Validation of tsunami inundation model TUNA-RP using OAR-PMEL-135 benchmark problem set

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Abstract. A standard set of benchmark problems, known as OAR-PMEL-135, is developed by the US National Tsunami Hazard Mitigation Program for tsunami inundation model validation. Any tsunami inundation model must be tested for its accuracy and capability using this standard set of benchmark problems before it can be gainfully used for inundation simulation. The authors have previously developed an in-house tsunami inundation model known as TUNA-RP. This inundation model solves the two-dimensional nonlinear shallow water equations coupled with a wet-dry moving boundary algorithm. This paper presents the validation of TUNA-RP against the solutions provided in the OAR-PMEL-135 benchmark problem set. This benchmark validation testing shows that TUNA-RP can indeed perform inundation simulation with accuracy consistent with that in the tested benchmark problem set.

1. Introduction
The 26 December 2004 Andaman tsunami caused the death of a quarter million people worldwide, mainly in Indonesia and Thailand, including 52 deaths in Penang, Malaysia. This catastrophic disaster serves as a wake-up call for all affected countries to develop programs capable of reducing the adverse impacts of tsunamis. These tsunami mitigation programs aim to develop societal resilience to tsunamis by cultivating awareness, education and preparedness regarding tsunami hazards, tsunamis risk zones and inundation maps. These tsunami mitigation programs invariably involve a sound understanding about the locations where tsunamis will most likely occur, as well as the extent of run-up and inundation distances. To provide information on run-up and inundation, tsunami run-up and inundation simulation models are normally used to forecast their occurrence. This paper begins with a brief literature review on four commonly used run-up and inundation algorithms. Their relative merits are briefly discussed, leading to a justification for choosing the wet-dry moving boundary algorithm that was incorporated into TUNA-RP. Allowable percentage errors for the three categories of problems listed in OAR-PMEL-135 used for benchmarking are then discussed. Benchmark testing is typically performed by fulfilling the criteria that the percentage errors incurred by a particular algorithm against the three categories of problems must not exceed the prescribed limits \[1\]. TUNA-RP simulation results are then tested against each of the four benchmark problems chosen for testing, the comparison of which indicates satisfactory performance of TUNA-RP. This paper ends with a conclusion that TUNA-RP is ready for performing run-up and inundation simulations. It is the wish of...
the authors that this paper will provide useful insights and easy-to-use guidelines on simulation of tsunami run-up.

2. Literature review on wetting-drying algorithms

One of the most challenging problems in tsunami research is modelling the physical process of wave run-up and inundation along shallow beaches. As the wave approaches the beach, it begins to inundate a previously dry area, turning the area into a wet area and continues to penetrate further into the dry land. If the area is flat or has very mild slope, the tsunami waves can indeed inundate further inland up to several km, causing potentially immense damages. The model algorithm must be able to dynamically track the constantly evolving wet-dry areas. As the tsunami wave recedes back to the seaward shoreline after reaching its maximum run-up height, the model must exclude the dry area from simulation. This addition of wet areas and removal of dry areas are named as wetting-drying (WD) algorithm in tsunami modelling terminology. The capability of WD algorithm is determined by the accuracy in simulating inundation distance and run-up height, as these are ultimately the most crucial information needed by tsunami hazard assessment.

A variety of numerical models has been developed to simulate tsunami inundation in one, two and three dimensions. These models employ a diverse set of innovative numerical schemes to solve the governing nonlinear shallow water equations (NSWE) and to optimally discretise the computational domains to suit the selected numerical schemes. This section reviews WD algorithms implemented in various numerical models reported in the literature with the aim of providing useful insights and easy to use guidelines to modellers interested in simulating tsunami run-up and wave inundation. Generally, WD algorithms fall into four broad frameworks: (a) thin film, (b) element removal, (c) depth extrapolation and (d) negative depth algorithms, as elaborated in the sections that follow.

2.1. Thin film algorithm

Thin film algorithm specifies a small artificial sublayer of fluid over the entire computational domain in order to give the computational domain a minimal depth at all time. This allows all nodes and all cells to be computed at each time step. There is typically a minimum threshold depth that defines the categories of being wet or being dry in the model. The Princeton Ocean Model (POM) employs this thin film algorithm into its finite difference scheme where the most landward cells must always remain “dry”, to ensure that water can never flow through the dry boundary located on dry land [2]. On the other hand, a 5 cm thin film of water is added on top of the seaward cells to mark it as “wet”. This allows the momentum equations to be solved in these seaward “wet” cells. If the depth is less than 5 cm, the velocity is set to be zero; otherwise, the velocity is computed by solving the momentum equations. The Finite Volume Community Ocean Model (FVCOM) is a finite volume coastal ocean model that also employs this similar thin-film WD algorithm [3]. Subsequently, Chen et al. [4] successfully employed the nested global-coastal FVCOM to simulate the 11 March 2011 Great Eastern Japan Earthquake and Tsunami (GEJET) event to assess the inundation in the central Sendai coastal region.

2.2. Element removal algorithm

Element removal algorithm employs a system of checks to determine if a node or cell is wet or dry. The wet nodes are included in the computational domain, while the dry nodes are excluded. For example, the TELEMAC-2D model is a finite element model [5,6] that employs the element removal algorithm by categorizing all elements as one of the three categories: (a) fully wet, (b) fully dry and (c) partially wet-dry. The categorization is performed at the beginning of each time step to include all fully wet and partially wet-dry elements in the computations and to exclude fully dry elements from the computations. Using this concept, Grilli et al. [7] coupled a tsunami generation and propagation model, codenamed FUNWAVE-TVD to the TELEMAC-2D to assess tsunami hazard along the north shore of Hispaniola due to tsunamis from far and near field Atlantic sources. In the TELEMAC-2D inundation simulation, fine-resolution unstructured meshes of 12 m to 30 m are used to accurately simulate run-up heights and inundation distances along the coast. One of the most commonly used model for simulating coastal hydrodynamics, Delft3D-FLOW also employs this element removal
algorithm in its finite difference scheme [8]. Apotsos et al. [9] validated the capability of Delft3D-FLOW in simulating tsunami inundation by comparing its simulation results to tsunami wave data obtained from standard benchmark problems.

2.3. Depth extrapolation algorithm
In depth extrapolation algorithm, the depth is extrapolated from a wet cell onto a dry cell if the depth in the wet cell is sufficiently high to inundate the dry cell. Furthermore, if the depth is able to be extrapolated from the wet cell into the dry cells, then the newly extrapolated depth is used to compute the velocity and the cell now becomes wet. For example, COULWAVE is a finite difference free surface wave model which employs depth extrapolation algorithm to simulate wave run-up [10]. The extrapolation process begins with locating the boundary between a wet cell and a dry cell. The free surface of the dry cell is then estimated using a one-dimensional linear extrapolation coupled with an averaging method. If the estimated free surface of the dry cell is above its land height, then the depth at the boundary is interpolated. The interpolated depth is in turn used to compute the velocity at the boundary. This model was adequately validated in both one-dimensional and two-dimensional space using an idealized computational domain and a sinusoidal wave forcing.

2.4. Negative depth algorithm
In the negative depth algorithm, the water surface is conceptually allowed to “exist” below the ground surface, allowing the governing equations to be computed over the entire domain, with cells having negative depth being considered dry. As the water depth gradually increases and eventually become positive, the waves begin the inundation of dry cells, which is then simulated. A finite element model RMA2 [11] employs this negative depth algorithm by introducing a porosity parameter, allowing the water to plunge below the ground surface and to flow in a low porosity medium. Nielsen and Apelt [12] applied the RMA2 model to four test cases and found that the selection of porosity parameter significantly influenced the simulation results. A major problem with this approach of using the concept of being “underground” is the difficulty in selecting an appropriate porosity parameter value as the physics involved is not real.

2.5. Choice of WD algorithm for TUNA-RP
In this subsection, we briefly discuss the relative merits of each of the four WD algorithms presented earlier and provide justification for the choice of the element removal algorithm used in TUNA-RP. The non-physics, thin film algorithm alters the mass conservation by the addition of an artificial sublayer of fluid on top of dry cells in order to allow the governing equations to be computed over the entire computational domain at every time step. Thin film algorithm tends to produce artificially smooth solutions at wetting-drying fronts, contrary to actual oscillations observed in real tsunami surveys. However, it is computationally less expensive at the expense of accuracy. The element removal algorithm that employs a system of checks to determine if a node or cell is wet or dry, thereby removing dry nodes or cells from computational domain, is the most commonly employed algorithm. Element removal algorithm saves computational cost by not computing the governing equations over the entire computational domain at each time step. However, it tends to perform better on advancing wetting fronts than on receding fronts. Depth extrapolation algorithm that extrapolates the water depth from wet nodes onto dry nodes and then computes the velocities using the newly extrapolated water depth also tends to produce artificially smooth solutions at the wetting front. Further, correction schemes are necessary to conserve mass balance because the extrapolation of water depth introduces new mass into the computational domain. Lastly, the negative depth algorithm that artificially allows the simulated water surface to penetrate below ground surface tends to be used exclusively in the finite element models. The inclusion of “appropriate” porosity parameter can capture the actual physical inundation process well. However, the choice of porosity is performed by trial and error, with the accuracy of simulation depends heavily on choice of good porosity parameter. Since the element removal algorithm saves computational cost and performs well in advancing wetting front, the algorithm is chosen for incorporation into TUNA-RP. Benefits and drawbacks in terms of accuracy,
robustness, computational efficiency, and conservation properties of these WD algorithms are detailed in [13].

3. Three categories of criteria for standard benchmarking
A group of tsunami simulation experts discussed key issues of accuracy for long-wave run-up models in the 2004 Third International Workshop on Long-Wave Run-Up Models held in Wrigley Marine Science Center, California, on 17-18 June 2004, focusing on inundation modelling and underwater landslide-generated tsunami modelling [14]. The workshop highlighted the importance of benchmarking any numerical model in producing practical and usable inundation maps, and proposed a list of benchmark problems for validating a numerical run-up model. Therefore, it is essential that all numerical models used in tsunami hazard assessment, particularly in producing inundation maps, be subjected to benchmark testing involving validation and verification. Validation is the process of ensuring that the model accurately solves the parent equations of motion, while verification is the process of ensuring that the model represents geophysical reality, following Synolakis et al. [15]. For this purpose, a standard set of benchmark problems, known as the OAR-PMEL-135, is developed by the National Tsunami Hazard Mitigation Program (NTHMP [16]. The benchmark problems include four cases: (1) the semi-analytical solution of run-up on a planar beach [17], (2) laboratory experiments of run-up on a circular island [18], (3) run-up on a simple sloping beach [19] and (4) run-up on a piecewise linear bathymetry ending in a vertical wall [20].

It should be noted that there is no absolute assurance that a numerical model that has passed the benchmark validation and verification test will always produce realistic inundation projections in a real tsunami event. The accuracy, spatial resolution and quality of input data driving the numerical model are the main determinant of the model accuracy. However, validated and verified models greatly reduce the level of uncertainty in their simulation results. The uncertainty in the initial conditions (tsunami generation) remains a major concern. Benchmark testing is typically performed by fulfilling the criteria that the percentage errors incurred by a particular algorithm against the three categories of problems must not exceed the prescribed limits as follows: (i) analytical solutions (< 5%), (ii) laboratory experiments (< 10%) and (iii) field measurements (< 20%). We will perform validation and verification test for our TUNA-RP model in the following section, using the criteria mentioned as a guide for allowable errors for three categories of problems used for benchmarking, as suggested by Horrillo et al. [1].

4. Results of TUNA-RP benchmarking
In this section, we will present TUNA-RP validation and verification test results by comparing its simulation results with four benchmark run-up solutions provided in OAR-PMEL-135 to demonstrate the satisfactory performance of TUNA-RP.

4.1. N-wave run-up on a planar beach
Carrier et al. [17] used the NSWE to simulate tsunami run-up and draw-down dynamics on a planar beach with a semi-analytical solution technique for examining shoreline movement and velocity field. In the 2004 Third International Workshop on Long-Wave Run-Up Models held in Wrigley Marine Science Center, California, the semi-analytical solution technique is used to produce the benchmark data for a leading-depression N-wave run-up on a planar beach. The benchmark problem setup and data are available in the Inundation Science & Engineering Cooperative (ISEC) website.

The computational domain of this benchmark problem is a uniformly sloping beach (planar beach) of 50 km with 1/10. The depth (bathymetry) increases linearly from the open sea (-5 km) to mean sea level (0 km), and the dry land (topography) elevates linearly from mean sea level (0 m) to 20 m in height. The initial free surface displacement is an N-wave composed of a leading depression followed by a smaller elevation, typically generated by an underwater landslide. The computation setup is then incorporated into TUNA-RP simulation to validate the capability of TUNA-RP in simulating the maximum run-up height. In this simulation, radiation boundary is employed at the seaward end to allow the wave to propagate out of the computational domain. In addition, a spatial step $\Delta x$ of 5 m is used, for the simulation to obtain more accurate run-up simulation, with a time step $\Delta t$ of 0.01 s.
ensure numerical stability. The Manning’s roughness coefficient \( n \) is set to zero, because the semi-analytical solution technique solves the NSWE without taking the friction term into account. It should be noted that the initial N-wave velocity is zero.

Figure 1 compares the simulation results of TUNA-RP with the semi-analytical solution plots available at the ISEC website for the benchmark problem. Simulation snapshots at time \( t = 160 \) s, 175 s and 220 s represent initial draw-down, maximum draw-down and maximum run-up, respectively. At \( t = 160 \) s, the wave is receding towards the seaward end (east boundary) with a positive velocity, indicating the wave will continue to recede until the maximum draw-down is achieved. At \( t = 175 \) s, maximum draw-down is achieved with a negative velocity, indicating the wave will soon propagate towards the dry land (west boundary). At \( t = 220 \) s, the simulated maximum run-up of 15.26 m is achieved. Compared to the analytical solution maximum run-up of 15.78 m, the relative error is 3 %, which is within the allowable error. The TUNA-RP simulated free surface displacement and velocity at \( t = 160 \) s, 175 s and 220 s agree well with the semi-analytical solution. Therefore, it can be concluded from this comparison that TUNA-RP satisfactorily simulates the run-up on a planar beach.

![Figure 1. (a) Free surface displacement and (b) velocity at time \( t = 160, 175, 220 \) s simulated by TUNA-RP compared to semi-analytical solution.](image)

4.2. Solitary wave run-up on a circular island

On 12 December 1992, an earthquake of magnitude \( M_w = 7.3 \) occurred near Flores Island, Indonesia. This tsunami generated a tsunami that struck the circular shaped island, named Babi Island, from the north, but inundated villages located in the southern or lee side of the island with unexpectedly high run-up [21]. Similar phenomenon was also observed on the pear shaped Okushiri island during the 12 July 1993 Hokkaido tsunami event with a measured run-up of 20 m at the lee side of the island [22]. Recognizing the need for a better understanding of this unexpected phenomenon, Briggs et al. [18] conducted a large-scale laboratory experiment that mimics the inundation inflicted on Babi Island.

The experiment was performed at the US Army Engineer Waterways Experiment Station, Coastal Engineering Research Center, Mississippi (USAEWES), in a 30 m wide and 25 m long wave basin. A circular island with 7.2 m base diameter, 0.625 m height and 14.04° slope was positioned in the basin with its center located at \( x = 15 \) m, \( y = 13 \) m. A Directional Spectral Wave Generator (DSWG) was installed along the \( x \)-axis to generate an initial solitary wave in a 0.32 m water depth. This experiment is recognized as one of the most important benchmark problems listed in OAR-PMEL-135 standard set by NTHMP to validate a numerical model. The physical experiment setup is then incorporated into TUNA-RP simulation to validate the capability of TUNA-RP in simulating the incident solitary wave refraction around the circular island and amplification of the run-up at the lee side of the island.

Figure 2 presents the time series of free surface displacement simulated by TUNA-RP in comparison with the experimental data at four wave gauge stations. The experimental data are available in the NOAA website (http://nctr.pmel.noaa.gov/benchmark/), including the locations of wave gauge stations. Good overall agreement is achieved between experimental data and TUNA-RP simulation results. In the front of the island, wave gauges 6 and 9 show an elevation wave followed by a depression wave, resembling the draw-down dynamics after the solitary wave reached the maximum run-up height. Note that, TUNA-RP does not produce the second elevation wave observed in the experimental data between \( t = 25 \) s and 30 s at wave gauges 6 and 9, because the second elevation wave is the reflected draw-down wave from the DSWG. In TUNA-RP simulation, radiation boundary is employed to allow the draw-down wave from the island to propagate out of the computational domain without being
reflected. On the lee side of the island, the reflection of solitary wave from the boundary of the basin at wave gauge 22 is also evident. However, this reflected wave does not affect the maximum run-up height observed in the laboratory experiment, because the first wave often generates the maximum run-up height and the subsequent reflected waves generate significantly smaller run-up height.

![Figure 2](image2.png)

**Figure 2.** Time series of free surface displacement at four different wave gauges simulated by TUNA-RP compared to experimental data.

### 4.3. Solitary wave run-up on a simple sloping beach

Synolakis [19] conducted a series of laboratory experiments to investigate the run-up dynamics of a solitary wave on a simple sloping beach. These experiments, which cover a wide range of non-breaking and breaking waves, have been cited extensively as a benchmark testing problem to check the accuracy of numerical solutions. TUNA-RP is used to reproduce this benchmark problem for the run-up of a non-breaking solitary wave. The run-up on a beach with 2.884° and relative wave height of 0.0185, for which experimental results are easily available, is examined. A sequence of three snapshots of the free surface elevation is presented in figure 3, showing the shoaling, run-up and rundown of the solitary wave. The simulated free surface profiles are compared with experimental data from Synolakis [19]. TUNA-RP correctly reproduces both the moving shoreline and the run-up, thereby reinforcing the validity of TUNA-RP in simulating tsunami run-up and inundation.

![Figure 3](image3.png)

**Figure 3.** Run-up of solitary wave with \( \eta / d = 0.0185 \) on 1:19.85 slope. The solid line shows the TUNA-RP results whereas circle symbols represent the experimental data from Synolakis [19].

### 4.4. Solitary wave run-up on a piecewise linear bathymetry ending in a vertical wall

Briggs et al. [20] conducted a laboratory experiment to examine solitary wave propagation and transformation over a compound slope and the subsequent run-up on a vertical wall. This laboratory data allows validation of model capabilities in handling nonlinearity, dispersion, breaking and run-up at the vertical wall. The model beach consists of three piecewise linear slopes of slopes 1:53 (first segment), 1:150 (second segment) and 1:13 (third segment) from seaward to shoreward. The vertical wall is located at the landward end of the 1:13 slope. A water gauge, which is used to measure time dynamics of the water surface height, is installed at the beginning and in the middle of each sloping segment. The locations of the gauges are available at the NOAA website. Three cases denoted by A, B and C with relative wave heights of 0.039, 0.264 and 0.696 respectively are considered in this experimental investigation. The solitary waves will propagate and transform over the three slopes before being reflected from the right wall.

Figure 4 displays the comparison of TUNA-RP simulated and experimentally-measured wave profiles for Case A, which involves a small amplitude non-breaking solitary wave. In both numerical simulation and the experiment, the wave propagates towards the vertical wall and is reflected by the
wall without any breaking. The numerical solution shows very good agreement of the amplitude and
waveform with the measurements despite slight phase lags in the reflected wave (the second peak in
the time series). Validations for Cases B and C are briefly tabulated in Table 1, which shows good
agreement between TUNA-RP and other NSWE run-up model simulation results.

![Comparison between the TUNA-RP simulated solutions (dotted line) and laboratory
measurements (solid line) for Case A.](image-url)

**Figure 4.** Comparison between the TUNA-RP simulated solutions (dotted line) and laboratory
measurements (solid line) for Case A.

**Table 1.** Validation of TUNA-RP for Cases A, B and C.

| Case | Relative wave height | Experimental result | Other NSWE model (Yeh et al. [23]) | TUNA-RP result |
|------|----------------------|---------------------|-----------------------------------|----------------|
|      | η₀/d | R | R/d | R | R/d |
| A    | 0.039 | 2.74 | 0.13 | 2.18 | 0.10 | 2.12 | 0.10 |
| B    | 0.264 | 45.72 | 2.10 | 8.81 | 0.40 | 8.43 | 0.39 |
| C    | 0.696 | 27.43 | 1.26 | 12.91 | 0.59 | 13.29 | 0.61 |

5. **Conclusion**

TUNA-RP is successfully validated by comparing its simulation results to four well acknowledged
international benchmark problems. In the first benchmark problem, TUNA-RP successfully
reproduces the semi-analytical solution to correctly demonstrate that the shoreline is drastically
drawn-down when a depression wave approaches. Then the wave propagates towards the planar beach
and inundates the dry land. Such a drastic wave draw-down accompanying a leading depression wave was
also observed in Penang and Langkawi during the 2004 Indian Ocean tsunami. Hence, it is important
for the coastal communities to keep in mind that this natural phenomenon of drastic wave draw-down
is a sure sign of tsunami arriving within minutes. On the second benchmark test problem to assess the
capability of TUNA-RP in simulating solitary wave run-up on a circular island, TUNA-RP correctly
shows that the front of the island is exposed to tsunami threats. More importantly, the lee side of the
island is exposed to tsunami threats too, due to wave refraction—waves wrapping around the island—
and amplification of run-up when waves collide behind an island. This observed phenomenon of wave
amplification behind an island, both in real wave basin experiment and in numerical simulations,
provides useful insights for authorities in planning evacuation routes and in coastal development
around islands. In particular, island residents should always seek high ground whenever a tsunami
warning is issued, regardless of the locations along the island, front or lee side. Finally, two additional
benchmark validation tests performed on TUNA-RP further confirm its satisfactory performance in
simulating run-up and inundation.

6. **References**

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