Frank copula on value at risk (VaR) of the construction of bivariate portfolio (Case Study: stocks of companies awarded with the IDX top ten blue with stock period of 20 October 2014 to 28 February 2018)

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Abstract. Value at Risk (VaR) is a method to estimate the worst risk of an investment. The stock data is one of the financial time series data which often has high volatility which causes inconstant residual variance. The combination between several stocks in the portfolio makes the assumption of residual normality of the joint distribution model difficult to fulfill. The previous research on VaR by Sofiana in 2011 [3] and Hermansyah in 2017 [4] found that VaR value was reliable only for the data fulfilling normality assumption. Therefore, it is necessary to estimate VaR without ignoring the presence of heteroscedasticity and unfulfilled residual normality of the joint distribution model. This research aims to measure the VaR using Frank Copula-GARCH method with stock return data of BBRI, TLKM and UNVR for the period of 20 October 2014 to 28 February 2018. The research found that a pair of bivariate portfolio was TLKM and UNVR because they had the highest residual correlation value of Rho Spearman of $\rho = 0.3204$. Based on the data generation obtained using Monte Carlo simulation, the results of the VaR were -0.027883; -0.01886425; -0.01403 with confidence level at 99%, 95%, and 90% respectively.

1. Introduction
Stock is a sign of the participation or ownership of a person or entity in a company or limited company [1]. A stock portfolio is a combination of two or more stocks selected as investments from investors in a certain period of time with certain conditions.

Investors can estimate the benefits that will be gained and the number of costs that will be incurred in the future to control the risk. The better the estimate, the smaller the variance will occur, so that the risk level will be smaller. The risk analysis method that can be used is Value at Risk. Value at Risk (VaR) is a statistical risk measurement method that estimates the maximum loss that may occur on a portfolio at a certain level of confidence [2].

Some researchers who have conducted research on VaR were measured VaR with Monte Carlo simulation of the stock of PT Telekomunikasi Indonesia Tbk and PT Unilever Indonesia Tbk [3] and estimated VaR with a normal distribution to predict investment returns [4]. The results of both studies are that VaR is good enough to be used for normally distributed data return. However, if the stock return data do not meet the assumption of normality, this can result in VaR estimates which are no longer valid. Therefore, a tool called Copula was introduced to overcome the problem [5].
The concept of copula was first introduced by Sklar in 1959. Copula theory is a powerful tool for modeling joint distribution because it does not require joint-normality assumption and it allows solutions on each n-dimensional joint distribution into an n-marginal distribution and a copula function. Copula produces multivariate joint distributions that combine marginal distribution and dependence among variables [6].

Archimedean Copula has several family members, namely Clayton Copula, Gumbel Copula, and Frank Copula. Copula Frank is well-known for some reasons such as that Frank Copula can express the relationship of positive or negative dependencies, Frank Copula has symmetrical dependency structures, and the allowed dependency distance range is very wide [7]. In addition, Frank Copula is a part of the Archimedean Copula which is most commonly used and is the most popular Archimedean Copula in solving empirical cases [8]. One of the copula methods that researchers often use is the Copula-GARCH (Generalized Autoregressive Conditional Heteroscedasticity) method. The GARCH method is used to model the data that has high volatility which then will be continued by an analysis using copula.

Therefore, in this research, the analysis will be conducted using Frank-GARCH Copula with the purpose of forming a portfolio of two between three stocks that have the highest GARCH residual correlation in order that the investments made will provide optimal return value with minimal risk.

2. Literature review

2.1. Stock return

The return of an asset is the level of return or the results obtained after conducting investment [9]. The return value can be calculated with Continuously Compounded Return (Log Return) formula as follows [10]:

$$ R_t = \ln \frac{P_t}{P_{t-1}} \tag{1} $$

with $P_t$ as the stock price in current period and $P_{t-1}$ as stock price in the previous period.

Mathematically, portfolio return from $n$ asset at $t$-time which can be written as follows:

$$ R_{pt} = \sum_{i=1}^{n} w_i . R_{lt} \tag{2} $$

$R_{pt}$ is portfolio return at $t$-time, $w_i$ is the quality of each fund allocation for $i$ single asset and $R_{lt}$ is $l$ asset return at $t$-time.

2.2. Time series analysis

2.2.1 Stationarity Test

Stationary in variance, it can be conducted through Box-Cox transformation, namely data transformation and a certain value of $\lambda$ will be generated according to the form of the transformation. The general form of Box-Cox transformation is:

$$ y = \begin{cases} \frac{x^{\lambda} - 1}{\lambda}, & \text{for } \lambda \neq 0 \\ \ln(x), & \text{for } \lambda = 0 \end{cases} \tag{3} $$

In the formal test, the stationarity test in the mean can be checked by searching whether the time series data contain the unit root using the Augmented Dickey-Fuller test.

2.2.2 Stationarity test

Box-Jenkins Model is a time series data model which has been introduced by Box and Jenkins in 1970. Below is the time series model:

- Autoregressive (AR) Model
  $$ Z_t = \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + \cdots + \phi_p Z_{t-p} + \alpha_t \tag{4} $$

- Moving Average (MA) Model
  $$ Z_t = \alpha_t + \theta_1 \alpha_{t-1} + \cdots + \theta_q \alpha_{t-q} \tag{5} $$

- Autoregressive Integrated Moving Average (ARIMA) Model
ARIMA modeling stage is identifying the model, estimating parameters, and then verifying the model by testing residual independence and residual normality.

2.3. Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model

This model is used to overcome the order which is too large in the ARCH model. The general form of the GARCH model \((p, q)\) is [10]:

\[
\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i \sigma_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2
\]

The coefficients of GARCH \((p, q)\) model are:

1. \(\alpha_0 > 0\)
2. \(\alpha_i \geq 0\) for \(i = 1, 2, \ldots, p\)
3. \(\beta_j \geq 0\) for \(j = 1, 2, \ldots, q\)
4. \(\sum_{i=1}^{p} \sum_{j=1}^{q} (\alpha_i + \beta_j) < 1\)

Condition 1, 2, and 3 are needed in order that \(\sigma_t^2 > 0\). Condition 4 is needed in order that the model is stationary [11].

2.3. ARCH-LM test

Lagrange Multiplier (LM) Test is a test conducted towards the presence of heteroscedasticity elements. This test is one way to find out the ARCH/GARCH effects introduced by Engle [10]. The following is a hypothesis testing step to find out whether there are ARCH / GARCH effects or not:

Hypothesis:

\(H_0\): \(\tau_1 = \tau_2 = \ldots = \tau_m = 0\)

(there is no ARCH/GARCH effect in the residual until \(m\) lag)

\(H_1\): there is at least one value of \(\tau_i \neq 0\), \(i = 1, 2, \ldots, m\)

(there is ARCH/GARCH effect in the residual until \(m\) lag)

Test statistics:

\[
LM = NR^2 \sim \chi_m^2
\]

\[
R^2 = \frac{\sum_{i=1}^{m}(a_i^2 - \bar{a}_i^2)^2}{\sum_{i=1}^{m}(a_i^2 - \bar{a}_i^2)^2}
\]

Test Criteria:

\(H_0\) is rejected if the value of LM > \(\chi_{(a,m)}^2\) or \(p\)-value < \(\alpha\)

2.4. ARCH-LM test

Best model selection is conducted by estimating the quality of the presumptive model, one of several ways which are conducted is using the value of Akaike’s Information Criterion (AIC). The best model is model which has the minimal value of AIC. The formula used to obtain the value of AIC is written as follows [12]:

\[
AIC(k) = n \ln \hat{\sigma}_n^2 + 2k
\]

2.5. Copula

Archimedean Copula has three important groups called Gumbel, Frank and Clayton Families. Archimedean Copula family is the most widely used in bivariate cases:

Archimedean Copula which has \(d\) dimension, can be defined [13] Nelsen (2006),

\[
C(u_1, \ldots, u_d) = \varphi^{-1}(\varphi(u_1) + \cdots + \varphi(u_d))
\]

The function of \(\varphi\) is called as copula generator, with an assumption that the generator has one parameter, namely \(\theta\).
Archimedean copula flexibility is given by the function of \( \phi \), for example from Clayton, Frank and Gumbel Copula [8] as follows:

\[
\phi(u) = u^{-\theta} - 1, \theta > 0 \quad \text{(Clayton)}
\]

\[
\phi(u) = \ln\left(\frac{e^{(-\theta u) - 1}}{e^{-\theta} - 1}\right), \theta > 0 \quad \text{(Frank)}
\]

\[
\phi(u) = (-\ln(u))^\theta, \theta > 1 \quad \text{(Gumbel)}
\]

For Archimedean copula function in bivariate cases, it can be written as follows:

\[
C(u_1, u_2) = \phi^{-1}(\phi(u_1) + \phi(u_2))
\]

2.5.1. Dependency structure test

Dependency Structure Test is conducted to find out the existence of dependence on each variable in a joint distribution modeling. If there is a variable \( X_1, ..., X_d \) and the data consist of \( x_i = (x_{i1}, ..., x_{id}) \) for \( i = 1, ..., n \) where the total sample is \( n \) and \( x_i \) is i.i.d distributed, then the dependence modeling step is constructing univariate model for each variable \( x_1, ..., x_d \), which then constructing copula model for dependence from \( d \) variable.

**Rho Spearman** correlation test is performed with hypothesis used as follows:

Hypothesis:

\[ H_0 : \rho = 0 \quad \text{(there is no correlation between \( X_1 \) and \( X_2 \))} \]

\[ H_1 : \rho \neq 0 \quad \text{(there is correlation between \( X_1 \) and \( X_2 \))} \]

Test Statistics:

The coefficient of *Rho Spearman* correlation is:

\[
\rho = 1 - \frac{6\sum d^2}{n(n^2-1)}
\]

Rejection Criteria:

Reject \( H_0 \) if \( p\)-value < \( \alpha \)

2.5.2. Uniform transformation \([0,1]\)

Transformation is the first step taken to analyze copula. The random variable is first transformed into the domain \([0,1]\). The transformation of the original data to the domain \([0,1]\) is done by creating transformation scatterplot \([0,1]\) by making the rank plot for each random variable as follows [8]:

\[
\left(\frac{\text{rank}_{i}^{(1)}}{n+1}, \frac{\text{rank}_{i}^{(2)}}{n+1}\right), 1 \leq i \leq n
\]

with

\[
\text{rank}_{1}^{(i)} = \text{rank} \ X_{1 \ ke-i}, i = 1,2, ..., n
\]

\[
\text{rank}_{2}^{(i)} = \text{rank} \ X_{2 \ ke-i}, i = 1,2, ..., n
\]

2.5.2. Frank Copula

The function of Frank Copula generator and the function inverse of the Frank Copula generator are known:

\[
\phi(u) = -\ln\frac{e^{\theta u_1 - 1}}{e^{-\theta} - 1}, \theta \in \mathbb{R} \setminus \{0\}
\]

with the function of generator above, it will generate each cumulative distribution function of Frank Copula as follows:

\[
C(u_1, u_2) = -\frac{1}{\theta} \ln\left(1 + \frac{(e^{-\theta u_1 - 1})(e^{-\theta u_2 - 1})}{e^{-\theta} - 1}\right)
\]

2.5.3. Frank copula estimation

The only one theory that can be used to gain the estimation of copula parameter namely by using Maximum Likelihood Estimation (MLE) method. The form of likelihood L function in bivariate cases can be written as follows [13]:

...
\[ L = \prod_{i=1}^{2} e_{u_1,u_2} \{ F_1(x_1), F_2(x_2) \} f_1(x_1)f_2(x_2) \]  

(19)

can be described into

\[
\ln f(x_1,x_2 ; \theta, \rho) = \ln c(F_1(x_1,\theta), F_2(x_2,\theta), \rho) + \ln f_1(x_1,\theta) + f_2(x_2,\theta)
\]

(20)

In solving equations (19) and (20), the closed form cannot be obtained, but it can be solved by numerical calculation with Rho Spearman approach.

2.5.4. Value at Risk (VaR)

Value at Risk (VaR) is a measuring tool which can calculate the worst losses that can occur by knowing the position of assets, the level of confidence in the occurrence of risk, and the time period of asset placement (time horizon) [14].

Calculating the VaR value at the level of confidence of \((1 - \alpha)\) in the time period of \(t\) day, is [9]:

\[ VaR_{(1-\alpha)}(t) = W_0R^* \sqrt{t} \]

(21)

where:

- \(W_0\) = initial investment fund of asset or portfolio
- \(R^*\) = \(\alpha\) quantile value of return distribution
- \(t\) = time period

3. Research methodology

3.1 Research variables

The data used in this study were 825 secondary data of daily stock closing price. The stock data used included 3 companies namely PT. Bank Rakyat Indonesia, Tbk (BBRI), PT. Telekomunikasi Indonesia, Tbk (TLKM), and PT. Unilever Indonesia, Tbk (UNVR) for the period of 20 October 2014 to 28 February 2018. Each data on the stock closing price can be accessed on the site of www.finance.yahoo.com.

3.2. Data analysis methods

1. Calculated the value of stock return from those three stock prices.
2. Analyzed those three stock return descriptively.
3. Conducted stationary data test of the variance by using Box-Cox test as well as stationary test of the mean by plot time series and Augmented Dickey-Fuller test.
4. After it was stated stationary, then it was continued by making ACF and PACF plot to determine the order to obtain temporary model.
5. Conducted estimation and testing of parameter significance.
6. Conducted assumption test which covers residual independence test, model normality test, and homoscedasticity test.
7. Chose ARIMA as the best model based on AIC criteria.
8. Conducted identification of ARCH/GARCH model, then normal distribution test on GARCH residual (1,1) was conducted.
9. Calculated all correlation between variables then the highest correlation was selected
10. Conducted and combined GARCH residual (1,1) to Frank Copula.
11. Conducted VaR estimation using data generation of Frank Copula and Monte Carlo simulation.
12. Drew conclusion from the results of analysis based on VaR, and from stock combination, VaR value with a confidence interval of \((1-\alpha)\) with \(\alpha\) level of significance.

4. Result and Discussion

4.1. Data Description

Descriptive analysis was carried out to find out the characteristics of stock assets return which includes PT. Bank Rakyat Indonesia, Tbk (BBRI), PT. Telekomunikasi Indonesia, Tbk (TLKM), and PT. Unilever Indonesia, Tbk (UNVR) for the period of 20 October 2014 to 28 February 2018. Figure 1 is a plot of the time series of those three stock returns.
Based on the graphic in figure 1 it shows that the return of the stocks of BBRI, UNVR, and TLKM have the constant fluctuations it indicates the stationary assumption in visual is fulfilled. Descriptive analysis for those three stock returns is presented in table 1.

| Variable     | BBRI      | UNVR       | TLKM       |
|--------------|-----------|------------|------------|
| Mean         | 0.000690  | 0.000664   | 0.000414   |
| Deviation Standard | 0.018118  | 0.015893   | 0.014925   |
| Minimum      | -0.074464 | -0.063776  | -0.056457  |
| Maximum      | 0.075986  | 0.087256   | 0.072925   |
| Skewness     | -0.048544 | 0.274754   | 0.173614   |
| Kurtosis     | 5.466944  | 5.483289   | 5.676425   |

Based on table 1, the return of BBRI, UNVR, and TLKM have positive mean that shows the third return will provide benefits to the investor. The highest standard deviation value is on the return stock of BBRI that show BBRI have the highest risk potential. The highest stock return is owned by UNVR with a rate of return 0.087256 and the lowest stock return is owned by TLKM with rate of return 0.074464.

4.2. Data stationary test
There are two steps of a stationary testing. Stationary tests in variance were carried out using Box-Cox transformation. The value of rounded value on stock returns of BBRI, UNVR, and TLKM was 1. Thus, it can be concluded that the three stock returns were stationary in variance. Stationary test in mean was performed using the Augmented Dickey-Fuller test. The t statistics value that was generated from BBRI, UNVR, and TLKM were -14.62457; -18.97874; and -17.90489 with a p-value of 0.0000. Therefore, it can be concluded that the three stock return data fulfilled the stationary assumptions both in variance and in the mean.

4.3. Model identification
The determination of AR and MA orders used a plot of autoregressive function (ACF) and partial autoregressive function (PACF). The ACF plot was disconnected at k lag if the value of the autocorrelation function (\( \hat{r}_k \)) exceeded the significance line, while the PACF plot was disconnected at the k lag if the value of the partial autocorrelation function was (\( \hat{r}_{kk} \)) The best ARIMA model can be determined by looking at the smallest AIC value of the ARIMA model in BBRI, UNVR, and TLKM were -5.190578; -5.477267; and -5.596269, the following is the best ARIMA model produced:

- **BBRI**: ARIMA ([1], 0, [2])
- **UNVR**: ARIMA (0,0, [1,3])
- **TLKM**: ARIMA (0,0, [1,2,4])

4.4. ARCH-LM test
Lagrange Multiplier (LM) test was used to find out whether there is ARCH/GARCH effect or not on the residual of ARIMA model. The results of the test are presented in table 2.

| Stock | Model            | P-value |
|-------|------------------|---------|
| BBRI  | ARIMA ([1], 0, [2]) | 0.000   |
| UNVR  | ARIMA (0, 0, [1,3]) | 0.000   |
| TLKM  | ARIMA (0, 0, [1,2,4]) | 0.000   |

With level of significance of $\alpha=5\%$, it rejects $H_0$ that it can be concluded that in the residual of ARIMA model on stock returns of BBRI, UNVR, and TLKM, there is ARCH/GARCH effect and GARCH model can be constructed.

4.5. GARCH (1,1) Modeling

The initial GARCH model formed on BBRI stock was ARIMA ([1], 0, [2]) GARCH (1,1), on UNVR stock was ARIMA (0,0, [1,3]) GARCH (1,1) and on TLKM stock was ARIMA (0,0, [1,2,4]) GARCH (1,1). After improving the model because there was no significant parameter, the optimal GARCH model for BBRI stock was ARIMA (0,0, [2]) GARCH (1,1), for UNVR stock was ARIMA (0,0, [1,3]) GARCH (1,1) and for TLKM stock was ARIMA (0,0, [1,2,4]) GARCH (1,1) with the mean and variance model equations as follows:

a. BBRI
   \[ Z_t = -0.08378 a_{t-2} + a_t \]
   \[ \sigma_t^2 = 0.00001 + 0.097774 a_{t-1}^2 + 0.874638 \sigma_{t-1}^2 \]

b. UNVR
   \[ Z_t = -0.144974 a_{t-1} - 0.101632 a_{t-3} + a_t \]
   \[ \sigma_t^2 = 0.000004 + 0.047253 a_{t-1}^2 + 0.936737 \sigma_{t-1}^2 \]

c. TLKM
   \[ Z_t = -0.085579 a_{t-1} - 0.156018 a_{t-2} - 0.075063 a_{t-4} + a_t \]
   \[ \sigma_t^2 = 0.000057 + 0.186539 a_{t-1}^2 + 0.553380 \sigma_{t-1}^2 \]

After obtaining the model from each stock, the next step was testing the normality of the residual generated from each model using the Jarque Bera test and it can be concluded that the residual of ARIMA GARCH (1,1) model on all stocks was not normally distributed.

4.6. Copula

The residual of ARIMA GARCH (1,1) model which was not normally distributed was then resolved by using copula before estimating the Value at Risk (VaR) value.

a. Determining Dependency Structure
   Dependency Structure test was conducted using Rho Spearman test. The pair of stock that will be used to model the copula was the residual of stock pair which has the highest correlation coefficient. TLKM and UNVR stocks had residual with the highest correlation coefficient of 0.3203765.

b. Uniform Transformation [0,1]
   Transformation was the first step that will be performed to conduct analysis towards copula. Firstly, plot of residual data that have been obtained was made to see the data distribution as it is presented in figure 2.
Figure 2. Scatter plot of GARCH (1,1) UNVR and TLKM model residual

In figure 2, it is shown that the plot does not show Frank copula due to the distribution of points concentrated at certain intervals. Then after transformation uniform [0,1] was conducted, the distribution of data points can be seen in figure 3.

Figure 3. Scatter Plot of GARCH (1,1) UNVR and TLKM Model Residual on Uniform Transformation [0,1]

In figure 3, the form of scatter plot after the transformation is shown and it demonstrates that there is distribution of points which are not only concentrated in the lower end but also concentrated in the upper end. It indicates the existence of tail dependence at the top and bottom of Frank copula.

c. Copula parameter estimation

Copula estimation was calculated using Maximum Likelihood Estimation (MLE) by using the value of Rho Spearman correlation of ARIMA GARCH (1,1) model residual of UNVR and TLKM stocks which is $\hat{\rho} = 0.3203765$. It was obtained that the Frank Copula parameter estimation was $\hat{\theta} = 2.025$. Therefore, Frank Copula model for ARIMA GARCH (1,1) residual of UNVR and TLKM stocks is

$$c_{1,183}^{Fr}(u_1u_2) = -\frac{1}{2.025}\ln\left(1 + \frac{(e^{-2.025u_1} - 1)(e^{-2.025u_2} - 1)}{e^{-2.025} - 1}\right)$$

The value of $\hat{\theta} > 0$ shows that copula has positive dependency which means that if the residual of GARCH (1,1) model of UNVR stock increased, then the residual of GARCH (1,1) model of TLKM stock will increase.

4.7. Value at Risk (VaR) estimation

Value at Risk (VaR) was calculated using Monte Carlo Simulation by generating 824 generation data which came from Frank Copula model which had been generated with 1000 times, 2000 times
and 3000 times repetitions and level of confidence of 90%, 95% and 99%. The results of VaR value estimation are presented in table 3.

Table 3. The value of VaR prediction with t = 1 day and $W_0 = 0.5$

| Number of Repetition | 99%            | 95%            | 90%            |
|----------------------|----------------|----------------|----------------|
| 1000                 | -0.02793519    | -0.01886483    | -0.01400994    |
| 2000                 | -0.02780772    | -0.01885294    | -0.01403064    |
| 3000                 | -0.02790626    | -0.018875      | -0.01405387    |
| Mean                 | -0.027883      | -0.01886425    | -0.01403       |

Based on mean of each level confidence, VaR value is -0.027883; -0.01886425; -0.01403 at level of confidence 99%, 95%, and 90% respectively.

5. Conclusion

Based on the results and discussion presented in this research, the conclusions can be drawn that the combination of UNVR and TLKM stocks are chosen because it has the highest residual correlation coefficient based on the Rho Spearman correlation test, namely $\hat{\rho}$ = 0.3204. Frank Copula parameter estimation is obtained based on the MLE method, and parameter $\hat{\theta}$ = 2.025 is obtained. Therefore, Frank Copula is found as follows:

$$C_{\text{Frank}}^{\text{GARCH}}(u_1, u_2) = -\frac{1}{2.025} \ln \left(1 + \frac{(e^{-2.025u_1} - 1)(e^{-2.025u_2} - 1)}{e^{-2.025} - 1}\right)$$

The risk obtained from the calculation of Value at Risk (VaR) with the Frank-GARCH copula method and Monte Carlo simulation were -0.027883; -0.01886425; -0.01403 and maximum loss in one-day prediction is IDR 2,788,300.00; IDR 1,886,425.00; and IDR 1,403,000.00 at a level of confidence of 99%, 95%, and 90% respectively.

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