Parity violation in pre-inflationary bounce

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Abstract

The power suppression at largest scale in the CMB TT power spectrum might imply the occurrence of a pre-inflationary bounce. We calculate the circularly polarized gravitational wave, leaded by the gravitational Chern-Simons term which universally appears in particle physics and string theory, in an inflationary background with the pre-inflationary bounce. The circularly polarized gravitational wave will induce TB and EB correlations at CMB last scattering surface. We find that although the BB power spectrum at largest scale is also suppressed, the TB and EB power spectra at corresponding scales may be enhanced.

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I. INTRODUCTION

Recently, the Planck collaboration has reported a power deficit in the CMB TT power spectrum at largest scale \cite{1,2}, which also was found in WMAP data, and is not concordant with the Planck bestfit model. Its statistical significance is about $3\sigma$. In addition, the Planck collaboration has also reported a hemispherical power asymmetry in CMB at low-$l$ \cite{2}, which conformed a similar observation of WMAP \cite{3,4}, see also \cite{5,6}.

These large-scale anomalies might be a hint of the pre-inflationary physics relevant with the initial singularity \cite{7,8}. In Refs. \cite{7,9,10,11,12}, the pre-inflationary universe is in a contracting phase and after the bounce it begins to inflate. In Refs. \cite{13,14,15}, the pre-inflationary universe is in a superinflationary phase, see also \cite{16} for the case with $\epsilon \ll -1$, like the emergent universe scenario\cite{17}. Above pre-inflationary evolutions generally will generate a large-scale cutoff in the primordial power spectrum, which may naturally suppress the CMB TT power spectrum at low-$l$, see e.g. \cite{11,13} for the details. In such scenarios, it is generally required that the slow-roll inflation lasts for just the minimal number of efoldings, i.e. just enough \cite{18}, and thus the power deficit at low-$l$ is attributed to the evolution of the pre-inflationary non-slow-roll background, e.g. see Ref.\cite{19} for a discussion only focusing on the pre-inflationary expanding phase, and also earlier Refs.\cite{20} and \cite{21}. See e.g. \cite{13,22,23,24,25} for some stringy embeddings.

The measure of large-scale E and B-mode polarizations \cite{26,27} will help to provide a unambiguous test for the pre-inflationary evolution. The scalar perturbation contributes the TT, TE and EE correlations in CMB. While the gravitational wave (GW), not only give them, but also the BB correlation in CMB.

However, if the gravity is chiral, we might have other channels to test the evolution of pre-inflationary universe. The gravitational Chern-Simons (gCS) term, motivated by the anomaly cancelation in particle physics and string theory \cite{28,29}, is parity-violating, see Ref.\cite{30} for a review, which will produce a difference in the amplitude of right-handed and left-handed circularly polarized GW \cite{31,32,33}. The primordial circularly polarized GW will induce TB and EB correlations at CMB last scattering surface \cite{34,35,36,37}.

The TB and EB correlations may be also brought by the electromagnetic CS term e.g.\cite{38,39,40}, which affects the CMB polarizations after the photon decoupling e.g.\cite{41}, see \cite{42} for the analysis with latest data. As a result, the shape of TB power spectrum is
the same with that of TE power spectrum. Thus such a power spectrum may be clearly distinguished from that contributed by circularly polarized GW, see e.g. [36].

In conventional slow-roll inflation scenario, the circular polarization of primordial GW from the gCS term is negligible [43], [44]. Thus the primordial TB and EB correlations in CMB is unseen [34], [36]. However, the significant circular polarization may be created in a string-inspired inflationary model with the GB term [45]. In this sense, it seems that the TB and EB correlations recording the chirality of primordial gravity might also encode the information of the evolution of primordial universe. The pre-inflationary bounce [7], [11] not only may account for the CMB anomalies at large scale, but also avoid the initial singularity problem of inflationary universe. Thus it is interesting to ask what about TB and EB correlations in such a scenario.

Here, we will calculate the circularly polarized GW, led by the gCS term, in an inflationary background with the pre-inflationary bounce. We find that if the pre-inflationary bounce actually occurs, although the CMB BB power spectrum is suppressed at large angular scales, the TB and EB power spectrum at corresponding scales may be enhanced.

II. PRE-INFLATIONARY GRAVITATIONAL WAVE

The gravitation action including the gCS term is

$$S = S_{Einstein} + \int (-g)^{1/2} d^4x \frac{f(\phi)}{8} R \wedge R, \quad (1)$$

in which $\phi$ is identified as the inflaton in the slow-roll inflation and as the background field in the pre-inflationary evolution, respectively.

The gCS term does not affect the scalar perturbation and the evolution of background and only affects the tensor perturbation, e.g. [31]. The tensor perturbation $h_{ij}$ obeys $\delta^{ij}h_{ij} = 0$ and $\partial_i h^{ij} = 0$, and its action is

$$S_2 = \frac{1}{8} \int d\eta d^3x \left[ a^2 M_P^2 \left( h_{ij}'' - (\partial h_{ij})^2 \right) - f' \epsilon^{ijk} \left( h_i'' \partial_j h_{kq} - \partial'' h_i^q \partial_j \partial_q h_{kq} \right) \right], \quad (2)$$

where $\epsilon^{ijk}$ is the Levi-Cevita symbol, and $'$ is the derivative with respect to $\eta = \int dt/a$.

Here, the universe is initially in a contracting phase and after the bounce it is in the inflationary phase. We, to investigate the evolution of $h_{ij}$, will adopt an instantaneous
matching between both phases \[7\], \[11\], i.e.

\[
a \simeq a_s (1 - 2\mathcal{H}_s \eta)^{1/2} \quad \text{for contracting phase,}
\]

\[
a \frac{a_s}{1 - \mathcal{H}_s \eta} \quad \text{for inflationary phase,}
\]

respectively, where \(\mathcal{H}_s\) sets the slow-roll inflationary scale by \(H_{inf} = \mathcal{H}_s / a_s\). Here, the pre-inflationary contraction is a kinetic-dominated phase, see Ref. \[11\] for a detailed model, and see \[46\], \[47\], \[48\] and \[49\] for the ghost-free bounces in Einstein gravity. In addition, the bounce can also be realized by modifying gravity \[50\], \[51\], \[52\]. Actually, lots of the bounce mechanisms have been argued, see \[53\], \[54\] for reviews and references. Generally the perturbation may continuously pass through the bounce, and its spectrum is insensitive with respect to the implementing detail of the bounce, e.g. \[55\]. See also \[56\] for other case.

We, following Ref. \[43\], define the left and right circular polarization modes \(h_s\) with the circular polarization tensor \(p^{s}_{ij}\), and expand \(h_{ij}\) as

\[
h_{ij}(t, \mathbf{x}) = \sum_{s=L,R} \int \frac{d^3k}{(2\pi)^3} h_s(t, k) p^{(s)}_{ij} e^{i\mathbf{k} \cdot \mathbf{x}}.
\]

where \(ik_q \epsilon^{rqj} p^{(s)}_{ij} = k\lambda s p^{(s)}_{i} \), and the modes with \(\lambda_{R,L} = 1, -1\) are called as the right-handed mode and the left-handed mode, respectively. 

Thus with \[2\], the equation of \(h_s(t, k)\) is

\[
v''_{sk} + \left( k^2 - \frac{z''_s}{z_s} \right) v_{sk} = 0,
\]

where \(v_{sk} \equiv z_s h_s\) is defined, and \(z_s = a_l \left( 1 - \frac{\lambda s k}{\alpha f} \right)^{1/2} f \). Here, \(f = \alpha \frac{\phi}{M_p}\) and \(\alpha\) is a parameter determined by the potential fundamental theory. Thus \(z_s\) equals to

\[
z_s = a_l \sqrt{1 - \lambda_s \Theta \left( \frac{k}{\mathcal{H}} \right)},
\]

where \(\Theta = \frac{\alpha H^2 \sqrt{2}}{M_p}\) and \(\epsilon = -\dot{H} / H^2\). When \(k^2 \simeq z''_s / z_s\), the perturbation mode is leaving the horizon. When \(k^2 \ll z''_s / z_s\), the solution of \(h_s\) given by Eq.(5) is

\[
h_s \sim C \quad \text{is constant mode}
\]

\[
or \quad D \int \frac{d\eta}{z_s^2} \quad \text{is decaying mode.}
\]
A. The unpolarized gravitational wave

When the gCS term is negligible, which implies $z_s = a$, both $h_R$ and $h_L$ will be equal and $v_{Rk} = v_{Lk} = v_k$. We firstly investigate this unpolarized case.

When $k^2 \gg \frac{a''}{a}$, i.e. the perturbation is deeply inside its horizon, $v_k$ oscillates with a constant amplitude,

$$v_k \sim \frac{1}{\sqrt{2k}} e^{-ik\eta}.$$  (9)

When $k^2 \ll \frac{a''}{a}$, i.e. the perturbation is far outside the horizon, in the pre-inflationary contracting phase, the solution of Eq.(5) is given by

$$v_k = \sqrt{\frac{\pi}{4}} \left( x + \frac{k}{H_*} \right) H_0^{(1)}(x + \frac{k}{H_*}),$$  (10)

where $x = k/H_* - k\eta$, $H_0^{(1)}$ is the 0th order Hankel function of the first kind. While in the slow-roll inflationary phase the solution of Eq.(5) is

$$v_k = x^{1/2} \left[ C_1 H_{3/2}^{(1)}(x) + C_2 H_{3/2}^{(2)}(x) \right],$$  (11)

where $H_{3/2}^{(1)}$ and $H_{3/2}^{(2)}$ are the 3/2th-order Hankel function of the first kind and the second kind, respectively, the parameters $C_1$ and $C_2$ are only dependent on $k$.

The continuity of $h_s$ around the bounce gives $C_1$ and $C_2$. Thus the GW spectrum is

$$\mathcal{P}_T = \sum_{s=L,R} \mathcal{P}_{T,s} = \mathcal{P}_{T,inf} \frac{2k}{\pi H_0} |C_1 - C_2|^2,$$  (12)

where $\mathcal{P}_{T,s} = \frac{k^3}{\pi^2} |h_s/a|^2$ is used and $\mathcal{P}_{T,inf} = \frac{2H_{inf}^2}{\pi^2 M_p^2}$ is that of the slow-roll inflation. We plot $\mathcal{P}_T(k)$ in Fig.1, in which for $k > H_*$, $\mathcal{P}_T(k)$ is almost scale-invariant, since the corresponding modes are produced in slow-roll inflationary phase, while for $k < H_*$ it gets a cutoff. The shape of $\mathcal{P}_T(k)$ is the same with that of the scalar spectrum in Ref. [11].

When $k \ll H_*$, we have approximately

$$\mathcal{P}_T^{k < H_*} \sim (2 + \ln \frac{4H_0}{k})^2 \frac{k^3}{\pi H_0^3} \mathcal{P}_{T,inf},$$  (13)

which is the usual output of original Pre-big bang scenario [57], [58], i.e. $n_T \simeq 3$. While for $k \gg H_*$, we have approximately

$$\mathcal{P}_T^{k > H_*} = \left( 1 + \frac{H_*}{4k} \sin \frac{2k}{H_*} \right) \mathcal{P}_{T,inf},$$  (14)

which is almost scale-invariant, but with a decaying oscillation. The result is consistent with the solid line in Fig.1.
B. The circularly polarized gravitational wave

When the gCS term is not negligible, it will produce a difference in the amplitude of \( h_R \) and \( h_L \). Here, to quantify this chirality, we define a chiral parameter \( \Delta \chi \) as

\[
P_{T,s} = \left( 1 - \lambda_s \Delta \chi \right) P_T / 2,
\]

in which \(-1 \leq \Delta \chi \leq 1\) reflects the magnitude of parity violation of primordial GW. This definition equals to that in [34], [36], also [45] but with inverse sign.

When \( k > H_* \), the modes are produced in the slow-roll inflationary phase. In slow-roll inflation, the power spectrum of \( h_s \) is, Ref. [43],

\[
P_T \sim \frac{H_{inf}^2}{\pi^2 M_P^2} \left( 1 - \lambda_s \frac{\pi \Theta}{2} \right),
\]

where the terms with higher order \( \Theta \) are neglected. We have, with Eq. (15),

\[
\Delta \chi_{inf} = 2 \frac{P_{T,L}}{P_{T,H_*}} - 1 \approx \frac{\alpha \pi H_{inf}^2 \sqrt{2 \epsilon_{inf}}}{M_P^2}.
\]

Here, \( H_{inf}^2 \ll M_P^2 \), which implies \( \Delta \chi_{inf} \) is negligible. However, in a stringy embedding, \( \alpha \sim \sqrt{g_{str} M_P^2 M_{10}} \), it has been argued in [43] that for the suitable values of the string scale \( M_{10} \) and the string coupling \( g_{str} \), we might have \( \alpha \frac{H_{inf}^2}{M_P^2} \sim 1 \). Thus Eq. (16) may be written as

\[
\Delta \chi_{inf} \lesssim \sqrt{\epsilon_{inf}}.
\]

We will estimate the chirality parameter \( \Delta \chi_{pre-inf} \) in the pre-inflationary contracting phase. Noting that, for the inflation, the constant mode (7) is dominated, while for the contraction, \( D \int d\eta / a^2 \) is dominated.

During the contraction, we have

\[
h_s^{k<H_*} \sim \int a^2 \left[ 1 - \lambda_s \Theta \left( -\frac{k}{H_*} \right) \right] \sim \int \frac{d\eta}{a^2} \left[ 1 + \lambda_s \Theta \left( -\frac{k}{H_*} \right) \right],
\]

where \( \Theta( -\frac{k}{H_*} ) \ll 1 \) is used since \( \Theta < 1 \) and the mode is far outside the horizon. Now, with (3) and \( H_* = H_{inf}/a_* \), we could obtain

\[
\Delta \chi_{pre-inf} = 2 \frac{P_{T,L}}{P_{T,H_*}} - 1 \approx 2 \alpha \sqrt{2 \epsilon_{pre-inf} \frac{H_{inf}^2}{M_P^2}} \left( \frac{k}{H_*} \right) \left( \int_0^{-} \frac{d\eta}{a_*^2 (1 - 2 H_* \eta)^3} \right) \left( \int_0^{-} \frac{d\eta}{a_*^2 (1 - 2 H_* \eta)} \right)^{-1}
\]

\[
\approx \alpha \sqrt{2 \epsilon_{pre-inf} \frac{H_{inf}^2}{M_P^2}} \left( \frac{k}{H_*} \right).
\]

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FIG. 1: In bounce inflation, $\mathcal{P}_T$ is plotted (solid line), which for $k > \mathcal{H}_*$ is almost scale-invariant but with a decaying oscillation and for $k < \mathcal{H}_*$ gets a cutoff. The dashed line is $\Delta \chi \mathcal{P}_T$, in which $\Delta \chi$ is the chirality parameter.

FIG. 2: Theoretical CMB BB power spectrum for the slow-roll inflation model (blue dashed line), in which $r = 0.05$ is set for our simulation, and the inflation model with the pre-inflationary bounce (yellow dashed line).

Thus

$$\Delta \chi_{pre-inf} \sim \frac{\sqrt{\epsilon_{pre-inf}}}{\sqrt{\epsilon_{inf}}} \left( \frac{k}{\mathcal{H}_*} \right) \Delta \chi_{inf},$$

which implies that for $k \ll \mathcal{H}_*$, $\Delta \chi_{pre-inf}$ is negligible, while around cutoff scale, i.e. $k \sim \mathcal{H}_*$, $\Delta \chi_{pre-inf}$ is far larger than $\Delta \chi_{inf}$, since $\sqrt{\epsilon_{pre-inf}} \sim 1 \gg \sqrt{\epsilon_{inf}}$.

In slow-roll inflationary scenario, $\Delta \chi$, which reflects the parity violation of primordial GW from the gCS term, is scale invariant, and also negligible. However, we see that if a pre-inflationary bounce occurs, $\Delta \chi$ will show itself a bump around the matching scale $k \sim \mathcal{H}_*$, see the dashed line in Fig.1 for a parameterized $\Delta \chi$ used in Sec.III. It could be imagined
that this bump would leave the imprint in TB and EB power spectrum of CMB at large angular scale. Here, in the leading order of $\Theta$, the polarized GW spectrum $P_T = \sum_{s=L,R} P_{T,s}$ is approximately Eq. (12).

III. CMB ANGULAR POWER SPECTRA

A. BB power spectrum

In Ref. [11], with the background (3), the primordial scalar spectrum was calculated to fit the Planck+WMAP data. The best-fit values of $H_*$ and $A_{R,inf}$ are $\ln \left( \frac{H_* \text{ Mpc}^{-1}}{1} \right) = -8.60$ and $\ln \left( 10^{10} A_{R,inf} \right) = 3.084$. We, with this result and Eq. (12), in which $P_{T,inf}$ is parameterized as

$$ P_{T,inf} = r A_{R,inf} \left( \frac{k}{k_*} \right)^{n_{T,inf}}, \quad (21) $$

plot the BB power spectrum in Fig.2 by modifying the CAMB [59], in which $r = 0.05$ is set for our simulation. We see that the large-scale cutoff of the primordial GW spectrum brings a BB power suppression at $l < 20$, which is a significant prediction of pre-inflationary bounce.

B. TB and EB power spectrum

The TB or EB power spectrum is

$$ C_{l T/E,B} \sim \int \frac{dk}{k} \Delta \chi P_T \left[ \Delta_{l T/E}^T(k) \Delta_{l B}^B(k) \right], \quad (22) $$

which relies on the difference between left-handed $h_L$ and right-handed $h_R$, i.e. $\Delta \chi P_T$, e.g. [34, 36]. Here, $\Delta \chi$ may be phenomenologically parameterized as

$$ |\Delta \chi| = \Delta \chi_{inf} + (\Delta \chi_{pre-inf} - \Delta \chi_{inf}) \left[ 1/2 - \frac{\text{Tanh}(\frac{1}{2} \text{Log}_{10} \frac{k}{2 H_*})}{2} \right], \quad (23) $$

where for $k > H_*$, the chiral parameter $\Delta \chi = \Delta \chi_{inf}$ is (16) in slow-roll inflation, and for $k < H_*$, $\Delta \chi = \Delta \chi_{pre-inf}$ is (19) in the pre-inflationary contraction. We plot $\Delta \chi P_T$ in Fig.1, in which since $\Delta \chi \sim \sqrt{\epsilon_{inf}}$ is negligible for $k > H_*$, while

$$ \Delta \chi \sim \left( \frac{k}{H_*} \right) \quad (24) $$
is large around $k \sim 2\mathcal{H}_*$ but is suppressed for $k \ll \mathcal{H}_*$, a bump appears around $k \sim 2\mathcal{H}_*$.

Usually, the TB and EB power spectrum should vanish. Here, since the gravity is chiral, the amplitudes of left-handed $h_L$ and right-handed $h_R$ of the primordial GWs are different, which straightly induce non-vanishing TB and EB correlations at CMB last scattering surface. We plot TB and EB in Fig.3. We see that for the same values of parameters with those in Fig.2, both are enhanced at low-$l$, compared with that in slow-roll inflationary model, due to the bump of $\Delta \chi$ at corresponding scale.

The results can be explained as follows. The bump height of the TB or EB spectrum at $l < 10$ is caused by the reionization of universe, which is mainly depicted by the optical depth to the beginning of reionization, $\tau$. The bump height of the BB and TB power spectra around $l \sim 2$ have been roughly estimated in Ref.[34], which are

$$C_{l,\sim 2}^{BB} \approx \frac{1}{100} (1 - e^{-\tau})^2 C_{T,l,\sim 2}^{TT},$$  \hspace{1cm} (25)

$$|C_{l,\sim 2}^{TB}| \approx \frac{|\Delta \chi|}{10} e^{-\tau} (1 - e^{-\tau}) C_{T,l,\sim 2}^{TT},$$  \hspace{1cm} (26)

where $C_{T,l}^{TT}$ stands for the TT power spectrum from the primordial GW without reionization. Thus since $\mathcal{P}_T$ is cut off at large scale, which lowers $C_{T,l,\sim 2}^{TT}$, the reionization bump in the BB power spectrum is suppressed. However, since the TB power spectrum relies on $|\Delta \chi|C_{T,l,\sim 2}^{TT}$, if $\Delta \chi_{pre-inf} \gg \Delta \chi_{inf}$, we may have an enhanced reionization bump, compared with that in slow-roll inflationary model.

However, it should be mentioned that for a different set of the values of parameters in Eq.(23), which reflects the details of different bounce mechanisms and different pre-inflationary evolutions, the possibility that $|\Delta \chi|$ dose not set off the effect of low $C_{T,l,\sim 2}^{TT}$ can not ruled out, though this case is quite finetuning. Thus, whether the TB and EB reionization bump are enhanced or not might be model-dependent. However, if the TB and EB reionization bump are actually enhanced, this will be an interesting signal of the pre-inflationary bounce, different from that in the TT power spectrum [11], and also the BB power spectrum in Fig.2.

IV. DISCUSSION

Recently, the Planck collaboration has showed a power deficit in the CMB TT power spectrum at low-$l$, which might imply the occurrence of a pre-inflationary bounce, a solution
FIG. 3: Theoretical CMB TB and EB power spectra for the slow-roll inflation model (blue dashed line), in which $r = 0.05$ and $\Delta \chi_{inf} = 0.01$, and the inflation model with the pre-inflationary bounce (brown dashed line), in which $\Delta \chi_{pre-inf} = 0.5$.

to the initial singularity problem of inflationary universe. In string landscape, the bounce will induce a AdS-dS transition [60], [61], [62], which is an efficient route to the slow-roll inflation, see the Appendix in [11], also see [63], [64], [65] for the effect of perturbation.

The gCS term universally appears in particle physics and string theory [28], [29], which will lead to the circular polarization of GW produced in primordial universe. This circular polarization will induce TB and EB correlations at CMB last scattering surface, which might provide an alternative test for the pre-inflationary bounce.

We have calculated the circularly polarized GW, leaded by the gCS term, in an inflationary background with the pre-inflationary bounce, and find that if the pre-inflationary bounce actually occurs, although the CMB BB power spectrum is suppressed at largest scale, the TB and EB power spectrum at corresponding scales may be enhanced. This result applies to the cases that the bounce can be realized without modified gravity, e.g. [46], [47], [48] and [49], [50].
and with modified gravity but the parity violation of gravity is mainly from the gravitational CS term.

However, it should be acknowledged that the amplitudes of both TB and EB are determined by the value of $r$, thus for negligibly small $r$ both TB and EB are actually undetectable, even if $\Delta \chi \simeq 1$. Thus rather than argue whether the detection is possible or not, we would like to conclude that if the gravity in primordial universe is chiral, or parity-violating, compared with those during slow-roll inflation, both TB and EB correlations around the pre-inflationary bounce may be overwhelmingly enhanced, which is a significant character of the pre-inflationary bounce.

Here, the enhancement of both TB and EB at largest scale actually is a reflection of the background evolution of pre-inflationary universe, since

$$C_{l=2}^{T,E,B} \sim \Delta \chi \sim \sqrt{|\epsilon_{\text{Pre-inf}}|}. \quad (27)$$

Thus if the anomalies at large scale in the TT power spectrum is attributed to a pre-inflationary evolution, it is interesting to show TB and EB for other non-slow-roll pre-inflationary background, and see whether the result is similar, which will be investigated afterwards. In addition, it is also interesting to consider other sources inducing chiral GW, e.g. non-Abelian gauge fields [66].

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