Modeling and Analyzing Adaptive User-Centric Systems in Real-Time Maude∗

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Pervasive user-centric applications are systems which are meant to sense the presence, mood, and intentions of users in order to optimize user comfort and performance or to assist people in their specific activities. Building such applications requires not only state-of-the-art techniques from artificial intelligence but also sound software engineering methods for facilitating modular design, runtime adaptation and verification of critical system requirements.

In this paper we focus on high-level design and analysis, and use the algebraic rewriting language Real-Time Maude for specifying applications in a real-time setting. We propose a component-based approach for modeling pervasive user-centric systems in a generic way and show how to instantiate the generic rules for a simple out-of-home digital advertising application and how to analyze and prove crucial properties of the system architecture through model checking and simulation. For proving time-dependent properties of systems we use Metric Temporal Logic (MTL) and present analysis algorithms for model checking two subclasses of MTL formulas: time-bounded response and time-bounded safety MTL formulas. The underlying idea is to extend the Real-Time Maude model with suitable clocks and to transform the MTL formulas into LTL formulas over the extended specification. This makes it possible to use the LTL model checker of Maude for verifying real time system properties. It is shown that component-based Real-Time Maude specifications as well as their extensions by clocks are time-robust and finite state; moreover, the above classes of formulas are tick-stabilizing if their atomic propositions are tick-stabilizing. As a consequence, model checking analyses are sound and complete for maximal time sampling.

The approach is illustrated by a simple adaptive advertising scenario in which an adaptive advertisement display can react to actions of the users in front of the display.

Keywords: Component-based software engineering, reconfiguration, algebraic specification, term rewriting, Real-Time Maude, real-time temporal logic

1 Introduction

As we are moving on from desktop computers to a pervasive computing intelligence interwoven in the “fabric of everyday life”, our environment is about to become enriched with more and more smart assistance systems.

Through this transformation, it becomes feasible for IT systems in our environment to measure responses of the user’s body through sensors and cameras and to influence our physical, emotional and cognitive state for our, the users, benefits. We call such systems pervasive user-centric applications [19]. Examples are a so called “mood player” which selects the music according to current mood of a person, a “driving assistant” which implements adaptive control in vehicles to achieve more secure, more pleasant

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and more effective driving, or “adaptive advertising” where the displayed content of an advertisement is dynamically adapted to the needs of the actual audience in front of the display [1].

Building pervasive user-centric applications is not easy, and requires state-of-the-art techniques from artificial intelligence including machine learning and probabilistic reasoning, as well as a lot of system calibration and experimental psychological research in order to determine the right sensor parameters for recognizing the mood or the cognitive state of a person. As a consequence, from the software engineering point of view it is important that such systems are easily changeable and adaptable at runtime; moreover, they need to react immediately to the behavior of the user and thus have to satisfy (soft) real-time constraints. In the REFLECT project [1] we have developed a component-based framework [4] which facilitates modular design, runtime adaptation and reconfiguration of systems and supports the implementation of pervasive user-centric applications (such as the ones mentioned above) in a flexible way.

In this paper we focus on the high-level design and analysis of pervasive user-centric applications in order to be able to make guarantees on the correct behavior of such systems in an early stage of development. We follow the algebraic paradigm based on term rewriting and use Real-Time Maude as a high-level formal modeling language for pervasive user-centric applications in a real-time setting.

In line with the REFLECT framework we propose a component-based approach for modeling pervasive user-centric applications in a generic way and show how to instantiate the generic rules for a simple out-of-home digital advertising application and how to analyze and prove crucial properties of the system architecture through model checking and simulation.

In our approach components are considered to be black boxes, making explicit only their communication requirements by means of required and provided ports. A system configuration comprises a number of components and connectors, which describe how the required ports are connected to suitable provided ports. We distinguish three kinds of components: basic components, timed components whose behavior is influenced by timers, and hierarchical components (often just called components) which typically contain other components and connectors as well as timers. Generic rules are defined for transmitting values along connectors as well as for time elapse and the specifications of different kinds of timers.

Individual components provide parts of the functionality required by the entire system. By changing connectors, adding and removing individual components, the system’s behavior can be changed at runtime. This process is called dynamic reconfiguration. Since entire components are replaced, little code needs to be added to the components to achieve this kind of adaptivity. Instead, it is attained on the level of the system architecture. In our approach, timed monitor components survey the behavior of the system and the environment, and trigger reconfigurations if necessary.

For proving properties of systems we use Metric Temporal Logic (MTL) [10]. This is an extension of Linear Temporal Logic (LTL) [11] for specifying timed properties. Currently, Real-Time Maude does not provide an MTL model checker. However, in previous works, cf., e.g. [15], Olveczky showed how to verify some simple MTL formulas by using the time-bounded search command of Real-Time Maude or the LTL model checker of Maude. In [13], Lepri et al. present an automatized analysis algorithm of two important classes of MTL formulas, namely the bounded response property $\square(p \rightarrow (\diamond_{\leq b}q))$ and the minimum separation property $\square(p \rightarrow (p W (\square_{\leq b}\neg p)))$. The underlying idea is to extend a Real-Time Maude model by a suitable clock and to transform the MTL formulas into LTL formulas over the extended specification. Then the LTL model checker of Maude can be used for performing the analysis.

In this paper, we extend these ideas and present analysis algorithms for two further and more general classes of MTL formulas:

1. Generalized time-bounded response: $\square(\bigvee_{i \in I}(\diamond_{\leq b}q_i))$ for $I = \{1, 2, \ldots, n\} \subset \mathbb{N}$ a finite set of indices, and
2. Time-Bounded safety: □(p ∨ □≤bq)

(Where qi, q, and p are all atomic propositions).

We show that component-based Real-Time Maude specifications as well as their extensions by clocks are time-robust and finite state; moreover, the above classes of formulas are tick-stabilizing if their atomic propositions are tick-stabilizing. As a consequence (cf. [16]), model checking analyses are sound and complete for maximal time sampling.

Throughout the paper we illustrate our modeling and analysis techniques by a simple scenario of adaptive advertisement.

The paper is organized as follows: In Section 2 we present the adaptive advertising case study which we use as running example. The following Section 3 contains a short introduction to Real-Time Maude. Sections 4 and 5 present our main results. In Section 4 we explain our generic framework for specifying component-based systems and the Real-Time Maude specification of the adaptive advertisement scenario. The transformation algorithms for the timebounded response and timebounded safety formulas are presented in Section 5, we show also completeness and termination of LTL model checking for our format of component-based specifications and illustrate our results by applying the Maude model checker successfully to the requirements of the adaptive advertisement scenario. In Sections 6 and 7 we discuss related work, summarize our results and discuss further work.

2 An adaptive advertising application

To showcase our approach to system verification, we consider a simple out-of-home digital advertising application [5]. The setup of this application consists of a large display screen and a camera monitoring the area in front of the display, and by this allowing interactions of passer-bys with the displayed content. The general idea of adaptive advertising is to adapt a displayed advertisement to the current situation in front of it – whether there are several people just passing by, a small group of persons watching the ad carefully, or just one person in front of it waiting for someone else [5]. In this simplified example, we consider the camera as a way to enable gesture-based interactions with a passer-bys and to discover their presence.

A simple scenario within the adaptive advertising setting is an adaptive car advertisement, reacting to gestures of users in front of the display: By moving around the display, pointing at items or looking at them, the users influence the contents of the ad. To function properly this system should satisfy the following two requirements: (G1) Being an interactive ad, the system should react to a user in front of the display. (G2) The content displayed must change at least every ten seconds: an advertising campaign using a large-scale display should not waste its capabilities by showing static content.

The realization shown in figure 1 (components that are present but inactive are shown in light-gray) is first deployed in interactive mode and monitors whether someone is interacting with the ad. If that is not the case, a reconfiguration is triggered which altering the system configuration so that it shows autoactive content generated by a presentation component, e.g. an advertising movie or predefined animation sequences. Introducing monitors allows a partial solution to assume that the environment exhibits certain features (e.g. always have someone interacting with the ad) that it does not exhibit in the general case. Note that the second system (figure 1 right) also needs monitoring, as it again does not satisfy (G1): The second system provides interactive content to its viewers, and therefore must be changed as soon as a person is in front of the display, interacting with it.

Reconfiguration leads to further requirements; in particular, system configurations should reasonably stable so that the system does not oscillate between several configurations. This can be expressed as
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follows: (G3) a reconfiguration should not happen instantaneously, but must take at least 200 ms to complete.

3 Real-Time Maude

Real-Time Maude [17] is a formal specification language based on Maude [7], a high-performance simulation and model checking tool which uses rewriting logic and membership equational logic for the specification of systems. Real-Time Maude extends Maude by supporting the formal specification of real-time system while benefiting from the expressiveness of the Maude language and the powerful analysis techniques like LTL model checking. In this section, we will briefly introduce the main concepts of specifications in Real-Time Maude; we refer to [17] for more details on the syntax and semantics of Real-Time Maude.

In Real-Time Maude, real-time systems are formally specified by a real-time rewrite theory of the form $R = (\Sigma, E, IR, TR)$ where $(\Sigma, E)$ is a membership equational logic [7] theory with $\Sigma$ a signature and $E$ a set of confluent and terminating conditional equations, and $IR$ is a set of instantaneous (rewrite)
rules specifying the system’s transitions which happen in zero time. Instantaneous rules are written
\[ \text{crl} \left[ l \right] : \ t \Rightarrow t' \text{ if } \text{cond} . \]

where \( l \) is a label, \( t, t' \) are terms, and \( \text{cond} \) is a condition on the terms \( t, t' \). Finally, \( TR \) is a set of tick (rewrite) rules which specify how the system behaves when time advances. Tick rules are written
\[ \text{crl} \left[ l \right] : \ \{i\} \Rightarrow \{i'\} \text{ in time } T \text{ if } \text{cond} . \]

where \( \{ \_ \} \) is a constructor of sort \text{GlobalSystem}, and \( T \) is a term of sort \text{Time} which denotes the duration of the tick rule. The form of the tick rules ensure that time advances uniformly in the whole system.

A one-step rewrite, written \( t \xrightarrow{R} t' \), is a single rewrite of a term \( t \) to a term \( t' \) (both of sort \text{GlobalSystem}) in time \( r \) (possibly zero time). We call \( t \) the source state of the rule \( t \xrightarrow{R} t' \), and \( t' \) the target state. A (timed) path in \( R \) is an infinite sequence \( \pi = t_0 \xrightarrow{r_0} t_1 \xrightarrow{r_1} t_2 \ldots \) where either for all \( i \in \mathbb{N} \), \( t_i \xrightarrow{r_i} t_{i+1} \) is a one-step rewrite of \( R \), or there exists a \( k \in \mathbb{N} \) such that for all \( 0 \leq i < k \), \( t_i \xrightarrow{r_i} t_{i+1} \) is a one-step rewrite in \( R \) and there is no one-step rewrite from \( t_k \) in \( R \), and \( t_j = t_k \) and \( r_{j-1} = 0 \) for each \( j > k \). The set of all timed paths of \( R \) starting in \( t \) is denoted by \( \text{Paths}(R) \). For \( k \in \mathbb{N} \), we define \( \pi^k \) to be the timed path starting after the \( k \)th one-step rewrite, i.e. \( \pi^k = t_0 \xrightarrow{r_0} t_1 \xrightarrow{r_1} \ldots \xrightarrow{r_k} t_{k+2} \ldots \). A term \( t' \) is reachable from a term \( t \) in \( R \) in time \( r \) if there is a path \( \pi = t_0 \xrightarrow{r_0} \ldots \xrightarrow{r_j} t_{j+1} \xrightarrow{r_{j+1}} \ldots \) such that \( t_k = t' \) and \( \sum_{i=0}^{j-1} r_i \).

Function symbols \( f \) are declared by the statement \( f : s_1 \ldots s_n \Rightarrow s \) with sorts \( s_1, \ldots, s_n, s \), and equations are written \( \text{eq } t = t' \). A variable \( x \) of sort \( s \) is declared by the statement \( \text{var } x : s \).

In \text{object-oriented} Real-Time Maude, classes are declared by
\[ \text{class } C \mid \text{att} \_ 1 : s_1, \ldots, \text{att} \_ n : s_n . \]

where \( \text{att} \_ 1, \ldots, \text{att} \_ n \) are attributes of sorts \( s_1, \ldots, s_n \), respectively. An object of class \( C \) is written as a term
\[ \text{o} : C \mid \text{att} \_ 1 : \text{val} \_ 1, \ldots, \text{att} \_ n : \text{val} \_ n > \]
of sort \text{Object}, where \( \text{o} \) is an object identifier of sort \text{Id} and \( \text{val} \_ i \) are the current values of attributes \( \text{att} \_ i (1 \leq i \leq n) \). A system state is a collection of objects and is of sort \text{Collection} which is a multiset equipped with an associative and commutative union operator with empty syntax, e.g.
\[ \text{o} : C \mid \text{att} \_ 1 : \text{val} \_ 1, \ldots, \text{att} \_ n : \text{val} \_ n > \text{o}' : C' \mid \text{att} \_ 1' : \text{val} \_ 1', \ldots, \text{att} \_ m' : \text{val} \_ m' > \ldots \]
represents a system state consisting of the objects \( \text{o}, \text{o}', \ldots \). Real-Time Maude supports \text{multiset rewriting}, i.e. rewrite rules are applied modulo associative and commutative rewriting of the system state. In object-oriented Real-Time Maude specifications, the time-dependent behavior is usually specified by a single \text{tick rule} of the form
\[ \text{var } C : \text{Configuration} \mid \text{var } T : \text{Time} . \]
\[ \text{crl} \left[ \text{tick} \right] : \{C\} \Rightarrow \{\text{delta}(C,T)\} \text{ in time } T \text{ if } T <= \text{mte}(C) \text{ /\ } \text{cond} \text{ [nonexec]} . \]

The function \( \text{delta} \) defines the effect of time elapse on a configuration, and the function \( \text{mte} \) defines the maximum amount of time that can elapse before some action must take place. These functions distribute over the objects in a configuration and must be defined for all single objects to defined the timed behavior of a system. The tick rule advances time nondeterministically by any amount \( T \) less than

\[ \text{In Real-Time Maude, states in object-oriented specifications usually contain messages which are used to model communication between objects. However, in our case study, we will not make use of messages and let objects “directly” communicate, i.e. the effect of communication between two objects is modeled by a rewrite rule having both objects in source and target state.} \]
or equal to $mte(C)$. To execute such rules, Real-Time Maude offers a number of time sampling strategies, so that only some moments in time are visited. In this paper, we will only make use of the maximal time sampling strategy which advances time to the next moment when some action must be taken, as defined by $mte$, i.e. when the tick rule is applied in a state $\{C\}$, time is advanced by $mte(C)$. The above form of the tick rule slightly differs from the usual form proposed in [17] by allowing an additional condition $cond$ ($T$ must not occur in $cond$).

A Real-Time Maude specification is executable and various formal analysis methods are supported. For a complete overview of these methods see, e.g., [17]. In this work we make use of the time-unbounded model checking command

\[
(mc \ t \ |=u \ \phi .)
\]

for an initial state $t$ and a temporal logic formula $\phi$.

In the rest of this paper, when we talk of a real-time rewrite theory $R$, we typically mean the real-time rewrite theory $R_{max}$ which is obtained from $R$ by applying the theory transformation corresponding to using the maximal time sampling strategy when executing the tick rules.

### 4 Modeling in Real-Time Maude

In this section we show how our case study, the digital advertising application, as described in Sect. 2, can be modeled in Real-Time Maude. For this purpose, we first present in Sect. 4.1 an implementation of a generic, port-based component model in object-oriented Real-Time Maude. Then we show in Sect. 4.2 how our case study described in Sect. 2 can be modeled as self-reconfiguring component-based system.

#### 4.1 Defining Components in Object-Oriented Real-Time Maude

Components are encapsulated entities with explicit ports over which communication take place. A port is modeled as an object instance of the class Port having one attribute value describing the current state of the port.

class Port | value : Bool.

The value of a port models the state of activity: a value true models the fact that at this port, a signal is received (or sent) via this port whereas false means that no signal is received (or sent).

A port always belongs to a unique component and may have two roles: either it is a provided port or a required port of this component. All provided ports are under the control of the owning component and therefore, the state (i.e. the value) of a provided port can only be changed by its owning component. In contrast, the value of a required port cannot be changed by the owning component but can only be changed by the environment – in this sense, a component can only react on different states if its required ports.

We introduce three different types of components: basic components, timed components, and hierarchical components (or just component). For these three different types of components we introduce the following (sub-)classes using inheritance, i.e. timed component inherits from basic component, and component inherits from timed component.

class ABasicComponent | prov : Configuration, req : Configuration.
class ATimedComponent | tstate : Configuration.
class AComponent | assembly : Configuration, innerreq : Configuration.
subclass ATimedComponent < ABasicComponent.
subclass AComponent < ATimedComponent.
The different types of components are also illustrated in Fig. 2. A basic component has a set of provided (prov) and required ports (req), and both attributes are modeled as instances of type Configuration. However, we assume that the multisets prov and req only contain object instances of type Port. A timed component inherits from basic component and has an additional attribute (tstate) which models a timed data state which need not be time invariant. For timed data states we refer the reader to the end of this section. Finally, hierarchical components embody an inner assembly (assembly) of connectors and components, and inner required ports (innerreq) that are connected to provided ports of components within the inner assembly.

For each type of component we actually introduce two classes, e.g. for basic components, we define an abstract class ABasicComponent and a concrete class BasicComponent and only allow object instances of the concrete class. This discrimination between abstract and concrete classes allows to define both common rewrite rules and equations applicable to all component types, and rules and equations applicable to a particular type of components only. The class definitions are hence as follows:

```
class BasicComponent .
class TimedComponent .
class Component .
subclass BasicComponent < ABasicComponent .
subclass TimedComponent < ATimedComponent .
subclass Component < AComponent .
```

For timed components (and hence for hierarchical components), a timed data state (attribute tstate) is a set of timers which decrement their value by the advanced time. In our case study later on, we need three different types of timers, all modeled as classes.

```
class Timer | value : TimeInf .
class OnOffTimer | value : TimeInf, active : Bool .
class DelayTimer | value : TimeInf, delay : TimeInf .
```

The class Timer models a simple timer which has an attribute value for the current time value of type TimeInf. The class OnOffTimer has an additional attribute active to switch the timer on and off. Finally, the third class DelayTimer is another timer class which contains – beside the timer value – an attribute delay. If the DelayTimer expires the timer value is reset to the fixed delay.

Components communicate over their required and provided ports which are connected by connectors. More precisely, we distinguish between two types of connectors: On the one hand, a class Connector models all connectors which link a provided port with a required port on the same level of the component hierarchy (i.e. not crossing component boundaries). On the other hand, a class DelegateConnector models all delegate connectors which link, within hierarchical components, ports in the assembly with either outer ports or inner required ports.

An overview of all types of connectors is given in Fig. 3.

(1) A connector links a provided and required port, on the same level of component hierarchy.
(2) A delegate connector links either

(a) a provided port of an inner component (source) with an outer provided port (target), or

(b) a provided port of an inner component (source) with an inner required port of the comprising
hierarchical component (target), or

(c) an outer required port (source) with a required port of an inner component (target).

Connectors are again modeled as object-oriented classes Connector and DelegateConnector, each having two attributes source and target which are object identifiers of instances of class Port.

class Connector | source : Oid, target : Oid.
class DelegateConnector | source : Oid, target : Oid.

Component behavior is modeled in an abstract way by defining an operation beh on configurations.

op beh : Configuration -> Configuration [frozen (1)] .

The behavior of a component can be defined then by introducing equations for beh. This abstract behavior operation is used in the following generic (instantaneous) rewrite rules.

We define generic, instantaneous rules for transmitting values along connectors. The following rule [transmit] assumes two (arbitrary) components with connected provided and required ports; if the value of the provided port is not equal to the required port value, then the latter value is changed accordingly such that afterwards, both connected ports have equal values. A necessary condition for this rule is that the target component which alters one of its ports is in a consistent state. A (hierarchical) component is called consistent if within its assembly, all connected ports are equal in value.

\[ \begin{align*}
\text{crl [transmit]} : &< o_1 : \text{ABasicComponent} | \text{prov} : < p_1 : \text{Port} | \text{value} : b > \text{PORTS} > \\
&< c : \text{Connector} | \text{source} : p_1, \text{target} : p_2 > \\
&< o_2 : \text{ABasicComponent} | \text{req} : < p_2 : \text{Port} | \text{value} : b’ > \text{PORTS’} > \\
=\rightarrow &< o_1 : \text{ABasicComponent} | \text{prov} : < p_1 : \text{Port} | \text{value} : b > \text{PORTS} > \\
&< c : \text{Connector} | \text{source} : p_1, \text{target} : p_2 > \\
&\text{beh}(< o_2 : \text{ABasicComponent} | \text{req} : < p_2 : \text{Port} | \text{value} : b > \text{PORTS’} >) \\
&\text{if } b \neq b’ \text{ and} \\
&\text{consistent}(< o_2 : \text{ABasicComponent} | \text{req} : < p_2 : \text{Port} | \text{value} : b’ > \text{PORTS’} >) .
\end{align*} \]

Note that in the above rule, the abstract operation beh modeling the component behavior is called on the receiving component which may react on its new state, more precisely, the altered state of its required ports. It is also worth mentioning that this rule is defined uniformly on all types of components by
using ABasicComponent; every “concrete” component (of type BasicComponent, TimedComponent, Component) is a subclass and hence inherits this rewrite rule.

For propagation of port values along connectors, each type of connector has its own behavior modeled as a rule. The rule [transmit] models the behavior of a connector, and since we have three different types of delegate connectors, we have also three more rules which do not described here in detail: A rule [delegateIn] equals inner required port with outer required port, a rule [delegateOut] equals outer provided port with inner provided port, and a rule [delegateInnerPort] equals an inner required port of the comprising hierarchical component with an inner required port (in the assembly).

Finally, we have the tick rule which advances time up to the maximal possible amount of time determined by the function mte.

\[
\text{var } C : \text{Configuration} . \quad \text{var } T : \text{Time} . \\
\text{crl} \ [\text{tick}] : \{C\} \Rightarrow \{\text{delta}(C,T)\} \text{ in time } T \text{ if } T \leq mte(C) /\ \text{consistent}(C) \ [\text{nonexec}] .
\]

It is important to point out that we only let time advance if the system is in a consistent state, i.e. the term \text{consistent}(C) evaluates to true if and only if all connected ports are equal in value. The functions \text{delta}, which models the effect of time elapse on the system state, and \text{mte}, which for a system state returns the maximal possible time elapse, are defined as usual in object-oriented specifications in Real-Time Maude (cf. [17]), e.g. for the class OnOffTimer, we have the following equations:

\[
\text{eq } \text{delta}(< o : \text{OnOffTimer} | \text{value} : t, \text{active} : b >, T) = < o : \text{OnOffTimer} | \text{value} : \text{if } b \text{ then } t \text{ minus } T \text{ else } t \text{ fi}, \text{active} : b > . \\
\text{eq } \text{mte}(< o : \text{OnOffTimer} | \text{value} : t, \text{active} : b >) = \text{if } b \text{ then } t \text{ else INF fi} .
\]

4.2 Modeling the Digital Advertising Application

We show now how the adaptive advertising application of Sect. 2 can be modeled as a component-based system in Real-Time Maude by extending the implementation introduced so far.

The system, cf. Fig. 1, is modeled as a hierarchical component with object identifier SYS. It has one outer provided port SYS.imgChange and one outer required port SYS.persThereIn. For receiving a reconfiguration signal from the inner assembly the system component contains an inner required port SYS.reconf. The (timed) state consists of a timer reconftimer which is used to trigger reconfiguration of the component after a (fixed) time delay. The inner assembly comprises delegate connectors (e.g. d1 connecting SYS.persThereIn and Camera.persThereIn), connectors (e.g. c1 connecting Camera.persThere and Interaction.persThere), basic components (e.g. Render), and timed components (MonitorOne and MonitorTwo). Each monitor is equipped with a timer, more precisely, an object instance of the class OnOffTimer. The purpose of these timers is to count the time when the condition for reconfiguration to the other configuration is true.

\[
\text{op } \text{SYS_in_C1} : \Rightarrow \text{Configuration} \ [\text{ctor}] . \\
\text{eq } \text{SYS_in_C1} = < \text{SYS} : \text{Component} | \\
\quad \text{prov} : < \text{SYS.imgChange} : \text{Port} | \text{value} : \text{true} >, \\
\quad \text{req} : < \text{SYS.persThereIn} : \text{Port} | \text{value} : \text{true} >, \\
\quad \text{tstate} : < \text{reconftimer} : \text{Timer} | \text{value} : \text{INF} >, \\
\quad \text{innerreq} : < \text{SYS.reconf} : \text{Port} | \text{value} : \text{false} >, \\
\quad \text{assembly} : \\
\quad \quad < \text{d1} : \text{DelegateConnector} | \text{source} : \text{SYS.persThereIn}, \text{target} : \text{Camera.persThereIn} > .
\]

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```
< c1 : Connector | source : Camera.persThere, 
        target : Interaction.persThere >
...
< Render : BasicComponent |
        prov : < Render.imgChange : Port | value : true >,
        req : < Render.alterContent : Port | value : true > >
< Presentation : BasicComponent |
        prov : < Presentation.alterContent : Port | value : true >,
        req : none >
...
< MonitorOne : TimedComponent |
        tstate : < m1timer : OnOffTimer | value : INF, active : true >,
        prov : < MonitorOne.reconf : Port | value : false >,
        req : < MonitorOne.gesture : Port | value : true > >
...
```

The component behavior is defined via equations for the operation `beh`. Note that `beh` applied to components which have altered the port values of required ports and have to react accordingly (e.g. `beh` is called in the rule `[transmit]`, cf. Sect. 4.1). For instance, for component `Render` we define `beh` by propagating the new value of the required port (which has been previously changed by rule `[transmit]`) to the provided port.

```
eq beh(< Render : BasicComponent |
        prov : < Render.imgChange : Port | >,
        req : < Render.alterContent : Port | value : b' > >)
   = < Render : BasicComponent |
        prov : < Render.imgChange : Port | value : b' >,
        req : < Render.alterContent : Port | > >.
```

The behavior of the monitor `MonitorOne` can be defined similarly, with a more involved behavior of the timer. `MonitorOne` is active in configuration C1 and surveys whether there is a person in front of the display. If there is no person in front (`MonitorOne.gesture` becomes false), the timer is initialised with 2000. Otherwise, if the valuation of port `MonitorOne.gesture` has changed from false to true and the timer has not expired already, the timer is set to infinite (INF), i.e. if time advances (by a finite amount of time), the timer is not decreased.

```
eq beh(< MonitorOne : TimedComponent |
        tstate : < m1timer : OnOffTimer | value : t, active : B >,
        req : < MonitorOne.gesture : Port | value : b > >)
   = < MonitorOne : TimedComponent |
        tstate : < m1timer : OnOffTimer | value : if (b or (not B)) and t /= 0
                                then INF else (if t == INF
                                           then 2000 else t fi) fi >.
```

Note that the timer `m1timer` of `MonitorOne` is not touched if the timer has expired, i.e. has 0 as value. In this case, for a period of 2000 ms, the port valuation of `MonitorOne.gesture` has been false. To guarantee the overall system guarantee (G2) which says that the display’s content should change at least every ten seconds, the system must be reconfigured. Reconfiguration is signaled to the component `SYS` by setting the value of the port `MonitorOne.reconf` to true.

```
rl [monitorOne-signal] :
   < MonitorOne : TimedComponent |
```

\(^2\)In Real-Time Maude, object instances in terms need not list all attributes; it is valid to omit attributes which are not relevant.
This value of the reconfiguration port of the monitor is propagated to the component SYS which then sets its reconfiguration timer to 250 ms which models the duration of the reconfiguration process.

eq \text{beh}(\langle \text{SYS} : \text{Component} | \text{tstate} : \langle \text{reconftimer} : \text{Timer} | \text{value} : t \rangle, \text{innerreq} : \langle \text{SYS}.\text{reconf} : \text{Port} | \text{value} : \text{true} \rangle \rangle) = \langle \text{SYS} : \text{Component} | \text{tstate} : \langle \text{reconftimer} : \text{Timer} | \text{value} : \text{if } t = \text{INF} \text{ then } 250 \text{ else } t \text{ fi} \rangle \rangle.

The reconfiguration is performed as soon as the timer expires: The reconfiguration timer is set to INF, the connectors of configuration 1 are replaced by the connectors configuration 2, and, moreover, the second monitor is activated which must observe the component to trigger a reconfiguration back to configuration 1 if the assumptions of configuration 2 are not met any more.

r1 [reconf-C1-to-C2] :
\langle \text{SYS} : \text{Component} | \text{tstate} : \langle \text{reconftimer} : \text{Timer} | \text{value} : 0 \rangle, \text{assembly} : \ldots \text{connectors of configuration 1} \ldots \rangle
\langle \text{MonitorTwo} : \text{TimedComponent} | \text{tstate} : \langle \text{m2timer} : \text{OnOffTimer} | \text{value} : \text{INF}, \text{active} : \text{false} \rangle, \ldots \rangle

\Rightarrow
\langle \text{SYS} : \text{Component} | \text{tstate} : \langle \text{reconftimer} : \text{Timer} | \text{value} : \text{INF} \rangle, \text{assembly} : \ldots \text{connectors of configuration 2} \ldots \rangle
\langle \text{MonitorTwo} : \text{TimedComponent} | \text{tstate} : \langle \text{m2timer} : \text{OnOffTimer} | \text{value} : \text{INF}, \text{active} : \text{true} \rangle, \ldots \rangle

\ldots \rangle.

We omit the rest of the rewrite rules for the remaining components and their equations for beh defining their behavior. They are in fact all very similar to the rules presented above. It is, however, worth mentioning that the timer of MonitorTwo is set to 500 ms as soon as there is a person detected in front of the display.

So far we have defined the term \text{SYS\_in\_C1} of sort Configuration and introduced appropriate rewrite rules and equations which model the behavior of the hierarchical component. However, it is an open component with a required port \text{SYS.persThereIn}; the overall behavior of component \text{SYS} depends on how the valuation of \text{SYS.persThereIn} evolve over time. To allow analysis of the system, we add a component \text{ENV} which models the environment; it has two ports which are connecting to their counterparts of the component \text{SYS}, thus yielding a closed system. The initial state is then defined as:

\begin{align*}
\text{op } \text{initial} & : \rightarrow \text{Configuration} [\text{ctor}] . \\
\text{eq } \text{initial} = \\
\langle \text{SYS\_in\_C1} \\
\langle \text{ENV} : \text{TimedComponent} | \text{prov} : \langle \text{ENV}.\text{persThereIn} : \text{Port} | \text{value} : \text{true} \rangle, \text{req} : \langle \text{ENV}.\text{imgChange} : \text{Port} | \text{value} : \text{true} \rangle, \\
\text{tstate} : \langle \text{envdtimer} : \text{DelayTimer} | \text{value} : 0, \text{delay} : 50 \rangle \rangle
\end{align*}
< CONN1 : Connector | source : ENV.persThereIn, target : SYS.persThereIn >
< CONN2 : Connector | source : SYS.imgChange, target : ENV.imgChange >.

The behavior of the environment is as follows: Every 50 ms the environment non-deterministically
choose whether to change the valuation of port ENV.persThereIn, or not. This recurring choice after
50 ms is modeled by a delay timer, always resetting the timer after each choice.

rl [env-true] :
< ENV : TimedComponent | tstate : < envdtimer : DelayTimer | value : 0, delay : 50 > >
=>
< ENV : TimedComponent | tstate : < envdtimer : DelayTimer | value : 50 >,
prov : < ENV.persThereIn : Port | value : true > >.

The rule [env-true] models the choice of ENV to set the value of ENV.persThereIn to true; the rule
[env-false] is analogous.

For all instantaneous rewrite rules (except those introduced in Sect. 4.1) we require that they are
triggered by the expiration of a timer which is indeed the case for all the rules in our example. The
advantage of this schema is that our specifications are time-robust [16] for which analysis techniques
with the maximal time sampling strategy is complete, i.e. if there is a counterexample of a property to be
analyzed we will actually find it with the analysis technique.

5 Analyzing in Real-Time Maude

Real-Time Maude provides a variety of analysis techniques including simulation through timed rewrit-
ing, untimed temporal logic model checking, or (unbounded or time-bounded) search for reachability
analysis. However, for real-time specifications, timed properties expressed in timed temporal logic are,
of course, of great relevance, e.g. for a flight control system, changes in sensor information must not only
be reported eventually, but within a specific time bound. Up to know, Real-Time Maude has lacked the
ability to model check any timed temporal logic formulas. In [13], Lepri et al. show how to model check
specific classes of timed temporal logic formulas, expressed in metric temporal logic. In the same line as
[13], we describe how to model check (different) classes of metric temporal logic which will be shown
useful for analyzing our real-time specification for our case study.

Metric Temporal Logic (MTL) [10] extends Linear Temporal Logic (LTL) [11] by allowing to de-
scribe timed properties of paths of a given system which is useful for to specify time-critical systems.
MTL is more expressive than LTL, for instance, we can state the timed property that some action should
happen within some time bounds, or that some property should always be satisfied within an interval.
Formally, the syntax of MTL formulas is the same as the syntax of LTL formulas, except for the until-
operator where a time interval is added. The formula \( p \cup_{[b_1,b_2]} q \) states that \( p \cup q \) holds, i.e. \( p \) holds
until \( q \) holds, and furthermore, \( q \) occurs within the time interval \([b_1,b_2]\). Thus, in MTL, time intervals
are added to all derived operators like \( \Box_{[b_1,b_2]} \) or \( \Diamond_{[b_1,b_2]} \).

MTL formulas \( \phi \) are inductively defined as follows:

\[
\phi ::= \text{true} \mid p \mid \neg \phi \mid \phi_1 \land \phi_2 \mid \phi_1 \cup_{[b_1,b_2]} \phi_2
\]

where \( p \) is a proposition and for time intervals \([b_1,b_2]\) (and a given time domain \( T \)) we allow either
\( b_1, b_2 \in T \), \( b_1 \leq b_2 \) and \( b_2 > 0 \), or \( t_1 \in T \) and \( t_2 = \infty \). Disjunction \( \lor \) and implication \( \rightarrow \) are defined as
usual. \( \Diamond_{[b_1,b_2]} \phi \) stands for \( \text{true} \cup_{[b_1,b_2]} \phi \), and \( \Box_{[b_1,b_2]} \) abbreviates \( \neg(\text{true} \cup_{[b_1,b_2]} \neg \phi) \). We will write
\( \cup_{b} \) (and \( \Diamond_{\leq b}, \Box_{\leq b} \)) if the lower bound is 0.
We follow [13] in the notational conventions for real-time rewrite theories: The set of states of a real-time rewrite theory $R = (\Sigma, E, IR, TR)$ is defined as the set of all terms (modulo the equations in $E$) of type $\text{GlobalSystem}$. A set $\Pi$ of atomic propositions can be defined equationally (in a protecting extension of $(\Sigma, E)$), and a labeling function $L_{\Pi}$ assigns to every state a finite set of propositions in $\Pi$ (cf. [17]).

Satisfaction of MTL formulas over timed paths of real-time rewrite theories is defined as follows:

**Definition 1** ([13]). Let $R$ be a real-time rewrite theory, $L_{\Pi}$ a labeling function on $R$, and let $\pi = t_0 \xrightarrow{r_1} t_1 \xrightarrow{r_2} \ldots$ be a timed path in $R$. The satisfaction relation of an MTL formula $\phi$ for the path $\pi$ in $R$ is then defined recursively as follows:

- $R, L_{\Pi}, \pi \models \text{true}$ always holds
- $R, L_{\Pi}, \pi \models p$ iff $p \in L_{\Pi}(t_0)$
- $R, L_{\Pi}, \pi \models \neg \phi$ iff $R, L_{\Pi}, \pi \not\models \phi$
- $R, L_{\Pi}, \pi \models \phi_1 \wedge \phi_2$ iff $R, L_{\Pi}, \pi \models \phi_1$ and $R, L_{\Pi}, \pi \models \phi_2$
- $R, L_{\Pi}, \pi \models \phi_1 \cup_{[b_1, b_2]} \phi_2$ iff there exists $j \in \mathbb{N}$ such that $R, L_{\Pi}, \pi^j \models \phi_2$
  and $R, L_{\Pi}, \pi \models \phi_1$ for all $i < j$, and $b_1 \leq \sum_{k=0}^{j-1} r_k \leq b_2$

For a state $t_0$ of sort $\text{GlobalSystem}$, the satisfaction relation of an MTL formula $\phi$ for the state $t_0$ in $R$ is defined as follows:

$$R, L_{\Pi}, t_0 \models \phi \iff \forall \pi \in \text{Paths}(R)_{t_0}. R, L_{\Pi}, \pi \models \phi$$

Real-Time Maude does currently not provide an MTL model checker. However, in previous works, cf., e.g. [13], some simple MTL formulas could already be model checked using the time-bounded search command or the LTL model checker of Maude. In [13], Lepri et al. present an automatized analysis algorithm of two important classes of MTL formulas, namely the bounded response property $\Box(p \rightarrow (\Diamond \leq b q))$ and the minimum separation property $\Box((p \rightarrow (p W (\Box \leq b \neg p))))$. We extend their ideas and present algorithms for two further and more general classes of MTL formulas:

1. Generalized time-bounded response: $\Box(\bigvee_{i \in I}(\Diamond \leq b_i q_i))$ for $I = \{1, 2, \ldots, n\} \subset \mathbb{N}$ a finite set of indices, and
2. Time-Bounded safety: $\Box(p \lor \Box \leq b q)$

where $q_i$, $q$, and $p$ are all atomic propositions of $\Pi$.

In the following sections 5.1 and 5.2 we will describe the algorithm of the transformation of the two classes of MTL formulas to corresponding LTL formulas which are then model checked over the transformed rewrite theory $\tilde{R}$. So in each case, for each formula $\phi$ belonging to one of the classes, we will show that $R, L_{\Pi}, \pi \models \phi$ if and only if $\tilde{R}, L_{\Pi}, \pi \models \phi$, hence it is shown how to modify the rewrite theory $\tilde{R}$ such that we can use the LTL model checker of Maude to verify $\phi$.

### 5.1 Model Checking MTL Formulas of the Form $\Box(\bigvee_{i \in I}(\Diamond \leq b_i q_i))$

For model checking MTL formulas of the form $\Box(\bigvee_{i \in I}(\Diamond \leq b_i q_i))$ for a finite set $I$ of indices, we add a single clock $c$ to the system state which will count the elapsed time after each state in which no $q_i$ is satisfied. It is indeed possible to restrict oneself to using one single clock by leveraging the observation that given a sequence of states $t_1, \ldots, t_n$ satisfying no $q_i$ within the time interval $\max\{b_i \mid i \in I\}$, the first state $s_1$ determines the deadlines $b_i$, and hence we can set the single clock $c$ to zero and start it in state $s_1$. ...
the first occurrence of a $q_i$-satisfying state $s_w$ will also witness the validity of $\square(\forall i \in I (\Diamond (\leq b_i q_i)))$ in all $s_i$ states between $s_1$ and $s_w$ (i.e. state $s_i$ satisfies the formula for all $1 \leq i \leq w$). The clock is switched off in state $s_w$, and can be switched back on if another state satisfying none of the $q_i$'s is found after the witness state $s_w$, and restart the counting process. In summary, to model check the MTL formula $\phi \equiv \square(\forall i \in I (\Diamond (\leq b_i q_i)))$ for a path $\pi$ in a real-time rewrite theory $\mathcal{R}$, the following steps are necessary:

1. A class modeling the clock is added:
   ```plaintext
   sort ClockStatus .
   ops on off : -> ClockStatus [ctor] .
   class Clock | clock : Time, status : ClockStatus .
   ```

2. The initial state $\{t_0\}$ is modified by adding a clock object such that the new initial state is
   ```plaintext
   \{t_0 < c : Clock | clock : 0, status : x \}
   ```
   where $c$ is a constant of sort $Oid$ and $x$ is $off$ if $\{t_0\} \models \forall i \in I q_i$, else $on$.

3. The functions $\delta$ (modeling the effect of time elapse on a configuration) and $mte$ (computing the maximum time elapse for a configuration) are extended for the newly introduced clock object as follows:
   ```plaintext
   eq \delta(< c : Clock | status : on, clock : T >, T') =
   \langle c : Clock | clock : if T <= b_{max} then min(T+T', b_{max}+1) else T fi > .
   eq \delta(< c : Clock | status : off >, T') = < c : Clock | > .
   eq mte(< c : Clock | >) = INF .
   ```
   where $b_{max} = \max\{b_i | i \in I\}$.

4. Instantaneous rewrite rules are modified such that the clock is switched on and off depending on the target state. Each instantaneous rule $t \Rightarrow t'$ if $cond$ or $\{t\} \Rightarrow \{t'\}$ if $cond$ in $\mathcal{R}$ is replaced by the following four rules (where $REST$ is a new variable of type $Configuration$):
   
   Rule (1): If the clock is off then the clock stays off if at least one of the $q_i$'s is satisfied.
   ```plaintext
   \{t REST < c : Clock | status : off >\}
   \Rightarrow \{t' REST < c : Clock | >\}
   if (modelCheck(\{t' REST\}, q_1) == true or ...
   modelCheck(\{t' REST\}, q_n) == true) and cond .
   ```

   Rule (2): If the clock is off then the clock is switched on if none of the $q_i$'s is satisfied.
   ```plaintext
   \{t REST < c : Clock | status : off >\}
   \Rightarrow \{t' REST < c : Clock | clock : 0, status : on >\}
   if (not (modelCheck(\{t' REST\}, q_1) == true or ...
   modelCheck(\{t' REST\}, q_n) == true)) and cond .
   ```

   Rule (3): If the clock is on then the clock stays on if for all $i \in I$, either $q_i$ is not satisfied or the time bound is already exceeded.
The new rules, and conversely, new rules yield the same result for the original part of the state. It follows that the original timed behavior is not modified, in particular, no original paths are blocked by its corresponding time bound \( b_i \).

Thus, by the above steps 1. to 4. we obtain a real-time rewrite theory \( \tilde{\mathcal{R}} \) which is adapted to the transformed state space while the labeling remains unchanged (i.e. \( L_{\Pi}(\{t\}) = L_{\Pi}(\tilde{\{t_{oclock}\}}) \) where \( oclock \) is the added clock), and \( \tilde{\{t_0\}} \) is the transformed initial state.

Finally, for model checking the MTL formula we need to add an atomic proposition stating that the current clock value is less or equal than a given time value \( r \).

The MTL formula \( \Box(\bigvee_{i \in I}(\diamond(q_i \land \text{clockLeq}(b_i)))) \) can then be model checked using Real-Time Maude’s untimed LTL model checking features, i.e. we check whether the transformed formula holds by invoking

\[
\text{mc} \{\tilde{t_0}\} \models u
\]

\[
\Box(\neg(q_1 \land \text{clockLeq}(b_1)))
\]

\[
\lor \ldots \lor
\]

\[
(\neg(q_n \land \text{clockLeq}(b_n)))
\]

which precisely is \( \tilde{\mathcal{R}}, \tilde{L}_{\Pi}, \tilde{t_0} \models \Box(\bigvee_{i \in I}(\diamond(q_i \land \text{clockLeq}(b_i)))) \).

**Proof of Correctness of the Transformation.**

**Lemma 1** (cf. [13]). Let \( \mathcal{R} \) be a real-time rewrite theory, \( L_{\Pi} \) with \( q_i \in \Pi \) for all \( i \in I \) a labeling function for \( \mathcal{R} \), and let \( \{t_0\} \) be an initial state for \( \mathcal{R} \). Let \( \tilde{\mathcal{R}}, \tilde{L}_{\Pi}, \) and \( \{\tilde{t_0}\} \) be the result of the \( \triangleright \)-transformation applied to \( \mathcal{R}, L_{\Pi}, \) and \( t_0 \). Then for each path \( \{t_0\} \tilde{\rightarrow} \{t_1\} \tilde{\rightarrow} \ldots \) in \( \tilde{\mathcal{R}} \) there is a path \( \{\tilde{t_0}\} \tilde{\rightarrow} \{\tilde{t_1}\} \tilde{\rightarrow} \ldots \) in \( \mathcal{R} \) such that, for all \( i \geq 0 \), there exists \( t'_i \) with \( \tilde{t}_i = t_i t'_i \), and vice versa.

**Proof.** We have to show that the transformation does not modify the original timed behavior. This is ensured by the following facts:

- Adding the clock class and a clock object to the initial state does not affect the original part of the state, and moreover, the timed behavior of the original system is not affected by the newly introduced clock since \( \text{mc} \) of the clock evaluates to \( \text{INF} \).

- The transformation replaces each rewrite rules by a number of rules with additional conditions. However, for each (extended) state to which the original rule is applicable, there is exactly one new rule applicable, and furthermore, the new rules treat the original state part as the original rule.

It follows that the original timed behavior is not modified, in particular, no original paths are blocked by the new rules, and conversely, new rules yield the same result for the original part of the state. \( \Box \)
Theorem 1. Let $\mathcal{R}$ be a real-time rewrite theory, $L_{\Pi}$ a labeling function for $\mathcal{R}$ with $q_i \in \Pi$ for all $i \in I$, and $\{t_0\}$ an initial state of $\mathcal{R}$. Let $\tilde{\mathcal{R}}, L_{\Pi}$, and $\{\tilde{t}_0\}$ be the result of the $\diamond$-transformation applied to $\mathcal{R}$, $L_{\Pi}$, and $\{t_0\}$. Then the following equivalence holds:

$$\mathcal{R}, L_{\Pi}, \{t_0\} \models \Box \bigwedge_{i \in I} \diamond (q_i \land \text{clock} \leq b_i) \iff \tilde{\mathcal{R}}, L_{\Pi}, \{\tilde{t}_0\} \models \Box \bigwedge_{i \in I} (q_i \land \text{clock} \leq b_i))$$

Proof. "$\Rightarrow$": Assume $\tilde{\mathcal{R}}, L_{\Pi}, \{\tilde{t}_0\} \not\models \Box \bigwedge_{i \in I} (q_i \land \text{clock} \leq b_i))$, we show $\mathcal{R}, L_{\Pi}, \{t_0\} \not\models \Box \bigwedge_{i \in I} (q_i \land \text{clock} \leq b_i))$. Let $\tilde{\pi} = \{\tilde{t}_0\} \stackrel{m_0}{\rightarrow} \{\tilde{t}_1\} \stackrel{m_1}{\rightarrow} \ldots$ be a path in $\tilde{\mathcal{R}}$ which does not satisfy $\Box \bigwedge_{i \in I} (q_i \land \text{clock} \leq b_i))$. By definition of $\models$ we know that there exists $j \geq 0$ such that $\tilde{\pi}^j \not\models \Box \bigwedge_{i \in I} (q_i \land \text{clock} \leq b_i))$, i.e.

$$\forall i \in I. \forall k \geq j. (\tilde{\pi}^k \not\models q_i) \lor (\tilde{\pi}^k \not\models \text{clock} \leq b_i). \quad (1)$$

Let $j \geq 0$ be the smallest index satisfying (1), and therefore, if $j = 0$ then the clock status is off, otherwise $j > 0$ and $\tilde{\pi}^{j-1} \models \bigwedge_{i \in I} \diamond (q_i \land \text{clock} \leq b_i))$. It follows that there exists $i \in I$ such that $\tilde{\pi}^{j-1} \models q_i \land \text{clock} \leq b_i$. Rule (4) ensures that as soon as this formula is satisfied, the clock status is off, hence the clock status in $\{\tilde{t}_{j-1}\}$ is off, too. It follows that the rewrite step from $\{\tilde{t}_{j-1}\}$ to $\{\tilde{t}_j\}$ is an instantaneous step of the form of rule (2) which sets the clock status to on and the clock value to 0. Furthermore, in both cases the clock can only be switched off by rule (4) which can never be applied because of the condition $\bigwedge_{i \in I} q_i \land \text{clock} \leq b_i$ which is $\text{by assumption}$ not satisfied. We can conclude that, from state $\{\tilde{t}_j\}$ on, the clock is continuously on and the clock value equals the elapsed time since $\{\tilde{t}_j\}$, i.e. the clock value is the sum of the durations of the applied tick rules since $\{\tilde{t}_j\}$. Therefore, for all $i \in I$ and for all $k \geq j$, $\tilde{\pi}^k \models \text{clock} \geq b_i$ if and only if $\sum_{i=1}^{j-1} r_i > b_i$. From (1) it follows

$$\forall i \in I. \forall k \geq j. (\tilde{\pi}^k \not\models q_i) \lor \left(\sum_{i=j}^{k-1} r_i > b_i\right). \quad (2)$$

Hence from (2) we can conclude that $\tilde{\pi}^k \not\models q_i$ for all $k \geq j$ such that $\sum_{i=j}^{k-1} r_i \leq b_i$. This implies $\tilde{\pi}^j \not\models \bigwedge_{i \in I} \diamond (q_i \land \text{clock} \leq b_i)$, and then $\pi \not\models \Box \bigwedge_{i \in I} \diamond (q_i \land \text{clock} \leq b_i)$. By Lemma 1 there exists a unique path $\pi$ with initial state $\{t_0\}$ for which $\pi \not\models \bigwedge_{i \in I} \diamond (q_i \land \text{clock} \leq b_i)$. Finally, it follows $\mathcal{R}, L_{\Pi}, \{t_0\} \not\models \Box \bigwedge_{i \in I} \diamond (q_i \land \text{clock} \leq b_i)$ which was to be shown.

"$\Leftarrow$": Assume $\mathcal{R}, L_{\Pi}, \{t_0\} \not\models \Box \bigwedge_{i \in I} \diamond (q_i \land \text{clock} \leq b_i)$, we show $\tilde{\mathcal{R}}, L_{\Pi}, \{\tilde{t}_0\} \not\models \Box \bigwedge_{i \in I} \diamond (q_i \land \text{clock} \leq b_i)$. Let $\pi = \{t_0\} \stackrel{m_0}{\rightarrow} \{t_1\} \stackrel{m_1}{\rightarrow} \ldots$ be a path in $\mathcal{R}$, by Lemma 1, we have that there exists a path $\tilde{\pi} = \{\tilde{t}_0\} \stackrel{m_0}{\rightarrow} \{\tilde{t}_1\} \stackrel{m_1}{\rightarrow} \ldots$ in $\tilde{\mathcal{R}}$. By assumption, $\pi$ and hence also $\tilde{\pi}$ do not satisfy $\Box \bigwedge_{i \in I} \diamond (q_i \land \text{clock} \leq b_i)$. Therefore, for all $i \in I$ and for all $k \geq j$, $\tilde{\pi}^k \models \text{clock} \geq b_i$ if and only if $\sum_{i=1}^{j-1} r_i > b_i$. From (1) it follows

$$\forall i \in I. \forall k \geq j. (\tilde{\pi}^k \not\models q_i) \lor \left(\sum_{i=j}^{k-1} r_i > b_i\right). \quad (3)$$

Let $j \geq 0$ be the minimal index satisfying (3). We show that in $\{\tilde{t}_j\}$ the clock status is on. If $j = 0$ then the clock status is on by definition of the initial state. Now assume $j > 0$. Then in state $\{\tilde{t}_{j-1}\}$ it must hold $\tilde{\pi}^{j-1} \models q_i$ for some $i \in I$. So the clock status in $\{\tilde{t}_{j-1}\}$ is off and the clock value is 0 because otherwise, if the clock status was on, there would exist a state before $j$ not satisfying (3) and hence contradicting our assumption. From (3) it follows that the rewrite step from $j - 1$ to $j$ is an instantaneous rewrite step, switching the clock on (with clock value 0). Since the clock cannot be switched off (the conditions of rule (4) are never met from $\tilde{\pi}^j$ on), the durations of the tick steps since $\tilde{t}_j$ and the clock value are equal. It follows that

$$\forall i \in I. \forall k \geq j. (\tilde{\pi}^k \not\models q_i) \lor (\tilde{\pi}^k \not\models \text{clock} \leq b_i)$$

which implies $\mathcal{R}, L_{\Pi}, \{t_0\} \not\models \Box \bigwedge_{i \in I} \diamond (q_i \land \text{clock} \leq b_i)$ which was to be shown. \qed

---

3The clock value will not be greater than $\max_{i \in I} b_i + 1$. 
5.2 Model Checking MTL Formulas of the Form $\Box (p \lor \square \leq b q)$

For model checking MTL formulas of the form $\Box (p \lor \square \leq b q)$, we add a single clock which counts the minimum time that $q$ needs to be true once $p$ became false. Here, we use the observation that if $p$ was false at $t_1$ and becomes false again between $t_1$ and $t_1 + b$, say at $t_2$, $q$ must additionally hold until $t_2 + b$. Hence, it is valid to reset the clock at $t_2$ and thereby enforce that $q$ must hold true for $b$ more time units. So to model check the MTL formula $\phi \equiv \Box (p \lor \square \leq b q)$ for a path $\pi$ in a real-time rewrite theory $\mathcal{R}$ with labeling function $L_{\Pi}$, the following steps are necessary: First, to $\mathcal{R}$ a class Clock, modeling the clock, and corresponding equations are added; second, $\phi$ is translated to $\tilde{\phi} \equiv \Box (p \lor (q \land (\text{clock} > b)))$ where $\text{clock}$ is an atomic proposition which refers to the current time value of the clock; the rewrite rules are transformed to adequately take into account the propositions and the clock behavior. The transformation, which we will call $\Box$-transformation in the following, proceeds as follows.

1. A class modeling the clock is added (analogous to the $\Diamond$-transformation):
   
   ```
   sort ClockStatus .
   ops on off : -> ClockStatus [ctor] .
   class Clock | clock : Time, status : ClockStatus .
   ```

2. The initial state $\{t_0\}$ is modified by adding a clock object such that the new initial state is
   
   $$\{t_0 < c : \text{Clock} | \text{clock} : 0, \text{status} : \text{off} >\}$$

3. The functions $\delta$ and $\text{mte}$ are extended for the newly introduced class Clock as follows (again analogous to the $\Diamond$-transformation):
   
   ```
   eq \delta(< c : \text{Clock} | \text{status} : \text{on}, \text{clock} : \text{T} >, \text{T}') =
   < c : \text{Clock} | \text{clock} : \text{if T <= b then min(T + T', b+1) else T fi} > .
   eq \text{mte}(< c : \text{Clock} | \text{status} : \text{off} >, \text{T}') = < c : \text{Clock} | > .
   ```

4. Instantaneous rewrite rules are modified such that the clock is switched on and off depending on the target state. Each instantaneous rule $t \Rightarrow t'$ if $\text{cond}$ or $\{t\} \Rightarrow \{t'\}$ if $\text{cond}$ in $\mathcal{R}$ is replaced by the following four rules (where $\text{REST}$ is a new variable of type $\text{Configuration}$):

   **Rule (1):** If in the next state the formula $\neg p \lor \neg q$ is satisfied, or in the previous state the formula $p \lor \neg q$ is satisfied, then the clock stays or is switched off.
   
   $$\{t \text{ REST} < c : \text{Clock} | >\}$$
   
   $$\Rightarrow \{t' \text{ REST} < c : \text{Clock} | \text{clock} : 0, \text{status} : \text{off} >\}$$
   
   if (modelCheck($\{t' \text{ REST}\}$, $p \lor \neg q$) == true or modelCheck($\{t \text{ REST}\}$, $p \lor \neg q$) == true) and $\text{cond}$.

   **Rule (2):** If the clock is off, it only gets switched on if in the previous state $\neg p \land q$ was satisfied and in the next state $p \land q$ is satisfied. So the clock begins to count if there was a state where $p$ was not true (so we need to look for an interval of length $\geq r$ where $q$ always holds) and in the next state $p$ is true (so the formula $\Box (p \land \square \leq b q)$ is satisfied).
   
   $$\{t \text{ REST} < c : \text{Clock} | \text{status} : \text{off} >\}$$
   
   $$\Rightarrow \{t' \text{ REST} < c : \text{Clock} | \text{clock} : 0, \text{status} : \text{on} >\}$$
   
   if (not modelCheck($\{t' \text{ REST}\}$, $\neg p \land \neg q$) == true or modelCheck($\{t \text{ REST}\}$, $p \lor \neg q$) == true) and $\text{cond}$.

   **Rule (3):** If the clock is on and in the next state the formula $p \land q$ is satisfied then the clock stays on. The clock is only on if we are looking for an interval of length $\geq b$ such that $q$ is satisfied, so we can safely go on with counting the advanced time since we do not “miss” any counterexample since $p$ is satisfied in the next state.
Proof. Thus, by the above steps 1. to 4. we obtain a real-time rewrite theory \( \mathcal{R} \), a labeling function \( L_{\Pi} \) which is adapted to the transformed state space while the labeling remains unchanged (i.e. \( L_{\Pi}(\{t\}) = \mathcal{R}(\{t, a_{\text{clock}}\}) \) where \( a_{\text{clock}} \) is the added clock), and \( \{i_0\} \) is the transformed initial state.

Finally, for model checking the MTL formula we need to add an atomic proposition stating that the current clock value is less or equal than a given time value \( b \).

\[
\text{op clockLeq : Time -> Prop [ctor].}
\]

\[
\text{eq } \langle c : \text{Clock} \mid \text{clock : t, status : s} > \text{REST}.
\]

The MTL formula \( \Box (p \lor \Box \leq b q) \) can then be model checked using Real-Time Maude’s untimed LTL model checking features, i.e. we check whether the transformed formula holds by invoking

\[
\text{(mc } \{i_0\} \Rightarrow \Box \langle p \lor q \text{ \& \& (not clockLeq(b))} \rangle).
\]

Proof of Correctness of the Transformation.

Lemma 2 (cf. [13]). Let \( \mathcal{R} \) be a real-time rewrite theory, \( L_{\Pi} \) with \( p, q \in \Pi \) a labeling function for \( \mathcal{R} \), and let \( \{i_0\} \) be an initial state for \( \mathcal{R} \). Let \( \mathcal{R}', L_{\Pi}, \) and \( \{i_0\} \) be the result of the \( \Box \)-transformation applied to \( \mathcal{R}, L_{\Pi}, \) and \( \{i_0\}. \) Then for each path \( \{i_0\} \overset{r_0}{\rightarrow} \{i_1\} \overset{r_1}{\rightarrow} \ldots \) in \( \mathcal{R} \) there is a path \( \{i_0\} \overset{\tilde{r}_0}{\rightarrow} \{\tilde{i}_1\} \overset{\tilde{r}_1}{\rightarrow} \ldots \) in \( \mathcal{R}' \) such that, for all \( i \geq 0 \), there exists \( \tilde{i}_i \) with \( \tilde{i}_i = t_1^{i-1}, \) and vice versa.

Proof. Very similar to the proof of Lem.1 \( \Box \)

Theorem 2. Let \( \mathcal{R} \) be a real-time rewrite theory, \( L_{\Pi} \) a labeling function for \( \mathcal{R} \) with \( p, q \in \Pi \), and \( \{i_0\} \) an initial state of \( \mathcal{R} \). Let \( \mathcal{R}', L_{\Pi}, \) and \( \{i_0\} \) be the result of the \( \Box \)-transformation applied to \( \mathcal{R}, L_{\Pi}, \) and \( \{i_0\}. \) Then the following equivalence holds:

\[
\mathcal{R}, L_{\Pi}, \{i_0\} \vDash \Box (p \lor \Box \leq b q) \iff \mathcal{R}', L_{\Pi}, \{i_0\} \vDash \Box (p \lor (q \text{ \& \& clock} > b)).
\]

Proof. “\( \Rightarrow \)” Let \( \tilde{\pi} = \{i_0\} \overset{\tilde{r}_0}{\rightarrow} \{\tilde{i}_1\} \overset{\tilde{r}_1}{\rightarrow} \ldots \) be a path in \( \mathcal{R}' \). Assume \( \mathcal{R}, L_{\Pi}, \{i_0\} \vDash \Box (p \lor (q \text{ \& \& clock} > b)), \) and we show \( \mathcal{R}, L_{\Pi}, \{i_0\} \vDash \Box (p \lor \Box \leq b q). \) By assumption,

\[
\exists j \geq 0, (\tilde{\pi}^j \vDash p) \land (\exists k \geq j, (\tilde{\pi}^k \vDash q) \land (\tilde{\pi}^k \vDash \text{clock} > b)) \land \forall l, j \leq l < k \Rightarrow (\tilde{\pi}^l \vDash q) \land (\tilde{\pi}^l \vDash \text{clock} > b)).
\]

Let \( j \in \mathbb{N} \) be the minimal index satisfying the above formula. We know \( \tilde{\pi}^i \vDash p \), if in addition \( \tilde{\pi}^i \vDash q \) then we are finished because obviously \( \tilde{\pi}^j \vDash p \lor \Box \leq b q \). So assume \( \tilde{\pi}^j \vDash q \). If \( j = 0 \) then the clock status is \( \text{off} \) and the clock value is 0; if \( j > 0 \) then the last rewrite step \( \{i_{j-1}\} \overset{r_{j-1}}{\rightarrow} \{i_j\} \) must have been the rule (1) since \( \neg p \) is satisfied in \( i_j \), i.e. the clock status in \( i_j \) is \( \text{off} \) and the clock value is 0. Hence, in any case, we know that in \( i_j \) the clock status is \( \text{off} \) and the clock value is 0. Let \( k > j \) be the minimal index such that \( \tilde{\pi}^k \vDash q, \tilde{\pi}^k \vDash \text{clock} > b, \) and \( \forall l, j \leq l < k \Rightarrow (\tilde{\pi}^l \vDash q) \land (\tilde{\pi}^l \vDash \text{clock} > b); \) such a \( k \) exists by assumption.

We know that \( p \) is not satisfied in \( \tilde{i}_j \). If \( p \) is not satisfied for all states \( \tilde{i}_m \) for \( j \leq m < k \) then we can just take the state \( \tilde{i}_{k-1} \) as a counterexample, that is \( \tilde{\pi}^{k-1} \vDash p \lor \Box \leq b q \). So assume that there exists a maximal \( m \) with \( j \leq m < k \) such that \( \tilde{i}_m \) does not satisfy \( p \). Furthermore we can assume that between \( m \) and \( k \) there is at least one tick rule since otherwise, in state \( \tilde{i}_m \) the proposition \( p \) is not satisfied, but also \( q \) is not satisfied in \( \tilde{i}_k \) which is reachable in zero time. It follows that in this case \( \tilde{\pi}^m \) does not satisfy \( p \lor \Box \leq b q \).
and hence there must be a tick rule between \( \bar{t}_m \) and \( \bar{t}_k \), and moreover, \( m \leq k - 3 \) (after the \( m \)th state there must be an instantaneous step changing \( \neg p \) to \( p \), one application of the tick rule, and an instantaneous step changing \( q \) to \( \neg q \)).

It follows that the rewrite step \( \{ \bar{t}_m \} \xrightarrow{\bar{r}} \{ \bar{t}_{m+1} \} \) switches the clock on (\( \bar{t}_m \) satisfies \( \neg p \) and \( \neg q \), and \( \bar{t}_{m+1} \) satisfies \( p \) and, by the above observation that \( m \leq k - 3 \), also \( q \)). From the state \( \bar{t}_{m+1} \) on the clock counts and for all subsequent states up to \( \bar{t}_k \) the clock value equals the duration of the tick rules between \( \bar{t}_m \) and \( \bar{t}_k \). Since, by assumption, \( \bar{\pi}^k \Vdash \text{clock} \leq b \), it follows \( \sum_{l=m}^{k-1} r_l \leq b \) and we can conclude that in \( \bar{\pi}^m \) the formula \( p \lor \Box \leq q \) is not satisfied: \( p \) is not satisfied in \( \bar{t}_m \) and moreover, \( q \) does not hold for all states reachable within time \( b \). Thus \( \bar{\pi} \not\Vdash \Box(p \lor \Box \leq q) \), and since \( \bar{\pi} \) was an arbitrary path in \( \mathcal{R} \) with initial state \( \{ \bar{t}_0 \} \) it follows from Lemma 2 that \( \mathcal{R}, L_{\Pi}, \{ \bar{t}_0 \} \not\Vdash \Box(p \lor \Box \leq q) \).

\[ \text{“} \Leftarrow \text{”}: \text{ Assume } \mathcal{R}, L_{\Pi}, \{ \bar{t}_0 \} \not\Vdash \Box(p \lor \Box \leq q) , \text{ and we show } \mathcal{R}, L_{\Pi}, \{ \bar{t}_0 \} \not\Vdash \Box(p \lor (q \models \text{clock} > b)) . \]

Let \( \pi = \{ \bar{t}_0 \} \xrightarrow{\bar{r}_1} \{ \bar{t}_1 \} \xrightarrow{\bar{r}_2} \ldots \) be a path in \( \mathcal{R} \). By assumption we know

\[ \exists j \geq 0 . \left( \bar{\pi}^j \not\Vdash \Box(p) \wedge \left( \exists k \geq j . \bar{\pi}^k \not\Vdash q \wedge \sum_{l=j}^{k-1} r_l \leq b \right) \right) . \tag{1} \]

By Lemma 2 there exists a (unique) path \( \bar{\pi} \) in \( \mathcal{R} \) satisfying formula (1) (where \( \pi \) is replaced by \( \bar{\pi} \)). Let \( j \geq 0 \) and \( k \geq j \) be the minimal indices satisfying (1). If \( \bar{\pi}^j \not\Vdash q \) then we are finished. Now assume \( \bar{\pi}^k \Vdash q \). In \( \bar{t}_k \) the proposition \( p \) is not satisfied implying that the clock status in \( \bar{t}_j \) is \( \sigma f l \) and the clock value is 0. It is clear that the clock value in all states between \( \bar{t}_j \) and \( \bar{t}_k \) is at most the sum of the duration of the tick steps between \( \bar{t}_j \) and \( \bar{t}_k \); moreover, by assumption, for all \( m \in \mathbb{N} \) with \( j < m \leq k \) it holds \( \sum_{l=j}^{m-1} r_l \leq b \). It follows that \( \bar{\pi}^l \not\Vdash \text{clock} > b \) for all \( j \leq l \leq k \). Hence \( \bar{\pi}^l \not\Vdash p \lor (q \models \text{clock} > b) \), and thus we have shown that \( \mathcal{R}, L_{\Pi}, \{ \bar{t}_0 \} \not\Vdash \Box(p \lor (q \models \text{clock} > b)) \).

\[ \square \]

### 5.3 Completeness and Termination

The strength of Real-Time Maude is clearly the expressiveness and the generality of the systems that can be specified, and moreover, powerful analysis techniques by simulation of specifications. However, the drawback of modeling in Real-Time Maude is the fact that, since we are dealing with general classes of infinite-state real-time systems, formal analyses are in general incomplete, and sometimes even unsound. In Real-Time Maude, on the one hand, an analysis method is called sound if any counterexample found by this method is a real counterexample in the system. On the other hand, an analysis method is called complete if the fact that no counterexample is found using this method actually implies that no such counterexample exists. For instance, the LTL model checking of a formula \( \phi \) is sound, if any counterexample found by the model checker is a real counterexample in the system. LTL model checking of a formula \( \phi \) is complete, if the fact that the model checker responds that the formula is satisfied, the formula is actually satisfied by the system, i.e., there exists no counterexample falsifying \( \phi \).

**Sound and complete model checking of time-bounded formulas**  
In [16] Olveczky and Meseguer have characterized easily checkable conditions for specifications in Real-Time Maude which imply soundness and completeness of LTL model checking under the maximal time sampling strategy. Given a real-time rewrite theory \( \mathcal{R} \), a labeling function \( L_{\Pi} \) with \( \Pi \) atomic propositions, then model checking an LTL formula \( \phi \) with the maximal time sampling strategy is sound and complete, if (1) \( \mathcal{R} \) is time-robust, and (2) all atomic propositions in \( \Pi \) are tick-stabilizing. Time-robustness of real-time rewrite theory intuitively means that time can either advance by any amount, by any amount up to and including a specific point in time, or not at all (and this property is not affected by advancing time unless we reach
the specific time bound in the second case), and instantaneous rules can only be applied when the system has advanced time by the maximal possible amount. The second condition for sound and complete model checking is that all atomic propositions are tick-stabilizing which means that they do not change arbitrarily during a maximal time step, more precisely, tick-stabilizing state propositions are allowed to change not at all during a maximal time step, or only once. For exact definitions see [16].

As our goal is to achieve soundness and completeness of model checking generalized time-bounded response and time-bounded safety MTL formulas, it is essential that time-robustness is preserved by both ♦- and □-transformation.

**Theorem 3.** Let $R$ be a real-time rewrite theory and let $\tilde{R}$ be the result of the ♦- or □-transformation applied to $R$. If $R$ is time-robust, then $\tilde{R}$ is time-robust.

**Proof.** This assertion is proved by the observation that, according to Lemma 1 and 2 both transformations do not change the original timed behavior of $R$.

We will now sketch a proof that model checking generalized time-bounded response and time-bounded safety MTL formulas, with tick-stabilizing atomic propositions, with the real-time system specification for our case study as described in the previous sections is indeed sound and complete.

**Theorem 4.** Let $R$ be a component-based real-time rewrite theory $R$ of the form described in Section 4 and let $\phi$ be a generalized time-bounded response MTL formula or a time-bounded safety MTL formula with tick-stabilizing atomic propositions. Then time-unbounded model checking of the transformed formula $\tilde{\phi}$ w.r.t. the transformed theory $\tilde{R}$ is sound and complete for the maximal time sampling strategy.

**Proof.** According to [16] it is sufficient to prove that $\tilde{R}$ is time-robust and $\tilde{\phi}$ only has tick-stabilizing atomic propositions.

A component-based real-time rewrite theory $R$ is time-robust since every instantaneous rewrite rule is triggered by the expiration of a timer, or by the fact that the system is inconsistent, i.e. at least two connected ports are not equal in value which can only happen after a previous instantaneous step. According to Theorem 3 our transformations described in Sect. 5 preserve time-robustness, so the transformed theory $\tilde{R}$ is time-robust as well.

The second condition requires that all atomic propositions are tick-stabilizing. By Lemma 1 and 2 both transformations do not change the original time behavior of $R$ hence all atomic propositions remain tick-stabilizing. Note that both transformations introduce a new (parameterized) atomic proposition clockLeq which, however, is tick-stabilizing, since the truth of clockLeq($b$) for $b$ a time bound is not changed during a maximal time step, or only once.

**Termination** In general, real-time rewrite theories are infinite-state systems for which model checking will not terminate. However, if we are dealing with finite-state systems, model checking will terminate. More precisely, in a real-time rewrite theory $R$ with a fixed time sampling strategy, if both the reachable state space of $R$ from an initial state $\{t_0\}$ and the number of different rewrite durations in all possible paths in $R$ from $\{t_0\}$ are finite, MTL model checking (of generalized time-bounded response MTL formulas or of time-bounded safety MTL formulas) terminates. So if the reachable state space in $R$ from an initial state $\{t_0\}$ (under a fixed time sampling strategy) is finite, then the reachable state space of the transformed real-time rewrite theory $\tilde{R}$ is finite. The main point in the proof of this fact is that

---

4Note that our tick rule deviates from other well-known examples of object-oriented real-time rewrite theories. Beside the condition that mte returns a value greater than 0 we require that the system is consistent, i.e. any connected ports are equal in value.
the clock value is never increased more than necessary: if it exceeds the upper bound \((b_{\text{max}}, b, \text{resp.})\) then it is not increased any more which does not change the truth of propositions of the form \(\text{clock} \geq b\), and ensures that the state space remains finite. For a detailed proof of this fact for a slightly different transformation (which however follows the same schema) we refer the interested reader to the work of Lepri et al. \([13]\). Thus, in our case study, model checking generalized time-bounded response MTL formulas or time-bounded safety MTL formulas with the maximal time sampling strategy will terminate.

5.4 Model Checking the Requirements of Digital Advertising

In this section, we briefly describe the analysis of our real-time specification of the digital advertising scenario in Real-Time Maude using the untimed LTL model checking command. The analysis has been performed on a single core processor (3.2GHz Intel\(^\text{®}\) Pentium 4) with 2 GB of RAM.

Note that the transformations described above require that the real-time object-oriented specifications are applied to are flat specifications in which rewrites happen only in the “outermost” configuration, and no rewrite is possible for attribute values. Our real-time specification, however, is non-flat as we are dealing with arbitrarily nested, hierarchical components. A simple solution to this problem is to adapt all rewrite rules such that they can only be applied at the outermost layer. For hierarchical components, this implies that the transmission rule must be duplicated for each layer of the component system (in our case study, for two layers). This replication is part of the future work on automatizing our analysis approach.

As all atomic propositions introduced in the following are tick-stabilizing and moreover, the real-time specification of our case study is time-robust, all analysis carried out (using the maximal time sampling strategy) are complete, i.e. if the model checking command of a temporal logic formula returns a positive result, then the formula is provably correct for all timed paths of the real-time specification.

Now, we discuss the analysis of our case study. To recall its basic functionality, the digital advertising system can be found in one of two configurations: in the first configuration, the system allows the user to interact with the displayed content, while the system displays autoactive content in the second configuration.

**Verification of the guarantees (G1) and (G2).** The contract to be satisfied by the digital advertising system consists of two guarantees: (G1) Being an interactive ad, the system should react to a user in front of the display. (G2) The content displayed must change at least every ten seconds: an advertising campaign using a large-scale display should not waste its capabilities by showing static content.

Verifying the system guarantee (G2) amounts to model check that always eventually the system changes the content of the display. This can be model checked by the command

\[
(mc \{\text{initial}\} |\equiv \[\] <> \text{imgChange} .)
\]

where \(\text{imgChange}\) holds if the value of the provided port \(\text{ENV.imgChange}\) of the environment component is true. However, the guarantee (G2) requires more: the displayed content must change at least every ten seconds, so the above LTL formula is obviously insufficient. Instead, the following formula must be used:

\[
\square(\diamond_{\leq 10000} "\text{image is changing"})
\]

This formula expresses that the image changes within ten seconds regardless of the current system configuration. It is worthwhile to investigate, since it is not immediately clear that the system guarantees this properties in both configurations and under arbitrary reconfigurations. This can be checked by applying
the transformations presented above and executing the command

\[
\text{mc \{initial\_MC\} |=u [] <> ( imgChange /\ clockLeq(10000) ) .}
\]

where initial\_MC is the transformed initial state. The model checking command took 8 minutes to complete, and did not find any counterexamples.

To verify (G1), we check the property

\[
□(□≤800 \text{“person is in front”} \rightarrow ◇≤1000 \text{“configuration 1”})
\]

using the transformations specified above. The property states that if a person stays in front of the display for at least 800 milliseconds, the system will be found in interactive mode within one second. Hence, the property specifies that the system always guarantees to react to a person in front of it. It can be model checked with the command

\[
\text{mc \{initial\_MC\} |=u [] ( (<> ( ¬ persThereIn /\ clockLeq(800))) \lor (<> ( in-C1 /\ clockLeq(1000))) ) .}
\]

Model checking this property took 6 minutes, and again no counterexample was found in the model of our case study.

**Verification of state steadiness (G3).** In addition to guaranteeing the system contract, we must assure that the system cannot exhibit a behavior in which reconfigurations are continuously performed and consume the available computing resources. In order to guarantee that configuration states are reasonably stable, two properties (G3) are checked with the help of the above transformations:

\[
□(\text{“reconfiguration triggered in conf. 1”} \rightarrow □≤200 \text{“configuration 1”})
\]

and similarly for configuration 2. These two properties state that the reconfiguration does not happen instantaneously, but must take at least 200 ms to complete. These properties guarantee that the system does not oscillate between configurations and that reconfigurations leave enough resources for the actual system operation.

The model checking command for the translated first property is the following:

\[
\text{mc \{initial\_MC\} |=u [] ( (¬ reconfTriggeredInC1) \lor (in-C1 W (¬ clockLeq(200)))) .}
\]

Executing this model checking command took 12 minutes; the second property can be translated and model-checked analogously.

Altogether, it is possible to check all real-time contract guarantees G1, G2, and G3 with the help of transformations and the built-in untimed Maude LTL model-checker.

### 6 Related Work

Pervasiveness and ubiquity of software systems is a topic that has been researched for about two decades [21, 19, 12]. A more recent stream of research is focusing on leveraging the new sources information becoming available through ubiquity of systems, i.e. bio-signals. The ultimate goal is to create biocybernetic loops [20] in which the system and the user create a feedback loop by influencing each other’s reactions, adapting the environment in an nonobtrusive way to the needs and ideas of the user without requiring explicit interaction. Emotional computing [18] is one of the most known manifestations of this
principle, but cognitive, and physical aspects can be considered as well in the creation of a biocybernetic loop.

Constructing pervasive user-centric applications [5] and, more general, the construction of self-adaptive applications have been a field of active research in recent years. In this work, we follow an approach based on reconfiguration of the system in order to achieve adaptability; [6] gives a decent overview of various approaches. Formal specification and verification of component-based systems and their reconfiguration is presented in several works, e.g. in [3], a logic-based approach to the specification of reconfiguration is developed, and in [2], reconfigurable components are verified by model checking formulas of the $\mu$-calculus. However, none of these frameworks for the verification of systems under reconfiguration use time semantics and therefore, only untimed properties can be verified.

Specifying systems with metric temporal logic goes back to the work of Koyman in [10] and Hooman in [9, 8]; in the latter work, a compositional approach to the verification of system components with metric temporal logic is presented. However, our work differs from the above works by using a dynamic architecture instead of a static one.

In a previous work [15] on specification and verification of systems in Real-Time Maude [7] already include ideas and methods how to verify timed temporal logic formulas using the LTL model checker of Maude. However, the first automatized transformational approach is presented in [13], which cover MTL formulas expressing the bounded response property or the minimum separation property. In this work, we have extended the ideas of [13] and presented analysis algorithms for two further and more general classes of MTL formulas.

7 Concluding Remarks

In the previous sections we have presented a new approach for formally modeling and analyzing pervasive user-centric applications at an early design stage. A system is modeled as a set of components which interact via connectors between provided and required ports. To allow adaptation of the system to new situations, the system can be dynamically reconfigured by changing the connections at runtime.

For specifying and prototyping such systems in a real-time setting, we use the algebraic rewriting language Real-Time Maude. Time-dependent system properties are expressed in Metric Temporal Logic (MTL). Real-Time Maude is also well-suited for model checking two practically important classes of formulas, the so-called generalized time-bounded response MTL formulas and the time-bounded safety MTL formulas. By extending the component-based Real-Time Maude models with suitable clocks and by transforming these kinds of MTL formulas into pure LTL formulas over the extended specification we have shown that these two classes of formulas can be analyzed with the (untimed) Maude LTL model checker, and that this analysis is sound, complete and terminating for the maximal time sampling strategy.

As case study we have specified a simple adaptive advertising scenario in Real-Time Maude and could automatically verify all three requirements (G1–G3) with the Maude model checker by using our analysis method. However, the execution of the model checking command took in all cases several minutes although we had already abstracted all values to boolean data. For more complex case studies, further optimizations will be necessary to make model checking a practically feasible analysis method. One simple, but efficient technique is to replace each model checking command, $\mathtt{mc}$ say, of form $\mathtt{modelCheck}(t'\ \mathtt{REST}, \ q)$ in a condition of a rule of the extended theory $\tilde{R}$ by a boolean expression; indeed, each $q$ is a state formula which can easily defined as a boolean function $\mathtt{is-q}$ such that $\mathtt{is-q}(t'\ \mathtt{REST})$ is true iff $\mathtt{mc}$ is true. Another technique is to reduce the nondeterminism in hierarchical components by directly connecting the ports of the environment with their corresponding ports of
the subcomponents (e.g. ENV.personThereIn with Camera.personThereIn). The resulting specification, \( \tilde{R} \), say, is stuttering equivalent (see e.g. [14]) with the original one; model checking \( \tilde{R} \) is a matter of seconds, not of minutes.

The metric temporal logic properties in this paper take only non-trivial upper bounds into account; the lower bound of any interval is 0. A “natural” extension of our work will be the study of metric properties over intervals with non-null lower bounds. Another interesting future work will be models with time-dependent probabilistic behavior. Pervasive user-centric applications interface with the real world through sensors and actuators, which may be unreliable. With a probabilistic real-time framework, it would be possible to model this uncertain behavior of the environment, and reason about the performance of pervasive user-centric applications in these environments.

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