Simulation on Rotation Curve of Spiral Galaxies
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Abstract. The problem of dark matter in galaxies is still one of the most important unsolved problems in the contemporary extragalactic astronomy and cosmology. The existence of a significant dynamic difference between the visible mass and the conventional mass of galaxies firmly establishes observational result. In this paper an unconventional explanation will be tested as an alternative to the cold dark matter hypothesis; which is called the modified Newtonian dynamics (MOND).

In this paper covers the simulation of galactic evolutions; where the two hypotheses are tested via the rotation curves. N-body simulation was carried adopting different configuration like a hot disk, elliptical (with arms and without arms) with a different parameter that covers the objects distribution, masses and velocities. It is shown from the simulation results that the MOND hypothesis has generated better rotation curves than the Newtonian theorem. Moreover, the appropriate configuration and parameters for spiral galaxy are investigated. It is shown that this assignment has not been easy; because the problem is very delicate and unstable.

Key words: rotation curve, spiral galaxy, N-body, disk, MOND.

1.  . Introduction
The fundamental building blocks of the Universe are galaxies. Some of them have a very simple structure, involving only normal stars and no particular individual features presence. On the other hand, others have complex systems, composite from many separate components; stars, neutral and ionized gas, dust, molecular clouds [1].

The flatness of spiral galaxy rotation curves was established through extended optical curve by Rubin in 1978 and more securely from extended HI rotation curves by Bosma in 1978 and 1981. HI observations have the usefulness that the gaseous disk expands much farther out than the optical disk, so the missing mass discrepancy, which increases with radius, is more pronounced [2,3].

In 2007 Paolo Salucci discovered that RC pursue their viral radius, a Universal Rotation Curve (URC) produced by gravitational potential of a Freeman stellar disk and the other related to that of a dark halo [4].

Milgrom M. and Robert H. Sanders in 2007 presented MOND analysis for several of the lowest mass disk galaxies currently amenable to such analysis, with (baryonic) masses below $4 \times 10^8$ Mʘ. The agreement is good, extending, the validity of MOND and its predicted mass velocity relation, to such low masses [5].

Yoshiaki S., Mareki H., and Toshihiro O. in 2008 introduced a united rotation curve by repeating the calculations of the distances and velocities for a set of galactic constants $R_0 = 8$ kpc and $V_0 = 200$ km.s$^{-1}$. Furthermore Yoshiaki Sofue in 2008 structure a Galacto-Local Group rotation curve and demonstrate the manner of some of the significant features of RC[6,7].
The aims of this paper is to solve the related issues for the N-body problem and describe it. Mathematically, the N-body problem improvement is achieved and proved to be an Ordinary Differential Equation (ODE), and solutions for the N-body ODE are structured by numerical analysis.

2. Theory of galaxy dynamics
This section introduce the theory of galaxy dynamics description; also it will give the simulation for the N-body problem simulate. The faster simulation of gravitational force were thought up.

2.1. The gravitational force for N-body problem
The N-body problem is known by former conditions i.e. initial positions and initial velocities of all bodies in the system. All bodies in the system are interacted and it is nessacery to assess the interaction forces to get new positions and velocities. This assessment is achieved repeatedly so the time evolution of the system is obtained. Mathematically and by the system of Ordinary Differential Equations (ODE) incoming from Newton’s laws of motion, the N-body problem is subedited as [8]

$$m_i \frac{d\vec{v}_i}{dt} = \sum_{j=1}^{N} \vec{F}_{ij}$$  \hspace{1cm} (1)

where $m_i$ and $\vec{v}_i$ are the parameters of mass and velocity for the $i$-th particle, respectively, and $i = 1, 2, ..., N$. The force $\vec{F}_{ij}$ is usually the sum of external forces. Newton’s gravitational force will be applied in case of stars or stellar systems

$$\frac{d\vec{r}_i}{dt} = G \sum_{j=1}^{N} \frac{m_j \vec{r}_{ij}}{r_{ij}^3}$$ \hspace{1cm} (2)

where $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ and $r_{ij} = |\vec{r}_i - \vec{r}_j|$. Generally, collisions between bodies are prevented in astrophysical simulation; this is sensible for galaxies that may pass right through each other. For this reason, a softening factor $\epsilon > 0$ is used, and the denominator is re-expressed as follows [8]

$$\vec{F}_{ij} = \frac{Gm_i \sum_{j=1}^{N} m_j}{(r_{ij} + \epsilon)^3} \vec{r}_{ij}$$ \hspace{1cm} (3)

In effect, the softening factor determines the force magnitude between the bodies, which is eligible for numerical integration of the system state. To integrate over time, the acceleration to update the position and velocity of body $i$ is required, thus the computation will be easier to the following form [9]

$$\vec{a}_i = G \sum_{j=1}^{N} \frac{m_j}{(r_{ij} + \epsilon)^3} \left( \vec{r}_j - \vec{r}_i \right)$$ \hspace{1cm} (4)

For updating the positions and velocities, the integrator is employed which is jump because it is applicable to this problem and is computationally effective. Usually for N-body problems, the selection of integration method based on the nature of a studied system.

2.2. Mathematics of MOND
The flatness of galaxies rotation curves at large radii was the first piece of observational evidence pointing out the possibility that there must be more mass in a galaxy than its luminous matter shows.
One interesting option is the MOND theory, which is based on a changing of Newton's second law ($\vec{F} = m\vec{\alpha}$) rewritten in the form [10].

$$m\mu \frac{\alpha}{a_o} \vec{\alpha} = \vec{F}$$  \hspace{1cm} (5)

$a_o$ : define a critical acceleration; which below it Newton's second law is not valid anymore. $a_o$ can take a universal value around ($a_o \sim 10^{-8}$ cm/s$^2$).

Where $\mu(x)$ is a function that can take the form

$$\mu(x) = \begin{cases} 1 & \text{for } x \gg 1 \\ x & \text{for } x \ll 1 \end{cases}$$  \hspace{1cm} (6)

where $x = \frac{a}{a_o}$

MOND has been very successful in explaining the rotation curve of the spiral galaxies [11].

The general form of the MOND theory is given by

$$m\mu(x)\vec{\alpha} = G\frac{M\mu}{r^3}$$  \hspace{1cm} (7)

$\mu(x)$ can take the form

$$\mu(x) = \frac{x}{\sqrt{1+x^2}}$$

For technical reasons, it is convenient to rewrite equation (7) as

$$G\frac{M\mu}{r^3} \vec{r} = m\mu \left( \frac{\alpha}{a_o} \right) \vec{\alpha}$$  \hspace{1cm} (8)

$$G\frac{M}{r^3} \vec{r} = \mu \left( \frac{\alpha}{a_o} \right) \vec{\alpha}$$  \hspace{1cm} (9)

taking into account that $a \ll a_o$ therefore $\mu \left( \frac{\alpha}{a_o} \right) = \frac{a}{a_o}$ then one get

$$G\frac{M}{r^2} = \frac{a^2}{a_o}$$  \hspace{1cm} (10)

From which can be expressed as [12]

$$|\vec{\alpha}| = \frac{GMa_o}{r^2}$$  \hspace{1cm} (11)

For a circular motion (centrifugal force)

$$\vec{F} = m \vec{a} = m\frac{v^2}{r} \vec{r}$$ \hspace{1cm} so \hspace{1cm} $|\vec{a}| = \frac{v^2}{r}$ \hspace{1cm} then

$$|\vec{a}| = \frac{v^2}{r} = \frac{\sqrt{GMa_o}}{r}$$  \hspace{1cm} (12)

Finally, the circular velocity is [13]

$$V(r) = \frac{\sqrt{GM(r)a_o}}{r}$$  \hspace{1cm} (13)
It can be seen that the circular velocity is independent on the distance $r$ from the center.

### 2.3 Leapfrog Method

At each leapfrog step, *Leapfrog Method* utilizes derivatives computed. In addition, it averts the requirement for the second derivative computing. The system preparing for the leapfrog method is needed extra actions as the position prediction of the body at the middle of time step according to [14]

$$\hat{\mathbf{r}}_i^{(t+\frac{1}{2})} = \hat{\mathbf{r}}_i^{(t)} + \hat{\mathbf{v}}_i^{(t)} \frac{\Delta t}{2}$$  \hspace{1cm} (14)

and the acceleration based on these positions at $\mathbf{r}_{ij} = \hat{\mathbf{r}}_i^{(t+\frac{1}{2})}$ computed as

$$\hat{a}_i^{(t+\frac{1}{2})} = G \sum_{j=1}^{N} \frac{m_j}{r_{ij}^3} \hat{\mathbf{r}}_j$$  \hspace{1cm} (15)

Then the i-th particle is advanced according to

$$\hat{\mathbf{v}}_i^{(t+1)} = \hat{\mathbf{v}}_i^{(t)} + \hat{a}_i^{(t+\frac{1}{2})} \Delta t$$  \hspace{1cm} (16)

$$\hat{\mathbf{r}}_i^{(t+1)} = \hat{\mathbf{r}}_i^{(t)} + \left( \hat{\mathbf{v}}_i^{(t)} + \hat{\mathbf{v}}_i^{(t+1)} \right) \frac{\Delta t}{2}$$  \hspace{1cm} (17)

![Figure 1](image-url)  
**Figure 1.** Schematic representation of a simple leapfrog method. $x_1$ is determined from $x_0$ and $v_{1/2}$. $v_{3/2}$ is determined by $v_{1/2}$ and the acceleration determined from $x_1$, and so forth [14].

### 3. Results and Discussion

The most popular tool used to study galaxy formation and evolution is a numerical simulation; Collision less N-body simulation that evolves an initial distribution of point masses according to the laws of gravity. Most astronomers and physicists prefer the alternative, modified Newtonian dynamics (MOND) for use over great distances, so MOND is a success in predicting the rotation curves in larger distance case.

This type of simulation governed by the Newtonian law (Newtonian gravity), requires solving $(6N)$ ordinary differential equations (three for object position and the other three for velocity) to describe the astronomical object in space. Particle-Particle (P-P) method will be adopted to achieve the desired simulation. The case of simulation will cover hot disk and elliptical distributions configuration.

#### 3.1 Disk distribution:

To generate a disk distribution; the polar coordinates will be used to distribute the object location as:
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\[ x = r \cos(\theta) \]
\[ y = r \sin(\theta) \]  

(18)

An initial angle \( \theta \) is drawn uniformly between 0 and 2\( \pi \).

Dispersion velocity can be simply represented by another velocity but with a small random magnitude relative to the main object velocity (circular velocity). So the cold disk has no dispersion velocity while the hot disk has it.

The object's velocity is governed by

\[ v_x = r_p \times v \times \cos(\theta) \]
\[ v_y = r_p \times v \times \sin(\theta) \]  

(19)

Where \( r_p \) object distance from the center of the simulated distribution and \( v \) is suggested initial velocity, while the mass distribution of the object will be unity but the central mass that will be big enough like \( 7e3M_\odot \).

3.2 Hot Disk Realizations

The hot disk has circular velocities. Its rotation is that obeyed by Keplerian laws. The hot disk is similar to cold disk except for the dispersion velocity that distributes many objects with random magnitude. The case is carried out with the following parameters.

**Table 1.** Parameters of simulation for hot disk model

| Number of star particles | Time step | Number of iteration | Radius | Total mass | Velocity | Velocity dispersion |
|--------------------------|-----------|---------------------|--------|------------|----------|---------------------|
| 3000                     | 0.00015   | 32128               | 5      | \( 1 \times 10^3 \) | 30       | random              |

Figure 2. Initial setup for the hot disk with the velocity dispersion added to orbital velocities.

Hot case output is given by figure 3 to figure 8.
Figure 3. A time sequence of the hot disk model in the xyz plane

Figure 4. A time sequence of the hot disk model in the x-y plane
Start

322.9 million years

626.28 million years

920 million years

1.272 billion years

1.572 billion years

Figure 5. A time sequence of the hot disk model in y-z plane

322.9 million years

626.28 million years

920 million years

1.272 billion years

1.572 billion years

Figure 6. Rotation curve of the hot disk model after many million years
No spiral shape is found at the end of test time, while the RC keeps the traditional behaviour with no significant different with respect to the previous cases. Thickness (dispersion in the z-axis) appears clearly in these cases in spite of the previous one (cold); this is due to the additional dispersion velocity figure 5. The final distribution leaves the plane by ejecting objects into third axis (z-axis); which increase in time. The hot reveal a significant behaviour, where they are started from continuous distribution into two high distinct regions, which represent the bulge and disk parts, i.e.in order to get a spiral galaxy configuration all is needed to generate arms.

3.3 Elliptical Distribution:
Elliptical distribution can be described by the same way of the disk distribution but with two descriptors: the major and minor radius. It is assumed here that the ratio between the major and minor radii is double. To start the simulation, the following parameters were taken.

| Number of star particles | Time step | Number of iteration | Radius | Total mass | Velocity | Velocity dispersion | Shape factor |
|-------------------------|-----------|---------------------|--------|------------|----------|--------------------|-------------|
| 3000                    | 0.00015   | 32128               | 5      | $1 \times 10^7$ | 30       | random             | 2           |

Figure 7. Velocity distributions in the xyz plane of the hot disk model after 1.572 billion years.

Figure 8. Velocity in the x-y plane of the hot disk model after 1.572 billion years.
Output simulation for this configuration shown by figure 9 to figure 11 in 3-D and 2-D coordinates respectively.

Figure 9. A time sequence of the spiral hot disk model in the xyz plane.
Figure 10. A time sequence of the spiral hot disk model in the x-y plane.

Figure 11. A time sequence of the spiral hot disk model in the y-z plane.
Figure 12. Rotation curve of the spiral hot disk model after many million years

Figure 13. Velocity distributions in the xyz plane of the spiral hot disk model after 626.28 million years

Figure 14. Velocity distributions in the x-y plane of the spiral hot disk model after 626.28 million years
From Figure 9 it's shown that there is an essential change in the objects distribution with respect to the previous distribution cases. A spiral galaxy at last, has been generated. The two distinct arms are generated after approximately 322 million years. But the problem is that those arms are not stable and vanish after approximately 1.2 billion years.

It seems that the shape factor of the ellipse is the key of spiral shape generation. One can connect this phenomenon to the gravitational force difference acting on the outer part of the initial distribution objects (objects with minor distance distribution affected by a larger force than those with major distance). This force (attraction towards the center); and rotation about the center generate arms.

This difference in acting force bends the nearest objects inside while the far objects outside the distribution center in which finally generate the spiral shape. But the problem is that those arms are not stable and will vanish after about 1.2 billion years because the gravitational force becomes stronger with time. The change in the shape distribution will affect the shape of RC as in figure 12. The change in RC behavior will be logically related to the arms formation. The RC is still obeying keplerian laws; where when these arms dissipated the RC plane restore its usual shape.

3.4 Modified Newtonian Dynamics:
After applying Newtonian theory with no good results; the attention is directed to other hypotheses, which is MOND, to test the modification considering the following cases taking into account the modification law of gravitational force (acceleration).

3.4.1 Elliptical Distribution:
Section 3.3 which rotates to elliptical distribution under Newtonian theorem were spiral distribution is reached. Below is the list the parameters that will be used to start the experiment and monitor the behaviour difference.

| Parameters of simulation for the elliptical distribution model |
|---------------------------------------------------------------|
| Number of star particles | Time step | Number of iteration | Radius | Total mass | Velocity | Velocity dispersion | Shape factor |
| 3000 | 0.00015 | 85676 | 5 | $1\times10^6$ | 80 | random | 2 |

Figure 15 to 21 display the output simulation results of this case.
Figure 15. A time sequence of the spiral hot disk model in the xyz plane

Figure 16. A time sequence of the spiral hot model in the x-y plane
Figure 17. A time sequence of the spiral hot disk model in the y-z plane

Figure 18. Rotation curve of the spiral hot disk model after many million years
A very clear and beautiful spiral distribution is gained. Sharper spiral shape and earlier is reached than Newtonian and even longer arms period time is reached (see also figure 19 and figure 20 for velocity distribution).

As expected those arms were not stable and finally will merge with the rest of the distribution into a disk-like. The corresponding RC (figure 18) shows the desired behaviour. This curve is seen to be shrinking with time, where the velocity distribution collapses to the inside according to increasing gravitational acceleration.

4. Conclusions
The following conclusions can be obtained:

1. The previous points are proved by simulation. It is very difficult to reach a specific distribution shape in spite of the big number of tries. This can be due to the complexity that covers the whole case of creation simulation. It is too difficult to try making the ALLAH creation whatever one can use. This note is tested in this work: in spite of this difficulty, a good result is reached using specific kind of distribution like elliptical distribution in very specific condition for objects numbers, distributions, masses and velocities.

2. It can be shown that every beginning event there is an end; for example, the spiral shape starts and end with time (birth and death). This phenomenon is very popular with humankind, and it's true for either creation like astronomical objects.

3. Combining multiple simulated distributions like bulge, disk and halo as is proposed finally gives best results for distribution and RC.
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