HELIOSEISMOLOGY AND ASTEROSEISMOLOGY: LOOKING FOR GRAVITATIONAL WAVES IN ACOUSTIC OSCILLATIONS
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ABSTRACT

Current helioseismology observations allow the determination of the frequencies and surface velocity amplitudes of solar acoustic modes with exceptionally high precision. In some cases, the frequency accuracy is better than one part in a million. We show that there is a distinct possibility that quadrupole acoustic modes of low order could be excited by gravitational waves (GWs), if the GWs have a strain amplitude in the range $10^{-20} h_{-20} \sim 1$ or $h_{-20} \sim 10^3$, as predicted by several types of GW sources, such as galactic ultracompact binaries or extreme mass ratio inspirals and coalescence of black holes. If the damping rate at low order is $10^{-3} \eta_N \mu$Hz, with $\eta_N \sim 10^{-3} \mu$1, as inferred from the theory of stellar pulsations, then GW radiation will lead to a maximum rms surface velocity amplitude of quadrupole modes of the order of $h_{-20} \eta^{-1}_N \sim 10^{-9} – 10^{-10}$ cm s$^{-1}$, on the verge of what is currently detectable via helioseismology. The frequency and sensitivity range probed by helioseismological acoustic modes overlap with, and complement, the capabilities of eLISA for the brightest resolved ultracompact galactic binaries.

Key words: cosmology: miscellaneous – gravitational waves – instrumentation: detectors – stars: black holes – stars: oscillations – Sun: helioseismology

Online-only material: color figures

1. INTRODUCTION

The rapid development of gravitational wave (GW) detection by either resonant mass detectors or ground-based and space interferometers give us hope that GW observations will very soon become a reality. If such a goal were to be achieved, a new window will open toward understanding the formation of many compact structures in the universe, most of which are still poorly understood, such as black holes and neutron star binaries (e.g., Gair et al. 2013; Sathyaprakash & Schutz 2009). Nevertheless, even if the detection of GW radiation can be achieved by these modern experiments, the goal will only be attained if the GW signal can be successfully separated from the background “noise.” Therefore, any prior information of incoming GW events (e.g., Abbott et al. 2009; Sathyaprakash & Schutz 2009) for the GW experimental research community is of great interest.

In this paper, we discuss an alternative method to probing for direct GW radiation. The Sun, as is the case for many other stars, is a natural massive GW detector with an isotropic sensitivity to GWs, able to absorb GWs from any direction of the sky. In recent years, this possibility of using stars as GW detectors has become very appealing, as current helioseismology and astroseismology observations allow the determination of the frequency and the velocity amplitude of many modes of vibrations with exceptional accuracy.

In the Sun, the acoustic modes have been continuously observed by the SOHO mission since 1996 (Turck-Chièze & Lopes 2012) and some of the low degree modes are measured with a precision of one part per million. The COROT (Michel et al. 2008) and Kepler (Chaplin et al. 2011) missions have discovered more than 500 pulsating stars, most of which are in the main sequence and sub-giant phase, and some of these stars have been observed over intervals of several months in the last four years (e.g., Chaplin & Miglio 2013). As with the Sun, the damping and excitation of the oscillation modes in these stars are attributed to turbulent convection in their upper layers. The continuous monitoring of pulsating modes in the Sun and many stars of different masses and sizes give us the possibility of surveying the local universe for GW radiation, either by probing for a stochastic background, for rare events, or for periodic signals (e.g., Sathyaprakash & Schutz 2009). Among other possible GW sources emitting in the frequency range of solar acoustic oscillations ($0.2 \text{mHz} \lesssim v \lesssim 5 \text{mHz}$), there are the occasional GW events occurring during the coalescence of massive black hole binaries and neutron star binaries (Lynden-Bell & Rees 1971), extreme mass ratio inspirals (Gair & Porter 2012), and the periodic GW signal of AM CVn stellar systems (Nelemans et al. 2004; Roelofs et al. 2007; Yu & Jeffery 2010). The strain amplitude of these GW events is in the range of $10^{-17} – 10^{-24}$ (e.g., Sathyaprakash & Schutz 2009; Moore et al. 2014).

Preliminary studies of the impact of incoming GW radiation on massive bodies, such as the Earth, Moon, planets, and stars were previously presented by several authors (e.g., Dyson 1969; Zimmerman & Hellings 1980; Boughn & Kuhn 1984; Khosroshahi & Sobouti 1997). Boughn & Kuhn (1984) were the first to compute the impact of GW on solar gravity and acoustic modes, for which they also put upper limits on the stochastic gravitational background from the observed solar oscillations. More recently Siegel & Roth (2011) used a hydrodynamical model to re-evaluate the excitation of solar oscillations by GWs (Siegel & Roth 2010). Equally, they have updated the previous stochastic gravitational background limits (Siegel & Roth 2014).
A complementary approach was performed recently (McKernan et al. 2014) in which the authors estimated that gravitational radiation that is absorbed by stars near black holes, and discuss how the absorption by the Sun of GWs from Galactic white dwarf binaries could be observed by a second generation of gravitational wave detectors.

Here, we show that GWs with a strain spectral amplitude of \(10^{-20} h_{-20} \) with \( h_{-20} \gtrsim 1 \) can lead to the excitation of low order quadrupole acoustic modes in the Sun, for which the rms surface velocity amplitudes could be as large as \( \sim h_{-20} \) cm s\(^{-1}\). These results use theoretical predictions of damping rates of acoustic modes consistent with current solar observations at high frequencies. Moreover, we discuss the strategy to search for GW events in stellar oscillations. Our theoretical model closely follows the GW model of resonant mass detectors. This approach facilitates the use of our work by the GW experimental community.

2. GRAVITATIONAL WAVES AND STELLAR OSCILLATIONS

In the presence of GWs, stars behave like resonant-mass spherical detectors. Accordingly, the oscillations of a star equally excited by convection and GWs can be accurately represented by the simplified wave equation (e.g., Chaplin et al. 2005; Samadi & Goupil 2001; Lopes 2001; Cox 1980):

\[
\frac{\partial^2 \xi}{\partial t^2} + 2\eta_N \frac{\partial \xi}{\partial t} + L \xi = \frac{1}{\rho} F_{\text{conv}} + F_{\text{gw}} \tag{1}
\]

for the displacement \(\xi(r, t)\) of a forced oscillation corresponding to a mode \(N\). In this equation, all the terms homogeneous in \(\xi\) have been put on the left-hand side, and the fluctuating terms arising from stochastic excitation by turbulent convection \(F_{\text{conv}}\) or by GW perturbations \(F_{\text{gw}}\) are on the right-hand side. \(\omega_N\) corresponds to the frequency of the mode \(N\) and \(\rho\) is the density of the star in equilibrium.\(^6\) Although to compute the excitation, damping, and propagation of acoustic and gravity waves inside stars it is necessary to resolve the full set of hydrodynamic equations, in the Sun and identical stars, the acoustic modes of oscillation are well represented by the linearized pulsation dynamics as described by the wave equation (1). This equation has been very successful in explaining the solar and stellar observational data (Chaplin et al. 2005).

The pulsation variations of the fluid caused by momentum and heat are included in the damping rate \(\eta_N\) and the linear spatial differential operator \(L\) (Unno et al. 1989). Moreover, these quantities are chosen in such a way that both the frequency \(\omega_N\) and eigenfunctions \(\xi_N(r)\) of the homogeneous equation

\[
L \xi_N = \omega_N^2 \xi_N \tag{2}
\]

are real. The set of eigenfunctions \(\xi_N\) can be shown to be orthogonal and form a complete set (e.g., Aizenman & Smeyers 1977). In particular, \(\xi_N\) has two eigenfunction components \(\xi_{lN}(r)\) and \(\xi_{hN}(r)\), the radial and horizontal surface displacements.

As already stated, we include as a source of excitation those fluctuations arising from turbulent convection \(F_{\text{conv}}(r, t)\) which have been widely reported in the literature (e.g., Goldreich & Keeley 1977; Goldreich et al. 1994; Belkacem et al. 2008), and \(F_{\text{gw}}(r, t)\) is the driving force related to GW fluctuations of the spacetime continuum\(^7\) where the star is located (Misner et al. 1973). \(F_{\text{gw}}(r, t)\) has the components

\[
[F_{\text{gw}}]_l = \frac{1}{2} \hbar_{ij} x^j. \tag{3}
\]

\(x^j\) are the spatial coordinates of index \(j\) and \(\hbar_{ij}\) is the second time derivative of the tensor \(\hbar_{ij}\). As usual, \(\hbar_{ij}\) is the spatial part of the tensor \(h_{ij}\) that describes a small perturbation relative to a flat spacetime universe (Minkowski space). Moreover, the \(h_{ij}\) deviation from a flat spacetime is solely attributed to GWs, for which the effects of curvature is neglected due to the mass of the star (e.g., Schutz 2009).

Adopting a standard procedure of normal analysis (e.g., Unno et al. 1989), we choose to represent any perturbation described by Equation (1) as a combination of the eigenfunctions such that

\[
\xi_N(r, t) = A(t) \xi_N(r) e^{-i \omega_N t}, \tag{4}
\]

where \(A(t)\) is the instantaneous amplitude of the mode (Chaplin et al. 2005; Belkacem et al. 2008). In \(\xi_N(r, t)\) we do not show the term related to the contribution of the temporal phase variation in the argument of \(e^{-i \omega_N t}\), as this quantity is negligible for the formation of standing acoustic waves (Chaplin et al. 2005). Equally, the complex conjugate is also not represented as this quantity is not relevant for our analysis (Samadi & Goupil 2001). This approximation is valid for modes for which the energy exchange between the stellar turbulent convection and the oscillations occurs in a timescale that is much longer than the oscillation period, i.e., \(\eta_N \ll \omega_N\), as is the case with acoustic modes. This result has been shown to be valid for current solar and stellar acoustic oscillations. By substituting this form of \(\xi_N(r, t)\) into Equation (1), multiplying both members by \(\xi_N^\ast\) (the complex conjugate of \(\xi_N\)), integrating this equation for the total mass of the star and keeping only the leading terms, the equation reduces to

\[
\frac{d^2 A}{dt^2} + 2\eta_N \frac{d A}{dt} + \omega_N^2 A = S_{\text{conv}}(t) + \delta^2 \omega \frac{d^2 S_{\text{gw}}}{dt^2}. \tag{5}
\]

where \(\delta^2 \omega\) is the Kronecker tensor. Wave motion is a complex process with many second order terms. Fortunately, these are very small when comparing with the leading terms, \(S_{\text{conv}}(t)\) or \(S_{\text{gw}}(t)\). Accordingly, the amplitudes of acoustic oscillations correspond to the solution of a damping harmonic oscillator as described by the previous equation. A detailed account of the nature of the second order terms neglected in this computation can be found in Chaplin et al. (2005).

\(S_{\text{conv}}\) and \(S_{\text{gw}}\) are respectively the excitation source terms related to turbulent convection and GWs. \(S_{\text{gw}}\) reads

\[
S_{\text{gw}}(t) = \frac{1}{I} \int_{0}^{R} F_{\text{gw}} \cdot \xi_N^\ast \rho r^2 \, dr, \tag{6}
\]

where \(R\) is the radius of the star and \(I\) is the mode inertia. \(I\) is an arbitrary constant which we choose to be equal to the mode of inertia, as is usually done in the theory of stellar oscillations (e.g., Aerts et al. 2010). \(I\) is given by

\[
I = 4\pi \int_{0}^{R} \xi_N^2 \cdot \xi_N^\ast \rho r^2 \, dr. \tag{7}
\]
It is convenient to introduce $M_N$, the so-called modal mass; thus $M_N = I/\zeta$, where $\xi \equiv \xi_2(R) + 6\xi(R)$.

In the eventuality of such a star having been perturbed by a passing GW, the response will be somehow identical to a tidal perturbation produced by a nearby object on the stellar modes. Following from the specific properties of gravitational systems as demonstrated in general relativity (Maggiore 2008), GW perturbations only have modes with $l \geq 2$. For convenience, we opt to study the leading order of the GW perturbation, i.e., the quadrupole modes ($l = 2$). This is the reason why we have introduced $\delta^2_l$ in Equation (4).

Equation (5) can be written in a more convenient form by using Equations (3) and (6) for which $S_{gw}(t)$ reads

$$S_{gw}(t) = L_n \tilde{h}_m(t),$$

where $L_n$ is the effective length that measures the sensitivity of a mode of order $n$ to a GW perturbation and $\tilde{h}_m$ are the spherical components of $h(t)$ for which the $m$ (azimuthal order) take one of the following integer values: $-2, -1, 0, 1, 2$. $L_n$ is given by

$$L_n = 1/2 R \chi_n,$$

where $R$ is the radius of the star and $\chi_n$ is the excitation that determines the efficiency of a mode of order $n$ to be excited by GWs. $\chi_n$ reads

$$\chi_n = \frac{3}{4 \pi \rho_s} \int_0^1 (r^2 + 3r^2) \rho_s^3 dr.$$

In the computation of Equation (7), as is usually done, we arbitrarily normalized the eigenfunctions to the average density of the star $\bar{\rho}_s$, such that $I \equiv (4\pi/3)\bar{\rho}_s^3$. In the case of the Sun, $\bar{\rho}_s$ is approximately 1.4 g cm$^{-3}$. Equation (9) is identical to others found in the literature, such as by Boughn & Kuhn (1984) and more recently by Siegel & Roth (2011). $\chi_n$ differ among these works only by the arbitrary normalization.

Figure 1 and Table 1 show the $\chi_n$ coefficients computed for the standard solar model (SSM; Turck-Chieze & Lopes 1993) with a stellar structure in very good agreement with helioseismology data. The difference between theoretical and observational frequencies is smaller than 0.1% (see Table 1). This solar model was computed using a modified version of the CESAM code (Morel 1997) for which the microphysics was updated. In particular, we have computed the so-called low-Z SSM (Haxton et al. 2013) for which the solar composition used corresponds to the one determined by Asplund et al. (2009). The CESAM nuclear physics network uses the fusion cross-sections recommended for the Sun by Adelberger et al. (2011) with the most recent coefficients. A detailed discussion about the physics of the current SSM can be found in the recent literature (e.g., Lopes & Silk 2013).

The values of $|\chi_n|$ in the Sun decrease with $n$ (see Table 1), a behavior identical to the one found for a resonance sphere of constant density. However, in the solar case, $\chi_n$ is two orders of magnitude smaller. This difference is related to the fact that the solar density decreases rapidly toward the Sun’s surface and eigenfunctions of acoustic modes are more sensitive to the external layers of the star. For instance, the largest of the $\chi_n$ coefficients, $\chi_1$ has a value of $-0.0011$ for the Sun and $-0.328$ in the case of a resonant sphere (Maggiore 2008). Moreover, $|L_n|$ takes values from $10^{-7}$ cm ($n = 0$) to 100 cm ($n = 18$). Solar low order modes have much larger values than the equivalent ones found in an experimental detector. A similar quantity to $\chi_n$...
was computed by Boughn & Kuhn (1984) and by Siegel & Roth (2011). Unfortunately, the comparison of $\chi_n$ with these models or a resonant-mass detector of constant mass as described by Maggiore (2008) is not trivial to make. Nevertheless, $\chi_n$ varies in a similar way to the $\chi_n$ factor found by Siegel & Roth (2011). In both cases these terms decrease as $n$ increases and by identical orders of magnitude.

3. EXCITATION OF STELLAR MODES BY GRAVITATIONAL WAVES

By taking the Fourier transform of Equation (4) and neglecting transient terms arising from the initial conditions on $A$, we obtain for the averaged power spectrum $P_N(\omega) = \langle |\tilde{A}^2(\omega)| \rangle$:

$$P_N(\omega) = \frac{P_{\mathrm{conv}}(\omega) + \delta^2_P P_{gw}(\omega)}{(\omega^2 - \omega_N^2)^2 + 4\eta_N^2 \omega^2},$$

(10)

where $P_{\mathrm{conv}}(\omega) = \langle |\tilde{S}_{\mathrm{conv}}^2(\omega)| \rangle$ and $P_{gw}(\omega) = \langle |\tilde{S}_{gw}^2(\omega)| \rangle$ are the average power spectrum due to forcing caused by turbulent convection and gravitational waves. $\tilde{f}(\omega)$ denotes the Fourier transform of $f(t)$. This previous result is obtained under the approximation that the damping rate is always much smaller than the frequency, i.e., $|\eta_N| \ll \omega$, as it is the case with acoustic oscillations of the Sun and Sun-like stars. In the derivation of the previous result, $P_{\mathrm{conv}}(\omega)$ and $P_{gw}(\omega)$ are assumed to vary slowly with $\omega_N$.

The power spectrum generated by stochastic excitation $P_{\mathrm{conv}}(\omega)$ is known to be caused by turbulent convection in the upper layers of the Sun and Sun-like stars just beneath the stellar photosphere (e.g., Belkacem et al. 2008). This term represents the random spectrum due to the turbulent convection: if the temporal series is very long, the Lorentzian profile of each acoustic mode becomes visible due to the systematic beating of the mode by a random process of excitation (Kosovichev 1995).

In the following, we compute the GW contribution to the power spectrum, i.e., $P_{N,gw}(\omega)$. From Equations (7) and (10), $P_{N,gw}(\omega)$ reads

$$P_{N,gw}(\omega) = T_{N,gw}^2(\omega) P_m(\omega),$$

(11)

where $T_{N,gw}(\omega)$ is the transfer function of mode $N$ and $P_m(\omega)$ the power spectrum of the GW source. The former depends uniquely on the properties of the star, and the latter on the source of GWs, $T_{N,gw}^2(\omega)$ reads

$$T_{N,gw}^2(\omega) = \frac{L_n^2 \omega^4}{(\omega^2 - \omega_N^2)^2 + 4\eta_N^2 \omega^2}.$$

(12)

The power spectrum of the GW source $P_m(\omega)$ is computed as $P_m(\omega) = \langle \tilde{h}_m^2 \rangle$. In the Sun, the propagation of forward and backward traveling waves originating in the internal differential rotation leads to the generation of acoustic modes of different $m$. The frequency of these $m$-modes (fix $l$ and $n$) differs only by a few $\muHz$ (Howe 2009). The solar magnetic field produces a similar effect leading to frequency differences of tens of $\muHz$ (Antia 2002). Thus, for convenience, we will consider that $h_m$ and $P_m$ are fiducial values(for $l = 2$ and $n$ fixed). This approximation is well justified as the different $h_m$ values mainly give us information about the direction of the GW source in the sky in relation to the star (Maggiore 2008).

In the following, we compute the rms surface velocity $V_N(\omega_N)$ of the $N$ mode, which is measured at a specific layer of the surface of the star (e.g., Samadi et al. 2001; Chaplin et al. 2005).

Thus, the energy absorbed by a mode with a velocity $\dot{\xi}_N$ subject to a force $F_{gw}(t) = M_N L_n h_m$ (Equation (4)), averaged over several cycles, reads

$$\frac{dE_{abs}}{dt} = \langle F_{gw}(t) \dot{\xi}_N \rangle = M_N h_m^2 \omega^2 \langle \xi_N^2 \rangle T_{N,gw}^2(\omega).$$

(13)

In this calculation, we consider that the gravitational wave source is monochromatic, $h_m = h_m N [e^{-i\omega t}]$, where $h_m = 10^{-17} h_{-17}$ is the strain sensitivity amplitude. In an experimental detector, $h_m$ is computed from the strain spectral amplitude $h_s = h_s \sqrt{T_s}$, where $T$ is the observation time for a GW source that evolves slowly with time (source approximately monochromatic), or the characteristic width in the case of a short-lived GW burst. In the case where $\omega \sim \omega_N$, Equation (13) approaches the result $dE_{abs}/dt = 2\eta_N E$, where $E$ is the energy of the mode. Therefore, the square of the surface rms velocity, $V_N^2(\omega) \equiv \langle \xi_N^2 \rangle / (2\eta_N I) dE_{abs}/dt$, when excited by a GW source, reads

$$V_{N,gw}^2(\omega) = \frac{1/2 \gamma_s h_s^2 L_n^2 \omega^6}{(\omega^2 - \omega_N^2)^2 + 4\eta_N^2 \omega^2},$$

(14)

where $\gamma_s$ is an additional parameter (dimensionless and of the order of unity), which relates to the surface layer where the velocity measurement is made.

The oscillation quantities, such as the acoustic eigenfunctions, strongly depend on the solar surface structure, especially the stellar atmosphere. Hence, to test the quality of our solar oscillation model, we computed the normalized inertia $C_m^\omega$ (with $l = 2$) for the quadrupole acoustics modes, which are very sensitive to the surface of the star. We found that $C_{m^2}$ varies from $5.8 \times 10^{-4}$ for $n = 0$ to $1.0 \times 10^{-9}$ for $n = 18$, these values are consistent with the results found in the literature (Provost et al. 2000). In the case where $\omega = \omega_N$, Equation (14) reduces to

$$V_{N,gw}^2(\omega) = \frac{\gamma_s h_s^2 R_n^2 X_{\omega N}^2 \omega^4}{32\eta_N^2}.$$

(15)

4. DISCUSSION

In the Sun, as in any spherical resonant-mass detector, the excitation of eigenmodes by an external GW source strongly depends on the internal structure of the star, and in particular on how these modes are damping in the stellar upper layers. As shown in Equation (14), $\eta_N$ is the leading coefficient that determines the capacity of solar acoustic oscillations to absorb GWs. Although $\eta_N$ is determined with precision from solar oscillations in the high-frequency range of the acoustic spectrum (above 1.5 mHz), this is not the case in the lower-frequency range. In this region of the spectrum, we only have a few theoretical predictions.

Figure 2 shows the damping rates obtained by different observational groups: Libbrecht (1988); Chaplin et al. (1997); Baudin et al. (2005); Garcia et al. (2011), as well as the theoretical predictions of Houdé et al. (1999); Grigahcène et al. (2005); Belkacem et al. (2009, 2012, 2013). The damping rate increases in a nonlinear way with the frequency of the modes, mostly due to the fact that $\eta_N$ is strongly dependent on the properties of the convection and the microphysics of the...
upper layers of the star (e.g., Lopes & Gough 2001; Brito & Lopes 2014). The current predictions of $\eta_N$ agree well with observations for modes with $\nu \geq 1.5$ mHz. Unfortunately, for modes in the lower frequency range, observational data is non-existent, and there are only a few theoretical predictions (Houdek et al. 1999; Belkacem et al. 2009). Estimating the damping rate for low frequencies is very difficult. We note that Equation (4), which describes the amplitude of acoustic oscillations, was obtained from the wave equation (1), which is a good approximation for most of the acoustic oscillations (Chaplin et al. 2005). Moreover, even for such low values of $\eta_N$, the steady state solution is reached, even if it is not strictly the case for a pure harmonic damped oscillator (e.g., Rathore et al. 2004; McKernan et al. 2014). Actually, an $\eta_N$ of $\sim 10^{-2}$ mHz is currently observed for global low degree modes (Chaplin et al. 1997; Baudin et al. 2005). In particular, in the case of the Sun, the damping rates of all radial low orders ($n \geq 1$ or $\nu \geq 250 \mu$Hz) have been successfully measured (e.g., Turck-Chieze & Lopes 2012). As acoustic modes with similar frequencies are equally damped in the convection zone, the damping rates of quadrupole modes can be estimated from the same quantities measured from radial modes.

This is due to the fact that hydrodynamic simulations of turbulent convection in stars are not able to accurately reproduce stellar convection. As a consequence, the prediction of damping and excitation of low order modes, including the damping of quadrupole acoustic modes, is not fully reliable. For future use, in Figure 2 we show a “comparison” model in which $\eta_N$ is almost constant for $\nu \leq 1$ mHz, and the damping rate of low order modes is assumed to be identical to the $\eta_N$ value for $\nu \sim 1$ mHz. The motivation for representing this “comparison” theoretical model is to show the importance of $\eta_N$ in the detection of GW events. In particular, the value of $\eta_N$ for low values of $\nu$ has a major impact on the transfer function.

Figure 2. Damping rates as a function of the frequency for the Sun. The magenta, cyan, and blue dots correspond to the measurements made by Baudin et al. (2005), Chaplin et al. (1997), and Libbrecht (1988), and the green and yellow dots correspond to the theoretical predictions (Houdek et al. 1999; Houdek & Gough 2002; Baudin et al. 2005). The yellow dots correspond to a “comparison” theoretical model for which the damping rate is considered constant for $\nu \leq 1.0$ mHz. The agreement between the theory and observation is very good for the high frequencies, but for the lower frequencies no observational data is available, and there are only a few theoretical predictions. The green and yellow dots correspond to the values adopted for calculation of the GW transfer function (see Figure 3).

Figure 3. Square of the transfer function $T_{N,gw}^2(\omega_N)$ for the acoustic quadrupole modes of different radial order. All the acoustic modes show a clear well-defined Lorentz profile. However, the low order modes have a larger FWHM than the high order modes. The red curve corresponds to the square transfer function of the combined quadrupole acoustic modes spectrum (yellow dots in Figure 2). The blue, green, magenta, and cyan curves correspond to $T_{N,gw}^2(\omega_N)$ for acoustic quadrupole modes of order $n = 0, 1, 2$ and 3. (A color version of this figure is available in the online journal.)
If \( \eta_N \) has values of the order of \( 10^{-6} \mu \text{Hz} \) or \( 10^{-3} \mu \text{Hz} \) as predicted by some theoretical damping oscillation models (cf. Figure 2), GW events with \( h_{-20} \) lead to \( V_{N, gw}(\omega) \) with \( 10^{-9} \text{ cm s}^{-1} \) (comparison model) or \( 10^{-6} \text{ cm s}^{-1} \) (theoretical model). In the case of an occurrence of GW events with \( h_{-20} \sim 10^3 \), \( V_{N, gw}(\omega) \) will have values of the order of \( 10^{-6} \text{ cm s}^{-1} \) or \( 10^{-3} \text{ cm s}^{-1} \). This latter result is relatively near the current helioseismology measurements.

In principle, it should be possible to separate the quadrupole excitation by gravitational waves from the excitation by convection. Current observational data from helio- and asteroseismology allow us to determine in great detail the properties of damping and excitation of acoustic oscillations by the turbulent motions in the stellar upper layers (e.g., Lopes & Gough 2001). In particular, the accurate measurement of frequencies, damping rates, and the maximum rms surface velocities of global acoustic modes (modes with \( l \leq 4 \)) can be used to separate the GW excitation of quadrupole modes from the excitation and damping due to the turbulent convection. This is possible because it has been shown both theoretically and observationally that the excitation and damping of global acoustic modes by convection (including quadruple \( l = 2 \)) depends only on the frequency of the mode (and is independent of the degree of the mode). As all the low degree modes are equally excited by convection, if a low order quadrupole is stimulated by a GW source, it will show a unique pattern in the pulsation spectrum, quite distinct from the other global acoustic modes (like radial, dipole, and octopole) with identical frequencies. This should be a strong hint of excitation of quadrupole modes by a GW source.

5. SUMMARY AND CONCLUSION

In this article, we calculated the excitation of acoustic quadrupole modes by GW in a star like the Sun by using a formulation identical to that used for the computation of eigenmodes in resonant-mass detectors. In this work, we used realistic theoretical predictions of damping rates for acoustic modes of low order which have been validated at high frequencies.

In particular, we find that the low order modes in the Sun have a quality factor an order of magnitude higher than those found in resonant-mass detectors. Moreover, the sensitivity of acoustic modes to GW perturbations is regulated by an effective length as in an experimental bar/spHERE detector which in the Sun takes values between \( 10^7 \text{ cm} \) and \( 10^9 \text{ cm} \). This large variation in the value of the effective length is related to the fact that in stars, the eigenfunctions of acoustic modes (increasing with the order of the mode) are mostly sensitive to the stellar envelope and less sensitive to the stellar core.

The helioseismological acoustic wave frequencies overlap with the gravitational radiation frequency range that will be probed by eLISA (Amaro-Seoane et al. 2013). One of the targets of eLISA will be nearby ultracompact binaries. The sensitivity \( h_f \) of eLISA will be only \( 10^{-18}(\text{Hz})^{-1/2} \) at 0.001 Hz, and a factor of 10 worse at 0.0003 Hz. The brightest nearby binaries have predicted strain spectral amplitudes in the range \( (3.10^{-15} \sim 3.10^{-17})(\text{Hz})^{-1/2} \) over frequencies 0.01 Hz to 0.001 Hz. The strongest binaries over two years of observation are predicted to have \( h_f \sim 10^{-13}(\text{Hz})^{-1/2} \) (or \( h_{18} \sim 10^{-20} \) and frequencies as low as 0.0003Hz. The helioseismological modes are excited over 300–3000 \( \mu \text{Hz} \) and could be up to a factor 100 more sensitive than eLISA.\(^{11} \)

Presently, the main caveat in this model is the damping rate, which in the case with modes with high frequencies is well determined \((v \geq 1.5 \text{ mHz}) \) from observations, but in the case with modes with low frequencies the damping rates are theoretical. Accordingly, with present damping rate estimates, we predict an rms square velocity on the solar surface of the order of \( (10^{-1} \sim 1) \cdot h_{-20} \text{ cm s}^{-1} \) for an GW event with a strain amplitude of \( 10^{-20} h_{-20} \). Some of these values are near the current rms surface velocity amplitudes measured in the Sun’s surface.

In principle, as in experimental detectors, the measurement of the maximum amplitude of the rms velocity of quadrupole eigenmodes excited by GW periodic or random events is very difficult. Nevertheless, this difficulty could in part be overcome by taking advantage of several aspects that are unique to stars: (1) stars (due to their very large masses) have a very high GW integrated cross-section; (2) a large number of stars of different masses have been found (presently more than 500) to oscillate in a manner identical to the Sun; (3) stellar seismology instruments are recording very long time series of seismic data, in some cases spanning over several years, and in the case of the Sun, more than two decades; (4) the possibility of looking simultaneously for the same single or periodic GW event in distinct stars (as GWs propagates between stars at the speed of light); and (5) the possibility of using radial and dipole acoustic modes to isolate the GW signal in the quadrupole mode, as the excitation and damping of acoustic modes depends uniquely on the frequency. In particular, oscillating stars can provide a unique way of looking for contemporaneous quadrupole mode excitations in different stars by a single GW event. As the distances between many of these stars are relatively small, as in the case with stellar clusters, this can be used advantageously to look for the same GW imprint on quadrupole modes of different stars. In these cases, the time-lag between the excitation of quadrupole modes of two distinct stars can be determined accurately from

\(^{11} \) We remind the reader that \( h_f = h_{18}/\sqrt{T} \), where \( T \) is the observation time (see Section 3).
the locations of the stars and the speed of propagation of the GWs.

Although the challenges are great, the discovery of GW via stellar acoustic oscillations by the current set-up of experiments on Earth and/or in space is such an exceptional outcome that all the effort toward accomplishing this goal is well worth the investment.

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