Are there bound states in the continuum in a dielectric ring?

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Abstract. Trapping and confining electromagnetic waves is important in both basic research and a variety of applications. For these purposes, various physical mechanisms are exploited including bound states in the continuum, which have been actively investigated recently. Bound states in the continuum have been observed in various objects consisting of both one and a number of dielectric structures. In particular, these photonic states were observed in high-contrast dielectric cylinders in the regime of strong eigenmode coupling, which leads to destructive interference in the far-field zone. In this article, we present the results of a study of bound states in a continuum in a dielectric ring, i.e. cylinder with coaxial air hole. The dependence of the quality factor Q on the normalized diameter of the hole is discussed.

1. Introduction

Bound states in the continuum (BIC) is a general wave phenomenon that was predicted in 1929 by von Neumann and Wigner [1] for electronic states and further considered for various types of waves such as electromagnetic, acoustic, elastic, and water waves. Photonic BIC coexist with extended electromagnetic waves and lie within the continuum, but theoretically remain completely confined without any radiation [2-4]. Theoretically, BICs are states with an infinite lifetime and can be realized if at least one dimension of the structure extends to infinity [2] or in systems of finite length, when the permittivity approaches zero [5, 6]. Unfortunately, in reality there are no such structures suitable for experiment. However, to simulate an ideal BIC one can use a resonator with a set of eigenmodes and change its geometric and dielectric parameters until the system most closely satisfies the condition required for the BIC in the corresponding infinite system. This quasi-BIC is achieved when the Q-factor no longer changes slowly as in a conventional resonator, but instead rises rapidly, following the BIC trend, before it reaches its maximum value, due to the finite size of the resonator [7].

To date, quasi-BICs have been observed in a large number of photonic objects, both single and consisting of a set of separate structures. In this case, of particular interest is the case when a quasi-BIC is observed in the simplest geometric object, for example, a single homogeneous finite-length dielectric cylinder [8, 9]. It was demonstrated that a quasi-BIC arises in a cylinder in accordance with the Friedrich-Wintgen theory [10], which implies a strong interaction of two eigenmodes of the structure. When changing the parameters of the cylinder, the damping rate of one mode increases, while the damping rate of the other mode decreases and at a minimum value a quasi-BIC is achieved. It has been demonstrated that a multitude of quasi-BIC associated with strong interactions between Mie and Fabry–Perot -like modes are observed in the cylinder at various geometric parameters [8, 9].

In this article, we continue the search for quasi-BIC in the simplest dielectric objects, which are bodies of revolution. We transform the cylinder into a ring by adding a coaxial air hole to the high-index dielectric cylinder and gradually increasing the radius of the hole. The results of numerical
calculations are presented that demonstrate the regularities of the change in the quasi-BIC with the expansion of the hole in the cylinder and the change in the eigenmodes of the dielectric ring.

2. Results and discussion
We calculate the scattering cross-section spectra of the dielectric cylinder with a coaxial air hole as a function of its aspect ratio R/L for a set of hole radius r normalized to the cylinder radius r/R starting from r = 0, that is, a solid cylinder. Here we analyze the modes excited by a TE-polarized plane wave and chose the value of the dielectric permittivity $\varepsilon = 80$ following Refs. [8,9].

**FIGURE 1.** Scattering cross-section spectra of the dielectric cylinder (a) and the ring (cylinder with a coaxial air hole, r/R=0.2) (b) as a function of aspect ratio R/L in the region of the avoided resonance crossing between the modes TE$_{1,1,0}$ and TM$_{1,1,1}$. The dielectric permittivity of both structures $\varepsilon = 80$.

The modes shown in Fig. 1 have the symmetry TE$_{1,1,0}$ (slow-varying Mie-like mode) and the symmetry TM$_{1,1,1}$ (rapidly-varying Fabry-Perot-like mode) [8,9]. Here, we employ the standard mode classification [11], the first index $m$ corresponds to the azimuthal number of the mode, whereas other two indices $k$ and $n$ enumerate the Mie and Fabry-Perot resonances, respectively. Due to different spectral shifts of the Mie and Fabry-Perot modes when the aspect ratio varies R/L, the modes with the same azimuthal index $m$ could interact, and they undergo coupling each other with the avoid crossing at special values of the parameter R/L with a dramatic narrowing of the high-frequency line, which corresponds to the quasi-BIC, as shown earlier [8,9].

The calculation results show that the TE$_{1,1,0}$ and TM$_{1,1,1}$ modes do not change dramatically in the presence of a hole in a sufficiently wide range of the parameter r/R ≤ 0.2, as can be seen from the comparison of the spectra in the panels (a) and (b) in Fig. 1, as well as from the comparison of the field distribution in the cylinder and the ring, Fig. 2. From Fig. 2, the type of the resonance mode is also well defined with a change in the parameter R/L. Below the region of the avoid crossing, the mode belongs to the Mie-type (the field changes in the xy plane), and above this region, the mode belongs to the Fabry-Perot type (the change in the field along the z axis). In the region of the quasi-BIC, the mode has a hybrid character, changing both in the xy plane and along the z axis.
FIGURE 2. The calculated distribution of the electric field amplitude $|E|$ in the $(x,z)$ cross sections (resonator side view) for the high-frequency branch of the interacting pair of modes $\text{TE}_{1,1,0}$ and $\text{TM}_{1,1,1}$. The left column is the field in the cylinder, the right column is the field in the ring, $r/R=0.2$. The parameter $R/L$ is indicated.

FIGURE 3. The evolution of the Q-factor for the high-frequency branch of the interacting pair of modes $\text{TE}_{1,1,0}$ and $\text{TM}_{1,1,1}$ in the region of the quasi-BIC. The dependence from the normalized hole size $r/R$.

Figure 3 contains the main result of this work, demonstrating the dependence of the Q-factor of the quasi-BIC on the hole size. With an increase in the parameter $r/R$, two effects are clearly observed: the shift in the position of the quasi-BIC and a decrease in the value of the Q-factor by only 20%. Thus, we have demonstrated a robust quasi-BIC state in a cylinder which can endure a loss of dielectric material of 20% or more.
3. Conclusion

Our results demonstrate that we can give a positive answer to the question: are there bound states in the continuum in a dielectric ring? Based on the calculations performed, we can draw a not the most obvious conclusion. Quasi-BIC turn out to be robust when a coaxial air hole is introduced into the structure of a high-contrast cylinder in a rather wide range of values of the normalized hole radius $r/R$. With an increase in the parameter $r/R$ to the value of 0.2, the Q-factor of the quasi-BIC decreases. The result obtained can find application in various resonant devices, the principle of operation of which is based on the use of the quasi-BIC.

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