Discrete symmetries and the muon’s gyro-gravitational ratio in $g - 2$ experiments

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We show that recent, persistent discrepancies between theory and experiment can be interpreted as corrections to the gyro-gravitational ratio of the muon and lead to improved upper limits on the violation of discrete symmetries in rotational inertia.

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In a recent comment one of us (G.P.) suggested that discrepancies between the experimental and standard model values of the muon’s anomalous magnetic moment $a_\mu (exp) - a_\mu (SM)$ lead to upper limits on the violation of discrete symmetries in the spin-perturbation coupling [1]. Improved values of $a_\mu (exp)$ for positive and negative muons and persistent, residual differences $a_\mu (exp) - a_\mu (SM)$ can now be used to reduce the extent of $P$ and $T$ violation in spin-perturbation coupling and also to determine the parameters $\epsilon_+ \epsilon_-$ of [1]. These quantities can be interpreted as corrections to the gyro-gravitational ratio of the muon. The argument is summarized below.

$g - 2$ experiments involve muons in storage rings [2]. As the muons decay, the angular distribution of those electrons projected forward in the direction of motion reflect the precession of the muon spin along the cyclotron orbits. The value of $a_\mu$ is determined experimentally from the number of decay electrons or positrons [3,4]

$$N(t) = N_0 e^{-t/\gamma \tau} \left[ 1 + C \cos \left( a_\mu \frac{eBt}{m} + \phi_0 \right) \right],$$

(1)

where $m$ is the mass of the muon and $\gamma \tau$ its dilated lifetime. $N_0$, $C$ and $\phi_0$ depend on the energy threshold selected. Equation (1) can be directly related to the muon Hamiltonian $H$ that follows from the Dirac equation in the muon’s rotating frame. $H$ can be split into a part $H_0$ that is diagonal and contributes only to the overall energy $E$ of the states, and a part

$$H' = \left( -\frac{\hbar}{2} \vec{\sigma} - \mu \vec{B} \right) \cdot \vec{\sigma},$$

(2)

that accounts for spin precession. The spin-perturbation coupling in (2) is also known as the Mashhoon term [5], $\mu = (1 + a_\mu) \mu_0$ represents the total magnetic moment of the muon. Before decay the muon states can be represented as

$$|\psi(t)\rangle = a(t)|\psi_+\rangle + b(t)|\psi_-\rangle,$$

(3)

where $|\psi_+\rangle$ and $|\psi_-\rangle$ are the right and left helicity states of the Hamiltonian $H_0$. For simplicity all quantities are taken to be time-independent. It is also assumed that the effects of electric fields used to stabilize the orbits and stray radial electric fields are cancelled by choosing an appropriate muon momentum [2]. The total Hamiltonian $H = H_0 + H'$ is most conveniently referred to a left-handed triad of axes rotating about the $x_2$-axis in the direction of motion of the muons. The $x_3$-axis is tangent to the orbits and in the direction of the muon momentum. The magnetic field is $B_2 = -B$. As shown in [6], $H'$ by itself accounts for the main features of (1).

Assume now that the coupling of rotation to $| \psi_+ \rangle$ differs in strength from that to $| \psi_- \rangle$ as in [1]. This can be accomplished by multiplying the Mashhoon term in (2) by the matrix $A = \begin{pmatrix} \kappa_+ & 0 \\ 0 & \kappa_- \end{pmatrix}$ that reflects the different coupling strength of rotation to the two helicity states. A violation of $P$ and $T$ in $H$ thus arises through $\kappa_+ - \kappa_- \neq 0$. The constants $\kappa_+$ and $\kappa_-$ are assumed to differ from unity by small amounts $\epsilon_+$ and $\epsilon_-$. Persistent, residual discrepancies in very high precision measurements of $a_\mu$ for both positive and negative muons [3,4] can now
be used to find the values of both \( \epsilon_+ \) and \( \epsilon_- \). The same discrepancies yield upper limits on \( P \) and \( T \) invariance violations in spin-rotation coupling.

The coefficients \( a(t) \) and \( b(t) \) in (3) evolve in time according to

\[
\frac{i}{\hbar} \frac{\partial}{\partial t} \left( \begin{array}{c} a(t) \\ b(t) \end{array} \right) = M \left( \begin{array}{c} a(t) \\ b(t) \end{array} \right),
\]

where

\[
M = \left( \begin{array}{cc} E - i \frac{\omega}{2} & i \left( \kappa_+ \frac{\omega_2}{2} - \mu B \right) \\ -i \left( \kappa_- \frac{\omega_2}{2} - \mu B \right) & E - i \frac{\omega}{2} \end{array} \right),
\]

and \( \Gamma \) represents the width of the muon. The spin-rotation term is off-diagonal in (5) and does not therefore couple to matter universally. It violates Hermiticity as shown in [1] and, in a general way, in [7]. It also violates \( T \), \( P \) and \( PT \), while nothing can be said about \( CPT \) conservation which requires \( H \) to be Hermitian [8,9]. Non-Hermitian corrections to the width of the muon are of second order in \( \epsilon_- 's \) and are neglected.

\( M \) has eigenvalues \( \hbar = E - i \Gamma / 2 \pm R \) and eigenstates

\[
\left| \psi_1 \right> = N \left[ \eta |\psi_+ > + |\psi_- > \right], \\
\left| \psi_2 \right> = N \left[ -\eta |\psi_+ > + |\psi_- > \right],
\]

where

\[
R = \sqrt{\left( \kappa_+ \frac{\omega_2}{2} - \mu B \right) \left( \kappa_- \frac{\omega_2}{2} - \mu B \right)},
\]

\( |N|^2 = 1/ (1 + \left| \eta \right|^2) \) and \( \eta = \frac{i}{\hbar} \left( \kappa_+ \frac{\omega_2}{2} - \mu B \right) \). Then the muon states (3) that satisfy the condition \( |\psi(0) >= |\psi_- > \) are

\[
|\psi(t) > = \frac{e^{-iE t - \Gamma t / 2}}{2} [-2i\eta \sin R t |\psi_+ > + 2 \cos R t |\psi_- >],
\]

and the spin-flip probability is

\[
P_{\psi_- \rightarrow \psi_+} = \left| < \psi_+ | \psi(t) > \right|^2 \\
= \frac{e^{-\Gamma t} \kappa_+ \omega_2 - 2 \mu B}{\kappa_- \omega_2 - 2 \mu B} \left[ 1 - \cos (2 R t) \right].
\]

When \( \kappa_+ = \kappa_- = 1 \), (9) reproduces the essential features of (1) [6].

Substituting \( \kappa_+ = 1 + \epsilon_+, \kappa_- = 1 + \epsilon_- \) into (9), one finds

\[
P_{\psi_- \rightarrow \psi_+} = \frac{e^{-\Gamma t} \epsilon_+ - a_\mu}{\epsilon_- - a_\mu} \left[ 1 - \cos (2 R t) \right].
\]

A similar expression for \( P_{\psi_+ \rightarrow \psi_-} \) can be obtained starting from the condition \( |\psi(0) >= |\psi_+ > \).

We attribute the discrepancy between \( a_\mu(exp) \) and \( a_\mu(SM) \) to a violation of the conservation of the discrete symmetries by the spin-rotation coupling term \( -\frac{1}{2} A \omega_2 \sigma^2 \). The upper limit on the violation of \( P, T \) and \( PT \) is derived from (10) assuming that the deviation from the current value of \( a_\mu(SM) \) is wholly due to \( \epsilon_\pm \). The most precise sets of data yet give \( a_{\mu_+}(exp) - a_{\mu_+}(SM) \equiv b = 26 \times 10^{-10} \) for positive muons [3] and \( a_{\mu_-}(exp) - a_{\mu_-}(SM) \equiv d = 33 \times 10^{-10} \) for negative muons [4]. These then are the upper limits to the violation of the discrete symmetries.

At the same time the two values of \( a_{\mu_+}(exp) - a_{\mu_+}(SM) \) are due, in the model, to the different coupling strengths between rotational inertia and the two helicity states of the muon. The values of \( \epsilon_+ \) and \( \epsilon_- \) can be determined from \( \cos (2 R t) \) in (9). The equations are

\[
(a_{\mu_+} - \epsilon_+) (a_{\mu_+} - \epsilon_-) = b^2
\]

and

\[
(a_{\mu_-} - \epsilon_+) (a_{\mu_-} - \epsilon_-) = d^2.
\]

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The reality condition that follows from the solutions of (11) and (12) is satisfied, for $b$ and $d$ fixed, by ranges of values of $a_\pm$ and $a_-$ that are compatible with present experimental accuracies. Equations (11) and (12) have the approximate solutions

$$
\epsilon_+ \simeq \frac{a_{\mu_+} + a_{\mu_-}}{2} - \frac{d^2 - b^2}{2 (a_{\mu_-} - a_{\mu_+})}
$$

(13)

and

$$
\epsilon_- \simeq a_{\mu_+} + \frac{2b^2 (a_{\mu_-} - a_{\mu_+})}{(a_{\mu_-} - a_{\mu_+})^2 + (d^2 - b^2)}
$$

(14)

More precise, numerical solutions give $\epsilon_+ \simeq 11659189 \cdot 10^{-10}$, $\epsilon_- \simeq 11659152 \cdot 10^{-10}$ and $\Delta \epsilon \equiv \epsilon_+ - \epsilon_- \simeq 37.65878 \cdot 10^{-10}$. These values are significant in view of the precision with which $a_{\mu\pm}, b, d$ have been determined. In our simple model, therefore, the coupling of rotation to positive helicity is larger than that to negative helicity.

In conclusion, muons in storage rings are rotating quantum gyroscopes that are sensitive probes of rotational inertia. Extremely precise $g - 2$ experiments also concern the violation of the equivalence principle in quantum mechanics [10,11] and the conservation of discrete symmetries. Possibly, the deviations of $\epsilon_\pm$ from unity that are consistent with $g - 2$ experiments are both of the order of $10^{-3}$, and differ from each other by $\Delta \epsilon \simeq 3.7 \cdot 10^{-9}$. While small values of $\epsilon_\pm$ do not give rise to measurable mass differences in macroscopic objects [1], violations of the discrete symmetries may have interesting astrophysical and cosmological implications.

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