Spacetime correlators of perturbations in slow-roll de Sitter inflation

Adrian del Rio and Jose Navarro-Salas
Departamento de Fisica Teorica and IFIC, Centro Mixto Universidad de Valencia-CSIC. Facultad de Fisica, Universidad de Valencia, Burjassot-46100, Valencia, Spain.
(Dated: January 27, 2014)

Two-point correlators and self-correlators of primordial perturbations in quasi-de Sitter spacetime backgrounds are considered. For large separations two-point correlators exhibit nearly scale invariance, while for short distances self-correlators need standard renormalization. We study the deformation of two-point correlators to smoothly match the self-correlators at coincidence. The corresponding angular power spectrum is evaluated in the Sachs-Wolfe regime of low multipoles. Scale invariance is maintained, but the amplitude of $C_\ell$ could change in a non-trivial way.

Key Words: quantum field theory in curved spacetime, correlators, asymptotic behaviour, renormalization, cosmology.

PACS numbers: 04.62.+v, 98.80.Cq

I. INTRODUCTION

In recent years de Sitter space has received considerable attention. Astronomical observations are pointing out that our universe has now a very tiny positive cosmological constant which, however, embody around three quarters of the energy of the observable universe. Moreover, according to inflationary cosmology the very early universe underwent a period of very rapid expansion powered by a large effective cosmological constant. The discovery of the anisotropies in the cosmic microwave background (CMB) constitutes a very sensitive probe of the primordial density perturbations and its quantum mechanical origin. The comparison of observations with theoretical predictions is currently a sharp tool to test inflation and the theory of quantized field in curved backgrounds. Therefore, a precise understanding of the quantum properties of fields in de Sitter space is fundamental for both the very early and the late-time universe.

In this note we will focus on the quantum treatment of primordial perturbations, which will be regarded as quantum fields $\phi$ living in a curved (quasi-de Sitter) spacetime. The two-point correlation function $\langle \phi(t, \vec{x})\phi(t, \vec{x}') \rangle$ exhibits scale invariance at large separations $|\vec{x} - \vec{x}'|H > 1$ or, equivalently, at late-time $Ht >> 1$. On the other hand, the amplitude of the perturbation at a given spacetime point can be quantified by the self-correlator $\langle \phi^2(t, \vec{x}) \rangle$, which requires to get rid off the corresponding ultraviolet (UV) divergences and renormalize the expectation value.

In the first part of this work we will compute and analyze the above quantities in a slow-roll de Sitter background. We will also project the large-distance behavior of the correlator of scalar perturbations on a sphere of fixed radius. This sphere is linked by time evolution to the last scattering surface, where the cosmic microwave background and its anisotropies are formed, and the angular power spectrum can be easily obtained within this spacetime picture. In the second part we will redo the calculation for the angular power spectrum by using a deformed two-point correlator. The new correlator is defined in such a way that it matches the self-correlator at coincidence. To this end we naturally use methods of renormalization in homogeneous backgrounds. The revised angular power spectrum maintains the nearly scale invariance, but the amplitude of the multipole coefficients $C_\ell$ may be altered in a non-trivial way. We will focus on the lower multipoles, where the Sachs-Wolfe effect dominates.

II. SPACETIME CORRELATORS IN SLOW-ROLL INFLATION

A. Correlator of tensorial perturbations in a slow-roll scenario

Tensorial perturbations can be described by two independent, massless scalar fields propagating in the unperturbed quasi de Sitter background. These two scalar fields represent the two independent polarization components of the fluctuation tensorial modes $D_{ij}$ in the inflationary universe: $ds^2 = dt^2 - a^2(t)(\delta_{ij} + D_{ij})dx^i dx^j$. Expanding the fluctuating fields $D_{ij}$ in plane wave modes $D_k(t)e_{ij}e^{ik}$, where $e_{ij}$ is a constant polarization tensor obeying the conditions $e_{ij} = e_{ji}, e_{ij} = 0$ and $k_i e_{ij} = 0$, one obtains the equation $\ddot{D}_k + 3H\dot{D}_k + \frac{k^2}{a^2}D_k = 0$, with $k \equiv |k|$ and $H = \dot{a}/a$. The conditions for the polarization tensor imply that the perturbation field $D_{ij}$ can be decomposed into two polarization states described by a couple of massless scalar fields $D_{ij} = D_+ e_{ij}^+ + D_\times e_{ij}^\times$, where $e_{ij}^+ e_{ij}^\times = 2\delta_{sr} (s = +, \times$ stands for the two independent polarizations), both obeying the above wave equation (see, for instance, [6]). For simplicity we omit the subindex $+$ or $\times$.

In the slow-roll approximation one assumes that the Hubble parameter $H(t)$ changes very gradually and the change is parameterized by a slow-roll parameter $\epsilon \equiv$
$-\dot{H}/H^2 \ll 1$. Within this approximation it is possible to solve the wave equation in a closed form in terms of the conformal time $\eta \equiv \int dt/a(t)$. Taking into account that $(1-\epsilon)\eta = -\frac{1}{aH}$, the wave equation for $D_k$ turns out to be of the form

$$\frac{d^2D_k}{d\eta^2} - \frac{2}{\eta(1-\epsilon)} \frac{dD_k}{d\eta} + k^2D_k = 0.$$  \hfill (1)

Treating now the parameter $\epsilon$ as a constant, one can univocally solve the above equation with the requirement of recovering, for $\epsilon \to 0$, the Bunch-Davies vacuum [5]. The properly normalized solutions for the modes are

$$D_k(t) = \frac{\sqrt{16\pi G}}{\sqrt{2(2\pi)^3 a^3}}(-\eta a \pi/2)^{1/2} H_\nu^{(1)}(-k\eta),$$  \hfill (2)

where $G$ is the Newton constant and the index of the Hankel function is exactly $\nu = \frac{3}{2} + \frac{1}{1-\epsilon}$. Having the explicit form of the modes we can now compute the two-point function. At equal times $t = t'$ we find

$$\langle D(t, \vec{x})D(t, \vec{x}') \rangle \sim \frac{4GT(3/2 - \nu)}{\pi^{3/2} a^3 \eta^2} \left(\frac{\Delta x}{-\eta}\right)^{2(\nu - 3/2)}.$$  \hfill (3)

One can immediately observe that the amplitude above is nearly scale-invariant. Moreover the term $(-\eta)^{1-2\nu}/a^2$ is time-independent, which allow us to evaluate it at the most convenient time. In fact, the correlator can be rewritten as

$$\langle D(t, \vec{x})D(t, \vec{x}') \rangle \sim -\frac{16\pi G}{2\epsilon} \left(\frac{H(t_{\Delta x})}{2\pi}\right)^2,$$  \hfill (4)

where the time $t_{\Delta x}$ is defined as $a(t_{\Delta x}) \Delta x = H^{-1}(t_{\Delta x})$. Note that there is an implicit $\Delta x$-dependence on $H(t_{\Delta x})$, given by the one in [4].

**B. Correlator of scalar perturbations in a slow-roll scenario**

Scalar perturbations can be studied through the gauge-invariant field $R$ (the comoving curvature perturbation; see for instance [7]). For single-field inflation, the modes of the scalar perturbation are given by

$$R_k(t) = (-\pi\eta/4(2\pi)^3 z^2)^{1/2} H^{(1)}_\nu(-\eta k),$$  \hfill (6)

where now $\nu = 3/2 + (2\epsilon + \delta)/(1-\epsilon)$ and $\delta \equiv H/2H\dot{H}$ is a second slow-roll parameter. Moreover, $z \equiv a\phi_0/H$, where $\phi_0(t)$ is the homogeneous part of the inflaton field. These modes determines the vacuum state of scalar perturbations. Such a state can also be regarded as the natural extension of the Bunch-Davies vacuum of de Sitter space. The corresponding two-point function $\langle R(t, \vec{x}), R(t', \vec{x}') \rangle$ is given by

$$\langle R(t, \vec{x})R(t, \vec{x}') \rangle \sim \frac{1}{16\pi^2 z^2 \eta^2} \Gamma\left(\frac{3}{2} + \nu\right) \Gamma\left(\frac{3}{2} - \nu\right) \left(\frac{\Delta x}{-\eta}\right)^{2(\nu - 3/2)}.$$  \hfill (7)

For separations larger than the Hubble radius $a|\vec{x} - \vec{x}'| \gg H^{-1}$ we get

$$\langle R(t, \vec{x})R(t, \vec{x}') \rangle \sim \frac{4\pi G}{(1-n)\epsilon} \left(\frac{H(t_{\Delta x})}{2\pi}\right)^2.$$  \hfill (8)

This expression can be rewritten, assuming $\nu - 3/2 \equiv (1-n)/2 \approx 0$ [n is the scalar spectral index], as

$$\langle R(t, \vec{x})R(t, \vec{x}') \rangle \sim -\frac{4\pi G}{(1-n)\epsilon} \left(\frac{H(t_{\Delta x})}{2\pi}\right)^2.$$  \hfill (9)

**C. Angular power spectrum**

Restricting the two-point function of scalar perturbations to points such that $|\vec{x}| = |\vec{x}'|$ we can further obtain

$$\Delta x^{1-n} = 2^{\frac{2-n}{2}} |\vec{x}|^{(1-n)(1-\cos \theta)^{(1-n)/2}}$$

where $\theta$ is the angle formed by $\vec{n} = \vec{x}/|\vec{x}|$ and $\vec{n}' = \vec{x}'/|\vec{x}|$. Then, taking $|\vec{x}| = r_L$, where $r_L$ is the comoving radial coordinate of the last scattering surface

$$r_L = H(t_0)^{-1}a(t_0)^{-1} \int_{\frac{1}{1+z_L}}^{1} \frac{dx}{\sqrt{\Omega_L x^4 + \Omega_M x + \Omega_R}},$$  \hfill (10)

with the standard cosmological values for $z_L$, $\Omega_L$, $\Omega_M$ and $\Omega_R$, [6], the correlator of scalar perturbations for large separations [6] shows exactly

$$\langle R(t, \vec{x})R(t, \vec{x}') \rangle \sim -\frac{4\pi G}{\epsilon} H^2(1-\epsilon)^2 \frac{H^2(1-\epsilon)^2}{16\pi^2} \frac{4\Gamma(2 - n)}{\sqrt{\pi}} \frac{\Gamma\left(n - \frac{1}{2}\right)}{\Gamma\left(n - \frac{1}{2}\right)} \left(\frac{r_L}{1+z_L}\right)^{1-n} \Gamma\left(\frac{1-n}{2}\right)^2$$  \hfill (11)

where we have defined the dimensionless quantity $\tilde{r}_L(t) \equiv H(1-\epsilon)a\tilde{r}_L$. This two-point function can be related to the temperature fluctuations in the CMB

$$\langle \Delta T(\vec{n})\Delta T(\vec{n}') \rangle = \sum_\ell C_\ell \frac{2\ell + 1}{4\pi} P_\ell(\cos \theta),$$  \hfill (12)

where $P_\ell$ is the Legendre polynomial, via the Sachs-Wolfe effect (see, e.g., [2])

$$\langle \Delta T(\vec{n})\Delta T(\vec{n}') \rangle_{SW} = \frac{T_d^2}{25} \langle R(r_L\vec{n})R(r_L\vec{n}') \rangle.$$  \hfill (13)
The above self-correlators quantify the amplitude of perturbations at a given space-time point.

IV. MODIFIED CORRELATORS AND ANGULAR POWER SPECTRUM

In previous sections we have studied the correlator \( \langle \phi(t, \vec{x}) \phi(t, \vec{x}') \rangle \) and self-correlator \( \langle \phi^2(t, \vec{x}) \rangle \) of tensorial and scalar perturbations in slow-roll inflation. For a ordinary quantum mechanical system, with a finite number \( N \) of degrees of freedom, expectation values of the form \( \langle \phi(i) \phi(j) \rangle \) and \( \langle \phi^2(i) \rangle \) match when \( j = i \) (for instance, in a chain of spins with \( \phi(i) \equiv S_i(i) \) and \( i = 1, \ldots, N \)). However, we are facing here a field theory (with an infinite number of degrees of freedom) and the above matching is not a priori guaranteed. This is so because the self-correlator requires renormalization. We may either

\[
\langle \phi(t, \vec{x}) \phi(t, \vec{x}') \rangle_{\text{ren}} \equiv \langle \phi(t, \vec{x}) \phi(t, \vec{x}') \rangle - G_{\text{Ad}}^{(2)}((t, \vec{x}), (t, \vec{x}'))
\]

and taking the limit \( \vec{x}' \to \vec{x} \). The method determines univocally the subtraction terms, which are found to be

\[
G_{\text{Ad}}^{(2)}((t, \vec{x}), (t, \vec{x}')) = \frac{H^2(1 - \epsilon)^2}{16\pi^2} \left\{ \frac{4}{\Delta \vec{x}^2} + \left( \frac{1}{4} - \nu^2 \right) \log \Delta \vec{x}^2 + \left( \frac{3}{16} - \nu^2 \right) \right\} ,
\]

where \( \mu \) is a renormalization scale and the corresponding prefactor mentioned above for scalar or tensorial perturbations must be considered. We observe immediately that the UV divergences cancel exactly and we are left with

\[
\langle \phi^2(t, \vec{x}) \rangle_{\text{ren}} = \frac{H^2(1 - \epsilon)^2}{16\pi^2} \left\{ \left( \frac{1}{4} - \nu^2 \right) \left( -1 + \psi(3/2 - \nu) + \psi(3/2 + \nu) - \log \frac{\mu^2}{H^2(1 - \epsilon)^2} \right) - \frac{2 - \epsilon}{3(1 - \epsilon)^2} \right\} .
\]
assume this discontinuity or modify the two-point correlation function to force it to match \( \langle \phi^2(t, \vec{x}) \rangle \) in the coincidence limit \( \vec{x} \rightarrow \vec{x}' \). This second possibility was indirectly explored in \cite{14} by analyzing the power spectrum of perturbations in momentum space. We further analyze here this possibility taking advantage of the spacetime viewpoint sketched above.

Therefore, we shall modify the correlators by adding the subtraction terms prescribed by renormalization and according to \cite{19}. We note that a distinguishing characteristic of adiabatic renormalization is that the subtraction terms \( G^{(2)}_{Ad}(t, \vec{x}, (t, \vec{x}')) \) are well-defined for arbitrary point separation. In general this is not possible for an arbitrary spacetime, but for the homogeneous spaces relevant in cosmology the adiabatic subtraction terms extent to arbitrary large distances. With this in mind, we will finally compute the angular power spectrum for primordial perturbations using the modified spacetime correlators.

As a previous step we will compute the two-point function at leading order in slow-roll.

### A. Two-point function at leading order in slow-roll

The procedure is similar for scalar and tensorial fluctuations, so we will do a general treatment. First, we start off splitting equations \cite{8} and \cite{7} as a combination of two hypergeometric functions. To this end we use the transformation properties of hypergeometric functions \cite{17}.

\[
F \left( \frac{3}{2} + \nu, \frac{3}{2} - \nu, 2, 1 - Z \right) = \frac{Z^{-\nu} \Gamma(-2\nu)}{\Gamma(\frac{3}{2} - \nu) \Gamma(\frac{3}{2} + \nu)} \text{Re} \left\{ F \left( \frac{3}{2} + \nu, 1 + 2\nu, 1 - Z \right) \right\} 
+ \frac{Z^{\nu} \Gamma(2\nu)}{\Gamma(\frac{3}{2} + \nu) \Gamma(\frac{3}{2} + \nu)} \text{Re} \left\{ F \left( \frac{3}{2} - \nu, 1 - 2\nu, 1 - Z \right) \right\} 
(22)
\]

with \( Z = \Delta \bar{x}^2 / 4 \geq 0 \). We now expand expression \cite{22} as a power series of the “slow-roll” parameter \( \nu \) around \( \nu = 3/2 \), and stay at first order (for details see the appendix). Grouping terms we arrive at the following expression for the two-point function

\[
\langle \phi(x) \phi(x') \rangle \approx \frac{H^2(1-\nu)^2}{16\pi^2} \left\{ \frac{4}{\Delta \bar{x}^2} - 2 \log \Delta \bar{x}^2 - 1 + \frac{2}{(3/2-\nu)} \left( \Delta \bar{x}^2 \right)^{-3/2} + 4 \text{Re} \left[ \log \left( \frac{\Delta \bar{x}}{2} + \sqrt{\Delta \bar{x}^2 - 1} \right) \right] \right\} 
(23)
\]

Notice that the UV divergences are just the same as those found in \cite{18}, but now they are obtained at leading order in the slow-roll expansion. We recover exactly expression \cite{18} taking the limit \( \Delta \bar{x} \rightarrow 0 \) and the slow-roll approximation.

### B. Modified two-point function

We can now proceed to do the subtraction. The modified two-point function then reads

\[
\langle \phi(x) \phi(x') \rangle_{\text{ren}} \approx \frac{H^2(1-\nu)^2}{16\pi^2} \left\{ \frac{2}{\Delta \bar{x}^2} \left( \Delta \bar{x}^2 \right)^{-3/2} + 4 \text{Re} \left[ \log \left( \frac{\Delta \bar{x}}{2} + \sqrt{\Delta \bar{x}^2 - 1} \right) \right] - \frac{\gamma}{2} + 4\gamma + 2 \log \frac{3}{4} \right\} 
. \quad (24)
\]

We remark that, at leading order in the slow-roll expansion, this is an expression valid for small and large separations. For scales larger than the Hubble horizon, \( \Delta \bar{x} >> 1 \), we can further take the approximation,

\[
4 \text{Re} \left[ \log \left( \frac{\Delta \bar{x}}{2} + \sqrt{\Delta \bar{x}^2 - 1} \right) \right] \approx 2 \log(\Delta \bar{x}^2).
\]

### C. Angular power spectrum

We now compute the corresponding angular power spectrum from the modified two-point function for scalar perturbations and for low multipoles

\[
C^{SW}_\ell \equiv \frac{2\pi T_e^2}{25} \int_{-1}^{1} \langle R(x)R(x') \rangle_{\text{ren}}(y)P_\ell(y)dy \quad . \quad (25)
\]

By construction this is a finite quantity, even without taking the large separation limit for the two-point function. To evaluate the logarithmic contributions of \cite{22} to \cite{25} we take into account that \( \int_{-1}^{1} dy \log(1 - y)P_\ell(y) = -2/\ell(\ell + 1), \ell = 1, 2, \ldots \). The final result for the angular power spectrum with the modified two-point function is very well approximated by the following analytical ex-
pression
\[ C_{l}^{SW} \approx \frac{4\pi G 8\pi T_{0}^{2}}{\epsilon} \frac{H^{2}(1-\epsilon)^{2}}{25} \times \left\{ \bar{r}_{L}^{3-n} \left( \frac{\Gamma(\ell + \frac{n-1}{2})}{\Gamma(\ell + 2 - \frac{n-1}{2})} + \frac{1}{\ell(\ell + 1)} \right) \right\} , \tag{26} \]

where we have used \( \nu - \frac{3}{2} = \frac{1-n}{2} \), and \( n \) represents the scalar index of inflation \( n = 1 - 4\epsilon - 2\delta + O(\epsilon, \delta)^{2} \). Also notice that expression \( \bar{r}_{L}^{3-n} \) is valid for \( \ell \geq 1 \), as for \( \ell = 0 \) there would be present all the constant contributions from the renormalized two-point function \( [23] \), including the one depending on the renormalization scale. In fact, the renormalization scale may be fixed by imposing the natural condition \( C_{0}^{SW} = 0 \).

Notice that the first term in \( \bar{r}_{L}^{3-n} \) reproduces the standard result \( \frac{3}{2} \nu - 1 \nu \). The second one comes from the subtraction terms that we have added to the two-point correlator to continuously match the self-correlator at coincidence, but it shows scale-invariance as well. Therefore, it is consistent with observations \([3]\).

The amplitude of the first term can be easily estimated by taking into account that \( H^{2}(t)_{\ell}^{(1-n)}(t)/\epsilon(t) \) is time-independent, and that \( \bar{r}_{L}^{3-n} \) is of order \( O(1) \). This implies that the second term in \( \bar{r}_{L}^{3-n} \) may correct the amplitude in an amount that depends on the period of cosmic evolution at which the primordial quantum perturbations behave as classical perturbations \([12]\). If the quantum-to-classical transition happens at late times in the cosmic evolution the contribution of the new term in \( \bar{r}_{L}^{3-n} \) becomes negligible at observational scales. However, if the transition happens during inflation, the correction to the amplitude could be significant. We can see this with a simple reasoning. The general condition \( C_{l} \geq 0 \) defines the instant of time \( \ell_{t} \) in the inflationary epoch \( a(t_{i})r_{L} \approx H^{-1}(t_{i}) \) at which quantum effects generate relevant cosmological perturbations. Taking into account that \( H(t_{i})/H(t) \) is of order unity for inflation, we have \( \bar{r}_{L}^{3-n}(t) \approx a(t)/a(t_{i}) \) so that \( \bar{r}_{L} \approx e^{N} \) at the end of inflation. Assuming that inflation last for around \( N = 60 \) e-foldings since the scale \( r_{L} \) exited the horizon at \( t_{i} \) we have \( r_{L}^{3-n} \approx 10 \) at the end of inflation. We have assumed that \( n \approx 0.96 \). Therefore, the new term in \( \bar{r}_{L}^{3-n} \) reduces the amplitude of (low) multipoles \( C_{l}^{SW} \) in at least a 10%. If the quantum-to-classical transition takes place at an earlier state the correction may be even more important. Within the current limits of our understanding of quantum gravity it is difficult to determine the effective state of the cosmic evolution where this transition happens and hence the relative strength between the two competing terms of \( \bar{r}_{L}^{3-n} \).

V. CONCLUSIONS

We have analyzed two-point correlators and self-correlators of primordial perturbations in quasi-de Sitter spacetime backgrounds. For large separations two-point correlators exhibit nearly scale invariance in a very elegant way. We have deformed the two-point correlators to smoothly match the self-correlators at coincidence. To this end we have used renormalization methods in homogeneous backgrounds. We have studied the physical consequences for the angular power spectrum at low multipoles. Scale-invariance is maintained, but the amplitude of \( C_{l} \) could change significantly. Obviously, if one accepts a mismatch between the standard two-point correlators and the self-correlators the conventional predictions remain unaltered.

Acknowledgements. J. N-S. would like to thank I. Agullo, A. Fabbri and G. Olmo for very useful discussions. This work is supported by the Spanish grant FIS2011-29813-C02-02 and the Consolider Program CPANPHY-1205388.

APPENDIX

In this appendix we give the basic steps to obtain the result \( \bar{r}_{L}^{3-n} \). We consider \( \frac{3}{2} - \nu \) firstly. Since the first prefactor is of order \( O((\frac{3}{2} - \nu)^{1}) \), we only need the corresponding hypergeometric function to be of order \( O((\frac{3}{2} - \nu)^{0}) \). One can see that

\[ Z^{-\frac{3}{2} - \nu} Re \left\{ F \left( \frac{3}{2} + \nu, \frac{3}{2}, 1 + 2\nu, \frac{1}{Z} \right) \right\} |_{\nu = 3/2} = \tag{27} \]

\[ 6 Re \{ \log(Z - 1) - 6 \log(Z) + \frac{3}{Z} - \frac{3}{1-Z} \} \]

On the other hand, the second prefactor of \( \frac{3}{2} - \nu \) is of order \( O((\frac{3}{2} - \nu)^{0}) \), so it is necessary to evaluate the second hypergeometric function at first order in the slow-roll series. To this end we will employ the following relation \([17]\).

\[ F \left( \frac{3}{2} - \nu, \frac{1}{2} - \nu, 1 - 2\nu, \frac{1}{Z} \right) = \tag{28} \]

\[ \left( 1 - \frac{1}{Z} \right)^{-3/4} P_{1/2}^{\nu} \left[ \frac{2Z - 1}{2\sqrt{Z(Z - 1)}} \right] 2^{-2\nu}(1 - \nu)Z^{\nu}, \]

together with

\[ P_{1/2}^{\nu}(Z) = \left( \frac{Z + 1}{Z - 1} \right)^{\nu/2} F \left( \frac{-1 + 3}{2}, \nu + 1 - \nu \frac{1-Z}{Z} \right) \Gamma(1 - \nu). \tag{29} \]

At this point one can expand

\[ F \left( \frac{-1}{2}, \frac{3}{2}, 1 - \nu, \frac{1-Z}{2} \right) \approx \tag{30} \]

\[ F \left( \frac{-1}{2}, \frac{3}{2}, \frac{1-Z}{2} \right) + \left( \nu - \frac{3}{2} \right) \frac{dF}{d\nu} \left( \frac{-1 + 3}{2}, 1 - \nu, \frac{1-Z}{2} \right) |_{\nu = 3/2}, \]

where the derivative can be performed using the representation series of the hypergeometric function. Doing
all the calculation properly one finally arrives at the following result

\[
\text{Re} \left\{ F \left( \frac{3}{2} - \nu, \frac{1}{2} - \nu, 1 - 2\nu, \frac{1}{Z} \right) \right\} \approx 1 + \left( \frac{3}{2} - \nu \right) \left[ \frac{1}{4Z} + \frac{1}{4(1-Z)} - \frac{1}{2} \text{Re} \left\{ \log(Z - 1) \right\} - \frac{1}{2} \log(Z) \\
+ 2 \text{Re} \left\{ \log \left( \sqrt{Z} + \sqrt{Z - 1} \right) \right\} \right].
\]

Taking all these results together for \( Z \equiv \Delta \bar{x}^2 / 4 \) in (22), we can approximate the two-point function as in (23). We have also checked numerically that this expansion works well irrespectively of the value of \( Z \).

[1] A.G. Riess et al., Astron. J. 516, 1009 (1998). S. Perlmutter et al. Astron. J. 517, 565 (1999).

[2] Smoot, G., et al. Astrophys. J. Lett. 396, L1-L4 (1992).

[3] Mukhanov V.F. and Chibisov G.V., JETP Letters 33, 532(1981). Hawking, S. W., Phys. Lett. B115, 295 (1982). Guth A. and Pi, S.-Y., Phys. Rev. Lett. 49, 1110 (1982). Starobinsky, A. A., Phys. Lett. B117, 175 (1982). Bardeen, J.M., Steinhardt, P.J. and Turner, M.S., Phys. Rev.D28, 679 (1983).

[4] Planck Collaboration, Planck 2013 Results. XV. CMB power spectra and likelihood, arXiv:1303.5075 (2013).

[5] Parker L. and Toms D.J., Quantum field theory in curved spacetime: quantized fields and gravity, Cambridge University Press, (2009).

[6] Birrell N.D. and Davies P.C.W., Quantum fields in curved space, Cambridge University Press, (1982).

[7] Weinberg S., Cosmology, Cambridge University Press, (1972).

[8] T.S. Bunch and P.C.W. Davies, Proc. P. Soc. London A 360, 117 (1978).

[9] L. H. Ford and L. Parker, Phys. Rev. D 16, 245 (1977). A. Vilenkin and L. H. Ford, Phys. Rev. D 26, 1231 (1982). B. Allen, Phys. Rev. D 32, 3136 (1985). B. Allen and A. Folacci, Phys. Rev. D 35, 3771 (1987).

[10] L. Parker and S. A. Fulling, Phys. Rev. D 9, 341 (1974).

[11] A. Landete, J. Navarro-Salas and F. Torrenti, arXiv:1311.4958 [gr-qc], Phys. Rev. D (2014); Phys. Rev. D 88, 061501 (2013).

[12] We note that that for a free conformal-invariant field (like the electromagnetic one), the logarithmic subtraction term is absent and hence also the renormalization scale \( \mu \). The two-point correlator is diluted by inflation and only the self-correlator survives as a quantity with non-negligible amplitude [19].

[13] I. Aguillo and J. Navarro-Salas, Conformal anomaly and primordial magnetic fields, arXiv:1309.3435 [gr-qc].

[14] L. Parker, Amplitude of perturbations from inflation, hep-th/0702216. I. Aguillo, J. Navarro-Salas, G. J. Olmo and L. Parker, Phys. Rev. Lett. 103, 061301 (2009).

[15] A. R. Liddle and D. H. Lyth, Cosmological inflation and large-scale structure, Cambridge University Press, (2006).

[16] Gradsteyn I. S. and Ryzhik I. M., Table of integrals, series, and products, Academic Press, (1994).

[17] M. Abramowitz and I.A. Stegun, Handbook of mathematical functions, National Buerau of Standards, (1972).