Nuclear $0\nu2\beta$ decays in B-L symmetric SUSY model and in TeV scale left–right symmetric model

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Abstract
In this paper, we take the B-L supersymmetric standard model (B-LSSM) and TeV scale left–right symmetric model (LRSM) as two representations of the two kinds of new physics models to study the nuclear neutrinoless double beta decays ($0\nu2\beta$) so as to see the senses onto these two kinds of models when the decays are taken into account additionally. Within the parameter spaces allowed by all the existing experimental data, the decay half-life of the nucleus $^{76}\text{Ge}$ and $^{136}\text{Xe}$, $T_{1/2}^{0\nu}$($^{76}\text{Ge}$, $^{136}\text{Xe}$), is precisely calculated and the results are presented properly. Based on the numerical results, we conclude that there is greater room for LRSM type models than for B-LSSM type models in foreseeable future experimental observations on the decays.

Keywords: B-LSSM, QCD correction, neutrinoless double beta decay, LRSM

(Some figures may appear in colour only in the online journal)

1. Introduction
Tiny but nonzero neutrino masses explaining neutrino oscillation experiments [1] are unambiguous evidence of new physics (NP) beyond the standard model (SM). It is because in SM there are only left-handed neutrinos so therefore the neutrinos can acquire neither Dirac masses nor Majorana masses. Hence to explore any mechanism inducing the tiny neutrino masses, as well as relevant phenomenology, is an important direction to search for NP. The simple extension of SM is to introduce three right-handed neutrinos in a singlet of the gauge group SU(2) additionally, where the neutrinos acquire Dirac masses, and to fit the neutrino oscillation and nuclear decay experiments as well as astronomy observations, the corresponding Yukawa couplings of Higgs to the neutrinos are requested so tiny as $\lesssim 10^{-12}$, that is quite unnatural.

However, neutrino(s) may acquire masses naturally by introducing Majorana mass terms in extended SMs. Once a Majorana mass term is introduced, certain interesting physics arise. One of the consequences is that the lepton-number violation (LNV) processes, e.g. the nuclear neutrinoless double beta decays ($0\nu2\beta$), etc may occur. Of them, the $0\nu2\beta$ decays are especially interesting, because they may tell us sensitively about the nature of the neutrinos, whether Dirac [2] or Majorana [3]. When the decays $0\nu2\beta$ are observed in experiments, most likely, the neutrinos contain Majorana components. Thus studying $0\nu2\beta$ decays is attracting special attention.

Nowadays there are several experiments running to observe the $0\nu2\beta$ decays, and the most stringent experimental bounds on the processes are obtained by GERDA [4, 5] and KamLAND-Zen [6, 7]. They adopt suitable approaches and nuclei such as $^{76}\text{Ge}$ and $^{136}\text{Xe}$ respectively. Now the latest experimental lower bound on the decay half-life given by GERDA experiments is $T_{1/2}^{0\nu} > 1.8 \times 10^{26}$ years (90% C.L.) for nucleus $^{76}\text{Ge}$ [8], and in the near future, the sensitivity can reach up to $10^{28}$ years [9]. For the nucleus $^{136}\text{Xe}$, the most stringent lower bound on the decay half-life is $T_{1/2}^{0\nu} > 1.07 \times 10^{26}$ years (90% C.L.) given by KamLAND-Zen [6],
and the corresponding future sensitivity can reach up to \(2.4 \times 10^{27}\) years [10]. Moreover, underground experiments PANDAX, CDEX etc. are originally designed for searching for WIMP dark matter, are also planning to seek the \(0\nu2\beta\) decays i.e. they may observe the \(0\nu2\beta\) decays with the sensitivity which at least will set a fresh lower bound.

In literature, there are a lot of theoretical analyses on the \(0\nu2\beta\) decays. The analyses are carried out generally by dividing the estimation of the \(0\nu2\beta\) decays into three ‘factors’: one is at quark level to evaluate the amplitude for the ‘core’ process \(d + d \rightarrow u + u + e + e\) of the decays; the second one is, from quark level to nucleon level, to involve the quark process into the relevant nucleon one i.e. the ‘initial’ quarks \(d\) and \(d\) involve into the two neutrons in the initial nucleus and the ‘final’ quarks \(u\) and \(u\) involve into the two protons in the final nucleus; the third one is, from nucleon level to nucleus level, the relevant nucleons involve in the initial nucleus and the final nucleus properly. For the ‘core’ process, in [11] a general Lorentz-invariant effective Lagrangian is constructed by dimension-9 operators, and in [12, 13] the QCD corrections to all of these dimension-9 operators are calculated. In [14] the short-range effects at quark level are considered, the analyses of the decay rates in the SM effective field theory are presented in [15–17], the \(0\nu2\beta\) decay rates are derived in [18], the corresponding nuclear matrix elements (NME) and phase-space factors (PSF) for the second and the third factors of the decays i.e. from quark level to nucleon and nucleus levels, are considered in [19–29], some theoretical predictions on the \(0\nu2\beta\) for certain models are presented in [30–34], and the theoretical analyses on the decays are reviewed in [35–37].

In this work, we are investigating the constrains from the \(0\nu2\beta\) decays for the B-L supersymmetric model (B-LSSM) and for the TeV scale left–right symmetric model (LRSM) comparatively. It is because the two models are typical: both have an LNV source but the mechanisms which give rise to the Majorana mass terms are different [38–51]. In the B-LSSM, the tiny neutrino masses are acquired naturally through the so-called type-I seesaw mechanism which is proposed firstly by Weinberg [52]. In the LRSM [53, 54], the tiny neutrino masses are acquired by both type-I and type-II seesaw mechanisms, in addition, the new right-handed gauge bosons \(W_R\) is introduced in this model, then both left-handed and right-handed currents cause the \(0\nu2\beta\) decays [55–72]. As a result, the computations of the decays are much more complicated in the LRSM than those in the B-LSSM. Hence these two models, being representatives of NP models, are typical for the \(0\nu2\beta\) decays, and one may learn the mechanisms in the models well via analyzing the \(0\nu2\beta\) decays comparatively.

In the study here, we will mainly focus on the first ‘factor’ about the quark level i.e. the core process, which relates to the applied specific model closely. We will evaluate the Wilson coefficients of the operators relevant to the core process \(d + d \rightarrow u + u + e + e\) etc. on the models, whereas the estimation of the other two ‘factors’, i.e. to evaluate ‘NME’ and ‘PSF’ etc, we will follow the literature [29–33]. With respect to the ‘core’ process \(d + d \rightarrow u + u + e + e\), all of the contributions in the B-LSSM can be deduced directly quite well, while the contributions in LRSM cannot be so. As shown in [73], the calculations in the LRSM are much more complicated and the interference effects are quite hard to be considered well. In this work, a new approximation, i.e. the momenta of the two involved quarks inside the initial or final nuclei is tried to be set equal, is made so as may reduce all contributions in the LRSM quite similar to the case of B-LSSM. Then the calculations in LRSM are simplified quite a lot and under the approximation, the interference effects can be treated comparatively well. Finally, for comparison, we also present the results obtained by the traditional method [65].

The paper is organized as follows: In section 2, for B-LSSM, the seesaw mechanisms which give rise to the tiny neutrino masses, the heavy neutral leptons as well, the relevant interactions etc, the calculations of the \(0\nu2\beta\) decay half-lives of the nuclei are given. Similarly, in section 3, for LRSM, the seesaw mechanisms which give rise to the tiny neutrino masses, as well the heavy neutral leptons, the relevant interactions, and the calculations of the \(0\nu2\beta\) decay half-lives of the nuclei are given. In sections 4.1 and 4.2 the numerical results for B-LSSM and LRSM are presented respectively. Finally, in section 5 brief discussions and conclusions are given. In the appendix, the needed QCD corrections to the effective Lagrangian which contains the dimension-9 operators in the region from the energy scale \(\mu \approx M_W\) to the energy scale \(\mu \approx 1.0\) GeV are collected.

2. The B-LSSM for \(0\nu2\beta\) decays

In the B-LSSM, the local gauge group is \(SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}\), where \(B, L\) denote the baryon number and lepton number respectively, and the details about the gauge fields, their breaking, extra lepton and Higgs fields etc can be found in [42–51]. In this model, tiny neutrino masses are acquired by the so-called type-I seesaw mechanism, when the \(U(1)_{B-L}\) symmetry is broken spontaneously by the two \(U(1)_{B-L}\) singlet scalars (Higgs). The mass matrix for neutrinos and neutral heavy leptons in the model can be expressed as

\[
\begin{pmatrix}
0 & M_D^T \\
M_D & M_R
\end{pmatrix}
\]

and the mass matrix can be diagonalized in terms of a unitary matrix \(U_v\), as follows:

\[
U_v^T \begin{pmatrix}
0 & M_D^T \\
M_D & M_R
\end{pmatrix} U_v = \begin{pmatrix}
\hat{m}_\nu & 0 \\
0 & \hat{M}_N
\end{pmatrix},
\]

where the neutrino masses \(\hat{m}_\nu = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})\), the masses of the heavy neutral leptons \(\hat{M}_N = \text{diag}(M_{N_1}, M_{N_2}, M_{N_3})\) and \(U_v\) is a matrix of \(6 \times 6\) which can be rewritten as

\[
U_v = \begin{pmatrix}
U & S \\
T & V
\end{pmatrix}
\]

where \(U, S, T, V\) are matrices of \(3 \times 3\).
The interactions, being applied later on, in the model are
\[
\mathcal{L}_I = \frac{ig_2}{\sqrt{2}} \sum_{j=e,\mu} | U_{\nu_j \nu} |^2 \bar{\nu}_j \gamma^\mu P_L \nu \frac{\not{e} \not{W}_L}{m_\nu} + S_j \bar{\nu}_j \gamma^\mu \nu \frac{\not{N} \not{W}_L}{m_N} + \text{h.c.}, \tag{4}
\]
where \( \nu, N \) are the four-component fermion fields of the light and heavy neutral leptons respectively.

The Feynman diagrams which are responsible for the dominated contributions to the 0\( \nu / 2\beta \) decays in the B-LSSM are plotted in figure 1. Note here that the contributions from charged Higgs exchange(s) are ignored safely as they are highly suppressed by the charged Higgs masses and Yukawa couplings, thus in figure 1 the charged Higgs exchange does not appear at all. To evaluate the contributions corresponding to the Feynman diagrams for the 0\( \nu / 2\beta \) decays, and to consider the roles of the neutral leptons (light neutrinos and heavy neutral leptons) in the Feynman diagrams, the useful formulae are collected below so as to deal with the neutrino propagator sandwiched by various chiral project operators \( P_{L,R} = \frac{1}{2}(1 \pm \gamma_5) \):

\[
\begin{align*}
P_L \frac{\not{f} \not{P}_L}{m} &= \frac{m}{k^2 - m^2} P_L, \\
P_R \frac{\not{k} \not{P}_R}{m^2} &= \frac{m}{k^2 - m^2} P_R, \\
P_L \frac{\not{f} \not{P}_R}{m} &= \frac{f}{k^2 - m^2} P_R, \\
P_R \frac{\not{k} \not{P}_L}{m^2} &= \frac{k}{k^2 - m^2} P_L.
\end{align*}
\]

Relating to the exchanges of the heavy neutral leptons (the virtual neutral lepton momentum \( k \) has \( |k| \approx 0.10 \text{ GeV} \ll M_N \)) for the decays, from figure 1(a) the effective Lagrangian at the energy scale \( \mu \approx M_N \) can be read out as

\[
\begin{align*}
\frac{2m_p}{G_F^2 \cos \theta_C} & \mathcal{L}^{\text{eff}}_{\beta}(N) = \sum_i \frac{2m_{\nu_i}}{M_N} \bar{\nu}_{\gamma} P_L d \bar{e} P e e^\nu \\
& \equiv C^{LL}_{N}(N) O^{LL}_{N}, \\
C^{LL}_{N}(N) &= \sum_i \frac{2m_{\nu_i}}{M_N} (S_{1i})^2, \\
O^{LL}_{N} &= 8 \bar{\nu}_{\gamma} P_L d \bar{e} P e e^\nu, \tag{7}
\end{align*}
\]
where \( X, Y, Z = L, R, \theta_C \) is the Cabibbo angle, \( m_p \) is proton mass introduced for normalization of the effective Lagrangian, and \( S_{1i} \) is the matrix elements in equation (3).

Since the nuclear 0\( \nu / 2\beta \) decays take place at the energy scale of about \( \mu \approx 0.10 \text{ GeV} \), obviously we need to consider the QCD corrections for the effective Lagrangian obtained at the energy scale \( \mu \approx M_N \) in equation (7) i.e. to evolve the effective Lagrangian in terms of renormalization group equation (RGE) method from the energy scale \( \mu \approx M_N \) to that \( \mu \approx 1.0 \text{ GeV} \) first, where the corrections are in perturbative QCD (pQCD) region. Thus, for completeness, the QCD corrections to all of the possible dimension-9 operators which may contribute to the nuclear 0\( \nu / 2\beta \) decays, are calculated by the RGE method, and the details of computations are collected in the appendix. Whereas the QCD corrections in the energy scale region \( \mu \approx 1.0 \text{ GeV} \sim \mu \approx 0.10 \text{ GeV} \), being in the non-perturbative QCD region, we take them into account by inputting the experimental measurements for the relevant current matrix elements of nucleons, which emerge when calculating the amplitude based on the effective Lagrangian at \( \mu \approx 0.10 \text{ GeV} \).

For the decays when considering contributions from the neutrino (\( m_\nu \ll |k| \)) exchanges as described by the Feynman diagram figure 1(b), and the interferences between the light neutrinos’ and heavy neutral leptons’ contributions, to derive the effective Lagrangian for the neutrino contributions is better at the energy scale \( \mu \approx 1.0 \text{ GeV} \) as heavy neutral leptons too. At this
energy scale the effective Lagrangian can be written down according to the Feynman diagram figure 1(b) as below:

\[
\frac{2m_{\nu}}{G^2 v^2} \rho_{\nu \nu}^{\text{eff}}(\nu) = \frac{m_{\nu}}{m_e} (U_1)_{ij}^2 \times \frac{2m_{\nu} m_e}{-k^2} \Omega_{3k}^{\nu}(\nu) \frac{2m_{\nu} m_e}{-k^2} \Omega_{3k}^{\nu},
\]

\[
C_{3k}^{\nu}(\nu) = \frac{m_{\nu}}{m_e} (U_1)_{ij}^2.
\]

(8)

Since the light neutrino exchange is of long range, the QCD corrections in the region \( \mu \approx 1.0 \) GeV \( \sim \mu \approx 0.10 \) GeV to the coefficients in equation (8) may be involved like in the above case for the heavy neutral lepton, via inputting the experimental measurements for the relevant current matrix elements of nucleons, which emerge when calculating the matrix elements in the amplitudes at \( \mu \approx 0.10 \) GeV.

To evaluate the half-life \( T^{0\beta}_{1/2} = \ln \frac{2}{\Gamma} \) of the \( 0\nu\beta \) decays, the contributions from the heavy neutral leptons (figure 1(a)) and those from the neutrinos (figure 1(b)) should be summed up for the amplitudes. The half-life \( T^{0\beta}_{1/2} = \ln \frac{2}{\Gamma} \) of the \( 0\nu\beta \) decays can be written as [28]

\[
\frac{1}{T^{0\beta}_{1/2}} = G^{0\nu}[M^{0\nu}]^2 \frac{m_{\nu}^{\text{BL}}}{m_e},
\]

(9)

where \( G^{0\nu} = 2.36 \times 10^{-15} \) (14.56 \times 10^{-15} yr \(^{-1} \) [28] for \( \gamma^2(\text{Ge}) \) for the PSF, \( M^{0\nu} = -6.64 \pm 1.06 \) (-3.60 \( \pm \) 0.58) [19, 28] for \( \gamma^2(\text{Ge}) \)) is the NME corresponding to the long range contributions which is defined as

\[
M^{0\nu} \equiv \langle \mathcal{O}_\nu^\gamma | \frac{2m_{\nu} m_e}{-k^2} | 4(\bar{u}\gamma_\mu P_L d) \times (\bar{u}\gamma_\nu P_L d) | \mathcal{O}_\nu^\gamma \rangle
\]

(10)

with |\( \mathcal{O}_\nu^\gamma \rangle\), |\( \mathcal{O}_\nu^\gamma \rangle\) denoting the initial and final nuclear states respectively. Note that in equation (10) that the factor \( \frac{2m_{\nu} m_e}{-k^2} \) in equation (8) is absorbed into the so-called ‘neutrino potential’ which is used to compute the long range NME. And

\[
m_{\nu}^{\text{BL}} \equiv U_{\text{eX}}^{\text{XX}} C_{3k}^{\nu}(N) \frac{M^{\text{XX}}(N)}{M^{0\nu}} + C_{3k}^{\nu}(\nu),
\]

(11)

where \( U_{\text{eX}}^{\text{XX}} \) is the QCD running factor from \( \mu \approx M_{\nu e} \) to \( \mu \approx 1.0 \) GeV (the numerical result of \( U_{\text{eX}}^{\text{XX}} \) can be found in equation (A60)), \( M^{\text{XX}}(N) = -200 \pm 56(-111 \pm 31.08) \) [19, 28] for \( \gamma^2(\text{Ge}) \)) is the NME corresponding to short range contributions which is defined as

\[
M^{\text{XX}}(N) \equiv \langle \mathcal{O}_\nu^\gamma | \{4(\bar{u}\gamma_\nu P_L d)(\bar{d}\gamma_\nu P_L d)\} | \mathcal{O}_\nu^\gamma \rangle.
\]

(12)

3. The LRSM

For the model LRSM, the gauge fields are \( SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \) and the details about the gauge fields and their breaking can be found in [38–41]. In this model, the tiny neutrino masses are obtained by both of type-I and type-II seesaw mechanisms due to introducing the right-handed neutral leptons and two triplet Higgs (scalars) accordingly. In the model, the mass matrix for the neutral leptons generally is written as

\[
\left( \begin{array}{c} M_e \cr M_\nu \cr M_R \end{array} \right)
\]

(13)

and the mass matrix equation (13) \(^5\) can be diagonalized in terms of a unitary matrix \( U_\nu \), whereas the matrix \( U_\nu \) can be expressed similarly as that in the case of the B-LSSM equation (3).

For the model LRSM, if the left–right symmetry is not broken manifestly but spontaneously, i.e. \( g_L = g_R \equiv g_3 \) and as one consequence, the mass terms of \( W \) bosons can be written as

\[
\mathcal{L}_{M_W} = \frac{g_3^2}{4} \left( W_+^\nu, W_-^\nu \right) \times \left( \begin{array}{c} v_1^2 + v_1^2 + 2 v_1 v_2 \cr 2 v_1 v_2 \cr v_1^2 + v_2^2 + 2 v_2^2 \end{array} \right) \left( \begin{array}{c} W_L^\pm \cr W_R^\pm \end{array} \right),
\]

(14)

where \( v_1, v_2, \nu_L, \nu_R (v_L \ll v_R) \) are the VEVs of new scalars (Higgs) in the LRSM. Then the physical masses of the \( W \) bosons can be obtained [72]

\[
M_{W_L} \approx \frac{g_3^2}{2} (v_1^2 + v_2^2)^{1/2},
\]

\[
M_{W_L} \approx \frac{g_3^2}{2} v_R.
\]

The mass eigenstates \( W_{L,R}^\pm \) are related to the interaction eigenstates \( W_{L,R}^\pm \) by \( \zeta \)

\[
\left( \begin{array}{c} W_{L}^\pm \cr W_{R}^\pm \end{array} \right) = \left( \begin{array}{cc} \cos \zeta, & \sin \zeta \cr -\sin \zeta, & \cos \zeta \end{array} \right) \left( \begin{array}{c} W_{L}^\pm \cr W_{R}^\pm \end{array} \right),
\]

(16)

where \( \tan \zeta = \frac{2v_0v_2}{v_1^2 - v_2^2} \).

The interactions, being applied later on, in the model are

\[
\mathcal{L}^{\nu_{LRSM}} = \frac{i g_3}{\sqrt{2}} \sum_{j=1}^3 e_j \langle \bar{\nu_j} (\cos \theta_j / \mu) p_L + \sin \theta_j \gamma_5 \nu_j / \mu \rangle \left( \begin{array}{c} \bar{e}_j \langle \cos \theta_j / \mu \rangle p_R - \sin \theta_j \gamma_5 \nu_j / \mu \rangle \langle \bar{p}_j \rangle \left( \begin{array}{c} \bar{w}_{j R}^+ \cr \bar{w}_{j L}^+ \end{array} \right)
\]

\[
+ \bar{e}_j \langle \cos \theta_j / \mu \rangle p_R + \sin \theta_j \gamma_5 \nu_j / \mu \rangle \langle \bar{p}_j \rangle \left( \begin{array}{c} \bar{w}_{j R}^- \cr \bar{w}_{j L}^- \end{array} \right)
\]

\[
+ \bar{e}_j \langle \cos \theta_j / \mu \rangle p_R + \sin \theta_j \gamma_5 \nu_j / \mu \rangle \langle \bar{p}_j \rangle \left( \begin{array}{c} \bar{w}_{j R}^+ \cr \bar{w}_{j L}^- \end{array} \right)
\]

\[
+ \bar{e}_j \langle \cos \theta_j / \mu \rangle p_R + \sin \theta_j \gamma_5 \nu_j / \mu \rangle \langle \bar{p}_j \rangle \left( \begin{array}{c} \bar{w}_{j R}^- \cr \bar{w}_{j L}^+ \end{array} \right) + \bar{u} \left( \langle \cos \theta_j / \mu \rangle p_R + \sin \theta_j \gamma_5 \nu_j / \mu \rangle \langle \bar{p}_j \rangle \right) \left( \begin{array}{c} \bar{w}_{j R}^+ \cr \bar{w}_{j L}^- \end{array} \right) + \bar{u} \left( \langle \cos \theta_j / \mu \rangle p_R + \sin \theta_j \gamma_5 \nu_j / \mu \rangle \langle \bar{p}_j \rangle \right) \left( \begin{array}{c} \bar{w}_{j R}^- \cr \bar{w}_{j L}^+ \end{array} \right) + \text{h.c.},
\]

(17)

\(^4\) The NMEs adopted here are obtained within the framework of the microscopic interacting meson model for nuclei [28], and the uncertainties for NMEs calculations from the various nuclear structure models are quite wild, here we remind only that the NMEs obtained by various approaches are varying by a factor of (2–3) roughly [74].

\(^5\) The matrix equation (13) with \( M_{\nu} = 0 \) indicates the masses are acquired by type-I seesaw mechanism; it with \( M_{\nu} = 0 \) indicates the masses are acquired by type-II seesaw mechanism; in general feature indicates the masses are acquired by type-I+II seesaw mechanism.
where the definitions for \( U, S, T, V, \nu, N \) are the same as the ones in the B-LSSM.

In the LRSM, the dominant contributions to the \( 0/2/3 \) decays are represented by Feynman diagrams figure 2.

In comparison with those of the B-LSSM, the contributions from the Higgs exchanges can be ignored so in figure 2 there is no Higgs exchange at all, but besides the left-handed gauge boson components \( W_L^\pm \), there are right-handed components \( W_R^\pm \) in \( W_L^\pm \) and \( W_R^\pm \) gauge bosons. Therefore the situation in determining the effective Lagrangian for the \( 0/2/3 \) decays is different and comparatively complicated than that for the B-LSSM. In the case of the heavy neutral lepton exchanges shown in figure 2 (a), considering the fact that the heavy neutral leptons propagator \( m_{\nu}^{-2} + \frac{V_{\nu}}{m_N} \approx \frac{1}{m_N} (M_N \geq M_W) \) and \( s_\nu, t_\nu \ll 1 \) in the decays, by using equations (5), (6), the effective Lagrangian at the energy scale \( \mu \approx M_W \) (\( W_1 \) is the lighter one boson between \( W_{1,2} \)) can be written down as follows

\[
\begin{align*}
\frac{2m_p}{G_F^2 \cos \theta_C^2} L^{LR}_{\nu}(N) &= C^{LL}_{\nu}(N) O^{LL}_{\nu} + C^{LR}_{\nu}(N) O^{LR}_{\nu}, \\
S^{LL}_{\nu}(N) &= \frac{2m_p}{M_N} \cos^2 \zeta S^{LL}_{\nu}, \\
S^{RR}_{\nu}(N) &= \frac{2m_p}{M_N} \cos^2 \zeta \sin^2 \zeta V^{LL}_{\nu}, \\
S^{RL}_{\nu}(N) &= \frac{2m_p}{M_N} \cos^2 \zeta \sin \zeta V^{RR}_{\nu}. 
\end{align*}
\]

In the literature, due to small \( \zeta \), the contributions corresponding to \( C^{RL}_{\nu}(N), C^{LL}_{\nu}(N) \) in equation (18) are neglected. However, since \( \tan 2\zeta = \frac{2 \sin \zeta}{\cos \zeta} \), i.e. \( \zeta \approx \frac{m_{\nu}}{M_W} / \frac{m_{\nu}}{M_W} \), and when \( x \equiv v_2/v_1 > 0.02 \) [75], the terms with \( C^{RL}_{\nu}(N) \) and \( C^{LL}_{\nu}(N) \) can also make essential contributions compared with the terms with \( C^{RL}_{\nu}(N), C^{LL}_{\nu}(N) \). Thus in this work, when evaluating the \( 0/2/3 \) decays we would like to keep the contributions from the terms of \( C^{RL}_{\nu}(N), C^{LL}_{\nu}(N) \), and consider the QCD corrections to the effective Lagrangian equation (18) in a similar way as that in the B-LSSM.

As the next step, when considering the contributions from the light neutrino exchanges as figure 2(b), owing to the fact that \( W_{1,2}^\pm \) contain both components \( W_{L,R}^\pm \) in LRSM, according to equations (5), (6), the light neutrino propagators with chiral project operators \( P_{L,R} \) are new and substantial, and in B-LSSM they do not appear at all. Moreover when the contributions from the ‘higher order’ terms for sine \( \zeta, S_\nu, T_\nu \), such as those small terms proportional to \( \sin^2 \zeta, S_\nu, T_\nu \) etc. are ignored, then the effective Lagrangian at the energy scale \( \mu \approx 1.0 \) GeV may be read out from figure 2(b) as

\[
\frac{2m_p}{G_F^2 \cos \theta_C^2} L^{LR}_{\nu}(\nu) = \cos^4 \zeta U_{11}^2 m_{\nu} \frac{2m_p m_{\nu}}{m_{\nu}} O^{LL}_{\nu} + \cos \zeta U_{11}^2 \frac{2m_p m_{\nu}}{m_{\nu}} \cos \zeta V^{LL}_{\nu} \times \cos \zeta U_{11}^2 \frac{2m_p m_{\nu}}{m_{\nu}} \cos \zeta V^{RR}_{\nu}.
\]

The QCD corrections in the energy scale region \( \mu \approx 1.0 \) GeV to \( \mu \approx 0.10 \) GeV, being of non-perturbative QCD, are taken into account by inputting in the experimental measurements for the relevant current matrix elements of nucleons, which emerge at the effective Lagrangian at \( \mu \approx 0.10 \) GeV.
In [55–66], the second term and the third term of equation (19) are defined as \(\eta, \lambda\) respectively. Extracting the factors

\[
C_\eta = \cos^3 \zeta \sin \zeta U_{1i} T_{1i}^i, \\
C_\lambda = \cos^4 \zeta U_{1i} T_{1i}^i \frac{M_{b_i}^2}{M_{u_i}^2},
\]

(20)

then the operators \([4(\bar{\nu}_\mu P_L d)(\bar{\nu}_\mu P_L d) e^{\mu \nu} \gamma^\mu \gamma^\nu e^e], [4(\bar{\nu}_\mu P_L d)(\bar{\nu}_\mu P_L d) e^{\mu \nu} \gamma^\mu \gamma^\nu P_R e^e]\) are attributed to the calculations of NME and PSF.

Whereas calculating the NMEs and PSF, the interference effects among the contributions, especially to consider the contributions from the factors \(\frac{m}{2} P_{LR}\) for the light neutrino exchanges, are complicated and hard (in the literature, to treat them even the Lorentz covariance is lost [76]). In this work to calculate NMEs and PSF, we try to make an additional approximation on the contributions relevant to the factors \(\frac{m}{2} P_{LR}\) for the light neutrino exchanges, which we call a ‘frozen approximation’. Under the approximation, the momenta of the two involved quarks inside the initial nucleus and two involved quarks inside the final nucleus (figure 2) are assumed to be equal approximately:

\[
p_1 \simeq p_2 \equiv \bar{p}, \quad k_1 \simeq k_2 \equiv \bar{k}.
\]

(21)

With the ‘frozen approximation’ and the ‘on-shell approximation’ onto the momenta for the out legs as well, all contributions corresponding to figure 2(b) can be well-deduced and the final results can be collected as

\[
\frac{2m_p}{G_F^2 \cos \theta_C^l} \epsilon_{\text{eff}}^{LR}(\nu) = \frac{2m_p m_e}{k^2} \left[ C_{53}^{LR}(\nu) \epsilon_{33}^{LR} + C_{63}^{LR}(\nu) \epsilon_{63}^{LR} \right]
\]

(22)

where

\[
\epsilon_{33}^{XY}(\nu) = 4(\bar{\nu}_\mu P_L d)(\bar{\nu}_\mu P_L d) e^{\mu \nu} \gamma^\mu \gamma^\nu e^e,
\]

\[
C_{53}^{LR}(\nu) = \frac{1}{m_e} \cos^3 \zeta U_{1i} T_{1i}^i (m_i \cos \zeta_{U_{1i}} - m_e \sin \zeta_{T_{1i}^i}),
\]

\[
C_{63}^{LR}(\nu) = -\cos^3 \zeta \sin \zeta (U_{1i} T_{1i}^i),
\]

\[
C_{53}^{RL}(\nu) = C_{63}^{RL}(\nu) = -\frac{1}{2} \cos^4 \zeta U_{1i} T_{1i}^i \left( \frac{M_{w}}{M_{u_i}} \right)^2,
\]

\[
C_{53}^{RR}(\nu) = C_{63}^{RR}(\nu) = -m_p m_e \cos^3 \zeta \sin \zeta U_{1i} T_{1i}^i \left( \frac{M_{w}}{M_{u_i}} \right)^2,
\]

\[
C_{53}^{LR}(\nu) = -C_{63}^{LR}(\nu) = m_p m_e \cos^3 \zeta \sin \zeta U_{1i} T_{1i}^i \left( \frac{M_{w}}{M_{u_i}} \right)^2,
\]

The half-life of 0\(\nu/\beta\) decays can be written as [28]

\[
\frac{1}{T_{1/2}^{0\nu}} = G_{0\nu}^2 |M_{0\nu}^R|^2 \left( \frac{m_{\nu e}^{LR}}{m_{\nu e}} \right)^2,
\]

(24)

where

\[
m_{\nu e}^{LR} = m_e \left[ C_{53}^{RR}(\nu) + C_{63}^{RR}(\nu) \right] + C_{33}^{LL}(\nu) U_{13}^{XY} M_{33}^{XY}(N) \frac{M_{0\nu}}{M_{0\nu}}
\]

\[
+ C_{53}^{RL}(\nu) U_{13}^{XY} M_{33}^{XY}(N) \frac{M_{0\nu}}{M_{0\nu}}
\]

\[
+ C_{63}^{RL}(\nu) \left[ C_{53}^{RR}(\nu) + C_{63}^{RR}(\nu) \right] \left( \frac{M_{0\nu}}{M_{0\nu}} \right)^2
\]

\[
+ \left[ C_{53}^{LL}(\nu) U_{13}^{XY} M_{33}^{XY}(N) + C_{63}^{LL}(\nu) U_{13}^{XY} M_{33}^{XY}(N) \right] \frac{M_{0\nu}}{M_{0\nu}}
\]

\[
+ C_{53}^{RL}(\nu) U_{13}^{XY} M_{33}^{XY}(N) \frac{M_{0\nu}}{M_{0\nu}}
\]

\[
+ C_{63}^{RL}(\nu) \left[ C_{53}^{RR}(\nu) + C_{63}^{RR}(\nu) \right] \left( \frac{M_{0\nu}}{M_{0\nu}} \right)^2
\]

\[
\left[ C_{53}^{RR}(\nu) U_{13}^{XY} M_{33}^{XY}(N) + C_{63}^{RR}(\nu) \right] \left( \frac{M_{0\nu}}{M_{0\nu}} \right)^2
\]

(25)

where \(U_{13}^{XY}(N)\) is the \(2 \times 2\) RGE evolution matrix from \(\mu \approx M_{33}\) to \(\mu \approx 1.0\) GeV (the numerical result of \(U_{13}^{XY}\) can be found in equation (A60), \(C_{53}^{XY}(N, \nu)\) is the coefficient defined in equation (23), \(G_{0\nu}^{(1)} = -0.28 \times 10^{-15}(-1.197 \times 10^{-15})\) years\(^{-1}\) [28] for \(76\)Ge(\(^{136}\)Xe) is the PSF, and \(M_{33}^{XY}(\nu) = 4.24 \pm 0.68(2.17 \pm 0.35)\), \(M_{33}^{XY}(N) = 99.8 \pm 27.94(51.2 \pm 14.34)\) [19, 28] for \(76\)Ge(\(^{136}\)Xe) are the NMEs corresponding to the exchange of light neutrinos, heavy neutral leptons respectively which are defined as

\[
M_{33}^{XY}(\nu) \equiv \langle O_{33}^{XY} | \frac{2m_p}{k^2} [4(\bar{\nu}_\mu P_L d)(\bar{\nu}_\mu P_L d)] \langle O_{33}^{XY} \rangle,
\]

\[
M_{33}^{XY}(N) \equiv \langle O_{33}^{XY} | [4(\bar{\nu}_\mu P_L d)(\bar{\nu}_\mu P_L d)] \langle O_{33}^{XY} \rangle.
\]

(26)

Similar to the case of B-LSSM, the factor \(\frac{2m_p m_e}{k^2}\) in equation (22) is absorbed into the ‘neutrino potential’ which is used to compute the long range NME. In addition, we should note that in equation (25) (the terms of \(C_{53}^{RR}(\nu), C_{53}^{LL}(\nu), C_{53}^{RL}(\nu)\) in equation (23) do not make contributions to the 0\(\nu/\beta\) decays for the reason below. They make contributions to 0\(\nu/3\) decays in the form

\[
M_{33}^{XY}(\nu) | C_{53}^{RR}(\nu) + C_{63}^{RR}(\nu) | + M_{33}^{XY}(\nu) [C_{53}^{RL}(\nu) + C_{63}^{RL}(\nu)],
\]

(27)
4. Numerical results

With the formulas obtained above, now in this section, we do the numerical calculations and present the results on the $0\nu2\beta$ decays for the nuclei $^{76}$Ge and $^{136}$Xe accordingly. In our numerical calculations, the parameters are taken as the weak boson mass: $M_W = 80.385$ GeV for B-LSSM and $M_W = 80.385$ GeV for LRSM, $m_b = 4.65$ GeV for b-quark mass, $m_c = 1.275$ GeV for c-quark mass, $\alpha(m_b) = 1/128.9$ for the coupling of the electromagnetic interaction, $\alpha_s(m_Z) = 0.118$ for the coupling of the strong interaction. The constraints from available experimental data, such as the most stringent upper limit on the sum of neutrino masses by PLANK [77] $\sum m_\nu < 0.12$ eV; the neutrino mass-squared differences obtained via analyzing the solar and atmospheric neutrino oscillation data at 3$\sigma$ deviations [77]

$$\Delta m^2_{12} \equiv m^2_{\nu_1} - m^2_{\nu_2} = (7.4 \pm 0.61) \times 10^{-5} \text{ eV}^2,$$

$$\Delta m^2_{23} \equiv m^2_{\nu_2} - m^2_{\nu_3} \approx (2.526 \pm 0.1) \times 10^{-3} \text{ eV}^2 \quad (\text{NH})$$

$$\Delta m^2_{31} \equiv m^2_{\nu_1} - m^2_{\nu_3} \approx (2.508 \pm 0.1) \times 10^{-3} \text{ eV}^2 \quad (\text{IH}),$$

and phase space factor of the chosen nuclei. However these factors appear in the expression of $m_{\nu_{\text{ee}}}^{LR}$ in the form of some ratios, and these ratios for $^{76}$Ge are equal to those for $^{136}$Xe roughly, hence we can use the same expression of $m_{\nu_{\text{ee}}}^{LR}$ for $^{76}$Ge, $^{136}$Xe in the analyses later on.

4.1. The numerical results for the B-LSSM

According to the above analysis, in B-LSSM the contributions from the heavy neutral leptons shown in equation (3) are highly suppressed ($S_{ij} \approx 10^{-7}$) for the TeV-scale heavy neutral leptons, hence the dominant contributions to the $0\nu2\beta$ decays come from the light neutrinos. Then with the neutrino mixing parameters

$$s^2_{12} = (0.275 \sim 0.35), \quad s^2_{23} = (0.02044 \sim 0.02437),$$

$$\alpha = (0 \sim 2\pi), \quad \beta = (0 \sim 2\pi)$$

and the neutrino mass-squared differences as those in equation (29) at 3$\sigma$ error level, $m_{\nu_{\text{ee}}}^{BL}$ versus $m_{\nu_{\text{lightest}}}$ (the lightest neutrino mass) for the B-LSSM is plotted in figure 3, where the
green (red) points denote the NH (IH) results, the blue (purple) solid line denotes the constraints from the lower $0\nu2\beta$ decay half-life bound of $^{76}\text{Ge}$ ($^{136}\text{Xe}$), the blue (purple) dashed line denotes the experimental ability of $^{76}\text{Ge}$ ($^{136}\text{Xe}$) for the next generation of experiments, the orange solid (dashed) line denotes the constraints from PLANK 2018 for NH (IH) neutrino masses (the meaning for the red points, green points, blue lines, purple lines, orange lines is also adopted accordingly in the figures later on). In figure 3, $m_{ee}^{BL}$ is well below the experimental upper bounds in the cases of NH and IH, and there is not tighter restriction on the $m_{ee}^{\text{lightest}}$ than that offered by PLANK. Additionally, the blue dashed line shows that there is certainly opportunity to observe the $0\nu2\beta$ decays in the next generation of experiments, whereas if any of the decays is not observed in the next generation of experiments, then the IH neutrino masses will be excluded completely by the blue dashed line in figure 3.

4.2. The numerical results for the LRSM

In the LRSM owing to the bosons $W_L$, $W_R$ and their mixing, the situation is much more complicated than that in the B-LSSM, and both the light neutrinos and heavy neutral leptons make substantial contributions to the $0\nu2\beta$ decays. For TeV-scale heavy neutral leptons in the LRSM, the consequences of the type II seesaw dominance are similar to those of the type I seesaw dominance, while the consequences are very different from the ones of type I+II seesaw dominance, because the light-heavy neutral lepton mixing is not much suppressed in this case. Hence we do the numerical computations for the type I and type I+II seesaw dominance cases in our analyses. For simplicity and not losing general features, we assume that there is only one heavy neutral lepton making substantial contributions to the model. It indicates that there are no off-diagonal elements in the matrix $M_R$ in equation (13), i.e. we have $M_R \approx M_N = \text{diag}(M_{N_1}, M_{N_2}, M_{N_3})$. Then under the dominance of either the type I seesaw or the type I+II seesaw mass mechanisms, we compute $m_{ee}$ numerically in turn.

4.2.1. The results under type I seesaw dominance. Under the type I seesaw dominance and due to the tiny neutrino masses, the sub-matrix $T$ in equation (3) has $T_{jj} \ll 1 \ (j = 1, 2, 3)$. It indicates that the contributions from the terms proportional to $T_{ij}$ as shown in equation (23) are highly suppressed. Then the scanning parameter space in equations (29), (32) and in the parameter space

$$x = (0.02 \sim 0.5), \quad M_{W_L} = (4.8 \sim 10.0)\text{TeV}, \quad M_{N_1} = (0.10 \sim 3.0)\text{TeV}, \quad (33)$$

we plot $m_{ee}^{LR}$ versus $m_{ee}^{\text{lightest}}$ in figure 4.

Comparing figure 4 with figure 3, the red points show that the range of $m_{ee}^{LR}$ is similar to the range of $m_{ee}^{BL}$ in the case of IH neutrino masses, but in the case of NH neutrino masses, there are points with $m_{ee}^{LR} > m_{ee}^{\text{lightest}}$. For IH neutrino masses, the contributions to $m_{ee}^{LR}$ are dominated by the terms proportional to light neutrino masses, hence $m_{ee}^{LR}$ depends on $x, M_{W_L}, M_{N_1}$ negligibly, which leads to the range of red points being similar to the results of $m_{ee}^{BL}$. Additionally, $m_{ee}^{LR} > m_{ee}^{BL}$ shown as green points indicate the contributions to $m_{ee}^{LR}$ can be dominated by heavy neutral leptons for appropriate values of $x, M_{W_L}, M_{N_1}$.

In order to see the effects of $x, M_{W_L}, M_{N_1}$ clearly, we take $m_{\nu_1} = 0.001 \text{eV}$ for the NH neutrino masses, $s_{12}, s_{13}, \Delta m^2_{21}, \Delta m^2_{31}$ at the corresponding center values and the CP violation phases $\alpha = \beta = 0$. Then taking $M_{W_L} = 5.0 \text{TeV}$, we plot $m_{ee}^{LR}$ versus $M_{N_1}$ in figure 5(a), where the solid, dashed, and dotted lines denote the results for $x = 0.10, 0.25, 0.40$ respectively. Similarly, $m_{ee}^{LR}$ versus $M_{W_L}$ for $M_{N_1} = 0.20 \text{TeV}$ is plotted in figure 5(b).

From figure 5, one may see the fact that the obtained $m_{ee}^{LR}$ increases with the increasing of $M_{N_1}, M_{W_L}$, then $m_{ee}^{LR}$ approaches to a constant when $M_{N_1}$ or $M_{W_L}$ becomes large. According to the definition of $W_L-W_R$ mixing parameter $\zeta$, the coefficient $C_{33}^{BL}(N)$ in equation (18) increases with increasing $x \equiv \frac{\zeta}{v_1}$, which leads to that $m_{ee}^{LR}$ increases with $x$ increasing as shown in the figures. Additionally one may see that $m_{ee}^{LR}$ depends on the values of $x, M_{W_L}, M_{N_1}$ mildly when $M_{N_1}$ or $M_{W_L}$ is large. It is because the contributions from heavy neutral lepton(s) are highly suppressed when its mass or right-handed boson mass becomes large, thus the contributions from the light neutrinos become dominant and proportional merely to the light neutrino masses in this case.

4.2.2. The results under the type I+II seesaw dominance. As pointed out above, the mixing parameters of the light neutrinos and the heavy neutral leptons are not tiny under the type I+II seesaw dominance, hence the Dirac mass matrix $M_D$ can also affect the numerical results via the mixing of the light neutrinos and the heavy neutral leptons. For simplicity and not losing general features, we assume that the mass matrix $M_D$ in equation (13) is diagonal as $M_D = \text{diag} (M_{D_{11}}, M_{D_{22}}, M_{D_{33}})$. Taking the assumption that only one generation of heavy neutral leptons makes substantial contributions, i.e. $M_{D_{22}}$ and $M_{D_{33}}$ do not affect the results. Scanning the ranges
The solid, dashed, dotted lines denote the obtained $m_{\nu_{e}}^{LR}$ for $x = 0.10, 0.25, 0.40$ respectively.

Figure 5. With $m_{\nu_{e}} = 0.001$ eV for NH neutrino masses, (a): $m_{\nu_{e}}^{LR}$ versus $M_{N}$ for $M_{W_{i}} = 5.0$ TeV, (b): $m_{\nu_{e}}^{LR}$ versus $M_{W_{i}}$ for $M_{N} = 0.2$ TeV.

The effects with the parameters being fixed above are presented a similar behavior as the case of type I seesaw dominance, hence we do not repeat the study of the effects on these parameters. With $M_{W_{i}} = 5.0$ TeV, in figure 7 (a) we plot $m_{\nu_{e}}^{LR}$ versus $S_{e}^{2}$ and the solid, dashed, dotted lines denote the results for $x = 0.02, 0.06, 0.1$ respectively. Similarly, with $x = 0.02$, $m_{\nu_{e}}^{LR}$ versus $S_{e}^{2}$ is plotted in figure 7(b), where the solid, dashed, and dotted lines denote the results for $M_{W_{i}} = 5.0, 6.0, 7.0$ TeV respectively.

From figures 7(a), (b) one may see the fact that with the mixing parameter $S_{e}^{2}$ increasing, $m_{\nu_{e}}^{LR}$ decreases to a minimum value and then increases. In the case of the type I+-II seesaw dominance, i.e. $S_{1i}$ and $T_{i1}$ are not too small, the contributions from the light neutrinos are dominated over those terms which do not depend on the neutrino mass $m_{\nu_{e}}$ in equation (23). When $S_{2}^{2}$ is small, $C_{SR}^{RR}(N)$ and $C_{IL}^{RR}(N)$ in equation (18) plays a dominant role, and as $S_{e}^{2}$ increasing, the contributions from $C_{IL/IR}^{RR}(N)$ in equation (23) become larger. The minimum values for $m_{\nu_{e}}^{LR}$ as shown in figure 7 is due to the opposite signs for $M_{3}^{RS}(N)$ and $M_{3}^{n}(\nu)$, i.e. it is caused by cancellation of the contributions from $C_{SR}^{RR}(N)$ and $C_{IL/IR}^{RR}(\nu)$. From the figures, the fact can be seen clearly that when $S_{e}^{2}$ is increasing, the contributions from $C_{IL/IR}^{RR}(\nu)$ become dominant which leads to the increasing of $m_{\nu_{e}}^{LR}$ as long as $S_{2}^{2}$ becomes large enough. The results indeed show that with $S_{e}^{2}$ varying, the cancellation takes place in a proper manner. When the cancellation takes place, the value of $S_{2}^{2}$ depends on $M_{W_{i}}$ and $x$ explicitly.

To compare the results obtained under the fresh approximation in this work with the ones obtained in the traditional way, we take the results from [65] and present $m_{\nu_{e}}^{LR}$ versus $M_{W_{i}}$ in figures 8(a), (b) for $S_{e}^{2} = 0.25 \times 10^{-8}$, $10^{-8}$ respectively, where the solid, dashed lines denote the results obtained under the fresh approximation in this work and those by the traditions way from in [65] respectively.

In figure 8, we take $M_{W_{i}} = 0.2$ TeV, $m_{\nu_{e}} = 0.010$ eV, $x = 0.020$, $s_{12}, s_{13}, \Delta m_{21}^{2}, \Delta m_{31}^{2}$ at the corresponding center values and the CP violation phases $\alpha = \beta = 0$. Note that in the figures the results obtained in this work coincide well with the traditional ones when $S_{2}^{2}$ is small or $M_{W_{i}}$ is large in the chosen parameter space. In addition, the results obtained in this work, being approximate ones but having the interference effects considered better, are smaller than the ones obtained in the traditional way when $W_{i}$ is not heavy enough, and the reduction factor depends on the parameters in LRSM completely and also
comes from the uncertainties of NMEs partly. Considering the difficulties and the uncertainties etc in computing NMEs and the interference effects among the various contributions by the way in the literature and the approximation approach in this work, from figure 8, it seems that the results with very heavy $W_2 (M_{W_2} \gtrsim 12\text{TeV})$ or small $S^2_e (\lesssim 10^{-9})$ may be convinced more as the results approach to coinciding with each other.

5. Summary and discussions

In the paper, we take the B-L supersymmetric standard model (B-LSSM) and TeV scale left–right symmetric model (LRSM) as two representations of two kinds of new physics models to study the nuclear neutrinoless double beta decays ($0\nu\beta\beta$). As stated in the Introduction, the calculations are those about the factor on the ‘core’ factor of the decays: evaluating the process $d + d \rightarrow u + u + e + e$ by considering the effective Lagrangian containing the operators with the Wilson coefficients or considering the relevant amplitude etc. Whereas here the estimations of the other necessary ‘factors’ for the decays, i.e. to evaluate relevant nuclear matrix element (NME) and phase space factor (PSF) etc, that do not relate to the specific models, are treated by the following literature.

In the B-LSSM, all of the calculations can be well deduced and the results are dominated by the neutrino mass terms. However, in the LRSM, owing to the existence of right-handed gauge boson $W_R$, the calculations are complicated, and the interference effects are hard and easily estimated. In this work, a new approximation, i.e. the momenta of the two involved quarks inside the initial nucleus and inside the final nucleus, are assumed to be equal approximately, is made, then all contributions in the LRSM can be well reduced and summed up all the contributions. Then the calculations in LRSM are simplified and the interference effects can be calculated comparatively easily. To see the consequences of the approximation, we compare the results obtained in this work with the results obtained by the traditional method numerically, and the results coincide with each other well when light-heavy neutral lepton mixing parameter $S^2_e$ is small or $M_{W_2}$ is large. For the effective dimension-9 contact interactions in these two models, the contributions from heavy neutral lepton exchange, the QCD corrections from the energy scale $\mu \simeq M_{W_2}$ (or $M_{W_1}$) to the energy scale $\mu \simeq 1.0$ GeV to all dimension-9 operators in the effective Lagrangian which is responsible for the $0\nu2\beta$ decays are calculated by the RGE method, and all the QCD corrections including the contributions from light neutrinos in the energy scale region

Figure 7. $m_{ee}^{1LR}$ versus $S^2_e$ with $M_{W_2} = 0.2$ TeV, $m_{\nu_e} = 0.001$ eV for the NH neutrino masses, $s_{12}, s_{13}, \Delta m^2_{21}, \Delta m^2_{31}$ at the corresponding center values and the CP violation phases $\alpha = \beta = 0$. (a): the results for $M_{W_1} = 5.0$ TeV, and the solid, dashed, dotted lines denote the results for $x = 0.02, 0.06, 0.10$ respectively. (b): the results for $x = 0.02$, and the solid, dashed, dotted lines denote the results for $M_{W_1} = 5.0, 6.0, 7.0$ TeV respectively.

Figure 8. (a): $m_{ee}^{1LR}$ versus $M_{W_2}$ for $S^2_e = 0.25 \times 10^{-9}$. (b): $m_{ee}^{1LR}$ versus $M_{W_2}$ for $S^2_e = 10^{-9}$. Both are with $M_{W_2} = 0.2$ TeV, $m_{\nu_e} = 0.01$ eV for the NH neutrino masses. The solid, dashed lines denote the results obtained under the new approximation in this work and the traditions method shown in [65] respectively.
\( \mu \simeq 1.0 \text{ GeV} \) to \( \mu \simeq 0.10 \text{ GeV} \), being of non-perturbative QCD, are taken into account alternatively by inputting in the experimental measurements for the relevant current matrix elements of nucleons, which emerge at the effective Lagrangian at \( \mu \simeq 0.10 \text{ GeV} \).

With necessary input parameters allowed by experimental data, the theoretical predictions on \( 0\bar{\nu}_2/\beta \) decay half-life \( ^{76}\text{Ge} \) and \( ^{136}\text{Xe} \) are obtained in these two models. In the B-LSSM, the contributions from heavy neutral leptons are highly suppressed by the tiny light-heavy neutral lepton mixing parameters for TeV-scale heavy neutral leptons. Hence \( m_{\nu_{ee}}^{\text{LR}} \) depends on the light neutrino masses mainly, and the numerical results show that the \( 0\bar{\nu}_2/\beta \) decays may be observed with quite a great opportunity in the near future. Whereas if thedecays are not observed in the next generation of \( 0\bar{\nu}_2/\beta \) experiments, the IH neutrino masses are excluded completely by the lower bound on \( m_{\nu_{ee}}^{\text{LR}} \) (as shown in figure 3).

In the LRSM, the situation is much more complicated than that in the B-LSSM, so we have gained a lot of experience in the study of the \( 0\bar{\nu}_2/\beta \) decays for the model. As for the type I seesaw dominance, the contributions from the terms proportional to \( G^{\nu}_{1j} \) (see equation (23)) are highly suppressed by the tiny neutrino masses. The numerical results of the contributions from light neutrinos are similar to the ones for B-LSSM, but the heavy neutral leptons can make comparatively large contributions through the right-handed current when \( M_{\bar{\nu}_1} (\lesssim 0.5 \text{ TeV}) \) and \( M_{\bar{\nu}_2} (\lesssim 7 \text{ TeV}) \) both are not too heavy. For the type I+II seesaw dominance, the terms which do not depend on the neutrino mass \( m_{\nu_3} \) in equation (23) play the dominant roles. In this case, the contributions from \( C_{1j}^{\text{LR}}(H) \) when the light-heavy mixing parameter \( S_x^2 \) is appropriate because the signs of the corresponding NMEs are opposite. Moreover, the effects of the cancellation are affected by \( S_x^2 \), \( x \) and the right-handed-W-boson mass \( M_{\bar{\nu}_R} \) etc in a complicated way. In addition, figure 6 shows that the points either in green (NH) or in red (IH) spread out a lot, and there are many 'exotic points', that cannot be reached for the cases of the B-LSSM and type I seesaw dominance LRSM. Thus the characteristic feature on the distribution of \( m_{\nu_{ee}}^{\text{LR}} \) versus \( m_{\nu_{ee}-\text{lightest}} \) may help to realize whether the decays are caused by the LRSM in the type I+II seesaw dominance or not, particularly when the \( 0\bar{\nu}_2/\beta \) decays are observed and the points for \( m_{\nu_{ee}}^{\text{LR}} \) versus \( m_{\nu_{ee}-\text{lightest}} \) just fall on the exotic points.

Finally, according to the numerical results of the present comparative studies on the \( 0\bar{\nu}_2/\beta \) decays for the two typical models LRSM and B-LSSM, it may be concluded that the room for LRSM type models for the foreseeable future decay experiments is greater than that of B-LSSM type models, and having the right-handed gauge bosons, the feature of the LRSM type models is more complicated than that of the B-LSSM type models.

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Appendix. QCD corrections to the relevant dimension-9 operators

The essential process \( d + d \rightarrow u + u + e + e \) for the \( 0\bar{\nu}_2/\beta \) decays involves six ‘fermion legs’, so the effective Lagrangian for the decays is composed generally of a set of independent dimension-9 operators as follows [14]:

\[
\mathcal{L}_{\text{DBD}}^{\text{eff}} = \frac{G_F^2 \cos^2 \theta_C}{2m_p} \sum_{X,Y,Z} \sum_{i=1}^{3} C_{ij}^{XY}(\mu) \cdot \mathcal{O}_{ij}^{XY}(\mu) \\
+ \left( \sum_{j=1}^{5} c_{ij}^{XY}(\mu) \cdot \mathcal{O}_{ij}^{XY}(\mu) \right), \quad (A1)
\]

where \( \theta_C \) is the Cabibbo angle, \( \mu \simeq 0.1 \text{ GeV} \) is the energy scale where the decays take place, and the independent dimension-9 operators \( \mathcal{O}_{ij}^{XY}(\mu) \) are defined as:

\[
\mathcal{O}_{ij}^{XY}(\mu) = 4 (\bar{u}_i P_X d_i)(\bar{u}_i P_Y d_j),
\]

where \( X, Y, Z = L, R \); \( P_{R/L} = (1 \pm \gamma^5)/2 \), the leptonic currents are defined as

\[
j_{R/L} = \bar{e}(1 \pm \gamma^5) e^c, \quad j_{\mu} = \bar{e}\gamma_\mu \gamma^5 e^c. \quad (A3)
\]

In the region from \( M_{\bar{\nu}_2} \) to 1.0 GeV for the energy scale \( \mu \), the pQCD is applicable, so the QCD corrections to the effective Lagrangian for the \( 0\bar{\nu}_2/\beta \) decays can be carried out by the renormalization group equation method [82, 83]. The corresponding QCD corrections for the \( 0\bar{\nu}_2/\beta \) decays were also calculated in [12], so here we describe how the corrections are calculated briefly. In the RGE method the renormalized operator matrix elements \( \langle \mathcal{O} \rangle^{(R)} \) (\( \mathcal{O} \) is defined in equation (A2)) for pQCD relate to their bare ones up to one-loop level generally as the following form:

\[
\langle \mathcal{O} \rangle^{(R)} = \left[ \delta_{ij} + \frac{\alpha_s}{4\pi} b_{ij} \left( \frac{1}{\epsilon} + \ln \frac{\mu^2}{-\epsilon} \right) \right] \langle \mathcal{O} \rangle^{\text{bare}}. \quad (A4)
\]

where \( \langle \mathcal{O} \rangle^{\text{bare}} \) are the ‘bare operator matrix elements’. The ‘renormalization’ for quark fields \( q \) and operator elements
where
\[ Z_q = 1 - C_F \frac{\alpha_s}{4\pi} + O(\alpha_s^2), \]
and \( C_F = (N^2 - 1)/(2N) \) is the SU(N) color factor \((N = 3)\). The singularities in equation (A4) are required to be cancelled, then we have
\[ Z_q Z_{q^{-2}} = \left[ \delta_q + \frac{\alpha_s}{4\pi} b_q \frac{1}{\varepsilon} \right]. \] (A7)

Then \( Z_q \) can be read out and written as
\[ Z_q = \delta_q + \frac{\alpha_s}{4\pi} (b_q - 2C_F \delta_q) \frac{1}{\varepsilon} + O(\alpha_s^2). \] (A8)

Considering the 4-quark-leg operators \( \mathcal{O}(q^{bare}) \) in the effective Lagrangian equation (A1) are constructed by the bare quark field \( q^{bare} \), and the corresponding coefficients \( c^{bare} \) are also bare. Then \( q^{bare}, c^{bare} \) relate to the renormalized ones as
\[ q^{bare} = Z_q^{1/2} q^R, \quad c_i^{bare} = Z_i^{C_i} c_i^R. \] (A9)

Hence we have
\[ c^{bare} \mathcal{O}(q^{bare}) = Z_q^2 Z_i^{C_i} c_i^R \mathcal{O}(q^R), \quad \mathcal{O}(q) = \frac{\alpha_s}{4\pi} \beta_0 \ln \left( \frac{\mu}{\mu_0} \right), \]
and the matrix elements for the QCD corrections are read as
\[ Z_q^2 Z_i^{C_i} c_i^R \langle \mathcal{O}(q^R) \rangle^{bare} = c_i^R \langle \mathcal{O}(q^{bare}) \rangle^R. \] (A11)
Combining equation (A11) and equation (A5), we can obtain
\[ Z_i^{C_i} = Z_i^{-1}. \] (A12)

Due to the fact that the bare quantities \( C_i^{bare} \) do not depend on the renormalization energy scale \( \mu \), we have
\[ \frac{d}{d\ln \mu} C_i^{bare} = \frac{d}{d\ln \mu} Z_i^{C_i} c_i^R = 0, \] (A13)
which can be rewritten as
\[ \frac{dC^R(\mu)}{d\ln \mu} = \hat{\gamma} C^R(\mu). \] (A14)

Equation (A14) is the RGE accordingly for the Wilson coefficients, where \( \hat{C} = (C_1, C_2, \ldots) \) is written as a vector form. Then the anomalous dimension matrix \( \hat{\gamma} \) can be written as
\[ \hat{\gamma} = \frac{1}{Z} \frac{d}{d\ln \mu} \hat{Z}. \] (A15)
Combining with the one-loop expression in the \( \overline{MS} \)-scheme
\[ \hat{\gamma}(\alpha_s) = -2\alpha_s \frac{\partial \hat{Z}(\alpha_s)}{\partial \alpha_s} \] (A16)
where \( \hat{Z} \) is the coefficient matrix of \( 1/\varepsilon \) in equation (A8), the ‘anomalous dimension matrix’ to the leading order can be written as
\[ \gamma(\alpha_s) = -\frac{\alpha_s}{4\pi} \gamma_0 \quad \text{with} \quad \gamma_0 = -2(b_q - 2C_F \delta_q). \] (A17)

Then solving equation (A14), the evolution of Wilson coefficients \( C^R(\mu) \) can be expressed by \( C^R(\Lambda) \) in terms of the \( \mu \)-evolution matrix \( \hat{U}(\mu, \Lambda) \):
\[ C^R(\mu) = \hat{U}(\mu, \Lambda) : C^R(\Lambda), \] (A18)
where precisely
\[ \hat{U}(\mu, \Lambda) = \hat{V} \text{Diag} \left\{ \frac{\alpha_s(\Lambda)}{\alpha_s(\mu)}, \frac{\alpha_s(\mu)}{\alpha_s(\Lambda)} \right\} \hat{V}^{-1}, \] (A19)
and
\[ \text{Diag} \{ \gamma_i \} = \hat{V}^{-1} \hat{\gamma} \hat{V}, \] (A20)
with \( \hat{\gamma} = \gamma_0 \hat{\gamma} \) in matrix form. The running coupling constant to one-loop level of QCD can be written as
\[ \alpha_s(\mu) = \frac{\alpha_s(\Lambda)}{1 - \beta_0 \frac{\alpha_s(\Lambda)}{2\pi} \ln \left( \frac{\mu}{\mu_0} \right)}, \] (A21)
with \( \beta_0 = (33 - 2f)/3 \), and \( f \) is the number of the active quark flavors which is varied with the energy scale \( \mu \), and only the quark \( q^i \) with mass \( m_i \) smaller than the upper bound of the considered energy scale region are ‘active’. Thus in the region from \( \sim 1.0 \) GeV to \( m_i \sim 1.3 \) GeV we have \( f = 3 \), in the region from \( m_i \) to \( m_b \sim 4.6 \) GeV we have \( f = 4 \) and in the region from \( m_b \) to \( M_W \) we have \( f = 5 \). As a result, the required matrix to describe the QCD RGE evolution from \( \mu = M_W \) to \( \mu \simeq 1.0 \) GeV is
\[ \hat{U}(\mu, \Lambda) = \hat{V} \text{Diag} \left\{ \frac{\alpha_s(\Lambda)}{\alpha_s(\mu)}, \frac{\alpha_s(\mu)}{\alpha_s(\Lambda)} \right\} \hat{V}^{-1} \text{Diag} \{ \gamma_i \} \] (A22)
\}[\text{Diag} \{ \gamma_i \}] = \hat{V}^{-1} \hat{\gamma} \hat{V}, \] (A20)

To the leading order, the QCD corrections of the operators in equation (A2) correspond to figure 9 as follows.

Calculating the diagrams respectively, the operator matrix elements corresponding to those in equation (A4) have the following structures

\[ \text{Figure 9(a)} \Rightarrow \mu^{4-\varepsilon} \int \frac{d^4k}{(2\pi)^D} \left( \bar{u} \gamma_3 \gamma_5 \gamma_0 \right) (\bar{u} \Gamma_3 d) \cdot \frac{i}{(k - p)^2} \left( ig s^2 C_F \right) = (\bar{u} \gamma_3 \gamma_5 \gamma_0 \gamma_3 \gamma_0 d)(\bar{u} \Gamma_3 d) \]
\[ = \frac{1}{4} C_F \alpha_s \frac{1}{4\pi} \frac{1}{\varepsilon} + \ln \frac{\mu^2}{\mu_0^2}, \] (A23)
where $\Gamma_i$ are the Lorentz structures of the operators in equation (A2), and $T^a$ are the generators of SU(N). Since the lepton sector is irrelevant with the QCD corrections, in the calculation the leptonic factor in the operators is irrelevant. According to equations (A4)–(A17), the anomalous dimension matrix elements can be extracted from equations (A23)–(A28).
Summarizing the obtained anomalous dimension matrix elements for $O_{24}^{XY}$, $O_{25}^{XY}$, $O_{26}^{XY}$ and $O_{27}^{XY}$, we have

\[
\begin{align*}
\chi_{\gamma}(12) &= -2 \left( 6C_{\gamma} - \frac{3}{2N} + \frac{1}{4} \right), \\
\gamma_{\gamma}(11) &= -2 \left( \frac{3}{N} - \frac{6}{N} \right), \\
\gamma_{\gamma}(12) &= -2 \left( \frac{3}{N} - \frac{6}{N} \right), \\
\gamma_{\gamma}(145) &= -2 \left( \frac{2}{N} - C_{\gamma} \frac{3}{2} + \frac{3}{2} \right). \\
\end{align*}
\]

(A29)

It is easy to realize that our results on the anomalous dimensions for the matrix elements $O_{24}^{XY}$ coincide with those in [13], but do not coincide with those in [12]. Hence the calculational details about the anomalous dimensions of $O_{24}^{XY}$, $O_{25}^{XY}$ are given below:

\textbf{•} $O_{24}^{XY}$:

(a) \(- (1): \frac{1}{4} C_{\gamma} (\bar{u}r_{\gamma} \gamma_{r} r_{\gamma} \gamma_{r} P X d) (\bar{u} \sigma_{\mu} P X d) \)
\(= C_{\gamma} (\bar{u}r_{\gamma} \gamma_{r} P X d) (\bar{u} \sigma_{\mu} P X d), \) \hspace{1cm} (A30)

(b) \(- (2): \frac{1}{4} C_{\gamma} (\bar{u}r_{\gamma} \gamma_{r} P X d) (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} P X d) = 0, \) \hspace{1cm} (A31)

\textbf{•} $O_{25}^{XY}$:

(a) \(- (1): \frac{1}{4} C_{\gamma} (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u} \sigma_{\mu} P X d) \)
\(= C_{\gamma} (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u} \sigma_{\mu} P X d), \) \hspace{1cm} (A32)

(b) \(- (2): \frac{1}{4} C_{\gamma} (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) \)
\(= (\bar{u}r_{\gamma} \gamma_{r} P X d) (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d), \) \hspace{1cm} (A33)

(c) \(- (1): \frac{1}{4} C_{\gamma} (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) \)
\(= 3(\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u} \sigma_{\mu} P X d) \)
\(= 3(\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u} \sigma_{\mu} P X d) \)
\(+ 3i(\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u} \gamma_{r} P X d), \) \hspace{1cm} (A34)

(c) \(- (2): \frac{1}{4} C_{\gamma} (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u} \sigma_{\mu} P X d) \)
\(= (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u} \sigma_{\mu} P X d) \)
\(= (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u} \sigma_{\mu} P X d), \) \hspace{1cm} (A35)

(b) \(- (1): \frac{1}{4} C_{\gamma} (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u} \gamma_{r} P X d) \)
\(= (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u} \gamma_{r} P X d) \)
\(= (\bar{u}r_{\gamma} \gamma_{r} \gamma_{r} \gamma_{r} \gamma_{r} P X d) (\bar{u} \gamma_{r} P X d), \) \hspace{1cm} (A36)
(c) \(-1): \frac{1}{4} (\bar{a} \gamma_{\nu} \gamma_{\mu} T^a P_X d (\bar{a} \gamma^\nu \gamma^\mu T^a P_X d)
= (\bar{a} \gamma_{\nu} T^a P_X d (\bar{a} T^a P_X d)
= i (\bar{a} \gamma^\nu T^a P_X d (\bar{a} \sigma_{\mu} T^a P_X d)
- i (\bar{a} \gamma^\nu T^a P_X d (\bar{a} \sigma_{\mu} T^a P_X d), \quad (A48)
\Rightarrow (a) + (b) + (c) = 5 C_F \mathcal{O}_S^{2X}
- 2 i (\bar{a} \gamma^\nu T^a P_X d (\bar{a} \sigma_{\mu} T^a P_X d)
= \left( 5 C_F - \frac{3}{2} \right) \mathcal{O}_S^{2X} + \left( \frac{1}{2} - \frac{1}{N} \right) \mathcal{O}_S^{2Y} . \quad (A50)
\end{align*}

\begin{align*}
\mathcal{O}_S^{2X} (X = Y):
(a) \(-1): \frac{1}{4} C_F (\bar{a} \gamma_{\nu} \gamma_{\mu} \gamma_{\beta} \gamma_{\alpha} P_X d (\bar{a} P_Y d)
= C_F (\bar{a} \gamma_{\nu} \gamma_{\mu} P_X d (\bar{a} P_Y d), \quad (A51)
(b) \(-1): \frac{1}{4} (\bar{a} \gamma_{\nu} \gamma_{\mu} T^a P_X d (\bar{a} \gamma^\nu \gamma^\mu T^a P_Y d)
= 4 C_F (\bar{a} \gamma_{\nu} \gamma_{\mu} P_X d (\bar{a} P_Y d), \quad (A52)
\Rightarrow (a) + (b) + (c) = 5 C_F \mathcal{O}_S^{2X}
- 2 i (\bar{a} \gamma^\nu T^a P_X d (\bar{a} \sigma_{\mu} T^a P_X d)
= \left( 5 C_F - \frac{3}{2} \right) \mathcal{O}_S^{2X} + \left( \frac{1}{2} - \frac{1}{N} \right) \mathcal{O}_S^{2Y} . \quad (A57)
\end{align*}

The Fierz transformation formalisms in the above calculation read

\begin{align*}
(P_X)_{22} (P_X)_{34} = \frac{1}{2} (P_X)_{14} (P_X)_{32} + \frac{1}{8} (\sigma_{\mu \nu} P_X)_{14} (\sigma_{\mu \nu} P_X)_{32},
(P_X)_{12} (P_Y)_{34} = \frac{1}{2} (\gamma_{\mu} P_X)_{14} (\gamma_{\mu} P_Y)_{32},
(\sigma_{\mu \nu} P_X)_{12} (\sigma_{\mu \nu} P_X)_{34} = 6 (P_X)_{14} (P_X)_{32} - \frac{1}{8} (\sigma_{\mu \nu} P_X)_{14} (\sigma_{\mu \nu} P_X)_{32},
(\gamma_{\mu} P_X)_{12} (\gamma_{\mu} P_X)_{34} = -2 (P_X)_{14} (P_Y)_{32} - \frac{1}{2} (\gamma_{\mu} P_X)_{14} (\gamma_{\mu} P_X)_{32},
(\gamma_{\mu} P_X)_{12} (\gamma_{\mu} P_X)_{34} = \frac{1}{2} (\gamma_{\mu} P_X)_{14} (P_X)_{32} - \frac{1}{8} (\gamma_{\mu} P_X)_{14} (\gamma_{\mu} P_X)_{32},
\end{align*}

and for the generators $T^a$ of $SU(N)$ we have

\begin{align*}
T^a_{ij} T^a_{jk} = N^2 - 1 \frac{\delta_{ij}}{2N} - \frac{\delta_{ik}}{N} \mathcal{O}_S^{2Y} . \quad (A59)
\end{align*}

Combining equations (A20)–(A22) with equation (A29), we can compute the numerical QCD RGE evolution matrices from $\Lambda = M_W$ to $\mu \approx 1.0$ GeV as

\begin{align*}
\hat{U}_{(12)} (\mu, \Lambda) = \begin{pmatrix} 1.96 & 0.01 \\
-2.82 & 0.45 \end{pmatrix}, \quad \hat{U}_{(31)} (\mu, \Lambda) = \begin{pmatrix} 0.87 & -1.4 \\
0 & 2.97 \end{pmatrix},
\hat{U}^{XY}_{(45)} (\mu, \Lambda) = \begin{pmatrix} 0.68 & -0.24i \\
0.023i & 1.4 \end{pmatrix}, \quad (A60)
\end{align*}
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