Guiding High-Performance SAT Solvers with Unsat-Core Predictions

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Abstract. The NeuroSAT neural network architecture was introduced in [36] for predicting properties of propositional formulae. When trained to predict the satisfiability of toy problems, it was shown to find solutions and unsatisfiable cores on its own. However, the authors saw “no obvious path” to using the architecture to improve the state-of-the-art. In this work, we train a simplified NeuroSAT architecture to directly predict the unsatisfiable cores of real problems. We modify several state-of-the-art SAT solvers to periodically replace their variable activity scores with NeuroSAT’s prediction of how likely the variables are to appear in an unsatisfiable core. The modified MiniSat solves 10\% more problems on SAT-COMP 2018 within the standard 5,000 second timeout than the original does. The modified Glucose solves 11\% more problems than the original, while the modified Z3 solves 6\% more. Our results demonstrate that NeuroSAT can provide effective guidance to high-performance SAT solvers on real problems.

1 Introduction

Over the past decade, neural networks have dramatically advanced the state of the art on many important problems, most notably object recognition [21], speech recognition [13], and machine translation [44]. There have also been several attempts to apply neural networks to problems in discrete search, such as program synthesis [32,7], first-order theorem proving [17,26] and higher-order theorem proving [43,41,16]. More recently, [36] introduce a neural network architecture designed for satisfiability problems, called NeuroSAT, and show that when trained to predict satisfiability on toy problems, it learns to find solutions and unsatisfiable cores on its own. Moreover, the neural network is iterative, and the authors show that by running for many more iterations at test time, it can solve problems that are bigger and even from completely different domains than the problems it was trained on. While these results may be intriguing, the authors’ motivation was to study the capabilities of neural networks rather than to solve real SAT problems, and they admit to seeing “no obvious path” to beating existing SAT solvers.

In this work, we make use of the NeuroSAT architecture, but whereas the original authors used it as an end-to-end solver on toy problems, we use it to help inform variable branching decisions within high-performance SAT solvers on real
problems. Given this goal, the main design decision becomes how to produce data to train the network. There are many possible choices. For example, one could create a dataset of satisfiable problems and their solutions, and train the network to predict the latter from the former. Or, one could use reinforcement learning, with [37] as a prominent example, and try to learn a policy that minimizes the running time of a hybrid solver on a collection of problems. Our approach is inspired by the fail first strategy articulated in [8] that advocates branching on variables most likely to cause conflicts. We approximate this strategy by training NeuroSAT to predict which variables will be involved in unsatisfiable cores. Note that perfect predictions would not always yield a useful variable branching heuristic: for some problems, every variable is in the core, and of course for satisfiable problems, there are no cores at all. Thus, our approach is pragmatic; we rely on NeuroSAT predicting imperfectly, and hope that the probability NeuroSAT assigns to a given variable being in a core correlates well with that variable being good to branch on.

The next biggest design decision is how to make use of the predictions inside a SAT solver. Even if we wanted to query NeuroSAT for every variable branching decision, doing so would have severe performance implications, particularly for large problems. A SAT solver makes tens of thousands of assignments every second, whereas even with an on-device GPU, querying NeuroSAT on an industrial-sized problem may take hundreds or even thousands of milliseconds. We settle for periodically querying NeuroSAT on the entire problem (i.e. not conditioning on the current trail), and setting all variable activity scores at once in proportion to how likely NeuroSAT thinks the variable is to be involved in a core. Between queries, the base solver adjusts the variable activity scores according to its own heuristic (EVSIDS). Although performance reasons compel us to only query periodically, this may actually be a superior approach in terms of the number of decisions made, since the base heuristics are already strong, and they may only need an occasional, globally-informed reprioritization to yield substantial improvements.

We summarize our pipeline:

1. Generate many unsatisfiable problems by decimating existing problems.
2. For each such problem, generate a DRAT proof, and extract the variables that appear in the unsat core.
3. Train NeuroSAT (henceforth NeuroCore) to map unsatisfiable problems to the variables in the core.
4. Instrument state-of-the-art solvers (MiniSat, Glucose, Z3) to query NeuroCore periodically (using the original and the learnt clauses), and to reset its variable activity scores according to NeuroCore’s predictions.

As a result of these modifications, the MiniSat solver solves 10% more problems on SAT-COMP 2018 within the standard 5,000 second timeout. The modified Glucose 4.1 solves 11% more problems than the original, while the modified Z3 solves 6% more. Our results demonstrate that NeuroSAT (and in particular, NeuroCore) can provide effective guidance to high-performance SAT solvers on
2 Data generation

As discussed in §1, we want to train our neural network architecture to predict which variables will be involved in unsat cores. Unfortunately, there are only about a thousand unsatisfiable problems across all SAT-COMP competitions, and a network trained on such few examples would be unlikely to generalize well to unseen problems. We overcome this limitation and generate a dataset containing over 150,000 different problems with labeled cores by considering unsatisfiable subproblems of existing problems.

Specifically, we generate training data as follows. We use the distributed execution framework ray [29] to coordinate one driver and hundreds of workers distributed over several machines. The driver maintains a queue of (sub)problems, and begins by enqueuing all problems from SAT-COMP (through 2017 only) as well as a few hundred scheduling problems. It might help to initialize with even more problems, but we did not find it necessary to do so. Whenever a worker becomes free, the driver dequeues a problem and passes it to the worker. The worker tries to solve it using Z3 with a fixed timeout (we used 60 seconds). If Z3 returns \texttt{sat}, it does nothing, but if Z3 returns \texttt{unsat}, it passes the generated DRAT proof [42] to DRAT-trim [42] to determine which of the original clauses were used in the proof. It then computes the variables in the core by traversing the clauses in the core, and finally generates a single datapoint by constructing the sparse adjacency matrix $A$ for the problem (§3) along with the core bitmask.

If Z3 returns \texttt{unknown}, the worker cubes [11] on a single variable (using Z3’s implementation of the March heuristic [28]) and returns the two subproblems to the driver to be added to the queue.

This process generates one datapoint roughly every 60 seconds per worker. Some of the original problems are very difficult, and so the process may not terminate in a reasonable amount of time; thus we stopped it once we had generated 150,000 datapoints.

Note that our data generation process is not guaranteed to generate diverse cores. To the extent that March is successful in selecting variables to cube on that are in the core, the cores of the two subproblems will be different; if it fails to do this, then the cores of the two subproblems may be the same (though the non-core clauses will still be different). Also note that predicting the (binary) presence of a variable in the core is simplistic and loses information. A more sophisticated approach might predict a distribution over the variables in the core, where variables that appear in more clauses (perhaps weighted inversely by their size) have higher mass.
3 Neural Network Architecture

We now describe our simplified version of the NeuroSAT architecture. Readers already familiar with NeuroSAT may choose to skip to the summary of key differences at the end of this section.

We represent a Boolean formula in CNF by an undirected graph with nodes for every literal and clause, with two different types of edges: occurrence edges between literals and the clauses they appear in, and flip edges between literals and their negations. For example, we represent the formula

\[(x_1 \lor x_2 \lor x_3) \land (x_1 \lor \overline{x}_2 \lor \overline{x}_3)\]

by the following graph:

\[
\begin{array}{ccccc}
  c_1 & & & & c_2 \\
\end{array}
\]

\[
\begin{array}{cccccc}
  x_1 & \overline{x}_1 & x_2 & \overline{x}_2 & x_3 & \overline{x}_3 \\
  c_1 & 1 & 1 & 1 & 0 & 0 & 0 \\
  c_2 & 1 & 0 & 0 & 1 & 1 & 1 \\
\end{array}
\]

where the solid lines denote occurrence edges and the dotted lines denote flip edges. Let \(n_v\) and \(n_c\) be the number of variables and clauses in the formula respectively. To pass as input to our neural network, we encode the graph of a Boolean formula as an \(n_c \times 2n_v\) sparse adjacency matrix \(A\), where (for reasons we’ll see shortly) the first \(n_v\) columns represent the positive literals and the last \(n_v\) columns represent the negative literals. For example, we represent the graph above as the following 2 \(\times\) 6 (sparse) matrix:

\[
A = \begin{pmatrix}
  x_1 & x_2 & x_3 & \overline{x}_1 & \overline{x}_2 & \overline{x}_3 \\
  c_1 & 1 & 1 & 1 & 0 & 0 & 0 \\
  c_2 & 1 & 0 & 0 & 1 & 1 & 1 \\
\end{pmatrix}
\]

Note that the flip edges are only implicit in this representation. Define the operation Flip to swap the first half of the columns with the second half, so that each literal’s column is swapped with its negation’s.

Our neural network itself is made up of three standard fully-connected, feed-forward networks:

\[
\begin{align*}
C_{\text{update}} : & \mathbb{R}^{2d} \rightarrow \mathbb{R}^d \\
L_{\text{update}} : & \mathbb{R}^{3d} \rightarrow \mathbb{R}^d \\
V_{\text{proj}} : & \mathbb{R}^{2d} \rightarrow \mathbb{R}
\end{align*}
\]

We use function-call notation to denote applying a feed-forward network to each row of its input as a batch. For example, if \(X \in \mathbb{R}^{k \times 2d}\), then \(C_{\text{update}}(X) \in \mathbb{R}^{k \times d}\) and its \(i\)th row is the result of applying \(C_{\text{update}}\) to the \(i\)th row of \(X\).

The network computes forward as follows. First, it initializes two matrices \(C \in \mathbb{R}^{n_c \times d}\) and \(L \in \mathbb{R}^{2n_v \times d}\) to all ones. Each row of \(C\) corresponds to a clause,
while each row of \(L\) corresponds to a literal. Next, it applies the following updates \(T\) times (we used \(T = 4\)):

\[
C \leftarrow C_{\text{update}} \left(\text{Concat}(C, AL)\right) \tag{1}
\]
\[
L \leftarrow L_{\text{update}} \left(\text{Concat}(L, A^\top C, \text{Flip}(L))\right) \tag{2}
\]

Note that passing \(\text{Flip}(L)\) as input to the update for \(L\) manifests the implicit flip edge in the problem graph. After each such update, it also normalizes each column of \(C\) and \(L\) to have mean zero and variance one. After \(T\) updates, we transform \(L \in \mathbb{R}^{2n_v \times d}\) into \(V \in \mathbb{R}^{n_v \times 2d}\) by concatenating the rows corresponding to the same variable. Finally, we project each 2d variable embedding into a single scalar by applying the third feed-forward network:

\[
\hat{V} \in \mathbb{R}^{n_v} \leftarrow V_{\text{proj}}(V) \tag{3}
\]

The vector \(\hat{V}\) is the output of NeuroCore, and its components are interpreted as the logits of a probability distribution over the variables. During training, we turn each labeled bitmask over variables into a probability distribution \(V^*\) by assigning uniform probability to each variable in the core and zero probability otherwise. We optimize the three feed-forward networks all at once to minimize the Kullback-Leibler divergence \(D_{KL}(V^* \parallel \text{Softmax}(\hat{V}))\).

Comparison to the original NeuroSAT. While the original NeuroSAT architecture was designed to solve small problems end-to-end, ours is designed to provide cheap, heuristic guidance on (potentially) large problems. Accordingly, our network differs from the original in a few key ways. First, ours only runs for 4 iterations at both train and test time, whereas the original was trained with 26 iterations and ran for upwards of a thousand iterations at test time. Second, our update networks are simple feed-forward networks, whereas the original used Long Short-Term Memories (LSTMs) [14]. Third, as discussed above, ours is trained with supervision at every clause and variable and outputs vectors \(\hat{C} \in \mathbb{R}^{n_c}\) and \(\hat{V} \in \mathbb{R}^{n_v}\), whereas the original is trained with only a single bit of supervision and accordingly only outputs a single scalar.

Training NeuroCore. As we discussed in [1] our goal is not to learn a perfect core predictor, but rather only to learn a coarse heuristic that broadly assigns higher score to more important variables. Thus, fine-tuning the network is relatively unimportant, and we only ever trained with a single set of hyperparameters. We used the AdamOptimizer [18] with a constant learning rate (1e-4), and trained asynchronously with 20 GPUs for under an hour, using distributed TensorFlow [1].

4 Hybrid Solving: Extending CDCL with NeuroCore

Background on CDCL. Modern SAT solvers are based on the Conflict-Driven Clause Learning (CDCL) algorithm [27,20]. Before explaining how we integrate
NeuroCore with CDCL solvers, we briefly summarize the parts of CDCL that are relevant to our work. At a high level, a CDCL solver works as follows. It maintains a trail of literals that have been given tentative assignments, and continues to assign variables and propagate the implications until reaching a contradiction. It then analyzes the cause of the contradiction and learns a conflict clause that is implied by the existing clauses and that would have helped avoid the current conflict. Finally, it pops variables off the trail until all but one of the literals in the learnt clause have been set to false, propagates the learnt clause, and continues from there. Most CDCL solvers also periodically restart (clear the trail), and also periodically simplify the clauses in various ways.

There are many crucial, heuristic decisions that a CDCL must make, such as which variable to branch on next, what polarity to set it to, which learned clauses to prune and when, and also when to restart. We only consider the first one: which variable to branch on next. The decision of which variable to branch on next has been the subject of intense study for decades and many approaches have been proposed. See [4] for a comprehensive overview. MiniSat, Glucose and Z3 all implement variants of the Variable State-Independent Decaying Sum (VSIDS) heuristic (first introduced in [30]) called Exponential-VSIDS (EVSIDS). The EVSIDS score of a variable $x$ after the $t$th conflict is defined by:

\[
\text{InConflict}(x, i) = \begin{cases} 
1 & \text{x was involved in the } i\text{th conflict} \\
0 & \text{otherwise}
\end{cases}
\]

\[
\text{EVSIDS}(x, t) = \sum_i \text{InConflict}(x, i) \rho^{t-i}
\]

Intuitively, it measures how many conflicts the variable has been involved in, with more recent conflicts weighted much more than past conflicts. As we will discuss in §4, our approach is to periodically reset these EVSIDS scores based on the outputs of NeuroCore.

Integrating NeuroCore. As discussed in §1, it is too expensive to query NeuroCore for every variable branching decision, and so we settle for querying periodically on the entire problem and replacing the variable activity scores with NeuroCore’s prediction. We now describe this process in detail.

Every 100 seconds, we build the sparse clause-literal adjacency matrix $A$ (see §3) as follows. First, we collect all non-eliminated variables that are not units at level 0. These are the only variables we tell NeuroCore about. Second, we collect all the clauses that we plan to tell NeuroCore about. We would like to tell NeuroCore about all the clauses, both original and learnt, but the size of the problem can get extremely large as the solver accumulates learnt clauses. At some point the problem would no longer fit in the GPU, and it might be undesirably expensive even before that point. After collecting the original clauses, we traverse the learned clauses in ascending size order, collecting clauses until the number of literals plus the number of clauses plus the number of cells exceed a fixed cutoff (we used 10 million). If a problem is so big that the original clauses already exceed this cutoff, then for simplicity we do not query NeuroCore at all,
although we could have still queried it on random subsets of the clauses. Finally, we traverse the chosen clauses to build the clause-literal adjacency matrix $A$. Note that because of the learned clauses, NeuroCore is shown a substantially different graph on each query even though we do not condition on the trail.

NeuroCore then returns a vector $\tilde{V} \in \mathbb{R}^{n_v}$, where a bigger element indicates that NeuroCore thinks the corresponding variable is more likely to be involved in a core. The numbers in $\tilde{V}$ are not a-priori comparable to the current EVSIDS scores of the variables. We first turn $\tilde{V}$ into a probability distribution by dividing it by a scalar temperature parameter $\tau$ (we used 0.25), and taking the softmax. We then scale this vector by the number of variables in the problem, and additionally by a fixed constant $\kappa$ (we used $10^4$). Finally, we replace the EVSIDS scores:

$$\forall i, \text{EVSIDS}(x_i, t) \leftarrow \text{Softmax}(\tilde{V}/\tau)[i]n_v\kappa$$

Note that MiniSat uses a decay factor of $\rho = 0.95$, and to a first approximation MiniSat averages ten thousand conflicts per second, so these scores decay to 0 in only a fraction of a second. However, such an intervention can still have a powerful effect by re-targeting EVSIDS on a more important part of the search space.

5 Solver Experiments

We evaluate the hybrid solver neuro-minisat (described in §4) and the original MiniSat solver minisat on the 400 problems from the main track of SATCOMP-2018, with the same 5,000 second timeout used in the competition. For each solver, we solved the 400 problems in 400 different processes in parallel, spread out over 8 identical 64-core machines, with no other compute-intensive processes running on any of the machines. In addition, the hybrid solver also had network access to 5 machines each with 4 GPUs, with the 20 GPUs split evenly and randomly across the 400 processes. We calculate the running time of a solver by adding together its process time with the sum of the wall-clock times of each of the TensorFlow queries it requests on the GPU servers. We ignore the network transmission times since in practice one would use an on-device hardware accelerator.

Note that although we did not train NeuroCore on any (sub)problems from SATCOMP-2018, we did perform some coarse tuning of hyperparameters (specifically $\rho$, which a-priori might reasonably span 100 orders of magnitude) based on runs of the hybrid solver on problems from SATCOMP-2018. In hindsight we regret not using alternate problems for this, and strongly suspect we would have found a similar ballpark by only tuning on problems from other sources.

Results. The main result, alluded to in §4 is that neuro-minisat solves 205 problems within the 5,000 second timeout whereas minisat only solves 187. This

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3 In MiniSat, this involves setting the activity vector to these values, resetting the variable increment to 1.0, and rebuilding the order-heap.
corresponds to an increase of 10%. Most of the improvement comes from solving more satisfiable problems: \textit{neuro-minisat} solve 125 satisfiable problems compared to \textit{minisat}'s 109, which is a 15% increase. On the other hand, \textit{neuro-minisat} only solved 3% more unsatisfiable problems (80 vs 78). Figure 1 shows a cactus plot of the two solvers, which shows that \textit{neuro-minisat} takes a substantial lead within the first minutes and maintains the lead until the end. Figure 2 shows a scatter plot of the same data, which shows there are quite a few problems that \textit{neuro-minisat} solves within a few minutes that \textit{minisat} times out on. It also shows that there are very few problems on which \textit{neuro-minisat} is substantially worse than \textit{minisat}.

\textbf{Fig. 1.} Cactus plot comparing NeuroCore-assisted MiniSat (\textit{neuro-minisat}) with the original (\textit{minisat}). It shows that \textit{neuro-minisat} takes a substantial within the first few minutes and maintains the lead until the end.

\textit{Analysis.} The results show that our hybrid approach is effective, but do not tell us much about why it is effective. We do not have a satisfying answer to this question yet. As a consolation, we report a few additional experiments that shed some light on why it may work.

First, we investigated whether using EVSIDS between queries was necessary, or whether NeuroCore’s predictions were sufficient on their own. Simply increasing $\rho$ from $10^4$ to $10^{40}$ (which only prevents EVSIDS from taking over for approximately 200ms following each query) already had an substantial negative effect: it solved less than half of the problems that \textit{minisat} solved. Thus NeuroCore is not a replacement to EVSIDS but only a complement to it. Second, we investigated whether NeuroCore’s predictions even mattered at all, or if the solver would benefit equally from just periodically setting the EVSIDS scores to random values. When we would otherwise call NeuroCore, we substituted $\tilde{V}$ with logits sampled uniformly between $(-1,1)$, and transformed them
Fig. 2. Scatter plot comparing NeuroCore-assisted MiniSat (neuro-minisat) with the original (minisat). It shows that there are quite a few problems that neuro-minisat solves within a few minutes that minisat times out on, and there are very few problems on which neuro-minisat is substantially worse than minisat.

to EVSIDS scores using the original $\tau$ and $\rho$ values. This change had an even more harmful effect: it only solved a handful of problems out of 400. Third, we considered that perhaps NeuroCore’s predictions are mostly irrelevant, and that the important part is that they are roughly the same at every query. We tried the same experiment with random logits but with the logits sampled uniformly once at the beginning of search and reused at every query. This did a little better than when the random logits change each time, but not by much. These experiments do not rule out the possibility that there is a simple, hardcodeable heuristic that could do just as well as NeuroCore, but they do suggest that there is substantial signal in NeuroCore’s predictions.

Glucose. As a follow-up experiment and sanity-check, we made the same modifications to Glucose 4.1 and evaluated in the same way on SATCOMP-2018. To provide further assurance that our findings are robust, we altered the NeuroCore schedule, changing from fixed pauses (100 seconds) to exponential backoff (5 seconds at first with multiplier $\gamma = 1.2$). The results of the experiment are very similar to the results from the MiniSat experiment described above. The number of problems solved within the timeout jumps 11% from 186 to 206. Figure 3 show the scatter plot comparing neuro-glucose to glucose. This comparison is even more favorable to the NeuroCore-assisted solver than Figure 2 as it shows that there are many problems neuro-glucose solves within seconds that glucose times out on. The cactus plot for the Glucose experiment is almost identical to the one in Figure 1 and so is not shown.
Fig. 3. Scatter plot comparing NeuroCore-assisted Glucose (neuro-glucose) with the original (glucose). It shows that there are quite a few problems that neuro-glucose solves within a few seconds that glucose times out on, and there are very few problems on which neuro-glucose is substantially worse than glucose.

Z3. Lastly, we made the same modifications to Z3, except we once again altered the NeuroCore schedule, this time from exponential backoff in terms of user-time to geometric backoff in terms of the number of conflicts. Specifically, we first query NeuroCore after $\kappa = 50,000$ conflicts, and then each time wait $\kappa$ more conflicts than the previous time before querying NeuroCore again. The modified Z3 solves 170 problems within the timeout, up from 161 problems, which is a 6% increase.

Note that for the Z3 experiment, to save on computational costs, we evaluated both solvers simultaneously instead of sequentially. To ensure fairness, we ordered the task queue by problem rather than by solver. The lower absolute scores compared to MiniSat and Glucose are partly the result of the increased contention.

6 Related Work

Machine learning in automated deduction has been pursued in several guises. Two established approaches are strategy selection [45] and axiom selection [40].

Strategy selection is to our knowledge mainly applied to setting configuration parameters for SAT, MIP (Mixed-Integer Programming), TSP (the Traveling Salesman Problem), and ATP (First-order Automated Theorem Proving) systems, and enjoy the additional advantage that they can be used in a setting where multiple systems are combined to approach a virtual best solver [4].

\[^{4}\text{http://ada.liacs.nl/events/sparkle-sat-18/documents/floc-18-sparkle-extended.pdf}\]
systems regularly use strategy selection especially when preparing for competitions, e.g., [35], ever since Gandalf [39] won the 1996 ATP competition (known as CASC [38]) by spending the first few minutes running a suite of different strategies before selecting one that appeared to make the most progress. Strategy selection as composing tactics [6] was pursued in [3] to speed up performance over baseline tactics.

Axiom selection methods help focus search on a subset of input clauses. The domain-specific Sine [15] method is a prominent example, and selects axioms that share infrequently-appearing symbols with the goal. ATP systems rely on clause selection for driving inferences, and a recent use of machine learning for clause selection [26] was integrated in the E theorem prover [34]. In SAT, CDCL solvers select clauses using unit propagation and conflict analysis, and rely on garbage collection of redundant clauses to balance available inferences from memory and propagation overhead. Carefully crafted methods have been introduced to balance different heuristics within SAT garbage collection, more recently by [31], combining glue levels with activity scores. Ostensibly as a reaction to the opacity and complexity of these heuristics, the CryptoMiniSat solver [5] has recently integrated machine learning to eliminate redundant clauses. Similar to our approach, their approach relies on information from DRAT proofs to relate features of learned clauses (for example, their glue levels) with their usefulness to a derivation. The CryptoMiniSat version of DRAT [6] indeed collects several more features than the original version of DRAT-trim [7] that we used in this work. Their approach then trains a succinct decision tree on this data, that is compiled into a specialized version of CryptominiSat.

Integration of machine learning techniques for branch selection in SAT is to our knowledge relatively unexplored. The VSIDS heuristic (and its descendants such as EVSIDS) presented a breakthrough in SAT solving as it amplified branching on variables that would maximize the conflict-to-branch ratio, thus focusing search within clusters of related clauses. Several refinements of VSIDS are used in newer SAT solvers, including CHB (Conflict History Based) [25] and VMTF (Variable Move-To-Front) [33, 4]. Branch selection heuristics within CDCL solvers are finely tuned for performance because they are invoked on every decision.

In contrast, lookahead solvers [9] afford higher overhead during a lookahead phase to identify branch literals, also known as so-called cubes when a lookahead solving is used in the cube-and-conquer paradigm [11]. Cubes are selected to optimize a carefully crafted metric on clause reduction, such as weighing variable occurrences inversely by the sizes of the clauses they appear in [12, 11, 10, 23]. Cubing is an appealing target for machine learning because it is used in phases where a global analysis on a problem is feasible.

In MIP solvers, branch-and-bound methods [24] share many similarities to cube-and-conquer methods in SAT. Branch operations split the search into

5 https://github.com/msoos/cryptominisat/
6 https://github.com/msoos/drat-trim/
7 https://github.com/marijnheule/drat-trim
Separate parts, and are relatively rare, as the main engine in state-of-the-art MIP solvers remains dual-Simplex, often augmented by interior point methods. Branching is applied when the linear programming optimization is unable to find integral values for integer variables. State-of-the-art branching methods in MIP solvers use heuristics that are related in spirit to lookahead heuristics: among a set of candidate branch variables they run a limited (cheap) form of linear programming and assemble progress metrics for each candidate variable, and branch on a variable that optimizes a selected metric. As a common trait, these metrics depend on finely tuned parameters, and are therefore ripe targets for machine learning techniques [2].

In the backdrop of the related work, the approach we pursue here is wedged between the fine-grained branching preferences of CDCL solvers and the single-step branch decisions of lookahead solvers. NeuroCore performs a global analysis to predict a good ordering among all unassigned variables, but does this only periodically to allow the fine-grained built-in heuristics to take over during inferences. It provides the capability to rehash a search into a different cluster of clauses where CDCL can perform local tuning.

7 Discussion

There is a vast design space for how to train NeuroSAT and how to use it to guide SAT solvers. In this work we have only considered one tiny point in that design space. We now briefly discuss some of the other possible directions.

For our first experiment predicting unsatisfiable cores, we trained NeuroSAT on a synthetic graph-coloring distribution that happened to have tiny cores. NeuroSAT was able to predict these cores so accurately that we could get almost arbitrarily big speedups over Z3 by only giving it the 0.5% of clauses that NeuroSAT thought most likely to be in the core (and doubling the number of clauses given as necessary until they included the core). Unfortunately, it is much harder to learn a general-purpose core predictor than one on a particular synthetic distribution for which instances may all have similar cores. Real problems also rarely have such tiny cores, so even a perfect core predictor might not be such a silver bullet. With that said, we do think NeuroCore may prove useful in eliminating unhelpful learned clauses, and we plan to pursue this direction in the future.

We have also experimented with training NeuroSAT to imitate the decisions of the March cubing heuristic. Based on preliminary experiments in a challenging scheduling domain, we found that NeuroSAT trained only to imitate March may actually produce better cubes than March itself, though it remains to be seen if this result holds up to greater scrutiny.

Lastly, inspired by the success of [37], we have experimented with various forms of Monte-Carlo tree search and reinforcement learning, though so far the only competitive heuristic we have been learn de novo is a cubing strategy for uniform random problems. There are two main challenges for learning variable branching heuristics by exploration alone: problems may have a huge number
of variables, and it may take substantial time to solve the (sub)problems in order to get feedback about a given branching decision. The former challenge can be mitigated by beginning with imitation learning (e.g. by imitating March). We tried to mitigate the latter by pre-training a value function based on data collected from solving a collection of benchmarks, and then using the value function estimates to make cheap importance-sampling estimates of the size of the search tree under different policies as described in [19]. We found that even in the supervised context, training the value function was difficult; without taking logs it is numerically difficult, and with taking logs, one can get very low loss while ignoring the relatively few hard subproblems towards the roots that make the most difference.

We have only scratched the surface of this design space. We hope that our promising initial results with NeuroCore inspire others to try leveraging NeuroSAT in other, creative ways.

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