Blur Length Estimation Based on Image Energy Spectrum Statistics

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Abstract. In order to resolve the non-smooth blur problem for coded exposure imaging, this paper proposes an automatically motion blur estimation method based on image energy spectrum statistics, and expands successfully this method to estimate motion blur length for traditional motion blurred images. This paper finds that the residual sums of squares of polynomial fitness result of the deblurred image energy spectrum statistics can reach the minimum, only using correct blur length in the deconvolution process. Given an initial value, using simulated annealing algorithm to implement the iteration, the correct blur length can be obtained automatically in a very short time. Both the real and simulated experiment results demonstrate the effectiveness and robustness of the proposed method.

1. Introduction

As the relative motion between the imaging device and the scene, the resulted image will appear motion blur. The motion blur can seriously reduce the quality of the image, which brings great difficulties to the follow-up researches and application of the image. The restoration of blurred image has always been a hot and difficult problem in the field of image processing [1].

From the imaging process, Raskar proposed a novel and effective coded exposure technology to solve the problem of large length motion blur [2]. Although the technique has many advantages, the deconvolution process of the coded exposure image requires firstly the accurate estimation of image blur length. Due to the exposure coded image blur information is discrete, the current motion blur estimation methods are less effective, such as the cepstrum method [3] and Radon transform method [4], etc, which are based on the analysis of the spectrum of the blurred image.

The researchers have done a lot of effective work on the motion blur estimation of the coded exposure images. Raskar et al. used artificial calibration methods to estimate manually the blur length of coded exposure images [1]. Subsequently, Agrawal and Xu extracted the motion information of the encoded exposure image from the blurred edges of the motion objects on the idea of Alpha cutgraph [5]. The Alpha cutgraph method requires the obvious contrast between the image foreground and background, which restricts its application. Based on motion blur projection model, Tai proposed a method for motion information estimation of the coded exposure image [6]. However, this method is a semi-automatic method, which requires the artificial participation of certain professional image processing knowledge. Xu Shukui constructed a mixture visual acquisition system [7], and used the
motion measurement technique to obtain the motion information of the object. This technology obviously increases a lot of hardware costs, and the multi-camera calibration itself is a problem.

Without loss of generality, this paper studies the blurred length estimation of image motion in the condition of uniform linear motion between imaging equipment and shooting scene. Based on the nature image energy spectrum statistics, an automatically motion blur estimation method is proposed for the coded exposure image. And simulation and real image experiment results verify the accuracy and robustness of this method. In addition, through a large number of experiments and the statistics analysis of the energy spectrum of traditional motion blurred image, a little adjustment to the statistical parameters in the proposed method, this method will be expand effectively to the traditional camera motion blur image blur length estimate.

2. Natural image energy spectrum statistics

The motion blur process of image can be expressed as the convolution of clear image and bluring kernel:

\[ g(x, y) = f(x, y) \ast h(x) + n(x, y) \] (1)

Here, \( g(x, y) \), \( f(x, y) \) and \( h(x) \) represents respectively the blurred image, clear image and bluring kernel function. \( \ast \) means the convolution operator, and \( n(x, y) \) is the additive noise. If the noise is ignored, according to the convolution theorem, the Fourier transform of the formula (1) can be obtained as follows:

\[ |G| = |FH| \quad |F| = |H| \] (2)

Where, \(|G|\), \(|F|\) and \(|H|\) represent respectively the powerspectrum of the Fourier transform of the blurred image, clear image and bluring kernel function.

The study of Van der Schaaf and Van Hateren found that the spectra of powerspectrum of clear natural images without motion blurring obey \( 1/\omega \) exponential distribution [8].

The natural image powerspectrum is converted to polar coordinates \( |F(\omega, \phi)| \), where, \( u = \omega \cos \phi \), \( v = \omega \sin \phi \), \( \omega \) is the radius of polar coordinates, \( \phi \) is polar Angle. The powerspectrum statistics of natural images is as follows:

\[ S_{\omega}(|F(\omega, \phi)|) = \frac{1}{360} \sum_{\phi=1}^{360} |F(\omega, \phi)| = \frac{A}{\omega^\gamma} \] (3)

Where, \( \omega = 1, 2, \ldots, M \), \( A \) and \( \gamma \) are constants.

In the actual operation, \( “M = 127” \) is usually set, because the high frequency part of the image powerspectrum is affected badly by the noise.

After taking the logarithm of the formula (3), the following linear expression can be obtained:

\[ \log \left( S_{\omega}(F) \right) = \log(A) - \gamma \log(\omega) = \log(A) - \gamma x \] (4)

where, \( x = \log(\omega) \).

![Energy spectrum statistics and polynomial fitness of latent sharp image and blurred image](image-url)
The formula (4) shows that the power spectrum of the clear image obeys the one-dimensional linear distribution (also known as the power law distribution) under the double logarithm axis. However, when the image appears motion blur, its energy spectrum statistics will deviate from the power law distribution, and show more divergence, as shown in Figure 1. It is found in this paper that the larger the image blurs length, the greater the divergence of the statistical data points of his energy spectrum.

3. Blur length automatic estimation

3.1. Automatic estimation of blur length of coded exposure image

Based on the analysis to the relation between the powerspectrum of natural image statistics and image blur length, the polynomial fitting is used in this paper to fit the energy spectrum statistics data, and the residual sum of squares is used to interpret the degree of divergence of statistics data points. Suppose the data points of the image power spectrum statistics in the double logarithmic coordinates are respectively \((\omega_i, S_i)\), \((\omega_j, S_j)\), \(\cdots\), \((\omega_n, S_n)\), and the corresponding polynomial fitting data points are \((\hat{\omega}_i, \hat{S}_i)\), \((\hat{\omega}_j, \hat{S}_j)\), \(\cdots\), \((\hat{\omega}_n, \hat{S}_n)\), then the residual sum of squares can be shown by the formula (5):

\[
RSS = \sum_{i=1}^{n} \left| S_i - \hat{S}_i \right|^2
\]  

A large number of experimental results showed that when the image was reconstructed using the correct blur length, the minimum RSS of the power spectrum statistical data points based on the reconstructed image was obtained.

In this paper, a coded exposure image with a blur length of 50 pixels is simulated, and the anti-convolution parameters, the blur length of 30 to 70 pixels, are used in turn to reconstruct the image. Then to do the polynomial fitting of the energy spectrum of the 41 reconstructed images statistics, and calculate the corresponding RSS of the corresponding image energy spectrum statistics. The results (Figure 2) show the reconstructed image with the right blur length has the minimum RSS.

![Figure 2. Relations between the RSS and blur length](image)

3.2. Blur length automatic estimation for the traditional motion blurred image

In this paper, an iterative optimization method is used to estimate the blur length of the coded exposure image. Because of the reversibility of the blur kernel function of the coded exposure image, this paper directly uses the generalized matrix inversion approach to recover the image, which greatly improves the efficiency of the algorithm, and makes the proposed method approach practical. Another important reason for using matrix inversion approach is that the recovery process doesn’t join any nonlinear optimization operation, which makes the restored image can accurately reflect the blur length is correct or not.
However, the direct matrix inverse approach has a severe ringing effect on the restoration of traditional motion blur images, as shown in Figure 3 (a), which affects its energy spectrum statistics. Because the blur kernel function of the traditional motion blurred image contains high frequency zeros. The bright stripes like "waves" will appear in the corresponding locations of these high frequency zeros of the energy spectrum of the restored images, as shown in Figure 3 (b), when the matrix inverse approach is used directly in the image restoration. These bright stripes can seriously interfere with the energy spectrum statistics of the reconstructed image.

Due to the bright stripes mainly deriving from the high-frequency information, in order to eliminate the ringing effect on the influence of the energy spectrum statistics of the restored image, this paper proposes, based on a large number of experimental analysis, that only using the data points with a frequency of less than 50 (above the dotted line as shown in Figure 3(b)) can get better results.

![Deblurred image](image1.png)

(a) Deblurred image  
(b) Energy spectrum statistics of deblurred image

**Figure 3.** Deblurred image using pseudo matrix inversion algorithm and the corresponding energy spectrum statistics in polar coordinates

A traditional motion blurred image with a blur length of 50 pixels is simulated to be used to test. In the same way, in order to show intuitively the RSS of the energy spectrum statistics in the different blur length, the anti-convolution parameters, the blur lengths of 30 to 70 pixels, are used in turn to reconstruct the image. The results (shown in Figure 4) also show that the reconstructed image with the right blur length have the minimum RSS.

![RSS graph](image2.png)

**Figure 4.** Relations between the RSS and blur length for traditional blurred image

### 3.3. Image cropping

Image restoration is a process of deconvolution. As a result, there will be a serious ringing effect on the edge parts of the restored image, which affects the energy spectrum statistics in the proposed algorithm. In order to eliminate the influence of the ringing effect, in this paper, a simple image cropping method, as shown in Figure 5, is used to cut the left and right sides of the restored image by 10% of the image length or make the cropped length equal to the blur length of the input image.
3.4. The process of algorithm

In this paper, the simulated annealing algorithm is used to automatically estimate the blur length of motion blurred images. The detailed Process of Algorithm is shown in Figure 6.

![Figure 5. Cropping operator](image)

![Figure 6. Flow of the proposed method](image)

First, an initial blur length is inputted, and then the generalized matrix inverse algorithm is used to obtain the restored image, and the Fourier energy spectrum of the restored image is obtained. Then, the energy spectrum data are counted and polynomial fitted under the double logarithmic polar coordinate, and the residual square sum is obtained. And after that, the simulated annealing algorithm (SAA) is used to determine whether the optimal residual square sum is obtained or the algorithm has reached the ending condition of the simulated annealing algorithm. If yes, end the algorithm. If no, the new blur lengths are given by the simulated annealing algorithm, and automatically loop iteration.

4. Results and analysis

The experiments are runned in the laptop with Intel Core(TM) i5-3230M 2.6GHz, dual-core processor, and 4G DDR3 RAM. The experiment platform is Matlab 2011a. The annealing algorithm used in this
paper is provided by Matlab global optimization toolbox. The end condition of the algorithm: the maximum iterations is less than 100 or the RSS is less than $5 \times 10^{-5}$.

4.1. Blur length estimation for real coded exposure image

In order to verify the validity of the proposed algorithm in this paper, two real coded exposure blurred images "Tomas" (830*613) and "CabCar" (992*366) provided by Amit Agrawal automatic, as shown respectively in Figure 7(a) and figure 8 (a), are used for the blur length estimation experiments [9]. These two images were all manually calibrated and validated by Agrawal. The blur length in figure 7(a) is 118 pixels and the blur length in figure 8(a) is 66 pixels.

![Blurred image “Tomas”](image1)

![Iteration process of simulated annealing algorithm and the corresponding result](image2)

(a) Blurred image “Tomas”

(b) Iteration process of simulated annealing algorithm and the corresponding result

**Figure 7.** Blur length estimation of “Tomas”

![Blurred image “CabCar”](image3)

![Iteration process of simulated annealing algorithm and the corresponding result](image4)

(a) Blurred image “CabCar”

(b) Iteration process of simulated annealing algorithm and the corresponding result

**Figure 8.** Blur length estimation of “CabCar”

Using the algorithm of this paper, the automatic estimation process and result of the blur length for Fig7(a) is shown in Fig7(b), which shows the estimated blur length is 118 pixels. The validity of the proposed algorithm is verified. The time of the proposed method for “Tomas” for running the algorithm 1~10 times is respectively shown in Table 1, which shows the high time efficiency of the proposed method.

**Table 1.** The time (unit:second) of the proposed algorithm for “Tomas” in 1~10 times

| No. | time  | No. | time  |
|-----|-------|-----|-------|
| 1   | 30.8617 | 6   | 27.1035 |
| 2   | 33.3275 | 7   | 28.4581 |
| 3   | 31.6435 | 8   | 28.1503 |
| 4   | 31.6796 | 9   | 33.0002 |
| 5   | 27.8680 | 10  | 37.0472 |
Using the algorithm of this paper, the automatic estimation process and result of the blur length for Figure 8(a) is shown in Figure 8(b), which shows the estimated blur length is 66 pixels. The validity of the proposed algorithm is also verified.

4.2. Blur length estimation for the traditional motion blurred image
Only a little adjustment to the parameters of the energy spectrum statistics, the proposed blur length estimation algorithm can be applied effectively to the traditional motion blur image. Figure 9(a) is a simulation image blurred by motion. Figure 9(c) is the automatic estimation process and the result of the proposed algorithm. The estimated blur length is 29.2 pixels, which has the error 2.67%. The deblurred result after 20 iterations is shown in Figure 9(b). Figure 9(c) shows that the proposed algorithm for the traditional motion blur image restoration need more time than for the coded exposure blurred image restoration.

4.3. Robust analysis of the algorithm
In order to verify the robustness of the algorithm in noise disturbance, A coded exposure blurred images with blur length 70 pixel is simulated, and the gaussian white noise with the mean value of 0 and the variance of 0.001 and 0.005 is added respectively, shown in Figure 10(a) and (b). Figure 10(d) is the original image. Figure 10 (c) is the reconstructed image of Figure 10(a).
Using the proposed algorithm, the estimated two blur length is respectively 68.69 pixels and 67.11 pixels. The errors are respectively 1.87% and 4.13%. Figure 10(e) and (f) are respectively the estimation processes and the corresponding results of Figure 10(a) and (b).

The above results show that the proposed algorithm has strong robustness. Although the estimation accuracy of is a little affected by noise interference, even under the strong noise interference with the variance of 0.005, the error of the algorithm is still in the reasonable range of 5% (proposed by McCloskey, etc. [10]).

5. Conclusions
The discontinuity of the blurred information of the coded exposure image makes the current popular motion blur length estimation method not applicable. In this paper, an automatic method of the blur length estimation is proposed based on the statistics of image energy spectrum statistics. The method can also be applied to the blur length estimation of the traditional motion blurred image. The accuracy of this method is reduced to a certain extent when the strong Gaussian noise is disturbed. The better robustness of the method will be further studied.

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