Two mechanisms for the elimination of pinch singularities in out of equilibrium thermal field theories

I. Dadić\(^{1,2}\) *

\(^{1}\) Ruder Bošković Institute, Zagreb, Croatia
\(^{2}\) Fakultät für Physik, Universität Bielefeld, Germany

Abstract

We analyze ill-defined pinch singularities characteristic of out of equilibrium thermal field theories. We identify two mechanisms that eliminate pinching even at the single self-energy insertion approximation to the propagator: the first is based on the vanishing of phase space at the singular point (threshold effect). It is effective in QED with a massive electron and a massless photon. In massless QCD, this mechanism fails, but the pinches cancel owing to the second mechanism, i.e., owing to the spinor/tensor structure of the single self-energy insertion contribution to the propagator. The constraints imposed on distribution functions are very reasonable.

* E-mail: dadic@faust.irb.hr
1 Introduction

Out of equilibrium thermal field theories have recently attracted much interest. From the experimental point of view, various aspects of heavy-ion collisions and the related hot QCD plasma are of considerable interest, in particular the supposedly gluon-dominated stage.

Contrary to the equilibrium case \cite{1,2}, where pinch, collinear, and infrared problems have been successfully controlled \cite{3,4,5,6}, out of equilibrium theory \cite{7,8,9} has suffered from them to these days. However, progress has been made in this field, too.

Weldon \cite{10} has observed that the out of equilibrium pinch singularity does not cancel; hence it spoils analyticity and causality. The problem gets worse with more than one self-energy insertions.

Bedaque has argued that in out of equilibrium theory the time extension should be finite. Thus, the time integration limits from \( -\infty \) to \( +\infty \), which are responsible for the appearance of pinches, have to be abandoned as unphysical \cite{11}.

Le Bellac and Mabilat \cite{12} have found that collinear singularities cancel in scalar theory, and in QCD using physical gauges, but not in the case of covariant gauges. Niégawa \cite{13} has found that the pinch-like term contains a divergent part that cancels collinear singularities in the covariant gauge.

In their discussion of the pinch-like term Le Bellac and Mabilat \cite{12} try to avoid the problems with pinching singularity by substituting the bare retarded photon (gluon) propagator with the resummed Schwinger-Dyson series calculated in the HTL \cite{14,15,16} approximation:

\[
\Delta_T(q_o, q) = \frac{-1}{q_o^2 - q^2 - m^2 \left( x^2 + \frac{1}{2} (1-x^2) \log \frac{x+1}{x-1} \right)} , \quad \Delta_T(q_o, q) = \frac{-1}{q^2 + 2m^2 \left( 1 - \frac{x}{2} \log \frac{x+1}{x-1} \right)},
\]

with the spectral function given by \( \rho_{T,L} = 2Im\Delta_{T,L}(q_o+i\eta, q) \). In the above expressions \( m \) is the thermal photon (gluon) mass given by \( m^2 = e^2 T^2 / 6 \) (\( m^2 = g^2 T^2 (N_c + N_f/2)/6 \) for the gluon; note that they assume a small deviation from equilibrium), and \( x = q_o/q \). In equilibrium, at high temperatures and low momenta, this substitution is necessary in order to obtain the results correct to the leading order in \( g^2 \). One expects that similar methods work also for a narrow class of particle distributions corresponding to "high temperatures" out of equilibrium. However, their expression for pinch-like contribution differs from the one following from our general expression for the resummed Schwinger-Dyson series (2.32). At medium and large photon momenta the elimination of pinching by the use of the HTL approximated propagator is no more justified.

Altherr and Seibert have found that in massive \( g^2 \phi^3 \) theory pinch singularity does not occur owing to the kinematical constraint \cite{17}. This result is restricted to the case of one-loop self-energies.

Altherr has suggested a regularization method in which the propagator is modified by the width \( \gamma \) which is an arbitrary function of momentum to be calculated in a self-consistent way. In \( g^2 \phi^4 \) theory, for small deviations from equilibrium, \( \gamma \) was found to be just the usual equilibrium damping rate \cite{18}.

This recipe has been justified in the resummed Schwinger-Dyson series in various problems with pinching \cite{15,20,21,22,23}. Baier, Dirks, and Redlich \cite{19} have calculated the \( \pi - \rho \) self-energy contribution to the pion propagator, regulating pinch contributions by the damping rate. In subsequent papers with
Schiff they have calculated the quark propagator within the HTL approximation; in the resummed Schwinger-Dyson series, the pinch is naturally regulated by $Im \Sigma_R$.

Carrington, Defu, and Thoma have found that no pinch singularities appear in the HTL approximation to the resummed photon propagator.

Niégawa has introduced the notion of renormalized particle-number density. He has found that, in the appropriately redefined calculation scheme, the amplitudes and reaction rates are free from pinch singularities.

By pinching singularity we understand the contour passing between two infinitely close poles:

$$\int \frac{dx}{(x + i\epsilon)(x - i\epsilon)}, \quad (1.2)$$

where $x = q^2 - m^2$. It is controlled by some parameter, e.g., $\epsilon$. For finite $\epsilon$, the expression is regular. However, when $\epsilon$ tends to zero, the integration path is "pinched" between the two poles, and the expression is ill-defined. Integration gives an $\epsilon^{-1}$ contribution plus regular terms. By performing a simple decomposition of $(x \pm i\epsilon)^{-1}$ into $PP(1/x) \mp i\pi\delta(x)$, one obtains the related ill-defined $\delta^2$ expression.

The following expression, which is similar to (1.2), corresponds to the resummed Schwinger-Dyson series:

$$\int dx \frac{\omega(x)}{(x - \Sigma_R(x) + i\epsilon)(x - \Sigma_R^*(x) - i\epsilon)}, \quad (1.3)$$

where $\omega(x)$ and $\bar{\omega}(x)$ (which appears in (1.4)) are, respectively, proportional to $\Omega(x)$ and $\bar{\Omega}(x)$, where $\Omega(x)$, $\Sigma_R(x)$, and $\bar{\Omega}(x)$ are the components of the self-energy matrix to be defined in Sec. III.

In expression (1.3), pinching is absent if $Im \Sigma_R(x_o) \neq 0$ at a value of $x_o$ satisfying $x_o - Re \Sigma_R(x_o) = 0$.

The expression corresponding to the single self-energy insertion approximation to the propagator is similar to (1.3):

$$\int dx \frac{\bar{\omega}(x)}{(x + i\epsilon)(x - i\epsilon)}, \quad (1.4)$$

One can rewrite the integral as

$$\int dx \frac{\bar{\omega}(x)}{2(x + i\epsilon)(x - i\epsilon)} + \frac{1}{x - i\epsilon} \frac{\bar{\omega}(x)}{x}. \quad (1.5)$$

If it happens that

$$\lim_{x \to 0} \frac{\bar{\omega}(x)}{x} = K < \infty, \quad (1.6)$$

then the integral (1.3) decomposes into two pieces that, although possibly divergent, do not suffer from pinching.

There are two cases in which the function $\bar{\omega}(x)$ is even identically zero in the vicinity of the $x = 0$ point: in thermal equilibrium, because of detailed balance relations; in massive $g^2 \phi^3$ theory out of equilibrium, owing to the mass shell condition. The latter mechanism also works in out of equilibrium QED if a small photon mass $m_{\gamma}$ is introduced. However, this elimination of pinching can be misleading: the domain of $x$, where $\bar{\omega}(x) = 0$, shrinks to a point as $m_{\gamma} \to 0$. We shall show that the elimination of pinching also occurs in the $m_{\gamma} = 0$ case.
In this paper we identify two mechanisms leading to relation (1.6). They are based on the observation that in the pinch-like contribution loop particles have to be on mass shell.

The first mechanism is effective in out of equilibrium QED: in the pinch-like contribution to the electron propagator, phase space vanishes linearly as $x \to 0$. In the pinch-like contribution to the photon propagator, the domain of integration is shifted to infinity as $x \to 0$. For distributions disappearing rapidly enough at large energies, the contribution again vanishes linearly in the $x \to 0$ limit. This mechanism is also valid in QCD in the cases with massive quarks.

In out of equilibrium massless QCD, phase space does not vanish, but there is an alternative mechanism: the spinor/tensor structure in all cases leads to relation (1.6).

Also, in out of equilibrium massless QCD, introduction of a small gluon mass does not help. In this case, processes like $q\bar{q} \to g$ are kinematically allowed, the spinor/tensor structure is modified, and $\Omega$ does not vanish in the $x \to 0$ limit.

In a few cases, none of the mentioned mechanisms works and one has to sum the Schwinger-Dyson series. This is the case of the $\pi - \rho$ loop in the $\pi$ self-energy. Even in the limit of zero pion mass, $\tilde{\omega}(x)$ vanishes only as $|x|^{1/2}$ and relation (1.6) is not fulfilled. A similar problem appears in electroweak interactions involving decays of $Z$ and $W$ bosons, decay of Higgs particles, etc.

Another important case is massless $g^2\phi^3$ theory. In contrast to massless QCD, massless $g^2\phi^3$ theory contains no spin factor to provide a $q^2$ factor necessary to obtain (1.6).

The densities are restricted only mildly: they should be cut off at high energies, at least as $|k_o|^{-3-\delta}$, in order to obtain a finite total particle density; for nonzero $k_o$, they should be finite; for $k_o$ near zero, they should not diverge more rapidly than $|k_o|^{-1}$, the electron (positron) distribution should have a finite derivative. Further restrictions may come from Slavnov-Taylor identities\cite{24,25,26}, but they are not crucial for our analysis.

When necessary we assume that the zero-temperature renormalization has already been performed. The "finite temperature"\cite{24,25,26,30} renormalization out of equilibrium\cite{31} may give rise to new problems formally similar to those treated here. Their treatment is beyond the scope of this paper.

The paper is organized as follows.

In Sec. II we analyze the Schwinger-Dyson equation in the Keldysh representation, solve it formally, and identify pinch-like expressions. For one-loop self-energy insertions, we find that the Keldysh component ($\tilde{\Omega}(q^2)$) of the self-energy is responsible for pinches. We further find that the nonzero Keldysh component requires loop particles to be on shell.

In Sec. III we analyze functions such as $\Omega$, $\tilde{\Omega}$, and $\text{Im}\Sigma_R$, and investigate their threshold properties.

In Sec. IV we show that the electron and photon propagators, calculated in the single self-energy insertion approximation, are free from pinching.

In Sec. V we analyze pinch-like expressions in the $q - \bar{q}$, $g - g$, and ghost-ghost contributions to the gluon propagator, the quark propagator and the ghost propagator in the single self-energy insertion approximation. We find that, in all the cases, the spinor/tensor factor $F$ contains a factor $q^2$ that is sufficient to eliminate pinching.

In Sec. VI we briefly recollect the main results of the paper.
2 Propagators and the Schwinger-Dyson equation

We start \[32,33\] by defining out of equilibrium thermal propagators for bosons, in the case when we can ignore the variations of slow variables in Wigner functions \[12,34\]:

\[
D = \begin{pmatrix}
D_{11} & D_{12} \\
D_{21} & D_{22}
\end{pmatrix},
\]

(2.1)

\[
D_{11}(k) = D_{22}^*(k) = \frac{i}{k^2 - m^2 + 2i|k_o|} + 2\pi \sinh^2 \theta \delta(k^2 - m^2),
\]

(2.2)

\[
D_{12}(k) = 2\pi \delta(k^2 - m^2)(\cosh^2 \theta \Theta(k_o) + \sinh^2 \theta \Theta(-k_o)),
\]

(2.3)

\[
D_{21}(k) = 2\pi \delta(k^2 - m^2)(\cosh^2 \theta \Theta(-k_o) + \sinh^2 \theta \Theta(k_o)).
\]

(2.4)

For particles with additional degrees of freedom, relations (2.1)-(2.4) are provided with extra factors \((k + m)/(k - m)\) for spin 1/2, \((g_{\mu\nu} - (1 - a)k_\mu k_\nu/(k^2 \pm 2i\epsilon k_o))\) for vector particle, etc., and similarly for internal degrees of freedom. To keep the discussion as general as possible, we show these factors explicitly only when necessary. The propagator defined by relations (2.1)-(2.4) satisfies the important condition

\[0 = D_{11} - D_{12} - D_{21} + D_{22}.\]

(2.5)

In the case of equilibrium, we have

\[
\sinh^2 \theta(k_o) = n_B(k_o) = \frac{1}{\exp \beta |k_o| - 1}.
\]

(2.6)

To obtain the corresponding relations for fermions, we only need to make the substitution

\[
\sinh^2 \theta(k_o) \rightarrow -\sin^2 \bar{\theta}(k_o).
\]

(2.7)

In the case of equilibrium, for fermions we have

\[
\sin^2 \bar{\theta}_{F,\bar{F}}(k_o) = n_{F,\bar{F}}(k_o) = \frac{1}{\exp \beta(|k_o| + \mu) + 1}.
\]

(2.8)

Out of equilibrium, \(n_B(k_o)\) and \(n_F(k_o)\) will be some given functions of \(k_o\).

To transform into the Keldysh form, one defines the matrix Q as

\[
Q = \frac{1}{\sqrt{2}} \begin{pmatrix}
-1 & 1 \\
1 & 1
\end{pmatrix}.
\]

(2.9)

Now

\[
\begin{pmatrix}
0 & D_R \\
D_A & D_K
\end{pmatrix} = QDQ^{-1},
\]

(2.10)

\[
D_R(k) = -D_{11} + D_{21} = \frac{-i}{k^2 - m^2 + 2i\epsilon k_o},
\]

(2.11)

\[
D_A(k) = -D_{11} + D_{12} = \frac{-i}{k^2 - m^2 - 2i\epsilon k_o} = -D_R^*(k) = D_R(-k),
\]

(2.12)
\[ D_K(k) = D_{11} + D_{22} = 2\pi \delta(k^2 - m^2)(1 + 2 \sinh^2 \theta). \] (2.13)

We need \( D_K \) expressed through \( D_R \) and \( D_A \):
\[ D_K = h(k_o)(D_R - D_A), \quad h(k_o) = -\epsilon(k_o)(1 + 2 \sinh^2 \theta). \] (2.14)

Again for fermions, \( D_K \) is equal to
\[ D_K(k) = D_{11} + D_{22} = 2\pi \delta(k^2 - m^2)(1 - 2 \sin^2 \bar{\theta}). \] (2.15)

The proper self-energy
\[ \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \] (2.16)

satisfies the condition
\[ 0 = \Sigma_{11} + \Sigma_{12} + \Sigma_{21} + \Sigma_{22}. \] (2.17)

It is also transformed into the Keldysh form (in Niemi’s paper there is a misprint using \( Q^{-1} \) instead of \( Q \)):
\[ \begin{pmatrix} \Omega & \Sigma_A \\ \Sigma_R & 0 \end{pmatrix} = Q \Sigma Q^{-1}, \] (2.18)
\[ \Sigma_R = - (\Sigma_{11} + \Sigma_{21}), \] (2.19)
\[ \Sigma_A = - (\Sigma_{11} + \Sigma_{12}), \] (2.20)
\[ \Omega = \Sigma_{11} + \Sigma_{22}. \] (2.21)

We also find
\[ \Sigma_A = \Sigma_A^*. \] (2.22)

The ”cutting rules” (refs. [33, 36], see also ref. [37] for application of the rules out of equilibrium) will convince us that only on-shell loop-particle momenta contribute to \( \text{Im} \Sigma_R \) and \( \Omega \).

The calculation of the \( \Sigma \) matrix gives (propagators \( S(k) \) and \( G(k) \) in the self-energy matrix and in the Schwinger-Dyson equation are also given by (2.1) to (2.14), with the spin indices suppressed to keep the discussion as general as possible):
\[ \Sigma_R = -i \frac{1}{2} g^2 \int \frac{d^4k}{(2\pi)^4} (D_R(k)(S_R(k - q) - S_K(k - q)) + (D_A(k) - D_K(k))S_A(k - q)), \] (2.23)
\[ \Sigma_A = -i \frac{1}{2} g^2 \int \frac{d^4k}{(2\pi)^4} (D_R(k) - D_K(k))S_R(k - q) + D_A(S_A(k - q) - S_K(k - q)), \] (2.24)
\[ \Omega = i \frac{1}{2} g^2 \int \frac{d^4k}{(2\pi)^4} ((D_R(k) + D_A(k))(S_A(k - q) + S_R(k - q)) + D_K(k)S_K(k - q)). \] (2.25)

The Schwinger-Dyson equation
\[ \mathcal{G} = G + iG\Sigma\mathcal{G}, \] (2.26)
can be written in the Keldysh form as
\[ \begin{pmatrix} 0 & \mathcal{G}_R \\ \mathcal{G}_A & \mathcal{G}_K \end{pmatrix} = \begin{pmatrix} 0 & \mathcal{G}_R \\ \mathcal{G}_A & \mathcal{G}_K \end{pmatrix} + i \begin{pmatrix} 0 & \mathcal{G}_A \Sigma_A \mathcal{G}_A \\ \mathcal{G}_A \Sigma_A \mathcal{G}_A & \mathcal{G}_A \Omega \mathcal{G}_R + \mathcal{G}_K \Sigma_R \mathcal{G}_R + \mathcal{G}_A \Sigma_A \mathcal{G}_K \end{pmatrix}. \] (2.27)
By expanding (2.27), we deduce the contribution from the single self-energy insertion to be of the form

\[ \mathcal{G}_R \approx G_R + iG_R \Sigma_R G_R, \quad \mathcal{G}_A \approx G_A + iG_A \Sigma_A G_A, \]  

which is evidently well defined, and the Keldysh component suspected for pinching:

\[ \mathcal{G}_K \approx G_K + iG_A \Omega G_R + iG_K \Sigma_R G_R + iG_A \Sigma_A G_K. \]  

(2.29)

It is easy to obtain a solution \[^ {33} \] for \( \mathcal{G}_R \) and \( \mathcal{G}_A \) using the form (2.27). One observes that the equations for \( \mathcal{G}_R \) and \( \mathcal{G}_A \) are simple and the solution is straightforward:

\[ \mathcal{G}_R = \frac{1}{G_R^{-1} - i\Sigma_R} = -\mathcal{G}_A^*. \]  

(2.30)

To calculate \( \mathcal{G}_K \), we can use the solution (2.26) for \( \mathcal{G}_R \) and \( \mathcal{G}_A \):

\[ \mathcal{G}_K = \mathcal{G}_A (G_A^{-1} G_K G_R^{-1} + i\Omega) \mathcal{G}_R. \]  

(2.31)

Now we eliminate \( \mathcal{G}_K \) with the help of (2.14):

\[ \mathcal{G}_K = \mathcal{G}_A \left( h(q_o)(G_A^{-1} - G_R^{-1}) + i\Omega \right) \mathcal{G}_R. \]  

(2.32)

The first term in (2.32) is not always zero, but it does not contain pinching singularities! The second term in (2.32) is potentially ill-defined (or pinch-like). The pinch-like contribution appears only in this equation; thus it is the key to the whole problem of pinch singularities. In the one-loop approximation, it requires loop particles to be on mass shell. This will be sufficient to remove ill-defined expressions in all studied cases.

Equation (2.32) differs from the one used in Ref. \[^ {12} \]. Indeed in Ref. \[^ {12} \] \( \text{Im} \mathcal{G}_R \) is used instead of \( \mathcal{G}_R \mathcal{G}_A \).

We start with (2.29). After substituting (2.14) into (2.29), we obtain the regular term plus the pinch-like contribution:

\[ \mathcal{G}_K \approx \mathcal{G}_{Kr} + \mathcal{G}_{Kp}, \]  

(2.33)

\[ \mathcal{G}_{Kr} = h(q_o) (G_R - G_A + iG_R \Sigma_R G_R - iG_A \Sigma_A G_A), \]  

(2.34)

\[ \mathcal{G}_{Kp} = iG_A \Omega G_R, \quad \Omega = \Omega - h(q_o)(\Sigma_R - \Sigma_A). \]  

(2.35)

For equilibrium densities, we have \( \Sigma_{21} = e^{-\beta q_o} \Sigma_{12} \) , and expression (2.35) vanishes identically. This is also true for fermions.

Expression (2.35) is the only one suspected of pinch singularities at the single self-energy insertion level. The function \( \Omega \) in (2.35) belongs to the type of functions characterized by the fact that both loop particles have to be on mass shell. It is analyzed in detail in Secs. III and IV (for threshold effect) and in Sec. V (for spin effect). With the help of this analysis we show that relation (2.35) transforms into

\[ \mathcal{G}_{Kp} = -i \frac{K(q^2, m^2, q_o)}{2} \left( \frac{1}{q^2 - m^2 + 2i\epsilon q_o} + \frac{1}{q^2 - m^2 - 2i\epsilon q_o} \right), \]  

(2.36)
where $K(q^2, m^2, q_o)$ is $\bar{\Omega}/(q^2 - m^2)$ multiplied by spinor/tensor factors included in the definition of $G_{R,A}$. The finiteness of the limit

$$\lim_{q^2 \to m^2 \mp 0} K(q^2, m^2, q_o) = K_\pm(q_o) < \infty$$

(2.37)

is important for cancellation of pinches. The index $\mp$ indicates that the limiting value $m^2$ is approached from either below or above, and these two values are generally different. To isolate the potentially divergent terms, we express the function $K(q^2, m^2, q_o)$ in terms of functions that are symmetric ($K_1(q^2, m^2, q_o)$) and antisymmetric ($K_2(q^2, m^2, q_o)$) around the value $q^2 = m^2$:

$$K(q^2, m^2, q_o) = (K_1(q^2, m^2, q_o) + \epsilon(q^2 - m^2)K_2(q^2, m^2, q_o))$$

(2.38)

These functions are given by

$$K_{1,2}(q^2, m^2, q_o) = \frac{1}{2}(K(q^2, m^2, q_o) \pm K(2m^2 - q^2, m^2, q_o))$$

(2.39)

Locally (around the value $q^2 = m^2$), these functions are related to the limits $K_\pm(q_o)$ by

$$K_{1,2}(q^2, m^2, q_o) = \frac{1}{2}(K_+(q_o) \pm K_-(q_o))$$

(2.40)

As a consequence, the right-hand side of expression (2.36) behaves locally as

$$\mathcal{G}_{Kp}(q^2, m^2, q_o) \approx -\frac{i}{2} \left( K_1(q_o) + \epsilon(q^2 - m^2)K_2(q_o) \right) \left( \frac{1}{q^2 - m^2 + 2i\epsilon q_o} + \frac{1}{q^2 - m^2 - 2i\epsilon q_o} \right)$$

(2.41)

and the term proportional to $K_2$ is capable of producing logarithmic singularity. Furthermore, we were unable to eliminate pinches related to the double, triple, etc., self-energy insertion contributions to the propagator.

### 3 Threshold factor

In this section we analyze the phase space of the loop integral with both loop particles on mass shell. Special care is devoted to the behavior of this integral near thresholds. In this analysis the densities are constrained only mildly: they are supposed to be finite and smooth, with a possible exception at zero energy. We also assume that the total density of particles is finite. The expressions are written for all particles being bosons, and spins are not specified; change to fermions is elementary.

To obtain the integrals over the products of $D_{R,A}$ and $S_{R,A}$, we start with a useful relation:

$$\int_{-\infty}^{+\infty} dk_o f(k, q) \frac{1}{(k^2 - m_D^2 + i\lambda k_o \epsilon)((k - q)^2 - m_S^2 + i\eta(k_o - q_o)\epsilon)}$$

$$= -i\pi \int_{-\infty}^{+\infty} dk_o f(k, q) \mathcal{P}\left( \frac{\lambda\epsilon(k_o)\delta(k^2 - m_D^2)}{(k - q)^2 - m_S^2} + \frac{\eta\epsilon(k_o - q_o)\delta((k - q)^2 - m_S^2)}{k^2 - m_D^2} \right)$$

$$+ 2\pi^2 \delta_{\lambda - \eta} \int_{-\infty}^{+\infty} dk_o f(k, q) \epsilon(k_o)\epsilon(k_o - q_o)\delta(k^2 - m_D^2)\delta((k - q)^2 - m_S^2)$$

(3.1)
where \( f(k, q) \), as a function of \( k_o \), is some polynomial of order 0, 1, 2, or 3.

Relation (3.1) is obtained as an average of the results obtained by closing the integration path through the upper and through the lower semi-plane.

In the defining relations (2.2) to (2.4), the particle distribution \( \sinh^2 \theta \) (and similarly for \( \sin^2 \bar{\theta} \)) appears only in the expression where it is multiplied by \( \delta(k^2 - m^2) \). Thus there is freedom to replace \( k_o \) by its on mass shell value \( \epsilon(k_o)\omega_D \), where \( \omega_D = (\bar{k} - m^2_D)^{1/2} \). The physical results should not be altered by this replacement. Then we write \( h(k_o) \), defined in (2.14), as \( -k_o/\omega_D(1 + 2 \sinh^2(\omega_D)) \), and similarly for \( h(k_o - q_o) \). This substitution matters when one wants to perform integrals in (2.23) to (2.25) with the help of (3.1). With this replacement, \( f(k, q) \) in (3.3) does not depend on densities and (3.1) can be proved as stated above, without need to discuss the analytic properties of \( \sinh^2 \theta \) and \( \sin^2 \bar{\theta} \). Without our replacement, one would be immediately stack with the question how these quantities (i.e., \( \sinh^2 \theta \) and \( \sin^2 \bar{\theta} \)), supposed to be known, within some uncertainty, along the real axis, behave in the complex plane very far from the real axis!

Similar relations could be obtained for higher powers of \( D_{R-A} \) and \( S_{R-A} \). For example, for the nth power of \( k^2 - m^2_D + i\lambda k_o\epsilon \), the real part of the integral will be obtained by substituting \( \delta(n)(k^2 - m^2_D)(-1)^{(n)} \) instead of \( \delta(k^2 - m^2_D) \). Now we easily calculate \( Re\Sigma_R \) as

\[
Re\Sigma_R = \frac{-g^2}{2(2\pi)^3} \int d^4k \bar{P} \left( \frac{\epsilon(k_o)\delta(k^2 - m^2_D)h_D(k_o)}{(k^2 - m^2_D)} + \frac{\epsilon(k_o - q_o)\delta((k - q)^2 - m^2_S)}{(k^2 - m^2_D)} \right) F.
\]

(3.2)

\( F \) is the factor dependent on spin and internal degrees of freedom. The thermal part of \( Re\Sigma_R \) is given by:

\[
Re\Sigma_R \theta_b = \frac{g^2}{(2\pi)^3} \int d^4k \bar{P} \left( \frac{\delta(k^2 - m^2_D)\sinh^2_{D}(k_o)}{(k^2 - m^2_D)} + \frac{\delta((k - q)^2 - m^2_S)\sinh^2_{S}(k_o - q_o)}{(k^2 - m^2_D)} \right) F.
\]

(3.3)

At equilibrium equation (3.3) (after necessary boson→fermion conversion) agrees with the known results.

Now, starting from (2.19) to (2.23), we calculate \( \Omega \) and \( Im\Sigma_R \).

\[
\Omega = 2iIm\Sigma_{11} = \frac{2ig^2}{2} \int \frac{d^4k}{(2\pi)^4} 4\pi^2\delta(k^2 - m^2_D)\delta((k - q)^2 - m^2_S)N_\Omega(k_o, k_o - q_o)F,
\]

(3.4)

where

\[
N_\Omega(k_o, k_o - q_o) = \frac{1}{2}(-\epsilon(k_o(k_o - q_o)) + (1 + 2 \sinh^2 \theta_D(k_o))(1 + 2 \sinh^2 \theta_S(k_o - q_o)),
\]

(3.5)

\[
Im\Sigma_R = \frac{g^2}{2} \int \frac{d^4k}{(2\pi)^4} 4\pi^2\delta(k^2 - m^2_D)\delta((k - q)^2 - m^2_S)N_R(k_o, k_o - q_o)F,
\]

(3.6)

and

\[
N_R(k_o, k_o - q_o) = (\sinh^2 \theta_D(k_o)\epsilon(k_o - q_o) + \sinh^2 \theta_S(k_o - q_o)\epsilon(-k_o) + \Theta(-k_o)\Theta(k_o - q_o) - \Theta(k_o)\Theta(q_o - k_o)).
\]

(3.7)
At equilibrium equations (3.3) to (3.7) agree (after setting \( F = -1 \) for scalar case) with the corresponding equations obtained for boson-boson intermediate state\(^\text{[20]}\).

It is useful to define \( N_\Omega(k_o, k_o - q_o) \) as

\[
N_\Omega(k_o, k_o - q_o) = N_\Omega(k_o, k_o - q_o) - h(q_o)N_R(k_o, k_o - q_o). \tag{3.8}
\]

After integrating over \( \delta \)'s, one obtains expressions of the general form

\[
I = \frac{1}{4|\vec{q}|} \int |\vec{k}| dk_o d\phi N(k_o, k_o - q_o) F(q_o, |\vec{q}|, k_o, |\vec{k}||q^2_o, ...)|(1 - z_o^2), \tag{3.9}
\]

where \( |\vec{k}| = (k_o^2 - m_D^2)^{1/2} \),

\[
q^2 = |\vec{q}|^2 = |\vec{k}||z_o|^2, \tag{3.10}
\]

\[
z_o = \frac{q^2 + \vec{k}^2 - (\vec{q} - \vec{k})^2}{2|\vec{k}||\vec{q}|}. \tag{3.11}
\]

\( \phi \epsilon(0, 2\pi) \) is the angle between vector \( \vec{k}_T \) and \( x \) axes.

Let us start with the \( q^2 > 0 \) case. Solution of \( \Theta(1 - z_o^2) \) gives the integration limits

\[
k_o = \frac{1}{2q^2} \left( q_o(q^2 + m_D^2 - m_S^2) \mp |\vec{q}|(q^2 - q_{+tr}^2)(q^2 - q_{-tr}^2)^{1/2} \right), \tag{3.12}
\]

or

\[
|\vec{k}| = \frac{1}{2q^2} \left( |\vec{q}|(q^2 + m_D^2 - m_S^2) \mp q_o((q^2 - q_{+tr}^2)(q^2 - q_{-tr}^2)^{1/2} \right), \tag{3.13}
\]

\[
q_{\pm tr} = |m_D \pm m_S|. \tag{3.14}
\]

Assume now that \( q_{tr} \neq 0 \). In this case, at the threshold, the limits shrink to the value

\[
k_{o tr} = \frac{q_o(q_{tr}^2 + m_D^2 - m_S^2)}{2q^2_{tr}}, \quad |\vec{k}|_{tr} = \frac{|\vec{q}|(q_{tr}^2 + m_D^2 - m_S^2)}{2q^2_{tr}}. \tag{3.15}
\]

Near the threshold, it is convenient to replace the integration variable by \( dk_o |k_o||\vec{k}| = d|\vec{k}|. \)

Now, for \( |q^2 - q_{tr}^2| \) such small, that the integration limits \( k_o 1, 2 \) are both of the same sign as \( k_{otr} \), we have \( k_o = \epsilon(k_{otr})(\vec{k}_2^2 + m_D^2)^{1/2} \).

We define the coefficient \( c_1 \) by

\[
c_1 = \frac{1}{4|\vec{q}|} \int d\phi N(k_{otr}, k_{otr} - q_o) F(q_o, |\vec{q}|, k_{otr}, |\vec{k}|_{otr}, q_{k_{otr}}, ...). \tag{3.16}
\]

Now the expression (3.9) can be approximated by

\[
I \approx c_1(|\vec{k}|_2 - |\vec{k}|_1)
\]

\[
\approx c_1(\Theta(q^2 - q_{+tr}^2) + \Theta(-q^2 + q_{-tr}^2)) \frac{q_o((q^2 - q_{+tr}^2)(q^2 - q_{-tr}^2)^{1/2}}{q^2}. \tag{3.17}
\]

Relation (3.17) is the key to further discussion of the threshold effect.
We obtain this also for higher dimension (D=6, for example).

Relation \( (3.17) \) put some limits on the behavior of density functions: they should not tend to infinity at any value of \( q_o \neq 0 \); near \( q_o = 0 \), owing to the presence of the factor \( q_o \), they should not rise more rapidly than \( q_o^{-1} \).

Owing to \( (3.12) \) and \( (3.14) \), the function \( \mathcal{I}(q^2, m_D^2, m_S^2) \) has the following properties important for cancellation of pinches.

It vanishes between the thresholds, i.e., the domain \((m_D - m_S)^2 < q^2 < (m_D + m_S)^2\) is forbidden \((\mathcal{I} = 0)\). If it happens that the bare mass \( m^2 \) belongs to this domain, the single self-energy insertion will be free of pinching. In this case, multiple (double, triple, etc.) self-energy insertions will also be free of pinching. Massive \( \lambda \phi^3 \) theory \([17]\) is a good example of this case.

It is (in principle) different from zero in the allowed domain \( q^2 < (m_D - m_S)^2 \) and \( (m_D + m_S)^2 < q^2 \). In this case, one cannot get rid of pinching. This situation appears in the \( \pi - \rho \) interaction \([19]\). An exception to this rule are occasional zeros owing to the specific form of densities.

The behavior at the boundaries (i.e., in the allowed region near the threshold) depends on the masses \( m_D \) and \( m_S \) and there are a few possibilities.

If both masses are nonzero and different \((0 \neq m_D \neq m_S \neq 0)\), then there are two thresholds and \( \mathcal{I} \) behaves as \((q^2 - q^2_{\pm tr})^{1/2}\) in the allowed region near the threshold \( q^2_{\pm tr} \). For \( m^2 = q^2_{tr} \), the power 1/2 is not large enough to suppress pinching.

If one of the masses is zero \((m_D \neq 0, m_S = 0 \text{ or } m_D = 0, m_S \neq 0)\), then \( (3.17) \) gives that the thresholds are identical (i.e., the forbidden domain shrinks to zero) and one obtains the \((q^2 - m_D^2)^{1/2}\) behavior near \( m_D^2 \). This case (for \( m^2 = m_D^2 \)) is promising. The elimination of pinching in the electron propagator, considered in Sec.IV, is one of important examples.

If the masses are equal but different from zero \((m_D = m_S \neq 0)\), then there are two thresholds with different behavior. The function \( \mathcal{I} \) behaves as \((q^2 - q^2_{+ tr})^{1/2}\) in the allowed region near the threshold \( q^2_{+ tr} = 4m_D^2 \). This behavior cannot eliminate pinching in the supposed case \( m^2 = 4m_D^2 \).

However, at the other threshold, namely at \( q^2_{- tr} = 0 \), the physical region is determined by \( q^2 < 0 \) and the above discussion does not apply. In fact, the integration limits \( (3.12) \) or \( (3.13) \) are valid, but the region between \( k_{o.1} \) and \( k_{o.2} \) is now excluded from integration. One has to integrate over the domain \((-\infty, k_{o.1}) \cup (k_{o.2}, +\infty)\). This leads to the limitation in the high-energy behavior of the density functions. An important example of such behavior, elimination of pinching in the photon propagator \((m_\gamma)\), is discussed in Sec.IV.

If both masses vanish \((m_D = m_S = 0)\), the thresholds coincide, there is no forbidden region and no threshold behavior. The behavior depends on the spin of the particles involved. For scalars, the leading term in the expansion of \( \mathcal{I} \) does not vanish. Pinching is not eliminated.

The case of vanishing masses \((m_D = m_S = 0)\) for particles with spin exhibits a peculiar behavior. In all studied examples (see Sec.V for details), \( \mathcal{I} \) behaves as \( q^2 \) as \( q^2 \to 0 \), which promises the elimination of pinching.

4 Pinch Singularities in QED
4.1 Pinch Singularities in the Electron Propagator

In this subsection we apply the results of preceding section to cancel the pinching singularity appearing in a single self-energy insertion approximation to the electron propagator. To do so, we have to substitute $m_D = m$, $m_S = 0$, $\sinh^2 \theta_D(k_o) \rightarrow -n_e(k_o)$, $\sinh^2 \theta_S(k_o - q_o) \rightarrow n_\gamma(k_o - q_o)$, and $h(k_o) = -\epsilon(k_o)(1 - 2n_e(k_o))$, where $n_e$ and $n_\gamma$ are given non-equilibrium distributions of electrons and photons in relations (3.5), (3.7), (3.8), and (2.14). The thresholds are now identical

$$q^2_{\pm tr} = m^2,$$  \hspace{1cm} (4.1)

and the integration limits are

$$k_{o \ 1, 2} = \frac{1}{2q^2} \left(q_o(q^2 + m^2) \mp |\vec{q}|((q^2 - m^2)) \right)$$  \hspace{1cm} (4.2)

or

$$|\vec{k}|_2 - |\vec{k}|_1 = \frac{q_o}{q^2}(q^2 - m^2)).$$  \hspace{1cm} (4.3)

At threshold the limits shrink to the value

$$k_{o \ tr} = q_o, \ \ |\vec{k}|_{tr} = |\vec{q}|.$$  \hspace{1cm} (4.4)

Then, with the help of (3.16), we define

$$K(q^2, m^2, q_o) = \frac{(\hat{q} + m)\bar{\Omega}(\hat{q} + m)}{(q^2 - m^2)} \approx \frac{1}{16\pi^2|\vec{q}|(q^2 - m^2)} \int d\phi N_\Omega(k_{otr} = q_o, k_{otr} - q_o = 0)$$

$$(\hat{q} + m)F(q_o, |\vec{q}|, k_{otr}, |\vec{k}|_{tr}, \bar{q}k_{tr}, ...) (\hat{q} + m)(|\vec{k}|_2 - |\vec{k}|_1).$$  \hspace{1cm} (4.5)

The trace factor $F$ is calculated with loop particles on mass shell:

$$F_{\gamma \gamma} = \left( g_{\mu \nu} - (1 - a)\frac{(k - q)_{\mu}(k - q)_{\nu}}{(k - q)^2 \mp 2i(k_o - q_o)\epsilon} \right) \gamma^\nu(k + m)\gamma^\nu$$

$$= \left( -2\hat{k} + 4m - (1 - a)(\hat{q} + m) - \frac{(\hat{k} - \hat{q})(-k^2)q^2}{(k - q)^2 \mp 2i(k_o - q_o)\epsilon} \right).$$  \hspace{1cm} (4.6)

In calculating the term proportional to $(1 - a)$, we have to use the trick

$$((k - q)^2 \pm i\epsilon)^{-2} = \lim_{m_z \rightarrow 0} \left[ \frac{\partial}{\partial m_z^2}((k - q)^2 \pm i\epsilon - m_z^2)^{-1} \right].$$  \hspace{1cm} (4.7)

For $q^2 \neq 0$, we can decompose the vector $k$ as

$$k = \frac{(k \cdot q)}{q^2} q + \frac{(k \cdot \tilde{q})}{q^2} \tilde{q} + k_T = (q - q_o)\frac{-m^2 + m^2 + q^2}{2q^2} + \frac{k_o}{|\vec{q}|} \tilde{q} + k_T,$$  \hspace{1cm} (4.8)

where, in the heat-bath frame with the $z$ axis oriented along the vector $\vec{q}$, we have

$$q = (q_o, 0, 0, |\vec{q}|), \ \ \tilde{q} = (|\vec{q}|, 0, 0, q_o), \ \ q\tilde{q} = 0, \ \ q^2 = -q^2.$$

$$(\hat{k} - \hat{q})(-k^2)q^2 = (4.1).$$  \hspace{1cm} (4.9)
to the photon propagator, we have to make the substitutions

\[(q^2 - m^2)\left(-\frac{q^2 - m^2}{q^2} \hat{q}^2 + \left(-\frac{q_o(q^2 + m^2)}{q^2|q|} + 2k_o \right)^2 + 2k^2_T\right)

\[-(1-a)\left(\frac{q^2 - m^2}{2q^2}\left(-\hat{q} + \frac{q_o}{|q|}\right)\right). \quad (4.10)\]

The transverse component of \(k, k_T\) vanishes after integration over \(\phi\).

Finally, we obtain

\[(\hat{q} + m)\tilde{F}(\hat{q} + m) = 2m(q^2 + m^2 + 2m\hat{q}) \]

\[+(q^2 - m^2)\left(-\frac{q^2 - m^2}{q^2} \hat{q}^2 + \left(-\frac{q_o(q^2 + m^2)}{q^2|q|} + 2k_o \right)^2 + 2k^2_T\right)\]

\[-(1-a)\left(\frac{q^2 - m^2}{2q^2}\left(-\hat{q} + \frac{q_o}{|q|}\right)\right). \quad (4.10)\]

Now we can study the limit

\[\mathcal{K}(q_o) = \lim_{q^2 \to m^2} K(q^2, m^2, q_o) \]

\[= (\hat{q} + m)\frac{q_o}{2\pi|q|m^2} N_{\Omega}(k_o, k_o - q_o). \quad (4.11)\]

It is easy to find that \(\mathcal{K}(q_o)\) is finite provided that \(m^2 \neq 0\) and \(N_{\Omega}(q_o, 0) < \infty\). The last condition is easy to investigate using the limiting procedure:

\[N_{\Omega}(q_o, 0) = \lim_{k_o \to q_o} N_{\Omega}(k_o, k_o - q_o) = \lim_{k_o \to q_o} 2n_{\gamma}(k_o - q_o)(n_e(q_o) - n_e(k_o)) \]

\[+ \lim_{k_o \to q_o} \left((n_e(q_o) - n_e(k_o)) - (e(q_o)e(k_o - q_o)(n_e(q_o) + n_e(k_o) - 2n_e(q_o)n_e(k_o))\right). \quad (4.12)\]

One should observe here that the integration limits imply that the limit \(k_o \to q_o\) is taken from below for \(q^2 > m^2\), and from above for \(q^2 < m^2\). The two limits lead to different values of \(N_{\Omega}(q_o, 0)\). Only the first term in (4.12) can give rise to problems. We rewrite it as \(\lim_{k_o \to 0} (2k_o n_{\gamma}(k_o) \frac{\partial n_e(k_o + q_o)}{\partial k_o})\). As relation \((4.11)\) should be valid at any \(q_o\), we can integrate over \(q_o\) to find that the photon distribution should not grow more rapidly than \(|k_o|^{-1}\) as \(k_o\) approaches zero, while the derivative of the electron distribution \(n_e(q_o)\) should be finite at any \(q_o:\)

\[\lim_{k_o \to 0} k_o n_{\gamma}(k_o) < \infty, \quad (4.13)\]

\[\left|\frac{\partial n_e(q_o)}{\partial q_o}\right| < \infty. \quad (4.14)\]

Under the very reasonable conditions \((4.13)\) and \((4.14)\) the electron propagator is free from pinches.

It is interesting to observe the discontinuity of \(K(q^2, m^2, q_o)\) at the point \(q^2 = m^2\). This feature will be repeated in massless QCD.

It is worth observing that \(K(q_o)\) is gauge independent, at least within the class of covariant gauges.

### 4.2 Pinch Singularities in the Photon Propagator

To consider the pinching singularity appearing in a single self-energy insertion approximation to the photon propagator, we have to make the substitutions \(m_D = m = m_S, \sinh^2 \theta_D(k_o) \to\)
\(-n_e(k_o), \sinh^2 \theta_S(k_o - q_o) \rightarrow -n_e(k_o - q_o), \) and \(h(k_o) = -\varepsilon(k_o)(1 + 2n_e(k_o)).\) There are two thresholds, but only \(q_{tr}^2 = 0\) and the domain where \(q^2 < 0\) are relevant to a massless photon. The integration limits are given by the same expression (1.2), but now we have to integrate over the domain \((-\infty, k_{o1}) \cup (k_{o2}, +\infty).\) As \(q^2 \rightarrow -0,\) we find \((k_{o1} \rightarrow -\infty)\) and \((k_{o2} \rightarrow +\infty).\) The integration domain is still infinite but is shifted toward \(\pm \infty\) where one expects that the particle distribution vanishes:

\[
K_{\mu\nu}(q^2, q^2) = \left( g_{\mu\rho} - (1 - a) \frac{q_{\rho}q_{\rho}}{q^2 - 2i q_0 \varepsilon} \right) \frac{\bar{\Omega}_{\rho\sigma}}{q^2} \left( g_{\sigma\nu} - (1 - a) \frac{q_{\sigma}q_{\nu}}{q^2 - 2i q_0 \varepsilon} \right)
= \frac{1}{16\pi^2 |q|^2} \left( \int_{-\infty}^{k_{o1}} + \int_{k_{o2}}^{\infty} \right) \frac{k_0dk_0}{|k|} \int d\phi N_{\Omega}(k_0, k_0 - q_0) \left( g_{\mu\rho} - (1 - a) \frac{q_{\rho}q_{\rho}}{q^2 - 2i q_0 \varepsilon} \right) \frac{\bar{\Omega}_{\rho\sigma}}{q^2} \left( g_{\sigma\nu} - (1 - a) \frac{q_{\sigma}q_{\nu}}{q^2 - 2i q_0 \varepsilon} \right) .
\]

To calculate \(F^{\mu\nu}\) for the \(e - \bar{e}\) loop, we parametrize the loop momentum \(k\) by introducing an intermediary variable \(l\) perpendicular to \(q, \) \(m\) is the mass of loop particles:

\[
k = \alpha q + l, \quad q.l = 0, \quad k^2 = (k - q)^2 = m^2, \quad l^2 = m^2 - \alpha^2 q^2, \quad \alpha = \frac{k^2 + q^2 - (k - q)^2}{2q^2}.
\]

At the end of the calculation we eliminate \(l\) in favor of \(k.\) After all possible singular denominators are canceled, one can set \(\alpha = 1/2.\)

\[
F^{\mu\nu}_{e\bar{e}} = -Tr(k + m)\gamma^\mu(k - q + m)\gamma^\nu
= 2q^2 g^{\mu\nu} - 2q^\mu q^\nu + 8l^\mu l^\nu = \left( \frac{4m^2q_0^2}{q^2} A^{\mu\nu}(q) \right.
+ \left. \frac{q^2}{q^2} \left( 4k_0(k_0 - q_0) - 4m^2 - q^2 \right) A^{\mu\nu}(q) + (-8(k_0 - \frac{q_0}{2})^2 + 2q^2) B^{\mu\nu}(q) \right).
\]

Using relation (7.21) we obtain

\[
K_{\mu\nu}(q^2, q_o) = \frac{1}{16\pi^2|q|^2} \left( \int_{-\infty}^{k_{o1}} + \int_{k_{o2}}^{\infty} \frac{k_0dk_0}{|k|} \right) \int d\phi N_{\Omega}(k_0, k_0 - q_0)
\left( \frac{4m^2q_0^2}{q^2} A^{\mu\nu}(q) \right.
+ \left. \frac{q^2}{q^2} \left( 4k_0(k_0 - q_0) - 4m^2 - q^2 \right) A^{\mu\nu}(q) + (-8(k_0 - \frac{q_0}{2})^2 + 2q^2) B^{\mu\nu}(q) \right).
\]

In the integration over \(k_0\) the terms proportional to \((k_0^2 q^2)^n\) dominate and \(\lim_{q^2 \rightarrow 0} |K_{\mu\nu}(q^2, q_o)| < \infty\) if

\[
\left| \frac{1}{16\pi^2|q|^2} \left( \int_{-\infty}^{k_{o1}} + \int_{k_{o2}}^{\infty} \frac{k_0dk_0}{|k|} \right) \left( \alpha + \beta k_0^2 q^2 \right) \int d\phi N_{\Omega}(k_0, k_0 - q_0) \right| < \infty.
\]
Here \( N_{\Omega}(k_o, k_o - q_o) \) is given by
\[
N_{\Omega}(k_o, k_o - q_o) = -2n_e(k_o - q_o)(-n_\gamma(q_o) - n_e(k_o)) - n_\gamma(q_o) - n_e(k_o) - \epsilon(q_o)\epsilon(k_o - q_o)(-n_\gamma(q_o) + n_e(k_o) + 2n_\gamma(q_o)n_e(k_o)).
\] (4.20)

Assuming that the distributions obey the inverse-power law at large energies \( n_e(k_o) \propto |k_o|^{-\delta_e} \) and \( n_\gamma(k_o) \propto |k_o|^{-\delta_\gamma} \), we find that the terms linear in densities dominate. Thus, for \( n = 0, 1 \), one finds
\[
-\frac{1}{q^2} \left( \int_{-\infty}^{k_o} + \int_{k_o}^{+\infty} \right) |k_o|dk_o |k_o|^{2n-\delta}(-q^2)^n \propto (\delta - 1 - 2n)^{-1}(|q|m)^{1+2n-\delta}(-q^2)^{(\delta-3)/2}.
\] (4.21)

It follows that (4.19) is finite (in fact, it vanishes) if \( \delta_\epsilon, \delta_\gamma > 3 \). Similar analysis for electron propagator at \( q^2 < 0 \) (thus outside of our analysis of pinch singularities) leads to \( \delta_\gamma > 3 \). This is exactly the condition
\[
\int d^3k n_{\gamma,e,e}(k_o) < \infty.
\] (4.22)

Thus the pinching singularity is canceled in the photon propagator under the condition that the electron and positron distributions should be such that the total number of particles is finite.

Also, in the photon propagator, the quantity \( \lim_{q^2 \to 0} K_{\mu\nu}(q^2, q_o) \) does not depend on the gauge parameter.

Expression (4.21) is not valid for \( m = 0 \).

5 Pinch Singularities in Massless QCD

In this section we consider the case of massless QCD. Pinching singularities, related to massive quarks, are eliminated by the methods used in the preceding section.

In self-energy insertions related to gluon, quark, and ghost propagators, the masses in the loop as well as the masses of the propagated particles are zero. Thus, the methods of the preceding section do not produce the expected result. Attention is turned to the spin degrees of freedom, i.e., to the function \( F \) of the integrand in (3.4) to (3.9). In the calculation of \( F \) it has been anticipated that the loop particles have to be on mass shell. In this case, \( F \) provides an extra \( q^2 \) factor in all the cases considered, in which not all particles are scalars. This \( q^2 \) factor suffices for the elimination of pinching singularities.

The integration limits are now
\[
k_{o \, 1, 2} = \frac{1}{2} (q_o \mp |\bar{q}|).
\] (5.1)

The difference \( |\bar{k}|_2 - |\bar{k}|_1 \) is finite and there is no threshold effect.

It is worth observing that for \( q^2 > 0 \), we have to integrate between \( k_{o1} \) and \( k_{o2} \), whereas for \( q^2 < 0 \), the integration domain is \( (-\infty, k_{o1}) \cup (k_{o2}, +\infty) \). This leads to two limits, \( \lim_{q^2 \to 0} K(q^2, q_o) = K_{\pm}(q_o) \), in all cases of massless QCD.

By inspection of the final results (2.4), (5.5), and (5.6), we find that the case \( q^2 < 0 \) requires integrability of the function \( k_o^2 N_{\Omega}(k_o, k_o - q_o) \) leading to the condition (4.22) on the quark, gluon, and ghost distribution functions.

By using (1.16), we again introduce the intermediary variable \( l \) perpendicular to \( q \); now we have to set \( m = 0 \).
5.1 Self-Energy Insertions Contributing to the Gluon Propagator

The function $K_{\mu\nu}(q^2, q_o)$ related to the gluon propagator is the sum

$$K_{\mu\nu}(q^2, q_o) = \Sigma_i K_{q_i\bar{q}_i, \text{massive,} \mu\nu}(q^2, q_o)$$

$$+ \Sigma_i K_{q_i\bar{q}_i, \text{massless,} \mu\nu}(q^2, q_o) + K_{ghgh, \mu\nu}(q^2, q_o) + K_{gg, \mu\nu}(q^2, q_o), \quad (5.2)$$

where the terms in the sum are defined as

$$K_{\mu\nu}(q^2, q_o) = (g_{\mu\rho} - (1 - \alpha)D_{R\mu\rho}(q))\frac{\bar{Q}_{\sigma\rho}}{q^2}(g_{\sigma\nu} - (1 - \alpha)D_{A\sigma\nu}(q)). \quad (5.3)$$

Pinching singularities, related to massive quarks, are eliminated by the methods used in the preceding section. The tensor $F$ related to the massless quark-antiquark contribution to the gluon self-energy is

$$F_{\mu\nu}^{\mu\nu} = -\frac{\delta_{ab}}{6}Trk^\mu(k - \hat{k})\gamma^\nu \equiv \frac{\delta_{ab}}{6}(2q^2g^{\mu\nu} - 2g^\mu q^\nu + 8l^\mu l^\nu)$$

$$= \frac{\delta_{ab}}{6}\left(\frac{q^2}{q^2} \left((4k_o(k_o - q_o) - q^2)A^{\mu\nu}(q) + (-8(k_o - \frac{q_o}{2})^2 + 2q^2)B^{\mu\nu}(q)\right) + O^{\mu\nu}(\hat{k}_T)\right). \quad (5.4)$$

As $F_{\mu\nu}$ contains only $A$ and $B$ projectors, relation (7.21) guarantees that the result does not depend on the gauge parameter.

Relation (5.4) contains only terms proportional to $q^2$, and $\lim_{q^2 \to 0} K_{\mu\nu}(q^2, q_o)$ is finite.

For the ghost-ghost contribution to the gluon self-energy, the tensor $F$ is given by

$$F_{ghgh}^{\mu\nu} = -\delta_{ab}N_c k^\mu(k - q)^\nu = -\delta_{ab}N_c \left(-\frac{q^2q^\nu}{4} + l^\mu l^\nu + \frac{q^\mu l^\nu - l^\mu q^\nu}{2}\right)$$

$$= -\delta_{ab}N_c \left(\frac{q^2}{q^2} \left(\frac{4k_o(k_o - q_o)}{8} + q^2A^{\mu\nu}(q) - (k_o - \frac{q_o}{2})^2B^{\mu\nu}(q) - \frac{q^2}{4}D^{\mu\nu}(q) + O^{\mu\nu}(\hat{k}_T)\right)\right). \quad (5.5)$$

The antisymmetric part vanishes after integration, so we have left it out from the final result in (5.3).

The tensor $F$ for the gluon-gluon contribution to the gluon self-energy is

$$F_{gg}^{\mu\nu} = \delta_{ab}N_c \left((g^{\mu\sigma}(q + k)^\tau - g^{\sigma\tau}(2k - q)^\mu + g^{\tau\mu}(k - 2q)^\sigma)\right)$$

$$\left(g_{\sigma\tau}^\prime - (1 - \alpha)\frac{(k - q)_\sigma(k - q)_\tau}{(k - q)^2 + 2i(k_o - q_o)\epsilon}\right)$$

$$\left((g^{\mu\sigma}(q + k)^\tau - g^{\sigma\tau}(2k - q)^\mu + g^{\tau\mu}(k - 2q)^\sigma)\right) \left(g_{\tau\sigma}^\prime - (1 - \alpha)\frac{k_\tau k_\sigma}{k^2 + 2ik_o\epsilon}\right)$$

$$= \delta_{ab}N_c \left(4q^2g_{\mu\nu} - \frac{9}{2}q_\mu q_\nu + 10l_\mu l_\nu - (1 - \alpha)(-5q_\mu q_\nu + 3q^2g_{\mu\nu})\right)$$

$$- \frac{1 - \alpha}{k^2 + 2ik_o\epsilon} \left(-\frac{q^2}{4}q_\mu q_\nu + 5q^2l_\mu l_\nu + q^2(l_\mu q_\nu + l_\nu q_\mu) + \frac{q^4}{4}g_{\mu\nu}\right)$$
\[-\frac{1-a}{(k-q)^2 + 2i(k_o - q_o)\epsilon}\left(-\frac{q^2}{4}q_\mu q_\nu + 5q^2 l_\mu l_\nu - q^2(l_\mu q_\nu + q_\mu l_\nu) + \frac{q^4}{4}g_{\mu\nu}\right)\]

\[+(1-a)^2\left(-2q_\mu q_\nu + \left(\frac{1-a}{k^2 + 2i k_o \epsilon}\right) - \frac{(1-a)^2}{(k-q)^2 + 2i(k_o - q_o)\epsilon}\right)2q^2(q_\mu l_\nu + l_\mu q_\nu)\]

\[+(1-a)^2\frac{(k^2 + 2ik_o \epsilon)((k-q)^2 + 2i(k_o - q_o)\epsilon)}{4q^4 l_\mu l_\nu}\]

\[\rightarrow \delta_{ab} N_c q^2 \left(\frac{1}{q^2} \left((10(k_o - \frac{q_o}{2})^2 + \frac{3}{2}q^2) A^{\mu\nu}(q)\right)\right.\]

\[\left.\ - (1-a)\left(\frac{1}{2} A^{\mu\nu} - B^{\mu\nu} - \frac{q_o}{q} C^{\mu\nu}\right)\right)\]

\[+(1-a)^2\left(-\frac{q^2}{q^2} A^{\mu\nu} + 2\frac{q_o}{q^2} B^{\mu\nu} - 2\frac{q_o}{q^2} C^{\mu\nu} - 2D^{\mu\nu} + O^{\mu\nu}(\vec{k}_T)\right)\]

Expressions (5.4), (5.5), and (5.6) have been obtained by substitution of (7.17), (7.19), and (7.20) and, finally, by eliminating the variable \(l\) in favor of \(k\). The tensor \(O^{\mu\nu}(\vec{k}_T)\) is linear in \(k\), thus it vanishes after integration over \(\phi\).

We note here that, in the Feynman gauge (\(a = 1\)), the operator \(C\) is absent from the gluon self-energy! Consequently, the relation originating from Slavnov-Taylor identities (proved in [24, 25, 26] for equilibrium densities), \(\Pi_L^2 = (q^2 - \Pi_L)\Pi_D\), is fulfilled at \(a = 0\) only if \(\Pi_D = 0\). Thus the contributions to \(\pi_D\) from the ghost-ghost and gluon-gluon self-energies mutually cancel, imposing restrictions on the densities related to unphysical degrees of freedom. As it does not interfere with the cancellation of pinches, the problem of unphysical degrees of freedom will be discussed elsewhere.

Finally, we need (7.21) in all three cases.

Expressions (5.4), (5.5), and (5.6) for the ghost-ghost, quark-antiquark, and gluon-gluon contributions to the gluon self-energy contain only terms proportional to \(q^2\). The function \(K^{\mu\nu}(q^2, q_o)\) approaches the finite value \(\bar{K}^{\mu\nu}(\pm, q_o)\).

Thus we have shown that the single self-energy contribution to the gluon propagator is free from pinching under the condition (4.22).

5.2 Quark-Gluon Self-Energy Contribution to the Quark Propagator

The \(K\) spinor for the quark-gluon contribution to the massless quark propagator is defined as

\[\bar{K}(q^2, q_o) = \bar{q} \frac{\Omega}{q^2} q\]  \hspace{1cm} (5.7)

In the self-energy of a massless quark coupled to a gluon the spin factor \(F\) is given by

\[F_{gq} = \delta_{ab} \frac{N_c^2 - 1}{2N_c} \left(g_{\mu\nu} - (1-a)\frac{(k - q)\mu(k - q)\nu}{(k-q)^2 \pm 2i(k_o - q_o)\epsilon}\right) \gamma^\mu \tilde{k}^\nu\]
\[ \delta_{ab} \frac{N^2 - 1}{2N_c} \left( -2\hat{k} - (1 - a)(\hat{q} - \frac{1}{2} - \frac{(\hat{k} - \hat{q})(q^2 - k^2)}{(k - q)^2 \pm 2i(k_o - q_o)\epsilon}) \right) \]

\[ \rightarrow \delta_{ab} \frac{N^2 - 1}{2N_c} \left( -\hat{q} + \frac{q_o}{|\hat{q}|}\hat{q} - 2\frac{k_o}{|\hat{q}|}\hat{k} - 2\frac{1 - a}{2}(-\hat{q} + \frac{q_o}{|\hat{q}|}\hat{q}) \right). \tag{5.8} \]

For our further discussion, we need the product

\[ \hat{q}\hat{F}_{qq}\hat{q} = \delta_{ab} \frac{N^2 - 1}{2N_c} q^2 \left( -\hat{q} + \frac{q_o}{|\hat{q}|}\hat{q} + 2\frac{k_o}{|\hat{q}|}\hat{k} + 2\frac{1 - a}{2}(-\hat{q} + \frac{q_o}{|\hat{q}|}\hat{q}) \right), \tag{5.9} \]

which contains the damping factor \( q^2 \). The term \( \hat{k}_T \) will be integrated out.

By inserting (5.9) into (5.7), we obtain (2.37) free from pinches.

To calculate \( K(q_o) \), we need the limit

\[ \lim_{q^2 \to 0} \frac{\hat{q}\hat{F}_{qq}\hat{q}}{q^2} = \delta_{ab} \frac{N^2 - 1}{2N_c} \frac{2(k_o - q_o)}{q_o} \hat{q}. \tag{5.10} \]

From (5.10) we conclude that \( K(q_o) \) does not depend on the gauge parameter.

Omitting details, we observe that pinching is absent from the quark propagator, also in the Coulomb gauge.

### 5.3 Ghost-Gluon Self-Energy Contribution to the Ghost Propagator

The \( K \) factor is defined as

\[ K(q^2, q_o) = \frac{\Omega}{q^2}. \tag{5.11} \]

The \( F \) factor for the ghost-gluon contribution to the ghost self-energy is

\[ F_{qgq} = \delta_{ab} N_c k^\mu q^\nu \left( g_{\mu\nu} - (1 - a)\frac{(k_\mu - q_\mu)(k_\nu - q_\nu)}{(k - q)^2 \pm 2i(k_o - q_o)\epsilon} \right) \]

\[ \rightarrow \delta_{ab} N_c \frac{q^2}{2}. \tag{5.12} \]

The factor \( q^2 \) ensures the absence of pinch singularity and a well-defined perturbative result.

### 5.4 Scalar-Photon Self-Energy Contribution to the Scalar Propagator

Although the massless scalar boson interacting with a photon is not part of massless QCD, it is treated using the same methods.

The \( K \) factor is defined as

\[ K(q^2, q_o) = \frac{\tilde{\Omega}}{q^2}. \tag{5.13} \]
The $F$ factor for the massless scalar-photon contribution to the scalar self-energy,

$$F_{s\gamma} = (q + k)^\mu(q + k)^\nu \left( g_{\mu\nu} - (1 - a) \frac{(k - q)_\mu(k - q)_\nu}{(k - q)^2 \pm 2i(k_o - q_o)\epsilon} \right) \frac{(k - q)_\mu(k - q)_\nu}{(k - q)^2 \pm 2i(k_o - q_o)\epsilon}$$

$$= q^2 \left( 2 - (1 - a) \frac{q^2 - k^2}{(k - q)^2 \pm 2i(k_o - q_o)\epsilon} \right) \to 2q^2,$$

(5.14)

clearly exhibits the $q^2$ damping factor!

6 Conclusion

Studying the out of equilibrium Schwinger-Dyson equation, we have found that ill-defined pinch-like expressions appear exclusively in the Keldysh component ($G_K$) of the resummed propagator (2.32), or in the single self-energy insertion approximation to it (2.35). This component does not vanish only in the expressions with the Keldysh component (2.21) ($\Omega$ or $\bar\Omega$ for the single self-energy approximation) of the self-energy matrix. This then requires that loop particles be on mass shell. This is the crucial point to eliminate pinch singularities.

We have identified two basic mechanisms for the elimination of pinching: the threshold and the spin effects.

For a massive electron and a massless photon (or quark and gluon) it is the threshold effect in the phase space integration that produces, respectively, the critical $q^2 - m^2$ or $q^2$ damping factors.

In the case of a massless quark, ghost, and gluon, this mechanism fails, but the spinor/tensor structure of the self-energy provides an extra $q^2$ damping factor.

We have found that, in QED, the pinching singularities appearing in the single self-energy insertion approximation to the electron and the photon propagators are absent under very reasonable conditions: the distribution function should be finite, exceptionally the photon distribution is allowed to diverge as $k_o^{-1}$ as $k_o \to 0$; the derivative of the electron distribution should be finite; the total density of electrons should be finite.

For QCD, identical conditions are imposed on the distribution of massive quarks and the distribution of gluons; the distributions of massless quarks and ghosts (observe here that in the covariant gauge, the ghost distribution is not required to be identically zero) should be integrable functions; they are limited by the finiteness of the total density.

In the preceding sections we have shown that all pinch-like expressions appearing in QED and QCD (with massless and massive quarks!) at the single self-energy insertion level do transform into well-defined expressions. Many other theories behave in such a way. However, there are important exceptions: all theories in which lowest-order processes are kinematically allowed do not acquire well-defined expressions at this level. These are electroweak interactions, processes involving Higgs and two light particles, a $\rho$ meson and two $\pi$ mesons, $Z$, $W$, and other heavy particles decaying into a pair of light particles, etc. The second important exception is massless $g^2\phi^3$ theory. This theory, in contrast to massless QCD, contains no spin factors to provide (1.7). In these cases, one has to resort to the resummed Schwinger-Dyson series.
The main result of the present paper is the cancellation of pinching singularities at the single self-energy insertion level in QED- and QCD-like theories. This, together with the reported\cite{12,13} cancellation of collinear singularities, allows the extraction of useful physical information contained in the imaginary parts of the two-loop diagrams. This is not the case with three-loop diagrams, because some of them contain double self-energy insertions. In this case, one again has to resort to the sophistication of resummed propagators.

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7 Appendix

We start by defining a heat-bath four-velocity $U^\mu$, normalized to unity, and define the orthogonal projector

$$\Delta_{\mu\nu} = g_{\mu\nu} - U_\mu U_\nu. \quad (7.1)$$

We further define spacelike vectors in the heat-bath frame:

$$\kappa_\mu = \Delta_{\mu\nu} q^\nu, \quad \kappa_\mu \kappa^\mu = \kappa^2 = -q^2. \quad (7.2)$$

There are four independent symmetric tensors (we distinguish retarded from advanced tensors by the usual modification of the $i\epsilon$ prescription) $A$, $B$, and $D$ (which are mutually orthogonal projectors), and $C$:

$$A_{\mu\nu}(q) = \Delta_{\mu\nu} - \frac{\kappa_\mu \kappa_\nu}{\kappa^2}, \quad (7.3)$$

$$B_{R,\mu\nu}(q) = U_\mu U_\nu + \frac{\kappa_\mu \kappa_\nu}{\kappa^2} - \frac{q_\mu q_\nu}{(q^2 + 2iq_\alpha \epsilon)}, \quad (7.4)$$

$$C_{R,\mu\nu}(q) = \frac{(-\kappa^2)^{1/2}}{U.q} \left( \frac{(U.q)^2}{\kappa^2} U_\mu U_\nu - \frac{\kappa_\mu \kappa_\nu}{\kappa^2} + \frac{q_\mu q_\nu}{q^2} \right), \quad (7.5)$$

$$D_{R,\mu\nu}(q) = \frac{q_\mu q_\nu}{q^2 + 2iq_\alpha \epsilon}. \quad (7.6)$$

In addition to the known multiplication properties

$$A(q)A(q) = A(q), \quad B_{R,A}(q)B_{R,A}(q) = B_{R,A}(q), \quad (7.7)$$

$$C_{R,A}(q)C_{R,A}(q) = -(B_{R,A}(q) + D_{R,A}(q)), \quad D_{R,A}(q)D_{R,A}(q) = D_{R,A}(q), \quad (7.8)$$

$$A(q)B(q) = B(q)A(q) = 0, \quad A(q)C(q) = C(q)A(q) = 0, \quad (7.9)$$

$$A(q)D(q) = D(q)A(q) = 0, \quad B(q)D(q) = D(q)B(q) = 0, \quad (7.10)$$

$$B_{R,A}(q)C_{R,A}(q) = (C_{R,A}(q)D_{R,A}(q))_{\mu\nu} = \frac{\tilde{q}_\mu q_\nu}{q^2 + 2iq_\alpha \epsilon}, \quad (7.11)$$

we need mixed products

$$B_{R,A}(q)B_{A,R}(q) = \frac{1}{2}(B_{R}(q) + B_{A}(q)), \quad (7.12)$$

$$C_{R,A}(q)C_{A,R}(q) = -\frac{1}{2}(B_{R}(q) + B_{A}(q) + D_{R}(q) + D_{A}(q)), \quad (7.13)$$

$$D_{R,A}(q)D_{A,R}(q) = \frac{1}{2}(D_{R}(q) + D_{A}(q)), \quad (7.14)$$

$$(B_{R,A}(q)C_{A,R}(q))_{\mu\nu} = (C_{R,A}(q)D_{A,R}(q))_{\mu\nu} = \frac{1}{2}\left(\frac{q_\mu \tilde{q}_\nu}{q^2 + 2iq_\alpha \epsilon} + \frac{q_\mu \tilde{q}_\nu}{q^2 - 2iq_\alpha \epsilon}\right), \quad (7.15)$$
\[(C_{R,A}(q)B_{A,R}(q))_{\mu\nu} = (D_{R,A}(q)C_{A,R}(q))_{\mu\nu} = \frac{1}{2}(\frac{q_\mu \bar{q}_\nu}{q^2 + 2i\epsilon} + \frac{q_\nu \bar{q}_\mu}{q^2 - 2i\epsilon}).\] (7.16)

By calculating the traces of the tensors \(l^\mu l^\nu\), \(q^\mu l^\nu + l^\mu q^\nu\), and \(q^\mu q^\nu\) with projectors, we find

\[l^\mu l^\nu = m^2 \frac{q^2}{2\bar{q}^2} A^{\mu\nu}(q) + \frac{q^2}{\bar{q}^2} \left(\frac{4l^2 - q^2}{8} A^{\mu\nu}(q) - l^2 B^{\mu\nu}(q)\right) + O^{\mu\nu}(\vec{k}_T),\] (7.17)

\[q^\mu l^\nu + l^\mu q^\nu = -\frac{q^2 l^2}{|\bar{q}|} C^{\mu\nu}(q) + O^{\mu\nu}(\vec{k}_T),\] (7.18)

\[q^\mu q^\nu = q^2 D^{\mu\nu}(q),\] (7.19)

\[g^{\mu\nu} = (A^{\mu\nu} + B^{\mu\nu} + D^{\mu\nu}).\] (7.20)

The tensor \(O^{\mu\nu}(\vec{k}_T)\) is linear in \(\vec{k}\), and vanishes after integration over \(\phi\). One should observe that (7.17) to (7.20) are valid for an arbitrary (but the same for \(B\) and \(D\)) \(R/A\) prescription, so we do not indicate it.

Using the multiplication rules one obtains

\[(g_{\mu\rho} - (1 - a)D_{R\mu\rho}(q))(f_a A^{\rho\sigma} + f_b B^{\rho\sigma} + f_c C^{\rho\sigma} + f_d D^{\rho\sigma})(g_{\sigma\nu} - (1 - a)D_{A\sigma\nu}(q))\]

\[= \frac{1}{2}\left(f_a A^{\rho\sigma}_R + f_b B^{\rho\sigma}_R - (1 - a) f_c C^{\rho\sigma}_R + (1 - a)^2 f_d D^{\rho\sigma}_R\right)\]

\[+ \frac{1}{2}\left(f_a A^{\rho\sigma}_A + f_b B^{\rho\sigma}_A - (1 - a) f_c C^{\rho\sigma}_A + (1 - a)^2 f_d D^{\rho\sigma}_A\right).\] (7.21)

It is important to observe that, owing to the properties of the mixed products (7.12) to (7.16), the \(R/A\) assignment of \(F\) does not influence the final result!
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