Inverse problem for transport equation of ultra-high energy cosmic rays

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Abstract. Propagation of ultra-high energy nuclei in expanding universe filled with background radiation is considered. We developed numerical code for solution of inverse problem for cosmic-ray transport equation that allows determination of source spectrum from the spectrum observed at the Earth. The injection spectra of protons and iron nuclei in extragalactic sources are found assuming that these species dominate in the source composition. Data from the Auger and Telescope Array experiments are used to illustrate the method.

1. Introduction
The origin of cosmic rays with energies $E > 10^{18}$ eV remains a key problem of cosmic ray astrophysics. The observed suppression of the cosmic ray flux at energies above $\sim 5 \times 10^{19}$ eV confirms the presence of the GZK cutoff although the suppression due to acceleration limits in cosmic ray sources can not be excluded. The occurrence of the GZK suppression and the high isotropy of the highest energy cosmic rays are indicative of their extragalactic origin. The list of potential sources which could give the observed cosmic ray flux includes AGNs, gamma-ray bursts, magnetars, interacting galaxies, large-scale structure formation shocks and other objects, see e.g. review by [1].

The present knowledge about the highest energy cosmic rays was mainly acquired from the experiments HiRes, Auger, and Telescope Array, see review [2]. The mass composition of these cosmic rays remains uncertain. The interpretation of HiRes and Telescope Array data favors the proton composition at energies $10^{18} - 5 \times 10^{19}$ eV, whereas the Auger data indicate that the cosmic ray composition is becoming heavier with energies changing from predominantly proton at $10^{18}$ eV to more heavy composition and probably reaching the pure Iron composition at about $5 \times 10^{19}$ eV. The mass composition interpretation of the measured quantities depends on the assumed hadronic model which is based on not well determined extrapolation of the physics from lower energies.

Usually, the energy spectrum in extragalactic sources is determined by the trail-and-error method when one makes the calculations of the expected at the Earth cosmic ray intensity assuming some shape of the source spectrum and the source composition. The calculations follow cosmic ray propagation from the source to the observer, see e. g. [3]. The standard assumption is that the source spectrum is a power law on magnetic rigidity.

In the present work we inverse the procedure and calculate the source function starting from the observed at the Earth cosmic ray spectrum without ad hoc assumptions about the shape
of source spectrum. Two simple cases of pure proton and pure Iron source composition are considered.

2. Transport equation of cosmic rays and solution of inverse problem

We use the following transport equation for cosmic ray nuclei with mass number \( A \) in an expanding Universe filled with the background electromagnetic radiation (see [4] for detail):

\[
-H(z)(1 + z) \frac{\partial}{\partial z} \left( \frac{F(A, \varepsilon, z)}{(1 + z)^3} \right) - \\
\partial \varepsilon \left( \varepsilon \left( \frac{H(z)}{(1 + z)^3} + \frac{1}{\tau(A, \varepsilon, z)} \right) F(A, \varepsilon, z) + \nu(A, \varepsilon, z) F(A, \varepsilon, z) \right) \\
= \sum_{i=1,2...} \nu(A + i - A, \varepsilon, z) F(A + i, \varepsilon, z) + q(A, \varepsilon)(1 + z)^m. \tag{1}
\]

The system of equations (1) for all kinds of nuclei with different \( A \) from Iron to Hydrogen should be solved simultaneously. The energy per nucleon \( \varepsilon = E/A \) is used here because it is approximately conserved in a process of nuclear photodisintegration, \( F(A, \varepsilon, z) \) is the corresponding cosmic-ray distribution function, \( q(A, \varepsilon) \) is the density of cosmic-ray sources at the present epoch \( z = 0 \), \( m \) characterizes the source evolution (the evolution is absent for \( m = 0 \)), \( \tau(A, \varepsilon, z) \) is the characteristic time of energy loss by the production of \( e^-e^+ \) pairs and pions, \( \nu(A, \varepsilon, z) \) is the frequency of nuclear photodisintegration, the sum in the right side of equation (1) describes the contribution of secondary nuclei produced by the photodisintegration of heavier nuclei, \( H(z) = H_0((1 + z)^3\Omega_m + \Omega_\Lambda)^{1/2} \) is the Hubble parameter in the flat expanding universe with the matter density \( \Omega_m(=0.3) \) and the \( \Lambda \)-term \( \Omega_\Lambda(=0.7) \).

The numerical solution of the cosmic-ray transport equations follows the finite differences method. The variables are the redshift \( z \) and \( \log(E/A) \).

Let us introduce solution \( G(A, \varepsilon; A_s, \varepsilon_s) \) of equations (1) at \( z = 0 \) for a delta-source \( q(A, \varepsilon) = \delta_{AA_s} \delta(\varepsilon - \varepsilon_s) \). This source function describes the emission of nuclei with mass number \( A_s \) and energy \( \varepsilon_s \) from cosmic ray sources distributed over all \( z \) up to some \( z_{\text{max}} \). The general solution of equations (1) at the observer location \( z = 0 \) can now be presented as

\[
F(A, \varepsilon, z = 0) = \sum_A \int d\varepsilon' G(A, \varepsilon; A', \varepsilon') q(A', \varepsilon'). \tag{2}
\]

The observed all-particle spectrum is determined by the summation over all types of nuclei \( \sum_A F(A, \varepsilon, z = 0) \).

The set of discrete values of particle energy \( \varepsilon_i \) is defined to solve the transport equation numerically. The grid with constant \( \Delta\varepsilon/\varepsilon \) and with 100 energy bins per decade is used in our calculations. Equation (2) in discrete form is

\[
F_i(A, z = 0) = \sum_{j, A'} (\Delta\varepsilon)_j G_{ij}(A; A') q_j(A'), \tag{3}
\]

where the subscript indexes \( i \) and \( j \) denote the corresponding energies \( \varepsilon_i \) and \( \varepsilon_j \). The all particle spectrum is \( \sum_A F_i(A, z = 0) \).

The source term \( q_j(A) \) for each type on nuclei can be derived from the system of linear equations (3) if the observed spectra \( F_i(A, z = 0) \) for all types of nuclei are known. If the information on observed spectra of individual types of nuclei \( F_i(A, z = 0) \) is not available and only the all particle spectrum \( \sum_A F_i(A, z = 0) \) is known, the source spectra can be found only if the source abundances of different types of ions are known. In the simple case, when only
nuclei with mass number $A = A_s$ are accelerated in the sources, equation (3) allows to find the following relation:

$$q_j(A_s)(\Delta \varepsilon)_j = (\sum_A G_{ij}(A, A_s))^{-1} \times (\sum_A F_i(A, z = 0)).$$

(4)

The calculations of inverse matrix in equation (4) is straightforward since the initial matrix $G_{ij}$ is the triangular ones.

To illustrate how the described procedure works, we made calculations of the source spectra using equation (4) in two simple cases of pure proton and pure Iron source composition. The Auger data and the Telescope Array data from [5] were used. The results are shown in Figure 1, see figure caption for the explanation. The difference in the source functions for proton and Iron composition is evident. The spread of data points results in irregularities of the derived source spectra. The analytical approximation of the observed Auger spectrum [5]

$$J(E) \propto E^{-3.27}, E < E_{\text{ankle}}; J(E) \propto E^{-2.63}(1 + \exp(\frac{\log E - \log E_{1/2}}{\log W_c}))^{-1}, E > E_{\text{ankle}}$$

(5)

(here $E_{\text{ankle}} = 10^{18.62}$eV, $E_{1/2} = 10^{19.63}$eV, $W_c = 10^{0.15}$eV) gives smooth source functions shown by dash-dot lines in Figure 1. Even roughly, the source spectrum can not be described by a power law in the energy range $10^{18}$ to $5 \times 10^{19}$ eV.

3. Discussion and Conclusion

We showed how one can find the spectrum of extragalactic sources from the cosmic ray spectrum observed at the Earth. This inverse problem is solved based on the system of transport equations (1) which describes the propagation of ultra-high energy cosmic ray in expanding universe filled with the background electromagnetic radiation. The purpose of the present paper is the demonstration of general approach to the problem. The source spectra were found from the Auger and Telescope data on the all particle spectra assuming pure proton or pure Iron.
composition at the sources. The derived source spectra show evidence of hardening at energies $\sim 5 \times 10^{19}$ eV for the proton source composition and $\sim 10^{19}$ eV for the Iron source composition. The knowledge of cosmic ray composition of the observed flux is required for more advanced consideration.

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References

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