Improving students’ mathematical representational ability through RME-based progressive mathematization

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Abstract. This study aims to determine the differences in the improving of mathematical representation ability based on progressive mathematization with realistic mathematics education (PMR-MP) with conventional learning approach (PB). The method of research is quasi-experiments with non-equivalent control group designs. The study population is all students of class VIII SMPN 2 Tangerang consisting of 6 classes, while the sample was taken two classes with purposive sampling technique. The experimental class is treated with PMR-MP while the control class is treated with PB. The instruments used are test of mathematical representation ability. Data analysis was done by t-test, ANOVA test, post hoc test, and descriptive analysis. The result of analysis can be concluded that: 1) there are differences of mathematical representation ability improvement between students treated by PMR-MP and PB, 2) no interaction between learning approach (PMR-MP, PB) and prior mathematics knowledge (PAM) to improve students’ mathematical representation; 3) Students’ mathematical representation improvement in the level of higher PAM is better than medium, and low PAM students. Thus, based on the process of mathematization, it is very important when the learning direction of PMR-MP emphasizes on the process of building mathematics through a mathematical model.

1. Introduction
Mathematics becomes one of the important knowledge to be mastered by students from elementary school to college level to achieve mathematical ability. In the Indonesian national curriculum, both in 2006 curriculum and the 2013 curriculum, mathematical ability is the target to be achieved in mathematics learning.

Although this mathematical ability is very important, the students’ achievement level in mathematical ability in Junior High School is still low. This is seen from the results reports on student achievement of Indonesia in the international scale TIMSS and PISA from the first participate until now, there are no significant change outcomes ranking and always occupy the 10 lowest among other participating countries [1]. The low mathematical ability of students according to [2] is influenced by several factors such as students' accustomed to acquire and use formal mathematics knowledge in the
classroom that is the teacher give without knowing how to get the formula, and evaluation system in Indonesia still use level low.

It is important to understand various approaches to learning mathematics that can be used in order to develop students’ skills, especially the ability of mathematical representation. One of the mathematical approaches that provide ample space for students to construct and discover mathematical concepts is realistic mathematics education. In Indonesian version, known as Pendidikan Matematika Realistik (PMR). Realistic learning is a learning approach that places mathematics as a human activity derived from the real world or the world that is close to the student and can be imagined by the students [3]. In this case, students are actively given the opportunity to reinvent a mathematical concept through teacher guidance [4]. The process of student activity is mainly to build a mathematical model or concept. In RME, the process of mathematization consists of horizontal and vertical mathematization.

The interesting thing about the mathematization process is that students should not be considered as passive receivers of ready-made mathematics, but students are considered as active individuals who are able to develop their own mathematical potential ([5]; [6]). In the process of mathematization, students are required to build or develop a mathematical model of a model that has been formed previously with the knowledge possessed. This is in line with the opinion of Freudenthal [4], mathematization in learning mathematics is as a process of improvement and development of mathematical ideas progressive mathematization process occurs gradually when the process of vertical mathematization forms formalization process of mathematical concepts of mathematical models derived from horizontal mathematization [1]. Based on the background of the problems that have been stated above, then this study aims to determine the differences in the improving of mathematical representation ability based on progressive mathematization with realistic mathematics education (PMR-MP) with conventional learning approach (PB).

2. Progressive Mathematization
The meaning of representation based on [7] mentions that the student’s mathematical representation is a form of ideas expressions or mathematical ideas that are displayed as an attempt to find a solution of the problem he was facing. To construct mathematical concept as a form of mathematical representation of a contextual problem hence required step or process of mathematization that consists of the process of horizontal mathematization and vertical mathematization. Freudenthal in [4], in the learning of mathematics, is a process of improvement and development of mathematical ideas gradually. In accordance with De Lange's opinion [1] which defines mathematization as organizing activities in finding order, relationships, and mathematical structures using the prior knowledge and skills built.

The importance of mathematization is also used by PISA to develop students’ mathematical abilities. In PISA’s explanation [8], states that mathematical processes are an activity that describes what someone does to connect the contextual problems with mathematics. The process of mathematizing in PISA’s version related to building mathematical models and concepts begins by identifying the contextual problems, looking for relationships with symbols, patterns relating to the problem so as to form a mathematical model. The staging process from contextual problem to the mathematical problem is done by the process of horizontal mathematization. While finding a mathematical problem-solving process in form of mathematical representation, formal mathematics and generalization is the process of vertical mathematization or model for and formal model.

The constructing or finding mathematical process concepts using mathematical models that have been formed before is called progressive mathematization process. For Trafer in [9], schematization was not the only argument in the aforementioned progressive schematization course, but be also emphasized other characteristics of informal strategies such as mental calculation, flexible arithmetic, and estimation. This is in accordance with Wijaya’s statement [1] that, using models, schemes, diagrams, and symbols for progressive mathematization is important for mathematical discovery and development concepts by students. The model, in this case, relates to the model of the situation and the
mathematical model developed by the students themselves. [4] and [10] mentions four levels or stages in the development of mathematical models, which are: 1) situational level, 2) referential level, at this level, 3) the general level, 4) formal level. The mathematization process is a key in the RME so that it is not only the main activity of math but also introduces students to situations close to the students’ daily lives in searching for mathematical models. In addition, at the final stage of mathematization is the formalization through the axiomatization mathematical concepts so that with this process the students’ mathematical ability, especially students’ mathematical representation can increase.

3. Method
The research method used in this research is quasi-experiment with the design of nonequivalent control group design [11], consisting of experimental group and control group. An experimental group is a group that gets PMR-MP, while the control group is a group that gets PB. The data to be analyzed in this research is N-Gain data (normalized gain) from students’ mathematical representation test. This normality and homogeneity test used the Kolmogorov-Smirnov and Levene tests. The difference test and interaction analysis using the t-test and ANOVA. All statistical calculations use the IBM SPSS 21 computer program.

4. Results and Discussion
4.1 Mathematical Representation Ability Description
The students’ mathematical representation improvement data analysis was done by using N-Gain data from mathematical representation test. The descriptive statistical is presented in Table 1

| Category of PAM | Average N-Gain on PAM Category |
|-----------------|--------------------------------|
| PMR-MP          | 0.53 7 0.39 21 0.30 8 0.39 36 |
| PB              | 0.41 6 0.27 24 0.12 6 0.27 36 |

Based on Table 1, the PAM category (high. medium. low) the average N-Gain of the students’ mathematical representation ability in the class by PMR-MP learning is greater than in the class taught by PB. This can be seen from the total N-Gain average of PMR-MP and PB is 0.39> 0.27. This result shows that the students’ mathematical representation abilities improvement based on the overall data and PAM categories (high. medium. low) in the classroom with the students by PMR-MP learning better than the control class with the students by PB.

4.2 Mathematical Representation Improvement Based on Learning Analysis and PAM
Based on the normality analysis data, the category PAM each data is normally distributed. Further analysis to determine the improvement of the ability of mathematical representation based on the overall data and category PAM (high, medium, low) between the experimental class and the control class is done by using the t-test. The different test results between the experimental and control classes are presented in Table 2.

| Category of PAM | t-test | Sig. (2-tailed) |
|-----------------|--------|----------------|
| Overall         | 3.262  | 0.002          |
| High            | 2.392  | 0.036          |
| Medium          | 2.653  | 0.011          |
| Low             | 2.633  | 0.026          |

Based on Table 2, the difference test between the two classes of experimental class and control class with the probability result is 0.001 < $\alpha = 0.05$. means reject the Ho hypothesis. These results indicate that students who have received PMR-MP learning have better mathematical representation
than students with PB learning in overall data. While to know the difference test between experimental class and control class in PAM (high, medium, low) category with result of probability value is $0.0175 < \alpha = 0.05, \ 0.0056 < \alpha = 0.05, \text{ and} \ 0.013 < \alpha = 0.05$ means reject $H_0$. These results indicate that students who have received PMR-MP learning have better mathematical representation ability than students receiving PB learning in the high PAM category, medium, and low PAM category.

The conclusion of different test analysis based on learning in the PAM category is there is an improvement of students’ mathematical representation ability in the experimental class where the students treated by PMR-MP learning better than the students in the control class who treated by PB.

4.3 Interaction Analysis of Mathematical Representation Ability Based on Learning and PAM

Interaction analysis between learning and PAM was done by two-path ANOVA analysis. The ANOVA Analysis results are presented in Table 3 below.

| Source       | Type III Sum of Squares | df | Mean Square | F    | Sig. |
|--------------|-------------------------|----|-------------|------|------|
| Corrected Model | .750^a                  | 5  | .150        | 8.346| .000 |
| Intercept     | 5.823                   | 1  | 5.823       | 324.134 | .000 |
| Learning      | .218                    | 1  | .218        | 12.144| .001 |
| PAM           | .492                    | 2  | .246        | 13.687| .000 |
| Learning * PAM| .003                    | 2  | .002        | .097 | .907 |
| Error         | 1.186                   | 66 | .018        |      |      |
| Total         | 9.913                   | 72 |             |      |      |
| Corrected Total | 1.935                | 71 |             |      |      |

From table 3 above, It shows that the value of significance $0.907 > 0.05$, so that the $H_0$ hypothesis is accepted. It means that there is no interaction between learning (PMR-MP, PB) and PAM (high, medium, low). In other words, this means that the difference in mean score of mathematical representation ability of mathematical knowledge students (high, medium, and low) who treated by PMR-MP does not differ significantly with students who have treated by PB. Graphically, the interaction can be seen in Figure 1 below.

![Figure 1](image1.png)

Based on figure 1 above, it can be explained that the learning with PMR-MP is appropriate for all categories of PAM (high, medium, low) in improving mathematical representation ability. This can be seen from the average students’ mathematical representation ability score who treated by PMR-MP higher than students who received treated by PB. From figure 1 above also indicates that students with low mathematical ability gain the greatest benefit in learning based on PMR-MP rather than students
with high and medium mathematical skills. This can be demonstrated through the difference of the average score of N-Gain ability of mathematical representation between students who learn through PMR-MP and PB in successive high-ability students (0.12), moderate (0.12), low (0.18).

From Table 3, it can be seen that the value calculation of F for the learning approach is 12.144 with a significance level of 0.001 < 0.05, so the Ho hypothesis is rejected. This means that there is a mathematical representation skill improvement difference based on the learning approach. In other words, there is a difference in the improvement of the significant mathematical representation ability among students who get the learning approach of PMR-MP and PB learning.

Referring to the research and discussion results above, it can be concluded that Freudenthal [6] further mentions that mathematization is a key process in RME. The reason is the first mathematization is not just a math-centered activity but also introduces students to situations close to the student’s daily life. Second, the final stage of mathematics is formalization through axiomatization. This final stage is not a starting point when mathematical concepts are considered as found in traditional mathematical learning. Therefore, mathematics learning should be delivered in the form by giving students an opportunity to reinvent ideas and mathematical concepts along with teacher guidance through exploration of various contextual issues. If we hold fast to the objectives of learning mathematics and apply the process standard properly with the RME approach then the students ‘mathematical abilities development in particular students’ mathematical representation may increase.

5 Conclusion

Based on the analysis results of the findings obtained in this study, it can be concluded several things as follows: 1) students’ mathematical representation improvement ability who received learning with the PMR-MP approach is better than students who received learning with PB 2) students’ mathematical representation improvement ability who received learning with the PMR-MP approach are better than students who received learning with PB in terms of PAM category (high, medium, low). 3) there is no interaction between learning approaches (PMR-MP and PB) with PAM (high, medium, low) to mathematical representation ability. and 4) student responses on learning by PMR-MP approach students give a positive response they feel happy with the implementation experienced learning.

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