Natural Quintessence?

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Abstract

We formulate conditions for the naturalness of cosmological quintessence scenarios. The quintessence lagrangian is taken to be the sum of a simple exponential potential and a non-canonical kinetic term. This parameterization covers most variants of quintessence and makes the naturalness conditions particularly transparent. Several “natural” scalar models lead, for the present cosmological era, to a large fraction of homogeneous dark energy density and an acceleration of the scale factor as suggested by observation.
1 Introduction

The phenomenology of the expanding universe appears to be converging on a set of fundamental parameters that includes a non-zero homogeneous energy density $\epsilon_{\text{vac}} \simeq 2 \times 10^{-123} M_P^2$ (see [1] and refs. therein). Furthermore, large redshift supernova observations suggest that the cosmological scale factor is accelerating at present [2]. Given the well-known difficulties in explaining why the cosmological constant should be zero (see, e.g., [3]), it appears to be even harder to understand a finite number that represents such a tiny fraction of the natural scale set by the Planck mass

$$M_P = \left(8\pi G_N\right)^{-1} = 1.22 \times 10^{19} \text{ GeV}.$$ 

The scale discrepancies introduced into physics by this large mass have triggered many attempts and speculations to interpret the Planck mass as a dynamical scale [4], to understand a possible time evolution of the cosmological constant [5], or to have the dynamics of a scalar field adjusting the cosmological constant to zero [6,7]. Quintessence as homogeneous dark energy of an evolving scalar field is partially successful in explaining a small present-day value of the homogeneous energy density [8 –12]. It can lead to a cosmology consistent with observation [13–19].

In this paper, we discuss the naturalness of different realizations of the quintessence scenario from the perspective of the scalar field lagrangian

$$\mathcal{L}(\varphi) = \frac{1}{2} \left(\partial \varphi\right)^2 k^2(\varphi) + \exp[-\varphi]. \quad (1)$$

Here and in what follows all quantities are measured in units of the reduced Planck mass $\overline{M}_P$, i.e., we set $\overline{M}_P^2 \equiv M_P^2 / (8\pi) \equiv (8\pi G_N)^{-1} = 1$. The lagrangian of Eq. (1) contains a simple exponential potential $V = \exp[-\varphi]$ and a non-standard kinetic term with $k(\varphi) > 0$. If one wishes, the kinetic term can be brought to the canonical form by a change of variables. Introducing the field

$$\chi = K(\varphi) \quad \text{with} \quad k(\varphi) = \frac{\partial K(\varphi)}{\partial \varphi} \quad (2)$$

one obtains

$$\mathcal{L}(\chi) = \frac{1}{2} \left(\partial \chi\right)^2 + \exp[-K^{-1}(\chi)] \quad (3).$$

Nevertheless, the important question whether a given quintessence model can be considered as natural from a field theory perspective can be discussed particularly simply in terms of the lagrangian of Eq. (1).

We restrict our discussions to potentials that are monotonic in $\chi$. (Otherwise, the value of the potential at the minimum must be of the order of today’s cosmological constant, with $V_{\text{min}} \approx 10^{-120}$. Cosmologies of this type are discussed in [14].) All monotonic potentials can be rescaled to the ansatz Eq. (1). An initial value of $\varphi$ in the vicinity of zero corresponds then to an initial scalar potential energy density of order one. We consider this as a natural starting point for cosmology in the Planck era. As a condition for naturalness we postulate that no extremely small parameters should be present in the Planck era. This means, in particular, that $k(0)$ should be of order one. Furthermore, this forbids a tuning to many decimal places of parameters appearing in $k(\varphi)$ or the initial
conditions. For natural quintessence all characteristic mass scales are given by $M_P$ in the Planck era. The appearance of small mass scales during later stages of the cosmological evolution is then a pure consequence of the age of the universe (and the fact that $V(\varphi)$ can be arbitrarily close to zero). In addition, we find cosmologies where the late time behaviour is independent of the detailed initial conditions particularly attractive. For such tracker solutions [8, 9, 12] no detailed understanding of the dynamics in the Planck era is needed. One of our main findings is the existence of viable cosmological solutions with high present-day acceleration which are based on functions $k(\varphi)$ that always remain $O(1)$.

Non-canonical kinetic terms have been considered in cosmology before. For example, they were used in models of inflation [20] and as tool for the adjustment of the cosmological constant [1, 21], most recently in the context of quintessence [22]. A non-canonical kinetic term appears in supergravity theories [23] and was also used in [19] to relate the present-day cosmic acceleration to the onset of matter domination at $a \simeq 10^{-4}$. In the context of higher dimensional unification the identification of $\ln \varphi$ with the volume of internal space or some appropriate dilaton-type field generically leads to a non-canonical kinetic term [24].

It is convenient to analyse the cosmological evolution using the scale factor $a$ instead of time as the independent variable. In this case, the evolution of matter and radiation energy density is known explicitly and one only has to solve the set of the two differential equations for the homogeneous dark energy density $\rho_\varphi$ and the cosmon field $\varphi$

$$\frac{d \ln \rho_\varphi}{d \ln a} = -3(1 + w_\varphi), \quad \frac{d \varphi}{d \ln a} = \sqrt{6\Omega_T/k^2(\varphi)},$$

(4)

with $\Omega_T = T/(3H^2)$ the fraction of kinetic field energy and $w_\varphi = p_\varphi/\rho_\varphi$. Here the cosmon kinetic energy is denoted by $T = \dot{\varphi}^2 k^2(\varphi)/2$ whereas $p_\varphi = T - V$ and $\rho_\varphi = T + V$ specify the equation-of-state of quintessence. Thus, more explicitly, the cosmology is governed by four equations for the different components of the energy density $\rho_m, \rho_r, \rho_\varphi$ and $\varphi$

$$\frac{d \ln \rho_m}{d \ln a} = -3(1 + w_m), \quad \frac{d \ln \rho_r}{d \ln a} = -3(1 + w_r),$$

$$\frac{d \ln \rho_\varphi}{d \ln a} = -6 \left(1 - \frac{V(\varphi)}{\rho_\varphi}\right), \quad \frac{d \varphi}{d \ln a} = \sqrt{\frac{6(\rho_\varphi - V(\varphi))}{k^2(\varphi)(\rho_m + \rho_r + \rho_\varphi)}},$$

(5)

where $w_m = 0$ and $w_r = 1/3$ for matter and radiation respectively.

For our exponential potential $V = \exp[-\varphi]$, the last equation can be rewritten as

$$\frac{d \ln V}{d \ln a} = -\sqrt{\frac{6(\rho_\varphi - V)}{k^2(-\ln V)(\rho_m + \rho_r + \rho_\varphi)}}.$$

(6)

We note that today’s value of $\rho_\varphi$ plays the role of $\epsilon_{vac}$ and $\Omega_\varphi = \rho_\varphi/(3H^2)$. For a rough orientation, today’s value of $\varphi$ must be $\varphi_0 \simeq 276$ for all solutions where the present potential energy is of the order of $\epsilon_{vac}$.
The simplest case, $k(\varphi) = k = \text{const.}$, corresponds to the original quintessence model [8] with a potential term $\exp[-\chi/k]$. If $k^2 < 1/n_b$ (with $n_b = 3(1 + w_b)$ and $b = r$ for radiation and $b = m$ for matter), then the scalar field energy $\rho_\varphi$ follows the evolution of the background component $\rho_b$ in a well-known manner. In this case, one finds a constant dark energy fraction

$$\Omega_\varphi = n_b k^2.$$ (7)

This attractor solution can be easily established from Eqs. (5) and (6) by noting the constancy of $\rho_\varphi/\rho_b$ and $V/\rho_b$. For $k^2 > 1/n_b$ the cosmological attractor is a scalar dominated universe [8, 10, 25] with $H = 2k^2 t^{-1}$, $w_\varphi = 1/(3k^2) - 1$.

If a solution obeying (approximately) Eq. (7) is valid during nucleosynthesis, the “right tuning of the clock” requires $\Omega_\varphi \lesssim 0.2$ [8,26]. Another constraint arises from structure formation since solutions with large constant $\Omega_\varphi$ slow down the growth of density fluctuations [11]. This is described by the simple relation [11]

$$\delta_c \sim a^{1-\epsilon/2} \quad \text{with} \quad \epsilon = \frac{5}{2} \left( 1 - \sqrt{1 - \frac{24}{25}\Omega_\varphi} \right),$$ (8)

where $\delta_c$ is the density contrast of cold dark matter. The formation of galaxies also requires $\Omega_\varphi \lesssim 0.2$ for a sufficiently long time after the onset of matter domination [1]. For building quintessence models, this constraint is the most stringent one because it requires a recent increase of $\Omega_\varphi$ that is relatively rapid on a cosmological scale.

It has been emphasized early [8] that there is actually no reason why $k(\varphi)$ should be exactly constant and that interesting cosmologies may arise from variable $k(\varphi)$. In particular, one may imagine an effective transition from small $k$ (small $\Omega_\varphi$) in the early universe (nucleosynthesis etc.) to large $k$ ($\Omega_\varphi \simeq 1$) today [13,18,27].

### 2 Leaping kinetic term

A particularly simple case of a $\varphi$ dependent kinetic coefficient $k(\varphi)$ is obtained if $k$ suddenly changes from a small number $k < 0.22$ (consistent with nucleosynthesis and structure formation bounds) to a number above the critical value $1/\sqrt{n_b}$. Consider, for example, the function

$$k(\varphi) = k_{\text{min}} + \tanh(\varphi - \varphi_1) + 1 \quad \text{(with} \quad k_{\text{min}} = 0.1, \quad \varphi_1 = 276.6),$$ (9)

that gives rise to the cosmological evolution of Fig. [1]. This model, which completely avoids the explicit use of very large or very small parameters, realizes all the desired features of quintessence. The homogeneous dark energy density tracks below the background component in the early universe ($k = 0.1$) and then suddenly comes to dominate the evolution when $k$ rises to a value $k = 2.1$ approximately today. With a tuning on the percent level (the value of $\varphi_1$ has to be appropriately adjusted) realistic present-day

\footnotetext[1]{{Our bound is very conservative. A more realistic limit is probably given by $\Omega_\varphi \lesssim 0.1 \ldots 0.15$.}}
values of $\Omega_\varphi$ and $w_\varphi$ can be realized. In the above example, one finds $\Omega_{\varphi,0} = 0.70$ and $w_{\varphi,0} = -0.80$. Note that, due to the extended tracking period, the late cosmology is completely insensitive to the initial conditions. In the example of Fig. 1, the evolution starts at the Planck epoch with a total energy density $\rho_{\text{tot}} = 1.0$, $\varphi = 2.0$ and $\dot{\varphi} = 0$ (corresponding to $\Omega_\varphi = 0.14$). We have checked explicitly other initial conditions, e.g., with $\Omega_\varphi$ near one.

![Figure 1: Cosmological evolution with a leaping kinetic term. We show the fraction of energy in radiation ($\Omega_r$) and matter ($\Omega_m$) with $\Omega_\varphi = 1 - \Omega_r - \Omega_m$. The equation of state of quintessence is specified by $w_\varphi$.](image)

The present day value $w_\varphi$ can be forced to be even closer to $-1$ if the leap of $k(\varphi)$ is made sharper or the final value of $k$ is made higher by a simple generalization of Eq. (9). Thus, all scenarios between a smoothly rising quintessence contribution and a suddenly emerging cosmological constant can be realized.

As a limiting case of the sudden increase of $k(\varphi)$, one can consider models where $k(\varphi)$ has a singularity at a certain value of $\varphi$. For example, the function

$$k(\varphi) = k_{\text{min}} + (\varphi - \varphi_1)^{-2} \quad \text{(with } k_{\text{min}} = 0.1, \ \varphi_1 = 277.5),$$

leads to a cosmology very similar to the one displayed in Fig. 1. Note, however, that the potential, when rewritten in terms of $\chi = K(\varphi)$, approaches a constant non-zero value at $\chi \to \infty$. Thus, one could argue that a cosmological constant has, after all, been introduced in a hidden way. Nevertheless, the lagrangian with non-canonical kinetic term may open up new perspectives on the problem of sudden cosmic acceleration. In particular, it appears possible that the sudden rise of the kinetic coefficient is the result of some transition in the cosmic evolution which has a natural reason to occur in the present epoch.
3 Runaway quintessence

A somewhat different realization of a cosmology with late-time acceleration is obtained if the early history of the universe includes a prolonged period with small $k$. To illustrate this, consider the particularly simple function

$$k(\varphi) = k_{\text{min}} + b (\tanh(\varphi - \varphi_1) \tanh(\varphi - \varphi_2) + 1)$$

( with $k_{\text{min}} = 0.15$, $b = 0.25$, $\varphi_1 = 50.0$, $\varphi_2 = 254.8$ ),

which leads to the cosmological evolution of Fig. 2. As $\varphi$ increases, the coefficient $k(\varphi)$ (cf. the almost piecewise constant curve in Fig. 3) changes from the large initial value 0.65 to a smaller intermediate value 0.15 and back to the large value.

![Figure 2](image2.png)

**Figure 2**: Cosmological evolution with cosmon field running to very large values and mimicking a small cosmological constant.

![Figure 3](image3.png)

**Figure 3**: Two different kinetic coefficients $k(\varphi)$ leading to the runaway quintessence scenario.

During the initial period with large $k$, the universe inflates and the background energy density (i.e. radiation) becomes very small. When $k$ drops, $\varphi$ accelerates and its kinetic energy dominates the universe ($w_\varphi = 1$). Since kinetic energy density decays faster than the background component, the universe becomes radiation dominated after a certain
time, from which point on the value of $\varphi$ remains essentially frozen. At this time $V(\varphi)$ has already become very small and the cosmic evolution (including nucleosynthesis and structure formation) proceeds in the conventional way. However, at a certain moment (which is chosen to be approximately now by a moderate tuning of the parameters in Eq. (11)) the potential $V(\varphi)$ becomes relevant again. Since in the meantime $k$ has returned to its large value, a new scalar dominated epoch starts.

In the example of Fig. 2, the evolution begins at $\rho_{\text{tot}} = 1.0$, $\varphi = 2.0$ and $\dot{\varphi} = 0$. These initial conditions lead to $\Omega_{\varphi,0} = 0.5$ and $w_{\varphi,0} = -0.76$ today. Note that, in this example, no extended period of tracking exists. Therefore today’s cosmological parameters depend on the initial conditions. However, no extreme tuning of the lagrangian parameters or the initial conditions is required. Complete independence of initial conditions could be realized by introducing a period of tracking before the sequence of events illustrated in Fig. 2.

The same qualitative scenario can also be realized without any abrupt changes of $k$. For example, the function

$$k(\varphi) = k_{\min} + \left(\frac{\varphi - \varphi_1}{\varphi_2}\right)^2 \quad \text{(with } k_{\min} = 0.1, \varphi_1 = 152.4, \varphi_2 = 170.0) \quad (12)$$

(cf. the parabola in Fig. 3) gives rise to a cosmological evolution that is very similar to the one of Fig. 2, if the same initial conditions at the Planck epoch are used. The essential qualitative feature of $k(\varphi)$ is its small value during an intermediate period, so that $\varphi$ can run away to the large values that correspond to a tiny $V(\varphi)$.

## 4 Smoothly changing kinetic terms

In Sect. 2, late time acceleration was achieved by a relatively sudden change of the kinetic term. In Sect. 3, it was realized by an essentially frozen dark energy contribution which suddenly becomes the dominant component. Even though the latter does not require abrupt changes of the kinetic term, the whole cosmic evolution is far from smooth. In this section, we want to explore whether an interesting cosmology can be realized with a smooth function $k(\varphi)$ and a smooth, tracking evolution of $\varphi$.

An obvious problem arises from the necessity to produce enough structure in our universe. Assuming the approximate validity of the relation Eq. (7) during structure formation, one needs a value $k < 0.26$ to fulfil the condition $\Omega_{\varphi} < 0.2$. By contrast, we need $k > 0.41$ today to have, say, $\Omega_{\varphi} > 0.5$. Thus, while $\log_{10} a$ grows by about 3 units or less (which is a small fraction of its whole evolution from $\log_{10} a \simeq -30$ to $\log_{10} a = 0$), a significant change of $k$ has to occur. An even stronger rise of $k$ is necessary to account for an appreciable acceleration of the expansion today.

Within the approximate validity of Eq. (7) (with $k \to k(\varphi)$) one has

$$\frac{d k(\varphi)}{d\varphi} = \frac{1}{3\sqrt{3}} \frac{d}{d\ln a} \sqrt{\Omega_{\varphi}} \gtrsim 0.007. \quad (13)$$
Here the bound relates to the time of structure formation and corresponds to the change of $\Omega_\phi$ from 0.2 to 0.5 mentioned before. Together with the value $\varphi \simeq 250$ and $k = 0.26$ (at the borderline allowed for structure formation) this makes it obvious that this increase cannot be achieved by a linear rise of $k$ with $\varphi$. Cosmologies saturating the bound Eq. (13) will not lead to an accelerating universe today.

Let us next consider an exponential form of $k(\varphi)$,

$$k(\varphi) = \exp\left[\frac{(\varphi - \varphi_1)}{\alpha}\right], \quad (14)$$

which allows for a strong growth of $k(\varphi)$ during the cosmic evolution. The phenomenology arising from this functional form is, in fact, well known because the corresponding canonical lagrangian possesses a simple power-law potential:

$$V(\chi) = A\chi^{-\alpha}, \quad A = \alpha^\alpha \exp[-\varphi_1]. \quad (15)$$

If $\alpha$ is large, $k$ varies smoothly and $\varphi$ follows the growth of $\ln a : \Delta \varphi \simeq \Delta (\ln a)$. Thus, if $k$ changed by, say, a factor of two between $\log_{10} a = -3$ and today, one may roughly expect that it has changed by a factor of $2^{10} \simeq 1000$ since the Planck epoch. This violates our naturalness assumption. Smaller values of $\alpha$ exacerbate this dilemma. Only for $\alpha < 6$ acceleration can be realized [13]. In this case $A$ is a very small parameter when expressed in units of the Planck mass. In our language this situation is highly unnatural because the initial value of $k(\varphi)$ is very small, i.e., $k(0) \approx 10^{-20}$ for $\alpha = 6$.

Let us briefly mention a further interesting aspect of the model with exponential kinetic coefficient (or, equivalently, the power law potential for $\chi$). With an initial condition $\varphi \ll \varphi_1$, the potential energy $V(\varphi)$ is far above the tracking value at the beginning of the evolution (in [13] this is justified by an equipartition requirement). Therefore, initially $\varphi$ runs to very large values (and correspondingly small $V(\varphi)$) as in our runaway-scenario of Sect. [3]. If $\alpha$ is very small, the age of the universe is insufficient for $\varphi$ to return to tracking. Thus, the effect of the potential energy in late cosmology is similar to a cosmological constant: $\varphi$ is almost constant and $V(\varphi)$ very suddenly becomes the dominant component. This last possibility, which is quite attractive phenomenologically, suffers, however, from an “unnatural” tiny parameter in the lagrangian and a tuning of the initial conditions. In our opinion, the runaway scenario of Sect. [3] represents a viable alternative with similar characteristics for late cosmology.

Let us finally note that replacing the exponential form of Eq. (14) by a different function, e.g., $k \sim \varphi^2$, does not solve the problem. Unless the present era is effectively singled out as in Sect. [4], a rapidly growing $k(\varphi)$ implies an unnatural situation in the Planck era while a relatively flat $k(\varphi)$ fails to produce acceleration today.

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2Even in the case of small $\alpha$ no extremely small numbers appear directly in Eq. (14). The smallness of $k(\varphi)$ at the initial point $\varphi \simeq 0$ arises from the exponential factor $\exp[-\varphi_1/\alpha]$. This seems, however, to be only an optical improvement.
We have formulated a condition for a natural quintessence scenario: in the Planck era no extremely small parameter should appear neither in the effective action nor in the initial conditions. We have presented examples that realize this scenario and are consistent with present cosmological observations of a large fraction in homogeneous energy density and acceleration in the scale factor. Translating to a standard kinetic term our examples correspond to a relatively mild modification of exponential potentials. Some other popular quintessence scenarios, like the ones based on power law potentials with moderate powers, do not obey our naturalness criterion.

Despite the consistency of these scenarios with present day observations, we feel that two issues are not yet understood in a completely satisfactory way. The first one concerns the value of the potential for $\varphi \to \infty$. A modification of the potential into $e^{-\varphi} + \lambda$ would introduce an asymptotic cosmological constant. A tiny value of $\lambda$ is consistent with cosmological observations but incompatible with our naturalness criterion. We mention two proposals to answer the question why $\lambda$ is precisely zero in the quintessence context. One invokes the dilatation anomaly [8] and the other is based on a dynamical tuning mechanism [22] (see also [28]).

The second issue concerns the particular role of the present epoch. In all realistic scenarios we have found, the present time is characterized by a relatively sharp transition to a scalar dominated universe. Our era is singled out by this transition. The question “why now” is much less dramatic than for a cosmological constant: within quintessence the parameters in $k(\varphi)$ have to be tuned on the percent level in contrast to $10^{-120}$ for the cosmological constant. Nevertheless, a natural explanation of the special role of “today” would be very welcome. We can imagine two solutions to this puzzle. Either the present phenomenological constraints weaken such that smaller values of $\Omega_{\varphi}$ and $|w_{\varphi}|$ are allowed. Or some particularities of the present epoch may affect the dynamics. Some proposals are based on the change of the effective equation of state of the clustering dark matter at the end of radiation domination [19]. Another possibility is the coupling of the cosmon to clustering dark matter [8, 29] that would be ineffective during radiation domination.

In this context it is worthwhile to recall that the cosmon field $\varphi$ represents the average value of a fluctuating scalar field in a nonequilibrium, inhomogeneous universe. Thus, the potential $V(\varphi)$ and the kinetic coefficient $k(\varphi)$ are, in principle, themselves time-dependent dynamical quantities. Their time evolution is described by the time-dependent effective action of nonequilibrium field theory [30] that accounts for the nonequilibrium values of higher correlation functions. The present epoch is characterized by the onset of strong nonlinearities in the density fluctuations and therefore large higher correlation functions. Could this be the origin of the effective dynamics discussed in this work?
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