Effects of Thermal Resistance on One-Dimensional Thermal Analysis of the Epidermal Flexible Electronic Devices Integrated with Human Skin

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Abstract. Nowadays, flexible electronic devices are increasingly used in direct contact with human skin to monitor the real-time health of human body. Based on the Fourier heat conduction equation and Pennes bio-heat transfer equation, this paper deduces the analytical solutions of one-dimensional heat transfer for flexible electronic devices integrated with human skin under the condition of a constant power. The influence of contact thermal resistance between devices and skin is considered as well. The corresponding finite element model is established to verify the correctness of analytical solutions. The results show that the finite element analysis agrees well with the analytical solution. With bigger thermal resistance, temperature increase of skin surface will decrease. This result can provide guidance for the design of flexible electronic devices to reduce the negative impact that exceeding temperature leave on human skin.

1. Introduction
In recent years, flexible electronic technology develops fast. Flexible electronic has become a hotspot in the fields of electronics, mechanics, materials and physics because of its unique stretch ability, high efficiency as well as its low manufacturing costs. Various researches have been developed in the bio-integrated applications of flexible electronic devices. Devices including health monitoring equipments [1-3], smart surgical gloves [4], epidermal electronics [5, 6] and flexible electrodes for brain surface recoding [7] present a promising trend of the next generation of clinical monitoring and wearable products based on flexible electronics.

Considering bio-integrated applications mentioned above, devices will have a direct contact with human skin without exception. In order to avoid the adverse thermal influences of human-integrated applications, it’s essential to investigate the thermal management of the device-skin system. The Fourier heat conduction equation is adopted to study the thermal properties of flexible electronic devices by many relevant researchers. Thermal analysis and design methods focusing devices are developed analytically and experimentally [8-14].

Different from devices, thermal properties of skin tissue should take the influence of blood perfusion and metabolism into account, which is described by Pennes bio-heat transfer equation [15-19]. Cui et al. [20] developed a one-dimensional analytical heat transfer model of device-skin system by coupling two methods mentioned above. The skin is modeled as a 4 layer-structure as shown in Figure 1a. However, Cui et al. [20] assumed that the device and human skin are perfectly bonding.
without considering thermal resistance. While in the practical applications, the contact thermal resistance between skin and device do exist [21, 22] and need to be considered in the analytical prediction model.

Aiming to have a better understanding of how thermal resistance influence the heat transfer, this paper establishes a one-dimensional analytical model under constant power considering thermal resistance between device and skin tissue, which is validated by FEA results. Section 2 and 3 present the mathematical modeling process and results, respectively. The conclusions are given in Section 4.

Figure 1. (a) Schematic illustration of multilayered skin tissue; and (b) schematic illustration of one-dimensional device-skin heat transfer model.

2. Mathematical Modeling
The one-dimensional geometry of the analytical model is shown in Figure 1b. It consists of six layers: encapsulation layer (e.g., SU8), substrate layer (e.g., polydimethylsiloxane (PDMS)), stratum corneum, epidermis, dermis and fat. The functional component planted between SU8 and PDMS can be regarded as a planar source [20]. The top surface of encapsulation layer has the natural convection boundary with \( h \) as the coefficient of heat convection. The bottom surface of fat layer has constant core body temperature \( T_{\text{core}} \). And contact thermal resistance \( R \) exists at the interface of device and skin. The origin of the \( x \) coordinate is on the top surface of encapsulation layer, and the positive direction of the axis points to the bottom of skin. \( H \) denotes the thickness of the whole model, and \( h_i \) is the thickness of each layer, where \( i \) (i=1,2,3,4,5,and 6) stands for the encapsulation, substrate, stratum corneum, epidermis, dermis and fat layer, respectively.

In this paper, we focus on the steady state results. Fourier equation illustrates the heat transfer in the first and the second layers as follows,

\[
k_i \frac{d^2 T_i}{dx^2} = 0 \quad (i = 1, 2). \tag{1}
\]

As for the skin tissue, the blood temperature is assumed to be the same as the core temperature [15], and the metabolism heat generation is constant for a healthy person [19]. Considering that the blood perfusion only exists in dermis layer [19], the Pennes bio-heat equation describes the thermal behavior of skin tissue as follows,
\[
\begin{align*}
\left\{ \begin{array}{l}
  k_i \frac{d^2 T_i}{dx^2} + q = 0 \quad (i = 3, 4, 6) \\
  k_s \frac{d^2 T_s}{dx^2} - (\rho c \omega)_s (T_s - T_{core}) + q = 0
\end{array} \right.
\end{align*}
\] (2)

Let \( \theta_i = T_i - T_{core} \) represents the temperature increase of the system. Then, equations (1) and (2) reduce to,

\[
\left\{ \begin{array}{l}
  k_i \frac{d^2 \theta_i}{dx^2} = 0 \quad (i = 1, 2) \\
  k_i \frac{d^2 \theta_i}{dx^2} + q = 0 \quad (i = 3, 4, 6) \\
  k_s \frac{d^2 \theta_s}{dx^2} - (\rho c \omega)_s \theta_s + q = 0
\end{array} \right.
\] (3)

Where, \( k_i \) is the thermal conductivity of relevant layers. \( \rho, c, \omega \) and \( q \) are the density, specific heat of blood, blood perfusion rate, and metabolism heat generation, respectively.

Then the solutions of equations (3) can be given as follows

\[
\begin{align*}
\theta_1 &= A_1 x + B_1 \\
\theta_2 &= A_2 x + B_2 \\
\theta_3 &= -q x^2 / 2 k_3 + B_3 x + C_3 \\
\theta_4 &= -q x^2 / 2 k_4 + B_4 x + C_4 \\
\theta_5 &= A_5 \exp \left( x \sqrt{\sigma / k_3} \right) + B_5 \exp \left( -x \sqrt{\sigma / k_4} \right) + q / \sigma \\
\theta_6 &= -q x^2 / 2 k_5 + B_6 x + C_6
\end{align*}
\] (4)

Where, \( \sigma = (\rho c \omega)_s \). In order to derive the unknown coefficients in the equation, we need to employ the boundary conditions and the interfacial continuity conditions. At the surface of encapsulation layer \((x=0)\), the natural convection condition gives,

\[
-k_1 \frac{d \theta}{dx} \bigg|_{x=0} = -h \cdot (\theta - \theta_0) \bigg|_{x=0},
\] (5)

Where, \( \theta_0 = T_0 - T_{core} \) while \( T_0 \) is the ambient temperature, \( k_1 \) is the thermal conductivity of encapsulation layer and \( h \) is the coefficient of heat convection. At the encapsulation/substrate interface \((x=h)\), the temperature is continuous,

\[
\theta_i \bigg|_{x=h} = \theta_i \bigg|_{x=h},
\] (6)

And the heat flux satisfies the face heat source condition,

\[
k_i \frac{d \theta_i}{dx} \bigg|_{x=h} = -k_s \frac{d \theta_s}{dx} \bigg|_{x=h} = Q,
\] (7)
Where $k_2$ is the thermal conductivity of substrate layer. At the skin/device interface ($x=h_1+h_2$), the heat flux is continuous and can be derived by the thermal resistance and temperature as follows [21],

$$-k_2 \frac{d\theta}{dx}igg|_{x=h_1+h_2} = -k_2 \frac{d\theta}{dx}igg|_{x=h_1+h_2} = \frac{1}{R} \left( \theta \bigg|_{x=h_1+h_2} - \theta \bigg|_{x=h_1+h_2} \right),$$

(8)

Where $k_3$ is the thermal conductivity of stratum corneum layer, $R$ is the contact thermal resistance between device and skin. At the interface between any other two layers, both temperature and heat flux satisfy the continuous condition,

$$\left. \theta \right|_{(\sum l_n R)} = \left. \theta \right|_{(\sum l_n R)} ,$$

(9)

$$\left. k_n \frac{d\theta}{dx} \right|_{(\sum l_n R)} = \left. k_n \frac{d\theta}{dx} \right|_{(\sum l_n R)} ,$$

(10)

Where $n=3,4,5$, and $k_n$ denotes the thermal conductivity of layer $n$, with $n=3$ for stratum corneum, $n=4$ for epidermis, and $n=5$ for dermis.

The temperature of the bottom of fat layer ($x=\sum_{i=1}^{6}h_i$) is the same as the core temperature, thus ambient temperature boundary condition gives,

$$\left. \theta \right|_{x=\sum_{i=1}^{6}h_i} = 0,$$

(11)

Usually, the stratum corneum has the same thermal property as that of epidermis, i.e. $k_3=k_4$, which means the thermal behavior of layer 3 and 4 are the same. Then coefficients in Eq. (4) can be given as follows

$$\begin{align*}
\frac{\delta \mu \left[ -\gamma \sin h(\mu) + \beta \cos h(\mu) \right] - \gamma \cos h(\mu) + \delta \sin h(\mu) - \epsilon}{\delta \mu \left[ \alpha \sin h(\mu) + \beta \cos h(\mu) \right] + \alpha \cos h(\mu) + \beta \sin h(\mu)} & \\
\frac{k_3}{k_3} A_l + B_1 & \\
(k_n A_1 - Q)/k_2 & \\
h_n A_1 + B_1 - h_n A_2 & \\
[k_1 A_1 - Q + q(h_1 + h_2)]/k_3 & \\
R_k A_1 + h_n A_1 + B_1 + h_n A_2 + q(h_1 + h_2)^2/(2h_3 - B_1(h_1 + h_2)) & \\
(k_n A_1 - Q + q(h_1 + h_2))/k_3 & \\
R_k A_1 + h_n A_1 + B_1 + h_n A_2 + q(h_1 + h_2)^2/(2h_3) - B_1(h_1 + h_2) & \\
[(\alpha + \beta) A_1 + \gamma - \delta]/2h_3 & \\
[(\alpha - \beta) A_1 + \gamma + \delta]/2h_3 & \\
[\alpha \mu (\alpha A_1 + \gamma) \sin h(\mu)] + q(\beta A_1 - \delta) \cos h(\mu)]/h_5 & \\
q(h_1 + h_2 + h_3 + h_4 + h_5)/k_5 & \\
(\alpha A_1 + \gamma) \cos h(\mu) + (\beta A_1 - \delta) \sin h(\mu) + q/\sigma & \\
q(h_1 + h_2 + h_3 + h_4 + h_5)^2/2k_5 - B_1(h_1 + h_2 + h_3 + h_4 + h_5) & \\
\end{align*}$$

(12)
Where,

\[
\begin{align*}
\alpha &= h_1 + k_1/h + k_2/k_1 + h_2/(h_1 + h_3) / k_1 + Rk_i \\
\beta &= k_i / \sqrt{k_i^2\sigma} \\
\gamma &= -(R + h_1/k_2 + (h_1 + h_3)/k_1)Q - q(h_1 + h_3)^2/(2k_1) - q/\sigma + \theta_i \\
\delta &= (Q + q(h_1 + h_3))/\sqrt{k_i^2\sigma} \\
\epsilon &= q/\sigma - qh_i^2/(2k_1) \\
\mu &= h_i/\sigma/k_i \\
\tau &= k_i h_i/k_1 h_1
\end{align*}
\]

In order to verify the correctness of the analytical solutions in Eqs. (4) and (12), a relevant finite element analysis is employed using ABAQUS software (6.13-1, Dassault Simulia, Waltham, MA, USA) so that we can better study the thermal properties of this system. Continuum element DC3D8 is used during our analysis. The thicknesses of encapsulation, substrate, stratum corneum, epidermis, dermis, and fat are taken as 7 µm, 2 mm, 0.02 mm, 0.08 mm, 1.5 mm and 4.4 mm, respectively [8,19]. And 0.2 W/(m·K), 0.15 W/(m·K), 0.21 W/(m·K), 0.21 W/(m·K), 0.37W/(m·K) and 0.16W/(m·K) are the thermal conductivities of encapsulation, substrate, stratum corneum, epidermis, dermis, and fat, respectively [8,19]. Heat convection coefficient \( h = 25\text{W/m}^2\cdot\text{K} \) [22]. We assume that the ambient temperature is 25°C, the core temperature is 37°C, and the metabolic heat generation of the skin tissue is 368W/m³ [19, 20]. The blood density multiplies the specific heat is 4.218 × 10⁶ J/(m³·K) and the blood perfusion rate is 0.03 mL/(mL·s) [20].

3. Results and discussion

Figure 2 shows the results of the comparison between finite element analysis (FEA) and analytical solutions. The input power density of the in-plane heat source is set to be 2500W/m² [20], and the resistance is 1 mm²·K/mW [21]. Besides, the result without considering thermal resistance of Ref. [20] is also given as a control group. Compared to the results of Ref. [20], contact thermal resistance between skin and device brings a gradient of temperature increase at the skin interface. Temperature increase is no longer continuous along the direction of thickness with thermal resistor considered. From the profile, we can conclude that the FEA models have a great agreement with the analytic model. The temperature increase decreases as the \( x \)-coordinate increases whether the resistance exists or not. Without considering resistance, the temperature increase of the heat source and skin surface can reach about 29°C and 9°C, respectively. When the resistance exists, the temperature increase of the heat source increases to about 30°C, while the temperature increase of the skin surface decreases to about 8°C.

Figure 3 shows how skin interface temperature increase and heat source temperature increase change with the resistance varies from 1 mm²·K/mW to 8 mm²·K/mW. The skin interface temperature increase decreases with the resistance increases. And the temperature increase of the heat source presents the opposite trend. This result reveals that thermal resistance is a controlling factor of temperature field of the whole system. Thermal design can consider to use different thermal resistance to achieve temperature control to avoid some adverse thermal response.
Figure 2. The comparison of temperature increase along the thickness direction for analytic prediction with resistor, analytic prediction without resistor, FEA with resistor.

Figure 3. Temperature increase comparison between analytical prediction and FEA of: (a) skin interface; and (b) heat source.

4. Conclusion
A one-dimensional heat transfer model considering contact thermal resistance is established by using Fourier heat conduction equation as well as the Pennes bio-heat transfer equation and validated by FEM. The results show that analytical model have a good agreement with the FEA. Compared with the situation that no resistance exists, there is a step of temperature increase at the skin interface. And the temperature increase of the skin interface decreases with the resistance increases. This result may be helpful to reduce the adverse thermal influence on human skin by increasing the surface contact thermal resistance.

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