The enhancement of mathematical reasoning and problem solving ability through metaphorical thinking approach

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Abstract. The objective of this study was to determine the effect of metaphorical thinking learning approach on mathematical reasoning and problem solving ability. This research used quasi-experimental method. The population of this study was the third-semester students of mathematics education study program at STKIP PGRI Lubuklinggau. The sample selection was done by using saturated samples. Based on the analysis of the t-test, the results of the study showed that 1) \( t_{obtained} (2.351) \) of the ability of mathematical reasoning was more than the \( t_{table} (2.06) \), so \( H_0 \) was rejected and \( H_a \) was accepted. It can be concluded that there was significant influence of the metaphorical thinking learning approach on the students' mathematical reasoning ability; and 2) \( t_{obtained} (2.311) \) of the problem solving skill was more than the \( t_{table} (2.06) \), so \( H_0 \) was rejected and \( H_a \) was accepted. It can be concluded that there was a significant effect of the metaphorical thinking approach on the students' problem solving skill. In short, metaphorical thinking approach has influenced the students' mathematical reasoning and problem solving ability significantly.

1. Introduction
The National Research Council states that "Mathematics is the key to opportunity." It is the key to the chances of success. The problem solving and mathematical reasoning abilities are the skills that have been done through the standard process of mathematics by the National Council of Teachers of Mathematics (NCTM), which include problem solving, reasoning and evidence, links, communication and representation [1,2].

One of the goals and principles of the implementation of higher education is interpreted that students must have the ability to solve various problems independently and be able to apply them to the community. The problem can be resolved if the students has the ability of reasoning and problem solving abilities [3]. Thus, the process of learning mathematics in higher education must be able to empower the ability of mathematical reasoning and problem solving in the teaching and learning process.

Holmes stated that "the background or the reason someone needs to learn to solve mathematical problems is the fact in this twenty-first century that people who are able to solve life's problems productively." He added that “those who are able to solve problems become more productive workers and understand complex issues related to the global society” [4]. One branch of science in mathematics is analytic geometry. It is a branch of mathematics which is a combination of algebra and geometry. Analytic geometry is divided into two stages, namely field analytic geometry and space analytic geometry. The concept of field analytic geometry usually uses Cartesian coordinates for plane, line, and geometrical equations which are often in two or dimensions [5].

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Field analytic geometry is one of the compulsory courses with 3 credits taken by the third semester students in the Mathematics Education Study Program of STKIP PGRI Lubuklinggau. This course discusses conic sections including equations of circles, parabolas, ellipses, and hyperboles to complete and depict angles and distances on planes, complete vector calculations and determine comparisons of lines and vector multiplication and draw solid figure. This course also aims to equip students with practical insight and abilities related to the field analytic geometry needed to identify, analyze and prove a field in the solid figure. Mathematics in higher education is very different from mathematics in the education unit level. The mathematics studied is more abstract and requires a high level of cognitive ability. According to Ruseffendi, mathematics in higher education includes 4 broad insights namely arithmetic, algebra, geometry and higher analysis [6].

Based on the results of the survey in the last two years on the final grades in the field of analytic geometry of the third semester students of Mathematics Education Study Program of STKIP PGRI Lubuklinggau, there were still some students who scored C or grades below 65. In addition, seen from the analysis of the results of the midterm exam, there were still errors experienced by the students when they solved problems related to conic sections. They still had difficulties in analyzing problems related to the equation in the cone section. Some students understood that it was like a nightmare since there were many questions, numbers and formulas and abstractions. However, they should have been able to analyze the problems in analytic geometry both from reasoning and problem solving abilities during lectures.

Through the implementation of the metaphorical thinking approach as an alternative solution to improve the lecturing process, it is expected to trigger the development of students’ mathematical reasoning and problem solving ability. The metaphorical thinking approach is a learning approach that uses metaphors to explain a concept systematically. The metaphor used in this approach is the process of transferring new meanings and associations from one abstract object or idea to another object or idea that is better known. Through the metaphorical process, students are trained to see the relationships between the knowledge (concepts) they have obtained and the knowledge (concepts) they will obtain. They are also trained to analogize a model and interpretation of the knowledge they build [7]. Furthermore, metaphors connect abstract ideas to concrete forms, giving rise to relations among previous experiences [8].

The characteristic of the metaphorical thinking approach is to bridge abstract concepts into more concrete things. These concepts are explained through metaphors or analogies by comparing two or more things with different meanings. Metaphorical thinking is a bridge between models and interpretations that provides great opportunities for students to explore their knowledge in learning mathematics. Then through this way, the learning process of students becomes more meaningful because they can see the relationship between the concepts they learn and the concepts they have known [9]. In brief, metaphorical processes are expected to improve students’ reasoning and mathematical problem solving ability.

2. Methods
This is an experimental research. The research was conducted on third semester students of mathematics education study program of STKIP PGRI Lubuklinggau in the academic year 2018/2019. Saturated sampling technique was used because all members of the population were used as samples. This is often done when a relatively small population only has fewer than 30 people or the research wants to make generalizations with very small errors [10]. The consideration to use this sampling technique was due to the same characteristics that can represent the characteristics of the population and in accordance with the objective of the researcher. In this case, the researchers took 13 students from III-A class and 14 students from III-B class. The design of this study is as follows.
Table 1. Research design.

| Class      | Pretest | Treatment | Posttest |
|------------|---------|-----------|----------|
| Experimental | T₁     | X        | T₂       |
| Control    | T₁     | Y        | T₂       |

Notes:
T₁: Pretest of mathematical reasoning and problem solving ability
T₂: Posttest of mathematical reasoning and problem solving ability
X: Methaporical Thinking Approach
Y: Conventional Approach

The instrument consisted of two mathematical reasoning ability test questions and two problem solving ability test questions arranged based on their mathematical ability indicators. The indicators of mathematical reasoning ability used in this research were by 1) presenting mathematical statements verbally, in writing, pictures and graphics; 2) doing mathematical manipulation; 3) compiling and providing reasons for several solutions; 4) drawing conclusions from statements; and 5) checking the validity of the arguments. On the other hand, the indicators of students' mathematical problem solving ability used in this research by being able to 1) understand the problem, 2) plan strategical solutions, 3) carry out the plan, and 4) explain and check the truth of the answers obtained [11]. The data collection techniques were through the reporting technique of reasoning ability with a maximum score of 15 and mathematical problem solving ability with a maximum score of 11. After the data were collected, testing of the research hypothesis that had been formulated was done to see the effect of the metaphorical thinking approach on the students’ mathematical reasoning and problem solving ability. In testing the hypothesis, the requirements consisted of 1) normal distribution of data; 2) homogeneity of variance; and 3) a subsequent t-test.

3. Results and discussion

3.1. Descriptive analysis

The following tables present the calculation of the final test recapitulation of mathematical reasoning and problem solving ability.

Table 2. Descriptive analysis of mathematical reasoning ability test.

| Class      | N  | X_{min} | X_{max} | \bar{x} | Sd  |
|------------|----|---------|---------|--------|-----|
| Experimental | 13 | 16      | 28      | 22.85  | 3.193 |
| Control    | 14 | 10      | 28      | 18.43  | 5.63 |

Table 3. Descriptive analysis of problem solving ability test.

| Class      | N  | X_{min} | X_{max} | \bar{x} | Sd  |
|------------|----|---------|---------|--------|-----|
| Experimental | 13 | 13      | 20      | 16.92  | 2.397 |
| Control    | 14 | 7       | 21      | 13.86  | 4.185 |

From tables 2 and 3, it can be seen that the experimental class has an average score that is higher than the control class. This is because the ability of mathematical reasoning and problem solving during the teaching and learning process was emphasized on the ability to connect between mathematical concepts and real phenomena around them. Through metaphorical thinking approach, a number of mathematical concepts learned from one's own experience will easily build a mathematical model with an accurate interpretation. Students experienced the integrative process between the mathematical model and its application, so the concept of thinking mathematically infinite students...
that can be compensated and given a concrete picture to understand the abstract concept of geometry [9,12].

3.2. Statistical analysis of mathematical reasoning ability

The results of the normality test of the posttest score from the experimental class and the control class on mathematical reasoning ability using SPSS 24.0 are presented below.

| Class     | Kolmogorov-Smirnov*a |
|-----------|-----------------------|
|           | Statistic | df | Sig.     |
| Experimental | .165      | 13 | 0.200*  |
| Control    | .170      | 13 | 0.200*  |

Based on the table 4 above, the significance level of experimental and control class is of \( p = 0.200 \), so \( p > 0.05 \). It means that mathematical reasoning ability of the experimental class and the control class came from a normally distributed population.

The homogeneity test results of posttest scores on mathematical reasoning ability by using SPSS 24.0 are shown in the following table.

| Levene's Test for Equality of Variances |
|---------------------------------------|
| Equal variances assumed               |
| \( F \) | \( 0.937 \) | \( 0.342 \) |

Based on the table 5 above, the significance level of experimental and control class is of \( p = 0.342 \), so \( p > 0.05 \). It means that mathematical reasoning ability of the experimental class and the control class came from homogenous population.

The t-test results of students' mathematical reasoning ability are shown in the following table.

| t-test for Equality of Means |
|-----------------------------|
| Equal variances assumed     |
| \( t \) | \( 2.351 \) | \( 25 \) | \( 0.027 \) |
| Equal variances not assumed |
| \( t \) | \( 2.382 \) | \( 23.239 \) | \( 0.026 \) |

From the table, the t-test results on the posttest scores of students’ mathematical reasoning ability show \( t \)-obtained is 2.351 and \( t \)-table is 2.06 with \( p \) (Sig. (2-tailed)) is 0.027. It means that \( t \)-obtained is more than \( t \)-table and \( p \)-obtained is more than 0.05. Therefore, \( H_0 \) is rejected, and \( H_a \) is accepted. From the two classical and probabilistic approaches, it can be concluded that the average posttest scores of students’ mathematical reasoning ability through metaphorical thinking approach is more than the average score of students’ mathematical reasoning ability through conventional approach. Because in the metaphorical thinking approach the learning process is carried out using metaphors to conceptualize abstract concepts into concrete concepts, learning how to build relationships between two different things in choosing, asserting and giving freedom to make questions or metaphorical parables based on experience. The situation of this learning activity can improve students' reasoning.
abilities and abilities much better than control class students who are only given conventional learning. [13].

3.3. Statistical test analysis final test of mathematical problem solving abilities
The results of the normality test of the posttest score from the experimental class and the control class on mathematical problem solving ability by using SPSS 24.0 are presented below.

| Class            | Kolmogorov-Smirnova Statistic | df | Sig.  |
|------------------|-------------------------------|----|-------|
| Experimental     | .135                          | 13 | 0.200*|
| Control          | .129                          | 13 | 0.200*|

Based on the table above, the significance level of experimental and control class is of p = 0.200, sop > 0.05. It means that mathematical problem solving ability of the experimental class and the control class came from a normally distributed population. Homogeneity test results of posttest scores on mathematical problem solving ability by using SPSS 24.0 are shown in the following table.

| Levene's Test for Equality of Variances | F    | Sig.  |
|---------------------------------------|------|-------|
| Equal variances assumed               | 2.461| 0.129 |

Based on the table above, the significance level of experimental and control class is of p = 0.129, so p > 0.05. It means that mathematical problem solving ability of the experimental class and the control class came from homogenous population. T-test results of students' mathematical problem solving ability are shown in the following table.

| t-test for Equality of Means | t     | df  | Sig. (2-tailed) |
|-----------------------------|-------|-----|-----------------|
| Equal variances assumed     | 2.311 | 25  | 0.029           |
| Equal variances not assumed | 2.356 | 20.969 | 0.028 |

From the table, the t-test results on the posttest scores of students’ mathematical reasoning ability show t-obtained is 2.311 and t-table is 2.06 with p (Sig. (2-tailed)) is 0.029. It means that t-obtained is more than t-table and p-obtained is more than 0.05. Therefore, H0 is rejected, and H1 is accepted. From the two classical and probabilistic approaches, it can be concluded that the average posttest scores of students' mathematical problem solving ability through metaphorical thinking approach is more than the average score of students' mathematical problem solving ability through conventional approach.

Thus, the ability to solve problems and mathematical reasoning in the experimental class is superior to the control class. The engagement of most students in the learning process through metaphorical thinking approach allows an increased ability in analyzing and solving abstract problems [14]. Abstract concepts that are organized through metaphorical thinking are expressed in concrete things based on structures and ways of reasoning based on sensory-motor systems called conceptual metaphors. The metaphorical conceptual form includes: (1) grounding metaphors that are the basis for
understanding mathematical ideas connected to everyday experiences; (2) linking metaphors that build relationships of two things, which are to choose, emphasize, give freedom, and organize the characteristics of the main topics to be supported by additional topics in the form of metaphorical statements; (3) redefinitional metaphors that redefines the metaphors and chooses the most suitable to the topics that will be taught [15].

Based on the results of testing the hypothesis, the pretest of reasoning ability and mathematical problem-solving ability between the experimental class and the control class got relatively similar averagescores. Whereas after being given treatment, the posttest of reasoning ability and mathematical problem-solving ability between the experimental class and the control class got higher score than the control class. This shows that learning mathematics with metaphorical thinking approach is better than conventional approach. The use of metaphorical thinking approach in the learning process makes students’ learning more meaningful because they can see the relationship between the concepts they learn and the concepts they know [16].

4. Conclusion
Based on the results of research and discussion of the metaphorical thinking approach to the reasoning ability and mathematical problem solving of students in the third semester of the Mathematics Education Study Program of STKIP PGRI Lubuklinggau, it can be concluded that: 1) the t-test results on the posttest scores of students’ mathematical reasoning ability show that $t_{obtained}$ is 2.351 and $t_{table}$ is 2.06. It means that $t_{obtained}$ is more than $t_{table}$. Therefore, $H_0$ is rejected, and $H_a$ is accepted. It can be concluded that there is a significant influence of the metaphorical thinking approach on the mathematical reasoning ability of the third semester students of the Mathematics Education Study Program of STKIP PGRI Lubuklinggau, 2) the t-test results on the posttest scores of students’ mathematical problem solving ability show that $t_{obtained}$ is 2.311 and $t_{table}$ is 2.06. It means that $t_{obtained}$ is more than $t_{table}$. Therefore, $H_0$ is rejected, and $H_a$ is accepted. It can be concluded that there is a significant effect of the metaphorical thinking approach on the mathematical problem solving ability of the third semester students of the Mathematics Education Study Program of STKIP PGRI Lubuklinggau, and 3) The development of modules or teaching materials is in accordance with the characteristics of the metaphorical thinking approach to facilitate students’ learning activities to make it easier for them to understand the concepts that will be conveyed, and optimization of the learning process can be achieved.

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References
[1] Hasratuddin 2014 Pembelajaran Matematika Sekarang dan yang akan Datang Berbasis Karakter Jurnal Didaktik Matematika 1 pp 30-42
[2] NCTM 2000 Principles and Standars for School Mathematics Reston The National Council of Teachers of Mathematics Inc
[3] Hapizah 2014 Pengembangan Instrumen Kemampuan Penalaran Matematis Mahasiswa pada Mata Kuliah Persamaan Diferensial Jurnal Kreano 5 pp 73-81
[4] Wardhani S, Purnomo S S, dan Wahyuningsih E 2010 Pembelajaran Kemampuan Pemecahan Masalah Matematika di SD Kementerian Pendidikan Nasional Badan Pengembangan Sumber Daya Manusia Pendidikan dan Penjaminan Mutu Pendidikan P4TK Matematika.
[5] Zaimil R dan Rosmiyati 2016 Tahap Design Pengembangan Modul Berbasis Masalah Pada Perkuliahan Geometri Analitik Bidang Di FKIP UMMY Solok. Jurnal Lemma 3 pp 59-77
[6] Nursuprianah I dan Sholikhah M 2009 Analisis Kesulitan Mahasiswa Dalam Memahami Mata Kuliah Aljabar Matriks (Studi Kasus Pada Semester IV Tadris Matematika Tahun Akademik 2008/2009 di STAIN Cirebon) Jurnal EduMa 1 pp 75-84
[7] Sunito I dkk 2013 Metaphorming: Beberapa Strategi Berpikir Kreatif pp 1-162
[8] Kilic C 2010 Belgian and Turkish Pre-Service Primary School Mathematics Teachers’ Metaphorical Thinking about Mathematics CERME 7 pp 1-10
[9] Hendriana H 2012 Pembelajaran Matematika Humanis dengan Metaphorical Thinking untuk Meningkatkan Kepercayaan Diri Siswa Jurnal Infinity 1 pp 1-14
[10] Sugiyono 2011 Metode Penelitian Kuantitatif Kualitatif dan R&D
[11] Sumarmo U 2014 Mengembangkan Instrumen untuk Mengukur High Order Mathematical Thinking dan Affective Behavior Handout disajikan pada Workshop Pendidikan Matematika UIN Jakarta
[12] Nurhikmayati I 2016 Pembelajaran dengan Pendekatan Metaphorical Thinking Untuk Meningkatkan Kemampuan Penalaran Siswa SMP Jurnal THEOREMS 1 pp 21-34
[13] Roesdiana L 2016 Pembelajaran Dengan Pendekatan Metaphorical Thinking Untuk Mengembangkan Kemampuan Komunikasi Dan Penalaran Matematis Siswa Jurnal Pendidikan UNSIKA (JUDIKA) 4 pp 169-184
[14] Wahyuni I, Noto M S dan Hikmah A N 2017 Pengaruh Pendekatan Metaphorical Thinking Terhadap Kemampuan Literasi Matematis Siswa Jurnal Euclid 3 pp 491
[15] Hendriana H, Rohaeti E E dan Hidayat W 2017 Metaphorical Thinking Learning and Junior High School Teachers’ Mathematical Questioning Ability Journal on Mathematics Education 8 pp 55-64
[16] Afrilianto M 2012 Peningkatan Pemahaman Konsep dan Kompetensi Strategis Matematika Siswa SMP dengan Pendekatan Methaporical Teaching. Jurnal Infinity 1 pp 192-202