Abstract—The recent research effort towards defining new communication solutions to guarantee high availability level with limited cabling costs and complexity has renewed the interest in ring-based networks. This topology has been recently used for industrial and embedded applications, with the implementation of many Real Time Ethernet (RTE) profiles. A relevant issue for such networks is handling cyclic dependencies to prove timing predictability, a key requirement for safety-critical systems, e.g., avionics and automotive. To deal with the performance evaluation of such networks, most relevant existing techniques are based on the Network Calculus framework, and consists in analyzing locally the delay upper bound in each crossed node, resulting in pessimistic end-to-end delay bounds. To overcome this limitation, an enhanced global timing analysis, accounting the flow serialization phenomena along the flow path, is proposed in this paper to improve the delay bounds tightness. The main contribution consists in defining and proving a closed form formula of the guaranteed end-to-end service curve of any flow of interest crossing a FIFO ring-based network. An extensive analysis of such a proposal has been conducted regarding the tightness of delay bounds and its impact on the system performance, in terms of system scalability and resource-efficiency. Results highlight the proposed approach efficiency to compute tight delay bounds, in comparison with conventional timing analysis and in reference with a worst-case delay lower bound.

Index Terms—Network Calculus, PMOO, Performance analysis, Ring topology, Cyclic dependencies, Delay bounds.

I. INTRODUCTION

The recent research effort towards defining new communication solutions to guarantee high availability level with limited cabling costs and complexity has renewed the interest in ring-based networks, which provide an implicit redundant path by introducing only one additional connection between the two end nodes, compared to line or star topologies [13]. The ring-based networks have been prominently used for industrial applications with the implementation of many Real Time Ethernet (RTE) profiles cited in IEC 61784-2 [4], e.g., EtherCAT [1], SERCOSIII [2] and Profinet-IRT [16], and recently in other application fields like automotive, e.g. RACE [20], and avionics, e.g. AeroRing [6]. A relevant issue for such networks is proving time predictability, i.e., limited cabling costs and complexity has renewed the interest in ring-based networks. To handle these limitations, a global timing analysis of ring-based networks is proposed in this paper to compute end-to-end delay bounds along the flow path. The main idea is to extend the most recent result of Network Calculus framework, the Pay Multiplex Only Once Principle (PMOO), to ring-based networks. The PMOO principle has been initially proposed in [9] for feedforward networks, i.e. networks without cyclic dependencies, and more recently adapted in [18] when considering different contention scenarios. This principle consists in paying the bursts of interfering flows only once, when accounting the flow serialization phenomena, to compute tight end-to-end delay bounds. In [5], we proposed as a first step the timing analysis of a specific ring-based network for avionics, called AeroRing, by adapting the end-to-end service curve formula in [9] to integrate the cyclic dependencies impact. However, we were not aware of the work in [15], which has proved that the formula in [9] may result in optimistic end-to-end delay bounds, i.e., less than the worst-case delays that may actually occur. Therefore, we propose in this paper a new closed form formula of the guaranteed end-to-end service curve in FIFO ring-based networks and provide the formal proof of its correctness.

Hence, the main contributions of this work are threefold:

• The analysis of conventional Network Calculus approaches for ring-based networks and the identification of their main limitations in terms of resource efficiency and system scalability;

• An enhanced worst-case timing analysis for ring-based networks, based on a global approach, through the definition of a new closed form formula of the guaranteed end-to-end service curve and the formal proof of its correctness;
Extensive analysis of the proposed timing analysis approach regarding the delay bound tightness and its impact on the system performance, in terms of system scalability and resource-efficiency. Results highlight the proposed approach efficiency to compute tight delay bounds, in comparison with conventional timing analysis and in reference with a worst-case delay lower bound.

In the next section, we present the basic concepts of the Network Calculus framework, used in this paper to conduct the worst-case timing analysis of ring-based networks. Then, we give an overview of the most relevant timing analysis approaches in this specific area, and relate them to our work in Section III. Afterwards, we detail the main assumptions, notations and system model in Section IV. In Section V, we revisit the conventional timing analyses of such networks and discuss their limitations. Our proposed approach is then detailed and proved in Section VI. Section VII presents the performance evaluation of our proposal, in comparison with conventional approaches, and we conclude the paper in Section VIII.

II. NETWORK CALCULUS BACKGROUND

The worst-case timing analysis proposed in this paper is based on Network Calculus formalism, providing deterministic upper bounds on delays and backlogs (queue sizes). Delay bounds depend on the traffic arrival described by the so called arrival curve \( \alpha \), and on the availability of the traversed node described by the so called minimum service curve \( \beta \). The definitions of these curves are explained as following.

**Definition 1.** (Arrival Curve) a function \( \alpha(t) \) is an arrival curve for a data flow with an input cumulative traffic function \( F(t) \), i.e., the number of bits received until time \( t \), iff:

\[
\forall t, F(t) \leq F \otimes \alpha(t)
\]

**Definition 2.** (Service Curve) The function \( \beta(t) \) is the minimum simple service curve for a data flow with an input cumulative traffic function \( F(t) \) and output cumulative traffic function \( F^*(t) \) iff:

\[
F^*(t) \geq F \otimes \beta(t)
\]

These definitions allow us to compute performance characteristics of flows, according to the following theorem.

**Theorem 1** (Performance Bounds). Consider a flow \( i \) constrained by an arrival curve \( \alpha \) crossing a system \( S \) that offers a service curve \( \beta \). The performance bounds obtained at any time \( t \) are given by:

Output arrival curve: \( \alpha^*(t) = \alpha \otimes \beta(t) \)

Backlog: \( \forall t : q(t) \leq (\alpha \otimes \beta)(0) =: v(\alpha, \beta) \)

Delay: \( \forall t : d(t) \leq \inf \{ t \geq 0 : (\alpha \otimes \beta)(-t) \leq 0 \} =: h(\alpha, \beta) \)

The computation of these bounds is greatly simplified in the case of leaky bucket arrival curve \( \alpha(t) = b + rt \), with \( b \) the maximal burst and \( r \) the maximum rate, i.e., the flow is \( \langle b, r \rangle \)-constrained; and the Rate-Latency service curve \( \beta_{R,L}(t) = [R \cdot t + T]_T \) with latency \( T \) and rate \( R \). In this case, the delay is bounded by \( \frac{b}{r} + T \), the backlog bound is \( b + r \cdot T \), and the output arrival curve is \( b + r(T + t) \).

Finally, to compute end-to-end delay bounds of individual traffic flows under FIFO policy, we need the following theorem and corollary.

**Theorem 2** (FIFO Minimum service curve). Consider a lossless node serving two flows, 1 and 2, in FIFO order. Assume that the node guarantees a minimum service curve \( \beta \) to the aggregate of the two flows, and flow 2 is constrained by the arrival curve \( \alpha_2 \). Define the family of functions \( \beta^1(t) \) by:

\[
\beta^1(t) = \left[ \beta(t) - \alpha_2(t - \tau) \right]^+ \cdot 1_{\{ t > 0 \}}
\]

if \( \beta^1(t) \) is wide-sense increasing, i.e., if \( s \leq t \) then \( f(s) \leq f(t) \), then it is a minimum service curve for the flow 1.

**Corollary 1** (Burstiness Increase due to FIFO). Consider a node serving two flows, 1 and 2, in FIFO order. Assume that flow 1 is constrained with a leaky bucket with a rate \( \rho_1 \) and a burst \( \sigma_1 \). Assume that the node guarantees to the aggregate of the two flows a rate latency service curve with a rate \( R \) and a latency \( T \), then flow 1 has a service curve equal to the rate latency service curve with a rate \( R - \rho_2 \) and a latency \( T + \frac{\sigma_2}{R} \); and at the output, the flow 1 is constrained by one leaky bucket with rate \( \rho_1 \) and burst \( \sigma_1 = \sigma_1 + \rho_1(T + \frac{\sigma_2}{R}) \).

III. RELATED WORK: TIMING ANALYSIS OF RING-BASED NETWORKS

The timing analysis of ring-based networks aims to compute the adequate temporal metric, e.g. minimum cycle duration or end-to-end delays, which will be compared to messages deadlines to verify the timing predictability.

For the most relevant ring-based Real Time Ethernet (RTE) profiles [4], adequate analytical approaches have been proposed to compute the minimum cycle time of the network communication, and an interesting overview of the most relevant ones is detailed in [17]. Conducting such approaches has been greatly simplified due to the time triggered communication scheme, e.g. Master/slave or TDMA, implemented by these RTE profiles. However, with an event-triggered communication scheme, the minimum cycle duration becomes no longer applicable and we need to compute worst-case end-to-end delays or at least upper bounds. In our case, we are interested in analyzing such event-triggered networks, which guarantee high resource utilization efficiency and configuration flexibility. However, the implementation of such a communication scheme on top of a ring topology induces cyclic dependencies, i.e., some transmitted flows are interdependent.
and their paths form cycles, which complicates the timing analysis compared to time-triggered solutions, e.g., Master/slave or TDMA.

A large body of work, based on Network Calculus [14], exists for timing analysis of networks with acyclic network graph, and an interesting overview of the most relevant approaches in this area is detailed in [10]. However, these approaches are not directly applicable for ring-based networks with cyclic dependencies. The fundamental problem to handle such dependencies consists in defining the input traffic upstream the node of interest, depending on the output of the node downstream, which in turn depends on its input.

To handle such cyclic dependencies, a first class of interesting approaches has been proposed to break the potential cycles through prohibiting the use of some links or sub-paths to ensure the feed-forward property [19][21]. Although these approaches simplify the timing analysis of such networks, they imply at the same time a reliability level deterioration, since the ring topology is transformed into line.

The second class of approaches introduces computation methods to support cycles using an iterative approach by successively analyzing the delay bound in each crossed node in the network, resulting in end-to-end delay bounds computation. The most relevant approaches are focusing on, either each crossed node delay bound, i.e., [8][7][12], or each crossed node backlog bound, i.e., [22][14]. However, these main conventional analysis methods result in pessimistic delay bounds, limiting the network performance in terms of resource-efficiency and system scalability, i.e. the guaranteed network utilization rate decreases dramatically when the network size increases.

To overcome these limitations, our main proposal in this paper consists in introducing an enhanced worst-case timing analysis of ring-based networks, based on a global method, when accounting the flow serialization phenomena along the flow path. A closed-form formula of the guaranteed end-to-end service curve is defined and proved herein; and the performance evaluation process of such a proposal shows its efficiency in terms of bound tightness and its impact on system performance, in comparison with the conventional analysis methods.

IV. SYSTEM MODEL

We consider the following assumptions and notations to compute the worst-case end-to-end delay bounds for a flow of interest \( f \) crossing the network:

- We consider a unidirectional ring topology, as shown in Fig. [1] connecting \( M \) nodes, labelled from 1 to \( M \), and serving a fixed number of flows \( I \). The notations \( l \oplus k \) and \( l \odot k \) designate \((l+k) \mod M\) and \((l-k) \mod M\) for the \( k \)-eth successor and \(-k\)-eth predecessor of node \( l \), respectively;
- Each flow \( i \in I \) follows a fixed path from its initial source until the final sink, defined as \( \text{path}_i = (0, i, \text{first}, i, \text{first} \oplus 1, \ldots, i, \text{first} \oplus (h_i - 1)) \), where 0 is a virtual node representing the source, \( i, \text{first} \)

V. CONVENTIONAL TIMING ANALYSIS OF RING-BASED NETWORKS AND LIMITATIONS

In this section, we detail the two main conventional local timing analysis techniques, based on Network Calculus. Then, we point out the limitations of each approach through an illustrative example.

A. Time Stopping Method

This approach has been proposed in [8] and consists of two steps. First, a finite burstiness bound for transmitted flows is assumed to obtain a set of equations to compute the delay bounds. Then, the feasibility conditions to solve these equations are defined. Therefore, we will first express all the equations to compute the upper bounds on bursts and delays in each crossed node. Then, we deduce the feasibility condition.
In [3], the burst propagation formula of a flow $i$ at the output of node $j \circ 1$ is given by:

$$\sigma_{j}^{i} = \sigma_{i}^{0} + \rho_{i} * D_{i}^{j \circ 1}$$

where $D_{i}^{j \circ 1}$ is the delay of the flow $i$ within the node $j \circ 1$.

Hence, at the input of node $j$, flow $i$ has already crossed $(j-1) \circ i$ nodes since node $i$. The input burst of flow $i$ at the node $j$ is given as follows:

$$\sigma_{i}^{j} = \sigma_{i}^{0} + \rho_{i} * \sum_{k=0}^{(j-1)\circ i} D_{i}^{j \circ k}$$

(1)

On the other hand, for any node $k$, the delay $D_{i}^{k}$ to process the flow $i$ is equal to the sum of its latency $T^{k}$ and the processing time of all the crossing bursts, i.e. $j \circ k$:

$$D_{i}^{k} = \sum_{j=0}^{(j-1)\circ k} \sigma_{j}^{k} + T^{k}$$

(2)

Equations (1) and (2) can be represented by the following matrix system:

$$\begin{cases}
D = A_{1} * B + C_{1} \\
B = A_{2} * D + C_{2}
\end{cases}$$

(3)

where $D$ is the vector of delays, $B$ is the vector of propagated bursts, and $C_{1}$ and $C_{2}$ are the constant vectors.

Thus, by propagating these constraints, we obtain:

$$D = [I - A_{1} * A_{2}]^{-1} * C_{3}$$

(4)

where $C_{3} = A_{1} * C_{2} + C_{1}$ and $I$ is the identity matrix.

The system admits a solution if the matrix determinant is not null. Then, the feasibility condition is $\rho < \frac{2C}{M(M-1)}$. Therefore, no feasible solution exists if the network utilization rate is greater or equal to $\frac{2}{M(M-1)}$. Figure 2 illustrates the maximum utilization rate according to the number of nodes. As we can notice, the utilization rate decreases dramatically when the network size increases, e.g. less than 0.1 for 20 nodes. This means that the network has to be under utilized to satisfy the feasibility condition, which limits the system resource-efficiency.

C. Limitations

Let’s consider an illustrative example to point out the limitations of these approaches, having the following characteristics:

- a ring-based network with $M$ nodes implementing FIFO service policy;
- we consider broadcast communications, i.e., each node transmits its traffic to all the network nodes;
- all nodes have the same transmission capacity $C = 1 Gbit/s$;
- each node generates a traffic with an arrival curve ($\alpha \sim (\sigma, \rho)$).

For time stopping method, the matrix $[I - A_{1} * A_{2}]$ is as follows:

$$\begin{pmatrix}
-C & \rho & 2\rho & \cdots & M\rho \\
M\rho & -C & \rho & \cdots & (M-1)\rho \\
(M-1)\rho & M\rho & -C & \cdots & (M-2)\rho \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\rho & 2\rho & 3\rho & \cdots & -C
\end{pmatrix}$$

(7)

The system admits a solution if the matrix determinant is not null. Then, the feasibility condition is $\rho < \frac{2C}{M(M-1)}$. Therefore, no feasible solution exists if the network utilization rate is greater or equal to $\frac{2}{M(M-1)}$. Figure 2 illustrates the maximum utilization rate according to the number of nodes.

As we can notice, the utilization rate decreases dramatically when the network size increases, e.g. less than 0.1 for 20 nodes. This means that the network has to be under utilized to satisfy the feasibility condition, which limits the system resource-efficiency.

B. Backlog-based Method

This second method has been initially proposed in [22] and more recently generalized in [14]. The authors provide the maximum backlog bound, when considering non work-conserving nodes. The maximum bound on the delay within each node $k$ for any flow $i$ is the processing time of the maximum backlogged traffic $Backlog_{k}^{i}$ served with a transmission capacity $R_{k}^{i}$, and it is as follows:

$$D_{i}^{k} = \frac{Backlog_{k}^{i}}{R_{k}^{i}}$$

(6)

The end-to-end delay communication bound still is computed using Eq. (5).

Fig. 2: The maximum utilization rate vs number of nodes for the Time Stopping Method
network, accounting the flow serialization phenomena and the cyclic dependencies along its path. First, we will present and prove a closed form service curve, guaranteed to any flow of interest $f$ on its subpath of $n \leq h_i$ hops, $\text{subpath}_f(n)$. Then, we detail the analysis approach to compute the worst-case end-to-end delay bound to deal with the cyclic dependencies.

### A. Closed Form End-to-end Service Curve in FIFO Ring-based Networks

In [18], the authors propose an innovative approach to compute tighter upper bounds on end-to-end delays than the classic iterative analysis approaches. However, this approach has been applied only for feedforward networks under blind multiplexing, when considering small scale network, e.g., three hops; thus could not be directly applied in our case because of cyclic dependencies, and need to be extended to large-scale networks. Hence, we prove herein the closed form service curve, detailed in Th. [3] for any flow of interest $f$ along its subpath $\text{subpath}_f(n)$ under the assumptions detailed in Section [IV]. It is worth noting that the burst of each flow in contention with the flow of interest is paid only once in Eq. [8].

**Theorem 3.** The service curve offered to a flow of interest $f$ along its subpath $\text{subpath}_f(n)$, in aggregate scheduling ring-based network with FIFO service curve nodes of the rate-latency type $\beta_{R,T}$ and leaky bucket constrained arrival curves $\alpha_{\sigma,\rho}$ is a rate-latency curve, $\beta_{R,\text{subpath}_f(n), T, \text{subpath}_f(n)}$ and defined as follows:

$$\beta_{f, \text{subpath}_f(n)}(t) = R_{\text{subpath}_f(n)}(t - T_{\text{subpath}_f(n)})^+$$

where,

$$\begin{align*}
R_{\text{subpath}_f(n)} &= \min_{k \in [0, n-1]} R_{f, \text{first}_k} - \sum_{i \in \mathbb{R}_f^n} \rho_i \\
+ &\sum_{i \in \mathbb{R}_f^n} \min_{k \in [0, n-1]} \frac{\sigma_i^k}{\min_{k \in [0, n-1]} k} \\
T_{\text{subpath}_f(n)} &= T_{f, \text{first}_k} \\
+ &\sum_{i \in \mathbb{R}_f^n} \min_{k \in [0, n-1]} \frac{\sigma_i^k}{\min_{k \in [0, n-1]} k} \\
\end{align*}$$

**Proof:** For any flow $i$, we consider the progress of its input and output cumulative traffic functions along its path $h_i$, as $(F_{i,0}^k(t_0), F_{i, \text{first}_k}^k(t_1), F_{i, \text{first}_k}^k(t_2), ..., F_{i, \text{first}_k}^k(t_{h_i-1}), F_{i, \text{first}_k}^k(t_{h_i}))$ and $(F_{i,0}^k(t_0), F_{i, \text{first}_k}^k(t_1), F_{i, \text{first}_k}^k(t_2), ..., F_{i, \text{first}_k}^k(t_{h_i-1}), F_{i, \text{first}_k}^k(t_{h_i}))$, respectively. The time indices are chosen to match the input/output node indices. It is worth noting that the output cumulative traffic function of flow $i$ from node $k$ at the instant $t_k$, $F_{i}^{k}(t_k)$, is its input cumulative function at node $k$ at the same instant, i.e., $F_{i}^{k}(t_k) = F_{i}^{k+1}(t_k)$. Moreover, in the particular case of the source node 0, $F_{i}^{0}(t_0) = F_{i}^{1}(t_0)$.

According to Def. [2] for any flow $f$ and any number of hops $n \in [1, h_f - 1]$, the following formula is verified for any
crossed node $f.first \oplus k$ along its subpath $\text{subpath}_f(n)$ with $k \in [0, n - 1]$:

$$F_{f.first \oplus k}(t_{f.first \oplus k}) - F_{f.first}(t_{f.first \oplus k})$$

Thus, obtaining the following equation when varying $k \in [0, n - 1]$:

$$
\sum_{k \in [0, n - 1]} (F_{f.first \oplus k}(t_{f.first \oplus k})) - F_{f.first \oplus k}(t_{f.first \oplus k}) \geq \sum_{k \in [0, n - 1]} \beta f.first(2, f.first) - t_{f.first \oplus k}) \geq \beta f.first(2, f.first) - t_{f.first \oplus k})$$

(10)

Since the output cumulative traffic function of flow $f$ at node $k$ at the instant $t_k$ is its input cumulative traffic function at node $k \ominus 1$ at the same instant, $F_{f.first \ominus 1}(t_k) = F_{f.first \ominus 1}(t_k)$, thus:

$$F_{f.first \ominus 1}(t_k) - F_{f.first}(t_k)$$

$$+ F_{f.first \ominus 1}(t_{f.first \ominus 1} - F_{f.first \ominus 1}(t_{f.first \ominus 1}))$$

$$+ F_{f.first \ominus 2}(t_{f.first \ominus 2} - F_{f.first \ominus 2}(t_{f.first \ominus 2}))$$

$$...$$

$$+ F_{f.first \ominus (n-1)}(t_{f.first \ominus (n-1)}) - F_{f.first \ominus (n-1)}(t_{f.first \ominus (n-2)})$$

$$= F_{f.first \ominus (n-1)}(t_{f.first \ominus (n-1)}) - F_{f.first}(t_{f.first \ominus 1})$$

(11)

Hence, Eq. (11) can be reformulated as follows, when considering for the flow of interest $f$, the defined set $\mathbb{K}_f^n$ and the parameters $M.first(i, f, n)$ and $M.last(i, f, n)$ for any flow $i \in \mathbb{K}_f^n$:

$$F_{f.first \ominus (n-1)}(t_{f.first \ominus (n-1)}) - F_{f.first}(t_{f.first \ominus 1})$$

$$\geq \sum_{k \in [0, n - 1]} \beta f.first(2, f.first) \geq \sum_{i \in \mathbb{K}_f^n} \beta f.first(2, f.first) \geq \sum_{i \in \mathbb{K}_f^n} \min_{k \in [0, n - 1]} R_{f.first \ominus k}(t_{f.first \ominus k} - t_{f.first \ominus k}) + T_{f.first \ominus k}$$

(13)

To substitute the cumulative traffic functions of flows in $\mathbb{K}_f^n$ by their arrival curves in Eq. (13), we use the same idea than in Eq. (12) and the causality constraint of cumulative traffic functions, i.e., $\forall t, F(t) \geq F^*(t)$. Moreover, since the input arrival curve of a flow $i$ at each crossed node $k$ is $\alpha_i^{\ominus 1}(t)$, we obtain the following:

$$\sum_{i \in \mathbb{K}_f^n} \sum_{l = M.first(i, f, n)} \alpha_i^{\ominus 1}(t_l) - F_{i.first \ominus 1}(t_{i.first \ominus 1})$$

(14)

$$= \sum_{i \in \mathbb{K}_f^n} (F_{i.first \ominus 1}(t_{M.first(i, f, n)})$$

$$- \sum_{i \in \mathbb{K}_f^n} (F_{i.first \ominus 1}(t_{M.first(i, f, n)}))$$

$$- \sum_{i \in \mathbb{K}_f^n} (F_{i.first \ominus 1}(t_{M.first(i, f, n)}))$$

$$- \sum_{i \in \mathbb{K}_f^n} (F_{i.first \ominus 1}(t_{M.first(i, f, n)}))$$

$$- \sum_{i \in \mathbb{K}_f^n} (F_{i.first \ominus 1}(t_{M.first(i, f, n)}))$$

(15)

Giving the ring-based topology, any flow $i \in \mathbb{K}_f^n$ will be multiplexed with the flow $f$ for the first time at $t_{M.first(i, f, n)} \geq t_{f.first \ominus 1}$ and leave the subpath $\text{subpath}_f(n)$ at an instant $t_{M.last(i, f, n)} \leq t_{f.first \ominus (n-1)}$. Hence, when substituting the cumulative traffic functions in Eq. (13), using Eq. (14) and according to Th. 2, we obtain the following equation:

$$F_{f.first \ominus (n-1)}(t_{f.first \ominus (n-1)}) - F_{f.first}(t_{f.first \ominus 1})$$

$$\geq \min_{k \in [0, n - 1]} R_{f.first \ominus k}(t_{f.first \ominus k} - t_{f.first \ominus k})$$

$$- \sum_{i \in \mathbb{K}_f^n} \alpha_i^{M.first(i, f, n)}(t_{M.last(i, f, n)} - t_{f.first \ominus (n-1)})$$

(15)
where \( \Delta = t_{f, \text{first}(n-1)} - t_{f, \text{first}}^\oplus 1 \).

According to Corollary [1] in the particular case of rate-latency service curves and leaky-bucket arrival curves, i.e.,
\[ \alpha_i^{M_{\text{first}}(i, f, n)^{\oplus 1}}(t) = \sigma_i^{M_{\text{first}}(i, f, n)^{\oplus 1}} + \rho_i \alpha \]
the residual service curve offered to flow \( f \) is wide-sense increasing rate-latency service curve as follows:

\[ F_f^v, f_{\text{first}}^{\oplus (n-1)}(t_{f, \text{first}}^{\oplus (n-1)}) - F_f^{f_{\text{first}}^\oplus 1}(t_{f, \text{first}}^\oplus 1) \]
\[ \geq \left[ \min_{k \in [0, n-1]} R_f f_{\text{first}}^{\oplus k} - \sum_{i \in \mathcal{K}_f} \rho_i \right] \times \left[ (t_{f, \text{first}}^{\oplus (n-1)}) - t_{f, \text{first}}^{\oplus 1} - \sum_{k \in [0, n-1]} T_f f_{\text{first}}^{\oplus k} \right] \]
\[ - \sum_{i \in \mathcal{K}_f} \sigma_i^{M_{\text{first}}(i, f, n)^{\oplus 1}} \min_{k \in [0, n-1]} R_f f_{\text{first}}^{\oplus k} \]
\[ - \sum_{i \in \mathcal{K}_f} \sigma_i^{f, \text{first}}^{\oplus 1} \min_{k \in [0, n-1]} R_f f_{\text{first}}^{\oplus k} \]  \tag{16} \]

Furthermore, we have two possibilities for the first multiplexing point of flows \( i \) and \( f \) due to the ring-based topology: the first node crossed by flow \( i \), i.e., \( i_{\text{first}} \); or the first node crossed by flow \( f \), i.e., \( f_{\text{first}} \). Hence, when substituting \( M_{\text{first}}(i, f, n)^{\oplus 1} \) in Eq. \ref{16}, we obtain the following equation:

\[ F_f^{v, f_{\text{first}}^{\oplus (n-1)}}(t_{f, \text{first}}^{\oplus (n-1)}) - F_f^{f_{\text{first}}^\oplus 1}(t_{f, \text{first}}^\oplus 1) \]
\[ \geq \left[ \min_{k \in [0, n-1]} R_f f_{\text{first}}^{\oplus k} - \sum_{i \in \mathcal{K}_f} \rho_i \right] \times \left[ (t_{f, \text{first}}^{\oplus (n-1)}) - t_{f, \text{first}}^{\oplus 1} - \sum_{k \in [0, n-1]} T_f f_{\text{first}}^{\oplus k} \right] \]
\[ - \sum_{i \in \mathcal{K}_f} \sigma_i^{M_{\text{first}}(i, f, n)^{\oplus 1}} \min_{k \in [0, n-1]} R_f f_{\text{first}}^{\oplus k} \]
\[ - \sum_{i \in \mathcal{K}_f} \sigma_i^{f, \text{first}}^{\oplus 1} \min_{k \in [0, n-1]} R_f f_{\text{first}}^{\oplus k} \]  \tag{17} \]

The Eq. \ref{17} defines the service curve of flow \( f \) on its subpath \( \text{subpath}_f(n) \), which finishes the proof.

\[ \frac{\text{between these bursts and the latency is obtained as follows:}} \]

\[ T_{\text{subpath}}(n) = \sum_{k \in [0, n-1]} T_{f, \text{first}}^{\oplus k} \]  \tag{18} \]

The arrival curve of the flow of interest \( f \) at the output of the last node of its subpath, \( \text{subpath}_f(n) \), is obtained throughout the application of Theorem \ref{1} as follows:

\[ \alpha_{f, \text{first}}^{\oplus (n-1)}(t) = \alpha_f^0 \oplus \sigma_{\text{subpath}_f(n)}^0(t) \]
\[ \Rightarrow \sigma_{f, \text{first}}^{\oplus (n-1)}(t) = \sigma_f^0 + \rho_f \times T_{\text{subpath}}(n) \]  \tag{19} \]

The interdependency due to the cycle can be seen between latency equation (18), depending on the propagated bursts, and the equation of propagated bursts (19), depending in its turn on the latency.

The main issue is to find the different latencies and bursts of any flow \( f \in I \) along any of its subpaths with a length \( n \in [1, M] \). Let \( T \) be the vector holding all the \( T_{\text{subpath}}(n) \) variables, for \( f \in I \) and \( n \in [1, M] \); and \( \sigma \) the one holding all the \( \sigma_{f, \text{first}}^{\oplus (n-1)} \) variables, for \( f \in I \) and \( n \in [1, M] \). From formula (18) and (19), we construct the following matrix system:

\[ \begin{align*}
T = C_1 + A_1 \times \sigma \\
\sigma = C_2 + A_2 \times T
\end{align*} \]  \tag{20} \]

where,  
- \( A_1 \) holds all the coefficients of the unknown bursts and \( C_1 \) the constants of formula \ref{18};  
- \( A_2 \) holds all the coefficients of the unknown latencies and \( C_2 \) the constants of formula \ref{19}.

Then, by propagating the constraints, we obtain the following relation:

\[ T = (Id - A_1 \times A_2)^{-1} \times C_3 \]  \tag{21} \]

where \( C_3 = C_1 + A_1 \times C_2 \).

The system admits a solution if the matrix \( (Id - A_1 \times A_2) \) is invertible, i.e., its determinant is not null. If this condition is verified, then we can compute the vectors \( T \) and \( \sigma \). Afterwards, the delay bound of any flow \( i \), after crossing \( n \in [1, M] \) nodes, can be computed as following (Theorem \ref{1}):

\[ \frac{\sigma_f^0}{R_{\text{subpath}}(n)} + T_{\text{subpath}}(n) \]
In the simple case of broadcast communication with one traffic class and with an utilization rate per node of \( x \), the determinant of the matrix \((1d - A_1 \times A_2)\) is a polynomial function of the variable \( x \) with a degree \( M \):

\[
(1 - M) \times (x + 1)^{(M-1)} \times \left(x - \frac{1}{M - 1}\right)
\]

This matrix system resolution is feasible for \( x \leq \frac{1}{M-1} \), which induces a network stability condition under a full network utilization, which will clearly enhance the resource-efficiency of the system, compared to conventional analytical approaches, as it will be illustrated in the next section.

VII. PERFORMANCE EVALUATION

In this section, we conduct performance analysis of the proposed approach to measure the obtained delay bound tightness and its impact on the system performance, in reference with conventional approaches based on Network Calculus, e.g. Backlog-based and Time stopping methods. Moreover, we consider an achievable worst-case delay bound, to have a more precise idea on the pessimism ratio of the computed upper bound delay, in comparison with the exact worst-case delay, i.e., if the gap between the upper and lower bounds is small, then the exact worst-case delay is not far away from the computed maximum delay bound. First, we describe the case of study and the considered scenarios. Then, we detail the tightness of bounds and its impact on the system performance, under the different analysis approaches.

A. Case of study

We consider the case study with the following assumptions:

- The topology is a unidirectional ring topology, connecting \( M \) nodes;
- The links speed is 1Gbit/s;
- All equipments are similar, implement FIFO and send the same traffic in broadcast mode;
- Technological latency within each node is 600\( ns \);
- Each equipment generates one type of traffic, representative of audio streaming flows, with an arrival curve \( \sim (166\text{bytes}, 128\text{Kbit/s}) \).

To conduct the performance analysis of our proposed timing analysis approach, we consider the three following scenarios:

- Scenario 1: to analyse the impact of the traffic bursts, we increase the burst size from 166 bytes until 1500 bytes for a network of 35 nodes.
- Scenario 2: to analyse the impact of increasing the network congestion, the upper bounds on end-to-end delays are computed when the number of nodes is fixed, \( M = 10 \), and the network load is increasing by a step of 10% until reaching 100%.
- Scenario 3: to analyse the impact of enlarging the network scalability, i.e., network size, the upper bounds on end-to-end delays are computed under the variation of the node number, from 10 to 100 nodes by a step of 10 nodes.

B. Tightness of Bounds

To investigate the delay bound tightness computed with the different approaches, we benchmark the delay bounds obtained with our proposed method, denoted as Ring-PMOO in the figures, against the two conventional analyses, i.e., Time Stopping and the Backlog-based methods, and in reference with the achievable worst-case delay, denoted as WCD lower bound. This latter is computed when considering an intuitive worst-case scenario, which consists in accounting for each flow of interest only the impact of direct interferences within each crossed node, and ignoring the impact of the upstream flows at its source node, i.e. it is considered as null. The gap between the computed upper and WCD lower bounds will give us an idea about the delay bound tightness. In fact, this interval includes necessarily the exact worst-case delay; thus if this interval duration is small, then the upper bound delay is tight.

First, we consider Scenario 1 to analyse the impact of the traffic bursts on the tightness of bounds. As illustrated in Fig 5, the delay bound increases when increasing the burst size, since the multiplexing delay within each crossed node increases. As we can notice, the conventional approaches lead to overly pessimistic bounds, in comparison with the proposed one. For example, for a burst of 1500 bytes, the upper bound on the end-to-end delay is almost equal to 1ms, 10ms and 100ms with Ring-PMOO, Time stopping and the backlog-based approaches, respectively. Moreover, the WCD lower bound is about 0.5ms, which yields to a low pessimism ratio of the computed upper bound with Ring-PMOO approach, i.e., \( \leq 0.5\text{ms} \). This fact proves the delay bound tightness obtained with the proposed approach under high bursty traffic, in comparison with the conventional methods.

![Fig. 5: Upper bounds on the end-to-end delays vs size of burst](image-url)

Then, we consider Scenario 2 to analyse the impact of increasing congestion on the tightness of bounds. As shown...
in Fig. 6 the Time Stopping method leads to infinite upper bounds under a total utilization rate higher than 22.22%. Furthermore, as we can notice, the Backlog-based method yields loose upper bounds in comparison with the proposed one. For example, under full utilization load, the delay bound is higher than 10s with the Backlog-based method, whereas is less than 1ms with the proposed one. Moreover, this latter is very close to the WCD lower bound, which proves its tightness under high network congestion.

![Fig. 6: Upper bounds on the end-to-end delay bounds vs network load](image)

Finally, we consider Scenario 3 to analyse the impact of enlarging the network scalability on the tightness of bounds. As shown in Fig. 7 the proposed approach still outperforms the conventional approaches in terms of delay tightness when the network scale increases. As an example, for a network size of 100 nodes, the upper bounds are more than 1s and 10ms with the Backlog-based and Time Stopping methods, respectively; whereas it is less than 1ms with the proposed approach. Moreover, the gap between the WCD lower bound and the computed upper bound is less than 0.1ms, which proves the bound tightness for large-scale networks.

**Discussion:** These analysis results under various scenarios show the tightness of the end-to-end delay upper bounds computed with the proposed approach (Ring-PMOO), in comparison with the conventional ones and in reference with the WCD lower bound.

### C. Sensitivity Analysis

We discuss herein the impact of the timing analysis method on the system performance, in terms of resource efficiency, i.e., the maximum achievable utilization rate guaranteeing the network stability condition, and system scalability, i.e. the network size guaranteeing the system schedulability. Hence, we reconsider the different scenarios to show their impact on both metrics.

**Impact of increasing congestion:** as illustrated in Figure 6 when considering Scenario 2, the time stopping method diverges for a global utilization rate around 22.22%, which corresponds to the feasibility condition of $\frac{2M}{M-1}$ detailed in Section V-A; whereas a full utilization rate still is achievable under the Backlog-based and Ring-PMOO approaches. It is worth noting that the tight delay bounds obtained with the proposed approach will necessarily enhance the system schedulability, since all the messages with a deadline less than 1ms are schedulable under full utilization rate.

**Impact of enlarging network scalability:** Figure 7 illustrates the impact of the network size on the end-to-end delay bounds using the different conventional methods and the proposed one, when considering Scenario 3. Obviously, the delay bounds increase with the network size, since the number of generated messages and crossed nodes increase. As we can notice, the proposed approach leads to tight delay bounds for large-scale networks, in comparison with the conventional methods. This fact enhances the system scalability, while still guaranteeing the system schedulability for all messages with a deadline less than 1ms for large-scale network, e.g., 100 nodes.

**Discussion:** this detailed sensitivity analysis shows that using the proposed timing analysis approach yields to enhance the guaranteed system schedulability for large-scale network under full utilization rate, in comparison with the conventional timing analyses. The Time Stopping method limits the network performance in terms of resource efficiency, i.e. the utilization rate decreases dramatically when the network size increases; whereas the Backlog-based method limits the system scalability, i.e. the nodes number is hardly constrained to guarantee hard temporal deadlines.

![Fig. 7: Upper bounds on the end-to-end delays vs number of nodes](image)
VIII. CONCLUSIONS

In this paper, an enhanced worst-case timing analysis, based on Network Calculus framework, has been proposed for FIFO ring-based networks. Unlike conventional approaches based on local analysis of the delay bound in each crossed node in the network, our proposed approach is based on a global analysis method, accounting the flow serialization phenomena along the flow path, to allow the computation of tighter end-to-end delay bounds. Hence, we contribute a new theorem, which defines the end-to-end service curve of any flow of interest crossing a FIFO ring-based network, and we provide formal proof of its correctness. Afterwards, the performance evaluation of such a proposal under various scenarios has been conducted, and results highlight the tightness of computed delay bounds, in contrast to conventional methods, i.e. Time Stopping and Backlog-based methods, and in reference with a lower bound of the exact worst case delay. Furthermore, the proposed method yields to guarantee enhanced system performance, in terms of resource efficiency and network scalability, i.e., guaranteed system schedulability for large-scale network under full utilization rate.

As a next step, we envision to extend this result to other service policies, such as Static Priority and Weighted Round Robin. Furthermore, the proved end-to-end service curve formula needs to be generalized for any non-feedforward network topology.

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