DISKS IRRADIATED BY BEAMED RADIATION FROM COMPACT OBJECTS

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ABSTRACT

We examine the reprocessing of X-radiation from compact objects by accretion disks when the X-ray emission from the star is highly beamed. The reprocessed flux for various degrees of beaming and inclinations of the beam axis with respect to the disk is determined. We find that, in the case where the beam is produced by a nonrelativistic object, the intensity of the emitted spectrum is highly suppressed if the beam is pointing away from the disk. However, for beams produced by compact objects, general relativistic effects cause only a small reduction in the reradiated flux even for very narrow beams oriented perpendicularly to the disk. This is especially relevant in constraining models for anomalous X-ray pulsars, whose X-ray emission is highly beamed. We further discuss other factors that can influence the emission from disks around neutron stars.

Subject headings: accretion, accretion disks — stars: neutron — X-rays: stars

1. INTRODUCTION

Reprocessing of X-rays by accretion disks is a common phenomenon in astrophysics. For example, reprocessed radiation is thought to be responsible for the optical emission observed in X-ray binaries (e.g., van Paradijs & McClintock 1994; de Jong, van Paradijs, & Augusteijn 1996), and the ratio between optical and X-ray flux has been connected with properties of the system itself, such as the orbital period (van Paradijs & McClintock 1994).

Another class of astrophysical objects for which reprocessing of radiation by an accretion disk could potentially be important is that of anomalous X-ray pulsars (AXPs). The luminosity of these objects, \( L_X \sim 10^{35} - 10^{36} \text{ erg s}^{-1} \), is far larger than their rotational energy loss, \( |E| \approx 4\pi^2 I P^2 \approx 10^{32.5} \text{ erg s}^{-1} \), and therefore their X-ray emission cannot be rotation-powered. Competing models explaining the origin of their X-ray luminosity invoke either the existence of neutron stars (NSs) with very large magnetic fields, \( B \sim 10^{14} - 10^{15} \ G \) (e.g., Duncan & Thompson 1992; Thompson & Duncan 1996; Heyl & Hernquist 1998a, 1998b), or accretion from either a companion (Mereghetti & Stella 1995) or a disk, where the latter situation could have resulted from the debris of a disrupted high-mass companion (van Paradijs, Taam, & van den Heuvel 1995; Ghosh, Angelini, & White 1997) or from material falling back after the supernova explosion (Corbet et al. 1995; Chatterjee, Hernquist, & Narayan 2000; Alpar 1999, 2000; Marsden et al. 2000). So far, no evidence for a companion has been found for any AXP, and the debate on whether AXPs are isolated NSs powered by internal energy or NSs accreting from a disk of some sort is still open.

The strongest constraint on the presence of a disk is given by its optical and longer wavelength emission, which, at the luminosities typical for AXPs, is dominated by reradiation. Phase observations of the X-ray luminosity show a high degree of pulsation, which cannot be accounted for by an isotropic X-ray emission but require some degree of beaming (Dedec, Psaltis, & Narayan 2000; Perna, Heyl, & Hernquist 2000b). However, computations of the reprocessed flux for disks around AXPs (Perna, Hernquist, & Narayan 2000a; Hulleman et al. 2000) have so far always assumed isotropic emission from the star. In this Letter, we consider the problem of disks irradiated by beamed radiation. For radiation produced by a compact object, such as an NS, the general relativistic effect of light bending is taken into account in transforming the beam pattern formed within the atmosphere of the NS into that which is seen by an observer at infinity. We show that the inclusion of these relativistic corrections has a significant influence on the reradiated spectrum when the beam points away from the disk. We finally discuss, more generally, other effects that can influence the spectrum of the reradiated flux, particularly in the context of the types of disks expected in accretion models for AXPs.

2. MODEL

2.1. Beaming Pattern

The radiation pattern emerging from accretion into the atmosphere of an NS has been computed in detail by Nagel (1981) and Mészáros & Nagel (1985). At low accretion rates, a good approximation is given by the function

\[
I(\delta) = I_0 \cos^n \delta,
\]

with \( n = 2-3 \) and where \( \delta \) is the angle that the emitted photons make with the normal to the local stellar surface. What we need to find is the pattern of radiation that an observer at infinity will detect. As a result of light-bending effects, a photon emitted at an angle \( \delta \) with respect to the normal to the surface comes from a colatitude \( \theta \) on the star given by (e.g., Page 1995)

\[
\theta(\delta) = \int_0^{R_s/2R} \frac{x \, du}{\sqrt{(1 - R_s/R)(R_s/2R)^2 - (1 - 2u)u^2x^2}},
\]

where \( x = \sin \delta \), \( R \) is the radius of the star, and \( R_s = 2GM/c^2 \) its Schwarzschild radius. If the beam pattern formed in the atmosphere of the NS is \( I(\delta) \), then an observer at infinity will see a beam pattern given by

\[
I(\theta) = I(\delta) \frac{\sin \delta \, d\delta}{\sin \theta \, d\theta}.
\]

Given a colatitude \( \theta_s \) on the star, the corresponding angle \( \delta \) at the emission point is computed by numerically solving the equation \( \theta_s - \theta(\delta) = 0 \). The intensity at that point is then found from

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equation (1) and converted to the observed radiation emitted at $\theta_\alpha$ with the transformation given in equation (3).

Figure 1 shows $I(\theta)$ for a star of mass $M = 1.4 M_\odot$ and different choices of radii. The more relativistic a star is (i.e., smaller $R/R_p$), the more spread out the radiation pattern becomes.

Now, let $\alpha$ be the angle that the axis of the beam makes with the plane of the disk; the axis is then described by the unit vector $\hat{O} = (\cos \alpha \cos \phi, \cos \alpha \sin \phi, \sin \alpha)$, where $\phi = \Omega t$, with $\Omega$ being the rotation rate of the star. Then, let $\hat{P} = (\sin \beta, 0, \cos \beta)$ be a point along a direction making an angle $\beta$ with the perpendicular to the disk in the upper plane. While the star rotates, the angle formed between the beam axis and the direction along $\hat{P}$ is

$$\theta_{up}(\phi, \beta; \alpha) = \arccos \left( \sin \beta \cos \alpha \cos \phi + \cos \beta \sin \alpha \right).$$  \hspace{1cm} (4)

As a result of the symmetry of the problem, two beams are expected from the NS, centered around the two poles. If the axis of the upper beam forms an angle $\alpha$ with the plane of the disk, the axis of the lower beam is at an angle $-\alpha$, and, while the star rotates, it makes an angle

$$\theta_{down}(\phi, \beta; \alpha) = \arccos \left( \sin \beta \cos \alpha \cos \phi - \cos \beta \sin \alpha \right).$$  \hspace{1cm} (5)

with the direction $\hat{P}$ defined above.

Figure 2 shows the intensity profiles averaged over a rotation period of the star, as a function of the angle $\beta$ of the observation point with respect to the normal to the disk,

$$\langle I(\beta; \alpha) \rangle = \frac{1}{2\pi} \int_0^{2\pi} d\phi I[\theta_{up}(\phi, \beta; \alpha)] + I[\theta_{down}(\phi, \beta; \alpha)].$$  \hspace{1cm} (6)

### 2.2. Temperature Profile and Emitted Spectrum

Following Vrtilek et al. (1990), the computation of the temperature profile is obtained by combining two equations for the disk structure. The equation for hydrostatic equilibrium in the $z$-direction is

$$\frac{k}{\mu m_p} T = \omega^2 h^2,$$  \hspace{1cm} (7)

where $\omega$ is the Keplerian rotation rate, $\omega = (GM/r^3)^{1/2}$, $h$ is the half-thickness of the disk, $k$ is the Boltzmann constant, $\mu$ is the mean molecular weight, and $m_p$ is the proton mass. The second equation for the disk structure states that the radiation emitted by a surface element, $\delta T_{\text{surf}}^4$, should be equal to the sum of the energy released locally and the absorbed radiation (Cunningham 1976; Pacharintanakul & Katz 1980),

$$\sigma T_{\text{surf}}^4 = \sigma T_0^4 + \frac{f L_\infty}{4\pi r} \frac{\partial h}{\partial r},$$  \hspace{1cm} (8)

where

$$\sigma T_0^4 = \frac{3GM M}{8\pi r^3} \left( 1 - \sqrt{\frac{R}{r}} \right).$$  \hspace{1cm} (9)
gives the effective temperature for a disk that is not irradiated (Shakura & Sunyaev 1973). In equation (8), \( f_i \) is the absorbed fraction of the radiation impinging on the surface (the quantity \( 1 - f_i \) is called the “albedo” of the disk), for which we assume \( f_i = 0.5 \) as in Vrtilek al. unless otherwise stated. The X-ray luminosity is given by

\[
L_X(\beta; \alpha) = L_X^0 f(\beta; \alpha),
\]

with \( f(\beta; \alpha) \) given by equation (6) and normalized so that \( \int_0^{\pi/2} f(\beta; \alpha) \sin \beta = 1 \); \( L_X^0 \) is the luminosity corresponding to isotropic emission, and it can be related to \( M \) via

\[
L_X = f_2 \frac{GM^2}{R},
\]

where a value of \( f_2 = 0.5 \) corresponds to the assumption that the X-ray luminosity produced in the disk is radiated mainly in a direction perpendicular to the orbital plane and that only the X-rays directly from the NS can impinge on the disk. Finally, the angle \( \beta \) in equation (10) is related to the half-thickness of the disk at position \( r \) by

\[
\beta = \frac{\pi}{2} - \arctan \left( \frac{h}{r} \right).
\]

Note that the vertical structure of the disk is assumed to be isothermal.

Equations (7)–(12) are solved numerically for the temperature profile \( T(r) \), and the flux to the observer is obtained by integrating the local emissivity \( B_r(T) \) (assumed blackbody) over the entire surface of the disk:

\[
F = \frac{1}{d^2} \int_{r_{\text{in}}}^{r_{\text{out}}} dr \int_0^{2\pi} dB_r(T) \times \left[ \cos \phi \sin \gamma(r) \sin i + \cos \gamma(r) \cos i \right].
\]

Here \( i \) is the angle between the line of sight and the normal to the disk at \( r = 0 \), and \( \gamma(r) = dh(r)/dr \) is the tilt angle of the surface of the disk at position \( r \). In our calculations we assume \( i = 60^\circ \). The inner edge of the disk \( r_{\text{in}} \) is taken at the magnetospheric radius \( R_m \); for a typical luminosity \( L_X \sim 10^{35} \) ergs s\(^{-1}\) and a magnetic field \( B \approx 8 \times 10^{12} \) G, as required by the accretion model of Chatterjee et al. (2000), one has \( R_m \approx 2 \times 10^5 \) cm. The outer edge \( R_{\text{out}} \) is determined according to the solution given by Cannizzo (1993); for a disk of about \( \approx 10^5 \) yr, as typical for AXPs, it is \( R_{\text{out}} \approx 5 \times 10^{14} \) cm.\(^2\) We show our results for an X-ray luminosity \( L_X = 10^{35} \) ergs s\(^{-1}\).

Figure 3 shows the emitted flux for the exponents \( n = 2 \) and \( n = 3 \) of the beaming function and for the limiting inclination angles of the beam axis \( \alpha = 0^\circ \) and \( \alpha = 90^\circ \). Each case is made for \( R/R_m \rightarrow \infty \) (nonrelativistic case), and for \( R/R_m = 3 \). The emission corresponding to the isotropic case is shown for comparison. The figure illustrates how, in the nonrelativistic case, the reprocessed flux is suppressed by several orders of magnitude if the beam axis is pointing away from the disk. However, in the case of radiation produced by a highly relativistic object such as an NS, even a beam perpendicular to the disk (\( \alpha = 90^\circ \)) gives a significant contribution to the spectrum. This reduction in the difference between spectra produced by beams pointing parallel and perpendicularly to the disk is due to the same general relativistic effects that cause a reduction in the pulsation amplitude of the radiation from the NS itself (Page 1995).

\(^2\) We also assume that the disk has a high opacity, as it is likely to be made of heavy elements if formed from the debris of a supernova explosion.
3. DISCUSSION

We have considered the effects that beaming of the X-ray luminosity impinging on a disk has on the flux that is reradiated by the disk itself. We have found that, because of general relativistic effects of light deflection, even narrow beams pointing away from the disk still give a significant contribution to the impinging flux and on the consequent intensity of the reprocessed radiation.

The problem we have considered is particularly important for AXPs, which, because of the high pulsation amplitude of their flux, must produce a highly beamed X-ray luminosity. The reradiated flux gives the dominant contribution at long wavelengths; this is produced in the bulk of the disk, whereas the optical emission is generated in its innermost part. If the inner edge of the disk were truncated at a radius larger than the magnetospheric radius (which could, for example, happen if the inner region of the disk made a transition to a different state), then the optical emission would be highly suppressed. This is shown in Figure 4, where the spectrum is computed for different choices of \( r_{\text{min}} \). Constraints from optical observations have already been made in a few cases. Hulleman et al. (2000) have used \( I \) - and \( R \)-band observations to rule out accretion scenarios for AXP 1E 2259. However, if this object had an accretion disk truncated at \( r_{\text{min}} \sim 10R_{\text{ms}} \), the observational constraints would not be violated any longer.3

On the other hand, we have shown that the longer wavelength emission is hardly suppressed, even if the X-ray emission is highly beamed and the beam axis points away from the disk. A reduction of the disk to 10% of its size (Fig. 4b) or an increase in the disk albedo from 50% to 95% (Fig. 4c) would not reduce the peak flux by more than an order of magnitude. Therefore, definite constraints on the nature of AXPs can only be made from multiwavelength observations.

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3 Note that these constraints are given assuming an inclination angle of 60° between the line of sight and the normal to the disk. Clearly, a higher inclination would not violate the observational constraints while allowing a larger \( r_{\text{ms}} \).

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