Effect of rear walls on the rocking response of rock blocks under seismic excitations

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Abstract. The rocking response of rigid free standing bodies subjected to seismic excitation has been studied by many researchers interested in different slender elements such as ancient stone columns, tombstones, rigid building structures. The extension of this model to rock mechanics has been proposed by a few authors. The rocking response of rectangular free standing bodies subjected to horizontal accelerations of natural recorded motions showed that the pseudo-static approach, based on Peak Ground Acceleration (PGA), permits only the determination of the uplift conditions and the beginning of rocking. It does not permit to evaluate the overturning of the blocks. The combined effect of vertical and horizontal seismic motions is negligible and, in some cases, beneficial. This paper presents a new mechanical model, called “one-sided rocking”, that takes into account the presence of a rear rigid wall, that is a typical scenario for the rock blocks completely detached from the cliff but close to it. The dynamic response of a great number of rectangular rigid blocks, subjected to 62 recorded earthquake motions on rock soil (from US, Europe and Asia), has been analyzed considering only the horizontal acceleration. The results show that the presence of the wall is detrimental for the rocking stability. However, there is still a safety reserve more significant for large blocks and rich frequency content time histories. This reserve could be taken into account in simplified (pseudo-static) analyses through reductive coefficient of PGA.

1. Introduction

During an earthquake, different slender elements such as ancient stone columns, tombstones, furniture, reservoirs, electrical equipment may slide, rock or slide-rock and overturn. Rocking motion of a rigid block on a rigid plane subjected to dynamic actions, presented in this work, has been focused on toppling.

Starting from the pioneering work of [1] a number of contribution may be found in literature. The first studies approached the problem by analyzing the dynamic behaviour of the slender elements defining the equations and the parameters affecting the motion by means of deterministic approaches, based on the integration of motion ([2], [3], [4], [5], and [6]) or probabilistic approaches, based on fragility curves ([7], [8], [9]). Other Authors proposed numerical and approximate closed form solutions limited to rectangular and sinusoidal pulses of half-cycle duration ([10], [11], [12], [6], [13]) and also analytical formulations based on natural recorded motions ([3], [5], [14], [15], [16], [17], [18], [19]). Just a few Authors extended this model to rock mechanics ([20], [21]).

Maiorano et al. [22] analysed the rocking response of rigid rectangular bodies free to rotate in both sides (two-sided rocking) and subjected both to simple pulses, such as sinusoidal pulses of one-cycle duration, and to natural earthquakes by considering only the horizontal accelerations. The analyses were carried out by means of a mechanical model developed in SIMULINK extension in Matlab [26]. The results showed that the dynamic response of a rigid body on a stiff horizontal plane is very complex and it is not directly related to the Peak Ground Acceleration (PGA), but it is strongly affected from both...
size and slenderness parameters of blocks. The authors proposed a seismic reduction coefficient $\beta$ for pseudo-static analysis, based on the results of dynamic analysis carried out for different rectangular blocks subjected to natural earthquakes. Maiorano et al. [22] defined the coefficient $\beta$ as the dimensionless block base ratio $b/b_{ps}$, where $b$ is the minimum stable base obtained from dynamic analysis and $b_{ps}$ is the minimum stable base obtained from pseudo-static analysis.

In the present paper the model developed by Maiorano et al. [22] is extended to consider the effects of the presence of a rear wall, that is the typical scenario for the rock blocks completely detached from the cliff but close to it.

2. Problem definition

2.1. Rocking motion of rigid block against the wall

Unlike free-standing blocks, which can oscillate on both sides at their base (two-sided rocking), there are some situations where the rocking occurs only on one side (one-sided rocking). This scenario is very common for the rock blocks adjacent to the cliff. Very few studies have addressed on one-sided rocking problem unlike to two-sided rocking.

A similar problem was studied by Hogan [23]: he investigated the rocking response of household objects, such as slender furniture placed against walls, during earthquakes. Winkler et al. [24] studied one-sided rocking by means a series of shaking table tests and numerical analysis with Distinct Element Method observing that for lower frequencies, one-sided rocking is more stable than two-sided, while for higher frequencies is the opposite. Sigurdsson et al. [25] resumed this study for household furniture, like the IKEA BRIMNESS bookshelf. They showed that the one-sided rocking is less safe than two-sided rocking. Other works on one-sided rocking are related to masonry structures to study the out-of-plane behaviour of masonry walls (Giresini et al. [9]).

The configuration of the problem is illustrated in figure 1. The rigid block on a stiff horizontal plane has a rectangular shape with uniformly distributed mass $m$ and dimension $2b$ times $2h$. The block is characterised by a dimensionless parameter $a=b/h$, or the equivalent critical angle of rotation $\alpha=\tan^{-1}(b/h)$, and a size parameter $R=(h^2+b^2)^{0.5}$ that is the radial distance from the centre of gravity. Depending on the characteristic of the horizontal acceleration of the stiff ground and the properties of the frictional interface, the block may rest, slide, rock, or slide-rock.

Assuming that the coefficient of friction is large enough to prevent sliding, the rigid block, subjected to a horizontal acceleration $\ddot{u}_g(t)$ of the stiff ground, rests until the overturning moment is less than the restoring moment and begin to rotate around points $O$ or $O'$ as soon as:

$$\left|\frac{\ddot{u}_g}{g}\right| \geq a$$

With reference to figure 1, the equations of motion for the rotation $\theta$ of a rigid block with rotational inertia $I_0$ subjected to a horizontal acceleration $\ddot{u}_g(t)$ when rocking respectively around $O$ and $O'$ are:

$$I_0\ddot{\theta}(t) + mgR\sin(-\alpha - \theta(t)) = -m\ddot{u}_g(t)R\cos(-\alpha - \theta(t)) \quad \theta(t) < 0$$

$$I_0\ddot{\theta}(t) + mgR\sin(\alpha - \theta(t)) = -m\ddot{u}_g(t)R\cos(\alpha - \theta(t)) \quad \theta(t) > 0$$
For a uniform rectangular block, the polar moment of inertia \( I_0 \) about corner point is:

\[
I_0 = \frac{4}{3} mR^2
\]  

(4)

Introducing the sign function, equations (2) and (3) can be expressed in the compact form:

\[
\dot{\theta}(t) = -p^2 \left\{ \sin \left[ \alpha \text{sgn}(\theta(t)) - \dot{\theta}(t) \right] + \frac{\ddot{u}_s(t)}{g} \cos \left[ \alpha \text{sgn}(\theta(t)) - \dot{\theta}(t) \right] \right\}
\]  

(5)

where:

\[
p = \sqrt{\frac{3g}{4R}}
\]  

(6)

is a frequency parameter (rad/s) of the block and is an expression of its size. During rocking motion, when the block hits on the ground, it loses a part of its kinetic energy. Its angular velocity after each impact (at time \( t_0^+ \)) is a fraction of that just prior to impact (at time \( t_0^- \)):

\[
\dot{\theta}^2(t_0^+) = r \cdot \dot{\theta}^2(t_0^-)
\]  

(7)

Conservation of angular momentum before and after the impact gives the maximum coefficient of restitution [1]:

\[
r = \left(1 - \frac{3}{2} \sin^2 \alpha \right)^2
\]  

(8)

When the block hits the rear wall, an elastic impact is assumed without the change of the centre of rotation. This implies a simple reverse of the velocity.

2.2. Numerical model

Maiorano et al. [22] developed a two dimensional mechanical model (RHA model: Rocking with Horizontal Accelerations) in the SIMULINK extension of Matlab extension [26] to analyze two-sided rocking. The non-linear equation (5) is solved with a numerical integration by means of standard Ordinary Differential Equations (ODE3) solver.
In the present paper the mechanical model was extended to include the presence of the adjacent wall with a limit rotation (with reference to figure 1, is a counterclockwise or negative rotation), related to the case study.

3. One-sided rocking response to a one-sine pulse

Analyses of one-sided rocking subjected to dynamic actions have been carried out varying the amplitude and the frequency of a simple one-sine pulse for a rigid rectangular block, of dimensions \(b=0.5\) m and \(h=2.5\) m. The excitation, modelled as a sinewave of one-cycle duration, is similar to a natural record because it starts from zero amplitude. Unlike two-sided rocking, in this case the problem is asymmetric and therefore all toppling simulations have been doubled by changing the sign of the initial acceleration in order to take into account the different position of the wall, in both sides.

With reference to figure 1, if the motion starts from positive accelerations, the block initially rotate with negative rotations and it always hits against the wall. If the motion starts from negative accelerations, the block rotate with positive rotations and four scenarios are possible:

- the block overturns without impact;
- the block has counterclockwise rotations without impact against the wall, the velocity decreases with time after each impact against the ground and it overtops;
- the block has counterclockwise rotations, impacts the wall and overturns;
- the block does not overturn.

Considering two distances, \(d\), between the block and the wall of 1 and 10 cm, a wide range of accelerations between 0 and 5 g and a range of frequencies between 0 and 2.5 Hz, the overturning acceleration spectra for the slender block investigated are illustrated in figures 2(a), 2(b), 3(a) and 3(b). The spectra in figures 2(a) and 2(b) report the toppling potential for initial positive accelerations. It is possible to define two different areas: “safe” region (blue area), where the rocking rotations are damped until expiring and “overturning with impact” region (yellow area). On the other hand, the spectra in figures 3(a) and 3(b) for initial negative accelerations show another region between the previous: the “overturning with no impact” region.

The results show that the overturning area increases when the block always hits against the wall (for positive accelerations) and in particular a distance of 10 cm between the block and the wall is detrimental for the block’s stability. It means the presence of wall makes less stable the blocks.

![Figure 2(a)](image1.png) Overturning accelerations spectrum of slender block \((p=1.70\text{ rad/s}, \alpha=0.197\text{ rad}, r=0.89, b=0.5\text{ m}, h=2.5\text{ m}, d=1\text{ cm})\) under one-sine pulse with positive acceleration

![Figure 2(b)](image2.png) Overturning accelerations spectrum of slender block \((p=1.70\text{ rad/s}, \alpha=0.197\text{ rad}, r=0.89, b=0.5\text{ m}, h=2.5\text{ m}, d=10\text{ cm})\) under one-sine pulse with positive acceleration
Figure 3(a). Overturning accelerations spectrum of slender block \((p=1.70 \text{ rad/s}, \alpha=0.197 \text{ rad}, r=0.89, b=0.5 \text{ m}, h=2.5 \text{ m}, d=1 \text{ cm})\) under one-sine pulse with negative acceleration

Figure 3(b). Overturning accelerations spectrum of slender block \((p=1.70 \text{ rad/s}, \alpha=0.197 \text{ rad}, r=0.89, b=0.5 \text{ m}, h=2.5 \text{ m}, d=10 \text{ cm})\) under one-sine pulse with negative acceleration

4. One-sided rocking response to earthquake ground motions

This section provides the numerical results of the one-sided rocking due to seismic excitation. The seismic database used in the analysis has been taken from Maiorano et al. ([22], [27]), considering the horizontal component of acceleration. These records are representative of significant events and cover a wide range of strong motion parameters [28] and are collected from International, European and Italian earthquakes.

For each earthquake the rocking response has been computed for 400 rectangular blocks, meaning for 20 values of slenderness parameter \(a\), ranging between 0.1 and 0.5, and 20 values of \(b\), ranging from 0.1 m to 1 m. The blocks investigated were placed at a distance \(d\) of 1 and 10 cm from the wall.

The stability analyses have been carried out for all the investigated blocks and the earthquakes of the seismic database of Maiorano et al. ([22], [27]). The results are reported on the \(\beta-T_p/T_m\) plane, where \(\beta\) is the reductive coefficient of the seismic actions, proposed by Maiorano et al. [22]:

\[
\beta = \frac{b}{b_{\mu}} = \frac{PGA}{b \cdot h}
\]

and \(T_p/T_m\) a dimensionless parameter, representative of frequency characteristics of the ground motion and block dimensions. \(T_p\) is the inverse of the frequency parameter \(p\):

\[
T_p = \frac{2\pi}{p} = 2\pi \sqrt{\frac{4R}{3g}}
\]

while \(T_m\) is the mean period proposed by [29] of the horizontal component of the acceleration time history.

The upper bound curve found by Maiorano et al. [22] for the two-sided rocking and given by the equation

\[
\beta = 1.35 \cdot \exp[-0.12 \cdot (T_p/T_m)]
\]

has been superposed on the rocking spectrum reported in figures 4 and 5.
The plot in figure 4 shows the results of the dynamic analyses for the blocks placed at a distance $d$ of 1 cm from the wall. The rocking spectrum highlights a reduction of the safety reserve compared to two-sided rocking, as shown by the points above the curve: this means that the presence of the wall is detrimental for the rigid block stability. In particular, we observe an increase of the reductive coefficient $\beta$ for the larger blocks ($T_p/T_m > 10$).

![Figure 4. Rocking spectrum of rigid blocks placed at a distance $d$ of 1 cm from the wall](image)

This effect becomes more important for the rigid blocks placed at a distance $d$ of 10 cm from the cliff. It can be observed an increase of the point above the upper bound curve that corresponds to a reduction of the safety reserve (figure 5). These results show a good agreement with those carried out by simple excitations. In particular, short distances between the rigid block and the wall are more safe than the greater ones. For higher distances the rocking tends to the two-sided case.

![Figure 5. Rocking spectrum of rigid blocks placed at a distance $d$ of 10 cm from the wall](image)

5. Concluding remarks
The paper investigates the rocking response of rigid rectangular blocks placed close to a rigid wall subjected to simple excitations and to natural earthquakes.

When the blocks are subjected to a simple sinewave of one-cycle duration, the distance between the block and the wall has a strong effect on the toppling potential. The stability of the blocks decreases...
when the distance increases from 1 to 10 cm. For larger distances, not illustrated in the present paper, the rocking tends to the two-sided case.

In order to compare the two-sided rocking with one-sided rocking, the upper bound curve, found by Maiorano et al. [22], has been superposed on the dynamic results of the analysis with natural records considering the horizontal component of the acceleration. The blocks investigated are placed at a distance \( d \) of 1 and 10 cm from the cliff and when they are subjected to natural earthquakes, the presence of the wall is detrimental for the rocking stability, in particular for larger blocks.

The negative effect of the wall is greater for the blocks placed at a distance \( d \) of 10 cm from the wall, as already observed in the analyses with simple sine excitations.

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