Students’ metacognitive abilities in solving derivative problems

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Abstract. Students need skills and solving strategies to solve mathematical problems. One of the strategies that students should have is metacognition. Metacognitive ability plays an important role in avoiding errors and allows students to use their prior knowledge and certain strategies in solving mathematical problems. This research is a preliminary study that aims to describe students’ metacognitive abilities in mathematical problem solving. The participants in this study were 28 students of an Islamic senior high school in Banda Aceh, Indonesia. Data collection conducted in this study was tests and interviews analysed descriptively. This study found that most students satisfied the planning indicator of metacognitive abilities in mathematical problem solving. Furthermore, some students met the monitoring indicator, even though their answers were not perfect, and some reached the evaluation indicator. Students’ metacognitive abilities in solving mathematical problems can be improved; one of the ways is applying problem-based learning model.

1. Introduction
Mathematics is a subject that requires students to be able to master mathematical problem solving. Problem solving is one of five mathematical skills students should have in learning mathematics [1]. Students should have a good strategy in doing mathematical problem solving to get the expected results; one of the strategies required for mathematical problem solving is metacognitive strategy [2]. Metacognition can be defined as one’s knowledge of cognitive processes and products resulting from the cognitive processes, including active monitoring, regulation and orchestration that are consequent of the processes [3]. Another defines metacognition as a person's awareness of knowledge or understanding of knowledge he possessed [4].

Students who have metacognitive abilities are aware of how to solve their learning problems. They become aware of their own learning styles and can apply the most effective strategy to solve a problem [5]. The ability of metacognition plays an important role in preventing students from doing trial and error, avoiding mistakes, and allowing students to use prior knowledge and certain strategies in solving mathematical problems [6].

Metacognitive abilities possessed by students can be measured through three indicators: planning, monitoring and evaluation [7]. First, planning is defined as being able to know how to achieve goals. Second, monitoring refers to being able to control cognition used, to identify problems, and to modify plans in solving problems. Third, evaluation means being able to assess the results obtained, whether they have met the desired goals or not. In learning mathematics, especially in the process of solving
mathematical problems, there are several activities carried out by students, namely starting from understanding the problem, making decisions about what should be done and implementing the decision [8]. During the process, students should monitor and evaluate what is done. If the selected decision is not appropriate, then try other possibilities. The process of monitoring work results and being aware of errors in work are some of the metacognitive aspects needed during the mathematical problem solving process.

In this study, the researcher focused on the derivative topic, because based on the results of interviews with the teacher that the ability of students to solve problems with derivative applications was still very low, students had difficulty in applying the concept of derivatives to the application of derivative materials. One of the reasons causing some students had low metacognitive abilities on the derivative topic is because they solve problems without understanding the logic behind it [9]. When solving mathematical problems, some students experience metacognitive difficulties in understanding the use of mathematical symbols and representing the symbols to model the mathematical problems, whilst some students have metacognitive abilities at ‘tacit use’ level [10].

Considering the gap between the importance of metacognitive abilities and the students' level, the researchers intend to analyse students' metacognitive abilities in solving mathematical problems, and also to find out learning designs that help students to improve metacognitive abilities. Thus, the research questions in this study are: 1) How are students' metacognitive abilities in solving mathematical problems on the derivative topic? and 2) What learning designs can improve students' metacognitive abilities?

2. Method

Based on the research questions, this study utilized a qualitative descriptive approach to build an understanding of a phenomenon based on participants' point of view [11]. The first purpose of this study is to investigate the level of students' metacognitive abilities in mathematical problem solving, specifically on the derivative topic. Furthermore, the second purpose is to find out learning designs that can improve students' metacognitive abilities in mathematical problem solving. This study was conducted in one of the Islamic senior high schools in Banda Aceh, involving 28 students taking the test. The instrument used in this study was a test on the application of derivative in which the problems are in the category of mathematical problem solving. The test consisting an item with three problems about the application derivative application is adapted from a thesis and has been validated by experts.

The indicators of the test are shown in Table 1. In addition to the test, the researchers also conducted an interview two days after the test. The interview involved three students with low, medium and high test results that satisfied indicators of metacognitive ability. It aims to support students' test results and to delve students' metacognitive abilities. The interview guide contains questions to cross-check students' test results. Students' answers were grouped based on the indicators of metacognitive ability, which are, planning, monitoring and evaluation. Thus, the percentage of students at each indicator could be made.

| Content | Indicator of mathematical problem solving | Question type |
|---------|------------------------------------------|---------------|
| Application of derivative on three dimensional objects, i.e., rectangular cuboid | Understanding the problem | Essay |
| | Planning strategies | |
| | Implementing the strategies | |
| | Reflection | |
3. Results and discussion
Table 2 presents the percentage of students’ metacognitive abilities in solving the problem in the test. This analysis is guided by three indicators of metacognitive ability by Flavell, which are planning, monitoring and evaluation [3]. Table 2 shows that 13 students (46.42%) were at the planning phase; they were only able to write down the important information known and asked in the problem. Moreover, approximately ten students (35.71%) reached the monitoring phase. They recognized the formulas or strategies used to solve the problem and were able to apply other concepts in solving the problem, even though some students did not apply the concepts well. Furthermore, five students (17.85%) were at the evaluation phase. They were able to solve the problem by giving conclusions for every solution and recognizing the rules that must be followed to get the correct solutions.

| Measured aspect | Percentage             |
|-----------------|------------------------|
| Planning        | 13 students (46.42%)   |
| Monitoring      | 5 students (35.71%)    |
| Evaluation      | 5 students (17.85%)    |

Further, to support the analysis on the test results, the researchers interviewed students to see conformity with the test results. The interview involved three students selected based on their results in the test. They are a low level student (i.e., JN), a medium level student (i.e., KAV), and a high level student (i.e., CM).

![Figure 1. JN’s solutions.](image1)

![Figure 2. KAV’s solutions.](image2)

First, the analysis of JN's solutions (see Figure 1) showed that JN did not write any information about what is known and asked in the problem. She also did not write the concepts used to solve the problem, but only wrote the formulas. She drew a graph, yet did not write the steps for applying the concepts completely and did not do calculations in her problem solving. Then, JN did not reflect on the solution she got. Based on those analyses, JN only satisfies the planning indicator.
Afterwards, the researchers interviewed JN. During the interview, JN explained, “The information known from the problem is the size of the box without a lid, that is, the length of the box is 24cm, and the width of the box is 9cm”. Therefore, it suffices to say that the information called by JN did not match what was known in the problem. Next, the student explained about the reason, “The equation $12x^2 - 132x + 216$ was obtained from $4 \times 4$ because of $4x^3$, I forgot”, which was considered as the wrong reason. Then the researcher asks “Is it necessary to check the points obtained?” and JN explained, “We need to check the points, but I don't know how to do it”.

The results of the interview revealed that JN did not understand the problem well, and she incorrectly gave the information known in the problem. Moreover, JN hesitated when mentioned the concepts she used, did not comprehend how to apply the concepts, and was unable to describe her graph. Consequently, JN was unable to elaborate on the solution she obtained. Hence, JN only met the planning indicator, which is the lowest level in the metacognitive ability.

Second, the analysis of KAV's solutions (see Figure 2) showed that he incorrectly wrote one of the three information given in the problem and incompletely mentioned what the problem asks. As a result, KAV made an error when applied the concepts in solving the first problem, although his final answer was correct. In the second problem, KAV’s answer was the same as JN’s answer, but KAV made a conclusion so that he got a correct answer in the third problem. Unfortunately, he did not evaluate his answer.

During the interview, KAV explained, “What is known from the problem is the size of the box without a lid, with the length $24 - 2x$, width $x$ and height $9 - 2x$”. KAV did not know that the width and height information he explained was reversed, so he was wrong when applied to the formula. Then, when the researcher asked “How do you get this $12x^2 - 132x + 216$ equation?”, KAV explained that “I got the equation $12x^2 - 132x + 216$ from the results of the volume and then I applied the derivative concept to the equation”. Next, the researcher asked KAV: “Is it necessary to check the points obtained by distributing these points to the second equation that has been derived?”, and KAV explained: “I simply looked at the graph of the up and down functions, and there is no need to test the points by lowering the previous second equation”. Thus, it suffices to say that he did not evaluate the point by substituting it to the second equation, even though it will be more convincing. Furthermore, KAV correctly explained the reasons for the answers written on the next problem (see Figure 2).

The results of the interview revealed that KAV recognized the concepts and understood how to apply them, yet he wrongly stated one of the three information embedded in the problem. Accordingly, he did not perform well at the planning phase. In the second problem, KAV perfectly demonstrated his understanding of the concepts and how to use them. Also, he was able to describe his graph, although he did not realize that he missed some steps. In the third problem, KAV was able to explicate the solution he obtained; however, he did not look back and reflect on his answer; he doubted his answer. Based on the analysis on the test and interview, KAV reached the second indicator, monitoring phase.

Third, the analysis of CM's solutions (see Figure 3) indicated that CM was able to write correctly the information about what is known and asked in the problem. In the first problem, CM was able to demonstrate the concepts she used and apply them correctly until she arrived at a correct answer. In the second problem, CM solved the problem correctly, even though she incompletely wrote her steps. Last, CM performed her answer correctly for the third problem and drew a conclusion for each answer.

Furthermore, the researcher interviewed the students of CM, and when interviewed she explained: “What is known from the problem is the size of the box without the lid, with the length of $24 - 2x$, width $9 - 2x$ and height $x$”. Next, the researcher asked “Where did you get this $12x^2 - 132x + 216$ equation?” and CM explained, “I got the equation $12x^2 - 132x + 216$ when I applied the concept of derivatives to the equation $216x - 66x^2 + 4x^3$”. Next, the researcher asked “Where did you get $x = 2$ and $x = 9$?”, and the student explained, “I just did factorization and applied the concept of derivatives to get this equation, then I tested the point so that it could be $x = 2$ and $x = 9$.”
During the interview, CM explained her answers confidently even though she did not write the process described. Then the researcher asked “Why did you write that $x = 2$ is the maximum value of $x$?” and she explained “The condition for the maximum value is less than 0, and when I test the point, I found that the result for $x = 2$ is less than 0, so the maximum value of $x$ is 2”. Furthermore, the student also explained the calculation operations she performed to obtain a maximum volume of 200 $cm^3$ (see Figure 3).

The results of the interview showed that CM correctly stated every information that is known and asked in the question. In the first problem, CM comprehended the concepts she used and was able to illustrate how to apply the concepts. In the second problem, CM was able to explain the concepts and to describe the graphs she drew. CM also realized that she wrote an incomplete answer about the test point concept. Nonetheless, CM was confident of her answers and able to draw conclusions from the solutions. As such, based on the analysis on the test and interview, CM had fulfilled planning and monitoring indicators and met the characteristics of evaluation indicator albeit not perfect.

Based on the results, the students mostly satisfied the first indicator of metacognitive ability, which is planning. Because it only met the first indicator, the students’ metacognitive ability to solve problems in the derivative topic is low. Thus, the researchers intend to improve students’ metacognitive abilities to achieve the third indicator of metacognitive ability, which is evaluation.

Furthermore, the improvement of students’ metacognitive abilities cannot occur spontaneously; it should be supported by continuous practice and effective learning methods [12]. In this case, to improve students’ metacognitive abilities in mathematical problem solving, the researchers suggested that teachers implement problem-based learning model. Problem-based learning (PBL) is a learning model that can help students improve their metacognitive abilities in mathematical problem solving [13]. PBL can also develop students’ metacognitive awareness [14]. Teachers can apply the PBL model in the classroom to produce students with strong metacognition [15]. In addition to that, PBL is recommended to be applied in the learning process because it helps students improve their conceptual understanding [16, 17].
4. Conclusion
Metacognitive abilities are one of the learning strategies that students can apply in solving mathematical problems. Having difficulties in understanding problems, and determining and applying concepts in solving mathematical problems is the result of weak metacognitive abilities. The analysis on the test and interview revealed that most of the students had metacognitive abilities in the first indicator, i.e., planning. Yet, there were also some students who met the monitoring indicator even though their answers were not perfect, and some reached the evaluation indicator. Students’ metacognitive abilities in solving mathematical problems are influenced by the design of learning implemented in the classroom. One of the appropriate learning designs that help to improve students’ metacognitive abilities in mathematical problem solving is problem based learning model.

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