Unified FSM treatment of CP physics extended to hidden sector giving (i) $\delta_{CP}$ for leptons as prediction, (ii) new hints on the material content of the universe

José BORDES$^{1}$
jose.m.bordes@uv.es
Departament Fisica Teorica and IFIC, Centro Mixto CSIC, Universitat de Valencia, Calle Dr. Moliner 50, E-46100 Burjassot (Valencia), Spain

CHAN Hong-Mo
hong-mo.chan@stfc.ac.uk
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom

TSOU Sheung Tsun
tsou@maths.ox.ac.uk
Mathematical Institute, University of Oxford, Radcliffe Observatory Quarter, Woodstock Road, Oxford, OX2 6GG, United Kingdom

Abstract

A unified treatment of CP physics for quarks and leptons in the framed standard model (FSM) is extended to include the predicted hidden sector giving as consequences: (i) that an earlier part-estimate of the Jarlskog invariant $J'$ for leptons is turned into a prediction for its actual value, i.e., $J' \sim -0.012$ ($\delta_{CP}' \sim 1.11\pi$), which is of the right order of magnitude, of the right sign, and in the range of values favoured by present experiment, (ii) some novel twists to the effects of CP-violation on the material content of the universe.

$^{1}$Work supported in part by Spanish MINECO under grant FPA2017-84543-P and PROMETEO 2019-113 (Generalitat Valenciana).
1 Preamble

1.1 CP in the FSM

The framed standard model (FSM) [1, 2], constructed initially for understanding the mass and mixing patterns of quarks and leptons, is found to provide also a neat unified treatment of CP physics in the world of quarks and leptons covered by the standard model (SM).

Briefly, this has come about as follows. The mass matrix for quarks and leptons in FSM turns out to be of the following form:

\[ m = m_T \alpha \alpha^\dagger, \]  

where the scalar \( m_T \) depends on fermion type, but the real unit vector \( \alpha \) in 3-dimensional generation space is the same for all quarks and leptons, and changes with scale (rotates) in a manner governed by renormalization group equations (RGE). And it is the rotation of this \( \alpha \), according to the FSM, which gives, and quite accurately, the intricate mass and mixing pattern for quarks and leptons which is observed in experiment.

As for CP and its violations, the usual SM treatment has some loose ends:

- [a] What is known as the strong CP problem, namely in the QCD action a theta-angle term of topological origin (instantons) [3, 4] which potentially can give CP-violating effects many orders of magnitude larger than is allowed by experiment [5].

- [b] A CP-violating Kobayashi-Maskawa (KM) phase [6] is formally allowed in the CKM mixing matrix for quarks but the size and physical origin of which are left unexplained.

- [c] Although a theta-angle term similar to [a] is in principle present also in the flavour action, there are suggestions that it can be trivially transformed away [3].

- [d] A CP-violating phase, similar to [b], can exist also in the PMNS mixing matrix for leptons, but again with size and origin equally unexplained.

However, the form of the mass matrix (1) in the FSM changes all that.

\[ ^2 \text{based on arguments we find insufficient, see [7]} \]
• [a′] The mass matrix (1) has a state in the direction of the Darboux binormal $\nu$ with zero mass eigenvalue. It follows then that the theta-angle term can be cancelled in Feynman path integrals by a chiral transformation on that zero mode, solving thus the strong CP problem at every scale $\mu$.

• [b′] But, since $\alpha$ rotates with scale, the direction of $\nu$ also changes with scale; the cancellation in [a′] at one scale does not guarantee the same at another scale. This scale-dependence figures in the CKM matrix (which compares the direction of up and down state vectors measured at different scales) and appears as the KM phase, providing thus not only an explanation of its origin but also a means to calculate its size [8, 9, 2].

• [c′] Parallel to the considerations [a′] for quarks but now applied instead to leptons cancels the effect of the theta-angle term in flavour.

• [d′] By the same token as [b′] the scale dependence of $\alpha$ gives a CP-violating phase in the PMNS matrix for leptons, explains its origin and gives its size.

Besides it has been shown [7] that the CP-violations as measured by the Jarlskog invariants $J$ [10] (equivalent in content to the phases $\delta_{CP}$ but easier to work with in FSM because parametrization invariant) so obtained in [b′] and [d′] are both of the same order of magnitude as those observed in experiment so that no other sources for CP-violation are needed for their explanation at present.

The result is, it seems,

• [CPU] A unified treatment of all at present known effects of CP-violation in particle physics, in which:

  - CP-violations for quarks and leptons are put on the same footing.

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3When we consider a curve embedded on a given surface, as in the case of the trajectory of $\alpha$ on the unit sphere, the radial vector $\alpha$, the tangent $\tau$ to the curve, and the binormal $\nu$ form an orthonormal triad which we call the Darboux triad. Note that the choice above of the zero eigenvalue direction along the $\nu$ direction is unique. Although (11) has a zero eigenvalue also in the direction of $\tau$, a chiral transformation in that direction at any $\mu$ would make the vector $\alpha(\mu + \delta\mu) = \alpha(\mu) + \delta\mu\tau$ complex at a neighbouring scale while the RGE governing its rotation would keep $\alpha$ real [3, 9].
They both have a topological origin in the theta-angle terms in respectively the colour and flavour theory.

In both cases, potentially huge violations in theta-angle terms are converted to CP-violating phases in the mixing matrices of the order seen in experiment.

If true, this is considerably neater than what we had for the SM in [a]—[d] above, thus tidying up the loose ends that have been left untied.

1.2 The hidden sector

However, neater as this might seem, it is for the FSM still incomplete because, in addition to the particles already known to us, designated here as the standard sector [SS], the FSM predicts another, what we might call

- [HS] the hidden sector, which communicates but little with the standard sector we know, and turns out to be related to the latter by having the roles of colour and flavour interchanged [11], as will be explained immediately below.

Given further that the stable neutral members of this hidden sector are all potential candidates for dark matter, it is clear that [HS], if true, would have far-reaching consequences, and any consideration in FSM of CP or its violation would be incomplete without taking account of it.

That a hidden sector [HS] should exist in FSM comes about as follows. The FSM is constructed by framing the standard model, meaning that the frame vectors (vielbeins) in the internal symmetry space are promoted into fields and treated as dynamical variables alongside the usual gauge boson and matter fermion fields. These new fields, called framons, though vectors in internal symmetry space, are Lorentz scalars. Specifically, they appear as:

- [FF] The flavour (“weak”) framon:
  \[ \alpha \Phi = \alpha \tilde{\phi}_r; \]  
  \[ (2) \]

- [CF] The colour (“strong”) framon:
  \[ \beta \Phi = \beta \phi_a; \]  
  \[ (3) \]
where:

\[ r = 1, 2, \quad \overline{r} = 1, 2, \quad a = 1, 2, 3, \quad \overline{a} = 1, 2, 3. \]  

(4)

The \( 2 \times 2 \) matrix \( \Phi \) and the \( 3 \times 3 \) matrix \( \Phi' \) are scalar fields depending on space-time points \( x \), while the 3 vector \( \alpha \) and 2 vector \( \beta \) are \( x \)-independent global quantities [12].

The flavour framons, of which only one column needs to be kept, as explained in for example [2], is, apart from the factor \( \alpha \), essentially just the Higgs field of the standard electroweak theory. But, the colour framons is entirely new with no analogue in the standard model. And the hidden sector we are after is basically just the manifestation of the new degrees of freedom that the colour framons [CF] represent.

These colour framons are colour triplets, and since colour is confined, they cannot propagate freely as particles in space, but have to combine with other coloured objects to form colour singlets. A colour framon can combine with its own conjugate to form scalars (generically denoted by H) or vectors (generically denoted by G), or with colour triplet fermions to form colour neutral fermion states (generically denoted by F) [11]. They are the analogues of respectively the Higgs boson \( h_W \), the vector bosons \( \gamma-Z, W \), and the quarks and leptons in the flavour theory when the latter is pictured as a confining theory, as suggested by 't Hooft [13]. So, as our standard sector is populated by \( h_W, \gamma-Z, W \) and quarks and leptons, the hidden sector is populated by the H, G, and F. Some details known so far of these predicted particles H, G and F are spelt out in [11], and possible evidences for the existence of some of them have been examined in [14, 15].

Given then that this hidden sector exists in the FSM, our consideration of CP and its violations as summarized in [CPU] would be incomplete unless extended to include the hidden sector as well.

2 Yukawa couplings for colour framons

An extension into the hidden sector of the analysis of CP and its violation, along the lines one did for the standard sector [7] as summarized in [CPU] above, is not going to be straightforward. The arguments for that analysis, one recalls, centre on the fermion mass matrix, that is, on the Yukawa coupling, but the Yukawa coupling for the hidden sector is unfortunately the weakest link in the theoretical structure of the FSM, as emphasized in [11].
The reason is the following. Of the 3 sets of fields functioning as dynamical variables of the FSM, the matter fermion fields are the ones of which we know the least. The vector bosons are gauge fields, or connections of fibre bundles, with a clear geometrical meaning and a particular function to discharge. What they are, and how they should enter in the FSM action harbour therefore little doubt. The framon scalars also, once ascribed the geometrical significance of frame vectors in internal symmetry space, possess likewise a geometrical meaning and function, and pose again few questions on what they should be and how they should enter in the FSM action. For the matter fermion fields, on the other hand, one does not know as yet of the geometrical role, if any, they play. One does not know a priori even what matter fermion fields should enter in the theory, let alone how they are coupled to one another. This difficulty was there, of course, already in the standard sector, but there one had the benefit of experimental knowledge, by dint of which, and of the ingenuity and patience of pioneer workers, one knows what fermion fields should enter in the theory, namely 3 generations each of flavour doublet left-handed and flavour singlet right-handed quarks, and the same for leptons, and hence how to construct the Yukawa coupling from them. But now in the hidden sector, there is of course no experiment to help. One’s sole guidance, apart from general invariance principles, is analogy with the standard sector. And with these alone in the hidden sector to explore CP and its violations, one is obviously treading on rather thin ice. Any findings achieved thereby can thus be taken as only tentative and are to be treated with caution. It is with this understanding that we proceed in what follows.

Now in [11], a working model was suggested for the Yukawa couplings in the hidden sector, that is, the couplings of the colour framon $[\text{CF}]$ to the following tentatively proposed list of fundamental fermion fields:

$$
\begin{align*}
\psi_L(1, 2, 3), & \psi_R(-\frac{1}{3}, 1, 3), \psi_R(\frac{2}{3}, 1, 3), \\
\psi_L(-\frac{1}{2}, 2, 1), & \psi_R(-1, 1, 1), \psi_R(0, 1, 1) \\
\psi_R(-\frac{1}{2}, 2, 1), & \psi_L(-1, 1, 1), \psi_L(0, 1, 1),
\end{align*}
$$

where the first argument inside the brackets denotes the $u(1)$ charge, the second the dimension of the $su(2)$ representation and the third that of the $su(3)$ representation. Of these, those in the first row carry colour and can combine with the colour framon via colour confinement to form compound Fs of the handedness indicated. From the first, $\psi_L(\frac{1}{6}, 2, 3)$, having neutralized
its colour by the framon, one obtains a bound state $F$, say $\chi_L(\pm \frac{1}{2}, 2, 1)$, carrying still local flavour, which we call a co-quark, this being the parallel of a left-handed quark in the standard sector under colour-flavour interchange.

Similarly, from the other two coloured fields on the list (5), one obtains two Fs, neither of which carries local flavour, and both, being parallels of the right-handed lepton in the standard sector, are called co-leptons.

The left-handed co-quark $\chi_L(\pm \frac{1}{2}, 2, 1)$ can then be partnered by two other members on the list (5), namely $\psi_R(\frac{1}{2}, 2, 1)$ and $\psi_L(\frac{1}{2}, 2, 1)$ (where superscript $C$ denotes charge conjugate) to form Yukawa couplings for co-quarks in analogy to those for quarks in the standard sector. Explicitly, in [11], equation (73), the following (with minor changes in notation) were suggested:

$$A_{YQ} = \left[ Z_1 \bar{\psi}_L(\frac{1}{6}, 2, 3)(\Phi \cdot \delta_1)\psi_R(\frac{1}{2}, 2, 1) + Z_2 \bar{\psi}_L(\frac{1}{6}, 2, 3)(\Phi \cdot \delta_2)\psi_R(\frac{1}{2}, 2, 1) + Z_3 \bar{\psi}_L(\frac{1}{6}, 2, 3)(\Phi \cdot \alpha)\psi_R(\frac{1}{2}, 2, 1) \right] + h.c. \quad (6)$$

Of these, the term proportional to $Z_3$ seems unambiguous, but for the other two, some questions remain, which we already noted at the time we wrote them down but had not then the incentive to sort out. Now however, with new incentive and hindsight, two shortcomings to these terms can be identified:

S1 Before these couplings (6) were inserted, the FSM vacuum was invariant under, say, an $\tilde{su}(2)_H$ symmetry in the two directions orthogonal to $\alpha$ [2]. This is broken by (6) explicitly along two arbitrarily chosen directions $\delta_1$ and $\delta_2$.

S2 The two right-handed fields, $\psi_R(\frac{1}{2}, 2, 1)$ and $\psi_R(\frac{1}{2}, 2, 1)$ could, as far as quantum numbers are concerned, be coupled to both the $\delta_1$ and $\delta_2$ terms, but had been taken arbitrarily in (6) to couple respectively to only one each of these two terms.

In view of these shortcomings in (6), we propose now to replace it by:

$$A_{YQ} = \left[ \bar{\psi}_L(\frac{1}{6}, 2, 3)(\Phi \cdot \omega)(Z \cdot \psi_R(\frac{1}{2}, 2, 1)) + Z_3 \bar{\psi}_L(\frac{1}{6}, 2, 3)(\Phi \cdot \alpha)\psi_R(\frac{1}{2}, 2, 1) \right] + h.c., \quad (7)$$

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4 The left-handed quark, we recall, is in 't Hooft's confinement picture of the electroweak theory [13], a bound state via flavour confinement of a fundamental fermion with a Higgs scalar—here a flavour framon—which carries still local colour.
where $\omega$ is a unit 3-vector in dual colour space (say, orthogonal to $\alpha$ and therefore a linear combination of $\delta_1, \delta_2$), while

$$ (Z \cdot \psi_R((\frac{1}{2}, 2, 1)) = Z_1\psi_{R1}(\frac{1}{2}, 2, 1) + Z_2\psi_{R2}(\frac{1}{2}, 2, 1). $$

Since both $\psi_{R1}$ and $\psi_{R2}$ can now in general be coupled to both $\delta_1$ and $\delta_2$, the new coupling (7) can claim to have avoided the shortcoming [S2] of the old (6). Part of [S1] seems to remain in that the $\tilde{su}(2)_H$ symmetry is still explicitly broken, now along the direction $\omega$, but if the latter rotates (changes in direction) with changing scale, as we shall argue below, then at least some democracy is restored between the various directions covered by $\tilde{su}(2)_H$, and [S1] therefore, in this aspect, alleviated. Besides, if $\omega$ can be assigned a physical significance, on which we have some idea not as yet substantiated, then the direction in which the $\tilde{su}(2)_H$ symmetry is broken is no longer arbitrary, as were $\delta_1$ and $\delta_2$ in [S1].

Whether these improvements of the new Yukawa coupling (7) on the old version (6) are sufficient to qualify it for being taken now as the Yukawa coupling for the co-quarks is not easy to ascertain, all our arguments being based just on analogy with the Yukawa couplings of the flavour framed in the standard sector, but these latter offered no parallel of the situation here of two $\chi_L$ states with identical quantum numbers.

However, we find (7) attractive because, as we shall show below:

- [P1] it gives a mass matrix for co-quarks with a zero mode,
- [P2] the direction of that zero mode might depend on scale,

which, as outlined in Section 1.1. and detailed in [7], were the conditions which cured the strong CP problem in the standard sector, and which we hope may do the same for the parallel problem that will later appear in the hidden sector as well. We accept (7) therefore as now our working model.

That [P1] attain can be seen as follows. The Yukawas coupling (7) gives for the 2 charged $\frac{1}{2}$ co-quarks a factorizable mass sub-matrix proportional to:

$$ m \sim \left( \begin{array}{c} \omega_1 \\ \omega_2 \end{array} \right) (Z_1, Z_2) \left( \frac{1}{2}(1 + \gamma_5) + h.c. \right), $$

where $(\omega_1, \omega_2)$ are the components of $\omega$ at any chosen scale along the two directions $\delta_1$ and $\delta_2$. This submatrix has a zero mode in the direction orthogonal to $(\omega_1, \omega_2)$, as can be seen as follows. By a suitable relabelling of
the right-handed fields, following a procedure made familiar already with the mass matrix \( m \) in (11) for quarks and leptons, \( \hat{m} \) of (9) can be cast into a Hermitian form independent of \( \gamma_5 \), thus:

\[
\hat{m} \rightarrow (Z_1^2 + Z_2^2)^{1/2} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} (\omega_1, \omega_2).
\]  

(10)

In this form, the mass spectrum can easily be read. There is a zero mode:

\[
\begin{pmatrix} -\omega_2 \\ \omega_1 \end{pmatrix}.
\]  

(11)

plus a massive eigenstate:

\[
\begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix}
\]  

(12)

with eigenvalue proportional to \( \sqrt{Z_1^2 + Z_2^2} \).

These observations on \( \hat{m} \) in (9) apply at every scale, but the value of \( (\omega_1, \omega_2) \) may differ at different scales. The reason is that the basis vectors \( \delta_1 \) and \( \delta_2 \) are only required in their definition to be orthogonal to \( \alpha \). Then, given the the symmetry \( \tilde{su}(2)_H \), there is in general an arbitrariness in choice of which two orthogonal vectors in the plane orthogonal to \( \alpha \) to be taken as \( \delta_1 \) and \( \delta_2 \). Having made a choice at one scale does not oblige one to make the same choice at some other scale. Hence, \( (\omega_1, \omega_2) \), the components of \( \omega \) in the directions of \( \delta_1 \) and \( \delta_2 \) can depend in general on scale, or \([P2]\), as claimed.

Whether it does change with scale, and if so how, depend first on what is taken as \( \omega \), namely on the physical meaning assigned to the direction in which the \( \tilde{su}(2)_H \) symmetry is explicitly broken. Secondly, its dependence on scale, if any, may be constrained by renormalization group equations in which \( \omega \) figures. These are questions needing closer study to which we have at present no clear answer.

Let us first suppose, however, that \( (\omega_1, \omega_2) \) does indeed depend on scale or, in the language adopted above, that the direction \( \nu' = (-\omega_2, \omega_1) \) in which the zero mode appears does rotate with changing scales and ask what will then result. We are then in a situation very similar to that encountered for quarks and leptons in the standard sector which we thought we knew how to handle. Only at the end shall we return to ascertain how the result will change when the rotation assumption is dropped.
The above comments, from (6) onwards, on the Yukawa coupling of co-quarks apply similarly to the Yukawa couplings of the co-leptons but these will not be needed in detail in what follows.

3 Extending CP considerations to include the hidden sector

Before we attempt to extend CP considerations to the whole of FSM, let us first systematize what was done in the standard sector, specifically with the view of extending it to include the hidden sector. The starting point there, we recall, was the strong CP problem, or rather the solution of it, from which the other results on the CP-violating phase in the CKM and PMNS matrices follow.

We start by noting the following points:

1. By a solution of the strong CP problem in the FSM, one means a rather different process than, say, in the Peccei-Quinn \[16\] approach. In the latter, one starts with the SM with all fermions taken massive so that CP is defined already for all fermion states. Adding then to the Lagrangian density a theta-angle term:

\[
-\frac{1}{16\pi^2}\theta_I \text{Tr}(H_{\mu \nu}H_{\mu \nu}^*)
\]

for the colour field \(H_{\mu \nu}\), with \(\theta_I\) of order unity, would violate CP even in strong interactions, in contradiction to the existing experimental bound on, for example, the neutron electric dipole moment which would require a value of \(|\theta_I| < 10^{-9}\) \[5\], so that a correction to the SM is needed. Thus, to this end, a new \(U(1)\) dynamics was introduced in \[16, 17\], to explain why the theta-angle prefers to be near vanishing. The FSM, on the other hand, though involving also new dynamics appended to the SM in the form of framon fields, these were introduced to understand the mass and mixing patterns of fermions, not specifically to solve the strong CP problem. The solution of the strong CP problem comes as a bonus since no extra new dynamics needs to be introduced to achieve that end. There is already inherent in the FSM at every scale a quark state with zero mass eigenvalue for which CP is not yet defined. A solution of the strong CP problem in the FSM means then only the
recognition that for an appropriate choice of phases in the definition of CP for that quark zero mode would automatically induce a chiral transformation to generate a factor in the measure of Feynman path integrals [18] to cancel the theta-angle term that one has started with in the action, leaving thus strong interactions CP-invariant.

2. By strong interactions in the standard sector we mean usually the soft interactions between hadrons resulting from the colour interactions between quarks. To isolate the latter, one ignores electroweak effects, that is, not only those interactions between quarks resulting from flavour and electromagnetism but also all those particles such as leptons which have no colour but only electroweak interactions. Thus to ascertain that strong interactions in the standard sector is CP-invariant, it suffices to show that QCD for quarks is CP-invariant.

3. Flavour-induced interactions in the standard sector, apart from electromagnetism, are termed weak not so much because of the smallness of the flavour gauge coupling $g_2$ (which, after all, is only some $\sim 2$ times smaller than the colour gauge coupling $g_3$), but because they are mediated by massive particles such as $W, Z$ and Higgs, resulting in a suppression of their effects by order $g_2^2/M^2$. It is partly because of this, and partly because hadron interactions are soft, that the hard (that is, perturbative) flavour-induced interactions of known particles are often negligible in hadron physics. For instance, flavour interactions between quarks, being CP-violating via the Kobayashi-Maskawa phase in the CKM matrix, do give contributions to the electric dipole moment of the neutron, but these are small and still much below the existing, already stringent, experimental bound.

With these remarks in mind, let us now formalize the FSM solution of the strong CP problem in the standard sector as follows, a procedure which we shall then extend to the whole of the FSM, including its hidden sector. We first identify and exhibit the approximate theory pertaining to strong interactions which conserves CP, and then show how CP is violated as that approximation is removed.

The part of the FSM action involving only those fields appearing in the standard sector may be represented schematically as:

$$\mathcal{A}_{SS} \sim FF + GG + HH + \theta_1^I GG^* + \theta_1 HH^* + (D\Phi)^\dagger D\Phi + V[\Phi] + \bar{\psi}_q D\psi_q + \bar{\psi}_e D\psi_e + \bar{\psi}_q \Phi \psi_q + \bar{\psi}_e \Phi \psi_e, \quad (14)$$
where $F^{\mu\nu}, G^{\mu\nu}, H^{\mu\nu}$ are respectively the $u(1), su(2), su(3)$ gauge fields. This part of the FSM action differs from the SM action only in replacing the Higgs scalar field in SM by the flavour framom field $[FF]$ where $\Phi$ is a $2 \times 2$ matrix, but of this matrix only one column is independent [2], and this column differs from the standard scalar Higgs field $\phi$ in SM only by a global vector $\alpha$ as seen in (2). The extra factor $\alpha$ does not figure in the potential $V[\Phi]$, which is still just the Mexican hat potential for $\phi$ familiar already in SM. As in SM then, the fact that $\phi$ has nonzero vacuum expectation value in $V[\Phi]$ gives mass to the Higgs boson $h_W$, and via the term $(D\Phi)^\dagger D\Phi$ also to the vector bosons $W-Z$. It also gives quarks and leptons a mass matrix of the form (1), which at tree-level has nonzero mass eigenvalue only for the top generation, and no up-down mixing. Nonzero masses for lower generations and mixing appear only as higher order effects when $\alpha$ rotates. All these features of the FSM are detailed in earlier publications, for example [2], and are briefly summarized above only for completeness.

Our present focus is on the theta-angle terms coming from instantons respectively in flavour $\theta_I^f \text{Tr}(G^{\mu\nu}G^{*}_{\mu\nu})$, and in colour $\theta_I \text{Tr}(H^{\mu\nu}H^{*}_{\mu\nu})$, terms which can lead to potentially huge CP-violations. Suppose for the moment that we are interested only in strong effects from quarks, such as the electric dipole moment of the neutron, these would be given in terms of Feynman path integrals of the form:

$$\frac{\int \delta\psi_q F[\psi_q] \exp i\mathcal{A}_{SS}}{\int \delta\psi_q \exp i\mathcal{A}_{SS}},$$

where $F[\psi_q]$ is some appropriate functional of the quark fields $\psi_q$. If then, to isolate and exhibit the part of the action pertaining only to strong interactions, we can ignore electromagnetic interactions because of the small coupling and weak flavour interactions as per the remark 3 above because of suppression by the mass of the flavour bosons. What we have left in (14) is only:

$$\mathcal{A}_{SSstr} \sim HH + \theta_I HH^* + \bar{\psi}_q D_C \psi_q + \bar{\psi}_q m_q \alpha \alpha^\dagger \psi_q$$

where $D_C$ denotes the covariant derivative with respect to colour only, and the last term is the mass matrix in (1) specialized now only to quarks which is obtained by expanding the Higgs field in the Yukawa coupling about its vacuum expectation value, keeping only its vacuum value and neglecting all higher terms mediated by exchanges of the massive Higgs, which would be suppressed. We note in particular that the instanton term $\text{Tr}(G^{\mu\nu}G^*_{\mu\nu})$ for
flavour in the numerator of (15), like many other terms in the action (14), being uncoupled to the $\psi_q$ in $\mathcal{F}$ in the approximation when flavour interactions are ignored, has been cancelled with the same term in the denominator of (15).

The remaining quantity (16), according to (2) above is what one would call the strong action in the standard sector, and this contains still the instanton term $\text{Tr}(H^{\mu\nu}H_{\mu\nu}^*)$ which can violate CP hugely. However, there being zero modes in the mass matrix for quarks in (16), an appropriate chiral transformation on which will cancel the term $\text{Tr}(H^{\mu\nu}H_{\mu\nu}^*)$ in the Feynman path integral (15), and this is what we called above the solution of the strong CP problem in the standard sector in FSM.

Let us turn now to tackle the same problem when extended to include the hidden sector. To do so, we shall need to add to (14) terms involving the new fields which had not figured in the standard sector before, thus (again schematically):

$$\mathcal{A} \sim \mathcal{A}_{SS} + V[\Phi] + \nu[\Phi, \bar{\Phi}] + (D\Phi)^\dagger D\Phi$$
$$+ \bar{\psi}_Q D\psi_Q + \bar{\psi}_L D\psi_L + \bar{\psi}_Q \Phi \psi_Q + \bar{\psi}_L \Phi \psi_L.$$  \hspace{1cm} (17)

In particular, the framon self-interaction potential constructed from the underlying invariance of FSM becomes much more intricate, involving now the colour framon in (3) which is a full $3 \times 3$ matrix with many more field degrees of freedom. Details of this can be found in, for example, [2]. Besides the Mexican hat potential $V[\Phi]$ for the standard Higgs field included already in (14), we have now also a $V$ for $\bar{\Phi}$, as well as a term $\nu[\Phi, \bar{\Phi}]$ which links the flavour and colour framon fields. As a result, the vacuum also becomes much more intricate, the details for which are given also in [2]. However, what matters here is just the fact that the colour framon has a nonzero vacuum expectation value, giving thus masses to the scalar bosons $H$, and also via the term $(D\Phi)^\dagger D\Phi$ masses to the vector bosons $G$. These masses, as suggested by the analysis in [11], are typically of order TeV. Hence, according to point 3 above, for co-quarks and co-leptons, it is now the colour interactions via the exchange of these massive $H$s and $G$s which will be suppressed and are to be considered “weak”, in the same spirit as the flavour interactions of quarks and leptons in the standard sector were considered weak. But it is the flavour interactions, coming from the local flavour that co-quarks still carry, which are to be taken as “strong” instead. Local flavour in the hidden sector, being unbroken and confining, can thus form, from co-quarks (and anti-co-quarks)
via flavour confinement, co-hadrons, which can have, among themselves, soft interactions similar in strength to the soft interactions we see among hadrons in the standard sector \[1, 11\]. In other words, as anticipated, the roles of colour and flavour are switched from the standard to the hidden sector.

In parallel then to \((15)\), strong effects from co-quarks would be given in terms of Feynman path integrals of the form:

\[
\frac{\int \delta \psi_Q F'[\psi_Q] \exp iA}{\int \delta \psi_Q \exp iA}, \tag{18}
\]

where \(F'[\psi_Q]\) is some appropriate functional, now of the co-quark field \(\psi_Q\). And if, to identify the strong action, as was done above for the standard sector, we omit for the moment the electromagnetic interactions of the co-quarks \(Q\) because of the small coupling, and their weak colour interactions because of their suppression by the \(H\) and \(G\) mass, as well as those particles which have only these interactions, what we have left of \(A\) which is relevant for the hidden sector in the path integral \((18)\) is just:

\[
A_{HS\text{str}} \sim GG + \theta'_i G G^* + \bar{\psi}_Q D_F \psi_Q + \bar{\psi}_Q m_Q \psi_Q, \tag{19}
\]

where \(m_Q\) is the mass matrix of the co-quark \(Q\):

\[
m_Q = \zeta_S \left( \begin{array}{cc} \frac{1}{3}(1 - R)\hat{m} & 0 \\ 0 & \frac{2}{3}(1 + 2R) \end{array} \right), \tag{20}
\]

with \(\hat{m}\) as given in \((11)\) of the preceding section, \(D_F\) the covariant derivative with respect to flavour only, and \(\zeta_S\) and \(R\) are scalar quantities given in \([2]\), for example, which, though important for consideration of the actual mass spectrum and so on, are not necessary for the discussion on CP here. There are terms in \(A\) other than those exhibited in \((19)\), but they are either negligible in the present approximation or else, being linked to \(Q\) only by the neglected interactions, would cancel in \((18)\) between the numerator and the denominator.

There is still in \((19)\) the term \(\theta'_i \text{Tr}(G^{\mu\nu} G_{\mu\nu}^*)\), which can potentially give large CP-violating effects to strong interactions of the co-quarks \(Q\), as represented by \((18)\). However, given that the mass matrix \(m_Q\), as shown in the analysis around \((10)\) in the last section, has a mode, say \(Q_0\), in the direction \(\nu'\) which has zero mass eigenvalue, a chiral transformation can be performed on that mode, thus:

\[
\psi_{Q_0} \to \exp(-i\frac{1}{2} \theta'_i \gamma_5) \psi_{Q_0}, \tag{21}
\]
to generate a factor from the measure of the integral (18) to cancel the term $\theta'_I \text{Tr}(G^{\mu\nu}G^{\ast \mu\nu})$ from (19), so as to leave the effects of strong (flavour) interaction of co-quarks CP-invariant.

At first sight, this result may not seem too surprising, given that the standard and hidden sectors in the FSM are related by having the roles of colour and flavour interchanged, and both quarks and co-quarks are known to have zero modes. Hence, if in the standard sector, the strong CP problem can be cured by a chiral transformation on the quark zero mode, one may not perhaps be too surprised to find that in the hidden sector, the parallel strong CP problem can be cured by a parallel chiral transformation on the co-quark zero mode. On closer look, however, one finds that the situation is in fact more intricate. For solving the strong CP problem in the standard sector, the chiral transformation is performed on the quark zero mode in a direction $\nu$ in generation or dual colour space. One would expect then that, in parallel under a colour-flavour interchange, the strong CP problem in the hidden sector would be cured by a chiral transformation on a co-quark zero mode in some direction in “co-generation” or dual flavour space. However, according to the analysis in [11], no “co-generation” exists in the hidden sector parallel to generation in the standard sector, because of a so-called minimal embedding condition imposed on the flavour frameon [FF] which was what led to one column in $\Phi$ in (2), as explained there, being made redundant. Instead, in the above treatment, the direction $\nu'$, containing the zero mode $Q_0$ on which the chiral transformation is performed to cure the strong CP problem in the hidden sector, is still in dual colour space, or rather in a subspace of that space orthogonal to the vector $\alpha$. Nevertheless, despite this difference in details, it seems that the parallel in result still holds.

For the FSM as a whole then, including both the standard and the hidden sectors, strong interactions can be made CP-invariant at tree-level (in fact even at one-loop level at any fixed scale) by appropriately defining CP for the zero modes of the quarks and co-quarks. This was done in the approximation when electromagnetic and weak interactions (that is, flavour in [SS] and colour in [HS]) were omitted.

Let us next put back these interactions so far ignored to consider CP for the FSM in full. Quarks having then recovered their weak flavour interactions, the chiral transformation (13) performed on the quark state in the $\nu$ direction to cure the strong CP-problem in [SS] will then generate in the
measure of the full Feynman path integrals a factor with exponent:

\[-\frac{\theta'_C}{16\pi^2} \text{Tr}(G_{\mu\nu}^* G_{\mu\nu}^*),\]  

(22)

with \(\theta'_C = -3\theta_1/4\), as noted in [7]. This can lead to large, experimentally unwanted, CP-violations in electroweak effects. However, as was shown in [7], (22) can again be cancelled by another factor generated in the measure of full Feynman path integrals by a chiral transformation on the lepton state in the \(\nu\) direction, making then the standard sector, including now all interactions, fully CP-invariant at any fixed scale.

Similarly, in the hidden sector, the co-quarks, having recovered their weak colour interactions, would generate, via their earlier chiral transformation to cure the strong CP problem in [HS], from the measure of full Feynman path integrals a factor with exponent:

\[-\frac{\theta_F}{16\pi^2} \text{Tr}(H_{\mu\nu}^* H_{\mu\nu}^*),\]

(23)

with \(\theta_F = -\theta'_1/2\). Such a term could in turn give large CP-violations to the weak colour interactions in the hidden sector but can again be cancelled by making the appropriate chiral transformations on the zero mode(s) of the co-leptons. However, our knowledge on the spectrum of co-leptons being so meagre, no attempt at further elucidation is at present possible.

The arguments above being quite involved and loaded by necessity with details, it is worthwhile for clarity to go succinctly over them once more, even at the cost of some repetitions, as follows.

Schematically, the strong part of the FSM action appears as:

\[\mathcal{A}_{\text{str}} = \mathcal{A}_{\text{SSstr}} + \mathcal{A}_{\text{HSstr}},\]

where

\[\mathcal{A}_{\text{SSstr}} \sim HH + \theta_1 HH^* + \bar{\psi}_q \slashed{D}_C \psi_q + \bar{\psi}_q m_q \alpha \alpha^{\dagger} \psi_q,\]

\[\mathcal{A}_{\text{HSstr}} \sim GG + \theta'_1 GG^* + \bar{\psi}_Q \slashed{D}_F \psi_Q + \bar{\psi}_Q m_Q \psi_Q.\]

There are two CP violating terms: \(\theta_1 HH^*\) involving colour for the SS and \(\theta'_1 GG^*\) involving flavour in the HS. They could produce large CP violation in respectively the SS and the HS.
[1] The $\theta_I HH^*$ term can be cancelled in Feynman integrals by a chiral transformation $\alpha$ on the 2 zero modes (1 up, 1 down) of the quark mass matrix, with
\[
\alpha = -\frac{1}{4} \theta_I.
\]

[2] Assuming that large CP violation in the HS is not acceptable, the $\theta'_I GG^*$ term can be cancelled by a chiral transformation $\alpha'_{HS}$ on the zero mode of the co-quark mass matrix, with
\[
\alpha'_{HS} = -\frac{1}{2} \theta'_I.
\]

These two steps will make the strong action manifestly CP invariant.

Next we put back the weak parts of the action, but now with two additional terms generated by the chiral transformations [1] and [2] above.

[3] The chiral transformation $\alpha$ on the quarks, the LH components only of which carry flavour, will generate from the measure of Feynman integrals an extra term
\[
-\frac{\theta'_C}{16\pi^2} \text{Tr} GG^* = -\frac{3\alpha}{16\pi^2} \text{Tr} GG^* = \frac{3}{2} \frac{\theta_I}{16\pi^2} \text{Tr} GG^*,
\]
with a factor of 3 because quarks come in 3 colours, as explained in [7]. This extra term can be cancelled in the Feynman integrals by a chiral transformation $\alpha'$ on the zero modes of the lepton mass matrix, with
\[
\alpha' = -\theta'_C = \frac{3}{4} \theta_I.
\]

[4] Similarly a chiral transformation $\alpha'_{HS}$ on the co-quarks will generate a term $-\frac{\theta_F}{16\pi^2} \text{Tr} HH^*$ in the action, with
\[
\theta_F = 2\alpha'_{HS} = -\theta'_I,
\]
with a factor of 2 because co-quarks come in 2 flavours. This additional CP violating term can also in principle be cancelled by a chiral transformation $\alpha_{HS} = -\theta_F = \theta'_I$ on the co-leptons, but this we leave open at present, lacking sufficient knowledge of the co-lepton spectrum.

Hence, we see that by suitable chiral transformations on the zero modes of the relevant mass matrix, all CP violating terms have been cancelled,
which means that CP is conserved at tree-level, both in the SS and the HS. However, when loop corrections are included, the zero modes, on which the above chiral transformations act, will change with scale and induce CP-violating phases in the CKM and the PMNS matrices. Both these effects are observed in experiments, the first well-established and the second a 3 σ effect more recently. For HS, on the other hand, though parallels can apply, little can be said at this stage for lack of experimental knowledge.

There are two points on the above treatment that we wish to highlight for later reference:

- (I) CP is conserved at every scale but violations set in when scale changes are involved;
- (II) These CP-violations come from radiative corrections and are thus bound to be perturbatively small.

These results have been deduced under the assumption that ω in the Yukawa coupling (7) rotates with scale. What will be changed if this assumption is dropped? Now it is clear that so long as the zero mode in the hidden sector exists, given (7), then the theta-angle term θ′ Tr(G^μν G^∗ μν) in (19) can be cancelled by a chiral transformation on that zero mode to keep strong interactions in the hidden sector CP invariant, independently of whether the state vector of that zero mode rotates with scale or not. With no rotation, however, one can no longer appeal to the “leakage mechanism”, used in for example [2] for quarks and leptons, to give nonzero physical masses or residual CP-violations in mixing matrices to co-quarks and co-leptons. Massless particles may then be unavoidable in the hidden sector, which last will then be CP-invariant at all scales. These changes would affect the physical consequences to be drawn in the next section, as we shall indicate at the end of that section.

4 Remarks

4.1 The CP-violating phase δ′_{CP} in the PMNS matrix

An immediate consequence of the above extension of CP to the hidden sector is to turn the following result of [7] from a part-estimate into a prediction of the actual value for the Jarlskog invariant of the PMNS matrix,

\[ J' \sim -0.012. \]  

(24)
We recall that there are actually two theta-angle terms for flavour, namely
\[
-\frac{1}{16\pi^2}(\theta'_I + \theta'_C) \text{Tr}(G^{\mu\nu}G^{\mu\nu}_*),
\]  
(25)
where the $\theta'_I$ term comes from instantons and appears in the original action while $\theta'_C$ comes from the measure in Feynman path integrals as the result of a chiral transformation made on the quark zero mode to cure the strong CP problem in QCD. The angle $\theta'_I$ is unknown, but an estimate for the value of $\theta'_C$ has been obtained from an earlier fit to data [2]. Now, to cancel both these terms so as to make the whole theory CP-invariant at a fixed scale, there was available in [7], where the discussion was restricted to the standard sector, only the one chiral transformation on the lepton zero mode. Hence, the Jarlskog invariant which ensues for leptons would depend on the yet unknown value of $\theta'_I$, resulting thus in only a part-estimate, or at best an order-of-magnitude estimate. However, now that the discussion has been extended to the hidden sector, one finds that in order to guarantee that there should be no strong CP problem there, the $\theta'_I$ term has to be cancelled by a chiral transformation on the co-quark zero mode. This leaves only the known term $\theta'_C$ to be cancelled by the chiral transformation on the lepton zero mode, giving then the result (24) as a prediction for the actual value of the Jarlskog invariant in the PMNS matrix for leptons.

And as shown already in [7], (24) is (i) has the right order of magnitude, (ii) has the right sign and (iii) has a value in the range favoured by present experiment, providing thus, it would seem, a nontrivial check on the FSM scenario above, somewhat fanciful though the latter might perhaps appear.

We should recall also that the turning of (24) into a prediction for the value of $J'$ depends on the condition that strong interactions in the hidden sector is CP-conserving. For this, of course, one has no direct evidence, since even the existence of the hidden sector has yet to be ascertained. One can cite, however, the following points in support:

- (a) One has, of course, first to define what CP is before one can speak of CP-violations. It seems both logically and aesthetically unsound thus not to make a theory CP-conserving at the strong interaction level where a simple choice of phase is enough to achieve that end. And one is free to do so in the definition of CP for certain zero modes in the FSM, as was done above.
• (b) Given that the bound on CP-violations in strong interactions in the standard sector, as provided by the experimental bound on the electric dipole moment of the neutron \cite{22}, is extremely stringent, and that the hidden sector is not entirely cut off from the standard one but is connected to it by some portal states, as shown in \cite{11}, strong CP-violations in the hidden sector, if allowed, would leak into the standard sector as well, which is likely to cause problems in satisfying the bounds set by the neutron dipole moment, although one is not in a position as yet to verify that this is indeed the case.

• (c) CP-violations at strong interaction level in the hidden sector might lead to a superabundance of dark matter in the universe much beyond what is needed. See note (c) of the subsection below.

4.2 Hints on the material content of the universe

Two outstanding and quite novel features [I] and [II] of CP in the FSM are listed near the end of the section above.

We recall now that in understanding the material content of the universe, CP-violations come in as a crucial factor, as pointed out by Sakharov already in 1967 \cite{23}. Roughly, the picture one has is the following. At very early times when the universe was very hot, masses were negligible and massive particles were present as abundantly as massless ones. When the universe cooled, however, massive particles like baryons started to annihilate with their C-conjugates, eventually almost all into photons, leaving only a small excess of baryons over anti-baryons, and this is most of the luminous matter that we now see. For this excess of baryons over anti-baryons to occur, CP had, according to Sakharov, to be violated at some stage. That CP-violations are weak effects then explain why this excess of baryons over anti-baryons is small and why photons occur as much as a billion times in numbers as baryons do in the universe today.

However, now that the FSM has injected the above two novel features into CP-violations, the above picture for the material content of the universe is also given some new twists.

• (a) In the FSM, (I) says that although the theory can be made at any scale to be CP-conserving, CP-violations would automatically develop as the scale changed (as the universe cooled), so that there seems to
be no need to assume any pre-existing (or primordial) CP-violation in the universe for the picture outlined above to materialize.

- (b) Most models for baryo- and lepto-genesis would take as input CP-violations from the measured CKM matrix (and presumably also the PMNS matrix now that it is beginning to be seen), but in the FSM the CKM and PMNS matrices are supposed to be themselves the consequence of (I) above. It would seem therefore, that a study of the effect directly from (I) on baryo- and lepto-genesis might give some new insight.

- (c) Parallel considerations to those above for the standard sector would suggest by (I) that massive particles in the hidden sector would also annihilate into photons when the universe cooled, leaving as residue by (II) only a small excess of particles over anti-particles. In other words, given that most hidden sector particles are expected to end up as dark matter, this would seem to mean that the overwhelming predominance in number of photons over massive matter particles would be maintained in the dark sector as it was in the luminous sector. But this might not be the case without (II), that is, if we had allowed CP to be violated at the strong interaction level in the hidden sector.

- (d) Given that the mechanisms suggested above for CP-violation are similar in the two sectors, it is tempting to assume that roughly the same amount of matter would survive annihilation when the universe cooled. Recall now that colour framons [\text{CF}] in (3) number more in degrees of freedom than flavour framons [\text{FF}] in (2) (of which, as noted there, only one column needs be kept [2]) by a ratio of 9:2; and that colour framons are what make up matter in the hidden sector as opposed to flavour framons which make up matter in our standard sector. Then, if we really assume that the same proportion in each sector would survive annihilation when the universe cooled, we would end up with the same ratio $9/2 = 4.5$ of dark matter in the hidden sector to luminous matter in our standard sector (i.e. dark matter making up 82 percent of the total). This is not far from what is given in [24] for $\Omega_{cdm}/\Omega_b \sim 5.3$ (or dark matter making up 84 percent of the total). The assumptions one started with, however, are much too simplistic for this coincidence, though interesting, to be taken seriously at present. First, CP-violation, though necessary, is not the only factor that would govern
the amount of excess matter that survived annihilation. And secondly, in the hidden sector, one does not even know the particle spectrum well enough to decide which particles are likely to dominate as dark matter. Even if one assumes that, in parallel to the standard sector where baryons dominate, co-baryons also dominate as dark matter in the hidden sector, sufficient differences in property exist between baryons and co-baryons to dissuade one from drawing too close a parallel between them.

5

Again, we need to ask what would change if the assumption of rotation for the zero mode $\nu'$ in the hidden sector is dropped. In view of the observations made in the last paragraph in the preceding section, one sees that everything in this section is left unchange except for (c) and (d) of subsection 4.2. Without this assumption, one will need another source for CP-violation in the hidden sector for massive particles to survive annihilation when the universe cooled. Moreover, co-quarks and co-leptons with vanishing physical masses may no longer be avoidable. Now although one has no direct evidence against zero mass co-quarks or co-leptons, their existence may give more hot dark matter in the universe than one would like.

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5For instance, co-baryons are binary objects, each formed from 2 co-quarks by flavour $su(2)$ confinement while baryons are trinary, each formed from 3 quarks by colour $su(3)$ confinement. This means that, if the formation of either had to compete with other processes, co-baryons could end up in larger numbers than baryons would. And besides, co-baryons are bosonic while baryons are fermionic, adding thus further food for thought.
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