Bridge Condition Assessment based on Vibration Responses of Passenger Vehicle

Ayaho Miyamoto¹) and Akito Yabe²)

¹) Prof., Dr. Eng., Graduate School of Science & Engineering, Yamaguchi University, Ube, Japan (miya818@yamaguchi-u.ac.jp)
²) Chief Engineer, Master of Eng., Seismic Engineering Dept., KOZO KEIKAKU Engineering Inc. Tokyo, Japan nagai@kke.co.jp

Abstract. In this paper, we propose a new method of assessing the condition of existing short- and medium-span reinforced/prestressed concrete bridges based on vibration monitoring data obtained from a public bus. This paper not only describes details of a prototype monitoring system that uses information technology and sensors capable of providing more accurate knowledge of bridge performance than conventional ways but also shows a few specific examples of bridge condition assessment based on vehicle vibrations measured by using an in-service public bus equipped with vibration measurement instrumentation. This paper also describes a sensitivity analysis of deteriorating bridges based on simulation of the acceleration response of buses conducted by the "substructure method" employing a finite element model to verify the above bridge performance results. The main conclusions obtained in this study can be summarized as follows: (1) Because the vibration responses of passenger vehicles, such as buses, have a good linear relationship with the vibration responses of the target bridges, the proposed system can be used as a practical monitoring system for bridge condition assessment. (2) The results of sensitivity analysis performed by the substructure method show that bus vibration responses are useful for evaluating target bridge performance. (3) The proposed method was applied to a network of real bridges in a local area to evaluate its effectiveness. The results indicate that the proposed method can be used to prioritize the repair/strengthening works of existing bridges based on various vibration information in order to help bridge administrators establish rational maintenance strategies.

1. Introduction

About 70 percent of a total of nearly 670,000 bridges in Japan were constructed in the 1960s and 1970s, and these bridges have been playing an important role as part of Japan's infrastructure for about 40 to 50 years. Many of the bridges constructed during those 20 or so years, however, are reaching the end of their service lives concurrently.¹) Therefore, it is necessary, before that point, to identify the state of deterioration (incipient stage, propagation stage, acceleration stage, or deterioration stage) and decide on actions to take. In order to ensure a minimum level of safety, it is important to monitor each bridge on a daily basis to determine whether the bridge has entered the acceleration or deterioration stage, and to take necessary actions on a priority basis.

In view of the fact that many of the bridges in Japan are short- or medium-span (10–20 m) bridges, this study aims to propose a rational and feasible method of assessing declines in safety performance due to structural damage, paying attention to the acceleration and deterioration stages, which are particularly important from the viewpoint of safety management. In this study, as one solution to the problem, a new monitoring method using a public bus as part of a public transit system (Fig. 1) is proposed, along with safety indices, namely, characteristic deflection, which is relatively free from the influence of dynamic disturbances due to such factors as the unevenness of the road surface, and a structural anomaly parameter.

A basic study was conducted by using the results of technical verification experiments and numerical analysis simulation. In this study, attention is paid only to changes in structural characteristics, which are basic performance attributes of a bridge. Local deterioration and factors contributing to such deterioration, therefore, are not identified.
It has been shown\(^4\) that the effects of structural anomalies of bridges and measured physical quantities show certain tendencies. If a bus is used, it is important to distinguish changes in information caused by structural anomalies from other changes. Changes in this category may include changes in the road surface profile and changes in structural characteristics of the bus\(^5\). The characteristic deflection and structural anomaly parameter proposed in this study are indices that are relatively free from the influence of external disturbances caused by the roughness and type of the road surface, as long as the structural conditions of the bus remain unchanged.

![Figure 1 Concept of a bridge-health monitoring system for short- and medium-span bridges based on the vibration response of a bus.](image)

2. **Conventional preventive maintenance methods**

Structures in the final phase of the acceleration stage or an early phase of the deterioration stage are at high risk of showing a sudden change in an anomaly, as shown in Fig. 2, because of the relationship between cumulative damage probability due to performance declines and changes over time.\(^6\) It is possible, therefore, that a sudden change in an anomaly that could cause fatal structural damage over time is overlooked. If such anomalies are not detected and left unattended, a serious management problem might result. An effective way to prevent such problems is to conduct inspections at shorter intervals, but it is difficult to do this in reality because of the difficulty in retaining specialists and economic limitations. In order to detect anomalies in a bridge at the final phase of the acceleration stage or an early phase of the deterioration stage, therefore, it is necessary to use a method that is simpler and easier to use than visual inspection.

![Figure 2 Reduction in safety level and coverage of proposed system.](image)

An effective means of doing so is continuous monitoring by use of Structural Health Monitoring (SHM) sensors directly installed on bridges. Research on this is currently underway in many countries. There are a number of problems, however.

A maintenance approach dependent solely on bridge maintenance based mainly on visual inspection or the SHM method, which requires installing sensors on each bridge, is not suitable for maintaining a large number of deteriorating bridges, since there are many problems to be solved in order to achieve long-term
continuous monitoring while meeting the technical and economical rationality requirements. Research results related to onboard monitoring systems include the detection of anomalies of railway tracks\textsuperscript{(7),(8),(9)} and the assessment of the soundness of road structures by use of a multifunctional inspection vehicle\textsuperscript{(10)}. The authors have also found many useful research results concerning dynamic bridge interactions, such as simulation-based studies on impact coefficients \textsuperscript{(11),(12),(13)}. Many other useful research results have also been reported regarding basic theories and experimental studies on bridge–vehicle interactions during the passage of a vehicle in connection with bridge health assessment.

3. Principle of anomaly detection based on bus wheel vibration

When a vehicle passes over a bridge, its mechanics model can be expressed as a dynamic interaction between the equation of motion of the bridge, expressed by Eq. (1), and the equation of motion of the vehicle, expressed by Eq. (2). The method of formulating a bridge structure model and a vehicle structure model as separate equations of motion and expressing their interaction at joints with input and output vectors is called the "substructure method"\textsuperscript{(19)}.

\[
\begin{align*}
M_m \ddot{u}_m + C_m \dot{u}_m + K_m u_m &= \begin{cases} R \end{cases}_m \quad (1) \\
M_s \ddot{u}_s + C_s \dot{u}_s + K_s u_s &= \begin{cases} 0 \end{cases} \quad (2)
\end{align*}
\]

where the parameters are as follows

- \(M_m, C_m, K_m, M_s, C_s, K_s\): mass/damping/stiffness matrix for bridge and for vehicle,
- \(\ddot{u}_m, \dot{u}_m, u_m\): response acceleration/velocity/displacement vector for bridge and for vehicle,
- \(u_s\): input forced displacement vector for vehicle, and
- \(R_m, R_s\): support reaction vectors for bridge and vehicle.

Figure 4  Substructures for modeling bridge and bus body.

For bridge–vehicle interactions, the vehicle wheel reaction to the bridge is input as the load vector \(R_s\), and the bridge deflection \((\delta(t) \in u_m)\) and road surface unevenness \((\lambda(t))\) are input to the vehicle as the force displacement vector \(u_s\). Changes in bridge stiffness \(K_m\) are reflected in the measurement results for both the bridge system and the vehicle system. Then, it can be concluded that, because structural anomalies of the bridge due to deterioration, etc. are reflected in changes in the nodal response \(\ddot{u}_s, \dot{u}_s, u_s\) of the vehicle system, anomalies of the bridge, in theory, can be detected from the vehicle.

Here, let us consider substructure segmentation based on the bus body–wheel–bridge concept, as shown in Fig. 4. First, let us take a look at the bus body–wheel system. In a mechanics problem in which the difference method approximation holds true, when forced displacement including the bridge deformation \(\delta(t)\) is input to the wheel of the bus, the equation of motion can be approximated, as shown in Eq. (3), by using the proportionality constant \(P\), which is dependent on physical quantities that remain unchanged in the system, such as time, stiffness, damping and mass, and the state constant \(C_0\) (known) before time \(t\) obtained by a Taylor expansion, etc.\textsuperscript{(22)} This means that the system's response to the input vector is allocated proportionately depending on the system–dependent constants.

\[
u_s' \cdot P(M_s, C_s, K_s, t) + C_0(M_s, C_s, K_s, t) = F(t) \quad (3)
\]
where the parameters are
- \( M, C, K \): mass/damping/stiffness matrix for the vehicle,
- \( u \): displacement vector,
- \( C_0 \): known value before time \( t \) obtained by Taylor expansion, etc.,
- \( F(t) \): input vector to vehicle system, and
- \( P \): proportionality constant dependent on system.

Next, let us consider wheel–bridge vibration transmitted from the bus body. The response of the wheel and the bridge to the input from the bus body is allocated proportionately depending on the physical constants of the system. Let \( A_b \) represent the response vector of the bridge, and \( A_s \), the wheel response vector of the bus. Then, we have matrix \( P \) expressed as shown below:

\[
A_b = P^{-1} A_s P
\]

This means that if Eq. (4) holds true in a mechanics problem in which the difference method approximation holds true, then the vibration behavior of the bridge is proportional to the wheel vibration of the bus, and changes in \( A_b \) due to bridge anomalies are reflected proportionately in changes in wheel vibration, \( A_s \), of the bus.

Next, let us explain the rationale behind the method for extracting damage/deterioration-related information from measured rear wheel vibrations of a bus without being affected by the dynamic characteristics of the bridge and the vehicle or the road surface profile. The rear wheel vertical vibration \( \delta(t) \) of a bus running at a constant speed can be expressed as the sum of static displacement \( \delta_s(t) \), which is dependent on the measured rear wheel vertical vibration of the bus, and dynamic displacement \( \delta_d(t) \), which is dependent on the road surface profile and the vibration characteristics of the bridge and the vehicle:

\[
\delta(t) = \delta_s(t) + \delta_d(t)
\]

If it is assumed that the unevenness of the road surface can be expressed as a steady-state random Gaussian process with an average value of 0 and that dynamic displacement including bridge–vehicle interaction is ergodic and Fourier-series-expandable, the dynamic displacement \( \delta_d(t) \) can be expressed as their sum:

\[
\delta_d(t) = S_r(\Omega, t) + \int_{-\infty}^{+\infty} X(f) \cdot e^{2\pi r t} df
\]

where the parameters are
- \( S_r(\Omega, t) \): density function for unevenness of road surface,
- \( \Omega \): spatial frequency of road surface profile, and
- \( X(f) \): Fourier coefficient.

In Eq. (6), the sample mean in the second term is 0. The mean of a sufficient number \( N \) of samples obtained from \( \delta_d(t) \) can be expressed as

\[
\frac{\sum_{i=1}^{N} \delta_d(t_i)}{N} = 0
\]

Because the sample mean follows a normal distribution in accordance with the central limit theorem, \( \bar{\delta}_d(t) \) converges to a certain value \( \mu_\delta \). For the sample mean \( \bar{\delta}_d(j) \) of a sufficient number \( n \) of samples, sampling is performed so that there is no overlap in the samples. If N is sufficiently large for \( nk = N \) and \( t = 1 \sim N \), the following equation holds true:

\[
\mu_\delta = \frac{\sum_{i=1}^{n} \bar{\delta}_d(t_i)}{n} = \frac{\sum_{i=1}^{nk} (\delta_d(j))_i}{nk} = \frac{\sum_{i=1}^{Nk} \delta_d(t_i)}{N} = \frac{\sum_{i=1}^{Nk} \delta_s(t)}{N}
\]
This means that the mean displacement of the rear axle of the bus passing over the bridge obtained from a sufficient number (\(N\)) of samples can be extracted as a value, \(\mu_a\) (referred to as characteristic deflection), that is relatively free from the influence of the vibration characteristics of the bridge and the vehicle and the dynamic displacement due to the unevenness of the road surface. On the basis of a similar assumption, the mean of the deflection \(\delta_b(t)\) at a given point on the bridge during the passage of the vehicle converges to a certain value (\(\mu_b\)) that is relatively free from the influence of dynamic deflection. By using static deflection \(\delta_{sb}(t)\), \(\mu_b\) can expressed. If similarity holds true between \(\delta_{sa}(t)\) and \(\delta_{sb}(t)\) as discussed earlier, and values extracted from measurements taken after the occurrence of deterioration or damage are represented by \(\mu_{a}'\) and \(\mu_{b}'\), then the following equation can be defined:

\[
\alpha = \frac{\mu_{b}'}{\mu_b} = \frac{\mu_{a}'}{\mu_a}
\]

The rate of change \(\alpha\) expressed by Eq. (9) is defined as the structural anomaly parameter. After a given value of \(\alpha\) is set, the characteristic deflection is monitored continuously. When the characteristic deflection has exceeded a certain limit, it can be judged that the bridge of interest has moved from the acceleration stage on to the deterioration stage (see Fig. 2).

4. Verification experiment to extract bridge vibration behavior by using bus wheel vibration data

(1) Experimental method using a real bridge and a public bus

In order to verify the hypothesis of the similarity between the vertical bus-wheel vibration and the vertical bridge vibration shown in Eq. (4), a verification experiment using a real bridge and a public bus was conducted. Repeatability of measurements and statistical characteristics of the data were also verified to evaluate the feasibility of the proposed monitoring system. The bridge (referred to in this study as the "KW Bridge"), shown in Fig. 5, spans the Hinuma River in Ibaraki Prefecture, and the bus has a vehicle weight of 95.5 kN. The single-span reinforced concrete bridge has a span of about 22 m and has four girders and five 0.21-meter-wide cross beams spaced 5.15 m apart. The bridge is 44 years old but has no major damage that would affect its structural performance. Fig. 6 shows a view of the underside (girders) of the bridge and the bus. The acceleration sensors were installed at the midspan location of an external girder directly under the path of the bus that passed over the bridge. The experiment was conducted three times each at vehicle speeds of 30 km/h and 40 km/h. The vehicle speed was controlled by visually monitoring the speedometer.
(2) Experimental results and discussion

Fig. 7 shows a time history of the bridge (midspan) and bus rear wheel response acceleration at 40 km/h, when the bus passed the midspan ("Passing midspan"). At that time, roughly similar vibrations were observed. Similar results were obtained in the others measurements. The distribution of vibration frequency components was determined by Gabor's continuous wavelet transform method. Fig. 8 shows a scalogram showing the midspan and bus rear wheel acceleration measurement data obtained at 40 km/h. These experimental results indicate the similarity between the vertical vibration acceleration of the bridge and the rear wheel vibration acceleration of the passing bus. In a mechanics problem with finite differences, the results indicate the high likelihood that the vertical vibration acceleration of the bridge will be approximated by the rear wheel vibration acceleration of the passing bus on the basis of the assumptions indicated in this study.

Figs. 9 (a) and (b) show a time history and the probability distribution of the rear wheel vibration acceleration response obtained from three sets of measurement results obtained when the bus passed the midspan zone at 40 km/h. Measurement errors were balanced out by averaging the results obtained from a number of measurement sessions. This indicates that the vibration of the bridge can be estimated from data obtained by averaging the results of a number of measurement sessions, and structural characteristics of the bridge can be identified.

Figure 7  Comparison of bridge acceleration (midspan) and bus wheel acceleration.

Figure 8  Comparison of Gabor wavelet transform scalogram between bridge girder (midspan) and bus wheel.

Figure 9  Differences among three measurements of bus wheel (rear axle) acceleration at 40 km/h.
5. Influence of structural behavior of bridge on characteristic values

Fig. 10 illustrates the finite element method (FEM), and Table 1 details its material specifications. For the road surface unevenness (bump) parameters, the average parameter values for asphalt pavements were used.

On the basis of the predetermined bus body dominant frequency (1.8 Hz) and the wheel dominant frequency (around 10.0 Hz) and the data indicated on the automobile inspection certificate, a four-degree-of-freedom spring–mass model having dominant frequencies roughly matching the predetermined bus body and wheel frequencies was constructed. Fig. 11 shows the natural frequencies and mode shapes of the bridge model. Fig. 12 shows the natural frequencies and mode shapes of the bus. The body vertical vibration modes and front and rear wheel vibration modes of the bus showed fair agreement with the experimental results (body frequency around 1.8 Hz, wheel frequency around 10 Hz). The measurement and simulation results for the midspan strain when the bus was moving at 30 km/h were compared in order to verify the validity of the analysis model. Strain was measured with a long-gauge optical fiber sensor. The comparison results are shown in Fig. 13. As shown, the simulation results showed slightly greater strains. Judging from the timing of the occurrence of the maximum strain and the tendency of increase or decrease in strain, however, it was thought that the degree of this tendency was low enough to ignore when comparing damage levels (damaged or not).

![Figure 10](image1.png)

Bridge width: 6.5 m  End point

Bridge length: 22 m  Traffic lane

Start point

![Figure 11](image2.png)

Figure 11 Target orders and modes of bridge vibration (eigen analysis).

(a) 1st mode: 4.8 Hz  (b) 2nd mode: 5.7 Hz  (c) 3rd mode: 13.0 Hz  (d) 4th mode: 15.9 Hz

![Figure 12](image3.png)

Figure 12 Target orders and modes of bus vibration (eigen analysis).
Figure 13 Comparison of strain behavior between measurements at midspan lower flange of main girder and simulation results obtained by FEM (30 km/h running speed).

Figure 14 Damage type and location of assumed severe damage area.

By referring to reported cases\(^1\),\(^2\),\(^3\) of damage to existing reinforced concrete bridges, two cases of damage at the final phase of the acceleration stage or an early phase of the deterioration stage were assumed: (1) end-to-end spalling at the lower flanges of all girders (Type A damage), and (2) midspan flexural yielding of one of the girders directly under the lane in which the bus passes (Type B damage). The assumed damage locations are shown in Fig. 14. According to the effective mass ratios in the vertical direction obtained from the eigenvalue analyses, the effective mass ratio in the 1-1st vibration mode was dominant (77%). Therefore, by applying the concept of a plastic hinge often used in seismic design, etc., to the midspan region located near the peak of the mode shape, the stiffness of the plastic hinge region was reduced to 1/100 of the normal value by the effect of damage due to flexural cracking, etc. The vehicle speeds assumed were 30 km/h and 40 km/h, as in the experiment. For the road surface condition (roughness), the five-level criteria defined by the ISO (from "very good" to "very poor") for road surface roughness evaluation was used.

For each evaluation criterion, 100 different patterns of road surface profile were generated by Monte Carlo simulation. Under these conditions, simulation was conducted for a total of 3,000 cases.

Fig. 15 shows a distribution of the characteristic deflection for different road surface profiles. Examination of the differences in the distribution of the characteristic values depending on the existence or nonexistence of damage reveals that deterioration-induced changes are overwhelmed by surface irregularities if the road surface is in a "poor" or better condition. The examination also reveals, however, that if the road surface is kept in a condition better than "average," deterioration-induced changes can be identified with relative clarity, regardless of surface irregularities and vehicle speed.

Table 2 shows the average values of characteristic deflection for different vehicle speeds and road surface profiles in the no-damage and damage (Type A and Type B) cases. As shown in the table, the structural anomaly parameter \(\alpha\) is 1.40 in the case of Type A damage and 1.55 in the case of Type B damage.
6. Calculation of characteristic deflection by use of measurement data and discussion on detection of structural anomalies

Vertical displacement was estimated by using acceleration wave integrals obtained, by the Fourier transform method, from the rear wheel vertical vibration acceleration of the bus passing over the bridge in the experiment in order to calculate the characteristic deflection. The average value of the characteristic deflection in the no-damage cases was $-0.4464$. Table 3 shows the characteristic deflection values. If, on the basis of the simulation described earlier, Type A damage ($\alpha = 1.40$) is used as a monitoring criterion, it can be judged that some kind of serious structural anomaly has occurred if the average value of the characteristic deflection is lower than $-0.625$ mm.

| Speed and road condition | Not damaged $\mu_a$ | Type A severe damage $\mu'_A (A)$ | Type B severe damage $\mu'_B (B)$ |
|--------------------------|---------------------|-------------------------------|-------------------------------|
| 30 km/h, Very good       | -1.55               | -2.17                         | -2.41                         |
| 30 km/h, Good            | -1.57               | -2.19                         | -2.43                         |
| 30 km/h, Average         | -1.53               | -2.15                         | -2.39                         |
| 40 km/h, Very good       | -1.56               | -2.18                         | -2.42                         |
| 40 km/h, Good            | -1.60               | -2.22                         | -2.46                         |
| 40 km/h, Average         | -1.54               | -2.16                         | -2.40                         |
| Average                  | -1.56               | -2.18                         | -2.42                         |

Table 3 Average values of characteristic deflection in KW Bridge experiment

| Speed | $\mu_a$ (not damaged) |
|-------|-----------------------|
| 30 km/h | -0.4568             |
| 40 km/h | -0.3078             |
| Average   | -0.4464             |

7. Conclusions

In this study, a new method of monitoring the structural health of bridges using a public bus has been proposed, along with safety indices, as a means of solving SHM-related problems, and a fundamental study of the technical problems associated with the use of the proposed method was conducted. The main conclusions reached through analysis and the results of verification experiments and simulation can be summarized as follows:

1) By using the similarity matrix $P$ under certain conditions, vertical vibration of a bridge can be estimated from the wheel vibration of a bus passing over the bridge.

2) Structural anomalies of a short- or medium-span bridge with a relatively damage-free ("average") road surface at the acceleration or deterioration stage can be detected by monitoring the characteristic deflection measured from wheel vibrations on the condition that the stiffness of the bus used for measurement remains unchanged.

3) In the event of short-span bridge damage as assumed in this study, an anomaly can be detected from the wheel vibration of the bus used for measurement. A monitoring system for detecting major signs of anomalies occurring around or during the transition from the acceleration stage to the deterioration stage by using a bus is feasible.

References

[1] Ministry of Land, Infrastructure, Transport and Tourism in Japan, 2006
[2] Furuta, H., Kayano, M., and Watanabe, E.: "Present state and future outlook of bridge maintenance and bridge management systems" *Doboku Gakkai Ronbunshuu F*, Vol. 63, No. 3, 287–294, July 2007.
[3] JSCE Joint Subcommittee on Maintenance Engineering: *Infrastructure Maintenance Engineering*, University of Tokyo Press, 2004.

[4] Nishimura, A., Fuji, M., Miyamoto, A., and Kagayama, T.: "Usefulness of mechanical behavior in bridge damage assessment", *Proceedings of the Japan Society of Civil Engineers*, No. 380/I-7, April 1987.

[5] Tamakoshi, T., Nakas, K., Ishio, M., and Nakatani, S.: "Research on the dynamic responses of bridges which are caused by large size vehicles (II)", *Technical Note of National Institute for Land and Infrastructure Management*, No. 180, May 2004.

[6] Japan Society of Civil Engineers: *Health monitoring technology for concrete structures*, Concrete Technology Series No. 76.

[7] Kojima, T., Tsunashima, H., Matsumoto, A., and Ogata, M.: "Rail track defect detection by use of onboard measurement data", *Transactions of the Japan Society of Mechanical Engineers (Series C)*, Vol. 72, No. 720, pp. 2447–2454, August 2006.

[8] Ishii, H., Fujino, Y., Mizuno, Y., and Kaito, K.: "Development of Train Intelligent Monitoring System (TIMS) using wheel vibration of in-service vehicle", *Doboku Gakkai Ronbunshuu F*, Vol. 64, No. 1, 44–61, February 2008.

[9] Mizuno, Y., Fujino, Y., Kataoka, K., and Matsumoto, Y.: "Development of a mobile sensing unit and its prototype implementation", *Tsinghua Science and Technology*, Vol. 13, No. S1, pp. 223–227, October 2008.

[10] Sugiura, K., Oshima, Y., Yamaguchi, T., Kobayashi, Y., Okano, H., and Okashiro, S.: "Structural health assessment of road structures by use of multifunction inspection vehicle", Report on results of research and development of technology contributing to improvement of quality of road policy measures, No. 17–8, Committee on Advanced Road Technology, July 2008.

[11] Kawatani, M. and Shimada, R.: "Nonstationary random response and impact of girder bridges under moving vehicles", *Proceedings of the Japan Society of Civil Engineers*, No. 398/I-10, pp. 385–394, October 1988.

[12] Nakajima, A. and Tsumura, N.: "A study on simulation method for vehicle-bridge system", *Journal of Structural Engineering*, Vol. 38A, pp. 781–788, March 1992.

[13] Lin C.W. and Yang Y.B. (2005) "Use of a passing vehicle to scan the fundamental bridge frequencies: an experimental verification. Engineering Structures, 27, 1865-1878.

[14] Kim, C. and Kawatani, M.: "Bridge health assessment using data on vibration induced by a single vehicle", *Steel Construction Engineering*, Vol. 15, No. 58, pp. 37–46, June 2008.

[15] Kitagaki, K., Yamaguchi, T., Kitada, T., Sugiura, K., and Kano, M.: "Analytical study on damage identification of bridge by dynamic vehicle response", *Journal of Construction Steel*, Vol. 15, Japan Society of Steel Construction, 585–590, 2007.

[16] Subcommittee on Monitoring of Bridge Vibration, Committee on Structural Engineering, JSCE: *Proceedings of the Colloquium on Bridge Vibration '97*, September 26, 1997.

[17] Health Monitoring Working Group, Subcommittee on Monitoring of Bridge Vibration and its Standardization, Committee on Structural Engineering, JSCE: *Monitoring technology for structural performance assessment*, April 22, 2005.

[18] Committee on Structural Engineering, JSCE: "Present state and problems of research on traffic-induced bridge vibration" (report of the subcommittee on traffic-induced bridge vibration), April 1994.

[19] Yabe, A.: "Development of method for analyzing dynamic moving body–structure interaction using substructure method", *Proceedings of the 61st Annual Conference of the Japan Society of Civil Engineers*, 1-424, pp. 845–846, September 2006.

[20] Bridge Vibration Study Group: *Measurement and analysis of bridge vibration*, Gihodo Shuppan, October 1993.

[21] Japan Society of Civil Engineers: *Guidelines for bridge vibration monitoring*, Structural Engineering Series, No. 10, 2000, pp. 110–119, 1996.

[22] Japan Society of Mechanical Engineering: *Computer analysis in vibration engineering*, Corona Publishing, 1987.