Bright solitons in cavity-QED arrays containing two-level atoms

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Abstract. We examine the problem of polariton soliton formation in the array of weakly coupled optical cavities, each containing an ensemble of ultracold non-interacting two-level atoms. An effective complex Ginzburg-Landau equation (GLE) is derived in the continuum limit taking into account the effects of cavity field dissipation and atomic polarization dephasing. Bright solitons are found to be supported under perturbations only in the upper (optical) branch of polaritons, for which the corresponding group velocity is controlled by tuning the atom-light detuning. With the help of perturbation theory for solitons, we also demonstrate that the group velocity of these polariton solitons is suppressed by the diffusion process.

1. Introduction
At present, photonic band-gap micro- and nanostructures, that is photonic crystals and photonic crystal fibres, cavity arrays, etc, are known to be important tools for controlling light propagation in media and for quantum optical information processing, see e.g. [1]. In particular, an effective reduction of the light group velocity is achieved in both hot and cold atomic ensembles [2,3], in resonant photonic band-gap structures, i.e. in coupled resonator optical waveguides (CROW) [4] and also in different types of solid-state multilayer semiconductor systems [5]. The physical reason for observable reduction of the group velocity of light propagation in a medium is determined by the so-called dark and bright polaritons. The objects are bosonic quasi-particles representing a linear superposition of photon states in the external electromagnetic field and the macroscopic (coherent) excitations in the two-level oscillator system.

It is known that quantum optical solitons are a natural candidate for quantum optical information processing purposes, see e.g. [6]. Studies of their transmission, formation, and transformation in coupled cavity-QED arrays represent important steps for achieving this aim. Polariton solitons have some advantages in this case. In particular, as demonstrated in [7], polaritonic nonlinearity can be high enough in comparison with pure optical nonlinearities achieved, e.g. with pure optical cavity solitons in VCSELs [8], and can permit to achieve a soliton regime for essentially smaller particle (polariton) number. More precisely, here we deal with the problem of polariton soliton formation occurring at quantum matter-filed interface in semiconductor quantum wells (QWs) embedded in Fabry-Perot microcavities. Nowadays polariton solitons and relevant superfluid behavior of nonequilibrium exciton-polaritons with
narrow-band (GaAs) semiconductor structures are observed in a few labs, [7, 9]. The main physical features of such solitons are connected with the balance between dissipation effects and external pumping that occurs for parametrical processes allowing to create non-equilibrium exciton-polariton Bose-Einstein condensate (BEC) at lower polariton branch under the temperatures of few kelvins. The solitons in experiments are excited within picoseconds time scale and localized in micrometer-scale sizes.

Recently, in [10] we proposed a 2D polaritonic crystal model that represents cluster material exhibiting high nonlinear properties due to the small but macroscopical number of two-level atoms strongly coupled with optical field in the cavity lattice. Basing on the Holstein-Primakoff approach, we have shown that such nonlinearity can basically be created by means of a saturation effects occurring due to localization of macroscopically small number of atoms at each cavity. Experimentally such a system can be designed by using 2D photonic crystal host with defect cavities [4].

In the paper we consider the problem of soliton formation in 1D weakly coupled cavity arrays containing two-level atoms that interact with single mode optical cavity fields – see Fig. 1. In the continuum limit, we use a multiple-scale envelope function (MSEF) method [11,12] to construct soliton solutions for considering nonlinear interactions among atoms as well as dissipation effects for them. In particular, we derive and examine a complex Ginzburg-Landau equation (GLE) that supports slow soliton propagations in the presence of polariton formation. Being a step in the studies of collective properties of light with interacting media, our results in the proposed coupled-cavity-QED system are shown to be connected with the formation of coupled matter-field states, upper (UB) and lower (LB) branch polaritons, which would be natural carriers for quantum information processing at matter-field interface.

2. The model of 1D cavity-QED array with two-level atoms

As illustrated in Fig. 1 we consider a one-dimensional (1D) array of optical cavities, each containing an ensemble of \(N_i\) of non-interacting two-level atoms under the extremely low temperatures, cf. [12–14]. For the configuration of an array of cavities, the atoms are assumed to be confined inside the cavities, while photons can tunnel from one cavity to another. We also assume that the tunneling process for photons is dominated by the hopping between adjacent sites. The value of hopping matrix elements is a function of the distance between cavities. If the distance between each cavity is sufficiently small, a cavity field will only hop to the nearest neighbors. Then the total Hamiltonian of interaction for this cavity-QED array is given as,

\[
\hat{H} = \hbar \sum_{i=1}^{M} \left[ \Delta_i \hat{a}_i^\dagger \hat{a}_i + \frac{g_i}{\sqrt{N_i}} \left( \hat{a}_i \hat{J}_{+,i} + \hat{a}_i^\dagger \hat{J}_{-,i} \right) - \alpha \left( \hat{a}_{i+1}^\dagger \hat{a}_i + \hat{a}_i^\dagger \hat{a}_{i+1} \right) \right].
\] (1)
the field is described by its creation (annihilation) operators, $a^\dagger_i (a_i)$: the collective atom-field coupling are represented by the constants $g_i$. In the present work, we assume that only one species of atoms is considered and that the number of ensemble atoms is the same in each cavity, i.e., $N_i = N$. For simplicity, the parameters are taken as real and homogeneous so that the detuning frequency, hopping and coupling strengths be all identical in each cavity site, i.e., $\Delta_i = \Delta$, $g_i = g$, and $\alpha_i = \alpha$, respectively.

If most of atoms stay in the ground state, we can apply Holstein-Primakoff transformation by mapping atomic excitation operator $\hat{J}_{i,i}$ on the Schwinger representation for a two-level system, i.e., $\hat{J}_{+i} = \sqrt{N}\hat{b}_i \left( 1 - \hat{b}_i^\dagger \hat{b}_i / N \right)^{1/2}$ and $\hat{J}_{zi} = \hat{b}_i^\dagger \hat{b}_i - N/2$, along with the commutation relation $[\hat{b}_i, \hat{b}_j^\dagger] = \delta_{ij}$. Here $\hat{b}_i^\dagger (\hat{b}_i)$ is creation (annihilation) operator of polarization (excitation) of $i$-th atom. Practically, the number of atoms $N$ is large enough and one can expand the orbital angular momentum representation for the atomic ensemble up to the order of $1/N$, i.e., $\left[1 - (\hat{b}_i^\dagger \hat{b}_i / N)\right]^{1/2} \approx 1 - \hat{b}_i^\dagger \hat{b}_i / 2N$. We are working within the so-called low-excitation-density limit when the atoms mostly populate their ground levels. At the same time low-excitation-density limit implies that the average photon number at each cavity should be essentially smaller than the average number of atoms at the excited state, that is $\langle \hat{a}_i^\dagger \hat{a}_i \rangle \ll \langle \hat{b}_i^\dagger \hat{b}_i \rangle$ – cf. [15].

Now, since number $N$ is assumed to be large enough, we are able to neglect higher-order terms in the expansion of $\hat{J}_{+i}$ operator. Then, we can transform the Hamiltonian in Eq. (1) into

$$\hat{H}_{HP} = \hbar \sum_{i=1}^{M} \left\{ -\alpha \left( \hat{a}_i^\dagger \hat{a}_{i+1} + \hat{a}_{i+1}^\dagger \hat{a}_i \right) + g \left( \hat{a}_i^\dagger \hat{b}_i + \hat{a}_i \hat{b}_i^\dagger \right) \right\} - \hbar g \frac{2N}{\beta} \left[ \hat{a}_i^\dagger \hat{b}_i^\dagger \hat{b}_i + \hat{a}_i \hat{b}_i^\dagger \hat{b}_i + \Delta \hat{a}_i^\dagger \hat{a}_i \right].$$

(2)

Based on this Hamiltonian, Heisenberg’s equations of motion for the involved two bosonic operators of cavity modes $\hat{a}_i$ and atomic polarization $\hat{b}_i$ are

$$i\partial_t \hat{a}_i = \Delta \hat{a}_i - \alpha (\hat{a}_{i+1} + \hat{a}_{i-1}) + g \left( \hat{b}_i - \frac{1}{2N} \left( \hat{b}_i^\dagger \hat{b}_i \right) \right),$$

(3)

$$i\partial_t \hat{b}_i = g \left[ \hat{a}_i - \frac{1}{2N} \left( \hat{a}_i^\dagger \hat{b}_i + 2\hat{a}_i \hat{b}_i^\dagger \right) \right].$$

(4)

Here, we have neglected quantum fluctuations due to a large number of atoms considered.

To have mean-field solutions, we replace the pair of operators $(\hat{a}_i, \hat{b}_i)$ by the corresponding expectation values $(\psi_i, \beta_i)$, and approximate this array configuration to a continuous model, i.e., $\psi_{i+1} + \psi_{i-1} \approx 2\psi(x, t) + d^2 \partial_x^2 \psi(x, t)$, with the distance between two adjoint optical cavities denoted by $d$. By considering the conservation of the total photon number and atomic excitations, we can re-normalize the variables with respect to the number of polaritons $N_{pol}$, defined as $N_{pol} = |\psi|^2 + |\beta|^2$. By replacing the cavity field and atomic excitation by $\psi \rightarrow \psi / \sqrt{N_{pol}}$ and $\beta \rightarrow \beta / \sqrt{N_{pol}}$, respectively, now we have a normalization condition for the cavity field and atomic excitation, $|\psi|^2 + |\beta|^2 = 1$. Thus, in this continuum limit, the equations of motion in Eqs. (3,4) become,

$$i\partial_t \psi = (\Delta - 2\alpha - i\gamma_c)\psi - \alpha d^2 \partial_x^2 \psi + g\beta - U_{sat} |\beta|^2 \beta,$$

(5)

$$i\partial_t \beta = g\psi - U_{sat} \left[ \beta^4 \psi^2 + 2|\psi|\beta^2 \right] - i\Gamma_d \beta,$$

(6)

where $U_{sat} \equiv \frac{gn_{pol}}{2}$; $n_{pol} = N_{pol}/N$ is a photonic number density. In Eqs. (5,6) we have a phenomenologically introduced decay rate for photonic field $\gamma_c$ that characterizes the leakage of
photon in the cavity and dephasing rate $\Gamma_d$ for the atomic system. Physically, parameter $U_{\text{sat}}$ characterizes a nonlinear saturation effect under the atom-light interaction.

Let us briefly discuss the applicability of our model (Eqs. (5,6)) for real atomic systems. In the paper we examine ultracold two-level rubidium atoms with resonance frequency $\omega_{ab}/2\pi = 382\text{THz}$ that corresponds to mean weighted rubidium D-lines [16]. The atomic polarization dephasing rate and the minimal value of each cavity field decay rate can be taken as several tens of megahertz's that corresponds to cavity quality factor $Q = \omega_{ab}/2\gamma_c \sim 10^6$. The strength of interaction of a single atom with a quantum optical field is taken as $g_0/2\pi = 89.5\text{MHz}$ at each cavity with the effective volume of atom-field interaction $V = 5000\mu\text{m}^3$. To achieve a strong atom-field coupling regime, see Eqs. (9) below, one can propose a macroscopically large number of atoms at each cavity, say $N = 5 \times 10^5$. This number is relevant to the density $\rho = 10^{14}\text{cm}^{-3}$ of ultracold atoms that implies a collective atom-field coupling parameter as $g \equiv g_0\sqrt{N} = 2\pi \times 63.3\text{GHz}$ at each cavity. Noticing, that the parameter that characterizes atom-atom interaction $\eta = 4\pi\hbar a_{\text{sc}}P/m$ in the Born approximation can be estimated as $\eta = 2\pi \times 0.73\text{kHz}$, where $a_{\text{sc}}$ is atomic scattering length that we choose as $a_{\text{sc}} = 5\text{nm}$. Thus, for atomic QED array we can neglect atom-atom interaction processes in Eqs. (5,6).

The set of coupled nonlinear equations (5,6) is the starting point of our work, and soliton solutions both for the wave-packet envelope of cavity field $\psi$ (order parameter) and for atomic excitations $\beta$ are considered analytically below.

3. Polariton solitons under perturbations

To construct solitons in this cavity-QED arrays, we seek for wave-packet solutions by the multiple-scale envelope function method [11, 12]. With the introduction of different length and time scales, i.e., $x_m = \lambda^m x$ ($\lambda \ll 1, m = 0, 1, 2 \ldots$) and $t_m = \lambda^m t$ ($\lambda \ll 1, m = 0, 1, 2 \ldots$), we can expand photon field $\psi = \sum_{n=1}^{\infty} \lambda^n \psi^{(n)}$ and atomic excitation $\beta = \sum_{n=1}^{\infty} \lambda^n \beta^{(n)}$. By substituting these expansions in Eqs. (5-6), we can gather all terms that are proportional to the first order of $\lambda$. This first-order expansion for the atomic excitation $\beta^{(1)}$ supports a plane wave solution, from which we can find a corresponding dispersion relation by using the solution in the form of $\psi^{(1)} = E^{(1)} e^{i(kx_0 - \omega t_0)}$ and $\beta^{(1)} = \frac{g}{\omega + \Gamma_d} E^{(1)} e^{i(kx_0 - \omega t_0)}$. The corresponding carrier frequencies are

$$\omega \equiv \omega_{\pm} = \frac{1}{2} \left[ \delta - i (2\gamma_c - \gamma) \pm \left( (\delta - i\gamma)^2 + 4g^2 \right)^{1/2} \right] , \quad (7)$$

where $\gamma = \gamma_c - \Gamma_d$ is an effective atom-field decay rate for the system and $\delta = \Delta - 2\alpha (1 - d^2k^2/2)$ is a total momentum dependent atom-light detuning, $k$ is a wave number. Physically, Eq. (7) reproduces a familiar result for frequencies of two branches of polaritons (cf. [10]), which are denoted as UB ($\omega_{+}$) and LB ($\omega_{-}$), respectively.

It is important to note that field ($\psi^{(1)}$) and atomic excitation ($\beta^{(1)}$) variables become exact solutions of Eqs. (5,6) neglecting nonlinear effects, i.e., setting $U_{\text{sat}} = 0$. Hence, it is easy to understand that MSEF method developed here is valid if dispersion characteristics of polaritons, see Eq. (7), cannot be essentially modified taking into account nonlinear effects. Strictly speaking, the condition $U_{\text{sat}} \ll 2g$ should be fulfilled for coupled atom-light systems described by Eqs. (5,6). In practice this condition implies achieving low-excitation density limit $n_{\text{pol}} \ll 1$ discussed above.

Next, for the second order of multiple-scales, i.e., $\lambda^2$, we can have a linear wave equation for a wave-packet envelope $(\partial_{t_1} + v_\pm \partial_{x_1}) E^{(1)} = 0$, from which one can find the corresponding group velocities,

$$v_\pm = \partial_h \omega_\pm = \frac{2\alpha kd^2 \Omega_{\pm}^2}{\Omega_{\pm}^2 + g^2} \quad (8)$$
that are defined for UB \((v_+\) and LB \((v_-)\) polariton wave-packets. Below we consider variable \(E^{(1)}\) in the moving frames \(\xi_{\pm} = x_1 - v_{\pm} t_1\). In Eq. (8) we have also introduced characteristic frequencies \(\Omega_{\pm} = \frac{1}{2} \left[ \delta - i\gamma \pm \left( (\delta - i\gamma)^2 + 4g^2 \right)^{1/2} \right]\).

It is noted that the group velocities \(v_{\pm}\) defined in Eq. (8) are complex numbers in general. It is known that this leads to the additional deformation of a pulse envelope propagated in the medium [17]. The above MSEF method works only for small decay rates if a strong coupling atom-light condition is valid. In particular, we require that conditions

\[ \gamma_c, \Gamma_d \ll 2g, \quad \Im(\omega_{\pm}) \ll \Re(\omega_{\pm}). \quad (9a, b) \]

should be fulfilled. The inequality (9b) means that dissipation effects are not dominant under atom-field interaction in the cavity array.

Now, we do a multiple-scale expansion up to the third order by collecting terms with \(\lambda^3\), and reach a complex GLE, i.e.,

\[ i\frac{\partial \Psi}{\partial t} + D_\pm \frac{\partial^2 \Psi}{\partial X_\pm^2} + C_\pm |\Psi|^2 \Psi = 0, \quad (10) \]

where variables \(\Psi = \lambda E^{(1)}, X_\pm = \xi_{\pm}/\lambda\), and \(t_2 = \lambda^2 t\) have been introduced. The coefficients \(D_\pm\) and \(C_\pm\), appearing in the second and third terms of Eq. (10) respectively, have the forms

\[ D_\pm = \frac{\alpha d^2 \Omega_\pm^3 + v_{\pm}^2 g^2}{\Omega_\pm (\Omega_\pm^2 + g^2)}, \quad C_\pm = \frac{n_{pol} g^4 \left[ (3\Omega_\pm^2 + \Omega_\pm^2) \right]}{2 (\Omega_\pm^2 + g^2)}. \quad (11a, b) \]

Coefficients defined in Eqs. (11a,b) are complex and can be evaluated as \(D_\pm = D_\pm^{(1)} + iD_\pm^{(2)}\), \(C_\pm = C_\pm^{(1)} + iC_\pm^{(2)}\). Real part \(D_\pm^{(1)}\) of \(D_\pm\)-coefficient describes diffraction effects occurring with the wave-packet; while the imaginary part \(D_\pm^{(2)}\) characterizes diffusion processes. Parameter \(C_\pm^{(1)}\) is responsible for a Kerr-like nonlinearity that occurs due to polariton-polariton scattering. At the same time an imaginary part, that is \(C_\pm^{(2)}\), is relevant to nonlinear absorption effects.

By taking into account Eq. (7), it is helpful to introduce convenient UB \((m_+)\) and LB \((m_-)\) polariton masses

\[ m_{\pm} = \hbar \left[ \frac{\partial_k \omega_{\pm}}{k} \right]_{k=0}^{-1} = \frac{m_{ph}}{\frac{\omega_\pm^3}{\Omega_\pm^2 + g^2}}, \quad (12) \]

where \(m_{ph} = \hbar/2\alpha d^2\) is an effective photon mass in the cavity, cf. [15]. Neglecting the kinetic energy of polaritons and supposing that the solutions are taken at the bottom of dispersion curve, one can approximate

\[ D_\pm \simeq \frac{\hbar}{2m_\pm}, \quad C_\pm \simeq \pm \frac{2n_{pol} g Y_{\pm}^3}{Y_{\pm}}, \quad (13a, b) \]

where we have introduced Hopfield coefficients such as \(Y_{\pm} = \frac{1}{\sqrt{2}} \left[ 1 \pm \frac{\delta}{\sqrt{\delta^2 + 4g^2}} \right]^{1/2}\) and for the sake of simplicity we suppose that \(g > 0\). The latter parameters determine the contributions from photons and atomic excitations to polaritons. In particular, we can express field variable \(\Psi\) as following

\[ \Psi = Y_+ \Xi_{UB} - Y_- \Xi_{LB}, \quad (14) \]
where $\Xi_{UB}$ and $\Xi_{LB}$ are new variables characterizing UB and LB polaritons respectively. They are defined in a convenient way by using Bogoliubov transformation, cf. [16].

In particular, for a positive and large frequency detuning $\delta$, that is $|\delta| \gg g$ and $\delta > 0$, we have $Y_+ \approx 1$ and $Y_- \approx \frac{g}{|\delta|}$, respectively, which corresponds to photon-like UB polaritons ($\Xi_{UB} \approx \Psi$) with mass $m_+ \approx m_{ph}$ and group velocity $v_+ = \hbar k/m_+$, see Eqs. (12-14). Remarkably, at the same time the group velocity of LB polaritons with mass $m_- \approx m_{ph} \frac{\delta}{\pi}$ approaches $v_- = \hbar k/m_- \ll c$, where $c$ is speed of light in vacuum. Now, we have a ”slow” (matter-like) soliton formation in the cavity array. For $|\delta| \gg g$ but $\delta < 0$, a physical picture becomes opposite and a propagating optical pulse relevant to LB polaritons ($\Xi_{LB} \approx -\Psi$) with mass $m_- \approx m_{ph}$ and group velocity $v_- = \hbar k/m_- \gg v_+$. In the presence of atom-light resonance condition, $\delta = 0$, UB and LB polaritons equally contribute to the photonic state, that is $Y_+ = 1/\sqrt{2}$. The masses of polaritons are equal to each other, i.e., $m_{pol} \equiv m_\pm / 2m_{ph}$. Their group velocities are $v_\pm = \hbar k/m_{pol}$.

It is useful to examine Eqs. (10-11) in the limiting (dissipationless) case for the system by setting $\gamma = \Gamma_d = 0$. In this limit, both coefficients $D_\pm$ and $C_\pm$ become real and GLE in Eq. (10) is reduced to a standard nonlinear Schrödinger equation, that possesses exact bright and dark soliton solutions depending on the sign of the coefficients $C_\pm = C_\pm^{(1)}$ and $D_\pm = D_\pm^{(1)}$. Here, we suppose that coefficients $D_\pm > 0$. Then, for the case of $C_+ > 0$, we have a bright soliton solution; while for the case of $C_- < 0$, dark solitons can be found [18]. By using Eq. (11b) it is possible to show that we can only have a bright soliton solution supported in UB; while a dark one in LB.

Now we are going to examine a coupled atom-light system in respect of bright soliton formation; the system being non-equilibrium. In this case we are interested in soliton solutions of Eq. (10) for optical field amplitudes $\Psi$ in the presence of polariton formation. It is useful to bring Eq. (10) to a dimensionless form, rewriting it as follows

$$\frac{i}{\tau} \frac{\partial \Psi}{\partial \tau} + \frac{1}{2} \frac{\partial^2 \Psi}{\partial x^2} + |\Psi|^2 \Psi = -i \varepsilon_1 |\Psi|^2 \Psi + i \varepsilon_2 \frac{\partial^2 \Psi}{\partial x^2}, \quad (15)$$

where $x = X_+/d$ and $\tau = \frac{2D_+^{(1)} t}{\alpha}$ are new dimensionless spatial and temporal coordinates respectively; $\varepsilon_1 = \frac{C_+^{(2)}}{C_+^{(1)}}$, $\varepsilon_2 = \frac{D_+^{(2)}}{2D_+^{(1)}}$ are perturbation coefficients ($\varepsilon_{1,2} > 0$). In Eq. (15) we also use the fact that the characteristic time scale of a dispersion action and the influence of

![Figure 2.](Color online) Perturbation coefficients $\varepsilon_{1,2}$ versus atom-light detuning $\delta$. The parameters are: $g = 2\pi \times 63.3$GHz, $\alpha = 2\pi \times 10$GHz, $\gamma_c = 2\pi \times 30$MHz, $\Gamma_d = 2\pi \times 6$MHz, $d = 5 \mu$m, $n_{pol} = 0.01.$
nonlinearity on wave packet spreading should be the same [18]. In particular, the condition

\[ n_{\text{pol}} = \frac{4D_+^{(1)} |\Omega_+|^2}{d^2g^4} \left| \Re \left[ \frac{3\Omega_+ + \Omega_+^*}{\Omega_+^2 + g^2} \right] \right|^{-1} \]  

should be met for polariton number density \( n_{\text{pol}} \) in this case.

Behavior of perturbation coefficients \( \varepsilon_{1,2} \) as a function of atom-light detuning \( \delta \) for soliton formation is shown in Fig. 2. It is important that parameters \( \varepsilon_{1,2} \ll 1 \) within wide range of variation of \( \delta \) for appropriate effective decay rate \( \gamma \). Noticing that for \( |\delta| \gg g \) and \( \delta < 0 \) the coefficient \( \varepsilon_2 \) becomes negative. However, in this limit inequality (9b) is violated and we are not able to consider dissipation effects as a small perturbations. Below we examine the GLE in Eq. (15) using perturbation theory for solitons by consulting the reference [19]. In particular, we are looking for the solution of Eq. (15) in the form

\[ \Psi(\tau, x) = 2\nu \sech \left[ 2\nu (x - \zeta(\tau)) \right] e^{i\varphi(\tau, x)}, \]  

where \( \nu, \zeta(\tau), \) and \( \varphi(\tau, x) \) are related soliton amplitude, position, and phase, respectively. In the absence of perturbation \( (\varepsilon_{1,2} = 0) \), the ansatz used in Eq. (17) represents an exact solution for Eq. (15) with the parameters evolving in time as \( \zeta(\tau) = \nu \tau/2 + \zeta_0, \varphi(\tau, x) = \nu (x - \zeta(\tau))/2 + \delta(t) \), where parameters \( \zeta_0 \) and \( \delta_0 \) represent the initial soliton position and phase respectively. Here, \( \nu = v_+ \) is soliton velocity that we can associate with the velocity of UB polaritonic wave-packet by setting \( \gamma = \Gamma_d = 0 \). In the presence of small perturbation \( (\varepsilon_{1,2} \neq 0) \), we consider the soliton amplitude \( \nu \) and velocity \( v \) to be time dependent. Applying soliton perturbation theory, we arrive at the set of equations for solitons parameters under the adiabatic approximation,

\[ \frac{d\nu}{d\tau} = -\frac{8}{3} (2\varepsilon_1 + \varepsilon_2) \nu^3 - \frac{2}{3} \varepsilon_2 \nu^2, \quad \frac{dv}{d\tau} = -\frac{16}{3} \varepsilon_2 \nu \nu', \quad \frac{d\delta}{d\tau} = \frac{v^2}{2} + 2\nu^2, \]  

where \( \dot{\nu} \) denotes derivative in respect of normalized time, i.e., \( d/d\tau \).

In Fig. 3, we represent time dependent solutions of Eqs. (18) (solid lines). In particular, Fig. 3a demonstrates soliton profile taken at different time moments. Based on real parameters of atomic system the initial shape of this soliton is characterized by the dashed (red) curve. From Fig. 3a, it is evident that a soliton amplitude vanishes due to effective nonlinear absorption. At the same time the group velocity of soliton is slightly modified due to diffusion processes characterized by parameter \( \varepsilon_2 \) shown in Eq. (18a). The characteristic time of disturbing soliton group velocity is tens of nanoseconds which seems to be reasonable taking into account polariton lifetime in atomic structures, cf. [10,11].
4. Conclusions
In this work, we consider the problem of soliton formation in the array of weakly coupled atomic ensembles interacting with the optical field in a tunnel-coupled cavity array. The effects of cavity field dissipation and atomic dephasing have been taken into account. We use multiple-scale envelope function method to obtain an effective complex Ginzburg-Landau equation, containing nonlinear absorption and diffusion terms. Mean-field soliton solutions for this coupled matter-field states are revealed in the representation of UB and LB polaritons. We have demonstrated that bright solitons are formed for UB polariton wave packets and characterized by slowly varying soliton amplitude, momentum, position, and phase. Notably, the group velocity of soliton decreases in time due to diffusion processes within a nanoseconds domain. Our analytical and numerical results represented here and connected with a cavity-QED array provide a platform for the studies of collective properties of light with interacting media.

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