X-RAY LUMINOSITY FUNCTION OF LOW-MASS X-RAY BINARIES IN GALAXIES

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Abstract

X-ray luminosity function derived from observations of X-ray sources in galactic bulges can be explained by principal evolutionary relations for mass accretion rate onto the compact object. The observed mean distribution of individual X-ray luminosities of galactic LMXB is satisfactorily described by a symmetric quasi-Lorentzian curve. The flux variance for bright sources is found to be proportional to the mean luminosity. Such a distribution does not change the slope of the power law luminosity function of the source population, which is expected from the dependence of the mass transfer rate on the mass of the Roche-lobe filling non-degenerate optical component.

1 Introduction

X-ray flux measurements from numerous point-like sources in other galaxies have recently become possible by Chandra and XMM-Newton X-ray telescopes. Most of these sources are accreting compact stars in binaries. These observations have important implications for population studies of X-ray binaries in galaxies. Grimm et al. (1993) and Gilfanov (2004) carried out a detailed statistical analysis of the observed X-ray luminosity function of X-ray sources observed by Chandra and XMM. They constructed X-ray luminosity function of X-ray binaries in individual galaxies, derived the mean luminosity function and discussed its astrophysical implications. In our previous paper (Postnov 2003) we noted that the universal power-law shape of X-ray luminosity function (XLF) of high-mass X-ray binaries obtained by these authors \( dN/dL_x \sim L^{-1.6} \) in a wide range of X-ray luminosities \( 10^{34} - 10^{40} \) erg/s can be readily explained by the universal properties of accretion from stellar wind
from the early-type component. Important points were the possibility to use the power-law dependence for mass-luminosity and mass-radius relations for early-type stars, as well as a natural assumption on the power-law initial stellar mass function (Salpeter law or Miller-Scalo law). We also assumed that the mass accretion rate onto the compact star is the principal parameter determining the X-ray luminosity: \( L_x \propto \dot{M}_a \).

New observational data were used by Gilfanov (2004) (see also Kim and Fabbiano 2004) to construct XLF of low-mass X-ray binaries (LMXB). It turned out that the mean XLF for these sources, too, can be approximated by a universal function, which is, however, not a single power-law, as in the case of HMXB.

It is well known that X-ray binaries in general are variable. Moreover, many of them are transient sources, with X-ray luminosity changing by several times and even orders of magnitudes (e.g., X-ray novae) over different time scales. Typical X-ray observations do not last for more than several tens kiloseconds, which in most cases less than the characteristic time of significant luminosity changes in individual sources. So the question arises as to what exactly the observed mean XLF reflects, the instantaneous luminosity distribution of a collection of sources or the luminosity function of individual sources.

In the present paper we first show that the observed dispersion of individual X-ray luminosities of galactic LMXB as obtained from the analysis of the RXTE ASM data can be on average described by a symmetric Lorentzian distribution with the variance proportional to the mean flux. Next we prove that such a distribution of accretion rate \( \dot{M}_a \) onto the compact object does not change a power-law character of XLF which is theoretically calculated from the analysis of mass exchange rate \( \dot{M}_o \) in close binaries. We arrive at the conclusion that the mean XLF of LMXB can be explained by evolutionary features of the mass transfer process in low-mass close binary systems: the mass transfer rate due to binary’s angular momentum removal by gravitational waves shapes the low-luminosity end of the observed XLF \( \propto L_x^{-1} \) \( (L_x < 2 \times 10^{37} \text{ erg/s}) \), while mass exchange rates driven by magnetic stellar wind from the late-type optical component can reproduce the high-luminosity end of the observed XLF of LMXB \( \propto L_x^{-2} \) \( (2 \times 10^{37} < L_x < 5 \times 10^{38} \text{ erg/s}) \).
2 Power-law luminosity functions

First of all, let us show on general grounds how power-law XLF in accreting binaries can be obtained. The basic assumption is the possibility to express the X-ray luminosity of a source through the mass exchange rate $\dot{M}_o$ which is a powe-law function of the optical star mass $M_o$ (Postnov 2003):

$$L_x \sim \dot{M}_o \sim M_o^\alpha$$ (1)

Below we shall consider the luminosity function per logarithmic interval $dN/d \ln L_x$. Under assumption (1) in the stationary situation we have:

$$\frac{dN}{d \ln L_x} = \frac{dN}{d \ln M_o} \frac{d \ln \dot{M}_o}{d \ln L_x} \propto L_x^{\beta_{st}-1}$$ (2)

Here the stationary mass function for optical components was utilized:

$$f_{st}(M) \equiv \frac{dN}{dM_o dt} \sim M_o^{-\beta_{st}}$$ (3)

(for example, $\beta_{st} = 2.35$ for the Salpeter mass function). Consequently, a power-law shape for XLF is recovered:

$$\frac{dN}{d \ln L_x} \sim L_x^{-\Gamma}$$

where the power-law index is

$$\Gamma = \frac{\beta_{st} - 1}{\alpha}$$

and depends only on the slope of the mass function and relationship (1).

2.1 Stationary mass function of optical components

Two cases should be distinguished: The case of young massive close binary systems in which the mass of the optical component virtually did not change from the original value, and teh case of old low-mass close binaries, where the mass of the optical star significantly diminished in the course of mass transfer. In the former case the mass function of the optical component does not alter significantly, so we can adopt its initial form:

$$f(M)_{st} = f_o(M) \propto M^{-\beta} \quad \beta_{st} = \beta$$ (4)
In the latter case the stationary optical star mass distribution can be derived from the one-dimensional kinetic equation with stationary source $f_o(M)$

$$\frac{\partial}{\partial M} \left[ f(M) \dot{M} \right] \propto f_o(M)$$

(5)

In our case $\dot{M} < 0$ and we obtain

$$\dot{f}_{st}(M) = \int_{M}^{M_{\text{max}}} \dot{f}_0(M')dM'$$

(6)

where $M_{\text{max}}$ is the upper mass limit for stars. For power-law mass functions its exact value is of minor importance.

Substituting the power-law mass function $f_o(M) \sim M^{-\beta}$ into Eq. (6), we find

$$f_{st}(M) \propto M^{-\beta + 1 - \alpha},$$

(7)

that is the power-law index in the stationary stellar mass distribution becomes dependent on the mass exchange rate:

$$\beta_{st} = \beta - 1 + \alpha$$

(8)

For example, for HMXB accreting from stellar wind $\alpha \approx 2.5$ (Postnov 2003) and for $\beta = 2.35...2.5$ we get $\Gamma = 0.54...0.6$ (the observed value is $\sim 0.6$).

3 Effect of the luminosity function of individual sources on the luminosity function of the source population

Before considering a more complicated case of low-mass close binary systems, let us discuss the influence of individual source XLF on the XLF of the entire population. Let the accretion rate onto the compact object and hence X-ray luminosity $L_x$ of a source changes according to some distribution function $F(x)$ (i.e., $F(x)$ gives the probability to detect the X-ray luminosity in the range $[F(x), F(x + \Delta x)]$). Here the mean mass exchange rate $\dot{M}_o$ is allowed to change or remain constant. Let $L_0$ be the X-ray luminosity corresponding to the mean accretion rate. We assume that this rate represents
Table 1: Low-mass X-ray binaries with short orbital periods. For each source given are the orbital period $P(h)$ (in hours), the mean ASM flux $F_x$ (in ASM count rate), the rms variance of the flux $\sigma$ (in ASM count rate), minimal ($F_{\text{min}}$) and maximal ($F_{\text{max}}$) ASM fluxes over the entire time interval used in the analysis.

| N% | Source     | $P(h)$ | $F_x$  | $\sigma$ | $F_{\text{min}}$ | $F_{\text{max}}$ |
|----|------------|--------|--------|----------|------------------|------------------|
| 1  | V1333 Aql | 18.97  | 2.31   | 7.85     | -13.85           | 49.68            |
| 2  | V821 Ara  | 14.86  | 6.80   | 13.51    | -13.92           | 72.57            |
| 3  | V1408 Aql | 9.33   | 1.89   | 1.23     | -17.07           | 6.95             |
| 4  | LU TrA    | 9.14   | 1.01   | 1.06     | -4.59            | 8.63             |
| 5  | V691 CrA  | 5.57   | 1.18   | 1.33     | -8.62            | 16.73            |
| 6  | V926 Sco  | 4.65   | 12.89  | 3.38     | -9.70            | 29.64            |
| 7  | V2216 Oph | 4.20   | 17.34  | 2.51     | 0.00             | 35.78            |
| 8  | GR Mus    | 3.93   | 1.93   | 1.04     | -6.02            | 23.36            |
| 9  | V801 Ara  | 3.80   | 10.90  | 4.88     | -5.63            | 34.92            |
| 10 | 1705-4402 | 1.31   | 11.67  | 8.85     | -3.89            | 35.94            |

The mass exchange rate between the components determined by the system’s evolutionary state. It is easy to see that the observed XLF is the convolution

$$\frac{dN/dL_x}{1/dL} \approx \int_{L_{\text{min}}}^{L_{\text{max}}} \frac{dN/dL_0}{F(L_x - L_0)} \frac{F(L_x - L_0) dL_0}{F(L_x - L_0) dL_0}$$

(9)

We restrict ourselves by considering power-law XLFs, i.e. $dN/d \ln L_0 \sim L_0^{-\Gamma}$. If the function $F(x - y)$ can be factorized in the form

$$F(x - y) = F_1(y) F_2(x/y - 1),$$

(10)

then the power-law XLF of the source population $dN/d \ln x \sim x^{-\Gamma}$ preserves when $F_1(y)$ is also a power-law function for arbitrary function $F_2$ (all necessary integration conditions are assumed). The proof readily follows from changing variables $t = x/y - 1$ in integrals (9) and setting $L_{\text{min}} \to 0$, $L_{\text{max}} \to \infty$.

Let us use the RXTE ASM data to find the shape of the function $F(L_x - L_0)$ for galactic LMXBs. First let us consider bright LMXBs with known orbital periods shorter than 20 hours and mean fluxes above 1 ASM count/s (Table 1). In close binaries with orbital periods below 20 hours the mass
Figure 1: XLF for 10 LMXBs from Table 1 (with known orbital periods shorter than 20 hours) constructed from the RXTE ASM data. The luminosity of each source is given in units of the mean luminosity $L_0$. Distributions are normalized to unity $\int dN/dL \, dL = 1$. 

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of the optical star filling Roche lobe is \( \lesssim 2M_\odot \) (we assume the compact star mass \( 1.4 \, M_\odot \) and main-sequence optical star with solar chemical abundance). The mass transfer is driven by the orbit angular momentum removal due to magnetic stellar wind or gravitational wave emission (for orbital periods shorter than several hours, see below). Table 1 lists the mean flux of the source \( F_x \) (averaged over several years of observations, typically \( 10^3 - 10^4 \) individual points per source) and rms deviation \( \sigma \) from the mean \( F_x \). Both \( F_x \) and \( \sigma \) are given in units of the ASM count rate (we remind that the ASM flux from Crab is 75 ASM counts/s).

In Fig. 1, we plot individual normalized XLF of sources from Table 1 calculated with the use of the ASM data and show the values of the corresponding orbital periods. Fig. 2 shows the mean XLF calculated for these systems (the histogram) and its approximation by a quasi-Lorentzian curve \( F(x - y) = \frac{a}{(x - y)^2 + by^2} \) \( (a, b = \text{const}) \), which obviously satisfies condition (10). The errors in each bin are calculated using the formula for variance \( \sigma_i = \sqrt{\sum_{j=1}^{n_i} (x_j - \bar{x})/n_i} \) \( (n_i \text{ is the number of points in the bin, } \bar{x} \text{ is the mean value}) \). The formal confidence level of the fit according to the \( \chi^2 \) criterion for 17 degrees of freedom is \( P(\chi^2_{17} \geq 8.3) \approx 0.96 \).

As is seen from Table 1, almost all sources with known orbital periods are weak, so individual XLFs (especially left wings corresponding to weak fluxes) are determined with substantial errors. So as an additional test we used a set of bright LMXBs with arbitrary (or unknown) orbital periods (Table 2). XLF of an accreting X-ray source is determined by properties of non-stationary...
Figure 3: The same as in Fig. 1 for 10 bright LMXBs from Table 2.
Table 2: Bright LMXBs. The mean fluxes $F_x$ and the rms flux variances $\sigma$ are given in units of the ASM count rate.

| N% | Source     | $F_x$  | $\sigma$  |
|----|------------|--------|------------|
| 1  | Cyg X2     | 37.22  | 8.32       |
| 2  | Sco X1     | 880.995| 114.68     |
| 3  | GX 9+9     | 17.22  | 2.42       |
| 4  | GX 9+1     | 39.24  | 4.57       |
| 5  | GX 5-1     | 70.75  | 9.53       |
| 6  | GX 3+1     | 22.34  | 5.99       |
| 7  | GX 17+2    | 44.86  | 5.33       |
| 8  | GX 13+1    | 22.45  | 2.93       |
| 9  | GX 340+0   | 29.39  | 4.34       |
| 10 | GX 349+2   | 51.40  | 12.19      |
| 11 | V926 Sco   | 12.89  | 3.38       |
| 12 | V2216 Oph  | 17.34  | 2.51       |
| 13 | V801 Ara   | 10.90  | 4.88       |
| 14 | 1705-4402  | 11.67  | 8.85       |

accretion on the compact object. The external radius of the accretion disk is moderately depends on the system’s orbital period ($a \propto P^{2/3}$). The mean flux is determined by the mass of the compact object. In our case compact objects are neutron stars of about the same mass. Individual XLFs of bright LMXBs from Table 2 are shown in Fig. 3, the mean XLF for these sources is presented in Fig. 4 (the histogram). As for LMXBs with short orbital periods shown in Fig. 2, the mean XLF for bright LMXBs is well approximated by the Lorentzian curve (confidence level $P(\chi^2_1 \geq 8.8) \approx 0.94$).

Fig. 5 demonstrates the variance of the observed XLF for bright sources from Table 2 as a function of the mean flux. The proportionality of the XLF variance to the mean flux holds over more than an order of magnitude of flux values.
Figure 4: The mean XLF for 14 bright LMXBs from Table 2 (the histogram). The confidence level is $P(\chi^2_{17} \geq 8.8) \approx 0.94$.

Figure 5: The variance of the X-ray flux $\sigma$ as a function of the mean flux $F_x$ for 14 bright LMXBs from Table 2. The solid line is the best-fit linear regression.
4 X-ray luminosity function of LMXB

The XLF of LMXB is derived from the analysis of observations of point-like sources in bulges of galaxies (Gilfanov 2004, Kim and Fabbiano 2004) to have the power-law from \( dN/d \ln L_x \sim L_x^{-\Gamma} \) with

\[
\Gamma \sim \begin{cases} 
0, & L_x < 2 \times 10^{37} \text{erg/s} \\
-0.9 \ldots -1.1, & 2 \times 10^{37} < L_x < 5 \times 10^{38} \text{erg/s} \\
-5, & L_x > 5 \times 10^{38} \text{erg/s}
\end{cases}
\] (11)

Now we wish to show that for systems with orbital periods shorter than \( \sim 20 \) hours the power-law form of XLF can be related to principal evolutionary factors driving the mass exchange in LMXBs – the magnetic stellar wind from optical star (MSW) and emission of gravitational waves (GW).

4.1 Magnetic stellar wind

Let the mass of the optical star and compact object in a binary be \( M_o \) and \( M_x \), respectively, the component mass ratio be \( q = M_o/M_x \), and the semi-major axis of the binary orbit be \( a \). The characteristic time scale of the orbital angular momentum loss due to MSW \( \tau_{\text{MSW}} \equiv J/\dot{J}_{\text{MSW}} \) is usually calculated using the empirical Skumanich law for rotation braking of single late-type main-sequence stars, according to which the surface angular velocity of a star decreases gradually with age \( t \) as \( \sqrt{t} \). Applying this law to the late-type low-mass optical component in a LMXB and assuming the tidal locking of the component’s spin to the binary orbital period (see e.g. Massevich and Tutukov 1988, van den Heuvel 1992 for more detail) yields

\[
\tau_{\text{MSW}} \sim \frac{a^5 M_x}{(M_x + M_o)^2 M_o^4}
\] (12)

In LMXBs the mass transfer is due to the Roche lobe overflow of the optical star so we can write

\[
R_o = a f(q)
\] (13)

where for the shape of the ROche lobe we can adopt

\[
f(q) \approx \left( \frac{q}{1+q} \right)^{1/3}, \quad q < 0.5
\]
Then the mean accretion X-ray luminosity is
\[ L_0 \sim \dot{M}_o \sim \frac{M_o}{\tau_{MSW}} \sim \frac{M_o^5 M_x (1 + q)^2 f(q)^5}{R_o^5} \]

Using the mass-radius relation for late-type main-sequence stars \( R_o \sim M_o^{0.9} \) we find
\[ L_0 \sim M_o^{2.17...2.67}, \quad \alpha \approx 2.17...2.67 \]

Substituting this into Eq. (8) we obtain for the stationary mass function of optical components in LMXBs \( \beta_{st} \approx 3.52...4 \) assuming the Slapeter initial mass function. Finally, we arrive at the luminosity function in the form:
\[ \frac{dN}{d\ln L_0} \sim L_0^{-1.6...-1.13}, \quad \Gamma_{MSW} \sim 1.6...1.13 \]

### 4.2 Gravitational radiation

In the dipole approximation, the time-scale for the angular momentum loss in a binary system reads
\[ \tau_{GR} \sim \frac{a^4}{(M_x + M_o)M_o M_x} \] (15)

Using the Roche lobe filling condition \( a = R_o/f(q) \) as above we get
\[ L_0 \sim \dot{M}_o \sim \frac{M_o}{\tau_{GR}} \sim \frac{M_o^2 M_x^2 (1 + q)^4 f(q)^4}{R_o^4} \]

Utilizing \( R_0 \sim M_o^{0.9} \) we find
\[ L_0 \sim M_o^{-0.27...-0.3}, \quad \alpha \approx -0.27... - 0.3 \]

Substituting this into Eq. (8) yields \( \beta_{st} \approx 1.05...1.08 \), so finally we obtain the luminosity function
\[ \frac{dN}{d\ln L_0} \sim L_0^{-0.16...}, \quad \Gamma_{GR} \sim 0.16...0.3 \] (16)
4.3 The break of the mean XLF of LMXBs

The break of the mean XLF of LMXBs is observed at X-ray luminosities $\sim 2 \times 10^{37}$ erg/s, which corresponds to the accretion rate on the neutron star $\dot{M} \sim 10^{-9} M_\odot / y$. In our interpretation, the break should correspond to the transition from the MSW scale to GW scale as the mass of the optical star decreases below a certain value where MSW becomes inefficient. From the simple equality $\tau_{\text{MSW}} = \tau_{\text{GR}}$ we find the stellar mass

$$M_o \approx 0.4 M_\odot$$

which is close to the generally accepted value $\sim 0.3 M_\odot$ (Spruit and Ritter, 1983). At such masses the mass exchange rate in LMXB drops to

$$\dot{M}(0.4 M_\odot) \sim 3 \times 10^{-10} M_\odot / y$$

(see, for example, Rappaport et al. 1984 and later calculations of LMXB evolution with non-degenerate companions), which roughly agrees with the observed value of the break.

5 Discussion and conclusion

The above analysis allows us to conclude that the observed mean XLF of LMXBs can be generally explained by accretion on neutron stars from Roche-filling non-degenerate component driven by gravitational wave emission (below $L_x \sim 2 \times 10^{37}$ erg/s) and by magnetic stellar wind (above $L_x > 2 \times 10^{37}$ erg/s). We have shown that these mechanisms, which determines the evolution of binaries with orbital periods shorter than 15-20 hours, lead to power-law dependences of the mass transfer rate $\dot{M}_o$ on the optical star mass $M_o$, which is the necessary condition for the power-law form of the population of such sources. In LMXBs with longer orbital periods, the Roche lobe is filling by a subgiant optical star. Using the analytical treatment of the mass transfer process in such binaries (Yungelson and Livio 1998), it is straightforward to show that in LMXBs with subgiants, too, the power-law dependence of the mass exchange rate on the optical star mass is recovered.

We should also note that a sizable fraction of LMXBs actually recides or was born in globular clusters due to effective dynamical interactions between stars. The evolution of LMXBs in dense stellar clusters is more complicated.
than outlined above. So a more accurate evolutionary treatment of XLF of such LMXBs is required. For example, the population synthesis method seems to be promising.

The contribution of ultrashort-period binaries dynamically formed in globular clusters (like X 1820-30) into XLF of LMXBs was discussed by Bildsten and Deloy (20). In these binaries, the Roche lobe is filled by a low-mass degenerate white dwarf with inverse mass-radius relation and mass transfer is controlled by gravitational wave emission. The analysis of these authors shows that a collection of such ultracompact binaries could significantly contribute to the XLF of LMXBs in the luminosity range $6 \times 10^{37} < L_x < 5 \times 10^{38}$ erg/s and give the slope of XLF as observed.

Our analysis indicates that the mean XLF of individual sources constructed from the RXTE ASM data can be satisfactorily fitted by a quasi-Lorentzian distribution with variance proportional to the mean luminosity in a wide range of X-ray fluxes. We have shown that such a shape of the mean individual XLF does not affect the power-law shape of the XLF formed by the entire population of the binary sources. The latter is determined by the dependence of the mean X-ray luminosity of each source on the mass of the optical star filling the Roche lobe.

We stress that it is the possibility to describe the mean XLF of individual LMXBs by a single law with specific symmetry (proportionality of the variance to the mean value) that allows us to recover the power-law form of the XLF of the entire source population from evolutionary properties of mass transfer in LMXBs. Observations show (Kim and Fabbiano 2004) that XLFs of individual galaxies are somewhat different, the unique power-law of XLF is obtained after averaging over many galaxies. This apparently reflects individual features of LMXBs in different galaxies (e.g., various contributions from ultra-compact binaries, globular clusters, etc.). These observations can be used to study formation and evolution of LMXBs in other galaxies.

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