The Phantom Bounce: A New Proposal for an Oscillating Cosmology

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**Abstract**

An oscillating universe cycles through a series of expansions and contractions. We propose a model in which “phantom” energy with a supernegative pressure \( p < -\rho \) grows rapidly and dominates the late-time expanding phase. The universe’s energy density is so large that the effects of quantum gravity are important at both the beginning and the end of each expansion (or contraction). The bounce can be caused by high energy modifications to the Friedmann equation, which make the cosmology nonsingular. The classic black hole overproduction of oscillating universes is resolved due to their destruction by the phantom energy.
I. INTRODUCTION

The arrow of time is intimately connected to the entropy of the universe. The second law of thermodynamics inexorably drives us to ever increasing entropy, yet we live in neither a situation of maximal entropy (a black hole) nor in a minimal entropy universe. Apparently we thrive in the current “medium entropy” universe. How is this possible? In this conference, two possible explanations for this homogenous and isotropic universe we live in have been discussed: special initial conditions or eternal inflation combined with anthropic arguments. In fact, there is a third option: cyclicity. Here the universe oscillates through a series of expansions and contractions. In a successful model of a cyclic nature, the entropy that has been created in each cycle must yet again be destroyed in order to reset the stage for the next oscillation. We discuss a “phantom bounce” [1] as a proposal for an oscillating universe in which we postulate violation of the weak energy condition as a mechanism to destroy the (high entropy) black holes that are produced during each cycle. A key advantage of our proposal is that the phantom component of our proposal is testable in astrophysical data soon.

We discuss a scenario in which the universe oscillates through a series of expansions and contractions. After it finishes its current expanding phase, the universe reaches a state of maximum expansion which we will call “turnaround”, and then begins to recollapse. Once it reaches its smallest extent at the “bounce”, it will once again begin to expand. This scenario is distinguished from other proposed cyclic universe scenarios [2, 3] in that cosmological acceleration due to “phantom” energy (i.e., dark energy with a supernegative equation of state, $p < -\rho$) [4] plays a crucial role. We originally proposed the phantom bounce in 2004 [1]. Subsequent related work includes [5, 6]; see also [7]. Perturbations in this cosmology were discussed in [8].

The idea of an oscillating universe was first proposed in the 1930’s by Tolman. Over the subsequent decades, two problems stymied the success of oscillating models. First, the formation of large scale structure and of black holes during the expanding phase leads to problems during the contracting phase [9]. The black holes, which cannot disappear due to Hawking area theorems, grow ever larger during subsequent cycles. Eventually, they occupy the entire horizon volume during the contracting phase so that calculations break down. (Only the smallest black holes can evaporate via Hawking radiation.) The second unsolved
problem of oscillating models was the lack of a mechanism for the bounce and turnaround. The turnaround at the end of the expanding phase might be explained by invoking a closed universe, but the recent evidence for cosmological acceleration removes that possibility. For the observationally favored density of “dark energy”, even a closed universe will expand forever. Thus, cyclic cosmologies appeared to conflict with observations.

Our scenario resolves these problems. Our resolution to the black hole overproduction problem is provided by a “phantom” component to the universe, which destroys all structures towards the end of the universe’s expanding stage. Phantom energy, a proposed explanation for the acceleration of the universe, is characterized by a component $Q$ with equation of state

$$w_Q = p_Q/\rho_Q < -1.$$

(1)

Since the sum of the pressure and energy density is negative, the dominant energy bound of general relativity is violated; yet recent work explores such models nevertheless. Phantom energy can dominate the universe today and drive the current acceleration. Then it becomes ever more dominant as the universe expands. With such an unusual equation of state, the Hawking area theorems fail, and black holes can disappear \cite{10}. In “big rip” scenarios \cite{11}, the rapidly accelerating expansion due to this growing phantom component tears apart all bound objects including black holes. (We speculate about remnants of these black holes below.)

The phantom energy density becomes infinite in finite time \cite{11, 12}. The energy density of any field described by equation of state $w_Q$ depends on the scale factor $a$ as

$$\rho_Q \sim a^{-3(1+w_Q)}.$$

(2)

Hence, for $w_Q < -1$, $\rho_Q$ grows as the universe expands. Of course, we expect that an epoch of quantum gravity sets in before the energy density becomes infinite. We therefore arrive at the peculiar notion that quantum gravity governs the behavior of the universe both at the beginning and at the end of the expanding universe (i.e., at the smallest and largest values of the scale factor). Here we consider an example of the role that high energy density physics may play on both ends of the lifetime of an expanding universe: we consider the idea that large energy densities may cause the universe to bounce when it is small, and to turn around when it is large. The idea is economical in that it is the same physics which operates at both bounce and turnaround.
We use modifications to the Friedmann equations to provide a mechanism for the bounce and the turnaround that are responsible for the alternating expansion and contraction of the universe. In particular, we focus on “braneworld” scenarios in which our observable universe is a three-dimensional surface situated in extra dimensions. Several scenarios for implementing a bounce have been proposed in the literature [13, 14]. As an example, we focus on the modification to the Randall-Sundrum [16] scenario proposed by Shtanov and Sahni [13], which involves a negative brane tension and a timelike extra dimension leading to a modified Friedmann equation. Another example is the quantum bounce in loop quantum gravity [6, 15]. Once the energy density of the universe reaches a critical value, cosmological evolution changes direction: if it has been expanding, it turns around and begins to recontract. If it has been contracting, it bounces and begins to expand.

We emphasize that the two components we propose here work together: we use a modified Friedmann equation as a mechanism for a bounce and turnaround, and we add a phantom component to the universe to destroy black holes. Due to the phantom component, the same high energy behavior that produces a bounce at the end of the contracting phase also produces a turnaround at the end of the expanding phase. In addition, the bounce and turnaround are both nonsingular, unlike the cyclic scenario proposed by Steinhardt and Turok [3], which is complicated by a number of physical singularities related to brane collisions near the bounce [17]. This is currently a very controversial topic.

II. THE BOUNCING COSMOLOGY

In an oscillating cosmology, what we observe to be “The Big Bang” really is the universe emerging from a bounce. The universe at this point has its smallest extent (smallest scale factor \(a\)) and largest energy density, somewhere near the Planck density. The universe then expands, its density decreases, and it goes through the classic radiation dominated and matter dominated phases, with the usual primordial nucleosynthesis, microwave background, and formation of large structure. A period of inflation may or may not be necessary to establish flatness and homogeneity. At a redshift \(z = O(1)\), the universe starts to accelerate due to the existence of a vacuum component or quintessence field \(Q\). We take a “phantom” component with \(w_Q < -1\). The energy density of this component grows rapidly as the universe expands. Any structures produced during the expanding phase, including
galaxies and black holes, are torn apart by the extremely rapid expansion provided by the phantom component. Any physics relevant at the high densities near the “Big Bang” again becomes important at the high densities near the end of the expanding phase. Modifications to the Friedmann equation become important at high densities, and cause the universe to turn around. The universe reaches a characteristic maximum density $2|\sigma|$ (which might be anywhere in the range from TeV to $M_p$), and starts to contract. As it contracts, at first its energy density decreases (as the phantom component decreases in importance), but then it again increases as matter and radiation become dominant. Eventually it reaches the high values at which the modifications to Friedmann equations become important. Once the energy density again reaches the same characteristic scale $2|\sigma|$, the universe stops contracting, bounces, and once again expands.

In the standard cosmology, there is no way to avoid a singularity for small radius or scale factor $a$. In the context of extra dimensions, however, one can have a bounce at finite $a$ so that singularities are avoided. A nonsingular bounce is obtained if the Friedmann equation is modified by the addition of a new negative term on the right hand side:

$$H^2 = \frac{8\pi}{3M_p^2} \left[ \rho - f(\rho) \right],$$

where the function $f(\rho)$ is positive. For a contracting universe to reverse and begin expanding again, we must have $\ddot{a} > 0$, which results in a condition on $f(\rho)$,

$$3 (1 + w) \rho f' (\rho) - 2 f(\rho) - (1 + 3w)\rho > 0.$$  

Similarly, for an expanding universe, $\ddot{a}$ must be negative for the expansion to reverse. A modified Friedmann equation of the form of Eq. (3) can be motivated in the context of braneworld scenarios, where our observable universe is a 3-dimensional surface embedded in extra dimensions. Ref. [18] showed that Einstein’s equations in higher dimensions, together with Israel boundary conditions on our brane, can give rise to an equation of the form of Eq. (3). Different values of energy/momentum in the extra dimensions (the bulk) can be responsible for different $f(\rho)$ in Eq. (3).

In particular, we focus on “braneworld” motivated modifications to the Friedmann equation, where the modification to the Friedmann equation for the brane bound observer [13, 18, 19] is

$$H^2 = \frac{\Lambda_4}{3} + \left( \frac{8\pi}{3M_p^2} \right) \rho + \epsilon \left( \frac{4\pi}{3M_5^2} \right)^2 \rho^2 + \frac{C}{a^4}.$$  

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where the last term \((C)\) is an integration constant) appears as a form of “dark radiation” (that is constrained like ordinary radiation), and \(\epsilon\) corresponds to the metric signature of the extra dimension \([13]\). We will also assume that the bulk cosmological constant is set so that the three-dimensional cosmological constant \(\Lambda_4\) is negligible\(^1\). Hence the relevant correction to the Friedmann equation is the quadratic term, \(f(\rho) = \rho^2/2|\sigma|\). For \(\epsilon < 0\), the Friedmann equation becomes

\[
H^2 = \frac{8\pi}{3M_p^2} \left[ \rho - \frac{\rho^2}{2|\sigma|} \right].
\]

One way to obtain \(\epsilon < 0\) corresponds to an extra timelike dimension: models with more than one time coordinate typically suffer from pathologies such as closed timelike curves and non-unitarity. We use the model in Ref. \([13]\) to motivate the choice of sign in the Friedmann equation, but a more detailed treatment would need to address these other issues to form a fully consistent picture.

Alternatively, in loop quantum gravity, there is a quantum bounce that takes place at Planck densities in lieu of the singularity in the standard classical Friedmann equation \([6, 15]\); if one couples this quantum bounce with a phantom component as in this paper, one would again obtain the same oscillating cosmology as discussed in this paper.

The expansion rate of the universe \(H = 0\) at \(\rho_{\text{bounce}} = 2|\sigma|\); it is at this scale that the universe bounces and turns around. For this choice of \(f(\rho)\),

\[
3 (1 + w) \rho f' (\rho) - 2 f (\rho) - (1 + 3w)\rho = 3 (1 + w) \rho,
\]

and the required condition on \(\ddot{a}\) is satisfied at both bounce \((w > 0)\) and turnaround \((w < -1)\). On one end of the cycle it goes from contracting to expanding (this bounce looks to us like the Big Bang), and then at the other end of the cycle it goes from expanding to contracting. This behavior is illustrated in the Figure. In models motivated by the Randall-Sundrum scenario, the most natural value of the brane tension is \(\sigma = M_p\), but we treat the problem generally for any value of \(\sigma > \text{TeV}\).

At scales above \(\rho > \sigma\), the validity of Eq. \([6]\) breaks down in detail. However, the approach to \(H = 0\) and thus the existence of a bounce and turnaround remain sensible. In any case, we use this braneworld model merely as an example of a correction to the

\(^1\) This fine-tuning is the usual cosmological constant problem, which is not addressed in this paper.
FIG. 1: Scale factor (left) and energy density (right) at the bounce and turnaround, plotted as functions of time. The dotted lines in the plot of the energy density show $\rho = \sigma$. We note that the energy density is large at the bounce due to radiation ($\rho_{\text{rad}} \propto a^{-4}$) and is large at turnaround due to phantom energy $\rho_Q \propto a^{3|1+w_Q|}$. The plots are presented for $w = -4/3$ to illustrate the basic behavior of the model (detailed numbers are irrelevant).

Friedmann equation. Other modifications to the Friedmann equation might work as well, as long as there is the requisite minus sign in the equation.

III. DESTRUCTION OF BLACK HOLES

Black holes pose a serious problem in a standard oscillating universe. However, the Hawking area theorems that guarantee the continued existence of black holes have been constructed in special settings and may not apply here; e.g., the same modifications to gravity that give a bounce rather than a singularity in the cosmology may avoid singularities in the black holes. Indeed, when $w_Q < -1$, Davies [10] has shown that the theorem does not hold. Recently, Caldwell, et al. [11] described the dissolution of bound structures in
the “big rip” towards the end of a phantom dominated universe. Any black holes formed in
an expanding phase of the universe are torn apart before they can create problems during
contraction.

When are the black holes destroyed? We want to be certain that they are torn apart
before turnaround. In general relativity, the source for a gravitational potential is the volume
integral of $\rho + 3p$. An object of radius $R$ and mass $M$ is pulled apart when

$$-\frac{4\pi}{3}(\rho + 3p)R^3 \sim M. \quad (8)$$

Writing $\rho + 3p = \rho(1 + 3w_Q)$ during phantom domination and taking $R = 2GM$ for the
black hole, we find that black holes are pulled apart when $-(4\pi/3)\rho(1 + 3w_Q)8M^3/M_p^6 \sim M$,
which happens when the energy density of the universe has climbed to a value

$$\rho_{BH} \sim M_p^4 \left(\frac{M_p}{M}\right)^2 \frac{3}{32\pi} \frac{1}{|1 + 3w_Q|}. \quad (9)$$

More massive black holes are destroyed at lower values of $\rho$, i.e. earlier. It is the smallest
black holes that get shredded last.

We must ensure that the black holes are destroyed before turnaround, so that $\rho_{BH} < \rho_{\text{turn}} = 2|\sigma|$. As an example, we can take $w_Q = -3$. Then $10^6$ solar mass black holes,
such as those at the centers of galaxies, get pulled apart when $\rho \sim 10^{-90}M_p^4$, which easily
satisfies the above condition. The most tricky case would be Planck mass black holes, which
either formed primordially or are relics of larger black holes that Hawking radiated. Even
these should still be disrupted. From Eq. ($9$) these will be shredded when $\rho \sim 10^{-2}M_p$, 
before turnaround if the brane tension $|\sigma| = M_p^4$. However, for GUT scale brane tension $|\sigma| = m_{\text{GUT}}^4$, only black holes with $M \geq 10^5M_p$ are disrupted. Fortunately these black holes
Hawking evaporate in a time $\tau \sim (25\pi M^3/M_p^4)$ where $M$ is the black hole mass. This occurs
in only $\sim 10^{-27}$ sec for a black hole with $M = 10^5M_p$. We also speculate that Planck mass
remnant black holes that cannot disappear (still containing the singularity) may be dark
matter candidates.

IV. DISCUSSION

Our proposal contains the novel feature that both bounce and turnaround are produced
by the same modification to the Friedmann equation. However, it does so at the price of
including more than one speculative element: the modified Friedmann equation requires a braneworld model to achieve, and the cosmology must be dominated by phantom energy. In many cases a phantom component is difficult to implement from a fundamental standpoint without severe pathologies such as an unstable vacuum (see, for example, Ref. [22].) However, Parker and Raval [23] have investigated a cosmological model with zero cosmological constant, but containing the vacuum energy of a simple quantized free scalar field of low mass, and found that it has $w < -1$ without any pathologies. Several additional areas also remain to be addressed. First, as the universe is contracting, those modes of the density fluctuations that we usually throw away as decaying (in an expanding universe) are instead growing. Hence dangerous structures may form during the contracting phase. At the end of the contracting phase, there is no phantom energy to wipe out whatever structure is formed. In this sense, the initial conditions for structure formation in this picture are set either during the phantom energy dominated epoch near turnaround or by the quantum generation of fluctuations in the collapsing phase [20]. Black hole formation could still kill the model. Second, it is not obvious that it is possible to create a truly cyclic (i.e. perfectly periodic) cosmology within the context of the “Phantom Bounce” scenario. The reason for this is entropy production. We speculate that it may be possible to create quasi-cyclic evolution by redshifting entropy out of the horizon during the period of accelerating expansion. Even more speculatively, we note that the special case of $w_Q = -7/3$, although disfavored by observation, possesses an intriguing duality between radiation ($\rho_{\text{rad}} \propto a^{-4}$) and phantom energy ($\rho_Q \propto a^4$). In this case, the behaviors of these components exchange identity under a transformation $a \rightarrow 1/a$ [21], effectively exchanging bounce for turnaround, a symmetry which might be exploited to achieve truly cyclic evolution.

On the observational side, our key ingredient is testable. The current expansion of the universe is the subject of much intense investigation. The universe is apparently accelerating, but the exact nature of the acceleration is not yet known. The previous value of the equation of state may be discovered over the next decade. The current uncertainty in the equation of state easily allows for the possibility of a phantom energy; some [24] have argued that $w_Q < -1$ is an excellent fit to the data. If upcoming observations discover that such a phantom energy indeed exists, then the community may be forced to conclude that the weak energy condition is violated and will need to rethink many basic assumptions. Phantom energy may be forced upon us, with the helpful consequence of permitting the “medium”
entropy universe we inhabit.

[1] M. G. Brown, K. Freese and W. H. Kinney, “The phantom bounce: A new oscillating cosmology,” arXiv:astro-ph/0405353 JCAP in press.
[2] R. Tolman, *Relativity, Thermodynamics and Cosmology* (Oxford: Oxford U. Press, 1934).
[3] P. Steinhardt and N. Turok, *Phys.Rev.* D65 126003 (2002); J. Khoury, P. Steinhardt, and N. Turok, *Phys.Rev.Lett.* 92 031302 (2004).
[4] S. M. Carroll, M. Hoffman and M. Trodden, *Phys. Rev.* D 68, 023509 (2003); A. Melchiorri, L. Mersini, C. J. Odman and M. Trodden, *Phys. Rev.* D 68, 043509 (2003).
[5] L. Baum and P. H. Frampton, arXiv:astro-ph/0608138. I. Aref’eva, P. H. Frampton and S. Matsuzaki, arXiv:0802.1294 [hep-th].
[6] J. Mielczarek, T. Stachowiak and M. Szydlowski, arXiv:0801.0502 [gr-qc].
[7] S. Nojiri and S. D. Odintsov, *Phys. Lett.* B 595, 1 (2004) arXiv:hep-th/0405078.
[8] T. J. Battefeld and G. Geshnizjani, *Phys. Rev.* D 73, 064013 (2006) arXiv:hep-th/0503160.
[9] R. Dicke and P.J.E. Peebles, in *General Relativity: An Einstein Centenary Survey*, ed. by S. Hawking and W. Israel (Cambridge: Cambridge Univ. Press, 1979).
[10] P. Davies, *Annales Poincare Phys.Theor.* 49 297 (1988); E. Babichev, V. Dokuchaev and Y. Eroshenko, arXiv:gr-qc/0402089.
[11] R. Caldwell, M. Kamionkowski, N. Weinberg, *Phys.Rev.Lett.* 91 071301 (2003).
[12] R. Caldwell, *Phys.Lett.* B545 23 (2002).
[13] Y. Shtanov and V. Sahni, Phys. Lett. B 557, 1 (2003)
[14] P. Kanti and K. Tamvakis, Phys. Rev. D 68, 024014 (2003); S. Foffa, Phys. Rev. D 68, 043511 (2003).
[15] A. Ashtekar, T. Pawlowski and P. Singh, Phys. Rev. D 74, 084003 (2006) arXiv:gr-qc/0607039.
[16] L. Randall and R. Sundrum, *Phys.Rev.Lett.* 83, 3370 (1999).
[17] D. Lyth, Phys. Lett. B526, 173 (2002); J. Martin, P. Peter, N. Pinto Neto and D. J. Schwarz, Phys. Rev. D 65, 123513 (2002); C. Gordon and N. Turok, Phys. Rev. D 67, 123508 (2003); J. Martin and P. Peter, Phys. Rev. Lett. 92, 061301 (2004); T. J. Battefeld, S. P. Patil and R. Brandenberger, arXiv:hep-th/0401010.
[18] D. Chung and K. Freese, *Phys.Rev.* D61, 023511 (2000)

[19] P. Binetruy, C. Deffayet, and D. Langlois, *Nucl.Phys.* B565, 269 (2000); E.E. Flanagan, S. Tye, and I. Wasserman, *Phys. Rev.* D62, 024011 (2000); C. Csaki, M. Graesser, C. Kolda, and J. Terning, *Phys. Lett.* B462, 34 (1999); J. Cline, C. Grojean, and G. Servant, *Phys. Rev. Lett.* 83, 4245 (1999); L. Mersini, *Mod. Phys. Lett.* A16, 1583 (2001); W. Goldberger and M. Wise, *Phys. Lett.* B475, 275 (2000).

[20] L. E. Allen and D. Wands, [arXiv:astro-ph/0404441](http://arxiv.org/abs/astro-ph/0404441).

[21] M. P. Dabrowski, T. Stachowiak and M. Szydlowski, Phys. Rev. D 68, 103519 (2003); L. P. Chimento and R. Lazkoz, Phys. Rev. Lett. 91, 211301 (2003); J. E. Lidsey, [arXiv:gr-qc/0405055](http://arxiv.org/abs/gr-qc/0405055); L. P. Chimento and R. Lazkoz, [arXiv:astro-ph/040551](http://arxiv.org/abs/astro-ph/040551).

[22] J. M. Cline, S. Jeon and G. D. Moore, Phys. Rev. D 70, 043543 (2004) [arXiv:hep-ph/0311312](http://arxiv.org/abs/hep-ph/0311312).

[23] L. Parker and A. Raval, Phys. Rev. Lett. 86, 749 (2001).

[24] A. Melchiorri, L. Mersini-Houghton, C. J. Odman and M. Trodden, Phys. Rev. D 68, 043509 (2003) [arXiv:astro-ph/0211522](http://arxiv.org/abs/astro-ph/0211522).