Centrality Dependence of Thermal Parameters in Heavy-Ion Collisions at CERN-SPS

J. Cleymans\textsuperscript{a}, B. Kampfer\textsuperscript{b} and S. Wheaton\textsuperscript{a},

\textsuperscript{a}Department of Physics, University of Cape Town, Rondebosch 7701, Cape Town, South Africa
\textsuperscript{b}Institut für Kern- und Hadronenphysik, Forschungszentrum Rossendorf, PF 510119 D-01314 Dresden, Germany

The centrality dependence of thermal parameters, characterizing the hadron multiplicities, is determined phenomenologically for lead-on-lead collisions at CERN-SPS for a beam energy of 158 AGeV. The strangeness equilibration factor shows a clear, approximately linear, increase with increasing centrality, while the freeze-out temperature and chemical potential remain constant.

Key Words: relativistic heavy-ion collisions, hadron yields, thermal model
PACS: 24.10.Pa, 25.75.Dw, 25.75.-q

It has been shown that the abundances of different hadronic species in the final state of relativistic heavy-ion collisions can be well described by statistical-thermal models (cf. \cite{1,2,3} and further references therein). In such a way, the observed multiplicities of a large number of hadrons can be reproduced by a small number of parameters \cite{2,3}, such as the temperature, the baryon chemical potential, the volume, and a parameter \cite{4} measuring the degree of equilibration of strange particles. Such a description can be justified for multiplicities measured over the whole phase-space, since many dynamical effects cancel out in ratios of hadron yields; in particular, effects due to flow can be shown to disappear if the freeze-out surface is characterized by a single temperature and chemical potential \cite{5,6}.

As a matter of fact we mention that electromagnetic signals, i.e. real and virtual photons, observed in central heavy-ion collisions at CERN Super Proton Synchrotron (SPS) energies can also be described astonishingly well within the framework of a thermal fireball model \cite{7}. This is not necessarily a proof for (global) thermalization \cite{6}, but shows that the use of thermal models allows a very economical description of a large set of observables by a few characteristic parameters.

It is the subject of the present paper to pursue this idea and to analyze the centrality dependence of the thermal parameters describing hadron multiplicities. This will provide further information about the effects of the size of the excited strongly interacting system and help in the systematic understanding of the experimental data. We will show that the thermal model is able to describe the available data for various centrality classes at one beam energy.

In order to have a sound basis for the application of the thermal model we rely as much as possible on fully integrated particle multiplicities. For this reason we concentrate our efforts on the analysis of results obtained by the NA49 collaboration \cite{10} using centrality selected fixed-target Pb + Pb collisions at a beam energy of 158 GeV per nucleon, which are here analyzed within the framework of the thermal model \cite{3}. The data \cite{10} consists of centrality binned multiplicities of the following hadronic species: $\bar{p}$, $\langle \pi \rangle = (\pi^+ + \pi^-)/2$, $K^+$ and $K^-$, as a function of the number of participants. Following \cite{10,11}, these centrality classes correspond to mean impact parameters (in units of fm) 2.2 (bin I), 4.3 (bin II), 6.0 (bin III), 7.4 (bin IV), 8.9 (bin V), and 10.7 (bin VI). The mean number of participants is listed in Table 1. Notice that the number of hadronic species at our disposal is quite restricted in comparison with other available data sets for central collisions. We investigate here the general trend for changing centrality, relying on the data set given in \cite{10}. It should be emphasized that we have not included in our analysis the multiplicity of protons. At larger impact parameters many protons are spectators which should not be included in the thermal model analysis.

For completeness we briefly recall the basic features of the statistical-thermal model used to extract the chemical freeze-out parameters. A more detailed description can be found in \cite{2,3}.

\begin{itemize}
  \item An ideal gas of all hadrons and hadronic resonances listed by the particle data group \cite{13} was used.
  \item The grand-canonical ensemble description was used throughout with the additional parameter $\gamma_s$ (cf. \cite{4}) included to account for the incomplete equilibration in the strange sector.
  \item The correct quantum statistics was used for each particle.
  \item Resonances were described with a Breit-Wigner distribution \cite{3} over an interval $[m - \delta m, m + 2\Gamma]$ where $\delta m = \min[m_{\text{threshold}}, 2\Gamma]$. The resonance widths $\Gamma$ were taken from \cite{13}.
  \item All resonances were allowed to decay with branching ratios taken from \cite{13}. Weak decays were excluded since the data in \cite{10} were corrected accordingly.
\end{itemize}
No excluded-volume corrections (cf. [14]) were incorporated. This does not affect the intensive thermal parameters \( T, \mu_B, \) etc...), however, corrections to the densities will be called for, in particular the baryon and the energy densities.

According to our proposition, at chemical freeze-out the system is completely specified by its temperature \( T \), effective volume \( V_{\text{eff}} \), strangeness suppression factor \( \gamma_s \) and chemical potentials \( \mu_B, \mu_s \) and \( \mu_Q \). The initial conditions of the collision fix \( \mu_S \) and \( \mu_Q \) for a given \( \mu_B; \mu_S \) is fixed by the requirement of strangeness conservation, while \( \mu_Q \) is determined by the constraint that the ratio of the total charge to total baryon number of the system is conserved.

The chi-square minimization routine MINUIT was used to fit the chemical freeze-out parameters using

\[
N_i = \sum_j Br(j \rightarrow i) n_j^{(\text{prim})} V_{\text{eff}}
\]

where \( Br(j \rightarrow i) \) is the branching ratio of the decay of hadron species \( j \) into species \( i \), and \( n_j^{(\text{prim})} \) denotes the primary density. The values of \( T, \mu_B, \gamma_s \) and \( V_{\text{eff}} \) were used as free fit parameters adjusted to the fully integrated yields \( \bar{p}, K^+, K^-, \langle \pi \rangle \) and \( N_{\text{part}} \) from [10]. The results obtained for each of the six centrality bins are listed in Table 1 and displayed in Figs. 1 - 4. The number of participants, \( N_{\text{part}} \), is identified with the baryon number of the hadron gas.

As seen in Table 1, good agreement exists between the trends in the data and our model results. As the number of pions has a very small experimental error, the minimization routine zooms in on them and fixes the parameters so as to reproduce this number as accurately as possible. On the contrary, the number of participants, \( N_{\text{part}} \), has a large error and the minimization routine gives much less weight to them, hence the agreement is not as good.

Confident that the thermal model reproduces the available hadron multiplicities in each of the centrality bins with sufficient accuracy, let us consider the trends of the thermal model parameters. The chemical freeze-out temperature and the chemical potential stay fairly constant for the various centralities, as seen in Figs. 1 and 2.

A comparison of \( T \) and \( \mu_B \) with the values obtained in the recent analysis of [3] for the most central collisions shows that they are consistent with the values of the present analysis. The results from [3] are shown as a band in Figs. 1 and 2. As proven in [3], the inclusion or omission of certain hadron species can change considerably the extracted values of \( T \) and \( \mu_B \). We stress however that our analysis, due to the restricted available data, focuses on the trends with changing centrality. The temperature, \( T \), and the baryon chemical potential, \( \mu_B \), do not show any noticeable dependence on centrality.

There are two thermal parameters which exhibit a pronounced dependence on centrality: the strangeness equilibration factor \( \gamma_s \) increases approximately linearly and then saturates with increasing centrality, as shown in Fig. 3; similarly, the radius \( R \) of the hadron gas, which can be readily compared to the radius of a static Pb nucleus increases approximately linearly with the number of participants, \( N_{\text{part}} \), as seen in Fig 4.

Fig. 5 shows the centrality dependence of various thermodynamic state variables, such as the energy per hadron \( \langle E \rangle / \langle N \rangle \), the energy density, the baryon density, and the entropy per baryon \( S/B \). They all remain fairly independent of the centrality. In contrast, the Wroblewski factor [15], defined as

\[
\lambda_s = \frac{2 \langle s \bar{s} \rangle}{\langle uu \rangle + \langle dd \rangle}
\]

increases nearly linearly with increasing centrality.

The predictions of other hadron multiplicities, such as \( K^0_s, \phi, \Xi^\pm, \) and \( \Lambda, \bar{\Lambda} \) can serve as a further test of the thermal model. These are not yet at our disposal in such a centrality binned way as the yields of \( \pi^\pm, K^\pm, \) and \( \bar{p} \) but, may become accessible in further data analyses. We therefore present our prediction in Table 1.

In summary, the analysis of the thermal parameters, describing the integrated yields of \( \pi^\pm, K^\pm, \) and \( \bar{p} \) as obtained by the NA49 experiment [10], shows that the radius of the fireball increases linearly with increasing centrality. Also, the strangeness parameter \( \gamma_s \) increases, i.e., strange particle multiplicities approach chemical equilibrium. In contrast, the temperature and the baryon chemical potential do not change with centrality. No in-medium modifications are needed to describe the above quoted hadron yields.

Acknowledgments

We acknowledge helpful correspondence and discussions with P. Jacobs, D. Röhrich, P. Seyboth, F. Sikler, and R. Stock on the NA49 data.

[1] K. Redlich, Rapporteur’s talk at QM2001, Stony Brook, hep-ph/0105104, to be published in the proceedings of the conference.
| Particle | BIN I   | BIN II  | BIN III  |
|---------|--------|--------|----------|
| ⟨π⟩    | 598    | 598    | 499.4    |
| K⁺     | 96.3   | 93.7   | 80.3     |
| K⁻     | 52.6   | 53.9   | 45.1     |
| ̄n     | 10.4   | 10.4   | 8.6      |
| N_{part} | 362   | 418    | 304      |

| Particle | BIN IV  | BIN V   | BIN VI   |
|---------|--------|--------|----------|
| ⟨π⟩    | 279    | 279    | 182.4    |
| K⁺     | 35.5   | 35.2   | 20.5     |
| K⁻     | 20.2   | 20.3   | 11.8     |
| ̄n     | 5.25   | 5.24   | 3.7      |
| N_{part} | 188   | 199    | 130      |

| Particle | BIN I   | BIN II  | BIN III  | BIN IV  | BIN V   | BIN VI   |
|---------|--------|--------|----------|--------|--------|----------|
| Λ       | 50.5   | 42.2   | 29.4     | 19.4   | 11.4   | 5.5      |
| ̄Λ      | 5.97   | 5.00   | 3.54     | 2.34   | 1.45   | 0.56     |
| Ξ⁻      | 3.66   | 3.11   | 1.90     | 1.13   | 0.58   | 0.24     |
| ̄Ξ⁻     | 0.78   | 0.66   | 0.43     | 0.25   | 0.14   | 0.05     |
| K^0     | 73.2   | 62.1   | 41.51    | 27.73  | 16.16  | 7.68     |
| φ       | 6.37   | 5.46   | 3.23     | 1.92   | 0.98   | 0.38     |

TABLE I: Comparison of experimental hadron yields with the results of the thermal model with parameters as displayed in Figs. 1 - 4.

TABLE II: Prediction of further hadron multiplicities using the parameters as displayed in Figs. 1 - 4.

[2] P. Braun-Munzinger, I. Heppe and J. Stachel, Phys. Lett. B 465, 15 (1999).
[3] F. Becattini, J. Cleymans, A. Keranen, E. Suhonen and K. Redlich, Phys. Rev. C 64, 024901 (2001).
[4] J. Letessier, J. Rafelski, A. Tounsi, Phys. Rev. C 50, 406 (1994); C. Slotta, J. Sollfrank, U. Heinz, Proc. of Strangeness in Hadronic Matter (Tucson), (Ed. J. Rafelski), AIP conference proc. 340 (1995) p. 462.
[5] J. Cleymans and K. Redlich, Phys. Rev. C 60, 054908 (1999).
[6] For a recent review see, e.g., D. Rischke, Rapporteur’s talk at QM2001, Stony Brook, to be published in the proceedings of the conference.
[7] K. Gallmeister et al., Phys. Rev. C 62, 057901 (2000), Phys. Lett. B 473, 20 (2000), Nucl. Phys. A 688, 933 (2001); B. Kämpfer et al., hep-ph/0102192, Nucl. Phys. A (2001) in print.
[8] E. Shuryak, Phys. Rev. Lett. 68, 3270 (1992).
[9] U. Heinz, Nucl. Phys. A 638, 357c (1998).
[10] F. Sikler (for the NA49 Collaboration), Nucl. Phys. A 661, 45c (1999).
[11] G.E. Cooper, Nucl. Phys. A 661, 362c (1999); “Baryon Stopping and Hadronic Spectra in Pb-Pb Collisions at 158 GeV/nucleon.”, LBL preprint LBNL-45467, April 2000.
[12] J. Cleymans and H. Satz, Z. Phys. C 57, 135 (1993).
[13] R.M. Barnett et al. (Particle Data Group), Phys. Rev. D 54, 1 (1996).
[14] D.H. Rischke, M.I. Gorenstein, H. Stöcker and W. Greiner, Z. Phys. C 51, 485 (1991).
[15] A. Wroblewski, Acta Phys. Pol. B 16, 379, (1985).
FIG. 1: Centrality dependence of the chemical freeze-out temperature $T$ as a function of the mean participant number. The result of [3] is indicated by the band: $T = 158.1 \pm 3.2$ MeV.
FIG. 2: Centrality dependence of the baryon chemical potential $\mu_B$. The result of [3] is indicated by the band: $\mu_B = 238 \pm 13$ MeV.
FIG. 3: Centrality dependence of the strangeness equilibration factor $\gamma_s$. 
FIG. 4: Centrality dependence of the fireball radius $R$. 
FIG. 5: Centrality dependence of various thermodynamic state variables. The entropy per baryon is scaled by a factor 1/20. The baryon density is in units of 1/fm$^3$, the energy density is in units of GeV/fm$^3$ and are both uncorrected for excluded volume effects. The energy per hadron $\langle E \rangle / \langle N \rangle$ is in units of GeV.