Born–Infeld AdS black holes as heat engines

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Abstract
We study the efficiency of heat engines that perform mechanical work via the $pdV$ terms present in the first law in extended gravitational thermodynamics. We use charged black holes as the working substance, for a particular choice of engine cycle. The context is Einstein gravity with negative cosmological constant and a Born–Infeld nonlinear electrodynamics sector. We compare the results for these ‘holographic’ heat engines to previous results obtained for Einstein–Maxwell black holes, and for the case where there is a Gauss–Bonnet sector.

Keywords: black holes, thermodynamics, quantum gravity

(Some figures may appear in colour only in the online journal)

1. Introduction

This paper presents a class of corrections to the efficiency of holographic heat engines which have charged black holes as the working substance. The corrections arise as a result of nonlinear extension of the electromagnetic sector, the Born–Infeld action. Holographic heat engines were defined in [1]. They are a natural concept in extended gravitational thermodynamics, which, in making the cosmological constant ($\Lambda$) dynamical in a theory of gravity, supplies a pressure variable $p = -\Lambda/8\pi$ and its conjugate volume $V$ (see [2–13]). One may extract mechanical work via the $pdV$ term in the first law of thermodynamics, and so it is possible to define a cycle in state space during which there is a net input heat $Q_H$ flow, a net output flow $Q_C$, and a net output work $W$, such that $Q_H = W + Q_C$. The efficiency is then $\eta = W/Q_H$. Its value is determined by the equation of state of the system and the choice of cycle in state space. The gravitational solution (a black hole, in our case) supplies the equation

\[ \text{(Some figures may appear in colour only in the online journal)} \]

1 Here we are using geometrical units where $G$, $c$, $\hbar$, $k_B$ have been set to unity.
of state: the temperature $T$, entropy $S$, enthalpy $H$ and other quantities can be defined [2, 14–17], and there are relations between them.

We will choose the cycle given in figure 1, following earlier work in [1, 18], where it is explained why that is a natural choice for static black holes, which we will study here. The Born–Infeld action [23–25] is a nonlinear generalization of the Maxwell action, controlled by a parameter $\beta$:

$$\mathcal{L}(F) = 4\beta^2 \left( 1 - \sqrt{1 + \frac{F^\mu\nu F^\mu\nu}{2\beta^2}} \right), \tag{1}$$

where in the limit $\beta \to \infty$ we recover the Maxwell action. The completion of Maxwell into Born–Infeld is quite natural in string theory [26], where $1/\beta \sim 2\pi\alpha'$, i.e. it is like the tension of the string. The corrections about the $\beta \to \infty$ expansion (the famous zero-slope limit) is an infinite family of $\alpha'$ corrections. In studying the efficiency of our black hole heat engine as a function of $\beta$, we can therefore think of it as capturing the effects of $\alpha'$-like corrections in some underlying string theory model, but it is not necessary to do so. It is interesting enough to consider the system in its own right.

Another possible context for this study is the fact that we will work in negative cosmological constant (defining a positive pressure), for which such physics has an holographic duality [27–31] to non-gravitational field theories in one-dimension fewer, at large $N$ (where $N$ is the rank of a field theory gauge group, or an analogue thereof). As pointed out in [1], since changing $\Lambda$ involves changing the $N$ (or analogues thereof) of the dual theory, the heat engine cycle is a kind of tour on the space of a family of field theories rather than staying within one particular field theory. It is possible that the efficiency, $\eta$, of such cycles may help characterize transport and response properties of the family of theories dual to a given class of black hole. Our studies therefore concern corrections to such physics as well, but our focus here will be to study the properties of our new Born–Infeld-corrected black hole engines for their own sake, leaving examination of the implications for those possible applications for another time.

Once we have extracted the efficiency of our engines in the presence of Born–Infeld (and we will do so working in a high temperature limit) we will compare the results to the Einstein–Maxwell case, and also contrast it with the results obtained in [18] for another class of $\alpha'$-like corrections, the Gauss–Bonnet case. In this way we will have studied two distinct classes of corrections to these engines’ efficiency: corrections to the Einstein sector, and corrections to the Maxwell sector.

2. The Black holes and the equation of state

Our Einstein–Hilbert–Born–Infeld bulk action in $D$-dimensions is:

$$I = \frac{1}{16\pi} \int d^D x \sqrt{-g} \left( R - 2\Lambda + \mathcal{L}(F) \right), \tag{2}$$

References [19–22] have since done further studies of such heat engines.

2 Here, we follow a common strand of terminology in the literature: strictly speaking, the displayed action (1) is due to Born [23, 24]. In the original $D = 4$ context, the full Born–Infeld action has under the square root a quartic term $(\mathbf{B} - \mathbf{E})^2$ as well as the quadratic term $\mathbf{E}^2 - \mathbf{B}^2$ shown. However, in the case of vanishing magnetic sector (as will be true in this paper), the actions have the same content. The action (1), in any number of dimensions, is often referred to as Born–Infeld in the literature—as is the $D$-dimensional version of the full action that Born and Infeld wrote [25] (with $\det((\eta_{\mu\nu} + F_{\mu\nu}/\beta)$ under the square root). Thanks to David Chow for prompting the clarification after an earlier version of this manuscript appeared.

3 As pointed out in [32] it also involves changing the size of the space the field theories live on.
with $\mathcal{L}(F)$ given in equation (1). The cosmological constant sets a length scale $l$ according to:

$$\Lambda = -\frac{(D - 1)(D - 2)}{2l^2}. \quad (3)$$

The black hole has mass and charge parameters $m$ and $q$, with metric [33–35]

$$ds^2 = -Y(r)dr^2 + \frac{dr^2}{Y(r)} + r^2d\Omega_{D-2}^2, \quad (4)$$

where $d\Omega_{D-2}^2$ is the metric on a round $D - 2$ sphere with volume $\omega_{D-2}$, and

$$Y(r) = 1 - \frac{m}{r^{D-3}} + \frac{r^2}{l^2} + \frac{4\beta^2}{(D - 1)(D - 2)}\left(1 - \sqrt{1 + \frac{(D - 2)(D - 3)q^2}{2\beta^2}}\right)$$

$$+ \frac{2(D - 2)q^2}{(D - 1)r^{2D-7}}\; _2F_1\left[\frac{D - 3}{2D - 4}, \frac{1}{2}; \frac{3D - 7}{2D - 4}; \frac{(D - 2)(D - 3)q^2}{2\beta^2}\right] \quad (5)$$

where $_2F_1$ is the hypergeometric function. The gauge potential is:

$$A_r = -\frac{q}{c} \frac{1}{r^{D-2}}\; _2F_1\left[\frac{D - 3}{2D - 4}, \frac{1}{2}; \frac{3D - 7}{2D - 4}; \frac{(D - 2)(D - 3)q^2}{2\beta^2}\right],$$

with $c = \sqrt{\frac{2(D - 3)}{D - 2}}$. \quad (6)

The mass and charge of the solution are given by:

$$M = \frac{(D - 2)\omega_{D-2}}{16\pi}m \quad \text{and} \quad Q = \sqrt{\frac{2(D - 2)(D - 3)}{8\pi}}\left(\frac{\omega_{D-2}}{8\pi}\right)q. \quad (7)$$

Given an horizon of radius $r_+$ (the largest root of $Y(r) = 0$), the temperature $T$, entropy $S$, and volume $V$ are given by [34–36]:

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**Figure 1.** Our engine.
\[ T = \frac{1}{4\pi} \left( 16\pi p r_+ + (D - 3) + \frac{4\beta^2 r_+^2}{(D - 2)} \left( 1 - \sqrt{1 + \frac{(D - 2)(D - 3)q^2}{2\beta^2 r_+^{2D-4}}} \right) \right), \quad (8) \]

\[ S = \frac{\omega_0}{4} r_+^{D-2}, \quad \text{and} \quad V = \frac{\omega_0}{(D - 1)} r_+^{D-1}, \quad (9) \]

where we have used that \( p = -\Lambda/8\pi \) and equation (3). The temperature expression (8) can be re-arranged into an equation of state (actually a family of equations of state parameterized by \( q \), which we will keep fixed) in the \( p - r_+ \) plane, or equivalently the \( p - V \) plane. As noted in [18], in the high temperature limit, the leading behaviour of this equation of state is:

\[ p^{1/(D-1)} \sim T, \quad (10) \]

a sort of ideal gas limit for our black holes. At lower temperatures there can be quite non-trivial behaviour as a coming from multivaluedness of the state curve (generalizing what was found for Reissner–Nordström black holes in [37–39]) giving rise to non-trivial phase transitions [40–43], which will not be our focus here.

### 3. The engine efficiency

#### 3.1. The specific heat

The specific heat at constant pressure \( C_p \equiv T\partial S/\partial T \) is the next quantity we need. It is most easy to compute it in terms of \( r_+ \), as discussed in [18], and the result is, for general \( D \):

\[ C_p = \frac{(D - 2)\omega_0}{4} r_+^{D-2} \]

\[ \times \left( \frac{16\pi}{(D - 2)(D - 3)p} r_+^{2D-4} + \frac{4\beta^2}{(D - 2)(D - 3)} \left( 1 - R^2 \right) r_+^{2D-4} \right) \]

\[ \times \left( \frac{16\pi}{(D - 2)(D - 3)} p r_+^{2D-4} - r_+^{2D-6} + \frac{4\beta^2}{(D - 2)(D - 3)} \left( 1 - R^2 \right) r_+^{2D-4} + 2(D - 2)q^2 R^{-\frac{1}{2}} \right), \quad (11) \]

where

\[ R \equiv 1 + \frac{(D - 2)(D - 3)q^2}{2\beta^2 r_+^{2D-4}}. \quad (12) \]

#### 3.2. The efficiency \( \eta \)

For our engine cycle defined in figure 1, we have

\[ W = (V_2 - V_1)(p_1 - p_3), \quad (13) \]

where the subscripts refer to the quantities evaluated at the corners labeled (1, 2, 3, 4). The heat flows take place along the top and bottom, with the upper isobar giving the net inflow of heat:

\[ Q_H = \int_{T_1}^{T_2} C_p(p, T) \, dT. \quad (14) \]

The efficiency is then \( \eta = W/Q_H \).
3.3. The high temperature limit

To get an explicit expression for $C_p$ in terms of $T$ so that we can integrate along the isobar, we take a high temperature limit, solving for $r_+$ perturbatively in a large $T$ expansion, using equation (8), and substituting into (11). We expand $V$ in the same way. For example, in $D = 4$:

$$\frac{r_+}{2} = \frac{T}{2p} - \frac{1}{4\pi T} + \frac{1}{8} \frac{p(8\pi pq^2 - 1)}{\pi^2 T^3} + \frac{1}{8} \frac{p^2(16 q^4 p^4 - 1)}{\pi^3 T^5}
+ \left( \frac{1}{32} \frac{p^3(5 - 12 q^2 p^2 + 192 q^6 p^2 \pi^2)}{\pi^4} - \frac{4}{\pi} \frac{p^6 q^4}{T^7} \right) + \cdots,$$

$$V = \frac{4\pi}{5} r_+^4 = \frac{\pi}{6p^3} T^3 - \frac{1}{4} \frac{T}{p^2} + 8 \frac{q^2}{T} + 1 \frac{48 q^4 p \pi - 1}{\pi^2 T^3}
- \frac{1}{32} \frac{p(1 - 48 q^2 p^2 + 128 q^4 p^2 \pi^2 + 128 p^3 q^4 \pi^3 / \beta^2)}{\pi^3 T^3} + \cdots,$$

$$\int C_p dT = \frac{\pi}{6p^3} T^3 + \frac{1}{8} \frac{16\pi pq^2 - 1}{\pi T} + 1 \frac{p(24 q^4 p \pi - 1)}{\pi^2 T^3}
- \frac{1}{32} \frac{p^2(5 - 160 q^2 p^2 + 320 q^4 p^2 \pi^2) + 24 p^5 q^4 \beta^2}{\pi^3} \frac{1}{T^5} + \cdots$$ (15)

and in $D = 5$:

$$\frac{r_+}{4} = \frac{3}{2} \frac{T}{p} - \frac{1}{2\pi T} + 4 \frac{p^2(32 q^2 \pi^2 p^2 - 9)}{81 \pi^2 T^3} + 3 \frac{p^4(128 q^2 p^2 \pi^2 - 15)}{729 \pi^2 T^3}
+ \frac{32}{19683} \frac{p^5(-34560 q^2 p^2 \pi^2 + 10240 q^4 p^5 \pi^4 + 1701)}{\pi^5}
- \frac{131072}{19683} \frac{p^{10} q^4}{\pi^5} \frac{1}{T^5} + \cdots$$ (16)

with

$$V = \frac{81}{2} \frac{r_+^4}{512} \frac{\pi^2 T^4}{p^4} - \frac{27}{64} \frac{\pi^2 T^2}{p^4} + \frac{9}{64 p^2} + 4 \frac{\pi pq^2}{3 \pi^2 T^2}
+ \frac{1}{96} \frac{(256 q^2 p^2 \pi - 3)}{\pi^2 T^4} + 1 \frac{p(464 q^2 p^2 \pi^2 - 9)}{108 \pi^2 T^6}
- \frac{1}{2916} \frac{p^5(-40320 \pi^2 p^2 q^2 + 28672 \pi^4 p^4 q^2 + 567 + 16384 q^4 p^5 \pi^5 / \beta^2)}{\pi^4 T^8} + \cdots,$$

$$\int C_p dT = \frac{81}{512} \frac{\pi^2 T^4}{p^3} - \frac{27}{128} \frac{\pi^2 T^2}{p^3} + \frac{1}{96} \frac{464 q^2 p^2 \pi^2 - 9}{\pi T^2}
+ \frac{5}{288} \frac{p(256 q^2 p^2 \pi^2 - 9)}{\pi^2 T^4} + 7 \frac{p^2(320 q^2 p^2 \pi^4 - 9)}{216 \pi^2 T^6}
- \frac{1}{324} \frac{p^3(-8064 \pi^2 p^2 q^2 + 4096 \pi^4 p^4 q^4 + 189 + 2048 q^4 p^5 \pi^6 / \beta^2)}{\pi^4 T^8} + \cdots$$ (17)

These expressions for $V$ and $\int C_p dT$ can now be used to compute the efficiency via equations (13) and (14), taking their ratio. Both quantities are evaluated at the pressure of the
upper isobar, \( p_1 \), and because \( pV \) and \( \int C_p dT \) have identical leading terms, we have \( \eta = (1 - p_2/p_1) + \cdots \) with corrections that can be evaluated readily by substitution (see [1, 18] for further discussion).

A striking feature of these expansions is how late \( \beta \) enters: it is at order \( T^{-5} \) for \( D = 4 \) and at order \( T^{-8} \) for \( D = 5 \). This will mean (as we shall see in the next section) that the variation in the efficiency as a function of \( \beta \) will be somewhat understated as compared to the variation with \( \alpha \) (the coefficient of the Gauss–Bonnet action) in the companion study of [18]. There, \( \alpha \) appears immediately at the next-to-leading term at order \( T^2 \).

4. Two studies of \( \eta(\beta) \)

Equipped with our high temperature expansion, we can now study the efficiency as a function of \( \beta \), seeing how \( \eta(\beta) \) behaves as we move away the \( \beta = \infty \) (Maxwell) limit. The efficiency in the Einstein–Maxwell limit will be denoted \( \eta_0 = \lim_{\beta \to \infty} \eta(\beta) \). We will study the two schemes that were defined in [18], determined by what parameters of the cycle we specify and hold fixed as we change \( \beta \).

4.1. Scheme 1

Here, for our engine cycle (see figure 1) we specify the two operating pressures \( (p_1, p_2) \) and the two temperatures \( (T_1, T_2) \). We can evaluate the efficiency in this scheme as a function of \( \beta \), seeing how it moves away from the benchmark \( \eta_0 \) of Maxwell electrodynamics.

Actually, at a given value of \( \beta \) we can compare to an important additional benchmark, the Carnot efficiency \( \eta_C = 1 - T_C/T_H \), where \( T_C \) and \( T_H \) are, respectively, the lowest and highest temperatures our engine can attain. This is the efficiency obtained with a reversible heat engine operating between those two temperatures. Although we’ve specified \( T_H \equiv T_2 \), \( \eta_C \) changes with \( \beta \) since \( T_C \) does: the equation of state must be used to determine \( T_1 \equiv T_C \). We observe that as \( \beta \) runs from \( \infty \) toward smaller values \( T_C \) increases slowly. Correspondingly, the Carnot efficiency decreases. See figure 2 for the exact \( T_C \) and \( \eta_C \) for a sample range \( 10^{-2} < \beta < 10^2 \). The Maxwell limit is to the right. (This is analogous to what was seen in scheme 1 in [18].) As already remarked at the end of the last section, the dependence on \( \beta \) is relatively weak, and for all quantities plotted in this section, the variation is small over this wide range, with most of the change beginning late in the range at a ‘turnaround region’ where, roughly, \( \beta \sim 10^{-1} \). All plots are against \( \log_{10}(\beta) \) to better display the features.

Figure 3(a) displays the ratio \( \eta/\eta_C \) and figure 3(b) shows that the ratio \( \eta/\eta_0 \), both plotted against \( \log_{10}(\beta) \). The Maxwell limit is to the right. In figure 3(a), it can be seen that the ratio grows slowly for a while and then rises more rapidly in the turnaround region. In figure 3(b) there is a very slow initial decline before the faster fall in the turnaround region. It is worth comparing this behaviour to that seen in the Gauss–Bonnet case for scheme 1, as exhibited in [18]. We will discuss the comparison further in section 5.

4.2. Scheme 2

In this scheme for our engine (again see figure 1) we instead specify the temperatures \( (T_2, T_1) \), equivalent to specifying \( (T_H, T_C) \), as well as the volumes \( (V_2, V_1) \) (which also gives the pair \( (V_3, V_4) \)). Now the Carnot efficiency \( \eta_C \) is fixed for all \( \beta \). Instead, however, the pressures \( p_1 = p_2 \) and \( p_3 = p_4 \) must be determined using the equation of state, and so are now \( \beta \)-dependent. (We checked that the pressures in the engine remained physical over the range of \( 10^{-2} < \beta < 10^2 \), which is to be expected since we have fixed our highest and lowest
temperatures to be far enough into the high temperature regime. In figure 4, we again plot the ratios $\eta / \eta_C$ and $\eta / \eta_0$, against $\log_{10}(\beta)$. Again, the Maxwell limit is to the right. We will discuss these results further in section 5.

**Figure 2.** (a) The exact temperature $T_C$ versus $\log_{10}(\beta)$, in scheme 1. (b) The exact Carnot efficiency $\eta_C$, versus $\log_{10}(\beta)$, also in scheme 1. These quantities were computed using the exact equation of state. See text. (Here, we have chosen the values $p_1 = 5$, $p_2 = 3$, $T_1 = 50$, $T_2 = 60$, and $q = 0.1$. The same key features were observed for a range of sample values, including even higher temperatures.)

**Figure 3.** (a) The engine efficiency $\eta / \eta_C$ versus $\log_{10}(\beta)$, in scheme 1. (b) The ratio $\eta / \eta_0$ versus $\log_{10}(\beta)$ over the same range, also in scheme 1. (See the caption of figure 2 for the parameter values chosen.)
It is worth noting that the $D = 4$ case was explored explicitly as well, for each scheme, and the qualitative structure of the results was found to be the same as for the $D = 5$ case explored here, so no detailed results are reported from that case. The features (a slow change followed by the characteristic elbow or knee in the turnaround region) are a bit more pronounced since $\beta$ appears at slightly higher order: $O(T^5)$. It is expected that higher $D$ will also work similarly.

5. Closing remarks

In [18], the corrections to the geometrical sector, from a Gauss–Bonnet term with coefficient $\alpha$, were studied for their effect on the efficiency of a heat engine in the same schemes 1 and 2 studied here. In the present study we looked instead at corrections to the Maxwell sector, controlled by parameter $\beta$ in the Born–Infeld action, examining their effects on the heat engine efficiency. These two studies reveal features that are in sharp contrast to each other.

The most striking contrast is how weak the $\beta$-corrections are compared to the $\alpha$-corrections, as noted at the end of section 3.3, and as can be seen in all the figures in section 4. The resulting magnitude of the variation in the efficiency with $\beta$ is of order $10^{-12}$. Even after restrictions to the relatively narrow physical window allowed for $\alpha$, the variations there are many orders of magnitude greater [18]. (Our examples used in each case have the same values for the fixed parameters to allow this comparison.)

There is a much larger window of available values of $\beta$ to explore, while $\alpha$ is tightly constrained by certain physical requirements. This difference may ultimately be traceable to the fact that $\alpha$ controls just one finite set of correction terms in the action, while in contrast $\beta$ can explore a much richer range of effects starting with an infinite family of terms (in the sense of expanding the action (1) around the $\beta \to \infty$ limit) all the way to the effects at small $\beta$ that generate the turnaround region seen in section 4.
The large (but not infinite) $\beta$ regime is where we can best compare directly to the small $\alpha$ regime that is the physical window in [18]. Here again, we see some contrasts. In scheme 1, the ratios $\eta/\eta_0$ and $\eta/\eta_0$ were seen to increase as the $\alpha$-corrections were turned on. The opposite is seen here for the ratio $\eta/\eta_0$. In scheme 2, the behaviour of the ratios are of opposite character for the $\beta$ variations versus the $\alpha$, variations, increasing for the former case, and decreasing for the latter.

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