The decays $B^+ \to D_s^{*+}\gamma$ and $B^+ \to D^{*+}\gamma$ can be used for an extraction of $|V_{ub}|$. When the \( b \) and \( c \) quarks are nearly degenerate the rate for these modes can be determined in terms of other observed rates, namely $B\bar{B}$ mixing and $D^* \to D\gamma$ decay. To this end we introduce a novel application of heavy quark and flavor symmetries. Although somewhat unrealistic, this limit provides us with a first estimate of these rates.

12.15.Hh, 12.39.Hg, 13.40.Hq, 13.25.Hw

Abstract

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I. INTRODUCTION

The extraction of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix element $V_{ub}$, theoretically and experimentally, stands out as one of the prominent challenges of particle physics. Both its magnitude (through decay rates) and its phase relative to other CKM elements (through CP violation) are of considerable interest. Nevertheless, its determination remains elusive, ultimately because the CKM matrix describes the mixing of quarks, whereas of course only hadrons are observed. In the case of $V_{ub}$, much of this impediment is overcome by the application of the heavy quark effective theory (HQET) [1,2], which in particular relates the strong interaction matrix elements of systems with $b$ and $c$ quarks, eliminating much of the strong interaction uncertainty. However, the $u$ quark is by no means heavy, and it is notoriously difficult to separate its CKM and strong interaction couplings. This is true even for the experimentally clean semileptonic modes, $B \rightarrow \pi \ell \bar{\nu}$ and $B \rightarrow \rho \ell \bar{\nu}$, although dispersive bounds on the shapes of the form factors help to restrict such strong interaction uncertainties [3]. Inclusive semileptonic rates for $B \rightarrow X_u \ell \bar{\nu}$ are theoretically under better control but are plagued, since $|V_{ub}| \gg |V_{ub}|$, by the preponderance of $B \rightarrow X_c \ell \bar{\nu}$ everywhere in kinematic space except in the endpoint region of maximal lepton energy, where a $c$ quark cannot kinematically be produced; however, in this small region it has proved necessary to include hadronic model dependence [4]. A technique involving invariant hadronic mass spectra for invariant mass below $m_D$ is under better theoretical control but requires neutrino reconstruction [5].

Hadronic modes, containing everywhere strong interaction uncertainties, are even more problematic. Even if these difficulties could be tamed, one would still encounter the mixing of different weak topologies. For example, $B^0 \rightarrow \pi^+ K^-$ may proceed through either $b \rightarrow u s \bar{u}$ or a penguin $b \rightarrow s g \rightarrow s u u$; the presence of spectator quarks complicates the analysis. One may probe $V_{ub}$ using the decays $B^+ \rightarrow D^{*+} \gamma$ ($\bar{b} u \rightarrow c \bar{s} \gamma$) and $B^+ \rightarrow D^{*+} \gamma$ ($\bar{b} u \rightarrow c d \gamma$) [3], collectively $B^+ \rightarrow D^{*+}_{(s)} \gamma$. Even though the second decay is Cabibbo-suppressed compared to the first, the lower reconstruction efficiency associated with $D^{*+}_{s}$ compared to $D^{*+}$ makes both processes worth studying. The hard, monochromatic photon in these decays, 2.22 and 2.26 GeV, respectively, provides a distinctive experimental signature.

As shown in Fig. 1, the invariant amplitude for these decays at $O(G^1_F)$ consists of only one weak topology, because four flavor quantum numbers change ($\Delta B = -1$, $\Delta C = +1$, $\Delta(S, D) = +1$, $\Delta U = -1$), thus fixing the quark couplings to the $W$ boson. The photon may couple to any of the quark lines, although one expects the dominant contributions from radiation emitted by the light $u$ or $s$. The diagram with the photon emitted from the $W$ is suppressed, of $O(G^2_F)$. On the other hand, since gluons may pair-produce quarks, these decays may be sensitive to multiparticle intermediate states, for example $B^+ \rightarrow D^0 K^+ \rightarrow D_{s}^{*+} \gamma$ [3], but the weak topology remains unchanged.

In this paper we study a theoretical limit in which the decays $B^+ \rightarrow D_{(s)}^{*+} \gamma$ are calculable from first principles in terms of other measured quantities. We assume both the $b$ and $c$ quarks crossed.

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1 At $O(G^2_F)$, the nontrivial diagrams include the di-penguin $\bar{b} \rightarrow \bar{s} g$, $u g \rightarrow c$ and a box diagram with the internal quarks crossed.
quarks are heavy and nearly degenerate, $\Lambda_{\text{QCD}} \ll m_b$, $\Lambda_{\text{QCD}} \ll m_c$ and $\delta m \equiv m_b - m_c \lesssim \Lambda_{\text{QCD}}$. We do not expect this limit to be a good approximation to reality. Our calculation is a starting point for further investigations on controlled approximations of this rate.
II. FORMALISM

The dominant contributions to the amplitude for $B^+ \rightarrow D^+\gamma$ are the long distance processes $B^+ \rightarrow D^+_s\gamma$ and $B^+ \rightarrow B^+\gamma \rightarrow D^+\gamma$. The premise of this calculation is that the matrix elements of $B^+ \rightarrow D^+_s\gamma$, $D^+_s \rightarrow D^+\gamma$, and $B^+ \rightarrow B^+\gamma$ can be related by heavy quark and $SU(3)$ flavor symmetries to the rates for $B\overline{B}$ mixing and $D^+_s \rightarrow D\gamma$. Alternately, we expect that lattice calculations similar to those that determine the $B\overline{B}$ mixing matrix element [8] can be employed to compute that of $B^+ \rightarrow D^+_s\gamma$ directly.

For the initial calculation presented here, we work in the generally-low (GL) velocity limit

$$\delta m \equiv m_b - m_c \lesssim \Lambda_{QCD} \ll m_b,$$

which means that the four-velocity of each heavy particle in the process is assumed to remain constant to lowest order. This limit is more restrictive than the slow-velocity (SV) limit considered by Shifman and Voloshin [2], $\Lambda_{QCD} \ll \delta m \ll m_b$. To indicate the quality of this assumption, we point out that the value of $\gamma_{D^+_s} = v_B \cdot v_{D^+_s}$ is 1.45 for the strange case and 1.50 for the nonstrange case. On the other hand, letting $m_B = m_b + \Lambda - 3\lambda^2/4m_b$, $m_{D^*_s} = m_c + \Lambda + \lambda^2/4m_c$, one finds that $\gamma_{D^*_s}$ is parametrically small,

$$\gamma_{D^*_s} = \frac{m^2_B + m^2_{D^*_s}}{2m_B m_{D^*_s}} = 1 + \frac{1}{2m^2_b} \delta m^2 + O\left(\frac{1}{m^3_b}\right).$$

Heavy quark symmetries may be used to relate the matrix element for $B \rightarrow D_{(s)}$ to the one for $B \rightarrow \overline{B}$ only if the velocity of the $D_{(s)}$ in the $B$ rest frame is parametrically small, say $O(\Lambda/m_b)$ or $O(\delta m/m_b)$. It is therefore sufficient to assume the weaker SV limit for this part of our argument. However, in order to relate the photon emission processes to the real decays $B^* \rightarrow B\gamma$ and $D^* \rightarrow D\gamma$, it is necessary to impose the GL condition to ensure softness of the photon, as we now argue.

The matrix element of the electromagnetic current $j_\mu$ between vector ($D^*$ or $B^*$) and pseudoscalar ($D$ or $B$) states is characterized by a single Lorentz invariant form factor
\[ \langle \vec{p}', \vec{\varepsilon} | j_\mu(0) | \vec{p} \rangle = -ig(q^2)\epsilon_{\mu\nu\lambda\sigma}\varepsilon^\nu p'^\lambda p'^\sigma, \]  
(2.3)

where \( q = (p - p') \). The form factor at \( q^2 = 0 \) governs the on-shell decay of the vector to the pseudoscalar, which is characterized by a transition magnetic moment, \( g(0) = e\mu \) [see below, Eq. (3.1)]. However, implicit in this definition is that the mesons are on-shell states; if the transition occurs with one of the states being virtual, as is the case here, then the \( q^2 \) argument of the form factor must be computed accordingly. The relevant scale over which a hadronic form factor changes is \( \Lambda_{\text{QCD}} \). For example, in the non-relativistic potential quark model of mesons, the scale that dictates the behavior of the form factor \( g \) is the constituent quark mass \( \sim \Lambda_{\text{QCD}} \). In \( B^+ \to D^+_{(s)} \gamma \) the form factor appears with argument \( q^2 \approx -\delta m^2 \), and in the GL limit one may approximate \( g(-\delta m^2) \approx g(0) = e\mu \).

The formalism of HQET combined with chiral symmetry, relevant to the physics of mesons containing heavy quarks, was developed in Refs. [9]. It was extended to radiative processes, such as \( D^* \to D\gamma \), in Ref. [10], while the Lagrangian for heavy quark-heavy antiquark mixing appeared in [11]. We use ingredients from all of these works, but for brevity include only notation relevant to the present process. While convenient, the formalism is not really a necessary framework for our calculation. In fact, light mesons enter diagrams only at higher order in the chiral expansion. By stating our calculation in the language of this effective Lagrangian we are setting the stage for further investigations of, for example, the effects of the finite strange quark mass.

Since chiral symmetry is one of the ingredients of the Lagrangian, we begin by including the octet of pseudo-Nambu–Goldstone bosons in the usual nonlinear form \( \xi \):

\[ \xi = \exp(i\Pi/f), \]
(2.4)

where

\[ \Pi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & K^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix}, \]
(2.5)

with \( f \approx 131 \text{ MeV} \). Under the chiral SU(3)\(_L\) \( \times \) SU(3)\(_R\) transformation \((L, R)\),

\[ \xi \mapsto L\xi U^\dagger = U\xi R^\dagger, \]
(2.6)

with \( U \) implicitly defined by the equality of these two forms.

The ground state vector \( P_{Aa}^* \) and pseudoscalar \( P_{Aa} \) fields that destroy mesons of heavy quark flavor \( A \) and light antiquark flavor \( a \) are incorporated into the \( 4 \times 4 \) bispinor

\[ H_{Aa} = \frac{(1 + \theta)}{2} (P_{Aa}^*\gamma_\mu - P_{Aa}\gamma_\nu), \]

\[ \overline{H}^{Aa \dagger} = \gamma^0 H_{Aa}^\dagger \gamma^0. \]
(2.7)

Under SU(4) heavy quark spin-flavor symmetry transformations \( S \) and SU(3)\(_L\) \( \times \) SU(3)\(_R\) transformations \( U, H \) transform as
Defining the chiral covariant derivative and axial current by

\[ D_\mu^{ab} = \delta_\mu^{ab} \partial_\mu - V_\mu^{ab}, \]
\[ A_\mu^{ab} = i \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right)_{ab} - \frac{1}{f} \partial_\mu \Pi_{ab} + O(\Pi^3), \]

one finds that \( D\xi, D\xi^\dagger, DH, \) and \( D\overline{H} \) have precisely the same transformation properties under chiral symmetry as their underlying fields, and

\[ (A_\mu^{a})^a_b \mapsto U^a_c (A_\mu^{b})^d_c U^\dagger_d b. \]

Thus, one obtains the heavy meson Lagrangian invariant under chiral and heavy quark symmetry with the minimum number of derivatives,

\[ \mathcal{L} = - \text{Tr} \overline{H}^{Aa} i\gamma_\mu \cdot D_\mu H_{Ab} + g \text{Tr} \overline{H}^{Aa} H_{Ab} A_\mu^b \gamma_5. \]

It should be noted that the kinetic term is canonically normalized when the field \( H \) has 2, rather than the usual \( 2M \), particles per unit volume.

Since lowest-order HQET integrates out heavy antiparticle degrees of freedom, it is necessary to include such fields explicitly when they can appear in the asymptotic states. One defines \( \overline{A} \)

\[ H^{\overline{A}a} = \left( \overline{P}^*_{\mu} A^a \gamma^\mu_{\overline{A}} - \overline{P}^*_{\mu} \gamma^5_{\overline{A}} \right) \frac{(1 - \gamma^5)}{2}, \]
\[ \overline{P}^{\overline{A}a} = \gamma^0 \overline{H}^{\overline{A}a} \gamma^0. \]

For example, while the field \( P_{\mu}^* A_a \) destroys a vector meson of flavor content \( A\bar{a} \), the field \( P_{\mu}^{\overline{A}a} \) destroys one of flavor content \( \overline{A}a \); the two are related by a chosen charge conjugation convention \( \overline{A} \). One demands that \( H_{Aa} \) and \( H_{\overline{A}a} \), or \( \overline{H}^{Aa} \) and \( \overline{H}^{\overline{A}a} \), have the same transformation properties under heavy quark symmetry and chiral transformations. Then the construction of the Lagrangian for mesons with heavy antiflavors is straightforward.

As for direct mixing between the two sectors, the standard model operator that destroys quarks of flavors \( \overline{A}, a \) and creates quarks of flavors \( B, b \), given charge constraints \( -Q_{\overline{A}} + Q_a = Q_B - Q_b \), is

\[ \mathcal{O}^{ab, \overline{AB}} = \overline{A} \gamma^\mu (1 - \gamma_5) a \cdot B \gamma_\mu (1 - \gamma_5) b. \]

For \( B\overline{B} \) mixing, one sees that \( A \) and \( B \) are \( b \) quarks, and \( a \) and \( b \) are \( d \) quarks; in the present case, \( A, B, a, \) and \( b \) are \( b, c, u, \) and \( (d \) or \( s) \) quarks, respectively. In the limit that HQET and SU(3) are good symmetries, both processes have the same strong matrix element. In Ref. \[11\] it was shown that (2.13) matches in the symmetry limit of the effective theory onto
\[ O^{ab,AB} = 4\beta C \left[ (\xi P^*P) A^a (\xi P^*) B^b + (\xi P^1) A^a (\xi P^2) B^b \right] + \cdots, \tag{2.14} \]

where \( C \) is the color vacuum saturation coefficient, 1 for \( B\overline{B} \) mixing and \( 3/8 \) for \( BD_s \).

Calculation of the process also requires understanding the coupling of heavy mesons to photons. Minimal substitution for the kinetic term in Eq. (2.11) provides no coupling to real transverse photons at lowest order, while minimal substitution applied to the second term in (2.11) requires including at least one pion for a nonvanishing term. The unique lowest-order electromagnetic term \[ \delta L = \frac{1}{4} e \mu_{Aa} \text{Tr} H^{Aa} H A_{\mu\nu} F_{\mu\nu}^{EM} + \text{charge conjugate}. \tag{2.15} \]

### III. Calculation

The photon coupling in Eq. (2.15) represents the lowest-order heavy-to-heavy electromagnetic transition. It does not couple pseudoscalars to each other, but provides for the decay of vector mesons:

\[ \Gamma(P_{Aa}^* \to P_{Aa} \gamma) = \frac{1}{3} \alpha |\mu_{Aa}|^2 E^3. \tag{3.1} \]

To lowest order in HQET and SU(3), one may write \( \mu_{Aa} = Q_a \mu \). For \( P_{Aa}^* = D^{**} \), the current experimental bound \[ \Gamma(D^{**}) < 0.131 \text{ MeV} \]

and electromagnetic branching fraction \( < 3.2\% \), lead to \( |\mu| < 2.5 \text{ GeV}^{-1} \).

The calculation of the rate is then straightforward in the combined HQET and chiral symmetry limit, and consists of the two diagrams depicted in Fig. 2. We use the GL limit to compute the amplitude, but retain full mass dependence in the phase space calculation. The result is

\[ \Gamma(B^+ \to D_s^{**} \gamma) = \frac{G_F^2}{64} \left| V_{ub} V_{cs} \right|^2 (3\beta/2)^2 \alpha |\mu|^2 m_{D_s}^2 (m_B - m_{D_s}) (m_B + m_{D_s})^3. \tag{3.2} \]

with \( V_{cs} \to V_{cd} \) and \( m_{D_s}^* \to m_D^* \) for \( B^+ \to D^{**} \gamma \).

**FIG. 2.** The two leading diagrams for \( B^+ \to D_s^{**} \gamma \) that occur in the heavy quark and chiral limits. The double line indicates a heavy meson, one containing a \( \bar{b} \) (\( c \)) quark on the left (right) of the flavor-changing vertex, indicated by a box.

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\[ ^2 \text{In Eq. (14) of Ref. [10], \( \mu \) is labeled } \beta. \]
The effective Lagrangian mixing parameter $\beta$ in (2.14) is related to familiar quantities of $B\bar{B}$ mixing via

$$\frac{1}{m_B} \langle B^0 | \mathcal{O}^{dd, \bar{B}B} | B^0 \rangle = \frac{8}{3} f_B^2 m_B B_B = 4\beta.$$  \hfill (3.3)

A full treatment requires including renormalization point dependence in $B_B$, but we neglect this here for simplicity. Now there are two obvious directions of analysis. One is to eliminate $\beta$ using its calculated value from lattice simulations. Using $f_B = 170 \pm 35$ MeV and $B_B = 0.98 \pm 0.06$ (renormalization point 2 GeV), and $m_B = 5.279$ GeV, $\tau_B = 1.6 \cdot 10^{-12}$ s, one finds $4\beta = 0.40 \pm 0.17$ GeV$^3$, and

$$\text{Br}(B^+ \to D_s^{*+}\gamma) \simeq 2 \cdot 10^{-7} \cdot \left( \frac{B_B}{0.98} \right)^2 \cdot \left( \frac{\Gamma(D_s^{*+})}{0.131 \text{ MeV}} \right) \cdot \left( \frac{\text{Br}(D^{*+} \to D^+\gamma)}{3.2\%} \right) \cdot \left[ \frac{V_{ub} V_{cs}}{3 \cdot 10^{-3}} \right]^2,$$  \hfill (3.4)

with $V_{cs} \to V_{cd}$ for $B^+ \to D^{*+}\gamma$, in which case the coefficient (taking $|V_{cd}| = 0.22$) is $7 \cdot 10^{-9}$. The other direction for analysis eliminates $\beta$ as a systematic strong interaction uncertainty between this process and $B\bar{B}$ mixing. In particular, $\Delta m_B \propto G_F^2 |V_{tb} V_{td}^*|^2 \beta$, which means that eliminating $\beta$ leads to values for the ratio $|V_{ub} V_{cs}|^2/|V_{tb} V_{td}^*|^4$, which is proportional to $(\rho^2 + \eta^2)/[(1-\rho)^2 + \eta^2]^2$ in the usual Wolfenstein space. Curves of this family tend to intersect the $\rho$ axis with vertical slope, which is useful since the current experimentally allowed region in $\rho-\eta$ space is broad in the $\rho$ direction.

### IV. PROSPECTS

The inclusion of HQET-violating, chiral symmetry-violating, and short-distance corrections is straightforward, and this program should certainly be carried out. Indeed, many of the necessary corrections already appear in the literature, including corrections to $\mu_{Aa}$ [10], meson decay constants [11,13], $B\bar{B}$ mixing [11], and a number of other SU(3)-symmetry corrections [11,14]. However, a number of other diagrams remain to be computed, because the inclusion of the photon changes the allowed quantum numbers of intermediate states. For example, there are new chiral loop diagrams for $BD(s)$ mixing compared to $B\bar{B}$ mixing where a single pion emerges directly from the mixing vertex.

More urgent and less tractable is understanding of the radiative form factor $g(q^2)$ at realistic momenta. Taking $g(q^2) \to g(0) = \epsilon\mu$ introduces large uncertainties into our estimate for the decay rate. Alternatively, one may use only the less extreme SV limit to relate the mixing amplitudes and express the rate in terms of the corresponding form factors. This is done by replacing $\epsilon\mu$ in Eq. (3.2) by $g_B(q^2) + g_D(q^2)$ at $q^2 = -\delta m^2$, which can be used a starting point for a calculation based on estimates using hadronic models for the form factors.

\[\text{Note that } \Gamma(B^+ \to D_{(s)}^{*+}\gamma) \propto G_F^2 \beta^2, \text{ while } \Delta m_B \propto G_F^2 \beta, \text{ due to the different weak topologies.}\]
We expect that the best determination of $V_{ub}$ will still require careful analysis of a number different decay modes; however, the decays $B^+ \rightarrow D^{*+}_{(s)} \gamma$ provide an important additional handle on the problem.

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REFERENCES

[1] N. Isgur and M. B. Wise, Phys. Lett. B 232, 113 (1989); B 237, 527 (1990); E. Eichten and B. Hill, Phys. Lett. B 234, 511 (1990).

[2] M. Shifman and M. B. Voloshin, Yad. Fiz. 47, 801 (1988) [Sov. J. Nucl. Phys. 47, 511 (1988)].

[3] C. G. Boyd, B. Grinstein, and R. F. Lebed, Phys. Rev. D 56, 6895 (1997); C. G. Boyd and M. J. Savage, Phys. Rev. D 56, 303 (1997).

[4] I.I. Bigi, M. A. Shifman, N. G. Uraltsev, and A.I. Vainshtein, Int. J. Mod. Phys. A 9, 2467 (1994); Phys. Lett. B 328, 431 (1994); T. Mannel and M. Neubert, Phys. Rev. D 50, 2037 (1994); M. Neubert, Phys. Rev. D 49, 3392 and 4623 (1994).

[5] V. Barger, Phys. Lett. B 251, 629 (1990); J. Dai, Phys. Lett. B 333, 212 (1994); A. F. Falk, Z. Ligeti and M. B. Wise, Phys. Lett. B 406, 225 (1997).

[6] The viability of this mode was suggested to us by the fine CLEO and BABAR experimentalists Vivek Sharma and Soeren Prell.

[7] We thank Nathan Isgur for this observation.

[8] For a summary, see Appendix C of The BABAR Physics Book, P. F. Harrison and H. R. Quinn, editors, Report No. SLAC-R-504.

[9] M. B. Wise, Phys. Rev. D 45, R2188 (1992); G. Burdman and J. F. Donoghue, Phys. Lett. B 280, 287 (1992); T.-M. Yan et al., Phys. Rev. D 46, 1148 (1992); ibid. D 55, 5851(E) (1997).

[10] J. F. Amundson et al., Phys. Lett. B 296, 415 (1992).

[11] B. Grinstein, E. Jenkins, A. V. Manohar, M. J. Savage, and M. B. Wise, Nucl. Phys. B380, 369 (1992).

[12] C. Caso et al. (Particle Data Group), Eur. Phys. J. C 3, 1 (1998).

[13] C. G. Boyd and B. Grinstein, Nucl. Phys. B442, 205 (1995).

[14] A. F. Falk and B. Grinstein, Nucl. Phys. B416, 771 (1994).