Abstract

We pursue the study of the type IIB matrix model as a constructive definition of superstring. In this paper, we justify the interpretation of space-time as distribution of eigenvalues of the matrices by showing that some low energy excitations indeed propagate in it. In particular, we show that if the distribution consists of small clusters of size $n$, low energy theory acquires local $SU(n)$ gauge symmetry and a plaquette action for the associated gauge boson is induced, in addition to a gauge invariant kinetic term for a massless fermion in the adjoint representation of the $SU(n)$. We finally argue a possible identification of the diffeomorphism symmetry with permutation group acting on the set of eigenvalues, and show that the general covariance is realized in the low energy effective theory even though we do not have a manifest general covariance in the IIB matrix model action.
1 Introduction

Several proposals have been given as constructive definitions of superstring [1, 4, 5, 6, 7, 8, 9, 10, 11, 12]. Type IIB matrix model [1, 2, 3], a large \( N \) reduced model of maximally supersymmetric Yang-Mills theory, is one of those proposals. It is defined by the following action:

\[
S = -\frac{1}{g^2} \text{Tr}(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]),
\]

(1.1)

where \( A_\mu \) and \( \psi \) are \( N \times N \) Hermitian matrices, \( \mu = 1, \ldots, 10 \) and \( \psi \) is a ten dimensional Majorana-Weyl spinor field. It is formulated in a manifestly covariant way, which is suitable for studying nonperturbative issues of superstring theory. Also since it is a simple model of an ensemble of zero dimensional matrices, it is particularly appropriate for numerical simulations. In fact we in principle predict dimensionality of space-time, low energy gauge group and matter contents by solving this model. In this paper, we pursue this line of analysis and report some results on possible structures of the low energy effective theory and the origin of local space-time gauge symmetry and diffeomorphism invariance. These analysis gives justification of our interpretation of space-time as distributed eigenvalues of the matrices \( A_\mu \).

We first list several important properties of the IIB matrix model. This model can be regarded as a large \( N \) reduced model of ten dimensional \( \mathcal{N} = 1 \) supersymmetric \( SU(N) \) Yang-Mills theory. It was shown [13] that a large \( N \) gauge theory can be equivalently described by its reduce model, namely a model defined on a single point. In this reduction procedure space-time translation is represented in the color \( SU(N) \) space and eigenvalues of matrices are interpreted as momenta of fields. As a constructive definition of superstring, on the other hand, we will see that we need to interpret eigenvalues of matrices as coordinates of space-time points, which interpretation is T-dual to the above.

Since our IIB matrix model is defined on a single point, the commutator of the supersymmetry

\[
\begin{align*}
\delta^{(1)} A_\mu &= i\bar{\epsilon}_1 \Gamma_\mu \psi \\
\delta^{(1)} \psi &= \frac{i}{2} \Gamma^{\mu\nu}[A_\mu, A^\nu] \epsilon_1
\end{align*}
\]

(1.2)

vanishes up to a field dependent gauge transformation and we can no longer interpret this supersymmetry as space-time supersymmetry in the original
sense. However, after the reduction, we acquire an extra bosonic symmetry

\[ \delta A_\mu = c_\mu \mathbf{1}, \]

(1.3)

whose transformation is proportional to a unit matrix \( \mathbf{1} \) and an extra supersymmetry

\[
\begin{aligned}
\delta^{(2)} A_\mu &= 0 \\
\delta^{(2)} \psi &= \epsilon_2.
\end{aligned}
\]

(1.4)

Combinations of these two supersymmetries (1.2) and (1.4)

\[ \tilde{Q}^{(1)} = Q^{(1)} + Q^{(2)}, \quad \tilde{Q}^{(2)} = i(Q^{(1)} - Q^{(2)}), \]

(1.5)

satisfy commutation relations

\[ [\bar{\epsilon}_1 \tilde{Q}^{(i)}, \bar{\epsilon}_2 \tilde{Q}^{(j)}] = -2\bar{\epsilon}_1 \gamma_\mu \bar{\epsilon}_2 p^\mu \delta^{(ij)} \]

(1.6)

where \( p^\mu \) is the generator of the translation (1.3) and \( i, j = 1, 2 \). Therefore, if we interpret eigenvalues of the matrices \( A_\mu \) as our new space-time coordinates, the above symmetries can be regarded as ten-dimensional \( \mathcal{N} = 2 \) space-time supersymmetries. Since the maximal space-time supersymmetry guarantees the existence of graviton, it supports the conjecture that the IIB matrix model is a constructive definition of superstring. This is one of the major reasons to interpret the eigenvalues of \( A_\mu \) as coordinates of the newly emerged space-time.

The second important and confusing property is that the model has the same action as the low energy effective action of D-instantons \(^{[14]}\). We should emphasize here differences between these two theories since we are led to different interpretations of space-time. From the effective theory point of view, the eigenvalues represent the coordinates of D-instantons in the ten dimensional bulk space-time which we have assumed a priori from the beginning of constructing the effective action. On the other hand, from the constructive point of view we cannot assume such a bulk space-time itself in which matrices live, since not only fields but also the space-time should be dynamically generated as a result of dynamics of the matrices and should be constructed only from the matrices. The most natural interpretation is that the space-time consists of \( N \) discretized points and the eigenvalues represent their space-time coordinates. One of the main purposes of this paper is to
confirm this interpretation of the space-time and make sure that low energy excitations propagate in this space-time.

Final important property is that the type IIB matrix model has no free parameters. The coupling constant $g$ can be always absorbed by field redefinitions:

$$\begin{align*}
A_\mu &\to g^{1/2}A_\mu \\
\psi &\to g^{3/4}\psi.
\end{align*}$$ (1.7)

This is reminiscent of string theory where a shift of the string coupling constant is always absorbed to that of the dilaton vacuum expectation value (vev). In the previous analysis of the Schwinger-Dyson equation of IIB matrix model [2], we have introduced an infrared cut-off $\epsilon$, which gives string coupling constant $g_{st} = 1/N\epsilon^2$. But through more careful analysis of dynamics of the eigenvalues [3], we have shown that there is no such infrared divergences associated with infinitely separated eigenvalues and the infrared cutoff $\epsilon$ we have introduced by hand can be determined dynamically in terms of $N$ and $g$. Roughly speaking, as we will study in the present paper, the density of the eigenvalues represents dilaton vev and gives the string coupling constant. More solid relation requires knowledge of double scaling limit and we hope to report in a separate paper.

There are several reasons we believe that IIB matrix model is a constructive definition of the type IIB superstring. First this action can be related to the Green-Schwarz action of superstring [15] by using the semiclassical correspondence in the large $N$ limit:

$$\begin{align*}
-i[\ , \] &\to \{ \ , \ }, \\
Tr &\to \int d^2\sigma \sqrt{\hat{g}}.
\end{align*}$$ (1.8)

In fact eq.(1.1) reduces to the Green-Schwarz action in the Schild gauge [16]:

$$S_{\text{Schild}} = \int d^2\sigma [\sqrt{\hat{g}}\alpha (\frac{1}{4}\{X^\mu, X^\nu\}^2 - \frac{i}{2}\bar{\psi}\Gamma^\mu\{X^\mu, \psi\}) + \beta \sqrt{\hat{g}}].$$ (1.9)

Through this correspondence, the eigenvalues of $A_\mu$ matrices are identified with the space-time coordinates $X_\mu(\sigma)$ of string world sheet. This is consistent with our interpretation that the eigenvalues of the matrices represent space-time coordinates.
The correspondence can go farther beyond the above identification of the model with a matrix regularization of the first quantized superstring. Namely, we can describe arbitrary number of interacting D-strings and anti-D-strings as blocks of matrices, each of which corresponds to matrix regularization of a string. Off-diagonal blocks induce interactions between these strings \[1, 17\]. Thus it must be clear that the IIB matrix model is definitely not the first quantized theory of a D-string but the full second quantized theory.

It has also been shown \[2\] that Wilson loops satisfy the string field equations of motion for the type IIB superstring in the light cone gauge, which is the second evidence for the conjecture that the IIB matrix model is a constructive definition of strings. We consider the following regularized Wilson loop \[2\] \[18\]:

\[
w(C) = \text{Tr} \left[ \prod_{n=1}^{M} \exp \left\{ i \epsilon (k_{n}^{\mu} A_{\mu} + \bar{\lambda}_{n} \psi) \right\} \right].
\]  
(1.10)

Here \(k_{n}^{\mu}\) are momentum densities distributed along a loop \(C\), and we have also introduced fermionic sources \(\lambda_{n}\). \(\epsilon\) in the argument of the exponential is an infrared cutoff associated to space-time extension. In the large \(N\) limit, \(\epsilon\) should go to 0 so as to satisfy the double scaling limit. There is still some subtly how to take this double scaling limit in which we get an interacting string theory.

Considered as a matrix regularization of the Green Schwartz IIB superstring, IIB matrix model describes interacting D-strings. On the other hand in the analysis of the Wilson loops IIB matrix model describes joining and splitting interactions of fundamental IIB superstrings as Wilson loops. From these considerations, it is plausible to conclude that if we can take the correct double scaling limit, IIB matrix model becomes a constructive definition of the type IIB superstring. Furthermore we believe that all string theories are connected by duality transformations, and once we construct a nonperturbative definition of any one of them, we can describe vacua of any other strings, in particular, the true vacuum in which we live.

Once we are convinced that IIB matrix model is a constructive definition of superstring, we can simply go on to do numerical calculations \[19\]. But before doing so we will clarify in this paper how we can read the space-time geometry from matrix configurations and also propose a scenario of obtaining local space-time gauge symmetry and diffeomorphism symmetry.
It is amusing that these fundamental symmetries can arise from a very simple matrix model defined on a single point.

Dynamics of eigenvalues, that is, dynamical generation of space-time was first discussed in our previous paper [3]. An effective action of eigenvalues can be obtained by integrating all the off-diagonal bosonic and fermionic components and then diagonal fermionic coordinates (which we call fermion zeromodes). If we quench the bosonic diagonal components \( x_i^\mu (i = 1...N) \) and neglect the fermion zeromodes \( \xi^i \), the effective action for \( x_i^\mu \) coincides with that of the \( D = 4 \mathcal{N} = 4 \) supersymmetric Yang Mills theory and vanishes respecting stability of supersymmetric moduli. Inclusions of fermion zeromodes and also of non-planar contributions lift the degeneracy and we can obtain a nontrivial effective action for space-time dynamics. In our previous paper [3] we estimated this effective action by perturbation at one loop, which is valid when all eigenvalues are far from one another \(|x_i - x_j| >> \sqrt{g} \). Of course this one-loop effective action is not sufficient to determine the full space-time structure, but we expect that it captures some of the essential points for the formation of the space-time. One of important properties of the effective action is that as a result of grassmannian integration of the fermion zeromodes, space-time points make a network connected locally by bond interactions. This becomes important when we extract diffeomorphism symmetry from our matrix model. This is discussed in section 4.

The organization of the paper is as follows. In section 2, we briefly review our previous results on the dynamics of the space-time and show how a network picture of the space-time arises. Here is an analogy with the dynamical triangulation approach to quantum gravity. In section 3 we derive a low energy effective theory in some particular eigenvalue distributions consisting of small clusters of size \( n \). The effective theory acquires local \( SU(n) \) gauge symmetry and we show that the associated gauge field indeed propagate in the distribution. This supports our interpretation of the space-time. Furthermore we show that a massless fermion field in the adjoint representation of the gauge group propagates in the distribution. In this paper we consider matrix configurations near simultaneously diagonalizable ones. Then the eigenvalues and the diagonal components of matrices coincide and we obtain the same effective action for them at least up to one-loop perturbation in off-diagonal components. We expect that the classical space-time picture is valid only in such a case. For more general configurations, the classical space-time picture is broken and non-commutativity of space-time will become important [20]. We leave it for future analysis.
tation appears with a gauge invariant kinetic term. In this way, low energy effective theory is formulated as a lattice gauge theory on a dynamically generated random lattice. In section 4, we study the origin of gravity and diffeomorphism symmetry in the space-time. We show that invariance under permutations of the eigenvalues leads to the diffeomorphism invariance. Background metric is shown to be encoded in the density correlation of the eigenvalues. Section 5 is devoted to conclusions and discussions.

2 Dynamics of Eigenvalues and Space-time Generation

In this section we briefly review our previous analysis [3] on the dynamics of the eigenvalues to clarify some of important properties. Let us consider expansion around the most generic classical moduli where the gauge group $SU(N)$ is broken down to $U(1)^{N-1}$. Then diagonal elements of $A_\mu$ and $\psi$ appear as zeromodes while the off-diagonal elements become massive. We may hence integrate out the massive modes first and obtain an effective action for the diagonal elements.

We thus decompose $A_\mu$ into diagonal part $X_\mu$ and off-diagonal part $\tilde{A}_\mu$. We also decompose $\psi$ into $\xi$ and $\tilde{\psi}$:

\[
A_\mu = X_\mu + \tilde{A}_\mu; \quad X_\mu = \begin{pmatrix} x^1_\mu \\ x^2_\mu \\ \vdots \\ x^N_\mu \end{pmatrix},
\]

\[
\psi = \xi + \tilde{\psi}; \quad \xi = \begin{pmatrix} \xi^1 \\ \xi^2 \\ \vdots \\ \xi^N \end{pmatrix},
\]

(2.1)

where $x^i_\mu$ and $\xi^i$ satisfy the constraints $\sum_{i=1}^{N} x^i_\mu = 0$ and $\sum_{i=1}^{N} \xi^i = 0$, respectively, since the gauge group is $SU(N)$. We then integrate out the off-diagonal parts $\tilde{A}_\mu$ and $\tilde{\psi}$ and obtain the effective action for supercoordinates of space-time $S_{\text{eff}}[X, \xi]$. The effective action for the space-time coordinates
\( S_{\text{eff}}[X] \) can be obtained by further integrating out \( \xi \):

\[
\int dA d\psi e^{-S[A,\psi]} = \int dX d\xi e^{-S_{\text{eff}}[X,\xi]} = \int dX e^{-S_{\text{eff}}[X]},
\]

(2.2)

where \( dX \) and \( d\xi \) stand for \( \prod_{i=1}^{N-1} \prod_{\mu=0}^{9} dx_{i}^{\mu} \) and \( \prod_{i=1}^{N-1} \prod_{\alpha=1}^{16} d\xi_{i}^{\alpha} \), respectively.

Perturbative expansion in \( g^2 \) around diagonal backgrounds \((X_{\mu}, \xi)\) is valid when all of the diagonal elements are widely separated from one another:

\[
|x^{i} - x^{j}| \gg g^{1/2}.
\]

At one loop, it is easy to integrate over the off-diagonal components.

After adding a gauge fixing and the Fadeev-Popov ghost term associated with the broken symmetry \( SU(N)/U(1)^{N-1} \)

\[
S_{g.f.} + S_{\text{F.P.}} = -\frac{1}{2g^{2}} Tr([X_{\mu}, A^{\mu}]^{2}) - \frac{1}{g^{2}} Tr([X_{\mu}, b][A^{\mu}, c]),
\]

(2.3)

the action can be expanded up to the second order of the off-diagonal components \( \tilde{A}_{ij}, \tilde{\psi} \) as

\[
S_{2} + S_{g.f.} = \frac{1}{2g^{2}} \sum_{i \neq j} ((x^{i}_{\nu} - x^{j}_{\nu})^{2} \tilde{A}_{ij}^{*} \tilde{A}_{ij}^{\mu} - \bar{\tilde{\psi}} \Gamma^{\mu} (x^{i}_{\mu} - x^{j}_{\mu}) \tilde{\psi}^{ij} \\
+ (\xi^{i} - \tilde{\xi}^{i}) \Gamma^{\mu} \psi^{ij} \tilde{A}_{ij}^{*} + \bar{\tilde{\psi}} \Gamma^{\mu} (\xi^{i} - \xi^{j}) \tilde{A}_{ij}^{*}).
\]

(2.4)

The first and the second terms are the kinetic terms for \( \tilde{A} \) and \( \tilde{\psi} \) respectively, while the last two terms are \( \tilde{A} \tilde{\psi} \xi \) vertices. A bosonic off-diagonal component \( \tilde{A}_{ij}^{\mu} \) is transmuted to a fermionic off-diagonal component \( \tilde{\psi}^{ij} \) emitting a fermion zeromode \( \xi^{i} \) or \( \xi^{j} \). This vertex conserves \( SU(N) \) indices \( i \) and \( j \). Note that the propagators for \( \tilde{A} \) and \( \tilde{\psi} \) damp as \( 1/(x^{i} - x^{j})^{2} \) or \( 1/(x^{i} - x^{j}) \) respectively.

Integration over all the off-diagonal components gives an effective action for the zeromodes, \( x^{i}_{\mu} \) and \( \xi^{i} \):

\[
\int d\tilde{A} d\tilde{\psi} db dc e^{-(S_{2} + S_{g.f.} + S_{\text{F.P.}})} = \prod_{i<j} \det_{\mu\nu}(\eta^{\mu\nu} + \delta^{\mu\nu})^{-1} \\
\equiv e^{-S_{\text{loop}}^{1}[X,\xi]},
\]

(2.5)
where

\[
S_{(ij)}^{\mu\nu} = \xi_{ij}^{\mu} \Gamma^{\mu\nu} \xi_{ij}^{\nu} \frac{x_{ij}^{\mu}}{(x_{ij}^{\mu})^4}.
\]

(2.6)

Here \(\xi_{ij}^{\mu}\) and \(x_{ij}^{\mu}\) are abbreviations for \(\xi^i - \xi^j\) and \(x_{\mu}^i - x_{\mu}^j\).

The effective action can be expanded as

\[
S_{1-\text{loop}}^{\text{eff}}[X,\xi] = \sum_{i<j} tr \ln(\eta_{\mu\nu} + S_{(ij)}^{\mu\nu})
\]

\[
= -\sum_{i<j} tr \left( \frac{S_{(ij)}^4}{4} + \frac{S_{(ij)}^8}{8} \right),
\]

(2.7)

which is a sum of all pairs \((ij)\) of space-time points. Here the symbol \(tr\) in the lower case stands for the trace for Lorentz indices. Other terms in the expansion vanish due to the properties of Majorana-Weyl fermions in ten dimensions. Note that, since \(S_{(ij)}^{\mu\nu}\) contains two fermion zeromodes, the first term \(S_{(ij)}^4\) contains 8 fermion zeromodes \(\xi_{ij}^{\mu}\) along a link \((ij)\) and the second term \(S_{(ij)}^8\) contains full 16 fermion zeromodes. Hence, when integrating the fermion zeromodes, the second term acts as a delta function for the grassman variables \(\delta_{16}(\xi_i - \xi_j)\). Integration over the fermion zeromodes gives the final effective action \(S_{1-\text{loop}}^{\text{eff}}[X]\) for the bosonic eigenvalues \(x_{\mu}^i\).

Several comments are in order. The first comment is about supersymmetry. The one-loop effective action \(S_{1-\text{loop}}^{\text{eff}}[X,\xi]\) has \(\mathcal{N} = 2\) supersymmetry which is a remnant of the original one:

\[
\begin{align*}
\delta x_{\mu}^i &= i\bar{\epsilon}_1 \Gamma_{\mu} \xi^i \\
\delta \xi^i &= \epsilon_2.
\end{align*}
\]

(2.8)

Transformations for \(\epsilon_1 = \epsilon_2\) and \(\epsilon_1 = -\epsilon_2\) correspond to those generated by \(\mathcal{N} = 1\) supersymmetry generator \(Q\) and its covariant derivative \(D\). In this sense zeromodes of \(x_i\) and \(\xi_i\) can be viewed as supercoordinates of \(\mathcal{N} = 1\) superspace.

The next comment is infrared convergence. The partition function is obtained by integrating over all the boson and fermion zeromodes. A naive expectation is that, since the bosonic coordinates originally parameterize the supersymmetric moduli, the integral over \(x_i\) diverges. However, as proved in [3], the integral converges for finite \(N\) to all orders of perturbation theory due to the damping property of the propagators. At one loop this infrared
convergence can be easily seen since two fermion zeromodes in $S_{ij}^{\mu}$ always appear with a damping factor $1/(x^i - x^j)^3$ and a condition for saturating 16 fermion zeromodes requires $1/r^{24}$ damping when two clusters of eigenvalues are put apart with distance $r$. This shows that all points are gathered as a single bunch and hence space-time is inseparable. This is consistent with the explicit calculations of the partition function [21]

The one-loop effective action for $x_i$ is given by further integrating out the fermion zeromodes $\xi_i$. Non-vanishing contributions in the expansion of $\exp(-S_{\text{eff}}^{1\text{-loop}}[X, \xi])$ come from terms which saturate all the fermion zero-mode integrals $d\xi_i$. Since the terms $tr(S_{ij}^4)$ or $tr(S_{ij}^8)$ in the action $S_{\text{eff}}^{1\text{-loop}}$ contains 8 or 16 fermion zeromodes $\xi_{ij}$ respectively along a link $(ij)$, this expansion can be visualized as summing over graphs in which some of the $N C_2$ pairs among the $N$ space-time points $x_i$ ($i = 1\ldots N$) are connected by a term with 8 fermion zeromodes $tr(S_{ij}^4)/4$ or a term with 16 fermion zeromodes $(tr(S_{ij}^8)/32 + tr(S_{ij}^8))/8$:

$$
\int dX d\xi e^{-S_{\text{eff}}^{1\text{-loop}}[x, \xi]} = \int dX d\xi \sum_{G: \text{graph}} \prod_{(ij): \text{bond of } G} \left[ \frac{tr(S_{ij}^4)}{4} \right] \text{ or } \left( \frac{1}{2} \left( \frac{tr(S_{ij}^4)}{4} \right)^2 + \frac{tr(S_{ij}^8)}{8} \right) \\
= \sum_{G: \text{graph}} \int dX W[X; G]. \tag{2.9}
$$

Here $W[X; G]$ is a Boltzmann weight for a graph $G$ and configuration $X$. In this way, we arrive at network picture of the space-time. The network is induced by fermion zeromode integrations.

Integrations over the fermion zeromodes give interactions along a link $(ij)$ of the graph, which is of order $1/(x^i - x^j)^3$ for each fermion zeromode between two points $(ij)$. That is, the terms with 8 or 16 fermion zeromodes give bond interactions of order $1/(x^i - x^j)^{12}$ or $1/(x^i - x^j)^{24}$ in this manner we obtain networks of space-time coordinates connected by bonds with these interactions. Since the bond interactions suppress connection between two distant points

\footnote{Note that the bond interactions induced by terms with 8 fermion zeromodes are not scalars and depend on relative angles between several links $x^i - x^j$ in a graph. On the other hand, interactions induced by terms with 16 zeromodes are scalars since $S_{ij}^8$ acts as a delta function for $\xi_{ij}$.}
at least by a factor of order $1/(x^{ij})^{12}$, only closer points tend to be connected and the network becomes local. Also to saturate the grassman integral, we need $16(N - 1)$ fermion zeromodes. Hence number of bonds is of order $N$ which is much smaller than possible number of pairs $N(N - 1)/2$ in the large $N$ limit. In this sense $N$ discretized space-time points are weakly bound. All the allowed networks must be summed in determining the dynamics of the eigenvalues. This reminds us of summation of all triangulations in the dynamical triangulation approach to quantum gravity. We come back to this analogy in section 4.

As shown in [3], if we neglect the first type of bond interactions with 8 fermion zeromodes in (2.9), the $\xi$ integration can be performed exactly and the dynamics of eigenvalues is governed by the statistical system of a branched polymer. Since the Hausdorff dimension of the branched polymer is four, we may conjecture that other interactions will make eigenvalue distribution a smooth four dimensional manifold, which may be classified as a new universality class near the branched polymer phase. Numerical simulation is still under investigation.

3 Local Gauge Invariance

Once we describe the space-time as dynamically generated distribution of the eigenvalues, low energy effective theory in the space-time can be obtained by solving dynamics of fluctuations around the background $X_\mu$. Both of the space-time $X_\mu$ and matters $\tilde{A}_\mu$ are unified in the same matrices $A_\mu$ and should be determined dynamically. Low energy fluctuations are in general composites of $A_\mu$ and $\psi$, and it is natural from the analysis of the Schwinger-Dyson equation for the Wilson loops that a local operator in the space-time is given by a microscopic limit of the Wilson loop operators, such as

$$w(k; O) = Tr[O(A, \psi) \exp(ik^\mu A_\mu)].$$

(3.1)

Here $O(A, \psi)$ is some operator made of $A_\mu$ and $\psi$. In order to identify the total momentum of this operator as $k$, the operator $O(A, \psi)$ should be invariant under a constant shift of $A_\mu$, that is, translation in the space-time coordinates.

In the first approximation around the diagonal background $X_\mu$, the coor-
The coordinate representation of this operator is given by

\[ \hat{w}(x; O) = \int \frac{d^{10}k}{(2\pi)^{10}} \exp(-ik^\mu x_\mu) \ w(k; O) \sim \sum_{i=1}^N O_{ii} \delta^{(10)}(x - x_i). \]  

(3.2)

Here we have replaced \( A_\mu \) by \( X_\mu + \tilde{A}_\mu \) and take the leading term. \( O_{ii} \) is the \( ii \) component of the operator \( O \). Due to the delta function, the operator has a support only on the area where eigenvalues distribute. Vanishing of the operator \( \hat{w}(x; O) \) outside of the area of the distributed eigenvalues supports our interpretation of the space-time. Of course if we take into account higher terms in the expansion, the operator becomes dim and extended around the background \( X_\mu \).

We can apply a similar analysis to strings which propagate in the space-time. In the \( 1/N \) expansion, correlation between Wilson loop operators can be evaluated by summing over all surfaces made of Feynman diagrams connecting the Wilson loops at the boundary. This surface is interpreted as string world sheet connecting strings at the boundaries. Each \( SU(N) \) index \( x_\mu \) of a loop in the diagrams represents a coordinate on the world sheet and it takes value in the eigenvalue distribution in the leading approximation around the diagonal background \( X_\mu \). Hence string world sheet evolves only in the space-time of the eigenvalue distribution and again supports our interpretation of the space-time.

It is generally difficult to obtain how fluctuations propagate in the eigenvalue distribution, which is reminiscent of the QCD effective theory: Excitations are expressed as composite operators of microscopic variables and their low energy dynamics can be discussed only through the symmetry argument, namely the argument based on the chiral symmetry. Also in our case we will show that there are eigenvalue distributions around which symmetry arguments allow us to discuss low energy dynamics for some excitations. Suppose that eigenvalue distribution forms clusters consisting of \( n \) eigenvalues. At length scale much larger than the size of each cluster, the \( SU(N) \) symmetry is broken down to \( SU(n)^m \) where \( m = N/n \). We can expand \( A_\mu \) and \( \psi \) around such a background \( X_\mu \) similarly to the analysis in the previous
section. First write $A_\mu$ and $\psi$ in block forms:

$$A_\mu = \begin{pmatrix} A_{\mu 1}^1 & A_{\mu 1}^2 & \cdots \\ A_{\mu 2}^1 & A_{\mu 2}^2 & \cdots \\ \vdots & \vdots & \ddots \\ A_{\mu m}^1 & A_{\mu m}^2 & \cdots & A_{\mu m}^m \end{pmatrix},$$

$$\psi = \begin{pmatrix} \psi_{11} & \psi_{12} & \cdots \\ \psi_{21} & \psi_{22} & \cdots \\ \vdots & \vdots & \ddots \\ \psi_{mm} \end{pmatrix}. \tag{3.3}$$

Each block $A_{\mu i}^j$ or $\psi_{ij}$ is an $n \times n$ matrix and the diagonal blocks can be further decomposed

$$A_{\mu i}^i = x_i 1 + \tilde{A}_{\mu i}^i,$$

$$\psi_{ii} = \xi_i 1 + \tilde{\psi}_{ii} \tag{3.4}$$

where $1$ is an $n \times n$ unit matrix and $\text{tr} A_{\mu i}^i = 0$. Here $\text{tr}$ means trace for the submatrix of $n \times n$. We interpret each cluster of the eigenvalues as a space-time point with an internal structure $SU(n)$. Since each $SU(n)$ symmetry acts on the variables at the position $i$ independently, the unbroken $SU(n)^m$ symmetry can be regarded as local gauge symmetry. Indeed under a gauge transformation $g$ of the unbroken $SU(n)^m$ symmetry

$$g = \begin{pmatrix} g_1 \\ g_2 \\ \vdots \\ g_m \end{pmatrix} \in SU(n)^m \subset SU(N), \tag{3.5}$$

the diagonal block fields, $\tilde{A}_{\mu i}^i$ and $\tilde{\psi}_{ii}$, transform as adjoint matters (i.e. site variables in the lattice gauge theory) while the off-diagonal block fields, $A_{\mu i}^j$ and $\psi_{ij}$, as gauge connections (i.e. link variables):

$$\tilde{A}_{\mu i}^i \to g_i \tilde{A}_{\mu i}^i g_i^\dag,$$

$$\tilde{\psi}_{ii} \to g_i \tilde{\psi}_{ii} g_i^\dag,$$

$$A_{\mu i}^j \to g_i A_{\mu i}^j g_j^\dag,$$

$$\psi_{ij} \to g_i \psi_{ij} g_j^\dag. \tag{3.6}$$
Some of the dynamics for low energy excitations is governed by this local gauge invariance. Gauge fields live on the links and transform as the link variables in the lattice gauge theory. In our case, we have too many such fields (at least 10 boson fields $A^i_j$ for a link $(ij)$ ) but only one unitary link variable is assured to be massless by the gauge symmetry and others will acquire mass dynamically. Therefore, in deriving low energy effective theory, we first apply polar decomposition to $A^i_j$ into unitary and hermitian degrees of freedom and identify all the unitary components in various off-diagonal block fields by setting them one common field $U^{ij}$ on each link.

Among various fields in the dynamically generated random lattice, some fields will survive in the low energy effective theory. Here let us consider simple fields made of a single microscopic variable $A^i_\mu$ or $\psi$. As we will see soon, the off-diagonal block fields become massive except the unitary component $U^{ij}$. The gauge field $U^{ij}$ is assured to be massless due to the gauge invariance. Next, stability of the cluster type eigenvalue distribution we have assumed requires that the diagonal bosonic fields $A^{ii}_\mu$, which break this distribution, should become massive dynamically. Finally, the diagonal fermionic block field $\psi^{ii}$ can be massless and we should investigate their low energy action.

We henceforth integrate out the off-diagonal blocks first while keeping the unitary degrees of freedom $U^{ij}$ and obtain an effective action for other variables which survive in the low energy effective theory.

We add a gauge fixing term for broken generators $SU(N)/SU(n)^m$

$$S_{g.f.} = \frac{1}{2g^2} \sum_{i \neq j} tr|x^{ij}_\mu A^{ij}_\mu|^2, \quad (3.7)$$

and also its associated Fadeev-Popov term. The action can be expanded as before. Lower order terms in the action are composed of the following terms:

$$S_2 + S_{g.f.} = \frac{1}{2g^2} \sum_{i \neq j} tr((x^{ij}_\nu)^2 |A^{ij}_\mu|^2 - \bar{\psi}^{ji} \Gamma^\mu x^{ij}_\mu \psi^{ij}$$

$$- 2\bar{\psi}^{ii} \Gamma^\mu (A^{ij}_\mu \psi^{ji} - \psi^{ij} A^{ii}_\mu)). \quad (3.8)$$

The first and the second terms are the kinetic terms for the off-diagonal block fields $A^{ij}_\mu$ and $\psi^{ij}$. The third term corresponds to the term of the zeromode $\xi$ insertion in the previous section. Integrating out the fermion fields in the
off-diagonal blocks $\psi_{ij}$, the third term in (3.8) becomes
\[
\frac{1}{g^2} \sum_{i \neq j} tr(\bar{\psi}^i \Gamma^{\mu \nu} A^i_{\mu} \frac{x_{ij}^{ij}}{(x_{ji})^2} (A^j_{\nu} \bar{\psi}^j - \bar{\psi}^{ij} A^i_{\nu})).
\] (3.9)

It consists of a mass-like term proportional to $\bar{\psi}^i \Gamma^{\mu \nu} \bar{\psi}^i$ and a hopping term $\bar{\psi}^i \bar{\psi}^j$ between $i$ and $j$. Integration over $A^i_{\mu}$ can be performed, with its unitary component $U^i_{ij}$ kept fixed, by replacing
\[
\frac{A^i_{\mu}}{A^i_{\mu} \otimes A^i_{\nu}} = 0
\]
\[
\frac{A^i_{\mu} \otimes A^i_{\nu}}{A^{ij}} = g^2 \frac{\delta^{\mu \nu}}{(x_{ij})^2} U^{ij} \otimes U^{ji}.
\] (3.10)

In the second equation, the damping factor $1/(x_{ij})^2$ corresponds to the propagation of the hermitian degrees of freedom while the appearance of the link variable $U^{ij}$ corresponds to keeping the unitary degrees of freedom. This replacing makes the mass-like term in (3.9) vanish and the hopping term
\[
- g^2 \sum_{i \neq j} tr(\bar{\psi}^i \Gamma^{\lambda} A^j_{\nu} \frac{x_{ij}^{ij}}{(x_{ji})^4} U^{ij} \bar{\psi}^{ij} U^{ji}).
\] (3.11)

This is a gauge invariant kinetic term for an adjoint fermion field $\bar{\psi}^i$ on a (random) lattice generated dynamically by the distributed eigenvalues. The fields $\bar{\psi}^i$ can hop between any pair of points in the space-time, but since the hopping parameter is suppressed by $1/(x_{ij})^3$, the propagation is expected to become local in the continuum limit.

Other terms in the action generate a plaquette action for the gauge field $U^{ij}$ as follows. Relevant terms in the action are
\[
S_4 = \frac{1}{g^2} \sum_{i \neq j \neq k \neq l} tr(A^i_{\mu} A^j_{\nu} - A^i_{\nu} A^j_{\mu}) A^k_{\mu} A^l_{\nu}.
\] (3.12)

By integrating out the hermitian degrees of freedom of the off-diagonal blocks with the procedure (3.10), this action itself vanishes $S_4 = 0$. However interactions generated by $(S_4)^2$ induce a kinetic term for the gauge field;

\[
\overline{(S_4)^2} \sim \sum_{i \neq j \neq k \neq l} \frac{g^4}{(x_{ij})^2(x_{jk})^2(x_{kl})^2(x_{li})^2} tr(U^{ij} U^{jk} U^{kl} U^{li}) tr(U^{il} U^{lk} U^{kj} U^{ji}).
\] (3.13)
This is the plaquette action generated by a Wilson loop for the adjoint representation and hence the gauge field $U^{ij}$ indeed propagates in the space-time of eigenvalue distribution. Again the gauge field can hop between any pair of space-time points, but the hopping is suppressed by $1/x^8$ for distant points and we will recover locality in the continuum limit.

To summarize this section, supposing that distribution of the eigenvalues consists of small clusters with size $n$, we have shown that the low energy effective theory contains several massless fields such as the gauge field associated with the local $SU(n)$ gauge symmetry and fermion field in the adjoint representation of $SU(n)$ gauge symmetry. Gauge invariant kinetic terms were also derived. Thus our system is a lattice gauge theory on a dynamically generated random lattice. It is invariant under a permutation for the set of the $N$ discrete space-time points, since the permutation group $S_N$ is a subgroup of the original $SU(N)$ symmetry. It is the most different point from the ordinary lattice gauge theory on a fixed lattice, which becomes important in deriving the diffeomorphism invariance of our model. We will come back to this point in the next section. Although the permutation invariance requires that all space-time points are equivalent, locality in the space-time will be assured due to the suppression of the hopping terms between distant points. In general, however, we need a sufficient power for the damping of the hopping terms in order to assure locality in the continuum limit. Though we do not yet know the real condition for locality, we expect that terms with lower powers are canceled due to supersymmetry or by averaging over gauge fields.

4 Gravity and Diffeomorphism Invariance

As we have seen in section 2, the one-loop effective action for the space-time points is described as a statistical system of $N$ points whose coordinates are $x^i_μ$. Integration over the fermion zero-modes $ξ$ gives the Boltzmann weight, which depends on a graph (or network) connecting the space-time points locally by order $N$ number of bond interactions;

$$Z = \sum_{G:\text{graph}} \int dX \ W[X; G]. \quad (4.1)$$
$W[X; G]$ is a complicated function of a configuration $X$ and a graph $G$. An important property is that the weight is suppressed at least by a damping $1/(x^i - x^j)^{12}$ when two points $i$ and $j$ are connected. This system is, of course, invariant under permutations $S_N$ of $N$ space-time points, which is a subgroup of the original symmetry $SU(N)$, while the Boltzmann weight for each graph $G$ is not. The invariance is realized by summing over all possible graphs. In other words the system becomes permutation invariant by rearrangements of bonds in the network of space-time points. This reminds us of the dynamical triangulation approach to quantum gravity \cite{22}, where the diffeomorphism invariance is believed to arise from summing all possible triangulations. It is amusing that our system satisfies both of locality and permutation invariance simultaneously by summing over all possible graphs.

In this section we see that the permutation invariance of our system actually leads to the diffeomorphism invariance. To see how the background metric is encoded in the effective action for low energy excitations, let us consider, as an example, a scalar field $\phi^i$ propagating in distributed eigenvalues. The effective action will be given by

\begin{equation}
S = \sum_{i,j} \frac{(\phi^i - \phi^j)^2}{2} f(x^i - x^j) + \sum_i m(\phi^i)^2 \tag{4.2}
\end{equation}

where $f(x)$ is a function decreasing sufficiently fast at infinity to assure locality in the space-time. Introducing the density function of the eigenvalues

\begin{equation}
\rho(x) = \sum_i \delta^{(10)}(x - x^i) \tag{4.3}
\end{equation}

and a continuous field $\phi(x)$ which satisfies $\phi(x^i) = \phi^i$, the action can be rewritten as

\begin{equation}
S = \int dx dy \langle \rho(x) \rho(y) \rangle \frac{(\phi(x) - \phi(y))^2}{2} f(x - y) + m \int dx \langle \rho(x) \rangle \phi(x)^2. \tag{4.4}
\end{equation}

Here the expectation $\langle ... \rangle$ for the density and the density correlation means that we have taken average over configurations $X$ and networks $G$ of

\footnote{In this section we consider general eigenvalue distributions in which all eigenvalues have indegenerate space-time coordinates. If we take the cluster type distribution considered in the previous section, the permutation symmetry responsible for the diffeomorphism invariance should be $S_{N/n}$.}
space-time points. Normalizing the density correlation in terms of the density
\[ \langle \rho(x)\rho(y) \rangle = \langle \rho(x) \rangle \langle \rho(y) \rangle (1 + c(x, y)) \] (4.5)
and expanding \( \phi(x) - \phi(y) = (x - y)_\mu \partial^\mu \phi(x) + \cdots \), the action becomes
\[ S = \frac{1}{2} \int dx \langle \rho(x) \rangle \left[ \int dy \langle \rho(y) \rangle (x - y)_\mu (x - y)_\nu f(x - y)(1 + c(x, y)) \right] \]
\[ \partial^\mu \phi(x) \partial^\nu \phi(x) + m \int dx \langle \rho(x) \rangle \phi(x)^2 \cdots . \] (4.6)

This expansion shows that the field \( \phi(x) \) propagating in the eigenvalue distribution feels the density correlation as the background metric while the density itself as vacuum expectation value of the dilaton field. Namely we can identify
\[ g_{\mu\nu}(x) \sim \int dy \langle \rho(y) \rangle (x - y)_\mu (x - y)_\nu f(x - y)(1 + c(x, y)) \] (4.7)
\[ \sqrt{g} e^{-\Phi(x)} \sim \langle \rho(x) \rangle . \] (4.8)

If the density correlation respects the original translational and rotational symmetry, that is, if they are not spontaneously broken, the metric becomes flat \( g_{\mu\nu} \sim \eta_{\mu\nu} \). (Normalization can be absorbed by the dilaton vev.) The fact that the background metric is encoded in the density correlations as above indicates that our system is general covariant even though the IIB matrix model action (1.1) defined in flat ten dimensions does not have manifest general covariance.

Then let us see how the diffeomorphism invariance is realized in our model. The action (1.1) is invariant under the permutation \( S_N \) of the eigenvalues, which is a subgroup of \( SU(N) \). Under a permutation
\[ x^i \rightarrow x^{\sigma(i)} \text{ for } \sigma \in S_N, \] (4.9)
the field \( \phi^i \) transforms into \( \phi^{\sigma(i)} \). Then, from the definition of the continuous field \( \phi(x) \), we should extend the transformation (4.9) into \( x \),
\[ x \rightarrow \xi(x) \] (4.10)
such that \( \xi(x^i) = x^{\sigma(i)} \). Under this transformation, the eigenvalue density transforms as a scalar density and the field \( \phi(x) \) as a scalar field. On the other
hand, the metric transforms as a second rank tensor, if the function \( f(x - y) \) decreases rapidly around \( x = y \) and the \( y \) integral in (1.7) has support only near \( y = x \). The tensor property of the metric is also required from the invariance of the action under the transformation (4.10). In this way, the invariance under the permutation of the eigenvalues leads to the invariance of the low energy effective action under general coordinate transformations.

Background metric is encoded in the density correlation of the eigenvalues. Since we have started from the Poincare invariant type IIB matrix model action (1.1), the density correlation is expected to be translational and rotational invariant and we may obtain low energy effective action in a flat background. A nontrivial background can be induced dynamically if Lorentz symmetry is spontaneously broken and the eigenvalues are non-trivially distributed.

A nontrivial background can be also described by condensing a graviton operator \([23]\). Bosonic parts of graviton and dilaton operators are given by

\[
S_{\mu\nu}(k) \sim Tr(F_{\mu\lambda}F^{\lambda\nu}e^{ik\cdot A}) + (\mu \leftrightarrow \nu) \tag{4.11}
\]

\[
D(k) \sim Tr(F^2 e^{ik\cdot A}). \tag{4.12}
\]

Condensation of them induces extra terms in IIB matrix model action;

\[
S_{\text{cond}} = \int dk(\sum h^{\mu\nu}(k)S_{\mu\nu}(k) + h(k)D(k)). \tag{4.13}
\]

We can similarly obtain an effective action for fluctuations around a diagonal background from this modified matrix model action. Condensation of dilaton changes the Yang-Mills coupling constant \( g \) locally in the space-time. Since \( g \) is the only dimensionful constant in our model and thus determines the fundamental length scale, a local change in \( g \) will lead to a local change in the eigenvalue density. This is consistent with our earlier discussion that the dilaton expectation value is encoded in the eigenvalue density. On the other hand, condensation of graviton induces asymmetry of the space-time. For condensation of \( k = 0 \) graviton mode, it is obvious that the condensation can be compensated by a field redefinition of matrices \( A_\mu \)

\[
A_\mu \rightarrow (\delta_\mu^\nu + h^\nu_\mu)A_\nu, \tag{4.14}
\]

and the two models, the original IIB matrix model and the modified one with the \( k = 0 \) graviton condensation, are directly related through the above field
rere definition. The density of the eigenvalues is mapped accordingly and the
density correlation is expected to become asymmetric in the modified matrix
model. For more general condensation, if the graviton operator $\hat{S}_{\mu\nu}(x)$ (co-
ordinate representation of (4.11)) changes only local property of dynamics of
the eigenvalues, the density correlation will become asymmetric locally in the
space-time around $x$ and therefore induces a local change in the background
metric.

Our low energy effective action is formulated as a lattice gauge theory
on a dynamically generated random lattice. Since the lattice itself is gener-
ated dynamically from matrices, we must sum over all possible graphs. In
this way, our system is permutation $S_N$ invariant, which is responsible for
the diffeomorphism invariance. The background metric is encoded in the
density correlation of the eigenvalues and the low energy effective action be-
comes manifestly general covariant. The graviton operator is represented
as fluctuation around the background space-time and constructed from the
off-diagonal components of the matrices. Microscopic derivation of the prop-
agation of the graviton is difficult to obtain, but once we have clarified the
underlying diffeomorphism symmetry, it is natural that the low energy ef-
fective action for the graviton is described by the Einstein Hilbert action.
Employing this diffeomorphism invariance and the supersymmetry, we will
be able to derive the low energy behavior of the graviton multiplets, which
will be reported in a separate paper.

5 Conclusions and Discussions

In this paper we have discussed a possibility to interpret the space-time in
type IIB matrix model as distributed eigenvalues. We have shown that, if
we suppose that eigenvalue distribution consists of small clusters with size
$n$, the low energy theory acquires $SU(N)$ local space-time gauge symmetry.
This gauge invariance can predict the existence of the gauge field propagating
in the space-time of distributed eigenvalues. Also we have obtained a gauge
invariant kinetic action for a fermion in the adjoint representation of $SU(N)$.
Low energy behavior for these fields is described in terms of a lattice gauge
theory on a dynamically generated random lattice and hence supports our
interpretation of the space-time.

Since the type IIB matrix model is proposed as a constructive definition
of superstring, we need to show the existence of the massless graviton and the
diffeomorphism invariance of the low energy effective theory. One plausible
argument for the existence of massless graviton is based on the maximal sup-
ersymmetry \( (D = 10, \mathcal{N} = 2 \text{ susy}) \). We have shown that the diffeomorphism
invariance of our model is originated in the invariance under permutations of
the eigenvalues. Our model realizes the invariance in an interesting way by
summing all possible graphs connecting the space-time points. The diffeo-
morphism invariance restricts the low energy behavior of the model and gives
another reasoning for the existence of massless graviton. Background metric
for propagating fields is encoded in the density correlation of the eigenvalues
while the dilaton vev is encoded in the eigenvalue density. Curved back-
ground can be described as a nontrivial distribution of eigenvalues whose
density correlation behaves inhomogeneously.

Both of these fundamental symmetries, local gauge symmetry and the
diffeomorphism symmetry, are originated in the \( SU(N) \) invariance of the
matrix model. These symmetries have been attempted to unify by many
physicists, beginning with Kaluza and Klein \[24\]. It is amusing that our
matrix model can unify them in a natural way and complicated structures
are hidden in such a simple matrix model.

There are still several problems unanswered. One is the low energy su-
persymmetry in the effective theory in a dynamically generated distribution
of eigenvalues. Since we have obtained a massless gauge boson and an ad-
joint fermion, we may expect \( \mathcal{N} = 1 \) low energy supersymmetry. We need to
clarify how \( \mathcal{N} = 2 \) supersymmetry in type IIB matrix model acts on the low
energy fields. Since \( \mathcal{N} = 1 \) ten dimensional Yang-Mills theory is anomalous
by itself, other fields (e.g. supergravity multiples) must be required massless
to cancel anomalies. Also the low energy gauge group might be restricted or
projected.

Next problem is locality in the low energy effective theory. The gauge
invariant kinetic terms (3.11) or (3.13) have hopping parameters suppressed
due to the inverse powers of distances, but the suppression seems insufficient
to assure locality or a well behaved infrared property. As for the action
(3.11), we may obtain more suppressed hopping terms since the link variable
\( U^{ij} \) between distant points can fluctuate much and averaging over the gauge
field may reduce the strength of effective hopping parameters.

We have also more fundamental or conceptual issues. We have obtained
nonabelian gauge symmetry from type IIB matrix model by assuming a par-
ticular eigenvalue distribution. This indicates that this vacuum is not a perturbative vacuum of type IIB superstring. Instead we may wonder if this is a perturbative vacuum of heterotic string or type I string realized in a nonperturbative way within type IIB matrix model. As we saw in the introduction, our matrix model contains both of the world sheets of the fundamental IIB string and the D-strings. By a semiclassical correspondence (1.8), we have identified IIB superstring in the Schild gauge where $\text{tr}$ is interpreted as integration over a D-string world sheet. We can also construct an F-string world sheet in terms of surfaces made of Feynman diagrams whose $SU(N)$ index represents a space-time coordinate of a world sheet point. In both cases, if we assume an eigenvalue distribution consisting of small clusters, an internal structure appears on the world sheet and hence current algebra may arise.

Another issue is how to describe global topology in type IIB matrix model. The simplest example is a torus compactification. A possible procedure of a torus compactification \cite{25} is to identify $A_\mu$ with $A_\mu + R_\mu$ by embedding a derivative operator into our matrix configuration. Therefore $N$ is taken infinity from the beginning. Since this procedure has a subtlety in the large $N$ limit, we need a careful examination of the double scaling limit.

We also do not yet know how we can describe chiral fermions in lower dimensions after compactification. Our compactification procedure looks different from the ordinary Kaluza-Klein compactification but there is some similarity. In the ordinary case, isometry of a compactified space leads to local gauge symmetry in four dimensional space-time while in our case the internal structure arises from assuming the cluster type eigenvalue distribution. In our description of four dimensional space-time, eigenvalues are confined in four directions and shrunk in the other six directions. Our compactification therefore can be regarded as a segment compactification in an ordinary picture. We need to investigate how chiral fermions can arise in such a compactified space-time.

These problems and issues are under investigations and we want to discuss more in the near future.

**Acknowledgments**

We would like to thank Y. Kitazawa, T. Tada and A. Tsuchiya for discussions and especially H. Aoki for the collaboration in the early stage.
References

[1] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B498 (1997) 467, hep-th/9612113.

[2] M. Fukuma, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl.Phys.B510 (1998) 158, hep-th/9705128.

[3] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa and T. Tada, Prog. Theor. Phys. 99 (1998) 713, hep-th/9802083.

[4] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, Phys. Rev. D55 (1997) 5112, hep-th/9610043.

[5] R. Dijkgraaf, E. Verlinde and H. Verlinde, Nucl.Phys. B500 (1997) 43.

[6] V. Periwal, hep-th/9611103.

[7] T. Yoneya, Prog.Theor.Phys. 97 (1997) 949, hep-th/9703078.

[8] J. Polchinski, talk at Nishinomiya Yukawa memorial symposium (Nov. 1998), to appear in Prog. Theor. Phys. Suppl., hep-th/9903165.

[9] H. Sugawara, hep-th/9708029.

[10] H. Itoyama, A. Tokura, Prog.Theor.Phys. 99 (1998) 129, hep-th/9708123; Phys.Rev. D58 (1998) 026002, hep-th/9801084; B. Chen, H. Itoyama, H. Kihara, hep-th/9810237; H. Itoyama and A. Tsuchiya, hep-th/9812177; K. Ezawa, Y. Matsuo and K. Murakami, Phys.Lett. B439 (1998) 29, hep-th/9802164.

[11] N. Kim and S. J. Rey, hep-th/9704158; T. Banks and L. Motl, hep-th/9703218; D. A. Lowe, hep-th/9704041; S. J. Rey, hep-th/9704158; P. Horava, hep-th/9705035; M. Krough, hep-th/9801034, hep-th/9803088.

[12] Several variants of type IIB matrix model have also been proposed; A. Fayyazuddin, Y. Makeenko, P. Olesen, D.J. Smith and K. Zarembo, hep-th/9703038; C. F. Kristjansen and P. Olesen, hep-th/9704017; J. Ambjorn and L. Chekhov, hep-th/9805212; S. Hirano and M. Kato, hep-th/9708039; I. Oda, hep-th/9806096; hep-th/9801083; Phys.Lett.B427 (1998) 267, hep-th/9801031.
N. Kitsunezaki and J. Nishimura, Nucl. Phys. B526 (1998) 351; T. Tada and A. Tsuchiya, hep-th/9903037.

[13] T. Eguchi and H. Kawai, Phys. Rev. Lett. 48 (1982) 1063.
G. Parisi, Phys. Lett. 112B (1982) 463.
D. Gross and Y. Kitazawa, Nucl. Phys. B206 (1982) 440.
G. Bhanot, U. Heller and H. Neuberger, Phys. Lett. 113B (1982) 47.
S. Das and S. Wadia, Phys. Lett. 117B (1982) 228.
J. Alfar and B. Sakita, Phys. Lett. 121B (1983) 339.

[14] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724; E. Witten, Nucl. Phys. B443 (1995) 85.

[15] M. Green and J. Schwarz, Phys. Lett. 136B (1984) 367.

[16] A. Schild, Phys. Rev. D16 (1977) 1722.

[17] I. Chepelev, Y. Makeenko and K. Zarembo, hep-th/9701151; A. Fayyazuddin and D.J. Smith, hep-th/9701168; I. Chepelev and A. A. Tseytlin, hep-th/9705120; B. P. Mandal and S. Mukhopadhyay, hep-th/9709098; N. D. Hari Dass and B. Sathiapalan, hep-th/9712179; Y. Kitazawa and H. Takata, hep-th/9810004; N. Hambli, hep-th/9812008.

[18] K. Hamada, Phys. Rev. D56 (1997) 7503, hep-th/9706187.

[19] See [3] and T. Hotta, J. Nishimura, A. Tsuchiya, hep-th/9811220; M. Nakajima and J. Nishimura, Nucl. Phys. B528 (1998) 355, hep-th/9802082; W. Krauth and M. Staudacher, hep-th/9902113 for numerical analysis of eigenvalue distribution.
For an evaluation of the partition function: W. Krauth, H. Nicoli and M. Staudache, Phys. Lett. B431 (1998) 31, hep-th/9803117; W. Krauth and M. Staudacher, Phys. Lett. B435:350-355, 1998, hep-th/9804199; S. Bal and B. Sathiapalan, hep-th/9902087.

[20] A. Connes, Comm. Math. Phys. 182 (1996) 155; hep-th/9603053; A. Connes, M. Douglas and A. Schwarz, hep-th/9711162.

[21] M. Green and M. Gutperle, JHEP 9801 (1998) 005, hep-th/9711107; Phys. Rev. D58 (1998) 046007, hep-th/9804123; G. Moore, N. Nekrasov and S. Shatashvilli, hep-th/9803263; V. A. Kazakov, I. K. Kostov and
N. A. Nekrasov, [hep-th/9810035]; P. Vanhove, [hep-th/9903050]. Also see papers in [19].

[22] J. Ambjorn, B Durhuus and T. Jonsson, *Quantum Geometry*, Cambridge (1997).

[23] Condensation of graviton is discussed in the paper by Yoneya from a slightly different point of view; T. Yoneya, talk at Nishinomiya Yukawa Memorial symposium (Nov. 1998), to appear in Prog. Theor. Phys. Suppl., [hep-th/9902200].

[24] T. L. Kaluza, Sitz, Bal. Akad., (1921) 966; O. Klein, Z. Phys. 37 (1926) 895. Also see recent discussions, F. Wilczek, Phys. Rev. Lett. 80(1998) 4851, [hep-th/9801184].

[25] Washington Taylor, Phys.Lett. B394 (1997) 283, [hep-th/9611042].