TIME-DEPENDENT MODELS FOR BLAZAR EMISSION WITH THE SECOND-ORDER FERMI ACCELERATION

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ABSTRACT

The second-order Fermi acceleration (Fermi-II) driven by turbulence may be responsible for the electron acceleration in blazar jets. We test this model with time-dependent simulations. The hard electron spectrum predicted by the Fermi-II process agrees with the hard photon spectrum of 1ES 1101−232. For other blazars that show softer spectra, the Fermi-II model requires radial evolution of the electron injection rate and/or diffusion coefficient in the outflow. Such evolutions can yield a curved electron spectrum, which can reproduce the synchrotron spectrum of Mrk 421 from the radio to the X-ray regime. The photon spectrum in the GeV energy range of Mrk 421 is hard to fit with a synchrotron self-Compton model. However, if we introduce an external radio photon field with a luminosity of \(4.9 \times 10^{38} \text{ erg s}^{-1}\), GeV photons are successfully produced via inverse Compton scattering. The temporal variability of the diffusion coefficient or injection rate causes flare emission. The observed synchronicity of X-ray and TeV flares implies a decrease of the magnetic field in the flaring source region.

Key words: acceleration of particles – BL Lacertae objects: individual (1ES 1101−232, Mrk 421) – radiation mechanisms: non-thermal – turbulence

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1. INTRODUCTION

Multi-frequency spectra of blazars are characterized by the double peaks of the synchrotron and inverse Compton (IC) components. They have been successfully fit with steady-state leptonic models (e.g., Kino et al. 2002; Celotti & Ghisellini 2008). In most models, non-thermal emission is presumed to be emitted by shock-accelerated electrons (the Fermi-I process; e.g., Kirk et al. 1998; Spada et al. 2001). The flare phenomena may be caused by internal shocks in the blazar outflows, as have been discussed in the models of the prompt emission of gamma-ray bursts (GRBs; Mészáros 2006).

However, the emission from blazars, especially in quiescent states, can be regarded as quasi-steady, which is different than that of GRBs. The existence of steady shocks in the outflows is non-trivial. This may imply a different acceleration process from the Fermi-I process. The electron energy distributions obtained from the photon-spectrum fits also cast doubt on the Fermi-I acceleration. The maximum electron energy is far below the Bohm limit (Inoue & Takahara 1996), while electrons accelerated by the shocks of supernova remnants attain energies close to the Bohm limit (Aharonian & Atoyan 1999; Yamazaki et al. 2004). The detections of very high-energy gamma rays (>10^{11} eV) from high-redshift blazars (Aharonian et al. 2006, 2007a, 2007b), despite the obligatory absorption due to extragalactic background light (EBL), indicate very hard photon spectra (photon index \(\lesssim 1.5\)). Those unusually hard spectra are supported by the non-detection of GeV photons with Fermi (Neronov & Vovk 2010). The implied electron spectra may be harder than the prediction of the simplest version of diffusive shock acceleration theory; the electron spectral index should be larger than 2. Several mechanisms to produce harder spectra for the shock accelerated particles have been proposed, although they are not yet well established. The non-linear back reaction of cosmic-ray pressure on the shock structure (Malkov & Drury 2001) has been frequently discussed. Alternatively, Vainio & Schlickeiser (1999) considered the particle acceleration in non-relativistic shocks and showed that the electron power-law spectral indices can be smaller than 2 when the scattering center compression ratio is larger than the gas compression ratio. Vainio et al. (2003) also demonstrated this for relativistic shocks.

Second-order Fermi acceleration (Fermi-II) is a promising process to make hard spectra (e.g., Schlickeiser 1984; Park & Petrosian 1995; Becker et al. 2006; Stawarz & Petrosian 2008). This slow acceleration process can naturally explain the lower maximum energy of electrons. Applications of the Fermi-II to active galactic nucleus (AGN) jet emission have been discussed by several authors (e.g., Böttcher et al. 1999; Schlickeiser & Dermer 2000; Katarzyński et al. 2006; Dahai et al. 2012). The turbulence responsible for the Fermi-II acceleration may be induced by the Kelvin–Helmholtz instability (Hardee 2004; Mizuno et al. 2007) or the current-driven instability (Lyubarskii 1999; Narayan et al. 2009; Mizuno et al. 2011). Such instabilities may be triggered by recollimation of the jet induced by a pressure gradient in the medium (Daly & Marscher 1988; Komissarov & Falle 1997; Agudo et al. 2001). Actually, the signature of the Fermi-II process has been explored in photon spectra. X-ray spectra have been fit with a curved function, such as a log-parabolic shape, which has been discussed in the theoretical context of the Fermi-II process (Massaro et al. 2004a, 2004b; Tramacere et al. 2009). Even for GRBs, the Fermi-II process has been considered (Asano & Terasawa 2009) to yield hard spectra below the spectral peak energy (~0.1–1 MeV).

Lefa et al. (2011) adopted the Fermi-II process to fit a hard blazar 1ES 0229+200, although their discussion focused on the
balance between the acceleration and cooling. In this paper, we further pursue the possibility of the Fermi-II process in blazar jets. Here, we use the time-dependent code of Asano & Mészáros (2011) developed for GRB studies (see also Asano & Mészáros 2012) to follow the evolution of the electron energy distribution and photon production. In our code, the electron distribution is obtained with the effects of the injection, acceleration, radiative cooling, adiabatic cooling, electron–positron pair production, and heating due to synchrotron self-absorption. Based on the photon production and escape from the source region, the code outputs photon spectra and light curves for an observer including the Doppler and curvature effects.

We try to fit the broadband spectra of 1ES 1101−232 and Mrk 421 with our simulations. This would be the first application of comprehensive Fermi-II models to current data of broadband blazar spectra with a time-dependent method. The temporal evolution of the electron and photon energy distributions will be explicitly shown, which will help us understand the roles of the temporal evolution of the Fermi-II process and particle injection rates on the photon spectra. We will show that the temporal evolution is important not only for the spectral variability in flares (Kusunose et al. 2000; Chen et al. 2011) but also for steady emission. Temporal evolution of the injection rate and the acceleration efficiency, etc. may play an important role in steady photon spectra (see, e.g., Becker et al. 2006).

In Section 2, we explain our model and numerical method. The results for the hard spectrum blazar 1ES 1101−232 are shown in Section 3. The results for the famous blazar Mrk 421 are divided into two parts: Section 4 for the steady photon spectrum and Section 5 for the spectral variability in flares. The summary and discussion are in Section 6. To provide spectra and light curves for an observer, we adopt the cosmological parameters $H_0 = 70\text{ km s}^{-1}\text{ Mpc}^{-1}$, $\Omega = 0.3$, and $\Lambda = 0.7$.

2. NUMERICAL METHODS

Our model is summarized in Figure 1. The calculation starts at a radius $R = R_0$, where high-energy electrons also start to be injected. The quasi-steady outflow is modeled by identical shells continuously ejected from $R = R_0$. We consider a shell region of a constant width $W = R_0/\Gamma^2$ ($R_0/\Gamma$ in the comoving frame) that is moving outward with a Lorentz factor $\Gamma = 1/\sqrt{1 - \beta^2}$. Our time-dependent numerical code (Asano & Mészáros 2011) will follow the evolution of the electron energy distribution and photon production in the shell with increasing radius $R$. Our numerical code was developed for GRBs so that the geometry of the jet is assumed to be a cone with a constant half-opening angle $\theta_j$, while the emission region of blazars has been frequently modeled as a spherical blob or cylindrical flow in previous studies. Here, we assume a narrow cone with $\theta_j = 1/\Gamma$. Given this opening angle, the transverse scale of the jet $R/\Gamma$ is comparable to the radial scale in the comoving frame. In this case, the curvature effect of the cone is not very important. This geometry is not significantly different from spherical or cylindrical emission zones.

We regard this thin shell as a homogeneous region and particle distributions are assumed to be isotropic in the comoving frame (one-zone approximation). Our one-zone numerical code includes the effects of electron cooling and injection to obtain the temporal evolution of the plasma and photon production. In this paper, we also add the acceleration and energy-diffusion effects due to plasma-wave turbulence.

The evolution of the electron momentum distribution is described by the Fokker–Planck equation as

$$\frac{\partial f_e(p,t)}{\partial t} = \frac{1}{p^2} \frac{\partial}{\partial p} \left[ p^2 D_{pp}(p) \frac{\partial f_e(p,t)}{\partial p} + p^2 \left( \frac{\dot{p}}{p} \right)_{\text{cool}} f_e(p,t) \right]$$

$$- \frac{\dot{V}}{V} f_e(p,t) + \dot{f}_{e,\text{inj}}(p,t),$$

where we have assumed an isotropic and homogeneous distribution for the distribution function $f_e(p,t)$. The electrons are assumed to be confined in a volume $V$. The term with $\dot{V}/V$ expresses the density decrease due to the volume expansion, where $V$ is the volume expansion rate. The effects of radiative/adiabatic cooling and particle injection are described using the momentum loss rate $(\dot{p})_{\text{cool}} > 0$ and $f_{e,\text{inj}}(p,t)$, respectively. For ultrarelativistic particles, their energies can be approximated as $\epsilon_e = cp$. The homogeneous approximation allows us to describe the total energy-distribution function as $N_e(\epsilon_e, t) = 4 \pi p^2 c^{-1} f_e(p,t)V$. Then, converting the diffusion coefficient $D_{pp}(p)$ into $D(\epsilon_e) = c^2 p^2 D_{pp}(p)$, Equation (1) becomes

$$\frac{\partial N_e(\epsilon_e, t)}{\partial t} = \frac{\partial}{\partial \epsilon_e} \left[ D(\epsilon_e) \frac{\partial N_e(\epsilon_e, t)}{\partial \epsilon_e} \right]$$

$$- \frac{\partial}{\partial \epsilon_e} \left[ \left( \frac{2D(\epsilon_e)}{\epsilon_e} - \langle \epsilon_e \rangle_{\text{cool}} \right) N_e(\epsilon_e, t) \right]$$

$$+ \dot{N}_{e,\text{inj}}(\epsilon_e, t),$$

where $\langle \epsilon_e \rangle_{\text{cool}}$ is the energy loss rate and $\dot{N}_{e,\text{inj}}(\epsilon_e, t)$ is the total electron injection rate. Electrons are gradually accelerated via scattering by turbulence. If the average scattering frequency $\nu$ and the fractional energy change per scattering $\tilde{\xi}$ are given, the diffusion coefficient can be written as

$$D(\epsilon_e) = \frac{\tilde{\xi}}{2 \epsilon_e^2} v.$$

A collision with a fluid element of velocity $\beta_d \ll 1$ yields $\tilde{\xi} \simeq 4 \beta_d^2/3$. The average velocity of turbulence may be determined by the Alfvén velocity or the sound velocity. A fluid with a relativistic temperature (the sound speed is $c/\sqrt{3}$) gives an extreme limit of $\xi \simeq 2/3$. Quasi-linear theory implies that the collision frequency $\nu$ is proportional to the gyration frequency $\Omega = eBc/\epsilon_e$ as

$$\nu \equiv \frac{\pi k|\delta B|^2}{4B^2} \frac{\epsilon_e}{\Omega},$$

where $k$ is the wave number. The linear gain rate is given by $A = c^2 (\epsilon_e^2 - \epsilon_e) / (2 \epsilon_e^2) \nu$. The linear gain rate $A \propto \nu \propto \epsilon_e^{-1/2}$ is dominant for $\epsilon_e > 1$.
where $k \simeq eB/e_c$ is the wavenumber of turbulence that resonates with the gyration frequency of the electrons (Blandford & Eichler 1987). The Fourier transform of the magnetic turbulence is assumed to be a power-law function given by $|\delta B|^2 \propto k^{-q}$. Then, as is well known, the diffusion coefficient becomes a power-law function given by

$$D(e_c) = \frac{\pi \epsilon e c k |\delta B|^2}{8B} \equiv K e_c^{q-1}.$$  \hspace{1cm} (5)

As shown in Dung & Schlickeiser (1990), the cross helicity state of the Alfvén waves can affect the momentum diffusion coefficient. However, here we simply extrapolate the above formula for isotropic turbulence in the shell.

In our numerical procedure, for each time step, after the calculation for electron cooling, the differential terms including $D(e_c)$ in Equation (2) are evaluated with the MUSCL scheme with second-order accuracy (van Leer 1979) for first-order differentiation and the central-difference method for second-order differentiation. In Figure 2, our test calculations neglecting the electron cooling are shown. Here, we continuously inject electrons at $10^7$ eV at a constant rate. The acceleration timescale can be roughly written as $t_{acc} \sim e_c^{q-2}/2D(e_c) \propto e_c^{-q}$. So, the steady-state solution provides a power-law distribution $N_c(e_c) \propto e_c^{-1-q}$. In Figure 2, we normalize time by the acceleration timescale for $10^{10}$ eV, $t_0 \equiv (10^{10}e^2/2D(10^{10} eV))$. The spectral evolution agrees with the acceleration timescale and spectral index estimated above. The cooling effect in this code is also checked. Schlickeiser (1985) provides time-dependent formulae of the electron distribution under the simultaneous action of synchrotron and IC radiation losses competing with Fermi-I and Fermi-II accelerations. Here, we simply show a steady-state case in Fermi-II models. When the Fermi-II acceleration is balanced by synchrotron energy losses, the electron distribution becomes a Maxwell-like function $N_c(e_c) \propto e_c^{q} \exp \left\{-(e_c/e_c)^{3-q}\right\}$ (Lefa et al. 2011), which is identical to the steady-state solution of Schlickeiser (1985). The inset figure in Figure 2 shows a quasi-steady distribution after switching off the injection but with acceleration for $q = 2$. In this test calculation, we consider only synchrotron cooling. The distribution agrees with the analytical one (dashed line).

The energy source of the turbulence may be dissipation of the bulk kinetic energy of the jet. In those cases, jets are expected to be decelerated. However, for simplicity, we assume a constant Lorentz factor $\Gamma$ throughout this paper. If internal fluid motions exist in the jet at $R < R_0$, the dissipation of the internal motions can be the energy source. This model may validate the constant Lorentz factor in our simulations. Regardless, we do not specify the dissipation source for the turbulence and the second-order Fermi acceleration is phenomenologically treated with the parameter $K$.

Hereafter, we fix the index $q$ in Equation (5) at the Kolmogorov value $5/3$ for simplicity. The diffusion coefficient should be determined by the characteristics of the turbulence. At the present time, we have no definite theory for the magnetic turbulence in blazar jets. To reproduce the observed spectra, we will adjust diffusion coefficients below. If the coefficient $K$ in Equation (5) is larger than the value $K_{max} \sim (\gamma_{max} m_e c^2)^{-1/3}$ estimated from Equation (4) with an extreme limit $k \delta B^2 \propto 1$ at $e_c \sim \gamma_{max} m_e c^2$, it is physically unrealistic. As will be seen below, the values of $K$ we adopt are safely smaller than $K_{max} \sim (\gamma_{max} m_e c^2)^{-2/3}eBc \sim 1.4 \times 10^4(B/0.1G)(\gamma_{max}/10^3)^{-2/3}eV^{1/3}s^{-1}$.

For simplicity, the electron injection is assumed to be monoenergetic; the electron Lorentz factor at injection will be fixed to $\gamma_{nj} = 100$. Hereafter, the quantities in the comoving frame are denoted with primed characters. As the shell outflows, the injected electrons are gradually accelerated following Equation (2). As the emission region flows outward, the volume increases as $V' \propto R^2$ in this conical geometry. Adiabatic cooling is taken into account with the same method as Asano & Mészáros (2011), in which the electron energy decreases as $e'_c \propto V'^{-1/3}$ in the ultrarelativistic limit. The electrons remaining in the shell cool adiabatically and the emission will cease as the shell expands even if electrons do not escape from the shell.

We do not include the effect of the electron escape in this paper, as shown in Equation (1). This is a critical process for obtaining the electron spectrum, as is well known. In our quasi-steady outflow model depicted in Figure 1, the electron escape is equivalent to the electron transfer between shells. Our one-zone approximation is not optimized for the electron transfer. However, the isotropic diffusion we assumed may allow us to neglect the escape, because the escape rate from a shell may be almost equal to the incoming rate from the adjoining shells. Given the mean free path $l_m = c/\nu$, the spatial diffusion coefficient can be approximated as $D_{xx} = l_m e_c/3$. Then, the diffusion length in the dynamical timescale $W'/c$ becomes $\langle L_s' \rangle = \sqrt{D_{xx} W'/c}$, which implies

$$\frac{\langle L_s' \rangle}{W'} = \sqrt{\frac{\xi}{6W'K_0}} \simeq 0.7 \xi^{1/2} \left( \frac{W'}{10^{16} \text{ cm}} \right)^{-1/2} \times \left( \frac{K_0'}{10^{-2} \text{ eV}^{1/3} s^{-1}} \right)^{-1/2} \left( \frac{e_c}{10^2 \text{ eV}} \right)^{1/6} \hspace{1cm} (6)$$

for $q = 5/3$. For a conservative value of $\xi \ll 1$, the spatial diffusion is not sufficient. This also supports neglecting the escape effect in our model.
The average magnetic field should decay with radius, unless some kind of amplification mechanism is at work. In this paper, we assume a power-law evolution as $B = B_0 (R/R_0)^{-\gamma}$, which implies conservation of magnetic energy.

In each time step, photons are produced in the shell with a rate that is consistent with the electron cooling rate. The photon production processes we adopt are synchrotron and IC emission. The Klein–Nishina effect on IC emission is fully included in our numerical method. The photon density is evaluated with the homogeneous approximation taking into account the photon escape from the shell (see Asano & Mészáros 2011 for details). We adopt this spectral density of photons to estimate the seed photons for IC scattering. Our one-zone approximation does not solve the radiative transfer in the steady outflow. Therefore, the photons that escape are not counted as the seed photons for IC scattering. Such photons may contribute to the seed photons in regions outside the original shell. However, we take into account only the photons remaining in the shell. This problem in our method may be absorbed by the uncertainty in the model setting (simplified geometry, electron injection, etc.). The photon absorption via $\gamma\gamma$ collision, secondary electron–positron pair injection, and synchrotron self-absorption are also included in our code. However, those effects are not so important in our examples below.

The evolution of accelerated particles and photon production in a shell are computed with the time-dependent method, as we explained above. Considering the curvature effect, Doppler boosting due to the relativistic bulk motion of the shell, and the opening angle $\theta$, we can estimate the arrival time and energy of photons escaping from the shell for an observer. Based on those outputs, the temporal evolution of the photon spectrum emitted from “one shell” can be obtained. High-energy photons can be absorbed via $\gamma\gamma$ collisions with the EBL during propagation in the intergalactic medium. To obtain the spectra seen by observers, we adopt the model in Kneiske et al. (2004) for the EBL evolution.

The central engine may continuously eject shells that emit photons. Photons escaping from different shells can arrive at an observer simultaneously. We can model the temporal evolution of blazar emission by adding the contributions of such shells with different launch times. If we change the model parameters for each shell, various models including steady emission will be realized. To model steady emission from a steady flow, we assume that identical shells are continuously ejected at $R = R_0$ with a time step of $R_0/(c\beta\Gamma^2)$. The steady spectrum for an observer is comprised of the contributions of all shells at $R \geq R_0$. The time-integrated spectrum from one shell provides the average spectral shape from the steady flow. The steady spectral flux is easily obtained by dividing the time-integrated spectrum emitted from one shell by the shell ejection time step $R_0/(c\beta\Gamma^2)$.

For the steady emission model, there are six model parameters: the initial radius $R_0$, the radius where the injection and acceleration cease $R_\epsilon$, the bulk Lorentz factor $\Gamma$, the initial magnetic field $B_0$, the injection rate $N_\epsilon$, and the diffusion coefficient $K'$. These are the minimum parameters required in our model. In Section 4, we will additionally consider the radial evolution of $N_\epsilon$ and $K'$. In this case, the power-law indices are introduced as two additional parameters. The number of parameters is not many compared with previous models. For example, the model parameters for Mrk 421 in Abdo et al. (2011), a one-zone synchrotron self-Compton (SSC) model, is 11.

The model parameters for this blazar are $\Gamma = 25, B_0 = 0.03$ G, and $W = R_0/\Gamma = 2.8 \times 10^{16}$ cm. We inject electrons with a constant rate $N_\epsilon = N_0 = 1.5 \times 10^{46}$ s$^{-1}$ in spherically symmetric evaluation over a timescale of $\Delta T_{\text{inj}} = W/c$ in the shell frame. This implies that the electron injection ceases at $R = 2R_0$. In this injection timescale, turbulence in the shell accelerates electrons with the diffusion coefficient $K' = 4.3 \times 10^{-3}$ eV$^{1/3}$ s$^{-1}$. After the end of the electron injection, we assume that the turbulence is terminated as well, so electrons cool monotonically via radiation and adiabatic expansion.

Figure 3 shows the evolution of the photon energy distribution in the shell frame. As the electron injection and acceleration proceed, the electron energy density grows and achieves a maximum at $R = 2R_0$. The $\nu\nu' n(\epsilon')$ spectrum has a maximum at $\sim 10^{11}$ eV, where $\nu(\epsilon') = N'(\epsilon')/V'$. This peak energy is determined by the duration time of the acceleration, which corresponds to the acceleration timescale of this energy. After the end of the electron injection and acceleration, the shell expansion causes the density drop and adiabatic cooling lowers the electron energy. Thus, the spectral peak energy shifts to lower energies as the shell expands. The effect of the radiative cooling is seen as the growth of the sharpness of the spectral cutoff above the peak energy.

In this parameter set, the radiative cooling is not so efficient that the photon energy density is always lower than the electron energy density, as indirectly shown in Figure 4. Nonetheless, the photon energy density overtakes the magnetic energy density in the later phase, which leads to sufficient SSC emission.

The steady-state spectrum obtained from our model is shown in Figure 5. Our simple assumption (constant injection and diffusion coefficient) succeeds in reproducing the observed hard spectrum and avoiding the Fermi upper limit. The hard electron spectrum due to the Fermi-II acceleration naturally leads to this hard spectrum.

3. IES 1101−232

First, we adopt the Fermi-II acceleration model for the TeV blazar IES 1101−232 (Aharonian et al. 2007b). The detection of TeV gamma rays from this high-redshift ($z = 0.186$) object piqued interest in light of the constraint on the EBL (Aharonian et al. 2006). The Fermi telescope provided a stringent upper limit in the GeV energy range (Neronov & Vovk 2010). This implies a very hard spectrum from GeV to TeV.

The model parameters for this blazar are $\Gamma = 25, B_0 = 0.03$ G, and $W = R_0/\Gamma = 2.8 \times 10^{16}$ cm. We inject electrons with a constant rate $N_\epsilon = N_0 = 1.5 \times 10^{46}$ s$^{-1}$ in spherically symmetric evaluation over a timescale of $\Delta T_{\text{inj}} = W/c$ in the shell frame. This implies that the electron injection ceases at $R = 2R_0$. In this injection timescale, turbulence in the shell accelerates electrons with the diffusion coefficient $K' = 4.3 \times 10^{-3}$ eV$^{1/3}$ s$^{-1}$. After the end of the electron injection, we assume that the turbulence is terminated as well, so electrons cool monotonically via radiation and adiabatic expansion.

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4. STEADY EMISSION IN Mrk 421

The Fermi-II acceleration model can naturally explain the hard-spectrum blazar as shown in the previous section. If this acceleration mechanism is universal in the quasi-steady emission from blazars, relatively softer spectra for other ordinary blazars should be also fit by this model. However, the hard spectra obtained from the simplest model apparently contradict the observed one. In order to overcome this problem, we consider the temporal (equivalently radial) evolution of the electron injection rate and diffusion coefficient.

As a representative example of blazars, we consider Mrk 421 at \( z = 0.031 \), whose broadband spectrum from the radio to TeV is one of the most precisely observed spectra. Here, we adopt the spectrum obtained from the 4.5 month long multi-frequency campaign (2009 January 19 to 2009 June 1; Abdo et al. 2011). During this campaign, Mrk 421 showed low activity and relatively small flux variations at all frequencies. Thus, this data set can be used to study steady emission from blazars. In Abdo et al. (2011), to fit the obtained spectrum by leptonic models, an electron distribution of three power-law functions (namely two breaks) is required. This may be because the spectral shape around the peak energy from optical to X-ray bands is too broad for single-break models. While the origin of such spectral breaks is unknown, the time-dependent model may provide us with a new possible picture for this blazar.

For electrons injected at a later phase, the effective duration of the acceleration becomes shorter than that of the electrons injected initially. Such electrons injected later remain in the low-energy regime. Therefore, an increase in the injection rate for a finite injection timescale leads to a softer electron spectrum than that with a constant injection rate. The diffusion coefficient may also evolve with time. A decrease of the diffusion coefficient makes electrons injected later remain in the low-energy regime. However, a too rapid decline of \( K' \) results in a too low maximum energy of the electrons. In order to reproduce the observed spectrum, hereafter, we adjust the evolution of \( N'_e \), while the evolution of \( K' \) is fixed as \( K' \propto R^{-1} \) for simplicity.

4.1. Simple SSC Model

Figure 6 shows the result obtained from our model with the temporal evolution of the injection rate and diffusion coefficient. The model parameters are \( \Gamma = 15 \), \( R_0 = 0.13 \) G and \( W' = R_0/\Gamma = 1.0 \times 10^{16} \) cm. The duration time of the electron injection and acceleration is assumed to be \( \Delta T'_{\text{inj}} = 2W'/c \) (end at \( R = 3R_0 \)), which is longer than the assumption in 1ES 1101–232 to enhance the effects of the temporal evolution. In this duration time, the injection rate is assumed to evolve as \( N'_e = N_0(R/R_0)^{\gamma} \), where \( N_0 = 9.8 \times 10^{35} \) s\(^{-1} \). Similarly, the diffusion coefficient evolves as \( K' = K_0(R/R_0)^{-3} \), where \( K_0 = 1.3 \times 10^{-2} \) eV\(^{1/3}\) s\(^{-1} \).

The synchrotron component is well reproduced by this model. An advantage of this model is that the curved spectral feature is naturally explained by the power-law evolution of the injection and diffusion, while the usual shock acceleration models need breaks at ad hoc energies in the injection spectrum.

The curved photon spectrum is a direct consequence of the curved electron spectrum, as shown in Figure 7. The electron spectra are softer than the case in 1ES 1101–232 owing to the temporal evolution of the electron injection. Just above \( \epsilon'_{\text{c}} = \gamma'_m m_e c^2 \), the electron spectral index is about 1.06, but the spectrum gradually becomes softer with increasing energy. After the acceleration ceases, the electron spectra show a sharper
Figure 6, the synchrotron spectrum becomes narrower than the
background spectrum rather than the absorption effect. The actual
injection rate is determined by short-wavelength turbulence that resonates with the gyro motion of
low-energy electrons. Hence, synchrotron self-absorption is negligible in our
model. So, synchrotron self-absorption is negligible in our
model.

We now confirm this statement analytically. At \( R = 3R_0 \), the
spectral density is \( n'(\epsilon_e) \approx 2200 \text{erg}^{-1} \text{cm}^{-3} \) at \( \epsilon_e \approx m_e c^2 \).
If we denote this as \( n'(\epsilon_e) = C\epsilon_e^{-3} \), \( \approx 0.18 \text{cm}^{-2} \), which is
comparable to the density \( \sim 1 \text{cm}^{-2} \), obtained with the time
integrated number of electrons, \( 3^5R_0N_0/8\pi \Gamma \) (note \( dt'/dR \approx 1/(cT) \) and volume, \( V = 4\pi(3R_0)^2W \).
The formula in Rybicki & Lightman (1979) gives the optical depth due to synchrotron
self-absorption as

\[
\tau_{SSA} = 1.3 \times 10^{-2} \left( \frac{C}{0.18 \text{ cm}^{-3}} \right) \left( \frac{B'}{0.1 \text{ G}} \right)^{3/2}
\times \left( \frac{\epsilon_e'}{10^{15} \text{ eV}} \right)^{-5/2} \left( \frac{W'}{10^{16} \text{ cm}} \right),
\]

or the break photon energy, defined as \( \tau_{SSA}(\epsilon_e') = 1 \), becomes

\[
\epsilon_e' = 1.8 \times 10^{-6} \left( \frac{C}{0.18 \text{ cm}^{-3}} \right)^{2/5} \left( \frac{B'}{0.1 \text{ G}} \right)^{3/5}
\times \left( \frac{W'}{10^{16} \text{ cm}} \right)^{2/5} \text{ eV}.
\]

As shown in Figure 7, the low-energy electron density in our
model is much less than the extrapolations of the analytic
models. As we have discussed, this is one of the reasons why
the self-absorption frequency is relatively low. Distinct from the
assumption in the above analytical estimate of Equation (9),
the electron distribution has a break at \( \epsilon_c = \gamma_c \epsilon_{J} = 100 \), so
that the above break energy would decrease. Thus, the spectral
break at \( \epsilon \sim 10^{-5} \text{ eV} \) is mainly due to the break in the electron
spectrum rather than the absorption effect. The actual \( \gamma_c, m \)
may be smaller than what we assumed (\( \gamma_c, m \) has been set as 100
to save computational costs). So, the electron spectrum in our
model can comprehensively explain the spectrum from the radio
to the X-ray without introducing a minimum Lorentz factor
\( \gamma_{c, \text{min}} \).

In our model, while the diffusion coefficient decreases, a sharp
rise in the injection rate (\( N_{\epsilon_e} \propto R^2 \)) is required, which may seem
unnatural. The electron injection rate is determined by short-wavelength turbulence that resonates with the gyro motion of
low-energy electrons. Such waves may have a different evolution
from the turbulence that accelerates high-energy electrons.
While the long waves are produced by large-scale instabilities,
such as the Kelvin–Helmholtz instability, etc., the origin of the
short waves may be the cascade of the long waves. In this
case, the efficiency of the electron injection may grow later relative to the development of the long waves. Another possibility
is that the low-energy threshold of the electrons in the acceleration
process decreases gradually. Let us consider electrons at energies of the cutoff tail in the Maxwellian distribution. When
the minimum wavelength is relatively long, only higher energy
electrons can be injected into the acceleration process. If the
minimum wavelength gradually decreases as the cascade proceeds, lower energy electrons are also injected. This mechanism
may cause a sharp rise in the injection rate, retracing the cutoff
shape in the Maxwellian distribution.

We have assumed the following evolutionary forms: \( N_{\epsilon_e} \propto R^2 \)
and \( K' \propto R^{-1} \). If either \( N_{\epsilon_e} \) or \( K' \) is constant, as shown in Figure
6, the synchrotron spectrum becomes narrower than the
observations. The broad peak represented by the X-ray and infrared–optical data points is achieved by the combination

\[ \gamma_c \geq 100 \]
of this evolution. Of course, our example of the parameter evolutions may not be a unique solution. On the other hand, we find that the X-ray spectrum shape can be solely fit without this evolution, if we neglect the infrared–optical and radio data. The X-ray spectral shape is determined by the high-energy cutoff shape of the electron spectrum, which may be controlled by the diffusion process in momentum space and radiative cooling rather than the parameter evolution.

Our parameter choice reproduces the flux level of the IC component as well. However, the observed flux at $\sim 100$ MeV is significantly higher than the model spectrum. The spectrum obtained with Fermi is relatively flat compared with the synchrotron spectrum. The steady SSC spectrum obtained with our time-dependent model is hard to reconcile with the Fermi data.

4.2. SSC+EIC Model

The simplest method to fit to the GeV flux is an introduction of another emission region that contributes to this energy range. Such two-zone models have been discussed by several authors such as Ghisellini et al. (2005).

Here, we consider another possibility, the effect of an external photon field, to reproduce the 100 MeV–GeV flux in Mrk 421. While external photons are indispensable to explain IC components of flat spectrum radio quasars (FSRQs), BL Lac objects have been fit without external photons. For FSRQs, optical photons from broad-line regions are a candidate for the external photon field. However, the average electron energy in BL Lac objects is much higher than that in FSRQs so that the Klein–Nishina effect makes the contribution of the external optical photons negligible. Moreover, the typical energy range of the IC-scattered optical photons becomes much higher than the GeV energy range. Here, we consider external radio photons, which may come from compact radio lobes as seen in young radio-loud AGNs (Snellen et al. 2004).

We consider an external photon field whose spectral peak in the $\varepsilon f(\varepsilon)$ diagram is $10^{-6}$ eV (240 MHz). This corresponds to $\sim 10^{-6} \Gamma$ eV $\sim 10^{-5}$ eV in the shell frame, which is safely high enough to avoid synchrotron self-absorption (see Equation (9)). Since we have no definite model for the spectrum, the Band function (Band et al. 1993), smoothly joined power laws, is adopted here. The low- and high-energy photon indices (defined as $-d \ln f(\varepsilon)/d \ln \varepsilon + 1$) are chosen to be $-1$ and 2.5, respectively. The total luminosity is $L_{\text{ex}} = 4.9 \times 10^{38}$ erg s$^{-1}$. When photons are isotropically distributed, the photon energy density in the comoving frame of the jet is $4U_{\text{ex}}\Gamma^2/3$ (Dermer & Schlickeiser 2002). However, the isotropic approximation may not be accurate. So, we neglect the numerical coefficient and assume the comoving energy density to be

$$U_{\text{ex}}' = \frac{\Gamma^2 L_{\text{ex}}}{\pi R^2 c}. \quad (10)$$

The spectral shape is simply shifted by a factor of $\Gamma$ in the shell frame. Based on this photon distribution in the shell frame, we calculate the contribution of external IC (EIC). Of course, our time-dependent code can wholly take into account the non-linearities of the cooling processes (Zacharias & Schlickeiser 2012). For simplicity, we assume isotropic emission in the shell frame, although the external photons may be beamed in this frame. Therefore, the contribution of the external photons is simply taken into account by adding the boosted external photons to the photon field in the shell frame. The external component is intrinsically indistinguishable from the internal synchrotron/IC photons.

As shown in Figure 8, the photon energy density is initially dominated by the external photons so that the ratio $U_{b}/U_{\gamma}$ is almost constant. As the electron injection proceeds, photons produced in the shell becomes predominant, as seen at $R > 2R_0$, and its energy density overtakes the magnetic one. Similarly to 1ES 1101–232, the emission efficiency is so low that most of the electron energy is not released as radiation.

The final results for our model with external photons are shown in Figure 9, where the model spectrum well agrees with observed spectra from the radio to the TeV. As we have explained in Section 2, the steady spectrum is a superposition of emission from multiple shells at different $R$. However, the steady photon spectrum is virtually determined by the electron spectral shape at $R = 3R_0$ because the rapid increase of the electron injection makes the electron density reach a maximum at the end point of the injection/acceleration (see Figure 7). However, the emission spectrum from one shell for an observer evolves as shown in Figure 9. The synchrotron component shows

![Figure 8](image_url)  
Figure 8. Evolution of the energy density ratios in a shell in the model for Mrk 421 with external photons (see Section 4.2). The label notations are the same as in Figure 4. (A color version of this figure is available in the online journal.)

![Figure 9](image_url)  
Figure 9. Steady photon spectrum (thick) for the model of Mrk 421 with external photons (see Section 4.2). Thin lines show the evolution of the photon spectrum emitted from one shell, neglecting the emission from the other shells. The steady spectrum can be interpreted as a superposition of those spectra. The time (ks) labeling each thin line is for observers at Earth. (A color version of this figure is available in the online journal.)
a hard-to-soft evolution. This may be due to the decay of the magnetic field. On the other hand, the SSC component, which has a peak around $10^{11}$ eV, does not show a drastic evolution in its hardness. The Klein–Nishina effect makes a peak at the energy that is determined by the maximum energy of the electrons. As a result, this peak energy is insensitive to the synchrotron peak energy. The EIC emission, whose spectral peak is clearly seen in the single-shell spectra, especially for the early period (10–32 ks), succeeds in reproducing the Fermi data.

While two-zone models are still promising, the success of the EIC model encourages single emission-region models. The EIC model needs another parameter set for the external photon field. The essential parameters are its luminosity and peak photon energy because the details of the photon spectral shape are not so important. Thus, the practical number of model parameters is 10 in this model (see the last part in Section 2). This number is still fewer than the DBP model in Abdo et al. (2011), although the DBP model is not designed to address the radio spectrum.

5. VARIABILITY IN Mrk 421

The MAGIC telescope reported day-scale flux variations and a clear correlation between TeV and X-ray fluxes of Mrk 421 (Albert et al. 2007). Fossati et al. (2008) claimed a possible lag (~2 ks) of TeV flares relative to soft X-ray flares, whereas TeV and hard X fluxes are well correlated (see also Acciari et al. 2011).

Spectral evolution obtained with Suzaku (Ushio et al. 2009) indicates that the spectral peak energy shifts to a higher energy with increasing flux in X-ray flares of Mrk 421. Another interpretation Ushio et al. (2009) claimed is that two components, “steady” and “variable,” coexist in X-ray flares. The “variable” component is described by a broken power law, while the “steady” component has an exponential cutoff at ~1 keV. In this section, based on the picture of the “steady” and “variable” components, we argue the spectral evolution in flares in Mrk 421. As shown in Tramacere et al. (2009), the flare spectra may provide a signature of Fermi-II acceleration. Note that we do not intend to fit individual flare spectra. We just probe the quantitative behaviors of the flare spectra with our time-dependent code.

In our steady flow approximation, the identical shells are continuously ejected from $R = R_0$, as shown in Figure 1. In order to simulate flares in Mrk 421, we replace one shell in the sequence of the shells with a shell that has a different parameter set from the other shells. Then, the time-dependent contribution from the replaced shell will produce a flare on the steady emission due to the other shells. In this section, the model for the steady emission is the same as the model with the external photons in Section 4.2.

5.1. Variable Plasma Parameters

First, we propose a model in which the replaced shell has a larger diffusion coefficient and lower magnetic field than those for the other shells. The other parameters are the same as those for the other shells except for $N_0'$. The magnetic field is taken to be $B_0 = 0.06$ G and $K_0$ is 1.5 times the value for the other shells. To harden the electron/photon spectrum of the flare, the injection rate is also changed to $N_0' = N_0(R/R_0)^5$ (remember that $N_0' \propto R^2$ for the other shells), where $N_0 = 4.9 \times 10^{44}$ s$^{-1}$.

As we have discussed previously, a flare in the high X-ray band is emitted by the highest-energy electrons, while the origin of the TeV flare is SSC emission from relatively lower-energy electrons. Because radiative cooling is very effective for the highest-energy electrons, the hard X-ray flare ceases faster than flares in other energy bands, as shown in Figure 11.
This tendency is not consistent with the observed synchronicity of hard X-ray and TeV flares or the hard lag between X-ray bands. The time-dependent simulations by Chen et al. (2011) also failed to reproduce this observed feature. The slight lag of 100 GeV–TeV light curves relative to the soft X-ray flare seems to be reproduced by our simulations. The model light curves show long tails, while typical light curves from blazars are almost symmetric in their rise and decay shape. This long tail is not due to the curvature effect, namely the contribution of off-axis emission. Since radiative cooling is inefficient for the electrons, the decay of the flares is regulated by adiabatic cooling. Unless a sudden shutdown of the emission is artificially adopted, emission from slowly cooling electrons yields long tails in their light curves.

The above two problems, the early termination of the hard X-ray flare and asymmetric light curves, are inevitable in our model. We have replaced only one shell, changing the physical parameters to produce a flare. This implies that a partial and discrete transition of the physical parameters occurs in the outflow. Realistic outflows may have a gradual parameter change in a wider spatial range. The symmetric light curve may be a result of this gradual parameter change. Moreover, if the onset of the magnetic field decay is faster than the increase of the diffusion coefficient, the observed delay of hard X-ray flares should be reproduced. Thus, the hard X-ray delay requires different evolution of the magnetic field and electron injection.

5.2. Variable Lorentz Factor

A fluctuation of the bulk Lorentz factor may cause a flare as well. Shifts of the spectral peak energies are naturally expected for a photon source with a higher \( \Gamma \). Strictly speaking, we cannot embed a faster shell in a steady flow of a constant \( \Gamma \). Such a shell interacts with the precedent shell and may be decelerated by shocks. Actual outflows may not be completely continuous. Hence, postulating a quasi-steady outflow as a background, we simply add the contribution of the faster shell to the emission discussed in Section 4 here.

Given a synchrotron luminosity, a higher \( \Gamma \) leads to a lower synchrotron photon density in the shell frame. In order to produce simultaneous X-ray and TeV flares, a weaker magnetic field is required even in this case. This means that a larger \( K' \) is also required to shift the spectral peaks higher. Here, a shell with \( \Gamma = 30 \) is assumed to be the origin of the flare. We adopt the same \( R_0 \) as before, but the high \( \Gamma \) leads to a narrower width \( W' = 5 \times 10^{15} \) cm. Other parameters are \( \Delta T'_{\text{inj}} = 2W'/c, B_0 = 0.03 \) G, \( \delta K' = K_0(R/R_0)^{-1} \) with \( K_0 = 3.9 \times 10^{-2} \) eV\(^1/3\) s\(^{-1}\), and \( N'_e = N_0(R/R_0)^{3} \) with \( N_0 = 4.9 \times 10^{44} \) s\(^{-1}\).

The obtained spectra are plotted in Figure 12. If we neglect the EIC emission in this model, flares are seen in only X-ray and TeV energy bands (solid lines). However, a single outflow model with the external photons imposes the EIC emission on the flare source. The higher \( \Gamma \) enhances the efficiency of the EIC; given the electron total number and energy distribution in the shell frame, the EIC luminosity is proportional to \( \Gamma^6 \) (a Doppler factor \( \delta \sim \Gamma \) is assumed), while the synchrotron luminosity is \( \propto \Gamma^4 \) (see, e.g., Dermer & Schlickeiser 2002). Hence, the amplification of the GeV flare due to the EIC emission is very large (the dashed line in Figure 12). If this huge GeV flare is not observationally favorable, the high-\( \Gamma \) model with the external photons will be rejected. In this case, \( \Gamma \) should be almost constant, or a different source for the GeV steady emission (no external radio source) may be required.

When shells with different values of \( \Gamma \) are injected, they collide and particles are accelerated by first- and second-order Fermi processes. Böttcher & Dermer (2010) elaborated on the emission properties from such collisions and showed that the various types of evolution of the emission spectrum are induced by such collisions. We have neglected such effects in the EIC models, which may be observationally constrained based on the high sensitivity to \( \Gamma \). This should be tested in future studies.

The light curves for the model without the EIC emission are plotted in Figure 13. The high \( \Gamma \) leads to a short variability timescale (\( \propto \Gamma^{-2} \)) compared with the model in Section 5.1. The qualitative behavior is similar to the case in Figure 11. The very weak magnetic field extends the cooling timescale for the highest-energy electrons. Thus, the early termination of the hard X-ray flare is not prominent compared with the model in Section 5.1.

5.3. Shock Acceleration

While the quasi-steady emission may be due to the Fermi-II acceleration, the flare phenomena may be attributed to shocks in
the outflow. The interpretation in Ushio et al. (2009) is compatible with such a picture. As a model with a combination of Fermi-I and Fermi-II processes, Weidinger & Spanier (2010a, 2010b) calculated electron and photon spectra, dividing the blazar region into acceleration and radiation zones. The accelerated electrons escape from the acceleration zone and are injected into the radiation zone. By changing the particle injection, they obtained the light curves for 1ES 1218+30.4 and PKS 2155−034.

Within our picture, we also test the Fermi-I model with our code. We inject shock-accelerated electrons of the single power law with an exponential cutoff into the flaring shell. The power-law index is $p = 2$ and the cutoff Lorentz factor is $\gamma_{e,\text{max}} = 10^7$. The minimum Lorentz factor is taken to be $\gamma_{e,\text{min}} = 15$. The bulk Lorentz factor is $\Gamma = 15$ and the shell width is $W = 1.0 \times 10^{16}$ cm, the same as those in the steady component. The injection is assumed to be constant over a timescale $\Delta T_{\text{inj}} = W/c$ and we neglect the reacceleration by turbulence. The total energy of electrons is $E_{e,\text{iso}} = 5 \times 10^{51}$ erg in spherically symmetric evaluation ($E_e = E_{e,\text{iso}} \theta_j^2/2 = 1.1 \times 10^{51}$ erg). Even in this model, a weak magnetic field is required ($B_0 = 0.06$ G) to produce a TeV flare (see the dashed line in Figure 14 for the model with $B_0 = 0.13$ G and $E_{e,\text{iso}} = 2 \times 10^{51}$ erg).

As shown in Figure 14, the synchrotron spectra show flat shapes (the photon index is $\sim 2$) in the X-ray band. These values are significantly different from the other models. The light curves in Figure 15 show coincident peaks at $\sim 20$ ks from keV to 100 GeV. This is due to the continuous injection of the high-energy electrons. The electron injection and cooling balance each other in the high-energy regions, so the electron energy distribution remains quasi-steady until the electron injection stops. The termination times of the emissions are controlled by the electron injection. The slight delay of the TeV light curve may come from the evolution of the seed X-ray photons.

6. SUMMARY AND DISCUSSION

In this paper, we have simulated the temporal evolution of high-energy electrons and photon production in relativistically outflowing shells. Our numerical code can follow the electron distribution with the effects of the electron injection, acceleration, synchrotron cooling, and IC cooling. The full non-linearities of IC cooling, including the Klein–Nishina effect, are taken into account. We have considered the Fermi-II process as the electron acceleration mechanism, while there are other candidates for the acceleration mechanism, such as Fermi-I. The Fermi-II process, driven by some kind of turbulence in the outflows, can naturally make electron spectra harder than those predicted by the simplest version of diffusive shock acceleration theory. As opposed to the shock acceleration in supernova remnants, the maximum energy of the electrons is expected to be far below that in the Bohm limit. Those characteristics are favorable to explain blazar photon spectra. In this method and model, the diversity in the temporal evolution of the electron injection and acceleration can be expected to generate a variety of photon spectral shapes.

We have modeled steady photon emission by superposition of time-evolving emission from continuously ejected multiple shells. The photon spectrum of the TeV blazar 1ES 1101−232 is well reproduced by a simple model with a constant injection rate and diffusion coefficient. For Mrk 421, which shows a softer
spectrum than that in IES 1101–232, we need to adjust the evolution of the electron injection rate, etc. to fit the spectrum. A power-law evolution of $N_e \propto R^7$ makes a curved electron spectrum, which produces a good fit to the observed synchrotron spectrum from the radio to X-ray bands. An advantage in our model is that we do not need to introduce unprescribed energy scales as break energies in the electron spectrum. However, the required rapid growth of the injection rate has not been theoretically justified yet. Future progress in the study of the injection processes with time-dependent ways will be important to examine the validity of the model.

Our Fermi-II model explains the radio data as well as the optical and X-ray data as the emission from a single source. In most of preceding models based on shock acceleration, which fit the optical and X-ray data of blazars, the electron density in the low-energy range is much higher than in our model (see Figure 7), so that the synchrotron self-absorption effect is much stronger. As a result, the radio data have difficulty explaining simultaneously the optical and X-ray data by one-zone shock acceleration models. These data are frequently explained by the superposition of multi-zone self-absorbed emission (e.g., Königl 1981). The picture we proposed is different from such models.

In this paper, we have conservatively assumed the Kolmogorov type of turbulence, $q = 5/3$. An alternative way to make a soft electron spectrum is to adopt a larger value of $q$. Even if $N_e$ and $K'$ are constant, the hard-sphere scattering ($q = 2$; see, e.g., Park & Petrosian 1995) leads to a soft electron spectrum, as shown in Figure 2. This case implies that the acceleration timescale is independent of the particle energy, which is similar to the original idea of Fermi (1949). Another possibility is the effect of particle escape (e.g., Becker et al. 2006); we have not included this effect. Especially for the model with $q = 2$, the escape timescale is independent of the particle energy so that the effect can be important. The escape effect will not only change the spectral index of the electron distribution, but in some cases may also introduce cutoffs in $N_e$. We may need at least two zones, an acceleration region and an emission region, to simulate such models without neglecting the contribution of the escaped particles. Note that the model of Weidinger & Spanier (2010a, 2010b) is a two-zone model. However, the accelerated electrons are injected in the emission zone uniformly and the effect of geometrical separation of the acceleration and emission zones has not been considered.

It is interesting that the obtained electron spectrum is close to the log-parabolic function in the most important energy range. In Massaro et al. (2004a), the origin of this shape is attributed to the energy dependence of the escape probability. However, our time-dependent calculations have not included the escape effect. The analytical study of the Fermi-II process by Park & Petrosian (1995) based on Green’s functions may be a meaningful hint for this spectral shape. Since most high-energy electrons are injected at early times, their spectral shape is primarily determined by the Green’s function for a single injected energy. The spectral shapes around the synchrotron peak predicted by the DBP and log-parabolic models are hard to distinguish from our model. Thus, to search for the signature of the electron minimum energy required in those analytical models, future infrared and submillimeter observations will be required.

In order to reproduce the GeV flux for Mrk 421 by our single emission-region model, an external radio photon field is needed. The radio photons interacting with high-energy electrons in the outflow can be upscattered to GeV energies. The required radio luminosity $4.9 \times 10^{38}$ erg s$^{-1}$ is far below the bolometric luminosity $1.4 \times 10^{43}$ erg s$^{-1}$ (assuming $\theta_e = 1/15$). Alternatively, an additional emission region may contribute as a GeV photon source. Correlation analyses of flux variabilities between GeV and another band may provide a clue to the GeV emission region. While significant variability (a factor of about three) in the GeV flux has been reported (Abdo et al. 2011), the correlations with X-ray or TeV variabilities seem still ambiguous for determining the model.

By replacing a shell in the sequence of the identical shells and changing the parameters, we simulate flare phenomena. In this method, the flare light curves show asymmetric shapes. The flare emission gradually fades out via adiabatic cooling. To reproduce the symmetric light curves as is frequently seen in blazar flares, gradual changes of the parameters may be required, while our models correspond to discrete changes of the parameters. The cooling time of the electrons that emit hard X-rays is quite short. Therefore, a gradual cessation of the electron injection or acceleration in this highest energy range may be required to synchronize the peak times of the hard X-ray and TeV light curves. A sudden shut down of acceleration/injection would lead to an early hard X-ray termination. The most critical aspect to produce simultaneous flares in the X-ray and TeV bands is to decrease the magnetic field. The Klein–Nishina effect prevents TeV flares caused by a growth of the diffusion coefficient that increases the electron maximum energy. An enhancement of the SSC emission efficiency by weakening the magnetic field is required to generate a TeV flare. The required anti-correlation between the fluxes and the magnetic field is a challenging problem.

For the EIC model, high variability of the bulk Lorentz factor $\Gamma$ is not favorable. Since the EIC emission is sensitive to $\Gamma$, the observed GeV variability strictly constrains the fluctuation of $\Gamma$ by about a factor of three.

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