The Interpretation of Geomagnetic Field Reversal on the Basis of Fluctuation Growth by Using Disk Dynamo Model

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Based on the viewpoint of fluctuation growth, we analyze and discuss the geomagnetic field reversal, focusing on a three-disk dynamo model, mainly on the effect of mutual inductance on dynamic behaviors. If mutual inductance is large, which means strong interaction between neighboring disks and their associated circuits, then a very slight fluctuation of electric current is found to be greatly enhanced after a short incubation time. On the contrary, after a long incubation time, fluctuation growth is very slow when mutual inductance is small. Therefore the degree of interaction is related to the length of incubation time: the stronger the interaction, the shorter incubation time. The analogy between the disk dynamos and the geomagnetic field reversal patterns implies slight fluctuations are greatly enhanced during the period for the past 25 Ma, whereas fluctuation growth is very slow during the period of the Cretaceous Superchron, implying that mutual inductance is dependent on time. Hence, disk dynamo calculations including time dependence of mutual inductance are found to explain geomagnetic field reversal pattern qualitatively. From the mathematical viewpoint, numerical solutions for time dependent mutual inductance is found to be obtained by frequency modulation (FM) over those for time independent mutual inductance.

1. Introduction

Beginning with the study of self-excited bistable disk dynamos by Rikitake (1958), the study of disk dynamo models employing two-disks (e.g., Allan, 1962; Cook and Roberts, 1970; Shimizu and Honkura, 1985; Kono, 1987; Hoshi and Kono, 1988) and those of higher complexity (e.g., Lebovitz, 1959; Miura and Kai, 1984; Shimizu and Honkura, 1985) have been successful in describing and explaining the geomagnetic field reversal phenomenon.

Certainly dynamo theory, including disk dynamo models, can successfully explain polarity reversals of Earth's magnetic field. However, as pointed out by Merrill and McFadden (1995), the existence of the Cretaceous and Kiaman Superchrons, during which the magnetic polarity remained constant, remains problematical. Traditional dynamo theory can not explain both very long and very short reversal periodicity.

Therefore, for the purpose of trying to explain the existence of the Cretaceous Superchron phenomenologically using disk dynamo, in this article we focus on the dynamic behaviors of three-disk dynamo systems with six degrees of freedom in which the resistance and driving couple can be ignored relative to the other terms. The reasons to examine three-disk dynamos with these conditions are as follows: (A) it is easy to evaluate precision for numerical calculations using energy conservation, (B) calculation of the Liapounov function is straightforward, and (C) it is easy to examine the steady state stability. In addition, analyses of dynamic equations for disk dynamos becomes remarkably simplified (i.e., easily analyzed mathematically, as shown later).

We primarily investigate and discuss the effect of $M$ (mutual inductance) which can be regarded as a coupling constant between neighboring dynamos, on the growth or enhancement of fluctuations. Furthermore, we discuss this effect from a mathematical viewpoint.
2. Calculation Results and Analysis

Figure 1 shows N-disk dynamos which are electromagnetically coupled (e.g., Lebovitz, 1959). The equations governing the system are given by

\[ L_n \frac{d}{dt} I_n(t) + R_n I_n(t) = M_n \Omega_n(t) I_{n-1}(t) \quad (n = 1, 2, ..., N) \]

\[ C_n \frac{d}{dt} \Omega_n(t) = G_n - M_n I_n(t) I_{n-1}(t) \]

where \( L_n \) and \( R_n \) represent the self-inductance and resistance for the \( n \)-th coil, and \( I_n \) and \( \Omega_n \) represent electric current and angular velocity. \( C_n \) and \( G_n \) are the moment of inertia and the driving couple corresponding to the \( n \)-th disk, \( 2\pi M_n \) is a mutual inductance between the \((n - 1)\)-th coil and the periphery of the \( n \)-th disk. We disregard the resistance and self-inductance of the disk. As stated in introduction, we focus on three-disk dynamos; therefore we fix \( N = 3 \).

If we set \( G_n \) and \( R_n \) to zero for all \( n \), then we obtain the following equation from Eq. (1)

\[ L_n I_n(t) \frac{d}{dt} I_n(t) = -C_n \Omega_n(t) \frac{d}{dt} \Omega_n(t) \quad (n = 1, 2, 3). \]

Hence by integrating Eq. (2), we get

\[ L_n [I_n(t)]^2 + C_n [\Omega_n(t)]^2 = (\varepsilon_n)^2 = \text{const} \]

where \( \varepsilon_n \) is a constant determined by the initial conditions of the system. Equation (3) means energy for each disk and associated circuit is conserved; \( (\varepsilon_n)^2 \) expresses energy for the \( n \)-th disk and its associated circuit. By combining Eq. (3) with the equation \( C_n d\Omega_n(t)/dt = -M_n I_n(t) I_{n-1}(t) \), then we obtain the following equation:

![Fig. 1. Illustration for N-disk dynamo system.](image-url)
\[ C_n \left( \frac{d\Omega_n(t)}{dt} \right)^2 = \frac{M_n^2}{I_n L_{n-1}} \left[ (\varepsilon_n)^2 - C_n (\Omega_n(t))^2 \right] \left[ (\varepsilon_{n-1})^2 - C_{n-1} (\Omega_{n-1}(t))^2 \right]. \]  

Consequently, as can be seen above, Eq. (4) can't be expressed as a one-body term; hence we can not usually obtain the solution analytically except for specific initial conditions and the case of \( N = 2 \) (i.e., Rikitake dynamo).

For simplifying the analysis and discussion, we will assume the system of three disk dynamos to be homogeneous, namely, \( C_1 = C_2 = C_3 = C, L_1 = L_2 = L_3 = L, M_1 = M_2 = M_3 = M \). Accordingly, the equations which we shall discuss and analyze are given as follows:

\[ \frac{d}{dt} I_n(t) = \frac{M}{L} \Omega_n(t) I_{n-1}(t) \]  

\[ \frac{d}{dt} \Omega_n(t) = -\frac{M}{C} I_n(t) I_{n-1}(t) \]

\((n = 1, 2, 3\) and \( I_0(t) = \Omega_0(t) = \Omega_3(t) \)).

Furthermore, to simplify calculations, we fix \( L = 1 \) H and \( C = 1 \) HA²sec² hereafter. If we replace \( I_n(t), \Omega_n(t) \) and \( t \) by introducing the dimensionless variables \( I'_n(t'), \Omega'_n(t') \) and \( t' \) as expressed below

\[ t = t_0 t', \]

\[ I_n(t) = I_0^n I'_n(t'), \]

\[ \Omega_n(t) = \Omega_0^n \Omega'_n(t') \]

where \( I_0^n, \Omega_0^n \) and \( t_0 \) are constants with dimensions of Amperes, rad/s and sec, respectively; we take \( I_0^n = 1 \) A, \( \Omega_0^n = 1 \) rad/sec for all \( n \) and \( t_0 = 1 \) sec. Then Eq. (5) is

\[ \frac{d}{dt'} I'_n(t') = m \Omega'_n(t') I'_{n-1}(t') \]  

\[ \frac{d}{dt'} \Omega'_n(t') = -m I'_n(t') I'_{n-1}(t') \]

where \( m = M/L \) is dimensionless. We find solutions for these equations numerically, using a fourth-order Runge-Kutta method with \( dt' = 0.001 \).

For convenience of subsequent calculations, we rewrite Eq. (5') in the following form:

\[ \frac{d}{dt'} J_n(t') = f_n(J_1(t'), J_2(t'), J_3(t'), J_4(t'), J_5(t'), J_6(t')) \]

where \( J_1(t') = I'_1(t'), J_2(t') = I'_2(t'), J_3(t') = I'_3(t'), J_4(t') = \Omega'_1(t'), J_5(t') = \Omega'_2(t'), J_6(t') = \Omega'_3(t') \).
First, we examine the dynamic behavior of \( J_n(t') \) around the steady point \( J^*(=J_1(t'), J_2(t'), J_3(t'), J_4(t'), J_5(t'), J_6(t')) = (0, 0, 0, 0, 0, 0) \), using the Liapounov function \( V(J_1(t'), J_2(t'), ..., J_6(t')) \) defined below

\[
V(J_1(t'), J_2(t'), ..., J_6(t')) = \sum_{n=1}^{6} [J_n(t')]^2. \tag{7}
\]

As should be expected from Eqs. (3), (5') and (7),

\[ V > 0 \] except at point \( J^* \)

and

\[
\frac{d}{dt'} V = 0. \tag{8}
\]

Consequently, the existence of a Liapounov function satisfying Eq. (8) indicates that the locus of \( J(t') \) \((=J_1(t'), J_2(t'), ..., J_6(t'))\), which begins in domain \( D \) in the phase space, remains in domain \( D \) for all time and that the steady point \( J^* \) is a center showing no convergence, even in the limit of \( t \to \infty \), because \( dV/dt' = 0 \). The existence of the Liapounov function is, as easily inferred, related to energy conservation as indicated in Eq. (3).

Next let us examine the dynamic behavior of \( J_n(t') \) when a slight deviation (fluctuation) is added onto another steady state \( J_0 = (0, 0, 0, \omega_1, \omega_2, \omega_3) \) (\( \omega_n = \) constant and non-zero \( n = 1, 2, 3 \)); therefore, state \( J_0 \) means that each disk rotates with constant angular velocity \( \omega_n \) and generates no electric current. If \( J_0 \) is stable, we can guess that the locus of \( J(t) \), starting from a point very close to \( J_0 \), will converge to \( J_0 \) gradually. Hence, for the purpose of investigating the stability of \( J_0 \), we apply linear stability theory (Minorsky, 1962; Saarty and Bram, 1964) to a series of differential equations as illustrated in Eq. (6).

Provided that slight deviations from the steady point \( J_0 \) are expressed as \( \delta J \), then \( \delta J \) satisfies the below equation under the linear approximation,

\[
\frac{d}{dt'} \delta J = A \delta J \tag{9}
\]

where the matrix element \( A_{kl} \) of \( A \) is given by

\[
A_{kl} = \left[ \frac{\partial}{\partial J_j} f_k \right]_{J_0}
\]

and \( \delta J = (\delta J_1, \delta J_2, ..., \delta J_6) \).

Unless all the real parts of the eigenvalues for matrix \( A \) are negative, then steady point \( J_0 \) turns out to be unstable. It is, therefore, necessary to determine the eigenvalues \( \lambda \) for matrix \( A \) which satisfies the following equation:
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\[
\det(A - \lambda E) = \begin{vmatrix}
-\lambda & 0 & m\omega_1 \\
m\omega_2 & -\lambda & 0 \\
0 & m\omega_3 & -\lambda \\
\end{vmatrix} = 0 \quad (10)
\]

where \( E \) is the identity matrix. Equation (10) can be easily expanded and \( \lambda \) is given by

\[
\lambda = 0 \quad \text{triplet}, \quad \lambda = m\sqrt{\omega_1\omega_2\omega_3} \quad \text{and} \quad \lambda = m\sqrt{\omega_1\omega_2\omega_3} \left( -1 \pm \frac{i\sqrt{3}}{2} \right). \quad (11)
\]

Because \( \omega_1\omega_2\omega_3 \neq 0 \) in Eq. (11), some of \( \text{Re}(\lambda) \) are positive; steady point \( J_0 \) is found to be unstable. Furthermore, it should be noted that \( \text{Re}(\lambda) \) is proportional to \( m \). In other words, a slight deviation from steady point \( J_0 \) is greatly enhanced when \( m \) is large. This is because of the term \( \exp(\lambda t) \).

Figures 2(A), (B) and (C) illustrate \( J_n(t') \) \((n = 1, 2, ..., 6)\) for \( m = 0.1, 0.5 \) and \( 1.0 \) with the initial conditions \((J_1(0) = 0.0, J_2(0) = 0.0, J_3(0) = 10^{-7}, J_4(0) = 1, J_5(0) = -2, J_6(0) = 3)\). As stated previously, these initial conditions mean that a very slight deviation \((-10^{-7})\) is added to the steady state \( J_0 = (0, 0, 0, 1, -2, 3) \). From the physical point of view, this initial condition implies that a very slight electric current of \( 10^{-7} \) A \((0.1 \mu A)\) is suddenly applied to the 3rd circuit at \( t = 0 \) whereas the system has held state \( J_0 \) from \( t = -\infty \) to \( 0 \) with no disturbance or fluctuation from the outside and/or inside. We can also consider the fluctuation of current \((0.1 \mu A)\) to be generated at \( t = 0 \) sec.

As illustrated in Figs. 2(A), (B) and (C), the appearance of a deviation from the steady state \( J_0 \) seems to start at about the same time, regardless of differences in the dynamic behavior of each \( J_n(t') \). This implies that \( J_n(t') \) are strongly coupled to each other through Eq. (6). As should be expected from the preceding discussion, even slight deviations are found to be greatly enhanced with increased \( m \); furthermore, the frequency of oscillations tends to become more marked with increasing \( m \). This supports the above conclusion of \( \text{Re}(\lambda) \) being proportional to \( m \). The incubation time, during which \( J_n(t) \) seems at a glance to be no different from steady state \( J_0 \), becomes shorter as \( m \) increases.

From the mathematical point of view, the appearance of a high frequency component and a shorter incubation time with increasing \( m \) can be easily understood by introducing a dimensionless time \( t'' \) defined as

\[
t'' = mt'.
\]

Then Eq. (5') can be rewritten as follows:

\[
\frac{d}{dt''} I'_n(t'') = \Omega'_n(t'') I'_{n-1}(t'')
\]

\[
\frac{d}{dt''} \Omega'_n(t'') = -I'_n(t'') I'_{n-1}(t''). \quad (12)
\]

From the above equations, the dynamical behaviors of \( I'_n \) and \( \Omega'_n \) during \( t'' \) from 0 to 1 are found to correspond to those during \( t' \) from 0 to \( 1/m \); hence the change of \( J_n(t') \) in \( t'' \) from 0 to 1 is contracted with an increases in \( m \). As a result, high frequency components appear with an increase in \( m \), due to the con-
Fig. 2. Numerical calculation results for $J_n(t')$ ($n = 1, 2, ..., 6$) when a very slight fluctuation is added to the steady state. (A) $m = 0.1$, (B) $m = 0.5$, and (C) $m = 1.0$. Initial conditions are given as $(J_1(0), J_2(0), J_3(0), J_4(0), J_5(0), J_6(0)) = (0, 0, 10^{-7}, 1, -2, 3)$.
traction of time; $m$ represents a scale factor for time extension ($m < 1$), contraction ($m > 1$) or no scaling ($m = 1$).

Provided that $I'_n(t'; m, J(0))$ and $\Omega'_n(t'; m, J(0))$ are the solutions for Eq. (5'), then we can derive the following equation from Eq. (12) and the above discussion,

$$I'_n(t'; m, J(0)) = I'_n(mt'; 1, J(0))$$

(13)

$$\Omega'_n(t'; m, J(0)) = \Omega'_n(mt'; 1, J(0))$$

where $J(0)$ is the initial condition for Eq. (5'). From Eq. (13), we can see that $J_n(t'; m, J(0)) = J_n(mt'; 1, J(0))$; it is, therefore, possible to evaluate $J_n(t'; m, J(0))$ analytically without numerical calculation if precise values of $J_n(t'; 1, J(0))$ can be obtained numerically. Equation (13) describes an operation for $J_n(t'; m, J(0))$ which is a frequency modulation (FM).

As stated in the previous section, even slight fluctuations from steady state are found to be greatly enhanced and eventually develop an irregular oscillation. Since a small deviation from the steady state can be regarded as a slight instability from a stable state, the appearance of high frequencies due to an increasing $m$ is interpreted as growth (or enhancement) of instability. Consequently, from the physical point of view, $m$ represents the degree of coupling or interaction between the neighboring disks and their associated circuits. On the contrary, if the system has very weak interactions between disks and associated circuits through a small value of $m$, slight deviations are not greatly enhanced and the system seems to be stable at a glance during the long period of incubation time. However, after the long incubation time, fluctuations are greatly enhanced and finally result in irregular oscillations already seen in the case of large $m$. Hence we can conclude that the variations in the value of $m$ mainly affect the incubation time: the larger the $m$, the shorter incubation time.

Based on the above conclusion, let us consider geomagnetic field reversals using the idea of fluctuation growth. Figure 3 shows the geomagnetic polarity time scale for the past 170 Ma (after Cox, 1982). The reversal pattern for the past 25 Ma shows a relatively high frequency of reversals. Since higher frequency parts appear with increasing $m$, this period corresponds to large $m$. However, the reversal frequency pattern is more relaxed from about 25 Ma to about 83 Ma, preceded by the 35-Ma-long Superchron. In addition, it should be noted that prior to the 35-Ma-long normal Cretaceous Superchron, there exists relatively long (about 4 to 6 myr) periods with no polarity reversals. Periods of constant polarity of this length are not present from now to about 25 Ma; the existence of relatively long periods prior to the Cretaceous Superchron seems to indicate the reversal process to be very slow around this period. Therefore the geomagnetic reversal pattern does not appear to be a random process, but rather an organized one. This could also be the result of insufficient sampling of older rocks, however.

From the perspective of fluctuation growth, we can consider the period of the 35-Ma-long normal Cretaceous Superchron to be the minimum period for the growth of fluctuation. During this long period, it is supposed that fluctuation (instability) grows very slowly without changing polarity. This state seems, at a glance, to be very stable; however, instability grows (but very slowly) during this time. Based on the linear stability theory as discussed previously, fluctuation is roughly estimated to be proportional to $\exp(C_0mt)$ ($C_0$ is constant and positive); growth of fluctuation can be easily expected to become very slow if $m$ is reduced to one-tenth of its present value. The point is that this long Superchron is not an equilibrium point during which slight instabilities (fluctuations) vanish so that fluctuations can be regarded as only noise (as has been observed in the spin fluctuations of a ferromagnet below its Curie point), but rather the period exhibiting a very slow increase in instability (fluctuation) without changing the polarity. In other words, the Cretaceous Superchron may be interpreted as a long incubation time caused by small $m$.

Accordingly, we consider it better to take into account the time dependence of $m$; namely, to regard $m$ as $m(t')$ for the purpose of simulating the geomagnetic field reversal patterns. Based on geomagnetic data and the above discussion, $m(t')$ is required to have a minimum value during the time corresponding
Fig. 3. Geomagnetic field reversal pattern for the past 170 Ma (after Cox, 1982).

to the Cretaceous Superchron; consequently, for the purpose of reproducing the geomagnetic field reversal pattern qualitatively, we take \( m(t') \) as the follows:

\[
m(t') = m_1(t') \quad 0 \leq t' \leq t_1
\]
\[
  = m_2(t') \quad t_1 \leq t' \leq t_2
\]
\[
  = m_3(t') \quad t_2 \leq t' \leq t_3.
\] (14)
Figure 4 illustrates the dynamic behavior of $J_n(t')$ together with the reversal pattern (=sign($J_1 + J_2 + J_3$), sign(x) = 1 for $x > 0$ and sign(x) = -1 for $x < 0$) on the basis of $m_1(t') = 1 - t'/30$, $m_2(t') = 1/30$ and $m_3(t') = 1/30 + (t' - 50)/20$ ($t_1 = 29$, $t_2 = 50$ and $t_3 = 130$). We take the initial condition $J(0) = (-2, -2, 3, 0, 0, 0)$. In Eq. (14), $m_1(t'), m_2(t'), m_3(t'), t_1, t_2, t_3$ and $J(0)$ are defined for the convenience sake in order to try to reproduce geomagnetic field reversal pattern qualitatively and, therefore, do not have specific meanings. Starting point $t' = 0$ corresponds to the time more than 120 Ma ago.

As shown in Fig. 4, a long periods with a constant sign of current appear, which correspond to the Superchrons. Reversal patterns resemble geomagnetic timescale reversals qualitatively. In addition, it is remarkable that each angular velocity ($\Omega_n(t'), n = 4, 5, 6$) seems to remain constant during the time when $m(t')$ is minimum, whereas all currents ($J_n(t'), n = 1, 2, 3$) can't maintain the same sings; hence this state may be regarded as a pseudo steady state.

Let us consider the time dependence of $m$ from a mathematical point of view. In Eq. (5'), we replace $m$ with $m(t')$, then

$$\frac{d}{dt'} I'_n(t') = m(t')\Omega'_n(t') I'_{n-1}(t')$$

(15)

$$\frac{d}{dt'} \Omega'_n(t') = -m(t')I'_n(t') I'_{n-1}(t')$$

here we introduce a new variable $\tau$ as defined below,

$$\tau = \int_0^{t'} m(x) dx.$$  

(16)

By substituting Eq. (16) into Eq. (15), we get the same equation as Eq. (12); we can write down solutions for Eq. (15) in the following form:

$$I'_n(t') = I'_n(\tau; 1, J(0)) = I'_n\left(\int_0^{t'} m(x) dx; 1, J(0)\right)$$

(17)

$$\Omega'_n(t') = \Omega'_n(\tau; 1, J(0)) = \Omega'_n\left(\int_0^{t'} m(x) dx; 1, J(0)\right).$$

Therefore, the above equations (i.e., solutions for Eq. (15)) means, as mentioned previously, frequency modulation (FM) of $I'_n(t'; 1, J(0))$ and $\Omega'_n(t'; 1, J(0))$. Accordingly, it may be possible to find a $m(t)$ which can explain actual geomagnetic reversal data quantitatively, on the basis of Eq. (17). The procedures to determine $m(t')$ from the geomagnetic field reversal pattern is supposed to correspond to those used to extract the sound signal from FM waves.

3. Discussion

In this paper we have analyzed the geomagnetic reversal pattern from the viewpoint of enhancement of fluctuation, primarily focusing on the relation between the dynamic behavior of $J_n(t')$ and the value of $m$. It is discovered that slight amounts of fluctuations (i.e., instabilities) are greatly enhanced when $m$ is large while the growth of instability is very slow when $m$ is small. The growth of instability is connected to the appearance of high frequency part: high frequency parts appear with increase in $m$.

From the physical model point of view, $m$ represents the extent of interaction between neighboring...
Fig. 4. Numerical calculation results for $I(x)$ ($n = 1, 2, \ldots, 6$) together with reversal pattern $I_{1} + I_{2} + I_{3}$, where $\text{sign}(x) = 1$ for $x > 0$ and $\text{sign}(x) = -1$ for $x < 0$. We take initial condition $(I(0), I_{1}(0), I_{2}(0), I_{3}(0), I_{4}(0), \ldots) = (2, 3, 0, 0)$. 

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disk and circuit; therefore, because $dV/dt' = 0$, the existence of strong interactions in the system enhances slight fluctuations greatly and easily leads to irregular oscillations without converging to a steady state.

The existence of a long periods with no polarity changes (Cretaceous Superchron) can be understood as the period when the growth of instability is very slow in comparison with other periods. However it
should be noticed that long periods such as the Cretaceous Superchron do not mean that the equilibrium (steady) state where a small fluctuations cannot grow or vanish is achieved; we consider it better to regard such a long period as a pseudo steady state or incubation time leading to irregular oscillations.

However, our analysis is based on calculation results from three-disk dynamos ignoring resistance and driving couple. The reasons why we select a three-disk dynamo and disregard resistance and driving couple for each disk dynamo and associated circuit are, as stated in the introduction: (A) ease of estimation of the precision of the solution for differential Eq. (5) using energy conservation as shown in Eq. (3), (B) ease in finding the Liapounov function as defined in Eq. (7), and (C) ease in examining the steady-state stability. Accordingly, the system we have analyzed and discussed seems rather idealistic and/or specific; a study including the influence of resistance, driving couple and dynamos with more than three disks will be needed for obtaining the generality. However, our proposed idea-growth of fluctuation-derives from the above system seems to have generality because the presence of Superchrons can be explained by this approach phenomenologically. In addition, the idea of growth of fluctuation has been generally accepted in other fields such as the Burridge-Knopoff model in seismology; therefore, the above idea we propose is not of such peculiarity.

As has been pointed out, approaches to explaining geomagnetic field reversals using disk dynamos are phenomenological. Hence, information obtained from studies of disk dynamo behaviors seems to be obscured in that there are no distinct correspondences to the interior of Earth. Certainly, difficulty in mathematical treatment of calculation is drastically reduced by use of disk dynamo models; however, disk dynamo models are considered to include essential aspects in very complex phenomena such as geomagnetic field reversals. Accordingly, it is not meaningless to study the interior of Earth by analogy, based on the above conclusion.

If we could assume that the core of Earth consists of several convection cells and each convection cell corresponds to each disk and associated circuit, then m seems to represent the degree of interaction among convection cells because m indicates the extent of interaction between neighboring circuits. Hence, based on the above assumption, we could infer that interaction among convection cells is rather small during the period of Cretaceous Superchron, and on the contrary, is relatively large for the past 25 Ma. According to the report by McFadden and Merrill (1984), the possibility of hotter outer core-mantle interface (OCMI) in the Cretaceous in comparison with that of today is pointed out; the degree of interaction among convection cells may be related to the temperature in the OCMI. However, the above description of the interior of the Earth seems to be very speculative and is based on a series of assumptions. Further investigations are needed for ascertaining how well it applies.

4. Conclusion

We examine and analyze geomagnetic field reversals on the basis of the fluctuation growth, using the three-disk dynamo model.

If there exists strong interactions between neighboring disks and their associated circuits, it is found that a very slight fluctuation is greatly enhanced after a short incubation time. On the contrary, weak interactions can't cause fluctuations to grow rapidly; it takes a longer time to grow fluctuation showing long incubation times.

From the analogy between the disk dynamos and the geomagnetic field reversal patterns, we can consider that even slight fluctuations have been greatly enhanced during the past 25 Ma while fluctuation growth was very slow during the Cretaceous Superchron; in other words, the Cretaceous Superchron could be interpreted as a long incubation time during which fluctuations grew very slowly with no polarity reversals. Hence it is found that disk dynamo calculations including time dependence the term m(t') can explain the geomagnetic field reversal pattern qualitatively.

From a mathematical point of view, the existence of a time dependent term m(t') means the operation of frequency modulation (FM) on the irregular oscillations which are the numerical solutions for the dynamic behavior of the disk dynamo with m(t') = constant.
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