Decision Conflict, Logit, and the Outside Option

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Abstract

Decision makers often opt for the deferral outside option when they find it difficult to make an active choice. Contrary to existing logit models with an outside option where the latter is assigned a fixed value exogenously, this paper introduces and analyzes a class of logit models where that option’s value is menu-dependent, may be determined endogenously, and could be interpreted as proxying the varying degree of decision difficulty at different menus. We focus on the power logit special class of these models. We show that these predict some observed choice-deferral effects that are caused by hard decisions, including non-monotonic “roller-coaster” choice-overload phenomena that are regulated by the presence or absence of a clearly dominant feasible alternative. We illustrate the usability, novel insights and explanatory gains of the proposed framework for empirical discrete choice analysis and theoretical modelling of imperfectly competitive markets in the presence of potentially indecisive consumers.

Keywords: decision difficulty; quadratic logit; power logit; outside option; choice deferral; estimation.

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1 Introduction

It is a well-established fact that people often opt for the choice-avoidance/deferral outside option when they find it hard to compare the active-choice alternatives available to them, even when all these alternatives are individually considered “good enough” to be chosen and are paid attention to. Real-world examples of such behaviour include: (i) employees who operated within an “active decision” pension-savings environment and did not sign up for one of the plans that were available to them within, say, a day, week or month of first notice, possibly even opting for indefinite non-enrolment;\(^1\) (ii) patients who, instead of choosing “immediately” one of the active treatments that were recommended to them against a medical condition, delayed making such a choice -often at a health cost- due to “facing a treatment dilemma”;\(^2\) (iii) doctors who were willing to prescribe the single available drug to treat a medical condition but were not prepared to prescribe anything when they had to decide from the expanded set that contained one more drug, because “the difficulty in deciding between the two medications led some physicians to recommend not starting either” (Redelmeier and Shafir, 1995).

Motivated by the relevance of opting-out decisions for understanding preferences and explaining behaviour, our goal in this paper is to model choice in the presence of a choice-avoidance/deferral outside option within a stochastic choice framework in ways that deviate as little as possible from existing well-understood modelling practices and, at the same time, make predictions that are in line with some findings from the empirical/experimental literature and evade existing models. We pursue this by extending in disciplined ways the foundational Luce (1959)/logit model and its econometric specification pioneered by McFadden (1973). Specifically, we propose and study the class of decision-conflict logit models which, in their most general form, are a straightforward but so far unexplored extension of the logit with an outside option that assign a menu-dependent utility to that option while retaining the menu-invariance assumption on all active-choice alternatives. The relative value of outside-option utility at a menu in turn determines the probability of avoiding/deferring choice and can be interpreted as proxying decision difficulty.

Despite its simplicity, this baseline general model can act as the starting point for many richly structured special cases. We introduce and focus on the broad class of power logit models that are examples of such cases where decision difficulty depends in intuitive ways on the logit values of all active-choice alternatives. In these models, decision difficulty could be thought of as driven by the agent’s noisy resampling of the menu’s elements. More specifically, in the quadratic logit special case of this class of models such resampling takes the form of the

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\(^1\)Such behaviour is documented in Carroll, Choi, Laibson, Madrian, and Metrick (2009), for example.

\(^2\)See Knops, Ubbink, Legemate, Stalpers, and Bossuyt (2013, p. 78).
choice probability of a market alternative emerging as the product of two logit probabilities according to a single value function/criterion. Intuitively, the agent is more likely to choose an active-choice alternative if and only if its value realizations according to this criterion are much larger than those of everything else feasible across both rounds of sampling. Conversely, the agent is more likely to avoid/defer choice when no alternative achieves such unanimous clear dominance. This model could therefore be thought of as capturing a hesitant decision maker who behaves as if they used an objective criterion to compare alternatives (e.g. sum or multiply each option’s values across all relevant attributes) but is aware that their subjective evaluation according to this objective criterion may be imperfect, possibly due to cognitive limitations, thereby leading them to performing this task twice. To our knowledge, this model and its power-logit generalization are the first to provide a theory where the no-choice outside option is feasible and has an endogenously determined, menu-dependent utility.

We further show that these structured models predict the following empirical phenomena that various studies in cognitive and consumer psychology have documented about decisions that allow agents to avoid/delay making an active choice:

(i) The dominance-driven non-monotonic relation between menu expansion and the tendency to opt out, which we refer to as the “roller-coaster” choice-overload effect. This has implications for the interpretation and policy responses to so-called “too-much-choice” phenomena.

(ii) “Relative-desirability” effects, whereby holding constant the total value in a menu while increasing the value differences between the menu’s alternatives increases the probability of an active choice.

Finally, we illustrate the applicability of our analysis both in theoretical and empirical settings. To this end, we first show how the classic assumptions and argument that underpin the discrete-choice formulation of the logit without an outside option (McFadden, 1973) must be modified and extended in order for both the quadratic logit and the more general power logit models to admit a similar discrete-choice formulation and be taken to the data for maximum-likelihood estimation of their respective parameters. We then show the potential fruitfulness of such analyses by estimating both the quadratic and power logit models on the deferral-permitting discrete-choice data with film decisions from the survey experiment of Bhatia and Mullett (2016), using the participants’ subjective ratings of the different films as the explanatory variable. To assess the added value of the hereby proposed models on these data, we use standard criteria to evaluate their goodness of fit and compare them to those of baseline logit models with a fixed or a random outside option. Our analysis suggests that both the power
and quadratic logit often perform better compared to either version of the baseline logit under these performance criteria, particularly in those situations where theory suggests they would do so. Hence, they could be considered in the analysis of similar datasets whenever the researcher suspects that the observed opting-out/deferring behaviour might be due to decision difficulty rather than to the relative unattractiveness of the available active-choice alternatives.

In our second application we analyse a duopolistic model where firms simultaneously compete in price and quality under logit and power-logit demand. We derive intuitive closed-form solutions for all equilibrium quantities in the model. A key feature of the equilibrium is that, as the power parameter capturing consumers’ decision difficulty/hesitation increases, both firms increase their products’ quality/price ratio and see their profits decreased, both because of the reduced profit margins and the lower share of consumers making buying any product. Intuitively, this is driven by each firm increasing its quality/price ratio in an effort to reduce the consumer’s decision difficulty and mitigate the risk of losing them to the rival firm or driving them out of the market altogether.

As far as the axiomatic analysis is concerned, we note that at the heart of the structure of all models that we analyse in this paper is the so far unexplored version of the Luce/Independence of Irrelevant Alternatives axiom where its odds-invariance restrictions are required to apply over pairs of active-choice alternatives but not on pairs that involve such an alternative and the outside option. The quadratic-logit special case, moreover, is characterized by means of novel and interpretable additional axioms that impose intuitive restrictions on the odds of deferring/not deferring at binary menus and/or their interplay with the relative choice probabilities of market alternatives at those menus. However, although we study its properties and predictions in some detail, we do not provide a characterization of the general class of power logit models, leaving it as an open problem.

The remaining parts of the paper are organized as follows. Section 2 introduces the notation and the general formulation of the model that will be studied in more structured ways in the sequel. Section 3 introduces the power-logit class of models and its quadratic-logit special case, studies their general properties, and provides an axiomatic characterization of the latter special case. Sections 4, 5 and 6, respectively, illustrate the models’ descriptive relevance and applicability in empirical and theoretical environments, as remarked above. Section 7 places the contribution to the existing literature and Section 8 concludes. Unless otherwise noted, all proofs appear in the Appendix.
2 Preliminaries

Let $X$ be the grand choice set of finitely many active-choice alternatives, with generic elements $a, b \in X$. Let $\mathcal{M} := \{A : \emptyset \neq A \subseteq X\}$ be the collection of all menus of such alternatives, and let $\mathcal{B}$ be its sub-collection that comprises all binary menus. The outside option is denoted by $o \notin X$. A random free-choice model on $X$ is a function $\rho : X \times \mathcal{M} \to \mathbb{R}_+$ such that $\rho(a, A) \in [0, 1]$ for all $A \in \mathcal{M}$ and all $a \in A$; $\rho(a, A) = 0$ for all $A \in \mathcal{M}$ and all $a \notin A$; and $\sum_{a \in A} \rho(a, A) \leq 1$, where $\rho(o, A) := 1 - \sum_{a \in A} \rho(a, A)$ is the probability of choosing the -always feasible– outside option at menu $A$. To simplify notation, for $A, B \in \mathcal{M}$ with $B \subseteq A$ we write $\rho(B, A) := \sum_{b \in B} \rho(b, A)$.

We start by introducing the logit with a general outside option as the model that comprises value functions $u : X \to \mathbb{R}^{++}$ and $D : \mathcal{M} \to \mathbb{R}_+$ such that, for every menu $A$ and alternative $a \in A$,

$$\rho(a, A) = \frac{u(a)}{\sum_{b \in A} u(b) + D(A)}, \quad (1)$$

where $(u, D)$ is unique up to a common positive linear transformation. Like the baseline Luce model, all active-choice alternatives here are assigned menu-independent values that determine their relative likelihood of being chosen. Unlike the baseline model–which will be recalled shortly–where this property also extends to the outside option, here the probability of making an active choice in the first place (equivalently, the probability of avoiding/deferring this decision) is determined by the menu-dependent utility of the outside option.

The following axioms characterize the class of models that can be represented in this way:

**A1 (Positivity).**

For all $A \in \mathcal{M}$ and all $a \in A$: $\rho(a, A) > 0$.

**A2 (The Active-Choice Luce Axiom).**

For all $A, B \in \mathcal{M}$ and all $a, b \in A \cap B$:

$$\frac{\rho(a, A)}{\rho(b, A)} = \frac{\rho(a, B)}{\rho(b, B)}$$

A1 is standard and allows for a crisper illustration of the main ideas that we put forward in this paper. A2 imposes the standard kind of IIA-consistency only in the odds of pairs of active-choice alternatives, while allowing odds that involve such an alternative and the outside option to deviate from it. That is, $\frac{\rho(o, A)}{\rho(o, A)} \neq \frac{\rho(o, B)}{\rho(o, B)}$ is possible here.
Proposition 1

\( \rho \) is a logit with a general outside option if and only if it satisfies A1-A2. Indeed, by adapting the arguments in Luce (1959) one obtains an equivalence between A1-A2 and the existence of a function \( u : X \to \mathbb{R}_{++} \) such that, for every \( A \in \mathcal{M} \) and \( a \in A \),

\[
\rho(a, A) = (1 - \rho(o, A)) \cdot \frac{u(a)}{\sum_{b \in A} u(b)},
\]

(2)

where

\[
u(a) := \alpha \cdot \frac{\rho(a, X)}{\rho(z, X)}
\]

(3)

for arbitrary and fixed \( \alpha > 0 \) and \( z \in X \). It follows then that for every \( A \in \mathcal{M} \) there is a unique \( D(A) \geq 0 \) that makes (1) true, with

\[
D(A) = \frac{\rho(o, A)}{1 - \rho(o, A)} \cdot \sum_{b \in A} u(b).
\]

(4)

Finally, it is immediate that \((u, D)\) and \((u', D')\) represent the same \( \rho \) if and only if \( u = \alpha u' \) and \( D = \alpha D' \) for some \( \alpha > 0 \).

We now compare (1) to the baseline logit with an outside option (Anderson, Palma, and Thisse, 1992; Hensher, Rose, and Greene, 2015) and to the one without such an option that is originally due to Luce (1959). To this end, recall that a random non-forced choice model \( \rho \) on \( X \cup \{o\} \) admits the former representation if there is a function \( u : X \cup \{o\} \to \mathbb{R}_{++} \) such that, for all \( A \in \mathcal{M} \) and \( a \in A \),

\[
\rho(a, A) = \frac{u(a)}{\sum_{b \in A} u(b) + u(o)}.
\]

(5)

On the other hand, \( \rho \) admits a logit representation without an outside option if there exists some \( u : X \to \mathbb{R}_{++} \) such that, for all \( A \in \mathcal{M} \) and \( a \in A \),

\[
\rho(a, A) = \frac{u(a)}{\sum_{b \in A} u(b)},
\]

(6)

The latter obviously implies \( \sum_{a \in A} \rho(a, A) = 1 \) for all \( A \in \mathcal{M} \), so that the opportunity to defer is either infeasible in this model or feasible but never acted upon. Thus, (1) includes (6) as a special case when \( o \not\in X \), which is our assumption here; and (1) extends (5) without nesting it unless A1-A2 and \( \rho \) operate on the enriched domain \( X \cup \{o\} \).
Despite the simplicity of (1) and the complete straightforwardness of the proof of Proposition 1, this extension of the baseline logit appears to be new in the theoretical and applied literature. As we show in the sequel, it gives rise to intuitive and non-trivially structured special cases that impose restrictions on $u$, $D$ and the relationship between the two, leading to several interesting new insights.

Before endogenizing $D$ in the models studied in the next two sections we will focus on an intuitive basic property of this function, and also point out some interesting behavioural implications. In particular, we will refer to both a decision-conflict logit $\rho = (u, D)$ and $D$ as monotonic if

$$A \supset B \implies D(A) \geq D(B).$$

(7)

If $D(A) > D(B)$ is always true when $A \supset B$, then $D$ and $\rho = (u, D)$ will be called strictly monotonic. In line with our intended interpretation of $D$ as a complexity/cost function, the total number of pairs of distinct alternatives increases as a menu expands, and therefore so does the expected number of comparisons between alternatives that a fully-attentive individual needs to make. In expectation, therefore, decision difficulty also goes up in absolute terms when more alternatives are added to a menu. Importantly, however, this does not imply that deferring always becomes more likely once a menu is expanded when $D$ is monotonic. We will return to this important point in Section 5.

Monotonicity, however, does have a familiar general implication for active-choice alternatives which in the standard forced-choice random-utility environments was originally stated in Block and Marschak (1960):

**Proposition 2**

If $\rho$ is a monotonic decision-conflict logit, then $a \in B \subset A$ implies $\rho(a, B) > \rho(a, A)$.

Thus, monotonic models satisfy what we will refer to as active-choice regularity, whereby the probability of such alternatives cannot increase when more options are added to a menu. Crucially, however, as we discuss and illustrate by example later, this property does not hold for the outside option.

When it comes to using this model in relevant applications, the analyst must first decide whether to employ a special case where function $D$ is set exogenously or one where it is determined endogenously instead. In the first case the choice might be dictated by the analyst’s a priori assessment of the specific environment in question and could include, for example, defining $D$ as the menu-cardinality function (Iyengar and Lepper, 2000; Iyengar, Huberman, and Jiang, 2004) or, if the alternatives have clearly identifiable attributes, some measure of
similarity in attribute space (Spektor, Gluth, Fontanesi, and Rieskamp, 2019). The analyst’s choice in the second case might instead be dictated by an agnosticism towards what is the most appropriate functional form for $D$, and by resorting instead to a general decision process that is in line with the model and where $D$ is determined endogenously and is a function of the feasible options’ $u$-values. We study such a structured special case in the next section.

3 Power and Quadratic Logit

3.1 Introduction

We define the power logit model by the existence of a menu-independent stimulus intensity value function $\hat{u}: X \rightarrow \mathbb{R}^{++}$ and a parameter $p \geq 1$ such that, for every menu $A$ and alternative $a$ in $A$,

$$\rho(a, A) = \left( \frac{\hat{u}(a)}{\sum_{b \in A} \hat{u}(b)} \right)^p$$  \hspace{1cm} (8)

Clearly, this model predicts $\rho(o, A) > 0$ at every menu $A$ if and only if $p > 1$, and reduces to (6) at $p = 1$.

The agent portrayed in (8) could be thought of as behaving according to the standard logit with a single valuation criterion but, possibly aware of their decision difficulty, also as if they sampled all alternatives more than once before making a decision. For example, in the quadratic logit case of special interest where $p = 2$, the agent might be thought of as sampling the same menu twice. Because the resulting value realizations generally differ across these two rounds of sampling due to the postulated randomness, this individual would be more likely to choose an active-choice alternative if its perceived signal/stimulus intensity from both inspections, captured by the two value realizations of $\hat{u}$, is relatively high, and as being more likely to avoid/defer choice when this is not true for any such alternative. When deciding which insurance plan to buy, for example, an agent whose behaviour is approximated by the quadratic logit may review the top-rated plans from a service comparison website in the morning, receive some value stimuli/signals from each of them, and then go back and repeat this process in the evening. Assuming that the two sampling rounds are independent (admittedly, a demanding assumption), an insurance plan is more likely to be chosen at the end of this two-stage process if its relative stimulus/signal intensity is sufficiently high to make the product stand out despite the agent’s hesitation.

The intuition in the more general case where $p \neq 2$ in (8) is analogous and admits a probabilistic explanation. Specifically, if the analyst a priori restricts $p$ to lie between 1 and
2, then \( p - 1 \) might be interpreted as the (exogenous) probability that the agent will engage in two rounds of sampling, equalling 1 in the limit where the quadratic logit decision process emerges with certainty. Similarly, if \( p \) is assumed to lie between 2 and 3, then \( p - 2 \) could be thought of as the probability that the agent will perform three rounds of sampling, conditional on the analyst expecting them to do at least two. More generally, the power parameter \( p \) in this model could be viewed as reflecting the agent’s propensity to engage in possibly multiple rounds of sampling.

### 3.2 Basic Properties

That the power logit is a decision-conflict logit may not be obvious at first glance but quickly becomes so upon noticing that one can write

\[
\begin{align*}
  u(a) & := \hat{u}(a)^p \\
  D(A) & := \left( \sum_{b \in A} \hat{u}(b) \right)^p - \sum_{b \in A} \hat{u}(b)^p
\end{align*}
\]

With \( p = 2 \) these expressions admit the simpler and more easily interpretable form

\[
\begin{align*}
  u(a) & := \hat{u}(a)^2, \\
  D(A) & := \left( \sum_{b \in A} \hat{u}(b) \right)^2 - \sum_{b \in A} \hat{u}(b)^2 \\
  & = 2 \sum_{a, b \in A, a \neq b} \hat{u}(a)\hat{u}(b) \\
  & = \sum_{a, b \in A} D(\{a, b\})
\end{align*}
\]

where the last step makes use of the notational convention

\[ D(\{a, a\}) \equiv D(\{a\}). \]

This clarifies that the quadratic logit \( \rho \equiv (\hat{u})^2 \) is an additive decision-conflict logit \((u, D)\) in the sense that the utility of the outside option at every menu depends additively on the utility of that option at each of its binary submenus. It also clarifies that the latter utility takes a symmetric Cobb-Douglas form with respect to \( \hat{u} \). We will return to additivity later in this section but note here that the quadratic case where \( p = 2 \) is the only one where the \((u, D)\) representation of (8) has this property.

Interestingly, the power-logit model’s predicted probability of deferring at a menu as a
function of the number of active-choice alternatives at that menu is bounded above in the following simple way.

**Proposition 3**

If $\rho$ is a power logit $(\hat{u}, p)$, then, for every menu $A$,

\[
\rho(o, A) \leq 1 - |A|^{1-p}, \\
\rho(o, A) = 1 - |A|^{1-p} \iff \hat{u}(a) = \hat{u}(b) \text{ for all } a, b \in A.
\]

Figure 1: Maximum probability of deferring as a function of menu size in the power logit model.

In this model, therefore, an agent’s decision difficulty at a menu, as revealed by the deferral probability at that menu, is maximized when all feasible active-choice alternatives are equally desirable, and this maximum difficulty is increasing in proportion to the total number of such alternatives at a decreasing rate (Figure 1).

Turning to the model’s comparative statics in the important class of binary menus, Figure 2 illustrates by example the general pattern in the behaviour of $\rho(a, \{a, b\})$ and $\rho(o, \{a, b\})$ as the stimulus intensity of $a$ changes while that of $b$ is held fixed. Interestingly, the monotonic increase of $\rho(a, \{a, b\})$ in $\hat{u}(a)$ occurs at an increasing rate as this value approaches the $\hat{u}(b)$ stimulus-intensity threshold from below than when $\hat{u}(a)$ increases monotonically beyond $\frac{\hat{u}(b)}{2}$.

Intuitively, the inflection-point stimulus intensity value $\frac{\hat{u}(b)}{2}$ that dissects $\rho(a, \{a, b\})$—viewed as a function of $\hat{u}(a)$—into convex and concave regions suggests that marginal improvements in the appeal of $a$ lead to more rapid market share increases when this alternative is still “catching up” with $b$ than when it has become sufficiently close to (or surpassed) it in attractiveness. On the other hand, $\rho(o, \{a, b\})$ is a strictly concave function of $\hat{u}(a)$ and, consistent with Proposition 3, attains its maximum value of $\frac{1}{2}$ when $\hat{u}(a) = \hat{u}(b)$. 
3.3 Characterization

We start the axiomatic analysis of this section by noting the following direct implication of the power logit model:

**A3 (Desirability & Complexity)**

For all $A \in \mathcal{M}$: $\rho(A, A) = 1 \iff |A| = 1$.

To motivate the intuition behind A3 we first recall that, as clarified early in this paper, our aim here is to model decision difficulty that is rooted in a fully attentive individual’s potential inability to make some preference comparisons between otherwise desirable options. If a single such option was feasible to such an individual, therefore, one might expect that person to immediately choose that one option. If on the other hand there are at least two available options and the individual is not forced to make a choice immediately, then the experimental/empirical evidence suggests that there is at least some probability that this person’s attempt to find a most preferred option and choose that option will not be fruitful reasonably quickly. To the extent that this is so, a legitimate approach from the analyst’s perspective would be to portray that decision maker as deferring choice with positive probability whenever at least one non-trivial comparison is required.

Imagine, for example, a patient as in Knops, Ubbink, Legemate, Stalpers, and Bossuyt (2013) who has been diagnosed with a life-threatening disease. Suppose that their doctor informs them that there is only one available treatment that can cure this disease, and asks whether they would like to sign up for this treatment. One would expect the patient to sign up immediately because there would be no benefit from delaying their only chance for a cure. Now suppose instead that the doctor tells the patient that there are two possible treatments: one with high efficacy but severe side effects, and another with milder side effects but lower cure rates. Although if either of these was the only feasible treatment it would have been chosen...
immediately, in this case one might expect the patient to delay making such an active choice, perhaps until they think about the conflicting pros and cons and then ultimately determine which treatment would be best for them. Situations of this kind are compatible with and, in fact, motivate the modelling framework and axioms in this study.

In the spirit of these examples, A3 postulates that an active choice is made with certainty only at singleton menus and, as such, it formalises the behavioural mechanisms outlined above. Of course, one can still think of situations where this axiom is descriptively invalid. Yet, for analytical purposes it is a useful property because it allows for completely isolating the decision-difficulty channel to deferrals from other potential channels such as undesirability of the available alternatives or limited attention, which have quite distinct behavioural origins.

In light of the analysis in the preceding subsection, the next result is immediate:

**Corollary 4**

\( \rho \) satisfies A1–A3 if and only if it is a \((u, D)\)-model with the property that

\[
D(A) = 0 \iff |A| = 1
\]  

We will refer to this special class of generalized logit models with a context-dependent outside option as the class of decision-conflict logit models, and to the menu function \( D \) that captures the varying appeal of opting out at different menus as the decision cost or decision complexity function. Justifying such a name for the function \( D \) given the requirement that it be zero-valued only at singletons may benefit from some explanation that would supplement the preceding discussion. When the decision environment is such that avoidance/deferral is caused solely by decision difficulty instead of other factors (e.g. none of the active-choice alternatives is good enough, or none is considered due to limited-attention constraints), our decision maker is portrayed as not having any problem deciding between deferring or choosing the only available active-choice option: they do the latter. By contrast, the decision between deferring or choosing from two or more such options is at least somewhat costly because of the effort that is necessary to make the relevant preference comparisons.

The next condition on choice probability distributions with a feasible outside option is new, testable and easily seen to be implied by every quadratic logit model.

**A4 (Symmetric Deferral Odds)**

*For all \( a, b \in X \):

\[
\frac{1}{2} \cdot \frac{\rho(o, \{a, b\})}{\rho(a, \{a, b\})} = \left( \frac{1}{2} \cdot \frac{\rho(o, \{a, b\})}{\rho(b, \{a, b\})} \right)^{-1}
\]
A4 requires that the odds of deferring relative to choosing \(a\) at menu \(\{a, b\}\) be inversely proportional to the odds of deferring relative to choosing \(b\) at that menu, with the specific shape of this symmetric non-linear relationship determined by the scalar \(\frac{1}{2}\) and depicted in the solid curve of the simplex shown in Figure 3.

As shown in that figure, in particular, under this axiom the probability of opting out attains its maximum value of 0.5 as \(a\) and \(b\) become equi-probable. This implication is our concrete formalization of the idea that decision difficulty is increased when the feasible market alternatives are more or less equally appealing. This discussion suggests that A4 is a potentially good approximation of an individual’s behaviour in cases where: (i) \(a\) and \(b\) are similarly attractive and this similarity translates into high decision conflict; (ii) \(a\) or \(b\) is the clearly superior option and choosing it is the most likely decision outcome.

**Figure 3:** The loci of binary choice probability distributions that are compatible with the power logit under different values of \(p\).

The next result provides a partial characterization of the quadratic logit in the class of binary menus \(B\) by means of A4. The general characterization is given later in this section.

**Proposition 5**

\(\rho\) is a quadratic logit on the binary menus of \(X\) if and only if it satisfies A1–A4.

Recall now that, as noted in (11), the quadratic logit predicts that decision complexity at any menu with more than two alternatives is additive in the complexity at each of its binary
D(A) = \sum_{a,b \in A} D(\{a,b\}). \tag{14}

Intuitively, for a decision maker like the one we are modelling in this paper who is paying full attention to all feasible options, all binary comparisons may turn out to be relevant in the search towards determining the overall best alternative in a menu. This is especially so if such a search is inefficient, which is not unlikely for human decision makers. In this case, therefore, the degree of difficulty in identifying such an alternative depends on how hard it is to make every such comparison. Additivity disciplines this monotonic relation in an analytically convenient way.

**A5 (Balancing Odds).**

For all $A, B \in \mathcal{M}$ such that $B \supset A$:

$$\frac{\rho(o, A)}{1 - \rho(o, A)} = \sum_{a,b \in A, a \neq b} \left( \frac{\rho(o, \{a,b\})}{1 - \rho(o, \{a,b\})} \cdot \frac{\rho(\{a,b\}, B)}{\rho(A, B)} \right)$$

In words, the odds of opting out at $A$ depend in an additively separable way on the weighted odds of opting out at every $\{a,b\} \subset A$. The weights on these odds in turn are given by the likelihood of choosing $a$ or $b$ relative to choosing any option in $A$ when deciding at some larger $B \supset A$. The binary-menu odds could be greater, equal or less than one, whereas their weights cannot exceed unity. Intuitively, the closer the weights are to this upper bound, the higher the appeal of $a$ and $b$ at $A$, and the greater the influence that the odds of opting at $\{a,b\}$ have on the respective odds at $A$.

**Proposition 6**

A decision-conflict logit $\rho$ on $X$ is additive if and only if it satisfies A5.

**Theorem 7**

$\rho$ is a quadratic logit on $X$ if and only if it satisfies A1–A5.

This general characterization is obtained as an implication of Propositions 1–6 once it is observed that an additive decision-conflict logit that is defined on the full domain of menus, $\mathcal{M}$, is a quadratic logit in that domain iff it is a quadratic logit in the domain of binary menus, $\mathcal{B}$.

A full list of necessary and sufficient conditions for the more general power logit is currently elusive. While desirable, arriving at such a characterization is complicated by two factors. First,
there is no formula through which terms such as \((y + z)^p\) can be expanded for non-integral values of \(p\). This prevents the derivation of closed-form expressions for the menu-dependent utility of the outside option. Second, unless \(p = 2\), the complexity function \(D\) in the power logit model is either sub-additive (when \(p \in (1, 2)\)) or super-additive (when \(p > 2\)). These facts, respectively, make it intractable to specify and solve for general values of \(p\) the system of equations that pins down the power-logit \(\hat{u}\)-values of alternatives and \(D\)-values of binary menus, and to extrapolate from the latter–using A1–towards calculating \(D\) at any larger menu.

4 Two Empirically Supported Behavioural Predictions

We proceed with an illustration of how a general \((u, D)\) model on the one hand and the more structured power/quadratic logit on the other make predictions that help explain intuitively some empirically documented choice-deferral phenomena.

Since, as was discussed previously, it is not generally true that avoiding/deferring becomes more likely as menus expand even for monotonic decision-conflict logit models, it is naturally of interest to understand when exactly such behaviour is to be expected in this environment. The general idea in answering this question is that, even if decision difficulty increases in absolute terms when new alternatives are introduced, when these new alternatives are sufficiently better than the pre-existing ones their added value will offset the elevated decision cost and will ultimately result in a higher probability of making an active choice at the larger menu. To state this more formally we will abuse notation slightly by letting

\[
u(S) := \sum_{s \in S} u(s)
\]

stand for the total Luce utility at menu \(S \in \mathcal{M}\).

Proposition 8

If \(\rho = (u, D)\) is a decision-conflict logit, then for any \(A, B \in \mathcal{M}\) such that \(A \supset B\):

\[
\rho(o, A) \leq \rho(o, B) \iff \frac{D(A) - D(B)}{D(B)} \leq \frac{u(A) - u(B)}{u(B)}
\]

This eloquent equivalence clarifies that the choice probability of opting out will decrease

---

3One might be tempted to invoke the uniqueness properties of the power-logit model toward a normalization that would enable rewriting this term without loss as \((1 + t)^p\) for \(t < 1\) and then applying the power-series formula \((1 + t)^p = \sum_{k=0}^{\infty} \binom{p}{k} t^k\), where \(\binom{p}{k}\) here is Newton’s general binomial coefficient. Doing so, however, does not help toward deriving an interpretable testable condition.
following menu expansion if and only if the marginal benefit of this expansion, as measured by
the percentage increase in total utility, exceeds its marginal cost, as measured by the percentage
increase in decision complexity. This is a distinctive property of decision-conflict logit models.
It clarifies that they do not belong to the random-utility class with an outside option, and
enables them to explain simply the non-monotonic and dominance-driven effect that menu
expansion has been known to exert on the probability of deferring (Scheibehenne, Greifeneder,
and Todd, 2010; Chernev, Böckenholt, and Goodman, 2015), which we will refer to as the
“roller-coaster choice overload” effect.

Table 1: Illustration of “roller-coaster” choice-overload predictions with the quadratic logit.

| Option | $\tilde{u}$ | $\rho(\cdot, \{a, b\})$ | $\rho(\cdot, \{a, b, c\})$ | $\rho(\cdot, \{a, b, c, d\})$ |
|--------|------------|----------------|----------------|----------------|
| $a$    | 10         | 0.980          | 0.250          | 0.007          |
| $b$    | 0.1        | 0.001          | 0.000          | 0.000          |
| $c$    | 9.9        | $-$            | 0.245          | 0.007          |
| $d$    | 100        | $-$            | $-$            | 0.694          |
| $o$    | $-$        | $-$            | $\uparrow$ 0.505 | $\downarrow$ 0.292 |

Indeed, citing several studies in consumer psychology, the meta-analysis in Chernev, Böck-
enholt, and Goodman (2015) notes that “it has been shown that consumers are more likely to
make a purchase from an assortment when it contains a dominant option than when such an
option is absent” (p. 338). This finding is important for the interpretation and policy responses
to choice-overload phenomena of the kind that were first reported in Iyengar and Lepper (2000).
To our knowledge, the decision-conflict logit is the first random-choice model that predicts this
dominance-driven emergence and disappearance of choice-overload effects, and it does so without
imposing any undesirability or inattention constraints. Table 1 illustrates an example such
effect that is predicted by the quadratic logit model.

As the next result establishes, the power-logit class further predicts another important
choice-deferral phenomenon that is known as the “relative-desirability” effect (Dhar, 1997;
White, Hoffrage, and Reisen, 2015; Bhatia and Mullett, 2016). This predicts that, choosing
the outside option becomes more likely in binary menus as the available options become more
equally desirable, other things equal.

Proposition 9
If $\rho = (u, D) = (\tilde{u}, p)$ is a power logit, then for any $a, b, c, d \in X$ where $\tilde{u}(a) + \tilde{u}(b) = \tilde{u}(c) + \tilde{u}(d)$
or \( u(a) + u(b) = u(c) + u(d) \) is true, the following is also true:

\[
\rho(o, \{a, b\}) > \rho(o, \{c, d\}) \iff |\hat{u}(a) - \hat{u}(b)| < |\hat{u}(c) - \hat{u}(d)| \tag{17}
\]

\[
|u(a) - u(b)| < |u(c) - u(d)| \tag{18}
\]

The result clarifies, therefore, that the power logit predicts relative-desirability effects both when the “other things equal” proviso applies to the total stimulus intensity values of the feasible alternatives and when it applies to their total logit values that emerge from the former via the (convex) power transformation (Figure 4). More strongly, equivalence (18) further clarifies that stimulus-intensity and logit-value differences are ordinally equivalent.

5 Econometric Estimation

It is often the case in empirical applications that the choice frequencies available to the analyst are obtained from the choices made by a cross section of individuals who are presented with the same menu, rather than from a single decision maker’s repeated choices at that menu. Random-utility based discrete choice estimation in those cases is often carried out under the assumption that the observable component of every individual’s utility coincides, and that the error term in that model’s formulation captures all individual heterogeneity that is unobserved
to the analyst. Adopting and adapting this assumption to our non-random-utility environment, in this section we first show how the other assumptions and formal argument that underpin the discrete-choice formulation of the logit model without an outside option that was pioneered by McFadden (1973) can be modified to arrive at a similar discrete-choice version of the quadratic- and power-logit models. We then estimate these models on the data from Bhatia and Mullett (2016) and compare their predictions to those of the baseline conditional logit with an inferior outside option. It is worth remarking that, as we show in Section 5.3, the use of otherwise standard discrete-choice datasets is sufficient towards estimating these models, as long as they are obtained from a “free choice” decision environment, i.e. one where individuals could choose the no-choice outside option, where the analyst observes both the active choices and those of the latter option.

5.1 Discrete Choice with the Quadratic Logit

We start by denoting the set of all quadratic-logit decision makers by \( \{1, \ldots, n, \ldots, N\} \). Keeping the menu \( A := \{a_1, \ldots, a_i, \ldots, a_k\} \subseteq X \) fixed throughout this and the next subsection, we proceed by recalling and breaking down the baseline assumptions of the discrete-choice formulation of the baseline logit in (6) as follows:

1. Random utility [structural assumption]: there is some function \( u_n: X \to \mathbb{R} \) such that

   \[
   u_n(a_i) = g(\beta; x_{ni}) + \epsilon_{ni},
   \]

   where \( x_{ni}, \beta \) are, respectively, \( m \)-vectors of observable product/consumer characteristics and unknown coefficients capturing their relative importance via the relationship specified by some mapping \( g: \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R} \) and \( \epsilon_{ni} \) is an error term. As is often done in applications, we further impose the additive-linear structure

   \[
   g(\beta; x_{ni}) = \beta \cdot x_{ni},
   \]

   where \( \cdot \) denotes the inner product.

2. Random utility maximization [behavioural assumption]: for all \( a_i \in A \),

   \[
   \rho_n(a_i, A) = Pr(u_n(a_i) \geq u_n(a_j) \text{ for all } j \leq k).
   \]

3. Gumbel noise [distributional assumption]: the error term \( \epsilon_{ni} \) is independently and identically distributed across \( i \) according to the standard Gumbel density

   \[
   f(\epsilon_n) = e^{-\epsilon_n} e^{-e^{-\epsilon_n}}.
   \]
As has been widely known since the seminal contribution of McFadden (1973), these assumptions jointly imply the analytically convenient and famous form

$$\rho_n(a_i, A) = \frac{e^{\beta \cdot x_{ni}}}{\sum_{j=1}^{k} e^{\beta \cdot x_{nj}}}. \quad (23)$$

We proceed by examining how the premises and conclusion of this classic discrete-choice logit model are affected and can be modified when we assume that decision maker $n$ uses the single but noisy utility criterion captured by $u_n$ to sample the values of the alternatives in $A$ twice, as per the the quadratic special case of the power logit (focusing on the quadratic case here is done for simplicity of the exposition; we deal with the general case later). To this end we note first that maintaining the additive and linear utility assumption implies that at the end of the second round of sampling the individual has perceived two values for each alternative $a_i \in A$,

$$u^1_n(a_i) = \beta \cdot x_{ni} + \epsilon^1_{ni},$$

$$u^2_n(a_i) = \beta \cdot x_{ni} + \epsilon^2_{ni}.$$

These generally distinct values across the two rounds will vary according to the distribution of $\epsilon_{ni}$. Such multiplicity of value realizations in turn implies that each alternative $a_i \in A$ is ultimately associated with a vector of values $(u^1_n(a_i), u^2_n(a_i))$. With utility now being vector-valued, however, the utility-maximization behavioural assumption that underpins (23) is no longer applicable in an obvious way. To break this impasse we assume that the random utility maximization behavioural assumption is replaced by a dominance assumption whereby

$$\rho_n(a_i, A) = Pr(u^l_n(a_i) \geq u^l_n(a_j) \text{ for all } j \leq k \text{ and for } l = 1, 2). \quad (24)$$

Turning, finally, to the modification of the distributional assumption (22), to make it operational in the quadratic-logit framework we assume that the random errors $\epsilon^1_{ni}$ and $\epsilon^2_{ni}$ are independent across all alternatives $i \leq k$ and across the two sampling rounds $l \leq 2$. 

\(^5\text{Luce and Suppes (1965) and, indeed, McFadden (1973) also credit Eric W. Holman and Anthony A. J. Marley for this discovery.}\)
With these assumptions in place we can now write

\[
\rho_n(a_i, A) = Pr(u_n^1(a_i) \geq u_n^1(a_j) \forall j \neq i) \cdot Pr(u_n^2(a_i) \geq u_n^2(a_j) \forall j \neq i)
\]

\[
= Pr(\beta \cdot x_{ni} + \epsilon_{ni}^1 \geq \beta \cdot x_{nj} + \epsilon_{nj}^1 \forall j \neq i) \times Pr(\epsilon_{nj}^1 \leq \beta \cdot x_{ni} + \epsilon_{ni}^1 - \beta \cdot x_{nj} \forall j \neq i)
\]

\[
\int_{-\infty}^{\infty} \left( \prod_{j \neq i} e^{-\epsilon_{ni}^1 + \beta \cdot x_{ni} - \beta \cdot x_{nj}} \right) e^{-\epsilon_{ni}^1} e^{-\epsilon_{ni}^2} d\epsilon
\]

\[
= \prod_{j \neq i} e^{-\epsilon_{ni}^1 + \beta \cdot x_{ni} - \beta \cdot x_{nj}} \int_{-\infty}^{\infty} \left( \prod_{j \neq i} e^{-\epsilon_{ni}^2 + \beta \cdot x_{ni} - \beta \cdot x_{nj}} \right) e^{-\epsilon_{ni}^2} e^{-\epsilon_{ni}^2} d\epsilon
\]

\[
= \left( \frac{e^{\beta \cdot x_{ni}}}{\sum_{j=1}^{k} e^{\beta \cdot x_{nj}}} \right)^2,
\]  

where each integral is \(k\)-dimensional, the first and second steps make use of the above behavioural, distributional and independence assumptions on \(\epsilon_{ni}^1\), while the last step follows from the derivation of the discrete-choice logit [see, for example, Train (2009, pp. 36-37 & 74-75)].

An important difference between the discrete-choice version of the logit with an outside option in (5) and its quadratic-logit counterpart is that in the former case the modeller specifies the utility of that option \textit{exogenously} (see Anderson, Palma, and Thisse, 1992; Hensher, Rose, and Greene, 2015), whereas in the latter case this utility emerges endogenously as a function of the observable characteristics of all active-choice alternatives. Indeed, upon rewriting (25) as

\[
\rho_n(a_i, A) = \frac{e^{2\beta \cdot x_{ni}}}{\sum_{j=1}^{k} e^{2\beta \cdot x_{nj}} + 2 \sum_{i \neq j} e^{\beta \cdot (x_{ni} + x_{nj})}},
\]  

one observes that

\[
u_n(a_i) \equiv e^{2\beta \cdot x_{ni}},
\]  

\[
D_n(A) \equiv 2 \sum_{i \neq j} e^{\beta \cdot (x_{ni} + x_{nj})}.
\]
By contrast, in the baseline model we have

$$u_n(a_i) \equiv e^{\gamma x_{ni}},$$ (29)
$$u_n(o) \equiv e^{\gamma x_{no}},$$ (30)

where $x_{no}$ is set by the analyst.

### 5.2 Maximum-Likelihood Estimation in the General Case

We proceed with an analysis of the properties and estimation of the discrete-choice version of the more general power-logit model, where

$$\rho_n(a_i, A) = e^{\beta \cdot x_{ni}} \left( \sum_{j=1}^{k} e^{\beta \cdot x_{nj}} \right)^p$$

$$\rho_n(o, A) = \left( \sum_{j=1}^{k} e^{\beta \cdot x_{nj}} \right)^p - \sum_{j=1}^{k} e^{p \beta \cdot x_{nj}}$$

Following McFadden (1973) and the ensuing literature, we now show how the vector $\beta$ and scalar $p > 1$ in (31)-(32) can be estimated by minimizing the log-likelihood function that emerges from this model. To this end, let us write

$$Pr_{ni} \equiv Pr_{ni}(\beta) := \rho_n(a_i, A), \quad i \leq k,$$

and

$$Pr_{no} \equiv Pr_{no}(\beta) := \rho_n(o, A) = 1 - \sum_{i=1}^{k} Pr_{ni} > 0.$$

Next, let us denote by $y_n$ the $n$-th individual’s observed decision at menu $A$. It is critical to distinguish between this decision being an active choice or choice of the outside option. To this end, we define the binary variables $y_{ni}, i = 1, \ldots, k,$ and $y_{no}$ by

$$y_{ni} := \begin{cases} 1, & \text{if } y_n = a_i \\ 0, & \text{otherwise} \end{cases} \quad \text{and} \quad y_{no} := \begin{cases} 1, & \text{if } y_n \neq a_i \text{ for all } a_i \in A \\ 0, & \text{otherwise} \end{cases}$$
to account for the former and latter cases, respectively. With these in place, the multinomial
density for a given active-choice or opt-out decision made by agent \( n \) can now be written as

\[
g_n(\beta, p) = P_{r_{n_{no}}} \prod_{i=1}^{k} P_{r_{n_{ni}}}.\]

Assuming an exogenous sample and covariates \( x_{ni} \) for every agent \( n \leq N \) and alternative \( i \leq k \),
the likelihood function that results from the \( N \) independent decisions is now given by

\[
L(\beta, p) = \prod_{n=1}^{N} g_n(\beta)
= \prod_{n=1}^{N} \left( P_{r_{n_{no}}} \prod_{i=1}^{k} P_{r_{n_{ni}}} \right)
\]

This leads to the log-likelihood function

\[
LL(\beta, p) = \sum_{n} \sum_{i} y_{ni} \ln P_{r_{ni}} + \sum_{n} y_{no} \ln P_{r_{no}}
= \sum_{n} \sum_{i} y_{ni} \ln \left( \frac{e^{\beta \cdot x_{ni}}}{\sum_{j} e^{\beta \cdot x_{nj}}} \right) + \sum_{n} y_{no} \ln \left( \frac{\left( \sum_{j} e^{\beta \cdot x_{nj}} \right)^{p} - \sum_{j} e^{\beta \cdot x_{nj}}} {\left( \sum_{j} e^{\beta \cdot x_{nj}} \right)^{p}} \right)
\]

Recalling that \( \beta = (\beta^1, \ldots, \beta^m) \) and \( x_{ni} = (x_{ni}^1, \ldots, x_{ni}^m) \), the first-order conditions of its
maximization with respect to \( p \) and \( \beta \) are

\[
\frac{\partial LL(\beta, p)}{\partial p} = \sum_{n} \sum_{i} y_{ni} \left[ \beta \cdot x_{ni} - \ln \left( \sum_{j} e^{\beta \cdot x_{nj}} \right) \right]
- \sum_{n} y_{no} \left[ \frac{\sum_{j} (\beta \cdot x_{nj}) e^{\beta \cdot x_{nj}} - \sum_{j} e^{\beta \cdot x_{nj}} \ln \left( \sum_{j} e^{\beta \cdot x_{nj}} \right)}{\left( \sum_{j} e^{\beta \cdot x_{nj}} \right)^{\tilde{p}}} \right]
= 0
\]

and
\[
\frac{\partial LL(\beta, p)}{\partial \beta_l} = p \sum_{n} \sum_{i} y_{ni} x_{ni}^l - p \sum_{n} \sum_{i} y_{ni} x_{ni}^l \left( \frac{e^{\beta x_{ni}}}{\sum_{j} e^{\beta x_{nj}}} \right) \\
- p \sum_{n} \sum_{i} y_{no} x_{ni}^l \left( \frac{e^{\beta x_{ni}}}{\sum_{j} e^{\beta x_{nj}}} \right) \\
+ p \sum_{n} y_{no} \left[ \left( \sum_{j} e^{\beta x_{nj}} \right)^{p-1} \left( \sum_{j} x_{nj}^l e^{\beta x_{nj}} \right) - \sum_{j} x_{nj}^l e^{p\beta x_{nj}} \right] \\
\left( \sum_{j} e^{\beta x_{nj}} \right)^p - \sum_{j} e^{p\beta x_{nj}} \\
= 0, \quad l = 1, \ldots, m
\]

Observing that \(\sum_{i} y_{ni} + y_{no} = 1\) holds by construction and that \(p > 0\) enters all their terms multiplicatively, the latter \(m\) first-order conditions simplify to

\[
\sum_{n} \sum_{i} y_{ni} x_{ni}^l = \sum_{n} \sum_{i} x_{ni}^l \left( \frac{e^{\beta x_{ni}}}{\sum_{j} e^{\beta x_{nj}}} \right) \\
- \sum_{n} y_{no} \left[ \left( \sum_{j} e^{\beta x_{nj}} \right)^{p-1} \left( \sum_{j} x_{nj}^l e^{\beta x_{nj}} \right) - \sum_{j} x_{nj}^l e^{p\beta x_{nj}} \right] \\
\left( \sum_{j} e^{\beta x_{nj}} \right)^p - \sum_{j} e^{p\beta x_{nj}}
\]

Thus, unlike the standard logit where the term appearing with a negative sign in the last equation is absent and where, by construction, the estimated \(\hat{\beta}\) ensures that empirical and average predicted frequencies of active-choice alternatives coincide (Train, 2009; Greene and Hensher, 2010), the presence of the said term here clarifies that this is no longer true in the power logit when deferral choices are present in the data.

5.3 Proof-of-Concept Illustration from a Survey Dataset with Film Choices

Data

For our application we use the survey-experiment data with film choices that were collected by Bhatia and Mullett (2016). In that study, 58 subjects were initially asked to rate from 1
(least desirable) to 9 (most desirable) the 100 films that, at the time, were ranked most popular by members of the IMDB (https://www.imdb.com) platform. Following that, subjects were presented with 100 distinct binary menus with films that were drawn from that list. In the free-choice treatment they were asked to choose either the film positioned on the left or on the right of each menu, or to defer the decision. In the forced-choice treatment, the same 100 menus were presented but deferral was not feasible. The study featured a within-subject design and subjects were randomly assigned to start the experiment in either of the two treatments.

Analysis
Although Bhatia and Mullett (2016) focused mainly on the relationship between choice deferral and response times, they also reported on the relationship between ratings and active-choice probabilities conditional on an active choice being made. Specifically, they found that the film with a higher rating, where relevant, is chosen 83% of the time (p. 137). Our focus here instead is on the unconditional analysis of the explanatory value of the subjects’ own ratings on their subsequent active-choice and deferral decisions, and on comparing the results from this analysis when it is carried out via existing or via our proposed modelling approach. To this end, on each of the 100 menus in this dataset we estimate and compare the goodness of fit of the following models:

Multinomial Logit with a Fixed Outside Option

In line with existing practices (see, for example, pp. 411-414 in Hensher, Rose, and Greene, 2015), to estimate this model we treat the outside option as an explicit alternative with a fixed value that is common to all subjects. Doing so leads to the following three-parameter multinomial logit specification:

\[
P_{n}^{ML}(l, A) = \frac{e^{\beta_{l,A}^{0} + \beta_{l,A}^{1} \text{rat.Left}_n}}{1 + e^{\beta_{l,A}^{0} + \beta_{l,A}^{1} \text{rat.Left}_n} + e^{\beta_{r,A}^{0} + \beta_{r,A}^{1} \text{rat.Right}_n}}
\]

\[
P_{n}^{ML}(r, A) = \frac{e^{\beta_{r,A}^{0} + \beta_{r,A}^{1} \text{rat.Right}_n}}{1 + e^{\beta_{l,A}^{0} + \beta_{l,A}^{1} \text{rat.Left}_n} + e^{\beta_{r,A}^{0} + \beta_{r,A}^{1} \text{rat.Right}_n}}
\]

\[
P_{n}^{ML}(o, A) = \frac{1}{1 + e^{\beta_{l,A}^{0} + \beta_{l,A}^{1} \text{rat.Left}_n} + e^{\beta_{r,A}^{0} + \beta_{r,A}^{1} \text{rat.Right}_n}}
\]

The left-hand-side terms denote the estimated probabilities of subject \( n \) choosing “left”, “right” or “defer” at binary menu \( A \). On the right hand side, \( \beta_{l,A}^{1} \) and \( \beta_{r,A}^{1} \), \( \beta_{l,A}^{0} \) and \( \beta_{r,A}^{0} \) are, respectively, the estimated slope and intercept coefficients at menu \( A \). The former captures the effect that a unitary

\[\text{Under these two conditions the exact value of the outside option’s “rating” is unimportant for this model’s maximized log-likelihood and estimate of } \beta_{l,A}^{1}, \text{ mattering only for the estimates of } \beta_{l,A}^{0} \text{ and } \beta_{r,A}^{0}.\]
increase in subject $n$’s rating of the left (right) film—denoted here by $\text{rat.Left}$ ($\text{rat.Right}$)—has on the log-odds of choosing that film over deferring when the latter option’s value is fixed. The option-specific intercepts $\beta_{l,A}^0$ and $\beta_{r,A}^0$ on the other hand capture the log-odds of choosing, respectively, the left and right film over deferring when the relevant film’s rating is zero. Hence, including these terms in the estimation is essential for otherwise the prediction would be equal choice probabilities for “left”, “right” and “defer” if both films had a zero rating. This, in turn, would go against the model’s treatment of the outside option as any other alternative that is more likely to be chosen as the other feasible options become worse.

**Multinomial Logit with a Random Outside Option**

We also consider the variant of the preceding model where, instead of assuming a fixed common value (“rating”) for the outside option, we allow it to vary across subjects and menus by randomizing over the permissible rating values.

**Quadratic Logit**

As discussed in the previous subsection, estimating the quadratic logit amounts to estimating the parameter $\gamma^A$ in

$$
P_{n}^{QL}(l, A) = \left( \frac{e^{\gamma^A \cdot \text{rat.Left}}}{e^{\gamma^A \cdot \text{rat.Left}} + e^{\gamma^A \cdot \text{rat.Right}}} \right)^2
$$

$$
P_{n}^{QL}(r, A) = \left( \frac{e^{\gamma^A \cdot \text{rat.Right}}}{e^{\gamma^A \cdot \text{rat.Left}} + e^{\gamma^A \cdot \text{rat.Right}}} \right)^2
$$

$$
P_{n}^{QL}(o, A) = 1 - P_{n}^{QL}(l, A) - P_{n}^{QL}(r, A)
$$

There are some important differences between this model and the multinomial logit with an outside option laid out above. First, unlike that model, the quadratic logit does not include any intercept terms. This is in line with the theoretical predictions of the general version of this model (Proposition 3), according to which all active-choice options are equally likely to be chosen when they have the same value. Including alternative-specific intercept terms here would go against this prediction as it would lead to generally distinct predicted probabilities for the left and right film when their ratings are identically equal to zero. Second, unlike $\beta^A$, the slope coefficient $\gamma^A$ here captures the log-odds of choosing one film over the other (i.e. not over deferring) following a unitary change in the former film’s rating. In particular, given (2), (11), (12) and (27), a more appropriate interpretation of this coefficient is that it captures the relevant change in the log-odds of choosing one film over the other following a unitary increase in the former’s rating \textit{conditional on an active choice having been made}, while the uncondi-
tional change in these log-odds is obtained by multiplying them by $1 - \rho(o, A)$. By contrast, (26) clarifies that the log-odds of choosing a film over deferring following a unitary increase in that option’s rating is captured by $2\gamma_A$ instead.

**Power Logit**

Estimating this more general model now involves finding simultaneously optimal values for the slope coefficient $\theta_A$ and the power parameter $p_A$ in

$$P_n^{PL}(l, A) = \left( \frac{e^{\theta_A \cdot \text{rat.Left}}}{e^{\theta_A \cdot \text{rat.Left}} + e^{\theta_A \cdot \text{rat.Right}}} \right)^{p_A}$$

$$P_n^{PL}(r, A) = \left( \frac{e^{\theta_A \cdot \text{rat.Right}}}{e^{\theta_A \cdot \text{rat.Left}} + e^{\theta_A \cdot \text{rat.Right}}} \right)^{p_A}$$

$$P_n^{PL}(o, A) = 1 - P_n^{PL}(l, A) - P_n^{PL}(r, A),$$

The parameter $\theta_A$ here admits an analogous interpretation to $\gamma_A$ in the quadratic logit, while the term $p_A \theta_A$ is interpretable as the effect that a unitary change in a film’s rating has on the log-odds of choosing that film over deferring.

**Model Estimation and Goodness-of-Fit Summary Comparisons**

We perform a goodness-of-fit analysis and comparison of the four models that aim to assess their explanatory and predictive performance on these data. To this end, we use the maximized log-likelihood value, the Akaike (AIC) and Bayesian (BIC) information criteria, and each model’s proportion of correct predictions. In particular, denoting by $\hat{L}_A$, $k$ and $N_A$, respectively, a model’s maximized log-likelihood value at menu $A$, the number of its parameters and its sample size, recall that $AIC = 2k - 2 \log(\hat{L}_A)$ and $BIC = k \log(N_A) - 2 \log(\hat{L}_A)$. The value of $k$ is 3 for the two multinomial logit models with a fixed and random outside option, 2 for the power logit and 1 for the quadratic logit. The sample size is $N_A = 58$ in all 4 models and for all 100 menus. In the prediction analysis we compared the three models’ predicted 5800 ($= 100 \times 58$) choices to subjects’ actual choices. A model was taken to make a correct prediction at a given menu and for a given subject if it predicted a weakly highest choice probability for the option that was actually chosen by that subject in that menu.

Figure 6 plots the 100 pairs of power- and slope-parameter estimates that emerge from the power-logit model. The mean, median and standard deviation of the $p$ estimates in those

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7 The results presented in this subsection were obtained with code written in the R programming language (R Core Team, 2024, v4.3.3) with RStudio (RStudio Team, 2020), and utilising the “mlogit” (Croissant, 2020), “optimx” (Nash and Varadhan, 2011), “plyr” (Wickham, 2011) and “tidyverse” (Wickham et al., 2019) packages/libraries.
Table 2: Goodness-of-fit comparison of the four models under different criteria.

| Model                        | Parameters | Log-Likelihood | AIC  | BIC  | Correct predictions with menu-specific estimates | Correct predictions with average menu estimates |
|------------------------------|------------|----------------|------|------|-------------------------------------------------|-----------------------------------------------|
| Logit with fixed outside option | 3         | 79             | 62   | 38   | 2067 35.6%                                      | 2113 36.4%                                     |
| Logit with random outside option | 3         | 14             | 10   | 7    | 2065 35.6%                                      | 2266 39.1%                                     |
| Power logit                  | 2         | 7              | 24   | 38   | 2414 41.6%                                      | 2627 45.3%                                     |
| Quadratic logit              | 1         | 0              | 4    | 17   | 1804 31.1%                                      | 1800 31.0%                                     |

Note: the random outside option in the second model was estimated on values drawn from a normal distribution with a mean and standard deviation of 5 and 1.5, and were restricted to lie in the 1–9 range and rounded to the nearest integer.

regressions are 1.51, 1.47 and 0.27, respectively. The slope-parameter estimates on the other hand have a mean, median and standard deviation of 0.43, 0.40 and 0.15, suggesting that the effect of a one-unit increase in a film’s rating is an approximately 53% increase in the odds of choosing that film over the alternative. For comparison, the mean/median and standard deviation in the slope estimates corresponding to the baseline logit with a fixed outside option are 0.58 and 0.15, respectively, pointing to an approximately 78% increase in the above-mentioned odds.

Interestingly, there is a negative correlation (Spearman $\rho = -0.33$) between the $\hat{p}$ and $\hat{\theta}$ estimates in these data. The fact that $\hat{p}$ tends to be lower at menus where $\hat{\theta}$ is higher, however, can indeed be interpreted intuitively through the lens of this model. Specifically, when $\hat{p}$ is high, the deferral frequency also tends to high. When deferrals are primarily caused by the relative undesirability of the two films, along the lines of the logit with an outside option, a higher value of the slope parameter would be expected, in line with the above finding whereby $\bar{\beta}_1 > \bar{\theta}$. This is so because, in this model, the marginal effect of a unitary change in a film’s rating is more likely to be high when both films have a low rating. But when deferrals are not primarily due to undesirability but, instead, are mainly caused by decision difficulty, then relatively low values of $\hat{\theta}$ could be observed not because of low but because of similar ratings and the harder comparison that such similarity entails.

The potential presence of such a channel is further supported by the negative correlation (Spearman $\rho = -0.17$) between the $\hat{p}$ estimates and average—across all subjects—absolute differences in ratings at the respective menus. The mean, median and standard deviation of this variable at the 100 menus are 2.25, 2.21 and 0.45, respectively. The bottom-right quarter of the scatter plot in Figure 5 reveals the presence of 31 menus with an estimated $p$ in excess of its median value of 1.47 and an average absolute difference in ratings between the two films at
each of these menus below its median of 2.25. The mean and median estimates of the power-logit slope parameter $\hat{\theta}$ at these 31 menus are 0.39, while the corresponding statistics in the remaining 69 menus are 0.46 and 0.42. The difference in the distribution of $\hat{\theta}$ between these two groups is statistically significant ($p = 0.044$; two-sided Mann-Whitney test) and corroborates this intuition and theoretical prediction.

We now turn to the results of the goodness-of-fit comparisons, which are summarized in Table 2. In particular, the logit with a fixed outside option performs better than the other three models in the majority of menus under both log-likelihood and AIC criteria, wile it is tied with the power-logit under the BIC criterion where each performs best at different sets of 38 menus. The quadratic logit on the other hand is best under BIC in nearly a fifth of all menus, followed by the logit with a random outside option in nearly a tenth. In terms of the proportion of correct predictions made at each menu under the different models’ corresponding estimates at that menu, the power logit is better (41.6%), followed by the baseline logit with a fixed or inferior outside option (both 35.6%) and by the quadratic logit (31.1%).

Importantly, the predictive ability of the power logit is even better than the other models’ in the “bird’s-eye view” specification where each model is compared against each other based on its average parameter estimates at the 100 menus. This is a relevant comparison if one is interested in making out-of-sample predictions for similar samples of decision makers at similar choice problems. Based on the average parameter values $\bar{p} = 1.51$ and $\bar{\gamma} = 0.43$, the power logit
now makes correct predictions 45.3% of the time. Notably, the respective predictions made by the quadratic and baseline logit with a fixed outside option are largely unchanged, but a notable improvement of 3.5 percentage points is seen in those made by logit with a random outside option (up to 39.1%).

Further light on the relevance of the behavioural channel that was discussed earlier in this section can now be shed by comparing the models’ fit in those menus where the average film ratings are high and low. This is relevant because the mechanism underpinning the logit with a fixed outside option suggests that choosing that option is more likely when the average rating is low. Intuitively, therefore, we would expect this model to provide a better fit in the latter group of menus compared to the power logit. To this end, we compare the two models’ AIC and BIC scores in the two groups of 50 menus with above- and below-median average total rating (the median value of this statistic is 11.44). In line with this intuition, the baseline logit performs better in a higher proportion of menus with a low than with a high rating under both criteria (AIC: 86% vs 60%; BIC: 52% vs 38%), with the difference in proportions being significant in the case of the former ($p = 0.006$; two-sided Fisher’s exact test).

The preceding analyses suggest that the proposed class of power-logit discrete-choice models with an endogenously determined menu-dependent value of the outside option can indeed provide meaningful explanatory gains relative to the baseline models with an exogenously or randomly set value for the outside option in free-choice datasets where decision makers were allowed to choose the avoidance/deferral outside option. At the same time, they also
demonstrate that the baseline model is descriptively relevant in many of the 100 decision problems in this dataset as well, and provide clarifications on which kinds of environments either model is likely to be more appropriate. We hope that these will be helpful to the empirical researcher who is interested in creating and analyzing similar datasets.

6 Power-Logit Duopolistic Competition in Price and Quality

We proceed with an illustration of the potential usefulness of the power-logit functional form in the analysis of oligopolistic markets when consumers potentially face comparison difficulties and may avoid/delay making an active choice.\textsuperscript{8} To this end, we consider a market where two profit-maximizing firms compete for a single consumer (equivalently, a unit mass of consumers) by offering a product that is differentiated in quality, \( q_i \), and price, \( p_i \). Producing a product of quality \( q_i \) costs \( q_i \) to firm \( i = 1, 2 \), while \( 0 \leq q_i \leq p_i \leq I \) and \( I > 0 \) denotes consumer income. Furthermore, utility from product \( (q_i, p_i) \) coincides with that product’s quality-price ratio:

\[
    u(q_i, p_i) = \frac{q_i}{p_i},
\]

This assumption further implies

\[
    u(q_i, p_i) \in [0, 1]
\]

for all \( (q_i, p_i) \). Such a “value-for-money” specification imposes intuitive positive and negative dependences of utility on quality and price, respectively, with the former being linear and the latter strictly convex. Moreover, while identifying utility with quality-price ratios as in (36) rather than with quality-price differences \( q_i - p_i \) appears to be a novel modelling assumption, it is consistent with some central implications of the behavioural choice model by Bordallo, Gennaioli, and Shleifer (2013) concerning consumer preferences for high quality-price ratio products, even though that model starts from very different primitives and features a quality-price difference utility function instead.

The two firms choose their products’ quality and price levels simultaneously and under complete information. The market share of product \( (q_i, p_i) \) at menu/strategy profile \( ((q_1, p_1), (q_2, p_2)) \) is determined by the power logit model

\[
    \rho((q_i, p_i), \{(q_j, p_j)\}_{j=1}^2) = \left( \frac{q_i}{p_i} \frac{q_i}{q_i + q_j} \right)\frac{1}{p_i + p_j},
\]

\textsuperscript{8}Piccione and Spiegler (2012), Spiegler (2015), Bachi and Spiegler (2018) and Gerasimou and Papi (2018) have recently suggested distinct approaches to study such markets.
where $s \geq 1$ and $s = 1$ in the baseline special case where there is no decision difficulty. Under the above assumptions, each firm $i = 1, 2$ solves

$$\max_{0 \leq q_i \leq p_i \leq I} \pi_i(q_i, p_i) := (p_i - q_i) \cdot \rho((q_i, p_i), \{(q_j, p_j)\}_{j=1}^2)$$

(38)

The strategic trade-off in this model, which applies both when $s = 1$ and $s > 1$, is that each firm wishes to increase its quality/price ratio in order to expand its market share, while at the same time also wishing to decrease it in order to enlarge its profit markup.

Turning to consumer welfare, taking into account that decision conflict can potentially drive the consumer out of the market altogether, and that -by A3- this would be undesirable, we consider a utilitarian welfare measure that weighs the possible utility levels at a given strategy profile by the probabilities that these utilities will actually be realized at that profile. We formalize this with the utilitarian consumer welfare function $W : \mathbb{R}^{4+} \to [0, 1]$ defined by

$$W((q_i, p_i), (q_j, p_j)) = \rho((q_i, p_i), \{(q_j, p_j)\}_{j=1}^2) \cdot u(q_i, p_i) + \rho((q_j, p_j), \{(q_j, p_j)\}_{j=1}^2) \cdot u(q_j, p_j).$$

This welfare indicator may be particularly relevant in cases where consumer surplus is equilibrium-invariant, as will turn out to be the case in the present environment.\footnote{A related measure that identifies welfare with the proportion of consumers who make an active choice was studied in Spiegler (2015), while Gerasimou and Papi (2018) introduced an index that is similar to $W$ but features instead the probability-weighted product variety that is associated with a strategy profile.}

Perhaps surprisingly, this duopolistic model leads to the following simple and intuitive equilibrium predictions:

**Proposition 10**

The power-logit equilibrium is $(q^*_1, p^*_1) = (q^*_2, p^*_2) = \left( \frac{sI}{2 + s}, I \right)$ and is associated with equilibrium expected profits $\pi^*_1 = \pi^*_2 = \frac{2^{1-s}}{2 + s} I$ and welfare $W^* = 2^{1-2s}$.

Thus, although the equilibrium pricing strategy features full surplus extraction irrespective of the value of the hesitation/resampling parameter $s$, the equilibrium quality level increases in $s$ at the rate $\frac{s}{2+s}$, starting at the low of $\frac{I}{2}$ in the baseline case of logit market shares and no consumer hesitation ($s = 1$), and approaching $I$ as $s$ becomes large. An intuitive interpretation of this fact is that decision conflict inevitably introduces a third “competitor” into the market, the outside option, that becomes more “powerful” as $s$ grows. The power logit predicts that the choice probability of the outside option goes down as the utility of one of the two products is unilaterally increased, while the choice probability of the comparatively more appealing product simultaneously goes up during the process. This in turn creates incentives for each firm to unilaterally increase its quality level relative to the baseline logit case. But
since increasing quality is costly, the above-mentioned strategic trade-off that is embedded in each firm’s profit function eventually kicks in and halts this increase at the above symmetric-equilibrium level.

Figure 7: Power-logit equilibrium quantities in the duopolistic game as the power parameter varies. (I normalized to 1).

Notably, while consumer surplus is zero in equilibrium because each firm’s profits turn out to be strictly increasing in its product’s price, consumer welfare changes in an interesting way as $s$ varies. In particular, despite the increase in the attainable utility level in equilibrium once firms best-respond to consumers’ hesitation and resampling, welfare decreases in $s$. This decrease is caused by the fact that in the power logit with two equally attractive products the consumer is more/equally/less likely to defer than to make an active choice when $s > 2/s = 2/s < 2$ and, conditional on doing the latter, equally likely to choose either of the two available products (Proposition 3). The implication of this in the present environment is that the higher utility level that the consumer receives in expectation under the equilibrium with some decision conflict ($s > 1$) is not sufficiently high to offset the lower utility level that they receive with certainty under the equilibrium with no conflict ($s = 1$). The firms’ profits, finally, also decrease when consumers are hesitant relative to the case where there they are not. This large decrease is intuitive and contributed by the reduced probability of the consumer choosing either product, as well as by the reduction in the firms’ profit margins that is brought about by the improvement in quality. Figure 7 illustrates these facts graphically when $I$ is normalized to 1.
7 Related Literature

As was also illustrated in the empirical application of Section 5, standard discrete choice models with an outside option that are based on random-utility maximization treat this option just like any other alternative and predict that it is more likely to be chosen when its utility is higher than that of all feasible active-choice options. Anderson, Palma, and Thisse (1992) and Hensher, Rose, and Greene (2015), for example, are textbook references that discuss this approach in detail. The class of models that we study in this paper differ radically from this (un-)desirability approach to modelling choice of the outside option. This is so because they predict that every active-choice alternative is always chosen when it is the only feasible one (cf the A3 axiom) and, in the structured models of Section 3, that the probability of opting out at larger menus increases as the feasible such alternatives become more equally appealing, in line with the relevant empirical evidence that was discussed.

Starting with Manzini and Mariotti (2014), moreover, several random choice models of limited attention that are also logically distinct from the modelling framework proposed in this paper have included an outside option as a model-closing assumption that requires this option to be chosen when no attention is paid to any of the feasible market alternatives. Because of this assumption, deferring/opting out becomes less likely in these models as menus become bigger. Horan (2019) recently clarified, however, how the deferral option can be removed from these models without affecting their general features and primary purpose, which is to explain active-choice decision making subject to cognitive/attention constraints.

Conceptually related to the sequential-sampling metaphor of Section 3 but formally distinct and with a different focus from that analysis are also the logit models with costly information sampling and rational inattention in Matějka and McKay (2015), Caplin, Dean, and Leahy (2022) and their extension to dynamic environments in Steiner, Stewart, and Matějka (2017). Important differences between this line of work and the present paper are the absence of an outside option in the former and the non-explicit accounting for a sampling cost in the quadratic/power-logit formulation in the latter.

Also distinct from the modelling framework of this paper in their foci, motivations, formal components and predictions are the “perception-adjusted” Luce model in Echenique, Saito, and Tserenjigmid (2018) and the “focal Luce” model in Kovach and Tserenjigmid (2022). In the former, active-choice probabilities are influenced by the alternatives’ position in a priority ordering. As the authors showed, the choice probability of the outside option is weakly higher in that model than what it would have been in the baseline Luce model with an outside option because the utility of that option in their model is the sum of a menu-independent and a menu-dependent part. The focal Luce model in on the other hand comprises a menu-independent...
utility function over alternatives, a menu-dependent focus function that assigns a consideration set to every menu, and a menu-dependent focality bias function that gives a “utility boost” to alternatives in the consideration set. Although not the paper’s main focus, an outside option—called “default” by the authors—can be introduced in that model and assigned a menu-independent value $u(o)$. This is an important difference to the modelling framework that we focus on in this paper where the outside option has a menu-dependent value. In addition, that modelling assumption leads to violations of the A3 axiom, thereby clarifying that the focal Luce model is also formally distinct from the decision-conflict logit class.

The deterministic choice-theoretic model that is most closely related to the decision-conflict logit class is that of dominant choice with incomplete preferences that was studied in a deferral-permitting deterministic environment in Gerasimou (2018, Section 2). This predicts that an active choice is made if and only if a most preferred feasible alternative exists according to a stable but generally incompletely preordered preference relation, and has found some empirical support in the experimental evidence reported in Costa-Gomes, Cueva, Gerasimou, and Tejišćák (2022). While this model’s predictions are in line with dominance-mediated “roller-coaster” choice-overload effects, however, it is unsuitable for thinking about relative-desirability effects, and is also less tractable than the class of decision-conflict logit for economic applications.

We note, finally, that this paper is related to a growing literature in decision theory and behavioural/experimental economics that studies the effects of complexity on decision-making quality in different choice domains. Several recent papers in this body of work primarily focus on the effects that complexity of the available alternatives or the general decision environment has on the emergence of behavioural deviations from rational choice models, such as choice reversals, time-inconsistent preferences, probability weighting, biased belief updating, imperfect perception of the options’ objective values, status quo bias, and on how agents might follow decision processes in such environments that might deviate from standard utility maximization and instead aim to minimize ex-post regret. This paper contributes to this literature by providing novel theoretical links between complexity and choice avoidance/deferral that is rooted in the potential difficulty to decide between the available active-choice alternatives that is modelled with an inflated/deflated relative appeal of the menu-dependent outside option, accompanied by empirical tests that point towards their potential descriptive relevance.

For example, Sarver, 2008; Fudenberg, Iijima, and Strzałeczki, 2015; Frick, 2016; Buturak and Evren, 2017; Cerreia-Vioglio, Dillenberger, Ortoleva, and Riella, 2019; Puri, 2024; Dean, Ravindran, and Stoye, 2022; Enke, Graeber, Oprea, and Yang, 2024.
8 Concluding Remarks

Understanding the “easy” and “hard” parts of people’s preference comparisons as these are revealed by their active-choice or choice-avoidance/delay decisions is important methodologically and also for practical applications such as effective choice architecture. The present paper contributes in this respect by introducing the tractable class of decision-conflict logit models and analysing an intuitively structured class of special cases thereof, namely the power logit and its quadratic-logit special member. These models assume that people can avoid/delay making an active choice and are more likely to select the choice-deferral outside option when it is harder for them to identify a best alternative from those available to them. This prediction is supported empirically and differs from the predictions of existing models where the outside option is chosen due to the undesirability of all feasible alternatives, limited attention, or other sources of bounded-rational behaviour. Thus, the class of models that we introduce and study in this paper complement existing ones in, as we demonstrated in Sections 5 and 6, empirically/theoretically relevant and applicable ways.

In conjunction with the insights from the relevant decision-making literature, our analysis suggests that decision-conflict logit models can help theoretical and applied empirical economists think formally and perhaps more realistically about strategic or non-strategic situations where decision makers: (i) are presented sufficiently small menus, so that limited-attention considerations are not pertinent; (ii) consider all feasible active-choice alternatives to be desirable/good enough, so that any one of them would be expected to be chosen if it were the only feasible item; (iii) find it difficult to compare these alternatives due to their complexity or due to potentially non-trivial trade-offs these generate; and (iv) are not forced to make an active choice.

Core References

Anderson, Simon P., Andrè de Palma, and Jacques-Francois Thisse (1992). Discrete Choice Theory of Product Differentiation. Cambridge, MA: MIT Press.

Apesteguia, Jose, Miguel A. Ballester, and Jay Lu (2017). “Single-Crossing Random Utility Models”. In: Econometrica 85, pp. 661–674.

Bachi, Benjamin and Ran Spiegler (2018). “Buridanic Competition”. In: Games and Economic Behavior 107, pp. 298–315.

Bhatia, Sudeep and Timothy L. Mullett (2016). “The Dynamics of Deferred Decision”. In: Cognitive Psychology 86, pp. 112–151.
Block, H. D. and J. Marschak (1960). “Random Orderings and Stochastic Theories of Response”. In: *Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling*. Ed. by I. Olkin. Stanford, CA: Stanford University Press, pp. 97–132.

Bordallo, Pedro, Nicola Gennaioli, and Andrei Shleifer (2013). “Salience and Consumer Choice”. In: *Journal of Political Economy* 121, pp. 803–843.

Buturak, Gökhan and Özgür Evren (2017). “Choice Overload and Asymmetric Regret”. In: *Theoretical Economics* 12, pp. 1029–1056.

Caplin, Andrew, Mark Dean, and John Leahy (2022). “Rationally Inattentive Behavior: Characterizing and Generalizing Shannon Entropy”. In: *Journal of Political Economy* 130, pp. 1676–1715.

Carroll, Gabriel D., James J. Choi, David Laibson, Brigitte C. Madrian, and Andrew Metrick (2009). “Optimal Defaults and Active Decisions”. In: *Quarterly Journal of Economics* 124, pp. 1639–1674.

Cerreia-Vioglio, Simone, David Dillenberger, Pietro Ortoleva, and Gil Riella (2019). “Deliberately Stochastic”. In: *American Economic Review* 109, pp. 2425–2445.

Chernev, Alexander, Ulf Bökenholt, and Joseph Goodman (2015). “Choice Overload: A Conceptual Review and Meta-Analysis”. In: *Journal of Consumer Psychology* 25, pp. 333–358.

Costa-Gomes, Miguel, Carlos Cueva, Georgios Gerasimou, and Matus Tejišćák (2022). “Choice, Deferral and Consistency”. In: *Quantitative Economics* 13, pp. 1297–1318.

Dean, Mark, Dilip Ravindran, and Jörg Stoye (2022). “A Better Test of Choice Overload”. In: *Working Paper*.

Dhar, Ravi (1997). “Consumer Preference for a No-Choice Option”. In: *Journal of Consumer Research* 24, pp. 215–231.

Echenique, Federico, Kota Saito, and Gerelt Tserenjigmid (2018). “The Perception-Adjusted Luce Model”. In: *Mathematical Social Sciences* 93, pp. 67–76.

Enke, Benjamin, Thomas Graeber, Ryan Oprea, and Jeffrey Yang (2024). “Behavioral Attenuation”. In: *Working Paper*.

Frick, Mira (2016). “Monotone Threshold Representations”. In: *Theoretical Economics* 11, pp. 757–772.

Fudenberg, Drew, Ryota Iijima, and Tomasz Strzalecki (2015). “Stochastic Choice and Revealed Perturbed Utility”. In: *Econometrica* 83, pp. 2371–2409.

Gerasimou, Georgios (2018). “Indecisiveness, Undesirability and Overload Revealed Through Rational Choice Deferral”. In: *Economic Journal* 128, pp. 2450–2479.

Gerasimou, Georgios and Mauro Papi (2018). “Duopolistic Competition with Choice-Overloaded Consumers”. In: *European Economic Review* 101, pp. 330–353.
Greene, William H. and David A. Hensher (2010). *Modelling Ordered Choices*. New York: Cambridge University Press.

Hensher, David A., John M. Rose, and William H. Greene (2015). *Applied Choice Analysis*. New York: Cambridge University Press.

Horan, Sean (2019). “Random Consideration and Choice: A Case Study of Default Options”. In: *Mathematical Social Sciences* 102, pp. 73–84.

Iyengar, Sheena S. and Mark R. Lepper (2000). “When Choice is Demotivating: Can One Desire Too Much of a Good Thing?” In: *Journal of Personality and Social Psychology* 79, pp. 995–1006.

Iyengar, Sheena Sethi, Gur Huberman, and Wei Jiang (2004). “How Much Choice is Too Much? Contributions to 401(k) Retirement Plans”. In: *Pension Design and Structure: New Lessons from Behavioral Finance*. Ed. by Olivia S. Mitchell and Stephen P. Utkus. New York: Oxford University Press, pp. 83–96.

Knops, Anouk M., Dirk T. Ubbink, Dink A. Legemate, Lukas J. Stalpers, and Patrick M. Bossuyt (2013). “Interpreting Patient Decisional Conflict Scores: Behavior and Emotions in Decisions about Treatment”. In: *Medical Decision Making* 33, pp. 78–84.

Kovach, Matthew and Gerelt Tserenjigmid (2022). “The Focal Luce Model”. In: *American Economic Journal: Microeconomics* 14, pp. 378–413.

Luce, R. Duncan (1959). *Individual Choice Behavior: A Theoretical Analysis*. New York, NY: Wiley.

Luce, R. Duncan and Patrick Suppes (1965). “Preference, Utility and Subjective Probability”. In: *Handbook of Mathematical Psychology, Volume 3*. Ed. by R. D. Luce, R. R. Bush, and E. H. Galanter. New York: Wiley, pp. 249–410.

Manzini, Paola and Marco Mariotti (2014). “Stochastic Choice and Consideration Sets”. In: *Econometrica* 83, pp. 1153–1176.

Matějka, Filip and Alisdair McKay (2015). “Rational Inattention to Discrete Choices: A New Foundation for the Multinomial Logit Model”. In: *American Economic Review* 105, pp. 272–298.

McFadden, Daniel (1973). “Conditional Logit Analysis of Qualitative Choice Behavior”. In: *Frontiers in Econometrics*. Ed. by P. Zarembka. New York: Academic Press, pp. 105–142.

Piccione, Michele and Ran Spiegler (2012). “Price Competition under Limited Comparability”. In: *Quarterly Journal of Economics* 127, pp. 1–39.

Puri, Indira (2024). “Simplicity and Risk”. In: *Journal of Finance*. forthcoming.

Redelmeier, D. A. and E. Shafir (1995). “Medical Decision Making in Situations that Offer Multiple Alternatives”. In: *Journal of the American Medical Association* 273, pp. 302–305.
Sarver, Todd (2008). “Anticipating Regret: Why Fewer Options May Be Better”. In: *Econometrica* 76, pp. 263–305.

Scheibehenne, Benjamin, Rainer Greifeneder, and Peter M. Todd (2010). “Can There Ever Be Too Many Options? A Meta-Analytic Review of Choice Overload”. In: *Journal of Consumer Research* 37, pp. 409–425.

Spektor, Mikhail S., Sebastian Gluth, Laura Fontanesi, and Jörg Rieskamp (2019). “How Similarity Between Choice Options Affects Decisions from Experience: The Accentuation-of-Differences Model”. In: *Psychological Review* 126, pp. 52–88.

Spiegler, Ran (2015). “On the Equilibrium Effects of Nudging”. In: *Journal of Legal Studies* 44, pp. 389–416.

Steiner, Jakub, Colin Stewart, and Filip Matějka (2017). “Rational Inattention Dynamics: Inertia and Delay in Decision Making”. In: *Econometrica* 85, pp. 521–553.

Stoye, Jörg (2019). “Revealed Stochastic Preference: A One-Paragraph Proof and Generalization”. In: *Economics Letters* 177, pp. 66–68.

Strzalecki, Tomasz (2024). *Stochastic Choice Theory*. Econometric Society Monographs. Cambridge: Cambridge University Press.

Train, Kenneth E. (2009). *Discrete Choice Methods with Simulation*. 2nd. Cambridge: Cambridge University Press.

White, Chris M., Ulrich Hoffrage, and Nils Reisen (2015). “Choice Deferral Can Arise from Absolute Evaluations or Relative Comparisons”. In: *Journal of Experimental Psychology: Applied* 21, pp. 140–157.

**Software References**

Croissant, Yves (2020). “mlogit: Random Utility Models in R”. In: *Journal of Statistical Software* 95.

Nash, John C. and Ravi Varadhan (2011). “Unifying Optimization Algorithms to Aid Software System Users: optimx for R”. In: *Journal of Statistical Software* 43.9, pp. 1–14. DOI: 10.18637/jss.v043.i09.

R Core Team (2024). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing. Vienna, Austria. URL: [https://www.R-project.org/](https://www.R-project.org/).

RStudio Team (2020). *RStudio: Integrated Development Environment for R*. RStudio, PBC. Boston, MA. URL: [http://www.rstudio.com/](http://www.rstudio.com/).

Wickham, Hadley (2011). “The Split-Apply-Combine Strategy for Data Analysis”. In: *Journal of Statistical Software* 40.1, pp. 1–29. URL: [https://www.jstatsoft.org/v40/i01/](https://www.jstatsoft.org/v40/i01/).
Appendix A: Proofs

Proof of Proposition 1.

In the main text. ■

Proof of Proposition 2.

If $D$ is monotonic and $a \in B \subset A$, then since $D(A) \geq D(B)$ holds by assumption and $u(A) > u(B)$ [see (15)] is also true by the postulated strict positivity of $u$, it immediately follows that

$$
\rho(a, A) = \frac{u(a)}{u(A) + D(A)} < \frac{u(a)}{u(B) + D(B)} = \rho(a, B)
$$

■

Proof of Proposition 3.

For the second claim, suppose $\widehat{u}(a) = \widehat{u}(b) := c$ for all $a, b \in A$. By (8), $\rho(o, A) = 1 - \sum_{a \in A} \left( \frac{\widehat{u}(a)}{\sum_{b \in A} u(b)} \right)^p = 1 - |A| \left( \frac{c}{|A|c} \right)^p = 1 - |A|^{1-p}$. Thus, $\rho(o, A)$ is independent of the specific $\widehat{u}$ values at $A$ whenever these values coincide. This readily implies that, viewed as the function

$$
\rho(o, A; \widehat{u}(a_1), \ldots, \widehat{u}(a_{|A|})) = \left( \frac{\left| A \right|}{\sum_{i=1}^{\left| A \right|} \widehat{u}(a_i)^p} \right) - \sum_{i=1}^{\left| A \right|} \widehat{u}(a_i)^p \left( \frac{\left| A \right|}{\sum_{i=1}^{\left| A \right|} \widehat{u}(a_i)^p} \right)^p,
$$

(39)

$\rho(o, A)$ has any $|A|$-vector of $\widehat{u}$ values ($c, \ldots, c$) as a critical point that trivially satisfies both the first- and second-order conditions of local optimality. Yet, because the determinant of the Hessian matrix at any such point is zero, it is not immediately clear if this point is a local maximizer. To show that this is indeed so, by symmetry it suffices to consider marginal deviations in a single direction; say, an $\epsilon$ increase or decrease in $\widehat{u}(a_1)$. Since $\widehat{u}(a_i) = \widehat{u}(a_j)$
\[ c > 0, \text{ by assumption, this and (39) yield} \]
\[ \rho(o, A; c + \epsilon, c, \ldots, c) = \frac{|A|c + \epsilon) - (|A| - 1)c - (c + \epsilon)}{(|A|c + \epsilon)^p} \]
\[ = 1 - \frac{|A| - 1)c + (c + \epsilon)}{(|A|c + \epsilon)^p}, \]

Suppose to the contrary that this weakly exceeds \(1 - |A|^{-p}\). Without loss of generality, write \(\epsilon := mc\) for some small \(m > 0\) or \(m < 0\). We have
\[ |A|^{-p} \geq \frac{|A| - 1)c + (c + \epsilon)}{(|A|c + \epsilon)^p} \]
\[ = \frac{|A| - 1)c + (c + mc)}{(|A|c + mc)^p} \]

To ease notation, write \(n := |A|\). Rearranging, observe that the above is true if and only if
\[ n^{1-p}(n + m)p \geq (n - 1)c + (1 + m)p, \]
which in turn is true if and only if
\[ n^{1-p}(n + m) \geq n - 1 + (1 + m)^p. \]

Rearranging further, we get
\[ n \left( \frac{n + m}{n} \right)^p \geq n + 1 + (m + 1)^p \]
from which we finally obtain
\[ \left( \frac{1 + m}{n} \right)^p - \frac{n + 1 + (m + 1)^p}{n} \geq 0 \]

Taking the limit as \(m \to 0\) and rearranging leads to \(n \geq n + 2\), which is impossible. We have therefore established that the above critical point is indeed a local maximizer of \(\rho(o, A)\).

We proceed to showing that it is in fact a global maximizer, thereby concluding the proof. To this end, notice first that \(\rho(o, A) < 1\), by strict positivity of \(\hat{u}\). Suppose to the contrary that there is a non-constant \(|A|\)-vector \((\hat{u}(a_1), \ldots, \hat{u}(a_{|A|}))\) that satisfies the first-order conditions of optimality that are derived from (39). Differentiating and rearranging pins down these
conditions to
\[
\hat{u}(a_i^*) = \left( \frac{\sum_{j \neq i} \hat{u}(a_j)}{\sum_{j \neq i} \hat{u}(a_j)^p} \right)^{1 \over 1 - p}, \quad i = 1, \ldots, |A| \tag{40}
\]
Solving this system leads to
\[
\hat{u}(a_1^*) = \hat{u}(a_2^*) = \ldots = \hat{u}(a_{|A|}^*),
\]
contradicting the supposed non-constancy of the postulated alternative local maximizer. It follows that \( \rho(o, A) \) is maximized at any constant \( |A| \)-vector only. From this and the second claim that was established earlier it now follows that this maximum is indeed given by \( 1 - |A|^{1-p} \), as per the first claim.

**Proof of Corollary 4.**

In the main text.

**Proof of Proposition 5.**

Let \( X := \{a_1, \ldots, a_k\} \) and suppose \( \rho = (u, D) \) is a decision-conflict logit on \( X \). Consistent with (3), and without loss of generality, we may let
\[
u(a_i) := \frac{\rho(a_i, X)}{\rho(a_1, X)}.
\]
Note that \( \rho \) has the quadratic-logit property at the binary menus of \( X \) if and only if there is a vector \((\hat{u}(a_1), \ldots, \hat{u}(a_k))\) that solves the system
which in turn imply (41) and (44) we have

\[ \begin{pmatrix} \hat{u}(a_1)^2 \\ \hat{u}(a_2)^2 \\ \vdots \\ \hat{u}(a_k)^2 \\ 2\hat{u}(a_1)\hat{u}(a_2) \\ 2\hat{u}(a_1)\hat{u}(a_3) \\ \vdots \\ 2\hat{u}(a_{k-1})\hat{u}(a_k) \end{pmatrix} = \begin{pmatrix} u(a_1) \\ u(a_2) \\ \vdots \\ u(a_k) \\ D(\{a_1, a_2\}) \\ D(\{a_1, a_3\}) \\ \vdots \\ D(\{a_{k-1}, a_k\}) \end{pmatrix} \equiv \begin{pmatrix} 1 \\ \rho(a_2, X) \\ \rho(a_1, X) \\ \vdots \\ \rho(a_k, X) \\ \rho(a_1, X) \\ \rho(a_1, \{a_1, a_2\}) \\ \rho(a_1, \{a_1, a_3\}) \\ \vdots \\ \rho(a_{k-1}, \{a_{k-1}, a_k\}) \end{pmatrix}, \tag{41} \]

where we’ve simplified the subset of equations that pertain to \( D \) in the last column vector of (41) by making a particular use of the fact that

\[ \frac{\rho(A, X)}{\rho(A, A)} = \frac{\rho(a, X)}{\rho(a, A)} \text{ for any } a \in A \in \mathcal{M}, \tag{42} \]

which, in turn, is a straightforward implication of A2.

Next, we observe that (41) reduces to

\[ \begin{align*}
 u(a_1) &= 1, \\
 u(a_i) &= \frac{D(\{a_1, a_i\})^2}{4}, \quad 1 \neq i \leq k,
\end{align*} \]

which in turn imply

\[ \begin{align*}
 \hat{u}(a_1) &= 1, \\
 \hat{u}(a_i) &= \frac{D(\{a_1, a_i\})}{2}, \quad 1 \neq i \leq k. \tag{43} \\
\end{align*} \]

By (41) and (44) we have

\[ D(\{a_1, a_j\}) = \frac{\rho(a, \{a_1, a_j\})}{\rho(a_1, \{a_1, a_j\})} \tag{45} \]

for all \( j \neq 1 \). By (45) and the above we also get

\[ \hat{u}(a_j) = \frac{1}{2} \frac{\rho(a_1, \{a_1, a_j\})}{\rho(a_1, \{a_1, a_j\})} \tag{46} \]
for all such $j$. By (41) and A2, moreover, we also have

$$
\hat{u}(a_j)^2 = \frac{\rho(a_j, X)}{\rho(a_1, X)} = \frac{\rho(a_j, \{a_1, a_j\})}{\rho(a_1, \{a_1, a_j\})}
$$

(47)

Therefore, by (46) and (47), consistency of (41) is achieved iff

$$
\frac{1}{4} \left( \frac{\rho(o, \{a_1, a_j\})}{\rho(a_1, \{a_1, a_j\})} \right)^2 = \frac{\rho(a_j, \{a_1, a_j\})}{\rho(a_1, \{a_1, a_j\})},
$$

which is easily seen to be equivalent to A4.

\[\square\]

**Proof of Proposition 6.**

Let $\rho = (u, D)$ be a decision-conflict logit. By (3) and (4), there are $\alpha > 0$ and $z \in X$ such that

$$
D(A) = \alpha \frac{\rho(o, A)}{1 - \rho(o, A)} \sum_{b \in A} \frac{\rho(b, X)}{\rho(z, X)} = \alpha \frac{\rho(A, X)}{\rho(z, X)} \frac{\rho(o, A)}{1 - \rho(o, A)},
$$

(48)

and

$$
D(\{a, b\}) = \alpha \frac{\rho(\{a, b\}, X)}{\rho(z, X)} \frac{\rho(o, \{a, b\})}{1 - \rho(o, \{a, b\})}.
$$

(49)

Now recall that A5 is satisfied if and only if, for all $A, B \in \mathcal{M}$ with $B \supset A$, and for all distinct $a, b \in A$,

$$
\frac{\rho(A, B)}{\rho(A, A)} = \sum_{a, b \in A} \frac{\rho(\{a, b\}, B)}{\rho(\{a, b\}, \{a, b\})} \frac{\rho(o, \{a, b\})}{\rho(o, A)}.
$$

We also have $\rho(A, X) = \kappa \rho(A, B)$ for some $\kappa > 0$. By A2 and (42), moreover, we have $\rho(\{a, b\}, X) = \kappa \rho(\{a, b\}, B)$ for all $\{a, b\} \subset B$ too. In light of this fact, the above can be written equivalently as

$$
\frac{\rho(A, X)}{\rho(A, A)} = \sum_{a, b \in A} \frac{\rho(\{a, b\}, X)}{\rho(\{a, b\}, \{a, b\})} \frac{\rho(o, \{a, b\})}{\rho(o, A)}.
$$
which, in turn, can be rewritten further as

\[ \rho(A, X) \cdot \frac{\rho(o, A)}{1 - \rho(o, A)} = \sum_{a,b \in A} \rho(\{a, b\}, X) : \frac{\rho(o, \{a, b\})}{1 - \rho(o, \{a, b\})}. \]  

(50)

Upon multiplying both sides of (50) by \( \frac{\alpha}{\rho(z, X)} \) and rearranging again, this becomes equivalent to

\[ \alpha \frac{\rho(A, X)}{\rho(z, X)} \cdot \frac{\rho(o, A)}{1 - \rho(o, A)} = \alpha \sum_{a,b \in A} \frac{\rho(\{a, b\}, X)}{\rho(z, X)} : \frac{\rho(o, \{a, b\})}{1 - \rho(o, \{a, b\})}. \]  

(51)

Substituting (48) and (49) into (51), finally, shows that (50) (hence A6) is equivalent to

\[ D(A) = \sum_{a,b \in A} D(\{a, b\}). \]

Proof of Theorem 7.

Recall that: (i) by Proposition 1, A1–A2 are equivalent to \( \rho \) being a Luce model \((u, D)\) with a general outside option; (ii) by Proposition 5, \( \rho = (u, D) \) is a quadratic logit on \( X \) and \( B \subset M \) if and only if it also satisfies A3-A4; (iii) by Proposition 6, \( \rho = (u, D) \) is additive if and only if it satisfies A5. Now, since \( \rho \) is additive by virtue of the postulated A5, we observe that \( \rho \) is, in fact, a quadratic logit on \( X \) and \( M \) because, by (4) and (12), expanding system (41) by including the equations corresponding to non-binary menus is redundant because these are linear combinations of the linearly independent equations in (41). Therefore, an additive \( \rho = (u, D) \) on \( X \) and \( M \) is a quadratic logit if and only if (41) is solvable. By Proposition 5 in turn, this is true if and only if \( \rho = (u, D) \) satisfies A4. Therefore, by Propositions 1–6 a random non-forced choice model \( \rho \) on \( X \) and \( M \) is a quadratic logit if and only if it satisfies A1–A5.
Proof of Proposition 8.

Suppose $A \supset B$. We have

$$\rho(o, A) \leq \rho(o, B) \iff \frac{D(A)}{u(A) + D(A)} \leq \frac{D(B)}{u(B) + D(B)} \iff \frac{D(A)}{u(B) + u(A \setminus B) + D(A)} \leq \frac{D(B)}{u(B) + D(B)} \iff \frac{D(A)}{D(B)} \leq \frac{u(B) + u(A \setminus B)}{u(B)} \iff \frac{D(A) - D(B)}{D(B)} \leq \frac{u(A \setminus B)}{u(B)} \iff \frac{D(A) - D(B)}{D(B)} \leq \frac{u(A) - u(B)}{u(B)}.$$

Proof of Proposition 9.

To dispense with the absolute value sign, assume without loss of generality that $\hat{u}(a) > \hat{u}(b)$ and $\hat{u}(c) > \hat{u}(d)$. We will first show that (17) holds under either of the postulated conditions. Following that, we will show that (17) $\iff$ (18), also under either condition.

Starting with (17), consider first the case where $\hat{u}(a) + \hat{u}(b) = \hat{u}(c) + \hat{u}(d)$. Denote this common sum by $s$. We have $\rho(o, \{a, b\}) > \rho(o, \{c, d\}) \iff \frac{\hat{u}(c) + \hat{u}(d)}{s^p} > \frac{\hat{u}(a) + \hat{u}(b)}{s^p}$. This is equivalent to

$$\hat{u}(c)^p + \hat{u}(d)^p > \hat{u}(a)^p + \hat{u}(b)^p \quad (52)$$

Suppose to the contrary that

$$\hat{u}(a) - \hat{u}(b) \geq \hat{u}(c) - \hat{u}(d). \quad (53)$$

This and the postulated equality yield $\hat{u}(a) \geq \hat{u}(c)$. Furthermore, this and (52) jointly imply $\hat{u}(a) > \hat{u}(c)$ and $\hat{u}(d) > \hat{u}(b)$. Thus,

$$\hat{u}(a) > \hat{u}(c) > \hat{u}(d) > \hat{u}(b) \quad (54)$$

In view of (54), observe that the terms $\frac{\hat{u}(a) - \hat{u}(c)}{\hat{u}(a) - \hat{u}(b)}$ and $\frac{\hat{u}(c) - \hat{u}(b)}{\hat{u}(a) - \hat{u}(b)}$ are convex weights. Hence, since
\( \hat{u}(\cdot) \mapsto \hat{u}(\cdot)^p \) is a strictly convex function, we have

\[
\left( \frac{\hat{u}(a) - \hat{u}(c)}{\hat{u}(a) - \hat{u}(b)} \right) \hat{u}(b)^p + \left( \frac{\hat{u}(c) - \hat{u}(b)}{\hat{u}(a) - \hat{u}(b)} \right) \hat{u}(a)^p > \left[ \frac{\hat{u}(a) - \hat{u}(c)}{\hat{u}(a) - \hat{u}(b)} \right] \hat{u}(b) + \left[ \frac{\hat{u}(c) - \hat{u}(b)}{\hat{u}(a) - \hat{u}(b)} \right] \hat{u}(a) \right]^p
\]

\[= \hat{u}(c)^p, \quad (55)\]

\[
\left( \frac{\hat{u}(a) - \hat{u}(d)}{\hat{u}(a) - \hat{u}(b)} \right) \hat{u}(b)^p + \left( \frac{\hat{u}(d) - \hat{u}(b)}{\hat{u}(a) - \hat{u}(b)} \right) \hat{u}(a)^p > \left[ \frac{\hat{u}(a) - \hat{u}(d)}{\hat{u}(a) - \hat{u}(b)} \right] \hat{u}(b) + \left[ \frac{\hat{u}(d) - \hat{u}(b)}{\hat{u}(a) - \hat{u}(b)} \right] \hat{u}(a) \right]^p
\]

\[= \hat{u}(d)^p \quad (56)\]

Adding (55) to (56) and recalling that \( \hat{u}(a) + \hat{u}(b) = \hat{u}(c) + \hat{u}(d) = s \) yields

\[
\hat{u}(c)^p + \hat{u}(d)^p < \left( \frac{2\hat{u}(a) - \hat{u}(c) - \hat{u}(d)}{\hat{u}(a) - \hat{u}(b)} \right) \hat{u}(b)^p + \left( \frac{\hat{u}(c) + \hat{u}(d) - 2\hat{u}(b)}{\hat{u}(a) - \hat{u}(b)} \right) \hat{u}(a)^p
\]

\[= \left( \frac{2\hat{u}(a) - s}{\hat{u}(a) - \hat{u}(b)} \right) \hat{u}(b)^p + \left( \frac{s - 2\hat{u}(b)}{\hat{u}(a) - \hat{u}(b)} \right) \hat{u}(a)^p
\]

\[= \hat{u}(b)^p + \hat{u}(a)^p,
\]

which contradicts (52). Thus,

\[
\hat{u}(a) - \hat{u}(b) < \hat{u}(c) - \hat{u}(d) \quad (57)
\]

holds. Conversely, suppose (57) is true. This and the postulated equality together imply

\[
\hat{u}(c) > \hat{u}(a) > \hat{u}(b) > \hat{u}(d) \quad (58)
\]

Applying the preceding convexity argument using (58) yields (52), thereby completing the proof that (17) holds under the first postulate.

We now show that (17) is true when \( u(a) + u(b) = u(c) + u(d) \) or, equivalently,

\[
\hat{u}(a)^p + \hat{u}(b)^p = \hat{u}(c)^p + \hat{u}(d)^p \quad (59)
\]

holds instead. Let \( t \) denote this common sum. We have \( \rho(o, \{a, b\}) > \rho(o, \{c, d\}) \iff \frac{\hat{u}(c)^p + \hat{u}(d)^p}{\hat{u}(a) + \hat{u}(b)} > \frac{\hat{u}(a)^p + \hat{u}(b)^p}{\hat{u}(a) + \hat{u}(b)} \iff \frac{\hat{u}(a) + \hat{u}(b)}{\hat{u}(a) + \hat{u}(b)} > \frac{\hat{u}(a) + \hat{u}(b)}{\hat{u}(a) + \hat{u}(b)} \iff (\hat{u}(a) + \hat{u}(b))^p > (\hat{u}(a) + \hat{u}(b))^p. \) This is true if and only if \( \hat{u}(a) + \hat{u}(b) > \hat{u}(c) + \hat{u}(d) \), which is equivalent to

\[
\hat{u}(a) - \hat{u}(c) > \hat{u}(d) - \hat{u}(b) \quad (60)
\]

Suppose to the contrary that

\[
\hat{u}(a) - \hat{u}(b) \geq \hat{u}(c) - \hat{u}(d) \quad (61)
\]
From (60) and (61) we get \( \hat{u}(a) > \hat{u}(c) \) and \( \hat{u}(b) < \hat{u}(d) \). Thus,

\[
\hat{u}(a) > \hat{u}(c) > \hat{u}(d) > \hat{u}(b) > 0
\]  

(62)

By (60), (61) and convexity of \( \hat{u}(\cdot) \mapsto \hat{u}(\cdot)^p \) we have

\[
\hat{u}(a)^p - \hat{u}(c)^p > \hat{u}(d)^p - \hat{u}(b)^p,
\]

which contradicts (59). Hence, (57) holds. Conversely, suppose (57) is true and assume to the contrary that (60) is violated, i.e.

\[
\hat{u}(a) - \hat{u}(c) \leq \hat{u}(d) - \hat{u}(b)
\]

(63)

Rearranging (60),

\[
\hat{u}(a) - \hat{u}(c) < \hat{u}(b) - \hat{u}(d)
\]

(64)

By (63) + (64) we obtain \( \hat{u}(a) < \hat{u}(c) \). This and (59) in turn imply \( \hat{u}(b) < \hat{u}(d) \). Hence,

\[
\hat{u}(c) > \hat{u}(a) > \hat{u}(b) > \hat{u}(d) > 0
\]

(65)

By (63) we have

\[
\hat{u}(c) - \hat{u}(a) \leq \hat{u}(b) - \hat{u}(d)
\]

(66)

Finally, (65), (66) and convexity of \( \hat{u}(\cdot) \mapsto \hat{u}(\cdot)^p \) jointly lead to the same contradiction as above. This completes the proof that (17) holds under the second postulate as well.

We now show that (18) holds under either of the postulated conditions. That is, we verify that \( \hat{u}(a) - \hat{u}(b) < \hat{u}(c) - \hat{u}(d) \Leftrightarrow \hat{u}(a)^p - \hat{u}(b)^p < \hat{u}(c)^p - \hat{u}(d)^p \). Suppose first that \( \hat{u}(a) + \hat{u}(b) = \hat{u}(c) + \hat{u}(d) \). Let \( \hat{u}(a) - \hat{u}(b) < \hat{u}(c) - \hat{u}(d) \) be true and assume to the contrary that

\[
\hat{u}(a)^p - \hat{u}(b)^p \geq \hat{u}(c)^p - \hat{u}(d)^p
\]

(67)

The former two assumptions imply \( \hat{u}(a) < \hat{u}(c) \), \( \hat{u}(b) > \hat{u}(d) \) and therefore

\[
\hat{u}(c) > \hat{u}(a) > \hat{u}(b) > \hat{u}(d)
\]

(68)

Using again the convexity argument that revolved around (55) and (56) we get

\[
\hat{u}(a)^p + \hat{u}(b)^p > \hat{u}(c)^p + \hat{u}(d)^p
\]

(69)
By (67) and (69) we now obtain \( \hat{u}(a) > \hat{u}(c) \), which is a contradiction. Conversely, suppose \( \hat{u}(a)^p - \hat{u}(b)^p < \hat{u}(c)^p - \hat{u}(d)^p \) and assume to the contrary that \( \hat{u}(a) - \hat{u}(b) \geq \hat{u}(c) - \hat{u}(d) \). This and \( \hat{u}(a) + \hat{u}(b) = \hat{u}(c) + \hat{u}(d) \) jointly imply \( \hat{u}(a) > \hat{u}(c) \) and \( \hat{u}(b) < \hat{u}(d) \). Thus, we have \( \hat{u}(a) > \hat{u}(c) > \hat{u}(d) > \hat{u}(b) \). Using the above convexity argument once again we obtain \( \hat{u}(c)^p + \hat{u}(d)^p < \hat{u}(a)^p + \hat{u}(b)^p \). Subtracting \( \hat{u}(a)^p - \hat{u}(b)^p < \hat{u}(c)^p - \hat{u}(d)^p \) from this inequality yields \( \hat{u}(b) > \hat{u}(d) \), a contradiction.

Finally, we establish (18) under the postulate

\[
\hat{u}(a)^p + \hat{u}(b)^p = \hat{u}(c)^p + \hat{u}(d)^p
\]  

(70)

Let

\[
\hat{u}(a) - \hat{u}(b) < \hat{u}(c) - \hat{u}(d),
\]

(71)

and again assume to the contrary that (67) is true. By (67) + (64) we get \( \hat{u}(a) \geq \hat{u}(c) \). This and (71) implies \( \hat{u}(b) > \hat{u}(d) \). But \( \hat{u}(a) \geq \hat{u}(c) \) and (70) also implies \( \hat{u}(b) \leq \hat{u}(d) \). This is impossible. Conversely, suppose \( \hat{u}(a)^p - \hat{u}(b)^p < \hat{u}(c)^p - \hat{u}(d)^p \). This and the postulated \( \hat{u}(a)^p + \hat{u}(b)^p = \hat{u}(c)^p + \hat{u}(d)^p \) jointly imply \( \hat{u}(c) > \hat{u}(a) \) and \( \hat{u}(b) < \hat{u}(d) \). Together with the without-loss initial assumption whereby \( \hat{u}(a) > \hat{u}(b) \) and \( \hat{u}(c) > \hat{u}(d) \), this in turn implies \( \hat{u}(c) > \hat{u}(a) > \hat{u}(d) > \hat{u}(b) \). Assume to the contrary that \( \hat{u}(a) - \hat{u}(b) \geq \hat{u}(c) - \hat{u}(d) \). This is equivalent to \( \hat{u}(d) - \hat{u}(b) \geq \hat{u}(c) - \hat{u}(a) > 0 \). Rearranging (70), we also have \( \hat{u}(b)^p - \hat{u}(d)^p = \hat{u}(c)^p - \hat{u}(a)^p \). Since \( \hat{u}(\cdot) \mapsto \hat{u}(\cdot)^p \) is a strictly increasing function, it follows from the above that the left hand side of this equation is negative while the right hand positive. This is a contradiction. Thus, (18) holds in this case too.

**Proof of Proposition 10.**

Firm \( i = 1,2 \) maximizes \( \pi_i \) with respect to \( q_i \) and \( p_i \) taking the choices of the other firm \( j \neq i \) as given. Differentiating \( \pi_i \) with respect to \( p_i, q_i \) and simplifying we get

\[
\frac{\partial \pi_i}{\partial p_i} = \left( \frac{pj_{ij}}{(pj_{ij} + p_iq_j)} \right)^{s} \frac{(pj_{ij} + q_j(p_i - p_is + q_is))}{pj_{ij} + p_iq_j},
\]

\[
\frac{\partial \pi_i}{\partial q_i} = \left( \frac{pj_{ij}}{(pj_{ij} + p_iq_j)} \right)^{(1+s)} \frac{(pj_{ij}q_i^2 + p_iq_j(q_i - p_is + q_is))}{pj_{ij}^2},
\]
Setting the two equations equal to zero yields the first-order conditions

\[ p_i^* = \frac{q_i(p_j + q_j s)}{q_j(s - 1)}, \quad (72) \]

\[ q_i^* = \frac{p_i \sqrt{q_j \sqrt{q_j + 4p_j s + 2q_j s + q_j s^2 - p_i q_j - p_i q_j s^2 - p_i q_j}}}{2p_j} \quad (73) \]

It can be checked upon rearranging these conditions in \( \frac{q_i}{p_i} \) form (which, in particular, is a non-negative term) and simplifying that they cannot be satisfied simultaneously under the assumption that \( p_i, q_i, s \geq 0 \) and \( I > 0 \). This implies that there is no equilibrium where firms choose interior strategies. Since \( q_i^* \leq p_i^* \) must hold, this fact and (72), (73) together imply either \( p_i^* = 0 \) or \( p_i^* = I \). Because the latter (former) case is associated with a strictly positive (zero) profit, it follows that

\[ p_i^* = I \]

for \( i = 1, 2 \). Since the problem is symmetric, by (73) and \( p_i^* = I \) we get

\[ q_i^* = \frac{1}{2} \left( \sqrt{q_j(4sI + q_j(1 + s)^2) - q_j(1 + s)} \right) \]

for \( i = 1, 2 \). Solving this system yields

\[ q_i^* = \frac{sI}{2 + s}, \]

as claimed. The remaining assertions are verifiable by simple substitution, hence omitted. ■
Appendix B: Monotonic Decision-Conflict Logit

This class of models, introduced in Section 2, is characterized by the following condition:

**AA1 (Active-Choice Lower Bounds)**

For all \( A, B \in \mathcal{M} \) such that \( A \supset B \):

\[
\rho(B, B) \geq \frac{\rho(o, B) - \rho(o, A)}{\rho(o, A)} - \frac{\rho(A, A) - \rho(B, A)}{\rho(B, A)}
\]

The numerator on the right hand side of this inequality is the percentage change in the probability of opting out when the agent moves from the larger menu \( A \) to the smaller menu \( B \). The denominator on the other hand is the percentage increase in the probability of making an active choice at menu \( A \supset B \) that is contributed by those alternatives that are available in \( A \) but not in \( B \). When \( \rho(o, B) \geq \rho(o, A) \) holds, then AA1 and \( \rho(B, B) \in (0, 1) \) together imply that the percentage decrease in the probability of deferring when moving from \( B \) to \( A \) is strictly lower—and in proportion to \( \rho(B, B) \)—than the percentage increase in the probability of making an active choice at \( A \) when moving from the submenu \( B \) to all of \( A \). When \( \rho(o, A) \geq \rho(o, B) \) holds instead, then AA1 is trivially satisfied. Thus, the axiom can be thought of as allowing for deferral to become less likely in larger menus while at the same time imposing an upper bound on how less likely it can become.

**Proposition 11**

A decision-conflict logit is monotonic if and only if it satisfies AA1.

Conditional on this result, it is also immediate that strictly monotonic models are characterized by the special case of AA1 where the inequality is always strict.

**Proof of Proposition 11.**

Let \( \rho = (u, D) \) be a decision-conflict logit and suppose \( A \supset B \). By (3) and (4), \( D \) is
monotonic iff

\[ D(A) \geq D(B) \iff \frac{\rho(o, A)}{1 - \rho(o, A)} \frac{\rho(A, X)}{\rho(z, X)} \geq \frac{\rho(o, B)}{1 - \rho(o, B)} \frac{\rho(B, X)}{\rho(z, X)} \]

\[ \iff \rho(o, A) \frac{\rho(A, X)}{\rho(A, A)} \geq \rho(o, B) \frac{\rho(B, X)}{\rho(B, B)} \]

\[ \iff \rho(o, A) \frac{\rho(A, X)}{\rho(B, X)} \geq \rho(o, B) \frac{\rho(A, A)}{\rho(B, B)} \]

\[ \iff \rho(o, A) \left( \frac{\rho(B, X) + \rho(A \setminus B, X)}{\rho(B, X)} \right) \geq \rho(o, B) \frac{\rho(A, A)}{\rho(B, B)} \]

\[ \iff 1 + \frac{\rho(A \setminus B, X)}{\rho(B, X)} \geq \frac{\rho(o, B)}{\rho(o, A)} \frac{\rho(A, A)}{\rho(B, B)} \]

\[ \iff \frac{\rho(A \setminus B, X)}{\rho(B, X)} \geq \frac{\rho(o, B) - \rho(o, A)}{\rho(o, A) \rho(B, B)} \]

\[ \iff \frac{\rho(A \setminus B, A)}{\rho(B, A)} \geq \frac{\rho(o, B) - \rho(o, A)}{\rho(o, A) \rho(B, B)} , \]

where the last step follows from A2 and the last inequality is equivalent to AA1.  

\[ \blacksquare \]