Online Route Planning for Cooperative Area Coverage Search of Aircraft Swarm

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Abstract. Aiming at the problem of cooperative search for aircraft swarm in an unknown environment without prior information, a cooperative search algorithm with the coverage rate as the optimization objective is proposed, based on the Model Predictive Control theory and the Differential Evolution algorithm. Firstly, the Area Coverage Map (ACM) is established to describe the mission area, and a rapid method of updating the ACM based on Hadamard product is given. Then, the search effect is measured by coverage rate calculated based on the ACM. We regard the aircraft swarm as a control system, and establish a systematic prediction model. The maximum coverage rate in the predicted period is defined as the optimization goal, and the DE is used to solve this problem. Finally, the simulation results verify the effectiveness of the proposed method.

1. Introduction
Aircraft swarm is a flight system composed of a number of manned and unmanned aerial vehicles with different functions, which can cooperate with each other and have the ability to emerge as a whole\textsuperscript{[1-4]}. Carrying out regional search missions using aircraft swarm has important value of application, such as searching the distribution of targets in the enemy area or searching and rescuing in the desert region.

In the traditional way, in \textsuperscript{[5-7]}, the mission area was divided in several parts and the search routes in each sub-area was designed. These methods are usually pre-calculation, offline planning, and the flight route is fixed. Due to the uncertainty of the environment and the unexpected situation, the cooperative area search is a dynamic process\textsuperscript{[8]}. Online route planning is needed according to real-time information and aircraft state in search process.

Ref. \textsuperscript{[9]} classified the environment based on a hexagonal grid, using information entropy to describe the environmental uncertainty. However, the establishment of search probability maps depends on prior information. In \textsuperscript{[10]}, the environment was described using Cartesian grids, and each grid was given a value to represent the uncertainty of the target distribution. The search reward function and the no-fly zone avoidance strategy were designed. Based on \textsuperscript{[10]}, Ref. \textsuperscript{[11]} considered communication constraints and analyzed the influence of different communication constraints on the efficiency of cooperative search. Although both of them consider the maneuvering limitations of the UAV, it is assumed that the flight of the UAV between the grids resulting in more turnovers and larger turn angles. Ref. \textsuperscript{[12]} effectively reduce the size of the solution of cooperative search decision-making problems based on distributed model predictive control framework, using the algorithm based on the combination of Nash
optimization and Particle Swarm Optimization (PSO). In general, the above method still has some limitations in practical application:

a) Relying on a priori information to establish environmental search map;
b) Aircraft moves across the grid, and every turn angle is fixed;
c) Coverage rate of the area is still to be improved.

In view of the above problems, this paper mainly studies: how to control the aircraft swarm effectively implementing online route planning in the unknown environment without any prior information, and search the mission area with the largest coverage rate.

2. Model of the problem

Without the priori information of the targets and their location within the region, aircraft swarm carrying communication equipments and sensors to search for a specific mission area, as shown in Fig. 1. In order to search all the targets in the area as soon as possible, the maximum coverage rate of the mission area needs to be achieved. Once an aircraft being shot down by enemy fire [8], traditional offline planning methods are not robust enough to perform adaptive search missions.

![Figure 1. Mission area](image1)

Let the mission area $\Omega$ be a rectangular area of $L_x \times L_y$, rasterize the area into $M_x \times N_y$ grids and construct the Area Coverage Map (ACM). As the search progresses, the ACM updated and shared among the aircraft swarm in real time, providing decision information for each aircraft. Each grid is given a value $\mu(i, j, k)$, which is used to describe whether or not the grid $(i, j)$ has been searched at time $k$, as shown in Fig. 2.

![Figure 2. Mission area rasterization](image2)
For the sake of research, we made the following assumption: Once the grid \((i, j)\) is within the scope of the aircraft’s sensor detection, it is considered that the grid has been searched and the target distribution within the grid is absolutely known. Based on this assumption, the mission area \(\Omega\) can be divided into two areas: searched area \(\Omega_c(k)\) and unsearched area \(\Omega_u(k)\). \(\mu(i, j, k)\) can be expressed as

\[
\mu(i, j, k) = \begin{cases} 
0, & (i, j) \in \Omega_c(k) \\
1, & (i, j) \in \Omega_u(k) 
\end{cases}
\]  

(1)

The ACM can be defined as

\[
ACM = \{\mu(i, j, k) | (i, j) \in \Omega\}
\]  

(2)

Represent the ACM as a matrix and establish an environment matrix

\[
C_e(k) = [\mu(i, j, k)]_{N \times N}
\]  

(3)

We assumed that the aircraft is flying at a constant altitude over the mission area and that the aircraft is considered as a mass of motion in a two-dimensional space whose motion equation is

\[
\begin{align*}
\chi(k+1) &= \chi(k) + v_x \Delta t \cos \phi_i(k+1) \\
y(k+1) &= y(k) + v_y \Delta t \sin \phi_i(k+1) \\
\phi_i(k+1) &= \phi_i(k) + \Delta \phi_i(k) \\
\Delta \phi_i(k) &= [\varphi_{max}, \varphi_{max}]
\end{align*}
\]  

(4)

Where, \([\chi(k), y(k)]\) is aircraft position; \(v_o\) is flying speed; \(\Delta t\) is time step; \(\phi_i\) is heading angle; \(\Delta \phi_i\) is heading angle increment for the decision input; \(\varphi_{max}\) is the maximum mobility under the turning angle. The state of the aircraft at time \(k\) is \(p(k) = (x(k), y(k), \phi_i(k))\). Aircraft swarm consisting of \(n\) aircrafts can be regarded as a control system, the state of the system is \(P(k) = (p_1(k), p_2(k), \ldots, p_n(k))\), the equation of state is

\[
P(k+1) = f(P(k), u(k))
\]  

(5)

Among them, \(u(k) = (\Delta \phi_1(k), \Delta \phi_2(k), \ldots, \Delta \phi_n(k))\) is the input of the system; \(f(\cdot)\) is the state transfer function of the system, determined by the equation of motion of the aircraft.

From the moment \(k\), the position of all aircraft can be predicted, given multiple predictive control inputs. By optimizing the control inputs, it leads the aircraft swarm to cover as many unsearchable areas as possible. According to the thought above and system state equation, a systematic prediction model is established

\[
P(k + j + 1|k) = f(P(k + j|k), u(k + j|k))
\]  

\(j = 0, 1, 2, \ldots, H - 1\)

(6)

Where \(H\) is the prediction period and \(P(k + j + 1|k)\) is the system state at \(k+j+1\) based on the prediction of the system state at time \(k\), the value of which depends on the system state \(P(k + j|k)\) and the control input \(u(k + j|k)\).

3. ACM Modeling and updating

Coverage rate is used to measure the pros and cons of search methods as an evaluation index of search mission. During the mission, the increase of coverage rate from \(k\) to \(k+1\) can evaluate the merits and demerits of decision-making course at time \(k\). Therefore, how to calculate coverage rate quickly and accurately is of great significance to mission evaluation and real-time decision-making.
In this paper, the environment matrix is updated rapidly by using the Hadamard product of the environmental sub-matrix and the detection matrix, so as to update the ACM. Through the perception of the ACM, the aircraft can draw the current and incremental coverage rate of the next decision-making, which can provide basis for the decision-making.

In this paper, the detection range of the sensor is simplified to a circular area with $R_s$ radius centered on the location of the aircraft. The circumscribed square area of the detection area is denoted by $SQ$ and the area $SQ$ is rasterized by $M_u \times M_u$ grids as shown in Fig. 3. $M_u$ can be expressed as

$$M_u = 2 \times \left\lfloor \frac{R_s}{\Delta d} \right\rfloor + 1$$  \hspace{1cm} (7)

Where, $R_s$ is sensor detection radius; $\Delta d$ is raster fixed interval; $\left\lfloor \cdot \right\rfloor$ is rounded up function.

![Detection area rasterization](image)

Each grid is assigned a value of $\eta(i, j)$, which describes whether the grid sensor can be detected. Assuming that the area $SQ$ is divided into two areas: detectable area $SQ_c$ and undetectable area $SQ_{nc}$. If the grid $(i, j)$ is within the scope of the aircraft sensor detection, $(i, j) \in SQ_c$, conversely, $(i, j) \in SQ_{nc}$. $\eta(i, j)$ can be expressed as:

$$\eta(i, j) = \begin{cases} 0, & (i, j) \in SQ_c, \\ 1, & (i, j) \in SQ_{nc}, \end{cases}$$  \hspace{1cm} (8)

Establishing the detection matrix $D_e = [\eta(i, j)]_{i,j=M_u}$, which is the capabilities of aircraft searching for adjacent grids. When the aircraft is orbiting at a fixed altitude, the sensor detection radius $R_s$ is constant. Therefore, the detection matrix $D_e$ does not change with time $k$ and is uniquely determined.

Define environment sub-matrix $C_{e^{(m,n)}}(k)$: In the environmental matrix $C_e(k)$, the sub-blocks with the element $\mu(m_u, n_u, k)$ as the central element and the dimension $M_u \times M_u$ are called the environmental sub-matrix $C_{e^{(m,n)}}(k) = [\mu^{(m,n)}(i,j,k)]_{M_u \times M_u}$, $(m_u, n_u) \in \Omega, \ (i, j = 1, 2, \cdots M_u)$. The relationship between the element $\mu^{(m,n)}(i,j,k)$ in $C_{e^{(m,n)}}(k)$ and the element $\mu(i,j,k)$ in $C_e(k)$ can be expressed as

$$\mu^{(m,n)}(i,j,k) = \mu(i+m_u-M_u-1, j+n_u-M_u-1, k)$$  \hspace{1cm} (9)
When the aircraft is within grid \((m, n)\), it is approximately assumed that the aircraft is in the center of the grid. In this case, the environment matrix \(C_{e}^{(m, n)}(k)\) overlaps with the detection matrix \(D_{u}\), and the dimensions are equal. Hadamard product of the above two matrices

\[
C_{e}^{(m, n)}(k + 1) = C_{e}^{(m, n)}(k) \circ D_{u} = \left[\mu^{(m, n)}(i, j, k) \times \eta(i, j)\right]_{M_{e} \times N_{e}}
\]

(10)

Where, \(\circ\) is Hadamard product. The Hadamard product of \(C_{e}^{(m, n)}(k)\) and \(D_{u}\) is multiplied by the corresponding elements \(\mu^{(m, n)}\) and \(\eta\). The updated environment sub-matrix \(C_{e}^{(m, n)}(k + 1)\) is replaced to the corresponding sub-block of the environment matrix \(C_{e}(k)\), that is, the update of the ACM is implemented.

The above method is extended to the case of multiple aircraft movements: Assuming that the detection period of the sensor is \(T_{s}\), the linear movement of \(n\) aircraft from time \(k\) to time \(k + 1\) is dispersed as a point trace with \(T_{s}\) as the time interval. For each point on the trace of doing the above operation, you can achieve time \(k + 1\) ACM updating.

4. Objective function and algorithm

The key to realize cooperative search is to design a search objective function to evaluate the pros and cons of each step decision. Therefore, in the route planning, each step of the decision mainly considers the increment of coverage rate as the search efficiency, and considers the boundary distance and turning angle as the search cost.

Based on the above considerations, the objective function can be expressed as

\[
J_{1}(P(k), u(k)) = \omega_{1} J_{1}(k) - \omega_{2} J_{2}(k) - \omega_{3} J_{3}(k)
\]

(11)

Where, \(J_{1}\) is the increment of coverage rate of all aircraft performing one step; \(J_{2}, J_{3}\) is the cost function of turning angle and boundary distance; \(\omega_{1}, \omega_{2}, \omega_{3}\) is the corresponding weight; \(\gamma\) is the importance factor, \(\gamma \geq 1\); Since the above revenue and cost functions have different dimensions, therefore, we need to standardize them.

Coverage rate at time \(k\) is the area percentage of \(\Omega_{e}(k)\) to \(\Omega\), approximately equal to the ratio of the number of searched grids to the total number of grids, which can be expressed as

\[
Cover(k) = \frac{M_{e} \times N_{e} - \sum_{i=1}^{M_{e}} \sum_{j=1}^{N_{e}} \mu(i, j, k)}{M_{e} \times N_{e}}
\]

(12)

The increment of coverage rate at moment \(k\) to moment \(k + 1\) is the difference between \(Cover(k + 1)\) and \(Cover(k)\). Search earnings can be expressed as

\[
J_{1}(k) = Cover(k + 1) - Cover(k) = \sum_{i=1}^{M_{e}} \sum_{j=1}^{N_{e}} [\mu(i, j, k) - \mu(i, j, k + 1)]
\]

(13)

If the turning angle is too large, fuel consumption will increase, which will affect the battery life. Therefore, a cost function is designed to minimize the turning angle of each aircraft and reduce the cost of fuel consumption caused by turning. The cost function of turning angle can be expressed as

\[
J_{2}(k) = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{\partial \phi^{(k)}}{\partial \phi_{\text{max}}} \right|
\]

(14)
In the search process, the closer to the boundary, the less effective area can be covered by the sensor. Drawing on the idea of virtual potential function, a cost function is designed. The aircraft will be subjected to the virtual "repulsion" of the boundary. The closer to the boundary, the larger the "repulsion" will be. Therefore, the cost function of the boundary distance can be expressed as

\[
J_i(k) = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{1}{x_i(k) - L_x} + \frac{1}{y_i(k) - L_y} \right)
\]

Model Predictive Control (MPC) is a method to design the optimal control input of the system in the prediction cycle. The core idea is rolling optimization. In this paper, we combine MPC and Differential Evolution (DE), and design a reasonable cooperative search algorithm.

Assuming that the system state at time \( k \) is \( P(k) \) and the input of \( H \)-step predictive control is \( U(k) = \{u(k), u(k+1), \ldots, u(k+H-1)\} \), the search performance of system after \( H \)-step prediction is recorded as

\[
J^{(H)}(P(k), U(k)) = \sum_{j=0}^{H-1} J(P(k+j|k), u(k+j|k))
\]

Then, the optimal model for solving system optimal control input at time \( k \) can be expressed as

\[
U^*(k) = \arg \max J^{(H)}(P(k), U(k))
\]

s.t. \( P(k+1+j|k) = f(P(k+j|k), u(k+j|k)) \)

WHERE: \( U^*(k) \) is the system optimal predictive control input. The first item \( u^*(k|k) \) of \( U^*(k) \) is a control input in time \( k \), that \( u(k) = u^*(k|k) \). At \( k+1 \), the aircraft repeats the above optimization process based on current environmental information and system state. Different from the greedy search algorithm, which only considers the current search revenue, the MPC-based rolling optimization method predicts the long-term search revenue and improves the overall search efficiency.

5. Simulation results and analysis

The cooperative search algorithm based on coverage rate is given in the previous section. In order to verify its feasibility, this section simulates it. Simulation environment for I7-4960, clocked at 2.60GHz, 16G memory, based on Matlab 2014a platform for simulation. The mission area for the rectangular area of 8000m*8000m, each grid size of 20m*20m, the implementation of the search mission of the four aircraft takeoff coordinates were \((200,0), (2700,0), (5200,0), (7800,0)\). Some other parameters are respectively set as \( v_0 = 30m/s \), \( R_s = 200m \), \( \phi_{max} = 60^\circ \), \( \Delta t = 10s \) and \( H = 2 \).

The coverage rate is greater than 90% for the termination conditions, the simulation results as shown in Fig. 4.

Observing the Fig. 4(a), we can intuitively see that the four aircraft keep the detection range of the sensor as non-overlapping as possible in the initial search phase, and achieve a higher coverage rate increment. Through simulation, the proposed method can make the coverage rate increase at a higher rate in the early stage of the search. The angle and the boundary cost function in the objective function better achieve the smoothing of the route and the boundary constraints. From the Fig. 4(b), we can see that the four aircrafts achieve a better cooperative search, the overlap area of the sensor detection area is less, and finally the area coverage rate of more than 90% is reached.
In order to further verify the advantages of this method in improving coverage rate, 100 simulations are performed. The curves of the coverage rate obtained with time are shown in Fig. 5.

It can be seen from the Fig. 5 that the proposed method has a good performance of cooperative search, and the coverage rate has maintained a relatively high growth rate in the initial search period. The search capability has declined in the middle and late stages due to the decentralization of unsearchable regions. When the search time $t=2000s$, the coverage rate of 92% or more, achieving the desired search purposes.

6. Conclusions
In order to improve the regional coverage rate of the aircraft swarm when performing unknown environment search mission, this paper proposes a fast ACM updating method based on Hadamard product, and designs the search objective function with the coverage rate as the main objective. Draw lessons from MPC idea, take DE algorithm to solve. Simulation results show that the cooperative search algorithm proposed in this paper can implement coverage search for unknown environments and maximize the search capability of each aircraft. In the case of a failure of some aircraft, the online planning method proposed in this paper can continue to achieve collaboration based on the current...
environment. Compared with other online planning methods, this method can achieve higher coverage rate.

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