The Deviation of the Size of the Broad-line Region between Reverberation Mapping and Spectroastrometry

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Abstract

The combination of the linear size from reverberation mapping (RM) and the angular distance of the broad-line region (BLR) from spectroastrometry (SA) in active galactic nuclei can be used as a “standard ruler” to measure the Hubble constant \(H_0\). Recently, Wang et al. successfully employed this approach and estimated \(H_0\) from 3C 273. However, there may be a systematic deviation between the response-weighted radius (RM measurement) and luminosity-weighted radius (SA measurement), especially when different broad lines are adopted for size indicators (e.g., \(H\beta\) for RM and Pa\(\alpha\) for SA). Here we evaluate the size deviations measured by six pairs of hydrogen lines (e.g., \(H\beta\), \(H\alpha\), and Pa\(\alpha\)) via the locally optimally emitting cloud (LOC) models of the BLR. We find that the radius ratios \(K = R_{SA}/R_{RM}\) of the same line deviated systematically from 1 (0.85–0.88) with dispersions between 0.063 and 0.083. Surprisingly, the \(K\) values from the Pa\(\alpha\)(SA)/\(H\beta\)(RM) and \(H\alpha\)(SA)/\(H\beta\)(RM) pairs not only are closest to 1 but also have considerably smaller uncertainty. Considering the current technology of infrared interferometry, the Pa\(\alpha\)(SA)/\(H\beta\)(RM) pair is the ideal choice for low-redshift objects in the SARM project. In the future, the \(H\alpha\)(SA)/\(H\beta\)(RM) could be used for high-redshift luminous quasars. These theoretical estimations of the SA/RM radius pave the way for future SARM measurements to further constrain the standard cosmological model.

Unified Astronomy Thesaurus concepts: Active galactic nuclei (16); Distance measure (395); Hubble constant (758); Reverberation mapping (2019); Interferometry (808); Standard candles (1563)

1. Introduction

The Hubble constant \(H_0\) is a fundamental parameter of cosmology. However, there is a significant difference (up to 4.4\(\sigma\)) between the Planck measurement from cosmic microwave background anisotropies, and supernovae Ia (SNe Ia) measurements calibrated with Cepheid distances (Freedman 2017; Riess et al. 2019; Planck Collaboration et al. 2020). Therefore, a better and independent measurement of \(H_0\) tension is urgently needed.

Active galactic nuclei (AGNs) are ubiquitous and the most luminous persistent celestial objects in the universe. They have the potentiality to become established as cosmological probes based on some of their features, such as the nonlinear relation between the UV and X-ray luminosities (Risaliti & Lusso 2019), the short-term UV/optical variability amplitude (Sun et al. 2018), the wavelength-dependent time delays of continuum flux variations (Collier et al. 1999; Cackett et al. 2007), and the lag–luminosity relationship for the broad-line region (BLR) (Watson et al. 2011; Czerny et al. 2013; Wang et al. 2020) or the dusty torus (Hoëníg & Kishimoto 2011; Hönik 2014; Koshida et al. 2014; Hönik et al. 2017; He et al. 2021).

Elvis & Karovska (2002) proposed a purely geometrical method to determine the distance to quasars with \(D_A = R_{BLR}/\theta\), where \(R_{BLR} = c\tau\), \(\tau\) is the light-travel time from the center to the BLR, and \(\theta\) is the resolved angular size of the BLR from interferometric observations. Compared with other tools to measure the cosmological distances, it has three primary advantages: (1) it is a model-independent method; (2) its uncertainties could be reduced by repeating observations of a median-size AGN sample (Wang et al. 2020; Songsheng et al. 2021); (3) AGNs are luminous and scattered in all directions over a wide range of redshift, which can be used to test the potential anisotropy of the accelerating expansion of the universe.

Recently, GRAVITY at the Very Large Telescope Interferometer (VLTI) successfully revealed the structure, kinematics, and angular sizes of the BLR of 3C 273 (an AGN at \(z = 0.15834\)) using the spectroastrometry (SA) method (Abuter et al. 2017; GRAVITY Collaboration 2018). Wang et al. (2020) firstly combined the SA measurement and the reverberation mapping (RM) measurement, namely the SARM project. Based on the SARM analysis, they were able to determine an angular distance of 551.5±97.2 Mpc to 3C 273, and thus to constrain \(H_0 = 71.5^{+11.9}_{-10.6} \text{ km s}^{-1} \text{ Mpc}^{-1}\) with the \(z-D_A\) relation (Peacock 1998).

However, the RM radius and SA radius could be different things: the RM radius represents the variable components of the BLR region and is known as the response-weighted (or flux variation-weighted) radius, while the SA radius represents the flux-weighted region. Furthermore, the RM and SA measurements adopted two different lines: Pa\(\alpha\) for SA and \(H\beta\) for RM measurements due to the difficulty that reverberation has with an infrared emission line with relatively weaker variability and longer lags than optical lines. Consequently, there may be a systematic deviation between these two types of radius.

In this work, we will systematically evaluate this probable deviation of SA/RM radius for several observable hydrogen lines (e.g., \(H\beta\), \(H\alpha\), and Pa\(\alpha\)) in the future SARM project based on the locally optimally emitting cloud (LOC) model of the BLR (Baldwin et al. 1995). The paper is organized as follows. In Section 2, we illustrate the setup of the LOC simulations.
The difference in the SA and RM sizes is calculated for several hydrogen lines in Section 3. We conclude in Section 4.

2. Photoionization Simulation

2.1. LOC Model

We use the photoionization code CLOUDY17.01 (Ferland et al. 2017) to carry out the simulations for the line emission of the BLR based on the LOC model, which is a physically motivated photoionization model for the BLR (Baldwin et al. 1995). In the LOC model, the BLR consists of clouds with different gas densities and distances from the central continuum source with an axisymmetric distribution. The total emission-line intensity we observe originates from the combination of all clouds but is dominated by those with the highest efficiency of reprocessing the incident ionizing continuum. As a result, the total emission-line intensity can be calculated from the formula (Baldwin et al. 1995)

\[ L_{\text{line}} \propto \int_{R_{\text{in}}}^{R_{\text{out}}} r^2 F(r) f(r) g(n) dn dr, \]

where \(F(r)\) is the emission intensity of a single cloud at radius \(r\), \(f(r)\) is the cloud covering factor, and \(g(n)\) is the cloud distribution function. The distribution functions of clouds, i.e., \(f(r)\) and \(g(n)\), can be specified by the observed emission-line properties. According to Baldwin et al. (1995), \(f(r)\) and \(g(n)\) can be simplified as power-law functions: \(f(r) \propto r^\Gamma\) and \(g(n) \propto n^\beta\) with \(\Gamma = -1\) and \(\beta = -1\). For the well-known NGC 5548, Korista & Goad (2000) constrained \(-1.4 < \Gamma < -1\) by using the observed time-averaged UV spectrum. Based on a large sample of 5344 quasar spectra taken from the Sloan Digital Sky Survey Data Release 2, the parameters \(\Gamma\) and \(\beta\) are constrained to \(\Gamma = -1.52 \pm 0.13\) and \(\beta = -1.08 \pm 0.05\) (Nagao et al. 2006). Here, we adopt a sufficiently wide parameter range of \(\Gamma\) from \(-1.5\) to \(-0.5\), and a fixed \(\beta = -1\) (since previous studies, e.g., Korista & Goad (2000), suggest that \(\beta\) should not be far from \(-1\)) to evaluate the deviation of SA/RM radius for several observable hydrogen lines.

We assume a typical AGN with the black hole mass \(M_{\text{BH}} = 10^8 M_\odot\) and bolometric luminosity \(L_{\text{bol}} = 10^{45}\) erg s\(^{-1}\) with respect to the Eddington ratio \(L_{\text{bol}}/L_{\text{edd}} \simeq 0.1\) in our simulation. Considering the generality, we use a typical radio-quiet AGN spectral energy distribution (SED) (Dunn et al. 2010) for the incident SED, resulting in an emission rate of hydrogen-ionizing photons of \(Q_H = 10^{55}\) s\(^{-1}\). The outer BLR boundary is determined by dust sublimation of the inner edge of the torus corresponding to a surface ionizing flux \(\Phi_H = 17.9\) cm s\(^{-1}\) (Nenkova et al. 2008; Landt et al. 2019), which is about 140 lt-day in the best-studied AGN NGC 5548 (e.g., Korista & Goad 2000), with an average \(Q_H = 10^{54.13}\) s\(^{-1}\) for the BLR. According to the definition of surface ionizing flux,

\[ \Phi_H = \frac{Q_H}{4\pi R^2}, \]

the outer BLR boundary is \(R_{\text{out}} = 10^{18}\) cm for \(Q_H = 10^{55}\) s\(^{-1}\). Furthermore, both LOC calculation of the most extended Mg II line (Guo et al. 2020) and near-IR reverberation measurements (Kishimoto et al. 2007) constrained that the outer BLR boundary or the innermost radius of the torus is a factor of \(\sim 3\) less than or approximately equal to \(10^{18}\) cm. Therefore, we assume the range of the outer BLR boundary is \(10^{17.5} - 10^{18}\) cm to evaluate the deviation of the SA/RM radius.

In our calculation, we set the inner BLR boundary to be two orders of magnitude smaller than the outer BLR boundary (Landt et al. 2014), i.e., \(R_{\text{in}} = 10^{16}\) cm. Guo et al. (2020) showed that the influence of the uncertainty of \(R_{\text{in}}\) on the final result can be negligible. We consider the range of the gas number density: \(10^8\) cm\(^{-3}\) \(\leq n_H \leq 10^{12}\) cm\(^{-3}\), since below \(n_H = 10^9\) cm\(^{-3}\) the clouds are inefficient in producing emission lines and above \(n_H = 10^{12}\) cm\(^{-3}\) the clouds mostly produce thermalized continuum emission rather than emission lines (Korista & Goad 2000). In addition, we adopt the metallicity \(Z = Z_\odot\) in our calculation. Note that, since we only focus on the hydrogen emission lines, the metallicity will not affect our calculations. The overall covering factor is set to \(CF = 50\%\), as adopted in Korista & Goad (2004).

As shown in Figure 1, we calculate the H\(\alpha\), H\(\beta\), and Pa\(\alpha\) emissions at different gas densities \(n_H\) and surface ionizing fluxes \(\Phi_H\) for a cloud with a hydrogen column density \(N_H = 10^{23}\) cm\(^{-2}\). For a fixed \(Q_H\), \(\Phi_H\) is inversely proportional.

![Figure 1](https://example.com/figure1.png)
Figure 2. (a) Radial emissivity function $F(r)$. (b) Radial responsivity function $\eta(r)$ calculated based on Equation (4). $R_{in}$ and $R_{out}$ are the inner and outer BLR boundaries in our calculation.

to the square of the distance, i.e., $\Phi(H) \propto 1/r^2$. For $Q_H = 10^{55}$ s$^{-1}$, $R_{in} = 10^{16}$ cm and $R_{out} = 10^{18}$ cm correspond to $\Phi(H) = 10^{21.9}$ cm$^{-2}$ s$^{-1}$ and $10^{21.9}$ cm$^{-2}$ s$^{-1}$, respectively. So, we integrate emission intensity to generate the total ionizing photon flux,

$$F(H) = \int_{R_{in}}^{R_{out}} r^2 F(r) f(r) g(n) dn dr.$$

2.2. The Definition of SA and RM Sizes

The spectroastrometry size of the BLR is actually the flux-weighted radius. As a result, the SA radius can be calculated as follows:

$$R_{SA} = \frac{\int_{R_{in}}^{R_{out}} r^3 F(r) f(r) g(n) dn dr}{\int_{R_{in}}^{R_{out}} r^2 F(r) f(r) g(n) dn dr}.$$  (3)

The $F(r)$ curves of H$\alpha$, H$\beta$, and Pa$\alpha$ are shown in Figure 2(a).

The RM size is actually the response-weighted radius. According to the previous works (Goad et al. 1993; Korista & Goad 2004; Goad & Korista 2014), the emission-line responsivity (in logarithmic space) to the changes in the incident hydrogen-ionizing photon flux can be written as

$$\eta(r) = \frac{d \log_{10} F(r)}{d \log_{10} \Phi_H} \propto -0.5 \frac{d \log_{10} F(r)}{d \log_{10} r}, \text{ since } \Phi_H \propto r^{-2}. \tag{4}$$

$\eta(r)$ of H$\alpha$, H$\beta$, and Pa$\alpha$ are shown in Figure 2(b). In linear space, the emission-line responsivity $dF(r)/d\Phi_H = [F(r)/\Phi_H] \eta(r) \propto F(r) \eta(r) r^2$. As a result, the RM radius can be calculated as follows:

$$R_{RM} = \frac{\int_{R_{in}}^{R_{out}} r^3 F(r) \eta(r) f(r) g(n) dn dr}{\int_{R_{in}}^{R_{out}} r^2 F(r) \eta(r) f(r) g(n) dn dr} = \frac{\int_{R_{in}}^{R_{out}} r^5 F(r) \eta(r) f(r) g(n) dn dr}{\int_{R_{in}}^{R_{out}} r^4 F(r) \eta(r) f(r) g(n) dn dr}.$$  (5)

3. The Deviation between SA and RM Sizes

3.1. The Simulation Results

We adopt a radius ratio $K = R_{SA}/R_{RM}$ to describe the deviation between SA and RM sizes. In this section, we will estimate the uncertainty of $K$ for the following parameter intervals, which have been mentioned in the previous section: $-1.5 < \Gamma < -0.5$, $z = -1$, $10^{3} cm^{-3} \leq n_H \leq 10^{12} cm^{-3}$, $N_H = 10^{23} cm^{-2}$, and $10^{17.5} cm < R_{out} < 10^{19} cm$. We define the average and half-range of $K$ as $\bar{K} = (K_{max} + K_{min})/2$ and $\Delta K = (K_{max} - K_{min})/2$, where $K_{max}$ and $K_{min}$ are the maximum and minimum of $K$ in the parameter intervals, respectively. We first calculate $K$ for the combination of the same lines. As shown in Figures 3(a)–(c), the radius ratios $K$ are $0.859 \pm 0.063$, $0.879 \pm 0.067$, and $0.851 \pm 0.083$ for the pairs H$\alpha$/H$\alpha$(RM), Pa$\alpha$/Pa$\alpha$(RM), and H$\beta$/H$\beta$(RM), respectively. Surprisingly, all the radius ratios $K$ obtained from these same lines are much lower than 1 ($\sim 0.85$–$0.88$). For the combinations of the different lines (Figures 3(d)–(f)), the radius ratios $K$ are $0.974 \pm 0.042$, $0.947 \pm 0.028$, and $0.894 \pm 0.061$ for the pairs Pa$\alpha$/H$\beta$(RM), H$\alpha$/H$\alpha$(RM), and Pa$\alpha$/H$\alpha$(RM), respectively. Consequently, the corrected angular diameter distance $D_A^{corr} = R_{SA} / \theta = K \times R_{RM} / \theta = K \times D_A$. According to the $z-D_A$ relation (Peacock 1998), the corrected Hubble constant $H_0^{corr} = H_0 / K$, where $H_0$ is measured by $D_A$. Interestingly, it clearly shows that not only is the half-range of $K$ from combinations of different lines less than that from combinations of the same lines, but also the values of $K$ from combinations of different lines are closer to 1. As a result, combinations of different lines are more suitable for the SARM project than combinations of the same lines. Among them, the $K$ values from the Pa$\alpha$/H$\beta$(RM) and H$\alpha$/H$\beta$(RM) pairs are closest to 1 and have the smallest uncertainty.

At present, the technology of interferometry has been achieved only in the infrared band K or at longer wavelengths and for bright sources. For example, GRAVITY of VLTI operates in the wavelength range 2.0–2.4 $\mu$m. For objects with a redshift $z < 0.28$ the Pa$\alpha$ line is the only accessible strong broad line. Thus the Pa$\alpha$/H$\beta$(RM) pair is the best choice for low-redshift objects in the SARM project. In the future, with telescopes that have a much larger collection area and with advanced technology, it will be possible to conduct
interferometric observations for faint sources and at shorter wavelengths, enabling SARM for intermediate-redshift quasars. In that case, a combination of $\text{H}α(\text{SA})/\text{H}β(\text{RM})$ would be fruitful.

3.2. Comparison between Simulation and Observation for the RM Measurements of Hα and Hβ

In order to test the reliability of our simulation, we compare the simulation with observation for RM measurements of Hα and Hβ. As shown in Figure 4(a), the radius ratio of Hα to Hβ of the RM measurements is $R_{\text{RM}(\text{H}α)}/R_{\text{RM}(\text{H}β)} = 1.10 \pm 0.04$. The definitions of the average and half-range of $R_{\text{RM}(\text{H}α)}/R_{\text{RM}(\text{H}β)}$ are the same as for $K$. The observational results from Kaspi et al. (2000), Bentz et al. (2010), and Grier et al. (2017) are shown in Figure 4(b). The black histogram in Figure 4(b) represents the sum of these three samples. The mean and standard deviation of the distribution of observational $R_{\text{RM}(\text{H}α)}/R_{\text{RM}(\text{H}β)}$ are $1.4 \pm 0.5$. Both this simulation and observational results suggest that the Hα region is slightly larger than the Hβ region. And our simulation result is roughly consistent with the observational results within a standard deviation. Therefore, our selection of parameters for the LOC model and the simulation results are in a reasonable range.

3.3. Discussion

In general, the range of the number density of BLR gas can be considered to be $10^8 \text{ cm}^{-3} \leq n_H \leq 10^{12} \text{ cm}^{-3}$. However, as...
shown in Figure 1, the locations of EW peaks of Hα, Hβ, and Paα lines are in the range $10^{12}$ cm$^{-3} < n_H < 10^{14}$ cm$^{-3}$. Furthermore, the maximum value of BLR gas density can reach $10^{14}$ cm$^{-3}$ under the model of radiation pressure confinement (RPC, i.e., equilibrium between the gas pressure and radiation pressure) (Baskin et al. 2014; Stern et al. 2014). In view of this, it is necessary to check the simulation results when the maximum value of $n_H$ reaches $10^{14}$ cm$^{-3}$. The radius ratios $K$ in the case of $10^{14}$ cm$^{-3} < n_H < 10^{16}$ cm$^{-3}$ are shown in Figure 5. Similar to the case of $10^8$ cm$^{-3}$, not only is the half-range of $K$ from combinations of different lines in (d)–(f) less than that from combinations of the same lines in (a)–(c), but also the values of $K$ from combinations of different lines are closer to 1. Similar to Figure 3, the $K$ values from the $\text{Pa}\alpha$($\alpha$)/$H\beta$($\beta$) and $H\alpha$($\alpha$)/$H\beta$($\beta$) pairs are also closest to 1 and have the smallest uncertainty. Furthermore, as shown in Figure 6, the radius ratio of $H\alpha$ to $H\beta$ of the RM measurements is $R_{\text{RM}}(H\alpha)/R_{\text{RM}}(H\beta) = 1.10 \pm 0.05$, which is similar to the case for $10^8$ cm$^{-3} < n_H < 10^{12}$ cm$^{-3}$ (Figure 4). As a result, no matter whether the maximum value of the number density of BLR gas is $10^{12}$ cm$^{-3}$ or $10^{14}$ cm$^{-3}$, the $\text{Pa}\alpha$($\alpha$)/$H\beta$($\beta$) and $H\alpha$($\alpha$)/$H\beta$($\beta$) pairs are always the best two choices for the SARM project.

As mentioned above, the RPC model (Baskin et al. 2014) is a more physically realistic BLR model and assumes that the gas pressure and radiation pressure are in equilibrium. In the RPC model, the cloud is no longer a slab of uniform density, but has a density structure. In our next work, we will calculate the deviation between SA and RM sizes under the RPC model and make a comprehensive and detailed comparison with the LOC model.

4. Conclusion

The combination of interferometry and RM for the BLR of AGNs can be used as a “standard ruler” to measure the Hubble constant $H_0$. However, there may be a systematic deviation between these two different radii $R_{\text{SA}}$ and $R_{\text{RM}}$. In this work, we calculate this systematic deviation for the three hydrogen lines $H\alpha$, $H\beta$, and Paα based on the LOC model of the BLR. We estimate the half-range of the radius ratio $K = R_{\text{SA}}/R_{\text{RM}}$ for sufficiently wide parameter ranges: $-1.5 < \Gamma < -0.5$, $\beta = -1$, $10^8$ cm$^{-3} < n_H < 10^{12}$ cm$^{-3}$, $N_{\text{HI}} = 10^{23}$ cm$^{-2}$, and $10^{17.5}$ cm $< R_{\text{out}} < 10^{18}$ cm. Our main results can be summarized as follows.

1. The ratios for the same line are systematically lower than unity by 10%–15% for all hydrogen lines considered here (i.e., $H\alpha$($\alpha$)/$H\beta$($\beta$), $\text{Pa}\alpha$($\alpha$)/$\text{Pa}\alpha$($\alpha$), and $H\beta$($\beta$)/$H\beta$($\beta$)) and the scatter in these ratios is typically 0.06–0.08.
2. The $K$ values from the Pa$\alpha$/H$\beta$(RM) and H$\alpha$(SA)/H$\beta$(RM) pairs are closest to 1 and have the smallest uncertainty.

Considering the current technology of infrared interferometry, the Pa$\alpha$/H$\beta$(RM) pair is the best choice for low-redshift objects in the SARM project. In the future, the H$\alpha$(SA)/H$\beta$(RM) pair could be used for high-redshift luminous quasars in the SARM project.

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