CIRCLE DETECTION
USING FAST FINDING AND FITTING (FFF) ALGORITHM

WEI Yi
S. Marshall

KEY WORDS  circle detection; geometrically symmetric axes; least-square estimation

ABSTRACT  This paper describes a Fast Finding and Fitting (FFF) Algorithm based on the geometric symmetry of the circle. A comparison with other circle detection approaches, in particular the Hough Transform and its extensions is presented. A method to determine a relevant estimation of accuracy is also given. Experiments show that FFF possesses advantages in memory usage, high speed, reliability and accuracy are also presented.

1 Introduction

The main objectives of this paper are circle recognition and extraction. Detection of basic geometric primitives, such as circles, is an important aspect of Image Analysis with applications in vision systems and industrial inspection.

The Hough Transform (HT) and its extensions have been widely used in circle detection and they are still the most commonly used methods because of their high resistance to noise. Unfortunately, the large memory requirement and low speed are two unavoidable shortcomings.

This paper describes an alternative method for circle detection called the Fast Finding and Fitting (FFF) algorithm. This paper is organized as follows: section 2 gives a brief introduction to five methods used in circle detection, together with their advantages and disadvantages. Section 3 introduces the FFF algorithm. Section 4 gives the accuracy estimation of the FFF. Section 5 shows some experimental results of FFF and finally, section 6 draws conclusions from this work.

2 Typical methods for circle detection

Five methods of circle detection together with their advantages and disadvantages are briefly summarized as follows.

2.1 Hough Transform (HT)\textsuperscript{[1,8,9]}

Principles: A circle can be expressed in the familiar way:
\[(x - x_0)^2 + (y - y_0)^2 = r^2\] (1)

Where \(x, y\) are the pairs of cartesian coordinates lying on the perimeter of the circle; \(x_0, y_0\) represent the center of the circle and \(r\) is its radius.

The method is detailed as follows:

Fix \(x, y\) and change \(x_0, y_0\) and \(r\) through all possible values. Thus \(x_0, y_0\) and \(r\) fall onto the surface of a cone named \(x_0-y_0-r\) cone. The final detected values lie at intersection of all cones.

Advantages: High disturbance resistance because there is only one true intersection of the cones.

Disadvantages: Low speed; Requirement of large memory; It cannot be used in complicated images because of false peak detection.

2.2 Improved HT based on tangent\textsuperscript{[1]}

Principles: The equation of a circle can also be expressed as follows:
\[(x - x_0) + (y - y_0)dy/dx = 0\] (2)
If \( dy/dx \) is known, the equation is linear. By conducting similar procedures as HT, the final results can be found.

Advantages: Speed is improved as comparing with HT since only \( x_0 \) and \( y_0 \) are changed.

Disadvantages: Memory usage is still large; Usually \( dy/dx \) is not known, and even if it is known, the accuracy cannot be guaranteed because of the errors in preprocessing of the image.

2.3 Improved HT based on geometric properties of the circle

Principles: According to the circle and points indicated in Fig. 1, the following equations can be formed:

\[
\begin{align*}
(x_{v1}, y_{v1}) &= (x_0, y_0 + r) \\
(x_{v2}, y_{v2}) &= (x_0 + r, y_0) \\
(x_{v3}, y_{v3}) &= (x_0, y_0 - r) \\
(x_{v4}, y_{v4}) &= (x_0 - r, y_0)
\end{align*}
\]

therefore

\[
\begin{align*}
x_0 &= (x_{v1} + x_{v2} + x_{v3} + x_{v4})/4 \\
y_0 &= (y_{v1} + y_{v2} + y_{v3} + y_{v4})/4 \\
r &= (x_{v2} - x_{v4} + y_{v1} - y_{v3})/4
\end{align*}
\]

Advantages: Speed is faster than HT and memory usage is reduced to only two dimensions.

Disadvantages: There is no strict theory of threshold setting to choose \( x_0, y_0 \) and \( r \). Due to the image preprocessing, different local peaks can match different values.

2.4 Probabilistic Hough Transform (PHT)

Principles: Select 3 pixels at a time and conduct a HT.

Advantages: Save much time in translation from parameter space to Hough space.

Disadvantages: A threshold for sample size must be selected; Suppose there are \( N \) pixels in all, if \( N \) is large, \( C_N^3 \) is also so large that it affects speed seriously.

2.5 Genetic algorithm

Principles: Choose two primitives from a region as parents and generate two children. Primitives could be any representative set of features such as shape parameters. Judge the cost value of two children and put them into the region. Compare cost values of every primitive and remove the least well fitted to keep the region size fixed. Repeat above steps until every primitive has similar cost value, then the result is optimal.

Advantages: Accurate; memory usage is not too large.

Disadvantages: It can extract only one circle at one time, so the efficiency is low, therefore it cannot be used in an image with many circles.

3 Fast Finding and Fitting (FFF) algorithm

3.1 Fast finding

Principles: Each circle has just one horizontal symmetrical axis and one vertical symmetrical axis. Given a complex image with many circles, we can extract all possible horizontal and vertical axes and group all the pixels symmetric to one pair axes into one subimage. Thus all the isolated points, lines and other irregular curves are removed at this stage. Then judge all subimages as to whether they are circles and remove those that are rectangles and ellipses. In general, the remaining subimages are all circles.

Diagram: Suppose \( f \) is the original image; \( F \) is all its edge pixels; \( A, A', B, B' \) are points in \( f \); \( M \) and \( N \) are sets of the middle points of \( AA' \) and \( BB' \) respectively; \( G \) and \( L \) are two accumulators. The flow diagram is shown in Fig. 2.

3.2 Fitting

Principles: Statistical methods are commonly used in circle estimation. But unfortunately, they often lead to nonlinear equations. Thus a correct initial value has to be chosen and the consideration of speed and convergence is unavoidable. Here a least-square approach based on estimation theory is rec-
recommended. It first converts the circle equation into a linear equation, so that there is no need for initial value setting or concern about convergence.

\[ \frac{\partial E}{\partial x_0} = 2 \sum_{i=1}^{n} (x_i^2 - 2x_0x_i + y_i^2 - 2y_0y_i + z) = 0 \]
\[ \frac{\partial E}{\partial y_0} = 2 \sum_{i=1}^{n} (x_i^2 - 2x_0x_i + y_i^2 - 2y_0y_i + z) = 0 \]
\[ \frac{\partial E}{\partial z} = 2 \sum_{i=1}^{n} (x_i^2 - 2x_0x_i + y_i^2 - 2y_0y_i + z) = 0 \]

Defining \( A, X \) and \( D \) as follows:

\[ A = \begin{bmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} y_i^2 & \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i & 1 \end{bmatrix} \]

\[ X = [\hat{x}_0 \hat{y}_0 \hat{z}]^T \]

\[ D = \sum_{i=1}^{n} (x_i^3 + x_i y_i^2) \]

Then there is \( AX = D \). Since it is a real full-order matrix, the result of this equation is:

\[ X = A^{-1}D \]

Therefore \( x_0, y_0 \) and \( z \) are estimated and \( r \) is obtained by \( r = (x_0^2 + y_0^2 - z)^{1/2} \) finally.

### 4 Accuracy Estimation

According to data processing theory, there are three steps in parameter estimation: (1) select a linear mathematical model; (2) find the result according to estimation theory; (3) estimation of accuracy. The first two steps are completed. The step 3 is described as follows.

#### 4.1 Pixel-position accuracy

If \((x'_1, y'_1)\) and \((x''_1, y''_1)\) correspond to the same pixel before and after detection, according to Bessel formula, the coordinate accuracy of a circle \( \sigma_x \) and \( \sigma_y \) can be written as:

\[ \sigma_x = \pm \sqrt{\frac{\sum_{i=1}^{n} (x_i' - x_i'')^2}{(n - 1)}} \]
\[
\sigma_y = \pm \sqrt{\sum_{i=1}^{n} (y_i' - y_i^n)^2 / (n - 1)}
\]  
(11)

where \(n\) is the total amount of pixels in the detected circle \((i = 1, 2, \ldots, n)\). Thus the pixel-position accuracy after fitting is:

\[
M_p = \pm \sqrt{\sigma_x^2 + \sigma_y^2}
\]  
(12)

**4.2 Accuracy of circle center \(m_c\)**

Firstly, the accuracy of \(x_0\) and \(y_0\) are:

\[
m_{x0} = \pm \sigma_x \sqrt{Q_{x0}}
\]  
(13)

\[
m_{y0} = \pm \sigma_y \sqrt{Q_{y0}}
\]  
(13)

\(Q_{x0}\) and \(Q_{y0}\) are the cofactors and they are equal to the top left elements on the diagonal of matrix \(A^{-1}\). Therefore \(m_c\) is:

\[
m_c = \pm \sqrt{m_{x0}^2 + m_{y0}^2}
\]  
(14)

**4.3 Accuracy of radius \(m_r\)**

Since \(r^2 = x_0^2 + y_0^2 - z^2\), \(z\) can be considered as a constant value after fitting. According to convergence spreading law, the accuracy of radius \(m_r\) is as follows:

\[
m_r = \pm \sqrt{m_{x0}^2 m_{x0}^2 + m_{y0}^2 m_{y0}^2} / r
\]  
(15)

**5 Result**

The methods described in this paper were applied to a number of test images containing circles set in various amounts of noise and clutter. The results are given in Fig. 3~Fig. 5. In Fig. 3(a), the circles are badly damaged, especially the four circles on the right side. They are very close to each other. As can be seen in Fig. 3(b), all the circles are extracted and the detection accuracy is still good as shown in Table 1. Fig. 4 and Fig. 5 show that FFF can detect circles with a large range of diameters. In general, FFF can recognize circles of poor quality with noise. Obviously the poorer the quality, the worse the accuracy of detection, but the method does appear to be particularly robust.

From a quantitative viewpoint, Table 1 gives the calculated measures of accuracy for these examples. It can be seen that the pixel-position error is generally less than 0.7 pixel and the center and radius accuracy is generally less than 0.1 pixel.

**6 Conclusion**

This paper has described the FFF and demonstrated that it has two chief properties: one is Fast speed. It removes other elements from the original image and at the same time divides the circles into different subimages. That is to say, finding and removal happen at the same time. Besides, since the estimation of circle parameters has been converted
Detecting a large number of circles of close and differing diameters simultaneously.

Table 1 Accuracy estimation

| Name of Circle | Pixel-position accuracy $M_p$ | Accuracy of circle center $m$ | Accuracy of radius $m_r$ |
|----------------|-------------------------------|-------------------------------|--------------------------|
| dd1            | 99                            | $6.15 \times 10^{-5}$         | $1.23 \times 10^{-3}$    |
| dd2            | 35                            | $9.65 \times 10^{-3}$         | $1.23 \times 10^{-1}$    |
| dd3            | 176                           | $2.69 \times 10^{-4}$         | $2.69 \times 10^{-4}$    |
| dd4            | 7                             | $1.51 \times 10^{-2}$         | $2.36 \times 10^{-1}$    |
| dd5            | 110                           | $6.92 \times 10^{-4}$         | $8.08 \times 10^{-4}$    |
| dd6            | 13                            | $7.64 \times 10^{-3}$         | $3.56 \times 10^{-2}$    |

to finding the result of a group of linear equations, there is no need for calculation and peak detection. The other property is high accuracy. It is a strict mathematical method based on estimation theory. There is no subjective stage in the detection, thus there is no loss in accuracy. As space is limited, only some final results are given here. Experiments on Octek 486 personal computer show that FFF is 75 times faster than HT in speed.

References

1. Kumar S, Ranganathan N, Goldgof D. Parallel algorithms for circle detection in images. *Pattern Recognition*, 1994, 27(8): 1019−1028
2. Raymond K. K YIP, Peter K. S Tam, Leung Dennis N. K. Modification of Hough Transform for circles and ellipse detection using a 2-Dimensional array. *Pattern Recognition*, 1992, 25(9): 1007−1022
3. Chun-ta ho and Chen K H A fast ellipse/circle detection using geometric symmetry. *Pattern Recognition*, 1995, 28(1): 117−124
4. Gerhard R, Dlevine M. Geometric primitive extraction using a genetic algorithm. *IEEE Trans. On PAMI*, 1994, 16(9): 901−905
5. Nakamura A, Aizawa K. Digital circles. *Computer Vision, Graphics and Image Processing*, 1984, 26: 242−255
6. Nakamura A, Aizawa K. Digital images of geometric pictures. *CVGIP*, 1985, 30: 107−120
7. Wang R. *Image Analysis*. Beijing: Press of Technical University of National Defense, 1995 (in Chinese)
8. Jing R, Ye X. *Computer Image Processing*, 1990 (in Chinese)
9. Leavers V F. The dynamic generalized Hough Transform: its relationship to circles and ellipses. *Image Understanding*, 1992, 56(3): 381−398