Parametrization of Born-Infeld Type Phantom Dark Energy Model

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Applying the parametrization of dark energy density, we can construct directly independent-model potentials. In Born-Infeld type phantom dark energy model, we consider four special parametrization equation of state parameter. The evolutive behavior of dark energy density with respect to red-shift \(z\), potentials with respect to \(\phi\) and \(z\) are shown mathematically. Moreover, we investigate the effect of parameter \(\eta\) upon the evolution of the constructed potential with respect to \(z\). These results show that the evolutive behavior of constructed Born-Infeld type dark energy model is quite different from those of the other models.

\textit{Keywords:} Dark energy; Born-Infeld Field; Parametrization; Potential.

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1. Introduction

Since the first observational data of SNe Ia was issued in 1998\cite{1}, the astrophysics and cosmology have undergone a deep change and were facing a grand challenge. Up to now, series of astrophysical observational data such as 5 year WMAP\cite{2}, SDSS\cite{3} together with SNe Ia data show us such a fact clearer and clearer: the Universe is spatially flat to high precision, \(\Omega_{\text{total}} = 0.99 - 0.03\)\cite{4}, and consists of about two thirds unknown energy density, one third dust matter including cold dark matters plus baryons, and negligible radiation, and that our universe is undergoing an accelerated expansion. This unknown energy density which is called ”dark energy” with negative pressure pervades the whole universe and is unchumped. We know little on the nature of dark energy thought many models of dark energy such as cosmological constant model\cite{5-10}, quintessence\cite{11-17}, tachyon\cite{18-21}, holographic dark energy\cite{22}, K-essence model\cite{23-25} and phantom model\cite{26}, have been proposed and investigated. Thus phenomenological investigation are good choices and have been widely conducted by parametrizing the dark energy equation of state(EOS) \(\omega\). To study the evolution of the universe, different kinds of potentials can be put into different model with scalar field and then the EOS \(\omega\) would be studied. Of course, potentials can also be reconstructed from a parametrization of the EOS \(\omega\) fitting current observational data. The latter looks more persuasive to find out a best-fit dark energy model, because it is independent on model.

Born-Infeld(shortened B-I) field was firstly introduced to explain the singularity in classical electromagnetic dynamics by Born and Infeld in 1934\cite{27}. So far, many authors have been studying the nonlinear B-I type string theory and cosmology. Their work show that the lagrangian density of this B-I type scalar field posses some interesting characteristics\cite{28}. Recently, we investigated the universe of B-I type scalar field with potential and find that this model can undergo a phase of accelerating expansion corresponding EOS \(-1 < \omega < -\frac{1}{3}\)\cite{29}. This model admits a late time attractor solution that leads to EOS \(\omega = -1\). The lagrangian of B-I type scalar field with negative kinetic energy also is considered by us. An interesting result is that weak energy condition and strong energy condition are violated for phantom B-I type scalar field. In this model, the EOS \(\omega\) is always smaller than -1, which meets the current observation data well.

Guo et al.\cite{30} have constructed a theoretical method of constructing the quintessence potential \(V(\phi)\) directly from the dark energy equation of state function \(\omega\). In Ref.\cite{31}, we apply the method of parametrization of dark energy density function to the dilaton coupled quintessence(DCQ) model. According to the comparison between the constructed DCQ potential and quintessence potential(\(\alpha = 0\)), we find that the shapes of the constructed DCQ potential quite different from the one of quintessence potential. An interesting result is DCQ potentials possess two different evolutive mode ”O” and ”E”.

In this paper we will investigate the parametrization of B-I type phantom dark energy model. Using the model-independent method, we construct potential which is best-fit observations. Many authors have
presented various parametrization of EOS $\omega_\phi$ of dark energy and investigated their interesting features. In this paper we choose four cases of them to reconstruct the potential of B-I type dark energy directly from the EOS $\omega_\phi$. The four typical parametric of EOS: $\omega_\phi = \omega_0$; Case II: $\omega_\phi = \omega_0 + \omega_1 z$; Case III: $\omega_\phi = \omega_0 + \omega_1 \ln(1 + z)$; Case IV: $\omega_\phi = \omega_0 + \omega_1 \ln(1 + \eta z)$, have been proved that they fit the observations well in different areas of red-shift $z$. The numerical results of the evolution of scalar potential with respect to scalar field $\phi$ and red shift $z$ will be shown in the plots correspondingly. The evolutions of the dark energy density $\rho_\phi$ with respect to $z$ are also shown mathematically. This paper is organized as follows: Section I is introduction. In Section II, we introduce the equations of B-I type field firstly, after reconstructing the scalar potential of B-I type dark energy model by parametrizing the EOS $\omega_\phi$, we consider four special cases and get the corresponding mathematically results. Section III is summary.

2. Basic Equations and Numerical Results

The lagrangian density for a B-I type phantom scalar field is

$$L_S = \frac{1}{\eta} \left[ 1 - \sqrt{1 + \eta g^{\mu\nu} \partial_\mu \phi, \partial_\nu \phi} \right]$$

(1)

Eq.(1) is equivalent to the tochoy lagrangian $[-V(\phi) \sqrt{1 + g^{\mu\nu} \partial_\mu \phi, \partial_\nu \phi} + \Lambda]$ if $V(\phi) = \frac{1}{\eta}$ and cosmological constant $\Lambda = \frac{1}{\eta} (\frac{\eta}{\eta_0} \text{ is two times as "critical" kinetic energy of } \phi \text{ field})$. Now we consider the Lagrangian with a potential $V(\phi)$ in spatially homogeneous scalar field, Eq.(1) becomes

$$L_S = \frac{1}{\eta} \left[ 1 - \sqrt{1 + \eta \phi^2} \right] - V(\phi)$$

(2)

In the spatially flat Robertson-Walker metric $ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2)$, Einstein equation $G_{\mu\nu} = KT_{\mu\nu}$, can be written as

$$H^2 = \frac{1}{3} (\rho_\phi + \rho_m)$$

(3)

$$\frac{\ddot{a}}{a} = -\frac{1}{6} (\rho_\phi + 3\rho_m + \rho_m)$$

(4)

$$\ddot{\phi} + 3H \dot{\phi}(1 + \eta \phi^2) - V'(\phi)(1 + \eta \phi^2)^{\frac{3}{2}} = 0$$

(5)

$$\dot{\rho}_m + 3H (\rho_m + p_m) = 0$$

(6)

$$\dot{\rho}_\phi + 3H (\rho_\phi + p_\phi) = 0$$

(7)

where $\rho_m$, $\rho_\phi$ and $\rho_\phi$ are the matter energy density, the effective energy density and effective pressure of the B-I type scalar field respectively, the prime sign denotes the derivative to $\phi$ and we work in units $8\pi G = 1$. The Energy-moment tensor is

$$T^\mu_\nu = -\frac{g^{\mu\rho} \phi, \rho, \phi, \nu}{\sqrt{1 + \eta g^{\mu\nu} \partial_\mu \phi, \partial_\nu \phi}} - \delta^\mu_\nu L_S$$

(8)

From Eq.(8), we have

$$\rho_\phi = \frac{1}{\eta \sqrt{1 + \eta \phi^2}} - \frac{1}{\eta} + V(\phi)$$

(9)

$$p_\phi = \frac{1}{\eta} - \frac{\sqrt{1 + \eta \phi^2}}{\eta} - V(\phi)$$

(10)

From Eqs.(9)(10), we get

$$\rho_\phi + p_\phi = \frac{-\dot{\phi}^2}{\sqrt{1 + \eta \phi^2}}$$

(11)

According to the Eq.(11), we can obtain such a result: when $\eta \rightarrow 0$, B-I type dark energy model reduces to the $\Lambda$CDM model. In order to construct $V(\phi)$ directly independent-model, we need to get the analytic expression of $V(\phi)$ with $\rho_\phi$ and $p_\phi$. We can obtain the analytic solutions of Eqs.(9) and (10) set,

$$V(\phi) = -p_\phi - \frac{1}{\eta} - \frac{1}{\eta \sqrt{1 + \frac{4}{2} (\rho_\phi + p_\phi) [\eta (\rho_\phi + p_\phi)] - \sqrt{4 + (\eta p_\phi)^2 + (\eta p_\phi)^2 + 2(\eta)^2 \rho_\phi p_\phi}$$

(12)
\[(\dot{\phi})^2 = \frac{1}{2}(\rho_\phi + p_\phi)[\eta(\rho_\phi + p_\phi) - \sqrt{4 + (\eta \rho_\phi)^2 + \eta \rho_\phi p_\phi} + 2(\eta)^2 \rho_\phi p_\phi]\]  

(13)

Obviously, when \(\eta \to 0\) we get
\[
\lim_{\eta \to 0} V(\phi) = \frac{\rho_\phi - p_\phi}{2} 
\]

(14)

\[
\lim_{\eta \to 0} (\dot{\phi})^2 = -(\rho_\phi + p_\phi) 
\]

(15)

which corresponds to linear phantom scalar field case. In this model, we consider dark energy and matter including baryon matter and cold dark matter, and neglect radiation. From Eqs. (3) and (4), we deduce
\[
\rho_\phi = 3H^2 - \rho_{m0}(1 + z)^3 
\]

(16)

\[
p_\phi = -2\dot{H} - 3H^2 
\]

(17)

We define the dimensionless dark energy function \(\zeta(z)\) as follows
\[
\zeta(z) = \frac{\rho_\phi}{\rho_{\phi0}} 
\]

(18)

where \(\rho_{\phi0}\) is dark energy density at red-shift \(z = 0\) (present). Using Eqs.(2)(18), we get
\[
\frac{dH}{dz} = \frac{\rho_{\phi0}}{6H} \frac{d\zeta}{dz} + \frac{\rho_{m0}(1 + z)^2}{2H} 
\]

(19)

\[
\dot{H} = -H(z + 1)\frac{dH}{dz} 
\]

(20)

\[
\dot{\phi} = -H(z + 1)\frac{d\phi}{dz} 
\]

(21)

Substituting Eqs.(16),(17) and (19-21) into Eqs.(12) and (13), we have
\[
V(z) = 2\dot{H} + 3H^2 + \frac{1}{\eta} - \frac{1}{\eta}\sqrt{1 + \eta \left\{ \frac{1}{2}(-2\dot{H} - \rho_m)\{ -\eta(2\dot{H} + \rho_m) - \sqrt{f(z)} \} \right\}} 
\]

(22)

\[
\left(\frac{d\phi}{dz}\right)^2 = -\frac{1}{2}(2\dot{H} + \rho_m)[-\eta(2\dot{H} + \rho_m) - \sqrt{f(z)}] 
\]

(23)

where
\[
f(z) = 4 + \eta^2(3H^2 - \rho_m)^2 + \eta^2(2\dot{H} + 3H^2)^2 - 2\eta^2(3H^2 - \rho_m) \times (2\dot{H} + 3H^2) 
\]

(24)

\[
H = H_0 E(z) = \sqrt{\frac{\rho_0}{3}}\left\{ [(\Omega_{m0}(1 + z)^3 + (1 - \Omega_{m0})(1 + z)^3]^{1/2} \right\} 
\]

(25)

\[
\dot{H} = -H(1 + z)\frac{dH}{dz} = \rho_0\left[ -\frac{1}{6} - \Omega_{m0}(1 + z)^3 \right] 
\]

(26)

\[
\rho_m = \rho_{m0}(1 + z)^3 
\]

(27)

\(\Omega_{m0} \equiv \frac{\rho_m}{\rho_0}\) is the matter energy density with \(\rho_0 = \rho_{\phi0} + \rho_{m0}\) being present total energy density, and \(E(z)\) is the cosmic expansion rate relative to its present value. Clearly, when \(\eta \to 0\), Eq.(22) and (23) can be expressed
\[
\lim_{\eta \to 0} V(\phi) = \dot{H} + 3H^2 - \frac{1}{2}\rho_m 
\]

(28)

\[
\lim_{\eta \to 0} (\dot{\phi})^2 = -2\dot{H} - \rho_m 
\]

(29)

which expresses the linear case. Next we investigate the evolutive properties of the constructed nonlinear B-I type scalar potential numerically. In this model, we take the simple dimension dark energy function \(\zeta(z) = (1 + z)^{3(1 + \omega_\phi)}\) where \(\omega_\phi < -1\) corresponding to phantom dark energy model.

Now let us consider four cases[32-37], which fit the observations well.

Case I: \(\omega_\phi = \omega_0[31]\)

Case II: \(\omega_\phi = \omega_0 + \omega_1 z[32]\)

Case III: \(\omega_\phi = \omega_0 + \omega_1 \frac{z}{1+z}[33-35]\)

Case IV: \(\omega_\phi = \omega_0 + \omega_1 \ln(1+z)[36]\)
Fig. 1 The evolution of B-I type phantom dark energy density $\rho_\phi$ with respect to $z$ in the four cases: Case I (real line), Case II (dot-dashed line), Case III (dot line), Case IV (dashed line) in B-I type phantom dark energy model. We set $\Omega_{m0} = 0.3$, $\eta = 0.01$, $\rho_0 = 1$, $\omega_0 = -1.1$, $\omega_1 = 0.16$.

Fig. 2 Constructed B-I type scalar potentials $V - \phi$ for Case I (real line), Case II (dot-dashed line), Case III (dot line), Case IV (dashed line). We set the same parameter values as those of Fig. 1.

Fig. 3 Constructed B-I type phantom scalar potentials $V - z$ for Case I (real line), Case II (dot-dashed line), Case III (dot line), Case IV (dashed line). We set the same parameter values as those of Fig. 1.

Fig. 4 Constructed B-I type scalar potentials for different $\eta$: $\eta = 0.0001$ (real line), $\eta = 5$ (dot line) and phantom case $\eta = 30$ (dot-dash line), $\eta = 8000$ (dash line). We set $\Omega_{m0} = 0.3$, $\rho_0 = 0.01$, $\omega_0 = -1.1$ and $\omega_1 = 0$.

We can see from Fig.1 that the evolutive behaviors of the dark energy density $\rho_\phi$ with respect to $z$ in the four cases tend to be one in the low red-shift area ($0 < z < 0.15$), on the contrary, very different in the higher red-shift area ($z > 0.15$). The constructed potentials $V(z)$ with respect to $\phi$ by four different parametrizations are mathematically shown in Fig.2. The evolutions of constructed potentials in Case I, case II and case IV basically keep in step in the low red-shift areas as not they are in high red-shift. The evolutions of B-I type scalar potentials with respect to red-shift $z$ are also plotted in Fig.3. We can see from this figure that in the case $\omega < -1$ the phantom potentials increases as the universe expands. This evolutive behavior is quite different from quintessence potentials whose curve goes downwards. In quintessence model the system tends to be stable when the potential arrives at its minimum value, on the contrary, in phantom model the system tends to be stable when the potential arrives at its maximum value. So, we see a upward curve before the B-I phantom potential arrives at its maximum value which corresponds to attractor solution. The evolutive behavior of constructed potentials for different $\eta$ (where we set $\eta = 0.0001$, 5, 30 and 8000) are shown mathematically in Fig.4. Obviously, the evolutive trend of constructed potentials with respect to red-shift $z$ in B-I type dark energy model are quite similar in low red-shift area ($-0.4 < z < 0.1$) and become different in higher red-shift area ($z > 0.1$).

3. Summary

Using the method of parametrization of dark energy density function $\zeta(z)$, we construct directly the independent-model phantom potentials of B-I type dark energy model. In this paper, we consider the simple parametrization $(1 + z)^{3(1+\omega_0)}$. Based on this, we investigate four special cases which have been widely studied. Comparing the evolutive orbit of constructed B-I type (nonlinear) potentials with the linear scalar potentials, we find that they are basically possess similar characters: the same behavior in low red-shift areas and quite different in high red-shift. The evolutive curve of B-I type phantom potentials go upwards before they
arrive at their maximum values, because phantom field possesses abnormal characters: the kinetic energy of phantom field is negative. The investigation of effect of different $\eta$ values upon the evolution of B-I type phantom potential with respect to $z$ shows that the effect is very small in low red-shift area but becomes larger with the increasing of red-shift values. In the case of $\eta \rightarrow 0$, the nonlinear B-I type phantom dark energy model reduces to be ordinary phantom model. Recent astrophysical data[38] show us the(constant) effective equation of state (EOS) parameter $\omega$ of dark energy should lie in the internal $-1.48 < \omega < -1.072$. For a phantom model the internal becomes $-1.48 < \omega < -1$, therefore our model fits the observational data well.

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