Stokes Profile Inversion in Meso-Structured Magnetic Atmospheres

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Abstract.
Based on the Meso-Structured Magnetic Atmosphere (MESMA) approximation (Carroll & Kopf 2006) we present first results of an inversion of spectropolarimetric observations obtained from internetwork regions. To cope with the inherent complexity of the mostly unresolved magnetic field in the solar photosphere the MESMA approach provides a statistical description of the underlying atmosphere in terms of a random Markov field. This statistical model allows us to derive a stochastic transfer equation for polarized light. The stochastic transfer equation explicitly accounts for the spatial correlation – the characteristic length scale – of the underlying magnetic and non-magnetic structures. We use this new diagnostic parameter in an inversion approach to demonstrate that the magnetic flux structures in the solar internetwork possess a finite correlation length which is not compatible with the classical flux tube picture.

1 Introduction

The entire solar photosphere exhibits a rich structure of large and small scale magnetic features like sunspots, pores faculae or plages. But except for sunspots and pores these magnetic fields cannot be spatially resolved with present telescopes, although these fields clearly manifest themselves in high resolution spectropolarimetric observations.

With the improvement of spectropolarimetric sensitivity and spatial resolution over the last years it became clear that these unresolved magnetic fields are much more ubiquitous than previously thought. This raises the question of the significance of these elusive and complex magnetic fields for the solar magnetism in general (Schrijver & Title 2003; Sánchez Almeida 2004) and how these magnetic fields can be appropriately investigated by spectropolarimetric observations.

The interpretation of Stokes profiles in the context of the thin flux-tube model relies on the basic picture of an embedded cylindrical magnetic structure surrounded by a quasi field-free medium. Based on that assumption a so called 1.5-dimensional radiative transfer is applied where a number of rays piercing through the underlying 2- or 3-dimensional geometry of the model to obtain the spectral ‘signature’ of the underlying magnetic structure (Solanki 1993). But if the underlying structures are much more dynamic, disrupted and intermittent, the conventional static flux-tube model will allow only a poor representation of the real magnetic field structure. In this sense the flux-tube modeling provides a rather macroscopic treatment of the problem – in the 1.5 dimensional sense – since the averaging process for of
all line-of-sights (LOS) is performed after the actual integration of the transfer equation. The other extreme, in contrast to the macroscopic view, is the MISMA approximation. The assumption here is that the atmospheric conditions along the line-of-sight are rapidly changing. The fluctuation of the atmospheric parameters occurs on very short scales, such that a micro-structured or micro-turbulent approach is justified. This allows an averaging over all atmospheric parameters at each spatial position before the actual transfer equation is integrated. Despite its appealing simplicity in the way this approach treats the radiative transfer the idealized assumptions about the underlying atmosphere strongly limits the application of this approach. Structures in the solar photosphere whether magnetic or non-magnetic are in general not in a microturbulent state. Magneto-convective simulations suggests that neither predefined static macro-structures nor pure micro-structures are present in the solar photosphere, the possible structuring seems much more to comprise a broad range of different scales. (Schaffenberger et al. 2005; Vögler et al. 2005; Stein & Nordlund 2006).

This paper is organized as follows: In Sect. 2 i briefly summarize the basic concept of line formation in stochastic media and present the stochastic polarized transfer equation. In Sect. 3 i give an overlook of the first results of an inversion of spectropolarimetric observations obtained from internetwork regions. Sect. 4 concludes with a summary of the here presented analysis.

2 The stochastic transfer equation for polarized light

The approach described here is based on a statistical model of the atmosphere in terms of a random Markov field, the MEso-Structured Magnetic Atmosphere (MESMA) which was introduced by Carroll & Kopf (2006). In this contribution i will just give a brief summary of the basic concept of the MESMA approach, for a more detailed presentation of the statistical model and derivation of the stochastic transfer equation the reader referred to Carroll & Staude (2003, 2005a) and Carroll & Kopf (2006).

The atmospheric volume of interest (the actual resolution element) is assumed to be characterized by an a-priori unknown structuring along the line-of-sight. The only assumption we make about the underlying atmosphere is that the structures have a finite spatial extent and can be described in terms of a Markov random field. This allows us to neglect all higher order spatial correlation effects to use a first order approximation to describe the spatial correlations.

We begin by introducing a random atmospheric vector $B$ which comprises all relevant atmospheric parameters such as temperature, pressure, velocity, magnetic field strength, magnetic field inclination, etc. If we move then along an arbitrary line-of-sight we obtain a series of realizations at different positions $s$ for the random vector $B$. This spatial dependency allows us to describe $B$ in terms of a stochastic process (a Markov process) along the line-of-sight and for which we can specify a suitable conditional probability density or transition probability from one spatial point $s$ to another $s + \Delta s$,

$$p(B_{s+\Delta s} \mid B_s) = e^{\frac{\Delta s}{\lambda}} \delta(B_s - B_{s+\Delta s}) + (1 - e^{\frac{\Delta s}{\lambda}}) p(B_{s+\Delta s}) .$$

(1)

This conditional probability which specifies the so called Kubo-Anderson process (Frisch & Frisch 1976) describes how the probability for a transition changes as we move along the line-of-sight from a given position $s$ and the associated atmospheric conditions $B_s$. 
at this position. The probability for staying in the same regime $B$, for the entire trajectory $\Delta s$ decays exponentially while the probability for a sudden jump into another atmospheric regime $B + \Delta s$ rapidly grows. The conditional probability density therefore describes the correlation of the individual structures between two spatial positions $s$ and $s + \Delta s$. The degree of correlation is controlled by the parameter $\lambda$, the characteristic length (correlation length) of the structures. Based on that particular Markov process we can derive the following stochastic transport equation (see Carroll & Kopf 2006), for the so called mean conditional Stokes vector $Y$

$$ \frac{\partial Y_B(s)}{\partial s} = -K Y_B + j + \int_B \lambda_B^{-1} Y_B \rho(B', s) dB' - \int_B \lambda_B^{-1} Y_B p(B'', s) d B'', $$

(2)

The mean conditional Stokes vector $Y_B$ is defined as

$$ Y_B(s) = \int_I I p(I, s | B, s) d I. $$

(3)

It is important to realize that $Y_B$ is a statistical equation and it is conditioned on one particular atmospheric regime $B$. The transport Eq. (2) describes the evolution of $Y_B$ through the atmosphere along the LOS. There are four basic processes that govern the transport of $Y_B$, the two processes of absorption and emission and two more processes that describe the statistical inflow and outflow of intensity to and from the regime $B$ under consideration. The degree of statistical scattering and absorption is controlled by the correlation length $\lambda$ of the particular atmospheric structures. The observable of our problem – the expectation value of the Stokes vector – at the top of the atmosphere $s_t$ can easily be obtained from a final integration of the mean conditional Stokes vector over the entire atmospheric state space $\hat{B}$,

$$ < I(s_t) > = \int_B Y_B(s_t) p(B, s_t) d B. $$

(4)

It is this extra degree of freedom in the stochastic transfer equation, given by the correlation length, which provides the additional diagnostic capability of the stochastic approach (Carroll & Staude 2005b, 2006; Carroll & Kopf 2006). As could be shown by Carroll & Kopf (2006) the asymmetry of the Stokes $V$ profiles directly depends upon the underlying correlation length and allows to estimate the characteristic length scale of the magnetic field from the net-circular-polarization (NCP) of Stokes $V$ profiles.

### 3 Analysis of internetwork magnetic fields

Based on the stochastic mesostructured approach we have analyzed observations of full Stokes profiles of the iron line pair Fe I at 630 nm, taken at the High Altitude Observatory/National Solar Observatory Advanced Stokes Polarimeter (ASP). The quiet sun data were obtained by B. Lites on 1994 September 29 (Lites et al. 1996). In the following we are in particular interested to gain some insight into the characteristic length scale of the magnetic structures in the internetwork. Our inversion routine is based on the Levenberg-Marquardt algorithm (Press et al. 1992) and incorporates the stochastic transport equation as the forward kernel. In a first step we analyzed granular and intergranular regions in order
to determine the convective characteristics of the non-magnetic components from Stokes $I$ profiles.

For the granular and intergranular components we adopted the granular and intergranular model atmospheres from Borrero & Bellot Rubio (2002). Based on the temperature and pressure stratification of these models we assumed a simple 3-type stochastic velocity field. Please note, that the stochastic approach makes no assumption about the numbers of individual structures in the resolution element, the only assumption made here is that there are three different types of structures present.

The free parameter of the fitting routine are the three single-valued velocities for each atmospheric component, the probability values of the individual types (comparable to the conventional filling factor) and the correlation lengths of the individual components. For an upflow (granular) region a fit of a Stokes $I$ profile is shown in Fig.(1). The remarkable result here is the fact that there is no need to use nonphysical parameters like micro- or macroturbulence to fit the profiles. The convective velocity structure exhibits a clear mesostructured behavior on scales between 200 km and 400 km. These results are in agreement with earlier investigations of mesoturbulent velocity fields in the solar photosphere by Gail et al. (1976).

As we could place tight constraints upon the characteristics of the ambient flow pattern we proceed by analyzing the magnetic field by inverting the respective Stokes $V$ profiles. We adopted a stochastic model of two different types of structures (non-magnetic and magnetic). The free parameter of the inversion are the velocities, magnetic field strengths as well as the correlation lengths of the individual structures. Two fits of the Stokes $V$ profiles are shown in Fig.(1) and Fig.(2). The observed Stokes $V$ profiles could be well reproduce by the stochastic two-ensemble model. In particular the asymmetries of the Stokes $V$ profiles are well reproduced. The magnetic structures in the analyzed internetwork region have surpris-

Figure 1. A fitted Stokes $I$ profile of the Fe I 6301.5 nm line. Diamonds represent the observation, the solid line the profile fit.
Figure 2. Fit of a typical Stokes $V$ profile of the Fe I 6301.5 nm line, again the diamonds represent the observation and the solid line the fit to the profile.

Figure 3. A typical fit for the Stokes $V$ profile of the Fe I 6302.5 nm line. Diamonds observations, solid line fit.

...ingly small correlation lengths between 50 km and 125 km for structures in upflow elements and 150 km to 230 km for structures in downflow elements. These results clearly indicate that the magnetic field – as well as the velocity field – in the internetwork can neither by described by macroscopic structures like flux tubes nor can they be described in terms of a microstructured or microturbulent field. A interesting result here is the clear trend for...
stronger field structures to have larger correlation lengths and for weaker structures to have shorter correlation lengths. This seems to be the result of the increased buoyancy forces of strong magnetic structures which results in a preferred vertical alignment and gives rise – for disc center observations – to an increase of the line-of-sight correlation lengths.

4 Summary

This analysis of magnetic field structures in the internetwork clearly demonstrate the feasibility of an inversion under the MESMA concept. Moreover, we found that the characteristic length scales in the solar internetwork are relatively small, but clearly far from being microstructured or microturbulent. However, the length scales found in this work are also not consistent with the classical flux tube picture [Stenflo 1994] which would require correlation lengths larger than 350 km. Another intriguing result here is the fact that the obtained correlation lengths of the magnetic structures agrees very well with the autocorrelation (along a vertical direction) of magnetic structures in magnetohydrodynamic simulations [Schaffenberger et al. 2005] which have a mean value of approximately 220 km. This surely deserves further investigations but one can already say that advanced magnetoconvecive simulations and (polarized) radiative transfer modeling provide a fruitful combination to gain further insight into the surface magnetism of the sun.

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