Results in the Spontaneous Annihilation of the Cosmological Constant

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We suggest a new formula, which allows the Schwarzschild’s solution and the Einstein radius to be applied to the dynamic universe, when our universe is hypothetically regarded as a single dynamic black hole. In this study, a cosmological constant problem is solved in the simplest manner, while we find excellent agreements with observation. We adopt a model, wherein k=0, Λ≠0, and Ω=1 to interlock Λ with critical density of the black hole of our universe ρ_{BH}, thereby presenting complimentary relation between Λ and Bekenstein-Hawking entropy S_{BH}.

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I. BACKGROUND

Unexpected results are discovered over a process to analyze and examine a hierarchical problem of elementary particles and their free parameters. We generate a large number of phenomenological models by applying an abduction method to a data set of Particle Data Group (PDG). Through the models, we acquire certain implications of the postulate can be stated as a ratio of dimensionless numbers. Namely, formulae can be regarded as physically identical when their calculated values are the same, i.e. 1 = 1s = 1Hz and 2 = 2s = 2Hz, albeit they have different structures. In cosmology, a dimensionless number ‘1’ is interpreted as a state, wherein the early universe had high order, at one second after the inflation.

The first fixed parameter, the Newtonian gravitational constant G, is presented as follows.

\[ G \approx 6.673 \, 384 \, 392 \, \cdots \times 10^{-11} \, \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2} \]  

Planck mass \( M_P \)

\[ (\hbar c/G)^{1/2} = \left( \frac{1}{2\pi G} \right)^{1/2} \approx 2.176 \, 508 \, 699 \, \cdots \times 10^{-8} \, \text{kg} \]  

Planck length \( L_P \)

\[ \left( \frac{\hbar G}{c^2} \right)^{1/2} = \left( \frac{G}{2\pi} \right)^{1/2} \approx 1.616 \, 199 \, 666 \, \cdots \times 10^{-35} \, \text{m} \]  

Planck energy density \( \rho_P \)

\[ \left( \frac{E_P}{L_P^3} \right) = \left( \frac{M_P}{L_P^3} \right) \approx 5.155 \, 554 \, 340 \, \cdots \times 10^{36} \, \text{kg} \cdot \text{m}^{-3} \]
\[ G = \left( \frac{L_p}{M_p} \right), \quad 2\pi L_p^2 = G, \quad \left( \frac{G}{L_p^2} \right)^2 = 2\pi G \quad (1.2.4) \]

II. RESCALED FRIEDMANN EQUATION

Friedmann considered the subsequent Einstein field equation, which includes the cosmological constant as a starting point.[4]

\[ G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} \quad (2.1) \]

Eq.(2.1) is simplified by

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k c^2}{a^2} + \frac{\Lambda c^3}{3} \quad (2.2) \]

For further simplification of Eq.(2.2), Friedmann formulated the following formulae, under the assumption that \( k = 0 \) and \( \Lambda = 0 \).

\[ H^2 = \frac{8\pi G}{3} \rho \quad (2.3) \]

Or,

\[ \rho = \frac{3H^2}{8\pi G} \quad (2.4) \]

In the Einstein-de Sitter Model, wherein \( E = 0 \), the flat universe, \( k = 0 \), corresponds to the Friedmann model. In this case \( \rho \rightarrow \rho_c \), so that an independent hypothesis is required.

III. HYPOTHESIS AND SPONTANEOUS ANNIHILATION MECHANISM

In the actual observation of the universe [3, 6], the cosmological constant, \( \Lambda \), is not equal to zero, \( \Lambda \neq 0 \). Therefore, in this study, we have two hypotheses to include \( \Lambda \) and satisfy the observation data and the Einstein field equation. The first one is that \( k = 0 \), \( \Lambda \neq 0 \), and \( \Omega = 1 \), since we adopt the theories [7, 8], which provide meaningful statements with regard to the rescaled Friedmann model and the density parameter, \( \Omega = 1 \). We come up with the second hypothesis in order to precisely match relation among actual density \( \rho \), critical density \( \rho_c \), and critical density of the black hole of our universe \( \rho_{cBH} \), each of which is a cosmological parameter. That is,

\[ \Omega = \frac{\rho}{\rho_c} = \frac{8\pi G}{3H^2\rho} = 1 \quad (3.1) \]

\[ \rho = \rho_c = \rho_{cBH} \quad (3.2) \]

In the Hubble’s law, which is expressed by \( v = Hr \), when velocity of a galaxy is \( v \rightarrow c \), in a cosmological horizon (particle horizon), the following transformation is possible.

\[ r = \frac{c}{H} = \frac{1}{H} \quad (3.3) \]

According to the hypothesis of Eq.(3.2), when we assume that our universe is the single dynamic black hole in Eq.(3.3), then moving time \( r \) in the Hubble time, \( t_H \), is closer to \( R_s \), having the maximum critical radius \( R_{smax} \), namely, \( r \rightarrow R_s \). Therefore, we have consistent relation among \( R_s, \rho_{cBH}, \rho_c \) and \( \rho \) in the hypothesized dynamic black hole of our universe. The FLRW cosmological model describes a boundary condition [9] with regard to a solution of the particle horizon \( H_p \). The particle horizon is similar to an event horizon, but has little difference. It is noteworthy that in this study, the Schwarzschild radius, \( R_s \), which refers to the event horizon is different from \( R_s \), which refers to the particle horizon. We will describe the difference, corresponding to an upper bound concept [10] of entropy in the black hole. All symbols in bold font are cosmological parameters related to the comoving universe, e.g. \( M \), and \( R_s \). Given aforementioned statements, Eq.(3.1) can be re-described by

\[ \Omega = \frac{\rho}{\rho_c} = \frac{8\pi G}{3H^2\rho} = \frac{8\pi G R_s^2}{3}\rho_c = 1 \quad (3.4) \]

In addition, Eq.(3.3) clarifies the meaning of Eq.(2.4), and relation between \( R_s \) and \( M \) is proportional. Therefore, \( \rho \) is transformed as below.

\[ \frac{\rho}{8\pi G} = \frac{3}{8\pi G R_s^2} = \frac{3}{32\pi G^3 M^2} = \rho_c = \rho_{cBH} \quad (3.5) \]

In Eq.(3.5), it is notable that

\[ \rho = \rho_c = \rho_{cBH} \propto \frac{1}{R_s^2} \propto \frac{1}{M^2} \]

After completing his field equation, Einstein acquired information of the Hubble’s expanding universe, and corrected his static universe concept in the following way. Namely, it is presented that the cosmological constant, \( \Lambda \), is proportional to the following terms.

\[ \Lambda \propto \left( \frac{1}{R^2} \right) \propto \left( \frac{\kappa \rho}{2} \right) \quad (3.6) \]

where

\[ \kappa = \text{Einstein’s constant} \]

Given Eq.(3.5),

\[ \left( \frac{1}{R_s^2} \right) \rightarrow \left( \frac{1}{R_s^2} \right) \]
in Eq. (3.6), while it seems that
\[ \rho \to \rho_c, \rho_{cBH} \]

If the implication of Eq. (3.6) is correct, it is expected that \( \kappa = 2 \). This case seems to be associated with the \( H_p \) boundary condition.

Generally, Planck energy density \( \rho_P \)
\[ \rho_P = \left( \frac{E_p}{L_p^3} \right) = \left( \frac{M_p}{L_p^3} \right) \]
is used in a unit of the cosmological constant. Considering observation data and Eqs. (3.5), (3.6), \( \Lambda \) can be expressed as \( \rho_c \) or \( \rho \) with regard to Planck energy density \( \rho_P \). In this case, if an annihilation factor is set as \( z^* \), then \( z^* \) is expressed as
\[ z^* \Lambda = \left( \frac{z^* \rho_{cBH}}{M_p/L_p^3} \right) = \left( \frac{z^* \rho_{c}}{M_p/L_p^3} \right) \]

In Eq. (3.7),
\[ \rho = \frac{3H^2}{8\pi G} = \frac{3}{8\pi G R_s^2} = \frac{3}{32\pi G^3 M^2} = \rho_c = \rho_{cBH} \]
Therefore, \( z^* \Lambda \) can be rewritten as follows.
\[ z^* \Lambda = \frac{z^* 3L_p^3}{8\pi G R_s^2 M_p} = \left( \frac{z^* 3L_p^3}{32\pi G^3 M^2 M_p} \right) \]

The second fixed parameter, the black hole radius \( R_s \) of our universe, is shown in the following numerical value.
\[ R_s \approx 1.721 \, 944 \, 132 \cdots \times 10^{26} \text{ m} \] (3.9)

The numerical value of \( R_s \) is of the order of \( 10^{19} \) light years

By using the numerical value (3.9), we can obtain the critical density of the black hole of our universe \( \rho_{cBH} \) as follows.
\[ \rho_{cBH} = \frac{3H^2}{8\pi G} = \frac{3}{8\pi G R_s^2} = \frac{3}{32\pi G^3 M^2} \approx 5.421 \, 365 \, 353 \cdots \times 10^{-27} \text{ kg} \cdot \text{m}^{-3} \] (3.10)

Given the value from Eq. (3.10), it is expected that the number of hydrogen atoms is around \( 3.241 \, 238 \, 054 \cdots \) per m³.

As such, mass of our universe \( M \) is multiplication of volume from the numerical value (3.9) and density from Eq. (3.10).
\[ M = \frac{4\pi R_s^3}{3} \times \rho_{cBH} \]
\[ \approx 1.159 \, 456 \, 489 \cdots \times 10^{53} \text{ kg} \] (3.11)

With regard to \( M \) in Eq. (3.11), if proton \( m_p \approx 1.672 \, 621 \, 776 \cdots \times 10^{-27} \text{ kg} \), then, the number of protons is around \( 6.931 \, 970 \, 548 \cdots \times 10^{79} \) in \( M \), and this shows that observation data and theoretical prediction are approximately identical.

As such, a calculated dimensionless value of \( \Lambda \) excepting for \( z^* \) is defined by
\[ \Lambda \approx 1.051 \, 558 \, 182 \cdots \times 10^{-123} \] (3.12)

When we transform the processes of \( M_p \to M \) and \( L_p \to R_s \) (max.) into calculated values, then it is possible to obtain the following relation.
\[ \frac{M}{M_p} = \frac{R_s}{2L_p} \approx 5.237 \, 139 \, 237 \cdots \times 10^{60} \] (3.13)

Eq. (3.12) implies that our universe was inflated more than \( 10^{60} \) times at one second after the inflation, based on the Plank scale. It seems that the early universe maintains such order with scale invariance.

\( \Lambda \) seems to be proportional to the critical density of the black hole \( \rho_c \), in Eqs. (3.7), (3.8). However, it is notable that, when density of black hole critical density \( \rho_c \) reaches a lower bound along with flow of time, \( \Lambda \) is inversely proportional to the maximum critical density radius \( R_{s \text{max}} \) and the maximum mass \( M_{\text{max}} \), thereby being minimized. This is because the black hole is the system, which has the maximum entropy of our universe \( S_{\text{max}} \) under the complementary relation, and it is acknowledged that the black hole has information analogous to that of thermodynamics [12]. Especially, with regard to the holographic principle, suggested by t’Hooft, Susskind [13] presented that all entities in a certain area of a space can be described by information pieces, distributed on a boundary surface. Briefly, Susskind and others suggested that a positive cosmological constant has surprising consequences, such as the finite maximum entropy of the observable universe. Consequently, in Eq. (3.7), the annihilation factor, \( z^* \), is regarded as the most important parameter, which mediates between gravitational force and quantum mechanics. The annihilation factor, indeed, is the final result of a large number of phenomenological models, and it is confirmed that \( z^* \) can be replaced by the Bekenstein-Hawking entropy formula \( S_{\text{BH}} \) in a precise manner. That is,
\[ z^* = S_{\text{BH}} = \frac{k_B c^3 A}{4\hbar G} \] (3.14)

In Eq. (3.14), \( A \) is an area of the event horizon of our universe, and \( A = 4\pi R_{s \text{BH}}^2 \). Here, we regard \( A \) as a cosmological horizon, instead of the event horizon. When \( \lvert c \rvert = \lvert k_B \rvert = 1 \) is applied, \( S_{\text{BH}} \) can be briefly expressed as follows.
\[ S_{BH} = \frac{\pi A}{2G} = \frac{\pi (4\pi R_s^2)}{2G} = \frac{4\pi^2(2GM)^2}{2G} = 8\pi^2GM^2 \]  
\[ (3.15) \]

\[ S_{BH} = \frac{A}{4L_p^2} = \frac{4\pi G^2M^2}{L_p^2} = 8\pi^2GM^2 \]  
\[ (3.16) \]

It is notable that, in comparison to Eq. (3.8), inverse relation is true: \( S_{BH} \propto R_s^2 \propto M^2 \).

In Eqs. (3.15), (3.16), it is possible to identify values of \( G \), \( M \), and \( L_p \), so that a calculated dimensionless value of the observable maximum entropy of the universe \( S_{BH} \) is expressed in the following formula.

Roger Penrose used a symbol, \( 'S_A' \), which refers to the ultimate entropy and calculated its value. Our calculated value is nearly identical to the Penrose’s result. Roger Penrose wrote, “With the observed value of \( \Lambda \), the temperature \( T_A \) would have the absurdly tiny value \( 10^{-30}K \), and the entropy \( S_A \) would have the huge value \( \sim 3 \times 10^{122} \).”

\[ S_{BH} = 3.566 \times 10^{122} \]  
\[ (3.16.1) \]

Interaction between \( \Lambda \) and \( S_{BH} \) from Eq. (3.16) is completely annihilated in a spontaneous manner. That is,

\[ \Lambda \cdot S_{BH} = \frac{3L_p^3}{32\pi G^3M^2M_p} \cdot \frac{4\pi G^2M^2}{L_p^2} = \frac{3L_p}{8GM} = \frac{3}{8} \]  
\[ (3.17) \]

or

\[ \frac{\rho_c}{M_p/L_p} = \frac{3}{8} \]  
\[ (3.17.1) \]

**IV. DISCUSSION AND CONCLUSION**

Did any special initial state exist in our universe? In the Eq. (3.17.1), if \( E = 0 \), and curvature \( k=0 \), then \( \rho \rightarrow \rho_c \). Namely, if we re-write the irreducible fraction, 3/8, as

\[ \rho = \rho_c = \rho_{cBH} = \frac{3}{32\pi G^3M^2} \]

then, Eq. (3.17) regresses to the original Eq. (3.5). It implies that the maximum entropy of our universe \( S_{max} \) encounters \( \Lambda \) expressed by \( \rho_{cBH} \), then this becomes an unobservable process. Namely, when entropy spontaneously collapses to satisfy Eqs. (3.1), (3.2) simultaneously, then this means that the special initial state exists with high order.

It shows agreements with the observation data and seems to contribute to find valuable information with regard to a question why entropy of our universe is low [13]. Namely, when the spontaneous annihilation mechanism of the cosmological constant is excluded, it is possible to come up with a question whether entropy can decrease, as the universe re-collapses to explain the low entropy of the early universe [16].

Followed by Eq. (3.17), we present relation between entropy and the temperature. That is, when a formula of the Bekenstein-Hawking entropy \( S_{BH} \) is combined with a formula of the Hawking radiation temperature \( T_{BH} \), the result is shown below.

\[ S_{BH} \cdot T_{BH} = \frac{k_Bc^3A}{4\hbar G} \cdot \frac{\hbar c^3}{8\pi GMk_B} = 8\pi^2GM^2 \cdot \frac{1}{16\pi^2GM} = \frac{M}{2} \]  
\[ (4.1) \]

Here, we can have a calculated value of \( T_{BH} \) by substituting \( M \) with \( T_{BH} = \frac{1}{cM_{BH}} \). When the calculated value is transformed into thermodynamic temperature, \( K \), then it is possible to calculate the lower bound temperature. Roger Penrose used the symbol, \( 'T_A' \) for his calculations, and our calculation is nearly equal to Penrose’s numerical factor [14].

\[ T_{BH} = \frac{\hbar c^3}{8\pi GM_{BH}k_B} = \frac{1}{16\pi^2GM} \]  
\[ (4.1.1) \]

\[ \approx 1.058 \times 10^{-30}K \]

Given Eqs. (3.17), (4.1), the final implication is that when entropy shows a waveform graph along with the flow of time, energy distribution, during the contraction/expansion phases, precisely becomes half, thereby generating a symmetrical pattern. However, during the contraction phase, energy is absorbed into an empty space and disappears. Therefore, only the expansion phase entropy can be observed, and thus it seems to show an asymmetrical structure in the direction of the arrow of time.

Eq. (3.17) implies the annihilation mechanism of \( \Lambda \), which is generated by quantum perturbation in a vacuum, and coexistence of the initial and the final conditions of our universe. This means that we expand the theory: the critical density and the actual density are precisely matched so as to be autonomously coordinated with one another [17]. Namely, the upper bound of the black hold radius \( R_{max} \), which corresponds to the maximum entropy \( S_{max} \), or its black hole volume is close to the lower bound, this means that a concept of the Bing Bang can be potentially replaced by Bouncing cosmology [18]. This idea provides a philosophical issue by referring to related articles. The final relation among \( S_{BH} \), \( T_{BH} \), \( \Lambda \), \( G \), and \( R_s \) is presented by

\[ \frac{S_{BH} \cdot T_{BH} \cdot \Lambda \cdot G}{R_s} = \frac{3}{32} \]  
\[ (4.2) \]

We verify Eq. (4.2) in the following way. If we express the formula in relation to \( \Lambda \), then the result is presented as below.
\[ \Lambda = \frac{3R_s}{32 \cdot S_{BH}^2 \cdot T_{BH} \cdot G} = \frac{6GM \cdot 16\pi^2GM}{32 \cdot 64\pi^4G^4M^4} = \frac{3}{64\pi^2GM^2} \]  

(4.2.1)

In the Eq.(3.3), when we exclude the annihilation factor, \( z^* \), and describe \( \Lambda \) only, then

\[ \Lambda = \left( \frac{\rho}{M_p / L_p^4} \right) = \left( \frac{\rho_c}{M_p / L_p^4} \right) \]

In addition,

\[ \rho = \rho_c = \rho_{cBH} = \frac{3}{32\pi G^3M^2} \]

Therefore, Eq.(4.2.1) can be described as follows.

\[ \frac{3}{64\pi^2GM^2} = \frac{3L_p^3}{32\pi G^3M^2M_p} \]  

(4.2.2)

Eq.(4.2.2) can be simply stated by

\[ \frac{1}{2\pi} = \frac{L_p^3}{G^2M_p} = \frac{L_p^2}{G} \left( \frac{c}{G} \cdot M_p = L_p \right) \]

The Einstein's static universe shows the following equation in the Friedmann equation with regard to the Einstein radius \( E_R \).

\[ E_R = \frac{c}{\sqrt{4\pi G \rho}} \]  

(4.3)

When we apply \( c = 1 \), relation between the dynamic universe \( R_s \) and the static universe of the Einstein radius \( E_R \) is expressed as below.

\[ \frac{R_s}{E_R} = \left( \frac{3}{2} \right)^{1/2} \]  

(4.4)

Eq.(4.4) is proven by the following procedure. As \( \rho = \rho_c = \rho_{cBH} \), when both sides of Eq.(4.4) are squared, the result is as follows.

\[ \left( \frac{R_s}{E_R} \right)^2 = \left( \frac{2GM}{32\pi G^3M^2} \right)^2 \cdot \frac{3}{2}(QED) \]

As a final result, extensively brief relation is produced between Eq.(4.2) and Eq.(4.4).

\[ \frac{3}{32} \cdot \left( \frac{R_s}{E_R} \right)^2 = \frac{3}{32} \cdot \frac{3}{2} = \frac{9}{64} = \left( \frac{3}{8} \right)^2 = (S_{BH} \cdot \Lambda)^2 \]  

(4.5)

CONCLUSION

In this study, we find excellent agreements between (a) the annihilation mechanism of the cosmological constant \( \Lambda \) and (b) a spontaneous decay process of the black hole maximum entropy \( S_{BH} \) itself in the consistent self-coordinating system, which makes \( \rho, \rho_c \) and \( \rho_{cBH} \) exactly identical at one second after the inflation of our universe. The procedure is a well-coordinated hidden mechanism, and it is unobservable. Therefore, the universe evolves from low entropy of our universe with regard to observation, so that the universe can be observed in the direction of the arrow of time. Despite this fact, energy and entire information are preserved.

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