The Hydrodynamical Limit of Quantum Hall system

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Abstract

We study the current algebra of FQHE systems in the hydrodynamical limit of small amplitude, long-wavelength fluctuations. We show that the algebra simplifies considerably in this limit. The hamiltonian is expressed in a current-current form and the operators creating inter-Landau level and lowest Landau level collective excitations are identified.
I. INTRODUCTION

Theoretical insights into the physics of Fractional Quantum Hall Effect (FQHE) systems was initially obtained by the study of the pioneering variational wavefunctions proposed by Laughlin \[1\] and by the exact diagonalisation of small systems. The concept of composite fermions, put forward by Jain \[2\] gave further insight into the physics and a unified picture of all the observed integer and fractional values of the filling factor, \(\nu\). The composite fermions can be modelled as fermions interacting with a Chern-Simons gauge field. The observed fractions can be obtained as mean field solutions of this theory.

While attempts to treat the fluctuations about the mean field state have had some success \[3\], there is really no systematic expansion as yet. Recently, G.Murthy and R. Shankar \[4\] have proposed a Hamiltonian formalism using which they develop an analytic approximation scheme in the long wavelength limit. Based on these results, R. Shankar \[5\] has formulated a calculational prescription that goes beyond the long wavelength limit. This prescription has been successful in analytically computing the excitation spectrum of the system with a fair degree of accuracy.

Our work is motivated by the success of the abovementioned works. The approximations used in reference \[4\] are valid for longwavelength, small amplitude density fluctuations, namely in the hydrodynamic limit. In this limit, current algebra techniques have been used with success in other systems of interacting fermions. These span a huge range in energy and length scales. From 1d electrons in solids (energies \(\sim\) meV and wavelengths \(\sim\) \(\AA\)) to interacting quarks (energies \(\sim\) MeV and wavelengths \(\sim\) fm). In these cases, the theory can be written in terms of the hydrodynamic variables, the current density and charge density operators. The spectrum of collective excitations (particle-hole pairs) is described by small amplitude harmonic oscillations of the fluid and the charged, fermionic quasiparticles manifest as topological solitons. Such a picture is physically well established in FQHE systems. The ground state is a an incompressible liquid, the low energy collective excitations are small amplitude density fluctuations and the charged quasiparticles (in this case fractionally
charged anyons) are vortices. Thus current algebra techniques should be useful here also and could possibly be used to develop a systematic long-wavelength expansion.

This work is our first step in this direction. We first develop a fully field theoretic version of the formalism in reference [4]. In section II, we begin with the system of electrons and construct a unitary flux attachment operator that transforms electrons into flux carrying composite fermions coupled to a Chern-Simons gauge field. Section III constructs the "mean field" states and shows that they correspond to the (unprojected) Jain states. We then derive the current algebra of the composite fermions and its hydrodynamic limit, in section IV. In this limit, section V expresses the hamiltonian in terms of the electron current and charge densities. The collective excitations and the lowest Landau level (LLL) projection is discussed in section VI. We summarise our results in section VII.

II. COMPOSITE FERMIONS

A. The electronic system

We start with the system of interacting electrons in a magnetic field. The electron annihilation and creation field operators, $C(x)$ and $C^\dagger(x)$ satisfy the anticommutation relations,

$$\{C(x), C(y)^\dagger\} = \delta^2(x - y)$$  \hspace{1cm} (1)

The Hamiltonian of the system is $H = H_0 + H_{\text{int}}$ where,

$$H_0 = \int d^2x C(x)^\dagger \left( -\frac{i}{2} \frac{\partial}{\partial x^j} + \frac{1}{2} \epsilon_{ij} x^j \right)^2 C(x)$$ \hspace{1cm} (2)

$$H_{\text{int}} = \frac{e^2}{2\kappa} \int d^2x d^2y U(x - y) C(x)^\dagger C(y)^\dagger C(y) C(x)$$ \hspace{1cm} (3)

We have expressed all lengths in units of $l_c$ ($\frac{1}{l_c} = \frac{eB}{\hbar c}$) and all energy scales in units of $\hbar \omega_c = \frac{eB}{m^* c}$. In these units we have,

$$A_i = -\frac{1}{2} \epsilon_{ij} x^j$$ \hspace{1cm} (4)

$$B = 1 \text{ and } \bar{\rho} = \frac{\nu}{2\pi}$$
The charge density and current operators are given by

\[ \rho(x) = C(x)\dagger C(x) \]  
\[ J_i(x) = \frac{1}{2i}(C\dagger\partial_i C - \partial_i C\dagger C) - A_i(x)\rho \]  

(5)

B. The extended Hilbert Space

We now introduce the Chern-Simons gauge fields, \( a'_i(x) \), obeying the commutation relations,

\[ [a'_i(x), a'_j(y)] = i4\pi k\epsilon_{ij}\delta^2(x - y), \quad (k = 1, 2, 3...) \]  

(6)
equivalently, \( a'(x) \equiv (a'_1 + ia'_2)/\sqrt{2\pi k} \) obey the oscillator algebra

\[ [a'(x), a'(y)\dagger] = \delta^2(x - y) \]  

(7)

We denote the Hilbert space of the Chern-Simons fields as \( \mathcal{H}_{CS}' \) and call the electronic Hilbert space \( \mathcal{H}_{el} \). We then define an extended Hilbert space,

\[ \mathcal{H}_{ex} \equiv \mathcal{H}_{el} \otimes \mathcal{H}_{CS}' \]  

(8)

A subspace \( \mathcal{H}_{phy} \) is defined as the set of states satisfying the constraint,

\[ \frac{1}{4\pi k}\epsilon_{ij}\partial_i a_j|\psi >_{phy} = 0 \]  

(9)

It can be shown that a unique solution exists to this constraint in \( \mathcal{H}_{CS}' \). Therefore we trivially have

\[ \mathcal{H}_{phy} = \mathcal{H}_{el} \]  

(10)
Next we construct an unitary operator, which attaches 2k units of Chern-Simons flux to the electrons. Namely, an unitary operator that transforms electrons into composite fermions. This operator, \( V \), is given by,

\[
V = e^{i\left(\int d^2x d^2y \frac{-k}{2}\rho(x)\theta(x-y)\rho(y) + \int \rho(x)d^2y \epsilon_{ij}\alpha^v_i(x-y)a_j(y)\right)}
\]

where,

\[
\alpha^v_i = \epsilon_{ij} \partial_j \Theta(x-y)
\]

\[
\epsilon_{ij} \partial_i \alpha^v_j(x-y) = 2\pi \delta^2(x-y)
\]

It can be shown that,

\[
VC(x)V^\dagger = e^{i2k(\Omega_L(x)-\Phi(x))}C(x)
\]

\[
\equiv \Psi(x)
\]

\[
V a'_i(x)V^\dagger = a'_i(x) + 2k \int d^2y (\alpha)^v(x-y)\rho(y)
\]

\[
\equiv a_i(x)
\]

where,

\[
\Omega_L(x) \equiv \frac{1}{4\pi k} \int_y \epsilon_{ij}\alpha^v_i(x-y)a_j(y)
\]

\[
\Phi(x) \equiv \int_y \theta(x-y)\rho(y)
\]

Since \( \Psi \) and \( a_i \) are unitary transforms of \( C \) and \( a'_i \), they satisfy the same commutation relations,

\[
\{\Psi(x),\Psi^\dagger(y)\} = \delta^2(x-y)
\]

\[
[a_i(x),a_j(y)] = i4\pi k\delta^2(x-y).
\]

Equations (17) and (20) imply that \( \Omega_L \) is conjugate to the Chern-Simons magnetic field i.e,
\[ [\Omega_L(x), \epsilon_{ij} \partial_i a_j(y)] = i2\pi \delta^2(x - y) \quad (21) \]

Therefore, \( e^{2iK\Omega} \) creates a vortex carrying 2k units of flux at x. \( e^{-2ik\Phi(x)} \) gives the corresponding Ahronov-Bohm Phase. Thus \( \Psi^\dagger(x) \) creates an electron and a vortex carrying 2k units of flux at x, namely it creates a composite fermion.

Thus if we define \( V\mathcal{H}_{CS}^\dagger \equiv \mathcal{H}_{CS} \) and \( V\mathcal{H}_{el}^\dagger \equiv \mathcal{H}_{CF} \), then we have

\[ \mathcal{H}_{ex} = \mathcal{H}_{CF} \otimes \mathcal{H}_{CS} \quad (22) \]

The subspace \( \mathcal{H}_{phy} = \mathcal{H}_{el} \) satisfies the constraint,

\[ (\frac{1}{4\pi K} \epsilon_{ij} \partial_i a_j - \Psi^\dagger(x)\Psi(x))|\Psi_{phy} > = 0 \quad (23) \]

The constraints in equation (23) are the generators of gauge transformations and we have,

\[ U(\Omega)\Psi(x)U^\dagger(\Omega) = e^{i\Omega(x)}\Psi(x) \quad (24) \]

\[ U(\Omega)\vec{a}(x)U^\dagger(\Omega) = \vec{a}(x) - \vec{\nabla}\Omega \quad (25) \]

where, \( U(\Omega) = e^{i\int \frac{1}{4\pi K} \epsilon_{ij} \partial_i a_j - \rho(x)\Omega(x)} \) is an element of the gauge group. Thus the physical subspace is the gauge invariant subspace of the composite fermion-Chern-Simons theory.

The electronic observables are hence mapped on to gauge invariant operators. The ones of interest to us can be explicitly computed to be,

\[ \rho(x) = \Psi^\dagger(x)\Psi(x) \quad (26) \]

\[ \vec{J}(x) = \frac{1}{2i} (\Psi^\dagger \vec{\nabla}\Psi - \vec{\nabla}\Psi^\dagger\Psi) - (\vec{A} - \vec{a})\rho(x) \quad (27) \]

\[ H_0 = \int d^2 x \Psi^\dagger(x)(\vec{p} - \vec{A} + \vec{a})^2\Psi(x) \quad (28) \]

\[ H_{int} = \frac{1}{2} \int U(x - y)\Psi^\dagger(x)\Psi^\dagger(y)\Psi(y)\Psi(x)d^2xd^2y \quad (29) \]

III. MEAN FIELD STATES
A. Physical States

As seen above, the physical states are the gauge invariant states. We can construct a projection operator, $P_G$ that projects out the gauge invariant part of any state.

$$P_G \equiv \int D[\Omega]e^{i\frac{1}{4\pi k}(\epsilon_{ij}\partial_i a_j - \rho)\Omega}$$ (30)

$P_G$ basically projects out the singlets of the gauge group. It is easy to show that,

$$\left(\frac{1}{4\pi k}\epsilon_{ij}\partial_i a_j - \Psi^\dagger(x)\Psi(x)\right)P_G\ket{\Psi} = 0$$ (31)

For example, the empty state of the electron theory is given by,

$$\ket{0}_e = P_G\ket{0}_{CF} \otimes \ket{0}_{CS}$$ (32)

where,

$$\Psi(x)\ket{0}_{CF} = 0 = a(x)\ket{0}_{CS}$$ (33)

B. Smoothening the flux: Coherent states

The mean field theory requires states with uniform Chern-Simons magnetic fields. A convenient way to construct such states is by constructing a coherent state basis for the Chern-Simons sector. The details of this construction can be found in reference [6], we outline it here.

The displacement operator $D(\alpha)$, is defined as,

$$D(\alpha) = e^{\frac{i}{4\pi k} \epsilon_{ij}\alpha_i a_j d^2 x}$$ (34)

The coherent states are obtained by the action of $D(\alpha)$ on the fluxless state, $\ket{0}_{CS}$ where $a(x)\ket{0}_{CS} = 0$,

$$\ket{\alpha} = D(\alpha)\ket{0}_{CS}$$ (35)
The states $|\alpha>$ satisfy the standard coherent state properties including,

$$<\alpha|a(x)|\alpha> = \alpha(x)$$

(36)

Thus they are minimum uncertainty wavepackets peaked around $a(x) = \alpha(x)$.

To get some more feel for these states, consider the state $|\bar{\alpha}>$ corresponding to a constant magnetic field $b = \nabla \times \vec{\alpha}$ and compute its overlap with the N-vortex state defined as,

$$|x_1..x_N> = \prod_{m=1}^{N} e^{i2k\Omega(x_m)} P_G |0>_{CS}$$

(37)

The overlap, which is the amplitude of finding the system in the N-vortex state, if it is in the constant field state, can be computed exactly using the techniques developed in reference [6] and is found to be,

$$<x_1..x_N|\bar{\alpha}> = \prod_{n>m}^{N} |x_n - x_m|^{2k} e^{-\frac{b}{2} \sum_{m=1}^{N} |x_m|^2}$$

(38)

Thus we see how the constant field state can give rise to the modulus of the Laughlin factor in the wavefunctions.

C. The Jain states

We are now ready to construct the gauge invariant mean field states of the composite fermion - Chern-Simons theory. Namely, take the direct product of a composite fermion state with $p$ filled Landau levels in a magnetic field $B^* = \frac{1}{2kp+1}$, which we denote by $|p,k>_{CF}$ and a constant field Chern-Simons state with magnetic field, $b = B - B^* = \frac{2kp}{2kp+1}$, denoted by $|p,k>_{CS}$ and gauge project it. We then obtain a state with the filling factor $\nu = \frac{p}{2kp+1}$.

$$|p,k> = P_G |p,k>_{CF} \otimes |p,k>_{CS}$$

(39)

The wave function of the above state is the overlap with the $N$ electron state,

$$|\{x_n\} >_{el} = \prod_{n=1}^{N} C^\dagger(x_n) |0 >_{el}$$

(40)

The overlap can be exactly calculated as shown in reference [6] to get,
\[ e_l < \{x_m\} | p, k > = \Phi_{(p,k)}(\{x_m\}) \prod_{n>m} (x_n - x_m)^{2k} e^{-\frac{\hbar}{2} \sum_{n=1}^{N} |x_n|^2} \]  

(41)

Where \( \Phi_{(p,k)}(\{x_n\}) \) is the Slater determinant of the first \( p \) Landau levels in a magnetic field \( B^* = \frac{1}{2kp+1} \). This is the Jain wave function, in which lowest Landau level projection is not made. We will therefore refer to the states defined in equation (33) as the unprojected Jain states. Note that here “unprojected” refers to the lowest Landau level and not to the gauge projection.

**IV. THE CURRENT ALGEBRA**

The electron current and density have been written down in terms of the composite fermion variables in equations (26) and (27). We defining the composite fermion current to be,

\[
\vec{J}^{CF}(x) = \frac{1}{2i}(\Psi^\dagger \vec{\nabla} \Psi - \vec{\nabla} \Psi^\dagger \Psi) + \vec{b}\rho(x)
\]  

(42)

\( \vec{b}(x) \) is the shifted Chern-Simons field,

\[
\vec{b}(x) = \vec{a}(x) + \vec{A}(x) - \vec{A}^*(x)
\]  

(43)

Where, \( \vec{\nabla} \times \vec{A}^*(x) = B^* \). The commutators between the components of the currents and the density can be computed. They form the following closed current algebra.

\[
[\rho(x), \rho(y)] = 0
\]  

(44)

\[
[J_i^{CF}(x), \rho(y)] = i\rho(x)\frac{\partial}{\partial y^i}\delta^2(x - y)
\]  

(45)

\[
[J_i^{CF}(x), J_j^{CF}(y)] = i\epsilon_{ij}B^* \rho(x)\delta^2(x - y) - iJ_i^{CF}(y)\frac{\partial}{\partial x^j}\delta^2(x - y) + iJ_j^{CF}(x)\frac{\partial}{\partial y^j}\delta^2(x - y)
\]  

(46)

In terms of the fourier transforms, the algebra can be written as,

\[
[\rho(q), \rho(q')] = 0
\]  

(47)

\[
[J_i^{CF}(q), \rho(q')] = q_i'\rho(q + q')
\]  

(48)

\[
[J_i^{CF}(q), J_j^{CF}(q')] = i\epsilon_{ij}B^* \rho(q + q') + q_i'J_j^{CF}(q + q') - q_j'J_i^{CF}(q + q')
\]  

(49)
In the hydrodynamic limit, we are interested in the small amplitude, long-wavelength fluctuations. So we put,
\[
\rho(q + q') = \bar{\rho}\delta^2(q + q') + \Delta \rho
\]
where $\bar{\rho} = \nu/2\pi$ and $\Delta \rho/\bar{\rho} \ll 1$. Thus in the hydrodynamic limit we replace $\rho$ by its average value and take only the leading terms in $q$. This is what we will mean by the term "hydrodynamic limit" from now. The algebra then simplifies considerably to give,
\[
[\rho(q), \rho(q')] = 0 \quad (51)
\]
\[
\left[ J_{i}^{\text{CF}}(q), \rho(q') \right] = q_i \bar{\rho}\delta^2(q + q') \quad (52)
\]
\[
\left[ J_{i}^{\text{CF}}(q), J_{j}^{\text{CF}}(q') \right] = i\epsilon_{ij} B^* \bar{\rho}\delta^2(q + q') \quad (53)
\]

If we define, $J^{\text{CF}}(x) = (J_{1}^{\text{CF}}(q) + iJ_{2}^{\text{CF}}(q'))/\sqrt{2B^*\bar{\rho}}$, then we have the composite fermion current components satisfying the oscillator algebra,
\[
\left[ J^{\text{CF}}(q), J^{\text{CF} \dagger}(q') \right] = \delta^2(q - q') \quad (54)
\]

In the hydrodynamic limit, the currents behave like the Chern-Simons gauge fields, they satisfy the same commutation relations.

V. THE HAMILTONIAN AND GROUND STATE

A. Current-current form of the hamiltonian

We will now show that the Hamiltonian can be expressed completely in terms of the currents and densities in the hydrodynamic limit. The interaction part of course is already expressed in terms of the charge densities. So we focus on the non-interacting part, $H_0$, which can be written as,
\[
H_0 = \int d^2 x \frac{1}{2}(\nabla \Psi \cdot \nabla \Psi + 2J^{\text{CF}} \cdot \vec{b} + \bar{\rho} \vec{b} \cdot \vec{b}) \quad (55)
\]
It is straightforward to compute the commutator of the first term in the above equation with the composite fermion current. In the hydrodynamic limit, we get,

\[
\left[ \int y \frac{1}{2} (\nabla \Psi, \nabla \Psi), J_i^{CF}(x) \right] = iB^* \epsilon_{ij} J_j^{CF}(x)
\]

\[
= \left[ \int y \frac{1}{2} \mathcal{J}_i^{CF}(y), J_i^{CF}(x) \right]
\]

(56)

Similarly, we can also show in the hydrodynamic limit that,

\[
\left[ \int y \frac{1}{2} (\nabla \Psi, \nabla \Psi), \rho(x) \right] = \left[ \int y \frac{1}{2} \mathcal{J}^{CF}(y), \mathcal{J}^{CF}(y), \rho(x) \right]
\]

(58)

Therefore, in the hydrodynamic limit, for the dynamics of the charge and current densities we can make the following replacement in the hamiltonian,

\[
\int y \frac{1}{2} (\nabla \Psi, \nabla \Psi) \rightarrow \int y \frac{1}{2} \mathcal{J}^{CF}(y), \mathcal{J}^{CF}(y)
\]

(59)

The hamiltonian can then be written as,

\[
H_0 = \int d^2x \frac{1}{2\rho} (\mathcal{J}^{CF} + \bar{\rho} \bar{b}).(\mathcal{J}^{CF} + \bar{\rho} \bar{b})
\]

(60)

\(\sqrt{1/(2kp + 1)}\), we can rewrite the hamiltonian up to a constant as,

\[
H_0 = \int d^2x (\cos \theta J^{CF\dagger} + \sin \theta b^\dagger)(\cos \theta J^{CF} + \sin \theta b)
\]

(61)

\[
\equiv \int d^2x J^{\dagger}(x)J(x)
\]

(62)

where, \(J \equiv (\cos \theta J^{CF} + \sin \theta b)\), \(\cos \theta \equiv \sqrt{1/(2kp + 1)}\) and \(\sin \theta \equiv \sqrt{2kp/(2kp + 1)}\).

**B. The ground state**

The ground states of \(H_0\) (lowest Landau level states) would be those that satisfy,

\[
J(q)\mid GS \geq 0
\]

(63)
It can be shown that,

\[ [J_{CF}(q), \Psi_{nl}] = \sqrt{n} \Psi_{n-1,l+1} + o(q) \]  (64)

It then follows that, to leading order in \( q \),

\[ J_{CF}(q)|p, k >_{CF} = 0 \]  (65)

since we also have,

\[ b(q)|p, k >_{CS} = 0 \]  (66)

we find that the unprojected Jain states are ground states of \( H_0 \). All zero energy eigenstates of \( H_0 \) should be lowest Landau level states, whereas the unprojected Jain states are not. However, we should remember that our results are valid only to leading order in \( q \). So these results imply that the non-lowest Landau level component of the unprojected Jain states does not contribute to matrix elements of the charge and current density operators to leading order in \( q \). This conclusion will be further bolstered by the consistency of the results in the next section.

VI. COLLECTIVE EXCITATIONS

A. Inter-Landau level excitations

\( J \)From equations (61,62), it follows that the states,

\[ |q; p, k > \equiv J^\dagger(q)|p, k > \]  (67)

are eigenstates of the hamiltonian with energy = 1 (in our units, \( \hbar \omega_c \)). These are therefore, magnetoplasmons corresponding to inter-Landau level particle-hole pairs. This is also indicated by equation (64).
B. Lowest Landau level excitations

Lowest Landau level excitations will be created by operators that commute with $H_0$. The following combination of $J^{CF}$ and $b$ that is "orthogonal" to $J$ does just that.

$$\tilde{J}(x) \equiv \kappa (-\sin \theta J^{CF}(x) + \cos \theta b(x))$$

(68)

Using equations (27), (42) and choosing $\kappa$ appropriately, we have,

$$\vec{\nabla} \times \vec{\tilde{J}} = \frac{1}{4\pi k} \vec{b}(x) - \vec{\tilde{J}}(x)$$

(69)

$\vec{\tilde{J}}(x)$ is not gauge invariant and thus is not a physical operator. But $\vec{\nabla} \times \vec{\tilde{J}}$ is gauge invariant and hence a physical operator. In the physical subspace, we can replace $\frac{1}{4\pi k} \vec{\nabla} \times \vec{b}$ by $\Delta \rho$. We then have,

$$\vec{\nabla} \times \vec{\tilde{J}} = \Delta \rho - \vec{\nabla} \times \vec{J}$$

(70)

Thus we have written $\vec{\nabla} \times \vec{\tilde{J}}$ in terms of the electronic charge and current densities. To further, discover the meaning of this quantity, we compute and find the commutators at two different momenta to be,

$$\left[i\vec{q} \times \vec{\tilde{J}}(q), i\vec{q}' \times \vec{\tilde{J}}(q')\right] = i\vec{q} \times \vec{q}' \left((\vec{q} + \vec{q}') \times \vec{\tilde{J}}(q + q')\right)$$

(71)

Note, that to get this result, one must use the exact form of the current algebra as in equations (47), (48) and (49) and use the hydrodynamic approximation only after the commutators are computed. Thus, $\vec{\nabla} \times \vec{J}$ obeys the GMP algebra [7] to leading order in $q$. It is therefore natural to identify it with the charge density operator, projected to the lowest Landau level. This can also be directly derived from equation (70). To do so, consider first the single particle charge and current density operators.

$$\hat{\rho}(x) = \delta^2(\hat{x} - x)$$

(72)

$$\hat{j}_i(x) = \frac{1}{2}(\hat{\Pi}_i \hat{\rho}(x) + \hat{\rho}(x) \hat{\Pi}_i)$$

(73)

where, $\hat{\Pi}_i = \hat{p}_i - A_i(\hat{x})$. Taking fourier transforms,
\[\hat{\rho}(q) = e^{i \vec{q} \cdot \vec{x}} \quad (74)\]
\[\hat{j}_i(q) = \frac{1}{2} (\hat{\Pi}_i \hat{\rho}(q) + \hat{\rho}(q) \hat{\Pi}_i) \quad (75)\]

The lowest Landau level projected density operator is obtained by expressing \( \hat{x}_i \) in terms of \( \hat{\Pi}_i \) and \( \hat{\bar{\Pi}}_i \). Where, \( \hat{\bar{\Pi}}_i = \hat{p}_i + A_i(\hat{x}) \).

\[\hat{\rho}(q) = e^{i \vec{q} \times \vec{\Pi}} \quad (76)\]

Expanding equations (74), (75) and (76) to leading order in \( q \) gives us,

\[\hat{\rho}(q) = \hat{\rho}(q) + i \vec{q} \times \vec{j}(q) \quad (77)\]

Since we have,

\[\rho(q) = \int_{x_1, x_2} \Psi^\dagger(x_1) < x_1 | \hat{\rho}(q) | x_2 > \Psi(x_2) \quad (78)\]
\[J_i(q) = \int_{x_1, x_2} \Psi^\dagger(x_1) < x_1 | \hat{j}_i(q) | x_2 > \Psi(x_2) \quad (79)\]
\[\hat{\rho}(q) = \int_{x_1, x_2} \Psi^\dagger(x_1) < x_1 | \hat{\rho}(q) | x_2 > \Psi(x_2) \quad (80)\]

it follows that,

\[\vec{\nabla} \times \vec{j} = \Delta \hat{\rho} \quad (82)\]

It is therefore no accident that it satisfies the GMP algebra to leading order in \( q \). It will therefore, create the lowest Landau level collective excitations.

Note however, that if we had taken the curl of equation (69) and had computed the commutators without using the constraint, then the GMP algebra would not have been obeyed in general but only in the physical subspace. Thus if in an approximation scheme, the constraint is not imposed exactly, as in reference [4], it is better to use the expression in equation (69) which is what is effectively done in reference [4]. However, if the constraint is imposed exactly, then of course either expression could be used.
VII. SUMMARY

As the first step in an attempt to develop a systematic analytic long-wavelength approximation for FQHE systems, we have studied the current algebra of the system in the hydrodynamic limit. Namely, in the limit of small amplitude, long-wavelength fluctuations. We develop a fully field theoretic Hamiltonian formalism for the Composite fermion theory following the ideas in reference [4]. In the hydrodynamical limit, we show that the components of the current density satisfy very simple commutation relations, the same as those satisfied by the components of the Chern-Simons fields. The Hamiltonian can be expressed in the current-current form and the unprojected Jain states are shown to be ground states. Operators that create inter-Landau level and lowest Landau level collective excitations are naturally identified using the current algebra. We show that the operator creating lowest Landau level collective excitations is precisely the projected charge density operator and that it satisfies the GMP algebra to leading order in $q$.

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