Within the framework of teleparallel equivalent of general relativity (TEGR) theory, calculation of the total energy and momentum of Kerr-NUT spacetimes have been employed using two methods of the gravitational energy-momentum, which is coordinate independent, and the Riemannian connection 1-form, $\tilde{\Gamma}^\beta_{\alpha\gamma}$. It has been shown that the two methods give the same an unacceptable result, i.e., divergent value. Therefore, a local Lorentz transformation that plays a role of a regularizing tool, which subtracts the inertial effects without distorting the true gravitational contribution, has been suggested. This transformation keeps the resulting spacetime to be a solution of the equations of motion of TEGR.

1. Introduction

The geometry of Einstein general relativity (GR) is based on the Riemannian geometry which can define both metric and connection uniquely. This geometry is the main reason responsible for the problem of defining a consistent expression of energy in GR in addition to the equivalence principle of the gravitational theory. Using the Lagrange-Noether approach, one can derive the conserved currents that arise from the invariance of the classical action under transformations of fields. In Riemannian geometry one can not find symmetries that can be used to generate the conserved energy-momentum currents, but one can only speak about the energy of asymptotically flat spacetime. Earlier analysis of this problem can be found in details in [ references 1)∼ 5) and references therein].
As is well known, there are two ways to describe the gravitational interaction: one by curvature and the other by torsion.\(^6\) According to GR, curvature is due to geometry from which a successful description of the gravitational interaction is carried out. On the other hand, teleparallelism attributes gravitation to torsion. In this case, torsion accounts for gravitation not by geometrizing the interaction, but by acting as a force. Accordingly, in the teleparallel equivalent of GR, there are no geodesics, but force equations are quite analogous to the Lorentz force equation of electrodynamics.\(^7\) Therefore, gravitational interaction, can be described either in terms of curvature, as is usually conducted in GR, or in terms of torsion, in which case we have the teleparallel gravity.

Teleparallel theories are interesting for many reasons: First GR can be viewed as a particular theory of teleparallelism and, thus, teleparallelism could be considered at the very least as a different point of view that can lead to the same results.\(^8\) Second, within this framework, one can define an energy-momentum tensor for the gravitational field that is a true tensor under all general coordinate transformations. This is the reason why teleparallelism was reconsidered by Møller when he was studying the problem of defining an energy-momentum tensor for the gravitational field.\(^9\) The idea was taken over by Pellegrini and Plebański who constructed the general Lagrangian for these theories.\(^10\) The third reason as to why these theories are interesting is that they can be seen as gauge theories of the translation group (not the full Poincaré group) and, thus, they give an alternative interpretation of GR.\(^11, 12\) Teleparallel theory is considered as an essential part of generalized non-Riemannian theories such as the Poincaré gauge theory\(^13\sim 20\) or metric-affine gravity.\(^11, 21\sim 25\) Within the framework of metric-affine gravity, a stationary axially symmetric exact solution of the vacuum field equation is obtained.\(^26\)

Geometrically, teleparallel models are described by the Weitzenböck spacetime which is characterized by the vanishing curvature (constructed from the connection of this geometry) and non-zero torsion. The tetrad (or a coframe) is the basic field variable which can be treated as the gauge potential corresponding to the group of local translations. Then the torsion is naturally interpreted as the corresponding gauge field strength. As a result, a gravitational teleparallel Lagrangian is straightforwardly constructed, in a Yang-Mills manner, from the quadratic torsion invariants.

Mathematically, there are infinitely many tetrads since a reference frame of an observer can obviously be constructed infinitely. In particular, from a given tetrad field \(h^\mu_i\), we can obtain a continuous family of tetrads by performing the local Lorentz transformation \(h'^\alpha_i = \Lambda^{\alpha}_\beta h_i^\beta\), where the elements of the Lorentz matrix \(\Lambda^{\alpha}_\beta(x)\) are arbitrary functions of the spacetime coordinates.\(^27\)

An important difference between Einstein GR theory and teleparallel theories is that it is possible to distinguish between gravitation and inertia.\(^27, 28\) Since inertia is in the realm of the pseudotensor behavior of the usual expressions for the gravitational energy-momentum density, it seems possible in teleparallel gravity to write down a tensorial expression for such density.\(^29\) With the purpose of getting a deeper insight into the covariant teleparallel formalism, as well as to test how it works Lucas et al.\(^27\) reanalyze the computation of the total energy of two examples. Recently Obukhov et al.\(^30\) computed the energy and momentum transported by exact plane gravitational-wave solutions of Einstein equations using the teleparallel equivalent formulation of Einsteins theory. It is our aim in this current work to calculate energy and momentum of Kerr-NUT spacetime using two methods: the gravitational energy-momentum which is a coordinate independent and the Riemannian connection 1-form, \(\tilde{\Gamma}^\beta_\alpha\). The value of energy of Kerr-NUT spacetime has been shown to have a divergent
result. Therefore, an appropriate Local Lorentz transformation has been introduced. Using
this transformation, it has been shown that the value of energy is always acceptable.

In §2, a brief review of the derivation of the TEGR field equations is given. A summary
of the derivation of energy and angular momentum using the Hamiltonian formulation in
TEGR is also given in §2. In §3, Kerr-NUT spacetime and calculation of its energy, using
the definition given in §2 has been presented and a divergent value is obtained. To make the
picture clearer we use another definition to calculate the energy. Therefore, we give a brief
review of the covariant formalism for the gravitational energy-momentum which is described
by the pair \((v^\alpha, \Gamma^\beta_\alpha)\) in §3. Using the Riemannian connection 1-form, \(\tilde{\Gamma}^\beta_\alpha\) we repeat the
calculation of energy and obtain the same divergent value. In §4, explicate and adequate
calculations show that due to an inconvenient choice of a reference system, traditional com-
putation of the total energy of Kerr-NUT spacetime are always divergent! Therefore, a new
local Lorentz transformation which when applied to the Kerr-NUT spacetime and repeat the
calculation either using the gravitational energy-momentum or the Riemannian connection
1-form \(\tilde{\Gamma}^\beta_\alpha\), we get finite and acceptable result. In section §5, we give another Kerr-NUT
spacetime and calculate the energy using the two definitions and got a non acceptable re-
sult. Using the local Lorentz transformation suggested in §4, we get an acceptable result
obtained in §4. Final section is devoted to main result and discussion.

2. The TEGR for gravitation

In a spacetime with absolute parallelism the parallel vector field \(h^\mu_a\) defines the nonsym-
metric affine connection

\[
\Gamma^\lambda_{\mu\nu} \overset{\text{def.}}{=} h^\lambda_a h^a_{\mu\nu},
\]

where \(h^a_{\mu\nu} = \partial^\nu h^a_{\mu}\). The curvature tensor defined by \(\Gamma^\lambda_{\mu\nu}\) is identically vanishing, however. The
metric tensor \(g_{\mu\nu}\) is given by

\[
g_{\mu\nu} = O_{ab} h^a_{\mu} h^b_{\nu},
\]

with the Minkowski metric \(O_{ab} = \text{diag}(+1, -1, -1, -1)\).

The Lagrangian density for the gravitational field in the TEGR, in the presence of matter
fields, is given by\(^{31,3}\)

\[
\mathcal{L}_G = \sqrt{-g} L_G = -\sqrt{-g} \left( \frac{T^{abc}T_{abc}}{4} + \frac{T^{abc}T_{bac}}{2} - T^a T_a \right) - L_m = -\frac{\sqrt{-g}}{16\pi} \Sigma^{abc} T_{abc} - L_m,
\]

where \(g = \text{det}(g_{\mu\nu})\). The tensor \(\Sigma^{abc}\) is defined by

\[
\Sigma^{abc} \overset{\text{def.}}{=} \frac{1}{4} \left( T^{abc} + T^{bac} - T^{cab} \right) + \frac{1}{2} \left( O^{ac} T^b - O^{ab} T^c \right).
\]

\(^{3}\)spacetime indices \(\mu, \nu, \cdots\) and \(\text{SO}(3,1)\) indices \(a, b \cdots\) run from 0 to 3. Time and space indices are
indicated to \(\mu = 0, i\), and \(a = (0), (i)\).

\(^{3}\)Latin indices are rasing and lowering with the aid of \(O_{ab}\) and \(O^{ab}\).

\(^{3}\)Throughout this paper we use the relativistic units , \(c = G = 1\) and \(\kappa = 8\pi\).
The torsion tensor and the basic vector field defined by

\[ T^a_{\mu\nu} \equiv h^a_{\lambda} T^\lambda_{\mu\nu} = \partial_\mu h^a_{\nu} - \partial_\nu h^a_{\mu}, \tag{5} \]

and

\[ T^a_{\mu} \equiv T^\nu_{\mu \nu}, \quad T^a_{\mu} \equiv h^a_{\mu} T^\mu = T^b_{\mu} a. \tag{6} \]

The quadratic combination \( \Sigma^{abc} T_{abc} \) is proportional to the scalar curvature \( R \), except for a total divergence term.\(^{32} \) \( L_m \) represents the Lagrangian density for matter fields.

The gravitational field equations for the system described by \( L_G \) are the following

\[ h_{a\lambda} h_{b\mu} \partial_\nu \left( \sqrt{-g} \Sigma^{b\lambda\nu} \right) - \sqrt{-g} \left( \Sigma^{b\nu} a T_{b\nu\mu} - \frac{1}{4} h_{a\mu} T_{bcd} \Sigma^{bcd} \right) = \frac{1}{2} \kappa \sqrt{-g} T_{a\mu}, \tag{7} \]

where

\[ \frac{\delta L_m}{\delta h_{a\mu}} \equiv \sqrt{-g} T_{a\mu}. \]

It is possible to prove by explicit calculations that the left hand side of the symmetric part of the field equations (7) is exactly given by\(^{31} \)

\[ \frac{\sqrt{-g}}{2} \left[ R_{a\mu} - \frac{1}{2} h_{a\mu} R \right]. \]

The axial-vector part of the torsion tensor \( A_\mu \) is defined by

\[ A_\mu \equiv \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma} = \frac{1}{3} \epsilon_{\mu\nu\rho\sigma} \gamma^{\nu\rho\sigma}, \quad \text{where} \quad \epsilon_{\mu\nu\rho\sigma} \equiv \sqrt{-g} \delta_{\mu\nu\rho\sigma}, \tag{8} \]

with \( \gamma_{\mu\rho\sigma} = O^{ab} h_{ab\mu} h_{b\rho} ; \sigma \) being the contortion tensor and \( \delta_{\mu\nu\rho\sigma} \) is completely antisymmetric and normalized as \( \delta_{0123} = -1. \)

The definition of the gravitational energy-momentum \( P^a \) four-vector has the form\(^{32} \)

\[ P^a \equiv - \int_V d^3x \partial_i \Pi^{ai}, \tag{9} \]

where \( V \) is an arbitrary volume of the three-dimensional space. In the configuration space we have

\[ \Pi^{ai} \equiv - \frac{2}{\kappa} \sqrt{-g} \Sigma^{a0i}, \quad \text{with} \quad \partial_\nu (\sqrt{-g} \Sigma^{a\lambda\nu}) \equiv \frac{\kappa}{2} \sqrt{-g} h^a_{\nu} (t^\lambda_{\mu} + T^\lambda_{\mu}), \]

\( t^\lambda_{\mu} \equiv \frac{1}{2\kappa} \left( 4 \Sigma^{bc\lambda} T_{bc\mu} - g^{\lambda\mu} \Sigma^{bcd} T_{bcd} \right). \tag{10} \]

Maluf et al.\(^{31}, \ 32 \) defined

\[ L^{ab} = 2 \int_V d^3x M^{[ab]}, \tag{11} \]

as the 4-angular-momentum of the gravitational field for an arbitrary volume \( V \) of the three-dimensional space.
3. First Kerr-NUT spacetime

The covariant form of the first Kerr-NUT tetrad field having axial symmetry in spherical coordinates \((t, r, \theta, \phi)\), can be written as

\[
(h^\alpha_i)_1 = \begin{pmatrix}
A_1 & 0 & 0 & 0 \\
0 & A_2 & 0 & 0 \\
0 & 0 & A_3 & 0 \\
A_4 & 0 & 0 & A_5 \sin \theta
\end{pmatrix},
\]

where \(A_i, i = 1 \cdots 5\) are functions of \(r\) and \(\theta\) having the form

\[
A_1 = -\sqrt{\frac{AB}{C}} \sin \theta, \quad A_2 = \sqrt{\frac{A}{B}}, \quad A_3 = \sqrt{A}, \quad A_4 = \frac{G}{\sqrt{AC}}, \quad A_5 = \sqrt{\frac{C}{A}},
\]

\[
A = r^2 + (L + a \cos \theta)^2, \quad B = r^2 - 2Mr + a^2 - L^2, \quad C = aB \cos^3 \theta(a \cos \theta - L) - \cos^2 \theta \left(r^4 + 8a^2L^2 + 6r^2L^2 - a^4 - 3L^4 - 8MrL^2 + 4Mr^2a^2\right) + 4aLB \cos \theta + 2Mr^2 + L^4 + r^4 + r^2a^2 + 2r^2L^2 + 3a^2L^2, \quad G = -2(BL \cos \theta + a[L^2 + Mr \sin^2 \theta]),
\]

where \(M\), \(a\) and \(L\) are the mass, the rotation and the NUT parameters respectively.\(^{33}\)

We consider a non asymptotically flat spacetime in this paper, and impose the boundary condition that for \(r \to \infty\) and \(L \to 0\) the tetrad \((12)\) approaches the tetrad of Minkowski spacetime, in Cartesian coordinate. The metric tensor \(g_{ij} \overset{\text{def}}{=} O_{\mu\nu}h^{\mu i}h^{\nu j}\) associated with the tetrad field \((12)\) has the form

\[
ds^2 = (A_1^2 - A_4^2) dt^2 - A_2^2 dr^2 - A_3^2 d\theta^2 - A_5^2 \sin^2 \theta d\phi^2 - A_5 A_4 \sin \theta dt d\phi,
\]

which is the Kerr-NUT spacetime written in Boyer-Lindquist coordinates.\(^{27}\)

The previously obtained solutions, Schwarzschild and Kerr spacetimes can be generated as special solutions of the tetrad \((12)\) using \((13)\) by putting \(a = 0\), \(L = 0\) and \(L = 0\) respectively.\(^{34},\ 35\)

Now we are going to calculate the energy content of the tetrad field \((12)\) using \((13)\). The non-vanishing components of the tensor \(\Sigma^{abc}\) needed to the calculation of energy have the form

\[
\Sigma^{301} \approx \frac{L \cos \theta}{r^3 \sin^2 \theta} + O \left(\frac{1}{r^4}\right), \quad \Sigma^{401} \approx \frac{1}{r^3 \sin \theta} \left\{2r^2 \sin^2 \theta - 2L^2 (1 + \cos^2 \theta) - 4La \cos \theta \sin^2 \theta + a^2 \sin^2 \theta (1 - 3 \cos^2 \theta)\right\} + O \left(\frac{1}{r^4}\right).
\]

Using Eq. \((15)\) in Eq. \((9)\) we finally obtain

\[
P^{(0)} = E = -\oint_{S \to \infty} dS_k \Pi^{(0)k} = -\frac{1}{4\pi} \oint_{S \to \infty} dS_k \sqrt{-g} h^{(0)}_{\mu} \Sigma^{0k} \approx \infty!
\]
It follows from Eq. (16) that the energy of the Kerr-Tube-NUT spacetime is divergent which is not an acceptable result.

Due to the above non physical result and to make the picture more clear we will use another method to calculate the energy of tetrad (12) to show if the divergent result will continue or not?

**Notation**

We use the Latin indices $i, j, \cdots$ for local holonomic spacetime coordinates and the Greek indices $\alpha, \beta, \cdots$ to label (co)frame components. Particular frame components are denoted by hats, $\hat{0}, \hat{1}, \cdots$ As usual, the exterior product is denoted by $\wedge$, while the interior product of a vector $\xi$ and a p-form $\Psi$ is denoted by $\xi \lrcorner \Psi$. The vector basis dual to the frame 1-forms $\vartheta^\alpha$ is denoted by $e_\alpha$ and they satisfy $e_\alpha \lrcorner \vartheta^\beta = \delta^\beta_\alpha$. Using local coordinates $x^i$, we have $\vartheta^\alpha = h^\alpha_i \, dx^i$ and $e_\alpha = h^\alpha_i \partial_i$ where $h^\alpha_i$ and $h^i_\alpha$ are the covariant and contravariant components of the tetrad field. We define the volume 4-form by

$$
\eta_{\alpha} \overset{\text{def}}{=} e_\alpha \lrcorner \eta = \frac{1}{3!} \epsilon_{\alpha \beta \gamma \delta} \vartheta^\beta \wedge \vartheta^\gamma \wedge \vartheta^\delta,
$$

where $\epsilon_{\alpha \beta \gamma \delta}$ is a completely antisymmetric tensor with $\epsilon_{0123} = 1$.

$$
\eta_{\alpha \beta} \overset{\text{def}}{=} e_\beta \lrcorner \eta_{\alpha} = \frac{1}{2!} \epsilon_{\alpha \beta \gamma \delta} \vartheta^\gamma \wedge \vartheta^\delta,
$$

$$
\eta_{\alpha \beta \gamma} \overset{\text{def}}{=} e_\gamma \lrcorner \eta_{\alpha \beta} = \frac{1}{1!} \epsilon_{\alpha \beta \gamma \delta} \vartheta^\delta,
$$

which are bases for 3-, 2- and 1-forms respectively. Finally,

$$
\eta_{\alpha \beta \gamma \mu} \overset{\text{def}}{=} e_\mu \lrcorner \eta_{\alpha \beta \gamma} = e_\mu \lrcorner e_\beta \lrcorner e_\alpha \lrcorner \eta,
$$

is the Levi-Civita tensor density. The $\eta$-forms satisfy the useful identities:

$$
\vartheta^\beta \wedge \eta_{\alpha} \overset{\text{def}}{=} \delta^\beta_\alpha \eta, \quad \vartheta^\beta \wedge \eta_{\mu \nu} \overset{\text{def}}{=} \delta^\beta_\mu \eta_{\nu} - \delta^\beta_\nu \eta_{\mu}, \quad \vartheta^\beta \wedge \eta_{\alpha \mu \nu} \overset{\text{def}}{=} \delta^\beta_\alpha \eta_{\mu \nu} + \delta^\beta_\mu \eta_{\alpha \nu} + \delta^\beta_\nu \eta_{\alpha \mu},
$$

$$
\vartheta^\beta \wedge \eta_{\alpha \gamma \mu \nu} \overset{\text{def}}{=} \delta^\beta_\nu \eta_{\alpha \gamma \mu} - \delta^\beta_\mu \eta_{\alpha \gamma \nu} + \delta^\beta_\nu \eta_{\alpha \mu \nu} - \delta^\beta_\nu \eta_{\alpha \nu \mu}.
$$

(17)

The line element $ds^2 \overset{\text{def}}{=} g_{\alpha \beta} \vartheta^\alpha \otimes \vartheta^\beta$ is defined by the spacetime metric $g_{\alpha \beta}$.

Teleparallel geometry can be viewed as a gauge theory of translation.\textsuperscript{11), 12), 36}\textsuperscript{~1) In this geometry the coframe $\vartheta^\alpha$ plays the role of the gauge translational potential of the gravitational field. GR can be reformulated as the teleparallel theory. Geometrically, teleparallel gravity can be considered as a special case of the metric-affine gravity in which the coframe 1-form $\vartheta^\alpha$ and the local Lorentz connection are subject to the distant parallelism constraint $R_{\alpha \beta} = 0$.\textsuperscript{12), 43}) In this geometry the torsion 2-form

$$
T^\alpha = D \vartheta^\alpha = d \vartheta^\alpha + \Gamma^\alpha_\beta \wedge \vartheta^\beta = \frac{1}{2} T_{\mu \nu}^\alpha \vartheta^\mu \wedge \vartheta^\nu = \frac{1}{2} T_{ij}^\alpha \, dx^i \wedge dx^j,
$$

(18)

arises as the gravitational gauge field strength, $\Gamma^\alpha_\beta$ being the Weitzenböck connection 1-form, $d$ is the exterior derivative and $D$ is the exterior covariant derivative. The torsion $T^\alpha$ can be decomposed into three irreducible pieces: the tensor part, the trace, and the axial trace, given respectively by\textsuperscript{12), 27}), for example

$$(1) \; T^\alpha \overset{\text{def}}{=} T^\alpha - (2) T^\alpha - (3) T^\alpha,
$$

with
\[ T^\alpha \overset{\text{def}}{=} \frac{1}{3} \theta^\alpha \wedge T, \quad \text{where} \quad T = (e_\beta]T^\beta), \quad e_\alpha]T = T_{\mu\alpha}^\mu, \quad \text{vectors of trace of torsion} \]

\[ T^\alpha \overset{\text{def}}{=} \frac{1}{3} e^\alpha]P, \quad \text{with} \quad P = (\vartheta^\beta \wedge T_\beta), \quad e_\alpha]P = T^{\mu\nu\lambda} \eta_{\mu\nu\lambda}, \quad \text{axial of trace of torsion.} \]  

The Lagrangian of the teleparallel equivalent using the language of forms has the form,

\[ V = -\frac{1}{2\kappa} T^\alpha \wedge^* \left( (1) T_\alpha - 2(2) T_\alpha - \frac{1}{2}(3) T_\alpha \right). \]  

\[ \kappa = \frac{8\pi G}{c^3}, \quad G \text{ is the Newton gravitational constant, } c \text{ is the speed of light and } * \text{ denotes the Hodge duality in the metric } g_{\alpha\beta} \text{ which is assumed to be flat Minkowski metric } g_{\alpha\beta} = O_{\alpha\beta} = \text{diag}(+1, -1, -1, -1), \text{ that is used to raise and lower local frame (Greek) indices.} \]

The variation of the total action with respect to the coframe gives the field equations in the form,

\[ DH_\alpha - E_\alpha = \Sigma_\alpha, \quad \text{where} \quad \Sigma_\alpha \overset{\text{def}}{=} \frac{\delta L_{\text{matter}}}{\delta \vartheta^\alpha}, \]  

is the canonical energy-momentum current 3-form of matter which is considered as the source. In accordance with the general Lagrange-Noether scheme\( ^{11, 38} \) one derives from (20) the translational momentum 2-form and the canonical energy-momentum 3-form:

\[ H_\alpha \overset{\text{def}}{=} -\frac{\partial V}{\partial T_\alpha} = \frac{1}{\kappa} * \left( (1) T_\alpha - 2(2) T_\alpha - \frac{1}{2}(3) T_\alpha \right), \quad E_\alpha \overset{\text{def}}{=} \frac{\partial V}{\partial \vartheta^\alpha} = e_\alpha]V + (e_\alpha]T^\beta) \wedge H_\beta. \]  

Due to geometric identities\( ^{43} \), the Lagrangian (20) can be recast as

\[ V = -\frac{1}{2} T^\alpha \wedge H_\alpha. \]

The presence of the connection field \( \Gamma^\alpha_\beta \) plays an important regularizing role due to the following:

\[ \text{i:} \quad \text{The theory becomes explicitly covariant under the local Lorentz transformations of the coframe, i.e., the Lagrangian (20) is invariant under the change of variables} \]

\[ \vartheta'^{\alpha} = \Lambda^{\alpha}_{\beta} \vartheta^{\beta}, \quad \Gamma^{'\beta}_{\alpha} = \Lambda^\mu_{\alpha} \Gamma^{\nu}_{\mu} (\Lambda^{-1})^{\beta}_{\nu} - (\Lambda^{-1})^{\beta}_{\gamma} \Lambda^\gamma_{\alpha}. \]  

Due to the non-covariant transformation law of \( \Gamma^\alpha_\beta \) as shown by Eq. (24), if a connection vanishes in a given frame, it will not vanish in any other frame related to the first by a local Lorentz transformation.

\[ \text{ii:} \quad \Gamma^\alpha_\beta \text{ plays an important role in the teleparallel framework. This role represents the inertial effects which arise from the choice of the reference system}^{27} \). \text{The contributions of this inertial in many cases lead to unphysical results for the total energy of the system. Therefore, the role of the teleparallel connection is to separate the inertial contribution from} \]
the truly gravitational one. Since the teleparallel curvature is zero, the connection is a pure
gauge, that is
\[ \Gamma_\alpha^\beta = (\Lambda^{-1})^\beta_\gamma d\Lambda^\gamma_\alpha. \] (25)
The Weitzenböck connection always has the form (25). The translational momentum has
the form\(^\dagger\)
\[ \tilde{H}_\alpha = \frac{1}{2\kappa} \tilde{\Gamma}^\beta_\gamma \wedge \eta_\alpha^\beta_\gamma, \quad \Gamma_\alpha^\beta \overset{\text{def}}{=} \tilde{\Gamma}_\alpha^\beta - K^\alpha_\beta, \] (26)
with \( \tilde{\Gamma}_\alpha^\beta \) is the purely Riemannian connection and \( K^{\mu\nu} \) is the contorsion 1-form which is
related to the torsion through the relation
\[ T^\alpha \overset{\text{def}}{=} K^\alpha_\beta \wedge \vartheta^\beta. \] (27)
Using the spherical local coordinates \((t, r, \theta, \phi)\) the Kerr-NUT, using Eq. (12), frame is
described by the coframe components:
\[ \vartheta^1_0 = A_1 c dt, \quad \vartheta^1_1 = A_2 dr, \quad \vartheta^2_0 = A_3 d\theta, \quad \vartheta^3_0 = A_4 dt + A_5 \sin \theta d\phi. \] (28)
If we take coframe (28), as well as the Riemannian connection \( \tilde{\Gamma}_\alpha^\beta \) and substitute into (26)
we finally get
\[ \tilde{H}_0 \approx -\frac{\sin \theta}{8r^2 \pi} \left[ 2Mr^2 - 2r^3 - 2Ma^2 \cos^2 \theta - rM^2 + M^3 - ra^2 \sin^2 \theta + 2rL^2 + (2raL - 6LMa) \cos \theta ight. \\
+ 4ML^2 \cot^2 \theta \left. \right] d\theta \wedge d\phi + \cdots + O \left( \frac{1}{r^2} \right). \] (29)
Using Eq. (29) to compute the total energy at a fixed time in the 3-space with a spatial
2-dimensional boundary surface \( \partial S = \{ r = R, \theta, \phi \} \) we finally obtain
\[ \tilde{E} = \int_{\partial S} \tilde{H}_0 = \infty! \] (30)
which is identical with Eq. (16).

4. On the choice of the frame

Let us consider the Lorentz transformation described by the matrix
\[ (\Lambda^\alpha_\beta) = \begin{pmatrix}
B_1 & B_2 & B_3 & B_4 \\
C_1 \sin \theta \cos \phi & C_2 \sin \theta \cos \phi & C_3 \cos \theta \cos \phi & C_4 \sin \theta \sin \phi \\
F_1 \sin \theta \sin \phi & F_2 \sin \theta \sin \phi & F_3 \cos \theta \sin \phi & F_4 \cos \phi \sin \theta \\
G_1 \cos \theta & G_2 \cos \theta & G_3 \sin \theta & G_4 \cos \theta
\end{pmatrix}, \] (31)
\(^*\cdots\) means terms which are multiply by \( \theta \wedge d\theta, \theta \wedge dt, dr \wedge d\phi \) ect.
where \( B_i, C_i, F_i \) and \( G_i \), \( i = 1 \cdots 4 \) are defined as:

\[
B_1 = \frac{\sin \theta}{\sqrt{ABL_5}} \left( aL^3 \cos \theta - a^2 L^2 (2 - \cos^2 \theta) + L^2 (Mr - r^2) - aL (3r^2 - 4Mr - 3a^2) \cos \theta - a^4 \cos^2 \theta - r^2 a^2 (1 + \cos^2 \theta) - r^4 - a^2 rM + r^3 M + 2Ma^2 r \cos^2 \theta \right), \\
B_2 = \frac{(Mr + LL_1)}{\sqrt{AB}}, \quad B_3 = 0,
\]

\[
B_4 = \frac{M_r + L (aL_1 [1 + \cos^2 \theta] + 2r^2 \cos \theta)}{\sqrt{AL_5}}, \quad C_1 = \frac{-2r (LB \cos \theta + M_r \chi + aL^2 \sin^2 \theta) \sin \phi + L_3 \cos \phi}{\sqrt{ABL_5}},
\]

\[
C_2 = \frac{r_1 \alpha - [Mr + LL_1] \cos \phi}{\sqrt{AB} \cos \phi}, \quad C_3 = \frac{\alpha}{A \cos \phi}, \quad C_4 = \frac{1}{\sqrt{AL_5 \sin \phi}} (r_1 \alpha \sin \phi - L_4 \cos \phi),
\]

\[
F_1 = \frac{r_1 (LB \cos \theta - M_r \chi - aL^2 \sin^2 \theta) \cos \phi + L_3 \sin \phi}{\sqrt{ABL_5}}, \quad F_2 = \frac{-1}{\sqrt{AB} \sin \phi} (r_1 \beta - [Mr + LL_1] \sin \phi),
\]

\[
F_3 = \frac{\beta}{A \sin \phi}, \quad F_4 = \frac{1}{\sqrt{AL_5 \sin \phi}} (-r_1 A \cos \phi + L_4 \sin \phi), \quad G_1 = \frac{-((Mr + LL_1)(r^2 + a^2 + L^2))}{\sqrt{ABL_5}},
\]

\[
G_2 = \frac{(Mr - r^2 - a^2 - aL \cos \theta)}{\sqrt{AB}}, \quad G_3 = \frac{-r_1}{A}, \quad G_4 = \frac{-M_r \chi + L [2L^2 \cos \theta - aL (1 - 3 \cos^2 \theta) - a^2 \cos \theta \sin^2 \theta]}{\sqrt{AL_5}},
\]

where \( \Omega, \ \Upsilon, \ L_1, \ L_3, \ L_4, \ \alpha, \ \beta, \ r_1 \) and \( \chi \) are defined by

\[
\Omega \overset{\text{def}}{=} r^2 + L_1^2, \quad \Upsilon \overset{\text{def}}{=} r^2 + a^2 - 2Mr - L^2, \quad L_1 \overset{\text{def}}{=} L + a \cos \theta,
\]

\[
L_3 \overset{\text{def}}{=} aL \cos^3 \theta (L^2 + r^2 + a^2) + \cos^2 \theta (L^4 + L^2 Mr - a^2 rM + L^2 r^2 - L^2 a^2 + M^3) + aL \cos \theta (a^2 + r^2 - 4Mr - 3L^2) + aL^2 + a^2 Mr - L^4 - a^3 M - rML^2 - r^2 L^2,
\]

\[
L_4 \overset{\text{def}}{=} La^2 \cos \theta (a \cos \theta - 3) - a^3 \cos^2 \theta + arM \cos^2 \theta + 3aL^2 \cos^2 \theta - rM \chi - r^2 a - 2aL^2,
\]

\[
L_5 \overset{\text{def}}{=} -aB \cos^3 \theta (a \cos \theta + 4L) - \cos^2 \theta \left( r^4 + 8aL^2 + 6r^2 L^2 - a^4 - 3L^4 - 8MrL^2 + 4Mr^2 \right) + 4aLB \cos \theta + 2Mr^2 + L^4 + r^4 + r^2 a^2 + 2rL^2 + 3aL^2, \quad \alpha \overset{\text{def}}{=} r_1 \cos \phi + a \sin \phi,
\]

\[
\chi \overset{\text{def}}{=} a \sin^2 \theta - 2L \cos \theta, \quad \beta \overset{\text{def}}{=} r_1 \sin \phi - a \cos \phi, \quad r_1 \overset{\text{def}}{=} \sqrt{r^2 + L(L + 2a \cos \theta)}.
\]

Using

\[
(h^{\alpha \gamma})_i = (\Lambda^{\alpha \gamma}_i) (h^{\gamma i})_1,
\]

in Eq. (10) to calculate the non-vanishing components needed to the calculations of energy, we finally get

\[
\Sigma^{101} \simeq \frac{a}{2r^5} \left( [aL \cos \theta + Mr + L^2] a \sin^2 \theta - 4aL^2 \cos^2 \theta + [L^2 + Mr] 2L \cos \theta \right) + O \left( \frac{1}{r^6} \right),
\]

\[
\Sigma^{201} \simeq \frac{-a}{2r^5 \sin \theta} \left( [Ma \cos \theta - L^2] \sin^2 \theta - 2LM \cos^2 \theta \right) + O \left( \frac{1}{r^6} \right),
\]

\[
\Sigma^{301} \simeq \frac{-L}{2r^5 \sin^2 \theta} \left( a^2 \cos \theta \sin^2 \theta + aL \sin^2 \theta - L^2 \cos \theta \right) + O \left( \frac{1}{r^6} \right),
\]

*Science the equations of motion (7) is just Einstein equations written in terms of tetrad fields \( h^a_i \) therefore, Eq. (34) is an exact solution to Eq. (7). This case is studied intensively by Hayashi and Shirafuji (cf., Ref. 12) Eqs. (7.2)~(7.11) and references therein.*
\[ \Sigma^{01} \approx -\frac{1}{2r^4} \left( 2a^2 r \cos^2 \theta + a^2 M \sin^2 \theta + 2aL \cos \theta (2r - M) + 2r[L^2 - r^2] \right) + O \left( \frac{1}{r^3} \right). \]  

(35)

Using Eq. (35) in Eq. (9) we finally obtain\(^1\)

\[ P^{(0)} = E = -\oint_{S \to \infty} dS_k \Pi^{(0)k} = -\frac{1}{4\pi} \oint_{S \to \infty} dS_k \sqrt{-g} h^{(0)}_\mu (\Sigma^{\mu 0 k} - \Sigma^{\mu 0 k}_{M=0,a=0,L=0}) \]

\[ \cong M + \frac{L^2}{r} - \frac{L^2 M}{r^2} - \frac{L^2(5a^2 + 6L^2)}{6r^3} + O \left( \frac{1}{r^4} \right). \]  

(36)

Eq. (36) is a satisfactory result.\(^3\) It is clear from (36) that the energy content is shared by both the interior and exterior of the Kerr-NUT spacetime. The total energy when \( r \to \infty \) gives the ADM (Arnowitt-Deser-Misner).

The non-vanishing components needed to calculate the spatial momentum have the form\(^2\)

\[ \Sigma^{(1)01} \cong -\frac{\sin^2 \theta \cos \phi}{8\pi r} \left( 2Mr + 2L^2 + 3aL \cos \theta \right) + O \left( \frac{1}{r^2} \right), \]

\[ \Sigma^{(2)01} \cong -\frac{\sin^2 \theta \sin \phi}{8\pi r} \left( 2Mr + 2L^2 + 3aL \cos \theta \right) + O \left( \frac{1}{r^2} \right), \]

\[ \Sigma^{(3)01} \cong -\frac{\sin \theta}{8\pi r} \left( 3aL \cos^2 \theta + 2L^2 \cos \theta + 2Mr \cos \theta - aL \right) + O \left( \frac{1}{r^2} \right). \]  

(37)

Using Eq. (37) in Eq. (9), we finally get the spatial momentum in the form

\[ P_1 = -\oint_{S \to \infty} dS_k \Pi^{(1)k} = -\frac{1}{4\pi} \oint_{S \to \infty} dS_k \Sigma^{(1)0k} = 0, \quad \text{by same method } P_2 = 0, \quad P_3 \cong \left( \frac{1}{r^2} \right). \]  

(38)

Now turn our attention to the calculation of angular-momentum. The non-vanishing components of the angular-momentum are given by

\[ M^{(0)(1)}(r, \theta, \phi) \cong -\frac{M \cos^2 \theta}{4\pi r^2} \left( \{L^2 + rM + aL \cos \theta + 2M^2 \} \cos \phi + aM \sin \phi \right) + O \left( \frac{1}{r^2} \right), \]

\[ M^{(0)(2)}(r, \theta, \phi) \cong -\frac{M^2 \sin \phi \cos^2 \theta}{4\pi r} + O \left( \frac{1}{r^2} \right), \quad M^{(0)(3)}(r, \theta, \phi) \cong \frac{M^2 \sin \theta \cos \theta}{4\pi r} + O \left( \frac{1}{r^2} \right), \]

\[ M^{(1)(2)}(r, \theta, \phi) \cong -\frac{M^2 a \sin \theta \cos^2 \theta}{4\pi r^2} + O \left( \frac{1}{r^2} \right), \]

\[ M^{(1)(3)}(r, \theta, \phi) \cong \frac{M \cos \theta [r + M] \cos \phi + a \sin \phi}{4\pi r} + O \left( \frac{1}{r^2} \right), \]

\[ M^{(2)(3)}(r, \theta, \phi) \cong \frac{M \cos \theta [r + M] \sin \phi - a \cos \phi}{4\pi r} + O \left( \frac{1}{r^2} \right). \]  

(39)

Using Eq. (39) in (11) we get

\[ L^{(0)(1)} = \int_0^\pi \int_0^{2\pi} \int_0^\infty d\theta d\phi dr \left[ M^{(0)(1)} \right] = 0, \]  

(40)

\(^1\)We introduce \( \Sigma^{\mu 0 k}_{M=0,a=0,L=0} \) in Eq. (36), to remove the divergence appearers from term like \( r \). It is worth to mention that we cannot use the expression \( \Sigma^{\mu 0 k}_{r \to \infty} \) because the spacetime we use is not asymptotically flat.

\(^2\)Terms like \( M^2, L^3, L^3M, M^2a, \cdots \) etc. are neglected in this calculations.
which is a consistent result. By the same method we finally obtain
\[ L^{(0)(2)} = L^{(0)(3)} = L^{(1)(2)} = L^{(1)(3)} = L^{(2)(3)} = 0. \] (41)

We show by explicit calculations that the energy-momentum tensor which is a coordinate independent does not give a consistent result of the angular momentum when applied to the tetrad field given by Eq. (12)!

To show if Eq. (34) continue to give acceptable result of energy we use the superpotential (26). The coframe of Eq. (34) takes the form
\[ \vartheta'_{\alpha} = (\Lambda^{\alpha}_{\beta}) \vartheta_1^\beta, \] (42)
with \((\Lambda^{\alpha}_{\beta})\) and \(\vartheta_1^\beta\) are given by Eqs (28) and (31) respectively. If we take coframe (42), as well as the trivial Weitzenböck connection, i.e., \(\Gamma^\alpha_{\beta\gamma} = 0\) and substitute into (26) we finally get the temporal component of the translation momentum in the form
\[ \tilde{H}_0 \simeq -\frac{\sin \theta}{8r^2} \left[ 3a^2 M \cos^2 \theta + 6aLM \cos \theta - 2ar L \cos \theta + 2ML^2 - 2rL^2 - a^2 M + 2r^2 - 2r^2 M \right] d\theta \wedge d\phi + \cdots + O \left( \frac{1}{r^3} \right). \] (43)
Computing the total energy up to order \(O \left( \frac{1}{r^2} \right)\) at a fixed time in the 3-space with a spatial 2-dimensional boundary surface \(\partial S = \{ r = R, \theta, \phi \} \) we obtain\(^5\)
\[ \tilde{E} = \int_{\partial S} \left( \tilde{H}_0 - \left\{ \tilde{H}_0 \right\}_{M=0,a=0,L=0} \right) \simeq M + \frac{L^2}{R} - \frac{L^2 M}{R^2} + O \left( \frac{1}{R^3} \right), \] (44)
which is the energy of Kerr black hole when \(L = 0^{33}\).

The necessary components needed to calculate the spatial momentum \(\tilde{H}_{\hat{\alpha}}, \hat{\alpha} = 1, 2, 3\) have the following components
\[ \tilde{H}_1 = \frac{2 \cos \phi \sin^2 \theta [3aL \cos \theta + 2L^2 + 2Mr + 4M^2]}{r} d\theta \wedge d\phi + \cdots, \]
\[ \tilde{H}_2 = \frac{2 \sin \phi \sin^2 \theta [3aL \cos \theta + 2L^2 + 2Mr + 4M^2]}{r} d\theta \wedge d\phi + \cdots, \]
\[ \tilde{H}_3 = \frac{2 \sin \theta (3aL \cos \theta + 2L^2 + 2Mr + 4M^2 \cos \theta - aL)}{r} d\theta \wedge d\phi + \cdots. \] (45)
Using Eqs. (45) in (26), we finally get the spatial momentum in the form
\[ P_1 = P_2 = P_3 \simeq O \left( \frac{1}{R^2} \right). \] (46)

\(\footnote{We introduce \(\left\{ \tilde{H}_0 \right\}_{M=0,a=0,L=0}\) to remove the divergence appearers from term like \(r\). It is worth to mention that we cannot use the expression \(\left\{ \tilde{H}_0 \right\}_{r \to \infty}\) because the spacetime we use is not asymptotically flat.}
5. Second Kerr-NUT spacetime

The covariant form of the second Kerr-NUT tetrad field having axial symmetry in spherical coordinates, can be written as

\[
(h^\alpha_i)_2 = \begin{pmatrix}
A_1 & 0 & 0 & 0 \\
-\sin \phi A_4 & \sin \theta \cos \phi A_2 & \cos \theta \cos \phi A_3 & -\sin \theta \sin \phi A_5 \\
\cos \phi A_4 & \sin \theta \sin \phi A_2 & \cos \theta \sin \phi A_3 & \sin \theta \cos \phi A_5 \\
0 & \cos \theta A_2 & -\sin \theta A_3 & 0
\end{pmatrix}
\]

where \( A_i, \ i = 1 \ldots 5 \) are defined by Eq. (13).

(47)

Tetrad field (47) has the same associated metric of tetrad (12), i.e., Kerr-NUT spacetime given by Eq. (14). Tetrad (47) is related to tetrad (12) through the relation

\[
(h^\alpha_i)_2 = (A_1^\alpha_\gamma) (h^\gamma_i)_1,
\]

where \((A_1^\alpha_\gamma)\) is the local Lorentz transformation given by

\[
(A_1^\alpha_\gamma) \overset{\text{def.}}{=} \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \sin \theta \cos \phi & \cos \theta \cos \phi & -\sin \phi \\
0 & \sin \theta \sin \phi & \cos \theta \sin \phi & \cos \phi \\
0 & \cos \theta & -\sin \theta & 0
\end{pmatrix},
\]

(49)

Following the same technics used in §3 to calculate energy we finally get the same non-vanishing components of \(\Sigma^{a01}\) asymptotically as given by Eq. (15) and the form of energy will be the same as given by Eq. (16). Repeat the same calculation done in §3, in which Riemannian connection 1-form is employed and using tetrad (47), we obtain same divergence value of energy given by Eq. (30).

To remove such unphysics results, let us consider the local Lorentz transformation described by Eq. (31) and follow the same procedure done in the previous section we can remove the divergence appears.

6. Discussion and conclusion

The tetrad field given by Eq. (12) whose associated metric gave the Kerr-NUT spacetime is used. From this spacetime one can generate Kerr and Schwarzschild spacetimes by putting \( L = 0 \) and \( a = 0 \), \( L = 0 \) respectively. Our results can be summarized as follows:

- The energy content of tetrad field (12) using the gravitational energy-momentum tensor

*Eq. (48) is an exact solution to the equations of motion (7) due to the reasons discussed for Eq. (34).
(9) was calculated and a divergent result was obtained. It is worth to mention that the value of energy did not depend on the radial coordinate $r$, i.e., the divergence is not related to the fact that when $r \to \infty$ the value of energy will be infinity. The source of divergence is related to the NUT parameter $L$, because when $L$ is nil the value of energy becomes finite and will be identical with the ADM which is the energy of Kerr [Ref. 46 Eq. (25)].

- To assure that the divergence of energy is not related to the expression which has been used for calculation, another expression to recalculate the energy has been applied, i.e., we have used the Riemannian connection 1-form, and have got the same divergent result.

- To remove such an unacceptable result, the local Lorentz transformation (31) has been suggested. This transformation when is multiplied by tetrad (12) and by repeating the same calculation of energy, using the gravitational energy-momentum tensor, we got the correct value of Kerr-NUT spacetime [Ref. 47 Eq. (44)]. Tetrad (34) gave a very satisfactory result of the spatial momentum, but for the angular momentum the result is not correct. We may claim that the unfamiliar result of the angular momentum are related to the expression used in calculation, Eq. (11). To assure that the result of energy is correct, the Riemannian connection 1-form (26) has been used and the same consistent result have been reached, Eq. (44).\(^3\)

- We have used another tetrad given by Eq. (47) and have calculated the energy using the two expressions: the gravitational energy-momentum Eq. (9) and the Riemannian connection 1-form (26) and got the same result, i.e., divergent value, which is given by Eq. (16). The divergence of the energy is due to the fact that tetrad (47) is related to tetrad (12) through the local Lorentz transformation (48). It is of interest to note that in the spherical symmetric case, i.e., when $L = 0$ and $a = 0$ is shown in\(^2\) that local Lorentz transformation (48) when applied to Eq. (12) the expression of energy became finite [Ref. 27 Eq. (4-13)]. However, this procedure did not work here because an axially symmetric tetrad is used.

- When local Lorentz transformation (31) is multiplied by Eq. (47) combined with a repetition of the same procedure to calculate the energy, a finite and consistent result have been obtained.

- It is of interest to note that expression (11) did not give the correct result of angular momentum. In our forthcoming research work, calculations of the total conserved quantities related to tetrad fields (12) and (47) will be dealt with.

**Acknowledgements**

The author would like to thank Professor A. Hussain; English Department college of Arts, King Faisal University, Saudi Arabia for taking the time in reviewing the language of the manuscript.

**References**

[1] A. Trautman, *Conservation laws in general relativity*, in: "Gravitation: An introduction to current research", L. Witten, ed. (John Wiley and Sons, New York, 1962) pp. 169-198.
[2] L.D. Faddeev, *Problem of energy in Einstein’s theory of gravity*, Sov. Phys. Usp. 25 (1982) 130 [Usp. Fiz. Nauk, 136 (1982) 435].

[3] L. B. Szabados, *Quasi-local energy-momentum and angular momentum in GR: A review article*, Living Rev. Rel. 7 (2004), 4; http://www.livingreviews.org/lrr-2004-4.

[4] M. Blagojević, *Gravitation and gauge symmetries* (Institute of Physics: Bristol, 2002).

[5] J. M. Nester, *Class. Quantum Grav.* 21 (2004), S261.

[6] Y. N. Obukhov and J.G. Pereira, *Phys. Rev.* D67 (2003), 044016.

[7] V. C. de Andrade and J.G. Pereira, *Phys. Rev.* D56 (1997), 4689.

[8] T. Ortín ”Gravity and Strings” Cambridge University Press (2004), P. 166.

[9] C. Møller, *Tetrad Fields and Conservation Laws in General Relativity*, Proceedings of the International School of Physics Enrico Fermi, edited by C. Møller (Academic Press, London, 1962); *Conservation Laws in the Tetrad Theory of Gravitation*, Proceedings of the Conference on Theory of Gravitation, Warszawa and Jabłonna 1962 (Gauthier-Villars, Paris, and PWN-Polish Scientific Publishers, Warszawa, 1964) (NORDITA Publications No. 136).

[10] C. Pellegrini and J. Plebański, *Mat. Fys. Scr. Dan. Vid. Selsk.* 2 (1963), no.4.

[11] F. W. Hehl, J. D. McCrea, E. W. Mielke and Y. Neeman, *Phys. Rep.* 258 (1995), 1.

[12] K. Hayashi, *Phys. Lett.* 69B (1977), 441; K. Hayashi and T. Shirafuji. *Phys. Rev.* D19 (1979), 3524; *Phys. Rev.* D24 (1981), 3312; M. Blagojević and M. Vasič *Class. Quant. Grav.* 5 (1988), 1241; T. Kawai, *Phys. Rev.* D62 (2000), 104014, T. Kawai, K. Shibata and I. Tanaka, *Prog. Theor. Phys.* 104 (2000), 505.

[13] Y. Itin, *Class. Quant. Grav.* 19 (2002), 173.

[14] G. G. L. Nashed, *Eur. Phys. J.*, C 54 (2008), 291.

[15] Y. Itin, *J. Math. Phys.* 46 (2005), 012501.

[16] F. W. Hehl, Y. Neeman, J. Nitsch and P. Von der Heyde, *Phys. Lett.* B78 (1978), 102.

[17] F. W. Hehl, *Four Lectures on Poincaré gauge theory*, in Proceedings of the 6th. school of cosmology and gravitation course on spin, Torsion rotation and supergravity, held at Eric, Italy, (1979), eds., P.G. Bergmann, V. De Sabbata (Plenum, N.Y. (1980)) P.5; F. Gronwald and F. W. Hehl, *On the gauge aspects of gravity*, in: International School of Cosmology and Gravitation: 14th. course: Quantum Gravity, Held May 1995 in Erice, Italy. Proceedings. P.G. Bergmann et al. (eds.) World Scientific, Singapore (1996) gr-qc/9602013.

[18] M. Leclerc, *Phys. Rev* D71 (2005), 027503.

[19] T. Kawai, *Phys. Rev.* D49 (1994), 2862; *Phys. Rev.* D62 (2000), 104014.

[20] W. Kopzyński, *J. Phys.* A15 (1982), 493.

[21] F. W. Hehl, G. D. Kerlick, P. von der Heyde, *Phys. Lett.* 63B (1976), 446.
[22] F. W. Hehl, E.A. Lord, L. L. Smalley, *Gen. Relat. Grav.* **13** (1981), 1037.

[23] F. W. Hehl, J. D. McCrea, *Found. Phys.* **16** (1986), 267.

[24] F. W. Hehl, J. Socorro, *Acta Phys. Polon.* **B29** (1998), 1113.

[25] J. Nitsch and F. W. Hehl, *Phys. Lett.* **B90** (1980), 98.

[26] P. Baekler and F. W. Hehl, *Int. J. Mod. Phys.* **D** **15** (2006), 635.

[27] T. G. Lucas, Y. N. Obukhov and J. G. Pereira, *Phys. Rev. D* **80** (2009), 064043.

[28] R. Aldrovandi, L. C. T. Guillen, J. G. Pereira, and K. H. Vu, *Bringing together gravity and the quanta*, contribution to the proceedings of the Albert Einstein Century International Conference, Paris, July 18-22, 2005 [gr-qc/0603122].

[29] R. Aldrovandi, T. G. Lucas, and J. G. Pereira, *Gravitational energy-momentum conservation in teleparallel gravity*, arXiv:0812.0034 [gr-qc].

[30] Y. N. Obukhov, J.G. Pereira and G. F. Rubilar, *Class. Quant. Grav.* **26** (2009), 215014. Lagrangian, Phys. Rev. D14 (1976), 2521.

[31] J. W. Maluf, J. F. da Rocha-neto, T. M. L. Toribio and K. H. Castello-Branco, *Phys. Rev. D65* (2002), 124001.

[32] J. W. Maluf, *J. Math. Phys.* **35** (1994), 335.

[33] M. Ahmed and S. M. Hossain 1995 *Prog. Theor. Phys.* **93** (1995), 901.

[34] N. Toma, *Prog. Theor. Phys.* **86** (1991), 659.

[35] G. G. L. Nashed, *Mod. Phys. Lett. A* **21** (2006), 2241.

[36] Y. N. Obukhov, and G. G. Rubilar, *Phys. Rev D73* (2006), 124017.

[37] Y. M. Cho, *Phys. Rev. D14* (1976), 2521.

[38] F. Gronwald, *Int. J. Mod. Phys. D* **6** (1997), 263.

[39] U. Muench, *Über teleparallele Gravitationstheorien*, Diploma Thesis, University of Cologne (1997).

[40] R. Tresguerres, *Int. J. Geom. Meth. Mod. Phys. 5* (2008), 905.

[41] G. G. L. Nashed, *Int. J. Mod. Phys. A* **25** (2010), 2883.

[42] Y. N. Obukhov and J. G. Pereira, *Phys. Rev. D69* (2004), 128502.

[43] Y. N. Obukhov, G. F. Rubilar and J. G. Pereira, *Phys. Rev. D74* (2006), 104007.

[44] Y. N. Obukhov, *Int. J. Geom. Meth. Mod. Phys. 3* (2006) 95; http://www.worldscinet.com/ijgmmp/.

[45] H. T. Nieh, *Int. J. Mod. Phys. A* **22** (2007), 5237; R. Jackiw and S.-Y. Pi, *Phys. Rev. D* **68** (2003), 104012; M. B. Cantcheff, *Phys. Rev. D* **78** (2008), 025002; E. W. Mielke, *Gen. Rel. Grav. 40* (2008), 1311; *Phys. Rev. D80* (2009), 067502.
[46] G. G. L. Nashed, *Mod. Phys. Lett.* A 22 (2007), 1047.

[47] G. G. L. Nashed, *Int. J. Mod. Phys.* A 23 (2008), 1903.