Tachyon Splits the \((d = 2\) String) Black Hole Horizon and Turns it Singular

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ABSTRACT. We present a static solution for \(d = 2\) critical string theory including the tachyon \(T\) but not its potential \(V(T)\). This solution thus incorporates tachyon back reaction and, when \(T = 0\), reduces to the black hole solution. When \(T \neq 0\) one finds that (1) the Schwarzschild horizon of the above black hole splits into two, resembling Reissner-Nordstrom horizons and (2) the curvature scalar develops new singularities at the horizons. These features, as we argue, will persist even with \(V(T)\) present. Some possible methods for removing these singularities are discussed.
It is a challenging task to resolve the puzzles of gravitating systems such as the nature of singularities, the end effect of Hawking radiation, and information lost(?) inside the black hole. Recently there has been a renewed effort [1, 2] to solve these problems in the simpler context of two dimensional \((d = 2)\) systems, using the string inspired toy models for quantum gravity the inspiration having come from the discovery of black holes in \(d = 2\) critical strings [3, 4, 5].

The \(d = 2\) black hole for graviton-dilaton system was discovered as an \(\text{SL}(2, \mathbb{R})/\text{U}(1)\) gauged Wess-Zumino-Witten model [6]; as a solution of \(\mathcal{O}(\alpha')\) -function equations [6, 7] for critical string theory with \(d = 2\) target space-time [4]; and in other forms [5]. Similar toy models in \(d = 2\) space-time for quantum gravity including matter were studied first by [1] and then by many others [2] with a view of solving various puzzles of gravitating systems in this simpler context. Most of these models have mainly studied the graviton-dilaton system.

For a more complete story, one should also include tachyon, the only remaining low energy degree of freedom for \(d = 2\) strings (and which is not really tachyonic for \(d = 2\)). Though important, its inclusion results in non linear equations which are solved only in a few asymptotic cases [8, 2]. The solution of -function equations for \(d = 2\) strings including tachyons is not known. However, solving for tachyon in the \(d = 2\) string black hole background leads to singular behaviour [4]. Thus it is important to
understand the back reaction of tachyons for \( d = 2 \) critical strings, especially because of its importance as a model for \( d = 2 \) quantum gravity.

In this work we describe a static solution of the \( \beta \)-function equations for the low energy \( d = 2 \) critical string theory including tachyon \( T \), but not its potential \( V(T) \). This solution thus incorporates tachyon back reaction. Though \( V(T) \) is taken to be zero in obtaining the solution, we argue that including it will not alter the qualitative features of the solution. The solution has, beside the “black hole mass” parameter, a new parameter \( \epsilon \) which is a measure of tachyon strength. We find that: (1) The Schwarzschild horizon of the previous black hole splits into two, resembling Reissner-Nordstrom horizons. However, the solution cannot be analytically continued in between the two “horizons”. (2) The curvature scalar \( R \) has a point singularity, as for \( \epsilon = 0 \), but now with \( \epsilon > 0 \) develops new singularities at the horizons. Hence this solution is not really a black hole solution in the usual sense, though for \( \epsilon = 0 \) it is.

The sigma model action of the \( d = 2 \) critical string theory for graviton \( (G_{\mu\nu}) \), dilaton \( (\phi) \), and tachyon, in a notation similar to that of [4], is given by \( (\mu, \nu = 0, 1) \)

\[
S_{\text{sigma}} = \frac{1}{8\pi \alpha'} \int d^2 \tilde{x} \sqrt{g} (G_{\mu\nu} \nabla x^\mu \nabla x^\nu + \alpha' R \phi + 2T)
\]

and the conformal invariance requires the following \( \beta \)-function equations to be satisfied:

\[
R_{\mu\nu} + \nabla_\mu \nabla_\nu \phi + \nabla_\mu T \nabla_\nu T = 0
\]
\[ R + (\nabla \phi)^2 + 2 \nabla^2 \phi + (\nabla T)^2 + 4\gamma K = 0 \]
\[ \nabla^2 T + \nabla \phi \nabla T - 2\gamma K_T = 0 \]  
(1)

where \( \gamma = -\frac{2}{\sigma} \), \( K = 1 + \frac{\gamma}{4\gamma} \), \( V = \gamma T^2 + \mathcal{O}(T^3) \) and \( K_T = \frac{dK}{dT} \). These equations also follow from the target space effective action
\[ S = \int d^2x \sqrt{G} e^\phi (R - (\nabla \phi)^2 + (\nabla T)^2 + 4\gamma K). \]  
(2)

We proceed to solve the equations (1) in the target space conformal gauge
\[ ds^2 = e^{\sigma} du dv, \text{ with } u = x^0 + x^1 \text{ and } v = x^0 - x^1, \] for the static case where the fields depend on \( \xi (= uv) \) but not on \( \chi (= u/v) \) [9]. Equations (1) then become, with \( e^\Sigma = \gamma \xi e^\sigma \) and \( \phi_n = (\xi \frac{d}{d\xi})^n \phi \), etc.,
\[ \Sigma_2 + \phi_1 \Sigma_1 = \Sigma_2 + \phi_2 + T^2_1 = 0 \]
\[ T_2 + \phi_1 T_1 - \frac{1}{2} e^\Sigma K_T = \phi_2 + \phi_1^2 + e^\Sigma K = 0 . \]  
(3)

The above equations obey one Bianchi Identity [8]; hence, the last equation above for example, need not be solved and can only be used to determine some constants. We expect the fields to evolve depending on the local metric and hence look for their solutions in terms of \( X(\xi) \equiv \Sigma_1 \). Letting \( F(X) \equiv \phi_1 \) and \( F' = \frac{dF}{dX} \), etc. equations (3), using e.g. \( F_1 = X_1 F' \), give \( \Sigma_2 = -XF; T^2_1 = XF(1 + F') \), and
\[ XFF'' + (XF' - F)(1 + F') + e^{\Sigma_1} K_T (XF)^{-1} = 0. \]

The curvature scalar is given by \( R = -4\gamma e^{-\Sigma} XF \). We cannot solve these equations for the general case. \( T = 0 \) will give the solutions of [4] but with
no tachyon back reaction. However, for \( V = 0 \) these equations can be solved. The solution incorporates tachyon back reaction and, as we argue below, has features that are not altered when \( V \neq 0 \).

Thus taking \( K = 1 \) the equation for \( F'' \) gives, after dividing it by \( XF(1 + F') \),

\[
F(1 + F') = \epsilon(1 + \epsilon)X
\]

where \( \epsilon(\geq 0) \) is an integration constant chosen so that \( T_1^0 \geq 0 \) (equivalently \( \epsilon \) can be \( \leq -1 \)). Taking \( X = e^s \) and \( F = e^sf \) the above equation can be solved to get

\[
(F - \epsilon X)^\epsilon (F + (1 + \epsilon)X)^{(1+\epsilon)} = \text{constant} .
\]  

In principle, this is the complete solution. But it is difficult to understand its implications. So we choose the parametrisation

\[
F - \epsilon X = lB\tau^{-1}; \quad F + (1 + \epsilon)X = B\tau^\delta - 1
\]

where \( \tau(\geq 0) \) is a new parameter, \( l = \pm 1 \), \( \delta = (1+2\epsilon)(1+\epsilon)^{-1} \), \( B = A(1+2\epsilon) \) and \( A \) is a constant. Note that (i) for \( l = -1 \) the constant in equation (4) is not real and hence the choice of \( l \) constitutes minimal analytic continuation; and (ii) by using residual conformal gauge transformation \[10\] we can set \( B = 1 \). The above equations then give

\[
X = A\tau^{-1}(\tau^\delta - l); \quad F = A\tau^{-1}(\epsilon\tau^\delta + l(1 + \epsilon)) .
\]

Denoting \( \dot{X} = \frac{dX}{d\tau} \), etc. and noting that \( \dot{X} = ((1+\epsilon)\tau)^{-1}F \), equations (3) give

\[
\dot{\phi} = -X^{-1}\dot{X}, \quad \dot{T} = -\sqrt{\delta - 1}\tau^{-1}, \quad \dot{\Sigma} = -((1 + \epsilon)\tau)^{-1}, \quad \text{and} \quad \tau_1 = -(1 + \epsilon)\tau X
\]
which can be integrated to obtain

\[ e^{\phi} = \beta_0 \tau (\tau^\delta - l)^{-1} \]
\[ T = -\sqrt{\delta - 1} \ln \tau \]
\[ e^{\Sigma} = -(ml)B^2 \tau^{-(1+\epsilon)^{-1}} \int_0^\tau d\tau (\tau^\delta - l)^{-1} = A(1 + \epsilon) \ln \left( \frac{\tilde{\alpha}_0}{m\xi} \right) \]  
(5)

where \( \beta_0 \) and \( \tilde{\alpha}_0 \) are constants and \( m = \pm 1 \). The curvature scalar is given by

\[ R = 4\gamma (1 + 2\epsilon)^{-2} \tau^{-\delta} (\tau^\delta - l)(\epsilon \tau^\delta + l(1 + \epsilon)) . \]  
(6)

Equations (5) and (6) form the solution of equations (3) with \( K = 1 \). Its features are as follows.

When \( \epsilon = 0 \), \( \tau \) can be expressed in terms of \( \xi \). The solution in (5) and (6) corresponds to that of [4] if one takes \( A = 1 \), \( \beta_0 = a \), and \( 2\tilde{\alpha}_0 = a\alpha' \) (\( a \) is related to black hole mass as in [4]) and further makes the branch choices, I : \( l = m = 1 \), \( 1 \leq \tau \leq \infty \) and II : \( l = m = -1 \), \( 0 \leq \tau \leq \infty \). The respective ranges of \( \xi \) are I : \( \infty \geq \xi \geq \xi_+ > 0 \) for branch I and II : \( -\tilde{\alpha}_0 \leq \xi \leq -\xi_- < 0 \) for branch II and

The explicit \( \tau \) integration is not possible when \( \epsilon \neq 0 \) (or \( \infty \)). However, one can easily read off the following important features of the solution.

(1) From the behaviour of the integrand for \( \epsilon > 0 \), it can be deduced that \( \infty \geq \xi \geq \xi_+ > 0 \) for branch I and \( -\tilde{\alpha}_0 \leq \xi \leq -\xi_- < 0 \) for branch II and
that $\xi_\pm \to 0$ as $\epsilon \to 0$ where $\xi_\pm$ are constants. From the zeroes of $G_{\mu\nu}$ in Schwarzschild coordinates \[12\] one sees that the horizon which was located at $\xi = 0$ for $\epsilon = 0$ now splits into two located at $\xi = \pm \xi_\pm$. Thus the horizon resembles Schwarzschild horizon for $\epsilon = 0$ and resembles Reissner-Nordstrom horizon for $\epsilon > 0$. 

(2) The curvature scalar $R$ is singular at $\tau = 0 (\xi = -\bar{\alpha}_0)$ as before, but now for $\epsilon > 0$ it has new singularities (with a strength proportional to $\epsilon$, for small $\epsilon$) at $\tau = \infty (\xi = \pm \xi_\pm)$. Thus the new horizons are singular. Also, the above solutions cannot be analytically continued into the region $-\xi_- < \xi < \xi_+$ between these two “horizons”. (Hence the above solution is not really a black hole solution in the usual sense, though for $\epsilon = 0$ it is).

(3) At the horizons $\xi_\pm$, the field $e^\phi$ becomes zero. This signals a strong coupling regime as can be seen by equation (2).

(4) Asymptotically (branch I, $\tau \to 1_+$) $G_{\mu\nu}$ and $\phi$ have the same behaviour as for the case $\epsilon = 0$. Hence, the “black hole mass” calculated asymptotically, using any of the methods of \[1.13\], is of the form $M(\epsilon) = M_0 + \mathcal{O}(\epsilon)$ where $M_0$ is the black hole mass when the tachyon is zero. (However, for the non zero tachyon case we could not find any conserved quantity except the trivial ones given in the equations of motion).

(5) Now consider the tachyon $T$ and its potential $V$. It is often convenient to view the tachyon equation in \[13\] as an equation for an (anti)damped oscillator where the tachyon potential $V$ provides the restoring force and the
couplings to graviton and dilaton provide the damping force. Physically, when the tachyon field acquires large kinetic energy due to gravitational interactions the potential may be neglected. That this is what happens can be seen by calculating the kinetic \((\nabla T)^2\) and potential \(V\) energy terms in the action (2) (or, equivalently, the damping \((\phi_1 T_1)\) and the potential \(e^{\Sigma K_T}\) terms in the tachyon equation in (3) ). From the expressions \((\nabla T)^2 = 4\gamma e^{-\sigma T_1^2, T = -\sqrt{\delta - 1}\ln \tau, \phi_1 T_1 = \sqrt{\epsilon(1 + \epsilon)}XF, e^{\Sigma K_T} = e^{\Sigma(T + \mathcal{O}(T^2))}\) one sees, for the solutions (5), that away from the asymptotic region (branch I, \(\tau >> 1\)) the tachyon potential can indeed be neglected.

However, this is not true asymptotically. All the above terms are of the same magnitude and tachyon potential cannot be neglected. But it is reasonable to expect that the correct asymptotic solution with \(V\) included can be matched at some point to (5) which becomes more and more valid as one nears the horizon. That this is likely to be the case can also be seen by: (i) taking into account the tachyon back reaction in the asymptotic region by starting with asymptotic tachyon solution with \(V\) present and evaluating graviton-dilaton equations including \(T_1^2\) terms and feeding the resulting values into tachyon equation again — one finds that the potential term \(e^{\Sigma K_T}\) is softened; (ii) the \(\mathcal{O}(T^3)\) term in the tachyon potential, \(V\) (see e. g. [14]) also softens the potential — (i) and (ii) imply that the tachyon potential is less steep; and (iii) recent works [8, 2] suggest instability due to tachyon back reaction.
Hence it seems that neglecting tachyon potential is a reasonable approximation and that the qualitative features of the solution presented here will persist even when $V(T)$ is properly taken into account.

We now discuss some possible interaction terms that may remove the singularities, atleast the new ones:

1. $V(T)$: This term is unlikely to do the job as discussed above.

2. Higher order $\alpha'$ corrections: Tseytlin [15] had shown that black hole solutions of [3, 4, 5] survive these corrections. Very likely, the solution given here will also survive these corrections since the tachyon can be thought of as an (anti)damped oscillator gaining energy by gravitational interactions — so that it would have grown strong before one reaches the region of strong curvature where $\alpha'$ corrections are deemed important [16]. Thus these corrections may not remove the singularities.

3. Antisymmetric tensor, $H$ (indices on $H$ suppressed): This field is not there for $d = 2$ space-time. However, for $d = 2$ toy models of a $D$ dimensional space-time, as considered in [1] for example, the resulting equations that include quantum effects will be similar to the $\beta$-function equations. $H$ field interactions present in such cases may possibly remove the singularity. Moreover, $H$ fields arise in any space-time obtained from string theory, so it is natural to include them.

4. Supersymmetry: This symmetry introduces fermions which may provide enough repulsive force to avoid the formation of the singularities. How-
ever, the fermions might instead form attractive condensates and not remove the singularities.

The above (and other) possibilities are worth pursuing. It is important to resolve this problem of new singularities — whether they are really generic (as it seems to be the case) and if so how to remove them. This issue is particularly relevant for the string inspired toy models of $d = 2$ quantum gravity that may answer the puzzles of $d = 4$ space-time. The removal of the singularities seen here might also suggest new interactions that could be important.

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[9] Equivalently one can take the static ansatz $ds^2 = -G(x^1)(dx^0)^2 + G^{-1}(x^1)(dx^1)^2$. However, the $\beta$-function equations for the fields $G = \ln(4\gamma) - \ln G$, $\psi$ (defined by $\frac{d}{dx^1}(\psi + G - \phi) = 0$), and $T$ turn out to be exactly the same as the equations (3) for the fields $\Sigma$, $\phi$, and $T$ respectively.

[10] Under the transformations $u = U^b$, $v = V^b$, which preserve the conformal gauge, the fields are still $\chi$ independent and transform as $(e^\Sigma, X, F, \phi, T)_{\text{new}} = (b^2e^\Sigma, bX, bF, \phi, T)$. 

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[11] It is useful to draw a graph of \((\tau^\delta - l)^{-1}\) for \(\epsilon = 0, \infty\) and \(0 < \epsilon < \infty\) for the branches I and II.

[12] The metric is \(ds^2 = e^{\Sigma}dt^2 - e^{-\Sigma}dr^2\) in the Schwarzschild coordinates \((r, t)\) given by \(d\chi = (2\sqrt{-\gamma})\chi dt\) and \(d\xi = (2\sqrt{-\gamma})\xi e^{-\Sigma}dr\). The asymptotic region is \(r \to \infty\) and the fields are indeed static.

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[16] For similar reasons, higher massive modes \(A_n\) are of no help. In fact, taking their effective action as given in [14] with zero potential, it is easily seen that \(A_n\)'s have solutions similar to that of tachyon \(T\) and hence they do not remove the singularities.