Abstract
The Gaussian process latent variable model (GP-LVM) is a popular approach to non-linear probabilistic dimensionality reduction. One design choice for the model is the number of latent variables. We present a spike and slab prior for the GP-LVM and propose an efficient variational inference procedure that gives a lower bound of the log marginal likelihood. The new model provides a more principled approach for selecting latent dimensions than the standard way of thresholding the length-scale parameters. The effectiveness of our approach is demonstrated through experiments on real and simulated data. Further, we extend multi-view Gaussian processes that rely on sharing latent dimensions (known as manifold relevance determination) with spike and slab priors. This allows a more principled approach for selecting a subset of the latent space for each view of data. The extended model outperforms the previous state-of-the-art when applied to a cross-modal multimedia retrieval task.

1. Introduction
Gaussian Process latent variable models (GP-LVM) reduce the dimensionality of data by establishing a mapping from a low dimensional latent space, $X$, to a high dimensional observed space, $Y$, through Gaussian Process (GP) (Lawrence, 2005; Titsias & Lawrence, 2010). The non-parametric nature of GP and the flexibility of using non-linear kernels enables GP-LVM to produce compact representations of data. GP-LVM has been successfully applied to various domains as a dimension reduction method. For example, Buettner & Theis (2012) used GP-LVM for resolving differences in single-cell gene expression patterns from zygote to blastocyst, and Lu & Tang (2014) developed a discriminative GP-LVM for face verification that was the first to surpass human-level performance.

When applying GP-LVM to dimension reduction problems, a key parameter of choice is the dimensionality of the latent space $Q$. A larger latent dimensionality can correspond to a significantly higher number of parameters, which potentially leads to overfitting. A standard approach for choosing the latent dimensionality of GP-LVM is to look at the values of the length-scale parameters of kernel functions. These parameters characterise the scales of individual latent dimensions. The underlying latent function can only vary “slowly” along the latent dimension with a high length-scale. When the length scale of a latent dimension is significantly larger than the ones of other dimensions, the influence of this latent dimension to overall (co)variances is negligible. Therefore, the number of latent dimensions is conventionally determined according to the length-scale parameters, typically by comparing to a manually chosen threshold. It closely relates to the idea of automatic relevance determination (ARD) regression (Rasmussen & Williams, 2005), where the ARD parameters are defined as one over the length scales. A limitation of this approach is that the choice of threshold could involve a lot of hand tuning. Furthermore, for non-linear kernels like the exponentiated quadratic form, it is difficult to figure out whether a relatively high length scale means that the latent dimension is “slowly” switched off or the latent dimension is very smooth (Vehtari, 2001).

A more principled approach is preferred for automating the selection of the number of latent dimensions. It means to let the model decide which latent dimensions it really needs. Driven by this idea, we introduce spike and slab prior (Mitchell & Beauchamp, 1988; George & McCul-
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loch, 1993; West, 2003) for the latent variable $X$. A spike and slab prior contains binary variables, which allows the model to probabilistically discard latent dimensions. Given observed data $Y$, the posterior probability of a latent dimension being used can be derived from the model definition. It offers a principled approach for selecting latent dimensions. However, the exact inference of the posterior distribution of latent variables is intractable, because there is no closed form solution for the integral of latent variables in the log marginal likelihood. In this paper, we derive a closed form variational lower bound of the log marginal likelihood for the spike and slab GP-LVM and develop an efficient inference method for posterior distributions of latent representations $X$ and switching variables. In spike and slab models standard mean field approximations are problematic due to the strong correlation between switch variables and input variables. Our variational approach assumes a conditional dependence between input variables and switch variables. It is closely related to the ideas like structured mean field (Saul & Jordan, 1995; Xing et al., 2003). This allows us to efficiently infer the latent representations of data while simultaneously determining the active latent dimensions.

In the literature, the spike and slab prior has been used for variable selection in various regression models. For instance, Carbonetto & Stephens (2012) developed a variational inference method with spike and slab prior for Bayesian variable selection in linear models. Savitsky et al. (2011) introduced spike and slab prior to length scale parameters in variable selection of GP regression models. They proposed inference through a Markov chain Monte Carlo (MCMC) scheme. In this paper, both switch variables and input variables $X$ are variationally integrated out. This enables us to infer latent representations for dimension reduction. Note this contrast with regression, where available inputs are only switched on or off. Here we are selecting both the number of available inputs and their nature (through the latent variable approach). Spike and slab priors have also been used in unsupervised learning for sparse linear models (i.e. sparse coding) with variational or truncated approximations (Titsias & Lazaro-Gredilla, 2011; Sheikh et al., 2014). The spike and slab GP-LVM we introduced is much more flexible because it allows for the encoding of non-linear relationships through appropriate Gaussian process covariance selection.

Through principled formulation of the selection of latent dimensions, our efficient variational approach allows us to extend the multi-view learning of GP with explicit separation of latent spaces for related views of a data set. The multi-view GP model, known as manifold relevance determination (MRD) in (Damianou et al., 2012), develops latent spaces that are shared amongst the different views and latent spaces that are particular to each given view. It formulation can be distilled as a set of inter-related GP-LVM models which share latent dimensions. Learning in the model consists of assigning each GP-LVM to a separate sub set of the latent dimensions through adjustment of length-scale parameters. Therefore, with applying a GP-LVM to each view of data, each view can “softly” decide which latent dimensions to use by variation of the length-scale parameter. However, this introduces the same ambiguity we referred to above. A threshold must be selected for deciding when a latent dimension is being ignored. With the spike and slab prior, different views can focus on a subset of the latent space by discrete switching of the unnecessary dimensions. This provides a more principled approach for multi-view learning with GPs. Our variational spike and slab GP-LVM is easily extended to handle this particular case.

Before introducing the new variational approximation, we first review the GP-LVM and its Bayesian counterpart. We then introduce our spike and slab GP-LVM, and the extension to MRD. Finally we empirically demonstrate the effectiveness of our model in selecting latent dimensions with both synthetic and real data. We demonstrate the new multi-view approach on an image-text dataset, in which gives significantly better results than the previous state-of-the-art.

2. Gaussian Process Latent Variable Model

For unsupervised learning, we typically assume a set of observed data $Y \in \mathbb{R}^{N \times D}$ with $N$ datapoints and $D$ dimensions for each datapoint. Our aim is to obtain a latent representations of the observed data, which we denote by $X \in \mathbb{R}^{N \times Q}$, where $Q$ is the number of latent space. In GP-LVM, the relationship between latent representations and data is given by a Gaussian process

$$p(f_d | X) = \mathcal{N}(f_d; 0, K),$$

and for simplicity we assume Gaussian noise,

$$p(y_d | f_d) = \mathcal{N}(y_d; f_d, \beta^{-1} \mathbb{1}),$$

where $y_d \in \mathbb{R}^N$ is the $d$th dimension of the observed data, $f_d$ is called the noise-free observation, and $K$ is the covariance matrix of $X$ computed according a kernel function $k(x, x')$. By maximizing the log likelihood $\log p(Y | X)$, the point estimates of latent representations $X$ can be obtained (Lawrence, 2005). However, the point estimation of $X$ implies fitting a lot of parameters, therefore, the resulting model is prone to overfitting. Titsias & Lawrence (2010) overcome this limitation by introducing a unit Gaussian prior for $X$ and deriving a variational lower bound of the log marginal likelihood (known as Bayesian GP-LVM). In their model, the sparse GP formulation (Titsias, 2009) is
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used. This relies on augmenting our variable space with a set of inducing variables, \( \mathbf{u} \), and the model becomes:

\[
p(f_d|\mathbf{u}_d, X, Z) = \mathcal{N}(f_d; K_{ff}K_{uu}^{-1}\mathbf{u}_d, K_{ff} - K_{ff}K_{uu}^{-1}K_{fu}^T),
\]

\[
p(\mathbf{u}_d|Z) = \mathcal{N}(\mathbf{u}_d; 0, K_{uu}),
\]

\[
p(X) = \prod_{n=1}^{N} \mathcal{N}(x_n; 0, I),
\]

where \( \mathbf{u}_d \) is the inducing variable for \( d \)th dimension, and \( Z \) is the inducing input. \( K_{ff} \) and \( K_{uu} \) are the covariance matrices for \( X \) and \( Z \) respectively, and \( K_{fu} \) is the cross covariance matrix between \( X \) and \( Z \). Then, the log marginal likelihood of the model is defined as:

\[
p(Y) = \int \prod_{d=1}^{D} p(y_d|X) p(X) \, dX.
\]

For tractability, the integral of \( X \) in the log marginal likelihood is approximated variationally. A variational posterior distribution of \( X \) is defined as:

\[
q(X) = \prod_{n=1}^{N} \mathcal{N}(x_n; \mu_n, S_n).
\]

With the assumption \( p(f_d|y_d, \mathbf{u}_d, X) = p(f_d|\mathbf{u}_d, X) \), a lower bound of log marginal likelihood can be obtained with Jensen’s inequality:

\[
\log p(Y) \geq \sum_{d=1}^{D} \tilde{F}_d(q) - \text{KL}(q(X)||p(X)),
\]

\[
\tilde{F}_d(q) = \log \left[ \frac{1}{2\pi} \frac{1}{2} |K_{uu}|^{-\frac{1}{2}} e^{-\frac{1}{2}y_d^T W y_d} \right]

- \frac{\beta \psi_0 + \beta}{2} \text{Tr}(K_{uu}^{-1} \Psi_2),
\]

\[
\text{KL}(q(X)||p(X)) = \int q(X) \log \frac{q(X)}{p(X)} \, dX,
\]

where \( W = \beta \mathbb{I} - \beta^2 \Psi_1 (\beta \Psi_2 + K_{uu})^{-1} \Psi_1^T \), \( \psi_0 \), \( \Psi_1 \) and \( \Psi_2 \) are the expectation of the covariance matrices w.r.t. \( q(X) \), i.e. \( \psi_0 = \text{Tr}(E_q(X)[K_{ff}]), \Psi_1 = E_q(X)[K_{fu}], \Psi_2 = E_q(X)[K_{fu} K_{fu}] \). With this formulation, the latent variable \( X \) is variationally integrated out. It leads to a closed-form lower bound of the log marginal likelihood.

In this formulation, the selection of latent dimensionality relies on parameters called length scales, each of which is applied to scale separately the input dimensions, e.g., \( l \) in the exponentiated quadratic kernel function \( k(x, x') = \sigma_f^2 \exp\left(-\frac{1}{2} \sum_{q=1}^{Q} (x_q - x_q')^2 / l_q^2 \right) \). If the length scale of a latent dimension is significantly lower than the other latent dimensions, the influence of this latent dimension to overall (co)variances is negligible. Therefore, a typical approach selects latent dimensions by thresholding their length scales. In this work, we propose a more principled approach, where each latent dimension is intrinsically considered whether it is used or not, by explicitly introducing a latent switching variable \( b \in \{0, 1\}^Q \) that consists of a set of binary variables, each controlling the usage of a latent dimension.

3. Spike and Slab GP-LVM

We introduce a switch variable \( b \in \{0, 1\}^Q \) that determines whether a particular latent dimension is used or not. The way that the switch variable controls the usage of a latent dimension is done by replacing the input variable \( x_n \) in the original Bayesian GP-LVM by \( x_n \circ b \), where \( \circ \) denotes the element-wise multiplication. If a binary variable in \( b \) is zero, the input of the corresponding dimension to the underlying GP becomes zero. As the input variable \( X \) has a Gaussian prior, the combination of \( X \) and \( b \) is known as a spike and slab prior. The prior distribution of the latent switch variables \( b \) are governed by Bernoulli distributions,

\[
p(b) = \prod_{q=1}^{Q} p^{b_q}(1 - \pi)^{(1 - b_q)},
\]

where \( \pi \) is the prior probability, which typically takes the value 0.5. Due to the introduction of \( b \), all the \( x_n \) in original covariance matrices are replaced with \( b \circ x_n \). This changes the form of the cross covariance matrix \( K_{fu} \). In the case of the exponentiated quadratic kernel it becomes

\[
(K_{fu})_{nm} = \sigma_f^2 \exp\left(-\sum_{q=1}^{Q} \frac{(b_q x_{nq} - z_{nq})^2}{2 l_q^2} \right),
\]

where \( (\cdot)_{nm} \) indicates the \( n \)th row and \( m \)th column of the matrix, while \( K_{uu} \) is not affected. To make inference tractable, we take a variational approach, where we assume a variational posterior distribution for the spike and slab model. As the switch variable and the slab variable are strongly correlated, the variational posterior is defined as a conditional distribution:

\[
q(b) = \prod_{q=1}^{Q} \gamma_q^{b_q} (1 - \gamma_q)^{(1 - b_q)},
\]

\[
q(x_{nq}|b_q = 1) = \mathcal{N}(x_{nq}; \mu_{nq}, s_{nq}),
\]

where \( \gamma_q \) is the posterior probability of the \( q \)th dimension being used (the overall probability of the slab part), and \( \mu_{nq} \) and \( s_{nq} \) are the mean and variance of the variational posterior distribution for the slab part. It is closely related to the ideas like structured mean field (Saul & Jordan, 1995; Xing et al., 2003). Therefore, the lower bound of log marginal
likelihood becomes
\[
\log p(Y) \geq \sum_{d=1}^{D} \tilde{F}_d(q) - \text{KL}(q(b, X)||p(b)p(X)), \tag{15}
\]
\[
\text{KL}(q(b, X)||p(b)p(X)) = \sum_b \int q(b, X) \log \frac{q(b, X)}{p(b)p(X)} dX, \tag{16}
\]

where \(\tilde{F}_d(q)\) keeps the same form as in (9), while \(\Psi_1\) and \(\Psi_2\) need to adapted according to the new \(K_{fu}\). Then, the new \(\Psi_1\) and \(\Psi_2\) are defined as:
\[
(\Psi_1)_{nm} = \sum_b q(b) \int k(b \circ x_n, z_m) q(x_n|b) dx_n, \tag{17}
\]
\[
(\Psi_2)_{nm'} = \sum_{n=1}^{N} \sum_b q(b) \int k(b \circ x_n, z_m) k(z_{m'}, b \circ x_n) q(x_n|b) dx_n. \tag{18}
\]

For some kernels, the closed-form solution for \(\psi_0\), \(\Psi_1\) and \(\Psi_2\) can be obtained. For example, \(\psi_0\), \(\Psi_1\) and \(\Psi_2\) of the exponentiated quadratic kernel are derived as:
\[
\psi_0 = N \sigma_f^2, \tag{19}
\]
\[
(\Psi_1)_{nm} = \sigma_f^2 \prod_{q=1}^{Q} \left[ \frac{\gamma_{nq}}{(s_{nq}/l_q + 1)^2} e^{-\frac{z_{nq}^2}{2s_{nq}/l_q}} \right], \tag{20}
\]
\[
(\Psi_2)_{nm'} = \sigma_f^2 \prod_{n=1}^{N} \prod_{q=1}^{Q} \left[ \frac{\gamma_{nq}}{(s_{nq}/l_q + 1)^2} \right. \left. e^{-\frac{(z_{nq} - z_{m'q})^2}{2s_{nq}/l_q}} e^{-\frac{(z_{nq} - z_{m'q} + s_{m'q} - s_{nq})^2}{2z_{m'q}/l_q}} \right] \tag{21}
\]

Note that the distribution \(q(x_{nq}|b_q = 0)\), because as the switch variable is zero, the slab variable does not influence the likelihood anymore, so that \(q(x_{nq}|b_q = 0)\) will only appear inside the KL divergence, which make it always equal to the prior distribution \(p(X)\).

4. Spike and Slab MRD

In manifold relevance determination (Damianou et al., 2012), multiple views of data are considered simultaneously. The model assumes that those views share some aspects and retain some aspects that are particular to each view. Each latent dimension can both relate to other views in some shared latent dimensions while keeping some private latent dimension for its own (see Fig. 1a). In the original MRD paper this effect was achieved through appropriate sharing of ARD parameters within each view. This leads to a soft sharing approach where a particular latent dimension can be used to a greater or lesser extent by each of the views. Spike and slab MRD allows for probabilistic selection of binary variables to perform the sharing (see Fig. 1b).

We wish to relate \(C\) views \(Y^{(c)} \in \mathbb{R}^{N \times D_c}\) of a dataset in our model. We assume the dataset can be represented as a latent variable \(X \in \mathbb{R}^{N \times Q}\), in which a different subset of the \(Q\) latent dimensions are used to represent each view.

The selection of latent dimensions for \(c\)th view is done by a vector of latent binary variable \(b_c \in \mathbb{R}^Q\) with the prior distribution in (11). As mentioned in previous section, each view can be modeled by a spike and slab GP:
\[
p(Y|X, B) = \prod_{c=1}^{C} p(Y^{(c)}|X, B). \tag{22}
\]

With the prior distribution of \(X\) and \(B\), the marginal distribution of our MRD model is
\[
p(Y) = \int \prod_{c=1}^{C} \sum_{B} p(B) \prod_{d=1}^{D} p(y^{(c)}_d|X, B)p(X) dX \tag{23}
\]

For a tractable inference algorithm, we wish to introduce a variational approximation for the latent variable \(X\) and \(B\). Similar to spike and slab GP-LVM, the latent variable \(X\)
and $B$ are closely correlated, so that we define a conditional variational distribution $q(X, B) = q(X|B) \prod_{c=1}^{C} q(b_c)$, in which there is a variational posterior distribution for each view representing the subset of latent dimensions used by that view, and the posterior distribution for the latent representation of data, which is the same for all the views. In order to be consistent with the choices $B$, $q(X|B)$ is defined as

$$q(b_c) = \prod_{q=1}^{Q} \gamma_q^{b_q}(1 - \gamma_q)^{(1 - b_q)}, \quad (24)$$

$$q(x_{nq} | \bigvee_{c=1}^{C} (b_{cq} = 1)) = \mathcal{N}(x_{nq}; \mu_{nq}, s_{nq}), \quad (25)$$

where $\bigvee$ is the or operation for binary variables. We denote the conditional variational posterior distribution $q(x_{nq} | \bigvee_{c=1}^{C} (b_{cq} = 1))$ as $q_c(x_{nq})$. It gives the variational posterior of $X$ for all the views if any of the views decide to use the latent dimension, otherwise the posterior distribution will fall back to the prior, therefore the posterior $q(x_{nq} | \bigvee_{c=1}^{C} (b_{cq} = 0))$ does not need to be explicitly defined. By variationally integrating out the latent variable $X$ and $B$, we obtain a lower bound for our MRD model:

$$p(Y) = \sum_B p(B) \int \prod_{c=1}^{C} p(Y^{(c)}|X, b_c) p(X|B) dX \quad (26)$$

$$\log p(Y) \geq \sum_{c=1}^{C} \sum_{d=1}^{D} F_q^{(c)}(q) - \text{KL}(q(B, X)||p(B)p(X)), \quad (27)$$

$$\text{KL}(q(B, X)||p(B)p(X)) = \sum_B \int q(B, X) \log \frac{q(B, X)}{p(B)p(X)} dX, \quad (28)$$

With this definition, we will have a new $\Psi_1$ and $\Psi_2$ correspondingly.

$$(\Psi_1^{(c)})_{nm} = \sum_B \prod_{c=1}^{C} q(b_c) \int k(b_c \circ x_n, z_m) q(x_n|B) dx_n, \quad (29)$$

$$(\Psi_2^{(c)})_{nm'} = \sum_{n=1}^{N} \sum_B \prod_{c=1}^{C} q(b_c) \int k(b_c \circ x_n, z_m) k(z_{m'}, b_c \circ x_n) q(x_n|B) dx_n, \quad (30)$$

however, they lead to exactly the same formulas as (19), (20) and (21). For efficient implementation, the computation of $\psi_0$, $\Psi_1$ and $\Psi_2$ which is usually the bottleneck can be easily parallelized by dividing data points into small groups and evaluating the results in a distributed way (Dai et al., 2014; Gal et al., 2014).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{(a) The three latent signals used for generating observed data. (b) Two observed data generated from the latent signals. The $y$-axis shows the different dimensions in the observed data, 12 dimensions for each. The $x$-axis shows the data in each dimensions, and 50 samples are drawn evenly.}
\end{figure}

5. Experiments

We first demonstrate our model with synthetic data. We aim to recover the latent signal from two sets of multidimensional observed data. Then, we show the effectiveness of the switch variable as a cue for choosing latent dimensions. After that, we apply our SSMRD model to a text-image dataset, where our model gives significantly better results than state of the art performance.

5.1. Synthetic Data

We first apply both of our models to a synthetic data, where that the nature of the latent representation is known and we can ascertain when it is correctly reconstructed. We introduced three artificial signal sources, which are $y = \sin(x)$, $y = -\exp(-\cos(2x))$ and $y = \cos(x)$, and drew 50 samples evenly from the interval between 0 and $2\pi$ (see Fig. 2(a)). The drawn samples are normalized to zero mean and unit variance. These are the latent signals that we would like to recover. We generated two sets of observed signals. For the first observed signal, we combined the 1st and 3rd signals, and transformed into 12 dimensional signals through a random linear transformation. The second ob-
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Figure 3. (a) The recovered latent signals according to our SSGP-LVM model from the first signal in Fig. 2(b). It shows the conditional variational posterior distribution \( q_\phi(X) \). Each row corresponds to a latent dimension, and the curve shows the mean, and the width of the colored region shows the variance. (b) The learned posterior probability for the switch variable \( b \). Different colors correspond to different views. (c) The learned length scales of latent dimensions.

Figure 4. (a) The recovered latent signals according to our SSMRD model by assigning two views to the two observed data respectively. (b) The learned posterior probability for the switch variable \( b \). Different colors correspond to different views. (c) The learned length scales of latent dimensions.

The observed signal are generated in the way combining the 2nd and 3rd signals (see Fig. 2(b)).

We first applied the spike and slab GP-LVM model to the first observed signal to see whether it can recover the latent signals and determine the used latent dimensions if you offer more latent dimensions than the number of underlying signals. We applied our model with a linear kernel and 5 latent dimensions. The recovered latent signals are shown in Fig. 3(a) and Fig. 3(b). The learned 1st and 2nd latent dimensions successfully recover the latent signals with very small posterior variances, and the posterior probabilities of these two latent dimensions are close to 1 while the rest are close to zero, which means the model only used the first two latent dimensions to explain the data. The lengthscale parameters are plotted in Fig. 3(c), where there are a big difference between the used and unused dimensions, which matches the observations with Bayesian GP-LVM. It perfectly matches with the information that we put into the data.

We then applied the spike and slab MRD model to both observed sets signals, taking each observed signal set as a different view of the data. The way the data was generated implies that each view has one private latent signal and also shares a common latent signal. We aim to recover all the latent signals with a correct assignment of latent dimensions. We applied a linear kernel for each view and 5 latent dimensions. The recovered latent signals are shown in Fig. 4(a) and Fig. 4(b). All the three latent signals are recovered with very small posterior variances. The 1st view takes the 1st latent dimension as its private space, which recovers its private latent signal, and the 2nd view takes the 4th latent dimension for its private signals, while the 5th latent dimension are shared by both views, which recovers the shared latent signal. The 2nd latent dimensions are used by both views to give some structured noise, in which the inferred variance of the signal is significantly higher than the true signals. The inferred length scale for the kernels of both views are shown in Fig. 4(c). Note that it is not instantly clear how to threshold the latent dimensions according to length scale parameters. For instance, for the 1st view, the length scales of the 1st and 5th dimensions are roughly at the same level which corresponds to the true signals, while the 2nd and 4th dimensions also have relatively low length scales. However, according to the posterior probabilities of its switch variable, the 2nd dimension is used by the model while the 4th dimension is the private space of the other view. In this case, thresholding the length scale parameters
is not able to give the same answer as observed according to the posterior probabilities of switch variable.

5.2. Classification data

We next considered a data set of hand written digits to quantify the number of latent dimensions required to represent the digits. We took a subset of images from the MNIST dataset (LeCun et al., 1998). We chose the images digits of “1”, “7” and “9”, and took 1,000 images for each character for training and 1,000 for testing.

We applied the spike and slab GP-LVM to the training set for dimensional reduction, where the initial number of latent dimensions was chosen to be 20. The optimization of our model parameters was done in a purely unsupervised fashion, where no label information was used. The resulting length scales of latent dimensions are shown in Fig. 5(a), where the latent dimensions are sorted according to their length scales. The posterior probabilities of the switch variable are shown in Fig. 5(b). From these values we can see that the model actively makes use of 10 latent dimensions.

After optimizing the model parameters, we use the learned model to infer the conditional variational posterior distributions of test images \(q_c(X_x)\), which encode the posterior mean and variance of a datapoint in latent space if the corresponding latent space are used. Afterwards, we apply the nearest neighbor classifier, which compares the posterior mean between training and testing data points, and predict the label of a test image according to the label of its nearest training image in the latent space. We test the classification accuracy for different latent space configurations. We chose the latent dimensions by thresholding their length scales at different values, by which we obtained 20 different choices of latent space. We compared the performance of these choices of latent space with choosing the latent space according to the posterior probability of the switch variable \(\gamma\). The comparison is shown in Fig. 5(c), where the different numbers in \(x\)-axis denotes the choices corresponding to different number of used latent dimensions, and \(\gamma\) denotes the choice according to the parameter \(\gamma\).

By comparing Fig. 5(a) and Fig. 5(c), we see that the best classification accuracy can be obtained by using only 6 latent dimensions, but we do not see a significant changes in length scale between the 6th and 7th latent dimension. A human tentative choice is to use 7 latent dimensions, which can give a similar level of performance, but is difficult to automate such kind of decisions. On the other hand, the choice according to \(\gamma\) has a slightly higher number of dimensions, but it is trivial to make automatic decision based on that.

![Figure 6. The precision-recall curve for both image and text queries on the Wiki dataset.](image-url)

5.3. Text-Image Retrieval

An interesting application of MRD models is to relate information from different domains. For example, relating image to text can potentially solve ambiguities by looking at only a single view of the data. The image representations of an object can have a lot of variances due to changing in location, viewing angle, illumination conditions, etc. Purely from image data, it might be difficult to figure out different variants of the same object, but with text such ambiguities may be resolved. Similarly, image representations can help to resolve ambiguities in text, e.g., a facial image can easily tell different people with the same name.

We show results on a text-image dataset collected from Wikipedia (Costa Pereira et al., 2014). The task here to perform multimedia information retrieval, i.e., given a text query, the algorithm needs to produce a ranking of the images in the training set, and similarly given an image query to produce a ranking of texts. The Wikipedia dataset consists of 2173/693(training/testing) image-text pairs associated with 10 different topics. We used the features for images and texts provided by the authors. It has a 10D text feature extracted from a LDA model() for each document and a 128D SIFT histogram image features for the corresponding image. The quality of the inferred ranking is assessed in terms of mean Average Precision (mAP) and precision-recall curves.

We apply our SSMRD model by assigning image features to a view and text features to another view. Both image and text features are normalized to zero mean and unit variance before inference. To overcome the unbalanced dimensionality between image and text features (128 v.s. 10), we replicate the text features 6 times, making it a 60D representation. An exponentiated quadratic kernel is used for each view and the number of latent dimensions is chosen to be 10. We initialize the mean of variational posterior \(\mu\) ac-
The performances of state of art algorithms are also shown in Tab. 1. All the results used the same feature set extracted with dataset. Correlation matching (CM), semantic matching (SM), and semantic correlation matching (SCM) are the methods proposed by the creators of the dataset (Costa Pereira et al., 2014), of which SCM gives the state of art performance. The results with generalized multi-view analysis (GMA) with LDA (GMLDA) and marginal Fisher analysis (GMMFA) are reported by (Sharma et al., 2012). The mAP measures are directly taken from their papers. Our performance for text queries is below the state of art performance. We suspect it is due to lack of enough inducing inputs (100 is used for the reported performance), which directly limits the modeling capability, and not introducing label information into learning. Note that all their algorithms except CM are supervised algorithms, while our model does not make use of label information during training except initializing the latent space.

6. Conclusion

Standard approaches to variable selection in Gaussian process latent variable models have relied on scaling priors to reduce the influence of particular dimensions. We have introduced switching variable and a spike and slab prior which allows us to explicitly model the switching on and off of particular latent dimensions. This provides a more principled approach for selecting latent dimensions. By variationally integrating out the spike and slab latent variable we derived a lower bound on the log marginal likelihood. For efficient implementation we used a parallel version of the algorithm with GPU acceleration. It is due to lack of enough inducing inputs (100 is used for the reported performance), which directly limits the modeling capability, and not introducing label information into learning. Note that all their algorithms except CM are supervised algorithms, while our model does not make use of label information during training except initializing the latent space.
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