Wirelessly Powered Backscatter Communications: Waveform Design and SNR-Energy Tradeoff

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Abstract—This paper shows that wirelessly powered backscatter communications is subject to a fundamental tradeoff between the harvested energy at the tag and the reliability of the backscatter communication, measured in terms of SNR at the reader. Assuming the RF transmit signal is a multisine waveform adaptive to the channel state information, we derive a systematic approach to optimize the transmit waveform weights (amplitudes and phases) in order to enlarge as much as possible the SNR-energy region. Performance evaluations confirm the significant benefits of using multiple frequency components in the adaptive transmit multisine waveform to exploit the nonlinearity of the rectifier and a frequency diversity gain.

Index Terms—Backscatter Communications, Waveform Design, SNR-Energy Tradeoff, Wireless Power Transfer

I. INTRODUCTION

The emergence of RFID technology in the last decade is the first sign of a serious interest for far-field wireless power transfer (WPT) and backscatter communications. RFID tags harvest energy from the transmit RF signal and rely on backscattering modulation to reflect and modulate the incoming RF signal for communication with an RFID reader. Since tags do not require oscillators to generate carrier signals, backscatter communications benefit from orders-of-magnitude lower power consumption than conventional radio communications [1]. Backscatter communication has recently received a renewed interest, in the context of the Internet-of-Things, with advances in backscatter communication theory and the development of sophisticated backscatter communication systems [2]–[5].

Backscatter communications commonly assume that the RF transmitter generates a sinusoidal continuous wave (CW). Significant progress has recently been made on the design of efficient signals for WPT [6]–[9]. In particular, multisine waveforms adaptive to the Channel State Information (CSI) have been shown particularly powerful in exploiting the rectifier nonlinearity and the frequency-selectivity of the channel so as to maximize the amount of harvested DC power [7].

In this paper, we depart from this traditional CW transmission and leverage those recent progress in WPT signal design, and in particular the adaptive multisine wireless power waveform design, to show that wirelessly powered backscatter communications is subject to a fundamental tradeoff between the harvested energy at the tag and the SNR at the reader. Indeed, the SNR at the reader is a function of the backscatter channel (concatenation of the forward channel from transmitter to tag and backward channel from tag to reader) while the harvested energy at the tag is a function of the forward channel only. Due to the difference between those two channels, the optimal transmit waveform design for SNR and energy maximization are different. This suggests that adjusting the transmit waveform leads to a SNR-energy tradeoff.

Specifically, assuming that the CSI is perfectly available to the RF transmitter, we derive a systematic and optimal design of the transmit multisine waveform in order to enlarge as much as possible the SNR-energy region. Due to the non-linearity of the rectifier, the waveform design and the characterization of the region results from a non-convex posynomial maximization problem that can be solved iteratively using a successive convex approximation approach. Simulation results highlight that increasing the number of sinewaves in the transmit multisine waveform enlarges the SNR-energy region by exploiting the non-linearity of the rectifier and a frequency diversity gain.

Notations: Bold letters stand for vectors or matrices whereas a symbol not in bold font represents a scalar. $\| . \|$ refers to the absolute value of a scalar and the 2-norm of a vector. $\mathcal{E} \{ . \}$ refers to the averaging operator.

II. SYSTEM MODEL

The overall system architecture is illustrated in Fig I (left).

A. Received Signal at the Tag

Consider a multisine signal (with $N$ sinewaves) transmitted by an RF transmitter at time $t$ over a single antenna

$$x(t) = \Re \left\{ \sum_{n=0}^{N-1} w_n e^{j2\pi f_n t} \right\},$$

(1)

with $w_n = s_n e^{j\phi_n}$ where $s_n$ and $\phi_n$ refer to the amplitude and phase of the $n^{th}$ sinewave at frequency $f_n$, respectively. We assume for simplicity that the frequencies are evenly spaced, i.e. $f_n = f_0 + n\Delta_f$ with $\Delta_f$ the frequency spacing. The magnitudes and phases of the sinewaves can be collected into vectors $s$ and $\Phi$. The $n^{th}$ entry of $s$ and $\Phi$ are written as $s_n$ and $\phi_n$, respectively. The transmitter is subject to a transmit power constraint $\mathcal{E} \left\{ |x|^2 \right\} = \frac{1}{2} \|s\|_F^2 \leq P$.

The transmit waveform propagates through a multipath channel and is received at the single-antenna tag as

$$y(t) = \sum_{n=0}^{N-1} s_n A_n \cos(2\pi f_n t + \psi_n)$$

$$= \Re \left\{ \sum_{n=0}^{N-1} h_n w_n e^{j2\pi f_n t} \right\}$$

(2)

(3)
where $h_n = A_n e^{j \psi_n}$ is the forward channel frequency response at frequency $f_n$. The amplitude $A_n$ and the phase $\psi_n$ are such that $A_n e^{j \psi_n} = A_n e^{(\phi_n + \psi_n)} = e^{\phi_n} h_n$.

**B. Tag’s Operation**

We assume the tag only performs binary modulation. Binary 0 corresponds to a perfect impedance matching that completely absorbs the incoming signal (i.e. the reflection coefficient is 0). The signal absorbed by the tag during binary 0 operation is conveyed to a rectifier that converts the incoming RF signal into DC current. Binary 1 corresponds to a perfect impedance mismatch that completely reflects the incoming signal (i.e. the reflection coefficient is 1). The signal reflected during binary 1 operation is backscattered to a reader, whose objective is to decide upon the sequence of transmitted bits (0 or 1).

**C. Rectenna Model and DC Current at the Tag**

We will assume the same rectenna model as in [6, 7]. The rectenna is made of an antenna and a rectifier. The antenna model reflects the power transfer from the antenna to the rectifier through the matching network. A lossless antenna can be modelled as a voltage source $v_{in}(t)$ followed by a series resistance $R_{ant}$. Let $Z_{in} = R_{in} + j X_{in}$ denote the input impedance of the rectifier with the matching network. Assuming perfect matching during binary operation 0 ($R_{in} = R_{ant}$, $X_{in} = 0$), all the incoming RF power $P_{in,av}$ is transferred to the rectifier and absorbed by $R_{in}$, so that $P_{in,av} = E \{ |v_{in}(t)|^2 \}/R_{in}$ with $v_{in}(t) = v_s(t)/2$ the input voltage to the rectifier as per Fig 1(right). Since $P_{in,av} = E \{ |y(t)|^2 \}$, $v_{in}(t) = y(t)\sqrt{R_{in}} = y(t)\sqrt{R_{ant}}$.

Consider a rectifier composed of a single diode followed by a low-pass filter with load ($R_L$). Denoting the voltage drop across the diode as $v_d(t) = v_{in}(t) - v_{out}(t)$ where $v_{out}(t)$ is the output voltage across the load resistor (see Fig 1), a tractable behavioural diode model is obtained by Taylor series expansion of the diode characteristic equation $i_d(t) = i_s(e^{v_d(t)/v_t} - 1)$ (with $i_s$ the reverse bias saturation current, $v_t$ the thermal voltage, $n$ the ideality factor equal to 1.05) around a quiescent operating point $v_d = a$, namely

$$i_d(t) = \sum_{i=0}^{\infty} k_i' (v_d(t) - a)^i,$$  

where $k_0' = i_s (e^{v_t/a} - 1)$ and $k_i' = i_s^{(i)}(v_t/a^{(i)})/i!$, $i = 1, \ldots, \infty$.

Assume a steady-state response and an ideal low pass filter such that $v_{out}(t)$ is at constant DC level. Choosing $a = E \{ v_{d}(t) \} = -v_{out}$, (4) can be simplified as $i_d(t) = \sum_{i=0}^{\infty} k_i' v_{in}(t)^i = \sum_{i=0}^{\infty} k_i' R_{ant}^{(i)} y(t)^i$. Truncating the expansion to order 4, the DC component of $i_d(t)$ is the time average of the diode current, and is obtained as $i_{out} \approx k_0' + k_2 R_{ant} E \{ y(t)^2 \} + k_4 R_{ant}^2 E \{ y(t)^4 \}$.

**D. Backscatter Signal and SNR at the Reader**

The backscatter signal received at the reader is given by

$$z(t) = m R \left\{ \sum_{n=0}^{N-1} h_{r,n} w_n e^{j \nu_f n t} \right\} + n(t)$$  

where $m$ equals 0 or 1 for binary operation 0 and 1, respectively. The quantity $n(t)$ is the AWGN and $h_{r,n} = A_{r,n} e^{j \psi_{r,n}}$ is the frequency response of the backward channel (from tag to reader) on frequency $n$.

After applying a product detector to each frequency and assuming ideal low pass filtering, the baseband signal on each frequency $n$ is given by

$$z_n = h_{r,n} w_n m + n_n$$

where $n_n \sim \mathcal{CN}(0, \sigma^2)$. The SNR after Maximum Ratio Combining (MRC) is finally given by

$$\rho(s) = \sum_{n=0}^{N-1} \frac{|h_{r,n} w_n|^2}{\sigma^2} = \sum_{n=0}^{N-1} \frac{A_{r,n}^2 A_{n}^2 s_n}{\sigma^2}.$$  

**E. CSIT Assumption**

We assume perfect CSIT, i.e. the forward $h_n$ and backscatter $h_n h_{r,n}$ channels are perfectly known $\forall n$ to the RF transmitter, so as to shape the transmit waveform dynamically as a function of the channel states to maximize $i_{out}$ and $\rho$. The backscatter channel $h_n h_{r,n}$ can be obtained at the RF transmitter by letting the reader send pilots, reaching the RF transmitter through backscattering. Backscatter and forward channels can then be estimated and obtained at the RF transmitter [3].

We also assume that the concatenated channel $h_{r,n} h_n w_n$ is perfectly known to the reader to perform MRC.

**III. WAVEFORM OPTIMIZATION AND SNR-ENERGY REGION CHARACTERIZATION**

**Subject to a transmit power constraint $\frac{1}{2} \|s\|^2 \leq P$ and under the assumption of perfect CSIT, the maximization of the SNR suggests an adaptive single-sinewave strategy (ASS) that consists in transmitting all power on a single sinewave, namely the one corresponding to the strongest channel $\hat{n} = \arg \max_n |h_{r,n}|$. On the other hand, the maximization of the harvested energy, namely $i_{out}$, is shown in [7] to be equivalent to maximizing the quantity

$$z_{DC}(s, \Phi) = k_2 R_{ant} E \{ y(t)^2 \} + k_4 R_{ant}^2 E \{ y(t)^4 \}$$  

where $k_i = \frac{1}{\|r_i\|^2}$, $i = 2, 4$. The maximization of (8) suggests allocating power over multiple sinewaves, and those with stronger frequency-domain channel gains are allocated more power, in order to exploit the non-linearity of the rectifier and the frequency diversity [7]. Hence the design of efficient waveforms for backscatter communication is subject to a tradeoff between maximizing received SNR at the reader and maximizing harvested energy at the tag. Characterizing this SNR-energy tradeoff and the corresponding waveform design is the objective of this section.

1 Assuming $i_s = 5 \mu A$, a diode ideality factor $n = 1.05$ and $v_t = 25.86 mV$, typical values are given by $k_2 = 0.0034$ and $k_4 = 0.3829$. 

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Fig. 1. System architecture (left) and single diode rectifier at the tag (right).
We can now define the achievable SNR-harvested energy (or more accurately SNR-DC current) region as

$$C_{SNR-IDC}(P) \triangleq \left\{ (SNR, IDC) : SNR \leq \rho(s), \right.$$  

$$I_{DC} \leq z_{DC}(s, \Phi^*), \frac{1}{2} \|s\|^2 \leq P \right\}. \quad (10)$$

Optimal values $s^*, \Phi^*$ are to be found in order to enlarge as much as possible $C_{SNR-IDC}$. The expression of $z_{DC}$ is provided in [6] after plugging [6] into [6].

We note that the phases of the waveform $\phi_n$ influence $z_{DC}$ but not the SNR. Hence we can choose the phases as in point-to-point WPT in [6], namely $\phi_n^* = -\psi_n$. This guarantees that all arguments of the cosine functions in $z_{DC}$ are equal to 0 in [9], which can simply be written as

$$z_{DC}(s, \Phi^*) = \frac{k_2}{2} R_{ant} \left[ \sum_{n=0}^{N-1} s_n^2 A_n^2 \right] + \frac{3k_4}{8} R_{ant}^2 \sum_{n_0+n_1+n_2+n_3} \prod_{j=0}^{3} s_{n_j} A_{n_j}, \quad (11)$$

$\Phi^*$ is obtained by collecting $\phi_n^* \forall n$ into a vector.

Recall from [11] that a monomial is defined as the function $g : \mathbb{R}^N_+ \to \mathbb{R} : g(x) = cx_1^{a_1} x_2^{a_2} \cdots x_N^{a_N}$ where $c > 0$ and $a_i \in \mathbb{R}$. A sum of $K$ monomials is called a posynomial and can be written as $f(x) = \sum_{k=1}^{K} g_k(x)$ with $g_k(x) = c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_N^{a_{Nk}}$ where $c_k > 0$. As we can see from [11], $z_{DC}(s, \Phi^*)$ is a posynomial.

In order to identify the achievable SNR-energy region, we formulate the optimization problem as an energy maximization problem subject to transmit power and SNR constraints

$$\max_s z_{DC}(s, \Phi^*) \quad (12)$$

subject to

$$\frac{1}{2} \|s\|^2 \leq P, \quad (13)$$

$$\rho(s) \geq SNR. \quad (14)$$

It therefore consists in maximizing a posynomial subject to constraints. Unfortunately this problem is not a standard Geometric Program (GP) but it can be transformed to an equivalent problem by introducing an auxiliary variable $t_0$

$$\min_{s,t_0} 1/t_0 \quad (15)$$

subject to

$$\frac{1}{2} \|s\|^2 \leq P, \quad (16)$$

$$t_0/z_{DC}(s, \Phi^*) \leq 1, \quad (17)$$

$$SNR \frac{\rho(s)}{t_0} \leq 1. \quad (18)$$

This is known as a Reversed Geometric Program. A similar problem also appeared in the WPT waveform optimization [6], [7] and the rate-energy region characterization of Simultaneous Wireless Information and Power Transfer [10]. Note that $1/z_{DC}(s, \Phi^*)$ and $1/\rho(s)$ are not posynomials, therefore preventing the use of standard GP tools. The idea is to replace the last two inequalities (in a conservative way) by making use of the arithmetic mean-geometric mean inequality.

Let $\{g_k(s, \Phi^*)\}$ be the monomial terms in the posynomial $z_{DC}(s, \Phi^*) = \sum_{k=1}^{K} g_k(s, \Phi^*)$. Similarly we define $\{f_n(s)\}$ as the set of monomials of the posynomial $\rho(s) = \sum_{n=0}^{N-1} f_n(s)$ with $f_n(s) = s_n^2 A_n^2 A_{n_0,n_1,n_2,n_3} / \sigma^2$. For a given choice of $\{\gamma_k\}$ and $\{\beta_n\}$ with $\gamma_k, \beta_n \geq 0$ and $\sum_{k=1}^{K} \gamma_k = \sum_{n=0}^{N-1} \beta_n = 1$, we perform single condensations and write the standard GP as

$$\min_{s,t_0} \frac{1}{t_0} \quad (19)$$

subject to

$$\frac{1}{2} \|s\|^2 \leq P, \quad (20)$$

$$t_0 \prod_{k=1}^{K} \left( \frac{g_k(s, \Phi^*)}{\gamma_k} \right)^{-\gamma_k} \leq 1, \quad (21)$$

$$SNR \prod_{n=0}^{N-1} \left( \frac{f_n(s)}{\beta_n} \right)^{-\beta_n} \leq 1. \quad (22)$$

It is important to note that the choice of $\{\gamma_k, \beta_n\}$ plays a great role in the tightness of the AM-GM inequality. An iterative procedure can be used where at each iteration the standard GP [19] is solved for an updated set of $\{\gamma_k, \beta_n\}$. Assuming a feasible set of magnitude $s^{(i-1)}$ at iteration $i-1$, compute at iteration $i$ $\gamma_k = g_k(s^{(i-1)}, \Phi^*) / z_{DC}(s^{(i-1)}, \Phi^*)$ $k = 1, \ldots, K$ and $\beta_n = f_n(s^{(i-1)}) / \rho(s^{(i-1)})$, $n = 0, \ldots, N-1$, and then solve problem [19] to obtain $s^{(i)}$. Repeat the iterations till convergence. The whole optimization procedure is summarized in Algorithm 1. The successive approximation method used in the Algorithm 1 is also known as a successive convex approximation. It cannot guarantee to converge to the global solution of the original problem, but yields a point fulfilling the KKT conditions [11].

Algorithm 1 Backscatter Communication Waveform

1: Initialize: $i \leftarrow 0$, $SNR, \Phi^*, s, z_{DC}^{(0)} = 0$
2: repeat
3: $i \leftarrow i + 1, \ s \leftarrow s$
4: $\gamma_k \leftarrow g_k(s, \Phi^*) / z_{DC}(s, \Phi^*), \ k = 1, \ldots, K$
5: $\beta_n \leftarrow f_n(s) / \rho(s), \ n = 0, \ldots, N-1$
6: $s \leftarrow \arg \min \left[ 19 - 22 \right]$
7: $z_{DC}^{(i)} \leftarrow z_{DC}(s, \Phi^*)$
8: until $\left| z_{DC}^{(i)} - z_{DC}^{(i-1)} \right| < \epsilon$ or $i = i_{\text{max}}$

IV. SIMULATION RESULTS

We consider a centre frequency of 5.18GHz, 36dBm EIRP, 2dBi receive and transmit antenna gain at the tag and 2dBi
receive antenna gain at the reader. The path loss between the transmitter and the tag and between the tag and the reader is $58\,\text{dB}$ for each link. A NLOS channel power delay profile is obtained from model B [13]. The channel taps each with an average power $\beta_l$ are independent, circularly symmetric complex random Gaussian distributed and normalized such that $\sum_l \beta_l = 1$. This leads to an average receive power of $-20\,\text{dBm}$ at the tag and $-74\,\text{dBm}$ at the reader. The noise power $\sigma^2$ at the reader is fixed to $-84\,\text{dB}$.

The simulation is run over a channel realization with a bandwidth $B = 1, 10\,\text{MHz}$. The frequency responses of the forward and backward channels are illustrated in Fig. 2. The channel frequency response within the $1\,\text{MHz}$ bandwidth is obtained by looking at Fig 1 between $-0.5$ MHz and $0.5$ MHz. The frequency spacing of the multisine waveform is fixed as $\Delta f = B/N$ and the $N$ sinewaves are centered around $5.18\,\text{GHz}$.

For the channel frequency responses of Fig. 2, Algorithm 1 is used, along with CVX [12], to compute the optimal waveform and the corresponding SNR-$I_{DC}$ tradeoff, illustrated in Fig 3 for $B=1\,\text{MHz}$ and $B=10\,\text{MHz}$. The extreme point on the $x$-axis (SNR maximization) is achieved using the ASS strategy. On the other hand, the maximum energy is in general achieved by allocating transmit power over multiple subcarriers (as a consequence of the non-linearity of the rectifier) [7]. A first observation from Fig 3 is that SNR and $I_{DC}$ are indeed subject to a fundamental tradeoff, i.e. increasing one of them is likely to result in a decrease of the other one. Nevertheless, as the channel becomes more frequency flat or the bandwidth decreases, the SNR-$I_{DC}$ appears more rectangular. A second observation is that an increase in the number of frequency components $N$ of the multisine waveform results in an enlarged SNR-energy region. Indeed, by increasing $N$, the waveform exploits the nonlinearity of the rectifier and a frequency diversity gain, the latter being beneficial to both SNR and energy. A third observation is that the shape of the SNR-energy region highly depends on the channel realizations and bandwidth. In particular, for the specific channel realization of Fig 2 we note that the $1\,\text{MHz}$ bandwidth favours higher $I_{DC}$ while the $10\,\text{MHz}$ bandwidth favours higher SNR. This can be explained as follows. Recall first that $I_{DC}$ is a function of the forward channel amplitudes $A_n \forall n$, while the SNR is a function of the backscatter channel $A_n A_{r,n}$. From Fig 2 $A_n$ reaches its peak for frequencies between -1 MHz and 0.5 MHz. Since the multisine waveform with a power allocation over multiple frequency components helps increasing $I_{DC}$, allocating the $N$ frequencies uniformly within the $1\,\text{MHz}$ bandwidth leads to higher $I_{DC}$ than that obtained with a $10\,\text{MHz}$ bandwidth (which exhibits deep fades). On the other hand, $A_n A_{r,n}$ exhibits its largest gain around 2MHz, which is outside the $1\,\text{MHz}$ bandwidth. Since ASS maximizes the SNR, larger SNRs are obtained on the $10\,\text{MHz}$ channel.

V. CONCLUSIONS

The paper derived a methodology to design adaptive transmit multisine waveforms for backscatter communications and characterize the fundamental tradeoff between conveying energy to the tag and enhancing the SNR of the backscatter communication link. Future interesting works consist in addressing the design of waveforms and the characterization of the SNR-energy region for more general setup including multiple antennas, multiple transmitters and multiple tags. The problem of CSI acquisition and its impact on the SNR-energy region is also of significant interest.

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