Knight Shift in the FFLO State of a Two-Dimensional D-Wave Superconductor

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Abstract. We report on the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state in two-dimensional $d$-wave superconductors with magnetic field parallel to the superconducting planes. This state occurs at high magnetic field near the Pauli-Clogston limit and is a consequence of the competition between the pair condensation and Zeeman energy. We use the quasiclassical theory to self-consistently compute the spatially nonuniform order parameter. Our self-consistent calculations show that the FFLO state of a $d$-wave order parameter breaks translational symmetry along preferred directions. The orientation of the nodes in real space is pinned by the nodes of the basis function in momentum space. Here, we present results for the Knight shift and discuss the implications for recent nuclear magnetic resonance measurements on CeCoIn$_5$.

Keywords: low-dimensional superconductivity, Fulde-Ferrell-Larkin-Ovchinnikov phase, $d$-wave superconductivity.

PACS: 75.60.-d, 76.60.Cq, 74.81.-g

LA-UR-05-5255

The Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state of spin-singlet superconductors is the compromise between the pairing condensate, favoring anti-parallel spin alignment, and the Zeeman effect, favoring parallel spin alignment along the field. This compromise leads to a spatially inhomogeneous state of “normal” and “superconducting” regions, where the “normal” regions are defined by a spectrum of spin-polarized quasiparticles.

The FFLO phase of $d$-wave superconductors is modified by the anisotropy of the order parameter in momentum space compared to $s$-wave superconductors. The upper critical transition line, $B_{c2}(T)$, has a kink at low temperatures, $T^* \sim 0.06T_c$, corresponding to the discontinuous change in the modulation of the order parameter. Recent calculations of the spatial modulation of the order parameter in 2D near $B_{c2}$ predicted that the energetically favored state at low $T$ and high $B$ forms a “square lattice” instead of the 1D stripe order.

Here, we restrict our study to temperatures above this structural phase transition, $T > T^*$, and address the quasiparticle response in the FFLO phase between the lower critical field $B_{c1}$ and the upper critical field $B_{c2}$. In addition, we assume that $B$ is parallel to the superconducting planes. In this geometry the magnetic field affects the superconducting condensate only through the Zeeman coupling of the quasiparticle spin to the field. Furthermore, we assume a cylindrical Fermi surface.

Within the quasiclassical theory of superconductivity we calculate self-consistently the order parameter $\Delta(R, \hat{p})$ and the quasiclassical Green’s functions by solving Eilenberger’s equation in a constant magnetic field $B$. The Zeeman coupling of the quasiparticle spin with magnetic field enters through $\mu B \cdot \sigma$, where $\sigma_i$ are Pauli spin matrices and $\mu = g/2\mu_B$ is the absolute value of the magnetic moment of a quasiparticle with negative charge $e$; $\mu_B = |e|/2mc$ is the Bohr magneton. Note that the $g$-factor is a free material parameter in this calculation.

From the solutions of Eilenberger’s equation we can calculate measurable quantities like the free energy, quasiparticle local density of states and local magnetization $M(R)$. Here, we consider spin-singlet order parameters that factorize into $\Delta(R, \hat{p}) = \Delta(R) \cos 2\phi$, with a spatially dependent amplitude, $\Delta(R)$, and an angular dependent $d_{\pm \pm \pm}$-wave basis function.

The local magnetization is given by the paramagnetic response of the medium and the spin-vector component of the quasiclassical Matsubara Green’s function:

$$M(R) = 2\mu N_f \left[ \mu B + T \sum_{\varepsilon_n} \int \frac{d\hat{p}}{2\pi} g(\hat{p}; R; \varepsilon_n) \right],$$

with the normal-state density of states $N_f$ at the Fermi level. The normal-state susceptibility, $\chi_N = 2\mu^2 N_f$, is defined by $M_N = \chi_N B$.

For comparison, we show in Fig. 1 the calculated temperature dependence of the magnetization in the uniform superconducting (USC) phase for three different values of $B$. Increasing $B$ changes the $T$-dependence of the magnetization from linear to quadratic with a residual zero-temperature value due to the field induced shift in the spin-split density of states of the gapless $d$-wave superconductor. This result is in agreement with scaling arguments by Yang and Sondhi.

In Fig. 2, we show temperature scans of the minimum, average and maximum local magnetization for the stable FFLO phase, with spatial order-parameter modulation along the $\langle 110 \rangle$ direction, i.e., along the nodal direction.
of the gap function. The Knight shift $K$ is proportional to the change in the local magnetic field at the nucleus, thus it is directly proportional to the local magnetization. Since $K$ is weighted by the field distribution, the largest contribution comes from areas where the derivative of $M$ vanishes, which are at the minimum and maximum locations of $M$. The calculated bifurcation between minimum and maximum local magnetization seen in Fig. 2 is in qualitative agreement with measurements of the Knight shift on CeCoIn$_5$ reported by Kakuyanagi et al. [11].

In Fig. 3, we show field scans of the local magnetization at $T/T_c = 0.1$ starting in the USC phase and into the FFLO phase. It illustrates the nonlinear magnetic response of the quasiparticles due to an external field and the continuous second order transition at the lower critical field $B_{c1}$, which also is signaled by the appearance of a single domain wall. This finding clearly contradicts the claim by Yang and Sondhi [10] about a first order transition at $B_{c1}$ between the USC and FFLO phase.

In addition, we calculated the spin-resolved local density of states in the FFLO state [6] (not shown). We found that the characteristic Andreev bound states, due to the periodic sign change of the order parameter, are responsible for the excess spin polarization of quasiparticles at the domain walls seen in Figs. 2 and 3. Therefore, the Andreev bound states should be clearly visible features in scanning tunneling spectroscopy measurements.

ACKNOWLEDGMENTS

We thank N. J. Curro, R. Movshovich and V. F. Mitrović for helpful discussions and A. V. Balatsky and J. A. Sauls for suggesting this problem. This research is supported by the Department of Energy, under contract W-7405-ENG-36, (MJG) and the LSU Board of Regents (ABV).

REFERENCES

1. P. Fulde and R. Ferrell, Phys. Rev. 135, A550–A563 (1964).
2. A. I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP 20, 762–769 (1965).
3. K. Maki and H. Won, Czech. J. Phys. 46, 1035–1036 (1996).
4. H. Shimahara and D. Rainer, J. Phys. Soc. Japan 66, 3591–3599 (1997).
5. U. Klein, D. Rainer, and H. Shimahara, J. Low Temp. Phys. 118, 91–104 (2000).
6. A. B. Vorontsov, J. A. Sauls, and M. J. Graf, cond-mat/0506257 (2005).
7. H. Shimahara, J. Phys. Soc. Japan 67, 736–739 (1998).
8. K. Maki and H. Won, Physica B 322, 315–317 (2002).
9. J. A. X. Alexander, T. P. Orlando, D. Rainer, and P. M. Tedrow, Phys. Rev. B 31, 5811–5825 (1985).
10. K. Yang and S. L. Sondhi, Phys. Rev. B 57, 8566 (1998).
11. K. Kakuyanagi et al., Phys. Rev. Lett. 94, 047602 (2005).