Frequency-dependent capacitors using paper

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Measurements of capacitors made with paper sheets reveal a significant decrease in capacitance with increasing frequency from 10 to 100,000 Hz, offering a simple demonstration of complex dielectric phenomena using common equipment. © 2021 American Association of Physics Teachers. https://doi.org/10.1119/10.0002655

Paper for printing and copying is a convenient material for exploring dielectrics and capacitors. In particular, its shape and uniform thickness are well suited for building parallel-plate capacitors, whose capacitance varies with the number of sheets between their plates. Paper is a complex anisotropic dielectric, though, with a variability and environmental sensitivity that can be troublesome for quantitative measurements. However, as Fig. 1 shows, paper often has an appreciable dependence on frequency that presents an opportunity to witness complex dielectric phenomena.

The capacitance $C$ of an ideal parallel-plate capacitor made from two identical plates is

$$C(f) = \text{Re}[\epsilon_r(f)]\epsilon_0 A/d,$$

where $\epsilon_0$ is the vacuum permittivity, $A$ is the plate area, and $d$ is the thickness of a dielectric between the plates. For polarizable dielectrics, the capacitance is enhanced by the real part of the permittivity $\epsilon_r$ (or dielectric constant), which is within roughly 1.3–4.0 for paper. The closely related imaginary part of $\epsilon_r$ produces loss. If a sinusoidal voltage with frequency $f$ is applied, then a frequency dependence in the permittivity will create a frequency dependence in the capacitance.

Many polarizable sources inside paper contribute including ions that move and dipoles that rotate, for example, in cellulose and absorbed water. Their responses are not instantaneous, however, and as the frequency $f$ increases, slow sources contribute less and less. For paper, the slowest sources are ionic (space charge) polarizations, and their relaxations dominate at low frequency. Overlapping of multiple sources blurs distinct features, leading $C(f)$ to gradually decrease with frequency as shown in Fig. 1(a) (cf. Fig. 3 of Ref. 4). This relaxation also produces loss as shown in Fig. 1(b) (cf. Fig. 4 of Ref. 4). Sharper features occur in other dielectrics or at higher frequencies.

To compare the frequency dependence of different capacitors, the data for each capacitor in Fig. 1(a) are normalized using the value at 100 Hz. The data for two paper capacitors show a similar variation with frequency, while a control capacitor shows no variation to within measurement uncertainty. The loss values in Fig. 1(b) are measurements of a dissipation factor (or loss tangent) $D(f) \approx -\text{Im}[\epsilon_r(f)]/\text{Re}[\epsilon_r(f)]$ for paper over this frequency range. The data for the paper capacitors again show a similar variation, while the data for the control show negligible loss.

To highlight the capacitance variation, the data for both paper capacitors were fit with the function $y(f) = C(f)/C(100 \text{ Hz})$ using the Cole-Cole empirical model.

$$\epsilon_r(f) = \epsilon_0 + \frac{\epsilon_1}{1 + (f/f_0)^\gamma},$$

$$= \epsilon_0 + \frac{1 + [\cos (\gamma \pi/2) - i \sin (\gamma \pi/2)](f/f_0)^\gamma}{1 + 2 \cos (\gamma \pi/2)(f/f_0)^\gamma + (f/f_0)^{2\gamma}},$$

with $1 \geq \gamma > 0$. The second line follows from substituting $\cos \phi/2$ for $i$. This model captures the broadened relaxation of many solid and liquid dielectrics. For $\gamma = 1$, it is equivalent to the Debye model for ideal dipolar relaxation. While there seems to be no standard model for paper, the fitted curve summarizes the data well. However, there was not enough low-frequency data to fully constrain the model. Least-squares fitting including uncertainty gave $\gamma(\infty) = 0.59 \pm 0.02$, $\gamma = 0.50 \pm 0.13$, and $f_0 = 24 \pm 94 \text{ Hz}$. The fit predicts a dissipation factor that matches the data well, as shown in Fig. 1(b).

The measurements in Fig. 1 used two paper capacitors made from compressing three aluminum-foil plates inside a

![Figure 1](image-url)

**Fig. 1.** (a) Measured capacitance $C(f)$ versus frequency $f$ for two paper capacitors and a nearly frequency-independent silver mica capacitor. Measurements used an impedance bridge or an LCR meter. A fitted curve highlights the appreciable variation of the paper capacitors. (b) Dissipation factor $D(f)$ from LCR-meter measurements. A curve shows the predicted $D$ from the fit of the capacitance data.
The dielectric properties of materials are fascinating. Other types of paper or different sheet materials could be explored with this approach. I observed similar results with most everyday paper products, from cardboard to a textbook and even a silicone placemat. Commercial paper film capacitors could be used, though their frequency variation is typically only a few percent. Alternatively, a phase-sensitive bridge could be used to separate loss from capacitance and reach lower frequencies. Frequency-dependent capacitors like those in Fig. 1 can be modeled as resistor-capacitor networks. One prediction of such models is dielectric absorption, an effect leading some capacitors to recharge over time, which is why capacitors must be left shorted after discharging to guarantee safe handling.

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13L. Yu, F. B. Madsen, S. Hvilsted, and A. L. Skov, “Dielectric elastomers, charges) may have contributed in the placemat (cf. Fig. 1).”

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