Clock Synchronization based on Second-Order Quantum Coherence of Entangled Photons

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We present an algorithm for synchronizing two clocks based on second-order quantum interference between entangled photons generated by parametric down-conversion. The procedure is distinct from the standard Einstein two-way clock synchronization method in that photon correlations are used to define simultaneous events in the frame of reference of a Hong-Ou-Mandel (HOM) interferometer. Once the HOM interferometer is balanced, by use of an adjustable optical delay in one arm, arrival times of simultaneously generated photons are recorded by each clock. Classical information on the arrival times is sent from one clock to the other, and a correlation of arrival times is done to determine the clock offset.

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High-accuracy synchronization of clocks plays an important role in fundamental physics and in a wide range of applications such as communications, message encryption, navigation, geolocation and homeland security. There are two classical methods of time synchronization of spatially separated clocks: Eddington slow clock transport[1], and Einstein synchronization[2] by exchange of light signals. In Eddington slow clock transport, two co-located clocks are initially synchronized, and then one of these clocks is slowly transported to another location to synchronize a distant clock. For most technological applications today, this method is not practical because it requires transport of hardware as well as conflicting requirements: on the one hand clock transport must be slow to reduce the effect of time dilation, but on the other hand it must be fast enough so that significant time differences do not accrue from unavoidable timing errors due to the limited frequency stability of the transported clock’s mechanism or due to gravitational potential differences along the path of the transported clock[2].

Einstein synchronization consists of a round-trip exchange of classical light signals between two spatially separated clocks[2]. A signal is sent from clock B (which is to be synchronized with clock A) and reflects off the face of clock A, and then returns to clock B. The signal that reflected off the face of clock A carries classical information, from clock A to clock B, consisting of the time on clock A at the event of signal reflection. In practical applications today, a satellite system, such as the Global Positioning System (GPS), is used for synchronizing two spatially separated clocks[3, 4, 5]. The GPS is a satellite system based on Einstein synchronization, since signals are sent from satellite-to-ground and from ground-to-satellite to synchronize the satellite clocks with a master clock on Earth[6]. The time-synchronization accuracy provided by a GPS receiver is on the order of 20 ns. However, there are many applications, such as coherent detection of electromagnetic signals, where time synchronization is required to an accuracy that cannot be provided by the GPS. Recently, there have been several clock synchronization schemes proposed that are based on quantum mechanical principles[7, 8, 9, 10, 11]. The hope is that quantum mechanical methods can provide a higher clock synchronization accuracy than classical methods.

In this letter, we present a simple quantum mechanical algorithm to synchronize two spatially separated clocks to an accuracy limited by the resolution of the Hong-Ou-Mandel (HOM) interferometer, which has been demonstrated to be better than one hundred femtoseconds[12]. Our algorithm for clock synchronization has four
key features. First, no information is needed on the geometric distances between clocks. Second, the algorithm works in vacuum because it does not rely on the presence of an optical medium, and hence is applicable to space-based systems. Third, rather than using massive particles, the algorithm is based on entangled photon pairs, which are weakly coupled to the environment so the synchronization can be carried out over large distances. Finally, the essential elements that are contained in the algorithm, such as single-photon detection, time of arrival tagging, and signal time delay techniques, have already been demonstrated in the laboratory at various levels of precision.

Our synchronization algorithm uses photon coincidence counting in the frame of reference in which the entangled photon source is at rest. The underlying method is based on second order quantum interference between two entangled photons, rather than on classical exchange of information, as used in classical Einstein synchronization. The synchronization algorithm is shown schematically in the Minkowski diagram in Figure 1. The goal is to synchronize clock B with clock A. Clocks A and B are assumed stationary in the frame of reference in which the source of correlated photons is at rest. The HOM interferometer is spatially co-located with the photon source, and both are on world line O. Also shown in Figure 1 in grey is the world volume of an optical medium which has an adjustable index of refraction, n, which is located between the world line O and world line of clock B.

We use “hardware time”, \( \tau^* \), to mean the elapsed time (since some epoch) kept by a real hardware clock. Proper time, \( \tau \), is the elapsed time that is kept by an ideal clock and it depends on the world line, or history, of the ideal clock. The difference between “hardware time” and proper time is that a real hardware clock can have mechanical imperfections that make “hardware time” deviate from proper time. On the other hand, coordinate time, \( t \), is a global quantity, which is associated with the metric of spacetime, \( g_{ij} \), and enters in the definition of the system of 4-dimensional coordinates. It is coordinate time that provides the connection between proper times on two different world lines. However, it is well-known that coordinate time is not a measurable quantity.

A synchronization algorithm must deal with synchronization of time kept by real hardware clocks, which means it must deal with “hardware time”. In what follows, we assume that our hardware clocks are sufficiently stable so that “hardware time” is a good approximation of proper time: \( \tau^* = \tau \), over the times of interest in the synchronization procedure. We also assume that both clocks A and B have the same rate with respect to coordinate time, and with respect to each other.
so we are neglecting the effect of gravitational potential differences between the locations of clock A and B during the synchronization time. Therefore, we assume that both clocks A and B keep proper time, but they have an unknown constant offset between them, which we seek to determine by our synchronization algorithm.

The algorithm is as follows. Correlated photon pairs are generated in a crystal by parametric down-conversion \[21\, \text{to} \, 24\] at event \(P\). The pairs are created simultaneously at a unique event in space-time, within the crystal to an accuracy of better than 100 femtoseconds \[12\]. One photon from the pair arrives at clock A at coordinate time \(t_o^{(A)}\), is reflected back toward the world line \(O\), and arrives at event \(M_o\). The other member of the photon pair travels through an adjustable optical medium to clock B, shown in grey in Figure 1, and is reflected back toward world line \(O\), again traveling through the optical medium. This procedure is continued while the index of refraction \(n\) of the optical medium is adjusted, until both photons from the pair arrive at a single event \(M_o\). The event \(M_o\) is identified by a minimum observed in the HOM interferometer, indicating that the interferometer is balanced. The balance condition means that the \textit{coordinate time} of photons arriving at clock A and B are equal,

\[ t_o^{(A)} = t_o^{(B)} \] (2)

indicating that these events are simultaneous in the system of coordinates in which the interferometer is at rest \[13\]. We define simultaneous events as events with the same coordinate time in the given system of inertial coordinates \[2\]. Once the interferometer is balanced, correlated photon pairs are emitted on the world line \(O\) at events \(M_1, \ldots, M_N\). Photons detected at clock B have passed through the optical device, whose delay has been adjusted to obtain the balanced interferometer condition. Data consisting of the arrival time of photons at clock A and B is recorded on the world lines of clocks A and B, consisting of the sets of numbers, \(\{\tau_i^{(A)}\}\) and \(\{\tau_i^{(B)}\}\), \(i = 1 \cdots N\), as measured with respect to clocks A and B, respectively. We use the notation \(\tau_i^{(A)}\) and \(\tau_i^{(B)}\) for the proper times of photon arrival at clock A and B, as distinct from coordinate time of photon arrival, \(t_i^{(A)}\) and \(t_i^{(B)}\).

Photons that are “coincident at clock A and clock B” are defined to be those that are simultaneous in the inertial system of space-time coordinates in which the HOM interferometer is at rest. Our definition of simultaneous photon arrival times on world lines A and B means that the coordinate times of photons arriving on world line A and B are the same

\[ t_i^{(A)} = t_i^{(B)} \] (3)

for photons \(i = 1, \ldots, N\). On the world line of clock A, the proper time elapsed between the events at coordinate time of reception \(t_i^{(A)}\) and \(t_1^{(A)}\) is given by

\[ \tau_i^{(A)} + \Delta \tau^{(A)} = t_i^{(A)} - t_1^{(A)} \] (4)

where \(\Delta \tau^{(A)}\) is the clock correction that relates coordinate time to proper time of clock A. A similar relation exists for clock B:

\[ \tau_i^{(B)} + \Delta \tau^{(B)} = t_i^{(B)} - t_1^{(B)} \] (5)

where \(\Delta \tau^{(B)}\) is the clock correction that relates coordinate time to proper time for clock B. It is clear from Eq. 4 and 5 that the proper times of clocks A and B are related by

\[ \tau_i^{(B)} - \tau_i^{(A)} = \Delta \tau^{(A)} - \Delta \tau^{(B)} \] (6)

for all received photon pairs \(i = 1, \ldots, N\).

The data taken by clock A and clock B is a series of \(N\) photon arrival times, tagged at the detector, with respect to each clock’s respective hardware times, \(\tau_i^{(A)}\) and \(\tau_i^{(B)}\), \(i = 1, \ldots, N\). Both clocks record photon arrival time data for a time \(T\) measured with respect to their local clock. The time \(T\) is long compared to the expected clock difference between clock A and clock B. The photon arrival time data, \(\{\tau_i^{(A)}\}\) and \(\{\tau_i^{(B)}\}\), are assembled into functions \(f_A(t)\) and \(f_B(t)\):
in the sums, for $i \neq j$, the same value of $\tau$, and $B$ means that $\tau_B = \Delta \tau_B - \Delta \tau(A)$, as given by Eq. (6), and the correlation function is of order unity at a value of $\tau = \tau_0 = \Delta \tau(B) - \Delta \tau(A)$. According to Eq. (6), if $\tau_0$ is added to the value of the current clock time $\tau(B)$, then clock B will be synchronized to clock A. Note that the clock time on clock B after synchronization is, in general, not equal to coordinate time in the chosen system of coordinates.

In summary, we have defined a simple algorithm to synchronize two real clocks that are spatially separated. The accuracy of the algorithm is based on accurate control of an optical delay line and on the accuracy of second order quantum interference exhibited by correlated photons in a HOM interferometer, which is below 100 femtoseconds [12]. Each experimental element in the algorithm, such as the source of correlated photons, photon detection, arrival time tagging, and the HOM interferometer, have been separately experimentally demonstrated during the past several years. Consequently, this algorithm is today a viable method of synchronizing two spatially separated clocks. This algorithm may be useful in applications, such as communications, message encryption, navigation, geolocation systems, to achieve synchronization accuracy that is beyond the current reach of classical synchronization methods.

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