Ultraintense attosecond pulse emission from relativistic laser-plasma interaction

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Abstract
We develop an analytical model for ultraintense attosecond pulse emission in the highly relativistic laser-plasma interaction. In this model, the attosecond pulse is emitted by a strongly compressed electron layer around the instant when the layer transverse current changes the sign and its longitudinal velocity approaches the maximum. The emitted attosecond pulse has a broadband exponential spectrum and a stabilized constant spectral phase. The waveform of the attosecond pulse is also given explicitly. We validate the analytical model via particle-in-cell simulations for both normal and oblique incidence. Based on this model, we highlight the potential to generate an isolated ultraintense phase-stabilized attosecond pulse.

Keywords: analytical model, attosecond pulse, plasma HHG, laser-plasma interaction, PIC simulation

(Some figures may appear in colour only in the online journal)

1. Introduction
An ultrashort pulse with atomic unit of timescale (24 as) can be used as a camera to capture ultrafast electron dynamics in atoms, molecules and condensed matters, enabling highly time-resolved studies of many fundamental physical processes [1, 2], such as Auger process, photoionization, and double ionization. Since the demonstration of attosecond pulse emission from high-order harmonic generation (HHG) in gas jets [3], remarkable developments in attosecond metrology like attosecond-pump attosecond-probe scheme have been utilized in extensive research areas from atomic physics to biology science [4]. However, due to small photon flux and low photon energy of attosecond pulses generated in gaseous HHG [5], the application of attosecond metrology is so far limited to low-energy physical processes in extreme ultraviolet regime. Besides, the poor phase matching in gaseous HHG results in attosecond pulses with broad duration [6] and uncontrollable waveforms, both of which restrict the temporal resolution of attosecond metrology [7]. In order to extend attosecond metrology to high-energy physical processes in x-ray regime with unprecedented temporal resolution, an attosecond pulse with ultrahigh intensity, ultrabroad bandwidth and stabilized spectral phase is required, which can be achieved from HHG via ultra-relativistic laser-plasma interaction.

Plasma HHG originates from the nonlinear interaction of the electron current with a linearly polarized laser pulse at the laser-plasma interface [8]. Contrary to gaseous HHG in which incident laser intensities are limited to be much lower than relativistic intensity, i.e. \( I_l \ll 10^{18} \text{ W cm}^{-2} \) corresponding to \( a_0 = \frac{eE_l}{mc\omega_l} \ll 1 \), due to the strong ionization of gaseous media [5], much brighter harmonic photon flux can be emitted in plasma HHG by employing highly relativistic laser pulses \( a_0 \gg 1 \), where \( I_l \), \( E_l \) and \( \omega_l \) are the laser intensity, electric field and frequency, \( e \) and \( m_e \) denote the electron charge and mass, respectively, \( c \) is the light speed in vacuum. As the strong laser ponderomotive force compresses the surface electrons into a layer with nanometer thickness \( \Delta x \sim 1 \text{ nm} \) [9], the collective electron current in the layer guarantees the coherence of harmonics up to \( \omega \sim 1/\Delta x \sim 1 \text{ keV} \). In the highly relativistic regime, plasma HHG is temporally locked around the so-called ‘\( \gamma \)-spike’ node where the plasma surface longitudinal velocity approaches the maximum [10], which insures the synchronization of the emission of different
harmonics and results in much smaller phase chirp than that in gaseous HHG [6, 11]. By filtering out low-order harmonics in the phase-locking spectrum, a temporally coherent attosecond pulse train or an isolated attosecond pulse can be obtained [12–14].

The physics behind plasma HHG has been extensively investigated with different models [15–19]: coherent wake emission (CWE), relativistically oscillating mirror (ROM), coherent synchrotron emission (CSE) and relativistic electron spring (RES). These models are associated and characterized by distinctive harmonic properties: spectrum, divergence and phase [10, 11, 17, 20, 21]. The relative dominance of each model in the interaction depends on the laser intensity, plasma density and interaction geometry. Generally, the CWE model dominates in the nonrelativistic or mildly relativistic regime ($a_0 \lesssim 1$) with oblique incidence on a plasma gradient, and in the relativistic regime ($a_0 > 1$), plasma HHG can be explained by the ROM model, while in the highly relativistic regime ($a_0 \gg 1$) the CSE and RES models are dominant. Recently, the CSE model has been employed in near-critical density plasmas to generate a single bright attosecond pulse in the laser transmitted direction [22]. Although the emission of attosecond pulse based on these models has been demonstrated, the explicit waveform of the attosecond pulse, which relates straightforwardly to the applications of attosecond metrology and the plasma dynamical processes of the emission, has not been thoroughly discussed in the literature.

In this paper, we develop a theoretical model for ultra-intense attosecond pulse emission in the ultra-relativistic laser-plasma interaction. In section 2, we first introduce the theoretical model and then derive the intensity spectrum, spectral phase and analytical waveform of the emitted attosecond pulse. In section 3, simulation results are provided to validate the theoretical model. At the end, a brief summary is given. Hereafter, unless specifically stated, dimensionless quantities are used: $n_e = n_e/n_n$, $t = \omega t$, $x = k x$, $\beta = v/c$, $\omega_i = \omega_0/\omega_l$, $I = I/I_s$, $J = J/(e n_n c)$, $E = eE/(m_e c \omega_0)$, $B = eB/(m_e \omega_0)$, where the plasma critical density $n_c = \omega_0^2 e_0 m_e c^2 = 1.742 \times 10^{21}$ cm$^{-3}$ and the relativistic laser intensity $I_c = c e_0 (m_e c \omega_0)/e^2 = 4.276 \times 10^{18}$ W cm$^{-2}$ for the laser wavelength $\lambda_0 = 0.8 \mu$m.

2. Theoretical model

2.1. Model description

In figure 1, we illustrate the evolution of the electron number density (a) and current density (b) at the plasma front surface irradiating by a highly relativistic laser pulse ($a_0 = 40$). As shown, the plasma electrons are extremely compressed by the laser ponderomotive force into an ultradense layer with nanometer thickness [9]. This electron layer is crucial for the physics of laser-plasma interaction [19] and dominates the plasma radiation. The strong charge separation field formed due to the electron compression would effectively accelerate the electron layer toward the incident field to emit an intense attosecond pulse [14, 18]. The pulse amplitude could be much stronger than the incident laser field $E_i$, as the laser energy is first accumulated in the charge separation field and then radiated in an attosecond period [18]. By tracing the pulse emission along the retardation relation $t' + x' = t + x$, where $(x, t)$ and $(x', t')$ denote the spatio-temporal points of the field detector and the plasma, two characteristics of the pulse emission can be highlighted:

(i) In figure 1(b), the pulse emission happens around the instant when the transverse current changes its sign, i.e. $J_x \approx 0$ corresponding to $\beta_x^d \approx 0$. At this instant, the layer longitudinal velocity $\beta_x^d$ approaches its maximum in figure 1(d). We wish to stress that the condition: $\beta_x^d \approx -1$ is very important for the simplification of the derivations below, but will not affect the waveform of the attosecond pulse. The transverse current changing its sign during the emission results in the emitted pulse with an odd-functioned waveform.

(ii) In figure 1(c), we find that the pulse is mainly emitted by the compressed electron layer. The layer spatial distribution $f[x' - x'_0(t')]$ can be represented by $\delta[x' - x'_0(t')]$ for coherent emissions with the wavelength $\lambda_0$ much larger than the layer thickness $\Delta x$, i.e. $\lambda_0 \gg \Delta x$. 

Figure 1. 1D PIC simulation of the pulse emission process. Contour of the evolution of the electron number density $n_e$ (a) and current density $J_x$ (b) at the plasma front surface overlaid with the retardation paths of the pulse centers $\phi' = x + t - x'$. Red and black lines for the 1st and 2nd pulses in figure 2 respectively. Along the paths, we can trace the formation of the pulses $E_i(x, t) \propto \int J(y', x + t - x')dy'$. (c) Electron density $n_e$ along the retardation paths. (d) Velocities $\beta_i = -J_y/(e n_n c)$, $\beta_e = -J_x/(e n_n c)$ of the electron currents along the retardation paths. At the emission node (green stars) where $\beta_e \approx 0$, the longitudinal velocity $\beta_i$ approaches the maximum ($\beta_i^m = -0.8518$ for the 1st pulse and $\beta_i^m = -0.8287$ for the 2nd pulse). The emission nodes in (a)–(c) are also labeled by green stars. In (a) and (b), the solid-line parts of the retardation paths represent the pulse propagation after emissions, and the dashed-line parts denote the formation of the pulses in the plasma. Normal incident geometry ($\theta = 0$) is employed. The laser has a step-like profile with the amplitude $a_0 = 40$, and the plasma has no density gradient with the constant density $n_n = 200n_c$. Ions are free. The laser arrives the plasma surface at $t = 0, x = 0$. 

S Tang and N Kumar
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We approximate the surface current for coherent emissions as
\[ J_s(x', t') \approx -n_e(t') \beta^\parallel(t') \delta[x' - x^\parallel(t')], \tag{1} \]
where \( x^\parallel(t') \), \( \beta^\parallel(t') \) and \( n_e(t') \) are the location, transverse velocity and areal density of the electron layer, respectively.

The reflected radiation field from a current distribution in 1D geometry can be obtained by solving the Maxwell equations with the help of the Green function formalism [23]:
\[ E^*_y(x, t) = -\frac{1}{2} \int_{-\infty}^{\infty} dx' J_y(x', t'), \tag{2} \]
where the subscript \( y \) denotes the direction of the laser electric field. Inserting equation (1) into (2), we obtain
\[ E^*_y(x, t) = \frac{1}{2} \int_{-\infty}^{\infty} dx' n_e(t') \beta^\parallel(t') \delta[x' - x^\parallel(t')] \]
\[ = \frac{1}{2} \int_{-\infty}^{\infty} dx' n_e(t') \beta^\parallel(t') \delta(x) \]
\[ = \frac{n_e(t') [1 - \beta^\parallel(t')] \beta^\parallel(t')}{2 \left[1 - (\beta^\parallel(t'))^2\right]} , \tag{3} \]
here we replace the argument of the \( \delta \)-function with \( \mathcal{X} = x' - x^\parallel(t') \). Because of the retardation relation \( t' + x' + t + x = \Delta t' \), we can have \( dx' = d\mathcal{X}/[1 + \beta^\parallel(t')] \), and the condition \( \mathcal{X} = 0 \) gives a new retardation relation:
\[ t + x = t' + x^\parallel(t'). \tag{4} \]
Making use of the general relation \( \gamma \equiv [1 - \beta_x^2 - \beta_y^2]^{-1/2} \Rightarrow (1 - \beta_y^2)^{-1} \equiv \gamma^2 [1 + (\gamma \beta_y)^2]^{-1} \), we can reach
\[ E^*_y(x, t) = \frac{n_e(t') [1 - \beta^\parallel(t')] \beta^\parallel(t')}{2 \left[1 - (\beta^\parallel(t'))^2\right]} \bigg|_{t' = t + x^\parallel(t')} , \tag{5} \]
where \( \gamma(t') \) and \( \beta^\parallel(t') \) are the Lorentz factor and transverse momentum of the electron layer, respectively.

Since the layer transverse current passes through the zero node during the pulse emission and the emission is on the attosecond time scale, the transverse momentum \( p^\parallel \) of the electron layer in the pulse emission process can be approximated in the first order:
\[ p^\parallel = \Delta t' \frac{dp^\parallel}{dt'} \bigg|_{t' = t_0} = \frac{\Delta t}{1 + \beta^\parallel(t_0)} \frac{dp^\parallel}{dt} \bigg|_{t' = t_0} , \tag{6} \]
where \( \Delta t' \) is the short time duration around the node where \( p^\parallel(t_0) = 0 \), and equation (4) is utilized for the relation:
\[ \Delta t = \Delta t' [1 + \beta^\parallel(t_0)], \tag{7} \]
\( \Delta t \) denotes the time duration around \( t_0 \) which fulfills the retardation relation \( x + t_0 = t_0 + x^\parallel(t_0) \). Hereafter, we label \( t_0 = 0 \) for convenience and replace \( \Delta t \) with \( t \).

Inserting equation (6) back into (5), we gain the pulse expression:
\[ E^*_y(t) = \hat{E} \hat{A} \frac{2\omega_d t}{1 + (\omega_d t)^2} , \tag{8} \]
where two crucial parameters are introduced:
\[ A_{\omega_d}(t') = \frac{n_e(t') \gamma(t') [1 - \beta^\parallel(t')]}{4} \bigg|_{t' = t + x^\parallel(t')} \approx \frac{n_e(t') \gamma(t')}{2} \bigg|_{t' = t + x^\parallel(t')} , \tag{9} \]
representing the pulse amplitude,
\[ \omega_d = \frac{1}{1 + \beta^\parallel(t_0)^2} \left| \frac{dp^\parallel}{dt} \bigg|_{t' = t_0} \right| \approx 2 \gamma_{\omega_d} \left| \frac{dp^\parallel}{dt} \bigg|_{t' = t_0} \right| , \tag{10} \]
scaling the pulse duration, i.e. \( T_{\omega_d} \sim 1/\omega_d \propto \gamma_{\omega_d}^{-3} \) [10]. We also introduce \( \hat{E} \hat{A} = \text{sign}(dp^\parallel/\text{d}t') \) denoting the sign of the reflected electric field, and \( \beta^\parallel(t') \approx -1 \) is used for simplifying equations (9) and (10).

In equation (9), the pulse amplitude \( A_{\omega_d}(t') \) depends on the retarded time. With the first order approximation:
\[ A_{\omega_d}(t') = A_{\omega_d}(t_0) + \Delta t' \frac{\text{d}A_{\omega_d}}{\text{d}t'} \bigg|_{t' = t_0}, \tag{11} \]
where \( A_{\omega_d} = A_{\omega_d}(t_0) \) is the constant pulse amplitude, and \( A_{\omega_d} = \text{d}A_{\omega_d}/\text{d}t'/[1 + \beta^\parallel(t')]\bigg|_{t' = t_0} \) denotes the first order temporal derivative of the pulse amplitude. The straightforward consequence of this amplitude variation is the pulse asymmetry and a constant shift of the pulse spectral phase as we will see later.

As we see, the pulse amplitude \( A_{\omega_d} \) depends on the product of the areal density \( n_e \) and the relativistic Lorentz factor \( \gamma_{\omega_d} \) of the electron layer, and \( \omega_d \) is determined by the layer transverse acceleration \( |dp^\parallel/\text{d}t'| \) and also the Lorentz factor \( \gamma_{\omega_d} \). These may point out the direction to generate a more intense pulse with shorter duration by tailoring the laser-plasma parameters to increase the value of \( n_e \gamma_{\omega_d} \) and \( |dp^\parallel/\text{d}t'| \).

The above derivations are based on the dynamic properties of the electron layer and do not take advantage of any specific effects, e.g. hole-boring effect, collision damping, temperature effect, or radiation reaction force etc. All of these effects may be taken into account qualitatively by considering their influence on the kinetic parameters \( n_e \gamma_{\omega_d} |dp^\parallel/\text{d}t'| \) of the electron layer. For example, collision effect can decrease the Lorentz factor \( \gamma_{\omega_d} \) of the electron layer by damping its motion, thus leading to smaller pulse amplitude and longer duration. Temperature effect can also reduce the layer backward acceleration, resulting in smaller \( \gamma_{\omega_d} \) impeding the plasma compression.
In the cold fluid approximation, and ignoring the collision damping, the temporal derivative of the transverse momentum at the emission instant can be approximated as [24]:

\[
\frac{d\mathbf{p}\perp}{dt}\bigg|_{t=t_0} = -(E_y - \beta_y^i B_z)|_{t=t_0}
\]

\[
= -[(E_y^r - \beta_y^i B_z^r) + (E_y^r - \beta_y^i B_z^r)]|_{t=t_0}
\]

\[
\approx -\mathbf{E}_0^r(2|E_y^r(t_0)|),
\]

where \(\mathbf{E}_0^r\) is the sign of the incident electric field at the emission instant \(t_0\), and \(E_y^r = -B_z^r\) is used because of the reflected pulse propagating in \(-x\) direction. Inserting equation (12) into (10), we can have

\[
\omega_d \approx 4\gamma^2(t_0)|E_y^r(t_0)|,
\]

and relate the sign of the reflected field to the sign of the incident field, i.e. \(E_y^r = -E_y^i\).

### 2.2. Spectral and phase properties

The spectral and phase properties relate closely to the applications of the emitted pulse in experiments. A pulse with an ultrabroad spectrum is always needed for inner shell electron cations of the emitted pulse in experiments. A pulse with an ultrashort duration.

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From equation (8), the pulse spectrum can be calculated via a simple Fourier transformation:

\[
\tilde{E}_y^r(\omega) = -\frac{\mathbf{E}_0^r}{2\pi} \int_{-\infty}^{\infty} A_m(t) \frac{2\omega_d t}{1 + (\omega_d t)^2} e^{i\omega t} dt
\]

\[
= -\frac{\mathbf{E}_0^r}{2\pi \omega_d} \int_{-\infty}^{\infty} \left( A_0^0 + \frac{A_1^1}{\omega_d} \right) \frac{2\omega}{1 + \omega^2} e^{i\omega t} d\omega,
\]

where the integral variable is replaced by \(\lambda = \omega_d t \Rightarrow dt = d\lambda/\omega_d\). If \(\omega > 0\),

\[
\tilde{E}_y^r(\omega) = -\frac{\mathbf{E}_0^r}{2\pi \omega_d} \int_{-\infty}^{\infty} \left( A_0^0 + \frac{A_1^1}{\omega_d} \right) \frac{1}{\lambda - i} e^{i\lambda \lambda} d\lambda
\]

\[
= \frac{\mathbf{A}_m}{\omega_d} e^{-\frac{\mathbf{E}_0^r}{2\pi} \gamma \frac{1}{\lambda} - \psi_{A_m}},
\]

if \(\omega < 0\),

\[
\tilde{E}_y^r(\omega) = -\frac{\mathbf{E}_0^r}{2\pi \omega_d} \int_{-\infty}^{\infty} \left( A_0^0 + \frac{A_1^1}{\omega_d} \right) \frac{1}{\lambda + i} e^{i\lambda \lambda} d\lambda
\]

\[
= \frac{\mathbf{A}_m}{\omega_d} e^{-\frac{\mathbf{E}_0^r}{2\pi} \gamma \frac{1}{\lambda} + \psi_{A_m}},
\]

and if \(\omega = 0\),

\[
\tilde{E}_y^r(0) = -\frac{\mathbf{E}_0^r}{\omega_d} \left[ 2\omega_d \delta(\omega)|_{\omega=0} - 1 \right],
\]

where

\[
\mathbf{A}_m = \sqrt{(A_0^0)^2 + (A_1^1/\omega_d)^2},
\]

\[
\cos(\psi_{A_m}) = \frac{A_0^0}{\mathbf{A}_m},
\]

\[
\sin(\psi_{A_m}) = \frac{A_1^1}{\mathbf{A}_m}.
\]

From this derivation, we can obtain the pulse spectrum:

\[
I(\omega) = (\tilde{E}_y^r(\omega))^2 = \frac{\mathbf{A}_m^2}{\omega_d^2} e^{-\frac{\mathbf{E}_0^r}{2\pi} \gamma}. \tag{18}
\]

The emitted pulse possesses an exponential spectrum with the spectral decay \(2/\omega_d\). As the Lorentz factor \(\gamma_d\) and the transverse acceleration \(d\mathbf{p}\perp/dt\) of the electron layer are boosted in the ultra-relativistic regime, the spectral decay \(2/\omega_d \propto \gamma_d^{-2} d\mathbf{p}\perp/dt\) would become very slow, implying an ultrabroadband pulse with ultrashort duration \(T_d \sim 1/\omega_d\).

Furthermore, we can also obtain the pulse spectral phase \(\psi(\omega)\):

\[
\psi(\omega) = \frac{\mathbf{E}_0^r}{2\pi} \gamma \psi_{A_m} \quad \text{for} \quad \omega > 0, \tag{19a}
\]

\[
\psi(\omega) = -\frac{\mathbf{E}_0^r}{2\pi} \gamma \psi_{A_m} \quad \text{for} \quad \omega < 0 \tag{19b}
\]

with the definition \(1 : E_y^r(\omega) = (\tilde{E}_y^r(\omega))^2 \exp(-i\psi(\omega))\). As we see, the pulse spectral phase is a constant and composed of two terms:

1. \(\pi/2\): This particular phase is the consequence of the transverse current changing its sign at the emission instant when \(\beta_y^i = 0\). This term regulates the pulse waveform and leads to a minimum at the pulse center, contrary to the synchrotron-like pulse [26]. We wish to stress that this particular phase does not depend on the carrier-envelope-phase (CEP) of the incident laser, but on the dynamics of the compressed electron layer during the emission. This property stabilizes the spectral phase of the generated attosecond pulses from the laser pulses with shot-to-shot unstabilized CEP.

2. \(\psi_{A_m}\): This term arises from the temporal variation of the pulse amplitude \(A_m\) and would induce the pulse waveform asymmetry. In principle, this phase depends on the CEP of the incident laser because the pulse amplitude \(A_m(t)\) relates to the processes of electron layer compression (\(r_{el}\)) and acceleration (\(\gamma_d\)), both can change in the interaction driven by the laser with different CEP. However, in the ultra-relativistic regime, the dependence is negligible as the phase \(\psi_{A_m}\) itself would become very small. In this regime, the duration \((\propto 1/\omega_d)\) of pulse emission is extremely short, the value of the temporal variation \(A_m/\omega_d\) would be much smaller than the constant value \(A_m\), i.e. \(A_m/\omega_d \ll A_m^0\).

\[1\] The sign of \(\psi(\omega)\) is chosen to be same with the linear term \(\omega t\) in \(E(t) = \int_{-\infty}^{\infty} E(\omega)|\omega|^{-i\omega t + \psi(\omega)}|d\omega\).
and thus $\psi_{A_n}$ can be approximated as
\[
\psi_{A_n} \sim A_n^1/(A_n^0 \omega_d).
\] (20)

We point out that the pulse spectral phase denotes the time-independent phase of the different frequency components in a single pulse, it relates directly to the dynamics of the surface layer during the pulse emission. By studying the spectral phase of each pulse, we can diagnose the plasma dynamics on attosecond time scale which may be hidden in the harmonic phase. The latter is the consequence of the interference among all the pulses in the whole reflection [6, 21, 11].

2.3. Finite distribution of the electron layer

If the emission wavelength $\lambda_e$ is close to or smaller than the layer thickness $\lambda_e \approx \Delta x$, the $\delta$-function approximation of the layer distribution (see equation (1)) can not be applied. The pulse spectrum and the spectral phase in this high-frequency region must be modiﬁed.

We now calculate the spectrum from equation (2) with the finite extension of the current density
\[
J_x(t', x') = -n_{el}(t') \beta_x(t') f[x' - x_0(t')],
\]
where we neglect the temporal dependence of the finite distribution: $f(x', t') \approx f(x')$. Because of the extremely short emission duration, the electron layer expansion is negligible. Thus we can have
\[
\tilde{E}_y(x, \omega) = -\frac{1}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx' J_x(x', t') \left|_{t = t' + \tau} \right.
\]
\[
= e^{-i\omega \Delta t} \tilde{F}(-\omega) \cdot \text{Const},
\]
where the Fourier expansion of the finite spatial distribution is used: $f(x') = \int_{-\infty}^{\infty} \tilde{F}(k) e^{ikx'}dk$, and Const is an integral constant:
\[
\text{Const} = \int_{-\infty}^{\infty} n_{el}(t') \beta_x(t') f[x_0(t') x'] \frac{1}{2} e^{-i\omega [t' + x_0(t')] dt'}.
\]
The integral constant can be evaluated by the substitution: $X = t' + x_0(t') \Rightarrow dx' = dX/[1 + \beta_x(t')]$,
\[
\text{Const} = \int_{-\infty}^{\infty} A_m(t') \frac{2p_y(t')}{1 + p_y(t')} e^{i\omega X/X_0} dX
\]
\[
\approx -\frac{\tilde{E}_y}{2} \int_{-\infty}^{\infty} A_m(X - X_0) \frac{e^{i\omega (X - X_0)/\omega_d^2}}{1 + \omega_d^2} dX
\]
\[
= -e^{-i\omega \Delta x} \frac{\tilde{E}_y}{2} \int_{-\infty}^{\infty} A_m(t') \frac{e^{i\omega dt'}}{1 + \omega_d^2} dt',
\]
where $X_0 = t_0' + x_0(t_0')$. We consider the main contribution around the emission instant when $\beta_x(t_0') = 0, \beta_y(t_0') \approx -1$. This is in line with the stationary phase approximation, as in [17]. During the emission, the phase term $\exp(i\omega x)$ is approximated to be constant because of $dX = dt'/(1 + \beta_x) \approx 0$, which mainly contributes to the integral. At the non-stationary phase point, the phase term results in rapid oscillations in the integral, especially for high frequencies, thus their contributions cancel each other and can be neglected [27].

After the same calculations in equations (14) and (15), we obtain
\[
\tilde{E}_y'(x, \omega) = 2\pi |\tilde{F}(\omega)| \frac{\tilde{A}_m e^{-i\varphi}}{\omega_d} \times e^{-i\varphi(\omega)} \begin{cases} e^{-i(\tilde{E}_y^2/2 + \psi_{\omega})}, & \omega > 0, \\ e^{-i(-\tilde{E}_y^2/2 + \psi_{\omega})}, & \omega < 0, \end{cases}
\]
and
\[
I(\omega) = |\tilde{E}_y'(x, \omega)\|^2 = 4\pi^2 |\tilde{A}_m|^2 \omega_d^2 |\tilde{F}(\omega)|^2 e^{-2i\varphi},
\]
where $\tilde{F}(-\omega) = \tilde{F}^*(\omega) = |\tilde{F}(\omega)| e^{-i\varphi(\omega)}$ is used, and $t_0 = t_0' + x_0(t_0') - x$ is taken to be zero. From equations (23) and (24), we see that the layer finite distribution can affect not only the pulse spectrum with $|\tilde{F}(\omega)|^2$ but also the spectral phase with $\psi(\omega)$.

To clearly see the influence of the layer finite distribution, we qualitatively express $\tilde{F}(\omega)$ as
\[
\tilde{F}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx 
\]
\[
\approx \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx.
\]
If the wavelength $\lambda_e \gg \Delta x$, the spectral phase $\tilde{F}(\omega) = \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx$ approaches to zero, we have
\[
\tilde{F}(\omega) \approx \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx = \frac{1}{2\pi}.
\]
which means that for low-frequency emissions ($\lambda_e \gg \Delta x$), the layer finite distribution does not affect the pulse spectrum or the spectral phase.

If $\lambda_e \approx \Delta x$, the phase $2\pi x/\lambda_e$ would be significant and result in rapid oscillation in the integral, thus
\[
\tilde{F}(\omega) \approx \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{i\omega x} dx \ll 1
\]
which indicates that the layer finite distribution could speed up the decay of the high-frequency spectrum ($\lambda_e \approx \Delta x$), and simultaneously, the distribution-induced phase $\psi(\omega)$ would also be very important for the spectral phase of the high-frequencies.

On the other hand, we assume that different parts of the electron layer contribute to the pulse emission coherently. Since the radiations from different parts may arrive at detector at different times, the time difference $\Delta t \propto \Delta x/e$ leads to a phase difference $\Delta \psi(\omega) \approx \omega \Delta x/c$, thus affecting the intensity of the pulse which actually is the superposition of the radiations from all parts of the electron layer. Obviously, this incoherence is negligible for radiations with $\lambda_e = 2\pi c/\omega \gg \Delta x$, but could induce significant phase fluctuation for emissions with $\lambda_e \approx \Delta x$. Hence, based on the layer thickness, we can qualitatively define a threshold for incoherent emission:
\[
\omega_{th} \approx \frac{2\pi c}{\Delta x},
\]
which actually truncates the region of coherent emission. Moreover, the phase space $f(\beta_x, \beta_y)$ of the electron layer is
expanded during the interaction, implying that only electrons in a narrow phase space element emit coherently.

2.4. Attosecond pulse generation

With the above discussions, we know that the coherent high-frequency emissions are bunched on attosecond time scale around the node where $\beta_t = 0$, $\beta_t \approx -1$. By filtering out the low-frequencies in the reflection, an attosecond pulse can be generated.

To obtain the analytical waveform of the attosecond pulse, we filter out the low-frequency components ($\omega < \omega_f$) in equations (14) and (15), and then inversely transform the coherent high-frequency components back in the time domain:

$$E_y^r(\omega_f, t) = \int_{-\omega_f}^{0} \tilde{E}_y^r(\omega)e^{-i\omega t}d\omega + \int_{0}^{\omega_f} \tilde{E}_y^r(\omega)e^{-i\omega t}d\omega$$

$$= \frac{-2E^r_\lambda A_m}{\sqrt{1 + (\omega_f t)^2}}[-\exp(-\frac{\omega}{\omega_{\text{th}}}) \cos[\omega_f t + \varphi(t) - \psi_{Am}] + \exp(\frac{\omega}{\omega_{\text{th}}}) \cos[\omega_{\text{th}} t + \varphi(t) - \psi_{Am}]]$$

(26)

with the definition of the temporal phase chirp $\varphi(t)$:

$$\cos[\varphi(t)] = \frac{\omega_f t}{\sqrt{1 + (\omega_f t)^2}}, \quad \sin[\varphi(t)] = \frac{-1}{\sqrt{1 + (\omega_f t)^2}}.$$

In order to obtain a strong attosecond pulse, the filtering frequency must be much smaller than the threshold of incoherent emission, i.e. $\omega_f \ll \omega_{\text{th}} = \omega_{\text{th}}^{\text{in}} \exp(-\omega_f/\omega_{\text{th}}) \gg \exp(-\omega_{\text{th}}^{\text{in}}/\omega_{\text{th}})$, thus we can write the attosecond pulse expression in a simplified form:

$$E_y^r(\omega_f, t) = \frac{-2E^r_\lambda A_m}{\sqrt{1 + (\omega_f t)^2}} \cos[\omega_f t + \varphi(t) - \psi_{Am}].$$

(27)

As we can see, the amplitude of the attosecond pulse:

$$A_{\text{atto}} = 2\hat{A}_m e^{-\text{\frac{\omega_f t}{\tau}}}$$

(28)

depends linearly on the original pulse amplitude $\hat{A}_m$ but exponentially on the ratio of the filtering frequency $\omega_f$ to the parameter $\omega_{\text{th}}$. The pulse temporal envelope:

$$f_{\text{atto}}(t) = \frac{1}{\sqrt{1 + (\omega_f t)^2}}$$

(29)

is determined only by $\omega_f$. The attosecond pulse duration can be calculated as the full width at half maximum of the intensity profile $[f_{\text{atto}}^2(t)]$ and is given as

$$T_{\phi} = \frac{2}{\omega_f}.$$  

(30)

From equation (27), we can also know that the oscillation of the electric field is controlled by $\omega_f$ and the temporal chirp $\varphi(t)$. If we set the filtering frequency $\omega_f \ll \omega_{\text{th}}$, the electric field oscillation mainly depends on $\varphi(t)$, leading to the generation of an ultrashort single-cycle attosecond pulse. The constant phase $\psi_{Am}$ can be regarded as the CEP of the attosecond pulse.

In the ultra-relativistic regime, $\omega_{\text{th}}$ becomes very large, leading to $\psi_{Am} \approx 0$ (see equation (20)), then the attosecond pulse expression can be further simplified as [14]

$$E_y^r(\omega_f, t) = \frac{-2E^r_\lambda A_m}{\sqrt{1 + (\omega_f t)^2}} e^{-\text{\frac{\omega_f t}{\tau}}} \cos[\omega_f t + \varphi(t)],$$

(31)

which demonstrates the possibility to generate an ultraintense, ultrabroadband and phase-stabilized attosecond pulse.

2.5. Comparison with the RES and CSE models

The exponential spectrum is same as in the RES model [18], but expressed in a different parametric form. This is apparent since the RES model considers the radiation from an ideal moving electron layer. However, as it ignores the temporal variation of the pulse amplitude during the emission, i.e. $A_m^1 = 0$, the RES model cannot explain the phase property [see equation (19)] of the emitted pulse. Moreover, the condition $\beta_t + \beta_n = 1$ utilized in the RES model, results in a singularity in equation (8) at the pulse emission node where $\beta_t = 0$. Thus it cannot describe the realistic waveform of the attosecond pulse.

In our model, the low-frequency emission during the layer backward acceleration is neglected, as we intend to study the properties of attosecond pulse which is synthesized by the emitted high-frequencies when $\beta_t$ approaches its maximum. Though the consideration of the layer acceleration, $\beta_n(t) = -\beta_0 + \alpha t^2$ in the CSE model [17], gives a power-law spectrum $[\eta(\omega) \propto \omega^{-3/2}]$ in low-frequency region, the emitted attosecond pulse cannot be changed as the low-frequencies have to be filtered off. Moreover, the analytical calculation of the pulse waveform becomes very challenging with the consideration of the layer acceleration because of the Airy function involved as discussed in the CSE model [17].

3. Simulation results

To confirm our analytical model, we consider a fully ionized carbon plasma ($Z/A = 6/12$) irradiated respectively by a highly relativistic laser pulse ($\theta_0 = 40$) with normal ($\theta = 0$) incidence and by an ultra-relativistic laser pulse ($\theta_0 = 100$) with oblique ($\theta = 45^\circ$) incidence, where $Z$ and $A$ denote the charge and mass number of the carbon ion, $\theta$ is the laser incidence angle. Both of the cases are simulated in 1D geometry with the EPOCH-particle-in-cell (PIC) code [28] including the effect of plasma collisions and with the spatial resolution $dx = \lambda_0/20000$. Four hundred electron and 200 ion quasi-particles per cell are taken. The plasma target with thickness $3.0\lambda_0$ is located between $3.5\lambda_0$ to the left boundary and $0.5\lambda_0$ to the right boundary. A 2D PIC simulation with normal incidence is also performed.
In figure 2, we show the obtained attosecond pulses by filtering out low-order harmonics ($\omega < \omega_p$) in the reflection from the laser-plasma parameters described in figure 1. The retardation paths of the 1st and 2nd pulses are shown in figures 1(a) and (b). As clearly shown, in each half laser cycle, one attosecond pulse is emitted by the backward accelerated electron layer around the instant when the transverse current changing sign.

In figure 3, we present the intensity spectra and the spectral phase of the 1st and 2nd pulses from figure 2. As we can see in figure 3(a), the emitted pulses have broad exponential spectra in the regions: $80 < \omega < 450$ for the 1st pulse, and $50 < \omega < 280$ for the 2nd pulse. The exponential regions are numerically confirmed by the linear-logarithm fitting [blue and green dashed lines in figure 3(a)]; $\log_10[I(\omega)] = \log_10(\delta_0) - S_\omega \omega$. Comparing with equation (18) and utilizing the spectral fitting slope $S_\omega$, we can obtain the spectral decay parameter $\omega_d$ of each pulse:

$$\frac{2\log_10(e)}{\omega_d} = 0.014 \Rightarrow \omega_d = 62.04, \text{ for 1st pulse,}$$

$$\frac{2\log_10(e)}{\omega_d} = 0.022 \Rightarrow \omega_d = 39.48, \text{ for 2nd pulse.}$$

Inserting $\omega_d$ into equation (29), we can exactly produce the envelopes of attosecond pulses in figures 2(c) and (d), which give the pulse duration: $T_p = 2/\omega_p = 13.6$ as for the 1st pulse and $T_p = 21.2$ as for the 2nd pulse. Corresponding to the exponential spectrum region, the pulse spectral phase is manifested to be constant as shown in figure 3(b). At the emission instant, the sign of the incident electric field is $E^I_0 = -1$ for the 1st pulse and $E^I_0 = 1$ for the 2nd pulse, and thus combining with equation (19), we can obtain:

$$\psi_A^I \approx 0.35\pi, \text{ for 1st pulse,}$$

$$\psi_A^I \approx 0.30\pi, \text{ for 2nd pulse.}$$

Making use of equation (27) with the results in equations (32) and (33), and choosing the pulse amplitude $A_m$ as

$$A_m = 5.4, \text{ for 1st pulse,}$$

$$A_m = 4.8, \text{ for 2nd pulse,}$$

we completely reproduce the waveforms of 1st and 2nd attosecond pulses in figures 2(c) and (d) respectively. Thus, we can verify the validity of our analytical model for the normal incidence case.

We try to estimate the pulse amplitude $A_m$ from the simulation. In figures 4(a)–(d), we present the phase space distribution $f(\beta_x, \beta_y)$ and the density $n_e$ of the electron layer at the emission instants. As shown, the phase space of the electron layer is expanded. We assume that the emitted pulse is synthesized by the radiations from the electrons in a narrow space: $\beta_x < -0.8$ and $|\beta_y| < 0.1$. By integrating the electron density distribution (red and black lines in figures 4(c) and (d)): $n_y = \int n_e(x) dx$, we obtain: $n_y = 6.58$ for the 1st pulse and $n_y = 7.75$ for the 2nd pulse, and we calculate the Lorentz factor: $\gamma_y \approx (1 - \beta_y^2)^{-1/2} = 1.90, 1.79$ for the 1st and 2nd pulses from the backward velocity $\beta_y$ in figure 1(d) at the emission instants (green stars). The layer velocity is calculated as $\beta_y = -J_y/(en_e c)$, $\beta_y = -J_y/(en_e c)$. Thus we can obtain the pulse amplitude from equation (9): $A_m = 6.25$ for the 1st pulse and $A_m = 6.93$ for the 2nd pulse, which qualitatively matches the values used in equations (34). The difference may come from the overestimation of the electron phase space. Here, we ignore the temporal variation of the pulse amplitude. As we can also see in figures 4(c), red line) and (d), black line), the thickness of the electron layer is about $\Delta x \approx 0.0025\lambda_i$ for the 1st pulse and $\Delta x \approx 0.0040\lambda_i$ for the 2nd pulse, which corresponds to the fluctuation thresholds: $\omega^{th}_f \approx 450\omega_i$ and $\omega^{th}_f \approx 260\omega_i$ for the 1st and 2nd pulses in figure 3, satisfying equation (25) qualitatively.

### 3.2. Oblique incidence

Now, we consider a more general situation where an ultraintense laser pulse, $a(t) = a_0 \sin(t + \phi) [\tanh((t - T)/W] -$
\[ \tanh \left( \frac{\theta}{2} \right) \]

is obliquely \( (\theta = 45^\circ) \) incident onto the pre-plasma, \( n_0 = n_e \exp(x/L)/2 \) present in front of a plasma bulk, where \( L = \lambda/8 \) is the scale length of the pre-plasma and \( n_e = 500 \) is the density of the bulk plasma. \( W = T_1 = \lambda/c \) is the laser ramping front, and the other laser parameters are: \( a_0 = 100, T_1 = 6T_0, T_0 = 14T_0, \lambda_1 = 0.8 \mu \text{m} \), and the CEP \( \phi = 0 \). For convenience, we treat this interaction geometry in a Lorentz boosted frame [29] in which the plasma target moves with the initial velocity \( \beta_0 = -\sin(\theta) \) along the plasma surface and is irradiated normally by the laser pulse with \( p \)-polarization.

In figure 5, we show the obtained attosecond pulses by filtering out low-order harmonics \( (\omega < \omega_b) \) in the reflected wave. For the oblique incidence \( (\theta = 45^\circ) \), only one attosecond pulse is emitted in each laser cycle because the laser electric field in one half of the cycle hinders the compression of the electrons. Here the 1st pulse arises due to the reflection of the laser ramp which is too weak to compress a well-defined electron layer, thus leading to weak spectral intensity in figure 6(a) and large phase fluctuations in figure 6(b). The 2nd pulse is emitted in the first cycle of the main laser pulse interacting with the bulk of the plasma target, and the 3rd pulse generated in the next laser cycle is emitted with much weaker intensity than the 2nd pulse. As seen in figure 6(a), the 2nd pulse has a slower intensity decay than the 1st and 3rd pulses, implying higher efficiency for high-frequency emission. In figure 6(b), the high-frequency components in the 1st and 3rd pulses display larger phase fluctuation than that in the 2nd pulse, which could further reduce the pulse intensity and extend the duration. In the following laser cycles, the generated pulses would have much faster spectral decay and strong phase fluctuation [14].

Figure 4. (a), (b) Phase space distributions \( f(\beta_x, \beta_y) \) of the electron layer at the emission instants; (c), (d) Density distributions \( n_L(x) \) of the plasma electrons at the emission instants (green line). The density of electrons in the phase space element: \( \beta_x < -0.8 \) and \( |\beta_y| < 0.1 \) are also shown (red and black lines); (e), (f) Evolution of the parameter \( \beta_y \) along the retardation paths as discussed in figure 1. Green stars denote the emission instants; (a), (c) and (e) are for the 1st pulse; (b), (d) and (f) are for the 2nd pulse.

Figure 5. Attosecond pulses with different filter frequencies: (a) \( \omega = 40\omega_c \), (b) \( \omega = 60\omega_c \), (c), (d) Electric field \( E_x \) of the 2nd and 3rd pulses compared with the analytical expressions. We label the pulse center at time \( t = 0 \) and zoom on the time axis in unit of ns. The temporal envelopes [see equation (29), magenta dashed-dotted lines] of each pulse are also shown. The incident laser profile (black dashed line, in a.u.) is also shown in (a) and (b). The detector is located at \( 3\lambda_f \) from the plasma surface.

Figure 6. Intensity spectra (a) and spectral phase (b) for the pulses in figure 5. The spectral fittings of the 2nd pulse \( |\log_{10}(I) = 0.1 - 0.0145\omega \) (blue dashed line) and 3rd pulse \( |\log_{10}(I) = 1.9 - 0.052\omega \) (red dashed line) are shown with the power-law spectral scaling \( |I(\omega) \propto \omega^{-8/3}, \text{magenta solid line} \) fitting the lower-frequency region in the spectra. (c) Spectral phase of the 2nd pulse with different CEP \( \phi \) of the incident laser pulses. The simulation in figure 5 is repeated with the same parameters, but with different CEPs \( (\phi = \pi/2, \pi) \).
Density distribution of the plasma electrons at the emission instant corresponding electron currents along the retardation paths. (the density of the electrons in the phase space:)

\[
\vec{D}_f = \frac{-2E^*_j\vec{A}_m e^{-\gamma_f}}{\sqrt{1 + (\omega_b^f t)^2}} \cos [\omega_f t + \varphi(t) - \psi_{\lambda_m}] \tag{35}
\]

but with \(\vec{A}_m^b\) and \(\omega_b^f\) calculated in the boost frame by equations (9) and (10).

In figure 6(a), the pulse exponential spectra are confirmed by the linear-logarithm fittings in the regions: \(50 < \omega < 400\) for the 2nd pulse (blue dashed line) and \(40 < \omega < 110\) for the 3rd pulse (red dashed line), and in figure 6(b) the pulse spectral phase in the corresponding regions are proved to be constant. The fitting slopes of the pulse spectra give the spectral decay:

\[
\omega_b^f \approx 58.82, \quad \text{for 2nd pulse}, \tag{36a}
\]

\[
\omega_b^f \approx 16.66, \quad \text{for 3rd pulse}, \tag{36b}
\]

which fix the pulse temporal envelopes in figures 5(c) and (d), and reveal the duration \(T_d = 2/\omega_b^f = 14.2\) as for the 2nd pulse and \(T_d = 50.8\) as for the 3rd pulse. At the emission instants, the sign of the incident electric field is \(E_1^i = -1\) for each pulse. With equation (19) and figure 6(b), we quantitatively obtain that:

\[
\psi_{\lambda_m} \approx 0.12\pi \quad \text{for 2nd pulse}, \tag{37a}
\]

\[
\psi_{\lambda_m} \approx 0.06\pi \quad \text{for 3rd pulse}. \tag{37b}
\]

Inserting equations (36) and (37) into (35), and choosing

\[
\vec{A}_m^b = 52 \quad \text{for 2nd pulse}, \tag{38a}
\]

\[
\vec{A}_m^b = 112 \quad \text{for 3rd pulse}, \tag{38b}
\]

we validate our pulse emission model for oblique incidence case by precisely reproducing the 2nd and 3rd attosecond pulses in figures 5(c), (d).

To access the pulse amplitude \(A_m^b\) from the simulation, we gather the electrons in the phase space: \(\beta_x < -0.99, |\beta_y| < 0.02\) for the 2nd pulse and \(\beta_x < -0.97, |\beta_y| < 0.06\) for the 3rd pulse, and plot the density distributions \(n_{e}\) respectively in figures 7(g), (h) and (j), (k), (l), (m). By integrating the density distributions, we get: \(n_{e} = 13.5, 36.7\) for the 2nd and 3rd pulses, and with the backward velocity \(\beta_m^b\) in figures 7(c) and (f) at the emission instants (green stars), we calculate the Lorentz factor: \(\gamma_{el} \approx 11.4, 7.2\) for the 2nd and 3rd pulses. Based on equation (9), we can obtain: \(A_m^b \approx 77\) for the 2nd pulse and \(A_m^b \approx 132.1\) for the 3rd pulse, which are close to the values used in equations (38). We can also see that the thickness of the electron layer is about \(\Delta x \approx 0.002\lambda_i^b\) for the 2nd pulse and \(\Delta x \approx 0.007\lambda_i^b\) for the 3rd pulse, which correspond to the phase fluctuation thresholds \(\omega_f^i \approx 400\omega_i\) and \(\omega_f^i \approx 150\omega_i\) for the 2nd and 3rd pulses as shown in figure 6(b).

3.3. 2D simulation

In figure 8, we perform a 2D PIC simulation to verify our analytical model. The plasma target is same as in figure 5, but is located between \(x = 11\lambda_i\) and \(x = 14.5\lambda_i\) and between lab reference:

\[
E'(\omega_f, t) = \frac{-2E^*_j\vec{A}_m e^{-\gamma_f}}{\sqrt{1 + (\omega_b^f t)^2}} \cos [\omega_f t + \varphi(t) - \psi_{\lambda_m}] \tag{35}
\]
Figure 8. 2D PIC simulation for the pulse emission processes. (a) Reflected attosecond pulses filtered with the frequency $\omega_l = 5\omega_l$. (b) Electric field $E_\perp$ of the 2nd pulse compared with the analytical expression with $\omega_l = 4\omega_l$. Intensity spectrum (c) and spectral phase (d) of the 2nd pulse in (a). The spectral fittings of the 2nd pulse

$\log_{10}(I) = 1.30 - 0.070\omega$, red dashed line is also shown.

Figure 8. 2D PIC simulation for the pulse emission processes.
(a) Reflected attosecond pulses filtered with the frequency $\omega_l = 5\omega_l$. (b) Electric field $E_\perp$ of the 2nd pulse compared with the analytical expression with $\omega_l = 4\omega_l$. Intensity spectrum (c) and spectral phase (d) of the 2nd pulse in (a). The spectral fittings of the 2nd pulse $\log_{10}(I) = 1.30 - 0.070\omega$, red dashed line is also shown.

\[ y = 0\lambda_{\perp} \text{ and } y = 14\lambda_{\perp} \text{ in the simulation box. The laser has a transverse Gaussian profile } g(y) = \exp[-(y - y_0)^2/\sigma_y^2] \text{ with } y_0 = 7\lambda_{\perp} \text{ and } \sigma_y = 4\lambda_{\perp} \text{ and the same longitudinal profile as that for figure 5, but with ramping front } W = 0.5T_l, T_l = 2T_l, \text{ and } T_e = 8T_l. \text{ The spatial resolution is } dx = dy = \lambda_{\perp}/2000. \text{ In each cell, 12 electron and 2 ion quasi-particles are taken.}

In figure 8(a), we show the attosecond pulses at $y = y_0$ and $t = 21T_l$ on employing the cut-off frequency $\omega_l = 5\omega_l$. As discussed in figure 5(a), the 2nd pulse emitted in the first cycle of the main laser pulse is the strongest. In figure 8(b), we zoom in on the spatial structure of the 2nd pulse with frequencies $\omega > \omega_l = 4\omega_l$ and show its spectrum and spectral phase in figures 8(c) and (d), respectively. As we can clearly see, the emitted 2nd pulse has an exponential spectrum $\omega_l = 2\log_{10}(c)\cdot0.070 \approx 12.41$ and a constant spectral phase $\psi_{\omega_l} \approx 0.1\pi$. With these simulation results and taking $A_{\omega_l} = 45$, we reproduce the 2nd pulse with equation (27) (red dashed line in figure 8(b)), which reasonably matches the simulation result.

3.4. Discussion

In figures 4(e) and (f), we compute the value of $\omega_{\psi}$ based on the cold fluid approximation in equation (13) along the pulse retardation paths in figure 1. At the emission instants (green stars), the obtained values qualitatively match the values in equation (32), but are larger. This may be because the plasma thermal pressure and collision friction decrease the transverse acceleration of the electron layer. In the ultra-relativistic case in figure 7, the calculations of the parameter $\omega_{\psi}$ in equation (13) (not shown) are much larger than the values obtained from the pulse spectra in equation (36), as the plasma target is strongly heated by the long ramping front of the laser pulse. Hence the theoretical prediction can deviate from the PIC simulation results. Here, we have to state that the accurate specification of the emission instant is difficult, which may also induce numerical error in the calculation of $\omega_{dL}$.

In figures 4(c) and (d), we find that the density distributions (red and black lines) of the electrons coherently contributing to the emitted pulse are narrower than the whole electron distributions (green lines) at the plasma front. This may also result from the thermal expansion of the electron phase space during the interaction. In figures 7(g) and (h) for the oblique incidence, this expansion becomes larger as the plasma is strongly heated by the ramping front of the laser pulse.

As we see in figures 3(a) and 6(a), the pulse intensity spectra are larger than the extension of the fitting spectra (dashed lines) outside the exponential region. In the lower-frequency region, this is because more low frequencies are radiated by the electron layer during its backward acceleration, leading to the power-law spectral scaling $I(\omega) \propto \omega^{-8/3}$ (magenta solid lines) [10]. In figures 3(b) and 6(b), phase-shift [6] is manifested in the corresponding frequency region resulting from the different Doppler frequency upshift at different times during the acceleration. In higher-frequency region, the more intense spectra come from the superposition of the radiations from all the electrons in the surface layer (green lines in figures 4(a), (b) and 7(g), (h)).

As discussed in section 2.2 and validated with the simulation results in figures 3(b) and 6(b), the emitted pulse has a constant spectral phase in a rather broad frequency region. The value of the constant phase $\psi(\omega) = \pm \pi/2 - \psi_{\omega_0}$ is determined by the dynamics of the well-defined electron layer and slightly perturbed by the temporal variation of pulse amplitude $A_{\omega_0}(t)$. In figure 6(c), we show the spectral phase of the emitted pulses in the cases driven by the lasers with different CEP. As we can see, with the laser CEP changing from $\phi = 0$ to $\phi = \pi$, the value of the spectral phase in the region $50 < \omega < 400$ displays a small change less than $\Delta\psi \approx 0.1\pi$. This indicates that with an appropriate frequency filter $\omega > 50$, the obtained attosecond pulse has a quasi-stable constant spectral phase independent on the laser CEP, which highlights the applications of the emitted attosecond pulse in experiments avoiding shot-to-shot changes due to the unstable laser CEP [30]. Moreover, we can also see that the coherent spectral interval can be extended to a higher-frequency region ($\omega_{\psi} \approx 450$) with a well controlled laser CEP ($\phi = \pi/2$).

In figures 7(a) and (d), we clearly see the double-pulse structure of the emitted pulse: a subpulse $E_2^\perp$ is emitted after the main pulse by the secondary electron bunch formed behind the electron layer [31] as shown by the intersection between the retardation path of the subpulse (magenta line) and the electron surface (back dashed line) in figures 7(b) and (c). The interference between the double pulses in same emission process would lead to the oscillations in the pulse spectra and spectral phase. Due to the less efficiency of the secondary bunch radiation [14], these oscillations can only be present in the low-frequency region as shown in figure 6. To get an attosecond pulse with a constant phase, a suitable frequency filter is needed to overcome the phase oscillation. We wish to stress that these oscillations are the consequence
of the interference between the double pulses, which is essentially different from the plasma-wave modulation in the harmonic spectrum of the total reflection [32].

Though we have mostly performed 1D and 2D PIC simulations in this paper, it is reasonable as long as the laser spot size is larger than its wavelength. Full 3D simulations (time-consuming for high resolution) are always required to capture the physical processes such as the deformation of the plasma surface, focusing of the emitted pulses, and the surface plasma wave.

4. Summary

We develop an analytical model which describes the attosecond pulse emission from a strongly compressed electron layer in the highly relativistic laser-plasma interaction. The physical properties of the emitted attosecond pulse are completely described: exponential spectrum, constant spectral phase and explicit waveform (see equations (18), (19) and (27) respectively). All of these results are validated by PIC simulations for both normal and oblique incidence cases.

From the simulation results, we clearly see that the emitted attosecond pulse possesses a phase-stabilized spectrum ranging from $\omega \approx 40–60$ eV to $\omega \approx 400–600$ eV and highly relativistic intensity $I \gg I_c$, which highlights the potential for the use of the generated pulse to attosecond-pump attosecond-probe measurement to detect the inner-shell electron excitation in high-Z atoms. We also see that, with an appropriate frequency filter $\omega_f$ (see figure 5(b)), the attosecond pulse emitted in the first cycle of the main laser pulse is order of magnitude stronger than those emitted in the following cycles, which is promising to generate an isolated ultraintense attosecond pulse. The choice of the frequency filter for the pulse isolation is conducted by the parameter maps in [13].

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