Neutrino induced meson productions in forward limit

S X Nakamura	extsuperscript{1}, H Kamano	extsuperscript{2}, T S H Lee	extsuperscript{3}, T Sato	extsuperscript{4}

\textsuperscript{1} Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8542, Japan
\textsuperscript{2} Research Center for Nuclear Physics, Osaka University, Ibaraki, Osaka 567-0047, Japan
\textsuperscript{3} Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA
\textsuperscript{4} Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

E-mail: nakamura@yukawa.kyoto-u.ac.jp

Abstract. We study neutrino-induced meson productions off the nucleon in the forward limit by applying the PCAC hypothesis to our dynamical coupled-channels (DCC) model. The DCC model reasonably describes $\pi N, \gamma N \rightarrow \pi N, \eta N, K \Lambda, K \Sigma$ data in the resonance region. We give a prediction for $\nu N \rightarrow \pi N, \pi \pi N, \eta N, K \Lambda, K \Sigma$ reactions cross sections. We compare our results with those from the Rein-Sehgal model, and find a significant difference.

1. Introduction

The last year has seen the discovery of non-zero $\theta_{13}$, and neutrino physics research entered a next stage. Next generation experiments will be targeting the leptonic CP violation and the mass hierarchy of the neutrino. To achieve this goal, it is essential to understand the neutrino-nucleus interaction more precisely, 10% or even better, over a rather wide kinematical region that covers quasi-elastic, resonance, and deep inelastic scattering (DIS) regions.

In this contribution, we are concerned with the resonance region, from the $\Delta(1232)$ through second and third resonance regions, up to $W \lesssim 2$ GeV. Several models have been developed for the neutrino-induced single pion production off the nucleon in the resonance region, and have been used as a basic ingredient to construct neutrino-nucleus interaction models \cite{1,2}. So far, most models deal with only the single pion production. However, the neutrino-nucleon interaction in the resonance region is a multi-channel reaction. Two-pion production has a contribution comparable to the single pion production. $\eta$ and kaon productions can also happen. In order to deal with this kind of multi-channel reaction, an ideal approach is to develop a unitary coupled-channels model; this is what we will pursue.

Recently we developed a unitary dynamical coupled-channels (DCC) model that can be extended to the neutrino reactions \cite{3}. Our DCC model is based on a comprehensive analysis of $\pi N, \gamma N \rightarrow \pi N, \eta N, K \Lambda, K \Sigma$ reactions in the resonance region, taking account of the coupled-channels unitarity including the $\pi \pi N$ channel. In this contribution, we report our first step of extending the DCC model to the neutrino reactions \cite{4}. For that, we invoke the Partially Conserved Axial Current (PCAC) hypothesis that allows us to relate cross sections of the pion-induced meson productions to those of the corresponding neutrino-induced meson productions in the forward limit.
Figure 1. Total cross sections (σ), unpolarized differential cross sections (dσ/dΩ) and photon asymmetry (Σ) for γp → π⁰p from the DCC model \[3\] are compared with data. The total energy is denoted by W, and the scattering angle of the pion by θ.

2. Dynamical coupled-channels model

In our DCC model \[3\], we consider 8 channels: γN, πN, ηN, π∆, ρN, σN, KΛ, KΣ. The ππN channel is included in the π∆, ρN, σN channels using Feshbach’s projection method, thus maintaining the three-body unitarity. Meson-exchange driving terms are derived from meson-baryon Lagrangian. The driving terms as well as bare N* excitation mechanisms are implemented in a coupled-channels Lippmann-Schwinger equation from which we obtain unitary reaction amplitudes. We analyzed πN, γN → πN, ηN, KΛ, KΣ reaction data simultaneously up to W = 2.1 GeV (W : total energy). The analysis includes fitting about 20,000 data points. To see the quality of the fit, we show in Fig. 1 the single pion photoproduction observables from the DCC model compared with data. As seen in the figure, our DCC model gives a reasonable description of meson production data in the resonance region. As a consequence, the DCC model contains all four-star resonances (and more) listed by the Particle Data Group \[5\]. Thus the DCC model provides a good basis with which we proceed to the neutrino reactions.

3. PCAC and neutrino-induced forward meson productions

Kinematic variables used in the following discussion are as follows. We consider the inclusive l(k) + N(p) → l′(k′) + X(p′) reactions (X = πN, ππN, ηN, ... etc.), where (l, l′) = (νₑ, e⁻), (¯νₑ, e⁺) for the charged-current (CC) reactions. Although we do not show a result, the neutral-current reactions can be studied in a similar manner. We assume that leptons are massless. In the laboratory frame, the four-momentum are defined to be k = (E, ⃗k), p = (m_N, 0, 0, 0), k′ = (E′, ⃗k′) and p′ = k + p − k′. With the momentum transfer between l and l′, q = k′ − k = (ω, ⃗q), we define the positive quantity Q² by Q² = −q² = 4EE′sin²θ/2, where θ is the scattering angle of l′ with respect to l in the laboratory frame.

For later use, we also define another frame where X is at rest. In this frame, q and p are denoted as q = (ω, ¯qₑ) and p = (E_N, − ¯qₑ), respectively, where E_N = √m_N² + | ¯qₑ|² and m_N is the nucleon mass. Also, we set ¯qₑ = (0, 0, | ¯qₑ|) so that ¯qₑ defines the z-direction of this frame.

The cross sections for the inclusive neutrino and anti-neutrino reactions are expressed as

\[
\frac{dσ_{\alpha}}{dE'd\Omega'} = \frac{G_F^2 |V_{ud}|^2}{2\pi^2 E'^2} \left[ 2W_{1,\alpha} \sin^2 \frac{\theta}{2} + W_{2,\alpha} \cos^2 \frac{\theta}{2} \pm W_{3,\alpha} \frac{E + E'}{m_N} \sin^2 \frac{\theta}{2} \right],
\]

with the label \(\alpha = \text{CCν} \text{ or CC} \bar{\nu}\); \(\Omega'\) is the solid angle of l′ in the laboratory frame; \(V_{ud}\) is the CKM matrix element; the sign in front of \(W_{3,\alpha}\) is taken to be + (−) for ν (¯ν) induced reactions.
The structure functions, $W_{i,\alpha}$ ($i = 1, 2, 3$), are Lorentz-invariant functions of two independent variables, $(Q^2, W)$, where $W$ is the total energy of $X$ at its rest frame. In the forward limit, $\theta \to 0$, only the $W_2$ term survives. The structure function $W_{2,\alpha}$ is expressed in terms of matrix elements of the weak current between the initial nucleon $N$ and the final state $X$, $\langle X|J_{\alpha}^I|N\rangle$, as

$$W_{2,\alpha} = \frac{Q^2}{d^2} \sum \frac{1}{2} \left[ (\langle X|J_{\alpha}^I|N\rangle)^2 + (\langle X|J_{\alpha}^I|N\rangle)^2 \right] + \frac{Q^2}{d^2} \left| \langle X|\left( J_{\alpha}^0 + \frac{\omega_c}{Q^2} \vec{q} \cdot J_{\alpha} \right)|N\rangle \right|^2,$$

where the summation symbol indicates all possible final states $X$, integration over momentum states of $X$, the average of initial nucleon spin state, and some kinematical factors including the phase-space factor. In the forward limit where $Q^2 = 0$, what survives in Eq. (2) is only the last term that contains the divergence of the current. The weak current consists of the vector $(V^\mu)$ and axial $(A^\mu)$ currents. Because of the vector current conservation $\langle X|q \cdot V|N\rangle = 0$ in the isospin limit, the divergence of the axial current remains. According to Refs. [6, 7, 8], we can define the pion field with the divergence of the axial currents as

$$\langle X(p')|q \cdot A^\mu|N(p)\rangle = f_\pi m_\pi^2 \langle X(p')|\hat{\pi}^\alpha|N(p)\rangle,$$

where $f_\pi$ is the pion decay constant (pion mass), and $\hat{\pi}^\alpha$ is the normalized interpolating pion field with the isospin state $\alpha$. Furthermore, the matrix element $\langle X(p')|\hat{\pi}^\alpha|N(p)\rangle$ at $Q^2 = 0$ can be expressed as

$$\langle X(p')|\hat{\pi}^\alpha|N(p)\rangle = \frac{\sqrt{2}\omega_c}{m_\pi^2} T_{\pi^\alpha N \to X}(0).$$

Here, $T_{\pi^\alpha N \to X}(q^2)$ is the T-matrix element of the $\pi^\alpha(q) + N(p) \to X(p')$ reaction in the $\pi N$ center-of-mass frame, and the incoming pion is off-mass-shell $q^2 = 0 \neq m_\pi^2$. Using Eqs. (3), (4) and $T_{\pi^\alpha N \to X}(q^2 = 0) \sim T_{\pi^\alpha N \to X}(q^2 = m_\pi^2)$, the structure function $W_{2,\alpha}$ is related to the total cross section for $\pi N \to X$. Now we can evaluate neutrino-induced forward meson production cross sections at $\theta = 0$ using the $\pi N \to X$ total cross sections of the DCC model. In the next section, we show the dimensionless structure function $F_2$ defined by $F_2 = \omega W_2$.

4. Result

We show the structure functions $F_2$ for the neutrino-induced meson productions off the nucleon in Fig. 2 (left). The figure shows $F_2$ for the CC neutrino-neutron or antineutrino-proton scattering where both $I = 1/2$ and $3/2$ states give contributions. While the $\pi N$ production is the dominant process up to $W = 1.5$ GeV, above that energy, the $\pi \pi N$ production becomes comparable to $\pi N$, showing the importance of the $\pi \pi N$ channel in the resonance region above $\Delta(1232)$. Also, we observe that the $\pi N$ and $\pi \pi N$ spectra above the $\Delta$ have rather bumpy structure, reflecting contributions from many nucleon resonances. This structure cannot be simulated by a naive extrapolation of the DIS model to the resonance region, as has been often done in previous analyses of neutrino oscillation experiments. Other meson productions, $\eta N$, $K \Lambda$, and $K \Sigma$ reactions have much smaller contribution, about $[O(10^{-1})-O(10^{-2})]$ of $\pi N$ and $\pi \pi N$ contributions. We remark that this is the first prediction of the neutrino-induced $\pi \pi N$, $\eta N$, $K \Upsilon$ production rates based on a model that has been extensively tested by data.

It is also interesting to compare our result with $F_2$ from the Rein-Sehgal (RS) model [9, 10] that has been extensively used in many Monte Carlo simulators for analyzing neutrino experiments. Such a comparison is shown in Fig. 2 (right). We can see that the RS model underestimates the $\Delta(1232)$ peak by $\sim 20\%$. On the other hand, in higher energies, the RS model significantly overestimates $F_2$. Our result is based on the DCC model tested by lots of data in the resonance region while the RS model has not but based on a quark model. Considering that, the current Monte Carlo simulators using the RS model should be improved.
Figure 2. (Left) The structure function $F_2(Q^2 = 0)$ for the neutrino-induced meson productions from the DCC model [4]. The solid (red), dashed (purple), dash-dotted (green), two-dotted dash (blue), and two-dash dotted (orange) curves are for the $\pi N$, $\pi \pi N$, $\eta N$, $K \Lambda$ and $K \Sigma$ reactions, respectively. The sum of them is given by the thick solid (black) curve. The SL model [1] is shown by the dotted (blue) curve. (Right) Comparison of $F_2(Q^2 = 0)$ between the DCC model and Rein-Sehgal (RS) model.

In this work, the comparison with the RS model is done only in the forward limit. More comparison for non-forward kinematics, as well as full description of neutrino reactions needs development of a dynamical axial current model. Such a development is currently underway.

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