Does the scale-free topology favor the emergence of cooperation?

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In a recent Letter [F.C. Santos and J. M. Pacheco Phys. Rev. Lett. 95, 098104 (2005)], the scale-free networks are found to be advantageous for the emergence of cooperation. In the present work an evolutionary prisoner’s dilemma game with players located on Barabási-Albert scale-free networks is studied in detail. The players are pure strategist and can follow two strategies: either to defect or to cooperate. Several alternative update rules determining the evolution of each player’s strategy are considered. Using systematic Monte Carlo simulations we have calculated the average density of cooperators as a function of the temptation to defect. It is shown that the results obtained by numerical experiments depend strongly on the dynamics of the game, which could lower the important of scale-free topology on the persistence of the cooperation. Particularly, the system exhibits a phase transition, from active state (coexistence of cooperators and defectors) to absorbing state (only defectors surviving) when allowing “worse” strategy to be imitated in the evolution of the game.

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Cooperation is widespread in many biological, social and economic systems. Understanding the emergence and persistence of cooperation in these system is one of the fundamental and central problems. In investigating this problem the most popular framework utilized is game theory together with its extensions to an evolutionary context. One simple game, the Prisoner’s Dilemma game (PDG), has attracted most attention in theoretical and experimental studies. In the standard PDG, the players can either defect or cooperate; two interacting players are offered a certain payoff, the reward $R$, for mutual cooperation and a lower payoff, the punishment $P$, for mutual defection. If one player cooperates while the other defects, then the cooperator gets the lowest sucker’s payoff $S$, while the defector gains the highest payoff, the temptation to defect $T$. Thus, we obtain $T > R > P > S$. It is easy to see that defect is the better choice irrespective of the opponent’s selection. For this reason, defection is the only evolutionary stable strategy in fully mixed populations.

Since the cooperation is abundant and robust in nature, considerable efforts have been expended trying to understand the evolution of cooperation on the basis of the PDG. These extensions include those in which the players are assumed to have memory of the previous interactions, or players are spatially distributed, or allowing the players to voluntarily participating. In addition, dynamic network model and dynamic payoff matrices were also introduced to sustain high concentration of cooperation in PDG. In a recent Letter, Santos and Pacheco have studied the PDG and another famous game, the snow-drift game (commonly known as the hawk-dove or chicken game), on scale-free networks and observed interesting evolutionary results: due to the underlying network generated by growth and preferential attachment (or the scale-free topology), the cooperation can be greatly enhanced and becomes the dominating trait throughout the entire range of parameters of both games.

In the present work, we study the PDG on Barabási-Albert (BA) scale-free networks. Several alternative update rules determining the evolution of each player’s strategy are considered in the following. Using systematic Monte Carlo (MC) simulations we have calculated the average density of cooperators as a function of the temptation to defect. It is shown that the results obtained by numerical experiments depend strongly on the dynamics of the game, which suggest that the scale-topology of the underlying interacting network may not be the crucial factor persisting the high density of the cooperators. Of particular interesting, we have found that the system undergoes a phase transition, from active state (coexistence of cooperators and defectors) to absorbing state (only defectors surviving) when allowing “worse” strategy (adopted by the player who gains a lower payoff) to be imitated in the evolution of the game.

The model and simulation. We consider the PDG with pure strategist: the players can follow two strategies, either to defect or to cooperate ($C$). Each player interacts with its neighbors and collects payoff depended on the payoff parameters. The total payoff of a certain player is the sum over all interactions. Following common studies, we also start by rescaling the game such that it depends on a single parameter, i.e., we can choose $R = 1$, $P = S = 0$, and $T = b$ ($1.0 \leq b \leq 2.0$) representing the advantage of defectors over cooperators (or the temptation to defect), without any loss of generality of the game. After each round, the players are allowed to inspect their neighbors’ payoffs and, according to the comparison, determine their strategies to be adopted in the next round. To investigate how the dynamics of the game affect the evolution of the cooperation, three kinds
of update rules which determine the transformation of each player’s strategy are considered in the following.

I) Best-takes-over. It is commonly observed that people try to imitate a strategy of their most successful neighbor [10]. Thus we first use a deterministic rule according to which the individual with the highest gain in a given neighborhood reproduces with certainty. Since the PDG performed on scale-free networks whose elements (or nodes) possess heterogeneous connectivity [12], To avoid an additional bias from the higher degree of some nodes, the gain of the certain player is calculated as the average payoff of the individual interactions: the sum of the payoff from each neighbor is divided by the number of the neighbors. It is important to note that this rule does not reduce to the replicator dynamics when applied to individual-based models of populations without spatial structure [10]. This update rule has also been widely adopted in the studying of PDG [3, 11, 13, 14].

II) Betters-possess-chance. Technically, the rule I is particularly simple to implement, but its biological relevance is rather limited because it assumes a noise free environment. Thus in the second case the stochasticity is add to the dynamics, and we adopt the update method just as the one used in Ref. [17] (the unique place different from Ref. [8] is to consider average payoff of the players rather than the total payoff), i.e., evolution is carried out implementing the finite population analogue of replicator dynamics [7, 17] by means of the following transition probabilities: in each generation, whenever a site i is updated, a neighbor j is drawn at random among all its neighbors; whenever \( E_j > E_i \) (i.e., only the better players have the chance to reproduce, and if \( E_i > E_j \) the player does not change its strategy), the chosen neighbor takes over site i with probability given by

\[
W = \frac{E_i - E_j}{T - S},
\]

where \( E_i \) and \( E_j \) correspond to the average payoffs accumulated by the player i and j in the previous round respectively.

III) Payoff-difference-dependent. One can see that, for both the above rules, the error mutation, which is very common in evolutionary systems, is not permitted, i.e., the players who gain lower average payoffs have no chance to replace a neighbor who does better than them. The update rule dependent on the payoff difference, which was adopted widely in literature [7, 12, 17, 18], can overcome this difficult. Given the average payoffs \((E_i \text{ and } E_j)\) from the previous round, the player i adopts the neighbor j’s strategy with probability

\[
W = \frac{1}{1 + \exp \left[ - (E_j - E_i)/\kappa \right]},
\]

where \( \kappa \) characterizes the noise introduced to permit irrational choices. This update rule states that the strategy of a better performing player is readily adopted, whereas it is unlikely, but not impossible, to adopt the strategies of worse performing payers. The parameter \( \kappa \) incorporates the uncertainties in the strategy adoption. In the limit of large \( \kappa \) values, all information is lost, that is, player i is unable to retrieve any information from \( E_j \) and switches to the strategy of \( j \) by tossing a coin [17].

Generate a random number \( r \) uniformly distributed between zero and one, if \( r < W \), the neighbor’s strategy is imitated.

Initially, the two strategies was randomly distributed among the players with equal probability 1/2. The above rules of the model are iterated with parallel updating by varying the value of b. The total sampling times are 11000 MC steps and all the results shown below are averages over the last 1000 steps.

Results and discussion. In the following we show the results of simulations performed in a system of \( N = 10000 \) players located on BA scale-free networks with average connectivity of the nodes fixed as 4 (the construction of BA network can refer to Refs. [11, 12]). Our key quantity is the average density of players adopting the strategy \( C \) in the steady-state. First let us consider the model driven by the rule I. The simulation results are shown in Fig. [4] The cooperators and defectors coexist and coevolution throughout the entire range of parameter \( b \). With the increasing of the temptation to defect, the average density of cooperators decreases monotonically; the cooperation is inhibited quickly and sustains a low level in a wide range of the parameter, which is clearly different from the results obtained in Ref. [3] using different dynamics where the cooperators dominate the whole range of parameter of the game.

The evolutionary results of the game under the update rule II are reported in Fig. [2] As compared to the former case, cooperators can exist and survive in the whole region of \( b \). However, the cooperation in the region of large values of \( b \), namely \( b > 1.4 \), is extremely inhibited
FIG. 2: As shown in Fig. 1, but for the system driven by the update rule II (better-possess-chance). For the sake of comparison, the evolutionary results adopting the original method of Ref. [8] are also given out using squares.

FIG. 3: As shown in Fig. 1 but for the system driven by the update rule III (payoff-difference-dependent). Squares, circles and triangles correspond to different noise intensity $\kappa = 0.04, 0.03, 0.02$ respectively. The lines are guides to the eye.

FIG. 4: Average density of cooperators as a function of the temptation to defect $b$ in a evolutionary PDG on a random regular graph with total nodes $N = 10000$. The number of neighbors is fixed as 4 for each node. From top to bottom, (a), (b) and (c) correspond to the evolutionary results of the game driven by the update rule I, II and III respectively. The data in (c) are obtained for $\kappa = 0.03$. Of particular interesting, one can observe that there arises two separate phases (coexistence phase and absorbing phase) of the evolution. As indicated, the average density of $C$ decreases monotonically with increasing $b$ until a certain threshold value $b_c$ where the cooperators vanish and an absorbing state (all defectors) forming. The threshold value depends on the level of the noise: the smaller intensity of the noise $\kappa$, the larger threshold $b_c$. These phenomena are reminiscent of the studies in Ref. [15], where the players are located on a two dimen-

when allowing more "better" players' strategies to be imitated in strategy updating of the players. The density of cooperators maintains a minor level and is almost invisible in Figure 2 (yet larger than zero). To compare distinctly with the results presented in Ref. [8], we also calculated the average density of $C$ by taking account into total payoff difference, just as what has been done in Ref. [8], instead of average payoff difference in the update rule II. As expected, we recover qualitatively the results of Ref. [8]: cooperation becomes the dominating trait throughout the entire range of parameter of the game. The minor difference comes from the average times of the results. Due to the computational resource limit, here the experiment results are averaged over 20 simulations for the same network of contacts (less than the 100 simulations in Ref. [8]). The difference between the two results is distinct: the cooperation is no longer dominating whenever the average payoff difference is considered in rule II.

Now let us consider the case of payoff-difference-dependent. Figure 3 shows the $b$ dependence of the average density of cooperators for different intensity of the noise $\kappa = 0.02, 0.03, 0.04$. Once again, the cooperation is not the favorable choice of the players in a wide range of $b$. Of particular interesting, one can observe that there arises two separate phases (coexistence phase and absorbing phase) of the evolution. As indicated, the average density of $C$ decreases monotonically with increasing $b$ until a certain threshold value $b_c$, where the cooperators vanish and an absorbing state (all defectors) forming. The threshold value depends on the level of the noise: the smaller intensity of the noise $\kappa$, the larger threshold $b_c$. These phenomena are reminiscent of the studies in Ref. [15], where the players are located on a two dimen-
sional square lattice with periodic boundary and interact with their four nearest neighbors.

From the above simulations, one can see that the dynamics (or at least the combination of the structure of the network and the dynamics), rather than only the scale-free topology, has an deterministic influence on the evolution of the PDG. In order to check this statement, we also investigated the case that all players are located on a random regular graph [19]. These two network models hold similar geometric characters: small average path length and small clustering coefficient, whereas expect for the connectivity distribution of the vertices (one behaves scale-free and the other displays peak distribution) [12]. In this way, we expect that the experimental results may give out some intrinsic view: whether the scale-free topology is crucial for the emergence of high density of cooperators? The simulation results obtained for random regular graph are plotted in Fig. 4. The qualitative behavior of the data are consistent, on the whole, with those that the PDG is played on BA scale-free networks. For best-takes-over rule, the lack of stochasticity magnifies the importance of certain local configurations, which results in discontinuous jumps and plateaus of the steady state density of cooperators as a function of the temptation to defect [8].

Whenever all better players are allowed to share a chance to reproduce, the cooperation is inhibited in the same way with the increasing of temptation and the average density of C maintain a very low level for large b values (larger than zero though invisible in Fig.4(b)); and if bad imitation is permitted (the rule III), absorbing phase arising when a certain value of b_c arrived (Fig.4(c)).

Conclusions. To sum up, we have explored the general question of cooperation formation and sustainment on BA scale-free networks based on the framework of PDG with different driving dynamics. The simulation results suggest that the topology of the underlying interacting network, i.e., the scale-free structure, may not be the crucial factor for the emergence and the persistence of the cooperation, whose evolutionary results depend strongly on the dynamics governing the game. These results are different from those obtained in a recent work Ref. [8] whose researches support that the scale-free networks are advantageous for the emergence of cooperation. Of particular interesting, we have found that the system undergoes a phase transition, from active state to absorbing state when allowing “worse ” strategy to be imitated in the evolution of the game. Comparisons between the evolution implemented on BA scale-free networks and on random regular graph also gave out the same hints: there is no obvious evidence supporting the scale-free topology possessing particular advantage for the emergence of cooperation.

A lots of things are waited to do further. Here we only considered the case of PDG. Are the results obtained in the present work also suitable for the case of the snowdrift game when considering different dynamics? how the fraction of cooperators goes to zero when taking account into the payoff-difference-dependent rule? What is the accurate diagrams between b_c and κ? What is the relationship between the extinction behavior of cooperators and the case studied by Szabó and Tőke [15] (there they found that the extinction behavior of the cooperators on square lattice when increasing b belongs to universality class of directed percolation)? Work along these lines is in progress.

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