EXPERIMENTAL DETECTION OF INTERACTIVE
PHENOMENA AND THEIR ANALYSIS

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The article is devoted to mathematical methods of experimental detection of interactive phenomena in complex systems and their analysis.

This article may be regarded as an account of conclusions from the ten year author’s practical researches on the joint of experimental mathematics, experimental psychophysics and computer science in the problems of visual perception in the interactive videosystems and from his parallel theoretical studies on interactivity and tactics. The current exposition is self-consistent and claims a minimal knowledge of the experimental or theoretical basis, which underlies it. The goal of the article is to discuss the methods of experimental detection of interactive phenomena and of their analysis. Though the author had deal presumably with the interactive phenomena in perception the resulted scheme is applicable to many systems of natural, behavioral, social and economical sciences and it is reasonable to think that many concrete specialists will find it useful for their own needs in the nearest future.

I. EXPERIMENTAL DETECTION OF INTERACTIVE PHENOMENA

Let us consider a natural, behavioral, social or economical system $S$. It will be described by a set $\{\varphi\}$ of quantities, which characterize it at any moment of time $t$ (so that $\varphi = \varphi_t$). One may suppose that the evolution of the system is described by a differential equation

$$\dot{\varphi} = \Phi(\varphi)$$

and look for the explicit form of the function $\Phi$ from the experimental data on the system $S$. However, the function $\Phi$ may depend on time, it means that there are some hidden parameters, which control the system $S$ and its evolution is of the form

$$\dot{\varphi} = \Phi(\varphi, u),$$

where $u$ are such parameters of unknown nature. One may suspect that such parameters are chosen in a way to minimize some goal function $K$, which may be an integrodifferential functional of $\varphi_t$:

$$K = K([\varphi_\tau]_{\tau \leq t})$$
(such integrodifferential dependence will be briefly notated as $K = K([\varphi])$ below). More generally, the parameters $u$ may be divided on parts $u = (u_1, \ldots, u_n)$ and each part $u_i$ has its own goal function $K_i$. However, this hypothesis may be confirmed by the experiment very rarely. In the most cases the choice of parameters $u$ will seem accidental or even random. Nevertheless, one may suspect that the controls $u_i$ are interactive, it means that they are the couplings of the pure controls $u^o_i$ with the unknown or incompletely known feedbacks:

$$u_i = u_i(u^o_i, [\varphi])$$

and each pure control has its own goal function $K_i$. Thus, it is suspected that the system $S$ realizes an interactive game. There are several ways to define the pure controls $u^o_i$. One of them is the integrodifferential filtration of the controls $u_i$:

$$u^o_i = F_i([u_i], [\varphi]).$$

To verify the formulated hypothesis and to find the explicit form of the convenient filtrations $F_i$ and goal functions $K_i$ one should use the theory of interactive games, which supplies us by the predictions of the game, and compare the predictions with the real history of the game for any considered $F_i$ and $K_i$ and choose such filtrations and goal functions, which describe the reality better. One may suspect that the dependence of $u_i$ on $\varphi$ is purely differential for simplicity or to introduce the so-called intention fields, which allow to consider any interactive game as differential. Moreover, one may suppose that

$$u_i = u_i(u^o_i, \varphi)$$

and apply the elaborated procedures of a posteriori analysis and predictions to the system.

In many cases this simple algorithm effectively unravels the hidden interactivity of a complex system.

II. Analysis of interactive phenomena

Below we shall consider the complex systems $S$, which have been yet represented as the $n$-person interactive games by the procedure described above.

2.1. Functional analysis of interactive phenomena. To perform an analysis of the interactive control let us note that often for the $n$-person interactive game the interactive controls $u_i = u_i(u^o_i, [\varphi])$ may be represented in the form

$$u_i = u_i(u^o_i, [\varphi]; \varepsilon_i),$$

where the dependence of the interactive controls on the arguments $u^o_i$, $[\varphi]$ and $\varepsilon_i$ is known but the $\varepsilon$-parameters $\varepsilon_i$ are the unknown or incompletely known functions of $u^o_i$, $[\varepsilon]$. Such representation is very useful in the theory of interactive games and is called the $\varepsilon$-representation.

One may regard $\varepsilon$-parameters as new magnitudes, which characterize the system, and apply the algorithm of the unraveling of interactivity to them. Note that $\varepsilon$-parameters are of an existential nature depending as on the states $\varphi$ of the system $S$ as on the controls.
The $\varepsilon$-parameters are useful for the functional analysis of the interactive controls described below.

First of all, let us consider new integrodifferential filtrations $V_\alpha$:

$$v_\alpha^0 = V_\alpha([\varepsilon],[\varphi]),$$

where $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$. Second, we shall suppose that the $\varepsilon$-parameters are expressed via the new controls $v_\alpha^0$, which will be called \textit{desires}:

$$\varepsilon_i = \varepsilon(v_1^0, \ldots, v_m^0, [\varphi])$$

and the least have the goal functions $L_\alpha$. The procedure of unraveling of interactivity specifies as the filtrations $V_\alpha$ as the goal functions $L_\alpha$.

\textit{Example.} Let us considered the interactive videosystem directed by the eye movements of an observer. The pure controls are the slow movements of eyes, whereas saccads are considered as a result of the unknown feedbacks (tremor is supposed to be random). Many classical and modern experiments clarifies the role of saccads in the formation of the stable and complete final image so such formation may be regarded as their goal function. The functional analysis of the eye movements extracts the parameters (the normal forms), which describe saccads in the concrete interactive videosystems. The normal forms are extremely interesting in the multi-user mode when the saccads of various observers begin to be correlated and synchronized.

2.2. The second quantization of desires. Intuitively it is reasonable to consider systems with a variable number of desires. It can be done via the second quantization.

To perform the second quantization of desires let us mention that they are defined as the integrodifferential functionals of $\varphi$ and $\varepsilon$ via the integrodifferential filtrations. So one is able to define the linear space $H$ of all filtrations (regarded as classical fields) and a submanifold $M$ of the dual $H^*$ so that $H$ is naturally identified with a subspace of the linear space $O(M)$ of smooth functions on $M$. The quantized fields of desires are certain operators in the space $O(M)$ (one is able to regard them as unbounded operators in its certain Hilbert completion). The creation/annihilation operators are constructed from the operators of multiplication on an element of $H \subset O(M)$ and their conjugates.

To define the quantum dynamics one should separate the quick and slow time. Quick time is used to make a filtration and the dynamics is realized in slow time. Such dynamics may have a Hamiltonian form being governed by a quantum Hamiltonian, which is usually differential operator in $O(M)$.

If $M$ coincides with the whole $H^*$ then the quadratic part of a Hamiltonian describes a propagator of the quantum desire whereas the highest terms correspond to the vertex structure of self-interaction of the quantum field. If the submanifold $M$ is nonlinear the extraction of propagators and interaction vertices is not straightforward.

2.3. SD-transform and SD-pairs. The interesting feature of the proposed description (which will be called the \textit{S-picture}) of an interactive system $S$ is that it contains as the real (usually personal) subjects with the pure controls $u_i$ as the impersonal desires $v_\alpha$. The least are interpreted as certain perturbations of the
first so the subjects act in the system by the interactive controls \(u_i\) whereas the desires are hidden in their actions.

One is able to construct the dual picture (the \(D\)-picture), where the desires act in the system \(S\) interactively and the pure controls of the real subjects are hidden in their actions. Precisely, the evolution of the system is governed by the equations

\[
\dot{\varphi} = \tilde{\Phi}(\varphi, v),
\]

where \(v = (v_1, \ldots, v_m)\) are the \(\varepsilon\)-represented interactive desires:

\[
v_\alpha = v_\alpha(v_\alpha^o, [\varphi]; \tilde{\varepsilon}_\alpha)
\]

and the \(\varepsilon\)-parameters \(\tilde{\varepsilon}\) are the unknown or incompletely known functions of the states \([\varphi]\) and the pure controls \(u_\alpha^o\).

\(D\)-picture is convenient for a description of systems \(S\) with a variable number of acting persons. Addition of a new person does not make any influence on the evolution equations, a subsidiary term to the \(\varepsilon\)-parameters should be added only.

The transition from the \(S\)-picture to the \(D\)-picture is called the \(SD\)-transform. The \(SD\)-pair is defined by the evolution equations in the system \(S\) of the form

\[
\dot{\varphi} = \Phi(\varphi, u) = \tilde{\Phi}(\varphi, v),
\]

where \(u = (u_1, \ldots, u_n), v = (v_1, \ldots, v_m)\),

\[
u_i = u_i(u_i^o, [\varphi]; \varepsilon_i)
\]

and \(v_\alpha = v_\alpha(v_\alpha^o, [\varphi]; \tilde{\varepsilon}_\alpha)\)

and the \(\varepsilon\)-parameters \(\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)\) and \(\tilde{\varepsilon} = (\tilde{\varepsilon}_1, \ldots, \tilde{\varepsilon}_m)\) are the unknown or incompletely known functions of \([\varphi]\) and \(v^o = (v_1^o, \ldots, v_m^o)\) or \(u^o = (u_1^o, \ldots, u_n^o)\), respectively.

Note that the \(S\)-picture and the \(D\)-picture may be regarded as complementary in the N.Bohr sense. Both descriptions of the system \(S\) can not be applied to it simultaneously during its analysis, however, they are compatible and the structure of SD-pair is a manifestation of their compatibility. The choice of a picture is an action of our attention: it is concentrated on the personal subjects in \(S\)-picture (the self-conscious attention) whereas it is concentrated on the impersonal desires in \(D\)-picture (the creative attention).

2.4. Verbalization of SD-pairs and synlinguism. The main problem is to interrelate the \(S\)- and \(D\)-pictures of the system \(S\). One way is a verbalization of SD-pairs. Let us remind a definition of the verbalizable interactive game.

An interactive game of the form

\[
\dot{\varphi} = \Phi(\varphi, u)
\]

with \(\varepsilon\)-represented couplings of feedbacks

\[
u_i = u_i(u_i^o, [\varphi]; \varepsilon_i)
\]
is called *verbalizable* if there exist *a posteriori* partition \( t_0 < t_1 < t_2 < \ldots < t_n < \ldots \)
and the integrodifferential functionals

\[
\omega_n(\vec{\varepsilon}(\tau), \varphi(\tau)|t_{n-1} \leq \tau \leq t_n),
\]
\[
u_n^*(u^o(\tau), \varphi(\tau)|t_{n-1} \leq \tau \leq t_n)
\]
such that

\[
\omega_n = \Omega(\omega_{n-1}, u_n^*; \varphi(\tau)|t_{n-1} \leq \tau \leq t_n),
\]
quantities \( \omega_n \) are called the *words*.

Let us now consider the SD-pair and suppose that both S- and D-pictures are verbalizable with the same \( \omega_n \). The fact that \( \omega_n \) are the same for both S- and D-pictures is called their *synlinguism*. One may characterize it poetically by the phrase that “the speech of real subjects is resulted in the same text as a whisper of the impersonal desires”. The existential character of the synlinguism should be stressed. Really it is not derived from the fact that the objective states \( \varphi \) of the system \( S \) are the same in the S- and D-pictures. The synlinguism interrelates the different \( \varepsilon \)-parameters of existential nature in both pictures.

The synlinguism is very important in the analysis of tactical phenomena, which essentially used the concept of verbalization in their definition. To my mind the synlinguism lies in the basis of psychophysical nature of mutual understanding of the independent subjects of a dialogue communication. In this situation it allows to identify the personal interpretations with the impersonal ones, unraveling the role of impersonal desires as bearers of the objective sense and its dynamics.

To the verbalizable SD-pairs some procedures of linguistic analysis are applicable. Some of them are inherited from the verbalizable interactive games (the grammatical analysis), some are specific (the explication and analysis of objective sense).

### III. Conclusions

Thus, mathematical procedures of the experimental detection of interactive phenomena in complex natural, behavioral, social and economical systems and their analysis are described. The special attention is concentrated on the role of desires and their second quantization as well as on the abstract structure of SD-pairs, their verbalization and the synlinguism.