Primordial magnetic fields with X-ray and S-Z cluster survey

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ABSTRACT

The effect of primordial magnetic fields on X-ray or S-Z galaxy cluster survey is investigated. After recombination, the primordial magnetic fields generate additional density fluctuations. Such density fluctuations enhance the number of galaxy clusters. Taking into account the density fluctuations generated by primordial magnetic fields, we calculate the number of galaxy clusters based on the Press-Schechter formalism. Comparing with the results of Chandra X-ray galaxy cluster survey, we found that the existence of primordial magnetic fields with amplitude larger than 10 n Gauss would be inconsistent. Moreover, we show that S-Z cluster surveys also have a sensitivity to constrain primordial magnetic fields. Especially SPT S-Z cluster survey has a potential to constrain the primordial magnetic fields with several nano Gauss.

1 INTRODUCTION

The origin of large-scale magnetic fields observed in galaxies and galaxy clusters still remains unclear. The most widely accepted theory is the astrophysical scenario in which seed magnetic fields are generated by the battery mechanism in astrophysical phenomena and amplified by the dynamo mechanism in the interstellar or intergalactic medium (Brandenburg & Subramanian 2005). However, there is uncertainty about the efficiency of the dynamo mechanism (Kulsrud et al. 1997; Giovannini 2004). The recent studies on Faraday rotation measurements of high redshift quasars suggest the existence of the \( \mu \) Gauss magnetic fields in high-redshift galaxies (Kronberg et al. 2008; Bern et al. 2008). The existence of such magnetic fields may also challenge the dynamo scenario.

Another candidate for the origin of the galactic magnetic fields is primordial magnetic fields which is generated in the early universe (Widrow 2002; Takahashi et al. 2005; Ichiki et al. 2006; Giovannini 2008). If primordial magnetic fields existed, Big Bang Nucleosynthesis (BBN) and Cosmic Microwave Background (CMB) anisotropy suffer the effect of primordial magnetic fields. Therefore, the constraint on the primordial magnetic fields through BBN and CMB anisotropy has been studied by many authors.

After the recombination epoch, primordial magnetic fields generate additional density fluctuations by the Lorentz force (Wasserman 1978; Kim et al. 1996) and many authors have studied their effects on the evolution of the large scale structures; the redshift-space matter power spectrum, the epoch of reionization, and 21 cm fluctuations (Gopal & Sethi 2003; Sethi & Subramanian 2004; Tashiro & Sugiyama 2006a,b; Schleicher et al. 2009; Sethi & Subramanian 2004; Shaw & Lewis 2010). It was found that magnetic fields as small as a few nano Gauss can give strong cosmological impacts. Therefore detailed observations planned in near future have the potential to set further constraints on primordial magnetic fields.

In this paper, we study the effect of primordial magnetic fields on the mass function of galaxy clusters. The additional density fluctuations generated by the primordial magnetic fields enhance the formation of galaxy clusters. Thus, number count of clusters can constrain their amplitude as well as the standard cosmological parameters such as the amplitude of density fluctuations (\( \sigma_8 \)) and energy density of matter (\( \Omega_m \)). Specifically we investigate the potential of the cluster number count by X-ray observations and Sunyaev-Zel'divich (S-Z) survey.

We can find massive clusters by the observation of the X-ray emitted from the hot intracluster gas. X-ray-flux-selected cluster samples with the calibration between the X-ray temperature and the cluster mass give the mass function of galaxy
clusters. Vikhlinin et al. (2009) and Mantz et al. (2010) applied it to give a constraint on the cosmological parameters such as the equation of state of dark energy.

The S-Z effect is the scattering of CMB photons by the hot intracluster electron gas (Sunyaev & Zeldovich 1972) and is also a powerful tool for detecting galaxy clusters at high redshifts. Combining the S-Z galaxy survey with X-ray or optical observations, we can obtain the mass- or redshift-abundance of the number of galaxy clusters. There are many observation projects carried out or planned. In particular, Planck and the South Pole Telescope (SPT) is expected to give large catalogs of S-Z galaxy clusters and the significant constraints on cosmological parameters.

The rest of the paper is organized as follows. In Sec. II, we give a description of the density fluctuation generation by primordial magnetic fields after the recombination epoch. In Sec. III, we study the effect of primordial magnetic fields on the mass function derived from Chandra observations and obtain a constraint on the strength of primordial magnetic fields. In Sec. IV, we calculate the number count for S-Z galaxy cluster surveys and discuss the potential of Planck and SPT to give constraints on the primordial magnetic fields. We conclude in Sec. V. Through this paper, we assume a ΛCDM cosmological model with $h = 0.72$, $\Omega_m = 0.3$, $\Omega_b = 0.05$ and $\sigma_8 = 0.75$.

## 2 DENSITY FLUCTUATIONS DUE TO PRIMORDIAL MAGNETIC FIELDS

In this section, we calculate the density fluctuations produced by primordial magnetic fields. First, we make some assumptions about primordial magnetic fields. Because our interest is in relatively large length scales, we can assume that the backreaction of the mass function derived from primordial magnetic fields after the recombination epoch can be related to the magnetic field strength $B$. Subramanian & Barrow (1998). As a result, the magnetic field power spectrum has a sharp cutoff around the damping scale $k_d$. The damping scale $k_d$ can be related to the interaction between ionized and neutral baryon is strong in redshifts considered here. Using the MHD approximation to the baryon fluid, we can write the evolution equations of density fluctuations with primordial magnetic fields as,

$$\frac{\partial^2 \delta}{\partial t^2} = -2 \frac{\dot{a}}{a} \frac{\partial \delta}{\partial t} + 4\pi G (\rho_0 \delta_B + \rho_d \delta_d) + S(t, x),$$

where $\delta_B$ and $\delta_d$ denote the density perturbations of the baryon and dark matter, respectively, $\rho_0$ and $\rho_d$ are the baryon and dark matter densities, $S(t, x)$ is the source term, and $a$ is the scale factor.

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\[ S(t, x) = \nabla \cdot \left( \left( \nabla \times B_0(x) \right) \times B_0(x) \right) / 4\pi \rho_0 a^3(t), \]

(7)

where \( \rho_b \) and \( \rho_d \) are the baryon density and the dark matter density, and \( \delta_b \) and \( \delta_d \) are the density contrast of baryon and dark matter, respectively. Solving Eqs. (9) and (3), we can obtain the power spectrum of the density fluctuations. With the assumption that there is no correlation between primordial magnetic fields and primordial density fluctuations, the density matter power spectrum can be separated into two parts as

\[ P(k, t) = P_V(k, t) + P_M(k, t), \]

(9)

where the first term \( P_V(k) \) is originated from the primordial density fluctuations, whose growth rate is proportional to \( t^{2/3} \), while the latter dominates for \( n < -1.5 \), the power-law index of density fluctuations depends on that of magnetic fields. Accordingly, if magnetic fields have a power-law index smaller than \( -1.5 \), the power-law index of density fluctuations depends on that of magnetic fields. However, if the power-law index of the magnetic fields is larger than \( -1.5 \), the power-law index of the density fluctuations is about 4 and the amplitude is decided by their damping scale.

We introduce an important scale for the evolution of density perturbations, i.e., magnetic Jeans length. Below this scale, the magnetic pressure gradients, which we do not take into account in Eq. (6), counteract the gravitational force and prevent further evolution of density fluctuations. The magnetic Jeans scale is evaluated as

\[ k_{\text{MJ}} = \left( 13.8 \left( \frac{B_3}{\ln G} \right)^{-1} \left( \frac{h^2 \Omega_m}{0.18} \right)^{1/2} \right)^{2/5} \text{Mpc}^{-1}. \]

(13)

For simplicity, we assume that the density fluctuations do not grow below the scale, although the density fluctuations below the scale are, in fact, oscillating like the baryon oscillation.

3 MASS FUNCTION AND X-RAY OBSERVATION

The additional density fluctuations produced by primordial magnetic fields enhance the number of dark matter halos. In order to estimate this enhancement, we use the mass function which is calibrated to fit the numerical simulation by Tinker et al. (2008).

\[ \frac{dn}{dM} = \frac{A \Omega_m \rho_0}{M} \frac{d \ln \sigma}{dM} \left( \frac{\sigma}{b} \right)^{-a} e^{-c/a^2}, \]

(14)

where

\[ A(z) = A_0 (1 + z)^{-0.14}, \quad a(z) = a_0 (1 + z)^{-0.06}, \quad b(z) = b_0 (1 + z)^{-a}, \quad \log \alpha(\Delta) = -\left( \frac{0.75}{\log(\Delta/75)} \right)^{1.2}, \]

(15)

\[ M_{\Delta} = \Omega_m 4\pi 3 R_\Delta^3 \rho_c \Delta. \]

(16)

For example, in the case of the halo virial mass, \( \Delta = 178 \) in the matter dominated epoch.
Figure 1. The integrated number counts of X-ray galaxy clusters. The left panel shows the number count for the low redshift bin $z = 0.025 - 0.25$, while the right panels shows that for the high redshift bins $z = 0.35 - 0.90$. In both panels, the solid, the dashed and the dotted lines represent the number counts for the case of $B_\lambda = 10$ nG, $B_\lambda = 8$ nG and $B_\lambda = 0$ nG, respectively. We also plot the results of Chandra in [Vikhlinin et al. (2009)]. For comparison, we put the number counts for the $B_\lambda = 0$ nG case with $\sigma_8 = 0.85$ as the thin solid line.

Our interest is to examine the potential of the X-ray galaxy cluster observation to give constraints on primordial magnetic fields through the halo mass function. In the X-ray observation, the halo mass for high $\Delta$ is more robust than for low $\Delta$. Therefore, according to Vikhlinin et al. (2009), we set $\Delta = 500$. The observational luminosity threshold gives the mass threshold for observed halos. Therefore, the number count of halos over the luminosity threshold corresponds to the integrated mass function,

$$N(> M_L) = \int_{M_L}^\infty dM \frac{dn}{dM}, \quad (17)$$

where $M_L$ is the mass threshold corresponding to the luminosity threshold.

In Fig. 1 we plot the integrated number count as a function of the mass threshold for each primordial magnetic field strength with the magnetic field spectral index $n = -2.8$. The left panel in Fig. 1 is for the low redshift bin, $z = 0.025 - 0.25$, while the right panel is for the high redshift bin, $z = 0.35 - 0.90$. Although the additional density power spectrum induced by primordial magnetic fields dominate the primordial power spectrum on smaller scales than 1 Mpc, the additional power spectrum can enhances $\sigma_8$ to 0.8 for $B_\lambda = 10$ nG. For reference, we put the integrated number count for the $\Lambda$CDM model with $B_\lambda = 0$ nG and $\sigma_8 = 0.85$. The existence of the primordial magnetic fields lift up the mass function on small scales, because the additional density fluctuations produced by primordial magnetic fields have a blue spectrum. As a result, compared with the case with $B_\lambda = 0$ nG and $\sigma_8 = 0.85$, while the number count in the case with primordial magnetic fields is small on large scales, it is more enhanced on small scales.

Vikhlinin et al. (2009) obtained galaxy cluster mass functions at two redshift range using Chandra observation data and concluded that the obtained mass functions are in good agreement with the cosmological model with $\sigma_8 = 0.75$. We compare the theoretical mass functions with the data with the error bars due to the Poisson uncertainties in Fig. 1. From this figure, we can conclude that Chandra observations rule out the primordial magnetic field strength, $B_\lambda \gtrsim 8$ nG at roughly one-sigma.

4 S-Z NUMBER COUNTS

The S-Z effect is caused by the scattering of CMB photons with electrons in hot gas in galaxy clusters. The change of the CMB intensity with the frequency $\nu$ by the S-Z effect is expressed, in the R-J limit, as (Sunyaev & Zeldovich 1972; Birkinshaw 1999)

$$I_\nu(\theta) = 2\nu^2T_{\text{CMB}}g(x)y(\theta), \quad (18)$$

where $T_{\text{CMB}}$ is the CMB temperature and $g(x)$ is the S-Z effect spectral shape given by $g(x) = x^2e^x/(\tanh(x/2) - 4)/(e^x - 1)^2$ with $x = 2\pi\nu/T_{\text{CMB}}$. The Compton y-parameter is given by the integral of the electron gas pressure along the line of sight at $\theta$

$$y(\theta) = \int dl \frac{T_e}{\mu_em_ne\sigma_T}, \quad (19)$$
where $T_e$ is the electron gas temperature, $n_e$ is the electron density, $\sigma_T$ is the Thomson scattering cross-section, $m_e$ is the electron mass, $\mu_e$ is the mean mass per electron $\mu_e = 1.143$ and $f_b$ is a gas fraction in a galaxy cluster which we set $f_b = 0.12$ (Mohr et al. [1999]). In the S-Z cluster survey, it is assumed that the S-Z cluster is a point like source within the telescope beam. Therefore, we consider the total flux density from a cluster at redshift $z$ by integrating the cluster surface,

$$S_v = 2\nu^2 T_{\text{CMB}} g(x) \frac{Y}{D_v^2},$$

(20)

where $D_v$ is the angular diameter distance to the cluster at $z$ and $Y$ is the integrated $y$-parameter over the cluster surface,

$$Y = \int d\Omega \ y(\theta).$$

(21)

In order to calculate Eq. (21), we need the electron density profile in a galaxy cluster. We take the assumption that the electron density profile is isothermal (Cavaliere & Fusco-Femiano [1976]),

$$n_e(r) = \begin{cases} n_0 \left( 1 + \frac{r^2}{\sigma^2} \right)^{-\frac{3\alpha}{2}} & r < R_c \\ 0 & r \geq R_c \end{cases}$$

(22)

where $R_c$ is the core radius of galaxy cluster which is related to the virial radius $R_v$ with the parameter $s(z)$ as $R_c = R_v/s(z)$.

Following Komatsu & Seljak (2002), we set

$$s(z) \approx \frac{10}{1 + z} \left[ \frac{M}{M_\odot}\right]^{-0.2},$$

(23)

where $M_\odot$ is the solar mass and $M$ is the virial mass. Taking the assumption that the galaxy clusters are spherical and in the hydrodynamical equilibrium, we can relate the virial mass $M_v$ with the virial radius,

$$M_v = \frac{4\pi}{3} \rho_m(z) \Delta_v(z) R_v^3,$$

(24)

where $\Delta_v(z)$ is the overdensity contrast for virialization (Nakamura & Suto [1997]).

We introduce the electron density weighted average temperature $(T_e)_n$, which is defined by $(T_e)_n = \int d\Omega n_e T_e / \int d\Omega n_e$. Battye & Weller (2003) has obtained $(T_e)_n$ under the isothermal assumption in the $\Lambda$CDM model,

$$(T_e)_n = T_\star \text{keV} \left[ \frac{M_{\text{vir}}}{10^{15}h^{-2}M_\odot} \right]^{2/3} \left[ \frac{\Delta_v(z) H(z)^2}{H_0^2} \right]^{1/3} \left[ 1 - 2 \left( 1 - \Omega_m(z) \right) \right]^1,$$

(25)

where $T_\star$ is the temperature normalization factor and they adopt $T_\star = 1.6$ for agreement with numerical simulation works in Bryan & Norman (1998) and Pierpaoli et al. (2001).

Using the $\beta$-profile assumption with the electron density weighted average temperature given by in Eq. (25), we can write the $y$-parameter as

$$y(\theta) = \frac{(T_e)_n f_{\text{gas}} M_{\text{vir}}}{m_e \mu_e} \sigma_T \zeta(\theta),$$

(26)

where $\zeta(\theta)$ is the projected profile of the electron density,

$$\zeta(\theta) = \left( 1 + \theta^2 \right)^{-\frac{1}{2}} \tan^{-1} \left( \frac{\theta^2}{1 + \alpha^2} \right)^{1/2} \tan^{-1} \frac{\alpha}{\theta}.$$  

(27)

In actual observations, the finite beam size of telescopes causes the beam-smearing effect. This effect can be accounted by modifying Eq. (21) to (Bartlett 2000)

$$Y = \int d\Omega' y(\theta) B(\theta).$$

(28)

where $B(\theta)$ is the beam profile described in a Gaussian form $B(\theta) = \exp[-\theta^2/(2\sigma_b^2)]$ with $\sigma_b = \theta_{\text{FWHM}}/\sqrt{8\ln2}$ where $\theta_{\text{FWHM}}$ is the full-width-half-maximum (FWHM).

The parameter $Y$ depends on the mass and the redshift of galaxy clusters. Therefore, giving the flux limit $S_v\text{lim}$ of the observation, we can obtain the limit mass of galaxy clusters $M_L$ at each redshift. We show $M_L$ for Planck and SPT in Fig. 3. We present the parameter value for each observation in Table 1. Planck will cover the full sky and the Planck sensitivity is 14 mJy at 100 GHz. SPT covers $\Delta\Omega = 4000$ degree square and the SPT sensitivity is 0.8 mJy at 150 GHz. SPT has a better sensitivity than Planck. As a result, $M_L$ for SPT is lower than for Planck.

The combination between the S-Z galaxy cluster survey and the follow-up optical observation enables us to obtain the cluster number count for redshift bins. In Fig. 3 we show the number count of galaxy clusters with mass higher than the limiting mass shown in Fig. 2

$$\Delta N(z) = \frac{dn}{dz} \Delta z = \Delta\Omega \int_{M_L} dM \frac{dV}{dzd\Omega} \frac{dn}{dM} \Delta z.$$  

(29)
In both panels of Fig. 3 we set $\Delta z = 0.1$ and the primordial magnetic field spectral index $n = -2.8$.

Fig. 3 shows that the predicted number counts of Planck and SPT are almost same in low redshifts. This is because, although Planck has less sensitive to small galaxy clusters than SPT, Planck’s full sky survey area increases the number of the observed galaxy clusters, and vice versa. However, in high redshift, since the number of large-mass clusters rapidly decreases, the number count for Planck become much lower than for SPT.

Primordial magnetic fields generate additional density fluctuations in small scales and bring the early structure formation. Therefore, the difference from ΛCDM cosmology due to the existence of primordial magnetic fields is emphasized in the SPT observation, especially in high redshifts. Even the primordial magnetic fields with $B_\lambda = 6$ nG amplifies the number count in high redshifts by 50 % for the SPT sensitivity, while, for Planck, such primordial magnetic fields cannot bring a significant amplification.

For reference, we put the number counts for the $B_\lambda = 0$ nG case with $\sigma_8 = 0.85$ as the thin solid line in both panels in Fig. 3. Although $\sigma_8$ in the case of $B_\lambda = 10$ nG corresponds to 0.8, the power spectrum in the case of $B_\lambda = 10$ nG has larger amplitudes in high $k$s than in the case of $\sigma_8 = 0.85$ without primordial magnetic fields. This results in the fact the number of galaxy clusters with small mass is larger in the case of $B_\lambda = 10$ nG than in the case of $\sigma_8 = 0.85$. Therefore, the number counts for $B_\lambda = 10$ nG exceed the ones for $\sigma_8 = 0.85$ in the low redshifts where both Planck and SPT are sensitive to low mass clusters as shown Fig. 3. In particular, the number counts of SPT for $B_\lambda = 10$ nG is almost same as for $\sigma_8 = 0.85$ even in high redshifts, because SPT has small limiting mass.

5 CONCLUSION

In this paper, we have studied the effect of primordial magnetic fields on the galaxy survey by X-ray and S-Z observation. The primordial magnetic fields generate additional density fluctuations which has a blue power spectrum. Therefore, the number of galaxy clusters, especially small ones, is enhanced. X-ray and S-Z survey can directly observe this enhancement. Nano-Gauss primordial magnetic fields bring observable enhancement of the number count by the order of factors.

For X-ray cluster surveys, we have used Chandra’s result to put a constraint on the amplitude of primordial magnetic fields. We have found that Chandra’s result rules out the existence of primordial magnetic fields with $B_\lambda \gtrsim 10$ nG at roughly one-sigma level.

| $S_{\text{lim}}$ | $\nu$ [GHz] | $\theta_{\text{fwhm}}$ | $\Delta \Omega$ [deg$^2$] |
|-----------------|-------------|-----------------|-----------------|
| Planck          | 14 mJy      | 100             | 9.'             |
| SPT             | 0.8 mJy     | 150             | 1.'             |

Table 1. The experimental parameters for S-Z surveys.
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Figure 3. The integrated number counts of S-Z galaxy clusters. The left panel is for Planck, and the right panels is for SPT. In both panels, the solid, the dashed, the dashed-dotted and the dotted lines represent the number counts for the case of $B_\lambda = 10$ nG, $B_\lambda = 8$ nG, $B_\lambda = 6$ nG and $B_\lambda = 0$ nG, respectively. For comparison, we put the number counts for the $B_\lambda = 0$ nG case with $\sigma_8 = 0.85$ as the thin solid line.

S-Z cluster surveys also have a sensitivity to constrain primordial magnetic fields. Especially the observation like SPT which has small limiting mass with 1 arcmin angular resolution is a good probe of primordial magnetic fields. We have found that the combination with high redshift optical surveys has the potential to put the constraint on the fields of nano Gauss order.

In this paper, we consider only primordial magnetic fields with $n = -2.8$. The power spectrum of the density fluctuations generated by primordial magnetic fields has a dependence on the spectral index of the primordial magnetic fields. The large spectral index induces the large amplification of the density fluctuations on small scales and increases the mass function for small-mass clusters. For example, $B_\lambda = 4$ nG and $n = -2.5$ amplifies the number count in high redshifts by 50 % for the SPT sensitivity, comparing with the number count without primordial magnetic fields. This amplification is same as in the case of $B_\lambda = 6$ nG with $n = -2.5$. Therefore, SPT has the potential to put the strong constraint on the primordial magnetic fields with large $n$.

In our calculation, we ignore the effect of primordial magnetic field on the structure of a halo. However, in order to obtain a highly accurate constraint on primordial magnetic fields, it is necessary to study the modification on the electron density profile and the relation between the X-ray temperature and the cluster mass by primordial magnetic fields. For example, Zhang (2004) and Gopal & Roychowdhury (2011) pointed out that magnetic fields with several $\mu$Gauss in a halo modify the electron density profile and this modification change the S-Z effect signal. The adiabatic contraction in the halo formation easily amplifies the order of nano Gauss of primordial magnetic field strength to the order of $\mu$ Gauss. Taking into account such effects, we will study the constraint on primordial magnetic fields through X-ray and S-Z surveys in the future.

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