On the $\rho$ and $\sigma_{\text{tot}}$ measurement by the TOTEM Collaboration: in the wake of recent discoveries.

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Abstract

We show that extraction of the quantities $\rho(s) = \text{Re}[T_N(s,0)]/\text{Im}[T_N(s,0)]$ and $\sigma_{\text{tot}}(s)$ from the data on the $pp$ differential cross-section at $\sqrt{s} = 13$ TeV obtained by the TOTEM Collaboration gives results essentially different from those presented in publication [1] if to use a modified formula for Coulomb-nuclear interference. The physical interpretation of these data changes accordingly.

The Problem.

The experimental data of the TOTEM group on differential, $(d\sigma/dt)(s,t)$, and total, $\sigma_{\text{tot}}(s)$, cross-sections in $pp$-scattering, as well as the parameter $\rho(s) \equiv \text{Re}[T_N(s,0)]/\text{Im}[T_N(s,0)]$ at the LHC energy $\sqrt{s} = 13$ TeV, were published in [1].

An unexpectedly small (compared with the predictions of most models) value of the parameter ($\rho = 0.1$ or even less) was interpreted - based on a comparison with one specific model [2] - as the discovery of the "maximal Odderon" supposedly in the form of a "3-gluon bound state" or, "at least", as slowing down of the growth of total cross sections in the LHC energy region. Also noteworthy is quite a categorical statement that "the unprecedented precision of the $\rho$ measurement, combined with the TOTEM total cross-section measurements in an energy range larger than 10 TeV (from 2.76 to 13 TeV) has implied the exclusion of all the models classified and published by COMPETE$^4$" (italics are ours).

Let us remind that the "maximal Odderon" doctrine was launched in Ref.[6] where it was assumed that the upper bound for high-energy behaviour of the crossing-odd forward scattering amplitude is functionally saturated and leads eventually to a maximal violation of the difference form of the Pomeranchuk theorem which predicted, in particular, that

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$^4$This refers to a theoretical collaboration that published in [11] a general parameterization of the total cross sections and parameters $\rho$ for all available initial channels.
\[ \sigma_{pp}^{\text{tot}} - \sigma_{\bar{p}p}^{\text{tot}} \rightarrow 0 \]
at high energies while the "maximal Odderon" option implies instead that at \( s \rightarrow \infty \)
\[ \sigma_{pp}^{\text{tot}} - \sigma_{\bar{p}p}^{\text{tot}} \sim + \ln s. \]

It is not surprising that the results \( [1] \) being related, as we see, to the most important conceptual problems of the theory of strong interactions at high energies naturally aroused considerable interest and a number of discussions, accompanied by several dozen publications\(^5\).

In this regard, we believe that a thorough analysis of the results obtained in \( [1] \) (and the physical conclusions made on their basis), undertaken in the present work (including their re-analysis on the base of different theoretical means), seems quite appropriate and at least not useless, especially on the eve of the upcoming measurements at the LHC energy of 14 TeV.

**Dealing with the TOTEM data.**

When acquainting with experimental data on the differential cross sections of \( pp \) scattering at 13 TeV obtained by the TOTEM collaboration \( [1] \) (Table 3 in that article), the fact is noteworthy that in many cases the systematic errors in determining the value of \( (d\sigma/dt)(s, t) \) exceed the statistical errors an order of magnitude or more. This fact is especially pronounced for small values of \(|t|\) as shown in Fig.\( [1] \).

Approximately half of the experimental data (low \(|t|\)) are indicated with a detailed indication of the sources of errors (see Table 3 in \( [1] \)), but the remaining (larger \(|t|\)) data (see Table 5 in \( [7] \)) contain only statistical and systematic errors as a whole. Therefore, it is impossible to build a correlation matrix between the measurement results. There is nothing else left but to assume, for a while, the total measurement error to be \( \sqrt{\text{stat}^2 + \text{syst}^2} \). The TOTEM collaboration obtained values of \( (d\sigma/dt)(s, t) \) down to very small \(|t| \geq 0.000879 \text{ GeV}^2 \) which allows one to estimate the value of the parameter \( \rho(s) = \frac{\text{Re}[T_N(s, 0)]}{\text{Im}[T_N(s, 0)]} \) when fitting experimental data only for very small values of the transferred momentum. To conduct further quantitative estimates, it is necessary to precise what is meant by "small values of the transferred momentum" (\(|t|\)). Specifically, all experimental values are being used at \(|t| \leq t_0\) while discarding all experimental values at \(|t| > t_0\). According to the set of experimental data selected in this way, fitting was carried out using the model for the strong interaction amplitude given in \( [1] \) (the "TOTEM model") and making sequential monitoring of the dependence on \( t_0 \).

\(^5\)These results made a fuss in the media \( [4] \) including the tabloid press \( [5] \).
Figure 1: Systematic and statistical errors of \(\frac{d\sigma}{dt}(s,t)\) in the TOTEM experiment at low momentum transfers (double logarithmic scale).

**Notations, normalization and theoretical requisite**

In general case, the differential cross section is expressed in terms of the scattering amplitude \(T_{N+C}(s,t)\) (in this paper we consider only \(pp\) scattering) which depends on the c.m.s. energy \(\sqrt{s}\) and the transferred momentum squared \(t\):

\[
\frac{d\sigma}{dt}(s,t) = \frac{g|T_{N+C}(s,t)|^2}{16\pi s(s - 4m_p^2)}.
\]

Index \(N + C\) means account both of strong (\(N\), "nuclear") and Coulombic (\(C\)) forces while \(g = (hc)^2 \text{ [GeV}^2\text{mb]}\) is the usual conversion factor (the amplitude is dimensionless). The total cross section \(\sigma_{\text{tot}}(s)\) (till the end of the paper we will omit the index "\(\text{tot}\)"") and \(\rho(s,t) = \cot^{-1}[\text{Arg}T_N(s,t)]\) are expressed in terms of the scattering amplitude in the standard way:

\[
\sigma_{\text{tot}}(s) = g \cdot \text{Im}[T_N(s,0)]/\sqrt{s(s - 4m_p^2)}, \quad \rho(s,t) = \text{Re}[T_N(s,t)]/\text{Im}[T_N(s,t)].
\]

In turn, the scattering amplitude itself is expressed in terms of the sum of the nuclear scattering amplitude \(T_N(s,t)\), the pure Coulomb component (taking into account the proton form factor \(F(t) = 1/(1 - t/(0.71 \text{ GeV}^2)^2)\) and two additional terms \(L_1(s,t)\) and \(L_2(s,t)\) as was obtained in [9] and then invariably used in the TOTEM publications:

\[
T_{N+C}(s,t) = T_N(s,t) + \frac{8\pi\alpha sF^2(t)}{t} + L_1(s,t) + L_2(s,t).
\]
The last two terms reflect the Coulomb-nuclear interference at the amplitude level and are expressed by the formulas:

\[
L_1(s,t) = \frac{i \alpha}{\pi} \int_{-\infty}^{0} dt' I(t, t') \left[ T_N(s, t) - T_N(s, t') \right],
\]

\[
L_2(s,t) = -i \alpha T_N(s, t) \int_{-\infty}^{0} dt' \ln \left( \frac{t'}{t} \right) \frac{dF^2(t')}{dt} 
\]

where

\[
I(t, t') = \int_{(\sqrt{-t} - \sqrt{-t'})^2}^{(\sqrt{-t} + \sqrt{-t'})^2} \frac{F^2(x)}{\sqrt{((\sqrt{-t} + \sqrt{-t'})^2 - x)(x - (\sqrt{-t} - \sqrt{-t'})^2)}} dx.
\]

Note that the above relations are of a general nature and do not imply any model expressions for the amplitude \( T_N(s, t) \) which can be selected in various ways already during the data processing. It must be pointed out that in expression (1), as shown in [3], the term \( L_2(s,t) \) is redundant. However, since the TOTEM group still uses an expression containing this term, we present fitting options with both the single term \( L_1(s,t) \) and with addition of \( L_2(s,t) \). The latter is necessary in order to trace the source of the numerical value for \( \rho \) obtained in [1]. It is interesting that if interference at the amplitude level is not taken into account at all, no terms \( L_{1,2}(s,t) \) (which is formally, of course, incorrect), then the result of the fit — in any case for those ”nuclear” amplitudes that are used below — is close to ”option 1” (only \( L_1(s,t) \)). In particular, this suggests that with the ”nuclear” amplitude according to the ”TOTEM model” (see the next Section), the interference term \( L_1(s,t) \) is numerically small in the region of low transfers and at \( \sqrt{s} = 13 \) TeV, which undoubtedly affects the numerical estimate of parameter \( \rho \). More on this in the next section [6].

”TOTEM model” and the experimental data at small t.

For small transfers, formula (1) is still valid with the refinement that the expression for the nuclear amplitude at \( t \to 0 \) can be significantly simplified, and we adopt the expression for \( T_N(s,t) \) in the form used by the TOTEM collaboration (”TOTEM model”) for processing experimental data at small \( t \), namely

\[
T_N(s,t) = 4s(i + \rho) \sqrt{\frac{\pi}{1 + \rho^2 g^{-1}} \frac{d\sigma}{dt}(s,0)} e^{(b_1|t| + b_2|t|^2 + b_3|t|^3)/2} 
\]

where the parameters \( \rho, \frac{d\sigma}{dt}(s,0), b_{1,2,3} \) are determined from fitting the experimental data. In [1], experimental data on differential cross sections are presented, including small values of the transferred momentum (down to \( 10^{-3} \) GeV\(^2\)). On this basis, parameters, such as \( \rho \), are determined when fitting the experimental data at the lowest possible values of \( t \). In fact, the experimental TOTEM data are presented in two versions (Table 3 in [1]):

\[\text{In the general case, for fitting experimental data on a complete set of experimental values of } t \]
\[\text{we use the tested model described in [10].}\]
1. Differential cross-sections with a systematic and experimental error (5 and 6 columns in Table 3 in [1]). It can be seen that systematic errors in most cases significantly exceed statistical errors - sometimes by more than an order of magnitude.

2. Differential sections with a weighted set of error sources of these values (7-11 columns in Table 3 in [1]). In this case, it is also easy to notice that the weighting coefficients in the first column sharply exceed the coefficients in the remaining columns.

Due to the described properties of the experimental data, we have analysed the experimental data in the following four options:

1. $d\sigma/dt$ with the systematic and experimental errors:
   - with the statistical error only;
   - with the total error.

2. $d\sigma/dt$ with a weighted set of error sources:
   - excluding the source of errors in normalization (8-11 columns in Table 3 in [1]);
   - taking into account all sources of errors (columns 7-11 in Table 3 in [1]).

However, before presenting the results of these four options, it is necessary to first consider the values of the interference components $L_1(s,t)$ and $L_2(s,t)$ in the main formula (1).

![Figure 2: Experimental data on $d\sigma/dt$ in $pp$ scattering at $\sqrt{s} = 13$ TeV, $|t| < 0.01$ GeV$^2$ and parameter values for three fitting variants. All curves are graphically indistinguishable. The graph shows only statistical errors (for systematic see the previous figure)](image-url)
Evaluation of various forms of CNI with the same TOTEM nuclear model.

We begin by examining the role played by the terms $L_1(s,t)$ and $L_2(s,t)$ in expression (1) for the full amplitude at small $t$. As mentioned above, all the data will be omitted for $|t|$ above some value $t_0$, and the remaining experimental points are fitted from the side of small values of $|t|$ for various values of $t_0$.

The issue of errors of the fit parameters is left for special consideration below.

The fit was carried out in the following three options for the full amplitude:

1. $T_{C+N}(s,t) = T_N(s,t) + \frac{8\pi\alpha s F^2(t)}{t} + L_1(s,t)$

   In accordance with the above, it is in this form that it is necessary to take the total scattering amplitude, where $L_1(s,t)$ takes into account the Coulomb-nucleus interference in the lowest order in $\alpha$.

2. $T_{C+N}(s,t) = T_N(s,t) + \frac{8\pi\alpha s F^2(t)}{t} + L_1(s,t) + L_2(s,t)$

   Here we intentionally hold the excess term $L_2(s,t)$ so that to trace the genesis of the $\rho$ value obtained in [1].

3. $T_{C+N}(s,t) = T_N(s,t) + \frac{8\pi\alpha s F^2(t)}{t}$

   This (formally incomplete) option was used to evaluate the numerical significance of the term $L_1(s,t)$ in option 1.

As an example, let us first give the graphic result of the fits that correspond to these three options (Fig 2).

The results of the fitting parameters are extremely close to each other 7, and the behaviour of the curves is visually indistinguishable for these three options.

The values of the fitting parameters behave identically for these three cases (depending on the cut-off value $t_0$), and they are presented in Fig 3.

It can be seen from the figure that if to go from the main option 1 (on the graphs it is indicated by the number 1) to the option with then term $L_2(s,t)$ (option 2 - on the graphs it is indicated by the number 2), then the parameter values increase significantly. In option 3, i.e. without using the terms $L_1(s,t)$ and $L_2(s,t)$, the parameter values practically return to the original version 1.

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7However, they cannot be considered definite until their errors are calculated - more on that below.
Results when considering only statistical and complete errors of experimental data

Due to the large magnitude of systematic errors for small $|t|$ the value $\chi^2$/NoF, when using such experimental data with a small number of points (as can be seen from the graph in Fig. 1), will be close to zero even with three fitting parameters (retaining only $b_1 \equiv b$ from three $b_i$), which negates the fit result. In addition, with a decrease in the number of experimental data, a larger number of fitting parameters does not allow one to approach small values of $t$ as close as possible. Three parameters $b_i$ (see Eq. (5)) are redundant, and it is more correct to use only one parameter $b_1 \equiv b$ (note that in the TOTEM publication [1] this minimization of parameters $b_i$ was also used when only statistical errors were taken into account and for $t_0 = 0.07$ GeV$^2$). In the case of higher energies, there is nothing else left but to use the complete errors of the experimental data $\sqrt{\text{stat}^2 + \text{syst}^2}$. Thus, from the foregoing, the question arises: how was it possible to obtain the “experimental value” of the $\rho$ parameter with an accuracy of about 10%, as stated in the publication [1]? As noted above, we investigated two options: one with only statistical errors and another, with complete errors in experimental data. The behaviour of the parameter $\rho$ with a decrease in the cut-off value $t_0$ is shown in Fig. 4 and Fig. 5. It can be seen that approaching the zero value of $t$ fails closer than to $|t| \approx 0.004$ GeV$^2$. In this case, it really turns out that $\rho \approx 0.09$. However, the total error of this value is almost 100%. The error $\mathcal{O}(10\%)$ (as can be seen from the results given in [1] $\rho = 0.09 \pm 0.01$ and $\rho = 0.10 \pm 0.01$) does not work even if only statistical errors are taken into account: in fact, they are no less than 20%.
Similar statements hold for the other fit parameters: \((d\sigma/dt)_{t=0}\) and \(b\). Their behaviour is presented in figs. 6, 7, 8, 9.

Having the values of these parameters, it is possible to estimate the total cross-section by the formula:

\[
\sigma_{\text{tot}} = 4\hbar c \sqrt{\pi (d\sigma/dt)_{t=0}}
\]

(3)

In the figures figs. 10 and 11 shows the \(\sigma_{\text{tot}}\) corresponding to these parameters.

The behaviour of the total cross section and its errors depending on the cut-off value \(t_0\) leads to the following values of the total cross sections:

\[
\sigma_{\text{tot}} = 110.0 \pm 8.0 \text{ [mb]} \quad \text{(including complete errors)},
\]

\[
\sigma_{\text{tot}} = 110.8 \pm 1.0 \text{ [mb]} \quad \text{(with account of statistical errors only)}.
\]

**Evaluation of parameters when accounting for correlations of experimental data.**

When fitting using error sources, it becomes possible to use all five parameters in formula (5). Fig. 8, 9, 10 show the results of fitting with three, two, and one parameter in the exponent, respectively. Blue color indicates the results with all error sources, and the red means all error sources except normalization (except for the first column in the error source). In these two cases, the results no longer differ so dramatically as in the previous paragraph, although of course one needs to trust the results when taking into account all the sources of errors. Plots showing \(\chi^2/\text{NoF}\) are also provided in order to understand which results should be discarded.

The final results of the description with error sources are summarized in tables to be shown below.

**Summary of the parameters from describing the data with the ”TOTEM model” for \(T_N(s, t)\) and modified account of CNI**

Although we present the values of the fitting parameters without taking into account normalization, we do not at all consider this method of action to be completely correct because if there is a source of errors, then it must be taken into account (why then do the experimenters bring them?). From the tables and graphs below, it also follows that the use of more than one term in the exponent (5) \((b_1|t| + b_2|t|^2 + b_3|t|^3)\) is redundant and one term \(b_1|t| \equiv b|t|\) is fairly enough to get the same value of \(\chi^2/\text{NoF}\). In addition, there is a strong correlation among all the three.

The smallest value for \(\rho\) is obtained for option 1 (see above: all three parameters in the exponent):

\[
\rho(3b_i) = 0.104 \pm 0.022.
\]
However, we believe that due to the excessive number of parameters $b_i$, there is no good reason for accepting such a result.

The totality of all fit results indicates that there is no statistically significant reasons to fall below $t_0 = 0.015$ GeV$^2$, because in these cases either $\chi^2$/NoF is too close to zero (which is statically unacceptable), or $\chi^2$/NoF > 1, which gives a statistical certainty of less than 50%. In Tables 1 and 2 we give a general summary of the parameters and their errors (as well as their correlation matrices) for cases of accounting the source of normalization errors (the largest contribution to the error in the experimental data table in $[1]$) and without it, respectively. All calculations are done for three options of the number of parameters $b_i$ in the exponent:

1. a single parameter $b_1$;
2. two parameters $b_1$ and $b_2$;
3. three parameters $b_1$, $b_2$ and $b_3$.

The results obtained without taking into account the source of normalization errors, despite that they give in some cases the expected values of $\rho < 0.1$, are hardly worth considering (Table 2). Since experimenters give this source of error, there is no reason not to use them when numerically processing the experimental results.

Thus, we are inclined to believe that parameter values should be taken from Table 2.

Once again, we emphasize, based on this table, that there is no need to use two (moreover, three) parameters in the exponent, because even without them we get a statistically significant $\chi^2$/ NoF. Besides, this would also result in a sharp increase in the error values of these parameters.

Thus, based on the considerations presented above, we believe that the correct result for the parameter $\rho$ obtained using all sources of errors, the modified formula for CNI and with only the linear term in the exponent ($b|t|/2$) of the "TOTEM" model for $T_N$ reads:

$$\rho(13 \text{ TeV}) = 0.123 \pm 0.010,$$

what corresponds to the following value of the total cross section:

$$\sigma_{\text{tot}}(13 \text{ TeV}) = 111.4 \pm 1.8 \text{ [mb]}.$$

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*In all these cases, the expected value of $\sigma_{\text{tot}}$ is given according to the formula (3).
Figure 4: Values of the $\rho$-parameter and its error depending on the cut-off value of the experimental data $|t_0|$ taking into account only statistical errors. Crossed out are results for which $\chi^2/\text{NoF} > 1$.

Figure 5: Values of the $\rho$-parameter and its error depending on the cut-off value of the experimental data $|t_0|$ with full errors. However, in all these results, $\chi^2/\text{NoF} \ll 1$, and these results should be regarded as evaluative.
Figure 6: The values of the parameter \((d\sigma/dt)_{t=0}\) and its error, depending on the cut-off value \(|t_0|\) of the experimental data taking into account only statistical errors. Crossed out are the results for which \(\chi^2/\text{NoF} > 1\).

Figure 7: The values of the parameter \((d\sigma/dt)_{t=0}\) and its error, depending on the cut-off value \(|t_0|\) of the experimental data with full errors. However, in all these results, \(\chi^2/\text{NoF} \ll 1\), and these results should be regarded as evaluative.
Figure 8: The values of the parameter $b$ and its error depending on the cut-off value $|t_0|$ of the experimental data taking into account only statistical errors. Crossed out are the results for which $\chi^2/\text{NoF} > 1$.

Figure 9: The values of the parameter $b$ and its error, depending on the cut-off value $|t_0|$ of the experimental data with full errors. However, in all these results, $\chi^2/\text{NoF} \ll 1$, and these results should be regarded as evaluative.
Figure 10: The values of $\sigma_{\text{tot}}$ and its error depending on the cut-off value $|t_0|$ of the experimental data taking into account only statistical errors. Crossed out are the results for which $\chi^2/\text{NoF} > 1$.

Figure 11: The values of $\sigma_{\text{tot}}$ and its error, depending on the cut-off value $|t_0|$ of the experimental data with full errors. However, in all these results, $\chi^2/\text{NoF} \ll 1$, and these results should be regarded as evaluative.
| $N_b$ | $\rho$ | $d\sigma/dt_{t=0}$ [mb/GeV$^2$] | $b_1$ [GeV$^{-2}$] | $b_2$ [GeV$^{-4}$] | $b_3$ [GeV$^{-6}$] | $\frac{\chi^2}{\text{NoF}}$ |
|------|-------|-----------------|-----------------|-----------------|-----------------|----------------|
| 1    | 0.123(10) | 634.1±20.1 | -21.0(3) | - | - | 0.99 |
|      | 100 | -70.1 | -81.4 | | | |
|      | 100 | 37.7 | 100 | | | |
|      | $\sigma_{\text{tot}} = 111.4 \pm 1.8$ | 100 | | | | |
| 2    | 0.117(15) | 635.55±20.24 | -20.07±1.73 | -43.34±82.08 | | 0.98 |
|      | 100 | -54.6 | -84.0 | 75.7 | | |
|      | 100 | 18.2 | 100 | -11.6 | | |
|      | $\sigma_{\text{tot}} = 111.53 \pm 1.78$ | 100 | | | | |
| 3    | 0.104(22) | 631.1±21.0 | -15.48±5.91 | -547.3±627.9 | 18008.499±22265.478 | 0.97 |
|      | 100 | -17.200 | -85.010 | 77.201 | -70.874 | |
|      | 100 | -22.278 | 26.923 | -28.635 | |
|      | 100 | -98.586 | 95.678 | 100 | -99.150 | |
|      | $\sigma_{\text{tot}} = 111.1 \pm 1.9$ | 100 | | | | |

Figure 12: Table 1. Parameter values with all error sources
| $N_b$ | $\rho$     | $\frac{d\sigma}{dt_{t=0}}$ [mb/GeV$^2$] | $b_1$ [GeV$^2$] | $b_2$ [GeV$^4$] | $b_3$ [GeV$^6$] | $\chi^2$/NoF |
|-------|------------|----------------------------------------|----------------|---------------|--------------|----------------|
| 1     | 0.118(6)   | 646.37±2.45                           | -20.9(3)       | -             | -            | 0.99          |
|       | 100        | -84.02                                 | -91.67         |               |              | $\tau_0=0.015$|
|       |            | $\sigma_{\text{tot}} = 112.48 \pm 0.21$ |                |               |              |               |
| 2     | 0.1123(101)| 642.68±5.68                           | -19.79±1.53    | 54.22±75.43   | -            | 0.99          |
|       | 100        | 93.2                                   | -85.8          | 78.3          |              | $\tau_0=0.015$|
|       | 100        | -95.8                                  | 90.3           |              |              |               |
|       | 100        |                                       | -98.4          |              |              |               |
|       |            | $\sigma_{\text{tot}} = 112.16 \pm 0.50$ |                |               |              |               |
| 3     | 0.1017(158)| 633.316±12.055                      | -15.21±5.48    | -570.940±597.075 | 18723.28±21451.17 | 0.97          |
|       | 100        | 95.703                                | -89.038        | 82.416        | -76.661      | $\tau_0=0.015$|
|       | 100        | -97.454                               | 93.103         | -88.484      |              |               |
|       | 100        |                                       | -98.714        | 95.993       | -99.194     |               |
|       |            | $\sigma_{\text{tot}} = 111.335 \pm 1.060$ |                |               |              |               |

Figure 13: Table 2. Parameter values with all error sources except normalization.
Whether the COMPETE parametrization is ruled out?

Here we would like to comment in passing the following categorical judgement made in [1]: "...none of the COMPETE models is compatible with the ensemble of TOTEM’s $\sigma_{\text{tot}}$ and $\rho$ measurements."

In simple words this means that the use of the COMPETE parametrizations as given in [11] cannot lead to simultaneous description of $\sigma_{\text{tot}}$ and $\rho$ which is consistent with the values of these quantities given in [1].

This issue has been addressed in [12] with the conclusion that the actual predictions of the COMPETE modelling set for $\sqrt{s} = 13$ TeV is the whole band:

\[ 86 \text{ mb} \leq \sigma_{\text{tot}} \leq 117 \text{ mb} \]

and

\[ 0.058 \leq \rho \leq 0.145. \]

The only thing that could be concluded (if we would take for granted the values of $\sigma_{\text{tot}}$ and $\rho$ given in [1]) is that the fit which was considered earlier as the best one, should be accommodated to new data. This opportunity is in no way excluded by the general framework of COMPETE. In terms of discussing of the presence or absence of the Odderon, mention should be made of the behaviour of the curves of the $\rho$-parameter in these two cases. We carried out a study of the description of all experimental data of differential cross sections in the range from 7 GeV to 13 TeV together with experimental data on $\sigma_{\text{tot}}$ and $\rho$ parameter between 5 GeV and 13 TeV [10]. Fig. [14] shows the behaviour of these curves. It is seen that in the case of the presence of the Odderon the curve of the $\rho$-parameter passes even higher than 0.123 at $\sqrt{s} = 13$ TeV. In addition, in the case of vanishing maximum Odderon contribution, the curves for $pp$ and $\bar{p}p$ converge at high energies as is observed in the COMPETE model. This fundamentally contradicts the behaviour of these curves in the case of the presence of the Odderon.

On slowing down of the total cross-section growth.

It was announced in [1] that ...the $\rho$ value determined by TOTEM would represent a first evidence of a slowing down of the total cross-section growth at higher energies. It would be instructive to try to understand whether some specific value of $\rho$ would imply a decrease in the rate of growth of $\sigma_{\text{tot}}$. For this purpose let us use the derivative forward dispersion relations. Differential dispersion relations read [13] (for more rigorous treatment see Ref.[14])

\[ \frac{\text{Re}[T(s,0)]}{s} = \frac{\pi}{2} \frac{d}{dy} \left( \frac{\text{Im}[T(s,0)]}{s} \right), \quad y = \ln s. \]

(6)
Figure 14: Evolution of the $\rho$ parameter with the maximal Odderon (left) and without it (right).

The "slowing down of the total cross-section growth" announced in [1]) means that at the LHC energy $\mathcal{O}(13 \text{ TeV})$ when $\rho = 0.1$ the acceleration $d^2\sigma_{\text{tot}}/dy^2$ vanishes and changes from positive to the negative one. It is not a hard task to derive from Eq.(9) that equation

$$d^2\sigma_{\text{tot}}/dy^2 = 0$$

is equivalent to equation

$$d(\rho \cdot \sigma_{\text{tot}})/dy = 0.$$ 

If to take the values of the product $(\rho \cdot \sigma_{\text{tot}})$ up to 13 TeV, as presented in the TOTEM publications, it is seen that the value of $\rho \cdot \sigma_{\text{tot}}$ really reaches its maximum in the vicinity of 7 TeV. This would mean that already at $\sqrt{s} \simeq 7 \text{ TeV}$ the total cross section started to decelerate (although still increasing).

It is clear that this conclusion is tightly related with the low value of $\rho$ announced by the TOTEM Collaboration in [1].

On the other hand, according to the COMPETE parametrization [11] the value $\rho = 0.1$ will be reached only at $\sqrt{s} = \mathcal{O}(10^6) \text{ TeV}$ while the value of $(\rho \cdot \sigma_{\text{tot}})$ steadily grows and at present energies does not reveal any sign of saturation.

The same conclusion follows from our estimations (4) and (5).

**Summary and conclusions.**

We have reanalyzed the TOTEM data at 13 TeV [1] on the basis of a new treatment of CNI but with the same nuclear amplitude. Our results significantly differ from those presented in [1]:

1. TOTEM [1]: $\rho = 0.09 \pm 0.01 (0.10 \pm 0.01)$; $\sigma_{\text{tot}} = 110.5 \pm 2.4 \text{ mb.}$;
2. This paper: $\rho = 0.123 \pm 0.010$; $\sigma_{\text{tot}} = 111.4 \pm 1.8 \text{ mb.}$

We also presented arguments against a supposed total failure of the COMPETE framework claimed in [1] and have shown that the conclusion about slowing down of the total cross-section growth critically depends on the values of the product $(\rho \cdot \sigma_{\text{tot}})$. According to our results such a slowing down does not take place at present energies.
Appendix.

**TOTEM data and the Bethe formula for CNI.**

Classical results on the $\rho$-parameter and the total cross section $\sigma_{\text{tot}}$ at the ISR were presented in [8]. However, in this work, to extract the $\rho$-parameter, a different (from that of used in the TOTEM publication) formula for account of CNI was used (the well known Bethe parametrization) which gives:

$$\frac{d\sigma}{dt} = \frac{4\pi g \alpha^2 F^4(t)}{t^2} + \sigma_{\text{tot}} \alpha \left[ \rho + \alpha \phi(t) F^2(t) \right] e^{\phi(t)/2} + \frac{(1 + \rho^2) \sigma_{\text{tot}}^2 e^{bt}}{16\pi g}$$

(7)

where

$$\phi(t) = \ln \left( \frac{0.08}{-t} \right) - 0.5772... .$$

The fitting parameters are $\sigma_{\text{tot}}$, $\rho$ and $b$.

We applied this formula to the TOTEM data at 13 TeV, slightly modifying its mathematical form and taking the expression for $\sigma_{\text{tot}}$ used in the "TOTEM model" in eq. (3).

So, we have got again three parameters: $(d\sigma/dt)_{t=0}$, $\rho$ and $b$.

Fig. [15] shows the results of processing the experimental data according to the procedure that we used earlier. It can be seen that the value $\chi^2/\text{NoF}$ stably lies in a small neighbourhood of unity (and remains less than this value) until the cut-off of the experimental points with $|t| > t_0 \approx 0.012 GeV^2$.

Based on this, we come to the following parameter values:

$$\rho = 0.0958 \pm 0.0113;$$

$$\sigma_{\text{tot}} = 110.3 \pm 1.9 \text{ [mb];}$$

$$(d\sigma/dt)_{t=0} = 632.6 \pm 20.9 \text{ [mb/GeV^2].}$$

These results are very close to the values declared by the TOTEM collaboration [1]. Note that we also tested the fitting procedures on experimental data obtained at ISR energies and at lower energies (the U-70 Serpukhov data [9]). The results are very close to the values presented in these works.

Thus, we see that the values of the $\rho$ parameter extracted from experimental data are significantly model-dependent. This alone negates the conclusion of [1] that the COMPETE model does not correctly describe the behaviour of the $\rho$ parameter.

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9 As was obtained earlier, the introduction of additional parameters $b_{2,3}$ in the exponent is redundant.
Figure 15: Parameters at $\sqrt{s} = 13$ TeV as function of the cut-off $t_0$. 
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