $R_{tr} \sim 10^4 R_g$, which is definitely in the very outer region of SSD. However, it is well known that the outer region of SSD is gas-pressure-supported and is both thermally and viscously stable. In our opinion, it is difficult to see from the theoretical point of view how to convert a gas-pressure-supported SSD directly into an ADAF. The apparent conflict between present observations and theories remains an unsolved issue.
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Fig. 1.— Two types of $\log(\Sigma) - \log(\dot{M})$ picture for accretion flows at a fixed radius. The solid lines mark the different branches of solutions. The arrows show the behavior of the flow resulting from the thermal instability of a radiation-pressure-supported SSD. (a) $\alpha < \alpha_{\text{crit}}$, a limit-cycle occurs. (b) $\alpha > \alpha_{\text{crit}}$, an SSD-ADAF transition occurs.

Fig. 2.— Dependence of the possible behavior of an original SSD flow on $\alpha$ and $\dot{m}$.

Fig. 3.— An example of global SSD-ADAF solution with $\alpha = 0.8$, and $\dot{m} = 0.1$. The $\log(\Sigma) - \log(\dot{m})$ picture is for the accretion flow at $R = 8.0R_g$, where the SSD becomes thermally unstable. The arrows show the transition from the SSD (filled circle) to an ADAF (filled square). The dotted line corresponds to the limit: $H/R = 0.63$. 
