On supersymmetric gauge theories with higher derivatives and nonlocal terms in the matter sector

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Abstract

In this work, we consider local and nonlocal higher-derivative generalizations of the super-Chern-Simons theory and four-dimensional SQED. In contrast to previous studies, the models studied here also have higher-derivative terms in the matter sector. For these models, we calculate the one-loop superfield effective potential.

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I. INTRODUCTION

Historically, higher-derivative theories have been introduced in an attempt to prevent singularities in a classical field theory \([1]\), and to avoid ultraviolet divergences of a quantum field theory \([2]\). In supersymmetric models, higher-derivative theories have been studied within different contexts. For example, the phenomenological implications of an extension of the Minimal Supersymmetric Standard Model with dimensions-five and six operators have been investigated in \([3]\). In \([4]\) supersymmetric versions of cubic and quartic Galileon theories were proposed. Nonlocal higher-derivative extensions for the scalar, super Yang-Mills, and supergravity theories have been constructed in \([5]\). In \([6]\), a new mechanism to construct ghost-free higher-derivative models was formulated. Recently, the higher-covariant-derivative regularization has successfully been applied in supersymmetric gauge theories \([7]\).

The effective potential is an important theoretical tool for studying the ground state of a theory and the phenomena related to it, such as the spontaneous breaking and restoration of symmetries \([8, 9]\). In the context of higher-derivative superfield theories, the effective potential has been investigated for different models \([10–12]\). In particular, in Ref. \([13]\), the one-loop effective potential has been explicitly calculated for the simplest higher-derivative extension of a abelian gauge superfield theory. In \([14]\) and \([15]\), the effective potential has been studied in higher-derivative gauge superfield theories defined on the \(N = 1\) and \(N = 2\) three-dimensional superspaces. More recently, a nonlocal higher-derivative extension of the supersymmetric gauge theory was proposed and the one-loop Kählerian effective potential has been explicitly calculated for this theory \([16]\).

One important limitation of the higher-derivative gauge superfield theories studied in \([13–16]\) is that they do not include higher derivatives in the matter sector. In particular, the four-dimensional theories studied in Refs. \([13–16]\) also do not contain chiral self-interaction terms. Since these terms give non-trivial contributions to the one-loop superfield effective potential, there is no reason (other than convenience for calculating the one-loop effective potential) to ignore higher-derivative and chiral self-interaction terms in the matter sector of a higher-derivative supersymmetric gauge theory. Thus, the aim of this paper is to formulate higher-derivative or nonlocal gauge covariant terms in the matter sector and to calculate the superfield effective potential at the one-loop level for local and nonlocal higher-derivative generalizations of the super-Chern-Simons theory and SQED by taking into account these new terms in the matter sector.
sector. In this regard, our work is a further development of the studies presented in \[13–16\].

This paper is organized as follows. In section II, we formulate a generic higher-derivative Super-Chern-Simons Theory coupled to matter and calculate the one-loop contribution to the superfield effective potential. In section III, we formulate a generic higher-derivative four-dimensional SQED and explicitly calculate the one-loop Kählerian effective potential in it. In section IV, we give a short summary of the results obtained and suggest a possible continuation of this study.

II. HIGHER-DERIVATIVE SUPER-CHERN-SIMONS THEORY

Our starting point is the following $\mathcal{N} = 1$, $d = 3$ higher-derivative action for the complex scalar multiplet:

$$
S_{HM} = \frac{1}{2} \int d^{5}z \left[ \bar{\Phi} \left( f(\Box) D^2 + mg(\Box) \right) \Phi + \text{h.c.} \right] + \int d^{5}z V(\bar{\Phi}\Phi),
$$

(1)

which is invariant under the global transformations $\delta \Phi = iK \Phi$ and $\delta \bar{\Phi} = -iK\bar{\Phi}$. This model was originally proposed in Ref. \[11\] and studied only in the context of local theories. The dimensionless operators $f(\Box)$ and $g(\Box)$ are assumed to be analytical functions of the d’Alembertian operator. Additionally, in order to reproduce the standard action for the complex scalar multiplet, we also suppose that $f(\Box)$ and $g(\Box)$ coincide with the unit operator in some suitable limit.

We are interested in coupling of the theory (1) to the abelian gauge superfield $A_{\alpha}$. In order to do this, we will use the identity $\Box = (D^2)^2$ and apply the minimal coupling prescription changing the simple covariant derivative by the gauge covariant one through the rule

$$
D_{\alpha} \Phi \rightarrow \nabla_{\alpha} \Phi \equiv D_{\alpha} \Phi - iA_{\alpha} \Phi.
$$

(2)

Thus, Eq. (1) can be rewritten as

$$
S_{HM} = \frac{1}{2} \int d^{5}z \left[ \bar{\Phi} \left( f(\nabla^4) \nabla^2 + mg(\nabla^4) \right) \Phi + \text{h.c.} \right] + \int d^{5}z V(\bar{\Phi}\Phi).
$$

(3)

Evidently, this gauged model is invariant under the local transformations

$$
[\Phi]' = e^{iK} \Phi ; \quad [A_{\alpha}]' = A_{\alpha} + D_{\alpha} K.
$$

(4)

Since $A_{\alpha}$ is a non-dynamical superfield in (3), to introduce a consistent dynamics for it we will add to (3) the following higher-derivative generalization of the supersymmetric Chern-Simons
theory
\[ S_{HCS} = \frac{1}{2e^2} \int d^5 z A^\alpha h(\Box) D^\beta D_\alpha A_\beta , \] (5)

which is also invariant under the transformations (4).

Finally, the higher-derivative version of the super-Chern-Simons theory coupled to matter superfields that we will study in this work has the following action
\[ S = S_{HCS} + S_{HM} + S_{GF} , \] (6)

where, to perform quantum calculations, we conveniently added the gauge-fixing functional
\[ S_{GF} = \frac{1}{2e^2} \int d^5 z A^\alpha h(\Box) D_\alpha D^\beta A_\beta . \] (7)

In order to carry out the calculation of the one-loop superfield effective potential in three dimensions [18], we will employ the background field method [19]. Making the background-quantum splitting \( \Phi \rightarrow \Phi + \phi \) in (6), assuming that the background superfield satisfies the condition \( D_\alpha \Phi = 0 \), and expanding the action to up to the second order in the quantum superfields, after some tedious but straightforward manipulations we obtain
\[ S_2 = \int d^5 z \left( \frac{1}{2} A^\alpha \hat{H}^{\alpha}_\beta A_\beta + A^\alpha \mathcal{F}_\alpha \right) + \frac{1}{2} \int d^5 z \phi^T \dot{\Phi} \dot{\phi} , \] (8)

where
\[ \hat{H}^{\alpha}_\beta \equiv \frac{h(\Box)}{e^2} \left( D^\beta D_\alpha + \frac{1}{\alpha} D_\alpha D^\beta \right) + \frac{\Phi^2}{2} \left( -D^\beta D_\alpha + f(\Box) D_\alpha D^\beta \right) \frac{D^2}{\Box} \]
\[ - \frac{m |\Phi|^2 [g(\Box) - 1]}{2\Box} D_\alpha D^\beta ; \] (9)
\[ \mathcal{F}_\alpha \equiv \frac{i}{2} \left\{ \Phi f(\Box) D_\alpha \phi - \bar{\Phi} f(\Box) D_\alpha \phi + m \Phi \frac{[g(\Box) - 1]}{\Box} D_\alpha D^2 \phi - m \bar{\Phi} \frac{[g(\Box) - 1]}{\Box} D_\alpha D^2 \phi \right\} , \]

and
\[ \phi \equiv \begin{pmatrix} \phi \\ \bar{\phi} \end{pmatrix} ; \quad \dot{\Phi} \equiv \begin{pmatrix} V_{\Phi\Phi} \\ \frac{f(\Box)D^2 + mg(\Box)}{V_{\Phi\Phi}} + V_{\Phi\bar{\Phi}} \\ \frac{f(\Box)D^2 + mg(\Box)}{V_{\Phi\Phi}} + V_{\Phi\bar{\Phi}} \end{pmatrix} . \] (10)

For present purposes, it is useful to diagonalize (8). To do this, let us consider the following nonlocal change of variables [20]
\[ A_\alpha(z) \rightarrow A_\alpha(z) - \int d^5 z' G_\alpha^\beta(z, z') F_\beta(z') , \] (11)

where \( G_\alpha^\beta(z, z') \) is the Green’s function of the operator \( \hat{H}_\alpha^\beta \) defined from the equation
\[ G_\alpha^\beta(z, z') = \left( AD^\beta D_\alpha + BD_\alpha D^\beta \right) \delta^5(z - z') , \] (12)
where the coefficients $A$ and $B$ are written in the Appendix.

Under the nonlocal transformation (11), the functional (8) assumes the diagonalized form

$$S_2 = \frac{1}{2} \int d^5 z A^\alpha \hat{H}_\alpha \beta A^\beta + \frac{1}{2} \int d^5 z \phi^T \hat{O} \phi - \int d^5 z d^5 z' F^\alpha(z) G^\beta_{\alpha \beta}(z, z') F^\beta(z') \, .$$

(13)

Since (11) is merely a shift by a constant, it leaves the integration measure in the path integral invariant. By integrating out the quantum superfields $A^\alpha$ and $\phi$, we get two contributions to the Euclidean one-loop effective action

$$\Gamma^{(1)} = \Gamma^{(1)}_A + \Gamma^{(1)}_\phi \, .$$

(14)

The first contribution $\Gamma^{(1)}_A$ is given by the trace:

$$\Gamma^{(1)}_A \bigg|_{\alpha = 0} = \frac{1}{2} \Tr \ln \hat{H}_\alpha \beta = \frac{1}{2} \Tr \ln \left\{ \frac{h(\Box)}{e^2} \left( D^\gamma D_\alpha + \frac{1}{\alpha} D_\alpha D^\gamma \right) \right\} + \frac{1}{2} \Tr \ln \left\{ \delta_\gamma \beta - \frac{M}{2 \Box h(\Box)} D^\gamma D_\gamma - \frac{\alpha M f(\Box)}{2 \Box h(\Box)} D\gamma D_\gamma + \frac{\alpha M \left[ g(\Box) - 1 \right]}{2 \Box^2 h(\Box)} D^\gamma D_\gamma \right\} \, .$$

(15)

where we factored out the inverse of the propagator of $A^\alpha$ and defined $M \equiv \frac{1}{2} e^2 |\Phi|^2$.

Since the first trace does not depend on the background superfield $\alpha = 0$, we can drop it out. The second trace can be simplified if we assume the Landau gauge $\alpha = 0$. Therefore, it follows from (15) that

$$\Gamma^{(1)}_A \bigg|_{\alpha = 0} = \frac{1}{2} \Tr \ln \hat{H}_\alpha \beta = \frac{1}{2} \Tr \ln \left\{ \delta_\gamma \beta - \frac{M}{2 \Box h(\Box)} D^\gamma D_\gamma \right\} = \frac{1}{2} \int d^5 z \int \frac{d^3 k}{(2\pi)^3} \frac{1}{|k|} \arctan \left[ \frac{M}{|k| h(-k^2)} \right] \, .$$

(16)

We now determine the second contribution $\Gamma^{(1)}_\phi$. If we impose the Landau gauge, then $G^\beta_{\alpha \beta}(z, z') = AD^\beta D_\alpha \delta^\beta(z - z')$ and the last term in Eq. (13) vanishes due to the identity $D^\alpha D_\beta D_\alpha = 0$. Therefore, we can write

$$\Gamma^{(1)}_\phi \bigg|_{\alpha = 0} = -\frac{1}{2} \Tr \ln \hat{O} = -\frac{1}{2} \Tr \ln \left( \begin{array}{cc} 0 & f(\Box) D^2 \\ f(\Box) D^2 & 0 \end{array} \right) - \frac{1}{2} \Tr \ln \left[ \hat{I}_2 + \mathcal{M} D^2 \frac{D^2}{\Box f(\Box)} \right] \, .$$

(17)

where we factored out the inverse of the $\phi$-propagator and defined

$$\mathcal{M} \equiv \left( \begin{array}{cc} mg(\Box) + V_{\Phi \Phi} & V_{\Phi \Phi} \\ V_{\Phi \Phi} & mg(\Box) + V_{\Phi \Phi} \end{array} \right) \, .$$

(18)

Again, we can drop the first trace out and the second one can be evaluated to give

$$\Gamma^{(1)}_\phi \bigg|_{\alpha = 0} = \frac{1}{2} \int d^5 z \int \frac{d^3 k}{(2\pi)^3} \frac{1}{|k|} \arctan \left[ \frac{\mathcal{M}}{|k| f(-k^2)} \right] \, .$$

(19)
Finally, substituting (16) and (19) into (14), we can infer that the superfield effective potential is given by the expression

\[ K^{(1)}(\Phi, \bar{\Phi}) = \frac{1}{2} \int \frac{d^3 k}{(2\pi)^3} \frac{1}{|k|} \left\{ \arctan \left( \frac{M}{|k|h(-k^2)} \right) + \sum_{i=+, -} \arctan \left( \frac{\lambda_i(-k^2)}{|k|f(-k^2)} \right) \right\}, \]  

(20)

where the \( \lambda \)'s are the eigenvalues of the matrix \( M \):

\[ \lambda_\pm(-k^2) = mg(-k^2) + \sqrt{\bar{V}_{\Phi\Phi} \pm \sqrt{(\bar{V}_{\Phi\Phi})^2 - 4\bar{V}_{\Phi\Phi}V_{\Phi\Phi}}}, \]  

(21)

The last step of our calculation is to evaluate the integrals in Eq. (20). However, to evaluate these integrals one must specify \( f(-k^2), g(-k^2), \) and \( h(-k^2) \). In this work, we will examine two higher-derivative models which lead to an improved ultraviolet behavior of the theory: one local and one nonlocal.

A simple local higher-derivative model is defined by

\[ f(\nabla^4) = g(\nabla^4) = 1 - \nabla^4 \frac{\Lambda^2}{L^2}; \quad h(\Box) = 1 - \Box \frac{\Lambda^2}{L^2}, \]  

(22)

where \( \Lambda_L \) is the mass scale at which the higher-derivative contributions begin to be pertinent. It follows from (22) that

\[ f(-k^2) = g(-k^2) = h(-k^2) = 1 + \frac{k^2}{\Lambda^2_L}. \]  

(23)

All integrals in this article will be evaluated approximately by employing the strategy of expansion by regions \([21]\). Therefore, substituting (23) into (20) and assuming that \( \Lambda_L \) is large, we find

\[ K^{(1)}_{L}(\Phi, \bar{\Phi}) \approx -\frac{M^2}{16\pi} \left( 1 + 2 \frac{M^2}{\Lambda^4_L} + 7 \frac{M^4}{\Lambda^4_L} + 30 \frac{M^6}{\Lambda^6_L} + \cdots \right) - \sum_{i=+, -} \frac{(m + \tilde{\lambda}_i)^2}{16\pi} \left[ 1 + 2 \frac{\tilde{\lambda}_i(m + \tilde{\lambda}_i)}{\Lambda^2_L} \right] \]

\[ + \frac{\tilde{\lambda}_i(m + \tilde{\lambda}_i)^2(2m + 7\lambda_i)}{\Lambda^4_L} + 2 \frac{\tilde{\lambda}_i(m + \tilde{\lambda}_i)^3(m^2 + 9m\lambda_i + 15\lambda^2_i)}{\Lambda^6_L} + \cdots, \]

(24)

where \( \tilde{\lambda}_\pm = \sqrt{\bar{V}_{\Phi\Phi} \pm \sqrt{(\bar{V}_{\Phi\Phi})^2 - 4\bar{V}_{\Phi\Phi}V_{\Phi\Phi}}} \).

On the other hand, a simple nonlocal model is defined by

\[ f(\nabla^4) = g(\nabla^4) = \exp \left( -\nabla^4 \frac{\Lambda^4_{NL}}{L^4} \right); \quad h(\Box) = \exp \left( -\Box \frac{\Lambda^2_{NL}}{L^2} \right), \]  

(25)

where, similar to the local model, \( \Lambda_{NL} \) describes the characteristic energy at which the nonlocal contributions become important. According to (25), we have

\[ f(-k^2) = g(-k^2) = h(-k^2) = \exp \left( \frac{k^2}{\Lambda^2_{NL}} \right). \]  

(26)
Therefore, substituting (26) into (20) and assuming that Λ_{NL} is large, we get
\[
K^{(1)}_{NL}(\Phi, \bar{\Phi}) \approx -\frac{M^2}{16\pi} \left(1 + 2\frac{M^2}{\Lambda_{NL}^2} + 6\frac{M^4}{3\Lambda_{NL}^4} + \cdots\right) - \\
- \sum_{i=+,-} \frac{(m + \tilde{\lambda}_i)^2}{16\pi} \left[1 + 2(m + \tilde{\lambda}_i) \right. \left. \frac{\tilde{\lambda}_i((m + \tilde{\lambda}_i)^2(m + 6\tilde{\lambda}_i)) + \frac{1}{3} \tilde{\lambda}_i(m + \tilde{\lambda}_i)^3(m^2 + 23m\tilde{\lambda}_i + 64\tilde{\lambda}_i^2) + \cdots}{\Lambda_{NL}^6} \right].
\]
Since Λ_{L} and Λ_{NL} are finite physical parameters, the one-loop effective potentials (24) and (27) are UV finite. Notice that this finiteness remains even if we set the parameters to be infinitely large Λ_{L} \to \infty and Λ_{NL} \to \infty while many higher-derivative or nonlocal theories turn out to be divergent in this limit which is equivalent to removing the higher-derivative term. Indeed, such one-loop finiteness is a characteristic feature of the three-dimensional theories. Moreover, we note that the expressions (24) and (27) coincide up to the orders Λ_{L}^{-2} and Λ_{NL}^{-2} in the approximations. This coincidence occurs because the operators (22) and (25) are identical in this particular order.

III. HIGHER-DERIVATIVE SQED

In the present section, we are interested in a more realistic theory. Thus, let us consider the following four-dimensional matter action:
\[
S_M = \int d^8 z (\bar{\Phi} \Phi + \bar{\Phi} \Phi) + \int d^6 z (m\Phi_- \Phi_+ + W (\Phi_+ \Phi_+)) + h.c.,
\]
which is invariant under the rigid U(1) transformations:
\[
[\Phi_+]' = e^{i\lambda} \Phi_+ ; \quad [\Phi_-]' = e^{-i\lambda} \Phi_- \quad [\bar{\Phi}_+]' = e^{-i\lambda} \bar{\Phi}_+ ; \quad [\bar{\Phi}_-]' = e^{i\lambda} \bar{\Phi}_- .
\]
A natural higher-derivative generalization of this model is the following
\[
S_{HM} = \frac{1}{2} \int d^8 z \left(\bar{\Phi}_+ f_+ (\square) \Phi_+ + (f_+ (\square) \bar{\Phi}_+) \Phi_+ + \bar{\Phi}_- f_- (\square) \Phi_- \right. \left. + \frac{1}{2} \int d^6 z (m\Phi_- g (\square) \Phi_+ + W (\Phi_- \Phi_+)) + h.c. \right),
\]
where again \(f_+ (\square), f_- (\square),\) and \(g(\square)\) are dimensionless analytical functions and coincide with the identity in some suitable limit. This model is essentially a two-superfield version of the one proposed in Ref. [12] which has been studied only in the context of nonlocal theories.
Due to the chirality of the superfields $\Phi_{\pm}$, we have $\bar{D}^2 D^2 \Phi_{\pm} = \Box \Phi_{\pm}$. Thus, the action (30) can be rewritten in a more convenient form

$$S_{HM} = \frac{1}{2} \int d^8 z \left( \Phi_+ f_+ (\bar{D}^2 D^2) \Phi_+ + (f_+ (\bar{D}^2 D^2) \Phi_+) \Phi_+ + \Phi_+ f_- (\bar{D}^2 D^2) \Phi_- 
+ (f_- (\bar{D}^2 D^2) \Phi_-) \Phi_- \right) + \left[ \int d^6 z \left( m \Phi_- g (\bar{D}^2 D^2) \Phi_+ + W (\Phi_- \Phi_+) \right) + h.c. \right].$$

In order to extend the transformations (29) to local $U(1)$ transformations, we define

$$\Phi_+ ' = e^{i\Lambda} \Phi_+ ; \quad \Phi_- ' = e^{-i\bar{\Lambda}} \Phi_- \quad \bar{\Phi}_+ ' = e^{i\bar{\Lambda}} \bar{\Phi}_+ ; \quad \bar{\Phi}_- ' = e^{-i\Lambda} \bar{\Phi}_-, \quad (32)$$

where the local parameter $\Lambda$ is chiral.

To extend (31) up to a form invariant under (32), we must put into use the minimal coupling prescription (22):

$$D_\alpha \Phi_\pm \rightarrow \nabla_\alpha \Phi_\pm \equiv D_\alpha \Phi_\pm \mp i \Gamma_\alpha \Phi_\pm ; \quad \Gamma_\alpha \equiv i D_\alpha \Lambda \quad (33)$$

where the gauge superfield $V$ and the connection $\Gamma_\alpha$ transform as

$$[V]' = V + i (\bar{\Lambda} - \Lambda) ; \quad [\Gamma_\alpha]' = \Gamma_\alpha + D_\alpha \Lambda \quad (34)$$

Therefore, Eq. (31) can be rewritten as

$$S_{HM} = \frac{1}{2} \int d^8 z \left( \Phi_+ e^V f_+ (\bar{\nabla}^2 \nabla^2) \Phi_+ + (f_+ (\bar{\nabla}^2 \nabla^2) \Phi_+) e^V \Phi_+ + \Phi_+ e^{-V} f_- (\bar{\nabla}^2 \nabla^2) \Phi_- 
+ (f_- (\bar{\nabla}^2 \nabla^2) \Phi_-) e^{-V} \Phi_- \right) + \left[ \int d^6 z \left( m \Phi_- g (\bar{\nabla}^2 \nabla^2) \Phi_+ + W (\Phi_- \Phi_+) \right) + h.c. \right]. \quad (35)$$

This model is invariant under the combined transformations (32) and (34). Notice that we introduced the factor $\exp(V)$ to change a $\Lambda$ representation to a $\bar{\Lambda}$ representation of the group $[17]$.

Since $V$ has no kinetic term in (35), we will add to (34) the following higher-derivative generalization of the supersymmetric abelian gauge theory

$$S_{HG} = \frac{1}{16 g^2} \left[ \int d^6 z W^\alpha h (\Box) W_\alpha + \int d^6 z \bar{W}^\dot{\alpha} h (\Box) \bar{W}_{\dot{\alpha}} \right], \quad (36)$$

where the superfield strengths are expressed in terms of the gauge superfield as

$$W_\alpha = i \bar{D}^2 D_\alpha V \quad \bar{W}_{\dot{\alpha}} = -i D^2 \bar{D}_{\dot{\alpha}} V \quad (37)$$
Finally, the higher-derivative version of the SQED that we will study in this paper is given by

\[ S = S_{HG} + S_{HM} . \]  

(38)

Here, our goal is to calculate the one-loop correction to the Kählerian effective potential \[23\].

Thus, as we have done in the last section, we will expand (38) around background superfields:

\[ \Phi_+ \rightarrow \Phi_+ + \phi_+ ; \quad \Phi_- \rightarrow \Phi_- + \phi_- . \]  

(39)

We will assume that the background superfields are subject to the constraints \[24\]. However, this replacement introduces a new gauge symmetry, namely \[\delta \psi_\pm = \tilde{D}^\alpha \omega_{\pm\alpha} \] and \[\delta \bar{\psi}_\pm = D^\alpha \omega_{\pm\alpha} \]. Therefore, in order to fix this gauge invariance and the one \[32\], we will add to \[\Pi_0 \] the following gauge-fixing functionals

\[ S_{GF1} = - \frac{1}{8e^2 \alpha} \int d^8 z V \Box h(\Box) \Pi_0 V ; \]  

(42)

\[ S_{GF2} = \int d^8 z \bar{\psi}_+ f_+(\Box) (\tilde{D}^2 D^2 - D^\alpha \tilde{D}^2 D_\alpha) \psi_+ ; \]  

(43)

\[ S_{GF3} = \int d^8 z \bar{\psi}_- f_-(\Box) (D^2 D^2 - D^\alpha \tilde{D}^2 D_\alpha) \psi_- . \]  

(44)
Therefore, it follows from (40-44) that
\[
\tilde{S}_2 \equiv S_2 + S_{GF1} + S_{GF2} + S_{GF3} = \int d^8z \left( \frac{1}{2} V \hat{H} V + V \mathcal{F} \right) + \frac{1}{2} \int d^8z \psi^T \hat{O} \psi ,
\] (45)
where
\[
\hat{H} \equiv -\frac{1}{4} e^2 \square h (\square) \left[ \Pi_2 + \frac{1}{\alpha} \Pi_0 \right] + \left( |\Phi_+|^2 + |\Phi_-|^2 \right) \Pi_2 + \left( |\Phi_+|^2 f_+(\square) \right)
+ \left( |\Phi_-|^2 f_-(\square) \right) \Pi_0 - 2m\Phi_-\Phi_+ [g(\square) - 1] \frac{D^2}{\square} - 2m\Phi_-\Phi_+ [g(\square) - 1] \frac{\bar{D}^2}{\square}
\] (46)
\[
\mathcal{F} \equiv \Phi_+ f_+(\square) \bar{D}^2 \psi_+ - \Phi_- f_-(\square) \bar{D}^2 \psi_- - m\Phi_- [g(\square) - 1] \Pi_0 \psi_+ + m\Phi_+ [g(\square) - 1] \Pi_0 \psi_- + h.c. ,
\] (47)
and
\[
\psi \equiv \begin{pmatrix}
\psi_+ \\
\psi_- \\
\bar{\psi}_+ \\
\bar{\psi}_-
\end{pmatrix}; \quad \hat{O} \equiv \begin{pmatrix}
\hat{W} D^2 & \square \hat{F} \\
\square \hat{F} & \hat{W} D^2
\end{pmatrix} .
\] (48)

Lastly, the matrices $\hat{W}$ and $\hat{F}$ are defined as
\[
\hat{F} \equiv \begin{pmatrix}
f_+(\square) & 0 \\
0 & f_-(\square)
\end{pmatrix}; \quad \hat{W} \equiv \begin{pmatrix}
\frac{\partial^2 W}{\partial \Phi_+^2} & mg(\square) + \frac{\partial^2 W}{\partial \Phi_+ \partial \Phi_-} \\
mg(\square) + \frac{\partial^2 W}{\partial \Phi_+ \partial \Phi_-} & \frac{\partial^2 W}{\partial \Phi_-^2}
\end{pmatrix} .
\] (49)

The mixing terms between the quantum superfields $V$ and $\psi$ can be eliminated by the following nonlocal change of variables in the path integral:
\[
V(z) \rightarrow V(z) - \int d^8z' G(z, z') \mathcal{F}(z') ,
\] (50)
where $G(z, z')$ is the Green’s function of the operator $\hat{H} = \hat{H}(z)$, namely $\hat{H}G(z, z') = \delta^8(z - z')$.

This equation has the solution
\[
G(z, z') = \left( X\Pi_2 + Y\Pi_0 + ZD^2 + \bar{Z}\bar{D}^2 \right) \delta^8(z - z') .
\] (51)
The coefficients $X$, $Y$, and $Z$ are written in the Appendix.

Therefore, after the change of variables (50), the $\tilde{S}_2$ can be put in the diagonalized form
\[
\tilde{S}_2 = \frac{1}{2} \int d^8z V \hat{H} V + \frac{1}{2} \int d^8z \psi^T \hat{O} \psi - \int d^8z d^8z' \mathcal{F}(z) G(z, z') \mathcal{F}(z') .
\] (52)
From $\tilde{S}_2$, we can compute the Euclidean one-loop effective action by formal integrating out the superfields $V$ and $\psi$. Therefore, we arrive at
\begin{equation}
\Gamma^{(1)} = \Gamma^{(1)}_V + \Gamma^{(1)}_\psi .
\end{equation}

The first contribution $\Gamma^{(1)}_V$ is given by the trace:
\begin{align}
\Gamma^{(1)}_V &= -\frac{1}{2} \text{Tr} \ln \hat{H} = -\frac{1}{2} \text{Tr} \ln \left\{ \frac{1}{4e^2} \Box h(\Xi) \left[ \frac{1}{2} \Pi^2 + \frac{1}{a} \Pi_0 \right] + \frac{1}{4e^2} M \Pi^2 + \frac{1}{4e^2} \tilde{f}(\Xi) \Pi_0 \\
&\quad - \frac{2m}{4e^2} \hat{g}(\Xi) \frac{D^2}{\Box} - \frac{2m}{4e^2} \tilde{g}(\Xi) \frac{\tilde{D}^2}{\Box} \right\},
\end{align}
where we introduced the definitions
\begin{align}
M &\equiv 4e^2 \left( |\Phi^2| + |\Phi^-|^2 \right) ; \\
\tilde{f}(\Xi) &\equiv 4e^2 \left( |\Phi^2| f_+(\Xi) + |\Phi^-|^2 f_-(\Xi) \right) ; \\
\hat{g}(\Xi) &\equiv 4e^2 \Phi^2 (\Xi - 1) ; \\
\tilde{g}(\Xi) &\equiv 4e^2 \Phi^2 (\Xi - 1) .
\end{align}

We can factor out the inverse of the $V$-propagator, which is independent of the background superfields, and subsequently drop it. Therefore, it follows from (54) that
\begin{equation}
\Gamma^{(1)}_V \big|_{\alpha=0} = -\frac{1}{2} \text{Tr} \ln \hat{I} = -\frac{1}{2} \text{Tr} \ln \left\{ 1 - \frac{M}{\Box h(\Xi)} \Pi^2 - \frac{2ma \hat{g}(\Xi)}{\Box h(\Xi)} \Pi_0 - \frac{2ma \tilde{g}(\Xi)}{\Box h(\Xi)} \frac{D^2}{\Box} \right\} .
\end{equation}

This trace assumes its simplest form in the Landau gauge $\alpha = 0$. Therefore, in this particular gauge, we find
\begin{equation}
\Gamma^{(1)}_V \big|_{\alpha=0} = -\frac{1}{2} \text{Tr} \ln \hat{I} = -\frac{1}{2} \text{Tr} \ln \left( W D^2 \quad \tilde{F} \right) .
\end{equation}

Let us move on to the calculation of the second contribution $\Gamma^{(1)}_\psi$. Notice that $G(z, z') = \lambda \Pi^2 \delta^2(z - z')$ in the Landau gauge. Since the Green’s function is transversal in this gauge, the last term in Eq. (52) vanishes. Therefore, we can write
\begin{equation}
\Gamma^{(1)}_\psi \big|_{\alpha=0} = -\frac{1}{2} \text{Tr} \ln \hat{O} = -\frac{1}{2} \text{Tr} \ln \left( \hat{W} D^2 \quad \hat{F} \right) .
\end{equation}

Again, we can factor out the inverse of the $\psi$-propagator and subsequently drop it out. Thus, we can rewrite (58) as
\begin{equation}
\Gamma^{(1)}_\psi \big|_{\alpha=0} = -\frac{1}{2} \text{Tr} \ln \left[ \hat{I} + \left( \begin{array}{cc} 0 & \hat{F} \hat{W} D^2 \\
\hat{F}^{-1} \hat{W} D^2 & 0 \end{array} \right) \right] .
\end{equation}
Only the even powers in the expansion of the logarithm give non-vanishing contributions to the trace. Therefore, we can show that

\[ \Gamma^{(1)}_{\psi} \big|_{\alpha=0} = -\frac{1}{4} \text{Tr} \ln \left[ \hat{I}_4 - \begin{pmatrix} \hat{F}^{-1} \hat{W} \hat{F}^{-1} \hat{W} \Pi_{0-} & 0 \\ 0 & \hat{F}^{-1} \hat{W} \hat{F}^{-1} \hat{W} \Pi_{0+} \end{pmatrix} \right] \]

\[ = -\frac{1}{4} \text{Tr} \ln \left( \hat{I}_2 - \frac{\hat{F}^{-1} \hat{W} \hat{F}^{-1} \hat{W}}{\square} \Pi_{0-} \right) - \frac{1}{4} \text{Tr} \ln \left( \hat{I}_2 - \frac{\hat{F}^{-1} \hat{W} \hat{F}^{-1} \hat{W}}{\square} \Pi_{0+} \right) . \quad (60) \]

These traces can be evaluated to give

\[ \Gamma^{(1)}_{\psi} \big|_{\alpha=0} = \frac{1}{2} \int d^8 z \int \frac{d^4 p}{(2\pi)^4 p^2} \text{Tr} \ln \left( \hat{I}_2 + \frac{\hat{W} \hat{W}^\dagger m=0}{p^2 f^2(-p^2)} \right) . \quad (61) \]

This integral is rather complicated. In order to obtain clear analytical results, we will assume that \( m = 0 \) and

\[ f_+(\nabla^2 \nabla^2) = f_-(\nabla^2 \nabla^2) \equiv f(\nabla^2 \nabla^2) . \quad (62) \]

Therefore, it follows that

\[ \Gamma^{(1)}_{\psi} \big|_{\alpha=0} = \frac{1}{2} \int d^8 z \int \frac{d^4 p}{(2\pi)^4 p^2} \text{Tr} \ln \left( \hat{I}_2 + \frac{\hat{W} \hat{W}^\dagger m=0}{p^2 f^2(-p^2)} \right) . \quad (63) \]

Finally, substituting (57) and (63) into (53), we can infer that the one-loop correction to the \( K \)ähler effective potential is given by

\[ K^{(1)}(\Phi, \bar{\Phi}) = \int \frac{d^4 p}{(2\pi)^4 p^2} \left\{ -\ln \left[ 1 + \frac{M}{p^2 h(-p^2)} \right] + \frac{1}{2} \sum_{i=+, -} \ln \left[ 1 + \frac{\lambda_i}{p^2 f^2(-p^2)} \right] \right\} , \quad (64) \]

where the \( \lambda \)'s are the eigenvalues of the matrix \( \hat{W} \hat{W}^\dagger m=0 \) and they are given by

\[ \lambda_{\pm} = \frac{1}{2} \left\{ \frac{\partial^2 W}{\partial \Phi_+^2} \pm 2 \left| \frac{\partial^2 W}{\partial \Phi_+ \partial \Phi_-} \right|^2 + \left( \frac{\partial^2 W}{\partial \Phi_+^2} \pm 2 \left| \frac{\partial^2 W}{\partial \Phi_- \partial \Phi_+} \right|^2 \right)^2 \right\} - 4 \left| \frac{\partial^2 W}{\partial \Phi_+ \partial \Phi_-} \right|^2 . \quad (65) \]

In the same manner as the previous section, it is necessary to specify the functions \( f(-p^2) \) and \( h(-p^2) \) in order to evaluate the integrals in (64). Thus, again, we will consider one local and one nonlocal higher-derivative model.

The local higher-derivative model is described by

\[ f(\nabla^2 \nabla^2) = 1 - \frac{\nabla^2 \nabla^2}{\Lambda_L^2} ; \quad h(\square) = 1 - \frac{\square}{\Lambda_L^2} . \quad (66) \]
Thus, we can infer that
\[ f(-p^2) = h(-p^2) = 1 + \frac{p^2}{\Lambda_L^2}. \] (67)

Therefore, replacing these functions in (64) and assuming that \( \Lambda_L \) is large, we obtain
\[
K^{(1)}_L(\Phi, \bar{\Phi}) \approx \frac{M}{16\pi^2} \left\{ -1 + \ln \left( \frac{M}{\Lambda_L^2} \right) + \frac{M}{2\Lambda_L^2} \left[ 1 + 2\ln \left( \frac{M}{\Lambda_L^2} \right) \right] + \frac{M^2}{6\Lambda_L^4} \left[ 10 + 12\ln \left( \frac{M}{\Lambda_L^2} \right) \right] \right. \\
+ \frac{M^3}{12\Lambda_L^6} \left[ 59 + 60\ln \left( \frac{M}{\Lambda_L^2} \right) \right] + \cdots - \sum_{i=+,-} \frac{\lambda_i}{32\pi^2} \left[ \ln \left( \frac{\lambda_i}{\Lambda_L^2} \right) + \frac{\lambda_i}{6\Lambda_L^2} \right] 13 \\
+ 12\ln \left( \frac{\lambda_i}{\Lambda_L^2} \right) \left[ 193 + 140\ln \left( \frac{\lambda_i}{\Lambda_L^2} \right) \right] + \frac{\lambda_i^3}{84\Lambda_L^6} \left[ 3825 + 2520 \times \ln \left( \frac{\lambda_i}{\Lambda_L^2} \right) \right] + \cdots \right\} \\
\times \ln \left( \frac{\lambda_i}{\Lambda_L^2} \right) + \cdots. \] (68)

On the other hand, the nonlocal higher-derivative model is described by
\[ f(\bar{\nabla}^2\nabla^2) = \exp \left( -\frac{\bar{\nabla}^2\nabla^2}{\Lambda_{NL}^2} \right); \quad h(\square) = \exp \left( -\frac{\square}{\Lambda_{NL}^2} \right). \] (69)

Evidently, Eq. (69) implies that
\[ f(-p^2) = h(-p^2) = \exp \left( \frac{p^2}{\Lambda_{NL}^2} \right). \] (70)

Therefore, replacing these functions in (64) and assuming that \( \Lambda_{NL} \) is large, we have
\[
K^{(1)}_{NL}(\Phi, \bar{\Phi}) \approx \frac{M}{16\pi^2} \left\{ \ln \left( \frac{M}{\Lambda_{NL}^2 e^{1-\gamma}} \right) + \frac{M}{\Lambda_{NL}^2} \ln \left( \frac{2M}{\Lambda_{NL}^2 e^{1-\gamma}} \right) + \frac{M^2}{4\Lambda_{NL}^4} \left[ -1 + 6\ln \left( \frac{3M}{\Lambda_{NL}^2 e^{1-\gamma}} \right) \right] \right. \\
+ \frac{4M^3}{9\Lambda_{NL}^6} \left[ -2 + 6\ln \left( \frac{4M}{\Lambda_{NL}^2 e^{1-\gamma}} \right) \right] + \cdots - \sum_{i=+,-} \frac{\lambda_i}{32\pi^2} \left[ \ln \left( \frac{2\lambda_i}{\Lambda_{NL}^2 e^{1-\gamma}} \right) + \frac{2\lambda_i}{\Lambda_{NL}^2} \right] \\
\times \ln \left( \frac{2\lambda_i}{\Lambda_{NL}^2 e^{1-\gamma}} \right) + \frac{\lambda_i^2}{\Lambda_{NL}^2} \left[ -1 + 6\ln \left( \frac{6\lambda_i}{\Lambda_{NL}^2 e^{1-\gamma}} \right) \right] + \frac{32\lambda_i^3}{288\Lambda_{NL}^6} \left[ -2 + 6 \times \ln \left( \frac{8\lambda_i}{\Lambda_{NL}^2 e^{1-\gamma}} \right) \right] + \cdots \right\} \\
\times \ln \left( \frac{8\lambda_i}{\Lambda_{NL}^2 e^{1-\gamma}} \right) + \cdots. \] (71)

Like the one-loop effective potentials obtained in the previous section, the Kähler effective potentials (68) and (71) are also UV-finite. However, when we set the parameters to be infinitely large \( \Lambda_L \to \infty \) and \( \Lambda_{NL} \to \infty \), such finiteness ceases to exist because of the leading term of the potentials. We know this must be so, because in the limits \( \Lambda_L \to \infty \) and \( \Lambda_{NL} \to \infty \) both potentials (68) and (71) must agree with the one for the standard SQED.

IV. SUMMARY

We considered the higher-derivative/nonlocal extensions of supergauge theories where, unlike previous papers on such theories [13,16], the higher derivatives or nonlocality are implemented
not only in the gauge sector, but also in the matter sector. It is important to note that within our approach, these kinds of theories are treated within the same methodology. Effectively, we introduced a new class of higher-derivative/nonlocal Abelian supergauge theories and new class of gauge-matter couplings.

We performed the one-loop calculations in these theories with use of the functional supertrace approach and explicitly demonstrated that this approach can be applied to these theories with the same degree of success that to other supergauge theories. In three-dimensional case, the result continues to be finite even when the characteristic scale \( \Lambda_L \) (or \( \Lambda_{NL} \)) goes to infinity, as it must be, since the one-loop effective action is finite in three-dimensional theories. This is not so in four-dimensional case since the one-loop effective potential in the usual SQED diverges \([24, 25]\), and the higher-derivative/nonlocal terms in various field theory models clearly play the role of the regularization.

The net result of our paper consists in formulating of new gauge-matter couplings. Therefore, it is natural to expect that these couplings can be generalized to other theories, especially to those ones interesting from the phenomenological viewpoint, and to various effective theories. The advantage of such theories consists in the fact that they are finite and ghost-free. We plan to study phenomenological impacts of new couplings in our next papers.

**APPENDIX**

Below the coefficients of the Green’s functions (12) and (51) are listed:

\[
A = -\frac{e^2 e^2 | \Phi |^2 D^2 + 2 \Box h(\Box)}{2 e^{4|\Phi|^2} - 4 \Box^2 h^2(\Box)} ;
\]

\[
B = \frac{e^2 \alpha \left[ e^2 | \Phi |^2 f(\Box) D^2 - 2 \Box h(\Box) + \alpha m e^2 | \Phi |^2 [g(\Box) - 1] \right]}{2 \alpha^2 e^{4|\Phi|^2} f^2(\Box) - (2 \Box h(\Box) - \alpha m e^2 | \Phi |^2 [g(\Box) - 1])^2} ;
\]

\[
X = \frac{4 e^2}{-\Box h(\Box) + M} ;
\]

\[
Y = \frac{4 e^2 \alpha \Box \left[-\Box h(\Box) + \alpha \tilde{f}(\Box)\right]}{\Box \left[-\Box h(\Box) + \alpha \tilde{f}(\Box)\right]^2 - 4 \alpha^2 m^2 \tilde{g}(\Box) \tilde{\gamma}(\Box)} ;
\]

\[
Z = \frac{2 m \alpha \tilde{g}(\Box)}{\Box \left[-\Box h(\Box) + \alpha \tilde{f}(\Box)\right]} Y .
\]

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