Temperature Dependence of Conductance and Thermopower Anomalies of Quantum Point Contacts

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It has been shown within the Landauer single-channel approach that the presence of the 0.7 anomaly in the conductance of a ballistic microcontact and the respective plateau in the thermopower implies unusual pinning of the potential barrier height $U$ at a depth of $k_BT$ below the Fermi level $E_F$. A simple way of taking into account the effect of electron-electron interaction on the profile and temperature dependence of a smooth one-dimensional potential barrier in the lower spin degeneracy subband of the microcontact has been proposed. The calculated temperature dependences of the conductance and Seebeck coefficient agree with the experimental gate-voltage dependences, including the emergence of anomalous plateaux with an increase in temperature.

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I. INTRODUCTION

Quantization of the conductance of submicron constriction in two-dimensional electron gas is described well by the Landauer formula under the assumption of spin degeneracy of one-dimensional single-particle subbands in zero magnetic field. The same approach explains the alternation of zero plateaux and peaks of thermopower (Seebeck coefficient $S$), which obeys the Mott formula, $S \propto \frac{\partial \ln G(V_g, T)}{\partial E_F}$. However, the dependence $G(V_g)$ of the conductance on the gate voltage exhibits a narrow region of anomalous behavior, the 0.7 - $2e^2/h$ plateau. This plateau broadens with an increase in temperature, but persists at a complete thermal spread of the conductance quantization steps and references therein. Numerous scenarios including spin polarization, Kondo effect, Wigner crystal, charge-density waves, and the formation of a quasi-localized state have been suggested. Calculations that reproduce the unusual temperature behavior of the 0.7 conductance anomaly have been performed so far only within phenomenological fitting models with spin subbands or beyond the Landauer formula. Although a particular mechanism of the appearance of the anomalous plateaux in conductance and thermopower remains unclear, their common reason is thought to be the electron-electron interaction, which should manifest itself most effectively at the onset of filling the first subband, where the electron system is one-dimensional.

In this work, we start from the standard Landauer approach to the description of conductance and thermopower of a single-mode ballistic quantum wire with spin degeneracy. This approach take into account interaction via the $T$-dependent one-dimensional reflecting barrier. First, we show that the appearance of the 0.7 anomaly implies pinning of the barrier height $U$ at a depth of $k_BT$ below the Fermi level $E_F$ and that this pinning yields the plateau $S \neq 0$. Next, motivated by description of Friedel oscillations surrounding a delta-barrier in a one-dimensional electron gas, we suggest a simple formula which reduces the $T$-dependent part of the correction to the interaction-induced potential to the temperature dependence of the one-dimensional electron density.

The behavior of the conductance and Seebeck coefficient calculated with the corrected potential agrees with the published experimental results.

II. BASIC FORMULAS AND UNUSUAL PINNING

Conductance and Seebeck coefficient of single mode ballistic channel can be written within the Landauer approach as follows:

$$G = \frac{2e^2}{h} \int_0^\infty D(E, U(x, V_g)) F(\epsilon) d\epsilon,$$

$$S = \frac{2e^2 k_B}{hG} \int_0^\infty D(E, U(x, V_g)) \epsilon F(\epsilon) d\epsilon,$$  \hspace{1cm} (1)

where $D$ is the transmission coefficient, $E$ is the energy of ballistic electrons, $U(x, V_g)$ is the effective $T$-dependent one-dimensional barrier, $\epsilon = (E - E_F)/k_BT$, $F(\epsilon) = 4k_BT \cosh^2(\epsilon/2)$ is the derivative of the Fermi distribution function with respect to $-E$. There is also the Mott approximation generalized for arbitrary temperatures $T$:

$$S^M = \frac{\pi^2 k_B^2 T}{3e} \frac{\partial \ln G(V_g, T)}{\partial E_F}.$$

If the barrier $U(x)$ in a one-dimensional channel is sufficiently wide (according to the three-dimensional electrostatic calculations of gate-controlled quantum wires the barrier half-width must be $\gtrsim 200$ nm), the step in the energy dependence $D(E)$ of the transmission coefficient is abrupt and the respective transition is much narrower in the energy $E$ than the thermal energy $k_BT$, at which the 0.7 plateau occurs. This condition is satisfied in many experiments. Then, Eq. (1) for $G$, and $S^M$.
S and Mott approximation gives

\[ G = \frac{2e^2}{\hbar} (1 + e^{-\eta})^{-1}, \]
\[ S = -\frac{k_B}{e} \left[ (1 + e^{-\eta}) \ln(1 + e^\eta) - \eta \right], \]
\[ S^M = -\frac{k_B}{e} \frac{\pi^2}{3} (1 + e^\eta)^{-1}, \]

where \( \eta = (E_F - U)/k_B T \) and \( U = U(x = 0) \) is the height of the barrier \( U(x, T, V_g) \). Clearly, the dependencies \( G(E_F - U) \) at different \( T \) are simply smooth steps of unit height with the fixed common point \( G = e^2/\hbar \). Curves \( S^M(\eta) \) and \( S(\eta) \) are numerically close to each other at the interval \( 0 < S < 2k_B/e \) (see Appendix A). The values \( G \approx 0.7 \cdot 2e^2/\hbar \) in \( G(\eta) \) are not particularly interesting except that they correspond to \( \eta \approx 1 \). However, in experiments, there appear plateaus of \( G(V_g) \) at these values, which implies pinning of \( U(V_g) \) at a depth of \( k_B T \) below the Fermi level (see Appendix A). According to Eq. (2), the discovered pinning can be expected to give the plateau \( S \approx -0.8k_B/e \) (\( S^M \approx -k_B/e \)) in the curve \( S(V_g) \). Appendix A compares the calculated plateaus with the experimental ones,17 and shows that parameter \( \eta \) obtained from \( G \) is equal to that from \( S \), which verifies applicability of Eq. (2) to the experiment. Notice that this pinning differs from the pinning discussed earlier17,20,22,24 by unusual temperature dependence and single-channel transmission. The pinning that we detected seems paradoxical and urges us to suggest that a probe ballistic electron at the center of the barrier is almost independent of \( T \) and the dependence \( V(T, V_g) \) computed self-consistently with the electron density (see Appendix B). In fact, similar to our previous calculations,25,26 we computed three-dimensional electrostatics of single-mode quantum wires using different kinds of self-consistency between the potential and the electron density within25 and without the inclusion of exchange interaction and correlations in the local approximation. These calculations show quite definitely that the one-dimensional electron density \( n_e \) in the center of the barrier is almost independent of \( T \) and is linear in \( V_g \) starting from small \( n_{e0} \) values; i.e., the electric capacitance between the gate and the quantum wire is conserved at \( G > e^2/\hbar \). In addition, since the density of states is positive, \( d(E_F - V)/dn_e > 0 \) and the dependence \( V(V_g) \) of the self-consistent barrier height on the gate voltage does not yield pinning even with the inclusion of the exchange-correlation corrections in the local approximation (Appendix B). Therefore, we suggest that the discovered pinning of the reflecting barrier height is due to the nonlocal interaction.

III. ESTIMATION OF NONLOCAL INTERACTION

It is well known in atomic physics and physics of metallic surfaces and tunneling gaps between two metals that the potential seen by a probe electron in a low-density region is different from the self-consistent potential found with the inclusion of interaction in the local approximation.25,26 This difference was attributed to nonlocal exchange and correlations, i.e., to the attraction of the electron to an exchange-correlation hole, which remains in a high-density region.29,30 Consideration of this phenomenon regarding a quantum point contact is currently unavailable, despite the obvious analogy between two metallic bars separated by a tunneling gap and two-dimensional electron gas bathed separated by a potential barrier. We suggest that a ballistic electron coming to the barrier region, where the density \( n_e \) is low, gets separated from its exchange-correlation hole, which is situated in the region of dense electron gas. As a result, the local description of the correction to the potential becomes inadequate. Although the hole has a complicated shape, it can be associated with the effective center. Then, a decrease in the potential for the ballistic electron in the center of the barrier is \( U - V \approx -\gamma e^2/(4\pi \epsilon_0 r) \), where \( r \) is the distance between the centers of the barrier and hole, whereas \( \gamma \approx 1 \) takes into account the shape of the hole and weakly depends on \( r \). In perturbation theory, we are interested in a small (i.e., \( T \)-dependent) part of the correction:

\[ \delta U \approx -\frac{e^2}{(4\pi \epsilon_0 r)} \gamma (r(T) - r(0))^{-1}, \]

and the correction at \( T = 0 \) is thought to be already included in the independent variable, which is the initial barrier \( U_0(x) \). Obviously, \( r \) decreases with an increase in \( n_e \), until the electron and hole recombine and the local approximation for the interaction term becomes valid in the center of the barrier. According to this tendency and the smallness of the \( T \)-dependent correction, we can write \( \gamma (r(T) - r(0))^{-1} \approx (n_e(T) - n_e(0))/(r^* n_e^*), \) where \( n_e(T) \) and \( n_e(0) \) are found perturbatively from the single-particle wavefunctions in the barrier \( U_0(x) \). In a certain range of the barrier height, we can also neglect a change in the positive phenomenological parameter \( \gamma r^* n_e^* \). Under these assumptions, Eq. (3) is formally a special case of the interaction-induced correction \( \delta U(x) \propto -\alpha \delta n(x) \), here, \( \delta n(x) \) stands for Friedel density oscillations. This correction results from calculation of the propagation through the delta barrier in a one-dimensional electron system,25 in which case \( \alpha = \alpha(0) - 2k_F \) is a result of the competition between the exchange \( (\alpha(0)) \) and direct \( (\alpha(2k_F)) \) contribution to the interaction. A similar correction was used to simulate multimode quantum wires.25,31 We can attempt to extend the range of this correction, with the respective change in the meaning of \( \alpha \) to the entire first subband of the quantum wire, including the top of the smooth barrier.

IV. CALCULATION OF 1D ELECTRON DENSITY

To find this correction perturbatively, we first compute the complete set of wavefunctions for the bare smooth barrier \( U_0(x) \) and find the electron density \( n(x) \) at a given temperature:

\[ n = \frac{1}{2\pi b} \int_0^\infty \frac{dE}{(E E_0)^{1/2}} \frac{|\psi_L(x, E)|^2 + |\psi_H(x, E)|^2}{1 + e^{(E - E_F)/k_B T}}, \]

where \( E_0 = \hbar^2/2m^* b^2 \), \( b = 1 \) nm is the length scale, \( m^* = 0.067m_e \) is the electron effective mass, and \( \psi_L(x, E) \), \( \psi_H(x, E) \), \( n(x) \), and \( n_e(x) \) are the wavefunctions, density, and electron density of the electron and hole, respectively.
ψ(x,E) is the wavefunction of electrons incident on the barrier from the left (right). The amplitude of the incident wave is set to unity. The bare potential is specified as \( U_0(x) = V_0 / \cosh^2(x/a) \). This form is quite appropriate for simulation of short ballistic channels, including escape to two-dimensional reservoirs.\(^2\) This potential does not yield any features in the transmittance \( D(E) \) except the steps of unit height.\(^3\) The values of the parameters were taken to be typical for ballistic quantum wires in a GaAs/AlGaAs two-dimensional electron gas. The calculated dependence \( n(x,T) \) is presented in Fig. 1. One can see that the density strongly changes with increasing temperature in the transition from the tunnel regime to the open one. At the lowest temperature there are Friedel oscillations (FOs) in the tunnel regime, while in the open regime they are suppressed. Calculated correction \( \delta n(x,T) \) is a wide perturbation of density across the whole barrier (Figs. 1 and 2). At \( V_0 > E_F \) one can see thermally activated increase \( n(x,T) \) at the barrier top; this temperature behavior inverts in the open regime \( V_0 < E_F \). Details of this temperature behaviour are best seen in \( \delta n = n(x) - n_0(x) \), where \( n_0(x = 0) = n_c(T = 0) \), and \( n_0(x \neq 0) = \langle n(x,T = 0) \rangle \) is the density averaged over the Friedel oscillations (Fig. 2).

The most interesting for the analysis of the consequences of Eqs. (2), (3) is the dependence of the electron density \( n_c \) in the center of the barrier on the bare height \( V_0 \) at various \( T \). Figure 3 shows quite clearly the details of the \( T \)-dependent behavior of \( n_c \) and \( \delta n_c \), when \( U_0(x) \) is an independent variable. In the experiment, on the contrary, the gate voltage \( V_g \) is varied independently and, according to the electrostatic calculation (see Appendix B and Refs. 4, 5), there is a linear relation between \( n_c \) and \( V_0 \) above some small \( n_{c0} \) value, so that the temperature dependence of \( n_c(V_g) \) can be neglected. According to Fig. 5 this contradiction is resolved under the assumption that the quantity \( V_0 \) actually depends on \( T \) at constant \( n_c \) or \( V_g \). The relation between \( V_g \) and the height of

\[ V_g = \frac{3 \text{ meV}}{5} \]

\[ V_0 \]

\[ n(x,T) \]

\[ n_0(x) \]

\[ \delta n(x) \]

\[ \langle n(x,T = 0) \rangle \]

\[ \delta n_c \]

\[ n_c(T = 0) \]

\[ n_0(x = 0) \]

\[ \delta n = n(x) - n_0(x) \]

\[ n(x,T) \]

\[ n(x,T = 0) \]

\[ \langle n(x,T = 0) \rangle \]
the bare barrier $V_0$ is mediated by the electron wavefunctions and the Fermi distribution. This relation and $\alpha(x, T)$ do not depend, in the first-order perturbation theory, on interaction. However, they do contribute into it.

V. CORRECTED POTENTIAL

According to Eq. (3) and similar to Ref. [21] the $T$-dependent part of the interaction-induced correction to the bare potential $U_0(x)$ was calculated with the use of the phenomenological formula:

$$\delta U(x) = -\alpha \pi \hbar v_F \delta n(x, E_F, T),$$ (5)

where $\alpha = \text{const} > 0$, $(\hbar v_F)^{-1}$ is the one-dimensional density of states far from the barrier, $\delta n = n(x) - n_0(x)$, where $n_0(x = 0) = n_c(T = 0)$, and $n_0(x \neq 0) = \langle n(x, T = 0) \rangle$ is the density averaged over the Friedel oscillations. The interaction-corrected potential $U(x)$ at various $V_0$ and $T$ values is shown in Fig. 4. At high $V_0$ values and low $T$ values, penetration of an incident electron to the classically forbidden region of the barrier is very low and the thermal perturbation of the electron density inside the barrier is negligible. Therefore, the potential barrier remains almost unchanged. At $V_0 = E_F$, the height of the barrier $U(x)$ is lowered considerably with an increase in temperature. At constant $V_0 < E_F$, the barrier $U(x)$ is raised with $T$, in contrast to the case of $V_0 \geq E_F$. The behavior of the barrier height $U$ is shown in more detail in Fig. 5. For $T = 0$ we have $U = V_0$, because $\delta n(x = 0) = 0$ by definition. One can see that $U$ becomes independent of $V_0$ near $E_F$ at $T > 0.1$ meV. There appears a plateau below $E_F$. It becomes broader and deeper with an increase in $T$. The relative correction $\Delta U(x)/V_0$ at the parameters specified in the figure caption reaches 10%. This is close to the limiting value for the present approximation, which implies that the correction is small. This limits the growth of $T$ and $\alpha$ in the model. Figure 5 in combination with Eq. (2) provides a qualitative understanding of the development of the 0.7 conductance anomaly and the thermopower plateau with an increase in temperature. As is seen, the height $U$ of the temperature-dependent barrier near $E_F$ is stabilized at about $T$ below $E_F$.

VI. CALCULATED TRANSPORT ANOMALIES

Conductance as a function of the barrier height $V_0$ was calculated with the aid of formulas (1), (4), (5). The result is shown in Fig. 6. There is an usual conductance step with unit height at $T = 0.01$ meV. However with increasing temperature additional 0.7-plateau is developed at $V_0 = E_F$. The width of these plateaux well corresponds to temperature. Notice that the height of corrected barrier is almost not changed at the lowest temperatures and only the distant Friedel oscillations can have influence on transmission. Indeed, similar to Ref. [21] scattering off Friedel oscillations leads to a small decrease in transmission coefficient at low but finite temperatures and a shift of the conductance step to the lower values of $V_0$. For comparison we show in Fig. 6 the curves $G(V_0, T)$ calculated for the bare potential $U_0(x) = V_0 \cosh^2(x/\alpha)$.

Figure 7 shows that conductance behavior is somewhat universal for elevated $T$ and $\alpha$, and it strongly differs from that for $T \to 0$. Conductance $G(T, \alpha)$ for corrected potential is plotted as a function of $G(T, \alpha = 0)$ in Figure 7. These conductances are related to each other via common parameter $V_0$. Conductance $G$ calculated at $E_F = V_0$ as a function of interaction parameter $\alpha$ shows that the height of the 0.7-plateau is saturated with increasing $\alpha$ (Fig. 7).
The dependence of the Seebeck coefficient on $E_F$ and $T$ (Fig. 8) was computed from Eqs. (4), (5), and (1). In this case, we used a larger barrier half-width and a lower $\alpha$ value than before. In agreement with the analysis within approximation (2) and similar to the experiment, $S(E_F)$ exhibits an anomalous step with a height of 0.9–1.0 of $-k_B/e$. As is clearly seen in Fig. 8, this step is formed with an increase in $T$ simultaneously with the 0.7 conductance anomaly. It is noteworthy that such a combined evolution with temperature has not yet been observed. Thus, we propose to make the respective experiment for the additional proof of the proposed model.

Though dependences $U(V_0)$, $G(V_0)$, $G(E_F)$ and $G(G_{\alpha=0})$ are easy to calculate, they can hardly be measured. However, we can compare with the experiment the respective dependences on the electron density $n_e$ in the center of the barrier, see Fig. 8.

In the figure, dependences (a)–(c) show the plateaux which appear and become more pronounced with increase of the temperature. The shape of the computed plateaux is almost the same as that of experimental ones (see Figs. 11 and 12 in Appendix A). The widths of the plateaux in Fig. 8, $\Delta n_e \approx 0.01 \, \text{nm}^{-1}$ and $\Delta V_g \approx 0.01 - 0.02 \, \text{V}$, agree in several experiments (Figs. 10–12), within small variations of the gate capacitance. Appendix B discusses the variations in more details. If correction Eq (5) is zero ($\alpha = 0$), then calculated dependence $G(V_0(n_e, T))$ is almost the same as for self-consistent potential obtained in 3D-electrostatic potential calculation (Fig. 13).

We made a number of simplifying assumptions in our 1D-model. Therefore, a detailed fit of the experimental data to the calculated curves is hardly appropriate. For example, it was checked that approximation (2) in the calculation of the conductance replaces quite well more general formula (1) (except the case of ultimately low $k_B T$ values). Thus, the discovered effect is unrelated to the details of the barrier profile $U(x)$ and is induced merely by the dependence of the difference $E_F - U$ on $n_e$; i.e., the correction $\delta U(T)$ in the center of the barrier, where Eq. (3) presumably holds, yielding $\alpha > 0$, is crucial. It is difficult to find $\alpha$ from theoretical considerations. However, similarity of the experimental and calculated curves persists under a 50% variation of $\alpha$ (Fig. 7). It is noteworthy that the effective value $\alpha = 0.2$ corresponds to $r^* n_e \approx 1.5$; i.e., the distance $r$ between the probe electron in the center of the barrier and the exchange-correlation hole is $1.5 \gamma / n_e$. On the
FIG. 9: (a,b) Calculated dependences $E_F - U, \eta(n_c) = (E_F - U)/k_B T$ for $E_F = 5$ meV, the interaction parameter $\alpha = 0.2$, and the bare potential $U_0(x) = V_0/\cosh^2(x/a)$ with $a = 200$ nm. (c,d) Calculated conductance of the one-dimensional channel with the corrected (b) and the bare (c) potential versus $n_c$.

VII. CONCLUSION

A simple model of anomalous plateaux in the conductance and thermopower of one-dimensional ballistic quantum wires has been proposed on the basis of the Landauer approach with spin degeneracy. The key points of the model are pinning of the effective one-dimensional barrier height $U$ at a depth of $k_B T$ below the Fermi level under a change in the one-dimensional density in the center of the barrier or the gate voltage and the inclusion of all (local and non-local) temperature-dependent interaction-induced corrections via phenomenological formula (5).

basis of the typical values $n_c \sim 0.01$ nm$^{-1}$ (see Figs. 2 and 3), we can conclude that the conditions used to Eq. (3) and Eq. (5) are fulfilled.

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Appendix A: Data processing

We tested the validity of Landauer formulas by the following way. Combining formulas in approximation (2) it is easy to write $S = -[(1 - G) \ln(1 - G) + G \ln G]/G$, $S^M = (\pi^2/3)(1 - G)$ where thermopower $S$ and conductance $G$ are measured in units of $-k_B/e$ and $2e^2/h$, respectively. Thus we can find thermopower from conductance data and compare it to the measured thermopower. We know about only one paper17 which reports anomalous plateaux for con-

FIG. 10: Data processing of the gate voltage dependences of conductance and thermopower from Ref. 17 for two samples with the same geometry of metal gates and two temperatures. Measured $S$-values are denoted by open blue and fill red circles, or fill olive square. Seebeck coefficient $S$ obtained from conductance is shown by green and cyan line, $S^M$ obtained from $G$ is shown by orange and yellow line (a,b). Curves $S(\eta)$ and $S^M(\eta)$ calculated by Eq. (3) and measured $S$-values as a function of $\eta$-values obtained from measured $G$ are shown in panel (c).
ductance and thermopower simultaneously. We extracted \( S \) and \( S^M \) from \( G(V_g) \) and plotted the reconstructed and measured points \( S(V_g) \) (scaled to common unit \(-k_B/e\)). The values corresponding to the first subband almost coincided (Fig. 10b). One can see that the height of the anomalous conductance plateau equals 0.7, and the corresponding height of the thermopower plateau is the same for two samples and agrees closely with values \( S \approx 0.8 - 0.9, S^M \approx 1 \). Above the first subband the thermopower behaves in accordance with the Mott law and with the calculations of the peak heights between zero plateaux. In Refs. 7 and 10 the height of the first peak was shown to be approximately equal to \(-0.5k_B/e\) if the conductance quantization plateaux are smoothed out, or less than this value if the plateaux are pronounced. On the other hand, experimental data in Ref. 17 was normalized to less than this value if the plateaux are pronounced. On the other hand, experimental data in Ref. 17 was normalized to less than this value if the plateaux are pronounced.

For additional verification of the described interpretation of the experimental data we extracted \( \eta \)-values from Eq. (2) and measured \( G \)-values for the first subband and plotted the points \((S, \eta)\) along with the calculated curves \( S(\eta) \) and \( S^M(\eta) \) using formulas (2). Figure 10 shows a good agreement between the experimental data and universal curves \( S(\eta), S^M(\eta) \), for different temperatures and devices. Therefore, we show that the plateaux are present in the \( V_g \) dependences but not in the dependences on \( E_F - U \). This means that the usual assumptions about \( U \) being independent of \( T \) and \( E_F - U \) being linear in \( V_g \) do not work when the first subband begins getting occupied in the microcontact. Nevertheless, the spin degeneracy Landauer approach and Mott approximation remain valid up to the temperature of 1K in the case of Fig. 10 including anomalous plateaux.

We used Eq. (2) to study the behavior of the reflecting barrier \( U = E_F + k_B T \ln(1/G(V_g, T) - 1) \) in different quantum point contacts (QPC). Figures 11 and 12 show that \( U(V_g) \) is pinned with increasing temperature. In addition, there is a strong temperature dependence of the quantity \( U(V_g) \), which determines the transport. It is usual to assume that the main temperature dependence of conductance at any fixed \( V_g \) is defined by the Fermi distribution, not the reflecting barrier. In our case this assumption is not valid, though in Fig. 12 one can notice common asymptotics \( U(V_g) \) at different \( T < 4 \) K when conductance approaches to \( 2e^2/h \).

![FIG. 11: Data processing of the gate voltage dependences of conductance from Refs. 15, 12. Top: Measured conductance for structures with split metal gate (left) and in-plane side gate (right). The insets show the shape of the gates and etching strips (gray regions) forming the channel in 2DEG. Bottom: dependences \( E_F - U(V_g) \) extracted from the measured \( G(V_g) \) with use of Eq. (2). The location of the anomalies in \( G \) and \( E_F - U \) is indicated with the dotted lines.](image1.png)

![FIG. 12: Data processing of the gate voltage dependences of conductance from Ref. 14. (a) Measured conductance for a split metal gate structure. (b,c) Dependences \( E_F - U(V_g) \) and \( \eta = (E_F - U(V_g))/k_B T \) extracted from the measured \( G(V_g) \) with help of Eq. (2). The dotted lines indicate the position of anomalies in \( G \) and \( \eta \).](image2.png)
We computed self-consistency the 3D-electrostatic potential and the 3D-electron density of a quantum point contact. We took into account quantization of transverse motion and exchange-correlation correction of interaction in the local approximation, determined by the volume electron density.\textsuperscript{5,12,14} Fig. 13 shows calculated gate voltage dependences of the 1D-electron density $n_c$, the first subband bottom $V_0 - E_F$ and $(V_0 - E_F)/k_B T$ at the narrowest place of the QPC ($V_0 = E_1(x = 0)$). Figure 13 shows that the one-dimensional electron density $n_c$ in the center of the barrier is almost independent of $T$ and is linear in $V_g$ starting from small values $n_{c0} \sim 10^{-3}$ nm$^{-1}$; i.e., the electric capacitance between the gate and the quantum wire is conserved at $G \gtrsim 0.1 e^2/h$; $C_g/ \epsilon \approx 1/3$ V$^{-1}$ nm$^{-1}$. Notice that for a QPC based on GaAs/AlGaAs heterostructure capacitance $C_g$ is a weak function of the distance between the gate and the center of the constriction. So a typical scale $n_c \approx 0.01$, corresponding to the width of the anomalous plateau, can be found from the calculated $C_g$ and the measured interval in $V_g$ ($\Delta V_g \approx 0.01 \div 0.02$ V in Figs. [10][12].

It is interesting to compare the behavior of the height of reflecting barrier $U(V_g)$ (Figs. [11][12]) discovered by this simple processing of experimental data to the behavior of a bottom of the first subband $V_0(V_g)$ at the narrowest place, obtained in the 3D electrostatic self-consistent calculations of potential and electron density (Fig. [13][c]). One may see a qualitative difference between $U(V_g, T)$ (Fig. [12][c]) and $V_0(V_g, T)$ (Fig. [13][c]). Calculated dependence $V_0(V_g)$ smoothes with increasing temperature and there is not any pinning. Naturally 0.7-anomaly does not appear if formula (1) is used for potential $V_0(x, T, V'_g)$ (Fig. [13][d]). Consequently, ballistic electron feels not self-consistent potential, but another one (see discussion of formula (3) in the main text).

\textbf{Appendix B: Self-consistent calculations}
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