Return of Harrison-Zeldovich spectrum in light of recent cosmological tensions

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ABSTRACT
The spectral index $n_s$ of scalar perturbation is the significant initial condition set by inflation theory for our observable Universe. According to Planck results, current constraint is $n_s = 0.965 \pm 0.004$, while an exact scale-invariant Harrison-Zeldovich spectrum, i.e. $n_s = 1$, has been ruled out at $8.4\sigma$ significance level. However, it is well-known that the standard ΛCDM model is suffering from the Hubble tension, which is at $\sim 5\sigma$ significance level. This inconsistency likely indicates that the comoving sound horizon at last scattering surface is actually lower than expected, which so seems to be calling for the return of $n_s = 1$. Here, in light of recent observations we find strong evidence for a $n_s = 1$ Universe. And we show that if so, it would be confirmed conclusively by CMB-S4 experiment.

Key words: cosmological parameters – cosmology: observations – inflation

1 INTRODUCTION
Half a century ago, Harrison (1970), Zeldovich (1972), and Peebles & Yu (1970) found that a scale-invariant power law spectrum of primordial density perturbation is consistent with the crude constraints available at that time, known as Harrison-Zeldovich spectrum, i.e. the spectral index $n_s = 1$. Recently, the standard ΛCDM model has been well inspected with the advent of the era of precise cosmology. According to Planck 2018 results, $n_s = 1$ has been ruled out at $8.4\sigma$ significant level (Akrami et al. 2020).

However, the case might be not so simple (Di Valentino et al. 2018). There are some inconsistencies (Verde et al. 2019; Riess 2019; Di Valentino et al. 2019; Handley 2021) in standard ΛCDM model, which are inspiring a re-interpretation of currently available data. Recent cosmic microwave background (CMB) observations showed Hubble constants $H_0 \approx 67$ km/s/Mpc. However, most local observations give higher values, e.g. $H_0 \approx 73$ km/s/Mpc using Cepheid-calibrated supernovae (Riess et al. 2022a,b), although some other measurements, e.g. (Kelly et al. 2023), show values that are compatible with CMB. This is well-known Hubble tension, which has reached $\sim 5\sigma$ significance level for many observations (Riess 2019), e.g. (Di Valentino et al. 2021; Abdalla et al. 2022) for recent reviews, see also (Dainotti et al. 2021, 2022) for the evolution of $H_0$. As pointed out in (Bernal et al. 2016; Aylor et al. 2019; Knox & Millea 2020), it is likely the lower sound horizon $r_s$ at last scattering surface that expected that results in higher $H_0$, since current observations require $r_s H_0 \sim$ const.. There have been some attempts to perform $r_s$-independent analysis (Baxter & Sherwin 2021; Philcox et al. 2021; Farren et al. 2022; Philcox et al. 2022; Madhavacheril et al. 2023), but not robust enough for the solution of Hubble tension (Smith et al. 2022a). It has been found that if the recombination physics is not modified, in such a lower-$r_s$ model $n_s$ must be proportionally raised (Ye et al. 2021; Jiang & Piao 2022)

\[ \delta n_s \approx 0.4 \frac{\delta H_0}{H_0} \approx -0.4 \left( \frac{\delta r_s}{r_s} \right), \]

which seems to suggest that the complete resolution $H_0 \sim 73$ km/s/Mpc of Hubble tension is pointing towards a scale-invariant Harrison-Zeldovich spectrum, i.e. $n_s = 1$ (Here $n_s = 1$ refers to a very small region near $n_s = 1$, contrasted with Planck result $n_s = 0.965 \pm 0.004$ on ΛCDM model), see also (Smith et al. 2022b). Recent large-scale structure observations is actually not conflicted with $n_s = 1$ (Simon et al. 2023; Ye et al. 2023).

As is well known, the shift of $n_s$ towards $n_s = 1$ would have significant and profound implications for our insight into the inflation theory. In this work, in light of recent observations, we investigate to what extent we might live with $n_s = 1$.

It is usually thought that the simplest and well-motivated $V_{\inf}(\phi) \sim \phi^p$ inflation ($p = 2$ for the chaotic inflation (Linde 1983) and $p = 2/3, 1$ for the monodromy inflation (Silverstein & Westphal 2008; McAllister et al. 2010; D’Amico et al. 2022)) have been ruled out by Planck+BICEP/Keck (Adh et al. 2021) due to their large tensor-to-scalar ratio $r$. However, the case might not be so. It is possible that initially the inflaton is at slow-roll region with $N_\epsilon \approx 60$, where $N_\epsilon$ is the efolds number before the slow-roll parameter $\epsilon \approx 1$. And if inflation ends prematurely at $N_\epsilon - 60$ at which $\epsilon \ll 1$ (Kallosh & Linde 2022; Ye et al. 2022) by e.g. waterfall instability, we will have $|n_s - 1| \approx \frac{p/2 + 1}{N_\epsilon} = O(0.001)$ (equivalently $n_s = 1$) at CMB.
observable band, so that both chaotic and monodromy inflation (also power-law inflation (D’Amico & Kaloper 2022) models will be perfectly compatible with the recent BICEP/Keck constraint (Ye et al. 2022), since the uplift to \( n_s = 1 \) markedly lowers \( r \sim |n_s - 1| \).

2 MODELS AND DATASETS

2.1 Injection of EDE

The energy injection at \( z \approx z_{eq} \) will lead to faster expansion of the universe during recombination, which helps to pull lower the sound horizon. A well-known possibility is EDE. In axion-like EDE (Poulin et al. 2019), an axion-like potential is

\[
V(\phi) = m^2 f^2 (1 - \cos(\phi/f))^3,
\]

while in AdS-EDE (Ye & Piao 2020), we have an AdS-like potential as

\[
V(\phi) = \begin{cases} 
V_0 \left( \frac{\phi}{M_{Pl}} \right)^4 - V_{\text{AdS}}, & \phi < V_{\text{AdS}} \frac{1}{V_0}^{1/4} \\
0, & \phi > V_{\text{AdS}} \frac{1}{V_0}^{1/4},
\end{cases}
\]

In corresponding models, \( \phi \) starts to roll around redshift \( z_c \sim 3000 \). However, their energies must diluted rapidly not to spoil the fit to CMB. The axion-like EDE achieves it through oscillation while the AdS-EDE achieves it through the AdS phase \( (w > 1) \).

There are also other possible EDE models, see e.g. (Agrawal et al. 2019; Lin et al. 2019; Niedermann & Sloth 2021; Sakstein & Trodden 2020; Lin et al. 2020; Karwal et al. 2022; Rezazadeh et al. 2022).

2.2 Datasets used

**Planck**: Planck 2018 low-\( \ell \) TT,EE,TE,EE Plik likelihoods (Aghanim et al. 2020).

**BAO** (Baryonic Acoustic Oscillations): BOSS DR12 (Alam et al. 2017), 6dF Galaxy Survey (Beutler et al. 2011) and Main Galaxy Sample of SDSS DR7 (Ross et al. 2015).

**SN** (Supernovae): The Pantheon Type Ia Supernovae observations (Scollnic et al. 2018).

**R21**: The measurement of \( H_0 \) reported by SHOES (Riess et al. 2022a) using Cepheid-calibrated Type Ia Supernovae is regarded as the Gaussian constraint on \( M_B \). We use it because it is the typical representative of the local measurements to the Hubble constant, which have been well studied and are consistent with many other local measurements.

**SPT-3G Y1**: The public SPT-3G likelihood 1, which includes TE and EE spectra within multipoles 300 < \( \ell \) < 3000 (Dutcher et al. 2021).

**ACT DR4**: The marginalized likelihood 2 from ACT DR4, which includes TE and EE spectra within multipoles 326 < \( \ell \) < 4325 and TT spectra within multipoles 576 < \( \ell \) < 4325.

**DES Y3**: The measurements of Dark Energy Survey Year 3 on galaxy clustering and weak lensing (Abbott et al. 2022).

1. https://github.com/SouthPoleTelescope/spt3g_y1_dist
2. https://github.com/ACTCollaboration/pyactlike

3 METHODOLOGY AND RESULTS

As (1) suggested, a pre-recombination injection, e.g. well-known early dark energy (EDE) (Karwal & Kamionkowski 2016; Poulin et al. 2019), might be required to pull lower \( r_s \). As examples, we consider the extension of \( n_s = 1 \) in \( \Lambda \)CDM model, i.e. the \( n_s = 1 \) model with axion-like EDE (Poulin et al. 2019; Smith et al. 2020) or AdS (anti-de Sitter) EDE (Ye & Piao 2020; Jiang & Piao 2021).

In what follows we evaluate the Bayesian evidences for such \( n_s = 1 \) models over the \( \Lambda \)CDM model. The widely-used Bayesian ratio is

\[
B = \frac{Z_{n_s = 1 \text{ (EDE)}}}{Z_{\text{standard } \Lambda \text{CDM model}}},
\]

And the Bayes evidence for model \( M \) is

\[
\mathcal{Z}_M \equiv P(D \mid M) = \int \mathcal{L}(D \mid \Theta, M) \pi(\Theta \mid M) d\Theta,
\]

where \( \mathcal{L} \) is the likelihood of the data \( D \) for model \( M \) and parameters \( \Theta \), and \( \pi \) is the prior for the model \( M \). However, Bayesian ratios are prior dependent. Thus we also consider another prior-independent quantity, called Suspiciousness (Lemos et al. 2020). Here, it is calculated based on the Markov chain Monte Carlo (details are presented in section A) results with (Heymans et al. 2021)

\[
\ln S = \left( \frac{x_{n_s=1 \text{ (EDE)}}}{\langle x_{\text{standard } \Lambda \text{CDM}} \rangle} \right)^2 - \left( \frac{x_{\text{standard } \Lambda \text{CDM}}}{\langle x_{\text{standard } \Lambda \text{CDM}} \rangle} \right)^2
\]

where \( \langle \cdot \rangle \) means weighted average according to the posterior distribution. The relevant \( p \)-value is

\[
p = \int_{d_{n_s=1 \text{ (EDE)}}}^{d_{\Lambda \text{CDM}}} -2 \ln S \int dx \mathcal{L}^2(x) dx,
\]

where \( d = 2 \left( \langle \ln \mathcal{L} \rangle \right)_\phi - \left( \langle \ln \mathcal{L} \rangle \right) \phi \) (Handley & Lemos 2019). As presented in Figure 1, our base dataset is Planck+BAO+SN+R21, and also they are fully compatible in \( n_s = 1 \) models. The results of Bayes ratio and Suspiciousness are shown in Table 1 and Figure 2. Both indicate that \( n_s = 1 \) models are favored. According to a revised version of the Jeffreys’s scale (Trotta 2008), the evidence is very strong. In \( B > 10 \) (note that if we exclude R21, \( n_s = 1 \) models will not be favored, however, such a compare without Hubble punishment is not fair). This conclusion can also be confirmed by quite negative \( \Delta \ln \mathcal{Z} \), see Table 2. As a supplement, we also show the results with base1000+ACT+SPT dataset, which is the combination of base dataset with ACT DR4 (Aiola et al. 2020) and SPT-3G Y1 (Dutcher et al. 2021) observations, while the small scale part (\( \ell > 1000 \)) of Planck TT is cut off, as in (Hill et al. 2022; Poulin et al. 2021; La Posta et al. 2022; Smith et al. 2022b; Jiang & Piao 2022). And it has similar conclusion.

It is well-known that weak lensing and galaxy clustering measurements for the growth of structure have 2 ~ 3\( \sigma \) level tension with CMB, quantified as \( S_8 = \sigma_8 (\Omega_m / 0.3)^{0.5} \), while the injection of EDE will worsen it (Hill et al. 2020; D’Amico et al. 2021; Ivanov et al. 2020), see also(Krishnan et al. 2020; Nunes & Vagnozzi 2021). As expected, for \( n_s = 1 \) models we find \( S_8 \approx 0.84 \), see also Figure 3. However, with the base dataset+DES-Y3 (Abbott et al. 2022), we still find strong evidence (\( \ln B > 5 \)) for \( n_s = 1 \) models. This might due to that at present DES-Y3 is not constrained well, although its uncertainty is already close to other recent observations, such as KIDS-1000 (Heymans et al. 2021).

However, we also need to note that \( n_s \) is inversely associated with \( S_8 \) through \( \Omega_m \), which is clearer in \( \Lambda \)CDM, as shown in Figure 4. Thus lower \( \Omega_m \) can satisfy \( n_s = 1 \) and a lower \( S_8 \) simultaneously, but at the cost of worse fit to BAO+SN. In fact, our \( \Omega_m \) is shifted from 0.2995 \( \pm 0.0054 \) to 0.2911 \( \pm 0.0039 \) for \( n_s = 1 \) model with axion-like EDE.
Table 1. Suspicousness and relevant $p$-values of $n_s = 1$ models over $\Lambda$CDM model for different datasets. Negative Suspicousness indicate the preference for $n_s = 1$ models.

| Dataset             | $n_s = 1$ (axion-like EDE) | $n_s = 1$ (AdS-EDE) |
|---------------------|---------------------------|-------------------|
|                     | Suspicousness  | $p$-value | Suspicousness | $p$-value |
| base                | -13.5         | $2 \times 10^{-7}$ | -8.5         | $2 \times 10^{-5}$ |
| base-R21            | 0.0022        | 0.37      | 6.44         | 1         |
| base1000+ACT+SPT    | -17.9         | $1 \times 10^{-9}$ | -13.0        | $5 \times 10^{-7}$ |
| base1000+ACT+SPT-R21| -5.18         | 0.00082   | -0.67        | 0.14      |
| base+DES            | -11.54        | $7 \times 10^{-6}$ | -4.33        | $6 \times 10^{-3}$ |

4 DISCUSSION AND CONCLUSION

In conclusion, in light of recent observations, we find strong evidence for a Harrison-Zeldovich Universe. In such $n_s = 1$ models with pre-recombination EDE, we naturally have $H_0 \sim 73$ km/s/Mpc without well-known Hubble tension. Though DES-Y3 weakens the evidence for $n_s = 1$ models, it is still critical to check if the joint analysis of further $S_8$ observations with CMB could show evidence in support of $n_s = 1$, or one might need to rethink the physics of dark matter. It should also be noted that the high-redshift halo abundances observed by James Webb Space Telescope (JWST) appears to require a higher $n_s$ (Klypin et al. 2021; Boylan-Kolchin 2023) while the Lyman-alpha forest seems to prefer lower values (Iršič et al. 2017a; Chabanier et al. 2019; Palanque-Delabrouille et al. 2020). More studies are needed for high redshift and the tension between CMB and large-scale structure observations.

It is also significant to see, as shown in Figure 5, that $n_s = 1$ will imprint unique signals in the CMB spectrum, especially in the small-scale part and the polarization spectrum, where the impacts of both $n_s = 1$ and EDE are hardly balanced by the shifts of other cosmological parameters. We make a mock data forecast using CMB-S4 (Abazajian et al. 2019)\(^3\). The results are shown in Table 3. The forecast with CMB-S4 indicates that if the $n_s = 1$ model is in reality,\(^3\)

\(^3\)Regarding the $\Lambda$CDM model as the real universe, we employed the model parameters from the baseline results (bestfit values) of Planck 2018 (Aghanim et al. 2020):

$$
H_0 = 67.32 \quad \Omega_b h^2 = 0.022383 \quad \Omega_c h^2 = 0.12011
$$

$$
\tau = 0.0543 \quad n_s = 0.96605 \quad A_s = 2.1005 \times 10^{-9}
$$

Regarding the $n_s = 1$ model as the real universe, we employed the bestfit values from the results of our base dataset:

$$
\log_{10} a_c = -3.858 \quad f_{\text{EDE}} = 0.155
$$

$$
H_0 = 72.49 \quad \Omega_b h^2 = 0.023270 \quad \Omega_c h^2 = 0.13439
$$

$$
\tau = 0.0626 \quad A_s = 2.1550 \times 10^{-9}
$$

The mock data for CMB-S4 is generated using [GitHub repository](https://github.com/misharash/cobaya_mock_cmb). The noise curves for CMB-S4 are taken from [SNS site](https://sns.ias.edu/~jch/S4_190604d_2LAT_Tpol_default_noisecurves.tgz) and details are avail-
Figure 2. Bayes ratio of $n_s = 1$ models over $\Lambda$CDM model for different datasets. Positive Bayes ratios indicate the preference for $n_s = 1$ models. A revised version of the Jeffreys’ scale (Trotta 2008) is also plotted as the background color.

Figure 3. Posterior distribution of model parameters under the base datasets, marginalized to the $S_8 - n_s$ plane. The constraints on $H_0$ from SH0ES (Riess et al. 2022a) (purple band) and $S_8$ from DES-Y3 (Abbott et al. 2022) (cyan band) are also plotted.

Figure 4. Posterior distribution of the $\Lambda$CDM parameters under the base dataset, marginalized to the $S_8 - n_s$ plane, and the relation with $\Omega_m$.

DATA AVAILABILITY
The data underlying this article will be shared upon request to the corresponding author(s).

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Table 3. Bayes ratios, Suspiciousness and relevant $p$-values of $n_s = 1$ model with axion-like EDE over the ΛCDM model for the CMB-S4 mock data tests. Positive Bayes ratios and negative Suspiciousness indicate the preference for $n_s = 1$ model.

| Data                                           | ln $B$  | Suspiciousness | $p$-value       |
|------------------------------------------------|--------|---------------|-----------------|
| Mock data from ΛCDM model                      | −32.3  | 29.3          | $1 \times 10^{-39}$ |
| Mock data from $n_s = 1$ (axion-like EDE) model| 96.7   | −86.2         |                 |

Figure 5. The residuals on the CMB power spectrum of the bestfit ΛCDM model, when fitting the mock data assuming a $n_s = 1$ (axion-like EDE) Universe. We also plot the binned CMB-S4 forecasted error bars ($\Delta t = 100$). The different colors represent different scaling shown in the corresponding vertical axes.

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Table A1. The prior range of parameters used in our analysis. Uniform priors are employed for all parameters.

| Parameter               | Prior range          |
|-------------------------|----------------------|
| $n_s$ (for standard $\Lambda$CDM model) | [0.8, 1.2] |
| log$(10^{10}A_s)$       | [1.61, 3.91]         |
| $H_0$                   | [20, 100]            |
| $\Omega_b h^2$         | [0.005, 0.1]         |
| $\Omega_c h^2$         | [0.05, 0.99]         |
| $\tau_{reio}$          | [0.01, 0.8]          |
| log$_{10}$($z_c$)       | [3, 4]               |
| $f_{EDE}(z_c)$          | [0, 0.3]             |
| $\Theta_{inj}$ (for axion-like EDE) | [0, 3.1] |

**APPENDIX A: DETAILS OF MCMC (MARKOV CHAIN MONTE CARLO) ANALYSIS**

The cosmological evolution simulations and the MCMC analyses are performed using CLASS (Blas et al. 2011) and cobaya (Torrado & Lewis 2021), respectively, and the Gelman-Rubin tests for all chains have been converged to $R-1 < 0.05$. The precision settings for CLASS are increased, especially for the calculation of the lensing effect since it has non-negligible effects on ground-based CMB observations, see also the appendix of (Hill et al. 2022). The injections of EDE are performed with the modified CLASS: https://github.com/PoulinV/AxiCLASS (Smith et al. 2020; Poulin et al. 2018) and https://github.com/genye00/class_multiscf. Then we use BOBYQA (Powell et al. 2009) to find the bestfit points. The prior range of parameters are summarized in Table A1 In our calculation, we fixed AdS depth to the value in (Ye & Piao 2020). And we have confirmed that our results do not strongly depend on the choice of AdS depth.