Fully-heavy hexaquarks in a constituent quark model

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In this work, we systematically investigate the mass spectra of fully-heavy hexaquarks within a constituent quark model by including the color Coulomb potential, linear confining potential, and spin-spin interactions. Our results show that all of the fully-heavy hexaquarks lie above the corresponding baryon-baryon thresholds, and thus no stable compact one exists. These states may exist as resonances and decay into two fully-heavy baryons easily through the fall-apart mechanism, which can be searched in future experiments.

I. INTRODUCTION

Theoretical and experimental studies on the exotic states beyond the conventional hadrons play essential roles in understanding the nonperturbative properties of quantum chromodynamics (QCD). In the past two decades, researchers have witnessed lots of new hadrons observed by large-scale scientific facilities, and many of them are difficult to interpreted as the conventional hadrons. With this remarkably experimental achievement, theorists also payed plenty of attentions and efforts to study the inner structures of these exotica and provided highly valuable information for experimental searches. More details of these discoveries and recent progresses can be found in the review articles [1–14].

Those new exotica are usually interpreted as compact multi-quark states, loosely bound molecules, kinematic effects, or even the conventional states. Actually, people often have difficulties distinguishing the various explanations if they lie in the similar energy region and have same quantum numbers. Fortunately, the fully-heavy exotics are particularly interesting and less troublesome, where they are far away form the scope of conventional hadrons. Also, without the light meson exchanges, the loosely bound molecular configurations seem to be disfavored. Moreover, the relativistic effects should be small enough owing to the absence of light quark, and the nonrelativistic quark model can describe these fully-heavy systems pretty well. Thus, it is a good place to hunt for the genuine compact exotic states.

Since the observations of $X(6900)$ by the LHCb Collaboration in 2020 [15], the fully-heavy exotics have attracted lots of interests in recent years [16–42]. Also, more structures in the $J/ψ – J/ψ$ and $J/ψ – ψ(2S)$ invariant mass spectrum very recently [43, 44]. These structures are good candidates for the fully-heavy tetraquarks, which have been suggested in the literature. Also, the masses of the ground states for fully-heavy pentaquarks are investigated within the framework of quark model [45, 46].

Unlike the thriving scene of the tetraquarks and pentaquarks, the experimental and theoretical progresses on dibaryons seem relatively few. The deuteron is certainly a well established six-quark state, which is interpreted as a proton-neutron molecular state. Another six-quark state is the $d^*(2380)$ observed by the WASA-at-COSY Collaboration [47, 48], and are investigated by several theoretical groups [49–62]. Moreover, other possible structures are also claimed experimentally, such as the isotensor $ΔN$ dibaryon resonance $D_{21}$ [63, 64] and proton–$Ω$ correlation in heavy-ion collisions [65], and the readers interested in light hexaquarks is referred to the recent review [66, 67]. Owing to the complexities of light few-body systems, the nature of $d^*(2380)$ is a long-standing problem, which seems intractable and will stay in dispute in the near future. A more realistic and intelligent way to looking for the hexaquarks is turning to the heavy systems, where the structures may be picked out more easily by experiments.

There have been several studies on the fully-heavy hexaquarks in the literature, which mainly concentrate on the $Ω_{ccc}Ω_{ccc}$, $Ω_{ccc}Ω_{bbb}$, and $Ω_{bbb}Ω_{bbb}$ systems. Within the lattice QCD method, the authors calculated the mass of various dibaryons and pointed out that the $Ω_{ccc}Ω_{bbb}$, $Ω_{ccc}Ω_{ccc}$, and $Ω_{bbb}Ω_{bbb}$ dibaryons should be below their respective two-baryon thresholds [68–70]. Also, the one-boson-exchange model and QCD sum rule approach suggested that there should exist weakly bound ones [71, 72]. However, several studies within quark models disfavored the bound hexaquarks [73–75] except the work of Ref. [76]. In short, the previous works between quark models and quantum field theory approaches show quite different conclusions. Thus, it is interesting for us to investigate this controversy and find out possible reasons. Moreover, a systematical and solid study in a constituent quark model with the same potential...
and parameters as conventional hadrons are essential to ensure the validity of our exploration.

In this work, we adopt a nonrelativistic constituent quark model to investigate the mass spectra of fully-heavy hexaquarks within compact configurations. This framework composed of potential and parameters have been widely employed to study the conventional and tetraquark states [16, 77–80], and have been proven effective enough for these fully-heavy systems. In the fully-heavy hexaquarks, the relativistic effects should be extremely small and no light meson exchange needs considering, which assures the reliability of results obtained by the nonrelativistic constituent quark model including Coulomb potential, linear confining potential, and spin-spin interactions. We find that all of the fully-heavy hexaquarks lie above the corresponding baryon-baryon thresholds, and thus no stable compact one exists. We hope our results can provide valuable information for future experimental and theoretical studies.

This article is organized as follows. In Section II, we introduce the framework of the nonrelativistic constituent quark model for hexaquarks. The results and discussions for the mass spectra of fully-heavy heavy hexaquarks are given in Section III. The last section is a summary.

II. FRAMEWORK

In the nonrelativistic constituent quark model, the Hamiltonian of the fully-heavy hexaquark systems can be expressed as

$$ H = \left( \sum_{i=1}^{6} m_i + T_i \right) - T_G + \sum_{i<j} V_{ij}(r_{ij}), $$

(1)

where $m_i$ and $T_i$ are the constituent quark mass and kinetic energy of the $i$th quark, respectively; $T_G$ stands for the center-of-mass kinetic energy of the hexaquark system; $V_{ij}(r_{ij})$ corresponds to the potential between the $i$-th and $j$-th quark, which includes short-range one-gluon-exchange interaction and long-range linear confinement. The explicit formula of $V_{ij}(r_{ij})$ can be written as [78]

$$ V_{ij}(r_{ij}) = V_{ij}^{OGE}(r_{ij}) + V_{ij}^{Conf}(r_{ij}), $$

(2)

with

$$ V_{ij}^{OGE} = \frac{\alpha_{ij}}{4} (\lambda_i \cdot \lambda_j) \left\{ \frac{1}{r_{ij}} - \frac{\sigma_i \cdot \sigma_j}{2 \pi^{3/2}} \cdot \frac{4}{3m_i m_j} (\sigma_i \cdot \sigma_j) \right\}, $$

(3)

and

$$ V_{ij}^{Conf}(r_{ij}) = -\frac{3}{16} (\lambda_i \cdot \lambda_j) \cdot b r_{ij}. $$

(4)

The relevant parameters are shown in Table I. With these parameters, the mass spectra of fully-heavy mesons, baryons, and tetraquarks have been described well in previous works [16, 77–80]. Also, some calculated masses of fully-heavy conventional hadrons are listed in Table II for reference. Thus, it is suitable to adopt the same framework to investigate the fully-heavy hexaquark systems.

To solve this Hamiltonian, we also need to construct the fully antisymmetric color-spin-flavor-orbital wave functions according to the Pauli exclusion principle. Since the charm and bottom quarks are not identical particles, the flavor parts of these systems are always trivial, that is, symmetric in the identical subsystems and no permuting symmetry between charm and bottom quarks. Also, the orbital parts are symmetric for the $S$-wave ground states. Then, the color-spin wave functions should be antisymmetric in the charm or bottom subsystem. All the possible configurations for the $S$-wave fully-heavy hexaquark systems are listed in Table III. The explicit forms for color-spin wave functions can be obtained with the help of Clebsch-Gordan coefficients of groups $SU(N)$ and $S_6$ after tedious calculations [57, 82–84]. With the color-spin-flavor wave functions, one can calculate the matrix elements of operators $\lambda_i \cdot \lambda_j$ and $\lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j$ in the Hamiltonian. Here, we present the final elements for all configurations in Table IV for reference.

The trail orbital wave function for a hexaquark state in the coordinate space can be expanded by a series of Gaussian functions,

$$ \psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \mathbf{r}_4, \mathbf{r}_5, \mathbf{r}_6) = \sum_{\ell} C_{\ell} \prod_{i=1}^{6} \left( \frac{m_i \omega_i}{\pi} \right)^{3/4} \exp \left[ -\frac{m_i \omega_i}{2 \pi} r_i^2 \right], $$

(5)

which is usually adopted to study the compact multiquark systems [78, 85, 86]. Some useful overlaps for Gaussian functions are shown as follows

$$ \left( \prod_{j=1}^{6} \phi(\omega_i, \mathbf{r}_j) \right) \sum_{i=1}^{6} T_i - T_G \left| \prod_{k=1}^{6} \phi(\omega_i, \mathbf{r}_k) \right| = 3840 \left( \frac{\omega_i \omega_e}{\omega_e + \omega_i} \right)^{11/2}, $$

(6)

\[
\begin{array}{l}
| T_i - T_G | \prod_{k=1}^{6} \phi(\omega_i, \mathbf{r}_k) \right| = 3840 \left( \frac{\omega_i \omega_e}{\omega_e + \omega_i} \right)^{11/2}.
\end{array}
\]

TABLE I: Relevant parameters in the constituent quark model.

| Parameter | Value     |
|-----------|-----------|
| $m_i$ (GeV) | 1.483     |
| $m_b$ (GeV) | 4.852     |
| $\alpha_{cc}$ | 0.5461    |
| $\alpha_{bb}$ | 0.4311    |
| $\alpha_{bc}$ | 0.5021    |
| $\sigma_{cc}$ (GeV) | 1.1384    |
| $\sigma_{bb}$ (GeV) | 2.3200    |
| $\sigma_{bc}$ (GeV) | 1.3000    |
| $b$ (GeV$^2$) | 0.1425    |

The explicit forms for color-spin wave functions can be obtained with the help of Clebsch-Gordan coefficients of groups $SU(N)$ and $S_6$. With the color-spin-flavor wave functions, one can calculate the matrix elements of operators $\lambda_i \cdot \lambda_j$ and $\lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j$ in the Hamiltonian. Here, we present the final elements for all configurations in Table IV for reference.
TABLE II: Masses of fully-heavy conventional hadrons. Experimental data are taken from PDG [81]. The units are in MeV.

| State | η_0 | J/ψ | B_0 | B_0’ | η_0 | Y | Ω_0c | Ω_0cb | Ω_0b | Ω_0bb | Ω_0bbb |
|-------|-----|-----|-----|------|-----|---|------|-------|------|--------|---------|
| Mass  | 2983 | 2984 | 3097 | 6271 | 6326 | 9390 | 9460 | 4828  | 8047  | 8070   | 11248   |
| Experiments | 2983 | 3097 | 6271 | 9390 | 9460 | 4828 | 8047 | 8070  | 11248 | 14432 |

TABLE III: All possible configurations for S-wave fully-heavy hexaquark systems. The subscripts and superscripts stand for the spin and color types, respectively. The braces { } are adopted for the subsystems with symmetric flavor wave functions.

| System | J^P | Configuration |
|--------|-----|---------------|
| cccccc | 0^+ | [ccc]_{i}^{3} [c]_{i}^{j=2} |
| ccccb  | 0^+ | [ccc]_{i}^{3} [bb]_{j}^{3} |
| cccbb  | 1^+ | [ccc]_{i}^{3} [bb]_{j}^{1} |
| cccbb  | 2^+ | [ccc]_{i}^{2} [bb]_{j}^{3} |
| cccbb  | 3^+ | [ccc]_{i}^{2} [bb]_{j}^{1} |
| bbbcc  | 0^+ | [bb]_{i}^{2} [cc]_{j}^{3} |
| bbbbc  | 1^+ | [bb]_{i}^{2} [cc]_{j}^{1} |
| bbbbc  | 2^+ | [bb]_{i}^{2} [cc]_{j}^{2} |
| bbbbc  | 3^+ | [bb]_{i}^{2} [cc]_{j}^{3} |

III. MASS SPECTRA AND DISCUSSIONS

In this work, we adopt the same variational parameters \( \omega_c \) and \( n \) as previous work [78] to solve the generalized eigenvalue problem numerically. With these parameters, we can obtain stable mass spectra of fully-heavy tetraquarks, and also fully-heavy hexaquarks here. Certainly, these variational parameters are introduced just for a numerical calculation and do not affect the final results of constituent quark model when the numerical procedure is convergent and stable enough [87]. The predicted masses for \( S \)-wave fully-heavy hexaquarks are listed in Table V and Figure 1.

It can be seen that all of the fully-heavy hexaquarks are predicted to lie above the corresponding thresholds. These results are consistent with other quark model calculations [73–75] but contradict with conclusions of lattice QCD [68–70], one-boson exchange model [71], and QCD sum rule [72]. Instead of the absolute masses, whether above or below the threshold is just a qualitative conclusion, which can not be simply boiled down to the uncertainties of models and numerical calculations. Our results together with previous quark model works indicate that the potential with Coulomb potential, linear confining potential, and spin-spin interactions can hardly give a stable compact fully-heavy hexaquark. Probably, more interactions or mechanism are included in lattice QCD approach, which lead to this difference. More theoretical works and future experimental data will help us to resolve this puzzle.

With the obtained wave functions, we can also estimate the expectations of \( \langle r_{ij}^2 \rangle^{1/2} \) for \( cc, cb, \) and \( bb \) subsystems. We find that these expectations are similar in different configurations, and are about 0.53 ~ 0.55, 0.43 ~ 0.44, and 0.29 ~ 0.30 fm for \( cc, cb, \) and \( bb \) subsystems, respectively. These relatively small distances show typical features of the compact multiquarks, which are significantly different from the loosely bound molecular picture. Moreover, for \( cccbb, cccbbb, \) and \( bbbbcc \) systems, some configurations with same

\[
\left( \psi(\omega_i, r_i, r_j) \left| \frac{1}{r_{ij}} \right| \psi(\omega_j, r_i, r_j) \right) = 2 \sqrt{\frac{m_{ij} (\omega_i \omega_j)^{3/2}}{\pi \left( \omega_i + \omega_j \right)^{5/2}}}, \quad (7)
\]

\[
\left( \psi(\omega_i, r_i, r_j) \left| e^{-\sigma^2 r^2} \right| \psi(\omega_i, r_i, r_j) \right) = \frac{m_i (\omega_i + \omega_j)}{m_i (\omega_i + \omega_j + \sigma_{ij})^{1/2}}, \quad (8)
\]

\[
\left( \psi(\omega_i, r_i, r_j) \left| r_{ij} \right| \psi(\omega_j, r_i, r_j) \right) = 2 \sqrt{\frac{m_{ij} (\omega_i \omega_j)^{3/2}}{\pi m_{ij} (\omega_i + \omega_j + \sigma_{ij})^{5/2}}}, \quad (9)
\]

where \( \psi(\omega_i, r_i, r_j) \equiv \psi(\omega_i, r_i)\phi(\omega_j, r_j), m_{ij} = m_i m_j/(m_i + m_j). \)

With the full wave functions and all the matrix elements involved in the Hamiltonian, the masses without a mixing mechanism can therefore be calculated by solving the generalized eigenvalue problem straightforwardly

\[
\sum_{\ell=1}^{n} (H_{\ell^P} - E N_{\ell^P}) C_{\ell^P} = 0, \quad (\ell = 1, 2, \ldots, n), \quad (10)
\]

where the \( H_{\ell^P} \) are the matrix elements of the total Hamiltonian, \( N_{\ell^P} \) are the overlap matrix elements of the Gaussian functions arising from their nonorthogonality, \( E \) stands for the mass, and \( C_{\ell^P} \) is the eigenvector corresponding to the coefficients of the orbital wave function for a fully-heavy hexaquark. Moreover, for a given hexaquark system, different configurations with same \( J^P \) can mix with each other in principle. The mixing effects are taken into account and discussed in present calculations, and then the final mass spectra and wave functions are obtained by diagonalizing the mass matrix of these configurations.
for several fully-heavy hexaquarks. Usually, one expects that the mass of higher $J$ state should be larger than that of the lower $J$ one for a given spin multiplicity, which is also confirmed by the spectroscopy of conventional hadrons both theoretically and experimentally. In the literature [88–91], the authors discussed the possibilities of inverted mass hierarchy for conventional hadrons by considering specific dynamical

$J^P$ can mix with each other. The mixing effects arise from the spin-spin interaction, and then are highly suppressed by the masses of heavy quarks. Hence, it is inevitable that these mixtures should be extremely small, and our numerical results confirm this expectation.

An interesting discovery is the inverted mass hierarchy
FIG. 1: Mass spectra for the fully-heavy hexaquarks. The solid lines stand for the predicted masses of different systems, and dashed lines are their corresponding thresholds.

mechanism, however, these predictions are not verified by experiments until now. In the fully-heavy hexaquarks, the lightest states for the $cccccb$, $ccccbb$, $bbbbbc$, and $bbbbcc$ systems have the highest total angular momentum. Moreover, for the pure configurations, when the total angular momentum increases, the masses decrease. This inverted mass hierarchy is due to the symmetry constraint, where the color-spin wave functions are limited in specific forms and jointly act on the fine splittings. Especially, the four pure configurations $|\frac{1}{2}; \frac{1}{2}\rangle$ with $S = 0, 1, 2, 3$ have the same mass because no residual interaction is left between the $ccc$ and $bbb$ subsystems. It can be seen that the exotic states have more complicated color structures than that of conventional hadrons, and thus can perform more particular spectrum. We hope that the future experiments can test this inverted mass hierarchy of fully-heavy hexaquarks.

We can also investigate the mass differences between hexaquark states and their corresponding thresholds versus different systems. The mass differences between the lowest hexaquarks and thresholds for different systems are plotted in Figure 2. It can be found that the $cccbab$ system has lowest mass difference while the $cccccc$ and $bbbbb$ systems have larger mass differences. Compared with the $cccccc$ and $bbbbb$ systems, the $cccbab$ system seems to be more asymmetric. Also, if one splits a fully-heavy hexaquark into two three-quark subsystems, it is easy to see that the $cccbab$ system has the largest mass ratio between these two subsystems. Actually, in previous study on tetraquarks [92], we also found that the systems with larger mass ratios tend to be stable. Our present results indicate that the fully-heavy hexaquarks with lower symmetry and larger mass ratios are more likely to stabilize. If one keeps increasing the mass ratios, the fully-heavy hexaquarks will become heavy-light systems. Thus, it is reasonable to speculate that there may be some stable heavy-light hexaquarks below the corresponding thresholds. More precise calculations for the heavy-light hexaquarks within our framework are needed to verify or deny this conjecture.

Furthermore, it should be emphasized that, even if a exotic state is not bound, it may also subsist as a resonance with finite decay width and be observed by future experiments. These fully-heavy hexaquarks can decay into the two fully-heavy baryons by fall-apart mechanism. The precise decay widths depend on the transition operator, wave functions of initial and final states, and phase space, which are all model dependent. Without any experimental data as a benchmark, it is hard to estimate the total widths of these resonance
theoretically at present.

IV. SUMMARY

In this paper, we adopt the nonrelativistic constituent quark model to investigate the S-wave fully-heavy hexaquarks systematically. The mass spectra are obtained by solving the Hamiltonian including the Coulomb potential, confining potential, and spin-spin interactions. All of the fully-heavy hexaquarks are predicted to lie above the corresponding baryon-baryon thresholds, and thus no stable binding one exists. Then, these fully-heavy hexaquarks can subsist as resonances and may easily decay into the fully-heavy baryons through the fall-apart mechanism. Our present results are consistent with other quark model calculations but different with the conclusions of lattice QCD, QCD sum rule, and consistent with other quark model calculations. We hope our present predictions can provide helpful information for future experimental searches.

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