SHAPE OF THE AMPLIFICATION LINE CORRESPONDING TO AN ADJACENT TRANSITION IN A STRONG FIELD

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An analysis is made of the effect of a strong field on the shape of the amplification line for monoenergetic atoms. There are three strong-field contributions, differing in their dependence on the set of relaxation parameters of the medium and on the differences among the populations corresponding to the Raman transitions. An analysis is made of the conditions under which each of the contributions is predominant. The change in the line shape by an intensified external field is found for each case. Neon atoms are discussed as an example. The results are compared with those corresponding to a Maxwellian velocity distribution.

1 Introduction

A strong field which is resonant for one of the allowed transitions in a medium can change the emission and absorption spectra corresponding to adjacent transitions [1-3]. This happens because the photons of a weak field may be emitted in a manner correlated with that in which photons of the strong field are emitted [4, 5]. In addition, a strong field changes the populations and splits the energy levels corresponding to the resonant transition [6].

Each of the three effects has a different dependence on the unsaturated-level populations and relaxation characteristics of the medium. These features are displayed most clearly for uniformly broadened optical transitions, because optical transitions are characterized by a larger number of relaxation constants than are microwave transitions, and the population differences corresponding to the various transitions may differ markedly. By choosing transitions appropriately one can suppress some effects while intensifying others.

Below we analyze the conditions for the appearance of each of these effects separately, and we analyze the associated changes in the spectral properties of the amplification-absorption coefficient for conditions typical of the optical range.

2 Equation for the Amplification Coefficient

We consider the amplification (or absorption) of a weak field at frequency \( \omega_\mu \), approximately equal to the frequency \( \omega_{gn} \) of the \( g-n \) transition, in the presence of a strong field \( E \) the frequency of which \( \omega \) is approximately equal to the transition frequency \( \omega_{mn} (E_m > E_g > E_n) \). The weak-field amplification coefficient \( \alpha_\mu (\Omega_\mu) \) is related in a simple manner to the emission power per unit volume \( w_{gn} (\Omega_\mu) \):

\[
\alpha_\mu (\Omega_\mu) = \left( \frac{c}{8\pi} |E_\mu|^2 \right)^{-1} w_{gn} (\Omega_\mu),
\]
where \( E_\mu \) is the amplitude of the "weak field", \( \Omega_\mu = \omega_\mu - \omega_{gn} \), and \( \omega_{gn} \) is calculated as in [3]. We thus have:

\[
\alpha_\mu = \alpha^0_\mu \Gamma_{gn} \left\{ \left[ \Gamma_{gn} + i \Omega'_{\mu} + |G|^2(\Gamma_{gn} + i(\Omega'_{\mu} - \Omega'))^{-1} \right] - 1 \right\} \times \left( 1 - |G|^2 \Delta n_{nm}^\alpha n_{ng} \right) \frac{1}{\Gamma^2(1 + \alpha') + \Omega'^2} \left( 1 - \frac{\gamma_{mn}}{\Gamma_m} \right) \frac{2\Gamma}{\Gamma_n} \left( 1 + \frac{\Gamma + i\Omega'}{\Gamma_n + i(\Omega'_{\mu} - \Omega')} \right) \right\}.
\]

(2.1)

Here \( G = -d_{nm}E/2\hbar \); \( \alpha^0_\mu \) is the coefficient at the line center in the absence of an external field (\(|G|^2 = 0\)); \( \Gamma_{ik}, \Gamma_i \) are the Lorentz broadenings of the lines and levels (\( \Gamma_{mn} = \Gamma \)); \( \gamma_m \) is the probability for a transition from level \( m \) to level \( n \), \( \Omega'_{\mu} = \omega_\mu - k_v v, \Omega' = \omega - k_v v \) is the atomic velocity, \( (n_m - n_n)/(n_g - n_n) = \Delta n_{nm}/\Delta n_{ng} \) is the ratio of the unsaturated population differences corresponding to the transitions \( m \leftrightarrow n \) and \( g \leftrightarrow n; \alpha = (\Gamma_m + \Gamma_n - \gamma_m)/n_m \). The population differences \( \rho_{nm} - \rho_{nn} \) and \( n_g - \rho_{nn} \) depend on the field in the following manner:

\[
\rho_{nm} - \rho_{nn} = (\Gamma_2 + \Omega'^2)\Delta n_{mn} \left[ |G|^2(1 + \alpha') + \Omega'^2 \right]^{-1},
\]

\[
n_g - \rho_{nn} = \Delta n_{gn} - \Delta n_{mn} \left( 1 - \frac{\gamma_{mn}}{\Gamma_m} \right) \frac{2\Gamma}{\Gamma_n} \left[ |G|^2(1 + \alpha') + \Omega'^2 \right]^{-1}.
\]

(2.2)

The term proportional to \(|G|^2\) in the common denominator in Eq. (2.1) reflects the broadening and splitting of the line by the strong field; expanding Eq. (2.1) in simple fractions, we can write the expression for \( \alpha_\mu \) as:

\[
\alpha_\mu = \alpha^0_\mu Re \left\{ \frac{\Gamma_{gn}(\alpha_1 - \alpha_2)}{\Gamma_{gn} - \alpha_2' + i(\Omega'_{\mu} - \alpha_2')} \left( \alpha_1 - |G|^2 \frac{\Delta n_{mn}/\Delta n_{ng}}{\Gamma^2(1 + \alpha') + \Omega'^2} \left( 1 - \frac{\gamma_{mn}}{\Gamma_m} \right) \frac{2\Gamma}{\Gamma_n} \frac{1}{(\Gamma_2 + \Omega'^2)\Delta n_{mn}} \left[ |G|^2(1 + \alpha') + \Omega'^2 \right]^{-1} \right) - \frac{\Gamma_{gn}(\alpha_1 - \alpha_2')}{\Gamma_{gn} - \alpha_1' + i(\Omega'_{\mu} - \alpha_1')} \right\}.
\]

(2.3)

where

\[
\alpha_{1,2}' + \alpha_{1,2}'' = \frac{1}{2} \left( \Gamma_{gn} - \Gamma_{gm} + i\Omega' \pm \sqrt{(\Gamma_{gn} - \Gamma_{gm} + i\Omega')^2 - 4|G|^2} \right).
\]

(2.4)

It follows from Eqs. (2.3) and (2.4) that it is easiest to achieve level splitting with \( \Gamma_{gn} = \Gamma_{gm} \), and this splitting can be observed most easily in its pure form in the case \( n_m = n_n \).

The terms proportional to \( 1 - (\gamma_{mn}/\Gamma_{mn}) \) describe the changes caused in the populations of levels \( m \) and \( n \) by the strong field, and the proportional quantities \( \Gamma + i\Omega' \) corresponds to nonlinear interference effects [2,3]. The relative weights of these effects depend on several factors: the relaxation properties of the system, the ratio \( \Delta n_{mn}/\Delta n_{ng} \), and the atomic velocity distribution. The case of a Maxwell velocity distribution was analyzed in [3]. Below we will omit the primes from \( \Omega' \) and \( \Omega'_{\mu} \) and use \( \Omega \) and \( \Omega_{\mu} \) to signify the deviation from resonance in the inertial reference system. A real or effective monoenergetic beam can be produced artificially; an effective beam can be produced, for example, by exciting atoms with a coherent field from the ground level to one of the higher-lying levels and by the subsequent relaxation of the atoms to the \( m, n \), and \( g \) levels. Conditions can be arranged such that for the levels of interest the projections of the atomic velocity on the direction of \( k_0 \) will lie in a very narrow velocity range \( \Delta v = \gamma_0/k_0 \ll \Gamma/k, \Gamma_{gn}/k_\mu \) near the velocity \( v_0 = \Omega_0/k_0 \). Here \( \gamma_0 \) is the line half-width, and \( k_0 \) and \( \Omega_0 \) are the modulus of the wave vector and the deviation from resonance for the exciting transition. Then we can neglect the atomic velocity distributions at the \( m, n \), and \( g \) levels. For a gas with nonuniform broadening these results can be used to show how the individual atoms interact with the field.

### 3 Spectral Properties of the Amplification Coefficient

We first take up the case \( \Delta n_{mn} = 0 \), in which the only effect of the field is to split the levels. Equation (2.3) becomes:

\[
\frac{\alpha_\mu}{\alpha^0_\mu} = Re \left\{ \frac{\Gamma_{gn}}{\alpha_1 - \alpha_2} \left[ \frac{\alpha_1}{\Gamma_{gn} - \alpha_2' + i(\Omega'_{\mu} - \alpha_2')} - \frac{\alpha_2}{\Gamma_{gn} - \alpha_1' + i(\Omega'_{\mu} - \alpha_1')} \right] \right\}.
\]

(3.1)
To determine how the line shape \( \alpha_\mu(\Omega_\mu) \) depends on \( |G|^2 \) it is sufficient to analyze the following limiting cases.

For the case \( \Omega = 0 \) the roots \( \alpha_1 \) and \( \alpha_2 \) can be written as:

\[
\alpha_1 \approx \begin{cases}
   (\Gamma_{gn} - \Gamma_{gm}) \left( 1 - \frac{|G|^2}{(\Gamma_{gn} - \Gamma_{gm})^2} \right), & 4|G|^2 \ll (\Gamma_{gn} - \Gamma_{gm})^2, \\
   (\Gamma_{gn} - \Gamma_{gm})/2 + i|G|, & 4|G|^2 \gg (\Gamma_{gn} - \Gamma_{gm})^2;
\end{cases}
\]

\[
\alpha_2 \approx \begin{cases}
   |G|^2/(\Gamma_{gn} - \Gamma_{gm}), & 4|G|^2 \ll (\Gamma_{gn} - \Gamma_{gm})^2, \\
   (\Gamma_{gn} - \Gamma_{gm})/2 - i|G|, & 4|G|^2 \gg (\Gamma_{gn} - \Gamma_{gm})^2.
\end{cases}
\]  

From Eqs. (3.1) and (3.2) we see that with \( 4|G|^2 \ll (\Gamma_{gn} - \Gamma_{gm})^2 \) the spectrum is a set of two components having half-widths \( \Gamma_{gn} - |G|^2/(\Gamma_{gn} - \Gamma_{gm}) \) and \( \Gamma_{gm} + |G|^2/(\Gamma_{gn} - \Gamma_{gm}) \). The maxima of the two components occur at the same frequency, but the maximum of the component having a half-width \( \Gamma_{gn} - |G|^2/(\Gamma_{gn} - \Gamma_{gm}) \) is higher than that of the second component by a factor of about \( \Gamma_{gm}(\Gamma_{gn} - \Gamma_{gm})^2/\Gamma_{gn}|G|^2 \). Accordingly, the overall effect of a low-intensity external field in the case \( \Omega = 0 \) is a slight broadening of the spectral line corresponding to the \( g \leftrightarrow n \) transition in the case \( \Gamma_{gm} > \Gamma_{gn} \). For a high-intensity external field \( |G|^2 \gg (\Gamma_{gm} - \Gamma_{gn})^2 \), the spectrum consists of two components having the same intensity and the same half-widths \( (\Gamma_{gm} + \Gamma_{gn})/2 \).

The components are centered at positions symmetric with respect to the frequency \( \Omega_\mu = 0 \) and are separated by \( \Delta \omega = 2|G| \).

In the other limiting case of \( \Gamma_{gm} = \Gamma_{gn}, \Omega \neq 0 \) we find:

\[
\alpha_1 \approx i(\Omega + 2|G|)/\Omega, \quad \alpha_2 \approx -i|G|^2/\Omega, \quad 4|G|^2 \ll \Omega^2;
\]

\[
\alpha_1 \approx i(\Omega + 2|G|)/2, \quad \alpha_2 \approx i(\Omega - 2|G|)/2, \quad 4|G|^2 \gg \Omega^2.
\]  

\[
\alpha_\mu/\alpha_\mu^0
\]

\[
\Omega_\mu/\Gamma_{gn}
\]

\[
\Delta n_{mn} = 0, \quad 1) \ x = 2, \ \Omega = 0; \quad 2) \ x = 2, \ \Omega = \Gamma; \quad 3) \ x = 8, \ \Omega = 0; \quad 4) \ x = 8, \ \Omega = \Gamma.
\]

\[
\alpha_\mu/\alpha_\mu^0
\]

\[
\Omega_\mu/\Gamma_{gn}
\]

\[
\Delta n_{mn} = 0, \quad 1) \ x = 2, \ \Omega = 0; \quad 2) \ x = 3, \ \Omega = \Gamma; \quad 3) \ x = 8, \ \Omega = 0; \quad 4) \ x = 8, \ \Omega = \Gamma.
\]

In this case the two spectral components have the same half-width, \( \Gamma_{gn} = \Gamma_{gm} \). For weak fields \( (|G|^2 < \Omega^2) \) one line has a maximum at \( \Omega_\mu = -|G|^2/\Omega \), while the other has a maximum at \( \Omega_\mu = \Omega + |G|^2/\Omega \); the intensity at the maximum of the first line is higher than that at the maximum of the center of the second by a factor of \( \Omega^2/|G|^2 \). In intense fields \( (|G|^2 \gg \Omega^2) \) the two lines have the same intensity and lie at symmetric positions with respect to frequency \( \Omega_\mu = \Omega/2 \), separated by \( 2|G| \).

We can draw the following conclusion regarding the change in the spectrum accompanying an increase of the intensity of the external field on the basis of these arguments. When the external field is applied, we find, in addition to the fundamental component, having a width of approximately \( 2\Gamma_{gn} \), an additional component, having a width approximately equal to that of the line corresponding to the Raman transition \( (2\Gamma_{gm}) \), the center of which is near \( \Omega_\mu = \Omega \). As the external field is intensified, the additional component becomes relatively more important. The width of each line changes in such a manner that the line width of high external field intensities the widths of both components become the same, equal to \( \Gamma_{gn} + \Gamma_{gm} \). The width change is accompanied by an increase in the separation between the centers of the
The relaxation properties of the neon 3s
This behavior is illustrated in Fig. 1, where the average values of $\Omega$ and $i\nu$ are proportional to $\Gamma + i\nu$.

The integral radiation intensity, on the other hand, is governed only by the change in the quantity $n_g - \rho_n(\nu)$ caused by the field and is independent of the external field with $\Delta n_{mn} = 0$. This behavior is illustrated in Fig. 1, where the average values of $\Omega$ and $\nu$ for the case of a model having the relaxation properties of the neon 3s$^2 - 2p_4$ and 2s$^2 - 2p_4$ transitions.

We turn now to an analysis of the spectral properties of $\alpha_\mu$, taking into account the interference term proportional to $\Gamma + i\Omega$. We restrict the discussion to the case in which the numerator in Eq. (2.1) is governed primarily by this term, i.e., to the case in which we have

$$\frac{\Gamma}{\Gamma_{gm}} \gg \left(1 - \frac{\gamma_{mn}}{\gamma_m}\right) \frac{2}{\Gamma_n}, \quad \frac{\Gamma}{\Gamma_{gm}} \cdot \frac{|G|^2\Delta n_{mn}/\Delta n_{gn}}{\Gamma^2(1 + \nu^2) + \Omega^2} \gg 1.$$  

*In this case amplification is possible even at $n_g - \rho_n(\nu) < 0$ and $\alpha_\mu$ may change sign as a function of $\Omega_\mu$. It follows from Eq. (2.1) that with $\Delta n_{mn}/\Delta n_{gn} > 0$, amplification occurs in the frequency band between $(\Omega_\mu)_1$ and $(\Omega_\mu)_2$, given by

$$(\Omega_\mu)_1 = \frac{1}{2G} \left\{ (\Gamma_{gm} + \Gamma + \Gamma_{gm})\Omega \pm \sqrt{2\Gamma_{gm}\Gamma + (\Gamma + \Gamma_{gm})^2\Omega^2 - 4\Gamma_{gm}\Gamma\Omega^2 + 4\Gamma^2(\Gamma_{gm}\Gamma_{gm} + |G|^2)} \right\}.$$  

It follows from this equation that the band width increases with increasing $\Omega^2$ and $|G|^2$. In the limiting cases in which the intense field is far from and close to resonance, we find from Eq. (2.4)

$$(\Omega_\mu)_1 \approx \frac{1}{2} \left(1 + \frac{\Gamma_{gm}}{\Gamma}\right) \Omega \pm \sqrt{\Gamma_{gm}\Gamma + |G|^2}, \quad \text{if} \quad \Gamma_{gm} \ll \Gamma, \Gamma_{gm}; \quad \Omega^2 \leq \frac{4\Gamma^2(\Gamma_{gm}\Gamma_{gm} + |G|^2)}{(\Gamma_{gm} - \Gamma)^2},$$

$$(\Omega_\mu)_2 \approx \frac{\Gamma_{gm}}{\Gamma}, \quad (\Omega_\mu)_2 \approx \Omega, \quad \text{if} \quad \Gamma_{gm} \ll \Gamma, \Gamma_{gm}; \quad \Omega^2 \approx \frac{4\Gamma^2(\Gamma_{gm}\Gamma_{gm} + |G|^2)}{(\Gamma_{gm} - \Gamma)^2}.$$  

With $\Omega = 0$ we have $(\Omega_\mu)_1 = \pm \sqrt{\Gamma_{gm}\Gamma_{gm} + |G|^2}$ for any values of $\Gamma_{gm}$, $\Gamma_{gm}$ or $|G|^2$. Here the half-width at half-height of the amplification line is

$$(\Delta \Omega_\mu)^2_{1,2} = \frac{1}{2} \left\{ \sqrt{(\Gamma_{gm} + \Gamma_{gm})^4 + 4(\Gamma_{gm}\Gamma_{gm} + |G|^2)^2} - (\Gamma_{gm} + \Gamma_{gm})^2 \right\},$$

$$(\Delta \Omega_\mu)^2_{1,2} \approx \frac{\Gamma_{gm}\Gamma_{gm} + |G|^2}{\Gamma_{gm} + \Gamma_{gm}}, \quad \frac{\Gamma_{gm}\Gamma_{gm} + |G|^2}{(\Gamma_{gm} + \Gamma_{gm})^2} \ll 1.$$  

The relaxation properties of the neon 3s
It follows from Eqs. (3.5) and (3.6) that with $|G|^2 \ll \Gamma_{\text{eff}} \Gamma_{\text{gm}}$ and $\Gamma_{\text{gm}} \ll \Gamma_{\text{gn}}$ the width of the amplification band is governed by the geometric average of $\Gamma_{\text{gn}}$ and $\Gamma_{\text{gm}}$, while the width of the amplification line is $2\Gamma_{\text{gm}}$ and may be much narrower than the natural line width corresponding to the $g \leftrightarrow n$ transition. When the frequency of the strong field is scanned, the amplification band of the weak field also shifts; the band width depends on both $\Omega^2$ and $|G|^2$.

We see from Eq. (2.1) that as field $E$ increases there are increases in the population and interference contributions to $\alpha_\mu$. On the other hand, there is a tendency for the amplification coefficient at the center of the line to fall off with increasing $|G|^2$ because of the level splitting. Analysis of Eq. (2.1) for $\alpha_\mu = \Omega = 0$ shows that the optimum value at $\kappa$, corresponding to the maximum value of $\alpha_\mu$ at the line center, is given by

$$\kappa_{\text{opt}} = \kappa_1(x)\left\{1 + \sqrt{1 + [x\kappa_1(x)]^{-1}(2\Gamma_{\text{gm}}\Gamma_{\text{gn}}\kappa_1(x) + 1)}\right\}, \quad x > x_1,$$

where $x = \Delta n_{mn}/\Delta n_{gm}$,

$$x_1 = [(\Gamma_m - \gamma_{mn} + \Gamma_n\Gamma_m)(2\Gamma_{\text{gm}})^{-1} - \Delta n_{mn}/\Delta n_{gm}], \quad \kappa_1(x) = (xx_1^{-1} - 1)^{-1}.$$

The spectral properties of the function $\alpha_\mu(\Omega_{\mu})/\alpha_\mu^0$ for the model discussed above are illustrated in Figs. 2-4, where $x$ is set equal to 4.14, corresponding to the optimum field $\kappa = 2$. Figure 2 corresponds to the case $\Omega = 0$ for values $\kappa > \kappa_{\text{opt}}$. The change in $\alpha_\mu$ at the maximum is very rapid while $\kappa$ varies near the optimum. (These cases are not illustrated in Fig. 2.) For example, with $\kappa = \kappa_{\text{opt}} = 2$ we have $\alpha_\mu(0)/\alpha_\mu^0 = -32$. As $\kappa$ falls off to half its value, $\alpha_\mu(0)/\alpha_\mu^0$ falls off to about one-third its value. As $\kappa$ increases to a value 50% above $\kappa_{\text{opt}}$, the value of $\alpha_\mu(0)/\alpha_\mu^0$ falls off by a factor of 60. As $\kappa$ changes from 2 to 3, the line half-width at half-height changes from $\approx 0.2\Gamma_{\text{gm}}$ to $\approx \Gamma_{\text{gn}}$. With $\kappa = \kappa_{\text{opt}}$ the $\alpha_\mu$ profile is symmetric and changes sign at $\Omega_{\mu}^2 \approx \Gamma_{\text{gn}}^2$. The absorption in the wings changes very slowly with increasing $|\Omega_{\mu}|$. The maximum absorption is roughly 1/30 the value of $|\alpha_\mu(0)|$.

Figure 2 shows the change in $\alpha_\mu(\Omega_{\mu})$ corresponding to a further increase in $\kappa$. The case $\kappa = 3$ corresponds to the vanishing of $\rho_{nn}(\kappa)$. The integral value of the coefficient $\alpha_\mu$ corresponding to $g \leftrightarrow n$ transition also vanishes. As the external field is intensified, the line splits, so that with $\kappa = 40$ a plateau appears on the amplification curve, about $6\Gamma_{\text{gn}}$ in width.

Figures 3 and 4 show the changes in the spectral properties of the amplification coefficient as the frequency and intensity of the strong field are changed. Figure 3 shows the line profile for $\Omega = \Gamma$, while figure 4 shows this profile for $\Omega = 10\Gamma$. These cases correspond to quasiresonant Raman scattering through a common lower level. We see from Figs. 3 and 4 that the frequency separation between the amplification and absorption maxima increases as the deviation of the strong field from resonance increases. For fixed $\Delta n_{gm} < 0$, an increase in $|\Omega|$ requires a more intense external field for appreciable amplification. This effect is accompanied by a change in $n_g - \rho_{nn}(\kappa)$ which is significant in comparison with $\Delta n_{gn}$.

### 4 Conclusion

In conclusion we will compare the results for the cases of a beam having a Maxwell atomic velocity distribution and a monoenergetic beam. With a Maxwell velocity distribution, peaks or troughs appear against the background of the $\alpha_\mu(\Omega_{\mu})$ Doppler profile under the influence of the traveling wave of the intense field; these peaks and dips are described by

$$\frac{k_\mu}{k} \frac{1}{\sqrt{1 + \kappa}} Re \left\{ (N_m - N_n)|G|^2 \left[ \frac{\Gamma_0 + i}{\Gamma_\pm + i} \left( \frac{\Omega_{\mu} + k_\mu}{k} \right) \frac{1}{\Gamma_\pm + i} \right] \right\},$$

where

$$k_\mu > k, \quad \Gamma_0 = \Gamma_{\text{gn}} + \frac{k_\mu}{k} \Gamma \sqrt{1 + \kappa}, \quad \Gamma_\pm = \Gamma_{\text{gm}} + \left(1 + \frac{k_\mu}{k}\right) \Gamma \sqrt{1 + \kappa}, \quad \tilde{\Gamma}_n = \Gamma_n(1 \pm \sqrt{1 + \kappa}).$$

The upper sign corresponds to the case $k_\mu k > 0$, and the lower sign corresponds to the case $k_\mu k < 0$ [3]. Comparing Eq. (4.1) with Eq. (2.1), we conclude that the line profile for the peaks (or dips) for a gas with a Doppler velocity distribution is the same as the line shape for an effective monoenergetic (atomic)
beam, for which we have \(|N_m - N_n| \gg |N_g - N_n|\), \(\Omega' = 0\), \(\Gamma_{gn} = \Gamma_0\), \(\Gamma_n = \tilde{\Gamma}_n\), \(\Gamma_{gm} = \Gamma_{\pm}\) and for which the resonant frequency is \(\omega_{gn} \pm (k_{\mu}/k) \cdot \Omega\). However, in the case of a Maxwell distribution the line shape has a completely different dependence on the magnitude of the external field, and there is no profile-asymmetry effect, as may be displayed in the case of a monoenergetic beam.

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