World Sheet Logarithmic CFT in AdS Strings, Ghost-Matter Mixing and M-theory.

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Abstract

We discuss several closely related concepts in the NSR formulation of superstring theory. We demonstrated that recently proposed NSR model for superstrings on $AdS_5 \times S^5$ is described by the world-sheet logarithmic conformal field theory (LCFT). The origin of LCFT on a world-sheet is closely connected to the matter-ghost mixing in the structure of a brane-like vortex operators. We suggest a dynamical origin of M theory as a string theory with an extra dimension given by bosonised superconformal ghosts.

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Introduction

The question of what is a proper non-perturbative formulation of String Theory is one of the most important. Last decade we had an enormous progress in our understanding of different aspects of non-perturbative formulation of String Theory (see for example very interesting lectures [1], [2] and references therein) - but we still walk along the shore of an ocean. One of the most remarkable suggestion was that instead of superstring theory in 10 dimensions we have so called M theory in 11 dimensions and instead of (super)string as a fundamental object we have a supermembrane - \( M_2 \) brane [3], [4]. These ideas lead to very elegant unification of all existing string theories (and 11-dimensional supergravity). However the complete firstly quantized theory of \( M_2 \) brane (and it dual \( M_5 \) brane) is still unknown - contrary to (super)strings [5], [6], [7]. Another remarkable development was the discovery of D-branes in superstring theory [8]. From the point of view of M theory one can get type IIA D-branes from \( M_2 \) and \( M_5 \) branes. The third remarkable progress was the discovery of AdS/CFT correspondence [9], [10], [11] and related picture of closed strings in \( d \)-dimensional gauge theories (loops of glue) propagating actually in a \( d + 1 \)-dimensional space [12]. The emergence of extra dimension is due to Liouville filed, i.e. one has to start with non-critical \( d \)-dimensional string and dynamical Liouville field plays the role of a new dimension transforming \( d \)-dimensional space into \( d + 1 \)-dimensional.

It will be nice if one can make such a trick and get extra dimension moving us from 10 to 11, it will be also nice to find inside string theory objects which can play the role of \( M_2 \) and \( M_5 \) branes and it will be nice to understand what kind of world-sheet dynamics we must have to describe these objects. In this paper we are addressing these questions. In the next section we are discussing the new brane vertex operators in NSR formulation of superstring theory which amusingly enough can actually play the role of creation operators of \( M_2 \) and \( M_5 \) branes. We show that world sheet conformal field theory describing them is rather unusual and finally we demonstrate that there is an extra bosonic field in superstring theory - superconformal ghost which indeed produces an extra eleventh dimension!

**Brane vertex operators in NSR superstring.**

NSR critical string theory includes a specific class of NS physical states (BRST invariant and BRST nontrivial vertex operators) which are not associated with any perturbative particle emission but appear to play an important role in the non-perturbative physics. For example in the previous works we have shown that inserting this new class of operators to NSR string theory is somewhat equivalent to introducing branes; In particular,
some of these operators dynamically deform flat ten-dimensional space time to $AdS_5 \times S^5$ background [13].

Generally, these states are described by vertex operators that exist at nonzero ghost pictures only, not admitting ghost number zero representation. This crucially distinguishes them from usual perturbative open or closed string states, such as photon, graviton or dilaton, which in principle are allowed to exist at any arbitrary ghost picture. This new class of states appears in both closed and open superstring theories and includes both massless and tachyonic states. In open string theory the massless ones are represented by two-form and five-form vertex operators; they are given by:

$$V^{(-3)}_{m_1...m_5}(z,k) = e^{-3\phi}\psi_{m_1}...\psi_{m_5}e^{ikX}$$
$$V^{(+1)}_{m_1...m_5}(k) = e^{\phi}\psi_{m_1}...\psi_{m_5}e^{ikX}(z) + \text{ghosts}$$
$$V_{m_1m_2} = e^{-2\phi}\psi_{m_1}\psi_{m_2}e^{ikX}$$

These dimension 1 primary fields must also be integrated over the worldsheet boundary. The five-form picture $-3$ operator $V^{(-3)}_{m_1...m_5}$ is BRST-invariant at any momentum, as is easy to see by simple and straightforward computations involving the BRST charge given by

$$Q_{brst} = \oint \frac{dz}{2i\pi} c(T_{\text{matter}} + \frac{1}{2}T_{\text{ghost}}) + \gamma(G_{\text{matter}} + \frac{1}{2}G_{\text{ghost}})$$

which may also be written as

$$Q_{brst} = \oint \frac{dz}{2i\pi} \{cT - \frac{1}{2}e^{\phi-\chi}\psi_{m}\partial X^{m} - \frac{1}{4}be^{2\phi-2\chi} + b : c\partial c\}$$

with T being the full matter + ghost stress-energy tensor of NSR string theory. However $V^{(-3)}_{m_1...m_5}$ is BRST non-trivial only if its momentum k is polarized along the 5 out of 10 directions orthogonal to $m_1,...,m_5$ (for any given polarization choice of $m_i$). Indeed, it is easy to see that the only BRST triviality threat for the 5-form operator may appear from the expression $\{Q_{brst},S_{m_1...m_5}\}$ where

$$S_{m_1...m_5} = e^{x-4\phi}\partial\chi(\psi\partial X)\psi_{m_1}...\psi_{m_5}e^{ikX}$$

- indeed, when $S_{m_1...m_5}$ is primary field of dimension 1 (note that this is the case only if there are no X’s with coincident indices in $(\psi_{s}\partial X^{s})$ and $e^{ikX}$, i.e. the index $s$ and the momentum polarization are chosen so that the $(\psi_{s}\partial X^{s})$ is directed orthogonally with respect to both $e^{ikX}$ and fermions $\psi_{m_1}...\psi_{m_5}$) we have

$$\oint \frac{dz}{2i\pi} cT(z)S_{m_1...m_5}(w) \oint \frac{dz}{2i\pi} \frac{1}{z-w}(c\partial S_{m_1...m_5} + \partial cS_{m_1...m_5}) = \partial(S_{m_1...m_5})$$
i.e. the full derivative which vanishes after integrating the vertex operator over the world-sheet boundary. Next, obviously, there is no pole in the O.P.E. between \( \oint b \partial c \) of \( Q_{\text{brst}} \) and \( S_{m_{1}...m_{5}} \) (since \( S \) contains no fermionic ghosts) and also there is no pole in the O.P.E. between \( \oint e^{2\phi-2\chi}b \) of \( Q_{\text{brst}} \) and \( S_{m_{1}...m_{5}} \) since

\[
e^{2\phi-2\chi}b(z)e^{\chi-4\phi}\partial\chi(\psi\partial X)\psi_{m_{1}}...\psi_{m_{5}}e^{ikX}(w) \sim
\]

\[
(z - w)be^{-2\phi-\chi}(\psi\partial X)\psi_{m_{1}}...\psi_{m_{5}}e^{ikX} + O(z - w)^{2}
\]

and therefore these terms do not contribute to the commutator of \( S \) with the BRST charge. But the commutator of \( S \) with the \( \oint \gamma(\psi\partial X) \) in the BRST charge does give the \( V_{5} \) operator as the relevant O.P.E. has a simple pole:

\[
e^{\phi-\chi}(\psi\partial X)(z)e^{\chi-4\phi}\partial\chi(\psi\partial X)\psi_{m_{1}}...\psi_{m_{5}}e^{ikX}(w) \sim \frac{1}{z - w}e^{-3\phi}\psi_{m_{1}}...\psi_{m_{5}}
\]

and therefore we have the BRST triviality

\[
\{Q_{\text{brst}}, S_{m_{1}...m_{5}}\} \sim e^{-3\phi}\psi_{m_{1}}...\psi_{m_{5}}
\]

However, it is clear that if the momentum \( k \) is orthogonal to \( m_{1}, \ldots, m_{5} \) directions the \( S_{m_{1}...m_{5}} \) is not a primary field: the \( (\psi\partial X) \) part of it always has internal O.P.E. singularities with either \( \psi_{m_{1}}...\psi_{m_{5}} \) or \( e^{ikX} \). As a result, whenever the momentum \( k \) is orthogonal to \( m_{1}, \ldots, m_{5} \) directions, the O.P.E. of the stress-energy tensor with \( S_{m_{1}...m_{5}} \) always has a cubic singularity. Therefore \( S_{m_{1}...m_{5}} \) does not commute with the \( \oint cT \) term of \( Q_{\text{brst}} \) and \( \{Q_{\text{brst}}, S_{m_{1}...m_{5}}\} \) does not reproduce the 5-form vertex operator \( V_{(-3)}^{m_{1}...m_{5}} \). However, in case if the momentum \( k \) of \( V_{m_{1}...m_{5}}^{(-3)} \) is longitudinal, i.e. is polarized along \( m_{1}...m_{5} \) directions it is easy to see that the vertex operator becomes BRST trivial: indeed, it can be written as a BRST commutator with the primary field: \( \{Q_{\text{brst}}, C_{m_{1}...m_{5}}\} \) with

\[
C_{m_{1}...m_{5}} = e^{\chi-4\phi}\partial\chi(\psi\partial X)^\perp \psi_{m_{1}}...\psi_{m_{5}}e^{ikX}
\]

with the supercurrent part \( (\psi\partial X)^\perp \) now polarized orthogonally to \( m_{1}, \ldots, m_{5} \), i.e. both to \( e^{ikX} \) and other world-sheet fermions of the 5-form.

So we see that BRST non-triviality condition imposes significant constraints on the propagation of the 5-form: namely, it is allowed to propagate in the 5-dimensional subspace transverse to its own polarization. This also is an important and remarkable distinction of this vertex operator from usual vertices we encounter in perturbative string theory; it
is well known that those are able to propagate in entire ten-dimensional space-time. The
two-form is also BRST-invariant at any k, as is easy to check using the above expression
for $Q_{brst}$ It is BRST-trivial at zero momentum as it can be represented as a commutator
\[
\{Q_{brst}, e^{\chi - 3 \phi \psi_{[m_1}} \partial X_{m_2]} \partial \chi \}
\]
but it becomes BRST non-trivial at non-zero momenta and again, in complete analogy
with the 5-form case, its momentum must be orthogonal to the $m_1, m_2$ two-dimensional
subspace, i.e. the two-form propagates in eight transverse dimensions. Constructing the
BRST-invariant version of the five-form at picture +1 is a bit more tricky since the straight-
forward generalization given by $e^\phi \psi_{m_1} ... \psi_{m_5}$ does not commute with two terms in the
BRST current given by $b \gamma^2$ and $\gamma \psi_{m} \partial X^m$. To compensate for this non-invariance one has
to add two counterterms, one proportional to the fermionic ghost number 1 field $c$ and
another to the ghost number $-1$ field $b$.

To construct these ghost counterterms one has to take the fourth power of picture-
changing operator $\Gamma^4 \sim : e^{4 \phi} G \partial G \partial^2 G \partial^3 G :$ with G being the full matter + ghost world-
sheet supercurrent and calculate its full O.P.E. (i.e. including all the non-singular terms)
with the picture $-3$ five-form operator. If the $-3$-picture vertex operator is at the point 0 then
\[
V_{5(+1)}(0) = e^\phi \psi_{m_1} ... \psi_{m_5} \Gamma_{m_1 ... m_5}^{\alpha \beta} F_{m_1 ... m_5}(k)
\]
This operator is BRST invariant by construction since both $\Gamma^4$ and picture $-3$ 5-form
operator are BRST invariant.

The BRST commutator with counterterms must be computed at a point $z$ and then
the limit $z \to 0$ is to be taken. Fortunately, due to the condition of fermionic ghost
number conservation this unpleasant non-local ghost part is unimportant in computations
of correlation functions and can be dropped at least in cases when not more than one
picture +1-operator is involved. For our purposes in this paper this shall be sufficient
and we will shall drop the non-local part elsewhere. The origin of these exotic 5-form
and 2-form operators is in fact closely related to Ramond-Ramond states at non-canonical
pictures.

Consider the Ramond-Ramond vertex operators at zero momentum in $(-1/2, -1/2)$
and $(-3/2, -3/2)$-pictures on a disc:
\[
V^{(-1/2,-1/2)}(k, z, \bar{z}) = e^{-\frac{i}{2} \phi \Sigma^\alpha (z)} e^{-\frac{i}{2} \bar{\phi} \Sigma^\beta (\bar{z})} e^{ikX(z, \bar{z})} \Gamma_{\alpha \beta}^{m_1 ... m_5} F_{m_1 ... m_5}(k)
\]
\[
V^{(-3/2,-3/2)}(k, z, \bar{z}) = e^{-\frac{3}{2} \phi \Sigma^\alpha (z)} e^{-\frac{3}{2} \bar{\phi} \times \Sigma^\beta (\bar{z})} e^{ikX(z, \bar{z})} \Gamma_{\alpha \beta}^{m_1 ... m_5} F_{m_1 ... m_5}(k)
\]
where \( \phi(z) \) and \( \chi(z) \) are free fields that appear in the bosonization of the NSR superconformal ghosts \( \beta, \gamma; \Sigma^\alpha \) is spin operator for NSR matter fields.

The crucial (and often neglected) point is that if a Ramond-Ramond vertex is placed on a disc and the boundary is present, the holomorphic and anti-holomorphic matter and ghost spin operators are no longer independent but they are related as:

\[
\bar{\phi}(\bar{z}) = \phi(\bar{z}), \quad \bar{\chi}(\bar{z}) = \chi(\bar{z}), \\
\bar{\Sigma}^\alpha(\bar{z}) = M^{(p)}_{\beta} \Sigma^\beta(z), \\
M_{\alpha\beta} \equiv (\Gamma^0...\Gamma^p)_{\alpha\beta}
\]

The expression for the matrix \( M^{(p)}_{\alpha\beta} \) implies that the Dirichlet boundary conditions are imposed on \( p \) out of 10 \( X^m \)'s while the Neumann conditions are imposed on the rest. As long as the vertices (1) are far from the edge of a D-brane (that is, \( z \neq \bar{z} \)) one may neglect the interaction between holomorphic and anti-holomorphic spin operators; however, as one approaches the boundary of the disc where \( z = \bar{z} \) the internal normal ordering must be performed inside the Ramond-Ramond vertex operators in order to remove the singularities that arise in the O.P.E. between the spin operators located at \( z \) and \( \bar{z} \). Adopting the notation \( F^{(q)}(k) \equiv \Gamma^{m_1}...\Gamma^{m_q}F_{m_1...m_q}(k) \) we find that the result of the normal ordering is given by:

\[
\lim_{z, \bar{z} \to s} : e^{-\frac{3}{2} \phi(\bar{z})} \Sigma(\bar{z}) : F^{(q)}_{\alpha\beta} : e^{-\frac{3}{2} \bar{\phi}(z)} \Sigma(z) : \\
\sim \frac{1}{z - \bar{z}} Tr(F^{(q)}(M^{(p)}(z) \Gamma^{m_1...m_5}) e^{-3\phi(\bar{z})} \psi_{m_1}...\psi_{m_5}(s) + ... \\
\lim_{z, \bar{z} \to s} : e^{-\frac{3}{2} \phi(\bar{z})} \Sigma(\bar{z}) : F^{(q)}_{\alpha\beta} : e^{-\frac{1}{2} \bar{\phi}(z)} \Sigma(z) :
\]

where we have dropped the less singular terms in the O.P.E. as well as full derivatives.

We see that due to the internal normal ordering at the boundary of the disc the Ramond-Ramond vertex operators degenerate into massless open – string vertices - the two-form \( Z_{mn} = e^{-2\phi} \psi_m \psi_n \) and the five-form \( Z_{m_1...m_5} = e^{-3\phi} \psi_{m_1}...\psi_{m_5} \). Giving a proper physical interpretation to these new massless states in the spectrum of a superstring obviously is a challenging puzzle. In our previous works [14] we have also shown that two-form and five-form vertices (3) appear as central terms in the space-time superalgebra for NSR superstring theory when the supercharges are taken in non-canonical pictures. The proof is quite analogous the above derivation of the brane-like states from the Ramond-Ramond insertions on a disc. Since p-form central terms in a SUSY algebra are always related to
the presence of p-branes, this leads us to conjecture that the 2-form and 5-form operators are related to non-perturbative dynamics of branes in string theory; so to speak they may be thought of as vertex operators creating extended soliton-like objects (unlike usual string vertex operators that correspond to emission of point-like particles)

The brane-like states (1) also have their analogues in the closed string-sector. To construct the closed string counterpart of the 5-form state (1) let us split the ten-dimensional space-time index $m$ in the 4+6 way: $X_m = (X_a, X_t)$ where $m = 0, \ldots, 9; a = 0, \ldots, 3; t = 4, \ldots, 9$ and similarly for the worldsheet fermions. Then the relevant closed string vertex operator is given by:

$$V_5(z, \bar{z}, k|X) = \lambda(k|X)e^{-3\phi - \bar{\phi}}\psi_{a_1} \cdots \psi_{a_4} \bar{\psi}_t \bar{\psi} z e^{ik|X}(z, \bar{z})$$

$$V_5^*(z, \bar{z}, k|X) = \lambda(k|X)e^{-3\phi - \bar{\phi}}\psi_{t_1} \cdots \psi_{t_6} \bar{\psi} e^{ik|X}\epsilon_{t_1 \ldots t_6}$$

with $k|X \equiv k_a X^a$ which we will also refer to as $V_5$-operator in the rest of the paper. The $V^*$ operator describes excitations of a D5-brane in 4 directions transverse to its worldvolume, while the $V_5$-operator effectively corresponds to a D3-brane (obtained by wrapping the 5-brane along the 2-cycle) and its excitations are confined to its 4-dimensional worldvolume. An important remark should be made here to avoid possible confusions: note that for the sake of compactness in our formulae we chose the worldvolume of a D5-brane to span the 6 directions $t_1, \ldots, t_6$ which on the other hand correspond to transverse coordinates of a D3-brane in our definition of $V_5$; but of course it should be understood that the physical implications of the four-dimensional momentum $k|X$ in $e^{ik|X}$ are significantly different in cases of $V_5$ and $V_5^*$: in the first case it corresponds to longitudinal oscillations in the D3-brane worldvolume while in the second case ($V_5^*$) $k|X$ accounts for transverse oscillations of the D5-brane.

The properties of the sigma-model with the $V_5$-operator have been studied in [13] where the relevance of this vertex operator to non-perturbative D3-brane dynamics has been shown (both $V_5$ and $V_5^*$ operators can be used in the sigma-model in an equivalent way) It is important that BRST invariance condition for the $V_5$ and $V_5^*$-operators (3) requires that their propagation is confined to four dimensions. Namely, to insure the BRST invariance, the momentum $k|X$ must be polarized along the $a_1, \ldots, a_4$ directions. Indeed, it is easy to see that the only way to avoid a cubic singularity in the O.P.E. between the antiholomorphic BRST current and $V_5$ (which arises from the $\bar{c}\bar{\partial} X_m \bar{\partial} X^m$ term in $j_{brst}$ and destroys the BRST invariance of $V_5$) one has to take the $X'$s in the exponent of $V_5$. 

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orthogonal to the $X^t$ in the antiholomorphic part, i.e. the momentum should be polarized along the longitudinal $a = 0, \ldots, 3$ four dimensional subspace; for the $V_5^*$ operator everything goes totally analogously. Again, as in the case of open string theory one is able to show BRST non-triviality of $V_5^*$ and $V_5$. Indeed, the only possible possible BRST triviality threat is once again coming from the commutator of $\int \gamma(\psi \partial X)$ in holomorphic part of $Q_{brst}$ with $C = e^{-4\phi - \bar{\phi}} \psi^{[t_1 \ldots \psi \bar{t}_5]}(\psi \partial X) \partial X e^{ik||X}$ but again C is not a primary field, having a cubic O.P.E. singularity with stress-energy tensor and therefore its commutator with $Q_{brst}$ does not reproduce $V_5^*$ - and similarly for $V_5$. Moreover, the condition of the world sheet conformal invariance (preserving the conformally invariant form of the O.P.E. between two stress-energy tensors corresponding to the action (2) or the vanishing of the beta-function in the lowest order of string perturbation theory) requires that the space-time scalar field $\lambda(k||)$, corresponding to the $V_5$-operator, should behave as

$$\lambda(k||) \sim \frac{\lambda_0}{k||^4}$$

(7)

where $\lambda_0$ is constant. The $V_5$-operator has manifest $SO(1, 3) \times SO(6)$ isometry and therefore NSR sigma-model with the $V_5$ operator has the same space-time Lorenz symmetry as the Green-Schwarz action of string theory on $AdS_5 \times S^5$ with the gauge kappa-symmetry fixed. Indeed, as it has been argued in [13], the role of the $V_5$ operator is that it transforms the flat ten-dimensional space-time vacuum into that of $AdS_5 \times S^5$, thus connecting two maximally supersymmetric backgrounds in ten dimensions. This is because adding the $V_5$-term to the sigma-model action (2) is in fact equivalent to introducing $D3$-branes in the theory. As a result, one may explore string theory in the AdS background (and consequently, the large N limit of gauge theory) by means of the brane-like sigma-model (2) which technically lives in flat ten-dimensional space-time. Using the AdS/CFT correspondence [9,10,11], i.e. the correspondence between local gauge invariant operators in the large N Yang-Mills theory and massless vertex operators in string theory one may obtain the large N correlators in gauge theory by computing scattering amplitudes of the BRST invariant vertices in the sigma-model (2). For example, as the dilaton vertex operator $V_\phi$ corresponds to the $TrF^2$ field in gauge theory, the generating functional for various correlation functions of the $TrF^2$ operators is given by:

$$Z(\lambda_0, \phi) = \int D[X]D[\psi]D[\text{ghosts}] f(\Gamma, N) \exp\left\{ \int d^2z \partial X^\mu \bar{\partial} X_\mu + \psi^\mu \bar{\partial} \psi_\mu + \bar{\psi}^\mu \partial \bar{\psi}_\mu + \lambda_0 \epsilon_{a_1a_2a_3a_4} \int \frac{d^4k}{k||^4} e^{-3\phi} \psi^{a_1} \psi^{a_2} \psi^{a_3} \psi^{a_4} \psi_\bar{t} \bar{\partial} X^t e^{ik||X} + \int d^{10}p V_\phi(p, z, \bar{z}) \phi(p) \right\}$$

(8)

$$+ \text{c.c.} + \text{ghosts}$$

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where $\varphi(p)$ is ten-dimensional space-time dilaton field. The “measure function” $f(\Gamma, N) \sim (1 + N^2\Gamma^4)^{-1} \times \text{c.c.}$ ( $\Gamma$ is picture-changing operator and $N$ corresponds to the gauge group parameter) needs to be introduced to the measure of integration to insure correct ghost number balance on the sphere and normalization of scattering amplitudes. The two-point dilaton correlation function, corresponding to the generating functional (5) is given by

$$
<V_\varphi(p_1)V_\varphi(p_2)>_{\sigma-model} = \left. \frac{\delta^2 Z(\lambda_0, \varphi)}{\delta\varphi(p_1)\delta\varphi(p_2)} \right|_{\varphi=0}
$$

(9)

To compute this correlator we have to expand the functional (5) in $\lambda_0$. The first non-trivial contribution has the order of $\lambda_0^2$ and it is given by

$$
A_{\lambda_0^2}(p_1, p_2) \sim \lambda_0^2 \int \frac{d^4k_1||}{k_1^4} \int \frac{d^4k_2||}{k_2^4} <V_\varphi(p_1)V_\varphi(p_2)V_5(k_1||)V_5(k_2||)>
$$

(10)

where the four-point amplitude should be computed in the usual NSR string theory in flat space-time. In other words, this is just the usual four-point closed string Veneziano amplitude which has to be integrated over internal momenta of the $V_5$-vertices, i.e. over two out of three independent momenta. The straightforward computation of the four-point amplitude and the integration over the $V_5$ momenta has been performed in refs[15] and the answer is given by:

$$
A_{\lambda_0^2} \sim \lambda_0^2(p_1||)^4|log(p_1||)|^2 \int d^2w \frac{log(|log|w||) + log(|log|1 - w||)}{|1 - w|^4}
$$

(11)

where $p_1||$ is the longitudinal projection of the dilaton momentum to four longitudinal directions; $(p||)^2 = p_\alpha p^\alpha$. It is remarkable that the amplitude (8) depends exclusively on four-dimensional longitudinal projection of the dilaton momentum; up to normalization it has the same form as the two-point correlator $<TrF^2(p_1||)TrF^2(-p_1||)>$ in the $N = 4$ super Yang-Mills theory in $D = 4$, computed in the approximation of dilaton s-wave [10]. Fourier transforming the amplitude (8), one recovers the well-known expression for the two-point amplitude in the $N = 4 D = 4$ SYM theory in the four-dimensional coordinate space: $TrF^2(x)TrF^2(y) \sim \frac{1}{|x-y|^4}$. Furthermore, the momentum structure of amplitudes with more $V_5$ insertions agrees with the form of the $<TrF^2TrF^2>$ correlators computed at higher values of the dilaton angular momentum; in other words, expansion in the $\lambda_0$ parameter in the brane-like sigma-model (2) accounts for higher partial waves of the dilaton field in the $AdS_5 \times S^5$ supergravity. Proceeding similarly, one can in principle compute
higher point correlation functions from the generating functional (5) to show their agreement with the known expressions for 3 and 4-point correlators in the $N = 4, D = 4$ SYM theory.

To explore the mechanism of the dynamical compactification of flat ten-dimensional space-time on $AdS_5 \times S^5$ due to presence of the $V_5$ vertex in the sigma-model action one has to study the modification of the dilaton’s beta-function in the $V_5$-background. Such an analysis has been carried out in \[13\]. The analysis of the dilaton’s beta-function shows that the compactification on $AdS_5 \times S^5$ occurs as a result of certain very special non-Markovian stochastic process. Namely, the $V_5$ background in the sigma-model has a meaning of a “random force” term with the $V_5$-operator playing the role of a non-Markovian stochastic noise, which correlations are determined by the worldsheet beta-function associated with the $V_5$ vertex. Indeed, The straightforward computation shows that the dilaton’s beta-function equation in the presence of the $V_5$-term has the form of the non-Markovian Langevin equation:

$$\frac{d\phi(p)}{d(log\Lambda)} = -\int d^{10}q C_{\phi}(q)\phi\left(\frac{p-q}{2}\right)\phi\left(\frac{p+q}{2}\right) + \eta_5(p\parallel, \Lambda)$$

(12)

where

$$\eta_5(p\parallel, \Lambda) \equiv -\lambda_0^2(1 + \lambda_0 \int d^4k_2\parallel \int_0^{2\pi} d\alpha \int_0^\infty dr \delta V_5(r + \Lambda, \alpha, k\parallel))$$

(13)

In this equation the role of the stochastic noise term being played by the truncated worldsheet integral of the $V_5$-vertex. The logarithm of the worldsheet cutoff parameter plays the role of the stochastic time in the Langevin equation. The noise is non-Markovian and it is generated by the $V_5$ operator, as was already noted above.

The noise correlations in stochastic time are given by the worldsheet correlators of the $V_5$ vertices (one has to take their worldsheet integrals at different cutoff values and to compute and to evaluate the cutoff dependence) Knowing the $V_5$-noise correlators it is then straightforward to derive the corresponding non-Markovian Fokker-Planck equation for this stochastic process and to show that the Fokker-Planck distribution solving this equation is given by the exponent of the ADM-type $AdS_5$ gravity Hamiltonian (computed from the $AdS_5$ gravity action at a constant radial AdS “time” slice using the Verlinde’s prescription \[18\]). Such a mechanism naturally relates the radial AdS coordinate, stochastic time and the worldsheet cutoff, pointing out an intriguing relation between holography principle, AdS/CFT correspondence and non-Markovian stochastic processes. Therefore from space-time point of view the $V_5$ insertion leads to non-Markovian stochastic process.
which deforms flat ten-dimensional space-time geometry the one of to $AdS_5 \times S^5$. At the same time, from the worldsheet point of view the situation looks as follows. In the beginning we have a critical NSR string theory in flat space-time with a simple 2d conformal field theory on a worldsheet. This $CFT$ is perturbed by the $V_5$ vertex operator and as a result the worldsheet theory flows to some new fixed point, i.e. new CFT. It is this new CFT which, in agreement with the arguments above, should constitute the worldsheet theory of NSR strings on $AdS_5 \times S^5$. Also one can consider the NSR theory perturbed by open string $V_2$-operator (two-form) which effectively corresponds to a D-string propagating in 8 dimensions (transverse to its worldsheet). Its $SO(1,1) \times SO(8)$ covariant form should be given by

$$\lambda(k^\perp)\epsilon_{\alpha\beta\gamma}e^{-2\phi}\psi_\alpha\overline{\psi}_\beta\overline{\psi}_\gamma e^{ik^\perp X(z)}$$.

An important problem to consider is to check (by analyzing appropriate correlation functions in this sigma-model) that the $V_2$ operator effectively curves the background to give the $AdS_3$ compactification (just like the $V_5$-perturbation gives us $AdS_5 \times S^5$). If the answer is positive that would mean that we can generate all the essential $AdS$ backgrounds in ten-dimensional superstring theory by merely perturbing the flat space-time theory by the $V_5$ and $V_2$ pair. To complete a brane zoology in terms of brane-like vertex operators one also needs to construct a closed string version of the brane-like two-form. The construction is completely analogous to the $V_5$ case and the BRST invariant closed-string partner of $V_2$ is given by:

$$V_2^* \sim \rho(k^\perp)\epsilon_{\alpha\beta\gamma}e^{-2\phi}\overline{\psi}_\alpha\psi_\beta\overline{\psi}_\gamma e^{ik^\perp X}$$

$$V_2 \sim \rho_\alpha(k^{||})e^{-2\phi}\overline{\psi}_\alpha\psi_t\overline{\psi}_t e^{ik^{||}X}$$

(9 indices $t$ are orthogonal to $\alpha$ in $V_2$, $k^{||} \equiv k_\alpha$). The $V_2$ operator describes the D0-brane whose momentum is directed along a given $\alpha$ direction ($\alpha$ also labels the 1-dimensional D0-brane worldline along which the momentum is polarized) while the dual $V_2^*$ vertex should account for the membrane (with $\epsilon^{\alpha\beta\gamma}$ spanning its three-dimensional worldvolume).

To summarize, we have the following classification:

- D0-brane is described by the closed string $V_2$-vertex;
- D1-brane by the open-string two-form;
- D2-brane by the closed string $V_2^*$-vertex;
- D3-brane by the closed string $V_5$-vertex;
- D4-brane by the open-string five-form;
D5-brane by the closed string $V_5^*$ vertex.

Let us ask now a natural question - we know that even branes, i.e. $D0, D2, D4$ branes exist in type A theory (either type IIA or 0A) and odd branes, i.e. $D1, D3, D5$ exist in type B theory. It is well known that difference between types A and B is due to fermion numbers in left and right Ramon sectors [7] - namely type A has $(R+, R-) \text{ or } (R-, R+)$ or both of them (for type 0A) and type B has either $(R+, R+) \text{ or } (R, R)$ or both of them. But by direct inspection of $V_2$ and $V_2^*$ one can see that the difference between numbers of left and right fermions is odd (namely one) and open-string five-form vertex operator also have odd (namely five) fermions - so they all must create branes in type A theory - and indeed in our table they correspond to even branes. By direct inspection of $V_5$ and $V_5^*$ one can see that the difference between numbers of left and right fermions is even (namely four) and open-string two-form vertex operator also have even (namely two) fermions - so they all must create branes in type B theory - and indeed in our table they correspond to odd branes.

For example to get $D1 - D5$ system we have to deform our sigma model by closed string $V_5^*$ and open string two-form $V_{m_1, m_2}$ vertex operators in which open string operator is polarized in transverse eight directions and $V_5^*$ is polarized in a four-dimensional subspace of this eight-dimensional space. Thus addition of $V_5^*$ vertex will deform symmetry group $SO(1,1) \times SO(8)$ we have discussed earlier down to $SO(1,1) \times SO(4) \times U(1)^4$ which is precisely the symmetry group of $D1 - D5$ system. Of course in the near-horizon limit it corresponds to the $AdS^3 \times S^3 \times T^4$ metric [9].

This completes our discussion of non-perturbative brane-like vertex operators. In the next section we shall attempt to analyze some properties of the new worldsheet CFT created by brane-like vertex operators.

NSR AdS Strings and Logarithmic Operators

Sometime ago [17] it was suggested that world sheet dynamics describing backgrounds with collective coordinates must be described by Logarithmic Conformal Field Theory (LCFT) [18]. The arguments were based on hidden symmetries in LCFT and an existence of a logarithmic zero norm state [19]. In [20] a logarithmic pair describing D-brane recoil was constructed explicitly. The logarithmic recoil operator (so called D operator) looks very similar to operator $V_5$ integrated over momenta. So it seems reasonable to suggest that we may deal with LCFT here too. There is another reason to suspect that worldsheet dynamics is given by LCFT in this theory. As we saw the brane vertex operators has a
very unusual feature - they mix matter and ghost fields. It was suggested recently [21] that in theories with matter-ghost mixing one has LCFT on a world sheet.

To prove that we have LCFT let us consider the following pair of operators:

\[
L_5 = \int \frac{d^4 p}{p^4} e^{-3\phi - \bar{\phi}} \psi_0 \psi_3 \bar{\psi}_t \bar{\psi}_t^t t e^{ipX} (z, \bar{z});
\]

\[
N_5 = \int \frac{d^4 k}{k^2} e^{3\phi - \bar{\phi}} \psi_0 \psi_3 \bar{\psi}_t \bar{\psi}_t^t t e^{ikX} (w, \bar{w})
\]

The operator product expansion of \(L_5\) with itself is given by:

\[
L_5(z, \bar{z}) L_5(w, \bar{w}) \sim \int d^4 p d^4 q \frac{1}{|z - w|^2} e^{-2(pq) \log |z - w|} V \phi (-2, -2) (p + q)
\]

where \(V \phi\) is dilaton vertex operator in the \((-2, -2)\) picture:

\[
V_{\text{varphi}} \sim e^{-2\phi - 2\bar{\phi}} \partial X_m \bar{\partial} X_n (\eta^{mn} - k^m \bar{k}^n - \bar{k}^m k^n)
\]

\[
k^2 = \bar{k}^2 = 0; (k \bar{k}) = 1
\]

In principle this O.P.E. also contains a quartic pole proportional to \(\sim \frac{V_T}{|z - w|^4}\) where \(V_T = e^{-2\phi - 2\bar{\phi}} e^{ik||X}\) is BRST-invariant operator of a clearly tachyonic nature but fortunately it is BRST trivial since

\[
e^{-2\phi} e^{ik||X} \sim \{Q_{\text{brst}}, e^{-3\phi} \partial X(k||\psi) e^{ik||X}\}
\]

and therefore can be dropped elsewhere. So the above O.P.E. may be written also as

\[
\int d^4 p d^4 q \frac{1}{p^4 q^4} e^{-2(pq) \log |z - w|} V \phi (-2, -2) (p + q)
\]

Performing the change of variables in the momentum space: \(l = 1/2(p + q), k = 1/2(p - q)\) we write the integral as

\[
L_5 L_5 \sim \frac{1}{|z - w|^2} \frac{\partial}{\partial \log |z - w|} \int d^4 p d^4 q \frac{1}{p^4 q^4} e^{-2(pq) \log |z - w|} V \phi (-2, -2) (p + q)
\]

Now we denoted \(i\alpha \equiv \log |z - w|\); we shall evaluate the momentum integral at real values of the \(\alpha\) parameter, performing afterwards the straightforward analytic continuation. Furthermore, in our evaluation of the momentum integral over \(d^4 k\) it is convenient to make
the translation \((k - l) \to k\) which does not change the Jacobian. So we have to evaluate the integral

\[
I(l, \alpha) = \int \frac{d^4k}{k^4(k + 2l)^4} e^{i\alpha(k^2 + 2kl)} = I_1 + I_2 + I_3 + I_4 = \\
\int_{-\infty}^{\infty} \! dk_0 \int d^3\vec{k} \frac{e^{i(k^2 + 2kl)\alpha}}{[(k_0 - |k|)(k_0 + |k|)(k_0 + 2l_0 - |k + 2l|)(k_0 + 2l_0 + |k + 2l|)]^2}
\]

The integral in \(k_0\) has four poles and \(I_1, \ldots, I_4\) are corresponding residues. The first residue, at \(|k| = k_0\) gives the three-dimensional spatial momentum integral

\[
I_1(l, \alpha) = \int d^3\vec{k} \partial_{|k|} \left\{ \frac{e^{i|k|(l_0 - |l|\cos\theta)\alpha}}{4(l_0 - |l|\cos\theta)^2|k|^4} \right\} = -\frac{1}{4} \int_0^\pi \! d\cos\theta \int_{-\infty}^{\infty} \! \frac{d|k|}{|k|^3} \frac{e^{i|k|\alpha(l_0 - |l|\cos\theta)}}{
\]

where \(\theta\) is the angle between the spatial parts of the \(l\) and \(k\) vectors; \(|l|, |k|\) are absolute values of the spatial parts. We used integration by parts and the on-shell condition \(l^2 = 0\).

Finally, integrating over \(-k-\) and using the formula

\[
\int_{0}^{\infty} \! \frac{dx}{x^3} e^{iax} \sim 1/2a^2 \log(a) - a^2/4
\]

we get

\[
I_1 = \frac{i}{8} \alpha^2 \int_0^\pi \! d(cos\theta) \{ \log(l_0 - |l|\cos\theta) + \log\alpha - 1/2 \} = \frac{i}{8} \alpha^2 (\log(l^2) + 2\log\alpha - 1)
\]

On the other hand, the second residue at \(k_0 = -|k|\) gives

\[
I_2 = \frac{i}{8} \alpha^2 (\log(l^2) - 2\log\alpha + 1)
\]

The third and the fourth residues are evaluated likewise and in fact \(I_1 + I_2 = I_3 + I_4\), as is easy to check. Summing all the four residues together and performing analytic continuation in \(\alpha\) we get

\[
I \equiv I_1 + I_2 + I_3 + I_4 = \frac{1}{4} \alpha^2 \log|l| = \frac{1}{4} \log^2|z - w| |\log l|^2.
\]

Substituting this result into the o.p.e. between to \(L_5\)'s (involving the differentiation with respect to \(\log|z - w|\)) we get

\[
L_5(z, \bar{z})L_5(w, \bar{w}) \sim \frac{\log|z - w|}{|z - w|^2} \int d^4k \log(k^2) V_\varphi(w, \bar{w})
\]
with $V_\varphi$ being again the dilaton vertex operator in the $(-2, -2)$ picture. Next, consider the second O.P.E. $L_5 N_5$ Everything goes quite similarly:

$$L_5(z, \bar{z})N_5(w, \bar{w}) \sim \frac{\partial}{\partial \log |z - w|} \int \frac{d^4 p d^4 q}{p^4 q^2} e^{-2(pq)\log|z - w|} V_\varphi(p + q) \frac{1}{|z - w|^2}$$

$$= \frac{i}{|z - w|^2} \frac{\partial}{\partial \log |z - w|} \int d^4 l V_\varphi(l) \int \frac{d^4 k}{k^4(k + 2l)^2} e^{i \log |z - w|(k^2 + 2kl)}$$

Again, splitting the momentum integral into spatial and $dk_0$ parts and evaluating the 4 residues at the poles in $k_0$ we find the momentum integral to give $I = 2(I_1 + I_2)$ where

$$I_1 = \int_0^\pi d\cos \theta \int_0^\infty \frac{d|k|}{|k|^2} (l_0 - |l| \cos \theta)^{-1} e^{2i|k|(l_0 - |l| \cos \theta)\log |z - w|}$$

$$= \log |z - w|(\log(l^2) + \log |z - w| - 1)$$

and likewise

$$I_2 = \log |z - w|((\log(l^2) - \log |z - w| + 1)$$

$$I = I_1 + I_2 \sim \log |z - w|\log(l^2)$$

Substituting into the O.P.E. we see that the logarithm drops out of the final answer (because of differentiating over log) and we get

$$L_5(z)N_5(w) \sim \frac{1}{|z - w|^2} \int d^4 l \log(l^2) V_\varphi(l)$$

Finally, consider the O.P.E. $N_5 N_5$.

Proceeding exactly as above we get

$$N_5(z, \bar{z})N_5(w, \bar{w}) \sim \frac{i}{|z - w|^2} \int d^4 l V_\varphi(l) \frac{\partial}{\partial \log |z - w|} \int \frac{d^4 k}{k^2(k + 2l)^2} e^{i(k^2 + 2kl)\log |z - w|}$$

Evaluating the residues in the integral over $d^4 k$ as above we see that the entire dependence on $\log |z - w|$ goes away in the final answer and the integral is proportional to $I = I_1 + I_2 + I_3 + I_4 \sim \log(l^2)$. Again, substituting into the O.P.E. we get zero after differentiating over $\log |z - w|$ (since $I$ does not depend on log). Hence $N_5(z, \bar{z})N_5(w, \bar{w}) \sim 0$ and therefore $L_5, N_5$ constitute a pair of logarithmic operators.

The last check is to see that we have indeed correct OPE in LCFT

$$T(z)L_5(w) \sim \frac{1}{(z - w)^2}(L_5 - N_5) + ...$$

$$T(z)N_5(w) \sim \frac{1}{(z - w)^2}N_5 + ...$$

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where $T$ is the full matter + ghost stress tensor. These relations are easy to check - they follow from the definitions of $L_5$ and $N_5$ and the operator product:

$$T(z)e^{-3\phi-\bar{\phi}\psi_0...\psi_3\psi_t\bar{\psi}_X_t e^{ik||X}(w,\bar{w})$$

$$\sim \frac{1}{(z-w)^2}(1-\frac{k||^2}{2})e^{-3\phi-\bar{\phi}\psi_0...\psi_3\psi_t\bar{\psi}_X_t e^{ik||X}(w,\bar{w})$$

$$+\frac{1}{z-w}\partial_w e^{-3\phi-\bar{\phi}\psi_0...\psi_3\psi_t\bar{\psi}_X_t e^{ik||X}(w,\bar{w})}$$

(34)

It is $k||^2$ term which cause the mixing between $L_5$ and $N_5$ the same way as it was in a brane recoil case [20].

In critical string theory we have, of course, $< TT > = 0$, therefore $T$ has a good behaviour as a partner in a logarithmic pair of the worldsheet LCFT [19]. One can assume that besides logarithmic (1,1) pair we have discussed there must be also logarithmic (2,0) and (0,2) pairs. The situation here is the same as in the two-dimensional models describing critical disorder which are described by $c = 0$ LCFT [22] in which one has a logarithmic pair of (2,0) operators. We shall discuss this issue in a separate publication.

**Matter-ghost mixing and M-theory**

Let us note that the fact that we had 2- and 5-form vertex operator in string theory is a puzzle. We have here objects which belong to M-theory. So it is very tempting to suggest that M-theory is nothing but string theory with new non-perturbative brane vertex operators included.

However the natural question emerge - where is the extra coordinate of M-theory. It seems to us that the natural candidate is the bosonised superconformal ghost. Indeed it looks like very similar to transition from D-dimensional non-critical string theory to the D+1 dimensional critical theory. Liouville field is playing the role of an extra dimension.

The same is going to happen here. The moment we introduced superconformal ghosts we have another field with positive signature. Actually it is the only extra field in string theory which can be interpreted as an extra dimension. And we see that precisely brane vertex operators depend on it. Moreover the analysis we performed in the introduction about the possible polarizations of momenta in brane operators are correctly identified them with 2-and 5- branes in M-theory. Let us note that we also can see the exponential $\phi$ dependence is nothing like an analog of gravitational dressing in non-critical string theory. It is interesting fact that gravitational dressing also leads to LCFT [23].
So it seems that it string theory there is actually additional field which can play the role of extra dimension - this is bosonized superconformal ghost. Let us note that this is the only field which can create a dimension with positive signature, because it has positive central charge. The picture is very similar to what we had for the Liouville filed when dynamical Liouville field played a role of a new dimension transforming $d$-dimensional space into $d+1$-dimensional. What was important of course was gravitational dressing - the fact that vertex operators depend on Liouville field as well as background did. Otherwise it would be sterile degree of freedom.

The same happens when we study non-perturbative string theory. At perturbative level we do not see superconformal ghost. Usual closed and open string vertex operators do not depend on it. However when the new non-perturbative brane vertex operators are introduced we can do it only by making them explicitly depending on superconformal ghost! This way it becomes as important as other coordinates.

Let us also note that by direct inspection all brane vertex operators (closed as well as open) are asymmetric with respect to superconformal ghosts from the left and right sectors! One can show that it is impossible to construct them without introducing different left and right momenta for $\phi$ and $\bar{\phi}$ - superconformal ghosts in left and right sectors. This means that the eleventh dimension must be compact - another interesting prediction of our conjecture.

The relation between superconformal ghost and the extra dimension of M-theory may also be given sense in the context of stochastic quantization. Indeed, in principle the $AdS_5 \times S^5$ geometry can be viewed as the infinite stochastic time limit of solution of an infinite order Fokker-Planck equation (which takes into account all the $V_5$ noise correlations while $AdS_5$ gravity is a solution of the Fokker-Planck equation truncated at quadratic order corresponding to dilaton s-wave approximation). Therefore the Fokker-Planck distribution solving this equation is effectively eleven-dimensional away from equilibrium point. The role of additional dimension is played by stochastic time but it is known that in stochastic quantization of gauge theories (with gauge fixing no longer necessary) stochastic time effectively replaces the ghost degrees of freedom [24]. At the same time, the appearance random force term in the RG equation for the dilaton field in the brane-like sigma-model (8) is closely to the superconformal ghost structure the $V_5$-operator, to emphasizing the connection between the eleventh dimension (stochastic time) and superconformal ghost degree of freedom.
Finally, we would also like to make another comment about the $V_5$-operator curving the background to $AdS_5 \times S^5$. It is known that, unlike a string theory in flat space-time, the $AdS_5 \times S^5$ string satisfies the loop equation (at least in WKB approximation) and possesses a zigzag symmetry which is necessary to insure confining properties of the string. Of course the confining properties of the $AdS_5 \times S^5$ string totally fit the context of AdS/CFT. In this respect, the $AdS_5 \times S^5$ compactification implemented by the $V_5$ operator may be seen as a restoration of this special worldsheet zigzag symmetry lacked by usual NSR string in flat space-time. This restoration may be understood as follows. It is known that necessary and sufficient condition for the zigzag symmetry is closeness of operator algebra of massless open string operators (i.e. gluons). Of course in a usual NSR model this operator algebra is not closed as, for example, the full O.P.E. between photons contains infinite tower of massive vertices. However, introducing the $V_5$-operators should cure the open string operator algebra in a sense that it would enable us to remove these undesired massive vertices. Namely, the operator algebra of two $V_5$-vertices would contain the same massive vertices but with opposite signs, so introducing the $V_5$-background would absorb the massive tower of vertices in the O.P.E. algebra of photons and therefore restore the zig-zag symmetry so that we get the confining (i.e. $AdS_5 \times S^5$ string theory). In the future work we hope to examine this hypothesis of a zigzag symmetry restoration in more details.

**Discussion and Conclusion**

Before making a conclusion let us make here several interesting observations about superconformal ghosts. The superconformal ghost $\beta - \gamma$ have central charge

$$C_{\beta\gamma} = 11$$

(35)

which is intriguing relation with the dimension of M theory. However this is not enough - when one bosonised this system $[7]$ one actually have two conformal field theories - one describing scalar field with positive norm and central charge

$$C_{\phi} = 13$$

(36)

and another is

$$C = -2$$

(37)

system of symplectic fermions. $C = -2$ system is an LCFT $[18]$ and plays very important role in recently discussed critical disordered systems (see for example recent paper $[25]$ and references therein). Using the fact that critical strings and disordered systems are both
systems with total central charge $C_{\text{total}} = 0$ and may have some similarity \[24\] it is very interesting that this sector is naturally incorporated in superstring theory. However it did not reveal itself so far even at level of brane vertex operators. Is it possible that at some other level (off shell strings ?) the $C = -2$ ghosts will be important ? And if yes - can we also take into account $bc$ ghosts with central charge $-26$ ? One can ask an interesting question - is it possible to get extra dimensions from $-26$ and $-2$ ghosts ? Obviously it will give us not 11-dimensional space but 12 or even 13-dimensional space, moreover they will have two times and symmetry group will be $SO(10, 2)$ or $SO(11, 2)$ ! But this is precisely what have been discussed recently in relation with F and S theories \[26\] !

It is tempting to suggest that the full non-perturbative formulation of String theory (M,F,S, etc) is nothing but string theory with ALL ghosts field playing dynamical role - and with a FULL matter-ghost mixing.

In conclusion we want to outline several important issues which have to be studied. First of all we have to understand how to calculate tensions of $D$ branes in our picture and how to reproduce all known results about D-brane interactions. It is necessary to study in all details the structure of BRST cohomologies in LCFT and prove unitarity of these theories - presence of logarithmic zero norm states will be very important. Relation between superconformal ghosts and Yang-Mills ghosts which emerges may explain amusing fact that one loop beta functions in YM theory have coefficients proportional to conformal anomalies in strings theory. How to get heterotic string - does the fact that bosonised superconformal ghost has central charge which is 1/2 of critical dimension of bosonic string is related that heterotic string is half bosonic ? Is it plausible that matter-ghost mixing may be important to understand the nature of famous $1 < C < 25$ barrier in non-critical strings ? Is it possible to imagine that due to stochastic description of extra dimension we shall have quantum mixing and there will be processes in which pure state will evolve into mixed one ? Etc, etc, etc....

Some of these questions sounds very strange but they definitely worth further analysis and we hope to return to them in future publications.

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