A monitoring framework for deployed machine learning models with supply chain examples

Bradley Eck
IBM Research Europe
Dublin, Ireland
bradley.eck@ie.ibm.com

Duygu Kabakci-Zorlu
IBM Research Europe
Dublin, Ireland

Yan Chen
IBM
San Francisco, CA, USA

France Savard
IBM
Montreal, QC, Canada

Xiaowei Bao
IBM
Seattle, WA, USA

Abstract—Actively monitoring machine learning models during production operations helps ensure prediction quality and detection and remediation of unexpected or undesired conditions. Monitoring models already deployed in big data environments brings the additional challenges of adding monitoring in parallel to the existing modelling workflow and controlling resource requirements. In this paper, we describe (1) a framework for monitoring machine learning models; and, (2) its implementation for a big data supply chain application. We use our implementation to study drift in model features, predictions, and performance on three real data sets. We compare hypothesis test and information theoretic approaches to drift detection in features and predictions using the Kolmogorov-Smirnov distance and Bhattacharyya coefficient. Results showed that model performance was stable over the evaluation period. Features and predictions showed statistically significant drifts; however, these drifts were not linked to changes in model performance during the time of our study.

Index Terms—MLops, hypothesis-testing, drift-detection, Spark

I. INTRODUCTION

Monitoring of machine learning (ML) models in industrial applications helps ensure prediction quality and integrity of business decisions made based on model predictions. Most models are trained under the assumption of stationarity: that data used to make predictions will have the same probability distribution as that used for model training [1]. However, industrial applications are often driven by non-stationary processes due to seasonality, changes in consumer behavior, or evolving operational conditions. High volume and velocity of data in these applications introduces additional challenges for model monitoring as the data of interest may only be available for a short time or become expensive to access.

Both the literature and commercial ML system developers recognize the need to monitor models during production. Synthesizing experience with numerous systems at Google, Breck et al. weight monitoring during production as one quarter of their overall score of model readiness [2]. Precisely which quantities to monitor depends on the application. Considering the stationarity assumption underlying many models, it is not surprising that modern modelling software supports, and recent studies investigate, several methods for detecting distribution shift. Also called, drift detection, these methods evaluate the stationarity assumption by carrying out hypothesis tests such as the Kolmogorov-Smirnov test or estimating the shared information content using information entropy related metrics such as the Kullback-Leibler divergence [3]. Several studies present comparisons of algorithmic approaches to drift detection for ML models [4] [5] [6].

The choice of which monitoring algorithms to apply for a given modelling task remains application dependant and so modelling packages have growing support for drift related algorithms. Tensorflow’s data validation component quantifies drift using the L-infinity distance for categorical features and approximate Jensen-Shannon divergence for numeric features [7]. With pytorch, torchdrift supports several methods of drift detection including the Kolmogorov-Smirnov and Max Mean Discrepancy tests [8]. Scikit-learn (version 1.1.2) provides a variety of metrics to evaluate pairwise distances and sample affinity [9]. Spark (version 3.3.1) provides the one-sample Kolmogorov-Smirnov test and several distance measures [10]. The alibi-detect package provides algorithms for outlier, adversarial and drift detection [5]. With this landscape, much model monitoring can use algorithms from existing packages but there is room to add methods tailored for big data use cases.

Several recent contributions examine strategies for implementing drift detection as part of the modelling life cycle. Klaise et al. discuss the challenges of drift detection in production systems [11]. The Augur framework [12] examines drift detection metrics and thresholds with a view to eventually triggering model retraining. The MLFlow [13] tracking module, provides logging for metrics computed as part of a run. The Castor time series forecasting system [14] [15] tracks performance of rolling predictions. Drift monitoring also appears in the model lifecycle proposed by Hummer et al. who describe a cloud-based framework for AI Application development and lifecycle management [16]. In the market, cloud-based ML platforms provide monitoring for deployed models. IBM’s Watson© OpenScale™ supports monitoring for bias, fairness, and drift [17]. Amazon Sagemaker™ provides a model monitor component that detects outliers and data drift [18]. Microsoft Azure™ also has drift detection for machine learning data sets [19]. At Google®, the Vertex AI model monitoring component handles drift detection for categorical and numerical features [20]. Although these platforms are feature rich, many existing software applications embed their machine learning workflows rather than use a cloud service.
Moreover, existing commercial and open source model monitoring tools target models that already use a related software stack; there is thus gap for a more loosely coupled approach to model monitoring for applications with existing modelling workflows.

Monitoring of production models thus requires both a framework suitable for the deployment environment and metrics suitable for the modelling problem. In the use cases that motivate our work, we sought to add drift and performance monitoring in parallel to, and without disturbing, existing training and inference workflows. On the algorithmic side, we sought metrics that could work from lightweight summaries of the data and use computing and storage infrastructure already available in the target environment. We also sought to explore the application of hypothesis test and information theoretic metrics for model monitoring in a supply chain use case with a view to understanding which metrics could anticipate changes in model performance.

This paper outlines two contributions to model monitoring. First, we propose a framework for monitoring deployed models, especially where monitoring functionality should be added to applications that already embed model training and deployment. We motivate the framework with applications from several domains using ML on big data. In contrast to existing frameworks, we emphasize the deployment phase of the model lifecycle. Second, we apply the framework to a supply chain use-case and present computational experiments on real world data. These experiments use novel variations of classical techniques to reduce computational and storage requirements for measuring drift in features and predictions.

The remainder of the paper is organized as follows. Section II outlines the big data use-cases that motivate our work. Section III summarizes the concepts and architectural design of our monitoring framework. Section IV describes our experiments leveraging big data tools including Spark, Parquet, and object storage to monitor three models in a real supply chain use case. Results results appear in Section V. Finally, we conclude the work and note some promising directions for future effort in section VI.

II. Motivating Applications

Diverse applications, each with different modelling scenarios and data types, motivate our work on monitoring ML models during production. We consider monitoring models that forecast sales in supply chains, predict failures of machinery, and classify objects on assembly lines. The following sections elaborate each application to inform requirements for model monitoring.

A. Supply chain

Sales forecasts help retailers order adequate quantities, analyze the effect of discounts and position inventory to meet demand. Sales of individual products are tracked by stock keeping unit (SKU) and location. Large retailers carrying many products at many locations have tens or hundreds of millions of potential SKU-location combinations. Since every location will not carry every product the SKU-location mapping typically has low density. Table I summarizes three such real-world data sets.

In our supply chain application, data scientists train new sales forecast models each month using features as recent as the previous day’s sales. Each day, the inference workflow updates the model features and issues a new forecast. However, a model may start producing poor predictions before the next scheduled training due to shifts in feature distributions, buying habits, or other circumstances. For example, COVID-19 restrictions significantly changed consumer behavior. Therefore, these models require monitoring throughout deployment to enable model retraining on a data-driven rather than scheduled basis. Since a data scientist manages many supply chain forecasting models among several customers, automation of monitoring across models is essential.

B. Equipment failure

Industrial machines need maintenance to achieve optimal service life and return on investment. The availability of sensor output from such equipment opens ML use cases such as failure prediction. In our application, sensors on the machines emit measurements which are then processed and used to predict asset health and reliability. ML models making predictions with this sensor data can encounter non-stationarity for example when the unit of sensor output or operating regime changes. Hence the models require monitoring especially to detect data drift.

C. Object classification

In manufacturing, assembly lines produce products which require inspection for quality control and assurance. Image processing models doing object classification support human operators to deliver products with fewer defects. In the application we study, training data sets for the model are typically small, perhaps only a fraction of a percent of the number of images the model will generate inferences on within its lifetime. Small variety in the training data means that the image processing models receive myriad production images that are very different to training. Model monitoring is thus crucial for delivering reliable predictions.

III. Monitoring Framework

Models for sales forecasting, equipment failure, and object classification appear within existing enterprise software applications to provide automation and decision support. Synthe-
sizing the steps needed to implement model monitoring, the following steps were consistent across applications:

1) Register the model with the monitoring system and specify how the model should be monitored.
2) Process or store inference features, predictions, and optionally ground truth.
3) Compute metrics to evaluate model behavior and performance.
4) Evaluate metrics to trigger further actions like model retraining, report generation, or alerting.

We describe the above steps with reference to the following concepts:
- **Model**: the name or identifier for an instance of a trained ML model that is monitored by the system.
- **Monitor**: a collection of metrics computed over the same data. For example, a performance monitor could compare predictions with ground truth using the mean average error, root mean square error, and other related metrics.
- **Metric**: a computed value that results from running a monitor.
- **Reaction**: post-processing of metrics including side-effects. For example, compare a metric to a threshold and send an alert.
- **Log**: a document that results from running a reaction

To enable use of the framework in applications with different data types and architectures, we designed a layered system to separate the re-usable and application specific logic. Highly re-usable components include the API that emerges from pairing the above concepts with verbs like get, set, run and delete. The key-value schema for storing data generated during the production phase is used to train and score the model. Each of these components are further described in the following paragraphs.

### A. System Design

A loosely coupled ML model monitoring framework should be applicable to many domains each with their own data types, computing environments, and storage infrastructure. The framework proposed here aims to be flexible and extensible so that existing enterprise applications with ML models can easily add monitoring capability with minimal disturbance to existing workflows. The system comprises layers for orchestration, monitoring logic, and data storage (Fig. 1). The host application orchestrates monitoring of its ML models by invoking the monitoring framework. The framework in turn calls the application specific monitoring logic and stores the results in the monitoring data store. The application package provides concrete implementations of the monitors and reactions needed for the target models. These implementations typically use statistical hypothesis testing, model evaluation, and dimensionality reduction algorithms provided by other libraries. To make these calculations, the application package also connects to the model data storage. The storage layer distinguishes data generated by monitoring system from data used to train and score the model. Each of these components are further described in the following paragraphs.

The **framework package** encapsulates common functionality that interacts with the application specific package to deliver a working monitoring system. Features of the framework package include the data schema, client interface, and abstract or base classes for the framework concepts of Monitors, Metrics, Reactions, and Logs. In our experience, these features are independent of the ML model or application specifications so that it is possible to reuse these components.

The **application package** addresses functional needs of monitoring models in a particular application. It provides concrete implementations of monitors, metrics, reactions and logs for the target modelling scenario. For example, a report reaction enables custom visualisations for big data that plot samples of the data instead of all points. This package also handles the connection to the model training and inference data of the application.

**Algorithm package(s)** contain core implementations for computing metrics. Monitoring metrics depend on the data characteristics of an application. For example, image data often requires dimension reduction while big data gets most benefit from parallelism or approximated algorithms. Such algorithms are usually implemented without reference to the compute or storage infrastructure used by the data so that they can be used in multiple situations.

Finally, in the **data layer**, the framework package manages the storage of monitoring data while the application package interacts with the model data. For monitoring data, a key-value design allows storage of monitoring configuration and results using a variety of storage technologies including IBM Cloud® object storage, IBM DB2®, MongoDB or a file system. Model data, such as training data sets, features used to make predictions, values of predictions and eventually ground truth values remain in the model data storage. Applications with machine learning workflows already store this information; this design allows reuse of the existing data to support monitoring.

### B. Monitoring API

A simple application programming interface emerges from combining the framework concepts with the verbs set, get, run, delete. Setting a monitor associates a monitor with a model. Running a monitor computes metrics for a particular model.

![Fig. 1. Architecture of monitoring framework.](image-url)
Getting metrics retrieves computed metrics from storage. Similarly, setting a reaction associates a reaction and a model. Running the reaction creates logs. Logs may be the only result of running a reaction or may document a side-effect such as sending an alert or triggering model retraining. Getting logs retrieves this information from the monitoring data storage.

Table II indicates the methods comprising our monitoring API. Most verb-object combinations are supported. Exceptions are setting and running metrics and logs; these objects result from running monitors and reactions and thus are not settable or runnable outside the framework. With a layered system design and this API, the monitoring framework supports the necessary workflow steps while allowing application-specific customization.

### IV. Experimental Methodology

We used the framework described above to carry out monitoring experiments for the ML models making sales forecasts in our supply-chain use case. These experiments had three related aims. First, we needed to quantify the performance of the sales forecast models during production. Second, we wanted to check for non-stationarity in model features. Third, we wanted to identify metrics that could be computed at forecast time that might indicate an upcoming change in model performance. Our hypothesis was that a statistical test for distribution shift among the features or predictions could be such an indicator. To facilitate the experiments, we created a supply-chain application package with monitors for drift detection and model performance.

#### A. Drift Monitor

We monitor features and predictions for distribution shift by comparing data from training time to production values using variations of the Kolmogorov-Smirnov test and the Bhattacharyya coefficient. As further explained below, our variations to these classical methods enable computation of these metrics in a distributed environment with Spark and generate summaries of the data for re-use and visualization.

The Kolmogorov-Smirnov test for two samples calculates the largest distance between empirical cumulative distribution functions of the samples.

\[
D_{KS} = \max |F_1(x) - F_2(x)|
\]  

The distribution of this test statistic is also well known and so p-values can be readily computed, for example using [21]:  

\[
P(D_{KS} > \text{observed}) = Q_{KS}\left(\sqrt{\frac{NM}{N+M} D_{KS}}\right)
\]  

where the quantile function is:

\[
Q_{KS}(\lambda) = 2 \sum_{k=1}^{\infty} (-1)^{k-1} e^{-2k^2\lambda^2}
\]  

In our variation of the test, we build an approximate cumulative distribution function \( \hat{F} \) for each sample. We construct \( \hat{F}_1 \) and \( \hat{F}_2 \) using approximate quantiles from the Greenwald-Khanna algorithm [22] as implemented in Spark’s approxQuantile method. We build \( \hat{F} \) from quantiles at a linearly spaced probabilities between 1/N and 1. Evaluation at intermediate points uses linear interpolation between the resulting quantiles. This approximation of \( F \) summarizes the data of interest in fewer points.

The Bhattacharyya coefficient is defined by [23] as

\[
BC(p, q) = \int \sqrt{p(x)q(x)}dx,
\]  

where \( p \) and \( q \) are density functions. \( BC \) represents the cosine of the angle between unit vectors representing distributions \( p \) and \( q \). As a cosine, \( BC = 0 \) indicates perpendicular unit vectors and hence probability distributions without overlap. Similarly \( BC = 1 \) corresponds to parallel unit vectors and distributions that fully overlap. Thus \( BC \) is a convenient similarity measure for distributions as it always falls between 0 and 1, with 0 indicating no similarity and 1 indicating complete similarity.

In our variation, we evaluate \( BC \) using estimated probability density functions, \( \hat{f} \), derived from the cumulative density estimates \( \hat{F} \) already computed for (Eq. 1). The density estimate is

\[
\hat{f}(x) = (\Delta \hat{F}(x_k) + w_k)/\Delta x
\]  

where \( \Delta \hat{F}(x_k) \) is the difference in cumulative frequencies between breaks of the \( k \)th bin; \( w_k \) is a small correction factor to ensure relative frequencies sum to 1; and \( \Delta x \) is the bin width. In this way, the data summary \( \hat{F} \) can be re-used to compute inputs for \( BC \).

#### B. Performance Monitor

We monitor the in-production performance of sales forecasts from our model by comparing predicted and actual values. The target variable of the forecasting model is the seven-day average sales volume (termed velocity, as in units per day) for a product at a location, \( v_i \). Sales data become available each day and so after seven days the true value can be computed.

We report absolute and relative error metrics as follows. The mean absolute error (MAE) is

\[
MAE = \frac{1}{N} \sum_{i}^{N} |\hat{v}_i - v_i|
\]  

where the forecast velocity for the \( i \)th sku-location is \( \hat{v}_i \); the true velocity is \( v_i \); and there are \( N \) sku-location combinations of interest. Absolute errors are informative when target values have similar scales. When this is not the case, dividing the error by the target provides another useful view of performance. When there are no sales of a product at a location for a week,
the actual velocity \( v_i \) is 0, in which case the traditional MAPE cannot be computed. Thus we add a weight of unity to the true value in the denominator and report a weighted mean absolute percentage error:

\[
wMAPE = \frac{1}{N} \sum_{i} \left| \frac{\hat{v}_i - v_i}{v_i + 1} \right| \cdot 100\%
\]  

(7)

Our workflow orchestrator runs the model performance monitor daily for deployed models. Values of \( \hat{v}_i \) are extracted from the model data store. Values of \( v_i \) are computed on the fly from daily values in storage. We persist the resulting statistics for further analysis and visualization.

C. Run-time Environment

Fig. 2 shows the parallel modelling and monitoring workflows in our supply chain use-case. Data scientists initiate model training and deployment and enable monitoring. For drift monitoring, the system evaluates training data and stores the approximate cumulative distribution function, \( \hat{F} \), of model features and predictions along with other parameters in the monitoring data store for re-use. Each day, new information becomes available and is used to make a new forecast; the monitoring system evaluates drift between training and production data. The resulting metrics are saved into the monitoring data store for later visualization and analysis. Performance monitoring follows the same steps, except training data is not evaluated because calculations of MAE and wMAPE need only predictions and ground truth values.

We run monitoring workflows on a compute cluster of 7 worker nodes each with 16 CPU cores and 58GB memory. Kubernetes version 1.22 allocates workload to compute nodes. We define and invoke model monitoring steps as Argo workflows. Our algorithms are implemented in Python 3.7 using pyspark [10], numpy [24], scipy [25], pandas [26] and matplotlib [27]. Separate buckets on IBM Cloud object storage contain the model data, in Parquet [28] format, and monitoring data storage. Stocator [29] provides the connection between Spark and object storage. Using the monitoring system in this runtime environment enables automation across model deployments.

V. RESULTS AND DISCUSSION

Our experiments aimed to quantify forecast model performance, check for non-stationary features, and explore metrics that could be calculated at forecast time but indicate an upcoming performance change. To address these questions we ran the drift and performance monitors outlined above on three supply chain examples. One model was trained for each data set. Six of the model features are evaluated for drift. Drift and performance metrics for data sets A, B, and C of Table I. during March 2022 appear in Figs. 3 - 8. The figures show the Bhattacharyya coefficient, \( BC \), (Eq. 4) and Kolmogorov-Smirnov distance \( D_{KS} \) (Eq. 1) for model predictions and features by evaluation date. For model performance, we plot MAE (Eq. 6) and wMAPE (Eq. 7) on the forecast date even though the computation was actually carried out a week later, once the data became available. This arrangement compares the information available at forecast time with the model performance eventually observed.

We compare metrics’ behavior across data sets using their coefficient of variation, \( CV_v \):

\[
CV_v = \frac{s}{\bar{x}}
\]

where the sample standard deviation is \( s \) and the sample mean is \( \bar{x} \). This ratio provides a convenient way of comparing dispersion between samples with different means.

For data set A, the model was trained with 3 months data on March 7th and used to make forecasts for the remainder of the month. Prediction performance (Fig. 3) was stable with MAE and wMAPE having similar coefficients of variation: 0.0483 and 0.047. The distribution of predictions was also quite stable, but the KS distance (\( CV_v = 0.22 \)) showed considerably more variation than the Bhattacharyya coefficient (\( CV_v = 0.0084 \)). For model features (Fig 4), some trends of increasing \( D_{KS} \) and decreasing \( BC \) are visually evident in f2 and f6 while the other features showed little shift from the training distribution. Values of \( BC \) and \( D_{KS} \) for model features showed more variation than predictions. Neither drift in features or predictions were associated with changes in model performance.

For data set B, the model was trained with 3 months data on March 9th. Data was available for metric computation until March 17 except for March 10 which is not available. Model performance (Fig. 5) was again stable with MAE and wMAPE showing similar coefficients of variation 0.0065 and 0.0069 to each other, and slightly more variation than data set A. For prediction drift, KS distance (\( CV_v = 0.98 \)) showed much more variation than the Bhattacharyya coefficient (\( CV_v = 0.014 \)). The high variation in \( D_{KS} \) was driven by one very low value on March 11. March 14th showed highest shift in prediction values, but no change in model performance. Model features (Fig. 6) did not show any drift; the March 14th movement in prediction values was not apparent in the features.

For data set C, the model was trained with 1 month data on March 1st and production data available for the following week except the 4th. A trend of increasing drift in predictions is visually evident in the declining BC and rising \( D_{KS} \) values (Fig. 7). Consistent with the other examples, that trend does not carry through to model performance, where MAE and wMAPE were stable and had similar coefficients of variation 0.057 and 0.055. KS distance (\( CV_v = 0.28 \)) again showed more variation than BC (\( CV_v = 0.018 \)). Drift metrics for model features (Fig. 8) show a similar trend as those for predictions with drift most visually evident in f2 and f6. Consistent with the other examples, variations in drift metrics of features and predictions were not associated with changes in model performance.

Across data sets A, B, and C, model performance, as measured by MAE and wMAPE were very stable for the time periods over which data was available for these experiments. While this result indicates models that perform well, the good model performance hampered our experimental objective of identifying metrics that might anticipate changes in model
performance. The fact that we did not observe non-stationarity in model performance may be due to the relatively short time period over which data were available. Some non-stationarity was evident in features of data sets A and C. However, the models coped with this variation well and there were no meaningful changes in performance.

Regarding the hypothesis that a statistical test for distribution shift could anticipate changes in model performance, our experimental evidence did not support this view. At the sample sizes considered here, even small values of $D_{KS}$ are highly significant. For example solving (Eq. 2) with $\alpha = 0.05$ and $N = M = 332,000$ as for data set A yields a critical distance $D_{KS} = 0.00333$. Similarly, using $N = M = 13,500,000$ as for data set C gives a distance of $D_{KS} = 0.0005227$. An example is instructive to see why a highly significant distribution shift may not effect model performance. Figure 9 shows cumulative distribution functions for model predictions in the training and production samples from data set A on March 20, where the KS distance is 0.065. This distance is highly statistically significant with a p-value $< 10^{-16}$. While this distance is visible on the plot, in this context it is understandable that the distribution has not changed in an operationally meaningful way. For this reason, we also report BC values to provide evidence that the shape of the distribution has not changed much.

VI. CONCLUSIONS

We presented and applied a framework for monitoring machine learning models during deployment to three supply chain examples. The framework enables adding model monitoring capability to an existing application that is already training and scoring of ML models. It uses the application’s storage
In our supply chain examples we analyze sales predictions and model features for distribution shift. Across three data sets in March 2022, the forecast model performance as measured by MAE and wMAPE was very stable. The distribution of predicted sales showed more variation than model performance but this variation did not translate into performance changes, suggesting that the models are performing as intended. Features showed more drift but this was also not connected with performance change. All of the KS tests reported here were highly statistically significant. The fact no tests were operationally meaningful sounds a cautionary note about the utility of hypothesis testing for drift detection in this application. As an additional metric, the Bhattacharyya coefficient provided...
useful confirmation that the distribution shape had not changed much, despite the KS distance.

Future work could apply the proposed monitoring framework to other applications and longer duration data sets to identify suitable thresholds for BC, KS or other metrics to support alerting and retraining.

REFERENCES

[1] G. Ditzler, M. Roveri, C. Alippi, and R. Polikar, “Learning in nonstationary environments: A survey,” Comp. Intell. Mag., vol. 10, no. 4, p. 12–25, nov 2015. [Online]. Available: https://doi.org/10.1109/MCI.2015.2471196

[2] E. Breck, S. Cai, E. Nielsen, M. Salib, and D. Sculley, “The ml test score: A rubric for ml production readiness and technical debt reduction,” in 2017 IEEE International Conference on Big Data (Big Data), 2017, pp. 1123–1132.

[3] C. Huyen, Designing Machine Learning Systems. O'Reilly Media, Inc., 2022.

[4] S. Rabanser, S. Gunnemann, and Z. C. Lipton, “Failing loudly: An empirical study of methods for detecting dataset shift,” in Advances in Neural Information Processing Systems 32: Annual Conference on Neural Information Processing Systems 2019, NeurIPS 2019, December 8-14, 2019, Vancouver, BC, Canada, H. M. Wallach, H. Larochelle, A. Beygelzimer, F. d’Alché-Buc, E. B. Fox, and B. Garnett, Eds., 2019, pp. 1394–1406.

[5] A. Van Looveren, J. Klaise, G. Vacanti, O. Cobb, A. Scillitoe, R. Samolescu, and A. Athorne, “Alibi detect: Algorithms for outlier, adversarial and drift detection,” 2019. [Online]. Available: https://github.com/SeldonIO/alibi-detect

[6] O. Cobb and A. V. Looveren, “Context-aware drift detection,” in Proceedings of the 39th International Conference on Machine Learning, 2022.

[7] M. Abadi, A. Agarwal, P. Barham, E. Brevdo, Z. Chen, C. Citro, G. S. Corrado, A. Davis, J. Dean, M. Devin, S. Ghemawat, I. Goodfellow, A. Harp, G. Irving, M. Isard, Y. Jia, R. Jozefowicz, L. Kaiser, M. Kudlur, J. Levenberg, D. Mané, R. Monga, S. Moore, D. Murray, C. Olaï, M. Schuster, J. Shlens, B. Steiner, I. Sutskever, K. Talwar, P. Tucker, V. Vanhoucke, V. Vasudevan, F. Viégas, O. Vinyals, P. Warden, M. Wattenberg, M. Wicke, Y. Yu, and X. Zheng, “TensorFlow: Large-scale machine learning on heterogeneous systems,” 2015, software available from tensorflow.org. [Online]. Available: https://tensorflow.org/

[8] T. Viehmann, L. Antiga, D. Cortinovis, and L. Lozza, “TorchDrift: drift detection for pytorch,” 2021. [Online]. Available: https://www.torchdrift.org/

[9] F. Pedregosa, G. Varoquaux, A. Gramfort, V. Michel, B. Thirion, O. Grisel, M. Blondel, P. Prettenhofer, R. Weiss, V. Dubourg, J. Vanderplas, A. Passos, D. Cournapeau, M. Brucher, M. Perrot, and E. Duchesnay, “Scikit-learn: Machine learning in Python,” Journal of Machine Learning Research, vol. 12, pp. 2825–2830, 2011.

[10] M. Zaharia, R. Xin, P. Wendell, T. Das, M. Armbrust, A. Dave, X. Meng, J. Rosen, S. Venkataraman, M. Franklin, A. Ghodsi, J. Gonzalez, S. Shenker, and I. Stoica, “Apache spark: A unified engine for big data processing,” Communications of the ACM, vol. 59, pp. 56–65, 11 2016.

[11] J. Klaise, A. V. Looveren, C. Cox, G. Vacanti, and A. Coca, “Monitoring and explainability of models in production,” in Workshop on Challenges in Deploying and Monitoring MachineLearningSystems(ICML 2020), 2020.

[12] G. A. Lewis, S. Echeverría, L. Pons, and J. Chrabaszcz, “Augur: A step towards realistic drift detection in production ml systems,” in Workshop on Software Engineering for Responsible AI (SERAAI 22 ), 2022.

[13] M. Zaharia, A. Chen, A. Davidson, A. Ghodsi, S. A. Hong, A. Konwinski, S. Murching, T. Nykodym, P. Ogilvie, M. Parkhe, P. Xie, and C. Zamar, “Accelerating the machine learning lifecycle with mlflow,” Bulletin of the IEEE Computer Society Technical Committee on Data Engineering, 2018.

[14] B. Chen, B. Eck, F. Fusco, R. Gormaly, M. Purcell, M. Sinn, and S. Tirupathi, “Castor: Contextual IoT time series data and model management at scale,” Proc. of the 18th ICDM 2018, pp 1487-1492, 2018.

[15] B. Eck, F. Fusco, R. Gormaly, M. Purcell, and S. Tirupathi, “Scalable deployment of AI time-series models for IoT,” in Workshop AI for Internet of Things (AI4IoT) at the 28th International Joint Conference on Artificial Intelligence (IJCAI), 2019.

[16] W. Hummer, V. Muthusamy, T. Rausch, P. Dube, K. El Maghraoui, A. Murthi, and P. Oum, “Modelops: Cloud-based lifecycle management for reliable and trusted ai,” in 2019 IEEE International Conference on Cloud Engineering (ICE2), 2019, pp. 113–120.

[17] “Ibm watson openscale,” accessed 1 Sept 2022. [Online]. Available: https://www.ibm.com/docs/en/cloud-paks/cp-data/3.5?topic=services-watson-openscale

[18] “Amazon sagemaker model monitor,” accessed 1 Sept 2022. [Online]. Available: https://docs.aws.amazon.com/sagemaker/latest/dg/model-monitor.html

[19] “Monitor azure machine learning,” accessed 1 Sept 2022. [Online]. Available: https://docs.microsoft.com/en-us/azure/machine-learning/monitor-azure-machine-learning

[20] “Vertex ai model monitoring,” accessed 1 Sept 2022. [Online]. Available: https://cloud.google.com/vertex-ai/docs/model-monitoring

[21] W. H. Press and S. A. Teukolsky, “Kolmogorov-smirnov test for two-dimensional data,” Computers in Physics, vol. 2, no. 74, 1988.

[22] M. Greenwald and S. Khanna, “Space-efficient online computation of quantile summaries,” SIAMG Soc., vol. 30, no. 2, p. 58–66, May 2001. [Online]. Available: https://doi.org/10.1137/0637628.375670

[23] T. Kailath, “The divergence and bhattacharyya distance measures in signal selection,” IEEE Trans. on Communication Technology, vol. 15, no. 1, pp. 52–60, 1967.

[24] C. R. Harris, K. J. Millman, S. J. van der Walt, R. Gommers, P. Virtanen, D. Cournapeau, E. Wieser, J. Taylor, S. Berg, N. J. Smith, R. Kern, M. Picus, S. Hoyer, M. H. van Kerkwijk, M. Brett, A. Haldane, J. F. del Río, M. Wiebe, P. Peterson, P. Gérard-Marchant, K. Sheppard, T. Reddy, W. Weckesser, H. Abbasi, C. Gohlke, and T. E. Oliphant, “Array programming with NumPy,” Nature, vol. 585, no. 7825, pp. 357–362, Sep. 2020. [Online]. Available: https://doi.org/10.1038/s41586-020-2649-2

[25] P. Virtanen, R. Gommers, T. E. Oliphant, M. Haberland, T. Reddy, D. Cournapeau, E. Burovski, P. Peterson, W. Weckesser, J. Bright, S. J. van der Walt, M. Brett, J. Wilson, K. J. Millman, N. Mayorov, A. R. J. Nelson, E. Jones, R. Kern, E. Larson, C. J. Carey, I. Polat, Y. Feng, E. W. Moore, J. VanderPlas, D. Laxalde, J. Perktold, R. Cimrman, I. Henriksen, E. A. Quintero, C. R. Harris, A. M. Archibald, A. H. Ribeiro, F. Pedregosa, P. Van Mulbregt, and SciPy 1.0 Contributors, “SciPy 1.0: Fundamental Algorithms for Scientific Computing in Python,” Nature Methods, vol. 17, pp. 261–272, 2020.

[26] Wes McKinney, “Data Structures for Statistical Computing in Python,” in Proceedings of the 9th Python in Science Conference, Stéfan van der Walt and Jarrod Millman, Eds., 2010, pp. 56 – 61.

[27] J. D. Hunter, “Matplotlib: A 2d graphics environment,” Computing in Science & Engineering, vol. 9, no. 3, pp. 90–95, 2007.

[28] D. Vohra, Apache Parquet. Berkeley, CA: Apress, 2016, pp. 325–335. [Online]. Available: https://doi.org/10.1007/978-1-4842-2199-0_\_8

[29] G. Vernik, M. Factor, E. K. Kolodner, P. Michiardi, E. Ofer, and F. Pace, “Stocator: Providing high performance and fault tolerance for apache spark over object storage,” in 2018 18th IEEE/ACM International Symposium on Cluster, Cloud and Grid Computing (CCGRID), 2018, pp. 462–471.