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Comparative Study of Modified Mueller Matrix Transformation and Polar Decomposition Parameters for Transmission and Backscattering Tissue Polarimetries

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Abstract: Mueller matrix polarimetry is widely used in biomedical studies and applications, for it can provide abundant microstructural information about tissues. Recently, several methods have been proposed to decompose the Mueller matrix into groups of parameters related to specific optical properties which can be used to reveal the microstructural information of tissue samples more clearly and quantitatively. In this study, we thoroughly compare the differences among the parameters derived from the Mueller matrix polar decomposition (MMPD) and Mueller matrix transformation (MMT), which are two popular methods in tissue polarimetry studies and applications, while applying them on different tissue samples for both backscattering and transmission imaging. Based on the Mueller matrix data obtained using the setups, we carry out a comparative analysis of the parameters derived from both methods representing the same polarization properties, namely depolarization, linear retardance, fast axis orientation and diattenuation. In particular, we propose several modified MMT parameters, whose abilities are also analyzed for revealing the information about the specific type of tissue samples. The results presented in this study evaluate the applicability of the original and modified MMT parameters, then give the suggestions for appropriate parameter selection in tissue polarimetry, which can be helpful for future biomedical and clinical applications.

Keywords: Mueller matrix polarimetry; optical properties; biomedical imaging

1. Introduction

As a non-invasive, non-contact and label-free tool, Mueller matrix polarimetry is widely used in biomedical studies and clinical applications, for it can provide abundant microstructural information about tissues and cells [1–4]. Specifically, the Mueller matrix measurement system can be used for abnormal tissue detection for both backward imaging of bulk tissue samples and forward imaging of thin tissue slices [5–7]. However, although a Mueller matrix contains a comprehensive description of the polarization-related structural and optical properties of the medium, it is often difficult to obtain intrinsic polarimetric characteristics such as diattenuation, retardance and depolarization encoded in the Mueller
matrix elements directly. To solve this problem, in the past decades several methods have been proposed to decompose the Mueller matrix into groups of parameters related to specific optical properties which can be used to reveal the microstructural information of tissue samples more clearly and quantitatively [8–14].

Until now, the Mueller matrix polar decomposition (MMPD) and Mueller matrix transformation (MMT) are two popular methods in tissue polarimetry studies and applications. Nevertheless, unlike the MMPD method, which decomposes the Mueller matrix into three sub-matrices with clear physical meanings of depolarization, retardance, and diattenuation [12,13], the MMT method directly combines groups of Mueller matrix elements to derive parameters related to certain characteristics of tissue-like media with an advantage of fast calculation speed [14]. Both the MMPD and MMT methods have demonstrated the diagnostic potential in various cancerous tissue detection, including breast cancer, liver cancer, inflammatory bowel diseases and cervical cancer [15–20]. However, it is worth noting that the parameters derived from the MMPD and MMT methods are based on specific deduction models and calculation assumptions. Therefore, for accurate diagnostic purposes, their applicability and interpretability regarding different types of samples requires further discussion and analysis. Recently, some efforts have been made in this area, and some useful conclusions have been drawn by Ahmad and us [21–24].

In this study, to compare the difference among the parameters derived from the MMPD and MMT methods more comprehensively while applying them on different types of tissue samples measured by both backscattering and transmission imaging setups, we prepare seven kinds of thin tissue samples for forward measurement and five kinds of bulk tissue samples for backward measurement. Based on the Mueller matrix data obtained using the setups, we carry out thoroughly comparative analysis of the parameters derived from both methods representing the same polarization properties, namely depolarization, retardance, fast axis orientation and diattenuation. Specifically, according to tissue structures, we further proposed several modified MMT parameters, whose abilities were also analyzed for revealing information with regard to specific types of tissue samples. We discuss the applicability of the original and modified MMT parameters based on the comparisons, and then give suggestions for appropriate parameter selection in tissue polarimetry.

2. Materials and Methods

2.1. Mueller Matrix Polar Decomposition (MMPD)

The Mueller matrix polar decomposition method proposed by Lu and Chipman describes the process of interaction between light and medium by decomposing a Mueller matrix into three sub-matrices with clear physical meanings: diattenuation ($M_D$), retardance ($M_R$) and depolarization ($M_\Delta$), as shown as Equation (1) [12].

$$M = M_\Delta M_R M_D$$  

(1)

Based on Equation (1), a group of MMPD parameters $D$, $R$ and $\Delta$ can be calculated by Equations (2)–(4), representing diattenuation, retardance and depolarization properties of the samples, respectively. It should be noted that the lower case $m_{ij}$ ($i,j = 1, 2, 3, 4$) in Equation (2) represents the corresponding Mueller matrix elements before performing any decomposition, while the lower case $m_\Delta$ in Equation (4) is the $3 \times 3$ depolarization sub-matrix from $M_\Delta$.

$$D = \frac{1}{m_{11}} \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2}$$  

(2)

$$R = \cos^{-1} \left[ \frac{\text{tr}(M_\Delta)}{2} - 1 \right]$$  

(3)

$$\Delta = 1 - \frac{|\text{tr}(m_\Delta) - 1|}{3}$$  

(4)

The parameters $\delta$ and $\theta$ derived from the retardance matrix $M_R$ are usually used to describe the value and fast axis orientation of linear birefringence induced by fibrous
tissue structures [13]. Since the birefringent fibrous structures including collagen, elastics, and muscle fibers that are prevalent in tissues, these two MMPD parameters can provide important information of the distribution behaviors of fibrous structures during many pathological processes [15–20,25–28]. It should be noted that compared with the original orientation parameters $\theta$, a modified MMPD parameter $\theta_e$ proposed in our recent study has been demonstrated as a more suitable indicator for describing the orientation of complicated tissue samples containing layered fibrous structures [26]. The parameters $\delta$ and $\theta_e$ can be calculated according to Equations (5)–(8) [13,26]. The $M_R(i,j)$ ($i,j = 2, 3$) in Equation (5) represents the corresponding elements of the $4 \times 4$ retardance matrix $M_R$, while $R$ is the total retardance, $\epsilon_{ijk}$ is the Levi-Civita permutation symbol, $\delta_{ij}$ is the Kronecker delta, and $m_R$ is the $3 \times 3$ retardance sub-matrix striking out the first row and the first column of $M_R$.

$$\delta = \cos^{-1}\left\{ \sqrt{[M_R(2,2) + M_R(3,3)]^2 + [M_R(3,2) - M_R(2,3)]^2} - 1 \right\}$$ (5)  
$$\theta_e = \frac{1}{2} \tan^{-1}\left( \frac{a_2}{a_1} \right)$$ (6)  
$$(m_R)_{ij} = \delta_{ij} \cos R + a_ia_j(1 - \cos R) + \sum_{k=1}^{3} \epsilon_{ijk}a_k \sin R(i,j = 1,2,3)$$ (7)  
$$a_i = \frac{1}{2 \sin R} \sum_{lj=1}^{3} \epsilon_{ijk}(m_R)_{jk}$$ (8)

### 2.2. Mueller Matrix Transformation (MMT) Parameters

The Mueller matrix transformation method was proposed to derive groups of quantitative parameters for describing the microstructural features of biomedical tissue samples by fitting the Mueller matrix elements to the trigonometric functions [14]. Several MMT parameters such as $t_2$, $t_3$, $x_3$ and $1 - b$ were introduced in our previous studies [14,29], where $t_2$ (also denoted as $t_{1213}$ shown in Equation (9)) reveals the linear diattenuation property, $t_3$ (also denoted as $t_{2434}$ and $t_{4243}$ shown in Equations (10) and (11)) is related to the linear retardance of the tissue samples, while $x_3$ (also denoted as $\phi_{2434}$ and $\phi_{4243}$ shown in Equations (12) and (13)) can characterize the orientation of the birefringence structure and $1 - b$ (shown in Equation (14)) represents the linear depolarization of the tissue samples.

$$t_{1213} = \sqrt{m_{12}^2 + m_{13}^2}$$ (9)  
$$t_{2434} = \sqrt{m_{24}^2 + m_{34}^2}$$ (10)  
$$t_{4243} = \sqrt{m_{42}^2 + m_{43}^2}$$ (11)  
$$\phi_{2434} = \frac{1}{2} \tan^{-1}\left( \frac{-m_{24}}{m_{34}} \right)$$ (12)  
$$\phi_{4243} = \frac{1}{2} \tan^{-1}\left( \frac{m_{42}}{m_{43}} \right)$$ (13)  
$$1 - b = 1 - \frac{m_{22} + m_{33}}{2}$$ (14)

However, as these parameters are derived based on specific models [14], when we apply these parameters on complex tissue samples, their applicability needs further discussion. Our previous studies showed that there is a deviation between the values of $t_{2434}$ and $t_{4243}$ when multiple crossing linear birefringence effects coexist, as the information of linear retardance are distributed among the $m_{24}$, $m_{34}$, $m_{42}$ and $m_{43}$ elements [14,26,30]. Such deviation also occurs in the birefringence orientation parameters $\phi_{2434}$ and $\phi_{4243}$. 
Here, according to tissue structures, we propose two modified MMT parameters \( t_{qr} \) and \( \phi_{qr} \) (shown in Equations (15) and (16)), which take \( m_{24}, m_{34}, m_{42} \) and \( m_{43} \) elements into account, to calculate the linear retardance and birefringence orientation of the layered tissue sample more accurately. Besides, we also propose another two MMT parameters \( t_{121314} \) and \( 1 - b_m \) (Equations (17) and (18)) to describe the overall diattenuation and depolarization properties of the tissue samples by taking the elements \( m_{14} \) and \( m_{44} \) into account, which contain the information of circular diattenuation and circular depolarization.

\[
t_{qr} = \sqrt{\frac{t_{2434}^2 + t_{4243}^2}{2}} = \sqrt{\frac{m_{24}^2 + m_{42}^2 + m_{34}^2 + m_{43}^2}{2}} \quad (15)
\]

\[
\phi_{qr} = \frac{1}{2} \tan^{-1}\left(\frac{m_{42} - m_{24}}{m_{34} - m_{43}}\right) \quad (16)
\]

\[
t_{121314} = \sqrt{m_{12}^2 + m_{13}^2 + m_{14}^2} \quad (17)
\]

\[
1 - b_m = 1 - \frac{m_{22} + m_{33} + |m_{44}|}{3} \quad (18)
\]

Among the available Mueller matrix analyzing methods, the calculation speed of MMT is relatively fast, which is advantageous for clinical applications [5,24]. However, the principle of MMT calculations is based on the fitting parameters obtained by experiments and simulations, and its accuracy needs to be further studied when applied to quantitative characterization of specific tissues and cells. On the other hand, the MMPD parameters are widely used in the field of biomedical and pre-clinical detection. The applicability of the MMPD method on tissue samples has been proven in various ex vivo and in vivo measurements [5,6].

### 2.3. Materials and Measurement Setup

In previous studies, we have developed the transmission Mueller matrix microscope for measuring thin tissue slices and backscattering Mueller matrix imaging set-ups for measuring bulk tissue samples [19,31–35]. The measurement schemes are based on the dual-rotating retarder method. For this method, the PSG (polarization state generator) and PSA (polarization state analyzer) consist of a pair of polarizers fixed in the horizontal direction, which is also parallel to the source-sample-camera triangle plane for the backscattering imaging, and two retarders rotating with a fixed ratio of angles. It should be noted that the source, sample and camera are colinear in transmission imaging geometry. Then the 16 Mueller matrix elements can be calculated by using the Fourier coefficients \( \alpha_n \) and \( \beta_n \) shown as Equation (19) [36].

\[
I = \alpha_0 + \sum_{n=1}^{12} \left( \alpha_n \cos 2n\theta_1 + \beta_n \sin 2n\theta_1 \right) \quad (19)
\]

where \( I \) is the intensity, \( \theta_1 \) is the rotation angle of the retarder 1 for the PSG. The two retarders (retarder 1 for PSG, retarder 2 for PSA) rotate with a fixed rate \( \theta_1 = 5\theta_2 \). The Mueller matrix measurement setups used in this study were calibrated to ensure that the maximum errors of the measured Mueller matrix elements are less than 1%. More details on this Mueller matrix imaging method and calibration process can be found in [32,37,38].

Here, we prepared two types of tissue samples \( (n = 149) \) for the comparative analysis: thin pathological tissue slices for transmission measurement and bulk tissue samples for backscattering measurement. The unstained thin tissue samples include human colorectal, intestinal tuberculosis (ITB), liver, Crohn’s disease (CD), bladder, breast, adenoma tissue slices whose thickness are 12 \( \mu \)m, while the bulk tissue samples include human breast, rat-skin, porcine intestine, porcine stomach, and porcine liver tissues. The H-E stained 4-\( \mu \)-thick tissue slices of the samples were also prepared for pathological observations. More details of the samples used in this study are shown in Table 1. It should be noted that
the signal-to-noise ratio for the camera is related to the energy obtained from the source, which depends on both the power of LED and the exposure of the camera. As the power of LED for our Mueller matrix imaging is commonly within 1–3 W [24,25,34], we have selected an appropriate exposure time for the camera to achieve a good signal-to-noise ratio. Our previous studies showed that the LED with the power of 1 W did not generate a lot of heat to the thick sample when an appropriate exposure time was selected [24,25,34]. In this work, the human tissue samples are provided by the Department of Gastroenterology, Zhujiang Hospital, Southern Medical University, and Shenzhen Sixth People’s (Nanshan) Hospital. This work was approved by the Ethics Committee of the Shenzhen International Graduate School, Tsinghua University.

Table 1. Description of the tissue samples (n = 149) and the experimental setup of the Mueller matrix polarimetry systems.

| Type  | Sample Description | Experimental Setup | Total |
|-------|--------------------|-------------------|-------|
|       | Sample | Thickness (µm) | Number | Light Source | λ (nm) | Power (W) | Geometry |
| Thin  | colorectal | 12 | 14 | LED | 633 | 1 |
|       | ITB     | 12 | 14 | LED | 633 | 1 |
|       | liver   | 12 | 6  | LED | 633 | 1 |
|       | CD      | 12 | 10 | LED | 633 | 1 | Forward | 86 |
|       | bladder | 12 | 14 | LED | 633 | 1 |
|       | breast  | 12 | 14 | LED | 633 | 1 |
|       | adenoma | 12 | 14 | LED | 633 | 1 |
| Bulk  | breast  | -  | 7  | LED | 633 | 1 |
|       | rat-skin| -  | 23 | LED | 633 | 1 |
|       | porcine-intestine | -  | 8  | LED | 633 | 1 | Backward | 63 |
|       | porcine-stomach | -  | 16 | LED | 633 | 1 |
|       | porcine-liver | -  | 9  | LED | 633 | 1 |

2.4. Quantitative Indicators for MMT and MMPD Parameters Comparison

Here, we evaluate the difference between MMT and MMPD parameters in two aspects: how their linear relationship is and whether there exits statistical significance. For the linear relationship comparison, we take the square of the Pearson correlation coefficient ($R^2$ or $r^2$) as a measure between two sets of parameters, as it can provide a quantitative value for their linear relationship evaluation (range 0–1). For a correlation between variables $x$ and $y$, the formula for calculating the sample Pearson’s correlation coefficient ($R$ or $r$) is given by Equation (20) [39,40].

\[
R = r_{xy} = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^{n} (y_i - \bar{y})^2}}
\]  

(20)

where $x_i$ and $y_i$ are the values of $x$ and $y$ for the $i$th individual. It is also worth mentioning that the larger the value is, the more similar the changing trends of MMT and MMPD parameters are. Based on the correlation analysis, we conduct the statistical analysis between MMT and MMPD with $p$-value, which can assess whether both parameters are interchangeable without linear fitting according to their statistical differences [41].

3. Results and Discussion

To compare the MMT and MMPD parameters more comprehensively, in the following sections we analyze the three main polarization properties (diattenuation, retardance, and depolarization) of both the transmission imaging results of thin tissue samples, and backscattering imaging results of bulk tissue samples.
3.1. Quantitative Comparison of Diattenuation Related MMPD and MMT Parameters for Transmission Imaging of Thin Tissue Samples

We measured all seven different kinds of thin pathological tissue slices (86 in total) using the forward transmission Mueller matrix microscope. Then, for each kind of tissue sample, we calculated the diattenuation related MMT and MMPD parameters according to Equations (2), (9) and (17). Specifically, to find out whether the modified MMT parameters can be used to accurately obtain the structural properties of tissue samples, the quantitative analyses were conducted to show the correlations and differences between the MMT and the corresponding MMPD parameters.

The correlation coefficient $R^2$ and statistical analysis $p$ values of each kind of thin tissue slice samples are presented in Table 2. The comparison of the MMPD and MMT diattenuation parameters was performed in two groups: $(D, t_{1213})$ and $(D, t_{121314})$. As we can see from Table 2, for the correlation analysis, compared with the group of $(D, t_{1213})$, the group $(D, t_{121314})$, for each kind of sample has a higher correlation coefficient value of 1. More thorough and intuitive comparisons are illustrated in Figure 1a, c by plotting out all the sample data, which confirm that the modified MMT diattenuation parameter $t_{121314}$ has a good accordance with the MMPD parameter $D$ in the transmission measurement. On the other hand, as can be noticed from Table 2 and Figure 1b, the ITB tissue samples show statistically significant differences, with $p < 0.01$ between the parameters $D$ and $t_{1213}$. While for the modified parameter $t_{121314}$, the $p$-values of all kinds of tissue samples are above 0.05, meaning that the modified MMT parameter $t_{121314}$ can be used as a precise indicator for the diattenuation property of thin tissue samples.

### Table 2. Correlation and statistical analysis comparison results between forward measured MMT and MMPD diattenuation parameters for each kind of thin tissue samples.

| Type   | Tissues | Diattenuation | $R^2$ | $p$ |
|--------|---------|---------------|-------|-----|
|        |         | $D$—$t_{1213}$| $D$—$t_{121314}$| $D$—$t_{1213}$| $D$—$t_{121314}$|
| Forward| colorectal | 0.99 | 1.00 | >0.05 | >0.05 |
|        | ITB     | 0.90 | 1.00 | <0.01 | >0.05 |
|        | liver   | 0.99 | 1.00 | >0.05 | >0.05 |
|        | CD      | 0.99 | 1.00 | >0.05 | >0.05 |
|        | bladder | 0.99 | 1.00 | >0.05 | >0.05 |
|        | breast  | 0.98 | 1.00 | >0.05 | >0.05 |
|        | adenoma | 0.99 | 1.00 | >0.05 | >0.05 |
|        | Total   | 0.99 | 1.00 | >0.05 | >0.05 |

3.2. Quantitative Comparison of Diattenuation Related MMPD and MMT Parameters for Backscattering Imaging of Bulk Tissue Samples

To study the relationship between the diattenuation related MMPD and MMT parameters for backscattering polarimetric measurement, we prepared five different kinds of bulk tissue samples (63 in total) to attain their backscattering Mueller matrix data. Then, for each kind of tissue sample, we also calculated the diattenuation-related parameters in Section 3.1 for the correlation and statistical analysis whose results are shown in Table 3 and Figure 2.
Figure 1. Comparison of forward measured MMT and MMPD diattenuation parameters of thin tissue samples. (a) MMPD parameter $D$ plotted against its counterpart MMT parameter $t_{1213}$ for correlation analysis with all 7 kinds of thin tissue samples. (b) Statistical analysis bar chart of MMPD parameter $D$ and MMT parameter $t_{1213}$ for each kind of thin tissue samples. (c) MMPD parameter $D$ plotted against its counterpart modified MMT parameter $t_{121314}$ for correlation analysis with all 7 kinds of thin tissue samples. (d) Statistical analysis bar chart of MMPD parameter $D$ and modified MMT parameter $t_{121314}$ for each kind of thin tissue samples.

Table 3. Correlation and statistical analysis comparison results between backward measured MMT and MMPD diattenuation parameters for each kind of bulk tissue samples.

| Type        | Tissues     | $R^2$  | $D$—$t_{1213}$ | $D$—$t_{121314}$ | $D$—$t_{1213}$ | $D$—$t_{121314}$ |
|-------------|-------------|--------|----------------|------------------|----------------|------------------|
| Backward    | breast      | 0.99   | 1.00           | >0.05            | >0.05          | >0.05            |
|             | rat-skin    | 0.94   | 1.00           | >0.05            | >0.05          | >0.05            |
|             | porcine-intestine | 0.99 | 1.00           | <0.01            | >0.05          | >0.05            |
|             | porcine-stomach | 0.92 | 1.00           | <0.05            | >0.05          | >0.05            |
|             | porcine-liver | 0.99  | 1.00           | >0.05            | >0.05          | >0.05            |
|             | Total       | 0.99   | 1.00           | <0.05            | >0.05          | >0.05            |
We can observe from Table 3 that, for the backscattering measurement results, the correlation coefficient values of parameters ($D$, $t_{121314}$) for each kind of bulk samples also have higher values of 1 compared with that of parameters ($D$, $t_{1213}$), meaning that the modified MMT diattenuation parameter $t_{121314}$ also has a good accordance with the MMPD parameter $D$ in backscattering measurement. Meanwhile, for the statistical analysis of each kind of sample, it can be seen from Table 3 and Figure 2b that the rat-skin, porcine-intestine, porcine-stomach and liver tissue samples show statistically significant differences with $p < 0.05$ between the parameters $D$ and $t_{1213}$. However, the $p$-values of all kinds of tissue samples are larger than 0.05 for parameters ($D$, $t_{121314}$) shown in Table 3 and Figure 2d, which also confirms that the modified MMT parameter $t_{121314}$ can be used to accurately characterize the diattenuation property of the bulk tissue samples measured by backscattering polarimetry.

In summary, we can see from the comparative analyzing results presented in Sections 3.1 and 3.2 that the modified MMT parameter $t_{121314}$ shows a better accordance with the MMPD parameter $D$ in both the transmission and backscattering measurements than the original parameter $t_{1213}$, which only reflects the linear diattenuation of the samples. Though the circular diattenuation effects of the tissue samples are often very limited, taking
the element \( m_{14} \) (encoded the circular diattenuation property) [42] into account for tissue polarimetric calculations can produce more accurate results.

### 3.3. Quantitative Comparison of Linear Retardance Related MMPD and MMT Parameters for Transmission Imaging of Thin Tissue Samples

Linear retardance is viewed as one of the most important polarization properties of tissue samples in terms of revealing the location and density of the birefringent fibrous structures [15–20]. Our previous studies showed that the MMT parameters \( t_{2434} \) and \( t_{4243} \) can be used to reflect the linear retardance of the tissues [14,29]. However, both of the parameters may slightly deviate from each other when measuring complicated tissue samples containing layered anisotropic structures, reducing the accuracy of Mueller matrix polarimetry for quantitative tissue evaluation. To deal with this problem, we proposed a modified MMT parameter \( t_{qr} \) based on the elements from both the fourth column and fourth row shown in Equation (15) to describe the linear retardance property. To verify its effectiveness, we conducted the comparisons among the linear retardance related MMPD and MMT parameters in three groups: \((\delta, t_{2434})\), \((\delta, t_{4243})\), and \((\delta, t_{qr})\). The correlation and statistical analysis values of MMT and MMPD parameters groups for each kind of thin tissue samples are shown in Table 4 and Figure 3. For more intuitive comparisons, all the parameters are normalized to the [0, 1] value range.

**Table 4.** Correlation and statistical analysis comparison results between forward measured MMT and MMPD linear retardance parameters for each kind of thin tissue sample.

| Type | Tissues    | \( R^2 \) | \( \delta - t_{2434} \) | \( \delta - t_{4243} \) | \( \delta - t_{qr} \) | \( p \) |
|------|------------|-----------|-----------------|-----------------|-----------------|------|
| Forward | colorectal | 0.97      | 0.94            | 0.99            | <0.01           | <0.01|
|       | ITB        | 0.93      | 0.93            | 0.97            | <0.01           | <0.01|
|       | liver      | 0.98      | 0.99            | 0.97            | <0.01           | <0.01|
|       | CD         | 0.91      | 0.93            | 0.93            | <0.01           | <0.01|
|       | bladder    | 0.99      | 0.99            | 0.99            | <0.01           | <0.01|
|       | breast     | 0.90      | 0.91            | 0.99            | <0.01           | <0.01|
|       | adenoma    | 0.97      | 0.94            | 0.99            | <0.01           | <0.01|
|       | Total      | 0.98      | 0.98            | 0.99            | <0.01           | <0.01|

For the correlation analysis, Figure 3a,c,e show that, compared with the parameter groups \((\delta, t_{2434})\) and \((\delta, t_{4243})\), the group \((\delta, t_{qr})\) for all the samples have higher correlation coefficient values. We can also see the differences more clearly in Table 4, where the correlation coefficient values of \((\delta, t_{2434})\) and \((\delta, t_{4243})\) deviate from each other ranging between 0.90 to 0.99, while the correlation coefficients of \((\delta, t_{qr})\) are more stable with higher values for all the samples. Moreover, for the statistical analysis, all kinds of thin tissue samples show statistically significant differences with \( p < 0.01 \) among the parameter groups according to Table 4 and Figure 3b,d,f, meaning that the MMT linear retardance parameters cannot be substituted for MMPD parameter \( \delta \) directly without linear fitting. However, the modified MMT linear retardance parameter \( t_{qr} \) shows a good accordance with the MMPD parameter \( \delta \), indicating that it can be used to well present linear retardance distributions of tissues by linear fitting with MMPD parameter \( \delta \).
Figure 3. Comparison of MMT and MMPD linear retardance parameters of thin tissue samples. (a) Linear retardance MMPD parameter $\delta$ plotted against its counterpart MMT parameter $t_{2434}$ for correlation analysis with all seven kinds of thin tissue samples. (b) Statistical analysis bar chart of linear retardance MMPD parameter $\delta$ and MMT parameter $t_{2434}$ for each kind of thin tissue samples. (c) Linear retardance MMPD parameter $\delta$ plotted against its counterpart MMT parameter $t_{4243}$ for correlation analysis with all seven kinds of thin tissue samples. (d) Statistical analysis bar chart of linear retardance MMPD parameter $\delta$ and MMT parameter $t_{4243}$ for each kind of thin tissue samples. (e) Linear retardance MMPD parameter $\delta$ plotted against its counterpart MMT parameter $t_{qr}$ for correlation analysis with all seven kinds of thin tissue samples. (f) Statistical analysis bar chart of linear retardance MMPD parameter $\delta$ and MMT parameter $t_{qr}$ for each kind of thin tissue sample.
3.4. Quantitative Comparison of Linear Retardance Related MMPD and MMT Parameters for Backscattering Imaging of Bulk Tissue Samples

As shown in Table 5, for the backscattering measurement results of bulk tissue samples, the linear retardance related parameters derived from MMT and MMPD methods do not show a good correlation. The correlation coefficient values of each kind of bulk tissue samples for parameter groups ($\delta$, $t_{2434}$), ($\delta$, $t_{4243}$) and ($\delta$, $t_{qr}$) are between 0.16 and 0.89, and we can also see the data points of the tissue samples distribute randomly in Figure 4a,c,e. Besides, Figure 4b,d,f show that the MMPD linear retardance parameters are larger than the MMT linear retardance parameters, and they all have statistically significant differences with the $p$-value smaller than 0.05. Therefore, it is worth mentioning that when we try to measure linear retardance property of bulk tissues sample using backscattering polarimetry, the MMPD parameter $\delta$ will provide the information more accurately, while the MMT linear retardance parameters can be suitable for quickly evaluating the existence and location of birefringent structures.

Table 5. Correlation and statistical analysis comparison results between backscattering measured MMT and MMPD linear retardance parameters for each kind of bulk tissue samples.

| Type     | Tissues         | Linear Retardance |          |          |          |          |
|----------|-----------------|-------------------|----------|----------|----------|----------|
|          |                 | $R^2$             | $\delta$ | $t_{2434}$| $\delta$ | $t_{4243}$| $\delta$ | $t_{qr}$|
| Backward | breast          | 0.65              | 0.76     | 0.73     | <0.05    | <0.05    | <0.05    |          |
|          | rat-skin        | 0.46              | 0.65     | 0.68     | <0.01    | <0.01    | <0.01    |          |
|          | porcine-intestine| 0.73              | 0.74     | 0.74     | <0.01    | <0.01    | <0.01    |          |
|          | porcine-stomach | 0.33              | 0.16     | 0.22     | <0.01    | <0.01    | <0.01    |          |
|          | porcine-liver   | 0.65              | 0.89     | 0.84     | <0.01    | <0.01    | <0.01    |          |
|          | Total           | 0.63              | 0.65     | 0.66     | <0.01    | <0.01    | <0.01    |          |

3.5. Quantitative Comparison of Linear Birefringence Orientation Related MMPD and MMT Parameters for Transmission Imaging of Thin Tissue Samples

Besides the linear retardance value, the fast axis orientation of linear birefringence is another important indicator to reveal the structural features of tissue, such as the distribution behaviors of collagen and elastic fibers in pathological tissue samples, which is helpful for clinical diagnosis. In our previous study, considering that the layered fibrous structures can result in deviations of MMPD linear birefringence orientation parameters, we proposed a modified MMPD parameter $\theta_e$ to more accurately represent the fast axis orientation of the complicated linear birefringent structures in tissue samples [26]. Meanwhile, it was also found that while applying the MMT parameters $\phi_{2434}, \phi_{4243}$ to the fibrous tissue samples, they exist with positive or negative deviations from the ideal values. Hence, according to the experimental results, we proposed a modified MMT parameter, $\phi_{qr}$ shown in Equation (16), for layered linear birefringent structures. To verify its effectiveness, the correlation and statistical analysis are conducted with the variance of parameters groups ($\theta_e$, $\phi_{2434}$), ($\theta_e$, $\phi_{4243}$) and ($\theta_e$, $\phi_{qr}$), whose results are shown in Table 6 and Figure 5.
Figure 4. Comparison of MMT and MMPD linear retardance parameters of bulk tissue samples. (a) Linear retardance MMPD parameter \( \delta \) plotted against its counterpart MMT parameter \( t_{2434} \) for correlation analysis with all five kinds of bulk tissue samples. (b) Statistical analysis bar chart of linear retardance related MMPD parameter \( \delta \) and MMT parameter \( t_{2434} \) for each kind of bulk tissue samples. (c) Linear retardance MMPD parameter \( \delta \) plotted against its counterpart MMT parameter \( t_{4243} \) for correlation analysis with all five kinds of bulk tissue samples. (d) Statistical analysis bar chart of linear retardance MMPD parameter \( \delta \) and MMT parameter \( t_{4243} \) for each kind of bulk tissue samples. (e) Linear retardance MMPD parameter \( \delta \) plotted against its counterpart MMT parameter \( t_{qr} \) for correlation analysis with all five kinds of bulk tissue samples. (f) Statistical analysis bar chart of linear retardance MMPD parameter \( \delta \) and MMT parameter \( t_{qr} \) for each kind of bulk tissue sample.
Table 6. Correlation and statistical analysis comparison results between forward measured MMT and MMPD linear birefringence orientation parameters for each kind of thin tissue samples.

| Type  | Tissues | \( R^2 \) | \( \theta_e - \phi_{2434} \) | \( \theta_e - \phi_{4243} \) | \( \theta_e - \phi_{qr} \) | \( p \) |
|-------|---------|----------------|----------------|----------------|----------------|-----|
| Forward | colorectal | 0.97 | 0.97 | 1.00 | >0.05 | >0.05 | >0.05 |
|        | ITB      | 0.94 | 0.82 | 0.99 | >0.05 | <0.01 | >0.05 |
|        | liver    | 0.95 | 0.44 | 0.99 | <0.01 | >0.05 | >0.05 |
|        | CD       | 0.97 | 0.95 | 0.99 | >0.05 | >0.05 | >0.05 |
|        | bladder  | 0.97 | 0.93 | 0.99 | >0.05 | >0.05 | >0.05 |
|        | breast   | 0.94 | 0.98 | 0.99 | >0.05 | >0.05 | >0.05 |
|        | adenoma  | 0.98 | 0.97 | 1.00 | >0.05 | >0.05 | >0.05 |
|        | Total    | 0.88 | 0.92 | 0.99 | >0.05 | >0.05 | >0.05 |

Figure 5. Comparison of linear birefringence fast axis orientation MMT and MMPD parameters of thin tissue samples. (a) Variance of linear birefringence orientation MMPD parameter \( \theta_e \) plotted against its counterpart MMT parameter \( \phi_{2434} \) for correlation analysis with all seven kinds of thin tissue samples. (b) Statistical analysis bar chart of linear birefringence orientation MMPD parameter \( \theta_e \) and MMT parameter \( \phi_{2434} \) for each kind of thin tissue samples. (c) Variance of linear birefringence orientation MMPD parameter \( \theta_e \) plotted against its counterpart MMT parameter \( \phi_{4243} \) for correlation analysis with all seven kinds of thin tissue samples. (d) Statistical analysis bar chart of linear birefringence orientation MMPD parameter \( \theta_e \) and MMT parameter \( \phi_{4243} \) for each kind of thin tissue samples. (e) Variance of linear birefringence orientation MMPD parameters \( \theta_e \) plotted against its counterpart MMT parameter \( \phi_{qr} \) for correlation analysis with all seven kinds of thin tissue samples. (f) Statistical analysis bar chart of linear birefringence orientation MMPD parameter \( \theta_e \) and MMT parameter \( \phi_{qr} \) for each kind of thin tissue sample.
For the correlation analysis, Figure 5a,c,e show that compared with the parameter groups \((\theta_e, \phi_{2434})\), \((\theta_e, \phi_{4243})\) and \((\theta_e, \phi_{qr})\), the correlation coefficient values of \((\theta_e, \phi_{qr})\) for all the samples are larger. We can also find the difference more clearly in Table 6, where the correlation coefficient values of \((\theta_e, \phi_{2434})\) and \((\theta_e, \phi_{4243})\) deviate with each other ranging between 0.44 to 0.98, while for \((\theta_e, \phi_{qr})\) the values are more stable and closer to 1 for all kinds of thin tissue samples. For the statistical analysis, the group \((\theta_e, \phi_{qr})\) shows no statistically significant difference with \(p > 0.05\) according to Table 6 and Figure 5b,d,f, which means that the modified MMT parameters \(\phi_{qr}\) has the similar ability as the MMPD parameter \(\theta_e\) for revealing the linear birefringence orientation information of complicated thin tissue samples.

### 3.6. Quantitative Comparison of Linear Birefringence Orientation Related MMPD and MMT Parameters for Backscattering Imaging of Bulk Tissue Samples

For the backscattering measurement results of bulk tissue samples, the modified linear birefringence orientation MMT parameter \(\phi_{qr}\) also shows a good fit with the MMPD parameter \(\theta_e\). As shown in Table 7, the correlation coefficient values of each kind of bulk tissue samples for the parameters group \((\theta_e, \phi_{qr})\) are all equal to 0.99, while those for \((\theta_e, \phi_{2434}), (\theta_e, \phi_{4243})\) range between 0.58 and 0.97. We can also see that the data points of all tissue samples are evenly distributed along a straight line in Figure 6e. Particularly, Table 7 and Figure 6f also confirm that the parameters \((\theta_e, \phi_{qr})\) show no statistically significant differences with \(p > 0.05\). Therefore, when measuring linear birefringence orientation of bulk tissue samples using backscattering polarimetry, both the parameters \(\phi_{qr}\) and \(\theta_e\) can provide quantitatively similar information, and \(\phi_{qr}\) can be calculated faster.

| Type   | Tissues      | Linear Birefringence Fast Axis Orientation | \(R^2\) | \(p\)       |
|--------|--------------|------------------------------------------|--------|------------|
|        |              | \(\theta_e-\phi_{2434}\) \(\theta_e-\phi_{4243}\) \(\theta_e-\phi_{qr}\) \(\theta_e-\phi_{2434}\) \(\theta_e-\phi_{4243}\) \(\theta_e-\phi_{qr}\) |
| Backward| breast       | 0.97 0.97 0.99 | >0.05 | >0.05 | >0.05 |
|        | rat-skin     | 0.58 0.82 0.99 | >0.05 | >0.05 | >0.05 |
|        | porcine-intestine | 0.75 0.97 0.99 | >0.05 | >0.05 | >0.05 |
|        | porcine-stomach | 0.96 0.97 0.99 | >0.05 | >0.05 | >0.05 |
|        | porcine-liver | 0.91 0.96 0.99 | >0.05 | >0.05 | >0.05 |
|        | Total        | 0.87 0.95 0.99 | >0.05 | >0.05 | >0.05 |

In summary, the results shown in Sections 3.5 and 3.6 demonstrate that among the available MMT linear birefringence orientation parameters, \(\phi_{qr}\) shows the best accordance with the MMPD parameter \(\theta_e\). The parameters \(\phi_{2434}\) and \(\phi_{4243}\) only contain partial linear birefringence orientation information of tissue samples with layered fibrous structures. Thus, when taking the elements \(m_{24}, m_{34}, m_{42},\) and \(m_{43}\) from both the fourth column and fourth row of the Mueller matrix into account, the modified parameter \(\phi_{qr}\) can well present the linear birefringence orientation information more thoroughly.

### 3.7. Quantitative Comparison of Depolarization MMPD and MMT Parameters for Transmission Imaging of Thin Tissue Samples

In previous studies, both the MMT parameter \(1-b\) shown in Equation (14) and MMPD parameter \(\Delta\) shown in Equation (4) have been used to represent the depolarization property of tissue samples \([5]\). However, there would be a deviation between these two parameters when the medium contains a circular depolarization property encoded in the element \(m_{44}\), which the parameter \(1-b\) does not take into account. To more accurately represent the overall depolarization of tissues, we proposed a modified parameter \(1-b_m\) as Equation (18) shows. The comparison results of the MMT and MMPD depolarization parameters for 7 kinds of thin tissue samples are shown in Table 8 and Figure 7.
Figure 6. Comparison of linear birefringence fast axis orientation MMT and MMPD parameters of bulk tissue samples. (a) Variance of linear birefringence orientation MMPD parameter $\theta_e$ plotted against its counterpart MMT parameter $\phi_{2434}$ for correlation analysis with all five kinds of bulk tissue samples. (b) Statistical analysis bar chart of linear birefringence orientation MMPD parameter $\theta_e$ and MMT parameter $\phi_{2434}$ for each kind of bulk tissue samples. (c) Variance of linear birefringence orientation MMPD parameters $\theta_e$ plotted against its counterpart MMT parameter $\phi_{4243}$ for correlation analysis with all five kinds of bulk tissue samples. (d) Statistical analysis bar chart of linear birefringence orientation MMPD parameter $\theta_e$ and MMT parameter $\phi_{4243}$ for each kind of bulk tissue samples. (e) Variance of linear birefringence orientation MMPD parameter $\theta_e$ plotted against its counterpart MMT parameter $\phi_{qr}$ for correlation analysis with all five kinds of bulk tissue samples. (f) Statistical analysis bar chart of linear birefringence orientation MMPD parameter $\theta_e$ and MMT parameter $\phi_{qr}$ for each kind of bulk tissue sample.
Table 8. Correlation and statistical analysis comparison results between forward measured MMT and MMPD depolarization parameters for each kind of thin tissue samples.

| Type  | Tissues      | Depolarization | \( R^2 \) | \( \Delta - 1 - b \) | \( \Delta - 1 - b_m \) | \( p \) |
|-------|--------------|----------------|-----------|----------------|----------------|--------|
| Forward | colorectal   | 0.53           | 0.50      | <0.01          | <0.01         |        |
|        | ITB          | 0.23           | 0.26      | <0.01          | <0.01         |        |
|        | liver        | 0.73           | 0.70      | <0.01          | <0.01         |        |
|        | CD           | 0.09           | 0.05      | >0.05          | <0.01         |        |
|        | bladder      | 0.43           | 0.44      | <0.01          | <0.05         |        |
|        | breast       | 0.49           | 0.70      | >0.05          | <0.01         |        |
|        | adenoma      | 0.17           | 0.11      | <0.01          | <0.05         |        |
|        | Total        | 0.49           | 0.49      | <0.01          | <0.01         |        |

Figure 7. Comparison of MMT and MMPD depolarization parameters of thin tissue samples. (a) Depolarization MMPD parameter \( \Delta \) plotted against its counterpart MMT parameter \( 1 - b \) for correlation analysis with all seven kinds of thin tissue samples. (b) Statistical analysis bar chart of depolarization MMPD parameter \( \Delta \) and MMT parameter \( 1 - b \) for each kind of thin tissue samples. (c) Depolarization MMPD parameter \( \Delta \) plotted against its counterpart MMT parameter \( 1 - b_m \) for correlation analysis with all seven kinds of thin tissue samples. (d) Statistical analysis bar chart of depolarization MMPD parameter \( \Delta \) and MMT parameter \( 1 - b_m \) for each kind of thin tissue sample.
For the correlation analysis, the values of parameters groups of \((\Delta, 1 - b)\) and \((\Delta, 1 - b_m)\) are close and much smaller than 1 for most of the tissue samples indicated in Table 8, which means both groups of parameters hardly exist in a linear relationship. More intuitive comparisons illustrated in Figure 7a,c confirm that all the sample data points distribute randomly. For the statistical analysis, most of the samples show statistically significant differences with \(p < 0.01\) for the parameters groups \((\Delta, 1 - b)\) and \((\Delta, 1 - b_m)\) shown in Table 8. From Figure 7b,d, we can find that the MMPD and MMT depolarization parameters values of thin tissue samples are smaller than 0.02. This is the reason why the parameters \((\Delta, 1 - b)\) and \((\Delta, 1 - b_m)\) show no significant difference in correlation analysis for the thin tissue samples with a limited thickness.

### 3.8. Quantitative Comparison of Linear Birefringence Orientation Related MMPD and MMT Parameters for Backscattering Imaging of Bulk Tissue Samples

For the bulk tissue samples, both the MMT parameters \(1 - b\) and \(1 - b_m\) show a good accordance with the MMPD parameter \(\Delta\). As shown in Table 9, the correlation coefficient values of all kinds of bulk tissue samples for parameters group \((\Delta, 1 - b_m)\) are equal to 0.99, which differ from that for the group \((\Delta, 1 - b)\) ranging between 0.96 and 0.98. Moreover, we can also see that the data points of the tissue samples evenly distributed along a straight line in Figure 8a,c. Besides, Figure 8b,d indicate that the MMPD depolarization parameter \(\Delta\) is approximately equal to the MMT parameters. Specifically, the parameters group \((\Delta, 1 - b_m)\) shows no statistically significant difference with \(p > 0.05\). Therefore, it is worth noting that when bulk tissue samples are measured using backscattering equipment, the parameter \(1 - b_m\) can be used to extract the depolarization information quickly and accurately.

![Figure 8](image_url)

**Figure 8.** Comparison of MMT and MMPD depolarization parameters of bulk tissue samples. (a) Depolarization MMPD parameter \(\Delta\) plotted against its counterpart MMT parameter \(1 - b\) for correlation analysis with all five kinds of bulk tissue samples. (b) Statistical analysis bar chart of depolarization MMPD parameter \(\Delta\) and MMT parameter \(1 - b\) for each kind of bulk tissue samples. (c) Depolarization MMPD parameter \(\Delta\) plotted against its counterpart MMT parameter \(1 - b_m\) for correlation analysis with all five kinds of bulk tissue samples. (d) Statistical analysis bar chart of depolarization MMPD parameter \(\Delta\) and MMT parameter \(1 - b_m\) for each kind of bulk tissue sample.
Table 9. Correlation and statistical analysis comparison results between backward measured MMT and MMPD depolarization parameters for each kind of bulk tissue samples.

| Type   | Tissues          | Depolarization |  |  |
|--------|------------------|----------------|---|---|
|        |                  | $R^2$           | $\Delta - 1 - b$ | $\Delta - 1 - b_m$ | $\Delta - 1 - b$ | $\Delta - 1 - b_m$ |
| Backward | breast            | 0.96           | 0.99 | >0.05 | >0.05 |
|         | rat-skin          | 0.98           | 0.99 | >0.05 | >0.05 |
|         | porcine-intestine | 0.98           | 0.99 | >0.05 | >0.05 |
|         | porcine-stomach   | 0.98           | 0.99 | >0.05 | >0.05 |
|         | porcine-liver     | 0.97           | 0.99 | <0.01 | >0.05 |
|         | Total             | 0.98           | 0.99 | >0.05 | >0.05 |

4. Conclusions

In this study we compared the parameters derived from the MMT and MMPD methods thoroughly by measuring two types of tissue samples: seven kinds of thin tissue slices and five kinds of bulk tissue samples, with a transmission Mueller matrix microscope and backscattering Mueller matrix measurement setup. After grouping the parameters with the same polarization properties, namely diattenuation, linear retardance, linear birefringence fast axis orientation and depolarization, we performed both correlation and statistical analysis. Our preliminary experimental results showed that: (1) for obtaining the diattenuation property of complex tissues, we can calculate it through the modified MMT parameter $t_{12,3,14}$, with a fast speed and a high correlation with the MMPD parameter $D$; (2) for revealing the linear retardance value related to the density of birefringent structures like layered fibers of thin tissues, the modified MMT parameter $t_{qr}$ can reach a high linear correlation with the MMPD parameter $\delta$. While for the bulk tissue samples, it is more reasonable to use the MMPD parameter $\delta$ to extract the linear retardance value accurately; (3) for characterizing the orientation distribution of the birefringent structures, the modified MMT parameter $\phi_{qr}$ and MMPD parameter $\theta_e$ can both reach similar accurate results for thin and thick tissues; (4) the modified MMT parameter $1 - b_m$ shows a high degree of consistency with the MMPD parameter $\Delta$ for the bulk tissue samples with strong depolarization, so it can be used to quickly evaluate the tissue depolarization property. In summary, based on the analysis and discussion regarding the applicability of the MMPD and MMT parameters, this study gave suggestions for the appropriate selection of parameters in Mueller matrix imaging for different types of tissue samples, which can be useful for biomedical and clinical polarimetry.

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