A comparative study of several boundary conditions on the body surface for the meshless electromagnetic scattering method

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Abstract. In order to accurately calculate the radar cross sections (RCS) of stealth aircrafts by using the meshless method, several typical body surface boundary conditions from finite-volume time-domain (FVTD) methods, which are suitable for the meshless method, have been compared. The bistatic RCS of a 2-D cylinder irradiated by transverse magnetic wave (TM wave) or transverse electric wave (TE wave) and a 3-D sphere irradiated by different polarization waves are calculated by the meshless method based on different boundary conditions. The numerical results show that the bistatic RCS calculated by the first boundary condition is closest to the series solution, which indicates that the scattering electric field on the surface of perfect conductors is only related to the incident electric field, and the scattering magnetic field is related to the scattering magnetic field near the body surface and the variation of the normal scattering electric field on the body surface. Finally, based on the selected optimal body surface boundary condition, the paper present the electromagnetic scattering field and the bistatic RCS for a 3-D stealth aircraft model, which shows the ability of the meshless method in dealing with 3-D practical problems to a certain extent.

1. Introduction

In the past 20 years, people in both academic and engineering fields have focused more and more attention on the research of meshless methods. In the computational domain, meshless methods only involve the distribution of points and do not need to connect the discrete points to form grids, which make them flexible to implement[1]. At present, meshless methods have been developed and applied to solid mechanics[2], fluid mechanics[1], electromagnetics[3] and other fields.

Representative meshless methods for computing electromagnetic fields contain the Element Free Galerking method[4], the meshless method based on radial basis functions[5] and the meshless method based on Computational Fluid Dynamics (CFD)[3]. The former two kinds usually need to choose basis functions or shape functions, which can affect the quality of the companion matrix as well as the implementation of the boundary condition[6]. The latter kind is from CFD for solving Euler equations[1], which is different from the former two kinds in computing spatial derivatives and physical flux related
to the governing equations. At present, this kind of meshless method has been developed to solve RCS
for 2-D and 3-D perfect conductors[3].

According to this meshless method[3], the incident wave is introduced by the boundary condition on
the body surface. In order to reduce the influence of the numerical dissipation on calculated results of
RCS, the scattering electromagnetic fields needed for the calculation of RCS are also extracted on the
body surface. Therefore, the boundary condition on the body surface for this meshless method can not
only affect the iterative computation of scattering electromagnetic fields, but also directly affect the
calculated results of RCS. Therefore, it is important and necessary to study the accurate body surface
boundary condition for developing the meshless method.

We notice that there are several typical body surface boundary conditions in FVTD methods for
calculating RCS of perfect conductors. Therefore, this paper intends to introduce these boundary
conditions into the above meshless method and apply them to the iterative calculation of scattering
electromagnetic fields. In order to make a quantitative analysis of the calculation error of RCS based
on different boundary conditions, a 2-D cylinder example and a 3-D sphere example which have series
solutions are selected for comparison. Finally, based on the selected optimal body surface boundary
condition, the electromagnetic stealth characteristics for a 3-D stealth aircraft model is studied in order
to show the ability of the meshless method in dealing with 3-D practical problems.

2. The meshless method

2.1. Maxwell's equations

In 3-D Cartesian coordinate system, the non-dimensional form of Maxwell's equations may be written
as:

$$\frac{\partial W}{\partial t} + \left( \frac{\partial F_1}{\partial x} + \frac{\partial F_2}{\partial y} + \frac{\partial F_3}{\partial z} \right) = S$$

(1)

Where $W = \begin{bmatrix} \varepsilon E_x & \varepsilon E_y & \mu H_x & \mu H_y & \mu H_z \end{bmatrix}^T$, $F_1 = \begin{bmatrix} 0 & H_z & -H_y & 0 & -E_z & E_y \end{bmatrix}^T$, $F_2 = \begin{bmatrix} -H_z & 0 & H_x & 0 & -E_z & E_y \end{bmatrix}^T$, $F_3 = \begin{bmatrix} H_y & -H_z & 0 & -E_y & E_x & 0 \end{bmatrix}^T$, $S = \begin{bmatrix} -\sigma E_x & -\sigma E_y & -\sigma E_z & -\sigma_m H_x & -\sigma_m H_y & -\sigma_m H_z \end{bmatrix}$, $E = (E_x, E_y, E_z)$ is the electric
field vector, $H = (H_x, H_y, H_z)$ is the magnetic field vector, $\varepsilon$ is the electric permittivity, $\mu$ is the
magnetic permeability, $\sigma$ is the conductivity, $\sigma_m$ is the magnetic resistivity.

2.2. Points distribution and generation of clouds of points

According to the meshless method, a set of points should be distributed in the computational domain
firstly. Sometimes points based on grids can be used for convenience, sometimes techniques of
distributing points directly can be used if needed. After points being distributed, clouds of points then
should be constructed. For the instance of 2-D situations, how the cloud of points $C_i$ constructed can
refer to reference [3]. In figure 1, the cloud of points $C_i$ includes central point $i$ and satellite points
1–6.
2.3. Approximation of spatial derivatives and computation of physical flux

Based on the cloud of points $C_i$, the spatial derivatives at the central point $i$ can be approximated as

$$
\begin{align}
  a_1 &= \sum \alpha_k (f_k - f_i), \\
  a_2 &= \sum \beta_k (f_k - f_i), \\
  a_3 &= \sum \gamma_k (f_k - f_i)
\end{align}
$$

or

$$
\begin{align}
  a_1 &= \sum \alpha_{ik} (f_{ik} - f_i), \\
  a_2 &= \sum \beta_{ik} (f_{ik} - f_i), \\
  a_3 &= \sum \gamma_{ik} (f_{ik} - f_i)
\end{align}
$$

where $f$ represents any component of the vector $W$, coefficients $\alpha_k$, $\beta_k$, $\gamma_k$, $\alpha_{ik}$, $\beta_{ik}$ and $\gamma_{ik}$ are only related to the coordinates of the central point and the satellite points, which can be calculated before time-marching.

According to the meshless method, a virtual interface is created at the midpoint between the central point and each satellite point (see figure 2) in order to compute the numerical flux $Q_k$.

![Figure 2. Schematic diagram of a virtual interface at the midpoint between the central point and the satellite point](image)

Then, $Q_k$ can be determined by solving the Riemann problem at the virtual interface. The approximate Riemann solution based on the cloud of points $C_i$ can be written as:

$$
\begin{align}
  -\hat{d}_{ik} \times H_{ik} &= -\hat{d}_{ik} \times \left[ (\mu c)_L H_L + (\mu c)_R H_R \right] + \left[ (\mu c)_L + (\mu c)_R \right] \\
  \hat{d}_{ik} \times E_{ik} &= \hat{d}_{ik} \times \left[ (e c)_L E_L + (e c)_R E_R \right] + \hat{d}_{ik} \times (H_R - H_L) \\
  \hat{d}_{ik} &\rightarrow \frac{1}{\sqrt{\varepsilon \mu}}, \quad H_L, \quad E_L, \quad H_R \quad \text{and} \quad E_R \quad \text{can be calculated by} \quad f_L = f_i + 0.5 \nabla f_i \cdot r_{ik}, \quad f_R = f_k - 0.5 \nabla f_k \cdot r_{ik}, \quad \nabla f_i \quad \text{and} \quad \nabla f_k \quad \text{can be calculated by equations (2)}.
\end{align}
$$

2.4. Time-marching and boundary conditions

After the spatial discretization, an explicit four-stage Runge-Kutta scheme is employed in time-marching. In order to obtain the solution of the Maxwell's equations, the perfectly matched layer boundary condition is imposed in the far field, and the perfect conductor boundary condition is imposed on the body surface. Several typical body surface boundary conditions from FVTD methods are discussed in the following text.

1. According to reference [7]:

$$
\hat{n} \times E' = 0
$$

2. $\hat{n} \times H = \hat{n} \times H_s - \hat{n} \times \left[ \hat{n} \times (E_s - E) \right]/(\mu c)_s
$$

where $\hat{n}$ is the unit normal vector of the body surface at the point $j$ (see figure 3), the superscript 't' represents 'total fields', the subscript '*' represents the point $m$, $H$ and $E$ represent the scattering magnetic field and the scattering electric field respectively at the point $j$, which need to be calculated. Equation (5) indicates that $E$ is only determined by the incident electric field, and equation (6) indicates that $H$ is related to $H_s$ and the variation of the normal scattering electric field.
Figure 3. Schematic diagram of body surface

(2) Reference [8] gives the equation (5) and other two equations:

\[ \hat{n} \cdot \mathbf{H}^i = 0 \quad (7) \]

\[ \hat{n} \cdot \nabla (\hat{n} \times \mathbf{H}^i) = 0 \quad (8) \]

According to equation (7) and equation (8), we can obtained:

\[ \mathbf{H} = \mathbf{H}_s - (\mathbf{H}^i \cdot \hat{n} + \mathbf{H}_s \cdot \hat{n}) \hat{n} \]

(9)

where the superscript 'i' represents 'incident fields'. Equation (9) indicates that \( \mathbf{H} \) is only related to \( \mathbf{H}_s \) and \( \mathbf{H}^i \).

(3) Reference [9] also gives the equation (5) and the other equation:

\[ \mathbf{H} = \mathbf{H}_s - (\mathbf{H}^i \cdot \hat{n} + \mathbf{H}_s \cdot \hat{n}) \hat{n} - \hat{n} \times (\mathbf{E}_s - \mathbf{E}) / (\mu \varepsilon) \]

(10)

Equation (10) indicates that \( \mathbf{H} \) is related to \( \mathbf{H}_s \) and \( \mathbf{H}^i \) as well as the variation of the normal scattering electric field.

(4) Reference [10] gives the equation (5), the equation (7), the equation (8) and the other equation:

\[ \hat{n} \cdot \nabla (\hat{n} \cdot \mathbf{E}^i) = 0 \]

(11)

According to equation (5) and equation (11), we can obtained:

\[ \mathbf{E} = -\mathbf{E}^i + (\mathbf{E}^i \cdot \hat{n} + \mathbf{E}_s \cdot \hat{n}) \hat{n} \]

(12)

Equation (12) indicates that \( \mathbf{E} \) is related to \( \mathbf{E}^i \) and \( \mathbf{E}_s \).

It is important to illustrate that the point \( m \) is a virtual point introduced in order to apply the boundary condition, \( \mathbf{E}_s \) and \( \mathbf{H}_s \) can be obtained by interpolation of \( \mathbf{E} \) and \( \nabla \mathbf{E} \) as well as \( \mathbf{H} \) and \( \nabla \mathbf{H} \), where \( \nabla \mathbf{E} \) and \( \nabla \mathbf{H} \) can be calculated by equations (2). Because the incident wave is introduced by the boundary condition, and the scattering electromagnetic fields needed for the calculation of RCS are also extracted on the body surface, whether the boundary condition is accurate or not will be shown significantly in the calculated results of RCS.

3. Numerical results and discussion

In this section, based on the meshless method, a 2-D cylinder example and a 3-D sphere example are given to compare the above four different boundary conditions. Then, based on the selected optimal boundary condition, a study on the electromagnetic scattering fields and the bistatic RCS for a 3-D stealth aircraft model is carried out.

3.1. A 2-D cylinder example

Firstly, a 2-D cylinder case is selected to simulate. The cylinder body is assumed to be perfect electric conductor with \( a = \lambda \), where \( a \) is the radius of the cylinder. The computational domain is set as \( 12\lambda \times 12\lambda \) with 14720 points distributed. The incident harmonic TM wave and TE wave are both propagating along the \( x \) coordinate axis. The calculated bistatic RCS is presented in figure 4. When the cylinder is irradiated by TM wave, the mean deviations of calculated RCS based on the four different boundary conditions are 0.062dB, 0.421dB, 0.165dB, 0.421dB respectively. When the cylinder is irradiated by TE wave, the mean deviations are 0.196dB, 0.415dB, 0.196dB, 0.361dB respectively. The results show that the bistatic RCS calculated by the first boundary condition is
closest to the series solution\cite{11}, and the third boundary condition is second only to the first one. Therefore, it indicates that the scattering electric field on the surface of perfect conductors is only related to the incident electric field, and the scattering magnetic field is related to the scattering magnetic field near the body surface and the variation of the normal scattering electric field on the body surface.

![Figure 4. Bistatic RCS for the 2-D cylinder](image)

(a) irradiated by TM wave                  (b) irradiated by TE wave

**Figure 4. Bistatic RCS for the 2-D cylinder**

3.2. \textit{A 3-D sphere example}

Then, a 3-D sphere case is further chosen to solve. The sphere body is also assumed to be perfect electric conductor with $b = \lambda$, where $b$ is the radius of the sphere. For the 3-D situation, considering the rapidly increasing in the amount of computation, a relatively small computational domain is used, which is $8\lambda \times 8\lambda \times 8\lambda$ with 95984 points. The incident harmonic electromagnetic wave is also propagating along the $x$ coordinate axis, and the polarization angle is set as $\alpha = 0^\circ$ and $\alpha = 90^\circ$ respectively. The calculated bistatic RCS are presented in figure 5. When $\alpha = 0^\circ$, the mean deviations of calculated RCS based on the four different boundary conditions are 0.176dB, 0.557dB, 0.181dB, 0.578dB respectively. When $\alpha = 90^\circ$, the mean deviations are 0.202dB, 0.561dB, 0.228dB, 0.585dB respectively. The results also show that the bistatic RCS calculated by the first boundary condition is closest to the series solution\cite{11}. Therefore, it indicates that the first boundary condition is the optimal one.

![Figure 5. Bistatic RCS for the 3-D sphere](image)

(a) polarization angle $\alpha = 0^\circ$                  (b) polarization angle $\alpha = 90^\circ$

**Figure 5. Bistatic RCS for the 3-D sphere**
3.3. A 3-D stealth aircraft model example

Finally, based on the optimal body surface boundary condition, a 3-D stealth aircraft model case is calculated by the meshless method. The aircraft model (see figure 6(a)) is a low detectable configuration with $3\lambda$ length from nose to tail and $3.4\lambda$ width between two wingtips. The computational domain is also set as $8\lambda \times 8\lambda \times 8\lambda$ with 85328 points. The incident wave is propagating along the $z$ coordinate axis with the polarization angle $\alpha = 0^\circ$. Contours of scattering field is presented in figure 6(b), and the calculated bistatic RCS is also presented in figure 6(c). It could be observed from figure 6(b) that the scattering field is strongest in the $z$-axis direction (corresponding to $\theta = 0^\circ$), which is consistent with the bistatic RCS distribution (see figure 6(c)).

![Figure 6](image)

(a) Surface points distribution  (b) Contours of scattering field  (c) Bistatic RCS

Figure 6. The 3-D stealth aircraft model example

4. Conclusions

Based on the meshless method, several typical body surface boundary conditions from FVTD methods are compared. Both the 2-D cylinder example and the 3-D sphere example show that the first boundary condition is the optimal one, which indicates that the scattering electric field on the surface of perfect conductors is only related to the incident electric field, and the scattering magnetic field is related to the scattering magnetic field near the body surface and the variation of the normal scattering electric field on the body surface. The last 3-D stealth aircraft model example shows the good performance of the meshless method in calculating electromagnetic stealth characteristics of aircrafts.

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