Titmouse Versus Robins’ Behavioral Model for Innovation Propagation

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Abstract

Inspired by the spectacular observation that non-territorial songbirds (titmouses) propagate innovation (i.e., how to drill a hole in the cover of a milk bottle) more efficiently than territorial fellows (robins), we propose a stylized mathematical model of knowledge propagation founded on an analytically solvable multi-agent system. Individual agent dynamical state is modeled via a scalar variable which aggregates the agent’s time-dependent state of knowledge. Agent’s information is enhanced via imitation process, the efficiency of which is tuned by the number of leading fellows (i.e., those with higher state of knowledge) that each agent finds in a neighboring imitation range. Territorial agents (the robins) deploy comparatively small imitation range, which leads to an evanescent diffusive propagation wave of knowledge. This vanishing propagation contrasts with the non-territorial fellows (the titmouses), which implement significantly longer imitation ranges. From these long range interactions, one observes the self-emergence of a stable, constant velocity, collective propagating wave of knowledge. These drastically different dynamic behaviors suggest the existence of a critical imitation range where a behavioral phase transition occurs.

1 Introduction

In a contribution entitled Ideas and Growth [11], R. E. Lucas addresses the following fundamental question: What is it about modern capitalist economies that allows them, in contrast to all earlier societies, to generate sustained growth in productivity and living standards? To shed light on this question, R. E. Lucas proposes a model of growth built on the premise that all knowledge resides in the individual, and assuming that knowledge of a firm or an economy consists of the knowledge of its members. In [11], a continuum of people interact with each other randomly and copy each other’s productive ideas when it is beneficial to do so. Each individual benefits from the knowledge of those she interacts with, and she in turn stimulates her fellows. In Lucas’ words: [...] patents and "intellectual property" more generally play a very modest role in the overall growth of production-related knowledge. I have sought a formulation that emphasizes individual contributions of large numbers of people [...]. In [12], M. Ridley expresses basically the same idea by stating that: Economic growth itself is the work of a collective mind.
Focusing on elementary models of diffusion of innovation, we have been struck by an ornithology illustration, originally studied by three American evolutionary biologists (see in particular examples in [14]) and exposed by A. de Geus in his book *The Living Company* [4]. In [14] the authors develop the idea that evolution is essentially driven by species behavior, rather than by the environment only. To support this view, A. C. Wilson considers the behavior of songbirds in Great Britain. According to [14], at the beginning of the 20th century, Great Britain’s milkmen used to leave milk bottles outside of people’s home without any cover. Two species of songbirds, the titmouse and the robin had learned to feed on cream from these milk bottles. Then came an innovation in the milk industry in the 1930’s, that is covering milk bottle with aluminum bottle seals. According to [14], the titmouse had learned to pierce the aluminum seals and, in a matter of two decades, had successfully spread this newly acquired technique across their entire species over all Great Britain estimated at about a million individuals. In contrast, the robin never learned the aluminum seal drilling technique, this because of its very territorial attitude and isolation mode that does not promote the spread of innovative ideas. It was found that, while the titmouse is mobile and has a social behavior which encourages and promotes the propagation of innovation, the robin is mainly acting in autarky and hence do not possess the capability to exploit opportunities appearing in its environment. Indeed, the non-territorial titmouse is able to learn and to adapt to its environment in an agile and quick manner only thanks to its natural and efficient group mobility. This fascinating ornithological illustration has then been sorted out by A. de Geus to suggest one determinant feature that explains the long term survival of companies. Indeed A. de Geus points out in [4] that the capability to learn and quickly adapt to new environments is an essential characteristics of companies that manage to survive in the long term.

In the sequel, we follow the idea that the nature of mutual interactions between economic actors play a capital role in the economic growth process. We strongly emphasize that the global innovation process cannot be understood by the sole linear superposition of individual actions. The presence of intrinsically nonlinear features, like imitation processes involving several agents simultaneously, is determinant for the emergence of resilient self-generated innovation propagation patterns ultimately leading to economic growth. One will therefore assume here that it is possible to synthetically understand some basic aspects of economic growth as being due to the superposition of spontaneously generated innovative ideas (i.e. a steady linear component of the dynamics) with an imitation mechanism which selects and copies the best among these innovative ideas (n.b. this introduces a strongly nonlinear component into the dynamics). To investigate such complex nonlinear dynamics, B. LeBaron advocates [9, 10] for an agent-based computational methodology in the context of economy, which generally is, in B. LeBaron’s terms, [...] freed from the binds of analytical tractability and from the need to rely on narrow fragmented taxonomies arising from artificial disciplinary boundaries. While it should not be seen as being the ultimate goal, we still believe that analytical tractability of multi-agent systems offers a truly engrossing insight on how emergent behaviors are generated and this has motivated our present paper.

Analytical tractability brings us back to R. E. Lucas’ work [11] and its recent ramifications, in particular the following contributions [1, 13]. In his recent and truly engrossing paper entitled *Growth and the Diffusion of Ideas* [13], M. Staley builds on the Lucas’ model and extends it by assuming that each individual’s productivity also experiences random shocks. Starting from this premise, M. Staley derives a nonlinear reaction-diffusion equation (RDE) to describe the evolution of the distribution of income. Staley’s RDE is solved
by a traveling wave solution representing a growing economy. Inspired by this work, we propose here a novel fully solvable analytical model, which complements both R. E. Lucas and M. Staley previous papers. Indeed, as in [11] and [13], we assume that different agents have different states of knowledge so that an aggregated description of an economy can be summarized by a knowledge distribution function over the individual agents. Furthermore, as in [11] and [13], economic ideas are ranked, according to their productive usefulness, along the rungs of a scalar quality ladder. Economic actors (n.b. we will speak of agents from now on) have an incentive to adopt a higher productive state (i.e. jump to a higher rung) already occupied by another fellow (i.e. imitation process) or simply jump to a higher rung without any side considerations, (i.e. pure innovation process). In our approach, we allow in addition for the possibility of spontaneous regression moves (i.e. jumps to a lower rung), reflecting the idea that sometimes promising innovative moves may ultimately lead to counter productive effects. Our modeling approach is closely related to the dynamics that B. Jovanovic’s introduces in [7], as each agent’s evolution obeys a stochastic differential equation driven by a White Gaussian Noise process. We will illustrate in this paper that, despite its simplicity, our mathematical model is already sufficient to explain the drastically different dynamical propagation patterns observed for both robin and titmouse, and consequently also the phenomena appearing in economy where, according to A. de Geus, strongly self-confident and little reactive agents (i.e. territorial) contrast with highly reactive and open-minded fellows. We will be able to observe that territoriality implies a propagation of ideas dominated by a diffusive behavior, while non-territorial interactions produce stationary patterns which propagate innovation like a stationary wave (i.e. solitons). Our model implicitly unveils the existence of a behavioral phase transition separating these two propagating modes. The critical parameter that controls this phase bifurcation is the number of neighbors that directly influence a representative agent. In our model specifically, the number of fellows found in a given observation range determines the imitation susceptibility of a representative agent. When this number exceeds a critical threshold, a self-organizing transition is triggered. Such behavior is typical of and observed for bird flocking transitions [6], and is also found in other contexts like the spectacular behavioral phase transitions displayed by migratory locusts (see The Egyptian Plague, Exodus, Chapter 10), occurring when the density of larvas exceeds a critical threshold [2]. The present paper also complements and extends former recent contributions where similar dynamics have been investigated in presence of correlated colored noise [5, 6].

2 A Stylized Mathematical Model for Range-Dependent Imitation Processes

Let the dynamical state of each economic actor be summarized by a scalar state variable \( X_k(t), k = 1, 2, \ldots, N \), with \( N \) being the number of cohabitant economic agents. The state variables evolve according to a coupled set of nonlinear stochastic differential equations (SDEs):

\[
\begin{align*}
\dot{X}_k(t) &= f_k(X_k; \tilde{X}(t)) + \xi_k(t), \quad k = 1, 2, \ldots, N \quad \text{and} \quad X_k(t) \in \mathbb{R}, \\
X_k(t = 0) &= x_{k,0},
\end{align*}
\]

where \( \xi_k(t) \) stand for stochastic processes modeling the random environment affecting the dynamics of agent \( k \). The general drift force \( f_k(X_k; \tilde{X}(t)) \) is itself assumed to be
decomposable into an individual and an interactive component:

\[
f_k(X_k; \vec{X}(t)) := \frac{\mathcal{L}_k [X_k(t)]}{\text{individual dynamics}} + \frac{\mathcal{J}_k \left[ X_k(t); \vec{X}(t) \right]}{\text{interactions kernel}}.
\]  

(2)

The dynamics given by Eq. (2) clearly offers, in its full generality, a deep and rich modeling potential. However, our aim here being to construct a class of analytically solvable models, we will from now on require the following simplifying assumptions, namely:

(a) **Agent Homogeneity.** The full heterogeneity between the agents’ dynamical rules reflected by the \( k \)-subscript in \( f_k(X_k; \vec{X}(t)) \) will be removed and we will therefore assume that: \( \mathcal{L}_k(\cdot) \rightrightarrows \mathcal{L}(\cdot) \) and \( \mathcal{J}_k(\cdot) \rightrightarrows \mathcal{J}(\cdot), \forall k. \)

(b) **Agent Imitation Process.** The nonlinear interactions kernel \( \mathcal{J}(\cdot) \) generates an imitation mechanism that we will stylize by the following elementary algorithmic rules.

(i) **Observation Capability of Agents.** We assume that agent \( \mathcal{A}_k \) is permanently able to observe the dynamical states of all her \( (N - 1) \) fellows, namely: \( X_k(t) \in \mathbb{R}, \ k = 1, 2, \cdots, k - 1, k + 1, \cdots, N. \)

(ii) **Rest on One’s Laurels.** At time \( t \), we consider a tagged agent \( \mathcal{A}_k \) and assume that \( X_k(t) \geq X_j(t), \) for an agent \( \mathcal{A}_j, \ j \neq k. \) Then, our algorithmic rule imposes that the presence of \( \mathcal{A}_j \) does not modify the drift of \( \mathcal{A}_k, \) i.e. \( \mathcal{J} \left[ X_k(t); \vec{X}(t) \right] = 0 \) in this configuration and the drift in Eq. (2) reads as:

\[
f(X_k; \vec{X}(t)) = \mathcal{L}[X_k(t)] \iff X_j(t) \leq X_k(t), \ \forall j \neq k.
\]

In words, this is reflected as: being the leader, agent \( \mathcal{A}_k \) feels no incentive to imitate the dynamics of \( \mathcal{A}_j, \ j \neq k. \) Accordingly, agent \( \mathcal{A}_k \) keeps her nominal drift unchanged.

(iii) **Avoid Being the Laggard.** At time \( t \), we consider a tagged agent \( \mathcal{A}_k. \) Whenever \( X_k(t) < X_j(t), \) for \( j \neq k, \) agent \( \mathcal{A}_k \) realizes that she is a laggard. This incites her to enhance her drift in order to reduce her lateness. We assume \( \mathcal{A}_k \) to be influenced by those neighboring fellows \( \mathcal{A}_j, \ j \neq k, \) having dynamical states \( X_j(t) \in \Omega_k(t) := [X_k(t), X_k(t) + U], \) where \( U \geq 0 \) defines an imitation range for the observing agent. Quantitatively, for each agent \( \mathcal{A}_j \) found in \( \Omega_k(t), \) agent \( \mathcal{A}_k \) will try to reduce her lateness by adding a quantum of drift \( \gamma/N > 0 \) to her nominal drift \( f(X_k; \vec{X}(t)), \) the parameter \( \gamma \geq 0 \) standing for the imitation sensitivity of the agent, thus having the physical dimension of a frequency:

\[
f(X_k; \vec{X}(t)) = \mathcal{L}[X_k(t)] + \frac{N_{k,U}}{N} \gamma,
\]

(4)

where \( N_{k,U} \) is the number of agents \( \mathcal{A}_j, \ j \neq k, \) with \( X_j(t) \in \Omega_k(t). \)

For large \( N, \) the natural way to describe the global dynamical state of the multi-agent system is to introduce a field variable \( \rho(x, t) \) which describes the empirical probability density of the agents:

\[
\rho(x, t)dx = \frac{1}{N} \sum_{k=1}^{N} [\Theta(x) \ X_k(t)] \cdot \Theta(x) := \begin{cases} 1 & \text{when } x \in [x, x + dx], \\ 0 & \text{when } x \notin [x, x + dx]. \end{cases}
\]

(5)
Homogeneity implies that agents are undistinguishable. This simplifies the analytical approach as one can tag at random a single agent in the population and assume that her behavior will be representative of the whole agent society. Furthermore, we aggregate the influence of the tagged agent’s fellows into an effective external field, i.e. we adopt the well-established mean-field approach (MF). Introduced in statistical physics, the MF approach offers a powerful analytical tool for the analysis of multi-agent models, and more specifically in the context of economy (see for example \[8\]). For \(N \rightarrow \infty\), the law of large numbers holds in order to express that the relative importance of the fluctuations will decrease with the number of agents in the society. Ultimately, for \(N \rightarrow \infty\), the MF approach converges to exactness and the density function \(\rho(x,t)\) obeys a purely deterministic evolution equation.

In Eq.(1), the only modeling element remaining to be specified is the characteristics of the noise sources \(\xi_k(t)\). Alternatively, this raises questions regarding the dynamical features satisfied by \(X_k(t)\). The mathematically simplest candidate is given by the White Gaussian Noise (WGN) \(\xi_k(t) := \sigma dW_k(t)\) (n.b. \(W_k(t)\) denotes the Wiener process), which jointly implies the absence of noise correlations, the continuity of the realizations, and the Markovian character of \(X_k(t)\). Hence in presence of WGN, the Markovian aspect of the evolution ensures that all information required to solve Eq.(1) is contained in the initial conditions \(x_k,0\). Choosing WGN sources together with the interaction rules given in Eqs.(3) and (4), gives as a result that the deterministic evolution takes the form of a nonlinear Fokker-Planck (NLFP) equation \[5, 6\]:

\[
\frac{\partial}{\partial t} [\rho(x,t)] = -\partial_x \left\{ \mathcal{L}(x) + \gamma \int_0^x \rho(y,t)dy \right\} \rho(x,t) + \frac{\sigma^2}{2} \partial_{xx} [\rho(x,t)],
\]  

\(\rho(x,t) \in [0,1]\) and \(\lim_{|x| \to \infty} [\rho(x,t)] = 0\).

where to write Eq.(6), we have used the continuous representation:

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{k=1}^N \left[ \Theta(x) X_k(t) \right] = \int_x^{x+U} \rho(y,t)dy.
\]

We now observe that when both \(U > 0\) and \(\gamma > 0\), Eq.(6) is a nonlinear and non-local partial differential equation for \(\rho(x,t)\). Hence, explicit solutions are not to be expected in full generality, but the two following limiting regimes can be explicitly worked out.

(A) Infinitesimal Imitation Range \((U << 1, \text{ Robin-Like Behavior})\)

This regime assumes that interactions are effective only in an infinitesimal spatial range \(U\). This allows us to Taylor-expand (n.b. up to first order in \(U\)) the integral term in Eq.(6) to obtain:

\[
\frac{\partial_t [\rho(x,t)]}{\partial x} = -\partial_x \left\{ [\mathcal{L}(x) + (\gamma U)\rho(x,t)] \rho(x,t) \right\} + \frac{\sigma^2}{2} \partial_{xx} [\rho(x,t)],
\]

\(\lim_{|x| \to \infty} [\rho(x,t)] = 0\).
(B) Infinite Imitation Range \((U \to \infty, \text{Titmouse-Like Behavior})\)

On the extreme opposite case to regime \((A)\), we can again explicitly work out the dynamics in a differential form. Instead of the density \(\rho(x, t)\) which is involved in Eq.\((8)\), let us introduce and focus here on the complementary distribution function \(G(x, t)\):

\[
G(x, t) = \int_x^\infty \rho(y, t) \, dy \quad \Rightarrow \quad \partial_x G(x, t) = -\rho(x, t). \tag{9}
\]

When \(U = \infty\), using the notation of Eq.\((9)\) allows us to rewrite Eq.\((6)\) as:

\[
\begin{align*}
\left\{ \begin{array}{l}
\frac{\partial^2}{\partial x^2} [G(x, t)] = -\partial_x \left\{ [\mathcal{L}(x) + \gamma G(x, t)] (\partial_x G(X, t)) \right\} + \frac{\sigma^2}{2} \partial^3_{xxx} [G(x, t)], \\
G(-\infty, t) = 1 \quad \text{and} \quad G(+\infty, t) = 0.
\end{array} \right.
\end{align*} \tag{10}
\]

By integrating once Eq.\((10)\) with respect to \(x\) and imposing a vanishing integration constant \((\text{n.b. we effectively assume as usual that no probability current flows at infinity})\), one immediately obtains:

\[
\begin{align*}
\left\{ \begin{array}{l}
\partial_t [G(x, t)] = -\{ [\mathcal{L}(x) + \gamma G(x, t)] \partial_x G(X, t) \} + \frac{\sigma^2}{2} \partial^2_{xx} [G(x, t)], \\
G(-\infty, t) = 1 \quad \text{and} \quad G(+\infty, t) = 0.
\end{array} \right.
\end{align*} \tag{11}
\]

For general drift \(\mathcal{L}(x)\), explicit time-dependent general solutions of the nonlinear PDEs \((8)\) and \((11)\) cannot be derived. However, for constant drifts \(\mathcal{L}(x) = \alpha\) with \(\alpha \in \mathbb{R}\), we observe that Eqs.\((8)\) and \((11)\) exhibit, except for their boundary conditions, a fully similar functional form. To now solve these PDEs, let us first introduce the following change of referential: \(x \mapsto z = [x - \alpha t]\). Thanks to this Galilean transformation, Eqs.\((8)\) and \((11)\) reduce to the common functional form:

\[
\left\{ \begin{array}{l}
\partial_t [\varphi(z, t)] = -\Gamma \partial_z [\varphi(z, t)]^2 + \frac{\sigma^2}{2} \partial^2_{zz} [\varphi(z, t)], \\
\lim_{|x| \to \infty} [\varphi(z, t)] = 0,
\end{array} \right.
\]

where the parameter \(\Gamma\) in Eq.\((12)\) is suitably identified as:

\[
\Gamma = \begin{cases} 
\gamma U & \text{and} \quad \varphi(z, t) := \rho(z, t) \quad \text{for the model given in Eq.\((8)\)}, \\
\gamma & \text{and} \quad \varphi(z, t) := G(z, t) \quad \text{for the model given in Eq.\((11)\)}. 
\end{cases} \tag{13}
\]

The time evolution of \(\varphi(z, t)\) as given by Eq.\((12)\) is the celebrated nonlinear Burgers’ equation which can be linearized by using a logarithmic transformation. The explicit solutions are well known and read as described in cases \((A)\) and \((B)\) below (see for instance sections 8.3 and 8.4 in \([3]\)).

(A) Infinitesimal Imitation Range \((U << 1, \text{Robin-Like Behavior}), \text{c.f. Eq.\((8)\)}\)

For the boundary condition \(\lim_{|x| \to \infty} [\varphi(z, t)] = 0\) and for the initial condition \(\varphi(z, 0) = F(z)\), the solution of Eq. \((12)\), and subsequently the agent density function solving the model described by Eq.\((8)\) is given by:

\[
\rho(z, t) = \varphi(z, t) = \frac{\int_{\mathbb{R}} \left( \frac{z - \zeta}{2\pi T} \right) e^{-\left( \frac{\zeta}{T} \right)} \, d\zeta}{\int_{\mathbb{R}} e^{-\left( \frac{\zeta}{T} \right)} \, d\zeta}, \tag{14}
\]
with the definitions:
\[ \nu = \frac{\sigma^2}{4\Gamma} \quad \text{and} \quad f = f(\zeta, z, t) = \int_0^\zeta F(y) dy + \frac{(z - \zeta)^2}{2\Gamma t}. \] (15)

In particular, in presence of small noise intensity and for initial condition \( \varphi(z, 0) = F(z) = \delta(z) \Theta(z) \), the asymptotic behavior \((i.e. \ t \to \infty)\) of the dynamics given by Eq.\((14)\) can be approximately written as:
\[ \rho(z, t) = \varphi(z, t) \simeq \begin{cases} \frac{z}{2\Gamma t} & \text{if } 0 < z < \sqrt{4\Gamma t}, \\ 0 & \text{otherwise}, \end{cases} \] (16)

which, for this vanishing noise regime, converges towards a shock wave like pattern, as it is displayed in Figure 1.

![Figure 1: Resulting agent dynamics observed for infinitesimal imitation range \((U < 1, \text{Robin-Like Behavior})\), as given by Eq.\((14)\), when \( \gamma = 1, \alpha = 1, U = 0.1 \) and \( \sigma = 0.2 \). The interactions between the agents produce an asymmetric shape for \( \rho(x, t) \), which propagates at speed \( \alpha t \). Diffusion precludes the formation of a stationary pattern and for \( t \to \infty \), the resulting agent density becomes flat while still remaining normalized. This dynamics exhibit hence a complete absence of flocking behavior, as the agents are ultimately fully dispersed spatially. Innovation propagation is hence poorly met in this case, as innovative agents and early adopters do not influence enough their fellows because of the low level of interaction and imitation observed in the society.](image)

\( (B) \) **Infinite Imitation Range** \((U \to \infty, \text{Titmouse-Like Behavior}), c.f. \text{Eq.\((11)\)}\)

In that case, the boundary conditions are equal to \( \varphi(-\infty, t) = 1 \) and \( \varphi(+\infty, t) = 0 \). For any arbitrary initial condition \( \varphi(z, 0) = F(z) \), the solution of Eq.\((12)\), and subsequently the agent probability distribution solving the model described by Eq.\((11)\) can be written for asymptotic time as the following traveling wave solution:
\[ \varphi(z, t) = \frac{1}{2} \left[ 1 - \tanh \left( \frac{\Gamma(z - \Gamma t)}{\sigma^2} \right) \right]. \] (17)
Using Eq. (9), by differentiating Eq. (17) we obtain that the agent density function solving the model described by Eq. (11) is a soliton like propagating wave, as it is displayed in Figure 2:

$$\rho(z,t) = \partial_z \varphi(z,t) = \frac{\Gamma}{\sigma^2 \cosh^2 \left( \frac{\Gamma(z-\Gamma t)}{\sigma^2} \right)}. \quad (18)$$

Figure 2: Resulting agent dynamics observed for infinite imitation range ($U \to \infty$, Titmouse-Like Behavior), as given by Eq. (18), when $\gamma = 1$, $\alpha = 1$ and $\sigma = 1$. The interactions between the agents ultimately produce a soliton wave, which propagates at constant velocity $\alpha t$ and without altering its shape (n.b. the transient evolution is not represented in this figure). With this dynamics, we are hence in presence of a full flocking behavior, as the agents remain spatially tuned together. This behavior is obtained as the innovative agents and early adopters can significantly influence their fellows and consequently one reaches a high enough level of interaction and imitation within the society. As a result, innovation propagation propagates like a wave without damping and is hence efficiently met among the group of individuals.

It is truly engrossing to observe the fundamentally different dynamical behaviors emanating in the two regimes (A) and (B) exposed above, the solutions of which are given by Eqs. (14) and (18). Indeed, the variances $s^2(t)$ associated with Eqs. (14) and (18) can be explicitly written as:

$$s^2(t) = \int_{\mathbb{R}} (z^2) \rho(z,t) \, dz = \begin{cases} \frac{1}{6} \Gamma t \quad & \text{for short imitation range, c.f. case (A)}, \\ \left[ \frac{\pi \sigma^2}{2} \right] \frac{\sigma^4}{\Gamma^2} \quad & \text{for large imitation range, c.f. case (B)}. \end{cases} \quad (19)$$

Eq. (19) therefore exhibits a structural change for agents behaving with short versus long imitation ranges. The present modeling framework shows that only long range imitation mechanisms (i.e. titmouse like agents) give rise, for the innovation propagation, to the emergence of a stable stationary traveling wave having a constant variance in time. For shorter range mimetism behavior (i.e. robin like agents), imitation produces an emergent diffusive structure which, for small noise amplitudes, converges to a shock-wave type behavior, as given by Eq. (16). However in this case, the diffusion mechanism dominates and the resulting pattern is evanescent (i.e. no soliton like innovation wave can survive).
To summarize, the two drastically different regimes observed for titmice and robins, and quantified in this contribution, clearly suggest that it exists a critical imitation range $U_c$ which controls a behavioral phase transition leading to the emergence of an imitation wave within a society of individuals, [5, 6].

3 Perspectives and Conclusions

While the stylized approach adopted here may have been on purpose oversimplified in several aspects (e.g. scalar dynamics for the individual agents, simplistic interactions processes, noise sources without correlations), it offers however the truly exceptional opportunity to unveil comprehensible and fully analytic expressions for the characterization of the waves of innovation that can be induced by imitation mechanisms. Our model explicitly shows that only relatively few comparisons to neighboring fellows will not alter individual agent evolutions significantly enough to create a resilient and stationary self-organized spatio-temporal propagation (i.e. soliton wave). General results derived in the context of phase transitions for statistical mechanics (e.g. think of the celebrated Ising spin model) enable us to strongly believe that, while naive in some aspects, the behavioral transition observed in this contribution is generic (i.e. it does not depend on refined modeling details) and hence is likely to survive in more refined and more realistic contexts.

References

[1] P. Aghion and P. Howitt. Model of Growth Through Creative Destruction. *Econometrica*, 60(2):323-351, 1992.

[2] J. Buhl, D. J. T. Sumpter, I. D. Couzin, J. J. Hale, E. Despland, E. R. Miller, and S. J. Simpson. From Disorder to Order in Marching Locusts. *Science*, 312(5778):1402-1406, 2006.

[3] L. Debnath. Nonlinear Partial Differential Equations for Scientists and Engineers. Birkhäuser, (second edition), 2005.

[4] A. de Geus. The Living Company: Habits for Survival in a Turbulent Business Environment. Harvard Business School Press, 2002.

[5] F. Hashemi, M.-O. Hongler and O. Gallay. Spatio-Temporal Patterns for Generalized Innovation Diffusion Model. *Theoretical Economic Letters*, 2:1-9, 2012.

[6] M.-O. Hongler, R. Filliger and O. Gallay. Local versus Nonlocal Barycentric Interactions in 1D Dynamics. *Mathematical Bioscience and Engineering*, 11(2):323-351, 2014.

[7] B. Jovanovic. Job Matching and the Theory of Turnover. *The Journal of Political Economy*, 87(5):972-990, 1979.

[8] I. Karatzas and R. Fernholz. Stochastic Portfolio Theory: an Overview. *Handbook of Numerical Analysis*, 15:89-167, 2009.

[9] B. LeBaron. Agent-Based Computational Finance. *Handbook of Computational Economics*, 2:1187-1233 (Chapter 24), 2006.

[10] B. LeBaron and L. Tesfatsion. Modeling Macroeconomies as Open-Ended Dynamic Systems of Interacting Agents. *American Economic Review*, 98(2):246-250, 2008.
[11] R. E. Lucas, Jr. Ideas and Growth. *Economica*, 76(301):1-19, 2009.

[12] M. Ridley. The Rational Optimist. How Prosperity Evolves. Harper-Collins, NY, 2010.

[13] M. Staley. Growth and the Diffusion of Ideas. *Journal of Mathematical Economics*, 47(4-5):470-478, 2011.

[14] J. S. Wyles, J. G. Kunkel and A. C. Wilson. Birds, Behavior, and Anatomical Evolution. *Proceedings of the National Academy of Sciences of the United States of America*, 80(14):4394-4397, 1983.