Probing Lorentz violation in $2\nu\beta\beta$ using single electron spectra and angular correlations

O.V. Nițescu$^{1,2,3}$, S.A. Ghinescu$^{1,2,3}$, M. Mirea$^{1,2,†}$ and S. Stoica$^{1,2}$

1 "Horia Hulubei" National Institute of Physics and Nuclear Engineering, 30 Reactorului, POB MG-6, RO-077125, Bucharest-Măgurele, Romania
2 International Centre for Advanced Training and Research in Physics, PO Box MG12, 077125-Magurele, Romania
3 Faculty of Physics, University of Bucharest, 405 Atomistilor, POB MG-11, RO-077125, Bucharest-Măgurele, Romania

(Dated: September 14, 2020)

We show that current investigations of Lorentz invariance violation (LIV) in $2\nu\beta\beta$ decay could be complemented by searches in the single electron spectra and angular correlations between emitted electrons. We find that the angular correlation and the ratio between the spectrum including contribution and the Standard Model one diverge near $Q$-value. We find that both the sign and magnitude of the strength coefficient ($\hat{a}^{(3)}_{\nu}$) play a paramount role in the identification of LIV signatures. We also raise the issue of negative decay rates in case of negative $\hat{a}^{(3)}_{\nu}$ values.

Keywords: $2\nu\beta\beta$-decay, Lorentz invariance violation, angular correlation

Introduction. Searching of evidence to probe the Lorentz invariance violation is a very current topic that joins the increasing effort to test the limits of the Standard Model (SM) [1, 2]. The theoretical basis of these searches is the SM extension (SME), an effective field theory including operators that break Lorentz invariance for all the particles in the SM [2, 3]. In particular, the neutrino sector of SME provides the theoretical framework for a rich phenomenology for searching evidence of LIV, for example, those that can be proved in neutrino oscillations experiments [4-7]. However, there are LIV signatures related to the so-called countershaded effects associated with the oscillation-free operators of mass dimension three, which cannot be investigated in such experiments. The study of beta and double-beta decays offers the possibility to investigate of LIV effects related to the time-like (isotropic) component of this oscillation-free operator whose size is controlled by the coefficient ($\hat{a}^{(3)}_{\nu}$). In ref. [8, 9] the LIV effects in $2\nu\beta\beta$ decay were calculated for the summed energy spectra of electrons, but employing a non-relativistic approximation for the electron radial wave functions. At present, the accuracy required by the DBD experiments far exceeds this approximation. Recently, experiments like EXO [10], CUPID-0 [11], SuperNEMO [12], CUORE [13, 14], GERDA [15] have provided limits of the $\hat{a}^{(3)}_{\nu}$ parameter through a careful analysis of the summed energy spectra of electrons in $2\nu\beta\beta$ decays, using theoretical spectra obtained with better but still approximate methods of calculation. In a recent paper [16], we examined the effects of LIV mainly on summed electron energy spectra and quantities related to them using Fermi functions built with exact electron wave functions obtained by numerically solving a Dirac equation in a realistic Coulomb-type potential, with the inclusion of finite nuclear size and screening effects [17, 18].

In this work, we show that LIV signatures can be searched as well in the single electron spectra and the angular correlation between the two electrons emitted in $2\nu\beta\beta$ decay. First, we derive the formulas for the LIV contributions to these spectra and employ them using an improved version of our method described in refs. [16, 17] to compute these contributions. Then, we discuss possible signatures that could be probed in experiments. We find a shift in the single electron spectrum due to LIV corrections similar to that reported for summed energy spectrum of the two electrons. Further, we show other LIV effects that may occur by analyzing the ratio of the total SME spectra to their SM counterparts in both single and summed electron energy cases. Another interesting quantity is the angular correlation spectrum on which LIV contributions also induce a distinct signature, as we will discuss. We find that the LIV contributions manifest differently for positive and negative values of the $\hat{a}^{(3)}_{\nu}$ coefficient, increasing in magnitude as the electron energy gets close to the $Q$-value. As expected, small $\hat{a}^{(3)}_{\nu}$ magnitudes produce less pronounced LIV effects. We perform this study for the nucleus $^{100}$Mo, but the results hold qualitatively for any other nuclei that undergo a $2\nu\beta\beta$ decay.

Formalism. The differential decay rate for the standard $2\nu\beta\beta$ process, $0^+ \rightarrow 0^+_1$ transitions, can be expressed as [19-21]:

$$d\Gamma^{2\nu} = [A^{2\nu} + B^{2\nu}\cos\theta_{12}] w^{2\nu} d\omega_1 d\varv_2 d(\cos \theta_{12})$$  \hspace{1cm} (1)

where $\varv_{1,2}$ are the electron energies, $\omega_{1,2}$ are the antineutrino energies, and $\theta_{12}$ is the angle between the two emitted electrons. In what follows, we adopt the natural units ($\hbar = c = 1$). Within the Standard Model framework, the

† Deceased, 28.08.2020
where \( g_A \) is the axial vector constant, \( G_F \) is the Fermi coupling constant, \( V_{ud} \) is the first element of the Cabibbo-Kobayashi-Maskawa matrix and \( p_{1,2} \) are the momenta of the electrons.

The quantities \( \mathcal{A}^{2\nu} \) and \( \mathcal{B}^{2\nu} \) can be expressed \cite{21}, to a good approximation, by

\[
\begin{aligned}
\mathcal{A}^{2\nu} &= \frac{1}{4} a(\varepsilon_1, \varepsilon_2) |M_{2\nu}|^2 \tilde{A}^2 \times \\
&\quad \left[ (\langle K_N \rangle + \langle L_N \rangle)^2 - \frac{1}{3} (\langle K_N \rangle - \langle L_N \rangle)^2 \right] \\
\mathcal{B}^{2\nu} &= \frac{1}{4} b(\varepsilon_1, \varepsilon_2) |M_{2\nu}|^2 \tilde{A}^2 \times \\
&\quad \left[ (\langle K_N \rangle + \langle L_N \rangle)^2 - \frac{1}{9} (\langle K_N \rangle - \langle L_N \rangle)^2 \right]
\end{aligned}
\]  

(3)

where \( M_{2\nu} \) are the nuclear matrix elements and \( a(\varepsilon_1, \varepsilon_2) \) and \( b(\varepsilon_1, \varepsilon_2) \) are products of the radial wave functions of the emitted electrons. \( \langle K_N \rangle \), \( \langle L_N \rangle \) are kinematic factors that depend on the electrons and antineutrinos energies, on the ground state energy \( E_I \) of the parent nucleus and on an averaged energy \( \langle E_N \rangle \) of the excited states in the intermediate nucleus (closure approximation). The expressions for the kinematical factors are given by \cite{19}

\[
\begin{aligned}
\langle K_N \rangle &= \frac{1}{\varepsilon_1 + \omega_1 + \langle E_N \rangle - E_I} + \frac{1}{\varepsilon_2 + \omega_2 + \langle E_N \rangle - E_I}, \\
\langle L_N \rangle &= \frac{1}{\varepsilon_1 + \omega_2 + \langle E_N \rangle - E_I} + \frac{1}{\varepsilon_2 + \omega_1 + \langle E_N \rangle - E_I}.
\end{aligned}
\]

(4)

Here, the difference in energy in the denominator can be obtained from the approximation \( \tilde{A}^2 = [W_0/2 + \langle E_N \rangle - E_I]^2 \), where \( \tilde{A} = 1.12A^{1/2} \) (in MeV) gives the energy of the giant Gamow-Teller resonance in the intermediate nucleus. The energy \( W_0 \) is defined as

\[
W_0 = Q + 2m_e = E_I - E_F,
\]

(5)

where \( Q \) is the kinetic energy available for the four leptons, \( m_e \) is the rest energy of the electron, and \( E_F \) is the ground state energy of the final nucleus.

The functions \( a(\varepsilon_1, \varepsilon_2) \) and \( b(\varepsilon_1, \varepsilon_2) \) are defined as

\[
\begin{aligned}
a(\varepsilon_1, \varepsilon_2) &= |\alpha^{-1} - 1|^2 + |\alpha_{11}|^2 + |\alpha_1^{-1}|^2 + |\alpha_{-1}^{-1}|^2 \\
b(\varepsilon_1, \varepsilon_2) &= -2R(\alpha^{-1} - 1, \alpha_{11}^* + \alpha_1^{-1}\alpha_{-1}^{-1})
\end{aligned}
\]

(6)

with

\[
\begin{aligned}
\alpha^{-1} &= g_{-1}(\varepsilon_1)g_{-1}(\varepsilon_2), \quad \alpha_{11} = f_1(\varepsilon_1)f_1(\varepsilon_2), \\
\alpha_1^{-1} &= f_1(\varepsilon_1)g_{-1}(\varepsilon_2), \quad \alpha_{-1}^{-1} = g_{-1}(\varepsilon_1)f_1(\varepsilon_2).
\end{aligned}
\]

(7)

The functions \( f_1(\varepsilon_1) \) and \( g_{-1}(\varepsilon_2) \) are the electron radial wave functions evaluated on the surface of the daughter nucleus:

\[
\begin{aligned}
g_{-1}(\varepsilon) &= \int_0^\infty g_{-1}(\varepsilon, r)\delta(r-R)dr \\
f_1(\varepsilon) &= \int_0^\infty f_1(\varepsilon, r)\delta(r-R)dr,
\end{aligned}
\]

(8)

where \( R = r_0A^{1/3} \), \( r_0 = 1.2 \) fm.

The derivation of the decay rate with respect to the cosine of the angle \( \theta_{12} \) can be expressed as \cite{21}

\[
\frac{d\Gamma^{2\nu}_\text{SM}}{d(\cos \theta_{12})} = \frac{1}{2} \frac{\Gamma^{2\nu}_\text{SM}}{\Gamma^{2\nu}_\text{SM}} [1 + \kappa^{2\nu}_\text{SM} \cos \theta_{12}]
\]

(9)

where \( \kappa^{2\nu}_\text{SM} \) is the angular correlation coefficient defined by

\[
\kappa^{2\nu}_\text{SM} = \frac{\Lambda^{2\nu}_\text{SM}}{\Gamma^{2\nu}_\text{SM}}
\]

(10)

The decay rates \( \Gamma^{2\nu}_\text{SM} \) and \( \Lambda^{2\nu}_\text{SM} \) are obtained by integrating Eq (1) over the lepton energies. In the closure approximation their formulas can be written in a factorized form as follows:

\[
\begin{aligned}
\frac{\Gamma^{2\nu}_\text{SM}}{\ln 2} &= g_A^4 |m_eM_{2\nu}|^2 G^{2\nu}_\text{SM}, \\
\frac{\Lambda^{2\nu}_\text{SM}}{\ln 2} &= g_A^4 |m_eM_{2\nu}|^2 H^{2\nu}_\text{SM},
\end{aligned}
\]

(11)

which are essentially products of nuclear matrix elements (NMEs) and the phase space factors (PSFs) \( G^{2\nu}_\text{SM} \) and \( H^{2\nu}_\text{SM} \).

Within the SME, the LIV effects in \( 2\nu\beta\beta \) decay can arise from the action of countershaded operators, namely changing each antineutrino 4-momentum from \( \bar{q}^a = (\omega, \bar{q}) \) to an effective 4-momentum \( \bar{q}^a = (\omega, q + a^{(3)}_\text{of} - a^{(3)}_\text{of} \bar{q}) \) \cite{9,22,23}, with \( a^{(3)}_\text{of} \) the isotropic component of \( (a^{(3)}_\text{of})^a \). Since the two antineutrinos are not measured, the integration over all orientations leaves only the isotropic component \( a^{(3)}_\text{of} \). This leads to a change in the form of the antineutrino differential phase space, from the standard one \( d^3\bar{q} = 4\pi\omega^2d\omega \) to the one containing the LIV effects \( d^3\bar{q} = 4\pi(\omega^2 + 2\omega a^{(3)}_\text{of})d\omega \). To the first order in \( a^{(3)}_\text{of} \), the term \( w^{2\nu}_\text{SM} \) in the differential decay rate, i.e., Eq. (1), acquires the form

\[
w^{2\nu}_\text{SM} = \frac{g_A^4 G_F^4 |V_{ud}|^4}{64\pi^2} p_{12} \varepsilon_1 \varepsilon_2 \times \left[ \omega_1^2 \omega_2^2 + 2a^{(3)}_\text{of} \left( \omega_1^2 \omega_2 + \omega_1 \omega_2^2 \right) \right].
\]

(12)

In the above expression the first term represents the SM contribution and the following two terms are the LIV contributions in the first order in \( a^{(3)}_\text{of} \). Following
the same steps as in the case of SM, we get the SME expression of the differential decay rate with respect to the cosine of the angle \( \theta_{12} \)
\[
\frac{d\Gamma^{2\nu}_{\text{SME}}}{d(\cos \theta_{12})} = \frac{1}{2} \Gamma^{2\nu}_{\text{SME}} \left[ 1 + \kappa^{2\nu}_{\text{SME}} \cos \theta_{12} \right] \tag{13}
\]
where the angular correlation coefficient \( \kappa^{2\nu}_{\text{SME}} \) is defined by
\[
\kappa^{2\nu}_{\text{SME}} = \frac{\Lambda^{2\nu}_{\text{SME}}}{\Gamma^{2\nu}_{\text{SME}}} \tag{14}
\]
The decay rates \( \Lambda^{2\nu}_{\text{SME}} \) and \( \Gamma^{2\nu}_{\text{SME}} \) can be expressed as sums of standard and LIV contributions:
\[
\Gamma^{2\nu}_{\text{SME}} = \Gamma^{2\nu}_{00} + \Gamma^{2\nu}_{01} + \Gamma^{2\nu}_{10}, \quad \Lambda^{2\nu}_{\text{SME}} = \Lambda^{2\nu}_{00} + \Lambda^{2\nu}_{01} + \Lambda^{2\nu}_{10}, \tag{15}
\]
where 00 stands for the SM contributions. The decay rates can be written in a good approximation in a factorized form as products NMEs and PSFs, as follows:
\[
\frac{\Gamma^{2\nu}_{mn}}{\ln 2} = g_A^4 |m_e M_{2\nu}|^2 G^{2\nu}_{mn}, \quad \frac{\Lambda^{2\nu}_{mn}}{\ln 2} = g_A^4 |m_e M_{2\nu}|^2 H^{2\nu}_{mn}, \tag{16}
\]
with \( mn = \{00, 10, 01\} \) and \( M_{2\nu} \), the NMEs. The PSF expressions can be written in a compact form:
\[
\left\{ \begin{array}{l}
G^{2\nu}_{mn} \\
H^{2\nu}_{mn}
\end{array} \right\} = (10\delta^{(3)}_{nf})^{m+n} \frac{C_{mn}}{m_{11}^{11-m-n}} \times \\
\int_{E_1 - E_F - m_e}^{E_1 - E_F - \varepsilon_1} d\varepsilon_1 \int_{E_1 - E_F - \varepsilon_2}^{E_1 - E_F - \varepsilon_1} d\varepsilon_2 \int_{m_e}^{E_1 - E_F - \varepsilon_1 - \varepsilon_2} d\varepsilon_3 \\
\int_{0}^{E_1 - E_F - \varepsilon_1 - \varepsilon_2} d\varepsilon_4 \Omega_{mn} \times \\
\left\{ \begin{array}{l}
a(\varepsilon_1, \varepsilon_2) \left[ (K_N)^2 + (L_N)^2 + (K_N)(L_N) \right] \\
b(\varepsilon_1, \varepsilon_2) \left[ \frac{2}{3} (K_N)^2 + (L_N)^2 + \frac{2}{3} (K_N)(L_N) \right]
\end{array} \right\} \tag{17}
\]
with \( \Omega_{mn} = \omega_1^{2-m} \omega_2^{2-n} \) and
\[
C_{00} = \frac{\pi^2 G_a^4 |V_{ud}|^4 m_e^4}{96 \pi^2 \ln 2}, \quad C_{10} = C_{01} = \frac{\pi^2 G_a^4 |V_{ud}|^4 m_e^4}{480 \pi^2 \ln 2}. \tag{18}
\]
In this study, we consider that LIV influences only the PSFs. In the PSF definitions, the LIV parameter (strength) \( \delta H^{(3)}_{10} \) is included in MeV and the energy \( \omega_2 \) of the antineutrino is determined as \( \omega_2 = E_F - \varepsilon_1 - \varepsilon_2 - \omega_1 \). We observe that the first order LIV contributions (10) and (01) are functions of \( \omega_2 \), symmetric to the center of the integration interval \([0, E_F - \varepsilon_1 - \varepsilon_2]\), and hence they are equal in value. So, in what follows, we consider the corrections to the standard PSFs as:
\[
\left\{ \begin{array}{l}
\delta G^{2\nu}_{10} \\
\delta H^{2\nu}_{10}
\end{array} \right\} = \frac{2\delta H^{(3)}_{10}}{\delta G^{(3)}_{10}} \left\{ \begin{array}{l}
\delta G^{2\nu}_{10} \\
\delta H^{2\nu}_{10}
\end{array} \right\}. \tag{19}
\]
We note that by making the approximation
\[
\langle K_N \rangle \simeq \langle L_N \rangle \simeq \frac{2}{E_I - \langle E_N \rangle - W_0/2}, \tag{20}
\]
and integrating over the energy of the antineutrino \( \omega_1 \), one retrieves simplified expressions of the PSFs which were used in many previous works (see for example [24] and references therein) and also in the previous LIV analyzes [10–12].

Deriving the decay rate expression versus the kinetic energy of one electron and to the total kinetic energy of the two electrons we get the single electron spectrum:
\[
\frac{d\Gamma^{2\nu}_{\text{SME}}}{d\varepsilon_1} = C \frac{dG^{2\nu}_{00}}{d\varepsilon_1} \left( 1 + \alpha^{(3)} \chi^{(1)}(\varepsilon_1) \right), \tag{21}
\]
and the summed energy spectrum of the two electrons:
\[
\frac{d\Gamma^{2\nu}_{\text{SME}}}{dK} = C \frac{dG^{2\nu}_{00}}{dK} \left( 1 + \alpha^{(3)} \chi^{(+)}/K \right) \tag{22}
\]
where \( C \) is a constant including NME, \( K \equiv \varepsilon_1 + \varepsilon_2 - 2m_e \) is the total kinetic energy of the two electrons and
\[
\chi^{(1)}(\varepsilon_1) = \frac{d(\delta G^{2\nu}_{10})/d\varepsilon_1}{d\delta G^{2\nu}_{00}/d\varepsilon_1} \chi^{(+)}/K \frac{dG^{2\nu}_{00}/d\varepsilon_1}{dG^{2\nu}_{00}/d\varepsilon_1} \tag{23}
\]
are factors that incorporate the deviations of the electron spectra from their SM forms. Deriving also the decay rate versus \( \varepsilon_1 \) and \( \cos(\theta_{12}) \), we get the expressions of the angular correlation and its deviation from the SM form:
\[
\frac{d\Gamma^{2\nu}_{\text{SME}}}{d\varepsilon_1 d(\cos \theta_{12})} = C \frac{dG^{2\nu}_{00}}{d\varepsilon_1} \times \\
\left[ 1 + \alpha^{(3)} \chi^{(1)}(\varepsilon_1) + \left( \alpha^{(3)} \chi^{(1)}(\varepsilon_1) + \frac{d(\delta H^{2\nu}_{10})/d\varepsilon_1}{dG^{2\nu}_{00}/d\varepsilon_1} \cos \theta_{12} \right) \right]. \tag{24}
\]
where \( \alpha_{\text{SM}} \equiv (dH^{2\nu}_{00}/d\varepsilon_1)/(dG^{2\nu}_{00}/d\varepsilon_1) \) is the SM angular correlation while its SME expression is:
\[
\alpha_{\text{SME}} = \alpha_{\text{SM}} + \alpha^{(3)} \frac{d(\delta H^{2\nu}_{10})/d\varepsilon_1}{dG^{2\nu}_{00}/d\varepsilon_1} \tag{25}
\]
Deriving the decay rate expression versus \( \cos(\theta_{12}) \)
\[
\frac{d\Gamma^{2\nu}_{\text{SME}}}{d(\cos \theta_{12})} = CG^{2\nu}_{00} \times \\
\left[ 1 + \alpha^{(3)} \frac{d(\delta G^{2\nu}_{10})}{G^{2\nu}_{00}} + \left( \kappa^{2\nu}_{\text{SME}} + \alpha^{(3)} \delta H^{2\nu}_{10} \right) \cos \theta_{12} \right], \tag{26}
\]
we can identify (in round brackets) the SME expression of the angular correlation coefficient \( \kappa^{2\nu}_{\text{SME}} \). For an independent treatment with respect to \( \alpha^{(3)} \), we define \( \xi^{2\nu}_{LV} \equiv \delta H^{2\nu}_{10}/G^{2\nu}_{00} \) in units of MeV\(^{-1}\).
Results and discussion. We employ the previously derived formulas to calculate the relevant observables in the case of the $^{100}\text{Mo}$ nucleus. The important quantities are the PSFs, and for their calculation, we used our method described in detail in refs. [17, 18]. Here, we use an improved version that better handles the accumulation of truncation errors.

The numerical results are obtained using the following physical constants: the electron mass $m_e = 0.5110\text{MeV}$, the CKM matrix element $V_{ud} = 0.9743$, the Fermi coupling constant $G_F = 1.1666 \times 10^{-11}\text{MeV}^{-2}$, the fine structure constant $\alpha = 1/137$ and the Q-value of $^{100}\text{Mo}$ $2\nu\beta\beta$ decay $Q = 3.0344\text{MeV}$. We note that this value is the statistical average of multiple experimental results as described in [16]. We use two sets of $\tilde{a}_\alpha^{(3)}$ limits in our calculations: one reported by the EXO collaboration ($-2.65 \times 10^{-4}\text{MeV} \leq \tilde{a}_\alpha^{(3)} \leq 7.6 \times 10^{-4}\text{MeV}$) [10] and one reported by the NEMO collaboration ($-4.2 \times 10^{-4}\text{MeV} \leq \tilde{a}_\alpha^{(3)} \leq 3.5 \times 10^{-4}\text{MeV}$) [12].

In Fig. 1 we illustrate the normalized single electron energy spectrum and its deviation due to LIV. The shift in the SM spectrum towards higher energy values is a consequence of neutrinos violating Lorentz invariance, similar to that previously found in the summed energy spectra of electrons [10, 16]. This feature opens the possibility for a correlated observation of LIV effects in the maxima of both single and summed energy electrons spectra, although we note that the shift introduced in the single electron spectrum is smaller than in the summed energy case.

In order to see other LIV effects, we analyze the ratio between the total single electron spectrum (including LIV contribution) and its SM form, namely $1 + \tilde{a}_\alpha^{(3)} \chi^{(1)}(\epsilon_1)$. We plot this quantity in Fig. 2. One observes that the curves are very close to each other, but they diverge at electron energy values approaching $Q$-value. This divergence is due to a slower descent (in absolute value) of the LIV spectrum with respect to the SM one at the end of the energy interval. This effect is quite visible for the set of $\tilde{a}_\alpha^{(3)}$ limits reported by the EXO collaboration. On the contrary, when $\tilde{a}_\alpha^{(3)}$ is more stringently constrained, such as the limits reported by NEMO collaboration, this effect is practically un-observable. The drawback of searching for this behavior in the vicinity of $Q$-value in the single electron spectrum is the small statistic reachable experimentally at present.

Then, we perform a similar study for the summed energy spectrum of the two electrons, plotting the quantity $1 + \chi^{(+)}(K)$. As seen in Fig. 3 (plotted using the same notations as in Fig. 2). The result is an effect similar to the one from single electron spectrum case and with the same explanation for the larger values of the LIV contribution at energies close to the $Q$-value. The summed energy spectrum presents the experimental advantage of higher statistic in the $Q$-value vicinity, where neutrinoless double-beta decay mode is expected to be observed.

It is also noted that for negative values of $\tilde{a}_\alpha^{(3)}$, the quantity $1 + \chi^{(+)}$ may cut the $K$-axis at a certain electron energy value. This feature is illustrated in Fig. 3 and is most visible only for $\tilde{a}_\alpha^{(3)}$ value reported by EXO collaboration. For yet larger $K$ values, the differential rate becomes negative (non-physical). This peculiar result can be interpreted either as a need for improving the LIV theory for $2\nu\beta\beta$, in the case of negative $\tilde{a}_\alpha^{(3)}$ values, to avoid this non-physical behavior or that the $\tilde{a}_\alpha^{(3)}$ parameter should have only positive values.

Further, we discuss the implications of LIV on the an-
FIG. 3. (Color online) The quantity $\chi^+(K)$ depicted for current limits of $\alpha^{(3)}_\text{of}$. The same conventions as in Fig. 2 are used.

As seen from Eqs. 25, 26, the SME contribution is represented, in both quantities, by a linear factor controlled through the $\alpha^{(3)}_\text{of}$ coefficient.

$\chi^+(K)$ is given by
\[\chi^+(K) = 1 + \sum_i \chi_i^+(K)\]
where $\chi_i^+(K)$ are the contributions of individual channels.

The SME contribution is given by
\[\chi_{\text{SME}}^+(K) = \frac{2}{\pi^2} \sum_{\text{LIV}} \frac{d^2}{dK^2} G_{00}(K)\]

FIG. 4. (Color online) The angular correlation spectrum plotted for the current limits of $\alpha^{(3)}_\text{of}$. The same conventions as in Fig. 2 are used.

The angular correlation spectrum $\alpha^{2\nu}$ is given by
\[\alpha^{2\nu} = \frac{1}{2} \sum_{\text{LIV}} \frac{d^2}{dK^2} G_{00}(K)\]
where $G_{00}(K)$ is the single electron spectrum.

$\alpha^{2\nu}_{\text{SM}}$ is calculated from
\[\alpha^{2\nu}_{\text{SM}} = \frac{1}{2} \sum_{\text{SM}} \frac{d^2}{dK^2} G_{00}(K)\]

$\alpha^{2\nu}_{\text{SME}}$ is calculated from
\[\alpha^{2\nu}_{\text{SME}} = \frac{1}{2} \sum_{\text{SME}} \frac{d^2}{dK^2} G_{00}(K)\]

For completeness, we also calculate the LIV effect on the $k^{2\nu}$ parameter. Our SM calculated value for $2\nu\beta\beta$ decay is
\[k^{2\nu}_{\text{SM}} = -0.6836\]
and differs from other values in literature by less than 5% [25]. This difference is partly due to employing the closure approximation and, to a lower extent, to the usage of less precise radial wave functions. Further, we calculate
\[\xi^{2\nu}_{\text{LV}} = -4.172 \text{ MeV}^{-1}\]
which can be plugged back into Eq. (26) to obtain the final dependence of $k^{2\nu}_{\text{SME}}$ on $\alpha^{(3)}_\text{of}$ as
\[k^{2\nu}_{\text{SME}} = -0.6836 - 4.172 \times \alpha^{(3)}_\text{of}\]
where the LIV parameter must be taken in units of MeV.

Conclusions. We investigated possible LIV effects in the single electron spectra and angular correlations between the two electrons emitted in $2\nu\beta\beta$ decay in the case of $^{100}$Mo nucleus. We derived the formulae of the LIV contributions to these spectra and calculated them to analyze different possible signatures that could be probed in experiments. Besides the shift in the single electron spectrum due to LIV corrections, we show some other possible effects that may occur if the ratio between the electron (single and summed) energy spectra and angular correlation spectra that include LIV corrections and their SM forms are analyzed. The features of the LIV contributions, especially at electron energy close to the $Q$-value, depend both on the sign and magnitude of the $\alpha^{(3)}_\text{of}$ coefficient. Investigation of the summed energy spectra is the most appealing experimentally. We also show that, from a certain electron energy onward, the decay rate may become negative (i.e., non-physical) for negative values of $\alpha^{(3)}_\text{of}$. This drawback could be avoided either by improving the LIV theory for DBD or by considering only positive values for this coefficient. The study on the angular correlation spectrum revealed a similar trend: for less stringent $\alpha^{(3)}_\text{of}$ limits, the LIV effect becomes stronger as the electron energy increases, while for more stringent values, this effect might pass un-observed.

Finally, we mention that we can provide, at request, experimentalists with numerical values of any kinematical quantity, possibly needed in experimental LIV analyses.

The figures for this article have been created using the SciDraw scientific figure preparation system [26].

This work has been supported by the grants of the Romanian Ministry of Education and Research through the projects UEFISCDI-18PCCDI/2018 and PN19-030102-INCDFM.
[1] V. A. Kostelecký and S. Samuel, Phys. Rev. D 39, 683 (1989).
[2] V. A. Kostelecký and R. Potting, Phys. Rev. D 51, 3923 (1995).
[3] V. A. Kostelecký, Phys. Rev. D 69, 105009 (2004).
[4] V. A. Kostelecký and M. Mewes, Phys. Rev. D 70, 031902 (2004).
[5] V. A. Kostelecký and M. Mewes, Phys. Rev. D 70, 076002 (2004).
[6] V. A. Kostelecký and M. Mewes, Phys. Rev. D 80, 015020 (2009).
[7] J. S. Díaz, Advances in High Energy Physics 2014, 962410 (2014).
[8] J. S. Díaz, V. A. Kostelecký, and R. Lehnert, Phys. Rev. D 88, 071902 (2013).
[9] J. S. Díaz, Phys. Rev. D 89, 036002 (2014).
[10] J. B. Albert et al. (EXO-200 Collaboration), Phys. Rev. D 93, 072001 (2016).
[11] O. Azzolini et al., Phys. Rev. D 100, 092002 (2019).
[12] R. Arnold et al., The European Physical Journal C 79, 440 (2019).
[13] C. Brofferio, O. Cremonesi, and S. Dell’Oro, Frontiers in Physics 7, 86 (2019).
[14] I. Nutini, The CUORE experiment: detector optimization and modelling and CPT conservation limit, Ph.D. thesis, INFN - Gran Sasso Science Institute (2019).
[15] L. Pertoldi, Search for Lorentz and CPT symmetries violation in double-beta decay using data from the GERDA experiment, Ph.D. thesis, Universita degli Studi di Padova (2017).
[16] O. Nițescu, S. Ghinescu, and S. Stoica, Journal of Physics G: Nuclear and Particle Physics 47, 055112 (2020).
[17] S. Stoica and M. Mirea, Phys. Rev. C 88, 037303 (2013).
[18] M. Mirea, T. Pahomi, and S. Stoica, Romanian Reports in Physics 67, 872 (2015).
[19] W. Haxton and G. Stephenson, Progress in Particle and Nuclear Physics 12, 409 (1984).
[20] M. Doi, T. Kotani, and E. Takasugi, Progress of Theoretical Physics Supplement 83, 1 (1985).
[21] T. Tomoda, Reports on Progress in Physics 54, 53 (1991).
[22] V. A. Kostelecký and N. Russell, Rev. Mod. Phys. 83, 11 (2011).
[23] V. A. Kostelecký and J. D. Tasson, Phys. Rev. Lett. 102, 010402 (2009).
[24] J. Suhonen and O. Civitarese, Physics Reports 300, 123 (1998).
[25] F. F. Deppisch, L. Graf, and F. Šimkovic, arXiv:2003.11836 (2020).
[26] M. Caprio, Computer Physics Communications 171, 107 (2005).