A NOTE ON HAYMAN’S CONJECTURE

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Abstract. In this paper, we will give suitable conditions on differential polynomials $Q(f)$ such that they take every finite non-zero value infinitely often, where $f$ is a meromorphic function in complex plane. These results are related to Problem 1.19 and Problem 1.20 in a book of Hayman and Liningham [9]. As consequences, we give a new proof of the Hayman conjecture. Moreover, our results allow differential polynomials $Q(f)$ to have some terms of any degree of $f$ and also the hypothesis $n > k$ in [11 Theorem 2] is replaced by $n \geq 2$ in our result.

1. Introduction and Main Results

In 1940, Milloux obtained the first result involving the value distribution of meromorphic functions together with their derivatives and called usually Milloux’s inequality. This result has led to a series of important research questions, for example Hayman [7] (or Problem 1.19 in [9]) proved that if $f$ is a meromorphic function in the plane omitting a finite value $a$, and if its $k$-th derivative $f^{(k)}$, for some $k \geq 1$, omits a finite nonzero value $b$, then $f$ is constant. This result is known as Hayman’s alternative. Closely linked to Milloux’s inequality and Hayman’s alternative is Hayman’s conjecture on the value distribution of certain differential monomials, if $f$ is a transcendental meromorphic function and $n \in \mathbb{N}$, then Hayman conjectured that $(f^n)'$ takes every finite nonzero value infinitely often for all $n \geq 2$. Hayman [7] himself proved this conjecture for $n \geq 4$, Mues [13] proved the case of $n = 3$. In 1995, Bergweiler and Eremenko [1], Chen and Fang [3], and Zalcman [15] independently proved the conjecture for $n = 2$. In [2, 5, 10, 13, 15], the authors generalized Hayman’s conjecture to $f^{(k)}$. The effective result in this direction was given in [11 Theorem 2]: if $f$ is a transcendental meromorphic function in the plane and $m > k \geq 1$ then $(f^m)^{(k)}$ assumes every finite non-zero value infinitely often. A closed connection was given in [9 Problem 1.20], which was proven for $n \geq 5$ in [7], $n = 4$ in [13] and $n = 3$ in [2]: If $f$ is...
non-constant and meromorphic in the plane and \( n \geq 3 \), then \( f' - f^n \) assumes all finite complex values.

A question arising in connection with Hayman’s alternative was given by Eremenko and Langley [6]: whether \((f^m)^{(k)}\) can be replaced by a more general term, such as a linear differential polynomial

\[
F = L[f] = f^{(k)} + a_{k-1}f^{(k-1)} + \cdots + a_0 f,
\]

where the coefficients \( a_j \) are small functions of \( f \). The following counterexample gives a negative answer to above question. Let \( f(z) = 1 + e^{-z^2} \) and \( F(z) = f'' + 2zf' + 3f = 3 + e^{-z^2} \). Then \( f(z) \neq 1 \) and \( F(z) \neq 3 \), but \( f \) is not constant.

Until now, most results related to Hayman’s conjecture and Hayman’s inequality were considered for non-constant differential polynomials \( Q \) in \( f \), where all of whose terms have degree at least 2 in \( f \) and its derivatives (see [4], [11]).

In this paper, we will generalize Hayman’s conjecture to differential polynomials. Here, terms of the polynomials are not necessary all to have degree at least 2.

Let \( P(z) \) and \( Q(z) \) be polynomials in \( \mathbb{C}[z] \) of degree \( p \) and \( q \) respectively. We write

\[
Q(z) = b(z - \alpha_1)^{m_1}(z - \alpha_2)^{m_2}\cdots(z - \alpha_l)^{m_l}, \quad \text{and} \quad P(z) = c(z - \beta_1)^{n_1}(z - \beta_2)^{n_2}\cdots(z - \beta_h)^{n_h},
\]

where \( b, c \in \mathbb{C}^* \).

Our results are as follows.

**Theorem 1.** Let \( k \) be an positive integer. If \( f \) is a transcendental meromorphic function and \( q \geq l + 1 \), then \([Q(f)]^{(k)}\) takes every finite non-zero value infinitely often.

As a consequence, when we consider \( k = 1 \) and \( Q(z) = z^n \), Hayman conjecture is obtained:

**Corollary 1** (Hayman conjecture [1, 7, 8, 13]). If \( f \) is a transcendental meromorphic function, then \( f^n f' \) takes every finite non-zero complex value infinitely often, for any integer \( n \geq 1 \).

More generally, for any \( n \geq 2 \):

**Corollary 2.** [1, Theorem 2] If \( f \) is a transcendental meromorphic function, and if \( n \geq 2 \) and \( k \) are positive integers, then \((f^n)^{(k)}\) takes every finite non-zero complex value infinitely often.
Note that in [1, Theorem 2], the authors need $n > k$. However, in Corollary 2 we only need $n \geq 2$.

**Corollary 3.** Let $k, l$ be positive integers and $a_1, \ldots, a_l$ be complex numbers. If $f$ is a transcendental meromorphic function then $|(f - a_1)^2(f - a_2)\ldots(f - a_l)|^{(k)}$ takes every finite non-zero value infinitely often.

**Theorem 2.** Let $P$ and $Q$ be polynomials of degree $p$ and $q$ respectively, and $k \geq 1$ be a positive integer. Let $f$ be a transcendental meromorphic function. If $q \geq (k + 1)p + l + 2$ then $Q(f) + P(f^{(k)})$ has infinitely many zeros.

In the special case that $P(z) = z$ and $Q(z) = -az^n + b$ where $a \neq 0, b$ are constants, we recover many known results, for example the results in [7], as special cases of our result.

**Corollary 4.** [7, Theorem 9] If $f$ is a transcendental meromorphic function, $n \geq 5$ and $a \neq 0$, then $f'(z) - af(z)^n$ takes every finite complex value infinitely often.

**Theorem 3.** Let $P$ and $Q$ be polynomials of degree $p$ and $q$ respectively. Let $\alpha \neq 0$ be a small function with respect to $f$. If $f$ is a transcendental meromorphic function of finite order and $p - q - h - 2l - 2 > 0$, then $P(f)Q(f(z+c_1)+c_2f(z)) - \alpha$ has infinitely many zeros, for any $c_1, c_2 \in \mathbb{C}$.

In particular, if $f$ is a transcendental entire function of finite order and $p - h - l > 0$, then $P(f)Q(f(z+c_1)+c_2f(z)) - \alpha$ has infinitely many zeros, for any $c_1, c_2 \in \mathbb{C}$.

**Corollary 5.** [12, Theorem 1.2] Suppose that $f$ is a transcendental meromorphic function, $\alpha$ is a small function with respect to $f$ and $n, s$ are integers. If $n \geq s + 6$, then $f(z)^n(f(z+c) - f(z))^s - \alpha$ takes every finite non-zero complex value infinitely often.

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2. **Proof of theorems**

Let $f$ be a meromorphic function on the complex plane $\mathbb{C}$. We use standard notations, definitions and results of Nevanlinna theory in [8, 14]. We denote...
by $S(r, f)$ any function satisfying $S(r, f) = o(T(r, f))$ as $r \to +\infty$ outside of a possible exceptional set with finite measure. We first recall the following lemmas

**Lemma 1.** [3 Theorem 2.1] Let $f(z)$ be a transcendental meromorphic function of finite order. Then, for any $c \in \mathbb{C}$,

(i) $m\left(r, \frac{f(z+c)}{f(z)}\right) = S(r, f)$ for all $r$ outside of a set of finite logarithmic measure.

(ii) $T(r, f(z+c)) = T(r, f(z)) + S(r, f)$.

**Lemma 2.** (Milloux, see Hayman [8 Theorem 3.2]) For $k \geq 1$,

$$T(r, f) \leq N(r, f) + N\left(r, \frac{1}{f}\right) + N\left(r, \frac{1}{f^{(k)}} - 1\right) - N\left(r, \frac{1}{f^{(k+1)}}\right) + S(r, f).$$

Recently, Yamanoi [16] proved the following result.

**Lemma 3.** [16] Let $f$ be a transcendental meromorphic function in the complex plane, $k \geq 1$ be an integer, and $\epsilon > 0$; let $A \subset \mathbb{C}$ be a finite set of complex numbers. Then we have

$$kN(r, f) + \sum_{a \in A} N_1\left(r, \frac{1}{f-a}\right) \leq N\left(r, \frac{1}{f^{(k)}}\right) + \epsilon T(r, f),$$

for all $r > e$ outside a set $E \subset (e, \infty)$ of logarithmic density $0$. Here, $E$ depends on $f, k, \epsilon$ and $A$, and

$$N_1\left(r, \frac{1}{f-a}\right) = N\left(r, \frac{1}{f-a}\right) - N\left(r, \frac{1}{f-a}\right).$$

**Proof of Theorem 1** We will prove $[Q(f)]^{(k)}$ takes a non-zero constant $a$ infinitely often. Without loss of generality, we may assume $a = 1$. Applying Lemma 3 to the transcendental meromorphic function $Q(f)$ and $k \geq 1$, set $A = \{0\} \subset \mathbb{C}$ and $\epsilon = \frac{1}{2q}$, we have

$$kN(r, f) + N\left(r, \frac{1}{Q(f)}\right) - N\left(r, \frac{1}{Q(f)}\right) \leq N\left(r, \frac{1}{Q(f)^{(k+1)}}\right) + \frac{1}{2q} T(r, Q(f))$$

$$\leq N\left(r, \frac{1}{Q(f)^{(k+1)}}\right) + \frac{1}{2} T(r, f) + O(1)$$
Together with Lemma 2, we get
\[
T(r, Q(f)) \leq kN(r, f) + N\left(r, \frac{1}{Q(f)}\right) - N\left(r, \frac{1}{Q(f)}\right) - (k - 1)N(r, f) + N\left(r, \frac{1}{Q(f)}\right) + N\left(r, \frac{1}{(Q(f))^{(k)} - 1}\right) - N\left(r, \frac{1}{(Q(f))^{(k+1)} - 1}\right) + S(r, f)
\]
\[
\leq N\left(r, \frac{1}{Q(f)}\right) + N\left(r, \frac{1}{(Q(f))^{(k)} - 1}\right) + \frac{1}{2}T(r, f) + S(r, f)
\]
\[
\leq \sum_{i=1}^{l} N\left(r, \frac{1}{f - \alpha_i}\right) + N\left(r, \frac{1}{(Q(f))^{(k)} - 1}\right) + \frac{1}{2}T(r, f) + S(r, f)
\]
\[
\leq \left(l + \frac{1}{2}\right)T(r, f) + N\left(r, \frac{1}{(Q(f))^{(k)} - 1}\right) + S(r, f).
\]

Hence,
\[
\left(q - l - \frac{1}{2}\right)T(r, f) \leq N\left(r, \frac{1}{(Q(f))^{(k)} - 1}\right) + S(r, f).
\]

Thus \((Q(f))^{(k)} = 1\) has infinitely many roots when \(q \geq l + 1\). It follows that \((Q(f))^{(k)}\) assumes every finite non-zero value infinitely often. The proof is complete. □

Proof of Theorem 2. Set
\[
F = Q(f) + P(f^{(k)}),
\]
\[
R(f) = \frac{[Q(f)]'}{Q(f)} - \frac{F'}{F}, \quad H(f) = P(f^{(k)})\left(\frac{F'}{F} - \frac{[P(f^{(k)})]'}{P(f^{(k))}}\right).
\]

We have
\[
m(r, R(f)) = S(r, f),
\]
\[
Q(f)R(f) = H(f),
\]
and
\[
T(r, F) \leq T(r, Q(f)) + T(r, P(f^{(k)})) + S(r, f) = qT(r, f) + pT(r, f^{(k)}) + S(r, f)
\]
\[
\leq (q + p(k + 1))T(r, f) + S(r, f).
\]

Hence \(S(r, F) = S(r, f)\).

If \(R(f) \equiv 0\), then \(H(f) \equiv 0\). This means
\[
Q(f) = (c - 1)P(f^{(k)}),
\]

(2.5)
where \( c \neq 0, 1 \) is a constant. Since \( q \geq (k + 1)p + l + 2 \), Equation (2.5) implies that \( f \) cannot have poles. On the other hand, we have
\[
qm(r, f) = m(r, Q(f)) \leq m(r, P(f^{(k)}) + O(1) \leq pm(r, f^{(k)}) + O(1)
\]
\[
\leq pm(r, f) + pm(r, \frac{f^{(k)}}{f}) + O(1) = pm(r, f) + S(r, f).
\]
Therefore
\[
m(r, f) = S(r, f), \tag{2.6}
\]
and
\[
T(r, f) = N(r, f) + m(r, f) = S(r, f),
\]
which is a contradiction and then \( R(f) \neq 0 \).

It is easy to see that
\[
m(r, H(f)) \leq m(r, P(f^{(k)})) + m\left(r, \frac{F'}{F} - \frac{[P(f^{(k)})]'}{P(f^{(k)})}\right) + O(1)
\]
\[
\leq pm(r, f^{(k)}) + S(r, f) \leq p\left(m(r, f) + m(r, \frac{f^{(k)}}{f})\right) + S(r, f)
\]
\[= pm(r, f) + S(r, f). \tag{2.7}
\]
We have
\[
qm(r, f) = m(r, Q(f)) + S(r, f) = m\left(r, Q(f)R(f) - \frac{1}{R(f)}\right) + S(r, f)
\]
\[
\leq m(r, Q(f)R(f)) + m\left(r, \frac{1}{R(f)}\right) + S(r, f)
\]
\[= m(r, H(f)) + m\left(r, \frac{1}{R(f)}\right) + S(r, f)
\]
\[
\leq pm(r, f) + m\left(r, \frac{1}{R(f)}\right) + S(r, f),
\]
hence
\[
(q - p)m(r, f) \leq m\left(r, \frac{1}{R(f)}\right) + S(r, f). \tag{2.8}
\]
By the first main theorem and (2.3), we have
\[
m\left(r, \frac{1}{R(f)}\right) = T(r, R(f)) - N\left(r, \frac{1}{R(f)}\right) + O(1) = N(r, R(f)) - N\left(r, \frac{1}{R(f)}\right) + S(r, f). \tag{2.9}
\]

From the definition of \( R(f) \), we see immediately that the possible poles of \( R(f) \) occur only at the poles of \( f \) and the zeros of \( Q(f) \) and \( F \). Note that \( R(f) \) can have only simple poles. Now, suppose that \( z_0 \) is a pole of \( f \) of order \( s \). Then \( z_0 \) is a pole of \( Q(f) \) of order \( qs \) and \( H(f) \) of order at most \( (s + k)p + 1 \). Since \( q \geq (k + 1)p + l + 2 \), we have
\[
qs - (s + k)p - 1 = (q - p)s - kp - 1 > 0.
\]
Thus, by (2.4), we deduce $z_0$ must be a zero of $R(f)$ of order at least
$$qs - ((s + k)p + 1) = (q - p)s - kp - 1.$$ Hence, we get
$$N(r, R(f)) \leq \overline{N}(r, \frac{1}{P}) + \sum_{i=1}^{l} \overline{N}(r, \frac{1}{f - \alpha_i})$$
and
$$N(r, \frac{1}{R(f)}) \geq (q - p)N(r, f) - (kp + 1)\overline{N}(r, f).$$
Combining (2.8), (2.9), (2.10), (2.11) and by the first main theorem, we get
$$T(r, f) = (q - p)N(r, f) + (q - p)m(r, f)$$
$$\leq \overline{N}(r, \frac{1}{F}) + lT(r, f) + (kp + 1)\overline{N}(r, f) + S(r, f)$$
$$\leq \overline{N}(r, \frac{1}{F}) + (l + kp + 1)T(r, f) + S(r, f).$$
Hence,
$$(q - (k + 1)p - l - 1)T(r, f) \leq \overline{N}(r, \frac{1}{F}) + S(r, f).$$
Thus $F = Q(f) + P(f^{(k)})$ assumes every finite value infinitely often when $q \geq (k + 1)p + l + 2$. 

**Proof of Theorem 3.** Denote by $G(z) = P(f(z))Q(f(z + c_1) + c_2f(z))$. Applying the Second Main Theorem for the meromorphic function $G$ and $0, \infty, \alpha$, we have
$$T(r, G) \leq \overline{N}(r, G) + \overline{N}(r, \frac{1}{G}) + \sum_{i=1}^{h} \overline{N}(r, \frac{1}{f - \beta_i}) + \sum_{i=1}^{l} \overline{N}(r, \frac{1}{f(z + c_1) + c_2f(z) - \alpha_i})$$
$$+ \overline{N}(r, \frac{1}{G - \alpha}) + S(r, f)$$
$$\leq (h + 2l + 2)T(r, f) + \overline{N}(r, \frac{1}{G - \alpha}) + S(r, f).$$
On the other hand, we have
$$\frac{1}{f^qP(f)} = \frac{1}{G} \frac{Q(f(z + c_1) + c_2f(z))}{f^q}$$
$$= \frac{1}{G} \prod_{i=1}^{l} \left[ \frac{f(z + c_1) + c_2f(z) - \alpha_i}{f(z)} \right]^{m_i}.$$
Therefore,

\[(p + q)T(r, f) = T(r, f^qP(f)) + O(1)\]

\[\leq T(G) + \sum_{i=1}^{l} m_i T\left(r, \frac{f(z) + c_1 + c_2 f(z) - \alpha_i}{f(z)}\right) + O(1)\]

\[\leq T(G) + \sum_{i=1}^{l} m_i T\left(r, \frac{f(z) + c_1 - \alpha_i}{f(z)}\right) + O(1)\]

\[\leq T(G) + 2 \sum_{i=1}^{l} m_i T(r, f) + O(1)\]

\[\leq T(G) + 2qT(r, f) + O(1),\]

which implies

\[T(r, G) \geq (p - q)T(r, f) + O(1).\]  \hspace{1cm} (2.14)

Combining (2.12) and (2.14), we have

\[(p - q - h - 2l - 2)T(r, f) \leq N(r, \frac{1}{G - \alpha}) + S(r, f).\]

Thus, \(P(f)Q(f(z + c_1) + c_2 f(z)) - \alpha\) has infinitely many zeros if \(f\) is a transcendental meromorphic function of finite order and \(p - q - h - 2l - 2 > 0\).

In particular, if \(f\) is an entire function then Lemma 1 (i) implies

\[T(r, f(z + c_1) + c_2 f(z)) - \alpha_i) = T(r, f(z + c_1) + c_2 f(z)) + O(1)\]

\[= m(r, f(z + c_1) + c_2 f(z)) + O(1)\]

\[= m\left(r, f(z)\left(\frac{f(z) + c_1}{f(z)} + c_2\right)\right) + O(1)\]

\[\leq m(r, f) + m\left(r, \frac{f(z) + c_1}{f(z)} + c_2\right) + O(1)\]

\[\leq m(r, f) + m\left(r, \frac{f(z) + c_1}{f(z)}\right) + O(1)\]

\[\leq T(r, f) + S(r, f).\]
Together with (2.12), we have
\[ T(r, G) \leq N(r, \frac{1}{G}) + N(r, \frac{1}{G - \alpha}) + S(r, f) \]
\[ \leq \sum_{i=1}^{h} N(r, \frac{1}{f - \beta_i}) + \sum_{i=1}^{l} N(r, \frac{1}{f(z + c_1) + c_2f(z) - \alpha_i}) \]
\[ + N(r, \frac{1}{G - \alpha}) + S(r, f) \]
\[ \leq \sum_{i=1}^{h} N(r, \frac{1}{f - \beta_i}) + \sum_{i=1}^{l} T(r, f(z + c_1) + c_2f(z) - \alpha_i) \]
\[ + N(r, \frac{1}{G - \alpha}) + S(r, f) \]
\[ \leq (h + l)T(r, f) + N(r, \frac{1}{G - \alpha}) + S(r, f). \quad (2.15) \]

On the other hand, by (2.13), we obtain
\[(p + q)T(r, f) = T(r, fP(f)) + O(1) \]
\[ \leq T(G) + \sum_{i=1}^{l} m_i T\left(r, \frac{f(z + c_1) + c_2f(z) - \alpha_i}{f(z)}\right) + O(1) \]
\[ \leq T(G) + \sum_{i=1}^{l} m_i T\left(r, \frac{f(z + c_1) - \alpha_i}{f(z)}\right) + O(1) \]
\[ \leq T(G) + \sum_{i=1}^{l} m_i T(r, f) + O(1) \]
\[ \leq T(G) + qT(r, f) + O(1), \]
where the fourth inequality follows from
\[ T\left(r, \frac{f(z + c_1) - \alpha_i}{f(z)}\right) = m\left(r, \frac{f(z + c_1) - \alpha_i}{f(z)}\right) + N\left(r, \frac{f(z + c_1) - \alpha_i}{f(z)}\right) \]
\[ \leq m\left(r, \frac{f(z + c_1)}{f(z)}\right) + m\left(r, \frac{1}{f(z)}\right) + N\left(r, f(z + c_1) - \alpha_i\right) \]
\[ + N\left(r, \frac{1}{f(z)}\right) + O(1) \]
\[ \leq T(r, f) + S(r, f). \]

Hence, we get
\[ T(r, G) \leq pT(r, f) + S(r, f). \quad (2.16) \]

Combining (2.15) and (2.16), we have
\[(p - h - l)T(r, G) \leq \bar{N}(r, \frac{1}{G - \alpha}) + S(r, f).\]
Hence, \( P(f)Q(f(z+c_1)+c_2f(z)) - \alpha \) has infinitely many zeros when \( p-h-l > 0 \) if \( f \) is a transcendental entire function of finite order.

\[ \square \]

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