On the Initial Conditions for Pre-Big-Bang Cosmology

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Abstract: The beautiful scenario of pre-big-bang cosmology is appealing not only because it is more or less derived from string theory, but also because it separates clearly the problem of the initial conditions for the universe from that of high curvatures. Recently, the pre-big-bang program was subject to attack from on the grounds that pre-big-bang cosmology does not solve the horizon and flatness problems in a “natural” way, as customary exponential “new” inflation does. In particular, it appears that an arbitrarily small deviation from perfect flatness in the initial state can not be accommodated. For this analysis, matter in the universe before the big bang was assumed to be radiation. We perform a similar analysis to theirs, but using the equation of state for “string matter” \( \rho = -3p \) which seems more appropriate to the physical situation and, also, is motivated by the scale factor duality (in the flat case) with respect to our expanding, radiation dominated, universe. For an open universe we find, exactly, the same time-dependence of the scale factor as in the Milne universe, recently found to represent the universal attractor at \( t = -\infty \) of all pre big bang cosmologies. We conclude that our radiation dominated universe comes from a flat rather than a curved region.

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1. Introduction

The standard cosmological model does not deal with the initial singularity problem. In order to solve this Veneziano and collaborators have developed a stringy cosmology [1][2][3][4][5][6], mainly based on a property of the low-energy effective action, called scale-factor-duality. It is now possible to think physically about the universe before the high curvature, high temperature regime called the big bang. Recently, the horizon problem in the pre-big bang scenario has attracted a great deal of attention [7][8][9][10][11] assuming that the universe, before the big bang was radiation dominated. There is, however, no reason to expect that. Rather, within the validity range of the effective action for the massless modes of the strings, we expect all matter to still behave stringly, effectively as large null strings.

In section 2, motivated by the scale-factor duality, we use the equation of state of string matter in a homogeneous universe. Section 3 shows how these solutions deal with the horizon problem, and we conclude in section 4.

2. Cosmic solutions

The equations describing the universe during the pre-big-bang phase ($t < 0$) are obtained from the tree-level low-energy effective action of strings [12]

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{|g|} e^{-\phi}(R + \partial_{\mu}\phi\partial^{\mu}\phi) + \sum_{\text{matter}} \int d^4x \sqrt{|g|} \mathcal{L}_{\text{matter}}$$

(2.1)

where $g_{\mu\nu}$ is the four dimensional space-time metric, $\phi$ is the dilaton field, and $\mathcal{L}_{\text{matter}}$ accounts for all other fields (including the Kalb-Ramond field).

In a Friedman-Robertson-Walker background, the metric is

$$ds^2 = dt^2 - a^2(t) \left\{ \frac{dr^2}{1-kr^2} + r^2 d\theta + r^2 \sin^2 \theta d\phi^2 \right\}$$

(2.2)

where $a(t)$ is the cosmic scale factor and $k = 0, \pm 1$. The equation of motion for a homogeneous dilaton $\phi(t)$ is

$$\ddot{\phi} - 2\dot{\phi} - 6\dot{\phi}H + 6\frac{\ddot{a}}{a} + 6H^2 + 6k \frac{k}{a^2} = 0$$

(2.3)

where $H = \dot{a}/a$ is the Hubble constant.
The Einstein equations are

\[ 2(R_{\mu\nu} + \nabla_{\mu} \nabla_{\nu} \phi) = 16\pi G e^{\phi} T_{\mu\nu} \]  

(2.4)

where \( T_{\mu\nu} \) is derived from \( \mathcal{L}_{\text{matter}} \). By combining the time component of (2.4) with (2.3), we get the equation for the energy density

\[ 6H^2 - 6\dot{\phi}H + \dot{\phi}^2 + 6 \frac{k}{a^2} = 16\pi G e^{\phi} \rho \]  

(2.5)

and from the space component of (2.4), the equation for the pressure

\[ 3H^2 + \dot{H} - H\dot{\phi} - 2 \frac{k}{a^2} = 8\pi G e^{\phi} p \]  

(2.6)

The three equations (2.3), (2.5) and (2.6) describe a homogenous and isotropic universe (we disregard throughout spatial fluctuations, otherwise important for structure formation). In order to solve them we need an equation of state. Using radiation \( \rho = 3p \), it is easy to see that one solution is given by our post big bang \( (t > 0) \) radiation dominated universe, \( k = 0 \), \( a(t) \sim t^{1/2} \), \( \phi = \text{constant} \). Using the cosmological form of T-duality dubbed scale-factor duality [1] this solution is mapped to a pre-big-bang universe with increasing Newton’s constant

\[ G_N = \ell_{\text{st}}^2 e^{\phi} \]  

(2.7)

which satisfies equations (2.3) - (2.4) with \( (t < 0) \),

\[ a(t) = a_0 \left( -\frac{t}{t_0} \right)^{-1/2} \]  

(2.8)

\[ \phi(t) = \phi_0 - 3 \ln \left( -\frac{t}{t_0} \right) \]  

(2.9)

This universe, with initially vanishing Newton’s constant and scale factor, i.e., initially flat and empty, is the reasonable initial condition for a stringy cosmology. As emphasized by Veneziano, this initial condition is no less reasonable than the usual one in standard inflationary cosmology, namely a very hot and curved one. The interest of the pre-big bang scenario is that the initial condition is a separate issue from the behavior of matter at high temperature and energy. Interestingly enough, the duality transformation not only takes \( t \) to \(-t\) and \( a \) to \( 1/a \), but it also changes the dilaton and, most curiously, changes also the equation of state [1]

\[ \rho = -3p \]  

(2.10)
This equation of state represents string matter [13].

We wish to investigate the stability under a variety of initial conditions of the above pre-big-bang scenario. In particular, we are interested in whether enough inflation can take place while \( t < 0 \) to solve the problems of standard non-inflationary cosmology. Following the ideas in [9], we study what happens when \( k \neq 0 \). Note that, in this case, there is no abelian T-duality, and the non-abelian duality exists but it has no spherical symmetry [14]. Thus, we cannot find pre-big-bang cosmological solutions from post big bang ones, but of course we can just solve the equations directly. Since the equation of state describing stringy matter is a local feature of the equations, it should be insensitive to the global nature of the universe. We thus use, for \( t < 0 \), the same equation of state (2.10) for \( k = \pm 1 \) as for \( k = 0 \). At very early times (\( t \to -\infty, \phi \to -\infty \)), all interactions are turned off, and thus what equation of state we use to simulate the dynamics of all the fields other than the metric and the dilaton is actually irrelevant. However, in later stages of the pre-big-bang evolution, when gravity starts inflating the universe, it should make a difference. We are not interested in periodic universes, so we will concentrate on \( k = 0 \) and \( k = -1 \).

For an open universe, with spatial curvature \( k = -1 \), we find two possible universes

\[
a(t) = \frac{1}{\sqrt{3}}(-t) \\
\phi(t) = \phi_0 + 6 \ln \left( -\frac{t}{t_0} \right)
\]

and

\[
a(t) = \sqrt{3}(-t) \\
\phi(t) = \phi_0 + 2 \ln \left( -\frac{t}{t_0} \right)
\]

As we can see, the dilaton field is singular at both ends of the time evolution, \( t \to -\infty \) and \( t \to 0^- \). However, this does not mean that there is a strong-coupling regime in the far past. At very early times the equations of motion are a bit different because spatial and time derivatives are of comparable importance and in this treatment we are neglecting spatial derivatives. As shown in [15] once the spatial gradients are taken into account there is a generic early-time attractor which corresponds to the Milne universe, which eventually (as \( t \) goes from \(-\infty\) to \(0\)) would turn in ours solution.
To map the above solutions to the Einstein frame, we use the conformal rescaling \cite{16}

\[
\tilde{g}_{\mu\nu} = g_{\mu\nu} e^{-\phi} \quad \tilde{\phi} = \phi \quad (2.13)
\]

whereby the scale factor for \( k = 0 \) takes the form

\[
\tilde{a}(\tilde{t}) \sim (-\tilde{t})^{2/5} \quad (2.14)
\]

whereas the scale factor for the solution (2.11) with \( k = -1 \) goes like

\[
\tilde{a}(\tilde{t}) \sim (-\tilde{t}) \quad (2.15)
\]

while for (2.12) is constant and thus all the dynamics are governed by the dilaton.

Note that neither in the string frame nor in the Einstein frame we do get an accelerated solution for \( k = -1 \) since \( \ddot{a}(t) = \ddot{\tilde{a}}(\tilde{t}) = 0 \), \cite{17}. This is of some relevance, because the pre-big-bang flat universe is accelerated corresponding to a post-big-bang decelerated universe (ours), but for an open universe the pre-big bang solution is linear in time. This is not very appealing, as we shall see. Still, due to the ugly behavior of the dilaton, equations (2.11) and (2.12), much care should be exercised to interpret the solution physically.

3. Constraints on initial conditions

A condition to solve the horizon problem (in the Einstein frame, thus the tildes) is \cite{18}

\[
d_{\text{HOR}}(\tilde{t}_f) = \tilde{a}(\tilde{t}_f) \int_{\tilde{t}_i}^{\tilde{t}_f} d\tilde{t}' / \tilde{a}(\tilde{t}') > \tilde{a}(\tilde{t}_f) H_0^{-1} / \tilde{a}_0 \quad (3.1)
\]

where \( H_0^{-1} \) is the size of the observed Universe \( (H_0^{-1} \sim 10^{28} \text{cm}) \), \( \tilde{t}_f \) is the time by which the horizon problem is solved and \( \tilde{t}_i \) the time when inflation began. In order to solve the horizon problem, this condition, equation (3.1), must be satisfied in the Einstein frame as well as in the string frame. We compute our calculations in the string frame. For us, \( t_i \) and \( t_f \) determine the time range when the pre-big-bang description we are using remains valid. There is little reason to expect \( t_i \) to be anything else other than \(-\infty\): one of the conceptual beauties of the pre big bang scenario is that the initial state is empty flat space, a very perturbative string vacuum. As for \( t_f \), it should be of order \(-t_{\text{Planck}}\), we will estimate it carefully below. Obviously, letting \( t_i \to -\infty \) we see from (3.1) that the
horizon problem is solved both for \( k = 0 \). Still, it is of some interest to ask how long did the universe have to behave stringly before the big bang in order for it to come out free of flatness and horizon problems from the high curvature epoch (around \( t = 0 \)).

Let us now compute the amount of expansion required to solve the horizon problem, which is given by the ratio [1]:

\[
Z = \frac{H(t_f)a(t_f)}{H(t_i)a(t_i)}
\]

(3.2)

Experimentally (or rather, observationally), we need

\[
Z > e^{60}
\]

(3.3)
in order to solve the horizon problem for our big universe.

In the string frame the amount of inflation for a flat space using (2.8) turns out to be

\[
Z = \left( \frac{-t_i}{-t_f} \right)^{3/2}
\]

(3.4)

using (2.9) it is also true that

\[
Z = \left( \frac{e^{-\phi(t_i)}}{e^{-\phi(t_f)}} \right)^{1/2}
\]

(3.5)

From (3.3) and (3.4), it follows that

\[
t_i < e^{40}t_f
\]

(3.6)

and

\[
e^{\phi(t_f)} > e^{120}e^{\phi(t_i)}
\]

(3.7)

Since we are dealing with low-energy effective action we have two constraints on the solutions obtained from for the time when inflation ends \( t_f \). Since our effective action stops being valid when gravity becomes strongly coupled, we expect the pre big bang inflationary epoch to be over by the time \( t_f \) when

\[
e^{\phi(t_f)} \leq 1
\]

(3.8)

Similarly, the same effective action remains valid only while the curvature is not too big:

\[
H^{-1}(t_f) \sim (-t_f) \geq l_{st}
\]

(3.9)
When \( k = 0 \), these two requirements coincide, and the amount of inflation is thus

\[
Z = \left( \frac{-t_i}{l_{st}} \right)^{3/2}
\]  

(3.10)

which means that \( t_i < -10^{17}l_{st} \) in order to solve the horizon problem. Note that as \( t_i < t < t_f \) it is true that \( e^{\phi(t_f)} < 1 \).

From the solutions for \( k = -1 \), equations (2.11) and (2.12), we notice that in both cases \( e^{\phi(t_i)} \) can be very large, but our equations remain valid only as long as \( e^{\phi(t_i)} \geq 1 \) which implies \( t_i > -10^{-9}l_{st} \) for the first solution (2.11) ridiculously small and even more ridiculously \( t_i > -10^{-27}l_{st} \) for the second one (2.12).

The calculation of the amount of inflation \( Z \) for negative curvature is difficult because there is no accelerated solution, that is why we would rather not use a naive one. In any case the interval of time in which our solutions is reliable, \( t_i < t < t_f \), is extremely small since \( G_N \) is decreasing with time, which means that the time by which the universe starts to expand \( t_i \) has to be close to the big bang \( t = 0 \). Because this is not an accelerated solution we do not have enough time to solve the horizon problem. It would seem that nothing works out for negative curvature, but this is actually good news. It is clear that, for some time, we can solve the horizon problem if we start with a flat region. Now, a region with negative/positive curvature between an infinitely big flat space cannot inflate and get into what our observable universe is today. In other words, it seems that our radiation dominated universe comes from a flat rather than a curved region. Of course we would rather not fall into an anthropic explanation, but initial conditions are rather hard to explain, unless we do it \textit{a posteriori}.

4. Conclusions

As (3.10) shows, there is a restriction on the beginning of the pre-big bang inflation in order to solve the horizon problem. For negative curvature, however, it is not possible to solve it. We attribute this to two reasons. First, we do not get an accelerated phase, and as standard cosmology says, we need an accelerated phase in order to solve the horizon problem. Secondly, as we go back in time, the dilaton increases (opposite to that for flat space) and our tree-level low-energy effective action (2.1) breaks down.

Pre-big-bang cosmology requires a flat space [19] in order to solve successfully the horizon problem, which is a much more appealing initial state than the hot and highly-curved space postulated by standard (inflationary) cosmology.
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