Rare Decays as a Probe for New Physics

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We discuss the indirect search for new degrees of freedom beyond the standard model, within flavour physics. In particular, we analyse the minimal flavour violation hypothesis and its phenomenological implications, especially the large-tan$\beta$ scenario in supersymmetric models, and also compare it with the constrained minimal flavour violation scenario. Moreover, we briefly discuss some recent progress in inclusive $b \rightarrow s$ transitions and present a status report of the so-called $K\pi$ puzzle.

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1. INTRODUCTION

With the running $B$-factory experiments Babar at SLAC and Belle at KEK, the forthcoming $B$-physics programmes of the LHC experiments, especially LHCb, at CERN and the future options of $B$ experiments at Super-$B$ factories and at future linear colliders, $B$ physics is one important focus in particle physics today.

It is well known that rare $B$ decays, as flavour-changing neutral currents (FCNCs), are particularly sensitive to new physics. The present data from the $B$-physics experiments already imply significant restrictions for the parameter space of new physics models and lead to important clues for the direct search for new particles and the model building beyond the standard model (SM). After new physics will have been discovered, especially when the mass scale of the new physics will have been fixed, such observables, and also correlations between collider and flavour observables, will play an important role in analyzing the underlying new dynamics.

In particular, there is a flavour problem to solve in any viable new physics model, namely why FCNCs are suppressed. In supersymmetry the flavour problem is directly linked to the crucial question of how supersymmetry is broken. Moreover, there is a corresponding CP problem: while the CKM prescription has passed its first precision tests, the problem arise of finding the mechanism by which the often numerous additional CP phases are suppressed in a new physics model. For example, there are very stringent bounds on the 44 phases of the minimal supersymmetric standard model (MSSM).

One of the main difficulties in examining the observables in $B$ physics is the influence of the strong interaction. If new physics does not show up in flavour physics through large deviations, as recent experimental data indicate, one has to focus on theoretically clean variables such as inclusive rare $B$ decays, which are dominated by perturbative contributions or specific ratios of exclusive modes such as CP or charge asymmetries. It is important to calculate those specifically suitable observables to very high precision in order to exploit their sensitivity to possible degrees of freedom beyond the SM.

In the indirect search for new physics, it is also mandatory to go beyond the analysis of branching ratios and to measure more complex kinematical distributions such as CP, forward-backward, and isospin asymmetries to detect subtle patterns and to distinguish between the various scenarios beyond the SM.

The paper is organized as follows. In the next section we discuss the minimal flavour violation hypothesis and its phenomenological implications. Moreover, two analyses of the large tan$\beta$ scenario in supersymmetric models are briefly reviewed. In section 3 we briefly discuss some specific opportunities for the new-physics search.

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within $b \to s$ transitions and give more details on the present status of the so-called $K\pi$ puzzle. The reader will find a detailed discussion of the $b \to s$ transitions in a forthcoming review \[1\].

2. MINIMAL FLAVOUR VIOLATION AND BEYOND

There are two general approaches to new physics, which are most suitable in the present situation where no direct evidence for new degrees of freedom beyond the SM exists.

While a model-independent analysis takes into account the possibility of new flavour structures, which are parametrized by model-independent parameters, an analysis within the minimal flavour violation (MFV) hypothesis assumes that the flavour and the CP symmetry are broken as in the SM; it essentially requires that all flavour- and CP-violating interactions be linked to the known structure of Yukawa couplings (called $Y_U$ and $Y_D$ in the following). A renormalization-group-invariant definition of MFV based on a symmetry principle is given in \[2,3,4\]; this is mandatory for a consistent effective field theoretical analysis of new physics effects. In fact, a low-energy effective theory with all SM fields including one or two Higgs doublets is constructed; as the only source of $U(3)^5$ flavour symmetry breaking, the ordinary Yukawa couplings are introduced as background values of fields transforming under the flavour group ('spurions') \[4\].

In the construction of the effective field theory, operators with arbitrary powers of the dimensionless $Y_U/D$ have to be considered in principle. However, the specific structure of the SM, with its hierarchy of CKM matrix elements and quark masses, drastically reduces the number of numerically relevant operators. For example, it can be shown that in MFV models with one Higgs doublet, all FCNC processes with external $d$-type quarks are governed by the following combination of spurions due to the dominance of the top Yukawa coupling $y_t$: 

$$(Y_U Y_U^T)_{ij} \approx y_t^2 V_{td}^* V_{sj},$$  \hspace{1cm} (1)$$

where a basis is used in which the $d$-type quark Yukawa is diagonal.

There are two strict predictions in this general class of models, which have to be tested. First, the MFV hypothesis implies the usual CKM relations between $b \to s$, $b \to d$, and $s \to d$ transitions. For example, this relation allows for upper bounds on new-physics effects in $B(\bar{B} \to X_d \gamma)$, and $B(\bar{B} \to X_s \nu \bar{\nu})$ using experimental data or bounds from $B(\bar{B} \to X_s \gamma)$, and $B(K \to \pi^+ \nu \bar{\nu})$ respectively. This emphasizes the need for high-precision measurements of $b \to s/d$, but also of $s \to d$ transitions such as the rare kaon decay $K \to \pi \nu \bar{\nu}$.

The second prediction is that the CKM phase is the only source of CP violation. This implies that any phase measurement as in $B \to \phi K_s$ or $\Delta M_{B_{d,s}}$ is not sensitive to new physics. Note that there is also a RG-invariant extension of the MFV concept allowing for flavour-blind phases \[5\]: these lead to non-trivial CP effects, which get, however, strongly constrained by flavour-diagonal observables such as electric dipole moments (see for an example \[5\]).

The usefulness of MFV-bounds/relations is obvious; any measurement beyond those bounds
Table 1

| Branching Ratios       | MFV (95%) | SM (68%) | SM (95%) | exp |
|------------------------|-----------|----------|----------|-----|
| $B(K^+ \rightarrow \pi^+\nu\bar{\nu}) \times 10^{11}$ | $< 11.9$  | $8.3 \pm 1.2$ | $[6.1, 10.9]$ | $(14.7_{-8.9}^{+13.7})$ |
| $B(K_L^0 \rightarrow \pi^0\nu\bar{\nu}) \times 10^{11}$ | $< 4.59$  | $3.08 \pm 0.56$ | $[2.03, 4.26]$ | $< 5.9 \times 10^4$ |
| $B(K_L^0 \rightarrow \mu^+\mu^-) \times 10^9$    | $< 1.36$  | $0.87 \pm 0.13$ | $\text{---}$ | $\text{---}$ |
| $B(B \rightarrow X_s\nu\bar{\nu}) \times 10^6$    | $< 5.17$  | $3.66 \pm 0.21$ | $[3.25, 4.09]$ | $< 64$ |
| $B(B \rightarrow X_d\nu\bar{\nu}) \times 10^6$    | $< 2.17$  | $1.50 \pm 0.19$ | $[1.12, 1.91]$ | $\text{---}$ |
| $B(B_d^0 \rightarrow \mu^+\mu^-) \times 10^9$    | $< 7.42$  | $3.67 \pm 1.01$ | $[1.91, 5.91]$ | $< 2.7 \times 10^4$ |
| $B(B_s^0 \rightarrow \mu^+\mu^-) \times 10^9$    | $< 2.20$  | $1.04 \pm 0.34$ | $[0.47, 1.81]$ | $< 1.5 \times 10^4$ |

indicate the existence of new flavour structures.

In Ref. [6], upper bounds for rare decays in constrained MFV (CMFV) models at 95% probability are presented, see Table[1]. Furthermore, an upper bound on the $B_q - \bar{B}_q \ (q = d, s)$ mixing in CMFV was established more recently [7]. However, there is a subtlety involved. Those bounds are based on a constrained version of MFV where the additional dynamical assumptions are used that the relevant operators in the electroweak effective Hamiltonian for weak decays are the same as in the SM and that new physics only leads to changes of the effective couplings, the Wilson coefficients, but not of the CKM factors of those operators. Clearly, those assumptions allow for additional relations between different $B$ and kaon observables (see also [8]), but their violation do not necessarily indicate the existence of new flavour structures. Moreover, they explicitly rule out – as most important difference to the general MFV concept – new scalar operators and, thus, large tan$\beta$ effects, which are not necessarily based on new flavour structures.

It is well known that scenarios including two Higgs doublets with large tan$\beta = O(m_t/m_b)$ allow for the unification of top and bottom Yukawa couplings, as predicted in grand-unified models (see [9]), and for sizable new effects in helicity-suppressed decay modes (see [10][11]). There are more general MFV relations existing in this scenario due to the dominant role of scalar operators. However, since tan$\beta$ is large, there is a new combination of spurious numerically relevant in the construction of higher-order MFV effective operators, namely

$$ (Y_D Y_D^T)_{ij} \approx y_d^2 \delta_{ij}, $$

which invalidates the general MFV relation between $b \rightarrow s/d$ and $s \rightarrow d$ transitions.

Such a large-tan$\beta$ scenario within the MSSM was recently studied in Ref. [12]. The additional supersymmetric structure leads to more correlations between the observables. It is shown that for large squark masses and trilinear couplings above 1 TeV this scenario explains the present data set naturally, including flavour-conserving observables: for example, the significant enhancement of the anomalous magnetic moment of the muon $a_\mu$ and the large scalar Higgs mass above 115 GeV, but also the quite modest non-standard contributions to $B_s^0 - \bar{B}_s^0$ mixing and to $B \rightarrow X_s \gamma$. However, it allows for a large enhancement of $B \rightarrow \mu\mu$ and for a significant suppression of $B^\pm \rightarrow \tau^\pm \nu$, features compatible with present data. Obviously, the future measurements of the latter two observables are crucial for the test of this attractive new-physics scenario.

In Fig. [1][12] the constraints of the various $B$ physics observables and of $a_\mu$ on the two parameters tan$\beta$ and the charged Higgs mass $M_H$ are shown in an exemplary mode for the Higgs mass parameter $\mu = 0.5$, the trilinear term $A_U = -1.0$, the average squark mass $M_{\tilde{Q}} = 1$ TeV, $M_1 = 0.3$ TeV, and $M_2 = 0.2$ TeV. For the exclusion regions for $B(B_s \rightarrow \mu^+\mu^-)$ and $B(B \rightarrow X_s \gamma)$ the bounds at 90% c.l. were taken into account and in the case of $B(B_s \rightarrow X_s \gamma)$ the NLL prediction
Figure 2. $B(B_s \to \mu^+ \mu^-)$ as a function of $\tan \beta$ in a MFV GUT scenario with $\mu > 0$.

Figure 3. As in Fig. 2 but $\mu < 0$.

was still used. Possible bounds from a future measurement of $B(B \to \mu \mu)$ and $R_{B \to \tau \nu} = B^{\text{exp}}(B \to \tau \nu)/B^{\text{SM}}(B \to \tau \nu)$ are also indicated.

Large-$\tan \beta$ effects in $B(B_s \to \mu \mu)$ have been analysed, assuming a supersymmetric minimal flavour violation structure at the GUT scale [13]. The correlations with other decay modes are shown in Figs. 2 and 3 with a mass of the pseudoscalar Higgs of $M_A = 150$ GeV, for a Higgs mass parameter $\mu$ negative and positive, respectively. The grey points do not survive the constraints from $\bar{B} \to X_s \gamma$ and $a_\mu$. Large enhancement effects are still possible for $\mu > 0$.

Flavour structures beyond the Yukawa couplings can be introduced into the effective field theory approach with the help of additional spurions of the flavour group. Practically, this allows for a different hierarchy between the various spurion terms compared with the one fixed by the structure of the SM. Quite recently such a general framework was discussed as next-to-minimal flavour violation scenario (NMFV) [14].

In Ref. [15], a special choice of additional spurions are introduced, which modifies only the couplings of the 3rd quark generation. This represents a phenomenologically well-defined class of models; at present those escape from the most stringent experimental bounds, which are mainly on the first two generations. How minimal flavour violation can be extended into the framework of GUT theories is analysed in Ref. [16].

3. OPPORTUNITIES WITHIN $b \to s$ TRANSITIONS

Data from $K$ and $B_d$ physics shows that new sources of flavour violation in $s \to d$ and $b \to d$ are strongly constrained, while the possibility of sizable new contributions to $b \to s$ still remains open [18,19]. We also have hints from model building; flavour models are not very effective in constraining the $b \to s$ sector [20]. Moreover, in supersymmetric grand-unified theories the large mixing angle in the neutrino sector relates to large mixing in the right-handed $b$–$s$ sector [21,22,23].

Squark decays are governed by the same mixing matrices as the contributions to flavour violating low-energy observables. This allows for possible direct correlations between flavour non-diagonal observables in $B$ and collider physics. The present bounds on squark mixing, induced by the low-energy data on $b \to s$ transitions, still allow for large contributions to flavour violating squark decays at tree level [19].

Among the flavour-changing current processes, the inclusive $b \to s\gamma$ and $b \to s\ell^+\ell^-$ modes are still the most prominent. The stringent bounds obtained from those modes on various non-standard scenarios are a clear example of the importance of clean FCNC observables in discriminating new-physics models. The branching
ratio of \( \bar{B} \to X_s \gamma \) has already been measured by several independent experiments \cite{24,25,26,27,28}, leading to the world average of those five measurements (performed by the Heavy Flavour Averaging Group \cite{29}) for a photon energy cut \( E_\gamma > 1.6 \text{ GeV} \):

\[
B(\bar{B} \to X_s \gamma) = (3.55 \pm 0.24 ^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4},
\]

where the first error is a combined statistical and systematical one, the second and third are additional systematical errors due to the extrapolation and to the \( b \to d \gamma \) fraction, respectively.

After a global effort, the first theoretical prediction of the branching ratio to \( O(\alpha_s^2) \) has been recently presented \cite{30}. For \( E_\gamma > 1.6 \text{ GeV} \) the new prediction reads:

\[
B(\bar{B} \to X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4}.
\]

The overall uncertainty consists of non-perturbative (5%), parametric (3%), higher-order (3%) and \( m_c \)-interpolation ambiguity (3%), which have been added in quadrature. Compared with the HFAG average given in Eq. \( \text{(3)} \), the NNLL prediction is 1.2σ below the experimental data.

The decay \( \bar{B} \to X_s \ell^+ \ell^- \) is particularly attractive because of kinematic observables such as the invariant dilepton mass spectrum and the forward–backward (FB) asymmetry. The recently calculated NNLL contributions \cite{31,32,33,34,35,36} have significantly improved the sensitivity of the inclusive \( \bar{B} \to X_s \ell^+ \ell^- \) decay in testing extensions of the SM in the sector of flavour dynamics; in particular, the value of the dilepton invariant mass \( q_h^2 \), for which the differential forward–backward asymmetry vanishes, is one of the most precise predictions in flavour physics with a theoretical uncertainty well below 10%.

A recent update of the dilepton mass spectrum, integrated over the low dilepton invariant mass region in the muonic case, leads to \cite{37}

\[
B(\bar{B} \to X_s \mu^+ \mu^-) = (1.59 \pm 0.11) \times 10^{-6},
\]

where the error includes the parametric and perturbative uncertainties only. The analogous update of the other NNLL predictions will be presented in a forthcoming paper \cite{38}.

The corresponding rare exclusive decays, such as \( B \to K^* \gamma \), \( B \to K^+ \mu^+ \mu^- \) or also \( B_s \to \phi \mu^+ \mu^- \), are experimentally distinguished observables at the forthcoming LHCb experiment. In contrast to the measurement of the branching ratios, measurements of CP, forward-backward, and isospin asymmetries are less sensitive to hadronic uncertainties. Particularly, the value of the dilepton invariant mass \( q_h^2 \), for which the differential forward–backward asymmetry vanishes, can be predicted in quite a clean way. In the QCD factorization approach, at leading order in \( \Lambda_{\text{QCD}}/m_b \), the value of \( q_h^2 \) is free from hadronic uncertainties at order \( \alpha_s^4 \), a dependence on the soft form factor \( \xi_\perp \) and the light-cone wave functions of the \( B \) and \( K^* \) mesons appear at order \( \alpha_s^3 \). The latter contribution, calculated within the QCD factorization approach, leads to a large shift (see \cite{39,40,42,41}). Nevertheless, there is the well-known issue of power corrections (1 \( m_b \)) within the QCD factorization approach. There are also certain transversity amplitudes in \( B \to K^+ \mu^+ \mu^- \), which are rather insensitive to hadronic uncertainties and in particular highly sensitive to non-standard chiral structures of the \( b \to s \) current \cite{34}. For more details on those observables and also on distinguished mixing-induced or direct CP asymmetries in \( b \to s \) transitions, the reader is referred to a forthcoming review \cite{1}.

4. PRESENT STATUS OF THE SO-CALLED Kπ PUZZLE

The \( B \to K \pi \) modes are well known for being sensitive to new electroweak penguins beyond the SM \cite{44,45}. The data on CP-averaged \( K \pi \) branching ratios can be expressed in terms of three ratios:

\[
R = \frac{\tau_{B^+}}{\tau_{B^0}} \frac{B[B^0 \to \pi^- K^+] + B[B^0 \to \pi^+ K^-]}{B[B^+_d \to \pi^+ K^0] + B[B^0_d \to \pi^- K^0]},
\]

\[
R_n = \frac{1}{2} \frac{B[B^0 \to \pi^- K^+] + B[B^0 \to \pi^+ K^-]}{B[B^0 \to \pi^0 K^+] + B[B^0 \to \pi^0 K^-]},
\]

\[
R_c = \frac{2}{B[B^+_d \to \pi^+ K^0] + B[B^+_d \to \pi^- K^0]} \frac{B[B^+_d \to \pi^0 K^+] + B[B^0_d \to \pi^0 K^-]}{B[B^+_d \to \pi^+ K^0] + B[B^0_d \to \pi^- K^0]}
\]

The actual data presented at ICHEP06 read \cite{47}.
Moreover, approximate flavour symmetries in a study [55,56] along these lines, the number of unknown hadronic parameters [53,54].

In a study in Refs. [48,49,50], which leads to the prediction, based on the QCD factorization techniques of the SM, cannot be ruled out yet [57].

The uncertainties reflect the experimental uncertainties of the $B \to \pi\pi$ data only. In the future the assumptions of the $SU(3)_F$ symmetry can be tested experimentally. Because of the large non-factorizable contributions identified in the $B \to \pi\pi$ channel, however, large non-factorizable $SU(3)_F$- or isospin-violating QCD and QED effects within the SM cannot be ruled out yet [57].

Nevertheless, the new data set [47] significantly moved into the ballpark of the SM estimates. In the previous analyses the radiative corrections to charged particles in the final state were not taken into account, as was emphasized in the past (see for example Ref. [58]). These corrections, worked out in Ref. [59], are now properly included in the analysis of both experiments and are partially responsible for the shifts in the central values in the $K\pi$ data.

A new, more complete $SU(3)_F$ analysis of the CKM fitter group is now accessible [60], in which all available $\pi\pi, K\pi, K\bar{K}$ modes are included.

\[ R = 0.92^{+0.05}_{-0.05}, R_n = 1.00^{+0.07}_{-0.07}, R_c = 1.10^{+0.07}_{-0.07}, \]

which is a significant change with respect to the pre-ICHEP04 data:

\[ R = 0.84^{+0.06}_{-0.06}, R_n = 0.82^{+0.08}_{-0.08}, R_c = 1.00^{+0.09}_{-0.09}, \]

or, with respect to the pre-ICHEP04 data:

\[ R = 0.91^{+0.07}_{-0.07}, R_n = 0.76^{+0.10}_{-0.10}, R_c = 1.17^{+0.12}_{-0.12}. \]

The previous data sets were often called anomalous in view of the approximate sum rule proposed in Refs. [48,49,50], which leads to the prediction $R_c = R_n$ and the available SM approaches to these data based on QCD factorization techniques and on $SU(3)_F$ symmetry assumptions. The corresponding BBNS predictions, based on the QCD factorization approach [51,52], are

\[ R = 0.91^{+0.13}_{-0.11}, R_n = 1.16^{+0.22}_{-0.19}, R_c = 1.15^{+0.19}_{-0.17}. \]

Moreover, approximate flavour symmetries (isospin or $SU(3)_F$) can also be used to relate different decay amplitudes and reduce the number of unknown hadronic parameters [53,54].

In a study along these lines, the $B \to \pi\pi$ data were used to make theoretical predictions on the $B \to K\pi$ modes. This specific approach leads to [50]

\[ R = 0.96^{+0.02}_{-0.02}, R_n = 1.12^{+0.05}_{-0.05}, R_c = 1.15^{+0.05}_{-0.05}. \]

Figure 4. Constraint in the $(\rho, \bar{\eta})$ plane induced by the $\pi\pi, K\pi, K\bar{K}$ data compared with standard CKM fit.

Figure 5. Comparison of direct measurements of $R_n$ and $S(K^{0}_S\pi^0)$ (1σ band) with indirect fit (2σ contour).
so-called annihilation/exchange topologies and factorizable $SU(3)_F$ breaking are taken into account. As shown in Fig. 4 [60], the constraint in the $(\bar{\rho}, \bar{\eta})$ plane induced by these data implies that the compatibility with the $SU(3)$ and SM hypothesis is very good (the so-called pValue of that SM analysis is of order $30 - 40\%$). But the $\chi^2_{\text{min}}$ is not always the best measure of the compatibility of the data with the theory. Among the main contributions to the $\chi^2$ there are the ratio $R_n$ and the CP asymmetry $S(K^0_S\pi^0)$, which are all very sensitive to new electroweak penguins.

After removing them from the global fit, Fig. 4 [60] shows the comparison of the indirect fit (2$\sigma$ contour), with $\bar{\rho}, \bar{\eta}$ from the CKM fit and all other available modes, with the direct measurements (1$\sigma$ band) using the new data. This can be compared with the analogous plot based on the pre-ICHEP2006 data, see Fig. 6. While the indirect prediction for $R_n$ is now in good agreement with the direct measurement, there is still a small ‘discrepancy’ in the case of the observable $S(K^0_S\pi^0)$.

Future data from the $B$ factories and LHCb will clarify the situation completely. There will be up to 38 measured observables depending on the same 13+2 theoretical parameters. This will allow for the study of $SU(3)$ breaking and new-physics effects.

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