EVALUATION OF SOIL SHEAR STRENGTH PARAMETERS VIA TRIAXIAL TESTING BY HEIGHT VERSUS DIAMETER RATIO OF SAMPLE

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Abstract. The triaxial test is a most widely used laboratory method for determining the soil shear strength. It is assumed that a soil sample deforms uniformly during triaxial testing. But one often faces a case when the sample in the triaxial apparatus deforms on the contrary. The non-uniformity can be caused by the end restraining effect, the sample height influence factor, the insufficient drainage, the membrane effect and the sample self-weight factor etc. An analysis of known investigations lead to the following tools that could be employed for reducing an inaccuracy related to the non-uniform stress-strain distribution per soil sample during triaxial testing: reducing the sample height/diameter ratio from 2 to 1, eliminating the friction between the sample ends and the plates. Having not eliminated the above-mentioned influence, factors during the testing procedure the angle of internal friction \( \varphi \) and the cohesion \( c \) for the sample of \( \varphi \neq 0 \) are determined larger than the actual ones. The method for determining the angle of internal friction \( \varphi \) and the cohesion \( c \) when testing the soil sample of height/diameter \( H/D \) = 1 is proposed.

Keywords: triaxial testing, soil shear strength parameters, angle of internal friction, cohesion, height/diameter ratio of sample.

1. Introduction

A necessary information on soil properties, requested by designers and constructors, is obtained on each construction site by examining the physical and mechanical properties of soils (Amšiejus, Dirgėlienė 2007; Dirgėlienė et al. 2007; Jukneviciūtė, Laurinavičius 2008; Vervečkaitė et al. 2007; Ždankus, Stelmokaitis 2008). The triaxial and the direct shear tests are at present the most common tests for determining the soil shear strength parameters in laboratory. The triaxial test is acknowledged to be the most widely employed method for evaluating the soil shear strength. The test is also acknowledged to be the most reliable method employed for simulating a stress and strain state of ground.

Two main assumptions are introduced for determining the shear strength parameters of soil by the triaxial compression testing, namely: the normal stress on soil sample surface is applied only; the soil sample deforms uniformly during testing. The latter assumption expresses the fundamental of triaxial testing. Actually, the sample in triaxial apparatus deforms non-uniformly. The non-uniformity can be caused due: the actual sample ends conditions, those restraining the free displacements in horizontal directions; the sample height; the insufficient drainage in sample; the sample rubber membrane, the specimen self-weight factor etc. The finite element method (FEM) analysis also shows the non-uniform distribution of stress and strain in the sample when modelling the triaxial testing (Airey 1991; Hinokio, Nakai 2005; Jeremic et al. 2004; Liyanapathirana et al. 2005; Peric, Su 2005; Sheng et al. 1997; Vervečkaitė 2004).

The ratio 2 of sample height/diameter (H/D) is commonly used for triaxial testing procedures. Actually, the triaxial sample end restraints do not allow a free moving of their parts sideways. The soil bulging deformation generate the tangential stresses in the failure plane, the soil properties change here and moving of sample ends begins. Thus one obtains the non-uniform distribution of stress and strain per sample volume. The latter leads to the difficulties when interpreting testing results aimed at identifying the actual soil properties.

An eliminating of the friction between the sample ends and the apparatus plates ensures an avoiding of “the dead zones” and protects from a wrong increase in measured strength due to restraining the sample ends. The sample height should be decreased from the standard ratio of
height and diameter of 2 by that of 1. For this decrease it is necessary to ensure an effective lubrication. It results a more uniform stress and strain distribution, the sample may retain its cylindrical shape even at large strains. An eliminating of the friction has an insignificant effect when the standard height is employed (Head 1986).

Hettler and Gudehus (1985) carried out the standard triaxial tests for samples of $H/D$ ratio $H/D = 21.1$ cm/10 cm using the non-guided cap and the non-lubricated ends. They determined the $\varphi$ to be less by 5° versus the sample of $H/D = 28/78$ cm.

Lade and Wasif (1988) performed tests by varying the dense sand samples of anisotropic fabric and square sections for the $H/D$ ratio of 1 and 2.5. The drained triaxial tests were carried out. The used samples were formed of several layers, being inclined by various angles in respect of a vertical. The authors investigated the influence of the sample boundary conditions (flexible membrane; lubricated, rigid end platens) with 2 different types of samples. The test results have shown that boundary conditions produced different impacts on the investigated samples $H/D$ ratio equal to 1 and 2.5. The tested samples of $H/D = 2.5$ and that of with the inclined and vertical layer planes yielded an obvious stress-strain curve drop of short duration at a pre-failure stage. The angle of internal friction of soil decreased when the angle inclination of layer platens increased. The stress-strain curve of samples with $H/D = 1$ was more even, i. e. uniform. The inclination of layer plane has not influenced significantly the angle of internal friction of the soil sample.

A generalized analysis of the known experimental investigations by triaxial testing and that of the numerical simulations clearly states that the stress-strain distribution in a soil sample is not uniform. Thus, soil strength parameters are identified with certain errors. Therefore the continued investigations, aimed to ensure reducing and/or overcoming the sources of this error origin for obtaining the more reliable soil strength parameters are of an actual necessity.

2. Theoretical analysis of sample $H/D$ ratio influence on soil strength parameters obtained by triaxial testing

The experimental investigations show that soil shear strength versus normal stresses, acting on a failure plane, is in linear relationship. The shear strength $\tau_u$ resists the deformation caused by shear stresses. The shear strength depends on friction between soil particles and cohesion, acting between the soil particles. The general Coulomb law for soil strength reads:

$$\tau_u = \tan \varphi \cdot \sigma + c,$$

where $\sigma$ – the normal stresses acting on the failure plane, kPa; $\tau$ – the angle of internal friction, in degrees; $c$ – the cohesion, kPa.

The normal component of stresses acting on the failure plane is:

$$\tau = \sigma_1 \sin \alpha \cos \alpha - \sigma_3 \sin \alpha \cos \alpha = 0.5 \sin 2\alpha (\sigma_1 - \sigma_3).$$

The shear component $\tau$ of the stresses, acting on the failure plane, is defined by:

$$\tau = \sigma_1 \sin \varphi \cos \varphi - \sigma_3 \sin \varphi \cos \varphi = 4\tan \alpha (\sigma_1 - \sigma_3).$$

The shear strength versus normal stresses, acting on a failure plane, is in linear relationship. The shear strength $\tau$ depends on friction between soil particles and cohesion, acting between the soil particles. The general Coulomb law for soil strength reads:

$$\sigma = (\sigma_1 - \sigma_3) \cos^2 \alpha + \sigma_3,$$

where $\sigma_1$ – the major principle stress, kPa; $\sigma_3$ – the minor principle stress, kPa; $\alpha$ – an angle of the failure plane in respect of the minor principle stress in degrees.

The relationship between principal stresses in the critical state is the soil shear strength condition expressed by the principal stresses.

$$\sigma_1 = \sigma_3 \tan^2 \left(45^\circ + \frac{\varphi}{2}\right) + 2 \times c \times \tan (45^\circ + \frac{\varphi}{2}).$$

Let us refer to the reader on investigations for identifying the vertical component of stresses $\sigma_u$ that corresponding the relevant failure angles $\alpha$ (Dirgėlienė et al. 2007). They yield that the soil sample $H/D$ ratio effect has no influence on testing results for height

$$H \geq D \times \tan \left(45^\circ + \frac{\varphi}{2}\right).$$

An expected failure plane angle for clay is $45^\circ$. An expected failure plane angle for sand is $45^\circ + \frac{\varphi}{2}$ (Fig. 1). When sample height is $H \geq D \times \tan (45^\circ + \frac{\varphi}{2})$ the effect of the $H/D$ ratio vanishes when identifying the sand shear strength parameters via the triaxial testing. The case ensures a sufficient height for unconstrained developing the failure plane of an inclination angle $45^\circ + \frac{\varphi}{2}$.

![Fig. 1. Failure schemes of sand sample: a – ratio of $H/D = 2$; b – ratio of $H/D = 1$](image-url)
inclination angle less by the angle $45° + \frac{\varphi}{2}$. The latter results are larger than $\sigma_1$, necessary to cut the sample. The performed analysis results are presented in Fig. 2. The sand strength parameters are sensitive to the sample $H/D$ ratio only within certain ratio variation bounds. One can find the reducing of the sand sample height is unexpected, as it yields the larger $\sigma_1$.

Let us analyze sand soil sample. Shear stresses $\tau$ obtain the max value on failure plane of $45°$ inclination angle in respect of minor principal stresses direction (Fig. 4). But the shear strength $\tau_u$ in this plane is larger. Thus, the limit state is achieved, i.e., $\tau = \tau_u$. Thus, the limit state is achieved only on the failure plane of inclination angle of in respect of the minor principal stresses direction. When increasing the angle $\alpha$ from $45°$ to $60°$, $\tau$ reduces slower versus the shear strength $\tau_u$ of the soil.

Fig. 3 is assigned to variation of shear stresses $\tau$ and that of shear strengths $\tau_u$ on eventual failure plane versus its inclination angle $\alpha$ for clay sample. One can find that the max shear stresses correspond to the failure plane of the $45°$ inclination angle. The limit state will be reached, i.e. the shear stress equals the soil shear strength only in this failure plane. The shear stresses are less for all other planes of $\alpha \neq 45°$, as the clay shear strength is constant.

3. Experimental analysis of soil shear strength parameters

3.1 $H/D$ ratio variation of sand samples

An experimental analysis was performed via testing the sand soil samples. A type of tested soil corresponds the poorly-graded sand with fine SP–SM according the Unified Soil Classification System (Fig. 5). Particles of the sand are rounded. The sand uniformity coefficient is 3.03, the curvature coefficient – 1.47, the specific gravity of soil particles – 2.67, the max void ratio – 0.745, the min void ratio – 0.502.

The disturbed samples of 6% water content have been prepared by employing the compacting procedure. Two cases of sand samples have been investigated, namely of dense and loose ones. Their densities $\rho$ and void ratios are: $\rho = 1.87 \text{ gr/cm}^3$ and $e = 0.51$ for the dense sand, and $\rho = 1.61 \text{ gr/cm}^3$, and $e = 0.74$ for the loose sand.

The consolidated-drained triaxial tests have been carried out by employing the Italian CONTROLS apparatus. The boundary conditions of samples were described as
follow: the sample top is free for rotation, the friction between the sample ends and the platens is not eliminated (regular ends).

Samples of ratios $H/D = 2$ (height $H = 10$ cm, diameter $D = 5$ cm) and $H/D = 1$ (height $H = 5$ cm, diameter $D = 5$ cm) were used for experiments. The tests were carried out under constant cell pressures $\sigma_3 = 50, 100, 200$ kPa ensuring the axial strain rate of 0.1 min/mm.

The axial strain and axial load were measured during the test. The samples of the same density under the same cell pressure have been sheared 3 times at least. The test proceeding was completed, when the relative axial strains $\varepsilon_1$ reached 15%.

The dense soil samples of $H/D = 2$ and that of $H/D = 1$ at the first stages of loading increment consolidate, subsequently the failure plane develops being accompanied by an increment of the vertical displacements. For dense sand one can clearly fix a peak strength, corresponding to the max $\sigma_1 - \sigma_3$ (Lade, Prabucki 1995). Only having reached this peak strength and then subsequently increasing the axial strains, one can see the following: the soil strength reduces, the sample bulges, slow reducing the deviator stress. When repeating the testing procedures under the larger $\sigma_3$, one can observe that the shear strength reaches the minimum value corresponding to the different values of axial strains. The min value of shear strength was reached faster when employing the smaller $\sigma_3$ (Figs 6, 7).

Having performed the analogous standard triaxial compression tests for dense sands, one can observe the forming of failure plane and the parted sample parts trying to move in opposite horizontal directions along this failure plane (Fig. 8). The friction between sample ends and apparatus plates resists to the displacement of the sample ends. The latter prescribes an employment of the larger values of vertical component of stresses required to shear the soil sample.

When testing the loose sand samples of $H/D = 2$ and $H/D = 1$, one obtains the shape of graphs $\varepsilon_1 = f(\sigma_1 - \sigma_3)$ to be similar for both cases under investigation (Figs 9, 10). The loose sand samples consolidate per whole loading range, one can not fix the clear peak shear strength.

Fig. 6. Stress-strain $\varepsilon_1 = f(\sigma_1 - \sigma_3)$ curves (dense sand, $H/D = 2$): $-50$ kPa; $-100$ kPa; $-200$ kPa

Fig. 7. Stress-strain $\varepsilon_1 = f(\sigma_1 - \sigma_3)$ curves (dense sand, $H/D = 1$): $-50$ kPa; $-100$ kPa; $-200$ kPa

Fig. 8. Triaxial test dense sand sample of $H/D = 2$

Fig. 9. Stress-strain $\varepsilon_1 = f(\sigma_1 - \sigma_3)$ curves (loose sand, $H/D = 2$): $-50$ kPa; $-100$ kPa; $-200$ kPa

Fig. 10. Stress-strain $\varepsilon_1 = f(\sigma_1 - \sigma_3)$ curves (loose sand, $H/D = 1$): $-50$ kPa; $-100$ kPa; $-200$ kPa
The stresses $\sigma_1 - \sigma_3$ increase up to the bounds of 6–12% of the axial strain and that of 5–15% for samples of $H/D = 2$ and $H/D = 1$, respectively. The peak shear strength of loose sand samples is reached for much larger axial strain values comparing with those of dense sands. One cannot observe visually a failure plane for loose sand samples of $H/D = 2$, but one can observe multiple planes for the ones of $H/D = 1$ (Fig. 11).

Fig. 11. Triaxial test dense sand sample of $H/D = 1$

4. Calculation results of triaxial test in samples of $H/D = 1$

When the failure plane of the sample of $H/D = 1$ is inclined in respect of the minor principal stresses $\sigma_3$ by an angle $\alpha = 45^\circ$, one obtains $\cos 45^\circ = 0.5$. Having substitute this result into the expression of Eq (2), assigned to normal component of stresses on failure plane, one obtains the following expression of the normal stresses:

$$\sigma_{\alpha = 45^\circ} = (\sigma_1 - \sigma_3) \times 0.5 + \sigma_3,$$

$$\sigma_{\alpha = 45^\circ} = \frac{(\sigma_1 + \sigma_3)}{2}.$$  \hspace{1cm} (5)

When $\alpha = 45^\circ$, then $\sin 2\alpha = \sin 90^\circ = 1$. Substituting this result into expression of Eq (3), assigned to shear component of stresses $\tau$ on failure plane, one finally obtains:

$$\tau_{\alpha = 45^\circ} = \frac{(\sigma_1 - \sigma_3)}{2}.$$ \hspace{1cm} (6)

As the sample of $H/D = 1$ is sheared by plane inclined in respect of the minor principal stresses $\sigma_3$ by the angle of $45^\circ$, one obtains $\tau = \tau_{\alpha}$.

Having substituted the expressions of Eqs (5) and (6) into the Eq (1), one obtains:

$$\frac{(\sigma_1 - \sigma_3)}{2} = \frac{(\frac{\sigma_1 + \sigma_3}{2}) \times \tan \varphi + c.\hspace{5cm} (7)$$

The sample of $H/D = 1$ will be in the critical state when the major principal stresses reach the largest value. From Eq (7) one obtains $\sigma_1$ (during testing) reading:

$$\sigma_1 = \frac{\sigma_3 (\sec \varphi + 1 + 2c)}{1 - \sec \varphi} \hspace{5cm} (8)$$

Analyse 2 soil samples A and B of $H/D = 1$, being tested by triaxial test apparatus. The cohesion $c$ and angle of internal friction $\varphi$ are derived from the following Eqs system:

$$\begin{align*}
(1 - \sec \varphi)\sigma_{1d} &= \sigma_{3d} (\sec \varphi + 1) + 2c; \\
(1 - \sec \varphi)\sigma_{1b} &= \sigma_{3b} (\sec \varphi + 1) + 2c; \\
\sec \varphi &= \frac{\Delta \sigma_1 - \Delta \sigma_3}{\Delta \sigma_1 + \Delta \sigma_3},
\end{align*}$$ \hspace{1cm} (9)

$$c = \frac{1}{4} (\sigma_{1d} + \sigma_{1b}) - (\sigma_{3d} + \sigma_{3b}) (1 + \tan \varphi)$$ \hspace{1cm} (10)

Having performed the triaxial tests for sand samples of $H/D = 1$ for lateral normal stresses $\sigma_3$ of 50 and 200 kPa, one can obtain the major principal stresses $\sigma_1$. The other values of $\sigma_1$ were calculated according to Eq (4) substituting $\varphi$ and $c$ values obtained from Eqs (9) and (10). The values of these stresses are presented in Figs 12–15 (the residual values of $\sigma_1$ when $\varepsilon_1 = 15\%$ are employed). The latter results of $\sigma_1$ are very close the ones that have been determined for the sand samples of $H/D$ ratios equal to 2. For dense samples of $H/D = 1$ the residual value of $\sigma_1$ is larger approx to 24%, for loose sands approx 16% when comparing with the values of $\sigma_1$ calculated according to Eq (4).

It were determined values of the angle of internal friction $\varphi$ and the cohesion $c$ via the obtained expressions (9) and (10) and by employing the triaxial test results of samples for $H/D = 1$. The analogous results of $H/D = 1$ tests have been processed for determining the values of $\varphi$ and $c$ according the standard methodology presented in ČНиП 2.02.02-85 Имущественные гидroteхнические сооружений [СНиП 2.02.02-85 Foundation Beds of
Hydraulic Structures]. The obtained results are presented in Figs 16, 17. The latter values of $\phi$ are higher in 13–17%, $c$ are higher in 9–21% for dense and loose samples than the values obtained by proposed method via expressions Eqs (9) and (10).

When comparing the values of the angle of internal friction $\phi$ and that of cohesion $c$, one can find them to be very close the ones that have been determined for the sand samples of $H/D = 2$ and $\phi, c$ obtained by proposed method, respectively.

5. Conclusions

Review of literature suggests methods for ensuring an obtaining of the more uniform stress-strain distribution in soil sample during triaxial testing: reduce the sample $H/D$ ratio from 2 to 1; eliminate friction between the sample ends and the plates. Angle of internal friction for soil increases from $1^\circ$ to $5^\circ$ in this case.

For dense samples of $H/D = 1$ the residual value of $\sigma_1$ is larger approx 24%, for loose sands approx 16% when comparing with the values of $\sigma_1$ calculated according to Eq (4).

It was proposed a method for determining the angle of internal friction $\phi$ and the cohesion $c$ for the samples of $H/D = 1$.

The values of the angle of internal friction $\phi$ and the cohesion $c$ were determined via expressions (9) and (10) by employing triaxial test results for samples of $H/D = 1$.

It was found that these values are different from the ones, identified via triaxial testing for samples of $H/D = 1$.

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