Bounding the tau neutrino magnetic moment from the process $e^+e^- \rightarrow \gamma \nu \bar{\nu}$

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June 15, 2018

Abstract

In a class of $E_6$ inspired models with a light additional neutral vector boson, we discuss the effects of the magnetic moment of the tau neutrino by analyzing $e^+e^- \rightarrow \gamma \nu \bar{\nu}$ at the Z-pole. We take into account present scenarios for the extra Abelian group and vary extra neutral gauge boson mass beyond the present experimental exclusion limits. The present LEP experimental results prove us to set an upper bound on the magnetic moment of the tau neutrino: $\kappa_{\nu_\tau} \leq 1.83 \times 10^{-6}$.

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1 INTRODUCTION

Neutrinos are weakly interacting particles. They have very long mean free path. So they immediately escape from the star and effect the evolution of the star drastically. In the detection of neutrinos emerging from the sun, the experimental results do not agree with the expected rates (\(\sim 1/3\) of the expected rates) \([1]\) obtained from theoretical calculations using standard model (SM). Hence this is considered as a solar neutrino problem. On the other hand, due to the emission of neutrinos, some of the stars could not be detected in their proper places \([2]\). Magnetic moment of the neutrino is one of the properties to solve these problems. Magnetic moment can be the reason of the deflection of the neutrinos by the magnetic field of the sun.

The possible electromagnetic properties of a massive Dirac neutrino are summarized in the current \([3,4,5]\)

\[
J_{\mu} = \pi_{\nu}(q_2)[i \frac{e}{2m_\nu} F_2(q^2) + eF_3(q^2)\gamma_5] \sigma_{\mu\nu} q^\nu u_\nu(q_1)
\]

where \(F_2\) and \(F_3\) are dimensionless structure functions, corresponding to magnetic moment and electric dipole moment respectively at \(q^2 = 0\).

On the other hand, some of the experimental results for the cross section of some processes at high energies deflect from the values obtained theoretically using standard model. One example is \(e^+e^- \rightarrow \mu^+\mu^-\) scattering cross section calculated at TRISTAN and LEP-I energies and the result is \(2\sigma\) different from the theoretical calculations \([6,7]\). This discrepancy could be removed if a new intermediate vector boson (IVB) which comes out in the extension of the standard model or in GUT’s is considered \([8,9,10]\). In the successive breaking of \(E_6\) as

\[
E_6- \rightarrow SO(10)\chi XU(1)_\chi XU(1)_\psi
\]

the standard elektroweak group \(SU(3)_cXSU(2)_LXU(1)_Y\) is embedded in \(SU(5)\) subgroup of \(SO(10)\). Therefore, we can write \(Q'\) charges in terms of \(Q_\chi\) and \(Q_\psi\) which are already orthogonal to symmetries of elektromagnetism and other known interactions which are buried into \(SU(5)\). Then we write \([8,11]\)

\[
\begin{align*}
Q' &= Q_\chi \cos \vartheta + Q_\psi \sin \vartheta, \\
Q'' &= -Q_\chi \sin \vartheta + Q_\psi \cos \vartheta.
\end{align*}
\]

This new IVB \(Z_\vartheta\) is considered in our calculations together with the magnetic moment of the neutrino.

Thus, the relevant neutral current Lagrangian becomes \([12,13]\)

\[
- L_{NC} = g_1 Z_0^\mu J_{Z_0\mu} + g_2 Z_0^\mu J_{Z_\mu
\}
\]
with the currents

\[ J_{Z_0\mu} = \sum_f \overline{f} \gamma_\mu [a + b \gamma_5] f \]
\[ J_{Z_\vartheta\mu} = \sum_f \overline{f} \gamma_\mu [a' + b' \gamma_5] f \]

(4)

where \( f \) representing fermions,

\[ g_1 = (g^2 + g'^2)^{1/2} = \frac{e}{2 \sin \theta_W \cos \theta_W} = (\sqrt{2} G_\mu M_{Z_0}^2)^{1/2} \]
\[ g_2 = g_\vartheta \]

(5)

\[ a = -\frac{1}{2} + 2 \sin^2 \vartheta_w \]
\[ b = \frac{1}{2} \]

(6)

\[ a' = X \cdot \left( \frac{\cos \vartheta}{\sqrt{6}} + \frac{\sin \vartheta}{\sqrt{10}} \right) \cdot \left( -\frac{\cos \vartheta}{\sqrt{6}} + \frac{3 \sin \vartheta}{\sqrt{10}} \right) \]
\[ b' = 2 \cdot X \cdot \frac{\sin \vartheta}{\sqrt{10}} \left( -\frac{\cos \vartheta}{\sqrt{6}} + \frac{3 \sin \vartheta}{\sqrt{10}} \right) \]

(7)

and

\[ X = \left( g_\vartheta \right)^2 \cdot \left( \frac{M_{Z_0}}{M_{Z_\vartheta}} \right)^2 \]

(8)

is a parameter depending on the coupling constant \( g_\vartheta \) and the mass of \( Z_\vartheta \), where \( \vartheta \) is the mixing angle in \( E_6 \)

\[ Z_\vartheta = Z_\psi \cos \vartheta + Z_\chi \sin \vartheta. \]

(9)

For the following \( \vartheta \) values the corresponding models emerge:

\( \vartheta = 0^\circ \), \( Z_\vartheta \rightarrow Z_\psi \),
\( \vartheta = 37.8^\circ \), \( Z_\vartheta \rightarrow Z' \),
\( \vartheta = 90^\circ \), \( Z_\vartheta \rightarrow Z_\chi \),
\( \vartheta = 127.8^\circ \), \( Z_\vartheta \rightarrow Z_I \) [7].
2 CALCULATION

We calculated the differential cross section for the scattering process

\[ e^+(p_1) + e^-(p_2) \rightarrow \gamma(k) + \nu(q_1) + \bar{\nu}(q_2) \]  

(10)

at LEP energies where only the relevant four Feynman diagrams are considered and given in Fig. 1. Using the experimental result on the cross section of abovementioned process we obtain an upper bound for the magnetic moment of the tau neutrino (\( \nu_\tau \)). Charged currents can produce only \( \nu_e \) and \( \nu_\mu \) due to lepton number conservation, and if considered in the calculations, can only decrease the magnetic moment of the tau neutrino. Therefore charged current contribution is not considered. The same process is calculated at center of mass energies where \( \sqrt{s} \leq M_{Z_0} \) by D. Fargion et al. for six Feynman diagrams [14].

The four momenta of the electron, positron, photon, neutrino, and antineutrino are \( p_1, p_2, k, q_1, \) and \( q_2, \) respectively. The four momenta of \( Z_0 \) and \( Z_\vartheta \) are \( s_1, \) and \( s_2, \) respectively and are taken equal to center of mass energy \( \sqrt{s}. \)

The total matrix element for the diagrams in Fig. 1, is

\[ M_{Tot} = M_a + M_b + M'_a + M'_b \]  

(11)

where

\[ M_a = \left. \frac{-ie}{s - M_{Z_0}^2 + i(s/M_{Z_0})M_{Z_0}\Gamma_{Z_0}} \right| \gamma_\mu \left( \frac{a + b\gamma_5}{2} \right) \bar{u}_e (p_1) \]

\[ \left\{ \begin{array}{l}
\sin \theta_w \cos \theta_w \\
\sin \theta_w \cos \theta_w
\end{array} \right. \frac{i}{s - M_{Z_0}^2 + i(s/M_{Z_0})M_{Z_0}\Gamma_{Z_0}} \left( -g_{\mu\nu} + \frac{s_\mu s_\nu}{M_{Z_0}^2} \right). \]

\[ \bar{\nu}_\alpha(q_1) \mu_B \sigma_{\alpha\beta} e^\alpha k^\beta (F_2 + F_3\gamma_5) \frac{i}{\mu_{F_1} + \mu_{F_2}} \right| \gamma_\nu \left( \frac{1 - \gamma_5}{4} \right) \bar{u}_\nu (q_2) \]  

(12)

\[ M_b = \left. \frac{-ie}{s - M_{Z_0}^2 + i(s/M_{Z_0})M_{Z_0}\Gamma_{Z_0}} \right| \gamma_\mu \left( \frac{a + b\gamma_5}{2} \right) \bar{u}_e (p_1) \]

\[ \left\{ \begin{array}{l}
\sin \theta_w \cos \theta_w \\
\sin \theta_w \cos \theta_w
\end{array} \right. \frac{i}{s - M_{Z_0}^2 + i(s/M_{Z_0})M_{Z_0}\Gamma_{Z_0}} \left( -g_{\mu\nu} + \frac{s_\mu s_\nu}{M_{Z_0}^2} \right). \]

\[ \bar{\nu}_\alpha(q_1) \left[ \begin{array}{c}
\sin \theta_w \cos \theta_w \\
\sin \theta_w \cos \theta_w
\end{array} \right] \gamma_\nu \left( \frac{1 - \gamma_5}{4} \right) \frac{i}{\mu_{F_1} + \mu_{F_2}} \mu_B \sigma_{\alpha\beta} e^\alpha k^\beta (F_2 + F_3\gamma_5) \bar{u}_\nu (q_2) \]  

(13)

and for \( M'_a \) and \( M'_b \)

\[ M'_a = M_a (a \rightarrow a', b \rightarrow b', M_{Z_0} \rightarrow M_{Z_\vartheta}) \]  

(14)

\[ M'_b = M_b (a \rightarrow a', b \rightarrow b', M_{Z_0} \rightarrow M_{Z_\vartheta}) \]  

(15)
\[ |M|^2 = |M_a|^2 + |M_b|^2 + |M'_a|^2 + |M'_b|^2 + (M_a M'_a^\dagger + M_b M'_b^\dagger + M'_a M_a^\dagger + M'_b M_b^\dagger). \]

Here only the terms different from zero are written. In Eq. (16), the first two terms are due to the intermediate vector boson \( Z_0 \) and are calculated by T. M. Gould and I. Z. Rothstein [4]. The second two terms are due to \( Z_\vartheta \) which arise in the extensions of the standard model. The last four terms in parentheses are due to the mixing of \( Z_0 \) and \( Z_\vartheta \). We calculate the last six terms. In the following equations \( \Gamma_{Z_i} \) should be replaced by \( \Gamma_{Z_i} \rightarrow \frac{s}{M_{Z_i}^2} \Gamma_{Z_i} \), where \( i = 0 \) and \( \vartheta \).

Taking the sum over final particle’s polarizations and the average over the initial particle’s polarizations, the square of the matrix elements becomes

\[
\frac{1}{4} \sum_{\text{pol, spins}} |M_a|^2 = \frac{1}{4} \cdot \frac{e^4 \mu_B^2 e^\alpha e^5 k^3 k^n}{64 \cdot \sin^4 \vartheta_w \cos^4 \vartheta_w (q_2 + k)^4 ((s - M_{Z_0}^2)^2 + M_{Z_0}^2 \Gamma_{Z_0}^2)} \cdot \left[ \text{Tr}[(\not{q}_2 - m)\gamma_\mu(a + b \gamma_5)(\not{p}_1 + m)\gamma_\nu(a + b \gamma_5)] \cdot \text{Tr}[\epsilon_2 \gamma^\nu(1 - \gamma_5)(\not{p}_1 + \not{k})(F_2 - F_3 \gamma_5) \sigma_{\xi \eta} \not{q}_1 \sigma_{\alpha \beta} (F_2 + F_3 \gamma_5) \cdot (\not{q}_2 + \not{k}) \gamma^\mu(1 - \gamma_5)] \right]
\]

and

\[
\frac{1}{4} \sum_{\text{pol, spins}} |M_b|^2 = \frac{1}{4} \cdot \frac{e^4 \mu_B^2 e^\alpha e^5 k^3 k^n}{64 \cdot \sin^4 \vartheta_w \cos^4 \vartheta_w (q_2 + k)^4 ((s - M_{Z_0}^2)^2 + M_{Z_0}^2 \Gamma_{Z_0}^2)} \cdot \left[ \text{Tr}[(\not{q}_2 - m)\gamma_\mu(a + b \gamma_5)(\not{p}_1 + m)\gamma_\nu(a + b \gamma_5)] \cdot \text{Tr}[\epsilon_2 (F_2 - F_3 \gamma_5) \sigma_{\xi \eta} (\not{q}_2 + \not{k}) \gamma^\nu(1 - \gamma_5) \not{q}_1 \gamma^\mu(1 - \gamma_5) \cdot (\not{q}_2 + \not{k}) \sigma_{\alpha \beta} (F_2 + F_3 \gamma_5)] \right].
\]

The trace calculations yield

\[
MK 1 = \frac{1}{4} \sum_{\text{pol, spins}} (|M_a|^2 + |M_b|^2) = \frac{1}{4} \cdot \frac{16 \cdot \kappa^2 \cdot e^4 \cdot \mu_B^2}{8 \sin^4 \vartheta_w \cos^4 \vartheta_w ((s - M_{Z_0}^2)^2 + M_{Z_0}^2 \Gamma_{Z_0}^2)} \cdot \left[ 4(p_1.q_1)(p_2.q_2) \sin^4 \vartheta_w + 4(p_1.q_2)(p_2.q_1) \sin^4 \vartheta_w - 4(p_1.q_2)(p_2.q_1) \sin^2 \vartheta_w + (p_1.q_2)(p_2.q_1) \right],
\]

\[
MK 2 = \frac{1}{4} \sum_{\text{pol, spins}} (|M'_a|^2 + |M'_b|^2) = \frac{1}{4} \cdot \frac{16 \cdot \kappa^2 \cdot e^4 \cdot \mu_B^2 \cdot X^2}{7200 \cdot \sin^4 \vartheta_w \cos^4 \vartheta_w ((s - M_{Z_0}^2)^2 + M_{Z_0}^2 \Gamma_{Z_0}^2)}.\]
\[ 729(p_1,q_1)(p_2,q_2) \sin^4 \vartheta - 270 \cdot (p_1,q_1)(p_2,q_2) \sin^2 \vartheta \cos^2 \vartheta + 25(p_1,q_1)(p_2,q_2) \cos^4 \vartheta - 72 \cdot \sqrt{15}(p_1,q_2)(p_2,q_1) \sin^3 \vartheta \cos \vartheta - 40 \cdot \sqrt{15}(p_1,q_2)(p_2,q_1) \sin \vartheta \cos^3 \vartheta + 81(p_1,q_2)(p_2,q_1) \sin^4 \vartheta + 330(p_1,q_2)(p_2,q_1) \sin^2 \vartheta \cos^2 \vartheta + 25(p_1,q_2)(p_2,q_1) \cos^4 \vartheta, \quad (20) \]

\[
MK3 = \frac{1}{4} \sum_{\text{pol},\text{spins}} (M_a M_a' + M_b M_b' + M_a' M_a + M_b' M_b) = \\
\frac{1}{2} \cdot \frac{16}{240} \cdot \kappa^2 \cdot \varepsilon^4 \cdot \mu_B^2 \cdot X \cdot f(s, M_{Z_0}, \Gamma_{Z_0}, M_{Z_0}, \Gamma_{Z_0}) \cdot \\
[54 \cdot (p_1,q_1)(p_2,q_2) \sin^2 \vartheta_w \sin^2 \vartheta - 10 \cdot (p_1,q_1)(p_2,q_2) \sin^2 \vartheta_w \cos^2 \vartheta + 8 \cdot \sqrt{15} \cdot (p_1,q_2)(p_2,q_1) \sin^2 \vartheta_w \sin \vartheta \cos \vartheta - 4 \cdot \sqrt{15} \cdot (p_1,q_2)(p_2,q_1) \sin \vartheta \cos \vartheta - 18 \cdot (p_1,q_2)(p_2,q_1) \sin^2 \vartheta_w \sin^2 \vartheta - 10 \cdot (p_1,q_2)(p_2,q_1) \sin^2 \vartheta_w \cos^2 \vartheta + 9 \cdot (p_1,q_2)(p_2,q_1) \sin^2 \vartheta + 5 \cdot (p_1,q_2)(p_2,q_1) \cos^2 \vartheta]. \quad (21) 
\]

where

\[
f(s, M_{Z_0}, \Gamma_{Z_0}, M_{Z_0}, \Gamma_{Z_0}) = \frac{2(s-M_z^2)(s-M_z^2) + M_{Z_0} + M_Z^2 \cdot \Gamma_{Z_0} \cdot \Gamma_{Z_0})}{(s-M_{Z_0}^2)(s-M_{Z_0}^2) + M_{Z_0} \cdot \Gamma_{Z_0} \cdot \Gamma_{Z_0})^{(s-M_{Z_0}^2) \cdot M_{Z_0} \cdot \Gamma_{Z_0} - (s-M_{Z_0}^2) \cdot M_{Z_0} \cdot \Gamma_{Z_0})^2}. \]

Since we are interested with the magnetic moment in all of the above calculations we considered $F_3 = 0$ and $F_2 = \kappa$ and $m_e^2 \ll s$.

Now, using the differential cross section

\[
d\sigma = \frac{|M|^2}{4(p_1 \cdot p_2)^2 - m_em_\gamma \gamma} \frac{d^3q_1}{2q_{10}(2\pi)^3} \frac{d^3q_2}{2q_{20}(2\pi)^3} \frac{d^3k}{2k_0(2\pi)^3} (2\pi)^4 \delta^4(p_1 + p_2 - q_1 - q_2 - k). \quad (22) 
\]

with Lenard’s formula

\[
\int \frac{d^3q_1}{2q_{10}} \frac{d^3q_2}{2q_{20}} q_{1\mu} q_{2\nu} \delta^4(P - q_1 - q_2) = \frac{\pi}{24} [2P_P + g_{\mu\nu} P^2], \quad (23) 
\]

we obtain

\[
d\sigma = d(\sigma_1 + \sigma_2 + \sigma_3) = E_\gamma dE_\gamma d(\cos\vartheta_\gamma) \cdot \kappa^2 \alpha^2 \mu_B^2 \cdot \\
\{ - \frac{1}{96\pi \sin^4 \vartheta \cos^4 \vartheta \cdot \frac{1}{[(s-M_{Z_0}^2)^2 + M_{Z_0}^2 \cdot \Gamma_{Z_0}^2]} \}
\frac{1}{4\sqrt{s}(8 \sin^4 \vartheta_w - 4 \sin^2 \vartheta_w + 1) E_\gamma + 
\]

5
Using $\sigma$ mixing of $Z \leq \kappa$ for $\vartheta$ various values of our results [7]. We have calculated the cross section due to their magnetic moments for $\vartheta$ numerical values, $\sin \vartheta = 0.2314$, $\mu_B = 5.85 \times 10^{-18}$ GeV/Gauss, $\alpha = \frac{1}{137}$, $M_{Z_0} = 91.187$ GeV, $s = M^2_{Z_0}$, $X = \frac{1.15}{3} \cdot (\frac{M_{Z_0}}{M_{Z_0}})^2$, $\Gamma_{Z_0} = \Gamma_{Z_0} = 2.49$ GeV,

we obtain the cross section

$$\sigma = \sigma(\kappa, M_{Z_0}, \vartheta).$$

(25)

Since we have calculated the cross section at the Z- pole, that is at $s = M^2_{Z_0}$, the value of $\sin^2 \vartheta = 0.2314$, is not affected by $Z_0$ physics [15,16], and we take $\sin^2 \vartheta = 0.2314$. And also we have calculated the cross section due to their magnetic moments for $\vartheta$ contribution is small at $s = M^2_{Z_0}$, taking $\Gamma_{Z_0} = \Gamma_{Z_0}$ or $\Gamma_{Z_0} = 0$ does not effect our results [7]. We have calculated the cross section corresponding to different models of new physics due to superstring-inspired $E_8$ theories [8] and the numerical results for the cross sections for all neutrino interactions are given in Table 1.

The cross sections $\sigma_1$, $\sigma_2$, $\sigma_3$, correspond to $Z_0$ (SM), new intermediate vector boson $Z_0$, mixing of $Z_0$ and $Z_0$, respectively. The total cross section is $\sigma = \sigma_1 + \sigma_2 + \sigma_3$.

Using $\sigma_{\text{exp}} L = N$ and taking the experimental results [17], $L=48.0$ pb$^{-1}$, $N=14$, we obtain a limit for the neutrino magnetic moment $\kappa$ in terms of $\mu_B$ as

$$\kappa \leq 1.83 \times 10^{-6}$$

for $\sigma$ for $M_{Z_0} = 7 \times M_{Z_0}$, and $\vartheta = 37.8$. 

6
3 DISCUSSION AND CONCLUSION

We have seen that, there is no much difference for various $M_{Z\vartheta}$ and $\vartheta$ values in the calculation of the cross section in the content of this study. The only important contribution from $Z\vartheta$ particle is at $\vartheta = 37.8^\circ$ which corresponds to $Z\vartheta \rightarrow Z'$. That is, it gives negative contribution to the cross section.

We conclude that there is no further constraint on the magnetic moment of the neutrino due to $Z\vartheta$ since its mass is higher than $Z_0$ at $\sqrt{s} = M_{Z_0}$. But at higher center-of-mass energies $\sqrt{s} \sim M_{Z\vartheta}$, the $Z\vartheta$ contribution to the cross section becomes comparable with $Z_0$.

ACKNOWLEDGMENT

We thank Professor T. Aliev for the suggestion of the problem, and also thank Dr. D. A. Demir for helpful discussions.
FIGURE CAPTIONS

FIGURE 1. The lowest order Feynman diagrams for the pair annihilation process.
TABLE CAPTIONS

TABLE 1. Scattering cross section for various $\vartheta$ and $M_{Z_0}$ values at $\sqrt{s} = M_{Z_0}$. The unit of $\sigma$ is cm$^2$.
References

[1] R. J. N. Phillips, "Solar Neutrinos" Rutherford Appleton Laboratory, (September 1987).

[2] E. D. Commins, "Weak Interactions" McGraw-Hill, Inc. (1973).

[3] R. N. Mohapatra, P. B. Pal, "Massive Neutrinos in Physics and Astrophysics" World Scientific Pub. (1991).

[4] T. M. Gould, I. Z. Rothstein, Phys. lett. B 333, 545 (1994).

[5] R. Escribano, E. Masso, hep-ph/9403304.

[6] P. Osland, A. A. Pankov, Phys. lett. B 403, 93 (1997).

[7] P. Osland, A. A. Pankov, Phys. lett. B 406, 328 (1997).

[8] London, Rosner, Phys. Rev. D 34, 1530 (1986).

[9] P. Langacker, M. Luo, Phys. Rev. D 45, 278 (1992).

[10] G. Altarelli, R. Casalbuoni, S. De Curtis, and et al., Phys. Lett. B 318, 139 (1993).

[11] S. Capstick, S. Godfrey, Phys. Rev. D 35, 3351 (1987).

[12] A. Leike, hep-ph/9805494.

[13] S. Alam, J. D. Anand, S. N. Biswas, and A. Goyal, Phys. Rev. D 40, 2712 (15 Oct 1989).

[14] D. Fargion, R. V. Konoplich, and R. Mignani, Phys. Rev. D 47, 751 (1993).

[15] P. Langacker, M. Luo, and A. K. Mann, Reviews of Modern Physics, 64, 105 (Jan 1992).

[16] D. A. Demir, hep-ph/9809361.

[17] P. Mättig, CERN-PPE/95-081.
Fig. 1- The lowest order Feynman diagrams for the pair annihilation process.
TABLE I. Scattering cross sections for various $\vartheta$ and $M_{Z_0}$ values at $\sqrt{s} = M_{Z_0}$. The unit of $\sigma$ is cm$^2$.

| $\frac{M_{\varphi}}{M_{Z_0}}$ | $\frac{\sigma_{\vartheta}}{\kappa^2}$ | $\frac{\sigma_{\varphi}}{\kappa^2}$ | $\frac{\sigma_{\varphi}}{\kappa^2}$ | $\frac{\sigma}{\kappa^2}$ |
|---|---|---|---|---|
| $\vartheta = 0$ | | | | |
| 5 | $8.682 \times 10^{-26}$ | $2.913 \times 10^{-36}$ | $1.696 \times 10^{-35}$ | $8.682 \times 10^{-26}$ |
| 6 | $8.682 \times 10^{-26}$ | $6.605 \times 10^{-37}$ | $4.615 \times 10^{-36}$ | $8.682 \times 10^{-26}$ |
| 7 | $8.682 \times 10^{-26}$ | $1.896 \times 10^{-37}$ | $1.545 \times 10^{-36}$ | $8.682 \times 10^{-26}$ |
| 8 | $8.682 \times 10^{-26}$ | $6.450 \times 10^{-38}$ | $6.009 \times 10^{-37}$ | $8.682 \times 10^{-26}$ |
| 9 | $8.682 \times 10^{-26}$ | $2.497 \times 10^{-38}$ | $2.617 \times 10^{-37}$ | $8.682 \times 10^{-26}$ |
| 10 | $8.682 \times 10^{-26}$ | $1.070 \times 10^{-38}$ | $1.246 \times 10^{-37}$ | $8.682 \times 10^{-26}$ |
| 11 | $8.682 \times 10^{-26}$ | $4.974 \times 10^{-39}$ | $6.372 \times 10^{-38}$ | $8.682 \times 10^{-26}$ |
| $\vartheta = 37.8$ | | | | |
| 5 | $8.682 \times 10^{-26}$ | $1.982 \times 10^{-36}$ | $-1.322 \times 10^{-35}$ | $8.682 \times 10^{-26}$ |
| 6 | $8.682 \times 10^{-26}$ | $4.495 \times 10^{-37}$ | $-3.597 \times 10^{-36}$ | $8.682 \times 10^{-26}$ |
| 7 | $8.682 \times 10^{-26}$ | $1.290 \times 10^{-37}$ | $-1.204 \times 10^{-36}$ | $8.682 \times 10^{-26}$ |
| 8 | $8.682 \times 10^{-26}$ | $4.390 \times 10^{-38}$ | $-4.683 \times 10^{-37}$ | $8.682 \times 10^{-26}$ |
| 9 | $8.682 \times 10^{-26}$ | $1.700 \times 10^{-38}$ | $-2.040 \times 10^{-37}$ | $8.682 \times 10^{-26}$ |
| 10 | $8.682 \times 10^{-26}$ | $7.281 \times 10^{-39}$ | $-9.710 \times 10^{-38}$ | $8.682 \times 10^{-26}$ |
| 11 | $8.682 \times 10^{-26}$ | $3.385 \times 10^{-39}$ | $-4.965 \times 10^{-38}$ | $8.682 \times 10^{-26}$ |
| $\vartheta = 90$ | | | | |
| 5 | $8.682 \times 10^{-26}$ | $3.675 \times 10^{-35}$ | $7.869 \times 10^{-34}$ | $8.682 \times 10^{-26}$ |
| 6 | $8.682 \times 10^{-26}$ | $8.333 \times 10^{-36}$ | $2.141 \times 10^{-34}$ | $8.682 \times 10^{-26}$ |
| 7 | $8.682 \times 10^{-26}$ | $2.391 \times 10^{-36}$ | $7.170 \times 10^{-35}$ | $8.682 \times 10^{-26}$ |
| 8 | $8.682 \times 10^{-26}$ | $8.138 \times 10^{-37}$ | $2.788 \times 10^{-35}$ | $8.682 \times 10^{-26}$ |
| 9 | $8.682 \times 10^{-26}$ | $3.151 \times 10^{-37}$ | $1.214 \times 10^{-35}$ | $8.682 \times 10^{-26}$ |
| 10 | $8.682 \times 10^{-26}$ | $1.350 \times 10^{-37}$ | $5.781 \times 10^{-36}$ | $8.682 \times 10^{-26}$ |
| 11 | $8.682 \times 10^{-26}$ | $6.275 \times 10^{-38}$ | $2.956 \times 10^{-36}$ | $8.682 \times 10^{-26}$ |
| $\vartheta = 127.8$ | | | | |
| 5 | $8.682 \times 10^{-26}$ | $3.135 \times 10^{-35}$ | $7.398 \times 10^{-34}$ | $8.682 \times 10^{-26}$ |
| 6 | $8.682 \times 10^{-26}$ | $7.108 \times 10^{-36}$ | $2.013 \times 10^{-34}$ | $8.682 \times 10^{-26}$ |
| 7 | $8.682 \times 10^{-26}$ | $2.040 \times 10^{-36}$ | $6.740 \times 10^{-35}$ | $8.682 \times 10^{-26}$ |
| 8 | $8.682 \times 10^{-26}$ | $6.942 \times 10^{-37}$ | $2.621 \times 10^{-35}$ | $8.682 \times 10^{-26}$ |
| 9 | $8.682 \times 10^{-26}$ | $2.688 \times 10^{-37}$ | $1.142 \times 10^{-35}$ | $8.682 \times 10^{-26}$ |
| 10 | $8.682 \times 10^{-26}$ | $1.151 \times 10^{-37}$ | $5.435 \times 10^{-36}$ | $8.682 \times 10^{-26}$ |
| 11 | $8.682 \times 10^{-26}$ | $5.353 \times 10^{-38}$ | $2.779 \times 10^{-36}$ | $8.682 \times 10^{-26}$ |