Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
Numerical computations and theoretical investigations of a dynamical system with fractional order derivative

Muhammad Arfan\textsuperscript{a}, Ibrahim Mahariq\textsuperscript{b}, Kamal Shah\textsuperscript{a,c}, Thabet Abdeljawad\textsuperscript{c,d,e,*}, Ghaylen Laouini\textsuperscript{b}, Pshtiwan Othman Mohammed\textsuperscript{f}

\textsuperscript{a} Department of Mathematics, University of Malakand, Chakdara Dir(L), 18000 Khyber Pakhtunkhwa, Pakistan
\textsuperscript{b} College of Engineering and Technology, American University of the Middle East, Kuwait
\textsuperscript{c} Department of Mathematics and General Sciences, Prince Sultan University, P.O. Box 66833, Riyadh 11586, Saudi Arabia
\textsuperscript{d} Department of Medical Research, China Medical University, 40402 Taichung, Taiwan
\textsuperscript{e} Department of Computer Science and Information Engineering, Asia University, Taichung, Taiwan
\textsuperscript{f} Department of Mathematics, College of Education, University of Sulaimani, Sulaimani, Kurdistan Region, Iraq

Received 29 April 2021; revised 1 June 2021; accepted 9 July 2021
Available online 03 August 2021

Abstract This manuscript is devoted to consider population dynamical model of non-integer order to investigate the recent pandemic Covid-19 named as severe acute respiratory syndrome coronavirus-2 (SARS-CoV-2) disease. We investigate the proposed model corresponding to different values of largely effected system parameter of immigration for both susceptible and infected populations. The results for qualitative analysis are established with the help of fixed-point theory and non-linear functional analysis. Moreover, semi-analytical results, related to series solution for the considered system are investigated on applying the transform due to Laplace with Adomian polynomial and decomposition techniques. We have also applied the non-standard finite difference scheme (NSFD) for numerical solution. Finally, both the analysis are supported by graphical results at various fractional order. Both the results are comparable with each other and converging quickly at low order. The whole spectrum and the dynamical behavior for each compartment of the proposed model lying between 0 and 1 are simulated via Matlab.

© 2021 THE AUTHORS. Published by Elsevier BV on behalf of Faculty of Engineering, Alexandria University. This is an open access article under the CC BY-NC-ND license (http://creativecommons.org/licenses/by-nc-nd/4.0/).

1. Introduction

Since December 2019, nearly all the countries of the world have been contexting with rapidly transmitting pandemic of COVID-19. The virous of this disease was first tested in the Wuhan city of China. About 140 million population of
humans being have infected, three millions were died and the remaining are recovered [1]. The ratio of infection, death and recovery are different in various territories of the globe. The three quantities depends on the health conditions, environmental factors, social contact, etc. Now the objective of each government and state is how to control and minimize the transmission of this effective disease. Scientist and researchers have pointed out that social contact or gathering is the main reason of this infection. Interaction of infectious one with the healthy peoples case the much more spreading of the said pandemic. From the very start of this infection up to date various strategies like lock downs on overcrowded areas, business affairs, courts hearing cases, huge buses and plane travelings, etc. have been applied. Mostly the learning institutions were effected very highly in the said lock down. Recently the law and forcemeat agencies were called to follow the SOP’s on peoples by force. In short the learning institutions were effected very highly in the said crowded areas, business affairs, courts hearing cases, huge up to date various strategies like lock downs on over-crowded areas, business affairs, courts hearing cases, huge.

For all the times humans beings are trying to find better facilities for comfortable life. One of the biggest threat to human lives is the pandemic which may take their lives as well as comport. It produces tension, depression and various other diseases in the societies of human population. To find the cure for all these uncomfortable situation, different scientist and scholars along with the support of government are trying to finish it from their societies. For curing the various pandemic, they produces vaccines, medicines and also find some precautionary measures. Along all these precautions, need of fitting statistical data of the epidemic in the mathematical formula will guide about the future policy of the said disease by providing the results of infectious peoples at any time. Therefore, the importance of mathematical formulation can be seen in the investigation of different diseases or other interaction of humans or animals. This idea was first given in 1927 by converting the real life problem into a mathematical problems. Therefore various epidemiological problems have been converted to the mathematical formula for further analysis [3–6].

The different problems in mathematical form for recent pandemic due to Corona virus have been investigated by different techniques which can be seen in [7–11]. The mathematical formulation predicted the future results of the said disease which can be then seen in the controlling process. The present and future data of this disease play important role in the construction of mathematical problem. The analysis of various mathematical modeling and their importance can be seen in [12]. Making different assumption, observation and output of the real problem will lead to the mathematical model. For this various factor can be included in form of parameters in the formula which greatly effects the diseases. The results of the formula are them compare with the real problem and will be accepted for future analysis if the are comparable with each other. The mathematical models that we used in this paper is inspired from the classic Lotka-Volterra model[13,14] for analyzing predator–prey dynamics. The classic model has been suitably modified to build the susceptible-infected individual population dynamics model. As COVID-19 can spread easily in social gathering, therefore some scholars have analyzed the dynamics of such type of disease for transmission due to immigration as given below

\[
\begin{align*}
\dot{S} &= S - bS + (d - e)I - \delta S, \\
\dot{I} &= bS - (d - e)I - cI - \delta I, \\
\dot{R} &= eI - (d + c)R, \\
\dot{H} &= H - \delta H, \\
\dot{U} &= U - \delta U,
\end{align*}
\]

where the susceptible individual population is given by \( S \) at time \( t \), infected individual population is given by \( I \) at time \( t \). The infection rate is given by \( b = (1 - \text{protection rate}) \). The immigration rate of susceptible individuals is given by \( d \). The immigration rate of infected individuals is given by \( c \). The death rate is given by \( e \) and the recovered rate is given by \( e \). Further we include \( R \) being natural death rate and third equation of recovered individuals to make another model. This model is just an indication to see what happen in a community, if immigration of individuals does not control in it.

Applying the aforementioned points, we are going to study the model given in (2) by including recovered individuals equation for fractional-order derivative with \( 0 < \sigma \leq 1 \) as given by

\[
\begin{align*}
D_\sigma S(t) &= S(t) - bS(t) + (d - e)I(t) - \delta S(t), \\
D_\sigma I(t) &= bS(t) - (d - e)I(t) - cI(t) - \delta I(t), \\
D_\sigma R(t) &= eI(t) - (d + c)R(t), \\
H(0) &= H_0, \quad R(0) = R_0
\end{align*}
\]

where \( S_0 \) and \( I_0, R_0 \), are starting values for susceptible, infected and recovered classes as population densities in percents. We are investigating the Eq. (2) in arbitrary order derivative of Caputo sense because it gives much well result than integer order. providing such type of results makes fractional calculus superior than integer order calculus. Therefore, the physical and biological problems may be investigated with the general fractional derivative operators on every very small order having more degree of choice. The researchers interested to work on fractional derivatives as compared to the classical differentiation. So for the analysis are composed of theory of existence and uniqueness, stability, feasibility and approximate solution of fractional order equations [13,15–20].

Fractional order calculus have provided the information of the whole spectrum lying between any two different integer values [21–24]. The representation of various real problems have been modeled by fractional order differential or integral equation like, mathematical fractional order model for microorganism population, logistic non-linear model for human population, tuberculosis model, dingy problem, hepatitis B, C models and the basic Lotka-Volterra models being the basics of all infectious problems [25–34]. The afore said problems have been analyzed for qualitative analysis with help of some well known theorems of fixed point theory [35–39]. The feasibility and stability analysis have also been done through various theorems. Further the FODEs have been checked for analytical, semi-analytical and approximate solution using different techniques. Some of the well known techniques were of Euler, Taylor, Adams–Bashforth, predictor–corrector, various transforms, etc [13,26,40–44]. Here we are going to analyze our proposed fraction order model in sense of Caputo derivative for existence, uniqueness of solution and approximate solution by theory of fixed point and Laplace transform along with some decomposition techniques. We also find the numerical solution of the proposed problem by Non-Standard Finite Difference Scheme (NSFD). We will also investigate the effect of immigration on the pandemic transmission in the society using different fractional or arbitrary orders for the considered model.
2. Basics of modern calculus

Now, we present famous definitions as in [13,15,26].

Definition 2.1. Let we have an operator $\mathcal{H}(t)$ to give the definition of non-integer order of integration w.r.t $t$ in Caputo sense as

$$\mathcal{D}^\alpha_0 \mathcal{H}(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\eta)^{n-\alpha-1} \frac{d^n}{dt^n} \mathcal{H}(\eta) d\eta,$$

having satisfy the condition of convergency.

Definition 2.2. Let we have a mapping $\mathcal{H}(t)$, to provide the definition fractional order differentiation w.r.t $t$ as

$$\mathcal{D}_0^\alpha \mathcal{H}(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\eta)^{n-\alpha-1} \frac{d^n}{dt^n} \mathcal{H}(\eta) d\eta,$$

with right side quantity is piece wise continuous on $\mathbb{R}^+$ and $n = [\alpha] + 1$. If $0 < \alpha < 1$, then we can write

$$\mathcal{D}_0^\alpha \mathcal{H}(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\eta)^{-\alpha-1} \frac{d}{dt} \mathcal{H}(\eta) d\eta.$$

Lemma 2.2.1. [13] The solution of

$$\mathcal{D}_0^\alpha \mathcal{H}(t) = h(t), \quad 0 < \alpha \leq 1$$

is

$$\mathcal{H}(t) = c_0 + \frac{1}{\Gamma(1-\alpha)} \int_0^t h(\eta)(t-\eta)^{-\alpha-1} d\eta.$$

Lemma 2.2.2. [13]The Laplace transform of $\mathcal{D}_0^\alpha \mathcal{H}(t)$ for $0 < \alpha \leq 1$ is provided by

$$\mathcal{L} \left( \mathcal{D}_0^\alpha \mathcal{H}(t) \right) = \mathcal{L}^{\alpha} \left( \mathcal{H}(t) \right) - \mathcal{L}^{\alpha-1} \left( \mathcal{H}(0) \right).$$

Lemma 2.2.3. The feasible and bounded solution of the proposed system (2) is

$$\mathbb{S} = \left\{ (\mathcal{H},\mathcal{I},\mathcal{R}) \in \mathbb{R}_+^3 : 0 \leq N(t) \leq \frac{a + c}{\delta} \right\}.$$

Proof. For derivation we have to add all the three equations of (2), as

$$\begin{align*}
\frac{dN}{dt} &= (a) \mathcal{H}(t) - (\mathcal{H}(t) + \mathcal{I}(t) + \mathcal{R}(t))\delta + \mathcal{I}(t)c \\
&= (a) \mathcal{H}(t) + (\delta) N(t) + c \mathcal{I}(t) \\
&= a + c - (\delta) N(t). \\
\end{align*}$$

Hence one has $\frac{dN}{dt} + (\delta) N(t) \leq a + c.$

Upon integration of (3) we obtain

$$N(t) \leq \frac{a + c}{\delta} + C \exp(-((\delta)t)).$$

Here $C$ is constant due to integration. In (4), as $t$ tends to $\infty$, then

$$N(t) \leq \frac{a + c}{\delta}.$$

The last result proved the boundedness and feasibility of solution. □

3. Main work

In this section of the paper we will check our consider model (2) for qualitative properties. These kinds of properties can be easily checked by fixed point theory. We will also uses some basic theorems like Banach contraction and Schauder theorems to receive our required result. Take $\mathcal{U} = \mathcal{H}(t) + \mathcal{I}(t) + \mathcal{R}(t)$.

$$\mathcal{D}_0^\alpha \mathcal{U}(t) = \mathcal{F}(t, \mathcal{U}(t)), \quad \sigma \in (0, 1], \tag{5}$$

Applying fractional integration of order $\sigma \in (0, 1]$ to (5), we obtain

$$\mathcal{U}(t) = \mathcal{U}_0 + \frac{1}{\Gamma(\sigma)} \int_0^t (t-\eta)^{\sigma-1} \mathcal{F}(\eta, \mathcal{U}(\eta)) d\eta. \tag{6}$$

Consider $\infty > T \geq t > 0$ with complete norm space as $\mathcal{C}_1 = \mathcal{C}[0, T]$ under the norm $\|\mathcal{U}\| = \max_{t \in [0, T]} |\mathcal{U}(t)|$.

For qualitative properties, we take the condition of growth on non-linear operator as:

(A1) $\exists L_\mathcal{F} > 0; \forall \mathcal{U}, \mathcal{V} \in \mathcal{E}$ we have

$$|\mathcal{F}(t, \mathcal{U}) - \mathcal{F}(t, \mathcal{V})| \leq L_\mathcal{F} |\mathcal{U} - \mathcal{V}|.$$

(A2) $\exists C_\mathcal{F} > 0 \& M_\mathcal{F} > 0$;

$$|\mathcal{F}(t, \mathcal{U}(t))| \leq C_\mathcal{F} |\mathcal{U}| + M_\mathcal{F}.$$

Theorem 3.1. Let $\mathcal{F}$ be continuous and satisfying (A2), model (5) has $\geq 1$ solution.

Proof. Applying “Schauder fixed point theorem”, let us define a closed subset $\mathcal{B}$ of $\mathcal{E}$ as

$$\mathcal{B} = \{ \mathcal{U} \in \mathcal{E} : \|\mathcal{U}\| \leq R, \; R > 0 \},$$

where

$$R \geq \max \left[ \frac{\|\mathcal{U}_0\| \Gamma(\sigma + 1) + T^* M_\mathcal{F}}{\Gamma(\sigma + 1) - T^* C_\mathcal{F}} \right].$$

Let we provide a mapping $\mathcal{T} : \mathcal{B} \rightarrow \mathcal{B}$ and using (6) as

$$\mathcal{T} \mathcal{U} = \mathcal{U}_0 + \frac{1}{\Gamma(\sigma)} \int_0^t (t-\eta)^{\sigma-1} \mathcal{F}(\eta, \mathcal{U}(\eta)) d\eta. \tag{7}$$

At any $\mathcal{U} \in \mathcal{B}$, we get

$$|\mathcal{T} \mathcal{U}(t)| \leq |\mathcal{U}_0| + \frac{1}{\Gamma(\sigma)} \int_0^t (t-\eta)^{\sigma-1} |\mathcal{F}(\eta, \mathcal{U}(\eta))| d\eta,$$

$$\leq |\mathcal{U}_0| + \frac{1}{\Gamma(\sigma + 1)} \int_0^T (t-\eta)^{\sigma-1} |\mathcal{C}_\mathcal{F} |\mathcal{U}| + M_\mathcal{F} d\eta,$$

$$\leq |\mathcal{U}_0| + \frac{\Gamma^*}{\Gamma(\sigma + 1)} |\mathcal{C}_\mathcal{F} |\mathcal{U}| + M_\mathcal{F},$$

which implies that
From (8), it implies that $\mathcal{V} \in \mathcal{B}$. Thus $T(\mathcal{V}) \subset \mathcal{B}$. This also proves that $T$ is closed. Next for completely continuous we go ahead as:

Take $t_1 < t_2 \in [0, T]$, assume

$$
\|T(\mathcal{V})(t_2) - T(\mathcal{V})(t_1)\| \leq \frac{T}{\Gamma(\sigma + 1)} \left[ C_\sigma \|\mathcal{V}\| + M_\mathcal{V} \right], 
$$

(8)

where

$$
\mathcal{V}(t) = \begin{cases} 
\mathcal{V}_0(t), & 0 < \sigma \leq 1, \\
\mathcal{V}_0(t) + \mathcal{V}_1(t) + \mathcal{V}_2(t) \ldots & n \geq 0.
\end{cases}
$$

To produce semi-analytical solution to the considered model, consider a general problem as

Now from (9), we get $t_1 \rightarrow t_2$, then the right side will approach to 0. So we say that

$$
\|T(\mathcal{V})(t_2) - T(\mathcal{V})(t_1)\| \rightarrow 0, \quad \text{as} \quad t_1 \rightarrow t_2.
$$

Consequently we claim that

$$
\|T(\mathcal{V})(t_2) - T(\mathcal{V})(t_1)\| \rightarrow 0, \quad \text{as} \quad t_1 \rightarrow t_2.
$$

Hence $T$ is equi-continuous operator. Applying Arzelà-Ascoli theorem, mapping $T$ is completely continuous and which is also uniform continuous and bounded. Now by Schauder’s fixed point theorem model (2) has $\geq 1$ solution.

Further for uniqueness we proceeds as:

**Theorem 3.2.** With (A1), the considered model has one solution if

$$
1 > \frac{T}{\Gamma(\sigma + 1)} L_\mathcal{V}.
$$

**Proof.** We have already define an operator as $T : \mathcal{B} \rightarrow \mathcal{B}$, consider $\mathcal{U}$ and $\mathcal{V} \in \mathcal{B}$ as

$$
\|T(\mathcal{U}) - T(\mathcal{V})\| = \max_{0 \leq t \leq T} \frac{1}{\Gamma(\sigma)} \int_0^t (t - \eta)^{\sigma - 1}\left[ F(s, \mathcal{U}(\eta))d\eta \right],
$$

(10)

$$
\frac{1}{\Gamma(\sigma)} \int_0^T (t - \eta)^{\sigma - 1}\left[ F(s, \mathcal{V}(\eta))d\eta \right],
$$

(11)

From (10), we have

$$
\|T(\mathcal{U}) - T(\mathcal{V})\| \leq \frac{T}{\Gamma(\sigma + 1)} L_\mathcal{V} \|\mathcal{U} - \mathcal{V}\|.
$$

Hence $F$ is contraction. So finally by the results of theorem of Banach contraction the considered problem has one solution.

4. General series solution of the considered System (2)

To produce semi-analytical solution to the considered model, consider a general problem as

4. General series solution of the considered System (2)

To produce semi-analytical solution to the considered model, consider a general problem as

5. Approximate solution and discussion for (1)

To compute the approximate results, we take some values for parameters of (1) under fractional order. Let us in community the susceptible population be $\mathcal{S}_0$, infected be $\mathcal{I}_0$, recovered be $\mathcal{R}_0$ and taking various values for rates we discuss certain cases as bellow:
We see that at various arbitrary order the declines in suscepti-
solutions of various compartment has presented in Figs. 1–3.
for the parameters in the given Table 1.

6. Graphical results and discussion

To present the concerned approximate solutions (16) of the
model under consideration, we recall some numerical values
for the parameters in the given Table 1. Case-I In this case
we take the values of Table 1 and simulate the three compart-
ments for t = 300 days in Figs. 1–3.

On small immigration rate, the behavior of approximate
solutions of various compartment has presented in Figs. 1–3.
We see that at arbitrary order the declines in suscept-
ble and infected class is different. The decay is more at small
non-integer order while the recovery growth is also high at
small fractional order. The concerned values are just assump-
tion to see the behaviors of non-integer order approximate
solution of the proposed arbitrary order model. We can say
that the results of fractional order derivatives are comparable
with that of integer order and fractional calculus can also well
describe the situation of dynamical systems.

Case-II: In this case we changing the values of δ = 0.019
and ε = 0.00023 and the remaining values are the same as
given in Table 1 in Figs. 4–6.

Case-III: In the third case we again changes the values of
δ = 0.00019 and ε = 0.000023 while the remaining values are
fixed of Table 1.

7. Numerical simulation by Non-Standard Finite Difference
Scheme (NSFD)

In this section we will analyze the considered model for
approximate solution using the Grünwald-Letnikov tech-
niques of approximation for the 1-D arbitrary order as given
\[ \mathcal{D}^\alpha Y(t) = \lim_{\Delta \to 0} \sum_{i=0}^{\Delta} (-1)^i \binom{\alpha}{i} Y(t - i\Delta), \] 
where \( t = n\Delta \) and \( \Delta \) is interval. Now, we take the problem as
\[ \mathcal{D}^\alpha Y(t) = g(t, Y(t)), \quad t \in [0, T], \quad T < \infty, \]
\[ Y(b) = y_0. \]

Now by (18), the left side of (19) is discretized and one may write as
\[ \sum_{i=0}^{\Delta} K^\alpha Y(t_{p-i}) = g(t, Y(t)), \quad p = 1, 2, 3, \ldots, \]
where \( t_p = p\Delta \) and \( K^\alpha \) are the Grünwald-Letnikov coefficients
defined as
\[ K^\alpha = (1 - \frac{1}{i}) K^\alpha_{i+1}, \quad i = 1, 2, 3 \ldots \]
and
\[ K^\alpha_0 = \Delta^{-\alpha}. \]

Next we get the non-standard finite difference scheme (NSFD)
by Grünwald-Letnikov method for the arbitrary order model
(2) as follows
\[ \mathcal{D}^\alpha Y(t_{p+1}) = \sum_{i=0}^{p+1} K^\alpha Y(t_{p+1-i}) + \mathcal{D}^\alpha Y(t) + (-\delta) \mathcal{H}(t). \]

We will now simulate the three compartments of the consid-
ered equations by (NSFD) using the same data of Table 1 as
in the cases of (LADM) (see Figs. 7–9).

Case-I: In the first case we obtain the simulation by
(NFSD) using the data of Table 1 in Figs. 10–12 at different
fractional order. Such type of simulation will also provide a
comparable results as by (LADM).

### Table 1 The physical interpretation of the parameter.

| Parameters | The physical interpretation | Numerical values |
|------------|----------------------------|-----------------|
| \( \mathcal{H}_0 \) | Susceptible compartment | 8.567761 millions [45] |
| \( \mathcal{I}_0 \) | Infected compartment | 0.567261 millions [45] |
| \( \mathcal{R}_0 \) | Recovered compartment | 0.530597 millions [45] |
| \( a \) | Immigration rate of susceptible class | 0.0018 |
| \( b \) | The infection rate | 0.003 |
| \( c \) | Immigration rate of infected class | 0.005 |
| \( d \) | Death rate due to infection | 0.019 |
| \( \delta \) | Natural death rate | 0.0019 |
| \( e \) | Cure rate from infection | 0.00005 |
Case-II: In the second case we obtain the simulation by (NFSD) by changing $d = 0.019$ and $e = 0.00023$ while the rest of the data is of Table 1 in Figs. 13–15 at different fractional order.

Fig. 1  Graphical representation of series solution for $H(t)$ corresponding to various arbitrary order of $\sigma$.

Fig. 2  Graphical representation of series solution for $I(t)$ corresponding to various arbitrary order of $\sigma$.

Fig. 3  Graphical representation of series solution for $R(t)$ corresponding to various arbitrary order of $\sigma$. 

Numerical computations and theoretical investigations of a dynamical system 1987
Case-III: In this case we have drawn the simulation by (NFSD) by changing \( \delta = 0.00019 \) and \( e = 0.000023 \) while the rest of the data is of Table 1 in Figs. 16–18 at different fractional order.

Fig. 4  Graphical representation of series solution for \( H(t) \) corresponding to various arbitrary order of \( \sigma \).

Fig. 5  Graphical representation of series solution for \( I(t) \) corresponding to various arbitrary order of \( \sigma \).

Fig. 6  Graphical representation of series solution for \( S(t) \) corresponding to various arbitrary order of \( \sigma \).
Fig. 7  Graphical representation of series solution for $H(t)$ corresponding to various arbitrary order of $\sigma$.

Fig. 8  Graphical representation of series solution for $I(t)$ corresponding to various arbitrary order of $\sigma$.

Fig. 9  Graphical representation of series solution for $R(t)$ corresponding to various arbitrary order of $\sigma$. 
Fig. 10  Graphical representation of numerical solution for $H(t)$ by (NFSD) corresponding at various arbitrary order of $\sigma$.

Fig. 11  Graphical representation of numerical solution for $I(t)$ by (NFSD) corresponding at various arbitrary order of $\sigma$.

Fig. 12  Graphical representation of numerical solution for $R(t)$ by (NFSD) corresponding at various arbitrary order of $\sigma$. 
Fig. 13  Graphical representation of numerical solution for $S(t)$ by (NFSD) corresponding at various arbitrary order of $r$.

Fig. 14  Graphical representation of numerical solution for $I(t)$ by (NFSD) corresponding at various arbitrary order of $r$.

Fig. 15  Graphical representation of numerical solution for $R(t)$ by (NFSD) corresponding at various arbitrary order of $r$. 
Fig. 16  Graphical representation of numerical solution for $H(t)$ by (NFSD) corresponding at various arbitrary order of $\sigma$.

Fig. 17  Graphical representation of numerical solution for $I(t)$ by (NFSD) corresponding at various arbitrary order of $\sigma$.

Fig. 18  Graphical representation of numerical solution for $R(t)$ by (NFSD) corresponding at various arbitrary order of $\sigma$. 

Here we take some real data of infected cases in Pakistan from First April 2021 to 20the May 2021 as in Fig. 19.

8. Concluding remarks

We have investigated a fractional order mathematical model of severe acute respiratory syndrome coronavirus-2 (COVID-19) by using the non–integer order derivative in Caputo sense. The results for boundedness and feasibility of solution have been provided. Some results regarding qualitative analysis of the model have also been proved by using non-linear functional analysis. By using the famous integral transform due to Laplace along with decomposition techniques and Adomian polynomial, we have computed series type solutions for the proposed problem. Taking some different numerical values for natural death rate $d$ and recovery rate $e$ we have graphically presented three different cases of the solution. Further the Non–Standard Finite Difference Scheme is also applied to the given problem for investigation of iterative approximate solution. The results obtained by LADM are comparable with the results of NSFD at different fractional orders. The simulation converges rapidly at low order lying between 0 and 1 as compared to higher order. We With very small immigration rate of infectious peoples and high protection rate, the current infection may be controlled up to certain extent. So we conclude that arbitrary order derivatives along with the effect some parameter can well explain the mathematical models of real world phenomena along with history information.

Declaration section

Availability of Data and Materials Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Funding There is no funding source available for this article.

Authors Contribution All authors equally contributed this manuscript and approved the final version.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

The third author would like to thank Prince Sultan University for funding this work through research group Nonlinear Analysis Methods in Applied Mathematics (NAMAM), Group Number RG-DES-2017-01-17.

References

[1] World Health Organization, Coronavirus disease 2019 (COVID-19) Situation Report-62, 2020.
[2] H. Lu, C.W. Stratton, Y.W. Tang, Outbreak of Pneumonia of Unknown Etiology in Wuhan China: the Mystery and the Miracle, J Med Virol. (2020), https://doi.org/10.1002/jmv.25678.
[3] M. Goyal, H.M. Baskonus, A. Prakash, An efficient technique for a time fractional model of lassa hemorrhagic fever spreading in pregnant women, Eur. Phys. J. Plus 134 (481) (2019) 1–10.
[4] W. Gao, P. Veeresha, D.G. Prakasha, H.M. Baskonus, G. Yel, New approach for the model describing the deathly disease in pregnant women using Mittag-Leffler function, Chaos, Solitons & Fractals 134 (2020) 109696.
[5] D. Kumar, J. Singh, M. Al-Qurashi, D. Baleanu, A new fractional SIRS-SI malaria disease model with application of vaccines, anti-malarial drugs, and spraying, Adv. Diff Eqs. 278 (2019) 1–10.
[6] K. Shah, M.A. Alqudah, F. Jarad, T. Abdeljawad, Semi-analytical study of Pine Wilt Disease model with convex rate under Caputo-Febrizio fractional order derivative, Chaos, Solitons Fractals 135 (2020) 109754.
[7] X. Tian et al, Potent binding of 2019 novel coronavirus spike protein by a SARS coronavirus-specific human monoclonal...
antibody, Emerg. Microbes Infect. 2020 (2019), https://doi.org/10.1002/22221751.2020.1729069.

[8] X.Y. Ge et al, Isolation and characterization of a bat SARS-like coronavirus that uses the ACE2 receptor, Nature 503 (2013) 535–538.

[9] F.W.C. Jasper et al, Genomic characterization of the 2019 novel human-pathogenic coronavirus isolated from patients with acute respiratory disease in Wuhan, Hubet, China, Emerg. Microbes Infect. (2020) 1–5.

[10] H. Lu et al, Outbreak of Pneumonia of Unknown Etiology in Wuhan China: the Mystery and the Miracle, J Med Virol. (2020), https://doi.org/10.1002/jmv.25678, Jan 16.

[11] J. Riuou, C.L. Althaus, Pattern of early human-to-human transmission of Wuhan 2019 novel coronavirus (2019-nCoV), December 2019 to January 2020, Eurosurveillance 25 (4) (2020).

[12] Q. Lin et al, A conceptual model for the coronavirus disease 2019 (COVID-19) outbreak in Wuhan, China with individual reaction and governmental action, Int. J. Infect. Dis. 93 (2020) (2019) 211–216.

[13] V. Lakshmikantham, S. Leela, Naguma-type uniqueness result for fractional differential Equations, Non-linear Anal. 71 (2009) 2886–2889.

[14] J.A. Lotka, Contribution to the theory of periodic reactions, J. Phys. Chem. (2002) 271–274.

[15] I. Podlubny, Fractional Differential Equations, Mathematics in Science and Engineering, Academic Press, New York, 1999.

[16] R. Hilfer, Applications of Fractional Calculus in Physics, World Scientific, Singapore, 2000.

[17] Y.A. Rossikhin, M.V. Shitikova, Applications of fractional calculus to dynamic problems of linear and nonlinear hereditary mechanics of solids, Appl. Mech. Rev. 50 (1997) 15–67.

[18] H. Singh, H.M. Srivastava, Numerical simulation for fractional-order bloch equation arising in nuclear magnetic resonance by using the jacobi polynomials, Appl. Sci. 10 (8) (2020) 2850.

[19] H. Singh, Analysis for fractional dynamics of Ebola virus model, Chaos Solitons & Fractals 138 (2020) 109992.

[20] H. Singh, C.S. Singh, A reliable method based on second kind Chebyshev polynomial for the fractional model of Bloch equation, Alexandria Eng. J. 57 (3) (2018) 1425–1432.

[21] H. Singh, Operational matrix approach for approximate solution of fractional model of Bloch equation, J. King Saud Univ.-Sci. 29 (2) (2017) 235–240.

[22] H. Singh, R. Pandey, H. Srivastava, Solving non-linear fractional variational problems using jacobi polynomials, Mathematics 7 (3) (2019) 224.

[23] H. Singh, H.M. Srivastava, Numerical investigation of the fractional-order lienard and duffing equations arising in oscillating circuit theory, Front. Phys. (2020).

[24] H. Singh, M.R. Sahoo, O.P. Singh, Numerical method based on Galerkin approximation for the fractional advection-dispersion equation, Int. J. Appl. Comput. Mathe. 3 (3) (2017) 2171–2187.

[25] Y. Zhang, Initial boundary value problem for fractal heat equation in the semi-infinite region by Yang-Laplace transform, Thermal Sci. 18 (2) (2014) 677–681.

[26] K.S. Miller, B. Ross, An Introduction to the Fractional Calculus and Fractional Differential Equations, Wiley, New York, 1993.

[27] H. Eltayeb, Hassan, A. Kilicman, A note on solutions of wave, Laplace's and heat equations with convolution terms by using a double Laplace transform, Appl. Mathe. Lett. 21 (12) (2008) 1324–1329.

[28] G. Spiga, M. Spiga, Two-dimensional transient solutions for crossflow heat exchangers with neither gas mixed, J. Heat Transfer-Trans. Asme 109 (2) (1987) 281–286.

[29] T. Khan, K. Shah, R.A. Khan, A. Khan, Solution of fractional order heat equation via triple Laplace transform in 2 dimensions, Mathe. Methods Appl. Sci. 4 (2) (2018) 818–825.

[30] K. Shah, H. Khalil, R.A. Khan, Analytical solutions of fractional order diffusion equations by natural transform method, Iranian J. Sci. Technol., Trans. A: Sci. 42 (3) (2018) 1479–1490.

[31] H. Singh, F.A. Ghasabzadeh, E. Tohidi, C. Cattani, Legendre spectral method for the fractional Bratu problem, Mathe. Methods Appl. Sci. 43 (9) (2020) 5941–5952.

[32] H. Singh, H.M. Srivastava, Jacobi collocation method for the approximate solution of some fractional-order Riccati differential equations with variable coefficients, Phys. A 523 (2019) 1130–1149.

[33] H. Singh, H.M. Srivastava, D. Kumar, A reliable algorithm for the approximate solution of the nonlinear Lane-Emden type equations arising in astrophysics, Num. Methods Partial Diff. Eqs. 34 (5) (2018) 1524–1555.

[34] J. Singh, H.K. Jassim, D. Kumar, An efficient computational technique for local fractional Fokker Planck equation, Phys. A 555 (1) (2020) 124525.

[35] B. Ahmad, S. Sivasundaram, On four-point nonlocal boundary value problems of nonlinear integro-differential equations of fractional order, Appl. Math. Comput 217 (2010) 480–487.

[36] Z. Bai, On positive solutions of a nonlocal fractional boundary value problem, Nonlinear Anal. 72 (2010) 916–924.

[37] R.A. Khan, K. Shah, Existence and uniqueness of solutions to fractional order multi-point boundary value problems, Commun. Appl. Anal. 19 (2015) 515–526.

[38] K. Shah, N. Ali, R.A. Khan, Existence of positive solution to a class of fractional differential equations with three point boundary conditions, Math. Sci. Lett. 5 (3) (2016) 291–296.

[39] J. Wang, Y. Zhou, W. Wei, Study in fractional differential equations by means of topological Degree methods, Numer. Funct. Anal. Opti. 33 (2012) 216–238.

[40] A.A. Kilbas, H. Srivastava, J. Trujillo, Theory and application of fractional differential equations, North Holland Mathematics Studies, vol. 204, Elseview, Amsterdam, 2006.

[41] J. Singh, Analysis of fractional blood alcohol model with composite fractional derivative, Chaos, Solitons & Fractals <texmath type="inline"/>

[42] J. Singh, D. Kumar, D. Baleanu, A new analysis of fractional fish farm model associated with Mittag-Leffler type kernel, Int. J. Biomathemat. (2020), https://doi.org/10.1142/S1793524520500102.

[43] J. Singh, A. Ahmadian, S. Rathore, D. Kumar, D. Baleanu, M. Salimi, S. Salahshour, An efficient computational approach for local fractional poisson equation in fractal media, Num. Methods Partial Diff. Eqs. (2020), https://doi.org/10.1002/num.22589.

[44] V.P. Dubey, S. Dubey, D. Kumar, J. Singh, A computational study of fractional model of atmospheric dynamics of carbon dioxide gas, Chaos, Solitons & Fractals (2020), https://doi.org/10.1016/j.chaos.2020.110375.

[45] www.worldometers.info, Current update in Pakistan about COVID-19, April 21, 2021.

[46] www.worldometers.info, Current update in Pakistan about COVID-19, April 21, 2021.