Two-loop RGEs with Dirac gaugino masses

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Abstract

The set of renormalisation group equations to two-loop order for general supersymmetric theories broken by soft and supersoft operators is completed. As an example, the explicit expressions for the RGEs in a Dirac gaugino extension of the (N)MSSM are presented.

1 Introduction

Models with Dirac gaugino masses are attractive for a number of reasons. From a top-down model-building perspective, this is because they preserve R-symmetry, and so allow for simple supersymmetry-breaking sectors. This has attracted much interest in the literature [1-29]. On the other hand, if gauginos are found at the LHC, it must be determined whether they are of Majorana or Dirac type [25,30–33]. Moreover, with the current results from the LHC, Dirac mass terms allow the preservation of naturalness by having lower bounds on the gluino and squark masses in the Majorana case (they are “super-safe” [27]); by increasing the Higgs mass due to additional couplings [4,5,7,19]; and because they do not cause the Higgs masses to run as strongly [4,14,34].

Particularly out of a desire to study the naturalness and Higgs sectors of such theories, it is important to know the renormalisation group equations (RGEs) for them. The purpose of this paper is to complete the set of RGEs to two-loop order.

The standard soft supersymmetry-breaking mass terms are well known:

\[
\mathcal{L}_{\text{standard}} = -\frac{1}{2}(m^2)^{ij}_{\phi_i \phi_j} - \frac{1}{2}M^A_{\lambda A} \phi_A - \frac{1}{2}B^{ij}_{\phi_i \phi_j} - \frac{1}{6}A^{ijk}_{\phi_i \phi_j \phi_k} + \text{h.c.}
\] (1.1)

where the \( \phi_i \) are the scalars of chiral multiplets \( \Phi_i = \phi_i + \sqrt{2}(\bar{\psi}_i + ...) \) and \( \phi^i \equiv (\phi_i)^\ast \); \( \lambda_A \) are gauginos. The above includes Majorana gaugino masses \( M \). However, Dirac gaugino masses fall under the category of “non-standard” soft terms:

\[
\mathcal{L}_{\text{non-standard}} = -t^i_{\phi_i} - m^A_{\psi_i \lambda A} - \frac{1}{2} r^{ij}_{\phi_i \phi_j \phi_k} + \text{h.c.}
\] (1.2)

General choices of these terms will lead to quadratic divergences in singlet tadpoles, and this has often led to the terms being neglected or only included in theories without singlets. However, to give a Dirac mass to the Bino, a singlet superfield must be included, so it is

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necessary to worry about this issue. On the other hand, when supersymmetry is spontaneously broken, the expectation is that no quadratic divergences should be generated, and indeed it is generically found that only supersoft operators are generated:

\[
L_{\text{supersoft}} \supset \int d^2\theta \sqrt{2} m_D^{iA} \theta^i \Phi_i W_{A\alpha} + \text{h.c.} \\
\supset - m_D^{iA} \psi_\alpha \lambda_A + \sqrt{2} m_D^{iA} \phi_i D_A + \text{h.c.},
\]

where \( D_A \) is the D-term of the gauge group to which the adjoint couples with adjoint index \( 'A' \). They lead to a particular structure of non-standard soft terms such that the quadratic divergences exactly cancel. These interestingly augment supersymmetric trilinear couplings; including a superpotential

\[
W = L^i \Phi_i + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j + \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k
\]

the non-standard trilinear couplings are

\[
\begin{align*}
  r_{ij}^k &= Y^{ijk} \mu_{mk} + \sqrt{2} g (m_D^{iA} (t^A)^k_j + m_D^{jA} (t^A)^k_i) \\
  r_{ij}^k &= Y^{ijk} \mu_{mk} + \sqrt{2} g (m_{DiA} (t^A)^k_j + m_{DjA} (t^A)^k_i)
\end{align*}
\]

where \((t^A)^i_j\) are the generators of the gauge group. Of course, the supersymmetric terms do not generate quadratic divergences – these are cancelled by fermion loops – and only receive wavefunction renormalisation. The would-be quadratic divergences from the non-supersymmetric, supersoft, piece are not cancelled by fermion loops but instead vanish when they are all summed.

If there is a Fayet–Iliopoulos (FI) term \( \xi_Y \) (where \( Y \) denotes a \( U(1) \) index) then a contribution to the tadpole is generated of

\[
\Delta t^a = \sqrt{2} m_D^{aY} \xi_Y.
\]

Typically, however, any FI term generated can be absorbed into the soft masses; this equation shows that in the presence of Dirac gaugino mass terms it should also be absorbed into a shift of the tadpole. Interestingly, there is also a supersymmetric term that emulates a tadpole equal to \( L \supset - \mu^{ij} L_j \phi_i + \text{c.c.} \); of course this only has wavefunction renormalisation, just as for the trilinear terms above.

An important point is that the supersoft operator also generates contributions to the standard soft breaking terms via the self-coupling: from integrating out the auxiliary \( D \)-field there are terms

\[
L \supset - (m_D^{iA} \phi_i + m_{DiA} \phi^i) (m_D^{jA} \phi_j + m_{DjA} \phi^j)
\]

and so

\[
\begin{align*}
\Delta (m^2)^i_j &= 2 m_D^{iA} m_{DjA} \equiv 2 (m_D^{ii})^i_j \\
\Delta B^{ij} &= 2 m_D^{iA} m_D^{jA} \equiv 2 (m_D^{jj})^{ij}.
\end{align*}
\]

The two-loop renormalisation group equations (RGEs) for standard SUSY-breaking terms were derived some time ago [35–37], and later the generic RGEs for non-standard soft
terms were calculated in the absence of singlets. They also found that restricting the non-standard terms to only be generated from supersoft operators defined a renormalisation group invariant trajectory, explained by the holomorphic nature of the supersoft operator. This means that equations (1.5) and (1.8) are true at any renormalisation scale. The supersoft operator only obtains wavefunction renormalisation, so its beta-function is

\[ \beta_{m_{D}} = \gamma_{j} m_{D} + \frac{\beta_{g}}{g} m_{D} \]  

where \( \gamma_{j} \) is the anomalous dimension of the adjoint superfield, and \( \beta_{g} \) is the beta-function for the gauge coupling. Thus in a theory with Dirac gaugino mass terms, the RGEs for the standard soft terms can be found and evolved ignoring the Dirac gaugino mass, and then at the scale of interest the shifts (1.8) can be applied to find the physical masses. In a theory without gauge singlets, this is then enough to determine all of the RGEs in the theory. However, when there are singlets – such as when there is a Dirac mass for the Bino – the RGE for the tadpole is also required, which is a non-standard term so may depend upon the Dirac gaugino mass, and not just via the above shifts. Considering that the singlet superfield may couple to the Higgs via a term

\[ W \supset \lambda_{S} S H_{u} \cdot H_{d} \]  

it is clear that knowing the size of the singlet tadpole (indeed, ensuring that it is not too large, since it is not protected by R symmetry for example) is vital in order to investigate electroweak symmetry breaking and determine the Higgs mass. The main result of this paper is to determine these RGEs to two-loop order.

In section 2 the result is presented, with an explanation of how the different terms arise. The method used is that of Martin and Vaughn, deriving the RGE for a tadpole in a general renormalisable theory from the expressions given in [40–43], then specialising to the softly broken supersymmetric case, transforming from \( \overline{\text{MS}} \) to \( \overline{\text{DR}}' \). To do this the rules given in [44] will be used, and augmented with a new rule for Dirac gaugino mass terms. In section 3 and the appendix the RGEs are derived for a minimal Dirac gaugino extension of the supersymmetric standard model with rather general couplings.

2 Tadpole RGEs

There are several ways to derive the RGEs for softly broken supersymmetric models: by diagrams in component fields; by supergraphs; by RG invariance of the effective potential; or by translating the results from a general renormalisable theory into the broken supersymmetric case. This last approach is the one adopted here, although the one-loop result was checked via the effective potential method.

2.1 Tadpole in non-supersymmetric theories

The first step in calculating the tadpole is to write down the expression in general non-supersymmetric theories. This can be derived using spurions from the RGEs for the quartic
coupling in a general renormalisable theory given in [40–43]. Such a theory with real scalars \( \phi_a \) and complex fermions \( \psi_i \) has couplings

\[
\mathcal{L} \supset -\frac{1}{24} \lambda_{abcd} \phi_a \phi_b \phi_c \phi_d - \frac{1}{6} h_{abc} \phi_a \phi_b \phi_c - \frac{1}{2} m_{ab}^\alpha \phi_a \phi_b - t_a \phi_a
\]

\[-\frac{1}{2} \left[ (m_f)_{ij} (\psi_i \psi_j) + (Y^a)_{ij} \phi_a (\psi_i \psi_j) + h.c. \right] \tag{2.1}\]

in addition to a gauge coupling \( g \).

The one-loop tadpole RGE is found to be

\[
(4\pi)^2 \beta_{t_a} = 2\kappa Y_2(S)^{ab} t_b + h_{ae} m_{ef}^2 \left( 8g^2 C_2^{fg} - 4\kappa Y_2(S)^{fg} \right) - \frac{g^2}{2} h_{ae} h_{eg} h_{fg} - g^2 m_{ef}^2 \lambda_{aeg} h_{fg} + 4\kappa \left( 2\Pi^\lambda_a + H^Y_a + 2\Pi^{Y^\dagger}_a + 2H^3_a - g^2 H^F_a \right) \tag{2.2}\]

where \( \kappa = 1/2 \) for Weyl fermions (or 1 for Dirac fermions) and

\[
Y_2(S)^{ab} \equiv \frac{1}{2} \text{Tr} (Y^a Y^b + Y^{\dagger b} Y^a). \tag{2.3}\]

The two-loop tadpole RGE is

\[
\beta_{t_a} = (\gamma_S^{(2)})^{b}_{a} t_b
\]

\[
+ h_{ae} m_{ef}^2 \left( 8g^2 C_2^{fg} - 4\kappa Y_2(S)^{fg} \right) - \frac{g^2}{2} h_{ae} h_{eg} h_{fg} - g^2 m_{ef}^2 \lambda_{aeg} h_{fg} + 4\kappa \left( 2\Pi^\lambda_a + H^Y_a + 2\Pi^{Y^\dagger}_a + 2H^3_a - g^2 H^F_a \right) \tag{2.4}\]

where now \( C_2^{fg} \) is the quadratic casimir of the gauge group for the representation carried by fields \( f \) and \( g \), and

\[
(\gamma_S^{(2)})^{b}_{a} \equiv \frac{1}{24} \lambda_{acde} \lambda_{bcde} - \frac{3}{2} \kappa \text{Tr} (Y^a Y^{\dagger b} Y^{\dagger c} Y^{\dagger c}) - \kappa \text{Tr} (Y^a Y^{\dagger b} Y^{\dagger c} Y^{\dagger c}) + 5\kappa g^2 \text{Tr} (C_2 Y^a Y^{\dagger b}) + c.c.
\]

\[
\Pi^\lambda_a = 2h_{ae} \text{Tr} (m_f Y^{\dagger e} m_f Y^{\dagger f}) + 4m_{ef}^2 \text{Tr} (Y^{\dagger e} Y^{\dagger f} m_f Y^{\dagger f}) + c.c.
\]

\[
H^Y_a = \text{Tr} (Y_2(F) m_f Y^{\dagger e} m_f Y^{\dagger f}) + 2Y^{\dagger e} m_f Y^{\dagger f} Y^a Y^{\dagger f} m_f + Y^{\dagger e} m_f Y^{\dagger f} m_f Y^{\dagger a} m_f + c.c.
\]

\[
H^3_a = \text{Tr} (Y^a m_f Y^{\dagger e} m_f Y^{\dagger e}) + c.c.
\]

\[
H^F_a = 4\text{Tr} (C_2 Y^a m_f m_f Y^{\dagger e}) + c.c. \tag{2.5}\]

Here

\[
Y_2(F)_{ij} = (Y^{\dagger a} Y^a)_{ij}. \tag{2.6}\]

### 2.2 Translating from \( \overline{\text{MS}} \) to \( \overline{\text{DR}} \)

To specialise the above expressions to the supersymmetric case, they must be transformed to a complex basis (by summing repeated indices over both raised and lowered indices alternately) and insert the SUSY couplings. These can be written as block diagonal matrices,
with the top row/left column corresponding to gauge indices, and bottom row/right column matter indices. The Yukawa matrices become

\[
Y_i = \sqrt{2}g \left( \begin{array}{cc} 0 & (t^A)^i_j \\ (t^A)_j^i & 0 \end{array} \right) \quad \quad Y_i^\dagger = \sqrt{2}g \left( \begin{array}{cc} 0 & (t^A)_j^i \\ (t^A)^i_j & 0 \end{array} \right)
\]

\[
Y^i = \left( \begin{array}{cc} 0 & 0 \\ 0 & Y^i_{jk} \end{array} \right) \quad \quad Y^i_i = \left( \begin{array}{cc} 0 & 0 \\ 0 & Y_{ij} \end{array} \right)
\]

(2.7)

where \((t^A)^i_j\) are the gauge generators. With the definitions

\[
(Y_2)^a_b \equiv Y^{acd}Y_{bcd}
\]

\[
S_{26}^{AB} = (t^A)^j_i(t^B)^i_j
\]

(2.8)

then

\[
Y_2(S)^a_b \rightarrow (Y_2)^a_b + 4g^2C_2 g^f
\]

\[
Y_2(F) \rightarrow \left( \begin{array}{ccc} 2g^2S_2 & 1 & 0 \\ 9 & 2g^2C_2 + Y_2 \end{array} \right).
\]

(2.9)

The fermion mass terms then become

\[
m_f = \left( \begin{array}{ccc} M & 1 & m^{iA}_{D} \\ m^{jB}_{D} & \mu^{ij} \end{array} \right).
\]

(2.10)

The one-loop corrections from translating from \(\overline{\text{MS}}\) to \(\overline{\text{DR}'}\) only modify the fermionic part. Specialising to the case of interest, where ‘\(a\)’ is a singlet index, the corrections to the couplings are [11]:

\[
\mu \rightarrow \mu + \frac{g^2}{32\pi^2}[C_2(i) + C_2(j)]\mu_{ij} = \frac{g^2}{32\pi^2}\{C_2, \mu\}
\]

\[
M \rightarrow M + \frac{g^2}{16\pi^2}C_2(G)M = \frac{g^2}{32\pi^2}\{C_2, M\}
\]

\[
Y^\dagger_a \rightarrow Y^\dagger_a \left[ 1 + \frac{g^2}{32\pi^2}[C_2(j) + C_2(k)] \right]
\]

\[
= Y^\dagger_a + \frac{g^2}{32\pi^2}\{C_2, Y^\dagger_a\}
\]

\[
Y_a \rightarrow 0.
\]

(2.11)

For general Yukawa couplings (not involving a singlet) there are additional contributions; there is also a shift for general quartic couplings. However, they will not be relevant here.

For the Dirac gaugino mass, there is a similar transformation derived via the same technique:

\[
m_{DiA} \rightarrow m_{DiA} + \frac{g^2}{16\pi^2}C_2(G)m_{DiA} = m_{DiA} + \frac{g^2}{32\pi^2}\{C_2, m_{DiA}\}
\]

(2.12)

and hence

\[
m_f \rightarrow m_f + \frac{g^2}{32\pi^2}\{C_2, m_f\}.
\]

(2.13)

Note that there is no difference for the scalar trilinear couplings \(r^{ij}_k, r_{ij}^k\) between \(\overline{\text{MS}}\) and \(\overline{\text{DR}'}\), just as there is none for the couplings \(A^{ijk}\).
2.3 Result

The expressions can now be transformed to the SUSY basis and the shifts applied as described in subsection 2.2 to the expressions for the non-susy tadpole RGEs in subsection 2.1 to obtain the RGEs for the tadpole in the theory described by equations (1.1), (1.2) and (1.3). After a large amount of tedious algebra, the result for the tadpole beta-functions can be simply written as

\[ \beta_{\eta_i}^{(i)} = X_{S}^{(i)} + X_{\xi}^{(i)} + X_{D}^{(i)} \]  

(2.14)

where \( i \) is the loop order, \( X_{S}^{(i)} \) is the tadpole beta-function involving only standard soft terms, given by

\[ (4\pi)^2 X_S^{(1)} = (4\pi)^2 (\gamma^{(1)})_b^j k^b \]

\[ + A_{aej}(B_{ef} + Y_{efp}L_{p}) + Y_{efk}\mu_{ah}B_{ef} + 2Y_{ajk}\mu_{jm}(m^2)^m_k \]  

(2.15)

and

\[ (4\pi)^4 X_S^{(2)} = (4\pi)^4 (\gamma^{(2)})_b^j k^b \]

\[ - A_{aej}A_{egb}Y_{ghm}A_{kme} - Y_{ajm}\mu_{kmg}A_{kme}A_{gf}\mu_{mfe} \]

\[ - 4g^2 A_{aej}(C_2)^i_k(m^2)^m_k \mu_{kfe} - 4g^2 Y_{efp}L_{p}(C_2)^i_k(p^{kj}Y_{ine}) + 8g^2 |M|^2(C_2)^i_k\mu_{jkl}Y_{jkl} \]

\[ - 4g^2 M(B_{ef} + Y_{efp}L_{p})(C_2)^i_kY_{ajf} \]

\[ - Y_{ajm}(Y_{2m}^k)(m^2)^f_k \mu_{kfe} - 2Y_{ajm}Y_{mef}Y_{mn}(m^2)^f_k \]

\[ + (4g^2 C_2 - Y_{2j})^f \left[ Y_{ajm}Y_{mef}Y_{mef} + Y_{ajm}Y_{mef}Y_{mef} \right] \]

\[ + (4g^2 C_2 - Y_{2j})^f \left[ A_{aej}(B_{ef} + Y_{efp}L_{p}) + Y_{efk}\mu_{ah}B_{ef} \right] \]

\[ - Y_{ajm}Y_{mef}Y_{mef} + Y_{efk}\mu_{ah}B_{ef}. \]  

(2.16)

In the above, \( (\gamma^{(1)})_b^j \) and \( (\gamma^{(2)})_b^j \) are the one- and two-loop chiral superfield anomalous dimensions for singlets respectively, given by

\[ (4\pi)^2 (\gamma^{(1)})_b^j = \frac{1}{2}(Y_{2j})^i_k - 2g^2(C_2)^i_k \]

\[ (4\pi)^4 (\gamma^{(2)})_b^j = 2g^2(C_2)^i_k Y_{ajm}Y_{bkl} - \frac{1}{2} Y_{ajm}(Y_{2j})^i_k Y_{mef} \]  

(2.17)

The new terms are

\[ (4\pi)^2 X_{\xi}^{(1)} = 2\sqrt{2g_\gamma}m_D^{\gamma}Tr(Ym^2) \]

\[ (4\pi)^4 X_{\xi}^{(2)} = 2\sqrt{2g_\gamma}m_D^{\gamma}Tr(Ym^2(4g^2C_2 - Y_{2j})) \]  

(2.18)

where \( Y \) is the charge operator of \( U(1)\gamma \), and

\[ (4\pi)^2 X_{D}^{(1)} = \left[ (m_D^2)_{ef}(A_{aej} + MY_{ajm}) + Y_{efk}\mu_{ah}B_{ef} \right] \]

\[ (4\pi)^4 X_{D}^{(2)} = \left[ (\beta_{m_D}^2/m_D^2)^j_g \left[ (m_D^2)_{ef}(A_{aej} + MY_{ajm}) + Y_{efk}\mu_{ah}B_{ef} \right] \right] \]

\[ (\beta_{m_D}^2/m_D^2)^j_g = \left( \frac{1}{2}(Y_{2j})^i_k + g^2(S_2 - 5C_2(G)) \right)^i_k. \]  

(2.19)
The new contributions can be explained as follows. Firstly, the presence of \( X_\xi \) is simply due to the renormalisation of the Fayet–Iliopoulos term. Since the FI term is absorbed as it is generated by the running into the soft term and the tadpole (so \( \xi = 0 \)), via equation (1.9), it is found that

\[
X_\xi = \frac{d}{d\log \mu} (\sqrt{2} m_D^a Y \xi) = \sqrt{2} m_D^a Y \beta Y. \tag{2.20}
\]

The result for the RGE of the FI term is then exactly as found in [15], without any additional piece coming from Dirac gaugino terms. They gave

\[
16\pi^2 \beta_\xi^{(1)} = 2g \text{Tr}[\mathcal{Y}m^2]
\]

\[
16\pi^2 \beta_\xi^{(2)} = -4g \text{Tr}[\mathcal{Y}m^2\gamma^{(1)}]
\]

\[
16\pi^2 (\gamma^{(1)})^j_i = \frac{1}{2} (Y_2)^j_i - 2g^2 (C_2)^j_i
\]

which clearly exactly agrees with the above.

Secondly, the \( X_D \) terms contain two terms mimicking \( B \) insertions – via equation (1.8) – in the supergraph when they do not involve a gauge line, but not \( m^2 \) insertions. The additional term proportional to \( M(m_D)^2 \) is new and could not be obtained from shifting a standard term. Then the two-loop terms involve just wavefunction renormalisation of these.

Since the tree-level RGE for the tadpole only includes a gauge coupling via the FI term \( X_\xi \), the two-loop contributions \( X_S^{(2)} \), \( X_D^{(2)} \) include only one gauge coupling, and so the interpretation of the above result for several gauge groups is straightforward.

3 RGEs of a Dirac Gaugino extension of the (N)MSSM

Here I present the full two-loop RGES for a Dirac gaugino extension of the (N)MSSM. For generality, both Dirac and Majorana masses are included, as are all of the phenomenologically interesting superpotential couplings involving the new adjoint superfields, even those that break R-symmetry (this is motivated by generality and the possibility of generating \( \mu \) and \( B_\mu \) terms [19]). Hence the model encompasses those studied in e.g. [4], [5] and [11,14,19] (the one-loop RGES were presented in [14] without including Majorana gaugino masses).

The particle content of the model is just that of the MSSM extended by adjoint chiral superfields \( S, T, O \) where \( S \) is a singlet, \( T = \frac{1}{2} \left( \begin{array}{cc} T_0 & \sqrt{2} T^+ \\ \sqrt{2} T^- & -T_0 \end{array} \right) \) an SU(2) triplet, and \( O \) a colour octet. The Dirac gaugino masses are denoted \( m_{D1}, m_{D2}, m_{D3} \) coupling the respective singlet, triplet and octet fermions to the gauginos of the corresponding group, with gauge couplings \( g_1, g_2, g_3 \) and Majorana masses \( M_1, M_2, M_3 \).

The superpotential of the model is:

\[
W = Y_u \hat{u} q H_u - Y_d \hat{d} q H_d - Y_e \hat{e} \ell H_d + \mu H_u \cdot H_d + \lambda_S S H_u \cdot H_d + 2\lambda_T T H_u \cdot H_d + L S + \frac{M_S}{2} S^2 + \frac{\kappa}{3} S^3 + M_T \text{tr}(T T) + M_O \text{tr}(O O), \tag{3.1}
\]
the usual scalar soft terms are

\begin{align*}
-\Delta L_{\text{scalar soft}}^{\text{MSSM}} &= [A_u \tilde{u} q H_u - A_d \tilde{d} q H_d - A_e \tilde{e} l H_d + h.c.] \\
&+ m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + |B_u H_u \cdot H_d + h.c.] \\
&+ \tilde{q}^i (m_q^2)^i_j \tilde{q}_j + \tilde{u}^i (m_u^2)^i_j \tilde{u}_j + \tilde{d}^i (m_d^2)^i_j \tilde{d}_j + \tilde{e}^i (m_e^2)^i_j \tilde{e}_j
\end{align*}

(3.2)

and there are soft terms involving the adjoint scalars

\begin{align*}
- \Delta L_{\text{adjoints}}^{\text{soft}} &= (t_S S + h.c.) \\
&+ m_S^2 |S|^2 + \frac{1}{2} B_S (S^2 + h.c.) + 2 m_T^2 \text{tr}(T^\dagger T) + (B_T \text{tr}(TT) + h.c.) \\
&+ [A_S \lambda_S S H_u \cdot H_d + 2 A_T H_d \cdot T H_u + \frac{1}{3} \kappa_A S^3 + h.c.] \\
&+ 2 m_O^2 \text{tr}(O^\dagger O) + (B_O \text{tr}(OO) + h.c.)
\end{align*}

(3.3)

with the definition

\[ H_u \cdot H_d = H_u^+ H_d^- - H_u^0 H_d^0. \]

The most general renormalisable Lagrangian would include additional superpotential interactions

\[ W_2 = \lambda_{ST} \text{Str}(TT) + \lambda_{SO} \text{Str}(OO) + \frac{\kappa_O}{3} \text{tr}(OOO). \]

(3.4)

as well as the equivalent adjoint scalar A-terms. It would be straightforward to add these (and indeed \( \lambda_{ST}, \lambda_{SO} \) would contribute to the singlet tadpole) but they violate R-symmetry (admittedly as do all the terms on the last line of equation 3.1) and, more importantly, are much less phenomenologically interesting than the other terms.

To determine the parameters of the model, the standard soft terms and Dirac gaugino masses can be run according to the RGEs given below, and then at the scale of interest determine the physical soft masses by applying the shifts

\begin{align*}
m_S^2 &\rightarrow m_S^2 + 2 |m_D|^2 \\
B_S^2 &\rightarrow B_S^2 + 2 m_D^2
\end{align*}

(3.5)

and similarly for \( m_T, B_T, m_Q, B_O \).

The standard soft and supersymmetric RGEs, presented in the appendix, were calculated by implementing the model in SARAH [46,47] version 3.0.41. The RGEs for the Dirac mass terms and tadpoles are presented below, using the results of the previous section and the anomalous dimensions of the adjoint superfields.

Note there are no terms \( \text{tr}(T), \text{tr}(O), \text{tr}(TT) \) since these vanish by gauge invariance.
3.1 Dirac gaugino masses

Note \( gy = \sqrt{\frac{3}{5}} g_1 \); then

\[
(4\pi)^2 \beta_{mD1}^{(1)} = m_{D1} \left[ 2 \left( |\kappa_S|^2 + |\lambda_S|^2 \right) + \frac{33}{5} g_1^2 \right]
\]

\[
(4\pi)^4 \beta_{mD1}^{(2)} = m_{D1} \left[ -8|\kappa_S|^4 - 8|\lambda_S|^2 |\kappa_S|^2 \right.
\]
\[
\left. - \frac{2}{5} |\lambda_S|^2 \left( 10 |\lambda_S|^2 - 15 g_2^2 + 15 \text{Tr} \left( Y_a Y_d^\dagger \right) + 15 \text{Tr} \left( Y_a Y_u^\dagger \right) + 30 |\lambda_T|^2 + 5 \text{Tr} \left( Y_e Y_e^\dagger \right) \right) \right]
\]
\[
+ \frac{1}{25} g_1^2 \left( - 130 \text{Tr} \left( Y_u Y_u^\dagger \right) + 135 g_2^2 + 199 g_1^2 + 440 g_3^2 - 70 \text{Tr} \left( Y_d Y_d^\dagger \right) \right)
\]
\[
\left. - 90 |\lambda_T|^2 - 90 \text{Tr} \left( Y_e Y_e^\dagger \right) \right] \right]
\]

\[
(4\pi)^2 \beta_{mD2}^{(1)} = m_{D2} \left[ 2 |\lambda_T|^2 - g_2^2 \right]
\]

\[
(4\pi)^4 \beta_{mD2}^{(2)} = m_{D2} \left[ 28 g_4^2 - 12 |\lambda_T|^4 \right.
\]
\[
\left. + \frac{2}{5} |\lambda_T|^2 \left( - 10 |\lambda_S|^2 - 15 \text{Tr} \left( Y_d Y_d^\dagger \right) - 15 \text{Tr} \left( Y_u Y_u^\dagger \right) + 3 g_1^2 - 5 g_2^2 - 5 \text{Tr} \left( Y_e Y_e^\dagger \right) \right) \right]
\]
\[
+ \frac{1}{5} g_2^2 \left( - 10 |\lambda_S|^2 - 10 \text{Tr} \left( Y_d Y_d^\dagger \right) + 120 g_3^2 + 245 g_2^2 - 30 \text{Tr} \left( Y_d Y_d^\dagger \right) \right)
\]
\[
\left. - 30 \text{Tr} \left( Y_u Y_u^\dagger \right) - 70 |\lambda_T|^2 + 9 g_1^2 \right]
\]

\[
(4\pi)^2 \beta_{mD3}^{(1)} = m_{D3} \left[ - 6 g_3^2 \right]
\]

\[
(4\pi)^4 \beta_{mD3}^{(2)} = m_{D3} \left[ 36 g_4^2 + \frac{1}{5} g_2^2 \left( 11 g_1^2 - 20 \text{Tr} \left( Y_d Y_d^\dagger \right) - 20 \text{Tr} \left( Y_u Y_u^\dagger \right) + 340 g_3^2 + 45 g_2^2 \right) \right]
\]

3.2 Tadpole equation

The one-loop contribution to \( X_S \) is given by

\[
(4\pi)^2 X_S^{(1)} = (2 |\kappa_S|^2 + 2 |\lambda_S|^2) t_S
\]
\[
+ 4 m_S^2 \kappa_S M_S + 2 M_S B_S \kappa_S + 4 M_S B_S \lambda_S + 4 \lambda_S \mu^* (m_H^2 d + m_H^2 u)
\]
\[
+ 4 L_S \kappa_S A_\kappa + 2 B_S^* A_\kappa + 4 L_S \lambda_S^* A_S + 4 B_\mu^* A_S.
\]

The two-loop contribution is much longer and is given in the appendix, equation (3.7).

For the Dirac gaugino contribution arising via the FI term define

\[
\sigma_{1,1} = \sqrt{\frac{3}{5}} g_1 \text{Tr} (Y m^2)
\]
\[
\sigma_{3,1} = \sqrt{\frac{3}{5}} g_1 \text{Tr} (Y m^2 (4 g^2 C_2 - Y_2)).
\]

(3.8)
\( \sigma_{1,1} \) and \( \sigma_{3,1} \) also appear in the scalar mass RGEs; the full expressions for these in this model are given in appendix equation (A.77). Then

\[
(4\pi)^2 X_{\xi}^{(1)} = 2\sqrt{2}m_{D1}\sigma_{1,1} \\
(4\pi)^4 X_{\xi}^{(2)} = 8\sqrt{2}m_{D1}\sigma_{3,1}.
\]

Finally, the Dirac gaugino parts are given by

\[
(4\pi)^2 X_{D}^{(1)} = 4(m_{D1}^*)^2 \left[ A_\kappa + M_1 \kappa_S \right] + 4m_{D1}^2 \kappa_S^* M_S \\
(4\pi)^4 X_{D}^{(2)} = 8 \left[ |\kappa_S|^2 + |\lambda_S|^2 \right] + \frac{33}{5} g_1^2 \left[ (m_{D1}^*)^2 \left( A_\kappa + M_1 \kappa_S \right) + m_{D1}^2 \kappa_S^* M_S \right].
\]

The conclusion is that for R-symmetric soft terms that obey \( \text{Tr}(\mathcal{Y} m^2) = 0 \) at some scale (such as, for example, in gauge mediation with R-symmetric F-terms [9,10]), the new Dirac gaugino mass-dependent contributions to the singlet tadpole will be safely negligible.

### 4 Conclusions

The set of two-loop RGEs for Dirac gaugino models is now complete, which opens up the possibility of implementing them in spectrum generators (such as by an extension of [46, 47]) and studying their precision phenomenology. Since Dirac mass terms only appear in supersoft operators (in spontaneously broken supersymmetric theories) their effect upon the RGEs is extremely mild. In particular, they remain on an RG invariant trajectory with respect to the mass squared and \( B \) terms, but the result of this paper is that, although they contribute new terms to the tadpole RGE, these are exhausted at one-loop and the two-loop correction is merely a particular form of wavefunction renormalisation of the one-loop term.

As mentioned in the introduction, the tadpole RGE is particularly important for studying the Higgs potential, and since the new Higgs couplings in Dirac gaugino models can allow a natural increase in the Higgs mass – alleviating fine-tuning – it is now imperative to study the Higgs sector of Dirac gaugino models including full loop corrections [48].

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\section*{A Two-loop RGEs for SUSY and standard soft terms}

In this appendix the RGEs for the standard soft terms in the model of section 3 are presented. For brevity, the factors of $(4\pi)^2$ and $(4\pi)^4$ for one- and two-loop quantities shall be dropped. All of the below were generated using SARAH \cite{sarah} version 3.0.41 and so are presented in the notation of that package.

\subsection*{A.1 Anomalous Dimensions}

\begin{align}
\gamma_{\text{q}}^{(1)} &= -\frac{1}{30} \left(45g_2^2 + 80g_3^2 + g_1^2\right) 1 + Y_d^T Y_d + Y_u^T Y_u \\  \gamma_{\text{q}}^{(2)} &= \left(8g_2^2g_3^2 + \frac{1}{90}g_1^4 \right) + \frac{199}{900} g_1^4 + \frac{27}{4} g_2^4 + \frac{64}{9} g_3^4 \right) 1 + \frac{4}{5} g_1^2 Y_d^T Y_u - |\lambda_S|^2 Y_d^T Y_u \\
&- 3|\lambda_T|^2 Y_d^T Y_u - 2Y_d^T Y_d Y_d Y_u - 2Y_u^T Y_u Y_u Y_u \\
&+ Y_d^T Y_d \left(-3|\lambda_T|^2 - 3 \text{Tr} \left(Y_d Y_d^T\right) + \frac{2}{5} g_1^2 - |\lambda_S|^2 - \text{Tr} \left(Y_u Y_u^T\right)\right) - 3Y_u^T Y_u \text{Tr} \left(Y_u Y_u^T\right) \tag{A.1}
\end{align}

\begin{align}
\gamma_{\text{l}}^{(1)} &= -\frac{3}{10} \left(5g_2^2 + g_1^2\right) 1 + Y_e^T Y_e \\  \gamma_{\text{l}}^{(2)} &= +\frac{9}{100} \left(10g_2^2g_3^2 + 23g_1^4 + 75g_3^4\right) 1 - 2Y_d^T Y_d Y_e Y_e \\
&+ Y_e^T Y_e \left(-3|\lambda_T|^2 - 3 \text{Tr} \left(Y_d Y_d^T\right) + \frac{6}{5} g_1^2 - |\lambda_S|^2 - \text{Tr} \left(Y_u Y_u^T\right)\right) \tag{A.2}
\end{align}

\begin{align}
\gamma_{\text{H}_d}^{(1)} &= 3|\lambda_T|^2 + 3 \text{Tr} \left(Y_d Y_d^T\right) - \frac{3}{10} g_1^2 - \frac{3}{2} g_2^2 + |\lambda_S|^2 + \text{Tr} \left(Y_e Y_e^T\right) \tag{A.3}
\end{align}

\begin{align}
\gamma_{\text{H}_d}^{(2)} &= -\frac{2}{5} g_1^2 \text{Tr} \left(Y_d Y_d^T\right) + 16 g_2^2 \text{Tr} \left(Y_d Y_d^T\right) + \frac{6}{5} g_3^2 \text{Tr} \left(Y_e Y_e^T\right) - 9 |\lambda_T|^2 \text{Tr} \left(Y_u Y_u^T\right) \\
&- 3|\lambda_S|^2 \left(2|\lambda_T|^2 + \text{Tr} \left(Y_u Y_u^T\right)\right) - 9 \text{Tr} \left(Y_d Y_d^T Y_d Y_d^T\right) - 3 \text{Tr} \left(Y_d Y_d^T Y_u Y_u^T\right) - 3 \text{Tr} \left(Y_e Y_e^T Y_e Y_e^T\right) \tag{A.4}
\end{align}

\begin{align}
\gamma_{\text{H}_u}^{(1)} &= 3|\lambda_T|^2 - \frac{3}{10} \left(-10 \text{Tr} \left(Y_u Y_u^T\right) + 5g_2^2 + g_1^2\right) + |\lambda_S|^2 \tag{A.5}
\end{align}

\begin{align}
\gamma_{\text{H}_u}^{(2)} &= +\frac{207}{100} g_1^4 + \frac{9}{10} g_1^2 g_2^2 + \frac{27}{4} g_2^4 + 12 g_3^2 |\lambda_T|^2 - 2 \lambda_S |\kappa_S|^2 |\lambda_S^T| - 3 |\lambda_S|^2 |\lambda_S^T| - 15 |\lambda_T|^2 |\lambda_S^T| \\
&- 9 |\lambda_T|^2 \text{Tr} \left(Y_d Y_d^T\right) - 3 |\lambda_T|^2 \text{Tr} \left(Y_u Y_u^T\right) - |\lambda_S|^2 \left(3 \text{Tr} \left(Y_d Y_d^T\right) + 6 |\lambda_T|^2 \text{Tr} \left(Y_u Y_u^T\right)\right) \\
&+ \frac{4}{5} g_1^2 \text{Tr} \left(Y_u Y_u^T\right) + 16 g_2^2 \text{Tr} \left(Y_u Y_u^T\right) - 3 \text{Tr} \left(Y_d Y_d^T Y_u Y_u^T\right) - 9 \text{Tr} \left(Y_u Y_u^T Y_u Y_u^T\right) \tag{A.6}
\end{align}

\begin{align}
\gamma_{\text{d}}^{(1)} &= 2Y_d^T Y_d^T - \frac{2}{15} \left(20g_3^2 + g_1^2\right) 1 \tag{A.7}
\end{align}

\begin{align}
\gamma_{\text{d}}^{(2)} &= +\frac{2}{225} \left(101g_1^4 + 800g_3^4 + 80g_1^2 g_3^2\right) 1 - 2 \left(Y_d^T Y_d^T Y_d^T Y_d^T + Y_u^T Y_u^T Y_u^T Y_u^T\right) \\
&+ Y_d^T Y_d^T \left(-2 |\lambda_S|^2 - 2 \text{Tr} \left(Y_u Y_u^T\right) + 6g_2^2 - 6 |\lambda_T|^2 - 6 \text{Tr} \left(Y_d Y_d^T\right) + \frac{2}{5} g_1^2\right) \tag{A.8}
\end{align}
\[ \gamma_\alpha^{(1)} = 2Y_\alpha^*Y_\alpha^T - \frac{8}{15}\left(5g_3^2 + g_1^2\right)1 \]  \hspace{1cm} (A.11)

\[ \gamma_\alpha^{(2)} = -\frac{8}{225} \left(107g_1^4 + 200g_3^4 + 80g_1^2g_3^2\right)1 \]
\[ - \frac{2}{5} \left(5\left(Y_\alpha^*Y_\alpha^T - Y_\alpha^*Y_\alpha^T + Y_\alpha^*Y_\alpha^T\right) + Y_\alpha^*Y_\alpha^T \right) \left( -15g_2^2 + 15|\lambda_T|^2 + 15 \text{Tr}\left(Y_aY_a^\dagger\right) + 5|\lambda_S|^2 + g_1^2 \right) \]  \hspace{1cm} (A.12)

\[ \gamma_\epsilon^{(1)} = 2Y_\epsilon^*Y_\epsilon^T - \frac{6}{5}g_1^21 \]  \hspace{1cm} (A.13)

\[ \gamma_\epsilon^{(2)} = -\frac{234}{25}g_1^41 \]
\[ - 2Y_\epsilon^*Y_\epsilon^T Y_\epsilon^*Y_\epsilon^T \]
\[ + Y_\epsilon^*Y_\epsilon^T \left( -2|\lambda_S|^2 - 2\text{Tr}\left(Y_\epsilon Y_\epsilon^\dagger\right) + 6g_2^2 - 6|\lambda_T|^2 - 6 \text{Tr}\left(Y_\epsilon Y_\epsilon^\dagger\right) - \frac{6}{5}g_1^2 \right) \]  \hspace{1cm} (A.14)

\[ \gamma_S^{(1)} = 2 \left(|\kappa_S|^2 + |\lambda_S|^2\right) \]  \hspace{1cm} (A.15)

\[ \gamma_S^{(2)} = -8\kappa_S^2\kappa_S^2 - 8\lambda_S|\kappa_S|^2\lambda_S^2 \]
\[ - \frac{2}{5}|\lambda_S|^2 \left(10\lambda_S\lambda_S^2 - 15g_2^2 + 15 \text{Tr}\left(Y_\epsilon Y_\epsilon^\dagger\right) + 15 \text{Tr}\left(Y_aY_a^\dagger\right) + 30\lambda_T\lambda_T^2 - 3g_1^2 + 5 \text{Tr}\left(Y_\epsilon Y_\epsilon^\dagger\right) \right) \]  \hspace{1cm} (A.16)

\[ \gamma_T^{(1)} = 2|\lambda_T|^2 - 4g_2^2 \]  \hspace{1cm} (A.17)

\[ \gamma_T^{(2)} = -28g_2^4 - 12\lambda_T^2\lambda_T^2 \]
\[ + \frac{2}{5}|\lambda_T|^2 \left( -10\lambda_S\lambda_S^2 - 15 \text{Tr}\left(Y_\epsilon Y_\epsilon^\dagger\right) - 15 \text{Tr}\left(Y_aY_a^\dagger\right) + 3g_1^2 - 5g_2^2 - 5 \text{Tr}\left(Y_\epsilon Y_\epsilon^\dagger\right) \right) \]  \hspace{1cm} (A.18)

\[ \gamma_O^{(1)} = -6g_3^2 \]  \hspace{1cm} (A.19)

\[ \gamma_O^{(2)} = 36g_3^4 \]  \hspace{1cm} (A.20)

**A.2 Gauge Couplings**

\[ \beta_{g_1}^{(1)} = \frac{33}{5}g_1^3 \]  \hspace{1cm} (A.21)

\[ \beta_{g_1}^{(2)} = \frac{1}{25}g_1^3 \left( -130 \text{Tr}\left(Y_aY_a^\dagger\right) + 135g_2^2 + 199g_1^2 - 30|\lambda_S|^2 + 440g_3^2 - 70 \text{Tr}\left(Y_\epsilon Y_\epsilon^\dagger\right) \right) \]
\[ - 90|\lambda_T|^2 - 90 \text{Tr}\left(Y_\epsilon Y_\epsilon^\dagger\right) \]  \hspace{1cm} (A.22)

\[ \beta_{g_2}^{(1)} = 3g_2^3 \]  \hspace{1cm} (A.23)

\[ \beta_{g_2}^{(2)} = \frac{1}{5}g_2^3 \left( -10|\lambda_S|^2 - 10 \text{Tr}\left(Y_\epsilon Y_\epsilon^\dagger\right) + 120g_3^2 + 245g_2^2 - 30 \text{Tr}\left(Y_aY_a^\dagger\right) \right) \]
\[ - 30 \text{Tr}\left(Y_aY_a^\dagger\right) - 70|\lambda_T|^2 + 9g_1^2 \]  \hspace{1cm} (A.24)

\[ \beta_{g_3}^{(1)} = 0 \]  \hspace{1cm} (A.25)

\[ \beta_{g_3}^{(2)} = \frac{1}{5}g_3^3 \left( 11g_1^2 - 20 \text{Tr}\left(Y_aY_a^\dagger\right) - 20 \text{Tr}\left(Y_aY_a^\dagger\right) + 340g_3^2 + 45g_2^2 \right) \]  \hspace{1cm} (A.26)
A.3 Gaugino Mass Parameters

\[ \beta^{(1)}_{M_1} = \frac{66}{5} g_1^2 M_1 \]  
(A.27)

\[ \beta^{(2)}_{M_1} = \frac{2}{25} g_1^2 \left( 398 g_1^2 M_1 + 135 g_2^2 M_1 + 440 g_3^2 M_1 + 440 g_3^2 M_3 + 135 g_2^2 M_2 \right) 
- 30 \lambda_5^2 \left( - A_S + M_1 \lambda_S \right) - 90 \lambda_7^2 \left( - A_T + M_1 \lambda_T \right) 
- 70 M_1 \text{Tr} \left( Y_d Y_d^\dagger \right) - 90 M_1 \text{Tr} \left( Y_e Y_e^\dagger \right) - 130 M_1 \text{Tr} \left( Y_u Y_u^\dagger \right) + 70 \text{Tr} \left( Y_d^\dagger A_d \right) + 90 \text{Tr} \left( Y_e^\dagger A_e \right) 
+ 130 \text{Tr} \left( Y_u^\dagger A_u \right) \]  
(A.28)

\[ \beta^{(1)}_{M_2} = 6 g_2^2 M_2 \]  
(A.29)

\[ \beta^{(2)}_{M_2} = \frac{2}{5} g_2^2 \left( 9 g_1^2 M_1 + 120 g_3^2 M_3 + 9 g_1^2 M_2 + 490 g_2^2 M_2 + 120 g_3^2 M_2 \right) 
- 10 \lambda_5^2 \left( - A_S + M_2 \lambda_S \right) - 70 \lambda_7^2 \left( - A_T + M_2 \lambda_T \right) 
- 30 M_2 \text{Tr} \left( Y_d Y_d^\dagger \right) - 10 M_2 \text{Tr} \left( Y_e Y_e^\dagger \right) - 30 M_2 \text{Tr} \left( Y_u Y_u^\dagger \right) + 30 \text{Tr} \left( Y_d^\dagger A_d \right) + 10 \text{Tr} \left( Y_e^\dagger A_e \right) 
+ 30 \text{Tr} \left( Y_u^\dagger A_u \right) \]  
(A.30)

\[ \beta^{(1)}_{M_3} = 0 \]  
(A.31)

\[ \beta^{(2)}_{M_3} = \frac{2}{5} g_3^2 \left( 11 g_1^2 M_1 + 11 g_1^2 M_3 + 45 g_2^2 M_3 + 680 g_3^2 M_3 + 45 g_2^2 M_2 - 20 M_3 \text{Tr} \left( Y_d Y_d^\dagger \right) 
- 20 M_3 \text{Tr} \left( Y_u Y_u^\dagger \right) + 20 \text{Tr} \left( Y_d^\dagger A_d \right) + 20 \text{Tr} \left( Y_u^\dagger A_u \right) \right) \]  
(A.32)

A.4 Trilinear Superpotential Parameters

\[ \beta^{(1)}_{Y_u} = 3 Y_u Y_u^\dagger Y_u + Y_u \left( - 3 g_f^2 + 3 |\lambda_T|^2 + 3 \text{Tr} \left( Y_u Y_u^\dagger \right) - \frac{13}{15} g_1^2 - \frac{16}{3} g_3^2 + |\lambda_S|^2 \right) + Y_d Y_d^\dagger Y_d \]  
(A.33)

\[ \beta^{(2)}_{Y_u} = \frac{2}{5} g_1^2 Y_u Y_u^\dagger Y_u + 6 g_1^2 Y_u Y_u^\dagger Y_u - 3 |\lambda_S|^2 Y_u Y_u^\dagger Y_u - 9 |\lambda_T|^2 Y_u Y_u^\dagger Y_u 
- 2 Y_u Y_d Y_d^\dagger Y_d - 2 Y_u Y_d Y_d^\dagger Y_u - 4 Y_u Y_u^\dagger Y_u Y_u^\dagger Y_u 
+ Y_u Y_u^\dagger Y_u \left( - 3 |\lambda_T|^2 - 3 \text{Tr} \left( Y_u Y_u^\dagger \right) + \frac{2}{5} g_1^2 - |\lambda_S|^2 - \text{Tr} \left( Y_u Y_u^\dagger \right) \right) 
- 9 Y_u Y_u^\dagger Y_u \text{Tr} \left( Y_u Y_u^\dagger \right) 
+ Y_u \left( \frac{2743}{450} g_1^4 + g_1^2 g_2^2 + \frac{27}{2} g_2^4 + \frac{136}{45} g_1 g_2 g_3 + 8 g_2^2 g_3 + \frac{128}{9} g_3^4 + 12 g_2^2 |\lambda_T|^2 - 2 |\lambda_S|^2 |\lambda_T|^2 \right) 
- 3 \lambda_5^2 |\lambda_S|^2 - 15 |\lambda_T|^2 |\lambda_S|^2 - 9 |\lambda_T|^2 \text{Tr} \left( Y_d Y_d^\dagger \right) - 3 |\lambda_T|^2 \text{Tr} \left( Y_e Y_e^\dagger \right) \]
\[
\beta_{\lambda_g}^{(1)} = 3Y_u Y_d^\dagger Y_d + Y_d \left( -3g_y^2 + 3|\lambda_T|^2 + 3\text{Tr}\left(Y_u Y_e^\dagger\right) - \frac{16}{3}g_y^2 - \frac{7}{15}g_1^2 + |\lambda_S|^2 + \text{Tr}\left(Y_e Y_e^\dagger\right) \right) + Y_d Y_u^\dagger Y_u
\]

\[
\beta_{\lambda_d}^{(2)} = -\frac{4}{5}g_1Y_d Y_u^\dagger Y_u - |\lambda_S|^2 Y_d Y_u^\dagger Y_u - 3|\lambda_T|^2 Y_d Y_u^\dagger Y_u - 4Y_d Y_u^\dagger Y_d Y_d Y_d Y_d
\]

\[
\beta_{\lambda_d}^{(1)} = 3Y_e Y_e^\dagger Y_e + Y_e \left( -9g_y^2 + 9|\lambda_T|^2 + 3\text{Tr}\left(Y_u Y_u^\dagger\right) - \frac{9}{5}g_1^2 + |\lambda_S|^2 + \text{Tr}\left(Y_e Y_e^\dagger\right) \right)
\]

\[
\beta_{\lambda_e}^{(2)} = -4Y_e Y_e^\dagger Y_e Y_e + Y_e Y_e^\dagger Y_e \left( -3|\lambda_S|^2 - 3\text{Tr}\left(Y_u Y_u^\dagger\right) + 6g_y^2 - 9|\lambda_T|^2 - 9\text{Tr}\left(Y_d Y_d^\dagger\right) \right)
\]

\[
\beta_{\lambda_T}^{(1)} = 2|\lambda_T|^2 + 3\lambda_T \text{Tr}\left(Y_d Y_d^\dagger\right) + 3\lambda_T \text{Tr}\left(Y_u Y_u^\dagger\right) - 7g_y^2 \lambda_T + 8g_1^2 \lambda_T - \frac{3}{5}g_1^2 \lambda_T + \lambda_T \text{Tr}\left(Y_e Y_e^\dagger\right)
\]

\[
\beta_{\lambda_S}^{(2)} = -\frac{1}{50} \lambda_T \left( -207g_y^4 - 90g_1^2g_2^2 - 2075g_y^2 - 60g_1^2|\lambda_T|^2 - 1100g_2^2|\lambda_T|^2 + 200|\lambda_S|^2|\lambda_S|^2 + 300\lambda_5^2\lambda_5^2 \\
+ 2100\lambda_5^2\lambda_5^2 + 20g_1^2 \text{Tr}\left(Y_d Y_d^\dagger\right) - 800g_1^2 \text{Tr}\left(Y_u Y_u^\dagger\right) + 750|\lambda_T|^2 \text{Tr}\left(Y_u Y_u^\dagger\right) \\
- 60g_1^2 \text{Tr}\left(Y_e Y_e^\dagger\right) + 250|\lambda_T|^2 \text{Tr}\left(Y_e Y_e^\dagger\right) - 40g_1^2 \text{Tr}\left(Y_u Y_u^\dagger\right) - 800g_1^2 \text{Tr}\left(Y_u Y_u^\dagger\right) \\
+ 750|\lambda_T|^2 \text{Tr}\left(Y_u Y_u^\dagger\right) + 50|\lambda_S|^2 \left( 16\lambda_T\lambda_T^2 + 3\text{Tr}\left(Y_d Y_d^\dagger\right) + 3\text{Tr}\left(Y_u Y_u^\dagger\right) + \text{Tr}\left(Y_e Y_e^\dagger\right) \right) \\
+ 450\text{Tr}\left(Y_u Y_d^\dagger Y_d Y_d^\dagger\right) + 300\text{Tr}\left(Y_u Y_d^\dagger Y_d Y_u^\dagger\right) + 150\text{Tr}\left(Y_u Y_u^\dagger Y_d Y_u^\dagger\right) + 450\text{Tr}\left(Y_u Y_u^\dagger Y_u Y_u^\dagger\right)
\]

\[
\beta_{\lambda_S}^{(1)} = -\frac{3}{5}g_1^2\lambda_S - 3g_2^2\lambda_S + 2\lambda_S|\lambda_T|^2 + 6\lambda_S|\lambda_T|^2 + 4\lambda_5^2\lambda_5^2 + 3\lambda_S \text{Tr}\left(Y_u Y_u^\dagger\right) + \lambda_S \text{Tr}\left(Y_e Y_e^\dagger\right)
\]
\[ + 3 \lambda_S \text{Tr} \left( Y_u Y_u^\dagger \right) \]  
\[ \beta_{\lambda_S}^{(2)} = \frac{1}{50} \lambda_S \left( -207 g_1^4 + 90 g_1^2 g_2^2 - 675 g_2^4 - 1200 g_2^2 |\lambda_T|^2 + 400 \kappa_S^2 \kappa_S^{*2} + 600 \lambda_S |\kappa_S|^2 \lambda_S^{*2} + 500 \lambda_S^{*2} \lambda_S^{*2} ight. \]
\[ + 1500 \lambda_T^2 \lambda_T^{*2} + 20 g_1^2 \text{Tr} \left( Y_d Y_d^\dagger \right) - 800 g_2^2 \text{Tr} \left( Y_u Y_u^\dagger \right) + 450 |\lambda_T|^2 \text{Tr} \left( Y_d Y_d^\dagger \right) \]
\[ - 60 g_2^2 \text{Tr} \left( Y_u Y_u^\dagger \right) + 150 |\lambda_T|^2 \text{Tr} \left( Y_e Y_e^\dagger \right) \]
\[ - 30 |\lambda_S|^2 \left( 10 g_2^2 - 15 \text{Tr} \left( Y_d Y_d^\dagger \right) - 15 \text{Tr} \left( Y_u Y_u^\dagger \right) + 2 g_1^2 - 40 \lambda_T \lambda_T^{*} - 5 \text{Tr} \left( Y_e Y_e^\dagger \right) \right) \]
\[ - 40 g_2^2 \text{Tr} \left( Y_u Y_u^\dagger \right) - 800 g_2^2 \text{Tr} \left( Y_u Y_u^\dagger \right) + 450 |\lambda_T|^2 \text{Tr} \left( Y_u Y_u^\dagger \right) + 450 \text{Tr} \left( Y_d Y_d^\dagger Y_d Y_d^\dagger \right) \]
\[ + 300 \text{Tr} \left( Y_d Y_d^\dagger Y_d Y_d^\dagger \right) + 150 \text{Tr} \left( Y_u Y_u^\dagger Y_u Y_u^\dagger \right) + 450 \text{Tr} \left( Y_u Y_u^\dagger Y_u Y_u^\dagger \right) \]  
\[ \beta_{\kappa_S}^{(1)} = 6 \kappa_S \left( |\kappa_S|^2 + |\lambda_S|^2 \right) \]  
\[ \beta_{\kappa_S}^{(2)} = - \frac{6}{5} \kappa_S \left( 20 \kappa_S^2 \kappa_S^{*2} + 20 \lambda_S |\kappa_S|^2 \lambda_S^{*} \right. \]
\[ + |\lambda_S|^2 \left( 10 \lambda_S \lambda_S^{*} - 15 g_2^2 + 15 \text{Tr} \left( Y_d Y_d^\dagger \right) + 15 \text{Tr} \left( Y_u Y_u^\dagger \right) + 30 \lambda_T \lambda_T^{*} - 3 g_1^2 + 5 \text{Tr} \left( Y_e Y_e^\dagger \right) \right) \]  
\[ \text{A.5 Bilinear Superpotential Parameters} \]
\[ \beta_\mu^{(1)} = 2 \mu |\lambda_S|^2 - 3 g_2^2 \mu + 3 \mu \text{Tr} \left( Y_d Y_d^\dagger \right) + 3 \mu \text{Tr} \left( Y_u Y_u^\dagger \right) + 6 \mu |\lambda_T|^2 - \frac{3}{5} g_1^2 \mu + \mu \text{Tr} \left( Y_e Y_e^\dagger \right) \]  
\[ \beta_\mu^{(2)} = + \frac{207}{50} g_1^4 \mu + \frac{9}{5} g_1^2 g_2^2 \mu + \frac{27}{2} g_2^4 \mu + 24 g_2^2 \mu |\lambda_T|^2 - 48 \lambda_S \mu |\kappa_S|^2 \lambda_S^{*} - 6 \lambda_S^{*} \mu \lambda_S^{*2} \]
\[ - 30 \lambda_T^2 \mu \lambda_T^{*2} - \frac{2}{5} \mu \text{Tr} \left( Y_d Y_d^\dagger \right) + 16 g_3^2 \mu \text{Tr} \left( Y_u Y_u^\dagger \right) + 9 \mu |\lambda_T|^2 \text{Tr} \left( Y_d Y_d^\dagger \right) \]
\[ + \frac{6}{5} g_2^2 \mu \text{Tr} \left( Y_e Y_e^\dagger \right) - 3 \mu |\lambda_T|^2 \text{Tr} \left( Y_e Y_e^\dagger \right) + \frac{4}{5} g_1^2 \mu \text{Tr} \left( Y_u Y_u^\dagger \right) + 16 g_3^2 \mu \text{Tr} \left( Y_u Y_u^\dagger \right) \]
\[ - 9 \mu |\lambda_T|^2 \text{Tr} \left( Y_u Y_u^\dagger \right) - \mu |\lambda_S|^2 \left( 12 \lambda_T \lambda_T^{*} + 3 \text{Tr} \left( Y_d Y_d^\dagger \right) + 3 \text{Tr} \left( Y_u Y_u^\dagger \right) + \text{Tr} \left( Y_e Y_e^\dagger \right) \right) \]
\[ - 9 \mu \text{Tr} \left( Y_d Y_d^\dagger Y_d Y_d^\dagger \right) - 6 \mu \text{Tr} \left( Y_u Y_u^\dagger Y_u Y_u^\dagger \right) - 3 \mu \text{Tr} \left( Y_u Y_u^\dagger Y_e Y_e^\dagger \right) - 9 \mu \text{Tr} \left( Y_u Y_u^\dagger Y_u Y_u^\dagger \right) \]  
\[ \beta_{M_T}^{(1)} = 4 M_T |\lambda_T|^2 - 8 g_2^2 M_T \]  
\[ \beta_{M_T}^{(2)} = \frac{4}{5} M_T \left( 70 g_2^4 - 30 \lambda_T^2 \lambda_T^{*2} \right. \]
\[ + |\lambda_T|^2 \left( - 10 \lambda_S \lambda_S^{*} - 15 \text{Tr} \left( Y_d Y_d^\dagger \right) - 15 \text{Tr} \left( Y_u Y_u^\dagger \right) + 3 g_1^2 - 5 g_2^2 - 5 \text{Tr} \left( Y_e Y_e^\dagger \right) \right) \]  
\[ \beta_{M_O}^{(1)} = -12 g_3^2 M_O \]  
\[ \beta_{M_O}^{(2)} = 72 g_3^4 M_O \]
\[ \beta^{(1)}_{M_S} = 4M_S \left( |\kappa_S|^2 + |\lambda_S|^2 \right) \]  
\[ \beta^{(2)}_{M_S} = -\frac{4}{5}M_S \left( 20\kappa_S^2 \kappa_S^* - 20\lambda_S |\kappa_S|^2 \lambda_S^* + |\lambda_S|^2 \left( 10\lambda_S \lambda_S^* - 15g_2^2 + 15\text{Tr} \left( Y_d Y_d^\dagger \right) + 15\text{Tr} \left( Y_u Y_u^\dagger \right) + 30\lambda_T \lambda_T^* - 3g_1^2 + 5\text{Tr} \left( Y_e Y_e^\dagger \right) \right) \right) \]  
(A.51)  
\[ \beta^{(1)}_{L_S} = 2L_S \left( |\kappa_S|^2 + |\lambda_S|^2 \right) \]  
\[ \beta^{(2)}_{L_S} = -\frac{2}{5}L_S \left( 20\kappa_S^2 \kappa_S^* + 20\lambda_S |\kappa_S|^2 \lambda_S^* + |\lambda_S|^2 \left( 10\lambda_S \lambda_S^* - 15g_2^2 + 15\text{Tr} \left( Y_d Y_d^\dagger \right) + 15\text{Tr} \left( Y_u Y_u^\dagger \right) + 30\lambda_T \lambda_T^* - 3g_1^2 + 5\text{Tr} \left( Y_e Y_e^\dagger \right) \right) \right) \]  
(A.52)  

A.6 Linear Superpotential Parameters

\[ \beta^{(1)}_{A_u} = +2Y_u Y_d^\dagger A_d + 4Y_u Y_u^\dagger A_u + A_u Y_d^\dagger Y_d + 5A_u Y_u^\dagger Y_u - \frac{13}{15}g_1^2 A_u - 3g_2^2 A_u - \frac{16}{3}g_3^2 A_u \]  
\[ + |\lambda_S|^2 A_u + 3|\lambda_T|^2 A_u + 3A_u \text{Tr} \left( Y_u Y_u^\dagger \right) \]  
\[ + Y_u \left( 2\lambda_S^* A_S + 6g_2^2 M_2 + 6\lambda_T^* A_T + 6\text{Tr} \left( Y_d Y_d^\dagger \right) + \frac{26}{15}g_1^2 M_1 + \frac{32}{3}g_3^2 M_3 \right) \]  
\[ + 4 \frac{T}{g_1^2} Y_u Y_d^\dagger A_d - 2|\lambda_S|^2 Y_u Y_d^\dagger A_d - 6|\lambda_T|^2 Y_u Y_d^\dagger A_d - \frac{4}{5}g_1^2 M_1 Y_u Y_u^\dagger Y_u \]  
\[- 12g_2^2 M_2 Y_u Y_u^\dagger Y_u + \frac{6}{2}g_1^2 Y_u Y_u^\dagger A_u + 6g_2^2 Y_u Y_u^\dagger A_u - 4|\lambda_S|^2 Y_u Y_u^\dagger A_u \]  
\[- 12|\lambda_T|^2 Y_u Y_u^\dagger A_u + \frac{2}{5}g_1^2 A_u Y_d^\dagger Y_d - |\lambda_S|^2 A_u Y_d^\dagger Y_d - 3|\lambda_T|^2 A_u Y_d^\dagger Y_d \]  
\[ + 12g_2^2 A_u Y_d^\dagger Y_u - 5|\lambda_S|^2 A_u Y_d^\dagger Y_u - 15|\lambda_T|^2 A_u Y_d^\dagger Y_u - 4Y_u Y_d^\dagger Y_d^\dagger Y_u \]  
\[ - 2Y_u Y_d^\dagger Y_d^\dagger A_u - 4Y_u Y_d^\dagger A_d Y_d^\dagger Y_u - 4Y_u Y_d^\dagger A_d Y_u^\dagger Y_u - 6Y_u Y_d^\dagger Y_u^\dagger Y_u^\dagger Y_u \]  
\[ - 8Y_u Y_d^\dagger A_u Y_d^\dagger Y_u Y_u - 2A_u Y_d^\dagger Y_d^\dagger Y_d^\dagger Y_u Y_u - 6A_u Y_d^\dagger Y_u^\dagger Y_u Y_u + \frac{2743}{450}g_1^4 A_u \]  
\[- g_1^2 Y_u Y_u^\dagger A_u + \frac{27}{2} g_1^2 A_u + \frac{136}{45} g_1^2 g_3^2 A_u + 8g_2^2 g_3^2 A_u + \frac{128}{9} g_1^4 A_u + 12g_2^4 |\lambda_T|^2 A_u \]  
\[- 2\lambda_S |\kappa_S|^2 \lambda_S^2 A_u - 3\lambda_T^2 |\lambda_T|^2 A_u - 6\lambda_T^2 |\lambda_T|^2 A_u - 15\lambda_S^2 \lambda_T^2 A_u - 6\lambda_T^2 Y_u Y_u^\dagger Y_u A_S \]  
\[- 18\lambda_T^2 Y_u Y_u^\dagger A_T - 6Y_u Y_d^\dagger A_d \text{Tr} \left( Y_d Y_d^\dagger \right) - 3A_u Y_d^\dagger Y_d^\dagger \text{Tr} \left( Y_d Y_d^\dagger \right) \]  
\[ - 3|\lambda_S|^2 A_u \text{Tr} \left( Y_d Y_d^\dagger \right) - 9|\lambda_T|^2 A_u \text{Tr} \left( Y_d Y_d^\dagger \right) - 2Y_u Y_d^\dagger A_d \text{Tr} \left( Y_e Y_e^\dagger \right) \]  
\[ - A_u Y_d^\dagger Y_d \text{Tr} \left( Y_e Y_e^\dagger \right) - |\lambda_S|^2 A_u \text{Tr} \left( Y_e Y_e^\dagger \right) - 3|\lambda_T|^2 A_u \text{Tr} \left( Y_e Y_e^\dagger \right) \]  
(A.55)  

A.7 Trilinear Soft-Breaking Parameters
\[-12Y_uY_u^\dagger A_u \text{Tr}(Y_uY_u^\dagger) - 15A_uY_u^\dagger Y_u \text{Tr}(Y_uY_u^\dagger) + \left( \frac{4}{9}g_2^2A_u \text{Tr}(Y_uY_u^\dagger) \right) + 16g_3^2A_u \text{Tr}(Y_uY_u^\dagger) - \frac{2}{5}g_2^2A_u \text{Tr}(Y_uY_u^\dagger) + \left( 15\lambda^*_T \text{Tr}(Y_uY_u^\dagger) + 2g_1^2M_1 + 5\lambda^*_S \text{Tr}(Y_u^\dagger A_e) \right) \]

\[-18Y_uY_u^\dagger Y_u \text{Tr}(Y_uY_u^\dagger) - 3A_u \text{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger) - 9A_u \text{Tr}(Y_uY_u^\dagger Y_uY_u^\dagger) \]

\[-\frac{2}{225}Y_u \left( 2743g_1^4M_1 + 225g_2^2g_2^2M_1 + 680g_1^2g_2^2M_1 + 680g_1^2g_2^2M_3 + 1800g_2^3g_3^3M_3 + 6400g_3^3M_3 + 225g_1^2g_2^2M_2 + 6075g_2^2g_2^2M_2 + 1800g_2^3g_2^3M_2 + 1350\lambda_S \lambda^*_S^2 A_S + 450\kappa_S \lambda^*_S \left( \kappa_S A_S + \lambda_S A_e \right) \right) \]

\[+ 6750\lambda^*_T \lambda^*_T \text{Tr}(Y_uY_u^\dagger) + 225\lambda^*_S A_S \text{Tr}(Y_uY_u^\dagger) + 180g_1^2M_1 \text{Tr}(Y_uY_u^\dagger) \]

\[+ 3600g_3^2M_3 \text{Tr}(Y_uY_u^\dagger) + 675\lambda_S |^2 \text{Tr}(Y_uY_u^\dagger) + 225\lambda_S |^2 \text{Tr}(Y_uY_u^\dagger) \]

\[+ 675\lambda^*_T \left( 2\lambda^*_S A_T + \lambda^*_T A_S \right) + 4\lambda_T \left( Y_uY_u^\dagger - 4g_3^2 + \text{Tr}(Y_uY_u^\dagger) \right) + 4g_2^2M_2 + \text{Tr}(Y_uY_u^\dagger) \]

\[-180g_1^2 \text{Tr}(Y_uY_u^\dagger) - 3600g_2^2 \text{Tr}(Y_uY_u^\dagger) + 675 \text{Tr}(Y_uY_u^\dagger) + 675 \text{Tr}(Y_uY_u^\dagger) \]

\[+ 4050 \text{Tr}(Y_uY_u^\dagger) \]  

\[\beta^{(1)}_{A_d} = \left( \frac{4}{5}g_1^2Y_dY_d^\dagger A_d + 2Y_dY_d^\dagger A_u + 5A_dY_dY_d^\dagger A_d + A_dY_d^\dagger Y_u - \frac{7}{15}g_1^2A_d - 3g_3^2A_d - \frac{16}{3}g_3^2A_d \right) \]

\[+ |\lambda_S|^2A_d + 3|\lambda_T|^2A_d + 3A_d \text{Tr}(Y_dY_d^\dagger) + A_d \text{Tr}(Y_uY_u^\dagger) \]

\[+ Y_d \left( 2\lambda^*_S A_S + 2\text{Tr}(Y_u^\dagger A_e) + 6g_3^2M_2 + 6\lambda^*_T A_T + 6\text{Tr}(Y_u^\dagger A_d) + \frac{14}{15}g_2^2M_1 + \frac{32}{3}g_3^2M_3 \right) \]  

\[\beta^{(2)}_{A_d} = \left( \frac{6}{5}g_1^2Y_dY_d^\dagger A_d + 6g_2^2Y_dY_d^\dagger A_d - 4|\lambda_S|^2Y_dY_d^\dagger A_d - 12|\lambda_T|^2Y_dY_d^\dagger A_d \right) \]

\[- \frac{8}{5}g_1^2M_1Y_dY_d^\dagger Y_u + \frac{8}{5}g_1^2Y_dY_d^\dagger A_u - 2|\lambda_S|^2Y_dY_d^\dagger A_u - 6|\lambda_T|^2Y_dY_d^\dagger A_u \]

\[+ \frac{6}{5}g_2^2A_dY_dY_d^\dagger Y_d + 12g_2^2A_dY_dY_d^\dagger Y_d - 5|\lambda_S|^2A_dY_dY_d^\dagger Y_d - 15|\lambda_T|^2A_dY_dY_d^\dagger Y_d \]

\[+ \frac{4}{5}g_2^2A_dY_dY_d^\dagger Y_u - |\lambda_S|^2A_dY_u^\dagger Y_u - 3|\lambda_T|^2A_dY_u^\dagger Y_u - 6Y_dY_d^\dagger Y_d \]

\[+ \frac{287}{90}g_1^2A_d + g_1^2g_2^2A_d + \frac{27}{2}g_2^2A_d + \frac{8}{9}g_1^2g_3A_d + 8g_2^2g_3A_d + \frac{128}{9}g_3^2A_d + 12g_2^2|\lambda_T|^2A_d \]

\[- 2\lambda_S |\lambda_S|^2\lambda^*_S A_d - 3\lambda_S^2\lambda^*_S A_d - 6\lambda_T |\lambda_S|^2\lambda^*_T A_d - 15\lambda^*_S \lambda^*_T A_d \]

\[- 2\lambda^*_S \lambda^*_S A_d - 6\lambda^*_T Y_u^\dagger Y_u A_T - 12Y_dY_d^\dagger A_d \text{Tr}(Y_dY_d^\dagger) \]

\[- 15A_dY_d^\dagger Y_d \text{Tr}(Y_dY_d^\dagger) - \frac{2}{5}g_1^2A_d \text{Tr}(Y_uY_u^\dagger) + 16g_3^2A_d \text{Tr}(Y_uY_u^\dagger) \]

\[- 4Y_dY_d^\dagger A_d \text{Tr}(Y_eY_e^\dagger) - 5A_dY_d^\dagger Y_d \text{Tr}(Y_eY_e^\dagger) + \frac{6}{5}g_2^2A_d \text{Tr}(Y_eY_e^\dagger) \]  

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\(-6Y_dY_u^\dagger A_u\text{Tr}(Y_uY_u^\dagger) - 3A_dY_u^\dagger Y_u\text{Tr}(Y_uY_u^\dagger) - 3\lambda_S^2 A_d\text{Tr}(Y_uY_u^\dagger) - 9\lambda_T^2 A_d\text{Tr}(Y_uY_u^\dagger)
\)
\(-\frac{2}{5} Y_dY_d^\dagger Y_d\left(15\lambda_S^2 A_S + 15\text{Tr}(Y_d^\dagger A_e) + 30g_2^2 M_2 + 45\lambda_T^2 A_T + 45\text{Tr}(Y_d^\dagger A_d) + 4g_1^2 M_1\right)
\)
\(-6Y_dY_u^\dagger Y_u\text{Tr}(Y_uY_u^\dagger) - 9A_d\text{Tr}(Y_dY_d^\dagger Y_d^\dagger) - 3A_d\text{Tr}(Y_dY_u^\dagger Y_uY_u^\dagger) - 3A_d\text{Tr}(Y_dY_u^\dagger Y_uY_u^\dagger)
\)
\(-\frac{2}{45} Y_d\left(287g_1^4 M_1 + 45g_1^2 g_2^2 M_1 + 40g_1^2 g_3^2 M_1 + 40g_1^2 g_2^3 M_3 + 360g_2^2 g_3^2 M_3 + 1280g_3^4 M_3 + 45g_1^2 g_2^2 M_2 + 1215g_2^4 M_2 + 360g_2^2 g_3^2 M_2 + 270\lambda_S\lambda_S^* A_S + 90\lambda_S^2 A_S + 90\lambda_S\lambda_S^* A_S + 18\lambda_T\lambda_T^* A_T - 18g_1^2 M_2\text{Tr}(Y_dY_u^\dagger) + 720g_3^3 M_3\text{Tr}(Y_dY_u^\dagger) + 54g_1^2 M_1\text{Tr}(Y_eY_e^\dagger)
\)
\(+135\lambda_S^2 A_S\text{Tr}(Y_uY_u^\dagger) + 18g_1^2\text{Tr}(Y_d^\dagger A_d) - 720g_3^3\text{Tr}(Y_d^\dagger A_d) - 54g_1^2\text{Tr}(Y_e^\dagger A_e)
\)
\(+135\lambda_S^2 A_S\text{Tr}(Y_uY_u^\dagger) + 18g_1^2\text{Tr}(Y_d^\dagger A_d) - 720g_3^3\text{Tr}(Y_d^\dagger A_d) - 54g_1^2\text{Tr}(Y_e^\dagger A_e)
\)

\[\alpha_A^{(1)} = +4Y_e^\dagger A_e + 5A_e Y_e^\dagger Y_e - \frac{9}{5} g_1^4 A_e - 3g_2^2 A_e + |\lambda_S|^2 A_e + 3|\lambda_T|^2 A_e + 3A_e\text{Tr}(Y_dY_d^\dagger)
\]
\[+ A_e\text{Tr}(Y_dY_d^\dagger) + Y_e\left(2\lambda_S^2 A_S + 2\text{Tr}(Y_e^\dagger A_e) + 6g_2^2 M_2 + 6\lambda_T^2 A_T + 6\text{Tr}(Y_d^\dagger A_d) + \frac{18}{5} g_1^2 M_1\right)\]

\[\alpha_A^{(2)} = +\frac{6}{5} g_1^2 Y_e^\dagger Y_e^\dagger A_e + 6g_2^2 Y_e^\dagger Y_e^\dagger A_e - 4\lambda_S^2 Y_e^\dagger Y_e^\dagger A_e - 12|\lambda_T|^2 Y_e^\dagger Y_e^\dagger A_e
\]
\[+ \frac{6}{5} g_1^2 A_e Y_e^\dagger Y_e + 12g_2^2 A_e Y_e^\dagger Y_e - 5|\lambda_S|^2 A_e Y_e^\dagger Y_e + 15|\lambda_T|^2 A_e Y_e^\dagger Y_e
\]
\[+ 6Y_e^\dagger Y_e^\dagger A_e - 8Y_e^\dagger Y_e^\dagger A_e Y_e^\dagger Y_e - 6A_e Y_e^\dagger Y_e^\dagger Y_e + \frac{27}{2} g_1^4 A_e + \frac{9}{5} g_2^2 g_2 A_e + \frac{27}{2} g_2^4 A_e
\]
\[+ 12g_2^2 |\lambda_T|^2 A_e - 2\lambda_S^2 |\kappa_S|^2 \lambda_S^2 A_e - 3\lambda_S^2 \lambda_S^* A_e - 6\lambda_T^2 |\lambda_S|^2 \lambda_T^2 A_e - 15\lambda_S^2 \lambda_T^2 A_e
\]
\[+ 12Y_e^\dagger Y_e^\dagger A_e\text{Tr}(Y_dY_d^\dagger) - 15A_e Y_e^\dagger Y_e \text{Tr}(Y_dY_d^\dagger) - \frac{2}{5} g_2^4 A_e\text{Tr}(Y_dY_d^\dagger)
\]
\[+ 16g_2^4 A_e\text{Tr}(Y_dY_d^\dagger) - 4Y_e^\dagger Y_e^\dagger A_e\text{Tr}(Y_eY_e^\dagger) - 5A_e Y_e^\dagger Y_e\text{Tr}(Y_eY_e^\dagger)
\]
\[+ \frac{6}{5} g_1^2 A_e\text{Tr}(Y_eY_e^\dagger) - 3|\lambda_S|^2 A_e\text{Tr}(Y_uY_u^\dagger) - 9|\lambda_T|^2 A_e\text{Tr}(Y_uY_u^\dagger)
\]
\[+ 6Y_eY_e^\dagger Y_e\left(2g_2^2 M_2 + 3\lambda_T^2 A_T + 3\text{Tr}(Y_d^\dagger A_d) + \lambda_S^2 A_S + \text{Tr}(Y_e^\dagger A_e)\right) - 9A_e\text{Tr}(Y_dY_d^\dagger Y_dY_d^\dagger)
\]
\[-3A_e\text{Tr}(Y_dY_u^\dagger Y_uY_d^\dagger) - 3A_e\text{Tr}(Y_e^\dagger Y_e^\dagger Y_e^\dagger)
\]
\[-\frac{2}{5} Y_e\left(135g_1^4 M_1 + 9g_1^2 g_2^2 M_1 + 9g_1^2 g_2^2 M_2 + 135g_2^4 M_2 + 30\lambda_S \lambda_S^2 A_S + 10\lambda_S \lambda_S^* \left(\kappa_S A_S + \lambda_S \lambda_S^* A_S\right)\right)\]
\[+150\lambda_T \lambda_S^2 A_T - 2g_1^2 M_1 \text{Tr}(Y_d Y_d^\dagger) + 80g_3^2 M_3 \text{Tr}(Y_d Y_d^\dagger) + 6g_1^2 M_1 \text{Tr}(Y_e Y_e^\dagger)
\]
\[+15\lambda_S^* A_S \text{Tr}(Y_u Y_u^\dagger) + 2g_1^2 \text{Tr}(Y_d^\dagger A_d) - 80g_3^2 \text{Tr}(Y_d^\dagger A_d) - 6g_1^2 \text{Tr}(Y_e^\dagger A_e)
\]
\[+15|\lambda_S|^2 \text{Tr}(Y_d^\dagger A_u)
\]
\[+15\lambda_T^* \left(2\lambda_S \text{Tr}(A_T + \lambda_T A_S) + 3A_T \text{Tr}(Y_u Y_u^\dagger) + 3\lambda_T \text{Tr}(Y_u^\dagger A_u) - 4g_2^2 A_T + 4g_2^2 M_2 \lambda_T
\]
\[+90 \text{Tr}(Y_d^\dagger A_d Y_d^\dagger) + 15 \text{Tr}(Y_d Y_u^\dagger A_u Y_d^\dagger) + 30 \text{Tr}(Y_e Y_u^\dagger A_e Y_e^\dagger) + 15 \text{Tr}(Y_u Y_d^\dagger A_u Y_u^\dagger) \right)
\]
\[(A.60)
\]
\[\beta_{AT}^{(1)} = 2\lambda_S^* \left(2\lambda_T A_S + \lambda_S A_T\right) + A_T \left(24|\lambda_T|^2 + 3 \text{Tr}(Y_d Y_d^\dagger) + 3 \text{Tr}(Y_u Y_u^\dagger) - 7g_2^2 - \frac{3}{5}g_1^2 + \text{Tr}(Y_e Y_e^\dagger)\right)
\]
\[+ \frac{2}{5} \lambda_T \left(15 \text{Tr}(Y_d^\dagger A_d) + 15 \text{Tr}(Y_u^\dagger A_u) + 35g_2^2 M_2 + 3g_1^2 M_1 + 5 \text{Tr}(Y_e^\dagger A_e)\right) \]  
\[(A.61)
\]
\[\beta_{AT}^{(2)} = -\frac{414}{25} g_1^4 \lambda_T A_T - \frac{18}{5} g_1^2 g_2 M_1 \lambda_T - \frac{18}{5} g_1^2 g_2^2 M_2 \lambda_T - 16g_1^2 g_2^2 M_2 \lambda_T - 24\lambda_S \lambda_T \lambda_S^2 A_S + \frac{207}{50} g_1^4 A_T
\]
\[+ \frac{9}{5} g_1^2 g_2^2 A_T + \frac{83}{2} g_2^2 A_T - 6\lambda_S^2 \lambda_S^2 A_T - 210\lambda_T^2 \lambda_T \lambda_T A_T
\]
\[-4\kappa_S^* \lambda_S^2 \left(2 \lambda_S \lambda_T A_S + 2 \lambda_S \lambda_T \lambda_S + \kappa_S \lambda_S A_S\right) + \frac{4}{5} g_1^2 M_1 \lambda_T \text{Tr}(Y_d Y_d^\dagger) - 32g_3^2 M_3 \lambda_T \text{Tr}(Y_d Y_d^\dagger)
\]
\[-6\lambda_T \lambda_S^* A_S \text{Tr}(Y_d Y_d^\dagger) - \frac{2}{5} g_1^2 A_T \text{Tr}(Y_d Y_d^\dagger) + 16g_3^2 A_T \text{Tr}(Y_d Y_d^\dagger) - 3|\lambda_S|^2 A_T \text{Tr}(Y_d Y_d^\dagger)
\]
\[-\frac{12}{5} g_1^2 M_1 \lambda_T \text{Tr}(Y_e Y_e^\dagger) - 2\lambda_T \lambda_S^* A_S \text{Tr}(Y_e Y_e^\dagger) + \frac{6}{5} g_1^2 A_T \text{Tr}(Y_e Y_e^\dagger)
\]
\[-|\lambda_S|^2 A_T \text{Tr}(Y_e Y_e^\dagger) - \frac{8}{5} g_1^2 M_1 \lambda_T \text{Tr}(Y_u Y_u^\dagger) - 32g_3^2 M_3 \lambda_T \text{Tr}(Y_u Y_u^\dagger)
\]
\[-6\lambda_T \lambda_S A_S \text{Tr}(Y_u Y_u^\dagger) + \frac{4}{5} g_1^2 A_T \text{Tr}(Y_u Y_u^\dagger) + 16g_3^2 A_T \text{Tr}(Y_u Y_u^\dagger) - 3|\lambda_S|^2 A_T \text{Tr}(Y_u Y_u^\dagger)
\]
\[-\frac{4}{5} g_1^2 \lambda_T \text{Tr}(Y_d^\dagger A_d) + 32g_3^2 \lambda_T \text{Tr}(Y_d^\dagger A_d) - 6\lambda_T |\lambda_S|^2 \text{Tr}(Y_d^\dagger A_d) + \frac{12}{5} g_1^2 \lambda_T \text{Tr}(Y_e^\dagger A_e)
\]
\[-2\lambda_T |\lambda_S|^2 \text{Tr}(Y_d^\dagger A_e) + \frac{8}{5} g_1^2 \lambda_T \text{Tr}(Y_u^\dagger A_u) + 32g_3^2 \lambda_T \text{Tr}(Y_u^\dagger A_u) - 6\lambda_T |\lambda_S|^2 \text{Tr}(Y_u^\dagger A_u)
\]
\[-\frac{1}{5} |\lambda_T|^2 \left(80\lambda_S^2 \left(2\lambda_T A_S + 3\lambda_S A_T\right)
\]
\[-3A_T \left(110g_2^2 - 25 \text{Tr}(Y_e Y_e^\dagger) + 6g_1^2 - 75 \text{Tr}(Y_d Y_d^\dagger) - 75 \text{Tr}(Y_u Y_u^\dagger)\right)
\]+ 2A_T \left(110g_2^2 M_2 + 25 \text{Tr}(Y_e Y_e^\dagger) + 6g_1^2 M_1 + 75 \text{Tr}(Y_d Y_d^\dagger) + 75 \text{Tr}(Y_u Y_u^\dagger)\right)
\]
\[-9A_T \text{Tr}(Y_d Y_d^\dagger Y_d Y_d^\dagger) - 36\lambda_T \text{Tr}(Y_d Y_d^\dagger A_d Y_d^\dagger) - 6A_T \text{Tr}(Y_d Y_d^\dagger Y_u Y_u^\dagger)
\]
\[-12\lambda_T \text{Tr}(Y_d Y_u^\dagger A_u Y_u^\dagger) - 3A_T \text{Tr}(Y_e Y_e^\dagger Y_e Y_e^\dagger) - 12\lambda_T \text{Tr}(Y_e Y_e^\dagger A_e Y_e^\dagger)
\]
\[-12\lambda_T \text{Tr}(Y_e Y_d^\dagger A_d Y_u^\dagger) - 9A_T \text{Tr}(Y_u Y_u^\dagger A_u Y_u^\dagger) - 36\lambda_T \text{Tr}(Y_u Y_u^\dagger A_u Y_u^\dagger)
\]
\[(A.62)
\]
\[\beta_{AS}^{(1)} = 2\kappa_S^* \left(2\lambda_S A_S + \kappa_S A_S\right)
\]
\[+ A_S \left(12|\lambda_S|^2 - 3g_2^2 + 3 \text{Tr}(Y_d Y_d^\dagger) + 3 \text{Tr}(Y_u Y_u^\dagger) + 6|\lambda_T|^2 - \frac{3}{5}g_1^2 + \text{Tr}(Y_e Y_e^\dagger)\right)
\]
\[ + \frac{2}{5} \lambda S \left( 15 g_2^2 M_2 + 15 \text{Tr} \left( Y^\dagger_d A_d \right) + 15 \text{Tr} \left( Y^\dagger_u A_u \right) + 30 \lambda_T^2 A_T + 3 g_1^2 M_1 + 5 \text{Tr} \left( Y^\dagger_e A_e \right) \right) \]  

(\text{A.63})

\[ \beta^{(2)}_{A_s} = -\frac{414}{25} g_1^4 M_1 \lambda S - \frac{18}{5} g_1^2 g_2^2 M_2 \lambda S - \frac{18}{5} g_1^2 g_2^2 M_2 \lambda S - 54 g_2^2 M_2 \lambda S - 32 \kappa_S \lambda S \kappa_S^2 A_\kappa + \frac{207}{50} g_1^4 A_S + \frac{9}{5} g_1^2 g_2^2 A_S + \frac{27}{2} g_2^4 A_S - 8 \kappa_S \lambda S \kappa_S^2 A_S - 50 \lambda^2 S \lambda S^2 A_S - 30 \lambda_T \lambda^2 T A_T + \lambda T A_S \]  

(\text{A.64})

\[ \beta^{(2)}_{A_n} = -\frac{6}{5} \left( 100 \kappa_S \lambda S \kappa_S^2 A_\kappa + 10 \lambda S \lambda S \kappa S^2 \left( 4 \kappa_S A_S + \lambda S A_\kappa \right) \right) \]

(\text{A.65})

\[ \beta^{(1)}_{A_n} = 6 \left( 3 |\kappa S| \lambda S A_\kappa + \lambda S \kappa S^2 A_\kappa + 3 \kappa S A_\kappa \right) \]

(\text{A.66})

\[ \beta^{(j)}_{R_\mu} = +\frac{6}{5} g_1^4 M_1 \mu + 6 g_2^2 M_2 \mu + 2 \lambda S B_S \kappa_S^2 + 4 \mu \lambda S A_S + 12 \mu \lambda_T^2 A_T \]
\[ \beta_{B\nu}^{(2)} = +B_{\mu} \left( \frac{207}{50} g_1^4 + \frac{9}{5} g_1^2 g_2^2 + \frac{27}{2} g_2^2 - 14 \lambda S^2 \right) + 16 g_2^2 Tr(Y_d Y_d^\dagger) + 6 \frac{g_1^2}{5} Tr(Y_e Y_e^\dagger) \]

\[ + \frac{4}{5} \lambda T \left( -60 \lambda T Y_d^\dagger Y_d + 60 \lambda T Y_d Y_d^\dagger - 15 \lambda T Y_u^\dagger Y_u + 15 \lambda T Y_u Y_u^\dagger \right) \]

\[ \beta_{B\nu}^{(1)} = 4 \left(-2 g_1^2 B_T + 2 M_T \lambda_T A_T + 4 g_2^2 M_2 M_T + |\lambda T|^2 B_T \right) \]

\[ \beta_{B\nu}^{(1)} = 4 \left( -2 g_1^2 B_T + 2 M_T \lambda_T A_T + 4 g_2^2 M_2 M_T + |\lambda T|^2 B_T \right) \]

\[ \beta_{B\nu}^{(2)} = -\frac{1}{5} \left( B_T \left( 30 \lambda T^2 \lambda_T^2 - 70 g_1^4 \right) + |\lambda T|^2 \left( 10 \lambda S^2 + 15 \lambda T Y_d^\dagger Y_d + 15 \lambda T Y_u Y_u^\dagger - 3 g_1^2 + 5 g_2^2 + 5 \lambda T Y_e Y_e^\dagger \right) \right) \]

\[ + 2 M_T \left( 140 g_1^2 M_2 + 60 \lambda T \lambda T^2 A_T \right) \]

\[ + \lambda T \left( 10 \lambda S^2 + 15 \lambda T Y_d^\dagger Y_d + 15 \lambda T Y_u Y_u^\dagger - 3 g_1^2 + 5 g_2^2 + 5 \lambda T Y_e Y_e^\dagger \right) \]

\[ + 2 M_T \left( 140 g_1^2 M_2 + 60 \lambda T \lambda T^2 A_T \right) \]

\[ + \lambda T \left( 10 \lambda S^2 + 15 \lambda T Y_d^\dagger Y_d + 15 \lambda T Y_u Y_u^\dagger - 3 g_1^2 + 5 g_2^2 + 5 \lambda T Y_e Y_e^\dagger \right) \]
\[\beta_{BO}^{(2)} = 72g_3^2 \left( -4M_3MO + B_O \right) \]  
\[\beta_{BS}^{(1)} = 4 \left( 2|\kappa_S|^2 + |\lambda_S|^2 \right) B_S + 8 \left( \kappa_S B_\mu \lambda_S^* + M_S \kappa_S A_\kappa + M_S \lambda_S A_S \right) \]  
\[\beta_{BS}^{(2)} = -\frac{4}{5} \left( B_S \left( 40 \kappa_S \lambda_S^* \right) + 40 \lambda_S \kappa_S \lambda_S^* \right) \]
\[+ |\lambda_S|^2 \left( 10 \lambda_S \lambda_S^* + 15 \lambda_S^2 \right) + 15 \lambda_S \lambda_S^* \left( Y_{\lambda_a} Y_{\lambda_a}^\dagger \right) + 30 \lambda_T \lambda_T^* - 3g_1^2 + 5 \lambda_T \lambda_T^* \]  
\[+ 2 \left( 50M_S \kappa_S \lambda_S^* \right) \left( 2M_S \lambda_S + \kappa_S \mu \right) A_S + \kappa_S \lambda_S B_\mu \]  
\[+ \lambda_S^2 \left( 9g_1^2 M_1 M_S \lambda_S + 15g_2^2 M_2 M_S \lambda_S + 9g_3^2 M_1 \kappa_S \mu + 45g_2^2 M_2 \kappa_S \mu - 3g_1^2 M_S A_S - 15g_2^2 M_S A_S \right) \]
\[+ 30M_S \lambda_T^2 A_S + 10M_S \kappa_S^2 \left( 2 \lambda_S A_\kappa + 3 \kappa_S A_S \right) + 30M_S \lambda_S \lambda_T^* A_T + 30 \kappa_S \lambda_T^* A_T \]
\[+ 15M_S A_S A_T \left( Y_{\lambda_a} Y_{\lambda_a}^\dagger \right) \]
\[+ 5M_S A_S \left( Y_{\lambda_a} Y_{\lambda_a}^\dagger \right) + 15M_S A_T \left( Y_{\lambda_a} Y_{\lambda_a}^\dagger \right) \]
\[+ \kappa_S B_\mu \left( 15 \left( Y_{\lambda_a} Y_{\lambda_a}^\dagger \right) + 15 \lambda_T^2 - 45g_2^2 + 5 \lambda_T^2 \right) \frac{g_1^2}{M_3} \left( Y_{\lambda_a} Y_{\lambda_a}^\dagger \right) - 9g_1^2 + 15M_S \lambda_S \lambda_T^* A_T \]
\[+ 15 \kappa_S \mu \left( Y_{\lambda_a} A_\mu + 5 \lambda_S A_T \left( Y_{\lambda_a} A_\mu + 5 \kappa_S \mu \right) Y_{\lambda_a} A_\mu + 15M_S \lambda_S \lambda_T^* A_T \right) \]
\[+ 15 \kappa_S \mu \left( Y_{\lambda_a} A_\mu \right) \]  

A.9 Linear Soft-Breaking Parameters

\[ (4\pi)^2 X_S^{(1)} = 2 \left( 2m_\kappa M_S^* + M_S B_\mu \lambda_S^* + 2m_\kappa B_\mu \lambda_S^* + 2m_0 \lambda_S \mu^* + 2m_0 \lambda_S \mu^* + |\kappa_S|^2 t_S + |\lambda_S|^2 t_S \right) \]
\[ + 2L_S \kappa_S \lambda_S^* A_\kappa + B_\mu \lambda_S^* A_\kappa + 2L_S \lambda_S^* A_S + 2B_\mu \lambda_S^* A_S \]  
\[ (4\pi)^4 X_S^{(2)} = -\frac{2}{5} \left( 6g_1^2 L_S M_1 |\lambda_S|^2 + 30g_1^2 L_S M_2 |\lambda_S|^2 + 20M_S \left( |\kappa_S|^2 + |\lambda_S|^2 \right) B_S \kappa_S^2 + 6g_1^2 M_1 M_S \mu \lambda_S^* \right) \]
\[ + 30g_1^2 M_2 M_S \mu \lambda_S^* - 6g_1^2 M_1 B_\mu \lambda_S^* - 30g_2^2 M_2 B_\mu \lambda_S^* + 60M_S |\lambda_T|^2 B_\mu \lambda_S^* + 20M_S \lambda_S B_\mu \lambda_S^* \]
\[ - 6g_1^2 L_S M_2 \lambda_S \mu^* - 30g_1^2 L_S M_1 \lambda_S \mu^* - 6g_1^2 L_S M_2 \lambda_S \mu^* - 12g_1^2 L_S \lambda_S |\lambda_T|^2 \mu^* + 120m_1^2 M_1 \lambda_S |\lambda_T|^2 \mu^* \]
\[ + 60m_1^2 \lambda_S |\lambda_T|^2 \mu^* + 40L_S |A_S|^2 \mu^* + 60L_S |A_T|^2 \mu^* + 40m_1^2 M_1 \lambda_S^2 \mu^* + 40m_1^2 M_1 \lambda_S^2 \mu^* \]
\[ + 20m_1^2 \lambda_S^2 \mu^* + 120m_1^2 M_2 \lambda_S^2 \mu^* + 30g_2^2 M_2 \lambda_S B_\mu^* - 3g_2^2 |\lambda_S|^2 t_S - 15g_2^2 |\lambda_S|^2 t_S \]
\[ + 20\kappa_S^2 t_S + 20L_S |\kappa_S|^2 t_S + 30L_S |\kappa_S|^2 t_S + 30 \lambda_S |\lambda_T|^2 t_S + 40L_S |\lambda_T|^2 t_S \]
\[ + 20L_S |\lambda_T|^2 t_S + 20L_S |\lambda_T|^2 t_S + 20L_S |\lambda_T|^2 t_S + 20L_S |\lambda_T|^2 t_S \]
\[ + 20L_S |\lambda_T|^2 t_S + 20L_S |\lambda_T|^2 t_S + 20L_S |\lambda_T|^2 t_S + 20L_S |\lambda_T|^2 t_S \]
\[ + 20L_S |\lambda_T|^2 t_S + 20L_S |\lambda_T|^2 t_S + 20L_S |\lambda_T|^2 t_S + 20L_S |\lambda_T|^2 t_S \]

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\[
\begin{align*}
+ 60L_S|\lambda_S|^2\lambda_T^2\lambda_T^*A_T + 60M_S\lambda_S^2\lambda_T^*A_T + 60\lambda_S\lambda_T^*B_{\mu}^*A_T + 30M_SB_{\mu}\lambda_T^*Tr\left(Y_dY_d^\dagger\right) \\
+ 60m_{H_d}^2\lambda_S\mu^*Tr\left(Y_dY_d^\dagger\right) + 30m_{H_s}^2\lambda_S\mu^*Tr\left(Y_dY_d^\dagger\right) + 15|\lambda_S|^2t_STr\left(Y_dY_d^\dagger\right) \\
+ 30L_S\lambda_S^*A_STr\left(Y_dY_d^\dagger\right) + 30B_{\mu}^*A_STr\left(Y_dY_d^\dagger\right) + 10M_SB_{\mu}\lambda_T^*Tr\left(Y_cY_c^\dagger\right) \\
+ 20m_{H_d}^2\lambda_S\mu^*Tr\left(Y_cY_c^\dagger\right) + 10m_{H_s}^2\lambda_S\mu^*Tr\left(Y_cY_c^\dagger\right) + 5|\lambda_S|^2t_STr\left(Y_cY_c^\dagger\right) \\
+ 10L_S\lambda_S^*A_STr\left(Y_cY_c^\dagger\right) + 10B_{\mu}^*A_STr\left(Y_cY_c^\dagger\right) + 30M_SB_{\mu}\lambda_T^*Tr\left(Y_uY_u^\dagger\right) \\
+ 30m_{H_d}^2\lambda_S\mu^*Tr\left(Y_uY_u^\dagger\right) + 60m_{H_s}^2\lambda_S\mu^*Tr\left(Y_uY_u^\dagger\right) + 15|\lambda_S|^2t_STr\left(Y_uY_u^\dagger\right) \\
+ 30L_S\lambda_S^*A_STr\left(Y_uY_u^\dagger\right) + 30B_{\mu}^*A_STr\left(Y_uY_u^\dagger\right) + 30L_S|\lambda_S|^2Tr\left(Y_uA_d\right) + 30M_S\lambda_S^*Tr\left(Y_a^\dagger A_d\right) \\
+ 30\lambda_SB_{\mu}^*Tr\left(Y_d^\dagger A_d\right) + 10L_S|\lambda_S|^2Tr\left(Y_d^\dagger A_e\right) + 10M_S\lambda_S^*Tr\left(Y_d^\dagger A_e\right) + 10\lambda_SB_{\mu}^*Tr\left(Y_c^\dagger A_e\right) \\
+ 30L_S|\lambda_S|^2Tr\left(Y_u^\dagger A_u\right) + 30M_S\lambda_S^*Tr\left(Y_u^\dagger A_u\right) + 30\lambda_SB_{\mu}^*Tr\left(Y_u^\dagger A_u\right) + 30\lambda_S^*Tr\left(A_u^\dagger Y_d^\dagger\right) \\
+ 30\lambda_S\mu^*Tr\left(A_u^\dagger A_d^\dagger\right) + 10\mu^*A_STr\left(A_u^\dagger Y_c^\dagger\right) + 10\lambda_S\mu^*Tr\left(A_e^\dagger A_e^\dagger\right) + 10\mu^*A_STr\left(A_u^\dagger Y_u^\dagger\right) \\
+ 30\lambda_S\mu^*Tr\left(A_u^\dagger A_u^\dagger\right) + 30\lambda_S\mu^*Tr\left(Y_u^\dagger m_q^2Y_u^\dagger\right) + 30\lambda_S\mu^*Tr\left(Y_a^\dagger m_q^2Y_u^\dagger\right) \\
+ 10\lambda_S\mu^*Tr\left(Y_c^\dagger m_e^2Y_c^\dagger\right) + 10\lambda_S\mu^*Tr\left(Y_u^\dagger m_q^2Y_u^\dagger\right) + 30\lambda_S\mu^*Tr\left(Y_a^\dagger m_q^2Y_u^\dagger\right) \\
+ 30\lambda_S\mu^*Tr\left(Y_a^\dagger m_q^2Y_u^\dagger\right)
\end{align*}
\] (A.76)

A.10  Soft-Breaking Scalar Masses

\[
\sigma_{1,1} = \sqrt{\frac{3}{5}}g_1\left(-2Tr\left(m_u^2\right) - m_{H_d}^2 - Tr\left(m_d^2\right) + m_{H_u}^2 + Tr\left(m_q^2\right) + Tr\left(m_d^2\right) + Tr\left(m_u^2\right)\right) \\
\text{Tr}2U1\left(1,1\right) = \frac{1}{10}g_1^2\left(2Tr\left(m_d^2\right) + 3m_{H_d}^2 + 3m_{H_u}^2 + 3Tr\left(m_l^2\right) + 6Tr\left(m_d^2\right) + 8Tr\left(m_u^2\right) + Tr\left(m_q^2\right)\right) \\
\sigma_{3,1} = \frac{1}{20}\sqrt{15}g_1\left(-9g_1^2m_{H_d}^2 - 45g_2^2m_{H_u}^2 + 9g_1^2m_{H_u}^2 + 45g_2^2m_{H_u}^2\right) \\
+ 30\left(-m_{H_u}^2 + m_{H_d}^2\right)|\lambda_S|^2 + 90\left(-m_{H_u}^2 + m_{H_d}^2\right)|\lambda_T|^2 \\
- 9g_1^2Tr\left(m_l^2\right) - 45g_2^2Tr\left(m_l^2\right) + g_1^2Tr\left(m_u^2\right) + 45g_2^2Tr\left(m_q^2\right) + 8g_1^2Tr\left(m_u^2\right) + 4g_2^2Tr\left(m_u^2\right) \\
+ 80g_1^2Tr\left(m_d^2\right) + 36g_2^2Tr\left(m_d^2\right) - 32g_1^2Tr\left(m_u^2\right) - 16g_2^2Tr\left(m_u^2\right) + 90m_{H_d}^2Tr\left(Y_dY_d^\dagger\right) \\
+ 30m_{H_d}^2Tr\left(Y_dY_d^\dagger\right) - 90m_{H_u}^2Tr\left(Y_uY_u^\dagger\right) - 60Tr\left(Y_dY_d^\dagger m_q^2Y_u^\dagger\right) - 30Tr\left(Y_a^\dagger m_q^2Y_u^\dagger\right) \\
- 60Tr\left(Y_c^\dagger m_e^2Y_c^\dagger\right) + 30Tr\left(Y_u^\dagger m_q^2Y_a^\dagger\right) + 120Tr\left(Y_a^\dagger m_q^2Y_u^\dagger\right) - 30Tr\left(Y_a^\dagger m_q^2Y_u^\dagger\right)
\]

\[
\sigma_{2,2} = \frac{1}{2}\left(3Tr\left(m_q^2\right) + 6m_l^2 + m_{H_d}^2 + m_{H_u}^2 + Tr\left(m_l^2\right)\right) \\
\sigma_{2,3} = \frac{1}{2}\left(16m_q^2 + 2Tr\left(m_q^2\right) + Tr\left(m_u^2\right) + Tr\left(m_d^2\right)\right)
\] (A.77)
\[
\beta^{(1)}_{m_q} = -\frac{2}{15} g_1^2 \left| M_1 \right|^2 - \frac{32}{3} g_2^2 \left| M_3 \right|^2 - 6 g_1^2 \left| M_2 \right|^2 + 2 m_{H_u}^2 Y_d^\dagger Y_d + 2 m_{H_u}^2 Y_u^\dagger Y_u + 2 A_d^\dagger A_d \\
+ 2 A_u^\dagger A_u + m_q^2 Y_d^\dagger Y_d + m_q^2 Y_u^\dagger Y_u + 2 Y_d^\dagger m_q^2 Y_d + Y_u^\dagger m_q^2 Y_u + 2 Y_d^\dagger m_q^2 Y_u \\
+ Y_u^\dagger m_q^2 + \frac{1}{\sqrt{15}} g_1 \sigma_{1,1} 
\]
\[
\beta^{(2)}_{m_q} = \frac{2}{5} g_1^2 g_2^2 \left| M_2 \right|^2 + 69 g_1^2 \left| M_2 \right|^2 + 32 g_2^2 g_3^2 \left| M_2 \right|^2 \\
+ \frac{16}{45} g_2^2 \left( 15 \left( 10 g_3^2 M_3 + 3 g_2^2 \left( 2 M_3 + M_2 \right) \right) + g_1^2 \left( 2 M_3 + M_1 \right) \right) 1 M_3^* \\
+ \frac{1}{5} g_1^2 g_2^2 M_1 M_2^* + 16 g_2^2 g_3^2 M_1 M_2^* \\
+ \frac{4}{5} g_2^2 m_{H_u}^2 Y_d^\dagger Y_d - 4 m_{H_u}^2 \left| \lambda_S \right|^2 Y_d^\dagger Y_d - 2 m_{H_u}^2 \left| \lambda_S \right|^2 Y_u^\dagger Y_u \\
- 2 m_3^2 \left| \lambda_S \right|^2 Y_d^\dagger Y_d - 12 m_{H_u}^2 \left| \lambda_T \right|^2 Y_d^\dagger Y_d - 6 m_{H_u}^2 \left| \lambda_T \right|^2 Y_u^\dagger Y_u \\
- 6 m_7^2 \left| \lambda_T \right|^2 Y_d^\dagger Y_d - 2 \left| A_S \right|^2 Y_d^\dagger Y_d - 6 \left| A_T \right|^2 Y_d^\dagger Y_d - 2 \lambda_S A_S^\dagger Y_d^\dagger Y_u \\
- 6 \lambda_T A_T^\dagger Y_u^\dagger Y_d + \frac{8}{5} g_1^2 m_{H_u}^2 Y_u^\dagger Y_u - 2 m_{H_u}^2 \left| \lambda_S \right|^2 Y_u^\dagger Y_u - 4 m_{H_u}^2 \left| \lambda_S \right|^2 Y_u^\dagger Y_u \\
- 2 m_5^2 \left| \lambda_S \right|^2 Y_d^\dagger Y_u - 6 m_{H_u}^2 \left| \lambda_T \right|^2 Y_d^\dagger Y_u - 12 m_{H_u}^2 \left| \lambda_T \right|^2 Y_u^\dagger Y_u \\
- 6 m_2^2 \left| \lambda_T \right|^2 Y_d^\dagger Y_u - 2 \left| A_S \right|^2 Y_d^\dagger Y_u - 6 \left| A_T \right|^2 Y_d^\dagger Y_u \\
+ \frac{1}{225} g_1^2 M_1^* \left( \left( 16 \left( 2 M_1 + M_3 \right) + 9 g_3^2 \left( 2 M_1 + M_2 \right) \right) + 59 g_1^2 M_1 \right) 1 \\
+ 180 \left( 2 M_1 Y_d^\dagger Y_d - 2 Y_u^\dagger A_u + 4 M_1 Y_u^\dagger Y_u - Y_u^\dagger A_d \right) \\
- 2 \lambda_S A_S^\dagger Y_u^\dagger Y_u - 6 \lambda_T A_T^\dagger Y_u^\dagger Y_u - \frac{4}{5} g_2^2 M_1 A_u^\dagger Y_d + \frac{4}{5} g_2^2 A_d^\dagger A_d \\
- 2 \lambda_S A_S^\dagger A_d^\dagger A_u - 6 \lambda_T A_T^\dagger A_d^\dagger A_u - \frac{8}{5} g_1^2 M_1 A_d^\dagger Y_u + \frac{8}{5} g_1^2 A_u^\dagger A_u \\
- 2 \lambda_S A_S^\dagger A_u^\dagger A_u - 6 \lambda_T A_T^\dagger A_u^\dagger A_u + \frac{2}{5} g_1^2 m_q^2 Y_d^\dagger Y_d - \left| \lambda_S \right|^2 m_q^2 Y_d^\dagger Y_d \\
- 3 \lambda_T^2 m_q^2 Y_d^\dagger Y_u + \frac{4}{5} g_2^2 m_q^2 Y_d^\dagger Y_u - \left| \lambda_S \right|^2 m_q^2 Y_u^\dagger Y_u \\
- 3 \lambda_T^2 m_q^2 Y_u^\dagger Y_u + \frac{4}{5} g_2^2 m_q^2 Y_d^\dagger Y_d - 2 \left| \lambda_S \right|^2 m_q^2 Y_d^\dagger Y_d \\
- 6 \lambda_T^2 Y_d^\dagger m_q^2 Y_d + \frac{2}{5} g_2^2 Y_d^\dagger Y_d m_q^2 - \left| \lambda_S \right|^2 Y_d^\dagger Y_d m_q^2 \\
- 3 \lambda_T^2 Y_d^\dagger Y_d m_q^2 + \frac{8}{5} g_1^2 m_q^2 Y_u^\dagger Y_u - 2 \left| \lambda_S \right|^2 Y_u^\dagger Y_d m_q^2 \\
- 6 \lambda_T^2 Y_u^\dagger m_q^2 Y_u + \frac{4}{5} g_2^2 Y_u^\dagger Y_u m_q^2 - \left| \lambda_S \right|^2 Y_u^\dagger Y_u m_q^2 \\
- 3 \lambda_T^2 Y_u^\dagger Y_d^\dagger Y_u m_q^2 - 8 m_{H_u}^2 Y_u^\dagger Y_d^\dagger Y_d - 4 Y_d^\dagger Y_d A_d^\dagger A_d - 4 Y_d^\dagger A_d A_d^\dagger A_d Y_d \\
- 8 m_q^2 Y_u^\dagger Y_u A_u^\dagger A_u Y_u - 4 Y_u^\dagger Y_u A_u^\dagger A_u Y_u - 4 A_u^\dagger A_u A_u^\dagger A_u Y_u - 4 A_d^\dagger A_d Y_d Y_d^\dagger A_d \\
- 4 A_d^\dagger A_d Y_d Y_d - 4 A_u^\dagger A_u Y_u Y_u - 4 A_d^\dagger A_u Y_u Y_u - 2 m_q^2 Y_u^\dagger Y_d^\dagger Y_u Y_d \\
- 2 m_q^2 Y_d^\dagger Y_u Y_u Y_u - 4 Y_d^\dagger m_q^2 Y_d^\dagger Y_u Y_u - 4 Y_d^\dagger Y_u m_q^2 Y_d^\dagger Y_u - 4 Y_d^\dagger Y_d m_q^2 Y_u Y_u \\
- 2 Y_d^\dagger Y_d Y_d m_q^2 - 4 Y_d^\dagger m_q^2 Y_d Y_u Y_u - 4 Y_d^\dagger Y_u m_q^2 Y_d Y_u - 4 Y_d^\dagger Y_d Y_u m_q^2 Y_u 
\]
\[ -2Y_d^\dagger Y_u^\dagger Y_u m_q^2 - 2\lambda_S A^\dagger_1 Y_d A_S - 2\lambda_S A^\dagger_1 Y_u A_S - 6\lambda_T A^\dagger_1 Y_AT \]
\[ - 6\lambda_T A^\dagger_1 Y_A Y_A + 6g_2^2\mathbf{1}_{2\times 2} + \frac{32}{3} g_1^2 1\sigma_{2,2} + \frac{2}{15} g_1^2 \text{Tr}2U1(1,1) + 4 \sqrt{\frac{2}{5}} g_1 1\sigma_{3,1} \]
\[ - 12m_{H\nu}^2 Y_d^\dagger Y_d \text{Tr}(Y_d Y_d^\dagger) - 6 A^\dagger_2 A^\dagger_1 \text{Tr}(Y_d Y_d^\dagger) - 3m_{H\nu}^2 Y_d^\dagger Y_d \text{Tr}(Y_d Y_d^\dagger) \]
\[ - 6 Y_d^\dagger m_1 Y_d \text{Tr}(Y_d Y_d^\dagger) - 3 Y_d^\dagger Y_d m_1^2 \text{Tr}(Y_d Y_d^\dagger) - 4m_{H\nu}^2 Y_d^\dagger Y_d \text{Tr}(Y_d Y_d^\dagger) \]
\[ - 2A^\dagger_2 A^\dagger_1 \text{Tr}(Y_e Y_e^\dagger) - m_2^2 Y_d^\dagger Y_d \text{Tr}(Y_e Y_e^\dagger) - 2Y_d^\dagger m_2^2 Y_d \text{Tr}(Y_e Y_e^\dagger) \]
\[ - Y_d^\dagger Y_d m_2^2 \text{Tr}(Y_e Y_e^\dagger) - 12m_{H\nu}^2 Y_u^\dagger Y_u \text{Tr}(Y_u Y_u^\dagger) - 6 A^\dagger_1 A^\dagger_1 \text{Tr}(Y_u Y_u^\dagger) \]
\[ - 3m_2^2 Y_d^\dagger Y_d \text{Tr}(Y_u Y_u^\dagger) - 6 Y_u^\dagger m_2^2 Y_u \text{Tr}(Y_u Y_u^\dagger) - 3Y_u^\dagger Y_u m_2^2 \text{Tr}(Y_u Y_u^\dagger) \]
\[ - 6 A^\dagger_1 Y_d \text{Tr}(Y_d A_d) - 2 A^\dagger_1 Y_d \text{Tr}(Y_d^\dagger A_d) - 6 A^\dagger_1 Y_u \text{Tr}(Y_d^\dagger A_u) \]
\[ - 6 Y_d^\dagger A_d \text{Tr}(Y_d^\dagger A_d) - 6 Y_d^\dagger Y_d \text{Tr}(Y_d^\dagger A_d) - 2 Y_d^\dagger A_d \text{Tr}(A_d^\dagger Y_d^\dagger) \]
\[ + 2 Y_d^\dagger Y_d \text{Tr}(A_d^\dagger A_d^\dagger) - 6 Y_d^\dagger A_d \text{Tr}(A_d^\dagger A_d^\dagger) - 6 Y_d^\dagger Y_d \text{Tr}(A_d^\dagger A_d^\dagger) \]
\[ - 6 Y_d^\dagger Y_d \text{Tr}(m_2^2 Y_d^\dagger Y_d^\dagger) - 2 Y_d^\dagger Y_d \text{Tr}(m_2^2 Y_e^\dagger Y_e^\dagger) - 2 Y_d^\dagger Y_d \text{Tr}(m_2^2 Y_e^\dagger Y_e^\dagger) \]
\[ - 6 Y_d^\dagger Y_d \text{Tr}(m_2^2 Y_d^\dagger Y_d^\dagger) - 6 Y_d^\dagger Y_d \text{Tr}(m_2^2 Y_u^\dagger Y_u^\dagger) - 6 Y_d^\dagger Y_d \text{Tr}(m_2^2 Y_u^\dagger Y_u^\dagger) \]

\[ \beta_{m_1}^{(1)} = -\frac{6}{5} g_1^2 1|M_1|^2 - 6g_2^2\mathbf{1}|M_2|^2 + 2m_{H\nu}^2 Y_e^\dagger Y_e + 2A^\dagger_1 A_e + m_2^2 Y_e^\dagger Y_e + 2Y_e^\dagger m_2^2 Y_e \]
\[ + Y_e^\dagger Y_e m_2^2 - \sqrt{\frac{3}{5}} g_1 1\sigma_{1,1} \]

\[ \beta_{m_1}^{(2)} = +\frac{3}{5} g_2^2 \left( 115g_2^2 M_2 + 3g_1^2 \left( 2M_2 + M_1 \right) \right) 1M_2^* + \frac{12}{5} g_1^2 m_{H\nu}^2 Y_e^\dagger Y_e - \frac{4m_{H\nu}^2}{5} |\lambda_S|^2 Y_e^\dagger Y_e \]
\[ - 2m_{H\nu}^2 |\lambda_S|^2 Y_e^\dagger Y_e - 2m_2^2 |\lambda_S|^2 Y_e^\dagger Y_e - 12m_{H\nu}^2 |\lambda_T|^2 Y_e^\dagger Y_e \]
\[ - 6m_{H\nu}^2 |\lambda_T|^2 Y_e^\dagger Y_e - 6m_2^2 |\lambda_T|^2 Y_e^\dagger Y_e - 2 |\lambda_S|^2 Y_e^\dagger Y_e - 6 |\lambda_T|^2 Y_e^\dagger Y_e \]
\[ + \frac{3}{25} g_1^2 M_1^* \left( - 20Y_e^\dagger A_e + 3 \left( 5g_1^2 \left( 2M_1 + M_2 \right) + 69g_1^2 M_1 \right) 1 + 40 M_1 Y_e^\dagger Y_e \right) - 2\lambda_S A^\dagger_1 Y_e^\dagger A_e \]
\[ - 6\lambda_T A^\dagger_1 Y_e^\dagger A_e - \frac{12}{5} g_1^2 M_1 A^\dagger_1 Y_e + \frac{12}{5} g_1^2 A^\dagger_1 A_e - 2 |\lambda_S|^2 A^\dagger_1 A_e \]
\[ - 6 |\lambda_T|^2 A^\dagger_1 A_e + \frac{6}{5} g_1^2 m_2^2 Y_e^\dagger Y_e - |\lambda_S|^2 m_2^2 Y_e^\dagger Y_e - 3|\lambda_T|^2 m_2^2 Y_e^\dagger Y_e \]
\[ + \frac{12}{5} g_1^2 m_2^2 Y_e^\dagger Y_e - 2 |\lambda_S|^2 m_2^2 Y_e^\dagger Y_e - 6 |\lambda_T|^2 m_2^2 Y_e^\dagger Y_e + \frac{6}{5} g_1^2 Y_e^\dagger Y_e m_2^2 \]
\[ - |\lambda_S|^2 Y_e^\dagger Y_e m_2^2 - 3 |\lambda_T|^2 Y_e^\dagger Y_e m_2^2 - 8m_{H\nu}^2 Y_e^\dagger Y_e Y_e^\dagger Y_e - 4Y_e^\dagger Y_e A^\dagger_1 A_e \]
\[ - 4Y_e^\dagger A_1^\dagger Y_e^\dagger Y_e - 4A_1^\dagger Y_e^\dagger Y_e - 4A_1^\dagger A_1^\dagger Y_e^\dagger Y_e - 2m_2^2 Y_e^\dagger Y_e Y_e^\dagger Y_e \]
\[ - 4Y_e^\dagger m_2^2 Y_e^\dagger Y_e Y_e - 4Y_e^\dagger Y_e m_2^2 Y_e^\dagger Y_e - 4Y_e^\dagger Y_e^\dagger Y_e m_2^2 Y_e - 2Y_e^\dagger Y_e Y_e^\dagger Y_e m_2^2 \]
\[ - 2\lambda_S A^\dagger_1 Y_e A_e - 6\lambda_T A^\dagger_1 Y_e A_e + 6g_2^2 1\sigma_{2,2} + \frac{6}{5} g_1^2 \text{Tr}2U1(1,1) + 4 \sqrt{\frac{3}{5}} g_1 1\sigma_{3,1} \]
\[ - 12m_{H\nu}^2 Y_e^\dagger Y_e \text{Tr}(Y_d Y_d^\dagger) - 6 A^\dagger_1 A_e \text{Tr}(Y_d Y_d^\dagger) - 3m_2^2 Y_e^\dagger Y_e \text{Tr}(Y_d Y_d^\dagger) \]
\[ -6Y_c^4m_2^2Y_c Tr\left(Y_d Y_d^\dagger\right) - 3Y_c^4Y_e m_2^2Tr\left(Y_d Y_d^\dagger\right) - 4m_2^2Y_c Y_c Tr\left(Y_d Y_d^\dagger\right) \\
- 2A_i^4 A_e Tr\left(Y_d Y_d^\dagger\right) - m_2^2Y_c Y_e Tr\left(Y_e Y_e^\dagger\right) - 2Y_e^4m_2^2Tr\left(Y_d Y_d^\dagger\right) \\
- Y_e^4m_2^2Tr\left(Y_e Y_e^\dagger\right) - 6A_i^4 Y_e Tr\left(Y_e^\dagger A_d\right) - 2A_i Y_e Tr\left(Y_e^\dagger A_e\right) \\
- 6Y_c^4 A_e Tr\left(A_d^2 Y_e^T\right) - 6Y_e^4 A_e Tr\left(A_d^2 A_d^T\right) - 2Y_c^4 A_e Tr\left(A_e^* Y_e^T\right) \\
- 2Y_e^4 A_e Tr\left(A_e^* A_e^T\right) - 6Y_e^4 Y_e Tr\left(m_2^2 Y_d Y_d^\dagger\right) - 2Y_e^4 Y_e Tr\left(m_2^2 Y_e Y_e^\dagger\right) \\
- 2Y_e^4 Y_e Tr\left(m_2^2 Y_d Y_d^\dagger\right) - 6Y_e^4 Y_e Tr\left(m_2^2 Y_e Y_e^\dagger\right) \quad (A.81) \\
\]

\[ \beta_{m_2^4 A_e}^{(1)} = -\frac{6}{5}g_1^2|A|^2 - 6g_2^2M_2^2 + 2m_2^2H_u^2|\lambda_s|^2 + 2m_2^2H_u^2|\lambda_s|^2 + 2m_2^2|\lambda_s|^2 + 6m_2^2|H|^2 + 6m_2^2|\lambda_T|^2 \\
+ 6m_2^2|\lambda_T|^2 + 2|A|^2 + 2|A|^2 - \sqrt{\frac{3}{5}g_1\sigma_{11}} + 6m_2^2 Tr\left(Y_d Y_d^\dagger\right) + 2m_2^2 Tr\left(Y_e Y_e^\dagger\right) + 6Tr\left(A_e^* A_e^T\right) \\
+ 2Tr\left(A_e^* A_e^T\right) + 6Tr\left(m_2^2 Y_d Y_d^\dagger\right) + 2Tr\left(m_2^2 Y_e Y_e^\dagger\right) + 2Tr\left(m_2^2 Y_e Y_e^\dagger\right) + 6Tr\left(m_2^2 Y_d Y_d^\dagger\right) \quad (A.82) \\
\]

\[ \beta_{m_2^4 A_e}^{(2)} = \frac{1}{25} \left( g_1^2 M_1^4 + 621g_2^2 M_1 + 90g_2^2 M_2 + 45g_2^2 M_2 - 40M_1 Tr\left(Y_d Y_d^\dagger\right) + 120 M_1 Tr\left(Y_e Y_e^\dagger\right) + 20 Tr\left(Y_d Y_d^\dagger\right) \\
- 60Tr\left(Y_e Y_e^\dagger\right) \right) \\
+ 5 \left( 3g_2^2 M_2^2 \left( 115g_2^2 M_2 + 3g_2^2 \left( 2M_2 + M_1 \right) + 40\lambda_e^* \left( 2M_2 - A_T \right) \right) \right) \\
- 2 \left( - 60g_2^2 M_2^2 |\lambda_T|^2 - 60g_2^2 M_2^2 |\lambda_T|^2 - 60g_2^2 |A|^2 \right) \\
+ 30 \left( m_2^2 + m_2^2 H_u^2 + \frac{3}{5}g_1\sigma_{11} \right) |\lambda_s|^2 + 150 m_2^2 |\lambda_T|^2 + 150 m_2^2 |\lambda_T|^2 + 150 m_2^2 |\lambda_T|^2 \\
+ 60g_2^2 M_2 |\lambda_T|^2 A^*_e + 30 |A|^2 A^*_e A^*_e + 10\lambda_e^* \left( 4m_2^2 + m_2^2 H_u^2 + m_2^2 H_u^2 \right) |\lambda_s|^2 + A^*_e \left( |\kappa_s A_s + \lambda_s A_e| \right) \\
+ 30 \lambda_e^* A^*_e A^*_e A^*_e + 300 |A|^2 A^*_e A^*_e A^*_e + 15g_2^2 \sigma_{21}^2 - 3g_2^2 Tr\left( U_{11}^2 \right) + 2\sqrt{5}g_1\sigma_{31} \\
+ 2g_2^2 m_2^2 Tr\left(Y_d Y_d^\dagger\right) - 80g_3^2 m_2^2 Tr\left(Y_d Y_d^\dagger\right) - 160g_3^2 m_2^2 Tr\left(Y_d Y_d^\dagger\right) \\
- 6g_2^2 m_2^2 Tr\left(Y_e Y_e^\dagger\right) + 45m_2^2 |\lambda_T|^2 Tr\left(Y_d Y_d^\dagger\right) + 90m_2^2 H_u^2 |\lambda_T|^2 Tr\left(Y_d Y_d^\dagger\right) \\
+ 45m_2^2 |\lambda_T|^2 Tr\left(Y_e Y_e^\dagger\right) + 15 |A|^2 Tr\left(Y_d Y_d^\dagger\right) + 45 |A|^2 Tr\left(Y_e Y_e^\dagger\right) + 80g_3^2 M_3^2 Tr\left(Y_d Y_d^\dagger\right) \\
+ 15 \lambda_e^* A^*_e A^*_e A^*_e A^*_e + 45 \lambda_T A^*_e A^*_e Y_A^* Y_A^* A^*_e + 2g_2^2 M_1 Tr\left(A_d^* ^T Y_d^T\right) + 80g_2^2 m_2^2 Tr\left(A_d^* ^T Y_d^T\right) \\
+ 2g_2^2 Tr\left(A_d^* ^T Y_d^T\right) - 80g_2^2 M_2 Tr\left(A_d^* ^T A_d^T\right) + 6g_2^2 M_2 Tr\left(A_d^* ^T A_d^T\right) - 6g_2^2 Tr\left(A_d^* ^T A_d^T\right) \\
+ 45 |\lambda_T|^2 Tr\left(A_d^* ^T Y_d^T\right) + 45 |\lambda_T|^2 Tr\left(A_d^* ^T A_d^T\right) + 45 |\lambda_T|^2 Tr\left(A_d^* ^T A_d^T\right) - 80g_2^2 Tr\left(m_2^2 Y_d Y_d^\dagger\right) \\
- 60g_2^2 Tr\left(m_2^2 Y_e Y_e^\dagger\right) - 60g_2^2 Tr\left(m_2^2 Y_e Y_e^\dagger\right) + 2g_2^2 Tr\left(m_2^2 Y_d Y_d^\dagger\right) - 80g_2^2 Tr\left(m_2^2 Y_d Y_d^\dagger\right) \\
+ 45 |\lambda_T|^2 Tr\left(m_2^2 Y_d Y_d^\dagger\right) + 45 |\lambda_T|^2 Tr\left(m_2^2 Y_d Y_d^\dagger\right) + 45 |\lambda_T|^2 Tr\left(m_2^2 Y_d Y_d^\dagger\right) \\
+ 5\lambda_e^* \left( 4 \left( m_2^2 H_u^2 + m_2^2 H_u^2 + m_2^2 H_u^2 \right) \lambda_s |\lambda_T|^2 + 2A_e^* \left( |\kappa_s A_s + \lambda_s A_e| \right) \right) \\
+ 5\lambda_e^* \left( 4 \left( m_2^2 H_u^2 + m_2^2 H_u^2 + m_2^2 H_u^2 \right) \lambda_s |\lambda_T|^2 + 2A_e^* \left( |\kappa_s A_s + \lambda_s A_e| \right) \right) \\
\]
\[ + 3 \left( 4\lambda_S |A_S|^2 + 2 A_T^* (\lambda_S A_T + \lambda_T A_S) + m_H^2 \lambda_S \text{Tr} (Y_a Y_u^\dagger) + 2m_{\bar{H}_u} \lambda_S \text{Tr} (Y_a Y_u^\dagger) \right) \\
+ m_S^2 \lambda_S \text{Tr} (Y_a Y_u^\dagger) \\
+ A_S \text{Tr} (A_u^\dagger Y^\dagger_T) + \lambda_S \text{Tr} (A_u^\dagger A_T^* + \lambda_S \text{Tr} (m_{\bar{Y}_u}^2 Y_u + \lambda_S \text{Tr} (m_{\bar{Y}_u}^2 Y_u)) \\
+ 90m_{\bar{H}_d}^2 \text{Tr} (Y_{\bar{d}} Y_{d}^\dagger) + 90 \text{Tr} (Y_{\bar{d}} Y_d A_d^\dagger) + 15m_{\bar{H}_d}^2 \text{Tr} (Y_{\bar{d}}^\dagger Y_d Y_{d}^\dagger) \\
+ 15m_{\bar{H}_d}^2 \text{Tr} (Y_{\bar{d}} Y_u^\dagger A_d) + 15 \text{Tr} (Y_d Y_u^\dagger A_d^\dagger) + 90 \text{Tr} (Y_d A_d^\dagger A_d Y_{d}^\dagger) \\
+ 15 \text{Tr} (Y_d A_u^\dagger A_u Y_{d}^\dagger) + 30m_{\bar{H}_d}^2 \text{Tr} (Y_{\bar{d}} Y_d^\dagger A_d^\dagger) + 30 \text{Tr} (Y_{\bar{d}} Y_d^\dagger A_d^\dagger) + 30 \text{Tr} (Y_d A_u^\dagger A_u Y_{d}^\dagger) \\
+ 15 \text{Tr} (m_{\bar{Y}_u}^2 Y_{\bar{d}} Y_d Y_{d}^\dagger) + 15 \text{Tr} (m_{\bar{Y}_u}^2 Y_{\bar{d}}^\dagger Y_d Y_{d}^\dagger) + 15 \text{Tr} (m_{\bar{Y}_u}^2 Y_{\bar{d}}^\dagger Y_d Y_{d}^\dagger Y_d Y_{d}^\dagger)) \]  

(A.83)

\[ \beta_{m_{\bar{H}_u}}^{(1)} = - \frac{6}{\sqrt{3}} g_1^2 (M_2^2 - 6g_2^2 |M_2|^2 + 2m_{\bar{H}_d}^2 |\lambda_S|^2 + 2m_{\bar{H}_u}^2 |\lambda_S|^2 + 6m_{\bar{H}_u}^2 |\lambda_T|^2 + 6m_{\bar{H}_u}^2 |\lambda_T|^2 \\
+ 6m_{\bar{H}_u}^2 |\lambda_T|^2 + 2|A_S|^2 + 6|A_T|^2 + \frac{3}{\sqrt{3}} g_1 \sigma_{1,1} + 6m_{\bar{H}_u}^2 \text{Tr} (Y_{a} Y_{u}^\dagger) + 6 \text{Tr} (A_u^\dagger A_T^*) + 6 \text{Tr} (m_{\bar{Y}_u}^2 Y_{\bar{d}} Y_{d}^\dagger) \\
+ 6 \text{Tr} (m_{\bar{Y}_u}^2 Y_{\bar{d}} Y_{u}^\dagger) \]  

(A.84)
\[ + 5\lambda_s^2 \left( 6(2m_{H_d}^2 + 2m_{H_u}^2 + m_S^2 + m_\nu^2)\lambda_s |\lambda_T|^2 + 12\lambda_s |A_S|^2 \right) \\
+ 6\lambda_s |A_T|^2 + 6\lambda_T A_T A_S + 2A_*^2 (\kappa_s A_S + \lambda_s A_\nu) \\
+ 6m_{H_d}^2 \lambda_s \text{Tr} \left( Y_d Y_d^\dagger \right) + 3m_{H_u}^2 \lambda_s \text{Tr} \left( Y_d Y_d^\dagger \right) + 3m_S^2 \lambda_s \text{Tr} \left( Y_d Y_d^\dagger \right) + 2m_{H_d}^2 \lambda_s \text{Tr} \left( Y_d Y_d^\dagger \right) \\
+ m_{H_u}^2 \lambda_s \text{Tr} \left( Y_d Y_d^\dagger \right) + m_S^2 \lambda_s \text{Tr} \left( Y_d Y_d^\dagger \right) + 3A_s \text{Tr} \left( A_d A_d^\dagger \right) + 3\lambda_s \text{Tr} \left( A_d A_d^\dagger \right) + A_s \text{Tr} \left( A_d A_d^\dagger \right) \\
+ \lambda_s \text{Tr} \left( A_*^2 A_*^\dagger \right) + 3\lambda_s \text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) + \lambda_s \text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) + \lambda_s \text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) \\
+ 3\lambda_s \text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) \\
- 4g_1^2 \text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) - 80g_2^2 \text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) - 4g_1^2 \text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) - 80g_2^2 \text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) \\
+ 15m_{H_d}^2 \text{Tr} \left( Y_d Y_d^\dagger \right) + 15m_{H_d}^2 \text{Tr} \left( Y_d Y_d^\dagger \right) + 15\text{Tr} \left( Y_d Y_d^\dagger \right) + 15\text{Tr} \left( Y_d Y_d^\dagger \right) \\
+ 15\text{Tr} \left( Y_d Y_d^\dagger \right) + 15\text{Tr} \left( Y_d Y_d^\dagger \right) + 90m_{H_d}^2 \text{Tr} \left( Y_d Y_d^\dagger \right) + 90\text{Tr} \left( Y_d Y_d^\dagger \right) + 90\text{Tr} \left( Y_d Y_d^\dagger \right) \\
+ 15\text{Tr} \left( Y_d Y_d^\dagger \right) + 15\text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) + 15\text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) + 90\text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) \\
+ 15\text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) + 90\text{Tr} \left( m_S^2 Y_d Y_d^\dagger \right) \right) \]
\[
- 4A_d Y_u^\dagger Y_u A_u^\dagger - 4A_d A_u^\dagger Y_u Y_u^\dagger - 4A_d A_u^\dagger Y_u Y_u^\dagger - 2m_q^2 Y_u Y_u^\dagger Y_u Y_u^\dagger \\
- 2m_q^2 Y_u Y_u Y_u Y_u^\dagger - 4Y_u m_q^2 Y_u Y_u Y_u^\dagger - 4Y_u Y_u m_q^2 Y_u Y_u^\dagger - 2Y_d A_d^\dagger A_d Y_d Y_d^\dagger \\
+ 4Y_d A_d^\dagger Y_d m_q^2 Y_u Y_u^\dagger - 4Y_d m_q^2 Y_u Y_u^\dagger Y_u Y_u^\dagger - 4Y_d Y_u^\dagger Y_u m_q^2 Y_u^\dagger \\
- 2Y_d A_d^\dagger Y_u Y_u^\dagger m_q^2 - 4\lambda_S^2 Y_d A_d A_S - 12\lambda_T Y_d A_d^\dagger A_T + \frac{32}{3} g_3^2 1\sigma_{2,3} + \frac{8}{15} g_1^2 1 T^2 U^1 (1, 1) \\
+ 8 \frac{1}{\sqrt{15}} g_1^1 \sigma_{3,1} - 24 m_2^2 Y_d Y_d^\dagger T^\dagger (Y_d Y_d^\dagger) - 12 A_d A_u^\dagger T^\dagger (Y_d Y_d^\dagger) \\
- 6m_q^2 Y_d Y_d^\dagger T^\dagger (Y_d Y_d^\dagger) - 12 Y_d m_q^2 Y_d^\dagger T^\dagger (Y_d Y_d^\dagger) - 6Y_d Y_d^\dagger m_q^2 T^\dagger (Y_d Y_d^\dagger) \\
- 8 m_q^2 Y_d Y_d^\dagger T^\dagger (Y_e Y_e^\dagger) - 4A_d A_d^\dagger T^\dagger (Y_e Y_e^\dagger) - 2m_q^2 Y_d Y_d^\dagger T^\dagger (Y_e Y_e^\dagger) \\
- 4Y_d m_q^2 Y_d^\dagger T^\dagger (Y_e Y_e^\dagger) - 2Y_d Y_d^\dagger m_q^2 T^\dagger (Y_e Y_e^\dagger) - 12 A_d A_d^\dagger T^\dagger (Y_e Y_e^\dagger) \\
- 4Y_d A_d^\dagger T^\dagger (Y_e Y_e^\dagger) - 12 A_d A_d^\dagger T^\dagger (A_u^\dagger A_u^\dagger) - 12 Y_d Y_d^\dagger T^\dagger (A_u^\dagger A_u^\dagger) \\
- 4A_d Y_d^\dagger T^\dagger (A_u^\dagger A_u^\dagger) - 4Y_d Y_d^\dagger T^\dagger (A_u^\dagger A_u^\dagger) - 12 Y_d Y_d^\dagger T^\dagger (m_q^2 Y_d Y_d^\dagger) \\
- 4Y_d Y_d^\dagger T^\dagger (m_q^2 Y_d Y_d^\dagger) - 4Y_d Y_d^\dagger T^\dagger (m_q^2 Y_e Y_e^\dagger) - 12 Y_d Y_d^\dagger T^\dagger (m_q Y_d Y_d^\dagger) \\
\]

(A.87)

\[
\beta_{m_u^2}^{(1)} = -\frac{32}{15} g_1^1 |M|_2^2 - \frac{32}{9} g_3^2 |M_3|^2 + 4m_2^2 Y_u Y_u^\dagger + 4A_u A_u^\dagger + 2m_q^2 Y_u Y_u^\dagger + 4Y_u m_q^2 Y_u^\dagger \\
+ 2Y_u Y_u^\dagger m_q^2 - 4 \frac{1}{\sqrt{15}} g_1^1 \sigma_{3,1} \\
\]

(A.88)

\[
\beta_{m_u^2}^{(2)} = +\frac{32}{15} g_1^1 \left(75 g_3^2 M_3 + 8g_1^2 (2 M_3 + M_1) \right) |M_3|^2 - \frac{4}{5} g_1^2 m_2^2 Y_u Y_u^\dagger + 12 g_3^2 m_2^2 Y_u Y_u^\dagger \\
+ 24 g_3^2 |M_2|^2 Y_u Y_u^\dagger - 4m_2^2 |\lambda_S|^2 Y_u Y_u^\dagger - 8m_2^2 |\lambda_S|^2 Y_u Y_u^\dagger \\
- 4g_3^2 |\lambda_S|^2 Y_u Y_u^\dagger - 12m_2^2 |\lambda_T|^2 Y_u Y_u^\dagger - 24 m_2^2 |\lambda_T|^2 Y_u Y_u^\dagger \\
- 12m_2^2 |\lambda_T|^2 Y_u Y_u^\dagger - 4|A_S|^2 Y_u Y_u^\dagger - 12 |A_T|^2 Y_u Y_u^\dagger - 4 \frac{g_1^2}{5} M_1 Y_u A_u^\dagger \\
- 12g_3^2 m_2^2 Y_u A_u^\dagger - 12 g_3^2 M_2 A_u Y_u^\dagger - 4\lambda_S A_S A_u Y_u^\dagger - 12 \lambda_T A_T A_u Y_u^\dagger \\
+ \frac{4}{225} g_1^1 M_1^\dagger \left(45 \left( - 2 M_1 Y_u Y_u^\dagger + A_u Y_u^\dagger \right) + 8 \left(321 g_1^2 M_1 + 40 g_1^2 (2 M_1 + M_3) \right) \right) - \frac{4}{5} g_1^2 A_u A_u^\dagger \\
+ 12g_3^2 A_u A_u^\dagger - 4 |\lambda_S|^2 A_u A_u^\dagger - 12 |\lambda_T|^2 A_u A_u^\dagger - \frac{2}{5} g_1^2 m_2^2 Y_u Y_u^\dagger \\
+ 6g_3^2 |m_u|^2 Y_u Y_u^\dagger - 2 |\lambda_S|^2 m_2^2 Y_u Y_u^\dagger - 6 |\lambda_T|^2 m_2^2 Y_u Y_u^\dagger - \frac{4}{5} g_1^2 m_2^2 Y_u Y_u^\dagger \\
+ 12g_3^2 Y_u m_2^2 Y_u^\dagger - 4 |\lambda_S|^2 Y_u m_2^2 Y_u^\dagger - 12 |\lambda_T|^2 Y_u m_2^2 Y_u^\dagger - \frac{2}{5} g_1^2 Y_u Y_u^\dagger m_2^2 \\
+ 6 g_3^2 m_2^2 Y_u ^\dagger m_2^2 Y_u^\dagger - 6 |\lambda_T|^2 m_2^2 Y_u Y_u^\dagger m_2^2 \\
- 4m_2^2 Y_u Y_u^\dagger m_2^2 Y_u Y_u^\dagger - 4m_2^2 Y_u m_2^2 Y_u Y_u^\dagger - 4Y_u Y_u^\dagger A_d A_d^\dagger \\
- 8 m_2^2 Y_u Y_u^\dagger A_d A_d Y_u^\dagger - 4Y_u Y_u^\dagger A_d A_d Y_u^\dagger - 4Y_u A_d^\dagger A_d Y_u^\dagger \\
- 4A_d Y_d^\dagger Y_d A_d^\dagger A_u Y_u^\dagger - 4A_u Y_u^\dagger Y_u A_u^\dagger A_d - 4A_u A_d^\dagger A_d Y_u^\dagger - 4 Y_u A_d^\dagger A_d Y_u^\dagger \\
- 2m_q^2 Y_d Y_d^\dagger Y_u^\dagger - 2m_q^2 Y_d Y_d^\dagger Y_u Y_u^\dagger - 4Y u m_q^2 Y_d Y_u ^\dagger Y_u Y_u^\dagger - 4Y u m_q^2 Y_u Y_u^\dagger Y_u Y_u^\dagger \\
- 4Y_u Y_u^\dagger m_q^2 Y_d Y_d^\dagger Y_u Y_u^\dagger - 2 Y_u Y_d^\dagger Y_d Y_d^\dagger m_q^2 
\]

29
\[
- 4Y_u Y_u^\dagger m_u^2 Y_u Y_u^\dagger - 4Y_u Y_u^\dagger Y_u m_u^2 Y_u^\dagger - 2Y_u Y_u^\dagger Y_u Y_u^\dagger m_u^2 - 4\lambda S Y_u A_u^\dagger A_S \\
- 12\lambda T Y_u A_u^\dagger A_T + \frac{32}{3} g_{1T}^2 15 \sigma_{2,3} + \frac{32}{15} g_{1T}^2 1 \text{Tr} U_1 (1, 1) - 16 \frac{1}{15} g_{1T}^2 1 \sigma_{3,1} - 24 m_H^2 S Y_u^\dagger \text{Tr} (Y_u Y_u^\dagger) \\
- 12 A_u A_u^\dagger \text{Tr} (Y_u Y_u^\dagger) - 6m_u^2 Y_u Y_u^\dagger \text{Tr} (Y_u Y_u^\dagger) - 12 Y_u m_u^2 Y_u^\dagger \text{Tr} (Y_u Y_u^\dagger) \\
- 6 Y_u Y_u^\dagger m_u^2 \text{Tr} (Y_u Y_u^\dagger) - 12 Y_u A_u^\dagger \text{Tr} (A_u^* Y_u^\dagger) - 12 A_u Y_u^\dagger \text{Tr} (A_u^* Y_u^\dagger) \\
- 12 Y_u Y_u^\dagger \text{Tr} (A_u^* A_u^\dagger) - 12 Y_u Y_u^\dagger \text{Tr} (m_u^2 Y_u^\dagger Y_u) - 12 Y_u Y_u^\dagger \text{Tr} (m_u^2 Y_u^\dagger Y_u^\dagger) \\
(A.89)
\]

\[
\beta^{(1)}_{m_S^2} = - \frac{24}{g_{1T}^2} 1 (M_1)^2 + 2 \left( 2 A_c A_c^\dagger + 2 m_H^2 Y_e Y_e^\dagger + 2 Y_e m_e^2 Y_e^\dagger + m_e^2 Y_e Y_e^\dagger + Y_e Y_e^\dagger m_e^2 \right) \\
+ 2 \sqrt{\frac{3}{5}} g_{1T}^2 1 \sigma_{1,1} \\
(A.90)
\]

\[
\beta^{(2)}_{m_S^2} = \frac{2}{25} \left( 6g_{1T}^2 M_1^2 \left( 234g_{1T}^2 M_1 + 5 \left( -2M_1 Y_e Y_e^\dagger + A_e Y_e^\dagger \right) \right) + 20 g_{1T} 1 (3g_{1T} 2U_1 (1, 1) + \sqrt{15} \sigma_{3,1}) \right) \\
- 5 \left( 30g_{1T}^2 M_2^2 A_e Y_e^\dagger + 10\lambda S A_e Y_e^\dagger + 30\lambda T A_T A_e Y_e^\dagger + 6g_{1T}^2 A_e A_e^\dagger \right) \\
- 30\lambda S A_e A_e^\dagger + 10|\lambda S|^2 A_e A_e^\dagger + 30|\lambda T|^2 A_T A_T A_e A_e^\dagger + 3g_{1T}^2 m_e^2 Y_e Y_e^\dagger \\
- 15g_{1T}^2 m_e^2 Y_e Y_e^\dagger + 5|\lambda S|^2 m_e^2 Y_e Y_e^\dagger + 15|\lambda T|^2 m_e^2 Y_e Y_e^\dagger + 6g_{1T}^2 m_e^2 Y_e Y_e^\dagger \\
- 30g_{1T}^2 m_e^2 Y_e Y_e^\dagger + 10|\lambda S|^2 m_e^2 Y_e Y_e^\dagger + 30|\lambda T|^2 Y_e Y_e^\dagger m_e^2 + 3g_{1T}^2 Y_e Y_e^\dagger m_e^2 \\
- 15g_{1T}^2 Y_e Y_e^\dagger m_e^2 + 5|\lambda S|^2 Y_e Y_e^\dagger m_e^2 + 15|\lambda T|^2 Y_e Y_e^\dagger m_e^2 + 20m_H^2 Y_e Y_e^\dagger Y_e Y_e^\dagger \\
+ 10Y_e Y_e^\dagger A_e Y_e^\dagger + 10Y_e Y_e^\dagger A_e Y_e^\dagger + 10A_e Y_e^\dagger A_e Y_e^\dagger + 10A_e A_e^\dagger Y_e Y_e^\dagger \\
+ 5Y_e Y_e^\dagger Y_e Y_e^\dagger m_e^2 + 30A_e A_e^\dagger \text{Tr} (Y_e Y_e^\dagger) + 15m_e^2 Y_e Y_e^\dagger \text{Tr} (Y_e Y_e^\dagger) \\
+ 30 Y_e m_e^2 Y_e^\dagger \text{Tr} (Y_e Y_e^\dagger) + 15 Y_e m_e^2 Y_e^\dagger \text{Tr} (Y_e Y_e^\dagger) + 10 A_e A_e^\dagger \text{Tr} (Y_e Y_e^\dagger) \\
+ 5 Y_e m_e^2 Y_e^\dagger \text{Tr} (Y_e Y_e^\dagger) + 10 Y_e m_e^2 Y_e^\dagger \text{Tr} (Y_e Y_e^\dagger) + 5 Y_e m_e^2 Y_e^\dagger \text{Tr} (Y_e Y_e^\dagger) \\
+ Y_e A_e^\dagger (10\lambda S A_e + 10 A_T (Y_e Y_e^\dagger A_e) + 30g_{1T}^2 M_2 + 30\lambda T A_T A_T + 30 \text{Tr} (Y_e^\dagger A_e) - 6g_{1T}^2 M_1) \\
+ 30 A_e \text{Tr} (A_e^* Y_e^\dagger) + 10 A_e Y_e^\dagger \text{Tr} (A_e^* Y_e^\dagger) \\
+ 2 Y_e Y_e^\dagger (3g_{1T}^2 m_H^2 - 15g_{1T}^2 m_H^2 - 30g_{1T}^2 |M_2|^2 + 5 \left( 2m_H^2 + m_H^2 + m_S^2 \right) |\lambda S|^2 \\
+ 30m_H^2 |\lambda T|^2 + 15m_H^2 |\lambda T|^2 \\
+ 15m_T^2 |\lambda T|^2 + 5|A_S|^2 + 15|A_T|^2 + 30m_H^2 \text{Tr} (Y_e Y_e^\dagger) + 10m^2_H \text{Tr} (Y_e Y_e^\dagger) + 15 \text{Tr} (A_T^* A_T) \\
+ 5 \text{Tr} (A_e^* A_e^\dagger) + 15 \text{Tr} (m_H^2 Y_e Y_e^\dagger) + 5 \text{Tr} (m_H^2 Y_e Y_e^\dagger) + 15 \text{Tr} (m_H^2 Y_e Y_e^\dagger)) \\
(A.91)
\]

\[
\beta^{(1)}_{m_S^2} = 4 \left( 3m_S^2 |\lambda S|^2 + (m_H^2 + m_H^2 + m_S^2) |A_e|^2 + |A_S|^2 \right) \\
(A.92)
\]

\[
\beta^{(2)}_{m_S^2} = - \frac{4}{5} \left( 120m_S^2 \lambda S^2 \lambda S^2 + 20 \left( m_H^2 + m_H^2 + m_S^2 \right) \lambda S^2 \lambda S^2 \\
+ 20 \kappa_S \left( 4 \kappa_S |A_e|^2 + \left( 4 m_S^2 + m_H^2 + m_H^2 \right) \kappa_S |A_e|^2 + A_S^2 \left( \kappa_S A_S + \lambda_S A_e \right) \right) \right) \\
30
\]
+ \lambda S \left( 15g_2^2M_2 + 15\text{Tr} \left( Y_d^1 A_d \right) + 15\text{Tr} \left( Y_u^1 A_u \right) + 30\lambda_T |A_T|^2 + 5\text{Tr} \left( Y_e^1 A_e \right) \right) \\
+ \lambda S \left( -3g_1^2m_{H_u}^2 \lambda_S - 15g_2^2m_{H_u}^2 \lambda_S - 3g_1^2m_{H_u}^2 \lambda_S - 15g_2^2m_{H_u}^2 \lambda_S - 15g_2^2m_{H_u}^2 \lambda_S \\
+ 60m_{H_u}^2 \lambda_S |A_T|^2 + 60m_{H_u}^2 \lambda_S |A_T|^2 + 30m_{H_u}^2 \lambda_S |A_T|^2 + 30m_{H_u}^2 \lambda_S |A_T|^2 + 20\lambda_S |A_u|^2 + 40\lambda_S |A_S|^2 \\
+ 30\lambda_S |A_T|^2 + 20\kappa_S A_e^* A_S + 30\lambda_T A_e^* A_S + 3g_1^2M_1^T \left( -2M_1 \lambda_S + A_S \right) + 15g_2^2M_2^T \left( -2M_2 \lambda_S + A_S \right) \\
+ 30m_{H_u}^2 \lambda_S \text{Tr} \left( Y_d^1 Y_d^1 \right) + 15m_{H_u}^2 \lambda_S \text{Tr} \left( Y_d^1 Y_d^1 \right) + 15m_{H_u}^2 \lambda_S \text{Tr} \left( Y_d^1 Y_d^1 \right) + 10m_{H_u}^2 \lambda_S \text{Tr} \left( Y_e^1 Y_e^1 \right) \\
+ 5m_{H_u}^2 \lambda_S \text{Tr} \left( Y_e^1 Y_e^1 \right) + 5m_{H_u}^2 \lambda_S \text{Tr} \left( Y_e^1 Y_e^1 \right) + 15m_{H_u}^2 \lambda_S \text{Tr} \left( A_d^* A_d \right) + 15\lambda_S \text{Tr} \left( A_d^* A_d \right) + 5\lambda_S \text{Tr} \left( A_e^* A_e \right) \\
+ 15\lambda_S \text{Tr} \left( A_e^* A_e \right) + 15\lambda_S \text{Tr} \left( m_d^2 Y_d^1 Y_d^1 \right) + 5\lambda_S \text{Tr} \left( m_e^2 Y_e^1 Y_e^1 \right) \\
+ 5\lambda_S \text{Tr} \left( m_e^2 Y_e^1 Y_e^1 \right) + 15\lambda_S \text{Tr} \left( m_d^2 Y_d^1 Y_d^1 \right) + 15\lambda_S \text{Tr} \left( m_d^2 Y_d^1 Y_d^1 \right) \right) \\
(A.93)

\beta^{(1)}_{m_1^2} = 4 \left( -4g_2^2M_2 |A_T|^2 + \left( m_{H_u}^2 + m_{H_u}^2 + m_T^2 \right) |A_T|^2 + |A_T|^2 \right) \\
(A.94)

\beta^{(2)}_{m_1^2} = -4 \left( -3g_1^2|A_T|^2 + 5g_2^2|A_T|^2 + 60 \left( m_{H_u}^2 + m_{H_u}^2 + m_T^2 \right) \lambda_T^2 |A_T|^2 + 3g_1^2M_1 |A_T|^2 - 5g_2^2M_2 |A_T|^2 \\
+ 10\lambda_T \lambda_S |A_T|^2 - 10|A_S|^2 |A_T|^2 - 5g_2^2 M_2 + \lambda_T^2 \left( -2M_2 \lambda_T + A_T \right) \right) - 20g_2^2 \sigma_{2.2} \\
+ 15|A_T|^2 \text{Tr} \left( Y_d^1 Y_d^1 \right) + 5|A_T|^2 \text{Tr} \left( Y_e^1 Y_e^1 \right) + 15|A_T|^2 \text{Tr} \left( Y_u^1 Y_u^1 \right) + 15\lambda_T |A_T|^2 \text{Tr} \left( Y_d^1 A_d \right) \\
+ 5\lambda_T |A_T|^2 \text{Tr} \left( Y_e^1 A_e \right) + 15\lambda_T |A_T|^2 \text{Tr} \left( Y_u^1 A_u \right) \\
+ \lambda_T \left( -3g_1^2m_{H_u}^2 \lambda_T - 5g_2^2m_{H_u}^2 \lambda_T - 5g_2^2m_{H_u}^2 \lambda_T - 5g_2^2m_{H_u}^2 \lambda_T + 5g_2^2m_{H_u}^2 \lambda_T \\
+ 10 \left( 2m_{H_u}^2 + 2m_{H_u}^2 + m_d^2 + m_e^2 \right) \lambda_T |A_T|^2 + 10\lambda_T |A_S|^2 + 120\lambda_T |A_T|^2 \\
+ 10\lambda_S A_e^* A_T + 3g_1^2M_1 \left( -2M_1 \lambda_T + A_T \right) \\
+ 30m_{H_u}^2 \lambda_T \text{Tr} \left( Y_d^1 Y_d^1 \right) + 15m_{H_u}^2 \lambda_T \text{Tr} \left( Y_d^1 Y_d^1 \right) + 10m_{H_u}^2 \lambda_T \text{Tr} \left( Y_d^1 Y_d^1 \right) \\
+ 5m_{H_u}^2 \lambda_T \text{Tr} \left( Y_d^1 Y_d^1 \right) + 5m_{H_u}^2 \lambda_T \text{Tr} \left( Y_d^1 Y_d^1 \right) + 15m_{H_u}^2 \lambda_T \text{Tr} \left( Y_d^1 Y_d^1 \right) + 30m_{H_u}^2 \lambda_T \text{Tr} \left( Y_d^1 Y_d^1 \right) \\
+ 15m_{H_u}^2 \lambda_T \text{Tr} \left( Y_d^1 Y_d^1 \right) + 15\lambda_T \text{Tr} \left( A_d^* A_d \right) + 15\lambda_T \text{Tr} \left( A_d^* A_d \right) + 5\lambda_T \text{Tr} \left( A_e^* A_e \right) \\
+ 15\lambda_T \text{Tr} \left( A_e^* A_e \right) + 15\lambda_T \text{Tr} \left( m_d^2 Y_d^1 Y_d^1 \right) + 5\lambda_T \text{Tr} \left( m_e^2 Y_e^1 Y_e^1 \right) \\
+ 5\lambda_T \text{Tr} \left( m_e^2 Y_e^1 Y_e^1 \right) + 15\lambda_T \text{Tr} \left( m_d^2 Y_d^1 Y_d^1 \right) + 15\lambda_T \text{Tr} \left( m_d^2 Y_d^1 Y_d^1 \right) \right) \\
(A.95)

\beta^{(1)}_{m_\sigma^2} = -24g_3^2 |M_3|^2 \\
(A.96)

\beta^{(2)}_{m_\sigma^2} = 24g_3^2 \left( 15|M_3|^2 + \sigma_{2.3} \right) \\
(A.97)
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