Optical Lattices with Micromechanical Mirrors

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(Dated: February 25, 2010)

We investigate a setup where a cloud of atoms is trapped in an optical lattice potential of a standing wave laser field which is created by retro-reflection on a micro-membrane. The membrane vibrations itself realize a quantum mechanical degree of freedom. We show that the center of mass mode of atoms can be coupled to the vibrational mode of the membrane in free space, and predict a significant sympathetic cooling effect of the membrane when atoms are laser cooled. The controllability of the dissipation rate of the atomic motion gives a considerable advantage over typical optomechanical systems enclosed in optical cavities, in that it allows a segregation between the cooling and coherent dynamics regimes. The membrane can thereby be kept in a cryogenic environment, and the atoms at a distance in a vacuum chamber.

PACS numbers:

Optical lattices for ultracold atoms are generated by standing laser light waves, obtained e.g. by retroreflecting a running wave laser beam from a mirror. The intensity variation in the standing wave gives rise to a position dependent AC Stark shift, thus providing a periodic array of optical microtraps for the atomic motion. In the description of atomic dynamics in far-off-resonant lattices, e.g. in deriving the Hubbard dynamics of bosonic or fermionic atoms, the optical potentials are typically represented as c-number fields. The backaction of the atomic motion on the laser fields and spontaneous emission into vacuum modes appear as an additional source of dissipation. In light of recent experimental progress in the field of optomechanics, a new generation of optical lattice experiments is in sight, where the standing light waves are generated by reflection from a micromechanical mirror or reflecting dielectric membrane, whose motion must be described quantum mechanically at low temperature, and which is coupled via the moving lattice to a distant ensemble of cold atoms. In a lattice with moderate detuning, the back-action of atomic motion on the light fields can be substantial. This results in a sizable coupling of atomic motion and mirror vibrations, mediated by the quantum fluctuations of the lattice laser light. Below we develop a fully quantum theoretical description of such a setup (cf. Fig. 1). In particular, we derive a Quantum Stochastic Schrödinger Equation (QSSE) and thus a Master Equation for a setup where a coherent coupling of the mirror to the collective center-of-mass motion of atoms in the lattice emerges, and where the mirror can be sympathetically cooled to the motional ground state by laser cooling the atoms. We envision that the proposed system presents only one instance where the well-developed tools for laser cooling of atoms are used to provide a mechanism for sympathetic cooling of polarizable particles (here the dielectric membrane).

Model: Consider the setup shown in Fig. 1. A laser frequency is shined on and partially retro-reflected from a dielectric membrane to form a standing wave that acts as a one-dimensional optical lattice trap for a cloud of atoms. The Hamiltonian for the system is $H_{\text{full}} = H + H_{3D}$, where $H$ contains in a one-dimensional treatment the coupling of atoms and membrane vibrations to the paraxial field along the $z$-direction as shown in Fig. 1

\[ H = \hbar \omega_m a_m + \sum_{j=1}^{N} \frac{p_j^2}{2m_{\text{lat}}} - \sum_{j=1}^{N} \frac{\hbar^2}{2m} E^-(z_j, t) E^+(z_j, t) + \epsilon_0 A (n^2 - 1) \left[ E^-\left(\frac{\epsilon}{4}\right) E^+\left(\frac{\epsilon}{4}\right) - E^-\left(-\frac{\epsilon}{4}\right) E^+\left(-\frac{\epsilon}{4}\right) \right] Z, \]

while coupling to all other field modes is subsumed in $H_{3D}$. The first two terms in Eq. 1 represent the energy of the fundamental vibrational mode of the membrane with frequency $\omega_m$ and annihilation operator $a_m$, and the kinetic energy of

\[ \begin{array}{c}
\text{FIG. 1: A laser field impinging from the right is partially reflected off a dielectric membrane and forms a standing wave optical potential for an atomic ensemble. Vibrations of the membrane’s fundamental mode will shift the standing wave field, shaking atoms in the optical lattice. Conversely, oscillations of the atomic cloud (center of mass motion) will change the intensity of left/right propagating field components, thus shaking the membrane via changing the radiation pressure on it. The membrane can be kept in a cryogenic environment, and the atoms at a distance in a vacuum chamber.}
\end{array} \]
FIG. 2: (a) R- and L-modes for fields impinging from the right and left, respectively. All but the laser driven \((\omega_l, R)\)-mode are initially in vacuum. (b) Mediated action of atomic COM fluctuations on the membrane: At the advanced time \(t^+\) atoms absorb a photon from the right propagating component of the laser field (with amplitude \(\sqrt{\alpha}\), straight arrow) and reemit a sideway photon to the R-field \(b_R(t^-)\), wavy arrow), receiving a momentum kick of \(2\hbar k_\ell\), cf. the term in Eq. 3a. At a later time \(t\) the sideway photon is annihilated \(b_R(t^+)\) and transferred back to the laser field giving a momentum kick to the membrane, cf. first term in Eq. 3b. (c) Reverse action: Via corresponding emission and reabsorption processes as given in Eqs. 3c and 3d membrane vibrations affect atomic COM fluctuations. In these processes both the R- (upper panel) and the L-field (lower panel) contribute, effectively mediating a stronger action than the process shown in (b).

Atoms with momentum \(p_t\) and mass \(m_{at}\). The third term is the level shift operator \(\delta\) describing the dispersive interaction of the \(N\) two level atoms with dipole element \(\mu\) with off-resonant field modes with detuning \(\delta\) from the atomic resonance within a frequency band \(\delta \ll \delta^\prime\) [11]. In this treatment electronically excited states are already adiabatically eliminated. The second line in Eq. 1 is the potential of the radiation pressure force on a slab of thickness \(l\), cross section \(A\), index of refraction \(n\), and a small excursus \(Z\) of the membrane from the equilibrium. The radiation pressure force is proportional to the difference of the intensities on either side (at \(\pm \frac{l}{2}\)) of the membrane. The electric field is \(E(z) = E_R(z) + E_L(z)\) and consists of the two 1D continua of plane wave modes impinging from the right \((R)\) and left \((L)\), respectively, cf. Fig. 2. The explicit mode functions can be found in [9], and will be denoted by \(A_{\omega, \ell}(k, z)\) \((\omega = L, R)\) consistent with the notation used there. Thus, \(E(z, t) = E [d\omega A_{\omega, \ell}(k, z)b_{\omega, \ell}e^{-i\omega t}]\) with \([b_{\omega, \ell}, b_{\omega', \ell}] = \delta_{\omega, \ell}\delta(\omega - \omega')\) and \(k = \omega/c\). We are using a narrow band approximation \(E = \sqrt{\frac{\hbar}{3\pi c\ell_{\omega0}a}}\) and take the fields in an interaction picture with respect to the free field Hamiltonian. The contribution of other than paraxial modes contained in \(H_{1D}\) can be largely suppressed in the following. Its effect will be relevant and addressed separately in the context of momentum diffusion of atoms in the lattice. The driving laser of frequency \(\omega_l\) is at this point included by assuming the field to be in a coherent state \(|\alpha\rangle = D|\alpha\rangle\) (with \(D = \exp(\alpha b_{\omega, \ell} - h.c.)\)), corresponding to a laser field of power \(P = \hbar\omega_l\alpha^2/2\pi\) (with photon flux \(\alpha^2/2\pi\)) impinging from the right.

Quantum Stochastic Schrödinger Equation: We are interested in treating those processes only whose amplitudes are enhanced by the laser amplitude \(\alpha\) in either first or second order. In order to identify those we apply a Mollow transformation \([10]\) and move to a displaced picture, where the field is effectively in vacuum \(|\psi_{vac}\rangle = D|\alpha\rangle\) and the new Hamiltonian is accordingly \(H' = D^\dagger HD\). In \(H'\) the second order terms in \(\alpha\) provide the latent potential (with trap depth \(V_0 = \frac{4\hbar^2\ell_0^2\alpha^2}{\hbar^2}\) and trap frequency \(\omega_{st}\) = \(\sqrt{\frac{2\alpha^2}{m_{at}\omega_{st}}}\). The result can be interpreted as a (Stratonovich) quantum stochastic Schrödinger equation (QSSE) \([7]\) with time delays

\[
\frac{d}{dt}|\Psi\rangle = -iH(t, t^+, t^-)|\Psi\rangle
\]

\[
= \left\{ -iH_{sys} + x_{at}\left(\sqrt{g_{at, R}} b_R(t^-) - h.c.\right) + x_m(i\sqrt{g_{m, R}} b_R(t^-) - h.c.) + x_{at}\left(\sqrt{g_{at, L}} b_L(t^-) + i\sqrt{g_{m, L}} b_L(t^-) - h.c.\right) \right\}|\Psi\rangle
\]

where \(H_{sys} = \omega_m a_{m}^\dagger a_m + \omega_{at} a_{at}^\dagger a_{at}\) denotes the system Hamiltonian for the free evolution of the membrane and the atomic center of mass (COM) mode, with annihilation operator \(a_{at}\) for the later. The remaining terms denote the coupling of position fluctuations \(x_{m(at)} = (a_{m(at)} + h.c.)/\sqrt{2}\) of these modes to vacuum fluctuations of the EM field described by slowly varying field operators \(b_{\omega}(t) = \int \frac{d\omega}{\sqrt{2\pi}} b_{\omega}(\omega) \exp[-i(\omega - \omega_l)t] (\omega = R, L)\), evaluated at time \(t\) as well as at advanced and retarded times \(t^\pm = t \pm \frac{\ell}{c}\), where \(d\) is the distance between the atomic ensemble and the mirror, cf. Fig. 2. In this Hamiltonian the linear extension of the ensemble is neglected and we used the approximation \(b(t \pm \frac{\ell}{c}) \simeq b(t \pm \frac{\ell}{c})\) for all atoms \([13]\). The coupling constants of membrane vibrations and atomic COM position fluctuations to the EM field are given by

\[
g_{m, R} = 2(\alpha k_\ell)^2 \ell_0^2 / \pi, \quad g_{at, R} = 2\pi N(\omega_{at}/4\alpha k_\ell)^2,\]

where \(\ell_0 = \frac{\hbar}{m_{at}\omega_{st}}\) (attractor mass of the membrane), and \(g_{m(at), L} = \frac{\ell}{c} g_{m(at), R}\) with the membrane transmittivity \(t = 1 - \ell\). In Fig. 2a and c we provide an intuitive interpretation of each term in the QSSE as an (Anti)Stokes-backscattering process, where a laser photon is absorbed or emitted together with, respectively, an emission or absorption of a sideband photon at frequency \(\omega_l \pm \omega_{at}(\omega_{st})\) in one of the initial empty modes, along with a momentum transfer of \(2\hbar k_\ell\) to either the membrane or the atomic COM mode. The effective mediated coupling between atoms and membrane, as well as radiation pressure noise on each system, will eventually result from second order processes, where sideband photons are emitted and reabsorbed, Fig. 2b and c.

Cascaded System Master Equation: The corresponding effects of these second order processes can be seen explicitly...
when Eq. (2) is transformed into an equivalent master equation (MEQ) without time delays in the limit $t^\pm \rightarrow t$, and under the Born-Markov approximation for the EM field in vacuum. The technicalities of this procedure, which requires some care regarding correct time ordering, are outlined below. The result is a master equation for the density operator $\rho$ of the membrane and the COM motion of the form

$$\dot{\rho} = -i[H_{\text{sys}} + gx_{\text{at}} x_m, \rho] + C\rho + L_m \rho + L_{\rho}. \quad (3)$$

This MEQ contains, firstly, an atom-membrane interaction with a coupling $g = 2\sqrt{\gamma_m \gamma_{\text{at}} R + \gamma_m L \gamma_{\text{at}} L} = \omega_{\text{at}} \sqrt{\frac{N_{\text{mat}}}{m_m}}$. Here we assume comparable oscillation frequencies $\omega_m \simeq \omega_{\text{at}}$. While the ratio of masses $\sqrt{\frac{m_{\text{at}}}{m_m}}$ is always small under reasonable conditions, the rate of coherent coupling $g$ can still be significant thanks to the collective enhancement by $\sqrt{N}$.

Secondly, we find a contribution

$$C\rho = i\hbar \left[ (x_m, x_{\text{at}} \rho) - [p x_{\text{at}}, x_m] \right],$$

which is proportional to the membrane transmittivity $t$. The peculiar form of this term is in fact generic for cascaded quantum systems \([2, 10]\). Its effect becomes clear e.g. when looking at the evolution of mean values according to Eq. (3). $\langle p_{\text{at}}(t) \rangle = g x_{\text{at}}(0) + \cdots \cdots$ while $\langle p_{\text{at}}(t) \rangle = g x_{\text{at}}(0) + \cdots \cdots$, where dots stand for contributions of $H_{\text{sys}}, L_m$ and $L_{\rho}$. Thus, the action of atoms on the membrane is reduced by a factor $\gamma$ as compared to the action of membrane fluctuations on atoms \([19]\). This can be understood from the fact that action of membrane on atoms is mediated through both fields, $R$ and $L$, while only the $R$ field contributes to the action of atoms on the membrane, see Fig. 2.

In the limiting case $\gamma = 1$, that is for an ideal mirror, the term $C\rho$ does not contribute and the interaction is only due to the Hamiltonian part of Eq. (3). The scaling of $g$ as given above can then be understood in terms of a simple example in terms of quasi-static field modes: The mirror displacement $Z(t)$ provides a time-dependent boundary condition for the EM field, such that the standing wave laser field is $E(z) \propto \sin[k_z(z - Z)]$. The corresponding lattice potential for the atoms is $V(z_i) = \sum V_0 \sin[k_z(z - Z)]$. The standing wave laser field is $E(z) \propto \sin[k_z(z - Z)]$. The corresponding lattice potential for the atoms is $V(z_i) = \sum V_0 \sin[k_z(z - Z)]$.

On top of these decoherence channels there will be thermal heating of the membrane due to clamping losses and laser absorption, which drives the vibrational mode under consideration to thermal equilibrium at temperature $T$, i.e. $L_m \rho = \frac{1}{2}\sum (|n\rangle \langle n| \rho + \langle n| \cdot nD[|n\rangle _{\rho}] \rho$, with thermal occupation $n = \frac{k_B T}{\hbar \omega_m}$, and $\gamma_m = \frac{\omega_m}{Q_m}$ for a mechanical quality factor $Q_m$. For atoms in turn, it is important to note that the dissipation processes $L_{\rho}$ can to a large extent be engineered. In particular, it is possible to use Raman sideband laser cooling to cool atoms individually close to their ground states in the optical lattice potential \([11]\). In the master equation we account for this by adding a Lindblad term for the COM mode, $L_{\text{cool}} \rho = \frac{1}{2}\sum a_{\text{cool}} D[a_{\text{cool}}] \rho$, with a Raman-cooling rate $\gamma_{\text{cool}}$. If laser cooling is switched off, only the background decoherence at a rate $\gamma_{\text{at}}$ remains. The dynamics of the master equation (3) thus provides two interesting regimes: with efficient laser cooling of atoms it is possible to sympathetically cool the mirror, while for $\gamma_{\text{at}} \ll g$ a regime of coherent evolution becomes accessible, limited only by the decoherence rate of the membrane $\gamma_m = \gamma_m n$.

### Case Study

We estimate the magnitude of the relevant processes as follows: For a small SiN membrane of dimensions $150 \mu m \times 150 \mu m \times 50 \mu m$ exhibiting a mechanical resonance at $\omega_m = 2\pi \times 0.86$ MHz with an effective mass of $m_m = 8 \times 10^{-13}$ kg and a high mechanical quality of $Q_m = 10^7$ at $T \leq 2$ K, a power reflectivity $\tau = 0.31$ can be achieved for the wavelength of $\lambda = 780 \mu m$ relevant for $^{87}$Rb \([4]\). A lattice beam with power $P = 7$ mW and a waist of $230 \mu m$ at the position of atoms, detuned by $\delta = 2\pi \times 1$ GHz, can provide a sufficient lattice depth such that the longitudinal trap frequency $\omega_\text{at} \simeq \omega_m$. For an ensemble of $N = 3 \times 10^8$ atoms \([11]\) a strength of coherent coupling $g = 40$ kHZ can be achieved. For the decoherence due to radiation pressure noise one finds a momentum diffusion at rate $\gamma_{\text{diff}} = 52$ Hz.
for the membrane, and an atomic momentum diffusion in the lattice at rate $\gamma_{at}^{diff} = 16$ kHz. For the membrane the dominating dissipative effect will clearly be thermal decoherence at rate $\gamma_{th}^{mem} = 4$ MHz (24 kHz) at room temperature (2 K). On the other hand, Raman sideband cooling of atoms at a fast rate $\gamma_{cool}^{at} \approx 20$ kHz is possible [11]. For these parameters, a regime where $\omega_m = \omega_{at} \gg \gamma_{cool}^{at} \approx \gamma_{th}^{mem} \gg \gamma_{at}^{diff}$ is accessible. To suppress trap loss due to light-assisted collisions, blue detuning of the lattice laser is advantageous [11]. Transversally, the atoms can be confined in a far-detuned 2D lattice, so that atomic densities of order $10^{12}$ atoms/cm$^3$ are realistic [12]. A concern is the spread of vibrational frequencies $\Delta \omega_{at} \simeq 2\pi \times 24$ kHz across the ensemble due to the Gaussian intensity profile of the laser. It can be reduced by using a beam with a flat top profile. Due to its modularity, the presented setup is very flexible, e.g. several atomic ensembles could be trapped in multiple foci of the same lattice laser, thereby enhancing $N$ and thus $\eta$.

Sympathetic Membrane Cooling In order to evaluate the corresponding efficiency of sympathetic cooling of the membrane via laser cooling of atoms, we solve the master equation (3) for the steady state occupation $\bar{n}_{ss} = \langle a^\dagger_m a_m \rangle_{ss}$. For the parameters given above one finds a cooling factor $f = \bar{n}/\bar{n}_{ss} \approx 2 \times 10^4$, which is sufficient for ground state cooling ($\bar{n}_{ss} \approx 0.8$) of the membrane when starting from 500 mK. In order to get some more insight into the cooling efficiency we consider the weak-coupling limit $\gamma_{cool}^{at} \gg g$. In this case we can eliminate the atomic COM mode adiabatically along the lines of the treatments of the equivalent problem of optomechanical laser cooling [2, 3]. For the mean occupation one finds a rate equation $\dot{\bar{n}}_{ss} = -\Gamma_m (\bar{n}_{ss} - \bar{n})$ with an effective cooling rate $\Gamma_m = \gamma_m + \frac{g^2}{2 \omega_{at}^2}$ and a final occupation $\bar{n}_{ss} \simeq \frac{\bar{n}}{1 + \frac{(\gamma_{cool}^{at})}{\omega_m}}$. For large enough cooling rate the thermal contribution can be suppressed and the limitation is only due to Stokes-scattering processes. As in laser cooling of ions or optomechanics, this can be suppressed in the resolved sideband limit, which amounts here to having $\gamma_{at} \ll \omega_{at}$. Under this condition ground state cooling is possible.

If laser cooling of atoms is switched off ($\gamma_{cool}^{at} = 0$), a regime of coherent coupling is accessible, at least for cryogenic temperatures, where $g \gtrsim \gamma_{th}^{mem}$, $\gamma_{at}^{diff}$. As compared to the usual optomechanical setup, where the equivalent parameter to $\gamma_{cool}^{at}$ is the cavity decay rate which is a fixed parameter, this is a qualitatively new feature of the setup considered here. It is well known, and has been extensively studied in various contexts [17], that the given coupling Hamiltonian $\sim \omega_{at} x_m$ which becomes $\sim \langle a_m a^\dagger_m + a^\dagger_m a_m \rangle$ in the rotating wave approximation ($\omega_m \gg g$) - allows for a coherent state exchange of the two systems.

Appendix: Conversion of the QSSE from Stratonovich to Ito form We still need to explain how to transform the QSSE with time delays to the Markovian master equation (3). We first transform the Stratonovich QSSE to an Ito QSSE by first integrating Eq. 2 up to a time $t + \Delta t$ (such that $\Delta t \gg \tau \gg 1/\delta$, where $\theta$ is the bandwidth of modes) and then expand to second order,

$$U(\Delta t)|\Psi(t)\rangle = \left\{ 1 - i \int_t^{t+\Delta t} dt_1 H(t_1, t', t') \right\} |\Psi(t)\rangle$$

We use here the Ito increments $\Delta B_{at}(t) = \int_t^{t+\Delta t} dt' b_{at}(t')$ and their property $\Delta B_{at}(t) \equiv 0$ for all times. In the double time integral care needs to be taken in dealing with the time delays. It is easy to check that $\int_t^{t+\Delta t} dt_1 \int_t^{t+\Delta t} dt_2 b_{at}(t_1) b_{at}(t_2 + \delta) = \theta(\delta) \delta_{0, \theta} \Delta t$ [21]. After applying this rule it is possible to take the delay to zero, $t \rightarrow t$, as has been done in the second step of Eq. (4). The last equation (5) can then be interpreted as an Ito QSSE, which can in turn be converted to the master equation (3) following standard procedures [7]. Note that this procedure results in a seemingly collectively enhanced atomic momentum diffusion, which is an artefact of the 1D model adopted in Eq. (1). A more careful treatment based on a 3D model for the coupling of atoms to the EM field correctly yields the well known individual momentum diffusion of atoms in an optical lattice at the rate given in Eq. (2), along with dipolar interactions which are suppressed at low densities [22].
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[18] This is well justified if the “sideband wavelength” $c/\omega_{\text{lat}}$ is smaller than the linear extension of the atomic ensemble.
[19] Note the coupling strength $g$ itself tends to zero for $r \to 0$, as the lattice depth and therefore also $\omega_{\text{lat}}$ vanish in this limit and $g \propto \omega_{\text{lat}}$.
[20] For $r < 1$ a more careful reasoning in terms of quasi-static field modes can in fact reproduce the correct equations of motion for mean values.
[21] $\theta(\delta) = 0, \frac{1}{2}, 1$ for $\delta \lesssim 0$ is the step function.
[22] A 1D treatment misses the fall-off of inter-atomic interactions with the cubed distance, giving rise to an entirely collective diffusion.