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The Gravity Field Variation Caused by Inner Core Super Rotation

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Abstract Due to the super rotation of the Earth’s inner core, the tilted figure axis of the inner core would progress with respect to the mantle and thus cause the variation of the Earth’s external gravity field. This paper improves the present model of the gravity field variation caused by the inner core super rotation. Under the assumption that the inner core is a stratifying ellipsoid whose density function is fitted out from PREM and the super rotation rate is 0.27~0.53°/yr, calculations show that the global temporal variations on the Earth’s surface have a maximum value of about $0.79\sim1.54\times10^{-3}\ \mu\text{Gal}$ and a global average intensity of about $0.45\sim0.89\times10^{-3}\ \mu\text{Gal}$ in the whole year of 2007, which is beyond the accuracy of the present gravimetry and even the super conducting gravimeter data. However, both the gravity variations at Beijing and Wuhan vary like sine variables with maximal variations around $0.33\ \mu\text{Gal}$ and $0.29\ \mu\text{Gal}$, respectively, in one cycle. Thus, continuous gravity measurements for one or two decades might be able to detect the differential motion of the inner core.

Keywords inner core; super rotation; gravity variation

Introduction

Scientists paid more extensive attention to the structure and motion of the inner core after the German scientist Lehmann[1] found the existence of a solid inner core inside the Earth’s fluid outer core. The solid inner core approximates to a rotationally symmetric ellipsoid with a flattening of about $1/400$[2], and it consists of nearly pure iron with a density of about $12.763 \ 60 \ \text{g/cm}^3$ at the inner core boundary (ICB) and $13.088 \ 48 \ \text{g/cm}^3$ at the center[3].

Since located in the liquid outer core which has a low viscosity close to water[4], the inner core might have a differential motion relative to the mantle[5]. According to numerical simulations of the three-dimensional self-consistent convective geodynamo which aims to explain the origin of geomagnetic field, Glatzmaier and Robert[6] predicted the inner core is rotating $2\sim3\ \degree/\text{yr}$ faster than the mantle. However, Kuang & Bloxham[7] adopted different inner boundary conditions in the numerical simulations of the geodynamo, and then concluded that the inner core rotates sometimes eastwards and sometimes westwards with respect to the mantle. Song & Richards[8] analyzed the differential travel time residual between the phase PKP(DF) and PKP(BC) covering a period of about 30 years, and first discovered the inner core super rotation (ICSR) at a rate of about $1.1\ \degree/\text{yr}$. Based on a lateral anisotropy gradient within the inner core rather than a tilted symmetry axis, Creager[9] suggested that the super rotation would not exceed $0.3\ \degree/\text{yr}$. Sharrock & Woodhouse[10] investigated the time dependent inner core structure by the analysis of
the free oscillation spectra, and found some clues supporting a westward inner core differential rotation (i.e. the inner core rotates slower than the mantle). Souriau et al.\[11\], Souriau et al.\[12\] and Poupinet et al.\[13\] even doubted the existence of the inner core differential rotation. Hence, the differential rotation becomes one of the most controversial subjects in geoscience. But in general, more evidence seem to support the super rotation of the inner core[14]. Especially since recent results from earthquake waveform doublets strongly suggest that the inner core rotates 0.27−0.53°/yr eastward with respect to the mantle[15].

Besides, the inner core seems to be a little tilted relative to the mantle which explains the decadal variation of the polar motion[16,17]. Due to the ICSR, the tilted inner core figure axis would progress relative to the mantle and give rise to a variation of the gravity field. Shen et al. [18] obtained that the gravity caused by the ICSR has an annual variation of about 0.37 μGal. However, they assumed that the inner core is uniform and the figure axis of the inner core deviates 10° from that of the mantle. A uniform inner core model doesn’t coincide with the fact that the inner core density increases with the depth, as described by PREM (Preliminary reference Earth model[3]). Furthermore, recent studies[16,17,19] suggest that the inner core tilt should not exceed 1°. Consequently, a more accurate model for the gravity variation caused by the ICSR is significant for providing a gravimetric method to detect or even determine the ICSR.

In this paper, the inner core, whose figure axis has a small tilt of 0.07°[17], is treated as a rotationally symmetric equi-elliptically stratified-uniform ellipsoid[20,21], and its density distribution is assumed to be a continuous function determined from PREM. Thus a more accurate model for the gravity variation caused by the ICSR is provided. Furthermore, the possibility of detecting the ICSR by using gravity data is investigated.

1 Theoretical model for gravity variation caused by the ICSR

1.1 Mechanism of gravity variation caused by the ICSR

The mechanism of gravity variation caused by the ICSR is described in \[18,20,21\]. In order to precisely determine the external gravitational field of the inner core, here we would first introduce the rotationally symmetric equi-elliptically stratified-uniform ellipsoid[20,21]. This ellipsoid can be divided into \(N\) ellipsoidal layers which have a common center and the same thickness \(d\). In addition, the inner and outer surfaces of all the layers have the same flattening. Every layer is homogeneous but different layers might have different densities. When \(N \rightarrow \infty\) or \(d \rightarrow 0\), the density of this ellipsoid can be expressed as a continuous function, and then we obtain a so-called “rotationally symmetric equi-elliptically stratified-uniform ellipsoid”.

Based on the above discussion, the following model is put forward (Fig.1).

1) The solid inner core, with a flattening \(f = 1/412.88\)[2], is a rotationally symmetric equi-elliptically stratified-uniform ellipsoid, whose density function is fitted out from PREM.

2) The inner core’s rotational axis coincides with that of the Earth, but its figure axis has a 0.07° tilt relative to the mantle[17].

3) The inner core rotates 0.27−0.53°/yr faster than the mantle[15] and thus its figure axis processes eastward around the \(z\) axis with the same rate.

4) The gravity variation caused by the inner core is actually the temporal influence of the equivalent inner core ellipsoid on the gravity field\[18,20,21\]. The equivalent inner core ellipsoid has the same figure and motion with those of the inner core, but its density (referred to as the equivalent density) is equal to the difference between the density of the inner core and that of the outer core at the ICB.

\[\text{Fig.1 Model for the inner core super rotation}\]
1.2 Gravitational field of the inner core

PREM has provided a discrete density model of the inner core with a spherical approximation. In fact, the inner core’s density changes gradually with depth and a continuous function fitted from PREM can describe the inner core’s density distribution better, that is[20,21]

where $r$ is the mean radius of the inner core and its unit is kilometer (km).

Assuming that the inner core is a rotationally symmetric equi-elliptically stratified-uniform ellipsoid (so is the equivalent inner core ellipsoid) with $a_0$ and $b_0$ as the lengths of its semi-major and minor axes, respectively, one gets

\[ b = \frac{r_0}{\sqrt{1 + \epsilon^2}}, \quad b \in [0, b_0] \]  

(2)

where $\epsilon' = \sqrt{a^2 - b^2}/b = \sqrt{2f - f^3}(1 - f)$ is the secondary eccentricity of the inner core and $a \in [0, a_0]$. Then the density function of the inner core can be expressed as

\[
\begin{align*}
\rho(b) = & 13 \ 088.48 - 1.599 \times 6 \times 10^{-8} b (1 + e^2)^{1/3} - 2.177 \times 10^{-10} b^2 (1 + e^2)^{1/3} (\text{kg/m}^3), \quad b \in [0, b_0] \\
& \times 0.6 \times 10^{-10} \ b^2 (1 + e^2)^{1/3} \ (\text{kg/m}^3), \quad b \in [0, b_0] \\
\end{align*}
\]

The equivalent density $\rho'$ of the equivalent inner core ellipsoid is the difference between the density of the inner core and that of the outer core at ICB, i.e.

\[ \rho'(b) = \rho(b) - 2 \ 166.30 (\text{kg/m}^3), \quad b \in [0, b_0] \]

(5)

In a coordinate fixed to the inner core figure axes (expressed as $o-x'y'z'$, and called inner core coordinate for short), the inner core’s external gravitational potential for a field point can be expressed as[20,21]

\[
V = \int_0^{b_0} f(b)b \, db
\]

where

\[
f(b) = \frac{\pi \rho Gb(1 + e^2)}{b_0}
\]

\[
\left\{ 4 \arctan \left[ \frac{A}{\sqrt{2}r \cos \theta} + \sqrt{2r \cos \theta} \left[ \frac{2b^2e^2 + r^2 (3 \cos^2 \theta - 1)}{B} \right] \right] - \frac{C}{A + 2r \cos^2 \theta + A^2} \left[ \frac{1}{A} - \frac{2A}{2r \cos^2 \theta + A^2} \right] \right\} \theta \neq \pi/2
\]  

(7a)

\[
\left\{ 2b^2e^2 (D + 2 \arctan D) - 2D r^2 \cos^2 \theta - r^2 \sin^2 \theta \cdot \right\}
\]

\[
\left[ \frac{1}{D} \left( \frac{2b^4 e^2}{r^4} - \frac{b^2 e^2}{r^2} \right) + \frac{b^2 e^2}{r^2 D} \left( \frac{2b^4 e^2}{r^4} - 1 \right) \right] \right\}, \theta = \pi/2
\]

(7b)

Replacing $\rho$ in Eq.(7) with $\rho'$, one gets the external gravitational potential $V'$ of the equivalent inner core ellipsoid, and then the gravitational force

\[
g' = \nabla V' = \left( \frac{\partial V'}{\partial r}, \frac{1}{r} \frac{\partial V'}{\partial \theta}, \frac{1}{r \sin \theta} \frac{\partial V'}{\partial \lambda} \right)
\]

\[
= (g'_r, g'_\theta, g'_\lambda)
\]

(9)

where $g'_r, g'_\theta$ and $g'_\lambda$ denote the components of the gravitational force in $r, \theta$ and $\lambda$ directions, respectively. Then, the external gravitational fields of the inner core and the equivalent inner core ellipsoid are established.

1.3 Gravity field variation caused by the super rotation

The external gravitational field of the equivalent inner core ellipsoid has been obtained in the inner core coordinate in section 2.2. In practice, however, the gravity field is usually studied in the Earth-fixed coordinate $o-x'y'z'$ (the corresponding spherical coordinate is denoted by $(r_E, \theta_E, \lambda_E)$). Hence, it is necessary to express Eq.(9) in the Earth-fixed coordinate. The coordinate transformation is expressed as:

\[
\begin{align*}
\begin{cases}
r = r_E \\
\theta = \arccos[\cos \theta_E \cos \theta_0 + \sin \theta_E \sin \theta_0 \cos(\lambda_E - \omega t)] \\
\lambda = \arctan \frac{\sin \theta_E \sin(\lambda_E - \omega t)}{\cos \theta_0 \sin(\lambda_E - \omega t) - \cos \theta_0 \sin \theta_E}
\end{cases}
\end{align*}
\]

(10)
where $\omega$ is the angular velocity of the super rotation, and $\omega t$ is the super rotation angular.

Noting that
\[
\begin{aligned}
\frac{\partial V_e}{\partial \theta_e} &= \frac{\partial V}{\partial \theta} = \frac{\partial V}{\partial \theta} = g_e', \\
\frac{\partial V_e}{\partial \lambda_e} &= \frac{1}{r_e} \frac{\partial V}{\partial \theta} \frac{\partial \theta_e}{\partial \lambda_e} + \frac{\partial V}{\partial \lambda} \frac{\partial \lambda_e}{\partial \theta_e} \\
&= \frac{1}{r_e} \frac{\partial V}{\partial \theta} \frac{\partial \theta_e}{\partial \lambda_e} + \frac{\partial V}{\partial \lambda} \frac{\partial \lambda_e}{\partial \theta_e} g_e', \\
\frac{\partial V_e}{\partial \phi_e} &= \frac{1}{r_e \sin \theta_e} \frac{\partial V}{\partial \theta} \frac{\partial \theta_e}{\partial \phi_e} = \frac{1}{r_e \sin \theta_e} \left( \frac{\partial V}{\partial \theta} \frac{\partial \theta_e}{\partial \lambda_e} + \frac{\partial V}{\partial \lambda} \frac{\partial \lambda_e}{\partial \theta_e} \right) \\
&= \frac{1}{r_e \sin \theta_e} \frac{\partial V}{\partial \theta} \frac{\partial \theta_e}{\partial \lambda_e} + \frac{1}{r_e \sin \theta_e} \frac{\partial \lambda_e}{\partial \theta_e} g_e'.
\end{aligned}
\]

the external gravitational field generated by the equivalent inner core ellipsoid in the Earth-fixed coordinate can be expressed as
\[
\begin{aligned}
g_{E_e} &= g_e', \\
g_{\theta_e} &= \frac{\cos \theta_e \sin \theta_e - \sin \theta_e \cos \theta_e \cos (\lambda_e - \omega t)}{\sqrt{1 - [\cos \theta_e \cos \theta_e + \sin \theta_e \sin \theta_e \cos (\lambda_e - \omega t)]^2}} g_e', \\
g_{\lambda_e} &= \frac{\sin \theta_e \sin \theta_e (\lambda_e - \omega t)}{\sqrt{1 - [\cos \theta_e \cos \theta_e + \sin \theta_e \sin \theta_e \cos (\lambda_e - \omega t)]^2}} g_e'.
\end{aligned}
\]

with the modulus of the gravitational force
\[
g_E = \sqrt{g_{E_e}^2 + g_{\theta_e}^2 + g_{\lambda_e}^2}. \tag{14}
\]

Assuming that the inner core figure axis is located in the prime meridian plane (in the Earth-fixed coordinate) at the initial time $t = 0$ (so the super rotation angle is 0), the corresponding gravitational force of the equivalent inner core ellipsoid is $g_{E_e}(r_e, \theta_e, \lambda_e)$; at the time $t$, the super rotation angle changes to $\omega t$ and the corresponding gravitational force is $g_{E_e}(r_e, \theta_e, \lambda_e, t)$. Hence, the gravity variation $\Delta g_{E_e}(r_e, \theta_e, \lambda_e, t)$ caused by the super rotation can be expressed as
\[
\Delta g_{E_e}(r_e, \theta_e, \lambda_e, t) = g_{E_e}(r_e, \theta_e, \lambda_e) - g_{E_e}(r_e, \theta_e, \lambda_e, t) \tag{15}
\]

## 2 Numerical results and analysis

All the discrete density values of the inner core given by PREM are used to determine the inner core density function (1). Other geophysical parameters used in the present study are listed in Table 1.

| Parameter | Value |
|-----------|-------|
| Gravitational constant | $G = 6.672 ~ 10^{-11}$ m$^3$ s$^{-2}$ kg$^{-1}$ |
| Flattening of inner core | $f = 1/412.88$ |
| Mean radius of inner core | $r_0 = 1221.5$ km |
| Density of inner core at ICB | $\rho_{inn} = 12763.60$ kg/m$^3$ |
| Density of outer core at ICB | $\rho_{out} = 12763.60$ kg/m$^3$ |
| Longitude of inner core’s figure axis (1980) | $280^\circ$ |
| Tilt of inner core figure axes | $0.07^\circ$ |
| Rate of inner core super rotation | $0.27 ~ 0.53^\circ$/yr |

The variation of the gravity field on the ground is of great significance since the gravity observations might provide another clue besides seismic observations to determine the differential rotation of the inner core. Here we will first work out the magnitude and distribution of the variation, and then give an estimation of the temporal variation ratio of the second order of the fully-normalized spherical harmonic coefficients.

The orientation of the inner core figure axis was $280^\circ$E in 1980[16], under the assumption that the super rotation rate is $0.27~0.53^\circ$/yr[15], and the inner core tilt $0.07^\circ$[17] holds invariable during the periods discussed. Then, only the longitude of the inner core figure axis varies with the super rotation angle, which leads to the corresponding gravity variation. Noting that the inner core’s figure axis pointed to $287.56 ~ 294.84^\circ$E at the beginning of 2007, then the super rotation would be $0.27~0.53^\circ$ at the end of 2007. The gravity variations in this period are described in Table 2.

The present study shows that the gravity variation on the ground is in the order of $10^{-7} ~ 10^{-8}$ μGal (Table 2) and the temporal variation ratios of the second-order fully-normalized spherical harmonic coefficients are around $10^{-13} ~ 10^{-12}$/yr compared with the gravity on ground (which is around 980 Gal).
For a given field point, the gravity variation will naturally have one complete cycle when the super rotation angle reaches 360°. Here, the gravity variations caused by the ICSR at Beijing (39.92°N, 116.46°E) and Wuhan (30.08°N, 114.03°E) are taken as examples and the numerical results are shown in Table 3 (the initial direction of the inner core figure axis is 0°E).

As shown in Table 3, the gravity variations at Beijing and Wuhan are naturally cyclical. The absolute values of gravity variations at Beijing and Wuhan reach the maximal values 0.238691 μGal and 0.204518 μGal when the super rotation angle amounts to 116.46° and 114.03°, respectively. Remembering that the orientation of the inner core figure axis is 280°E in 1980, Table 3 also shows that the gravities at Beijing and Wuhan both have decreasing tendencies in recent years under the impact of the ICSR. Numerical computations show that there was a decrease of about 0.6 ~ 1.3×10⁻³ μGal/yr at Beijing and about 0.6 ~ 1.2×10⁻³ μGal/yr at Wuhan during 1980 ~ 2007, respectively, and the decreasing rate will be larger in the next 200 years.

By reviewing Tables 2~3, the following conclusions can be drawn:

1) Due to the ICSR, the gravity has an annual variation with a magnitude in the order of 10⁻⁴~10⁻³ μGal in most parts but with zero as the mean value. Apparently, even the superconducting gravimeters with accuracy as high as 0.01 μGal, are difficult to detect in the mentioned gravity variation.

2) Globally, the gravity variation caused by the ICSR is closely related to the position (coordinates), and the maximal or minimal gravity variation appears at the position with the co-latitude 46° or 136°. The gravity variation at Beijing is larger than that at Wuhan, that’s because Beijing is closer to the latitude 46° compared with Wuhan.

3 Discussions

Since its discovery, the ICSR is perhaps one of the most controversial subjects in geophysics, partly due to the limited evidence such as seismic observations. In fact, with a unique structure, the inner core’s differential motion (including the inner core super rotation, wobble/nutation, oscillation and even some other unknown behaviors) will give rise to many detectable geo-phenomena. Mathews et al. [2,22] pointed out that the Earth’s rotation would be impacted by the inner core nutation. Buffett [23] and Buffett & Creager[24] declared that the coupling between the inner core and mantle will lead to the decadal variation of the LOD. Greiner-Mai & Barthelmes[16], and Dumberry & Bloxham[17] showed that the inner core wobble can influence polar motion. Glatzmaier and Roberts [6], Kuang and Bloxham[7] and Buffett[4] proposed that the inner core motion play a key role in the geodynamo which excites and sustains the geomagnetic field. The inner core motion might be inversely determined through relevant observations when the above geophysical progresses are properly modeled.

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