A review of Hybrid/combined methods for trajectory optimization of flight vehicles

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Abstract. This work aims to review the proposed trajectory optimization algorithms and their applications in solving flight trajectory planning and optimal control problems. Beside the classification of trajectory optimization algorithms listed in Chai R, Savvaris A, Tsourdos A and Chai S such as the deterministic; the stochastic; and the multi-objective algorithms, this work adds in reviewing the effective and frequently-used approach for solving optimal problems—the hybrid/combined methods. The results of related works show that this hybrid approach is effective and can provide feasible solutions for solving the flight vehicle trajectory optimal design and control problems.

1. Introduction
The trajectory optimization problems are very important and necessary to be solved in many fields of the real-world. In aviation techniques, particularly in the modern automatic Air Traffic Management (ATM) systems, the flight trajectory becomes the fundamental element of a new set of operating procedures collectively referred to as Trajectory Based Operations (TBO). Trajectory Based Operations (TBO) is an ATM method for strategically planning, managing and optimizing flights by using time-based management, using information exchange between air and ground systems, the aircraft’s ability to fly precise paths in time and space. TBO calls for full integration of flight information in order to create a synchronized view of flight data by all stakeholders involved. This method has shown significant benefits in terms of enhanced safety; enhanced security; enhanced predictability; reduced fuel consumption and emissions. This has encouraged a great interest in the application of trajectory optimization techniques in commercial aviation.

In recent years, trajectory optimization problems have attracted significant attention. A large number of models and algorithms for solving these problems have been published in many works and researches.

2. Trajectory Optimization algorithms and applications
Some of the most effective trajectory optimization approaches from related literature are mentioned in following subsections. In this work don’t list all the trajectory optimization algorithms under each class, and only some important examples in application are reviewed.
2.1. Deterministic optimization algorithms and applications

A deterministic algorithm is an algorithm that always produce the same output for a particular input, if machine always passing through the same sequence of states. Some popular deterministic algorithms used in recent publics and works are listed as following: Sequential quadratic programming (SQP); Interior-point method (IP); Interior-point sequential quadratic programming (IPSQP); Linear programming (LP); Second-order cone programming (SOCP); Semi-definite programming (SDP); Dynamic programming (DP); Differential dynamic programming (DDP); Stochastic differential dynamic programming (SDDP).

The application of these stochastic algorithms can be found in many published works: Ong, Shaw Yang; Liu W, Ma S, Sun M, Yi H, Wang Z and Chen Z; Graham K F, Rao A V chose SQP for the computationally accurate and inexpensive numerical method in trajectory optimization by changing the performance index, model, and constraints with relatively little reprogramming. Laurent-Varin J, Bonnans F, Berend N, Haddou M, Talbot C applied the IP algorithm to solve the flight trajectory design problems. Chai R, Savvaris A, Tsourdos A, Chai S, Xia Y [1] proposed the two-step IPSQP approach to control the iteration of the inner loop so that the QP sub-problem does not need to be exactly solved. Liu X, Lu P, Pan B applied LP to increase the effectiveness of their optimization solution. Liu X, Shen Z, Lu P used SOCP with a combination of successive linearization and relaxation techniques to provide a good balance between the approximation accuracy and the computational complexity. Mueller J B, Griesemer P R, Thomas S J presented the SOCP method to produce the optimal trajectory of the aircraft by reformulating the non-convex collision avoidance constraints, the navigation uncertainties into convex constraints during the optimization phase. Helton J, Nie J applied the SDP to solve the trajectory optimization and optimal control problems. Hongying W, Nayibe Chio C, Bonadi H, Lumlong Z, Mora-Camino F [2] proposed a reverse DP based on the Bellman’s principle solution technique to develop a new trajectory management capability for an engine-out transportation aircraft. Colombo C, Vasile M, Radice G; Aziz J D, Parker J S, Scheeres D J and Englander J A applied the DDP method to calculate the rendezvous trajectory to near-Earth objects and solve the optimal problem relatively. Ozaki N, Campagnola S, Funase R, Yam C H [3] introduced a modified version of SDDP to compute nominal trajectories with disturbances.

2.2. Stochastic Optimization Algorithms and applications

Some popular stochastic algorithms used to solve trajectory optimization problems: Genetic algorithm (GA); Differential evolution (DE); Violation learning differential evolution (VLDE); Particle swarm optimization (PSO); Predator–prey pigeon-inspired optimization (PPPIO); Ant colony (AC); Artificial bee colony (ABC); Simulate annealing (SA); Tabu search (TS).

The application of these stochastic algorithms can be found in many published works, such as: Englander J A, Conway B A applied an automated method based on GA and monotonic basin hopping to solve a launch aircraft interplanetary trajectory problem. Patrón R S F and Botez R M implemented GA to reduce the total number of calculated trajectories compared to an exhaustive search. Elsayed S M, Sarker R A, Essam D L proposed an improved DE algorithm that uses a mix of different mutation operators and a covariance adaptation matrix evolution strategy algorithm for solving well-known benchmark and various real-world optimization problems. Chai R, Savvaris A, Tsourdos A used VLDE to enhance the quality of the solution and reduce the number of Newton iteration and computational time. Pontani M, Conway B A applied PSO technique to determining optimal continuous-thrust rendezvous trajectories in a rotating Euler–Hill frame. Zhang B, Duan H proposed a novel PPPIO to solve the uninhabited combat aerial vehicle (UCAV) three-dimension path planning problem in the dynamic environment. Ceriotti M, Vasile M established an AC based optimization algorithm similar to plan a multi-phase space aircraft orbital flight trajectory. Radice G, Olmo G presented
an initial analysis of the effectiveness and usefulness of applying an AC algorithm to a simple, two-impulse Earth–Mars transfer in identifying local and global optima. Duan H, Li S proposed an ABC–based direct collocation method for reentry trajectory optimization. Dai W, Zhang J, Delahaye D, Sun X used an improved SA algorithm to optimize aircraft trajectories. Lu P, Khan M A introduced a continuous SA algorithm as a new global trajectory optimization tool for non-smooth dynamic systems. Nobahari H, Haeri A [4] introduce the combination of continuous ant colony system and Tabu search optimization algorithms to generate the optimal predictive commands of line-of-sight (LOS) guidance law.

2.3. Multi-objective Optimization Algorithms and applications
Multi-objective trajectory optimization (MOTO) algorithms can be grouped into evolutionary and transcription methods.

2.3.1. Multi-objective evolutionary algorithms
Multi-objective evolutionary algorithms: Non-dominated sorting genetic algorithm-II (NSGA-II); Improved non-dominated sorting genetic algorithm-II (I-NSGA-II); Non-dominated sorting genetic algorithm-III (NSGA-III); Multi-objective evolutionary algorithm Based on decomposition (MOEA/D); Multi-objective particle swarm optimization (MOPSO); Trajectory competitive multi-objective particle swarm optimization (TCMOPSO); Multi-objective adaptive particle swarm optimization (MOAPSO); Multi-objective adaptive gradient particle swarm optimization (MOAGPSO); Multi-objective cellular particle swarm optimization (MOCPSO); Multi-objective artificial bee colony (MOABC); Niched Pareto genetic algorithm (NPGA); Strength Pareto particle swarm optimization (SPPSO); Adaptive differential evolution and modified game theory (ADEMG).

The application of these stochastic algorithms can be found in many published works, such as: Chai R, Savvaris A, Tsourdos A, Chai S applied an extended NSGA-II algorithm by introducing a new constraint handling strategy to solve the trajectory planning optimization problems. Chai R, Savvaris A, Tsourdos A, Chai S, Xia Y used the I-NSGA-II for comparing multi-objective problem (MOP) to multi-objective evolutionary algorithms (MOEAs). Chai R, Savvaris A, Tsourdos A, Chai S, Xia Y presented a specific multiple-shooting discretization technique with the newest NSGA-III optimization algorithm and constructs a new evolutionary optimal control solver. Jiang S, Yang S proposed a new Evolutionary Dynamic Multi-objective Optimization that is able to tune a number of challenging characteristics in many real-world optimization problems. Xue B, Zhang M, Browne W N presented the first study on MOPSO for feature selection. Antonakis A, Nikolaidis T and Pilidis P proposed a new MOPSO to solve the aircraft climb path optimization problem. Han H, Lu W, Qiao J developed an AMOPSO algorithm, based on a hybrid framework of the solution distribution entropy and population spacing (SP) information to improve the search performance in terms of convergent speed and precision. Han H, Wu X, Zhang L, Tian Y, Qiao J [5] designed an AGMOPSO-SORBF algorithm to find a trade-off between the accuracy and the complexity of radial basis function (RBF) neural networks. Zheng J, Luand C, Gao L developed MOCPSO with an adaptive neighborhood function and the three objective functions to gain a set of Pareto optimal solutions that are beneficial for a less risky and less costly wellbore trajectory design option. Akbari R, Hedayatzadeh R, Ziarati K, Hassanizadeh B proposed the MOABC algorithm to optimize problems with multiple objectives by using a grid-based approach to adaptively assess the Pareto front maintained in an external archive. Adibo M A presented a NPGA-based method to solve the non-linear multi-objective environmental/economic dispatch (EED) problem with competing, non-commensurable cost and emission objectives. Elhossini A, Areibi S, Dony R [used SPPSO to handle multi-objective
optimization problems by building three hybrid EA-PSO algorithms. Chai R, Savvaris A, Tsourdos A, Xia Y, Chai S [6] formulated an ADEMGT to generate the Pareto front of the multi-objective trajectory optimization problem.

2.3.2. Multi-objective transcription algorithms

Multi-objective transcription algorithms: Weighted sum (WS), Physical programming (PP), Fuzzy physical programming (FPP), Interactive physical programming (IPP), Interactive fuzzy physical programming (IFPP), Goal programming (GP), Fuzzy goal programming (FGP), Fuzzy satisfactory goal programming (FSGP), Adaptive surrogate model (ASM).

The application of these stochastic algorithms can be found in many published works, such as: Hu X, Wong K K, Yang K, Zheng Z aimed to minimize the weighted sum (WS) energy consumption of the unmanned aerial vehicle (UAV) and user equipment. Messac A developed a new effective and computationally efficient PP method for design optimization. Chai R, Savvaris A, Tsourdos A introduced a FPP method for solving multi-objective Space Maneuvre Vehicles (SMV) skip trajectory optimization problem based on hp-adaptive pseudo-spectral methods. Tappeta R V, Renaud J E, Messac A, Sundararaj G J developed an IPP framework that takes into account the designer’s or the decision maker’s (DM’s) preferences during the optimization process, and allows for design exploration at a given Pareto design. Chai R, Savvaris A, Tsourdos A, Xia Y provided an effective IFPP algorithm to solve the trajectory hopping problem. Suzuki S and Yoshizawat T applied the GP formulation to modify ill-defined problems as multi-objective design problems. Chen L H, Tsai F C formulated FGP incorporating different importance and preemptive priorities by using an additive model to maximize the sum of achievement degrees of all fuzzy goals. Chai R, Savvaris A, Tsourdos A, Chai S, Xia Y [7] introduced an extended optimization approach, named the FGP-GHGA that not rely on the designer’s physical understanding of the problem. Hu C F, Xin Y used a FSGP method to deal with the multi-objective reentry trajectory optimization problem. Wang W, Peng H proposed an ASM-based multi-objective optimization method as a fast multi-objective design method for emergency spacecraft trajectory design mission.

2.4. Hybrid/Combined optimization methods and applications

In reviewed published papers and works, the Combined/Hybrid optimization methods are shown to increase effectiveness and accuracy in optimization of dynamic flight vehicles trajectory during the whole flight phase with uncertainties and nonlinearities. Some of the most popular Combined/Hybrid optimization methods applied in publishes works are:

- **Combined method of EMD and ICA**: Yang D, Wang L, Hu W, Ding C, Gan W, Liu F [8] processed the signals from Fiber-Optic Gyro (FOG) by using median filtering (MF), Empirical Mode Decomposition (EMD) and the combined method of EMD and Independent Component Analysis (ICA). The combined EMD and ICA method has shown the best performance on trajectory optimization and error suppression with almost no distortion.

- **Combined method of GPM and GA**: Peng Q [9] proposed an optimization method combined a collocation method-Gauss Pseudospectral Method (GPM) and Genetic Algorithm (GA) to solve the problem.

- **Combined method of Genetic algorithm and Calculus of variations**: Esmaeelzadeh R [10] proposed a combination between a genetic algorithm with calculus of variations to optimize an interplanetary trajectory for the Bryson-Ho Earth-to-Mars orbit transfer problem.

- **Combined method of UB-LSTM and behavior recognition**: Xiao H, Wang C, Li Z, Wang R, Bo C, Sotelo M A and Xu Y [11] proposed a Unidirectional and Bidirectional LSTM (UB-LSTM) vehicle trajectory prediction model combined with behavior recognition to make an accurate prediction of vehicle trajectory in dynamic environment.
• **Hybrid method of GPM and natural computation algorithm:** A hybrid trajectory optimization method consisting of Gauss pseudo-spectral method (GPM) and natural computation algorithm has been developed and utilized by Yang Y and Nan Y [12] to solve multiphase return trajectory optimization problem, where a phase is defined as a subinterval in which the right-hand side of the differential equation is continuous.

• **Hybrid combination of indirect and direct optimization methods:** Jimenez-Lluva D, Root B [13] proposed an optimization method of low-thrust many-revolution trajectories, employing a hybrid combination of indirect and direct optimization methods. This research constitutes a significant advancement for space mission design and satellite operations, because it simultaneously harnesses the advantages of indirect and direct methods with broader flexibility than the popular indirect approaches and enhanced functionality than the former hybrid methods published in literature.

• **Hybrid genetic algorithm collocation method:** Subbarao K, Shippey B M [7] applied a hybrid GA collocation method for direct optimization of trajectories using the Hermite–Legendre–Gauss–Lobatto collocation method based on a simple genetic algorithm.

• **Hybridization of a stochastic based search approach:** Vasilie M [14] presented a hybridization of a stochastic based search approach for multi-objective optimization with a deterministic domain decomposition of the solution space. The proposed general formulation of the optimization problem is suitable to describe both single and multi-objective problems. The stochastic approach, based on behaviorism, combined with the decomposition of the solutions pace was tested on a set of standard multi-objective optimization problems and on a simple but representative case of space trajectory design. This hybridization increases the robustness (i.e. repeatability of the result) and the convergence accuracy at the same time.

• **Hybrid Multi-Objective method:** Zhang H, Peng Y, Hou L, Tian G and Li Z [15] formulated a Hybrid multi-objective method by combining multi-objective artificial bee colony (MOABC), best worst (BW) method, grey relational analysis (GRA) and visekriterijumsko kompromisno rangiranje (VIKOR) to solve the structural optimization problem for energy-absorbing structures in train collisions.

• **Combination of the Collocation and Control Parameterization approaches:** The proposed hybrid method in [16] is a combination of the Collocation and Control Parameterization approaches for obtaining solutions to a class of trajectories that are typical to the Mars Sample Return mission.

• **Combination of the trajectories based on GA:** Xu L, Liu H, Yan X, Liao S and Zhang X [17] presented a combined method to optimize trajectory in generating video synopses. The methodology applies genetic algorithm (GA) and the temporal combination ways of trajectories to deal with the optimization problem of motion trajectory combination. Experiment results verify the superiority of the trajectories combination based on GA is better than simulated annealing both in information loss, time consumption and easy manipulation by computers.

3. Conclusion

The main characteristics of some deterministic, stochastic algorithms, multi-objective optimization methods and different types of hybrid/combined techniques were reviewed and presented. Different types of hybrid/combined techniques were outlined and verified to be effective for solving the flight vehicles optimal control problems. Finally, a focus was given on the recent development of the combined/hybrid optimization methods, which more accurately and effectively solve the real-world nonlinear dynamic flight vehicles trajectory optimization problems with uncertainties. According to the fast development in computational methods...
using powerful machines, the combined/hybrid optimization methods will take advantages for improvement in terms of applying these techniques in flight vehicles trajectory optimization problems.

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