Solar and Atmospheric Neutrinos

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ABSTRACT

Possible explanations of solar neutrino and atmospheric neutrino anomalies are summarized and future tests discussed.

1 Introduction

In the standard model (SM) with no singlet right-handed $\nu's$ and a single Higgs field, all neutrino masses are zero and lepton number (as well as individual flavor quantum numbers) are exactly conserved. It follows that the charged leptonic current is diagonal in both mass and flavor basis and the mixing angles are zero. Hence any evidence for non-zero neutrino masses or for non-trivial mixings is evidence for physics beyond the Standard Model. This makes the search for neutrino masses and mixings doubly important: as measurement of fundamental parameters of intrinsic interest and as harbingers of new physics. In this talk I will concentrate on hints from solar and atmospheric neutrino observations that suggest non-zero neutrino masses and mixings.

2 Solar Neutrinos

The current status of the data on solar neutrino observations from the four on-going experiments is summarized in the table 1.

Table I
The solar neutrino data [1, 2, 3, 4] compared to the SSM[5]

| Experiment   | Data/SSM |
|--------------|----------|
| Kamiokande   | 0.51 ± 0.07 |
| Gallex       | 0.66 ± 0.12 |
| Sage         | 0.44±0.17  |
| Homestake    | 0.28 ± 0.04 |
The Kamiokande detector is sensitive only to $^8B$ neutrinos; and the Homestake detector is sensitive to $^8B$ (77%) as well as $^7Be$ (14%), pep (2%) and CNO (6%) neutrinos [5]. If the observations need no new neutrino properties, then the $^8B$ neutrinos are not distorted in their spectrum and the flux seen by Kamiokande (over a limited energy range), can be assumed uniform and hence applicable to Homestake as well. In that case a minimum of $(38 \pm 8)\%$ of SSM counting rate is contributed by $^8B$ neutrinos alone and adding pep neutrinos it is $(40 \pm 8)\%$ to be compared to the observed $(28 \pm 4)\%$. It is obvious that something must reduce the $^7Be$ neutrino flux drastically to obtain agreement. Since the effective temperature dependence of $^7Be$ neutrino flux is much weaker than for $^8B$ flux [6], it is difficult to arrange for a stronger suppression for $^7Be$ than for the $^8B$ flux. This is borne out in calculations where the core temperature is allowed to be a free parameter and it is found that a good fit to all the data cannot be obtained [7]. Furthermore, no solar model has been found which can reproduce the Chlorine rates even with the reduced $^8B$ flux or even come close [8]. There is a general agreement that with the Chlorine data averaged over the whole period some neutrino properties are called for [9].

Even if the remaining uncertainties in the solar modelling (or very low energy nuclear cross-sections) and difficulties inherent in pioneering experiments may cloud the interpretation of solar neutrino data in terms of neutrino properties [10]; it is important to keep in mind that there is no question that neutrinos from the sun have been detected: both at high energies - 10 MeV (Kamiokande, Homestake) and at low energies - 1 MeV (Gallex, SAGE). Hence a powerful neutrino beam with sensitivity to $\delta m^2 \geq 10^{-10}eV^2$ and $\sin^2 2\theta \geq 0.1$ is available, free of charge. It behooves us to utilise this beam maximally; and future upcoming experiments will do just that. They have rates of order $10^4$ per year; in real time, spectrum measurement, flux monitoring (via NC/CC in SNO) and low threshold (in Borexino). If the neutrino parameters lie in this region we will definitely know the answer by 1996.
Table II
Future Detector Characteristics

| Status  | Detector       | Size | $E_{th}$ | Ev/yr | Reaction Features       |
|---------|----------------|------|----------|-------|-------------------------|
| Constr. | SNO($\hat{c}$) | 1KT  | 5 MeV    | 10,000| $\nu_e D \rightarrow e p p$ spectrum |
| Constr. | SuperK($\hat{c}$) | 22KT | 5 MeV    | 3000  | $\nu D \rightarrow \nu p n$ NC/CC |
| Test    | Borexino (LS)  | 0.2KT| 0.25 MeV | 500   | $\nu_e$ spectrum         |
| Test    | ICARUS (IC)    | 3KT  | 5 MeV    | 8000  | $\nu_e$ spectrum         |
| Test    | Borexino (LS)  | 0.2KT| 0.25 MeV | 3000  | $\nu_e$ $^7$Be spectrum  |
| Prop.   | Bellaz         | 12T  | 0.1 MeV  | 12000 | $\nu_e$ pp               |

Assuming that neutrino properties are the culprit, I will summarize the solutions to the solar neutrino deficit with emphasis on the non-MSW options. For definiteness and simplicity I will assume (i) SSM fluxes of Bahcall and Pinosoument, (ii) two flavor mixing, (iii) and ignore mixing with sterile neutrinos and neutrino flavor changing neutral currents. I will briefly discuss the solutions and how each may be distinguished in future experiments; especially in Borexino, SNO, Superkamiokande and ICARUS [11].

MSW:
This is the case in which $\delta m^2$ and $\sin^2 2\theta$ lie in the range in which the solar matter effects are very important [12]. A fit to all four experiments leaves three allowed regions [13]. One is the small angle ($\sin^2 2\theta \sim 4.1 \times 10^{-3}, \delta m^2 \sim 10^{-5} eV^2$) region; in this region the rate for $^7$Be $\nu_e$ scattering in Borexino varies rapidly between 0.2 and 0.5 of SSM and $^8$B spectrum as seen in SNO or Superkamiokande will show distortion. Another is the large angle large $\delta m^2$ region ($\sin^2 2\theta \sim 1, \delta m^2 \sim 10^{-5} eV^2$); in this region $^7$Be is suppressed between 0.35 and 0.7 and there is no distortion of $^8$B spectrum. Finally there is a small region at large angle small $\delta m^2$ ($\sin^2 2\theta \sim 1, \delta m^2 \sim 10^{-6} eV^2$); here there is a strong day-night variation in $^7$Be line as seen in Borexino [14].

Large Angle Long Wavelength:
The large angle long wavelength ("just so") [15] continues to fit all the data [10] with $\delta m^2 \sim 10^{-10} eV^2$ and $\sin^2 2\theta \gtrsim 0.8$. Matter effects are negligible. This has striking predictions testable with future detectors: (i) suppression of $^7$Be in Borexino between 0.2 and 0.5, (ii) sharp distortion of $^8$B spectrum and most importantly, (iii) visible oscillations of $^7$Be line with time of the year with up to factor of 2 variations. This maybe the only chance [17] to see true quantum mechanical neutrino
oscillations [Fig. 1] and can be easily seen in Borexino and distinguished from the $1/r^2$ variation.

Akhmedov et al. [18] have given an interesting possible justification of such a scenario. They suppose that (i) there are only LH $\nu'$s, (ii) lepton number is conserved except by gravity; then at Planck scale there may be lepton number violating terms such as

$$\frac{g_{ij}}{m_p} \overline{\psi}^{\nu'}_{L_i} \psi_{L_j} \cdot \overline{\phi}^\tau \phi$$  \hspace{1cm} (1)

where $\phi$ is the standard Higgs doublet, $m_p$ Planck mass, i and j are family indices. Then the neutrino masses are Majorana and the mass matrix is

$$M_{\nu_{ij}} = \frac{g_{ij}v^2}{m_p}$$  \hspace{1cm} (2)

If one makes the further assumption that gravity is flavor-blind and $g_{ij} = g$ and $g \sim 0(1)$ then the matrix is

$$m_\nu = \frac{v^2}{m_p} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$  \hspace{1cm} (3)

which has as mass eigenvalues $m_1 = 0, m_2 = 0$ and $m_3 = m = 2v^2/m_p \approx 10^{-5}eV$. Hence $\delta m^2$ is about $10^{-10}eV^2$. The mixing matrix is easily calculated and it can be shown that

$$P(\nu_e \to \nu_e, L) = 1 - \frac{8}{9} \sin^2 \frac{m^2L}{4E}$$  \hspace{1cm} (4)

which corresponds to an effective $\sin^2 2\theta$ of 0.89.

Decay with Mixing:

A very old proposal is to have the neutrinos decay on the way to the earth [19]. The SN1987A observation of $\bar{\nu}_e'$s require that there be a stable component in $\nu_e$ and the mixing be not too small [20]. There must be also some new physics for the decay into another neutrino and a light or massless boson. In any case, phenomenologically, with the most recent Kamiokande and Gallex data in hand, the decay solution is ruled out at 98% C.L. [21].

Matter induced Decay:

There is another kind of decay that has been discussed in the literature; this is the matter induced or MSW- catalyzed decay [22]. The basic idea is that in matter the effective mass for $\nu_e$ is greater than for $\bar{\nu}_e$ and if a coupling to a scalar (e.g. Majoron) $\chi$ existed than the decay $\nu_e \to \bar{\nu}_e + \chi$ could occur in matter (but not in vacuum). Similarly, in presence of a flavor changing coupling the more relevant decay $\nu_e \to \bar{\nu}_\mu + \chi$ can also occur in matter. This matter induced decay lifetime (Lab) behaves as a constant rather than $E_\nu$ and hence this remains a viable solution.
Flavor Violating Gravity:

If gravitational interaction of neutrinos is not diagonal in flavor then even for massless neutrinos there are oscillations induced by this flavor dependent gravitational potential \[23\]. The survival probability for $\nu_e$ is given by

$$P(\nu_e \rightarrow \nu_e, L) = 1 - \sin^2 (2\theta_G) \sin^2[\tilde{\phi}EL]$$

(5)

where $\tilde{\phi}$ is the gravitational potential averaged over the neutrino pathlength, $\delta$ is the departure from flavor independence of gravity: $\delta = f_e - f_\mu$.

The quantity $(\delta\phi EL)$ can be written as $(\pi L/\lambda_G)$ where $\lambda_G = 6km(10^{-20}/[\pi L/\text{km}])$. The precise value of $\phi$ at the earth and the sun is very uncertain due to potentially large contributions from ”nearby” large masses such as the Virgo cluster or the local super cluster. Current limits on $\delta\phi$ from re-interpretations of $\delta m^2 - \sin^2 2\theta$ bound are (for $\nu_e - \nu_x$) $10^{-19}$ for $\sin^2 2\theta_G \sim 1$. It turns out that $\delta\phi$ in the range $10^{-20} - 10^{-21}$ and $\sin^2 2\theta_G \sim 1$ can provide a simultaneous good fit to all the solar neutrino data as well as the atmospheric anomaly. Future long-baseline experiments can extend the bounds on $\delta\phi$ to $10^{-22}$ or better and test this hypothesis as proposed by Pantaleone et al \[23\].

To summarize, future detectors such as SNO, Superkamiokande, Borexino, and ICARUS will have real time event rates of several thousand per year. They will measure the $^8B$ neutrino energy spectrum accurately, $^7Be$ line rate and the ratio of NC/CC in $\nu_e D$ reaction. With this information at hand it should be possible to establish that (a) neutrino properties are relevant, (b) distinguish between MSW, long wavelength, decay etc., (c) pin down the parameters narrowly and (d) deduce more precise information about the sun such as the core temperature.

3 Atmospheric Neutrinos

Neutrinos are produced by cosmic rays interacting in the atmosphere. A primary (P) reacts with “air” nucleus as:

“P” + “air” → $\pi + x$

(6)

The $\pi$ may interact or decay; if it decays:

$$\pi \rightarrow \mu + \nu_\mu$$

(7)

and at low energies (few GeV) the $\mu$ can also decay before it hits the ground:

$$\mu \rightarrow e + \nu_e + \nu_\mu$$

(8)
If all the $\mu$'s decay we are led to expect $N(\nu_\mu)/N(\nu_e)$ of 2 (ignoring the distinction between $\nu$ and $\bar{\nu}$). This ratio, furthermore, is expected to be essentially independent of the zenith angle at low energies. Neutrinos of energies below 2 GeV give rise to “contained” events in typical kiloton underground detectors. The results from the two large water-Cerenkov detectors suggest that the ratio $R = N(\nu_\mu)/N(\nu_e)$ is smaller than expected by almost a factor of two. Kamiokande finds (based on 6.1 Kton yr) for the ratio of ratios[24].

$$R_{\text{obs}}/R_{\text{MC}} = 0.60 \pm 0.07$$

(9)

while IMB finds (based on 7.7 Kton yr)[25]

$$R_{\text{obs}}/R_{\text{MC}} = 0.54 \pm 0.07$$

(10)

The result of Frejus (for contained events) and Nusex is respectively 0.87 $\pm$ 0.21 (based on 1.56 Kton yr) and 0.99 $\pm$ 0.40 (based on 0.4 Kton yr)[26, 27]. Finally, very recently SOUDAN II has found a result of 0.69 $\pm$ 0.19 based on a 1 Kton yr- exposure[28].

The ratio $N(\nu_\mu)/N(\nu_e)$ is considered more reliably calculated than the individual fluxes: the ratio is stable to about 5% amongst different calculations whereas the absolute fluxes vary by as much as 20 to 30%[29].

The most important question is whether there is a “mundane” explanation for the deficit or is a new physics explanation called for? Let us consider the mundane explanations. (i) Perhaps the $e/\mu$ identification in the water Cerenkov detectors is simply wrong. In response to this, Kamiokande has made a very convincing case for the correctness of their $e/\mu$ identification by showing how it works very well in finding the expected number of $\mu \rightarrow e$ decays in their contained events[30]. Also, the fact that Soudan-II sees the same deficit (in a non-water-Cerenkov detector) is encouraging. Finally, the upcoming (underway?) beam test at KEK should settle the issue once and for all. (ii) There is the question of low energy $\nu$-nuclear cross-sections and lepton energy distributions in the region $E_\nu \sim 200$ MeV to 1 GeV. Ideally we would like to have these ($\nu^{16}O \rightarrow e^{16}F$) measured experimentally. Even though $e/\mu$ universality is not expected to be violated except kinematically (and hence in a known manner) the difference between $\nu$ and $\bar{\nu}$ cross sections is important. This is unlikely to be the explanation[31]. (iii) It has been pointed out by Volkova[32] that if $\pi^+$ at low energies dominates over $\pi^-$ then (because $\sigma_{\nu_e} < \sigma_{\nu_\mu}$) the effect is to enhance $e/\mu$ signal. She finds that with a $\pi^+/\pi^- \sim 2.5$ (compared to values in the range 1.1-1.3 used to by others) the effect is only about 10% of the observed.
The importance of knowing the relative amount of $\bar{\nu}_e/\nu_e$ has also been stressed by Suzuki[30].

Now we turn to new physics solutions for the anomaly. The first two will depend on assuming some absolute flux calculation which accounts for the $\nu_\mu$ flux correctly; e.g. the flux calculated by Bugaev and Naumov[33]. (i) The simplest explanation[34] is that there is a universal $\nu_e$ excess of flux $10^{-3}cm^{-2}GeV^{-2}sr^{-1}sec^{-1}$. Its spectrum may be falling like, say, $E^{-2}$ or $E^{-3}$; in that case the energy density is about $1/100$ of that in cosmic rays and is quite “safe”. These must be isotropic (according to observations) and could be galactic or more likely extragalactic. Could they be from AGN’s? (ii) A very interesting proposal by Mann et al.[35] is that the excess $\nu_e$ events are not due to $\nu_e'$s at all but due to proton decay mode $P \rightarrow e^+\nu\nu$. In this case, the energy spectrum of the excess $e$ events should end at $E_e \sim 600$ MeV. Both Kamiokande and IMB have a few events beyond 600 MeV but the errors are large and the hypothesis can not be ruled out with the present data. The rate corresponds to a $\tau/BR$ of $4.10^{31}$ years.

To account for such a mode, the theoretical model has to forbid other decay modes (in addition to predicting this mode). The simplest operator for such a decay mode (in absence of a light $\nu_R$) is $f/M6 < \phi >$ $uud\ e^+\nu\nu$ and hence $M \sim 10^5$ GeV. In some typical models this implies Leptoquarks in mass range below 1 TeV and discoverable at LHC or Linear colliders[36].

A very recent calculation of the atmospheric neutrino flux by Perkins[37] uses new muon measurements in the atmosphere by MASS collaborations[37]. He finds that the absolute fluxes tend to agree with Barr et al.[29] but do not support the Bugaev-Naumov[33] low fluxes needed for the interpretations given above. For absolute fluxes somewhere between Bugaev-Naumov and Barr et al., another interpretation is possible viz. a universal isotropic source of equal number of $\nu_e'$s and $\nu_\mu'$s, as suggested e.g. by Tomozawa[38].

We now turn to the flux independent explanation in terms of neutrino oscillations. The deviation of $R_{obs}/R_{MC}$ from 1 is fairly uniform over zenith angle and is most pronounced in the charged lepton energy range 200-700 MeV which corresponds to neutrino energies from 300 MeV to 1.2 GeV. If we are to interpret this deficit of $\nu_e'$s (and/or excess of $\nu_\mu'$s) as being due to neutrino oscillations, the relevant parameters are determined rather easily[39]. The typical height of production, $h$, is about 15-20 km above ground and for a zenith angle $\theta$ the distance travelled by the neutrino before reaching the detector is

$$L(\theta) = R \left[ \sqrt{(1 + h/R)^2 - \sin^2 \theta - \cos \theta} \right]$$ (11)
where \( R \) is the radius of the earth. Allowing for angular smearing due to the scattering and finite angular resolution one finds that neutrino path lengths can vary between 30km to 6500 km, and hence \( L/E \) can vary between 25 km/GeV and 20,000 km/GeV. Since the data do not show any \( L \) (i.e. \( \theta \)) or \( E \) dependence we may infer that the oscillations have already set in at \( E_\nu \sim 1 \) GeV and \( L \sim 30 \) km and hence \( \delta m^2 \) cannot be much smaller than \( 3 \times 10^{-2} eV^2 \). As for the mixing angle \( \theta \), if \( P \) denotes the average oscillation probability i.e. \( P = \sin^2 2\theta < \sin^2 \delta m^2 L/4E > = \frac{1}{2} \sin^2 2\theta \); then \( R = 1 - P \) in case of \( \nu_\mu - \nu_\tau \) oscillations and for \( \nu_\mu - \nu_e \) oscillations

\[
R = \frac{1 - (1 - r)P}{1 + (1/r - 1)P} \quad (12)
\]

where \( r = N(\nu_e)/N(\nu_\mu) \) in absence of oscillations and most flux calculations yield \( r \sim 0.45 \). Since \( R \) is nearly 0.6, large mixing angles of order 30° to 45° are called for, \( \nu_\mu - \nu_e \) mixing needing somewhat smaller ones. Detailed fits by Kamiokande and more recently IMB, bear these expectations out although somewhat bigger range of parameters (\( \delta m^2 \) up to \( 4 \times 10^{-3} eV^2 \) and mixing angles up to 20°) are allowed [24].

There are also higher energy muons in the underground detectors. Typically in IMB and Kamiokande detectors events are classified as thrugoing muons and stopping muons. The average \( \nu_\mu \) energy for these correspond to about 100 GeV and 10 GeV respectively. These events are expected to have the famous \( \sec \theta \) zenith angle distribution due to the competition between \( \pi \) decay and interaction and the \( \nu_e \) flux is very small since the high energy \( \mu' \)s do not have time to decay in 20 km [40]. If the above explanation of the low energy anomaly is correct then for the thrugoing events (a) the zenith angle distribution should be distorted since for horizontal events oscillations will not have set in (\( \delta m^2 L/4E \ll 1 \)) but for vertical events there should be depletion (b) the total muon event rate itself should be decreased by the depletion and (c) in case of \( \nu_\mu - \nu_e \) oscillations there should be an enhancement of \( \nu_e \) (and hence showering) events especially at energies where there might be matter enhancement [39, 41]. Several detectors, IMB, Kamiokande, Baksan, KGF (and now MACRO) have data of the order of a few hundred events each [42, 43, 44, 45, 46]. There is no clear distortion of the zenith angle distribution or depletion of the total rate seen in any data. However, since the comparison has to be made to absolute flux calculations, the limits on \( \delta m^2, \theta \) derived are not yet strong enough to rule out the values needed to explain the low energy anomaly [17]. IMB has derived forbidden regions [42] by taking ratio of stoppers/thruogoers which is largely flux independent and which rules out the large angle region (\( \sin^2 2\theta \sim 0.6 \) to 1) for \( \delta m^2 \sim 3 \times 10^{-3} \) to \( 8 \times 10^{-3} \). The same data when used to constrain \( \nu_\mu - \nu_e \) mixing yield very weak constraints [48]. In any
case, neutrino oscillation explanation of the contained event anomaly is not ruled out.

4 Simultaneous Explanations for Solar and Atmospheric Anomalies:

(i) If $\nu_\mu - \nu_\tau$ mixing is responsible for atmospheric deficit with $\delta m^2 \sim 10^{-2}eV^2$ and large mixing; and if the see-saw mechanism is operative, with $m_{\nu_\tau} \sim 0.1 \, eV$ one expects a $m_{\nu_\mu}$ in the range $(\frac{m_{\tau}}{m_t})^2 m_{\nu_\tau} \sim 3.10^{-3}eV$ and leads to $\delta m^2$ for $\nu_\mu - \nu_e$ of about $10^{-5} \, eV^2$, making it just right for an explanation of solar neutrino data via MSW. In this case neutrino masses are all less than 0.1 eV and there is no possibility to account for any hot dark matter without sterile neutrinos.

(ii) If the atmospheric anomaly is accounted by $\nu_\mu - \nu_e$ mixing with $\delta m^2 \lesssim 0(10^{-2})eV^2$ and the see-saw mechanism is operative; then the $\nu_\tau$ mass is of order of $(m_t/m_\tau)^2 m_{\nu_\tau} \sim 10 \, eV$. This is in the right range to account for hot dark matter in the mixed (30% HDM, 70% CDM) DM scenario to account for the large scale structure which has been proposed recently. The implications for solar neutrinos are a uniform energy dependent depletion by about $(1 - \frac{1}{2} \sin^2 2\theta_\mu_e) \sim 0.5$ to 0.75. This is completely consistent with Kamiokande and Gallium results but not with $^{37}Cl$ results. In any case it can be verified in the future solar neutrino detectors. Both of the above scenarios were first discussed by Learned et al. in 1988.

(iii) Three flavor mixing:

A very interesting possibility is that $\delta m^2_{21} \sim 10^{10}eV^2$ and $\delta m^2_{31} \sim \delta m^2_{32} \sim 10^{-2}eV^2$. In this case “just so” is relevant for solar neutrinos and for atmospheric neutrinos it is a general 3-flavor mixing that is operative. The range of mixing angles allowed has been determined by Acker et al. A possible allowed matrix for example is:

$$U = \begin{pmatrix}
0.64 & 0.48 & 0.6 \\
-0.76 & 0.32 & 0.56 \\
0.08 & -0.81 & 0.57
\end{pmatrix}$$

There can be no see-saw mechanism since there is near degeneracy. It is even more interesting if the degenerate mass $m$ is of order of a few ($\sim 3$) eV. Then the effective mass for HDM is $3m \sim 10eV$ and $\nu_e$-mass is tantalizing close to the current $\beta$–decay mass limits. Furthermore, in neutrinoless double beta decay, the effective neutrino mass (assuming Majorana neutrinos) is

$$< m^{eff}_\nu > = \sum_i m_i \left| U_{ei} \right|^2 \eta_i$$

$$= \frac{\sum_i \left| U_{ei} \right|^2 \eta_i}{m_\Sigma \left| U_{ei} \right|^2 \eta_i}$$

(14)
where $\eta_i = \pm 1$ are the CP eigenvalues for $\nu_i$ and $m \sim 3$eV. For example if $\eta_1 = \eta_2 = \eta_3 = +1$ then $< m_{\nu}^{\text{eff}} > \sim 0.2m \sim 0.7$eV for the matrix above. In any case, in general $< m_{\nu}^{\text{eff}} >$ may be no smaller than a fraction of 1eV and within reach in next generation of double beta decay experiments\[53]. Search for interesting models which yield such interesting near degeneracy, mixing patterns and mass ranges is under way. It has been long known that exact degeneracy is easy to obtain\[54] by imposing symmetries; the trick is to find the correct breaking pattern.

(iv) Another possibility which has been discussed in the literature is the mixing of one sterile with the three flavor neutrinos\[55]. The viable scenarios are: (a) $\nu_e - \nu_s$, MSW with small mixing for solar; and $\nu_\mu - \nu_\tau (\delta m^2 \sim 10^{-2} eV^2)$ for atmospheric with $m_{\nu_\mu} \sim m_{\nu_\tau} \sim 5eV$ to give effective HDM mass of 10eV; (b) $\nu_e - \nu_\mu$, MSW for solar; $\nu_\mu - \nu_\tau$ for atmospheric, all with masses less than 0.1 eV as discussed earlier; supplemented by $\nu_s$ of mass of $0(10 eV)$ for HDM.

(v) It should be mentioned for completeness that the flavor violation by gravity has the amusing feature that the same parameter range can account for solar as well as atmospheric anomalies at the same time.

Conclusion:
We obviously need more data! Future solar neutrino detectors will measure the $^8B$ spectrum and the $^7B/CC$ which will be acid tests of neutrino oscillation hypothesis; measurement of $^7Be$ will also be crucial and the large rates will reduce statistical errors. For the atmospheric neutrinos; the most important milestones are: the KEK beam test and further results from Soudan II. Long Baseline and reactor experiments can also test neutrino oscillation hypothesis\[56]. The BNL Proposal 889 is very elegant and impressive and can test $\nu_\mu - \nu_\tau$ as well as $\nu_\mu - \nu_e$ hypothesis. Experiments such as CHORUS, NOMAD and P803 will determine whether $\delta m_{\mu\tau}^2$ is in the range 100 to 1000 $eV^2$ with even very small mixing. The next five years should bring many new exciting results.

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