Present direct detection sensitivities to WIMP–quark and WIMP–gluon effective interactions

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Abstract. Assuming for Weakly Interacting Massive Particles (WIMPs) a Maxwellian velocity distribution in the Galaxy we explore in a systematic way the relative sensitivity of an extensive set of existing Dark Matter (DM) direct detection (DD) experiments to operators up to dimension 7 of the relativistic effective field theory describing dark matter interactions with quarks and gluons. We discuss how the mixing of dimension–6 operators due to the running between different energy scales affects the DD bounds and include QCD effects and pion poles that arise in the nonperturbative matching of the effective field theory to the low–energy Galilean–invariant nonrelativistic effective theory describing DM–nucleon interactions. We also show how in most cases a single non–relativistic operator dominates the expected rate at the nucleon scale. This allows to obtain approximate constraints valid for any choice of the parameters of the relativistic theory in terms of model–independent limits on the Wilson coefficients of the non–relativistic theory expressed in terms of the WIMP mass and of the neutron–to–proton coupling ratio $c^n/c^p$. To perform such task we provide a simple interpolating interface in Python.
1 Introduction

One of the most popular scenarios for the Dark Matter (DM) which is believed to contribute to up to 27% of the total mass density of the Universe [1] and to more than 90% of the halo of our Galaxy is provided by Weakly Interacting Massive Particles (WIMPs) with a mass in the GeV-TeV range and weak–type interactions with ordinary matter. Such small but non vanishing interactions can drive WIMP scatterings off nuclear targets, and the measurement of the ensuing nuclear recoils in low–background detectors (direct detection, DD) represents the most straightforward way to detect them. Indeed, a large worldwide effort is currently under way to observe WIMP-nuclear scatterings, but, with the exception of the DAMA collaboration [2–5] that has been observing for a long time an excess compatible to the annual modulation of a DM signal, many other experiments using different nuclear targets and various background–subtraction techniques [6–18] have failed to observe any WIMP signal so far.

The calculation of DD expected rates is affected by large uncertainties, of both astrophysical and particle–physics nature. For instance, most of the explicit ultraviolet completions of the Standard Model that stabilize the Higgs vacuum contain WIMP exotic states that are viable DM candidates and for which detailed predictions for WIMP–nuclear scattering can be worked out, leading in most cases to either a Spin Independent (SI) cross section proportional to the square of the target mass number, or to a Spin–Dependent (SD) cross section proportional to the product of the WIMP and the nucleon spins. Crucially, this allows to determine how the WIMP interacts with different targets, and to compare in this way the sensitivity of different detectors to a given WIMP candidate, with the goal of choosing the most effective detection strategy. However, the non–observation of new
physics at the Large Hadron Collider (LHC) has prompted the need to go beyond such top–down approach and to use either “effective” or “simplified” models to analyze the data, implying a much larger range of possible scaling laws of the WIMP–nucleon cross section on different targets. Moreover, the expected WIMP–induced scattering spectrum depends on a convolution on the velocity distribution \( f(\vec{v}) \) of the incoming WIMPs, usually described by a thermalized non–relativistic gas described by a Maxwellian distribution whose root–mean–square velocity \( v_{\text{rms}} \simeq 270 \text{ km/s} \) is determined from the galactic rotational velocity by assuming equilibrium between gravitational attraction and WIMP pressure. Indeed, such model, usually referred to as Isothermal Sphere, is confirmed by numerical simulations [19], although the detailed merger history of the Milky Way is not known, allowing for the possibility of the presence of sizable non–thermal components for which the density, direction and speed of WIMPs are hard to predict [20].

As far as the latter issue is concerned, for definiteness, in the following we will adopt for the velocity distribution \( f(\vec{v}) \) of the incoming WIMPs a standard thermalized non–relativistic gas described by a Maxwellian distribution. On the other hand the goal of the present paper is to focus on the former issue of the scaling law in direct detection.

As far as the scaling law is concerned, since the DD process is non–relativistic (NR) it has been understood some time ago [21, 22] that the most general one besides the SI and the SD cross sections can be parameterized with an effective Hamiltonian that complies with Galilean symmetry, containing at most 15 terms in the case of a spin–1/2 particle:

\[
\mathcal{H}(r) = \sum_{\tau=0,1} \sum_{j=1}^{15} c^\tau_j \mathcal{O}_j(r) t^\tau, \tag{1.1}
\]

where the \( \mathcal{O}_j \) operators are listed in [22] and \( t^0 = 1, t^1 = \tau_3 \) denote the the 2×2 identity and third Pauli matrix in isospin space, respectively, and the isoscalar and isovector coupling constants \( c^0_j \) and \( c^1_j \), are related to those to protons and neutrons \( c^p_j \) and \( c^n_j \) by \( c^\tau_j = (c^0_j + c^1_j)/2 \) and \( c^\tau_j = (c^0_j - c^1_j)/2 \).

Indeed, the NR couplings \( c^\tau_j \) represent the building blocks of the low–energy limit of any ultraviolet theory, so that an understanding of the behaviour of such couplings is crucial for the interpretation of more general scenarios. As a consequence of this, in Ref. [23] we provided an assessment of the overall present and future sensitivity of an extensive list of both present and future WIMP direct detection experiments assuming systematically dominance of one of the 14 possible terms of the NR effective Hamiltonian in the calculation of the WIMP–nucleon cross section. Such approach has the advantage that the best available constraint on the WIMP–proton cross section is only function of two parameters, the WIMP mass \( m_\chi \) and the ratio of the WIMP–neutron and WIMP–proton couplings \( c^n/c^p \), and, with the exceptions of cancellations among different NR operators, can be easily applied to to get approximate constraints for any DM scenario.
However, in spite of its generality, such approach presents some drawbacks: in particular, the interference of different NR operators and especially the sensitivity of such effect to the running of the couplings from the energy scale of the ultraviolet theory to the nucleon scale [24, 25] are difficult to include in a model–independent way, as well as a possible momentum dependence of the Wilson coefficients of the NR theory. In particular, the latter can arise in the case of a long–range interaction such as for electric–dipole or magnetic–dipole DM [26, 27]. Moreover an additional momentum dependence arises when one needs to include the light-meson poles in the case the DM couples to the axial quark current [28].

For the reasons listed above in the present paper we wish to extend the analysis of Ref. [23] considering instead a relativistic effective theory for which, if not specified otherwise, we conventionally fix the Wilson parameters at the Electroweak (EW) scale (that we identify with the $Z$ boson mass), assessing the sensitivity of present DD experiments to a set of operators up to dimension 7 describing dark matter interactions with quarks $q$ and gluons

$$ \mathcal{L}_X = \sum_q \sum_{a,d} c^{(d)}_{a,q} Q^{(d)}_{a,q} + \sum_{b,d} C^{(d)}_{b} Q^{(d)}_{b}, $$

where the $c^{(d)}_{a,q}$, $C^{(d)}_{b}$ are dimensional Wilson coefficients. The sums run over the dimensions of the operators, $d = 5, 6, 7$ and the operator labels, $a$ and $b$. The operators $Q^{(d)}_{a,q}$, $Q^{(d)}_{b}$ that we will analyze are listed in Eqs. (2.1, 2.2, 2.3).

In particular, once, for each of the relativistic models we consider, the NR Wilson coefficients $c^{(d)}_j$ at the nucleon scale are obtained from the $C^{(d)}_{a,q}$’s, the expected rate calculation as well as the treatment of the experimental limits follow closely the analysis of Ref. [23], so that we address the reader to that paper for the formulas that we use to calculate the expected rates for WIMP–nucleus scattering as well as for the details of how each experimental limits has been obtained.

The approach of the present analysis is complementary to that of Ref. [23], but itself not devoid from drawbacks. The most obvious one is that, much in the same way of NR operators, in principle also in the relativistic case considering one operator at a time is not in general justified. However, a more serious problem is represented by the fact that the matching of the Wilson coefficients $c^{(d)}_{a,q}$’s of the WIMP–quark relativistic interaction into the $c^{(d)}_j$’s of the NR WIMP–nucleon Hamiltonian is highly degenerate, since, in principle, the relativistic effective theory requires to fix one coupling for each quark flavor $q$, while the NR theory contains only protons and neutrons. In other words, some assumptions must be made on how the $c^{(d)}_{a,q}$’s scale with the flavor $q$. For instance, the operator mixing between vector and vector–axial couplings calculated in [24, 25] induced by the running above the EW scale and discussed in Section 3 is driven only by the couplings between the WIMP and the quarks of the third family while, on the other hand, the DD expected rate does not depend on the corresponding values of the same couplings at the nucleon scale: as far as the DD scaling law among different targets is concerned the low–energy WIMP–top and
WIMP–bottom couplings can in principle assumed to be vanishing. A frequent approach in the literature is to parameterize the theory in terms of a single coupling $C_{a,q}^{(d)}$ common to all quarks \cite{29,30}, and in our analysis we will do the same. However it is worth pointing out that this assumption would not be applicable to the case of the supersymmetric neutralino, for which, for instance, $C_{4,q}^{(6)}$ scales as the $Z$–boson coupling in the case of a Higgsino, or $C_{5,q}^{(7)}$ depends on the mass and the weak isospin of the quark for a Gaugino–Higgsino mixing. Indeed, in the case of a generic scaling of the WIMP–quark couplings the only possible way to obtain a consistent limit is to calculate the ratio $c^n/c^p$ from the $C_{a,q}^{(d)}$’s and use the NR results of \cite{23}. Since the limit obtained in this way is only valid if one NR coupling dominates the predicted rate and in absence of cancellations among the NR couplings, to assess the validity of such a procedure a specific goal of our analysis is also to discuss the impact of such cancellations in the different relativistic models we consider.

The paper is organized as follows. In Section 2 we list the relativistic Effective Field Theory (EFT) terms that we consider in our analysis and we summarize how we calculate the NR Wilson coefficients starting from each of them; Section 3 is devoted to our quantitative analysis, where we will provide updated exclusion plots for each relativistic model assuming a common coupling $C_{a,q}^{(d)}$ for all quarks; in Section 4 we discuss the impact of interferences among different NR couplings, showing that in most cases only one non–relativistic operator dominates the expected rate and the bounds. We will provide our conclusions in Section 5. In Appendix A we specify the experimental data that we use in our analyses. Finally, in Appendix B we provide a simple interpolation code written in Python that, based on the conclusions of Section 4, allows to reproduce most of the results of Section 3 and to generalize them to other choices of the $C_{a,q}^{(d)}$ couplings assuming that one non–relativistic operator dominates the expected rate.

2 Relativistic effective models

In the present analysis we use the code DirectDM \cite{28,31} to calculate the nonperturbative matching of the effective field theory describing dark matter interactions with quarks and gluons at the EW scale to the effective theory of nonrelativistic dark matter interacting with nonrelativistic nucleons. For this reason we follow closely the notation of Ref.\cite{28} and consider the same relativistic operators.

In particular, we consider the two dimension-five operators:

$$Q_1^{(5)} = \frac{e}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}\chi)F_{\mu\nu}, \quad Q_2^{(5)} = \frac{e}{8\pi^2}(\bar{\chi}\sigma^{\mu\nu}i\gamma_5\chi)F_{\mu\nu}, \quad (2.1)$$

where $F_{\mu\nu}$ is the electromagnetic field strength tensor and $\chi$ is the DM field, assumed here to be a Dirac particle. Such operators correspond, respectively, to magnetic–dipole and electric–dipole DM and imply a long–range interaction \cite{32}. The dimension-six operators

1The anapole coupling $(\bar{\chi}\gamma^\mu\gamma_5\chi)\partial_\mu F_{\mu\nu}$ leads instead to an effective contact interaction. A recent discussion is provided in \cite{33}
are
\[
Q^{(6)}_{1,q} = (\bar{\chi} \gamma_\mu \chi)(\bar{q} \gamma^\mu q), \\
Q^{(6)}_{3,q} = (\bar{\chi} \gamma_\mu \chi)(\bar{q} \gamma^\mu \gamma_5 q), \\
Q^{(6)}_{2,q} = (\bar{\chi} \gamma_\mu \gamma_5 \chi)(\bar{q} \gamma^\mu q), \\
Q^{(6)}_{4,q} = (\bar{\chi} \gamma_\mu \gamma_5 \chi)(\bar{q} \gamma^\mu \gamma_5 q),
\]
(2.2)
and we also include the following dimension-seven operators: namely:
\[
Q^{(7)}_1 = \frac{\alpha_s}{12\pi} (\bar{\chi} \chi) G^{a\mu\nu} G^a_{\mu\nu}, \\
Q^{(7)}_3 = \frac{\alpha_s}{8\pi} (\bar{\chi} \chi) G^{a\mu\nu} \tilde{G}^a_{\mu\nu}, \\
Q^{(7)}_{5,q} = m_q (\bar{\chi} \chi)(\bar{q} q), \\
Q^{(7)}_{7,q} = m_q (\bar{\chi} \chi)(\bar{q} i \gamma_5 q), \\
Q^{(7)}_{9,q} = m_q (\bar{\chi} \sigma^{\mu\nu} \chi)(\bar{q} \sigma_{\mu\nu} q), \\
Q^{(7)}_{10,q} = m_q (\bar{\chi} i \sigma^{\mu\nu} \gamma_5 \chi)(\bar{q} \sigma_{\mu\nu} q).
\]
(2.3)
In the equations above \( q = u, d, s \) denote the light quarks, \( G^a_{\mu\nu} \) is the QCD field strength tensor, while \( \tilde{G}^a_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G^a_{\rho\sigma} \) is its dual, and \( a = 1, \ldots, 8 \) are the adjoint color indices. In the following we will also assume that all the operators listed in Eqs. (2.1)–(2.3) conserve flavor.

A potentially sizeable mixing effect among the vector and axial–vector currents of Eq. (2.2) is known to be induced by the running of the couplings above the EW scale \([24, 25]\). In particular, this may induce a quark vector coupling at the low scale relevant for DD even if the effective theory contains only an axial coupling at the high scale, changing dramatically the DD cross section scaling with the nuclear target and the ensuing DD constrains. For this reason, in the case of the operators of Eq. (2.2) with a vector–axial quark current, besides the results valid for a given effective operator at the EW scale we also show the corresponding ones when the same operator is defined at the scale \( \Lambda = 2 \) TeV, using the code runDM \([34]\) to evaluate the running from \( \Lambda \) to \( m_Z = 91.1875 \) GeV. In order to do so we assume that the axial–vector coupling is the same for all quarks at the high scale \(^2\). We then use the output of runDM as an input for directDM to perform the remaining running from \( m_Z \) to the nucleon scale, where the hadronization of the operators \( Q^{(d)}_{a,q} \) in eqs. (2.1, 2.2, 2.3) leads at leading order in the chiral expansion only to single-nucleon \((N=p,n)\) currents, i.e., schematically:

\[
\langle N | q \Gamma q | N \rangle = \sum_{\Gamma'} \Omega_N^{\Gamma'} \bar{\Psi}_N \Gamma' \Psi_N, \\
\langle N | G^{a\mu\nu} G_{a,\mu\nu} | N \rangle = \Omega_N^{i} \bar{\Psi}_N \Psi_N, \\
\langle N | G^{a\mu\nu} \tilde{G}_{a,\mu\nu} | N \rangle = \Omega_N^{i} \bar{\Psi}_N \tilde{\gamma}_5 \Psi_N.
\]
(2.4)

\(^2\)In particular, we assume the benchmark “QuarksAxial” in \([34]\), with a vanishing DM-Higgs coupling.
with $\Gamma, \Gamma' = 1, \gamma^\mu, \gamma^\mu \gamma_5, \gamma_5, \sigma^{\mu\nu}$ and $\Psi_N$ the nucleon field. Also for the quantities $\Omega, \Omega'$ and $\Omega''$ (which in general can depend on external momenta) we rely on the output of directDM (see appendix A of [28]). In particular, the matching of the axial-axial partonic level operator, as well as that of the coupling between the DM particle to the QCD anomaly term leads to pion and eta poles that can be numerically important, and that we include in our analysis. Specifically [28]:

$$\langle N'\mid q\gamma^\mu\gamma_5q\mid N \rangle = \bar{\Psi}_N \left[ F^{q/N}_{A}(q^2)\gamma^\mu\gamma_5 + \frac{1}{2m_N} F^{q/N}_{P\mu}(q^2)\gamma^\mu \right] \Psi_N, \quad (2.5)$$

$$\langle N'\mid m_q\bar{q}i\gamma_5q\mid N \rangle = F^{q/N}_{P}(q^2) \bar{\Psi}_N i\gamma_5\Psi_N, \quad (2.6)$$

$$\langle N'\mid \frac{\alpha_s}{8\pi} G^{a\mu\nu} \tilde{G}_{\mu\nu}\mid N \rangle = F_N^{G}(q^2) \bar{\Psi}_N i\gamma_5\Psi_N, \quad (2.7)$$

with:

$$F^{q/N}_{P,P'}(q^2) = \frac{m_N^2}{m_\pi^2 - q^2} a^{q/N}_\pi + \frac{m_N^2}{m_\eta^2 - q^2} a^{q/N}_\eta + b^{q/N}, \quad (2.8)$$

$$F^{N}_{G}(q^2) = \frac{q^2}{m_N^2 - q^2} a^{N}_{G,\pi} + \frac{q^2}{m_N^2 - q^2} a^{N}_{G,\eta} + b^{N}_G. \quad (2.9)$$

Finally, taking the non–relativistic limit, we obtain the coefficients $c_\tau$ of the effective Hamiltonian (Eq. 1.1), which turn out to be proportional to the initial relativistic dimensional coupling $C_{a,q}^{(d)}$, and, in general, depend on the WIMP mass $m_\chi$ and on the exchanged momentum $q$ (the latter dependence both through the poles of Eqs.(2.8,2.9) and because of the photon propagator induced by the dim–5 magnetic and electric dipole operators of Eq. (2.1)).

For the details of the expression to calculate the expected rate in a DD experiment we refer, for instance, to Section 2 of [23]. In particular, the differential rate is proportional to the squared amplitude:

$$\frac{d\sigma_T}{dE_{R}} = \frac{2m_T}{4\pi v_T^2} \left[ \frac{1}{2j_T + 1} \right] \left[ \frac{1}{2j_T + 1} \right] |M_T|^2, \quad (2.10)$$

with $v_T \equiv |\vec{v}_T|$ the WIMP speed in the reference frame of the nuclear center of mass, $m_T$ the nuclear mass, $J_T, J_\chi$ are the spins of target nucleus and WIMP, and [22]:

$$\frac{1}{2j_T + 1} |M_T|^2 = \frac{4\pi}{2j_T + 1} \sum_{\tau=0,1} \sum_{\tau'=0,1} \sum_k R^{\tau\tau'}_k \left[ c_j^\tau, (v_T^\tau)^2, \frac{q^2}{m_N^2} \right] W_T^{\tau\tau'}(y). \quad (2.11)$$

In the above expression the $R^{\tau\tau'}_k$'s are WIMP response functions which depend on the couplings $c_j^\tau$ as well as the transferred momentum $\vec{q}$, while:
\[(v_T^\perp)^2 = v_T^2 - v_{\text{min}}^2,\]  
\[v_{\text{min}}^2 = \frac{q^2}{4\mu_T^2} = \frac{m_T E_R}{2\mu_T^2},\]

represents the minimal incoming WIMP speed required to impart the nuclear recoil energy \(E_R\). Moreover, in equation (2.11) the \(W_{T_k}^{\tau\tau'}(y)\)'s are nuclear response functions and the index \(k\) represents different effective nuclear operators, which, under the assumption that the nuclear ground state is an approximate eigenstate of \(P\) and \(CP\), can be at most eight: following the notation in [21, 22], \(k=M, \Phi'', \Phi'M, \tilde{\Phi}', \Sigma', \Sigma''\). The \(W_{T_k}^{\tau\tau'}(y)\)'s are function of \(y \equiv (q b/2)^2\), where \(b\) is the size of the nucleus. For the target nuclei \(T\) used in most direct detection experiments the functions \(W_{T_k}^{\tau\tau'}(y)\), calculated using nuclear shell models, have been provided in Refs. [22, 35]. Details about the definitions of both the functions \(R_{\tau\tau'}^{\tau\tau'}\)'s and \(W_{T_k}^{\tau\tau'}(y)\)'s can be found in [22]. In particular, using the decomposition:

\[R_k^{\tau\tau'} = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \left(\frac{(v_T^\perp)^2}{c^2}\right) = R_{0k}^{\tau\tau'} + R_{1k}^{\tau\tau'} \left(\frac{v_T^2 - v_{\text{min}}^2}{c^2}\right),\]

the correspondence between each term of the NR effective interaction in (1.1) and the \(W_{T_k}^{\tau\tau'}(y)\) nuclear response functions is summarized in Table 1. Notice that \(W_M\) corresponds to the standard SI interaction, while \(W_{\Sigma''} + W_{\Sigma'}\) (with \(W_{\Sigma'} \simeq 2W_{\Sigma''}\)) to the standard SD one.

Finally, for the WIMP local density we take \(\rho_{\text{loc}}=0.3\) GeV/cm\(^3\) and for the velocity distribution we assume a standard isotropic Maxwellian at rest in the Galactic rest frame boosted to the Lab frame by the velocity of the Sun, \(v_\odot=232\) km/s, with root-mean-square velocity \(v_{\text{rms}}=270\) km/s and truncated at the escape velocity \(u_{\text{esc}}=550\) km/s.

| coupling | \(R_{0k}^{\tau\tau'}\) | \(R_{1k}^{\tau\tau'}\) | coupling | \(R_{0k}^{\tau\tau'}\) | \(R_{1k}^{\tau\tau'}\) |
|---------|-----------------|-----------------|---------|-----------------|-----------------|
| 1       | \(M(q^0)\)      |                  | 3       | \(\Phi''(q^1)\) | \(\Sigma'(q^2)\) |
| 4       | \(\Sigma''(q^0), \Sigma'(q^0)\) |                  | 5       | \(\Delta(q^1)\) | \(M(q^2)\) |
| 6       | \(\Sigma''(q^0)\) |                  | 7       |                   | \(\Sigma'(q^0)\) |
| 8       | \(\Delta(q^2)\) | \(M(q^0)\)      | 9       | \(\Sigma'(q^2)\) |                  |
| 10      | \(\Sigma''(q^0)\) |                  | 11      | \(M(q^2)\)      |                  |
| 12      | \(\Phi''(q^2), \tilde{\Phi}'(q^2)\) | \(\Sigma''(q^0), \Sigma'(q^0)\) | 13      | \(\tilde{\Phi}'(q^1)\) | \(\Sigma''(q^2)\) |
| 14      | \(\Sigma'(q^2)\) |               | 15      | \(\Phi''(q^0)\)  | \(\Sigma'(q^4)\) |

Table 1. Nuclear response functions corresponding to each coupling, for the velocity–independent and the velocity–dependent components parts of the WIMP response function, decomposed as in Eq.(2.14). In parenthesis is the power of \(q\) in the WIMP response function.
3 Analysis

In this Section for each of the models \( Q_{a.q}^{(d)} \), \( Q_{b}^{(d)} \) listed in Eqs. (2.1–2.3) we show the present constraints on the correspondent dimensional coupling \( C_{a.q}^{(d)} \) (assumed to be the same for all flavors) and \( C_{b}^{(d)} \) from an extensive list of experiments (see appendix A) in terms of lower bounds on the effective scale \( \tilde{\Lambda} \) defined through:

\[
C_{a.q}^{(d)}, C_{b}^{(d)} = \frac{1}{\Lambda^{d-4}}. \tag{3.1}
\]

As a default choice in all cases we fix \( C_{a.q}^{(d)} \), \( C_{b}^{(d)} \) at the EW scale, identified as the Z–boson mass, \( \mu_{\text{scale}} = m_Z \). Only for the 6–dimensional interaction terms \( Q_{3,q}^{(6)} \) and \( Q_{4,q}^{(6)} \) we also show in Fig. 4 the result obtained when the \( C_{a.q}^{(d)} \) coupling is fixed at the scale \( \mu_{\text{scale}} = 2 \text{ TeV} \) and run down to the EW scale using runDM and assuming that the axial–vector coupling is the same for all quarks at the high scale. One can notice that in the case of \( Q_{3,q}^{(6)} \) passing from \( \mu_{\text{scale}} = m_Z \) to \( \mu_{\text{scale}} = 2 \text{ TeV} \) the experimental bound is strengthened by more than two orders of magnitude. This is a well–known effect\[24, 25\] due to the mixing between \( Q_{3,q}^{(6)} \) and \( Q_{1,q}^{(6)} \) induced by the running from 2 TeV to \( m_Z \). In particular, without such mixing \( Q_{3,q}^{(6)} \) gives rise to two NR operators that have both a spin–dependent type scaling with the target, \( O_9 \) and \( O_7 \), the latter also velocity suppressed \[23\], while the mixing due to running induces a \( Q_{1,q}^{(6)} \) component leading to the SI \( O_1 \) operator (see Tables 1 and 2) that overwhelms the other contributions in spite of the loop–suppressed Wilson coefficient. Such effect is also present for the \( Q_{4,q}^{(6)} \) operator due to the mixing with \( Q_{2,q}^{(6)} \), although in this case the effect on the exclusion plot is less sizable. It is worth pointing out here that the mixing between vector and vector–axial currents is driven by the coupling between the DM particle and the quarks of the third family, so it is not present if the latter is assumed to vanish. In such case the results of Fig.4 would coincide to those of Fig. 3.

In all the plots the data are analyzed in the same way of the experimental collaborations. This implies that, for instance, in the case of the constraints from DarkSide–50 \[18\] and KIMS \[8\] we subtract the estimated background \( \lambda b_i \) and the expected signal \( \sigma S_i \) (with \( i \) the energy bin) by fitting the data to the sum \( \sigma S_i + \lambda b_i \) in terms of the two free parameters \( \sigma \) and \( \lambda \) \[23\]. This procedure is particularly effective when the spectral shapes of the signal and of the background are different, and since both for DarkSide–50 and KIMS the estimated spectrum of the background is rising with the recoil energy, it yields a weaker constraint for interactions types with an explicit momentum dependence that lead to a signal rising with energy in a way similar to the background. This loss of constraining power is the reason of the peculiar shapes of some of the exclusion plots for DarkSide–50 and KIMS in Figs. 1–9.

The exclusion plots of Figs. 1–9 can be roughly devided in two classes: in the case of models \( Q_{1,q}^{(5)} \), \( Q_{2,q}^{(5)} \), \( Q_{1,q}^{(6)} \), \( Q_{2,q}^{(6)} \), \( Q_{1}^{(7)} \), \( Q_{2}^{(7)} \), \( Q_{5,q}^{(7)} \), \( Q_{6,q}^{(7)} \) and \( Q_{10,q}^{(7)} \) the most constraining...
Figure 1. Lower bound on the effective scale $\Lambda$ defined in Eq. (3.1) for the operators $Q_{1,q}^{(5)}$ (left) and $Q_{2,q}^{(5)}$ (right). In both cases the dimensional couplings $C_{1,q}^{(5)}$ and $C_{2,q}^{(5)}$ are fixed at the EW scale $\mu_{\text{scale}}=m_Z$.

Figure 2. The same as in Fig. 1 for $Q_{1,q}^{(6)}$ (left) and $Q_{2,q}^{(6)}$ (right).

experiments are DarkSide–50 at low WIMP mass and XENON1T at larger $m_\chi$. As can be seen by combining Table 2 (that allows to see the correspondence between each $Q_{a,q}^{(d)}$ term and NR operators) and Table 1 (where the correspondence between each NR operator $O_i$ and the nuclear response functions $W_{TM}^{\tau\tau'}(y)$’s is shown) one can see that all such interactions take contributions from the $W_M$ nuclear response function, leading to a SI scaling of the cross section (possibly combined with explicit dependence from the exchanged momentum $q^2$ and from the WIMP incoming speed $v_3$). Indeed, due to its very low threshold DarkSide–50

\[ Q_{1,q}^{(5)} = \frac{s}{4m^2} (q^2 \gamma^5 \gamma^5 q) \]

\[ Q_{2,q}^{(5)} = \frac{s}{4m^2} (q^2 \gamma^5 \gamma^5 q) \]

\[ \mu_{\text{scale}} = m_Z \]
drives the exclusion plot at low mass, but only for interactions that do not require a nuclear spin (its target is $^{40}$Ar), while at larger masses the SI coupling enhances the sensitivity for scatterings off xenon in XENON1T. The second class of exclusion plots is represented by the models $Q_{3,q}^{(6)}$, $Q_{4,q}^{(6)}$, $Q_{3}^{(7)}$, $Q_{3,q}^{(7)}$, $Q_{8,q}^{(7)}$, and $Q_{9,q}^{(7)}$, for which the exclusion plot is driven by PICASSO and PICO60 (and, sometimes, by CDMSLite) at low WIMP mass, and by XENON1T at larger WIMP masses. In such cases, as can again be seen from Tables 2 and 1, the operator $M$ is always missing in the NR limit, while the response functions $\Sigma'$ and/or $\Sigma''$ are always present, leading to a SD–type scaling of the cross section for which large detectors containing fluorine are competitive with xenon.

Figure 4. The same as in Fig. 3 for $\mu_{\text{scale}}=2$ TeV.

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usually dominates in spite of the fact that it is velocity suppressed [23].
In Appendix B we introduce NRDD\_constraints, a simple interpolating code written in Python that can reproduce most of the results of this Section by assuming that one NR coupling dominates in the low–energy limit of the interactions of Eqs.\((2.1)–(2.3)\). In Figs. 1–3 and Figs. 5–9 the output of such code is indicated by “max NR op.” and represented by the red–dashed curve.

4 Interference and momentum effects in the NR theory

No matter which among the relativistic interactions listed in Eqs.\((2.1),2.2,2.3\) is generated at a higher scale by some beyond–the–standard–model scenario, Dark Matter DD scattering is a low–energy process completely described by the NR effective theory of (Eq. 1.1).
implies that the limits discussed in the previous Section can be expressed in terms of NR operators only. In particular, in the case of interactions between the DM particle and the quark current, this would have the advantage to present the limits from existing experiments in a way independent from the choice of the $C_{a,q}^{(d)}$ couplings for each flavor $q$, since the NR effective theory depends only on WIMP mass and on the ratio between the WIMP–neutron and the WIMP–proton couplings $r \equiv c^n / c^p$. Indeed, in ref. [23] we obtained updated upper bounds on the effective cross section:

$$
\sigma_i^N = \max(\sigma_i^p, \sigma_i^n),
$$

with:

\begin{figure}
\begin{center}
\includegraphics[width=0.4\textwidth]{fig7a.png}
\includegraphics[width=0.4\textwidth]{fig7b.png}
\end{center}
\caption{The same as in Fig. 2 for $Q_5^{(7)}$ (left) and $Q_6^{(7)}$ (right).}
\end{figure}

\begin{figure}
\begin{center}
\includegraphics[width=0.4\textwidth]{fig8a.png}
\includegraphics[width=0.4\textwidth]{fig8b.png}
\end{center}
\caption{The same as in Fig. 2 for $Q_7^{(7)}$ (left) and $Q_8^{(7)}$ (right).}
\end{figure}
Figure 9. The same as in Fig. 2 for $Q_{9}^{(7)}$ (left) and $Q_{10}^{(7)}$ (right).

$$\sigma_{i}^{p,n} = (\tilde{c}_{i}^{p,n})^{2} \mu_{\chi N}^{2} \pi,$$

(with $\mu_{\chi N}$ the WIMP–nucleon reduced mass) systematically assuming dominance of one of the 14 possible NR interaction terms $O_i$ of Eq. (1.1), providing for each of them a two–dimensional plot where the contours of the most stringent 90% upper bounds to $\sigma_i^N$ where shown as a function of the two parameters $m_{\chi}$ (WIMP mass), and $c^\alpha/c^\beta$. One possible drawback of this approach is however that, in general, a given relativistic coupling leads to more than one NR operator. In addition to that, as explained in Section 2, the NR coefficients $c_i^\tau$ may depend explicitly on the exchanged momentum, leading, in practice, to contributions which are equivalent to including additional NR operators of the type $F_\alpha(q^2)O_i$ (where, for each operator $O_i$ different momentum dependences are possible, as for instance in Eqs.(2.8, 2.9)). In fact, setting:

$$c_i^\tau(m_{\chi}, q^2) \equiv \tilde{c}_i^{\tau,\alpha}(m_{\chi}) F_\alpha(q^2),$$

$$R_{\tau\tau'}^{ij} \equiv c_i^\tau c_j^\tau' \tilde{R}_{\tau\tau'}^{ij} = \tilde{c}_i^{\tau,\alpha} \tilde{c}_j^{\tau',\beta} \tilde{R}_{\tau\tau'}^{\alpha \beta} F_\alpha(q^2) F_\beta(q^2),$$

the squared amplitude (2.11) can be rewritten as:

$$\frac{1}{2j_{\chi} + 1} \frac{1}{2j_T + 1} |M|^2 = \frac{4\pi}{2j_T + 1} \sum_{ij} \sum_{\alpha\beta} \tilde{c}_i^{\tau,\alpha} \tilde{c}_j^{\tau',\beta} \left[ \sum_k \sum_{\tau,\tau'} \tilde{R}_{\tau\tau'}^{\alpha \beta} W_{\tau\tau'}(q^2) \right] F_\alpha(q^2) F_\beta(q^2),$$

so that the expected rate $R$ can be expressed as a sum over all possible interferences among the contributions from each generalized NR term $F_\alpha^{\alpha}O_i$: 

---

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Figure 10. Parameter $\epsilon$ defined in Eq. (4.6) as a function of the WIMP mass $m_\chi$ for all the experiments, energy bins and operators considered in Section 3, with the exception of models $Q_{7,q}^{(7)}$ and $Q_{8,q}^{(7)}$. A value of $\epsilon$ close to one signals that a single NR operator dominates the WIMP–nucleus scattering rate. Conversely, assuming dominance of a single NR operator leads to an inaccuracy within $\epsilon^{-1}$.

$$R = \sum_{ij} \sum_{\alpha \beta} \hat{c}_{i,\alpha}^\tau \hat{c}_{j,\beta}^{\tau'} \langle O_i^\tau O_j^{\tau'} F_\alpha^\tau (q^2) F_\beta^{\tau'} (q^2) \rangle.$$ (4.5)

The terms $\langle O_i^\tau O_j^{\tau'} F_\alpha^\tau (q^2) F_\beta^{\tau'} (q^2) \rangle$ contributing to each of the interactions discussed in Section 3 are listed in Table 2.

To discuss whether it is correct to assume dominance of one effective operator $F_i^\alpha (q^2) O_i$ at a time, in Figs. 10, 11 and 12 we introduce the parameters:

$$\epsilon_{ij}^{\alpha \beta} = \frac{\hat{c}_{i,\alpha}^\tau \hat{c}_{j,\beta}^{\tau'} \langle O_i^\tau O_j^{\tau'} F_\alpha^\tau (q^2) F_\beta^{\tau'} (q^2) \rangle}{\sum_{lm} \sum_{\rho \sigma} \hat{c}_{l,\rho}^\tau \hat{c}_{m,\sigma}^{\tau'} \langle O_l^\tau O_m^{\tau'} F_\rho^\tau (q^2) F_\sigma^{\tau'} (q^2) \rangle}, \quad \epsilon \equiv \max(|\epsilon_{ij}^{\alpha \beta}|),$$ (4.6)
Table 2. Non–relativistic interaction terms contributing to the direct detection expected rate for each of the interactions listed in Eqs. (2.1–2.3).

and plot $\epsilon$ as a function of the WIMP mass $m_\chi$ for all the experiments and operators considered in Section 3. In particular, in Fig. 10 we do not include the two operators $Q_{7,q}^{(7)}$ and $Q_{8,q}^{(7)}$, which are shown separately in Figs. 11 and 12. Moreover, in the plots of the $\epsilon$
For model $Q_{7,q}^{(7)}$, one can see that, with the exception of the operators $\epsilon^0_2$, the $\epsilon$ parameter never departs from unity by more than $\simeq 50\%$ with the exception of the largest values $\simeq 1.7$, which correspond to the highest energy bins of DAMA0 where the rate is exponentially suppressed by the velocity distribution and irrelevant for the

**Figure 11.** The same as in Fig. 10 for model $Q_{7,q}^{(7)}$ (left) Parameter $\epsilon$ as a function of the WIMP mass $m_\chi$ for all the experiments and energy bins considered in Section 3. (right) Contributions $\epsilon^\alpha_\beta$ (each arising from one of the terms listed in Table 2) for the specific example of the XENON1T experiment.

![Diagram](image1)

**Figure 12.** The same as in Fig. 11 for model $Q_{8,q}^{(7)}$.
constraint. This shows that assuming dominance of one of the combinations $F_{i}^{\alpha}(q)\mathcal{O}_{i}$ in the calculation of the expected rate in the determination of the exclusion plot implies an inaccuracy within $\pm \approx 40-50\%$, but in most cases much smaller. In particular, as far as $\mathcal{Q}_{3}^{(7)}$ and $\mathcal{Q}_{4}^{(7)}$ are concerned, the dominant contribution corresponds to the constant term in Eq. (2.9). As shown in Figs. 11 and 12 the situation is different for $\mathcal{Q}_{7}^{(7)}$ and $\mathcal{Q}_{8}^{(7)}$, where each of the terms $\mathcal{O}_{n_{1},n_{2}}\frac{m_{Z}^{2}}{m_{Z}^{2}-q^{2}}\mathcal{O}_{n_{3},n_{4}}\frac{m_{Z}^{2}}{m_{Z}^{2}-q^{2}}$ (with $n=10,6$) is of the same order with large cancellations among them (as indicated by $\epsilon_{ij} \gg 1$ values in the left–hand plot). This is confirmed by the right–hand plot of each of the two figures, where we show explicitly the $\epsilon_{ij}^{\alpha\beta}$ contributions for the specific example of the XENON1T experiment. Indeed for the interaction terms $\mathcal{Q}_{7}^{(7)}$ and $\mathcal{Q}_{8}^{(7)}$ such effect is natural since, as explained in [28], in this particular case the momentum–independent term $\mathcal{O}_{n_{1},n_{2}}\frac{m_{Z}^{2}}{m_{Z}^{2}-q^{2}}$ and $\mathcal{O}_{n_{3},n_{4}}\frac{m_{Z}^{2}}{m_{Z}^{2}-q^{2}}$. 

Indeed, the result of Fig. 10 is not unexpected, since the scaling of the rate for different NR operators is very different, and depends also on experimental inputs, so barring accidental cancellations or clear–cut situations, like the one of axial operators in the $\mathcal{Q}_{7,q}^{(7)}$ and $\mathcal{Q}_{8,q}^{(7)}$, dominance of one NR operator appears natural. The numerical tests in this Section confirm this. In Appendix B we introduce NRDD_constraints, a simple code that exploits this feature to calculate approximate bounds on the couplings $C_{\alpha,q}^{(d)}$ and $C_{b}^{(d)}$. In particular, while the results of Section 3 have been obtained by assuming a single coupling $C_{\alpha,q}^{(d)}$ common to all quarks, using NRDD_constraints such constraints can be generalized to a generic dependence of such couplings on the flavor $q$.

5 Conclusions

Assuming for Weakly Interacting Massive Particles (WIMPs) a Maxwellian velocity distribution in the Galaxy we have explored in a systematic way the relative sensitivity of an extensive set of existing Dark Matter (DM) direct detection (DD) experiments to each of the operators $\mathcal{Q}_{a,q}^{(d)}$, $\mathcal{Q}_{b}^{(d)}$ listed in Eqs. (2.1–2.3) up to dimension 7 describing dark matter effective interactions with quarks and gluons. For all the operators we have fixed the corresponding dimensional coupling $C_{\alpha,q}^{(d)}$ at the scale $\mu_{\text{scale}}=m_{Z}$ and used the code DirectDM [31] to perform the running from $m_{Z}$ to the nucleon scale and the hadronization to single-nucleon (N=p,n) currents, including QCD effects and pion poles that arise in the nonperturbative matching of the effective field theory to the low–energy Galilean–invariant nonrelativistic effective theory describing DM–nucleon interactions. For operators $\mathcal{Q}_{3,q}^{(6)}$ and $\mathcal{Q}_{4,q}^{(6)}$ we have also used the runDM code [34] to discuss the mixing effect among the vector and axial–vector currents induced by the running of the couplings above the EW scale, when the DM vector–axial coupling is assumed to be the same to all quarks.
we provide a simple interpolating interface in Python. On the pion and eta propagators $Q$ are next-to-leading order compared to the terms $WIMP$ mass, and by XENON1T at larger $WIMP$ mass. On the other hand for models $Q_{3,q}$, $Q_{4,q}$, $Q_{5,q}$, $Q_{7,q}$, $Q_{8,q}$, and $Q_{9,q}$ the cross section scaling law is of the Spin–Dependent type, leading to an exclusion plot driven by PICASSO and PICO60 (and, sometimes, by CDMSLite) at low $WIMP$ mass, and by XENON1T at larger $WIMP$ masses.

The matching between the relativistic effective theory to the NR one implies a redundancy of the parameters $C_{a,q}^{(d)}$, implying that in many cases the DD constraints, that only depend on the ratio $c_i^1/c_i^0$ between the $WIMP$–neutron and the $WIMP$–proton couplings, can only be discussed for specific benchmarks. In particular in our exclusion plots we have assumed a flavor–independent coupling, $C_{a,q}^{(d)}=C_{d}^{(d)}$. However, we have shown how, once the $WIMP$ mass $m_X$ and the $c_i^1/c_i^0$ ratio are fixed, for all the $Q_{a,q}^{(d)}$ models with the exception of $Q_{7,q}^{(7)}$ and $Q_{8,q}^{(7)}$ the expected rate is naturally driven by a dominant contribution from one of the NR operators $O_i$ (possibly modified by a momentum–dependent Wilson coefficient, $O_i \rightarrow O_i F_i^{\alpha}(q^2)$) without large cancellations. This implies that the bounds directly obtained within the context of the NR theory by assuming dominance of one of the $F_i^{\alpha}(q^2)O_i$ can be used as discussed in Ref. [23] to obtain approximate constraints valid for any choice of the $Q_{a,q}^{(d)}$ parameters, with an inaccuracy that we estimate within 60%. To perform such task in Appendix B we provide a simple interpolating interface in Python. On the other hand, in the case of $Q_{7,q}^{(7)}$ and $Q_{8,q}^{(7)}$ the momentum–independent terms $O_{10}$ and $O_6$ are next-to-leading order compared to the terms $F_i(q^2)O_{10}$ and $F_i(q^2)O_6$ which depend on the pion and eta propagators $F_i(q^2) = 1/(m_i^2 - q^2)$, $1/(m_j^2 - q^2)$, so that they cannot be assumed to dominate. Indeed, in this case all the terms $F_i(q^2)O_n$ are naturally of the same order with large cancellations among them.

A Experiments

In our analysis we have included an extensive list of updated constraints from existing DM direct-search experiments: CDEX [16], CDMSLite [9], COUPP [11], CRESST-II [15, 36], DAMIC [17], DAMA (modulation data [2–5] and average count rate [37], indicated as DAMA0 in the plots), DarkSide–50 [18] (indicated as DS50 in the plots), KIMS [8], PANDAX-II [7], PICASSO [12], PICO-60 (using a CF$_3$I target [13] and a CF$_3$F$_8$ target [14]), SuperCDMS [10] and XENON1T [6]. With the exception of the latest result from XENON1T [38], the details of the treatment of the other constraints are provided in the Appendix of [23]. For XENON1T (2018 analysis), we have assumed 7 $WIMP$ candidate events in the range of $3PE \leq S_1 \leq 70PE$, as shown in Fig. 3 of Ref. [38] for the primary scintillation signal $S_1$ (directly in Photo Electrons, PE), with an exposure of 278.8 days and a fiducial volume of 1.3 ton of xenon. We have used the efficiency taken from
Fig. 1 of [38] and employed a light collection efficiency $g_1=0.055$; for the light yield $L_y$ we have extracted the best estimation curve for photon yields $⟨n_{ph}⟩/E$ from Fig. 7 in [39] with an electric field of 90 V/cm.

B  The program

The NRDD_constraints code provides a simple interpolating function written in Python that for a given generalized NR diagonal term $(F_1^α)^2O_iO_i$ among those listed in Table 2 (with the exception of those proportional to a meson pole) calculates the most constraining limit on the effective cross section:

$$\sigma_{i,α}^N = \max(\sigma_{i,α}^p, \sigma_{i,α}^n),$$

(B.1)

(for $F_1^α=1,(q^2)^{-1}$) with:

$$\sigma_{i,α}^{p,n} = (c_{i,α}^{p,n})^2 \frac{\mu^2}{\chi N},$$

(B.2)

as a function of the WIMP mass $m_χ$ and of the ratio $r_i = c_{i,α}^n/c_{i,α}^p$. The $c_{i,α}^{p,n}$ coefficients are defined in Eq. (4.5). The code requires the NumPy package and contains only three files, the code NRDD_constraints.py, a data file NRDD_data.npy and a driver template NRDD_constraints-example.py. The module can be downloaded from https://github.com/NRDD-constraints/NRDD or cloned by

```
git clone https://github.com/NRDD-constraints/NRDD
```

By typing:

```
import NRDD_constraints as NR
```

two functions are defined. The function `sigma_nucleon_bound(inter,mchi,r)` returns the upper bound $(\sigma_{i,α}^N)_{lim}$ on the effective cross section of Eq.(B.1) in cm$^2$ as a function of the WIMP mass $m_χ$ and of the ratio $r=r$ in the ranges $0.1 \text{ GeV} < m_χ < 100 \text{ GeV}$, $-10^4 < r < 10^4$. The `inter` parameter is a string that selects the interaction term and can be chosen in the list provided by the second function `print_interactions()`:

```
NR.print_interactions()
['01_01', '03_03', '04_04', '05_05', '06_06',
 '07_07', '08_08', '09_09', '010_010', '011_011',
 '012_012', '013_013', '014_014', '015_015'
 '05_05_qm4', '06_06_qm4', '011_011_qm4']
```
The list above includes all the $\mathcal{O}_i \mathcal{O}_j F_i^2(q^2)$ terms in Table 2 with the exception of those depending on pion and eta poles which, as explained in Section 4, are either subdominant in the case of models $Q_3^{(7)}$ and $Q_4^{(7)}$, or, in the case of $Q_7^{(7)}$ and $Q_8^{(7)}$, imply absence of a dominant term altogether (see Figs. 11 and 12) so that the procedure described in this Section leads to an inaccurate estimation of the constraints. This can be seen explicitly in Figs. 8, where the red–dashed curve approximates poorly the bound obtained with a full calculation. The upper bound returned by $\text{sigma_nucleon_bound}(\text{inter,mchi,r})$ corresponds to the results of [23] (updated to the latest XENON1T bound [38]) with the exception of the interaction terms with momentum dependence in the Wilson coefficient, that have been added to include the long–range interactions of Eq. (2.1).

The driver $\text{NRDD\_constraints\_example.py}$ calculates the exclusion plot on the reference cross section:

$$\sigma_{\text{ref}}^{\text{rel}} = C^2 \frac{\mu_{\chi N}^2}{\pi},$$

assuming for the interaction between the DM particle $\chi$ and the nucleon $\Psi_N$ the relativistic effective Lagrangian:

$$\mathcal{L} = C \bar{\chi} \gamma^\mu \chi \Psi_N \gamma^\rho \gamma_5 \Psi_N \rightarrow C \sum_{N=p,n} \left( c_7^N \mathcal{O}_7^N + c_9^N \mathcal{O}_9^N \right),$$

with $c_7^p = -2$, $c_9^p = 2m_N/m_\chi$ and $r_i = c_i^p/c_i^n$ two free parameters. Assuming dominance of one operator $\mathcal{O}_i$ at a time the driver implements a function $\text{coeff}(\text{inter,mchi,r})$ that calculates the largest Wilson coefficient between proton and neutron in absolute value $\max(|c_i^p(m_\chi,r)|, |c_i^n(m_\chi,r)|)$ and plots the bound $(\sigma_{\text{ref}}^{\text{rel}})_{\text{lim}}$ on $\sigma_{\text{ref}}^{\text{rel}}$ as [23]:

$$(\sigma_{\text{ref}}^{\text{rel}})_{\text{lim}} = \min_{i=7,9} \left( \frac{(\sigma_{\text{ref}}^{\text{rel}})_{\text{lim}}(m_\chi, r_i)}{\max(c_i^p(m_\chi, r_i)^2, c_i^n(m_\chi, r_i)^2)} \right),$$

for the specific choice $r_7 = r_9 = 1$. In particular, for a given value of the WIMP mass $m_\chi$:

```python
sigma_lim_rel_min=large_number
for inter in ['07_07', '09_09']:
    c=coeff(inter,mchi,r)
    sigma_lim_nucleon_NR=NR.sigma_nucleon_bound(inter,mchi,r)
    sigma_lim_rel=sigma_lim_nucleon_NR/c**2
    sigma_lim_rel_min=min(sigma_lim_rel_min,sigma_lim_rel)
```

Such limit is also converted into an upper bound $\hat{\Lambda}_{\text{lim}}$ in GeV on $\hat{\Lambda}$ using:

$$(\hat{\Lambda})_{\text{lim}} = \left( \frac{\mu_{\chi N}(\hbar c)}{\sqrt{(\sigma_{\text{ref}}^{\text{rel}})_{\text{lim}}/\pi}} \right)^{\frac{1}{4}},$$
with $\hbar c = 1.97 \times 10^{-14}$ cm GeV and $d = 6$ the dimension of the considered operator. An analogous procedure allows to obtain the red–dashed curve in Figs. 1–3 and Figs. 5–9 that reproduce accurately most of the results of Section 3.

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