Form Factors of the Nucleon in a Selfconsistent Chiral Model

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Abstract

In the present paper we extend the Schwinger model of the nucleon to the inclusion of loop corrections. Starting from a chiral model of nucleon and delta for the mesons $\pi, \rho$ and $A_1$ we extend our non-perturbative selfconsistent regularisation scheme to the calculation of the nucleon form factors. Chiral symmetry requires loops according to the interactions $L_{\pi\pi\rho}, L_{\rho\rho\rho}, L_{\rho\pi A_1}, L_{\rho A_1 A_1}$. The coupling constants are strictly introduced by chiral symmetry. The use of low energy theorems and SU(3) relations allow an otherwise parameterfree calculation of the form factors of the nucleon.

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The determination and understanding of nucleon form factors belong to the fundamental problems in hadron physics. As form factors are dynamical quantities they are exceptionally suited for obtaining information on the underlying strong interaction of the hadronic subcomponents. As the nucleon resp. pion are the lightest quark systems their study is of greatest importance. While there is already overwhelming information on the form factors of the proton there is upcoming information on the neutron form factors. In the present paper we are interested in a description of form factors at low momentum transfer i.e. $Q^2 < 0 - 1 GeV^2$.

QCD is thought to be the correct theory of the strong forces although several problems related to its application remain without solution:

- inability to solve dynamics of QCD at hadronic scales because of the large coupling constant.
- natural degrees of freedom of QCD can not be directly related with physical observables.

In the last decade Chiral Perturbation Theory\cite{1,2} has been applied to many problems in nuclear systems for low-energy regime, namely nucleon-nucleon potential, deuteron properties, few-body forces etc. Chiral Perturbation Theory combines a chiral lagrangian involving pions, nucleons and deltas with the underlying symmetries of QCD and a power counting scheme based on a natural expansion parameter $Q/M$ where $Q$ is the three-momentum of the particle and $M$ is the nucleon mass. In this approach it is considered the most general lagrangian which involves pions, nucleons and deltas and transforms under chiral symmetry as the QCD lagrangian. The parameters that appear in this lagrangian have to be related to the measured low-energy quantities. These undetermined coefficients will be fixed by fitting data from low-energy processes. This approach gets in trouble when the enery is increased because the number of unknown parameters grows with the order of the expansion. In the chiral perturbation approach, the coefficients of the lagrangian are all unknown phenomenological parameters which have to be determined in a fit to the experiments. The number of unknown parameters increase rapidly with increasing precision of the momentum expansion.

The intention of the present work is to develop an approach within there is no parameters to play with. Our aim is to go beyond the perturbation approach avoiding the introduction of new parameters each time we increase
the order of the expansion. A non-perturbative method is developed and used to take into account higher orders of the expansion.

In the present paper we argue that a "parameter free" calculation of the electromagnetic and strong formfactors is possible. We mean "parameter free" in the following sense. Starting from an effective Lagrangian which has the required invariance properties, such as gauge invariance, chiral invariance etc., we aim at a calculation of the currents which involves otherwise no additional free parameters. There should be no additional cut off parameters which allow us to "correct" our calculated momentum dependence of the form factors. We refine the description of refs. [18]. Schwinger considers a model consisting of nucleon, $\pi$, $\rho$ and $A_1$. The mesons rho and $A_1$ appear as gauge particles. The difficulty in such an approach is that including particles $\pi$, $\rho$, $A_1$ and one must simultaneously deal with the interactions like $L_{\pi\pi\rho}$, $L_{\rho\rho\rho}$, $L_{\rho\pi A_1}$, $L_{\rho A_1 A_1}$ (see also Weinberg [19], Wess and Zumino [20] and Gasiorowicz and Geffen [21]). The calculation of form factors therefore must include these meson-meson interactions. The consideration of these effects leads directly to the inclusion of loop corrections. Schwinger did not include loop corrections in his work as in a perturbation approach the loops are leading to undefined quantities. Using however the nonperturbative and selfconsistent approach of ref. [24], we are able to include these corrections in a parameterfree way.

We have extended the original Lagrangians of Schwinger to the inclusion of delta degrees of freedom which are known to be of considerable importance and have to be included when we want to compare with experiments.

Note that the symmetries are violated only due to the finite mass of the mesons. The imposition of current field identity on the lagrangian relates the $A_1$ mass to the rho mass, i.e. $m_{A_1} = \sqrt{2}m_\rho$. 

\[ m_{A_1} = \sqrt{2}m_\rho \]
The interaction terms in the effective Lagrangian are given as follows:

nucleon-nucleon-meson

\[ L_{NN\pi} = -\frac{g_{NN\pi}}{2M} \bar{\psi} \gamma^5 \gamma^\mu \psi \partial_\mu \pi^i \]

\[ L_{NNA_1} = -\frac{g_{NNA_1}}{2M} \bar{\psi} \gamma^5 \gamma^\mu \psi A_1^i \]

\[ L_{NN\rho} = -\frac{g_{NN\rho}}{2M} \bar{\psi} \gamma^5 \gamma^\mu \partial_\mu \rho \tau^i \]

\[ L_{NN\pi\rho} = +\frac{g_{NN\pi\rho}}{2M} \bar{\psi} \gamma^5 \gamma^\mu \psi \epsilon^{ijk} \tau_j \pi^k \rho^i \]

nucleon-delta-meson and delta-delta-meson

\[ L_{N\Delta\pi} = -\frac{g_{N\Delta\pi}}{2M} \bar{\Psi} \gamma^5 \gamma^\mu \gamma^\nu \gamma^5 \gamma_5 \bar{N} \gamma^\mu \pi \]

\[ L_{N\Delta A_1} = -\frac{g_{N\Delta A_1}}{2M} \bar{\Psi} \gamma^5 \gamma^\mu \gamma^\nu \gamma^5 \gamma_5 \bar{N} \gamma^\mu A_1^i \]

\[ L_{N\Delta \rho} = g_{N\Delta \rho} \bar{\Psi} \gamma^5 \gamma^\mu \gamma^\nu \gamma^5 \gamma_5 \bar{N} \gamma^\mu \rho^i \]

meson-meson-interaction

\[ L_{\pi p A_1} = -\frac{3}{2} m_{A_1} g_{\rho} \rho^i \cdot (\pi \times A_1^i) \]

\[ L_{\rho p p} = -g_\pi \epsilon^{ijk} \rho^i \rho^j \rho^k \]

\[ L_{\pi \pi p} = -g_\rho \epsilon^{ijk} \pi^i \pi^j \rho^k \]

\[ L_{\rho A_1 A_1} = g_\rho (A_1^i \times \partial_\mu A_1^i) \]

The coupling constants are strictly related by symmetries. The values are based on the Goldberger-Treiman relation: \( f_\pi g_{NN\pi} F_{NN\pi}(Q^2 = 0) = m_n g_A \) and the KSFR formula \( m_\rho^2 = 2g_\rho^2 f_\pi^2 \) as well as SU(3) model predictions. The relation of the different coupling constants are summarized in table 1.

The electromagnetic (e.m.) isovector current plays a special role through the relation to the \( \rho \)-nucleon form factor \( F_{NN\rho} \). The form factors in turn are
Table 1: Summary of the coupling constants used in the present paper. The values are strictly related by symmetries and low energy theorems (vector and axial vector gauge, SU(3),KSFR ,Goldberger Treiman and Weinberg’s sum rule. The imposition of current field identity on the broken symmetry of the Lagrangian leads to $m_{A_1} = \sqrt{2}m_\rho$. $f_\pi = 92.6 \pm 0.2MeV$, $g_A = 1.261 \pm 0.004$.

defined by the current matrixelement

$$< p_f | J_\mu(Q^2)_{(iv)} | p_i > =$$

$$\chi^\dagger \bar{u}(p_f) \frac{\tau^3}{2} \left\{ \gamma_{\mu} F_1^{(iv)} + i \frac{\sigma_{\mu\nu} q^\nu}{2M} \kappa_{iv} F_2^{(iv)} \right\} u(p_i) \chi$$

Due to vector dominance we have the following relation between electromagnetic and rho form factors:

$$G_{\gamma,magnetic}^{(iv)}(Q^2) = F_1^{(iv)}(Q^2) + \kappa_{iv} F_2^{(iv)}(Q^2)$$

$$F_\gamma^{(iv)}(Q^2) = \Delta_\rho F_\rho(Q^2)$$

$$\rightarrow G_{\gamma,magnetic}^{(iv)}(Q^2) = \Delta_\rho G_{\rho,magnetic}^{(iv)}(Q^2)$$

where $\Delta_\rho$ is the $\rho$ propagator.

The axialvector form factor is given by:

$$G_A(Q^2) = g_A \Delta_{A_1} F_{NNA_1}(Q^2),$$
where $\Delta_{A_1}$ is the $A_1$ propagator.

In the description of the nucleon form factors we consider a one loop topology. The actual calculation are performed by the method of unitary transformation. This method has several advantages one of which is the treatment of renormalization contributions. For details see refs. [16], [22]. In addition we use a nonrelativistic formulation of the effective Lagrangians. Off-shell effects in the form factors are neglected, as they are known to be small.

Let us discuss first a system which involves only $\gamma, N$ and $\rho$. The arising meson loops to the e.m.form factor $F_{NN\gamma}(Q^2)$ include different types of contributions:
i) there is a $N\rho$-loop with a coupling of the photon to the nucleon (L1), ii) there is a loop arising from the $\gamma - \rho$ interaction (T1). iii) In addition we have a wavefunction reorthonormalization contribution which involves a simple $N\rho$-loop (R1+R2). Compare from Fig.1.

We realize that a conventional renormalization procedure will fail to remove the divergencies in our loop integrations. We regularize the loop integral by replacing each bare vertex by the sum of all reducible higher order vertex corrections. Higher orders in the coupling constants are included in ladder approximation by retaining dressed vertices within a one-loop topology.

For the $\rho N$ system we end up with a condition for the $\gamma$-nucleon form factor $F_{NN\gamma}(Q^2)$ as illustrated in Fig.1:

The form factor is determined by the following equation:

$$F_{NN\gamma}(Q^2) = -\frac{\Delta_{\rho} T1(Q^2, F_{NN\rho}(k^2))}{L1(Q^2, F_{NN\rho}(k^2)) + R1(Q^2, F_{NN\rho}(k^2))} + R2(Q^2, F_{NN\rho}(k^2))$$

(4)

This relation tells us that starting the calculations with a form factor which fulfills the above equation, the meson corrections are included in a one-loop topology.

Current field identity relates this expression to the $\rho$ nucleon form factor $F_{NN\rho}$. We realize that the $F_{NN\rho}(Q^2)$ obeys a nonlinear equation of the
Figure 1: Illustration of the condition on the e.m. nucleon form factors. This relation is used to define the form factor. The black circles denote the e.m. nucleon form factors. The shaded circle denote the $\rho$ nucleon form factors. Vector dominance leads to the respective equation for the $\rho$ nucleon form factor.

\begin{align}
F_{NN\rho}(Q^2) &= -\frac{T1(Q^2, F_{NN\rho}(k^2))F_{NN\rho}(k^2))}{L1(Q^2, F_{NN\rho}(k^2)) + R1(Q^2, F_{NN\rho}(k^2)) + R2(Q^2, F_{NN\rho}(k^2))} \\
&= 0
\end{align}

Actually for a meson baryon system of N, $\Delta$, $\pi$, $\rho$ and $A_1$ (including the interactions $\pi\pi\rho$, $\rho\rho\rho$, $\rho A_1 A_1$), one has a set of coupled integral equations of the form equivalent to equ.(5). The $F_{NN\rho}$ form factor depends in the general case on all other form factors $F_{NN\pi}$, $F_{NNA_1}$, $F_{N\Delta\pi}$, $F_{N\Delta\rho}$, ...$F_{\Delta\Delta A_1}$. Similar relations hold for the other form factors. In general the form factors F depend on the form factors on other interactions F and on form factors belonging to the same interaction. Formally this can be writ-
where $\mathcal{K}$ denotes the meson loop contributions. Here we discuss the expressions for Dirac and Pauli form factors. Actually the system is solved for $F_{\mathcal{A}}$ and the magnetic form factor $G_{\mathcal{M}}$.

We realize that the solution of the system of coupled equations provides a physical self-consistent regularization and gives a parameter free and nonperturbative solutions for the meson nucleon form factors $F_{\mathcal{A}}$ within a one loop ladder approach.

The coupled system of nonlinear integral equations for the meson-baryon form factors are solved with an ansatz of monopol form: $F(Q^2) = \Lambda^2 + Q^2$. This ansatz turns out to be a very good approximation at low $Q^2$. A monopol ansatz simplifies the solution of the coupled integral equations and is good enough for the present purposes. In many cases a parameterization in terms of the monopol form is desired for comparison. This approximation is also consistent with the approximations of Schwinger in deriving the lagrangian functions. The results for the form factors are summarized in table 1. Note that for the tensor couplings constants we have the relation: $\kappa_\rho = \kappa_{(iv)} = 3.706$. This is the result of selfconsistency and the condition on the form factors. Not surprising the $\pi$ turns out to give the most important contribution to all form factors. The contribution of the $\rho$ is important. The contribution of $A_1$ is generally small, however, not negligible for the isovector form factor. We note the similarity between the $\pi$ and the $A_1$ form factor scales. A closer look at the equations for $\pi$ and $A_1$, tells us that this is an expected behaviour and reflects the relations of the resp. lagrangians.

It is interesting to compare the present results with some observed quantities. Among the possible choices there seems one of special impor-
Table 2: Solutions of the coupled equations for the system N, Δ, π, ρ, A1.
Form factors are calculated for space-like momentum transfer. They are parametrized in the form: $F(Q^2) = \frac{\Lambda^2}{\Lambda^2 + Q^2}$.

| Scales | $\Lambda_{NN\pi}$ | fm$^{-1}$ | GeV |
|--------|-------------------|----------|-----|
| $\Lambda_{NN\pi}$ | 4.06 | 0.801 |
| $\Lambda_{NNA_1}$ | 4.10 | 0.809 |
| $\Lambda_{NN\rho\text{Dirac}}$ | 5.95 | 1.170 |
| $\Lambda_{NN\rho\text{Magnetic}}$ | 4.410 | 0.870 |
| $\Lambda_{N\Delta\rho\text{Magnetic}}$ | 4.3 | 0.840 |
| $\Lambda_{N\Delta\pi}$ | 4.00 | 0.78 |
| $\Lambda_{N\Delta A_1}$ | 4.10 | 0.809 |
| $\Lambda_{\Delta\Delta\rho\text{Dirac}}$ | 4.6 | 0.9 |
| $\Lambda_{\Delta\Delta\rho\text{Magnetic}}$ | 4.5 | 0.880 |
| $\Lambda_{\Delta\Delta\pi}$ | 4.10 | 0.809 |
| $\Lambda_{\Delta\Delta A_1}$ | 4.20 | 0.82 |

tance. As we have very little information on the e.m. neutron form factors we discuss that choice as it is in addition the most sensitive quantity.

The neutron form factors can be expressed through measured e.m. form factors of the proton and the calculated isovector form factors of the nucleon.

$$G_n^E(Q^2) = G_p^E(\text{measured})(Q^2) - (F_1^{(iv)}(Q^2) - \frac{Q^2}{4M^2}\kappa^{iv}F_2^{(iv)}(Q^2))$$  \hspace{1cm} (7)$$

$$G_n^M(Q^2) = G_p^M(\text{measured})(Q^2) - (F_1^{(iv)}(Q^2) + \kappa^{iv}F_2^{(iv)}(Q^2))$$  \hspace{1cm} (8)$$

These expressions are used to obtain the theoretical "Ruhredata" for the neutron magnetic and electric formfactors. There are two possibilities in arriving at a description of the neutron form factors. One uses the widely discussed dipol fit ($G(Q^2)_{\text{Dipole}} = \frac{0.71[GeV^2]}{0.11[GeV^2]+Q^2}$) of the experimental proton magnetic and electric formfactors. Another possibility is to use directly the experimental data. Both possibilities are shown in Figs. (2,3) together with the most recent measurements. Concerning the proton data we use the Mainz analysis [17] of the world data. We realize that the dipol fit used as representative for the proton data is different for the electric and
magnetic neutron form factor. This is due to the smallness of the electric neutron form factor at low $Q^2$.

At momenta $Q^2 = 0.4 \, GeV^2$ one feels already the deviation from the monopolar form from the exact form. This is especially true for the Pauli contribution. The electric neutron form factor is extremely dependent on the shape of the isovector form factor. One realizes a much stronger dependence of the neutron form factors as compared to the axial form factor. While the axial form factor is directly given by the $A_1$ form factor (times $\Delta A_1$) the neutron form factors are obtained from a difference of proton and vector form factors. As already mentioned the axial form factor is in agreement with the world data in the considered momentum range.

To summarize we have shown that the extended chiral Schwinger model ($\Delta, \pi, \rho, A_1, \gamma$) together with the nonperturbative selfconsistent calculation of the mesonic loop corrections leads to an interesting description of the nucleon formfactors. As the calculations are obtained with no additional parameters, one expects a similar quality of the isoscalar current. In this case the model has to be extended to the inclusion of $\omega$ and $\phi$ as well as strange particles like K, $\Lambda$... We expect no significant changes from the isoscalar contribution in the coupling to the isovector current. There is however, hope that an extension of the present treatment to a calculation of the nucleon-nucleon potential will be manageable. The use of scales in the region of the obtained values for the meson nucleon form factors have already been shown to be possible. [23].
Figure 2: Predictions for the electric neutron form factor through the relation to the measured electric proton form factor. Two choices are shown: a) The dots ("Ruhrdata") are obtained with the use of the measured $G_E^p$ data from the analysis of [17]. b) the line shows the predictions when the dipole fit i.e. $G_E^p = G_{Dipole}$ is used. (The derivative of $G_E^p$ at $Q^2 = 0$ is $(G_E^p)'=0.35$ ) Experimental data for the electric neutron form factor are from: O M.Ostrick et al[10], ◇ D.Rohe et al[11], ♦ M.Meyerhoff et al[12], △ T.Eden et al[13], ♠ E.Bruins et al[9], ♣ C.Herberg et al[14], ∞ J.Becker et al[15].
shown: a) The dots ("Ruhrdata") are obtained with the use of the measured $G_M^p$ data [17]. b) the line shows the predictions when we use the dipole fit i.e. $\frac{G_M^p}{\mu} = G_{Dipole}$ is used. Experimental data for the magnetic neutron form factor are from: H.Anklin et al[3], P.Markowitz et al[5], H.Gao et al[6], H.Anklin et al[7], E.E.W.Bruins et al[9].
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