Abstract

Trajectory optimization of low-thrust perturbed orbit rendezvous is a crucial technology for space missions in low Earth orbits, which is difficult to solve due to its initial value sensitivity, especially when the transfer trajectory has many revolutions. This paper investigated the time-fixed perturbed orbit rendezvous between low-eccentricity orbits and proposed a priori quasi-optimal thrust strategy to simplify the problem into a parametric optimization problem, which significantly reduces the complexity. The optimal trajectory is divided into three stages including transfer to a certain intermediate orbit, thrust-off drifting and transfer from intermediate orbit to the target orbit. In the two transfer stages, the spacecraft is assumed to use a parametric law of thrust. Then, the optimization model can be then obtained using very few unknowns. Finally, a differential evolution algorithm is adopted to solve the simplified optimization model and an analytical correction process is proposed to eliminate the numerical errors. Simulation results and comparisons with previous methods proved this new method’s efficiency and high precision for low-eccentricity orbits. The method can be well applied to pre-military analysis and high-precision trajectory optimization of missions such as in-orbit service and active debris removal in low Earth orbits.

Keywords: Low-thrust perturbed rendezvous; Trajectory optimization; Parametric optimization model; Numerical error correction

1. Introduction

Trajectory optimization of perturbed orbit rendezvous is a crucial technology for space missions in low Earth orbits (LEOs) (Bonnal et al., 2013; Shen & Tsiotras, 2005). Low-thrust electrical propulsion is usually preferred in such missions because of its high efficiency (Moghaddam & Chhabra, 2021; Ruggiero et al., 2015; Leomanni et al., 2020). As the thrust is insignificant compared with the Earth’s gravity, the transfer trajectory usually has many revolutions, which brings additional difficulty for trajectory optimization. In this condition, existing numerical methods (Jiang et al., 2012; Li et al., 2018; Zhao et al., 2017; Guo et al., 2018; Neves & Sanchez, 2020) are very sensitive to the initial values in the shooting process and thus easily converge to local solutions of different revolutions. Moreover, the numerical orbit propagation including the non-linear perturbations is also time-consuming when the transfer duration is very long. Averaging methods (Gao, 2007; Tarzi et al., 2013; Kelchner & Kluever, 2020; Pontani & Pustorino, 2021) can be applied to replace the time-consuming orbit calculation. However, special assumptions are required and it’s hard to find a general optimization method applicable for all type of orbits.

This paper mainly studies the fuel-optimal trajectory opti-
mization of time-fixed orbit rendezvous in LEOs with low eccentricity. The major effect of the perturbations on a spacecraft is from the \( J_2 \) term of the Earth’s non-sphere perturbation, which drifts the rising node right ascension (RAAN) and the argument of periapsis with constant velocities (P, 2004). Therefore, instead of correcting the derivations brought by \( J_2 \) perturbation, one can utilize the natural drift of orbit elements actively to save the propellant.

Several studies simplified the problem to quickly evaluate the approximate propellant consumption for mission analysis and the global optimization of multi-target rendezvous problems. Cerf (2014) analyzed the feasibility of actively changing the semimajor axis and inclination to use the natural drift of the right ascension of ascending node and reduce the propellant and studied its application in target selection for an active debris removal mission. Berend & Olive (2016) discretized the semimajor axis and inclination to search for an optimal RAAN drift rate, which can partly reduce the complexity of global optimization. Shen (2021) established an approximate model of low-thrust orbit rendezvous for circular orbits and derived the analytical expression of velocity increment. Huang et al. (2020, 2022a) proposed an equality constraint optimization model of impulsive rendezvous and designed an iterative method to expand the impulsive solution to an equivalent low-thrust solution with high precision for orbits with small eccentricity. However, these methods cannot obtain the law of thrust and transfer trajectory.

To obtain the approximate thrust law, Cerf (2016) proposed an optimization model that only considered semimajor axis, inclination, and RAAN based on the minimum principle. Wen et al. (2021) improved Cerf’s method by introducing the yaw switch strategy and reduced the propellant consumption in some cases. Such idea could be also found in (Barea et al., 2022; Casalino & Forestieri, 2022). These methods introduced an intermediate drift orbit and let the spacecraft transfer to the drift orbit using Edelbaum’s time-optimal strategy (Edelbaum, 2003). As an improvement, Huang et al. (2022b) designed a parametric thrust strategy that allows the thrust to periodically switch between on and off when transferring to the drift orbit. Then, an equality constraint optimization model can be obtained and quickly solved. Moreover, in (Huang et al., 2022b), an analytical correction process was introduced to obtain the high-precision trajectory that considered full perturbations. These methods can only adapt to circular orbits. However, most of the debris and satellites in LEO are in elliptical orbits of small eccentricities, which should be considered in trajectory optimization. Therefore, this study investigates the fast optimization model of low-thrust rendezvous between elliptical orbits.

We propose a novel simplified parametric thrust strategy to approximate the optimal control law for fuel-optimal low-thrust rendezvous between low-eccentricity orbits, significantly improving the efficiency of the trajectory optimization. The major contribution can be summarized as three points:

1. Based on the three-stages near-optimal strategy for rendezvous with circular orbits (Huang et al., 2022b), an approximate optimization model incuding the radial component of thrust and allowing the length of two thrust-on arcs of each revolution to be asymmetric is proposed in this study. Thus, the obtained trajectory could satisfy the constraints on the six-dimensional orbit elements.

2. A fast solving process to judge the feasibility of a low-thrust single-revolution transfer and obtain the thrust parameters is proposed to reduce the dimensionality of the optimization model and improve the efficiency.

3. A fast analytical correction process is proposed to obtain high-precision trajectory using the numerical errors between predicted orbit and target orbit. Simulation results proved that the proposed method could quickly obtain the optimal high-precision transfer trajectory. The calculation is much smaller than indirect methods that often obtain local optimal solutions and thus need to repeat the shooting process with different initial values of costate for selecting the best solution. Compared with existing approximate methods, the proposed parametric optimization method considers constraints on eccentricity and is more precise for elliptical orbits.

2. Problem description

This study focuses on the fuel-optimal trajectory of time-fixed orbital rendezvous. Assuming \( \sigma_0 = [a_0, e_0, i_0, \Omega_0, \omega_0, M_0] \) represents the initial orbit elements (\( a \): semimajor axis, \( e \): eccentricity, \( i \): inclination, \( \Omega \): RAAN, \( \omega \): argument of periapsis, \( M \): mean anomaly, and subscript ‘0’ means initial orbit) for the spacecraft at \( t_0 \) and \( \sigma_f = [a_f, e_f, i_f, \Omega_f, \omega_f, M_f] \) (subscript ‘f’ means target orbit) presents the target orbit elements at \( t_f \), the trajectory optimization is a typical optimal control problem that need to solve the optimal thrust acceleration \( a(t) \) that satisfy the terminal constraint and minimize the propellant cost. Let \( \sigma(t) = [a, e, i, \Omega, \omega, M] \) denote the orbit elements during the transfer, the dynamics equations are

\[
\begin{align*}
\frac{da}{dt} &= \frac{2}{\mu} \left[(a_i + a_f^2)e \sin f + (a_i + a_f^2)(1 + e \cos f)\right] \\
\frac{de}{dt} &= \frac{\mathcal{M}}{\mu a_n} \left[(a_i + a_f^2) \sin f + (a_i + a_f^2) \cos E \cos f\right] \\
\frac{di}{dt} &= \frac{\mathcal{M}}{\mu a_n} \left[\alpha (a_n + a_i^2)\right] \\
\frac{d\Omega}{dt} &= \frac{n \mathcal{M}}{\mu a_n} \left[\alpha (a_n + a_i^2)\right] \\
\frac{d\omega}{dt} &= \frac{n \mathcal{M}}{\mu a_n} \left[\alpha (a_n + a_i^2)\right] \\
\frac{dM}{dt} &= n - \frac{1}{\mu a_n} \\left[(a_i + a_f^2) \cos f + (a_i + a_f^2)(1 + \frac{f}{2}) \sin f\right] - \cos \left(\frac{\mathcal{M}}{\mu a_n}\right)
\end{align*}
\]

where \( f \) is the true anomaly, \( p = a(1 - e^2) \) is the semi-latus rectum, \( u \) is the argument of latitude, \( n = \sqrt{\frac{\mu}{a^3}} \) is the orbital angular velocity, and \( a_i^2 = [a_i^2, a_i^3, a_i^5] \) are the three acceleration components due to the \( J_2 \) perturbation in the local vertical/horizontal (LVLH) reference frame. \( \alpha_i = c(t) \alpha \cos \beta, \alpha_n = c(t) \alpha \sin \beta \cos \phi, \alpha_e = c(t) \alpha \sin \beta \sin \phi \) are the three components of \( a(t) \) in the LVLH reference frame, where \( \beta \) is the angle between \( a(t) \) and the tangential direction, \( \phi \) is the angle between the projection of \( a(t) \) in the normal-radial plane and the radial direction, \( \alpha = \frac{F_{\text{max}}}{m} \) is the max acceleration expressed by the maximum thrust \( F_{\text{max}} \) and mass \( m \), and \( c(t) \in [0, 1] \) is engine throttling function representing magnitude of the thrust acceleration.
The objective function can be written as
\[ J = m \int_{0}^{\alpha} c(t) dt \]  
where \( m = \frac{F_{p} \Delta v}{g} \) is the constant mass flow rate, \( I_{sp} \) is the constant specific impulse, and \( g \) is the standard gravitational acceleration at sea level (9.80665 m/s²).

In this study’s investigation, when the propellant cost is small compared to the mass (for example, when \( I_{sp} = 1000 \) s and \( \Delta v = 200 \) m/s, the fuel cost is approximately 2%), it can be supposed that the mass is constant during the orbit transfer. Then, we can let \( \alpha = \frac{F_{p} \Delta v}{m_{0}} \) denote the maximum acceleration (\( m_{0} \) is the initial mass). Since \( \alpha \) is much smaller than the Earth’s gravity, completing the orbit rendezvous requires a long time. The number of orbit revolutions would be significant, and the orbit propagation would be time-consuming. In the next section, a simplified thrust strategy will be proposed to express \( c(t), \beta(t), \) and \( \phi(t) \) by a few parameters to reduce the complexity.

3. Methodology

This section first designs a parametric thrust strategy for orbit rendezvous with elliptical orbit and then establishes the simplified optimization model. A sub boundary value problem that solves the optimal parameters corresponding to orbital element changes in a single revolution is embedded in the optimization model to resolve terminal constraints. Finally, a differential evolution algorithm is employed to solve the optimal trajectory.

3.1. Near-optimal thrust strategy

A near-optimal thrust strategy (Huang et al., 2022b) is proposed for transfer between circular orbits and has shown to be efficient. It divides the transfer duration into three stages: transfer to arrive at an intermediate orbit with certain RAAN drift, natural drift duration with no thrust, and transfer from intermediate orbit to the target orbit. Thus, most of the non-coplanar maneuvers required for RAAN control can be avoided with the help of natural RAAN drift caused by J2 perturbation. In the first and third stages, it’s assumed that the near-optimal Bang-Bang control law switches the thrust on and off periodically in each revolution. When the thrust is on, its direction is assumed to be fixed in the LVLH reference frame. Then, the low-thrust optimization problem is simplified using very few parameters (Huang et al., 2022b).

In this study, to satisfy the eccentricity constraints for orbit rendezvous, we expand the thrust strategy by including the radial thrust component to change the eccentricity jointly with the tangential component. As illustrated in Fig. 1, there are two thrust arcs in each revolution, and their middle points are symmetric. When the thrust is on, the acceleration keeps the maximum value. The lengths of the two arcs could not be equal to change eccentricity by the tangential component of thrust while keeping the semimajor axis unchanged. Let \( u \) and \( u + \pi \) denote the arguments of altitude, and \( \pi k_{1} \) and \( \pi k_{2} \) denote the lengths of the two arcs. The three thrust components are fixed (\( \beta \) and \( \phi \) are constant) in the LVLH reference frame, and the values of radial and normal components are opposite in the two arcs. The sign of the tangential components may be the same or opposite, which is expressed by the unknown coefficient \( \eta = -1 \) or 1. Then, the trajectory of one revolution can be determined using the six parameters \( \eta, k_{1}, k_{2}, u, \beta, \) and \( \phi \).

Then, based on the three-stage near-optimal thrust strategy (Huang et al., 2022b) and the improvement above for eccentricity control, the whole transfer trajectory can be expressed by 14 parameters: \( \Delta t_{1} \) and \( \Delta t_{2} \) denote the duration of the first and third stages; \( \eta_{1}, k_{11}, k_{12}, u_{1}, \beta_{1}, \) and \( \phi_{1} \) denote the parameters of thrust in the first stage (using additional subscript ‘1’); and \( \eta_{2}, k_{21}, k_{22}, u_{2}, \beta_{2}, \) and \( \phi_{2} \) denote the parameters of thrust in the third stage (using additional subscript ‘2’). The parametric orbit rendezvous problem is detailed in the following subsections.

3.2. Assumptions declaration

To obtain a simplified model, several assumptions should be declared.

(1) First, the initial and target orbits are near-circular orbits, and \( p \approx a \approx r \). In addition, the changes in the semimajor axis and inclination are small enough, thus the \( a, i, \) and \( n \) in the right function of Eq. (1) are approximately constant and equal to their initial values \( a_{0}, i_{0}, \) and \( n_{0} \).

Here, \( dt \) in Eq. (1) can be replaced by \( du = n dt \) because \( n \) is also near constant. \( e \) and \( \omega \) are replaced by \( e_{x} = e \cos \omega \) and \( e_{y} = e \sin \omega \) to avoid a singularity. Then, Eq. (1) can be divided into two independent parts: effects of thrust and \( J_{2} \) perturbation. The changes in orbit elements by the thrust (without
perturbation are:

\[
\begin{align*}
\frac{dc}{dt} &= \frac{2a_0\alpha \cos \beta}{n_0 V_0} \frac{\sin \phi \cos \phi}{\cos u} + \frac{\alpha \sin \phi \cos \phi}{\cos u} \\
\frac{dc}{dt} &= \frac{\alpha \sin \phi \cos \phi}{\cos u} + \frac{\alpha \sin \phi \cos \phi}{\cos u} + \frac{\alpha \sin \phi \cos \phi}{\cos u} + \frac{\alpha \sin \phi \cos \phi}{\cos u} \\
\frac{de}{dt} &= \frac{2a_0\alpha \cos \beta}{n_0 V_0} \frac{\sin \phi \cos \phi}{\sin u} - \frac{\alpha \sin \phi \cos \phi}{\sin u} - \frac{\alpha \sin \phi \cos \phi}{\sin u} - \frac{\alpha \sin \phi \cos \phi}{\sin u} \\
\frac{de}{dt} &= \frac{\alpha \sin \phi \cos \phi}{\sin u} - \frac{\alpha \sin \phi \cos \phi}{\sin u} - \frac{\alpha \sin \phi \cos \phi}{\sin u} - \frac{\alpha \sin \phi \cos \phi}{\sin u}
\end{align*}
\]

(3)

where \( V_0 = n_0 a_0 \) is the mean orbital velocity. The effect of \( J_2 \) perturbation is expressed by the analytical form in (Vallado, 2007) as follows:

\[
\begin{align*}
\frac{dc}{dt} &= \frac{3n_0 a_0^2 \cos u}{2\pi r_e^2} \\
\frac{dc}{dt} &= \frac{3n_0 a_0^2 \cos u}{2\pi r_e^2} \\
\frac{dc}{dt} &= -e \cos \omega \frac{d\omega}{dt} \\
\frac{dc}{dt} &= e \cos \omega \frac{d\omega}{dt} \\
\frac{dc}{dt} &= \frac{3n_0 a_0^2 (2-2\sin^2 u)}{2\pi r_e^2}
\end{align*}
\]

(4)

where \( r_e \) is the mean equator radius. Note that according to Eq. (2) and Eq. (3), when calculating the effects of thrust and perturbations, we assumed the orbit is circular. The bias of this assumption can be evaluated by the terms in Eq. (1) that include \( e \) and \( p \). Assume \( e = 0.1 \) (for most near-circular satellites in LEO), then, \( p = 0.99a \) and the gap can be ignored. Meanwhile, the maximum error of \( e \cos f \) and \( e \sin f \) is 10% and the accumulative error would be much less after an integral from 0 to \( 2\pi \). The impulsive trajectory optimization in (Huang et al., 2020) used the same assumption and the simulation indicated that although the eccentricity difference is a little greater than 0.1, the relative error is less than 5%. Moreover, we will also provide iteration process in the next section to correct the law of thrust and eliminate the deviations of terminal states. Therefore, the assumption would be reasonable and applicable for most cases in LEO.

(2) Second, as illustrated in Fig.1, \( c(u) \), \( \alpha_n \), \( \alpha_r \), and \( \alpha_e \) are defined as in Eqs. (5) and (6), when \( \eta, k_1, k_2, u, \beta, \) and \( \phi \) are given.

\[
c(u) = \begin{cases} 
1, & \text{if } u \in \left[-\frac{k_1\pi}{2} + u_1, \frac{k_1\pi}{2} + u_1\right] \\
1, & \text{if } u \in \left[-\frac{k_2\pi}{2} + u_1 + \pi, \frac{k_2\pi}{2} + u_1 + \pi\right] \\
0, & \text{else}
\end{cases}
\]

(5)

\[
\alpha_r = \begin{cases} 
\alpha \cos \beta, & \text{if } u \in \left[-\frac{k_1\pi}{2} + u_1, \frac{k_1\pi}{2} + u_1\right] \\
\eta \alpha \cos \beta, & \text{if } u \in \left[-\frac{k_2\pi}{2} + u_1 + \pi, \frac{k_2\pi}{2} + u_1 + \pi\right] \\
-\alpha \sin \beta, & \text{if } u \in \left[-\frac{k_1\pi}{2} + u_1 + \pi, \frac{k_1\pi}{2} + u_1 + \pi\right] \\
-\alpha \sin \beta, & \text{if } u \in \left[-\frac{k_2\pi}{2} + u_1 + \pi, \frac{k_2\pi}{2} + u_1 + \pi\right]
\end{cases}
\]

(6)

where \( k_1 \) and \( k_2 \) should be positive and \( k_1 \pi + k_2 \pi \leq 2\pi \). Then, according to Eq. (3), the changes in elements after one revolution is the definite integral from 0 to \( 2\pi \), which is an extension of Eq. (3) in (Huang et al., 2022b) by involving the orbital period is \( T \). Then, the average change rate of \( a \) and \( \dot{i} \) are \( \dot{a} = \frac{2a_0}{n_0} \) and \( \dot{i} = \frac{\alpha}{n_0} \). Although we assume \( u \) moves at a constant velocity \( n_0 \) when calculating other orbit elements by Eq. (7), the effect of \( \Delta a \) on \( u \) after a long duration must be considered for accurate orbit rendezvous, as illustrated in Fig. 2(\( \dot{a} \)) is the relative drift rate of \( u \) compared with the initial orbit). The change in \( u \) after a given duration \( \Delta t \) (including \( T \)) can be expressed by the definite integral as:

\[
\Delta u_d = \int_0^{\Delta t} \dot{u} dt = \int_0^{\Delta t} \int_0^T \dot{u} dt + (\Delta t - T) \int_0^T \frac{\dot{u}}{T} dt
\]

(8)

where we assume \( \dot{u} \approx \frac{d\Delta \alpha}{dt} \dot{u} \). \( \frac{d\Delta \alpha}{dt} = \frac{d}{dt} \Delta \alpha = -\frac{3a_0}{2\pi} \) represents the derivative of \( u \) with respect to semimajor axis. \( n_0 \) and \( n_r \) are the angular velocities of initial orbit and orbit after one-revolution

\[
\Delta u_d = \int_0^{\Delta t} \dot{u} dt + (\Delta t - T) \int_0^T \frac{\dot{u}}{T} dt
\]

Fig. 2. Drift of argument of latitude.
control. Similarly, the change in RAAN after $\Delta t$ is

$$
\Delta \Omega_d = \int_0^{\Delta t} \frac{d\Omega}{dt} \, dt = \int_0^{\Delta t} \left( \frac{d\Omega}{d\eta} \Delta \eta + \frac{d\Omega}{d\omega} \Delta \omega \right) \, dt + (\Delta \Omega - T)(\frac{d\Omega}{d\eta} \Delta \eta + \frac{d\Omega}{d\omega} \Delta \omega)
$$

$$
= \Delta \Omega \Delta t - \frac{\Delta \Omega}{2} T
$$

(9)

where we assume $\Delta \Omega \approx \frac{d\Omega}{d\eta} \Delta \eta + \frac{d\Omega}{d\omega} \Delta \omega$ and $\frac{d\Omega}{d\eta}$ and $\frac{d\Omega}{d\omega}$ are the derivative of $\Omega$ with respect to semimajor axis and inclination by Eq.(4). $\Delta \Omega$ is the relative drift rate of RAAN. $\Omega_0$ and $\Omega$ are the RAAN drift rates of initial orbit and the orbit after $T$, respectively. The change in $\omega$ after $\Delta t$ is

$$
\Delta \omega = \int_0^{\Delta t} \frac{d\omega}{dt} \, dt = \int_0^{\Delta t} \left( \frac{d\omega}{d\eta} \Delta \eta + \frac{d\omega}{d\omega} \Delta \omega \right) \, dt + (\Delta \omega - \omega)(\frac{d\omega}{d\eta} \Delta \eta + \frac{d\omega}{d\omega} \Delta \omega)
$$

$$
= \Delta \omega \Delta t - \frac{\Delta \omega}{2} T
$$

(10)

where we assume $\Delta \omega \approx \frac{d\omega}{d\eta} \Delta \eta + \frac{d\omega}{d\omega} \Delta \omega$ and $\frac{d\omega}{d\eta}$ and $\frac{d\omega}{d\omega}$ are the derivative of $\omega$ with respect to semimajor axis and inclination by Eq.(4). $\Delta \omega$ is the relative drift rate of argument of perigee. $\omega_0$ and $\omega$ are the drift rates of initial orbit and the orbit after $T$, respectively.

(4) Four, the propellant cost is sufficiently small compared to the spacecraft’s mass, thus the fuel-optimal objective function is equal to minimizing the actual time of thrust-on arcs. The length of thrust-on arcs in one revolution is calculated as

$$
\Delta t_{\text{thrust}} = \frac{(k_1 + k_2)}{2} T
$$

(11)

3.3. Optimization model

According to Eqs. (3) (11), the terminal constraints $\sigma(t_f) = \sigma_f$ and objective function can be analytically expressed by the 14-dimensional unknowns ($\Delta t_1, \Delta t_2, \eta_1, \eta_2, \beta_1, \phi_1, \Delta u_1, \Delta u_2, k_1, k_2, \beta_2, \phi_2,$ and $u_0$). First, the changes in orbit elements during the first stage $\Delta t_1$ (transfer from the initial orbit to the intermediate drift orbit) are small

$$
\Delta t_1 = \int_0^{\Delta t_1} \frac{d\tau}{dt} \, dt = \int_0^{\Delta t_1} \left( \frac{d\tau}{d\eta} \Delta \eta + \frac{d\tau}{d\omega} \Delta \omega \right) \, dt + (\Delta \tau - \tau)(\frac{d\tau}{d\eta} \Delta \eta + \frac{d\tau}{d\omega} \Delta \omega)
$$

$$
= \Delta \tau \Delta t_1 - \frac{\Delta \tau}{2} T
$$

(12)

Similarly, the changes in orbit elements during $\Delta t_2$ and $\Delta t_3$ can be calculated. To complete the orbit rendezvous, the constraints are:

$$
\Delta t_2 + \Delta t_3 = \Delta t_0
$$

$$
\Delta \tau_2 + \Delta \eta_1 = \Delta \tau_0
$$

$$
\Delta \Omega_2 + \Delta \eta_2 + \Delta \Omega_d = \Delta \Omega_0
$$

$$
\Delta e_3 + \Delta e_3' = \Delta e_0
$$

$$
\Delta \omega_2 + \Delta \omega_2' = \Delta \omega_0
$$

$$
\Delta \dot{u}_d = \Delta \dot{u}_0
$$

(13)

where $\Delta \tau_0, \Delta \tau_1, \Delta \Omega_0, \Delta \tau_0, \Delta e_0, \Delta e_0'$ represent the orbit differences between the initial and target orbit. $\Delta e_1$ and $\Delta e_1'$ are the corrected changes in eccentricity when considering the drifting of argument of perigee after $\Delta t$ (as illustrated in Fig. 3):

$$
\Delta e_1' = \left[ \frac{k_1}{V_1} \sin \phi_1 \sin \Delta \omega_0 \right] \Delta t_1
$$

$$
\Delta e_1' = \left[ \frac{k_2}{V_1} \sin \phi_1 \sin \Delta \omega_0 \right] \Delta t_1
$$

(14)

where $\Delta \omega_0$ means the rotate angle of $\Delta e_1$ and $\Delta e_1'$ from beginning to $\Delta t$ (as illustrated in Fig. 4) and can be calculated by substituting $\Delta t_1$ and $\Delta t_2$ to Eq. (10) to replace $T$ and summarizing the results:

$$
\Delta \omega_d = \Delta \omega_0 + \Delta \omega_d
$$

$$
= (\omega_d - \omega_0)(\Delta t_2 - \frac{\Delta t_1}{2}) + (\omega_f - \omega_0)(\Delta t_2 - \frac{\Delta t_1}{2})
$$

(15)

where $\omega_0, \omega_d$, and $\omega_f$ are the drift rates of $\omega$ of the initial orbit, intermediate orbit, and target orbits. $\Delta e_2$ and $\Delta e_2'$ do not need to be corrected because the third transfer stage is close to the
rendezvous time and the drift of \( \omega \) can be ignored.

In the same way as Fig. 3, \( \Delta \Omega_d \) in Eq. (13) represents the change in \( \Omega \) by the perturbation and can be calculated by Eq. (9):

\[
\Delta \Omega_d = (\Omega_d - \Omega_0)(\Delta t - \frac{\Delta t_1}{2}) + (\Omega_f - \Omega_d)(\Delta t_2 - \frac{\Delta t_2}{2})
\]

(16) where \( \Omega_0, \Omega_d, \) and \( \Omega_f \) are the drift rates of \( \Omega \) of the initial orbit, intermediate orbit, and target orbits.

In the same way, \( \Delta u_d \) in Eq. (13) represents the change in the argument of latitude caused by the semimajor axis and can be calculated by Eq. (8):

\[
\Delta u_d = (n_d - n_0)(\Delta t - \frac{\Delta t_1}{2}) + (n_f - n_d)(\Delta t_2 - \frac{\Delta t_2}{2})
\]

(17) where \( n_0, \) \( n_d, \) and \( n_f \) are the angular velocities of the initial orbit, intermediate orbit, and target orbits, respectively. Since \( n_f \) is only determined by \( \Delta t_1 \) \( (n_d), \) when \( \eta_1, u_1, k_{11}, k_{12}, \beta_1, \phi_1 \) are given, \( \Delta u_d \) can be directly calculated by Eq. (17) and may be not equal to \( \Delta u_0. \) Then, we can always add a correction term to \( k_{11} \) by Eq. (18) to ensure \( \Delta u_d = \Delta u_0:\)

\[
\Delta k_{11} = \frac{-2}{3} \frac{\Delta u_0 - \Delta u_d}{\Delta t_0} \frac{V_0}{a \cos \beta \Delta \Omega_1}
\]

(18) where \( \Delta k_{11} \) is the correction to \( k_{11}. \) Thus, the constraint of orbit rendezvous on \( u \) is automatically satisfied.

The objective function is written as:

\[
J = \frac{1}{2} \frac{k_{11} + k_{12}}{\Delta t_1} + \frac{k_{21} + k_{22}}{2} \frac{\Delta t_2}{\Delta t_2}
\]

(19) Above all, Eqs. (13) to (19) form the optimization model of 14 parameters and 5 constraints. According to the possible values of \( \eta_1 \) and \( \eta_2, \) there are 4 conditions \( (\eta_1 = 1, \eta_2 = 1; \eta_1 = 1, \eta_2 = -1; \) and \( \eta_1 = -1, \eta_2 = -1) \) that can be solved separately and the solution that minimizes \( J \) can be chosen as the optimal solution.

3.4. Dimensionality reduction via a sub boundary value problem

Note that \( \Delta \alpha_2, \Delta i_2, \Delta \Omega_2, \Delta e_{z2}, \Delta e_{y2} \) can be analytically obtained by Eq. (13) when \( \eta_1, u_1, k_{11}, k_{12}, \beta_1, \phi_1 \) are given. Meanwhile, \( \eta_2, u_2, k_{21}, k_{22}, \beta_2, \phi_2 \) and \( \Delta \alpha_2, \Delta i_2, \Delta \Omega_2, \Delta e_{z2}, \Delta e_{y2} \) correspond by Eq. (12). Thus, if we can obtain the inverse solution of Eq. (12), the constraint of Eq. (13) can be eliminated from the optimization model and only \( \Delta t_1, \Delta t_2, \eta_1, u_1, k_{11}, k_{12}, \beta_1, \) and \( \phi_1 \) will be retained as unknown parameters.

Solving \( \eta_2, u_2, k_{21}, k_{22}, \beta_2, \phi_2 \) by \( \Delta \alpha_2, \Delta i_2, \Delta \Omega_2, \Delta e_{z2}, \Delta e_{y2} \) is a boundary value problem like the Lambert’s problem. First, \( u_2 \) can be directly obtained by Eq. (12):

\[
u_2 = \arctan(\Delta \Omega_2 \sin i_0, \Delta i_2)
\]

(20) Then, defining \( C \) and \( D \) as temporary variables, we get:

\[
C = \frac{(k_{21}^2 - n_2 k_{21}^2 \sin \beta_2 \Delta \Omega_2)}{V_0} = \Delta e_{z2} \cos u_2 + \Delta e_{y2} \sin u_2
\]

\[
D = \frac{(k_{22}^2 + n_2 k_{22}^2 \sin \beta_2 \sin \phi_2 \Delta \Omega_2)}{2 V_0} = \Delta e_{z2} \sin u_2 - \Delta e_{y2} \cos u_2
\]

(21) Then, \( \phi_2 \) can be obtained:

\[
\phi_2 = \arctan(D, \sqrt{D^2 + C^2})
\]

(22) To solve \( \eta_2, \beta_2, k_{21}, k_{22}, \) two conditions are considered, and the best solution will be reserved as the optimal solution:

(1) \( \eta_2 = -1 \)

In this condition, \( \beta_2 \) can be directly obtained by Eq. (21):

\[
\beta_2 = \arctan\left(\frac{2D}{\sin \phi_2}, C\right)
\]

(23) Thus, the equations of \( k_{21}, k_{22} \) in Eq.(21) are

\[
k_{21} = \frac{k_{21}^2}{\Delta \alpha_2 \cos \beta_2} = \frac{C V_0}{\Delta \alpha_2 \cos \beta_2} = E
\]

\[
k_{22} = \frac{k_{22}^2}{\Delta \alpha_2 \sin \phi_2} = \frac{D V_0}{\Delta \alpha_2 \sin \phi_2} = F
\]

(24) where \( E \) and \( F \) are temporary variables and Eq. (24) can be rewritten as

\[
\sin \frac{C}{2} k_{21} \sin \frac{C}{2} (k_{21} - F) = \frac{E}{2 F}
\]

\[
(1 + \cos \frac{C}{2} F) \sin \frac{C}{2} k_{21} - \sin \frac{C}{2} F \cos \frac{C}{2} k_{21} = \frac{E}{2 F}
\]

\[
\sin\left(\frac{C}{2} k_{21} - G\right) = \frac{E}{\sqrt{2 + \cos \frac{C}{2} F}}
\]

(25) where \( G = \arctan(\sin \frac{C}{2} F, 1 + \cos \frac{C}{2} F) \) is a temporary variable.

Hence, when \( \left|\frac{C}{2}\right| F \geq 1 \), there is no solution of \( k_{21} \) and \( k_{22} \). Otherwise, there are two solutions:

\[
\frac{C}{2} k_{21} = \arcsin\left(\frac{E}{\sqrt{2 + \cos \frac{C}{2} F}}\right) + G
\]

\[
\frac{C}{2} k_{21} = \pi - \arcsin\left(\frac{E}{\sqrt{2 + \cos \frac{C}{2} F}}\right) + G
\]

(26) One can calculate the objective function of two solutions and validate the constraint \( k_{12} + k_{22} \leq 2 \), and then choose the feasible solution that minimizes \( J \).

(2) \( \eta_2 = 1 \)

In this condition, \( \beta_2, k_{21} \) and \( k_{22} \) should be solved jointly by Eq.(12) and Eq.(21):

\[
k_{21} = \arctan\left(\frac{2D}{\Delta \alpha_2 \cos \beta_2}, C\right)
\]

\[
k_{22} = \frac{k_{22}^2}{\Delta \alpha_2 \sin \phi_2} = \frac{C V_0}{\Delta \alpha_2 \sin \phi_2} = F
\]

(27) The nonlinear solving package Minpack (Moré et al., 1980) is adopted to solve the equations, and the approximate initial values can be set by four cases:

When \( k_{21} \leq 1 \) and \( k_{22} \leq 1 \), one can assume \( \sin \frac{C}{2} k_{21} \approx \frac{C}{2 k_{21}}, \sin \frac{C}{2} k_{22} \approx \frac{C}{2 k_{22}} \). Then, Eq.(27) can be simplified and solved as:

\[
k_{21} = \frac{\Delta \alpha_2 \cos \beta_2}{2V_0 \Delta \alpha_2 \cos \beta_2} = \frac{C V_0}{2V_0 \Delta \alpha_2 \cos \beta_2} = E
\]

\[
k_{22} = \frac{k_{22}^2}{\Delta \alpha_2 \sin \phi_2} = \frac{C V_0}{2V_0 \Delta \alpha_2 \sin \phi_2} = F
\]

(28)
When \( k_{21} > 1 \) and \( k_{22} \leq 1 \), one can assume \( \sin \left( \frac{\pi k_{21}}{2} \right) = \sin \left( \frac{\pi k_{22}}{2} \right) \). Then, Eq. (27) can be simplified and solved as:

\[
\begin{align*}
(2 - k_{21} - k_{12}) & \cos \beta_2 = \frac{CV_0}{\Delta v(0 backbone)} \Rightarrow \\
(2 - k_{21} + k_{12}) & \sin \beta_2 = \frac{\Delta v_{(0,0,0,0,0,0)} \sin \phi_2}{2} \\
\beta_2 & = \arccos \left( \frac{CV_0}{\Delta v(0 backbone)} \right) \\
k_{21} & = \frac{ CV_0 }{ \Delta v_{(0,0,0,0,0,0)} \sin \phi_2 - 2 + \frac{ CV_0 \Delta v_{(0,0,0,0,0,0)} }{ \Delta v_{(0,0,0,0,0,0)} \sin \phi_2 } } / 2
\end{align*}
\]

\( \text{(29)} \)

When \( k_{21} \leq 1 \) and \( k_{22} > 1 \), one can assume \( \sin \left( \frac{\pi k_{21}}{2} \right) \approx \sin \left( \frac{\pi k_{22}}{2} \right) \approx \sin \left( \frac{\pi k_{21} - k_{22}}{2} \right) \). Then, Eq. (27) can be simplified and solved as:

\[
\begin{align*}
(2 - k_{21} - 2 + k_{12}) & \cos \beta_2 = \frac{CV_0}{\Delta v(0 backbone)} \Rightarrow \\
(2 - k_{21} + 2 - k_{12}) & \sin \beta_2 = \frac{\Delta v_{(0,0,0,0,0,0)} \sin \phi_2}{2} \\
\beta_2 & = \arccos \left( \frac{CV_0}{\Delta v(0 backbone)} \right) \\
k_{21} & = \frac{ CV_0 }{ \Delta v_{(0,0,0,0,0,0)} \sin \phi_2 - 2 + \frac{ CV_0 \Delta v_{(0,0,0,0,0,0)} }{ \Delta v_{(0,0,0,0,0,0)} \sin \phi_2 } } / 2
\end{align*}
\]

\( \text{(30)} \)

When \( k_{21} \geq 1 \) and \( k_{22} \geq 1 \), \( k_{21} + k_{22} > 2 \) cannot be satisfied and there is no solution for this case.

Eq. (28), Eq. (29), and Eq. (30) can be used as initial values to solve \( \beta_2 \), \( k_{21} \) and \( k_{22} \) separately. Further, the constraints on \( k_{21} \) and \( k_{22} \) should be validated, and the feasible solution that minimizes \( J \) will be retained.

Above all, the solving process from \( \Delta \alpha_2, \Delta \Omega_2, \Delta \varpi_2, \Delta \Omega_2 \), \( \Delta \varpi_2 \) to \( \eta_2, u_2, k_{21}, k_{22}, \beta_2, \phi_2 \) is obtained. If there is no feasible solution obtained after this process, the given \( \Delta \alpha_2, \Delta \Omega_2, \Delta \varpi_2 \), \( \Delta \varpi_2 \) are out of the reachable domain of low-thrust, and a punish term should be added to the objective function (Eq. (18)).

3.5. Optimization and parameters correction

After applying the algorithm in Section III.D, there are 8 unknowns. Differential evolution (DE) algorithm (Price et al., 2005) can be used to solve this optimization problem. DE is an efficient evolutionary algorithm for the optimization of continuous variables. Details of DE are beyond the scope of the study and one can refer to (Price et al., 2005). The process is illustrated in Fig. 5.

It should be noted that due to the approximations in the optimization model if we substitute the optimal law of thrust in the numerical dynamic model (Eq. (1), Eq. (5) and Eq. (6)), there may be errors between given target orbit and the predicted orbit after control. Let \( \Delta \alpha_1, \Delta \varpi_1, \Delta \varpi_2, \Delta \varpi_2, \Delta \Omega_1, \Delta \Omega_2, \Delta \varpi_2 \) denote the numerical errors of semimajor axis, inclination, RAAN and argument of latitude, an correction to the solution obtained by DE is proposed as follows. We first calculate the analytical corrections to the changes in orbit elements \( \Delta \alpha_1, \Delta \varpi_1, \Delta \varpi_2, \Delta \Omega_1, \Delta \Omega_2, \Delta \varpi_2 \) and then update the parameters of thrust \( \eta_1, u_1, k_{11}, k_{12}, \beta_1, \phi_1 \) and \( \eta_2, u_2, k_{21}, k_{22}, \beta_2, \phi_2 \) by Eqs. (19) to (30), respectively.

First, let \( \Delta \alpha_1^c \) and \( \Delta \alpha_2^c \) denote the corrections to \( \Delta \alpha_1 \) and \( \Delta \alpha_2 \). According to Eq. (13) and (17), we can obtain:

\[
\begin{align*}
\Delta \alpha_1^c + \Delta \alpha_2^c & = \Delta \alpha_1^p \\
- \frac{3 \mu}{a_0} \Delta \alpha_1^p (\Delta t - \frac{\mu t}{2}) - \frac{3 \mu}{a_0} \Delta \alpha_2^p (\Delta t - \frac{\mu t}{2}) & = \Delta \mu_p
\end{align*}
\]

(31)

where \( - \frac{3 \mu}{a_0} \Delta \alpha_1^c \) and \( - \frac{3 \mu}{a_0} \Delta \alpha_2^c \) are changes in angular velocity by \( \Delta \alpha_1 \) and \( \Delta \alpha_2 \). Then, \( \Delta \alpha_1^c \) and \( \Delta \alpha_2^c \) can be solved by Eq.(31):

\[
\begin{align*}
\Delta \alpha_1^c & = \frac{\mu \Delta \alpha_1^p + \frac{3 \mu}{a_0} \Delta \alpha_2^p (\Delta t - \frac{\mu t}{2})}{\Delta t} \\
\Delta \alpha_2^c & = \Delta \alpha_1^p - \Delta \alpha_2^c
\end{align*}
\]

(32)

Second, let \( \Delta \alpha_1^c \) and \( \Delta \alpha_2^c \) denote the correction to \( \Delta \alpha_1 \) and \( \Delta \alpha_2 \). \( u_1 \) and \( u_2 \) are assumed unchanged after the correction, which is more convenient to derive analytical equations of \( \Delta \alpha_1^c \) and \( \Delta \alpha_2^c \) (Huang et al., 2022b). According to Eq. (13) and (17), we can obtain:

\[
\begin{align*}
\Delta \alpha_1^c + \Delta \alpha_2^c & = \Delta \alpha_1^p \\
\Delta \Omega_1^c + \Delta \Omega_2^c + \Omega_0(-3.5 \Delta \alpha_1^c - \tan \Omega_0 \Delta \varpi_1^c) (\Delta t - \frac{\mu t}{2}) & = \Delta \Omega_0
\end{align*}
\]

(33)

Thus, \( \Delta \alpha_1^c \) and \( \Delta \alpha_2^c \) are also obtained. Third, divide \( \Delta \alpha_{e1} \) and \( \Delta \alpha_{e2} \) into two parts as the corrections to \( \Delta \alpha_{e1} \) and \( \Delta \alpha_{e2} \) and \( \Delta \alpha_{e2} \) according to the ratio of thrust time in the first and third stages:

\[
\begin{align*}
\Delta \alpha_{e1}^c & = \chi \Delta \alpha_{e1} \\
\Delta \alpha_{e2}^c & = \chi \Delta \alpha_{e2} \\
\Delta \alpha_{e1}^c & = (1 - \chi) \Delta \alpha_{e1} \\
\Delta \alpha_{e2}^c & = (1 - \chi) \Delta \alpha_{e2}
\end{align*}
\]

(36)

where the superscript ‘c’ represents corretion term and \( \chi = \frac{k_{11} + k_{12}}{k_{11} + k_{12} + k_{21} + k_{22}} \) represents the ratio of thrust time in the first transfer stage and total thrust time. Eq. (36) means the corrections of numerical eccentricity error that approximately allocated to the first and third transfer stage are proportionate to their thrust time. Note that the correction to \( \Delta \alpha_{e1} \) and \( \Delta \alpha_{e1} \) should consider the natural drift of argument of perige during the transfer. Therefore, according to Fig. 3, \( \Delta \alpha_{e1} \) and \( \Delta \alpha_{e1} \) should be recalculated to correct the drift angle \( \Delta \alpha_{e2} \) by Eq.
4. Simulation Result

In this study, a transfer between two space debris is analyzed. The initial and target orbits of the spacecraft are listed in Table 1. The transfer duration is 20 days and the constant acceleration is $6 \times 10^{-4} m/s^2$. The population of DE is set to 50, and the max generation is 800. Meanwhile, the crossover operator is 'Ran2Bestexp' (Price et al., 2005) with a probability of 0.8 and the mutation probability is set to 0.8.

(15):

$$\begin{align*}
  u_1^e &= \arctan(\Delta e_{11}^e, \Delta e_{11}^e) \\
  \Delta e_{11}^e &= \sqrt{(\Delta e_{11}^e)^2 + (\Delta e_{11}^e)^2} \\
  \Delta e_{11}^e &= \Delta e_{11}^e \cos(u_1^e - \Delta \omega_d) \\
  \Delta e_{11}^e &= \Delta e_{11}^e \cos(u_1^e - \Delta \omega_d)
\end{align*}$$

Fig. 6. Flowchart of the correction process.

where $u_1^e$ is the argument of latitude of the eccentricity correction and $\Delta e_{11}^e$ is the magnitude.

Above all, $\Delta a_1, \Delta \Omega_1, \Delta e_{11}, \Delta e_{11}$ and $\Delta a_2, \Delta \Omega_2, \Delta e_{12}, \Delta e_{12}$ have been correct and the parameters $\eta_1, u_1, \beta_1, k_{11}, k_{12}$ and $\eta_2, u_2, \beta_2, k_{21}, k_{22}$ can be updated by solving Eqs. (20) (30), respectively. The correction process can be repeated multiple times to obtain high-precision trajectory. The flow chart is illustrated in Fig.6.
The errors after each correction step are also detailed in Table 2. After five steps, the errors are close to zero and can be ignored (the position error is less than 0.03 m and the velocity error is less than 0.03 m/s). Finally, the optimal 14 parameters are \{7.8908, 5.2863, -1, -2.7473, 1.3809, 0.0906, 0.3748, 0.4650, 1, -2.4307, 2.4632, 0.1846, 0.0636, 0.1239\}. The optimal thrust is illustrated in Fig. 8. Differing from the symmetrical thrust strategy in (Huang et al., 2022b), the lengths of two thrust-on arcs in each revolution are not the same to make use of the tangential acceleration to change eccentricity. The history of the orbit elements is illustrated in Fig. 9, which indicates that the optimal law of thrust firstly should partly decrease the semimajor axis and the equivalent velocity increment is 196.35 m/s. The calculation takes less than 2 s on a personal computer (CPU: AMD Ryzen7 4.2 GHz).

The process of the sub boundary value problem in Section III.D corresponding to the optimal solution is illustrated in Fig. 7. In summary, the algorithm can correctly check the low-thrust reachability and quickly obtain the equivalent velocity increment is 196.35 m/s. The calculation takes less than 2 s on a personal computer (CPU: AMD Ryzen7 4.2 GHz).

### 4.1. Optimal solution

The optimal \(\Delta t_1, \Delta t_2, \eta_1, u_1, \beta_1, a_1, k_{11}, k_{12}, \eta_2, u_2, \beta_2, \phi_2, k_{21}, \) and \(k_{22}\) obtained by DE are \{7.8908, 5.2863, -1, -2.7460, 1.3779, 0.0141, 0.3691, 0.4660, 1, -2.4267, 2.4632, 0.1654, 0.0567, 0.1297\}. The total length of thrust-on arcs is 3.78 days, and the equivalent velocity increment is 196.35 m/s. The calculation takes less than 2 s on a personal computer (CPU: AMD Ryzen7 4.2 GHz).

The errors after each correction step are also detailed in Table 2. After five steps, the errors are close to zero and can be ignored (the position error is less than 10 m and the velocity error is less than 0.03 m/s). Finally, the optimal 14 parameters are \{7.8908, 5.2863, -1, -2.7473, 1.3809, 0.0906, 0.3748, 0.4650, 1, -2.4307, 2.4632, 0.1846, 0.0636, 0.1239\}. The optimal thrust is illustrated in Fig. 8. Differing from the symmetrical thrust strategy in (Huang et al., 2022b), the lengths of two thrust-on arcs in each revolution are not the same to make use of the tangential acceleration to change eccentricity. The history of the orbit elements is illustrated in Fig. 9, which indicates that the optimal law of thrust firstly should partly decrease the semimajor axis and the equivalent velocity increment is 196.35 m/s. The calculation takes less than 2 s on a personal computer (CPU: AMD Ryzen7 4.2 GHz).

### 4.2. Analysis of different thrust accelerations

To validate the applicability of the proposed method for different thrust levels, four cases of thrust acceleration (1 ×...
Fig. 8. Fuel-optimal law of thrust.

Fig. 9. History of orbit elements during the transfer.
10^{-3} m/s^2, 8 \times 10^{-4} m/s^2, 6 \times 10^{-4} m/s^2, 4 \times 10^{-4} m/s^2) are tested and the law of thrust are illustrated in Fig.10. When the thrust acceleration is larger, the optimal solution prefers longer natural drift duration to save fuel because the spacecraft can be transferred to the drift orbit faster. When the thrust acceleration is smaller, it’s more difficult to transfer to the drift orbit and thus larger velocity increment is required for direct non-coplanar control. However, it can be seen that coast arcs in the transfers arriving and departing the drift orbit are usually necessary to save propellant. It’s also obtained that the orbit transfer problem in Table 1 will be infeasible when the acceleration is smaller than 2.7 \times 10^{-4} m/s^2.

The equivalent velocity increments of different transfer durations and different thrust accelerations are illustrated in Fig.11, which indicates the natural drift of RAAN and argument of perigee can greatly decrease the propellant when the transfer duration is long enough. The method in this paper can well adept with trajectory optimization of different conditions. Optimization by DE could ensure the thrust parameters achieve the balance between direct control of orbit elements and indirect control via natural drift.

### 4.3. Comparison with previous methods and discussion

In this study, the thrust strategy is just approximately optimal because the direction of thrust acceleration is fixed and the thrust keeps the same in different revolutions. However, the solutions obtained from repeated calculations are consistent and are proved to be always very close to the best results of the indirect methods with numerical dynamics (Jiang et al., 2012; Li et al., 2018; Zhao et al., 2017) after a lot of simulations with different orbits and transfer durations. By contrast, the methods in (Jiang et al., 2012; Li et al., 2018; Zhao et al., 2017) may easily converge to locally optimal solutions when solving such long-duration perturbed orbit rendezvous problems because the number of revolutions is large and the effect of perturbations cannot be globally considered. Fig. 12 shows examples of several local optimal solutions (Δτ = 10 days) obtained by an indirect method corresponding to very large velocity increments. The approximate methods in (Huang et al., 2022a; Cerf, 2016; Wen et al., 2021; Huang et al., 2022b) are also tested using the same orbits in Table 1 with different transfer durations and different thrust accelerations. Taking the best results of the indirect method as a benchmark, the comparisons of precision and efficiency between different methods (using the same computer) are detailed in Table 3, which demonstrates the performance of
the proposed method. For the case in Table 1, the relative errors of methods in (Cerf, 2016; Wen et al., 2021; Huang et al., 2022b) are greater than 30% because the eccentricity difference is about 0.01, which requires additional velocity increment for orbit rendezvous. The calculation of the proposed method is much less than existing indirect methods requiring hundreds of shooting processes and the thrust law is also much simpler for engineering practice. Besides, the results are also closer to the global optimal solution than existing approximate methods.

5. Conclusion

A priori fuel-optimal thrust strategy is proposed to simplify the trajectory optimization of orbit rendezvous with low eccentricities into a parametric optimization problem, which significantly reduces the solving complexity. By parameterizing the switch strategy and direction of the thrust, the analytical expression of the rendezvous constraint and objective function are obtained and thus a sub boundary value problem is introduced to further reduce the number of unknowns. Finally, a differential evolution algorithm is adopted to solve the simplified optimization model and an analytical correction process is proposed to eliminate the numerical errors. Simulation results and comparisons with previous methods proved this new method’s efficiency and high precision for low-eccentricity orbits. The method can be well applied to premission analysis and high-precision trajectory optimization of missions such as in-orbit service and active debris removal in low Earth orbits.

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