The role of compressibility in solar wind plasma turbulence.

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Incompressible Magnetohydrodynamics is often assumed to describe solar wind turbulence. We use extended self similarity to reveal scaling in structure functions of density fluctuations in the solar wind. Obtained scaling is then compared with that found in the inertial range of quantities identified as passive scalars in other turbulent systems. We find that these are not coincident. This implies that either solar wind turbulence is compressible, or that straightforward comparison of structure functions does not adequately capture its inertial range properties.

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The supersonic and super-Alfvénic flow of the solar wind offers a unique opportunity to investigate the properties of the magnetized and turbulent plasma. The transition from a laminar to turbulent flow requires large Reynolds number \(Re = UL/\nu\), and its magnetic counterpart \(R_m = LU/\eta\), where \(L\) is the energy injection scale length, \(U\) is the velocity difference on scale \(L\), \(\nu\) is the viscosity and \(\eta\) is the magnetic diffusivity. Estimates of the hydrodynamic and magnetic Reynolds numbers in the solar wind exceed \(10^8\) compared with \(Re \approx R_m \approx 10^4\) obtained in direct numerical simulations (DNS) and just few hundreds in some magnetized liquid laboratory experiments. The presence of turbulence in the solar wind is strongly suggested by numerous observations. These include power law power spectra with \(-5/3\) Kolmogorov-like slopes in the kinetic and magnetic energy densities (e.g., [8, 9, 10]) and non-Gaussian Probability Density Functions (PDFs) (e.g., [11, 12, 13, 14]) found for fluctuations in the velocity and the magnetic field.

These observations imply that solar wind turbulence shares many of its statistical properties with incompressible isotropic hydrodynamic turbulence [15, 16, 17, 18]. As a result, the turbulent dynamics of the solar wind is often modelled assuming that the plasma density is constant. This assumption of incompressibility is particularly convenient in analytical and numerical studies of magnetohydrodynamic (MHD) turbulence. The assumption of incompressibility also appears to be in good agreement with the results of compressive MHD simulations where generation of compressive modes from Alfvénic turbulence was found to be suppressed [2]. In the context of the solar wind, incompressibility has been suggested to be a reasonable approximation for plasma in fast wind streams [16, 17]. Considerable progress has been made by treating the solar wind as dominated by Alfvénic fluctuations (e.g., [21, 22, 23, 24]) and with nearly incompressible magnetohydrodynamic theory [26].

However, statistical features associated with turbulence have also been identified in the density fluctuations derived from the solar wind observations. A \(k^{-5/3}\) scaling in the omnidirectional wave number spectrum of electron and proton density has been reported as early as the 1970s (see [24] and the references therein). Models of the solar wind can offer possible mechanisms for generating strong density fluctuations close to the heliospheric current sheets [27]. Recent studies suggest that, at least in the slow solar wind, the coupling of Alfvénic and magnetosonic MHD modes could not fully describe the nature of these fluctuations. For example, the ratio \(\delta \rho / \rho\) was found to be nearly constant and independent from the amplitude of the magnetic field fluctuations \(\delta B / B\) in the slow solar wind [25]. More recently the density fluctuations were also found to exhibit non trivial, approximately self-affine scaling [13] similar to that found in other solar wind bulk plasma parameters.

In this Letter we examine the scaling properties of the proton density \(\rho\) in fast and in slow solar wind. We focus on a comparison of the scaling properties of fluctuations in density with that of various passive scalars identified in experiments, and direct numerical simulations, of fluid turbulence. Previously, these passive scalars have been shown to exhibit scaling that corresponds closely to that of the magnitude of the magnetic field \(B\) in the solar wind, although the data interval was not ordered by solar wind speed [26]. By assuming incompressibility it can be argued [26] that \(B\) should act as a passive scalar. We...
and the magnetic field magnitude $B$ scalar in the turbulent solar wind flow: $\rho$

Given the assumption of incompressibility ($\nabla \cdot \mathbf{v} = 0$), it immediately follows that $\rho$ should behave as a passive scalar in the turbulent solar wind flow:

$$\partial_t \rho = - (\mathbf{v} \cdot \nabla) \rho.$$  

To investigate the scaling properties of the density $\rho$ and the magnetic field magnitude $B$ we use 64 seconds averaged data from the ACE spacecraft [30] set spanning from 01/01/1998 to 12/31/2001. This interval includes dates previously considered in Ref. [28]. The slow and fast solar wind are known to exhibit distinct phenomenology (e.g., [13, 18]). We thus split the data into slow and fast solar wind sets using 450 km/s wind speed as a separation criteria. The resulting data sets consist of $\sim 1 \times 10^6$ samples for the slow wind and $\sim 0.6 \times 10^6$ for the fast wind. We apply structure function [31] analysis to fluctuations in density for slow and fast wind separately. Generalized structure functions $S_m$ of fluctuations in say, the density $\rho(t)$, on timescale $\tau$ are defined as moments $m$ through $S_m(\tau) = \langle |\rho(t + \tau) - \rho(t)|^m \rangle$ where the ensemble $\langle \ldots \rangle$ is taken in the time domain [32]. If scaling is present in the time series we expect these to show a power law dependence on the temporal scale $\tau$, i.e., $S_m \propto \tau^{\zeta(m)}$.

Finite, experimental data sets include a small number of extreme events (outliers) that, due to poor statistics, may obscure the correct scaling of the high order moments. Here, we will exclude these events by the use of conditioning [17]. This approach puts a limit on the range of fluctuations used in computing structure functions. This limit is varied with the temporal scale $\tau$ to account for the growth of range with temporal scale in the signal. In our case we defined this threshold as $15\sigma(\tau)$, where $\sigma(\tau)$ is a standard deviation of fluctuations on temporal scale $\tau$. We stress that conditioning improves the scaling where it already exists but does not enforce it on the investigated data, if the applied threshold is sufficiently large. In practice, for the limit chosen here, we eliminate less than 1% of the data points.

Figure 4 shows the structure functions $S_m$ plotted versus $\tau$ on logarithmic axes for orders $1 \leq m \leq 4$ for fluctuations in density in the slow solar wind. The plot shows a scaling region extending from $\tau \sim 10$ minutes to $\tau \sim 3$ hours ($\sim 1.5$ decades on the logarithmic scale). The quality of the scaling deteriorates when fast wind streams are considered. The scaling regions can be still identified but they extend only $< 1$ decade from $\sim 10$ minutes to 1 hour.

It has been empirically shown that Extended Self Sim-
ilarity (ESS) approach can considerably extend the region of scaling in structure functions [33]. The method is based on the assumption that $S_m(\tau) \propto S^p_m(\tau)$. This suggests that scaling should emerge when the quantity $S_m$ is plotted as a function of $S_p$. The scaling exponent $\zeta$ can then be obtained straightforwardly from the relation $\zeta(m) = \zeta(p)\eta(m)$. For the density fluctuations in the solar wind, we find that $S_3$ is close to unity and we plot $S_m$ versus $S_3$ on logarithmic axes for fluctuations in the density in slow and fast solar wind, in figures 2 and 6 respectively. These figures demonstrate that we can extend scaling in the density in both the slow and the fast wind streams to over 2 decades when ESS is applied.

ESS was previously applied to the magnitude of magnetic field in an undifferentiated interval of solar wind [28] and for completeness we give the analysis in slow and fast wind here. Figures 4 and 5 show $S_m$ versus $S_3$ on logarithmic axes for fluctuations in the magnitude of magnetic field. Previously, the scaling exponents were obtained by considering $S_m$ versus $S_1$ [28] and we have verified that these closely correspond to the values found here. The local slopes of the third order structure functions $S_3$ then yield an estimate of the exponent $\zeta(3)$ and these are detailed in Table 1.

We can now directly compare the scaling found for the density $\rho$ with that identified for quantities acting as passive scalars and that of the magnetic field magnitude $B$. The resulting functional form of the scaling exponents $\zeta(m)$ for the density and magnetic field magnitude in slow and fast solar wind are shown in figures 3 and 7 respectively. For comparison, the scaling exponents obtained for passive scalars from the DNS [34] and the wind tunnel experiment [35] are also shown. We immediately see that whereas the exponents for magnetic field magnitude and the passive scalars fall close to each other on these plots (as also reported in [28]) they are distinct from those obtained for the solar wind density in both slow and fast solar wind. The fluctuations in $\delta B$, for slow and fast wind, and that of the passive scalars exhibit multi-fractal scaling. Intriguingly, the density fluctuations are nearly self-affine with scaling exponent $\alpha$ that differs in slow and fast wind. The values of these exponents are: $\alpha_{\rho_{\text{slow}}} = 0.39 \pm 0.03$ and $\alpha_{\rho_{\text{fast}}} = 0.33 \pm 0.03$.

This suggests one of two possible conclusions. The first of these is that the turbulent solar wind is compressible and equation (3) does not hold, so that the density is an active quantity. This is rather surprising and calls into question the significant body of work on MHD turbulence in the solar wind that relies on the assumption of incompressibility (for example, [20, 21, 22, 23]). This also invalidates the arguments in Ref. [28] required to cast the advection equation for $B$ in the form of a passive scalar which require incompressibility, yielding:

$$\partial_t B = -(\mathbf{v} \cdot \nabla)B + \eta \nabla^2 B + \lambda B,$$

(4)

The second possibility is that commonality of scaling (as measured by structure functions) does not imply shared phenomenology, and as a corollary, an absence of such a commonality does not imply distinct phenomenology. The fact that several quantities share the same structure functions is then coincidental rather than expressing some universality of fluid turbulence. This implies that the structure functions of a single quantity do
not fully capture the phenomenology of a given turbulent system. This has implications for analysis of turbulence which has been achieved through comparison between (multi-fractal) models of turbulence and the data, via the structure functions (see, for example, [32]).

A possible resolution may be found in the form of equations (3) and (4). It is known that the molecular diffusivity can change the inertial range scaling properties of a passive scalar [36]. The observed differences could then be due to the presence of the diffusive term in the magnetic field equation (4) and its absence in the density equation (3). This may account for the different scaling found in these quantities.

Intriguingly, equation (3), when written for the moments of fluctuations in $\rho$, has no explicit dependence on the order of the moment $m$.

\[ \langle \delta \rho \rangle^m \equiv \langle \rho(x, t) - \rho(x', t) \rangle^m. \]

In this case fluctuations in density should be simply those imposed on the initial condition $\rho(x, 0)$. These may be mediated via large scale coherent structures (shocks and coronal mass ejections). This may suggest a solar origin of these fluctuations in the density. It may thus be informative to attempt to relate the scaling found in the density fluctuations in the solar wind, with that of the solar corona. The power law scaling of X-ray flux from solar flares [37] is intriguing in this regard.

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### TABLE I: Exponents $\zeta(3)$ obtained from figures 3

| Quantity   | Slow/Fast | $\zeta(3)$ | Range [min] |
|------------|-----------|------------|-------------|
| $\delta B$ | Slow      | $0.85 \pm 0.06$ | 10 – 500    |
| $\delta \rho$ | Slow    | $1.16 \pm 0.08$ | 10 – 120    |
| $\delta B$ | Fast      | $0.79 \pm 0.05$ | 8 – 200     |
| $\delta \rho$ | Fast    | $1.01 \pm 0.07$ | 6 – 100     |