Application of Tukey’s Test for Statistical Measurement of Percentage Indexes Derived from the Difference Between Predicted Crystallographic Data in Five NiTiHf Alloys Obtained by Two Processes

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Abstract — The present paper uses the Tukey test to measure the minimal significant difference (MSD) between crystallographic data predicted by non-linear mathematical models generated from graphics published by Zarinejad (2008) and Potapov (1997) on 5 nominal compositions of Ni_{50}Ti_{50-x}Hf_{x} .at% substitutional alloys fused by arc melting and melt spinning processes. Results show that the calculated MSD do not represent a statistically significant difference. Therefore, it is concluded that the independent variable (Hafnium content) does not change the analysed dependent variables (lattice parameters, crystalline structure volume and monoclinic angle β) for any fusing processes.

Keywords — ANOVA; Monoclinic; NiTiHf; Statistical test; Tukey.

I. INTRODUCTION

The production of shape memory effect (SME) alloys depends sensibly on particular processes that, based on the variation of parameters involved in the equipment used (maximum melting temperature, vacuum magnitude, purge intensity, purity of the metallic charges, torch exposition time, etc), can or cannot generate different crystallographic features for samples which theoretically have the same chemical compositions. In this sense, it is important to compare the obtained results through predictive modelling functions to observe whether there are any significant statistical difference capable of indicating a great variability in crystallographic characteristics, such as: lattice parameters (a b c [Å]), cell volume (VOL [Å³]) and characteristic angle β (beta [°]) of the monoclinic structure (space group P12/m1). In this work, they are studied as variables dependent on hafnium content, which is considered an independent variable in the nominal composition. The focus of the study is on room temperature B19’ martensitic phase B19’, which has a monoclinic structure (Pearson symbol mP8) and is considered by the literature as an unstable phase. In addition, the five substitutional compositions Ni_{50}Ti_{50-x}Hf_{x}.at% considered in this instance are X = 8, 11, 14, 17 and 20 .at%, which were obtained by two different processes: Arc Melting (Zarinejad, 2008) [1] and Melt Spinning (Potapov, 1997) [2].

II. PREDICTION OF THE B19’ PHASE OF Ni_{50}Ti_{50-x}Hf_{x}.AT% ALLOYS

Based on the graphs by Zarinejad (2008) [1], which measurements were obtained by XRD [3] refinement method, it was possible to calculate, through numerical proportions, the equivalence of the lattice parameters (a, b, c), volume of the monoclinic structure B19’ and axial angle β, as shown in table 1.

| Hf (at.%) | a (Å) | b (Å) | c (Å) | V (Å³) | β (°) |
|----------|-------|-------|-------|--------|-------|
| 5        | 2.8228476 | 4.1605263 | 4.6628158 | 54.195804 | 98.068493 |
| 10       | 2.904966 | 4.1210526 | 4.7220216 | 55.758741 | 98.49315 |
| 15       | 2.9980132 | 4.192105 | 4.7707581 | 57.972027 | 99.189041 |
| 20       | 3.0814569 | 4.0503947 | 4.8754512 | 59.807692 | 100.04657 |

From the obtained results, the same variables were estimated for the five alloys of study (X = 5, 10, 15 and 20 at.%). From the third order nonlinear regression (shown in Figure 1) it was adopted the polynomial function ŷ = β_0 + β_1X + β_2X^2 + β_3X^3 ± ε for each parameter, which resulted in: ŷ_a = 2.77219 + 0.00562X + 0.00104X^2 − 2.73732E−5X^3 (R²=1); ŷ_b = 4.30224 − 0.04354X + 0.000354X^2 − 9.94737E−5X^3 (R²=1) and ŷ_c = 4.52671 + 0.03934X − 0.00287X^2 + 8.85679E−5X^3 (R²=1). The determination coefficient R² are equal to 1. This indicates that the proposed models are suitable to describe the phenomenon in which the parameters a and c increase, while the parameter b decreases. Equally, the models adopted for volume and β angle for b19’ phase presented 100% fitting.
Figure 1 - Scatter diagrams & Cubic regression curves: with the values of lattice parameters (a, b, c) determined for the Ni$_{50}$Ti$_{50}$X$_{X}$.at% alloys proposed by Zarinejad (2008)

All the values estimated for the five dependent variables are organized in table 2.

| Hf (at.%) | a (Å)     | b (Å)     | c (Å)     | VOLreg (Å$^3$) | $\beta$ (°) |
|----------|-----------|-----------|-----------|----------------|-------------|
| 8        | 2.897725  | 4.129549  | 6         | 4.703067       | 54.99855    |
| 11       | 2.996283  | 4.119240  | 6         | 4.730063       | 56.18361    |
| 14       | 3.129822  | 4.113564  | 6         | 4.757980       | 57.52923    |
| 17       | 3.302774  | 4.096405  | 6         | 4.811940       | 58.81437    |
| 20       | 3.519575  | 4.051650  | 6         | 4.874053       | 59.81439    |

Table 2 - Values of the parameters predicted by the 3rd order polynomials: lattice parameters (a, b, c), monoclinic volume and $\beta$, for Ni$_{50}$Ti$_{50}$X$_{X}$.at% alloys (X = 8, 11, 14, 17 and 20 at.%)

A comparison between the values of table 1 and table 2, confirms a coherence in the direct proportion between the percentage of hafnium and the volume of the monoclinic structure (phase B19'). The lattice parameters for the five Ni$_{50}$Ti$_{50}$X$_{X}$.at% Ni-rich alloys of study are plotted in figure 2:

Figure 2 - Scatter Plot with estimated values of the lattice parameters of the crystal structure B19' in Ni$_{50}$Ti$_{50}$X$_{X}$.at% alloys (X = 8, 11, 14, 17, 20 at.%) According to Figure 3(a), from the regressive function $\hat{y}_{\text{VOLreg}} = 54.31119 - 0.25944x + 0.05413x^2 - 0.00137x^3$ ($R^2 = 1$), adopted for all four Zarinejad (2008) alloys, in which $X = 5, 10, 15$ e $20$ at.%, it is possible to predict the volume (Å$^3$) of the B19' monoclinic martensitic structure for the five compositions of the alloy Ni$_{50}$Ti$_{50}$X$_{X}$.at% (X = 8, 11, 14, 17 and 20 at.%), shown in Figure 3(b).

Similarly, using the cubic polynomial function $y_{\beta} = 98.02467 - 0.03662x + 0.00981x^2 - 1.46128E-4x^3$ ($R^2 = 1$) adopted for Zarinejad alloys (2008), as seen in Figure 4(a), the $\beta$ angles of the B19' phase for the five Ni$_{50}$Ti$_{50}$X$_{X}$.at% (Hf = 8, 11, 14, 17 e 20 .at%) alloys of study were predicted and plotted in figure 4(b).

Figure 3 - Nonlinear regression (order 3) from which the volumes of the martensite phase (B19') of the five substitutive Ni$_{50}$Ti$_{50}$X$_{X}$.at% (X = 8, 11, 14, 17 and 20 .at%) alloys were calculated
For the functions used as statistical models it was calculated the analysis of variance in each polynomial, according to table 3.

| Variables | ANOVA | DF | SQ     | MQ     | F     | Significance F |
|-----------|-------|----|--------|--------|-------|---------------|
| Regression | 3     | 0.03776864 | 0.012589547 | —      | —     |               |
| Residue   | 0     | 0 |        |        |       |               |
| Total     | 3     | 0.03776864 |        |        |       |               |
| Regression | 3     | 0.006228132 | 0.002076044 | —      | —     |               |
| Residue   | 0     | 0 |        |        |       |               |
| Total     | 3     | 0.006228132 |        |        |       |               |
| Regression | 3     | 0.024311803 | 0.008103934 | —      | —     |               |
| Residue   | 0     | 0 |        |        |       |               |
| Total     | 3     | 0.024311803 |        |        |       |               |
| Angle β   | Regression | 3 | 2.245370993 | 0.748456998 | —      | —     |               |
| Residue   | 0     | 0 |        |        |       |               |
| Total     | 3     | 2.245370993 |        |        |       |               |
| Volreg    | Regression | 3 | 18.21455606 | 6.071518687 | —      | —     |               |
| Residue   | 0     | 0 |        |        |       |               |
| Total     | 3     | 18.21455606 |        |        |       |               |
| Volcalc   | Regression | 3 | 18.46596165 | 6.155320551 | —      | —     |               |
| Residue   | 0     | 0 |        |        |       |               |
| Total     | 3     | 18.46596165 |        |        |       |               |

Table 3 - ANOVAS calculated for each model \( \hat{y} = \beta_0 + \beta_1x + \beta_2x^2 + \beta_3x^3 \) referring to the 5 variables of the B19’ martensitic phase of the 5 Ni_{50}Ti_{50}XHf {X\at\% } (X = 8, 11, 14, 17 and 20 at\%) alloys rich in nickel

To make explicit the coherence of the regressive model adopted for the volume, the calculated volume (\( V_{\text{Vol,calc}} = 53.5806847 + 0.018979973x + 0.033586976x^2 + 8.39734E-04x^3 \)) was created.

All of the \( R^2 \) values were equal to 1. This makes unnecessary the calculation of Adjusted-\( R^2 \), normally used to measure the real degree of modeling reliability. According to Table 3, since there was no standard error in the coefficients, the general error of the models is 0. For this reason, F value is not presented in table 3. Thus, it can be stated that the mathematical models adopted here are statistically significant as well as predictive for all five dependent variables observed.

Taking the four Zarinejad alloys (2008) as a reference, the residues found for each regression, in each variable of the monoclinic structure B19’, are presented in table 4 and figure 5.

| Hf (at\%) | a (Å) | b (Å) | c (Å) | β (°) | Vol_{\text{reg,calc}} | Vol_{\text{vol,calc}} |
|----------|-------|-------|-------|-------|------------------------|------------------------|
| 5        | -1.78E-15 | 8.88E-16 | 1.78E-15 | -2.84E-14 | 7.1E-15 | -2.84E-14 |
| 10       | -8.88E-16 | 1.78E-15 | 2.66E-15 | 1.42E-14 | 1.42E-14 | 0 |
| 15       | -8.88E-16 | 8.88E-16 | 1.78E-15 | 1.42E-14 | 1.42E-14 | 0 |
| 20       | -8.88E-16 | 0 | 2.66E-15 | 1.42E-14 | 1.42E-14 | 0 |

Table 4 - Residues of each variable calculated for phase B19’ of the 4 substitutive Ni_{50}Ti_{50}XHf {X\at\% } (X = 5, 10, 15 and 20) alloys from Zarinejad (2008)
III. PREDICTION OF THE PHASE B19'ON Ni50Ti50-xHfx .AT% RIBBONS QUICKLY SOLIDIFIED BY MELT SPINNING

Based on the Potapov plots (1997) for the six ribbons of substitutional compositions Ni49.8Ti50.2-xHfx .at% (x = 8, 9.5, 11, 15, 20 e 25 .at%) obtained by *melt spinning*, the calculation of lattice parameters (a, b, c), B19' monoclinic crystal volume and characteristic angle \( \beta \) are shown in Table 5:

| Hf (at%) | a (Å)     | b (Å)     | c (Å)     | VOL (Å³) | \( \beta \) (°) |
|---------|-----------|-----------|-----------|----------|----------------|
| 8       | 2.985357  | 4.100357  | 4.725     | 56.961414| 99.992857     |
| 9.5     | 2.989285  | 4.096428  | 4.755642  | 57.337329| 100.07142     |
| 11      | 3.001071  | 4.094857  | 4.779214  | 57.769323| 100.38571     |
| 15      | 3.027785  | 4.0925    | 4.827142  | 58.499286| 102.03571     |
| 20      | 3.06000   | 4.084642  | 4.89000   | 59.539866| 103.05714     |
| 25      | 3.099285  | 4.072857  | 4.937142  | 60.25763  | 104.78571     |

Table 5 - Crystallographic characteristics of the martensitic phase obtained by XRD refinement for the six ribbons with Potapov (1997) nominal compositions Ni49.8Ti50.2-xHfx .at% obtained by fast melt spinning solidification

Considering the similarity of the composition used by Potapov (1997) regarding the composition Ni50Ti50-xHfx .at%, adopted in this work (difference of 0.2 .at% in the contents of Ni e Ti), the values of table 5 were used to proportionally calculate the same variables having as reference an alloy rich in Ni of nominal composition Ni50Ti50-xHfx .at%, quickly solidified by *melt-spinning*. The results are shown in Table 6:

| Hf (at%) | a (Å) | b (Å) | c (Å) | VOL (Å³) | \( \beta \) (°) |
|---------|-------|-------|-------|----------|----------------|
| 8       | 2.99735 | 4.08402 | 4.74398 | 57.19017 | 100.39443     |
| 9.5     | 3.00129 | 4.08011 | 4.77474 | 57.5676  | 100.47331     |
| 11      | 3.01312 | 4.07854 | 4.79841 | 58.00133 | 102.44549     |
| 15      | 3.03994 | 4.07637 | 4.84053 | 58.73422 | 103.47102     |
| 20      | 3.07229 | 4.06837 | 4.90964 | 59.779   | 105.20654     |
| 25      | 3.11173 | 4.05663 | 4.95697 | 60.49963 | 106.52065     |

Table 6 - Crystallographic characteristics of the martensitic phase calculated proportionally from Potapov's ribbons Ni49.8Ti50.2-xHfx .at% (1997) for the theoretical ribbons Ni50Ti50-xHfx .at% (x = 8, 9.5, 11, 15, 20 and 25 .at%) obtained by fast melt spinning solidification
Based on the data in table 6, these five variables were calculated with the nonlinear cubic model, which is composed by the third order function \( y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + e_i \) (i = 1, 2, ... n). The values of R² are, for the dependent variables a, b, c, volume and \( \beta \), respectively, 0.9986, 0.9938, 0.9987, 0.9986 and 0.9885. This certifies a large percentage of correct answers. The estimated quantities for the five compositions are presented in table 7.

For each of the cubic functions, the values of Adjusted-R² computed are, in the decrescent order, in table 8: 0.99643, 0.98445, 0.99682, 0.99658 e 0.97115.

For a graphic visualization, figure 6 presents the values calculated for the six Potapov ribbons (1997) plotted in the left column (figure 6(a), figure 6(c) and figure 6(e)) in a way that they are compared with the values predicted for the five ribbons studied here, plotted in the right column (figure 6(b), figure 6(d) and figure 6(f)).

According to the obtained numbers, the values of \( \beta \) angle, volume and "a" and "c" are directly proportional to the atomic percentage of hafnium from the alloy, except in the lattice parameter "b", which is inversely proportional to the content of Hf (at%).

### IV. COMPARISON OF CRYSTALLOGRAPHIC CHARACTERISTICS OF THE MARTENSITIC PHASE IN ZARINEJAD’S ALLOYS AND POTAPOV’S RIBBONS

Due to the fact that these variables are different, i.e., one-dimensional (lattice parameters) two-dimensional (\( \beta \) angles) and three-dimensional (monoclinic structures volumes) entities. To enable a dimensionless comparison between the predicted data from Zarinejad (2008) and Potapov (1997), it was necessary to consider the percentage indexess of the differences between the estimated values for each parameter (a, b, c, Vol e \( \beta \) ) of the five compositions \( \text{Ni}_{50}\text{Ti}_{50-x}\text{Hf}_x \).at% . This makes them arbitrary data. Therefore, it was necessary to organize the sample series from table 11 with the dimensionless format of table 9. Percentage indices have arbitrary units.
Using descriptive statistics, the measures of central tendency and dispersion were summarized in Table 10. The normal distribution (differences percentage indexes versus 𝑥̇s’ density) is represented by equation 2 and plotted in figure 7. It is composed of: 𝜎 (standard deviation), 𝜋 (Constante de Euler-Mascheroni: 2.718), 𝜋 (proportion) and 𝜇 (population mean).

\[
f(\chi) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{\chi - \mu}{\sigma}\right)^2}, \quad \chi \in (-\infty, \infty) \tag{2}
\]

It is noticeable, in figure 7, that there is a concentration of minor differences close to bigger densities. This certifies that the values from both authors are close.

However, there is a clear comprehension that the crystallographic characteristics depend not only on the hafnium content, but they are also sensitive to the peculiarities of the two production processes as well as the typical variables of each one of them: electric arc melting (Arc Melting) and quick solidification Melt Spinning.

Table 10 - Descriptive summary of the main statistical measurements

![Gaussian curve](image-url)
Depending on how the peculiarities involving each one of the processes are treated, one can directly influence the main properties of the samples such as: homogeneity, amount of residual stress and minimization of oxidation, among others. These physical and mechanic properties, among others are derived from atomic arrangement known as crystallographic structures (or crystalline structures).

It can be mentioned as variables of these processes: the number of times that the bulk was melted, exposure time to the torch, type of material of the mold, efficiency of the applied vacuum, rotating speed of the copper flywheel in quickly solidified ribbons etc. Hence, all the possibilities of variation and instrumental errors generate conditions of solidification which interfere in the micro structure of the alloys and obtained ribbons.

V. CONCLUSIONS

The analysis of variance (ANOVA) is an unilateral test based on the F-Snedecor table, which after calculating the F-value, it measures whether it is inside or outside of the acceptance area, according to the following hypothesis test:

\[
\begin{align*}
H_0 &: \mu_{Potapov} = \mu_{Zarinejad} \\
H_1 &: \text{there is at least one difference between the means}
\end{align*}
\]

As it is an OneWay ANOVA, the only factor (independent variable) is the atomic percentage of Hf (at%). As a rule, in terms of null hypothesis (H₀), the calculated F-value is inside the area of acceptance, i.e., minor than critical F. Otherwise, if H₀ is rejected, the alternative hypothesis (H₁) will be accepted having necessarily, the F-value calculated outside the area of acceptance, i.e., bigger than the critical F.

In this case, we considered that F(5%); DF₁ (DFW); DF₂ (DFR) = 3.12 (tabulated), according to the standard table for Tukey’s Test (α=0.05) [5]. According to Table 12, as the calculated F = 3.13 is bigger than the critical tabulated value, we concluded that there is a difference between the averages of both treatment groups: Potapov and Zarinejad. This conclusion is also confirmed by the P-value, which is smaller than 0.05. The lower the P-value, lower is the possibility for H₀ be true. It is important to note that DF₁ [horizontal = numerator] is the degree of freedom between groups (DFW) and that DF₂ [vertical = denominator] is the degree of freedom of the residues (DFR).

| Variation source | Sum of sQs | DF | Mean square | Fcalc | value-P | critical F tabulated |
|------------------|------------|----|-------------|-------|---------|---------------------|
| Between groups:  | 6739.09    | 2  | 3369.55     | 3.131 | 0.04968 | 3.123907449         |
| Within groups:   | 77488      | 72 | 1076.22     |       |         | Permutation p (n=99999) |
| Total:           | 84227.3    | 74 | 0.04909     |       |         | Table F-Snedecor - F(5%): 2.74 = 3.12 |

Table 12 - OneWay ANOVA calculated for two series of samples of the crystalline dimensions of the B19' monoclinic phase in Ni50Ti50, Hf₅ алloys and ribbons obtained by conventional and rapid solidifications (Zarinejad and Potapov, respectively)

(1) Between groups (W) = Treatment = Between treatments = Between (2) Within groups (R) = Residue = Error = Within (3) DF = degree of freedom

Therefore, the null hypothesis is rejected (H₀). However, to ascertain whether this difference is statistically significant, it is necessary to perform a parametric test to certify this significance. This decision was taken based on the normality test of Anderson-Darling [6], which predicts as “normal” the set of 25 data organized in table 9, as shown in figure 8:

A comparative analysis was made between two different solidification methods (arc melting e melt spin), when applying the TUKEY’s Test (through table 13) [7] to, in a complementary form to ANOVA, calculate the minimum significant difference (m. s. d.) there is between the averages in the martensitic crystallographic characteristics, according to the original dimensional data predicted by the adjustment models, shown in table 2 and table 7. It is concluded that the method to obtain the alloys doesn’t interfere in the dimensions here evaluated, that is, there is no significant statistical difference. This is due to the fact that the calculated difference (|µ_Potapov - µ_Zarinejad|) was of 0.75. According to equation 3, the m.s.d [Δ] necessary for both groups (data series) to have a significant difference is 19.027.

\[
\Delta = q \times \sqrt{\frac{\text{Mean Square (residues)}}{\text{Number of treatment repetitions}}} 
\]

In this calculation, q [q(5%); k; DF of residues = 2.899943] is a tabulated value (F-Snedecor table, α=0.05), the QMR is 1076.22 (highlighted in table 12) and the number of repetitions is equal to 25. This count of 25 is contained in table 13, before the summary (resume).
| Hf (at.% ) | Dimensions | Zarinejad (2008) | Potapov (1997) |
|-----------|------------|-----------------|----------------|
|           | a (Å)      | 2.897725078     | 2.995688       |
| 8         | b (Å)      | 4.129549466     | 4.083342       |
|           | c (Å)      | 4.703096765     | 4.746384       |
|           | Volume (Å³) | 54.99855        | 57.208961      |
|           | β (°)      | 98.28473246     | 100.237795     |
| 11        | a (Å)      | 2.996283729     | 3.012892       |
|           | b (Å)      | 4.119240505     | 4.07906        |
|           | c (Å)      | 4.730063875     | 4.795981       |
|           | Volume (Å³) | 56.18361        | 57.93075       |
|           | β (°)      | 98.6143636      | 100.997707     |
| 14        | a (Å)      | 3.129822061     | 3.031679       |
|           | b (Å)      | 4.113564167     | 4.076048       |
|           | c (Å)      | 4.757980318     | 4.838146       |
|           | Volume (Å³) | 57.52923        | 58.594993      |
|           | β (°)      | 99.0337747      | 101.835911     |
| 17        | a (Å)      | 3.302774532     | 3.051887       |
|           | b (Å)      | 4.096405712     | 4.07311        |
|           | c (Å)      | 4.801194093     | 4.87463        |
|           | Volume (Å³) | 58.81347        | 59.199742      |
|           | β (°)      | 99.51929314     | 102.726977     |
| 20        | a (Å)      | 3.5195756       | 3.073354       |
|           | b (Å)      | 4.0516504       | 4.06905        |
|           | c (Å)      | 4.8740532       | 4.907188       |
|           | Volume (Å³) | 59.81439        | 59.743049      |
|           | β (°)      | 100.047246      | 103.645474     |

Resume

|       | Sum        | 843.0616395    | 861.829798     |
| Mean  | 33.72246558 | 34.47319192    | 103.645474     |
| Variance | 1560.507263 | 1649.408532    |

Difference between means  
\[39.05 - 40.61] = 0.75072634

Table 13 - Paired values of the crystallographic characteristics of the B19' martensitic phase in Ni₅₀Ti₅₀X₇Hf at% alloys predicted by the fitting

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ATTACHMENTS

DF (n-k)  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10
--------|----|----|----|----|----|----|----|----|----
5       | 3.64 | 4.6 | 5.22 | 5.67 | 6.03 | 6.33 | 6.58 | 6.8 | 6.99
6       | 3.46 | 4.34 | 4.9 | 5.3 | 5.65 | 5.9 | 6.12 | 6.32 | 6.49
7       | 3.34 | 4.16 | 4.68 | 5.06 | 5.36 | 5.61 | 5.82 | 6 | 6.16
8       | 3.26 | 4.04 | 4.53 | 4.89 | 5.17 | 5.4 | 5.6 | 5.77 | 5.92
9       | 3.2 | 3.95 | 4.41 | 4.76 | 5.02 | 5.24 | 5.43 | 5.59 | 5.74
10      | 3.15 | 3.88 | 4.33 | 4.65 | 4.91 | 5.12 | 5.3 | 5.46 | 5.6
11      | 3.11 | 3.82 | 4.26 | 4.57 | 4.82 | 5.03 | 5.2 | 5.35 | 5.49
12      | 3.08 | 3.77 | 4.2 | 4.51 | 4.75 | 4.95 | 5.12 | 5.27 | 5.39
13      | 3.06 | 3.73 | 4.15 | 4.45 | 4.69 | 4.88 | 5.05 | 5.19 | 5.32
14      | 3.03 | 3.7 | 4.11 | 4.41 | 4.64 | 4.83 | 4.99 | 5.13 | 5.23

ATACHMENTS

|        |   |   |   |   |   |   |   |   |   |
|--------|---|---|---|---|---|---|---|---|---|
| DF     |   |   |   |   |   |   |   |   |   |
| k levels |   |   |   |   |   |   |   |   |   |

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Table 14 - Tukey test table ($\alpha = 0.05$): degrees of freedom and levels [studentized]

| DF in denominator | Degrees of Freedom (DF) in the numerator |
|-------------------|-----------------------------------------|
| 1                 | 161.45 199.5 215.71 224.58 230.16 233.99 236.77 238.88 240.54 241.88 |
| 2                 | 18.51 19 19.16 19.25 19.3 19.33 19.35 19.37 19.38 19.4 |
| 3                 | 10.13 9.55 9.28 9.12 9.01 8.94 8.89 8.85 8.81 8.79 |
| 4                 | 7.71 6.94 6.59 6.39 6.26 6.16 6.09 6.04 6 5.96 |
| 5                 | 6.61 5.79 5.41 5.19 5.05 4.95 4.88 4.82 4.77 4.74 |
| 6                 | 5.99 5.14 4.76 4.53 4.39 4.28 4.21 4.15 4.1 4.06 |
| 7                 | 5.39 4.74 4.35 4.12 3.97 3.87 3.79 3.73 3.68 3.64 |
| 8                 | 5.32 4.46 4.07 3.84 3.69 3.58 3.5 3.44 3.39 3.35 |
| 9                 | 5.12 4.26 3.86 3.63 3.48 3.37 3.29 3.23 3.18 3.14 |
| 10                | 4.96 4.1 3.71 3.48 3.33 3.22 3.14 3.07 3.02 2.98 |
| 11                | 4.84 3.98 3.59 3.36 3.2 3.09 3.01 2.95 2.9 2.85 |
| 12                | 4.75 3.89 3.49 3.26 3.11 3 2.91 2.85 2.8 2.75 |
| 13                | 4.67 3.81 3.41 3.18 3.03 2.92 2.83 2.77 2.71 2.67 |
| 14                | 4.6 3.74 3.34 3.11 2.96 2.85 2.76 2.72 2.66 2.62 |
| 15                | 4.54 3.68 3.29 3.06 2.9 2.79 2.71 2.64 2.59 2.54 |
| 16                | 4.49 3.63 3.24 3.01 2.85 2.74 2.66 2.59 2.54 2.49 |
| 17                | 4.45 3.59 3.2 2.96 2.81 2.7 2.61 2.55 2.49 2.45 |
| 18                | 4.41 3.55 3.16 2.93 2.77 2.66 2.58 2.51 2.46 2.41 |
| 19                | 4.38 3.52 3.13 2.9 2.74 2.63 2.54 2.48 2.42 2.38 |
| 20                | 4.35 3.49 3.1 2.87 2.71 2.6 2.51 2.45 2.39 2.35 |
| 21                | 4.32 3.47 3.07 2.84 2.68 2.57 2.49 2.42 2.37 2.32 |
| 22                | 4.3 3.44 3.05 2.82 2.66 2.55 2.46 2.4 2.34 2.3 |
| 23                | 4.28 3.42 3.03 2.8 2.64 2.53 2.44 2.37 2.32 2.27 |
| 24                | 4.26 3.4 3.01 2.78 2.62 2.51 2.42 2.36 2.3 2.25 |
| 25                | 4.24 3.39 2.99 2.76 2.6 2.49 2.4 2.34 2.28 2.24 |
| 26                | 4.23 3.37 2.98 2.74 2.59 2.47 2.39 2.32 2.27 2.22 |
| 27                | 4.21 3.35 2.96 2.73 2.57 2.46 2.37 2.31 2.25 2.2 |
| 28                | 4.2 3.34 2.95 2.71 2.56 2.45 2.36 2.29 2.24 2.19 |
| 29                | 4.18 3.33 2.93 2.7 2.55 2.43 2.35 2.28 2.22 2.18 |
| 30                | 4.17 3.32 2.92 2.69 2.53 2.42 2.33 2.27 2.21 2.16 |
| 35                | 4.12 3.27 2.87 2.64 2.49 2.37 2.29 2.22 2.16 2.11 |
| 40                | 4.08 3.23 2.84 2.61 2.45 2.34 2.25 2.18 2.12 2.08 |
| 45                | 4.06 3.2 2.81 2.58 2.42 2.31 2.22 2.15 2.1 2.05 |
| 50                | 4.03 3.18 2.79 2.56 2.4 2.29 2.2 2.13 2.07 2.03 |
| 100               | 3.94 3.09 2.77 2.46 2.31 2.19 2.1 2.03 1.97 1.93 |

Table 15 - F distribution of Snedecor ($\alpha = 0.05$)