A Membrane Action for OM Theory

J. Antonio García, Alberto Güijosa, and J. David Vergara

Departamento de Física de Altas Energías
Instituto de Ciencias Nucleares, UNAM
Apdo. Postal 70-543, México, D.F. 04510

garcia, alberto, vergara@nuclecu.unam.mx

Abstract

Through direct examination of the effect of the OM limit on the M2-brane worldvolume action, we derive a membrane action for OM theory, and more generally, for the eleven-dimensional M-theoretic construct known as Galilean or Wrapped M2-brane (WM2) theory, which contains OM theory as a special class of states. In the static gauge, the action in question implies a discrete spectrum for the closed membrane of WM2 theory, which under double dimensional reduction is shown to reproduce the known NCOS/Wound closed string spectrum. We examine as well open membranes ending on each of the three types of M5-branes in WM2 theory (OM theory arising from the ‘longitudinal’ type), and show that the ‘fully transverse’ fivebrane is tensionless. As a prelude to the membrane, we also study the case of the string, where we likewise obtain a reparametrization-invariant action, and make contact with previous work.

January 2002
1 Introduction

In recent times interesting limits of string/M theory have been discovered which give rise to decoupled open brane theories exhibiting some form of noncommutativity. Best understood among these are the $(p + 1)$-dimensional Noncommutative Open String (NCOS) theories \[1, 2\], defined as a low-energy limit of a stack of $Dp$-branes in the presence of a near-critical worldvolume electric field (or, equivalently, a near-critical bulk Kalb-Ramond field $B_{01}$). While decoupling the usual closed string modes, the limit in question remarkably manages to retain the whole tower of open string excitations; the result is a non-gravitational open string theory which displays noncommutativity between space and time \[2\].

Shortly after the formulation of NCOS theories, generalizations based on other types of open branes were found \[4, 5, 6, 7, 8\]. Foremost among these is Open Membrane (OM) theory \[4, 5\], defined as a low-energy limit of a stack of $M5$-branes with a near-critical worldvolume ‘electric’ field strength (or, equivalently, in a near-critical bulk gauge field $C_{012}$). The limit yields a $(5+1)$-dimensional theory decoupled from gravity, which reduces to the $(2, 0)$ superconformal theory at low energies. OM theory is expected to admit a description in terms of open $M2$-branes ending on the fivebranes, and to possess a generalized form of noncommutativity. This $M$-theoretic structure plays a role analogous to that of $M$ theory itself: OM theory underlies and unifies all of the noncommutative theories which originate from string theory, be they of the open brane \[1, 2, 4, 6, 7, 8\] or of the purely field-theoretic \[9, 10\] type.

The nature of NCOS theories came to be better understood through the work of Klebanov and Maldacena \[11\], who discovered that, upon compactification of the electric field direction, the NCOS spectrum contains not only open strings but also closed strings with strictly positive winding number. After that, it was shown in \[12, 13\] that the NCOS limit may be taken with or without D-branes, and consequently defines a $D$-dimensional string theory ($D = 10$ for the superstring). In more detail: starting with any of the conventional string theories, one can single out a spatial direction (we will take it to be $x^1$, and refer to it as the longitudinal direction), compactify it on a circle of radius $R$, and consider an $\epsilon \to 0$ limit where the coupling constant, string length, (closed string) metric, and Kalb-Ramond field scale as

\[
g_s = \frac{G_s}{\sqrt{\epsilon}}, \quad l_s = L_s \sqrt{\epsilon}, \quad g_{\mu\nu} = (-1, 1, \epsilon, \epsilon, \ldots), \quad B_{01} = 1 - \lambda \epsilon, \quad (1)
\]

with $G_s, L_s, R, \lambda$ fixed. This procedure yields a consistent $D$-dimensional theory, known as Wound \[12\] or Non-relativistic \[13\] string theory, in which all objects carry strictly positive F-string winding along the longitudinal direction and are essentially non-relativistic. The parameters $G_s$ and $L_s$ introduced in (1) are the effective coupling constant and string length of the theory, and $\lambda$ is essentially a free (and for the most part physically irrelevant) parameter.\footnote{See \[12, 16\] for further characterization of these parameters and their relation to the NCOS parameters $G_o^2, \alpha'$.} Since (1) is the NCOS limit, NCOS theory
evidently corresponds to the class of states in Wound string theory which contain D-branes extended along the longitudinal direction (with the theory on the branes decoupling from the bulk as $R \to \infty$). Other states in Wound string theory contain closed strings, transverse D-branes, and NS5-branes [12, 13, 14]. Gravity also turns out to be present, but in a vastly simplified form: it is Newtonian when the theory is formulated on a flat background [13, 14], and ‘asymptotically Newtonian’ in a more general background [11, 13, 17].

Given the relation between NCOS and OM theory, the embedding of NCOS into Wound string theory implies an analogous embedding for the OM case. Indeed, Wound IIA string theory can be lifted to eleven dimensions to obtain what is known as Wrapped [12] or Galilean [13] M2-brane theory, an M-theoretic construct which contains OM theory as a special class of states [12]. Wrapped M2-brane (WM2) theory is defined as M theory compactified on a rectangular two-torus with radii $R_1, R_2$, in an $\epsilon \to 0$ limit where the Planck length, metric, and three-form gauge field scale as

$$l_P = L_P \epsilon^{1/3}, \quad g_{MN} = (-1, 1, 1, \epsilon, \epsilon, \ldots), \quad C_{012} = 1 - \lambda \epsilon,$$

with $L_P, R_1, R_2, \lambda$ fixed. All objects in this eleven-dimensional theory carry strictly positive M2 wrapping number on the 1-2 torus, and are in effect non-relativistic. OM theory corresponds to those states of WM2 theory which contain M5-branes extended along the ‘longitudinal’ directions 1-2. WM2 theory contains in addition (partially or fully) transverse M5-branes, closed M2-branes, and Newtonian gravity [12, 13, 16], and includes all Wound string and Wrapped brane theories [12, 13] (and consequently all noncommutative open brane theories) as special limits. It is clearly desirable to increase our knowledge about this rich theoretical structure, which constitutes a simplified model of M theory.

OM theory is of course the most interesting subsector of WM2 theory. To date, information about it has been gathered either through direct examination of the effect of its defining limit, or by exploiting its connection to better understood theories. Attempts have been made to understand its geometry [18, 5, 19] and noncommutative structure [18, 5, 20], and its dual supergravity formulation has been scrutinized [21] (other work may be found in [22]). In the NCOS case, the properties of the theory have for the most part been deduced in a similar manner, by focusing on the corresponding aspect of the parent string theory and then studying the effect of the limit. An alternative approach would be to take the limit, once and for all, at the level of the worldsheet action. This was in fact accomplished by Gomis and Ooguri [13], and has the advantage of producing a finite worldsheet Lagrangian which serves as a more explicit definition of the full Wound string theory. It is the purpose of this paper to derive an analogous membrane worldvolume action for WM2/OM theory.

Our work is in consonance with the idea that it should be possible to capture

---

2 An intriguing fact, with implications that remain to be determined, is that the transverse NS5-branes of Wound string theory are tensionless [13]. In Section 4.3 we will discuss these objects from an eleven-dimensional perspective.

3 One may also consider more general compactifications [13, 17].
the physics of WM2/OM theory through an appropriate membrane action (or some regularized version thereof). This would certainly have to be the case if the same statement were true for M theory itself, as has been advocated over the years by a number of authors (see [23] for some interesting recent developments). In our more restricted setting, the question is to what extent a membrane formulation of WM2/OM theory will be afflicted by the same problems as the supermembrane. We will adopt a pragmatic attitude in this regard, and simply try to see how far one can get in developing such a formulation.

The approach of Gomis and Ooguri [13] for deriving the Wound string theory action made use of the standard Polyakov action for the string in conformal gauge, and involved as an essential ingredient a correlation between the worldsheet and longitudinal spacetime light-cones. It is not evident how one might generalize their approach to the membrane case, so we will develop an alternative procedure. For simplicity, we will begin by examining the point particle case in Section 2, where we will find that the Nambu-Goto type description allows for a much simpler derivation of the limiting form of the action, and one that can be easily generalized to the higher-dimensional cases. We will then apply this approach in Section 3 to derive a reparametrization-invariant action for Wound string theory, which can be gauge-fixed to obtain the known closed and open string spectra.

In Section 4 we proceed to the membrane case, our main interest. We restrict our attention to the bosonic part of the system, leaving its supersymmetric completion for future work. We obtain a reparametrization-invariant action for WM2 theory, and examine the description it provides for the various objects of the theory. For closed membranes or open membranes ending on a fully transverse M5-brane, a static gauge choice reduces the action to free-field form, and leads to a discrete non-relativistic spectrum. Upon double dimensional reduction, the closed membrane spectrum is shown to yield the correct Wound closed string spectrum. The cases where the open membrane ends on a partially transverse or longitudinal fivebrane are incompatible with the choice of static gauge, and therefore qualitatively different (remember that the latter is the OM theory setup). We show that in these cases the action can be simplified by choosing an orthonormal gauge, and determine the relevant boundary conditions. The potential which follows from this action is seen to possess flat directions, implying instabilities. In addition, we compute the tensions of each of the three types of M5-brane in WM2 theory, and find that the fully transverse fivebrane is tensionless. Our conclusions are summarized and discussed in Section 5. We also include an Appendix with a more careful analysis, carried out within the Hamiltonian formalism, of the limiting and gauge-fixing procedures, which provides a useful complement to the perspective of the main text.
2 The Point Particle

A relativistic point charge of mass $m$ in a background gauge field $A_\mu$ can be described with the standard action

$$S = -m \int d\tau \left[ \sqrt{-\dot{X}^\mu \dot{X}_\mu} - A_\mu \dot{X}^\mu \right]$$  \hspace{1cm} (3)

(where dots stand for $\tau$-derivatives), or equivalently, with

$$I = -\frac{m}{2} \int d\tau \left[ \sqrt{-\dot{\gamma} \left( \gamma^{-1} \dot{X}^\mu \dot{X}_\mu + 1 \right) - 2A_\mu \dot{X}^\mu} \right],$$  \hspace{1cm} (4)

where an intrinsic worldline metric $\gamma$ has been introduced as an auxiliary variable. The equivalence of (3) and (4) can be made manifest by eliminating $\gamma$ from $I$ using its equation of motion, which sets $\gamma$ equal to the induced metric $\dot{X}^\mu \dot{X}_\mu$.

Let us now study the effect of the $\epsilon \to 0$ limit

$$m = M \epsilon^{-1}, \quad g_{\mu\nu} = (-1, \epsilon, \epsilon, ...), \quad A_0 = 1 - \lambda \epsilon, \quad M \text{ and } \lambda \text{ fixed},$$  \hspace{1cm} (5)

which is the particle analog of (1) and (2). Consider first the effect on $I$. Inserting (3) in (4), and choosing for convenience the gauge $\gamma = -1$, we have

$$I = -\frac{M}{2\epsilon} \int d\tau \left[ (\dot{X}^0)^2 + 1 - 2A_0 \dot{X}^0 \right] + \frac{M}{2} \int d\tau \dot{X}_\perp^2.$$  \hspace{1cm} (6)

The relative scaling in (5) between $m$ and the spatial components of the metric has been chosen so as to make the action for the spatial coordinates $X_\perp$ manifestly finite. To deal with the divergence seen in the temporal term, we proceed in analogy with (3), rewriting the action by means of a Lagrange multiplier $l$,

$$I = -\int d\tau \left[ l(\dot{X}^0 - A_0) - \frac{\epsilon}{2M} l^2 + 1 - (A_0)^2 - \frac{M}{2} \dot{X}_\perp^2 \right].$$  \hspace{1cm} (7)

The limit $\epsilon \to 0$ can then be taken without any difficulty, yielding

$$I_W = -\int d\tau \left[ l(\dot{X}^0 - 1) - \frac{M}{2} \dot{X}_\perp^2 + \lambda M \right].$$  \hspace{1cm} (8)

Notice that, in the end, $l$ is nothing but $p_0$, the momentum conjugate to $X^0$ (see the Appendix). Its equation of motion implies the `static gauge’ condition $\dot{X}^0 = 1$, which is simply our original gauge condition $\gamma = \dot{X}^\mu \dot{X}_\mu = -1$, in the limit (5). $I_W$ is thus seen to be the action for a non-relativistic particle of mass $M$. Indeed, demanding that the associated worldline Hamiltonian vanish (just as it did for the original system, as a result of reparametrization invariance), one finds the energy spectrum $p_0 = p_\perp^2 / 2M + \lambda M$. Its non-relativistic nature is a result of the scaling of the metric in (5), which takes the speed of light to infinity. The presence of $A_0$ shifts the momentum conjugate to $X^0$; the role of the gauge field is thus merely to subtract a (dynamically irrelevant) divergent contribution to $p_0$, leaving behind a finite energy.
shift controlled by the free parameter $\lambda$. A more general gauge field configuration
\[ A_0(x) = 1 - \epsilon a_0(x), \quad A_i(x) = \epsilon a_i(x), \]
results in a coupling of the non-relativistic particle to the gauge field $a_\mu(x)$.

Now consider the effect of the limit (5) on the action (3). With the metric scaling
according to (5), the square root can be expanded in powers of $\epsilon$. The leading term,
\[ \dot{X}^0, \]
is finite, so multiplied by $m \propto \epsilon^{-1}$ it would imply a divergent contribution to the
action. It is precisely cancelled, however, by the leading contribution of the gauge
field. The subleading terms lead then to the finite action
\[ S_W = -\int d\tau \left[ -\frac{M \dot{X}^2}{2 X^0} + \lambda M \dot{X}^0 \right], \tag{9} \]
which can be recognized as the reparametrization-invariant version of (8), and in
particular leads to the same non-relativistic spectrum. It is clear then that $S$ is a
more convenient starting point than $I$ for the purpose of taking the limit (5): it
allows for a succinct and transparent derivation of the limiting form of the action.
Equally important for our purposes is the fact that the Nambu-Goto-based approach
is readily generalizable to the string and membrane cases. As shown in the Appendix,
the Hamiltonian version of the limit is also completely straightforward.

3 The Wound (Non-relativistic) String

3.1 The Wound string action

As noted in the Introduction, the authors of [13] were able to derive an action for
Wound string theory by starting with the Polyakov action for the string, in conformal
gauge, and then taking the limit (1). By analogy with the particle case studied in
the previous section, we expect the analysis of the limit to be more transparent if we
start instead with the Nambu-Goto action
\[ S = -t_1 \int d^2 \sigma \left[ \sqrt{-\det g_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu - B_{01} \varepsilon^{\alpha\beta} \partial_\alpha X^0 \partial_\beta X^1} \right], \tag{10} \]
where $t_1 = 1/2\pi l_s^2$ is the string tension, $\alpha, \beta = 0, 1$ are worldsheet indices associated
with the Lorentzian coordinates $\sigma^0 \equiv \tau, \sigma^1 \equiv \sigma$, and $\varepsilon^{01} = +1$. As a result of the
two-dimensional reparametrization invariance of $S$, the worldsheet energy-momentum
tensor (the Noether current for translations) vanishes identically.

For the purpose of taking the limit (1), it is convenient to denote the longitudinal
and transverse variables by $X^a (a = 0, 1)$ and $Y^i (i = 2, \ldots, D-1)$, respectively. With
the metric scaling according to (1), the longitudinal coordinates give the dominant
contribution to the Nambu-Goto determinant. This leading term is in fact a perfect
square,
\[ -\det \eta_{ab} \partial_a X^a \partial_b X^b = \left( \varepsilon^{\alpha\beta} \partial_\alpha X^0 \partial_\beta X^1 \right)^2. \tag{11} \]
Just like in the point-particle case, then, the leading term in the expansion of the
Nambu-Goto square-root in (10) in powers of $\epsilon$ (which would imply a divergent action,
due to $t_1 \propto \epsilon^{-1}$,) is precisely cancelled by the leading gauge-field contribution. Notice this cancellation takes place only if $\dot{X}^0 X'^1 - \dot{X}^1 X'^0 > 0$, where primes denote $\sigma$-derivatives, i.e., if the string is oriented along $+x^1$: only in this case can its diverging tensile energy be compensated by its coupling to the background $B$-field. As long as this condition is satisfied, the subleading terms yield the finite action

$$S_W = -T_1 \int d^2 \sigma \left[ \frac{2 \dot{X} \cdot X' \dot{Y} \cdot Y' - \dot{X}^2 Y'^2 - \dot{X}'^2 Y^2}{2 (\dot{X}^0 X'^1 - \dot{X}^1 X'^0)} + \lambda (\dot{X}^0 X'^1 - \dot{X}^1 X'^0) \right],$$

where $T_1 = 1/2 \pi L_s^2$ is the effective string tension, and the longitudinal and transverse coordinates are respectively contracted with $\eta_{ab}$ and $\delta_{ij}$. This is then the reparametrization-invariant worldsheet action of Wound string theory. Again, due to this invariance, the worldsheet energy-momentum tensor associated with (12) is identically zero.

### 3.2 Gauge-Fixing and Spectrum

If it is true that the action (12) completely captures the dynamics of Wound string theory, then in particular it should be able to reproduce the spectrum of excitations of the various objects in the theory. Besides closed strings, which require enforcing the periodicity conditions\(^4\)

$$X^a(\sigma + 2\pi, \tau) = X^a(\sigma, \tau) + 2\pi w R \delta_1^a, \quad Y^i(\sigma + 2\pi, \tau) = Y^i(\sigma, \tau), \quad (13)$$

there may be D-branes in the theory, whose excitations will be described as usual by open strings attached to them. The action (12) implies that boundary conditions must be chosen for these open strings such that

$$\delta X^a \left[ \frac{\dot{X}_a \dot{Y}' - X'_a \dot{Y}^2}{\dot{X}^0 X'^1 - \dot{X}^1 X'^0} - \epsilon_{ab} \dot{X}^b \left( \frac{2 \dot{X} \cdot X' \dot{Y} \cdot Y' - \dot{X}^2 Y'^2 - \dot{X}'^2 Y^2}{2 (\dot{X}^0 X'^1 - \dot{X}^1 X'^0)} + \lambda \right) \right] + \delta Y^i \left[ \frac{\dot{Y}_i \dot{X}' - Y_i' \dot{X}^2}{\dot{X}^0 X'^1 - \dot{X}^1 X'^0} \right] = 0. \quad (14)$$

For Dp-branes transverse to $x^1$, (14) implies the standard boundary conditions

$$X^0 = 0, \quad X^1(\sigma = 0) = 0, \quad X^1(\sigma = \pi) = 2\pi w R, \quad Y^i_N = 0, \quad Y^k_D = 0, \quad (15)$$

where $Y^i_N$ ($Y^i_D$) denotes the $p-1$ ($D-p-1$) transverse Neumann (Dirichlet) directions.

The simplicity of (13) and (15) allows for a straightforward derivation of the corresponding spectra. The quickest route is to note that, since in these cases the string is necessarily wound around the longitudinal direction, we can work in the

\(^4\)We assume for simplicity that only $x^1$ is compact.
The static gauge $X^0 = ct, X^1 = \zeta w R \sigma$, with $c$ an arbitrary constant and $\zeta = 1 \ (\zeta = 2)$ for the closed (open) string. In this gauge, (12) reduces to the free-field action

$$S^{(s)}_W = -T_1 \int d^2 x \left[ \frac{1}{2} \partial_a Y \cdot \partial^a Y + \lambda \right].$$

This is of course the action for a non-relativistic string. The usual mode expansions then lead to the expected energy spectra:

$$p_0 = \lambda \frac{w R}{L_s^2} + \frac{L_s^2 p^2}{2 w R} + \frac{N + \tilde{N}}{w R} \tag{17}$$

for the closed strings [11, 12, 13], and

$$p_0 = \lambda \frac{w R}{L_s^2} + \frac{L_s^2 p^2}{2 w R} + \frac{N}{2 w R} \tag{18}$$

for the open strings [10].

The case of a longitudinal D-brane (i.e., NCOS theory) is qualitatively different. This is to be expected, for it is known to lead to a relativistic open string spectrum [1, 2, 12, 13]. The essential difference is that in this case the endpoint variations $\delta X^0$ and $\delta X^1$ are both arbitrary, so (14) implies non-linear boundary conditions for $X^a$ (namely, the expression inside the first pair of brackets must vanish). These boundary conditions are in fact incompatible with the static gauge choice: physically, the point is that the length of the string along direction 1 is not fixed.

To deal with these complicated boundary conditions, it is convenient to work in conformal gauge. The resulting formalism will of course also be able to cover the closed string and transverse D-brane cases. In the standard Polyakov action approach, fixing the conformal gauge means requiring the intrinsic worldsheet metric to be conformally flat, i.e., $\gamma_{01} = 0, \gamma_{00} = -\gamma_{11}$. What is not usually stressed, however, is that, through the equation of motion for $\gamma_{\alpha \beta}$, 5 this ultimately entails that the induced metric is also conformally flat:

$$g_{\mu \nu} \dot{X}^\mu X'^\nu = 0, \quad g_{\mu \nu} \ddot{X}^\mu \dot{X}^\nu = -g_{\lambda \rho} X'^\lambda X'^\rho. \tag{19}$$

In the Nambu-Goto treatment of the string, then, fixing the conformal gauge means enforcing (19). The virtue of these conditions is of course that they linearize the equations of motion and boundary conditions following from (10), while maintaining spacetime covariance. In addition, the gauge transformations that conditions (19) leave unfixed, i.e., conformal reparametrizations, play a central role in the development of the formalism (e.g., they allow the calculation of scattering amplitudes in terms of vertex operators).

Upon taking the limit (1), the conformal gauge conditions (19) involve only the longitudinal coordinates $X^a$:

$$\dot{X} \cdot X' = 0, \quad \dot{X}^2 = -X'^2 \quad \Longrightarrow \quad X'^0 = \dot{X}^1, \quad X'^1 = \dot{X}^0. \tag{20}$$

5 $T_{\alpha \beta} = 0$, which in the conformal approach is enforced as a constraint on the physical states.
In this gauge, the equations of motion following from (12) become simple wave equations, and the boundary conditions implicit in (14) reduce to

\[
\dot{Y}^2 + Y''^2 = \lambda \left(-\dot{X}^2 + X'^2\right) . \tag{21}
\]

At this point one must remember the fact that, for longitudinal D-branes, the possible presence of a worldvolume electric field \( F_{01} \), together with the flux quantization condition for \( B_{01} + 2\pi l_s^2 F_{01} \), effectively fixes \( \lambda = 1/2\nu^2G_s^2 \), where

\[
\nu \equiv \frac{NL_p^{p-1}}{R_2 \cdots R_p} \tag{22}
\]

is the number density of fundamental strings bound to the Dp-brane, and \( G_s \) is the Wound string coupling [10]. This is in sharp contrast with the closed string and transverse D-brane cases, where \( \lambda \) is a strictly free (and physically irrelevant) parameter which may for instance be set equal to zero.

Using (20), the Wound string theory action (12) can be rewritten as

\[
S_W^{(c)} = -T_1 \int d^2\sigma \left[ -\frac{1}{2} \dot{Y}^2 + \frac{1}{2} Y'^2 + \lambda \left( \dot{X}^0 X'^1 - \dot{X}^1 X'^0 \right) + l_0 \left( \dot{X}^0 - X'^1 \right) + l_1 \left( \dot{X}^1 - X'^0 \right) \right] , \tag{23}
\]

where Lagrange multipliers \( l_a \) have been introduced to appropriately enforce the gauge conditions (20). A more careful Hamiltonian justification of this gauge-fixing procedure is given in the Appendix. The action (23) yields the equations of motion

\[
\ddot{Y} = Y'' , \quad X'^0 = \dot{X}^1 , \quad X'^1 = \dot{X}^0 , \quad l'_0 = \dot{l}_1 , \quad l'_1 = \dot{l}_0 \tag{24}
\]

(so, in particular, \( X^a, l_a \) satisfy the wave equation), and requires boundary conditions such that

\[
\delta Y_i^j Y'_i + \delta X^a \varepsilon_{ab} \left[ \lambda \dot{X}^b + l^b \right] = 0 . \tag{25}
\]

For the longitudinal D-brane (i.e., NCOS) case, this translates into the linear boundary conditions

\[
Y_{N}^i = 0 , \quad \dot{Y}_{D}^k = 0 , \quad l_0 = \lambda \dot{X}^0 , \quad l_1 = -\lambda \dot{X}^1 . \tag{26}
\]

For points on the boundary, the action (23) implies the two-point function

\[
\langle X^a(\tau) X^b(0) \rangle = -\nu^2 G_s^2 L_s^2 \eta^{ab} \log |\tau|^2 + \pi \nu^2 G_s^2 L_s^2 \varepsilon^{ab} \text{sgn}(\tau) , \tag{27}
\]

which exhibits the expected noncommutativity between space and time, with noncommutativity parameter \( \theta^{ab} = 2\pi \nu^2 G_s^2 L_s^2 \varepsilon^{ab} \) [10, 11, 12, 13]. Other ways to make this noncommutativity apparent have been explored in [24].

\footnote{In this connection, we would like to stress that regarding boundary conditions as Hamiltonian constraints is a procedure whose consistency remains to be established.}

8
In order for (23) to be equivalent to (12), we must require the worldsheet energy-momentum tensor $T_{\alpha\beta}$ which follows from $S_W^{(c)}$ to vanish, just as it did for the original system. This translates into the two independent constraints

$$
\dot{Y}' \cdot Y' - l_a X'^a = 0, \tag{28}
$$
$$
\dot{Y}' + 2 \left( l_0 X'^0 + l_1 X'^1 \right) = 0
$$

Equivalence to the original system (in conformal gauge) can be shown explicitly by solving the constraints (28) for the Lagrange multipliers. After a slight rewriting using (24), this leads to

$$
l_a = \frac{\dot{Y}' \cdot Y' X'_a - \frac{1}{2} \left( \dot{Y}' + 2 \right) X'_a}{X'^0 X'^1 - X'^1 X'^0}. \tag{29}
$$

Direct substitution of these solutions into (26) takes us back to the non-linear boundary condition (21). It is much simpler, however, to retain the auxiliary variables $l_a$ in the description, expand all variables into modes which satisfy (24) and (26), and enforce the constraints (28) only a posteriori, as physical state conditions. The zero-mode of the Hamiltonian constraint can then easily be seen to yield the expected NCOS spectrum [16]

$$
\nu^2 G_s^2 \left[ (p_0)^2 - (p_1)^2 \right] - p_\perp^2 = \frac{N}{L_s^2}. \tag{30}
$$

That (23) leads to the correct spectrum for open strings attached to longitudinal branes is not a new result: letting $2\beta \equiv -l_0 - l_1$, $2\tilde{\beta} \equiv -l_0 + l_1$, $\gamma \equiv X'^0 + X'^1$, and $\tilde{\gamma} \equiv -X'^0 + X'^1$, $S_W^{(c)}$ is seen to be just a (Lorentzian) rewriting of the Gomis-Ooguri action [13], which has been previously shown to yield the right spectra for closed strings [13] as well as longitudinal [13, 16] and transverse D-branes [16].

Although in (23) we have arrived at a known result, we believe our approach sheds some additional light on the worldsheet-level effect of the Wound string theory limit. Most importantly, as in the case of the point particle, we have seen in Section 3.1 that the use of the Nambu-Goto action (10) as a starting point makes the cancellation between tensile and Kalb-Ramond potential energies which is the physical essence of the Wound string theory limit completely transparent. At a more technical level, the analysis of this subsection clarifies a couple of aspects of the Gomis-Ooguri action: besides showing that the respectively purely left- and purely right-moving character of $\gamma = X^+$ and $\tilde{\gamma} = X^-$ [13] is not a dynamical effect, but a direct consequence of the conformal gauge conditions (20), it gives a physical meaning (through Eq. (29)) to the Lagrange multipliers $\beta, \tilde{\beta}$ appearing in the formalism of [13].
4 The Wrapped (Galilean) Membrane

4.1 The WM2 action

The bosonic part of the action for an M2-brane in a background $C_{012}$ field can be written in the Nambu-Goto form:

$$S = -t_2 \int d^3\sigma \left[ \sqrt{-\det g_{\alpha\beta} \partial_\alpha X^0 \partial_\beta X^1 \partial_\gamma X^2} - C_{012} \epsilon^{\alpha\beta\gamma} \partial_\alpha X^0 \partial_\beta X^1 \partial_\gamma X^2 \right],$$  \hspace{1cm} (31)

where $t_2 = 1/(2\pi)^2 l_P^2$ is the membrane tension, the worldvolume coordinates $\sigma^\alpha \equiv (\tau, \sigma, \rho)$, $\epsilon^{012} = +1$, and the spacetime indices $M, N = 0, \ldots, 10$. With this action as our starting point, the limit (2) can be taken in complete parallel with the particle and string cases analyzed in Section 3. It is convenient again to make a notational distinction between longitudinal and transverse coordinates: $X^a$ ($a = 0, 1, 2$), $Y^i$ ($i = 3, \ldots, 10$). The key point is once more that the Nambu-Goto square-root is dominated by the longitudinal piece of the determinant,

$$\sqrt{-\det \eta_{ab} \partial_a X^a \partial_b X^b} = |\epsilon^{\alpha\beta\gamma} \partial_\alpha X^0 \partial_\beta X^1 \partial_\gamma X^2|. \hspace{1cm} (32)$$

If the expression within the absolute value is positive (i.e., if the membrane is positively oriented on the $x^1\cdot x^2$ torus), these terms cancel against the leading gauge-field terms, and we are left with the finite action

$$S_W = -T_2 \int d^3\sigma \left[ -\epsilon^{\alpha\beta\gamma} \epsilon^{\alpha'\beta'\gamma'} \partial_\alpha X \cdot \partial_\alpha' X \partial_\beta X \cdot \partial_\beta' X \partial_\gamma Y \cdot \partial_\gamma' Y \right. \frac{1}{4\epsilon^{\alpha\beta\gamma} \partial_\alpha X^0 \partial_\beta X^1 \partial_\gamma X^2} \left. + \lambda \epsilon^{\alpha\beta\gamma} \partial_\alpha X^0 \partial_\beta X^1 \partial_\gamma X^2 \right], \hspace{1cm} (33)$$

where $T_2 = 1/(2\pi)^2 L_P^3$ is the effective membrane tension, and the longitudinal and transverse coordinates are respectively contracted with $\eta_{ab}$ and $\delta_{ij}$. Eq. (33) is the reparametrization-invariant worldvolume action of Wrapped membrane theory. As we will explain later, for the OM theory case (where the membrane is open and ends on a longitudinal fivebrane) the non-linear self-duality constraint on the M5 worldvolume leads to an additional term in the action, Eq. (55). The invariance of $S_W$ under diffeomorphisms implies an identically vanishing worldvolume energy-momentum tensor.

Just like in the analysis of the string, there are several cases to consider, depending on whether the membrane is closed or open, and if open, whether it ends on a longitudinal, partially transverse, or fully transverse M5-brane. In the following subsections we will analyze these cases separately.

---

7 Supermembrane reviews may be found in [25, 26].
8 It is also straightforward to generalize to arbitrary $p$-branes.
4.2 Closed membrane spectrum

For the closed membrane (parametrized such that $\sigma, \rho$ range from 0 to $2\pi$), the simplest approach is again to note that, since in this case the membrane is necessarily wrapped around the longitudinal directions, we can work in the static gauge

$$X^0 = c\tau, \quad X^1 = \zeta w_1 R_1 \sigma, \quad X^2 = \zeta w_2 R_2 \rho,$$

with $c$ an arbitrary constant and $\zeta = 1$, thereby reducing (33) to the free-field action

$$S_W^{(s)} = -T_2 \int d^3x \left[ \frac{1}{2} \partial_a Y \cdot \partial^a Y + \lambda \right],$$

which describes a non-relativistic membrane. The orientation requirement on the membrane (necessary to arrive at the finite action (33)) translates into the condition $w \equiv w_1 w_2 > 0$: the membrane must have positive wrapping number $w$ on the longitudinal torus [12].

It is straightforward to expand in Fourier modes:

$$Y^i(x^a) = \tilde{y}^i + \frac{L^3_y}{w R_1 R_2} p_i x^0 + \sqrt{\frac{L^3_p}{w R_1 R_2}} \sum_{\tilde{n} \neq 0} \left( a^i_{\tilde{n}} \frac{e^{-i q a x^a}}{\sqrt{2q_0}} + a^i_{\tilde{n}}^\dagger \frac{e^{i q a x^a}}{\sqrt{2q_0}} \right),$$

where $\tilde{n} \equiv (n_1, n_2)$, $q_a \equiv n_a/w_a R_a$ for $a = 1, 2$, and $q_0 \equiv \sqrt{q_1^2 + q_2^2}$. The resulting energy spectrum is

$$p_0 = \lambda \frac{w R_1 R_2}{L^3_p} + \frac{L^3_y p^2_{\tilde{n}}}{w R_1 R_2} + \frac{\tilde{N}}{w R_1 R_2},$$

where we have defined a number operator

$$\tilde{N} \equiv \sum_{\tilde{n}} \sqrt{(n_1 w_1 R_1)^2 + (n_2 w_2 R_2)^2} a_{\tilde{n}}^\dagger \cdot a_{\tilde{n}},$$

omitting a possible ordering constant.

It is well-known that a relativistic membrane leads to a continuous spectrum [27]. This can be understood intuitively from the fact that the Nambu-Goto action (31) implies an energy proportional to the membrane area, and therefore allows the membrane to develop arbitrarily long spikes of infinitesimal area, at zero energy cost.\footnote{There is some disagreement in the literature as to whether or not this instability is eliminated upon wrapping the membrane on a torus [28, 29, 30].}

The formulation of the Matrix model of M-theory [31], originally obtained as a discretization of the supermembrane action [32], turned this membrane instability into a virtue: it is a sign that the quantized membrane yields a second-quantized description, with a spectrum that includes multiple-particle states.\footnote{For reviews on the supermembrane-Matrix connection see, e.g., [26].}

An $n$-particle state is obtained by deforming the membrane into $n$ blobs connected by infinitesimally thin
tubes, which carry no energy. In this way, a single membrane leads to configurations which are indistinguishable from multiple-membrane states. In contrast with this standard case, we have found here that the Wrapped membrane action (33) implies a discrete spectrum. This is of course due to its non-relativistic character: as is evident in (34), the membrane potential does not have flat directions. What has happened, then, is that, as a consequence of the limit (and, as we will see later, our choice of gauge), the connecting tubes mentioned above carry a high energy cost, and are therefore forbidden, implying that the multi-particle states are removed from the spectrum. As stated before, all states in the resulting spectrum carry strictly positive M2-brane wrapping number \( w \), which in particular means that the limit (2) decouples the massless supergravity modes \([4, 5]\) (although these remain in the theory as carriers of Newtonian interactions, as described in \([16]\)).

As a check on the closed membrane spectrum (37), notice that under (double) dimensional reduction along \( x_2 \) (i.e., letting \( n_2 = 0, w_2 = 1 \)) it agrees with the closed string spectrum (17), with the expected identifications

\[
L_s^2 = \frac{L_P^3}{R_2}, \quad N = \sum_{n_1 > 0 \atop n_2 = 0} n_1 a_\eta^\dagger \cdot a_\bar{\eta}, \quad \tilde{N} = \sum_{n_1 < 0 \atop n_2 = 0} |n_1|^a a_\eta^\dagger a_\bar{\eta}.
\]  

(39)

4.3 Fivebranes and open membranes

It is well-known that open M2-branes can terminate consistently on M5-branes \([33, 34]\). By analogy with the case of open strings ending on D-branes, it is natural to expect the fivebrane dynamics to be describable through an appropriate open membrane quantization \([35, 36, 37, 38, 39]\). In the present subsection we will address this issue in the more restricted WM2 theory context. Before doing that, however, it is worth establishing some basic M5-brane properties which follow directly from the nature of the WM2 limit.

The scaling of the metric seen in (2) introduces a distinction between the \( x^1, x^2 \) (longitudinal) and \( x^3, \ldots, x^{10} \) (transverse) directions, so the M5-brane attributes become embedding-dependent. There are three qualitatively distinct cases to consider: the fivebrane may be fully transverse (e.g., spanning directions 034 567), partially transverse (e.g., 023456), or longitudinal (e.g., 012345). In the first case, the usual formula for the M5 tension implies in the limit (2) an energy per unit coordinate volume (i.e., proper volume in the WM2 metric)

\[
T_{M5}^{\perp \perp} = \frac{(g_{\perp \perp})^{5/2}}{(2\pi)^5 L_P^6} = \frac{\sqrt{\varepsilon}}{(2\pi)^5 L_P^6} \to 0.
\]

(40)

That is to say, fully transverse M5-branes in WM2 theory are tensionless! This is consistent with the fact that, upon reduction along \( x^2 \) (say), a fully transverse M5-brane becomes a transverse NS5-brane in Wound IIA (WIIA) string theory, an object which, as mentioned in the Introduction, is also known to be tensionless.\footnote{Through duality one can similarly infer that transverse Kaluza-Klein fivebranes in DLCQ string}
For a partially transverse M5-brane, the same reasoning implies a finite tension
\[ T_{M5}^{\parallel \perp} = \frac{(g_{\perp \perp})^{4/2}}{(2\pi)^5 l_P^4} = \frac{1}{(2\pi)^5 L_P^6}. \] (41)

Reducing along \( x^2 (x^1) \), an M5-brane oriented along 023456 becomes a transverse D4-brane (longitudinal NS5-brane) in WIIA string theory. Remembering that the WM2 and WIIA parameters are related through
\[ L_P = G_s^{1/3} L_s, \quad R_2 = G_s L_s \quad (R_1 = G_s L_s), \] (42)
one can verify that indeed
\[ T_{M5}^{\parallel \perp} 2\pi R_2 = \frac{1}{(2\pi)^4 G_s L_s^2} = T_{D4}^{\perp}, \] (43)
the tension of a transverse D4-brane [12], and
\[ T_{M5}^{\parallel \perp} = \frac{1}{(2\pi)^5 G_s^2 L_s^6} = T_{NS5}, \] (44)
the tension of a longitudinal NS5-brane [14, 15].

In the longitudinal case, the formula directly analogous to (40), (41) would yield a divergent fivebrane tension: \( (g_{\perp \perp})^{3/2}/(2\pi)^5 l_P^3 \propto 1/\sqrt{\epsilon} \). This just says that an isolated longitudinal M5-brane does not survive the WM2 limit. To remain in the spectrum of the theory, the fivebrane must be bound to some number \( w > 0 \) of longitudinal M2-branes. In other words, it must carry a positive \( F_{012} \) field, where \( F_{mnp} \equiv C_{mnp} + H_{mnp} \) (with \( C_{mnp} \) the pull-back of the bulk gauge field, \( H_{mnp} \) the worldvolume field-strength, and \( m, n, p = 0, \ldots, 5 \)) is the gauge invariant three-form field. \( F_{012} \) and \( w \) are related through a flux-quantization condition. For fixed \( w \), and with the metric and Planck length scaling as in (2), this condition implies that \( F_{012} \) must become near-critical, as seen for \( C_{012} \) in (2) (we work in the gauge \( H_{mnp} = 0 \)). In short, longitudinal M5-branes in WM2 theory give rise to the standard OM theory setup [12]. For later use, it is important to remember that the non-linear self-duality constraint [10] for \( F \) implies that the ‘electric’ component \( C_{012} \) seen in (2) must be accompanied by a ‘magnetic’ counterpart [1, 3, 4]
\[ C_{345} = -\frac{\epsilon}{\sqrt{2\lambda}}. \] (45)

The bottom line of the preceding discussion is that, for longitudinal fivebranes, one must examine the behavior of the M5-M2 bound state tension in the limit (2):
\[ \left( \frac{1}{(2\pi)^2 l_P^2} \right)^{w} \left( \frac{(g_{\perp \perp})^{3/2}}{(2\pi)^5 l_P^5} \right)^2 \to \frac{1}{(2\pi)^5 L_P^6} \left( \frac{|v| + 1}{2\epsilon} \right)^{w}, \] (46)
theory, and transverse D(6−p)-branes in Wrapped Dp-brane theory (which is related to the (p+3)-dimensionalNCYM and 6-dimensional ODp theories [12]), among others, are also tensionless [15].

The entire analysis here is a direct analog of the Dp-F1 (NCOS) case, discussed in more detail in Sec. 3.2.1 of [1].

13

13
where
\[ v \equiv \frac{w L_p^5}{R_3 R_4 R_5} \]  
(47)
is the number of M2-branes (oriented along 012) per unit transverse volume on the M5-brane (oriented along 012345). The leading term in (46) is divergent, but the divergence is proportional to the membrane wrapping number \( w \) carried by the five-brane. Since the total wrapping number (including both free M2 and bound M2 contributions) is conserved by interactions, the above \( \epsilon^{-1} \) divergence will be dynamically irrelevant as long as all objects carry strictly positive \( w \) \[12\]. In fact, with our choice of gauge for \( \mathcal{F} \) (i.e., when it is \( C \) and not \( H \) that becomes near-critical), this divergence is automatically subtracted by the coupling to the \( C_{012} \) field \[2\]. From the subleading term in (46) we conclude then that the tension of a longitudinal M5-brane in WM2 theory is given by
\[ T_{M5}^{||} = \frac{1}{2(2\pi)^3 v L_P^6} . \]  
(48)
If we reduce to ten dimensions along \( x^2 \) (say), then using \[22\], \[12\] and (47) we can readily verify that
\[ T_{M5}^{||} 2\pi R_2 = \frac{1}{2(2\pi)^4 v G_s^2 L_s^5} = T_{D4}^{||} , \]  
(49)
which is the correct tension for a longitudinal D4-brane in Wound IIA string theory \[12\].

Let us now proceed to determine what information may be extracted from the WM2 action \[33\] when the membrane is open and ends on one of the above types of M5-brane. We should note that, as has been emphasized in \[18\], the absence of a dimensionless coupling constant in M-theory implies that in general there is no sense in which the M2-brane tension may be regarded as small compared to the M5-brane tension (which one can do for F1 vs. D\( p \) in string theory, as long as \( g_s \ll 1 \)). This means that, \textit{a priori}, it is far from clear whether the M5-brane may be considered a rigid wall on which the open M2-brane terminates. Still, if one adopts the view that the physics of M theory is completely captured by membrane quantization— as we do here for WM2 theory— then the question is simply what boundary conditions are consistently allowed by the relevant action.\[14\] This is the issue which we now address.

We take \( \sigma, \rho \) to range from 0 to \( \pi \). For an open membrane ending on a fully transverse fivebrane, variation of (33) leads to a boundary term which can be seen to allow the ‘obvious’ boundary conditions
\[ \sigma = 0, \pi : \quad X^{a0} = 0, \quad X^{1,2} = c^{1,2}, \quad Y^\beta_N = 0, \quad Y^k_D = d^k , \]  
\[ \rho = 0, \pi : \quad \hat{X}^0 = 0, \quad \hat{X}^{1,2} = c^{1,2}, \quad \hat{Y}^\beta_N = 0, \quad \hat{Y}^k_D = d^k , \]  
(50)
where primes and hats denote \( \sigma \)- and \( \rho \)-derivatives, respectively, and \( c^a, d^k \) are constants. Our bosonic analysis does not constrain the number of transverse Neumann

\[14\] Notice also that, in contrast to M theory, WM2 theory does include the dimensionless parameters \( R_1/L_P, R_2/L_P \) and \( v \), which enter into the comparison of the M2 and M5 tensions.
(Y^N_M) and Dirichlet (Y^K_D) directions. For a fivebrane in ten spatial dimensions we would normally expect five of the Y^i to be Neumann, and three of them (together with X^{1,2}) to be Dirichlet. We have seen above, however, that the M5-brane in question is tensionless, which means that it can be deformed to any shape in the transverse space, at zero energy cost. We should perhaps anticipate then that this object will be effectively eight-dimensional, and all of the Y^i will satisfy Neumann boundary conditions.\(^{15}\) Setting this issue will require an analysis of the supersymmetric completion of the WM2 action, and in particular the compatibility between boundary conditions and \(\kappa\)-symmetry, a question which has been studied in the full M theory setting in \([36, 37, 38]\). We leave this interesting problem for future work.

Irrespective of that, we can progress further by noting that the boundary conditions (50) are compatible with the static gauge (34), with \(\zeta = 2\). The open membrane action can therefore be cast again in the free-field form (35), implying a discrete non-relativistic spectrum essentially identical to (37). Reduction to ten dimensions yields a prediction for the excitation spectrum of a transverse NS5-brane in WIIA theory (since this brane is also tensionless, it could again be expected to be effectively eight-dimensional).

Unsurprisingly, the static gauge turns out to be inadequate for dealing with the partially transverse and longitudinal fivebrane cases. Just like in the corresponding string case, the static gauge conditions are incompatible with the relevant boundary conditions. Physically, the issue is that, in these two cases, the length of the membrane along \(x^1\) and/or \(x^2\) is not fixed. These setups are thus qualitatively different from the closed membrane and transverse fivebrane, which are manifestly non-relativistic. Given the relation between OM and 4+1 NCOS theory, we expect the system to have a relativistic structure in the longitudinal fivebrane case. As noted before, the partially transverse M5-brane gives rise either to a transverse D4-brane (known to possess a non-relativistic spectrum) or a longitudinal NS5-brane in WIIA string theory.

A convenient gauge choice which is able to cover all cases is the membrane analog of the conformal gauge (19) for the string,

\[
g_{MN}\dot{X}^M X'^N = 0, \quad g_{MN}\dot{X}^M \dot{X}^N = 0, \quad L^2 g_{MN}\dot{X}^M \dot{X}^N = \frac{\epsilon_{PQ} X'^P \dot{X}^Q - g_{MN} X'^M X'^N g_{PQ} \dot{X}^P \dot{X}^Q}{g_{PQ} X^P \dot{X}^Q},
\]

where \(L\) is an arbitrary constant with dimension of length. We will refer to conditions (51) as orthonormal gauge; they are advantageous because they eliminate the square-root in (31). In the limit (2), the orthonormal gauge conditions reduce to

\[
\dot{X} \cdot X' = \dot{X} \cdot \dot{X} = 0, \quad L^2 \dot{X}^2 = (X' \cdot \dot{X})^2 - X'^2 \dot{X}^2 \implies L \dot{X}^a = \epsilon^{abc} X'_b \dot{X}_c .
\]

\(^{15}\)A similar situation arises in the analysis of open string field theories for unstable D-brane systems: at the closed string vacuum the branes are tensionless, and so effectively become space-filling, which explains why closed strings (understood to be flux tubes of the D-brane worldvolume gauge field) can move about freely in the nine-dimensional bulk \([11]\).

\(^{16}\)It was found in that context that the M2-brane may terminate on a p-brane only if \(p = 1(!), 5,\) or 9.
In this gauge, the WM2 action (33) simplifies to

\[ S^{(o)}_W = -T_2 \int d^3 \sigma \left[ -\frac{1}{2} L \dot{Y}^2 + \frac{1}{2} L^{-1} (X'^2 \dot{Y}^2 - 2X' \cdot \dot{X} \dot{Y}' \cdot \dot{Y} + \dot{X}^2 Y'^2) \right. \]

\[ + l_a (L \dot{X}^a - \varepsilon^{abc} X'_b \dot{X}_c) + \lambda \varepsilon^{\alpha \beta \gamma} \partial_\alpha X^0 \partial_\beta X^1 \partial_\gamma X^2 \]

where \( l_a \) are Lagrange multipliers enforcing the gauge conditions (the Appendix shows how this result can be justified using the Hamiltonian formalism). To ensure equivalence with (33), we must demand that the energy-momentum tensor \( T_{\alpha\beta} \) following from (53) vanish. This leads to the three independent constraints

\[ \dot{Y} \cdot Y' - l_a X'^a = 0, \]

\[ \dot{Y} \cdot \dot{Y} - l_a \dot{X}^a = 0, \]

\[ L^2 \dot{Y}^2 + Y'^2 \dot{Y}^2 - (Y' \cdot \dot{Y})^2 - 2L \varepsilon_{abc} l^a X'^b \dot{X}^c = 0. \]

For the OM theory (i.e., longitudinal fivebrane) case, the presence of the ‘magnetic’ field (45) implies an additional contribution

\[ -T_2 \int d^3 \sigma \frac{1}{\sqrt{2}\lambda} \varepsilon^{\alpha \beta \gamma} \partial_\alpha Y^3 \partial_\beta Y^4 \partial_\gamma Y^5 \]

(55) to the action (53). Moreover, in this case the flux quantization condition on the fivebrane effectively determines \( \lambda \) in terms of the number \( w \) of M2-branes in the M2-M5 bound state that the flux-carrying M5-brane represents. Indeed, by reducing to ten-dimensions along \( X^2 \), one can use the quantization condition for \( B_{01} \equiv C_{012} \) (see, e.g., [4, 16]) to show that \( \lambda = 1/2\nu^2 \), where \( \nu \) is the M2 brane number density defined in (47). This is completely unlike the closed membrane and (fully or partially) transverse fivebrane cases, where \( \lambda \) is a free (and dynamically inconsequential) parameter.

As a check on (53), notice that under the double dimensional reduction \( X^2 = \zeta \rho R_2, \dot{X}^0, = \dot{Y}^i = 0, \) and making use of (12), \( S^{(o)}_W \) coincides with the Wound string theory action (23), as long as we set \( L = \zeta R_2 \). As usual, this formal reduction does not necessarily imply that the membrane will dynamically reduce to a string when the circle is shrunk to zero size [28, 29, 30]. This is closely related to the question of whether or not the membrane displays instabilities.

As mentioned before, the potential for the standard supermembrane allows arbitrarily long spikes to develop at zero energy cost. To examine this issue here, consider a static configuration where, up to small corrections, all of the spatial variables depend on the same linear combination of \( \sigma \) and \( \rho \) (i.e., \( X'^r = c \dot{X}^r + \delta \) for \( r = 1, 2; Y'^i = c \dot{Y}^i + \delta \)), describing a spike of infinitesimal width. It is then easy to see that the potential energy implied by either (53) or (33) is of order \( \delta \), meaning that there are flat directions and instabilities. At the quantum level, and for the fully supersymmetric system, we would expect these classical instabilities to translate into a continuous

\[ \text{This is recognized to be the condition under which the orthonormal and conformal gauges (51) and (19) agree.} \]
spectrum, indicative of multi-particle states. This seems to be in conflict with our derivation of a discrete spectrum, Eq. (37), for the closed membrane. The origin of the discrepancy is the fact that the static gauge (34) employed there is incompatible with the condition $X^\sigma = c\hat{X}^\sigma + \delta$, which as we saw above defines the flat directions of the potential. We thus see that this gauge choice removes from the theory the modes responsible for the instability, and consequently, the possibility to have multi-particle states.

A variation of (53) + (55) produces a boundary term which on the $\sigma = 0, \pi$ edges reads

$$
\delta X^a \left[ L^{-1}(X'_a \hat{Y}^2 - \hat{X}_a Y' \cdot \hat{Y}) + \varepsilon_{abc}^{\prime lb} \hat{X}^b \hat{X}^c + \lambda \varepsilon_{abc} \hat{X}^b \hat{X}^c \right] + \delta Y^d \left[ L^{-1}(Y'_d \hat{X}^2 - \hat{Y}_d X' \cdot \hat{X}) - \left\{ \frac{1}{\sqrt{2\lambda}} \varepsilon_{def} \hat{Y}^e \hat{Y}^j \right\} \right]
$$

$$
+ \delta Y^l \left[ L^{-1}(Y'_l \hat{X}^2 - \hat{Y}_l X' \cdot \hat{X}) \right] = 0,
$$

where $a, b, c = 0, 1, 2$; $d, e, f = 3, 4, 5$; $l = 6, \ldots, 10$. The term inside the curly braces comes from (55), and is therefore only present in the OM theory case.

For the case of a fully transverse M5-brane, we expect $X^{1,2} = c^{1,2}$ at $\sigma = 0, \pi$, which of course implies that $\delta X^{1,2} = 0$ and $\hat{X}^{1,2} = 0$ there. We then see that (56) (together with the analogous term at the $\rho$-edges) implies the boundary conditions (50), as was claimed previously. We have already determined the excitation spectrum for this case using the static gauge (34) (which is of course a subgauge of the orthonormal gauge (51)), so we will not pursue it further here, except to note that, in an orthonormal gauge treatment, the boundary conditions for the auxiliary fields $l_a$ would be read off from the constraints (54).

For a partially transverse M5-brane we expect $X^1$ (but not $X^2$) to be Dirichlet. The boundary term (56) can then be seen to imply the conditions

$$
\sigma = 0, \pi : \quad X^0 \hat{Y}^2 = Ll_1 X^2, \quad X^1 = c^1, \quad X^2 \hat{Y}^2 = Ll_1 \hat{X}^0, \quad Y^i_N = 0, \quad Y^k_D = d^k, (57)
$$

$$
\rho = 0, \pi : \quad \hat{X}^0 Y^r = Ll_1 X^2, \quad X^1 = c^1, \quad \hat{X}^2 Y^r = Ll_1 X^0, \quad \hat{Y}^i_N = 0, \quad Y^k_D = d^k.
$$

It is easy to check that these are compatible with the gauge conditions (52). As was noted before for (56), to determine the ranges of $j$ and $k$ (i.e., the number of transverse Dirichlet and Neumann directions) we would need to carry out a supersymmetry analysis; but, given that the partially transverse fivebrane has a finite tension (Eq. (41)), our expectation for this case is simply that $j = 2, \ldots, 6$ and $k = 7, 8, 9, 10$. The nonlinearity of the boundary conditions (57) (and of the equations of motion that follow from (53)) makes it extremely difficult to proceed further. The situation is even worse in the case of a longitudinal M5-brane (OM theory), where we expect $Y^l = d^l$, and are then left with the highly non-linear requirement that each of the six expressions inside the $\delta X^a$ and $\delta Y^d$ square brackets in (56) vanish.
5 Conclusions

The main aim of this work has been to formulate OM theory in terms of a membrane action. Our approach in deriving this action has been to directly examine the effect of the OM theory limit on the worldvolume action for an M2-brane, an idea which was pursued for NCOS theory by Gomis and Ooguri [13]. Their results for the string case served as a general motivation for our work; their method, however, is not easily carried over to the membrane case, so we have developed an alternative approach.

Our strategy is to employ a Nambu-Goto (as opposed to Polyakov) type description for the object in question. As shown in the point-particle setting in Section 2, this allows a straightforward derivation of the limiting form of the action, and makes the crucial cancellation between tensile and ‘electric’ potential energies completely transparent. Using this strategy we have reexamined the string case in Section 3.1, obtaining a reparametrization-invariant worldsheet action for Wound string theory, Eq. (12). As explained in [12, 13] and reviewed in the Introduction, besides closed strings and ‘transverse’ D-branes this ten-dimensional structure contains the various NCOS theories as states associated with ‘longitudinal’ D-branes. The description of each of these objects in terms of the Wound string theory action was studied in Section 3.2. For closed strings and transverse D-branes, a static gauge choice was shown to reduce the action to free-field form, and lead to the expected energy spectra. The static gauge is on the other hand incompatible with the boundary conditions relevant to the longitudinal D-brane (i.e., NCOS) case. To obtain a formalism capable of describing all cases, it is convenient to work instead in conformal gauge. Doing so the action simplifies to the form (23), which is nothing but a rewriting of the Gomis-Ooguri action. Even though this result is not new, we believe our approach sheds additional light on the results of [13].

In Section 4 we turned our attention to the membrane case, our main interest. The Nambu-Goto-based treatment was shown in Section 4.1 to yield a reparametrization-invariant action, Eq. (33), for the eleven-dimensional M-theoretic construct known as Wrapped M2-brane (WM2) theory, within which OM theory appears as the class of states associated with a ‘longitudinal’ M5-brane [12, 13]. Just like its string counterpart, the closed membrane allows a choice of static gauge, and in Section 4.2 this was seen to reduce the WM2 action to free-field form. Contrary to the standard case, where flat directions in the potential give rise to an instability, the WM2 closed membrane spectrum, Eq. (37), was found to be discrete, and to possess a non-relativistic structure, as expected from the nature of the OM theory limit. Under double dimensional reduction it was seen to reproduce the closed string spectrum of Wound string theory, just as it should.

Besides closed membranes, WM2 theory contains three qualitatively distinct types of M5-branes, the properties of which were analyzed at the beginning of Section 4.3. It was found in particular that ‘fully transverse’ M5-branes are tensionless, an intriguing property (noted already in [17]) which in our opinion merits further study. We then proceeded to take some steps towards determining whether and how the dynamics of
each type of fivebrane is encoded in the WM2 action appropriate for the corresponding open membrane. For the fully transverse M5-brane, it is possible again to work in static gauge, thereby obtaining a discrete open membrane spectrum very similar to the closed membrane one. Reducing to ten dimensions, this leads to an interesting prediction for the excitation spectrum of a transverse NS5-brane in Wound IIA string theory (also known to be tensionless [13]). For partially transverse and longitudinal M5-branes, the relevant open membrane boundary conditions make it necessary to seek an alternative to the static gauge. Use of an ‘orthonormal’ gauge allows the WM2 action to be cast in the simplified form (53) (with the additional term (55) in the OM theory—i.e., longitudinal fivebrane—case). This is then our main result: the desired membrane action for WM2/OM theory. Although much simpler than the usual Nambu-Goto action, (53) is still power-counting non-renormalizable and leads, as we have noted, to complicated equations of motion and non-linear boundary conditions. Under double dimensional reduction, it was demonstrated to reduce as expected to the Wound string theory action (23).

As in the standard case, this formal reduction does not necessarily imply that the membrane will dynamically reduce to a string when the circle is shrunk to zero size [28, 29, 30]. This is closely related to the possible existence of membrane instabilities. As we saw in Section 4.3, the potential which follows from the action (53) has flat directions, which leads us to expect a continuous spectrum, indicative of multi-particle states. This seems to be in conflict with our derivation of a discrete spectrum, Eq. (57), for the closed membrane. As we explained, the origin of the discrepancy is the fact that the static gauge employed there removes from the theory the modes responsible for the instability, and consequently, the possibility to have multi-particle states.

Our analysis has been restricted to the bosonic part of the WM2/OM theory action, so an important pending task is to work out its supersymmetrized version. Knowledge of the full action will be essential for a more careful study of the question of nonrenormalizability, and more generally, to establish whether the membrane formalism we have developed here provides a useful handle on the dynamics of WM2/OM theory. We should also ask if there exists a regularized version of this membrane action which facilitates the extraction of physical information. A successful formulation should, among other things, permit the calculation of the fivebrane tensions (10), (11), and (18), yield the expected ‘Newtonian supergravity’ interactions [13, 16], and, in the OM theory case, allow a derivation from first principles (e.g., through the calculation of $X^M$ correlation functions) of the expected open membrane metric and ‘noncommutativity’ parameter [18, 13, 13, 20]. Such a formulation would hopefully bring us closer to the underlying structure of M theory.

6 Acknowledgements

AG would like to thank Ulf Danielsson and Martín Kruczenski for collaboration on the issue of the existence of tensionless branes in Wound/Wrapped theories, reported on
Appendix: Hamiltonian Analysis

The aim of this appendix is to rederive, by using the systematics of the Hamiltonian Dirac method, our basic results: the first order action (23) for Wound String theory, and the corresponding result (53) for the Wound Membrane case. We will carry out the discussion for the membrane, which includes the string and the particle as special cases. Our analysis will be done in a completely gauge-independent way and we will be able to recover these results as particular gauge-fixings from a basic first-order Lagrangian action, up to trivial redefinitions of the Lagrange multipliers. We will also analyze the role played by the boundary term (14) in this first order theory.

Our starting point is the Nambu-Goto membrane action (31)

\[ S = -T \int d^3 \sigma \left[ \sqrt{-\det g_{\alpha \beta}} - \frac{1}{6} C_{NML} \varepsilon^{\alpha \beta \gamma} \partial_\alpha X^N \partial_\beta X^M \partial_\gamma X^L \right], \]  

where \( g_{\alpha \beta} = g_{MN} \partial_\alpha X^M \partial_\beta X^N \), the world-sheet space-time labels \( \alpha, \beta, \gamma = 0, 1, 2 \), and the space-time labels \( N, M, L = 0, ..., 10 \). From here the canonical momenta \( P_N \) are given by

\[ P_N = \frac{T}{\sqrt{-\det g_{\alpha \beta}}} g_{NM} \varepsilon^{\alpha \beta \gamma} \partial_\alpha X^M (\partial_1 X \cdot \partial_\beta X)(\partial_2 X \cdot \partial_\gamma X) + \frac{T}{2} C_{NML} \varepsilon^{rs} (\partial_r X^M \partial_s X^L), \]

where \( r, s, t = 1, 2 \) are spatial world-sheet labels. As a result of the three-dimensional reparametrization invariance of (58), the momenta (59) conjugate to \( X^N \) satisfy the three primary first-class constraints

\[ \mathcal{H}_r \equiv P_M \partial_r X^M, \quad r = 1, 2 \]

\[ \mathcal{H} \equiv \frac{1}{2T} (P_N - \frac{T}{2} C_{NKL} \varepsilon^{rs} \partial_r X^K \partial_s X^L)^2 + \frac{T}{2} \det g_{rs}. \]

The first order Lagrangian action associated to the Dirac total Hamiltonian is

\[ S = \int d^3 \sigma (\dot{X}^N P_N - N\mathcal{H} - N^r \mathcal{H}_r), \]

where \( N \) and \( N^r \) are arbitrary Lagrange multipliers. This action can be rewritten in the form

\[ S = \int d^3 \sigma \left\{ P_K (\dot{X}^K + Ng^{NK} C_{NML} \varepsilon^{rs} \partial_r X^M \partial_s X^L - N^r \partial_r X^K) \right. \]

\[ - \frac{N}{2} \left[ \frac{g^{NK} P_N P_K}{T} + \frac{T}{4} (g^{IJ} C_{INM} C_{JKL} + 2g_{NK} g_{ML}) \varepsilon^{rs} \varepsilon^{tu} \partial_r X^N \partial_s X^M \partial_t X^K \partial_u X^L \right] \}. \]
Now it is easy to see the effect of the limit (2) in the first order action (62): it cancels the quadratic terms in the longitudinal momenta $P_a (a = 0, 1, 2)$, rendering the auxiliary fields $P_a$ as new Lagrange multipliers. Indeed, upon taking the limit we have

$$S = \int d^3 \sigma \left\{ P_a (\dot{X}^a + \frac{N}{2} \varepsilon^{a}_{bc} \varepsilon^{rs} \partial_r X^b \partial_s X^c - N^r \partial_r X^a) + P_i (\dot{Y}^i - N^r \partial_r Y^i) - \frac{N}{2} \left[ P_i P_j \delta^{ij} - \frac{T}{2} \varepsilon^{rs} \varepsilon^{tu} \partial_r X \cdot \partial_t X (\partial_s Y \cdot \partial_u Y + \lambda \partial_s X \cdot \partial_u X) \right] \right\}, \quad (63)$$

where $i, j, k, l = 3...10$ are transverse spatial indices. The Hamiltonian constraints that follow from this action are

$$H_r \equiv P_N \partial_r X^N, \quad (64)$$

$$H \equiv \frac{\delta^{ij} P_i P_j}{2T} - \frac{1}{2} \varepsilon^{a}_{bc} \varepsilon^{rs} P_a \partial_r X^b \partial_s X^c + \frac{T}{4} \varepsilon^{rs} \varepsilon^{tu} \partial_r X \cdot \partial_t X (\partial_s Y \cdot \partial_u Y + \lambda \partial_s X \cdot \partial_u X)$$

which can be recognized as the limiting form of the original Hamiltonian constraints (61) associated with the action (58). The structure of the Hamiltonian constraint $H$ makes the non-relativistic nature of the theory apparent (linear in $P_a$).

We can now eliminate the auxiliary variables $P_i$ using their equations of motion, to obtain the Lagrangian action

$$S = \int d^3 \sigma \left\{ P_a (\dot{X}^a + \frac{N}{2} \varepsilon^{a}_{bc} \varepsilon^{rs} \partial_r X^b \partial_s X^c - N^r \partial_r X^a) + \frac{T}{2N} (\dot{Y}^i - N^r \partial_r Y^i)^2 - \frac{N}{4} \left[ T \varepsilon^{rs} \varepsilon^{tu} \partial_r X \cdot \partial_t X (\partial_s Y \cdot \partial_u Y + \lambda \partial_s X \cdot \partial_u X) \right] \right\}, \quad (65)$$

where the variables $P_a$ are seen to play the role of Lagrange multipliers enforcing the Lagrangian constraint

$$\dot{X}^a + \frac{N}{2} \varepsilon^{a}_{bc} \varepsilon^{rs} \partial_r X^b \partial_s X^c - N^r \partial_r X^a = 0. \quad (66)$$

The Lagrangian action (65) is the main result of this Appendix. As we will see below, the actions (53) and (23) can be recovered as particular gauge-fixings of this general result.

Variation of the Hamiltonian action (65) leads to the boundary term

$$n_r \frac{\partial \mathcal{L}}{\partial X^N} \delta X^N = 0, \quad (67)$$

where $n_r$ is a unit vector normal to the boundary surface. There are several cases to consider, depending on whether the membrane is closed or open, and if open, whether it ends on a longitudinal, partially transverse, or fully transverse five-brane.
The above boundary term can be expanded into

\[
\begin{align*}
&n_r \left[ (N \epsilon_{bc} \epsilon^{rs} P_a \partial_s X^c - \frac{T}{2} N \eta_{ab} \epsilon^{rs} \epsilon^{tu} \partial_t X^a (\partial_s Y \cdot \partial_a Y - 2 \lambda \partial_s X \cdot \partial_a X) - N^r \partial_s X^b ) \right] \right. \\
&\left. + \left( \frac{T}{2} N \delta_{ij} \epsilon^{rs} \epsilon^{tu} \partial_t Y^j \partial_s X \cdot \partial_a X - N^r \delta Y^i \right) \right] = 0.
\end{align*}
\]

(68)

It is a difficult task to solve the equations of motion with these boundary conditions for any gauge. We therefore fix a particular gauge compatible with the closed, transverse, longitudinal or mixed cases.

In the closed membrane case we have only the periodicity requirement, which allow the static gauge choice

\[ X^0 = c \sigma^0, \quad X^1 = 2 w_1 R_1 \sigma^1, \quad X^2 = 2 w_2 R_2 \sigma^2. \]

This gauge is canonical and implies \( N = c/4 w_1 w_2 R_1 R_2 \) and \( N^r = 0 \). These fixed Lagrangian multipliers and the static gauge solve (63). The reduced action can be obtained by implementing this gauge in the action (65), and the result is the non-relativistic membrane given by (35).

The corresponding analysis for the open membrane case can be simplified in the orthonormal gauge. This gauge is noncanonical, and is implemented by imposing the conditions \( N = 1 \) and \( N^r = 0 \) on the Lagrange multipliers. A general property of a constrained system is that the Lagrange multipliers associated with primary first class constraints can always be obtained as Lagrangian functions using the equations of motion for the auxiliary fields \( P_N \),

\[
\dot{X}^K + \frac{1}{2} N g^{NK} C_{NML} \epsilon^{rs} \partial_r X^M \partial_s X^L - N^r \partial_r X^K - N^r \frac{g^{NK} P_N}{T} = 0,
\]

(69)

the constraints and the definition of the momenta (59). This procedure is the inverse of the Legendre transformation in the extended phase space where the Lagrange multipliers are promoted to dynamical variables. In our case

\[
N^r \partial_r X^N \partial_1 X^K g_{NK} = \partial_1 X^N \dot{X}^K g_{NK}, \quad N^r \partial_r X^N \partial_2 X^K g_{NK} = \partial_2 X^N \dot{X}^K g_{NK}
\]

(70)

and

\[
N = \sqrt{-\frac{\det g_{\alpha \beta}}{\det g_{rs}}},
\]

(71)

By implementing the noncanonical gauge \( N = 1/L, N^r = 0 \) on these relations we obtain the corresponding relations (51) of the main text.

When evaluated in the appropriate gauge, the results in this Appendix reduce to the ones presented in the main text. In particular, up to a trivial redefinition of the Lagrange multipliers, the first order action (53) can be obtained by enforcing the above noncanonical gauge in the action (53). The corresponding boundary terms are gauge-fixed versions of the general phase-space boundaries given by (68).
References

[1] N. Seiberg, L. Susskind and N. Toumbas, “Strings in background electric field, space/time noncommutativity and a new noncritical string theory,” JHEP 0006 (2000) 021 [arXiv:hep-th/0005040].

[2] R. Gopakumar, J. Maldacena, S. Minwalla and A. Strominger, “S-duality and noncommutative gauge theory,” JHEP 0006 (2000) 036 [arXiv:hep-th/0005048].

[3] N. Seiberg, L. Susskind and N. Toumbas, “Space/time non-commutativity and causality,” JHEP 0006 (2000) 044 [arXiv:hep-th/0005015];
J. L. Barbon and E. Rabinovici, “Stringy fuzziness as the custodian of time-space noncommutativity,” Phys. Lett. B 486 (2000) 202 [arXiv:hep-th/0005073].

[4] R. Gopakumar, S. Minwalla, N. Seiberg and A. Strominger, “OM theory in diverse dimensions,” JHEP 0008 (2000) 008 [arXiv:hep-th/0006062].

[5] E. Bergshoeff, D. S. Berman, J. P. van der Schaar and P. Sundell, “Critical fields on the M5-brane and noncommutative open strings,” Phys. Lett. B 492 (2000) 193 [arXiv:hep-th/0006112].

[6] T. Harmark, “Open branes in space-time non-commutative little string theory,” Nucl. Phys. B 593 (2001) 76 [arXiv:hep-th/0007147].

[7] J. X. Lu, “(1+p)-dimensional open D(p-2) brane theories,” JHEP 0108 (2001) 049 [arXiv:hep-th/0102056];
H. Larsson and P. Sundell, “Open string/open D-brane dualities: Old and new,” JHEP 0106 (2001) 008 [arXiv:hep-th/0103188].

[8] U. Gran and M. Nielsen, “Non-commutative open (p,q) string theories,” JHEP 0111 (2001) 022 [arXiv:hep-th/0104168].

[9] A. Connes, M. R. Douglas and A. Schwarz, “Noncommutative geometry and matrix theory: Compactification on tori,” JHEP 9802 (1998) 003 [arXiv:hep-th/9711162];
M. R. Douglas and C. M. Hull, “D-branes and the noncommutative torus,” JHEP 9802 (1998) 008 [arXiv:hep-th/9711165].

[10] N. Seiberg and E. Witten, “String theory and noncommutative geometry,” JHEP 9909 (1999) 032 [arXiv:hep-th/9908142].

[11] I. R. Klebanov and J. Maldacena, “1+1 dimensional NCOS and its U(N) gauge theory dual,” Int. J. Mod. Phys. A 16 (2001) 922 [Adv. Theor. Math. Phys. 4 (2001) 283] [arXiv:hep-th/0006083].

[12] U. H. Danielsson, A. Güijosa and M. Kruczenski, “IIA/B, wound and wrapped,” JHEP 0010 (2000) 020 [arXiv:hep-th/0009182].

23
[13] J. Gomis and H. Ooguri, “Non-relativistic closed string theory,” arXiv:hep-th/0009181.

[14] J. L. Barbon and E. Rabinovici, “On the nature of the Hagedorn transition in NCOS systems,” JHEP 0106 (2001) 029 arXiv:hep-th/0104169.

[15] A. Güijosa, talk at the 13th Nordic Network Meeting “Fields, Strings and Branes,” Uppsala, Sweden, May 2001.

[16] U. H. Danielsson, A. Güijosa and M. Kruczenski, “Newtonian gravitons and D-brane collective coordinates in Wound string theory,” JHEP 0103 (2001) 041 arXiv:hep-th/0012183.

[17] V. Sahakian, “The large M limit of non-commutative open strings at strong coupling,” arXiv:hep-th/0107180.

[18] E. Bergshoeff, D. S. Berman, J. P. van der Schaar and P. Sundell, “A non-commutative M-theory five-brane,” Nucl. Phys. B 590 (2000) 173 arXiv:hep-th/0005026.

[19] J. P. Van der Schaar, “The reduced open membrane metric,” JHEP 0108 (2001) 048 arXiv:hep-th/0106046; D. S. Berman, M. Cederwall, U. Gran, H. Larsson, M. Nielsen, B. E. Nilsson and P. Sundell, “Deformation independent open brane metrics and generalized theta parameters,” arXiv:hep-th-0109107; E. Bergshoeff and J. P. Van der Schaar, “Reduction of open membrane moduli,” arXiv:hep-th/0111061.

[20] S. Kawamoto and N. Sasakura, “Open membranes in a constant C-field background and noncommutative boundary strings,” JHEP 0007 (2000) 014 arXiv:hep-th/0005123; J. S. Park, “Topological open p-branes,” arXiv:hep-th/0012141; I. Rudychev, “From noncommutative string/membrane to ordinary ones,” JHEP 0104 (2001) 015 arXiv:hep-th/0101039; C. M. Hofman and W. K. Ma, “Deformations of closed strings and topological open membranes,” JHEP 0106 (2001) 033 arXiv:hep-th/0102201; A. K. Das, J. Maharana and A. Melikyan, “Open membranes, p-branes and noncommutativity of boundary string coordinates,” JHEP 0104 (2001) 016 arXiv:hep-th/0103229.

[21] D. S. Berman and P. Sundell, “Flowing to a noncommutative (OM) five brane via its supergravity dual,” JHEP 0010 (2000) 014 arXiv:hep-th/0007052; R. G. Cai, J. X. Lu, N. Ohta, S. Roy and Y. S. Wu, “OM theory and V-duality,” JHEP 0102 (2001) 024 arXiv:hep-th/0101069; D. S. Berman and P. Sundell, “AdS(3) OM theory and the self-dual string or membranes ending on the five-brane,” arXiv:hep-th/0105288.
U. Gran and M. Nielsen, “On the equivalence of bound state solutions,” arXiv:hep-th/0108113.

[22] T. Kawano and S. Terashima, “S-duality from OM-theory,” Phys. Lett. B 495 (2000) 207 [arXiv:hep-th/0006225];
J. G. Russo and M. M. Sheikh-Jabbari, “Strong coupling effects in noncommutative spaces from OM theory and supergravity,” Nucl. Phys. B 600 (2001) 62 [arXiv:hep-th/0009141];
B. Kors, D. Lust and A. Miemiec, “Non-commutative D- and M-brane bound states,” Fortsch. Phys. 49 (2001) 869 [arXiv:hep-th/0103203].

[23] A. Dasgupta, H. Nicolai and J. Plefka, “Vertex operators for the supermembrane,” JHEP 0005 (2000) 007 [arXiv:hep-th/0003280];
B. Pioline, H. Nicolai, J. Plefka and A. Waldron, “R**4 couplings, the fundamental membrane and exceptional theta correspondences,” JHEP 0103 (2001) 036 [arXiv:hep-th/0102123].

[24] F. Ardalan, H. Arfaei and M. M. Sheikh-Jabbari, “Noncommutative geometry from strings and branes,” JHEP 9902 (1999) 016 [arXiv:hep-th/9810072];
C. S. Chu and P. M. Ho, “Noncommutative open string and D-brane,” Nucl. Phys. B 550 (1999) 151 [arXiv:hep-th/9812219];
F. Ardalan, H. Arfaei and M. M. Sheikh-Jabbari, “Dirac quantization of open strings and noncommutativity in branes,” Nucl. Phys. B 576 (2000) 578 [arXiv:hep-th/9906161];
C. S. Chu and P. M. Ho, “Constrained quantization of open string in background B field and noncommutative D-brane,” Nucl. Phys. B 568 (2000) 447 [arXiv:hep-th/9906192];
M. M. Sheikh-Jabbari and A. Shirzad, “Boundary conditions as Dirac constraints,” Eur. Phys. J. C 19 (2001) 383 [arXiv:hep-th/9907053].

[25] E. Bergshoeff, E. Sezgin and P. K. Townsend, “Properties Of The Eleven-Dimensional Super Membrane Theory,” Annals Phys. 185 (1988) 330;
M. J. Duff, “Supermembranes,” in Fields, Strings and Duality: TASI 96 Proceedings, Costas Efthimiou and Brian Greene (eds), Singapore, World Scientific, 1997 [arXiv:hep-th/9611203].

[26] H. Nicolai and R. Helling, “Supermembranes and M(atrix) theory,” arXiv:hep-th/9809103;
B. de Wit, “Supermembranes and super matrix models,” arXiv:hep-th/9902054;
W. I. Taylor, “The M(atrix) model of M-theory,” arXiv:hep-th/0002016.

[27] B. de Wit, M. Luscher and H. Nicolai, “The Supermembrane Is Unstable,” Nucl. Phys. B 320 (1989) 135.

[28] J. G. Russo, “Supermembrane dynamics from multiple interacting strings,” Nucl. Phys. B 492 (1997) 205 [arXiv:hep-th/9610018].
[29] J. G. Russo and A. A. Tseytlin, “Waves, boosted branes and BPS states in M-theory,” Nucl. Phys. B 490 (1997) 121 [arXiv:hep-th/9611047].

[30] B. de Wit, K. Peeters and J. Plefka, “Supermembranes with winding,” Phys. Lett. B 409 (1997) 117 [arXiv:hep-th/9705225].

[31] T. Banks, W. Fischler, S. H. Shenker and L. Susskind, “M theory as a matrix model: A conjecture,” Phys. Rev. D 55 (1997) 5112 [arXiv:hep-th/9610048].

[32] B. de Wit, J. Hoppe and H. Nicolai, “On The Quantum Mechanics Of Supermembranes,” Nucl. Phys. B 305 (1988) 545.

[33] A. Strominger, “Open p-branes,” Phys. Lett. B 383 (1996) 44 [arXiv:hep-th/9512059].

[34] P. K. Townsend, “D-branes from M-branes,” Phys. Lett. B 373 (1996) 68 [arXiv:hep-th/9512062].

[35] K. Becker and M. Becker, “Boundaries in M-Theory,” Nucl. Phys. B 472 (1996) 221 [arXiv:hep-th/9602017].

[36] P. Brax and J. Mourad, “Open supermembranes in eleven dimensions,” Phys. Lett. B 408 (1997) 142 [arXiv:hep-th/9704163];
P. Brax and J. Mourad, “Open supermembranes coupled to M-theory five-branes,” Phys. Lett. B 416 (1998) 295 [arXiv:hep-th/9707246].

[37] K. Ezawa, Y. Matsuo and K. Murakami, “Matrix regularization of open supermembrane: Towards M-theory five-brane via open supermembrane,” Phys. Rev. D 57 (1998) 5118 [arXiv:hep-th/9707200].

[38] B. de Wit, K. Peeters and J. C. Plefka, “Open and closed supermembranes with winding,” Nucl. Phys. Proc. Suppl. 68 (1998) 206 [arXiv:hep-th/9710213].

[39] C. S. Chu and E. Sezgin, “M-fivebrane from the open supermembrane,” JHEP 9712 (1997) 001 [arXiv:hep-th/9710223].

[40] P. S. Howe and E. Sezgin, “Superbranes,” Phys. Lett. B 390 (1997) 133 [arXiv:hep-th/9607227];
P. S. Howe and E. Sezgin, “D = 11, p = 5,” Phys. Lett. B 394 (1997) 62 [arXiv:hep-th/9611008];
P. S. Howe, E. Sezgin and P. C. West, “Covariant field equations of the M-theory five-brane,” Phys. Lett. B 399 (1997) 49 [arXiv:hep-th/9702008];
P. S. Howe, E. Sezgin and P. C. West, “The six-dimensional self-dual tensor,” Phys. Lett. B 400 (1997) 255 [arXiv:hep-th/9702111].

[41] P. Yi, “Membranes from five-branes and fundamental strings from Dp branes,” Nucl. Phys. B 550, 214 (1999), hep-th/9901159.
O. Bergman, K. Hori and P. Yi, “Confinement on the brane,” Nucl. Phys. B 580, 289 (2000), hep-th/0002223.
G. Gibbons, K. Hori and P. Yi, “String fluid from unstable D-branes,” Nucl. Phys. B 596, 136 (2001), hep-th/0009061.
A. Sen, “Fundamental strings in open string theory at the tachyonic vacuum,” hep-th/0010240.
M. Kleban, A. E. Lawrence and S. Shenker, “Closed strings from nothing,” hep-th/0012081.