$p$-Branes and the GZK Paradox

Luis A. Anchordoqui,Jonathan L. Feng, and Haim Goldberg

$^1$Department of Physics, Northeastern University, Boston, MA 02115
$^2$Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139
$^3$Department of Physics and Astronomy, University of California, Irvine, CA 92697

Abstract

In spacetimes with asymmetric extra dimensions, cosmic neutrino interactions may be extraordinarily enhanced by $p$-brane production. Brane formation and decay may then initiate showers deep in the Earth’s atmosphere at rates far above the standard model rate. We explore the $p$-brane discovery potential of cosmic ray experiments. The absence of deeply penetrating showers at AGASA already provides multi-TeV bounds on the fundamental Planck scale that significantly exceed those obtained from black hole production in symmetric compactification scenarios. This sensitivity will be further enhanced at the Auger Observatory. We also examine the possibility that $p$-brane formation resolves the GZK paradox. For flat compactifications, astrophysical bounds exclude this explanation. For warped scenarios, a solution could be consistent with the absence of deep showers only for extra dimensions with fine-tuned sizes well below the fundamental Planck length. In addition, it requires moderately penetrating showers, so far not reported, and $\sim 100\%$ modifications to standard model phenomenology at 100 GeV energies.
A spectacular prediction of scenarios with strong gravity and large (or warped) extra dimensions \[1\] is the production of microscopic black holes (BHs) in particle collisions with center-of-mass energies larger than a TeV \[2\]. Cosmic neutrinos with energies above \(10^6 \text{ GeV}\) that strike a nucleon in the Earth’s atmosphere may then create BHs with cross sections two or more orders of magnitude above their standard model cross sections \[3\]. Criticisms \[4\] of the assumptions leading to these cross sections have been addressed \[5\]. These BHs are expected to decay promptly, initiating spectacular quasi-horizontal air showers deep in the atmosphere. The distinctive features of BH evaporation allow BHs to be differentiated from background \[6\], and the production and subsequent evaporation of such BHs may be studied in detail at cosmic ray observatories \[3, 7, 8\]. Additionally, neutrinos that traverse the atmosphere unscathed may produce BHs through interactions in the Earth; detailed simulations \[9\] of these BH events also find observable rates at neutrino telescopes.

Recently, based on the absence of a significant signal of deeply developing showers reported by the AGASA Collaboration \[10\], we derived new limits on the fundamental Planck scale in spacetimes with extra dimensions of equal length \[8\]. More recently, it was pointed out that for TeV-scale gravity with asymmetric large extra dimensions \[11\] the formation of \(p\)-branes could be competitive with black hole production \[12\]. The decay of \(p\)-branes is not well-understood. One possibility is that they may decay into lower dimensional brane-antibrane pairs, leading to a cascade of branes \[13\]. In any case, there is no reason for them to decay only to invisible particles, and it is reasonable to expect their decays, as with BH decays, to be dominated by visible quanta observable at cosmic ray observatories \[12\]. With this in mind, we study the implications of \(p\)-brane showers for cosmic ray physics.

Once one entertains the notion of asymmetric compactifications, a wide variety of possibilities arise, as one can consider the possibility of several compactification scales. We consider first the simplest example in which \(n\) flat extra dimensions are divided into two sets, with \(m\) dimensions of length \(L\), and \(n - m\) larger dimensions of length \(L'\). Brane production will be significant only in the presence of Planckian extra dimensions, and so we assume \(L \sim L_s \equiv M_s^{-1}\), where \(L_s\) and \(M_s\) are the fundamental Planck length and mass. \(M_s\) and the four-dimensional Planck mass \(M_{Pl} \simeq 1.2 \times 10^{19} \text{ GeV}\) are related by \[1\]

\[
M^2_{Pl} = M_s^{2+n} L^m L'^{n-m}.
\]

For simple toroidal compactifications, \(L\) and \(L'\) are related to radii by factors of \(2\pi\). Motivated by string/M theory, we will focus on the cases \(n = 6, 7\). To facilitate comparison with our earlier analysis and collider data, we will give results in terms of both \(M_s\) and \(M_D = [(2\pi)^n/8\pi]^{1/(n+2)} M_s\). For \(n = 6\) (7), \(M_D = 2.65 M_s\) (2.92\(M_s\)).

Scenarios with low values of \(n - m\) are already tightly constrained. Sub-millimeter tests of the gravitational inverse-square law show no deviation from Newtonian gravity \[14\], yielding \(L'/2\pi \lesssim 0.2 \text{ mm}\). For \(n - m = 1\) (2) and \(L \leq L_s\), this implies \(M_s \geq 2 \times 10^9\) (1.4) TeV. Additionally, in the presence of large extra dimensions, the usual four-dimensional graviton is complemented by a tower of Kaluza-Klein (KK) states, corresponding to the new available phase space in the bulk. The requirement that the neutrino signal of supernova 1987A not be unduly shortened by the emission of KK modes into the part of the bulk with large extra dimensions also bounds the compactification scale \[15\]. Such limits are further strengthened by constraints on KK graviton decay in typical astrophysical environments, yielding \(M_s \gg 10\) TeV for \(n - m \leq 3\) \[16\]. For \(n - m \geq 4\), bounds from colliders imply \(M_s \gtrsim 300\) GeV \[8\]. All of these bounds are for flat compactifications. Warped compactifications, in which bounds for small \(n - m\) are much less restrictive, will be discussed below.
We now consider an uncharged, non-rotating \( p \)-brane with mass \( M_p \) that lives in this \((4+n)\)-dimensional spacetime and wraps \( r \) Planckian dimensions and \( p-r \) large extra dimensions. Such a \( p \)-brane is described by the metric \([17]\)

\[
d s^2 = R^{\Delta/(p+1)} (-d\tau^2 + dz_i^2) + R^{(2-q-\Delta)/(q-1)} dr^2 + r^2 R^{(1-\Delta)/(q-1)} d\Omega_q^2 ,
\]

where \( z_i \) \((i = 1, \ldots , p)\) are the brane coordinates, \( d\Omega_q^2 \) \((q = 2 + n - p)\) is the metric of the \( q \)-dimensional unit sphere, \( \Delta = [q(p+1)/(p+q)]^{1/2} \), and \( R = 1 - (r_p/r)^{q-1} \). The radius \( r_p \) is given by

\[
r_p = \frac{1}{\sqrt{\pi M_*}} \gamma(n,p) \left( \frac{M_p}{M_* V_p} \right)^{\frac{1}{1+n-p}} ,
\]

where \( V_p = (L/L_*)^r (L'/L_*)^{p-r} \) is the volume wrapped by the \( p \)-brane in fundamental Planck units, and

\[
\gamma(n,p) = \left[ 8 \Gamma \left( \frac{3+n-p}{2} \right) \sqrt{\frac{1+p}{(2+n)(2+n-p)}} \right]^{\frac{1}{1+n-p}} .
\]

For \( p = 0 \), Eq. \((3)\) reduces to the metric of a \((4+n)\)-dimensional black hole and \( r_p \) becomes the Schwarzschild radius \([18]\). For \( p \geq 1 \), Eq. \((2)\) has a naked singularity at \( r_p \). Following Ref. \([12]\), we assume that this curvature singularity is smoothed out by the core of the \( p \)-brane. We also assume that a BH or \( p \)-brane is formed when two partons \( i, j \) with center-of-mass energy \( \sqrt{s} \) scatter with impact parameter \( b \leq r_p \), leading to the geometric cross section

\[
\hat{\sigma}_{ij \rightarrow p\text{-brane}}(\sqrt{s}) = \pi r_p^2 ,
\]

where \( r_p \) is given by Eq. \((3)\) with \( M_p = \sqrt{s} \).

Of interest for this work is the parameter space for which a \( p \)-brane cross section dominates the BH cross section. The ratio of these is \([12]\)

\[
\Sigma(\hat{s}; n, m, p, r) \equiv \frac{\hat{\sigma}_{ij \rightarrow p\text{-brane}}}{\hat{\sigma}_{ij \rightarrow BH}} = \left( \frac{M_{\text{Pl}}}{M_*} \right)^{-\alpha} \left( \frac{L}{L_*} \right)^{-\beta} \frac{\gamma(n,p)^2}{\gamma(n,0)^2} \left( \frac{\hat{s}}{M_*^2} \right)^{\frac{p}{1+n-1+p-n}} ,
\]

where

\[
\alpha = \frac{4(p-r)}{(n-m)(1+n-p)} \geq 0 , \quad \beta = \frac{2(nr-mp)}{(n-m)(1+n-p)} \geq 0 .
\]

For TeV-scale gravity with \( M_* \ll M_{\text{Pl}} \), \( p \)-brane production is negligible relative to BH production unless \( p = r \), i.e., the \( p \)-brane wraps only Planck size dimensions. (It is also suppressed for symmetric compactifications with \( L = L' \).

In this case, Eq. \((3)\) simplifies to

\[
\Sigma(\hat{s}; n, m, p, p) = \left( \frac{L}{L_*} \right)^{-\beta} \frac{\gamma(n,p)^2}{\gamma(n,0)^2} \left( \frac{\hat{s}}{M_*^2} \right)^{\frac{p}{1+n-1+p-n}} .
\]

As can be seen from Eq. \((3)\), and as noted in \([12]\), \( p \)-brane production significantly enhances BH production only if \( L \ll L_* \). The enhancement results from wrapping on small dimensions and is a consequence of the dependence of \( r_p \) solely on the density of the \( p \)-brane; thus, for a given mass, the density and radius \( r_p \) increase with decreasing \( L \) \([14]\). On the other hand, \( L \) cannot be much smaller than \( L_* \): in the string-based low energy Lagrangian, the gauge coupling squared is inversely proportional to the compactification volume. A small
FIG. 1: Total cross section $\sigma(\nu N \to \text{brane})$ for $n = 6$ (left), $n = 7$ (right), $L/L_{\ast} = 0.25$, $M_D = M_p^{\text{min}} = 1 \text{ TeV}$, and $m = 0, \ldots, n - 1$ from below. The standard model cross section $\sigma(\nu N \to \ell X)$ (dotted) is also shown.

volume corresponds to strong coupling and introduces low mass winding modes. In certain explicit models, these small volumes can be removed from the gauge sector via a $T$-duality transformation [20]. Below, we avoid reference to specific models, and present results for the generous range $0.1 < L/L_{\ast} < 10$.

The cross section for $p$-brane production from neutrino-nucleon scattering is

$$\sigma(\nu N \to p\text{-brane}) = \sum_{i} \int_{M_p^{\text{min}}/s}^{1} dx \hat{\sigma}_i(\sqrt{x}s) f_i(x, Q), \quad (9)$$

where $s = 2m_N E_\nu$, the sum is over all partons in the nucleon, and the $f_i$ are parton distribution functions. As in [3, 8], we set the momentum transfer $Q = \min\{M_p, 10 \text{ TeV}\}$, where the upper limit is from the CTEQ5M1 distribution functions [21]. Finally, $M_p^{\text{min}}$ is the minimum $p$-brane mass required for production, which we assume equal to $M_D$.

To obtain the total cross section for brane production, we assume that $p$-brane production is possible for all $p$, and so the total cross section is

$$\sigma(\nu N \to \text{brane}) = \sum_{p=0}^{m} \sigma(\nu N \to p\text{-brane}). \quad (10)$$

Total cross sections for brane production by cosmic neutrinos are given in Fig. 1 for $L/L_{\ast} = 0.25$ and $M_D = 1 \text{ TeV}$. The lowest solid curves for $m = 0$ are for BH production only, and are greatly enhanced relative to the standard model. We see, however, that for small values of $L/L_{\ast}$, even larger cross sections are possible for $p$-branes, especially for low $n - m$.

It has recently been proposed [22] that ultra-high energy neutrinos interacting via $p$-brane production may provide a solution to the puzzle of the observed cosmic rays with energies above $10^{11} \text{ GeV}$, i.e., above the Greisen-Zatsepin-Kuzmin (GZK) limit [23]. These cosmic ray showers begin high in the atmosphere, and so require $\nu N$ cross sections of order 100 mb. We see from Fig. 1, however, that such cross sections are approached only for one or two large extra dimensions ($n - m = 1, 2$) and $M_D \approx 1 \text{ TeV}$, a region of parameter space excluded by the sub-mm gravity experiments and astrophysical constraints discussed above.
FIG. 2: 95% CL lower bounds on $M_D$ in asymmetric compactification scenarios from the absence of $p$-brane-induced deep showers at AGASA. Bounds are given for various Planckian compactification lengths $L$, $n = 6$ (left) and $n = 7$ (right), $m = 0, \ldots, n - 3$ from below, and $M_p^{\text{min}} = M_D$. For each $m$, contributions from $p = 0, \ldots, m$ are summed.

Cosmic neutrinos with interaction strengths enhanced by $p$-brane production cannot resolve the GZK paradox in flat compactification scenarios. We will return to the possibility of warped compactifications below.

While $p$-brane cross sections for $n - m \geq 3$ are irrelevant for the GZK paradox, they may nevertheless enhance deep shower rates, with strong implications for cosmic ray experiments. For asymmetric spacetimes with $n - m \geq 3$, the event rate for deep showers is

$$N = \int dE_\nu N_A \frac{d\Phi}{dE_\nu} \sigma(\nu N \rightarrow \text{brane}) A(E_\nu) T,$$

where $N_A = 6.022 \times 10^{23}$ is Avogadro’s number, $d\Phi/dE_\nu$ is the neutrino flux, $A(E_\nu)$ is the acceptance for quasi-horizontal showers in cm$^3$ water equivalent steradians, and $T$ is the experiment’s running time. For the neutrino flux, we consider the conservative cosmogenic flux produced by interactions of the observed ultra-high energy protons with the cosmic microwave background. Specifically, as in our previous paper [8], we adopt the estimates of Protheroe and Johnson with an injection spectrum with cutoff energy $3 \times 10^{12}$ GeV [24]. Additional fluxes are possible and would only strengthen the conclusions below.

The AGASA Collaboration has searched for deeply penetrating showers [10]. An estimate of the AGASA acceptance for deeply penetrating events is given in [3]. In $T = 1710.5$ days of data taking, they find 1 event with an expected background of 1.72, leading to a 95% CL limit of 3.5 events from $p$-brane creation.

The absence of evidence for deeply penetrating showers then places bounds on the parameter space of asymmetric compactifications. These bounds are given in Fig. 2, and the results can be summarized as follows: (1) For $m = 0$, only 0-branes (BHs) are produced. The bounds on $M_D$ are therefore independent of $L/L_*$, and we recover the constraint $M_D \gtrsim 1.6$ TeV, first given in [3]. (2) For $L/L_* \to \infty$, the contribution from $p \neq 0$ vanishes, and all limits asymptotically approach the BH bound. (3) Even for $L \approx L_*$, the limits on $M_D$ from $p$-brane production are significantly enhanced above limits from BH production alone, and are as large as 3 TeV. (4) For smaller values of $L/L_*$, the lower bounds on $M_D$
rise dramatically. (5) The Auger Observatory, scheduled for completion by 2004 with an acceptance roughly 30-100 times that of AGASA, will provide an extremely sensitive probe of asymmetric compactifications.

We now return to scenarios with \( n - m = 1 \) or 2, but consider the possibility that these dimensions are warped \[25\]. Although no explicit models are available, these scenarios may evade the stringent constraints on \( M_D \) from astrophysics and Newtonian gravity. At the same time, the cross sections for \( p \)-brane production may be as in the flat compactification case if the curvature length scales are large compared with \( r_p \). In these scenarios, can \( p \)-brane production provide an explanation for cosmic rays above the GZK cutoff?

This possibility is constrained by at least three considerations. First, as in the \( n - m \geq 3 \) cases, these scenarios are limited by the absence of deeply penetrating showers at AGASA. The expected deep shower event rate is determined essentially as before, but now cross sections may be so large that showers begin high in the atmosphere and so do not contribute to deep shower rates. The atmospheric depth for quasi-horizontal showers with zenith angle 70° is about 3000 g/cm². This interaction length corresponds to a cross section of \( \sigma_{\nu N} = 0.56 \) mb. We determine deep shower rates assuming conservatively that only neutrinos with total cross sections below 0.56 mb contribute \[24\].

Second, if \( \sigma(\nu N) \gtrsim 100 \) mb for \( E_\nu > 10^{11} \) GeV, then one expects \( \sigma(\nu N) \sim 1 \) to 10 mb for \( E_\nu \sim 10^9 \) GeV. (See Fig. 1.) This implies that cosmic neutrinos should produce \( p \)-brane showers (akin to black hole showers \[3\]), but with primaries with mean free paths of \( \lambda_{\nu\text{-air}} \sim 4 - 30 \) times larger than \( \lambda_{p\text{-air}} \). Such moderately penetrating showers were discussed in \[27\]. Because this cross section would occur near the peak of the cosmogenic flux \[24\], such showers will be copiously produced and should be observed at cosmic ray detectors. This is an important test for these scenarios — an abundance of such moderately penetrating showers have not been reported to date.

Third, very large cross sections lead, via a dispersion relation, to large deviations in standard model predictions at lower energies \[28\]. With the cross sections of Fig. 1, it is straightforward to apply the results of \[28\] to show that cross sections

\[
\sigma_{\nu N}(10^{11} \text{ GeV}) \geq 300 \text{ mb}
\]  

lead to \( \sim 100\% \) corrections to, e.g., neutrino properties at energies \( \sim 100 \) GeV.

Cross sections \( \sigma(\nu N) \) at \( E_\nu = 10^{11} \) GeV are given in Fig. 3 for two scenarios with \( n - m = 1, 2 \). [Results for \((n, m) = (7, 6)\) are very similar to those for \((6, 5)\).] The shaded area is excluded by the AGASA bound on deeply penetrating showers. For large \( M_D \), cross sections are sufficiently suppressed to eliminate large deep shower rates. The upper boundary of this shaded region agrees with existing limits \[28\]. The AGASA constraint may also be evaded in the lower left corners, where cross sections are so large that \( p \)-brane showers develop high in the atmosphere and appear hadronic. These regions predict moderately penetrating showers. In addition, cross sections in this region are typically extremely large, and so require modifications to standard model physics at lower energies as discussed above. The region satisfying Eq. (12) is cross-hatched in Fig. 3.

As can be seen in Fig. 3, in some regions of parameter space, cross sections of \( \sim 100 \) mb are sufficient to mimic the highest energy cosmic rays. However, all of the desired parameter space with \( \sigma(\nu N) < 100 \) mb is excluded by the non-observation of deeply penetrating showers at AGASA. Regions with \( \sigma(\nu N) > 100 \) mb evade this constraint, but predict moderately penetrating showers, and large corrections to standard model physics at \( \sim 100 \) GeV energies. A GZK solution also requires small extra dimensions with size considerably below the fundamental Planck length, as well as low \( M_D \) values subject to collider probes.
In summary, we have considered the implications of $p$-brane production by ultra-high energy neutrinos. Current AGASA data imply multi-TeV bounds on $M_D$, the strongest bounds on asymmetric compactifications for $n - m \geq 4$. Auger, with a projected sensitivity 30 to 100 times that of AGASA, will either discover $p$-brane showers or significantly strain attempts to identify the weak and fundamental Planck scales in these scenarios. For flat compactifications, astrophysical and sub-mm gravity constraints exclude a $p$-brane explanation of super-GZK events. For warped compactifications, much of the potential parameter space is excluded by AGASA data. The remaining scenarios require low $M_D$ and small extra dimensions $L < 0.2L_*$, leading to strong coupling effects in the underlying stringy regime. These solutions also predict $\sim 100\%$ corrections to standard model neutrino physics at the 100 GeV scale, and moderately penetrating showers, not reported to date. These considerations leave very little room for explaining super-GZK events with $p$-brane physics.

Acknowledgments

We thank Eun-Joo Ahn, Marco Cavaglià, Carlos Nuñez, Angela Olinto, Al Shapere, and Tom Taylor for informative discussions. The work of LAA and HG has been partially supported by the US National Science Foundation under grants No. PHY–9972170 and No. PHY–0073034, respectively. The work of JLF was supported in part by the Department of Energy under cooperative research agreement DF–FC02–94ER40818.

[1] I. Antoniadis, Phys. Lett. B 246, 377 (1990); J. D. Lykken, Phys. Rev. D 54, 3693 (1996) [hep-th/9603153]; N. Arkani-Hamed, S. Dimopoulos and G. R. Dvali, Phys. Lett. B 429, 263 (1998) [hep-ph/9803315]; L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) [hep-ph/9905221].
[2] T. Banks and W. Fischler, hep-th/9906038; R. Emparan, G. T. Horowitz and R. C. Myers, Phys. Rev. Lett. 85, 499 (2000) hep-th/0003118; S. B. Giddings and E. Katz, J. Math. Phys. 42, 3082 (2001) hep-th/0009170; S. B. Giddings and S. Thomas, hep-ph/0106219; S. Dimopoulos and G. Landsberg, Phys. Rev. Lett. 87, 161602 (2001) hep-ph/0106293.
[3] J. L. Feng and A. D. Shapere, Phys. Rev. Lett. 88, 021303 (2001) hep-ph/0109106.
[4] M. B. Voloshin, Phys. Lett. B 518, 137 (2001) hep-ph/0107119; Phys. Lett. B 524, 376 (2002) hep-ph/0111099.
[5] S. Dimopoulos and R. Emparan, Phys. Lett. B 526, 393 (2002) hep-ph/0108060; S. B. Giddings, hep-ph/0110127; D. M. Eardley and S. B. Giddings, gr-qc/0201034; S. N. Solodukhin, hep-ph/0201248.
[6] L. Anchordoqui and H. Goldberg, Phys. Rev. D 65, 047502 (2002) hep-ph/0109242.
[7] J. L. Feng and A. D. Shapere, Phys. Rev. Lett. 88, 021303 (2001) hep-ph/0109106.
[8] M. B. Voloshin, Phys. Lett. B 518, 137 (2001) hep-ph/0107119; Phys. Lett. B 524, 376 (2002) hep-ph/0111099.
[9] S. Dimopoulos and R. Emparan, Phys. Lett. B 526, 393 (2002) hep-ph/0108060; S. B. Giddings, hep-ph/0110127; D. M. Eardley and S. B. Giddings, gr-qc/0201034; S. N. Solodukhin, hep-ph/0201248.
[10] L. Anchordoqui and H. Goldberg, Phys. Rev. D 65, 047502 (2002) hep-ph/0109242.
[11] J. L. Feng and A. D. Shapere, Phys. Rev. Lett. 88, 021303 (2001) hep-ph/0109106.
[12] M. B. Voloshin, Phys. Lett. B 518, 137 (2001) hep-ph/0107119; Phys. Lett. B 524, 376 (2002) hep-ph/0111099.
[13] S. Dimopoulos and R. Emparan, Phys. Lett. B 526, 393 (2002) hep-ph/0108060; S. B. Giddings, hep-ph/0110127; D. M. Eardley and S. B. Giddings, gr-qc/0201034; S. N. Solodukhin, hep-ph/0201248.
[14] L. Anchordoqui and H. Goldberg, Phys. Rev. D 65, 047502 (2002) hep-ph/0109242.