Quantum Ergoregion Instability

Gungwon Kang

Raman Research Institute, Bangalore 560 080, India; kang@rri.ernet.in

Abstract: We have shown that, as in the case of black holes, an ergosphere itself with no event horizon inside can evaporate spontaneously, giving energy radiation to spatial infinity until the ergoregion disappears. However, the feature of this quantum ergoregion instability is very much different from black hole radiation. It is rather analogous to a laser amplification. This analysis is based on the canonical quantization of a neutral scalar field in the presence of unstable modes characterized by complex frequencies in a simple model for a rapidly rotating star.

In general relativity, inertial frames around a rotating object are dragged in the sense of the rotation. If the object is rotating rapidly, this dragging effect can be so strong that in some region no physical object can remain at rest relative to an inertial observer at spatial infinity. This region of spacetime is called an ergoregion or an ergosphere. The most common example of ergoregions would be the outside of the event horizon of any rotating black hole. Ergoregions also arise in models of dense, rotating stars, in which cases no event horizon exists inside the ergosurface [1, 2]. The appearance of any ergoregion causes the well-known classical amplification phenomena such as Penrose process for particles and superradiance for waves. Let us consider a regular matter distribution with non-zero total angular momentum in the past such that the spacetime is almost flat. As the gravitational collapse occurs, the system may end up into a stationary rotating black hole in the future. As is well-known, quantum field theory in this evolving background spacetime shows that the black hole formed radiates spontaneously to infinity. In other words, there is no stable vacuum for matter fields in the presence of a rotating black hole which contains both event horizon and ergosphere. On the other hand, suppose the collapsing matter ends up into a stationary rotating star, which has an ergoregion but no horizon in the future. Now one may ask whether quantum field theory in this background spacetime also gives rise to a spontaneous radiation of the ergosphere as in the case of a collapsing black hole. As argued in Ref. [3], an ergosphere may not evaporate spontaneously since the transmitted part of an ingoing spherical wave packet in a superradiant mode, which carries a negative energy with respect to an observer at infinity, will come out again if there is no event horizon inside.

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the ergosurface after passing the center of a rotating object, leaving no net gain of positive energy at infinity. Recently, Matacz, Davies, and Ottewill [4] have considered a quantum scalar field in a simple model describing such spacetimes. One of the motivations of their study is to isolate the role of the ergoregion in order to resolve whether the Starobinskii-Unruh effect [3, 4, 5, 6] is primarily due to the existence of an event horizon or an ergoregion. They showed that the quantum vacuum for a scalar field is stable if all mode frequencies are real, indicating no Starobinskii-Unruh effect in the presence of ergoregion only. This result, however, contradicts with some general results of Ashtekar and Magnon [3, 5] obtained in the algebraic approach to quantization. In addition, spacetimes with ergoregions but no horizons such as rapidly rotating stars are known to be unstable to classical scalar, electromagnetic, and gravitational perturbations [9, 10, 11]. Such classical instability, the so-called ergoregion instability, can be intuitively explained as follows. After passing through the center of the rotating object, the transmitted part, carrying negative energy, of an incident wave packet in a “superradiant” mode will scatter at the ergosurface, giving transmission as well as reflection there with energies amplified, respectively. This process will repeat as long as the ergoregion remains, resulting in presumably “exponential” radiation of positive energy to infinity and accumulation of negative energy within the ergoregion in such a way that the total energy is conserved. It turns out that this instability is characterized by complex frequency modes in normal mode solutions of classical fields. Therefore, not all mode frequencies are real in such a background spacetime.

In this paper, we incorporate such unstable complex frequency modes into the field quantization and show that they lead to a spontaneous evaporation of an ergoregion. This quantum ergoregion instability is very much analogous to a laser amplification. The extended version of this article can be found in Ref. [12].

Now let us consider a system of a massless real scalar field $\phi(x)$ minimally coupled to gravitational fields satisfying the Klein-Gordon equations given by

$$\Box \phi = \frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu \nu} \partial_{\nu} \phi) = 0.$$  

(1)

We assume that the background spacetime possesses a Killing vector field $\xi = \partial_t$ in some regions, for instance, in the early and late stages of its evolution. Then normal mode solutions can be defined by $L_\xi \phi = -i \omega \phi$. Here $L_\xi$ is the Lie-derivative along a Killing vector $\xi$. Since $L_\xi \phi = \partial \phi / \partial x^0$, the time dependence of normal mode solutions is $\sim e^{-i \omega t}$. Given the usual Klein-Gordon inner product

$$\langle \phi_1, \phi_2 \rangle = \frac{i}{2} \int \phi_1^* \partial_{\mu} \phi_2 d\Sigma^\mu$$  

(2)

at an $x^0 = t = \text{const}$ spacelike hypersurface, one obtains

$$\langle \omega_2 - \omega_1^*, \phi_1, \phi_2 \rangle = 0$$  

(3)

for any given two normal mode solutions $\phi_1$ and $\phi_2$; see Ref. [12] for details. Thus the inner product is zero unless $\omega_2 = \omega_1^*$. Since our inner product defined in Eq. (4) is not
positive definite in general, the normal mode frequency $\omega$ is not necessarily always real. It is indeed possible that there exist bounded solutions with complex frequencies as shown in Refs. [11, 10, 9] for certain cases of spacetimes with ergoregions. Then, from Eq. (3), the norm of such complex frequency modes should be zero.

We assume that our background spacetime is described by the Minkowski flat metric in the past infinity and by the Kerr metric with mirror boundary condition on the field $\phi$, which is used in Refs. [4, 11], in the future infinity: Instead of considering the detailed dynamics inside the star, we simply assume that all classical solutions $\phi$ of Eq. (1) vanish on the surface of some sphere inside the ergoregion, e.g., a totally reflecting mirror boundary. The quantization of the field in the past infinity will be straightforward; it will have a Fock representation with a vacuum state $|0\rangle$ in $\mathcal{H}$. To carry out the canonical quantization in the future infinity, let us first construct normal mode solutions of Eq. (1).

As is well known, the Klein-Gordon equation is separable [13] and, in Boyer-Linquist coordinates, admits a complete set of normal mode solutions of the form

$$\phi(x) = \frac{R(r)}{\sqrt{r^2 + a^2}}S(\theta)e^{-i\omega t + im\varphi},$$

where $a$ is the angular momentum per unit mass of the star with mass $M$, and $m$ is an integer. Defining a “tortoise” coordinate $\tilde{r}$ by $d\tilde{r}/dr = (r^2 + a^2)/(r^2 + a^2 - 2Mr)$, the radial part of Eq. (4) becomes

$$\frac{d^2R}{d\tilde{r}^2} - V_{\omega lm}(\tilde{r})R = 0.$$ (5)

From the mirror boundary condition, the radial function vanishes at some $r = r_0$ (accordingly, $\tilde{r} = \tilde{r}_0$) inside the ergoregion: $R(r_0) = 0$. For simplicity, we assume that $r_0$ is very near the “horizon” radius $r = r_H = M + \sqrt{M^2 - a^2}$. That is, $\tilde{r}_0 \sim -\infty$. We also require that the field is not singular at spatial infinity, $\tilde{r} \sim \infty$. The asymptotic behavior of the effective potential $V_{\omega lm}$ induced through the interaction with gravitational fields is [14]

$$V_{\omega lm}(r) \sim \begin{cases} - (\omega - m\Omega_H)^2 & \text{as } \tilde{r} \to \tilde{r}_0, \\ -\omega^2 & \text{as } \tilde{r} \to \infty, \end{cases}$$ (6)

where $\Omega_H = a/2Mr_H$. We have $V_{\omega lm} = \infty$ at $\tilde{r} = \tilde{r}_0$, corresponding to the mirror boundary condition. Since $\omega$ could be complex in our model, $V_{\omega lm}(r)$ is a complex potential in general.

Let us now consider normal mode solutions $u_{\omega lm}(r)$ to Eq. (5) whose asymptotic forms are

$$u_{\omega lm}(r) \sim \begin{cases} B_{\omega lm}(e^{i\tilde{\omega}_0\tilde{r}} + A_{\omega lm}e^{-i\tilde{\omega}_0\tilde{r}}) & \text{as } \tilde{r} \to \tilde{r}_0, \\ e^{i\tilde{\omega}_0\tilde{r}} + C_{\omega lm}e^{i\tilde{\omega}_0\tilde{r}} & \text{as } \tilde{r} \to \infty, \end{cases}$$ (7)

where $A_{\omega lm} = -e^{2i\tilde{\omega}_0\tilde{r}_0}$ from the mirror boundary condition at $\tilde{r} = \tilde{r}_0$. If $\omega$ is complex, $u_{\omega lm}$ becomes exponentially divergent at spatial infinity and so we exclude this class of solutions from our construction. Thus $u_{\omega lm}$ represents real frequency normal mode solutions. Now the Wronskian relations from Eq. (3) with the mirror boundary condition give $|A_{\omega lm}| = |C_{\omega lm}| = 1$. Therefore, $u_{\omega lm}(r)$ is a stationary wave without any net ingoing or outgoing.
of solutions has not been included in the quantization procedure in Ref. [4]. Let complex frequencies which describe unstable modes in the presence of ergoregions. This class of solutions has not been included in the quantization procedure in Ref. [4] as a complete basis.

As mentioned above, however, there exists another class of normal mode solutions with complex frequencies which describe unstable modes in the presence of ergoregions. This class of solutions has not been included in the quantization procedure in Ref. [4]. Let \( v_{\omega lm}(r) \) be normal mode solutions in such class whose asymptotic behaviors are

\[
v_{\omega lm}(r) \sim \begin{cases} e^{\tilde{\omega} \tilde{r}} + R_{\omega lm} e^{-i\tilde{\omega} \tilde{r}}, & \text{as } \tilde{r} \to \tilde{r}_0, \\ T_{\omega lm} e^{i\tilde{\omega} \tilde{r}}, & \text{as } \tilde{r} \to \infty. \end{cases}
\]

The mirror boundary condition is satisfied if

\[
R_{\omega lm} = -e^{2i\tilde{\omega} \tilde{r}_0} = -e^{2i\tilde{\omega} R \tilde{r}_0} e^{-2\tilde{\omega} \tilde{r}_0},
\]

where \( \tilde{\omega} = \tilde{\omega}^R + i\tilde{\omega}^I \). Thus, \( \omega^I = \tilde{\omega}^I = -\ln |R_{\omega lm}|/2\tilde{r}_0 \). Note that, since the potential \( V_{\omega lm}(r) \) in Eq. (3) is complex, the Wronskian relation does not necessarily give \( |R_{\omega lm}| = 1 \).

If \( |R_{\omega lm}| > 1 \), \( \omega^I > 0 \) and so \( v_{\omega lm}(r) \sim e^{-\omega^I \tilde{r}} \) as \( \tilde{r} \to \infty \) and is regular at spatial infinity. From the time dependence of this solution, i.e., \( \sim e^{-i\omega t} = e^{-i\omega^H t} e^{\omega^H t} \), we also notice that it represents an outgoing mode which is exponentially amplifying in time but is exponentially decreasing as \( \tilde{r} \to \infty \). By making a wave packet, as suggested in Ref. [11], we may regard this solution as an outgoing wave packet with \( \tilde{\omega}^R = \omega^R - m\Omega_H < 0 \) starting from near the mirror surface, which will bounce back and forth within the ergoregion, and a part of which is repeatedly transmitted to infinity, resulting in exponential amplification in time in the inside as well as on the outside of the ergosurface. If \( |R_{\omega lm}| < 1 \), \( \omega^I < 0 \) and so this solution corresponds to an outgoing decaying mode in time. However, since its radial behavior becomes singular at spatial infinity, we do not include this mode in our construction of normal mode solutions.

For any given solution

\[
\phi_{\omega lm}(x) = \phi_{\omega lm}(r, \theta) e^{-i\omega t + im\phi} = v_{\omega lm}(r) \sqrt{\frac{\omega^H t + im\phi}{\omega^H t}},
\]

with \( \omega^I > 0 \), we find that there are three linearly independent solutions:

\[
\phi^*_{\omega lm}(x) = \frac{v^*_{\omega lm}(r)}{\sqrt{\frac{\omega^H t - im\phi}{\omega^H t}}} S_{\omega lm}(\theta) e^{i\omega^H t - im\phi} \sim e^{\omega^H t},
\]

\[
\phi_{\omega^* lm}(x) = \frac{v_{\omega^* lm}(r)}{\sqrt{\frac{\omega^H t + im\phi}{\omega^H t}}} S_{\omega lm}(\theta) e^{-i\omega^H t + im\phi} \sim e^{-\omega^H t},
\]

\[
\phi_{\omega^* lm}(x) = \frac{v_{\omega^* lm}(r)}{\sqrt{\frac{\omega^H t - im\phi}{\omega^H t}}} S_{\omega lm}(\theta) e^{i\omega^H t - im\phi} \sim e^{-\omega^H t}.
\]

\( \phi_{\omega^* lm} \) represents an exponentially decaying wave in time which originates at infinity. In other words, this mode is the same as \( \phi_{\omega lm}(x) \) but backward in time. For these non-stationary
modes, $\omega$ is a discrete complex number which is determined by the details of the potential and the boundary condition. In fact, by finding poles of the scattering amplitude for a more realistic model of rotating stars, Comins and Schutz [10] have shown that the imaginary part of the complex frequency for a purely outgoing mode is discrete, positive, and proportional to $e^{-2\beta m}$, where $\beta$ is of order unit. For our model, it also can be shown that complex eigenfrequencies are confined to a bounded region.

From our definition of the inner product in Eq. (2), we find

$$\langle \phi_{\omega\ell m}, \phi_{\omega'\ell' m'} \rangle = \frac{1}{2} \int \left[ (\omega' + \omega^*) - \Omega(m' + m) \right] \phi_{\omega\ell m}^* \phi_{\omega'\ell' m'} N^{-1} d\Sigma, \quad (12)$$

where we have used $d\Sigma^\mu = N^{-1} (\partial_t + \Omega \partial_\phi)^\mu d\Sigma$, $\Omega(r, \theta) = -g_{t\phi}/g_{\varphi\varphi}$, and $N = (-g^{tt})^{-1/2}$. For real frequency modes $u_{\omega\ell m}(x) = u_{\omega\ell m}(r)/\sqrt{r^2 + a^2 S_{\omega\ell m}(\theta)e^{-i\omega t + im\varphi}}$ with $\omega > 0$, we have the orthogonality relations

$$\langle u_{\omega\ell m}, u_{\omega'\ell' m'} \rangle = \delta(\omega - \omega') \delta_{\ell \ell'} \delta_{m m'} \quad \text{for} \quad u_{\omega\ell m} \notin N^-, \quad (13)$$

after suitable normalizations. Here $N^-$ is defined as a set consisting of mode solutions $u_{\omega\ell m}$ with $\omega > 0$ whose norms are negative. For complex frequency normal mode solutions $v_{\omega\ell m}(x) = v_{\omega\ell m}(r)/\sqrt{r^2 + a^2 S_{\omega\ell m}(\theta)e^{-i\omega t + im\varphi}}$, we have

$$\langle v_{\omega\ell m}, v_{\omega'\ell' m'} \rangle = 0, \quad (14)$$

All other inner products vanish.

Finally, it should be pointed out that it may be possible that any normal mode solution with complex frequency is expressed by linearly combining the real frequency normal modes $\{u_{\omega\ell m}(x)\}$. There is no disproof for this possibility yet. But, we assume that the set of complex frequency normal mode solutions represents new independent degrees of freedom of the system, which can describe field solutions carrying arbitrary values of energy including negative ones by linear combinations.

Based on the analysis of normal mode solutions for the classical scalar field above, we now apply the quantization methods developed in Refs. [16, 17, 18, 19] in the presence of complex frequency modes in the Minkowski flat spacetime. The neutral scalar field can be expanded in terms of normal mode solutions as

$$\phi(x) = \sum_{\ell m} \int_{gN-} d\omega \frac{1}{\sqrt{2}} [a_{\lambda\ell} u_{\lambda\ell}(x) + a_{\lambda\ell}^\dagger u^*_{\lambda\ell}(x)] + \sum_{\ell m} \int_{N} d\omega \frac{1}{\sqrt{2}} [a_{-\lambda\ell} u_{-\lambda\ell}(x) + a_{-\lambda\ell}^\dagger u^*_{-\lambda\ell}(x)]$$

$$+ \sum_{\omega_{\ell m}} \frac{1}{\sqrt{2}} [b_{\lambda\ell} v_{\lambda\ell}(x) + b_{\lambda\ell}^\dagger v^*_{\lambda\ell}(x) + b_{-\lambda\ell} v_{-\lambda\ell}(x) + b_{-\lambda\ell}^\dagger v^*_{-\lambda\ell}(x)], \quad (15)$$

where $\lambda$ denotes $(\omega, l, m)$, $-\lambda$ denotes $(-\omega, l, -m)$, and $\bar{\lambda}$ denotes $(\omega^*, l, m)$. Assuming the equal-time commutation relations for $\phi(x)$ and its momentum conjugate $\pi(x)$, we find commutation relations among mode operators

$$[a_{\lambda\ell}, a_{\lambda'\ell'}^\dagger] = \delta_{\lambda\lambda'}, \quad [b_{\lambda\ell}, b_{\lambda'\ell'}^\dagger] = \delta_{\lambda\lambda'}, \quad [b_{\lambda\ell}, b_{\lambda'\ell'}] = [b_{\lambda\ell}, b_{\lambda'\ell'}^\dagger] = [b_{\lambda\ell}, b_{\lambda'\ell'}^\dagger] = 0. \quad (16)$$
All others vanish. Note that the real frequency mode operators satisfy the usual commutation relations whereas mode operators for complex frequencies have unusual commutation relations. In particular, \( b_\lambda \) does commute with \( b_\lambda^\dagger \).

Now the Hamiltonian operator can be expressed in terms of mode operators

\[
H = \frac{1}{2} \sum_{lm} \int d\omega (a_\lambda^\dagger a_\lambda + a_\lambda a_\lambda^\dagger) + \frac{1}{2} \sum_{lm} \int d\omega (-\omega) (a_{-\lambda}^\dagger a_{-\lambda} + a_{-\lambda} a_{-\lambda}^\dagger)
\]
\[
+ \frac{1}{2} \sum_{\omega_{lm}} [\omega (b_\lambda b_\lambda^\dagger + b_\lambda^\dagger b_\lambda) + \omega^* (b_\lambda^\dagger b_\lambda + b_\lambda b_\lambda^\dagger)],
\]

(17)

where \( \omega > 0 \) for real frequency modes and \( \omega^I > 0 \) for complex frequency modes. The Hamiltonian for real frequency modes has a representation of a set of attractive harmonic oscillators as usual. Interestingly, although a vacuum state can be defined such that \( a_\pm \lambda |0_R \rangle = 0 \) for all \( \lambda \), it is not the lowest energy state since the energy associated with the second term in Eq. (17) is not bounded below but bounded above. Therefore, real frequency mode operators possess the usual symmetrized Fock representation \( H^R_\lambda \) as well as the particle interpretation. For complex frequency modes, let

\[
H^C_\lambda = \frac{1}{2} (p_\lambda^2 - (\omega^I)^2 q_{\lambda}^2) + \frac{1}{2} (p_\lambda^2 - (\omega^I)^2 q_{\lambda}^2) + \omega^R (q_{1\lambda} p_{2\lambda} - p_{1\lambda} q_{2\lambda}).
\]

(18)

By using linear transformations into Hermitian operators \( q \) and \( p \) satisfying \([q, p] = i\)

\[
b_\lambda = \frac{1}{2} \left[ i (\sqrt{\omega} q_{1\lambda} + \frac{1}{\sqrt{\omega}} p_{1\lambda}) + (\sqrt{\omega} q_{2\lambda} + \frac{1}{\sqrt{\omega}} p_{2\lambda}) \right],
\]
\[
b_\lambda^\dagger = \frac{1}{2} \left[ (\sqrt{\omega} q_{1\lambda} - \frac{1}{\sqrt{\omega}} p_{1\lambda}) + i (\sqrt{\omega} q_{2\lambda} - \frac{1}{\sqrt{\omega}} p_{2\lambda}) \right],
\]

(19)

one finds

\[
H^C_\lambda = \frac{1}{2} (p_{1\lambda}^2 - (\omega^I)^2 q_{1\lambda}^2) + \frac{1}{2} (p_{2\lambda}^2 - (\omega^I)^2 q_{2\lambda}^2) + \omega^R (q_{1\lambda} p_{2\lambda} - p_{1\lambda} q_{2\lambda}).
\]

(20)

Thus this is a system of two coupled inverted harmonic oscillators with the same frequency \( |\omega| \). Consequently, the energy spectrum for \( H^C_\lambda \) is \( E_{\epsilon, k_\lambda} = \omega^I \epsilon_\lambda + \omega^R k_\lambda \) where \( \epsilon_\lambda \) is any continuous real number and \( k_\lambda \) any integer. It shows that the energy eigenvalue is continuous for given \( k_\lambda \) and unbounded below. This unboundedness of the energy indicates that energy eigenstates are not normalizable. However, we can construct normalizable wave packets from them. These square integrable wave packets will form a Hilbert space \( \mathcal{H}^C_\lambda \) which is isomorphic to \( L^2(\mathbb{R}^2) \).

Any quantum state of the field which is in this Hilbert space will give rise to instability. It follows because, although the total energy of this state is definite and time independent, the energy density outside the ergoregion will be positive and have exponential time dependence whereas the energy density within the ergoregion will have the same behavior but with negative energy, keeping the total energy over the whole space fixed. Therefore, an observer...
sitting outside the ergoregion will measure time-dependent radiation of positive energy. In addition, since the energy spectrum is unbounded below, some external interaction with this system can give energy extraction from the system without bound. Note that mode operators no longer have particle interpretation as in those for real frequency modes. Finally, we complete our quantization of the field \( \phi(x) \) by constructing the total Hilbert space as follows,

\[
\mathcal{H} = \mathcal{H}^R \otimes \prod_\lambda \mathcal{H}_\lambda^C.
\]

Here \( \mathcal{H}^R \) is the usual symmetrized Fock space generated by real frequency modes and \( \prod_\lambda \mathcal{H}_\lambda^C \) is the infinite number of products of non-Fock-like Hilbert spaces \( \mathcal{H}_\lambda^C \) generated by complex frequency modes.

Now let us see how the appearance of an ergoregion at the late stage of a dynamically evolving background spacetime starts to give a spontaneous radiation of energy. We expect this spontaneous quantum radiation if the initial vacuum state \( |0\rangle \) in of the field in the past falls in any state in \( \prod_\lambda \mathcal{H}_\lambda^C \) in the remote future. To see this effect let us consider a “particle” detector linearly coupled to the field near \( t \sim \infty \) placed in the in-vacuum state \( |0\rangle \). Then, the transition rate of the detector integrated over angular variables \( \theta \) and \( \varphi \) is proportional to

\[
\begin{align*}
\frac{F(E)}{T} &\sim \frac{1}{2} \sum_{\sigma} \left\{ \sum_{\lambda, \omega} \int_{\mathbb{R}^-} d\omega |\beta_{\lambda\sigma}|^2 \frac{|u_{\lambda}(r)|^2}{\sqrt{r^2 + a^2}} \right\}^2 \delta(E - \omega) \\
&\quad + \sum_{\lambda, \omega} \int_{\mathbb{R}^-} d\omega |\alpha_{-\lambda\sigma}|^2 \frac{|u_{-\lambda}(r)|^2}{\sqrt{r^2 + a^2}} \delta(E - \omega) \\
&\quad - \sum_{\lambda, \omega} 2\text{Re}[\gamma_{\lambda\sigma}^* \gamma_{\lambda\sigma} (\frac{u_{\lambda}(r)}{\sqrt{r^2 + a^2}})^2 e^{i(E + \omega^R)T} (E + \omega)^2] \\
&\quad + \sum_{\lambda, \omega} \eta_{\lambda\sigma}^* \eta_{\lambda\sigma} (\frac{u_{\lambda}^*(r)}{\sqrt{r^2 + a^2}})^2 e^{-i(E - \omega^R)T} (E - \omega^*)^2 \frac{T}{(E - \omega^*)^2}
\end{align*}
\]  

(22)

for large \( T \gg 1 \). Here \( \alpha, \beta, \gamma, \) and \( \eta \) are Bogoliubov coefficients between in- and out-modes. This result in general shows nonvanishing excitations of the particle detector related to complex frequency modes as well as the usual contributions due to the mode mixing in real frequency modes. In particular, the contributions related to complex frequency modes are not stationary, but exponentially increasing in time \( T \) [21]. The \( \delta \)-function dependence in the first two terms implies the energy conservation; that is, only the real frequency mode, whose quantum energy is the same as that of the particle detector \( (\omega = E) \), can excite the detector. For complex frequency modes, however, all modes contribute to the excitation possibly because the energy spectrum for any complex frequency mode is continuous. From the relations among Bogoliubov coefficients (see Ref. [22]), one can see that it is impossible for \( \gamma_{\lambda\sigma}^* \gamma_{\lambda\sigma} \) and \( \eta_{\lambda\sigma}^* \eta_{\lambda\sigma} \) to vanish for all \( \lambda \) and \( \sigma \). Therefore, the result obtained above strongly implies that a rotating star with ergoregion but without horizon has the quantum instability as well, leading exponentially time-dependent spontaneous energy radiation to spatial infinity. This quantum instability is possible because negative energy could be accumulated.
within the ergoregion. Accordingly, our result resolves the contradiction between conclusions in Ref. [4] and Ref. [8].

It will be straightforward to extend our formalism to other matter fields such as massive charged scalar and electromagnetic fields. For spinor fields, however, it is unclear at the present whether or not rotating stars with ergoregions spontaneously radiate fermionic energy to infinity as well. It is because spinor fields do not give superradiance in the presence of an ergoregion. In fact, we find that the inner product defined in Ref. [6] for spinor fields is still positive definite in our spacetime model. Then, as explained below Eq. (3), there exists no complex frequency mode and hence no unstable mode for spinor fields classically. However, as rotating black holes give fermion emissions in the quantum theory inspite of no superradiance at the classical level, there might be some quantum process through which the ergoregion gives fermionic spontaneous energy radiation. In addition, the main result obtained in the algebraic approach [8, 7] does not seem to depend on which matter field is considered.

As shown in preceding sections, the Hamiltonian operators associated with unstable modes do not admit a Fock-like representation, or a vacuum state, or the particle interpretation of mode operators. Accordingly, the conventional analysis of the vacuum instability based on the uses of asymptotic vacua and appropriately defined number operators no longer applies to our case. However, we have shown that Unruh’s “particle” detector model, which indeed does not require the particle interpretation of the field, is still applicable for extracting some useful physics in our case. In fact, our case serves as a good example illustrating the point of view that the fundamental object in quantum field theory is the field operator itself, not the “particles” defined in a preferred Fock space [22]. The expectation value of the energy-momentum tensor operator, which is defined by field operators only, should also be a useful quantity in our case. To obtain meaningful expectation value, however, renormalization of the energy-momentum tensor would have to be understood first in the presence of such instability modes [23]. As far as we know, this interesting issue has never been addressed in the literature.

Based on the analysis in our paper, exponentially time-dependent spontaneous energy radiation will occur as soon as an ergoregion is formed. Then the back reaction of the quantum field on the metric will change the gravitational fields of the evolving rotational object itself, depending on the strength and the time scale of the spontaneous radiation. Since the wave trapped within the ergoregion carries negative energy and the angular momentum in the opposite sense of the rotation, the rotating object will loose its angular momentum and so the ergoregion can disappear at some point of its evolution. Then the spontaneous radiation will also stop occurring. The corresponding Penrose diagram is shown in Fig. 1. Here the curved solid line is the trajectory of the surface of a rotating object. The dotted lines denote the boundaries of the ergoregion.

It will also be very interesting to see how our quantization procedure in this paper can be translated into algebraic approach. The mode decomposition constructed in our canonical quantization procedure suggests that there may exist some way to construct a corresponding
complex structure in the algebraic approach. Finally, it should be pointed out that there are many other physical systems such as plasma, laser cavity, and gravitationally colliding system producing quasinormal modes where complex frequency modes play important roles. Generically, if a system stores some “free” energy which can be released through interactions, then some amplifications occur, revealing complex frequency modes classically. Therefore, the quantization formalism described in the present work may be useful to understand such systems in the context of quantum field theory.

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[14] For the detailed functional form of $V_{\omega m}(r)$, see Ref. [3].

[15] ZAMO’s are locally nonrotating observers whose trajectories are tangent to $\partial_\tau = \partial_t + \Omega \partial_\phi$ where $\Omega = -\partial_t \cdot \partial_\phi/\partial_\phi \cdot \partial_\phi$ and $\partial_\phi$ is the rotational Killing vector field of stationary axisymmetric spacetimes. The trajectories of Killing observers are tangent to $\partial_t$ which becomes spacelike inside ergoregions.

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