3-D Regularized focusing migration method for gravity data

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Abstract. Potential field migration of gravity data is a kind of method using the migration concept to efficiently obtain the density distributions subsurface. To get migration imaging results with the high resolution, we propose the regularized focusing migration method that adopts the regularization theory to realize focusing migration imaging and the conjugate migration direction method to solve the optimal solution of model parameters, which effectively improves the divergence of the conventional migration imaging. Synthetic model tests demonstrate that the proposed advanced method improves the imaging resolution significantly while guaranteeing the computational efficiency, and can bring about the relatively convergent results in a random Gaussian noise environment. This method is applied to the interpretation of the gravity anomaly from Huayangchuan, Shaanxi Province, which predicts the spatial distribution of uranium deposits.

1. Introduction
Gravity survey can effectively obtain the comprehensive response of the uneven density distributions, which is widely used in mining target prediction and mineral resources exploration. Tikhonov and Arsenin established the regularization theory[1], and on this basis a variety of 3D regularized inversion methods are developed for gravity data, such as regularized smooth inversion[2] and regularized focusing inversion[3], which have different application effects and scopes.

Considering the calculation amount and time, the inversion method will face the limitation of low computing efficiency in the actual data processing and cannot complete the overall 3-D density imaging over large areas. Improving computing efficiency is an urgent problem for geophysicists. Zhdanov proposed 2-D and 3-D potential field migration methods based on the migration concept[13], which greatly improves the computational efficiency. The migration imaging manifests itself as an integral transformation of gravity and its gradient data applying adjoint operator so as to directly obtain the image of density distributions subsurface. The potential field migration method is fast and stable, and can be used for real-time imaging. The migration imaging method is also extended to iterative migration[4] resulting in an improvement in imaging accuracy to some extent when the adjoint operator is iteratively used during migration. Then joint iterative migration of surface and borehole gravity gradiometry data[5]is employed to improve the vertical resolution. It is not difficult to see that the idea of iterative migration algorithm is essentially similar to inversion.

In this paper, we propose the regularized focusing migration method based on the conjugate migration direction, which introduces a parameter functional similar to that of regularized focusing inversion to minimize along the conjugate migration direction to solve the model parameters. The proposed imaging method can provide real-time high-resolution subsurface density distributions with sharp boundaries. We use a set of synthetic models to gauge the rationality and performance of the method. Then the method is applied to the interpretation of gravity data collected in Huayangchuan uranium deposit in Shaanxi, China, which helps to identify the potential mining targets associated with betafite and galenite.
2. Methodology

2.1. Forward modeling
The purpose of gravity forward modeling is to calculate the ground response of the subsurface density distributions. It can be regarded as a linear problem whose function satisfies the following equation:

\[ d = A \mathbf{m} \]  

(1)

where \( d \) is a vector of the observed data of the order \( n \); \( \mathbf{m} \) is a vector of the model parameters of the order \( m \), \( A \) is called the sensitivity matrix that is a matrix of a size \( n \times m \), formed by the corresponding gravity field kernels.

According to [6], the gravity anomaly generated by any geometric cuboid at the observation point can be expressed as:

\[ d = \gamma \rho_M \left\{ (\varepsilon - x) \ln |R - (\eta - y)| + (\eta - y) \ln |R - (\varepsilon - x)| \right\} \left[ \frac{\tan^{-1} \frac{(\varepsilon - x)(\eta - y)}{(\zeta - z) R}}{\varepsilon - \eta} \right] \]

\[ = \frac{\varepsilon_2 - \varepsilon_1}{\eta_2 - \eta_1} \left[ \frac{(\varepsilon - x)(\eta - y)}{(\zeta - z) R} \right] \]

(2)

Here, \( R = \sqrt{(\varepsilon - x)^2 + (\eta - y)^2 + (\zeta - z)^2} \), \( \gamma \) is the gravitational constant and \( \rho_M \) is the density of the geometric cuboid completely described by two triples \((\varepsilon_1, \eta_1, \zeta_1)\) and \((\varepsilon_2, \eta_2, \zeta_2)\).

2.2. Migration concept of gravity field
Zhdanov proposed the migration method whose imaging principle is described as follows [7].

\[ P(m) = \| W_d (A \mathbf{m} - d) \|^2 + \lambda \| W_m (\mathbf{m} - \mathbf{m}_{app}) \|^2 \rightarrow \min \]  

(3)

In the framework of the regularization theory, the solution of the inverse problem is simplified to solve the minimum of the Tikhonov parameter functional, one form of which is:

\[ \mathbf{m} = \frac{1}{\lambda} (W_{m}^* W_m)^{-1} A^* d^{\text{obs}} \]  

(4)

where \( A^* \) is the adjoint operator, which can be expressed as:

\[ A^* = \gamma \iiint \frac{d}{|r' - r|^2} (z' - z) \mathrm{d}s \]  

(5)

If \( \frac{1}{\lambda} = k_{\alpha} \), the migration imaging results can be expressed by the following equation:

\[ \mathbf{m} = k_{\alpha} (W_{m}^* W_m)^{-1} A^* d^{\text{obs}} = k_{\alpha} \omega_\alpha^2 A^* d^{\text{obs}} \]  

(6)

where

\[ \omega_\alpha = \sqrt{S_\alpha} \]  

(7)

\[ k_{\alpha} = \frac{\|A_{\alpha} d^{\text{obs}}\|^2}{\|A_{\alpha}^2 d^{\text{obs}}\|^2} \]  

(8)

2.3. The regularized focusing migration method
We can consider \( k_{\alpha} \) in equation (6) as the step size and \( \omega_\alpha^2 A_{\alpha}^* \) as the migration direction, in such a manner the migration is essential to an iteration of a general regularized inversion problem. It is obvious that a single migration cannot obtain relatively accurate imaging results, although it has great advantages in terms of efficiency. For this reason, the regularized focusing migration method is introduced in this paper.

The parameter function is weighted by focusing the weighting matrix \( W_{e} \), data weighting matrix \( W_{d} \), and model parameters weighting matrix \( W_{m} \). The weighted parameter functional can be rewritten as:
\[ P_\alpha (m^\omega, \ d^\omega) = \| (W_d A W_m^{-1} W_e^{-1})(W_e W_m) - (W_d d) \|^2 + \lambda \| W_e W_m m \|_{2}^2 \tag{9} \]

where

\[ W_e = \text{diag} \left( \frac{1}{\sqrt{(m)^2 + e^2}} \right) \tag{10} \]
\[ W_d = \text{diag}(AA^T)^{\frac{1}{2}} \tag{11} \]
\[ W_m = \text{diag}(A^T A)^{\frac{1}{2}} \tag{12} \]

When solving the model parameters, different from the conjugate gradient method used in the general inversion problem, we provide the conjugate migration direction method, that is, the search direction of each iteration is along the conjugate migration direction. Therefore, according to equation(6) we define the migration direction for the \( n \)th iteration (\( n = 0,1,\ldots \)) as

\[ I_m^\alpha_n = A\alpha_n^m \cdot A\alpha_n^m \]

where

\[ \beta_n = \frac{\| I_m^\alpha_n \|}{\| I_m^{\alpha_{n-1}} \|} \tag{14} \]

The successive line search in the conjugate gradient direction can be written as:

\[ m_{n+1}^{\omega_n} = m_{n+1}^{\omega_n} - k_n I_m^\alpha_n \tag{15} \]

where \( k_n \) is the iterative step size obtained by the linear line search criteria.

The actual model parameters can be converted as:

\[ m_{n+1}^{\omega_n} = W_m^{-1} W_e^{-1} m_{n+1}^{\omega_n} \tag{16} \]

Then \( m_{n+1}^{\omega_n} \) is reweighted to obtain \( m_{n+1}^{\omega_{n+1}} \). After reweighting, we estimate the regularization parameter by\[8\] to ensure that the misfit functional is focused to the smallest in the whole region. Iteration process terminates when the misfit reaches a given range and then output \( m_{N} \).

3. Synthetic models
The To test the application effect of the proposed method, we consider a model formed by two cubes underground with side length of 100m and residual density of 1 g/cm\(^3\) located at the same depth of 100m below the ground, and the spacing is 100 m. The subsurface region is divided into 40×40×20 regular cuboids for imaging, and the point spacing is 25m. The gravity anomalies of the model and 3-D perspective are shown in Figure 1 (a) and (b).
Figure 1 (c) and (d) show the imaging results from the regularized focusing migration. It can be seen that the density extremum corresponds well to the center of the geological bodies even when the underlying geological bodies are close to each other, and the imaging results have a good resolution, which can define a clear and accurate boundary. Figure 1 (e) and (f) show the imaging results from the conventional migration. This method cannot distinguish two geological bodies with poor convergence, and there is excess density distribution around the bottom of and between the geological bodies. The regularized focusing migration method recovers the real model in terms of position and density with high resolution.

Next, we assume a model formed by two cubes with different buried depths. The buried depths of the top of the two cubes are 70m and 100m respectively. Figure 2 (a) and (b) respectively show gravity anomalies of the model and 3-D perspective.
Figure 2. (a) Gravity anomalies on the ground generated from the model; (b) The 3-D diagram of the subsurface model; (c) The vertical cross section of $y=500m$ for the gravity data without noise; (d) The horizontal cross section of $z=150m$ for the gravity data without noise; (e) The volume image with a density cut-off greater than 0.3 g/cm$^3$ of the gravity data without noise; (f) The volume image with a density cut-off greater than 0.3 g/cm$^3$ of the gravity data with 5% random Gaussian noise added.

Figure 2 (c) and (d) show the imaging results obtained from the regularized focusing migration. Although the density is different from the real value especially for the deeper geological body, the extremum of the density distribution corresponds well to the center of the geological bodies, and the approximate location can be delineated.

The observed data generated for this model are contaminated by 5% random Gaussian noise to simulate the actual collected data. The blocks with a density greater than 0.3 of the results from noise data are 3D mapped and compared with that from noiseless data, as shown in figure 2 (e) and (f). The black lines represent the actual model position. One can see that the direct processing of the noise-containing data hardly changes the imaging results compared with that of the noiseless data, indicating a good noise resistance.

4. Case study for the U-polymetallic deposit in Huayangchuan, Shaanxi
The Huayangchuan U-polymetallic deposit is the first ultra-large carbonate uranium metallic deposit discovered in China, which fills the blank of this type of uranium deposit in China and is of great significance to the research. We use the proposed method to model the density distribution of the survey area in Huayangchuan and determine the mining target area based on the comprehensive analysis of previous geological research results.
The Huayangchuan U-polymetallic deposit is located in Luofu town, Huayin city, Shaanxi province, and its tectonic position is located in the Xiaoqinling mountain range, controlled by Xiaofuyu-Huayangchuan-Huanglongpu NW-trending deep fault zone[9]. The main geological bodies of this deposit are Taihua Group metamorphic rocks, quartz syenite, granitic pegmatite and carbonate. Regional fault structures are developed, mainly trending NW and NE. The main mineral types of Huayangchuan deposits are betafite and galenite and the genetic type is coalmagma-hydrothermal complex genesis type (Figure 3). The ore deposit generally shows a large network vein in space, and the mineralized vein rocks are mainly quartz-bearing carbonic acid vein and pegmatite vein, controlled by faults and not restricted by surrounding rocks. All kinds of ore-bearing vein rocks are interspersed into the cracks of Archean gneiss in a network[10]. Niobium and uranium are closely associated with betafite. The spatial distribution of lead ore bodies and betafite ore bodies basically coincides, and most of the lead ore bodies are distributed in or on both sides of uranium ore bodies. The occurrence of uranium ore bodies are basically the same as that of NW-trending dense vein rock zones, with inclination of NW. Uranium ore bodies are irregular lenticular, plate-like and lentil-shaped, and are concentrated above the elevation of 1300 m with a thickness range of 2 ~ 94 m[11].

![Figure 3. Sketch map showing the tectonic position of the study area. The Figure is referenced from[12].](image)

In view of the great difference in density between betafite along with galenite and gneiss host rocks, gravity data is of great importance in the exploration of Huayangchuan uranium-niobium-lead deposit. A 1:50,000 airborne gravity survey was performed in this area. The Bouguer gravity anomaly is separated by matched filtering to obtain the local gravity anomaly resulting from the shallow part, as shown in Figure 4. The high value in the local gravity anomaly is caused by uranium-niobium-lead ore bodies, so the high density distribution can be regarded as a prospecting indicator. The regularized focusing migration imaging of local gravity anomaly is carried out and the local gravity data are inverted on a horizontally uniform grid with 500m by 500m cell sizes in the x and y directions. Horizontal cross-sections with different depths are shown in Figure 5.

![Figure 4. Local gravity anomaly.](image)
We also use the Tilt angle method to identify the boundary. The zero value contour of the Tilt angle of the local gravity anomaly corresponds roughly to the geological body boundary. Based on the results of boundary recognition and migration imaging, we show the orebody range interpretation in Figure 6. The background map is gravity anomaly, and the red area is orebody range.

5. Conclusions
We put forward the regularized focusing migration method aiming at solving the divergence of conventional gravity migration imaging results. The spatial resolution of the results is greatly improved, and better imaging effect can be obtained for geological bodies with small spacing or large buried depth. Moreover, the method is robust to noises. This method shows a practical effectiveness in the study of deep mineral deposits that are mostly concentrated in high density in space. We applied it to the interpretation of the local gravity anomaly data of the Huayangchuan deposit. The imaging results help identify several potential mining targets which typically have no surface expression and can only be detected using modern high-resolution geophysical methods.

We will apply drilling data collected in Huayangchuan for joint migration to reduce the non-uniqueness of the results. The joint migration of multi-geophysical data will be the focus of our future research which can be seen in our subsequent papers.

6. Acknowledgement
This research was funded by the National Key Research and Development Program of China, grant numbers 2017YFC0602203 and 2017YFC0602000.

References
[1] Tikhonov A N 1977 Solutions of Ill-posed Problems W. H. Winston & Sons New York
[2] Li Y G, Oldenburg D W 1996 3-D inversion of magnetic data Geophysics 61(2) 394-408
[3] Portniaguine O, Zhdanov M S 1999 Focusing geophysical inversion images Geophysics 64(3) 874-887
[4] Wan L and Zhdanov M S 2013 Iterative migration of gravity and gravity gradiometry data SEG Technical Program Expanded Abstracts 1211-1215

[5] Wan L, Han M and Zhdanov M S 2016 Joint iterative migration of surface and borehole gravity gradiometry data SEG Technical Program Expanded Abstracts 1607-1611

[6] Forsberg, R., Tscherning, C. C., 1981, The use of height data in gravity field approximation by collocation: Journal of Geophysical Research Atmospheres, 86(B9).

[7] Zhdanov M S 2002 Geophysics Inverse Theory and Regularization Problems Elsevier Amsterdam

[8] Portniaguine O, Zhdanov M S 2002 3-D magnetic inversion with data compression and image focusing Geophysics 67(5) 1532-1541

[9] He S, Hui X, Guo J, 2018 U-Nb-REE mineralization characteristics of Huayangchuan uranium polymetallic deposit Shaanxi province World Nuclear Geoscience 35(04) 203-209+248

[10] Dong Q, Kang Q and Jiang H 2019 Analysis on metallogenic geological regularity of Huayangchuan uranium-polymetallic Deposit World Nonferrous Metals 2019(02) 175-176

[11] Hui X, Li Z, Feng Z and Cheng J 2014 Research on the Occurrence State of U in the Huayangchuan U-Polymetallic Deposit Shanxi Province Acta Mineralogica Sinica 34(04) 573-580

[12] Xue S, Xu Y and Ling M X 2018 Geochemical constraints on genesis of Paleoproterozoic A-type granite in the south margin of North China Craton Lithos S0024493718300720