Generalized generating functional for mixed-representation Green’s functions: Coherent-state path integral representation

M Blasone\textsuperscript{1}, P Jizba\textsuperscript{2} and L Smaldone\textsuperscript{1}

\textsuperscript{1} Dipartimento di Fisica, Universit`a di Salerno, Via Giovanni Paolo II, 132 84084 Fisciano, Italy & INFN Sezione di Napoli, Gruppo collegato di Salerno, Italy
\textsuperscript{2} FNSPE, Czech Technical University in Prague, Bˇrebov`a 7, 115 19 Praha 1, Czech Republic

E-mail: blasone@sa.infn.it
E-mail: petr.jizba@fjfi.cvut.cz
E-mail: luca.smaldone@studenti.unina2.it

Abstract. The aim of this paper is to study mixed-representation Green’s functions from the point of view of coherent-state path integrals. This is achieved by using the machinery of generalized generating functionals of Green’s functions, previously introduced by the present authors in the context of standard phase-space path integrals. The obtained results are illustrated in the context of the linear harmonic oscillator.

1. Introduction

Coherent states (CS’s) were first introduced by E. Schrödinger in 1926 [1], to describe a dispersionless wave packet of the linear harmonic oscillator (LHO). J. von Neumann [2] used similar ideas in order to investigate the measurement of coordinates and momenta in quantum mechanics (QM). With the advent of laser physics in early 60’s this subject firmly established itself through a number of pioneering works [3, 4, 5]. An excellent review, complemented by many original papers, can be found in Ref. [6].

A path integral (PI) formulation of QM in terms of CS’s were first proposed by J. R. Klauder in Ref. [3]. As shown in Ref. [7], CS PI’s are better mathematically behaved than the more conventional phase-space (PS) PI’s [8, 9]. For these reasons, CS PI’s found a wide applicability, for example, in quantum field theory (QFT) [9, 10, 11, 12] and theoretical chemistry [13, 14].

The present authors proved in Ref. [15] the usefulness of CS PI’s in connection with the study of canonical transformations in the context of quantum field theoretical functional integral (FI). In particular, it was found that the CS PI’s are instrumental in helping to exhibit features related to the existence of unitarily inequivalent representations of canonical commutation relations. In addition, in Ref. [16], it was introduced the concept of generalized generating functional of Green’s functions (GGF), in order to evaluate mixed representations Green’s functions, i.e. Green’s functions constructed on different vacua, connected with each other via unitary canonical transformations. There, it was found that the latter is intimately connected with the Schwinger...
closed-time-path formalism [17]. In Ref. [18] we applied this method to evaluate the generating functional of Green’s functions on the flavor vacuum, for a two-flavor bosonic system [19].

In this paper, GGF’s are used to provide a new viewpoint on CS PI’s that were otherwise treated only within the standard PS PI’s. More specifically, we will see that the CS evolution kernel can be regarded as a generalized generating functional, where the defining unitary transformation belongs to the quotient group $W_1/U(1)$, where $W_1$ is the one-dimensional Weyl–Heisenberg group [20].

The paper is organized as follows: in Section 2 we briefly remind some essentials of canonical coherent states. To this end we will employ the notation of Ref. [12]. In Section 3 we review key results of Ref. [16] and, in particular, the concept of GGF. In Section 4 we show how to connect CS PI’s with GGF’s. We apply this result to evaluate some correlation functions of the position operator. Various remarks and generalizations are addressed in the concluding Section 5.

2. Coherent state PI’s

A canonical (or Glauber) CS in the Heisenberg picture is defined as [6, 12]

$$|pq,t\rangle \equiv \exp[-iq\hat{P}(t)] \exp[ip\hat{Q}(t)] |0\rangle,$$

where $|0\rangle$ satisfies the equation

$$\left(\hat{Q}(t) + i\hat{P}(t)\right) |0\rangle = 0,$$

and hence it can be identified with the ground state of the LHO, with Hamiltonian

$$\hat{H} \left(\hat{P}(t), \hat{Q}(t)\right) = \frac{\hat{P}^2(t)}{2} + \frac{\hat{Q}^2(t)}{2}.$$

In analogy with QFT it is common to call $|0\rangle$ the vacuum state. The unitary operator

$$\hat{G}_{pq}(t) = \hat{G}^1_p(t) \hat{G}^2_q(t),$$

(with $\hat{G}^1_p(t) = e^{-ip\hat{Q}(t)}$ and $\hat{G}^2_q(t) = e^{ip\hat{P}(t)}$) belongs to the quotient group $W_1/U(1)$.

If we now employ the Baker–Campbell–Hausdorff formula we obtain:

$$\langle pq,t | \hat{Q}(t) | pq,t \rangle = q,$$

$$\langle pq,t | \hat{P}(t) | pq,t \rangle = p.$$

One can easily check that the resolution of unity can be written in terms of CS’s as

$$\mathbb{I} = \int \frac{dp \, dq}{2\pi} |pq,t\rangle \langle pq,t|.$$

This form can be now used to derive a CS PI form for the evolution kernel, namely (see, e.g. Ref. [6])

$$\langle q_f p_f, t_f | q_i p_i, t_i \rangle = \int Dp \, Dq \, \exp(iS_{cs}[q,p]),$$

where $q(t_f), p(t_f) = q_f, p_f$, $q(t_i), p(t_i) = q_i, p_i$, and

$$S_{cs}[q,p] = \int_{t_i}^{t_f} dt \, [p(t)\dot{q}(t) - H(p(t), q(t))].$$
is the classical action. Note that, as shown in Eqs. (5)-(6), \( p \) and \( q \) can be specified at the equal time. So, in a sense, CS’s provide a PS representation of the QM. The precise meaning of Eq. (8) together with a precise definition of the measure can be found, e.g., in Ref. [12].

For our purposes is also interesting to note that the Feynman–Matthews–Salam formula, i.e.

\[
\langle q_f p_f, t_f | T \left[ \hat{Q}(t_n) \ldots \hat{Q}(t_1) \right] | q_i p_i, t_i \rangle = \int \mathcal{D}p \mathcal{D}q q(t_n) \ldots q(t_1) \exp \left( i S_{cs} [q, p] \right),
\]

holds without changes. Here \( T[\ldots] \) denotes the time-ordered symbol and the boundary conditions are the same specified above. The PI on the RHS of Eq. (10) can be evaluated with the Schwinger trick, i.e. by introducing

\[
\langle q_f p_f, t_f | q_i p_i, t_i \rangle = \int \mathcal{D}q \mathcal{D}p \exp \left( i S_{cs} [p, q] \right),
\]

where

\[
S_{cs} [p, q] = \int_{t_i}^{t_f} dt \left[ p(t) \dot{q}(t) - H(q(t), p(t)) + J(t)q(t) \right],
\]

and \( J(t) \) is the Schwinger-type current (i.e., it has a compact support). With this we can write

\[
\langle q_f p_f, t_f | T \left[ \hat{Q}(t_n) \ldots \hat{Q}(t_1) \right] | q_i p_i, t_i \rangle = \left\{ (-i)^n \frac{\delta \langle q_f p_f, t_f | q_i p_i, t_i \rangle}{\delta J(t_1) \ldots \delta J(t_n)} \right\}_{J=0},
\]

or, in other words

\[
\langle q_f p_f, t_f | q_i p_i, t_i \rangle = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int \cdots \int dt_1 \ldots dt_n \langle q_f p_f, t_f | T \left[ \hat{Q}(t_n) \ldots \hat{Q}(t_1) \right] | q_i p_i, t_i \rangle J(t_1) \ldots J(t_n). \]

We say that \( \langle q_f p_f, t_f | q_i p_i, t_i \rangle \) is the generating functional of the correlation functions (10).

We now ask the question: how are Eqs. (8) and (11) related to the standard phase-space PI?

3. Generalized generating functional of Green’s functions

Let us denote the PS evolution kernel in presence of currents as\(^1\)

\[
\langle Q_f, t_f | Q_i, t_i \rangle_{Q,JQ,JP} = \int_{Q(t_i)}^{Q(t_f)} DQ \mathcal{D}P \exp \left( i S_{ps} [P, Q] \right),
\]

where the action functional has the form

\[
S_{ps} [P, Q] = \int_{t_i}^{t_f} dt \left[ P(t) \dot{Q}(t) - H(Q(t), P(t)) + J_Q(t)Q(t) + J_P(t)P(t) \right],
\]

and \( J_Q(t) \) and \( J_P(t) \) are again Schwinger-type currents.

We now perform a canonical transformation of the operators \( \hat{Q}(t) \) and \( \hat{P}(t) \):

\[
\hat{Q}(t; \alpha) \equiv \hat{G}_\alpha(t) \hat{Q}(t) \hat{G}_\alpha(t),
\]

\[
\hat{P}(t; \alpha) \equiv \hat{G}_\alpha(t) \hat{P}(t) \hat{G}_\alpha(t),
\]

\(^1\) We denote by \( Q, P \) the eigenvalues of the operators \( \hat{Q}, \hat{P} \), to distinguish them from \( q, p \) which are just expectation values (see Eqs.(5),(6)).
where
\[ \hat{G}_\alpha(t) = \exp \left[ i\alpha \hat{K} \left( \hat{P}(t), \hat{Q}(t) \right) \right], \]  
(19)
is a unitary continuous transformation which leaves invariant the canonical commutation relations
\[ [\hat{Q}(t;\alpha), \hat{P}(t;\alpha)] = i. \]  
(20)
At this stage we introduce
\[ |0(\alpha,t)\rangle = \hat{G}_\alpha^{-1}(t)|0\rangle, \]  
(21)
and we call \(|0(\alpha,t)\rangle\) the transformed vacuum state, satisfying
\[ \left( \hat{Q}(t;\alpha) + i\hat{P}(t;\alpha) \right) |0(\alpha,t)\rangle = 0. \]  
(22)
It can be proved \([16]\) that the correlation function
\[ i\mathcal{G}_{\beta 0} (t' - t) = \langle 0(\beta, t)| T \left[ \hat{Q}(t')\hat{Q}(t) \right] |0(\beta, t)\rangle, \]  
(23)
is indeed a Green’s function. The aim is now to derive a simple formula, similar to Eq. (13), which will allow to evaluate Eq. (23). To this end, we introduce the \(q\)-ordering as
\[ \mathcal{O}^q \left[ \exp \left( iK \left( \hat{P}(t), \hat{Q}(t) \right) \right) \right] = \sum_{k,l=0}^{\infty} K_{kl} \hat{Q}^k(t)\hat{P}^l(t). \]  
(24)
The latter orders the operator in such a way that all \(\hat{Q}(t;\alpha)\)’s are on the left and \(\hat{P}(t;\alpha)\)’s are on the right. Then we define the “classical” \(q\)-ordering as
\[ \mathcal{O}^q_{cl} \left[ \exp \left( iK \left( \hat{P}(t), \hat{Q}(t) \right) \right) \right] = \frac{\langle Q, t| \mathcal{O}^q \left[ \exp \left( iK \left( \hat{P}(t), \hat{Q}(t) \right) \right) \right]|P, t \rangle}{\langle Q, t| P, t \rangle}. \]
By following the usual route (see, e.g., Ref [9]) we arrive at
\[ \langle 0(\gamma,t_+)| T \left[ \hat{Q}(t_n) \cdots \hat{Q}(t_1) \right]|0(\beta, t_-)\rangle = \langle 0| T \left[ \hat{G}_\gamma(t_+)\hat{Q}(t_n) \cdots \hat{Q}(t_1)\hat{G}_{-\beta}^{-1}(t_-) \right]|0\rangle \]
\[ = \lim_{t_1 \to +\infty} \lim_{t_2 \to -\infty} \int_{Q(t_1) = Q_1}^{Q(t_f) = Q_f} \int_{Q(t_1) = Q_1}^{Q(t_f) = Q_f} DQ DP \mathcal{O}^q_{cl} \left[ \hat{G}_\gamma(t_+) \right] \mathcal{O}^q_{cl} \left[ \hat{G}_{-\beta}(t_-) \right] Q(t_n) \cdots Q(t_1) e^{iS(P,Q)}. \]  
(25)
Following Ref. [16], we introduce\(^2\)
\[ Z^{+ -}_{\gamma,\beta 0}[J_Q] = \left\{ e^{i \langle \gamma \rangle K \left( \frac{\gamma}{2}, \frac{\gamma}{2}, \frac{\gamma}{2}, \frac{\gamma}{2}, \frac{\gamma}{2} \right)} Z[J_Q, J_P] e^{i \langle -\beta \rangle K \left( \frac{-\beta}{2}, \frac{-\beta}{2}, \frac{-\beta}{2}, \frac{-\beta}{2}, \frac{-\beta}{2} \right)} \right\}_{J_P = 0}, \]  
(26)
where
\[ Z[J_Q, J_P] = \lim_{t_f \to +\infty} \lim_{t_i \to -\infty} \frac{\langle Q_f, t_f| Q_i, t_i \rangle_{J_Q, J_P}}{\langle Q_f, t_f| Q_i, t_i \rangle_{J_Q, J_P = 0}}. \]  
(27)
\(^2\) This is not the most general definition but fully sufficient for our purposes. For more technical details and ensuing discussion see Ref. [16].
and $f(\alpha)$ would be determined by imposing $\hat{G}_\alpha(t)$ to through the $q$–ordering as

$$\mathcal{O}_d^q \left[ \hat{G}_\alpha(t) \right] = \mathcal{O}_d^q \left[ e^{i\alpha K(P(t),Q(t))} \right] = e^{if(\alpha) K(Q(t),P(t))}. \quad (28)$$

Eq. (26) defines a generalized generating functional of Green’s functions. In fact, it can be shown that [16]:

$$\langle 0(\gamma,t_+) | T \left[ \hat{Q}(t_n) \ldots \hat{Q}(t_1) \right] | 0(\beta,t_-) \rangle = \left\{ (-i)^n \frac{\delta^n}{\delta J_Q(t_n) \ldots \delta J_Q(t_1)} Z_{\gamma \beta 0}^+ \right\}_{J_Q=0}, \quad (29)$$

where $t_+ > \{t_i\}_{i=1,...,n} > t_-$. At the end one can consider the limit $t_+ \to t_- \to t$:

$$\langle 0(\gamma,t) | T \left[ \hat{Q}(t_n) \ldots \hat{Q}(t_1) \right] | 0(\beta,t) \rangle = \lim_{t_+ \to t_- \to t} \left\{ (-i)^n \frac{\delta^n}{\delta J_Q(t_n) \ldots \delta J_Q(t_1)} Z_{\gamma \beta 0}^+ \right\}_{J_Q=0}, \quad (30)$$

This procedure is formally identical to the so-called Schwinger closed-time-path integral [17], namely

$$\langle 0(\beta,t) | T \left[ \hat{Q}(t_n) \ldots \hat{Q}(t_1) \right] | 0(\beta,t) \rangle = \langle 0 | T \left[ \hat{G}_{\beta}(t_+) \hat{Q}(t_n) \ldots \hat{Q}(t_1) \hat{G}^\dagger_{\beta}(t_-) \right] | 0 \rangle_{t_+ \to t_- = t}$$

$$= \lim_{t_f \to +\infty} \lim_{t_i \to -\infty} \left[ \int_{Q(t_i)=Q_i} \mathcal{D}Q \mathcal{D}P \mathcal{O}_d^q [G_{\beta}(t_+)] \mathcal{O}_d^q [G_{-\beta}(t_-)] Q(t_n) \ldots Q(t_1) e^{iS(P,Q)} \right]_{t_+ \to t_- = t}, \quad (31)$$

where the contour $C$ is shown in Figure 1.

Finally, the Green’s function (23) can be now easily evaluated:

$$G_{\beta 0}(t_2 - t_1) = \lim_{t_+ \to t_- = t} \left\{ (-i)^2 \frac{\delta^2}{\delta J_Q(t_2) \delta J_Q(t_1)} Z_{\beta 0}^+ \right\}_{J_Q=0}. \quad (32)$$
4. Coherent states PIs as generalized generating functionals

Let us perform the canonical transformation:

\[ \hat{P}(t; p q) = \hat{G}_{pq}^{-1}(t) \hat{P}(t) \hat{G}_{pq}(t), \]  
\[ \hat{Q}(t; p q) = \hat{G}_{pq}^{-1}(t) \hat{Q}(t) \hat{G}_{pq}(t), \]  

(33)

(34)

where \( \hat{G}_{pq}(t) \) was defined in Eq. (4). From Eqs. (1) and (21), it is thus evident that

\[ \langle q_f p_f, t_f | q_i p_i, t_i \rangle = \langle 0 | (q_f p_f, t_f) | 0 | (q_i p_i, t_i) \rangle. \]  

(35)

In the present case, Eq. (26) reduces to

\[ Z_f[J_Q] = \left\{ e^{\frac{i}{\hbar} \int_{t_f}^{t_i} f(t) \bar{J} Q(t)} e^{-\frac{i}{\hbar} \int_{t_f}^{t_i} p(t) \bar{Q} f(t)} Z[J_P, J_Q] e^{\frac{i}{\hbar} \int_{t_f}^{t_i} p(t) \bar{J} Q(t)} e^{-\frac{i}{\hbar} \int_{t_f}^{t_i} f(t) \bar{Q} J(t)} \right\}_{f=0}, \]  

(36)

where, in order to simplify the notation, we introduced \( Z_f[J_Q] \equiv Z_{f |q_f p_f,q_i p_i}^+ [J_Q] \). Hence, from Eqs. (30) and (35) follows that

\[ \langle q_f p_f, t_f | q_i p_i, t_i \rangle = Z_f[0]. \]  

(37)

That is the first result we were looking for. On the LHS we have the CS kernel Eq. (8) that can be evaluated as CS PI. On the RHS we have a GGF, which can be evaluated as a PS PI (see Eq. (25)). Moreover, when applying Eq. (29) to the present case, we get

\[ \langle q_f p_f, t_f | q_i p_i, t_i \rangle = \left\{ (-i)^n \frac{\delta^n}{\delta J_Q(t_n) \cdots \delta J_Q(t_1)} Z_f[J] \right\}_{J=0}. \]  

(38)

This means that

\[ Z_{f |q_f p_f,q_i p_i} = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int \cdots \int dt_1 \cdots dt_n \langle q_f p_f, t_f | T \left[ \hat{Q}(t_n) \cdots \hat{Q}(t_1) \right] | q_i p_i, t_i \rangle J_Q(t_1) \cdots J_Q(t_n). \]  

(39)

Because the currents are arbitrary, we can compare Eq. (14) with Eq. (39), deducing that

\[ \langle q_f p_f, t_f | q_i p_i, t_i \rangle = Z_f[J], \]  

(40)

which completes our result.

Let us now consider, for example, the \( n \)–point correlation functions

\[ i G^n_{qp}(t_1, \ldots, t_n) = \langle q_p, t | T \left[ \hat{Q}(t_n) \cdots \hat{Q}(t_1) \right] | q_p, t \rangle. \]  

(41)

These can be evaluated (see Eq. (30)) as

\[ i G^n_{qp}(t_1, \ldots, t_n) = \lim_{t_f \rightarrow t_i = t} (-i)^n \left\{ \frac{\delta^n}{\delta J_Q(t_n) \cdots \delta J_Q(t_1)} Z_{f |q_f p_f,q_i p_i} \right\}_{J=0}. \]  

(42)

We treat the explicit case of a LHO (see Eq. (3)). The generating functional Eq. (27) then reads [9]:

\[ Z[J_Q, J_P] = e^{-\frac{i}{2} \int_{-\infty}^{+\infty} d \tau' \int_{-\infty}^{+\infty} d \tau J_Q(\tau) \bar{g}_0(\tau - \tau') J_Q(\tau')} \times e^{-i \int_{-\infty}^{+\infty} d \tau' \int_{-\infty}^{+\infty} d \tau J_Q(\tau) \bar{g}_0(\tau - \tau') J_P(\tau') - \frac{i}{2} \int_{-\infty}^{+\infty} d \tau J_P(\tau) \bar{g}_0(\tau - \tau') J_P(\tau')}, \]  

(43)
where \( G_0(\tau - \tau') \) is the Feynman propagator:

\[
i G_0 (t' - t) \equiv i G_{00} (t' - t) = \langle 0| T \left[ \dot{Q}(t') \dot{Q}(t) \right] |0\rangle. \tag{44}
\]

By using the well known formula

\[
\exp \left( \frac{\alpha}{\delta J(x)} \right) F[J] = F[J + \alpha_x], \tag{45}
\]

with \( \alpha_x = \alpha \delta(x - y) \), we arrive at

\[
Z_{f_i}|Q\rangle = e^{-\frac{i}{2} \int_{-\infty}^{\infty} \text{d}t' \int_{-\infty}^{\infty} \text{d}r' J_Q(\tau - t') J_Q(\tau) e^{-i \int_{-\infty}^{\infty} \text{d}r J_Q(r) F(\tau ; t_f, t_i)} Z_{f_i}|0\rangle, \tag{46}
\]

where

\[
F(\tau; t_f, t_i) \equiv p_i G_0 (\tau - t_i) - p_f G_0 (\tau - t_f) + q_i \partial_t G_0 (\tau - t_f) - q_i \partial_t G_0 (\tau - t_i), \tag{47}
\]

and \( Z_{f_i}|0\rangle \) is nothing but the coherent state kernel of the LHO:

\[
Z_{f_i}|0\rangle = e^{-\frac{1}{2} (q_i^2 + p_i^2) + \frac{1}{2} q_i q_f e^{-i(t_f - t_i)} - \frac{1}{2} (p_i^2 + p_f^2) + \frac{1}{2} p_i p_f e^{-i(t_f - t_i)} + \frac{1}{2} (p_i q_f - p_f q_i)} \times e^{\frac{i}{2} (p_i q_f e^{-i(t_f - t_i)} - p_f q_i e^{-i(t_f - t_i)}) - \frac{1}{2} (p_i q_f - p_f q_i)}. \tag{48}
\]

In particular, performing the limit \( t_f \to t_i \), we get the well known result [12]:

\[
\lim_{t_f \to t_i} Z_{f_i}|0\rangle = \langle q_f p_f | q_i p_i \rangle = e^{-\frac{i}{2} [(q_f - q_i)^2 + (p_f - p_i)^2]} e^{\frac{i}{2} (p_f + p_i) (q_i - q_f)}. \tag{49}
\]

Passing to one-point correlation functions we get, by using Eq. (42),

\[
\langle q_f p_f, t_f | \dot{Q}(t) | q_i p_i, t_i \rangle = \frac{1}{2} \left\{ i \left[ p_i e^{-i(t_f - t_i)} - p_f e^{i(t_f - t_i)} \right] + \left[ q_i e^{-i(t_f - t_i)} + q_f e^{i(t_f - t_i)} \right] \right\} \times \langle q_f p_f, t_f | q_i p_i, t_i \rangle. \tag{50}
\]

Performing the limit \( t_f \to t_i = t \), we get the integral kernel of the operator \( \hat{Q} \)

\[
\langle q_f p_f, t | \dot{Q}(t) | q_i p_i, t \rangle = \frac{1}{2} \left[ i \langle p_i - p_f \rangle + q_i + q_f \right] \langle q_f p_f | q_i p_i \rangle. \tag{51}
\]

This coincides with the result in Ref. [12]. Moreover, one can now easily derive Eq. (5) by considering diagonal elements.

Passing to two-point functions, we get

\[
\langle q_f p_f, t_f | T \left[ \dot{Q}(t_2) \dot{Q}(t_1) \right] | q_i p_i, t_i \rangle = \left[ i G_0(t_2 - t_1) + F(t_2, t_f, t_i) F(t_1, t_f, t_i) \right] Z_{f_i}|0\rangle. \tag{52}
\]

In particular, if \( t_f \to t_i = t = t_1 \), \( t_2 = t' \), \( p_f = p_i = p \) and \( q_f = q_i = q \) we get the Green’s function (32):

\[
i G_{p,q_0} (t' - t) = i G_0(t_2 - t_1) + q \left[ p \sin (t' - t) + q \cos (t' - t) \right] . \tag{53}
\]

Note that, if \( q = p = 0 \) we can recover the standard Feynman propagator Eq. (44) and

\[
\mathcal{H} (t' - t; p, q) = q \left[ p \sin (t' - t) + q \cos (t' - t) \right], \tag{54}
\]

is a solution of the homogeneous equation

\[
\left( \partial_{t_2}^2 + 1 \right) \mathcal{H} (t' - t; p, q) = \left( \partial_{t_1}^2 + 1 \right) \mathcal{H} (t' - t; p, q) = 0. \tag{55}
\]

Therefore, the result of Eq. (53) differs from the Feynman propagator Eq. (44) just by the choice of boundary conditions.
5. Conclusions

In this paper we have proved that coherent-state path integrals can be viewed as GGF’s (see Eqs. (37) and (40)). As an illustrative toy model system we considered the LHO and performed the ensuing computations of the 1-point and 2-point mixed-representation correlation functions. All results obtained proved to be in agreement with known results from the literature.

Our present analysis focused only on canonical (Glauber) coherent states. It would be clearly interesting to extend our study to other coherent states such as Perelomov group-related CS’s [20] and Barut–Girardello CS’s [21]. Particularly, Perelomov’s CS’s are an attractive option in view of their relevance in QFT [22, 23].

Finally, the issue of boundary conditions, touched upon in Section 4, will be discussed in more detail in our future work.

Acknowledgments

P.J. was supported by the Czech Science Foundation Grant No. 17-33812L

References

[1] Schrödinger E 1926 Naturwissenschaften 14 664
[2] von Neumann J 1932 Mathematische Grundlagen der Quantenmechanik (Berlin: Springer)
[3] Klauder J R 1960 Ann. Phys. 11 123
[4] Glauber R J 1963 Phys. Rev. 130 2529
[5] Glauber R J 1963 Phys. Rev. 131 2766
[6] Klauder J R and Skagerstam B Coherent States—Applications in Physics and in Mathematical Physics (New York: World Scientific)
[7] Schweber S S (1962) J. Math. Phys. 3 831
[8] Schulman L S 1985 Techniques and Applications of Path Integration (New York: John Wiley and Sons)
[9] Kleinert H 2009 Path Integrals in Quantum Mechanics, Statistics, Polymer Physics, and Financial Markets (London: World Scientific)
[10] De Facio B, Hammer C L and Shrauner J E 1978 Phys. Rev. D 18 373
[11] Faddeev L D and Slavnov A A 1980 Gauge Fields—Introduction to Quantum Theory (New York: The Benjamin Cummings Publishing Company)
[12] Klauder J R 2011 A Modern Approach to Functional Integration (London: Birkhäuser)
[13] Heller E J 1977 Journ. Chem. Phys. 66 5777
[14] Weissman Y 1982 Journ. Chem. Phys. 76 4067
[15] Blasone M, Jizba P and Smaldone L 2017 Ann. Phys. 383 207–238; J. Phys: Conf. Ser. 804 012006
[16] Blasone M, Jizba P and Smaldone L 2017 eprint arXiv:1709.00461 [hep-th]
[17] Schwinger J 1961 J. Math. Phys. 2 407
[18] Blasone M, Jizba P and Smaldone L 2017 J. Phys: Conf. Ser. 880 012051
[19] Blasone M and Vitiello 1995 Ann. Phys. 244, 283 ; Blasone M, Capolupo A, Romei O and Vitiello G 2001 Phys. Rev. D 63 125015
[20] Perelomov A 1986 Generalized Coherent states and Their Applications (Berlin: Springer-Verlag)
[21] Barut A O and Girardello L 1971 Commun. Math. Phys. 21 41
[22] Blasone M and Jizba P 2012 J. Phys. A Math. Theor. 45 244009
[23] Blasone M, Jizba P and Vitiello G 2011 Quantum Field Theory and its Macroscopic Manifestations (London: World Scientific & ICP)