Properties of drip-line nuclei with an m-scheme cluster-orbital shell model approach

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Abstract. In the drip-line region of oxygen isotopes, an abrupt increase of the r.m.s.radius of 23O is observed from the analysis of the reaction cross section. We develop an m-scheme approach of COSM and perform calculations for oxygen isotopes. We examine the interaction dependence to the calculated energies and r.m.s.radii. Further, the relation between the density and nucleon-nucleon interaction is discussed.

1. Introduction

Recent remarkable progress on both experimental techniques and theoretical approaches enables us to widen the area of the nuclear physics. Nuclei near the neutron and proton drip-lines are investigated not only in the light mass region but also in the middle mass region [1]. For example, an abrupt increase of the r.m.s.radius of 23O has been observed as one of the typical phenomena in the drip-line region [1]. However, in the theoretical study, however, it is not so straightforward to reproduce such the abrupt increase [2, 3, 4].

The cluster-orbital shell model (COSM) approach has been developed to study light neutron rich nuclei [5]. By the COSM approach with the Gaussian basis set, many-body resonant states can be calculated exactly by making use of the complex scaling method [6]. As shown in Ref. [6], the COSM approach with the Gaussian basis set can describe the halo structure, which is a typical property of weakly bound systems. We developed an extension of the COSM approach in order to treat dynamics of the core nucleus [7] and showed the importance of the modification of the radius of the core. We also discussed the role of unbound states in the COSM approach [8]. In Ref. [8], we investigated the contribution of continua and resonant poles in the helium isotopes and compared our results with those obtained by the Gamow shell model approaches[9, 10].

Recently, we proposed an m-scheme approach of COSM [11]. With the help of the m-scheme type calculation, the number of valence nucleons can be extended to the drip-line region. In this paper, we show the m-scheme approach of COSM and calculate the binding energies and r.m.s.radii of the oxygen isotopes.

In Sect. 2, we briefly show the formalism of our m-scheme COSM approach. In Sect. 3, we show the calculated results. Summary and discussion are given in Sect. 4.
2. Formalism and interaction

2.1. M-scheme cluster-orbital shell model

We briefly explain the formalism of the cluster-orbital shell model (COSM) [5]. In COSM, coordinates of valence nucleons are spanned from the center of mass of the core nucleus. The kinetic energy of the center of mass motion \( T_C \) is subtracted from the total \( A \)-body Hamiltonian as follows:

\[
\hat{H} = \sum_{i=1}^{A} \hat{h}_i - \hat{T}_C + \sum_{i<j}^{A} \hat{v}_{ij} = H_C + \sum_{i \in V} (\hat{t}_i' + \hat{v}_i') + \sum_{i<j \in V} (\hat{T}_{ij} + \hat{v}_{ij}) .
\] (1)

Here, \( \hat{t}_i' \) is the kinetic energy operator between the core and an \( i \)th valence nucleon. \( \hat{T}_{ij} = (\hbar^2/m) \nabla_i \cdot \nabla_j \) comes from the subtraction of the center of mass motion. The potential between the core and the \( i \)th valence nucleon is defined by taking the sum for nucleons in the core part as \( \hat{V}_i' = \sum_{k \in C} \hat{v}_{ik} \).

The core and valence-nucleon wave functions, \( |\Phi_C\rangle \) and \( |\Phi_V\rangle \), are anti-symmetrized individually and the total wave function is anti-symmetrized as \( |\Psi\rangle = A' \{ |\Phi_C\rangle |\Phi_V\rangle \} \). For the valence nucleons, the radial function is defined by the products of the Gaussians as follows:

\[
F(r_1, \ldots, r_N) \equiv g_1(r_1) \cdots g_N(r_N) ,
\] (2)

where \( g_i(r_i) \) is the Gaussian for the \( i \)th valence nucleon, \( g_i(r) = N_i \exp(-r^2/(2a_i)) \).

We employ the basis set so that the \( z \)-components of the total angular momentum and total isospin are fixed as \( |MT_z\rangle = \{ \phi_{a_1} \phi_{a_2} \cdots \phi_{a_N} \}_{MMT_T} \). Here, \( \{ \cdots \}_{MT_z} \) indicates the \( z \)-component of the total spin and isospin; \( M \) and \( T_z \) are fixed. Hence, the wave function for the valence part is determined by using the basis set as follows:

\[
\Phi_V = \sum_m c^{(m)} A' \{ F^{(m)}(r_1, \ldots, r_N) \cdot |MMT_z^{(m)}\rangle \} .
\] (3)

If the basis size is sufficiently large, eigen vectors have a good quantum number for the total spin and isospin, and the coefficients \( c^{(m)} \) play the role of the Clebsch-Gordan coefficients.

2.2. Model and Interaction

We construct the interaction between the core and an \( i \)th valence nucleon by taking into account the structure of the core nucleus and the exchange effect as follows:

\[
\hat{V}_i'|\Phi_V\rangle \equiv \sum_{k \in C} \langle \Phi_C | \hat{v}_{ik} | \Phi_V \rangle A' \left\{ |\Phi_C\rangle |\Phi_V\rangle \right\} \simeq \hat{V}_i'^{d} + \hat{V}_i'^{ex} .
\] (4)

The direct part from the folding procedure gives the \( \hat{V}_i'^{d} \) term in Eq. (4). From the anti-symmetrization between a nucleon in the core and valence one, the exchange term \( \hat{V}_i'^{ex} \) appears in the interaction. We introduce an approximation, which is proposed by Kaneko et al. [14] that a recoil effect and other exchange kernels except for the knock-on exchange term are omitted [14].

Due to the Pauli principle between a nucleon in the core and a valence nucleon, it is necessary to eliminate spurious states in the model space of the valence-nucleon system. We employ the
orthogonality condition model (OCM) [15] in order to treat the Pauli principle and eliminate the spurious states.

To summarize, the Hamiltonian \( \hat{h}_i \) with an effective LS-force, \( \hat{V}_{ls}^i \), for the \( i \)th valence nucleon becomes as follows:

\[
\hat{h}_i = \hat{t}^i + \hat{V}^d_i + \hat{V}^{ex}_i + \hat{V}_{ls}^i.
\] (5)

The parameters in the potentials are adjusted to three positive-parity states, \( 5/2^+, 1/2^+ \) and \( 3/2^+ \), by regarding them as the single-particle states of the \( ^{16}\text{O}+n \) system, \( 0d_{5/2}, 1s_{1/2} \) and \( 0d_{3/2} \), respectively.

For the nucleon-nucleon interaction, we use the Volkov No.2 [18] (VN2) and Minnesota [19] (MN) interactions. Furthermore, we modify the potential parameter in Volkov No.2 potential (VN2'), so as to reproduce the drip-line of the oxygen isotopes at \( ^{24}\text{O} \). The potential parameter for VN2' gives a reasonable unbound nature for \( ^{25}\text{O} \) and \( ^{26}\text{O} \).

3. Results

We show the results of the energies and r.m.s. radii of the oxygen isotopes from \( ^{17}\text{O} \) to \( ^{26}\text{O} \) calculated by using the potentials, VN2, MN and VN2'. In the calculations, we take the maximum angular momentum as \( L_{\text{max}} = 2 \) for each partial wave.

Calculated energies of oxygen isotopes are shown in Table 1. The energies are measured from the \( ^{16}\text{O}+Xn \) threshold. Note that the parameters in VN2 are adjusted to the \( ^{18}\text{O} \) ground state in the \( L_{\text{max}} = 5 \) case. Therefore, the energy of \( ^{18}\text{O} \) for VN2 in this calculation becomes slightly smaller than the experimental value. The energies for VN2 and MN show an over-binding nature as the number of valence neutron increases, and the neutron drip-line of the oxygen isotopes is reproduced in the VN2' case.

Figure 1 shows the calculated and experimental values of the r.m.s. radii for the oxygen isotopes. For VN2 and MN, the r.m.s. radius is not enhanced at \( ^{23}\text{O} \). A small change of the radius at \( ^{23}\text{O} \) is due to the presence of the s-wave component in the wave function of the valence nucleons. For the VN2' case, the calculated r.m.s. radii are the same with the results of the VN2.
with VN2' is smaller than that obtained with VN2, the r.m.s. radius of interaction between the core and valence nucleons. Therefore, even if the binding energy of interaction for the valence part, the main contribution of the binding energy comes from the d-wave component in the shell model picture. As shown in Table 2, the dominance of the s-wave component increases in the VN2' case. Due to the weakness of the nucleon-nucleon interaction for the valence part, the main contribution of the binding energy comes from the interaction between the core and valence nucleons. Therefore, even if the binding energy of 22O with VN2' is smaller than that obtained with VN2, the r.m.s. radius of 22O is not so large as the experimentally observed value. On the other hand, for 23O and 24O, the valence nucleons occupy the 1s1/2-orbit in the shell model picture. As shown in Table 2, the dominance of the s-wave states of the VN2' case is much larger than that of the VN2 case. This makes the enhancement of the radius at 23O. However, the effect is still too small to reproduce the experimental value [1].

Next, we proceed to investigate the components of partial waves in oxygen isotopes. We compare the components of the VN2, MN and VN2' cases. Results are shown in Table 2. For 22O, in which the lowest configuration for neutron is (0d5/2)6 of the shell model picture, the d-wave component increases in the VN2' case. Due to the weakness of the nucleon-nucleon interaction for the valence part, the main contribution of the binding energy comes from the interaction between the core and valence nucleons. Therefore, even if the binding energy of 22O with VN2' is smaller than that obtained with VN2, the r.m.s. radius of 22O is not so large as the experimentally observed value. On the other hand, for 23O and 24O, the valence nucleons occupy the 1s1/2-orbit in the shell model picture. As shown in Table 2, the dominance of the s-wave states of the VN2' case is much larger than that of the VN2 case. This makes the enhancement of the radius at 23O. However, the effect is still too small to reproduce the experimental value [1].

Even under the situation that the small binding energies of 23O and 24O and the drip-line of the oxygen isotopes are reproduced by using VN2', the abrupt increase of r.m.s. radii of 23O and 24O are still smaller than that of the experiments as shown in Fig. 1. This result suggests that other mechanisms are necessary to be introduced in order to make a consistent understanding.

Table 1. Calculated energy of oxygen isotopes measured from the 16O+Xn threshold. All units except for the mass number A are in MeV.

| A   | VN2 | MN | VM2' | Expr. |
|-----|-----|----|------|-------|
| 18  | −11.68 | −10.31 | −9.98 | −12.19 |
| 19  | −16.24 | −15.16 | −14.19 | −16.14 |
| 20  | −23.33 | −21.41 | −19.50 | −23.75 |
| 21  | −28.35 | −26.94 | −23.68 | −27.56 |
| 22  | −35.72 | −33.32 | −28.68 | −34.41 |
| 23  | −40.80 | −39.06 | −32.56 | −37.15 |
| 24  | −47.98 | −45.65 | −36.67 | −40.85 |
| 25  | −49.61 | −47.14 | −35.75 | —     |
| 26  | −53.39 | −49.98 | −34.93 | —     |

and MN cases until 22O, and an enhancement arises at 23O. We consider that one of the reason of the enhancement is the weak attraction of the nucleon-nucleon interaction, which reproduce the drip-line at 24O. Another reason is the presence of the large s-wave component in 23O and 24O.

Table 2. Calculated components of partial waves in the oxygen isotopes for VN2, MN and VN2'.

| Components                      | VN2 (%) | VN2' (%) |
|--------------------------------|---------|----------|
| 22O  (d5/2)^6                  | 78.7    | 95.0     |
| (s1/2)^2(d5/2)^4               | 15.9    | 3.1      |
| (d5/2)^4(d3/2)^2               | 4.2     | 1.7      |
| 23O  (s1/2)(d5/2)^6             | 91.2    | 97.0     |
| (s1/2)(d5/2)^4(d3/2)^2         | 2.1     | 0.1      |
| (s1/2)(d5/2)^5(d3/2)           | 5.6     | 2.1      |
| 24O  (s1/2)^2(d5/2)^6           | 94.6    | 98.5     |
| (s1/2)^2(d5/2)^4(d3/2)^2       | 4.3     | 1.2      |
| (d5/2)^6(d3/2)^2               | 0.6     | 0.1      |
for the binding energies and r.m.s. radii of the oxygen isotopes in the drip-line region.

4. Summary and Discussions

We showed an m-scheme approach of the cluster-orbital shell model (COSM). In our COSM formalism, the interaction between the core nucleus and a valence nucleon is constructed based on a semi-microscopic way. The parameters in the potentials are determined so as to reproduce the $^{17}$O and $^{18}$O for the VN2 case. The VN2 and MN potentials cannot describe the neutron drip-line correctly, since the attraction of the interaction is too strong in the drip-line region. Therefore, we modify the exchange parameters in Volkov potential, VN2', in order to reproduce the drip-line.

Even though we use a potential VN2', in which the attraction is weaker than VN2, the r.m.s.radii of $^{23}$O and $^{24}$O do not become large values as the experiment shows. We consider the abrupt increase of the r.m.s.radius of $^{23}$O can not be explained only by employing a simple picture such that one or two neutrons are loosely bound around the core such as $^{11}$Be, $^{6}$He, $^{11}$Li.

Therefore, in order to reproduce the drip-line of the oxygen isotopes at $^{24}$O and the abrupt increase of the radius at $^{23}$O simultaneously, not only the loosely bound picture, but also other mechanisms, such as a core-excitation or modification due to the presence of many valence neutrons are necessary to be introduced. Further, a sophisticated potential which is based on the realistic nucleon-nucleon force, and a three-body interaction are also important to understand the physics of the oxygen isotopes near the drip-line.

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