A method to calibrate the absolute energy scale of air showers with ultra-high energy photons

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Calibrating the absolute energy scale of air showers initiated by ultra-high energy cosmic rays is an important experimental issue. Currently, the corresponding systematic uncertainty amounts to 14-21% using the fluorescence technique. Here we describe a new, independent method which can be applied if ultra-high energy photons are observed. While such photon-initiated showers have not yet been identified, the capabilities of present and future cosmic-ray detectors may allow their discovery. The method makes use of the geomagnetic conversion of UHE photons (preshower effect), which significantly affects the subsequent longitudinal shower development. The conversion probability depends on photon energy and can be calculated accurately by QED. The comparison of the observed fraction of converted photon events to the expected one allows the determination of the absolute energy scale of the observed photon air showers and, thus, an energy calibration of the air shower experiment. We provide details of the method and estimate the accuracy that can be reached as a function of the number of observed photon showers. Already a very small number of UHE photons may help to test and fix the absolute energy scale.

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Measuring the cosmic-ray flux and spectral features gives important clues on the origin of ultra-high energy (UHE) cosmic rays. For this, a sufficiently good energy reconstruction of the air showers initiated by UHE cosmic rays is needed which is an experimental challenge. Using the fluorescence technique, ideally in coincidence with a detector array, giant air shower observatories achieve a systematic uncertainty of 14–21% in determining the absolute energy scale. This is based on a piece-by-piece calibration of all relevant components in the reconstruction chain which includes the fluorescence yield in air, the transparency of the atmosphere, optical properties and the electronic response of the detector, and a correction for the invisible shower energy.

Comparing the present flux spectra from different experiments to each other, one finds a fairly good agreement when allowing for shifts in energy within the quoted systematic uncertainties. This shows that relative energies can be determined quite well by shower experiments. However, for getting better clues on the origin of UHE cosmic rays, a direct comparison of data to theoretical predictions in terms of an absolute energy scale is needed. This comparison is limited by the experimental systematic uncertainty quoted above.

As an example, a currently open question is whether cosmic rays at highest energy are (mostly) protons and whether the observed flux suppression above $4 \times 10^{19}$ eV is due to photo-pion production with the CMB during propagation (GZK effect). Based on this ansatz, very detailed theoretical predictions about the energy spectrum above $\sim 10^{18}$ eV were made, including the absolute energies of prominent features in the spectrum like a “dip” (due to pair production) or a “cutoff energy $E_{1/2}$” (due to photo-pion production). A confirmation of these predictions to high precision would constitute significant evidence for the existence of UHE protons and for their interaction with the CMB. This, in turn, would have profound implications for future research in terms of cosmic-ray astronomy at UHE and searches for GZK neutrinos and photons. In fact, a dip-like feature (ankle) and a suppression at UHE are observed. However, alternative, completely different scenarios can also explain the observed spectra within the given uncertainties.

Reducing the experimental systematic uncertainty of the absolute energy scale could significantly help to test the different scenarios and clarify the situation.

Here we present and study the potential of a new, completely independent method that could allow a direct end-to-end calibration of the absolute energy scale at highest energy above $\sim 2 \times 10^{19}$ eV. It makes use of the quite sharp threshold behaviour of pair production by UHE photons in the magnetic field of the Earth (geomagnetic conversion, or preshower). This is a well-known process of standard physics that can be calculated accurately by QED.

For the new method to work, one needs (1) a sufficiently good determination of the relative shower energy, of the shower direction and a sufficiently good separation between three classes of events (converted photons, unconverted photons, non-photons) and (2) the observation of UHE photon events. While the techniques for (1) are available as also discussed below, an observation of
UHE photons is still lacking (see 13, though). However, current experiments have a growing sensitivity for the detection of UHE photons (e.g. 14–17) both due to increasing data sets and due to an improved measurement of air showers by upgrading the detectors. And, as we will see, already a very small number of UHE photon events may help to test and fix the absolute energy scale.

The total probability \( P_{\text{conv}} \) of the UHE photon conversion is closely related to the primary photon energy \( E \). It can be concluded from e.g. 13 that \( P_{\text{conv}}(E) \) is a strictly increasing function of \( E \) for terrestrial geomagnetic conditions up to the maximum cosmic-ray energies observed by now. Particularly, \( P_{\text{conv}}(E) \) shows a quite sharp threshold behaviour, with \( P_{\text{conv}} \) increasing from 10% to 90% within a factor \( \sim 2 \) in energy. The key idea here is that one can determine in turn the absolute photon energy \( E = E(P_{\text{conv}}) \) provided \( P_{\text{conv}} \) can be measured to sufficient precision.

Fig. 1 shows examples of this inverse relation \( E(P_{\text{conv}}) \) for random arrival directions at two different observatory locations: the Pierre Auger Observatory 19 in Argentina, and the Tunka experiment 20 in Russia. The calculations were performed with the program PRESHOWER 21. The local geomagnetic field is significantly different at the two sites: \( \sim 0.23 \) G at Auger and \( \sim 0.58 \) G at Tunka. This is reflected by the energy shift in lines in Fig. 1. On the other hand, it can be seen that the relations plotted in Fig. 1 have similar shapes in terms of the slope steepness \( s \equiv \frac{dE}{dP_{\text{conv}}} \). The slope is important for the proposed method, as it affects the precision of finding \( E \): for a given uncertainty \( \Delta P_{\text{conv}} \) in measuring \( P_{\text{conv}} \), a small value of \( s \) results in a small uncertainty of \( E \), since \( \Delta E \approx s \Delta P_{\text{conv}} \). For instance, assuming an uncertainty \( \Delta P_{\text{conv}} = 0.1 \) one can estimate the relative energy uncertainty for different values of \( P_{\text{conv}} \): \( \Delta E(P_{\text{conv}} = 0.5)/E \approx 0.05, \Delta E(P_{\text{conv}} = 0.1 \) or \( 0.9)/E \approx 0.2 \) and \( \Delta E(P_{\text{conv}} = 0.01 \) or \( 0.99)/E \approx 1.0 \).

It turns out that the range \( 0.1 \leq P_{\text{conv}} \leq 0.9 \) where the slope \( s \) is sufficiently small, is most sensitive for an accurate energy determination.

We now show how to determine \( P_{\text{conv}} \) and \( \Delta P_{\text{conv}} \) from photon observations so that \( E \) and \( \Delta E \) can be concluded. As an illustration of the method, let us first consider the artificial case of observing \( n \gg 1 \) photon air showers of same primary energy and arrival direction. Out of these, \( k \) are observed to be initiated by preshivers (primary photon converted), \( 0 \ll k \ll n \). The probability of observing \( k \) converted photon events out of \( n \) photon showers is given by a binomial probability distribution with the maximum at \( P_{\text{conv}} = k/n \). In turn, the observed ratio \( k/n \) is the best estimate for \( P_{\text{conv}} \), and \( E(P_{\text{conv}}) \) can be concluded. The uncertainty \( \Delta P_{\text{conv}} \) can be found by checking the cumulative binomial distributions for different values of \( P_{\text{conv}} \) and finding the distributions for which the observed \( k \) can be excluded at a specified confidence level. For example, considering 10 simulated photon events arriving at the Pierre Auger Observatory from geographical South at a zenith angle of 13°, out of which 3 are classified as converted, one gets an absolute energy of \( E(n = 10, k = 3) \approx 8.4^{+11\%}_{-8\%} \times 10^{19} \text{eV} \).

In a realistic scenario, the photon events (in total perhaps just a few) have different energies and arrive from different directions. In this case a measurement of a single value of \( P_{\text{conv}} \) for a certain direction is not possible anymore. Nevertheless, the energy calibration based on a measurement of just one number describing a “global” photon conversion rate is feasible as long as the shower experiment provides a sufficiently good measurement of relative shower energies. All shower energies are affected by the same factor when adjusting the absolute energy scale, and it is this factor that needs to be determined.

Consider \( n \) photon events numbered by index \( i = 1 \ldots n \) out of which \( k \) were identified as converted. Based on current reconstruction methods (using e.g. signal strength in fluorescence or ground detectors), an initial estimate of energies \( E_{\text{ini}}(i) \) is obtained. While the relative energies are determined sufficiently good, the values \( E_{\text{ini}}(i) \) might differ from the true primary energies, but all of them differ by the same factor \( f_{\text{opt}} \). The purpose of the following method is to determine \( f_{\text{opt}} \).

We define the set \( \{C\} \) of photon event classes: \( (i) = 0 \) for an unconverted photon and \( (i) = 1 \) for a converted photon. We have \( \sum_i C(i) = k \) for given conversion probabilities \( P_{\text{conv}}(i) = k \). For given conversion probabilities \( P_{\text{conv}}(i) \) the probability of observing a specific set \( \{C\} \) is given by:

\[
Q(C) = \prod_{i_{\text{conv}} = 1}^{k} P_{\text{conv}}(i_{\text{conv}}) \prod_{i_{\text{unconv}} = k+1}^{n} (1 - P_{\text{conv}}(i_{\text{unconv}}))
\]

where \( i_{\text{conv}} \) and \( i_{\text{unconv}} \) number the converted and unconverted events, respectively.

To find \( f_{\text{opt}} \) and its uncertainty and, thus, to get the
absolute energy of the photon events, the following procedure is adopted (“bootstrapping” approach): (1) The initially estimated photon energies \( \{ E_{\text{ini}}(i) \} \) are multiplied by a factor \( f_j \) to generate a set of shifted energies \( \{ E_{\text{shift}}(i, j) \} \): 
\[
E_{\text{shift}}(i, j) = f_j \cdot E_{\text{ini}}(i).
\]
Having the set of shifted energies we use the relation \( P_{\text{conv}}(E) \) to compute the corresponding values of conversion probabilities \( \{ P_{\text{conv}}(i, j) \} \) and \( Q_j \) with Eq. (1). (2) We repeat step 1 with different factors \( f_j \) (e.g. \( f_j \approx 0.7 \ldots 1.3 \) given the present experimental uncertainties of \( 14 \ldots 21\% \)). The optimum shift \( f_{\text{opt}} \) is the one which maximizes \( Q_j(\{ P_{\text{conv}}(i, j) \}) \) given by Eq. (1). This energy shift fits best the observation (i.e. \( \{ C \} \)). (3) We proceed and estimate \( \Delta f_{\text{opt}} \). The found \( f_{\text{opt}} \) determines a set of conversion probabilities \( \{ P_{\text{opt}}(i) \} \). Each of the probabilities \( P_{\text{opt}}(i) \) determines the expected class of the observed photon event: converted (probability of occurrence \( P_{\text{conv}}(i) \) or unconverted (probability of occurrence: \( 1 - P_{\text{conv}}(i) \)). We generate \( n_r \) random sets \( \{ C_r \} \) of “conversion flags” (i.e. photon class “converted” or “unconverted”), with \( r = 1 \ldots n_r \) numbering the sets. (4) We repeat steps 1 and 2 for each set \( \{ C_r \} \) and get the distribution of the corresponding optimum energy shifts \( \{ f_{\text{opt}} \} \). The width of this distribution is used to estimate the uncertainty (resolution) \( \Delta f_{\text{opt}} \).

We note that a two-sided confidence interval is obtained only if both \( i_{\text{conv}} > 0 \) and \( i_{\text{unconv}} > 0 \) which is the case we focus on in this paper. For \( i_{\text{conv}} = 0 \) or \( i_{\text{unconv}} = 0 \) (i.e. all photons either converted or unconverted), a one-sided confidence interval results. This can still serve to place interesting limits to the energy scale e.g. when just two or three unconverted photons at the lower end of the preshower energy range are observed. We leave this special case of just one event class in the data set, which is increasingly unlikely for growing \( n \), for a further study.

We now evaluate the performance of the method (resolution and bias) for data sets of size \( n = 3 \ldots 100 \) adopting realistic experimental conditions (see e.g. \([1, 22, 27]\)). We take a resolution of 8\% in energy and 1° in direction. Furthermore, we account for uncertainties in classifying the events (converted photons, unconverted photons, non-photons) by means of the shower observables \( X_{\text{max}} \) (depth of shower maximum) and \( N_\mu \) (number of ground muons) and take a resolution of \( \Delta X_{\text{max}} = 20 \text{ g cm}^{-2} \) and \( \Delta N_\mu/N_\mu = 0.2 \). We note that alternative observables might be used as well, as for instance signal risetime and curvature of shower front \([15]\).

Based on simulations using CONEX \([28]\) with QGSJet II.04 \([29]\), we determined the relevant misclassification rates of (i) non-photons misclassified as photons, (ii) photons misclassified as non-photons, (iii) converted (unconverted) photons misclassified as unconverted (converted) photons. In essence, a simple but effective classification can be done by using \( N_\mu \) to separate photons from non-photons (typical difference of factor \( \sim 6 \) in \( N_\mu \)) and by using \( X_{\text{max}} \) to separate converted from unconverted photons (typical difference of \( \sim 200 \text{ g cm}^{-2} \) in \( X_{\text{max}} \)). From this approach, which could be improved to further optimize the classification, we obtain the following rates: (i) Based on \( N_\mu \), the fraction of protons misclassified as photons is \( \sim 10^{-4} \) (even smaller for heavier nuclei). Thus, the contamination of the photon sample by hadrons can be kept sufficiently small for cosmic-ray photon fractions down to the percent level or below. (ii) For the same \( N_\mu \), the cut of photons misclassified as non-photons is below 10\% which implies a minor loss of photon event statistics. As the misclassification rates are somewhat different for unconverted photons (\( \sim 6\% \)) and converted photons (\( \sim 2\% \)), this could lead to a slight overestimation of the energy scale (\( \sim 5\% \) for \( n = 5 \) and \( \sim 1\% \) for \( n \geq 10 \)). However, this bias can be corrected for, or reduced with more sophisticated cuts. (iii) Based on \( X_{\text{max}} \), the misclassification rate between converted and unconverted photons is below 10\%. The specific cut values can be adjusted in such a way that the expected number of misclassified converted photons equals that of misclassified unconverted photons. In this way, no bias is introduced.

Taking these experimental uncertainties into account, the performance of the method (steps 1–4) to calibrate the absolute energy of UHE photons has been determined by Monte Carlo simulations. The resulting resolution is shown in Fig. 2 as a function of the size of the data set. The simulations were performed for the conditions of the Pierre Auger Observatory for photons above \( 4 \times 10^{19} \text{ eV} \) with spectral index \( -4.2 \) (according to Ref. \([30]\)) and shower zenith angle \(< 80^\circ \) \([10]\)). As including events with \( P_{\text{conv}} \) values close to 0 or 1 would vary the total probability (Eq. (1)) only slightly within a wide range of energy shifts and would not improve the accuracy of determining the optimum shift \( f_{\text{opt}} \), only events with \( 0.1 \leq P_{\text{conv}}(f_j = 1.0) \leq 0.9 \) are counted. As mentioned earlier, we restrict our analysis to data sets which contain both classes of photon events (relevant only for very small \( n \)). As can be seen from Fig. 2, the resolution improves \( \propto (1/\sqrt{n}) \). It is \( \sim 20\% \) for \( n = 5 \), \( \sim 14\% \) for \( n = 10 \), and \( < 10\% \) for \( n > 20 \). Already with few events, an accuracy comparable to current approaches is reached.

We checked that the residual bias of the method, after correcting for the small effect from different misclassifications rates of converted and unconverted photons as non-photons discussed before in (ii), is below \( \sim 1\% \) for \( n \geq 10 \). For smaller \( n \), an overestimation of the energy scale of a few percent appears (up to \( \sim 10\% \) for \( n = 3 \)). This, however, can be understood due to the requirement of having both classes of photons in the sample, which leads to a small bias against unconverted photons (more numerous at lower energy). Again, a correction could be applied to account for and reduce this bias. In any
In summary, we presented a new method to determine the absolute energy scale of air showers initiated by UHE cosmic rays. It is an end-to-end calibration working directly in the interesting regime of highest energy. The method exploits the one-to-one relation between the probability for pair production by UHE photons in the geomagnetic field and the primary energy of these photons. The method is statistics-limited. Already with a small number of events, the accuracy of the method is comparable or superior to current approaches. Present and planned giant air shower experiments offer an unprecedented sensitivity to detect UHE photons. There is no guarantee that indeed UHE photons will be observed in future. But if so, they may become the “standard candle” for calibrating air shower energies.

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FIG. 2. Resolution of the energy calibration method as a function of the number $n$ of photon events, taking experimental uncertainties into account (see text). The resolution is well represented by the function $0.48/\sqrt{n}$ (solid line).
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