Half-Duplex or Full-Duplex Communications:  
A Capacity Analysis under Self-Interference  

Nirmal V. Shende, Kudret Akcapinar, Ozgur Gurbuz, and Elza Erkip

Abstract

In-band full-duplex (FD) communication provides a promising alternative to half-duplex (HD) for wireless systems, due to increased spectral efficiency and capacity. In this paper, HD and FD radio implementations of two way, two hop and two way two hop communication are compared in terms of degrees of freedom (DoF) and achievable rate, under a realistic residual self-interference (SI) model. DoF analysis is carried out for each communication scenario, and achievable rates are computed at finite SNR levels for HD, antenna conserved (AC) and RF chain conserved (RC) FD radio implementations. The DoF analysis indicates that for the two way channel, AC FD performs strictly below HD with imperfect SI cancellation, and RC FD DoF trade-off is superior, when the SI can be sufficiently cancelled. For the two hop channel, FD is better when the relay has large number of antennas and enough SI cancellation. For the two way two hop channel, when both nodes require similar throughput, HD is generally better than FD; for asymmetric traffic, FD can achieve better rate pairs than HD, provided the relay has sufficient number of antennas and SI suppression. Computed achievable rates for each scenario indicate that DoF results also extend to finite SNR.

Index Terms

full-duplex wireless, self-interference, simultaneous transmit and receive, two way communications, relaying

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I. INTRODUCTION

In almost all networks, a communicating device has a dual task of reception and transmission of data. This is commonly achieved via half-duplex (HD) operation, where the channel is time shared between transmission and reception, so that a node can either transmit or receive at a given time. Full-duplex (FD) operation provides a promising alternative, where both of these activities are implemented simultaneously. However, a FD node suffers from high amount of self-interference (SI), since typically the transmitted signal is about 100 dB stronger than the received signal. Recently FD has gained considerable interest due to promising results on practical implementations [1]–[5], as can be seen in the recent review article [6] and references therein.

Ideally, FD implementation uses the channel for transmitting and receiving simultaneously, and hence it is likely to give higher throughput. On the other hand, FD requires hardware resources, such as antennas to be divided between transmission and reception, in order to accomplish this with as little SI as possible. However, since SI cannot be suppressed completely, the residual SI reduces the received signal-to-interference-noise ratio (SINR), resulting in reduced data rates. Hence, how much improvement can FD communication in the presence of SI can provide over HD, considering similar hardware resources is an important question that needs to be investigated thoroughly for viability of FD. In order to address this problem, in this paper we compare wireless HD and FD communication in three communication scenarios, two way, two hop (relaying), and two way two hop (two way relaying) systems, illustrated in Figures 1-4 from degrees of freedom (DoF) and capacity points of view. The system models considered in this paper arise naturally in modern communication scenarios, such as cellular, WiFi, mesh or ad-hoc networks, which would particularly benefit from FD implementations.

One of the challenges in analytical study of the FD systems is the modeling of the residual SI. The model should be accurate, so that it captures the effect of SI, and also simple enough, so that it is useful for analysis and design. Some works assume constant increase in the noise floor due to SI [5], [7]. However, it is reasonable to expect that SI will depend on the transmit power. Other works assume linear increase in SI with transmit power [8]–[10], but this model fails to capture the effect, in which increased transmission power actually enhances SI suppression, since a better estimate of the SI signal is obtained. In our analysis in this paper, we use the experimentally validated SI model from [11], which shows that average residual SI power after
cancellation can be modeled as $P^{1-\lambda}$, where $0 \leq \lambda \leq 1$ is a constant that depends on the transceiver’s ability to mitigate SI and $P$ is the transmit power. This model, also used in [12], not only captures the effect of the practical SI cancellation mechanisms employed, but it is analytically tractable as well.

In order to provide a fair comparison of FD and HD implementations, it is important to keep the hardware resources fixed. For this purpose, we follow two approaches as in [13]: For each node, we either keep the total number of antennas or we keep the total number of RF chains of FD mode the same as that of HD mode, considering antenna conserved (AC) and RF chain conserved (RC) implementations of FD, respectively. The AC FD scenario is motivated by the recent FD implementations [1], [5]); the notion of keeping the number of RF chains equal is also reasonable from a practical perspective, since RF chains are the components that dominantly increase the total cost of a radio [14].

Our comparison of the performances of HD and FD, made under the above SI and hardware resource models, include both the high-SNR DoF analysis [15] and finite SNR behavior of achievable rates. The DoF metric admits simple analytical characterization facilitating the comparisons, and the trends predicted by the DoF comparison are also observed at finite SNRs, suggesting the value of the DoF analysis. Our main observations can be summarized as follows:

- For the two way channel (Figure 1), we show that in presence of SI ($\lambda < 1$), the FD DoF region, which shows the simultaneously achievable DoF pairs by both users for the AC scenario lies strictly inside the HD trade-off. For the RC scenario, however, with “good” SI suppression (typically $\lambda > 0.75$), FD can achieve certain DoF pairs which are not achievable by the HD implementation. Finite SNR analysis confirms the relative performance benefits of FD and HD. However, actual value of $\lambda$ at which FD performs better in the RC scenario depends on system parameters such as transmission power and number of antennas.

- For the two hop channel (a relay channel without a direct link between the source and destination) as shown in Figure 2, we compare the FD and HD DoFs for the symmetric case (when both source and destination have equal number of antennas) and the asymmetric case (when source has a single antenna, and destination has multiple antennas). We find that, for given number of source and destination antennas, and SI parameter $\lambda$, the FD implementation outperforms HD if the relay has sufficient number of antennas, otherwise HD is better. Number of antennas required at the relay for this crossover is lower for the
RC scenario, than that of the AC scenario, and depends on the SI mitigation level $\lambda$ and the number of source and destination antennas. When the number of antennas at each node is fixed, then there exists threshold value of $\lambda$, below which HD achieves higher throughput than FD. Finite SNR results agree with those obtained for the DoF.

- For the two way two hop channel (two way relay channel without a direct link between the communicating nodes, as shown in Figures 3 and 4), with only the relay having FD capability, in both AC and RC scenarios, if the symmetric DoF is to be maximized, then generally HD performs better than FD. For the asymmetric case however, provided the SI suppression is high enough (in terms of $\lambda$), FD can achieve certain DoF pairs which are not achievable by the HD. These pairs generally correspond to the extreme asymmetric DoF, when one node’s DoF requirement is significantly higher than the other one. At finite SNR levels, we compare the sum rate of the HD implementation and the FD implementation, and observe that HD always outperforms FD.

A. Related Literature

Recently, there has been a significant body of work on FD communications, and here, we briefly summarize the most relevant papers. In [16], the achievable sum rates in a two way channel for FD and HD are compared assuming perfect SI cancellation for AC implementation. Reference [17] compares the FD and HD two way channel in the presence of channel estimation errors, and provides an outer bound for the region over which FD is better than HD, depending on the level of SI and channel estimation errors. An outage analysis for FD two way communication under fading can be found in [18]. In [19], our earlier results on the sum rate performance of two way HD and FD communication are presented considering the FD implementations from [13]. In [20], a study on FD Multiple Input Multiple Output (MIMO) system is presented, basically showing how a common carrier based FD radio with a single antenna, as in [2], can be transformed into a common carrier FD MIMO radio.

In [21], two hop communication is studied with channel estimation errors in the presence of loop-back interference in order to come up with capacity cut-set bounds for both HD and FD relaying. An effective transmission power policy is proposed for the relay to maximize this bound, and performance of FD relaying with optimal power control is compared with HD relaying. Two hop communication in a cellular environment is investigated in [22], where a hybrid scheduler
that is capable of switching between HD and FD in an opportunistic fashion is proposed, for maximizing the system throughput. Reference [9] has shown that, in order to control the SI, the relay should employ power control and the proposed relaying scheme allows to switch between HD and FD modes in an opportunistic fashion, while transmit power is adjusted to maximize spectral efficiency. In [23], our earlier results in the relaying scenario are presented, comparing FD and HD relaying under the empirical residual SI model from [11]. In that work, power control is used asymptotically, so that the relay scales its power with respect to the source to achieve maximum DoF, when the relay operates in decode and forward mode. Similar asymptotic power control was also observed to give higher DoF in amplify and forward in mode in [12]. In [24], two way relaying HD and FD systems are analyzed, where source and destination nodes are assumed to hear each other. A survey on FD relaying can be found in [25].

The current literature does not contain a detailed investigation of the DoF and the supporting finite SNR rate analysis for the three communication scenarios under realistic SI and hardware constraints, as considered in this paper. This paper not only presents the results and guidelines for those scenarios, but it also provides the models and analyses that can be extended for studying more complex communication scenarios.

II. System Models

In the following, we describe the three different scenarios, two way, two hop and two way two hop communication, in which FD can be implemented. We start by providing a wireless channel model between two nodes, as a generalized point-to-point channel model that will be used throughout the paper. Then, for each communication model, we present the information flow for both HD and FD implementations.

A. Generic Channel Model Between Two Nodes

Consider a scenario where node $A$ is transmitting to node $B$, where node $B$ is operating either in HD mode or FD mode depending upon scenario being investigated. Let $P_A$ denote the average transmit power at node $A$, $\sigma_B^2$ is the average power of the AWGN at node $B$. Nodes are assumed to have multiple antennas with $H_{AB}$ denoting the channel matrix between nodes $A$ and $B$. We assume Rayleigh fading channel, so the entries of $H_{AB}$ are taken as circularly symmetric complex Gaussian random variables with unit variance [26]. Channel state information
is assumed to be available only at the receiver. The size of the channel matrices depends on the number of the transmit and receive antennas employed at the nodes. Then, the received signals at node $B$ is

$$y_B = \frac{1}{\sqrt{K}} H_{AB} x_A + w_B + i_B.$$ 

Here, $x_A$ denotes the vector of the transmitted symbol, $w_B$ denotes the noise term, and $i_B$ is the SI term if node $B$ is operating in FD mode. We assume entries of $i_B$ are Gaussian distributed with variance equal to average SI power. This assumption makes analysis tractable and can be viewed as the worst case scenario, since Gaussian distribution gives worst case capacity [27], [28]. Clearly, in the case of HD, this term is set to zero. $K$ is the parameter that characterizes the path loss between nodes. The SINR at the receiver is,

$$\Gamma_{AB} = \frac{P_A}{K (\sigma_B^2 + I_B(P_B))},$$

where we have explicitly showed the dependence of the average SI, $I_B$ on $P_B$, the transmit power of $B$ according to [11]. Details of the SI model will be described in Section II. Then, assuming that node $A$ transmits using $N_A$ antennas and node $B$ receives using $N_B$ antennas, the average achievable rate $R_{AB}$ is [15]

$$R_{AB} = \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{AB}}{N_A} H_{AB} H_{AB}^T \right) \right].$$

Degrees of Freedom (DoF) analysis characterizes the achievable rate at high SNR. For a point to point MIMO AWGN Rayleigh fading channel with $N_A$ antennas at $A$ and $N_B$ antennas at $B$, the largest DoF is given by [15]

$$\text{DoF}_{AB} = \lim_{P_A \to \infty} \frac{R_{AB}}{\log(P_A)} = \min(N_B, N_A).$$

B. Communication Scenarios

1) Two Way Channel: Two way communication channel was introduced by Shannon in [29]. Here, we consider a two way wireless channel, where node $A$ and $B$ have $N_A$ and $N_B$ antennas respectively, and wish to communicate with one another. This channel, for example may model a WiFi router communicating with a wireless device, which is simultaneously uploading and downloading data. In the HD mode, the nodes time share the wireless medium, taking turns transmitting as shown in Figures 1a and 1b. In this case, nodes use all of their antennas either
for transmission or for reception. If both $A$ and $B$ are FD capable, then they can use the channel simultaneously for transmission and reception, as shown in Figure 7. Here, $t_A$ and $r_A$ denote the number of transmit and receive antennas at node $A$, respectively. Similarly, $t_B$ and $r_B$ denote the number of antennas at node $B$. Dotted arrows in each direction represent the SI channels. The choice of $t_A, r_A, t_B$ and $r_B$ based on hardware constraints will be discussed in Section II-D.

Fig. 1: Two way channel

2) Two Hop Channel: In this scenario, node $A$ communicates with node $B$ through a relay node, $R$. We assume that there is no direct link between nodes $A$ and $B$, hence the relay assists in forwarding the packets from $A$ to $B$. The relay is assumed to operate according to decode and forward protocol, [30]. Nodes $A$ and $B$ have $N_A$ and $N_B$ antennas, respectively. Total number antennas employed in $R$ in the HD mode is denoted by $N_R$. When the relay operates in FD mode, then $t$ and $r$ denote the number of transmit and receive antennas respectively.

When the relay is in HD mode, the information flow takes place in two phases: First, $A$ transmits to $R$ as shown in Figure 2a, and then $R$ decodes the packets and forwards to $B$, as shown in Figure 2b. In the case of FD relaying, $R$ can receive from $A$ and simultaneously transmit to $B$, as shown in Figure 2c. It allocates its resources (antennas or RF chains) so as to increase the data rate from $A$ to $B$.

3) Two Way Two Hop Channel: This channel models a two way relay channel without a direct link between communicating nodes. Here, two nodes $A$ (with $N_A$ antennas) and $B$ (with $N_B$ antennas) wish to communicate with each other through a relay $R$ (with $N_R$ antennas in HD mode). Only $R$ is assumed to have FD capability and uses $t$ antenna for transmission and $r$ antenna for reception in FD mode. A motivating example for such scenario is two stations on earth communicating via a satellite, with no direct link between the stations.
When $R$ is operated in HD mode, we consider an effective communication strategy, such as [31]–[33], which takes place in two phases, as shown in Figures 3a and 3b. During the first phase, also known as the multiple access (MAC) phase, nodes $A$ and $B$ simultaneously transmit to $R$. During second phase, called broadcast (BC) phase, $R$ simultaneously transmits to $A$ and $B$, and both nodes can extract their desired signal by the virtue of analog coding techniques.

For the FD case, only $R$ is assumed to have FD capability, and two way FD communication occurs in two phases, as shown in Figures 4a and 4b. During the first phase node $A$ transmits to $B$ via $R$, and since $R$ is FD, it can receive from node $A$ and transmit simultaneously to $B$. During the second phase, direction of information flow is reversed, as node $B$ transmits to $A$ via $R$.  

Fig. 3: HD two way two hop channel
C. SI Cancellation Model

The major challenge of FD communication is SI cancellation. As discussed in detail in [34], the simplest SI cancellation technique is the passive one, obtained by the path-loss due to the separation between the transmit and receive antennas. More sophisticated active techniques, namely, analog cancellation and digital cancellation reduce the self-interference further. In analog cancellation, the FD node uses additional RF chains to estimate the channel between the transmitting and receiving antennas and then to subtract the interfering signal at the RF stage. In the digital cancellation, the self-interference is estimated and canceled in the baseband. Despite consecutive application of these three cancellation techniques, SI cannot be completely eliminated. In [11], the average power of the residual SI is modeled as

$$I = \frac{P_T(1-\lambda)}{\beta \mu \lambda}.$$  \hspace{1cm} (2)

Here, $P_T$ denotes the transmission power of the FD node, $\beta$, $\mu$ and $\lambda$ are the system parameters which depend on the cancellation technique employed, with $0 \leq \lambda \leq 1$. Note that, $\lambda = 1$ corresponds to increased noise floor SI model used in the literature.

D. Hardware Resources in HD and FD

For a fair comparison of HD and FD communications, hardware resources must be equalized. We investigate two conservation scenarios: antenna conservation, where the number of antennas is kept equal, and RF chain conservation, where the number of RF chains is kept equal [13]. For instance, if a node has $N$ antennas in HD mode, then it would have total $2N$ RF chains ($N$ each for up-converting and down-converting). While considering AC FD, we take total number of antennas to be $N$, i.e., if $r$ antennas are used for reception then remaining $(N-r)$ antennas are used for transmission. Whereas for the RC FD implementation, the total number of RF chains
is kept same as that in HD case, which is $2N$. Hence if $r$ antennas are used for reception, then in addition to $r$ down-converting RF chains, $r$ RF chains are used in analog cancellation, and remaining $2N - 2r$ RF chains can be used for up-converting in transmission, resulting in $2N - 2r$ transmit antennas. Note that, RC FD increases the total number of antennas in the FD mode. Comparison of the number of antennas is summarized in Table I.

|                | number of RX antennas | number of TX antennas | Total number of antennas |
|----------------|------------------------|-----------------------|--------------------------|
| HD             | $N$                    | $N$                   | $N$                      |
| AC FD          | $r$                    | $N-r$                 | $N$                      |
| RC FD          | $r$                    | $2N-2r$               | $2N-r$                   |

**TABLE I: Number of antennas in HD, AC FD and RC FD implementations**

III. TWO WAY CHANNEL

As the first scenario, we consider two way channel between nodes $A$ and $B$, as illustrated in Figure [I]. In this section, we formulate, calculate and compare the achievable rates as well as DoF of two way communication in HD and FD modes.

A. Half-Duplex Mode

When nodes $A$ and $B$ communicate in HD mode in the same band, they need to employ time sharing. Hence, the nodes alternate for transmission, as depicted in Figures [Ta] and [Tb]. Defining $\tau$ as the fraction of time, in which node $A$ transmits while node $B$ receives, the remaining fraction, $(1 - \tau)$ is utilized by node $B$ for transmission while node $A$ receives.

1) Achievable Rate: Recalling that the SINR at node $B$, $\Gamma_{AB}$ is calculated via equation (1), with the SI term $I_B(P_B)$ as zero in HD mode, the average achievable rate from $A$ to $B$ $R_{AB}^{HD}$ can be obtained as [15],

$$R_{AB}^{HD} = \tau \mathbb{E} \left[ \log \det \left( \mathbf{I} + \frac{\Gamma_{AB}}{N_A} \mathbf{H}_{AB} \mathbf{H}_{AB}^* \right) \right], R_{BA}^{HD} = (1 - \tau) \mathbb{E} \left[ \log \det \left( \mathbf{I} + \frac{\Gamma_{BA}}{N_B} \mathbf{H}_{BA} \mathbf{H}_{BA}^* \right) \right],$$

where $\mathbf{I}$ denotes the identity matrix, with $\mathbf{I} \in \mathbb{C}^{N_B \times N_B}$ and $\mathbf{I} \in \mathbb{C}^{N_A \times N_A}$, and $N_X$ is the total number of antennas for node $X$. 

2) Degrees of Freedom: The DoF characterizing the performance at high SNR, for the two-way channel considering HD communication is obtained as follows:

\[
\text{DoF}_{AB}^{HD} = \lim_{P_A \to \infty} \frac{R_{AB}^{HD}}{\log(P_A)} = \tau \min(N_A, N_B),
\]

\[
\text{DoF}_{BA}^{HD} = \lim_{P_B \to \infty} \frac{R_{BA}^{HD}}{\log(P_B)} = (1 - \tau) \min(N_A, N_B).
\]

This results in the following DoF trade-off,

\[
\{\text{DoF}_{AB}^{HD}, \text{DoF}_{BA}^{HD}\} = \{\tau, (1 - \tau)\} \min(N_A, N_B), \quad 0 \leq \tau \leq 1. \tag{3}
\]

B. Full-Duplex Mode

In this case, both nodes A and B are assumed to have FD capability, so that they can transmit to each other simultaneously in the same band. Then the SINR per node, \( \Gamma \) is calculated from (I) with the residual SI model in (2). Below, \( t_X \) denotes the number of antennas used for transmission, and \( r_X \) is the number of antennas used for reception at node \( X \) in FD mode, as illustrated in Section II-D.

1) Achievable Rates: The average achievable rates can be calculated as,

\[
R_{AB} = \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{AB}}{t_A} H_{AB} H_{AB}^* \right) \right], \quad R_{BA} = \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{BA}}{t_B} H_{BA} H_{BA}^* \right) \right],
\]

where \( I \in \mathbb{C}^{r_B \times r_B} \) and \( I \in \mathbb{C}^{r_A \times r_A} \).

2) Degrees of Freedom: The DoF trade-off for FD mode is achieved through the following power scaling approach,

\[
\frac{\log(P_B)}{\log(P_A)} = \gamma,
\]

for some \( \gamma > 0 \). Thus, the achievable DoF from node A to B are calculated as

\[
\text{DoF}_{AB}^{FD} = \lim_{P_A \to \infty} \frac{R_{AB}^{FD}}{\log(P_A)} = \left[ 1 - \gamma(1 - \lambda) \right]^+ \min(r_B, t_A).
\]

Similarly, from node B to A,

\[
\text{DoF}_{BA}^{FD} = \lim_{P_B \to \infty} \frac{R_{BA}^{FD}}{\log(P_B)} = \left[ 1 - \frac{(1 - \lambda)}{\gamma} \right]^+ \min(r_A, t_B).
\]

The DoF trade-off region is given by

\[
\{\text{DoF}_{A}^{FD}, \text{DoF}_{B}^{FD}\} = \bigcup_{1 - \lambda \leq \gamma \leq 1/(1-\lambda)} \left\{ \left[ 1 - \gamma(1 - \lambda) \right]^+ \min(r_B, t_A), \left[ 1 - \frac{(1 - \lambda)}{\gamma} \right]^+ \min(r_A, t_B) \right\}.
\]

\( (5) \)
Here $\bigcup$ denotes the convex-hull over the admissible parameters. The possible ranges for $r_A, r_B, t_A$ and $t_B$ depend on the hardware constraints as shown in Table I.

C. Comparison of the HD and FD Modes

Below, we evaluate and compare the achievable DoF and finite SNR rates of two way communication in HD and FD modes, considering different transmission power levels, number of antennas and SI cancellation levels. For the FD mode, we refer to the two implementation models considering the allocation of radio resources, namely, the AC FD and RC FD, as described in Section II-D.

1) Degrees of Freedom Comparison: From the DoF perspective, the following proposition shows that in the AC case, FD cannot perform better than HD.

**Proposition 1:** When $\lambda < 1$, the DoF region for AC FD implementation of the two way channel lies strictly inside the HD implementation.

**Proof:** The DoF region in (3) is equivalent to the following

$$\text{DoF}^{HD}_{AB} + \text{DoF}^{HD}_{BA} = \min(N_A, N_B).$$

Hence, in order to show that the FD DoF region lies strictly inside the HD one for $\lambda < 1$, it suffices to show that

$$\text{DoF}^{FD}_{AB} + \text{DoF}^{FD}_{BA} < \min(N_A, N_B).$$

Note that, (5) for the AC scenario becomes

$$\text{DoF}^{FD}_{AB} = (1 - \gamma(1 - \lambda)) \min(t_A, N_B - t_B),$$
$$\text{DoF}^{FD}_{BA} = \left(1 - \frac{1 - \lambda}{\gamma}\right) \min(N_A - t_A, t_B),$$

(6)

for some $\gamma \in [1 - \lambda, (1 - \lambda)^{-1}]$ and $0 < t_A < N_A, 0 < t_B < N_B$. Since $1 - \lambda \leq \gamma \leq (1 - \lambda)^{-1}$, we have

$$1 - \gamma(1 - \lambda) \leq 1 - (1 - \lambda)^2,$$

(7)

and also

$$1 - \frac{1 - \lambda}{\gamma} \leq 1 - (1 - \lambda)^2.$$
Then from (6), (7) and (8)

\[
\text{DoF}_{AB}^{FD} \leq (1 - (1 - \lambda)^2) \min(t_A, N_B - t_B),
\]

\[
\text{DoF}_{BA}^{FD} \leq (1 - (1 - \lambda)^2) \min(N_A - t_A, t_B).
\]

(9)

Thus,

\[
\text{DoF}_{AB}^{FD} + \text{DoF}_{BA}^{FD} \leq (1 - (1 - \lambda)^2)(\min(t_A, N_B - t_B) + \min(N_A - t_A, t_B))
\]

\[
\leq (1 - (1 - \lambda)^2) \min(N_A, N_B)
\]

\[
< \min(N_A, N_B),
\]

where the last equation follows, since \( \lambda < 1 \) implies \( (1 - (1 - \lambda)^2) < 1 \). □

The next proposition shows that, unlike AC FD case, for RC FD implementation, some part of FD DoF region lies outside the HD region.

**Proposition 2:** When \( \lambda > \frac{3}{4} \frac{\min(N_A, N_B)}{\min(N_A, N_B) - 6} \), there exists a point in FD DoF region, which is not achievable by HD transmission for the RF conserved scenario. If \( N_A \) and \( N_B \) are divisible by 3 then the condition becomes \( \lambda > 3/4 \).

**Proof:** Considering \( \gamma = 1 \), \( r_A = \lfloor 2N_A/3 \rfloor \) and \( r_B = \lfloor 2N_B/3 \rfloor \) in (5), results in \( t_A = \lfloor 2N_A/3 \rfloor \) and \( t_B = \lfloor 2N_B/3 \rfloor \) for the RC implementation, and

\[
\text{DoF}_{A}^{FD} \lambda = \min([2N_A/3], [2N_B/3]), \text{DoF}_{B}^{FD} \lambda = \min([2N_A/3], [2N_B/3]).
\]

(10)

Thus,

\[
\text{DoF}_{A}^{FD} + \text{DoF}_{B}^{FD} = 2\lambda \min([2N_A/3], [2N_B/3])
\]

\[
\geq 2\lambda \min(2N_A/3 - 1, 2N_B/3 - 1)
\]

\[
= \frac{4}{3} \lambda \min(N_A, N_B) - 2\lambda
\]

\[
> \min(N_A, N_B)
\]

(11)

where the last inequality can be shown to be true after some algebraic manipulation, when

\[
\lambda > \frac{\min(N_A, N_B)}{\frac{4}{3} \min(N_A, N_B) - 2}.
\]

When both \( N_A \) and \( N_B \) are divisible by 3, there is no flooring operation, and hence the condition simplifies to \( \lambda > 3/4 \). □
Fig. 5: Degrees of Freedom region for two way channel, $N_A = 4, N_B = 6, \lambda = 0.9$

Figure 5 shows the DoF region for HD, AC FD and RC FD scenarios, where for the FD case we have plotted the convex hull in (5). The corner point of the FD trade-off occurs when SI at one of the node is so high (due to high transmission power at that node) that the DoF it receives is effectively zero, even though the other node is transmitting to it. Note that while RC FD can achieve DoF pairs not possible with HD, its DoF region does not contain that of HD.

Fig. 6: Sum rate of two way communication for HD and FD implementations

2) Finite SNR Comparison: Figure 6 shows the sum rate performance of HD, AC FD and RC FD implementations as a function of $P_A = P_B$ under different SI suppression levels for the two way channel, while the number of antennas is set as, $N_A = N_B = 3$ for HD, $t_A = t_B = 1$, $r_A = r_B = 2$ for AC FD, and $t_A = t_B = r_A = r_B = 2$ for RC FD. Since $\lambda$ is a parameter that captures
the quality of SI cancellation, the performance of both AC FD and RC FD implementations are improved with higher $\lambda$ values. AC FD performs only slightly better than HD at low transmit power levels, while it performs strictly below HD at high transmit power levels, even for the case of perfect SI cancellation (i.e., when $I$ in equation (2) is taken as 0). The RC FD implementation provides superior performance over HD, when the SI is perfectly suppressed; however it performs below HD even for the case of low residual SI (i.e., $\lambda = 0.8$). Note that, the case of perfect SI cancellation presents the upper bound for FD performance, which is quite loose, since the actual sum rate is considerably lower. Since better performance is observed with RC FD relative to AC FD, we focus on the RF chain conserved FD implementation in the remaining numerical results.

![Graph](image)

(a) Effect of SI cancellation level, $\lambda$, $P_A = P_B = 10\, dB$

![Graph](image)

(b) $\lambda_{thr}$ vs transmission power

Fig. 7: Threshold $\lambda$ over which RC FD outperforms HD, $r_A = t_A = r_B = t_B = 2, N_A = N_B = 3$

In Figure 7, we investigate the effect of SI suppression on the sum rate of the two way channel, when $P_A$ and $P_B$ are both kept constant, as $P_A = P_B = 10$ dB. This figure allows us to find the threshold $\lambda$ value, $\lambda_{thr}$, over which RC FD performs better than HD. Naturally, the performance of HD mode does not change with $\lambda$, as shown by the red line in the figure, and the sum rate of RC FD mode increases with increasing $\lambda$ as depicted by the blue curve. The intersection of the two curves correspond to the so-called threshold, which is found to be $\lambda_{thr} \approx 0.95$ for this setting. Although this value is higher than 0.75, the threshold calculated in the DoF analysis in Proposition 2, from Figure 7b, we see that the value of $\lambda_{thr}$ decreases with the transmission power of nodes. By further experiments, by increasing the power to unrealistically high levels
(a) Poor SI cancellation, $\lambda = 0.2$

(b) Good SI cancellation, $\lambda = 0.8$

Fig. 8: Sum rate of RC FD for discrete values of $P_A$ and $P_B$

(such as 100 dB, which is not included in Figure 7b), we have actually observed that $\lambda_{thr}$ does converge to 0.75, the threshold of DoF analysis. Figure 7b also implies that a desired FD gain can be obtained by increasing transmission power of nodes.

In the finite SNR results presented above, unlike the DoF analysis, power control was not applied and $P_A$ was always kept equal to $P_B$. We next investigate the sum rate of RC FD for discrete values of $P_A$ and $P_B$ to observe the power levels that maximize the sum rate for poor ($\lambda = 0.2$) and good ($\lambda=0.8$) SI cancellation as shown in Figure 8. For poor SI cancellation, the sum rate is maximized when only one node is allowed to transmit (with maximum power level), and the other node is kept silent (with zero power), implying one way communication; while for good SI cancellation, both nodes should transmit at their maximum power levels to maximize the sum rate. Further investigating the power levels that yield the maximum sum rate for different $\lambda$ values, we observe that one way communication is preferred up to some $\lambda$ noted within $[0.7, 0.8]$, and two way communication with equal power allocation is favored afterwards. Since in this paper, we consider duplexing in two way communication, omitting the one way solution and setting equal power levels for the two end nodes in our finite SNR FD analysis is well-justified.

IV. TWO HOP CHANNEL: RELAYING

As the second scenario, we consider the two hop channel, where $A$ communicates with $B$ through $R$, as illustrated in Figure 2. Assuming that $B$ does not hear $A$, we formulate, calculate
and compare the achievable rates and DoF of two hop communication, i.e., relaying, in HD and FD modes.

A. Half-Duplex Mode

In the HD mode, A first transmits to \( R \) for a fraction of time, \( \tau \), \( R \) decodes the received bits and forwards them to \( B \) for the remaining fraction, \( 1 - \tau \) of time.

1) Achievable Rate: The average rate achievable from \( A \) to \( R \) is calculated as

\[
R_{AR}^{HD} = \tau \mathbb{E} \left[ \log \det \left( \mathbf{I} + \frac{\Gamma_{AR}}{N_A} \mathbf{H}_{AR} \mathbf{H}_{AR}^* \right) \right],
\]

and the rate achievable over \( R \) to \( B \) is given by

\[
R_{RB}^{HD} = (1 - \tau) \mathbb{E} \left[ \log \det \left( \mathbf{I} + \frac{\Gamma_{RB}}{N_R} \mathbf{H}_{RB} \mathbf{H}_{RB}^* \right) \right].
\]

By optimizing over \( \tau \), the end-to-end average achievable rate for HD relaying can be found as

\[
R_{AB}^{HD} = \max_{0 \leq \tau \leq 1} \min \left( R_{AR}^{HD}, R_{RB}^{HD} \right).
\]

2) Degrees of Freedom: For the DoF of the two hop channel, as in Section III-B2 we assume that the relay scales its power with respect to the transmission power of node \( A \), according to

\[
\frac{\log(P_R)}{\log(P_A)} = \gamma, \quad 0 < \gamma \leq 1.
\]

Then, the DoF of the relay network in HD mode is given by

\[
\text{DoF}_{HD} = \sup_{0 < \gamma \leq 1} \lim_{P_A \to \infty} \frac{R_{HD}}{\log(P_A)}
\]

DoF\(_{HD}\). Depending on the values of \( N_A, N_R, \) and \( N_B, \) and using optimal \( \tau \) denoted as \( \tau_{opt} \), following DoF values for the HD mode as shown in the Table II [23] are obtained.

| \( N_R \) | \( \tau_{opt} \) | DoF\(_{HD} \) |
|----------|----------------|------------|
| \( N_R \leq \min(N_A, N_B) \) | \( \frac{1}{2} \) | \( \frac{N_R}{2} \) |
| \( N_R \geq \max(N_A, N_B) \) | \( \frac{N_R}{N_B+N_A} \) | \( \frac{N_A N_B}{N_R+N_A} \) |
| \( N_A \leq N_R \leq N_B \) | \( \frac{N_R}{N_B+N_A} \) | \( \frac{N_R N_A}{N_R+N_A} \) |
| \( N_B \leq N_R \leq N_A \) | \( \frac{N_B}{N_B+N_R} \) | \( \frac{N_B N_A}{N_R+N_B} \) |

TABLE II: Degrees of Freedom for HD relaying
B. Full-Duplex Mode

In FD relaying, $R$ is able to receive and transmit simultaneously in the same band, however, it is subject to SI. In order to maintain causality, the relay node transmits $(i-1)^{th}$ symbol, while it receives the $i^{th}$ symbol.

1) Achievable Rate: When the relay node operates in FD mode, the rates are calculated as follows:

$$R^{FD}_{AR} = \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{AR}}{N_A} H_{AR} H_{AR}^* \right) \right], \quad R^{FD}_{RB} = \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{RB}}{t} H_{RB} H_{RB}^* \right) \right].$$

(14)

Recall that $t = (N_R - r)$ for AC FD, and $t = (2N_R - 2r)$ for RC FD. Depending on the average SINR at the relay node and SNR at $B$, the excess power at the relay can have a negative impact on the achievable rate due to increased SI. In fact, the SINR at the relay node is decreased as the relay power $P_R$ is increased, while $P_A$ is held constant. Thus, with the increase in $P_R$ for a constant $P_A$, the rate of the channel from node $A$ to $R$ is decreased, while the rate of the channel from $R$ to node $B$ is increased. Therefore, by letting $P_{R_{max}}$ denote the maximum average power at the relay, the achievable rate for FD relaying can be written as

$$R^{AD}_{FD} = \max_{0 < r < N_R} \min_{P_R \leq P_{R_{max}}} \left( R^{FD}_{AR}, R^{FD}_{RB} \right).$$

(15)

2) Degrees of Freedom: Assuming power scaling as in (13), the DoF of FD relaying is obtained as

$$\text{DoF}^{FD}_{AB} = \sup_{0 < \gamma \leq 1} \lim_{P_A \to \infty, P_R = P_A} \frac{R^{FD}_{FD}}{\log(P_A)},$$

where in order to control the SI, the relay scales its power with respect to the transmission power of node $A$, through

$$\frac{\log(P_R)}{\log(P_A)} = \gamma, \quad 0 < \gamma \leq 1.$$ 

Then, the achievable DoF for FD relaying can be computed as

$$\text{DoF}^{FD}_{AB} = \max_{0 < r < N_R} \min_{0 < \gamma \leq 1} \min((1 - \gamma(1 - \lambda)) \min(N_A, r),$$

$$\gamma \min(t, N_B))$$

$$= \max_{0 < r < N_R} \min_{0 < \gamma \leq 1} \min((1 - \gamma(1 - \lambda))N_A, (1 - \gamma(1 - \lambda))r, $$

$$\gamma t, \gamma N_B).$$

(16)
Here, $t = (N_R - r)$ for the AC FD implementation and $t = (2N_R - 2r)$ for the RC FD implementation. We explicitly compute the $\text{DoF}_{AB}^{FD}$ for symmetric and asymmetric cases and compare it with $\text{DoF}_{AB}^{HD}$ in the next subsection.

C. Comparison of HD and FD Relaying

1) Degrees of Freedom Comparison:

- **Symmetric Case ($N_A = N_B$):**

To compare the DoF of HD relaying and FD relaying, we first consider a symmetric case when the $A$ and $B$ have same number of antennas, i.e., $N_A = N_B = N$, and $N_R$ is even. Using the Table II, one can obtain

$$\text{DoF}_{AB}^{FD} = \min\left(\frac{N}{2}, \frac{N_R}{2}\right). \quad (17)$$

For AC FD implementation (16) can be written as

$$\text{DoF}_{AB}^{FD,AC} = \max_{0 \leq r < N_R} \min((1 - \gamma(1 - \lambda))N, (1 - \gamma(1 - \lambda))r, \gamma(N_R - r), \gamma N) \quad (18)$$

Since the minimum of the two terms in (18) is maximized when both are equal, (19) can be obtained by setting $\gamma = \frac{1}{2 - \lambda}$. Similarly,

$$\text{DoF}_{AB}^{FD,AC} = \max_{0 \leq r < N_R} \min((1 - \gamma(1 - \lambda))r, \gamma(N_R - r)), \quad (19)$$

$$= \frac{N_R}{2(2 - \lambda)}. \quad (20)$$

where (20) is obtained by setting $\gamma = \frac{1}{2 - \lambda}$ and $r = \frac{N_R}{2}$.

From (19) and (20), we can write

$$\text{DoF}_{AB}^{FD,AC} \leq \frac{1}{2 - \lambda} \min\left(\frac{N}{2}, \frac{N_R}{2}\right). \quad (21)$$

Equality in (21) can be achieved when $\gamma = \frac{1}{2 - \lambda}$ and $r = \frac{N_R}{2}$, leading to

$$\text{DoF}_{AB}^{FD,AC} = \frac{1}{2 - \lambda} \min\left(\frac{N}{2}, \frac{N_R}{2}\right). \quad (22)$$
Similarly, for the RC FD implementation we have,

\[
\text{DoF}_{AB}^{FD,RC} = \frac{1}{2 - \lambda} \min \left( N, \left\lfloor \frac{2N_R}{3} \right\rfloor \right). \tag{23}
\]

The DoF results for this case are plotted in Figure 9. As seen from the figure, when \( N_R \) is small, HD relaying performs better than FD relaying, and the situation is reversed when \( N_R \) gets larger, with the RC FD implementation always dominating AC FD implementation. Comparing (17) and (22), \( \text{DoF}_{AB}^{FD,AC} > \text{DoF}_{AB}^{HD} \) if

\[
N_R > N(2 - \lambda) \quad \text{and} \quad \lambda > 0. \tag{24}
\]

Similarly from (17) and (23), if \( N_R \) is divisible by 3, then \( \text{DoF}_{AB}^{FD,RC} > \text{DoF}_{AB}^{HD} \) provided

\[
N_R > \frac{3}{4} N(2 - \lambda) \quad \text{and} \quad \lambda > 0. \tag{25}
\]

If \( N_R \) is not divisible by 3, then (17) and (23) can be evaluated to compare \( \text{DoF}_{AB}^{FD,RC} \) and \( \text{DoF}_{AB}^{HD} \).

- **Asymmetric Case** \((N_A = 1)\):

Now, we consider the asymmetric case, where node \( A \) has a single antenna, i.e., \( N_A = 1 \), whereas the relay and node \( B \) have multiple antennas, \( N_R \) and \( N_B \geq 1 \), respectively. This could model a cellular phone, which cannot afford multiple antennas, communicating with an access point through a relay.
From the general expression for the DoF for the FD relaying channel for both AC and RC implementation, (16) with \( N_A = 1 \)

\[
\text{DoF}_{AB}^{FD} = \max_{0 < N_R \leq 1, 0 < r < N_R} \min((1 - \gamma(1 - \lambda)), (1 - \gamma(1 - \lambda)) r, \\
\gamma(N_R - r), \gamma N_B) 
\]

\[= \max_{0 < \gamma \leq 1} \min((1 - \gamma(1 - \lambda)), \gamma(N_R - 1), \gamma N_B) \]  \hspace{1cm} (26)

\[= \frac{\min(N_R - 1, N_B)}{\min(N_R - 1, N_B) + 1 - \lambda} \]  \hspace{1cm} (27)

where we have set \( r = 1 \) in (27) since the minimum of first two terms in (26) does not depend on \( r \) and the third term is decreasing in \( r \).

The DoF for HD relaying yields

\[
\text{DoF}_{AB}^{HD} = \frac{\min(N_R, N_B)}{\min(N_R, N_B) + 1},
\]

It can be seen that \( \text{DoF}_{AB}^{FD} > \text{DoF}_{AB}^{HD} \) if

\[
\lambda > 1 - \frac{\min(N_R - 1, N_B)}{\min(N_R, N_B)}.
\]

Hence, for both AC FD and RC FD implementations, the following holds: If \( N_R > N_B \), then DoF of FD relaying is strictly larger than DoF of HD relaying for both AC FD and RC FD implementations, provided \( \lambda > 0 \). If \( N_R \leq N_B \), then FD relaying performs better than HD relaying if \( \lambda > \frac{1}{N_R} \). Thus the FD implementation is better than the HD if

\[
N_R > \min\left( N_B, \frac{1}{\lambda} \right) \]  \hspace{1cm} (29)

It is interesting to note that, for the case \( N_A = N_B = 1 \), and \( N_R = 2 \), [12] obtained the DoF (referred to as multiplexing gain in [12]) for FD relaying channel, with the relay node operating in amplify and forward mode, using a similar SI model as in this paper. Their multiplexing gain term of \( \frac{1}{2 - \lambda} \) matches with our DoF, when evaluated at \( N = 1 \), and \( N_R = 2 \) in (22).

2) \textit{Finite SNR Comparison:} Here, we compare the end-to-end throughput performance of HD relaying with FD relaying, via numerical evaluations considering different system parameters. In these simulations, we consider a simple scenario, where nodes \( A \) and \( B \) have a single antenna (\( N_A = N_B = 1 \)). The relay node has three antennas (\( N_R = 3 \)) in HD mode, two receive (\( r = 2 \)) and two transmit antennas (\( t = 2 \)) in RC FD implementation, and two receive antennas (\( r = 2 \))
Fig. 10: Achievable end-to-end rates of HD relaying and FD relaying

and one transmit antenna ($t = 1$) in AC FD implementation, consistent with Table 1 in Section II-D. The relay node is assumed to perform power control, so that the $R_{AR}^{FD}$ and $R_{RB}^{FD}$ are kept equal.

Figure 10 shows the achievable rates from node $A$ to node $B$ as a function of $P_A = P_{R_{max}}$. As anticipated and also shown by Figure 10, the throughput performance of the network is improved as $\lambda$ gets higher. Figure 10 also shows that both AC FD and RC FD relaying have a superior performance over HD relaying even with imperfect SI cancellation (i.e., $\lambda = 0.8$). For the channel parameters in Figure 10 the HD relaying DoF is calculated as $1/2$, whereas the FD relaying DoF for both AC FD and RC FD implementations is $\frac{1}{2-\lambda}$. Hence, for example, for $\lambda = 0.2$, the FD relaying DoF is approximately 0.56, which is only marginally higher than the DoF of HD relaying. The DoF analysis does not take into account the constants in the residual SI model, as they do not scale with the power; however those constants are considered in finite SNR analysis, where FD relaying performs worse than HD relaying for this value of $\lambda$. For a higher value of $\lambda = 0.8$, FD relaying DoF is about 0.83, which is much larger than HD relaying DoF, dominating the performance in achievable rate as well. Hence, at this level of SI cancellation ($\lambda$), FD relaying is observed to provide higher achievable rate than HD relaying.

V. TWO WAY TWO HOP CHANNEL: TWO WAY RELAYING

As the third scenario, we consider the two way two hop channel, where $A$ and $B$ communicate with each other through $R$, performing two way relaying, as illustrated in Figures 3 and 4 representing HD and FD modes, respectively. Again, there is no direct link between nodes $A$
and $B$. We formulate, calculate and compare the achievable rates as well as DoF of two way two hop communication in HD and FD modes.

### A. Half-Duplex Mode

1) **Achievable Rates:** In HD mode, with transmission strategies described in the corresponding system model in Section II-B, the average achievable rates can be calculated as follows: During the MAC phase, both nodes $A$ and $B$ transmit their messages to the relay node with achievable rates calculated as [15],

$$
R_{AR}^{HD} \leq \tau \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{AR}}{N_A} H_{AR} H_{AR}^* \right) \right], 
R_{BR}^{HD} \leq \tau \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{BR}}{N_B} H_{BR} H_{BR}^* \right) \right],
$$

$$
R_{AR}^{HD} + R_{BR}^{HD} \leq \tau \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{AR}}{N_A} H_{AR} H_{AR}^* + \frac{\Gamma_{BR}}{N_B} H_{BR} H_{BR}^* \right) \right].
$$

Here we assume that MAC phase lasts for $\tau$ fraction of the time. During the BC phase, the relay node broadcasts a message to both of the nodes, such that each node can retrieve the other node’s message by subtracting its own data. The achievable rates are obtained as [15]

$$
R_{RA}^{HD} \leq (1 - \tau) \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{RA}}{N_R} H_{RA} H_{RA}^* \right) \right], 
R_{RB}^{HD} \leq (1 - \tau) \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{RB}}{N_R} H_{RB} H_{RB}^* \right) \right].
$$

Then, the end-to-end rates are obtained as

$$
R_{AB}^{HD} = \min (R_{AR}, R_{RB}), \quad R_{BA}^{HD} = \min (R_{BR}, R_{RA}).
$$

The BC phase is assumed to last $(1 - \tau)$ fraction of the time. The achievable two way sum rate is obtained by optimizing over the fraction of time spent between the two phases:

$$
R_{sum}^{HD} = \max_{0 \leq \tau \leq 1} \left( R_{AB}^{HD} + R_{BA}^{HD} \right).
$$

(30)

Note that, dropping the sum rate constraint from the MAC phase gives the cut-set upper bound for the HD two way two hop channel [35].

2) **Degrees of Freedom:** We compute an upper bound for DoF for HD two way two hop channel, which we then compare with the performance of FD two way two hop channel. We also compare FD with the achievable MAC-BC HD scheme introduced above.

During the first phase, which is assumed to be used for the fraction $\tau$ of time, the $\text{DoF}_{AR}$ and $\text{DoF}_{BR}$ are upper bounded by the respective point-to-point DoFs, i.e.,

$$
\text{DoF}_{AR} \leq \tau \min (N_A, N_R), \quad \text{DoF}_{BR} \leq \tau \min (N_B, N_R).
$$
Similarly, during the second phase, the DoF expressions are
\[
\text{DoF}_{RA} \leq (1 - \tau) \min(N_A, N_R), \quad \text{DoF}_{RB} \leq (1 - \tau) \min(N_B, N_R).
\]

The achievable DoF with MAC-BC scheme has an additional sum constraint
\[
\text{DoF}_{RA} + \text{DoF}_{RB} \leq (1 - \tau) \min(2N_A, N_R).
\]

Hence, the upper bound on end-to-end DoF is
\[
\text{DoF}_{AB} \leq \min(\text{DoF}_{AR}, \text{DoF}_{RB}) = \min(\tau N_A, \tau N_R(1 - \tau)N_R, (1 - \tau)N_B), \\
\text{DoF}_{BA} \leq \min(\text{DoF}_{BR}, \text{DoF}_{RA}) = \min(\tau N_B, \tau N_R(1 - \tau)N_R, (1 - \tau)N_A).
\]

For the symmetric case \(N_A = N_B = N\), \(\tau = \frac{1}{2}\) maximally enlarges the outer bound region, i.e.,
\[
\text{DoF}_{UD}^H = \{(\text{DoF}_{AB}, \text{DoF}_{BA}) \in \mathbb{R}^2 : \quad \text{DoF}_{AB} \leq \min \left( \frac{N}{2}, \frac{N_R}{2} \right), \\
\text{DoF}_{BA} \leq \min \left( \frac{N}{2}, \frac{N_R}{2} \right) \}
\]

Similarly, taking \(\tau = \frac{1}{2}\) gives the following achievable DoF region with MAC-BC scheme
\[
\text{DoF}_{DF}^H = \{(\text{DoF}_{AB}, \text{DoF}_{BA}) \in \mathbb{R}^2 : \quad \text{DoF}_{AB} \leq \min \left( \frac{N}{2}, \frac{N_R}{2} \right), \\
\text{DoF}_{BA} \leq \min \left( \frac{N}{2}, \frac{N_R}{2} \right), \quad \text{DoF}_{AB} + \text{DoF}_{BA} \leq \min \left( N, \frac{N_R}{2} \right) \}.
\quad (31)
\]

B. Full-Duplex Mode

In this section, we assume only \(R\) is FD enabled, and \(A\) and \(B\) are HD nodes. As described in the corresponding system model in Section II-B, FD two way two hop communication takes place in two phases assigned for each direction, where \(A\) and \(B\) send data to each other, as \(R\) performs FD relaying. Again, it is assumed that the fraction of time devoted to first phase is denoted by \(\tau\), and the remaining fraction, \(1 - \tau\), is assigned to the second phase.

1) Achievable Rates: According to SINR expressions at the nodes, the average rates are calculated through the following expressions:
\[
R_{AR}^{FD} \leq \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{AR}}{N_A} H_{AR} H_{AR}^* \right) \right], \quad R_{RB}^{FD} \leq \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{RB}}{\ell} H_{RB} H_{RB}^* \right) \right], \\
R_{BR}^{FD} \leq \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{BR}}{N_B} H_{BR} H_{BR}^* \right) \right], \quad R_{RA}^{FD} \leq \mathbb{E} \left[ \log \det \left( I + \frac{\Gamma_{RA}}{\ell} H_{RA} H_{RA}^* \right) \right].
\]
The end-to-end average rate is given by the following expressions,

\[ R_{FD}^{AB} = \tau \min(R_{AR}, R_{RB}) ,
R_{FD}^{BA} = (1 - \tau) \min(R_{BR}, R_{RA}) . \]

Then the two way relaying sum rate is given by,

\[ R_{sum}^{FD} = \max_{0 \leq \tau \leq 1} \left( R_{FD}^{AB} + R_{FD}^{BA} \right) . \tag{32} \]

where \( P_{RB} \) and \( P_{RA} \) denote the transmission power of the relay.

2) Degrees of Freedom: Since only \( R \) is FD capable, the DoF achievable from node \( A \) to node \( B \) is given as in Section [IV-B] Letting \( D_{FD}^{AB} \) and \( D_{FD}^{BA} \) be DoFs obtained when communication takes place from the node \( A \) to the node \( B \), and in the reverse direction respectively, the DoF trade off region is obtained via time sharing as follows:

\[ \text{DoF}_{FD} = \{ (\text{DoF}_{FD}^{AB}, \text{DoF}_{FD}^{BA}) \in \mathbb{R}^2 : \text{DoF}_{AB} \leq \tau D_{FD}^{AB}, \text{DoF}_{BA} \leq (1 - \tau) D_{FD}^{BA}, 0 \leq \tau \leq 1 \} . \]

Here, \( D_{FD}^{AB} \) is given by (16),

\[ D_{FD}^{AB} = \max_{0 < r < N_R} \min((1 - \gamma(1 - \lambda))(1 - \gamma(1 - \lambda))r, \gamma t, \gamma N_B) . \]

A similar expression holds for \( D_{FD}^{BA} \). We explicitly evaluate and compare the DoF regions of HD and FD two way relaying for some specific cases in the next subsection.

C. Comparison of HD and FD Two Way Relaying

1) Degrees of Freedom Comparison: Again, we consider the symmetric and asymmetric scenarios as follows.

- **Symmetric Case** \((N_A = N_B)\):

  If the nodes \( A \) and \( B \) have same number of antennas \((N_A = N_B = N)\), and \( N_R \) is even then the DoF region for the AC FD implementation is obtained as from Section [IV-C1]

\[ \text{DoF}_{FD} = \{ (\text{DoF}_{FD}^{AB}, \text{DoF}_{FD}^{BA}) \in \mathbb{R}^2 : \text{DoF}_{AB} \leq \frac{\tau}{2 - \lambda} \min\left(\frac{N_R}{2}, N\right), \text{DoF}_{BA} \leq \frac{1 - \tau}{2 - \lambda} \min\left(\frac{N_R}{2}, N\right), 0 \leq \tau \leq 1 \} . \]
Note that, the corner point of this trade-off is \( \left( \frac{1}{2-\lambda} \min \left( \frac{N_R}{2}, N \right), 0 \right) \), which is better than the corner point of for the HD case \( \left( \min \left( \frac{N_R}{2}, \frac{N}{2} \right), 0 \right) \) provided

\[ N_R > (2 - \lambda)N \text{ and } \lambda > 0. \]

Hence, if \( R \) has sufficient number of antennas, then some part of the FD DoF region lies outside the HD DoF upper bound.

Similarly, for the RC FD implementation, the DoF region is given by

\[
\text{DoF}^{FD} = \left\{ (\text{DoF}_{AB}, \text{DoF}_{BA}) \in \mathbb{R}^2 : \text{DoF}_{AB} \leq \frac{\tau}{2 - \lambda} \min \left( \left\lfloor \frac{2N_R}{3} \right\rfloor, N \right), \text{DoF}_{BA} \leq \frac{1 - \tau}{2 - \lambda} \min \left( \left\lfloor \frac{2N_R}{3} \right\rfloor, N \right), 0 \leq \tau \leq 1 \right\}. \tag{33}
\]

Hence the condition for some part of the FD DoF region for RC FD implementation to lie outside the upper bound of HD one is (provided \( N_R \) is divisible by 3) (see (25))

\[ N_R > \frac{3(2 - \lambda)}{4}N \text{ and } \lambda > 0. \tag{34} \]

An example plot comparing the DoF trade off for the symmetric case can be seen in Figure 11 where we have plotted the upper bound for the HD trade-off, an achievable HD trade-off through MAC-BC scheme, and the FD AC and RC trade-off. It can be observed that near the corner points, where one of the node’s DoF is small, the AC trade-off is better than the HD trade-off. However near the central region when both of the node’s DoF is nearly equal HD trade-off is better.

Fig. 11: DoF trade-off for HD and FD two way relaying, \( N_A = N_B = N = 4, N_R = 6. \)
• **Asymmetric Case** ($N_A = 1$):

Using the time sharing and expressions obtained for DoF for the asymmetric case in Section IV-C1, the DoF region for FD (both AC and RC) can be written as,

$$
\text{DoF}_{FD}^A = \{(\text{DoF}_{AB}, \text{DoF}_{BA}) \in \mathbb{R}^2 : \text{DoF}_{AB} \leq \tau \frac{\min(N_R - 1, N_B)}{\min(N_R - 1, N_B) + 1 - \lambda}, \\
\text{DoF}_{BA} \leq (1 - \tau) \frac{\min(N_R - 1, N_B)}{\min(N_R - 1, N_B) + 1 - \lambda}, 0 \leq \tau \leq 1\}.
$$

Thus some part of the FD DoF region lies outside the HD one provided (see (29))

$$N_R > \min \left( N_B, \frac{1}{\lambda} \right).$$

2) **Finite SNR Comparison**: Lastly, we provide numerical evaluations for two way two hop channel under finite SNR, and we compare the total achievable rate when $R$ operates in HD mode and FD mode. For the test scenario, nodes $A$ and $B$ are equipped with one antenna each and the relay node has multiple antennas: For HD two way relaying, $N_R = 3$ and in FD two way relaying, $r = 2$, $t = 1$ for AC FD implementation, and $r = 2$, $t = 2$ antennas are used for RC FD implementation.

![Graph](image.png)

**Fig. 12**: Sum rate of two way relaying with respect to $P_A = P_{R\text{max}} = P_B$

As seen from Figure 12, even for perfect SI cancellation, neither of FD two way relaying implementations shows a satisfactory performance over HD with RCF FD producing almost the same sum rate as HD does and AC FD falling short of HD. For RC FD and AC FD with realistic residual SI levels ($\lambda < 1$), FD two way relaying performs below HD.
For the same settings \( N_A = N_B = 1 \) and \( N_R = 3 \), we can observe that the sum DoF achievable for HD two way relaying via MAC-BC scheme is 1 (see (31)), and the DoF achievable on the FD two way relaying for the RC FD implementation, with perfect SI cancellation, i.e., \( \lambda = 1 \), is also 1 (see (33)). This is consistent with the finite SNR results in Figure 12, where the RC FD implementation does not provide any sum rate advantage over the HD two way relaying. Since with imperfect SI cancellation \( (\lambda < 1) \), the performance deteriorates and HD mode always performs better than FD in two way relaying, as predicted by the DoF calculations.

VI. Conclusions

In this paper, we have compared DoF and achievable rates for three communication scenarios: two way, two hop, and two way two hop. Using the antenna conserved and RF chain conserved implementations of FD with a realistic residual SI model, we have investigated the conditions under which FD can provide higher throughput than HD. Through detailed DoF analysis, for the two way channel, we have found that AC FD performs strictly below HD with imperfect SI cancellation. For the RC FD case, however, FD DoF trade-off can be better, when the SI cancellation parameter \( \lambda \) is high enough. The cross over point depends on various system parameters. In case of the two hop channel, FD is better when the relay has sufficient number of antennas and \( \lambda \) is high enough. For the two way two hop channel, when both of nodes require similar throughput, the HD implementation is generally better than FD. However, when one of the terminal’s data rate requirement is significantly higher than the other’s (e.g., when data flow occurs mostly in one direction, and the other direction is only used for feedback and control information, or in the case of asymmetric uplink and downlink data rates), then FD can achieve better rate pairs than HD, provided the relay has sufficient number of antennas and the SI suppression factor \( \lambda \) is high enough. Through numerical evaluations observing achievable rates, we have shown that the DoF results also extend to finite SNR.

The presented results in this paper provide guidelines for choosing HD or FD implementation in practical systems. Future research directions include studying more complex communication scenarios with inter-node interference and different relaying protocols, where the model and techniques used in this paper may provide a useful foundation.
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