WHICH CONSTITUENT QUARK MODEL IS BETTER?

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A comparative study has been done by calculating the effective baryon-baryon interactions of the 64 lowest channels consisting of octet and decuplet baryons with three constituent quark models: the extended quark gluon exchange model, the Goldstone boson exchange model and the quark gluon meson exchange hybrid model. We find that these three models give similar results for 44 channels. Further tests of these models are discussed.

1. A Debate on Which Constituent Quark Model is Better

For low energy quantum chromodynamics (QCD), especially for complicated quark gluon systems, nonperturbative QCD calculation is still very difficult, if not impossible, and one must rely on QCD models. The constituent quark model is quite successful in understanding hadron spectroscopy and even hadron interactions. However there has been a debate on which constituent quark model is better.\(^1\)\(^2\)\(^3\)\(^4\) There is a consensus that the constituent quark is a useful effective degree of freedom for low energy hadron physics. Different authors have radically different viewpoints on what other proper effective degrees of freedom may be.

Glozman and Riska proposed that the Goldstone boson is the only other proper effective degree of freedom (GR).\(^1\) Isgur insisted that the gluon is the proper one (GI).\(^2\) Manohar and Georgi argued that in between the spontaneously broken chiral symmetry and confinement energy scales, both Goldstone boson and gluon effective degrees of freedom survive (MG).\(^3\) Georgi also did an effective matrix element analysis of P-wave baryon spectroscopy with both the quark-Goldstone boson coupling and the quark-gluon coupling models, finding that the former fits the data with a smaller \(\chi^2\), but noted: “our results should not be interpreted, by themselves, as evidence in favor of the chiral quark model picture over the nonrelativistic quark model.”\(^5\) K.F. Liu produced a valence lattice QCD result which supports the Goldstone boson exchange picture, but Isgur pointed out that this is unjustified.\(^2\)\(^4\)
M. Furuichi and K. Shimizu calculated the baryon spectroscopy further with both
the GR and MG quark gluon meson exchange hybrid models and concluded that
"for the moment we do not have a definite answer which model is better than the
other." H. Garcilazo et al. checked the MG hybrid and GR models too and con-
cluded that "the simultaneous description of the one and two-baryon systems still
remains an open problem."5

If these three constituent quark models (GR, GI, MG) are really quite dif-
ferent, differences should also appear in their predictions for baryon-baryon (BB)
interactions which are more sensitive to these model details. Therefore, we did a
comparative study of the predictions of these three models for BB interactions in
the lowest 64 channels consisting of octet and decuplet baryons. Our results show
that 44 channels have similar effective BB interactions.

2. A Comparative Study of Three Constituent Quark Models

There are different versions of each model. We choose Glozman’s parametrization
of the Goldstone boson exchange, given in Ref.6, as a typical example of the GR
model. The hybrid MG model is widely used in studies of the BB interaction. We
choose the Fujiwara version7 as the MG example. It is well known that the quark-
gluon coupling model, GI, provides only a repulsive core for the NN interaction but
no intermediate range attraction. This attraction is attributed to σ exchange or to
two pion exchange in the hybrid model. Nor does the GI model include any one
pion exchange long range interaction.

We have extended the GI model: We keep as the three quark Hamiltonian
that used by Isgur, i.e., a harmonic confinement plus a color magnetic hyperfine
interaction, making two modifications to extend it to the BB interaction. First,
instead of the usual two centered single quark orbital wave function, we introduce a
delocalized quark orbital wave function, borrowed from the description of molecular
orbitals. It incorporates the mutual distortion of the interacting baryons or the
internal excitation of the baryon in the course of interaction, which enlarges the
variational Hilbert space simply. Second, we use a new parametrization of the
confinement potential to take into account nonperturbative QCD effects which could
not have been parametrized by a two body confinement and color magnetic hyperfine
interaction. It may not even be possible to check, by hadron spectroscopy, QCD
features such as the three gluon interaction, three body instanton interaction, etc.
(A detailed discussion can be found in Ref.8.) With these additions, the extended GI
model produces a quantitatively correct NN intermediate range attraction without
invoking σ or two pion exchange. More importantly, this is the unique model which
explains the long known resemblance of the nuclear force to the molecular force
(except for the obvious length and energy scale differences). We use this extended
quark gluon coupling model, which has been called the quark delocalization and
color screening model (QDCSM), as the example of GI.

We are interested in general features of the BB interactions which characterize
each individual constituent quark model. Therefore, we calculated the adiabatic
effective BB interactions for the 64 lowest channels consisting of octet and decuplet
baryons with these three constituent quark models.9 Typical results are shown in
Figs. 1-3. In Fig. 1, two (SIJ=022, -403) of the 17 pure repulsive core channels are shown. In Fig. 2, two weak attraction channels (SIJ=001, 010) are shown. Altogether there are 13 channels where all three models give such similar effective BB interactions. In Fig. 3, two other weak attraction channels (SIJ=-1/2, 0,-223) are shown. Altogether there are 14 channels where two of the three models give very similar results; the third gives a little different, but still similar, effective BB interaction.

For the H particle channel, the three models all give somewhat different results but are all weakly attractive with (GR and MG) or without (QDCSM) a repulsive core. However the dynamical calculations of the QDCSM and hybrid models yield a state with binding energy around zero. This is radically different from the naive bag model estimate and some earlier hybrid model estimates.

Only in a few channels do the three models give really different results. For example, the QDCSM predicts a strong attraction in the di-Δ (SIJ=003) channel, while the hybrid model predicts a strong attraction in the di-Ω (SIJ=-600) channel.

The general features can be summarized as: The three constituent quark models, although they appear to be very different, give similar effective BB interactions.
Returning to hadron spectroscopy, based on the debate\textsuperscript{1,2,3,5}, it is fair to say that while different constituent quark models each have their own advantages and disadvantages, none gives a perfect description of hadrons. For example, within the pure valence $q^3$ configuration, no model explains the nucleon spin and flavor structure and the spin-orbit splitting of the strange and nonstrange mesons and baryons together.\textsuperscript{12} The GI and MG models have the advantage of a unified description of the meson and baryon spectrum and the one and two-baryon systems simultaneously but the disadvantage of predicting the wrong order for the P and D wave excited baryon states. The GR model obtains the correct order for the P and D wave excited states but loses the unified manner of describing the meson and baryon spectrum and might find it difficult to describe the one and two baryon systems simultaneously.

3. A Tentative Explanation of the Origin of the Similarity

The same pure repulsive core obtained in 17 channels from each of these three constituent quark models can be understood in that they are due to Pauli forbidden combinations common to all quark models. The similar weak attraction obtained in the deuteron channel (SIJ=001) can be attributed to the adjustment of model parameters because a realistic BB interaction model should fit the deuteron properties first. However, there should be physical reasons for the fact that more than 2/3 of the channels have similar effective BB interactions in all models.

3.1. Antisymmetrization effect

Antisymmetrization might be one of the physical reasons for the similarity between the models. To examine this, we neglect the details of the orbital form and concentrate on the spin-flavor-color structure of the gluon and Goldstone boson exchange q-q interactions. (The hybrid model includes both parts so it does not need to be studied separately.) The structures are

$$V_{ij}(GE) = -V(GE)\lambda^c_i \cdot \lambda^c_j \sigma_i \cdot \sigma_j, \quad V_{ij}(BE) = -V(BE)\lambda^f_i \cdot \lambda^f_j \sigma_i \cdot \sigma_j, \quad (1)$$

respectively. Recalling the Dirac identity and the antisymmetrization condition, we
have,
\[ \lambda_i \cdot \lambda_j = 2P_{ij} - 2/3, \quad \sigma_i \cdot \sigma_j = 2P_{ij}^s - 1, \] (2)

orbo al symmetric orbital antisymmetric

\[
V_{ij}(GE)/V(GE) = 4P_{ij}^f + 4/3P_{ij}^s + 2P_{ij}^c - 2/3, \quad -4P_{ij}^f + 4/3P_{ij}^s + 2P_{ij}^c - 2/3, \text{ (3)}
\]

\[
V_{ij}(BE)/V(BE) = 4P_{ij}^c + 4/3P_{ij}^s + 2P_{ij}^f - 2/3, \quad -4P_{ij}^c + 4/3P_{ij}^s + 2P_{ij}^f - 2/3. \text{ (4)}
\]

Eqs. (3, 4) show that due to antisymmetrization, the color dependent gluon exchange interaction turns out to be flavor dependent and the flavor dependent Goldstone boson exchange interaction turns to be color dependent, also. No significant difference appears in the symmetric orbital case. However, there is a real difference in the antisymmetric orbital case, where the color and flavor dependent terms have opposite signs. Therefore we examine this case one step further.

We reexpress these two interactions using Casimir operators,

\[ -\sum_{i<j} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j = -4C_{SU(6)}^2 + 2C_{SU(3)}^2 + 4/3C_{SU(2)}^2 + 8N, \] (5)

where \(C_{SU(n)}^2\) is the second rank Casimir operator of the SU(n) group and \(N\) is the particle number of the system. Next, we obtain the eigenvalue, \(\Delta E\), of the gluon and boson exchange interactions as shown in Table 1, which is calculated for orbital symmetry [6]. The color part must always be [222] due to color confinement. The spin-flavor symmetry is restricted to be [33] due to antisymmetrization. It is then easy to see from Table 1 that the eigenvalues of these two interactions both decrease with the flavor symmetry but with slightly different slopes and that they end with the same minimum at the flavor singlet, namely the H particle state.

### Table 1. \(\Delta E\) of the gluon and boson exchange interactions.

| \(|f|\) | \(|6|\) | \(|51|\) | \(|42|\) | \(|33|\) | \(|41|\) | \(|32|\) | \(|222|\) |
|---|---|---|---|---|---|---|---|
| \(|\sigma|\) | \(|33|\) | \(|42|\) | \(|51|, |33|\) | \(|6|, |42|\) | \(|42|\) | \(|51|, |42|\) | \(|33|\) |
| \(\Delta E_{c-s}\) | 48 | 80/3 | 16,8 | 16,8/3 | 8/3 | -4,-28/3 | -24 |
| \(\Delta E_{f-s}\) | 12 | 8/3 | 0,-8 | -4,-28/3 | -28/3 | -10,-46/3 | -24 |

### 3.2. Nonperturbative QCD basis

Having an understanding of the QCD basis of these constituent quark models is helpful in judging these models. So far, there have been only approximate nonperturbative QCD ”derivations”.

Cahill and Roberts ”derived” a Goldstone boson-constituent quark coupling model under the approximation of only keeping the current-current interaction.\(^{13}\) The current quark is dressed to become the constituent quark in the spontaneously broken chiral QCD vacuum due to \(\bar{q}q\) condensation and the Goldstone boson is identified as due to quantum fluctuations of the boson field originating from the \(\bar{q}(x)\gamma_5 q(y)\) bilocal operator.
Based on this, and the results of Negele et al. on the dilute instanton model of Shuryak\textsuperscript{14}, we developed an approximate QCD "derivation" of the MG model. The gluon field is separated into two components: an instanton part and a quantum fluctuation part. The instanton part is assumed to be the classical part of the gluon field which induces spontaneously chiral symmetry breaking and dresses the current quark into constituent quark, while the quantum fluctuation part of the gluon field is kept as a perturbative gluon contribution.

In a Schwinger-Dyson equation approach\textsuperscript{15}, under a reasonable truncation, one obtains a dressed quark, gluon propagators and the dressed quark-gluon vertex which can be translated into an effective quark-gluon coupling model.

In summary, all three constituent quark models have an arguable basis in QCD and it is hard to say which one is better inspired by QCD.

4. An Outlook

Hadron spectroscopy will be improved by new experimental efforts. Hadronic transition matrix elements are more sensitive to model details; the internal structure of hadrons tests models more stringently. There are already candidates for exotic quark-gluon systems and pursuit of dibaryons by various groups continues. New facilities will provide more precise hyperon-nucleon, and even hyperon-hyperon, interaction data. Since each model has its strengths and weaknesses, the different model approaches to a unified description of hadron internal structure and hadron interactions are needed to teach us about hadron physics from different perspectives.

Nonperturbative QCD approaches have shown that the different quark models may well be related to QCD in different ways. It is highly desirable to continue this effort to clarify the relation between QCD theory and QCD models and, in turn, to understand the relation between different models, especially if they are contradictory, as has led to strong debate in recent years. It might be worthwhile to recall that a similar history occurred in the study of nuclear structure.

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1. L.Ya. Glozman and D.O. Riska, Phys. Rep. \textbf{268}, 263 (1996).
2. A.De Rujula, H. Georgi and S. Glashow, Phys. Rev. \textbf{D12}, 147 (1975). N. Isgur and G. Karl, Phys. Rev. \textbf{D18}, 4187 (1978); \textbf{D19}, 2653 (1979); \textbf{D20}, 1191 (1979). N. Isgur, Phys. Rev. \textbf{D61}, 118501; \textbf{D62}, 054026 (2000).
3. A. Manohar and H. Georgi, Nucl. Phys. \textbf{B234}, 189 (1984). H. Collins and H. Georgi, Phys. Rev. \textbf{D59}, 094010 (1999).
4. K.F. Liu, et al., Phys. Rev. \textbf{D59}, 112001 (1999); \textbf{D61}, 118502 (2000).
5. M. Furuchi and K. Shimizu, Phys. Rev. \textbf{C65}, 025201 (2002). H. Garcilazo, A. Valcarce and F. Fernandez, Phys. Rev. \textbf{C63}, 035207 (2001).
6. L.Ya. Glozman, et al., Phys. Rev. \textbf{D58}, 094030 (1998).
7. Y. Fujiwara, C. Nakamoto and Y. Suzuki, Phys. Rev. Lett. \textbf{76}, 2242 (1996); Phys. Rev. \textbf{C54}, 2180 (1996).
8. F. Wang, et al., Phys. Rev. Lett. \textbf{69}, 2901 (1992); J.L. Ping, F. Wang and T. Goldman, Phys. Rev. \textbf{C62}, 054007 (2000); Nucl. Phys. \textbf{A688}, 871 (2001); J.L. Ping, H.R. Pang,
F. Wang and T. Goldman, Phys. Rev. C65, 04403 (2002).
9. H.R. Pang, J.L. Ping, F. Wang and T. Goldman, Phys. Rev. C65, 014003 (2002); Commun. Theor. Phys. 37, 193 (2002).
10. R.L. Jaffe, Phys. Rev. Lett. 38, 471 (1977).
11. Y. Koiki, K. Shimizu and K. Yazak, Nucl. Phys. A513, 653 (1990).
12. F. Wang and C.W. Wong, Nucl. Phys. A467, 685 (1987); D. Qing, X.S.Chen and F. Wang, Phys. Rev. C57, R31 (1998); D58, 114032 (1998); Commun. Theor. Phys. 32, 403 (1999); Chin. Phys. Lett. 16, 403 (1999).
13. R.T. Cahill and S.M. Gunner, Fizika B7, 171 (1998).
14. J.W. Negele, Nucl. Phys. A670, 14c (2000).
15. C.D. Roberts and S.M. Schmit, Prog. Part. Nucl. Phys. 45, 1 (2000).