Bound States in the Vortex Core

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Abstract

The quasiparticle excitation spectrum of isolated vortices in clean layered \(d\)-wave superconductors is calculated. A large peak in the density of states in the "pancake" vortex core is found, in an agreement with the recent experimental data for high-temperature superconductors.

1 Introduction

The vortex core in classical type-II superconductors can be treated as the normal metal area with radius of the order of the coherence length \(\xi(0) \sim 10\) nm.\(^1\), \(^2\) The spectrum of the bound states near the Fermi surface, formed by the constructive interference between the incident and the Andreev reflected quasiparticles,\(^3\) is quasicontinuous (gapless). High-temperature superconductors (HTS) are layered, having a cylindrical Fermi surface, the superconductivity is of the strong and \(d\)-wave coupling type, and the vortex core radius is much smaller, \(\xi(0) \sim 1\) nm.

For a two-dimensional (2D) vortex and \(s\)-pairing, Rainer et al. have shown, using the Andreev quasiclassical theory in the analytical, and Eilenberger’s in the numerical part of their study, that bound states exist in the vortex core.\(^4\) Similar conclusions have also been obtained by Maki and coworkers for \(d\)-pairing, within the Bogoliubov-de Gennes approach.\(^5\)

In this paper, the Eilenberger quasiclassical equations\(^6\), in the case of both \(s\) and \(d\)-pairing, are solved analytically, within the model considering the spatial variation of the order parameter in the vortex core as for a normal metal cylinder of radius \(r_c \sim \xi\). A crucial difference is found in the quasiparticle spectra below the bulk energy gap, between the classical superconductors with spherical Fermi surface, \(s\)-pairing, large \(\xi\), and HTS with cylindrical Fermi surface, \(d\)-pairing, small \(\xi\). Our results confirm that the quasiparticle density of states (DOS) has one large maximum, recently observed in YBCO by scanning tunneling microscopy.\(^7\)

2 Model of the Vortex Core

An efficient method for calculating local spectral properties is the quasiclassical theory of superconductivity, which gives the Eilenberger equations

\[
\left[2\hbar \omega_n + \hbar \mathbf{v} \cdot \left(\nabla - \frac{2e}{\hbar c} \mathbf{A}\right)\right] f = 2\Delta g, \quad \left[2\hbar \omega_n - \hbar \mathbf{v} \cdot \left(\nabla + \frac{2e}{\hbar c} \mathbf{A}\right)\right] f^\dagger = 2\Delta^* g, \quad (1)
\]

\[
\hbar \mathbf{v} \cdot \nabla g = \Delta^* f - f^\dagger \Delta. \quad (2)
\]
Here \( g = g_{\downarrow\uparrow}(\mathbf{r}, \mathbf{v}, \omega_n) \) and \( f = f_{\downarrow\uparrow}(\mathbf{r}, \mathbf{v}, \omega_n) \) represent the normal and the anomalous Green function respectively, \( \Delta = \Delta(\mathbf{r}, \mathbf{v}) \) is the gap function, \( \omega_n = \pi k_B T (2n + 1), \) \( n = 0, \pm 1, \pm 2, \ldots \), are the Matsubara’s frequencies, and \( \mathbf{v} \) is the Fermi velocity vector. The function \( f^\dagger \) is defined by \( f^\dagger_{\downarrow\uparrow}(\mathbf{r}, \mathbf{v}, \omega_n) = f_{\downarrow\uparrow}(\mathbf{r}, -\mathbf{v}, \omega_n) \). \( \Delta \) and \( f \) are connected by the self-consistency equation.

For a homogeneous and isotropic superconductor, solutions of the Eilenberger equations are

\[
\langle f \rangle = \frac{\Delta}{\varepsilon_n}, \quad \langle f^\dagger \rangle = \frac{\Delta^*}{\varepsilon_n}, \quad \langle g \rangle = \frac{\hbar \omega_n}{\varepsilon_n}, \quad \left( \varepsilon_n^2 = |\Delta|^2 + (\hbar \omega_n)^2 \right).
\] (3)

In the cylindrical coordinates \((r, \varphi, z)\), with the origin situated on the vortex axis, the vortex magnetic field is \( \mathbf{h} = \hbar \mathbf{e}_z \). At distances \( r \sim \xi \ll \lambda \) from the vortex axis, \( \hbar \) is approximately constant, and the gauge can be chosen in the form \( \mathbf{A} = (\mathbf{h} \times \mathbf{r}) / 2 \). Taking the same vortex gap function as for a normal metal cylinder embedded in a superconductor

\[
\Delta = \Delta(r, \theta) e^{-i\varphi}, \quad \Delta(r, \theta) = \begin{cases} 0, & r \leq r_c \\ \Delta(\theta), & r > r_c \end{cases}.
\] (4)

Here, \( r_c \) is the vortex core radius, and \( \theta \) is the polar angle in \( \mathbf{k} \)-space. For \( d \)-pairing \( \Delta(\theta) = \Delta_0 \cos 2\theta \), and for \( s \)-pairing \( \Delta(\theta) = \Delta_0 \). The gauge in Eq. (4) is due to the flux quantization.

For a pancake vortex in \((r, \varphi)\) plane, denoting the coordinate along \( \mathbf{v} \) by \( s \), and along \( \mathbf{h} \times \mathbf{v} \) by \( p \) (Fig.1.), in the gauge with real gap, Eqs. (1) and (2) can be rewritten in the form

\[
2\hbar \omega_n + \hbar v \left( \frac{\partial}{\partial s} + i \frac{p}{\hbar^2} + i \frac{p}{r^2} \right)f = 2\Delta(r, \theta)g,
\] (5)

\[
2\hbar \omega_n - \hbar v \left( \frac{\partial}{\partial s} - i \frac{p}{\hbar^2} - i \frac{p}{r^2} \right)f^\dagger = 2\Delta(r, \theta)g,
\] (6)

\[
\hbar v \frac{\partial}{\partial s}g = \Delta(r, \theta)(f - f^\dagger),
\] (7)

where \( r^2 = p^2 + s^2 \) and \( \hbar^2 = \hbar c / e h \).

For a normal metal cylinder and zero magnetic field, Eqs. (5)-(7) with \( p = 0 \), the solution is of the form

\[
f = \sum_i f_i(p) e^{\kappa_i s}, \quad g = \sum_i g_i(p) e^{\kappa_i s}.
\] (8)

For \( r \leq r_c \), with \( \kappa_0 = 2\omega_n / v \),

\[
f = F e^{-\kappa_0 s}, \quad g = G.
\] (9)

For \( r > r_c \) and \( \kappa = 2\varepsilon_n / \hbar v \),

\[
f = \langle f \rangle + \Phi_1 e^{-\kappa_0 s}, \quad g = \langle g \rangle + \Gamma_1 e^{-\kappa_0 s}, \quad \text{for } s > 0,
\] (10)

\[
f = \langle f \rangle + \Phi_2 e^{\kappa_0 s}, \quad g = \langle g \rangle + \Gamma_2 e^{\kappa_0 s}, \quad \text{for } s \leq 0.
\] (11)

Eqs. (5)-(7) imply

\[
\frac{\Phi_1}{\Gamma_1} = \frac{\Delta(\theta)}{\hbar \omega_n - \varepsilon_n}, \quad \frac{\Phi_2}{\Gamma_2} = \frac{\Delta(\theta)}{\hbar \omega_n + \varepsilon_n}.
\] (12)

Using the continuity condition for \( f \) and \( g \) at \( \pm s_0 = \pm \sqrt{r_c^2 - p^2} \), for \( r \leq r_c \) the normal Green function is

\[
G = \frac{\hbar \omega_n \cosh(\kappa_0 s_0) + \varepsilon_n \sinh(\kappa_0 s_0)}{\hbar \omega_n \sinh(\kappa_0 s_0) + \varepsilon_n \cosh(\kappa_0 s_0)}.
\] (13)
For a vortex, approximating \( p/r^2 \) by \( p/r_c^2 \), the solution of Eq. (3)-(4) can be obtained from Eq. (13), by changing \( \omega_n \to \omega_n' \),

\[
G \approx \frac{\hbar \omega_n' \cosh(\kappa_0' s_0) + \varepsilon_n' \sinh(\kappa_0' s_0)}{\hbar \omega_n' \sinh(\kappa_0' s_0) + \varepsilon_n' \cosh(\kappa_0' s_0)},
\]

where

\[
\omega_n' = \omega_n + i \frac{p_v}{2} \left( \frac{1}{r_c^2} + \frac{1}{r_H^2} \right),
\]

and \( \kappa_0' = 2\omega_n'/v \). In this case, the magnetic flux quantization leads to [1, 5]

\[
p_i = \left( i + \frac{1}{2} \right) \frac{\hbar}{m v}, \quad i = 0, \pm 1, \pm 2, \ldots
\]

Since for an isolated vortex \( l_H \gg r_c \), the direct influence of the field can be neglected, and the only relevant contribution is due to the screening supercurrent flow, \( \eta p v/2r_c^2 \) term in Eq. (15).

3 Bound States

Performing an analytical continuation of \( G \) by \( \hbar \omega_n \to -iE + \eta \), \( E \) being the quasiparticle energy with respect to the Fermi level, the retarded propagator \( g^R(E,p,\theta) \) is obtained. In terms of reduced variables \( E/\Delta_0 \to E, \sqrt{\Delta^2(\theta) - E^2}/\Delta_0 \to \varepsilon, \sqrt{E^2 - \Delta^2(\theta)}/\Delta_0 \to e, p/\xi_0 \to p, 2s_0/\pi \xi_0 \to s_0, \xi_0 = h\nu/\pi \Delta_0 \) being the BCS coherence length, angle resolved partial DOS (PDOS) is obtained from \( N(E,p,\theta) = \text{Reg}^R(E,p,\theta) \).

For the normal metal cylinder

\[
N(E,p,\theta)/N(0) = \Theta \left( e^2 \right) \frac{|E|\epsilon}{e^2 \cos^2(Es_0) + E^2 \sin^2(Es_0)} + \Theta \left( e^2 \right) \frac{\pi |\Delta(\theta)|}{\Delta_0} \delta \left( E \sin(Es_0) - \varepsilon \cos(Es_0) \right),
\]

where \( \delta \) is the Dirac function, \( \Theta \) is the step-function, and \( N(0) = m/2\pi \hbar^2 \) is the normal metal density of states at the Fermi surface for one spin orientation. For s-wave pairing, PDOS does not depend on \( \theta \), while for d-wave pairing, averaging over the cylindrical Fermi surface leads to

\[
N(E,p)/N(0) = \frac{1}{2\pi} \int_0^{2\pi} N(E,p,\theta)/N(0) d\theta =
\]

\[
= \frac{2}{\pi} \int_{\max[0,E^2-1]}^{E} |E|e^2 \frac{de}{\sqrt{E^2 - e^2} \sqrt{1 - E^2 + e^2 \left( e^2 \cos^2(Es_0) + E^2 \sin^2(Es_0) \right)}} + \frac{(E \tan(Es_0) + |E \tan(Es_0)|)}{\sqrt{\cos^2(Es_0) - E^2}} \Theta \left( \cos^2(Es_0) - E^2 \right).
\]

Finally, after spatial averaging over the cylinder area \( \pi r_c^2 \), DOS is

\[
N(E) = \frac{4}{\pi} \int_0^1 N(E,p) \sqrt{1 - p^2} dp.
\]

For the vortex, in Eqs. (17) and (18), \( E \to E + E_0 \) sign \( E, E_0 = \hbar |p_i| \sqrt{2\Delta_0 r_c^2} \), Eq. (15), with \( p_i \) from Eq. (16), and \( \sum p_i \) instead of integration in Eq. (19). Here, signs of \( p \) and \( E \) are connected, because the magnetic field causes a difference in propagation of particles and holes.
For small radius vortices in HTS, $\xi_0 m v / h \sim 1$ ($\sim 10$ in classical superconductors), only one trajectory through the vortex core is allowed, with $p = p_0$, Eq. (16). Taking for YBCO $r_c = \xi_0$, $p_0 = 1/3$, $\Delta_0 / E_F = 0.424$, only one peak in DOS in the vortex core around $E / \Delta_0 \approx 0.3$ is obtained (Fig. 2). For comparison, DOS of normal metal cylinder embedded in the same superconductor and with the same radius $r_c = \xi_0$, but in the zero magnetic field, is shown. In this case, a large energy gap is found in DOS, due to formation of lowest bound state at high energy, of the order of $\Delta_0$. This is not the case in classical superconductors, where $\Delta_0 / E_F \ll 1$.

In conclusion, cylindrical Fermi surface, $d$-wave pairing and small $\xi_0$, large $\Delta_0 / E_F \sim 0.1$, make DOS of a normal metal cylinder embedded in HTS and a pancake vortex different from DOS of a normal cylinder and a vortex in classical superconductors. Since the Andreev bound states can transport charge currents, unlike the bound states in a potential well, supercurrents can flow through the vortex without losses, strongly influencing its dynamics. This could be very important for transport properties of HTS, especially for understanding the unusual magnetic-field dependence of the electrothermal conductivity, which was observed experimentally and awaits explanation.

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Figure 1
Figure 2