Analysis of the vortex characteristics during roadway ventilation

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Abstract. The ventilation in the roadway is in a turbulent state. As the main body of the ventilation, the vortex rotating produces energy dissipation under viscous force which in turn creates air resistance, and it plays an important role in heat and mass transfer. Considering the importance of the vortex in the roadway ventilation, a series of analyses were made by the Taylor model from the perspective of microscopic to explore the vortex characteristics based on the theory of viscous fluid mechanics and vortex motion. The phenomenon that the radii was increasing as vortex diffusing to the roadway center and the energy dissipation was the biggest near the wall was verified. The survival period of vortex was clarified which has an important effect on analyzing the formation of the turbulence. The paper also established the model of energy dissipation rate unit volume which is helpful to the theory of roadway ventilation. In the final, a proposal was offered that it was better to avoid small scale vortex in the roadway ventilation.

1. Introduction
The ventilation in the roadway is always in a turbulent state. Turbulence used to be considered as the disordered motion in the past and people obtained new understanding towards fluid until the discovery of the coherent structure [1-2]. The core of the coherent structure is vortex [3] and the ventilation in the roadway is full of all kinds of vortices [4]. Due to the viscous force, the energy dissipation is generated during the rotational process of vortex [5]. The essence of resistance [6-7] produced in roadway ventilation is energy dissipation [8], so that the resistance of roadway ventilation comes from vortex.

The walls in roadway are rough because of the production process. Via experiment, Nikuradse [9] proved that the rougher the walls were, the larger flow resistance was, so that the roughness of the walls in roadways played a significant role in the formation of vortex. According to literature [10], it is known that the near wall region is the main area that turbulent kinetic is generated, that is, vortex is produced in the near wall region with the largest kinetic energy. After the formation, vortex diffuses to the center of roadway under the effect of various force. Vortex also plays an important part in heat and mass transfer in the roadway [11-12]. Therefore, it is of great significance to study the characteristics of vortex for the research of roadway ventilation theory.
2. Energy dissipation rate unit volume of vortex layer

The vortex in ventilation could be regarded to be composed of a series of connected vortex layers with different semidiameter, whose thickness is \( dr \) and rotation direction is clockwise. It is assumed that external boundary of the vortex layer a is semidiameter (represented by \( r + dr \)), angular velocity is represented by \( \omega + d\omega \); semidiameter of inner boundary is \( r \) and its angular velocity is \( \omega \), as shown in Figure 1.

![Figure 1. Vortex layer structure diagram](image)

Based on the relationship between stress and change of strain, the shearing stress of plane fluid perpendicular to the central axis in the column coordinate system is represented as follows:

\[
\tau = \mu \frac{d\omega}{dr} \quad \frac{u}{r} \quad \text{(1)}
\]

The relationship between linear velocity and angular velocity could be expressed as follows:

\[
u = \omega r \quad \text{(2)}
\]

Combining Formula (1) and Formula (2), the formula of the shearing stress of vortex could be obtained, as shown in Formula (3).

\[
\tau = \mu \frac{d\omega}{dr} \quad \text{(3)}
\]

From the formula above, it is seen that the shearing stress of vortex has a positive relationship with the gradient of angular velocity. Taking the inner boundary of vortex layer as research object, the parameters of inner boundary were put into Formula (3), thus obtaining the shearing stress of the inner boundary of the vortex layer (Formula (4)).

\[
\tau_b = \mu \frac{d\omega}{dr} \quad \text{(4)}
\]

Based on the same principle, the shearing stress of the outer boundary of the vortex layer could be obtained.

\[
\tau_a = -\mu (r + dr) \frac{d\omega}{dr} \quad \text{(5)}
\]

It is assumed that the length of vortex is \( h \), friction moment of inner boundary of the vortex is \( M_b \), the friction moment of outer boundary of the vortex is \( M_a \). According to the formula of friction moment, the frictional moments on the inner and outer boundaries of the fluid in the vortex layer are expressed by Formula (6) and (7) respectively.

\[
M_b = 2\pi r \tau r^2 h \quad \text{(6)}
\]
\[
M_a = 2\pi r \tau (r + dr)^2 h \quad \text{(7)}
\]

The energy dissipating rate \( dN \) refers to the power consumed by the fluid in the vortex to overcome the friction moments acting on the inner and outer boundary layers.

\[
dN = -\omega M_b - (\omega + d\omega)M_a \quad \text{(8)}
\]

Combining Formula (4) and Formula (8), the energy dissipating rate of vortex layer could be obtained.
After sorting out Formula (9), the energy dissipation rate unit volume of vortex layer \( N_r \) could be obtained.

\[
N_r = \mu \left( r \frac{d\omega}{dr} \right)^2
\]  

According to the formulas above, it is found that the energy dissipation rate unit volume is related to the gradient of angular velocity, which could be obtained by vortex model.

3. Angular velocity gradient of vortex

There are lots of models to describe vortex and the Taylor vortex model is closer to the physical reality. According to this model, the linear velocity of vortex could be expressed as follows:

\[
u = \frac{H \omega}{4\pi^2 \mu^2} e^{-\frac{r^2}{2\mu^2}}
\]  

where \( H \) represents momentum moment of vortex and its calculation formula is as follows:

\[
H = \int_0^r \omega_0 2\pi r^3 dr = \frac{\pi \omega_0 r_0^4}{2}
\]  

where \( r_0 \) represents original semidiameter of vortex; \( \omega_0 \) represents original angular velocity .

After the derivation of Formula (11), the velocity gradient of vortex could be obtained:

\[
\frac{dv}{dr} = \frac{H}{4\pi^2 \mu^2} e^{-\frac{r^2}{2\mu^2}} \left( 1 - \frac{r^2}{2\mu^2} \right)
\]  

It is found that, when the semidiameter of vortex is \( \sqrt{2\mu} \), the velocity gradient is 0 and the velocity of the vortex reaches its maximum, that is, the semidiameter of vortex core is \( \sqrt{2\mu} \). The longer vortex rotation time, the larger semidiameter of vortex, that is, during the transfer process of vortex from the vicinity of the wall in roadway to the center of the roadway, the semidiameter of vortex is continuously increasing, which conforms to the previous research conclusion.

Based on Formula (11) and the relationship between angular velocity and velocity of vortex, the gradient of angular velocity of vortex could be obtained:

\[
\frac{d\omega}{dr} = \frac{\omega_0 r_0^4}{16\pi^2 \mu^2} e^{-\frac{r^2}{2\mu^2}}
\]  

The semidiameter of vortex is determined by its rotation time. Form the Formula (14), the gradient of angular velocity of vortex depends on its rotation time too.

4. Survival period of vortex

Once the vortex is formed near the roadway wall, the distance of vortex diffusion to the roadway center is closely related to the survival period. If the vortex has a short survival period, it will disappear before it spreads to the center of the roadway, then it is not easy to form turbulence. This paper analyzes the survival period of vortex from the angle of energy conservation.

The original vortex kinetic energy \( E_i \) is determined by original angular velocity and original radius of vortex, which can be expressed as:

\[
E_i = \int_0^{r_0} \frac{1}{2} \rho \omega_0^2 r^2 2\pi r h dr = \frac{r_0^4 \rho \omega_0^2 rh}{4}
\]  

Combining Formula(10) and Formula (14), the energy dissipation rate unit volume could be obtained:
The vortex energy dissipation rate $N$ can be obtained by integrating Formula (16):

$$N = \int_0^\infty N_r 2\pi \rho dr = \frac{h\mu \omega^2 r_0^2}{16\nu^3 t}$$ \quad (17)

It can be seen from the above equation that the longer the vortex exists, the smaller the energy dissipation rate is. Compared with the entire roadway section, the vortex energy dissipation rate near the roadway wall is the largest, which is consistent with the conclusion in literature [13]. Integrate Formula (17) with time to get the total energy consumption in the survival period of vortex.

$$W = \int_0^t \frac{\rho \pi \omega^2 r_0^4}{16\nu^2 t} dt = \frac{h\mu \omega^2 r_0^2}{32\nu^2 t_0^2}$$ \quad (18)

where $t_0$ represents original time.

The dissipated energy of vortex around the roadway comes from the original kinetic energy of vortex generated near the roadway wall, that is, the original kinetic energy of the vortex is equal to the total energy dissipation of the vortex in the survival period.

By combining Formula (15) and Formula (18), the original moment of vortex can be obtained.

$$t_0 = \frac{\sqrt{2}r_0^2}{4\nu}$$ \quad (19)

From the formula above, it is seen that the larger the original semidiameter of vortex is, the larger the corresponding original time is.

The stored kinetic energy of vortex will be dissipated due to the viscous force. Based on the energy dissipation rate of vortex (Formula (17)), when $r_0^2/t_0$ reaches its minimum, most of the energy in vortex will be dissipated and the responding time is the moment $T$ that all energy of vortex is dissipated, that is, the survival period of vortex is within the range of $t_0-\Delta t$.

5. Model of energy dissipation rate unit volume

Roadway is filled with vortex and the energy dissipation rate unit volume could reflects the energy dissipation rate of ventilation. Based on Formula (18), the energy dissipation rate unit volume $N_r$ is expressed by Formula (20).

$$N_r = \frac{W}{\pi^2 h(T-t_0)} = \frac{\rho \omega^2 r_0^4}{8\nu (T-t_0)^2}$$ \quad (20)

From the formula (20), it is found that the energy dissipation rate unit volume of vortex is related to the original angular velocity, the survival period of the vortex and the original semidiameter of vortex. The original angular velocity of vortex is determined by wind speed. Generally, the wind speed of roadway is certain so that the original angular velocity of vortex is a certain value too. For convenience, this paper introduced energy consumption index $nt$ to express the changing rules of energy dissipation rate unit volume.

$$nt = 30 + \ln \left[ \frac{N_r}{\left( \frac{\rho \omega^2}{8\nu} \right)} \right]$$ \quad (21)

If energy dissipation rate at a given moment $r_0^2/t_0 = 1.4 \times 10^{-39}$ is relatively small, the disappearing period of vortex $\Delta t$ could be obtained. Based on Formula (19), the original time of the vortex is related to the original semidiameter of vortex. By analysis above, the energy dissipation rate unit volume of vortex is only related to the parameter connected with the original semidiameter.

Combing Formula (19) and (21), the relational expression between energy consumption index and original semidiameter of vortex could be obtained.
Based on the formula above, when the value of the original semi-diameter of vortex is within the range 0.002m-0.12m, the congruent relationship between energy consumption index and original semi-diameter of vortex $r_c$ could be acquired (as shown in Figure 2).

![Figure 2. The energy consumption index varies with the original semidiameter](image)

The energy consumption index represents the energy dissipation per unit volume of vortex. From figure, it is seen that the unit volume of energy dissipation of vortex increases with the decreasing of original semidiameter, which is basically the same as the previous research conclusion [14-15]. The smaller the original semidiameter of vortex is, the larger wind resistance in roadway is. Therefore, it is necessary to avoid small scale vortex in the roadway ventilation.

### 6. Conclusions

As the basic energy consumption unit, vortex is the main reason that wind resistance in roadway is produced. It is great of significance to study the characteristics of vortex, which is beneficial for the research of roadway ventilation theory. By analyzing the vortex, this paper has made the following conclusions:

1) During the transfer process of vortex from wall to the center of roadway, the semidiameter of vortex is increasing. The phenomenon that the energy dissipation rate around the roadway wall reached its maximum was confirmed.

2) The survival period of vortex was clarified, which plays an important role in analyzing whether the roadway ventilation produces the turbulent flow.

3) The model of energy dissipation rate unit volume was established, thus laying the theoretical basis for studying roadway ventilation.

4) The smaller the original semidiameter of vortex, the larger wind resistance in the roadway. Therefore, it is necessary to avoid small scale vortex in the roadway ventilation.

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