Connecting the Reentrant Insulating Phase and the Zero Field Metal-Insulator Transition in a 2D Hole System

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We present the transport and capacitance measurements of 10nm wide GaAs quantum wells with hole densities around the critical point of the 2D metal-insulator transition (critical density \( p_c \) down to \( 0.8 \times 10^{10} \) cm\(^{-2} \), \( r_s \sim 36 \)). For metallic hole density \( p < p_c + 0.15 \times 10^{10} \) cm\(^{-2} \), a reentrant insulating phase (RIP) is observed between the \( \nu = 1 \) quantum Hall state and the zero field metallic state and is attributed to the formation of pinned Wigner crystal. Through studying the evolution of the RIP versus 2D hole density by transport and capacitance experiments, we show that the RIP is incompressible and continuously connected to the zero field insulator, suggesting a similar origin for these two phases.

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Strongly correlated electron systems have offered numerous interesting topics for mankind to explore, e.g. Fermi liquid, Wigner crystal (WC), fractional quantum Hall (QH) effect \(^1\), and high temperature superconductivity \(^2\). In the two dimensional (2D) case, a possible phase transition between metal and insulator, which contradicts the scaling theory of localization for non-interacting 2D fermions \(^3\), was discovered by Kravchenko et al. \(^4\). The origin of this metal-insulator transition (MIT) in strongly correlated 2D electron or hole systems has been of great interest \(^5\). \(^6\). A relevant question is how the zero field (\( B = 0 \)) MIT is related to the 2D QH states in the presence of a perpendicular magnetic field (\( B \perp \)). In early experiments, the phase diagram maps against density \( p \) and magnetic field \( B \) obtained from both transport \(^7\) and thermodynamic compressibility \(^8\) measurements suggested that \( B_1 \) transforms the \( B = 0 \) insulator into the QH liquid, somewhat similar to the conventional Anderson insulator-QH transition \(^9\). Yet this \( B = 0 \) insulator to QH transition moves to lower field and eventually terminates at \( p_c \) of the \( B = 0 \) MIT as carrier density \( p \) is increased \(^7\). Besides the zero field MIT and \( B = 0 \) insulator to \( \nu = 1 \) QH transition, 2D electron or hole systems are known to exhibit rich transitions between the so called high field insulating phases (HFIPs) \(^10\), reentrant insulating phases (RIPs) \(^11\) \(^14\), and the fractional QH states at \( \nu < 1 \). All the HFIPs and RIPs among fractional QH states in GaAs/AlGaAs systems are widely believed to be caused by the forming/melting of Wigner crystal \(^11\) \(^18\) due to the strong Coulomb interaction. Moreover, the observations of pinning mode of the WC oscillating in the disorder potential in microwave transmission experiments \(^19\) \(^22\) offer more direct evidence for the WC interpretation.

On the theory side, quantum Monte Carlo calculations \(^23\) \(^26\) had located that the liquid-Wigner crystal transition should happen around a critical value of \( r_s \sim 37 \) at \( B = 0 \); the simulations even determined the phase boundary \(^26\) starts from \( \nu = 1/6.5 \) and moves to higher \( \nu \) when \( r_s \) increases (density decreases). The results, together with other variational studies \(^27\), supported the interpretation of the observed HFIPs and RIPs as pinned Wigner solids. However, the validity of the phase diagram proposed by Ref.\(^26\) remains unsettled \(^28\) \(^29\), especially in the range from \( \nu = 1 \) to the \( B = 0 \) limit, since a reentrant WC that was expected at \( \nu > 1 \) has not been observed experimentally. Therefore, the relation between those HFIPs, RIPs and zero field insulating phase in 2D systems with large \( r_s \) is still unclear.

In this Letter, we report the first observation of a reentrant insulating phase between the \( \nu = 1 \) quantum Hall state and \( B = 0 \) metallic state in dilute 2D GaAs/AlGaAs hole system with stronger quantum confinement (narrower quantum well). We suggest that this RIP is a pinned Wigner crystal since it is incompressible \(^8\) \(^30\) and its peak resistance \( R_{\text{RIP}} \times \exp (\Delta_{\text{RIP}} / 2T) \) (\( \Delta_{\text{RIP}} \) up to 0.52K at \( p = 0.55 \times 10^{10} \) cm\(^{-2} \) for our highest mobility sample), resembling the previously observed low \( \nu \) RIPs in GaAs dilute 2D electron \(^11\) \(^12\) or hole \(^13\) \(^14\) \(^17\) systems. Our transport and compressibility phase diagrams at \( T = 70 \) mK indicate a unified phase boundary linking zero field MIT with the RIP between \( \nu = 1 \) and \( \nu = 2 \) QH states and the HFIP, consistent with the qualitative phase diagram proposed by Ref.\(^26\) (b). The results presented here suggest that the zero-field MIT is a liquid-WC transition.

Our transport measurements were performed down to 50mK on four high mobility 10nm wide p-GaAs quantum well (QW) samples (5A, 5B, 11C, and 11D) with low 2D hole density. Thermodynamic compressibility measurement was done on QW 11D by measuring the capacitance between the 2D channel and top gate. All of the four GaAs QWs are similar to those used in our previous studies \(^31\) \(^34\), which were grown on (311)A GaAs wafer using Al\(_1\)Ga\(_9\)As barriers and Si delta-doping lay-
ers placed symmetrically at a distance of 195 nm away from the 10nm GaAs QW. QW 5A and 5B were taken from wafer 5-24-01.1, and sample 11C and 11D were taken from another wafer 11-29-10.1. All the samples have density $p \sim 1.3$ (the unit for $p$ is $10^{10}/\text{cm}^2$ throughout this Letter) from doping. Without gating, QW 5A and 5B have a mobility $\simeq 5 \times 10^4 \text{cm}^2/\text{Vs}$, while QW 11C and 11D’s mobility $\simeq 2 \times 10^4 \text{cm}^2/\text{Vs}$. The back gates used to tune the hole density are approximately 150μm away from the well for QW 5A and 5B (300μm spacing for 11C), to avoid the screening of the Coulomb interaction between holes by the back gate. Sample 5A and 5B were fabricated into Hall bars with approximate dimensions: 5A:2.5mm×9mm; 5B:2mm×6mm, and six diffused In(1%Zn) contacts; sample 11C is a square one (5mm×5mm) with diffused In(1%Zn) contacts in Van de Pauw configuration. The measurement current was along the high mobility [23] direction and stayed low so the heating power is less than 3f Watt/cm² to prevent overheating [25] the holes. The compressibility measurement sample QW 11D has a size of 4.5mm×5mm, with four diffused In(1%Zn) contacts in Van de Pauw configuration and coated with Ti/Au top gate, which is 400nm away from the well. Capacitance was measured by applying a low frequency ($f$=3-13 Hz) excitation voltage $V_{ex}$ (typically 200 $\mu$V) to the top gate and monitoring the 90° out of phase current $I_q$, obtaining the capacitance $C=I_q/2\pi fV_{ex}$. A DC voltage $V_g$ was superimposed to tune the density via top gate. The measured capacitance can be viewed as due to two capacitors in series: $1/C = 1/C_0 + 1/C_g$, where $C_0$ is geometric parallel plate capacitance between the top gate and the 2D hole system, and $C_g$ is the quantum capacitance of charging the 2D hole system. The 2D hole system’s quantum capacitance $C_q$ is proportional to the compressibility $\kappa$: $C_q = e^2 dp/d\mu = e^2 p^2 \kappa$ (here $\mu$ is the chemical potential, $dp/d\mu$ is the thermodynamic density of states.).

Fig.4 is shown to establish the existence of metal-insulator transition (MIT) with a critical density ($p_c \sim 0.8 \times 10^{10}/\text{cm}²$) in a 10nm wide GaAs QW sample 5B. (b) Magneto-resistivity of sample 5B when $p=0.86$ at the temperatures of 50mK, 65mK, 80mK, 0.1K, 0.15K, and 0.2K. The reentrant insulating phase (RIP) resides between the $\nu=1$ QH state and the zero field metallic state. (c) Arrhenius plot of the resistance of the peak of the RIP from sample 11C at densities of 0.9, 0.96, 1.03, 1.09, 1.15 and 1.21. (d) Fitted thermal activation gap $\Delta_{RIP}$ Vs. hole densities in sample 5B and 11C.

![Graphical Abstract](image.png)

FIG. 1: (color online) (a) Zero field metal-insulator transition (MIT) with a critical density ($p_c \sim 0.8 \times 10^{10}/\text{cm}²$) in a 10nm wide GaAs QW sample 5B. (b) Magneto-resistivity of sample 5B when $p=0.86$ at the temperatures of 50mK, 65mK, 80mK, 0.1K, 0.15K, and 0.2K. The reentrant insulating phase (RIP) resides between the $\nu=1$ QH state and the zero field metallic state. (c) Arrhenius plot of the resistance of the peak of the RIP from sample 11C at densities of 0.9, 0.96, 1.03, 1.09, 1.15 and 1.21. (d) Fitted thermal activation gap $\Delta_{RIP}$ Vs. hole densities in sample 5B and 11C.

From 50mK to 0.2K. At small magnetic fields, the sample exhibits metallic behavior. However, it is striking to see an unexpected insulating peak below 150mK between the $\nu=1$ and the $\nu=2$ quantum Hall resistivity dips. This RIP is observed in all of our four 10nm QWs. Another example of the RIP between $\nu=1$ and $\nu=2$ QH states can be seen in Fig.1(c) of our previous paper [34]. The resistance at the peak of the RIP follows exponentially activated temperature dependence (Fig.1C) and greatly exceeds the sample’s zero field resistivity. Apparently the temperature dependence of the insulating peak is far stronger than the temperature dependence of the ordinary SdH oscillation amplitude, which is determined by the thermal damping factor, $X_T/\sinh(X_T)$, where $X_T=2\pi^2 k_BT/\hbar$. As another example, the peak resistivity for sample QW 5A exceeds the value of 12K Ohms/square, six times larger than $\rho$ at $B=0T$, at a temperature of 18mK and a density of $p=1.13$ (Ref.34). Fig.4 illustrates the temperature-dependent resistance of QW 11C at the peak of RIP for densities from $p=1.21$ to 0.9 in an Arrhenius plot. Similar to the RIPs between fractional QH states in GaAs/AlGaAs 2D electron [11] or hole [13] systems, the $R_{RIP}$ vs. $T$ of QW 5B and QW 11C is exponential, $R_{RIP} \propto \exp(\Delta_{RIP}/2T)$. The fittings according to the thermal activation model give activation gaps $\Delta_{RIP}$ up to 0.52K, shown in Fig.4 for QW 5B and QW 11C. For both QWs, the value of $\Delta_{RIP}$ appears to increase exponentially as the density decreases.
Besides, the insulating phase emerges at higher density in the lower mobility samples (QW 11C and 11D) than higher mobility samples (QW 5A and 5B), suggesting that disorder also contributes to the formation of this RIP. The Hall resistance data [34, 36] indicate that the ν=1 quantum Hall state is well formed but Hall resistance is suppressed significantly (∼20 – 30%) than the classical value 1/pe at the RIP and lower magnetic fields.

The similarity in the transport properties between the observed insulating phase at ν >1 and the previously studied RIPS between fractional QH states suggests that the insulating phase observed here is the RIP at ν>1 in the phase diagram proposed by Zhu and Louie (FIG.14 in Ref[26(b)]). Moreover, we performed capacitance measurements to elucidate the thermodynamic compressibility of the RIP observed here. Our experiment indicates that this RIP is more incompressible than both the ν=1 QH state and the B=0 state, offering further evidence supporting the pinned Wigner crystal as the interpretation according to the phase diagram in Ref[26(b)]. Fig.2a presents the magnetic-field-dependent top gate capacitance (symbol) C(B) for several densities (p=1.73, 1.42, 1.25, 1.12, 1.02, 0.95, 0.86) in QW 11D at 70mK, together with the corresponding magneto-resistance curves (solid lines). In the lower three panels where p>1.2, the capacitance value drops strongly at the ν=1 QH state, reflecting the 2D system’s small thermodynamic density of states (compressibility) when the Fermi level resides in the Landau level energy gap at low enough temperature. At low magnetic fields, the gate capacitance remains constant around the geometric capacitance value (marked as dashed horizontal lines in Fig.2a) due to the fact that quantum capacitance C_q of 2D systems is very large and does not contribute much in the measured capacitance until strong Coulomb interactions reduce the compressibility significantly in the low density regime. What is most interesting in Fig.2 is, however, the behavior of capacitance as 2D hole density p decreases towards the critical density (p_c ∼0.95). As Fig.2a shows, the capacitance dip at ν=1 weakens as density decreases (dotted grey arrow). But a new dip in the capacitance vs. B curve emerges at the position where transport data show the reentrant insulating behavior (light grey arrow). It is remarkable to note that the compressibility of the RIP is not only smaller than the zero field state, but also the ν=1 QH state. Moreover, since capacitance values smaller than the geometric capacitance was observed, the sign of the compressibility for the RIP is positive, in contrast to the well known negative compressibility of 2D electron liquids. In addition to the capacitance dip at the RIP between ν=1 and 2, the sample’s capacitance also drops at ν<1, corresponding to the HFIP.

Previously, it is known that the 2D system’s compressibility approaches zero in the insulating phase of MIT [3]. This behavior is also observed in our sample, as shown by the reduced capacitance value of our sample at B=0 when the carrier density is decreased to below p_c ∼0.95 (top two panels in Fig.2a). But for hole densities around the critical density, the application of perpendicular magnetic field induces RIP with strongly insulating transport behavior as well as lower capacitance (compressibility) than the zero field state. This observation implies that the RIP is somehow related to the zero field insulating phase of 2D MIT, yet perpendicular B helps the insulating phase forming more easily. To clarify the relation between the new RIP and the zero MIT, we examine the 2D hole system’s phase diagram against p and B⊥ by both transport and capacitance measurements. Figures

![FIG. 2: (color online) (a) Capacitance (symbol) and resistance (line) vs. perpendicular magnetic field (0T-0.8T) for QW 11D at 70mK with densities of p=0.86, 0.95, 1.02, 1.12, 1.25, 1.42 and 1.73. The ν=1 QH state gradually weakens as the density decreases, and eventually disappears, followed by the emergence of the new RIP. The dash line is the estimated value of geometric capacitance C_o. (b) Longitudinal resistance map of QW 11C in p-B⊥ plane at 70mK. The color varies from red to violet, the resistance increases from 300Ω up to 300kΩ in a log scale. The area with resistance larger than 300kΩ is filled with violet. (c) Capacitance of QW 11D in p-B⊥ plane at 70mK. The capacitance drops from 2.7nF down to 2.2nF as the color changes from red to violet. The area with capacitance less than 2.2nF is colored with violet.]
plot the colored contour maps of longitudinal resistance and capacitance for QW 11C and QW 11D from 0T up to 0.8T over the density range $0.8 < p < 1.5$ at 70mK. In Fig 2, the three red finger-shaped areas pointing towards origin are the $\nu = 1, 2$, and 3 QH states. Similarly, the violet finger in Fig 2 represents $\nu = 1$ QH state. Due to the dominance of $C_0$ in measured $C$, the $\nu = 2$ and 3 QH states were not resolved in our capacitance data. In Fig 2b of transport map, the insulating phase is shown in violet at the bottom, which also has low capacitance (compressibility) as seen in Fig 2c. Through comparing the two maps, it is significant to see a unified phase boundary linking zero field MIT with the new RIP and HFIP. First, the two different methods give the same phase boundary of the MIT from the $B=0$ limit up to $\nu=1$. Both maps indicate that the zero field insulator is linked with the new RIP, suggesting a common origin. It is necessary to point out that our phase diagram differs from previous study\cite{7} in that there is a RIP protruding into the high density metallic regime while previous finding in higher density samples showed a phase boundary line moving down monotonically as $B_⊥$ increases towards the $\nu=1$ QH \cite{7}. Our phase boundary coincides with the phase diagram in FIG.14 of Ref.\cite{26}(b), which predicts an insulating solid phase between the $B=0$ liquid state and $\nu=1$ QH state.

We also studied the temperature evolution of the phase boundary of the metal-insulator transition in the $p-B_⊥$ map via transport measurement. Figure 3 shows the longitudinal resistance maps of QW 5B from $B=0T$ to 0.5T and a density range $0.5 < p < 1.2$ at 150mK, 80mK and 50mK. This evolution depicts an interesting competition between the zero field MIT and the formation of integer QH and the RIP. First, as $T$ decreases, both the low field metallic state and $\nu=1$ QH become better formed as shown by the growing low resistivity (red) areas in the phase diagram towards lower density. At the same time, the RIP starts to form and protrude towards higher density in the diagram. Eventually, at lowest $T$, there is a RIP separating the low field metal and $\nu=1$ QH liquid for certain density range above the critical density $p_c=0.8$. Then the phase boundary becomes closer to the theoretical result predicted in Ref\cite{26}. Naturally, the implication is that the new RIP and $\nu=1$ QH state are the ground states in the $T=0K$ limit. In addition, by comparing Fig 2d and c with Fig 3 it is clear to see that the phase boundary shifts to lower density as the sample mobility increases (less disorder). Obviously, further study is necessary to understand better the evolution of the liquid-WC in 2D as a function of $T$ and the role of disorder.

The phase diagram of our narrow QWs is distinct from those of wide QWs or heterstructures \cite{1,8,17}. The RIP between the $\nu=1$ and $\nu=2$ quantum Hall states in strongly interacting 2D systems, even proposed decades ago \cite{20}, has never been reported in literature. We believe that the narrow QW width is a key in the formation of this RIP at $\nu>1$. Empirically, it has already been noticed that the RIP in GaAs 2D electron system shifts to higher filling factor as QW width decreases \cite{20}. Another argument is in GaAs hole system, Landau-level mixing is significant and the well width will affect the hole effective mass, which is essential to the Monte Carlo calculation \cite{26}. Furthermore, spin-polarization was seen to induce a more dramatic effect in driving metallic state into insulating phase in narrow QW system \cite{26}. Therefore, the strong spin-polarization close to the $\nu=1$ QH state may have also helped the formation of the RIP.

In summary, we have observed a new reentrant insulating phase (RIP) between the $\nu=1$ and $\nu=2$ QH states in a dilute p-GaAs quantum well system with narrow width and found that this RIP is connected with the zero field insulating phase of the 2D metal-insulator transition. The RIP is similar to those observed among the fractional QH states in other GaAs hetero-structures or QWs with wider width. Based on transport and thermodynamic compressibility experiments, we propose its origin to be related to the formation of disorder pinned Wigner crystal of low density 2D holes. Both our transport and thermodynamic phase diagrams match the liquid-Wigner crystal phase diagram proposed by Zhu and Louie \cite{20}, leading to the suggestion that the zero field MIT is a
liquid-WC transition \[39\], \[40\].

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