Superfluid states with moving condensate in nuclear matter

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(Dated: March 30, 2022)

Superfluid states of symmetric nuclear matter with finite total momentum of Cooper pairs (nuclear LOFF phase) are studied with the use of Fermi-liquid theory in the model with Skyrme effective forces. It is considered the case of four-fold splitting of the excitation spectrum due to finite superfluid momentum and coupling of \( T = 0 \) and \( T = 1 \) pairing channels. It has been shown that at zero temperature the energy gap in triplet-singlet (TS) pairing channel (in spin and isospin spaces) for the SkM* force demonstrates double-valued behavior as a function of superfluid momentum. As a consequence, the phase transition at the critical superfluid momentum from the LOFF phase to the normal state will be of a first order. Behavior of the energy gap as a function of density for TS pairing channel under increase of superfluid momentum changes from one-valued to universal two-valued. It is shown that two-gap solutions, describing superposition of states with singlet–triplet (ST) and TS pairing of nucleons appear as a result of branching from one-gap ST solution. Comparison of the free energies shows that the state with TS pairing of nucleons is thermodynamically most preferable.

PACS numbers: 21.65.+f; 21.30.Fe; 71.10.Ay

In this report we shall study superfluid states of nuclear matter with nonzero total momentum of Cooper pairs. First the states with moving condensate were considered in Refs. [1, 2] with respect to metallic superconductors. In this case the superconducting condensate has a spatially periodic structure. Corresponding phase is called the Larkin–Ovchinnikov–Fulde–Ferrel (LOFF) phase. Recent upsurge of interest to the LOFF phase is natural to develop some kind of a phenomenological theory, where a phenomenological pairing interaction is employed. As such a theory, we shall use the Fermi-liquid (FL) approach [3]. In the Fermi-liquid model the normal and anomalous FL interaction amplitudes are taken into account on an equal footing [4]. This will allow us to consider consistently influence of the FL amplitudes on superfluid properties of the nuclear LOFF phase. Besides, as a potential of NN interaction we choose the density dependent Skyrme effective forces, used earlier in a number of contexts for description of superfluid properties both finite nuclei [5, 6] and infinite nuclear matter [7, 8].

Superfluid states of nuclear matter are described by the normal \( f_{\kappa_1\kappa_2} = \text{Tr} \varphi a_{\kappa_1}^\dagger a_{\kappa_2} \) and anomalous \( g_{\kappa_1\kappa_2} = \text{Tr} \varphi a_{\kappa_1} a_{\kappa_2} \) distribution functions of nucleons (\( \kappa \equiv (p, \sigma, \tau) \), \( p \) is momentum, \( \sigma(\tau) \)) is the projection of spin (isospin) on the third axis, \( \varphi \) is the density matrix of the system. We shall study two-gap superfluid states in symmetric nuclear matter, corresponding to superposition of states with total spin \( S \) and isospin \( T \) of a pair \( S = 1, T = 0 \) (triplet-singlet (TS) pairing) and \( S = 0, T = 1 \) (singlet-triplet (ST) pairing) with the projections \( S_z = T_z = 0 \) (TS–ST states). In this case, assuming that a condensate moves with the finite momentum \( q \), the normal \( f \) and anomalous \( g \) distribution functions read [1, 2, 3]:

\[
\begin{align*}
f_{\kappa_1\kappa_2} &= f_{00}(\mathbf{p}_1)(\sigma_0\tau_0)_{\kappa_1\kappa_2}\delta_{p_1,p_2}, \\
g_{\kappa_1\kappa_2} &= (g_{00}(\mathbf{p}_1)\sigma_3\sigma_2\tau_2 + g_{03}(\mathbf{p}_1)\sigma_2\tau_3\tau_2)_{\kappa_1\kappa_2}\delta_{p_1,-p_2+q},
\end{align*}
\]

where \( \sigma_i \) and \( \tau_h \) are the Pauli matrices in spin and isospin spaces, respectively. For the energy functional, being invariant with respect to rotations in spin and isospin spaces, the quasiparticle energy and the order parameter have the similar structure:

\[
\begin{align*}
\varepsilon_{\kappa_1\kappa_2} &= \varepsilon_{00}(\mathbf{p}_1)(\sigma_0\tau_0)_{\kappa_1\kappa_2}\delta_{p_1,p_2}, \\
\Delta_{\kappa_1\kappa_2} &= (\Delta_{00}(\mathbf{p}_1)\sigma_3\sigma_2\tau_2 + \Delta_{03}(\mathbf{p}_1)\sigma_2\tau_3\tau_2)_{\kappa_1\kappa_2}\delta_{p_1,-p_2+q},
\end{align*}
\]

In terms of the 

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\end{align*}
\]
If to take into account the antisymmetry properties $\Delta^r = -\Delta$, $q^r = -q$ and to set $p_1 = p + q/2$, $p_2 = -p + q/2$ ($q$ is total momentum of a pair, $p$ is momentum of one of nucleons in the center of mass frame of a pair), then we obtain

$$
\Delta_{30}(p + \frac{q}{2}) = \Delta_{30}(-p + \frac{q}{2}) \equiv \Delta_{30}(p, q), \quad (5)
$$

$$
\Delta_{03}(p + \frac{q}{2}) = \Delta_{03}(-p + \frac{q}{2}) \equiv \Delta_{03}(p, q),
$$

and analogous relationships hold for the functions $g_{30}, g_{03}$. Further we shall write the self–consistent equations for the quantities $\Delta_{30}(p, q)$, $\Delta_{03}(p, q)$. Using the minimum principle of the thermodynamic potential and procedure of block diagonalization [3], one can express evidently the distribution functions $f_{00}, g_{30}, g_{03}$ in terms of the quantities $\varepsilon$ and $\Delta$:

$$
f_{00} = \frac{1}{4} \left[ (1 + n_+ - n_-) - \frac{\varepsilon}{E_+} (1 - n_+ - n_-) \right]^2 + (1 + n_+ - n_-) - \frac{\varepsilon}{E_-} (1 - n_+ - n_-), \quad (6)
$$

$$
g_{30} = -\frac{\Delta_+}{4E_+} (1 - n_+ - n_-) - \frac{\Delta_-}{4E_-} (1 - n_+ - n_-), \quad (7)
$$

$$
g_{03} = -\frac{\Delta_+}{4E_+} (1 - n_+ - n_-) - \frac{\Delta_-}{4E_-} (1 - n_+ - n_-). \quad (8)
$$

Here $f_{00} = f_{00}(p + \frac{q}{2})$, $n_\pm = n(\pm p + \frac{q}{2})$ and

$$
E_{\pm} = \sqrt{\xi^2 + |\Delta_{\pm}|^2}, \quad \Delta_{\pm} = \Delta_{30} \pm \Delta_{03},
$$

$$
\xi_+(p + \frac{q}{2}) = \frac{1}{2} \left( \xi(p + \frac{q}{2}) + \xi(-p + \frac{q}{2}) \right),
$$

$$
\xi_0(p + \frac{q}{2}) = \frac{1}{2} \left( \xi(p + \frac{q}{2}) - \xi(-p + \frac{q}{2}) \right),
$$

$$
\xi(p) = \varepsilon_{00}(p) - \mu^0, \quad n_\pm = \{ \exp Y_0(\xi_\pm + E_{\pm}) + 1 \}^{-1},
$$

$\mu^0$ is chemical potential, which should be determined from the normalization condition

$$
\frac{4}{3} \sum_p f_{00}(p + \frac{q}{2}) = \varrho,
$$

$\varrho$ is density of symmetric nuclear matter. As follows from the structure of the distribution functions $f_{00}, g_{30}, g_{03}$, the quantity

$$
\omega_{\pm, \pm} = \xi_\pm(\pm p + \frac{q}{2}) + E_{\pm},
$$

being the exponent in Fermi distribution functions $n_\pm(\pm p + \frac{q}{2})$, plays the role of the quasiparticle excitation spectrum. In the considering case the spectrum is four–fold split due to 1) finite superfluid momentum ($q \neq 0$), 2) coupling of TS and ST pairing channels ($\Delta_{30} \neq 0, \Delta_{03} \neq 0$).

To obtain the closed system of equations for the quantities $\Delta$ and $\xi$, it is necessary to set the energy functional of the system. In the case of symmetric nuclear matter with TS and ST pairings of nucleons the energy functional is characterized by one normal $U_0$ and two anomalous $V_1, V_2$ FL amplitudes [3, 13]. Then one can obtain the self–consistent equations in the form

$$
\xi(p) = \varepsilon_{00}^0(p) - \mu^0 + \xi_{00}(p), \quad \varepsilon_{00}^0(p) = \frac{p^2}{2m_0}, \quad (10)
$$

$$
\xi_{00}(p) = \frac{1}{2V} \sum_{p'} U_0(k)f_{00}(p'), \quad k = \frac{p - p'}{2},
$$

$$
\Delta_{30}(p, q) = \frac{1}{V} \sum_{p'} V_1(p, p')g_{30}(p', q), \quad (11)
$$

$$
\Delta_{03}(p, q) = \frac{1}{V} \sum_{p'} V_2(p, p')g_{03}(p', q), \quad (12)
$$

where $m_0$ being the mass of a bare nucleon.

Further for obtaining numerical results we shall use the Skyrme effective interaction. In the case of Skyrme forces the normal and anomalous FL amplitudes read [3, 14] potentials.

$$
U_0(k) = 6t_0 + 6t_3 g^3 + \frac{1}{\hbar^2} [6t_1 + 2t_2 (5 + 4x^2)] k^2 \equiv d_0 + e_0 k^2
$$

$$
V_{1,2}(p, p') = t_0 (1 \pm x_0) + \frac{1}{6} t_3 g^3 (1 \pm x_3) + \frac{1}{2\hbar^2} t_4 (1 \pm x_1) (p^2 + p'^2),
$$

where $t_i, x_i, \beta$ are some phenomenological constants, characterizing the given parametrization of Skyrme forces (we shall use the SkP [10], SkM* [14] potentials). With account of Eq. (13) we obtain

$$
\xi_\pm(p + \frac{q}{2}) = \frac{p^2}{2m_1} + \frac{\rho^2}{8m_2} - \mu, \quad (15)
$$

where

$$
\frac{\hbar^2}{2m_{1,2}} = \frac{\hbar^2}{2m_0} \pm \frac{\rho}{16} \left[ 3t_1 + t_2 (5 + 4x_2) \right], \quad (16)
$$

and the effective chemical potential $\mu$ should be determined from the normalization condition [3]. Besides, expression for the quantity $\xi_\pm$ reads

$$
\xi_\pm(p + \frac{q}{2}) = \frac{pq}{2m_0} - \frac{e_0}{4} \sum_{p'} f_{00}(p' + \frac{q}{2}) pp', \quad (17)
$$

The normal distribution function $f_{00}$ in turn depends on the quantity $\xi_\pm$ and, hence, expression (17) represents an equation for determining the quantity $\xi_\pm$. Since the second term in Eq. (17) is proportional to $\frac{pq}{2m_0}$, solution of Eq. (17) should be found in the form $\xi_\pm(p + \frac{q}{2}) = \frac{pq}{2m_0}$, where $m^*$ is some effective mass.
effective mass $m^*$ as
\begin{equation}
\Delta_{10} = - \frac{1}{4V} \sum_{p'} V_1(p, p') \left\{ \frac{\Delta_+}{E_+} \left( \tanh \frac{\omega'_+}{2T} \right) \\
+ \tanh \frac{\omega'_-}{2T} \right\},
\end{equation}
\begin{equation}
\Delta_{03} = - \frac{1}{4V} \sum_{p'} V_2(p, p') \left\{ \frac{\Delta_+}{E_+} \left( \tanh \frac{\omega'_+}{2T} \right) \\
+ \tanh \frac{\omega'_-}{2T} \right\},
\end{equation}
\begin{equation}
\frac{1}{V} \sum_p \left\{ 2 - \frac{\xi_s}{2E_+} \left( \tanh \frac{\omega'_+}{2T} + \tanh \frac{\omega'_-}{2T} \right) \\
- \frac{\xi_s}{2E_-} \left( \tanh \frac{\omega'_-}{2T} + \tanh \frac{\omega'_-}{2T} \right) \right\} = \theta,
\end{equation}
\begin{equation}
\frac{pq}{m_0} + \frac{c_0}{16} \sum_{p'} p p' \left\{ \left( \tanh \frac{\omega'_+}{2T} - \tanh \frac{\omega'_-}{2T} \right) \\
+ \left( \tanh \frac{\omega'_-}{2T} - \tanh \frac{\omega'_-}{2T} \right) \right\} = \frac{pq}{m^*}
\end{equation}
Here
\begin{equation}
\Delta_{\pm} = \Delta_{\pm}(p', q), \quad \omega'_{\pm} = \frac{p'q}{2m^*} + E_{\pm}(p', q)
\end{equation}
Eqs. (18)–(21) describe two–gap superfluid states of symmetric nuclear matter with moving condensate and contain one–gap solutions with $\Delta_{30} \neq 0, \Delta_{03} \equiv 0$ (TS pairing) and $\Delta_{30} \equiv 0, \Delta_{03} \neq 0$ (ST pairing) as some particular cases. We shall analyze Eqs. (18)–(21) using the simplifying assumption, that FL amplitudes $V_1, V_2$ are not equal to zero only in a narrow layer near the Fermi surface: $|\xi_s|/\theta, \theta \ll \varepsilon_F$ (further we set $\theta = 0.1 \varepsilon_F$).

First we shall find the dependence of the order parameters $\Delta_{30}(p = p_F), \Delta_{03}(p = p_F)$ from the superfluid momentum at zero temperature. We begin our analysis with finding one–gap solutions of the self–consistent equations. Results of numerical determination of the energy gap are presented in the Fig. (a). It is seen, that a general tendency is quite clear: at low superfluid momenta the energy gap retains its constant value and then rapidly decreases and vanishes at some critical point. This means, that one–gap superfluid states with moving condensate will disappear at large enough superfluid momenta. However, for TS pairing of nucleons, interacting via SkM* effective potential, the phase curve has an interesting peculiarity, namely, the energy gap demonstrates nontrivial double–valued behavior in the region, where it sharply falls. Such behavior differs from the ordinary one–valued behavior of the energy gap, as, e.g., in the case of TS pairing and SkP effective interaction. To understand this difference, we have determined the effective mass $m^*$ as a function of superfluid momentum, Fig. (b).

For the SkP potential the mass $m^*$ is equal to the bare mass $m_0$ of a nucleon practically for all superfluid momenta. Unlike to this behavior, the effective mass $m^*$ for the SkM* potential rapidly decreases close to the region of phase transition to the normal state. Since the mass $m^*$ enters into Eqs. (18)–(21) only through the ratio $q/m^*$, descent of the effective mass leads to the increase of the effective shift between the centers of proton and neutron Fermi spheres, on which the paired proton and neutron lie. This gives the possibility to the appearance of the second solution for the energy gap. According to Eq. (14), the parameter $c_0$ of the normal FL amplitude $U_0$ determines the sign before the sum in the r.h.s. of Eq. (21) and, hence, determines whether the mass $m^*$ will be greater (if $c_0 < 0$) or less (if $c_0 > 0$), than the bare mass $m_0$ (if corresponding sum is nonzero). For the SkM* potential $c_0 > 0$ while for the SkP potential $c_0 < 0$, that explains the difference in the behavior of the effective mass for these two forces. However, diminishing behavior of the effective mass $m^*$ does not guarantee that the energy gap will have two–valued behavior as a func-

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Energy gap (top) and effective mass $m^*$ (bottom) vs. superfluid momentum for different types of pairing and Skyrme forces at $\theta = 0.03$ fm$^{-3}$ and zero temperature.}
\end{figure}
tions of self–consistent equations at fixed superfluid momentum are presented in Fig. 3. As one can see, superfluidity with finite superfluid momentum exists in finite density region \((q_1(q), q_2(q))\), excluding some vicinity of the point \(q = 0\) (for TS pairing the left point \(q_1(q)\) is very close to \(q = 0\)). The most important peculiarity, i.e., the double–valued behavior of the energy gap for TS pairing in the case of SkM* potential is preserved for the given ratio \(q/p_F\). For other types of pairing and Skyrme forces, considered earlier (including the SkP potential, which is not shown in Fig. 3), the order parameter for one–gap solutions has one–valued behavior. In the case of two–gap solutions the mechanism of their appearance is similar to the considered above, i.e., it is branching from one–gap ST solution.

However, in general case behavior of the energy gap as a function of density is more complicated. In Fig. 4 we plot the dependence of the energy gap in the case of TS pairing and SkM* interaction for the set of fixed values of the ratio \(q/p_F\). It is seen, that behavior of the phase curves may be one–valued or two–valued, that depends on the value of the ratio \(q/p_F\). At small enough superfluid momentum (e.g., for \(q = 0.009p_F\)) the gap behaves as a one–valued function of density; for the ratios \(q/p_F\), larger than some critical value \(q_1(p_F)\), the gap has two–valued behavior in the region close to the right critical point \(q_2(q)\). Further increase of the ratio \(q/p_F\) leads to formation of the second part of the phase curve with double–valued behavior in the region close to the left critical point \(q_1(q)\) (e.g., for \(q = 0.1p_F\)). When \(q\) increases, the regions with double–valued behavior of the gap begin to approach and at \(q = q_c\) \((q_c \approx 0.106p_F)\) it takes place contiguity of the regions with two solutions. For \(q > q_c\) the phase curves are separated from the density axis and turn into the closed oval curves. Under further increase of \(q\) the dimension of the oval curves is reduced and at some

FIG. 3: Order parameters \(\Delta_{30}, \Delta_{03}\) vs. density for SkM* force at \(q/p_F = 0.04\) and zero temperature.

![Graph](image)

FIG. 2: Order parameters \(\Delta_{30}, \Delta_{03}\) vs. superfluid momentum for SkM* force at \(q = 0.03\) fm\(^{-3}\) and zero temperature.

are notations for the dependencies of TS and ST order parameters in the TS–ST solution of the self–consistent equations. As seen from Fig. 2, TS–ST solutions appear as a result of branching from one–gap ST solution (in the branching point \(\Delta_{30} = 0, \Delta_{03} = \Delta_{30}^{st}, \Delta_{03}^{st}\) being one–gap ST solution). Note that the self–consistent equations have two–gap solutions in the case of SkM* potential, but have no such solutions for the SkP potential. As clarified in the Ref. [2], for the existence of TS–ST solutions it is necessary that TS and ST coupling constants must be of the same order of magnitude. However, this condition is broken for the SkP potential, where TS coupling constant is much larger than ST one.

Since we use density dependent effective interaction, it allows us to study also the dependence of the order parameters from density of nuclear matter. The results of numerical determination of one–gap and two–gap solutions of self–consistent equations at fixed superfluid momentum are presented in Fig. 3. As one can see, superfluidity with finite superfluid momentum exists in finite density region \((q_1(q), q_2(q))\), excluding some vicinity of the point \(q = 0\) (for TS pairing the left point \(q_1(q)\) is very close to \(q = 0\)). The most important peculiarity, i.e., the double–valued behavior of the energy gap for TS pairing in the case of SkM* potential is preserved for the given ratio \(q/p_F\). For other types of pairing and Skyrme forces, considered earlier (including the SkP potential, which is not shown in Fig. 3), the order parameter for one–gap solutions has one–valued behavior. In the case of two–gap solutions the mechanism of their appearance is similar to the considered above, i.e., it is branching from one–gap ST solution.

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The phase curve $\Delta$ valued behavior, for $q$ parts with double–valued behavior and for channel, then the branch with larger size of the gap will double–valued behavior of the energy gap in TS pairing free energies of two different branches, corresponding to partition for the thermodynamic stability. If to compare the tum or density much smaller then for the case of ST and and density, it is necessary to check, which solution is thermodynamically favorable. Calculations show that due to the large size of the gap in TS pairing channel the free energy gap as a function of momentum or density changes from one–valued to universal two–valued character (until superfluidity disappears at some critical momentum). Two–gap solutions of self–consistent equations, corresponding to the case when both TS and ST order parameters are not equal to zero, appear as a result of branching from one–gap ST solution. Calculation of the free energy shows that TS superfluid state is thermodynamically most preferable state. In the case of double–valued behavior the gap changes in the critical point by a jump and, hence, the phase transition from the LOFF phase to the normal state will be of a first order (in superfluid momentum or density). Since the possible two–valued behavior of the gap will be preserved for small asymmetry, this will be also true for weakly asymmetric nuclear matter, that differs from the picture of a second order phase transition, considered in [10].

Since we have a few solutions of self–consistent equations, it is necessary to check, which solution is thermodynamically favorable. Calculations show that due to the large size of the gap in TS pairing channel the free energy of the corresponding state as a function of momentum or density much smaller then for the case of ST and TS–ST pairing. Hence, TS superfluid state wins competition for the thermodynamic stability. If to compare the free energies of two different branches, corresponding to double–valued behavior of the energy gap in TS pairing channel, then the branch with larger size of the gap will be thermodynamically favorable.

In summary, we studied superfluidity of symmetric nuclear matter with moving condensate in the FL model with density dependent Skyrme effective interaction (SkM*, SkP forces). It has been considered the case, where the quasiparticle excitation spectrum is four–fold split due to finite superfluid momentum and coupling of $T = 0$ and $T = 1$ pairing channels. Apart from the renormalization of the chemical potential and bare mass of a nucleon, taking into account of the normal FL amplitude leads to appearance of additional effective mass $m^*$ in the linear on superfluid momentum term in single particle energy. It is shown that at zero temperature the energy gap in TS pairing channel for SkM* potential demonstrates two–valued behavior as a function of superfluid momentum. This is caused by the decreasing behavior of the effective mass $m^*$ close to the region of the phase transition to the normal state and strong enough interaction in TS pairing channel. The behavior of the energy gap as a function of density in TS pairing channel in general case is more complicated and under increase of superfluid momentum it changes from one–valued to universal two–valued character (until superfluidity disappears at some critical momentum). Two–gap solutions of self–consistent equations, corresponding to the case when both TS and ST order parameters are not equal to zero, appear as a result of branching from one–gap ST solution.

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\[ \Delta = \Delta_m \text{ (MeV)} \]

FIG. 4: Energy gap vs. density in the case of TS pairing and SkM* force for different values of the ratio $q/p_F$. 

$q = q_m$ the oval curves shrink to a point ($q_m \approx 0.108 p_F$). For the values $q > q_m$ TS superfluidity of nuclear matter vanishes. Thus, in the range $0 < q < q_1$ the gap is one–valued function of density, in the range $q_1 < q < q_2$ the phase curve $\Delta_3 = \Delta_3(q)$ has one part with double–valued behavior, for $q_2 < q < q_c$ it contains two distinct parts with double–valued behavior and for $q_c < q < q_m$ the gap has a universal double–valued behavior.

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