Curvilinear Aperture Monopulse

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Abstract—By a symmetry argument, a synthetic aperture radar collection along a linear path does not collect three-dimensional information about the scene. However, it is known that vertical curvature can be used to derive some vertical position information. This paper approaches the problem from a monocycle perspective, resulting in a non-iterative computation that commutes with efficient image formation algorithms.

Index Terms—synthetic aperture radar, sar processing, array processing, monopulse

I. INTRODUCTION

Synthetic Aperture Radar (SAR) depicts radar reflectivity as a two-dimensional image in range and azimuth relative to the aperture traced out by the path of the platform. A three-dimensional point cloud can be created from two or more appropriately matched SAR images, using interferometric SAR or radargrammetric stereo (as in, for example, [1]). But for a SAR image from a linear aperture, no information is present about the position of a scatterer along a circle centered on a line through that aperture and lying on a plane perpendicular to it. This follows from simple symmetry arguments.

However, when the aperture is not linear, the symmetry is broken. Formation of a sharply-focused two-dimensional image requires compensating for out-of-plane motion using assumed height of scatterers, typically on the ground plane [1]. Conversely, objects in an image with height differing from the assumed ground plane may appear to be out of focus.

This depth of focus issue for curvilinear apertures, together with its implication for height estimation, has been most recently studied by [2], [3]. The use of quadratic phase errors for this purpose appears in [4], for a particular maneuver. More recent approaches include “CLEAN” [5], [6], [7]; Basis Pursuit Denoising [8]; and $\ell_1$-regularized sparse reconstruction followed by an iterative model and subtract (IMAS) step [9]. These use computationally-intensive iterative algorithms that rely on strong implicit or explicit assumptions that the radar reflections are caused by a few discrete or canonical scatterers. In [10], 3D backprojection imaging is applied to discrete scatterers. And in [11], the shape of a curvilinear aperture is found which optimizes the horizontal and vertical beampattern of the aperture, under assumptions related to practicalities of drone operation.

In this paper, we instead approach a given curvilinear aperture as a phased-array monopulse antenna, using techniques related to [12] and [13]. We derive the curvilinear aperture monopulse (CLAM) computations for vertical offset in Section II. Specifically, we compute the offset in the vertical dimension of a single scatter near a focus point. The technique commutes with efficient image formation algorithms (much as monopulse commutes with beamforming), potentially allowing for fast computation. Section III demonstrates the computation for an aperture with a simple third-order polynomial vertical component and a monotonic horizontal component. A restriction analogous to focus arises in the context of interference in Section III-C. Section IV summarizes conclusions.

II. DERIVATION

This section develops CLAM equations for a curvilinear aperture, in a single-frequency setting. Of course, practical radar pulses have finite bandwidth to permit range discrimination. But many radar applications are sufficiently narrowband that within a compressed pulse envelope (perhaps with time-delay focusing), the single-frequency approximation is valid.

A. Single frequency signal model

A receiver moves along an aperture defined by

$$\begin{align*}
x &= x_0 + x_\tau + \Delta x \\
y &= y_0 + \Delta y \\
z &= z_0 + z_\tau + \Delta z,
\end{align*}$$

(1)

where $x_0, y_0, z_0$ are known fixed offsets, and where $x_\tau, z_\tau$ describe the aperture as $\tau$ varies. We require that $x_\tau$ and $z_\tau$ have three derivatives. $\Delta x, \Delta y, \Delta z$ are unknown fixed offsets that we intend to measure. Our final objective, elevation, is the dot product of $[\Delta x, \Delta y, \Delta z]^T$ with the ground plane normal unit vector. This normally depends primarily on $\Delta z$, so we will focus on $\Delta z$ from this point on. (Of course, the radar directly measures $\Delta y$, but $\Delta y$ is needed in the computation for reasons that appear in Section III-B.) Nonlinear $z_\tau$ specifies a curvilinear aperture. Nonlinear $x_\tau$ does not significantly complicate the following derivation, but we focus on the case of monotonic or linear $x_\tau$. To simplify this derivation, $y$ does not vary with $\tau$.

Along this aperture, we measure a single-frequency radiating scalar field given by

$$E = Ae^{jk(\frac{x}{R_\tau} - R_\tau)},$$

(2)

where $R_\tau = \sqrt{x^2 + y^2 + z^2}$ is the range from $x = y = z$ to the origin. Here, $E$ is the horizontal or vertical polarization of

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the electric or magnetic field, \( t \) is “fast time”, and \( \tau \) is “slow time”. Coupling between \( t \) and \( \tau \) is ignored, and we assume the platform does not move during the travel time of the radar pulse. Choose \( p = 1 \) if the transmitter is stationary or \( p = 2 \) if the transmitter moves along the aperture with the receiver.

For typical SAR applications, the variation in \( x_\tau \) is large enough relative to \( y \) that the plane wave approximation is not valid. However, the first order Taylor approximation \( \sqrt{1 + a} \approx 1 + \frac{1}{2} a \) is often sufficient, leading to

\[
2y R_\tau \approx 2y^2 + x^2 + z^2. \tag{3}
\]

From here forward, we accept (3) as sufficient and drop the approximation symbol.

B. System of equations

From (2), the derivative of \( E \) with respect to \( \tau \) is

\[
E' = -jkR'_\tau E. \tag{4}
\]

(The “prime” symbol will be used throughout to denote differentiation with respect to \( \tau \).) Taking the derivative of (3) with respect to \( \tau \) and expanding using (1),

\[
(y_0 + \Delta y) R'_\tau = x'_\tau (x_0 + x_\tau + \Delta x) + z'_\tau (z_0 + z_\tau + \Delta z). \tag{5}
\]

Define \( Q_\tau \) as the value of \( R_\tau \) if the unknown \( \Delta x, \Delta y, \Delta z \) are set to zero, and then

\[
y_0 Q'_\tau = x'_\tau (x_0 + x_\tau) + z'_\tau (z_0 + z_\tau). \tag{6}
\]

Substituting (6) into (5),

\[
(y_0 + \Delta y) R'_\tau = x'_\tau \Delta x + z'_\tau \Delta z + y_0 Q'_\tau, \tag{7}
\]

and with (4),

\[
(y_0 + \Delta y) E' = -jkE(x'_\tau \Delta x + z'_\tau \Delta z + y_0 Q'_\tau), \tag{8}
\]

which is linear in \( \Delta x, \Delta y, \) and \( \Delta z \):

\[
y_0 (E' + jkQ'_\tau E) = -jkE x'_\tau \Delta x - E' \Delta y - jkE z'_\tau \Delta z. \tag{9}
\]

Differentiating (9),

\[
y_0 (E'' + jk(Q'_\tau E)') = -jkE x'_\tau \Delta x - E'' \Delta y - jk(E z'_\tau)' \Delta z, \tag{10}
\]

and differentiating (10),

\[
y_0 (E''' + jk(Q'_\tau E)')' = -jkE x'_\tau \Delta x - E''' \Delta y - jk(E z'_\tau)'' \Delta z. \tag{11}
\]

Now (9), (10), and (11) are a system of three linear equations in \( \Delta x, \Delta y, \) and \( \Delta z \). If we know the aperture, then \( y_0 \), as well as \( x_\tau, z_\tau, Q_\tau \), and their derivatives are known. It remains to compute or measure \( E', E'', \) and \( E''' \).

C. Scalar field derivatives

Direct measurement of the first three derivatives of the scalar field is unreasonable in many settings. But we can estimate the derivative across the aperture as follows.

Let \( w \) be a finite-length discrete window function with support over the slow-time aperture. Let \( h \) be the single-frequency backprojection function for a hypothesized scatterer at \( (x_0, y_0, z_0) \):

\[
h(\tau) = e^{-jkQ_\tau}. \tag{12}
\]

In the azimuthal direction, backprojection image formation of a pixel at \( (x_0, y_0, z_0) \), with a window, is given by the integral

\[
\int_{-T}^{T} w(\tau) h(\tau) E d\tau,
\]

where \( \pm T \) are the values of \( \tau \) specifying the beginning and end of the aperture.

For convenience, we will write \( h_w(\tau) \equiv w(\tau) h(\tau) \). Using integration by parts, if \( w(\tau) \) is zero except on \( [-T, T] \), and if its first three derivatives exist everywhere, then

\[
\begin{align*}
\int_{-T}^{T} h_w(\tau) E' d\tau &= -\int_{-T}^{T} h'_w(\tau) E d\tau \\
\int_{-T}^{T} h_w(\tau) E'' d\tau &= +\int_{-T}^{T} h''_w(\tau) E d\tau \\
\int_{-T}^{T} h_w(\tau) E''' d\tau &= -\int_{-T}^{T} h'''_w(\tau) E d\tau.
\end{align*}
\tag{13}
\]

Following [13], if \( w \) is chosen to have the form

\[
w(\tau) = w_{\text{base}}(\tau) * (\delta_{-\frac{1}{2}s} + \delta_{\frac{1}{2}s})
\]

(where \( \delta_s \) is the Dirac delta shifted to \( u \)), then for small \( s \), its derivative is approximated by

\[
w_1(\tau) = w_{\text{base}}(\tau) * \frac{1}{s^3} (\delta_{-\frac{1}{2}s} - \delta_{\frac{1}{2}s}).
\]

Applying this approximation multiple times, if

\[
w_0(\tau) = w_{\text{base}}(\tau) * (\delta_{-\frac{1}{2}s} + 3\delta_{-\frac{1}{2}s} + 3\delta_{\frac{1}{2}s} + \delta_{\frac{3}{2}s})
\]

\[
w_1(\tau) = w_{\text{base}}(\tau) * \frac{1}{s^3} (\delta_{-\frac{3}{2}s} + \delta_{-\frac{1}{2}s} - \delta_{\frac{1}{2}s} - \delta_{\frac{3}{2}s})
\]

\[
w_2(\tau) = w_{\text{base}}(\tau) * \frac{1}{s^7} (\delta_{-\frac{3}{2}s} - \delta_{-\frac{1}{2}s} - \delta_{\frac{1}{2}s} + \delta_{\frac{3}{2}s})
\]

\[
w_3(\tau) = w_{\text{base}}(\tau) * \frac{1}{s^9} (\delta_{-\frac{3}{2}s} - 3\delta_{-\frac{1}{2}s} + 3\delta_{\frac{1}{2}s} - \delta_{\frac{3}{2}s})
\]

(where * denotes convolution), then the \( i \)th derivative of \( w \) is approximated by \( w_i \) for small \( s \), for \( i = 0, 1, 2, 3 \). Turning attention to derivatives of the narrowband backprojection function,

\[
h'(\tau) = -jkQ_\tau h(\tau)
\]

\[
h''(\tau) = ((jkQ_\tau)^2 - jkQ_\tau^2) h(\tau)
\]

\[
h'''(\tau) = -(jkQ_\tau)^3 + 3(jk)^2 Q_\tau^2 Q_\tau^2 - jkQ_\tau^3) h(\tau)
\]
and inserting the approximations in (14),

\[
\begin{align*}
&h_w'(\tau) \approx w_1(\tau)h(\tau) + w_0(\tau)h'(

&h_w''(\tau) \approx w_2(\tau)h(\tau) + 2w_1(\tau)h'(\tau) + w_0(\tau)h''(\tau) \\
&h_w'''(\tau) \approx w_3(\tau)h(\tau) + 3w_2(\tau)h'(\tau) + 3w_1(\tau)h''(\tau) + w_0(\tau)h'''(\tau).
\end{align*}
\]

We can then compute integrals of each side of (9), (10), and (11), and

\[
M \Delta r = \begin{bmatrix}
M_{00} & M_{01} & M_{02} \\
M_{10} & M_{11} & M_{12} \\
M_{20} & M_{21} & M_{22}
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix} =
\begin{bmatrix}
b_0 \\
b_1 \\
b_2
\end{bmatrix} = b, \quad (17)
\]

where

\[
M_{00} = -jk \int_{-T}^{T} h_w(\tau)x'_r E d\tau,
\]

\[
M_{01} = - \int_{-T}^{T} h_w'(\tau) E d\tau,
\]

\[
M_{02} = -jk \int_{-T}^{T} h_w(\tau)z'_r E d\tau,
\]

\[
M_{10} = -jk \int_{-T}^{T} h_w'(\tau)x'_r E d\tau,
\]

\[
M_{11} = - \int_{-T}^{T} h_w''(\tau) E d\tau,
\]

\[
M_{12} = -jk \int_{-T}^{T} h_w'(\tau)z'_r E d\tau,
\]

\[
M_{20} = -jk \int_{-T}^{T} h_w''(\tau)x'_r E d\tau,
\]

\[
M_{21} = - \int_{-T}^{T} h_w'''(\tau) E d\tau,
\]

\[
M_{22} = -jk \int_{-T}^{T} h_w''(\tau)z'_r E d\tau,
\]

\[
b_0 = y_0 \int_{-T}^{T} (h_w(\tau) + jk h_w(\tau)Q') E d\tau,
\]

\[
b_1 = y_0 \int_{-T}^{T} (h_w''(\tau) + jk h_w'(\tau)Q') E d\tau,
\]

\[
b_2 = y_0 \int_{-T}^{T} (h_w'''(\tau) + jk h_w''(\tau)Q') E d\tau.
\]

For apertures critically sampled uniformly in \( \tau \) at a sample rate of \( s \), the integrals above can be computed as sums. When \( M \) is invertible, \( \Delta r = M^{-1}b \). The third element of \( \Delta r \) is \( \Delta z \).

D. Computation using efficient image formation algorithms

The integral in each \( M_{ij} \) term and each \( b_i \) term in (17), together with \( h_0 \), is the azimuth portion of a backprojection image computation, for a single pixel, for the scalar field \( E \), under some window function. For example, for \( M_{00} \), the window is \(-jk w_0(\tau)x'_r\). An efficient image formation technique like Polar Format Algorithm commutes with the operations above, and is a good approximation for backprojection. So in principle, \( M \) and \( b \) can be efficiently computed for all pixels in a scene, by performing an image formation operation for each \( M_{ij} \) and each \( b_i \). We will see in Section III-C that interference can corrupt pixels that are not notably brighter than their neighbors.

For sufficiently large images, the approximation of (3) may become invalid for large \( x \) or \( z \). In this case, it may be necessary to vary \( x_0 \) and \( z_0 \) to keep the approximation valid. Handling this situation, while leveraging an efficient image formation technique, is beyond the scope of this paper.

III. EXAMPLES

A. Asymmetric third-order polynomial aperture

Consider the third-order polynomial aperture pictured in Fig. 1. Its length is 55.5m, its height is 0.5m, and it has zero crossings at -25.54m, 5.55m, and 25.54m. Assume a center frequency of 9 GHz, and a range of 1km. A linear aperture of this length would have a horizontal resolution of 0.3m. Choose a hann window squared for \( w_{\text{base}}(\tau) \). Using a single-frequency point-scatterer simulation, setting \( \Delta x = \Delta y = 0 \), the computed vertical position matches the correct position with \( \Delta z \) out to and somewhat beyond the dashed lines at \( \pm 16.7m \), which are the edges of vertical resolution of a hypothetical rectangular aperture circumscribing the curve. Noise was added to each time sample, at an amplitude of 10% of that of the signal. Varying \( \Delta x \) over the horizontal resolution of a linear aperture of this length (\( \pm 0.15m \)), and varying \( \Delta y \) over the same amount (as might be expected for a SAR system), the error in computed \( \Delta z \) is shown in Fig. 3. So the approximations we have introduced are capable of measuring \( \Delta z \) to a high degree of accuracy in this setting, varying \( \Delta x, \Delta y \) over the space of a SAR pixel, for a single scatterer, with a small amount of noise.

B. Parabolas aperture ambiguities

For a symmetric second-order polynomial aperture, either a height offset or a small range offset causes similar wavefronts to reach the aperture. For the full computation of (17), the ambiguity causes an ill-conditioned matrix. But if we make the assumption that \( \Delta y \) is zero, we can modify (17) to remove both \( \Delta y \) and (11), leaving

\[
\begin{bmatrix}
M_{00} & M_{02} \\
M_{10} & M_{12}
\end{bmatrix}
\begin{bmatrix}
\Delta x \\
\Delta z
\end{bmatrix} =
\begin{bmatrix}
b_0 \\
b_1
\end{bmatrix}. \quad (18)
\]

A symmetric second-order polynomial aperture is shown in Fig. 4. The aperture height and length are the same as that of Fig. 1. Using (18), measured \( \Delta z \) varies with small range offsets, as shown in Fig. (5). So the impact of dropping \( \Delta y \).
Fig. 1. A third-order polynomial aperture of length 55.5m and height 0.5m; with zeros at -25.54m, 5.55m, and 25.54m.

Fig. 2. Computed $\Delta z$ vs. correct $\Delta z$. The computation is accurate, and is not unreasonably subject to small amounts of noise.

Fig. 3. Error in computed $\Delta z$, varying $\Delta x$ and $\Delta y$, using (17). The computation is accurate within the vertical resolution of a circumscribing rectangle (marked by dashed lines), and is not unreasonably subject to small amounts of noise or to variations in $\Delta x$ and $\Delta y$.

Fig. 4. A second-order polynomial aperture of length 55.5m and height 0.5m; with zeros at $\pm$ 19.63m.

Fig. 5. Error in computed $\Delta z$, for the second-order polynomial aperture of Fig. 4, caused by small offsets in actual $\Delta y$, using (18). Small errors due to noise are also present.

is that the vertical position estimate varies with small range offsets. The full CLAM computation using the aperture of Fig. (1) is immune to this ambiguity, as shown in (6). In certain applications and for certain apertures, the advantage of using a parabolic aperture may outweigh the impact of this ambiguity. In the case pictured, the vertical error is comparable to the size of the range resolution, which is typically comparable to the azimuthal resolution. But for some applications, this may be acceptable. Larger vertical apertures yield smaller ambiguities, for the same horizontal aperture. See Fig. (7) for results varying $\Delta x$, $\Delta y$ as in Fig. (3) The size of the errors are comparable for these parameters.

C. Interference and glint

The computation for $\Delta x$, $\Delta y$, $\Delta z$ in (17) is nonlinear in the scalar field $E$, because $E$ is present in all terms of $M$ and $b$ in (17). For this reason, if multiple scatterers contribute to $E$, the computed values of $\Delta x$, $\Delta y$, $\Delta z$ are not a linear combination of the values that would be computed in the single-scatterer
Fig. 6. Error in computed $\Delta z$, for the third-order polynomial aperture of Fig. 1, using (17). Small errors are present due to noise, but offsets in actual $\Delta y$ do not correlate with errors in measured $\Delta z$.

Fig. 7. Error in computed $\Delta z$, for the second-order polynomial aperture of Fig. 4, using (18), varying $\Delta x$ and $\Delta y$. The computation is accurate within the vertical resolution of a circumscribing rectangle, and is not unreasonably subject to small amounts of noise. But some systematic error is present.

Fig. 8. Error in computed $\Delta z$ due to confuser scatterers. Both the central scatterer and the confuser scatterer are at $\Delta z = 0$, but the confuser scatterer position varies over half the wavelength. The dashed line indicates the horizontal resolution of the aperture. The confuser scatterer can cause large errors.

Consider a scatterer at $\Delta x = 0, \Delta y = 0, \Delta z = 0$. If a “confuser” scatter is placed at the same $\Delta z = 0$, but at some other $\Delta x, \Delta y$, then the measured $\Delta z$ is not necessarily near 0. For the aperture of Fig. (1), the size of the error is shown in Fig. 8. $\Delta y$ is varied over half of a wavelength, because the phenomenon appears to be caused by the phase difference of the confuser. The horizontal resolution width is marked by dashed lines. The error is small for most phases, for most confuser scatterers within the resolution width. So a confuser scatterer in the same pixel is likely to result in the correct measurement. For confuser scatterers several resolution widths away, the directional effects of the backprojection function $h(\tau)$ rejects the energy from the confuser, and the measurement is approximately correct. However, for confuser scatterers only a few resolution widths away, the error can be quite large. This is because the beampatterns of many of the terms in (17) are wider than a resolution cell. Information from adjacent resolution cells are needed to make the measurement correctly. These computations will therefore not be valid in the sort of broad areas of uniform backscatter that are characterized by coherent speckle, and that perform so well under interferometric processing of pairs of images [1].

However, SAR images often have scatterers that are notably brighter than their azimuthal surroundings. Autofocus algorithms depend on this characteristic of SAR images. And the prevalence of such scatterers is what necessitates apodization methods (e.g. linear windowing or nonlinear spatially-variant apodization [14]). The vertical position of these bright pixels may potentially be computed using CLAM processing, and only a single image is required.

It may be difficult to guarantee that only a single scatterer is represented in a location, in a sensing context. We need a method to detect when a measurement is corrupted by “glint” from other scatterers. The determinant from the left side of (17) is shown in Fig. 9. Where this determinant is large, the glint error in Fig. 8 is small. This suggests that small determinants may be used to detect measurements corrupted by interference from other scatterers.

IV. Conclusion

Using computations given in this paper, with a curvilinear aperture, one can compute elevation of a single scatterer, provided the matrix $M$ from (17) is invertible. The monopulse-based computations are non-iterative, and commute with efficient image formation algorithms, potentially permitting fast computation. A nearby confuser scatterer of similar amplitude can corrupt the measurement, so elevation will only be available for pixels that are notably brighter than their neighbors. This is analogous to the focus assumption in other work on
curvilinear apertures, in that the concept of “focus” implies that for the correct image, certain pixels are notably brighter than their neighbors. For a pixel subject to this corruption, the determinant of $M$ is small, serving as a warning.

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