Ways to Minimize Volume (Weight) and Increase the Bearing Capacity of Rigid Pavement

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Abstract

The objective of research is finding of a possibility economy of rigid pavement weight and volume of material. The subject of the research is a mathematical model of rigid pavement in the form of a multilayer structure on an elastic foundation. The method of a research consists in modeling the behavior of rigid pavement in the form of a set of equations. These equations reflect the change in the stress-strain state of such structures. The system of equations takes into account the geometric nonlinearity of the work of materials and makes it possible to investigate the influence of various parameters on the values of stresses and displacements. Critical force coefficient and stress of shells are calculated by Bubnov-Galerkin. The formation way of the elastic foundation allows modeling the spreading layers with various characteristics. Use of two-layer model allows considering of a surface course and base course of road pavement designing (for example concrete and crushed stone). The graphs show the patterns of change of the stress of rigid pavement when changing the characteristics. The form of rigid pavement allowing to maintain big loadings is exposed to improvement. Findings shows the possibility of optimizing the geometric parameters of the design and achieving the savings in weight and volume of the consumable material.

Keywords: Rigid Pavement; Nonlinearity; Elastic Foundation; Critical Force; Strength; Variable Form.

1. Introduction

Structures in the form of plates on an elastic foundation have found wide application as designs of rigid pavements. Rigid pavement [1-6] differ from flexible pavement [7-10] in existence of a concrete layer in a surface course. Such constructions perceive big loading. One more advantage of such constructions is the speed of mounting and lack of the special mounting equipment. For this reason, such constructions are considered in our work. However, it is more expedient to use shallow shells that have a definite rise in the center. This makes it possible to provide the necessary transverse slope at the manufacturing stage. In addition, to reduce material costs. The work is necessary as it provides a technique for determining the stress-strain state of such structures. The difficulty is that it is necessary to consider a multilayer structure on an elastic foundation.

Different properties of the foundation must be taken into account. Optimization of the shape of the shallow shells on the elastic foundation and the characteristics of the elastic foundation will allow achieving greater effect. Currently, the optimization of rigid pavements is on the way to improving the material. Various plastics and recycling elements are mainly considered. However, savings can be achieved regardless of material. This can only be obtained by changing the shape of the construction. The innovation of the work is to develop a form of rigid pavement that reduces production costs and increases bearing capacity. Taking into account the geometric nonlinearity of the material makes it possible

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to approximate the calculated model to the actual work of the structures.

2. Research Methodology

The construction of rigid pavement loaded with a vertical load can be described by a system of equations [11, 12]:

\[
\begin{aligned}
\frac{1}{Eh} \nabla^2 \varphi + \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \\
+ \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - 2 \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 = 0;
\end{aligned}
\]

\[
\begin{aligned}
\frac{\partial^2 \varphi}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 \varphi}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 \varphi}{\partial x^2} \left( \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial y} \right) + \\
+ 2 \frac{\partial^2 \varphi}{\partial x \partial y} \left( \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) = Z + 2t \nabla^2 w - kw.
\end{aligned}
\]  

(1)

Where;

\( \varphi \): Effort function;

\( w \): Deflection function;

\( F = F(x,y) \): function of the shape of the structures, including the dimensions in the plan a and b, the rise \( f \) and the shape parameter \( \xi \);

\( Z \): Function loading on rigid pavement;

\( K \): Modulus of elastic foundation reaction;

\( t \): Soil shear parameter.

\[
k = \frac{E_0}{1 - \nu_0} \int_0^H \Psi^2 dz, \quad (2)
\]

\[
t = \frac{E_0}{4(1 - \nu_0)} \int_0^H \Psi^2 dz, \quad (3)
\]

\[
E_0 = \frac{E_s}{1 - \nu_s}, \nu_0 = \frac{\nu_s}{1 - \nu_s}, \quad (4)
\]

Where;

\( E_s \) and \( \nu_s \): Elastic module and Poisson's ratio of the soil;

\( H \): Depth of the soil stratum;

\( \Psi(z) \): function of cross distribution displacements.

Function of cross distribution displacements [13, 14]:

\[
\Psi(z) = \frac{shy H - z}{shy H - \frac{a}{H}}.
\]  

(5)

Equivalent stresses in the rigid pavement are found by the fourth stress hypothesis. Stress coefficient can be described by the equation [15-17]:

\[
\sigma = \frac{1}{\sqrt{2}} \left[ (\sigma_1 - \sigma_2) \right]^2 + (\sigma_3 - \sigma_1 \sigma_2 - \sigma_3) + (\sigma_2 - \sigma_1 \sigma_3)]^2.
\]  

(6)
Where:
\[
\sigma_1 = \left[ \frac{6}{t} BB \left( \frac{\partial^2}{\partial x^2} Z_x Z_y + \nu \frac{\partial^2}{\partial y^2} Z_x Z_y \right) \right] + \left[ \frac{A}{t^2} \frac{\partial^2}{\partial y^2} Z_x Z_y t \right] + \left[ \frac{g}{t^2} BB \left( \frac{\partial^3}{\partial x^3} Z_x Z_y + \frac{\partial}{\partial x} Z_x \frac{\partial^2}{\partial y^2} Z_y \right) \right].
\]

(7)

\[
\sigma_2 = \left[ \frac{6}{t} BB \left( \nu \frac{\partial^2}{\partial x^2} Z_x Z_y + \frac{\partial^2}{\partial y^2} Z_x Z_y \right) \right] + \left[ \frac{A}{t^2} \frac{\partial^2}{\partial y^2} Z_x Z_y t \right] + \left[ \frac{g}{t^2} BB \left( \frac{\partial^3}{\partial y^3} Z_x Z_y + \frac{\partial}{\partial y} Z_y \frac{\partial^2}{\partial x^2} Z_x \right) \right],
\]

(8)

\[
\sigma_3 = \left[ \frac{6}{t} (1 - \nu) BB \frac{\partial}{\partial x} Z_x \frac{\partial}{\partial y} Z_y \right] + \left[ \frac{A}{t^2} Z_x \frac{\partial}{\partial y} Z_y t \right].
\]

(9)

\[
\bar{B} = \frac{D}{E h^3}, \quad D = \frac{E h^3}{12(1 - \nu^2)},
\]

(10)

\[
\bar{A} = \frac{A \cdot a^2}{E f_0^5}, \quad \bar{B} = \frac{B \cdot a^2}{f_0^3}, \quad g = \frac{f_0}{a}.
\]

(11)

\(h\): Shell thickness;
\(E\): Elastic module;
\(\nu\): Poisson's ratio;
\(A, B\): Unknown of Bubnov-Galerkin method;
\(Z_x, Z_y\): Beam function.

The construction of rigid pavement can be considered as two-layer (Figure 1). The bottom layer is the foundation; the upper layer is a rigid shallow shell. The modulus of elasticity of the foundation is determined using a Nomogram [18], constructed on the basis of the solution of the elasticity problem for a two-layer system. The transition from a multilayer structure to an equivalent single-layer construction is carried out from the bottom to the top, beginning with the underlying soil.

The distance between the upper surface and the neutral of the layered shell can be determined by the equation [19, 20]:
\[
z_0 = \frac{E_1 h_1^2 + 2E_2 h_1 h_2 + E_2 h_2^2}{2(E_1 h_1 + E_2 h_2)}.
\]

(12)

Where;
\(E_1\) and \(E_2\): Elastic module of the upper and lower layer of rigid pavement.

Rigidity of the two-layer rigid pavement can be described by the equation:
\[
B = \frac{E_1 h_1 + E_2 h_2}{1 - \nu_1 \nu_2},
\]

\[
D = \frac{E_1 (z_0^3 - (z_0 - h_1)^3)}{3(1 - \nu_1 \nu_2)} + \frac{E_2 (h - z_0)^3 + (z_0 - h_1)^3}{3(1 - \nu_1 \nu_2)}.
\]

(13)

Where;
$h_1$ and $h_2$: Thickness of the upper and lower layer of rigid pavement.

![Figure 1. Two-layer rigid pavement](image)

3. Results and Discussion

With this technique it is possible to analyse the stress change in the construction of rigid pavement. Figure 3 depicts the dependence of the stress on the thickness of shell (Figure 3a) and on the form parameter (Figure 3b) for simple support and fixed support (Figure 2) [21].

![Figure 2. Road boundary: a) without road border (simple support); b) road border (fixed support)](image)

![Graph showing the dependence of stress on $x_i$](image)
The graphs show that the stress functions are smooth and unimodal. This simplify the optimum forms of rigid pavement analysis. The stress increases when the values of the thickness decrease and value of the form parameter 0.75. Type of support has an effect on the value of the stress. Fixed supported of construction of rigid pavement (installation of border stones) significantly reduces the stress value.

In order to obtain the optimal shape of the rigid pavement, it is necessary to reduce its volume (the amount of material expended) while maintaining the bearing capacity. The performance characteristics (especially the transverse slope) should be satisfactory.

\[
\begin{cases}
V(\xi, t) \to V_{\text{min}}, \\
\sigma(\xi, t) - \sigma_0 \leq 0.
\end{cases}
\]  \hspace{1cm} (14)

Where:

\( V(\xi, t) \): volume function;

\( \sigma(\xi, t) \): stress function.

On the results of the optimization procedure it is offered to use road plates-shells with median surface:

\[
F(x, y) = f \left[ 1 - \frac{f_1}{f} \left( \frac{x}{a} \right)^{2\xi} - \frac{f_2}{f} \left( \frac{y}{b} \right)^{2\xi} \right]
\]  \hspace{1cm} (15)

Where:

\( a, b \): Dimensions in the plan;

\( f \): Rise;

\( \xi \): Shape parameter;

\( f \): Rise of arch.

Optimum value of parameters: \( a = 1.5 \text{ m}, \ b = 3.5 \text{ m}, \ f = 0.15 \text{ m}, \ f_1 = 0.15 \text{ m}, \ f_2 = 0, \ \xi = 0.73 \).

The optimization algorithm is built on a combination of techniques gradient method and random search method. Penalty functions method of constrained function minimization is used [22].

The optimization algorithm shows a possibility of economy of material to 20%. It is reached due to rational distribution of thickness, a ratio of layers thickness and a form of rigid pavement.
Values of parameters at which volume or value of stresses is reduced can be selected. The specified function uses all design shape parameters that can be changed for designing process needs. The specified numerical values provide a necessary lateral slope and consider convenience of mounting of constructions.

4. Conclusion

This methodology can be applied to determine stresses for rigid pavement of variable shape of the middle surface, thickness of layers and the characteristics of the elastic foundation. In some problems, the numerical method provides greater accuracy and adaptability compared with the finite element method. The formation way of the elastic foundation allows modeling the spreading layers with various characteristics. Use of two-layer model allows considering of a surface course and base course of road pavement designing. The graphs show the patterns of change of the stress of rigid pavement when changing the characteristics. Findings shows the possibility of optimizing the geometric parameters of the design and achieving the savings in weight and volume of the consumable material. The optimization algorithm shows a possibility of economy of material. The results of design optimization are given. Shows an example of specific design parameter values that minimize the amount of material consumed.

5. Conflicts of Interest

The authors declare no conflict of interest.

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