Simulation-based process windows simultaneously considering two and three conflicting criteria in injection molding

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Process windows in injection molding are habitually built with only one performance measure in mind. A more realistic picture can be obtained when considering multiple performance measures at a time, especially in the presence of conflict. In this work, the construction of process windows for injection molding is undertaken considering two and three performance measures in conflict simultaneously. The best compromises between the criteria involved are identified through the direct application of the concept of Pareto-dominance in multiple criteria optimization. The aim is to provide a formal and realistic strategy to set processing conditions in IM operations. The resulting optimization approach is easily implementable in MS Excel. The solutions are presented graphically to facilitate their use in manufacturing plants.

Keywords: injection molding; design of experiments; multiple criteria optimization concepts; process windows; manufacturing process setup

1. Introduction

Injection molding (IM) is the most important process for the manufacture of polymer products (Bryce, 1996; Pötsch & Michaeli, 2008). Injection-molded parts are commonly used in daily life, including in automotive parts, packing products, household articles, electronics, and toys, amongst many other items. A critical challenge in IM is the optimal setting of molding conditions to meet multiple quality criteria (Chen, Chuang, Hsiao, Yang, & Tsai, 2009; Chen, Fu, Tai, & Deng, 2009; Chen, He, & Xu, 2010; Chen, Lai, Fu, & Chen, 2008; Deng, Chen, Sun, Chen, & Chen, 2008; Erzurumlu & Ozcelik, 2006; Huang & Lin, 2008a, 2008b; Kramschuster, Cavitt, Ermer, Chen, & Turng, 2005; Loera, Castro, Díaz, Mondragón, & Cabrera-Ríos, 2008; Ozcelik, & Erzurumlu, 2006; Yin, Mao, & Hua, 2011). This challenge is further complicated by the fact that the flow of molten polymers is influenced simultaneously by multiple variables. Setting IM process variables is not trivial. The physics and chemistry of polymers are tightly coupled and one change in a particular controllable variable, geared to improve one quality criterion, is often detrimental to another one.

A process window is a graphical representation that typically involves two controllable variables associated with the process, and typically a single response or criterion. The objective is to develop a map that will determine processing conditions in order to achieve a desired level of performance by the selected criterion. In this work, two and

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three conflicting criteria are considered simultaneously in order to determine a process window for injection molding. The best compromises between the criteria involved are identified through the direct application of Pareto-optimality conditions. A process window that considers several criteria and their conflicts simultaneously will provide a more realistic picture of process performance, and thus lead to a more effective IM control through the setting of optimal molding conditions. The construction of multiple criteria process windows is, then, the main contribution of this work.

Out of the many possible criteria to consider, cycle time is important in terms of economics, while others like part shrinkage, part thickness, and total weight are common proxies for part quality. The effect of different processing conditions on shrinkage, warpage, cycle time, and weight of the part has been reviewed in (Chang, Hioe, Villarreal, & Castro, 2009; Chen, Chuang et al., 2009; Chen et al., 2008; Chen, Fu et al., 2009; Kuo & Su, 2007; Mathivanan, & Parthasarathy, 2008; Ozcelik & Erzurumlu, 2006; Li, Zhou, & Zhao, 2009). These works primarily consider a single criterion for process improvement. This, however, is not realistic in the presence of conflict, such as that which occurs in IM and several other manufacturing processes. Although it is possible to establish a path from one attractive processing region to another, the number of possible combinations is infinite if the controllable variables are continuous, or quite large in number if they are discrete. In this work, the task of solving multicriteria optimization problems associated with IM to determine their efficient frontier was undertaken with the aim to make this information available for pertinent decision-making regarding process control.

The results associated with the efficient frontier need to be easily available to the molder for them to be usable. The typical representation of a process window is a graph where one can observe the response surface of a performance measure (PM) of interest and the associated controllable variable values. The representation of a multiple criteria process window in IM is not common in the literature base (Fernandes, Pontes, Viana, & Gaspar-Cunha, 2010; Urbano, Villarreal-Marroquin, Castro, Peña, & Cabrera-Ríos, 2011). In this work, the process window is presented considering two controllable variables, and two and three performance measures as a first approach.

An important contribution of this work is the direct use of Pareto-optimality conditions, which yields optimal compromises among multiple conflicting criteria with certainty. This means that the method proposed here belongs to the category of exact optimization techniques. A different category is that of heuristic methods, where competitive solutions are provided with the understanding that their optimality cannot be established with certainty. Metaheuristic techniques, which marshal several heuristics in a strategic manner, also lack certainty in optimality. In fact, it is often the case that the performances of heuristics and metaheuristics are measured through the comparison of their results to the known optimality of exact solutions. Furthermore, the proposed use of Pareto-optimality conditions permits a complete characterization of the Pareto-efficient frontier of the problem, as opposed to identifying only those that are in the convex region of such frontier. Finally, the method proposed in this work does not require any preference structure among the different performance measures in conflict a priori, providing objectivity to the analysis.

2. Literature review
Many process variables affect the quality characteristics of an IM part. When varied simultaneously, not all of these effects are desirable, thereby creating conflicts. The four
basic categories in which IM process variables are usually grouped are: temperature-related, pressure-related, time-related, and distance-related (Bryce, 1996). Several works in the literature present interesting analyses of IM conditions as discussed next.

Design of experiments (DOE) has become a popular tool to set IM process conditions. DOE is a methodology aimed to carry out an experiment in a way such that appropriate data can be collected to be analyzed through statistical methods, resulting in valid, repeatable, and objective conclusions (Montgomery, 2008). A DOE can be used to determine the effect of process settings in different IM performance measures (PMs), such as shrinkage, warpage, process cycle time, maximum injection pressure inside the mold, and total part weight, amongst others.

The fractional factorial design, for example, has been applied as an initial screening approach to determine which process variables are important to control relevant PMs in IM (Kramschuster et al., 2005; Mathivanan, & Parthasarathy, 2008; Zheng, Wu, Xia, & Chen, 2005), as well as in several other manufacturing processes (Kim, Son, Yang, & Yaragada, 2003; Santilli, Puente, & Tanco, 2011). To the same end, the orthogonal array design has been used by the practitioners of Taguchi methods in IM (Barzegari & Rodriguez, 2009; Chen, Chuang et al., 2009; Chen, Fu et al., 2009; Kuo & Su, 2007; Ozcelik & Erzurumlu, 2006). DOE has also been used to carry out material characterization as in the studies presented in (Barzegari & Rodriguez, 2009; Pantani, Cocomullo, Speranza, & Titomanlio, 2005). It is sound to use DOE as a means to detect important process variables. Special care, however, must be exercised in selecting an adequate design for a particular objective, because it plays an important role in the capabilities of any DOE study.

Optimization is another concern associated with IM process. Optimization, in the related literature, is widely understood as the manipulation of controllable variables to improve the performance of a system. In its most complete sense, optimization entails providing evidence of the dominance of a particular configuration of the system over all possible configurations (global optimality). Several IM optimization endeavors can be found in the literature, utilizing different kinds of techniques (Chen et al., 2008; Deng et al., 2008; Erzurumlu & Ozcelik, 2006; Fernandes et al., 2010; Huang & Lin, 2008b; Loera et al., 2008; Ozcelik, & Erzurumlu, 2006). These works are further categorized below.

IM optimization is often approached by focusing on a single criterion or PM. Using a sequence of an initial experimental design and an empirical model such as a regression equation, an artificial neural network (ANN), or a kriging model are also frequently used strategies. In (Ozcelik & Erzurumlu, 2006), an optimization strategy based on ANNs and genetic algorithms (GA) aims to minimize the warpage of an injection molding part using Moldflow simulations. The ANN is used to model part warpage as a function of mold and melt temperature, packing pressure, packing time, cooling time, as well as gate location and runner type. A GA is then used to find the minimum warpage based on the ANN approximation. A similar approach using ANNs and GAs was presented in (Chen et al., 2008), focusing on the optimization of part weight. Minimizing a part’s volumetric shrinkage, through the use of regression and steepest descent, was the subject of a study presented in (Deng et al., 2008) based on simulations and experimental validation runs. A strategy using a kriging model and global optimization techniques is presented in (Gao & Wang, 2009) to reduce part warpage. From these works, it is notable how the use of empirical models – fit through a well-crafted DOE – can be advantageous to expedite an optimization procedure. The fact that IM involves many
PMs that exhibit conflict, however, renders the use of a single criterion insufficient to realistically represent the decision-making involved in setting process conditions.

When considering multiple conflicting criteria in simultaneous optimization (Custódio, Madeira, Vaz, & Vicente, 2011; Takbiri & Afshar, 2012), there is not a single optimum solution, but rather a series of best compromises or trade-off solutions (Deb, 2001). These results are called Efficient solutions (Pareto-efficient, formally), and form the efficient frontier of a multiple criteria optimization problem. Multiple criteria optimization has actually been considered in various works, including (Branke, Kauler, & Schmeck, 2001; Castro, Cabrera-Ríos, & Mount-Campbell, 2004; Cheng, Tan, & Jin, 2009; Ehrgott & Klamroth, 1997; Erzurumlu, & Ozcelik, 2006; Fernandes et al., 2010; Ferreira, De Weck, Saraiva, & Cabral, 2010; Huang & Lin, 2008b; Loera et al., 2008; Villarreal-Marroquín, Cabrera-Ríos, & Castro, 2011).

One particularly interesting coincidence in many of these works is the use of artificial intelligence methods to approach the optimization task. Several variations of multiple objective GAs and Evolutionary Algorithms (EAs) are used across these works to determine the best compromises on a number of criteria (Ganesan, Elamvazuthi, & Vasant, 2011; Mateo & Alberto, 2012). GAs and EAs fall into the category of metaheuristics (Ganesan et al., 2011; Mateo & Alberto, 2012) and have, indeed, proven to be very competitive in finding efficient solutions. Their effective use, however, does involve the setting of a series of parameters by the user that affect the final solution. The challenge of choosing one particular efficient solution over another is not approached in many of these works, irrespective of the technique used. A process window that captures at least two PMs and helps to choose a particular set of conditions would be an initial way to address this challenge.

In light of the limitations faced by several multiple criteria optimization representations, (Faulkenberg, & Wiecek, 2009) and the incompletely solved challenge of selecting among several efficient solutions, Sayarshad and Marler (2009) developed an approach that incorporates preferences in an evolutionary algorithm. This algorithm is then combined with Pareto-optimality conditions (Mateo & Alberto, 2012) to arrive to a final decision. Because the output of a multiple criteria optimization problem is multidimensional, its representation is difficult and thus its use might not be straightforward. For this reason, a multiple criteria process window could prove useful here as well.

Our research group has advocated the use of Data Envelopment Analysis (DEA) for multiple criteria optimization in polymer processes (Cabrera-Ríos, Castro, & Mount-Campbell, 2004; Castro et al., 2004; Loera et al., 2008; Urbano et al., 2011; Villarreal-Marroquin et al., 2011). Our analysis strategies capitalize on DOE, empirical modeling, and DEA. DEA has the advantage of finding efficient solutions through the use of a series of linear optimization problems, which is very convenient, computationally speaking. Furthermore, the solutions found through DEA are undoubtedly Pareto-efficient, which makes it an exact method (non-heuristic). The efficient frontier identified through DEA, however, is oftentimes constrained to convexity. If the true efficient frontier of a multiple criteria optimization problem is not convex, then several efficient solutions might escape detection through DEA. An idea proposed previously in our group entailed using additional optimization problems to detect the efficient solutions lying in non-convex regions with success; however, it is still a somewhat elaborate process (Urbano et al., 2011). The work in (Urbano et al., 2011), along with its graphical representation of a multiple criteria process window in the IM process, laid the groundwork for the proposed method in our approach. For completeness, a review of single-criterion process window works can be found in (Hessel, Cortese, & Croon, 2011).
It is clear, from the review of literature, that setting process conditions in IM is truly a multiple criteria optimization problem that requires a user-friendly representation for its application by molders in manufacturing plants.

3. Proposed method
The method proposed to build a multiple criteria process window for the IM process is shown in Figure 1. This method integrates different techniques such as Design of Experiments and Multiple Criteria Optimization.

**Step 1. Initial experimental design**
The initial experimental design must be able to provide data to correctly build a second order regression model. For the purpose of this method, the use of a central composite design is highly advised.

**Step 2. Scale the controllable variables (CVs) and performance measures (PMs) to fall within 1 and −1**
Each CV and PM must be in the same scale. This is accomplished through suitable affine transformation of the form:

$$x^{\text{trans}} = ax + b$$

where $a$ and $b$ are scalar constants and $x$ is the variable subject to transformation.

![Figure 1](attachment:image.png)

Figure 1. Proposed method for the construction of the multiple criteria process window for injection molding process based on simulation.
Step 3. Fit a metamodel per PM

The scaled data are used to generate second-order regression metamodels. It is important that a competitive fit is sought. An R–Sq ≥ 90% on each metamodel, for example, can be used. R–Sq is the coefficient of determination calculated as:

\[ R^2 = 1 - \frac{SSE}{SST} \]  \hspace{1cm} (2)

where SSE is the sum of squared prediction errors and SST is the sum of squared deviations of each datum to the overall mean.

Step 4. Use the metamodels to generate PMs’ predictions in the experimental region of the CVs

PMs’ predictions are obtained following a factorial grid on the experimental region of the CVs. Each point in the grid corresponds to a particular combination of CVs’ values. Figure 2 shows the process used in this work to map out the feasible region as explained below:

(i) Vary from −1 to 1 in equally sized increments as to discretize the experimental region. In this work, an increment of .01 on each CV for a total number of combinations of 441 (21 values per CV).
(ii) The values of each combination are used to generate PM predictions.
(iii) The predictions, which are also pairs of values, (\( \hat{y}_1, \hat{y}_2 \)) can be plotted in a scatter diagram when limited to two performance measures.

It is clear that different step sizes can be used as necessary to accommodate different degrees of precision.

Step 5. Find the efficient frontier using the PM’s predictions

The best compromises between the two criteria are identified through the application of two conditions as described next:

(i) Condition 1: The solution X(1) is no worse than X(2) in all criteria.
(ii) Condition 2: The solution X(1) is strictly better than X(2) in at least one criterion.

The points that meet both conditions are considered Pareto-efficient, and are thus the best compromises between the PMs. The combinations of values of the CVs associated with the efficient solutions are said to dominate the rest of the set. After this step, the Efficient Frontier and their associated processing conditions are identified.

To illustrate the application of the concept represented by conditions (1) and (2), Figures 3 and 4 are considered. Seven solutions are presented in the space defined by two criteria, F1 and F2, respectively, the first of which is to be maximized, while the second one must be minimized. In Figure 3, the solution with maximum value in F1 and the solution in Figure 4 with the minimum value in F2 are found. The process to find these solutions is to perform an exhaustive pairwise comparison among all solutions in the grid.
In order to find the best compromises between F1 and F2, the cone defined by the optimization directions of these criteria is considered. This cone is illustrated in Figure 5. A solution resting at the origin of this cone is dominated by any solution within the

Figure 2. Diagram of the process to map out the feasible region.

Figure 3. Individual pairwise comparison per objective function (F1).

In order to find the best compromises between F1 and F2, the cone defined by the optimization directions of these criteria is considered. This cone is illustrated in Figure 5. A solution resting at the origin of this cone is dominated by any solution within the
cone. A solution resting at the origin of this cone is Pareto-efficient only if this cone is empty. Thus, this cone is, conceptually, one of dominance.

Figure 6 shows the application of definitions (1) and (2). If solution 2 is placed at the origin of a dominance cone, many solutions are found within it, and therefore solution 2 is a dominated solution. On the other hand, when solution 5 is analyzed, an empty dominance cone is found. Solution 5 is, then, an efficient solution. Figure 7 shows the sets of dominated and efficient solutions in this example. Once the efficient set has been identified, these solutions can be traced back to their corresponding processing conditions. This process effectively ties IM multicriteria performance to the controllable variables.

**Step 6. Validation**

The settings prescribed in the previous step should be validated through simulation to establish a truer frontier.

**Step 7: Multiple criteria process window**

The final step shows the results graphically; in the process window, one can observe the best compromises among the PMs of interest and their associated controllable variable values.

Figure 4. Individual pairwise comparison per objective function (F2).

Figure 5. Dominance cone of the optimization direction for the considered criteria.
The application of the proposed method is illustrated through two case studies in the following sections: one with two performance measures and another one with three performance measures. After the case studies, a comparison of the proposed method vs. the use of DEA is discussed. Do notice that DEA is also an exact method that does not require a preference structure among the multiple criteria to be defined a priori. Therefore, the comparison is a suitable one.

4. Injection molding case 1: minimize total weight and minimize volumetric shrinkage
This case study demonstrates the capability of the proposed method. A mold to produce ASTM testing specimens (Figure 8), as well as its associated finite element mesh were used for this purpose. The finite element mesh was used for simulation with the software MOLDFLOW 2011, installed in a MacPro workstation with an architecture of
64 bits. Moldflow has been a standard for research and industry applications in the last decade for polymer processing simulation, and is therefore considered in this work. However, the proposed method can be used with other simulation packages.

The PMs considered in this first case are Total Weight (g) and Volumetric shrinkage (%). Both PMs impact dimensional stability and the quality of molded parts, and both must be minimized. The CVs are Melt Temperature and Packing Pressure. These are varied in the ranges [227, 260] °C and [13.78, 41.36] MPa, respectively. The material chosen was D21148 Lyondel Bassel Advanced Polyolefin USA Inc. For the case 1, Mold temperature is left constant at 32 °C, injection time at 1 s, Velocity/Pressure Switch at 80%, packing time at 10 s, and cooling time at 20 s. The resulting optimization problem in its most basic form is as follows:

\[
\begin{align*}
\text{Find } Tm, Pp \text{ to} \\
\text{Minimize } TW(Tm, Pp) \\
\text{Minimize } VS(Tm, Pp) \\
\text{Subject to} \\
227 \leq Tm \leq 260 \\
13.78 \leq Pp \leq 41.36
\end{align*}
\]

where \( TW \) is the Total Weight, \( VS \) is the Volumetric Shrinkage, \( Tm \) is Melt Temperature, and \( Pp \) is Packing Pressure. The application of the proposed method is detailed next.

**Steps 1 and 2: Perform an initial experimental design and scale the variables**

A central composite design involving both variables and both PMs was carried out. The initial experimental design is shown in Table 1. The results of experimental simulation runs were carried out using Polymer Simulator Insight Moldflow software 2011. The data was transformed using Equation (1).
Step 3: Fit a metamodel per PM

The initial scaled data generated in the previous step were used to create metamodels. One second-order regression metamodel per performance measure was generated as shown below following the same notation as in P1:

\[ TW = 0.1158 - 0.5190Tm + 0.4640Pp + 0.0046Tm^2 - 0.0901Pp^2 + 0.0318TmPp \quad (3) \]

\[ VS = -0.0693 + 0.8430Tm - 0.1704Pp + 0.0568Tm^2 + 0.0568Pp^2 + 0.0426TmPp \quad (4) \]

Regressions (3) and (4) had R-sq values of 99.93 and 99.96%, respectively.

Step 4: Use the metamodels to generate PMs predictions in the experimental region of the CVs

The metamodels and the 441 combinations of CV’s shown in Figure 9 as a grid were used to predict the values of the PMs to find Predicted Feasible Region (PFR) in the space of the criteria. The previous second-order regression metamodels and the 441 combinations of CVs were used to predict the values per PM and find PFR. The PFR is show in Figure 10.

Table 1. Initial experimental design for the study case 1.

| Run | Melt temperature (°C) | Packing pressure (MPa) | Total weight (g) | Volumetric shrinkage (%) |
|-----|------------------------|------------------------|-----------------|------------------------|
| 1   | 227                    | 13.79                  | 33.6981         | 15.57                  |
| 2   | 227                    | 27.58                  | 33.9983         | 15.24                  |
| 3   | 227                    | 41.36                  | 34.2365         | 15.10                  |
| 4   | 243                    | 13.79                  | 33.3561         | 16.40                  |
| 5   | 243                    | 27.58                  | 33.6962         | 16.13                  |
| 6   | 243                    | 41.36                  | 33.9141         | 16.01                  |
| 7   | 260                    | 13.79                  | 33.0058         | 17.33                  |
| 8   | 260                    | 27.58                  | 33.3884         | 17.17                  |
| 9   | 260                    | 41.36                  | 33.6225         | 17.05                  |

Figure 9. Experimental region of CV’s for case 1.
Step 5: Find the efficient frontier using the PMs’ predictions

In order to find the Predicted Efficient Solutions (PES) and their associated values of CVs, conditions (1) and (2) are applied. The pairwise comparisons were carried out using an Excel spreadsheet. Figure 11 shows the predicted Non-Dominated set of solutions, while Figure 12 shows the shape of the PFR and the location of PES.

Step 6: Validation

All PES were used to perform validation runs. As it can be observed in Figure 13, several simulated volumetric shrinkage values are higher than predicted. This performance was caused by the use of a cooling time set at 20 s in the simulation. If the efficient solutions are considered alone, Figure 14 results.

Step 7: Multiple criteria process window

The final step is the representation of results graphically. In our case, this is called a Multiple Criteria Process Window. Figure 15 shows the efficient set classified in three areas, depending on the different level of compromise between PMs with the associated IM settings.

Figure 10. Predicted Feasible Region for the performance measures of the study case 1.

Figure 11. Predicted non-dominated (squares) and dominated sets (empty circles) for case 1.
Figure 12. Predicted Efficient Solutions in the Predicted Feasible Region.

Figure 13. Predicted values (empty circles) vs. simulated values (full circles) for case 1.

Figure 14. Graph with the predicted efficient values (triangle) and the simulated values classified in efficient (full circles) and dominated values (empty circles).
5. Injection molding case 2: minimize total weight, minimize volumetric shrinkage, and minimize cycle time

Case 2 considers the simultaneous optimization of three PMs. All of them must be minimized: Volumetric shrinkage (%), Total Weight (g), and Cycle Time (s). Two CVs were selected: packing pressure and melt temperature. These were varied in the ranges [13.79, 41.36] MPa and [227, 260] °C. Experimental ranges for the CVs were determined using the recommended values from Moldflow 2011 material’s database. The chosen material was: D21148 Lyondel Bassel Advanced Polyolefin USA Inc. The settings were: Mold temperature at 32 °C, Injection time at 1 s, V/P Switch-over at 80%, packing time at 10 s, and cooling time was set in automatic as in the previous case. The optimization problem, then, is as follows using the same notation as in P1:

\[
\begin{align*}
\text{Find } Tm, Pp & \text{ to } \\
\text{Minimize } TW(Tm, Pp) & \\
\text{Minimize } VS(Tm, Pp) & \\
\text{Minimize } CT(Tm, Pp) & \\
\text{Subject to} & \\
227 \leq Tm \leq 260 & \\
13.79 \leq Pp \leq 41.36 & 
\end{align*}
\]

where \(CT\) stands for cycle time. The application of the proposed method for this case is described next.

**Steps 1 and 2: Initial experimental design and scaling of all variables involved**

A face-centered central composite design involving both variables and both PMs was carried out as shown in Table 2. All results were scaled through using Equation (1).
Step 3: Fit a metamodel per PM

Metamodels were fit for each of the PMs. These are shown below following the same notation used for P1 and P2:

\[
TW = 0.1147 - 0.5128Pp + 0.4685Tm + 0.0121Pp^2 - 0.0843Tm^2 + 0.0456PpTm \quad (5)
\]

\[
VS = -0.0475 + 0.8176Pp - 0.1911Tm + 0.0545Pp^2 + 0.0119Tm^2 + 0.0217PpTm \quad (6)
\]

\[
CT = 0.0799 + 1.0000Pp + 0.0000Tm - 0.0799Pp^2 + 0.0000Tm^2 + 0.0000PpTm \quad (7)
\]

The R-sq values for regressions (5), (6), and (7) were 99.98, 99.52, and 100%, respectively.

Step 4: Use the metamodels to generate PM Predictions in the experimental region of the CVs

Figure 16 shows the grid used to predict the values of the PMs, and thus to find the PFR in the space of the criteria (Figure 17).

Steps 5 and 6: Find the efficient frontier using the PMs’ predictions and validate

In order to find the PES and their associated CVs’ values conditions (1) and (2) were applied as before. Figure 18 shows the shape of the PFR and the location of PES. All PES were validated with the simulation software. The results of this case (Figure 19) show that the proposed method is effective to predict the efficient frontier. The approximation in this case is less than 1% of error and all predicted solutions were indeed in the true efficient frontier.

Step 7: Multiple criteria process window

In Figure 20, the efficient set is classified in three areas, depending on the different level of compromise between PMs. The associated final process settings for case 2 are shown in this figure.

The best compromises for case 2 are found in two areas: (1) setting Melt Temperature at 227 °C and varying Packing Pressure in the range of [13.79, 41.36] MPa; and

| Melt temperature (°C) | Packing pressure (MPa) | Total weight (g) | Volumetric shrinkage (%) | Cycle time (s) |
|-----------------------|------------------------|-----------------|--------------------------|---------------|
| 227                   | 13.79                  | 33.6978         | 15.64                    | 254.3238      |
| 227                   | 27.58                  | 33.9739         | 15.24                    | 254.3238      |
| 227                   | 41.36                  | 34.2130         | 15.11                    | 254.3238      |
| 243                   | 13.79                  | 33.3561         | 16.31                    | 259.5714      |
| 243                   | 27.58                  | 33.6953         | 16.16                    | 259.5714      |
| 243                   | 41.36                  | 33.9110         | 15.99                    | 259.5714      |
| 260                   | 13.79                  | 33.0058         | 17.34                    | 264.3239      |
| 260                   | 27.58                  | 33.3910         | 17.21                    | 264.3239      |
| 260                   | 41.36                  | 33.6307         | 16.91                    | 264.3239      |
(2) setting Packing Pressure in 13.79 MPa while varying Melt Temperature in the range of [227, 260] ºC. The predicted values for the associated PMs are all less than 1% error of the simulated values.

Figure 16. Experimental region of CVs for case 2.

Figure 17. Predicted Feasible Region for case 2.

Figure 18. Predicted efficient set (full circles) and dominated set (empty circles) for case 2.
6. Comparison with data envelopment analysis

As discussed in the literature review section of this manuscript, there are many different methods to find the efficient frontier in multiple criteria problems. A fair comparison of the method would be against an exact method that did not require a preference structure a priori. In our research group, DEA has been used to this end in polymer processes (Cabrera-Ríos et al., 2004; Castro et al., 2004; Loera et al., 2008; Urbano et al., 2011; Villarreal-Marroquin et al., 2011). DEA is an exact and convenient computational method that has the advantage of finding efficient solutions with certainty through the

Figure 19. Predicted values (empty circles) vs. simulated values (full circles) for case 2.

Figure 20. Multiple criteria process window classified in three areas for Case 2.
use of a series of linear optimization problems. The efficient frontier identified through DEA, however, is oftentimes constrained to convexity. If the true efficient frontier of a multiple criteria optimization problem is not convex, then several efficient solutions might escape detection through DEA as shown in Figure 21, where only two efficient solutions are detected when using a convex approximation as in most DEA models. Through application of multiple criteria optimization concepts, as shown in the proposed method, the completed characterization of EF is made, as illustrated in Figure 22.

Strengths of both methods could be combined using DEA to find the location of the efficient frontier with computational convenience, to then finely characterize the efficient frontier through the use of the method proposed here. This integration will be pursued in our research group in the future.

7. Discussion

The strategy proposed in this work can be applied to the solution of problems similar in structure and size to P1 and P2 in injection molding. These types of problems commonly occur in the industry, giving considerable practical value to feasible solution

Figure 21. Efficient frontier using a DEA approach.

Figure 22. Efficient frontier using proposed method.
methods. It is possible to involve more than three performance measures conveniently; however, a more suitable representation would be necessary for cases with more than two controllable variables, since a graph with three dimensions could be difficult to interpret.

The underlying optimization method is basically one of complete pairwise comparison and direct application of the Pareto-optimality conditions. The strategy provides (Pareto) optimal solutions with certainty in spite of the convexity of the true Pareto frontier of the problem. Current MATLAB coding in our group can handle close to 10,000 solutions for pairwise comparison, and clustering schemes have been used to solve problems with up to 30,000 solutions with relative convenience. Thus, precision scalability is feasible for the proposed method.

Finally, it is important to note that the number of simulations required to effectively build a processing window with this strategy is kept to a minimum, thanks to the use of statistically designed experiments, as well as metamodels. This is particularly important for finite element simulations that take a considerable amount of computing time, which, in the experience of the authors, can reach up to several weeks for a single simulation run.

8. Conclusions and future work

In this work, a strategy to find the best compromises between criteria in conflict, based on multiple criteria optimization concepts, is presented. The performance of the strategy was demonstrated through two case studies: the first one with two performance measures and the second one with three performance measures. Both cases evidence the capabilities of the strategy in finding efficient frontiers.

The strategy is feasible, effective, and efficient in prescribing competitive processing conditions in injection molding operations, and provides important information on the trade-offs of quality criteria in conflict with a modest number of runs.

When compared with DEA, it is clear that the proposed strategy provides a better characterization of the efficient frontier. This is, perhaps, however, at the cost of a larger computational burden, primarily stemming from the requirement for extensive pairwise comparisons in cases with more than three performance measures of interest. A future strategy, combining the strength of DEA with the method proposed here, will be developed by our group for such cases.

Future work will also explore the use of clustering techniques to make pairwise comparisons less numerous, thereby providing the possibility of expanding the application of the proposed method to several additional PMs. Finally, transitioning the use of multiple criteria process window analysis to the manufacturing site will be critical.

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References

Barzegari, M. R., & Rodriguez, D. (2009). The effect of injection molding conditions on the morphology of polymer structural foams. Special Issue: POLYCHAR 16 Special Issue, 49, 111 (River Street, Hoboken, NJ 07030-5774, United States).

Branke, J., Kauler, T., & Schmeck, H. (2001). Guidance in evolutionary multi-objective optimization. Advances in Engineering Software, 32, 499–507.

Bryce, D. M. (1996). Plastic injection molding: Manufacturing process fundamentals. Dearborn, MI: SME.

Cabrera-Ríos, M., Castro, J. M., & MountCampbell, C. A. (2004). Multiple quality criteria optimization in reactive in-mold coating with a data envelopment analysis approach II: A case with more than three performance measures. Journal of Polymer Engineering, 24, 435–450 (Department of Industrial, Welding, and Systems Engineering, The Ohio State University, 210 Baker Systems, 1971 Neil Avenue, USA).

Castro, J. M., Cabrera-Ríos, M., & Mount-Campbell, C. A. (2004). Modeling and simulation in reactive polymer processing. Modeling and Simulation in Materials Science and Engineering, 12, S121–S149.

Chang, K. C., Hioe, Y., Villarreal, M. G., & Castro, J. M. (2009). Minimum ‘safe cycle time’: Selecting the frozen layer thickness. Polymer Engineering & Science, 49, 2320–2328.

Chen, C. P., Chuang, M. T., Hsiao, Y. H., Yang, Y. K., & Tsai, C. H. (2009). Simulation and experimental study in determining injection molding process parameters for thin-shell plastic parts via design of experiments analysis. Expert Systems with Applications, 36, 10752–10759.

Chen, W. C., Fu, G. L., Tai, P. H., & Deng, W. J. (2009). Process parameter optimization for MIMO plastic injection molding via soft computing. Expert Systems with Applications, 36, 1114–1122.

Chen, J. W., He, L., & Xu, B. P. (2010, June). The application of cavity pressure profile in the injection molding process parameters optimization. International Conference on Mechanic Automation and Control Engineering, MACE2010, Wuhan, China, 5350–5353.

Chen, W. C., Lai, T. T., Fu, G. L., & Chen, C. T. (2008). A systematic optimization approach in the MISO plastic Injection molding process. IEEE International Conference on Service Operations and Logistics, and Informatics, 2, 2741–2746.

Cheng, J., Tan, J. R., & Jin, L. L. (2009). Multi-objective intelligent optimization of injection molding parameters based on BP-NSGA. Jisuanji Jicheng Zhizao Xitong/Computer Integrated Manufacturing Systems, CIMS, 15, 1900–1906.

Data base Moldflow Insight. (2011).

Custódio, A. L., Madeira, J. F. A., Vaz, A. I. F., & Vicente, L. N. (2011). Direct multisearch for multiobjective optimization. SIAM Journal on Optimization, 21, 1109–1140.

Deb, K. (2001). Multi-objective optimization using evolutionary algorithms. New York, NY: John Wiley.

Deng, W. J., Chen, C. T., Sun, C. H., Chen, W. C., & Chen, C. P. (2008). An effective approach for process parameter optimization in injection molding of plastic housing components. Polymer – Plastics Technology and Engineering, 47, 910–919.

Ehrgott, M., & Klamroth, K. (1997). Connectedness of efficient solutions in multiple criteria combinatorial optimization. European Journal of Operational Research, 97, 159–166.

Erzurumlu, T., & Ozcelik, B. (2006). Minimization of warpage and sink index in injection-molded thermoplastic parts using Taguchi optimization method. Materials and Design, 27, 853–861.

Faulkenberg, S. L., & Wiecek, M. M. (2009). On the quality of discrete representations in multiple objective programming. Optimization and Engineering, 11, 423–440.

Fernandes, C., Pontes, A. J., Viana, J. C., & Gaspar-Cunha, A. (2010). Using multi objective evolutionary algorithms in the optimization of operating conditions of polymer injection molding. Polymer Engineering & Science, 50, 1667–1678.

Ferreira, I., De Weck, O., Saraiva, P., & Cabral, J. (2010). Multidisciplinary optimization of injection molding systems. Structural and Multidisciplinary Optimization, 41, 621–635.

Ganesan, T., Elamvazuthi, I., & Vasant, P. (2011). Evolutionary normal boundary intersection (ENBI) method for multi-objective optimization of Green Sand Mould system. IEEE International Conference on Control System, Computing and Engineering, 86–91.

Gao, Y., & Wang, X. (2009). Surrogate-based process optimization for reducing warpage in injection molding. Journal of Materials Processing Technology, 209, 1302–1309.
Hessel, V., Cortese, B., & de Croon, M. H. J. M. (2011). Novel process windows – Concept, proposition and evaluation methodology, and intensified superheated processing. Chemical Engineering Science, 66, 1426–1448.

Huang, M. S., & Lin, T. Y. (2008a). An innovative regression model-based searching method for setting the robust injection molding parameters. Journal of Materials Processing Technology, 198, 436–444.

Huang, M. S., & Lin, T. Y. (2008b). Simulation of a regression-model and PCA based searching method developed for setting the robust injection molding parameters of multi-quality characteristics. International Journal of Heat and Mass Transfer, 51, 5828–5837.

Kim, I. S., Son, K. J., Yang, Y. S., & Yaragada, P. K. (2003). Sensitivity analysis for process parameters in GMA welding processes using a factorial design method. International Journal of Machine Tools and Manufacture, 43, 763–769.

Kramschuster, A., Cavitt, R., Ermer, D., Chen, Z., & Turng, L. S. (2005). Quantitative study of shrinkage and warpage behavior for microcellular and conventional injection molding. Polymer Engineering & Science, 45, 1408–1418.

Kuo, C. F., & Su, T. L. (2007). Optimization of injection molding processing parameters for LCD light-guide plates. Journal of Materials Engineering and Performance, 16, 539–548.

Li, D., Zhou, H., Zhao, P., & Li, Y. (2009). A Real-time process optimization system for injection molding. Polymer Engineering & Science, 49, 2031–2040.

Loera, V. G., Castro, J. M., Diaz, J. M., Mondragon, O. L., & Cabrera-Rios, M. (2008). Setting the processing parameters in injection molding through multiple-criteria optimization: A case study. IEEE Transactions on Systems, Man, and Cybernetics, Part C: Applications and Reviews, 38, 710–715.

Mateo, P. M., & Alberto, I. (2012). A mutation operator based on a pareto ranking for multi-objective evolutionary algorithms. Journal of Heuristics, 18, 53–89.

Mathivanan, D., & Parthasarathy, N. S. (2008). Prediction of sink depths using nonlinear modeling of injection molding variables. The International Journal of Advanced Manufacturing Technology, 43, 654–663.

Montgomery, D. C. (2008). Design and analysis of experiments. New York, NY: John Wiley.

Ozcelik, B., & Erzurumlu, T. (2006). Comparison of the warpage optimization in the plastic injection molding using ANOVA, neural network model and genetic algorithm. Journal of Materials Processing Technology, 171, 437–445.

Pantani, R., Cocco, I., Speranza, V., & Titomanlio, G. (2005). Modeling of morphology evolution in the injection molding process of thermoplastic polymers. Progress in Polymer Science, 30, 1185–1222.

Pötsch, G., & Michaeli, W. (2008). Injection molding: An introduction. Munich: Carl Hanser Verlag.

Santilli, A., Puente, I., & Tanco, M. (2011). A factorial design study to determine the significant parameters of fresh concrete lateral pressure and initial rate of pressure decay. Construction and Building Materials, 25, 1946–1955.

Sayarshad, H. R., & Marler, T. (2009). A new multi-objective optimization formulation for railcar fleet sizing problems. Operational Research, 10, 175–198.

Takbiri, Z., & Afshar, A. (2012). Multi-objective optimization of fusegates system under hydrologic uncertainties. Water Resources Management, 26, 2323–2345.

Urbano, M. A., Villarreal-Marroquin, M. G., Castro, J. M., Peña, M. S., & Cabrera-Rios, M. (2011). Setting process conditions under multiple criteria. [Online]. Retrieved November 26, from http://www.highbeam.com/doc/1P3-1848735301.html

Villarreal-Marroquin, M. G., Cabrera-Rios, M., & Castro, J. M. (2011). A multicriteria simulation optimization method for injection molding. Journal of Polymer Engineering, 31, 397–407.

Yin, F., Mao, H., & Hua, L. (2011). A hybrid of back propagation neural network and genetic algorithm for optimization of injection molding process parameters. Materials and Design, 32, 3457–3464.

Zheng, Z. X., Wu, Y. B., Xia, W., & Chen, W. P. (2009). Process optimization of mid-density tungsten alloy for injection process in metal injection molding based on numerical simulation. Materials Science Forum, 628–629, 599–604.