Two-Stage Optimal Location Allocations of DPFC Considering Wind and Load Uncertainty

Xuedong Zhu, Liu Dichen and Jun Wu*

School of Electrical Engineering and Automation, Wuhan University, Wuhan, China

Distributed power flow controller (DPFC) has a considerable potential to regulate the power flow and generator rescheduling continuously. This study presents a novel two-stage stochastic model for optimal location allocations of the DPFC coupled with the interactions of DPFC to search for the optimal solutions. The Benders decomposition is utilized to reformulate the two-stage problem into the master problem and the subproblem. The optimal solution can be easily obtained with the master problem and subproblem iteratively. The relaxed DC power flow with a DPFC in the master problem accelerates the efficiency of optimal locations under a base condition. Slack variables are incorporated in the subproblem to check the feasibility of relaxed AC power flow. The optimal compensation levels of DPFC at different load/wind scenarios are optimized in the subproblem. The IEEE 118 bus system is conducted to verify the performance of the proposed procedure. The DPFC has positive impacts on unit costs, voltage performance, wind absorption, and power losses. Detailed simulation results illustrate the effect of the proposed approach.

Keywords: distributed power flow controller, relaxed AC-SOCP, Benders decomposition, uncertainty, optimal FACTS

1 INTRODUCTION

The existing transmission network can be challenging under the increasing growth of load. There is a congestion problem of power flow that should be mitigated because of the transfer capability limit of transmission lines (Hemmati et al., 2013). Transmission expansion planning (TEP) is one of the effective ways to alleviate congestions (Jabr, 2013). However, there are the characteristics of higher investment and occupation of transmission corridors (Ugranli et al., 2016). It is well known that the flexible AC transmission system (FACTS) can significantly influence the performance of power flow. With the rapid development of electronic technology, the FACTS is considered a strongly effective device to manage power flow (Yuan et al., 2010). It has the capability to control the voltage magnitude or phase angle and provide controllable active or reactive power compensation independently (Khanchi and Garg, 2013). The DPFC (Dai et al., 2019; Tang et al., 2020; Li et al., 2021; Zhang et al., 2021) is derived from the UPFC, which has the same external characteristics, such as voltage support, control of real power flow, and other functions. Compared to the UPFC device, the DPFC shows a great superiority in economy and reliability.

The optimal location and allocations of FACTS have been studied extensively. This research focused on three topics: equivalent injection model, optimization goals, and solution approach. The equivalent injection model is the key to implementing control strategies and improving the solution efficiency. Many exertions have been made in the last few years to establish effective injection models
of FACTS. Based on the voltage source model of unified power flow controller (UPFC), two approaches have been proposed to solve the optimal problem combined with the power injection model (Orfanogianni and Bacher, 2003; Tripathy et al., 2006; Shen et al., 2021a; Shen and Raksincharoensak, 2021a; Shen et al., 2021b; Shen and Raksincharoensak, 2021b), sensitivity analysis–based methods are the first choice to be employed to obtain the candidate of location, and another choice is the optimal power flow (OPF) method. Muwaffaq (Alomoush, 2004) proposed the II-model of UPFC to maintain the diagnose features of Jacobian matrix based on the port equivalent. A direct model has been proposed (Bhowmick et al., 2008) to simplify the difficulties of UPFC, the existing power system–installed UPFC is transformed into an augmented equivalent network without any UPFC, and the difficulty of Newton Raphson power flow diminishes dramatically due to the absence of UPFC. Many optimization goals with the FACTS injected have been researched intensively. Alomoush (2003), Alomoush (2004), Yang et al. (2021a), Yang et al. (2021b), Yang et al. (2021c), and Yang (2021) leveraged DC power flow to minimize the operating cost with the injection II-model of UPFC. Sarker and Goswami (2014) minimized the operating cost combined with sensitivity analysis–based methods, and the control values of UPFC and SVC can be directly obtained under the location of PI sensitivity. Several researchers (Verma and Gupta, 2006; Tiwari and Sood, 2012; Tiwari and Sood, 2013; Dawn and Tiwari, 2016) optimized the location allocations of FACTS into social welfare; this goal is to maximize the benefits of all anticipants, that is, to maximize the benefit of power sales and minimize the operating cost of the generator. Furthermore, some researchers use the FACTS to improve the performance of power flow, such as voltage stability (Singh, 2016; Zhang et al., 2020), power loss (Tripathy and Mishra, 2007; Sarker and Goswami, 2014), and transfer capability enhancement (Prasad et al., 2011; Rajabi-Ghahnavieh et al., 2015).

The mathematical formulations of FACTS location allocation are originally non-linear and non-convex because of its mixed integer non-linear programming (MINLP) model. The OPF method is the first approach to solving the MINLP problem. It can be solved by MATPOWER iteratively with the changing Jacobian matrix based on matrix block decompose technology and the injection model of FACTS. Noroozian et al. (1997) reconstructed the modified Jacobian matrix by correlating Jacobian’s matrix elements with the control variables of the UPFC load injection model. Pereira and Zanetta (2012) proposed the OPF approach with the control modes based on the voltage source model and power injection model of UPFC. Ebeed et al. (2019) and Vo Tien et al. (2019) established the modified matrix with installing variables of STATCOM or TCSC based on its shunt or series reactance model. The speed of the OPF method is questionable because of iteratively updating the Jacobian matrix. Another popular procedure is the heuristic method of solving the MINLP problem. Saravanan et al. (2007) presented the particle swarm optimization (PSO) technique to search the optimal solution of MINLP with a minimum investment cost of FACTS devices. Hooshmand et al. (2015) proposed a hybrid method that combines the bacterial foraging algorithm with a Nelder–Mead method to solve the MINLP problem. Ranganathan et al. (2016) proposed the self-adaptive firefly algorithm (SAFA) to optimize the power flow performance, such as voltage stability and power loss. However, the difficulties of FACTS injected into the system still exist, and its solving accuracy is difficult to guarantee because of the non-linear and non-convex characteristics of AC power flow. Linear approximation of AC power flow also has been utilized for the MISOCOP problem of FACTS. Nikoobakht et al. (2018) proposed a PWL approximation method to transform the MISOCOP problem to the MILP problem. Ding et al. (2015), Sahraei-Ardakani and Hedman (2015), and Sang and Sahraei-Ardakani (2017) developed an MILP model due to the robustness and high speed efficiency of the DC power flow. Second-order conic programming (SOCP) (Tang et al., 2018) is another method to solve the MISOCOP problem, and the optimal solution of convex optimization is easily obtained despite its non-linearity. However, the characteristic of reactive power is ignored in the DC power flow, whereas the SOCP model (Tang et al., 2018) with the DPFC hardly obtains the optimal solution whose decision variables belong to the open interval.

This article develops an equivalent power injection model (PIM) of DPFC considering its active compensation, which not only holds the external characteristic but also can be easily injected to the system. A two-stage MISOCOP problem consisting of the operating cost and investment of DPFC is formulated to optimize the location and compensation level of DPFC. The main contributions of this paper can be summarized as follows:

1) The optimization method holds the internal characteristics of DPFC, maintaining the interactions and increasing the consistent performance of scheduling planning.

2) A nested method consisting of the reactive model and the PIM has been developed to optimize the locations and allocations of DPFC simultaneously, where the efficiency and accuracy have been increased.

We demonstrate the effectiveness of the proposed two-stage stochastic problem in the IEEE 118 bus system and insight into the influence on the performance of DPFC. This paper is organized as follows. Section 2 introduces the equivalent reactive model of DPFC and its operating principle. Section 3

![Figure 1: Configuration and principle of the DPFC.](image-url)
presents the two-stage stochastic model of optimal location-allocation problem. **Section 4** describes the two-stage procedure of Benders decomposition method. **Section 5** shows the results and discussion, while the conclusion is presented in **Section 6**.

### 2 DPFC STEADY-STATE MODEL

#### A. DPFC Configuration and Principle

The general configuration of the DPFC device includes a shunt converter and multiple series converters, as shown in **Figure 1**. The shunt converter is similar to the shunt component of UPFC, injection power flow into the linked bus. Unlike the unified series component of UPFC with a larger rated capacity, the independent distributed lower capacity series converters of DPFC can provide similar effects based on the superposition theorem. Furthermore, there is a huge difference between the third harmonic characteristics of DPFC and the fundamental wave of UPFC on the principle of power flow control. The UPFC absorbs the fundamental frequency power flow on the shunt side and directly injects it into the series side through VSC1 and VSC2. However, the shunt converter of DPFC absorbs the fundamental frequency power flow and converts it into the third harmonic and then converts it back to the fundamental frequency power flow through the series converters, injected into the system.

Based on the configuration and principle of DPFC, the independent capacity of a single series converter is small, and only after multiple series converters are added, power system requirements can be satisfied. A simplified DPFC circuit diagram can be derived, composed of a shunt inverter and multiple series inverters, as shown in **Figure 2**.

**Figure 3** illustrates the cascade series inverters of DPFC superimposed into equivalent unified inverters based on the superposition theorem. The cascade DPFC on the series side can be modeled as an independent voltage source. The third harmonic power is nested in the operating condition, and only the base power flow is reflected in the static perspective. Referring to the equivalent voltage source model of UPFC, the following equation describes the equivalent process clearly for the DPFC series side:

\[
V_T e^{j\theta_w} = V_{T1} e^{j\theta_{w1}} + \cdots + V_{Tn} e^{j\theta_{wn}} = \sum_{i=1}^{n} V_{T_i} e^{j\theta_{wi}}. \tag{1}
\]

As shown in **Figure 3**, the equivalent variables \( V_T, I_m, I_T \) are the injected series voltage, the series current, and the shunt current. They can be decomposed into an in-phase voltage/current and quadrature voltage/current as follows:

\[
\begin{align*}
\bar{V}_T &= (V_p + jV_q) e^{j\delta_n}, \\
\bar{I}_T &= (I_p + jI_q) e^{j\delta_p}.
\end{align*}
\]

For the KCL and KVL, the terminal voltage and current can be explained as follows:

\[
\begin{align*}
\bar{V}_m &= \bar{V}_k + \bar{V}_T = V_ke^{j\delta_k} + V_pe^{j\delta_m} + V_qe^{j\delta_q}, \\
\bar{I}_m &= \bar{I}_k + \bar{I}_T = I_ke^{j\delta_k} - I_pe^{j\delta_m} - I_qe^{j\delta_q}.
\end{align*}
\]

The complex power of both DPFC series inverters and shunt inverters from **Figure 3** is illustrated in Eqs 6–7:

\[
\begin{align*}
S_w &= \bar{V}_m \cdot \bar{I}_m = V_p \cdot I_m + jV_q \cdot I_m, \\
S_h &= \bar{V}_k \cdot \bar{I}_m = V_k \cdot I_m + jV_q \cdot I_m.
\end{align*}
\]

where \( S_w \) and \( S_h \) are the complex power of series/shunt side of the DPFC device.

There is a common similarity between UPFC and DPFC with its external feature of active power balance (Dai et al., 2019), and the active power flow from the shunt side to the series side holds conservation characteristics, as shown by

\[
V_k I_p = V_p I_m. \tag{5}
\]

Together with Eqs (4), (5).

Combined with the complex power of the DPFC, in both the shunt and the series side with conservative characteristics, reactive power complies with the following equation, reflecting that the DPFC may generate or absorb reactive power after its injection into the power system.
\[
S_{ih} - S_{ke} = f(V_i I_q - V_q I_m).
\] (6)

Due to the conservation characteristic of active power in the DPFC device, a power injection model (PIM) can be conducted as depicted in Figure 4:

\[
P_{ij}^* = P_{ij} - P_{ij}^{DPFC},
\]
\[
P_{ij,rev}^* = P_{ij,rev} + P_{ij}^{DPFC},
\]

where \(P_{ij}\) and \(P_{ij,rev}\) are the line power and reverse line power and \(P_{ij}^{DPFC}\) is the DPFC compensation level.

### 3 PROBLEM FORMULATION

#### A. The Relaxed AC-SOCP Model

The AC power flow model can be represented as

\[
P_{ij}(\theta, V) = V_i^2 g_{ij} - V_i V_j (g_{ij} \cos(\theta_i - \theta_j) + b_{ij} \sin(\theta_i - \theta_j)),
\]
\[
Q_{ij}(\theta, V) = -V_i^2 b_{ij} - V_i V_j (g_{ij} \sin(\theta_i - \theta_j) - b_{ij} \cos(\theta_i - \theta_j)).
\]

The above traditional model is non-linear. Therefore, the equivalent transformation is introduced to cope with the difficulties of the non-linear problem. Several variables are defined in the following equations:

\[
U_i = V_i^2; U_j = V_j^2,
\]
\[
R_{ij} = U_i U_j \cos(\theta_i - \theta_j); R_{ij} \geq 0,
\]
\[
T_{ij} = U_i U_j \sin(\theta_i - \theta_j).
\]

The combined AC power flow model with Eqs 8, 9–11 is relaxed as follows:

\[
P_{ij} = g_{ij} U_i - g_{ij} R_{ij} - b_{ij} T_{ij},
\]
\[
Q_{ij} = -b_{ij} U_i - g_{ij} T_{ij} + b_{ij} R_{ij},
\]
\[
P_{ij,rev} = g_{ij} U_i - g_{ij} R_{ij} + b_{ij} T_{ij},
\]
\[
Q_{ij,rev} = -b_{ij} U_i + g_{ij} T_{ij} + b_{ij} R_{ij}.
\]

According to Eqs 10–11, a constraint between \(R_{ij}\) and \(T_{ij}\) must be satisfied as follows:

\[
R_{ij}^2 + T_{ij}^2 = V_i^2 V_j^2 = U_i U_j.
\]

The above equation is still non-linear due to the quadratic form, and we relax the equality constraint to inequality format which can be transformed into an SOCP form:

\[
\left\| \begin{array}{c}
2R_{ij} \\
2T_{ij} \\
U_i - U_j
\end{array} \right\|_2 \leq U_i + U_j.
\]

Thus, the relaxed AC-SOCP model is transformed into an SOCP model with Eqs 12, 14, which can be solved by the commercial solvers such as CPLEX.

#### B. Two-Stage Stochastic MISOCP Model

The power system planners aim to determine the location allocations of DPFC, which can enhance the management efficiency of power flow and decrease the investment of DPFC. However, the operators desire to minimize the operation cost of injected DPFCs. Therefore, optimal location allocations of DPFC in the power system must consider the operational cost and investment of installing DPFCs. The optimal model is represented by

\[
\begin{align*}
\min & \sum_{i \in \mathbb{W}_G} c_i P_i^G + \sum_{i \in \mathbb{G}_{Di}} \lambda_i P_{Di}^{DPFC}, \\
& \sum_{i \in \mathbb{W}_G} P_{Wi} + \sum_{i \in \mathbb{G}_{Di}} P_{Di}^{DPFC} - \sum_{i \in \mathbb{G}_{Di}} P_{Di} - \sum_{i \in \mathbb{G}_{Di}} P_{Di}^D, \\
& = \sum P_{ij}(\theta, V, \lambda),
\end{align*}
\]

\[
0.95^* \sum_{i \in \mathbb{W}_G} P_{Wi} + \sum_{i \in \mathbb{G}_{Di}} Q_{Gi} - \sum_{i \in \mathbb{G}_{Di}} Q_{Di} = \sum Q_{ij}(\theta, V, \lambda),
\]

\[
P_{Di}^{min} \leq P_{Di} \leq P_{Di}^{max},
\]

\[
Q_{Di}^{min} \leq Q_{Di} \leq Q_{Di}^{max},
\]

\[
V_{Di}^{min} \leq V_{Di} \leq V_{Di}^{max},
\]

\[
\theta_{Di}^{min} \leq \theta_{Di} \leq \theta_{Di}^{max},
\]

\[
P_{ij} = g_{ij} U_i - g_{ij} R_{ij} - b_{ij} T_{ij},
\]

\[
Q_{ij} = -b_{ij} U_i - g_{ij} T_{ij} + b_{ij} R_{ij},
\]

\[
P_{ij,rev} = g_{ij} U_i - g_{ij} R_{ij} + b_{ij} T_{ij},
\]

\[
Q_{ij,rev} = -b_{ij} U_i + g_{ij} T_{ij} + b_{ij} R_{ij},
\]

\[
T = \theta_i - \theta_j,
\]

\[
\left\| \begin{array}{c}
2R_{ij} \\
2T_{ij} \\
U_i - U_j
\end{array} \right\|_2 \leq U_i + U_j,
\]

\[
\left\| \begin{array}{c}
P_{ij} - P_{ij}^{DPFC} \\
Q_{ij}
\end{array} \right\|_2 \leq S_{ij},
\]

\[
\left\| \begin{array}{c}
P_{ij,rev} + P_{ij}^{DPFC} \\
Q_{ij,rev}
\end{array} \right\|_2 \leq S_{ij},
\]

\[
0 \leq P_{ij}^{DPFC} \leq \delta_{ij} P_{ij}^{DPFC, max}.
\]

\[
N_{DPFC} \in \mathbb{N}_+.
\]
4 SOLUTION APPROACH

A. DC Power Flow With Reactance Model of DPFC

Based on the conservation of active power and unbalance of reactive power in the DPFC, a reactance model of DPFC can be shown in Figure 5.

The equivalent reactance $x_{ij}^{DPFC}$ can be transformed into a function of the connected line's reactance where the DPFC is located:

$$x_{ij}^* = x_{ij} + x_{ij}^{DPFC}, \quad (27)$$
$$x_{ij}^{DPFC} = y_{ij} \cdot x_{ij}, \quad (28)$$

where $y_{ij}$ is the compensation level of line between buses $i$ and $j$ and $y_{ij} = \lambda_{ij}/(1 - \lambda_{ij})$. Therefore, the reactance with DPFC between buses $i$ and $j$ can be updated as follows:

$$x_{ij}^* = x_{ij}/(1 - \lambda_{ij}). \quad (29)$$

According to Eq. 6, the DPFC can absorb or generate reactive power, and the equivalent compensation level of the series reactance of the DPFC bounds the value between $-0.2$ and $0.7$. Consequently, the corresponding linear interval is between $-0.17$ and $2.33$.

The DC power flow approximation is widely used in power planning, which supposes voltage magnitude equal to 1 p.u. and ignores reactive power and resistance of lines because of $r_{ij} \ll x_{ij}$. This can be shown as

$$P_{ij} = \theta_{ij}/x_{ij}. \quad (30)$$

Once the DPFC is injected to the system, the reactance of the two adjacent buses is changed from $x_{ij}$ to $x_{ij}^*$. Furthermore, all the phase angles $\theta$ and injected compensation level $\lambda$ are decision variables; once $x_{ij}^*$ is substituted to form the DC active power with DPFCs, both $\theta$ and $x_{ij}^*$ are incorporated in a multiplied form, which is still non-linear.

The active power of the transmission line can be varied with DPFC injection with the updated reactance $x_{ij}^*$. The DC power flow with the DPFC is illustrated as follows:

$$P_{ij}(\theta, \lambda, \delta) = \theta_{ij}/x_{ij} = \theta_{ij}/x_{ij} - \delta_{ij}\lambda_{ij}/x_{ij}, \quad (31)$$
$$\lambda_{ij}^{\min} \leq \lambda_{ij} \leq \lambda_{ij}^{\max}. \quad (32)$$

There is a non-linear variable term $\delta_{ij}\lambda_{ij}/x_{ij}$ in Eq. 31, and a virtual variable $\phi_{ij}$ is introduced to linearize the non-linear term:

$$\phi_{ij} = \delta_{ij}\lambda_{ij}/x_{ij}. \quad (33)$$

The active power in Eq. 31 can be rewritten as

$$P_{ij}(\theta, \lambda, \delta) = \theta_{ij}/x_{ij} - \phi_{ij}. \quad (34)$$

Combining Eqs 32, 33, we multiply both sides of the equation with a voltage angle difference of $\delta_{ij}$:

$$\delta_{ij}\lambda_{ij}^{\min} \leq \phi_{ij}/x_{ij} \leq \delta_{ij}\lambda_{ij}^{\max}. \quad (35)$$

Therefore, the reactance with DPFC a reactance model of DPFC. This can be shown as

$$(\lambda_{ij} + \phi_{ij}/x_{ij}) \leq \lambda_{ij}^{\max} \theta_{ij} \leq \lambda_{ij}^{\max} \theta_{ij} + M_{ij} y_{ij}, \quad (36)$$

$$-M_{ij} (1 - y_{ij}) + \delta_{ij}\lambda_{ij}^{\max} \theta_{ij} \leq \phi_{ij}/x_{ij} \leq \delta_{ij}\lambda_{ij}^{\max} \theta_{ij} + M_{ij} (1 - y_{ij}). \quad (37)$$

In the optimization process, one of Eqs 36, 37 is valid, and another one is always useful because of the large number $M_{ij}$.

Note that the bilinear term $\delta_{ij}\theta_{ij}$ is still non-linear and another dummy variable $U_{ij}$ is introduced and linearized by applying the big-$M$ method repeatedly as follows:

$$U_{ij} = \delta_{ij}\theta_{ij}, \quad (38)$$
$$-\delta_{ij}\theta_{ij}^{\min} \leq U_{ij} \leq \delta_{ij}\theta_{ij}^{\max}, \quad (39)$$
$$\theta_{ij} = (1 - \delta_{ij})\theta_{ij}^{\max} \leq U_{ij} \leq \theta_{ij} + (1 - \delta_{ij})\theta_{ij}^{\max}. \quad (40)$$

Combining Eqs 36–37 with the dummy variable $U_{ij}$, the active power of line can be replaced as follows:

$$-M_{ij} y_{ij} + U_{ij}\lambda_{ij}^{\min} \leq \phi_{ij}/x_{ij} \leq U_{ij}\lambda_{ij}^{\max} + M_{ij} y_{ij}, \quad (36)$$
$$-M_{ij} (1 - y_{ij}) + U_{ij}\lambda_{ij}^{\max} \leq \phi_{ij}/x_{ij} \leq U_{ij}\lambda_{ij}^{\max} + M_{ij} (1 - y_{ij}). \quad (41)$$

Hence, the DC power flow with the DPFC, including Eqs 31, 39–41, is reformulated into an MILP problem.

B. Two-Stage Stochastic Optimal Location Allocations of DPFC

Due to the unrepeated features of DPFC planning, we develop a two-stage optimization method based on Benders decomposition to solve the MISOCP problem under different wind–load scenarios. The original MISOCP problem can be decomposed into an MILP master problem and an SOCP subproblem. The MILP problem is to solve the optimal locations of DPFC under the base-loadd case, and the relaxed DC power flow based on the reactance model of DPFC accelerates its efficiency. In contrast, the optimal ratings of DPFC with various scenarios are obtained in the SOCP subproblem.
The master problem is represented as

\[
\min \Phi_{\text{down}} = \sum_{i \in G(i)} c_i P_g^i + \alpha \tag{42}
\]

\[
\alpha \geq z + \sum_{ij} \mu_{ij} (\delta_{ij} - \delta_{ij}^\ast) + \sum_i \sigma_i (P_{i\alpha} - P_{i\alpha}^\ast) \tag{43}
\]

Equation 42 is the objective function of master problem, which is explicitly reflected in the lower bound. In the objective function, the first term is the generation cost, while the latter is the investment cost of DPFC. The relax DC active power balance is constrained in the second column of Eq. 42, whose non-linearized term is linearized using the big-M method. The second column in Eq. 42 is the Benders cuts generated in the subproblem to accelerate the solution efficiency.

The active power balance of the SOCP subproblem may be challenging because of wind–load uncertainty. Slack variables are incorporated into the power balance equations to relax and ensure the feasibility of the subproblem. The stochastic SOCP subproblem is represented by Eq. 44.

The subproblem is as follows:

\[
\min z = \sum_i \rho_i \left[ \sum_{ij} \pi_{ij} P_{ij}^\text{DPFC} + c_i \sum_{i \in G(i)} \left( \Delta k_{ij}^+ + \Delta k_{ij}^- \right) \right] \tag{43}
\]

\[
\left\{ \begin{array}{l}
\sum_{i \in G(i)} P_{ij}^D - \sum_{i \in D(i)} P_{ij}^D + \sum_{j \in \Omega(i)} P_{ij}^\text{DPFC} - \sum_{i \in G(i)} P_{ij}^\text{DPFC} + \Delta k_{ij}^+ \\
\quad - \Delta k_{ij}^- = \sum_{j \in \Omega(i)} P_{ij} (\theta, V, \lambda), \\
\sum_{i \in G(i)} Q_{ij}^D - \sum_{i \in D(i)} Q_{ij}^D = \sum_{j \in \Omega(i)} P_{ij} (\theta, V, \lambda), \\
P_{ij}^\ast - \Delta k_{ij}^- P_{ij}^\ast \leq P_{ij}^D \leq P_{ij}^\ast + \Delta k_{ij}^+, \\
Q_{ij}^\ast \leq Q_{ij}^D \leq Q_{ij}^\ast, \\
\Delta k_{ij}^+ \geq 0, \Delta k_{ij}^- \geq 0, \forall l \in \Omega_l, \\
(16) - (17), \\
(20 - 24), \\
0 \leq P_{ij}^\text{DPFC} \leq P_{ij}^\text{max}, \\
P_{ij}^D = P_{ij}^D : \mu_{ij}, \\
\delta_{ij} = \delta_{ij}^\ast : \sigma_{ij}.
\end{array} \right. \tag{44}
\]

Equation 44 represents the subproblem objective function, which consists of the investment cost of DPFC and the sum of relaxing slack variables. The constraints are updated under various scenarios, and some slack variables are introduced into the power flow constraints to ensure the convergence of power flow. Hence, the constraints of active and reactive power are also rewritten with the slack variables. The dual of Benders cuts is obtained from the latter columns of Eq. 44. However, the duals of cuts need to be reconstructed because of load/wind uncertainty. We reformulate the expected value of duals associated with numerous scenarios, as shown in the following equations:

\[
\mu_{ij} = \sum_s \rho_s \mu_{ij,s} \tag{45}
\]

\[
\sigma_i = \sum_s \rho_s \sigma_{ij,s} \tag{46}
\]

Based on the Benders decomposition method, the two-stage problem has a lower bound and upper bound. The Benders cuts accelerate the optimization efficiency iteratively and move the solution toward optimality. A stop criterion is justified as the optimal solution to the original problem. The upper bound of MISOCP and stop criterion is established, as shown in the following equations:

\[
\Phi^\up = z + \sum_{i \in G(i)} c_i P_{i\alpha}^D + \alpha \tag{47}
\]

\[
\left| \Phi^\up - \Phi^\text{down} \right| \leq \epsilon. \tag{48}
\]

The flowchart of two-stage stochastic optimization is depicted in Figure 6.

For a given gap \(\epsilon\), the complete procedure of solving the two-stage stochastic can be described as follows:

Step 1: Let \(\Phi^\text{down} = -\infty, \Phi^\up = +\infty, \text{iter}=0\);

Step 2: Solve the MP which is modeled in Eq. 42.
Obtain the output of generators \( P_i^{G_{i}} \) and location of DPFC \( \delta_{ij} \) under the base case,
- Update the lower bound \( \Phi^{\text{down}} \);

Step 3: Fix the location of DPFC and output of thermal units and solve the SP considering various wind–load scenarios,
- Obtain compensation levels \( P_{ij}^{DPC_{i,j}} \) and slack variables \( \Delta k_{ij}, \Delta k_{ij}^{+} \) under each scenario,
- Update the upper bound \( \Phi^{\text{up}} \);

Step 4: If \( \frac{|\Phi^{\text{up}} - \Phi^{\text{down}}|}{|\Phi^{\text{down}}|} \leq \epsilon \), return the optimal solutions and stop.
Otherwise, add the Benders cut into a master problem and go to step 2.

5 CASE STUDY
A. Verification of the Relaxed AC-SOCP Model
In this section, several power flow cases are utilized to illustrate the effectiveness of the proposed model. The numerical cases are tested on the IEEE 118 bus system. The data of IEEE 118 are obtained from MATPOWER 6.0.

Case 1: the proposed relaxed DC power flow in Section 3.1A, which is solved by GAMS/Cplex.
Case 2: the traditional non-linear power flow in Eq. 8, which is solved by GAMS/CONOPT.
Case 3: the proposed SOCP model in Eq. 22, which is solved by GAMS/Cplex.

As for the power flow analysis, we only consider the original power flow without DPFC. Compared to Case 2 and Case 3, Case 1 cannot simulate the AC power characteristic. Figure 7 depicts apparent power of lines in Case 2 and Case 3, and the difference of the two solutions is less than 1%. Figure 8 also compares the bus voltage performance between Case 2 and Case 3, and the voltage magnitude of the two cases is fairly close. Figure 9 shows significant differences of generation output in the three cases. The generation dispatch solution of Case 1 shows a different trend because of ignoring reactive power constraints, whereas the dispatch solutions show highly consistent characteristics between Case 2 and Case 3.

To illustrate the accuracy of the relaxed AC-SOCP model, we define a deviation index stated in (), which depicts the gap of line constraints between the non-linear AC power flow and the relaxed AC-SOCP model. Figure 10 shows the gap performance, which is almost zero for all lines:

\[
DI = U_iU_j - R_{ij}^2 - T_{ij}^2. \tag{49}
\]
B. Effect of the Optimal DPFC With High Penetration of Wind Power

To verify the proposed method, we conducted case studies on the modified IEEE 118 bus system. The baseload is 4242 MW, and the capacity of the total generator is 5,859.2 MW. The load uncertainty is statistically based on the Latin hypercube sampling (LHS) (Le and Wu, 2021) and K-means clustering method (Toyoda and Wu, 2021; Wu, 2021), as shown in Table 1. There is an artificially decreased capacity to create congestion with the thermal limits of transmission lines. GAMS implements the procedure, the MILP master problem is solved by GAMS/CPLEX, and GAMS/CPLEXD solves the SOCP subproblem. The threshold values of the stop criterion are set to be 1e-4.

A. The Performance With Different Numbers and Ratings of DPFCs

There are three huge impacts with different numbers of optimal DPFCs’ planning. Table 2 shows the total operation cost with different numbers of optimal location allocation. The operating cost of power systems shows a downward trend as the numbers of installed DPFCs increase because of their power flow management of DPFC. Compared to the optimal locations, there is a continuous trend, which verifies the robustness of the optimal planning program and overcomes the drawbacks of the iterative planning method. The level of wind absorption has also been improved. However, the increment level is not obvious between the two-DPFC and three-DPFC planning, which closely achieves the extreme in the system (Table 3):

\[ V_{vio} = \sum_{i \in G} \| V_i - V_{ref} \|. \]  

With different installing numbers of DPFCs, the voltage violation and power loss of the system are shown in Figure 11. The system voltage fluctuations gradually decrease as the number of DPFCs increases, whereas the increment of power loss has a positive trend.

Figure 12 shows a great advantage of voltage stability with three DPFCs installed over the others.

### Table 1 | Load and wind scenarios and probabilities.

| Scenarios | \( P_{W,s} \) | \( P_{D,s} \) | \( \rho_s \) |
|-----------|----------------|----------------|----------|
| s1        | 0.3023         | 0.4858         | 0.0555   |
| s2        | 0.8007         | 0.6916         | 0.0446   |
| s3        | 0.6263         | 0.7338         | 0.0412   |
| s4        | 0.0825         | 0.5919         | 0.0788   |
| s5        | 0.1846         | 0.4796         | 0.064    |
| s6        | 0.5815         | 0.487          | 0.0516   |
| s7        | 0.26           | 0.7026         | 0.0488   |
| s8        | 0.3488         | 0.6036         | 0.0868   |
| s9        | 0.0844         | 0.4701         | 0.0574   |
| s10       | 0.653          | 0.5936         | 0.0732   |

### Table 2 | Solution of optimal location allocations of DPFC.

| DPFC number | Optimal DPFC planning | Wind output (MW) | Generation cost value | Wind penetration |
|-------------|-----------------------|------------------|-----------------------|-----------------|
|             | Location | Capacity (MW) | 5 | 26 | 61 | 95 | 60070  | 57722  | 58623  | 56124  | 27.6% | 32.9% | 35.19% | 35.33% |
| 0           | -        | -               | 2 | 3.25 | 2.39 | 1.20 | 60070 | 57722 | 58623 | 56124 | 27.6% | 32.9% | 35.19% | 35.33% |
| 1           | L147     | 0.45           | 2.28 | 3.25 | 2.86 | 1.61 | 60070 | 57722 | 58623 | 56124 | 27.6% | 32.9% | 35.19% | 35.33% |
| 2           | L89      | 45.5           | 2.27 | 3.46 | 3.03 | 1.78 | 60070 | 57722 | 58623 | 56124 | 27.6% | 32.9% | 35.19% | 35.33% |
| 3           | L89      | 42.5           | 2.26 | 3.44 | 2.99 | 1.89 | 60070 | 57722 | 58623 | 56124 | 27.6% | 32.9% | 35.19% | 35.33% |

A voltage violation index is established to evaluate the stability, as is shown.
To evaluate the effects of DPFC on generator rescheduling, the output of generators under a loading operation with scenario s15 is shown in Figure 13. The generator dispatch has a considerable difference from the DPFC under the load level. Comparing the no-DPFC and one-DPFC solutions, the absorption of wind power in this scenario has little change. However, the output of thermal unit is significantly different because more economical units participate in more dispatch plans, which verifies the management efficiency of DPFC to the dispatch solution of thermal generators. Once two or three DPFCs are injected into the system, the wind absorption has an obvious increment, which illustrates the power flow shiftable capability of DPFC.

### The Performance With Certain Compensation Level of DPFC

When the compensation level of DPFC is equal to 5 MW, the performance is different from that in Table 2. It is observed that the expected cost decreases slightly as the number of DPFCs increases. The wind penetration also shows little changes. This result also confirms the superiority of the planning method, in which the location and allocation are optimized simultaneously.
C. The Performance With Variable Wind Location With Optimal DPFC

To assess the impact of optimal DPFC solutions with varied wind locations, we transfer the wind location to buses [3, 50, 80, 118]. Comparing Tables 2, 4, the overall decline in wind power penetration is relatively obvious, which only can illustrate the manage ability of DPFC is subjected to the structure of generators. At the same time, it can be observed that the operating cost and wind penetration also show a positive trend when DPFC numbers increased.

### 6 CONCLUSION

This work presents a novel two-stage stochastic optimization model, which simultaneously optimizes the location and compensation level of DPFCs considering various wind–load scenarios. Case studies are performed to demonstrate the effectiveness of the proposed method. The conclusions are summarized as follows:

1) The relaxed AC-SOCP model can easily simulate the non-linear AC power flow and has an advantage of solving speed and difficulties.

2) The proposed two-stage method has a consistent scheduling plan of DPFC, which maintains the non-linear internal characteristics of DPFC and overcomes the drawback of iterative scheduling planning.

3) The power flow management of DPFC on the network side plays an important role in system operation. The operating cost, power flow performance, and wind absorption have a positive trend as the numbers of DPFCs increased.

### DATA AVAILABILITY STATEMENT

The original contributions presented in the study are included in the article/Supplementary Material, and further inquiries can be directed to the corresponding author.

### AUTHOR CONTRIBUTIONS

XZ wrote the manuscript and performed the method. LD provided the financial support for this research. JW provided the support for solving method.

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### TABLE 4 | Solution of optimal DPFC with variable wind power.

| DPFC number | Optimal DPFC planning | Wind output (MW) | Generation cost value | Wind penetration |
|-------------|-----------------------|------------------|-----------------------|------------------|
|             | Location | Capacity (MW) | 3 | 50 | 80 | 118 |                           |       |               |
| 0           | -       | -              | 1.48 | 1.47 | 3.76 | 1.37 | 63287 | 27.3% |
| 1           | L120    | 74.5           | 1.53 | 1.54 | 3.86 | 1.50 | 62023 | 29.1% |
| 2           | L106    | 99.67          | 1.55 | 1.59 | 3.87 | 1.51 | 62372 | 29.5% |
| 3           | L106    | 13.13          | 1.52 | 1.59 | 3.84 | 1.47 | 61610 | 29.2% |
|             | L120    | 64.18          |         |       |       |       |         |       |
|             | L150    | 29.69          |         |       |       |       |         |       |
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GLOSSARY

Sets and indices

\(i/j\) Bus index

\(ij\) Line index connected bus i and j

\(s\) Load scenario index

\(G(i)\) Sets of generator located bus i

\(D(i)\) Sets of load located bus i

\(\delta(i)\) Sets of lines connected bus i DPFC variables

\(V_T/V_{Ti}\) Unified/distributed series voltage magnitude of DPFC

\(\theta_{se}/\theta_{sei}\) Unified/distributed series voltage angle phase of DPFC

\(S_{se}/S_{sh}\) Complex power of series or shunt side

\(x_{ij}^{DPFC}\) Equivalent reactance of DPFC located line ij

\(n_{ij}^{DPFC}\) Amortized cost of DPFC located line ij

\(\lambda_{ij}\) Compensation level of DPFC

\(N_{DPFC}\) Total numbers of DPFC Variables

\(r_{ij}/x_{ij}\) Resistance or reactance of line ij

\(P_i^G/Q_i^G\) Active or reactive power of generator located at bus i

\(P_i^D/Q_i^D\) Active or reactive load located at bus i

\(\Delta k_{p,ij}^+/\Delta k_{p,ij}^−\) Positive slack variable

\(\delta_{ij}\) Binary variable indicating DPFC located

\(y_{ij}\) Binary variable indicating the direction of power flow of line ij

\(c_i\) Coefficient of generator cost located bus i

\(P_{ij}\) Active power of line ij

\(V_i\) Voltage magnitude of bus i

\(\theta_i\) Voltage angle of bus i

\(\theta_{ij}\) Angle difference between bus i and j

\(\alpha_L\) Constant variable

\(P_{ij}^{max}\) Thermal limit of line ij

\(\lambda_{ij}^{max}/\lambda_{ij}^{min}\) Lower or upper bound of compensation level

\(\theta_i^{max}/\theta_i^{min}\) Lower or upper bound of voltage angle

\(V_i^{min}/V_i^{max}\) Lower or upper or lower bound of voltage magnitude

\(P_i^{G,min}/P_i^{G,max}\) Lower or upper and lower bound of active power supplied by generator

\(Q_i^{G,min}/Q_i^{G,max}\) Lower or upper bound of reactive power

\(\Phi^{down}\) Lower bound of original problem

\(\Phi^{up}\) Upper bound of original problem

\(M_{ij}\) Penalty coefficient

\(\rho_s\) Probability of scenarios.