WMAP 5-year constraints on time variation of $\alpha$ and $m_e$
in a detailed recombination scenario

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A B S T R A C T
We study the role of fundamental constants in an updated recombination scenario. We focus on the
time variation of the fine structure constant $\alpha$, and the electron mass $m_e$ in the early Universe, and put
bounds on these quantities by using data from CMB including WMAP 5-yr release and the 2dFGRS power
spectrum. We analyze how the constraints are modified when changing the recombination scenario.
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Time variation of fundamental constants is a prediction of theories that attempt to unify the four interactions in nature. Many
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where

\[ C_{\text{HI}} = \frac{1 + K_{\text{HI}} A_{\text{HI}} n_{\text{HI}} (1 - x_p)}{1 + K_{\text{HI}} (\alpha_{\text{II}} + \beta_{\text{II}}) n_{\text{II}} (1 - x_p)}, \]

(3)

\[ C_{\text{HeH}} = \frac{1 + K_{\text{HeII}} A_{\text{HeH}} n_{\text{HeII}} (f_{\text{HeH}} - x_{\text{HeH}}) e^{\hbar \nu_p / k T_m}}{1 + K_{\text{HeH}} (\alpha_{\text{II}} + \beta_{\text{II}}) n_{\text{II}} (f_{\text{HeH}} - x_{\text{HeH}}) e^{\hbar \nu_p / k T_m}}, \]

(4)

\[ C_{\text{C}} = \frac{1}{1 + K_{\text{C}} \beta_{\text{II}} n_{\text{II}} (f_{\text{HeH}} - x_{\text{HeH}}) e^{\hbar \nu_p / k T_m}}. \]

(5)

The last term in Eq. (2) accounts for the recombination through the triplets by including the semi-forbidden transition \( 2^1p \rightarrow 1^1s \). As remarked in [15], \( \alpha_{\text{HeII}} \) is fitted with the same functional form used for the \( \alpha_{\text{HI}} \) singlets, with different values for the parameters, so the dependences on the fundamental constants are the same, being proportional to \( \alpha^2 m_e^{-3/2} \). The two photon transition rates \( \Lambda_{\text{HI}} \) and \( \Lambda_{\text{HeH}} \) depend on the fundamental constants as \( \alpha^2 m_e \). The photoionization coefficients \( \beta \) are calculated as usual from the recombination coefficients \( \alpha_c \) (with \( c \) standing for \( \text{H}, \text{HeII} \), and \( \text{HeH} \)), so their dependences are known (see for example [4]).

The \( K_{\text{HI}}, K_{\text{HeII}} \) and \( K_{\text{C}} \) quantifies are the cosmological red-shifting of the H Ly \( \alpha \), \( \text{HeII} 2^1p \rightarrow 1^1s \) and \( \text{HeII} 2^1p \rightarrow 1^1s \) transition line photons, respectively. In general, \( K \) and the Sobolev escape probability \( p_s \) are related through the following equations (taking HeII as an example):

\[ K_{\text{HeII}} = \frac{\beta_{\text{HeII}}}{\beta_{\text{HeII}} + \beta_{\text{II}}}, \]

(6)

\[ K_{\text{HeII}} = \frac{\beta_{\text{HeII}}}{\beta_{\text{HeII}} + \beta_{\text{II}}}, \]

(7)

where \( A_{\text{HeII}2^1p\rightarrow1^1s} \) and \( A_{\text{HeII}2^1p\rightarrow1^1s} \) are the Einstein A coefficients of the HeII \( 2^1p \rightarrow 1^1s \) and HeI \( 2^1p \rightarrow 1^1s \) transitions, respectively. To include the effect of the continuum opacity due to H, based on the approximate formula suggested by Ref. [11], \( p_s \) is replaced by the new escape probability \( p_{\text{esc}} = p_s + p_{\text{con},\text{H}} \) with

\[ p_{\text{con},\text{H}} = \frac{1}{1 + \alpha_{\text{HeII}} \gamma_{\text{H}}}, \]

(8)

and

\[ \gamma' = \frac{\beta_{\text{HeII}} A_{\text{HeII}2^1p\rightarrow1^1s} (f_{\text{HeII}} - x_{\text{HeII}}) e^{\hbar \nu_p / k T_m}}{8 \pi \Delta \nu_{\text{HeII}2^1p} \Delta \nu_{\text{D,HeII}2^1p} \Delta \nu_{\text{D,HeII}2^1p} (1 - x_p)} \]

(9)

where \( \sigma_{\text{HeII}2^1p} \) is the H ionization cross section at frequency \( \nu_{\text{HeII}2^1p} \) and \( \Delta \nu_{\text{D,HeII}2^1p} = \frac{2 k T_m}{m_e c^2} \) is the thermal width of the HeII \( 2^1p \rightarrow 1^1s \) line. The cross section for photo-ionization from level \( n \) is [16]:

\[ \sigma_n(Z, h \nu) = \frac{2 \pi \alpha \sqrt{n^2 - 1}}{3 \sqrt{3}} \frac{n}{Z^2} (1 + n^2 \epsilon)^{-3} \ g_n(n, \epsilon), \]

(10)

where \( g_n(n, \epsilon) \approx 1 \) is the Gaunt-Kramers factor, and \( a_0 = \hbar / (m_e c \alpha) \) is the Bohr radius, so \( \sigma_{\text{HeII}2^1p} \) is proportional to \( \alpha^{-3} m_e^{-2} \).

The transition probability rates \( A_{\text{HeII}2^1p\rightarrow1^1s} \) and \( A_{\text{HeII}2^1p\rightarrow1^1s} \) can be expressed as follows [17]:

\[ A_{\text{HeII}2^1p\rightarrow1^1s} = \frac{4 \pi \alpha^2 \alpha_0^2 J_{\text{HeII}2^1p\rightarrow1^1s}}{3 \sqrt{3}} (\langle |\psi_1| r_1 + r_2 |\psi_2| \rangle)^2, \]

(11)

where \( \alpha_0 \) is the frequency of the transition, and \( J_{\text{HeII}2^1p\rightarrow1^1s} \) refers to the initial (final) state of the atom. First we will analyze the dependence of the bra-ket. To first-order in perturbation theory, all wavefunctions can be approximated to the respective wavefunction of hydrogen. Those can be usually expressed as \( \exp(-qr/a_0) \) where \( a_0 \) is the Bohr radius and \( q \) is a number. It can be shown that any integral of the type of Eq. (11) can be solved with a change of variable \( x = r/a_0 \). If the wave functions are properly normalized, the dependence on the fundamental constants comes from the operator, namely \( r_1 + r_2 \). Thus, the dependence of the bra-ket goes as \( a_0 \). On the other hand, \( \alpha_0 \) is proportional to the difference of energy levels and thus its dependence on the fundamental constants is \( \alpha_0 \approx m_e \alpha^2 \). Consequently, the dependence of the transition probabilities of HeII on \( \alpha \) and \( m_e \) is

\[ A_{\text{HeII}2^1p\rightarrow1^1s} \approx m_e \alpha^5. \]

(12)

In Fig. 1 we show how a variation in the value of \( \alpha \) at recombination affects the ionization history, moving the redshift at which recombination occurs to earlier times for larger values of \( \alpha \). The
difference between the functions when the two different recombination scenarios are considered, for a given value of \( \alpha \), is smaller than the difference that arise when varying the value of \( \alpha \). Something similar happens when varying \( m_e \).

With regards to the fitting parameters \( a_{\text{He}} \) and \( b_{\text{He}} \), since detailed calculation of their values are not available yet, it is not possible to determine the effect that a variation of \( \alpha \) or \( m_e \) would have on these new parameters. Wong et al. [15] have shown that they must be known at the 1% level for future Planck data. In this Letter, however, we are dealing with the 5 yr data from WMAP satellite and this accuracy is not required. To come to this conclusion, we have calculated the temperature, polarization and cross correlation CMB spectra, allowing the parameters \( a_{\text{He}} \) and \( b_{\text{He}} \) to vary at the 50% level. We found that for the temperature and polarization spectra, the variation is always lower than the observational error (1% for temperature and almost 40% in polarization). The largest variations occur in the cross correlation CMB spectra \( \langle C_l^T \rangle \). In this case, we have calculated the observational errors divided by the value of the \( C_l \)'s of all measured \( C_l^{TE} \) and compared them with the relative variation in the \( C_l \)'s induced when changing \( a_{\text{He}} \) and \( b_{\text{He}} \) by a 50%. In all of the cases the first quantity is several orders of magnitude greater than the variation of the \( C_l \)'s. Therefore, in order to analyze WMAP5 data, there is no need to modify these parameters.

To put constraints on the variation of \( \alpha \) and \( m_e \) during recombination time in the detailed recombination scenario studied here, we introduced the dependencies on the fundamental constants explicitly in the latest version of RECFAST code [18], which solves the recombination equations. We performed our statistical analysis by exploring the parameter space with Monte Carlo Markov chains generated with the publicly available CosmoMC code of Ref. [19] which uses the Boltzmann code CAMB [20] and RECFAST to compute the CMB power spectra. We modified them in order to include the possible variation of \( \alpha \) and \( m_e \) at recombination. We ran eight Markov chains and followed the convergence criterion of Ref. [21] to stop them when \( R - 1 < 0.0180 \). Results are shown in Table 1 and Fig. 2.

| Parameter | wmap5 + NS | wmap5 + PS | wmap3 + PS |
|-----------|------------|------------|------------|
| \( \Omega_b h^2 \) | 0.02241 ± 0.00084 | 0.02242 ± 0.00086 | 0.0218 ± 0.0010 |
| \( \Omega_{\text{CDM}} h^2 \) | 0.1070 ± 0.0030 | 0.1071 ± 0.0030 | 0.106 ± 0.0011 |
| \( \Theta \) | 1.033 ± 0.021 | 1.03261 ± 0.021 | 1.032 ± 0.021 |
| \( \tau \) | 0.0807 ± 0.0007 | 0.0863 ± 0.0004 | 0.090 ± 0.004 |
| \( \Delta \alpha / \alpha_0 \) | 0.004 ± 0.001 | 0.003 ± 0.001 | 0.002 ± 0.002 |
| \( \Delta m_e / m_e \) | −0.019 ± 0.003 | −0.017 ± 0.001 | 0.03 ± 0.004 |
| \( n_s \) | 0.96 ± 0.03 | 0.96 ± 0.03 | 0.97 ± 0.03 |
| \( H_0 \) | 70 ± 1.5 | 70 ± 1.6 | 70.4 ± 1.6 |

Table 1: Mean values and 1σ errors for the parameters including \( \alpha \) and \( m_e \) variations. NS stands for the new recombination scenario, and PS stands for the previous one.
Fig. 3. One-dimensional likelihood for $\Delta_\alpha/\alpha_0$ (upper row) and $\Delta m_e/(m_{e0})$ (lower row). Left: for WMAP5 data and two different recombination scenarios. Right: comparison for the standard recombination scenario, between the WMAP3 and WMAP5 data sets.

In Fig. 2 we show the marginalized posterior distributions for the cosmological parameters, $\Delta \alpha/\alpha_0$ and $\Delta m_e/(m_{e0})$, which are the variation in the values of those fundamental constants between recombination epoch and the present. The three successively larger two-dimensional contours in each panel correspond to the 68%-,$ 95%$, and 99%-probability levels, respectively. In the diagonal, the one-dimensional likelihoods show the posterior distribution of the parameters.

In Table 1 we show the results of our statistical analysis, and compare them with the ones we have presented in Ref. [4], which were obtained in the standard recombination scenario (i.e. the one described in [27], which we denote PS), and using WMAP3 [28,29] data. The constraints are tighter in the current analysis, which is an expectable fact since we are working with more accurate data from WMAP. The bounds obtained are consistent with null variation, for both $\alpha$ and $m_e$, but in the present analysis, the 68% confidence limits on the variation of both constants have changed.

In the case of $\alpha$, the present limit is more consistent with null variation than the previous one, while in the case of $m_e$ the single parameters limits have moved toward lower values. To study the origin of this difference, we perform another statistical analysis, namely the analysis of the standard recombination scenario (PS) together with WMAP5 data and the other CMB data sets and the 2dFGRS power spectrum. The results are also shown in Table 1. We see that the changes in the results are due to the new WMAP data set, and not to the new recombination scenario.

In Fig. 3 we compare the probability distribution for $\Delta \alpha/\alpha_0$ and also for $\Delta m_e/(m_{e0})$, in different scenarios and with different data sets.

In Fig. 4 we compare the 95%-probability contour level for the parameters, and their one-dimensional distributions, for two different analysis in the standard recombination scenario, namely the one with WMAP5 data (dashed lines) and the one with WMAP3 data (solid lines). The contours are smaller in the former case, which is expectable since that data set is more accurate. For the fundamental constants, the contours notably shrink. Moreover, the constraints are shifted to a region of the parameter space closer to that of null variation in the case of $\alpha$. On the other hand, limits on the variation of $m_e$ are shifted to negative values, but still consistent with null variation. From the one-dimensional likelihoods we see that the peak of the likelihood has moved for $\Omega_B h^2$. The obtained results for the cosmological parameters are in agreement within $1\sigma$ with the ones obtained by the WMAP Collaboration [30], without considering variation of fundamental constants.
Fig. 4. Comparison between the 95%-confidence levels of WMAP3 (solid line) with those of WMAP5 (dashed line). In the diagonal, we compare the one-dimensional likelihoods in these two cases.

References

[1] C.J. Martins, A. Melchiorri, R. Trotta, R. Bean, G. Rocha, P.P. Avelino, P.T. Viana, Phys. Rev. D 66 (2002) 023505.
[2] G. Rocha, R. Trotta, C.J.A.P. Martins, A. Melchiorri, P.P. Avelino, P.T.P. Viana, New Astron. Rev. 47 (2003) 863.
[3] K. Ichikawa, T. Kanzaki, M. Kawasaki, Phys. Rev. D 74 (2006) 023515.
[4] S.J. Landau, M.E. Mosquera, C. Scóccola, H. Vucetich, Phys. Rev. D (2008), in press.
[5] J. Yoo, R.J. Scherrer, Phys. Rev. D 67 (2003) 043517.
[6] M.R. Nolta, et al., arXiv: 0803.0593.
[7] V.K. Dubrovich, S.I. Grachev, Astron. Lett. 31 (2005) 359.
[8] E.R. Switzer, C.M. Hirata, Phys. Rev. D 77 (2008) 083006.
[9] C.M. Hirata, E.R. Switzer, Phys. Rev. D 77 (2008) 083007.
[10] E.R. Switzer, C.M. Hirata, Phys. Rev. D 77 (2008) 083008.
[11] E.E. Kholupenko, A.V. Ivanchik, D.A. Varshalovich, Mon. Not. R. Astron. Soc. 378 (2007) L39.
[12] M. Kaplinghat, R.J. Scherrer, M.S. Turner, Phys. Rev. D 60 (1999) 023516.
[13] S. Hannestad, Phys. Rev. D 60 (1999) 023515.
[14] J. Kujat, R.J. Scherrer, Phys. Rev. D 62 (2000) 023510.
[15] W.Y. Wong, A. Moss, D. Scott, Mon. Not. R. Astron. Soc. 386 (2008) 1023.
[16] M.J. Seaton, Mon. Not. R. Astron. Soc. 119 (1959) 81.
[17] C.W.F. Drake, D.C. Morton, Astrophys. J. Suppl. Ser. 170 (2007) 251.
[18] S. Seager, D.D. Sasselov, D. Scott, Astrophys. J. Lett. 523 (1999) L1.
[19] A. Lewis, S. Bridle, Phys. Rev. D 66 (2002) 103511.
[20] A. Lewis, A. Challinor, A. Lasenby, Astrophys. J. 538 (2000) 473.
[21] S.M. Raftery, A.E. Lewis, in: J.M. Bernardo (Ed.), Bayesian Statistics, Oxford Univ. Press., 1992, p. 765.
[22] A.C.S. Readhead, et al., Astrophys. J. 609 (2004) 498.
[23] C.L. Kuo, et al., Astrophys. J. 600 (2004) 32.
[24] F. Piacentini, et al., Astrophys. J. 647 (2006) 833.
[25] W.C. Jones, et al., Astrophys. J. 647 (2006) 823.
[26] S. Cole, et al., Mon. Not. R. Astron. Soc. 362 (2005) 505.
[27] S. Seager, D.D. Sasselov, D. Scott, Astrophys. J. Supp. Ser. 128 (2000) 407.
[28] G. Hinshaw, et al., Astrophys. J. Supp. Ser. 170 (2007) 288.
[29] L. Page, et al., Astrophys. J. Supp. Ser. 170 (2007) 335.
[30] J. Dunkley, et al., arXiv: 0803.0586.