3D printing technology as innovative tool for math and geometry teaching applications

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Abstract. The industrial revolution and automation of production processes have changed the face of the world. Three dimensional (3D) printing has the potential to revolutionize manufacturing and further change methods of production toward allowing in increasing number of people to produce products at home.

According to a recent OECD (see Backer [1]) publication, “…tapping into the next industrial revolution requires actions on many levels and in many different areas. In particular, unlocking the potential of emerging and enabling technologies requires policy development along a number of fronts, from commercialization to regulation and the supply of skills through education.”

In this paper we discuss the role of schools and their responsibility to act as quickly as possible to design a plan of action that will prepare the future citizens to deal with this new reality. This requires planning of action in different directions and on different planes, such as labs, teachers, and curricula.

3D printing requires higher levels of thinking, innovation and creativity. It has the power to develop human imagination and give students the opportunity to visualize numbers, two-dimensional shapes, and three-dimensional objects. The combination of thinking, design, and production has immense power to increase motivation and satisfaction, with a highly probable increase in a student’s math and geometry achievements. The CAD system includes a measure tool which enables and alternative way for calculating properties of the objects under consideration and allows development of reflection and critical thinking.

The research method was based on comparison between a reference group and a test group; it was found that intervention significantly improved the reflection abilities of 6th grade students in mathematics.

Keywords: education, mathematical thinking, additive manufacturing, 3D printing.

1. Introduction
In the 21st century, challenges to humanity are diverse. New generations are subject to fast advances in science, technology, and technological systems. These advances expose barriers to human development. The educated citizen should be able to adapt accordingly in order to find a direction to his life.

Perception of the real world depends on the senses. Mathematics and geometry require imagination and abstract thinking, making it very difficult for newcomers to advance in these fields. This (almost pre-ordained) barrier discourages students from pursuing these tracks.
Although, additive manufacturing, such as three-dimensional printing, requires multidisciplinary skills, e.g., critical thinking, problem solving, and computer-based skills, the final product is a physical object that was designed by the student, one that will add another dimension to the learning experience, and increase motivation and curiosity.

What makes 3D printing attractive to teaching mathematics and geometry? What is 3D printing? What are the types and applications of 3D printing? How could 3D printing be implemented in teaching?

In this article, answers to these questions are given by reviewing the literature and by proposing a plan of action for teachers and students.

Intuitively thinking, 3D printing requires skills from different disciplines and skills, mathematics (numbers, operators, and relations), geometry (plane and solid), problem solving, imagination, innovation, design [2], material science, and production processes. Putting these together is a challenge full of motivation and curiosity.

According to Wohlers and Gornet [3], the method of manufacturing by adding layers of material was initiated in 1987 with stereo lithography (SL). A material in the liquid state which is sensitive to light was used and the product was produced by solidifying the layers of the sensitive material to light.

As was described in Ventola [4], the produced object by the 3D printing manufacturing method is obtained by adding (printing) multiple layers of materials.

According to Gross et al. [5], different methods exist in the market. These include several manufacturing methods. The most popular method is fused deposition modelling (FDM) which was developed by Scott Crump of Stratasys. FDM is widely used manufacturing technology for rapid prototyping to fabricate a 3D model by extending (extruding) thermoplastic materials and depositing the semi molten materials onto a stage, layer by layer [6-7].

According to the Leading Edge Forum (LEF) [7], it was found that 3D printing is evolving rapidly, with practical examples in numerous fields and industries including defence, aerospace, automotive, and healthcare.

As was reported by Gross [5], 3D printing technology is suitable for industries such as automotive and aerospace (printing prototypes of car and airplane parts), architecture (printing structural models), medical advances, applications in the food industry, in fashion design, and in other private, and government applications.

As was addressed in [4], the method of 3D manufacturing is expected to make a revolution in medicine and other fields. 3D printing technology is developing continuously and new applications constantly appear in the market. As was reported in Zoran [8], 3D printing technology has the potential to revolutionize the production of acoustic instruments.

Some of the materials used by FDM to fuse or deposit layers to produce 3D objects are plastic, metals, ceramics, powders, liquids, and even living cells. [4].

Although 3D printers were known for many years, they were unaffordable for everyone, which explains why these printers were used mostly for industrial applications. What makes these printers more affordable is their significant growth in the last few years, as a result of which their prices have fallen more than tenfold, which makes them available for almost everyone (personal usage, educational usage, small businesses usage, etc …) [6].

Berman [9] looked into the properties and fields of applications and compared them with other manufacturing methods. Among many others, 3D printing is superior in low volume production, cheaper products, in the production of replacement parts, dental crowns, and artificial limbs, as well as in bridge manufacturing [9]. Another observation was reported by Szulżyk-Cieplak et al. [6], 3D printing can produce products that cannot be produced by other methods. This is due to the fact that material is added layer by layer which could trace almost any design and any shape.

Another important point was stated by Cambel et al. [10]. As the researchers claimed, the additive manufacturing or the 3D printing technology is about to make new revolution in manufacturing and production of objects.
2. 3D printers in education

Additive manufacturing is an attractive tool for teaching and education. As was reported by Szulżyk-Cieplak et al. [6], 3D printing technology is undoubtedly a new chapter in teaching. 3D printers directly support the teaching process. Computer aided designed (CAD) and printed physical models facilitate better understanding of the creation process. They also enhance students’ involvement in the classes. 3D printing enables students to transfer their ideas into reality. Students who can physically examine their projects will more enthusiastically participate in classes, and their spatial imaginations are effectively stimulated. Printed models let students learn about the strengths and weaknesses of their projects [6]. It is important to add that at early ages, teaching starts with physical objects, and only later moves towards abstraction (3D model). 3D printing works the other way around, starting from CAD design and ending with physical objects. Actually, it is possible to link both processes during the learning stage, which aids in developing creativity and imagination.

3D printing in education is expanding rapidly. The following is not intended to be comprehensive, and merely serves to provide a few examples and outline the directions being taken in this developing field.

In her article, Loy [11] addressed the suitability of 3D printing as environment of education and e-learning. The researcher highlights the importance of the virtual and physical worlds in design and education.

Irwin et al. [12] reported and discussed the issue of the 3D printing revolution in the science, technology, engineering, and mathematics (STEM) curriculum, which should have activities with an iterative process that involves statement of the problem, suggesting methods of solution, and optimizing creative solutions and designs. The article discussed the complicated role of educators, who are struggling with how to implement next generation science standards (NGSS) [12].

In his article, Eisenberg [13] addresses several points regarding 3D printing, pointing out that nowadays 3D printers are cheap and affordable, this make them available to young children. This fact raises a challenge of educating the young students to use 3D printers. The education process is a complex of several fields of design and manufacturing, which is not intuitive.

In his study, Segerman [14] discusses the importance of 3D printing for mathematical visualization, comparing 3D printing (additive manufacturing) with “subtractive” manufacturing techniques, in which material must be cut and removed from initially solid objects and objects with complex design can be very hard to manufacture. With the additive procedure of 3D printing, these difficulties are greatly removed. The procedure includes three steps: mathematical concept → computer model → 3D printed object [14].

In a 2013 article, DFE [15] discusses the issue of 3D printers in schools and its uses in a curriculum focused on enhancing the teaching of STEM and design projects. It was pointed out in the study that the 3D printer is greatly suited to project work, where studying stems naturally as part of an analysis or a design project. It was found that the printer encourages cross-curricular thinking [15].

In an article by Rainone et al. [16] entitled “IMAGINARY Math Exhibition using Low-cost 3D Printers,” they reported that they attempted to reproduce and construct 17 shapes of the IMAGINARY Open Mathematics Exhibition (www.imaginary.org) by means of cheap, desktop 3D printers.

In Slavkovsky’s [17] “Feasibility Study for Teaching Geometry and Other Topics Using Three-Dimensional Printers” thesis, she focused on using 3D printers for teaching mathematics as an illustrating tool. She relied on the work of George Hart (http://www.georgehart.com/sculpture/sculpture.html) regarding some thoughts about design and teaching material. According to her expectations, 3D printers will be needed by students nowadays and in the future in order to enhance their studies and understanding. 3D printers will help students to increase their spatial intuition, evaluate production cost and understand the need of optimization through the process of design and producing (printing) objects [17].
3. Materials and methods
As was discussed in the previous section, the processes of additive manufacturing account for three major steps: Design on paper (mathematical concept), CAD, and printing (manufacturing).

Each step includes several sub-steps, skills, and the existence of a body of knowledge acquired from different disciplines. This might be a barrier to newcomers.

Today, students at early ages are being exposed to computers, almost since the time of their birth. We can easily assume that they learn using technology systems faster than their older peers do, or even their teachers. This might suggest that the teachers and students could learn some subjects and/or disciplines together. In order to break symmetry, teachers are usually pre-prepared for teaching. Due to our fast-changing world, science and technology advances, there might be because of that, some overlapping in the process of teaching of teachers and students. One might suggest in order to minimize overlapping to teach teachers first and together with them develop activities for the students.

In order to be able to teach mathematics (or STEM activities with project-based learning) by using additive manufacturing, several actions should be performed. These include training the teachers to use 3D printing, and then teaching them how to transfer their newly acquired knowledge to their students. The teachers will be able to use this new information to teach mathematics and geometry in middle school. These new techniques can be used to provide instruction in almost any subject of a teacher’s specialty. (In fact, it is possible to begin such a program even earlier, with the only barriers being the availability or affordability of 3D printers. The Arab sector in general is poor in budgeting and labs.)

The following subsections summarize the basic concepts and skills required for 3D design and printing.

3.1. Unit 1 – Technical engineering drawings
It is important to teach this unit especially because most of mathematics and science teachers do not study this subject at college or university. The focus in this unit is on projection, front, top, side, isometric view, orthographic view, and perspective view. It should be possible for the learner to work both ways—from the physical body to the projections, and vice versa. Attention must also be given to teach or reinforce the ability to imagine or predict the combination, subtraction, and intersection of bodies.

3.2. Unit 2 – Computer aided design
There are many types of CAD programs available in the market. Some programs are commercial and professional, which are not freeware (such as SolidWork, AutoCAD, Maya, and similar programs), and some are freeware and suitable for school students and children who have computers available at home (such as Tinkercad and 123d design). In these activities, the teaching will be based on TinkerCad (a web-based program) for elementary school and 123d design for middle school. These choices are mainly because these platforms are freeware, thereby reducing the cost barrier confronting students.

3.3. 3D printing and methods of manufacturing
This last step or unit gives the student the opportunity to see the printed body and check the result. After testing the result, it is possible to improve. Students will practice saving or exporting the design into different file formats, including STL, the format suitable for 3D printers.

Teaching will be conducted using a hands-on approach and systematic procedures, in order to make the process of teaching as smooth as possible.

3.4. Audience
The action plan will address two populations: teachers and students. Usually, the teachers are specialists in one scientific subject (chemistry, biology, physics, mathematics, and similar fields). The students come from different schools and grade levels, ranging from fourth to ninth grade.
3.5. **Location**  
An experiment is planned to be presented in the sixth grade, to teach the very basics of solid geometry and to calculate volumes and surface areas on paper and by using a CAD system (see appendix).

3.6. **Start of action**  
The aforementioned activities and unit plans should take place during the opening term of the next school year.

3.7. **Measuring changes**  
In order to measure the effect of usage of 3D printing, the following actions should take place.  
- Define experimental and reference groups.  
- Test both groups before taking action.  
- Teach the reference group following the usual curriculum, without any changes.  
- Teach one unit of the new curriculum.  
- Test both groups using the same test.  
- Analyze results and draw some conclusions.

3.8. **Hypothesis and variables**  
Hypothesis:  
The use of 3D printing to teach math and geometry will improve student’s achievement (test score).  
The use of 3D printing includes the choice of an object to print (cube, sphere, box, and cylinder), choice of the dimensions of the object, representing the object in CAD program, and sending the object to the printer. The CAD program has the measure tool which allows measuring the surface area and the volume of the object. This tool allows an alternative method to find the aforementioned parameters for comparison purposes which in turn enhances reflection and critical thinking.  
Variables:  
Affecting variable (independent variable) – use of 3D printing (measure tool in CAD system).  
Affected Variable (dependent variable) – test score.  
These variables will be used to compare between the test group and reference group. Both group should take the same exam pre and post intervention.

4. **Data collection and analysis**  
Data were collected during the last academic year to test the effect of reflection on math achievements in the sixth grade. Two groups took part in the project: a reference group and an intervention group. Table 1 shows results from both groups at three times.  

| Term | Reference Group | Intervention Group |
|------|-----------------|--------------------|
| A    | 50.3            | 56.5               |
| B    | 54.7            | 61.5               |
| C    | 55.2            | 75.0               |

At the start of the new academic year, two additional groups (one reference group and one experiment group) will be tested: pre-intervention and post intervention. The scores of the tests will allow a t-test comparison to be made, where the variables can be reduced through the use of the same population.

5. **Summary and conclusion**  
In this article 3D printing or additive manufacturing were considered as tools to enhance teaching math and geometry in the middle school. The model of action described in the text includes three steps or three teaching units: design on paper, CAD design, and manufacturing.
The interdisciplinary features of the manufacturing process make it possible to address math and geometry. Design on paper, and the features and tools of CAD systems, make it possible to teach mathematical concepts and skills through geometrical construction and modeling in both two and three dimensions. Printing the designed object makes it possible to grasp and evaluate the design through checking the product. Thus, 3D printing enables mathematical thinking development.

According to Slavkovsky [17], the initial focus on the pure mathematics of basic geometric shapes was not as interesting as some later experiments. Her findings showed that it did not take very much time for students to learn to design such simple primitive structures as a cube, sphere, or cone, and then print them out. Another observation of her research was that both teachers and students wanted to learn more about 3D printing, which required patience, imagination, and a considerable amount of curiosity [17].

It is important to mention that these 3D basic concepts are taught at the sixth grade level and the students should learn to calculate volumes and surface areas. CAD systems give students the opportunity to perform calculations both on paper and by using the program. This enables them to use both their pencil and paper calculations and their physical printed objects to compare methods and reach the appropriate conclusions.

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Appendix
Solid entities: names and formulae.

| Shape # | Name                      | Volume          | Lateral area | Surface area |
|---------|---------------------------|-----------------|--------------|--------------|
|         | Cube Side a               | a*a*a           | 4*a*a        | 6*a*a        |
|         | Rectangular prism (Box) Sides a, b, c | a*b*c       | 2*a*c+2*b*c  | 2*a*c+2*b*c+2*a*b |
|         | Cylinder Radius r Height h | π*r*h           | 2*π*r*h     | 2*π*r*h+2*π*r*r |

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