Jet Tomography of Hot and Cold Nuclear Matter

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Medium modification of parton fragmentation functions induced by multiple scattering and gluon bremsstrahlung is shown to describe the recent HERMES data in deeply inelastic scattering (DIS) very well, providing the first evidence of $A^{2/3}$-dependence of the modification. The energy loss is found to be $(dE/dL) \approx 0.5 \text{ GeV/fm}$ for a 10-GeV quark in an Au nucleus. Including the effect of expansion, analysis of the $\pi^0$ spectra in central Au + Au collisions at $\sqrt{s} = 130 \text{ GeV}$ yields an averaged energy loss equivalent to $(dE/dL) \approx 7.3 \text{ GeV/fm}$ in a static medium. Predictions for central Au + Au collisions at $\sqrt{s} = 200 \text{ GeV}$ are also given.

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Energetic partons produced via hard processes provide an excellent tool that enables topomographic studies of both hot dense and cold nuclear matter. By measuring the attenuation [1–5] of these partons as they propagate through the medium, one would be able to study the properties such as the geometry [6,7] and the gluon density of the medium. The attenuation will suppress the final leading hadron distribution giving rise to modified parton fragmentation functions [8]. Such a modified fragmentation function inside a nucleus has been derived recently [9] in a pQCD approach with a systematic expansion of higher-twist corrections to the fragmentation processes. In this letter, we will compare the predicted nuclear modification to the recent HERMES experimental data [10] and extract the effective parton energy loss. We will further extend the study to the case of a hot QCD medium including the dynamics of expansion. We will then analyze within this framework the $\pi^0$ spectra as measured by the PHENIX experiment [11] in central Au + Au collisions at $\sqrt{s} = 130 \text{ GeV}$, which have shown significant suppression at large transverse momentum. The extracted effective parton energy loss will be compared with that in a cold nucleus, and discussions will be given about the implications of the PHENIX data on the gluon density in the early stage of central Au + Au collisions at the RHIC energy. We will also provide predictions for $\pi^0$ spectra in central Au + Au collisions at $\sqrt{s} = 200 \text{ GeV}$.

In deeply inelastic scatterings (DIS), quark fragmentation functions are factorizable in leading twist from the parton distribution functions and photon-quark ($\gamma^*q$) scattering cross section. In a nucleus target, the quark struck by the virtual photon suffers multiple scattering and induced bremsstrahlung before hadronization. Extending the generalized factorization [12] to the semi-inclusive process, $\ell(L_1) + A(p) \rightarrow \ell(L_2) + h(t_h) + X$, one can define a modified fragmentation function as [9],

$$E_{L_2} \frac{d\sigma_{\text{DIS}}}{d^3L_2dz_h} = \frac{\alpha^2_{\text{EM}}}{2\pi s} \frac{1}{Q^2} L_{\mu\nu} \times \sum_q \int dx f_q^A(x,Q^2) H_{\mu\nu}^{(0)}(x,p,q) D_q \rightarrow h(z_h,Q^2),$$  \hspace{1cm} (1)

where $f_q^A(x,Q^2)$ is the quark distribution function in the nucleus, $s = (p + L_1)^2$ and $p = [p^+,0,\vec{0}_L]$ is the momentum per nucleon inside the nucleus. The momentum of the virtual photon $\gamma^*$ is $q = [-Q^2/2q^-,q^-,\vec{0}_L]$ and the momentum fraction carried by the hadron is $z_h = \ell_h^+/q^-$.

The hard part of the $\gamma^* q$ scattering $H_{\mu\nu}^{(0)}$ is the same as in $ep$ scattering [9] and $L_{\mu\nu} = \frac{1}{2} \text{Tr}(\vec{\gamma}_{\mu} \vec{E} \gamma_{\nu})$.

Including the leading twist-4 contributions from double scattering processes, the modified effective quark fragmentation function can be obtained as [9]

$$D_q \rightarrow h(z_h,Q^2) = D_{q \rightarrow h}(z_h,Q^2) + \Delta D_{q \rightarrow h}(z_h,Q^2);$$  \hspace{1cm} (2)

$$\Delta D_{q \rightarrow h}(z_h,Q^2) = \int_0^{Q^2} \frac{d\ell_2^+}{2\pi} \alpha_s \int_0^{\ell_2^+} \frac{dz}{2\pi} \left[ \Delta \gamma_{q \rightarrow gg}(z,x,x_L,\ell_2^+) D_{q \rightarrow h}(z_h/z) + \Delta \gamma_{q \rightarrow gg}(z,x,x_L,\ell_2^+) D_{g \rightarrow h}(z_h/z) \right];$$  \hspace{1cm} (3)

$$\Delta \gamma_{q \rightarrow gg}(z,x,x_L,\ell_2^+) = \frac{1 + z^2}{(1 - z)^2} T_{gg}^A(x,x_L) + \delta(1 - z) T_{gg}^A(x,\ell_2^+),$$  \hspace{1cm} (4)

where, $C_A = 3$, $N_c = 3$, $\Delta \gamma_{q \rightarrow gg}(z,x,x_L,\ell_2^+) = \Delta \gamma_{q \rightarrow gg}(1 - z,x,x_L,\ell_2^+)$ are the modified splitting functions, $x_L = \ell_2^+/2p^+q(z(1-z))$, and $D_{q \rightarrow h}(z_h,Q^2)$ are the normal twist-2 parton fragmentation functions in vacuum. The $\delta$-function part in the modified splitting function is from the virtual-2 parton fragmentation functions in vacuum, with $\Delta T_{gg}^A(x,\ell_2^+)$ defined as
\[
\Delta T^A_{Qg}(x, \ell_T^2) = \int_0^1 \frac{dz}{1-z} \left[ 2T^A_{Qg}(x, x_L)|_{z=1} - (1+z^2)T^A_{Qg}(x, x_L) \right].
\]

Such virtual or absorptive corrections are important to ensure the infrared safety of the modified fragmentation function and the unitarity of the gluon radiation processes. The quark-gluon correlation function

\[
T^A_{Qg}(x, x_L) = \int \frac{dy^-}{2\pi} dy_1^- dy_2^- e^{i(x+x_L)p^+y^- + i x_T p^+(y_1^- - y_2^-)} (1 - e^{-ix_L p^+(y^- - y_1^-)})(1 - e^{-ix_L p^+(y^- - y_1^-)})
\]

\[
\frac{1}{2} \langle A|v_q(0) \gamma^+ F_{\sigma^+}(y_2^-) F^{\alpha^+}(y_1^-) \psi_q(y^-)|A\rangle \theta(-y_2^-) \theta(y^- - y_1^-)
\]

contains essentially four independent twist-4 parton matrix elements in a nucleus \([x_T = (k_T^2)/2p^+q^-(1-z)]\). The dipole-like form-factor \((1 - e^{-ix_L p^+(y^- - y_1^-)})(1 - e^{-ix_L p^+(y^- - y_1^-)})\) arises from the interference between the final state radiation of the \(\gamma^q\) scattering and the gluon bremsstrahlung induced by the secondary quark-gluon scattering. By generalizing the factorization assumption [12] to these twist-four parton matrices, we have

\[
T^A_{Qg}(x, x_L) \approx \widetilde{C}(Q^2)m_N R_A f_q^A(x)(1 - e^{-z^2/x_A^2}),
\]

with a Gaussian nuclear distribution \(\rho(r) \sim \exp(-r^2/2R_A^2)\), \(R_A = 1.12A^{1/3}\) fm. Here, \(x_A = 1/m_N R_A\), and \(m_N\) is the nucleon mass. We should emphasize that the parameter \(\widetilde{C}(Q^2)\) should in principle depend on the renormalization scale \(Q^2\) among other kinetic variables as shown recently in a detailed analysis of the twist-four nuclear matrix elements [13]. Therefore, it can take different values in different processes.

\[
\nu \langle GeV \rangle \quad \langle z \rangle = 0.34 - 0.42, \quad \langle Q^2 \rangle = 2.2 - 3.5 \text{ GeV}^2
\]

FIG. 1. Predicted nuclear modification of jet fragmentation function is compared to the HERMES data [10] on ratios of hadron distributions between A and D targets in DIS.

Since the two interference terms in the dipole-like form-factor involve transferring momentum \(x_L p^+\) between different nucleons inside a nucleus, they should be suppressed for large nuclear size or large momentum fraction \(x_L\). Notice that \(\tau_f = 1/x_L p^+\) is the gluon’s formation time. Thus, \(x_L/x_A = L_A/\tau_f\), with \(L_A = R_A m_N p^+\) being the nuclear size in the infinite momentum frame. The effective parton correlation and the induced gluon emission vanishes when the formation time is much larger than the nuclear size, \(x_L/x_A \ll 1\), because of the Landau-Pomeranchuk-Migdal (LPM) interference effect. Therefore, the LPM interference restricts the radiated gluon to have a minimum transverse momentum \(\ell_T^2 \sim Q^2/m_N R_A \sim Q^2/A^{1/3}\). The nuclear corrections to the fragmentation function due to double parton scattering will then be in the order of \(\alpha_s A^{1/3}/\ell_T^2 \sim \alpha_s A^{2/3}/Q^2\), which depends quadratically on the nuclear size. For large values of \(A\) and \(Q^2\), these corrections are leading; yet the requirement \(\ell_T^2 \ll Q^2\) for the logarithmic approximation in deriving the modified fragmentation function is still valid.

With the assumption of the factorized form of the twist-4 nuclear parton matrices, there is only one free parameter \(\widetilde{C}(Q^2)\) which represents quark-gluon correlation strength inside nuclei. Once it is fixed, one can predict the \(z\), energy and nuclear dependence of the medium modification of the fragmentation function. Shown in Figs. 1 and 2 are the calculated nuclear modification factor of the fragmentation functions for \(^{14}N\) and \(^{84}Kr\) targets as compared to the recent HERMES data [10]. There are strong correlations among values of \(Q^2, \nu\) and \(z\) in the HERMES data which are also taken in account in our calculation. The predicted shape of the \(z\)- and \(\nu\)-dependence agrees well with the experimental data. A remarkable feature of the prediction is the quadratic \(A^{2/3}\) nuclear size dependence, which is verified for the
first time by an experiment. This quadratic dependence comes from the combination of the QCD radiation spectrum and the modification of the available phase space in $\ell_T$ or $x_L$ due to the LPM interferences. The observed attenuation of the leading hadrons could also be attributed phenomenologically to hadron absorption inside the nucleus. This is, however, achieved only via some ad hoc assumption of the hadron formation time [10]. Considering the hadronization process as regeneration of gluon field within a spatial region of a hadron size $r_h$, the hadron formation time for a light quark will be $t_f^L \approx \nu r_h^2$ [14]. Taking $r_h \approx 1$ fm, $t_f^L$ is about 40 fm for $\nu = 8$ GeV, the lower limit of the HERMES experiment. This is much larger than the size of the heaviest nuclei available. We therefore assume that the nuclear attenuation in this energy region is mainly caused by induced gluon radiation and multiple parton scattering.

By fitting the overall suppression for one nuclear target, we obtain the only parameter in our calculation, $\bar{C}(Q^2) = 0.0060$ GeV$^2$ with $\alpha_s(Q^2) = 0.33$ at $Q^2 \approx 3$ GeV$^2$. This parameter is also related to nuclear broadening of the transverse momentum of the Drell-Yan dilepton in $pA$ collisions [15]. $\langle \Delta q^2 \rangle \approx \bar{C}\pi\alpha_s/N_{x,A}$. With an experimental value of $\langle \Delta q^2 \rangle = 0.016 A^{1/3}$ GeV$^2$ [16] and $\alpha_s(M_H^2) = 0.21$ ($< M_H^2 \approx 40$ GeV$^2$), one finds $\bar{C}(M_H^2) = 0.013$ GeV$^2$, which is about a factor 2 larger than the value obtained in our fit to the HERMES data. The value of $\bar{C}$ determined from nuclear broadening in photo-production of di-jet is even larger [12] at $Q^2 = 4p_T^2 \approx 64$ GeV$^2$. Such a strong scale dependence of $\bar{C}(Q^2)$ is in line with one’s expectation since it is related to gluon distribution $xg(x,Q^2)$ at small $x$ in nuclei [13].

If one defines theoretically the quark energy loss as that carried by the radiated gluons, then the averaged total fractional energy loss is [cf. Eq. (3)],

$$\langle \Delta z_q \rangle = \int_0^1 \frac{d^2q_T}{\ell_T^2} \int_{\ell_T} d\rho_q 1 + (1 - z)^2 \int_0^1 \frac{d\rho_q d\rho_g}{\ell_T^2 - \ell_T^2 + \langle k_T^2 \rangle} C_{Aq}^2 T_{gq}^A(x_B, x_L) \approx \bar{C}(Q^2) \alpha_s^2(Q^2) \frac{R_A}{N_c} \frac{x_B}{Q^2 x_A} 6 \ln \left( \frac{1}{2x_B} \right).$$

In the rest frame of the nucleus, $p^+ = m_N, q^- = \nu$, and $x_B = Q^2 / 2 p^+ q^- = Q^2 / 2 m_N \nu$. One can get the averaged total energy loss as $\Delta E = \nu \langle \Delta z_q \rangle \approx \bar{C}(Q^2) \alpha_s^2(Q^2) m_N R_A^2 (C_A/N_c) 3 \ln(1/2x_B)$. With the determined value of $\bar{C}$, $\langle x_B \rangle \approx 0.124$ in the HERMES experiment [10] and the average distance $\langle L_A \rangle = R_A \sqrt{2/\pi}$ for the assumed Gaussian nuclear distribution, one gets the quark energy loss $dE/dL \approx 0.5$ GeV/fm inside a Au nucleus.

To extend our study of modified fragmentation functions to jets in heavy-ion collisions, we assume $\langle k_T^2 \rangle \approx \mu^2$ (the Debye screening mass) and a gluon density profile $\rho(y) = (\tau_0/\tau) \theta(R_A - y) \rho_0$ for a 1-dimensional expanding system. Since the initial jet production rate is independent of the final gluon density which can be related to the parton-gluon scattering cross section [2] $\alpha_s x_T G(x_T) \sim \mu^2 g$, one has then

$$\frac{\alpha_s T_{gq}^A(x_B, x_L)}{f_{gq}^A(x_B)} \sim \mu^2 \int dy \rho(y)[1 - \cos(y/\tau_f)],$$

where $\tau_f = 2Ez(1 - z)/\ell_T^2$ is the gluon formation time. One can recover the form of energy loss in a thin plasma obtained in the opacity expansion approach [2,7],

$$\langle \Delta z_g \rangle = \frac{C_A \alpha_s}{\pi} \int_0^1 \frac{dz}{\ell_T^2} 1 + (1 - z)^2 \int_0^\tau u(1 + u) \times \int_0^{\tau_A} \frac{d\tau g}{\tau g} \phi(\tau) \left[ 1 - \cos \left( \frac{(\tau - \tau_0) u \mu^2}{2Ez(1 - z)} \right) \right].$$

Keeping only the dominant contribution and assuming $\sigma_g \approx C_2^A 2\pi \alpha_s^2/\mu^2$ ($C_2^A = 1$ for $gg$ and $9/4$ for $gg$ scattering), one obtains the averaged energy loss,

$$\frac{dE}{dL} \approx \frac{\pi C_A \alpha_s}{\pi} \int_0^{\tau_A} \frac{d\tau g}{\tau g} \phi(\tau) \left[ 1 - \cos \left( \frac{(\tau - \tau_0) u \mu^2}{2Ez(1 - z)} \right) \right].$$

Neglecting the logarithmic dependence on $\tau$, the averaged energy loss in a 1-dimensional expanding system can be expressed as

$$\langle dE/dL \rangle_{1d} \approx \frac{dE_0}{dL} \frac{2\tau_0}{R_A},$$

where $dE_0/dL \propto \rho_0 R_A$ is the energy loss in a static medium with the same gluon density $\rho_0$ as in a 1-d expanding system at time $\tau_0$. Because of the expansion, the averaged energy loss $(dE/dL)_{1d}$ is suppressed as compared to the static case and does not depend linearly on the system size. This could be one of the reasons why the effect of parton energy loss is found to be negligible in $AA$ collisions at $\sqrt{s} = 17.3$ GeV [20].

An effective model of modified fragmentation functions was proposed in Ref. [8]:

$$D_{a \rightarrow h}(z) \approx \frac{1}{1 - \Delta z} D_{a \rightarrow h} \left( \frac{z}{1 - \Delta z} \right),$$

with $\Delta z$ to account for the fractional parton energy loss. This effective model is found to reproduce the pQCD result from Eq. (3) very well, but only when $\Delta z$ is set to be $\Delta z \approx 0.6(z_q)$. Therefore the actual averaged parton energy loss should be $\Delta E/E = 1.62z$ with $\Delta z$ extracted from the effective model. The factor 1.6 is mainly caused by unitarity correction effect in the pQCD calculation. A similar effect is also found in the opacity expansion approach [17].

The PHENIX experiment has reported [11] a strong suppression of high $p_T$ hadrons in central $Au + Au$ collisions at $\sqrt{s} = 130$ GeV. To extract the parton energy
loss, we compare the data with the calculated hadron \( p_T \) spectra in heavy-ion collisions using the above effective model for medium modified jet fragmentation functions [18]. Shown in Fig. 3 are the nuclear modification factor \( R_{AA}(p_T) \) as the ratio of hadron spectra in \( AA \) (\( pA \)) and \( pp \) collisions normalized by the number of binary collisions [19]. Parton shadowing and nuclear broadening of the intrinsic \( k_T \) are also taken into account in the calculation which describes \( pA \) data for energies up to \( \sqrt{s} \approx 40 \text{ GeV} \) [18]. The nuclear \( k_T \)-broadening gives the so-called Cronin enhancement at large \( p_T \) in \( pA \) collisions, where there is no parton energy loss induced by a hot medium. Fitting the PHENIX data yields \( \langle dE/dL \rangle_{1d} \approx 0.34 \ln E/\ln 5 \text{ GeV/fm} \), including the factor of 1.6 from the unitarity correction effect. We consider only \( \pi^0 \) data here, since at large \( p_T \) the charged hadrons are dominated by baryons, which could be influenced mainly by non-perturbative dynamics [7].

In summary, the nuclear modification of parton fragmentation function predicted in a pQCD study describes well the HERMES experimental data. The extracted energy loss is \( dE/dL \approx 0.5 \text{ GeV/fm} \) for a quark with \( E = 10 \text{ GeV} \) in a Au nucleus. Analysis of the PHENIX data of \( \pi^0 \) spectra suppression in central \( Au + Au \) collisions yields an averaged parton energy loss in an expanding system that would be equivalent to \( (dE/dL)_0 \approx 7.3 \text{ GeV/fm} \) in a static medium.

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![FIG. 3](image_url)  
**FIG. 3.** Calculated nuclear modification factor of \( \pi^0 \) \( p_T \) spectra for \( d + Au \) and central \( Au + Au \) collisions at \( \sqrt{s} = 130 \) (solid) and 200 GeV (dashed) as compared to PHENIX data [11] (solid circles are measured spectra normalized by PHENIX parameterization of the \( pp \) spectra while open circles are normalized by our calculated \( pp \) spectra).

Taking into account the expansion, the averaged parton energy loss extracted from the PHENIX data would be equivalent to \( (dE/dL)_0 = 0.34(R_A/2\tau_0) \ln E/\ln 5 \) in a static system with the same gluon density as the initial value of the expanding system at \( \tau_0 \). With \( R_A \approx 6 \text{ fm} \) and \( \tau_0 \approx 0.2 \text{ fm} \), this would give \( (dE/dL)_0 \approx 7.3 \text{ GeV/fm} \) for a 10-GeV parton, which is about 15 times of that in a cold \( Au \) nucleus as extracted from the HERMES data. Since the parton energy loss is directly proportional to gluon density of the medium, we can predict the \( \pi^0 \) spectra at \( \sqrt{s} = 200 \text{ GeV} \) as given by the dashed lines in Fig. 3, assuming that the initial parton density in central \( Au + Au \) collisions at \( \sqrt{s} = 200 \text{ GeV} \) is about 10% higher than at 130 GeV.

In summary, the nuclear modification of parton fragmentation function predicted in a pQCD study describes well the HERMES experimental data. The extracted energy loss is \( dE/dL \approx 0.5 \text{ GeV/fm} \) for a quark with \( E = 10 \text{ GeV} \) in a Au nucleus. Analysis of the PHENIX data of \( \pi^0 \) spectra suppression in central \( Au + Au \) collisions yields an averaged parton energy loss in an expanding system that would be equivalent to \( (dE/dL)_0 \approx 7.3 \text{ GeV/fm} \) in a static medium.

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