Magnetic and electric AdS solutions in string- and M-theory

Aristomenis Donos, Jerome P Gauntlett and Christiana Pantelidou
Blackett Laboratory, Imperial College, London SW7 2AZ, UK
E-mail: j.gauntlett@imperial.ac.uk

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Abstract
The stability properties of a family of magnetic AdS$_3 \times \mathbb{R}^2$ solutions of $D = 5$, SO(6) gauged SUGRA are investigated in more detail. We construct an analogous family of magnetic AdS$_2 \times \mathbb{R}^2$ solutions of $D = 4$, SO(8) gauged SUGRA, including a family of supersymmetric solutions, and also investigate their stability. We construct supersymmetric domain walls that interpolate between AdS$_5$ and an AdS$_3 \times \mathbb{R}^2$ solution and also between AdS$_4$ and an AdS$_2 \times \mathbb{R}^2$ solution which provide stable zero temperature ground states for the corresponding dual CFTs. We also construct new families of electric AdS$_2 \times \mathbb{R}^3$ and AdS$_2 \times \mathbb{R}^2$ solutions.

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(Some figures may appear in colour only in the online journal)

1. Introduction

With a view towards condensed matter applications, there have been several holographic investigations into the behaviour of strongly coupled gauge theories in the presence of magnetic fields, starting with [1–4]. The main focus of this paper will be on AdS$_3 \times \mathbb{R}^2$ and AdS$_2 \times \mathbb{R}^2$ solutions of string or M-theory that are supported by purely magnetic fields in the $\mathbb{R}^2$ directions.

Such solutions are of interest because they provide candidate holographic dual descriptions of the IR limit of the zero temperature ground states of field theories in $d = 4$ and $d = 3$, respectively. For this to be the case, it is certainly necessary that the solutions are stable and, in particular, do not contain any modes that violate the AdS$_n$ BF bound. Such stability is guaranteed if the solutions are supersymmetric.

As far as we are aware, the first constructions of such supersymmetric AdS$_3 \times \mathbb{R}^2$ solutions were presented in [5]. The solutions were obtained by uplifting a ‘magnetovac’ solution of Romans’ $D = 5$ gauged supergravity [6] either on an $S^5$, to obtain a solution of type IIB supergravity, or on the general class of $M_6$ [7, 8] associated with AdS$_5 \times M_6$ solutions of $D = 11$ supergravity dual to $N = 2$ SCFTs in $d = 4$ [5].

Subsequently, a non-supersymmetric magnetic AdS$_3 \times \mathbb{R}^2$ solution of minimal gauged supergravity was shown to arise as the near-horizon geometry of magnetic black brane solutions...
at zero temperature\textsuperscript{1} \cite{12}. These black brane solutions can again be uplifted on an $S^5$, to obtain a solution of type IIB supergravity, but also on general $X_5$ \cite{13} and $N_6$ \cite{14} associated with $\text{AdS}_5 \times X_5$ and $\text{AdS}_5 \times N_6$ solutions of type IIB and $D = 11$ supergravity, respectively, dual to $N = 1$ SCFTs in $d = 4$ \cite{15, 16}. One of the conclusions of this paper is that for the special case of uplifting on the $S^5$, for which the dual field theory is just $N = 4$ SYM, the $\text{AdS}_3 \times \mathbb{R}^2$ solution studied in \cite{12} is not stable and hence does not describe the zero temperature ground state.

More recently, it was found that these supersymmetric and non-supersymmetric magnetic $\text{AdS}_3 \times \mathbb{R}^2$ solutions are members of a more general two-parameter family of solutions \cite{17}, with the supersymmetric solution of [6, 5] part of a one-parameter sub-family, which can be constructed within a $U(1)^3$ truncation of $D = 5 \text{SO}(6)$ gauged supergravity\textsuperscript{2}. An initial investigation into the stability of the non-supersymmetric $\text{AdS}_3 \times \mathbb{R}^2$ solutions of \cite{17} was undertaken in \cite{18}, within the context of $\text{SO}(6)$ gauged supergravity. Here we will re-examine the analysis of \cite{18} finding results which differ in some respects due to a mixing of modes that was overlooked in [18]. In addition, we will show that a large class of the $\text{AdS}_3 \times \mathbb{R}^2$ solutions also suffer from a new kind of instability involving neutral scalar fields that are spatially modulated in the $\mathbb{R}^2$ directions similar to \cite{19–22}.

Our results, and those of \cite{18}, imply that when the solutions are uplifted on $S^5$ to give type IIB solutions dual to $N = 4$ SYM theory, the parameter space of potentially stable $\text{AdS}_3 \times \mathbb{R}^2$ solutions is now very small, but still non-zero (see figures 1 and 3). While we think it is unlikely that there are any further instabilities within $\text{SO}(6)$ gauged supergravity, it is still possible that there are others within the full KK spectrum. Note that if we consider the one-parameter family of solutions that lie within Romans’ theory, some of the instabilities that we discuss here, but not all, involve fields lying outside of Romans’ theory. This is relevant when we uplift the relevant solutions not on $S^5$ to $D = 10$, but on the $M_6$ of [7] to $D = 11$ \cite{5}. Within minimal gauged supergravity, we do not find any instabilities for the unique magnetic $\text{AdS}_3 \times \mathbb{R}^2$ solution.

We will also construct a supersymmetric domain wall solution that interpolates between $\text{AdS}_5$ in the UV and a particular supersymmetric $\text{AdS}_3 \times \mathbb{R}^2$ solution in the IR. The example we choose lies within Romans’ theory so this can be uplifted both on $S^5$ to type IIB and also on the class of $M_6$ of [7] to $D = 11$. This solution, being supersymmetric, should describe the stable ultimate zero temperature ground state of the corresponding $d = 4$ CFTs when they are placed in a magnetic field.

The instabilities that we find for the non-supersymmetric $\text{AdS}_3 \times \mathbb{R}^2$ solutions show that these solutions cannot provide the ultimate IR ground states of dual $d = 4$ field theories when held in a magnetic field. Nevertheless, they are still physically interesting. In general, extending the work of \cite{12}, we expect to be able to construct finite temperature black hole solutions which at zero temperature interpolate between $\text{AdS}_5$ in the UV\textsuperscript{3} and a given non-supersymmetric $\text{AdS}_3 \times \mathbb{R}^2$ solution in the IR. The instability of the latter indicates that there will be a phase transition at finite temperature described by new types of black hole solutions and the instabilities that we discuss suggest the types of modes that will be involved in constructing

\footnote{Dyonic extensions were studied in [9–11].}

\footnote{Recall that $D = 5$ minimal $U(1)$ gauged supergravity is a truncation of Romans $D = 5 U(1) \times SU(2)$ gauged supergravity which in turn is a truncation of $\text{SO}(6)$ gauged supergravity. Correspondingly the $U(1)^3 \subset \text{SO}(6)$ truncation can be truncated to a $U(1)^2 \subset U(1) \times SU(2)$ truncation of Romans theory which can also be truncated to minimal gauged supergravity. Generically, a solution of $\text{SO}(6)$ gauged supergravity, and also the $U(1)^3$ truncation, can only be uplifted on $S^5$ to obtain type IIB solutions while, as noted in the text, solutions of Romans and minimal gauged supergravity can be uplifted to both type IIB and $D = 11$.}

\footnote{In general, in addition to the magnetic field, we expect that some of the scalar fields will also give rise to deformations of operators of the UV CFT.}
Figure 1. The moduli space of magnetic AdS$_3 \times \mathbb{R}^2$ solutions. Any point in the ($f_1, f_2$) plane, combined with a set of signs for the $q_i$, gives rise to an AdS$_3 \times \mathbb{R}^2$ solution. The red lines correspond to the locus of solutions that can preserve supersymmetry, for particular choices of the signs. The dashed lines correspond to solutions that can be embedded into Romans’ theory and the origin corresponds to solutions that can be embedded in minimal gauged supergravity.

them. Some specific magnetic AdS$_3 \times \mathbb{R}^2$ solutions have several different kinds of instabilities including spatially modulated instabilities, driven by neutral scalars, and superconducting instabilities driven either by charged scalars, charged vectors$^4$ or combinations thereof. Our results suggests that there is a rich story involving competing phases that would be interesting to elucidate. It is also worth emphasizing that since the supersymmetric AdS$_3 \times \mathbb{R}^2$ solutions are at the boundary of unstable non-supersymmetric solutions (see figure 3), the corresponding dual ground states must abut different phases and hence have the nature of quantum critical points.

We now turn to top down solutions containing AdS$_2$ factors that are supported by magnetic fields. Such solutions are particularly interesting since they might provide dual descriptions of locally quantum critical points, which have been shown to be associated with interesting non-Fermi liquid behaviour [29–32] (see [33, 34] for the inclusion of magnetic fields). For $D = 5$ it was shown in [17] that the $U(1)^3$ truncation of SO(6) gauged supergravity admits a non-supersymmetric magnetic AdS$_2 \times \mathbb{R}^3$ solution which can be uplifted on an $S^5$ to type IIB. However, it was subsequently shown in [35] that, within the same truncation, this suffers from an instability involving neutral scalar fields that are spatially modulated in the $\mathbb{R}^3$ directions.

It has long been known that minimal $D = 4$ gauged supergravity admits a non-supersymmetric magnetic AdS$_2 \times \mathbb{R}^2$ solution (the near horizon limit of the standard magnetic AdS–RN black brane solution) and that this can be uplifted to obtain solutions of $D = 10, 11$

$^4$ The possibility of charged vectors producing superconducting instabilities in the presence of magnetic fields has been recently discussed in [23, 24] based on the older work of [25, 26]. This is also reminiscent of ‘reentrant superconductivity’, reviewed in [27], that is seen in URhGe [28].
supergravity in a variety of ways [15, 16]. The concluding section of [18] briefly discussed how this solution can be generalized to form part of a larger family of magnetic AdS$_2 \times \mathbb{R}^2$ solutions of the $U(1)^4$ truncation of $SO(8)$ gauge supergravity which can be uplifted to $D = 11$ on $S^7$. Here we will flesh out these constructions in a little more detail. We will find a two-parameter locus of supersymmetric solutions that includes the specific examples already mentioned in [18], as special cases. We will also show that a large class of the non-supersymmetric solutions suffer from similar instabilities that we find for the magnetic AdS$_3 \times \mathbb{R}^2$ solutions. In particular we explicitly discuss an instability involving neutral scalar fields that are spatially modulated in the $\mathbb{R}^2$ directions and a simple instability involving charged scalars.

We will also construct a supersymmetric domain wall solution that interpolates between AdS$_4$ in the UV and a representative magnetic AdS$_2 \times \mathbb{R}^2$ solution in the IR. This solution, being supersymmetric, should describe the stable ultimate zero temperature ground state of the corresponding $d = 3$ CFTs when they are placed in a magnetic field. This provides the first top down holographic description of a locally quantum critical point where stability is guaranteed by supersymmetry, and it will be interesting to investigate the behaviour of fermion response functions for this background.

In the last part of the paper, we will briefly discuss AdS$_2$ solutions that are supported by electric fields. Indeed an electric–magnetic duality transformation for the $U(1)^4$ truncation of $D = 4$ $SO(8)$ gauged supergravity provides a simple way to obtain a three-parameter family of electric AdS$_2 \times \mathbb{R}^2$ solutions. We find that none of them preserve supersymmetry. Furthermore, we also can use the duality transformation to find a domain wall solution, solving first order equations, that interpolates between AdS$_2$ and a non-supersymmetric AdS$_2 \times \mathbb{R}^2$ solution. We do not investigate instabilities for these solutions here, but we expect that there will be many: for the special case of the electric AdS$_2 \times \mathbb{R}^2$ solution of minimal gauged supergravity see [40].

In addition, we also construct a two-parameter family of electric AdS$_2 \times \mathbb{R}^3$ solutions of the $U(1)^3 \subset SO(6)$ gauged supergravity. These solutions generalize the solutions of Romans’ theory given in [6]. Once again, none of these solutions are supersymmetric. We will again leave a detailed analysis of instabilities to future work, but we note that it was already shown in [22] that the electric AdS$_2 \times \mathbb{R}^3$ solutions of Romans’ theory suffer from helical $p$-wave superconducting instabilities.

2. Magnetic AdS$_3 \times \mathbb{R}^2$ solutions

In this section we will review the magnetic AdS$_3 \times \mathbb{R}^2$ solutions of [6, 17, 18]. We will also construct a supersymmetric domain wall solution that interpolates between AdS$_5$ in the UV and the supersymmetric AdS$_3 \times \mathbb{R}^2$ solution of [6] in the IR.

2.1. $U(1)^3 \subset SO(6)$ gauged supergravity

We start with the $U(1)^3$ truncation of $D = 5$ $SO(6)$ gauged supergravity [41] that keeps two neutral scalar fields $\phi_a$. It is convenient to package the two scalars in terms of three constrained scalars $X_i$ via

$$X_1 = e^{-\frac{1}{\sqrt{6}} \theta_1} e^{\frac{1}{\sqrt{2}} \theta_2}, \quad X_2 = e^{\frac{1}{\sqrt{6}} \theta_1 + \frac{1}{\sqrt{2}} \theta_2}, \quad X_3 = e^{\frac{2}{\sqrt{6}} \theta_1},$$

(2.1)

Note that in the context of $N = 2$ $D = 4$ gauged supergravity, related solutions have been discussed in [36–38].

6 In the context of AdS$_4 \times SE_7$ solutions, a non-supersymmetric flow solution that interpolates between AdS$_3$ in the UV and an electric AdS$_2 \times \mathbb{R}^2$ in the IR was constructed in [39] and, as yet, has not been shown to suffer from any instabilities.
with $X_1X_2X_3 = 1$. The Lagrangian is then given by
\[
\mathcal{L} = (R - V) * 1 - \frac{1}{2} \sum_{a=1}^{2} * d\phi_a \wedge d\phi_a - \frac{1}{2} \sum_{i=1}^{3} (X_i)^{-2} * F^i \wedge F^i + F^1 \wedge F^2 \wedge A^3, \tag{2.2}
\]
where
\[
V = -4 \sum_{i=1}^{3} (X_i)^{-1}. \tag{2.3}
\]
Any solution of this theory can be uplifted on an $S^5$ to obtain an exact solution of type IIB supergravity using the formulæ in [41].

This theory can be further truncated to obtain a sector of Romans’ $SU(2) \times U(1)$ gauge supergravity theory [6]. This is significant because any solution of Romans’ theory can also be uplifted to $D = 11$ supergravity using the general class of $M_6$ [7] associated with supersymmetric $AdS_5 \times X_5$ solutions of $D = 11$ supergravity that are dual to $N = 2$ SCFTs in $d = 4$ [5]. Specifically, if we set $X_1 = X_2 \rightarrow X$, $F^1 = F^2 \rightarrow F^3 / \sqrt{2}$, $F^3 \rightarrow -G$ we obtain Romans’ theory as in [5], after setting the two-form to zero and identifying $F^3$ with one of the $SU(2)$ gauge-fields. There are two other ways of obtaining Romans’ theory: one by setting $X_2 = X_3 = 1$, and also $F^1 = F^2 = F^3$ we obtain minimal $D = 5$ gauged supergravity. Recall that any solution of this theory can be uplifted to type IIB supergravity using the general class of $X_5$ [13] associated with supersymmetric $AdS_5 \times X_5$ solutions of type IIB supergravity that are dual to $N = 1$ SCFTs in $d = 4$ [15] or to $D = 11$ using the general class of $N_6$ [14] associated with supersymmetric $AdS_5 \times N_6$ solutions of $D = 11$ supergravity also dual to $N = 1$ SCFTs in $d = 4$ [16].

2.2. The $AdS_3 \times \mathbb{R}^2$ solutions

We now consider the family of magnetic $AdS_3 \times \mathbb{R}^2$ solutions to the equations of motion for (2.2) found in [17], and studied further in [18], given by
\[
dx_i^2 = L^2 \, d\tau^2(AdS_3) + dx_1^2 + dx_2^2, \\
F^i = 2q^i \, dx_1 \wedge dx_2, \quad \phi_1 = f_1, \quad \phi_2 = f_2, \tag{2.4}
\]
where $f_a$ are constants,
\[
L^{-2} = \sum_{i=1}^{3} (\bar{X}_i)^{-1}, \quad (q^i)^2 = \bar{X}_i, \tag{2.5}
\]
and $\bar{X}_i$ are the on-shell values
\[
\bar{X}_1 = e^{-\frac{\phi}{f_1 - f_2}}; \quad \bar{X}_2 = e^{-\frac{\phi}{f_1 + f_2}}; \quad \bar{X}_3 = e^{\frac{\phi}{f_1}}. \tag{2.6}
\]
The $q^i$ can be chosen to have either sign. Notice that when $f_2 = 0$, for example, these are solutions to Romans’ theory, and actually were already presented in [6] and uplifted to $D = 10, 11$ supergravity in [5]. When $f_1 = f_2 = 0$ they are solutions of minimal gauged supergravity.

The supersymmetry of these solutions was analysed in [18] where it was shown that the sum of the $q^i$, with suitable signs, must vanish. We will review this analysis in the next subsection where we also show that the locus of supersymmetric solutions is given by
\[
2 \sum_{i} \bar{X}_i^2 = \left( \sum_{i} \bar{X}_i \right)^2. \tag{2.7}
\]
We have summarized the moduli space of solutions in figure 1.
The AdS$_3$ solutions, assuming that they are stable, are dual to $d = 2$ CFTs, and the radius, $L$, is proportional to the central charge. We find that $L$ has a global maximum for the solution with $f_1 = f_2 = 0$. Along the supersymmetric branches, it is a maximum for the three solutions that can be embedded into Romans’ theory and then decreases monotonically away from them.

2.3. Supersymmetric AdS$_5$ to AdS$_3 \times \mathbb{R}^2$ domain wall

It was already shown in [18] that the magnetic AdS$_3 \times \mathbb{R}^2$ solutions (2.4),(2.5) can be supersymmetric, preserving two Poincaré supersymmetries (i.e. $(0, 2)$ in $d = 2$). Here we would like to show that there are supersymmetric domain wall solutions that interpolate between AdS$_5$ in the UV and AdS$_3 \times \mathbb{R}^2$ in the IR. We thus consider the ansatz

\[
\begin{align*}
    ds^2 &= e^{2W} (-dt^2 + dy^2) + d\rho^2 + e^{2U}(dx_1^2 + dx_2^2), \\
    F_i &= 2q_i \, dx_1 \wedge dx_2, \\
    \phi_a &= \phi_a(\rho),
\end{align*}
\]

where $W$ and $U$ are functions of $\rho$. We will consider the $N = 1$ supersymmetry transformations as given in [42]

\[
\begin{align*}
    \delta \psi_\mu &= \nabla_\mu \epsilon - \frac{i}{2} \sum_i A_i^\mu \epsilon + \frac{i}{6} \sum_j X_j \gamma_\mu \epsilon + \frac{i}{24} \sum_i X_i^{-1} [\gamma_\mu, \gamma_\nu] F_{\nu \epsilon} \epsilon, \\
    \delta \lambda_a &= \left[ -\frac{i}{4} \gamma_a \phi + \frac{i}{2} \sum_j \partial_\rho X_j + \frac{1}{8} \sum_j \partial_\rho X_j^{-1} F^j_{\mu \nu} \gamma^{\mu \nu} \right] \epsilon.
\end{align*}
\]

To preserve these supersymmetries we require that $\sum_i A_i^\mu = 0$ and hence the magnetic charges $q_i$ should satisfy

\[
\sum_i q_i = 0.
\]

As noted in [18] there are another three $N = 1$ supersymmetries with different sign choices for the gauge fields and hence the charges. We expect these to correspond to the conditions $q_1 + q_2 - q_3 = 0, q_1 - q_2 + q_3 = 0$ and $-q_1 + q_2 + q_3 = 0$. As noted in [18], this means that extra supersymmetry can be preserved only if one of the charges is zero: however from (2.5) we see that this is not possible, since $X_i > 0$.

Turning now to the specific ansatz (2.8), choosing $q_i$ to satisfy (2.10) and imposing the projection conditions

\[
\begin{align*}
    \gamma_\mu \epsilon &= -\epsilon, \\
    \gamma_\mu \gamma_\nu \epsilon &= i \alpha \epsilon, \\
    \alpha &= \pm 1,
\end{align*}
\]

we obtain

\[
\begin{align*}
    -W' + \frac{1}{3} \sum_i X_i - \frac{\alpha}{3} e^{-2U} \sum_i X_i^{-1} q_i &= 0, \\
    -U' + \frac{1}{3} \sum_i X_i + \frac{2\alpha}{3} e^{-2U} \sum_i X_i^{-1} q_i &= 0, \\
    \phi_a' + 2 \sum_j \partial_\rho X_j + 2\alpha e^{-2U} \sum_j q_j \partial_\rho X_j^{-1} &= 0, \\
    \left[ \partial_\rho - \frac{1}{6} \sum_i X_i + \frac{\alpha}{6} e^{-2U} \sum_j q_j X_j^{-1} \right] \epsilon &= 0.
\end{align*}
\]

Note that we should set their $g = 1$ and identify our $X_i$ with their $X^i = 1/(3X_i)$.
From the first and the last equation in (2.12) we derive that $\epsilon = e^{W/2}\eta$, with $\eta$ a constant spinor satisfying the projection conditions (2.11).

For the supersymmetric $\text{AdS}_3 \times \mathbb{R}^2$ solutions, we should set $W = L^{-1}\rho$, $U = 0$ and $X_i = \tilde{X}_i$ in (2.11). We then find the conditions

$$L^{-1} = \frac{1}{2} \sum_i \tilde{X}_i, \quad -2\alpha q' = \tilde{X}_i \left(-2\tilde{X}_i + \sum_j \tilde{X}_j\right).$$

(2.13)

The latter condition combined with (2.10) leads to the condition (2.7), that we mentioned earlier. For the $\text{AdS}_3$ vacuum solution we set $W = U = R^{-1}\rho$, $\phi_\alpha = 0$ and find $R = 1$.

We now show that there exists supersymmetric solutions that interpolate between $\text{AdS}_3$ in the UV and a supersymmetric $\text{AdS}_3 \times \mathbb{R}^2$ solution in the IR. We do this for just one representative solution in the IR, namely the one that exists inside Romans’ theory. In fact the entire domain wall solution lies within Romans’ theory and so we set $\phi_\alpha = 0$ and consider the supersymmetric solution with $\phi_1 = (2\sqrt{6}/3)\log 2$ and $q^1 = q^2 = -\frac{2}{3}$ and $q^3 = \frac{2}{3}$ (we have chosen $\alpha = 1$). Within this truncation, the flow equations (2.12) for the non-trivial functions are given by

$$W' = \frac{1}{3} \left(2 e^{-\frac{1}{2}\phi} + e^{-\frac{1}{2}\phi}\right) - \frac{2}{3} e^{-2L} \left(e^{\frac{1}{2}\phi} - e^{-\frac{1}{2}\phi}\right) = 0,$$

$$U' = \frac{1}{3} \left(2 e^{-\frac{1}{2}\phi} + e^{-\frac{1}{2}\phi}\right) + \frac{25}{3} e^{-2L} \left(e^{\frac{1}{2}\phi} - e^{-\frac{1}{2}\phi}\right) = 0,$$

$$\phi'_1 = \frac{4}{\sqrt{6}} \left(e^{-\frac{1}{2}\phi} - e^{-\frac{1}{2}\phi}\right) - \frac{5}{3} e^{-2L} \left(e^{\frac{1}{2}\phi} + 2e^{-\frac{1}{2}\phi}\right) = 0.$$ 

(2.14)

Close to the $\text{AdS}_3 \times \mathbb{R}^2$ solution in the far IR the system of equations (2.14) admits the expansion

$$W = w_0 + L^{-1}\rho + \frac{1}{16}(-29 + 3\sqrt{3})c_{IR} e^{L^{-1}\rho} + \cdots,$$

$$U = c_{IR} e^{L^{-1}\rho} + \cdots,$$

$$\phi_1 = 2\sqrt{2} \ln 2 - \sqrt{2/3}(5 + \sqrt{3})c_{IR} e^{L^{-1}\rho} + \cdots,$$

(2.15)

with $w_0$ and $c_{IR}$ two constants while $L^{-1} = 3/2^{2/3}$ and $\delta = \frac{1}{4}(-1 + \sqrt{33})$. In the far UV we would like to approach the unit radius $\text{AdS}_3$ vacuum. One can easily see that equations (2.14) admit the expansion

$$U = \rho + \mathcal{O}(e^{-4\rho}),$$

$$W = \rho + \mathcal{O}(e^{-4\rho}),$$

$$\phi_1 = 27/6 \sqrt{3} e^{-2\rho} \rho + c_{UV} e^{-2\rho} + \mathcal{O}(e^{-4\rho}),$$

(2.16)

where $c_{UV}$ is a constant of integration. From the above expansion, we see that the scalar $\phi_1$, which is dual to an operator with conformal dimension $\Delta = 2$ in the UV CFT, has both a VEV and a deformation. Using a shooting method, we find that there is a solution to (2.14) with boundary conditions (2.15) and (2.16) with $w_0 \approx -0.10$, $c_{IR} \approx 0.31$ and $c_{UV} \approx -1.97$ as we have indicated in figure 2. This is the supersymmetric domain wall solution.

This solution can be uplifted on $S^5$ to type IIB or on the class of $M_6$ [7] to $D = 11$ [5]. The solutions then describe the corresponding dual $d = 4$ CFTs deformed by the presence of the magnetic field and also by the operator dual to $\phi_1$. In particular, the uplifted supersymmetric $\text{AdS}_3 \times \mathbb{R}^2$ solutions describe the IR ground state at zero temperature.
3. Instabilities of magnetic $\text{AdS}_3 \times \mathbb{R}^2$ solutions

In this section we analyse various instabilities of the magnetic $\text{AdS}_3 \times \mathbb{R}^2$ solutions given in (2.4),(2.5) within $SO(6)$ gauged supergravity.

3.1. Spatially modulated instabilities of the neutral scalars

Possible instabilities of the two neutral scalars appearing in the $U(1)^3$ truncation (2.2) of $SO(6)$ gauge supergravity were investigated in [18] and none were found. In that analysis only fluctuations independent of the $x_1$ and $x_2$ direction were considered. Here we relax this assumption and find that there are spatially modulated modes violating the BF bound hence leading to instabilities, as summarized in figure 3(a).
3.1.1. Instabilities of the solutions existing in Romans’ theory. Let us first discuss the instabilities of the three lines of solutions that are solutions of Romans’ theory (the dashed blue lines in figure 1). We should emphasize at the outset that the unstable modes that we find involve fields lying outside of Romans’ theory and hence the instability is not relevant when we uplift such solutions on $M_6$ to $D = 11$ but only when we uplift them on $S^3$ to type IIB.

To illustrate, we will consider the line of AdS$_3 \times \mathbb{R}^3$ solutions with $f_2 = 0$ and $q^1 = q^2$, which arise in Romans’ theory. We consider the field perturbation

$$
\delta A^1 = a(t, y, \rho) \sin(k x_1) \, dx_2,
\delta A^2 = -a(t, y, \rho) \sin(k x_1) \, dx_2,
\delta \phi_2 = w(t, y, \rho) \cos(k x_1),
$$

(3.1)

where $a$ and $w$ are functions of the AdS$_3$ coordinates and $k$ is a constant. We find that the linearized equations of motion imply that

$$
(\Box_{\text{AdS}_3} - L^2 M^2) v = 0,
$$

(3.2)

where $v = (a, w)$, $\Box_{\text{AdS}_3}$ is the Laplacian of the unit radius AdS$_3$ and the mass matrix is given by

$$
M^2 = \begin{pmatrix}
\frac{k^2}{4 \sqrt{2} q^1 k} & \frac{2 \sqrt{2} q^1}{4 (X_1)^{-1} + k^2} \\
\frac{2 \sqrt{2} q^1}{4 (X_1)^{-1} + k^2} & 0
\end{pmatrix}.
$$

(3.3)

Notice for $k = 0$ there is no mixing at all as seen in [18]. After diagonalizing the mass matrix we find the two eigenvalues

$$
m^2_{\pm} = \frac{1}{X_1} (2 + k^2 X_1 \pm 2 \sqrt{1 + 4k^2 X_1}).
$$

(3.4)

The minimum mass is achieved on the $m^2_-$ branch when $k_{\text{min}} = \pm \frac{1}{2} \sqrt{15} (X_1)^{-1/2}$ giving

$$
m^2_{\text{min}} = -\frac{9}{4 (2 + X_1)}.
$$

(3.5)

The AdS$_3$ BF bound $L^2 m^2 > -1$ is violated for $f_1 > 2 \sqrt{2} \ln 2$. Note that when $f_1 = 2 \sqrt{2} \ln 2$, we have $X_1 = 2^{-2/3}$ and the solution will satisfy the supersymmetry condition (2.10) provided that $q^1$ has an opposite sign to that of $q^2$. In other words, the supersymmetric solutions are located right at the boundary of the set of solutions where the spatially modulated instabilities set in. It is also worth noting that for the supersymmetric solution, the static mode that saturates the BF bound, given explicitly by

$$
w(\rho) = c_1 e^{-\frac{\rho}{2}} \cos(|k_{\text{min}}| x_1), \quad a(\rho) = c_2 e^{-\frac{\rho}{2}} \sin(|k_{\text{min}}| x_1),
$$

(3.6)

where $|k_{\text{min}}| = \sqrt{15}/2^{2/3}$ and $c_1/c_2 = 2^{7/3} / 5$, preserves the supersymmetries of the background. The sign choice of $c_1/c_2$ depends on the choice of $a$ in the projector (4.11) and we note that for this solution the sign of $a$ is opposite to that of $q^1$.

3.1.2. More general analysis. We now consider perturbations about the full two-parameter family of AdS$_3 \times \mathbb{R}^3$ solutions (2.4), (2.5). In general we cannot decouple the metric perturbations and so we consider the time independent perturbation

$$
\delta g_{tt} = -\delta g_{\varphi \varphi} = L^2 r^2 h_3(r) \cos(k x_1),
\delta g_{t x_1} = h_a(r) \cos(k x_1), \quad a = 1, 2,
\delta A^i = a_i(r) \sin(k x_1) \, dx_2, \quad i = 1, 2, 3,
\delta \phi_2 = w_a(r) \cos(k x_1), \quad a = 1, 2,
$$

(3.7)
containing eight independent functions, which we take to be functions of the radial coordinate, \( r \), of \( \text{AdS}_3 \) space when written in Poincaré coordinates (with boundary located at \( r \to \infty \)). Expanding the equations of motion of (2.2) around the solutions (2.4) we find a total of eleven differential equations. After a little algebra we can show that for \( k \neq 0 \) the independent equations for the radial functions consist of two first order equations for \( h_1 \) and \( h_3 \) and six second order equations for each of the \( a_i, w_a \) and \( h_2 \). Note that the equations governing the perturbation (3.7) are independent of the sign choices in (2.5).

To find the scaling dimensions of the dual conformal field theory we look for solutions where the eight functions, as a vector, are of the form \( v r^\delta \) where \( v \) is a constant vector and \( \delta \) is a constant that is related to a scaling dimension in the two-dimensional conformal field theory dual to the \( \text{AdS}_3 \) solution. The system of equations then takes the form \( M v = 0 \) where \( M \) is an \( 8 \times 8 \) matrix. Demanding that non-trivial values of \( v \) exist implies that \( \det M = 0 \) and this specifies the possible values of \( \delta \) as a function of \( k \). In figure 3(a) we have shaded the region of the \( f_1 - f_2 \) plane for which we find a mode with complex scaling dimension. All the unstable modes that we find are spatially modulated with \( k \neq 0 \).

It is interesting to note that the boundary of the region plotted in figure 3(a) (red curve) is the set of points \( (f_1, f_2) \) for which there exists a choice in the signs of (2.5) satisfying the supersymmetry condition. In other words, in the supersymmetric solutions there always exists a mode which saturates the \( \text{AdS}_3 \) unitarity bound at finite \( k \). Based on the Romans’ case, we expect that this mode is always supersymmetric.

At the end of the last section we constructed a supersymmetric domain wall solution interpolating between \( \text{AdS}_5 \) in the UV and \( \text{AdS}_3 \times \mathbb{R}^2 \) in the IR. These solutions describe the zero temperature ground states of the dual \( d = 4 \) CFTs when held in a magnetic field and also deformed by the operator dual to the scalar field. The fact that the supersymmetric \( \text{AdS}_3 \times \mathbb{R}^2 \) solutions are at the boundary of unstable \( \text{AdS}_3 \times \mathbb{R}^2 \) solutions indicates that the supersymmetric ground states must adjoin different phases and hence have the nature of quantum critical points.

3.2. Instabilities of some charged scalars

We now want to consider possible instabilities of the magnetic \( \text{AdS}_3 \times \mathbb{R}^2 \) solutions with respect to other fields within \( SO(6) \) gauged supergravity. We first recall that \( SO(6) \) gauged supergravity has 42 scalar fields, parametrizing the coset \( E_{6(6)}/USp(8) \) and transforming as \( 20 + 10 + \bar{10} + 1 + 1 \) of \( SO(6) \). The scalars in the \( 20 \) irrep are described by a unimodular \( 6 \times 6 \) matrix \( T \) which, as we will discuss, contains the two scalars in the \( U(1)^3 \) truncation (2.2) that we have been discussing. In this subsection and the next, we will show that there are additional instabilities involving the scalar fields in \( T \). Our most general analysis will utilize the consistent truncation [44] that keeps \( T \) and the 15 \( SO(6) \) gauge-fields. It is worth noting that when uplifting to type IIB, only the \( D = 10 \) metric and five-form are involved in this truncation.

As the general analysis using the truncation [44] is rather involved we first investigate possible instabilities of three of the 20 scalars, \( \phi_i \), appearing in \( T \), using the consistent truncation of \( SO(6) \) gauge supergravity [45]. The Lagrangian is given by

---

Footnotes:

8 In appendix A we show that there are no instabilities in a truncation of [43] that keeps four complex scalars in the \( 10 + 10 \) irreps (we thank N. Bobev for suggesting this calculation). We also note that the two singlet scalars comprise the axion and dilaton; the stability of the dilaton was discussed in [18]. We have not investigated the stability of these or the 12 two-forms of \( SO(6) \) gauged supergravity.
\[
\mathcal{L} = (R - V) \ast 1 - \frac{1}{2} \sum_{a=1}^{2} \ast \phi_{a} \wedge \ast \phi_{a} - \frac{3}{2} \sum_{i=1}^{3} (X^i)^{-2} \ast F^i \wedge F^i + F^1 \wedge F^2 \wedge A^3
\]

\[
- \frac{1}{2} \sum_{i=1}^{3} \ast \phi_i \wedge \ast \phi_i - 2 \sum_{i=1}^{3} \sinh^2 \phi_i \ast A^i \wedge A^i, \quad (3.8)
\]

with

\[
V = -2[2X^2X^3 \cosh \varphi_2 \cosh \varphi_3 + 2X^2X^4 \cosh \varphi_4 + 2X^3X^2 \cosh \varphi_1 \cosh \varphi_2

- (X^1)^2 \sinh^2 \varphi_1 - (X^2)^2 \sinh^2 \varphi_2 - (X^3)^2 \sinh^2 \varphi_3]. \quad (3.9)
\]

After expanding around the background (2.4), the equation of motion for the charged scalar \( \varphi_1 \) gives

\[
\Box_{\text{AdS}_3} \varphi_1 + L^2 \Box_{\text{R}^3} \varphi_1 + 4L^2 (\dddot{X}_1 \dddot{X}_3 + \dddot{X}_1 \dddot{X}_2 - \dddot{X}_1^2 - (A^4)^2) \varphi_1 = 0, \quad (3.10)
\]

where, again, \( \Box_{\text{AdS}_3} \) is the Laplacian of the unit radius AdS3. The equations for \( \varphi_{2,3} \) are given by cyclic permutations of indices. We now choose a gauge such that \( A' = \varphi' \), \( (x_1 \ dx_2 - x_2 \ dx_1) \) while for the scalar we consider the lowest Landau level 'ground state'

\[
\varphi_1 = e^{-|q_1| (t^2 + z^2)} \psi_1(t, r, z), \quad (3.11)
\]

giving

\[
(\Box_{\text{AdS}_3} - L^2 m_{\psi_1}^2) \psi_1 = 0, \quad (3.12)
\]

where

\[
m_{\varphi_1}^2 = -4(\dddot{X}_1 \dddot{X}_3 + \dddot{X}_1 \dddot{X}_2 - \dddot{X}_1^2 - |q_1|), \quad (3.13)
\]

which agrees with the last line of equation (6.7) in [18] (after setting their \( g = 1 \)).

In figure 3(b) we have indicated where these modes violate the BF bound. As one can see from figure 3(b), these modes intersect the locus of supersymmetric solutions in six places. Using the results of [46] (see equation (2.24) and set \( g = 1 \)) one can check that at these points the modes saturating the BF bound are supersymmetric, just as we saw in section 3.1.1. The higher Landau levels have mass

\[
m_{\psi_1}^2 = -4(\dddot{X}_1 \dddot{X}_3 + \dddot{X}_1 \dddot{X}_2 - \dddot{X}_1^2 - |q_1|)(2n + 1), \quad (3.14)
\]

again as in [18]. These are unstable in sub-regions of figure 3(b), and in particular do not intersect the supersymmetric locus.

It is interesting to note that all of these instabilities involve electrically charged fields and hence are associated with new branches of finite temperature superconducting black brane solutions. As the superconductivity is being driven by a magnetic field, it would be interesting to construct and study them further.

### 3.3. Instabilities of charged scalars and vectors

We will now examine perturbations of SO(6) gauged SUGRA contained in the truncation [44]. This contraction contains twenty scalar fields arranged in a unimodular, 6 \times 6 symmetric matrix \( T_{ij} \) and keeps all of the SO(6) gauge fields. The vector and scalar equation of motion we would like to perturbatively expand are

\[
-D \ast DT_{ij} + T^{-1k_1k_2} DT_{ik_1} \wedge \ast DT_{kj_2} = -2T_{ik_1} T^{k_1k_2} T_{kj_2} - T_{ik_1} T^{k_1k} T_{k}^j

+ T_{lm}^{-1} F^i \wedge F^m - \frac{1}{6} T_{ij} \left[ -2T_{ik} T^k - \left( T_{ik}^k \right)^2 + T_{pk}^{-1} T_{lm}^{-1} F^{pk} \wedge F^{lm} \right],
\]

\[
D(T_{ik}^{-1} T_{ij}^{-1} F^k) = -2T_{ik}^{-1} \ast DT_{1j}^k, \quad (3.15)
\]
where
\[ DT_{ij} = dT_{ij} + A_i^k T_{kj} - T_{ik} A^k_j, \]
\[ F_{ij} = dA_{ij} + A_i A^i_j, \]
and we note that the first line of (3.15) corrects a sign in [44].

We find it convenient to switch to a complex notation which just keeps the\(SU(3) \subset SO(6)\) symmetry manifest. We will write the magnetic \(AdS_3 \times \mathbb{R}^2\) solutions (2.4),(2.5) as
\[ \tilde{T}_{ij} = X_i, \quad \tilde{A}_{ij} = q_I(\tilde{\omega} \, d\tilde{z} - z \, d\tilde{\omega}), \]
where \(I, J = 1, \ldots, 3\), are \(SU(3)\) indices and \(z = \frac{1}{\sqrt{2}}(x^1 + i \, x^2)\). Note that \(X_i\) are the on-shell values, \(X_i = \tilde{X}_i\), and also \(q_I = q'\) (hence \(q_I = \pm X_I^{1/2}\)). Consider the scalar perturbation \(T = \tilde{T} + t\) where \(t\) is a complex matrix. The perturbations \(t_{IJ}\) correspond to perturbations of the neutral scalar fields that we considered in section 3.1, while the perturbations \(t_{IJ}\) correspond to the charged scalars\(^9\) that we considered in section 3.2. Thus we now consider perturbations \(t_{IJ}\) and \(t_{IJ}\) with \(I \neq J\). We note that these modes were considered in [18] but the mixing between these modes and the charged modes in the gauge fields was overlooked and we will obtain different results for the spectrum.

We thus consider
\[ T = \tilde{T} + t, \quad A = \tilde{A} + a, \]
where we are expanding around the background (3.17). Furthermore, we find that it is consistent to set the components of the one-form \(a\) along the \(AdS_3\) directions to vanish and so we write
\[ a = a^i \, d\tilde{z} + a^2 \, d\tilde{\omega}, \]
where \(a^i\) and \(t\) are complex matrices that are functions of both the \(AdS_3\) coordinates and also \((z, \tilde{\omega})\).

We will first consider the modes \(a_{IJ}, t_{IJ}, I \neq J\). Linearizing the equations of motion and introducing an appropriate set of ladder operators we are led to the following spectrum (see appendix B for details). Here we define
\[ \omega_{IJ} = q_I + q_J, \quad W_{IJ} = \text{sign}(\omega_{IJ})(q^{-1}_I - q^{-1}_J), \quad V_{IJ} = X_I - X_J. \]

(3.20)

(note that \(W\) is defined slightly differently in the appendix). When \(\omega_{IJ} \neq 0\), the independent modes are labelled by two integers \(n, m > 0\). There is a tower of modes with \(n = 0\), just involving the charged vector fields \(a_{IJ}\), which have \(AdS_3\) mass given by
\[ m_{0,m}^2 = -2|\omega_{IJ}| - 2W_{IJ}V_{IJ} + V_{IJ}^2. \]

(3.21)

Also, for each \(m\) and \(n\) there are mixed modes, involving both \(a_{IJ}\) and \(t_{IJ}\), with mass matrix
\[ M_{n,m}^2 = \begin{pmatrix} 2|\omega_{IJ}|(2n + 1) - 4q^{-1}_I q^{-1}_J & 2W_{IJ}V_{IJ} + V_{IJ}^2 & 4\sqrt{2|\omega_{IJ}|W_{IJ}}(n + 1)^{1/2} \\ 2\sqrt{2|\omega_{IJ}|W_{IJ}}(n + 1)^{1/2} & 2|\omega_{IJ}|(2n + 1) - 2W_{IJ}V_{IJ} + V_{IJ}^2 \end{pmatrix}. \]

(3.22)

In figure 3(c) we have indicated where the zero modes (3.21) can violate the \(AdS_3\) BF bound leading to an instability. The modes arising from diagonalizing (3.22) can also violate the BF bound, but only in the region outside of the three supersymmetry lines (in particular there is no overlap with the zero mode instabilities in figure 3(c)). It is straightforward to determine, numerically, which of the diagonalized modes has the largest violation of the BF bound. The fact that \(n\) is an integer leads to a more elaborate structure as compared to the spatially modulated modes labelled by a continuous variable in figure 3(a).

\(^9\) Note that there are three more scalar fields complementing the real \(\varphi_i\) that we did not explicitly consider.
When $\omega_{IJ} = 0$, which occurs along the three lines with $X_I = X_J$ (the dashed lines in figure 1), we are essentially led back to the mass matrix that we saw for the spatially modulated neutral scalars in (3.3).

The story for the modes $a^I_{ij}, t_{ij}$, with $I \neq J$ is very similar. We now define
\begin{equation}
\omega_{ij} = q_i - q_j, \quad W_{ij} = \text{sign}(\omega_{ij})(q_i^{-1} + q_j^{-1}).
\end{equation}

When $\omega_{ij} \neq 0$, the independent modes are again labelled by two integers $n, m > 0$. Again there is a tower of modes with $n = 0$, again just involving the vector fields, with AdS$_3$ mass given by
\begin{equation}
m_{0,m}^2 = -2|\omega_{ij}| - 2W_{ij}V_{JI} + V_{IJ}^2.
\end{equation}

Then for each $m$ and $n$ there are mixed modes with mass matrix
\begin{equation}
M_{n,m}^2 = \left(\frac{2|\omega_{ij}|(2n + 1) + 4q_I^{-1}q_J^{-1} + 2W_{ij}V_{JI} + V_{IJ}^2}{2\sqrt{2|\omega_{ij}|W_{ij}((n+1)^{1/2})}} \right)^2.
\end{equation}

The instabilities are similar to those we saw above. In particular, the zero modes (3.24) can violate the AdS$_3$ BF bound leading to an instability as indicated in figure 3(c)), and there can also be BF violating modes in (3.25). When $\omega_{IJ} = 0$ the situation is again analogous to the spatially modulated neutral scalars in (3.3).

Note that the instabilities of these charged modes, mixing vectors and scalars, are again associated with holographic superconductivity. These instabilities are somewhat similar to those involving only charged gauge fields in the presence of a magnetic field that were studied in [23], building on [24–26]. We also note that there are various modes which we have studied in this section that saturate the BF bound and intersect with the locus of supersymmetric solutions. We expect them to preserve the supersymmetry but we have not checked the details.

### 3.4. Discussion

Figure 3 summarizes most$^{10}$ of the instabilities that we have found within SO(6) gauge supergravity which are thus relevant to $N = 4$ SYM theory after uplifting on $S^5$. We see that apart from the supersymmetric solutions, there is only a very small range of parameters for which we have not found an instability. It would be interesting to know whether or not those solutions are in fact stable within type IIB supergravity. The general picture that has emerged here and in [18] is that studying $N = 4$ SYM in a magnetic field using holography is not a straightforward proposition.

It is worth discussing which of the instabilities that we have discussed reside within the truncation to Romans’ theory. This is relevant if we uplift the solutions not on $S^5$ to type IIB but on the general class $M_6$ of [7] to $D = 11$ [5]. To be specific we consider the truncation $\phi_3 = 0$, i.e. $X_1 = X_2 \equiv X$ and $A^1 = A^2$. As we already mentioned there is no longer spatially modulated instabilities of the neutral scalar $X$. In the language of this section, the three SU(2) gauge fields of Romans’ theory can be identified with the real $A_{1+} = A_{2-}$ and the complex $A_{12}$. Recall that Romans’ theory does not have charged scalar fields. Putting together, we find that instabilities only arise in (3.21) with $I1 = 12$ after noting that $q^1 = q^2, W_{12} = V_{12} = 0$. Indeed we find instabilities for the solutions in the range $-2.00 \lesssim f_1 \lesssim 0.87$ (recall that the supersymmetric solution has $f_1 \sim 1.13$).

Finally, we note that none of the instabilities that we have discussed appear in minimal gauged supergravity. This is relevant when we uplift the single magnetic AdS$_3 \times \mathbb{R}^2$ solution

$^{10}$ It does not include the instabilities arising from (3.22) and (3.25) which, as we discussed, lie in a subset of the cyan coloured regions.
on the general class $X_4$ [13] to type IIB supergravity [15] or on the general class of $N_6$ [14] to $D = 11$ supergravity [16].

4. Magnetic AdS$_2 \times \mathbb{R}^2$ solutions

In this section we construct magnetic AdS$_2 \times \mathbb{R}^2$ solutions of $D = 4$ SO(8) gauged supergravity, which are analogous to the $D = 5$ solutions of section 2. We also construct a supersymmetric domain wall solution that interpolates between AdS$_4$ in the UV and a supersymmetric AdS$_2 \times \mathbb{R}^2$ solution in the IR.

4.1. $U(1)^4 \subset SO(8)$ gauged supergravity

We consider the $U(1)^4$ truncation of $D = 4$ SO(8) gauged supergravity that keeps three neutral scalar fields $\phi_a$ [41]. The Lagrangian is given by

$$\mathcal{L} = \frac{1}{2} R - \frac{1}{4} \sum_{a=1}^{3} (\partial \phi_a)^2 - \sum_{i=1}^{3} X_i^{-2} (F^i)^{\mu \nu} - V(X_i), \quad (4.1)$$

where

$$X_1 = e^{\frac{i}{2}(-\phi_1-\phi_2-\phi_3)}, \quad X_2 = e^{\frac{i}{2}(-\phi_1+\phi_2+\phi_3)}, \quad X_3 = e^{\frac{i}{2}(\phi_1-\phi_2+\phi_3)}, \quad X_4 = e^{\frac{i}{2}(\phi_1+\phi_2-\phi_3)}, \quad (4.2)$$

and we note $X_1X_2X_3X_4 = 1$. Any solution of this theory that satisfies $F^i \wedge F^j = 0$ can be uplifted$^{11}$ to $D = 11$ using the formulae in [41]; all of the solutions and the linearized modes that we consider in this section satisfy this condition.

Note that it is consistent to further truncate by setting $X_2 = X_3 = X_4$ along with $F^2 = F^3 = F^4$ to obtain a sector of the SU(3) invariant subsector of SO(8) gauged supergravity [48, 49] and the corresponding uplifted solutions will have SU(3) × $U(1)^2$ symmetry. This is a case that we will sometimes focus on in the sequel. Alternatively it is also consistent to set $X_1 = X_2, X_1 = X_4$ along with $F^1 = F^2$ and $F^3 = F^4$ and the corresponding uplifted solutions will have SU(2)$^2 \times U(1)^2$ symmetry. Both of these theories can be further truncated to minimal gauged supergravity by setting all of the scalars to zero, $X_1 = X_2 = X_3 = X_4 = 1$, and $F^1 = F^2 = F^3 = F^4$. Solutions of minimal $D = 4$ gauged theory can be uplifted to $D = 10$ and $D = 11$ using manifolds associated with general classes of AdS$_4 \times M_7$ solutions dual to $N = 2$ SCFTs in $d = 3$ [16], including $SE_7$ and also those of section 7.2 of [50].

We now look for the most general class of AdS$_2 \times \mathbb{R}^2$ solutions to the equations of motion of (4.1) that are supported by magnetic fluxes. We thus consider

$$\text{dx}^2 = L^2 \text{ds}^2 \text{(AdS}_2) + \text{dx}_1^2 + \text{dx}_2^2, \quad (4.3)$$

$$F^i = \frac{1}{2} q^i \text{dx}_1 \wedge \text{dx}_2,$$

$$\phi_1 = f_1, \quad \phi_2 = f_2, \quad \phi_3 = f_3,$$

where $q^i, f_a$ are constants and $L$ is the AdS$_2$ radius. If we define the on-shell quantities

$$\tilde{X}_1 = e^{\frac{i}{2}(-f_1-f_2-f_3)}, \quad \tilde{X}_2 = e^{\frac{i}{2}(-f_1+f_2+f_3)}, \quad \tilde{X}_3 = e^{\frac{i}{2}(f_1-f_2+f_3)}, \quad \tilde{X}_4 = e^{\frac{i}{2}(f_1+f_2-f_3)}, \quad (4.4)$$

$^{11}$To do this we should set $\alpha' = 1/2$ in equation (3.8) of [41] and identify $(F^i)^{\text{there}} = 2\sqrt{2}(F^i)^{\text{here}}$. It is also worth noting that we are using the same conventions as in [47] setting $g = 1$ there.
we find that there is a three-parameter family of solutions specified by arbitrary values of $(f_1, f_2, f_3)$ with
$$
(q_i')^2 = \frac{\bar{X}_i^2}{2} \sum_{j \neq k \neq l} \bar{X}_j \bar{X}_k, \quad L^{-2} = -2V(\bar{X}_i),
$$
(4.5)
and we note that the $q_i'$ can be chosen to have either sign.

For the $SU(3) \times U(1)_2$ symmetric subspace of solutions we take $f_1 = f_2 = f_3$ and $q_i^2 = q_j^2 = q_k^3$. This gives rise to a one-dimensional family of solutions labelled by $\bar{X} \equiv \bar{X}_2 = \bar{X}_3 = \bar{X}_4 = (\bar{X}_1)^{-1/3}$ with
$$
(q_1')^2 = \frac{3}{\bar{X}_4}, \quad (q_2')^2 = (2 + \bar{X}_4), \quad L^2 = \frac{\bar{X}_2^2}{6(1 + \bar{X}_4)}. \quad (4.6)
$$

### 4.2. SUSY fixed points

We next want to investigate which of these magnetic $AdS_2 \times \mathbb{R}^2$ solutions are supersymmetric. In the next subsection we will also investigate the possibility of supersymmetric flows that interpolate between $AdS_4$ in the UV and $AdS_2 \times \mathbb{R}^2$ in the IR. We thus consider the ansatz
$$
\begin{align*}
\text{d}s^2 &= -e^{2W} \text{d}t^2 + \text{d}\rho^2 + e^{2U}(\text{d}x_1^2 + \text{d}x_2^2), \\
F_i^j &= \frac{i}{2} q_i' \text{d}x_1 \wedge \text{d}x_2, \\
\phi_a &= \phi_a(\rho).
\end{align*}
$$
(4.7)
where $W, U$ are functions of $\rho$.

The supersymmetry variations for the $U(1)_4$ truncation of $SO(8)$ gauged supergravity were analysed in [47] and it was shown that it is convenient to break up the $N = 8$ real Killing spinors into four pairs. We would like to preserve some of the Poincaré supersymmetries of $AdS_4$ and a consideration of equations (2.15)–(2.16) in [47] implies that depending on which of the four pairs of supersymmetries that we want to preserve we should impose one of the conditions
$$
\begin{align*}
q_1 + q_2 + q_3 + q_4 &= 0, & q_1 + q_2 - q_3 - q_4 &= 0, \\
q_1 - q_2 + q_3 - q_4 &= 0, & q_1 - q_2 - q_3 + q_4 &= 0.
\end{align*}
$$
(4.8)

For definiteness, let us choose to preserve the supersymmetries corresponding to
$$
\sum_i q_i' = 0. \quad (4.9)
$$

The associated supersymmetry variations were written down in (4.3) of [47]. After suitably comparing our notation with that of [47], and switching from two real spinor parameters to a complex spinor parameter $\epsilon$, we have
$$
\begin{align*}
\frac{i}{2} \delta \psi_\mu &= \nabla_\mu \epsilon + i \sum_i A^{I}_i \epsilon + \frac{1}{4\sqrt{2}} \sum_i X_i Y_{I_i} \epsilon - i \frac{1}{4\sqrt{2}} \sum_i X_i^{-1} F^i Y_{I_i} \epsilon, \\
2 \delta \chi_a &= \left[ i\sqrt{2} \phi_a - i2 \sum_j \partial_{\phi_a} X_j + 2 \sum_j \partial_{\phi_a} X_j^{-1} F^j \right] \epsilon.
\end{align*}
$$
(4.10)

Turning now to the specific ansatz (4.7), choosing $q_i'$ to satisfy (4.9) and imposing the projection conditions
$$
\gamma_5 \epsilon = -\epsilon, \quad \gamma_5 \gamma_i \epsilon = i\alpha \epsilon, \quad \alpha = \pm 1,
$$
(4.11)
we obtain

\[-W' + \frac{1}{2\sqrt{2}} \sum_i X_i + \frac{\alpha}{2\sqrt{2}} e^{-2\rho} \sum_i X_i^{-1} q_i' = 0,\]

\[-U' + \frac{1}{2\sqrt{2}} \sum_i X_i - \frac{\alpha}{2\sqrt{2}} e^{-2\rho} \sum_i X_i^{-1} q_i' = 0,\]

\[-\sqrt{2} \phi' = 2 \sum_j \partial_\phi X_j + 2\alpha e^{-2\rho} \sum_j q_j \partial_\phi X_j^{-1} = 0,\]

\[\left[ \partial_\rho - \frac{1}{4\sqrt{2}} \sum_i X_i - \frac{\alpha}{4\sqrt{2}} e^{-2\rho} \sum_i q_i X_i^{-1} \right] \varepsilon = 0. \quad (4.12)\]

From the first and the last equation in (4.12) we derive that \(\varepsilon = e^{W/2} \eta\) with \(\eta\) a constant spinor satisfying the projection conditions (4.11).

To determine which of the \(\text{AdS}_2 \times \mathbb{R}^2\) solutions, summarized in (4.3), (4.4), (4.5), are supersymmetric, we set \(W = L^{-1} \rho, U = 0\) and \(X_i = \bar{X}_i\) in (4.11). We find that the magnetic charges are given by

\[2\alpha q_i = \bar{X}_i \left( -2 \bar{X}_i + \sum_j \bar{X}_j \right), \quad (4.13)\]

and (4.9) then gives the condition

\[2 \sum_i \bar{X}_i^2 = \left( \sum_i \bar{X}_i \right)^2, \quad (4.14)\]

analogous to what we saw in the \(D = 5\) case (2.7). One can directly check that the conditions in (4.5) are satisfied (as expected) and that the radius of the \(\text{AdS}_2\) factor can now also be written

\[L^{-1} = \frac{1}{\sqrt{2}} \sum_i \bar{X}_i. \quad (4.15)\]

At this point we have shown that any solution to the flow equations (4.12) preserves 1/16 of the supersymmetries, i.e. two Poincaré supersymmetries, which is enhanced to 1/8 supersymmetry for the \(\text{AdS}_2\) fixed points. For there to be supersymmetry enhancement, one needs to have solutions to another of the conditions in (4.8), but this is not compatible with (4.13), (4.14).

In the three-dimensional moduli space of solutions, labelled by \((f_1, f_2, f_3)\), we have a two dimensional locus of supersymmetric solutions fixed by (4.14), which we have plotted in figure 4. Let us discuss a few special cases. Firstly, there are supersymmetric solutions when one of the \(f_a\) is set to zero. However, there are no supersymmetric solutions when two of the \(f_a\) are set to zero. In particular, there are no supersymmetric solutions with \(U(1)^2 \times SU(2)^2\) symmetry that have e.g. \(X_1 = X_2, X_3 = X_4, q^1 = q^2\) and \(q^3 = q^4\), as noted in [18]. The \(\text{AdS}_2\) solutions of minimal gauged supergravity with all \(f_a\) zero are not supersymmetric, as is well known.

Secondly, there are supersymmetric solutions when we set two of the \(f_a\) equal. Furthermore, there is a single supersymmetric solution when we set all of them to be equal, \(f_1 = f_2 = f_3\). Specifically, in the \(SU(3) \times U(1)^2\) invariant class solutions given in (4.6) we should take \(q^1 = -3q^2\) (a condition that was also noted in [18]) with

\[\bar{X} = \left( -1 + \frac{2}{\sqrt{3}} \right)^{1/4}, \quad q^2 = \frac{1}{3\alpha} \sqrt{9 + 6\sqrt{3}}, \quad L^{-1} = 2(9 + 6\sqrt{3})^{1/4}. \quad (4.16)\]
We show in appendix C that the uplifted $D = 11$ metric for this solution, using the formulae in [41], can be recast in the formalism of [51]. This provides a direct and very satisfying check on the supersymmetry of the solution.

4.3. Supersymmetric AdS$_4$ to AdS$_2 \times \mathbb{R}^2$ domain wall

We will be interested in constructing a supersymmetric domain wall solution that describes a flow from the AdS$_4$ vacuum to the fixed point (4.16) which preserves $U(1)^2 \times SU(3)$ in eleven dimensions. To construct the flow we truncate to the $SU(3)$ invariant sector by setting $\phi_a = \phi$ in (4.12) to obtain the first order system of equations

\begin{align}
W' &= - \frac{1}{2\sqrt{2}} e^{-2U - \frac{1}{2}\phi}(e^{2U}(1 + 3e^{2\phi}) + \sqrt{9 + 6\sqrt{3}(e^\phi - e^{3\phi})}) = 0, \\
U' &= - \frac{1}{2\sqrt{2}} e^{-2U - \frac{1}{2}\phi}(e^{2U}(1 + 3e^{2\phi}) - \sqrt{9 + 6\sqrt{3}(e^\phi - e^{3\phi})}) = 0, \\
\phi' &= - \frac{1}{3\sqrt{2}} e^{-2U - \frac{1}{2}\phi}(-3e^{2U}(1 - e^{2\phi}) + \sqrt{9 + 6\sqrt{3}(e^\phi + 3e^{3\phi})}) = 0.
\end{align}
The expansion close to the AdS$_2 \times \mathbb{R}^2$ fixed point in the far IR is

\begin{align*}
W &= w_0 + L^{-1} \rho - 2 \frac{3 + \sqrt{3}}{3 + 2\sqrt{3}} c_{IR} e^{L^{-1} \rho} + \cdots, \\
U &= c_{IR} e^{L^{-1} \rho} + \cdots, \\
\phi &= -\frac{1}{4} \ln[3(7 + 4\sqrt{3})] + \frac{2}{\sqrt{3}} c_{IR} e^{L^{-1} \rho} + \cdots,
\end{align*}
(4.18)

with $c_{IR}$ and $w_0$ being constants of integration. Setting the magnetic charges $q_i = 0$ in the flow equations (4.12), we recover the AdS$_4$ solution

\begin{align*}
W &= U = R^{-1} \rho, \\
\phi &= 0, \\
R^{-1} &= \sqrt{2}.
\end{align*}
(4.19)

Turning on a non-zero $q_i$ triggers the following asymptotic expansion to the equations (4.17) given by

\begin{align*}
W &= R^{-1} \rho - \frac{3}{16} c_{UV} e^{-2R^{-1} \rho} + \cdots, \\
U &= R^{-1} \rho - \frac{3}{16} c_{UV}^2 e^{-2R^{-1} \rho} + \cdots, \\
\phi &= c_{UV} e^{-R^{-1} \rho} + \left( 2 \sqrt{1 + \frac{2}{\sqrt{3}} - \frac{c_{UV}^2}{2}} \right) e^{-2R^{-1} \rho},
\end{align*}
(4.20)

where $c_{UV}$ is a constant of integration. This expansion corresponds to the operator dual to $\phi$ having a deformation as well as a VEV, and we see that both the deformation and the VEV are fixed by $c_{UV}$.

Using a shooting method we find that there is a solution to (4.12) with boundary conditions (4.18) and (4.20) with

\begin{align*}
w_0 &= -0.47 \ldots, \\
c_{IR} &= 0.26 \ldots, \\
c_{UV} &= -1.71 \ldots,
\end{align*}
(4.21)

as we have indicated in figure 5. This is the supersymmetric domain wall solution. This solution can be uplifted on $S^7$, or an orbifold thereof, to $D = 11$ supergravity using the formulae in
The uplifted solutions then describe the dual $d = 3$ SCFTs deformed by the presence of the magnetic field and also by the operators dual to $\phi$. In particular, the supersymmetric $\text{AdS}_2 \times \mathbb{R}^2$ solutions describes the IR ground state at zero temperature.

5. Instabilities of magnetic $\text{AdS}_2 \times \mathbb{R}^2$ solutions

The instabilities for the magnetic $\text{AdS}_2 \times \mathbb{R}^2$ solutions that we constructed in section 4 are very similar to those that we have discussed for the $\text{AdS}_3 \times \mathbb{R}^2$ solutions in section 3. In this section, we will just present some illustrative calculations.

5.1. Spatially modulated instabilities of neutral scalars

We first investigate the possibility of spatially modulated instabilities of the neutral scalars $\phi_a$ for the magnetic $\text{AdS}_2 \times \mathbb{R}^2$ solutions given in (4.5). For simplicity we just analyse the one-dimensional subspace of $SU(3) \times U(1)^2$ solutions given in (4.6). In particular, we focus on the perturbation with

$$
\delta \phi_1 = 0, \quad \delta \phi_2 = -\delta \phi_1 = \phi(t, \rho) \cos(kx_1),
\delta A^1 = \delta A^2 = 0, \quad \delta A^3 = -\delta A^4 = a(t, \rho) \sin(kx_1) \mathrm{d}x_2.
$$

(5.1)

Defining the vector $v = (\phi, a)$, the equations of motion for the Lagrangian (4.1) imply, at linear order,

$$
(\Box_{\text{AdS}_2} - L^2 M^2)v = 0,
$$

(5.2)

where the Laplacian is with respect to a unit radius $\text{AdS}_2$ and the mass matrix is

$$
M^2 = \begin{pmatrix}
k^2 - 2X^2 + 6X^{-2} & 8q^2 X^{-2}k \\
k^2 & k^2
\end{pmatrix}.
$$

(5.3)

The matrix $M^2$ has eigenvalues

$$
m^2_{\pm} = \frac{1}{X^2}[3 + k^2 X^2 + X^4 \pm \sqrt{8k^2 X^2(2 + X^4) + (3 + X^4)^2}].
$$

(5.4)

The branch $m^2_-$ develops a minimum at

$$
k^2_{\text{min}} = \frac{1}{8X^2} \frac{55 + 58X^4 + 15X^8}{2 + X^4},
$$

(5.5)

with the corresponding minimum satisfying

$$
L^2 m^2_{\text{min}} = -\frac{1}{48} \frac{(5 + 3X^4)^2}{2 + 3X^4 + X^8}.
$$

(5.6)

For $X < (-1 + \frac{2}{\sqrt{3}})^{1/4}$ the mass minimum violates the $\text{AdS}_2$ BF bound of $-1/4$ making the solution unstable.

It is worth noting that for the supersymmetric solution with $X = (-1 + \frac{2}{\sqrt{3}})^{1/4}$, the static mode given by

$$
\phi(r) = c_1 e^{-\frac{r}{2}} \cos(|k_{\text{min}}|x_1), \quad a(r) = c_2 e^{-\frac{r}{2}} \sin(|k_{\text{min}}|x_1).
$$

(5.7)

with $c_1^2/c_2^2 = 8 \sqrt{-9 + 6 \sqrt{3}}$, which saturates the BF bound, preserves the supersymmetries of the background. The sign choice of $c_1$ depends on the choice of $\alpha$ in the projector (4.11).
5.2. Instabilities of charged scalars

Recall that $SO(8)$ gauged supergravity has 70 scalar fields, parametrizing the coset $E_{7(7)}/SU(8)$ and transforming as two 35 irreps of $SO(8)$. The scalars in one of these 35 irreps can be described by a unimodular $8 \times 8$ matrix $T$. The three neutral scalars we have been considering lie in this irrep. We next investigate possible instabilities of four charged fields lying in this irrep using the consistent truncation of $SO(8)$ gauge supergravity discussed in [45]. We should recall that $A^\text{here} = 2\sqrt{2}A^\text{here}$ and set their $g = 1/\sqrt{2}$. We then follow the earlier analysis in section 3.2 and for the scalar $\varphi_1$ (say) we find Landau levels with corresponding $\text{AdS}_2$ mass given by

$$m^2 = -2T^2\left(X_1X_2 + X_1X_3 + X_1X_4 - X_1^2 - (2n + 1)|q_1|\right).$$

These modes violate the BF bound for a large parameter space of solutions. Indeed, for the lowest level, $n = 0$, the unstable regions are, roughly, the obvious generalization of figure 3(b) to figure 4. In particular, these modes now intersect the locus of supersymmetric solutions in a one-dimensional sub-locus.

Finally, we note that there will be additional instabilities for the other scalars in the $8 \times 8$ matrix $T$ and these will mix with the gauge fields and the analysis will mirror the analysis that we carried out in section (3.3).

6. Electric Solutions

In this section, we construct new electric $\text{AdS}_2 \times \mathbb{R}^2$ and $\text{AdS}_2 \times \mathbb{R}^3$ solutions.

6.1. Electric $\text{AdS}_2 \times \mathbb{R}^2$ solutions

The equations of motion of the $U(1)^4$ truncation of $D = 4$ $SO(8)$ gauged supergravity (4.1) are invariant under the electric–magnetic duality transformation

$$F^i \rightarrow X^{-2}_i F^i, \quad \phi_a \rightarrow -\phi_a,$$

with the metric unchanged. We can use this symmetry to immediately obtain electric analogues of the magnetic solutions that we presented in section 4.

Starting with (4.3), (4.5) we obtain electric $\text{AdS}_2 \times \mathbb{R}^2$ solutions, which we can write as

$$dx^2 = L^2\left(dx_1^2 + dx_2^2 + dx_3^2\right),$$

$$F^i = \frac{i}{2}Q(L^2\text{Vol}(\text{AdS}_2)),\quad \phi_1 = f_1, \quad \phi_2 = f_2, \quad \phi_3 = f_3,$$

where

$$(Q')^2 = \sum_{j\neq i} \tilde{X}_j, \quad L^{-2} = -2V(\tilde{X}_i),$$

and, as before,

$$\tilde{X}_1 = e^{\frac{i}{2}(f_1 - f_2 + f_3)}, \quad \tilde{X}_2 = e^{\frac{i}{2}(f_1 + f_2 - f_3)}, \quad \tilde{X}_3 = e^{\frac{i}{2}(f_1 - f_2 - f_3)}, \quad \tilde{X}_4 = e^{\frac{i}{2}(f_1 + f_2 - f_3)}.$$

None of the solutions preserve the supersymmetry transformations given in (4.10). For the special case of the electric $\text{AdS}_2 \times \mathbb{R}^2$ solution of minimal gauged supergravity see [40] for a discussion on instabilities. An analysis for other solutions will be carried out elsewhere.

Starting with the supersymmetric domain wall solution that we presented in section 4.3, we can use the duality transformation (6.1) to immediately obtain an electric domain wall solution that interpolates between $\text{AdS}_3$ in the UV and an electric $\text{AdS}_2 \times \mathbb{R}^2$ solution in the IR. Note that despite the domain wall not preserving supersymmetry it solves first order flow equations.
6.2. Electric AdS$_2 \times \mathbb{R}^3$ solutions

We now consider electric AdS$_2 \times \mathbb{R}^3$ solutions of $D = 5$ $SO(6)$ gauged supergravity. By direct construction we find

$$\begin{align*}
\text{d}s^2 &= L^2 \text{d}s^2_{(\text{AdS}_2)} + \text{d}x_1^2 + \text{d}x_2^2 + \text{d}x_3^2, \\
F_i &= 2Q L^2 \text{Vol}(\text{AdS}_2), \\
\phi_1 &= f_1, \\
\phi_2 &= f_2,
\end{align*}$$

(6.5)

where $f_a$ are constants,

$$\begin{align*}
(Q')^2 &= \tilde{X}_i^2 \sum_{j \neq i} \tilde{X}_j, \\
L^{-2} &= -V(\tilde{X}_i),
\end{align*}$$

(6.6)

and $\tilde{X}_i$ are the on-shell values

$$\begin{align*}
\tilde{X}_1 &= e^{-\frac{1}{\sqrt{2}}(\gamma_1 + \gamma_2)}, \\
\tilde{X}_2 &= e^{-\frac{1}{\sqrt{2}}(\gamma_1 + \gamma_2)}, \\
\tilde{X}_3 &= e^{2\sqrt{6} \gamma_1}.
\end{align*}$$

(6.7)

When $f_2 = 0$, for example, these are solutions to Romans’ theory, and actually were already presented in [6] and further discussed in appendix B of [22]. When $f_1 = f_2 = 0$ we obtain the standard AdS$_2 \times \mathbb{R}^3$ solution of minimal gauged supergravity which is the near horizon limit of the usual electrically charged AdS-RN black brane solution. Note that the solutions do not preserve the supersymmetry (2.9); within Romans’ theory this was shown in [6].

It was shown in [22] that the electrically charged AdS$_2 \times \mathbb{R}^3$ solutions in Romans’ theory all suffer from instabilities corresponding to helical $p$-wave superconductors. A more detailed stability analysis of all solutions will be carried out elsewhere.

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Appendix A. Charged mode analysis for the truncation [43]

A consistent truncation that supplements the $U(1)^3 \subset SO(6)$ truncation (2.2) with four complex scalars $\zeta_m$, $m = 1, 2, 3, 4$ was given in [43]. After writing $\zeta_m = \tanh(\gamma_m) e^{i \theta_m}$, the Lagrangian is given in equation (2.7) of [43] and, to make contact with our notation, one should set $\theta_m^{\text{there}} = \gamma_m^{\text{here}}$, $g^{\text{there}} = 2$ and also identify the gauge fields via $A^{i(\text{there})} = A^{i(\text{here})}/2$.

It is straightforward to see that after expanding the equations of motion around the AdS$_3 \times \mathbb{R}^2$ solutions (2.4), the fields $\theta_m$ are all massless. The analysis for the charged modes $\gamma_m$ is very similar to that in section 3.2. For example, for $m = 1$ we find that the lowest mass mode is obtained by writing

$$\gamma_1 = e^{-\frac{1}{\sqrt{2}}(\gamma_1 + \gamma_2)} \sigma_1 (t, r, z),$$

(A.1)

giving

$$\square_{\text{AdS}_3} - L^2 m_{\sigma_1}^2) \sigma_1 = 0,$$

(A.2)

with

$$m_{\gamma_1}^2 = |q_1 + q_2 - q_3| + \sum_i \tilde{X}_i^2 - 2 \sum_i \tilde{X}_i^{-1}.$$  

(A.3)

Over the moduli space of AdS$_3 \times \mathbb{R}^2$ solutions we find that the minimum value is $L^2 m_{\gamma_1}^2 \approx -0.704$ and does not violate the BF bound.
Appendix B. The mixed charged modes

Here we provide some details of the calculations we carried out for section 3.3. We consider the perturbation \( t, a \) about the background AdS\(_3 \times \mathbb{R}^2 \) solution (2.4), (2.5) defined by

\[
T = \tilde{T} + t, \quad A = \tilde{A} + a,
\]

with

\[
\tilde{T}_{ij} = X_{ij}, \quad \tilde{A}_{ij} = q_I (\tilde{z} \, d\tilde{z} - z \, d\tilde{z}).
\]

It is useful to note that at leading order in the perturbation we have \( T^{-1} = \tilde{T}^{-1} - \tilde{T}^{-1} \tilde{T}^{-1} \). Furthermore, the linearized expression of the field strengths and some covariant derivatives are given by

\[
\begin{align*}
\delta(F_{IJ}) &= da_{IJ} + (\tilde{A}_{ij} + \tilde{A}_{ij}) \wedge a_{IJ}, \\
\delta(F_{Ij}) &= da_{Ij} + (\tilde{A}_{ij} - \tilde{A}_{ij}) \wedge a_{IJ}, \\
\delta(DT_{IJ}) &= dt_{IJ} + (\tilde{A}_{ij} + \tilde{A}_{ij}) t_{IJ} + g(X_J - X_I) a_{IJ}, \\
\delta(DT_{Ij}) &= dt_{Ij} + (\tilde{A}_{ij} - \tilde{A}_{ij}) t_{IJ} + g(X_J - X_I) a_{IJ}, \\
\delta(DF_J) &= -\delta(F_{IJ}) = (\tilde{A}_{ij} - \tilde{A}_{ij}) t_{IJ} + *\tilde{F}_{IJ} + \tilde{F}_{IJ} \wedge a_{IJ}, \\
\delta(DF_I) &= -\delta(F_{IJ}) = (\tilde{A}_{ij} + \tilde{A}_{ij}) t_{IJ} + *\tilde{F}_{IJ} + \tilde{F}_{IJ} \wedge a_{IJ}.
\end{align*}
\]

We will only provide details concerning the \( IJ \) components of the equations of motion (3.15), with \( I \neq J \). The case of \( IJ \) is very similar. At linearized order we have

\[
\begin{align*}
\delta(D \ast F_{IJ}) &= (X^{-1}_I \ast \tilde{F}_{IJ} - X^{-1}_J \ast \tilde{F}_{IJ}) \wedge \delta(DT_{IJ}) = -(X_J - X_I) \ast \delta(DT_{IJ}), \quad \text{(B.4)} \\
\delta(D \ast DF_I) &= 4 q_I^{-1} a_{IJ} \ast \delta(DT_{IJ}) + \ast(X_J^{-1} \tilde{F}_{IJ} - X_J^{-1} \tilde{F}_{IJ}) \wedge \delta(F_{IJ}). \quad \text{(B.5)}
\end{align*}
\]

For the gauge fields we take

\[
a_{IJ} = a_{IJ}^1 \, dz + a_{IJ}^2 \, d\bar{z},
\]

and, after defining

\[
\omega_{IJ} = q_I + q_J,
\]

we find

\[
\begin{align*}
\delta(F_{IJ}) &= da_{IJ} \wedge dz + da_{IJ} \wedge d\bar{z} - (\partial_I - \omega_{IJ}) a_{IJ} dz \wedge d\bar{z} + (\partial_{J} + \omega_{IJ}) a_{IJ} dz \wedge d\bar{z}, \\
\delta(DT_{IJ}) &= dt_{IJ} + (\partial_I - \omega_{IJ}) t_{IJ} dz + (\partial_{J} + \omega_{IJ}) t_{IJ} d\bar{z} + (X_J - X_I) (a_{IJ}^1 dz + a_{IJ}^2 d\bar{z}),
\end{align*}
\]

where \( \partial \) is the exterior derivative on AdS\(_3\). We can obtain analogous expressions for \( \delta(D \ast F_{IJ}) \) and \( \delta(D \ast DF_I) \) using B.3, which we then substitute into the equations of motion (B.4) and (B.5).

From (B.4) we are led to impose the constraint

\[
(\partial_I - \omega_{IJ}) a_{IJ}^1 \quad + \quad (\partial_J + \omega_{IJ}) a_{IJ}^2 \quad = \quad -(X_J - X_I) \, t_{IJ}.
\]

We also obtain

\[
- L^{-2} \Box_{dAdS} a_{IJ}^1 + 2 \left( -\partial_I \partial_I + \omega_{IJ}^2 \bar{z} + 2 \omega_{IJ} (z \partial_I - \bar{z} \partial_I) \right) a_{IJ}^1
\]

\[
+ \left( q^{-1}_I - q^{-1}_J \right) \left[ (\partial_I + \omega_{IJ}^1) t_{IJ} + (X_J - X_I) a_{IJ}^1 \right] = \quad -(X_J - X_I)^2 a_{IJ}^1,
\]

and

\[
L^{-2} \Box_{dAdS} a_{IJ}^2 + 2 \left( \partial_J \partial_J - \omega_{IJ}^2 z \right) + 2 \omega_{IJ} - \omega_{IJ} (z \partial_I - \bar{z} \partial_I) \right) a_{IJ}^2
\]

\[
+ \left( q^{-1}_I - q^{-1}_J \right) \left[ (\partial_I + \omega_{IJ}^1) t_{IJ} + (X_J - X_I) a_{IJ}^2 \right] = \quad (X_J - X_I)^2 a_{IJ}^2.
\]
where \( \Box_{\text{AdS}} \) is the Laplacian on a unit radius \( \text{AdS}_3 \). Similarly, from (B.5) we obtain
\[
L^{-2} \Box_{\text{AdS}} \tilde{t}_{IJ} + 2 \left( \partial_z \tilde{z} - \omega (z \partial_z - z \partial \tilde{z}) \right) \tilde{t}_{IJ} - (X_J - X_I)^2 \tilde{t}_{IJ} = -4 q_i^{-1} q_j^{-1} \tilde{t}_{IJ} + 2 \left( q_i^{-1} - q_j^{-1} \right) \left[ -\partial_z + \omega \tilde{z} \right] a^J_I + 2 \left( \tilde{z} \partial_z + \omega z \right) a^J_I. \tag{B.12}
\]
We now observe that because of the constraint (B.9), the three equations (B.10)–(B.12) are not independent. Indeed acting on equation (B.10) by \(-\partial_z + \omega \tilde{z}\), on equation (B.11) by \(\tilde{z} \partial_z + \omega z\) and adding one can show that equation (B.12) is satisfied.

To continue with the analysis, we need to fix the sign of \(\omega \equiv \omega_{\text{AdS}}\). We first take \(\omega > 0\). For this case, we can keep equation (B.11) and (B.12) which we write as
\[
L^{-2} \Box_{\text{AdS}} \tilde{t}_{IJ} + 2 \left( \partial_z \tilde{z} + \omega (z \partial_z - z \partial \tilde{z}) \right) \tilde{t}_{IJ} - (X_J - X_I)^2 \tilde{t}_{IJ} = -4 q_i^{-1} q_j^{-1} \tilde{t}_{IJ} + 2 \left( q_i^{-1} - q_j^{-1} \right) \left[ (X_J - X_I) \tilde{t}_{IJ} + 2 (\tilde{z} \partial_z + \omega z) a^J_I \right]. \tag{B.13}
\]
Next we introduce the ladder operators
\[
a = \frac{1}{\sqrt{2\omega}} (\partial_z + \omega z), \quad a^J = \frac{1}{\sqrt{2\omega}} (\tilde{z} \partial_z + \omega z), \tag{B.14}
\]
which can be checked to satisfy the algebra
\[
[a, a^J] = 1, \quad [b, b^J] = 1, \tag{B.15}
\]
and the rest of the commutators being trivial. Note that we have
\[
-\partial_z \tilde{z} + \omega^2 z \tilde{z} = \omega (a^J a + b^J b + 1),
\]
\[
\tilde{z} \partial_z - z \partial \tilde{z} = a^J a - b^J b. \tag{B.16}
\]
In terms of these operators, equations (B.11) and (B.13) take the form
\[
L^{-2} \Box_{\text{AdS}} a^J_I - 2 (2b^J b + 1) a^J_I + 2 (q_i^{-1} - q_j^{-1}) \left[ -\sqrt{2 \omega} b^J b + (X_J - X_I) a^J_I \right] - (X_J - X_I)^2 a^J_I = 0 \tag{B.17}
\]
and
\[
L^{-2} \Box_{\text{AdS}} t_{IJ} - 2 \omega (2b^J b + 1) t_{IJ} - (X_J - X_I)^2 t_{IJ} + 4 q_i^{-1} q_j^{-1} t_{IJ} - 2 (q_i^{-1} - q_j^{-1}) \left[ (X_J - X_I) t_{IJ} + 2 \sqrt{2 \omega} b^J b \right] = 0. \tag{B.18}
\]
To reduce the problem to modes on the \(\text{AdS}_3\) space we introduce the ground state
\[
L_{0,0} = \left( \frac{\omega}{\pi} \right)^{1/2} e^{-\omega z}, \tag{B.19}
\]
and the complete set of functions
\[
L_{m, n}(z, \bar{z}) = \frac{(b^J)^n (a^J)^m}{\sqrt{n!} \sqrt{m!}} L_{0,0}(z, \bar{z}), \quad m, n > 0. \tag{B.20}
\]
We use these to write the expansions
\[
t_{IJ} = \sum_{m,n} l^{m,n}_{IJ} L_{m,n}(z, \bar{z}), \quad a^J_I = \sum_{m,n} g^{m,n}_{IJ} L_{m,n}(z, \bar{z}), \tag{B.21}
\]
with \(f\) and \(g\) defined on \(\text{AdS}_3\). From equations (B.17) and (B.18) we see that the modes \(g^{0,0}_{IJ}\) decouple and they have an \(\text{AdS}_3\) mass
\[
m_{0,0}^2 = -2 \omega - 2 \left( q_i^{-1} - q_j^{-1} \right) (X_J - X_I) + (X_J - X_I)^2. \tag{B.22}
\]
For the rest of the modes, we see that $\delta^{n+1,m}_{IJ}$ mix with $f^{n,m}_{IJ}$ for $n \gg 0$ with mass matrix

$$M^2 = \begin{pmatrix}
2\omega (2n + 1) - 4q_i^{-1}q_j^{-1} + 2W_{IJ}V_{J} + V_{I}^2 & 4\sqrt{2}\omega W_{IJ} (n + 1)^{1/2} \\
2\sqrt{2}\omega W_{IJ} (n + 1)^{1/2} & 2\omega (2n + 1) - 2W_{IJ}V_{J} + V_{I}^2
\end{pmatrix},$$

(B.23)

where we set

$$W_{IJ} = q_i^{-1} - q_j^{-1}, \quad V_{IJ} = X_I - X_J.$$  

(B.24)

Note that our results differ from those presented in the first two lines of equation (6.7) and equation (6.12) of [18] because the mixing of the charged scalars and vectors was not taken into account in that reference\(^{12}\).

When $\omega < 0$, in order to get the zero modes, we should keep equations (B.10) and (B.12). For the ladder operators we should take $\sqrt{2}\omega \rightarrow -\sqrt{2}|\omega|$ and $\omega \rightarrow -\omega$ in (B.14). A very similar analysis then ensues and we obtain (B.22) and (B.23) after substituting $\omega \rightarrow |\omega|$ and also $W_{IJ} \rightarrow -W_{IJ}$.

Finally, let us consider $\omega = 0$. This occurs along the three lines in figure 1 with $X_I = X_J$. The equations (B.9)–(B.12) then simplify and we are essentially led back to the mass matrix that we saw for spatially modulated neutral scalars in (3.3).

**Appendix C. Construction of SUSY AdS$_2 \times \mathbb{R}^2$ solutions**

Recall [51] that supersymmetric AdS$_2$ solutions of $D = 11$ supergravity with purely electric four-form flux, generically dual to CFTs with two (Poincaré) supersymmetries, can be obtained from an eight dimensional Kähler metric, $d\tilde{s}^2$, whose Ricci tensor satisfies

$$\Box \tilde{s}^2 - \frac{1}{4} R^2 + R_{ij} R^{ij} = 0.$$  

(C.1)

The $D = 11$ metric has the form

$$d\tilde{s}^2 = e^{{2A}} [d\tilde{\tau}^2 (\text{AdS}_2) + e^{-3A} d\tilde{s}^2_{\text{CP}^2} + (dz + P)^2],$$

(C.2)

where $dP = \mathcal{R}$, where $\mathcal{R}$ is the Ricci-form, and $e^{-3A} = \frac{1}{4} \tilde{R}$.

Following [5] we start with an ansatz for an eight dimensional Kähler metric given by

$$d\tilde{s}^2_{\text{CP}^2} = \frac{dy^2}{U} + y^2 U (D\phi + A)^2 + y^2 d\bar{z}^2 + (ay^2 + b) dz^2 (\mathbb{R}^2),$$

(C.3)

with $dD\phi = 2J_{\text{CP}^2}$, where $J_{\text{CP}^2}$ is the Kähler form on $\mathbb{CP}^2$, and $dA = 2a J_{\mathbb{R}^2}$, where $J_{\mathbb{R}^2} = Vol(\mathbb{R}^2)$ and $a$ is a constant, and $U = U(y)$. For the corresponding holomorphic 2-form and 1-form on $\mathbb{CP}^2$ and $\mathbb{R}^2$ we have

$$d\Omega_{\text{CP}^2} = iP_{\text{CP}^2} \wedge \Omega_{\text{CP}^2}, \quad dP_{\text{CP}^2} = 2J_{\text{CP}^2},$$

$$d\Omega_{\mathbb{R}^2} = 0,$$

(C.4)

where $l$ is another (positive) constant. The Kähler-form and holomorphic 4-form for the eight-dimensional space can now be written

$$J = y dy \wedge (D\phi + A) + y^2 J_{\text{CP}^2} + (ay^2 + b) J_{\mathbb{R}^2},$$

$$\Omega_4 = e^{i\phi} y^2 \sqrt{ay^2 + b} \left[ \frac{dy}{\sqrt{U}} + iy\sqrt{U} (D\phi + A) \right] \wedge \Omega_{\text{CP}^2} \wedge \Omega_{\mathbb{R}^2}.$$  

(C.5)

We can easily show that

$$d\Omega_4 = iP \wedge \Omega_4, \quad P = ID\phi - g (D\phi + A),$$

(C.6)

12 Our zero modes (B.22) only involve the gauge fields and the fact that our results differ from equation (6.12) of [18] after setting their $n = 0$, which also just involve the gauge-fields, can be traced back to the fact that our equation of motion (B.4) for $t_{IJ} = 0$ does not come from the Lagrangian given in equation (6.11) of [18].
where
\[ g = 3U + \frac{ay^2 U}{ay^2 + b} + \frac{yU'}{2}. \] (C.7)

The Ricci form for the eight-dimensional space is given by
\[ R = \frac{2}{P + 2} (l - g) J_{CP^2} - 2ag J_{R^2} - g' dy \wedge (D\phi + A). \] (C.8)

In order to get an AdS\(_2 \times \mathbb{R}^2\) factor in (C.2) we now require that the Ricci scalar of the eight-dimensional space satisfies
\[ R = \frac{W}{ay^2 + b}, \] (C.9)
for some constant \( W \). The resulting second order equation for \( U \) gives the solution
\[ U = \frac{1}{48} \frac{1}{ay^2 + b} \left( 16bl + 8adl^2 - Wy^2 + \frac{c_1}{y} + \frac{c_2}{y^2} \right), \] (C.10)
where \( c_i \) are two constants of integration. One can check that in order to solve (C.1) we need to set \( c_1 = c_2 = 0 \) and we then find two solutions
\[ W = -4 \left( 1 \pm \sqrt{3} \right) a l \Rightarrow U = \frac{1}{12} \frac{l}{b + ay^2}. \] (C.11)

Let us now continue with the solution with the lower sign. We take \( a < 0 \) and change coordinates via \( y = \frac{2}{\sqrt{3} + \sqrt{3}} \sqrt{-\frac{b}{a}} \sin \xi \) and we will take \( 0 < \xi < \pi/2 \). We find
\[ U = l \left( 1 + \frac{1}{\sqrt{3}} \right) \frac{\cos^2 \xi}{1 + \sqrt{3} + 2 \cos 2\xi}, \]
\[ R_8 = -8al(3 + 2\sqrt{3}) \frac{1}{b} \frac{1}{1 + \sqrt{3} + 2 \cos 2\xi}, \]
\[ g = \frac{3 + \sqrt{3}}{3} \frac{1 + 2 \cos 2\xi}{1 + \sqrt{3} + 2 \cos 2\xi}. \] (C.12)

We can now assemble the \( D = 11 \) metric using (C.2) and, after setting \( l = 3 \), find
\[ ds^2 = e^{2A} L^{-2} \left\{ L^2 d\bar{x}^2 (AdS_2) + \frac{L^2 W}{2} ds^2 (\mathbb{R}^2) + 2\bar{X}^2 \left( ds^2 + \frac{\sin^2 \xi}{\bar{X}^2} \frac{1}{\Delta} \left[ ds^2 (CP^2) + \left( D\bar{\phi} + \frac{1 + \sqrt{3}}{4} A \right)^2 \right] + \frac{\bar{X} \cos^2 \xi}{\Delta} \frac{1}{8\sqrt{3}\bar{X}^4} (dz - 3\bar{A})^2 \right) \right\} \] (C.13)
where \( \bar{X} \) is as in (4.16) and
\[ \Delta = \frac{1}{\bar{X}^3} (\cos^2 \xi + \bar{X}^4 \sin^2 \xi). \] (C.14)

One can now check that if we set
\[ \frac{-a}{b} = \frac{4}{3^{1/4}(1 + \sqrt{3})} \] (C.15)
and rescale \( (L^2 W/2) ds^2 (\mathbb{R}^2) \rightarrow ds^2 (\mathbb{R}^2) \) then we precisely obtain the uplift of the solution (4.16) using the formulae in [41] (setting \( g^2 = 1/2 \) in equation (3.8) of [41] and identifying \( (F^2)_{\text{here}} = 2\sqrt{2}(F^2)_{\text{there}} \)).
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