State Transition Analysis of Time-Frequency Resource Conversion-based Call Admission Control for LTE-Type Cellular Network

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Abstract

To address network congestion stemmed from traffic generated by advanced user equipments, in [1] we propose a novel network resource allocation strategy, time-frequency resource conversion (TFRC), via exploiting user behavior, a specific kind of context information. Considering an LTE-type cellular network, a call admission control policy called double-threshold guard channel policy is proposed there to facilitate the implementation of TFRC. In this report, we present state transition analysis of this TFRC-based call admission control policy for an LTE-type cellular network. Overall, there are five categories of events that can trigger a transition of the system state: 1) a new call arrival; 2) a handoff user arrival; 3) a handoff user departure; 4) a call termination; and 5) a periodic time-frequency resource conversion. We analyze them case by case in this report and the validation of the analysis has been provided in [1].

Keywords: Time-frequency resource conversion, context-aware resource allocation, call admission control, state transition analysis.

I. STATE TRANSITIONS

The events that trigger a transition of the system state defined in [1] can be classified into five categories: 1) a new call arrival; 2) a handoff user arrival; 3) a handoff user departure; 4) a call termination; and 5) a periodic time-frequency resource conversion. In the following, we describe each category case by case. The important symbols defined in [1] and re-utilized in this report are summarized in Table I.
### TABLE I

**Summary of important symbol.**

| Symbol | Definition | Symbol | Definition |
|--------|------------|--------|------------|
| $T_1$ ($T_2$) | Wide-band (narrow-band) connection | $U_{T_1,T_2}$ | User set in which users with two simultaneous $T_1$ connections, one in state $S_1$ and the other in state $S_2$ |
| $\lambda_u$ | Connection arrival rate per user | $U_{T_1,T_2}$ | User set in which users with two simultaneous connections, one $T_1$ connection in state $S_2$ and one $T_2$ connection in state $S_1$ |
| $r_1$ ($r_2$) | Number of subchannels used by a $T_1$ ($T_2$) connection | $U_{T_2,T_1}$ | User set in which users with two simultaneous connections, one $T_2$ connection in state $S_2$ and one $T_1$ connection in state $S_1$ |
| $P_1$ ($P_2$) | Probability of any user initiating a $T_1$ ($T_2$) connection | $U_{T_2,T_1}$ | User set in which users with two simultaneous $T_2$ connections, one in state $S_1$ and the other in state $S_2$ |
| $1/\mu_1$ ($1/\mu_2$) | Mean of exponential distributed $T_1$’s ($T_2$’s) connection duration | $m_I$ | The maximum number of report times before the subchannel number of a user’s background $T_1$ connection decreases to 0 while the foreground connection is another $T_1$ connection |
| $1/\eta$ | Mean of exponentially distributed cell residual time | $m_{II}$ | The maximum number of report times before the subchannel number of a user’s background $T_1$ connection decreases to 0 while the foreground connection is a $T_2$ connection |
| $C$ | Subchannel number in a cell | $m_{III}$ | The maximum number of report times before the subchannel number of a user’s background $T_2$ connection decreases to 0 while the foreground connection is a $T_1$ connection |
| $C_R$ ($C_{HR}$) | Number of subchannels reserved for recovering (both recovering and handoff) calls | $m_{IV}$ | The maximum number of report times before the subchannel number of a user’s background $T_2$ connection decreases to 0 while the foreground connection is another $T_2$ connection |
| $K_A$ | Average user number per cell | $U_{S_1}^{(i)},T_1$, $0 \leq i \leq m_I$ | Set of users who have two simultaneous connections in which a $T_1$ connection is in TFRC and thus being withdrawn $r_{p_1}^{(i)}$, $T_2$, subchannels |
| $R_b$ | Average data rate per subchannel | $U_{S_2}^{(i)},T_2$, $0 \leq i \leq m_{II}$ | Set of users who have two simultaneous connections in which a $T_1$ connection is in TFRC and thus being withdrawn $r_{p_2}^{(i)}$, $T_2$, subchannels |
| $\tau$ | Feedback period of context information | $U_{S_2}^{(i)},T_2$, $0 \leq i \leq m_{III}$ | Set of users who have two simultaneous connections in which a $T_2$ connection is in TFRC and thus being withdrawn $r_{p_2}^{(i)}$, $T_2$, subchannels |
| $S_1$ ($S_2$) | Connection state in which the connection is in data transmission and in the foreground (background) of the screen | $U_{S_1}^{(i)},T_2$, $0 \leq i \leq m_{IV}$ | Set of users who have two simultaneous connections in which a $T_2$ connection is in TFRC and thus being withdrawn $r_{p_2}^{(i)}$, $T_2$, subchannels |
| $S_T$ | System state | $n_{1}(n_2)$ | Number of users in $U_{T_1}(U_{T_2})$ |
| $R(S_T)$ | Cell load measured by used subchannel number given system state $S_T$ | $N_{R}^{(i)}$, $0 \leq i \leq m_I$ | Number of users in $U_{S_1}^{(i)},T_1$ |
| $R_{r_t}(j_1,j_2,m)$ | Data amount connection $j_1$ received between the time that user $j$ pays attention to new connection $j_2$ and the time that the resource manager detects that the user focuses on $j_1$ again | $N_{R}^{(i)}$, $0 \leq i \leq m_{II}$ | Number of users in $U_{S_2}^{(i)},T_2$ |
| $U_{idle}(S_T)$ | Number of idle users given state $S_T$ | $N_{I}^{(i)}$, $0 \leq i \leq m_{III}$ | Number of users in $U_{S_2}^{(i)},T_1$ |
| $U_{T_1}$ ($U_{T_2}$) | User set in which users with only one $T_1$ ($T_2$) connection in state $S_1$ | $N_{T}^{(i)}$, $0 \leq i \leq m_{IV}$ | Number of users in $U_{S_2}^{(i)},T_2$ |
A. A New Call Arrival

1) A new $T_1$-type call arrival: When a new $T_1$-type call arrives in the cell, it will be accepted if $R(S_T) + r_1 \leq C - C_{HR}$; Otherwise, the call is blocked. Depending on the user who initiates the new call, there are three types of state transitions:

- When the call is initiated by a user without any connection, $n_1 \rightarrow n_1 + 1$, with a rate of $U_{idle}(S_T) \lambda_u P_1$;
- When the call is initiated by a user with only a single $T_1$-type connection, $n_1 \rightarrow n_1 - 1$, $N_{I}^{(0)} \rightarrow N_{I}^{(0)} + 1$, with a rate of $n_1 \lambda_u P_1$;
- When the call is initiated by a user with only a single $T_2$-type connection, $n_2 \rightarrow n_2 - 1$, $N_{III}^{(0)} \rightarrow N_{III}^{(0)} + 1$, with a rate of $n_2 \lambda_u P_1$.

2) A new $T_2$-type call arrival: Similarly, when a new $T_2$-type call arrives in the cell, it will be accepted if $R(S_T) + r_2 \leq C - C_{HR}$; Otherwise, the call is blocked. Again, depending on the user who initiates the new call, there are three types of state transitions:

- When the call is initiated by a user without any connection, $n_2 \rightarrow n_2 + 1$, with a rate of $U_{idle}(S_T) \lambda_u P_2$;
- When the call is initiated by a user with only a single $T_1$-type connection, $n_1 \rightarrow n_1 - 1$, $N_{II}^{(0)} \rightarrow N_{II}^{(0)} + 1$, with a rate of $n_1 \lambda_u P_2$;
- When the call is initiated by a user with only a single $T_2$-type connection, $n_2 \rightarrow n_2 - 1$, $N_{IV}^{(0)} \rightarrow N_{IV}^{(0)} + 1$, with a rate of $n_2 \lambda_u P_2$.

B. A Handoff User Arrival

1) A handoff user arrival for acceptance in $U_{T_1}$: When a handoff user with a single $T_1$ connection arrives, $n_1 \rightarrow n_1 + 1$ if $R(S_T) + r_1 \leq C - C_R$; Otherwise, the handoff requester fails to be accepted in the cell. According to Appendix A, handoff rate for this type of user is given by $\lambda_{h_1} = \bar{K}_A \cdot \eta \cdot \pi(S_u = r_1)$, where $\pi(S_u = x)$ is the steady-state probability of a user in the cell exactly occupying $x$ subchannels.

2) A handoff user arrival for acceptance in $U_{T_2}$: When a handoff user with a single $T_2$ connection arrives, $n_2 \rightarrow n_2 + 1$ if $R(S_T) + r_2 \leq C - C_R$; Otherwise, the handoff requester fails to be accepted in the cell. Again, according to Appendix A, handoff rate for this type of user is given by $\lambda_{h_2} = \bar{K}_A \cdot \eta \cdot \pi(S_u = r_2)$. 
3) A handoff user arrival for acceptance in \( U_{T_1,T_1} \): A handoff user may have multiple simultaneous connections. Such handoff user is individually considered according to the type of connections in service and the progress of the TFRC applied to the user. Let \( \lambda_{h(i)}^{(i)} \) denote the arrival rate of handoff users who have two \( T_1 \) connections in which one is in state \( S_1 \) and the other is in state \( S_2 \) and has been taken away \( r_I^{(i)} \) subchannels (i.e., belonging to \( U_{T_1^{(i)},T_1} \)), where \( i \in \{0,1,...,m_I\} \). According to Appendix A, \( \lambda_{h(i)}^{(i)} \) is equal to \( K_A \cdot \eta \cdot \pi (S_u = 2r_1 - r_I^{(i)}) \). When this type of user moves inward:

- \( N_I^{(i)} \rightarrow N_I^{(i)} + 1 \) if \( R(S_T) + 2r_1 - r_I^{(i)} \leq C - C_R \) (i.e., there exist sufficient resources to admit the user’s both connections);
- \( N_I^{(m_i)} \rightarrow N_I^{(m_i)} + 1 \) if \( R(S_T) + r_1 \leq C - C_R < R(S_T) + 2r_1 - r_I^{(i)} \) and \( 0 \leq i \leq m_I - 1 \) (i.e., the available resource can only afford one of the user’s connections, thus admitting the connection with user’s focus and freezing the other);
- Otherwise, the two connections are both dropped if \( R(S_T) + r_1 > C - C_R \).

4) A handoff user arrival for acceptance in \( U_{T_1,T_2} \): Similarly, handoff users belonging to \( U_{T_1^{(i)},T_2} \) arrive with a rate of \( \lambda_{h(I)}^{(i)} = K_A \cdot \eta \cdot \pi (S_u = r_1 - r_{II}^{(i)} + r_2) \), where \( i \in \{0,1,...,m_{II}\} \). When this type of user moves inward:

- \( N_{II}^{(i)} \rightarrow N_{II}^{(i)} + 1 \) if \( R(S_T) + r_1 - r_{II}^{(i)} + r_2 \leq C - C_R \) (i.e., there exist sufficient resources to admit the user’s both connections);
- \( N_{II}^{(m_{II})} \rightarrow N_{II}^{(m_{II})} + 1 \) if \( R(S_T) + r_2 \leq C - C_R < R(S_T) + r_1 - r_{II}^{(i)} + r_2 \) and \( 0 \leq i \leq m_{II} - 1 \) (i.e., the available resource can only afford one of the user’s connections, thus admitting the connection with user’s focus and freezing the other);
- Otherwise, the two connections are both dropped if \( R(S_T) + r_2 > C - C_R \).

5) A handoff user arrival for acceptance in \( U_{T_2,T_1} \): Handoff users belonging to \( U_{T_2^{(i)},T_1} \) arrive with a rate of \( \lambda_{h(I)}^{(i)} = K_A \cdot \eta \cdot \pi (S_u = r_2 - r_{III}^{(i)} + r_1) \), where \( i \in \{0,1,...,m_{III}\} \). When this type of user moves inward:

- \( N_{III}^{(i)} \rightarrow N_{III}^{(i)} + 1 \) if \( R(S_T) + r_2 - r_{III}^{(i)} + r_1 \leq C - C_R \) (i.e., there exist sufficient resources to admit the user’s both connections);
- \( N_{III}^{(m_{III})} \rightarrow N_{III}^{(m_{III})} + 1 \) if \( R(S_T) + r_1 \leq C - C_R < R(S_T) + r_2 - r_{III}^{(i)} + r_1 \) and \( 0 \leq i \leq m_{III} - 1 \) (i.e., the available resource can only afford one of the user’s connections, thus admitting the connection with user’s focus and freezing the other);
• Otherwise, the two connections are both dropped if $R(S_T) + r_1 > C - C_R$.

6) A handoff user arrival for acceptance in $U_{T_2}$. Handoff users belonging to $U_{T_2}^{(i)}$ arrives with a rate of
   $$\lambda_{hi}^{(i)} = \bar{K}_A \cdot \eta \cdot \pi (S_u = 2r_2 - r_{IV}^{(i)}),$$
   where $i \in \{0, 1, \ldots, m_{IV}\}$. When this type of user moves inward:
   • $\mathcal{N}_{IV}^{(i)} \rightarrow \mathcal{N}_{IV}^{(i)} + 1$ if $R(S_T) + 2r_2 - r_{IV}^{(i)} \leq C - C_R$ (i.e., there exist sufficient resources to admit the user’s both connections);
   • $\mathcal{N}_{IV}^{(m_{IV})} \rightarrow \mathcal{N}_{IV}^{(m_{IV})} + 1$ if $R(S_T) + r_2 \leq C - C_R < R(S_T) + 2r_2 - r_{IV}^{(i)}$ and $0 \leq i \leq m_{IV} - 1$ (i.e., the available resource can only afford one of the user’s connections, thus admitting the connection with user’s focus and freezing the other);
   • Otherwise, the two connections are both dropped if $R(S_T) + r_2 > C - C_R$.

C. A Handoff User Departure

When a user with connection(s) moves outward, a handoff user departs.
   • If the user is in $U_{T_1}$, $n_1 \rightarrow n_1 - 1$, with a rate of $n_1 \eta$;
   • If the user is in $U_{T_2}$, $n_2 \rightarrow n_2 - 1$, with a rate of $n_2 \eta$;
   • If the user is in $U_{T_1}^{(i)}$ where $i \in \{0, 1, \ldots, m_I\}$, $\mathcal{N}_{I}^{(i)} \rightarrow \mathcal{N}_{I}^{(i)} - 1$, with a rate of $\mathcal{N}_{I}^{(i)} \eta$;
   • If the user is in $U_{T_1}^{(i)}$ where $i \in \{0, 1, \ldots, m_{II}\}$, $\mathcal{N}_{II}^{(i)} \rightarrow \mathcal{N}_{II}^{(i)} - 1$, with a rate of $\mathcal{N}_{II}^{(i)} \eta$;
   • If the user is in $U_{T_2}^{(i)}$ where $i \in \{0, 1, \ldots, m_{IV}\}$, $\mathcal{N}_{IV}^{(i)} \rightarrow \mathcal{N}_{IV}^{(i)} - 1$, with a rate of $\mathcal{N}_{IV}^{(i)} \eta$.

D. A Call Termination

1) Call terminated by users in $U_{T_1}$: When a call belonging to a user in $U_{T_1}$ terminates, $n_1 \rightarrow n_1 - 1$. As proved in Appendix B, for a connection that has been applied time-frequency resource conversion, as long as it is successfully recovered, its call holding time after spectrum recovery follows the same distribution as the one that has not been applied TFRC. Therefore, call termination associated with users in $U_{T_1}$ occurs with a rate of $n_1 \mu_1$.

2) Call terminated by users in $U_{T_2}$: Similarly, when a call belonging to a user in $U_{T_2}$ terminates, $n_2 \rightarrow n_2 - 1$. And call termination associated with this type of user occurs with a rate of $n_2 \mu_2$. 
3) Call terminated by users in $U_{T_1,T_1}$: Specifically, when a call belonging to a user in $U_{T_1^{(i)},T_1}$ ($i \in \{0, 1, ..., m_I\}$) terminates, $N_I^{(i)} \rightarrow N_I^{(i)} - 1$, $n_1 \rightarrow n_1 + 1$. Since this type of user has two connections simultaneously, the call termination may be caused by either of the two connections. As shown in Appendix A of [1], if the terminated connection is the one in TFRC, its service rate is equal to $\mu_1(1-r_I^{(i)}/r_1)$. On the other hand, if the terminated connection is the other connection, its service rate is $\mu_1$. Therefore, call termination associated with users in $U_{T_1^{(i)},T_1}$ happens with a rate of $N_I^{(i)}[\mu_1(1-r_I^{(i)}/r_1) + \mu_1]$. Notice that call recovery can always be successful in this case, since we utilize the recovery protection mechanism.

4) Call terminated by users in $U_{T_1,T_2}$: For call termination associated with users in $U_{T_1^{(i)},T_2}$ ($i \in \{0, 1, ..., m_{II}\}$), we also should consider two cases. When the ended call is the one in TFRC, $N_{II}^{(i)} \rightarrow N_{II}^{(i)} - 1$, $n_2 \rightarrow n_2 + 1$, with a rate of $N_{II}^{(i)} \mu_1(1-r_{II}^{(i)}/r_1)$ (derived with the same approach used in Appendix A of [1]). When the ended call is the one without TFRC (i.e., the $T_2$-type connection), the released subchannel number is $r_2$. However, to restore the spectrum supply to the connection in TFRC, $r_{II}^{(i)}$ subchannels are needed. Note that $r_{II}^{(i)}$ may be larger than $r_2$ since $r_1 > r_2$. It is thus possible that in this case the user cannot recover the recycled bandwidth (i.e., a recovering call dropping occurs) if $R(S_T) + r_{II}^{(i)} - r_2 > C$. Otherwise, call recovery can be successful, and we have $n_1 \rightarrow n_1 + 1$. The second case should occur with a rate of $N_{II}^{(i)} \mu_2$.

5) Call terminated by users in $U_{T_2,T_1}$: Specifically, when the ended call is associated with users in $U_{T_2^{(i)},T_1}$ ($i \in \{0, 1, ..., m_{III}\}$) and is the one in TFRC, $N_{III}^{(i)} \rightarrow N_{III}^{(i)} - 1$, $n_1 \rightarrow n_1 + 1$, with a rate of $N_{III}^{(i)} \mu_2(1-r_{III}^{(i)}/r_2)$ (derived with the same approach used in Appendix A of [1]). On the other hand, when the ended call is the one without TFRC (i.e., the $T_1$-type connection), $N_{III}^{(i)} \rightarrow N_{III}^{(i)} - 1$, $n_2 \rightarrow n_2 + 1$, with a rate of $N_{III}^{(i)} \mu_1$. However, as $r_1 > r_2$, call recovery can always be successful in this case.

6) Call terminated by users in $U_{T_2,T_2}$: When the ended call is associated with users in $U_{T_2^{(i)},T_2}$ ($i \in \{0, 1, ..., m_{IV}\}$) and is the one in TFRC, $N_{IV}^{(i)} \rightarrow N_{IV}^{(i)} - 1$, $n_2 \rightarrow n_2 + 1$, with a rate of $N_{IV}^{(i)} \mu_2(1-r_{IV}^{(i)}/r_2)$ (derived with the same approach in Appendix A of [1]). On the other hand, when the ended call is the one without TFRC, $N_{IV}^{(i)} \rightarrow N_{IV}^{(i)} - 1$, $n_2 \rightarrow n_2 + 1$, with a rate of $N_{IV}^{(i)} \mu_2$. Therefore, call termination associated with users in $U_{T_2^{(i)},T_2}$ happens with a rate of $N_{IV}^{(i)}[\mu_2(1-r_{IV}^{(i)}/r_2) + \mu_2]$. However, call recovery can always be successful in this case.
E. A Periodic Time-Frequency Resource Conversion

System state also changes with the periodic time-frequency resource conversion. Specifically, at each feedback of context information, resource consumption of users with multiple connections can change. Thus, when a new feedback instant arrives, the system state changes according to the TFRC strategy designed in Section III of [1]:

1) the users of multiple connections but without TFRC apply their first round of spectrum reduction; and 2) the users of multiple connections with TFRC keep reducing spectrum supply until all subchannels of the connection in TFRC are withdraw, leading to

\[ N_j^{(0)} \rightarrow 0, N_j^{(i)} \rightarrow N_j^{(i-1)}, N_j^{(m_j)} \rightarrow N_j^{(m_j)} + N_j^{(m_j-1)} \]

where \( j \in \{I, II, III, IV\} \) and \( i \in \{1, 2, ..., m_j - 1\} \) correspond to one of the four types of users of multiple simultaneous connections and the state of the users’ time-frequency resource conversion, respectively, all defined in [1].

APPENDIX A: HANDOFF RATE ANALYSIS

In our model, call handoff can associate with different kinds of user. To derive handoff rate for each type of user, we analyze the user state transitions occurred in the system. Let us use the number of subchannels a user occupies to characterize the user type. Then the user state space can be given by

\[ \Upsilon_u = \left\{ 0, r_1, r_2, 2r_1 - r_1^{(0)}, 2r_1 - r_1^{(1)}, ..., 2r_1 - r_1^{(m_1)}, \\
  r_1 - r_1^{(0)} + r_2, r_1 - r_1^{(1)} + r_2, ..., r_1 - r_1^{(m_1)} + r_2, \\
  r_2 - r_2^{(0)} + r_1, r_2 - r_2^{(1)} + r_1, ..., r_2 - r_2^{(m_2)} + r_1, \\
  2r_2 - r_2^{(0)}, 2r_2 - r_2^{(1)}, ..., 2r_2 - r_2^{(m_2)} \right\} \]  

where without confusion we reuse the subchannel number to denote the user state. Specifically, here 0, \( r_1 \), and \( r_2 \) denote the idle user, the user with a single \( T_1 \)-type connection, and the user with a single \( T_2 \)-type connection, respectively. And \( 2r_1 - r_1^{(i)}, r_1 - r_1^{(i)} + r_2, r_2 - r_1^{(i)} + r_1, \) and \( 2r_2 - r_2^{(i)} \) respectively denote the user with two simultaneous \( T_1 \)-type connections in which one is in state \( S_1 \) (foreground of the screen) and the other is in state \( S_2 \) (background of the screen), the user with two simultaneous connections in which a \( T_1 \)-type connection is in state \( S_2 \) and a \( T_2 \)-type connection is in state \( S_1 \), the user with two simultaneous connections in which a \( T_2 \)-type connection is in state \( S_2 \) and a \( T_1 \)-type connection is in state \( S_1 \), and the user with two
$T_2$-type connections in which one is in state $S_1$ and the other is in state $S_2$, with $i$ representing the progress of TFRC on the connection in state $S_2$.

Similar to system state, user state changes not only with call arrival and call termination but also with the periodic time-frequency resource conversion. Similar to [1], by dividing any state $S_u(\in \Upsilon_u)$ into $M$ substates $S_u^{(m)}$, where $m = 1, 2, \ldots, M$, we can resort to multiple-stair Markov model to approximate the mixed continuous-discrete Markov process of user state change [2,3]. Detailed analysis is omitted here, but the derived user state transitions are listed in Table II. Based on the transition results we can obtain the set of steady-state probabilities with respect to user state $\{\pi(S_u^{(m)})\}$, and further obtain $\{\pi(S_u)\}$ because $\pi(S_u) = \sum_{m=1}^{M} \pi(S_u^{(m)})$.

As considered in [1], the total average user number in the cell is $\bar{K}_A$, and the residual time of user in the cell is exponentially distributed with mean $1/\eta$, then the handoff rate of the whole users in the cell is $\bar{K}_A \eta$. Thus, the handoff rate of user with state $S_u$ can be found by taking the obtained steady-state probability of the related user state into account, i.e., given by $\bar{K}_A \eta \cdot \pi(S_u)$. It is noteworthy that, if there is only a single connection type in the system without applying TFRC, the handoff rate derived here can be proved consistent with the result obtained in [4].

APPENDIX B: DERIVATION OF CALL HOLDING TIME DISTRIBUTION OF A CONNECTION APPLIED TFRC

Without loss of generality, take connection $j_1$ of Fig. 2 in [1] as an example. Let $l_{j_1}$ denote the call hold time of connection $j_1$ before applying TFRC, and $l'_{j_1}$ denote the call hold time of the connection successfully resumed full spectrum supply. Without loss of generality, we suppose that the spectrum recovery for $j_1$ occurs at the $(m+1)^{th}$ context information feedback. Since the data amount of the connection to be delivered remains the same with and without TFRC, we have

$$r_1 \cdot R_b \cdot (t_2 - t_1) + R_{rv}(j_1, j_2, m) + r_1 \cdot R_b \cdot l'_{j_1} = r_1 \cdot R_b \cdot l_{j_1}$$

(2)

where the three items of the left side here are referred to as the delivered data amounts of connection $j_1$ in the following three disjointed durations, namely, before the user gives his/her attention to the new connection $j_2$, until the resource manager detects that the user refocuses on connection $j_1$, and after the spectrum recovery for the connection. With some manipulation, it is clear that

$$l'_{j_1} = l_{j_1} - (t_2 - t_1) - R_{rv}(j_1, j_2, m)/(r_1 R_b).$$

(3)
| State transition | Relevant event | Transition rate |
|------------------|---------------|----------------|
| $0 \rightarrow r_k, k = 1, 2$ | The user without connection initiates a new $T_k$ connection | $\lambda_u P_k$ |
| $r_k \rightarrow 2r_k, k = 1, 2$ | The user with a $T_k$ connection initiates a new $T_k$ connection | $\lambda_u P_k$ |
| $r_1 \rightarrow r_1 + r_2$ | The user with a $T_1$ connection initiates a new $T_2$ connection | $\lambda_u P_2$ |
| $r_2 \rightarrow r_2 + r_1$ | The user with a $T_2$ connection initiates a new $T_1$ connection | $\lambda_u P_1$ |
| $r_k \rightarrow 0, k = 1, 2$ | The single $T_k$ connection that the user owns ends | $\mu_k$ |
| $2r_1 - r_1^{(i)} \rightarrow r_1$ | If $i \neq m_1$, one of user's two $T_1$ connections (in which the one in TFRC has been withdrawn $r_1^{(i)}$ subchannels) ends; otherwise, the connection without TFRC ends | $\mu_1 (2 - r_1^{(i)}/r_1)$ |
| $r_1 - r_1^{(i)} + r_2 \rightarrow r_1$ | The $T_2$ connection that the user owns ends when the $T_1$ connection in TFRC has been withdrawn $r_1^{(i)}$ subchannels | $\mu_2$ |
| $r_1 - r_1^{(i)} + r_2 \rightarrow r_2$ | If $i \neq m_1$, the $T_1$ connection ends when it has been withdrawn $r_1^{(i)}$ subchannels; otherwise, no state change occurs due to connection end | $\mu_1 (1 - r_1^{(i)}/r_1)$ |
| $r_2 - r_1^{(i)} + r_1 \rightarrow r_2$ | The $T_1$ connection that the user owns ends when the $T_2$ connection in TFRC has been withdrawn $r_1^{(i)}$ subchannels | $\mu_1$ |
| $r_2 - r_1^{(i)} + r_1 \rightarrow r_1$ | If $i \neq m_1$, the $T_2$ connection ends when it has been withdrawn $r_1^{(i)}$ subchannels; otherwise, no state change occurs due to connection end | $\mu_2 (1 - r_1^{(i)}/r_2)$ |
| $2r_2 - r_2^{(i)} \rightarrow r_2$ | If $i \neq m_1$, one of user's two $T_2$ connections (in which the one in TFRC has been withdrawn $r_2^{(i)}$ subchannels) ends; otherwise, the connection without TFRC ends | $\mu_2 (2 - r_2^{(i)}/r_2)$ |
| $(2r_1 - r_1^{(i)})^{(M)} \rightarrow (2r_1 - r_1^{(i+1)})^{(i)}$ | Inter-state transition occurs at the beginning of a new TFRC period thus more subchannels are withdrawn from a $T_1$ connection while the user focuses on the other $T_1$ connection | $M/\tau$ |
| $(r_1 - r_1^{(i)} + r_2^{(i)})^{(M)} \rightarrow (r_1 - r_1^{(i+1)} + r_2^{(i)})^{(i)}$ | Inter-state transition occurs at the beginning of a new TFRC period thus more subchannels are withdrawn from a $T_1$ connection while the user focuses on a $T_2$ connection | $M/\tau$ |
| $(r_2 - r_1^{(i)} + r_1^{(i)})^{(M)} \rightarrow (r_2 - r_1^{(i+1)} + r_1^{(i)})^{(i)}$ | Inter-state transition occurs at the beginning of a new TFRC period thus more subchannels are withdrawn from a $T_2$ connection while the user focuses on a $T_1$ connection | $M/\tau$ |
| $(2r_2 - r_2^{(i)})^{(M)} \rightarrow (2r_2 - r_2^{(i+1)})^{(i)}$ | Inter-state transition occurs at the beginning of a new TFRC period thus more subchannels are withdrawn from a $T_2$ connection while the user focuses on the other $T_2$ connection | $M/\tau$ |
| $(2r_1 - r_1^{(i)})^{(M)} \rightarrow (2r_1 - r_1^{(i+1)})^{(i)}$ | Intra-state transition occurs at the beginning of a new TFRC period when the user has two $T_1$ connections in which the one in TFRC has been withdrawn all subchannels | $M/\tau$ |
| $(r_1 - r_1^{(i)})^{(M+1)} + r_2^{(M)} \rightarrow (r_1 - r_1^{(i+1)})^{(i)}$ | Intra-state transition occurs at the beginning of a new TFRC period when the user has a $T_1$ connection and a $T_2$ connection in which the former is in TFRC and has been withdrawn all subchannels | $M/\tau$ |
| $(r_2 - r_2^{(i)})^{(M)} + r_1^{(M)} \rightarrow (r_2 - r_2^{(i+1)})^{(i)}$ | Intra-state transition occurs at the beginning of a new TFRC period when the user has a $T_1$ connection and a $T_2$ connection in which the latter is in TFRC and has been withdrawn all subchannels | $M/\tau$ |
| $(2r_2 - r_2^{(i)})^{(M)} \rightarrow (2r_2 - r_2^{(i+1)})^{(i)}$ | Intra-state transition occurs at the beginning of a new TFRC period when the user has two $T_2$ connections in which the one in TFRC has been withdrawn all subchannels | $M/\tau$ |
| $0^{(M)} \rightarrow 0^{(1)}$ | Intra-state transition occurs at the beginning of a new TFRC period when the user has no connection | $M/\tau$ |
| $r_1^{(M)} \rightarrow r_1^{(1)}$ | Intra-state transition occurs at the beginning of a new TFRC period when the user has only a $T_1$ connection | $M/\tau$ |
| $r_2^{(M)} \rightarrow r_2^{(1)}$ | Intra-state transition occurs at the beginning of a new TFRC period when the user has only a $T_2$ connection | $M/\tau$ |
| $S_u^{(m)} \rightarrow S_u^{(m+1)} \forall S_u \in \mathcal{Y}_u$, $m = 1, 2, ..., M - 1$ | Intra-state transition of any user state occurs in a temporal sequence in a period of TFRC but before the end of a period of TFRC | $M/\tau$ |
Due to a positive value of call hold time, we have \( l_{j_1} > (t_2 - t_1) + R_{rv}(j_1, j_2, m) / (r_1 R_b) \). Thus, by applying the memoryless property of the exponential distribution, we have

\[
P\{ l'_{j_1} > x \mid l'_{j_1} > 0 \} = P \left\{ l_{j_1} - (t_2 - t_1) - \frac{R_{rv}(j_1, j_2, m)}{r_1 R_b} > x \mid l_{j_1} > (t_2 - t_1) + \frac{R_{rv}(j_1, j_2, m)}{r_1 R_b} \right\} = P\{ l_{j_1} > x \} \tag{4}
\]

which implies that given exponential distribution for the call hold time \( l_{j_1} \) of a connection without applying TFRC, as long as connection \( j_1 \) is successfully recovered, its call holding time after spectrum recovery \( (l'_{j_1}) \) follows the same distribution as the one that has not been applied TFRC.

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