Radially Excited States of 1P Charmonia and X(3872)

Ying Chen \textsuperscript{ab}, Chuan Liu \textsuperscript{c}, Yubin Liu \textsuperscript{d}, Jianping Ma \textsuperscript{e}, Jianbo Zhang \textsuperscript{f}

(\textsuperscript{a} Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, P.R. China \textsuperscript{b} Theoretical Center for Science Facilities, Chinese Academy of Sciences, Beijing 100049, P.R. China \textsuperscript{c} School of Physics, Peking University, Beijing 100871, P.R. China \textsuperscript{d} School of Physics, Nankai University, Tianjin 300071, P.R. China \textsuperscript{e} Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, P.R. China \textsuperscript{f} Department of Physics, Zhejiang University, Hangzhou, Zhejiang 310027, P.R. China

The excited states of charmonia are numerically investigated in quenched lattice QCD with improved gauge and Wilson fermion actions formulated on anisotropic lattices. Through a constrained curve fitting algorithm, the masses of the first excited states in 0^{++}, 1^{++}, and 1^{-+} channels are determined to be 3.825(88), 3.853(57), and 3.858(70) GeV, respectively. Furthermore, a node structure is also observed in the Bethe-Salpeter amplitude of the 1^{++} first excited state. These observations indicate that X(3872) could be the first radial excitation of \(\chi_{c1}\).

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The narrow charmonium-like X(3872) (with width \(\Gamma < 2.3\) MeV) was first observed by Belle in the exclusive decay \(B^\pm \to K^\pm X \to K^\pm\pi^+\pi^- J/\psi\) \cite{1}, and has been confirmed by CDFII \cite{2}, DO \cite{3}, and BaBar \cite{4} in three decay and two production channels. Even though its \(J^{PC}\) quantum numbers have not been finally established, the present experimental data strongly favor that it is a 1^{++} state \(\chi_{c1}\) for the following reasons. First, the decay mode \(X(3872) \to \gamma J/\psi\) observed recently by Belle requires the charge conjugation of \(X(3872)\) to be positive \cite{5}. The possibility with \(J^{PC} = 0^{++}\) and \(0^{-+}\) can be ruled out based on the angular correlations in the \(\pi^+\pi^- J/\psi\) system \cite{1} and that of \(2^{++}\) or \(1^{-+}\) is also strongly disfavored according to the dipion mass distribution \cite{5}. Second, the bound \(\Gamma(e^+e^-) Br(X \to \pi^+\pi^- J/\psi) < 10\) eV at 90\% C.L., obtained with the data collected by BES at \(\sqrt{s} = 4.03\) GeV \cite{6}, implies that it is unlikely a 1^{-+} vector state.

Since the discovery of \(X(3872)\), there have been many theoretical studies. Given its quantum number \(J^{PC} = 1^{++}\), a natural assignment of the state is the first radial excitation of 1P charmonium state \(\chi_{c1}\). However, there are two main difficulties for this interpretation. One is its tiny decay width relative to other charmonium states, another is that it lies roughly 100 MeV lower in mass than the prediction of the quark model \cite{8}. These difficulties motivate many non-charmonium explanations of \(X(3872)\), such as hybrids \cite{9}, glueballs \cite{10}, diquark clusters \cite{11}, and molecular states \cite{12}.

Although the non-relativistic quark model is successful for heavy quark bound states, it is known that relativistic effects can be important for charmonia, since the charm quark is not heavy enough. In particular, these effects, which are not taken into account in the model, can be more important for higher excited states. It is therefore more desirable to study charmonia with a relativistic lattice QCD formalism, which includes all the relativistic effects. In contrast to the study of the ground states of charmonia, such as the 1S and 1P states, their excited states have not been investigated as much in the formalism of lattice QCD, even though they are more interesting in the present era when many new heavy mesons of open-charm and closed-charm are observed. The major obstacle is that the extraction of the excited states remains a challenge in Monte Carlo simulations. In this work, we investigate the relevant charmonium spectra in quenched lattice QCD and focus on the derivation of the first (radially) excited states through the sequential empirical Bayes method (SEB) \cite{13} advocated by the \(\chi QCD\) collaboration, which is in the spirit of the constrained curve fitting algorithm and has been successfully applied to the study of nucleon excited states \cite{14} and pentaquarks \cite{15}.

\begin{table}[h]
\centering
\caption{The input parameters for the calculation. Values for the coupling \(\beta\), anisotropy \(\xi\), the lattice spacing \(a_s\), lattice size, and the number of measurements are listed.}
\begin{tabular}{|c|c|c|c|c|}
\hline
\(\beta\) & \(\xi\) & \(a_s\) (fm) & \(L^3 \times T\) & \(N_{\text{conf}}\) \\
\hline
2.4 & 5 & 0.222 & 3.55 & 16^3 \times 80 & 200 \\
2.6 & 5 & 0.176 & 2.82 & 16^3 \times 80 & 200 \\
2.8 & 5 & 0.139 & 2.22 & 16^3 \times 80 & 800 \\
\hline
\end{tabular}
\end{table}

We use the quenched approximation in this study. The gauge configurations are generated by the tadpole improved gauge action \cite{16} on anisotropic lattices with the temporal lattice much finer than the spatial lattice, say, \(\xi = a_s/a_t \gg 1\), where \(a_s\) and \(a_t\) are the spatial and temporal lattice spacing, respectively. Each configuration is separated by 2000 heat-bath updating sweeps to avoid the autocorrelation. The much finer lattice in the temporal direction gives a high resolution to hadron correlation functions, such that masses of heavy particles can be tackled on relatively coarse lattices. The relevant input parameters are listed in Table \ref{tab:input}, where \(a_s\)'s are determined from \(r_0^{-1} = 410(20)\) MeV.

For fermions we use the tadpole improved clover action for anisotropic lattices \cite{17}. The parameters in the
action are tuned carefully by requiring that the physical dispersion relations of vector and pseudoscalar mesons are correctly reproduced at each bare quark mass 18. The conventional interpolation operators $\bar{u} \Gamma \psi$ are used for meson states with different gamma matrices $\Gamma$ for the specific spin-parity quantum numbers. After the Coulomb gauge fixing, the wall-source quark propagators are calculated under the anti-periodical boundary condition and are used to construct the correlation functions (the point-source correlation functions are found to be very noisy in $0^{++}$, $1^{++}$, and $1^{+-}$ channels). The bare charm quark masses at different $\beta$ are determined by the physical mass of $J/\psi$ $m_{J/\psi} = 3.097$ GeV.

It is obvious in Eq. (1) that the contribution of the lowest state dominates the correlation function when $t$ is large enough, because the relative contribution of higher states to the ground state damps exponentially as $\propto \exp(-\Delta M_j t)$, where $\Delta M_j = M_j - M_1$. Intuitively, if $W_j$ and $M_1$ of the first state are correctly derived in a time region $[t_1, t_{\text{max}}]$, one can treat them as priors for the ground state and then do the two-mass-term fit in a larger time range $[t_2, t_{\text{max}}]$ with $t_2 < t_1$. The criterion for including the second mass term is the observation of a sharp jump of $\chi^2/n.o.f$, which is a signal that a single exponential cannot describe the data well, when $t_1$ is decreased further. This procedure is repeated by adding more states until the data points are exhausted. This is the basic fitting procedure of SEB method. Generally speaking, the last state can not be taken as the realistic one because it includes almost all the contaminations from higher states. Figure 1 illustrates the SEB fitting procedure in the vector channel. The upper panel shows the fitted masses using 1-4 mass terms in the fit function, while the lower panel shows the $\chi^2/d.o.f$ with the change of fitting range.

A typical correlation function takes the function form,

$$C(t) = \sum_{i=1}^{\text{term}} W_i e^{-M_i t},$$  \hspace{1cm} (1)

where $W_i$ and $M_i$ are the spectral weight and the mass of the $i$-th state, respectively. If one fits the correlation function using this function form with multiple mass terms directly through the conventional maximal likelihood method, usually one gets a poor result due to the complicated parameter space and the result depends crucially on the the choice of the initial values of the parameters. A constrained curve fitting algorithm can help if one can obtain reasonable priors of the parameters that will be fitted 19. In this work, we apply the sequential empirical Bayes method (SEB) in the data analysis. We will outline the main ideas of this method here and more details can be found in Ref. 13.

Now we come to the analysis of the correlation functions in $0^{++}$, $1^{++}$, and $1^{+-}$ channels. It is found that these correlation functions are much noisier than those in vector (V) and pseudoscalar (PS) channels, such that we can only carry out maximally three-mass-term fits to them in the practical data analysis. Figure 2 shows the fitting procedure for the $1^{++}$ charmonium at $\beta = 2.8$. The upper panel is similar to that of Figure 1 but shows the three-mass-term fit, where one can see that the second state is very stable after the inclusion of the third state in the fitting model. The lower panel illustrates the effective mass plateau $m_{\text{eff}} = \ln C(t)/\ln C(t+1)$, where the data points are the simulation results while the curve is the three-mass fitting function with the best-fit param-

FIG. 1: The SEB fitting procedure of the vector charmonium correlation function at $\beta = 2.8$. The upper panel shows the fitted masses using fitting models with 1-4 mass terms. The lower panel shows the $\chi^2/d.o.f$ with the change of fitting range.
FIG. 2: The fitting procedure in $1^{++}$ channel. The upper panel is similar to that of Figure 3 but shows the three-mass-term fit. The lower panel illustrates the effective mass plateau $m_{\text{eff}} = \ln C(t)/\ln C(t + 1)$, where the data points are the simulations results while the curves are the fitting function plotted using the best fitted parameters.

The masses of the excited states obtained at $\beta = 2.8$ are relatively more reliable and can be treated as approximations of their continuum value with the awareness that the uncertainties from the finite lattice spacing are not properly tackled. Comparing with the predictions by the non-relativistic quark model [8] (the last column of Table II), the mass of the $1^{++}$ first excited state at $\beta = 2.8$ is lower than quark model expectation but consistent with the experimental value of $X(3872)$. Even though our result is not decisive to the assignment that $X(3872)$ be the first radial excitation of $\chi_{c1}$ due to the large statistical error, it indicates that the possibility that $X(3872)$ is a conventional charmonium state can not be simply ruled out.

In order to see if the extracted masses are really those of the first excited states, we also investigate the Bethe-Salpeter amplitudes of charmonium states at $\beta = 2.8$. In the Coulomb gauge, we split the sink operator into two parts, with each quark field residing on different spatial sites, namely, $\mathcal{O}_\Gamma(x,y) = \bar{\psi}(x)\Gamma\psi(y)$, where $\Gamma$ represents the various gamma matrices corresponding to specific $J^{PC}$ quantum numbers and $x = (x,t)$ and $y = (y,t)$.

In the practical calculation, all the two-point functions with $|x-y| \leq 6\sqrt{\Delta a}$ are calculated. It is found that the two-point functions exhibit a spherical symmetry with respect to the relative displacement between the quark and anti-quark fields, $r = x - y$. Therefore the two-point functions with the same spatial separation $|r|$ are averaged to increase the statistics.

The two-point functions calculated with the Coulomb wall sources are actually $C_{\Gamma}(r,t) = \sum_{n}(\mathcal{O}_\Gamma(x,t; x, r,t)\mathcal{O}_{\Gamma}(y,0)\langle 0 | \psi(y,0) \Gamma \psi(z,0) \rangle |x, z \rangle$. After the integration over $x$, we get

$$C_{\Gamma}(r,t) = \sum_{n} \frac{1}{2M_{n}} |\langle n | \mathcal{O}_{\Gamma}(0, 0; r, 0) | n \rangle| e^{-M_{n}t}$$

$$= \sum_{n} \Phi_{n}(r) e^{-M_{n}t} \quad (n = 1, 2, \ldots,)$$

where $M_{n}$ is the mass of the $n$-th state and $\Phi_{n}(r)$ the BS amplitude (up to an irrelevant prefactor) we would like to extract.

The $r$-dependent BS amplitudes $\Phi_{n}(r)$ have direct connections with the non-relativistic wave functions of heavy quarkonia [22, 23]. Specifically, for the $P$-wave charmonia, $\Phi_{n}(r)$ is related approximately to the radial wave

| $\beta$  | 2.4 | 2.6 | 2.8 | cont. | BGS[8] |
|---------|-----|-----|-----|------|--------|
| $0^{++}(1P)$ | 3.460(19) | 3.437(21) | 3.410(18) | 3.376(6) | 3.424 |
| $1^{++}(1P)$ | 3.520(19) | 3.507(17) | 3.477(14) | 3.453(6) | 3.505 |
| $0^{++}(2P)$ | 3.816(57) | 3.865(90) | 3.825(88) | 3.857(55) | 3.852 |
| $1^{++}(2P)$ | 3.937(56) | 3.887(54) | 3.853(57) | 3.800(2) | 3.925 |
| $1^{--}(2P)$ | 3.955(53) | 3.964(71) | 3.858(70) | 3.835(88) | 3.834 |

TABLE II: Best-fit masses of the ground and the first excited states of parity-positive charmonia at different $\beta$. All errors are statistical. The errors of the continuum limit values are from the linear extrapolation in terms of $a_{s}^{2}$. The continuum limits that the fitted first excited states may have significant extrapolations. The extrapolation values of the $P_{21}$ but lower than the experiment values. The results of the extrapolated masses of the first excited states should be taken with caution because the masses at the two small $\beta$’s can have large systematic errors as mentioned above.
function $R_n(r)$ as

$$\Phi_n(r) \sim CR'_n(r),$$  \hspace{1cm} (3)

where $R'_n(r)$ is the $r$-derivative of $R_n(r)$ and $C$ a factor irrelevant to the discussion here. In this work, $\Phi_1(r)$ and $\Phi_2(r)$ are obtained from a constrained-curve-fitting algorithm by assuming the fitted masses of the ground and the first excited states at $r = 0$ as known parameters, say, priors. Plotted in Figure 3 are $\Phi_n(r)$’s of the ground $(n = 1)$ and the first excited state $(n = 2)$ in the $1^{++}$ channel. It is obviously seen in the figure that $\Phi_1(r)$ has one node at $r/a \sim 4.8$ and $\Phi_2(r)$ has two nodes at $r/a \sim 2.5$ and $r/a \sim 8$. These behaviors are qualitatively in agreement with the theoretical anticipation described by Eqn. (3) if both states correspond to the $1P$ and $2P$ states of $1^{++}$ charmonium: the unique node of $\Phi_1(r)$ corresponds to the unique maximum of $1P$ radial wave function, the two nodes of $\Phi_2(r)$ manifest the two extrema, and necessarily a radial node in-between, of the $2P$ radial wave function.

To summarize, we have carried out a quenched lattice study of the excited states of charmonia on anisotropic lattices. In our study, a relativistic formalism of lattice QCD is employed and hence the relativistic effects are included. Using SEB, which is in the spirit of the constrained-curve-fitting method, we can derive reliably the masses of the ground states and the first excited states of charmonia in the $0^{++}$, $1^{++}$, and $1^{−−}$ channels. We obtain a mass of 3853(57) MeV for the first excited state of the $1^{++}$ charmonium, which is lower than the conventional quark model predictions but consistent with the measured mass of $X(3872)$ within statistical errors (systematic errors from the quenched approximation have not been considered yet). We have also observed a radial node of the wave function of the $1^{++}$ first excited state. All these results support the possibility that $X(3872)$ be the first radially excited state of $\chi_{c1}$.

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