The Fatigue Life Prediction of Train Wheel Rims Containing Spherical Inclusions

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Abstract. It is a common phenomenon that fatigue crack initiation occurs frequently in the inclusions of wheel rims. Research on the fatigue life of wheel rims with spherical inclusions is of great significance on the reliability of wheels. To find the danger point and working condition of a wheel, the stress state of the wheel rim with spherical inclusions was analyzed using the finite element method. Results revealed that curve conditions are dangerous. The critical plane method, based on the cumulative fatigue damage theory, was used to predict the fatigue life of the wheel rim and whether it contained spherical inclusions or not under curve conditions. It was found that the fatigue life of the wheel rim is significantly shorter when the wheel rim contains spherical inclusions. Analysis of the results can provide a theoretical basis and technical support for train operations and maintenance.

1. Introduction

Wheels are a key component to guarantee vehicle dynamics performance and the safety of trains. Compared with trucks and medium or low speed trains, Electric Multiple Units (EMU) can easily have safety problems when running because of their higher speed and more intense vertical vibration. To ensure the safe operation of trains, it’s essential to put forward higher requirements on wheel fatigue life and reliability. At present, the estimation methods for fatigue life mainly include the nominal stress method [1], the local stress strain method [2,3] and the critical plane method [4,5].

Numerous researchers have studied fatigue life prediction methods and related theories, and achieved a lot of results. In order to calculate the fatigue life of the train wheel, Li Shulin [6] proposed a new method based on the static strength analytical method and the nominal stress method to analyze the strength of train wheels. The main purpose is to calculate the fatigue life of the EMU wheels more realistically. Ekberg [7] studied rolling wheel fatigue and pointed out that rim crack can occur whether it contains inclusions or not, but the probability of rim crack occurrence was bigger when it contained inclusions. He proposed that the fatigue life of the railway wheel included crack initiation life and crack propagation life. American Brant Stratman [8] proposed a method to estimate the high cycle fatigue life of railway wheels. He applied the cumulative fatigue damage theory to the problem of wheel rail contact and established a 3D elastic-plastic finite element model which considers rail wheel contact. Fatigue wheel damage was calculated by the time load history of 360 degree wheel rotation, which was loaded into the finite element analysis.

Although there are many efficiency and high precision analytical methods for estimating contact fatigue life of wheels, there is a of lack of related guides for research on the life of wheel rims,
especially in estimation of the fatigue life of wheel rims containing inclusions. Firstly, a finite element model was created to get the stress of the wheel rim around spherical inclusions under different working conditions. Then the dangerous working condition was found. Lastly, fatigue life of the wheel rim containing spherical inclusions under dangerous working conditions based on the fatigue prediction model was predicted.

2. Fatigue life prediction model

For the three methods to provide for the fatigue life of parts, the premise of the nominal stress method and the local stress strain method requires that the material is uniform with no defects and no initial cracks. The object of this paper is a wheel rim containing spherical inclusions 10 mm below the tread. Inclusions in the rim are defective. So the life of the wheel rim with inclusions can be predicted by using the critical plane method. Among the many critical plane criteria, McDiarmid’s criterion [9,10], which considers two forms of fatigue crack initiation proposed by Brown and Miller is the most widely used.

McDiarmid proposed that the maximum fatigue damage is in the maximum shear stress amplitude plane. The McDiarmid model suggests that the shear stress amplitude and the maximum normal stress at the critical plane are two parameters that affect the fatigue crack growth. The McDiarmid model criterion is expressed as follows.

$$\tau_{ns} + \frac{1-\tau}{2\sigma_n} \sigma_{n,max} = \tau_f (2N_f)^b$$

On the wheel rim material, \(\tau_{ns}\) is the shear stress amplitude on the critical plane and the unit is MPa; \(\tau_{s}\) is the torsional fatigue limit strength of the material under a symmetrical load and the unit is MPa. When the torsion fatigue limit of the material is unknown, it can be estimated by using the empirical formula \(\tau = 0.58\sigma_s\), where \(\sigma_s\) is the symmetric bending fatigue limit which can be obtained according to the material list. \(\sigma_{n,max}\) is the normal maximum stress of the critical plane and the unit is MPa; \(\tau_f\) indicates the torsional fatigue strength coefficient and the unit is MPa. Normally \(\tau_f = \sigma_t / \sqrt{3}\) and the tensile strength coefficient is \(\sigma_t = \sigma_n + 350\). The high cycle fatigue life of the wheel rim with spherical inclusions can be predicted by McDiarmid’s critical plane criterion. The procedure is expressed as follows in Fig. 1.

![Figure 1. Steps of fatigue life prediction based on the McDiarmid model](image-url)
3. Compiling the wheel load spectrum under dangerous conditions

3.1. Determine the dangerous conditions
According to the symmetry characteristics of an axle-wheel system, and the load and constraint condition of the axle-wheel, the finite element model of the 1/4 axle-wheel is established and shown in Fig. 2. Using 3D solid unit SOLID187 when meshing. In order to calculate the accuracy of the results, refine the inclusion and surrounding basal material which are shown in Fig. 3[11].

CRH5 EMU usually run on straight, curved or turnout rails, so the contact forces on the wheel are quite different according to different conditions. The finite element method is used to analyze the distribution of stress around the spherical inclusions in the wheel rim under three different operating conditions. The analysis results are presented in Table 1.

| Load level | Load ratio | Recycle ratio |
|------------|------------|---------------|
| 1          | 1          | 1             |
| 2          | 0.950      | 106           |
| 3          | 0.850      | 6834          |

The results show that the value of maximum equivalent stress and maximum principal stress is highest under curve conditions, and smallest under straight conditions. Under straight conditions, the wheel runs smoothly, compared with the other two conditions. Therefore curve conditions for wheel rims are relatively harsh operating conditions.

3.2. Compile the load spectrum
The general maximum axle load of a CRH5 type EMU guide wheel reaches 17 tons. The axle weight is 1.47 tons. Depending on statistics, the running mileage of a train is about $6 \times 10^6$ kilometers per year, and 350 load cycles occur per kilometer. Moreover, a maximum load occurs for each of the $10^6$ cyclic loads. Tests by the Academy of Railway Sciences indicate that an eight level load spectrum can represent actual working conditions [12]. The literature [13] gives the eight level load ratio, the ratio of each load cycle, and lateral and vertical load spectra under curve conditions as shown in Table 2 and Table 3 respectively.

| Load level | Load ratio $F_i/F_l$ | Recycle ratio $n/n_i$ |
|------------|----------------------|-----------------------|
| 1          | 1                    | 1                     |
| 2          | 0.950                | 106                   |
| 3          | 0.850                | 6834                  |
4. Confirming the critical plane of the danger point and calculating its stresses

Based on the McDiarmid model, the plane of the maximum shear stress amplitude at the danger point is a critical plane. The maximum shear stress amplitude is equivalent to the absolute value of the difference of the maximum shear stress and the minimum shear stress. The minimum shear stress in the plane of a load cycle is approximately equal to zero. So it is considered that the maximum shear stress amplitude plane can be determined directly by the critical plane of the maximum shear stress. The steps for determining the critical plane, shear stress and normal stress are as follows.

1) Finite element analysis is carried out on the wheel rim, and the stress nephogram is shown in Fig. 4. The stress tensor $\sigma$ and the transformation matrix $\beta$ for the danger point can be extracted by analyzing the stress nephogram. The relationship between the stress tensor and the transformation matrix is $\sigma' = \beta \sigma \beta'$, and the principal stress tensor $\sigma'$ is

$$\sigma' = \begin{bmatrix} \sigma_1 & \tau_{12} & \tau_{13} \\ \tau_{12} & \sigma_2 & \tau_{23} \\ \tau_{13} & \tau_{23} & \sigma_3 \end{bmatrix} = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$$

2) The maximum shear stress $\tau_{\text{max}}$ at the danger point is in parallel with the intermediate principal stress $\sigma_2$, but which has 45 degrees between the maximum principal stress $\sigma_1$ and the minimum principal stress $\sigma_3$ respectively. Therefore, the direction cosine of the critical plane is $[a, b, c] = \left[ \frac{\sqrt{2}}{2}, 0, -\frac{\sqrt{2}}{2} \right]$ in the plane in which the principal stress is the reference coordinate. So the direction cosine $[a b c]$ of the critical plane in the global coordinate system is $[a b c]' = [a' b' c']$. Then we can calculate the direction cosine of the danger point in the critical plane under the global coordinate system.

3) According to the Cauchy formula, the stress state on the plane of the global coordinate system of the wheel rim under each load level is calculated by $[T; T; T]' = [\sigma][a b c]'$. Therefore, the shear stress and normal stress on the critical plane can be obtained by (2).

| Load level $i$ | Vertical load amplitude $F_{Zi}/N$ | Lateral load amplitude $F_{Y2i}/N$ | Frequency |
|---------------|----------------------------------|----------------------------------|-----------|
| 1             | 115437.500                       | 64645.00                         | 105       |
| 2             | 109665.625                       | 61412.75                         | 11130     |
| 3             | 98121.875                        | 54948.25                         | 717570    |
| 4             | 83692.188                        | 46867.63                         | 1.8335×10^7 |
| 5             | 66376.5                          | 37170.88                         | 6.4214×10^7 |
| 6             | 49060.938                        | 27474.13                         | 2.1203×10^7 |
| 7             | 31745.313                        | 17777.38                         | 517335    |
| 8             | 14429.688                        | 8080.63                          | 1365      |

Figure. 4. The stress nephogram of the wheel rim with spherical inclusions

Table 3. Vertical and lateral load spectrum
According to (2), the stress state at the danger point of the wheel rim under each load level can be transformed to the critical plane by coordinate transformation. The shear stress and normal stress on the critical plane under each load level is also obtained.

According to the above mentioned method, analyze the wheel rim 10 mm below the tread to confirm whether it contains spherical inclusions or not. The value of normal stress and shear stress at the critical plane of the wheel rim danger point is obtained under the eight level load spectrum as shown in Table 4.

Table 4. Normal stress spectrum and shear stress spectrum

| Stress level i | No inclusion 10 mm below the tread | The spherical inclusion located 10 mm below the tread | Frequency |
|----------------|-----------------------------------|-----------------------------------------------|-----------|
|                | Normal stress value/MPa | Shear stress value/MPa | Normal stress value/MPa | Shear stress value/MPa |
| 1              | 159.9981                  | 131.1945                   | 182.4679                  | 153.1566                  | 105 |
| 2              | 137.5671                  | 112.3342                   | 174.4935                  | 144.5594                  | 11130 |
| 3              | 124.7819                  | 101.6324                   | 157.5493                  | 129.3526                  | 717570 |
| 4              | 112.3956                  | 91.5843                    | 141.7427                  | 110.1936                  | 1.8335×10^7 |
| 5              | 101.7859                  | 79.0685                    | 123.7724                  | 98.109                    | 6.4214×10^7 |
| 6              | 87.2134                   | 61.2481                    | 101.8344                  | 76.1831                   | 2.1203×10^7 |
| 7              | 62.0159                   | 49.0032                    | 82.0274                   | 64.7765                   | 517335 |
| 8              | 51.2379                   | 34.7645                    | 69.9137                   | 41.8139                   | 1365 |

5. Estimating the fatigue life of the wheel rim when containing spherical inclusions
Firstly, based on the shear stress and normal stress spectrum at the critical plane of the danger point, and combined with the McDiarmid model, the fatigue life of the wheel rim under each level of cyclic load is calculated. Then, the fatigue damage under each level of the cyclic life load is calculated. The total fatigue damage is obtained by linear cumulative calculation, and lastly the total fatigue life is obtained.

5.1. Fatigue life under each load level in curve conditions
Firstly, get the shear stress and normal stress of the wheel rim danger point. Then according to the McDiarmid criterion we estimate the fatigue life under each stress level. Lastly, the fatigue life of the wheel rim danger point (whether containing spherical inclusions or not) can be obtained for curve conditions. The results are expressed in Table 5.

Table 5. Fatigue life of danger points under all levels of stress

| Stress level i | Fatigue life without inclusions(cycle time N) | Fatigue life with spherical inclusions(cycle time N) |
|----------------|-----------------------------------------------|---------------------------------------------------|
| 1              | 1.567301×10^8                                | 8.153729×10^7                                    |
| 2              | 2.376982×10^8                                | 9.143913×10^7                                    |
| 3              | 4.461356×10^8                                | 2.38399×10^8                                     |
| 4              | 5.19804×10^8                                 | 3.578407×10^8                                    |
| 5              | 6.99815×10^8                                 | 4.399726×10^8                                    |
| 6              | 8.724591×10^8                                | 6.411495×10^8                                    |
| 7              | 1.058579×10^8                                | 9.172023×10^8                                    |
| 8              | 1.40217×10^9                                 | 1.820413×10^9                                    |

5.2. Estimate the fatigue life under curve conditions
The so-called damage refers to the slight structural changes in the early stage of the fatigue process and the formation and extension of late cracks. One of the most widely used linear cumulative damage theories is the Miner rule. The rule stipulates that if the loading history of the specimen is made up of
different stress levels \( \sigma, \sigma_2, \ldots, \sigma_l \). Then the fatigue life of each stress level is \( N_1, N_2, \ldots, N_l \) and the number of cycles under each stress level is \( n_1, n_2, \ldots, n_l \). The specimen is damaged when the damage reaches the value \( D = \sum _{i=1}^{l} \frac{n_i}{N_i} = 1 \).

Based on the cumulative fatigue damage theory, combined with fatigue life under cyclic loading at all levels, the total damage \( D \) of the wheel rim can be obtained under curve conditions.

The total damage for the current demand is a load spectrum. That is the total damage of the car in actual work for one year. At the same time, according to the formula \( B = \frac{1}{D} = 1/ \sum _{i=1}^{l} \frac{n_i}{N_i} \), we can obtain the total number of load spectrum blocks until the wheel rim becomes invalid. Accordingly, the total operating life of the wheel is achieved.

The total damage \( D \) under the action of a load spectrum and load spectrum blocks \( B \) are obtained through the above method regardless of whether the wheel rim contains spherical inclusions or not as shown in Table 6.

| Table 6. Total damage under the action of a load spectrum and load spectrum blocks |
|-----------------------------------------------|
| No inclusions under curve conditions | Containing spherical inclusions under curve conditions |
| Total damage \( D \) | 0.153479554 | 0.233955917 |
| Load spectrum blocks \( B \) | 6.51552584 | 4.274309506 |

From the above table, we can see that the total damage at the danger point under the action of a load spectrum is \( D = 0.153 \) when the wheel rim without inclusions is in curve conditions. But the total damage is \( D = 0.234 \) when there are spherical inclusions. Because the load spectrum used in this paper is the load spectrum for a train running for a whole year, the train running time is equal to the load spectrum block, which is calculated from the total damage \( D \). The fatigue life of the train wheel rim without inclusions is 6.515 years when the is train running 300 thousand kilometers per year. The fatigue life of the train wheel rim is 4.274 years when it contains spherical inclusions. The fatigue life of the wheel rim is significantly shorter when spherical inclusions exist.

6. Conclusions

The finite element method is used to analyze the equivalent stress value and the maximum principal stress value of CRH5 train wheel rims containing spherical inclusions in straight, curve and turnout conditions. The value of maximum equivalent stress and maximum principal stress is the largest under curve conditions, and smallest under straight conditions. Compared with straight and turnout conditions, the influence of curve conditions on the fatigue life of the wheel rim is relatively larger. Therefore, the fatigue life of the wheel rim with spherical inclusions in curve conditions is estimated using the McDiarmid criterion, and compared with the fatigue life of the wheel rim without inclusions. It is found that the fatigue life of the wheel rim is significantly shorter when spherical inclusion exists. The results of this analysis can provide a theoretical basis and technical support for further analysis of reliability and sensitivity and is also important for reliability analysis models of wheel shaft systems.

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