Observational Evidence for Negative-Energy Dust in Late-Times Cosmology

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Abstract

We perform fits of unconventional dark energy models to the available data from high-redshift supernovae, distant galaxies and baryon oscillations. The models are based either on brane cosmologies or on Liouville strings in which a relaxation dark energy is provided by a rolling dilaton field (Q-cosmology). An interesting feature of such cosmologies is the possibility of effective four-dimensional negative-energy dust and/or exotic scaling of dark matter. An important constraint that can discriminate among models is the evolution of the Hubble parameter as a function of the redshift, \( H(z) \). We perform fits using a unifying formula for the evolution of \( H(z) \), applicable to different models. We find evidence for a negative-energy dust at the current era, as well as for exotic-scaling \((a^{-\delta})\) contributions to the energy density, with \( 3.3 \lesssim \delta \lesssim 4.3 \). The latter could be due to dark matter coupling with the dilaton in Q-cosmology models, but it is also compatible with the possibility of dark radiation from a brane Universe to the bulk in brane-world scenarios, which could also encompass Q-cosmology models. The best-fit model seems to include an \( a^{-2} \)-scaling contribution to the energy density of the Universe, which is characteristic of the dilaton relaxation in Q-cosmology models, not to be confused with the spatial curvature contribution of conventional cosmology. We conclude that Q-cosmology fits the data equally well with the \( \Lambda \)CDM model for a range of parameters that are in general expected from theoretical considerations.
1 Introduction and summary

There is a plethora of astrophysical evidence today, from supernovae measurements [1–5], the spectrum of fluctuations in the Cosmic Microwave Background (CMB) [6], baryon oscillations [7] and other astrophysical data, indicating that the expansion of the Universe is currently accelerating. The energy budget of the Universe seems to be dominated at the present epoch by a mysterious dark energy component, but the precise nature of this energy is still unknown. Many theoretical models provide possible explanations for the dark energy, ranging from a cosmological constant [8] to super-horizon perturbations [9] and time-varying quintessence scenarios [10], in which the dark energy is due to a smoothly varying (scalar) field which dominates cosmology in the present era.

The current astrophysical data are capable of placing severe constraints on the nature of the dark energy, whose equation of state may be determined by means of an appropriate global fit. Most of the analyses so far are based on effective four-dimensional Robertson-Walker Universes, which satisfy on-shell dynamical equations of motion of the Einstein-Friedman form. Even in modern approaches to brane cosmology [11], which are described by equations that deviate during early eras of the Universe from the standard Friedman equation (which is linear in the energy density), the underlying dynamics is assumed to be of classical equilibrium (on-shell) nature, in the sense that it satisfies a set of equations of motion derived from the appropriate minimisation of an effective space-time Lagrangian.

However, cosmology may not be an entirely classical equilibrium situation. The initial Big Bang or other catastrophic cosmic event, which led to the initial rapid expansion of the Universe, may have caused a significant departure from classical equilibrium dynamics in the early Universe, whose signatures may still be present at later epochs including the present era. In this context, there has been proposed a specific model for the cosmological dark energy, being associated with a rolling dilaton field that is a remnant of this non-equilibrium phase, described by a generic non-critical string theory [12–15]. We call this scenario ‘Q-cosmology’.

Since such a non-equilibrium, non-classical theory is not described by the equations of motion derived by extremising an effective space-time Lagrangian, one must use a more general formalism to make predictions that can be confronted with the current data. The approach we favour is formulated in the context of string/brane theory [16, 17], the best candidate theory of quantum gravity to date. Our approach is based on non-critical (Liouville) strings [18–20], which offer a mathematically consistent way of incorporating time-dependent backgrounds in string theory.

The basic idea behind such non-critical Liouville strings is the following. Usually, in string perturbation theory, the target space dynamics is obtained from a stringy $\sigma$-model [16] that describes the propagation of strings in classical target-space background fields, including the space-time metric itself. Consistency of the theory requires conformal invariance on the world sheet, in which case the target-space physics is independent of the scale characterising the underlying two-dimensional dynamics. These conformal invariance conditions lead to a set of target-space equations for the various background fields, which correspond to the Einstein/matter equations derived from an appropriate low-energy effective action that is invariant under general coordinate transformations. Unfortunately, one cannot incorporate in this way time-dependent cosmological backgrounds in string theory, since—to low orders in a perturbative expansion in the Regge slope $\alpha'$—the conformal invariance condition for the metric field would require a Ricci-flat target-space manifold, whereas a cosmological background necessarily
has a non-vanishing Ricci tensor.

To remedy this defect, and thus be able to describe a time-dependent cosmological background in string theory, the authors of Ref. [18] suggested that a non-trivial rôle should be played by a time-dependent dilaton background. This approach leads to strings living in numbers of dimensions different from the customary critical number, and was in fact the first physical application of non-critical strings [19]. The approach of Ref. [18] was subsequently extended [12–15, 20] to incorporate off-shell quantum effects and non-conformal string backgrounds describing other non-equilibrium cosmological situations, including catastrophic cosmic events, such as the collision of two brane worlds.

In a recent work [21], we have presented preliminary constraints on Q-cosmology by means of supernova data. An important result of our analysis was the unavoidable presence of negative-dust-like contributions to the energy budget of the Universe. Such negative-energy dust terms arise naturally in Q-cosmology [21], as a result of the existence of non-equilibrium dark energy contributions. However, a similar feature may also characterise certain brane cosmologies as a result of effective four-dimensional Kaluza-Klein (KK) graviton mode contributions to the energy density of the brane [22]. Such brane models are also characterised by dark radiation $a^{-4}$ terms, as a result of non-trivial gravitational bulk dynamics [11]. As discussed in Ref. [21], an important distinguishing feature of Q-cosmology is an $a^{-2}$-scaling contribution to the dark energy as a result of dilaton relaxation terms [12]. Such a term is distinct from the spatial curvature contribution as we shall review below. The astrophysical data analysis of Ref. [21] yields a non-zero value of such dark energy dilaton terms.

It is the purpose of the current work to update and complete such analyses by taking into account the very recent supernova data [4, 5], as well as data from (luminous red) galaxies [7, 23, 24] on the evolution of the Hubble parameter as a function of the redshift, which, as we shall see, turns out to be an important constraint for model building. In the same manner as in the analysis of Ref. [21], we also perform a comparison of the results of the fit for the Q-cosmology model with those of the conventional ΛCDM model and a rival model with super-horizon perturbations [9] superposed on an underlying Einstein-Friedman-Robertson-Walker Universe. We find that the current data are consistent with both the Q-cosmology and ΛCDM models. On the other hand, for the super-horizon model there appears a $2\sigma$ incompatibility between the best-fit values for the $H(z)$ galactic data and the supernova analysis. As in Ref. [21], the current analysis indicates the existence of negative-energy dust and non-trivial $a^{-2}$ dark-energy contribution attributed to the dilaton relaxation terms. Our results in this work should be considered as complementary to other similar studies of unconventional cosmologies that exist in the current literature [25].

The structure of the article is as follows. In Sec. 2 the most crucial phenomenological aspects of brane and Q-cosmology models are outlined, and a generic parametrization for the expansion rate of the Universe, to be used in the experimental fits, is developed, which is capable of capturing, in a single formula, the most important features of the various models, namely negative-energy dust, exotic scaling components to the energy density and dilaton dark energy relaxation terms. The cosmological data analyses are discussed in Sec. 3 for each of the three observational sources under study, namely the supernovae, the $H(z)$ measurement from distant galaxies observations and the baryon acoustic oscillations. In Sec. 4 the astrophysical constraints on the cosmological models are combined, leading to the determination of the model parameters, and implications on possible unconventional cosmological scenarios are discussed. Finally, conclusions and outlook are presented in Sec. 5.
2 Theoretical models

2.1 Brane Universes

We commence our discussion by summarising the basic features of brane cosmology [11]. According to this picture, matter fields are confined on three-brane worlds, while fields from the gravitational multiplet (dilaton, graviton, etc.) are allowed to propagate in the bulk. For definiteness, in this work we consider five-dimensional models. In such a case, the analogue of the effective four-dimensional Friedman equation on the brane reads [11]:

\[ H^2 = \frac{8\pi G}{3} \rho_M \left( 1 + \frac{\rho_M}{2\sigma} \right) + \frac{\Lambda_4}{3} + \frac{\mu}{a^4}, \]

where \( \rho_M \) is the matter density on the brane, and we have identified the brane tension \( \sigma \) with the Newton’s constant \( 8\pi G/3 = \sigma/18 \) and the four-dimensional cosmological constant \( \Lambda_4 \) is related to the AdS-bulk (negative) cosmological constant \( \Lambda_5 \) as follows:

\[ \frac{\Lambda_4}{3} = \frac{\sigma^2}{36} + \frac{\Lambda_5}{6}. \]

For our purposes, we shall assume a fine tuning such that \( \Lambda_4 = 0 \). The term \( \mu/a^4 \) in (1) is known as dark radiation, and stems from the non-trivial bulk dynamics and energy conservation. Quadratic terms in \( \rho_M \) do not play a crucial rôle in late times, and hence can be ignored when discussing phenomenology for redshifts below three, which will be of interest to us in this work.

An interesting feature of brane models has been pointed out in Ref. [22] and concerns the contributions to the effective four-dimensional energy density on the brane world by KK graviton modes. Such modes appear as massive gravitons on the brane with masses \( m > 3H/2 \), where \( H \) represents the expansion rate of a de Sitter brane world we assume here for concreteness. The analysis of Ref. [22] has shown that a single very massive KK mode, corresponding to a particle with high momentum along the bulk direction, exerts pressure on the brane pushing it outwards. As a result, the energy in the bulk decreases leading to a decrease in the dark energy term, and therefore an effective negative contribution to the dark radiation term. In this way, the KK mode behaves like a negative-energy dust. It should be noted at this stage that this analysis is still incomplete and cannot be trivially extended to realistic cosmological models, where one should integrate over the entire spectrum of the KK modes. Nonetheless, for our phenomenological purposes in this work we assume that KK modes behave like negative-energy dust. It should be noted that there is no conflict with positive-energy theorems, since the bulk energy density contribution of the KK modes is positive [22].

2.2 Q-cosmology

Next we discuss briefly Q-cosmologies, which is our second class of models to be constrained by the data. Details on the dynamical equations describing Q-cosmological models with matter and dark energy contributions have been reviewed in Ref. [21], where we refer the interested reader. In the analysis that follows, we will only outline the basic predictions for the Hubble rate \( H(z) \), which will be used in our fits.

For completeness, however, we stress again that the most important feature of Q-cosmology is the off-shellness of the appropriate equations describing the dynamics of the fields in the gravitational string multiplet (graviton and dilatons). The target-space effective action variations
with respect to the gravitational field are non zero, and the respective values are determined by the requirement that the Liouville mode restores the world-sheet conformal invariance of the non-critical string. The identification of the Liouville mode with the target time leads to a set of dynamical equations and constraints [12–14], whose consistent solution determines the Liouville string Universe, under the standard assumptions of homogeneity and isotropy.

There are several versions of the Liouville theory, which in general lead to different predictions. There may be models in which all matter and gravitational fields are non conformal on the world sheet, or others in which only the gravitational multiplet deviate from criticality. For our purposes in this work and in Ref. [21], we restrict ourselves to the latter category. After the identification of the Liouville mode with the target time, which can be explained dynamically in such models [26], the pertinent equations for gravitons, for instance near a fixed point in theory space (and thus for sufficiently long cosmic times), read:

\[
0 
eq \frac{\delta S_{G,\text{matter}}}{\delta g_{\mu\nu}} = \ddot{g}_{\mu\nu} + Q(t)\dot{g}_{\mu\nu} + \mathcal{O}(g^2),
\]

where \( S_{G,\text{matter}} \) denotes the gravitational and matter effective low-energy action in the target space and the dot is a Liouville world-sheet zero-mode derivative, which at the very end is identified (dynamically) with the cosmic time. The central charge deficit \( Q^2(t) \) describes the microscopic non-critical model, and is expressed as a functional of graviton \( g_{\mu\nu} \) and dilaton \( \Phi \) fields of the gravitational string multiplet. To lowest order in the Regge slope \( \alpha' \), it is given by:

\[
0 < Q^2(t) = 3g^{00}(\ddot{\varphi} - (\dot{\varphi})^2 + \ldots), \quad \varphi \equiv 2\Phi - \log\sqrt{g}.
\]

We note at this stage that the positive definiteness of \( Q^2(t) \) (supercritical string [18]) is a model-dependent feature, but it is essential in yielding a time-like Liouville mode, thus allowing its interpretation as a target time field. Such supercritical strings may describe, for instance, the stringy excitations on the three brane describing the observable Universe in a colliding-brane-world scenario [12], with the collision providing the main cause for departure from criticality.

The off-shell terms of the Eqs. (3) imply highly non trivial modifications of the analogue of the (effective, four-dimensional) Friedman equation for the Q-cosmology, which now includes, apart from the standard matter and dark energy contributions denoted collectively by \( \rho \), also the above-mentioned Liouville-string off-shell corrections, \( \Delta \rho \), which are not positive definite in general (without affecting, however, the overall positive-energy conditions of this non-equilibrium theory):

\[
H^2(z) = \frac{8\pi G_N}{3}\rho + \Delta \rho.
\]

The appropriate parametrisation for \( H(z) \) in the Q-cosmology framework at late eras, such as the ones pertinent to the supernova and other data (\( 0 < z < 2 \)), where some analytic approximations are allowed [21], reads:

\[
H(z) = H_0 \left( \Omega_3(1+z)^3 + \Omega_\delta(1+z)^\delta + \Omega_2(1+z)^2 \right)^{1/2}, \quad \Omega_3 + \Omega_\delta + \Omega_2 = 1,
\]

with the densities \( \Omega_{2,3,\delta} \) corresponding to present-day values (\( z = 0 \)). It is important to remark that, as discussed in detail in Ref. [21], the densities \( \Omega_i \) involve mixed contributions from both, off-shell dark energy and (dark) matter. In this respect, \( \Omega_2 \) denotes the dilaton dark energy contribution at late eras, not to be confused with spatial curvature contributions. Similarly,
\( \Omega_3 \) does not denote matter dust, but it includes (negative) dark energy contributions scaling with the redshift like dust. This is a result of the coupling of the dilaton as well as higher string loop corrections, which, depending on the model, could be significant at late epochs of the Universe. In general, there is no conflict with positive energy theorems, in a similar spirit to negative-energy dust contributions in brane models, as mentioned previously, which can be due to appropriately compactified KK graviton modes [22]. We must note at this point that Q-cosmologies can indeed be accommodated in brane-world scenarios. This may occur, for instance, in colliding branes, where the initial brane collision constitutes an Early-Universe cosmically catastrophic event, responsible for the departure of the string theory on the brane from criticality [12]. In such brany Q-cosmologies, KK graviton modes appear naturally, contributing negative-energy dust components to the effective four-dimensional energy density on the brane. Due to the dilaton dark-energy-relaxation (quintessence) terms, however, in such models, one may also obtain in general additional scaling contributions of the type \( a^{-2} \) at late eras [12, 21], which are absent in conventional brane models with constant dilatons (c.f. (1)).

Moreover, in Q-cosmologies there are dark matter contributions with exotic scaling, denoted by \( \Omega_\delta \), where the exponent \( \delta \) is theoretically close to, but not quite, four; such contributions are due to the coupling of dark matter species with the (non-constant) dilaton quintessence field. We stress once more that in all the above terms, the off-shell Liouville corrections play an important rôle [14, 15].

A complete analysis of the non-critical-string and dilaton effects, which turn out to be important in the present era after the inclusion of matter, requires a numerical treatment [14,15]. The energy budget of the Q-cosmology Universe is at present largely phenomenological, since the theoretical analysis, involving string loops, is not sufficiently developed to allow for concrete predictions insofar as the values of \( \Omega_3 \) and \( \delta \) are concerned. For instance, in Ref. [15] it was assumed, rather ad hoc, that there is no exotic matter present today \((z = 0)\), although this is not true in the past. An important constraint is, of course, nucleosynthesis, which in the analysis of Ref. [15] has been left essentially undisturbed, occurring at MeV scales.

In general, in our analysis the three parameters to be determined by the fits are \( \Omega_3 \), \( \Omega_\delta \) and \( \delta \). As we shall see, the data seem to point towards the fact that the amount of exotic-scaling dark matter today is non zero, in contrast to the assumption of Ref. [15]. This, in turn, implies the need for revisiting the phenomenological constraints on supersymmetric particle physics models at future colliders, such as the LHC, derived in Ref. [15] within the Q-cosmology framework. We shall do so in a future publication.

### 2.3 Formulae for data analysis

In our phenomenological fitting procedure, \( \delta \) is left as a free parameter, whose best fit value, however, is found, as we shall discuss below, close to four, thus in good agreement with theoretical expectations [13–15]. It is important to notice that such exotic matter scaling has important consequences on the relaxation of some of the stringent constraints imposed on supersymmetric models of particle physics from thermal dark matter analyses [15] in standard Friedman-Robertson-Walker cosmologies. On the other hand, a value \( \delta = 4 \) may correspond to dark radiation in brane models [22]. In this sense, our parametrisation (6) may be considered as a unifying formula, which can be used for fitting various Q-cosmologies and brane models:

\[
H(z) = H_0 \left( \Omega_3 (1 + z)^3 + \Omega_\delta (1 + z)^\delta + \Omega_2 (1 + z)^2 \right)^{1/2}
\] (7)
with $\Omega_3 + \Omega_5 + \Omega_2 = 1$ and $\Omega_3$, $\Omega_5$, $\delta$ as free parameters.

Before closing this section, we mention that we shall compare our results with two other cosmological models: a $\Lambda$CDM cold dark matter model with a cosmological constant [8], where the Hubble parameter is expressed as:

$$H(z) = H_0 \left( \Omega_M (1 + z)^3 + \Omega_\Lambda (1 + z)^{3(1+w_0)} \right)^{1/2},$$

and the super-horizon model, in which the Universe is assumed to be filled with non-relativistic matter only [9] and there is no dark energy of any sort:

$$H(z) = \frac{\pi_0}{1 - \Psi_{\ell 0}} \left( a^{-3/2} - a^{-1/2} \Psi_{\ell 0} \right),$$

with $\Psi(\vec{x}, t)$ the gravitational potential, $\Psi_{\ell 0}$ a free parameter, $1 + z = a^{-1}(t)e^{(a(t)-1)\Psi_{\ell 0}}$ and $a(t)e^{-\Psi_{\ell}(t)+\Psi_{\ell 0}}$ the scale factor of the Robertson-Walker Universe.

### 3 Data analysis

Observational data from three different astrophysical sources—namely supernovae, distant galaxies and baryon acoustic oscillations—are analysed in order to set constraints on the aforementioned cosmological models. The data fits and all analyses involved are performed within the ROOT [27] analysis framework and the Minuit [28] minimisation and error computation code.

In what follows, we shall use ‘generically’ the terminology ‘Q-cosmology’ whenever we use formula (7) as a fitting function to refer to both brane models with constant dilatons (1) and dilaton-quintessence Q-cosmology models. The reader should bear in mind that the distinguishing features of dilaton-quintessence Q-cosmology models, as compared to brane models with constant dilatons, are the $a^{-2}$-scaling contributions to the energy density of the Universe, as well as the exotic dark matter scaling $a^{-\delta}$, with $\delta \neq 4$.

Some important remarks are now in order. In the discussion of $\Lambda$CDM model, we have allowed, for the sake of completeness, for non-zero spatial curvature contributions. On the other hand, for simplicity, when we discuss Q-cosmology and super-horizon models, we assumed a spatially flat Universe. This is reflected in the relevant relations between observables and the Hubble parameter to be discussed in the following sections. This should make even clearer the distinct role played by the above mentioned dilaton dark energy $a^{-2}$ contributions to $H(z)$, as compared to spatial curvature terms of conventional cosmology.

#### 3.1 Supernovae

We analyse recent type-Ia supernovae (SN) data released by the Hubble Space Telescope (HST) [4] and the ESSENCE collaboration [5]. Among 16 newly discovered high-redshift SNe [4], the so-called ‘gold’ dataset (Riess07:gold) includes SNe from other sets: 14 SNe discovered earlier by HST [2], 47 SNe reported by SNLS [3] and 105 SNe detected by ground-based discoveries, amounting a total of 182 data points. An additional set of 77 SNe tagged as ‘silver’ due to lower quality of photometric and spectroscopic record is also listed, constituting the Riess07:gold+silver dataset. The ESSENCE dataset (WV07), on the other hand, consists out
of 60 SNe of $0.015 < z < 1.02$ discovered by ESSENCE [5], 57 high-$z$ SNe discovered during the first year of SNLS [3] and 45 nearby SNe. Besides these SN sets, a compilation of the aforementioned data (WV07+HST) [29], normalised in order to account for the different light-curve-fitters employed, is eventually analysed. The latter sample, listing 192 SNe, includes all high-redshift supernovae ($z > 1$) observed so far.

Supernova data are given in terms of the distance modulus $\mu = 5 \log d_L + 25$, where the luminosity distance $d_L$ (in megaparsecs) for a spatially flat universe is related to the redshift $z$ via the Hubble rate $H(z)$:

$$d_L(z) = c(1 + z) \int_0^z \frac{dz'}{H(z')}$$

whereas in the general case of a non-zero spatial curvature contribution $\Omega_k$ the luminosity distance is given by [2]:

$$d_L(z) = \frac{c(1 + z)}{\sqrt{|\Omega_k|}} \times \left\{ \begin{array}{ll} \sin \left( \sqrt{|\Omega_k|} \int_0^z \frac{dz'}{H(z')} \right), & \text{for a closed Universe,} \\ \sqrt{|\Omega_k|} \int_0^z \frac{dz'}{H(z')}, & \text{for a flat Universe,} \\ \sinh \left( \sqrt{|\Omega_k|} \int_0^z \frac{dz'}{H(z')} \right), & \text{for an open Universe.} \end{array} \right.$$  \hspace{1cm} (11)

We note that this observable depends on the expansion history of the Universe from $z$ to the present epoch, and recall that, although most of the available supernovae have $z < 1$, there is a handful with values of redshift up to $z \simeq 1.8$. For illustration purposes, henceforth both data and predictions of cosmological models will be expressed as residuals, $\Delta \mu$, from the empty-Universe prediction (Milne’s model, $\Omega_M = 0$). The analysis involves minimisation of the standard $\chi^2$ function with respect to the cosmological model parameters, as they are introduced via the Hubble expansion rate in Eqs. (7), (8) and (9), for the Q-cosmology, $\Lambda$CDM and super-horizon models, respectively.

The WV07+HST dataset, which amounts to a sample of 192 supernovae in total, is shown in Fig. 1 with the respective measurement errors. This sample combines all observed high-redshift supernovae, most of them detected by the HST. The predictions of the cosmological models under study are also displayed for the best-fit parameter values.

For the standard $\Lambda$CDM model, two cases are investigated: a spatially flat Universe, where (10) is used to calculate the luminosity distance and the general case of a curved Universe, where (11) is employed. The best-fit parameter values, the $1\sigma$ errors and the corresponding $\chi^2$ values for the Riess07 (gold and gold+silver) and the WV07+HST datasets are listed in Table 1. The confidence limits obtained in the ($\Omega_M, \Omega_A$) plane are shown in Fig. 2 together with other astrophysical constraints which will be discussed in the following sections. The fits to the three datasets are compatible and in agreement with those presented elsewhere [4, 5, 29]. In both cases investigated —for a spatially flat and for a curved Universe— the SN data provide stringent constraints on the cosmological parameters.

For the super-horizon model [9], the results of the fits to the three supernova datasets are presented in Table 1. The favoured parameter values for these datasets are compatible and in full agreement with those found in an earlier analysis [21]. The goodness of fit for this model is comparable with the one for the standard $\Lambda$CDM model, as shown by comparing the ratio of the $\chi^2$ to the number of degrees of freedom ($dof$).
Figure 1: Residual magnitude of the WV07+HST dataset supernovae [29] versus the redshift. Cosmological model predictions are drawn: (i) Milne’s Universe (black solid line); (ii) Matter-only Universe, $\Omega_M = 1$ (black dashed-dotted line); (iii) $\Lambda$CDM model for $(\Omega_M, \Omega_\Lambda) = (0.33, 0.85)$ (red long-dashed line); (iv) super-horizon model for $\Psi_{\ell 0} = -0.90$ (blue short-dashed line); and (v) Q-cosmology for $\delta = 4$, $\Omega_\Lambda = -2.8$ and $\Omega_\delta = 0.86$ (green thick solid line).

| Dataset          | Flat Universe        | Curved Universe       |
|------------------|----------------------|-----------------------|
|                  | $\Omega_M$           | $\chi^2$              | $\chi^2$/dof |
| Riess07:gold     | $0.308 \pm 0.020$    | 160                    | 0.88        |
| Riess07:gold+silver | $0.295 \pm 0.017$   | 369                    | 1.43        |
| WV07+HST         | $0.259 \pm 0.019$    | 196                    | 1.02        |
|                  | $(\Omega_M, \Omega_\Lambda)$ | $\chi^2$              | $\chi^2$/dof |
|                  | $(0.48, 0.95)$       | 156                    | 0.87        |
|                  | $(0.55, 1.10)$       | 356                    | 1.39        |
|                  | $(0.33, 0.85)$       | 195                    | 1.03        |

Table 1: Fits to the $\Lambda$CDM model parameter $\Omega_M$, assuming a flat Universe, and to parameters $\Omega_M$ and $\Omega_\Lambda$, when a curved one is considered. The values favoured by the Riess07:gold, the Riess07:gold+silver and the WV07+HST datasets are listed, together with the corresponding $\chi^2$ values.

| Dataset          | $\Psi_{\ell 0}$      | $\chi^2$              | $\chi^2$/dof |
|------------------|-----------------------|-----------------------|--------------|
| Riess07:gold     | $-0.71 \pm 0.05$      | 167                    | 0.92         |
| Riess07:gold+silver | $-0.78 \pm 0.05$    | 383                    | 1.48         |
| WV07+HST         | $-0.90 \pm 0.07$      | 200                    | 1.05         |

Table 2: Fits to the super-horizon model parameter $\Psi_{\ell 0}$. The values favoured by the Riess07:gold, the Riess07:gold+silver and the WV07+HST datasets are listed, together with the corresponding $\chi^2$ values.

In the same manner, Q-cosmology models are compared against the recent supernova data, including the off-shell effects, which are parametrised in the Hubble rate [7]. In general, this
The parametrisation of the model is described by three parameters, $\delta$, $\Omega_\delta$ and $\Omega_3$, which serve as free parameters of the fitting function to be determined by the data analysis. The results for a simple case where the parameter $\delta$ is kept fixed and equal to four are listed in Table 3. The value $\delta = 4$ is chosen to match the results of similar previous studies in Q-cosmology [21] for the exotic scaling of dark matter contributions, induced by dilatons, and dark radiation in brane models without dilatons. The current analysis shows that the Q-cosmology model fits equally well the supernova data as the $\Lambda$CDM model. The results for the Riess07:gold and the WV07+HST datasets are consistent at the 1$\sigma$ level. The Riess07:gold+silver dataset yields results compatible to the Riess07:gold sample, but slightly different to the WV07+HST. Nevertheless, this does not constitute a significant discrepancy, since the ‘silver’ sample comprises astrophysical objects whose classification as type-Ia supernovae is uncertain [4]. The estimated parameters are compatible with those derived in previous studies [21].

| Dataset          | $\Omega_3$   | $\Omega_4$  | $\Omega_2$  | $\chi^2$ | $\chi^2$/dof |
|------------------|--------------|--------------|--------------|-----------|-------------|
| Riess07:gold     | $-3.4 \pm 0.8$ | $1.2 \pm 0.3$ | $3.2 \pm 0.9$ | 157       | 0.87        |
| Riess07:gold+silver | $-4.5 \pm 0.6$ | $1.61 \pm 0.25$ | $3.9 \pm 0.7$ | 357       | 1.39        |
| WV07+HST         | $-2.8 \pm 0.5$ | $0.86 \pm 0.22$ | $2.9 \pm 0.5$ | 195       | 1.02        |

Table 3: Fits to the Q-cosmology parameters $\Omega_3$ and $\Omega_4$ for a fixed value $\delta = 4$. The values favoured by the Riess07:gold, the Riess07:gold+silver and the WV07+HST datasets are listed, together with the corresponding $\chi^2$ values. $\Omega_2$ is determined by the other densities so that $\Omega_4 + \Omega_3 + \Omega_2 = 1$.

The analysis was subsequently performed treating $\delta$ as a free parameter. In Fig. 3, the
minimum $\chi^2$ achieved for fixed values of $\delta$ leaving $\Omega_3$ and $\Omega_4$ unconstrained is plotted versus the chosen $\delta$. Despite the fairly low $\chi^2$ value achieved ($\chi^2$/dof = 1.029), the goodness of fit does not vary strongly with $\delta$. For the WV07+HST dataset, the minimum value of $\chi^2$ is $\sim 194.5$, hence the 68% confidence interval is defined by the $\chi^2 < 198$ condition, corresponding to a quite large range of favoured values for $\delta$. This feature (the weak dependence of the fit results on $\delta$) is also evident when the analysis is repeated for the Riess07:gold sample (cf. Fig. 3 inset), where the best-fit value for $\delta$ shifts from $\delta \simeq 7$ (WV07+HST dataset) to $\delta \simeq 3.1$. The special case of $\delta = 3$, yielding a $\chi^2$ of $\sim 181$ (Riess07:gold dataset), is of no relevance to the Q-cosmology model, corresponding to a generic matter-dominated Universe with no cosmological constant, already disfavoured by astrophysical data. To recapitulate, Q-cosmology models fit the supernovae data very well, with the analysis yielding a strong correlation between the cosmological parameters rather than providing a concrete determination of them. The latter may be achieved in conjunction with other astrophysical data, as we shall discuss in Section 4 where confidence intervals for this analysis will be presented.

![Figure 3: Minimum $\chi^2$ and $\chi^2$/dof for the Q-cosmology model and the WV07+HST SN dataset for various values of the $\delta$ parameter. Inset: the same for the Riess07:gold dataset.](image)

### 3.2 $H(z)$ constraints from distant galaxies

We now attempt to further constrain the cosmological models under study by performing fits to the Hubble parameter dependence on redshift, $H(z)$, as determined [23, 24] using measurements of differential ages of passively evolving galaxies following the method described in Ref. [30]. This method was applied on a sample of distant galaxies consisted by the Gemini Deep Deep Survey (GDDS) [31] and archival data [32], spanning a redshift range of $0 < z < 2$. The ratio $H(z)/H_0$ of these Hubble-rate measurements to the Hubble constant $H_0 = (73.5 \pm 3.2) \text{ Mpc}^{-1} \text{ km s}^{-1}$, measured by WMAP3 [6], is shown in Fig. 4. The point at $z \sim 0.1$ was derived [24] by the Luminous Red Galaxy (LRG) sample reported in the Sloan Digital Sky Survey (SDSS) [33]. For comparison, the points derived by the latest supernovae data released by the HST [4] are also plotted, even though they were not taken into account in the fits discussed in the following.

Here, the Hubble parameter as a function of the redshift, given in Eqs. (7), (8) and (9) for
Figure 4: Reduced Hubble parameter $H(z)/H_0$ as measured by SDSS (square) [24] and distant galaxies (triangles) [23] and for the $H_0$ value measured by WMAP3 [6]. The predictions for the following cosmological models are drawn: (i) $\Lambda$CDM model for $(\Omega_M, \Omega_\Lambda) = (0.2, 0.7)$ (red long-dashed line); (ii) super-horizon model for $\Psi_{\ell 0} = -0.56$ (blue short-dashed line); and (iii) Q-cosmology model for $\delta = 4$, $\Omega_3 = -1$ and $\Omega_\delta = 0.25$ (green thick solid line). The measurements from supernovae data [4] are also shown (circles).

the three cosmological models, is directly fitted to the aforementioned data points. The model predictions for the best-fit parameter values are given in Fig. 4 for the three models.

The $\Lambda$CDM model fits the data very well: if a spatially flat Universe is assumed, a value of $\Omega_M = 0.27 \pm 0.04$ is derived with $\chi^2 = 6.47$ ($\chi^2$/dof = 0.809), while in the general case of a curved Universe, the best-fit values are $\Omega_M = 0.2 \pm 0.3$ and $\Omega_\Lambda = 0.7 \pm 0.6$ with $\chi^2 = 6.46$ ($\chi^2$/dof = 0.923). The $H(z)$ curve for the latter parameters is shown in Fig. 5 and the fit result with the corresponding $1\sigma$-error is superimposed on the SN results in Fig. 2. It is evident that this measurement provides a quite loose constraint on the $\Lambda$CDM model with errors of the order of 15% (100%) for a flat (curved) Universe and it merely confirms the supernova analysis results.

On the other hand, the $H(z)$ measurement provides a stringent constraint on the super-horizon model parameter: $\Psi_{\ell 0} = -0.56 \pm 0.09$ with a fairly good fit of $\chi^2 = 6.98$ ($\chi^2$/dof = 0.872). The respective $H(z)$ curve is drawn in Fig. 5. This $\Psi_{\ell 0}$ value, however, is more than $2\sigma$ higher than the one favoured by the high-redshift supernovae of the WV07+HST dataset, presented in Table 2. An elaborate discussion on this model follows in the next section and in Sec. 4 in view of the baryonic oscillations data.

For the Q-cosmology model, the same procedure as for the supernovae is followed. For a fixed value of $\delta = 4$, the best-fit parameters are $\Omega_3 = 0.2 \pm 0.3$ and $\Omega_\delta = -1.0 \pm 1.0$ with $\chi^2 = 7.57$ ($\chi^2$/dof = 1.08). The fit is not as good as for the other two models, nevertheless it is still acceptable with a ratio of $\chi^2$ over number of degrees of freedom near unity (see also the Q-cosmology prediction in Fig. 4). The errors on the parameters are large, but—as will shall see in Sec. 4—they are correlated and the parameters are eventually sufficiently constrained especially when combined with other observational sources.

Likewise, the analysis was repeated treating $\delta$ as a free parameter. The evolution of the minimum $\chi^2$ obtained for various values of $\delta$ (and the corresponding best-fit values for $\Omega_3$ and
The global minimum value of $\chi^2 = 7.35$ ($\chi^2$/dof = 1.2) occurs at $\delta \approx 3$, however as for the supernovae, the quality of fit does not change dramatically with $\delta$, the 68% confidence level being at $\chi^2 = 10.9$. This outcome is further discussed in Sec. 4 in conjunction with other astrophysical data.

**Figure 5:** Minimum $\chi^2$ and $\chi^2$/dof for the Q-cosmology model and the galactic $H(z)$ determination for various values of the $\delta$ parameter.

The fits to all three models were likewise repeated for the same set of $H(z)$ points with the addition of the three points measured by HST. The derived model parameters and the goodness of fit were not significantly different than the ones reported above, since the HST points and their errors lie along the line connecting the galactic measurements.

### 3.3 Baryon acoustic oscillations

Complementary information on cosmological models is provided by data on Baryon Acoustic Oscillations (BAO), which show up in the patterned distribution of galaxies on very large scales ($\gtrsim 100$ Mpc). The SDSS [7] survey covers the region $0.16 < z < 0.47$, i.e. the typical redshift is $z_{\text{BAO}} = 0.35$. Among other observables, the SDSS collaboration has measured the dilation scale, $D_V$, defined as [7]:

$$D_V(z) = \left[ \frac{D_M^2(z) c z}{H(z)} \right]^{1/3},$$

(12)

where $D_M$ is the co-moving angular diameter distance at redshift $z$. Equation (12) presupposes a single-scale approximation in the redshift-to-distance conversion and an equivalent treatment between the line-of-sight and the transverse dilation. These assumptions are valid for the fiducial $\Lambda$CDM model for $(\Omega_M, \Omega_\Lambda) \simeq (0.3, 0.7)$ for redshifts around 0.35 [7]. They equally apply to the Q-cosmology model under discussion, as the Hubble parameter at $z \simeq 0.35$ does not deviate substantially from the fiducial $\Lambda$CDM, as shown in Fig. 4. The measured value of the dilation scale at $z_{\text{BAO}} = 0.35$ is $D_V(0.35) = 1370 \pm 64$ Mpc [7].

Additionally, the observable $\Omega_M h^2$, where $h$ is the reduced Hubble constant, has been measured by SDSS: $\Omega_M h^2 = 0.130 \pm 0.010$. Cosmological models predictions are consequently performed by the SDSS collaboration [7] against the observable $A$, defined as:

$$A \equiv D_V(z_{\text{BAO}}) \sqrt{\frac{\Omega_M H_0^2}{c z_{\text{BAO}}}},$$

(13)
with a measured value of $A = 0.469 \pm 0.017$. For a constant equation of state of dark energy, the quantity $A$ is expressed as [34]:

$$A = \sqrt{\Omega_M} \left[ \frac{z_{BAO}}{\Omega_k E(z_{BAO})} \right]^{1/3} \times \begin{cases} \sin^{2/3} \left( \sqrt{|\Omega_k|} \int_0^{z_{BAO}} \frac{dz}{E(z)} \right), & \text{for a closed Universe}, \\ \left[ \sqrt{|\Omega_k|} \int_0^{z_{BAO}} \frac{dz}{E(z)} \right]^{2/3}, & \text{for a flat Universe}, \\ \sinh^{2/3} \left( \sqrt{|\Omega_k|} \int_0^{z_{BAO}} \frac{dz}{E(z)} \right), & \text{for an open Universe}. \end{cases}$$

(14)

where $E(z) \equiv H(z)/H_0$ is the reduced Hubble parameter.

The impact of the measurement of observable $A$ on the $\Lambda$CDM model is depicted at 1σ level on top of other constraints in Fig. 2. As shown also in Ref. [34], BAO impose a further constraint (besides the observations of type-Ia supernovae) on the $\Lambda$CDM model parameters, $\Omega_M$ and $\Omega_\Lambda$. If a spatially flat geometry for the Universe is assumed, the matter density is determined by observable $A$ to be $\Omega_M = 0.273 \pm 0.025$ [7].

In the Q-cosmology case, as mentioned in Sec. 2, the ordinary and dark matter component appears in the Hubble parameter $H(z)$ mixed with exotic contributions. Therefore, the measurement of $\Omega_M h^2$ —and of the observable $A$ consequently— does not constrain the Q-cosmology model parameters $\Omega_3$ and $\Omega_\delta$. Nonetheless, it does set limits to the sum of the matter constituents of present-day densities $\Omega_3$ and $\Omega_\delta$. The predictions of Q-cosmology are hence compared against another observable, $B$, involving the dilation scale $D_V$ reduced so as to remove the dependence on the current value of the Hubble constant, $H_0$:

$$B \equiv \frac{H_0}{c} D_V(z_{BAO}) = \left[ \left( \int_0^{z_{BAO}} \frac{dz}{E(z)} \right)^2 \frac{z_{BAO}}{E(z_{BAO})} \right]^{1/3}.$$  

(15)

For a Hubble constant $H_0 = (73.5 \pm 3.2) \text{ Mpc}^{-1} \text{ km s}^{-1}$, as measured by WMAP3 [6] —also employed in the distant galaxies analysis—, we derive a measured value of $B = 0.334 \pm 0.021$. Although, the BAO observations may not provide a determination of the Q-cosmology parameters, nevertheless, as we shall see in Sec. 4 they constrain such models when combined with other astrophysical data.

As far as the super-horizon model is concerned, the independent measurement of $\Omega_M h^2$ [7] by BAO, implying a matter density $\Omega_M \approx 0.27$, practically excludes it, since it predicts a matter density equal to unity [9]. Yet the model parameter space allowed by observable $B$ (devoid of any dependence on $\Omega_M$), shown in Fig. 6 (blue curve), marginally agrees with the supernovae-favoured value for $\psi_{\ell 0}$.

### 4 Discussion and interpretation

For the conventional $\Lambda$CDM model, where a redshift-independent equation of state $w_{DE} = -1$ for the dark energy component is assumed, the constraints imposed by supernovae, distant galaxies and baryonic oscillations are reviewed in Table 4 and in Fig. 2 as well. The determination of the Hubble parameter by high-redshift galaxies loosely constrain the model parameters. The other two astrophysical observations, on the other hand, set a combined parameter determination around $(\Omega_M, \Omega_\Lambda) \approx (0.28, 0.75)$, as shown in Fig. 2.
Figure 6: Observable $B$ prediction versus $\Psi_{10}$ for the super-horizon model. The horizontal lines show the measured value of $B$ ($\pm 1\sigma$); the left-hand (right-hand) vertical band shows the value and $\pm 1\sigma$ error favoured by the supernovae (distant galaxies) data.

| Data                          | Flat Universe | Curved Universe |
|-------------------------------|---------------|-----------------|
|                               | $\Omega_M$    | $\chi^2$ $\chi^2$/dof | $(\Omega_M, \Omega_\Lambda)$ | $\chi^2$ $\chi^2$/dof |
| Supernovae (WV07+HST)        | 0.259 $\pm$ 0.019 | 196            | 1.02                         | (0.33, 0.85) | 195            | 1.03                         |
| Distant galaxies ($H(z)$)    | 0.27 $\pm$ 0.04 | 6.47           | 0.809                        | (0.2, 0.7)   | 6.46           | 0.923                        |
| Baryon oscillations ($A$)     | 0.273 $\pm$ 0.025 | $-$           | $-$                          | see Fig. [2] | $-$           | $-$                          |

Table 4: $\Lambda$CDM model parameter $\Omega_M$, assuming a flat Universe, and parameters $\Omega_M$ and $\Omega_\Lambda$ for a curved one resulting from type-Ia supernovae (WV07+HST), distant galaxies ($H(z)$) and baryon acoustic oscillations ($A$).

The results for the super-Hubble model, which is based on cosmological perturbations without introducing any dark energy component, are summarised in Table 4 and Fig. 6. The type-Ia supernovae data and the galactic $H(z)$ determination set incompatible constraints at $2\sigma$ level on the model sole parameter, $\Psi_{10}$. Furthermore, baryon oscillation observations lead to an independent measurement of $\Omega_M \approx 0.27$, as opposed to the model-predicted unity value for the matter density. The $\Psi_{10}$ values which are consistent with the observable $B$ measurement, on the other hand, are barely in agreement with the supernova fit result. In conclusion, the super-horizon model appears over-constrained when confronted by different sources of astrophysical observations and it is excluded by this analysis.
We now proceed on discussing the (off-shell) Q-cosmology models. The allowed parameter regions are shown in Fig. 7 for various values of the parameter $\delta$ related to the exotic scaling of dark matter: $\delta = 3.7, 4.1, 4.3$. All three astrophysical constraints are presented: the WV07+HST supernovae (68.3% and 99.7% C.L.), the distant galaxies (68.3% and 99.7% C.L.) and the observable $B$ of BAO (1$\sigma$). The $(\Omega_3, \Omega_\delta)$ region for which the parametrisation (7) for the Hubble parameter is not valid (i.e. $H^2(z) < 0$) for $0 < z < 2$ is also indicated. The constraints from supernova data largely overlap and lie in parallel with those from baryon acoustic oscillations. This stems from the fact that the corresponding observables —luminosity distance $d_L$ and dilation scale $D_V \propto B$— are both sensitive to $\int_0^z \frac{dz}{H(z)}$. However, the typical redshifts $z$ for these measurements are quite different: $z_{SN} = 2$ for the supernovae as opposed to $z_{BAO} = 0.35$ for BAO, rendering independent bounds.

The parameter $\delta$ may be restricted by considering the overlap of all three cosmological constraints. For values $\delta \lesssim 3.3$, the 68% confidence intervals for BAO and SN do not overlap. Furthermore, the 1$\sigma$ contours for the SN and the galaxies partially coincide only for $\delta \lesssim 4.3$. Hence, the allowed range for $\delta$ is fairly wide: $3.3 \lesssim \delta \lesssim 4.3$. The approximate values for the other two parameters, $\Omega_3$ and $\Omega_\delta$, as defined by the overlapping regions of the 1$\sigma$ intervals for all three astrophysical data are given in Fig. 8 as a function of $\delta$. These values represent the trend of the densities $\Omega_i$ with varying $\delta$ rather than a precise determination of model parameters (hence the thick curves). To recapitulate, the allowed ranges for the Q-cosmology parameters defined in the parametrisation (7), as restricted by current type-Ia supernovae, high-redshift galaxies and baryon acoustic oscillations are: $3.3 \lesssim \delta \lesssim 4.3$, $-7 \lesssim \Omega_3 \lesssim -1.5$ and $0.2 \lesssim \Omega_\delta \lesssim 4.5$ with the correlations defined in Fig. 8.

The negative-energy dust contribution appears essential in order to provide fits to the astrophysical data, and, as we have stressed previously, it may not be inconsistent with positive-energy theorems of the underlying microscopic cosmological model. In brane-world scenarios, for instance, such negative-energy dust might represent effective four-dimensional contributions to the energy density of KK-compactification (massive) graviton modes, as measured by an observer on the brane, while the corresponding bulk energy-density contributions are perfectly positive definite. In Q-cosmology models, such negative energies might also be due to the coupling with the non constant dilaton-quintessence field, including higher-string-loop terms, again without implying a conflict with positive-energy conditions. In addition to this negative dust energy, the data also provide evidence for (positive) exotic-scaling contributions to the energy density of the form $a^{-\delta}$. The value of $\delta = 4$ is included in the phenomenologically allowed range. Such terms may therefore be interpreted as either dark radiation, in the context of brane models [11], or exotic dark-matter contributions in Q-cosmologies [14, 15, 20], due to the coupling with non-trivial quintessence dilatons, or even both types in the case of brany

| Data                        | $\Psi_{i0}$ | $\chi^2$ | $\chi^2$/dof |
|-----------------------------|-------------|----------|--------------|
| Supernovae (WV07+HST)       | $-0.90 \pm 0.07$ | 200      | 1.05         |
| Distant galaxies ($H(z)$)   | $-0.56 \pm 0.09$ | 6.98     | 0.872        |
| Baryon oscillations ($B$)    | $< -0.8$    | -        | -            |

Table 5: Super-horizon model parameter $\Psi_{i0}$, rendered by type-Ia supernovae (WV07+HST), distant galaxies ($H(z)$) and baryon acoustic oscillations ($B$).
Figure 7: Observational constraints on Q-cosmology for $\delta = 3.7, 4.1, 4.3$ from top to bottom, respectively: (a) 68.3% and 99.7% C.L. contours for the WV07+HST supernova dataset (blue solid line); (b) 68.3% and 99.7% C.L. contours for the galactic $H(z)$ (red dashed line); and (c) 1σ region from the observable $B$ of baryon acoustic oscillations (yellow area).

Figure 8: Astrophysical data-favoured values of Q-cosmology parameters as defined from the overlap of the 1σ regions from supernovae, distant galaxies and baryon oscillations data.

non-critical-string dilaton-quintessence models [12].

Stringent constraints on these brane and off-shell Q-cosmological models may be set by the
WMAP measurements [6] of the cosmic microwave background (CMB), especially the position of the (acoustic) peaks in the spectrum. Nevertheless, since CMB provides evidence on the last scattering surface at a redshift of $z \sim 1100$, more phenomenological input is required in order to embark on the pertinent analysis. Reversely, astrophysical data analysis provides a handle to exclude regions in parameter space and to guide the phenomenological studies.

5 Conclusions and outlook

In this paper, we have performed an analysis on data collected from three distinct astrophysical sources, namely type-Ia supernovae ($z \lesssim 2$), distant galaxies ($z \lesssim 2$) —via the determination of the Hubble expansion rate— and baryon acoustic oscillations observed in luminous red galaxies ($0.1 \lesssim z \lesssim 0.5$). We compared them with the respective predictions of the conventional ΛCDM model [8], the super-Hubble model [9] and the Q-cosmology [12–15, 21]. The ΛCDM model, as expected, fits the data very well and is mainly constrained by the supernova and BAO data. The super-horizon model, on the other hand, is excluded by the BAO-measured matter density and yields contradictory (at 2σ level) parameter determinations by SN and galactic data.

Finally, the spatially-flat Q-cosmology predictions, as parametrised in (7) for $z \lesssim 2$, fits the data very well providing an alternative scenario to account for the dark energy component of the Universe. The three model parameters are not univocally determined; their allowed range and the correlation between them is rather defined. The parameter $\delta$, associated to the exotic scaling of dark matter in the context of Q-cosmology, is restricted in the range $3.3 \lesssim \delta \lesssim 4.3$, whilst the present values of (exotic) matter densities, $\Omega_3$ and $\Omega_\delta$, vary with $\delta$ as shown in Fig. S.

The fact that the value of $\delta = 4$ is included in the allowed range, also points towards dark radiation terms in brane models [11]. The data also point towards negative-energy dust contributions, which could be due to either dark-energy dilaton terms in Q-cosmologies [12,21], or Kaluza-Klein compactification (massive) graviton modes in brane-inspired modes [22].

Further detailed studies, such as the theoretical determination of the position of the acoustic peaks in the CMB spectrum, using the underlying formalism of the above models, or the measurement of the pertinent (complicated, in general $z$-dependent) equation of state, as, for instance, in the analysis of Refs. [23,35], are certainly required in order to provide further evidence that might discriminate among the various models. The important point that our analysis, however, brought hopefully forward, is the fact that alternative, unconventional Cosmologies are certainly compatible with the current data, and appear on an equal footing to the standard ΛCDM cosmological model. Thus, surprises on the possibility of discovering completely new and unconventional physical phenomena in the foreseeable future cannot be excluded.

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