Supplementary Information

Mechanically diverse gels with equal solvent content

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Figure S1: $^1$H-NMR of unpurified $n$-butyl acrylate at completion of ATRP reaction. (400 MHz, CDCl$_3$): 6.3, 5.77 (CH$_2$=C(H)C=O, d, 1H), 6.06 (CH$_2$=C(H)C=O, dd, 2H), 4.1 (-O-CH$_2$-(CH$_2$)$_2$-CH$_3$, t, 2H) 4.00 (O-CH$_2$-(CH$_2$)$_2$-CH$_3$, s, 10.3H), 0.88 ((-O-CH$_2$-(CH$_2$)$_2$-CH$_3$, t, 16H). The extent of reaction was taken to be the area ratio of peak $c/(b + c) = 0.84$. 
**Figure S2:** $^1$H-NMR of functionalized poly($n$-butyl acrylate) macromonomer. (400 MHz, CDCl$_3$): 6.14, 5.60 (CH$_3$-CH=CH=C=O, s, 1H), 4.01 (-O-CH$_2$-(CH$_2$)$_2$-CH$_3$, s, 22H) 4.01 (O-CH$_2$-CH$_2$, s, 2H), 0.88 ((-O-CH$_2$-(CH$_2$)$_2$-CH$_3$, t, 33H). The $n_{sc}$ was taken as $(c + b)/2 = 11$.

**Figure S3:** Gel permeation chromatographs of the synthesized poly($n$-butyl acrylate) macromonomers. The dispersity of the corresponding macromonomers was found to be 1.12, 1.08, and 1.07 for the corresponding $n_{sc} = 11, 23, and 41.$
Figure S4: (A) Time-resolved $^1$H-NMR of copolymerization of PEG ($n_{sc}$9): PBA ($n_{sc}$10) macromonomers at their initial feed ratio of 80:20 mol.% or 60:40 wt.% ([PEG]/[PBA]= 4.45). (B) Conversion of PEG and PBA macromonomers as a function of accumulative conversion. (C) Molar fraction of PEG macromonomer in the brush copolymer chains ([PEG]/([PBA]+[PEG])) as a function of conversion.

Figure S5: Differential scanning calorimetry of PEG, PBA, and 60:40 PEG-c-PBA elastomers. The later shows a distinct glass transition at a temperature between the glass transitions of the PEG and PBA samples.
2. Mechanical properties

**Stress-strain data analysis.** The mechanical stress in polymer networks in the dry (melt) state made of entangled strands undergoing uniaxial deformation with the elongation ratio \( \lambda \).

\[
\sigma_{\text{true}}(\lambda) = (\lambda^2 - \lambda^{-1}) \left( \frac{G_e}{\lambda} + \frac{G}{3} \left( 1 + 2 \left( 1 - \frac{\beta I_1}{3} \right)^{-2} \right) \right)
\]

(S1)

where \( I_1 = \lambda^2 + 2/\lambda \) is the first deformation invariant, which describes the relative elongation of the mean-square end-to-end distance of the network strands between crosslinks \( \langle R^2 \rangle \) with respect to their dimensions in the undeformed state \( \langle R^2 \rangle_{\text{in}} \) as \( I_1 = 3 \langle R^2 \rangle / \langle R^2 \rangle_{\text{in}} \). The three parameters in eq S1 characterizing response of the network to deformation are: entanglement shear modulus, \( G_e \), associated with network elasticity provided by trapped entanglements, structural shear modulus, \( G \), describing elastic response of the stress supporting crosslinked strands, and strand elongation ratio, \( \beta \), which controls the shape of the stress-deformation curve (network firmness parameter). The strain-stiffening parameter, \( \beta \), quantifies by how much a network strand with the degree of polymerization between crosslinks \( n_x \) can be stretched from its initial (undeformed) state with the mean-square end-to-end distance \( \langle R^2 \rangle_{\text{in}} \) to its fully extended state with the size equal to \( R_{\text{max}} = n_x l \) and monomer projection length \( l \)

\[
\langle R^2 \rangle_{\text{in}} = \frac{\alpha R_{\text{max}}^2}{2} \left( 1 - \frac{\alpha}{2} \left( 1 - \exp \left( -\frac{2}{\alpha} \right) \right) \right)
\]

(S2)

where parameter \( \alpha^{-1} \equiv n_x l / b_K = R_{\text{max}} / b_K \) defines the number of Kuhn segments per network strand.

The structural shear modulus for networks with crosslink functionality \( f \) made of brushlike strands having fraction of the backbone monomers \( \varphi = (1 + n_{sc} / n_{g})^{-1} \) is

\[
G = (1 - 2/f) \frac{k_B T \rho \varphi}{n_{x,\text{app}} b_K R_{\text{max}}} \langle R^2_{\text{in}} \rangle \approx (1 - 2/f) \frac{k_B T \rho \varphi}{n_{x,\text{app}}} \beta \alpha^{-1}
\]

(S3)

where \( k_B = 1.38 \cdot 10^{-23} \text{J/K} \), \( T \) - absolute temperature, \( \rho \) - monomer number density and \( n_{x,\text{app}} \) is apparent degree of polymerization between crosslinks accounting for network defects.

The Young’s modulus, \( E_0 \), corresponds to the slope of the stress-strain curve at \( \lambda \to 1 \)

\[
E_0 = 3 G_e + G (1 + 2(1 - \beta)^{-2})
\]

(S4)

For flexible strands \( n_x \gg n_{sc} \), the backbone adopts Gaussian conformation with \( \langle R^2 \rangle_{\text{in}} \approx b_K R_{\text{max}} \), and \( \alpha^{-1} \gg 1 \), and \( \beta \alpha^{-1} \approx 1 \). In this case, parameters \( G \) and \( \beta \) reduce to their conventional forms:
\[ G \approx \frac{k_B T}{vn_{x,app}(1 + n_{sc}/n_g)} \]  
\[ (S5) \]

\[ \beta = \frac{\langle R_{in}^2 \rangle}{R_{max}^2} \approx b_K \frac{n_x}{n} \approx \alpha \]  
\[ (S6) \]

We can estimate the degree of polymerization between crosslinks \( n_{x,G} \) from the deformation response of elastomers defined by \((\alpha, \beta, G)\) defined by eq S3 for \( f = 4 \) as

\[ n_{x,G} = 0.5k_B T \rho \phi \beta \alpha^{-1}/G \]  
\[ (S7) \]

**Figure S6.** True stress vs elongation curve profiles of linear poly(n-butyl acrylate) elastomers with different degrees of polymerizations, \( n_x \), of the network strands. The overlapping red lines correspond to the best fit line with eq S1 using \( G \) and \( \beta \) as fitting parameters (Table S1).²
Figure 7. True stress vs elongation curve profiles of brush poly(butyl acrylate) elastomers with different degrees of polymerizations, $n_x$, of the network strands. The overlapping red lines correspond to the best fit line with eq S1 using $G$ and $\beta$ as fitting parameters (Table S1). The Combs, SBB, and SSC legends correspond to the brush regimes in Figure S8.

Figure S8. The diagram of states of PBA elastomers brush-like strands defined by the degree of polymerization of side chains ($n_{sc}$) and grafting density given by the brush parameter $\varphi^{-1} = 1 + n_{sc}/n_g$. The diagram shows three regimes: comb, SBB (stretched backbone), SSC (stretched side chain) regimes. Logarithmic scales. For more details see ref.2.
Table S1. Mechanical properties of PBA elastomers.

| $n_{sc}$ | $n_g$ | $\varphi^{-1}$ | $n_x$ | $n_{x,G}$ | $G$ (kPa) | $G_e$ (kPa) | $\beta$ | $E_0$ (kPa) | $\lambda_{max}$ |
|----------|------|----------------|------|----------|-----------|------------|-------|----------|-------------|
| 0        | 1    | 1              | 50   | 63       | 134       | 42         | 0.1   | 591      | 1.9         |
| 100      |      |                |      |          |           |            |       |          |             |
| 200      |      |                |      |          |           |            |       |          |             |
| 11       | 1    | 12             | 50   | NA       | 10.3      | 0          | 0.21  | 43       | 2.4         |
| 100      |      |                |      |          |           |            |       | 21       | 2.9         |
| 200      |      |                |      |          |           |            |       | 8.3      | 4.5         |
| 11       | 3    | 4.7            | 50   | 42       | 32.9      | 0          | 0.14  | 122      | 1.9         |
| 100      |      |                |      |          |           |            |       | 66       | 2.3         |
| 200      |      |                |      |          |           |            |       | 0.07     | 35          |
| 11       | 5    | 3.2            | 100  | 138      | 7.6       | 4.1        | 0.06  | 37       | 4.7         |
| 200      |      |                |      |          |           |            |       |          |             |
| 11       | 10   | 2.1            | 50   | 103      | 47        | 0          | 0.11  | 166      | 2           |
| 100      |      |                |      |          |           |            |       | 0.06     | 111         |
| 200      |      |                |      |          |           |            |       | 0.03     | 72          |
| 23       | 2    | 12.5           | 50   | 57       | 12.3      | 0          | 0.14  | 46       | 2.6         |
| 100      |      |                |      |          |           |            |       | 0.12     | 20          |
| 200      |      |                |      |          |           |            |       | 0.08     | 7.8         |
| 23       | 4    | 6.8            | 50   | 37       | 19.4      | 0          | 0.2   | 80       | 2.1         |
| 100      |      |                |      |          |           |            |       | 0.12     | 35          |
| 200      |      |                |      |          |           |            |       | 0.07     | 12.6        |
| 23       | 10   | 3.3            | 50   | 32       | 47.3      | 0          | 0.17  | 185      | 1.7         |
| 100      |      |                |      |          |           |            |       | 0.11     | 94          |
| 200      |      |                |      |          |           |            |       | 0.07     | 44          |
| 41       | 2    | 21.5           | 50   | 78       | 4.2       | 0          | 0.23  | 18.4     | 2           |
| 100      |      |                |      |          |           |            |       | 0.14     | 7           |
| 200      |      |                |      |          |           |            |       | 0.07     | 2.1         |
| 41       | 5    | 9.2            | 50   | 34       | 6.1       | 0          | 0.15  | 23       | 2.7         |
| 100      |      |                |      |          |           |            |       | 0.09     | 8.2         |
| 200      |      |                |      |          |           |            |       | 0.09     | 33          |
| 41       | 10   | 5.1            | 50   | 41       | 17.4      | 0          | 0.15  | 66       | 2.1         |
| 100      |      |                |      |          |           |            |       | 0.09     | 33          |

(1) Degree of polymerization of side chains from NMR. (2) DP of backbone spacer between neighboring side chains defined by $n$-BA molar fraction. (3) Brush parameter $\varphi^{-1} = 1 + n_{sc}/n_g$. (4) Targeted DP of brush backbone between crosslinks defined by molar fraction of crosslinker, e.g., $n_x = 200$ corresponds to 0.25 mol%. (5) DP of brush backbone between crosslinks determined from the deformation response of elastomers (eq S7). (6) Structural shear modulus, (7) entanglement plateau modulus, and (8) strain-stiffening parameter from fitting stress-elongation curves with eq S1. (9) Young’s modulus as a stress-elongation slope at $\lambda \to 1$ (eq S4).
Figure S9. Stress-strain curves from tensile testing ($\dot{\varepsilon} = 0.005\, s^{-1}$, 22$^\circ$C) of a) PIB 2$^2$ and b) PDMS bottlebrush elastomers. As $n_x$ increased, the Young’s modulus (slope at $\lambda \to 1$), $E_0$, and $\beta$ (slope increase with $\lambda$) decrease. The dashed lines in a) correspond to eq S1.

Table S2: Structural parameters, mechanical and swelling properties of PIB bottlebrush elastomers.

| $n_{sc}$ | $n_g$ | $n_x$ | $\varphi^{-1.4}$ | $G$ (kPa) | $\beta$ | $E_0$ (kPa) | $Q_{eq}$ |
|---------|-------|-------|-------------------|-----------|---------|-------------|---------|
| 18      | 1     | 100   | 3.0               | 0.27      | 14.3    | 13          |
|         |       | 150   | 1.4               | 0.19      | 5.7     | 20          |
|         |       | 200   | 1.2               | 0.14      | 4.4     | 25          |
|         |       | 300   | 0.64              | 0.1       | 2.2     | 34          |

(1) Degree of polymerization (DP) of side chains. (2) DP of backbone spacer between neighboring side chains defined by $n$-BA molar fraction. (3) Targeted DP of brush backbone between neighboring crosslinks defined by molar fraction of crosslinker. (4) Brush parameter $\varphi^{-1} = 1 + n_{sc}/n_g$. (5) Structural shear modulus and (6) firmness parameter from fitting the stress-elongation curves (Fig. S9a) with eq S1. (7) Young’s modulus - stress-elongation slope at $\lambda \to 1$ (eq S4 at $G_e = 0$). (8) Equilibrium swelling ratio (toluene, 12 hrs).

Table S3: Structural parameters, mechanical and swelling properties of the PDMS bottlebrush elastomers (see the Table S2 legends).

| $n_{sc}$ | $n_g$ | $n_x$ | $G$ (kPa) | $\beta$ | $E_0$ (kPa) | $Q_{eq}$ |
|---------|-------|-------|-----------|---------|-------------|---------|
| 14      | 1     | 150   | 6.3       | 0.195   | 25.7        | 7.3     |
|         |       | 300   | 3.4       | 0.074   | 11.3        | 12.6    |
|         |       | 600   | 0.9       | 0.066   | 3.0         | 20.0    |
|         |       | 1200  | 0.5       | 0.039   | 1.6         | 24.0    |
Figure S10. Correlation between the experimentally measured and theoretically predicted elongations-at-break $\lambda_{\text{max}} = L_{\text{break}}/L_0$ for the PBA brush elastomers (Table S1). The theoretical values $\lambda_{\text{max,th}} \equiv 1/\sqrt{\beta}$ correspond to the maximum elongation of networks strands given by the strain-stiffening parameter $\beta \equiv \langle R_i^2 \rangle / R_{\text{max}}^2$.

3. Swelling properties

| $[n_{sc}, n_g, n_x]$ | $G_0, \text{gel} (kPa)$ Tensile | $G', \text{gel} \text{(kPa)}$ Shear | $Q_{eq}$ |
|-----------------------|--------------------------|------------------|--------|
| [41, 2, 100]            | 0.54±0.02                | 0.50 ± 0.04      | 18.4   |

Figure S11. Agreement between tensile and shear modulus measurements for brush elastomers. From A) tensile to B) shear stress measurement, the modulus for gel samples deviated less than 10 percent (Table). The precision of shear modulus for the gel sample was consistent within 7.3 percent error in triplicate. Shear measurements were performed at $\varepsilon = 0.05$ along the elastic plateau at $\dot{\varepsilon} = 0.1s^{-1}$ to minimize vaporization of solvent. It is important to note that in comparing modulus mechanical testing methods both experiments were performed at $\dot{\varepsilon} = 0.1s^{-1}$, however, further measurements were performed at $\dot{\varepsilon} = 0.005s^{-1}$ to ensure that the dry elastomer is measured at the elastic plateau. All measurements were performed at 22°C.
Figure S12. Effect of solvent evaporation on modulus. A single strip of elastomer, [11,10,100], was subjected to tensile testing at small strains after being swollen to $Q_{eq}$. Subsequently, the strip was re-submerged to $Q_{eq}$ and removed to evaporate solvent to variable $Q$. The process was repeated for six different $Q$. A) Tensile true stress measurements of brush gel at variable $Q$. Experiments were performed at $\dot{\varepsilon} = 0.005\, s^{-1}$. B) Gel modulus dependence on time. Modulus increased exponentially after 10 min as solvent was expelled from the gel network. Evaporation of solvent (toluene) occurred at 22°C.

Figure S13. Verification of isotropic swelling by comparing the experimentally measured areal expansion $S_{gel}/S_{dry}$ and volume expansion $Q = V_{gel}/V_{dry}$ ratios. The observed 2/3 scaling behavior is consistent with $S_{gel}/S_{dry} = Q^{2/3}$ expected for isotropic swelling.
**Figure S14.** Construction path for Figure 3a. a) True stress-elongation of representative PBA elastomers. b) Instantaneous modulus (eq 2 in the main text) as a function of the first deformation invariant of the [11,1] sample series. c) For all samples, the modulus values at networks deformation corresponding to $Q = Q_{eq}$ are determined and plotted versus experimentally measured $Q_{eq}$. The slope of modulus decay versus $Q_{eq}$ follows the theoretical prediction $Q_{eq} = KG(I_1)^{-3/8}$, where $I_1 = 3Q_{eq}^{2/3}$ and $K_{PBA} = 65.9 \pm 1.1$ kPa$^{3/8}$ is determined from the intercept 4.85.

**Figure S15.** a) Equilibrium swelling ratio, $Q_{eq}$, vs the instantaneous modulus $G(I_1)$ of PDMS elastomers (Table S3) at $I_1 = 3Q_{eq}^{2/3}$. b) Strain-stiffening parameter, $\beta$, is plotted against the structural modulus, $G$, of brush elastomers with different chemical compositions of side chains. The numbers next to the symbols indicate the corresponding $Q_{eq}$ values.
**Figure S16.** Theoretical $\beta(G)$ lines run diagonally to intersect with the iso-choric swelling lines for different swelling ratios, $Q_{eq}$, as depicted by the numbers inside the symbols. The symbol size correlates with the $Q_{eq}$ values. Each intersection point gives a set of three properties $[G, \beta, Q_{eq}]$ that belongs to a certain brush architecture characterized by $n_{sc}$ and $n_g$ used to draw the fitted line.

**S4. Nonlinear Swelling of Polymer Networks**

In order to describe the swelling of networks with brush-like strands, we modify the classical approach developed by Flory and Rehner$^3,4$ to account for nonlinear network strand deformation at equilibrium swelling$^5$, which results in the following expression for gel modulus

$$G_{gel}(Q) \equiv \frac{G}{3Q^{1/3}} \left(1 + 2\left(1 - \beta Q^{2/3}\right)^{-2}\right) = Q^{-1/3} G(Q)$$  \hspace{1cm} (S8)

To test the validity of the expression for the modulus of the swollen network given by eq S8, we performed coarse-grained molecular dynamics simulations of the deformation of swollen networks (**S5 Simulation Details**). The gel shear modulus was obtained from a uniaxial deformation of the swollen network at constant volume as shown in **Figure S17**. A network with initial dimensions $L_{0,i}$ in the dry (reference) state swells, reaching dimensions $L_{s,i}$, where $i=x, y$, and $z$. After that it is uniaxially deformed at a constant volume $V$ with the deformation ratios

$$\alpha = \frac{L_z}{L_{s,z}} = \left(\frac{V}{V_0}\right)^{-1/3} \lambda = Q^{-1/3} \lambda \quad \text{and} \quad \frac{L_y}{L_{s,y}} = \frac{L_x}{L_{s,x}} = \alpha^{-1/2}$$  \hspace{1cm} (S9)

where the deformation ratio $\lambda$ is defined with respect to the dry state as follows

$$\lambda = \frac{L_z}{L_{0,z}} \quad \text{and} \quad \frac{L_y}{L_{0,y}} = \frac{L_x}{L_{0,x}} = \left(\frac{Q}{\lambda}\right)^{1/2}$$  \hspace{1cm} (S10)
Figure S18: (a) Dependence of the strain-dependent shear modulus of the network in a dry state $G(Q_{eq}) = G_{dr} \left( 1 + 2 \left( 1 - \beta Q_{eq}^{2/3} \right)^{-2} \right) / 3$ on the swelling ratio $Q_{eq}$. (b) Dependence of the gel modulus at small deformations $G_{gel}(Q_{eq})$ on the equilibrium swelling ratio $Q_{eq}$. Data points correspond to brush gels with $n_g=0.5$ (blue symbols), $n_g=2$ (green symbols), $n_g=4$ (orange symbols), $n_g=8$ (red symbols), $n_g=16$ (violet symbols) and $n_{sc}=2$ (circles), $n_{sc}=4$ (triangles), $n_{sc}=8$ (inverted triangles), $n_{sc}=16$ (squares), $n_{sc}=32$ (rhombs).

For gels deformed at a constant equilibrium volume $V_s$, shear modulus $G_{gel}(Q_{eq})$ is obtained from the slope of the stress-deformation plots. Figure S18a shows the dependence of the strain-dependent shear modulus $G(Q_{eq})$ on the network equilibrium swelling ratio $Q_{eq}$. The observed scaling dependence $G(Q_{eq}) \propto Q_{eq}^{-2.6\pm0.1}$ agrees with $G(Q_{eq}) \propto Q_{eq}^{-8/3}$ derived in ref.5 for
all studied networks collapsing together demonstrating a weak dependence on the network strand architecture. However, by plotting \( G_{\text{gel}}(Q_{eq}) \) vs \( Q_{eq} \) (Figure S18b), while observing the expected scaling dependence \( G_{\text{gel}}(Q_{eq}) \propto Q_{eq}^{-3.5} \) (eq S8), we also see a significant effect of the strands’ architecture on the gel modulus, manifested in a systematic shift of the \( G_{\text{gel}}(Q_{eq}) \) lines. These dependences are consistent with the ones shown in Figures 3a and 3e of the main text.

To address this issue and introduce additional effect of strand architecture into the gel modulus, we use a scaling approach. This model takes into account that as a polymer network swells in a selective solvent, the network strands adopt conformations of stretched chains in a semidilute polymer solution as illustrated in Figure S19. In particular, the swelling of the side chains significantly renormalizes strand Kuhn length and excluded volume altering local brush conformations.

**Figure S19:** Schematic representation of a swollen network of bottlebrush strands as a semidilute solution of filaments with correlation length \( \xi \) and filament thickness \( D_s \) which determines the Kuhn length and excluded volume.

In equilibrium, the shear modulus of a swollen network is balanced by the polymer osmotic pressure of a semidilute polymer solution such that

\[
G_{\text{gel}}(Q_{eq}) \approx \frac{k_B T}{\xi^3} \approx \frac{k_B T}{l^3} \left( \frac{\nu_s}{l^3} \right)^{6\nu-3 \over 3\nu-1} \left( \frac{l}{b_{K,s}} \right)^{9\nu-6 \over 3\nu-1} \left( Q_{eq} \phi^{-1} \right)^{-3\nu \nu-6 \over 3\nu-1} \tag{S11}
\]

where \( b_{K,s} \) is the effective Kuhn length of the backbone, \( \nu_s \) is the effective excluded volume per repeat unit of the polymer backbone and \( \nu_0 \) is monomer volume in a dry state. The exponent \( \nu = 0.588 \) and \( \nu = 0.5 \) for a good and \( \theta \)-solvent respectively.

These calculations result in the following expression for gel modulus as a function of the equilibrium swelling ratio \( Q_{eq} \) and molecular parameters of the brush-like strands.
\[ G_{\text{gel}}(Q_{\text{eq}}) \approx \frac{k_B T}{v_0} \left((Q_{\text{eq}} \varphi^{-1})^{-2.31} (l b_{K,s})^{0.93} v_s^{0.69} v_0^{-1.31}, \text{good} \right) \]

In the linear chain networks \( \varphi^{-1} = 1 \), \( b_{K,s} \) and \( v_s \) are determined by the polymer-solvent interactions and can only be varied by changing polymer chemistry or solvent type. This makes it impossible to have gels of linear chains with iso-\( Q_{\text{eq}} \) but different values of \( G_{\text{gel}}(Q_{\text{eq}}) \) for a given polymer/solvent pair. However, already for gels of combs we can achieve iso-\( Q_{\text{eq}} \) gels with different values of shear modulus \( G_{\text{gel}}(Q_{\text{eq}}) \) by simply grafting more side chains and changing \( \varphi^{-1} \). In the case of bottlebrush gels, the situation becomes even more complex, since both \( b_{K,s} \) and \( v_s \) depend on molecular architecture of network strands. This offers additional architectural control over gel’s elastic properties as confirmed in Figure 3e of the main text and Figure S18b.

**S5 Simulation Details**

We performed coarse-grained molecular dynamics simulations of uniaxial deformation of brush gels which equilibrium swelling properties were studied in ref 5. The brush networks were prepared by crosslinking ends of side chains of precursor graft polymers with \( n_{bb} = 129 \) backbone monomers in a melt state and an average degree of polymerization of the backbone between crosslinks, \( n_x \approx 16 \). The side chains with \( n_{sc} \) monomers were grafted to the backbone with \( n_g \) backbone bonds between the adjacent grafting points. We have performed simulations of strands with \( n_{sc} \) varied between 2 and 32, and \( n_g \) having values between 0.5 and 16. Note that both chain ends are capped by linear chain segments with \( n_g \) bonds.\(^5\) All simulations were performed using LAMMPS\(^7\) under 3-D periodic boundary conditions.

The initial gel volumes were set to the equilibrium size such that \( L_{s,x} = L_{s,y} = L_{s,z} = Q_{\text{eq}}^{1/3} L_0.\(^5\) The stress-deformation curves in the swollen state were obtained from a set of uniaxial deformation simulations. In these simulations a new deformation state was obtained by a series of small affine deformations \( \{x_i, y_i, z_i\} \rightarrow \{(1+\Delta \alpha)^{-1/2} x_i, (1+\Delta \alpha)^{-1/2} y_i, (1+\Delta \alpha) z_i\} \) with an increment \( \Delta \alpha = 0.02 \). Each small incremental deformation is achieved by deforming the network at a constant rate within \( 1.25 \times 10^3 \tau_{LJ} \), followed by a \( 1.25 \times 10^3 \tau_{LJ} \) run for equilibration and a \( 2.5 \times 10^4 \tau_{LJ} \) run for the calculation of the average stress.\(^5,8\) Simulation results for gel shear modulus obtained from analysis of stress-deformation curves are summarized in Figure S18b.
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