Phase and spin dynamics in a superconductor/ferromagnet/superconductor junction

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Abstract. We study a phase dynamics induced by a spin dynamics in a ferromagnetic Josephson junction, in which two superconductors (SC's) are separated by a ferromagnetic layer. A new phenomenological model for the phase variable is proposed by including the spin dynamics excited by the ferromagnetic resonance (FMR) in the gauge invariant phase of s-wave SC's. We found that the current-voltage characteristics show step structures by tuning the microwave frequency to FMR. The result originates from the coupling between the spin and the phase dynamics, and provides a new route to observe the spin wave excitation by using the Josephson effect.

The Josephson effect is a macroscopic quantum phenomenon regarding phase coherence of superconductor (SC). The dc Josephson effect is characterized by a current through a thin barrier without voltage drop [1]. When a finite voltage-drop (V) appears in the junction, the phase difference, $\theta$, evolves with time, $t$, and $V \propto Vt$, which induces an alternating current, i.e. ac Josephson effect. When an alternating electric field generated by microwave irradiation is applied to the junction, the current-voltage ($I$-$V$) characteristics show step structures called Shapiro step [2]. The phase dynamics providing the $I$-$V$ characteristics is described by the resistively shunted junction (RSJ) model, which is an equivalent circuit composed of resistance, capacitor and Josephson current[3, 4].

A ferromagnetic Josephson junction, in which two SC's are separated by a ferromagnetic metal (FM), is of considerable interest in recent years [5, 6, 7, 8]. One of the unique phenomena is the sign reversal of critical current with thickness of FM and temperature. The origin of this sign reversal is the exchange splitting of the conduction band in FM. On the other hand, ferromagnetic materials possess dynamic properties such as spin waves, which can be excited by ferromagnetic resonance (FMR). Hence, the interaction between Cooper pairs and spin waves in the FM is expected to create a new effect in transport properties. In this paper, we study the effect of coupling between phase and spin dynamics in a ferromagnetic Josephson junction. The RSJ model is extended to include the effect of spin wave excitations taking account of the gauge invariance.

In a ferromagnetic Josephson junction composed of s-wave SC's separated by FM, we choose a coordinate such that the cross section of the junction is parallel to the $yz$ plane and the current flows along $x$. The Josephson junction is characterized by the phase difference $\theta(y, z, t)$ between
SCs, which is described by the RSJ model given by,

\[ i = i_c^0 \sin \theta (y, z, t) + \frac{1}{R} \frac{\Phi_0}{2\pi} d\theta (y, z, t) + C \frac{\Phi_0}{2\pi} d^2\theta (y, z, t), \tag{1} \]

where \( i \) is an applied current density and \( i_c^0 \) is the critical current density. The flux quantum is denoted by \( \Phi_0 \). The resistance, \( R \), and the capacitance, \( C \), in the Josephson junction are normalized by the junction area, \( S_{yz} \), as \( R = R_0 / S_{yz} \), \( C = C_0 / S_{yz} \). \( R_0 \) and \( C_0 \) are the resistance and the capacitance in the Josephson junction, respectively.

We adopt the gauge such that the superconducting phase difference is equal to \( \Phi_0 \). The microwave induces the precessional motion of the magnetization, which corresponds to a uniform mode of spin wave. The motion of magnetization due to the microwave radiation is given by the Landau-Lifshitz-Gilbert (LLG) equation [9],

\[ \frac{dM}{dt} = -\gamma M \times H_{\text{eff}} + \frac{\alpha}{M} \left[ M \times \frac{dM}{dt} \right], \tag{2} \]

where \( M(t) \) is the magnetization in the FM, \( \gamma \) is the gyromagnetic ratio, and \( \alpha \) is the Gilbert damping. Below, \( \alpha \) is set to 0.01 [9]. The effective field acting on \( M(t) \) is given by \( H_{\text{eff}} = H_0 + h_{\text{ac}}(t) \), where \( H_0 \) is an uniaxial magnetic anisotropic field, which is parallel to the \( z \) axis and \( h_{\text{ac}} = (h_{\text{ac}} \cos \Omega t, h_{\text{ac}} \sin \Omega t, 0) \) is the microwave driving field with frequency \( \Omega \). The linearized solution of Eq. (2) is given by,

\[ M^\pm(t) = \frac{\gamma M_z h_{\text{ac}}}{\Omega - \Omega_0 \mp i\alpha \Omega} e^{\pm \imath \theta}, \tag{3} \]

which describes the precession of \( M^\pm(t) = M_x(t) \pm i M_y(t) \) around \( H_0 \) with frequency \( \Omega \) and shows a resonance at \( \Omega_0 = \gamma H_0 \). The rotating magnetic field is induced in the FM through the magnetic flux density \( B(t) = 4\pi M(t) \). Here, it is noted that the superconducting phase difference is not gauge invariant in the magnetic field. The gauge invariant phase difference must satisfy the following differential equation [10],

\[ \nabla_{y,z} \theta(y, z, t) = -\frac{4\pi d}{\Phi_0} M(t) \times n, \tag{4} \]

where \( \nabla_{y,z} = (0, \partial / \partial y, \partial / \partial z) \), \( n \) is the unit vector perpendicular to the \( yz \) plane. Equation (4) is satisfied by the following solution,

\[ \theta(y, z, t) = \theta(t) - \frac{4\pi M_x d}{\Phi_0} y + \frac{4\pi M_y(t) d}{\Phi_0} z. \tag{5} \]

We adopt the gauge such that the superconducting phase difference is equal to \( \theta(t) \) without magnetic field. Introducing Eq. (5) in Eq. (1) and integrating over the junction area, we obtain the extended RSJ model that includes the spin dynamics in the FM,

\[ I / I_c = \frac{\sin \left[ \Phi_s M_y(\tau) \right]}{\Phi_s M_y(\tau)} \sin [\theta(\tau)] + \frac{d\theta(\tau)}{d\tau} + \beta_c \frac{d^2\theta(\tau)}{d\tau^2}, \tag{6} \]

with

\[ \Phi_s = \frac{\Phi_0}{\Omega_0} \frac{\gamma h_{\text{ac}} L_z}{L_y}, \tag{7} \]

\[ M_y(\tau) = \frac{(\Omega / \Omega_0 - 1) \sin (\Omega z \tau)}{((\Omega / \Omega_0 - 1)^2 + (\alpha \Omega / \Omega_0)^2)} + \frac{\alpha \Omega / \Omega_0 \cos (\Omega z \tau)}{((\Omega / \Omega_0 - 1)^2 + (\alpha \Omega / \Omega_0)^2)}, \tag{8} \]
Figure 1. Current-voltage characteristics. The solid and dashed lines show the I-V characteristics microwave frequency for $\Omega/\Omega_0 = 1$ and $\Omega/\Omega_0 = 0.9$, respectively. We choose $\Phi_s = 0.02$. (a) $I_{ac}/I_c = 0$. (b) $I_{ac}/I_c = 0.6$. $I_{ac}$ is ac current across the junction due to the ac electric field.

where $I$ is an external current and $I_c = I_0^J (\pi \Phi_0/\Phi_z) \sin(\pi \Phi_z/\Phi_0)$. $I_0^J$ is the Josephson critical current, and $\Phi_z$ is the total magnetic flux parallel to the $z$-axis in the FM. $L_y$ and $L_z$ are the width of the junction in the $y$-and $z$-direction, respectively. The other variables are given by, $\tau = t/\tau_j$, $\tau_j = \Phi_0 / (2\pi I_c R)$, and $\beta_e = RC/\tau_j$. Below, we numerically solve Eq. (6) for $\beta_e = 0$ (overdamped junction) and calculate the I-V characteristic using the relation, $\langle \partial \theta/\partial t \rangle = 2e \langle V \rangle / \hbar$, where $\langle \cdots \rangle$ denotes a time average.

We consider that only the magnetic field component of microwave can be applied to the junction and induces the FMR in the FM. This condition is realized by using a microwave cavity [11]. The I-V characteristics are calculated by using Eq. (6) and the result is shown in Fig. 1(a) by solid and dashed lines for on- and off-resonant cases, respectively. The vertical axis is the normalized current, $I/I_c$, and the horizontal axis is the normalized voltage, $V/I_c R$. When the microwave frequency ($\Omega$) deviates from the ferromagnetic resonance frequency ($\Omega_0$), i.e. off-resonance, the I-V characteristic is in agreement with that of a conventional Josephson junction as shown by the dashed line. On the other hand, when the microwave frequency is in the condition $| 1 - \Omega/\Omega_0 | \leq \alpha$, i.e. on-resonance, at which FMR occurs in the FM, the I-V characteristic shows the step structure as shown by the solid line. In these results, the FMR induces the step structures in the I-V characteristics.

Second, we consider the case that the electromagnetic field due to microwave irradiation is applied to the junction. For the electric field component, we use the ac current source model in which $I$ is replaced by $I + I_{ac} \sin(\Omega t)$ in Eq. (6), where $I_{ac}$ is the amplitude of ac current [12]. Figure 1(b) shows the I-V characteristics for on-resonance cases. The solid line and the dashed line represent the I-V characteristic in the $I_{ac}/I_c = 0.6$ and the $I_{ac}/I_c = 0$, respectively. In the solid line, we find the additional step structures compared with the dashed line. These additional step structures are caused by the ac electric field similar to the Shapiro step.

To elucidate the origin of the step structure, we analyze the first term of the Josephson current $I_J(t)$ in Eq. (6) in the case that the FMR occurs. Using the generating function of Bessel functions, we can obtain the following equation,

$$I_J(t) = -\frac{I_c}{\xi s} \sum_{m=0}^{\infty} \sum_{n=-\infty}^{\infty} J_{2m+1} (\xi s) e^{i[\theta(t)+2(n+m+1)\Omega_0 t + (n+m+1)\pi]}$$
\[ + I_c \sum_{n=0}^{\infty} \sum_{m=-\infty}^{\infty} J_{2m+1} (\zeta_s) e^{-i[\theta(t)-(n+m+1)\Omega_0t+(n+m+1)\pi]} , \tag{9} \]

where \( J_m (\zeta_s) \) is the first kind Bessel function and \( \zeta_s = \Phi_s/\alpha \). In Eq. (9), when \( \theta(t) = \pm 2(n + m + 1)\Omega_0t \), the time averaged \( I_J(t) \) has a nonzero component of the dc Josephson current. As a result, the \( I-V \) characteristics represent the step structure without voltage change. Using the relation, \( \partial \theta(t)/\partial t = \omega_J \), we can find the voltages at which the step structures occur,

\[ \omega_J = 2(n + m + 1)\Omega_0 , \tag{10} \]

where \( n \) and \( m \) are integers. From eq. (10), it is found that the step structures only appear at even number unlike the case of Shapiro step.

In conclusion, we have studied the effect of the spin dynamics on the ac Josephson effect in the ferromagnetic Josephson junction. The resistively shunted junction (RSJ) model has been extended to include the spin wave excitations induced by the ferromagnetic resonance (FMR) and the equations of motion for the phase difference between SCs have been derived by taking account of the gauge invariance. We find that the current-voltage characteristics show step structures when the frequency of applied magnetic field \( (\omega_J) \) is equal to the product of FMR frequency \( (\Omega) \) and even integer \( (2\ell) \) as, \( \omega_J = 2\ell\Omega \). The results provide a new route to observe the spin waves using the Josephson current in SC/FM/SC junctions.

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