Remarks on Adjoint QCD with \( k \) Flavors, \( k \geq 2 \).

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**Abstract**

I summarize what we know of adjoint QCD. Some observations (albeit very simple) are new.

**Introduction**

Currently we observe a certain revival of interest to QCD with fermions in the adjoint representation of the SU(\( N \)) gauge group, the so called “adjoint quarks.” This is due to a provocative claim [1] that this theory (to be referred to as adjoint QCD, or AQCD) with more than one flavor (\( k \geq 2 \)), being non-supersymmetric at the Lagrangian level, develops a supersymmetric spectrum of color-singlet hadrons at \( N = \infty \), possibly, with the exception of a few low-lying states. Supersymmetric spectrum is defined as follows [1]: each color-singlet hadron with integer spin (boson) is accompanied by a hadron with half-integer spin (fermion) of exactly the same mass \( M \) (in the limit \( N = \infty \)). The overall number of bosonic color-singlet degrees of freedom at each level \( M \) matches that of fermionic degrees of freedom, with a possible exception of several states whose number does not grow with \( M \).

I revisit adjoint QCD with the goal to summarize what we know about AQCD with \( k \geq 2 \), to see whether or not such scenario is possible. No indications on infrared (IR) large-\( N \) “accidental” supersymmetry are detected. Moreover, at \( k = 5 \) adjoint QCD is in the conformal regime in the IR, with small anomalous dimensions, and the number of the fermion degrees of freedom certainly does not match the number of the boson degrees of freedom. Most probably, the conformality extends to \( k = 4 \). At \( k = 2 \) one observes
a certain symmetry breaking pattern for a continuous chiral symmetry, with
two massless “pions” emerging as a result of this breaking. At the same
time, massless pions certainly cannot appear at weak coupling. This implies
a phase transition in passage from weak to strong coupling.

**Coupling constant**

The Lagrangian of AQCD has the form

\[
\mathcal{L} = -\frac{1}{4g^2} G_{\mu\nu}^a G^{\mu\nu,a} + \sum_k \left( \bar{\lambda}_a^\alpha \right)_k i D_{\dot{\alpha}_\alpha} \left( \lambda_a^\alpha \right)^k
\]

where \( k \) can be 1, 2, 3, 4 or 5. If \( k > 5 \), asymptotic freedom is lost, the theory
becomes IR free and uninteresting. The indices \( \alpha, \dot{\alpha} \) in (1) are Lorentz-
spinorial, \( a \) is the index of the adjoint representation of SU(\( N \)), and \( g^2 \) is the
gauge coupling,

\[
\alpha \equiv \frac{g^2}{4\pi}
\]

If I write, say, \( \lambda \) without the adjoint index, this will mean

\[
\lambda \equiv \lambda^a T^a
\]

where \( T^a \) stand for the generators of SU(\( N \)) in the fundamental representa-
tion.

If \( k = 1 \) the Lagrangian (1) represents \( \mathcal{N} = 1 \) supersymmetric Yang-Mills
(SYM) theory. Needless to say that supersymmetry is exact not only at the
Lagrangian level, but in the hadronic spectrum and scattering amplitudes
too. We will discuss \( k = 2 \) or larger.

The two-loop \( \beta \) function of AQCD with arbitrary \( k \) can be extracted from
\[2\],

\[
\beta = \mu \frac{\partial}{\partial \mu} \alpha(\mu) \equiv -\beta_0 \frac{\alpha^2}{2\pi} + \beta_1 \frac{\alpha^3}{2(2\pi)^2},
\]

where

\[
\beta_0 = \left( \frac{11}{3} - \frac{2}{3} k \right) N, \quad \beta_1 = \left( -\frac{34}{3} + \frac{16}{3} k \right) N^2.
\]

At \( k = 5 \) AQCD, still being asymptotically free, develops an IR fixed
point in the perturbative domain, at

\[
\frac{N\alpha_*}{4\pi} = \frac{1}{46}.
\]
Indeed, at $k = 5$ the value of $\beta_0$ is abnormally small, while the value of $\beta_1$ is positive and is of the normal order of magnitude. As a result, the $\beta$ function (2) has a reliable zero at a small value of $\alpha$, see (4). The latter is smaller than the value of the QCD coupling constant $\alpha_s$ at the Z peak, where perturbation theory is applicable beyond any doubt. Thus, here we encounter the regime first described by [3, 4] which goes under the name of the Banks-Zaks phenomenon. The above IR fixed point implies conformally symmetric theory in the infrared, with small anomalous dimensions. Neither confinement nor spontaneous breaking of the chiral symmetry ($\chi_{SB}$, see below) are implemented.

The same probably applies to four flavors, $k = 4$, too. Indeed, the would-be position of the zero of the $\beta$ function is given below, in the upper line,

$$\frac{N\alpha_\ast}{4\pi} = \begin{cases} 
\frac{1}{10}, & k = 4, \\
\frac{5}{14}, & k = 3,
\end{cases}$$

(5)

while nothing can be said about the existence (or nonexistence) of the IR fixed point at $k = 3$. At $k = 2$ the coefficient $\beta_2$ becomes negative.

AQCD with $k = 1, 2$ is believed to be confining. Thus, in our discussion we focus on these two cases.

1Beyond two loops the coefficients of the $\beta$ function are scheme-dependent. In the first and second loop only planar graphs contribute. This is not the case in higher orders, and e.g. at the four-loop order the right-hand side would contain $1/N^2$ corrections. Calculations including the third and fourth loops in a reasonable scheme performed in [5] indicate that the actual value of $\frac{N\alpha_\ast}{4\pi}$ for $k = 4$ is in fact somewhat lower than that indicated in [3] enhancing the probability that $k = 4$ belongs to the conformal window. The same conclusion is supported by lattice data [6].

It would be very interesting to reliably determine whether the left edge of the conformal window lies at $k = 3$ or $k = 4$. Note that in the model at hand, if we limit ourselves to the first and second loops, both edges of the conformal window depend only on the number of flavors, rather than on the ratio $N_f/N_c$ as in Seiberg’s supersymmetric conformal window [7]. It is worth adding that in three-color QCD with $N_f = 15$ widely believed to be conformal, $\frac{N\alpha_\ast}{4\pi} = \frac{2}{15} \sim \frac{1}{10}$. In the Intriligator-Seiberg-Shenker model (supersymmetric SU(2) Yang-Mills with the chiral quark field in the 3/2 representation of SU(2)), which was argued to be conformal [8, 9], $\frac{N\alpha_\ast}{4\pi} = \frac{2}{15} \sim \frac{1}{10}$ [9].
Chiral properties

Let us start from SYM theory which is a particular case of AQCD with \( k = 1 \). The only fermion current in this theory is

\[
R^\mu = \bar{\lambda}_a^\alpha (\bar{\sigma}^\mu)^{\dot{\alpha}\alpha} \lambda^a_\alpha .
\]

(6)

Classically it is conserved; however, at the quantum level it is internally anomalous,

\[
\partial_\mu R^\mu = \frac{N}{16\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu,a} .
\]

Moreover, \( \partial_\mu R^\mu \neq 0 \) even in the limit \( N \to \infty \), in contradistinction with the singlet fermion current in QCD.

Thus, in \( k = 1 \) AQCD there is no conserved fermion charge. However, the conserved operator \((-1)^F\), distinguishing fermions from bosons can be introduced. Indeed, the classical conservation of the current (6) leaves a remnant in the form of the discrete \( Z_{2N} \) symmetry which is dynamically broken down \([10, 11]\) to \( Z_2 \) by the gluino condensate \( \langle \text{Tr}\lambda^2 \rangle \neq 0 \) (for more details see \([12]\)). The existence of this operator is needed e.g. for the Witten index determination \([10]\). Considering a given color-singlet (hadronic) state, say, bosonic, one cannot say, however, whether it contains zero, two, four, six and so on “fermion quanta.” In this sense, unlike QCD in which the flavor charge is well defined, the notion of a “constituent” quark in SYM theory is meaningless. The operators \( \text{Tr}\lambda^2 \) and \( \text{Tr} G^2 \) have the same value of \((-1)^F\), while the operators \( \text{Tr} \lambda^2 \) and \( \text{Tr} \lambda G \) have the values +1 and −1, respectively, and produce degenerate spin-0 and spin-1/2 states.

In \( k = 2 \) AQCD the chiral symmetry of the Lagrangian is \( \text{SU}(2) \). Indeed, the Lagrangian stays invariant under arbitrary rotations

\[
\left( \begin{array}{c} \lambda^1 \\ \lambda^2 \end{array} \right) \to U \left( \begin{array}{c} \lambda^1 \\ \lambda^2 \end{array} \right), \quad U \in \text{SU}(2),
\]

(7)

where the superscripts 1 and 2 denote the value of the flavor index \( k \). In addition, at the classical level there exists the \( U(1) \) symmetry generated by the current (6) with summation over two flavors. This symmetry is anomalous; there is no need to consider it here.\(^2\)

\(^2\)As in supersymmetric Yang-Mills theory (or, which is the same, \( k = 1 \) AQCD), the anomalous current \( R^\mu = (\bar{\lambda}_1^a \bar{\sigma}^\mu \lambda^a_1 + \bar{\lambda}_2^a \bar{\sigma}^\mu \lambda^a_2) \) still can be used to define an opera-
Needless to say, the chiral symmetry (7) is spontaneously broken. The pattern of this breaking can be obtained from the ’t Hooft matching [14]. Assuming confinement and the large-$N$ limit, as it had been done in [15], one can conclude that

$$\text{SU}(2) \rightarrow \text{U}(1).$$

(8)

The corresponding analysis in AQCD with arbitrary $k$ was carried out in [16]. The order parameter triggering the above $\chi$SB can be chosen as follows:

$$\langle \text{Tr} \lambda^1 \lambda^2 \rangle + (1 \leftrightarrow 2) \neq 0$$

(9)

with the same convention on superscripts as in Eq. (7). Then the conserved unbroken U(1) current has the form

$$j^\mu_{\text{U}(1)} = 2 \left( \text{Tr} \bar{\lambda}_1 \sigma^\mu \lambda^1 - \text{Tr} \bar{\lambda}_2 \sigma^\mu \lambda^2 \right).$$

(10)

It generates rotations of two Weyl spinors $\lambda^{1,2}$ in the opposite direction. We could have rewritten $k = 2$ AQCD as a gauge theory of a single adjoint Dirac spinor. Then the current (10) will obvious become the vector fermion current. Thus, in this theory the fermion charge $F$ is perfectly defined,

$$F = \int d^3 x \ j^0_{\text{U}(1)},$$

(11)

unlike the SYM theory.

The $\chi$SB pattern (8) gives rise to two massless “pions” coupled to two broken currents; the U(1) charges of these pions are +2 and −2, respectively.

Note that $k = 2$ AQCD considered on the small-$L$ cylinder as in [1] (i.e. at weak coupling) can never produce massless pions. Thus, as we change $L$ from $L \ll \Lambda^{-1}$ to $L \gg \Lambda^{-1}$ (strong coupling) a phase transition is inevitable.

The existence of the conserved charge $F$ in $k = 2$ AQCD splits the Hilbert space of physical states (hadrons) into sectors with the given value of $F$,

$$F = 0, \pm 1, \pm 2, ...$$

The eigenvalue of $(-1)^f$ is 1 for any operator with the odd number of the $\lambda$ and $\bar{\lambda}$ fields and −1 for any operator with the even number of the $\lambda$ and $\bar{\lambda}$ fields. Moreover, $k = 2$ AQCD supports topologically stable solitons with mass scaling as $N^2$ [13], see below. Topological stability is due to the existence of a nontrivial Hopf invariant in the Skyrme-Faddeev model. All “normal” hadrons, with mass $O(N^0)$, are characterized by $(-1)^f(-1)^F = 1$, while for the Skyrmion states with mass $O(N^2)$ the value of $(-1)^f(-1)^F = -1$.

3For $k = 5$ (and, probably, $k = 4$) the ’t Hooft matching should be trivial since there is no confinement, see above.
This allows for a meaningful introduction of a “constituent” adjoint quark. Producing two extra adjoint quarks moves us from the $F = 0$ sector to the $F = 2$ sector, from $F = 1$ to $F = 3$, and so on. The operators $\text{Tr} \lambda^1 \lambda^1$ and $\text{Tr} G^2$ become distinguishable, and so are $\text{Tr} \bar{\lambda}^2 \lambda^1$ and $\text{Tr} G^2$. At the same time $\text{Tr} \lambda^2 \lambda^1$ and $\text{Tr} G^2$ both have vacuum quantum numbers.

Returning to the chiral properties, I should mention that the chiral symmetry breaking and the emergence of two pions have an impact not only on the low-lying states, but on high excitations too. The linear realization of the chiral symmetry is not restored in the highly excited states, unless the Regge trajectories intersect, which is unlikely [17]. If so, the Goldberger-Treiman relation should take place for (infinitely many) spin-$1/2$ states. In addition, the mesons and baryons forming chiral pairs must be split (non-degenerate in masses).

In the hadronic spectrum there are infinitely many sectors characterized by $F = 0, \pm 1, \pm 2, \ldots$. At $N = \infty$ not only the lowest-lying states in each sector (these sectors extend all the way up to the Skyrmion sector) are stable, but so are all excitations.

**Multiquark states in AQCD**

In multicolor QCD (i.e. in the 't Hooft limit [18]) exotic mesons with more than one quark-antiquark pair are not bound and split into a number of noninteracting nonexotic mesons, each of which contains exactly one $q\bar{q}$ pair connected by a gluon string [18, 19]. The string does not break at $N = \infty$.

This is *not* the case in $k = 2$ AQCD. It is easy to see that unbreakable color-singlet states with as many quarks as one wants do exist. For instance, consider the string operator

$$\text{Tr} \left[ \lambda^1(x_1) \exp \left( i \int_{x_2}^{x_3} dx_\mu A^\mu(x) \right) \bar{\lambda}_2(x_2) \exp \left( i \int_{x_2}^{x_4} dx_\mu A^\mu(x) \right) \lambda^1(x_3) \exp \left( i \int_{x_3}^{x_4} dx_\mu A^\mu(x) \right) \right] (12)$$

(see Fig. 1). It has $F = 4$ and, at the same time, cannot be split into a product of two color-singlet operators with $F = 2$ each. I will return to this point later in the context of planar equivalence [20, 21].

In pure Yang-Mills theory ($k = 0$) all color-singlet states are represented by excitations of a closed string, which can be written as the following Wilson
Figure 1: Graphic representation for the integration contour in the operator (12).

operator:

\[ W_C = \frac{1}{N} \text{Tr} \exp \left( i \int_C dx_\mu A_\mu(x) \right) \]  

(13)

where the integration contour can be chosen, for instance, as in Fig. 1. In QCD the “meson” string must be open, with (anti)quarks attached to its endpoints (Fig. 2). Glueballs are still produced by closed strings.

Figure 2: Open string corresponding to mesons in QCD with fundamental quarks.

Since the notion of the constituent quark is well-defined in \( k = 2 \) AQCD, it is natural to expect that the mass of a given hadron for not too large and fixed angular momentum \( L \) (i.e. \( L \ll F \)) depends on \( F \) as follows

\[ M_F = \Lambda (a + bF), \]  

(14)

where \( a \) and \( b \) are \( F \) independent constants. At the very least, I would say that in the large-\( F \) limit \( \frac{\partial M_F}{\partial F} = \text{const} \) is practically unavoidable. The color-singlet hadrons in the model at hand resemble nuclei in conventional QCD.
Planar equivalence

Because of the planar equivalence [20, 21] one can relate AQCD to its orientifold daughter: Yang-Mills theory with Dirac fermions $\psi$, each in the two-index antisymmetric representation of the color $SU(N)$ group,

$$\lambda_{\alpha,j}^i \leftrightarrow \{\chi_{\alpha}^{ij}, \eta_{\alpha,ij}\}, \quad \psi^{ij} = \{\chi_{\alpha}^{ij}, \bar{\eta}_{\alpha,ij}\}. \quad (15)$$

For $k = 2$ it is convenient to define two Dirac fermions of the daughter theory as follows:

$$\psi^1 = \begin{pmatrix} \chi^1 \\ \bar{\eta}_2 \end{pmatrix}, \quad \psi^2 = \begin{pmatrix} \chi^2 \\ \bar{\eta}_1 \end{pmatrix}. \quad (16)$$

Then

$$\text{Tr} \lambda^1 \lambda^2 + (1 \leftrightarrow 2) + \text{h.c.} \rightarrow \bar{\psi}_1 \psi^1 + \bar{\psi}_2 \psi^2. \quad (17)$$

The conserved $U(1)$ current from Eq. (10) takes the form

$$j_{U(1)}^\mu = \left( \bar{\psi}_1 \gamma^\mu \psi^1 - \bar{\psi}_2 \gamma^\mu \psi^2 \right). \quad (18)$$

The equivalence holds in the common sector. At $N = \infty$ three domains of the cylinder depicted in Fig. 3 become dynamically disconnected. To pass from $k = 2$ AQCD to the daughter theory one must cut out the middle sector, flip it around the vertical axis, and glue back. The opposite arrows
indicating color flow on the fermion lines become aligned. In this passage

terms of the relative order $1/N$ must be ignored.

As was mentioned in [20, 21], the two-index antisymmetric Dirac fermions
present a different way of the large-$N$ extension of bonafide QCD, sometimes
referred to as the ASV continuation (different compared to ’t Hooft’s con-
tinuation [18], with fermions in the fundamental for any $N$). At $N = 3$ both
are equivalent, since at $N = 3$ the two-index antisymmetric quark is exactly
the same as the fundamental antiquark. The ’t Hooft line of reasoning pre-
dicts that exotic (multiquark) mesons do not exist at $N = \infty$. The ASV
procedure, with the Wilson operator defined in (12) and conserved fermion
number $F$, yields stable mesons with arbitrary $F$ in the limit $N = \infty$.

Certainly, $N = \infty$ is not the same as $N = 3$. However, in other as-
pects of phenomenology both alternative continuations – that of ’t Hooft
and the ASV procedure – lead to results of comparable quality [22], even for
baryons. One can view this fact as an indication that multicolor QCD gen-
erally speaking does not disfavor exotic or cryptoexotic (four-quark) mesons
in bonafide QCD. They are likely to be implemented in the form of a bound
diquark-antidiquark pair [23, 24, 25]. Needless to say, there are no traces of
supersymmetry in multicolor QCD.

Regge trajectories

Noncritical string theory describing a “real” pure Yang-Mills theory in four
dimensions does not exist, let alone Yang-Mills theory with fermions, such as
SYM theory or QCD with massless quarks and $\chi$SB. Therefore, exact pre-
dictions for the Regge trajectories are unavailable. There are all reasons to
hope, however, that for large excitation numbers the quasiclassical approxi-
mation for the Regge trajectories must work well. Quasiclassical calculations
reproduce the famous Chew-Frautschi formula [26], with the linear depen-
dence of the meson and baryon masses squared on the angular momentum
$L$ and the excitation number $n$ (the so-called primary and daughter Regge
trajectories). In fact, the linear dependencies are clearly seen in experiment
even for the lowest-lying states (see e.g. [17]) in all cases where data are
available, with the exception of the Pomeron trajectory.

If Eq. (14) is valid I do not expect linear Regge trajectories in AQCD for
high-lying states, $F, L \gg 1$ (but $L \lesssim F$) because of the interplay of the linear
in mass dependence in (14) and quadratic in mass in the Chew-Frautschi
A parallel (perhaps, rather remote)

Equation (12) with $\lambda$ insertions in the closed loop, non-factorizable at $N = \infty$, resembles the construction worked out in [27,28] where confined monopoles were identified as kinks in the string world-sheet theory. Then the $\lambda$ insertion in the supersymmetric case ($k = 1$) can be viewed as a massless “kink” while at $k = 2$ the kinks acquire a mass.

Conclusion

Physics of AQCD is such that the claim [1] seems implausible, although this conclusion – I must admit – is not at the level of a mathematical theorem.

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