Possible peculiarities of synchrotron radiation in a strong magnetic field

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Abstract

Relativistic quantum effects on physical observables of scalar charged particles are studied. Possible peculiarities of their behavior that can be verified in an experiment can confirm several fundamental conceptions of quantum mechanics. For observables independent of charge variable, we propose relativistic Wigner function formalism that contains explicitly the measurement device frame. This approach can provide the description of charged particles gas (plasma). It differs from the traditional one but is consistent with the Copenhagen interpretation of quantum mechanics. The effects that are connected with this approach can be observed in astrophysical objects - neutron stars.

I. INTRODUCTION

The wave function nature has been considered as philosophical rather than physical question for a long time. However it is very actual now because of the recent theoretical and experimental progress in quantum information [12].

In 1980, the specific behavior of the quantum systems that are described by Einstein-Podolsky-Rosen (EPR) paradox was confirmed in an experiment [1]. It is very important that EPR correlation "spread" in a space instantly. Nevertheless, if one adheres to the Copenhagen interpretation there is no a casualty principle breaking.

In a contrast if one can try to hold an objective and deterministic description of quantum mechanics then classical understanding of the casualty principle should be broken, because of the conflict between quantum mechanics and special relativity [2,6]. In [4] the relativistic classical and quantum mechanics that generalizes the relativity principle was constructed. Such an approach follows from the Eberhard and Bell idea that correct description of quantum mechanics should contain a preferred frame.

This theory has a well definite position operator. In the preferred frame it coincides with the Newton-Wigner coordinate [7]. It means that measurement of the coordinate does not create a particle-antiparticle couple because of there is no an odd part.

Hence, it is very important to find situations when the odd part of the position operator could manifest itself. But unfortunately on the Earth, such experiments are very difficult due to a very strong field needed. There are such fields near the astrophysical objects - neutron stars. Therefore, it is interesting to find how the odd part of the position operator can influence the observable variables in a many-body system (in a gas of charged particles or plasma).

The Wigner function formalism [14] is a convenient method to describe such systems. But there are several problems to generalize it to the relativistic case. The first problem is that the time is not a dynamical variable in the Weyl rule. In [5] this problem was resolved by the generalization of the spatial integration over the whole space-time. The Wigner function formulation in a framework of the stochastic formulation of quantum mechanics is Lorentz invariant too [10].

Formalism of the matrix-valued Wigner function for spin 1/2 particles was developed in [8] with usual Weyl rule. Certainly, such equations are not Lorentz invariant.
Next problem is the absence of well-defined position operator. In [13] the Wigner function formalism was developed using the Newton - Wigner coordinate. The results in this approach differ from the standard one. However one can connect they with [4] where the correctly definition of the position operator is possible.

The aim of this work is a formulation of the Wigner formalism for scalar charged particles in the approach [8]. In addition we try to find several peculiarities of the behavior of relativistic quantum system including those ones with the complicated structure of the position operator.

II. WIGNER FUNCTION FOR A FREE PARTICLE

To develop the Wigner function formalism one needs to formulate the Weyl rule. Following [8] should take into account that classical variables are matrixes. But we restrict ourselves with those proportional to the identity matrix. For a convenience we shall use the Feshbach - Villars representation [7]. The Weyl rule is defined by usual way:

\[ \hat{A}_\alpha^\beta = \int A(p, q) \hat{W}_\alpha^\beta(p, q) dp dq, \]

where \( \alpha, \beta = \pm 1 \), \( A(p, q) \) is the classical variable, \( \hat{A}_\alpha^\beta \) is the corresponding classical variable, \( \hat{W}_\alpha^\beta \) is the operator of quasi-probability density that can be presented via the displacement operator:

\[ \hat{W}_\alpha^\beta(p, q) = \frac{1}{(2\pi\hbar)^2} \int \hat{D}_\alpha^\beta(P, Q) e^{i\hat{\mathbf{q}}\cdot\hat{\mathbf{P}} - \frac{\hbar}{2}(Qp - Pq)} dQdP. \]  \( (1) \)

In this representation the displacement operator can be expanded by eigenvectors of the momentum operators:

\[ \hat{D}_\alpha^\beta(P, Q) = \int \left| p + \frac{P}{2} \right> R_\alpha^\beta(p + \frac{P}{2}, p - \frac{P}{2}) e^{-\frac{\hbar}{2}(QP - Pq)} dp \left< p - \frac{P}{2} \right|. \]  \( (2) \)

In contrast to [13] and non-relativistic case there is a matrix-valued variable in (2):

\[ R_\alpha^\beta(p_1, p_2) = \varepsilon(p_1, p_2)\delta_\alpha^\beta + \chi(p_1, p_2)\tau_\alpha^\beta. \]

It contains even and odd parts and is expressed via the relativistic energy of a free particle \( E(p) \):

\[ \varepsilon(p_1, p_2) = \frac{E(p_1) + E(p_2)}{2\sqrt{E(p_1)E(p_2)}} \]
\[ \chi(p_1, p_2) = \frac{E(p_1) - E(p_2)}{2\sqrt{E(p_1)E(p_2)}}. \]  \( (3) \)

Now combining (1) and (2) we obtain the formula for the operator of quasi-probability density expansion:

\[ \hat{W}_\alpha^\beta(p, q) = \frac{1}{(2\pi\hbar)^2} \int \left| p + \frac{P}{2} \right> R_\alpha^\beta(p + \frac{P}{2}, p - \frac{P}{2}) e^{-\frac{\hbar}{2}(Pq)} dp \left< p - \frac{P}{2} \right|. \]  \( (4) \)
The Wigner function is the average of this operator on an arbitrary state:

\[ W(p,q) = \sum_{\alpha,\beta} \langle \psi_\beta | \hat{W}_{\alpha \beta}(p,q) | \psi_\alpha \rangle. \]  

(5)

This expression contains four terms. Two of them are the average of the even part of the operator of quasi-probability density and two others are the average of the odd part. Now one can introduce the symbols:

\[ W_{\alpha \beta}(p,q) = \langle \psi_\beta | \hat{W}_{\alpha \beta}(p,q) | \psi_\beta \rangle. \]  

(6)

It should be noted that \( W_{\alpha \beta}(p,q) \) is not the matrix-valued Wigner function in the sense [8].

Using the expressions (4) and (5) the components of the Wigner function are obtained in the form:

\[
W_{\alpha \alpha}(p,q) = \frac{1}{(2\pi \hbar)^2} \int \varepsilon(p + \frac{P}{2}, p - \frac{P}{2}) \psi^{*}_\alpha(p + \frac{P}{2}) \psi^{\alpha}(p - \frac{P}{2}) e^{-2iPq} dP
\]

\[
W_{\alpha -\alpha}(p,q) = \frac{1}{(2\pi \hbar)^2} \int \chi(p + \frac{P}{2}, p - \frac{P}{2}) \psi^{*}_\alpha(p + \frac{P}{2}) \psi^{\alpha}(p - \frac{P}{2}) e^{-2iPq} dP.
\]  

(7)

One can obtain the quantum Liouville equation by standard way [14]:

\[
\frac{\partial W_{\alpha \alpha}(p,q,t)}{\partial t} = \alpha^2 \frac{E(p)}{\hbar} \sin\left\{ -\frac{\hbar}{2} \frac{\partial}{\partial p} \frac{\partial}{\partial q} \right\} W_{\alpha \alpha}(p,q,t)
\]

\[
\frac{\partial W_{\alpha -\alpha}(p,q,t)}{\partial t} = i \alpha^2 \frac{E(p)}{\hbar} \cos\left\{ -\frac{\hbar}{2} \frac{\partial}{\partial p} \frac{\partial}{\partial q} \right\} W_{\alpha -\alpha}(p,q,t).
\]  

(8)

The equation for the even part of the Wigner function coincides with the similar expression in the Newton - Wigner coordinate approach [13]. Hence the dynamics of the distribution function in both cases are identical. The difference is in the constraints of the initial conditions. The physical variables that contain higher moments of the coordinate (for example dispersion) differs from those in [13]. For one particle problem these peculiarities were developed in [3].

\section*{III. PARTICLES IN A HOMOGENEOUS MAGNETIC FIELD}

The particles in external electromagnetic fields are more sensitive to the odd part of the coordinate. For example, in a uniform electric field the odd part of the position in the Hamiltonian of interaction results in the effects of particles creation from the vacuum [9]. The origin of this peculiarity is that even and odd parts of the Wigner function are entangled in equations like (8).

Here we shall study the behavior of particles in a time-independent and homogeneous magnetic field that is more typical for astrophysical objects. Following to [11] we shall use the energy representation and so we have to consider quasi-particles rather than particles. Both the position and momentum operators have odd parts in this approach.

Further we do not take into account the particle motion along the magnetic field and consider only relativistic rotator. Following to the previous paragraph one can write the displacement operator in the energy representation:

\[
D_{n,m;\alpha \beta}(P,Q) = (\varepsilon_{n,m} \delta_{\alpha \beta} + \chi_{n,m} \tau_{1 \alpha \beta}) D_{n,m}(P,Q),
\]  

(9)
where $D_{n,m}(P,Q)$ are the matrix elements of the usual displacement operator on the eigenfunctions of the harmonic oscillator [13] and $\varepsilon_{n,m}, \chi_{n,m}$ are defined like (8) but with the spectrum of the relativistic rotator in place of the energy of a free particle. Then the operator of quasi-probability density and the Wigner function are defined in the way presented in the previous paragraph. The final expressions for the even and odd components of the Wigner function are

$$W_{\alpha \alpha}(p,q) = \sum_{m,n} \varepsilon_{m,n} C_{n;m}^\alpha C_{m}^{* \alpha} T_{m,n}(p,q),$$
$$W_{\alpha -\alpha}(p,q) = \sum_{m,n} \chi_{m,n} C_{n;m}^\alpha C_{m}^{*-\alpha} T_{m,n}(p,q).$$

(10)

Here $C_{n;\alpha}$ is the wave function in the energy representation, $T_{m,n}(p,q)$ is the matrix elements of the usual operator of quasi-probability density

$$\hat{T}(p,q) = \frac{1}{(2\pi\hbar)^2} \int \hat{D}(P,Q) e^{-\hat{\pi}(Pq-Qp)} dPdQ.$$

(11)

The equations for the Wigner function (12) can be obtained in the standard way too. Here the different components are not entangled. Hence there are no any effects connected with vacuum instability [9]:

$$\frac{\partial W_{\alpha \alpha}(p,q,t)}{\partial t} = \alpha^2 \hbar E(p,q) \sin\left\{\frac{\hbar}{2}(\overleftarrow{\partial}_q \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \overrightarrow{\partial}_q)\right\} W_{\alpha \alpha}(p,q,t),$$
$$\frac{\partial W_{\alpha -\alpha}(p,q,t)}{\partial t} = i\alpha^2 \hbar E(p,q) \cos\left\{\frac{\hbar}{2}(\overleftarrow{\partial}_q \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \overrightarrow{\partial}_q)\right\} W_{\alpha -\alpha}(p,q,t).$$

(12)

In this expression we introduce the Weyl symbol for the Hamiltonian of the relativistic rotator in the energy representation (it should be redefined):

$$E(p,q) = mc^2\sqrt{1 + \frac{2}{mc^2}\left(\frac{\hbar^2}{2m}\omega_c^2 + \omega_c^2 m^2\right)},$$

where $\omega_c = \frac{eB}{m}$ is the cyclotron frequency, and the square root is defined in the sense of star-product $\star \equiv e^{i\frac{\hbar}{2}(\overleftarrow{\partial}_q \overrightarrow{\partial}_p - \overleftarrow{\partial}_p \overrightarrow{\partial}_q)}$.

The odd part in the Wigner function definition describes interference effects between particles and antiparticles. Furthermore the value $\varepsilon_{nm}$ defines the specification of the initial conditions.

In [11] we studied dynamics of the one particle problem. It was shown that in fields less than critical ($\hbar\omega_c < mc^2$) the mean radius of the trajectory oscillates with the frequency $\Omega = \frac{\hbar\omega_c}{mc^2}$. This is essentially quantum relativistic effect. It can be observed as a low frequency modulation of synchrotron radiation. But unfortunately such dynamical process is identical for both approaches and does not reveal the complicated structure of the position operator.

Now let us consider dispersion of the orbit radius for the nonlinear coherent state [11]:

$$\Delta R^2 = a^2 - \overline{R}^2 \left[1 - |C_o|^2 \sum_n \frac{\overline{R}^{2n}}{2^n n! \varepsilon_{n+1,n+2}^2}\right],$$

where $C_o$ is the normalization factor, $a^2 = \frac{\hbar}{m\omega_c}$. It contains both the standard and additional terms. They result in the appearance of the states with formally broken uncertainty relation. One can expect that such effects take place for many-body systems too.
IV. CONCLUSION

The odd part of the position operator results in the non-standard behavior of the physical observables. The whole system has peculiarities too. However it is observed not for all physical variables. For example, behavior of energy does not contain such peculiarities. Hence one can expect these effects for the quadratic and higher moments of the coordinate and momentum.

Especially one should notice the effects connected with interference between particles and antiparticles. They result from the odd part of the Wigner function and can be observed in systems of particles with opposite charge signs.

Finally we briefly note that relativistic quantum mechanics in a Wigner formulation contain the measurement device frame. Actually one can write the equation (8) and (12) using only four dimensional Lorentz invariant symbols. To make it possible one should incorporate into equations certain time-like vector. It can be interpreted as a four-velocity of the frame where a wave packet reduction happens relatively to the second (immobile) observer. It is very important that quantum mechanics equations contain explicitly the observer characteristics. This fact can serve as an additional argument in favor of the Copenhagen interpretations of quantum mechanics.
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