Conjecturing via analogical reasoning constructs ordinary students into like gifted student

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Abstract. The purpose of this study is to reveal the development of knowledge of ordinary students to be like gifted students in the classroom based on Piaget’s theory. In exposing it, students are given an open problem of classical analogy. Researchers explore students who conjecture via analogical reasoning in problem-solving. Of the 32 students, through the method of think out loud and the interview was completed: 25 students conjecture via analogical reasoning. Of the 25 students, all of them have almost the same character in problem solving/knowledge construction. For that, a student is taken to analyze the thinking process while solving the problem/construction of knowledge based on Piaget’s theory. Based on Piaget’s theory in the development of the same knowledge, gifted students and ordinary students have similar structures in final equilibrium. They begin processing: assimilation and accommodation of problem, strategies, and relationships.

1. Introduction
Today education emphasizes the need for students to develop the ability to think and solve problems. There is much hope to promote this thinking by using constructivist lessons or problem-based learning in the classroom [1]. Also in the learning process; learners are aware of the cognition process and can control and manage their cognition [2]. Despite the fact that the process of teaching and learning in Indonesia, there are many math teachers who teach the procedure without explaining why a particular procedure is used. As a result, students believe that in solving the problem, it is enough to choose the solution procedure according to the given problem. In this case, the focus of learning is not why certain procedures are used to solve problems. However, students choose the procedure to solve the problem, in addition, students argue: how to solve the problem using the procedure. The process of emphasizing "learning" leads to student behavior only "copying" the procedures undertaken by the teacher, without understanding why they should use this procedure. Therefore, if the problem is slightly changed or slightly modified, students cannot solve the problem. In learning, emphasis on the use of procedures without giving the right reasons is wrong. Teacher actions like this are the beginning of the formation of the pseudo-thinking process in students. Teachers sometimes do not realize that students already have the basic knowledge that can be used to build new knowledge. In addition, teachers sometimes forget that one student with another has a different conceptual understanding process [3]. Such learning will produce students in the lowest level of thinking below is seen from the hierarchy of thinking [4].

To avoid the occurrence of pseudo-thinking, then Students need to be given the opportunity to conjecture in learning when delivering new material through problem-solving. By reason of
conjecturing in solving problems using current knowledge to build new knowledge for him [1]. Conjecturing is one of the activities of students in active learning. However, it is possible to solve mathematical problems in which students find hunches and guesses. As stated [2] to solve problems, students do one of three possibilities, namely through hunches, guessing, and conjecturing.

Furthermore, Polya says that conjecturing is a part of mathematics and suspect's one of the branches of mathematics. Similarly [3] explains that when learning mathematics in a classroom, students are encouraged to create and test conjectures. In addition, students are helped to develop their reasoning skills. In the evaluation standard [5], the core of students doing mathematics is conjecturing by demonstrating the logical validity of conjecturing. J. Mason, at all [4] says that basically, the students' competence in mathematics includes two (2) things i.e. conjecturing and convincing.

In the implementation of the student, estimation cannot be separated using existing knowledge, both to solve problems and for construction of new knowledge. This suggests that students conjecturing via analogical reasoning as revealed [6]. Conjecturing via analogical reasoning is a learning process that emphasizes a scientific approach to active student learning in mathematics. Analogical reasoning according to [7] refers to the ability to understand and build similar structural similarities to objects whose surface features are always the same. This is called conjecture through analogy as revealed [8].

Analogical reasoning for the construction of knowledge [6] analogical reasoning in classroom math learning [9] analogical reasoning refers to the ability to understand and build structural similarities according to objects whose surface features are not always the same [9], conjecturing research through analogical reasoning in mathematical discovery [10], occurs in the math class although this is more the exception than the rule [6], building creative thinking [8]. Thus, mathematical education will be a very useful study of the use of conjecturing through analogical reasoning as a learning device.

Conjecturing via analogical reasoning can be used in learning for a scientific approach [6]. In addition, conjecturing via analogical reasoning is necessary to enhance the learning of active students in mathematics. Thus, avoid the pseudo thinking in constructing new knowledge [3]. Many junior high schools in Indonesia are learning gifted students and regular students are in one class, so classroom learning makes it difficult for teachers to apply learning models and approaches. Seeing these conditions requires wise treatment from the teacher so that all students are served to construct new knowledge/problem-solving.

The problem is "can ordinary students be constructed like gifted students?"

In this section, we combine several main theories that have examined conjecturing through analogical reasoning, as well as those who have investigated the process of abstraction in mathematical thinking and learning [2, 11-16] used more sociocultural and contextual systems approach to abstraction. We know that reification is the dominant mode to explain the process of abstraction, in which the process becomes an object of manipulation in itself as terminology [12]. In a sociocultural and contextual approach [17-18] reveal a shift in the construction of concepts occurring in diverse environmental discourses, through figures and reflections in the classroom facilitates reflecting abstractions. Radford [19] reveals despite the inherent tension in both abstractions, namely logical-deductive, and socio-discursive, but both can be seen as complementary components of the theory. That is, in turn can provide different meanings, connections, or explanations of the phenomenon under study. Our conceptual framework consists of Piaget and contextual (socio-cultural) views, analogical thinking, and abstraction. Specifically using conjecturing via analogical reasoning as instructional tools for use in constructing new knowledge can be developed so on.

2. Literature review
Some of the theories underlying this research are as follows:

2.1. Conjecturing via analogical reasoning
According to [20] conjecturing can be done in five ways, including conjecturing by analogy. In addition, according to English [21], the analogy consists of a pedagogical analogy, a problem of classical analogy and analogy (using A: B: C: D). Krulik in [22] argue that hierarchical thinking
consists of: (1) recall, (2) basic thinking, (3) critical thinking, and (4) creative thinking. Furthermore, Krulik et al. say that reasoning is part of the thinking process. This thinking included in reasoning is basic thinking, critical thinking, and creative thinking. Thus conjecturing through analogical reasoning is a conjecture to solve the "analogy target" based on "basic analogy" (knowledge already possessed/mastered). The invention [14] solves a number of interior corners of the tetrahedron and pentagon (the target analogies to the C: D relationship) before the student has mastered the 180° triangle's interior angle (the analogy/knowledge base already possessed by A: B relationship). Furthermore, the discovery of Supratman [8] for the construction of a cone equation (as the analogy target) is based on the construction of the angle bisector equation (as the basis of the analogy).

2.2. Conjecturing via analogical reasoning of students
Findings of Supratman [8] the students in solving the problem are divided into 6 groups, ie (1) the students do not do conjecture, (2) the students do conjectures, but the wrong conjecturing, and not via analogical reasoning, (3) the students do conjecture and the results of conjecturing they are true but not via analogical reasoning, because some students have learned from books and some are from the internet, (4) students make conjectures via analogical reasoning but their conjectures are wrong, (5) students make conjectures via analogical reasoning and their conjecture results true, but did not develop their conjecture. so that, they produce only one conjecture, and (6) the student does conjecture via analogy reasoning and his conjecture results correctly and the students are able to develop their conjecturing. So, they produce some conjecturing.

2.3. Construction of new knowledge
When a person learns, They will be even disequilibrium. That leads to assimilation and accommodation processes. Suppose the structure of the problem is not shared in depend of the student's cognitive structure. With this process, the cognitive structure evolves through a process of altering, combining, or forming a new scheme until this final equilibrium condition is said to be the construction of new knowledge or adaptation to the environment [23] Adaptation process begins to absorb the structure of the problem (intelligent behavior) which is the input to the cognitive structure in the initial level equilibrium condition, which goes beyond the initial cognitive structure. It results in disequilibrium between the problem structure and the cognitive structure. Therefore, there is an adaptation of the cognitive structure through assimilation and accommodation. After assimilation and accommodation, there will be a cognitive structure in the new level equilibrium, as it is constructed by new knowledge. Illustration of Piaget's theory of modification of [24] as shown in Figure1.

2.4. Ordinary student
In Indonesia, many schools that do not appreciate gifted students are supported by the low pedagogical mastery of teachers [1] so that the learning of one student with the other is the same. This resulted in learning; students only touched the recall and basic thinking only. In order not to be the wrong perception then we distinguish between ordinary students with mental students (mentally retarded / mentally retarded). As for the meaning of ordinary children does not mean belonging to children with IQ under 84 as expressed in American Association on Mental Deficiency (AAMD). The regular students in this study were officially registered students and participated in regular classes affiliated with the Ministry of Education at its level [25] Students in the same grade but for one reason or another, students are not able to solve problems or are unable to construct new knowledge directly for complex problems.

2.5. Gifted students
Although gifted students can cause problems for teachers because they are not a homogeneous group. Gifted students according to [26] students who have unlimited problem-solving abilities. an image of a gifted student according to the National Association for Gifted Children (NAGC) as "someone who shows, or has the potential to perform, a remarkable level of performance in one or more areas of
expression” [27]. Students who are gifted mathematically include those who have a high ability or have a high interest. Some students in this group may be gifted with intuitive knowledge of mathematical concepts, while others have a passion for the subject although they may have to work hard to learn it. Many gifted students make themselves obvious to parents, guardians, and teachers by understanding and articulating math concepts at an earlier age than expected [3]. They are often found to easily make connections between study topics and often cannot explain how they get answers quickly [28] can be seen in Figure 1.

![Diagram](image)

**Figure 1.** Occurrence of Assimilation process and accommodation (modification of Subanji and Supratman [24])

### 3. Method

#### 3.1. Research subject

The authors conducted a study of 52 students of Junior High School from the district of Ciamis, which is considered to have an ordinary ability. Comparative subject (S1) was taken from a class of accelerated students from SMP in Tasikmalaya, to solve the problem solved by the research subject (S2).

#### 3.2. Research methodology

This research is a qualitative-explorative research in taking verbal data, because the data to be collected and used is verbal data resulting from the exploration of students who are able to solve problems such as gifted students. The study was conducted at SMP in district of Ciamis. To see the construction of new knowledge is seen based on Piaget Theory. Piaget theory consists of assimilation and accommodation end then assisted premises by Think out loads. The research instrument is a researcher as an instrument, supported by a sheet of instrument task and completed by interview in accordance with [29] opinion. The reason of the researcher is as an instrument, because the researcher is the research manager as well as one of the instruments in data collection that cannot be replaced by other instruments. The initial task sheet instrument is as follows.

- The first problem if you have two points let A and B, A not equal B. What would you do with those two points?
- Next, there is another point suppose C, and $A \neq B \neq C$. What would you do with that point C? And what do you find?
- Next, there is another point let D, and $A \neq B \neq C \neq D$. What would you do with that point D? And what do you find?
- Next, there is another point suppose E, and $A \neq B \neq C \neq D \neq E$. What would you do with that E point? And what do you find?
- Next, there is another point suppose F, and $A \neq B \neq C \neq D \neq E \neq F$. What would you do with that F point? And what do you find?
- Next, there is another points suppose G, H, and $A \neq B \neq C \neq D \neq E \neq F \neq G \neq H$. What would you do with those points of G and H? and what do you find?

The problem is to construct the concept of the number of interior of corners $n = (n - 2) \times 180^0$

3.3. Data collection procedures

The study examines conjecturing via analogical reasoning to build students of ordinary ability to become like gifted students in constructing new knowledge based on Piaget's adaptation framework. Data collection, researchers provide problems related to the analogy to a student to complete. While doing the conjecturing process through analogy reasoning, the students express out loud what he is thinking. Researchers record verbal expressions and record the expressions (behaviors) and unique things that students do when conjecturing through analogical reasoning [30]. If you have finished one student, then do the same thing to the other students, to get the student in accordance with the desired. Such data retrieval, according to [31] is said in the Think Out Loud method, though [27] says think aloud (think aloud). In this method is done to ask the research subject to solve the problem as well as tell the thinking process.

4. Result and Discussion

4.1. Result

Of the 32 students who participated, found the following: 2 students did not conjecture only able to make a straight line and did not want to continue. 5 students do not conjecture via analogical reasoning, because they have read books related to the problem. The remaining 25 students were able to find the concept of the number of interior angles for the corner $n = (n - 2)180^0$. From the 25 students initially: 3 students are only able to make various triangles and the number of interior corners of the triangle. They are named students type A (StA); 7 students are only able to create rectangles and the number of rectangular interior corners. They are named students type B (StB); 8 students are only able to create pentagonal and the number of pentagonal interior angles. They are named students type C (StC); 5 students are only able to make hexagons and the number of interior corners. they are named students type D (StD); The remaining 2 students are able to make the octagonal and the number of interior corners, even able to construct the concept of the number of n-corner interior angles is $n = (n - 2)180^0$. They are named students type E (StE). StA, StB, StC and StD were able to develop their conjecture like StE. Before that, the researcher reminds StA, StB, StC and StD, that you can develop such conjecture as you did before. After that, they can to the construction of concept the number of an interior of corner $n = (n - 2)180^0$. Because they all share the same characteristics students are taken to see the new knowledge construction process, who more consistently explains a question and is named S2. While for the results of interviews with students as follows:

Researcher (R): Why do not you continue your work?
StA: Ehmm (while he toyed with his pencil). oh yes I can sir, that is making a rectangle. Then They did the next conjecture to find the number of interior of n-angles is $2 \times 180^0 = 360^0$

then researchers combine like StB

R : do you give up making a rectangle? I added a point. What will you make?
**StB:** he paused, and then he replied I can make a pentagon and the number of angles in it. eeh eeh (while he made a line connecting point A to point D and point B to point D) oooh there are 3 triangles. Means the number of angles is $3 \times 180^0 = 540^0$.

In fact, they make a line segment by connecting two points so they get 3 triangles. As for the form as can be seen in Figure 2.

![Figure 2](Image)

**Figure 2.** The students' work in drawing the pentagon and making it into several triangles

Then researchers combine like StC

R : Can you make further conjectures if I give one more point?
StC : I will try it sir. (They make a hexagon and divide into four triangles) as soon as he answers the result is $4 \times 180^0 = 720^0$

In fact, they make a line segment by connecting two points so they get 4 triangles. As for the form as can be seen in Figure 3.

![Figure 3](Image)

**Figure 3.** The students' work in drawing the pentagon and making it into several triangles

Then researchers combine like StD

R : Can you do this (while the researcher shows the task sheets instrument)?
StD : I will make an octagon so that the number of interior corners is $6 \times 180^0 = 1,080^0$

so the StD as S1 to find the number of interior angles for the n-segment is $(n - 2)180^0$. As for researchers interviewed S1 as follows.

R : Can you do this (while the researcher shows the task sheets instrument)?
S1 : oh sir I do not think I can (while he was scratching his head)
R : Have you got this problem before?
S1 : Not yet, sir
R : what do you know with two points A and B?
S1 : these are the two points representing the line or segment AB
R : after added point C why do you make a triangle?
S1 : because if I put on the elongation of line AB, point C is only a point located on line AB which can be before A, can be between A and B, and can after B. For that, I make triangle, and triangle the number of angles is $180^0$ (while he shows the number of angles from the three angles to the angle of the straightener by tearing the triangle into an alignment angle.
R : After added point D. Why do you make a rectangle?
S1 : Actually I want to create a line for line, line and line. But I did not bring a run so I made things easier. Actually I want to create a line for line, line and line. But I did not bring a run so I made things easier. The number of inner corners I can guess is $360^0$ because the rectangle is twice the triangle.
R : After added point E. Why do you make a quadrilateral?
S1 : Actually I want to make a line for a triangle and the other. But I did not bring a run for drawing, finally I made a rectangle. The number of angles in a quadrilateral I can guess is 360° because the rectangle is twice the triangle.

R : After added point F. Why do you make a hexagonal?
S1 : I draw a hexagon has an interior corner is $4 \times 180^\circ = 620^\circ$.

R : After added point G and H. Why do you make a octagonal?
S1 : I draw a octagonal has an interior corner is $(8 - 2) \times 180^\circ = 1080^\circ$.

R : What if added point again so as to form "n-rectangle". What is the n-corner interior angle?
S1 : the number of n-corner interior angles is $(n - 2) \times 180^\circ = 180^\circ n - 360^\circ$

When researchers interviewed the S2 as follows.

R : Can you do this? and what is your conclusion?
S2 : yes I try first sir (as well as when the researcher offered to S2.

R : Can you do this? and what is your conclusion?
S2 : yes I try first Pak (he did it straight away and within 23 minutes have found the number of interior corners in terms of \(-n = (n - 2) \times 180^\circ\))

The conjecture of S1 is the same as the conjecture rather than S2, but for S1 must be step by step different with S2. Because S2 directed conjecture for solve problem and his conjecture is thru. Process of thinking of S1 and S2 can be look at in Figure 4.

(a) The structure of the problem

(b) Structure of thinking

(c) Accomodation

(d) Complete Structure of S1 and S2

Figure 4. Process of thinking of S1 and S2
4.2. Discussion
S1 guessed through analogical reasoning to solve problems with the division of the problem structure. This shown when the problem presented simultaneously to the S1, S1 looks confusion. But after the question is given little by little as the stage of building the concept of looking for the number of interior corners of an aspect- \(n = (n-2) \times 180^\circ\). But S1 does conjecture via analogical reasoning. The reason the problem has never been S1 found, but S1 uses the knowledge that he previously knew.

Why say S1 and S2 do conjecture? Because S1 and S2 do not know the problem before, so really natural conjecturing occur. Why are S1 and S2 supposed to be conjecturing via analogical reasoning? because S1 and S2 do allegations through the use of concepts and events that have been previously owned. For example: S1 and S2 to estimate the number of interior angles by tearing the triangle into 3 parts. Then he suspects the number of interior angles of the triangle is to form a straight angle. S1 and S2 never saw that straight angle was 180\(^\circ\). Further, the quadrilateral consists of two triangles; the pentagon consists of three segments and so on so that S1 and S2 have the same conjecture for the number of n-corner interior angles \((n-2) \times 180^\circ\).

As Supratman pointed out in [31] that the events experienced by S1, who experienced category stages of StA, StB, StC, and StD, beginning with accommodation of problem, accommodation of strategic and accommodation of relationships. And then followed by assimilation of problems, assimilation of strategy and assimilation of relations. As for the S2, begins with assimilation of the problem, assimilation of strategy and assimilation of the relationship. Then it followed by accommodation of the problem back to the assimilation of the strategy and assimilation of the relationship [32-33]. But in the construction of new knowledge has a common structure of thinking between ordinary students with gifted students.

5. Conclusion
Conjecturing via analogical reasoning constructs ordinary students into like gifted student, but this study required further verification of other materials.

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