Kondo Scaling in the Optical Response of YbIn_{1−x}Ag_{x}Cu_{4}

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Theoretical work on Kondo systems predicts universality in the scaling of observable quantities
with the Kondo temperature, $T_K$. Here we report infrared-frequency optical response measurements
of the correlated system YbIn_{1−x}Ag_{x}Cu_{4}. We observe that $x$-dependent variations in the
frequency and strength of a low-energy excitation are related to the $x$-dependent Kondo temperature.
Comparison of the inferred trends with existing theory and a model calculation provides
a framework in which to view these experimental results as scaling phenomena arising from local-moment/conduction electron hybridization.

The study of the Kondo problem has contributed much
in the way of theoretical technology (e.g., renormalization group) as well as insight into the possible manifestations
and phenomenologies of interacting systems with many
degrees of freedom. One unifying effect born from the
years of investigation is the Kondo resonance—a many-body collective excitation responsible for much of the interesting
low energy behavior displayed by rare-earth and transition metal systems.

YbAgCu_{4} provides an example of a moderately heavy fermion system. At low temperatures, the $j = \frac{7}{2}$ Yb
moment is screened by conduction electrons giving an enhanced Pauli paramagnetic susceptibility.[1, 2]. Enhancement is also found
in the Sommerfeld coefficient $\gamma \sim 250 mJ/molK^2$, indicating a large effective carrier mass[2], or equivalently, a large density of states near
the chemical potential. At high temperature, the Yb moment acts independently of the carriers, which show unenhanced mass behavior.

Substituting In for Ag gives very different behavior.
YbInCu_{4} displays a first-order isostructural electronic phase transition at $T_c \simeq 42 K$ from a free-moment
semimetal to a low-temperature metallic mixed-valent phase.[3, 4]. This phase transition begins a phase boundary in the $x-T$ plane of YbIn_{1−x}Ag_{x}Cu_{4} which
increases in temperature as $x$ is increased and ends in a critical point around $x \simeq 0.3$. This phase boundary is of considerable interest[1, 2, 3, 4, 5, 6, 7] because it involves mainly
electronic degrees of freedom and because there is as yet no real understanding of or agreement regarding its fundamental origin. When approaching an understanding, one must address the periodic Anderson model (PAM), which provides a basic framework for interpreting the phenomenology of heavy fermion and mixed-valent systems, where crossover behavior is the norm. While much of the phenomenology of YbIn_{1−x}Ag_{x}Cu_{4} is consistent with the PAM, the occurrence of an unexpected phase transition at low $x$ could indicate the need for an additional interaction term in its minimal model Hamiltonian. It is natural in this context to consider whether adding conduction electron interactions (e.g., Giamarchi et al.[8] and Freericks et al.[9]) which are not generally included in the PAM, could provide competing influences necessary to understand the origin of this phase transition. Further exploration in this area may help establish a connection between the domain of Mott-Hubbard physics, where strong conduction electron interactions lead to a phase transition, and the moment-compensation physics of the Anderson and Kondo models.

In low-temperature YbInCu_{4}, where the Kondo scale is large, there appears a distinct feature in the mid-infrared
conductivity that is not present in the high temperature phase.[5]. It has been suggested that similar mid-infrared
features found in other heavy fermion materials[5, 9, 10] are a manifestation of local moment (Kondo lattice/periodic Anderson model) phenomena[11, 12, 13]. Here, by studying the Ag concentration dependence of the dynamical conductivity in YbIn_{1−x}Ag_{x}Cu_{4}, we effectively vary $T_K$ within the low temperature phase and observe systematic changes in characteristics of the mid-infrared excitation. We then explore the scaling behavior of this excitation in the context of a simple local-moment hybridization picture, and establish both the identification and phenomenology of the optical signature of the Kondo resonance.

We have measured the reflectivity of single crystals of a sequence of samples of YbIn_{1−x}Ag_{x}Cu_{4} with $x = 0, 0.3, 0.5, 0.75$ and 1.0. Our measurements cover the frequency range from $5 meV$ to $6.2 eV$ with detailed temper-
FIG. 1: The real part of the low-temperature optical conductivity of YbIn$_{1-x}$Ag$_x$Cu$_4$ ($T = 20 K$) is shown for five values of $x$. The open triangles mark peak frequencies. The dotted curves refer to a PAM-based joint density-of-states calculation in the text and the dark vertical bars indicate a threshold frequency from that calculation.

FIG. 2: a) Characteristic frequencies, $\omega_{\Delta}$ ($\triangle$) and $\omega_{th}$ ($\blacktriangle$) are shown as a function of $x$ (inset) and the Kondo temperature, $T_K$. b) Spectral weight is shown versus $x$ (inset) and $T_K$. $T_K$ is obtained from low-temperature magnetic susceptibility measurements. Spectral weight is calculated using a bare band mass $m = 4m_e$. Dotted lines show fits to the data based on model equations in the text.

$\sigma_1(\omega)$ and $T_K$ dependence of $\sigma_1(\omega)$ below 1 eV are not significantly influenced by any of the extrapolations above 6.2 eV.

In Figure 2a the low temperature conductivity is shown for the five $x$ values studied. A prominent feature centered around $\frac{1}{4} eV$ for $x=0$ (YbInCu$_4$) changes only slightly as $x$ is increased to 0.3, however further doping to $x=0.5$ and 0.75 causes both a red shift and strengthening of this feature. The triangles mark the frequency of the peak of $\sigma_1(\omega)$ and the vertical bars mark the location of a threshold frequency obtained from a model calculation described below.

In Figure 2b an integral of $\sigma_1(\omega)$ in the vicinity of this excitation is shown as a function of $x$ and $T_K$. This
spectral weight, or strength, is defined by the integral,
\[ n(\omega) = \frac{2m}{\pi e^2} \int_0^\infty \sigma_1(\omega')d\omega', \] (1)
where the limits of integration, 6 meV and 0.5 eV, encompass the \( \frac{1}{4} eV \) excitation and exclude a very small Drude peak at very low frequency. Neither integration limit is critical; in fact a lower limit of 0 and upper limits between anywhere between 0.4 eV and 1 eV produce the same \( x \) dependence which makes one confident that the \( n \) versus \( x \) dependence shown here is an essential characteristic of the data, and independent of any of the details of the choices we have made in the analysis.

Our discussion of the data centers on the PAM dispersion relations illustrated in the inset of Fig. 3a which address the essential physics of hybridization of conduction electrons with local moments and the appearance of the Kondo resonance at \( E_F \). At energies far from the chemical potential, the upper and lower bands, \( \epsilon^+ \) and \( \epsilon^- \), follow closely the unrenormalized free carrier dispersion. Near \( E_F \), the Fermi surface opens (and the bands flatten) to accommodate the \( f \)-electron weight projected up to the Fermi level as a result of hybridization. This reorganization of the bands in the vicinity of the Fermi level is due to many-body interactions, the strength of which is characterized by the parameter \( V \). The resultant narrow peak in the density of states, called the Kondo, or Abrikosov-Suhl, resonance is central to the understanding of heavy fermion and mixed-valent phenomenology \([11, 12, 13, 14, 18]\).

The hybridization-induced splitting creates the possibility of vertical transitions from filled states below \( E_F \) to unoccupied levels above \( E_F \), as illustrated by the arrows of Figure 3a. The threshold for these transitions occurs at a frequency \( \omega = 2V \), where \( V \) is the hybridization strength renormalized by the on-site \( f \)-electron repulsion. This energy scales with the Kondo temperature as \([11, 12, 13, 14, 18]\):
\[ \tilde{V} = \sqrt{T_K B} \] (2)
where \( B \) is related to the conduction electron bandwidth \([13]\). At threshold, the nesting of the coherent(lower) and incoherent(upper) bands leads to a very high joint density of states for vertical transitions and hence a strong peak in the conductivity, which we show below can be related with the width of the Kondo resonance, \( T_K \).

To proceed with the analysis, we use the Kubo-Greenwood formula \([10]\):
\[ \sigma_1(\omega) = \frac{\pi e^2}{m^2 \omega} \sum_{\ell, \ell'} JDOS_{\ell, \ell'}(\omega) |p_{\ell, \ell'}|^2 \] (3)
where \( |p_{\ell, \ell'}| \) denotes the dipole matrix element connecting electronic bands \( \ell \) and \( \ell' \), and \( JDOS_{\ell, \ell'}(\omega) \) is the corresponding joint density of states. Applying this formula to hybridizing quasiparticles as though they were

\[ \sigma_{\text{pam}}(\omega) = \frac{e^2 |p_{\perp, \perp}|^2}{4\pi^2 m^2 \omega} \sum_{k, k'} \left| \frac{4\pi k^2}{|\partial_\omega (\epsilon^+ - \epsilon^-)|}_{k=k'} \right|^{4/3} \] (4)

where \( |p_{\perp, \perp}| \) is the matrix element for the one remaining term in the \( JDOS_{\ell, \ell'} \) sum of Eq. (3).

For each value of \( \omega > 2\tilde{V} \), there are two contributions to the integral: one originating from levels inside the unrenormalized Fermi surface \( (k_\perp) \); the other from the states occupied as a result of renormalization, i.e., outside the unrenormalized Fermi surface \( (k_\parallel) \). An example of two such transitions with the same \( \omega \) are indicated by vertical arrows in the inset of Figure 3.

Simplifying equation (4) by making an isotropic approximation and the approximation of a constant \( |p_{\perp, \perp}| \) (discussed further below), one obtains

\[ \sigma_{\text{pam}}(\omega) = \frac{e^2 |p_{\perp, \perp}|^2}{4\pi^2 m^2 \omega} \sum_{k, k'} \left| \frac{4\pi k^2}{|\partial_\omega (\epsilon^+ - \epsilon^-)|}_{k=k'} \right|^{4/3} \] (5)

Model spectra can then be calculated using the explicit solution.
PAM dispersion relation \[ \epsilon_{\pm} = \frac{E_F + \epsilon_f + \epsilon_k \pm \sqrt{(E_F + \epsilon_f - \epsilon_k)^2 + 4V^2}}{2}, \] (6)

where \( E_F \) is the Fermi level, \( \tilde{V} \) is the renormalized hybridization strength, introduced above, and \( \epsilon_f \) is the \( f \)-level position renormalized by on-site \( f \)-electron repulsion which defines the scale of the low-energy physics and is commonly identified with the impurity Kondo temperature, \( T_K \).

An example of such a calculation for \( \tilde{V} = 41 \text{ meV} \) and \( \epsilon_f = 2.5 \text{ meV} \), is shown in Figure 3a. The onset for optical transitions appears as a cusp at \( 2\tilde{V} \). While the strength of this cusp is an important feature associated with nesting, the extreme sharpness is an artifact into theory can be done by allowing a dispersive \( \tilde{V}_k \), which would tend to restore some of the high frequency conductivity that is underestimated in our simplified approach. A more sophisticated approach to \( n(T_K) \) would be valuable in the ongoing effort toward a quantitative understanding of strongly correlated electrodynamics.

Equations (9) and (2) do, however, qualitatively explain the observed dependence of the frequency and strength on \( T_K \). In addition these equations show an interesting relationship between the scaling of \( n(T_K) \) and \( \omega_{th}(T_K) \). Using the impurity model result for \( c_{\text{pam}} \), fitting the measured \( n(T_K) \) to Eq. (9), one gets a value for the band parameter \( B = 0.36 \text{ eV} \). This value is determined solely by the extrapolation of the data to the \( T_K \) axis, and so does not depend on \( k_F \) or the matrix elements present in the coefficient of the logarithm. One can independently obtain a value for \( B \) from fitting the \( T_K \) dependence of the excitation frequency (Figure 2a), \( \omega_{th} = 2\tilde{V} = 2\sqrt{T_K B} \), which leads to \( B = 0.30 \text{ eV} \). The observation that these independently obtained values of \( B \) are comparable shows a non-trivial relationship between the scaling behavior of strength and characteristic frequency in our optical data. Overall our results indicate that \( T_K \) scaling is present in the low temperature finite-frequency dynamics of YbIn\( _{1-x} \)Ag\( _x \)Cu\( _4 \) and can be addressed in the context of local-moment models. Further work may be directed toward greater understanding of this low-\( T \) scaling behavior as well as unexplained temperature dependent behavior of YbIn\( _{1-x} \)Ag\( _x \)Cu\( _4 \).

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[21] For $N = 2j + 1 = 8$, the Fermi liquid theory of the Anderson impurity model predicts $\tilde{\epsilon}_f = k_BT_K/N^2\sin(2\pi/N)/w_N\pi(N - 1) \simeq 1.517T_K$. 
