Reduction and equalization of thermal loading of contact pairs of the friction unit

P A Polyakov¹, A E Litvinov¹, E A Polyakova¹, E S Fedotov¹, A A Golikov¹, A E Zadayanchuk²

¹ Kuban State Technological University, Krasnodar, 2a, Moskovskaya st., Krasnodar, 350020, Russia
² Kuban State University, Krasnodar, 149 Stavropol'skaya, st., Krasnodar, 350040, Russia
E-mail: artstyleone@mail.ru

Abstract. Controlling the processes of interaction of the working pairs of the friction unit is possible when solving the problem of determining the parameters of influence. As the main criterion of the process, it is necessary to highlight the temperature on the surface of the friction unit. This paper presents a method for modeling the magnitude and distribution of temperature over the surface of a metal friction element depending on the nanogeometry of the surface profile of contact pairs of friction units. The ability to control the process of interaction of contacting pairs of the friction unit will further predict the life of the part and avoid such phenomena as burns and the formation of foci of microcracks.

1. Introduction
In the works of researchers [2, 3, 4], the effect of temperature on the tribocontact of various friction units is indicated. In the literary source [7], modified friction units are proposed based on new polymer materials used in braking devices of various types. The paper [6] proposes a method for controlling the energy-loading of the friction unit using the example of a drum-shoe type brake. The work [8] investigated the developed thermal model of the brake disc with the use of surface layers on the working and matt surfaces of the brake disc. The literary source [1] describes the interaction of the electric fields of the surfaces of the contact pads of the friction units. From the analysis of works [9, 10], devoted to the wear of the working surfaces of friction units, it can be seen that the specific contact pressures of the working surfaces of friction pairs are unevenly distributed over the area of the metal element. These studies allow us to say that the interaction of the working pairs of the friction unit is a controlled process that depends on many factors and can and should be influenced. In [5], the authors proposed the results of modeling the contact pressure on an elementary area of the friction unit depending on the nanogeometry of the surface. This paper provides further research on the effect of surface nanogeometry on other parameters such as the temperature of contacting pairs.

In this regard, to control the processes of friction and wear in the friction unit, it is necessary to study the effect of temperature and temperature gradient on the process of destruction of the material of the parts of the friction unit. The criterion for the thermal destruction of the material is the limiting value of the temperature $T^*$ for the materials of the friction lining and the metal element when cracking occurs. Consequently, the most important criterion is the maximum temperature reached in the material $T^*$. 

[The rest of the document continues with the content related to the research and results discussed above.]
Thermal destruction in the development of braking devices at the design stage can be controlled using the nanogeometry of the surface of the contact area of the rubbing elements of the friction unit. It is important to minimize the maximum temperature of the contact area during braking. In the process of braking, an external heat source arises on the contact area of the rubbing elements, caused by external friction of the non-metallic material, the working surface of the metal element. As a result of conductive heat exchange, the temperature of the non-metallic and metallic friction elements increases. The control variables are taken to be the parameters of the nano-geometry of the surface of the metal element.

For a mathematical model to reduce the thermal state of the rubbing elements of the friction unit, using the example of a disc-shoe brake, we take the differential equation of thermal conductivity in a polar coordinate system:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = 0.$$  \hfill (1)

The metal friction element is assigned to the polar coordinate system \( r, \theta \), and the origin of coordinates is chosen at the center of concentric circles \( L_0 \) and \( L \) with radii \( R_0 \) and \( R \), respectively. We simulate the brake disc as a homogeneous isotropic body.

The sought function \( H(\theta) \), which describes the nanogeometry of the surface of an elementary area of the friction lining of the brake device, can be represented as a segment of the Fourier series:

$$H(\theta) = \sum_{k=0}^{\infty} \left( a_k^0 \cos k\theta + b_k^0 \sin k\theta \right).$$  \hfill (2)

where \( a_k^0, b_k^0 \) are control parameters.

The function describing nano-geometry is assumed to be given. Consequently, the problem of optimizing the thermal state is reduced to determining the control coefficients of the Fourier series.

The first step in solving the optimization problem is the need to determine the thermal state of the elementary surface of the friction unit, and then to find the temperature on the contact surface.

Based on this condition, we write down the heat conduction equation and its boundary conditions:

$$\Delta t = 0, \quad A_{t1} \frac{\partial t}{\partial n} - A_{t2} \alpha_1 t = -Q_b(\theta), \quad (r = \rho);$$

$$\lambda \frac{\partial t}{\partial r} - \alpha_1 t = 0, \quad (r = R_0);$$

where \( \Delta \) is the Laplace operator;

\( A_{t1} \) — heat absorption surface, m²;

\( A_{t1} \) - cooling surface, m²;

\( Q_b(\theta) \) is the amount of heat supplied within the boundaries of the contact area, J;

\( \lambda \) — coefficient of thermal conductivity of the disc material, W / (m K);

\( \alpha_1 \) is the heat transfer coefficient between the outer surface of the brake disc and the environment (inside the perimeter of the contact pad), W / (m²K).

For the distribution of excess temperature on the surface of the brake disc, we use an expansion in a small parameter, in which we neglect the terms of \( \varepsilon \) to a power higher than the first:

$$t_{\text{пов}} = t_{|r=\rho} = t^{(0)}_{|r=R} + \varepsilon \left[ \frac{\partial t^{(0)}}{\partial r} H(\theta) + t^{(1)} \right]_{|r=R},$$  \hfill (3)

where \( t(0) \), \( t(1) \) are the temperatures of the brake disc in zero and first approximations.

The temperature of the brake disc in zero and first approximations will be as follows:

$$t^{(0)}_{|r=R} = C_{10} + C_{20} \ln R + \sum_{k=1}^{\infty} \left( C_{10}^{(k)} R^k + C_{20}^{(k)} R^{-k} \right) \cos k\theta$$

$$+ \sum_{k=1}^{\infty} \left( A_{10}^{(k)} R^k + A_{20}^{(k)} R^{-k} \right) \sin k\theta.$$
\[ t^{(1)}_{r=R} = C_{11} + C_{21} \ln R + \sum_{k=1}^{\infty} \left( c^{(k)}_{11} R^k + c^{(k)}_{21} R^{-k} \right) \cos k\theta + \sum_{k=1}^{\infty} \left( A^{(k)}_{11} R^k + A^{(k)}_{21} R^{-k} \right) \sin k\theta; \]

where \( C_{10}, C_{20}, C_{(k)}^{10}, C_{(k)}^{20}, A_{10}, A_{20}, C_{11}, C_{21}, C_{(k)}^{11}, C_{(k)}^{21}, A_{(k)}^{11}, A_{(k)}^{21}\) - permanent; 
\( \partial t^{((0))}/\partial r \) - temperature gradient along the radius of the brake disc, K / m.

The gradient can also be expanded in a Fourier series:

\[ \frac{\partial t^{((0))}}{\partial r} = C_{20} \frac{1}{R} + \sum_{k=1}^{\infty} k \left( c^{(k)}_{10} R^{k-1} + c^{(k)}_{20} R^{-k-1} \right) \cos k\theta + \sum_{k=1}^{\infty} k \left( A^{(k)}_{10} R^{k-1} + A^{(k)}_{20} R^{-k-1} \right) \sin k\theta \]

To determine the constants \( C_{10}, C_{20}, C_{(k)}^{10}, C_{(k)}^{20}, A_{10}, A_{20}, C_{11}, C_{21}, C_{(k)}^{11}, C_{(k)}^{21}\), it is necessary to use the optimization boundary conditions in the zero approximation:

\[ C_{10} = \frac{\Delta_1}{\Delta}; \quad C_{20} = \frac{\Delta_2}{\Delta}; \quad \Delta = -A_{t2} \alpha_1 \left( \frac{\lambda}{R_0} + \alpha R_0 \right) - \alpha_2 \left( \frac{A_{11} \lambda}{R} - A_{t2} \alpha_1 \ln R \right); \]

\[ \Delta_1 = -\sigma f \alpha_0^0 \left( \frac{\lambda}{R_0} + \alpha_2 R_0 \right); \quad \Delta_2 = -\alpha_2 \sigma f \alpha_0^0; \quad \Delta_{10} = \frac{\Delta_1}{\Delta}; \quad \Delta_{20} = \frac{\Delta_2}{\Delta}; \]

\[ \Delta_k = \left( A_{t1} \lambda k R^{k-1} - A_{t2} \alpha_1 \lambda k R^{k-1} \right) \left( \alpha_2 R^k - \lambda k R_0^{k-1} \right) \]

\[ + \left( \lambda k R_0^{k-1} + \alpha_2 R_0^k \right) \left( A_{t2} \alpha_1 R^{k-1} + A_{t1} \lambda k R^{k-1} \right); \]

\[ \Delta_{1k} = -\sigma f \alpha_0^0 \left( \alpha_2 R^{k-1} - \lambda k R_0^{k-1} \right); \quad \Delta_{2k} = \sigma f \alpha_0^0 \left( \lambda k R_0^{k-1} + \alpha_2 R_0^k \right); \]

\[ A_{10}^{(k)} = \frac{\Delta_{10}^{(k)}}{\Delta_k}; \quad A_{20}^{(k)} = \frac{\Delta_{20}^{(k)}}{\Delta_k}; \quad \Delta_{1k}^{0} = -\sigma f \beta_0^0 \left( \alpha_2 R_0^k - \lambda k R_0^{k-1} \right); \quad \Delta_{2k}^{0} = \sigma f \alpha_0^0 \left( \lambda k R_0^{k-1} + \alpha_2 R_0^k \right); \]

\[ \alpha_0^0, \alpha_1 \alpha_2^0, \alpha_2, \alpha_0^1 \alpha_1 \beta_0^1 \alpha_2 \beta_0^1 \alpha_2 \] are the coefficients of the Fourier series.

\[ \alpha \] is the heat transfer coefficient between the outer surface of the friction pad and the environment, W / (m²K);

\( \alpha_2 \) is the heat transfer coefficient between the outer surface of the brake disc and the environment (outside the perimeter of the contact area), W / (m²K);

\( \sigma \) is the heat flow distribution coefficient;

\( f \) is the coefficient of friction;

\( v \) - average speed during the braking period, m / s;

\( p_{\theta}^{((0))}(\theta) \) - specific pressure on the friction contact area of the friction unit, MPa;
Let us write down the heat conduction equation (2) and its boundary conditions in the form of a Fourier series:

\[ A_{t2} \alpha \frac{\partial^2 t}{\partial r^2} - \lambda A_{t1} \frac{\partial^2 t}{\partial r^2} = \sum_{k=1}^{\infty} \left( A_k^0 \cos k\theta + B_k^0 \sin k\theta \right), \quad (r = R); \quad (7) \]

The amount of heat from the source is determined based on the dependence:

\[ Q_b^{(1)}(\theta) = \sigma f v \rho p(1)(\theta). \quad (8) \]

For the function of the surface temperature of the brake disc, we find the maximum value:

\[ t_{\text{пов max}} = t^{(0)}(\theta_{1}) + \varepsilon \left[ \frac{\partial t^{(0)}(\theta_{1})}{\partial r} H(\theta_{1}) + t^{(1)}(\theta_{1}) \right]_{r=R}. \quad (9) \]

The value of \( \theta_{1} \) is determined from the following equation:

\[ \frac{dt_{\text{пов}}}{d\theta} = 0. \quad (10) \]

The function \( H(\theta) \) must be chosen so as to ensure the minimization of \( t_{\text{пов max}} \). Since the temperature on the surface \( t_{\text{пов}} \) is an indicator of the quality of control, and also equation (9) linearly depends on the desired coefficients (2), the posed optimization problem is reduced to a linear programming problem. The main condition is the limitation:

\[ t_{\text{пов max}} \leq [t], \quad (11) \]

where \([t]\) is the permissible excess temperature, which is a characteristic of the material, K.

In addition to constraint (11), the following condition must be satisfied:

\[ \sum_{i=1}^{n} H(\theta_{i}) = n R_{a}, \quad a_{0}^{0} \geq 0, a_{k}^{0} \geq 0, b_{k}^{0} \geq 0. \quad (12) \]

where \( n \) is the number of sites;

\( R_{a} \) is the arithmetic mean deviation of the nanogeometric profile of the contact area of the friction unit.

For solving the optimization problem, the simplex algorithm is the most effective method, because it leads to an optimal solution in a finite number of iterations.

In the expansion (2), the functions \( H(\theta) \) were limited to \( k = 5 \) terms. The values of the control parameters (coefficients \( a_{k}^{0} \geq 0 \) and \( b_{k}^{0} \geq 0 \)) are found depending on the thermophysical and geometric parameters of the contact area of the friction unit. The results of preliminary computer simulation are presented in tables 1 and 2.

| Table 1. Calculation results (subject to \( v = 0.2 \text{ m/s} \)) |
|---|---|---|---|---|---|
| \( a_{0}^{0} \) | \( a_{1}^{0} \) | \( a_{2}^{0} \) | \( a_{3}^{0} \) | \( a_{4}^{0} \) | \( a_{5}^{0} \) |
| 0.642 | 0.598 | 0.522 | 0.355 | 0.248 | 0.218 |
| \( b_{1}^{0} \) | \( b_{2}^{0} \) | \( b_{3}^{0} \) | \( b_{4}^{0} \) | \( b_{5}^{0} \) |
| 0.627 | 0.563 | 0.312 | 0.288 | 0.175 |

| Table 2. Calculation results (subject to \( v = 1.0 \text{ m/s} \)) |
|---|---|---|---|---|---|
| \( a_{0}^{0} \) | \( a_{1}^{0} \) | \( a_{2}^{0} \) | \( a_{3}^{0} \) | \( a_{4}^{0} \) | \( a_{5}^{0} \) |
| 0.788 | 0.734 | 0.622 | 0.545 | 0.423 | 0.312 |
| \( b_{1}^{0} \) | \( b_{2}^{0} \) | \( b_{3}^{0} \) | \( b_{4}^{0} \) | \( b_{5}^{0} \) |
| 0.78 | 0.721 | 0.544 | 0.41 | 0.233 |

In the optimization problem, the parameters of the brake disc diameter were determined. Modeling has shown that solving the optimization problem, i.e. the temperature on the surface of the elementary area of the contact surfaces of the friction unit depends on its nanogeometry of the surfaces of the metallic \( H(\theta) \) and non-metallic \( H_{1}(\theta) \) friction elements. The final stage after determining the nanogeometry of the surface of the rubbing elements of the friction unit, it is necessary to find the condition for the uniform distribution of the surface temperature. Thus, when choosing the
nanogeometry of the friction surface, it is possible to reduce the temperature level. According to
dependence (3), it can be concluded that the temperature on the surface linearly depends on the
coefficients $a_k^0$ and $b_k^0$ of the Fourier series for the function $H(\theta)$ and on the coefficients $a_k^1$ and $b_k^1$ of the Fourier series for the function $H1(\theta)$. The temperature function on the surface of
the elementary contact pad will look like this:

$$t_{пов}(\theta, \tau) = F(\theta, \tau, a_0^0, a_k^0, b_k^0, a_1^0, a_k^1b_k^1), (i = 1, 2 \ldots m).$$

(13)

where $\tau$ is the braking time, s.

The optimization task is to determine the values of unknown parameters that will provide the
following condition:

$$U = \sum_{i=1}^{M} [F(\theta, \tau, a_0^0, a_k^0, b_k^0, a_1^0, a_k^1b_k^1) - \bar{t}]^2 \rightarrow \min;$$

(14)

where $\bar{t}$ is the optimal temperature value on the friction surface, K.

2. Materials and methods

The principle of least squares states that the most likely values of the parameters will be those at which
the sum of the squares of the difference between the surface and optimal temperatures will be the
smallest. The final solution to the optimization problem is presented in tables 3 and 4.

Table 3. Calculation results (subject to $v = 0.2$ m / s)

|     | $a_0^0$ | $a_1^0$ | $a_2^0$ | $a_3^0$ | $a_4^0$ | $a_5^0$ |
|-----|---------|---------|---------|---------|---------|---------|
| $a_0^0$ | 0.685   | 0.601   | 0.584   | 0.331   | 0.206   | 0.185   |
| $b_1^0$ | 0.422   | 0.344   | 0.129   | 0.065   | -0.173  |

Table 4. Calculation results (subject to $v = 1.0$ m / s)

|     | $a_0^0$ | $a_1^0$ | $a_2^0$ | $a_3^0$ | $a_4^0$ | $a_5^0$ |
|-----|---------|---------|---------|---------|---------|---------|
| $a_0^0$ | 0.702   | 0.655   | 0.598   | 0.433   | 0.389   | 0.241   |
| $b_1^0$ | 0.549   | 0.533   | 0.322   | 0.211   | 0.095   |

Figure 1 shows a graphical representation of the temperature distribution on the surface of an
elementary contact pad of the friction unit at different sliding speeds. The solid line is shown for a
smooth contour, and the dashed line is for the law of roughness distribution, described by the function
$H(\theta)$. The modeling results clearly show that when choosing the nanogeometry of the surface of an
elementary contact area, the surface temperature decreases and is distributed over the contact surface,
in contrast to a smooth contour. The exception is the boundaries of the elementary area, where, as a
result of the redistribution, the surface temperature of the elementary area increases in comparison with
the smooth contour of the brake disc. The figure may indicate that the optimization problem on the
example of an elementary area of the contacting surfaces of the friction unit of the brake disc performs
its functions and minimizes and evens out the temperature over the surface.
Figure 1. Dependences of the surface temperature of the contact pad of the friction unit on the value of the polar angle: curves 1 and 2 for a sliding speed $v = 0.2 \text{ m/s}$, curves 3 and 4 for a sliding speed $v = 1.0 \text{ m/s}$.

3. Conclusion
The optimization problem has shown that the surface temperature of an elementary area depends both on the nanogeometry of the working surfaces of the friction unit and on the contact pressure. The obtained control parameters make it possible to simulate the temperature distribution on the surfaces of the friction unit. Close to uniform temperature distribution on the surface allowed to reduce its value and distribute over the entire surface of the metal friction element. This fact is explained by the fact that with this choice of the nano-geometry of the profile, the entire surface of the contact area interacts and there are no foci of huge energy release on the surface.

The developed optimization problem makes it possible to design a friction unit with a predicted thermal load, which will further allow to control the process of minimization and distribution of the friction unit over the surface of the working platforms.

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