Modal analysis of a wind turbine tower by digital image correlation

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Abstract. Operational Modal Analysis is a method widely used in the wind industry to characterize structures and perform structural health monitoring or numerical model calibration. To reach this goal, optical techniques are becoming appealing because of their non-intrusiveness and their full-field feature in contrast to sensor-based methods. An innovative method is proposed herein for modal measurement with a single camera via Digital Image Correlation. The first resonance frequency of the tower is well captured from images of a shut down turbine.

1. Introduction

Operative Modal Analysis (OMA) aims at identifying the modal properties of a structure based on vibration data collected when it is in service, without any knowledge about the excitation. Such analysis can, for instance, help monitoring the structure health [1]. In the wind turbine industry, such analysis is required because of the loading complexity undergone by the structure, originating from aeroelastic phenomena. Several methods have been proposed in the literature, which can be classified into two categories, namely, intrusive and non-intrusive approaches. Intrusive methods consist in placing sensors on the structure, such as accelerometers [2, 3], strain gauges [4] or optical fibres [5]. These methods present as main drawback their intrusiveness (i.e., they are demanding for installation and maintenance, which imply additional costs) and their restricted scope (i.e., point measurements).

Non-intrusive methods are much less common. Micro-wave radar systems [6] and Light Detection And Ranging (LIDAR) [7] techniques have been proposed but their cost remains prohibitive for a larger implementation. To tackle the above-mentioned drawbacks, photogrammetric techniques appear as promising for operative modal analysis. Very few works have been proposed until now. One notable exception is Ref. [8], where the author proposed photogrammetry for such a purpose. The principle was to carry out an OMA of a wind turbine tower using two cameras. Reflective patches were placed on the top of the tower and on the blades. The rigid body motions of these patches were tracked over time. The difficulties of this method are the installation of patches, the required camera calibration for stereo-imaging, and finally the acquisition of images which are to be performed at night to exploit patch reflectiveness, namely, the turbine was illuminated with an LED based flash system.

A simpler way of performing a modal analysis is presented herein with a single camera. More precisely, global digital image correlation [9] is used with an integrated approach for the full-field
displacement measurement of a wind turbine tower in its fore-aft direction. Modal analysis is then performed based on displacement measurements.

2. Methodology

2.1. Digital Image Correlation

Digital Image Correlation (DIC) is a displacement field measurement technique that allows the image of a surface in its reference state to be registered with that of the deformed configuration. These two images are respectively described by matrices $f$ and $g$ representing the gray levels associated with the pixel at position $x$. By noting $u$ the displacement field between the two images, the following equation

\[ f(x) = g(x + u(x)) \]  

(1)

accounts for gray level conservation between the two images. The objective is to find the displacement field that minimizes the sum of quadratic gray level differences. Integrated DIC [9] allows for the addition of kinematic hypotheses to help solving the inverse (ill-posed) problem by adapting to the conditions of the experiment. It is convenient to discretize the displacement field via finite element shape functions $n_i(x)$ with unknown nodal displacements $a_i$

\[ u(x) = \sum_i a_i n_i(x) \]  

(2)

Based on the minimization of the quadratic differences between the two members of Equation (1), displacements are determined via a modified Gauss-Newton algorithm where incremental corrections to the displacement is obtained from the tangent (linear) problem

\[ [M] \{\delta a\} = \{b\} \]  

(3)

with

\[ M_{ij} = \sum_x (n_i(x) \cdot \nabla f(x))(n_i(x) \cdot \nabla f(x)) \]  

(4)

and

\[ b_i = \sum_x (n_i(x) \cdot \nabla f(x)) (f(x) - \tilde{g}_{\{a\}}(x)) \]  

(5)

where $[M]$ is the DIC matrix and $\{b\}$ the second-member. In the above equation, $\tilde{g}_{\{a\}}$ is the deformed image $g$, corrected by the current determination of the displacement field

\[ \tilde{g}_{\{a\}}(x) = g(x + a_i n_i(x)) \]  

(6)

2.2. Integrated Approach

The uncertainty of displacement measurement depends on several parameters: the quality of the camera (and more specifically the “noise” of the acquired images), the contrast of the observed surface, which is generally marked by a random speckle pattern with high contrast on a small scale, and the “complexity” of the displacement field to be measured (the more numerous the degrees of freedom, the higher the displacement uncertainty). Under classical conditions, typically in the laboratory, the uncertainty of displacement measured by DIC is of the order of $10^{-2}$ pixel. For the present case, due to very low contrast and very small motion amplitudes, the number of degrees of freedom must be drastically lowered. One possible way is to integrate a priori knowledge from a numerical model of the tower and lower the dimension of the subspace of sought solutions. This approach is called integrated DIC [10].

The choice of modes is made according to the considered problem. In the present case, the displacement of a wind turbine tower in the fore-aft direction depends on several mechanical
parameters, such as for instance, its moment of inertia as a function of the height, mass and inertia of the rotor, and the unknown external loading. The final objective is to capture resonance frequencies of a wind turbine tower. Thus, an appropriate basis appears to be the finite element modal basis. The model of the chosen tower is based on beam elements with variable cross-sections (i.e., 100 beam elements are used in the present situation). The Rotor-Nacelle Assembly (RNA) is modeled as an added mass with inertia and stiffness. The approximate values of these parameters are obtained with a 3D model from the DIEGO tool [11], which is the aeroelastic code for wind turbines developed at EDF R&D. Thus, the geometry of the tower is a mandatory input for this approach. The whole modelling is done with a multiple beams finite element code in Matlab [12]. The first three modes, determined by a modal analysis with a Matlab code, are shown in Figure 1.

![Figure 1. Three first modes of the wind turbine tower model calculated with a multiple beams FE code. The y-axis is dimensionless with respect to the tower height.](image)

The frequency calculation will be carried out from the mode weighting of the temporal signal. With this method the mode shapes are inputs of the problem, not outputs like other classical methods. This knowledge integration in the calculation is of paramount importance to lower measurement uncertainty.

2.3. Displacement measurement and experimental set up
Pictures of the wind turbine tower are acquired with a camera positioned 170 m away from the structure (Figure 2(a)). The camera is positioned along the perpendicular axis to the RNA orientation. One of the resulting pictures (cropped here to the region of interest) acquired by the camera is displayed in Figure 2(b). The hardware parameters of the optical setup are reported in Table 1.

![Figure 2(a).](image)

And the turbine parameters of the wind turbine are reported in Table 2.
Figure 2. (a) Set-up for image acquisition of the wind turbine tower. (b) Picture of the wind turbine tower taken by the camera.

Table 1. DIC hardware parameters.

| Parameter          | Value               |
|--------------------|---------------------|
| Camera             | PCO-edge CMOS 5.5   |
| Stand-off distance | 180 m               |
| Frame rate         | 56 fps              |
| Number of pictures | 1150                |
| Lens               | 50 mm               |

Table 2. Turbine parameters.

| Parameter        | Value            |
|------------------|------------------|
| Height           | 76 m             |
| Diameter         | 2.96-4.3 m       |
| Thickness        | 1.8-2.3 cm       |
| Rated power      | 3 MW             |

Generally, the structures of interest for DIC are speckled in order to provide high contrast at a fine scale and thus reduce the uncertainty of displacement measurements. However, this is not feasible here. It is proposed to exploit the only accessible information, namely, the boundary between the tower and the sky, where gray level gradients are the highest and well oriented to measure transverse displacements.

Acquisitions are carried out under in-service condition. Thus rigid body translations of the camera may occur during the test. A mode corresponding to horizontal translation is considered
to take into account this phenomenon. The Supervisory Control And Data Acquisition (SCADA) system enables the RNA to always be in the wind direction thanks to data collected from anemometers. Since it is proposed to measure the displacements of a wind turbine tower from a single point of view in order to access vibration frequencies, it is necessary that RNA remain fixed during the acquisition in the camera field of view. A change in orientation of the rotor would alter the modal analysis due to inertia changes with respect to the field of view plan. It is therefore proposed, for the sake of robustness, to perform the measurements on a shut down wind turbine with blocked rotor, pitch and yaw. Keeping the yaw fixed for a wind turbine in operation is not allowed by park operators. In this situation, the operational excitation acting on the wind turbine is likely to be white. With a rotating rotor, it can be expected that the harmonics would dominate the response and alter the frequency analysis. In [8] it has been shown with multiple OMAs that the first turbine frequency is independent from wind velocity, and if the turbine is on operation or not. Indeed, the gyroscopic effects are very small and their impact on the first frequency is negligible. This assertion is not true for high order modes. Even if it were possible, difficulties are likely to occur if the turbine was functioning. The brightness conservation must be ensured at best when using DIC. Depending on the sun orientation, the blade shadow could appear on the tower and may alter the DIC results. Although brightness and contrast corrections could be designed to mitigate these effects, they would increase the number of unknowns and in turn the uncertainties. Last, the expected vibration amplitude is of the order of 1 mm for an 80 m high tower. The size of one pixel is about 3 cm. Thus the motion amplitude to be captured (at most 3 centipixel) is very challenging.

In the present study, only two degrees of freedom are used: one related to rigid body translations and one associated with the first vibration mode obtained from the numerical model. In DIC, it is possible to determine directly from the DIC matrix \( [M] \) the covariance matrix \( [C] \) associated with the measurement uncertainty of all considered degrees of freedom of DIC

\[
[C] = 2\sigma^2 [M]^{-1}
\]  

where \( \sigma \) is the standard deviation of acquisition noise, which is assumed to be white and Gaussian. Thus, the a priori uncertainty estimates are the following. For the first mode, it corresponds to a maximum displacement of 0.15 mm at the tower top. In comparison, the method proposed in Ref. [8] reported a measurement uncertainty of 5 mm. The developed technique enables very low measurement uncertainty levels to be reached as compared to common techniques. Four sets of acquisitions are performed with 1150 pictures acquired for each sequence.

3. Results

This section discusses the results obtained with the previously described methodology. First, the time evolution of the amplitude of each considered degree of freedom is studied, then the frequency analysis is performed.

3.1. Displacement measurement

The amplitudes of both degrees of freedoms are plotted as functions of time in Figure 3(a) for a single series of pictures. The results obtained with two other series of pictures are shown for comparison purposes in Figure 3(b). A significant slow drift of the camera is seen, thereby confirming the need for including such rigid body translations as relevant degree of freedom. For the first vibration mode, oscillating amplitudes are clearly visible. Depending on the image series, the amplitudes of oscillations differ. This change is attributed to the fluctuating wind excitation during the 20 s acquisitions. However, the vibration frequencies look similar. To further investigate this point, the displacement is band-pass filtered in the range 0.15-3 Hz in order to suppress very low and very high frequency components that are irrelevant (reflecting respectively unsteady wind conditions and measurement uncertainties).
Figure 3. (a) Time history of the amplitudes of the two degrees of freedom (translation in blue, and first vibration mode in orange) considered in the DIC analysis for series # 1. (b) Time amplitude of the first vibration mode for series # 1 and # 3 showing similar oscillations although with different amplitudes depending on series.

3.2. Frequency analysis
For the four series, the power spectra of the vibration mode amplitude are plotted in Figure 4(a). The peak frequency (at about 0.35 Hz) is in the expected range. Indeed it has not been possible to perform a measurement with accelerometers. The turbine provider estimates the first frequency at 0.4 Hz. Differences between design and real on-site frequencies are common, the cause can be foundation particularities.

Figure 4(b) shows the four peak frequencies as measured independently for each series. Despite the variable in-service conditions, and the small duration of each series (no more than 7 periods are captured) the standard deviation of the peak frequency is 0.006 Hz, which is remarkably low. Such scatter is greater than what would be expected from an accelerometer (i.e., typically of the order of 0.001 Hz), but this level is regarded as satisfactory given the equipment used herein. It is expected that the scatter would be lowered if acquisitions were performed for a longer duration. Such a tool can thus be used for rapid estimations of a turbine first resonance frequency with very minimal instrumentation.

3.3. Model uncertainty
The considered vibration mode was directly extracted from a simple finite element model. Several parameters are not known with high accuracy. In this subsection, it is proposed to study the influence of the most uncertain parameter, namely, the RNA stiffness. Five values are tested around the initial guess.

Large variations of the RNA stiffness are tested and the impact on the first mode shape (as shown in Figure 5) is visible but modest. In turn, the resonance frequencies measured with DIC display very minute variations (i.e., less than 1 mHz over the entire range of variation for the RNA stiffness). Because the chosen displacement basis has been restricted to two modes only,
Figure 4. Power spectrum (a) and estimated peak frequency values with corresponding error bars (b) for the four pictures sets. The power spectrum is the squared Fast Fourier Transform.

Figure 5. Influence of the RNA stiffness on the first mode shape. The y-axis is dimensionless with respect to the tower height.
even an approximate mode shape is enough to capture its contribution in the displacement field over time. Moreover, even if the amplitude of variation of the vibration mode is affected by the spatial shape of the mode, it does not necessarily impact the estimate of the peak frequency. These results show that the proposed methodology for measuring the frequency of the first vibration mode is extremely robust.

4. Conclusion and Perspectives
In this work, an innovative approach was developed to identify the first resonance frequency of a wind turbine tower with a single camera when the turbine was shut down. Integrated digital image correlation was conducted with the help of a numerical model of the tower to estimate the spatial shape of the fundamental vibration mode. It enables very low measurement uncertainty levels to be reached (i.e., less than one millimeter). From 4 sets of images, a very small scatter (i.e., less than 0.01 Hz) was found on the estimate of the fundamental frequency, which is considered as very low given the extreme simplicity of the optical set-up. This method relies on prior knowledge of geometrical structure characteristics and approximate mechanical parameters such as rotor-nacelle assembly inertia, mass and stiffness. Yet, it was shown that the measurement was very robust to even important parameter changes of the models since only the spatial shape of the fundamental vibration mode matters.

In this work, only the first resonance frequency of the tower was measured. Calculations were also performed including the second vibration mode. They resulted in increased uncertainties associated with the different degrees of freedom. The first mode was still identifiable. However, the amplitude of the second mode was significantly lower, and revealed to be of the order of magnitude of noise. A longer time series, under presumably stronger wind excitation, could possibly make this second vibration frequency accessible. Temporal regularization of the DIC scheme [13, 14] could also be a perspective to enhance the sensitivity and robustness of the method.

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