Challenges of Lattice Calculation of Scalar Mesons

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Abstract. I review a proposed pattern of the light scalar mesons with $q\bar{q}$ mesons and glueball above 1 GeV and tetraquark mesoniums below 1 GeV. Several challenges and caveats of calculating these light scalar mesons with dynamical fermions are discussed.

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INTRODUCTION

The pseudoscalar, vector, axial, and tensor mesons with light quarks (i.e. $u, d$ and $s$) are reasonably well known in terms of their $SU(3)$ classification and quark content. The scalar meson sector, on the other hand, is much less understood in this regard. There are 19 experimental states below 1.8 GeV which are more than twice the usual $q\bar{q}$ nonet in other sectors. We show in Fig. 1 the experimentally known scalars including $\sigma(600), \kappa(800)$, and $f_0(1710)$ which are better established experimentally nowadays [1, 2]. The recent theoretical advance [3] in identifying $\sigma(600)$ as a $\pi\pi$ resonance by solving the Roy equation has settled the question about the existence of $\sigma(600)$. Nevertheless, there are still a number of puzzling features regarding the ordering of $a_0(1450)$ and $K^*_0(1430)$ with respect to their counterparts in the axial-vector and tensor sectors, the narrowness of $a_0(980)$ and $f_0(980)$ in contrast to the broadness of $\sigma(600)$ and $\kappa(800)$, etc [4]. We shall first review an emerging pattern of the scalar mesons below 1.8 GeV based on quenched lattice calculation and phenomenology and then discuss the challenges and caveats of full QCD calculation of these scalar mesons on the lattice.

PATTERN OF LIGHT SCALAR MESONS

The unsettling features regarding the nature of $a_0(1450)$ and $K^*_0(1430)$ are tentatively resolved in a recent quenched lattice calculation [5] with overlap fermions for a range of pion masses with the lowest one at 180 MeV. When the quenched ghost states, which correspond to $\pi\eta$ and $\pi\eta'$ scattering states in the dynamical fermion case are removed, it is found that $a_0$ is fairly independent of the quark mass. In other words, below the strange quark mass, $a_0$ is very flat and approaches $a_0(1450)$ in the chiral limit. This suggests that $SU(3)$ is a much better symmetry in the scalar meson sector than the other meson sectors and that both $a_0(1450)$ and $K^*_0(1430)$ are $q\bar{q}$ states. Furthermore,
Figure 1. Spectrum of scalar mesons together with $\pi$, $\rho$, $a_1$ and $a_2$.

$f_0(1500)$, by virtue of the fact that it is close by, should be a fairly pure $SU(3)$ octet state, i.e. $f_{\text{octet}} = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$.

Based on the lattice findings, a mixing scheme for the isoscalar $f_0(1370), f_0(1500)$ and $f_0(1710)$ – a glueball candidate, with slight $SU(3)$ breaking was developed and successfully fit to the decays of pseudoscalar meson pairs as well as various decays from $J/\Psi$ [6]. Some of the robust and conspicuous features of this mixing scheme are the following:

- $f_0(1500)$ is indeed a fairly pure octet with very little mixing with the flavor singlet and the glueball. $f_0(1710)$ and $f_0(1370)$ are dominated by the glueball and the $q\bar{q}$ singlet respectively, with $\sim 10\%$ mixing between the two. This is consistent with the experimental result $\Gamma(J/\Psi \rightarrow \gamma f_0(1710)) \sim 5\Gamma(J/\Psi \rightarrow \gamma f_0(1500))$ [2] which favors $f_0(1710)$ to have a larger glueball content.

- The ratio $\Gamma(f_0(1500) \rightarrow K\bar{K})/\Gamma(f_0(1500) \rightarrow \pi\pi) = 0.246 \pm 0.026$ is one of the best experimentally determined decay ratios for these mesons [1]. If $f_0(1500)$ is a glueball (i.e. a flavor singlet) or $s\bar{s}$, the ratio will be 0.84 or larger then unity. Either one is much larger than the experimental result. On the other hand, if $f_0(1500)$ is $f_{\text{octet}}$, then the ratio is 0.21 which is very close to the experimental value. This further demonstrates that $f_0(1500)$ is mainly an octet and its experimental decay ratio can be well described with a small $SU(3)$ breaking [3].

- Because the $n\bar{n}$ content is more copious than the $s\bar{s}$ in $f_0(1710)$ in this mixing scheme, the prediction of $\Gamma(J/\Psi \rightarrow \omega f_0(1710))/\Gamma(J/\Psi \rightarrow \phi f_0(1710)) = 4.1$ is naturally large and consistent with the observed value of $6.6 \pm 2.7$. This ratio is not easy to accommodate in a picture where the $f_0(1710)$ is dominated by $s\bar{s}$. One may have to rely on a doubly OZI suppressed process to dominate over the singly OZI suppressed process to explain it [7].
The mesons below 1 GeV were suggested to be tetraquark mesoniums from the MIT bag model and potential model studies. A recent lattice calculation with the overlap fermion on $12^3 \times 28$ and $16^3 \times 28$ quenched lattices with the two-quark-two-antiquark interpolation field $\Psi \gamma_5 \Psi \gamma_5 \Psi$ has confirmed the existence of such low-lying scalar tetraquark mesonium at $\sim 550$ MeV. This strongly suggests that it is the $\sigma(600)$.

Combining the lattice calculations of $a_0(1450), K^*_0(1430)$ and $\sigma(600)$ and the mixing study of $f_0(1370), f_0(1500)$ and $f_0(1710)$, a classification of the scalar mesons below 1.8 GeV was proposed. Those below 1 GeV, i.e. $\sigma(600), a_0(980), f_0(980)$ and $\kappa(800)$ form a nonet of tetraquark mesoniums; those above 1 GeV, i.e. $a_0(1450), K^*_0(1430)$ and $f_0(1500)$ form a fairly sure SU(3) octet; and $f_0(1370)$ and $f_0(1710)$ are good SU(3) singlet and glueball respectively, with $\sim 10\%$ mixture between the two.

We should stress that this is not finalized. It should be scrutinized in future experiments, such as high statistics $J/\Psi, D, B$ decays and $p\bar{p}$ annihilations. Furthermore, most of the the lattice results which led to the above proposed pattern were based on quenched calculations. There are loose ends that need to be tightened, come dynamical fermion calculations. In the following, we shall enumerate a number of challenges and the associated caveats in calculations with dynamical fermion configurations.

CHALLENGES AND CAVEATS OF FUTURE LATTICE CALCULATIONS WITH DYNAMICAL FERMIONS

In the quenched lattice calculation of $a_0$ with light quarks which correspond to $m_\pi < 500$ MeV, the quenched $\pi \eta$ ghost states are lower than the $a_0(1450)$ and, thus, dominate the long time behavior in the scalar correlator with a non-unitary negative tail. This has to be removed before the physical $a_0(1450)$ is revealed. These ghost states turn into

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1 These are two-quark and two-antiquark mesons which have been referred to as four-quark states, meson molecular, mesoniums, and tetraquark states. We shall call them tetraquark mesoniums so as to avoid implication on the nature of possible spatial and color clustering.
physical two meson scattering states in a full QCD calculation with the same valence and sea quark masses. This causes some difficulty in the fitting of scalar meson correlators and has been mentioned by S. Prelovsek [13] in this workshop. In the following, we shall point out several caveats and challenges facing the scalar meson calculation with light dynamical fermions.

There are several $N_f = 2$ dynamical fermion calculations of $a_0$ with the $\overline{\Psi}\Psi$ interpolation field [12, 14, 15, 16, 17]. Save for Ref. [12] which, upon removing the partially quenched ghost $\pi\eta$ state, found $a_0$ to be at 1.51(19) GeV, the others [15, 16, 17] found the lowest states at the chiral limit to be $\sim 1$ GeV, suggesting that $a_0(980)$ is the $q\bar{q}$ state. As pointed out in Ref. [4], this is most likely an untenable interpretation. If $a_0(980)$ is indeed a $q\bar{q}$ state, or has a sizable coupling to the $\overline{\Psi}\Psi$ interpolation field, then replacing the $u/d$ quark in the $a_0$ interpolation field with $s$ will place the corresponding $s\bar{u}$ at $\sim 1100$ MeV. This is far (i.e. 300 MeV) away from each of the two experimental states $K^*_0(1430)$ and $\kappa(800)$ (see Fig. 1). The likely resolution, we think, is that the state found at $\sim 1$ GeV is the $\pi\eta_2$ scattering state. Since $\eta_2$, the $\eta(\eta')$ in the two-flavor case is predicted to be $\sqrt{2}/3 m_{\eta'} = 782$ MeV in the large $N_c$ analysis with $U(1)$ anomaly, the weakly interacting $\pi\eta_2$ will be near the state seen at $\sim 1$ GeV. In other words, this $\pi\eta_2$ scattering state is the dynamical fermion realization of the corresponding ghost state in the quenched approximation. Parallel to the lesson learned in lattice calculations of pentaquark baryons [18], one has to include the multi-hadron states in addition to the physical resonances when fitting the two-point correlators for the excited spectrum. In the case of $a_0$ in the realistic $N_f = 2 + 1$ case, one needs to include $\pi\eta, \pi\eta'$, in addition to the physical $a_0(980)$, and $a_0(1450)$. This can be achieved with the sequential empirical Bayes method for curve-fitting [22] or the variational approach. Furthermore, one needs to distinguish the two-particle scattering states from the one-particle resonances. One way to distinguish a two-particle scattering state from a one-particle state is to examine the 3-volume dependence of the fitted spectral weight [19, 20, 5]. Another way is to impose a ‘hybrid boundary condition’ on the quark propagators [21]. No attempt has been made to identify the scattering $\pi\eta(\eta')$ states so far. This has to be carried out before one can reasonably reveal the quark content of $a_0(980)$ and $a_0(1450)$.

$$a_0(1450) \text{ and } K^*_0(1430)$$

$$f_0(980), f_0(1370), f_0(1500) \text{ and } f_0(1710)$$

In addition to the complication of two-meson scattering states (in this case $\pi\pi, K\bar{K}, \eta\eta, \eta\eta'$), one needs to calculate the correlators with disconnected insertions (D.I.) in addition to the connected insertions (C.I.) as in the $a_0$ case. This is to reflect the fact that these isoscalar mesons have annihilation channels. The usual

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2 Otherwise, it is considered to be a partially quenched calculation.
approach of adopting the noise \cite{23} to estimate the quark loops in the disconnected insertion makes the calculation much more expensive than the connected insertion one. One caveat with the noise estimator is that the signal falls exponentially in the meson correlator; whereas, the variance of the noise estimator approaches a constant at certain time separation \cite{23}. If one were to fit the time window where the variance of the noise levels off, the shoulder effect of the correlator could result in an unphysically light effective mass. In view of this, the very light mass from the D.I. part of the correlator in the $f_0$ calculations \cite{14, 24} should be subjected to the examination as to whether it is the $\pi\pi$ scattering state or due to the shoulder effect.

**Glueball**

The continuum and large volume limits of the quenched calculation places the scalar glueball at 1710(50)(80) MeV \cite{25}. This is very close to the viable experimental glueball candidate $f_0(1710)$. To verify this in the full QCD calculation is, however, non-trivial. Whatever interpolation one adopts, one has to disentangle the glueball from all the lower-lying $f_0$ states and the $\pi\pi, K\bar{K}, \eta\eta$ and $\eta\eta'$ two-meson states.

**Tetraquark Mesonoums**

If the nonet below 1 GeV in Fig. 1 are indeed dominated by the $q^2\bar{q}^2$ tetraquark mesonoums, one can access them through the $\bar{\psi}\gamma_5\psi\bar{\psi}\gamma_5\psi$ operator or other four-quark operators with the same quantum number. In the case of $a_0(980)$ and $f_0(980)$, the two-meson threshold, i.e. $K\bar{K}$ is close by. One may need a good variational method in order to disentangle them. By virtue of the fact that $a_0(980)$ and $f_0(980)$ are nearly degenerate, the D.I. should be small compared to the C.I. It should be confirmed in full QCD calculation.

$q\bar{q}$ meson vs $q^2\bar{q}^2$ tetraquark mesonium

The notion of $q\bar{q}$ or $q^2\bar{q}^2$ meson is primarily a quark model concept of the valence quark content. How does one distinguish them in lattice QCD with interpolation fields? So far, neither $a_0(980)$ nor $\sigma(600)$ is coupled to the $\bar{\psi}\psi$ interpolation field in the quenched approximation with discernable signal \cite{5}. If this is not true in full QCD calculation with light dynamical fermions, this will complicate matters substantially. One will need both $q\bar{q}$ and $q^2\bar{q}^2$ types of operators with a large basis in the variational calculation to identify states and; moreover, to distinguish the one-particle states from the multi-meson scattering states.
CONCLUSION

We summarized the pattern of light scalar mesons emerged from quenched lattice calculations and the study of mixing and decays of $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. We have discussed the subtlety and challenges of calculating them in full QCD with light dynamical fermions. In particular, if they couple strongly to both $q\bar{q}$ and $q^2\bar{q}^2$ types of interpolation operators, the interpretation of the underline pattern will be considerably more complex. We hope that Nature is only subtle but not malicious.

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