Strongly First Order Electroweak Phase Transition
induced by Primordial Hypermagnetic Fields

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Abstract
We consider the effect of the presence of a hypermagnetic field at the electroweak phase transition. Screening of the $Z$-component inside a bubble of the broken phase delays the phase transition and makes it stronger first order. We show that the sphaleron constraint can be evaded for $m_H$ up to 100 GeV if a $B_Y \gtrsim 0.3T^2$ exists at the time of the EW phase transition, thus resurrecting the possibility for baryogenesis within the minimal standard model (provided enough $CP$ violation can be obtained). We estimate that for $m_H \gtrsim 100$ GeV the Higgs condensate behaves like a type II superconductor with $Z$-vortices penetrating the bubble. Also, for such high Higgs masses the minimum $B_Y$ field required for a strong first order phase transition is large enough to render the $W$-field unstable towards forming a condensate which changes the simple picture of the symmetry breaking.

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It is by now generally accepted that baryogenesis is not possible in the minimal Standard
Model (MSM). The reason is that the sphaleron constraint, which is needed to ensure the
survival of the baryon number against sphaleron induced erasure, cannot be satisfied in the
MSM, no matter what the mass of the Higgs boson is [1], because the radiative correction
induced to the effective action by the top quark makes the transition too weakly first order.
While this result was first borne out from a detailed lattice calculation, it can actually be
robustly seen already at the level of the simple 1-loop effective potential.

However, it is conceivable that the presence of a large scale homogenous magnetic field
could change the situation. Magnetic fields are a generic feature in the very early universe,
and there has been several suggestions for mechanisms giving rise to strong fields (for a recent
review, see [2]). For instance, certain types of inflationary models can produce magnetic
fields extending over horizon distances [3]. These models must be able to break the conformal
invariance to escape the flux freezing constraint, which would dictate that \( B \sim R^{-2} \). Another
possibility [4] could be that the Yang-Mills vacuum is unstable and the true ground state
has a non-zero \( \vec{B} \), as suggested by Savvidy [3]. It however appears unlikely that the original
argument by Savvidy can be carried over to a finite temperature environment [3]. It has
also been proposed that a right-handed electron asymmetry, generated at the GUT scale and
conserved down to TeV-range in temperature [7], could give rise to a hypercharge magnetic
field \( B_Y \) via a Chern-Simons term [8].

No matter what the origin of the primordial magnetic field, it could have interesting
consequences for physics at the electroweak scale. For example, a hypermagnetic field could
have a pronounced effect on the expansion of the phase transition bubble which would have
to push against the field lines frozen in the plasma outside the bubble. In this letter we
concentrate on another interesting issue, namely whether a magnetic field could make the
electroweak phase transition more strongly first order by way of increasing the (Gibbs)
free energy difference between the broken and unbroken phases (this effect was recently
discussed also by Giovannini and Shaposnikov [9]). Naively the argument goes as follows:
Above the EW transition temperature any primordial large-scale field must be aligned in
the hypercharge direction, because in the symmetric phase the transverse \( SU(2) \) fields are
screened over distances larger than the inverse of the magnetic mass scale \( m_g \sim g^2 T \). In
the broken phase however, only the projection of the original hypermagnetic field onto the
Maxwell direction (corresponding to the usual photon) is unscreened, while the \(Z\)-component becomes screened by surface currents. Expelling the \(Z\)-component costs in energy, leading to a higher Gibbs free energy in the broken phase. The situation is similar to a superconductor, and since the mass of the Higgs field is relatively small compared to the \(Z\)-mass around the phase transition, one expects it to be typically of type I, i.e. without vortex formation. The \(B_Y\)-field is locally conserved and the expelled component has to go outside the bubble. The expelled field can obviously have a profound effect on the completion of the phase transition. However, we do not expect that to be the case for the bubble nucleation problem.

Because the early universe is an excellent conductor \([10]\), the magnetic flux was frozen in the plasma on large scales. There was however some magnetic diffusion, leading to a straightening out of the entangled field lines at small scales. From the MHD equation one finds that inhomogeneities with scales less than \(L_0\) have decayed before the onset of the EW phase transition, with \(L_0\) given by

\[
L_0 \simeq (t_{\text{EW}}/\sigma)^{1/2} \simeq 2 \times 10^5 \text{GeV}^{-1} ,
\]

where \(\sigma \simeq 10T\) is the conductivity, and here we take \(T \simeq 100\text{GeV}\). A typical size of the EW bubble is expected to be roughly of the order \(100/T - 1000/T\). Therefore the bubble formation takes place in the background of essentially constant field (apart from shortlived microscopic fluctuations), no matter how random it was originally, and for our purposes we may assume \(B_Y^{\text{ext}} \sim \text{const.}\) We parametrize

\[
B_Y^{\text{ext}} = b(T)T^2 ,
\]

and study the consequences as a function of \(b\).

The constraints on \(b\) are uncertain and follow mainly from the microwave background and primordial nucleosynthesis considerations. Assuming that the whole anisotropy measured by COBE is due to magnetic stresses one obtains \([11]\) \(B \leq 3.4 \times 10^{-9}\text{G} (\Omega_0 h_{50}^2)^{1/2}\) if the field can be taken homogeneous at the recombination. This limit implies a superhorizon size coherence length for \(B\) and hence must refer to a mean-root-square field \(\sqrt{\langle B^2 \rangle}\), which is expected to be much smaller than the small scale field and thus does not constrain \(b\) directly. The observed Helium abundance implies \([12]\) that \(B \lesssim 10^{12}\text{G}\) at \(T \simeq 0.1\text{ MeV}\) and at length
scales greater than 10 cm. If the flux is covariantly conserved, the nucleosynthesis limit implies that $B \lesssim T^2$ at scales greater than $10^{-12}L_{\text{horizon}}$. However, due to turbulence in the primordial magnetoplasma we also expect an inverse cascade of magnetic energy to take place, whereby power from small distance scales is transferred into larger scales. This means that the simple scaling law $B \sim B_0 T^2 / T_0^2$ may not hold and that the field at earlier times could at small scales have been even larger than $T^2$. Since the uncertainties are large we shall assume that $b(T) \lesssim 1$, with a slowly varying $b$ just before the time of EW phase transition. There is however no direct bound on the magnetic energy densities as long as it is much smaller than the radiation energy density which only implies $b(T) \ll 10$.

Let us first consider the sphaleron erasure constraint in the MSM in the absence of any external fields. Including the radiative corrections from all the known Standard Model particles, one obtains the effective potential

$$V_{\text{eff}}(\phi, T) \simeq -\frac{1}{2}(\mu^2 - \alpha T^2)\phi^2 - T\delta\phi^3 + \frac{1}{4}(\lambda - \delta\lambda_T)\phi^4 .$$

To the one loop order in the MSM the potential parameters are

$$\mu^2 = \left( \lambda - \frac{44}{3} B \right) v^2 , \quad \lambda = \frac{m_H^2}{2v^2} + \frac{32}{3} B ,$$

where

$$B \equiv \frac{1}{64\pi^2v^4} \left( 3M_Z^4 + 6M_W^4 - 12m_t^4 \right) ,$$

and

$$\alpha = \frac{1}{4v^2} \left( M_Z^2 + 2M_W^2 + 2m_t^2 \right) ,$$

$$\delta = \frac{1}{6\pi v^3} (M_Z^3 + 2M_W^3) ,$$

$$\delta\lambda_T = \frac{1}{16\pi^2v^4} \left( 3M_Z^4 f_B(M_Z, T) + 6M_W^4 f_B(M_W, T) - 12m_t^4 f_F(m_t, T) \right) ,$$

with

$$f_X(M, T) = \ln \frac{M^2}{T^2} + \frac{25}{6} - c_X ,$$

and $c_B \simeq 5.41$ and $c_F \simeq 2.64$. 

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From (3) one can easily find that at the critical temperature, when the two minima of the potential become degenerate
\[ \frac{\phi}{T} = \frac{2\delta}{\lambda - \delta\lambda_T}. \] (8)
In MSM \( \phi/T \) has a maximum value of 0.55 when \( m_H = 0 \) and decreases monotonically as \( m_H \) increases. However, avoiding the sphaleron wash-out requires that
\[ \left( \frac{\phi}{T} \right)_{\text{min}} \simeq 1.0 - 1.5, \] (9)
and therefore it follows from (8) that baryogenesis is not possible in MSM. Note that this conclusion is robust, because one expects that for accepted values of \( \phi/T \gtrsim 1 \) in Eq. (9), the effective potential approach is quite reliable.

Let us now address the question of how the presence of a homogenous large scale magnetic field affects the order of the transition. Consider first the MSM with a background field at the tree level and at the zero temperature. We induce the coupling to the external hyper electromagnetic field by adding a source term for the hyper field tensor \( f_{\mu\nu} \) to the Lagrangian
\[ \mathcal{L} = \mathcal{L}_{\text{MSM}} + \frac{1}{2} f_{\mu\nu} f^{\mu\nu}_{\text{ext}}, \] (10)
where
\[ \mathcal{L}_{\text{MSM}} = |D\Phi|^2 - V(\Phi^\dagger \Phi) - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} + \ldots \] (11)
Here \( D_\mu = \partial_\mu + i \frac{g}{2} \sigma^a A_\mu^a + i \frac{g'}{2} A_\mu^Y \) is the usual covariant derivative, \( F_{\mu\nu}^a \) is the \( SU(2) \) gauge field strength, and we have left out parts corresponding to fermions. They are not relevant here since at the EW scale there are no large scale fermionic condensates.

The vacuum expectation values of the fields are determined from the requirement that the modified action be stationary with respect to variations in \( \Phi \), the \( SU(2) \) gauge fields \( A_\mu^a \) and the hypercharge field \( A_\mu^Y \):
\[ D^2 \Phi - V''(\Phi^\dagger \Phi) \Phi = 0, \]
\[ (D_\mu F^{\mu\nu})^a - \frac{ig}{2} \left( \Phi^\dagger \sigma^a D_\mu \Phi - (D_\mu \Phi)^\dagger \sigma^a \Phi \right) = 0, \]
\[ \partial_\nu (f^{\mu\nu} - f^{\mu\nu}_{\text{ext}}) - \frac{ig'}{2} \left( \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \right) = 0. \] (12)
One can see that there is a simple solution for these equations:

\[
\begin{align*}
    f_{\mu\nu} &= f_{\text{ext}}^{\mu\nu}, \\
    F^{\alpha,\mu\nu} &= 0, \\
    |\Phi| &= 0,
\end{align*}
\]  

which clearly describes the conventional unbroken, or symmetric phase. In the broken phase the neutral Higgs field gets an expectation value \( \Phi = \frac{1}{\sqrt{2}}(0, \phi_0)^T \) and the weak gauge bosons become massive. The situation is more transparent when \( A_3^{\mu} \) and \( A_Y^{\mu} \) are written in terms of the photon field \( A_\mu \) and the neutral gauge boson field \( Z_\mu \). Here we leave out the charged \( W \)-fields; we will however discuss their role later. The equations of motion now become

\[
\begin{align*}
    (\partial_\mu - \frac{ig}{2} Z_\mu)^2 \phi_0 &= V'(\frac{v^2}{2}) \phi_0, \\
    \partial_\nu f_Z^{\mu\nu} - \sin \theta_W \partial_\nu f_{\text{ext}}^{\mu\nu} &= -\frac{ig}{4} \left( \phi^*_0 (\partial_\mu - \frac{ig}{2} Z_\mu) \phi_0 - (\partial_\mu - \frac{ig}{2} Z_\mu)^* \phi_0 \right), \\
    \partial_\nu f_A^{\mu\nu} - \cos \theta_W \partial_\nu f_{\text{ext}}^{\mu\nu} &= 0,
\end{align*}
\]

where \( \tilde{g} = \sqrt{g^2 + g'^2} \). Assume that \( \phi_0 = ve^{i\xi(x)} \), where \( v \) is the constant field corresponding to the broken minimum of \( V \). From Eq. (14) and \( V'(v^2/2) = 0 \) one sees that the \( Z \)-field must be a pure gauge: \( Z_\mu = (2/\tilde{g}) \partial_\mu \xi \), and hence \( f_Z^{\mu\nu} = 0 \). Note that this is a direct consequence of the condensation of a field with a \( Z \)-charge; the Maxwell field does not couple to \( \phi_0 \) and therefore is not similarly constrained. Since \( \partial_\nu f_{\text{ext}}^{\mu\nu} = 0 \), Eq. (16) only tells us that there are no currents for the Maxwell field strength inside the bubble. To find out which value \( f_A^{\mu\nu} \) takes we have to minimize Gibbs free energy. Assuming a pure magnetic field (electric fields are screened inside the plasma so \( E = 0 \)), we obtain in the broken phase

\[
G(\phi, B) = -\mathcal{L} = V(\phi) + \frac{1}{2} B^2 - \cos \theta_W B \cdot B_Y^{\text{ext}},
\]

where \( B_Y^{\text{ext}} \) is the external classical background hypermagnetic field and \( B \) is the Maxwell magnetic field. This is minimized for \( B = \cos \theta_W B_Y^{\text{ext}} \). The Gibbs free energy in the broken and unbroken phases are thus

\[
\begin{align*}
    G_b &= V(\phi) - \frac{1}{2} \cos^2 \theta_W (B_Y^{\text{ext}})^2, \\
    G_u &= V(0) - \frac{1}{2} (B_Y^{\text{ext}})^2.
\end{align*}
\]
It is clear that for very large external fields the unbroken solution (13) will be favoured because there the \( Z \)-component of the field is nonzero. The critical external field \( B_{Y,\text{crit}}^{\text{ext}} \), which marks the transition from one phase to the other is obtained by setting \( G_u = G_b \), that is

\[
\frac{1}{2} \sin^2 \theta_W (B_{Y,\text{crit}}^{\text{ext}})^2 = V(0) - V(\phi_c).
\] (19)

The magnetic fields strength needed for symmetry restoration in vacuum would of course be very large. However, close to the critical temperature the situation is dramatically different, because the free energy difference between the symmetric and broken minimum can be arbitrarily small due to the finite temperature corrections to the effective action. Moreover, since the critical temperature is very sensitive to changes in the free energies, a relatively small magnetic field can induce a large change in \( \phi_c/T_c \). For small magnetic fields the most important corrections to the effective potential are just the usual thermal corrections listed in Eqs. (6 - 7) above. With these corrections only, the analysis above leading to Eq. (19) remains unchanged except for replacing the potential by the corrected form given by Eq. (3).

The polarization effects of the plasma are typically small in comparison with the tree level terms and they have been neglected in the discussion so far. For example, polarization due to one Dirac fermion with charge \( e \) induces a potential term [14]

\[
\delta V_B^f = -\frac{(eB)^2}{12\pi^2} \ln \left( \frac{T}{m_f} \right),
\] (20)

which is a small correction to the tree level energy \( B^2/2 \), given that \( T^2 \gg m_f^2 \gg eB \), which is satisfied with a reasonable margin even for the relatively strong field with \( b \approx 0.3 \). The potentially most important effect is caused by polarization of \( W \)-bosons which become perturbatively unstable for \( eB > M_W^2 \phi^2/v^2 \). At one loop \( W \)-polarization gives rise to the term (see also [15])

\[
\delta_{\text{eff}} \phi^3 \equiv \frac{M^3W^3}{6\pi v^3} + \frac{eB}{2\pi} \left( \sqrt{\frac{M^2W^2\phi^2}{v^2}} + eB - \sqrt{\frac{M^2W^2\phi^2}{v^2}} - eB \right) - \frac{(2eBT)^3}{2\pi} \zeta \left( -\frac{1}{2}, \frac{1}{2} + \frac{M^2W^2\phi^2}{2eBv^2} \right),
\] (21)

where \( \zeta(a, x) \) is the generalized zeta-function. In contrast to the result in [15] we only include corrections from the unscreened transverse modes since the longitudinal modes are
Debye screened. The first and the third terms correspond to the terms in Eq. (6); they represent a small correction, smooth in $B$. The square root terms arise from the two lowest Landau levels, and the second of these contains the potentially dangerous instability [16]. Its existence signals the breakdown of perturbation theory due to appearance of a new massless mode. However, it is known that above the critical field $eB > M_W^2 \phi^2 / v^2$ a $W$-condensate is formed [16] and these terms are stabilized when the theory is expanded around the correct minimum. In that case the one-loop corrections will again be found to be small and smooth in $B$. We conclude that at least for field strengths below the critical value, one only need to retain the leading corrections, all of them small, and not include the square root terms in the computation.

We thus obtain the contours shown in Fig. (1) of how strong field is needed for $\phi_c / T_c$ to equal 1.0 and 1.1. Note that up to $m_H \simeq 100$ GeV the required field strengths are indeed smaller than the corresponding critical fields $b_{\text{inst}}$ leading to $W$-condensation, and
our analysis is expected to be valid. Given the constraint (9) we then conclude that a strong enough first order phase transition for electroweak baryogenesis cannot be ruled out for \( m_H \lesssim 100 \text{ GeV} \), given primordial hypermagnetic field strengths \( B_Y \gtrsim (0.3 - 0.5)T^2 \). Indeed, for a Higgs mass of 80 GeV our result agrees well with [9]. Above \( m_H = 100 \text{ GeV} \) our calculations (nor those of ref. [9]) cannot be trusted.

In our analysis we have implicitly assumed that MSM corresponds to the type I superconductor, i.e. no vortex solutions arise. This conclusion is supported by a crude argument using the correlation lengths of the fields. Indeed, one expects that the vortex phase can be formed only if the \( Z \)-component of the magnetic field can penetrate inside the bubble to a distance greater than the wall width, given by the inverse of the scalar mass in the broken phase. That is, a vortex phase (type II) can form during the transition only if \( m_H(T_c) > m_Z(T_c) \). The border line between the two phases is displayed in Fig. (1). Interestingly, this phase boundary for the vortex phase is very close to the region of instability towards \( W \)-condensation.

Another qualitative criteria for finding the type II region is to consider the stability of scalar field configurations against infrared perturbations in the unbroken phase. During the transition an instability manifests itself in a tachyonic mode of the scalar propagator

\[
D^{-1} = \omega^2 - k^2 - (2n + 1)\frac{\mu^2}{2} B_Y - m_H^2(\phi = 0, T).
\]

Clearly it arises if the critical temperature is lower than the ‘vorticity temperature’ \( T_v \), obtained from

\[
\frac{\mu^2}{2} b T_v^2 = m_H^2(\phi = 0, T) = -\mu^2 + \alpha T_v^2.
\]

As shown in Fig. (1), both this criteria and the one above lead to qualitatively similar solutions for the boundary between the type I and type II behaviour. They both allow for a sizable region in the parameter space, up to \( m_H \lesssim 100 \text{ GeV} \), where our analysis is valid and which is not constrained by the experiment. Of course, these are rough estimates and a numerical analysis of the surface energy is required to find out the phase diagram accurately.

In conclusion, the most obvious effect on the electroweak phase transition from a large scale hyper magnetic background field, namely the screening of the \( Z \)-component in the broken phase, has been shown to strengthen the phase transition considerably. A small change in the background field strength makes a relatively large change in \( \phi_c/T_c \) which is the quantity that determines the sphaleron transition rate. In order to have a strong enough phase transition to make baryogenesis possible we need to assume background field strength
of $B_Y \approx 0.3T^2$. This is a large field but nevertheless still not ruled out by observations. Thus we may conclude that baryogenesis could still be viable within the MSM, provided large primordial fields existed at the time of EW phase transition and that there is large enough CP violation.

One should also bear in mind that the actual transition temperature is not determined by $T_c$ as we assumed above. The action for a finite size bubble includes the bubble wall and it is necessary to find the bounce action in the presence of the background field to predict the nucleation rate. Depending on whether the system acts like a type I or type II superconductor the energy in the wall is either positive or negative. We have argued that the MSM is a type I superconductor for the interesting range of $B$ and $m_H$; therefore the bubble wall energy is positive which will lower the actual transition temperature and make it substantially more strongly first order.

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