Exploring masses and CNO surface abundances of red giant stars

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ABSTRACT

A grid of evolutionary sequences of stars in the mass range 1.2-7M⊙, with solar-like initial composition is presented. We focus on this mass range in order to estimate the masses and calculate the CNO surface abundances of a sample of observed red giants. The stellar models are calculated from the zero-age main sequence till the early asymptotic giant branch (AGB) phase. Stars of M \lessless= 2.2M⊙ are evolved through the core helium flash. In this work, an approach is adopted that improves the mass determination of an observed sample of 21 RGB and early AGB stars. This approach is based on comparing the observationally derived effective temperatures and absolute magnitudes with the calculated values based on our evolutionary tracks in the Hertzsprung-Russell diagram. A more reliable determination of the stellar masses is achieved by using evolutionary tracks extended to the range of observation. In addition, the predicted CNO surface abundances are compared to the observationally inferred values in order to show how far standard evolutionary calculation can be used to interpret available observations and to illustrate the role of convective mixing. We find that extra mixing beyond the convective boundary determined by the Schwarzschild criterion is needed to explain the observational oxygen isotopic ratios in low mass stars. The effect of recent determinations of proton capture reactions and their uncertainties on the 16O/17O and 14N/15N ratios is also shown. It is found that the 14N(p, γ)15O reaction is important for predicting the 14N/15N ratio in red giants.

Key words: convection, nuclear reactions, nucleosynthesis, abundances - stars: evolution - stars: low-mass

1 INTRODUCTION

After the main sequence evolutionary phase, stars evolve to the red giant branch (RGB). This evolution is initiated by the ignition of shell H-burning surrounding the He core, whose energy flux causes the envelope to expand and the star evolves to the RGB. The expansion increases the opacity and leads to the development of a deep convective envelope. This is the first dredge up event (FDUP), as convection mixes up the products of H-burning to the surface altering the surface composition of the star.

In the mass range (4-7)M⊙, stars exhibit blue loops at the beginning of core He burning (see [Halabi, El Eid & Champagne (2012), and references therein]). The main phase of core He-burning is completed before the track evolves back to the RGB. This leads again to the deepening of envelope convection. For solar metallicity stars of M \gapprox= 4M⊙, a second dredge up (SDUP) can reach deeper regions, which leads to further changes in the surface abundances.

This work uses observations obtained for a sample of red giants by Tsuji (2008), hereafter Tsuji08, in order to achieve two goals: (a) to estimate the masses of these giants by matching their observationally derived effective temperatures and bolometric magnitudes to the values obtained from our evolutionary tracks in the HR diagram. This is possible since the stars are not pulsating Mira variables (see Section 3.1 for details). We are able to improve the mass determination done by Tsuji08 by using more extended evolutionary tracks to avoid the extrapolation that he partially relied on to determine the masses of some giants, (b) to compare the calculated CNO abundances of these models to those inferred from observations in the light of recent determinations of key reaction rates.

A large body of observational data is available for the surface CNO abundances in RGB stars [Lambert & Reis 1981; Harris & Lambert 1984; Harris et al. 1985; Lambert et al. 1986; Gilroy & Brown 1991; Tsuji 1991].

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2 MODEL CALCULATIONS

The evolutionary sequences presented in this work are obtained using the stellar evolution code HYADES as described in El Eid et al. (2004), with the recent modifications outlined in Halabi, El Eid & Champagne (2012). This code is a one-dimensional implicit Lagrangian code based on a hydrodynamical method which solves the stellar structure equations on an adaptive grid. Mass-loss is included using semi-empirical rates adjusted to the global parameters of the star. The rates are used according to a Mira pulsation period (P) (Vassiliadis & Wood 1992). For P < 100 d, (Reimers 1975) mass-loss rate is used, with \( \eta = 1 \). For 100 < P < 500 d we use a more effective rate according to Bowen (1988). For P > 500 d, the supervwind mass-loss rate during the AGB is used as suggested by Vassiliadis & Wood (1992).

2.1 Analyzing the overshooting region

In the context of the Mixing Length Theory (MLT), the extension of a convective zone is determined by the Schwarzschild criterion, that is when \( \nabla_{\text{rad}} > \nabla_{\text{ad}} \), where \( \nabla_{\text{rad}} \) and \( \nabla_{\text{ad}} \) are the radiative and adiabatic temperature gradients, respectively.

Within this framework, a long-standing issue is to fix the boundary of the convective zone. In the local description of the MLT, mixing beyond the Schwarzschild boundary is introduced in a parameterized way. According the multi-dimensional hydrodynamic simulations by Freytag et al. (1999), a local description of an extra mixing (or overshooting) may be introduced in terms of an exponentially decaying diffusion coefficient:

\[
D(z) = D_\infty \exp(-z/F)
\]

where \( z = |r_{\text{boundary}} - r| \) is the overshoot distance, \( D_\infty \) is the diffusion coefficient at the boundary of the convective envelope obtained from the Mixing Length Theory (see Langer et al. 1985), \( H_\rho \) is the pressure scale height and \( f \) is a free parameter which is a measure of the efficiency of this extra partial mixing. It is clear from Eq. 1 that for smaller values of \( f \), \( D_\infty \) has a steeper profile, or equivalently less extra mixing. As \( f \) increases, this extra mixing extends further beyond the formal convective boundary. The numerical simulations by Freytag et al. (1999) find \( f = 0.25 \pm 0.05 \) and 1.0 \( \pm 0.1 \) for A-stars and DA white dwarfs, respectively. We will use the observationally inferred oxygen isotopic ratio in red giants to constrain the value of \( f \) (see Section 3.2).

The mixing of chemical elements is achieved by solving the diffusion equation given by:

\[
\frac{dX_i}{dt} = \frac{\partial}{\partial M_r} ((4\pi r^2 \rho)^2 D \frac{\partial X_i}{\partial M_r})
\]

where \( r \) is the radius, \( \rho \) is the mass density and \( D \) is the diffusion coefficient given by Eq. 1 when used in the overshoot region, otherwise it is equal to \( D_\infty \) in a convective zone according to the Schwarzschild criterion.

To illustrate the effect of the treatment of mixing described above, Fig. 1 shows the behavior of \( \nabla_{\text{rad}} \) and \( \nabla_{\text{ad}} \) as well as the profiles of the diffusion coefficient and hydrogen (XH) as a function of interior mass. This is done for a 1.2M_\odot model during the first dredge up phase (FDUP) after the star has evolved to the red giant branch.
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Figure 1. Physical quantities related to the condition of convection at the bottom of the convective envelope in a 1.2M\(_{\odot}\) star during FDUP, for (a) the standard stellar model and (b) the model with overshooting using \(f=0.125\). (c) and (d) are the same as (a) and (b), respectively, but \(3\times10^5\) years later. \(\nabla_{\text{rad}}\) and \(\nabla_{\text{ad}}\) are the radiative and adiabatic temperature gradients, respectively. Also shown are the hydrogen profile (left scale) and diffusion coefficient profile (right scale).

The left-hand panel of Fig. 1 shows the extension of the convective envelope in the standard models, i.e. without overshooting where the abrupt drop of the diffusion coefficient \(D\) is visible along with the vertical drop of \(X_{\text{H}}\). The right panel shows the behavior with overshooting. It is seen that in the latter case, mixing is extended into the radiative region where \(\nabla_{\text{rad}} < \nabla_{\text{ad}}\). The lower panels show the profiles at the next time-step, \(3\times10^5\) years later. In both cases, the convective envelope deepens in mass as FDUP proceeds, but it is deeper in the case with overshooting, where the change in composition alters the opacity so that \(\nabla_{\text{rad}} > \nabla_{\text{ad}}\) becomes satisfied deeper in mass. Therefore, overshooting does not only induce extra mixing but also drives convective instability. Details of the present calculations with this overshooting or extra diffusive mixing are provided in Section 3.2.2.

2.2 Observational data

A sample of red giant stars has been observed by Tsujii08 as given in Table 1. The effective temperatures were obtained using the infrared flux method (Blackwell et al. 1980), while the absolute bolometric magnitudes were determined from the bolometric luminosities obtained by integrating the spectral energy distributions and the Hipparcos parallaxes. The uncertainty on the effective temperature is estimated to be 100K, and the error on the bolometric magnitude is mainly due to the error on the parallaxes. In the next section, the effective temperatures and bolometric magnitudes are used to determine the stellar masses of the observed giants using our evolutionary tracks, which cover the whole range of observations.

3 EVOLUTIONARY RESULTS

Evolutionary sequences of stars in the mass range \((1.2-7.3)M_{\odot}\) are calculated from the zero-age main sequence till the early AGB phase. Stars of \(M \geq 3M_{\odot}\) exhibit blue loops starting at the onset of central helium burning, which become more extended for stars of mass \(\geq 4M_{\odot}\). A detailed discussion on this evolutionary phase has been presented in Halabi, El Eid & Champagne (2012). Stars of masses \(<2.2M_{\odot}\) and solar-like initial composition evolve through the core He-flash (Kippenhahn & Weigert 1990; Mocák et al. 2010; Bildsten et al. 2012). We find that for \(M < 2M_{\odot}\), the helium flash starts off-center owing to the cooling via the plasma and photo neutrino energy losses. In the mass range \(2 \leq M/M_{\odot} \leq 2.2\) the helium flash starts at the center since these stars evolve at relatively lower central densities so the cooling by neutrino energy losses is less efficient.

The core helium flash requires very short time steps to
Table 1. Spectral type, effective temperatures and bolometric magnitudes (Tsuji08).

| Object(BS/HD) | Spectral type | $T_{\text{eff}}$ (K) | $M_{\text{bol}}$ (mag) |
|---------------|---------------|----------------------|------------------------|
| δ Vir (4910)  | M3III         | 3643                 | −2.4 ± 0.3             |
| α Tau (1457)  | K5+III        | 3874                 | −1.7 ± 0.2             |
| RRUmi (5589)  | M4.5III       | 3397                 | −3.4 ± 0.3             |
| RZ Ari (687)  | M6III         | 3341                 | −3.5 ± 0.6             |
| δ Oph (6056)  | M0.5III       | 3790                 | −2.2 ± 0.3             |
| ν Vir (4517)  | M1III         | 3812                 | −2.2 ± 0.4             |
| τ4 Eri (1003) | M3+IIIa       | 3712                 | −2.9 ± 0.4             |
| 10 Dra (5226) | M3.5III       | 3730                 | −2.9 ± 0.3             |
| β Peg (8775)  | M2.5III-MIII  | 3606                 | −3.3 ± 0.2             |
| 30g Her (6146)| M6-III        | 3298                 | −4.2 ± 0.4             |
| σ Lib (5603)  | M2.5III       | 3596                 | −3.4 ± 0.5             |
| R Lyr (7157)  | M5III         | 3313                 | −4.3 ± 0.3             |
| μ Gem (2286)  | M3III         | 3643                 | −3.3 ± 0.3             |
| OP Her (6702) | M5II          | 3325                 | −4.4 ± 0.8             |
| ρ Per (921)   | M4II          | 3523                 | −4.1 ± 0.4             |
| α Cet (911)   | M1.5IIIa      | 3909                 | −3.2 ± 0.3             |
| λ Aqr (8698)  | M2.5III       | 3852                 | −3.4 ± 0.7             |
| XY Lyr (7009) | M5II          | 3300                 | −5.1 ± 1.1             |
| δ7 Lyr (7139) | M4II          | 3420                 | −5.5 ± 0.8             |
| α Her (6406)  | M5lb-II       | 3293                 | −5.8 ± 1.6             |
| BS6861(6861)  | M4            | 3600                 | −5.2 ± 2.0             |

accommodate the rapidly changing variables. In our calculation, the time step is of the order of less than a year during the core helium flash. For stars of masses in excess of 2.2$M_{\odot}$, no significant degeneracy effects occur and core He-burning proceeds under hydrostatic conditions.

### 3.1 Mass evaluation

In the work by Tsuji08, the masses of the red giant stars listed in Table 1 are derived using the evolutionary tracks by Claret (2004). A main difference between our code and that of Claret (2004) is that the latter used Caughlan & Fowler (1988) rates for the basic nuclear reactions in the network, while our used reaction rates are updated according to the JINA REACLIB database (Cyburt et al. 2010). This is expected to introduce modifications on the evolutionary tracks. Another difference is that Claret assumes core overshooting, with an overshooting distance of 0.2$H_P$ for the whole mass range and ignores envelope overshooting. Core overshooting results in a bigger core mass and affects the evolutionary tracks and the stellar lifetimes. In the sample considered here, most of the stars are low-mass stars which have very small or no convective core at all, thus applying the same amount of overshooting at the convective core boundary would result in a large amount of mixing that yields results which are inconsistent with observations (Woo & Demarque 2001). In order to avoid this artifact, core overshooting is not included in our calculation.

Moreover, the tracks by Claret (2004) are not extended enough, so that extrapolations were needed at temperatures below 3200K during the RGB phase for about half of the stars in the sample studied by Tsuji08. In this calculation, the stars are evolved until the early AGB phase without relying on extrapolation. Moreover, mass-loss is included, and we report the masses of the evolved stars rather than their initial masses, which improves the mass determination, particularly for the higher masses where mass-loss becomes more effective.

Adopting the values of $M_{\text{bol}}$ and $T_{\text{eff}}$ given in Table 1 the evolutionary tracks shown in Fig. 3 are used to evaluate the stellar masses. It is important to note here that a direct comparison of the theoretical temperature to that inferred observationally wouldn’t have been possible if the stars are pulsating Mira variables, in which case a radius cannot be strictly defined and any comparison wouldn’t hold. Even the term effective temperature may become questionable for the very evolved AGB stars featuring strong pulsations and mass loss rates (Baschek et al. 1991, Lebzelter et al. 2010). However, in this sample, the stars are on the RGB or early AGB phase, and thus, haven’t yet experienced any thermal pulsations. This allows a reliable comparison between the predicted effective temperatures and the observationally inferred ones. The theoretical bolometric magnitude is obtained using the well known relation: $M_{\text{bol}} = 4.75 - 2.5 \log (L/L_{\odot})$.

As seen in Fig. 3, our evolutionary tracks describe well the advanced evolutionary stage of these stars. Having obtained the mass of every star using these tracks, it is possible to identify its evolutionary stage and compare its CNO surface abundances with the observational data. This will be described in Section 3.2.

The stellar masses are given in Table 2. For completeness, masses obtained by other works for some stars are included (Maillard 1974, Smith & Lambert 1985, Harris et al. 1988; Decin et al. 1997). The error on the mass is determined from the error bars on the observational $M_{\text{bol}}$ and $T_{\text{eff}}$. Table 2 shows that our values are systematically lower than those by Tsuji08. We attribute this mainly to two reasons:

(a) The stars are evolved to the stage where they are observed, that is, we do not use any extrapolated tracks as done in Tsuji08.

(b) Mass-loss is taken into consideration, which becomes significant for the more massive stars.

Moreover, it is clear from Table 2 that our errors on the masses are also lower. Calculating the tracks up to advanced stages helps to get better evaluation of the masses of red giants.

### 3.2 CNO Surface Abundances in Red Giants

#### 3.2.1 Predictions of surface abundances with standard mixing

This section presents the results for the surface CNO abundances of the studied sample of stars. These are then com-

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Figure 2. Bolometric magnitude ($M_{\text{bol}}$) versus effective temperature ($T_{\text{eff}}$) showing the present evolutionary tracks of stars of masses (1.2-7)$M_\odot$. Also shown is the observed sample listed in Table 1.

Figure 3. The evolutionary tracks together with the observed red giants listed in Table 1. Note that the observed data points have errors as indicated in Table 1.
Table 2, $a$: present work ($M_{\text{RGB}}$ is the mass on the RGB, including mass-loss), $b$: Tsuji08, $c$: Smith and Lambert (1985), $d$: Harris et al. (1988), $e$: Decin et al. (1997), $f$: El Eid (1994). The masses are given in solar units. Note that the error on the mass by Tsuji08 increases for more massive stars, which may be related to the inaccuracy in the parallax measurements.

| Object  | BS/HD | $M^a_{\text{initial}}$ | $M^b_{\text{RGB}}$ | $M^b$ | $M($others$)$ |
|---------|-------|------------------------|---------------------|-------|--------------|
| $\delta$ Vir (4910) | 1.2 ± 0.2 | 1.19 ± 0.2 | 1.4 ± 0.3 | 2.0 | c |
| $\alpha$ Tau (1457) | 1.2 ± 0.2 | 1.20 ± 0.2 | 1.5 ± 0.3 | 1.5 | c,d |
| RRU MI (5589) | 1.3 ± 0.1 | 1.15 ± 0.1 | 1.6 ± 0.3 | 1.7 | c,d,e,f |
| RZ Ari (687) | 1.3 ± 0.2 | 1.14 ± 0.2 | 1.5 ± 0.4 | 1.7 | c,d,e,f |
| $\delta$ Oph (6056) | 1.4 ± 0.2 | 1.39 ± 0.2 | 1.7 ± 0.4 | 2.0 | c |
| $\nu$ Vir (4517) | 1.4 ± 0.4 | 1.39 ± 0.4 | 1.7 ± 0.4 | 2.0 | c |
| $\tau^4$ Eri (1003) | 1.8 ± 0.3 | 1.73 ± 0.3 | 2.0 ± 0.4 | 2.0 | c |
| 10 Dra (5226) | 1.8 ± 0.3 | 1.74 ± 0.3 | 2.1 ± 1.8 | 2.1 | |
| $\beta$ Peg (8775) | 1.8 ± 0.3 | 1.70 ± 0.3 | 2.2 ± 0.3 | 1.7 | c,d,e,f |
| 30g Her (6146) | 1.8 ± 0.3 | 1.65 ± 0.3 | 2.0 ± 0.6 | 4.0 | |
| $\sigma$ Lib (5603) | 2.0 ± 0.2 | 1.90 ± 0.2 | 2.2 ± 0.5 | 2.0 | |
| R Lyr (7157) | 2.0 ± 0.2 | 1.80 ± 0.2 | 2.1 ± 0.5 | 2.0 | |
| $\mu$ Gem (2286) | 2.0 ± 0.5 | 1.90 ± 0.5 | 2.3 ± 0.5 | 2.0 | d,f |
| OP Her (6702) | 2.1 ± 0.4 | 1.90 ± 0.4 | 2.3 ± 1.0 | 2.3 | |
| $\rho$ Per (921) | 2.5 ± 0.4 | 2.40 ± 0.5 | 3.2 ± 0.5 | 3.2 | |
| $\alpha$ Cet (911) | 3.0 ± 0.5 | 2.96 ± 0.5 | 3.6 ± 0.4 | 3.6 | |
| $\lambda$ Aqr (8698) | 3.0 ± 0.5 | 2.96 ± 0.5 | 3.7 ± 1.2 | 3.7 | |
| XY Lyr (7009) | 4.0 ± 1.0 | 3.0 ± 1.0 | 3.7 ± 1.5 | 3.7 | |
| $\delta^2$ Lyr (7139) | 5.0 ± 1.0 | 4.5 ± 1.0 | 5.5 ± 2.0 | 5.5 | |
| $\alpha$ Her (6406) | 5.5 ± 1.5 | 4.0 ± 1.0 | 5.0 ± 2.0 | 5.0 | 7.0 | f |
| BS 6861(6861) | 5.5 ± 1.5 | 5.35 ± 1.5 | 6.3 ± 4.0 | 6.3 | |

pared to the CNO isotopic ratios inferred from observations. The abundance profiles of the CNO isotopes and those of H and He prior to the FDUP are shown in Fig. 4. In order to study the variation of these abundance profiles as a function of the initial stellar mass, two sample stellar masses are shown: 1.4M⊙ (Fig. 4h) and 5M⊙ (Fig. 4i). Several features can be identified in these figures:

(a) The isotope $^{13}$C is produced near the middle of the star by the CN cycle in all cases. This reflects the relatively low temperatures required for the production of this isotope by the CN-cycle.

(b) The $^{17}$O profile is remarkable, showing a steep gradient in the central region. This is because $^{17}$O is produced by the ON cycle which requires higher temperatures to become effective.

(c) The isotope $^{18}$O is fragile and effectively destroyed by the $^{18}$O(p,$\alpha$)$^{15}$N reaction.

These results are well known in the literature, but it is important to understand the surface abundances resulting after FDUP and SDUP for different stellar masses. The efficiency of FDUP in modifying the surface abundances is related to the maximum penetration of the convective envelope on the RGB. In Fig. 4, the solid vertical line marks the convective boundary as determined by the Schwarzschild criterion and the dashed vertical line marks the position of the boundary when extra mixing beyond the formal convective boundary is considered (see Section 3.2.2 for details).

After FDUP, the change in the surface composition of a certain isotope depends on the shape of its profile inside the star. For example, the peak of the $^{13}$C profile is located near the middle part of the star, so that envelope convection is able to smear out the profile, causing an increase in the $^{13}$C surface abundance, or a decrease in the $^{12}$C/$^{13}$C ratio. For the isotope $^{17}$O, the situation is highly dependent on the stellar mass because the main production of $^{17}$O is concentrated in the inner part. Fig. 4b, which shows the case of a 1.4M⊙ as an example, illustrates that in low mass stars (M ≤ 2M⊙) envelope convection does not completely smear out the $^{17}$O peak. This makes the $^{17}$O profile sensitive to mixing in stars of M ≤ 2M⊙, so that any additional mixing below the envelope will increase its surface abundance. This effect is less pronounced in stars with M ≥ 3M⊙ (Fig. 4b), shows the case of a 5M⊙ as an example in this mass range), where the $^{17}$O bump is fully engulfed by the formal convective envelope, so extra mixing will not significantly alter its surface abundance in this mass range.

Table 3 summarizes the values of $^{16}$O/$^{17}$O, $^{12}$C/$^{13}$C and $^{14}$N/$^{15}$N after the FDUP and SDUP (if any, as we show later) in the case of standard convective mixing, together with those inferred from observations by Tsuji08 and other independent field star observations. The stars in the sample of Tsuji08 are advanced in evolution, but did not experience the third dredge up during the AGB phase. This is evident from the carbon surface abundances (Tsuji 2014, private communication). Thus, the comparison can be restricted to the effects of FDUP and SDUP only.

It is not easy to make a direct comparison between predicted surface abundances and observations, since this comparison is model-dependent on theoretical and observational grounds. Deriving the isotopic ratios observationally involves several sources of uncertainty like the dispersion in the ratios obtained from different lines, and inaccuracy in the atmospheric model parameters. Systematic errors may also be present, such as the uncertainty in the continuum position and departures from local thermodynamic equilibrium (LTE) (Abia et al. 2012), in addition to the difficulties that are inherent in the spectral analysis of very cool stars. The fact that atmospheric values are model-dependent causes discrepancies between observational results among different groups, and consequently, affect the subsequent discussion (Ramstedt &_OFOLSSON 2014). On the other hand, theoretical models are also challenged by uncertainties in convective mixing, mass loss and nuclear reaction rates, where standard FDUP models often face difficulties in explaining carbon and oxygen surface abundances, particularly in low-mass stars. While being vigilant to these limitations, a careful comparison is useful for the sake of a better understanding.

Fig. 4 shows the $^{16}$O/$^{17}$O ratios as a function of stellar mass, along with the theoretical predictions by Boothroyd & Sackmann (1999), Abia et al. (2012), Karakas & Lattanzio (2014) and the available observations. The predicted $^{16}$O/$^{17}$O shows a distinct behavior between
The overproduction of $^{16}\text{O}/^{17}\text{O}$ in low mass stars in our model shows that more $^{17}\text{O}$ needs to be mixed to the surface in order to lower the $^{16}\text{O}/^{17}\text{O}$ ratio. This may be achieved by extra mixing or overshooting at the bottom of the convective envelope. In order to constrain this overshooting, a better understanding is required for the role of first and second dredge up for the whole range of masses under consideration.

It is known that after the star leaves the main sequence, FDUP alters significantly the surface abundances as it enriches the envelope with $^4\text{He}$, $^{13}\text{C}$, $^{17}\text{O}$ and $^{14}\text{N}$ and reduces its $^{12}\text{C}$, $^{15}\text{N}$ and $^{16}\text{O}$ abundance. Every star that evolves to the AGB experiences the FDUP episode, and starts its early AGB phase with a sharp composition discontinuity at the point of maximum penetration of the FDUP. The H-burning shell in low-mass stars represents an entropy bar-
convection to deepen in a second dredge up (SDUP) and the increase in the opacity (Iben & Renzini 1983), coupled to the drop in the temperature of the expanding layer, results in a temporary extinction in the H-burning shell. This situation is pushed outwards in mass to low temperatures causing a barrier that prevents any deeper penetration of the convective envelope, and thus no further change in the surface abundances takes place. However, for solar metallicity stars of masses above 3-4M⊙, the gravitational energy release due to the contracting core and the increased energy flux from the burning He-shell lead to an expansion so that the H-shell is pushed outwards in mass to low temperatures causing a temporary extinction in the H-burning shell. This situation, coupled to the drop in the temperature of the expanding layers and the increase in the opacity (Iben & Renzini 1983), causes convection to deepen in a second dredge up (SDUP) event and mixes out the composition discontinuity left over by the FDUP. Therefore, SDUP introduces further changes in the surface abundances of stars of masses >3M⊙.

Fig. 8 shows the maximum penetration of the convective envelope at FDUP and SDUP as a function of the initial stellar mass. It is clear from the figure that for M< (4-5)M⊙, SDUP doesn’t penetrate deeper than the FDUP; and thus, it does not introduce significant change to the surface composition. This is in agreement with Karakas & Lattanzio (2014), for their solar metallicity case. Their Fig. 7 exhibits similar general features and also indicates a deepest penetration of
FDUP in $\approx 2.5M_\odot$ stars. For stars of $M > 6M_\odot$, the expansion is strong and convection extends deeper inwards. Since low mass stars do not experience SDUP, this implies that their observed surface abundances on the AGB phase are actually “inherited” from the RGB phase. Therefore, one way to account for the discrepancy between calculated and observationally inferred values of $^{16}O/^{17}O$ in these stars is to invoke extra mixing below the convective envelope on the RGB phase.

3.2.2 Predictions of surface abundances with extra mixing

Overshooting is applied as outlined in Section 2.1 and we find that $f = 0.125$ provides the best estimation for its efficiency in low mass stars. Fig. 7 shows the $^{16}O/^{17}O$ ratios of our star sample in the standard case and with overshooting. The low observational $^{16}O/^{17}O$ in low mass stars can now be fitted quite well within the error bars on the RGB phase. Therefore, one way to account for the discrepancy between calculated and observationally inferred values of $^{16}O/^{17}O$ in these stars is to invoke extra mixing below the convective envelope on the RGB phase.

3.3 Effect of modified nuclear reaction rates

3.3.1 The $^{16}O/^{17}O$ ratio

In order to investigate the effect of the $^{17}O$ production and destruction reaction rates on the $^{16}O/^{17}O$ ratios, four different evaluations of the proton-capture reactions $^{16}O(p,\gamma)^{17}F$, $^{17}O(p,\gamma)^{18}F$ and $^{17}O(p,\alpha)^{14}N$ are used. In particular, we use the compilations by Sergi et al. (2014) (SE14), Sallaska et al. (2013) (SA13), Hladis et al. (2010) (IL10) and Chafa et al. (2007) (CH07). The $^{16}O/^{17}O$ ratios obtained are shown in Fig. 8 for both cases: standard mixing (Fig. 8a) and envelope overshooting with $f = 0.125$ (Fig. 8b). The four sets of rates give very similar $^{16}O/^{17}O$ values in the considered mass range. None provides a reasonable agreement between model predictions and observations unless overshooting is included. This consistency shows that the existing discrepancy cannot be removed without invoking deeper mixing. Fig. 8b shows a better fit of the observed data within the error bars in the low mass stars.

The effect of the reaction rates uncertainties is also worth exploring. Since the SA13 compilation is based on a Monte Carlo simulation and the rates have statistically well-defined uncertainties, Exploring masses & abundances of red giants
Figure 8. The predicted surface abundances of (a) carbon and (b) nitrogen with observations as shown. The initial solar values of carbon and nitrogen are 8.5601 and 8.0499, respectively and are indicated by a star symbol.

Figure 9. The $^{16}\text{O}/^{17}\text{O}$ ratios calculated for the 21 giants, with four different sets of compilations of relevant reaction rates (see text for details). Left panel shows the case with standard mixing and the right panel is that with overshooting ($f = 0.125$). Observational data are also included.

2012 Iliadis et al (2014), the $^{16}\text{O}/^{17}\text{O}$ ratios are calculated using the recommended, high and low rates, where the rate boundaries correspond to a 95% coverage probability. The rates uncertainty has a very minor effect on the tracks during H-shell burning but none along the RGB, and hence it does not affect our mass determination. However, the $^{16}\text{O}/^{17}\text{O}$ ratios show a larger sensitivity to these uncertainties, as shown in Fig. 10. It is found that the observational data can be better explained when the whole range of uncertainty on the involved reaction rates is considered since the discrepancy between predictions and observations becomes less pronounced. On the other hand, the $^{12}\text{C}/^{13}\text{C}$ ratios are found to be immune to the uncertainties on the considered rates. The difference in $^{12}\text{C}/^{13}\text{C}$ obtained with the high rates and low rates does not exceed 3%.

3.3.2 The $^{14}\text{N}/^{15}\text{N}$ ratio

The surface $^{14}\text{N}/^{15}\text{N}$ ratio is worth considering in connection with the latest evaluation of the $^{14}\text{N}(p, \gamma)^{15}\text{O}$ rate (Marta et al 2011). This ratio is calculated for the mass range under consideration after FDUP and SDUP. It is found that our values are higher by $\sim 20\%$ relative to those by El Eid (1994) in stars of masses $\geq 3\text{M}_\odot$. This is expected and due to the NACRE $^{14}\text{N}(p, \gamma)^{15}\text{O}$ rate (Angulo et al. 1999) used in that work, which is almost a factor of 2 higher than the revised rate at stellar temperatures (see Halabi, El Eid & Champagne 2012 for the expression of this rate and explicit discussion). However, our $^{14}\text{N}/^{15}\text{N}$ is about 12% lower in the low-mass stars.

It is quite unfeasible to verify our findings since $^{14}\text{N}/^{15}\text{N}$ ratios are difficult to measure in RGB stars from CN lines because they are too weak even in very high resolution spectra. In fact, the $^{14}\text{N}/^{15}\text{N}$ ratio is difficult to measure di-
the revision of the spheric ratios (Hedrosa et al. 2012). Theoretical predictions in chemistry and thus, might not represent the stellar photo-
cumstellar N ratios might be affected by the incoming UV
cumstellar envelope of the star not the photosphere. Cir-
but in this case one would probably be looking at the cir-
cumstellar envelope of the star not the photosphere. Circ-
sumstellar N ratios might be affected by the incoming UV
radiation from the ISM triggering non-kinetic equilibrium
chemistry and thus, might not represent the stellar photo-
spheric ratios (Hedrosa et al. 2012). Theoretical predictions
of the $^{14}$N/$^{15}$N ratios, particularly the changes induced by
the revision of the $^{14}$N(p, $\gamma$)$^{15}$O reaction rate may benefit
from future observations or one may resort to the chemical
analysis of pre-solar grains originating from the envelopes
of late AGB stars.

4 CONCLUSIONS

A sample of observed RGB and early AGB stars was con-
sidered and their masses were obtained using extended ev-
olutionary tracks. This was done without the need of extrap-
olating the evolutionary tracks at lower effective tempera-
tures as it has been done in the work by Tsuji08. The present
evolutionary tracks include the effect of mass-loss, which be-
comes important during the red giant phases, especially for
the higher stellar masses under consideration. This investiga-
tion includes an analysis of the physical conditions in the
overshooting region and the effect of this overshooting on
the abundance yields. We find that overshooting is needed
to reconcile observational oxygen abundances with model
predictions particularly in low mass red giants. Although

Tsujio8 has interpreted the discrepancies between the predic-
ted and observational abundances as an effect of extra
mixing, he did not provide models including this extra mix-
ing to see how the discrepancies may be understood. We
showed that extra mixing based on the mixing of chemical
elements by diffusion explains reasonably the observation-
ally inferred oxygen isotopic ratios. It is important to realize
the challenges facing such observations and the uncertain-
ties involved in the available data. In this regard, the spread
in the observational data in the low and high mass ranges
was discussed in connection with the inherent difficulties in
analyzing the spectra of these relatively cool stars and the
uncertainties involved in measuring faint lines like $^{12}$C$^{16}$O.
Our overshooting treatment cannot, however, explain the
low surface carbon abundances in low mass stars. Another
mixing mechanism seems to be required during the long ev-
olutionary time needed for the low-mass stars to ascend the
red giant branch.

Furthermore, the present calculations were carried out
using recent determinations of proton-capture rates which
have reliable statistical error bars. This allows us to draw conclu-
sions on the uncertainties involving CNO surface
abundances. In particular, the effect of recent evaluations
of the reaction rates on the production and destruction of
$^{17}$O was explored. The experimentally suggested uncertainty
of these rates provides a better fit of the $^{16}$O/$^{17}$O observed
in low mass stars yet does not exclude the need to invoke
overshooting. Additional mixing beyond the convective en-
velope as determined by the Schwarzschild criterion is found
to be necessary to better explain the observational $^{16}$O/$^{17}$O
surface abundances, especially in low mass stars. Moreover,
the effect of $^{14}$N(p, $\gamma$)$^{15}$O rate on the $^{14}$N/$^{15}$N ratios was
studied in the considered mass range.

As a final remark, our approach in the present study
was to consider a sample of red giant stars to see how far
standard calculation predictions agree with observations. A
comprehensive comparison between stellar models and ob-
servations based on the analysis of the effect of extra mixing
was presented and linked to the uncertainties in key nu-
clear reaction rates affecting the CNO abundances in red
giants. Comparing theoretical predictions of stellar models
to available observations is required in order to constrain
parametrized approaches in determining the efficiency and
extension of mixing at convective boundaries. Future multi-
dimensional simulations of convection may introduce an im-
proved local description of this mixing and provide insight
towards a better understanding of the physical processes in-
volved.

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