Distribution of valence quarks and light-cone QCD sum rules.

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Abstract

A method for calculating the pion structure function directly in terms of light-cone wave functions is suggested. Taking twist-2 and twist-4 pion light-cone wave functions into account, it is shown that the QCD sum rule prediction is in agreement with quark distribution obtained from analysis of the Drell-Yan process. Twist-4 quark-gluon light-cone wave functions give a large positive contribution to the pion structure function for $x_B < 0.2$. A new constraint on the twist-2 pion light-cone wave function is obtained. We argue that the leading twist pion wave function $\varphi_\pi(u) \simeq 1$ for $u = 0.3$ with an accuracy of about 20-30%.

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1 Introduction

The calculation of structure functions for deep-inelastic lepton-hadron scattering, which is equivalent to finding the quark and gluon distributions in hadrons, is one of the most important problems of quantum chromodynamics (QCD). Perturbative treatments of deep-inelastic scattering give important information about structure functions as a function on $Q^2$ (where $Q^2$ is the momentum squared of the virtual photon) and, under some assumptions, can produce region behavior at small $x_B = Q^2/(2pq)$ [1]. At the present time there is significant activity in the application of the perturbative expansion for structure functions [3]. These perturbative investigations are very important. However, since quark and gluon distributions can not be determined from perturbative calculations, these distributions have been taken from experiment.

Theoretical calculations in QCD have achieved some progress in determining the second moments of the quark distribution functions for the nucleon [4] and pion [5] from QCD sum rules. The first moment of the chiral-odd structure function $h_1$ was evaluated in [6]. Recently, higher moments of quark distribution functions have also been considered in the QCD sum rule approach [7].

In ref. [8], Ioffe proposed a four-point correlator for theoretical calculations of quark distribution functions in the QCD sum rule approach suggested by Shifman, Vainshtein and Zakharov [9]. This method was applied for calculations of nucleon structure functions, such as $F_2(x_B)$ [10], $g_1(x_B)$ and $g_2(x_B)$ [11], $h_1(x_B)$ [12]. In the present paper we use a light-cone QCD sum rule [13]. Recently, it was suggested to apply this version of QCD sum rules to the problem of calculating the deep-inelastic structure function [14]. This light-cone QCD sum rule is based on the fact (which was noted by Ioffe in [8]) that if $x_B$ is not close to the boundary $x_B = 0$ and $x_B = 1$, then the imaginary part of the deep-inelastic scattering amplitude is determined by small distances in the $t$-channel. In the case of the pion structure function, the nearest singularity in the $t$-channel for the correlator of two vector currents and one axial current with a pion in the initial state is at $t = -\frac{x_B p^2}{1-x_B}$ for highly virtual photons, where $p$ is a momentum of the axial current. This means that in the case of intermediate $x_B$ we can apply operator product expansion (OPE) for the calculation of the correlator in the Euclidean region. Then, we can use a dispersion relation to construct QCD sum rules.
These light-cone QCD sum rules are formulated in terms of so-called light-cone wave functions of hadrons introduced in perturbative QCD to describe hadron form factors at large $Q^2$ \cite{15,16,17}.

This new version of QCD sum rules can be a useful tool for investigations of quark and gluon distributions and light-cone wave functions in hadron physics. Furthermore, in principle, it is possible to incorporate perturbative corrections to improve the calculations.

In the present paper we do not take into consideration perturbative corrections. So, our results can be considered as an initial condition for the evolution equation \cite{1}. For this reason, at the end of this paper we compare our results with the quark distribution function for low $Q^2$ \cite{18} obtained from the the analysis of the Drell-Yan process.

We consider the limit of massless quarks $m_q = 0$. In this limit, the pion is also massless, $m_\pi = 0$.

\section{Correlator}

As in \cite{14}, we consider the correlator

$$T_{\mu\rho\lambda}(p, q, k) = -i \int d^4x d^4ze^{ipx+iqz} <0|T\{j^5_{\mu}(x), j^d_{\rho}(z), j^d_{\lambda}(0)|\pi^-(k)>$$

(1)

for our calculation of the pion structure function. Here $k$ is the pion momentum,

$$j^5_{\mu} = \bar{u} \gamma_\mu \gamma_5 d, \quad j^d_{\rho,\lambda} = \bar{d} \gamma_\rho \gamma_\lambda d,$$

(2)

and the following kinematics is used:

$$k^2 = 0; \quad q^2 = (p + q - k)^2; \quad t = (p - k)^2 = 0; \quad s = (p + q)^2;
Q^2 = -q^2; \quad (2k, p + q) = s + Q^2; \quad (2pk) = p^2.$$  

(3)

The discontinuity in $s$ at fixed $p^2$ and $Q^2$ of the correlator \cite{1} is calculated from

$$ImT_{\mu\rho\lambda} = \frac{1}{2i} \left[T_{\mu\rho\lambda}(p^2, q^2, s + i\varepsilon) - T_{\mu\rho\lambda}(p^2, q^2, s - i\varepsilon)\right],$$

(4)

where $p^2$ and $q^2$ are space-like vectors, $p^2 < 0$, $q^2 < 0$, such that $|p^2|$, $|q^2| \gg \Lambda_{QCD}$. In the scaling limit, we assume that $|p^2| \ll |q^2|$ and keep only the first nonvanishing terms in an expansion in powers of $p^2/q^2$. 

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\(ImT_{\mu\rho\lambda}\) is calculated in the physical region of the \(s\)-channel, and the pion contribution in this amplitude has the following form,

\[
ImT_{\mu\rho\lambda} = p_\mu \frac{f_\pi}{p^2} \text{Im} \left\{ i \int d^4 z e^{iqz} < \pi(p)|T\{j_\rho^d(z), j_\lambda^d(0)\}|\pi(k) > \right\}.
\] (5)

On the other hand, the general form for the imaginary part in (5) is

\[
\text{Im} \left\{ i \int d^4 z e^{iqz} < \pi(p)|T\{j_\rho^d(z), j_\lambda^d(0)\}|\pi(k) > \right\} = A_1(s, Q^2)(q^2 g_{\rho\lambda} - q^{(2)}_{\rho} q^{(1)}_{\lambda}) + A_2(s, Q^2)(q^2 g_{\rho\lambda} - q^{(1)}_{\rho} q^{(1)}_{\lambda} - q^{(2)}_{\rho} q^{(2)}_{\lambda} + q^{(1)}_{\rho} q^{(2)}_{\lambda}) + B_1(s, Q^2) \left( p - \frac{pq^{(1)}}{q^2} q^{(1)} \right) \rho \left( p - \frac{pq^{(2)}}{q^2} q^{(2)} \right) \lambda + B_2(s, Q^2) \left( p - \frac{pq^{(1)}}{q^2} q^{(1)} \right) \rho \left( p - \frac{pq^{(2)}}{q^2} q^{(2)} \right) \lambda + B_3(s, Q^2) \left( p - \frac{pq^{(1)}}{q^2} q^{(2)} \right) \rho \left( p - \frac{pq^{(2)}}{q^2} q^{(1)} \right) \lambda + B_4(s, Q^2) \left( p - \frac{pq^{(1)}}{q^2} q^{(2)} \right) \rho \left( p - \frac{pq^{(2)}}{q^2} q^{(2)} \right) \lambda,
\] (6)

where \(q^{(1)} = q\) and \(q^{(2)} = p + q - k\) are the momenta of the virtual photons; \((q^{(1)})^2 = (q^{(2)})^2 = q^2\).

The structure function of deep-inelastic scattering is defined when \(q^{(1)} = q^{(2)}\), and it is clear in this case that

\[
4x_B q^d(x_B) = \frac{Q^2}{\pi} \left( B_1(s, Q^2) + B_2(s, Q^2) + B_3(s, Q^2) + B_4(s, Q^2) \right) |_{Q^2 \rightarrow \infty},
\] (7)

where

\[
Im \left\{ i \int d^4 z e^{iqz} < \pi(p)|T\{j_\rho^d(z), j_\lambda^d(0)\}|\pi(p) > \right\} = 4\pi x_B q^d(x_B) \left( p - \frac{pq}{q^2} q \right) \rho \left( p - \frac{pq}{q^2} q \right) \lambda + \ldots.
\] (8)

Here \(q^d(x_B)\) is \(d\)-quark distribution function of a pion.
In this paper the tensor structure $p_\mu p_\rho p_\lambda$ in correlator (1) is considered. We define the imaginary part of the correlation function for these tensor structures as $f_\pi \frac{4\pi x_B^2}{Q^2} t(p^2, x_B)$. Then, the dispersion relation for the function $t$ has the following form,

$$t(p^2, x_B) = \left( \frac{q^d(x_B)}{p^2} + \int \frac{\rho(s, x_B)}{s - p^2} ds \right),$$

where the last term corresponds to the higher-states contribution.

To suppress the contribution of exited states, as usually done in QCD sum rules, we will consider instead of $t(p^2, x)$ its Borel transform in $p^2$,

$$t(M^2, x_B) = \lim_{Q^2, n \to \infty, Q^2/n = M^2} \left( -\frac{p^2}{n!} \frac{d^n}{dp^2} \right) t(p^2, x_B)$$

$$= -\left( x_B^2 q^d(x_B) + \int \rho(s, x_B) e^{-s/M^2} ds \right).$$

The left-hand side will be calculated in terms of the light-cone pion wave function by using the operator product expansion (OPE). Every new term of the OPE will be suppressed by a factor $(\Lambda_{QCD}/M)^{t-2}$, where $t$ is the twist of light-cone wave function. For the right-hand side, we will use the standard continuum model for higher states, whose contribution is suppressed exponentially. As usual in the QCD sum rule approach, we can find the desired physical quantity (here it will be the quark distribution function) by using a fitting procedure in the region for the parameter $M^2$ where the higher states (which are estimated in the continuum model) and higher-twist pion wave function contributions are small.

### 3 Twist-2 light-cone wave function.

In this section we consider the contribution of the leading twist-2 pion light-cone wave function in the QCD sum rule (10).

It is clear that in the formal limit when $|p^2|, Q^2 \to \infty$, we can use a free $d$-quark propagator. All interactions with soft nonperturbative gluon fields are suppressed by a factor $\Lambda_{QCD}^2/p^2$ or $\Lambda_{QCD}^2/Q^2$. Perturbative contributions should be taken into account. However, as pointed out in the introduction, we do not take these corrections into consideration, assuming that our results
can be compared directly to the quark distribution function at low $Q^2$, where these contributions are small.

The result of a very simple calculation with free $d$-quark propagators in (11) gives

$$T_{\mu\rho\lambda}(p, q, k) = i \int d^4x d^4z e^{ipx+iqz} \frac{(x-z)_{\alpha}z_{\beta}}{4\pi^4(x-z)^4 z^4} \langle 0|\bar{u}(x)\gamma_5\gamma_\alpha\gamma_\rho\gamma_\beta\gamma_\lambda d(0)|\pi(k)\rangle.$$  \hspace{1cm} (11)

According to eqs. (6,7), the interesting contribution corresponds to the symmetric part of the amplitude $T_{\mu\rho\lambda}$. This symmetric part of the product of $\gamma$-matrices in (11) has the following form,

$$\gamma_\mu\gamma_5\gamma_\alpha\gamma_\rho\gamma_\beta\gamma_\lambda = g_{\rho\beta}(g_{\mu\alpha}\gamma_\lambda + g_{\lambda\alpha}\gamma_\mu) + g_{\beta\lambda}(g_{\mu\alpha}\gamma_\rho + g_{\rho\alpha}\gamma_\mu) + \text{(terms with } g_{\rho\lambda}, g_{\rho\mu}, g_{\lambda\mu}).$$  \hspace{1cm} (12)

Clearly the terms with $g_{\rho\lambda}, g_{\rho\mu}, g_{\lambda\mu}$ do not contribute to the $B_i$ that are defined in eq. (6).

From eq. (12) it is easy to see that the amplitude (11) is determined by only one matrix element,

$$\langle 0|\bar{u}(x)\gamma_5\gamma_\alpha\gamma_\mu d(0)|\pi(k)\rangle = i k_\mu f_\pi \int_0^1 dv e^{-i(kx)v}(\varphi_\pi(v) + x^2 g_1(v) + O(x^4))$$

$$+ f_\pi \left( x_\mu - \frac{x^2}{k_x k_\mu} \right) \int_0^1 dv e^{-i(kx)v}(g_2(v) + O(x^2)),$$  \hspace{1cm} (13)

where $\varphi_\pi(v), g_1(v)$ and $g_2(v)$ are twist-2 and twist-4 light-cone pion wave functions. In the formal limit $|p^2| \to \infty$ the leading contribution is determined by the twist-2 pion wave function, $\varphi_\pi(v)$. All other terms in the expansion of the amplitude (13) are suppressed by $(\Lambda_{QCD}^2/p^2)^{t-2}$ ($t$ is the twist of a light-cone wave function).

Using the definition (13), we obtain the following expression for the contribution of the leading twist-2 wave function,

$$- f_\pi \int_0^1 dv \int d^4x d^4z e^{i(p-k)z} \frac{\varphi_\pi(v)}{4\pi^4 x^4 z^4} e^{i(p-k)z + i(p+q-k)v z}$$

$$[z_\rho(x_\mu k_\lambda + x_\lambda k_\mu) + z_\lambda(x_\mu k_\rho + x_\rho k_\mu)].$$  \hspace{1cm} (14)
Here we have made the shift $x \to x + z$. After integration over the coordinates $x$ and $z$ we obtain

$$\int \frac{\varphi_\pi(v) dv}{(p - kv)^2 (p + q - kv)^2} \left[ (p + q - kv)_{\mu}(p - kv)_{\lambda}k_{\lambda} + (p - kv)_{\lambda}k_{\mu} \right] + (p + q - kv)_{\lambda}(p - kv)_{\mu}k_{\rho} + (p - kv)_{\rho}k_{\mu}).$$  \hspace{1cm} (15)

Using the following relations, which hold for the kinematics (3),

$$(p - kv)^2 = p^2(1 - v); \quad (p + q - kv)^2 = s - (s + Q^2)v, \hspace{1cm} (16)$$

we obtain an expression for the coefficient of the tensor $p_\mu p_\rho p_\lambda$ in the basis $(p, q^{(1)}, q^{(2)})$ (see 3), namely,

$$-4f_\pi \int_0^1 \frac{(1 - v)\varphi_\pi(v)}{p^2(s - (s + Q^2)v)} dv.$$ \hspace{1cm} (17)

From eq. (17), it is easy to find the imaginary part of this amplitude, $t(p^2, x_B)$, as

$$\frac{t(p^2, x_B)}{Q^2} = \frac{(1 - v)\varphi_\pi(v)}{p^2(s + Q^2)x_B^2} = \frac{\varphi_\pi(1 - x_B)}{Q^2p^2}.$$ \hspace{1cm} (18)

In eq. (18) we have used the relationships

$$\text{Im} \frac{1}{s - (s + Q^2)v} = -\pi\delta(s - (s + Q^2)v); \quad \frac{Q^2}{s + Q^2} = x_B.$$ \hspace{1cm} (19)

Comparing the result (18) with the dispersion relation (2), and using isotopic symmetry, $\varphi_\pi(v) = \varphi_\pi(1 - v)$, one may be tempted to claim that

$$q(x_B) = \varphi_\pi(x_B).$$ \hspace{1cm} (20)

Relation (20) corresponds to a pure parton picture when the pion consists of two free quarks, since

$$\int_0^1 q(x) dx = 1; \quad \int_0^1 xq(x) dx = 0.5,$$ \hspace{1cm} (21)
which follow from normalization ($\int_0^1 \varphi_\pi(v)dv = 1$) and the symmetry $\varphi_\pi(v) = \varphi_\pi(1-v)$ of the twist-2 pion wave function. Note, however, the identification (20) can not actually be valid, since the light-cone wave function is not a positive definite function. From the point of view of the QCD sum rules that are being constructed in this paper, (19) means that the leading-twist pion wave function determines the quark distribution function only in the region where $\varphi_\pi(x_B)$ has a positive value.

Note that in the parton model, the quark distribution function is equal to $\varphi_\pi^2(x_B)$. Thus, we can expect that the minimal corrections to the relation (20) (due to the higher-twist light-cone wave functions) will occur in the region where $\varphi_\pi(u) \approx \varphi_\pi^2(u) \approx q(u) \approx 1$. From the quark distribution function obtained in [18], one can find that $q(x_B) \approx 1$ for $0.2 < x_B < 0.3$ with an accuracy of about 20%. So, we can expect that the QCD sum rules will be the most reliable in that region. To check this assumption, we will have to evaluate the contribution of twist-4 light-cone wave functions.

### 4 Twist-4 Wave Functions

The easiest part of the calculation is the contribution of the two-particle twist-4 wave functions $g_1$ and $g_2$ (see [13]). The calculations with these wave functions are technically the same as those presented in the previous section. This correction was calculated in [14], and the corresponding contribution to $t(p^2, x_B)$ has the following form,

$$
t_4(p^2, x_B) = \frac{4}{p^4} \left( \frac{g_1(x_B) + G_2(x_B)}{x_B} + \frac{1}{2} g_2(x_B) - \frac{dg_1(x_B)}{dx_B} \right)
$$

$$
= \frac{1}{p^4} f_4(x_B). \tag{22}
$$

There are additional corrections corresponding to three-particle twist-4 wave functions. The general form of these three-particle wave functions can be written as follows:

$$
<0|\bar{u}(x)g_\alpha G_{\alpha\beta\gamma\delta}(z)d(0)|\pi(k)> = f_\pi(k_\alpha g_{\beta\gamma} - k_\beta g_{\alpha\gamma}) \int D\alpha_i e^{-ik_\alpha \gamma - ik_\alpha} f(\alpha_i)
$$

$$
+ \frac{f_\pi}{(k\cdot x)} k_\gamma(x_\alpha k_\beta - x_\beta k_\alpha) \int D\alpha_i e^{-ik_\alpha \gamma - ik_\alpha} f_x(\alpha_i)
$$

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with gluon field between points $x_d$ The contribution of the term (26) to the diagram where a 
Here and below we have used the notation $z$ (when $\alpha_m$ formula for the quark propagator in the presence of the gluon field [13], 
For practical calculations, it is very convenient to use the following f or-
$\alpha - \beta - \alpha - \beta = \alpha$. Comparing with the standard form for the light-cone wave funct ions 
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For practical calculations, it is very convenient to use the following f or-
$\alpha_m$ formula for the quark propagator in the presence of the gluon field [13], we obtain the following relations,

$$f(\alpha_i) = \varphi(\alpha_i),$$

$$f_x(\alpha_i) + f_z(\alpha_i) = \varphi(\alpha_i) + \varphi(\alpha_i),$$

$$\tilde{f}(\alpha_i) = \varphi(\alpha_i),$$

$$\tilde{f}_z(\alpha_i) + \tilde{f}_x(\alpha_i) = \varphi(\alpha_i) + \varphi(\alpha_i).$$

For practical calculations, it is very convenient to use the following formula for the quark propagator in the presence of the gluon field [13],

$$<0| T \{q(x)^a_{\alpha}, q^b_\alpha(z)\}|0> = \frac{\delta_{ab}}{2\pi^2 x^4}$$

$$+ \frac{g}{16\pi^2} \int_0^1 du G_{\mu\nu}(ux + (1 - u)z)$$

$$\left( (1 - u)(\hat{x} - \hat{z})\gamma_\mu\gamma_\nu + u\gamma_\mu\gamma_\nu(\hat{x} - \hat{z}) \right)$$

$$\delta_{\alpha\beta} \frac{\hat{x} - \hat{z}}{(x - z)^2} + O(g^2).$$

The contribution of the term [20] to the diagram where a $d$-quark interacts 
with gluon field between points $x$ and $z$ can be written in the following form,

$$\frac{1}{32\pi^4} \int_0^1 du \int \frac{d^4x d^4z}{(x - z)^2} e^{i p x + i q z}$$

$$<0| \bar{u}(x)\gamma_\mu\gamma_5 G_{\alpha\beta}(ux + (1 - u)z)((1 - u)(\hat{x} - \hat{z})\gamma_\alpha\gamma_\beta$$

$$+ u\gamma_\alpha\gamma_\beta(\hat{x} - \hat{z}))\gamma_\rho\gamma_\chi d(0)|\pi(k)>.$$

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The result of our calculations with (28) is
\[ t_g(x_B, p^2) = -\frac{Q^2}{p^4 x_B^2} \operatorname{Im} \left\{ \int \frac{\alpha_2 du D\alpha_i}{(1 - \alpha_1 - u\alpha_3)(s - (s + Q^2)(\alpha_1 + \alpha_3))} \right\} \]
\[ \left[ (1 - 2u)(2\varphi_\perp(\alpha_i) + \varphi_\parallel(\alpha_i)) - (2\tilde{\varphi}_\perp(\alpha_i) + \tilde{\varphi}_\parallel(\alpha_i)) \right] \]
\[ \int\limits_{Q^2 \to \infty}^\infty \]
\[ = -\frac{1}{p^4} \int_0^1 du \int_0^{1-x_B} \frac{d\alpha_3}{x_B + (1 - u)\alpha_3} [ (1 - 2u)(2\varphi_\perp(\alpha_i) + \varphi_\parallel(\alpha_i)) - (2\tilde{\varphi}_\perp(\alpha_i) + \tilde{\varphi}_\parallel(\alpha_i))] \]
\[ \alpha_2 = x_B, \alpha_1 = 1 - \alpha_2 - \alpha_3 = f \]
\[ g(x) = \frac{p^4}{p^4}. \] (29)

In this equation we have used the fact that our result depends on the functions \( f_x, f_z, \tilde{f}_x \) and \( \tilde{f}_z \) in the combination (25) only.

The contribution of the diagram in which a quark interacts with soft gluon fields in the propagator \( \langle q(z)\bar{q}(0) \rangle \) is suppressed by a factor \( 1/Q^2 \).

The contribution of the last term (27) of the quark propagator is
\[ \int e^{i\kappa x + i\kappa z} d^4xd^4z < 0|\bar{u}(x)\gamma_\mu\gamma_5 \frac{x - \hat{z}}{2\pi^2(x - z)} \gamma^\rho \frac{\hat{z}}{2\pi^2 z^4} \gamma^\lambda \]
\[ \int_0^1 dv (x - z)_\nu A_\nu(xv + (1 - v)z) d(0)|\pi(k) >. \] (30)

In the Fock-Schwinger gauge \( (x_\mu A_\mu(x) = 0) \), the gauge field \( A_\nu \) can be expressed in terms of \( G_{\mu\nu} \) as
\[ A_\nu(xv + (1 - v)z) = \int_0^1 u du (vx + (1 - v)z)_\delta G_{\delta\nu}(u(xv + (1 - v)z)). \] (31)

The result of our calculations for (31) is
\[ t_{g_1}(x_B, p^2) = -2\frac{Q^2}{x_B^2 p^4} \operatorname{Im} \left\{ \int \frac{Q^2 (udu D\alpha_i \varphi_\parallel(\alpha_i))}{(s - (s + Q^2)(\alpha_1 + u\alpha_3))^2} \right\} Q^2 \to \infty. \] (32)

In eq.(32) we did not include the part that could not be represented in terms of the light-cone wave functions \( \varphi_\parallel, \varphi_\perp, \tilde{\varphi}_\parallel, \tilde{\varphi}_\perp \). This contribution comes from the following integral,
\[ \int \frac{f_\pi}{\pi^4} \int ududv D\alpha_i d^4x d^4z e^{ix(p-k(\alpha_1+u\alpha_3))} e^{iz(p+k(\alpha_1+u\alpha_3))} \]
\[ (kz) \frac{x_\mu z_\rho k_\lambda}{x^2 z^4} \left( \frac{f_x(\alpha_i)}{(k, x + z)} + \frac{v f_\tilde{x}(\alpha_i)}{(k, vx + z)} \right). \] (33)
Numerical calculations show that the contribution of twist-4 quark-gluon wave functions (without the contribution of (33)) cancels the contribution of the two-particle wave functions in the QCD sum rule for the first moment of quark distribution function when we use a self-consistent set of twist-4 light-cone wave functions (they will be defined in the next Section) with the parameter $\varepsilon = 0$. It can be proved that there are no higher-twist corrections to the sum rule for the first moment of the quark distribution function. This indicates that the term (33) should be small, and we can not exclude the possibility that its contribution is equal to zero for this particular set of light-cone wave functions. In any case, we do not have a model for the wave functions $f_x$ and $f_z$ that we can use to make realistic numerical estimates for the contribution (33).

The remainder of the contribution (30) has the following form,

$$f_{g1}(x_B) = \frac{2}{p^4} \int_0^{1-x_B} \frac{d\alpha_3}{\alpha_3} \varphi(||(\alpha_i)|_{\alpha_1=\alpha_2=\alpha_3};\alpha_2=x_B)$$

$$- \frac{2}{p^4} \int_0^{1-x_B} \frac{d\alpha_3}{\alpha_3} \int_{1-x_B-\alpha_3}^{1-x_B} \varphi(||(\alpha_i)|_{\alpha_1=1-\alpha_1-\alpha_3})$$  \hspace{1cm} (34)

5 Light-Cone QCD Sum Rule.

Now we can begin our analysis using the QCD sum rule for the quark distribution function (10). This QCD sum rule has the following form,

$$\varphi_{\pi}(x_B) = \frac{1}{M^2}(f_4(x_B) + f_g(x_B) + f_{g1}(x_B)) = q^d(x_B) + c(x_B)e^{-m_{A1}^2/M^2}. \hspace{1cm} (35)$$

Here we use the results (18,22,29,34) obtained in the previous sections. The last term on the right-hand side of (34) imitates the higher-states contribution, where $m$ is the mass of a resonance. Below, we will assume that $m \sim m_{A_1} = 1.25GeV$. To find the quark distribution function $q(x_B)$ we have to analyze the QCD sum rules for all values of $x_B$.

In Fig.1, the dependence of functions $(f_4(x_B) + f_g(x_B) + f_{g1}(x_B))$, $f_4(x_B)$ and $(f_g(x_B) + f_{g1}(x_B))$ on $x_B$ is shown. The QCD sum rule is reliable in the region $0.2 < x_B < 0.6$, where the contribution of the twist-4 wave functions is small. Recall from our earlier discussion that we expected the minimal contribution of higher-twist wave functions to occur when $0.2 < x_B < 0.3$. 

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Figure 1: Twist-4 contributions to the d-quark distribution function as given in Eq. (35), evaluated with the pion wave functions given in Eq. (36).

For our numerical estimates, we have used the following set for light-cone wave functions:

\[ \varphi_{\pi}(u) = 6u(1 - u), \]
\[ g_1(u) = \frac{5}{2} \delta^2 \bar{u}^2 u^2 + \frac{1}{2} \epsilon \delta^2 [\bar{u}u(2 + 13\bar{u}u) + 10u^3(2 - 3u + \frac{6}{5}u^2)\ln(u) + 10\bar{u}^3(2 - 3\bar{u} + \frac{6}{5}\bar{u}^2)\ln(\bar{u})]; \]
\[ g_2(u) = \frac{10}{3} \delta^2 \bar{u}u(u - \bar{u}), \]
\[ G_2(u) = \frac{5}{3} \delta^2 \bar{u}^2 u^2, \]
\[ \varphi_{\perp}(\alpha_i) = 30\delta^2(\alpha_1 - \alpha_2)\alpha_3^2[\frac{1}{3} + 2\epsilon(1 - 2\alpha_3)], \]
\[ \varphi_{\parallel}(\alpha_i) = 120\delta^2 \epsilon(\alpha_1 - \alpha_2)\alpha_1\alpha_2\alpha_3, \]
\[ \tilde{\varphi}_{\perp}(\alpha_i) = 30\delta^2\alpha_3^2(1 - \alpha_3)[\frac{1}{3} + 2\epsilon(1 - 2\alpha_3)]. \]
\[
\hat{\varphi}_\parallel(\alpha_i) = -120\delta^2 \alpha_1 \alpha_2 \alpha_3 \left[ \frac{1}{3} + \epsilon(1 - 3\alpha_3) \right].
\] (36)

We have used the notation \( \bar{u} = (1 - u) \). The wave functions of twist-4 are very numerous. Here we use the twist-4 wave functions with leading and next-to-leading conformal spin [19] (see also [20]).

One of the parameters in (36) is defined by the matrix element
\[
\langle \pi | g_s \bar{d} \tilde{G}_{\alpha \mu} \gamma_\alpha u | 0 \rangle = i\delta^2 f_\pi q_\mu.
\] (37)

The QCD sum rule estimate of ref.[21] yields \( \delta^2 = 0.2 GeV^2 \) at \( \mu = 1 GeV \) (\( \mu \) is the renormalization scale). The last parameter is associated with the deviation of twist-4 wave functions from their asymptotic form. Here we used the set with \( \epsilon = 0 \). It was noted in the previous Section that we can expect that the contribution of (33) is small or even equal to zero. To use the set with \( \epsilon = 0.5 \) (see Ref. [19]) we have to take into account the term (33).

Note that there is large uncertainty in the choice for the form of these light-cone wave functions. Even the twist-2 pion wave function \( \varphi_\pi(x) \) is not known well enough, and there are different models for it. The most peculiar is the light-cone wave function with a two-humped profile, which was suggested by Chernyak and Zhitnitsky [15].

The asymptotic form for the wave functions can be taken as a starting point for a detailed investigation of these QCD sum rules. The results thus obtained from the sum rule (33) are shown in Fig.2. A comparison of these results with the ”experimental data”, which were obtained from Drell-Yan process and extrapolated to the low normalization point [18], shows good agreement. Agreement with the ”experimental data” can be improved by using a different form for the light-cone wave functions.

It is interesting to estimate the second moment of the quark distribution functions. Assuming that the region near the end points \( x_B = 0,1 \) (where our considerations are not valid) gives a small contribution to the second moment, we obtain the following sum rule from eq.(35),

\[
\frac{1}{2} - \frac{\delta^2}{M^2} + O(M^{-4}) = M_2^d + (\text{higher resonances}),
\] (38)

where

\[
M_2^d = \int_0^1 x_B q^d(x_B) dx_B.
\] (39)
Figure 2: Theoretical results for the d-quark distribution function of the pion compared to experimental results. The solid curve is the complete result appearing in Eq. (35), evaluated with the wave functions given in Eq. (36).
Note that the contribution of the quark-gluon wave function is small. It is about 10% of the contribution of twist-4 two particle wave functions. This means that our QCD sum rule is almost the same as the one obtained in [14]. The numerical evaluation of this sum rule gives $M_2^d = 0.27 \pm 0.05$, which is in good agreement with "experimental data": $M_2^d \simeq 0.3$ (see [18]).

It was noted above that from the point of view of the partonic model we can expect minimal higher-twist corrections to occur in the region where $q(x_B) \simeq 1$. The "experimental data" presented in Fig.2 show that for $x_B \simeq 0.3$, $q(x_B) = 1$. The twist-4 correction is small for $x_B = 0.3$. Thus, we can conclude that our consideration is self-consistent and leads to a new constraint on the pion twist-2 light-cone wave function, namely,

$$\varphi_\pi(u) \simeq 1 \ (u = 0.3).$$

This constraint follows from the "experimental data" [18] and from the sum rule (35). The estimate of higher-twist effects confirms our earlier argument in favor of such a constraint.

At the present time there are three different models for the twist-2 pion wave functions. The first is asymptotic wave function,

$$\varphi_\pi^{as.}(u) = 6u(1 - u) \quad \varphi_\pi^{as.}(0.3) = 1.26.$$ (41)

The second is Chernyak-Zhitnitsky wave function [15],

$$\varphi_\pi^{CZ}(u) = 30u(1 - u)(1 - 2u)^2; \varphi_\pi^{CZ}(0.3) = 1.01.$$ (42)

And, the third was suggested by Braun and Filyanov [19],

$$\varphi_\pi^{BF}(x) = 6x(1 - x)(1 + a_2(\mu)3/2(5(2x - 1)^2 - 1) + a_4(\mu)15/8(21(2x - 1)^4 - 14(2x - 1)^2 + 1)) \quad \varphi_\pi^{BF}(0.3) = 0.72$$ (43)

The coefficients $a_2 \simeq 0.41$, $a_4 = 0.23$ correspond to the normalization point $\mu = 1.3GeV$ (see [24]).

According to our findings, the most preferable model for the pion wave function is asymptotic one (41).
6 Conclusion

In this paper we have presented a new version of the light-cone QCD sum rule that can be used for investigating the structure functions of deep-inelastic scattering. We demonstrated that this approach gives predictions for the quark distribution function that are in reasonable agreement with present "experimental data".

We noted that the present QCD sum rule provides the possibility to determine the numerical value of the light-cone wave functions in the region $0.2 < x < 0.6$, where it is expected that the QCD sum rule should work. However, we have to emphasize that the perturbative corrections may give a significant contribution. These perturbative corrections can be evaluated in formalism of string operators [22].

Our analysis using the sum rule (35) can give important information about pion wave functions that are currently widely used in calculations of different hadronic processes [23]. A new constraint for the twist-2 light cone wave functions was formulated from this sum rule. This constraint indicates that the twist-2 pion light-cone wave function is not very different from its asymptotic form.

The agreement with "experimental data" can be significantly improved by changing the parameter set for the the light-cone wave functions. It is important to evaluate perturbative corrections.

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