RELATIVISTIC PSEUDOSPIN SYMMETRY IN NUCLEI

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1. Introduction

Peter Ring was one of the first to really grasp the significance of pseudospin symmetry as a relativistic symmetry [1, 2, 3, 4]. Originally, pseudospin doublets were introduced into nuclear physics to accommodate an observed near degeneracy of certain normal-parity shell-model orbitals with non-relativistic quantum numbers \((n_r, \ell, j = \ell + 1/2)\) and \((n_r - 1, \ell + 2, j = \ell + 3/2)\) where \(n_r, \ell,\) and \(j\) are the single-nucleon radial, orbital, and total angular momentum quantum numbers, respectively [5, 6]. The doublet structure, is expressed in terms of a “pseudo” orbital angular momentum \(\tilde{\ell} = \ell + 1\) coupled to a “pseudo” spin, \(\tilde{s} = 1/2\). For example, \((n_r s_{1/2}, (n_r - 1)d_{3/2})\) will have \(\tilde{\ell} = 1\), \((n_r f_{3/2}, (n_r - 1)f_{5/2})\) will have \(\tilde{\ell} = 2\), etc. Since \(j = \tilde{\ell} \pm \tilde{s}\), the energy of the two states in the doublet is then approximately independent of the orientation of the pseudospin. Some examples are given in Table 1. In the presence of deformation the doublets persist with asymptotic (Nilsson) quantum numbers \([N, n_3, \Lambda, \Omega = \Lambda + 1/2]\) and \([N, n_3, \Lambda + 2, \Omega = \Lambda + 3/2]\), and can be expressed in terms of pseudo-orbital and total angular momentum projections \(\tilde{\Lambda} = \Lambda + 1, \Omega = \tilde{\Lambda} \pm 1/2\). This pseudospin “symmetry” has been used to explain features of deformed nuclei [7], including superdeformation [8] and identical bands [9, 10, 11]. While pseudospin symmetry is experimentally well corroborated in nuclei, its foundations remained a mystery and “no deeper understanding of the origin of these (approximate) degeneracies” existed [12]. In this contribution we review more recent developments that show that pseudospin symmetry is an approximate relativistic symmetry of the Dirac Hamiltonian with realistic nuclear mean field potentials [1, 2].
TABLE 1. Experimental (Exp) and relativistic mean field (RMF) pseudospin binding energy splittings $\epsilon_{j'=\ell+1/2} - \epsilon_{j=\ell-1/2}$ for various doublets in $^{208}$Pb.

| $\ell$ | $(n_r-1, \ell+2, j'=\ell+3/2)$ | $\epsilon_{j'=\ell+1/2} - \epsilon_{j=\ell-1/2}$ (Exp) (MeV) | $\epsilon_{j'=\ell+1/2} - \epsilon_{j=\ell-1/2}$ (RMF) [3] (MeV) |
|-------|---------------------------------|-------------------------------------------------|-------------------------------------------------|
| 4     | 0h9/2 - 1f7/2                  | 1.073                                          | 2.575                                          |
| 3     | 0g7/2 - 1d5/2                  | 1.791                                          | 4.333                                          |
| 2     | 1f5/2 - 2p3/2                  | -0.328                                         | 0.697                                          |
| 1     | 1d3/2 - 2s1/2                  | 0.351                                          | 1.247                                          |

2. Pseudospin Symmetry of the Dirac Hamiltonian

The Dirac Hamiltonian, $H$, with an external scalar, $V_S$, and vector, $V_V$, potentials is invariant under a SU(2) algebra for $V_S = -V_V$ leading to pseudospin symmetry in nuclei [2]. The pseudospin generators, $\hat{S}_\mu$, which satisfy $[H, \hat{S}_\mu] = 0$ in the symmetry limit, are given by

$$\hat{S}_\mu = \left( \begin{array}{cc} \hat{s}_\mu & 0 \\ 0 & \hat{s}_\mu \end{array} \right) = \left( \begin{array}{cc} U_p \hat{s}_\mu U_p & 0 \\ 0 & \hat{s}_\mu \end{array} \right)$$

(1)

where $\hat{s}_\mu = \sigma_\mu / 2$ are the usual spin generators, $\sigma_\mu$ the Pauli matrices, and $U_p = \frac{\mathbf{r} \cdot \mathbf{p}}{p}$ is the momentum-helicity unitary operator introduced in [13]. If, in addition, the potentials are spherically symmetric, the Dirac Hamiltonian has an additional invariant SU(2) algebra, $[H, \hat{L}_\mu] = 0$, with the pseudo-orbital angular momentum operators given by $\hat{L}_\mu = \left( \begin{array}{c} \hat{l}_\mu \\ 0 \\ 0 \end{array} \right)$, where $\hat{r}_\mu = U_p \hat{l}_\mu U_p$, $\hat{l}_\mu = \mathbf{r} \times \mathbf{p}$. The pseudo spin $\tilde{s}$ and pseudo orbital angular momentum $\tilde{\ell}$ are seen to be the ordinary spin and orbital angular momentum respectively of the lower component of the Dirac wave function. The upper components of the two states in the doublet have orbital angular momentum $\tilde{\ell} \pm 1$ for $j = \tilde{\ell} \pm 1/2$ in agreement with the spherical pseudospin doublets originally observed. The corresponding radial quantum numbers are discussed in [14, 15]. In the pseudospin symmetry limit the two states in the doublet $j = \tilde{\ell} \pm 1/2$ are degenerate, and are connected by the pseudospin generators $\hat{S}_\mu$ of Eq. (1). This implies relationships between the doublet wave functions. In particular, since $\hat{S}_\mu$ have the spin operator $\hat{s}_\mu$ operating on the lower component of the Dirac wave function, it follows that the
Figure 1. The lower components of Dirac eigenfunctions ($2s_1/2, 1d_3/2$) in $^{208}\text{Pb}$ [3].

Figure 2. The lower components of Dirac eigenfunctions $[400]1/2$ (— solid line, — short-dash line) and $[402]3/2$ (— dash line, — dash-dot line) at $z = 1 \text{ fm}$ [16].

The spatial part of these components will be equal for the two states in the doublet within an overall phase, as can be seen in Fig. 1.

For axially deformed potentials satisfying $V_S = -V_V$, there is, in addition to pseudospin, a conserved $U(1)$ generator, corresponding to the pseudo-orbital angular momentum projection along the body-fixed symmetry axis, $\hat{\mathbf{\Lambda}} = \left(\hat{\Lambda}_0 0 0 \hat{\Lambda}\right)$, where $\hat{\mathbf{\Lambda}} = U_p \hat{\mathbf{\Gamma}} U_p$. In this case the Dirac wave functions are eigenstates of $\hat{\mathbf{\Lambda}}$ and both components have the same total angular momentum projection $\Omega$. The lower component has pseudo-orbital angular momentum projection $\hat{\Lambda}$ while the upper component has $\hat{\Lambda} \pm 1$ for $\Omega = \hat{\Lambda} \pm 1/2$, in agreement with the deformed pseudospin doublets mentioned in Section 1. For axially deformed nuclei the eigenfunctions depend on two spatial variables, $z$ and $\rho = \sqrt{x^2 + y^2}$, and there are two upper $g_{\pm}(\rho, z) \chi_{\pm 1/2}$ and lower $f_{\pm}(\rho, z) \chi_{\pm 1/2}$ components where $\chi_{\pm 1/2}$ is the spin wavefunction. Pseudospin symmetry predicts that, for the pseudospin eigenfunction with pseudospin projection $1/2$ ($-1/2$), the lower component $f_-(\rho, z)$ [$f_+(\rho, z)$] is zero. In addition, the lower component $f_+(\rho, z)$ for the pseudospin eigenfunction with pseudospin projection $1/2$ is equal to the the lower component $f_-(\rho, z)$ for the pseudospin eigenfunction with pseudospin projection $-1/2$ up to an overall phase. These relations are illustrated in Fig. 2.
However, in the exact pseudospin limit, $V_S = -V_V$, there are no bound Dirac valence states. For nuclei to exist the pseudospin symmetry must therefore be broken. Nevertheless, realistic mean fields involve an attractive scalar potential and a repulsive vector potential of nearly equal magnitudes, $V_S \sim -V_V$, and recent calculations in a variety of nuclei confirm the existence of an approximate pseudospin symmetry in both the energy spectra and wave functions [3, 4, 16, 17]. In Table 1 pseudospin-orbit splittings calculated in the RMF [3] are compared with the measured values in the spherical nucleus $^{208}$Pb and are seen to be larger than the measured splittings which demonstrates that the pseudospin symmetry is better conserved experimentally than mean field theory would suggest. Figures 1 and 2 show that in realistic RMF calculations, the expected relations between the lower components of the two states in the doublets are approximately satisfied both for spherical and axially deformed nuclei. The behavior of the corresponding upper components which dominate the Dirac eigenstates is discussed in the next section.

3. Test of Nuclear Wave Functions for Pseudospin Symmetry

Since pseudospin symmetry is broken, the pseudospin partner produced by the raising and lowering operators acting on an eigenstate will not necessarily be an eigenstate. The question is how different is the pseudospin partner from the eigenstate with the same quantum numbers? A recent study [17] addressing this question has shown that the radial wave functions of the upper components of the $j = \tilde{\ell} - 1/2$ pseudospin partner of the eigenstate with $j = \tilde{\ell} + 1/2$ is similar in shape to the $j = \tilde{\ell} - 1/2$ eigenstate but there is a difference in magnitude. This is shown in Fig. 3 where we compare the $s_{1/2}$ pseudospin partners (denoted by $[P(0d_{3/2})]s_{1/2}$, $[P(1d_{3/2})]s_{1/2}$) of the Dirac eigenstates $0d_{3/2}$, $1d_{3/2}$ ($\tilde{\ell} = 1$, $j = 3/2$), for $^{208}$Pb [3]. The lower components agree very well, which was noted previously, except for some disagreement on the surface. For the upper components the agreement is not as good in the magnitude but the shapes agree well, with the number of radial nodes being the same. The agreement improves as the radial quantum number increases, which is consistent with the observed decrease in the energy splitting between the doublets [1, 3].

On the other hand, the radial wave functions of the upper components of the $j = \tilde{\ell} + 1/2$ pseudospin partner of the eigenstate with $j = \tilde{\ell} - 1/2$ is not similar in shape to the $j = \tilde{\ell} + 1/2$ eigenstate. In fact these wave functions approach $r^{\tilde{\ell}+1}$ rather than $r^{\tilde{\ell}+3}$ for $r$ small, do not have the same number of radial nodes as the eigenstates, and do not fall off exponentially as do the eigenstates, but rather fall off as $r^{-(\tilde{\ell}+2)}$. Furthermore, the pseudospin partners of the “intruder” nodeless eigenstates, also fall off as as $r^{-(\tilde{\ell}+2)}$. As an
Figure 3. a) The upper component \([g(r)]\) and b) the lower component \([f(r)]\) in (Fermi)\(^{-3/2}\) of the \([P(0d_{3/2})s_{1/2}]\) partner of the 0\(d_{5/2}\) eigenstate compared to the 1\(s_{1/2}\) eigenstate. c) The upper component and d) the lower component of the \([P(1d_{3/2})s_{1/2}]\) partner of the 1\(d_{5/2}\) eigenstate compared to the 2\(s_{1/2}\) eigenstate for \(^{208}\)Pb \([3, 17]\).

example of this category we show in Fig. 4a,b the radial wavefunction of the \([P(0f_{7/2})h_{9/2}]\) partner of the 0\(f_{7/2}\) intruder state (\(\ell = 4, j = 7/2\)). There is no quasi-degenerate \(h_{9/2}\) eigenstate to compare to. The upper component has the \(r^{-6}\) falloff alluded to above. Although both components have zero radial quantum number, they do not compare well with the \(0h_{9/2}\) eigenstate shown in Fig. 4c,d. In Fig. 4c,d we show also the radial wavefunction of the \([P(1f_{7/2})h_{9/2}]\) partner of the 1\(f_{7/2}\) state (\(\ell = 4, j = 7/2\)) and compare it to the \(0h_{9/2}\) eigenstate. The upper component has again the \(r^{-6}\) falloff and therefore does not compare well on the surface. Also the number of radial quantum numbers differ. The lower components agree better. Work is in progress to study the goodness of pseudospin symmetry in the upper components of Dirac eigenfunctions in deformed nuclei.
Figure 4. a) The upper component \([g(r)]\) and b) the lower component \([f(r)]\) in (Fermi)\(^{-3/2}\) of the \([P(0f_{7/2})]h_{9/2}\) partner of the \(0f_{7/2}\) eigenstate. c) The upper component and d) the lower component of the \([P(1f_{7/2})]h_{9/2}\) partner of the \(1f_{7/2}\) eigenstate compared to the \(0h_{9/2}\) eigenstate for \(^{208}\)Pb [3, 17].

4. Implications for M1 and Gamow Teller Transitions

Because the lower components are small, in order to test the pseudospin symmetry prediction that the lower components are almost identical we must observe transitions for which the upper components are not dominant. Magnetic dipole and Gamow-Teller transitions between the states in the doublet are forbidden non-relativistically since the orbital angular momentum of the two states differ by two units, but are allowed relativistically. Pseudospin symmetry predicts that, if the magnetic moments, \(\mu\), of the two states are known, the magnetic dipole transition, \(B(M1)\), between the states can be predicted. Likewise if the Gamow - Teller transitions between the states with the same quantum numbers are known, the transition between the states with different quantum numbers can be predicted [18].
For example for neutrons, the M1 transition is given by

\[
\sqrt{B(M1 : n_r - 1, \ell + 2, j' \rightarrow n_r, \ell, j)_\nu} = -\sqrt{\frac{j + 1}{2j + 1}} (\mu_{j',\nu} - \mu_{A,\nu})
= \frac{j + 2}{2j + 3} \sqrt{\frac{2j + 1}{j + 1}} (\mu_{j',\nu} + \frac{j + 1}{j + 2} \mu_{A,\nu}),
\]

(2)

where \(j' = \ell + 3/2, j = \ell + 1/2\) and \(\mu_{A,\nu} = -1.913\mu_0\) is the anomalous magnetic moment. A survey of forbidden magnetic dipole transitions taking into account the single-particle corrections by using spectroscopic factors shows a reasonable agreement with these relations, an example of which is given in Fig. 5 [19].

5. Possible Connection to Chiral Symmetry Breaking in QCD

Applying QCD sum rules in nuclear matter [20], the ratio of the scalar and vector self-energies were determined to be \(\frac{V_S}{V_V} \approx -\frac{\sigma_N}{m_q}\) where \(\sigma_N\) is the sigma term which can be measured in pion-nucleon scattering and \(m_q\) is the current quark mass. For reasonable values of \(\sigma_N\) and quark masses, this ratio is close to -1. The significance of the sigma term is that it is a measure of chiral symmetry breaking and vanishes if chiral symmetry is
conserved. The implication of these results is that chiral symmetry breaking is responsible for the scalar field being approximately equal in magnitude to the vector field, thereby producing pseudospin symmetry.

6. Future Outlook

We have reviewed the foundations and implications of the relativistic pseudospin symmetry in nuclei. Although not discussed here, pseudospin symmetry has been shown to be approximately conserved in medium energy nucleon-nucleus scattering from nuclei [21, 22] but badly broken in low energy nucleon-nucleus scattering [23]. A recent study [24] suggests that pseudospin symmetry will improve for neutron rich nuclei. This needs further investigation. Finally, the link between pseudospin symmetry and chiral symmetry breaking in nuclei needs to be explored at a fundamental level.

7. Acknowledgements

D. G. Madland and P. von Neumann Cosel collaborated on different aspects of the work reported in this survey. This research was supported in part by the United States Department of Energy under contract W-7405-ENG-36 and by the U.S.-Israel Binational Science Foundation.

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