Electric Vehicle Battery SOC Estimation based on GNL Model Adaptive Kalman Filter

YAN Xiang-Wu, GUO Yu-Wei, CUI Yang, WANG Yu-Wei and DENG Hao-Ran

1State Key Laboratory of Alternate Electrical Power System with Renewable Energy Sources, North China Electric Power University, Beijing 102206, China
2CATARCAutomotive Engineering Research Institute, Tianjin 300300, China

Abstract. With the efficient development of the electric vehicle, it is urgent to recycle and utilize the decommissioned power battery which increases rapidly in quantity year by year. Accurate and reliable state of charge (SOC) estimation of the battery is the key technology to realize the battery cascade utilization. The traditional estimation methods do not take the self-discharge factors into account which affect the aging battery to a great extent. This research adopts the GNL circuit equivalent model, which considers the self-discharge factor and discretizes its state space equation by matrix quadratic form. The adaptive unscented Kalman filter algorithm (AUKF) is used to estimate and update the SOC in time. The experimental comparison verifies the effectiveness of AUKF for aging batteries. The results show that the proposed method can obtain less error of the state estimated value and fast following features which meets the actual demand of SOC estimation.

1. Introduction
Power battery pack with advanced manufacturing process can maintain rather high safety and electrical performance even after decommission. Cascade utilization of the decommissioned power battery is a vital part of low-cost battery achievement and has become an urgent task with efficient development of electric vehicle, which increases the amount of the decommissioned power battery year by year. The state of charge (SOC) of the lithium battery as the main basis for energy management and control strategy plays an important role in such as extending battery lifespan and achieving battery cascade utilization.

The premise of correct estimation of SOC is to establish a precise and reliable lithium battery model. The equivalent circuit model is composed of ideal circuit components (resistors, capacitors, etc.). It has been widely used to simulate the nonlinear operating characteristics of lithium batteries with the advantages of easy calculation and being appropriate for real-time systems. Common equivalent circuit models are as follows: 1) the Rint model, also known as the internal resistance model. It does not take into account the internal dynamic characteristics of the battery, which can only be used for simple circuit simulation but fails to achieve the accurate SOC estimation; 2) the Thevenin model. It connects the Rint model and an RC parallel circuit in series for predicting the battery's response to instantaneous load, better reflecting the internal dynamic characteristics of the battery; 3) the Second-order RC model. On the basis of the Thevenin model, it is connected in series with one more RC parallel circuit. The two RC parallel circuits represent electrochemical polarization and concentration polarization respectively,
further improving the practicability and accuracy of the model; 4) the PNGV model. It is a series connection between the Second-order RC model and a capacitor which aims to simulate how the variations of SOC caused by the time integral of current influence the open-circuit voltage of the battery. This model principally considers the issues of the battery open-circuit voltage $U_{OC}$ as a function of SOC; 5) the GNL model. Based on the former, the GNL model takes the influence of self-discharge into considerations. Therefore, various influence factors brought by the actual working state of the battery are fully considered, therefore the battery characteristics simulated by the model can be preferably approximated to its actual properties. The self-discharge rate of the decommissioned battery is relatively high\cite{2}, hence the influence of self-discharge resistance needs to be considered in SOC estimation. Nevertheless, the above-mentioned GNL model has a complicated structure and numerous parameters, which perplexes the simulation verification and parameter identification. Consequently, to simplify the GNL model, OCV-SOC curve is used to signify the impact of SOC changes on the open-circuit voltage of the battery instead of the capacitance added in the PNGV model making model more suitable for experimental simulation and application of Li-ion batteries SOC estimation.

The SOC estimation methods commonly used in engineering are mainly as below\cite{3}: 1) Ampere-Hour method. This approach not only requires high accuracy of the initial SOC value and measuring current of the battery but also is prone to cumulative error; 2) Open circuit voltage method (OCV). The lithium battery needs standing for a while during the open-circuit voltage test, which obviously cannot meet the requirements of SOC real-time online detection of the electric vehicle; 3) Neural network method. There is an indispensable large amount of experimental data in this method as a result of the estimation accuracy and computational load that depend on training method and training data; 4) Extended Kalman filter (EKF) algorithm\cite{3}. Based on the battery state space model, the SOC of the battery is estimated by recursion and iteration, of which the estimation accuracy is greatly affected by the model accuracy. Meanwhile, the EKF linearization process lowers the estimation accuracy and enlarges the amount of calculation; 5) Unscented Kalman filter (UKF) algorithm\cite{4-5}. UKF uses a statistical linearization method to reduce errors and computation. Unfortunately, the precision of the battery model still impacts SOC estimation accuracy. In allusion to addressing the current issues of battery state-of-charge estimation accuracy and unsatisfactory practicality, an adaptive unscented Kalman filter (AUKF) is used for SOC estimation in this study. During being used, the battery is a time-varying system, in which AUKF estimates better than UKF\cite{6-8}. AUKF estimates battery SOC by means of loop iterations, tracks and updates model parameters in real time, which avoids the influence of time-varying model parameters on the accuracy of the calculation. Moreover, applying Kalman filtering requires the discretization of continuous systems\cite{9}. In the past, SOC estimation based on equivalent circuit models directly used the substitution method based on the mapping relationship from S domain to Z domain in the system control theory. Increasing complexity of GNL model state equation leads to the formula derivation becoming complicated exponentially and it is not conducive to subsequent programming. For this reason, the general formula of discretization of continuous systems deduced from the matrix quadratic principle is used in this research to facilitate the structural design of computer programs, which helps to realize the integration of the Kalman filtering process in time continuous system on the computer.

2. Lithium battery model

2.1. GNL equivalent circuit model

The convergence and accuracy of the battery SOC estimation depends on the accurate establishment of the battery model. The GNL equivalent circuit model used in this study considers the self-discharge internal resistance based on the second-order RC model (as shown in Figure 1.(a)) and accurately reflects the dynamic characteristics of battery. Circuit diagram is shown in Figure 1.(b). $U$ and $U_{OC}$ are the terminal voltage and the open circuit voltage of the battery; $R_e$ is the ohmic resistance; $R_s$ is the self-discharge internal resistance; $I$ is the discharge current, which is negative when charging; $I_m$ is the current flowing through $R_e$; $R_1$, $C_1$ are the concentration polarization resistance and capacitance respectively,
characterizing the rapid internal electrode reaction of the battery; Similarly, \( R_2 \) and \( C_2 \) are the electrochemical polarization resistance and capacitance respectively, representing the slow electrode reaction inside the battery.

\[ R_1 R_2 C_1 C_2 UOC \]

(a) Second-order RC model of li-ion battery.  
(b) GNL model of li-ion battery.  

Figure 1. Second-order RC model and GNL model.

According to the GNL model, result is shown as follows.

\[
\begin{align*}
I_n &= \frac{U_{IC}}{R_1} + C_1 \frac{dU_{IC}}{dt} \\
I_n &= \frac{U_{IC}}{R_2} + C_2 \frac{dU_{IC}}{dt} \\
U &= U_{OC} - I_n R_1 - U_{IC} - U_{OC} \\
U &= (I - I_n) R_1 \\
SOC(t) &= SOC(t_0) - \frac{1}{Q_0} \int_0^t \eta I(t) dt
\end{align*}
\]

Where \( SOC(t) \) represents the state of charge of the battery at time \( t \). \( Q_0 \) is the capacity of the battery which is derived from the Ampere-Hour integral method; \( \eta \) is the Coulombic efficiency referring to the ratio of the discharge capacity to the charge capacity of the battery during one charge and discharge, whose default value is 1 for little concentration being put on it in this study.

According to equation (1), the state space equation of the GNL model is

\[
\begin{bmatrix}
U_1 \\
U_2 
\end{bmatrix} = \begin{bmatrix}
\frac{-R_1}{\tau_1 (R_1 + R_2)} & \frac{-R_1}{\tau_2 (R_1 + R_2)} \\
\frac{-R_2}{\tau_1 (R_1 + R_2)} & \frac{-R_2}{\tau_2 (R_1 + R_2)} 
\end{bmatrix} \begin{bmatrix}
U_1 \\
U_2 
\end{bmatrix} + \begin{bmatrix}
\frac{R_1 R_2}{\tau_1 (R_1 + R_2)} & \frac{R_1}{\tau_1 (R_1 + R_2)} \\
\frac{R_2 R_3}{\tau_2 (R_1 + R_2)} & \frac{R_2}{\tau_2 (R_1 + R_2)} 
\end{bmatrix} \begin{bmatrix}
I \\
U_{OC} 
\end{bmatrix}
\]

(3)

\[
U = \begin{bmatrix}
\frac{-R_1}{(R_1 + R_2)} & \frac{-R_2}{(R_1 + R_2)} 
\end{bmatrix} \begin{bmatrix}
U_1 \\
U_2 
\end{bmatrix} - \frac{R_1 R_2}{(R_1 + R_2)} I + \frac{R_2}{(R_1 + R_2)} U_{OC}
\]

(4)

Where \( \tau_1 \) is the time constant of \( R_1 \) and \( C_1 \); \( \tau_2 \) is the time constant of \( R_2 \) and \( C_2 \).

2.2. Relations between open circuit voltage and SOC

The functional relation between open-circuit voltage \( U_{OC} \) and the battery SOC is \( U_{OC} = F(SOC) \) (as shown in Figure 2. After measuring the data of the open-circuit voltage and state of charge of the battery, Gregory L. Plett’s “Composite Model” is used for fitting the OCV and the SOC. This method solves the issues that a large estimation error is brought by the small-scale variation of the high-order polynomial fitting coefficient. Besides, what this model attracts more are the less fitting coefficient, initial values that are easy to determine and good practicability. Fitting expression is shown as equation (4), and table 1 shows the fitting results.

\[
U_{OC} = k_0 - k_1 \frac{SOC}{k_2} - k_3 \ln(SOC) - k_4 \ln(1 - SOC)
\]

(5)

| Table 1. Composite model parameters |
|-----------------------------------|
| \( K_0 \) | \( K_1 \) | \( K_2 \) | \( K_3 \) | \( K_4 \) |


3. Estimating battery SOC by AUKF algorithm

3.1. Discretization of state space equation

General system discretization is based on the mapping relationship from S domain to Z domain in the system control theory, which is only appropriate for simple mathematical models. And it needs to be transformed into the time domain when Kalman filtering is applied. Undoubtedly, the calculation is rather complex. Consequently, the state space equation of the continuous system in this study is directly discretized in the time domain using the matrix quadratic type, which is also convenient for matlab programming.

Equation (6) shows the general continuous system state equation:

$$X(t) = F(t)X(t) + B(t)U(t) + G(t)W(t)$$ (6)

Where $X(t)$ is the state vector at time $t$; $X'(t)$ is the derivative of $X(t)$; $U(t)$ is the time input vector; $F(t)$ is the coefficient matrix of the state differential equation; $B(t)$ is the input control matrix; $G(t)$ is the noise distribution matrix; $W(t)$ is the noise vector.

The general expression of discretization is shown as equation (7):

$$X_{k+1} = \Phi_{k+1}X_k + \Gamma_{k+1}U_k + \Upsilon_{k+1}W_k$$ (7)

Where:

$$\Phi_{k+1} = \sum_{i=0}^{k} \frac{1}{i!} F^{i+1}(t_i) \Delta t^{i+1}$$

$$\Gamma_{k+1} = \sum_{i=0}^{k} \frac{1}{i!} F^{i+1}(t_i) B(t_i) \Delta t^{i+1}$$

$$\Upsilon_{k+1} = \sum_{i=0}^{k} \frac{1}{i!} F^{i+1}(t_i) G(t_i) \Delta t^{i+1}$$

Discretizing equation (3) obtains the discrete form of the GNL equivalent circuit model:

$$\begin{bmatrix}
S_{oc,k+1} \\
U_{1,k+1} \\
U_{2,k+1}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & X_{11} & X_{12} \\
0 & X_{21} & X_{22}
\end{bmatrix}
\begin{bmatrix}
S_{oc,k} \\
U_{1,k} \\
U_{2,k}
\end{bmatrix} +
\begin{bmatrix}
-\eta \Delta t / C & 0 \\
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}
\begin{bmatrix}
I_k \\
U_{oc,k}
\end{bmatrix} + w_k$$ (8)

Where:

$$\Phi_{k+1} = \begin{bmatrix}
X_{11} & X_{12} \\
X_{21} & X_{22}
\end{bmatrix}$$

$$\Gamma_{k+1} = \begin{bmatrix}
B_{11} & B_{12} \\
B_{21} & B_{22}
\end{bmatrix}$$

$$U_{k+1} = \frac{R_1}{(R_1 + R)} \times U_{k} - \frac{R_1}{(R_1 + R)} \times U_{z,k} = \frac{R_1 R_2}{(R_1 + R_2)} I_k + \frac{R_1}{(R_1 + R_2)} \times U_{oc,k}$$ (9)
The state space variable is \( x_k = [\text{SOC}_k, U_{1,k}, U_{2,k}]^T \); Control variable is \( I_1 \); Observation variable is \( y_k = U_1 \); \( w_k = [w_{1,k}, w_{2,k}, w_{1,1}]^T \) is the system noise whose covariance is \( Q \); \( v_k \) is the observation noise whose covariance is \( R \); \( \Delta t \) is the sampling interval; \( \text{SOC}_k, U_{1,k}, U_{2,k} \) refer to the SOC of the battery and two RC parallel circuit terminal voltages at the \( k \)-th sampling point.

### 3.2. Estimating battery state of charge by AUKF algorithm

Instead of doing Taylor series expansion at the estimation point and then the first-order approximation, the adaptive unscented Kalman filtering algorithm performs two lossless transformations at the sampling point to obtain the Sigma point set that is directly mapped in nonlinear then to get a state probability density function approximatively and a cyclic iterative relation is established when processing nonlinear filtering.

The steps for estimating the battery state using the adaptive unscented Kalman filter algorithm are as follows:

1. Initialize the state variables and their covariance:
   \[
   \begin{align*}
   k = 0; X_k^0 &= E(X_0^0); R_k^0 &= E(R_0) \\
   P_k^0 &= E\left[(X_0^0 - X_k^0)(X_0^0 - X_k^0)^T\right] \\
   P_k^0 &= E\left[(R_0 - R_k^0)(R_0 - R_k^0)^T\right]
   \end{align*}
   \]
   (10)

2. Utilize UT transform to construct Sigma point set and corresponding weight \( \omega \) with respect to state parameter. Set the extended state variable \( X^0_k = [x_k, w_k, \xi_k]^T \), the covariance \( P_{X,k} = \text{diag}\{P_{x,k}, Q, R\} \).

   Build the sigma point set by the extended state variables and set the estimation value of the last cycle state parameter as \( X_{k-1} \). Then the method of Sigma point set selection is shown as equation (11).

   \[
   \begin{align*}
   X_{k,i-1} &= \bar{X}_{k,i-1} \\
   X_{k,i} &= \bar{X}_{k,i} + \left((n + \lambda)P_{X,k-1}\right)^{1/2}, i = 1, \ldots, n \\
   X_{k,i} &= \bar{X}_{k,i} - \left((n + \lambda)P_{X,k-1}\right)^{1/2}, i = n + 1, \ldots, 2n
   \end{align*}
   \]

   Where \( \left((P_{X,k-1})^{1/2}\right)^T \left((P_{X,k-1})^{1/2}\right) = P_{X,k-1} \). And \( \left((P_{X,k-1})^{1/2}\right) \) represents the i-th column of the square root of the matrix. The number of the selected Sigma points is \( 2n + 1 \) and the Sigma point weight algorithm is as follows.

   \[
   \begin{align*}
   a_0^{m} &= \bar{\lambda}(n + \lambda) \\
   a_0^{c} &= \bar{\lambda}(n + \lambda) + (1 - \alpha^2 + \beta) \\
   a_i^{m} &= a_i^{c} = \bar{\lambda}/[2(n + \lambda)] \\
   &\quad i = 1, 2, \ldots, 2n
   \end{align*}
   \]

   In the equations, \( \omega^m \) and \( \omega^c \) represent the weight corresponding to the mean value estimation and the covariance estimation; \( n \) is the dimension of the expanded state variable, which equals 7 in this model; \( \alpha \) describes the extent that Sigma point deviating from the estimated state value, which satisfies inequality \( 1e^{-4} \leq \alpha < 1 \); \( \bar{\lambda} = \alpha n + \kappa \) \( n \) is the Sigma point scaling parameter, which is used to reduce the total prediction error. And \( \kappa \) is an optional parameter. To ensure \( (n + \lambda)P_{X,k-1} \) to be a positive semidefinite matrix, the value of \( \kappa \) satisfies \( \kappa \geq 0 \); \( \beta \) is a quantity related to the distribution of Sigma points. What is verified from experiment is that the most accurate result is obtained when \( \beta = 2 \).

3. Sigma point set obtained by equation (11). Set

   \[
   X_{k,i,j-1} = \begin{bmatrix}
   \bar{X}_{k,i-1} \\
   \bar{X}_{k,i} - \left((n + \lambda)P_{X,k-1}\right)^{1/2} \\
   \bar{X}_{k,i} + \left((n + \lambda)P_{X,k-1}\right)^{1/2}
   \end{bmatrix}^T
   \]

   (13)

4. Calculate one-step prediction of \( 2n + 1 \) Sigma point set.

   \[
   X_{k,i,j}^k = F(X_{k,i,j-1}, I_{k-1}) \quad i = 1, 2, \ldots, 2n + 1
   \]

(14)
(5) Weight the sum of the predicted values of the Sigma point set using the weight $\omega$ in equation (14) to obtain the one-step prediction and the covariance of the battery state parameter:

$$
\hat{x}_{k|k-1} = E\left[F(X^I_{k|k-1})\right]_{i=0,..,2n} = \sum_{i=0}^{2n} w_i^{(m)} X^I_{k|k-1,i}
$$

$$
P_{x,k|k-1} = E\left[(x_k - \hat{x}_{k|k-1})(x_k - \hat{x}_{k|k-1})^T\right] = \sum_{i=0}^{2n} w_i^{(m)} (X^I_{k|k-1,i} - \hat{x}_{k|k-1})(X^I_{k|k-1,i} - \hat{x}_{k|k-1})^T + Q
$$

(6) According to the one-step prediction value, use UT transformation again to generate a new Sigma point set.

$$
X^I_{k|k-1,i} = \left[\begin{array}{c}
\hat{x}_{k|k-1} + (\sqrt{n + \lambda} P_{x,k|k-1})_i \\
\hat{x}_{k|k-1} - (\sqrt{n + \lambda} P_{x,k|k-1})_i
\end{array}\right]
$$

(7) Substitute the Sigma point set obtained in 6) into the observation equation to obtain a one-step prediction of the observed quantity.

$$
y_{k|k-1} = G(X^I_{k|k-1,i}, I_k) \quad i = 1,2,..,2n+1
$$

(8) The observed values and the covariance matrix are given by the weighted sum of the one-step predicted value:

$$
\hat{y}_k = E\left[G(X^I_{k|k-1,i}, I_k)\right]_{i=0,..,2n} = \sum_{i=0}^{2n} w_i^{(m)} y_{k|k-1,i}
$$

$$
P_{y,k} = \sum_{i=0}^{2n} w_i^{(c)} (y_{k|k-1,i} - \hat{y}_k)(y_{k|k-1,i} - \hat{y}_k)^T + R
$$

$$
P_{y,k} = \sum_{i=0}^{2n} w_i^{(c)} (X^I_{k|k-1,i} - \hat{x}_{k|k-1})(X^I_{k|k-1,i} - \hat{x}_{k|k-1})^T + Q
$$

Unlike the dual Kalman filter algorithm, the adaptive unscented Kalman filter algorithm uses one group of Sigma point predictions and sets appropriate weights to get the weighted values of the Sigma points. Sequentially a one-step prediction of the battery state is obtained.

(9) Calculate the Kalman gain matrix:

$$
K_k = P_{y,k}^{-1} P_{y,k}^T
$$

(10) Calculate updates of the battery status and covariance:

$$
\hat{x}_k = \hat{x}_{k|k-1} + K_k (y_k - \hat{y}_k)
$$

$$
P_{x,k} = P_{x,k|k-1} - K_k P_{y,k} K_k^T
$$

Combined with the state space model (8) and (9), the corresponding computational load in the state space expression is substituted into the AUKF algorithm. And the SOC of the battery can be estimated in real time by loop iteration calculation.

### 4. Experimental testing and verification

In this study, the lithium iron phosphate battery pack produced by Xiangyang Camel Co., Ltd was selected as the study object, of which the single-cell battery with the highest degree of aging was chosen to be experimented. The superiority of the GNL circuit model and the AUKF algorithm in battery state estimation is verified under the conditions of intermittent constant current discharge and variable current conditions. This experiment was based on the current condition of the battery equivalent circuit characteristic parameters identification. The experimental process is as follows. A current with the discharge rate of 0.3C was applied to intermittently constant-current discharge of the battery at room temperature. After 800s of constant current discharge, the current was interrupted and the battery was left to stand for 100s before continuing to discharge. This cycle (the battery charges and discharges) kept going until the battery cell voltage reduced to the discharge cut-off voltage 2.6 V. The total test
duration was 11,000 s. Figure 3 shows the terminal voltage curve of Li-ion battery for once pulse discharge.

Figure 3. Curve of intermittent constant pulse.

Figure 4 shows the comparison between the actual voltage and the terminal voltage estimated by the AUKF algorithm and the estimation error. It can be seen that the error estimated by this algorithm is very small when estimating the battery terminal voltage with only two relatively large sampling points error at the end of discharge. The error is within ±0.5%V for most of the time. Obviously, the parameter results meet the accuracy requirements, which indicates the algorithm provides good support for SOC estimation.

Figure 4. Model output voltage and error curve.

Figure 5 represents the comparisons between the SOC reference value and the estimated values by using EKFSOC, the second-order RC model AUKF algorithm and the GNL model AUKF algorithm, respectively. And the SOC reference value is calculated by the ampere-hour counting method. The initial estimated value of the SOC of each single battery is set to be 85%. As shown in Fig. 7, the adaptive unscented filter algorithm SOC can quickly converge to the true value even if the initial value of the battery SOC has a large error. When the SOC is less than 0.01, the GNL model, with the estimated error between ±1%, has better equivalent accuracy contrasting with that of the second-order RC model.
5. Conclusion
The GNL model more comprehensively reflects the actual situation where battery performance declines gradually, being conducive to reflecting the actual state of charge of the aging battery, which lays a good foundation for battery health evaluation. At the same time, the AUKF algorithm is applied to estimate battery SOC contrasting with the second-order RC model and EKF algorithm. The results show that the AUKF algorithm based on the GNL model reduces the estimation error of the aging battery SOC, which improves the accuracy of the estimation result, possessing high applicability and practicality.

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