Impact of Semi-Rigid Joints in Light-Wood Framed Structures on the Serviceability Limit State

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Abstract. Timber structure especially light-wood framed structures depending on semi-rigid behaviour of its work have got lower spatial stiffness than theoretical model with full-fixed joints. Semi-rigid behaviour of the joints is a function of material and structural stiffness. Joints deformation are the results of material properties of wood as well as internal force redistribution onto individual elements intersected in analyzed joint. The force distribution between individual components depends on the direction of the grain in analyzed parts of the structure. Joints deformation as the result of its semi-rigid characteristics cause the reduction of the stiffness so the values of the displacements rise. Increase of the displacement allows possibility to exceed the limits imposed by the requirements of Serviceability Limit State. The exceeding of the safe values causes the structure lost its function due to the safety and comfortable use. The paper will present main concepts connected with non-linear and semi-rigid behavior of the joints as well the models and static schemes of joints work in the light-wood framed structures. The experimental researches were conducted in Bialystok University of Technology. The experiments were carried out on the specimen in micro-scale 1:3. The translational stiffness $k$ and rotational stiffness $\kappa$ were analyzed. The researches were conducted for stud-to-bottom plate joint as well floor joist-to-top plate of wall joint in the floor to wall connection. There were observed dependencies between the loads applied to the joint and the translational and rotational deformation of the connection. Selected results of these experimental tests present the behavior of the analyzed joints and expose its semi-rigid manner of work. For single-span beam with semi-rigid joints the calculation formulas presenting the change of extreme values of the displacement in relationship to the translational and rotational stiffness of the support joints is presented. The paper will be summarized by conclusions coming from the carried out analyses.

1. Introduction

Wood becomes a more important structural material on building market. After the years of domination of steel or reinforcement concrete a lot of construction is erected as timber or glued timber structures [1]. From small residential houses through public buildings to sport halls or bridges wood is used as constructional material [2,3].

Wood is ecological and biodegradable material, harvested from forest resources of many countries in the world. The use of timber as structural material fits the ecological trends of many areas of human life not only in civil engineering [4]. Additionally, the work behaviour of wood, despite the thousands year of using as structural material, is not well recognized as steel or concrete structures.

More secure and at the same time more economical designing of the structures is possible if the manner of work of the structure is known due to the type of the structural material used. The specifics
of the work of natural and glued timber structures impose good recognition of the joint behaviour because of its deformation. Deformation of timber elements affects not only load capacity but also spatial stiffness of the structure, particularly visible in case of light wood-framed constructions [5].

2. Construction of the joints in timber structures

All the structures regardless of structural material have to spend the requirements of Ultimate Limit State and Serviceability Limit State [6]. The structural components of light wood-framed buildings are often designed as single beam with pinned connection. In this case decisive to the selection of cross-section is its stiffness not load capacity. Meeting the requirements of SLS is very important in the design process.

Any connection of light wood-framed structures should be treated as flexible (semi-rigid). Stiffness of the joint is the function of material and structural stiffness. The material stiffness results from the physical and mechanical properties of wood. The structural stiffness depends on construction of the joint - the manner of structural components connection [7]. An example of joints construction in light wood-framed structures is shown in figure 1.

![Figure 1. Wall-to wall joint in light-wood framed structures](image)

Stiffness of the joint depends on its construction. Connections, made especially on the site, do not meet the requirements of fixed nodes. Connections between individual components of timber structures are provided by the traditional joinery, glue, punched metal plate fasteners or dowel type fasteners [8-9]. Nails, screws, staples or bolts are commonly used fasteners in light wood-framed structures depending on the place of its application. Due to the type and cross-section area of the fastener the behaviour of connection could vary but always is non-linear. Non-linear behaviour of the joints is caused by the non-linear behaviour of the fasteners [10]. Steel fasteners despite the high material strength have small cross-section area what affect on its non-linear behaviour. Increase of the shear and bending moment in the fasteners generate the decrease of the joint stiffness.

The deformation of wood under the forces applied to the joint is an additional factor affecting the semi-rigid behaviour of the joint. Wood is material with different values of modulus of elasticity and axial or bending strength depending on the direction of the grains in the component. Stiffness or load capacity is several times higher for the direction parallel to the grain in opposition to the direction perpendicular to the grain.

Structural elements are shaped with the longitudinal axis of the component parallel to the grain. The formation of the structural components this way allows to obtain the greatest capacity in direction of axial forces but also high load capacity and stiffness in case of bending. The arrangement of the grain in the components intersecting in the joint is presented in figure 2a. Forming all components in this manner causes the axial force acting in one component parallel to the grain is applied onto another component perpendicular to the grain. The result of this situation is significant deformation in loaded component, shown in figure 2b.
Deformation of the joint could be realized because of vertical and horizontal loads. The displacement on horizontal and vertical direction are larger than assumed in the adopted calculation models. Overall spatial stiffness of the light wood-framed building is smaller than assumed. Semi-rigid behaviour of the joints is connected not only with the applied loads but also with deformation depending on the rheological phenomena as well the environmental influences. The real scheme of the work in case of floor-to-wall connection is presented in figure 3. The significant values of the deformation could exceed the limit conditions of SLS.

3. Serviceability Limit States in timber structures

According to Eurocode 5 [6] in case of timber structures three types of Serviceability Limits States are distinguished: joint slip, limiting values for deflections of beam components and vibrations. In this paper authors focused on the second case, the limiting values of deformation of the structure.

For the timber structures the limitations of SLS are spent if the final deformation not exceed limit values. According to Eurocode 5 [6] final deformation is the sum of final deformation for permanent action (G) and variable action (Q). The final deformation is calculated using formula (1).

\[ u_{\text{fin}} = u_{\text{fin},G} + u_{\text{fin},Q} \leq u_{\text{lim}} = \frac{1}{300} l_{\text{eff}} \] (1)

Final deformation for permanent or variable action is a function of instantaneous deformation \( u_{\text{inst}} \) and deformation factor \( k_{\text{def}} \). The value of deformation is calculated using formula:

\[ u_{\text{fin},G(Q)} = u_{\text{inst},G(Q)} (1 + k_{\text{def}}) \] (2)
Instantaneous deformation for permanent or variable action of single-span beam with both semi-rigid supports of the same stiffness \( \kappa \) is calculated as [11]:

\[
\begin{align*}
    u_{\text{inst},P} &= \frac{\varepsilon \ p_d^{1/2}}{384 \ E_0,\text{mean}^3} \\
\end{align*}
\]

where:

- \( P, p \) – characteristic values of permanent (G, g) or variable (Q, q) action,
- \( l_{\text{eff}} \) – effective length of the floor beam,
- \( E_0,\text{mean} \) – modulus of elasticity in direction of grain,
- \( I \) – second moment of inertia,
- \( \varepsilon = 5 - 4\alpha \) – factor depending on the rotational stiffness \( \kappa \) of both semi-rigid joints, calculated as:

\[
    \alpha = \frac{\kappa \ l_{\text{eff}}^2}{2E_0,\text{mean}^3 + \kappa l_{\text{eff}}^2} \quad (4)
\]

In case of semi-rigid joint with both rotational \( \kappa \) and translational stiffness \( k \) the instantaneous deformation taken from formula (3) is increase by additional deformation related to translational stiffness. The value of the instantaneous deformation is described as:

\[
    u_{\text{inst},P} = \frac{\varepsilon \ p_d^{1/2}}{384 \ E_0,\text{mean}^3} + \frac{q \ k l_{\text{eff}}^2}{2\xi} (1 + \xi) \quad (5)
\]

where \( \xi \) is the ratio of stiffness \( k_1 \) of one support to stiffness \( k_2 \) of another one.

4. Results and discussions

4.1. Experiments

During the experimental researches [12,13] carried out in Bialystok University of Technology deformation of joints of light wood-framed structures were observed. The axial deformations as well the rotations were analyzed. The experiments were realized on specimen in micro-scale. Dimension scale was assumed as 1:3. Joints simulating stud-to-bottom plate joint were axially loaded to obtain transitional stiffness [13], while joints simulating floor joist-to-top plate of wall were loaded to obtain rotational stiffness [12]. The configurations of analyzed joints in the light wood-framed structures are presented in figure 4.

To obtain P-\( \Delta \) relationship the joint simulating stud-to-bottom plate connection was loaded by force applied in axis of vertical element. The testing specimen was realized as vertical component supported by horizontal one. The horizontal component had got restricted vertical displacements of bottom surface. The configuration of the specimen is shown in figure 5a. The tests were conducted to the loss of the load capacity of the joint. The experiment was interrupted at the moment the yield occur in compressed horizontal component.

The tests provided to recognize the rotational stiffness were carried out on joints simulating floor joist-to-top plate of wall. Testing specimen was realized as horizontal component lying on two supporting vertical elements. The configuration of the specimen is shown in figure 5b. There were occurred a problem to define value of moment in semi-rigid joints connecting horizontal beam and supporting elements. The connection of the elements was provided by the pressure between components in the joint.

The configuration of the joint assumed quasi-pinned connection and not allowed directly determine the value of rotational stiffness \( \kappa \) as the dependence of relationship M-\( \varphi \). During the experiments load-rotation relationship was observed but in place of joint moment M external force P was taken. The P-\( \varphi \) relationship was set instead M-\( \varphi \) relationship.
During the experiments the existence of fasteners in the joints were omitted. This simplification was assumed to avoid disturbance of the joint behaviour by local increase of capacity and stiffness due to the fasteners existence.

4.2. Results discussion
The results obtained from the experiments [12,13] show the semi-rigid behaviour of the joints working in non-linear range.

Figure 6 presents P-Δ relationship obtained from the experimental test for stud-to-bottom plate joint of the wall loaded by axial force [13]. The results show non-linear work of the joint in range of high loads near the level of destructive action. Due to high compressive stress and lower capacity in direction perpendicular to grain the significant deformation of the horizontal element occurred.

Figure 7 presents P-φ relationship obtained from the tests [12]. In opposition to above presented P-Δ relationship, the P-φ relationships are non-linear from low values of external load. Recorded displacements present higher rotational deformation than expected from assumed static scheme.

The results taken from experimental tests present that values of recorded rotation are greatest than obtained from analytical pattern. The value of the rotation of support joint is calculated using formula (6). The angle of rotation is linearly dependent on magnitude of the load P.

\[ \varphi = \frac{1}{16} \frac{P_{D}}{E_{w, mean}} \]  

(6)

Observed rotations in tested specimen differed to calculated using formula (6). The values changed in non-linear manner that proves the joints had been working as semi-rigid. Cooperation of the components in the joints depends on deformation of the elements as the result of translational and rotational actions.
Rotational stiffness of the joint could be calculated on the basis of rotation of the joint. Using Direct Displacement Method, the formula to calculate the rotation angle \( \phi \) was set. To simplify the calculation was assumed the same stiffness \( \kappa \) for both semi-rigid joints over the supporting elements.

The rotation angle \( \phi \) of semi-rigid supporting joint for single-span beam loaded by single force in mid-span of the component is calculated using formula:

\[
\phi = \frac{1}{8} \frac{P^2_{\text{eff}}}{(2E_{\text{mean}} + \kappa_{\text{eff}})}
\]

(7)
Converting the formula (7) to estimate the rotational stiffness $\kappa$ the following formula was obtained:

$$\kappa = \frac{P_{\text{eff}}}{8} - \frac{E_{\text{mean}}}{1}$$

Table 1 presents selected values of rotational stiffness calculated using formula (8). The results for specimen S-1 and S-2 were omitted due to low value of destructive load.

| Specimen | Load $P$ [kN] | Stiffness $\kappa$ [kNm] | Load $P$ [kN] | Stiffness $\kappa$ [kNm] |
|----------|---------------|--------------------------|---------------|--------------------------|
| S-3      | 4.00          | 2.845                    | 6.75          | 0.289                    |
| S-4      | 4.00          | 3.263                    | 7.50          | 1.474                    |
| S-5      | 4.00          | 3.022                    | 7.50          | 2.033                    |
| S-6      | 4.00          | 2.656                    | 7.50          | 1.908                    |
| S-7      | 4.00          | 1.899                    | 7.50          | 0.031                    |

The rotational stiffness presented above show its decrease according to increase of the external load. Due to heterogeneity of wood obtained values of stiffness vary. At level of 4kN force the stiffness is at similar level, but near to the destructive load the stiffness is significantly different.

5. Conclusions
Presented results show semi-rigid behavior of joints in light wood-framed structures. Translation and rotation of the component in analyzed joints are larger than values obtained using generally known analytical models assuming the work in elastic range, without deformation of the joints.

Deformations of the joints were progressive in non-linear manner. In range of destructive loads, the significant rise of the displacement appeared. Destruction of stud-to-bottom plate joint occurred due to the plasticity of upper surface of compressed horizontal component. In case of floor joist-to top plate of wall joint the destruction was progressing as the result of breakage of the horizontal component in area of the applied external force.

The proposed method of calculation of rotational stiffness using the formula developed on the basis of Direct Displacement Method shows the joints in fact work in different manner to assumed for simple bar system. Exact solution cannot be met using simple bar model. Accurate solution can be obtained in FEA. The model of joint should be described by plane strain or shell finite elements.

In structure with semi-rigid joints should be considered the risk of exceeding the limit conditions of SLS by the real final deformation. That situation disturbs the comfort of use of the building and in extreme cases may cause the damage of the structure.

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