THE CHIRAL ODD NUCLEON TENSOR CHARGE

LEONARD GAMBERG
Department of Physics and Astronomy, University of Pennsylvania, David
Rittenhouse Labs, 209 South 33rd Street, Philadelphia, PA 19194
E-mail: gamberg@dept.physics.upenn.edu

GARY R. GOLDSTEIN
Department of Physics and Astronomy, Tufts University, Medford, MA 02155
E-mail: ggoldst@tufts.edu

Exploiting the phenomenological symmetry of the $J^{PC} = 1^{+-}$ light axial vector
mesons and using pole dominance, we calculate the flavor contributions to the
nucleon tensor charge.

1 Introduction

The subject of the nucleon’s spin composition has been intensely studied and
has produced important and surprising insights, beginning with the revelation
that the majority of its spin is carried by quark and gluonic orbital angular
momenta and gluon spin rather than by quark helicity. In addition, con-
siderable effort has gone into understanding, predicting and measuring the
transversity distribution, $h_1(x)$, of the nucleon. Transversity, as combina-
tions of helicity states, $|\perp/T> \sim (|+> \pm |->)$, for the moving nucleon is a
variable introduced originally by Moravcsik and Goldstein to reveal an un-
derlying simplicity in nucleon–nucleon spin dependent scattering amplitudes.
In their analysis of the chiral odd distributions, Jaffe and Ji related the
first moment of the transversity distribution to the flavor contributions of the
nucleon tensor charge:

$$\int_0^1 (\delta q^a(x) - \delta q^{a'}(x)) \, dx = \delta q^a$$

(for flavor index $a$). This leading twist transversity distribution function,
$\delta q^a(x)$, is as fundamental to understanding the spin structure of the nucleon
as its helicity counterpart $\Delta q^a(x)$. Yet, while the latter in principle can be
measured in hard scattering processes, the transversity distribution (and thus
the tensor charge) decouple at leading twist in deep inelastic scattering since
it is chiral odd. Additionally, the non-conservation of the tensor charge makes
it difficult to predict. While bounds placed on the leading twist quark distribu-
tions through positivity constraints suggest that they satisfy the inequality
of Soffer,

\[ |2\delta q^a(x)| \leq q^a(x) + \Delta q^a(x) \]  

(where \( q^a \) denotes the unpolarized quark distribution), there are no definitive theoretical predictions for the tensor charge. Among the various approaches, from the QCD sum rule to lattice calculations models,

there appears to be a range of expectations and a disagreement concerning the sign of the down quark contribution. We present a new approach to calculate the tensor charge that exploits the approximate mass degeneracy of the light axial vector mesons (\( a_1(1260) \), \( b_1(1235) \) and \( h_1(1170) \)) and uses pole dominance to calculate the tensor charge. Our motivation stems in part from the observation that the tensor charge does not mix with gluons under QCD evolution and therefore behaves as a non-singlet matrix element. In conjunction with the fact that the tensor current is charge conjugation odd (it does not mix quark-antiquark excitations of the vacuum, since the latter is charge conjugation even) suggests that the tensor charge is more amenable to a valence quark model analysis.

## 2 The Tensor Charge and Pole Dominance

The flavor components of the nucleon tensor charge are defined from the forward nucleon matrix element of the tensor current,

\[
\langle P, S_T | \overline{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi | P, S_T \rangle = 2\delta q^a(\mu^2)(P^\mu S_T^\nu - P^\nu S_T^\mu).
\]  

We adopt the model that the nucleon matrix element of the tensor current is dominated by the lowest lying axial vector mesons

\[
\langle P, S_T | \overline{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi | P, S_T \rangle = \lim_{k^2 \to 0} \sum_M \frac{\langle 0 | \overline{\psi} \sigma^{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \psi | M \rangle \langle M, P, S_T | P, S_T \rangle}{M^2 - k^2}.
\]  

The summation is over those mesons with quantum numbers, \( J^{PC} = 1^{+-} \) that couple to the nucleon via the tensor current; namely the charge conjugation odd axial vector mesons – the isoscalar \( h_1(1170) \) and the isovector \( b_1(1235) \). To analyze this expression in the limit \( k^2 \to 0 \) we require the vertex functions for the nucleon coupling to the \( h_1 \) and \( b_1 \) meson and the corresponding matrix elements of the meson decay amplitudes which are related to the meson to vacuum matrix element via the quark tensor current. The former yield the

\[ ^1 \text{In contrast to the axial vector isovector charge, no sum rule has been written that enables a clear relation between the tensor charge and a low energy measurable quantity.} \]
nucleon coupling constants $g_{MNN}$ defined from the matrix element

$$\langle MP|P = \frac{i g_{MNN}}{2 M_N} \pi(P, S_T) \sigma^\mu\nu \gamma_5 u(P, S_T) \epsilon_\mu k_\nu, \quad (5)$$

and the latter yield the meson decay constant $f_M$ defined in

$$\langle 0|\bar{\psi} \sigma^\mu\nu \gamma_5 \lambda^a_\mu \frac{1}{2} \psi |M\rangle = i f_M \langle \epsilon_\mu k_\nu - \epsilon_\nu k_\mu \rangle. \quad (6)$$

Here $P_\mu$ is the nucleon momentum, and $k_\mu$ and $\epsilon_\nu$ are the meson momentum and polarization respectively.

Taking a hint from the valence interpretation of the tensor charge, we exploit the phenomenological mass symmetry among the lowest lying axial vector mesons that couple to the tensor charge; we adopt the spin-flavor symmetry characterized by an $SU(6) \otimes O(3)$ multiplet structure. Thus, the $1^{+ -} h_1$ and $b_1$ mesons fall into a $(35 \otimes L = 1)$ multiplet that contains $J^{PC} = 1^{++}, 0^{+-}, 1^{++}, 2^{++}$ states. This analysis enables us to relate the $a_1$ meson decay constant measured in $\tau^- \to a_1^- + \nu_\tau$ decay, $f_{a_1} = (0.19 \pm 0.03)\text{GeV}$, and the $a_1 NN$ coupling constant $g_{a_1 NN} = 7.49 \pm 1.0$ (as determined from $a_1$ axial vector dominance for longitudinal charge as derived in[3] but using $g_A/g_V = 1.26$[13]) to the meson decay constants and coupling constants. We find

$$f_{b_1} = \frac{\sqrt{2}}{M_{b_1}} f_{a_1}, \quad g_{b_1 NN} = \frac{5}{3\sqrt{2}} g_{a_1 NN}, \quad (7)$$

where the $5/3$ appears from the $SU(6)$ factor $(1 + F/D)$ and the $\sqrt{2}$ arises from the $L = 1$ relation between the $1^{++}$ and $1^{+-}$ states. Our resulting value of $f_{b_1} \approx 0.21 \pm 0.03$ agrees well with a sum rule determination of $0.18 \pm 0.03$[14]. The $h_1$ couplings are related to the $b_1$ couplings via $SU(3)$ and the $SU(6)$ $F/D$ value,

$$f_{b_1} = \sqrt{3} f_{h_1}, \quad g_{b_1 NN} = \frac{5}{\sqrt{3}} g_{h_1 NN} \quad (8)$$

For transverse polarized Dirac particles, $S^\mu = (0, S_T)$ these values, in turn, enable us to determine the isovector and isoscalar parts of the tensor charge,

$$\delta q^v = \frac{f_{b_1} g_{h_1 NN} (k_2^2)}{\sqrt{2} M_N M_{b_1}^2}, \quad \delta q^s = \frac{f_{h_1} g_{h_1 NN} (k_2^2)}{\sqrt{2} M_N M_{h_1}^2}, \quad (9)$$

respectively (where, $\delta q^v = (\delta u - \delta d)$, and $\delta q^s = (\delta u + \delta d)$). Transverse momentum appears in these expressions because the tensor couplings involve helicity flips that carry kinematic factors of 3-momentum transfer, as required by rotational invariance. The squared 4-momentum transfer of the external hadrons...
goes to zero in Eq. (4), but the quark fields carry intrinsic transverse momentum. This intrinsic $k_\perp$ of the quarks in the nucleon is determined from Drell-Yan processes and from heavy vector boson production. We use a Gaussian momentum distribution, and let $\langle k_\perp^2 \rangle$ range from $(0.58 \text{ to } 1.0 \text{ GeV}^2)$. 

3 Mixing and Results

In relating the $b_1(1235)$ and $h_1(1170)$ couplings in Eq. (8) we assumed that the latter isoscalar was a pure octet element, $h_1(8)$. Experimentally, the higher mass $h_1(1380)$ was seen in the $K^+K^-\pi^0$ decay channel while the $h_1(1170)$ was detected in the multi-pion channel. This decay pattern indicates that the higher mass state is strangeonium and decouples from the lighter quarks—the well known mixing pattern of the vector meson nonet elements $\omega$ and $\phi$. If the $h_1$ states are mixed states of the $SU(3)$ octet $h_1(8)$ and singlet $h_1(1)$ analogously, then it follows that

$$f_{h_1(1170)} = f_{b_1}, \quad g_{h_1(1170)NN} = \frac{3}{5}g_{h_1NN},$$

with the $h_1(1380)$ not coupling to the nucleon (for $g_{h_1(1)NN} = \sqrt{2}g_{h_1(8)NN}$). These symmetry relations yield the results

$$\delta u(\mu^2) = (0.58 \text{ to } 1.01) \pm 0.20, \quad \delta d(\mu^2) = -(0.11 \text{ to } 0.20) \pm 0.20. \quad (11)$$

These values are similar to several other model calculations: from the lattice; to QCD sum rules; the bag model; and quark soliton models. The calculation has been carried out at the scale $\mu \approx 1 \text{ GeV}$, which is set by the nucleon mass as well as being the mean mass of the axial vector meson multiplet. The appropriate evolution to higher scales (wherein the Drell-Yan processes are studied) is determined by the anomalous dimensions of the tensor charge, which is slowly varying function of $Q^2$.

It is interesting to observe that the symmetry relations that connect the $b_1$ couplings to the $a_1$ couplings in Eq. (5) can be used to relate directly the isovector tensor charge to the axial vector coupling $g_A$. This is accomplished through the $a_1$ dominance expression for the isovector longitudinal charges derived in

$$\Delta u - \Delta d = \frac{g_A}{g_V} = \frac{\sqrt{2}f_{a_1}g_{a_1NN}}{M_{a_1}^2}. \quad (12)$$

Hence for $\delta q^t$ we have

$$\delta u - \delta d = \frac{5}{6}g_A \frac{M_{a_1}^2}{g_V M_{b_1}^2} \frac{\langle k_\perp^2 \rangle}{M_N M_{b_1}}, \quad (13)$$
It is important to realize that this relation can hold at the scale wherein the couplings were specified, the meson masses, but will be altered at higher scales (logarithmically) by the different evolution equations for the $\Delta q$ and $\delta q$ charges. To write an analogous expression for the isoscalar charges $(\Delta u + \Delta d)$ would involve the singlet mixing terms and gluon contributions, as Ref. [12] considers. However, given that the tensor charge does not involve gluon contributions (and anomalies), it is expected that the relation between the $h_1$ and $b_1$ couplings in the same $SU(3)$ multiplet will lead to a more direct result

$$\delta u + \delta d = \frac{3}{5} \frac{M^2_{h_1}}{M^2_{b_1}} \delta q^v,$$

for the ideally mixed singlet-octet $h_1(1170)$.

4 Conclusions

Our axial vector dominance model with $SU(6)_W \otimes O(3)$ coupling relations provide simple formulae for the tensor charges. We obtain the same order of magnitude as many other calculation schemes. These results support the view that the underlying hadronic physics, while quite difficult to formulate from first principles, is essentially a $1^{++}$ meson exchange process. Forthcoming experiments will begin to test this notion.

Acknowledgments

L.G. thanks the organizers of the 6th Workshop on Non-Perturbative Quantum Chromodynamics for the wonderful atmosphere in which to present our work. In particular I thank Herb Fried for the invitation to speak and Mary Ann Rotondo for her tireless organizational work during and after the conference. This work is supported in part by Grant No. DE-FG02-92ER40702 from the U.S. Department of Energy.

References

1. J. Ashman et al. Nucl. Phys. B328, 1 (1989); B. Adeva et al., Phys. Lett. B 302, 533 (1993); P.L. Anthony et al., Phys. Rev. Lett. 71, 959 (1993); K. Abe et al., Phys. Rev. Lett. 72, 25 (1995).
2. X. Ji, Phys. Rev. Lett 78, 610 (1997).
3. V. Barone , A. Drago and P. G. Ratcliffe, arXive: hep-ph/0104283 and references contained therein .
4. G.R. Goldstein and M.J. Moravcsik, Ann. Phys. (NY) 98, 128 (1976); Ann. Phys. (NY) 142, 219 (1982); Ann. Phys. (NY) 195, 213 (1989).
5. R. L. Jaffe and X. Ji, Phys. Rev. Lett. 67, 552 (1991); Nucl. Phys. B375, 527 (1992).
6. J. Soffer, Phys. Rev. Lett. 74, 1292 (1995); G. R. Goldstein, R. L. Jaffe and X. Ji, Phys. Rev. D 52, 5006 (1995).
7. H. He and X. Ji, Phys. Rev. D 52, 2960 (1995); Phys. Rev. D 54, 6897 (1996); S. Aoki, et al., Phys. Rev. D 56, 433 (1997); L. Gamberg, H. Reinhardt and H. Weigel, Phys. Rev. D 58, 054014 (1998); L. Gamberg, “Structure Functions and Chiral Odd Quark Distributions in the NJL Soliton Model of the Nucleon”, Proceedings of Future Transversity Measurements, RIKEN-BNL Workshop (2000).
8. L. Gamberg and G. R. Goldstein, arXive: hep-ph/0106178.
9. L. Gamberg and G. R. Goldstein, Phys. Rev. Lett. 87, 242001 (2001).
10. B. Sakita, Phys. Rev. 136, B1756 (1964); F. Gürsen and L.A. Radicati, Phys. Rev. Lett. 13, 173 (1964).
11. W.-S. Tsai, Phys. Rev. D 4, 2821 (1971).
12. M. Birkel and H. Fritzsch, Phys. Rev. D 53, 6195 (1996).
13. D.E. Groom, et al. (Particle Data Group), Eur. Phys. Jour. C15, 1 (2000).
14. V.M. Belyaev and A. Oganesian, Phys. Lett. B395, 307 (1997).
15. R.K. Ellis, W.J. Stirling and B.R. Webber, QCD and Collider Physics (Cambridge University Press, Cambridge, U.K. 1996), p.305.
16. A. Abele, et al., Phys. Lett. B415, 280 (1997); D. Aston, et al., Phys. Lett. B201, 573 (1988).
17. A. Ando, et al., Phys. Lett. B291, 496 (1992); and references contained therein.
18. X. Artu and M. Mekhfi, Z. Phys. C45 (1990) 669.