We examine the $Q^2$ evolution of gluon polarization in polarized nucleons. As is well known, the evolution of $\alpha_s \Delta G(Q^2)$ is negligible for typical momentum transfer variations found in experimental deep inelastic scattering. As $\alpha_s$ increases, however, the leading nonzero term in the evolution equation for the singlet first moment reduces the magnitude of the gluon spin. At low $Q^2$ the term $\alpha_s \Delta G$ can vanish, and ultimately become negative. Thus, low energy model calculations yielding negative $\Delta G$ are not necessarily in conflict with experimental evidence for positive gluon polarization at high $Q^2$. 
I. INTRODUCTION

Polarized deep inelastic scattering (DIS) has proven to be an important new tool for studying the structure of the nucleon. Through sum rules, the experiments have provided us with values of various operator matrix elements; for instance, the singlet sum rule indicates that quark spin accounts for only a small fraction \((\approx 1/3)\) of the nucleon’s total spin \(\Delta s\). One possible resolution of this apparently surprising result is that the gluon fields in the nucleon bound state may carry a significant fraction of the nucleon’s spin. Orbital angular momentum of quarks and gluons may also be significant.

Recently, various authors \(^{[3]}\) have attempted to make estimates of the size of the gluon contribution to the spin of the nucleon,

\[
\Delta G(Q^2) = \int_0^1 dx \Delta g(x, Q^2) = \int_0^1 dx (g_1(x, Q^2) - g_1(x, Q^2)).
\]

In at least one case \(^{[3]}\), this quantity was evaluated in low energy models, and turned out to be negative. This is in apparent conflict with extractions, primarily at DIS scales, which find \(\Delta G\) positive \(^{[3]}\). Indeed, to leading order (LO) in perturbative QCD, \(\alpha_s(Q^2)\Delta G(Q^2)\) is renormalization group invariant, thus the claim has been made that at least the sign of \(\Delta G\) should be a reliable prediction of the models \(^{[3]}\).

It is well known \(^{[4]}\) that the singlet part of the first moment of the spin structure function \(g_1\) has an anomalous \(Q^2\) evolution. It is also well known \(^{[4]}\) that the leading term in \(\Delta \alpha_s\) in the axial anomalous dimension vanishes, and it is for this reason that many authors dismiss this \(Q^2\) evolution as insignificant. Roughly speaking, corrections to the singlet first moment arising from the anomalous dimension can be argued to behave like \(\alpha_s \log Q^2\), and hence appear approximately \(Q^2\) independent. This is, however, not precisely true at higher order in \(\alpha_s\). In an earlier paper, \(^{[1]}\) it was argued that comparing momentum sum rules from unpolarized electroproduction and the spin sum rule for \(g_1\), including their \(Q^2\) evolution, showed that DIS spin measurements are consistent with a low energy valence quark picture, where the valence quarks carry a substantial part of the spin of the proton. In a later paper, \(^{[10]}\), those calculations were extended to consistently include the next higher order QCD corrections in leading twist. Here, we will explicitly examine the effects on gluon spin. We evaluate the leading non-zero QCD corrections to the gluon spin evolution, and find that the sign of the first correction term forces one in the direction of smaller (more negative) \(\alpha_s \Delta g\) at lower \(Q^2\). Application of these perturbative formulae show that a sign change in \(\alpha_s \Delta G\) thus need not be such a surprise.

There are several caveats to this analysis which must be made at the outset. First, we apply perturbative formulae down to a regime where they cannot rigorously apply \((\alpha_s / \pi \approx 1)\) and thus do not expect to have quantitative predictive power. However, the sign change in \(\Delta G\) is significant, and if the perturbative result goes in this direction, there is certainly no reason why the full nonperturbative result might not behave similarly. Furthermore, one can point to the poorly understood successes of the one-gluon exchange mechanism in the hadron mass spectrum \(^{[1]}\), or of the evolution of unpolarized structure functions \(^{[6]}\), which provide some faith in at least the qualitative predictions of this method.

The second issue is one of scheme dependence. The decomposition of the singlet sum into terms involving \(\Delta \Sigma\) and \(\Delta G(Q^2)\) depends on renormalization and factorization scheme choice, and is gauge dependent \(^{[12]}\). Indeed, there is no unique way to define quark and gluon distributions beyond LO. When in a perturbative QCD regime, we work in the Adler-Bardeen (AB) factorization scheme \(^{[13]}\), with \(\overline{MS}\) renormalization conventions, which defines \(\Delta G\) and its contribution to the singlet first moment as well as its \(Q^2\) evolution to LO. However, if one wants to work beyond leading order, in particular to evolve to low scales to compare with model predictions, one must know in addition the connection with the choice of scheme (and thus the underlying operator matrix element definition) of e.g. \(\Delta G\) in the low energy model calculations. Thus, we really cannot make any direct quantitative comparisons with the numerical values extracted in quark model calculations. We can, nevertheless, unambiguously examine the leading evolution of the gluon spin downwards in \(Q^2\) given our choice of scheme, which shows a clear trend towards zero and eventually negative values. Ultimately, one must understand both the renormalization scale and scheme dependence of the quark model calculations in order to decide if the contribution of gluon spin is quantitatively compatible at low and high energy scales.

II. FORMALISM

Working at leading twist, the contributions to the hadronic tensors involved in polarized DIS, via the operator product expansion, are due to singlet and non-singlet axial currents. The first moment of the polarized spin structure function \(g_1(x, Q^2)\) is given by
Eq. (4), the evolution of these quantities are given by [8–10] and the beta function is [8,10].

Order in $\alpha$, the evolution of truncated at order $\alpha$, vanishes at order $\alpha$. Note that the difference $a_0(Q^2)$ with the coefficient function $C^S(Q^2)$ factored out, as shown above, but because of the $U(1)$ anomaly in QCD, $a_0$ is itself not scale independent. The QCD evolution of $a_0$ has been worked out to NNLO [8], and is given below. For our purposes, we also wish to consider a decomposition of $a_0$ into separated quark and gluon contributions. This is a renormalization scheme-dependent, and gauge dependent, separation. Some authors e.g. choose a so called gauge-invariant scheme in which the gluons by definition do not contribute to $a_0$ [12]. In the Adler-Bardeen factorization scheme [13], sometimes called a chirally invariant scheme, the anomalous gluon contribution is explicitly separated out as follows:

$$a_0(Q^2) = \Delta \Sigma - f \alpha_s(Q^2) \Delta G(Q^2)/2\pi$$

The singlet matrix element is obtained from

$$a_0(Q^2) s_{\mu} = \langle p, s | J^S_{\mu} | p, s \rangle = \langle p, s | \sum_{i=1}^{n_f} \bar{q}_i \gamma_{\mu} \gamma_5 q_i | p, s \rangle,$$

which in a naive quark model picture is just the total spin of the quarks. Experimental results are generally given for values of $a_0(Q^2)$ with the coefficient function $C^S(Q^2)$ factored out, as shown above, but because of the $U(1)$ anomaly in QCD, $a_0$ is itself not scale independent. The QCD evolution of $a_0$ has been worked out to NNLO [8], and is given below. For our purposes, we also wish to consider a decomposition of $a_0$ into separated quark and gluon contributions. This is a renormalization scheme-dependent, and gauge dependent, separation. Some authors e.g. choose a so called gauge-invariant scheme in which the gluons by definition do not contribute to $a_0$ [12]. In the Adler-Bardeen factorization scheme [13], sometimes called a chirally invariant scheme, the anomalous gluon contribution is explicitly separated out as follows:

$$a_0(Q^2) = \Delta \Sigma - f \alpha_s(Q^2) \Delta G(Q^2)/2\pi$$

where $f$ is the number of active flavors at this $Q^2$. The evolution of the full singlet first moment, $a_0$, is known to next to leading order (NLO) in $\alpha_s$ [14],

$$a_0(Q^2) = a_0(Q^0) \exp \left( - \int_{\alpha_s(Q^0)}^{\alpha_s(Q^2)} d\alpha' \frac{\gamma^S(\alpha')}{2\beta(\alpha')} \right)$$

where $\gamma^S(\alpha_s) = \gamma^S_1(\alpha_s/4\pi)^2 + \gamma^S_2(\alpha_s/4\pi)^3 + \cdots$ is the singlet anomalous dimension, with $\gamma^S_1 = 16f$, and $\gamma^S_2 = (944f/3 - 32f^2/9)$ with our choice of renormalization conventions, and $\beta$ is the beta function of QCD,

$$\beta(\alpha_s) = d\alpha_s/d\tau = -\beta_0 \alpha_s^2/4\pi - \beta_1 \alpha_s^3/16\pi^2 + \cdots$$

where $\tau = \ln(Q^2/\Lambda_{QCD}^2)$, and $\beta_0 = 11 - 2f/3, \beta_1 = 102 - 38f/3$ [13]. To leading non-zero order, this gives

$$a_0(Q^2) \approx a_0(Q^0) \left( 1 + \frac{\gamma^S_1}{8\pi \beta_0} (a_s(Q^2) - a_s(Q^0)) + \mathcal{O}(\alpha_s^3) \right)$$

Note that the difference $a_s(Q^2) - a_s(Q^0)$ appearing in the formulae above is itself of order $\alpha_s^2$, thus naturally the evolution of $a_0$ vanishes at order $\alpha_s$. The solution for $a_0$ including NLO corrections in the singlet anomalous dimension and the beta function is [10]

$$a_0(Q^2) = a_0(Q^0) \left( 1 + \frac{\gamma^S_1}{8\pi \beta_0} (a_s(Q^2) - a_s(Q^0)) + \left( \frac{\beta_0 \gamma^S_2 - \beta_1 \gamma^S_1}{64\pi^2 \beta_0^2} \right) (a_s^2(Q^2) - a_s^2(Q^0)) \right) + \frac{(\gamma^S_1)^2}{128\pi^2 \beta_0^2} (a_s(Q^2) - a_s(Q^0))^2 + \mathcal{O}(\alpha_s^4),$$

truncated at order $\alpha_s^3$.

It is important to emphasize that the evolution of $a_0(Q^2)$ is well defined, and is not itself gauge dependent. It is the separation of $a_0$ into “spin” and “gluon” terms which is scheme and gauge dependent. Using the separation choice of Eq. (4), the evolution of these quantities are given by [8,10]

$$\frac{d}{d\tau} \left( \frac{\Delta \Sigma(\tau) \Delta G(\tau)/2\pi}{a_s(\tau)} \right) = \alpha_s^2(\tau) \left( \gamma_{\Sigma\Sigma} \gamma_{\Sigma G} \gamma_{G G} \right) \left( \Delta \Sigma(\tau) \Delta G(\tau)/2\pi \right)$$

To leading order, $\gamma_{\Sigma\Sigma} = 1/2\pi^2, \gamma_{\Sigma G} = 0, \gamma_{G G} = 0, \gamma_{TF} = -f/2\pi^2$ [14]. The evolution manifestly begins only at second order in $\alpha_s$. To leading non-zero order, solving Eq. (4) directly (or alternatively, combining Eqs. [4] with [7]) we see
\[ \alpha_s(Q^2) \Delta G(Q^2) = \alpha_s(Q_0^2) \Delta G(Q_0^2) - \frac{\gamma_i}{4f/\beta_0} \left( \alpha_s(Q^2) - \alpha_s(Q_0^2) \right) a_0(Q^2). \] (10)

Of course, to first order in \( \alpha_s \), the above equation states that \( \alpha_s \Delta G \) is a renormalization group invariant, the usual result. This last equation already shows qualitatively the results we have claimed; namely, if \( Q^2 \) is some large scale appropriate to DIS, and \( Q_0^2 \) is some low scale appropriate to a quark model calculation, the leading nonzero correction to \( \alpha_s \Delta G \) is large and positive, and there is no reason the sign need be preserved between low and high \( Q^2 \) scales. In the Adler-Bardeen scheme, Eq. (4) does not receive higher order corrections \( \Delta \), and one can combine Eq. (4) with Eq. (8) to examine the NLO modifications to \( \alpha_s \Delta G \).

### III. RESULTS

For definiteness, we use the following experimental values \( \Delta \): \( \alpha_s(5 \text{ GeV}^2) = 0.287 \pm 0.02, \quad a_0(5 \text{ GeV}^2) = 0.37 \pm 0.11 \).

The evolution equation, (3) evaluated to LO (NLO) yields

\[ a_0(5 \text{ GeV}^2)/a_0(10 \text{ GeV}^2) = 1.008(1.010), \] (11)

which is a negligible change, well below the present limits of experimental observability. In this sense, the fact that to leading order there is no \( Q^2 \) evolution is born out. Leaving the region of small \( \alpha_s \), however, yields some nontrivial evolution. As discussed above, the separation of \( a_0 \) into \( \Delta \Sigma \) and \( \Delta G \) requires additional experimental and theoretical assumptions; recent analyses \( \Delta \) consistent with the scheme conventions of this work finds \( \Delta G(5 \text{ GeV}^2) \) ranging from 0.8 to 2.6, and thus \( \alpha_s \Delta G(5 \text{ GeV}^2) \) ranging from 0.2 to 0.75, and \( \Delta \Sigma \) (which is scale independent here) ranging from 0.4 to 0.65, depending on how the analysis and extraction are done. The most naive predictions originally gave \( \Delta \Sigma = 1 \), but many effective low energy quark models are consistent with a lower value, around \( \Delta \Sigma \approx 0.65 \). (The reduction from unity might come e.g. from the lower components of the relativistic quark spinors, the same source which reduces the axial charge in the bag model from 5/3) Using any such prediction in Eq. (3), however, assumes a partonic interpretation of \( \Delta \Sigma \), which is by no means required by QCD, and may be entirely incorrect.

Starting from the above experimental values, \( \alpha_s \Delta G \) will evolve from somewhere in the range \( (2 \leftrightarrow 75) \) at \( 5 \text{ GeV}^2 \) down to zero at a scale where \( \alpha_s/\pi \) is in the range \( (3 \leftrightarrow 1.5) \), i.e. where nonperturbative physics should just begin to become significant. Going further down in \( Q^2 \), \( \alpha_s \) increases and \( \alpha_s \Delta G \) becomes negative. A smaller assumed value for \( a_0(5 \text{ GeV}^2) \) results in less rapid downward evolution. In Figs. 1-3, we show the result of evolving the singlet moment as well as the gluon spin up in \( \alpha_s \) (down in \( Q^2 \)). If one applies the NLO corrections of Eq. (8) to \( a_0 \), this makes the evolution slightly more rapid. That is, one need not go so far in \( \alpha_s/\pi \) to get to the “turnover” point \( (\Delta G = 0, \text{ or } a_0 \approx 0.5) \). Explicitly, \( a_0 \) reaches 0.5 at \( \alpha_s \approx 2 \) using LO evolution for \( a_0 \), but \( \alpha_s \approx 1.3 \) using NLO. It is encouraging that the trend to smaller (more negative) \( \Delta G \) at low \( Q^2 \) is enhanced (rather than cancelled, as it certainly might have been) by considering the NLO corrections of Eq. (8).

Moving the other direction, one can ask how far the trend of increasing \( \alpha_s \Delta G \) with increasing \( Q^2 \) continues. From Eq. (7), as \( Q^2 \) increases, \( a_0(Q^2) \) (slowly) decreases and this in turn reduces the rate of change of \( \alpha_s \Delta G \). In this way, \( \alpha_s \Delta G \) inevitably increases, but assuming that \( \Delta \Sigma \) is in the range \( (5 \leftrightarrow 1) \), Eq. (10) shows that if \( \alpha_s \Delta G \) reaches around \( (1 \leftrightarrow 2) \), the increase with scale effectively halts, and \( \alpha \Delta G \) becomes renormalization group invariant beyond second order in \( \alpha_s \), as well.

Following the logic of refs \( \Delta \) and \( \Delta \), we might argue that evolution of both unpolarized momentum fractions and polarized spin fractions are consistent with a nonrelativistic quark picture at a low energy scale of \( \alpha_s(\mu_0^2)/\pi \approx 1 \pm 0.2 \), where the gluonic contributions vanish. Thus, with \( \Delta G(\mu_0^2) = 0 \), and \( \Delta \Sigma = 0.6 \text{ GeV}^2 \), we would predict, to leading non-zero order,

\[ \alpha_s \Delta G(5 \text{ GeV}^2) = 0 - (4/\beta_0)(\alpha_s(5 \text{ GeV}^2) - \alpha_s(\mu_0^2))(\Delta \Sigma) \approx 0.6 \pm 0.1, \] (12)

i.e. \( \Delta G(5 \text{ GeV}^2) \approx 2.1 \pm 0.4 \), a result which is certainly compatible with recent high energy experimental estimates \( \Delta \), even though we begin with \( \Delta G = 0 \) at the low energy scale.

Jaffe provides us with more sophisticated model predictions for \( \Delta G \) at the low energy scale \( \Delta \), namely for the non-relativistic quark model \( \Delta G(\text{NRQM}) = -0.8 \) and \( \alpha_s(\text{NRQM}) = 0.9 \), while for the bag model \( \Delta G(\text{bag}) = -0.2 \), and \( \alpha_s(\text{bag}) = 2.0 \). His paper makes it clear that these numbers are not to be taken too seriously, but the sign of \( \Delta G \) is taken as an interesting and surprising result. However, if we evolve these as above (bearing in mind the caveats outlined in the introduction) we would predict \( \alpha_s \Delta G(5 \text{ GeV}^2) \) (from NRQM) = \( -0.5 \), while \( \alpha_s \Delta G(5 \text{ GeV}^2) \) (from bag) = \( +1 \). The latter has shown a sign change, with \( \Delta G(5 \text{ GeV}^2) = +0.35 \) only slightly below the low end of the experimental range. Assuming a (scale independent) value of \( \Delta \Sigma = 0.7 \), rather than \( +0.5 \), brings these predictions even higher, e.g. from the bag model result it predicts \( \Delta G(5 \text{ GeV}^2) = 1 \). We see that the sign can indeed change, although it depends
on both the starting values for $\alpha_s$ and $\Delta G$, and thus on the quantitative details of the model. This is also clear from the evolution shown in Figs. 2 and 3, where the location (in $\alpha_s$) of the sign change is a strong function of the starting values.

We conclude that the sign of $\alpha_s \Delta G$ at low energy scales is ill-determined simply based on $\mathcal{O}(\alpha_s^2)$ (which is leading non-vanishing order) perturbative QCD arguments, starting from experimental values at moderate $Q^2$. Both LO and NLO perturbative evolution tend to decrease $\alpha_s \Delta G$, even to negative values, depending on starting conditions. The spin content of the glue remains an important aspect of nucleon structure, one whose measurements should certainly be improved at high $Q^2$, and which may play a key role in understanding and interpreting low energy models, but the connection between the two scales involves nonperturbative physics and is thus not trivial to predict, and one should not be particularly surprised if $\alpha \Delta G$ is found to be negative in a quark model calculation.

ACKNOWLEDGMENTS

The author acknowledges useful discussions with E. Kinney. This work is supported by U.S. Department of Energy grant DOE-DE-FG0393ER-40774. The author acknowledges the support of a Sloan Foundation Fellowship.

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FIG. 1. Plot of the singlet first moment $a_0(Q^2)$, as a function of $\alpha_s/\pi$. Solid (dashed) curve shows leading (next to leading) order calculations. We started from experimental values [3] at 5 GeV$^2$ and evolve downwards in $Q^2$. The two lines represent a band of possible results, beginning at $a_0(Q^2 = 5 \text{ GeV}^2) = .37 \pm .11$. The horizontal dotted line shows $a_0 = 0.6$, which would be the rough expectation in the naive quark model picture, assuming that $\Delta \Sigma$ (scale independent) corresponds to the “spin on the quarks”, when there is no significant contribution from the gluon anomaly term in Eq. (1).

FIG. 2. Same as previous figure, but showing $\alpha_s \Delta G$, as calculated in Eq. (10), as a function of $\alpha_s/\pi$. The dashed curve corresponds to using $a_0$ as computed in Eq. (8), to NLO. The horizontal dotted line is $\alpha_s \Delta G = 0$, where the gluon anomaly term makes no contribution to Eq. (1).
FIG. 3. Same as previous figure, but showing just $\Delta G$ as a function of $\alpha_s/\pi$, evaluated using Eq. (10). The dashed curve corresponds to using $a_0$ as computed in Eq. (8) to NLO.