Gravity with Perimeter Action 

and

Gravitational Singularities

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Abstract

We consider the perturbation of the Schwarzschild solution by the perimeter action. The asymptotic behaviour of the solution at infinity and at the horizon are calculated and analysed in the first approximation. In the regions far from the matter sources the perturbations are characterised by the ratio of the Plank length to the Schwarzschild radius and are infinitesimally small. At short distances the perturbation is large and there appears a spacetime region of the Schwarzschild radius scale that is unreachable by test particles. These regions are located there where the standard theory of gravity has singularities.
1  Perimeter Action

Unification of gravity with other fundamental forces within the superstring theory stimulated the interest to the quantum gravity and physics at Planck scale [1, 2, 3, 4, 5]. In particular, string theory predicts a modification of the gravitational action at Planck scale with additional high-derivative terms. This allows to ask fundamental questions concerning physics at Planck scale referring to these effective actions and, in particular, one can try to understand how they influence the gravitational singularities [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] and the black hole physics [23, 24, 25, 26, 27]. The classical and quantum gravity theories with generic higher-curvature terms were considered in [28, 29, 30] and recently in [31, 32]. The theories with limited-curvature hypothesis were considered in [33, 34, 35].

It is appealing to extend this approach to different modifications of general relativity that follow from the string theory and also to develop an alternative approach based on new geometrical principles [36, 37, 38, 39, 40, 41, 42, 43, 44]. Here the idea is to extend the Feynman path integral to an integral over the space-time manifolds in a way that makes the quantum-mechanical amplitudes proportional to the "linear size" or "perimeter" of the four-dimensional universe [44]. That will suppress the growth of the lower-dimensional spikes out of a 4-D manifold [36, 37, 38, 39, 40].

The suggested "perimeter" action can be considered as a "square root" of the Regge area action in discretised gravity [45, 46]. In the Regge action the area $\sigma_{ijk}$ of the triangle $<ijk>$ of the four-dimensional simplex is multiplied by the corresponding deficit angle $\omega^{(2)}_{ijk}$ and a summation is over all triangles:

$$S_A = \sum_{<ijk>} \sigma_{ijk} \cdot \omega^{(2)}_{ijk}. \quad (1.1)$$

The formula represents the discretised version of the continuous area action in gravity:

$$S_A = -\frac{c^3}{16\pi G} \int R \sqrt{-g} d^4x. \quad (1.2)$$

In the alternative "perimeter" action [36, 37, 38, 39, 40] now the perimeter $\lambda_{ijk}$ of the triangle $<ijk>$ is multiplied by the corresponding deficit angle:

$$S_P = \sum_{<ijk>} \lambda_{ijk} \cdot \omega^{(2)}_{ijk}. \quad (1.3)$$

The Regge action measures the "area" of the universe, the perimeter action measures the "linear size" of the universe and requires the introduction of the fundamental length scale.
It is unknown to the author how to derive a continuous limit of the perimeter action (1.3) in a unique way. In these circumstances one can try to construct a possible perimeter action for a smooth manifold of a space-time universe by using the available geometrical invariants. Any expression which is quadratic in the curvature tensor and includes two derivatives can be a candidate for the perimeter action. These invariants have the dimension of $1/cm^6$:

$$I_1 = -\frac{1}{80\pi} R_{\mu\nu\lambda\rho;\sigma} R^{\mu\nu\lambda\rho;\sigma}, \quad I_2 = +\frac{1}{16\pi} R_{\mu\nu\lambda\rho} \Box R^{\mu\nu\lambda\rho},$$  

(1.4)

and we will consider a linear combination of the above invariants:

$$S_P = M c \int \sqrt{I_1 + (1 - \epsilon) I_2} \sqrt{-g} d^4 x.$$  

(1.5)

The dimension of the integrant invariant is $1/cm^3$, and the above four-dimensional space-time integral has the dimensionality of $cm$. We have to introduce a mass parameter $M$ to get a correct dimensionality ($g \text{ cm}^2/\text{sec}$) of the action (1.5). The mass parameter can be expressed in terms of the Planck mass $M_P$ leading to the appearance of the Planck constant $\hbar$ in the action. We also introduced a dimensionless coupling constant $\gamma$ expressing the mass parameter in terms of Planck mass units,

$$M = \gamma M_P, \quad M_P = \sqrt{\hbar c/16\pi G},$$  

(1.6)

thus the perimeter action is:

$$S_P = \gamma \sqrt{\hbar} \sqrt{\frac{c^3}{16\pi G}} \int \sqrt{I_1 + (1 - \epsilon) I_2} \sqrt{-g} d^4 x.$$  

(1.7)

The action (1.7) fulfils our basic physical requirement on the action that it should have the dimension of length and should be similar to the action of the relativistic particle [36, 37, 38, 39, 40]:

$$S = -mc \int ds = -mc^2 \int \sqrt{1 - \frac{v^2}{c^2}} dt.$$  

(1.8)

Both expressions contain the geometrical invariants that are not in general positive-definite under the square root operation. In the relativistic particle case (1.8) the expression under the root becomes negative for a particle moving with a velocity that exceeds the velocity of light. In that case the action develops an imaginary part, and the quantum-mechanical superposition of the amplitudes prevents a particle from exceeding the velocity of light [47, 48, 49] as it is demonstrated in Fig. 1. A similar mechanism was implemented in the Born-Infeld modification of electrodynamics with the aim to prevent the appearance of infinitely
Figure 1: The graphic of the real and imaginary parts of the amplitude $\Delta K = e^{i\frac{mc^2}{\hbar}}\sqrt{1-v^2/c^2}\Delta t$.

large electric fields [50, 33, 34, 35]. One can expect that in the case of the perimeter action (1.5) there may appear space-time regions that are unreachable by the test particles as far as in that regions the action develops an imaginary part and the amplitude is exponentially small as it can be seen in Fig.2. If these space-time regions happen to appear and if space-time regions include singularities, then one can expect that the gravitational singularities are naturally excluded from the theory due to the fundamental principles of the quantum mechanics. The question of consistency of the new action principle, if it is the right one, can only be decided by its physical consequences.

In the next sections we will consider the black hole (BH) singularities and the physical effects that are induced by the inclusion of the perimeter action (1.5). As we will see, the expression under the root in (2.12) becomes negative in the region that is smaller than the Schwarzschild radius $r_g$ and includes the singularities. For the observer that is far away from the BH horizon the perimeter action induces a tiny advance precession of the perihelion but has a profound influence on the physics near the horizon. We are confronted here with space-time regions that are unreachable by the test particles and the expectation value of any observable $\langle O \rangle$ in that region will be exponentially suppressed (2.12), (2.14) and (4.66). If one accepts this concept, then it seems plausible that the gravitational singularities are excluded from the modified theory. In this letter we have only taken the first steps to describe the phenomena that are caused by the additional perimeter term in the gravitational action.

*The general form of the action is presented in the Appendix (3.45).
2 Perimeter Perturbation of Schwarzschild Solution

The modified action that we will consider is a sum

\[ S = -\frac{c^3}{16\pi G} \int R\sqrt{-g}d^4x + \gamma \sqrt{\hbar} \int \sqrt{\frac{c^3}{16\pi G}} \int \sqrt{I_1 + (1 - \epsilon)I_2} \sqrt{-g} d^4x. \]  

(2.9)

In the limit \( \hbar \to 0 \) the action reduces to the classical one\(^\dagger\). The additional perimeter term has high derivatives of the space-time metric, and the equations of motion are much more complicated than in standard gravity. We were unable to find exact solutions of these equations and we are suggesting that the equations can be solved by using a perturbation theory. The classical solutions of general relativity will be modified in the regions of the space-time where the gravitational field is changing at the short-scale distances.

Here we consider the perturbation of the Schwarzschild solution that is induced by the additional term in the action and try to understand how it influences the black-hole physics and the singularities. The Schwarzschild solution has the form

\[ ds^2 = (1 - \frac{r_g}{r})c^2dt^2 - (1 - \frac{r_g}{r})^{-1}dr^2 - r^2d\Omega^2, \]

(2.10)

where \( g_{00} = 1 - \frac{r_g}{r} \), \( g_{11} = -(1 - \frac{r_g}{r})^{-1} \), \( g_{22} = -r^2 \), \( g_{33} = -r^2 \sin^2 \theta \), and

\[ r_g = \frac{2GM}{c^2}, \quad \sqrt{-g} = r^2 \sin \theta. \]

The quadratic invariant in this case has the form \( I_0 = \frac{1}{12} R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho} = (\frac{r_g}{r})^2 \) and shows the location of the curvature singularity at \( r = 0 \). The event horizon is located where the metric component \( g_{rr} \) diverges, that is, at \( r_{\text{horizon}} = r_g \). The expressions for the two curvature polynomials \(^\dagger\dagger\) of our interest are\(^\dagger\dagger\)

\[ I_1 = \frac{9}{4\pi} \frac{r_g^2(r - r_g)}{r^9}, \quad I_2 = \frac{9}{4\pi} \frac{r_g^3}{r^9}, \]

(2.11)

and on the Schwarzschild background the action acquires additional term of the form

\[ S_P = \gamma \sqrt{\frac{hc^5}{G}} \int \frac{3}{2} \sqrt{1 - \epsilon r_g} \frac{r_g dr}{r} dt. \]

(2.12)

As one can see, the expression under the square root in \( 2.12 \) becomes negative at

\[ r < \epsilon r_g, \quad 0 \leq \epsilon < 1 \]

(2.13)

\(^\dagger\)The general form of the perimeter action is presented in the Appendix (3.45).

\(^\dagger\dagger\)It should be stressed that on a Schwarzschild solution all other invariant polynomials presented in the Appendix of the same dimensionality can be expressed in terms of \( I_1 \) and \( I_2 \) (3.47).
and defines the region where the action becomes complex and seems unreachable by the test particles. The size of the region depends on the parameter \( \epsilon \) and is smaller than the gravitational radius \( r_g \). This observation seems to have profound consequences on the gravitational singularity at \( r = 0 \). In a standard interpretation of the singularities, which appear in spherically symmetric gravitational collapse, the singularity at \( r = 0 \) is hidden behind the event horizon. In that interpretation the singularities are still present in the theory. In the suggested scenario it seems possible to eliminate the singularities from the theory based on the fundamental principles of the quantum mechanics. The quantum-mechanical amplitude is proportional to exponent \( K \sim e^{\frac{i}{\hbar} S[g]} \), and for the Schwarzschild space-time one can find the following expression for the action:

\[
S_P = \gamma \sqrt{\frac{\hbar c^5}{G}} \int_r^\infty \frac{3}{2} \sqrt{1 - \frac{\epsilon r_g}{r}} \frac{r \, dr}{r} \Delta t = \gamma \sqrt{\frac{\hbar c^5}{G}} \frac{1}{\epsilon} \left( 1 - \left( 1 - \frac{\epsilon r_g}{r} \right)^{\frac{3}{2}} \right) \Delta t.
\]  

(2.14)

The action is proportional to the length \( \Delta t \) of the world trajectory, as it should be for a relativistic particle at rest. In the neighbourhood of the Schwarzschild space-like singularity the action develops a large imaginary value that exponentially suppresses the propagation amplitude for the particles to enter the singularity region \( \text{(4.66)} \).

It is important therefore to find the perturbation of the Schwarzschild solution that is induced by the presence of the additional perimeter term in the action. The full equation has the following form:

\[
\frac{\delta S}{\delta g^{\mu\nu}} = \frac{\delta S_A}{\delta g^{\mu\nu}} + \gamma \frac{\delta S_P}{\delta g^{\mu\nu}} = -\frac{c^3}{16\pi G} (R^{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) + \gamma \sqrt{\hbar} \sqrt{\frac{c^3}{16\pi G}} \left( \frac{1}{2} \frac{1}{I_1 + (1 - \epsilon)I_2} \left( \frac{\delta I_1}{\delta g^{\mu\nu}} + (1 - \epsilon) \frac{\delta I_2}{\delta g^{\mu\nu}} \right) - \frac{1}{2} \frac{1}{I_1 + (1 - \epsilon)I_2} g_{\mu\nu} \right) = \frac{c^3}{16\pi G} (R^{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) + \gamma \sqrt{\hbar} \sqrt{\frac{c^3}{16\pi G}} \Lambda_{\mu\nu} = 0,
\]  

(2.15)

where \( \Lambda_{\mu\nu} \) is a new “energy-momentum” like term induced by the perimeter perturbation. We will search the solution of these equations in the following standard spherically symmetric form \( \text{[6]} \):

\[
ds^2 = e^{\nu(r)} c^2 dt^2 - e^{\lambda(r)} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\]  

(2.17)

At the zero order in \( \gamma \) the equations are:

\[
R_0^0 - \frac{1}{2} R g_0^0 = \frac{e^{-\lambda(r)}(r \lambda'(r) + \epsilon \lambda(r) - 1)}{r^2} = 0,
\]

\[
R_1^1 - \frac{1}{2} R g_1^1 = \frac{e^{-\lambda(r)}(-r \nu'(r) + \epsilon \lambda(r) - 1)}{r^2} = 0,
\]  

(2.18)
and the solution representing the Schwarzschild metric (2.10) is:

\[ \nu_0(r) = \log(1 - \frac{r_g}{r}), \quad \lambda_0(r) = -\log(1 - \frac{r_g}{r}), \] (2.19)

In order to solve the equations in the first order in \( \gamma \) we will represent the matrix \( g_{\mu\nu} \) in the form:

\[ g_{\mu\nu} = g_0^{\mu\nu} + \gamma g_1^{\mu\nu}, \quad \nu \to \nu_0 + \gamma \nu_1, \quad \lambda = \lambda_0 + \gamma \lambda_1, \] (2.20)

where \( g_0^{\mu\nu} \) is the Schwarzschild solution. The expansion of the equations has the form:

\[ \frac{\delta S_A[g_0 + \gamma g_1]}{\delta g^{\mu\nu}} + \gamma \frac{\delta S_P[g_0 + \gamma g_1]}{\delta g^{\mu\nu}} = \frac{\delta^2 S_A[g_0]}{\delta g^{\mu\nu} \delta g^{\lambda\rho}} g_1^{\lambda\rho} + \gamma \frac{\delta^2 S_P[g_0]}{\delta g^{\mu\nu}} + \mathcal{O}(\gamma^2) + ... = 0, \] (2.21)

and in the first order the equation is:

\[ \frac{\delta^2 S_A[g_0]}{\delta g^{\mu\nu} \delta g^{\lambda\rho}} g_1^{\lambda\rho} + \frac{\delta S_P[g_0]}{\delta g^{\mu\nu}} = 0. \] (2.22)

The equation is linear in \( g_1^{\lambda\rho} \) and requires the calculation of the second and first variational derivatives of the actions \( S_A \) and \( S_P \). The second variation of the \( S_A \) on the Schwarzschild solution (2.19) gives:

\[ -\frac{c^3}{16\pi G} \frac{e^{-\lambda}(1 - r\lambda_1') \lambda_1 + r\lambda_1'}{r^2} = -\frac{c^3}{16\pi G} \frac{1}{r} \left( \frac{1 - r_g}{r} \right) \lambda_1' + \frac{1}{r^2} \lambda_1, \]

\[ -\frac{c^3}{16\pi G} \frac{e^{-\lambda}(1 + r
u_1') \lambda_1 - r\nu_1'}{r^2} = -\frac{c^3}{16\pi G} \left[ -\frac{1}{r} \left( \frac{1 - r_g}{r} \right) \nu_1' + \frac{1}{r^2} \lambda_1 \right]. \] (2.23)

The first variation of the perimeter action \( S_P \) on the Schwarzschild solution takes the form:

\[ \gamma \sqrt{\frac{\hbar c^3}{16\pi G}} \left( a_1 \frac{1}{r^3} - a_2 \frac{r_g}{r^4} + a_3 \frac{r_g^2}{r^5} \right) \frac{1}{\sqrt{1 - \frac{r_g}{r}}}, \]

\[ \gamma \sqrt{\frac{\hbar c^3}{16\pi G}} \left( b_1 \frac{1}{r^3} - b_2 \frac{r_g}{r^4} + b_3 \frac{r_g^2}{r^5} \right) \frac{1}{\sqrt{1 - \frac{r_g}{r}}}, \] (2.24)

where the coefficients \( a_1, a_2, a_3, b_1, b_2, b_3 \) are given in the Appendix. By inserting these expressions into the equation (2.22) we will get:

\[ \frac{1}{r} \left( 1 - \frac{r_g}{r} \right) \lambda_1' + \frac{1}{r^2} \lambda_1 = \gamma \sqrt{\frac{\hbar c^3}{16\pi G}} \left( a_1 \frac{1}{r^3} - a_2 \frac{r_g}{r^4} + a_3 \frac{r_g^2}{r^5} \right) \frac{1}{\sqrt{1 - \frac{r_g}{r}}}, \]

\[ -\frac{1}{r} \left( 1 - \frac{r_g}{r} \right) \nu_1' + \frac{1}{r^2} \lambda_1 = \gamma \sqrt{\frac{\hbar c^3}{16\pi G}} \left( b_1 \frac{1}{r^3} - b_2 \frac{r_g}{r^4} + b_3 \frac{r_g^2}{r^5} \right) \frac{1}{\sqrt{1 - \frac{r_g}{r}}}. \] (2.25)
where

\[ l_P = \sqrt{\frac{\hbar 16\pi G}{c^3}} \]  \hspace{1cm} (2.26)

is the Planck length. Multiplying the equations by \( r^2 \) yields:

\[
(r - r_g)\lambda_1' + \lambda_1 = \gamma l_P \left( a_1 \frac{1}{r} - a_2 \frac{r_g}{r^2} + a_3 \frac{r_g^2}{r^3} \right) \frac{1}{\sqrt{1 - \varepsilon^2}};
\]

\[-(r - r_g)\nu_1' + \lambda_1 = \gamma l_P \left( b_1 \frac{1}{r} - b_2 \frac{r_g}{r^2} + b_3 \frac{r_g^2}{r^3} \right) \frac{1}{\sqrt{1 - \varepsilon^2}}. \]  \hspace{1cm} (2.27)

The first equation can be integrated:

\[
(r - r_g)\lambda_1 = \gamma l_P \int dr \left( a_1 \frac{1}{r} - a_2 \frac{r_g}{r^2} + a_3 \frac{r_g^2}{r^3} \right) \frac{1}{\sqrt{1 - \varepsilon^2}} \]  \hspace{1cm} (2.28)

and gives:

\[
\lambda_1 = \gamma \frac{l_P}{r - r_g} \left[ a_1 \log \frac{1 + \sqrt{1 - \varepsilon^2}}{1 - \sqrt{1 - \varepsilon^2}} + 2 a_3 - a_2 \varepsilon \right] + \frac{2a_3 - a_2 \varepsilon}{\varepsilon^2} \sqrt{1 - \varepsilon} \frac{r_g}{r} - \frac{2a_3 - a_2 \varepsilon}{3\varepsilon^2} (1 - \varepsilon^3) + \text{Const.} \]  \hspace{1cm} (2.29)

\[
\equiv \gamma \frac{l_P}{r - r_g} \left[ f \left( \frac{r_g}{r} \right) + \text{Const.} \right]
\]

At \( r \to \infty \) \( \lambda_1 \) has the following asymptotic:

\[
\lambda_1 \approx \frac{\gamma l_P}{r} \left[ a_1 \log \frac{4r}{\varepsilon r_g} - \frac{2a_2}{\varepsilon} + \frac{4a_3}{3\varepsilon^2} \right] + \mathcal{O}\left( \frac{1}{r^2} \right), \]  \hspace{1cm} (2.30)

and at the horizon \( r \to r_g \) it has the form:

\[
\lambda_1 \approx \frac{\gamma l_P}{r - r_g} \left[ a_1 \log \frac{1 + \sqrt{1 - \varepsilon}}{1 - \sqrt{1 - \varepsilon}} - 2 a_3 - a_2 \varepsilon \right] \frac{\sqrt{1 - \varepsilon}}{\varepsilon^2} + \frac{2a_3}{3\varepsilon^2} (1 - \varepsilon^3) + \mathcal{O}(r - r_g) \]  \hspace{1cm} (2.31)

In order to get a standard behaviour of the solution near the horizon and at infinity one should subtract the last term appropriately choosing the integration constant in (2.29)\(^8\). The \( g_{11} \) component of the metric now becomes equal to the following expression:

\[
g_{11} = e^{\lambda_0 + \lambda_1} = \frac{-1}{1 - \frac{r_g}{r}} e^{\lambda_1}, \]  \hspace{1cm} (2.32)

where

\[
\lambda_1 = \gamma \frac{l_P}{r - r_g} \left[ f \left( \frac{r_g}{r} \right) - f \left( 1 \right) \right]. \]  \hspace{1cm} (2.33)

\(^8\)I would like to thank Konstantin for suggesting the above boundary condition.
With this choice of the integration constant we will get the following leading behaviour of the metric $g_{11}$ at infinity:

$$g_{11} \simeq \frac{-1}{1 - \frac{r_g}{r}} \exp \left[ \gamma \frac{l_P}{r} \left( a_1 \log \frac{r}{r_g} + a \right) \right], \quad (2.34)$$

where the coefficients are $a_1 = \frac{82}{5\sqrt{\pi}} + \mathcal{O}(\varepsilon)$, $a = -\frac{33}{\sqrt{\pi}} + \mathcal{O}(\varepsilon)$. The behaviour near the horizon is:

$$g_{11} \simeq \frac{-1}{1 - \frac{r_g}{r}} \exp \left[ \gamma \frac{l_P}{r} b \right], \quad (2.35)$$

where $b = (a_1 - a_2 + a_3)/\sqrt{1 - \varepsilon} = \frac{11}{10\sqrt{\pi}} + \mathcal{O}(\varepsilon)$. In order to find the time component of the metric $g_{00}$ we subtract the second equation from the first one in (2.27):

$$(r - r_g)(\lambda_1' + \nu_1') = \gamma l_P \left( \frac{1}{r} - \frac{r_g}{r^2} + c_3 \frac{r_g^2}{r^3} \right) \frac{1}{\sqrt{1 - \varepsilon r_g^2}}, \quad (2.36)$$

where the coefficients $c_1, c_2, c_3$ are given in Appendix. Thus

$$\nu_1 + \lambda_1 = \gamma l_P \int \left( \frac{1}{r} - \frac{r_g}{r^2} + c_3 \frac{r_g^2}{r^3} \right) \frac{dr}{(r - r_g)\sqrt{1 - \varepsilon r_g^2}}, \quad (2.37)$$

and the integration gives

$$\nu_1 + \lambda_1 = \gamma l_P \frac{1}{r_g} \left[ \frac{2 c_1 \varepsilon - c_3}{\varepsilon^2} \sqrt{1 - \frac{r_g^2}{r}} + \frac{2 c_3}{3 \varepsilon^2} \left( 1 - \frac{r_g^2}{r} \right)^{3/2} + \text{Const} \right] \equiv \gamma l_P \left[ \frac{r_g}{r} \right] + \text{Const}. \quad (2.38)$$

where we used the important relation $c_1 - c_2 + c_3 = 0$ that eliminates a logarithmic singularity in $\nu_1$ (see Appendix). The $\lambda_1$ is given in (2.33). We should choose the integration constant equal to the asymptotic value of the integral at the infinity:

$$\nu_1 + \lambda_1 \simeq \gamma \frac{l_P}{r_g} \left[ \frac{2 c_1 \varepsilon - c_3}{\varepsilon^2} + \frac{2 c_3}{3 \varepsilon^2} \right].$$

Thus the time component of the metric is equal to the following expression:

$$g_{00} = e^{\nu_1 + \nu_1} = (1 - \frac{r_g}{r})e^{\nu_1}, \quad (2.39)$$

$$\nu_1 = -\gamma \frac{l_P}{r - r_g} \left[ f \left( \frac{r_g}{r} \right) - f(1) \right] + \gamma \frac{l_P}{r_g} \left[ g \left( \frac{r_g}{r} \right) - g \left( 0 \right) \right].$$

With this choice of the integration constant we will get the following leading behaviour of the metric $g_{00}$ at infinity:

$$g_{00} \simeq (1 - \frac{r_g}{r}) \exp \left[ -\gamma \frac{l_P}{r} \left( a_1 \log \frac{r}{r_g} + a + c_1 \right) \right], \quad (2.40)$$
Figure 2: The graph of the real and imaginary parts of the amplitude in gravity with perimeter action (2.9) $\Delta K = e^{i\gamma \sqrt{\frac{c_5}{\pi}} \sqrt{1-\epsilon} \frac{r_g}{r} \frac{\Delta r}{\Delta t}}$, here $\epsilon = 0.1$ and $\gamma \sqrt{\frac{c_5}{\hbar^2}} \frac{3 \Delta r}{r_g} \Delta t = 0.01$.

where $c_1 = \frac{41}{3\sqrt{\pi}} + \mathcal{O}(\varepsilon)$ and near the horizon as

$$g_{00} \simeq (1 - \frac{r_g}{r}) \exp \left[ - \gamma \frac{l_p}{r_g} (b + d) \right],$$

(2.41)

where $d = -\frac{2}{3\sqrt{\pi}} + \mathcal{O}(\varepsilon)$.

In summary, the spherically symmetric metric (2.17) is given by the formulas (2.32) and (2.39). The asymptotics of the metric components at infinity are given by (2.34) and (2.40), and the asymptotics of the metric components near the horizon are given in (2.35) and (2.41). From the obtained solution it is clearly seen that the characteristic behaviour of the corrections to the Schwarzschild solution is defined by the irrational functions of the form (2.29) and (2.38)

$$\sum_{n=0}^{\infty} \alpha_n (1 - \varepsilon \frac{r_g}{r})^{n/2}$$

(2.42)

that are developing the imaginary parts when $r < \varepsilon r_g$ and generate a something like a firewall [51] prohibiting particles entering the singularity. Let us compare this behaviour with the behaviour of the amplitude (3.56) for a relativistic particle $\Delta K = e^{i\frac{\pi}{m c^2} \sqrt{1-\frac{v^2}{c^2}}} \Delta t$. For the velocities larger than the velocity of light the amplitude is exponentially decreasing and the propagation of a particle outside of the light-cone is suppressed as one can see in Fig.1. The evaluation of the path integral leads to the Feynman propagator for relativistic scalar particle [48, 49]. In the case of gravity with perimeter action the same phenomena appears in the region $r < \varepsilon r_g$ where the action (2.9), (2.12) is developing an imaginary part shown in Fig.2 and the expectation value of any observable $\langle \mathcal{O} \rangle$ in the region $r < \varepsilon r_g$ is exponentially suppressed.
Let us also consider the behaviour of the solution at infinity and near the horizon. As we saw, the perturbation (2.9) generates a deformation to the distance invariant $ds$ (2.10) and allows to calculate the correction to the temporal component of the metric tensor caused by the additional term in the action at infinity and near the horizon. From (2.40) we will get the behaviour of the temporal component of the metric at large distances $r \gg r_g$:

$$g_{00} = 1 + 2\frac{\phi}{c^2} \approx 1 - \frac{r_g}{r} - \gamma \frac{l_P}{r} \left( a_1 \log \frac{r}{r_g} + a + c_1 \right),$$

(2.43)

with the additional logarithmic correction to the gravitational potential. The parameters appearing in this expression are:

$$l_P = \sqrt{\frac{\hbar}{16\pi G c^3}} \approx 11.3 \times 10^{-33} \text{cm}, \quad a_1 = \frac{82}{5\sqrt{\pi}} + \mathcal{O}(\varepsilon), \quad a = -\frac{33}{\sqrt{\pi}} + \mathcal{O}(\varepsilon), \quad c_1 = \frac{41}{3\sqrt{\pi}} + \mathcal{O}(\varepsilon).$$

The additional attractive term in the potential $\phi$ is logarithmically increasing with distance. The correction $\gamma \frac{l_P}{r} \log \frac{r}{r_g}$ is tiny because the action (1.7) contains the Planck constant $\hbar$ in front of the action and the mass parameter is proportional to the Planck mass (1.6). For the most astrophysical bodies the ratio $l_P/r_g \ll 1$ is very small. The potential of Sun on the Earth orbit will receive the following correction: $g_{00}(M_\odot) \approx 1 - 10^{-8} - \gamma \times 10^{-43}$, and the potential generated by Milky Way on the Sun orbit will be of the order $g_{00}(M_{MW}) \approx 1 - 10^{-5} - \gamma \times 10^{-54}$, where $\gamma$ is the coupling constant in (2.9). The gravitational time dilation near a massive body also receives a tiny correction, and therefore $d\tau \leq dt$ as in the standard gravity. The advance precession of the perihelion $\delta \phi$ expressed in radians per revolution is

$$\delta \phi = \frac{3\pi m^2 c^2 r^2}{2L^2} \left( 1 + 2\gamma \frac{l_P}{r_g} (8a_1 + a + c_1) \right) = \frac{6\pi GM}{c^2 l (1 - e^2)} \left( 1 + 2\gamma \frac{l_P}{r_g} (8a_1 + a + c_1) \right),$$

(2.44)

where $l$ is the semi-major axis and $e$ is the orbital eccentricity. The precession is advanced by the additional factor $\gamma \frac{l_P}{r_g}$ is tiny compared with the experimental uncertainty in the observational data for the advanced precession of the Mercury perihelion, which is $42.98 \pm 0.04$ seconds of arc per century. From the equation for light-like geodesics and from (2.41), (2.35) it follows that the horizon remains undisturbed and is at $r = r_g$.

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\footnote{One can consider a larger mass parameter $M$ in (1.5) in order to accommodate a flat rotation curve of spiral galaxies. The logarithmically growing potential in (2.43) will increase the rotation velocities.}
3 Appendix

The general expression for the perimeter action has the form:

\[ S_P = \sqrt{\hbar} \sqrt{\frac{c^3}{16\pi G}} \int \sqrt{\sum_{i=1}^{3} \eta_i K_i + \sum_{i=1}^{4} \chi_i J_i + \sum_{i=1}^{9} \gamma_i I_i} \sqrt{-g} d^4 x, \]  

(3.45)

where the curvature invariants have the form

\[ I_0 = \frac{1}{12} R_{\mu\nu\lambda\rho} R^{\nu\mu\lambda\rho}, \quad I_1 = -\frac{1}{8\pi} R_{\mu\nu\lambda\rho\sigma} R^{\mu\nu\lambda\rho\sigma}, \quad I_2 = \frac{1}{16\pi} R_{\mu\nu\lambda\rho} \Box R^{\mu\nu\lambda\rho}, \]

\[ I_3 = -\frac{1}{32\pi} \Box (R_{\mu\nu\lambda\rho} R^{\mu\nu\lambda\rho}), \quad I_4 = -\frac{1}{40\pi} R_{\mu\nu\lambda\rho\sigma} R^{\mu\nu\lambda\rho\sigma}, \quad I_5 = -\frac{1}{8\pi} (R^{\mu\nu\lambda\rho} R_{\mu\nu\lambda\rho})_{\mu;\alpha}, \]

\[ I_6 = -\frac{1}{8\pi} (R^{\alpha\nu\lambda\rho} R_{\nu\lambda\rho;\alpha})_{\mu;\mu}, \quad I_7 = \frac{1}{8\pi} R^{\alpha\nu\lambda\rho} R_{\nu\lambda\rho;\alpha\mu}, \]

\[ I_8 = \frac{1}{8\pi} R_{\nu\lambda\rho\mu} R^{\alpha\nu\lambda\rho}_{;\sigma}, \quad I_9 = \frac{1}{8\pi} R^{\alpha\nu\lambda\rho} R_{\nu\lambda\rho\mu;\alpha}, \]

\[ J_0 = R_{\mu\nu} R^{\mu\nu}, \quad J_1 = R_{\mu\nu;\lambda} R^{\mu\nu;\lambda}, \quad J_2 = R^{\mu\nu} \Box R_{\mu\nu}, \quad J_3 = \Box (R^{\mu\nu} R_{\mu\nu}), \quad J_4 = R_{\mu\nu;\alpha} R^{\nu\mu;\alpha}, \]

\[ J_5 = R^{\mu\nu} \Box R_{\mu\nu}, \quad J_6 = \Box R^{\mu\nu}, \quad J_7 = \Box R, \quad J_8 = R^{\mu\nu} \Box R_{\mu\nu}. \]

The integrand of the perimeter action (1.7) on the Schwarzschild solution has the form:

\[ \sqrt{I_1 + (1 - \epsilon)I_2|_{(\nu_0, \lambda_0)}} = \frac{3}{2\sqrt{\pi}} \sqrt{1 - \frac{r_g}{r}} \frac{r_g}{r^4}, \]  

(3.48)

and the variational derivatives of the invariants I_1 and I_2 on the Schwarzschild solution appearing in the equation (2.22) are:

\[ \frac{\delta I_1}{\delta \nu(r)} |_{g_{\mu\nu}} = \frac{3r_g \left(24r_g^2 - 77rr_g + 54r_g^2\right)}{10\pi r^9}, \quad \frac{\delta I_1}{\delta \lambda(r)} |_{g_{\mu\nu}} = \frac{3r_g \left(8r_g^2 - 39rr_g + 33r_g^2\right)}{20\pi r^9}, \]

\[ \frac{\delta I_2}{\delta \nu(r)} |_{g_{\mu\nu}} = \frac{3r_g \left(56r_g^2 - 175rr_g + 120r_g^2\right)}{4\pi r^9}, \quad \frac{\delta I_2}{\delta \lambda(r)} |_{g_{\mu\nu}} = \frac{r_g \left(28r_g^2 - 118rr_g + 93r_g^2\right)}{4\pi r^9}. \]

The above expressions allow to calculate the coefficients appearing in (2.24)

\[ a_1 = \frac{328 - 280\varepsilon}{20\sqrt{\pi}}, \quad a_2 = \frac{1014 - 875\varepsilon}{20\sqrt{\pi}}, \quad a_3 = \frac{708 - 615\varepsilon}{20\sqrt{\pi}}, \]

\[ b_1 = \sqrt{\frac{82 - 70\varepsilon}{30\sqrt{\pi}}}, \quad b_2 = \frac{331 - 295\varepsilon}{30\sqrt{\pi}}, \quad b_3 = \frac{282 - 255\varepsilon}{30\sqrt{\pi}}. \]  

(3.49)
The integration of the equation (2.37) defining the $\nu_1$ function gives:

$$\nu_1 = -\lambda_1 + \gamma \int_P \left[ -2 \frac{(c_1 - c_2 + c_3)}{\sqrt{1 - \varepsilon}} \log \frac{\sqrt{1 - \varepsilon} + \sqrt{1 - \varepsilon \frac{r_g}{r}}}{\sqrt{1 - \varepsilon} - \sqrt{1 - \varepsilon \frac{r_g}{r}}} + 4 \frac{c_2 \varepsilon - c_3 \varepsilon - c_3}{\varepsilon^2} \frac{2 c_3}{3 \varepsilon^2} (1 - \varepsilon \frac{r_g}{r})^{3/2} \right],$$

(3.50)

where the coefficients $c_1, c_2, c_3$ are:

$$c_1 = a_1 - b_1 = \frac{41 - 35 \varepsilon}{3 \sqrt{\pi}}, \quad c_2 = a_2 - b_2 = \frac{476 - 407 \varepsilon}{12 \sqrt{\pi}}, \quad c_3 = a_3 - b_3 = \frac{104 - 89 \varepsilon}{4 \sqrt{\pi}}, \quad c_1 - c_2 + c_3 = 0.$$

(3.51)

The last relation allows to eliminate the singular logarithmic term in (3.50) and equation reduces to the (2.38). The coefficient $a$ in the formula (2.34) was obtained in the expansion:

$$\frac{4 a_3}{3 \varepsilon^2} - \frac{2 a_2}{\varepsilon} - a_1 \log \frac{1 + \sqrt{1 - \varepsilon}}{1 - \sqrt{1 - \varepsilon}} - \frac{2 a_3 - 2 a_2 \varepsilon}{\varepsilon^2} \sqrt{1 - \varepsilon} + \frac{2 a_3}{3 \varepsilon^2} (1 - \varepsilon)^{3/2} + a_1 \log \frac{4 r}{\varepsilon r_g} =$$

$$= a_1 \log \frac{r}{r_g} + a + O(\varepsilon),$$

(3.52)

where

$$a_1 = \frac{82}{5 \sqrt{\pi}} + O(\varepsilon), \quad a = \frac{a_3 - 2 a_2}{2} = \frac{-33}{\sqrt{\pi}} + O(\varepsilon)$$

(3.53)

The coefficient $d$ in (2.41) is:

$$d = 2 \frac{c_1 \varepsilon - c_3}{\varepsilon^2} (\sqrt{1 - \varepsilon} - 1) + \frac{2 c_3}{3 \varepsilon^2} ((1 - \varepsilon)^{3/2} - 1) \simeq \frac{c_3 - 2 c_1}{2} + O(\varepsilon) = \frac{-2}{3 \sqrt{\pi}} + O(\varepsilon).$$

(3.54)

The field equations (2.15) can be represented also in the standard form:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \gamma \sqrt{\hbar \frac{16 \pi G}{c^3}} A_{\mu\nu} + \frac{16 \pi G}{c^4} T_{\mu\nu},$$

(3.55)

where $A_{\mu\nu}$ is a new \textquotedblright energy-momentum\textquotedblright like term induced by the perimeter perturbation and the $T_{\mu\nu}$ is the standard energy-momentum tensor of matter.

The path integral for a relativistic particle amplitude can conveniently be represented in the form:

$$K(t_b, x_b; t_a, x_a) = \int_{x_a}^{x_b} e^{ \int_{t_a}^{t_b} \frac{mc^2}{\varepsilon} \sqrt{1 - \frac{\varepsilon^2}{c^2}} dt} Dx(t).$$

(3.56)

4 Note Added

In order to investigate the perturbation of the Schwarzschild solution induced by the perimeter action in its general form (3.45) one should calculate the variational derivatives of the
where these invariants on the Schwarzschild solution are:

\[ I_3 = 5I_1 - I_2, \quad I_6 = I_5 = 5I_4 - I_7 - I_8 - I_9, \]  

(4.57)

therefore the independent invariants are: \( I_1, I_2, I_4, I_7, I_8, I_9 \). Now the field equation (2.22) will take the form:

\[
\frac{\delta S}{\delta g_{\mu\nu}} = \frac{\delta S_A}{\delta g_{\mu\nu}} + \gamma \frac{\delta S_P}{\delta g_{\mu\nu}} = -\frac{c^3}{16\pi G} (R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}) + \\
+\sqrt{\hbar} \sqrt{\frac{c^3}{16\pi G}} \left( \frac{1}{2} \frac{1}{\sqrt{\gamma}_I I} \frac{\delta I_j}{\delta g_{\mu\nu}} - \frac{1}{2} \sqrt{\gamma}_I I g_{\mu\nu} \right) = 0, \quad i, j = 1, 2, 4, 7, 8, 9,
\]

where the summation is over all possible invariants \( I_i \). The first variational derivatives of these invariants on the Schwarzschild solution are:

\[
\left. \frac{\delta I_4}{\delta \nu(r)} \right|_{g_{\mu\nu}^0} = \left. \frac{\delta I_1}{\delta \nu(r)} \right|_{g_{\mu\nu}^0}, \quad \left. \frac{\delta I_4}{\delta \lambda(r)} \right|_{g_{\mu\nu}^0} = \left. \frac{\delta I_1}{\delta \lambda(r)} \right|_{g_{\mu\nu}^0}, \\
\left. \frac{\delta I_7}{\delta \nu(r)} \right|_{g_{\mu\nu}^0} = \left. \frac{\delta I_2}{\delta \nu(r)} \right|_{g_{\mu\nu}^0}, \quad \left. \frac{\delta I_7}{\delta \lambda(r)} \right|_{g_{\mu\nu}^0} = \left. \frac{\delta I_2}{\delta \lambda(r)} \right|_{g_{\mu\nu}^0}, \\
\left. \frac{\delta I_8}{\delta \nu(r)} \right|_{g_{\mu\nu}^0} = 0, \quad \left. \frac{\delta I_8}{\delta \lambda(r)} \right|_{g_{\mu\nu}^0} = 0, \\
\left. \frac{\delta I_9}{\delta \nu(r)} \right|_{g_{\mu\nu}^0} = \frac{3\pi g (112r^2 - 301r^3g + 192r^2)}{8\pi r^9}, \quad \left. \frac{\delta I_9}{\delta \lambda(r)} \right|_{g_{\mu\nu}^0} = \frac{g (56r^2 - 143r^2g + 96r^2)}{8\pi r^9}.
\]

The invariants \( J_i \) and \( K_i \) do not contribute. Taking into account the relations (3.47) and (4.59) one can represent the second term of the perturbation equation (2.22) on the Schwarzschild solution in the form:

\[
\gamma \sqrt{\hbar} \sqrt{\frac{c^3}{16\pi G}} \left( \frac{1}{2} \frac{1}{\sqrt{\gamma}_I I} \delta I_2 \left( \frac{\delta I_1}{\delta g_{\mu\nu}} + (1 - \varepsilon) \frac{\delta I_2}{\delta g_{\mu\nu}} + \eta \frac{\delta I_9}{\delta g_{\mu\nu}} \right) - \frac{1}{2} \frac{1}{\sqrt{\gamma}_I I} \delta I_2 g_{\mu\nu} \right),
\]

where

\[
\gamma = \sqrt{\gamma_1 + \gamma_4}, \quad 1 - \varepsilon = \frac{\gamma_2 + \gamma_7}{\gamma_1 + \gamma_4}, \quad \eta = \frac{\gamma_6}{\gamma_1 + \gamma_4}, \quad \gamma_1 + \gamma_4 > 0.
\]

(4.60)

Thus compared to the perimeter perturbation that we considered in the main part of the article we have here an additional perturbation term with the coupling constant \( \eta \). As one can see from the last relation in (4.59), this term has the same dependence on the radial coordinate \( r \) as the terms associated with \( I_1, I_2 \), therefore the consideration of the general perimeter perturbation reduces to the following redefinition of the coefficients \( a_i, b_i \) and \( c_i \):

\[
a_1 = \frac{328 - 280\varepsilon + 280\eta}{20\sqrt{\pi}}, \quad a_2 = \frac{1014 - 875\varepsilon + 1505\eta/2}{20\sqrt{\pi}}, \quad a_3 = \frac{708 - 615\varepsilon + 480\eta}{20\sqrt{\pi}}, \\
b_1 = \frac{82 - 70\varepsilon + 70\eta}{30\sqrt{\pi}}, \quad b_2 = \frac{331 - 295\varepsilon + 715\eta/4}{30\sqrt{\pi}}, \quad b_3 = \frac{282 - 255\varepsilon + 120\eta}{30\sqrt{\pi}}.
\]

(4.61)
Figure 3: The Penrose-Carter diagram of the Schwarzschild space-time [53]. The horizontal lines \( r = 0 \) are the future and past singularities. The diagonal lines bounding the diagram on the right-hand side are past and future null infinity of asymptotically flat space. The diagonals are the past and future horizons. The propagator (4.62) represents the propagation from point \( x \) on the surface \( C_+ \) of \( r = \text{constant} < r_g \) in the future horizon \( II \) to the point \( x' \) on the world line of the observer \( O \) on the surface \( r' = \text{constant} > r_g \). The integration over time coordinate can be shifted by the amount \(-2\pi i r_g\) in the complex \( t \) plane (4.64) and corresponds to the reflection of the point \( x \) in the origin to the point \( x'' \) in the region \( III \) on the surface \( C_- \) of \( r'' = \text{constant} < r_g \). The resulting amplitude describes the propagation to the point \( x' = (0, \vec{r}') \) outside the black hole from the surface \( C_- \) in the past horizon region \( III \). The time reversal transformation of the amplitude (4.64) represents the propagation of massless particle from the region \( I \) into the interior region \( II \) inside the black hole.

Considering the massless scalar particles in the perturbed Schwarzschild background we can ask: What is the amplitude of finding the particles at location \( x \) inside the future horizon? To describe this situation we shall follow the Hartle and Hawking path integral derivation of the amplitude for a particle that propagates in the Schwarzschild space-time background [52, 53, 54, 55]. Let us consider the amplitude for a massless scalar particle of defined energy \( E \) to propagate from the space-time point \( x^\mu = (t, \vec{r}) \) inside the region \( II \) of the future horizon to the point \( x'^\mu = (t', \vec{r}') \) outside the black hole in the region \( I \) shown in Fig.3:

\[
K_E(\vec{r}', \vec{r}) = \int_{-\infty}^{+\infty} dt e^{-iEt} K(0, \vec{r}; t, \vec{r}),
\tag{4.62}
\]

where the integration is over time \( t \) inside the future horizon and \( \vec{r} \) is on a surface \( C_+ \) of \( r = \text{constant} < r_g \) [53]. Because the propagator is symmetric with respect to the coordinate exchange \( K(x', x) = K(x, x') \) the amplitude can be represented in the form [53]:

\[
K_E(\vec{r}, \vec{r}') = \int_{-\infty}^{+\infty} dt e^{-iEt} K(t, \vec{r}; 0, \vec{r}').
\tag{4.63}
\]

The propagator \( K(t, \vec{r}; 0, \vec{r}') \) is analytic in the coordinate \( t \) in the strips of the width \( 2\pi r_g \) except for the singularities that are below the real \( t \) axis and correspond to propagation along the future-directed null geodesics, while those corresponding to propagation along the
past-directed null geodesics lie above the real $t$ axis \[53\]. By distorting the contour of the $t$ integration downward by amount $-2\pi i r_g$ in the complex $t$ plane, one can represent the amplitude in the form \[53\]:

$$K_E(\vec{r}', \vec{r}) = \int_{-\infty}^{+\infty} dt e^{-iE(t-2\pi i r_g)} K(t-2\pi i r_g, r_i; 0, r_i'). \tag{4.64}$$

Since the displacement in $t = \tau + i\sigma$ by $\sigma \rightarrow -\sigma - 2\pi i r_g$ is equivalent to the reflection $u \rightarrow -u$ and $v \rightarrow -v$ of the Kruskal coordinates ($u = |u| e^{-i\sigma/2\pi r_g}$ and $v = |v| e^{i\sigma/2\pi r_g}$) the last integral can be interpreted as the amplitude to propagate to the point $x' = (0, \vec{r}')$ outside the black hole, but now from the surface $C_-$ in the past horizon III that is the reflection of the $C_+$ surface in the origin of the Kruskal $(u, v)$ coordinates defined above. Therefore the last amplitude may be written as \[53\]:

$$K_E(\vec{r}, \vec{r'}) = e^{-2\pi E r_g} \int_{-\infty}^{+\infty} dt e^{-iEt} K(t, \vec{r}; 0, \vec{r}') \tag{4.65},$$

where now the integration is over time $t$ inside the past horizon III and the $\vec{r}$ is on the reflected surface $C_-$ of $r = constant < r_g \[53\]$. It represents the amplitude to propagate to the point $x' = (0, \vec{r}')$ outside the black hole from the surface $C_-$ in the past horizon region III of the Penrose-Carter diagram Fig.3. By time-reversal invariance this amplitude is exactly equal to the amplitude for a particle that starts at $x' = (0, \vec{r}')$ and arrives at $C_-$ inside the black hole. This is exactly the amplitude of finding the particles at the location $x$ inside the future horizon.

Now we are able to calculate the amplitude of a particle to reach the region $r < \varepsilon r_g$ inside the future horizon under the perimeter perturbation of the Schwarzschild space-time. This amplitude is represented by the path integral

$$A_E(r < \varepsilon r_g) = \int_{r < \varepsilon r_g} d^3 \vec{r} \int_{x'} e^{i(S[g]+S[g,x])} Dg Dx \simeq \int_{r < \varepsilon r_g} d^3 \vec{r} \int_{x'} e^{iS[g_0+\gamma g_1]+S[g_0,x]} Dg Dx =$$

$$= \int_{r < \varepsilon r_g} d^3 \vec{r} e^{i S_p} K_E(\vec{r}, \vec{r'}) = \int_{r < \varepsilon r_g} d^3 \vec{r} e^{-\gamma \sqrt{\frac{\varepsilon c}{\pi \tau}} (\frac{\varepsilon c}{r})^3/2} \Delta t K_E(\vec{r}, \vec{r'}) \tag{4.66}$$

and due to the exponential factor $\exp \left( -C \left( \frac{\varepsilon}{\tau} \right)^3 \right)$ the amplitude tends to zero when a massless scalar particle approaches the singularity $r = 0$. The $S_p$ is given in (2.12) and (2.14). The Feynman propagators are known for the photons and fermions \[53\] and for the different choices of boundary conditions \[56\]. These results allow to calculate the corresponding amplitudes and to conclude that the same phenomenon will take place in the case of other elementary particles as well.
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