Einstein and the Early Theory of Superconductivity, 1919–1922

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Abstract
Einstein’s early thoughts about superconductivity are discussed as a case study of how theoretical physics reacts to experimental findings that are incompatible with established theoretical notions. One such notion that is discussed is the model of electric conductivity implied by Drude’s electron theory of metals, and the derivation of the Wiedemann-Franz law within this framework. After summarizing the experimental knowledge on superconductivity around 1920, the topic is then discussed both on a phenomenological level in terms of implications of Maxwell’s equations for the case of infinite conductivity, and on a microscopic level in terms of suggested models for superconductive charge transport. Analyzing Einstein’s manuscripts and correspondence as well as his own 1922 paper on the subject, it is shown that Einstein had a sustained interest in superconductivity and was well informed about the phenomenon. It is argued that his appointment as special professor in Leiden in 1920 was motivated to a considerable extent by his perception as a leading theoretician of quantum theory and condensed matter physics and the hope that he would contribute to the theoretical direction of the experiments done at Kamerlingh Onnes’ cryogenic laboratory. Einstein tried to live up to these expectations by proposing at least three experiments on the phenomenon, one of which was carried out twice in Leiden. Compared to other theoretical proposals at the time, the prominent role of quantum concepts was characteristic of Einstein’s understanding of the phenomenon. The paper concludes with comments on Einstein’s epistemological reflections on the problem.
Introduction

The history of superconductivity constitutes an example of conceptual change in physics where unexpected experimental discoveries have preceded theoretical analyses more than once. The very discovery of superconductivity in 1911 itself is a case in point. No definite theoretical expectations could be formulated as to how the electric resistance would behave in the very low temperature regime on the basis of contemporary theories and models of electrical resistance. The sudden loss of resistivity of some metals within a very small temperature interval at liquid helium temperatures over several

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\(^1\)For general historical accounts of the experimental and theoretical developments associated with the phenomenon of superconductivity, see [Dahl 1992] and [Matricon and Waysand 2003].
orders of magnitude to a value that was below any experimentally observable threshold was a novelty, not expected and not to be foreseen. The same would hold true, one may argue, for the discovery of the Meissner effect in 1933, the second of the two fundamental features that constitute, according to today’s understanding, the phenomenon of superconductivity. The discovery that superconductors are perfect diamagnets and expel any magnetic fields reversibly when entering the superconducting state, hence rendering the superconducting state a true thermodynamic state, was made in the context of theoretical speculations about the magnetic behavior of superconductors, but the effect itself was unexpected as well. More recently, the discovery of high-temperature superconductors by Bednorz and Müller in 1986 was again an unforeseen experimental discovery. Although by now many more superconductors of high transition temperature have been identified and a wealth of detail is known about these materials, the precise mechanism of high-temperature superconductivity is still not yet fully understood.

We now know that superconductivity is a genuine macroscopic quantum phenomenon. It defied theoretical understanding until first successfully interpreted in terms of a macroscopic wave function by V.L. Ginzburg and L.D. Landau in 1950. Similar to non-relativistic Schrödinger quantum mechanics, the square of the wave function is interpreted as a probability density for the superconducting electrons. The wave function in the Ginzburg-Landau theory also acts as a thermodynamic order parameter, such that the transition from normal conductivity to the superconducting state in the absence of magnetic fields is interpreted as a phase transition of second order. A microscopic justification of the phenomenological Ginzburg-Landau theory was given seven years later by J. Bardeen, L.N. Cooper and J.R. Schrieffer. In the so-called BCS theory which constitutes today’s standard explanation of superconductivity, the macroscopic wave function is accounted for by a microscopic theory in which an effective attractive interaction between electrons arises from lattice phonons so that electrons associate to pairs and condense to the macroscopic wave function. It should therefore be clear that in the very beginning of the history of superconductivity, i.e. long before the discovery of the Meissner effect and long before concepts such as a quantum-mechanic wave function and a phase transition of second order were available, an explanation of the phenomenon that in any sense might come close to our

\[2\] For an overview of the available experimental data and further references, see, e.g. [Poole 2000].
modern understanding was well out of reach of contemporary theoreticians.

As a study of how theoretical physics is being done in practice, it is interesting then to take a closer historical look at how physicists have interpreted experimental data that clearly challenged the validity of well-established concepts and theories by a phenomenon and that were, at the same time, well out of the horizon of what could possibly be understood at the time in any reasonable way. Gavroglu and Goudaroulis have coined the term “concepts out of context(s)” to capture the peculiar situation of theoretical attempts to come to grips with the phenomenon of superconductivity.

This paper examines one such reaction to this phenomenon, namely Albert Einstein’s. Given Einstein’s characteristic awareness of foundational problems that allowed him not only to overcome the limits of classical mechanics and electrodynamics with his theories of relativity, but also to be one of the first who perceived most clearly the limits of classical mechanics with respect to the quantum phenomena, a reconstruction of Einstein’s interpretation of the phenomenon of superconductivity promises insights into the theoretical horizon of the time. Indeed, as I will show, it was Einstein who not only most clearly recognized the challenge posed by the phenomenon to classical concepts but who also most explicitly advocated and actively explored the use of quantum concepts for a theoretical understanding of superconductivity.

In 1922 Einstein wrote a paper, entitled “Theoretical remarks on the superconductivity of metals,” which has received comparatively little attention from historians of science. One reason for its neglect in the historical literature might be that it appeared to be a quite isolated episode within Einstein’s published oeuvre, unconnected to his more prominently figuring concerns. Recently, however, the editorial project of the Collected Papers of Albert Einstein has brought to the fore some evidence in Einstein’s correspondence and unpublished manuscripts that not only allows us to get a better picture of Einstein’s thoughts on the problem of superconductivity, but also...

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3 Gavroglu and Goudaroulis 1984, Gavroglu and Goudaroulis 1989, see also Gavroglu 1985.
4 Einstein 1922. B.S. Schmekel recently published an English translation at http://www.arxiv.org/physics/0510251 (In my quotations from this paper, I will occasionally differ from Schmekel’s translation. Except for correspondence and writings published in the Collected Papers of Albert Einstein where English translations were taken from the translation volumes of this series, all other English translations are mine.). The paper is discussed in Yavelov 1980 and Dahl 1992 pp. 105–106; see also Matricon and Waysand 2003 p. 42, Kragh 1999 p. 86, Renn 1997 p. 335 for brief mentions of this paper.
would have us revise, or at least nuance to some extent, our understanding of Einstein’s preoccupations.

Einstein’s visit to Leiden in the fall of 1920 is mainly known for his famous inauguration lecture on “Ether and Relativity,” delivered on the occasion of his appointment as special visiting professor at Leiden. The lecture was given on October 27, 1920, and published separately as a little booklet [Einstein 1920]. Less known but equally important at the time was the fact that Einstein spent most of his time during this stay in Leiden in late October and early November 1920 participating in a meeting devoted to recent developments in low temperature physics, specifically about the problems of magnetism at low temperatures. Other participants in these discussions included Paul Ehrenfest, Heike Kamerlingh Onnes, Willem H. Keesom, Johannes P. Kuenen, Paul Langevin, Hendrik A. Lorentz, and Pierre Weiss.  

In fact, the initiators of Einstein’s appointment in Leiden specifically were hoping for his input in discussions of problems in low temperature physics. Thus Lorentz wrote to Einstein, almost a year earlier, on 21 December 1919:

Our Berlin colleagues will undoubtedly understand that we would like to have you here from time to time and that, for inst., Kamerlingh Onnes would put great store in discussing problems being addressed at his Cryogenic Laboratory with you.

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5 [Einstein 1920]. The text of the printed lecture was completed before 7 April 1920, and the title page of the printed version states that the lecture was given on 5 May 1920. Due to delays in his appointment (see [CPAE10, pp. xliii–xlvi]), the lecture was, however, given only on 27 October 1920, see [CPAE7, p. 321].

6 On this meeting, see [Matricon and Waysand 2003, p. 42], [CPAE10, p. xlvi–xlviii]. See also [Kamerlingh Onnes 1921a, p. 3], where the November meeting is explicitly mentioned in an introductory footnote. In the evening edition of 25 October 1920 of the Dutch daily Nieuwe Rotterdamsche Courant an announcement of the meeting was published in which Einstein’s role was especially emphasized: The discussions were to center on “the phenomena of paramagnetism at low temperatures and the peculiarities of superconductivity. [...] The attraction of these discussions, that will take place this week, is greatly enhanced by the participation of Prof. Einstein from Berlin [...] He is in particular participating with regards to the application of the quanta to the explanation of the mentioned phenomena.” The newspaper report also mentioned that Onnes hoped to host similar meetings “at times that Prof. Einstein is also in Leiden because of his professorship.” I wish to thank Jeroen van Dongen for alerting me to this newspaper article and for providing an English translation.

7 [CPAE9 Doc. 229]. For a very similar comment in this spirit, see also Lorentz to Einstein, 16 January 1920 [CPAE9 Doc. 264].
And the champion of low temperature physics himself, Kamerlingh Onnes wrote to Einstein on 8 February 1920:

Thus, best conditions are made for stimulating investigations, guiding ongoing analyses onto better paths, as well as exchanging fruitful ideas of every kind. Thus, with your Leiden professorship I also cherish the finest hopes for a flowering of the cryogenic laboratory. Virtually no one is so closely affiliated to it as you are. Many of the investigations performed there regard phenomena whose relevance to quantum theory you have recognized and for whose analysis this laboratory is somewhat of an international institution, insofar as the area of low temperatures is concerned. So your help can bring about much that is of benefit. You will perhaps find me very egoistical if I already immediately ask you to make available to me some of your precious time for devising strategies and identifying problems. But I take that risk, dear friend! And I assure you that I find it just as great a fortune for the Cryogenic Laboratory as for theoretical physics that you will be connected with Leiden as one of our own.\[8\]

Although outshone by the stellar success of the 1919 confirmation of gravitational light bending by the British eclipse expedition, there are thus a number of indications that Einstein at the time was indeed considered in Leiden a leading theoretician of low temperature physics.\[9\] What follows is an account of what we know about Einstein’s concerns with superconductivity until 1922 and an analysis of his interpretation of this phenomenon.

We have little evidence of Einstein’s thoughts on the subject before 1919 and, in spite of some efforts, I did not find any comments by him from later than 1922. Nevertheless, Einstein’s apparent silence on the subject may well be attributed to our as yet insufficient knowledge of the documents in the Einstein Archives. It is hence possible that material of interest may come to light, e.g. in the preparation of further volumes of Einstein’s Collected...
Papers. I will not address Einstein’s thoughts and ideas on other related phenomena, such as the behavior of specific heats at low temperature, or his statistical work that led to the identification of what we now call the Bose-Einstein statistics. The relative weight and significance of ideas about superconductivity was determined to some extent by the conceptualization of normal electric conductivity and of phenomena associated with it. But a thorough discussion of the issue of normal metallic resistivity, or of low temperature phenomena in general, or of those phenomena that were at the basis of the emergence of the new quantum theory is beyond the scope of the present paper which, focusses exclusively on the problem of accounting for the phenomenon of superconductivity.

My account will be organized as follows. In order to address the methodological difficulty just mentioned, my starting point will be the canonical conceptualization of electrical and thermal conductivity of the early twentieth century, which was based on Drude’s electron theory of metals and culminated in a quantitative formula for the Wiedemann-Franz law. I will discuss the standard derivation of this law within the electron theory of metals by annotating Einstein’s own derivation as written down sketchily in his course notes for a lecture course on the kinetic theory of heat held in 1910. I will then give a synchronic characterization of the state of knowledge about the phenomenon of superconductivity ca. 1920, followed by a discussion of our evidence that Einstein was, in fact, well informed about these experimental data through his strong professional and personal ties to the physicists at Leiden. I will then discuss the phenomenological theory of infinite or perfect conductivity, as expounded in an influential contribution by Gabriel Lippmann. Investigations of the Maxwell equations for infinite conductivity can be found in Paul Ehrenfest’s diaries. They were also the basis for a consideration by Einstein that was intended as background theorizing for proposed experimental investigations of the particular features of a Hall effect for superconductors, should such an effect exist. I will then turn to a discussion of microscopic models of infinite conductivity. In order to provide some necessary context for Einstein’s own theory, I will discuss several contemporary proposals of microscopic charge transport that were advanced specifically in order to meet the challenge posed by the phenomenon of superconductivity to the kinetic electron theory of metals and to account for the phenomena associated with it. Among these are models by Johannes Stark, Frederick A. Lindemann, Heike Kamerlingh Onnes, Joseph John Thomson, Fritz Haber, and finally by Einstein himself. Against the background of this
horizon of theoretical responses to the available experimental data, Einstein’s own theoretical speculations, as expounded in his only published paper on the subject, appear as an innovative and original contribution, not the least because he employed concepts from the emerging quantum theory. Einstein derived testable consequences of his specific microscopic assumptions about superconductive currents, at least one of which was tested in Leiden by an experiment specifically designed for this purpose. I will conclude with a discussion of Einstein’s epistemological reflections on the problem and some remarks on Einstein’s contributions.

Drude’s electron theory of metals

At the time of the discovery of superconductivity, the electron theory of metals was a highly developed and sophisticated theory. Its most impressive success was a theoretical justification of the so-called Wiedemann-Franz law. This law asserts that for many metals the ratio of thermal and electrical conductivity only depends on temperature and not on any specific properties of the metal. Part of the success of the electron theory of metals was the fact that it seemed to provide a well-founded and unambiguous way to also quantitatively compute the coefficient of the temperature dependence of the Wiedemann-Franz law also quantitatively, and that the theoretical values agreed with reasonable accuracy with the observed values.

The model itself was extremely simple, although more detailed theoretical discussions of its features could become quite involved. For our purposes it will suffice to discuss its basic features. We will do so by paraphrasing and commenting on Einstein’s own notes on a derivation of the Wiedemann-Franz law in the context of the electron theory of metals. A brief, “back-of-an-envelope” derivation of this law is written down in Einstein’s lecture notes for a course on kinetic theory, held in the summer semester 1910 at the University of Zurich. In these notes, Einstein sketched standard theoretical predictions for the thermal and electrical conductivities of metals.

10 For contemporary reviews, see [Seeliger 1921], [Suter 1920], [Meißner 1920]. For a historical discussion, see [Kaiser 1987] and also [Hoddeson and Baym 1980] and [Hoddeson et al 1987]. For a historical discussion of Einstein’s concerns with an electron theory of metals, see [Renn 1997].

11 The course notes are published as [CPAE3 Doc. 4]. For a facsimile of the course notes, see Einstein Archives Online (http://www.alberteinstein.info), Call Nr. 3-003. The page dealing with the electron theory of metals is [p. 49], i.e. [CPAE3 pp. 232–233]. For a very similar example of the following “back-of-an-envelope” calculation, including the factor-of-
cal considerations he had obtained from his readings of Boltzmann, Riecke, Drude, and others, as preparation for his classes, and without explicit reference to his sources.\textsuperscript{12}

The basic idea was to apply the concepts of the kinetic theory of gases to a gas of electrons in the metal. Electrons were conceived of as particles with inertial mass and electric charge that were moving about with random thermal motion in the metal. More specifically, it was assumed that the electrons would not interfere or interact with each other, and that they would only interact with the positive ions upon collision. After colliding with an ion, an electron would proceed on its path again freely, but with new energy and momentum whose statistical distribution would only depend on the place of the last collision.

The model allowed for a straightforward conceptualization of transport phenomena such as heat conduction or electrical conduction. In order to derive more specific relations for the quantities of interest, further simplifications were usually made. Thus, in the beginning of his course notes, Einstein sketched the derivation of a general relation in the kinetic theory of gases that is applicable for generic transport phenomena ("Transport of any Molecular Quantity through the Gas.") under the assumptions that all molecules at the same location have the same mean velocity \( c = \sqrt{c^2} \). He considered a molecular function \( G \) of an arbitrary quantity that is being transported through the gas:

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\text{Each molecule carries along a certain quantity of something, with this amount depending only on where the molecule's last collision}
\]

\textsuperscript{1/2-problem discussed below, see the first page of notes by Niels Bohr for a lecture course on the Electron Theory of Metals, held in 1914 at the University of Copenhagen, [BCW1, p. 446].}

\textsuperscript{12} For further evidence that Einstein was well acquainted with, and critical of, contemporary research in the electron theory of metals, see Einstein to Mileva Marić, 28? May 1901. In this letter, he reports about having read [Reinganum 1900], a paper, in which Drude's derivation of the Wiedemann-Franz law is reviewed and discussed with respect to its underlying assumptions. To Mileva Marić, he wrote: "I found there a numerical confirmation [...] for the fundamental principles of the electron theory, which filled me with real delight and completely convinced me about the electron theory." [CPAE1, Doc. 111]. Ten years later, Einstein expressed himself rather critical about Reinganum whose works he then characterized as "rather unclean" (Einstein to Alfred Kleiner, 3 April 1912 [CPAE5, Doc. 381]). See also Einstein to Hans Tanner, 24 April 1911 [CPAE5, Doc. 265] for another critical comment on Reinganum's work and, for a general discussion of Einstein's early appreciation and later criticism of Drude's electron theory, see [Renn 1997].
And he computed the flux $F$ of the molecular function $\mathcal{G}$ by considering all molecules that contribute to the transport and by integrating over all directions. The result was

$$F = -\frac{1}{3} n c \lambda \frac{\partial \mathcal{G}}{\partial z},$$

(1)

where $n$ is the number of molecules per unit volume, $\lambda$ the mean free path, and the partial derivative is taken arbitrarily with respect to the $z$-direction.

This relation is then quoted many pages later, when Einstein set out to discuss the “electron theory of metals.”[14] He first applied it to derive an expression for the thermal conductivity. Here the molecular function is taken to be the kinetic energy of a “molecule,” i.e. an electron of mass $\mu$,

$$\mathcal{G} = \frac{1}{2} \mu c^2 = \frac{3RT}{2N},$$

(2)

which he relates to the temperature $T$ using the equipartition theorem. $R$ is the gas constant and $N$ is Avogadro’s number. One has

$$\frac{\partial \mathcal{G}}{\partial z} = \frac{1}{2} \frac{\partial \mu c^2}{\partial T} \frac{\partial T}{\partial z} = \frac{3R}{2N} \frac{\partial T}{\partial z},$$

(3)

and hence

$$F = -\frac{1}{2N} n c \lambda \frac{\partial T}{\partial z},$$

(4)

from which one can readily read off the thermal conductivity $\kappa$ as the (negative) coefficient in front of $\partial T/\partial z$,

$$\kappa = \frac{1}{2N} n c \lambda.$$

(5)

Note that the thermal conductivity still depends on the electron density $n$ and the mean free path $\lambda$ that are specific to individual metals.

The next step then is to obtain an expression for the electric conductivity. Here the argument does not go back to the general formula (1) of the flux

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[13] CPAE3 p. 183.

[14] In spite of differences in notation, Einstein’s derivation closely followed the one given in Drude 1900. There Drude, too, began by quoting eq. (1) from Boltzmann as his starting point.
for the molecular function $G$. Instead, Einstein’s derivation started from the concept of a mean collision time $\tau$, taken to be the quotient of the mean free path and the mean velocity,$$
abla = \frac{\lambda}{c} = \tau. \quad (6)$$

In the absence of an external electric field $\mathcal{E}$, it is assumed that the electrons are flying in different directions by equal fractions and hence have no mean drift velocity $\mathcal{C}$. If, however, an external electric field $\mathcal{E}$ is applied, it is assumed that the electrons of charge $-e$ are accelerated during the time of their free flight by a constant acceleration $-e\mathcal{E}/\mu$. The mean drift velocity $\mathcal{C}$ was then obtained by averaging over the mean free flight time as

$$\mathcal{C} = -\frac{1}{\tau} \int_0^\tau \frac{e\mathcal{E}}{\mu} \, dt = -\frac{e\mathcal{E}}{\mu} \frac{\tau^2}{2} \cdot \frac{1}{\tau} = -\frac{1}{2} \frac{e}{\mu} \frac{\mathcal{E}}{c}. \quad (7)$$

Since a finite mean drift velocity $\mathcal{C}$ gives rise to a current density $-n\mathcal{C}e$, one has

$$-n\mathcal{C}e = +\sigma \mathcal{E} \quad (8)$$

and thus obtains the electric conductivity $\sigma$ as

$$\sigma = \frac{1}{2} \frac{e^2 n \lambda}{\mu c}. \quad (9)$$

Before discussing this expression let us complete the derivation by forming the quotient of the thermal and electric conductivities to obtain the Wiedemann-Franz law as

$$\frac{\kappa}{\sigma} = \frac{R}{N\epsilon^2 \mu c^2} = 3 \frac{R^2}{N^2 \epsilon^2 T}. \quad (10)$$

The remarkable feature of this derivation of the Wiedemann-Franz law is that it produces an expression for the Lorenz number $L$, i.e. the coefficient in front of $T$,

$$L \equiv \frac{\kappa}{\sigma T} = 3 \frac{R^2}{N^2 \epsilon^2}, \quad (11)$$

that is in fairly good numerical agreement with the experimental values.\footnote{\cite{Meissner1920, Ashcroft1976} For $R = 8.31$ J/mol·K, $N = 6.02 \times 10^{23}$/mol, $e = 1.6 \times 10^{-19}$C, we find $L$ to be $L \approx 2.2 \times 10^{-8}$ (J/mol·K), a value which is within 10–20% of the experimentally observed value for many elements, see e.g. [Meißner 1920, Tables I–VIII], or [Ashcroft and Mermin 1976, Table I.6].}
Historically, this quantitative agreement was of great significance, since it
convinced most theoreticians, including Einstein, that there was some truth
to the underlying model assumptions of the electron theory of metals. As it
turned out, however, this quantitative agreement is wholly fortuitous. In our
modern understanding of the issues at hand, it arises from the cancellation
of two factors of about one hundred.\footnote{See \cite{Ashcroft and Mermin 1976, p. 23}.} The electronic specific heat \( c_v \) turns
out to be a factor of 100 smaller than the classical prediction \( c_v = (3/2)n k_B \),
where \( k_B = R/N \). The mean square velocity of the electrons at room
 temperature, on the other hand, is about a factor of 100 larger.

Note, however, that already the numerical factor of 1/2 in expression (\ref{eq:drude})
for \( \kappa \), and hence also the numerical factor in the Wiedemann-Franz law (\ref{eq:wiedemann-franz}),
is an artifact, arising (among other things) from the simplification that all
molecules in the same place would have the same mean velocity. A more
careful derivation of (\ref{eq:drude}) would have to start from the full Maxwell
distribution, as was pointed out already by Drude himself.\footnote{[Drude 1900, p. 569].} Such a refinement was
carried out by Lorentz in 1905 who obtained a factor of 2 instead of 3 in the
Wiedemann-Franz law (\ref{eq:wiedemann-franz}). Other refinements of the derivation were also
discussed in the sequel and produced yet other numerical factors.\footnote{See, e.g., the discussion in \cite{Seeliger 1921, pp. 785–791].}

One other problem needs to be mentioned here. It was pointed out, in
a widely read modern textbook on solid state physics, that eq. (\ref{eq:wiedemann-franz}), as it
stands, is wrong by a factor of 1/2, since the electric conductivity \( \sigma \) should
actually be a factor of two larger than that given in eq. (\ref{eq:drude}).\footnote{See \cite{Ashcroft and Mermin 1976, p. 23 and prob. 1]. See also \cite{Seeliger 1921} note 16] and references cited therein for a contemporary discussion.} The claim
here is that Drude’s erroneous result arises from an inconsistent application
of the underlying statistical assumptions. The crucial point concerns the
assumptions about the statistical distribution of the times between successive
collisions. From a modern understanding, a natural assumption would be
a Poissonian statistics, where the probability for any electron to undergo a
collision in the infinitesimal time interval \( dt \) is proportional to \( dt/\tau^* \). Here
\( \tau^* \) is the mean collision time, or more precisely the mean time between col-
lisions in the trajectory of a single electron. However, it also follows from
the assumption of a Poissonian statistics that the mean time elapsed after
the last collision for an electron picked at random is also equal to \( \tau^* \), as is
the mean time until the next collision of any such electron picked at ran-
Hence the mean time between successive collisions averaged over all electrons is equal to $2\tau^*$. The averaging in eq. (7) should therefore be over $(1/2\tau^*)\int_0^{2\tau^*}\!$, or else by arguing that an electron picked at random has been flying, on average, for a time $\tau^*$ thus producing a mean drift velocity of $-e\tau^*/\mu$. A similar error was not made, however, in the derivation of the thermal conductivity (5). Hence, the theoretical account of the Wiedemann-Franz law in eq. (11) should have been off by a factor of 2 compared to the experimental data already on grounds of internal consistency of applying the model assumptions.

Drude’s result for the electric conductivity (9) is thus incorrect if we assume a Poissonian statistics for the collisions of the electrons in the metal. It is correct under the different and rather restrictive assumption that the time $\tau$ between collisions is always the same. In this case, and only in this case, eq. (7) still holds. Although the assumption of a constant mean collision time was not made explicit in Drude’s original paper, it seems to me that it does not contradict any of his explicit assumptions either, and the same holds for Einstein’s derivation in his kinetic theory lecture notes. After all, a similar simplifying assumption was made quite explicitly about the mean electronic velocity. Nevertheless, any non-trivial probability distribution for $\tau$ would lead to numerical factors in eq. (9) that would be different from $1/2$, and that would hence jeopardize the numerical agreement of the Lorenz number $L$ in eq. (11) with the experimental data.

Drude’s electron theory of metals thus had a curious epistemological status. Its model assumptions were extremely simple and intuitive. It allowed a more or less straightforward derivation of qualitatively correct results about what quantities play a role in such phenomena as electric conductivity. Some of these results turned out to be completely independent of any microscopic details of the substance at hand. The latter fact was in remarkable analogy to results in the kinetic theory of gases, which had also quite successfully been able to account for general regularities such as, e.g., the Dulong-Petit law. Nevertheless, the quantitative, numerical results, although in surprisingly good agreement with the available experimental data, were somewhat fragile in the sense that modifications of the model or of details of calcu-

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20Roughly speaking, the difference between the mean collision time of a single electron and the mean free flight time of an electron picked at random arises from the fact, that the probability distribution for the mean free flight time is invoked twice in the computation of the latter case.
lating the numerical results were not guaranteed to maintain the agreement between theory and experiment.

With this general statement in mind, let us now comment more specifically on the implications of expression (9) for the electric conductivity $\sigma$ in this model. For the purposes of our present account, two things need to be pointed out. First, in contrast to the Wiedemann-Franz law, the electric conductivity does depend on material-specific quantities. Specifically, the result states that the conductivity is proportional to the density $n$ of conduction electrons and their mean free path $\lambda$. It was also seen to be inversely proportional to the mean electronic velocity $c$, a quantity that was naturally assumed to be only temperature dependent. Other than that, the charge of the conduction electrons $-e$ was a constant, as was their inertial mass $\mu$, as long as relativistic effects were irrelevant.\footnote{Relativistic effects were, of course, irrelevant for Drude but recall that we are here discussing Einstein’s lecture notes of 1910 as background for a contemporary understanding of electric conduction.} The only quantities that could therefore affect the temperature dependence of the conductivity and account for its material specific features seemed to be the density of conduction electrons and their free mean path. The historical significance of this conclusion is illustrated in the following comment that Einstein made in a letter to Lorentz written shortly after his first visit to Leiden in 1911:

What I heard from Mr. Kamerlingh Onnes and Mr. Keesom was also very important. It seems that the relationships between electrical conductivity and temperature are becoming extremely important. If only there would not always crop up the difficulty of one’s not knowing whether the change in the electrical conductivity should be attributed mainly to the change in the number of the electrons or to the change in their free path length, or to both. But I hope and am confident that you will soon succeed in overcoming these difficulties.\footnote{Einstein to Lorentz, 15 Feb 1911, \cite[Doc. 254]{CPAE5}.}

Einstein had visited Leiden just a few weeks before superconductivity was seen for the first time in the cryogenic laboratory. His remark therefore reflects very precisely the assumptions and expectations to which theoretical physicists at the time, working as they did with a specific model of an electron gas, would assimilate the discovery of a sudden loss of resistivity.
A second comment on the significance of Drude’s expression (9) for the electrical conductivity follows from the first. The experimental fact that in certain situations the conductivity drops to an exceedingly small value, if not to 0 altogether, immediately leads to a paradoxical situation when one tries to assimilate the drop to the Drude model. In the case of the phenomenological theory the defining relation of the electrical conductivity (see eq. (12) below) degenerates for infinite $\sigma$. In the model, too, the basic conceptualization of electrical conductivity fails in such a limit: if, as seemed necessary, the temperature dependence of the conductivity arises only from the number of available conduction electrons and from their mean free path, it is immediately clear that Drude’s model cannot account for infinite conductivity. Given the unambiguous experimental result that the loss of resistivity is at least ten orders of magnitude compared to the resistance at room temperature, it is clear that with the sample sizes at hand neither the number of free electrons nor the available space for a large mean free path would permit an even roughly quantitative account of superconductivity.

One more comment may be in order before we proceed to discuss concrete proposals of models for charge transport to account for superconductive currents. While we are focusing for the purpose of the present account on the theory of electrical conductivity, it should be emphasized that the theoretical concepts and ideas that are being invoked have more or less immediate implications for other physical phenomena as well. The model’s assumptions are accordingly constrained by experimental knowledge that is directly relevant for other consequences of the theory, such as specific heats, magnetic properties, and the like. Conversely, the experimental fact of a superconductive state of some metals at very low temperatures poses constraints on theoretical considerations of other phenomena. Einstein had had a long-standing interest in the theory of specific heats, ever since his famous 1907 paper in which he applied Planck’s quantum hypothesis to the problem. Because of the connections implied by the theoretical assumptions between different areas, it was natural for Einstein to invoke the phenomenon of superconductivity in a consideration about the existence of zero point energy:

There are serious doubts about the assumption of zero-point energy existing in elastic oscillations. For if at falling temperatures the (thermal) elastic vibrational energy does not drop to zero but only drops to a finite positive value, then an analogous behav-

\[\text{[Einstein 1907].}\]
ior must be expected of all temperature-dependent properties of solids, i.e., the approach toward constant finite values at low temperatures. But this contradicts Kamerlingh Onnes’s important discovery, according to which pure metals become “superconductors” on approaching absolute zero.24

The experimental discovery of superconductivity thus posed a challenge to account for this phenomenon by modifying or substituting model assumptions inherent in Drude’s electron theory of metals. Before proceeding to discuss these theoretical responses, we will briefly summarize what was known about superconductivity ca. 1920.

Superconductivity around 1920

By 1920, superconductivity was an anomalous and isolated, albeit well-established phenomenon of cutting-edge technology. It was in Leiden that Kamerlingh Onnes had discovered the phenomenon in 1911, three years after he succeeded in liquifying helium.25 In fact, Onnes’s cryogenic laboratory was the only laboratory in the world able to achieve the liquefaction of helium at the time. It retained this status until 1923, when the cryogenic laboratory in Toronto liquified helium with a copy of the Leiden cryogenic apparatus. In 1925, the low temperature laboratory of the Physikalisch-Technische Reichsanstalt in Berlin began to produce liquid helium as well, and another such laboratory was established in Charkov, Ukraine, in 1930.26

Helium liquifies at atmospheric pressure at 4.22K. Since most metallic superconductors have a transition temperature that is below the boiling point of helium, it was only in Leiden that the phenomenon could be, and was, found. It was observed first for mercury, which has a transition temperature of 4.2K. Measurements of the electrical resistance of mercury at low temperatures were initially performed in order to find a thermometric device for low temperatures, that would replace thermometric measurements using

24 The comment was published as a discussion remark to Laue’s presentation at the second Solvay Congress [CPAE4, p. 553], as a revised version of an original text that is no longer available, see Einstein to Lorentz, 2 August 1915 [CPAE8, Doc. 103].
25 For general historical accounts of the discovery and early developments in the theory of superconductivity, see Gavroglu and Goudaroulis 1989, Dahl 1992, Matricon and Waysand 2003.
26 Matricon and Waysand 2003, p. 47.
the resistance of platinum. Mercury, the only metal that is liquid at room temperatures, was chosen because it was easiest to purify.

After establishing that the electrical resistance of mercury drops very suddenly to a very low value at a certain temperature, the phenomenon was further investigated. Around 1920, the following facts about superconductivity had been established at the Leiden laboratory.\(^{27}\)

First of all, mercury was not the only substance that showed the phenomenon. Four other metals were known in the early twenties to exhibit superconductivity. Tin (Sn), was discovered to be superconducting in 1912\(^{28}\) with a transition temperature of 3.72K. Lead (Pb), which has a transition temperature of 7.19K, was also found to be superconducting in 1912. However, here the precise temperature of the transition was not explicitly observed or determined because its transition temperature is in the temperature range between the melting point of hydrogen at 13K and the boiling point of helium where temperatures were not easily determined. Thallium (Tl) was discovered to be superconducting in 1919 with a transition temperature of 2.32K.\(^{30}\) In December 1922, indium (In) was found to be superconductive at 3.41K.\(^{31}\) However, gold (Au), iron (Fe), platinum (Pt), cadmium (Cd), and copper (Cu) showed a finite and constant electrical resistance at liquid helium temperatures.

As to the features of the superconductive transition, the following facts had been established. The resistivity below the transition temperature dropped to a value of order $10^{-10}$ as compared to that at room temperatures. Upper limits on the residual resistance were first determined by measuring potential drops along filaments carrying large currents, later by the lifetime of persistent currents induced in superconducting rings. The transition occurred within a narrow temperature interval of the order of $10^{-3}$K. The superconducting state was destroyed by critical currents of a certain value that depended on the temperature. It was also destroyed by magnetic fields, and it was determined that the threshold values were dependent on the temperatures. The latter two features were thought to be related, in that it was thought that the critical current is reached when the induced magnetic field

\(^{27}\)For contemporary reviews, see Crommelin 1920, Meißner 1920, Kamerlingh Onnes 1921b.

\(^{28}\)See, e.g., Kamerlingh Onnes 1924.

\(^{29}\)Dahl 1992, p. 73.

\(^{30}\)ibid., p. 99–100.

\(^{31}\)ibid., p. 106.
reaches a critical value.\footnote{This hypothesis was known as the Silsbee hypothesis, see Silsbee\cite{Silsbee1916}.}

A controversial question concerned the influence of impurities. The drop of resistivity in Mercury seemed to be independent of impurities, but the purity of non-superconducting metals influenced the electric resistance at low temperatures. The issue of impurities was a critical one, given their significance in the theoretical account for electric resistivity in the Drude model as well as in other models. In general, the role of impurities remained an open issue due to difficulties in controlling and determining the degree of purity. In particular, the available data did not allow for an unambiguous decision as to whether “really pure” metals like gold, iron, etc. would be superconducting at “very low” temperatures. As to the latter point, temperatures below $1.5\text{K}$ were very difficult to achieve, since the vapor pressure of helium decreases rapidly with temperature. The low temperature record was $0.8-0.9\text{K}$ and was attained by Onnes in 1921.\footnote{\cite{Dahl1992}.}

Many properties relating to superconductivity had already been established before the outbreak of World War I. During the war, low temperature research in general and further research into the phenomenon itself was stalled, due both to shortage of personnel\footnote{The lack of personnel is mentioned by Onnes who, himself almost 70 years of age, responds in a letter, dated 13 August 1921, to Einstein’s question about the empirical data on the equations of state: “We would have been further if only we had more collaborators in order to undertake the numerous time-consuming measurements that are necessary.” (Albert Einstein Archives, The Hebrew University, (AEA) Call Number 14-381). And at the end of that letter, Onnes asked Einstein directly: “How nice it would be if you could enthuse a well-trained experimenter to come to Leiden in order to learn the determination of equations of state at low temperatures and to continue these investigations as a collaborator.”} and of material resources, most importantly of sufficient supplies of helium gas. But after the end of the war, low temperature research was quickly resumed in Leiden with some significant experimental advances, most notably an improvement of the cryogenic apparatus that allowed the experimenter to physically remove the liquified helium from the liquifier and transport it to experimental designs that no longer had to be integrated with the liquifier.
Einstein’s professional and personal ties to the Leiden physicists

Einstein was well informed about the work and experiments that were being done in Leiden. As early as 1901, the 22-year-old ETH graduate had sent a postcard to Kamerlingh Onnes, who was looking for an assistant, and applied for the position. Along with the postcard, Einstein sent an offprint of his first published paper and a reply postcard which, however, is still contained in the Kamerlingh Onnes papers.

Ten years later, Einstein and Onnes exchanged offprints of their respective recent publications, this time as colleagues, since Einstein had recently been appointed associate professor at the University of Zurich. In his letter to Kamerlingh Onnes, on 31 December 1910, sending his own publications, Einstein also announces an imminent visit to Leiden:

In about a month’s time I will have the extraordinary pleasure of getting acquainted with you and your highly esteemed friend, Prof. Lorentz; for at that time I will deliver a lecture to the Leiden Student Association.

The lecture took place on 10 February 1911, and Einstein met Kamerlingh Onnes just a few weeks before the discovery of superconductivity. Appar-ently, the first encounter was congenial. A few weeks later, after Einstein had accepted an offer at the German University of Prague and had announced his resignation from the University of Zurich, Einstein and Kamerlingh Onnes corresponded about Albert Perrier, a Swiss physicist then working as Onnes’ assistant who was being considered as Einstein’s successor.

More important was the next encounter between Einstein and the Leiden physicists at the first Solvay congress that took place from 27 October to 3 November 1911 in Brussels. At the meeting, Kamerlingh Onnes gave an account of the experiments concerning electric conductivity at low tem-

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35 For a discussion of Einstein’s ties with Leiden, see also CPAE10 pp. xliii–xlviii.
36 van Proosdij 1959 and CPAE1 Doc. 98.
37 CPAE5 p. 623.
38 Einstein to Kamerlingh Onnes, 31 December 1910, CPAE5 Doc. 242.
39 For the chronology of the discovery, see Dahl 1992 ch. 3.
40 See Einstein to Hans Schinz, 10 March 1911 CPAE5 Doc. 259.
41 For a historical discussion of the first Solvay congress, see Kormos Barkan 1993.
His participation at the first Solvay congress firmly established Einstein as a peer and congenial colleague of the Leiden physicists and, in fact, as one of the leading theoretical physicists of the time. Just a few weeks later, Einstein was asked for an opinion on the work of Keesom, a student of Lorentz who was being considered for a vacant position in Utrecht. And in a letter of 13 February 1912, Lorentz himself asked Einstein whether he would consider becoming his successor in Leiden. 1912 is also the year of the first personal encounter with Paul Ehrenfest, who would instead become Lorentz’s successor in Leiden, when Ehrenfest visited Einstein in Prague. Ehrenfest soon became one of Einstein’s closest friends.

In August 1913, Einstein and Onnes met again when the latter spent some time in a resort hotel in Baden (Switzerland) and in March 1914 Einstein made another weeklong visit to Leiden on his way from Zurich to Berlin. By then he was on a first-name basis with Ehrenfest who in turn paid him another visit in Berlin in May 1914. During the war, Einstein at first declined an invitation to visit Leiden in December 1915 because of family obligations but then visited for two weeks in late September and early October 1916. When Ehrenfest invited him again in late 1917, he was unable to come due to severe health problems and the difficult travelling conditions.

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42 Kamerlingh Onnes 1912
43 See Willem Julius to Einstein, 25 November and 29 December 1911 [CPAE5 Docs. 314, 334].
44 Hendrik A. Lorentz to Einstein, 13 February 1912 [CPAE5 Doc. 359]. Einstein declined immediately with some formal and polite excuses but added a comment on his “feeling of intellectual inferiority with regard to you” that may well have been the true reason for his decision: “However, to occupy your chair would be something inexpressibly oppressive for me. I cannot analyze this in greater detail but I always felt sorry for our colleague Hasenöhrl for having to occupy Boltzmann’s chair.” (“Auf Ihrem Lehrstuhl zu sitzen, hätte etwas unsagbar Drückendes für mich. Ich kann dies nicht weiter analysieren, aber ich bemitleidete immer den Kollegen Hasenöhrl, dass er auf dem Stuhl Boltzmann’s sitzen muss.” [CPAE5 Doc. 360]). In a letter to Ehrenfest, Einstein even said that the offer “had given him the creeps” (“empfand ich ein unerträgliche Gruseln”), Einstein to Ehrenfest, between 20 and 24 December 1912 [CPAE5 Doc. 425].
45 See Klein 1970 chap. 12 and also Einstein 1934.
46 Einstein to Kamerlingh Onnes, 16 August 1913, and Kamerlingh Onnes to Einstein, 18 August 1913, [CPAE8 Docs. 469, 471].
47 [CPAE8] p. 990.
48 Einstein to Mileva Einstein-Maric, 2 April 1914, [CPAE8 Doc. 1].
49 [CPAE8] p. 991.
50 Einstein to Paul Ehrenfest, 26 December 1915, [CPAE8 Doc. 173].
51 [CPAE8] p. 1003.
conditions:

[...] you can believe me that nothing is more appealing to me than a trip to my dear Dutch friends, with whom I share such close and kindred feelings in everything.\textsuperscript{52}

Einstein’s next visit to Leiden took place in October 1919, when Einstein spent two weeks in the Netherlands where, among other things, he attended a meeting of the Amsterdam Academy on 25 October 1919 in which Lorentz informally announced results of the British eclipse expedition. By this time, his Leiden colleagues had already been trying to get Einstein to come to Leiden as a special professor.\textsuperscript{53} He spent three weeks in Leiden in May 1920, was inducted as foreign member into the Royal Dutch Academy of Sciences on May 29, and also saw Onnes’s laboratory:

Yesterday I visited Kamerlingh Onnes in his institute and attended a nice lecture of his, saw interesting experiments.\textsuperscript{54}

A second trip that same year took place in late October and early November, during which he delivered his inaugural lecture and participated in the meeting on magnetism mentioned above.

Except for Zurich, where he travelled frequently to see his sons, Einstein visited no other place so frequently during those years. We may thus assume that Einstein had regular, first-hand information about what was going on in the Leiden cryogenic laboratory.\textsuperscript{55}

We finally remark that a low temperature laboratory had also been established in 1908 in the Physikalisch-Technische Reichsanstalt (PTR) in Berlin. Einstein had been appointed member of the Kuratorium of the PTR in late 1916, regularly attended its annual meetings and actively participated in

\textsuperscript{52}Einstein to Paul Ehrenfest, 12 November 1917, \textsuperscript{[CPAE8 Doc. 399]}.
\textsuperscript{53}See Paul Ehrenfest to Einstein, 21 September 1919 and 24 November 1919 \textsuperscript{[CPAE9 Docs. 109, 175]}. See also \textsuperscript{[CPAE10 pp. xliii–xlviii]}.
\textsuperscript{54}Einstein to Elsa Einstein, 9 May 1920, \textsuperscript{[CPAE10 Doc. 9]}.
\textsuperscript{55}Thirty years later, Einstein would remember his relationship with Kamerlingh Onnes mainly as a personal friendship: “I also knew Kamerlingh Onnes quite well but mainly personally. Behind his warm and agreeable personality there was a tenacity and energy that you only find very rarely. He was naturally not so close to me in scientific matters, so that there were rarely points for debate. Discussions with him were in general not easy since he was extraordinarily precise in his intuitive thinking but could not easily express himself clearly conceptually and was not easily accessible to considerations of others, [...].” Einstein to M. Rooseboom, 27 February 1953 (AEA 14-396).
discussions about its research. Although not producing liquid helium temperatures until 1925, the experimental and theoretical expertise of his Berlin colleagues associated with this laboratory—Emil Warburg, Walther Nernst, Eduard Gruneisen, Walther Meißner, and others—gave Einstein further first-hand information about ongoing experimental research in the field of low temperature physics.

Phenomenological theory of infinite conductivity

The fact that superconductors showed zero electric resistance was experimentally well-established. Theoretically, this finding posed a challenge since the notion of infinite or perfect conductivity is conceptually problematic. The concept of electrical conductivity is defined by the proportionality of current density \( \vec{j} \) and electric field \( \vec{E} \),

\[
\vec{j} = \sigma \vec{E}.
\]  

(12)

To the extent that such a proportionality relation holds, the constant \( \sigma \) defines the electrical conductivity. Setting aside complications such as anisotropies of the conducting material, that render \( \sigma \) a tensorial quantity, frequency dependencies of \( \sigma \) in the case of alternating currents, or modifications of (12) in the presence of magnetic fields, the simple equation (12) is nevertheless constitutive of the very concept of conductivity. We see immediately that this relation seems to lose all practical meaning in the limit of \( \sigma \rightarrow \infty \).

Lippmann’s theorem

The concept of infinite or perfect conductivity is nevertheless a natural starting point for a theoretical analysis of the phenomenon of superconductivity. Consequences of Maxwell’s equations for metallic conductors of vanishing resistivity were investigated by Gabriel Lippmann (1845–1921) well before the discovery of superconductivity. In 1889, Lippmann, professor of physics at the Sorbonne who received the Nobel prize in 1908 for producing the

\[56\text{See}\ \text{Hoffmann 1980}.\]

\[57\text{For a brief discussion of Lippmann’s considerations of perfect conductivity, see}\ \text{Dahl 1992 pp. 102–103}.\]
first color photographic plate, had published a short note in the *Comptes rendus* on the law of induction in electric circuits of vanishing resistance [Lippmann 1889].

Contrary to what we have just said, in Lippmann’s understanding, the very notion of finite electrical conductivity was alien to the fundamental laws of electrodynamics. He compared the concept of conductivity to the notion of friction in analytical mechanics, where frictional forces are also not to be counted among the fundamental concepts. For him it was hence rather natural to address the case of perfect electrical conductivity if one wanted to come to an understanding of the fundamental laws of electromagnetism.

Lippmann considered a conducting circuit where, in the absence of external sources of voltage, the electromotive force $e$ is related to the electric current $i$ through

$$e - L \frac{di}{dt} - ri = 0,$$

with $L$ denoting the circuit’s coefficient of self-induction and $r$ the resistance.

For such a conducting loop, the electromotive force $e$ is equal to the change $dN$ in the number of magnetic flux lines per time due to external sources passing through the loop,

$$e = \frac{dN}{dt}.$$  \hfill (14)

In addition, any induced currents will produce a change in the total magnetic flux through the loop,

$$L \frac{di}{dt} = - \frac{dN'}{dt},$$  \hfill (15)

and so Ohm’s law can be written as

$$ri - \frac{dN}{dt} + \frac{dN'}{dt} = 0.$$  \hfill (16)

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58: The very concept of electric conduction was a problem for British field theoreticians but not so much for the Continental tradition of electrodynamics. My discussion of Lippmann’s theorem is not meant as implying that the notion of electric resistivity, and more specifically of vanishing electric resistivity, had not been a topic of theoretical discussion before. I discuss it here only as the most explicit discussion of the implications of superconductivity available for Einstein and his contemporaries at the time. For general accounts of the history of late nineteenth-century electrodynamics, see, e.g., [Whittaker 1951], [Whittaker 1953], [Buchwald 1985], and [Darrigol 2000].
Setting now $r$ equal to 0, one obtains after integration,

$$N + N' = \text{const},$$

(17)

an equation that expresses the conservation of the magnetic flux through the loop:

Put into words: *In a circuit devoid of any resistance, the intensity of the induced current is always such that the magnetic flux passing through the circuit remains constant.*

In the remainder of his note, Lippmann then discussed implications of equation (17) for superconducting coils and briefly observed that an approximate analog of infinite conductivity would be given experimentally for the rapidly oscillating Hertzian waves, pointing to the fact that in this case the fields only penetrate into a small surface layer of a metallic conductor.

In 1919, Lippmann took up his investigations of infinite conductivity again, with explicit reference to Kamerlingh Onnes' discovery of superconductivity. In three only slightly differing versions published in three different journals, he referred to his earlier paper and its original motivation to investigate electromagnetism without the friction-like concept of finite electrical conductivity. He proudly pointed out that the “fine experiments of Kamerlingh Onnes have brought about a physical justification of the hypothesis of vanishing resistance.”

Recapitulating the argument of his 1889 paper, Lippmann again considered Ohm’s law (13) which, for vanishing $r$, gives

$$e = L \frac{di}{dt},$$

(18)

from which it follows immediately that a finite current density $i \neq 0$ can be maintained in the wire even in the absence of an electromotive force $e$.

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59 [Lippmann 1889, p.252].

60 Lippmann obviously here refers to what is commonly known as the skin effect.

61 One version, [Lippmann 1919a], appeared in his own *Annales des physiques*, a journal he was editing together with E. Bouty, another version, [Lippmann 1919b], appeared in the *Comptes rendus* (Lippmann being a member of the French Académie des Sciences since 1886), and a third version, [Lippmann 1919c], was published in the *Journal de physique théorique et appliquée*.

62 [Lippmann 1919b, p. 73].
Specializing to the case of a thin, closed, homogeneous wire without any external sources of voltage or soldered joints that might produce thermoelectric voltages, Lippmann rederived his ‘théorème’ of conservation of flux lines or, equivalently, of the impenetrability for flux lines for an infinitely conductive ring. He noted that this theorem applies in particular to the experiments on superconductors performed by Kamerlingh Onnes. Curiously, Lippmann here cited nickel as a typical example of a metal that loses its resistance at liquid helium temperatures.\(^\text{63}\)

While the previous argument held for loops of thin superconducting wire, the general conclusions, argued Lippmann, remain true for three-dimensional conductors such as a metallic bulk cylinder of length $L$ and cross section $S$. A uniform magnetic field $H$ parallel to the axis of such a cylinder would penetrate the cylinder in the case of finite conductivity, creating a magnetic moment of size $SHL$. After cooling to the superconducting state, the flux would remain frozen in, and the cylinder’s magnetic moment would remain the same. Similar conclusions would hold true for a hollow cylinder, where the flux line distribution inside the cylinder would remain the same, but one would find the lines slightly distorted.

From another point of view, the difference between perfect and finite conductors could be interpreted as follows: In normal conductors, the electromotive forces that induce the currents are proportional to the relative velocity of field and conductor or to the temporal change of an external magnetic field. In the case of perfect conductivity, on the other hand, the electromotive forces only depend on the relative displacement of field and conductor. The forces in the former case are similar to viscous forces, while the electromotive forces in the superconducting situation behave like elastic forces. They try to keep the conductor at a fixed position which appears as a position of equilibrium.

In the dynamical case of electromagnetic waves, Lippmann repeated his observation about the known fact that electromagnetic waves do not penetrate into the bulk of metallic conductors of high conductivity. This behavior, he remarked, carries over to the superconducting case. Here again, electromagnetic waves cannot penetrate into the superconducting bulk substance.

\(^{63}\)”A partir du moment où le nickel est devenu hyperconducteur, le nombre de lignes de force reste invariable.” [Lippmann 1919a, p. 248]. In [Lippmann 1919a], but not in the other two versions of his paper, Lippmann also mentions gold, along side with lead, as one of the “various metals” whose resistance drops by a factor of at least $10^{10}$ [Lippmann 1919a, p. 246].
Lippmann concluded his note with comments on the peculiarities of the transmission of forces between two superconducting rings, and on interpreting Ampèrian molecular currents in terms of perfect conductivity.

**Ehrenfest’s diaries**

We have no direct evidence that Einstein was aware of Lippmann’s papers, but we do have some indirect evidence that he knew about Lippmann’s considerations. Lippmann’s name is mentioned in Ehrenfest’s diaries in an entry “Supraleiter-Hall-Effect (Lippmann)” found next to other entries dated April 1920. Since Einstein visited Leiden from 7 to 27 May 1920, we may assume that the topic was discussed by Ehrenfest and Einstein during that visit.

Indeed, Ehrenfest’s diaries contain a more elaborate entry on this topic. Entry 5548 is found next to an entry that describes Einstein’s visit to Leiden in November. The entry itself is then dated 2 November and entitled “Hall-Effect im Supraleiter.” The consideration and equations of this entry actually closely parallel those found on the blackboard on a photograph taken, in all probability, during the “Magnet-Woche” and showing (from left to right) Einstein, Ehrenfest, Langevin, Kamerlingh Onnes and Pierre Weiss, see Fig. (68). The equations on the blackboard appear to be written by Ehrenfest, who also poses in the photograph as if he were the one writing on the blackboard. Let us briefly review the consideration in Ehrenfest’s diary with cross-reference to the blackboard image.

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64 Ehrenfest Archive, Museum Boerhaave (Rijksmuseum voor de Geschiedenis van de Natuurwetenschappen en van de Geneeskunde), Leiden, Notebooks, ENB:1-26/2.

65 See [CPAE10, pp. 570, 572] and Einstein to Elsa Einstein, 27 May 1920 [CPAE10, Doc. 32].

66 Einstein’s name is mentioned frequently in Ehrenfest’s diaries, as are mentions of the problem of superconductivity: Entry 5463, following an entry explicitly dated to 31 May 1920, reads: “Precessionsbeweg. von Strömen in Supra-leitern - Kugel wegen a.) Trägheit der Elektronen b.) Hall-Effect.” (ENB 1-26/6); entry 5470, found between entries dated 14 June 1920 and 2 July 1920, again says: “Supraleiter mit Hall-Effect.” (ENB 1-26/7). See also the discussion of the “Magnet-Woche” below.

67 The diary entry on Einstein’s visit reads: “Magnet-Woche: Einstein allein Ankunft am Abend[—] kleines Fenster[—] alle jubelnd hinaus. Wandert zu Onnes — Spaziergang Haagsche Weg Goldnebel (Ruhe, Weide, Kirchhoff) Triospielen bei Maler Onnes. Ein Abendessen in grossem dunklen Esszimmer Einstein mit [–] Langevin rauchend auf eiskalter Nachtstrasse Weiss Langevin, Lorentz, Einstein, Taniz, Woltjer-Sonne.”

68 The photo is also shown on the jacket cover of [CPAE10].
Ehrenfest began by writing down the following condition

\[ \vec{E} + \alpha \frac{\vec{\nu}}{c} \times \vec{H} = 0. \]  

(19)

Here \( \vec{E} \) and \( \vec{H} \) denote the electric and magnetic field vectors, \( \vec{\nu} \) the local velocity of the current carrying charges, \( c \) the speed of light (using c.g.s. units), and \( \alpha \) is a numerical parameter which is included only in the diary version and whose significance will become clearer below. The square brackets denote the vectorial cross product. Equation (19) captures the condition of perfect conductivity. Since it will be the basis for much of the following let us discuss its significance in some more detail. Consider first a resistor for which Ohm’s law holds between the voltage \( U \) and the current \( I \) in its integral form,

\[ U = R \cdot I. \]  

(20)

\[ ^{69} \text{In the following, I will translate the equations as they appear in Ehrenfest’s and Einstein’s manuscripts into a unified notation, substituting, e.g., } \vec{E} \text{ for } \xi, \text{ and expressing vector analytic expressions throughout in terms of the Nabla-operator } \nabla = (\partial_x, \partial_y, \partial_z). \]
Here \( R \) is the total electrical resistance of a piece of current carrying matter of, say, cylindrical shape with length \( L \) and cross-section \( S \). If we assume homogeneity along the cylinder, we can relate the voltage drop \( U \) to the local electric field strength \( E \) along the wire as \( U = EL \), the total resistance \( R \) to a local resistivity \( \rho \) as \( R = \rho L/S \), and the total current \( I \) to a local current density \( j \) as \( I = jS \). We thus obtain a local version of Ohm’s law,

\[
E = \rho j,
\]

(21)

that is independent of the geometric shape of the resistor. The latter equation turns into equation (12) if we identify the local resistivity \( \rho \) as the reciprocal of the conductivity,

\[
\sigma = 1/\rho,
\]

(22)

and take into account the vector character of the current density and the electric field. Such a distinction between an integral and a local version of Ohm’s law was standard textbook knowledge of the time, as witnessed, e.g., in [Föppl 1907, § 53.] where the integral version is said to reflect directly an empirical fact whereas the differential law would be more suitable for theoretical analysis. Recalling now that the Lorentz force expression reads

\[
\vec{F} = \rho_e \left( \vec{E} + \frac{\vec{v}}{c} \times \vec{H} \right),
\]

(23)

where \( \vec{F} \) is the force density and \( \rho_e \) the electrical charge density moving with velocity \( \vec{v} \), it is natural to add a term proportional to the cross-product of the velocity \( \vec{v} \) of the charge carriers and the magnetic field \( \vec{H} \), to obtain a generalized and local version of Ohm’s law in the form

\[
\vec{j} = \sigma \left( \vec{E} + \frac{\vec{v}}{c} \times \vec{H} \right),
\]

(24)

where we have again introduced an arbitrary numerical factor \( \alpha \). In this version of Ohm’s law, one can now take again the limit of infinite conductivity \( \sigma \to \infty \) in a sensible way to obtain Ehrenfest’s condition of infinite conductivity in the form of equation (19).

Lippmann’s condition of perfect conductivity (18) was obtained in a somewhat analogous manner from Ohm’s law (13) for vanishing resistance \( r \). However, Lippmann worked with the total current in a circuit rather than a local
version of Ohm’s law valid at any point within a conductor. Hence, his induction term \( L\frac{di}{dt} \) is different from the magnetic term \( \vec{v} \times \vec{H} \) in Ehrenfest’s version.

Equation (19) is to be investigated in order to understand infinite conductivity. Ehrenfest does so by invoking Ampère’s law,

\[
\beta \vec{v} = \vec{\nabla} \times \vec{H},
\]

where \( \beta \vec{v} = (4\pi/c)\rho_e \vec{v} \) would be the current density\(^{70}\) and any displacement current terms are taken to be negligible. Ampère’s law allows him to eliminate \( \vec{v} \) from the condition of infinite conductivity. He also assumes that the magnetic field does not change with time\(^{71}\), which, by virtue of Faraday’s law, implies that the electric field is irrotational and hence has a potential \( \varphi \) as

\[
\vec{E} = -\vec{\nabla} \varphi.
\]

Using (25) and (26), we can hence write (19) as

\[
- \vec{\nabla} \varphi + \frac{\alpha}{\beta c} (\vec{\nabla} \times \vec{H}) \times \vec{H} = 0 \quad \text{(27)}
\]

In a final step, Ehrenfest now takes the rotation of (27) and obtains the condition

\[
\vec{\nabla} \times [(\vec{\nabla} \times \vec{H}) \times \vec{H}] = 0 \quad \text{(28)}
\]

as a characteristic condition for the magnetic field in superconductors in time-independent situations.

In order to see the consequences of (28), Ehrenfest rewrote it, using standard equations of vector calculus, more explicitly as

\[
[(\vec{\nabla} \times \vec{H}) \cdot \vec{\nabla}] \cdot \vec{H} - (\vec{\nabla} \cdot \vec{H})(\vec{\nabla} \times \vec{H}) + \vec{H} \cdot [\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H})] - (\vec{\nabla} \times \vec{H}) \cdot (\vec{\nabla} \cdot \vec{H}) = 0. \quad \text{(29)}
\]

The third term vanishes identically because it is the divergence of a rotation, and the fourth term vanishes on account of Maxwell’s equations. The

\( ^{70}\)On the blackboard, Ehrenfest used \( \alpha \) instead of \( \beta \).

\( ^{71}\)The time independence of \( \vec{H} \) is written down as a condition explicitly on the blackboard.

\( ^{72}\)This equation is not written down in the diary, but it is written on the blackboard with a different notation for the constant in front of the second term.
The remaining first two terms were then written out explicitly as
\[ 
\begin{align*}
\left[ \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) \frac{\partial}{\partial x} + \left( \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \right) \frac{\partial}{\partial y} + \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) \frac{\partial}{\partial z} \right] \cdot \\
\cdot (iH_x + jH_y + kH_z) - \\
- \left( H_x \frac{\partial}{\partial x} + H_y \frac{\partial}{\partial y} + H_z \frac{\partial}{\partial z} \right) \left\{ i \left( \frac{\partial H_z}{\partial z} - \frac{\partial H_y}{\partial z} \right) \ldots + \ldots \right\} = 0. 
\end{align*}
\] (30)

with orthogonal unit vectors \(i, j, k\). At this point in his diary, Ehrenfest adds the comment “discuss!” (“discutieren!”), and obviously this is also what Einstein, Ehrenfest, Langevin, Onnes, and Weiss are posing to do on their photograph. But instead of discussing (30) any further, Ehrenfest goes back to the original expression for perfect conductivity (19) and rewrites it, using (26) as well as \(\vec{j} \equiv \rho_e \vec{v}\) in the form
\[ 
- \nabla \varphi + \frac{\alpha}{\rho_e c} \times \vec{H} = 0. 
\] (31)

He immediately concludes that it follows that the electrostatic potential \(\varphi\) is constant along lines parallel to either the current density \(\vec{j}\) or the magnetic field lines \(\vec{H}\).

A discussion of expression (30), e.g. by specializing to certain symmetries, fields, etc., would be the natural thing to do, and, in fact, Ehrenfest began to simplify (30) for the case where all derivatives with respect to \(z\) would vanish. But this calculation breaks off. Indeed, an exploration of equation (30) or even of (28) would not be too enlightening in the end since all time dependence had been assumed absent from the outset anyway.

The calculation on the Hall effect in superconductors in Ehrenfest’s diary proceeded on the basis of the classical Maxwell equations and explored the implications of perfect conductivity. The latter condition was expressed in terms of equation (19). In his first calculation, Ehrenfest deduced from this ansatz a vector differential equation for the magnetic field (28) that does not contain any sources or currents. The equation was not explored any further, and it is unclear what conclusion Ehrenfest may have drawn at this point. However, we do have indications that these issues were further pursued in the discussions between Ehrenfest and Einstein and possibly other participants of the “Magnet-Woche.”

\footnote{On an earlier but closely related page (ENB 1-26/6), Ehrenfest is also concerned with a discussion of (28) but again does not proceed any further than by looking at components of (28) written out explicitly.}
Theorizing about experiments on the Hall effect for superconductors

Einstein, too, thought along this line of exploring consequences of Maxwell’s equations for infinite conductivity. Three different and independent sources all document the very same consideration. One source is another entry in Ehrenfest’s diary, in which he excerpted an argument from a (non-extant) letter by Einstein, dated 9 December 1920. Calculations by Einstein along the same line are also found on a single manuscript page, dated in an unknown hand to 12 December 1920, located at the Burndy library. And the very same argument is finally also laid out in a letter by Einstein to Lorentz, dated 1 January 1921.

I will here give a presentation of the argument that is not literally faithful to the originals but is in itself complete and notationally consistent. Special features of the individual source documents will be pointed out along the way.

Einstein works out on consequences of the condition for perfect conductivity (19), which we will rewrite here in the form

\[
\vec{E} = -\frac{\tilde{\alpha}}{c} \vec{j} \times \vec{H}
\]

for electric and magnetic fields \(\vec{E}\) and \(\vec{H}\) and current density \(\vec{j}\). \(\alpha = \tilde{\alpha}\rho_e\)

---

74 “from letter by Einstein 9 XII 1920.” ENB 1-26/46 and 1-26/47. The entry is numbered as 5559 but this is actually the second number with this entry since on the previous page, ENB 1-26/45, Ehrenfest had already recorded (unrelated) entries 5559 to 5564. Quite possibly Ehrenfest had opened his notebook on p.44 which ends with an entry 5558, then mistakenly turned over two pages at once and continued on p. 46 with another entry 5559. The entry with Ehrenfest’s excerpt is published as [CPAE10, Doc. 227].

75 The manuscript page is extant in the Burndy Library, Cambridge, Ma. A pencil note on the back reads: “Manuskript und Zeichnungen von Prof. Albert Einstein 12 XII 20.” The manuscript is published as [CPAE10, Appendix]. I wish to thank P. Cronenwett for providing the Einstein Papers Project with high-quality scans of the Burndy library manuscript.

76 AEA 16 533.

77 The three sources differ among each other in notation, in the degree to which the relevant equations were written out and commented on, as well as in the existence of illustrative figures. None of the three sources present the argument more comprehensively than any of the other. The Burndy manuscript is a little more complete in the equations that Einstein actually wrote down but Ehrenfest’s letter excerpt and Einstein’s letter to Lorentz are more explicit about the meaning of the calculations.
is again a numerical parameter to be discussed below. Instead of invoking Ampère’s law at this point (see (25) above), as Ehrenfest had done, Einstein started from Faraday’s law of induction
\[ \vec{\nabla} \times \vec{E} + \frac{1}{c} \partial_t \vec{H} = 0. \] (33)

Taking the rotation of (32) and substituting \(-\frac{1}{c} \partial_t \vec{H}\) for \(\vec{\nabla} \times \vec{E}\) then yields
\[ \tilde{\alpha} \vec{\nabla} \times (\vec{j} \times \vec{H}) + \partial_t \vec{H} = 0. \] (34)

which, written explicitly in components, reads
\[ \tilde{\alpha} \partial_y (j_x H_y - j_y H_x) - \tilde{\alpha} \partial_z (j_z H_x - j_x H_z) + \partial_t H_x = 0, \] (35)
\[ \tilde{\alpha} \partial_z (j_y H_x - j_x H_y) - \tilde{\alpha} \partial_x (j_x H_y - j_y H_x) + \partial_t H_y = 0, \] (36)
\[ \tilde{\alpha} \partial_x (j_z H_x - j_x H_z) - \tilde{\alpha} \partial_y (j_y H_z - j_z H_y) + \partial_t H_z = 0. \] (37)

Einstein’s consideration now rests on an interpretation of the characteristic equation (34) viz. (35)–(37) just as Ehrenfest was trying to interpret his (28). In his letter to Lorentz, Einstein wrote eqs. (32), (33), and (34), and continued “to investigate the evolution of the phenomenon in a slab that carries a current in the \(x\)-direction.” For illustration, Einstein included a sketch of a thin superconducting slab, see Fig. 2.

![Figure 2: A slab of superconducting metal extending along the \(x\)-axis.](image)

He first assumed that there is no component of the current in the vertical or \(z\)-direction,
\[ j_z \equiv 0, \] (38)
and that only $j_x$ and $j_y$ have non-vanishing values. Second, he assumed that the magnetic field only has a vertical component $H_z$, and the $x$- and $y$-components vanish,

$$H_x \equiv H_y \equiv 0.$$  \hfill (39)

Third, if moreover none of the fields and quantities (in the slab) change along the $z$-direction, i.e.

$$\partial_z \equiv 0,$$  \hfill (40)

eqs. (35)–(37) reduce to

$$- \tilde{\alpha} \partial_x(j_x H_z) - \tilde{\alpha} \partial_y(j_y H_z) + \partial_t H_z = 0.$$  \hfill (41)

Conditions (38)–(40) and the specification to a flat current carrying slab are very suggestive of an experiment designed to investigate the transverse Hall effect. And this is, in fact, what Einstein here had in mind. He continued by stating that the $y$-components of the current are “induced by the Hall effect”, and that one may assume, with good approximation, that

$$\partial_y(j_y H_z) \equiv 0.$$  \hfill (42)

With this assumption, eq. (41) further simplifies and its solutions are of the form (due to the continuity equation and since the slab is assumed to be flat, we have $\partial_x j_x = 0$)

$$H_z = f(x - \tilde{\alpha} j_x t),$$  \hfill (43)

for some arbitrary function $f$ which he interpreted as follows:

The magnetic field is hence dragged along by the current with velocity $\tilde{\alpha} j_x$.

This consequence is, in fact, a general property of the condition of perfect conductivity. In ideal magnetohydrodynamics, e.g., it is shown on similar grounds that the magnetic field lines move along with the current in an ionized plasma. We now also see the significance of the numerical parameter $\alpha$. If $\alpha$ is smaller than 1, the field lines are being “dragged along” with a velocity that is reduced by a factor of $\alpha$ compared to the velocity of the current.

Einstein continued in his letter to Lorentz:

\footnote{For a historical discussion of the Hall effect, see \cite{Buchwald1985, Beckman1922}. For a contemporary discussion, see, e.g., \cite{Buchwald1985}.}

\footnote{See the discussion below on p. 37.}
For a discontinuous change of the slab’s thickness, \([j_x H_z]\) is continuous or also \(\frac{1}{\delta} \cdot H_z\), where \(\delta\) is the slab’s thickness.

He concluded by suggesting an experimental investigation:

Thus we sufficiently understand what processes to expect in order to be able to decide experimentally whether the Hall effect exists at low temperatures.

For the last step, Einstein clearly assumed that the term \(\partial_t H_z\) vanishes, which leaves us with

\[
  j_x H_z = \text{const.} \quad (44)
\]

Consider then a slight variant of the slab, like the one in fig. 3 where the thickness varies along the \(x\)-direction. Since we would naturally assume

![Figure 3: A slab of superconducting metal of varying thickness extending along the \(x\)-axis.](image)

charge conservation,

\[
  \vec{\nabla} \times \vec{j} \equiv 0, \quad (45)
\]

for the superconducting current, the \(x\)-component of the current would vary in proportion to the thickness \(\delta\). In order to satisfy (44), the magnetic field component \(H_z\) would therefore have to vary in inverse proportion to the thickness \(\delta\), as stated by Einstein. Einstein seems accordingly that the transverse Hall voltage along a slab of varying thickness \(\delta\) should vary inversely as \(\delta\), and that this hypothesis should be put to experimental test.
With slight variations, Ehrenfest’s excerpt notes present Einstein’s argument in a similar manner to our presentation above. But his discussion of \( (44) \) is a little different from the one that Einstein gave in his letter to Lorentz. With reference to the situation of a slab of varying thickness as depicted in fig. 3, Ehrenfest argued as follows. Let the magnetic field \( H_z \) be constant at some initial point at time \( t = 0 \). If one now turns on the current, the current thus creates at its onset at first a point \( A \) (or \( B \) for negative \( \tilde{\alpha} \)) where the field is smaller, which point then runs with velocity \( [\tilde{\alpha} j_x] \) along the thin part of the slab. The field in the thick part remains permanently constant.

The Burndy manuscript version of the argument is less explicit and more sketchy. It also shows a few fragmentary equations involving the current four-vector, the electromagnetic field tensor and a stress-energy tensor in four-dimensional, Lorentz covariant notation.

I have not found any indication that Einstein’s argument was discussed anywhere in print, nor did I find any indication that the hypothesis of a varying magnetic field in superconducting slabs whose thickness changes from point to point was ever tested directly and explicitly experimentally. It seems likely that the technological possibilities of the Leiden cryogenic laboratory at the time were inadequate to produce superconducting slabs of varying and controllable thickness and to measure a Hall voltage with sufficient spatial resolution. In the concluding section of his 1921 Solvay report on superconductivity, Kamerlingh Onnes points out that the investigation of the phenomenon of superconductivity is complicated enough without external fields:

By introducing an external field, every question is doubled, as it were. Others are added. We would enter here into a vast terrain, where almost all experimental investigations are wanting.

And referring back to pre-war experiments on the Hall effect, Kamerlingh Onnes and Hof 1914, Onnes continues

It is only the Hall phenomenon on which investigations have been made, which have shown that the electromotive force that is ob-

\[^{80}\text{[CPAE10, Doc. 227].}\]
\[^{81}\text{Kamerlingh Onnes 1921b, p. 50].}\]
served in the usual way disappears with the resistance as soon as superconductivity appears.

The experiments referred to had been done in order to investigate the influence of a magnetic field on the electric conductivity. Onnes and Hof had investigated plates of tin and lead and found that a Hall effect was observed at liquid helium temperatures for magnetic fields that were high enough to destroy the superconductivity. But for low magnetic fields the Hall voltage was found to vanish just as did the electrical resistance. It is clear that those experiments were not sophisticated enough to provide the kind of spatial and temporal resolution that Einstein’s idea would require.

The situation might have changed in the late twenties or early thirties with other cryogenic laboratories capable of investigating superconductivity entering the scene. But then again, an experiment such as the one envisaged here would not have made much sense after the discovery of the Meissner effect in 1933. Once it was realized that superconductivity is a thermodynamic state characterized not only by infinite conductivity, but also by perfect diamagnetism, it would have become clear that the magnetic field would be expelled from the superconducting slab rather than be dragged along with the current flowing inside it.

After 1933, phenomenological theories of superconductivity also needed to account for perfect diamagnetism. This task was successfully achieved in 1935 through a modification of Maxwell’s equations proposed by the brothers Fritz and Heinz London. In this theory, the current \( \vec{j} \) is supposed to consist of two components, a normal component \( \vec{j}_n \) and a superconducting component \( \vec{j}_s \). For the superconducting component, one still has infinite conductivity \( \sigma_s = \infty \) but for the normal component one has a modification of the Maxwell equations, given by the so-called London equations,

\[
\lambda \partial_t \vec{j}_n = \vec{E}, \tag{46}
\]

and

\[
\lambda \vec{\nabla} \times \vec{j} = -\vec{H}. \tag{47}
\]

\[82\]ibid., p. 50. For an account of the Leiden experiments on the Hall effect at low temperatures, see also [Beckman 1922].

\[83\]In actual experiments of the kind suggested by Einstein, other effects may play a role, too, e.g. intermediate states of only partially expelled magnetic fields, see [Huebener 2001] for a discussion of magnetic flux effects in superconductors. Note also that since the Meissner effect concerns only bulk properties, Lippmann’s theorem of conservation of magnetic flux through a looped circuit still holds good.
On the basis of these equations, it can be shown that magnetic fields may only penetrate into the superconductive bulk matter up to a distance of order $\lambda^{84}$.

The condition of perfect conductivity (19) or (32) which was at the core of the phenomenological theory described in this section thus was no longer valid in the theory of superconductivity after 1933. Yet, the condition of infinite conductivity still plays a role today in the context of plasma physics, more specifically in the conceptual framework of ideal magnetohydrodynamics. The theory of an ionized plasma at low frequencies is again given by Maxwell’s equations plus the condition of infinite conductivity. Indeed, the general conclusion of Lippmann and Einstein of a freezing in of the magnetic flux lines carries over to the case of a magnetohydrodynamic liquid. The difference here is that the positive ions now also come into play, leading to the possibility of an energy transfer between electromagnetic field energy and kinetic energy of the positive ions. Adding an equation of motion for a charged liquid with mass density of the distributed positive ions, Hannes Alfvén first showed the possibility of the existence of so-called magnetohydrodynamic waves. In these waves, magnetic flux lines perform an oscillatory motion with the charged liquid, much like vibrating strings. Alfvén initially believed that these waves played a role in the solar sun spot cycle. While this expectation has not been confirmed, these kind of magnetohydrodynamic waves derived for charged liquids of infinite conductivity nevertheless do play a role in plasma physics.

**Microscopic theory of charge transport mechanism**

In the preceding section, we have addressed superconductivity on a phenomenological level exclusively as a special case of infinite conductivity, i.e. as far as its implications in the framework of Maxwell’s equations go. But physicists at the time also entertained speculations on a microscopic level, i.e. on the level of model assumptions about superconductive mechanisms of electric charge transport. In fact, investigations of the Hall effect were done

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84For historical discussion of the London theory of superconductivity, see Gavroglu 1995 and Dahl 1992 chap. 11.

85See Alfvén 1942 and Jackson 1975 chap. 10.
to some extent because the magnitude and sign of the Hall voltage carries information on the charge carriers and especially on their sign. If specific experimental data had been available on a Hall effect for superconductors, this would have had direct implications for speculations on the microscopic level.

The discussion of microscopic models of electric conductivity that we are going to discuss in the following were prompted by the phenomenon of superconductivity. To be sure, some of the models, or at least certain aspects of them, were not necessarily new. But for the purposes of the present account, I will discuss the contemporary microscopic speculations only to the extent that is needed to establish the historical horizon for Einstein’s own contributions in the period under consideration. In particular, I will refrain from making any claims about the prehistory of individual models of charge transport.

Stark’s model of thrust planes

 Alternatives to Drude’s electron theory of metals were advanced in order to account for a number of unexplained experimental facts. Foremost among them was the problem that the observed electric conductivities of metals would imply an electron density that would also give an appreciable electronic contribution to the specific heat. No such contribution, however, was seen experimentally. With reference to this problem, Johannes Stark published an alternative theory of electric conductivity in 1912. It is of interest here because Stark also alluded to the recently discovered superconductivity for its justification. His theory is based on what he called a “valence hypothesis” (“Valenzhypothese”) according to which “point-like separable, negative electrons are situated at the surface of the chemical atoms vis-a-vis extended, inseparable positive spheres.” In metallic conductors, these valence electrons are located at some distance away from the positive spheres. For monovalent metals the negative electrons and the positive spheres may crystallize into a regular lattice, as shown in Fig. 1, where the solid lines indicate lines of force between the electrons and the positive spheres. A single electron cannot move about easily within such a lattice aggregate, since local forces would immediately pull it back to its equilibrium position. But

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86 See also footnote 58 above.
87 Stark 1912.
88 ibid., p. 191.
there are certain directions along which electrons may move without doing work; these directions are parallel to planes located symmetrically between the positive spheres, such as those planes whose intersection with the paper plane is indicated by the dashed lines in Fig. 4. Stark calls such planes “thrust planes” (“Schubflächen”). Along those planes a valence electron may be moved “together with many other valence electrons by arbitrarily small forces.”\textsuperscript{89} Electric resistivity for those collective motions then would arise from thermal vibration of the positive spheres as well as from lattice defects. It follows that perfect conductivity is possible in the limit of zero absolute temperature. A metal that allows for such motion and electric conduction at absolute zero temperature is called by Stark a “whole metal” (“Ganzmetall”). Its resistance vanishes at zero temperature and increases “with increasing number of valence fields that are momentarily in a vibrating state and with increasing amplitude of these vibrations.”\textsuperscript{90} In contrast to the standard Drude model, this theory accounts, at least qualitatively, for the possibility of perfect conductivity but does not account for the sudden loss of resistivity at a low but finite low transition temperature. Accordingly, Stark refers to the recent Leiden findings on the conductivity for mercury not as a sudden

\textsuperscript{89}ibid., p. 193.

\textsuperscript{90}ibid., p. 194.
loss of resistivity but as a limiting phenomenon for temperatures approaching absolute zero\textsuperscript{91}.

**Lindemann’s model of electron space-lattices**

A similar model of collective motion of electrons that move about by preserving some lattice structure was proposed a few years later, in 1915, by F.A. Lindemann. Lindemann also pointed to the difficulties of the free electron model, in particular to the problem of the electronic specific heat. With reference to the magnitude of the Coulomb forces that act between electrons at a typical density in the metal, Lindemann argued that it is impossible to ignore the interaction between the electrons:

> The expression free electron, suggesting, and intending to suggest, an electron normally not under the action of any force, like an atom in a monatomic gas, might almost be called a contradiction in terms.\textsuperscript{92}

Instead, he put forth the hypothesis that “the electrons in a metal may be looked upon as a perfect solid.”\textsuperscript{93} Lindemann argued that in addition to their mutual repulsion, electrons are attracted by electrostatic forces to the positive ions up to a certain radius $r_0$ where a repulsive force between the core of the ions and the electron sets in. His model then amounted to the assumption that a “metal crystal would consist of two interleaved space-lattices, one consisting of atoms or ions, one of electrons.”\textsuperscript{94} The details and quantitative mathematical consequences of his model would be “a matter of great difficulty,” Lindemann conceded. But he indicated that he imagined the whole electron space lattice to shift with respect to the atomic lattice when an external field is applied, and that the electron space lattice may move continuously, with electrons at one end leaving the lattice structure which would continuously be filled up again at the other end, when a source of electrons is applied.

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\textsuperscript{91}“As they [i.e. the Leiden investigations] have shown, the conductivity, especially that of mercury, does not increase towards a maximum and then decreases again for decreasing temperature as a result of the smaller number of free electrons, but it approaches even infinity when one goes to absolute zero temperature.” \textsuperscript{[Stark 1912, p. 191].}

\textsuperscript{92}[Lindemann 1915, p. 129].

\textsuperscript{93}ibid.

\textsuperscript{94}ibid., p. 130.
In other words, the electron space-lattice or crystal may be said to melt at the one end and fresh layers may be said to freeze on at the other end when a current flows.²⁵

In order to account for superconductivity, Lindemann then argued that as long as the radius of the repulsive ion core, \( r_0 \), is less than half the distance between the centers of the atoms, “the electron space-lattice can move unimpe-ded through the atom space-lattice.” Again, just as in Stark’s theory, electric resistivity would set in through thermal vibrations of the positive ions. But in contrast to Stark’s theory, this state of superconductivity would certainly be possible at a low but finite temperature.

**Thomson’s model of electric dipole chains**

Another early reaction to Onnes’ discovery of superconductivity was also published in 1915 by J.J. Thomson. According to Thomson, Onnes’ experiments, showing that the specific resistivity of some metals drops to “less than one hundred thousandth millionth part of that at 0°C,” were of “vital importance in the theory of metallic conduction.”²⁶ Thomson was especially intrigued by the demonstration of the existence of persistent currents. In addition to the apparently complete loss of resistivity, Thomson emphasized the fact that the transition takes place at a definite temperature and that the loss of resistivity seemed to occur almost instantaneously. This fact seemed to him to be another “fatal objection” to the model of free electrons. With reference to his earlier work, Thomson now advanced a theory of electric conduction based on the assumption that the main mechanism of current transport is due to the existence of electric dipoles or, in Thomson’s words “electrical doublets, i.e. pairs of equal and opposite charges at a small distance apart.”²⁷ The existence of these doublets renders the substance polarizable, and Thomson proceeded to develop a quantitative theory of the temperature dependence of the electric polarization, in direct analogy to Langevin’s calculation of the magnetization on the basis of the kinetic theory of gases.

For any finite value of the electric polarization, Thomson argued, we may assume that some of the doublets are pointing into the same direction, while

²⁵ [Lindemann 1915, p. 130].
²⁶ [Thomson 1915, p. 192].
²⁷ ibid., p. 193.
the rest of them are pointing in random directions. Furthermore, Thomson suggested, “picture the substance as containing a number of chains of polarized atoms whose doublets all point in the direction of the electric force.”

To illustrate his model, Thomson included a sketch of one such chain, as shown in Fig. 5. These considerations were valid generically for both insulators and conductors. The crucial point of Thomson’s model was the assumption that the motion of the conducting electrons is not affected by the external electric force but rather by the local electric forces of the atoms in the chain of doublets.

On this theory the peculiarity of metals is that electrons, not necessarily nor probably those in the doublets, are very easily attracted by these forces from the atoms when these are crowded together. Thus we may suppose that under these forces an electron is torn from A and goes to B, another from B going to C, and so on along the line,—the electrons passing along the chain of atoms like a company in single file passing over a series of stepping-stones.

The conceptual distinction between the external electric force and the local forces exerted by the doublets, which are the forces that are actually acting on the conduction electrons, allows Thomson also to account for the phenomenon of superconductivity:

... the part played by the electric force in metallic conduction is to polarize the metal, i.e. to form chains: when once these are

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98 ibid., p. 195.
99 ibid., p. 195.
formed the electricity is transmitted along them by the forces exerted by the atoms on the electrons in their neighbours. Thus if the polarization remains after the electric force is removed the current will remain too, just as it did in Kamerlingh Onnes’ experiment with the lead ring.\(^{100}\)

A strong point of Thomson’s theory thus is the analogy to the ferromagnetic phase transition of paramagnets. Since it is the polarization that accounts for the electric conductivity, the model can explain, at least in principle, why the transition to the superconducting state happens discontinuously.

**Kamerlingh Onnes’s model of superconducting filaments**

Thomson’s model was received favorably by Kamerlingh Onnes who slightly modified it. At the 1921 Solvay Congress, Onnes gave a report on the state of knowledge about superconductivity, in which he also included a discussion of microscopic electronic theories.\(^{101}\) As in Thomson’s analysis, Onnes emphasized two features of the phenomenon which he singled out as fundamental: the complete loss of resistivity and the discontinuity of the transition. In view of the latter, Onnes asked whether there would be any other quantity that would undergo a sudden change at the superconductive transition and emphasized that there appeared to be none. In particular, he emphasized that no corresponding change of thermal conductivity was observed, and that in the superconducting state there would be “no longer any trace of the law of Wiedemann and Franz.”\(^{102}\) An attractive feature of Thomson’s model was that it could account for the discontinuity of the transition. But, wrote Onnes, with his hypothesis of an alignment of the doublets and the molecular field thus created Thomson went “perhaps a bit too far” in specializing his assumptions than would be necessary to explain the discontinuous transition.\(^{103}\) Instead, Onnes wondered whether the conduction electrons had, in general, “two ways of moving about in the atomic lattice.” One way, above the transition temperature, would be less ordered with frequent collisions.

\(^{100}\)ibid., p. 198.

\(^{101}\)Kamerlingh Onnes 1921b § 5.]. Einstein had been invited to attend the 1921 Solvay Congress and to talk about recent experiments on the gyromagnetic effect (see Lorentz to Einstein, 9 June 1920, [CPAE10 Doc. 49]) but decided instead to travel to the U.S. on a fundraising mission for the Hebrew University.

\(^{102}\)Kamerlingh Onnes 1921b p. 45].

\(^{103}\)ibid., p. 46.
with the atoms, and another one more ordered would take place below the transition temperature. Here the conduction electrons would “slide, by a sort of congelation, through the metallic lattice without hitting the atoms.” But Thomson’s general idea was still good, i.e. the idea “of a discontinuity determined by the temperature where some process has the character of an alignment.” Onnes discussed the difficulty of explaining the large mean free paths needed to account for the loss of resistivity according to the standard theory. He concluded that the notion of a mean free path has to be abandoned and replaced by a related concept:

We assume that under certain circumstances filaments of great length are being formed, along which an electron, that takes part in the conduction, can glide on the surface of the atoms and pass from one atom to the other without transmitting any energy to those degrees of freedom that contribute to the statistical equilibrium of the thermal motion.¹⁰⁴

Such motion would hence be called “adiabatic” (“adiabatique”). Those filaments need not, in contrast to Thomson’s model, be rectilinear but could be curved or twisted; they need not be made up necessarily from the same sort of atoms and could have ramifications everywhere, so that the electron might pass back and forth along these filaments throughout its path, always following the conditions of the superconductive state.

The adiabatic motion would have to be complemented by some non-adiabatic process at the ends of the filaments. As to the precise nature of those non-adiabatic events, Onnes only ventured a few conjectures in a footnote:

This could be the ejection of an electron from the atom, its passage in the state of free motion, and its collision with another atom, or else the immediate transport to an atom that comes into collision with the end of the filament, or the rupture of the filament by thermal agitation, if one lets oneself be guided by the old images, or else some other process of transmitting the ordered energy of the electrons to the thermal motion, if one strives to approach the theory of quanta.¹⁰⁵

¹⁰⁴ibid., p. 47.
¹⁰⁵ibid., p. 48.
Kamerlingh Onnes also observed that the notion of a collision of an electron would have to be generalized. The generalized notion would mainly have to render understandable how an electron can pass on its kinetic energy (“quantité de mouvement”) to the thermal energy of an atom. At this point, he added a footnote, alluding to a kind of billiard ball mechanism of electronic collisions:

As soon as the superconducting state was discovered, one had observed the analogy between the way in which the electricity is transported in a superconductor and that in which, in a common experience that one can do with a row of billiard balls suspended one next to each other, the momentum propagates from the first ball to the last.

Onnes remained vague at this point as to the precise mechanism that would be responsible for superconductivity. He referred in the end to the new theory of quanta, and formulated as a task for research to find a model of the atom that would allow a precise understanding of “this sort of electromagnetic crystallization, that, below a certain temperature, brings together all of a sudden the outer electrons of a huge number of atoms into filaments of a macroscopic order [...]”.

Haber’s model of osculating quantum orbits

The models of mechanisms of electric conduction discussed so far were based exclusively on classical concepts and did not invoke any of the new concepts associated with the emerging quantum theory. But by 1919, the success of the Bohr-Sommerfeld model of the atom suggested that these concepts should also be exploited for an understanding of the open problems in the theory of electric conductivity. This is what Kamerlingh Onnes had asked for in his contribution to the 1921 Solvay Congress. Before proceeding to discuss Einstein’s views on these matters, we will discuss one such proposal to make use of the new quantum theory of the atom for a new understanding of the phenomenon of superconductivity made by Fritz Haber in an addendum to the second of two communications devoted to the theory of metallic structure.

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106 ibid.
107 ibid., p. 49.
108 Haber 1919a, Haber 1919b.
Haber attempted to come to a better understanding of the structure and properties of metals by conceiving them as being made up of regular lattice structures where the lattice sites are occupied by positive ions and negative electrons and where the lattice energy is computed taking into account both van-der-Waals forces between the ions and the electrostatic forces between electrons and ions. More specifically, Haber computed quantitative relations between volume and compressibility on the one hand, and ionization energy of the metallic atoms and the heat of sublimation on the other hand and compared the theoretical values with observational data in order to test his general hypothesis. Haber’s second communication on the subject was presented to the Prussian Academy for publication on 27 November 1919, and an addendum to the second communication was written after Peter Debye had presented results about X-ray diffraction studies of lithium to the German Chemical Society on 29 November. Debye had shown that only core electrons were detected at the lattice sites of a body-centered cubic lattice, and that no valence electrons were observed that would be located at fixed lattice sites or on fixed orbits around lattice sites. As a consequence of these findings, Haber modified his original proposal to the effect that only positive ions make up the regular lattice structure, and that the outer electrons orbit around the positive cores in the interstitial spaces. Haber called the original model with both electrons and ions at the lattice points the “static picture of the metal” (“das statische Bild des Metalls.”). The case where the lattice is only made up of positive core ions is called a “dynamic lattice” (“Bewegungsgitter”). Conceiving of metals as dynamic lattices also solved, according to Haber, the difficulty posed by the phenomenon of superconductivity:

If the electrons were sitting fixed in the lattice sites, it could not be understood how the superconductivity at absolute zero temperature came about without violation of Ohm’s law. In this case a minimal force would be needed to effect their translation from one lattice point to the other.\textsuperscript{110}

But if metals were “dynamic lattices,” one would also be able to account for superconductivity. The idea was to invoke Bohr’s concept of stationary electron orbits around positive cores and to assume that electrons may both

\textsuperscript{109}[Haber 1919b, p. 1002].

\textsuperscript{110}ibid.
move on these orbits and also, under certain conditions, pass easily from one orbit to the other:

The point of view that naturally comes to mind is to conceive of superconductivity as a state in which the valence electrons of the metal move in orbits that have common tangents in points of equal velocity.\footnote{\textit{ibid.}, p. 1003.}

Since, according to Bohr’s quantum hypothesis, the electrons move around the atom cores on stationary orbits without radiating off electromagnetic energy, they can thus move along from atom to atom and give rise to a conduction current without electric resistivity. Therefore, in a dynamic lattice, an electric current may flow if one applies “an ever so weak field.”

Haber proceeded one step further to put his hypothesis to a quantitative test. In a body-centered cubic lattice half the distance between nearest neighbouring lattice sites is $r = \delta \sqrt{3}/4$ where $\delta$ is related to the molecular volume $V/N$ by $\delta^3 = 2V/N$. Circular orbits around the lattice sites that would have “common tangents” would hence have a radius of this value. Haber now invoked Bohr’s quantum condition for circular orbits, i.e. $\int pdq = mvr = nh/2\pi$ where $m$ is the electron’s mass, $v$ its speed, $h$ is Planck’s constant, and $n$ the quantum number. From this quantization condition, it follows that the electrons would have a kinetic energy $mv^2/2$ that could be seen as the frequency $\nu_s$ needed to kick out the electron in the photoelectric effect. Haber thus wrote the quantum condition as\footnote{\textit{ibid.}, p. 1004.}

$$\frac{mv^2}{2} \cdot 2mr^2 = \frac{n^2h^2}{4\pi^2} = h\nu_s \frac{2^{2/3}V^{2/3}m^3}{N^{2/3}8^{3/8}}, \quad (48)$$

where the second equation now expresses a testable relation between the empirically accessible quantities $\nu_s$, $V/N$, and $m$. Taking $n = 2$ for monovalent metals, Haber found “a reasonable representation of our experience for all monovalent metals, except for lithium and sodium, where our idealized model obviously does not suffice.”

\subsection*{Einstein’s model of conduction chains}

Einstein’s reaction to this kind of speculation about charge transport mechanisms on a microscopic level was characteristically twofold. He was a party
to the debate and contributed an idea that was actually put to an empirical test by Kamerlingh Onnes. He also reflected on the theoretical situation from an epistemological point of view. Let us discuss Einstein’s own model first.

We have some indirect evidence that the phenomenon of superconductivity was discussed not only phenomenologically but also on the microscopic level during the Leiden “Magnet-Woche” in early November 1919. The blackboard shown in Fig. 4 appears to hold sketches of what may well be models of electron trajectories. We also have some brief and sketchy notes by one of the participants, Willem H. Keesom, that have been discussed and partly reproduced in facsimile in [Matricon and Waysand 2003, pp. 41–42], see Fig. 6. The sketches in those figures suggest that the participants discussed mod-

Figure 6: Notes by Willem H. Keesom about models of superconductivity taken during discussions at the “Magnet-Woche” in Leiden in November 1919 (from [Matricon and Waysand 2003, p. 41]).

...els of superconductivity similar to Fritz Haber’s theory. Indeed, the notes by Keesom indicate that Einstein seems to have been debating whether an electron would revolve many times around an atom before making the next jump, or whether it would revolve only once. It seems that he was inclined
toward the former case in view of the sharpness of the transition between
normal conductivity and superconductivity.

A year later, Einstein entertained some concrete ideas along these same
lines. In a letter to Paul Ehrenfest, dated 2 November 1921, Einstein again
picked up the topic of a microscopic theory of superconductivity.

Do you remember our discussions about the superconductor? I
am getting back to this again. If there are no free electrons in
the metals, then an electric current means that there are elec-
trons whose well-ordered trajectory goes from atom to atom, and
in the case of superconductivity it does so in a stationary way.
But it cannot be single electrons because of the electric incom-
pressibility. Hence it must be electron chains that are formed by
atom-electrons marching in single file as it were. These chains are
permanent and undisrupted in the state of superconductivity.\footnote{Einstein to Paul Ehrenfest, 2 September 1921, AEA 9-566.}

So far, Einstein’s idea is strongly reminiscent of J.J. Thomson’s model, al-
though he did not invoke the idea of electric dipoles but instead referred
to “atom-electrons” (“Atom-Elektronen”). Einstein continued to draw some
immediate consequences from his hypothesis. He assumed that an electric
current is only possible through a chain that extends over the entire sub-
stance between two points. Each chain of conduction electrons extending
between two points contributes one unit of current.

The current is proportional to the number of such chains, hence
it can take on only discrete values.\footnote{ibid.}

One such unit would be given by the charge of an electron times the velocity
with which it is moving in those chains:

The discrete quantity of current is of the order $\nu e$ (opt[ical] fre-
quency $\cdot$ charge of the electron)\footnote{ibid.}

This suggests that Einstein was thinking more along the lines of Haber’s
model. The optical frequency refers to the circular frequency of an electron
travelling around the atom on a quantum orbit. Einstein now invoked a genuinely non-classical feature of the new quantum model, i.e. the assumption that electrons move on quantum orbits with discretely defined momenta.

If this is correct, then a superconducting coil would not respond to arbitrarily small electromotive forces, hence would not screen magnetic fields that are brought about sufficiently slowly (and that are weak enough so as not to destroy the superconductivity). The expression “superconductivity” would then be misleading.

The point is that since the superconducting current can only flow along the chains, and since the electrons travel on the quantized orbits, their velocity is fixed by the quantum conditions of Bohr’s atomic model. Consequently, there should be a finite minimal electric current that must be excited. Einstein suggested that this consequence should be put to experimental test in Leiden:

Such an experiment should be performed by you. [...] The superconducting coil could not carry currents below $10^{-4}$ up to $10^{-5}$ Ampère. Stronger magnetic fields destroy the chains.

More concretely, Einstein suggested measuring the self-induction of a non-superconducting coil that is placed next to a superconducting one, see Fig. (7). If the superconducting coil could take on only discrete and finite values of current, this feature should show up in the apparent self-induction of the non-superconducting coil. The minimal value of a superconducting current quoted by Einstein follows readily from his assumption that the circular frequency of the orbiting electrons is in the optical range. Indeed, the product of $\nu \cdot e$ evaluates to $\approx 1.5 \cdot 10^{-5}$ A if we take $\omega \approx 10^{16}/s$.

The occasion for Einstein’s returning again to the problem of superconductivity may well have been an invitation to contribute to a Gedenkboek to be published on the occasion of the fortieth anniversary of Onnes’s appointment as professor in Leiden. A direct response by Ehrenfest to Einstein’s letter is missing or not extant but he may well have alerted Einstein to the

\[116\] In Bohr’s atomic model the circular frequency $\omega$ is of order $\omega = h/(2\pi m_e r^2) = 2\pi \nu$ where $h = 6.6 \cdot 10^{-34}$Js, $m_e = 9.1 \cdot 10^{-31}$kg, and $r \geq 5.0 \cdot 10^{-10}$m, hence $\nu \lesssim 7.3 \cdot 10^{15}/s$, for ground-state hydrogen and smaller for outer orbits of larger atoms. The human eye is sensitive to electromagnetic radiation in the frequency range $\nu \approx 0.75\ldots0.43 \cdot 10^{15}/s$.

\[117\] ibid.

\[118\] ibid.

\[119\] Gedenkboek 1922.
fact that his model was reminiscent of some ideas existing in the literature. In a letter to Ehrenfest, written about two months later, Einstein referred to what is probably his contribution to the Onnes Gedenkboek:

[I am] Citing Haber in my article on superconductivity. He had developed a similar conception a few years ago in an Academy paper, albeit without “snakes.”

Einstein’s published contribution to the Gedenkboek contains an explicit reference to Haber’s 1919 paper discussed in the previous section. After arguing that there cannot be any free electrons in a metal, he continued with his hypothesis about metallic conduction.

Then metallic conductivity is caused by atoms exchanging their peripheral electrons. If an atom received an electron from a neighboring atom without giving an electron to another neighboring atom at the same time it would suffer from gigantic energetic changes which cannot occur in conserved superconducting currents without expenses in energy. It seems unavoidable that superconducting currents are carried by closed chains of molecules (conduction chains) whose electrons endure ongoing cyclic changes.

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120 Einstein to Paul Ehrenfest, 11 January 1922, AEA 10-004.
121 [Einstein 1922].
122 The reference was made in the postscript and refers to the first page of Haber 1919a, rather than more specifically to the addendum to Haber 1919b.
123 [Einstein 1922, pp. 433–434].

51
In contrast to Haber’s discussion, Einstein here emphasized that the electrons would have to move collectively in “conduction chains,” much like in Thomson’s model. This in any case seems to be the sense of his remark that Haber did not have the idea of “snakes.” In the published version, Einstein hardly was any more specific about his model of electric conduction. But he did repeat his suggestion to test the implication of a finite current threshold for superconductors.

..., there is the possibility that conduction chains cannot carry arbitrarily small currents but only currents with a certain finite value. This would also be accessible to experimental verification.\(^\text{124}\)

This experiment seems not to have been done in Leiden. But another consequence of his model that he proposed for experimental investigation was tested explicitly in an experiment done by Kamerlingh Onnes. Einstein’s idea of “conduction chains” along atomic quantum orbits was restrictive not only because it allowed only for quantized units of current. It was also restrictive in the sense that it did not allow for chains to be made up of different atoms, since the orbital velocities around different atoms would differ, and hence would not allow for smooth transitions of the conduction electrons from orbit to orbit.

It may be seen unlikely that different atoms form conduction chains with each other. Perhaps the transition from one superconducting metal to a different one is never superconducting.\(^\text{125}\)

Einstein further argued for this model of conduction chains by pointing out that it was quite natural that these chains would be destroyed by large magnetic fields, as well as by thermal motion “if it is strong enough and if the \(h\nu\) energy quanta that are being created are big enough.” Hence, it would also be understandable why superconductors turn into normal conductors by raising the temperature, and one could understand “maybe even the sharp temperature limit of superconductors.” Indeed, Einstein conjectured that normal electric conductivity may perhaps be nothing else but superconductivity that is constantly being destroyed by thermal motion. This conjecture, he concluded, would be suggested by the “consideration that the frequency of the

\(^{124}\)ibid., p. 434.
\(^{125}\)ibid.
transition of the electrons to the neighboring atom should be closely related to the circulation frequency of electrons in the isolated atom.\textsuperscript{126} The very last sentence of his paper then repeats the hypothesis that superconductors must necessarily be homogeneous:

\begin{quote}
If this idea of elementary currents caused by quanta proves correct it will be evident that such chains can never contain different atoms\textsuperscript{127}
\end{quote}

We have reason to believe that Einstein was eager to see whether these consequences would actually be observed. On 21 January 1922, he wrote to Ehrenfest:

\begin{quote}
Nurture Onnes about those superconductivity-experiments\textsuperscript{128}
\end{quote}

Indeed, a few weeks later, Ehrenfest reported back to Einstein that Onnes had investigated the issue of whether the interface between different superconducting materials would still be superconductive, and that he had found that no resistance was observed for a contact between tin and lead\textsuperscript{129}. Ehrenfest added that Onnes would write to Einstein himself about these findings, but that letter seems to have been lost. In any case, Einstein added a postscript to his Gedenkboek contribution. Referring to his final remark on the impossibility of having conduction chains contain different atoms, he added:

\begin{quote}
The last speculation (which by the way is not new) is contradicted by an important experiment which was conducted by Kamerlingh Onnes in the last couple of months. He showed that at the interface between two superconductors (lead and tin) no measurable Ohm resistance appears\textsuperscript{130}
\end{quote}

It appears that the results of these experiments were never published. But two years later, the very same experiment was repeated with greater accuracy by Kamerlingh Onnes together with his student Willem Tuyn. The better

\begin{flushleft}
\textsuperscript{126}ibid., p. 435.
\textsuperscript{127}ibid.
\textsuperscript{128}“Schüre Onnes wegen der Supraleitungs-Versuche.” Einstein to Paul Ehrenfest, 21 January 1922, AEA 10-011.
\textsuperscript{129}Paul Ehrenfest to Einstein, 11 March 1922, AEA 10-025.
\textsuperscript{130}[Einstein 1922, p. 435].
\end{flushleft}
accuracy was made possible by two modifications of the experimental setup. For one, Onnes and his collaborators had succeeded in isolating the liquid helium in a cryostat that could be physically removed from the liquifier and transported to a different location. They also employed a new method of determining residual resistances by looking at persistent currents in rings, rather than measuring the resistivity of filaments by directly observing the potential difference for strong currents. Details of these experiments were presented by Kamerlingh Onnes to the fourth Solvay Congress in April 1924, and to the Fourth International Congress of Refrigeration, held in London in June 1924. In the published report, Onnes gave an overview of recent experiments and investigations into superconductivity and discussed in section § 5. “diverse issues” (“questions diverses”).

One of these was Einstein’s hypothesis. Onnes began by mentioning that he had shown “with the method of filament” that the resistance of the “soldered interface” (“soudoure”) between lead and tin was below what could be determined with the given limits of experimental accuracy. Referring to Einstein’s contribution to the Gedenkboek, he remarked that Einstein had given up on “his idea that superconducting circuits cannot be constituted by different atoms.” He continued

Now that we have at our disposal a method for measuring these small resistances with a much larger precision, it was of highest interest to repeat these experiments. The microresidual resistance was measured by the lifetime of persistent currents in superconducting rings. Specifically, Onnes used a ring of lead suspended on a torsion rod within another slightly larger lead ring. The whole setup was isolated against mechanical vibrations by mounting on a shock absorber and immersed into liquid helium. Currents were induced by an external magnetic field perpendicular to the plane of the rings, and the inner ring was rotated out of its equilibrium position by an amount of 30°. Afterwards, the motion of the inner ring was monitored by light reflected from a mirror fixed to the torsion rod. Since the currents were persistent, no rotational motion was observed, and the setup gave an upper limit to the resistivity that was determined by the time that the experiment could be run

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131 Kamerlingh Onnes 1924.
132 ibid., p. 15.
133 ibid., p. 16.
before the liquid helium evaporated, a time that according to Onnes’s report took some six hours or so.

In order to test Einstein’s hypothesis, Tuyn and Onnes now used a ring consisting of 24 alternating sectors of lead and tin, see Fig. 8. More pre-

![Figure 8: Kamerlingh Onnes’ experimental setup to test Einstein’s hypothesis that junctions between two different superconducting metals are not superconductive (from [Kamerlingh Onnes 1924, p. 16]).](image)

cisely, the sectors consisted of bands of tin or lead wrapped around a ring of ivory. Care was taken to keep the middle of each sector cooled when soldering the sectors together in order to avoid any diffusion of lead into tin. A current was induced in the ring by a magnetic field, and the ring was displaced by an angle of 30°. The expectation according to Einstein’s hypothesis was that it would take a certain amount of time for the current to die down. However, the results did not accord with expectations.

But the experiment has shown that the currents continue to flow in the ring and when the experiment was repeated when the ring
was cut it showed the same magnetic moment.\textsuperscript{134}

This result was puzzling.\textsuperscript{135} Onnes presented his experiment as work in progress. Otherwise, he argued, Einstein’s hypothesis would have been proven:

Otherwise, one would already be driven to the conclusion that the 24 points of contact between the sectors have a resistance that is too big to be measured by this method, since the current induced in the complete circuit of the ring decays too rapidly alongside the persistent currents induced in the individual sectors.\textsuperscript{136}

Unfortunately, Onnes’s description is not sufficiently detailed to allow an interpretation of the outcome of his experiments from our modern understanding. If the interfaces between the sectors were clean, the sectored ring should have shown a persistent current. If isolating material had been added between the sectors, these would, in principle, become tunneling barriers for the superconducting wave function and the setup might perhaps have exhibited Josephson current effects. As described by Onnes, the experiments remain inconclusive.\textsuperscript{137}

In the last section, Onnes discussed “the structure of superconductors,” and again referred to Einstein:

\textsuperscript{134}[Kamerlingh Onnes 1924, p. 16].

\textsuperscript{135}Fig. 8 seems to show a slightly different setup than was described earlier for the persistent current measurements. Here only one half of the outer ring is shown. This different is not commented on in Onnes’s paper.

\textsuperscript{136}[Kamerlingh Onnes 1924, p. 16].

\textsuperscript{137}In fact, in 1926 Einstein suggested to investigate this question once more in the low temperature laboratory of the Physikalisch-Technische Reichsanstalt (PTR) in Berlin. In the discussion in its Kuratorium following the presentation of the annual report of the PTR for the year 1925 (when experiments at liquid helium temperatures had finally become possible), Einstein remarked that “the question is of particular interest whether the interface between two superconductors would be superconductive as well.” See “Bericht über die Tätigkeit der Physikalisch-Technischen Reichsanstalt im Jahre 1925,” copy deposited in the Library of the PTR, and minutes of the meeting of the Kuratorium of the PTR of 11 March 1926, Library of the PTR, sign. 240.2-241 (AEA 81-887), see also [Hoffmann 1980, p. 95]. Einstein’s suggestion apparently was followed up on, but met with difficulties. In the report for the following year (1926), the authors wrote: “The fact that alloys become superconductive, makes it more difficult to decide experimentally the question, posed by Einstein, whether a resistance appears at the interface of two superconductors due to a breaking up of the superconductive conduction chains.” [Tätigkeitsbericht 1926, p. 234].
I have accepted Einstein’s idea that the electrons that take part in the conductivity of a solid metal have velocities of the same order as the valence electrons in the free atoms [...].

As an immediate consequence of this assumption, the melting transition of a metal should have little influence on the conductivity, as Einstein had conjectured. But on a more general level, it meant that the atomic model of the emerging quantum theory had to be taken seriously for a theory of superconductivity. For this reason, Onnes turned to Hendrik Anton Kramers in Copenhagen, who provided him with a graphic visualization of the electronic structure of some of the metals under consideration, e.g. of Indium, as shown in Fig. 9. Although Onnes went into some detail regarding the atomic structure of metals and the consequences for a theoretical understanding of superconductivity.

Figure 9: Graph of the electronic and lattice structure of Indium, according to the Bohr-Sommerfeld quantum theory (from Kamerlingh Onnes 1924, p. 28).

138 Kamerlingh Onnes 1924, p. 26.
139 Einstein 1922, p. 433.
superconductivity, his results remained inconclusive as far as any quantitative results are concerned. Anything else would have been rather surprising from our modern understanding of the phenomenon. Nevertheless, it is remarkable not only that the phenomenon of superconductivity was perceived as a genuine quantum phenomenon, but also that Einstein was among those, who like Haber and Onnes, clearly advocated making use of the Bohr-Sommerfeld theory for an understanding of superconductivity.

In this context, another entry of around June 1922 in Ehrenfest’s diaries is of some interest, see Fig. 10, which suggests that Ehrenfest had talked to Bohr himself about the issue. The entry says: “Bohr: “Don’t know” - but conductivity! — idea:” and is accompanied by a small sketch strongly reminiscent of Einstein’s conduction chains. Ehrenfest added in brackets the names of J.J. Thomson and Einstein. Whatever the context of this entry, it supports the general conclusion that superconductivity was not only investigated experimentally in Leiden, but also interpreted as part of a larger attempt to come to an understanding of the new quantum theory.

It is in this sense that Onnes concluded his 1924 report by writing:

For the moment, in view of the state of the theory of quanta, it seems that it would be utterly premature if one wanted to form more detailed images, as I had in mind, of the motion of conduction electrons.

But one sees the dawning of the light that the application of this theory will bring.

\[^{140}\text{Kamerlingh Onnes 1924, p. 34}.\]
Einstein’s epistemological reflections

So far, we have only discussed Einstein’s comments on and considerations about superconductivity as an attempt of a contemporary physicist to come to a theoretical understanding of the new phenomenon. But his published paper on the subject also carries a distinctly and characteristically different overtone. In addition to presenting and defending his own model speculation on a conduction mechanism, it also offered quite explicit epistemological reflections on the status of physical theory. Indeed, it begins like this:

The theoretically working scientist is not to be envied, because nature, or more precisely: the experiment, is a relentless and not very friendly judge [Richterin] of his work. In the best cases, she only says “maybe” to a theory, but never “yes,” and in most cases she says “no.” If an experiment agrees with a theory it means “perhaps” for the latter. If it does not agree, it means “no.” Almost any theory will experience a “no” at one point in time - most theories very soon after they have been developed.141

Einstein had expressed similar falsificationist views in a little piece on “Induction and Deduction in physics”142 published in the daily Berliner Tageblatt just some two years earlier in late 1919, after the observational confirmation of gravitational light bending. There he argued that progress in physical theory usually does not occur by induction from empirical data but rather along some kind of hypothetico-deductive reasoning. The researcher, he wrote,

...does not find his system of ideas in a methodical, inductive way; rather, he adapts to the facts by intuitive selection among the conceivable theories that are based upon axioms.143

The experiment then appears, indeed, as a judge, and Einstein had continued in a very similar way as in 1922 by expressing his falsificationist leanings:

Thus, a theory can very well be found to be incorrect if there is a logical error in its deduction, or found to be off the mark if a fact is not in consonance with one of its conclusions. But the truth

141 [Einstein 1922, p. 429].
142 [Einstein 1919].
143 [CPAE7, p. 219].
of a theory can never be proven. For one never knows if future experience will contradict its conclusion; [...]\(^{144}\)

However, there is a subtle difference between Einstein’s 1919 reflections and those of 1922. In 1919, he was under the spell of the spectacular confirmation of his most significant theoretical achievement, the observation of the gravitational light bending, predicted by general relativity\(^{145}\). In 1922, Einstein reflected, as we will see, on the failure of Drude’s electron theory of metals in light, or should one say, in darkness of the fact that no convincing alternative was available to account for superconductivity. Hence, in 1919 he wrote that one never knows whether a theory will be proven wrong by contradicting experience, while in 1922 he asserted that “almost any theory will be proven wrong at some time.”

The justification for his epistemological pessimism was given in Einstein’s reflections on the present state of the theoretical understanding of metallic conductivity. His point of departure is Drude’s electron theory of metals. He quoted Drude’s formula for the specific resistance \(\omega\) of metals, i.e. the inverse of eq. (9)\(^{146}\)

\[
\omega = \frac{2m}{\epsilon u^2} \frac{1}{nl}, \tag{49}
\]

where \(m\) is the electron’s mass, \(\epsilon\) its charge, \(u\) its mean velocity, \(n\) the electron density, and \(l\) the mean free path, and proceeded to discuss the evidence against Drude’s theory.

The difficulties arise from the implicit consequences of the temperature dependencies of the mean velocity \(u\), the electron density \(n\), and the mean free path \(l\). The temperature dependence of the mean velocity is determined by the equipartition theorem

\[
mu^2 = 3kT \tag{50}
\]

where \(k\) is Boltzmann’s constant, and \(T\) the absolute temperature.\(^{147}\) Einstein now argued that one might expect the electron density \(n\) to increase

\(^{144}\)ibid.

\(^{145}\)For an account of the expedition, its results, and Einstein’s reaction to it, see \[CPAE9\] pp. xxxi–xxxvii].

\(^{146}\)With the same problematic factor of 2, that was discussed above (see the discussion following eq. (11)).

\(^{147}\)In [Einstein 1922], the factor of 3 was written erroneously on the left hand side of the equation (cp. eq. (2) above).
with temperature on the assumption that free conduction electrons are created by thermally enhanced dissociation. But the resistance of metals typically increases with temperature, rather than decreases. Hence, one might be tempted to assume that $n$ is roughly temperature independent, and that some temperature dependence of the mean free path arises from the thermal lattice vibrations. But the first hypothesis would be problematic, and the second one might be hard to justify quantitatively. Moreover, if the mean free path is determined by the thermal energy of the metal, one should expect that the resistance of non-superconducting metals tends to zero for decreasing temperature, while in fact it remains constant. The residual resistance might be explained by impurities, but the effect of impurities on the mean free path would be to add a constant to $1/l$. This, however, would change the resistance by an amount proportional to $u$. But since the effect of impurities is to change the resistance by a constant amount, one would have to assume that $u$ does not depend on temperature. But, concluded Einstein,

under no circumstances can $u$ be assumed to be temperature-independent, because otherwise the only success of the theory, i.e. the explanation of the Wiedemann-Franz law, would have to be sacrificed.\(^{148}\)

The bottom line of Einstein’s reflections on the implications of Drude’s result is that the thermal electron theory already fails to account for the empirical facts of normal electric conductivity.

The breakdown of the theory became entirely obvious after the discovery of the superconductivity of metals.\(^{149}\)

But since it was conceivable that the Wiedemann-Franz law might be explained also by some other theory, Einstein retracted his pessimistic epistemologic turn, if only vaguely.

No matter how the theory of electron conductivity may develop in the future, one main aspect of this theory may remain valid for good, namely the hypothesis that electric conductivity is based on the motion of electrons.\(^{150}\)

\(^{148}\)Einstein 1922, p. 432.
\(^{149}\)Ibid.
\(^{150}\)Ibid., p. 430.
Einstein’s discussion of the epistemological status of a physical theory against its empirical content may have been motivated only by the wish to justify the putting into print of theoretical speculations that until now he had aired only in personal discussions, correspondence, and unpublished manuscripts. In any case, he went on to present and justify his model of conduction chains, based on conduction electrons that move on quantized atomic orbits. He did emphasize that he considered these ideas little more than speculations:

Given our ignorance of the quantum mechanics of composite systems we are far away from being able to convert this vague idea into a theory.\textsuperscript{151}

It is interesting that Einstein referred to the emerging quantum mechanics of composite systems (“Quanten-Mechanik zusammengesetzter Systeme”) in this caveat. As we have seen, his approach to a microscopic theory of superconductivity was characteristically bold in putting these new concepts to use. Incidentally, as conjectured by H. Kragh, this may well be the first time ever that the term “quantum mechanics” appeared in print.\textsuperscript{152} In any case, Einstein’s 1922 contribution encouraged the exploration of new paths in the theoretical understanding of superconductivity.

This phantasizing can only be excused by the momentary quandary of the theory. It is obvious that new ways of doing justice to the facts of superconductivity have to be found.\textsuperscript{153}

\textbf{Concluding remarks}

In this paper, I have argued that Einstein’s appointment as a special visiting professor at the University of Leiden in 1920 was motivated to a considerable extent, if not primarily, by the fact that his Dutch colleagues perceived him to be a leading theoretician of condensed matter physics, and especially of low temperature physics. It was expected that he would contribute to the theoretical understanding of new phenomena observed in the low temperature regime, and that he would provide theoretical guidance to experimental

\textsuperscript{151}ibid., p. 434.
\textsuperscript{152}[Kragh 1999, p. 86].
\textsuperscript{153}[Einstein 1922, pp. 434–435].
investigations undertaken in Leiden. It has also become clear that Einstein himself tried to live up to these expectations, at least during the period of time that we have been considering, 1919–1922.

In his theoretical analyses of superconductivity, Einstein proposed at least three experiments to be done in Leiden. His exploration of the implications of Maxwell’s equations for the case of perfect conductivity led him to suggest a Hall experiment on a superconducting slab of varying thickness. His proposal of conduction chains as a microscopic mechanism of superconducting charge transport implied that superconductive currents were quantized in magnitude and, in particular, would show a minimal threshold value. He suggested that this implication be tested by measuring the effective self-induction of a coil of non-superconductive metal that was in inductive contact with a superconducting coil. Another consequence of his model was the implication that the interface between two different superconductors would not be superconducting. This latter hypothesis was explicitly tested by Onnes, with a negative result. The experiment was repeated two years later with an experimental setup that allowed for better accuracy but then produced results that were inconclusive.

It seems also fair to say that in the context of contemporary theorizing about superconductivity, Einstein’s considerations and ideas were rather sophisticated and advanced. His exploration of the implications of Maxwell’s equations for perfect conductivity went well beyond Lippmann’s investigations and also proved to be more successful and insightful than explorations along the same lines done by Ehrenfest. Similarly, his microscopic model of conduction chains was distinguished from alternative theories in that it went farthest in the application of concepts of the emerging quantum theory for an understanding of superconductivity.

One may regret that Einstein’s thoughts about superconductivity produced only one publication. But, from today’s point of view, it is also clear that, in spite of Einstein’s insights and creativity, none of his ideas would have brought about a better understanding of superconductivity or of quantum physics for that matter. The story of Einstein’s concerns with the phenomenon of superconductivity is hence neither one of failure, nor is it one of success. It is rather a reflection of a peculiar situation of the state of theoretical physics at the time that was characterized by an emerging division between theory and experimental practice, and the fact that the emerging quantum theory had not yet reached a stable and convincing status. Einstein’s falsificationist reflections on physical theory vis-à-vis experimental
observation seem to reflect the division of labor that was embodied in his own status as a theoretician for the Leiden cryogenic laboratory. His epistemological pessimism was justified at the time in view of the weakness of quantum theory. It is all the more surprising that he advocated so expressly an application of quantum concepts for the theoretical understanding of superconductivity.

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67
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