Kondo screening in high-spin side-coupled two-impurity clusters

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Abstract
We study the system of two magnetic impurities described by a two-impurity Kondo model where only the first impurity couples directly to the conduction band, while the second impurity interacts with the first through Heisenberg exchange coupling in a ‘side-coupled’ configuration. We consider various choices of the impurity spins (\(S_1 < S_2\), \(S_1 = S_2\) and \(S_1 > S_2\)) and we contrast the regimes where the inter-impurity exchange coupling \(J\) is either lower or higher than the Kondo temperature \(T_K^{(0)}\) of the first impurity in the absence of the second. This model is a high-spin generalization of the two-impurity model for side-coupled double quantum dots which corresponds to the simplest \(S_1 = S_2 = 1/2\) case, where the moments are Kondo-screened successively in two stages for \(J < T_K^{(0)}\) (the ‘two-stage Kondo effect’). We show that the two-stage Kondo screening occurs generically for \(S_2 \geq S_1\). For \(S_1 \geq 1\), the second Kondo temperature \(T_K^{(2)}\) is not exponentially reduced, as for \(S_1 = 1/2\), but is approximately a power-law function of the coupling \(J\). Furthermore, for \(S_1 \geq 1\) all three scales \((T_K^{(0)}, J, T_K^{(2)})\) explicitly appear in the temperature dependence of the thermodynamic properties. For \(S_1 > S_2\), there is no second stage of screening for \(J < T_K^{(0)}\). However, in the opposite limit \(J > T_K^{(0)}\) the Kondo screening of the effective spin \(S_1 - S_2\) is found.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
When magnetic impurities, such as substitutional defects in the bulk or adatoms on the surface, couple with the conduction band electrons through an antiferromagnetic exchange interaction, their spin is screened in the Kondo effect and the level degeneracy is effectively lifted [1–7]. When the separation between two such impurities is small, the impurities interact through the RKKY interaction [8], which may lead to critical behaviour in some parameter regimes [9, 10]. A simplified description of such systems is the two-impurity Kondo model [11–16]: the two magnetic atoms are represented by quantum spin operators which are coupled by some exchange interaction \(J\), and each furthermore interacts with the conduction band electrons through an effective Kondo exchange coupling. With few exceptions [17, 18], most studies of such models focus on spin-1/2 impurities, while real impurities may actually have higher spins [4]. The same may also be the case in artificial atoms, i.e. quantum dots [19], and in molecules with embedded magnetic ions [20]. Due to competing interactions and the vastness of the parameter space, a great variety of different types of magnetic behaviour are expected. In this work we discuss a sub-class of high-spin two-impurity models in which only one of the spins (\(S_1\)) couples to the conduction band, while the second spin (\(S_2\)) is ‘side-coupled’ to the first one. Only the \(S_1 = S_2 = 1/2\) limit of this family has been studied so far [18] and some results are known for the case of \(S_1 = 1/2\) and arbitrary \(S_2\) in the related Anderson–Kondo model [21]. It is shown that the two-stage Kondo screening [18, 22, 23] found in the \(S_1 = S_2 = 1/2\) model is a generic feature of all \(S_2 \geq S_1\) models, although for \(S_1 \geq 1\) some qualitative differences arise.

2. Model and method
We consider the two-impurity Kondo model:

\[
H = \sum_{k\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J_K \mathbf{S}_1 \cdot \mathbf{S}_2 + \sum_{\sigma} (S^\sigma_1 c_{\sigma}^\dagger c^\sigma_\sigma + \text{H.c.}).
\]
Operators $c_{i,\sigma}$ create conduction band electrons with momentum $k$, spin $\sigma \in \{\uparrow, \downarrow\}$ and energy $\epsilon_{i,\sigma}$, while $s = \{s_1, s_2, s_3\}$ is the spin density of the conduction band electrons at the position of the first impurity. Operators $S_1 = \{S_{1,x}, S_{1,y}, S_{1,z}\}$ and $S_2 = \{S_{2,x}, S_{2,y}, S_{2,z}\}$ are the quantum-mechanical impurity spin operators. Furthermore, $J_K$ is the effective Kondo exchange coupling constant and $J$ is the inter-impurity Heisenberg coupling constant. The models considered are the simplest generalization of the spin-1/2 two-impurity models for the side-coupled impurity configuration exhibiting the two-stage Kondo effect [18, 22–24], which can also be found in other multi-impurity problems [25, 26]. Two-stage Kondo screening occurs when the exchange coupling between the impurities is weaker than the energy scale of the Kondo screening of the directly coupled impurity. In such a situation, the moment on the first impurity is Kondo-screened at the Kondo temperature, which corresponds to a decoupled situation, the moment on the first impurity is Kondo-screened Kondo screening of the directly coupled impurity. In such a stage Kondo screening occurs when the exchange coupling $S$ discretization schemes [31, 32], without the we use the convention that $k$ numbers $m_1$ and $m_2$. The inner product of spin objects on the constituent spin $\langle S_1, S_2 \rangle$ operators can be easily computed as the expectation values of the $S_{c,i}$ operators in the maximum weight states $|S, S_i\rangle$:

\[ p_1 = \langle S, S|S_{c,1}\rangle |S, S \rangle \]

\[ = \sum_{m_1} |\langle S_1, m_1, S_2, S = S_1 - m_1 |S_2 - S_1, S_2 - S_1\rangle|^2 m_1 \]

\[ = \frac{S_1(S_1 - S_2)}{S_1 - S_2 - 1} \]

and

\[ p_2 = \langle S, S|S_{c,2}\rangle |S, S \rangle \]

\[ = \sum_{m_2} |\langle S_1, m_1, S_2, S = S_1 - m_1 |S_2 - S_1, S_2 - S_1\rangle|^2 \times (S_2 - S_1 - m_1) \]

\[ = \frac{(S_1 - S_2)(1 + S_2)}{S_1 - S_2 - 1}. \]

If the Hamiltonian describing the coupling of the impurities with the host conduction band is of the form

\[ H_C = J_1 S_1 \cdot s + J_2 S_2 \cdot s, \]

then the coupling of the effective spin takes the following form:

\[ H_{C_{eff}} = J_{eff} S \cdot s, \]

with

\[ J_{eff} = r_1 J_1 + r_2 J_2, \]

where $r_i = p_i/(S_2 - S_1)$. The ratios $r_i$ are thus the multiplicative factors which determine the effective Kondo exchange coupling of the composite object; they are tabulated in table 1. Note that the sign of $r_1$ is always negative, while the sign of $r_2$ is always positive. This implies that, in the side-coupled configuration discussed in this work, the Kondo screening of the effective spin in the $J \rightarrow \infty$ limit occurs only if the impurity which couples to the conduction band is the one with larger spin; in this case the impurity ground state multiplet will have spin $|S_2 - S_1| - 1/2$. In the opposite case, the exchange coupling to the conduction band is ferromagnetic and the ground state multiplet will have spin $|S_2 - S_1|$. 

Table 1. Multiplicative factors which determine the effective Kondo exchange coupling of the composite object made of the two spins locked into a $S = S_2 - S_1$ antiferromagnetically aligned state.

| $S_1$ | $S_2$ | $r_1$ | $r_2$ |
|-------|-------|-------|-------|
| 1/2   | 1     | -1/3  | 4/3   |
| 1/2   | 3/2   | -1/4  | 5/4   |
| 1/2   | 2     | -1/5  | 6/5   |
| 1     | 3/2   | -2/3  | 5/3   |
| 1     | 2     | -1/2  | 3/2   |
| 3/2   | 2     | -1    | 2     |
4. Results

We fix $\rho J_K = 0.1$ throughout this work. The Kondo temperature of a decoupled impurity is thus $T_K^{(0)} = 1.16 \times 10^{-5} W$ (Wilson’s definition) and it is the same for any value of spin $S_1 \{33–35\}$. The thermodynamic properties of the system for the case when the first spin is $S_1 = 1/2$ are shown in figure 1. For $S_2 = 1/2$ we recover exactly the prototypical two-stage Kondo screening where the second Kondo temperature is given by [18, 22]

$$ T_K^{(2)} = c_1 T_K^{(1)} e^{-c_2}, \quad (13) $$

where $c_1$ and $c_2$ are some numerical constants; the scaling of $T_K^{(2)}$ with $T_K^{(0)} / J$ is shown in the subfigure in the bottom panel.

For $S_2 \geq 1$, we observe very similar behaviour: for $J < T_K^{(0)}$, after the initial screening of the first impurity, the second impurity undergoes spin-$S_2$ Kondo screening which reduces its spin by one half-unit, as if it were coupled directly to the conduction band. The only effect of the first impurity is thus to induce a much lower effective bandwidth $D_{\text{eff}} \propto T_K^{(0)}$ and increased density of states $1/\rho_{\text{eff}} \propto T_K^{(0)}$. This picture is confirmed by the scaling of $T_K^{(2)}$ with $T_K^{(0)} / J$, which is very similar for all three values of $S_2$ in figure 1.

We now consider the case when the first spin is $S_1 = 1$. We remind the reader that, when a single impurity with spin $S \geq 1$ couples to the conduction band, one half-unit of the spin is screened in a spin-$S$ Kondo effect, giving rise to a spin $S - 1/2$ composite object which couples with the conduction band electrons with a ferromagnetic effective exchange coupling.

Thus it remains unscreened in the ground state [36–40, 34, 41]. The situation becomes more involved in the presence of an additional impurity, see figure 2.

For $S_1 = 1$ and $S_2 = 1/2$, in the limit $J \gg T_K^{(0)}$, the two spins couple into a spin-$1/2$ object which couples to the conduction band with an antiferromagnetic effective Kondo exchange coupling $J_{\text{eff}} = r_1 J$ with $r_1 = 4/3$ (see table 1) and undergoes the usual spin-$1/2$ Kondo effect, which results in fully compensated impurity spins and a non-degenerate singlet ground state. The expression for $J_{\text{eff}}$ holds strictly only when $J$ is much larger than any other scale in the problem (in particular $J_K$ and the bandwidth $W$); for $J \approx 60 J_K$ we indeed find that the Kondo temperature is $T_K = 1.7 \times 10^{-4} W$, which agrees with the expected scale of $T_K \approx W \sqrt{\rho J (4/3) \exp[-1/(\rho J (4/3))] \approx 2 \times 10^{-4} W}$. For lower $J$, the effective bandwidth is of the order of $J$ rather than $W$ and thus the Kondo temperature is reduced accordingly. An example of such behaviour is shown in figure 2 for $J/T_K^{(0)} = 100$. The initial free-spin $\ln 6$ entropy is reduced to $\ln 2$ at $T \sim J$ upon formation of the effective composite spin. This is followed by the conventional spin-1/2 Kondo screening (see the fit of the magnetic susceptibility with the universal Kondo curves).

In the opposite limit $J \ll T_K^{(0)}$ we observe the initial spin-1 Kondo screening of the first spin: the magnetic susceptibility goes toward 1/2 and the entropy toward 2 ln 2. The screening is, however, abruptly interrupted at $T \sim J$. This can be interpreted as the formation of a spin-singlet object composed from the residual spin-1/2 resulting from the Kondo screening and the side-coupled spin-1/2 impurity. The end result is the

![Figure 1](image-url)
same in both large-\( J \) and small-\( J \) limits; the crossover between the two is smooth as a function of \( J \).

For \( S_1 = 1 \) and \( S_2 = 1 \), the behaviour for \( J \gg T_K^{(0)} \) is particularly simple, since the two spins bind at the temperature \( T \sim J \). For \( J < T_K^{(0)} \), the Kondo screening of the \( S_1 = 1 \) spin into a residual spin-1/2 is interrupted at the temperature \( T \sim J \). The residual spin-1/2 then binds with the side-coupled \( S_2 = 1 \) into a new spin-1/2 composite object. Unlike the residual spin-1/2 resulting from the incomplete screening of a spin-1 Kondo impurity, which remains uncompensated since it couples to the conduction band ferromagnetically, the spin-1/2 composite object that emerges in this case couples with the conduction band antiferromagnetically. Thus at some lower temperature which we again denote \( T_K^{(2)} \), it is compensated in a spin-1/2 Kondo effect. This thus constitutes a non-trivial generalization of the two-stage Kondo screening phenomenology encountered in the \( S_1 = 1/2 \) case. The differences, however, are notable: (1) there are not two, but three, energy scales: \( T_K^{(0)} \), where the spin-1 Kondo screening takes place; \( J \), where this screening is abruptly interrupted, and \( T_K^{(2)} \), where the second Kondo screening occurs; and (2) the scaling of the second Kondo temperature \( T_K^{(2)} \) is not exponential with \( 1/J \). In the conventional two-stage Kondo effect with \( S_1 = 1/2 \), the only role of the coupling \( J \) is to set the lower Kondo temperature; no feature is observed there in the thermodynamic properties of the system at \( T \sim J \). Here, the coupling \( J \) is essential to produce a composite spin object which then couples antiferromagnetically with the rest of the system. Thus this scale is directly observable as a sharp change in the effective impurity degrees of freedom at \( T \sim J \). The Kondo temperature is defined by a power law with the exponent near 3, with some corrections (see the lower panels in figure 2).

For \( S_1 = 1 \) and \( S_2 = 3/2 \) the results for \( J \gg T_K^{(0)} \) are trivial: at the temperature \( T \sim J \), the spins lock into a spin-1/2 object which couples ferromagnetically with the conduction band; thus the composite spin remains unscreened. This is in accord with the expected behaviour in this limit (see section 3). For \( J \ll T_K^{(0)} \), the results are, however, very intriguing: the Kondo screening of the \( S_1 = 1 \) spin is interrupted at the temperature \( T \sim J \). At this point, the residual spin-1/2 couples antiferromagnetically with the \( S_2 = 3/2 \) spin into a spin-1/2 composite object. This composite object, interestingly, couples antiferromagnetically with the conduction band electrons, which leads to Kondo screening of one half-unit of spin at some lower temperature which we denote, yet again, as \( T_K^{(2)} \). The final residual spin-1/2 is not compensated, since it couples ferromagnetically with the conduction band. Thus we again observe a two-stage Kondo effect of the same universality class as in the \( S_1 = S_2 = 1 \) case. This result may, in fact, be generalized: for any \( S_2 \geq S_1 \), the impurity spins will be compensated for in two screening stages (the compensation is only partial for \( S_2 \neq S_1 \)).

To further substantiate the claim that the results are generic, we show in figure 3 the results for the \( S_1 = 3/2 \) case. For \( S_2 = 1/2 \), the only role of the coupling \( J \) is to set the lower Kondo temperature; no feature is observed there in the thermodynamic properties of the system at \( T \sim J \). Here, the coupling \( J \) is essential to produce a composite spin object which then couples antiferromagnetically with the rest of the system. Thus this scale is directly observable as a sharp change in the effective impurity degrees of freedom at \( T \sim J \). The second Kondo temperature is defined by a power law with the exponent near 3, with some corrections (see the lower panels in figure 2).
find the emergence of the Kondo screening of the rigidly antiferromagnetically bound \( S_2 - S_1 \) spin in the large-\( J \) limit and the formation of a \( S = (S_2 - 1/2) - S_1 \) bound state at \( T \sim J \) in the small-\( J \) limit. Furthermore, in the case of \( S_2 \geq S_1 \), the two-stage Kondo screening is again observed in the small-\( J \) limit, again with the power-law dependence of \( T_{K(2)}^\alpha \) on \( J \).

5. Discussion and conclusion

We have shown that, when a second impurity is side-coupled to a Kondo impurity with sufficiently small Heisenberg coupling \( J \), the spin will be screened in two stages for all systems, where the spin of the side-coupled impurity \( S_2 \) is equal to or greater than the spin of the directly coupled one, \( S_1 \). When \( S_1 = 1/2 \), the second Kondo temperature is exponentially reduced, while for \( S_1 \geq 1 \), it is a power-law function of the coupling \( J \). The difference stems from the fact that, for \( S_1 = 1/2 \), the second stage of the Kondo screening occurs with a local spin \( S_2 \) which interacts with a Fermi liquid of heavy electrons resulting from the first screening stage, while for \( S_1 \geq 1 \) the first screening stage leaves behind a residual uncompensated spin \( S_1 - 1/2 \), which is an extended object. This residual spin then rigidly binds with the spin of the side-coupled impurity at the temperature scale of \( T \approx J \) to produce a new extended spin object which then undergoes Kondo screening. Similar behaviour is found in the anisotropic single-impurity Kondo model, where an easy-plane anisotropy leads to a formation of an extended effective spin-1/2 degree of freedom which is Kondo-screened [35]; in this problem, the second Kondo temperature is a power-law function of the longitudinal magnetic anisotropy constant \( D \). No theory has been devised yet to map this class of problems with effective extended spin degrees of freedom onto the conventional Kondo model with a localized spin operator. Thus there is at present no analytical account of these power-law dependences. Nevertheless, it is clear that a power-law dependence of the second Kondo temperature, i.e. \( T_{K(2)}^\alpha \propto \exp(-1/\rho_{\text{eff}} J_{\text{eff}}) \propto J^\alpha \), implies an inverse logarithmic dependence of the effective impurity parameters, i.e. \( \rho_{\text{eff}} J_{\text{eff}} \propto -1/\ln J \). This form is suggestive of the energy dependence of the renormalized ferromagnetic exchange coupling of the residual spin in the underscreened Kondo model, \( J(\omega) = 1/\ln(\omega/T_0) \), where \( T_0 \) is some low energy scale [39]. This indicates that the ferromagnetic residual coupling might play a decisive role in determining the total effective antiferromagnetic exchange coupling of the composite spin object. In this scenario, the bare parameter \( J \) leads to the emergence of the composite spin object by antiferromagnetic binding of the residual spin with the side-coupled impurity spin, which occurs on the energy scale of \( \omega = J \), while the coupling of this object with the surrounding electron liquid is controlled solely by \( J(\omega) = J/\ln(J/T_0) \). This may be explained by the fact that the side-coupled impurity interacts with the electron liquid only indirectly through the first impurity and that the sign of the relevant multiplicative factor \( r_1 \) is always negative (see table 1). Thus the sign of the effective exchange interaction is flipped.

In conclusion it may also be remarked that a common feature of all models considered in this work is that the ground state in no way depends on the \( J/T_K^{(0)} \) ratio; for any non-zero Heisenberg coupling between the impurities we always end up in the same fixed point, only the temperature dependence of the spin dependence differs greatly. This no longer holds for problems with additional magnetic anisotropy terms (i.e. two-impurity extensions of models studied in [35, 42]), where level crossings may also occur as a function of \( J \). This behaviour will be addressed in future works.

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References

[1] Hewson A C 1993 The Kondo Problem to Heavy-Fermions (Cambridge: Cambridge University Press)

[2] Anderson P W 1970 A poor man’s derivation of scaling laws for the Kondo problem J. Phys. C: Solid State Phys. 3 2436

[3] Nozières P 1974 Fermi-liquid description of Kondo problem at low temperatures J. Physique 35 131

[4] Nozières P and Blandin A 1980 Kondo effect in real metals J. Physique 41 193

[5] Andrei N, Furuya K and Lowenstein J H 1983 Solution of the Kondo problem Rev. Mod. Phys. 55 331

[6] Tsvelick A M and Wiegmann P B 1983 Exact results in the theory of magnetic alloys Adv. Phys. 32 453

[7] Gunnarsson O and Schönhammer K 1983 Photoemission from Ce compounds: exact model calculation in the limit of large degeneracy Phys. Rev. Lett. 50 604

[8] Ruderman M A and Kittel C 1954 Indirect exchange coupling of nuclear magnetic moments by conduction electrons Phys. Rev. 96 99

[9] Jones B A and Varma C M 1989 Critical point in the solution of the two magnetic impurity problem Phys. Rev. B 40 324

[10] Affleck I and Ludwig A W W 1992 Exact critical theory of the two-impurity Kondo model Phys. Rev. Lett. 68 1046

[11] Jayaprakash C, Krishna-murthy H R and Wilkins J W 1981 Two-impurity Kondo problem Phys. Rev. Lett. 47 737

[12] Jones B A and Varma C M 1987 Study of two magnetic impurities in a Fermi gas Phys. Rev. Lett. 58 843

[13] Jones B A, Varma C M and Wilkins J W 1988 Low-temperature properties of the two-impurity Kondo Hamiltonian Phys. Rev. Lett. 61 125

[14] Jones B A, Kotliar B G and Millis A J 1989 Mean-field analysis of two antiferromagnetically coupled Anderson impurities Phys. Rev. B 39 3415

[15] Sakai O and Shimizu Y 1992 Excitation spectra of two impurity Anderson model. i. Critical transition in the two magnetic impurity problem and the roles of the parity splitting J. Phys. Soc. Japan 61 2333

[16] Silva J B, Lima W L C, Oliveira W C, Mello J L N, Oliveira L N and Wilkins J W 1996 Particle–hole asymmetry in the two-impurity Kondo model Phys. Rev. Lett. 76 275

[17] Le Hur K and Coqblin B 1997 Underscreened Kondo effect: a two-s = 1 impurity model Phys. Rev. B 56 668

[18] Vojta M, Bulla R and Hofstetter W 2002 Quantum phase transitions in models of coupled magnetic impurities Phys. Rev. B 65 140405(R)

[19] Sasaki S, de Franceschi S, Elzerman J M, van der Wiel W G, Eto M, Tarucha S and Kouwenhoven L P 2000 Kondo effect in an integer-spin quantum dot Nature 405 764

[20] Tsukahara N, Noto K, Ohara M, Shikari S, Takagi N, Takata Y, Miyawaki J, Taguchi M, Chainani A, Shin S and Kawai M 2009 Adsorption-induced switching of magnetic anisotropy in a single iron(ii) phthalocyanine molecule on an oxidized Cu(110) surface Phys. Rev. Lett. 102 167203

[21] Peters R and Pruschke T 2006 Relevance of quantum fluctuations in the Anderson–Kondo model New J. Phys. 8 127

[22] Cornaglia P S and Grempel D R 2005 Strongly correlated regimes in a double quantum dot device Phys. Rev. B 71 075305

[23] Žitko R and Bonča J 2006 Enhanced conductance through side-coupled double quantum dots Phys. Rev. B 73 035332

[24] Žitko R and Bonča J 2007 Correlation effects in side-coupled quantum dots J. Phys.: Condens. Matter 19 255205

[25] Žitko R and Bonča J 2007 Fermi-liquid versus non-Fermi-liquid behavior in triple quantum dots Phys. Rev. Lett. 98 047203

[26] Žitko R and Bonča J 2008 Numerical renormalization group study of two-channel three-impurity triangular clusters Phys. Rev. B 77 245112

[27] Wilson K G 1975 The renormalization group: critical phenomena and the Kondo problem Rev. Mod. Phys. 47 773

[28] Krishna-murthy H R, Wilkins J W and Wilson K G 1980 Renormalization-group approach to the Anderson model of dilute magnetic alloys. i. static properties for the symmetric case Phys. Rev. B 21 1003

[29] Bulla R, Costi T and Pruschke T 2008 The numerical renormalization group method for quantum impurity systems Rev. Mod. Phys. 80 395

[30] Krishna-murthy H R, Wilkins J W and Wilson K G 1980 Renormalization-group approach to the Anderson model of dilute magnetic alloys. ii. Static properties for the asymmetric case Phys. Rev. B 21 1044

[31] Campo V L and Oliveira L N 2005 Alternative discretization in the numerical renormalization group Phys. Rev. B 72 104432

[32] Žitko R and Pruschke T 2009 Energy resolution and discretization artefacts in the numerical renormalization group Phys. Rev. B 79 085106

[33] Rajan V T, Lowenstein J H and Andrei N 1982 Thermodynamics of the Kondo model Phys. Rev. Lett. 49 497

[34] Žitko R and Bonča J 2006 Multi-impurity Anderson model for quantum dots coupled in parallel Phys. Rev. B 74 045312

[35] Žitko R, Peters R and Pruschke T 2008 Properties of anisotropic magnetic impurities on surfaces Phys. Rev. B 78 224404

[36] Mattis D C 1967 Symmetry of ground state in a dilute magnetic alloy Phys. Rev. Lett. 19 1478

[37] Cragg D M and Lloyd P 1979 Kondo hamiltonians with a s–d exchange Kondo Hamiltonians J. Phys. C: Solid State Phys. 13 803

[38] Koller W, Hewson A C and Meyer D 2005 Singular dynamics of underscreened magnetic impurity models Phys. Rev. B 72 045117

[39] Mehta P, Andrei N, Coleman P, Borda L and Zarand G 2005 Regular and singular Fermi-liquid fixed points in quantum impurity models Phys. Rev. B 72 014430

[40] Borda L, Garst M and Kroha J 2009 Kondo cloud and spin–spin correlations around a partially screened magnetic impurity Phys. Rev. B 79 100408(R)

[41] Žitko R, Peters R and Pruschke T 2009 Splitting of the Kondo resonance in anisotropic magnetic impurities on surfaces New J. Phys. 11 053003