Analytic structure of the Landau gauge gluon propagator

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Abstract. The results of different non-perturbative studies agree on a power law as the infrared behavior of the Landau gauge gluon propagator. This propagator violates positivity and thus indicates the absence of the transverse gluons from the physical spectrum, i.e. gluon confinement. A simple analytic structure for the gluon propagator is proposed capturing all of its features. We comment also on related investigations for the Landau gauge quark propagator.

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In this talk a study of the analytic properties of the gluon propagator in Landau gauge QCD will be presented. Hereby results from non-perturbative calculations of this propagator are employed. A detailed account of this investigation can be found in ref. [1], for three-dimensional Yang-Mills theory see also ref. [2].

In the following we will confirm previous results [3, 4] on positivity violation for the gluon propagator. We will also provide a parameterisation of the gluon propagator that is analytic everywhere in the complex $p^2$ plane except on the real timelike axis and decreases to zero in every direction of the complex $p^2$ plane. Such a behaviour satisfies the standard axioms of local quantum field theory except positivity.

In Landau gauge the gluon propagator can be generically written as

$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}.$$  (1)

In Euclidean quantum field theory, positivity of a propagator can be tested by performing a Fourier transformation with respect to Euclidean time. A violation of the condition

$$\Delta(t) := \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(p_4 + \vec{p} \cdot \vec{x})} \frac{Z(p^2)}{p^2} = \frac{1}{\pi} \int dp_4 \cos(tp_4) \frac{Z(p^2)}{p^2} \geq 0,$$  (2)

1 We have also provided parameterisations of the quark propagator [3], some of which are analytic everywhere in the complex $p^2$ plane except the timelike real half-axis.
for the Schwinger function then proves violation of positivity. The Dyson–Schwinger equations (for recent reviews see e.g. [5]) for the ghost, gluon and quark propagators in the Landau gauge have been solved recently in a self-consistent truncation scheme [6, 7]. Especially, one analytically obtains

\[ Z(p^2) \sim (p^2)^{2\kappa}, \quad G(p^2) \sim (p^2)^{-\kappa}, \]  

(3)

for the gluon and ghost dressing function with exponents related to each other. In this particular truncation \( \kappa \) is an irrational number, \( \kappa = (93 - \sqrt{1201})/98 \approx 0.595 \) [8, 9]. This result depends only slightly on the employed truncation scheme: Infrared dominance of the gauge fixing part of the QCD action [10] implies infrared dominance of ghosts which in turn can be used to show [8] that the infrared exponents depend only weakly on the dressing of the ghost-gluon vertex [11] and not at all on other vertex functions [12]. Furthermore, investigations based on the Exact Renormalisation Group Equations find the relations (3) with an identical or slightly lower value for \( \kappa \) [13].

The running coupling as it results from numerical solutions for the gluon and ghost propagators can be accurately represented by [6]

\[ \alpha_{\text{fit}}(p^2) = \frac{1}{1 + (p^2/\Lambda_{\text{QCD}}^2)} \left( \alpha(0) + \frac{p^2}{\Lambda_{\text{QCD}}^2} \frac{4\pi}{\beta_0} \left( \frac{1}{\ln(p^2/\Lambda_{\text{QCD}}^2)} - \frac{1}{p^2/\Lambda_{\text{QCD}}^2 - 1} \right) \right), \]  

(4)

with \( \beta_0 = (11N_c - 2N_f)/3 \). The expression (4) is analytic in the complex \( p^2 \) plane except the negative real axis \( p^2 < 0 \), i.e. timelike momenta, where the logarithm produces a cut.

The fact that the exponent \( \kappa \) in eq. (3) is an irrational number has an important consequence: the gluon propagator possesses a cut on the negative real axis. It is possible to fit the non-perturbative solution for the gluon propagator very well without introducing further singularities. The fit to the gluon renormalization function

\[ Z_{\text{fit}}(p^2) = w \left( \frac{p^2}{\Lambda_{\text{QCD}}^2 + p^2} \right)^{2\kappa} \left( \alpha_{\text{fit}}(p^2) \right)^{-\gamma} \]  

(5)

with \( w = 2.65 \) and \( \Lambda_{\text{QCD}} = 520 \text{ MeV} \) is shown in fig. 1. Similar parametrizations have been explored in ref. [1]. Hereby \( w \) is a normalization parameter, and \( \gamma \) is the one-loop value for the anomalous dimension of the gluon propagator.

The Schwinger function \( \Delta(t) \) based on the fit (5) is compared to the one of the numerical DSE solution in fig. 1. To enable a logarithmic scale the absolute value is displayed. \( \Delta(t) \) has a zero for \( t \approx 5/\text{GeV} \) and is negative for larger Euclidean times: One clearly observes positivity violations in the gluon propagator. The overall magnitude \( w \) is arbitrary due to renormalization properties. The infrared exponent \( \kappa \) is determined analytically, and for the gluon anomalous dimension \( \gamma \) the one-loop value is used. Thus the parameterization of the gluon propagator has effectively only one parameter, the scale \( \Lambda_{\text{QCD}} \). This and the relatively simple analytic structure gives us confidence that the important features of the Landau gauge gluon propagator are given by (5).

Finally, we want to mention that the regular infrared behaviour of the quark propagator found from the Dyson–Schwinger equations [6] and on the lattice [15] complicates the
issue of determining the analytic properties of the quark propagator. Nevertheless there is some evidence that the Schwinger functions related to the quark propagator are positive definite. E.g. the quark Schwinger functions can be described accurately by a cut on the negative real axis to the left of a singular point at $p^2 = -m^2$ where $m = 350\ldots390$ MeV might be attributed the meaning of an infrared constituent mass.

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