Thermal evolution of rotating hybrid stars

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Accepted 0000, Received 0000

Abstract

As a neutron star spins down, the nuclear matter continuously is converted into quark matter due to the core density increase and then latent heat is released. We have investigated the thermal evolution of neutron stars undergoing such deconfinement phase transition. We have taken into account the conversion in the frame of the general theory of relativity. The released energy has been estimated as a function of change rate of deconfinement baryon number. Numerical solutions to cooling equation are obtained to be very different from the without heating effect. The results show that neutron stars may be heated to higher temperature which is well-matched with pulsar’s data despite onset of fast cooling in neutron stars with quark matter core. It is also found that heating effect has magnetic field strength dependence. This feature could be particularly interesting for high temperature of low-field millisecond pulsar at late stage. The high temperature could fit the observed temperature for PSR J0437-4715.

Key words: stars: neutron — dense matter — stars: rotation

1 Introduction

The composition of Neutron star (NS) interior is still poorly known due to the uncertainties of nuclear physics. NS cooling is an important tool for the study of dense matter. By comparing cooling models with thermal emission data from observations, we can gain insight into the equation of state (EOS) of dense matter inside NSs. After a supernova explosion a newly formed NS first cools via various neutrino emission mechanisms before the surface photon radiation take over. Only slower neutrino cooling mechanisms such as the modified Urca, plasmon neutrino and bremsstrahlung.
processes, should occur when the interior density is not high. However, for higher core density the more ’exotic’ extremely fast cooling processes take over. Cooling of hybrid stars (NSs with quark matter core) is just one of those examples.

However, heating effect on the cooling of compact stars is an important factor. Several heating mechanisms have been extensively discussed, for example, the dissipation of rotational energy due to viscous damping (Zheng & Yu (2006)) and mutual friction between superfluid and normal components of the star (Shibazaki & Lamb (1989)) and release of strain energy stored by the solid crust due to spin-down deformation (Cheng et al. (1992)). These heating processes are closely related with the rotation evolution of a star. It is well-known that a neutron star spin-down due to magnetic dipole radiation. Because of spin-down compression, the interior density of the star will gradually increase. For hybrid stars, this results in, little by little, the transformation of hadron matter into quark matter in the interior. It will lead to the release of latent heat if the transition is the first-order one. The generation of the energy increases internal energy of the star. It will be called deconfinement heating (DH). Deconfinement process has ever been investigated in strange stars, where neutron drops at bottom of a crust drip on quark matter surface to be instantaneously dissolved into quark matter (Haensel & Zdunik (1991), Yuan & Zhang (1999), Yu & Zheng (2006)). What we will here deliberate is slow transition process: nuclear matter undergoing from hadron matter phase (HP) to mixed hadron-quark matter phase (MP) and then to quark matter phase (QP) with gradually increasing density.

From the point of view above, the change of the internal structure of the compact star due to rotation has to be evaluated within general relativity theory. Using the method of perturbation theory (Hartle (1967), Chubarian et al (2000)), we are going to investigate evolution of phase transition region during the spin-down evolution of the rotating star. And then we also calculate changes in confined baryon numbers and quark numbers. The heat luminosity can be estimated with proportion to the change rates.

Of course, the star is not quite in weak interaction equilibrium state during spin down. The departure from the chemical equilibrium indeed lead to the rotochemical heating in a rotating neutron star (Reisenegger (1995), (2006)), as well as a hybrid star. The direct Urca processes would be triggered and dominates the rotochemical heating in our models. We find, however, the effect of rotochemical heating is much smaller than the deconfinement heating by estimating the both net heating rates for the sequences of different parameters. Hence, we don’t consider the effect of rotochemical heating in following study.

In this work, we take Glendenning’s hybrid stars model (Glendenning (1997)). The hadronic...
mater equations of state (EOS) in the framework of the relativistic mean field theory and MIT bag model of quark matter are used to construct the model of hybrid stars, but medium effect of quark matter has been considered in quasiparticle description (Schertle et al. (1997)). We choose the simplest possible nuclear matter composition, namely neutrons, protons, electrons, and muons (npeμ matter) and ignore superfluidity and superconductivity.

The plan of this paper is as follows. In Sec. 2 we introduce rotating hybrid star model. The DH effect is considered in Sec. 3. The cooling curves and the corresponding explanations are presented in Sec. 4. The conclusion and discussions are summarized in Sec. 5.

2. ROTATING HYBRID STARS MODEL

Quark deconfinement phase transition is expected to occur in neutron matter at densities above the nuclear saturation density \( n_b = 0.16 \text{fm}^{-3} \). Since many theoretical calculations have suggested that deconfinement transition should be of first order in low-temperature and high-density area (Pisalski & Wilczek (1984) and Gavai et al. (1987)), one may expect a MP during the transition. Most of the approaches to deconfinement matter in neutron star matter use a standard two-phase description of EOS where the HP and the QP are modelled separately and resulting EOS of the MP is obtained by imposing Gibbs conditions for phase equilibrium with the constraint that baryon number as well as electric charge of the system are conserved (Glendenning (1992), (1997)).

The Gibbs condition for mechanical and chemical equilibrium at zero temperature between the HP and the QP reads

\[
p_{HP}(\mu_n, \mu_e) = p_{QP}(\mu_n, \mu_e), \tag{1}
\]

where \( p_{HP} \) is pressure of HP and \( p_{QP} \) is the pressure of QP. We use the EOS of the relativistic mean field model (Glendenning (1997)) for hadron matter and employ a effective mass bag-model EOS for quark matter (Schertle et al. (1997)). Only two independent chemical potentials remain according to the corresponding two conserved charges of the \( \beta \)-equilibrium system. The total baryon number \( N_B \) as well as electrical charge \( Q \)

\[
n_B = \frac{N_B}{V} = \chi n_{QP} + (1 - \chi)n_{HP}, \tag{2}
\]

\[
0 = \frac{Q}{V} = \chi q_{QP} + (1 - \chi)q_{HP}. \tag{3}
\]

where \( \chi = V_Q/V \) is the volume fraction of quark matter in the MP. Taking the charge neutral EOS of the HP, Eq.(1), (2) and (3) for MP and the charge neutral EOS of the QP, we can construct the full hybrid star EOS. In Fig.1 we show the model EOS with deconfinement transition which is
the typical scheme of a first order transition at finite density with MP. We choose the parameters for hadronic matter EOS which have given by Glendenning (1997) and quark matter EOS with s quark mass $m_s = 150$ MeV, bag constant $B^{1/4} = 160$ MeV, coupling constant $g = 3$.

With the evaluated hybrid star EOS presented above we now turn to analyse the structure of the corresponding rotating hybrid stars. Using the Hartle’s perturbation theory (1967), Chubarian et al (2000) have studied the change of the internal structure of the hybrid stars due to rotation. In this paper, we also apply Hartle’s approach to investigate the structure of rotating hybrid stars. Hartle’s formalism is based on treating a rotating star as a perturbation on a non-rotating star, expanding the metric of an axially symmetric rotating star in even powers of the angular velocity $\Omega$. The metric of a slowly rotating star to second order in the angular velocity $\Omega$, can be written as

$$ds^2 = -e^{\nu(r)}[1 + 2(h_0 + h_2 P_2)]dt^2 + e^{\lambda(r)}[1 + \frac{2(m_0 + m_2 P_2)}{(r - 2M(r))}]dr^2 + r^2[1 + 2(v_2 - h_2)P_2][d\theta^2 + \sin^2 \theta(d\phi - w(r, \theta)dt)^2] + O(\Omega^3)$$

Here $e^{\nu(r)}, e^{\lambda(r)}$ and $M(r)$ are functions of $r$ and describe the non-rotating star solution of the Tolman-Oppenheimer-Volkov (TOV) equations. $P_2 = P_2(\theta)$ is the $l = 2$ Legendre polynomials. $\omega$ is the angular velocity of the local inertial frame and is proportional to the star’s angular velocity $\Omega$, whereas the perturbation functions $h_0, h_2, m_0, m_2, v_2$ are proportional to $\Omega^2$. we assume that matter in the star is described by a perfect fluid with energy momentum tensor

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + Pg^{\mu\nu}$$

The energy density and pressure of the fluid are affected by the rotation because the rotation deforms the star. In the interior of the star at given $(r, \theta)$, in a reference frame that is momentarily moving with the fluid, the pressure and energy density variation is respectively

$$\delta P(r, \theta) = [\epsilon(r) + P(r)][p_0^* + p_2^* P_2(\theta)]$$

$$\delta \epsilon(r, \theta) = \frac{d\epsilon}{dP}[\epsilon(r) + P(r)][p_0^* + p_2^* P_2(\theta)]$$

here, $p_0^*$ and $p_2^*$ are dimensionless functions of $r$, proportional to $\Omega^2$, which describe the pressure perturbation. The rotational perturbations of the star’s structure are described by the functions $h_0, m_0, p_0^*, h_2, m_2, v_2, p_2^*$. These functions are calculated from Einstein’s field equations. The effect of rotation described by the metric on the shape of the star can be divided into contributions: A spherical expansion which changes the radius of the star, and is described by the functions $h_0$ and $m_0$. The other part is a quadrupole deformation, described by functions $h_2, v_2$ and $m_2$. As a consequence of these contributions, the difference between the gravitational mass of the rotating star and the non-rotating star with the same central pressure is
\[ \delta M_{\text{grav}} = m_0(R) + \frac{J^2}{R^3} \]  

(8)

The change in the radius of the star is given by

\[ \delta R = \xi_0(R) + \xi_2(R)P_2(\theta) \]  

(9)

We wish to study sequences of the rotating stars with constant total baryon number at variable spin frequency \( \nu = \Omega/2\pi \). The expansion of total baryon numbers in powers of \( \Omega \) is

\[ N_B = N^0_B + \delta N_B + O(\Omega^4) \]  

(10)

where

\[ N^0_B = \int_0^R n_B(r)[1 - 2M(r)/r]^{-1/2}4\pi r^2 dr \]  

(11)

is the total baryons number of non-rotating star and

\[ \delta N_B = \frac{1}{m_N} \int_0^R (1 - \frac{2M(r)}{r})^{-1/2} \left[ \frac{m_0(r)}{r - 2M(r)} + \frac{1}{3} r^2 [\Omega - \omega(r)]^2 e^{-\epsilon} \right] m_N n_B(r) \] 

\[ + \frac{d m_N n_B(r)}{dP}(\epsilon + P)p^*_0(r)4\pi r^2 dr \]  

(12)

here \( m_N \) is the rest mass per baryon. To construct constant baryon number sequences, we first solve the TOV equations to find the non-rotating configuration for giving a central pressure \( P(r=0) \). And then, for an assigned value of the angular velocity \( \Omega \) the equations of star structure are solved to order \( \Omega^2 \), imposing that the correction to the pressure \( p^*_0(r=0) \) being not equal to zero. The value of \( p^*_0(r=0) \) is then changed until the same baryon number as that non-rotating star is obtained.

The results for the stability of rotating hybrid star configurations with possible deconfinement phase transition according to the EOS described above are shown in Fig.2, where the total gravitation mass is given as functions of the equatorial radius and the central baryon number density for static stars as well as for stars rotating with the maximum rotation frequency \( \nu_k \). The dotted lines connect configuration with the same total baryon number and it becomes apparent that the rotating configurations are less compact than the static ones. In order to explore the increase in central density due to spin down, we create sequences of hybrid star models. Model in a particular sequence have the same constant baryon number, increasing central density and decreasing angular velocity. Fig.3 displays the central density of rotating hybrid stars with different gravitational mass at zero spin, as a function of its rotational frequency. In the interior of these stars, the matter can be gradually converted from the relatively incompressible nuclear matter phase to more compressible quark matter phase.
3. DECONFINEMENT HEATING

As the star spins down, the centrifugal force decreases continuously, increasing its internal density. At this occurrence, the nuclear matter continuously converts into quark matter in an exothermic reaction, i.e. $n \rightarrow u + 2d$, $p \rightarrow 2u + d$, $s$ quarks immediately appear after weak decay. Fig. 3 identifies the fact that quarks are accumulating in the interior of the star with decreasing rotation frequency $\nu$. In a particular sequence with constant baryon number, we can calculate the deconfinement baryon number $N_q$ of the star with the varying rotation frequency $\nu$. The expression of $N_q$ is similar to that the total baryon number $N_B$, except that the baryon number density $n_B$ is displaced with the deconfinement baryon number density $n_q$. Deconfinement baryon number of $1.4M_\odot$ star, for example, is plotted in Fig. 4. The analytic expression fits as

$$N_q = N_q^0 (1 - 0.716\nu_3^2 + 0.055\nu_3^3 - 0.032\nu_3^4)$$

(13)

where $N_q^0 \approx 0.22N_\odot$ is the baryon number of quarks for the static configuration and $\nu_3 = \nu/10^3$ Hz.

We can also derive similar expressions for sequences of hybrid star models. Left panel in Fig. 1 indicates release of latent heat because there is the reduction in enthalpy per baryon at transition. Assumed the average value of release energy per nucleon that is transforming into quarks $q_n$, the total latent heat per time can be written as

$$H_{dec}(t) = q_n \frac{dN_q}{d\nu} \dot{\nu}(t)$$

(14)

with

$$\dot{\nu} = -\frac{16\pi^2}{3Ic^2}\mu^2\nu^3 \sin^2 \theta$$

(15)

is induced by magnetic dipole radiation (MDR), where $I$ is the stellar moment of inertia, $\mu = \frac{1}{2}BR^3$ is the magnetic dipole moment, and $\theta$ is the inclination angle between magnetic and rotational axes, $q_n$ can be estimated as the order of 0.1 MeV through contrast of HP enthalpy to corresponding MP enthalpy for same region of baryon number density, which is uniquely defined for given stellar mass in our work.

4. COOLING CURVES

The cooling is realized via two channels - by neutrino emission from the entire star body and by transport of heat from the internal layers to the surface resulting in the thermal emission of photons. Neutrino emission is generated in numerous reactions in the interiors of neutron stars, as reviewed, by Page et al. (2005). For the calculation of cooling of the hadron part of the hybrid
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We use the main processes which are nucleon direct Urca (NDU) and nucleon modified Urca (NMU) and nucleon bremsstrahlung (NB). If the proton and electron Fermi momenta are too small compared with the neutron Fermi momenta, the NDU process is forbidden because it is impossible to satisfy conservation of momentum. Under typical conditions one finds that the ratio of the number density of protons to that of nucleons must exceed about 0.11 for the process to be allowed. Medium effects and interactions among the particles modify this value only slightly but the presence of muons raise it to about 0.15. We apply the second condition in our calculations.

For the calculation of cooling of the quark matter we consider the most efficient processes: the quark direct Urca (QDU) processes on unpaired quarks, the quark modified Urca (QMU) and the quark bremsstrahlung (QB). Nucleon superfluidity and quark superconductivity are not included in the model. Considering the energy equation of the star, the cooling equation can be written as

\[ C_V \frac{dT}{dt} = -L_\nu - L_\gamma + H \]  

where \( C_V \) is the total stellar heat capacity, it incorporates neutron contribution, quark contribution and electron contribution. The term \( H \) indicates the heating energy per unit time, in our work \( H = H_{\text{dec}} \), \( L_\nu \) is the neutrino luminosity, and \( L_\gamma \) is the surface photon luminosity given by

\[ L_\gamma = 4\pi R^2 \sigma T_s^4, \]  

here \( \sigma \) is the Stefan-Boltzmann constant and \( T_s \) is the surface temperature. The surface temperature is related to internal temperature by a coefficient determined by the scattering processes occurring in the crust. We apply an formula which is demonstrated by Gudmundsson et al. (1983),

\[ T_s = 3.08 \times 10^6 g_{s,14}^{1/4} T_9^{0.5495} \]  

where \( g_{s,14} \) is the proper surface gravity of the star in units of \( 10^{14} \text{cms}^{-2} \). The gravitational red-shift is also taken into account. Then the effective surface temperature detected by a distant observer is

\[ T_s^\infty = T_s \sqrt{1 - R_g/R}, \]  

where \( R_g \) is the gravitational stellar radius. In our calculation, we choose the initial temperature \( T_0 = 10^9 \text{K} \), and the magnetic tilt angle \( \theta = 45^\circ \).

Fig.5 shows the cooling behavior of a 1.4\(M_\odot \) hybrid star for different magnetic fields (10⁹ – 10¹³ G). In this figure, \( q_n \) is taken to be 0.1 MeV. It is evident that the DH increase the surface temperature dramatically. This is extremely different from fast cooling scenario (solid curve in Fig.5). We find that there is a quite clear magnetic-field dependence of the curves. It is determined by the properties of MDR. Solutions to equation (15) show the magnetic-field dependence of the spin frequencies. The strong field strength induces a rapid spin-down at the beginning while the low field strength leads to only obvious spin-down at the older ages (see Fig.3 in Zheng et
Hence, a large quantity of nuclear matter is deconfined into quark matter during the earlier and shorter period for strong-field case. On the contrary, the plateau can form at late period for weak-field case.

In Fig.6 we present the cooling behavior of different mass stars for $B=10^{12}$ with and without DH. The observational data, taken from tables 1 and 2 in Page et al.(2004), have been shown in the figure. The theoretical curves are consistent with the observational data. The heating rate has a regular mass dependence. More massive neutron star have larger heating rate for the neutron stars gravitational mass $M \geq 1.2M_{\odot}$.

Fig.7 shows the evolution of the surface temperature of different mass stars for the weak ($B=10^{9}$G) magnetic field. In the cases of weak field, stars could maintain high temperatures even at older ages ($> 10^6$yrs). This feature may give an illustration of the inferred temperature for PSR J0437-4715 (Kargaltsev et al. (2004)), although the quantitative analysis needs future study.

5. CONCLUSIONS AND DISCUSSIONS

The thermal evolution of rotating hybrid stars with DH have been investigated in this work. Using Hartle’s perturbative approach, we have calculated the change of internal structure of rotating hybrid stars. The nuclear matter can continuously be converted into quark matter to release latent heat during the spins down of star. The heat luminosity can be estimated as proportion to the change rates of quark number. The results show the DH of rotating stars leads to good agreement with the observed data in the case of enhanced cooling (That is the onset of direct Urca process). We also found that heating effect has magnetic field strength dependence. For those stars with weak fields ($< 10^{10}$G), our results show that they can maintain a high temperature ($> 10^5$K) at older ages ($t \sim 10^{10}$yrs) such as PSR J0437-4715. It may be particularly interesting for high temperature of weak-field stars at late stage.

We here calculate the released latent heat by regarding the heat release per nucleon as a parameter. However, the heat release per nucleon should be a density-dependent quantity. Numerical calculations are necessary for heat luminosity. Precise fittings need consider improvement of the star model, the inclusions of such superfluidity, superconductivity and tension effect in MP. These will be the future works.

ACKNOWLEDGMENTS

This work is supported by NFSC under Grant Nos.90303007 and 10373007.
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Figure 1. Model EOS for energy per baryon and the pressure of hybrid star matter as a function of the baryon number density. The HP EOS is a relativistic mean-field model, the quark matter is effective mass MIT bag model with $m_s = 150\text{MeV}$, $B^{1/4} = 160\text{MeV}$, coupling constant $g = 3.0$.

Figure 2. Gravitational mass $M$ as a function of the equatorial radius (left figure) and the central density (right figure) for rotating hybrid stars configurations with a deconfinement phase phase transition. The solid curves correspond to static configurations, the dashed ones to those with maximum rotation frequency $\nu_{\text{max}}$. The lines between both extremal cases connect configurations with the same total baryon number.
Figure 3. Central density as a function of rotational frequency for rotating hybrid stars of different gravitational mass at zero spin. All sequences are with constant total baryon number. Dash horizontal lines indicate the density where quark matter is produced.

Figure 4. The number of converting quark into baryon as a function of rotational frequency for 1.4 $M_\odot$ rotating hybrid star.
Figure 5. Cooling curves of 1.4 $M_\odot$ hybrid star with DH for various magnetic fields (curve a: $10^{13}$, b: $10^{12}$, c: $10^{11}$, d: $10^{10}$, e: $10^9$) and the curves without DH (solid curve).

Figure 6. Cooling curves of neutron stars with DH for different star mass and $B=10^{12}$G (upper curves) and the curves without DH (lower curves).
Figure 7. Cooling curves of neutron stars with DH for different star mass and $B=10^9\,\text{G}$ (upper curves) and the curves without DH (lower curves).