A POSSIBLE CORRELATION BETWEEN THE GASEOUS DRAG STRENGTH AND RESONANT PLANETESIMALS IN PLANETARY SYSTEMS

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ABSTRACT

We study the migration and resonant capture of planetesimals in a planetary system consisting of a gaseous disk analogous to the primordial solar nebula and a Neptune-like planet. Using a simple treatment of the drag force, we find that planetesimals are mainly trapped in the 3:2 and 2:1 resonances and that the resonant populations are correlated with the gaseous drag strength in a sense that the 3:2 resonant population increases with the stronger gaseous drag, but the 2:1 resonant population does not. Since planetesimals can lead to the formation of larger bodies similar to asteroids and Kuiper Belt objects, the gaseous drag can play an important role in the configuration of a planetary system.

Subject headings: celestial mechanics — circumstellar matter — planetary systems

1. INTRODUCTION

The discovery of more than 160 planets orbiting other stars, extrasolar planets, opened a window of astronomy that could eventually provide insight to understanding the fundamental questions about our own solar system. Most of these extrasolar planets (exoplanets) were detected by the measurements of stellar radial velocities of their host star via the Doppler effect, and several more recently detected exoplanets were discovered through transit events. For example, the Optical Gravitational Lensing Experiment (OGLE) project has detected five transit exoplanets (Konacki et al. 2003; Bouchy et al. 2005), and the Trans-Atlantic Exoplanet Survey (TrES) team (using 10 cm telescopes) has discovered one transit planet (Alonso et al. 2004). All of these exoplanets were later confirmed by spectroscopic follow-up. Future observations using both methods will lead to the discoveries of additional systems, thereby improving the statistical significance of the population.

Among the many interesting properties exhibited by these planetary systems are the large range of masses, orbital periods, and orbital eccentricities (Jiang et al. 2006). In particular, the relations between the orbital periods of different planets have been noted and are usually connected with the mean motion resonances. As an example of extrasolar multiple planetary systems that show the resonances, Ji et al. (2003) confirmed that the 55 Cancri planetary system is indeed in 3:1 mean motion resonance by both numerical simulations and secular theory. The GJ 876 and HD 82943 planetary systems are probably in 2:1 resonance, as studied by Laughlin & Chambers (2001), Kinoshita & Nakai (2001), Gozdziewski & Maciejewski (2001), and also Ji et al. (2002). Moreover, the periods of small bodies in the solar system, such as asteroids and Kuiper Belt objects (KBOs), are also seen to have similar connections with resonances. Thus, such types of dynamics may also play a role in the configuration of KBOs.

On the other hand, it has been shown that disks can affect the orbital evolution of test particles within planetary systems (Jiang & Yeh 2004a, 2004b; Yeh & Jiang 2006). In particular, Jiang & Yeh (2004c) proposed a possible model of resonant capture for proto-KBOs driven by the gaseous drag, finding that many test particles can be captured into the 3:2 resonance, consistent with the observational results (Luu & Jewitt 2002).

To ensure that the gaseous drag influences the dynamics of planetesimals, sufficient gas must be present (about 0.01 $M_\odot$ as suggested in Nagasawa et al. 2000) when the planetesimals are already formed. It is likely that molecular gas is present around the nearby star epsilon Eridani as found by Greaves et al. (1998), however, only an upper limit of 0.4 $M_\odot$ in molecular gas is inferred from CO observations. This is to be compared to the primordial solar nebula, where the minimum-mass solar nebula is about 0.026 $M_\odot$.

Although there is evidence for a small amount of gas present in planetary systems, there may have been much more gas in the past. The kilometer-sized planetesimals representing protoasteroids were formed and likely influenced by the gaseous drag. During this process, the gaseous component is gradually depleted from a more massive primordial nebula to the current limited molecular gas.

Whether the above scenario is viable would be related to the formation timescale of kilometer-sized planetesimals and the depletion timescale of the gaseous disks. Cuzzi et al. (1993) argued that 10–100 km sized objects can be formed in about $10^6$ yr. Furthermore, observations by Kenyon & Hartmann (1995) and Haisch et al. (2001) show that at an age of ~$10^6$ yr, most low-mass stars are surrounded by optically thick disks. However, by the age of $10^7$ yr, no such disks are detected. It is therefore possible that there is a phase in the evolution of the system in which planetesimals are already formed, while the gaseous disk is not yet depleted. We investigate this phase to examine the effect of gaseous drag for the planetesimal dynamics in this paper.

Since the kilometer-sized planetesimals will grow into asteroids, KBOs, or even planets, their distribution and orbital evolution are extremely important for understanding the history of planetary systems. Based on a disk model analogous to the primordial solar nebula, we study the resonant capture of planetesimals under the influence of a gaseous disk for a given planetary system. In particular, we study the possibility for correlations between the gaseous drag strength and the resonant populations and examine the possible parameter space for which the drag-induced resonant capture could explain the resonant populations of a planetary system.

We present our model and assumptions in § 2. Section 3 describes the evolution of planetesimals and the stability tests. Finally, we discuss the results and conclude in § 4.

2. THE MODEL

We consider the motion of a test particle influenced by the gravitational force from the central star, a planet, and the protostellar
disk. This disk, which is mainly composed of gas, exerts a drag on the planet and the test particles. These test particles are envisioned to represent planetesimals such as protoasteroids and proto-KBOs. We assume that the mass of the central star \( \mu_1 = 1 M_\odot \), and the planet’s mass is taken to be similar to Neptune’s mass, i.e., \( \mu_2 = 5 \times 10^{-3} M_\odot \). We assume the test particle represents a planetesimal with a radius of ~10 km, with a mass assumed to be about \( \mu_3 = 4.3 \times 10^{-15} M_\odot \) (when the density is similar to Pluto’s). The coordinates of the central star, the planet, and the planetesimal are \((0,0),(x,y),\) and \((\xi,\eta)\), respectively. In this paper we consider the general situation in which the planet is free to move on any non-circular orbit.

### 2.1. The Units

In this paper, the unit of mass is \( M_\odot \) and the unit of length is 30 AU. Since we set the gravitational constant \( G = 1 \), the total simulation time is \( 3.8 \times 10^3 \), which would correspond to \( 10^7 \) yr. All simulations start from \( t = 0 \) and terminate at \( t = t_{\text{end}} = 123,200\pi = 3.8 \times 10^7 \).

### 2.2. The Equations of Motion

In this paper we only consider the coplanar case, so all motions are in a two-dimensional plane. The equations of motion are

\[
\frac{d^2x}{dt^2} = -\frac{\mu_1 x}{r^3} - \frac{\alpha}{\mu_2} \left( \frac{dx}{dt} - v_x \right) \rho_N - \frac{dV_N \cdot x}{dr \cdot r},
\]

\[
\frac{d^2y}{dt^2} = -\frac{\mu_1 y}{r^3} - \frac{\alpha}{\mu_2} \left( \frac{dy}{dt} - v_y \right) \rho_N - \frac{dV_N \cdot y}{dr \cdot r},
\]

\[
\frac{d^2\xi}{dt^2} = -\frac{\mu_1 \xi}{r^3 \cdot r_1} + \frac{x - \xi}{r^2 \cdot r_1^2} - \frac{\alpha}{\mu_3} \left( \frac{d\xi}{dt} - v_\xi \right) \theta_T - \frac{dV_T \cdot \xi}{dr_1 \cdot r_1},
\]

\[
\frac{d^2\eta}{dt^2} = -\frac{\mu_1 \eta}{r^3 \cdot r_1} + \frac{y - \eta}{r^2 \cdot r_1^2} - \frac{\alpha}{\mu_3} \left( \frac{d\eta}{dt} - v_\eta \right) \theta_T - \frac{dV_T \cdot \eta}{dr_1 \cdot r_1},
\]

where

\[
r^2 = x^2 + y^2,
\]

\[
r_1^2 = \xi^2 + \eta^2,
\]

\[
r_2^2 = (x - \xi)^2 + (y - \eta)^2.
\]

The equations of motion in terms of \( x \) and \( y \) describe the planet’s orbit, and the equations of \( \xi \) and \( \eta \) give the orbit of a test particle. In equation (1), \( V_N \) is the disk potential for the planet. In other words, we define \( V_N = V(r) \) to be the disk potential at the planet’s location \((x,y)\), where \( r \) is given by equation (2) and \( \rho_N = \rho(r) \) is the disk density at the planet’s location. Similarly, \( V_T = V(r_1) \) is the disk potential at the test particle’s location \((\xi,\eta)\), where \( r_1 \) is given in equation (3) and \( \rho_T = \rho(r_1) \) is the disk density at the location of the test particle.

The disk is represented by an annulus with inner edge \( r_i \) and outer edge \( r_o \), where \( r_i \) and \( r_o \) are assumed to be constants. We choose \( r_i = 1.5 \) and \( r_o = 5/3 \) in this paper. Since we set the unit of length to be 30 AU, \( r_i \) corresponds to the location where Jupiter was approximately formed, and \( r_o \) corresponds to the outer edge of the Kuiper Belt. Thus, our choice of inner and outer edges for the gaseous disk is based on the possible properties of the protoplanet.

The density profile of the disk is taken to be of the form \( \rho(r) = c/r^p \), where \( r \) is the radial coordinate as in equation (2), \( c \) is a constant completely determined by the total mass of the disk, and \( p \) is a natural number. In this paper we set \( p = 2 \) based on the theoretical work by Lizano & Shu (1989). This assumption is consistent with one of the models of the Vega debris disk (Su et al. 2005). Hence, the total mass of the disk is

\[
M_d = \int_0^{2\pi} \int_{r_i}^{r_o} \rho(r') r' dr' d\phi = 2\pi c(\ln r_o - \ln r_i).
\]

In this paper, the disk mass is assumed to be \( M_d = 0.01 M_\odot \), which is the same order as the minimum-mass solar nebula (0.026 \( M_\odot \)). It is also consistent with the observations by Beckwith et al. (1990) in which the disks have masses ranging from 0.001 to 1 \( M_\odot \). The disk’s gravitational potential and force can be calculated by elliptic integrals as in Jiang & Yeh (2004).

In equation (1), the Keplerian velocity of the gaseous material \( \sqrt{\frac{g}{r}} \) is given in equation (3) and \((0,0),(x,y),\) and \((\xi,\eta)\) correspond to the location where Jupiter corresponds to the location where Jupiter was approximately formed, and \( r_o \) corresponds to the outer edge of the Kuiper Belt. Thus, our choice of inner and outer edges for the gaseous disk is based on the possible properties of the protoplanet.

\[
F = kVg,
\]

where \( k < 0 \), \( V \) is the particle’s velocity in the inertial frame, and \( g \) is a scalar function of its position and velocity.

For the aerodynamic friction force, we use (see Youdin & Shu 2002; Youdin & Chiang 2004) and is given by

\[
F = \frac{1}{2m} C_D A \rho v^2,
\]

where \( C_D \) is the drag coefficient, \( m \) and \( A \) are the particle’s mass and cross section, \( \rho \) is the gaseous density, and \( V \) is the magnitude of the particle’s velocity relative to the gas.

The Epstein drag force per unit mass has also been adopted (see Youdin & Shu 2002; Youdin & Chiang 2004) and is given by

\[
F = \frac{4\pi}{3m} \rho g c_s v^2,
\]

where \( \rho_g \) is the mass density of the gas and \( c_s \) is the sound speed. The particle’s mass and size are \( m \) and \( a \), respectively. The relative velocity between a particle and the gas is \( v \).

Our disk is mainly composed of gas, and we wish to approximate the disk’s gaseous drag acting on the planetesimals and planets. Motivated by the above expressions for the drag, we employ a unified formula, which we use for both planetesimals and the planet. Thus, we assume that the drag force per unit mass is

\[
F = -\frac{\alpha \rho}{m} V,
\]

where \( \alpha \) is assumed to be a constant, \( V \) is the particle’s velocity relative to the Keplerian motion of the gaseous disk, and \( \rho \) is the gas density.

Although our drag differs from the Epstein drag, we can see that \( \alpha \) plays the role of the \( 4\pi c_s a^2/3 \) term from the Epstein drag. To obtain an estimate of the value of \( \alpha \), we (i) use equation (2) of Youdin & Chiang (2004) to calculate \( c_s \) and (ii) use the radius of our planetesimal particle, 10 km, as the value of \( a \) and find that \( 4\pi c_s a^2/3 \) is about \( 8 \times 10^{-16} \). We thus define
α_E ≡ 8 × 10^{-16}. Because of the simple treatment of the disk drag and the uncertainty in α, we adopt three different values of α: α_E, 5α_E, and 25α_E.

For the planet, the drag force in the x-direction is -(α/μ_L)(dx/dt - v_x)ρ_N, and thus this term appears in equation (1a). Similarly, there is a term -(α/μ_L)(dy/dt - v_y)ρ_N that appears in equation (1b). For the planetesimals, the drag force in the ξ- and η-directions are -(α/μ_L)(dξ/dt - v_ξ)ρ_T and -(α/μ_L)(dη/dt - v_η)ρ_T in equations (1c) and (1d), respectively.

2.4. The Simulations

A number of planetesimals (300) are randomly distributed in a belt region 1.1 ≤ r ≤ 1.8 with a uniform number density, where r is the radial coordinate. Assuming that one planetesimal is located at (ξ, η), then we set its initial velocity (v_ξ, v_η) to be v_ξ = -(1/r_0)^1/2 sin θ_T and v_η = (1/r_0)^1/2 cos θ_T, where θ_T = tan^{-1}(η/ξ). Thus, the planetesimals are initially in circular motion.

To study the outcome of different drag strengths, we adopt three different values of the drag coefficient in our simulations, that is, α = α_E in model A, α = 5α_E in model B, and α = 25α_E in model C.

The planet is always located at (1, 0) initially and was set in circular motion initially for models A, B, and C. However, in order to test the influence of the planet’s orbital eccentricity, we also consider a model (D) in which the planet moves on an initial eccentric orbit of e = 0.3 with the drag coefficient chosen to be α = 5α_E.

To study the resonance captures, the orbital semimajor axis and eccentricity for all the planetesimals and the planet are calculated. To make it clear, x, y, represent the coordinates of any particle. From Murray & Dermott (1999), we have

\[ r^2 = x^2 + y^2, \]
\[ v^2 = \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2. \]

Let \( h^2 = (x, dy/dt - y, dx/dt)^2 \); then semimajor axis \( a \) and eccentricity \( e \) are defined by

\[ a = \left( \frac{2}{r} - \frac{v^2}{\mu_1} \right)^{-1}, \]
\[ e = \sqrt{1 - \frac{h^2}{\mu_1 a}}. \]

Based on the formula in Murray & Dermott (1999) and Fitzpatrick (1970), the 2:1 resonant argument \( \phi_{2:1} \) and 3:2 resonance argument \( \phi_{3:2} \) are also calculated as

\[ \phi_{3:2} = 3λ_t - 2λ_N - ω_t, \]
\[ \phi_{2:1} = 2λ_t - λ_N - ω_t, \]

where \( λ_t \) is the mean longitude of a planetesimal’s orbit, \( λ_N \) is the mean longitude of the planet’s orbit, and \( ω_t \) is the longitude of the pericenter of a planetesimal’s orbit.

The number of particles in a particular resonance at time \( t_i \) is defined to be the total number of particles with the difference between the maximum and minimum resonance arguments less than 180° during \( t_{i-1} < t < t_i \). We set \( t_0 = 0 \) and \( t_i = t_{i-1} + 800π \), where \( i = 1, 2, \ldots, 154 \).

3. NUMERICAL RESULTS

3.1. Evolution of Planetesimals

The evolution of planetesimals on the x-y plane in the simulation of model A is illustrated in Figure 1. The panel labeled 0 in Figure 1 shows the initial positions of all 300 planetesimals. Based on this representation, it is difficult to discern the change in the distribution until the tenth panel. However, the histograms of particle number versus the radial distance reveal the variation more clearly (see Fig. 2). In the second panel of Figure 2, a gap starts to develop, and this gap becomes deeper and wider in the following panels. Finally, the gap encompasses the range from 1.5 to 1.7 after the eighth panel. The gap can also been seen in panels 10 and 11 of Figure 1 and separates the planetesimal distribution into two rings. The outer ring (from \( r = 1.7 \) to 1.8) is thinner than the inner one (from \( r = 1.1 \) to 1.5), as one can see from Figures 1 and 2.

In Figure 3 we plot the final planetesimal distribution of model A in the a-e plane in the left panel and the color contour of it in the right panel. There is a population with nearly circular orbits from \( a = 1.7 \) to 1.8. They are the main population of the outer ring of Figures 1 and 2, as one can estimate the total number from the color bar. From both panels, as well as the color bar, we find that some planetesimals are associated with 2:1, 3:2, 7:5, 4:3, 5:4, and 6:5 resonances and that there are more planetesimals in 3:2 (at \( a = 1.33 \)) than those in 2:1 resonance (at \( a = 1.6 \)). However, in order to confirm whether a given planetesimal is captured into a particular resonance, the particular resonance argument has to be calculated during the whole simulation. Because the influence of the first-order resonances, 2:1 and 3:2, are much stronger than the others, we calculate the 2:1 resonance argument.
Fig. 2.—Model A: histograms of particle distributions in the radial coordinate. The time between successive panels corresponds to $11,200\pi$. 

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Fig. 2.—Model A: histograms of particle distributions in the radial coordinate. The time between successive panels corresponds to $11,200\pi$.
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and 3:2 resonance arguments, i.e., \( \phi_{2:1} \) and \( \phi_{3:2} \). Figure 3 also shows that there is a nonresonant population (from \( a = 1.4 \) to 1.5) with nearly circular orbits. This population, together with all the resonant populations between 1.1 and 1.5, becomes the main population of the inner wider ring of Figures 1 and 2. The orbital eccentricities of the 2:1 resonant population are larger, and thus these planetesimals have a wider range of semimajor axes. Although this range might cover both rings and also the gap of Figures 1 and 2, this population is small, so it does not change the distribution.

The evolution of the planetesimals' distribution for model B is plotted in Figure 4. The evolution proceeds more rapidly than in model A. This could be due to a larger drag force for this model. For example, in the second panel there is already a gap in the belt. The planetesimals continue to migrate inward, and the gap becomes wider as shown in panels 3 and 4. In the fifth panel, the planetesimals are seen to distribute into two parts, i.e., an inner ellipse and an outer ring. The inner ellipse precesses slowly starting from panel 6 until the end of the simulation. This behavior is confirmed in the histograms in Figure 5 as the gap begins to form in panel 2. The inward migration is also evident as the particle number increases between \( r = 1 \) and 1.5.

It can be seen in Figure 6 that some planetesimals associated with the 2:1 and 3:2 resonances cluster around \( a = 1.6 \) and 1.33. All the planetesimals associated with the 3:2 resonance have almost exactly the same semimajor axes, and moreover, there is no other nonresonant planetesimals around \( a = 1.33 \), explaining why the inner ellipse is so thin in Figure 4.

In model C, the largest drag force is considered. The planetesimals' inward migrations are so fast that there is already a gap in the first panel of Figure 7. A clear inner ellipse is formed in panel 2, and it precesses slightly in the following panels. With that small precession, the whole distribution seems to be in a steady state. The histograms of Figure 8 also confirm the existence of the gap. We point out that there are no planetesimals in the 2:1 resonance for this model (see Fig. 9).

Although the planet's orbital eccentricity is taken to be zero initially, its eccentricity is about 0.02 during the simulations for models A, B, and C. In model D, the strength of the drag force is assumed to be moderate, i.e., the same as the one in model B, but the initial orbital eccentricity of the planet is 0.3. Because of the drag, the inward migrations are still significant, and the gap immediately appears in the first and second panels of Figure 10. However, planetesimals continue to migrate inward from panels 3.
Fig. 5.—Model B: histograms of particle distributions in the radial coordinate. The time between successive panels corresponds to 11,200\pi.

Fig. 6.—Model B: particle distribution in the $a$-$e$ plane at the end of simulation, i.e., $t = 123,200\pi$. The right color panel shows the number of particles in the particular area.
one to carry out many tests quickly. In this standard simulation, all 30 planetesimals migrate inward and are captured into the 3:2 resonance.

To check the stability of the orbital evolution to the initial orbital eccentricities of planetesimals, a simulation was performed in which all planetesimals have initial orbital eccentricities \( e = 0.01 \) with all other settings remaining the same as the standard one. These 30 planetesimals migrate inward, of which 27 are captured into the 3:2 resonance. Because the difference with the standard case is only three planetesimals, which is one order of magnitude less than the total number 30, we conclude that the system is stable in terms of the initial orbital eccentricities of planetesimals. For the results in which 300 planetesimals are used, we are confident that the result would be similar if the planetesimals’ initial orbital eccentricities were changed slightly.

To test the stability in terms of the disk mass, i.e., the value of \( M_d \), we ran two simulations, one with \( M_d = 0.009 \, M_\odot \), and another one with \( M_d = 0.011 \, M_\odot \), while keeping all other settings the same as the standard one. Note that \( M_d = 0.01 \, M_\odot \) in the standard case. For both simulations, all the 30 planetesimals migrate inward and are captured into the 3:2 resonance. Thus, the system is stable to perturbations in the disk mass.

To check the stability in terms of the location of the disk’s edges, i.e., the value of \( r_i \) and \( r_o \), four simulations were carried out with \( r_i = 1/5 - 0.01, r_i = 1/5 + 0.01, r_o = 5/3 - 0.01 \), and \( r_o = 5/3 + 0.01 \), while all other settings remain the same as the standard case. All the 30 planetesimals migrate inward and are captured into the 3:2 resonance for all four simulations. Hence, the system is stable to perturbations to the disk inner and outer edges.

4. DISCUSSIONS AND CONCLUSIONS

In this paper we have investigated the effect of different strengths of the gaseous drag on the resonant capture into the 3:2 and 2:1 resonances. For a small drag force as in model A, there are about 17\% of planetesimals that are captured into the 3:2 resonance and about 13\% into the 2:1 resonance. For a moderate drag force as in model B, the fraction of planetesimals captured into 2:1 is still about 13\%, but the fraction for the 3:2 resonance increases to 47\%. When a stronger drag force is used as in model C, the fraction of planetesimals captured into the 3:2 resonance greatly increases up to about 60\%. In contrast, the number for the 2:1 resonance becomes zero. Therefore, the numerical results of the resonant capture process reveal that it is very sensitive to the strength of the gaseous drag. Since the planetesimals are captured into resonances during their inward migrations, the stronger drag increases the speed of inward migration, so equivalently, the resonant population in capture processes is correlated with the speed of inward migration.

For a model in which the planet has an initially eccentric orbit, less than 3\% of the planetesimals are trapped into the 3:2 and 2:1 resonances. Hence, the assumption of a large finite eccentricity nearly destroys the possibility of resonant captures.

To understand the difference between the 3:2 and 2:1 resonant captures, there are two main possibilities:

1. The details of capture processes for the 3:2 and 2:1 resonances are fundamentally different and strongly depend on the migration speed of the planetesimals.
2. Under our assumptions, there are more planetesimals passing through the 3:2 resonant region during the simulations, so that more planetesimals are captured into the 3:2 resonance,
Fig. 8.—Model C: histograms of particle distributions in the radial coordinate. The time between successive panels corresponds to 11,200π.

Fig. 9.—Model C: particle distribution in the $a$-$e$ plane at the end of simulation, i.e., $t = 123,200\pi$. The right color panel shows the number of particles in the particular area.
For model B, the capture probability for the 2:1 resonance is still about 40%; however, the total number of planetesimals captured into the 3:2 resonance increases significantly up to about 140. This could be due to the fact that some planetesimals initially located out of \( r = 1.6 \) but \( \text{not} \) captured into the 2:1 resonance are also captured into the 3:2 resonance in time due to fast inward migrations.

Since the potential number of planetesimals to be captured into the 3:2 resonance is again 177, the capture probability for the 3:2 resonance is 140/177, which is about 79%. Therefore, not only is the number of planetesimals passing into the 3:2 resonant region larger than the one for the 2:1 resonance, but the capture probability of the 3:2 resonance is also larger than that of the 2:1 resonance.

For the largest gaseous drag considered (model C), the speed of inward migration is highest and even more planetesimals are in the 3:2 resonance. However, in this case, the number of particles in the 2:1 resonance vanishes.

From the above analysis for models A, B, and C, we confirm that the details of the 3:2 and 2:1 resonant capture are fundamentally different. As discussed in Peale (1976) and Murray & Dermott (1999), the resonant relations are determined by the influence of the planet on the planetesimals. In particular, during the conjunctions the net tangential force experienced by the planetesimal is key, because it can change the planetesimal’s angular momentum.

In the case in which the planet moves on a circular orbit and the planetesimals are assumed to move on eccentric orbits, there is no net tangential force if the conjunctions occur exactly at the pericenter or apocenter. When the conjunctions occur at any other point on the orbit, the symmetry is destroyed, and thus, there is a net tangential force. For a dense population of planetesimals moving on the same eccentric orbit, the net tangential force would cause about half of them to gain angular momentum and another half to lose angular momentum. If all these planetesimals were driven to migrate inward due to the drag, (1) those that gain angular momentum would expand their orbits a bit and could be captured into the resonance, and (2) those that lose angular momentum would continue to migrate inward. Since (1) the planetesimals might move on different orbits and (2) the exact locations of the repeated conjunctions are not known, it is difficult to estimate the capture probability.

The symmetry is further destroyed when the planet moves on an orbit with large eccentricity 0.3. As a result, it is more difficult for those conjunctions, which lead to planetesimals gaining angular momentum, to take place repeatedly. Thus, there are few planetesimals captured into the resonances in model D.

For the Kuiper Belt in the outer solar system, objects are known to occupy the 3:2 and 2:1 resonant regions. The conventional mechanism explaining these resonances relies on the resonant capture by an outwardly migrating Neptune with a migration timescale \( \tau = 2 \times 10^6 \text{yr} \) (Malhotra 1995). This migration was assumed to start in the late stages of the genesis of the solar system when the formation of the gas giant planet was largely complete, the solar nebula had lost its gaseous component, and the evolution was dominated by the gravitational interactions. However, this mechanism is based on an assumption of pure radial orbital migrations. The generality of this mechanism is unclear if a more realistic orbit of Neptune were chosen. As shown by the numerical simulations in Thommes et al. (1999) and the analytic calculations in Yeh & Jiang (2001), Neptune’s orbital eccentricity shall not be zero during the outward migration. Because Neptune is currently moving on a circular orbit, a massive disk is needed to circularize Neptune’s orbit if the outward migration did happen.
Fig. 11.—Model D: histograms of particle distributions in the radial coordinate. The time between successive panels corresponds to 11,200π.

Fig. 12.—Model D: the particle distribution in the \( \sigma - \epsilon \) plane at the end of simulation, i.e., \( t = 123,200\pi \). The right color panel shows the number of particles in the particular area.
On the other hand, our results show that the drag-induced resonant capture can explain the existence of objects in both 3:2 and 2:1 resonances, but the ratio of these two populations will depend on the gaseous drag strength. The similarity between the conventional picture and our mechanism reflects the fact that both captures are due to the relative motions between the planet and the small bodies. The main difference is the causes of the relative motions.

Finally, our results (model D) also show that the resonant capture occurs provided that the planet's orbital eccentricity is not too large. This result could place some constraint on the possible orbital history of the planet. For example, from this point of view in the conventional picture, Neptune will be able to capture the KBOs into the resonances only when its eccentricity is reduced to less than 0.3. This further constrains the orbital history and the timing of Neptune’s migration if it did significantly contribute to the resonant capture of 3:2 and 2:1 resonant KBOs.

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Fig. 13.—Number of resonant particles as a function of time for (a) the 3:2 resonance and (b) the 2:1 resonance. The solid curves are for model A, the dotted curves are for model B, the dashed curve is for model C, and the long-dashed curves are for model D. Please note that there are only three curves in (b), because there is no particle in 2:1 resonance in model C.