Best Image with Elliptical Synthetic Obscured Apertures

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Abstract:

It is well established that synthetic apertures of various shapes are superior to a single aperture in improving the overall performance of optical systems. In this research, a set of elliptical synthetic obstructed apertures was used in place of the ellipse synthetic obstructed apertures that was used previously and the calculation of the point spread function to assess the performance of the optical system and reduce the diffraction values from it previously, according to the results, an improvement was observed in the optical system with array of elliptical synthetic obstructed apertures.

Key words: - Point Spread Function, Elliptical Synthetic Apertures, Obscured Apertures.

Introduction:

Most optical systems used to compose images, such as the eye, camera, microscope, telescope, etc., use lenses and mirrors [1]. The lens is a tool for creating the image, and the single lens is used very little in the composition of the image as it suffers from various defects or aberrations [2], but optical devices use a group of lenses in the composition of the image, where these lenses are coaxial and have specific distances between them, and the pictures formed by the group of lenses are good and almost free of aberration [2].
Most of the initial checkups that are performed for optical systems are mainly based on optical interference and assessing the efficiency of the resulting wave front, and thus measuring the degree of aberration occurring in it. Some important functions such as diffusion (spread, linear, edge) functions [3].

One of the most important factors that affect the distribution of intensity at the image level is the shape of the optical system aperture as the aperture determines the amount of light that reaches the image plane in addition to that its size affects the depth of field as small holes give a depth of a longer field, also the size of the aperture determines the aberration image is distorted, and the larger apertures need an optical system with a larger diameter [4].

The image formed by a point source in a specific optical system is not just a point, the wave theory of light from the aperture, aberration, and dispersion makes the point body image extend to a diffuse spot or distribute a certain distribution such as Gauss distribution [5].

The spread function depends on diffraction that produces by the lens aperture, the amount of the aberrations and its type in lens or in the optical system. [6]

Because of the significance of the point spread function (PSF) in measuring and evaluating optical systems, it has become a dependent calculation for determining the efficiency of optical systems [7]. The basic unit of any image is the point spread function, which is the image of a point source of light [8].

**Materials and Methods:**

The point spread function is one of the allowed measures to know the efficiency of an optical device by setting the image efficiency of a point object [9]. It is defined as the distribution of intensity at the image plane of a raster object using different aperture shapes.

Was derived the equation for array of ellipse synthetic apertures and having the formula:
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = \left( \frac{R}{\sqrt{N}} \right)^2
\]

Where \( R=1 \), when \( R \) represented Radius for array of elliptical synthetic apertures

\[
\therefore x = \pm a\sqrt{\frac{1}{N} - a^2 y^2} \quad \text{and} \quad y = \pm \frac{1}{a\sqrt{N}}
\]

\[\text{……………….. (1)}\]

The equation for array of obscured elliptical synthetic apertures is given by the relation

\[
\frac{x'^2}{a'^2} + \frac{y'^2}{b'^2} = \left( \frac{\varepsilon R}{\sqrt{N}} \right)^2
\]

\[
\Rightarrow x' = \pm \sqrt{\frac{\varepsilon^2}{N} - a^2 y'^2} \quad \text{and} \quad y' = \pm \frac{\varepsilon}{a\sqrt{N}}
\]

\[\text{……………….. (2)}\]

The point spread feature amplitude for an array of obscured elliptical synthetic apertures can be determined using the relation [10].

\[
PSF = |F|^2 = |F_1 - F_2|^2 \quad \text{……………….. (3)}
\]

Where \( F_1 \) represented the point spread function of elliptical synthetic apertures and \( F_2 \) the point spread function of obscured having a elliptical shape.
We can express the complex amplitude in the point \((u', v')\) in the image plane by using Fourier transform to pupil function \([11,12,13,14,15]\).

\[
f(x', y') = \tau(x', y')e^{ikw(x', y')}
\] ........................(4)

\[
F_1(u', v') = n \int \int f(x, y)e^{i2\pi(u'x + v'y)} \, dx \, dy
\] ........................(5)

And for \(N\) from obscured ellipse apertures (Figure(1)), we get:

\[
\therefore x = x' + x_j \quad \text{and} \quad y = y' + y_j \quad ..............(6)
\]

By substituting the equation (4) by equation (5), we obtain:
The complex amplitude for an array of ellipse synthetic apertures is represented by equation (9.).

\[ F_1(u', v') = n \int \int \tau(x, y) e^{i2\pi(u'x + v'y)} \sum_{j=1}^{N} e^{i2\pi(u'x_j + v'y_j)} dx dy \]

\[ \sum_{j=1}^{N} e^{i2\pi(u'x_j + v'y_j)} \]

Let \( \tau(x, y) = 1 \) for Pupil Transparency

and \( k = \frac{2\pi}{\lambda} \)

\[ F_1(u', v') = n \int \int e^{i2\pi[w(x,y) + (u'x + v'y)]} dx dy \sum_{j=1}^{N} e^{i2\pi(u'x_j + v'y_j)} \]

By substituting the integral boundary from equation (1) the area of synthetic aperture includes

\[ F_1(z') = n \int \int \left[ \cos\{2\pi w(x, y) + z'x'\} + i\sin\{2\pi w(x, y) + z'x'\} \right] dx dy \]

\[ \sum_{j=1}^{N} \left[ \cos(z'x_j) + i\sin(z'x_j) \right] \]

The complex amplitude for an array of ellipse synthetic apertures is represented by equation (9.).

From equation (5), the complex amplitude function for an array
The normalizing factor calculated when the system diffraction limit and its value become.

\[ n \cdot f = \frac{1}{(\pi - \varepsilon^2 \pi)^2} \]

By substituting the value of Normalizing factor, and using the physical conception of the equation (3).

Equation the point spread function for array of obscured ellipse synthetic apertures,

Give as follows.
\[
PSF = \frac{1}{(\pi s^2 \pi')^2} \left( \frac{2}{s} \int_{-\frac{s}{2}}^{\frac{s}{2}} \int_{-\frac{a}{s} \sqrt{\frac{s^2}{s^2 - s' \pi}}}^{\frac{s}{2}} \cos \left(2\pi w (x, y) + z' x' \right) dx dy \sum_{j=1}^{N} e^{i z' y_j} \right)^2 \\
- \frac{2}{s} \int_{-\frac{s}{2}}^{\frac{s}{2}} \int_{-\frac{a}{s} \sqrt{\frac{s^2}{s^2 - s' \pi}}}^{\frac{s}{2}} \cos \left(2\pi w (x', y') + z' x' \right) dx' dy' \sum_{j=1}^{N} e^{i z' y_j} \\
+ \int_{-\frac{s}{2}}^{\frac{s}{2}} \int_{-\frac{a}{s} \sqrt{\frac{s^2}{s^2 - s' \pi}}}^{\frac{s}{2}} \sin \left(2\pi w (x, y) + z' x' \right) dx dy \sum_{j=1}^{N} e^{i z' y_j} \\
- \int_{-\frac{s}{2}}^{\frac{s}{2}} \int_{-\frac{a}{s} \sqrt{\frac{s^2}{s^2 - s' \pi}}}^{\frac{s}{2}} \sin \left(2\pi w (x', y') + z' x' \right) dx' dy' \sum_{j=1}^{N} e^{i z' y_j}
\]

\[\text{(12)}\]
N=8, w20=0, e=0, a=1,2,3,4

N=12, w20=0, e=0.5, a=1,2,3,4

N=4, w20=0, a=4, e=0,0.25,0.5,0.75

N=8, w20=0, a=4, e=0,0.25,0.5,0.75
Results and Discussion

The aim of using obscured apertures is to improve image quality and minimize aberration effects. This improves the optic system's dealing power; on the other hand, the blurred aperture can be used in speech telescopes. To investigate the effect of ellipse obscured synthetic apertures, MathCAD programs were used to simulate the equation (12).

We observe from the figures that the curve of the point spread function (distribution of intensity in the plane of the image versus the optical axis) and of a diffraction limited system and devoid of focus error, that the most curve from the axis of the point spread function is for the circular apertures (a=1) and the curves of the ellipse apertures (a=2,3,4) are closer to the axis Point spread function and with values that vary depending on the dimensions of the ellipse (the length of the major axis (a) and the length of the minor axis(b)), where we notice that when increasing the length of the larger axis and reducing the length of the smaller axis the curve becomes more impulsive towards the axis the point spread function and this means obtaining a very high resolving power compared to Circular apertures resolving power.

when the obscured ratios changed (ε=0.25,0.5,0.75), the results showed that the elongation of the ellipse apertures led to the surge of the curve the point spread function inward, and this led to a clear improvement in the concentration of energy within a small area.

The increase in the number of apertures leads to the crawl of the site of the secondary peaks inward and this means an increase in the intensity of the secondary peaks.

Conclusion

The change in the shape of the obscured circular apertures to the obscured ellipse apertures led to a clear improvement in the values of the point spread function, the increase in the elongation of the obscured ellipse apertures, i.e. the increase in the length of the major axis of the obscured ellipse apertures led to a greater improvement in the values of the point spread function.
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