Preheating and entropy perturbations in axion monodromy inflation

Evan McDonough, Hossein Bazrafshan Moghaddam and Robert H. Brandenberger

Department of Physics, McGill University, Montréal, QC H3A 2T8, Canada
Institute for Theoretical Studies, ETH Zürich, CH-8092 Zürich, Switzerland
E-mail: evanmc@physics.mcgill.ca, bazrafshan@physics.mcgill.ca, rhb@physics.mcgill.ca

Received February 10, 2016
Accepted April 24, 2016
Published May 4, 2016

Abstract. We study the preheating of gauge fields in a simple axion monodromy model and compute the induced entropy perturbations and their effect on the curvature fluctuations. We find that the correction to the spectrum of curvature perturbations has a blue spectrum with index $n_s = 5/2$. Hence, these induced modes are harmless for the observed structure of the universe. Since the spectrum is blue, there is the danger of overproduction of primordial black holes. However, we show that the observational constraints are easily satisfied.

Keywords: axions, cosmological perturbation theory, primordial black holes, string theory and cosmology

ArXiv ePrint: 1601.07749
1 Introduction

Axion monodromy inflation [1] (see also [2]) may be the most promising way to obtain large field inflation in the context of superstring theory.\(^1\) Large field inflation models have the advantage over most small field models in that the inflationary slow-roll trajectory is a local attractor in initial condition space [13–19] (see e.g. [20] for a recent review of this issue).

Axion monodromy models contain, in addition to the axion field (which plays the role of the inflaton), gauge fields to which the axion couples via a Pontryagin term in the effective action. As a consequence, during the post-inflationary phase when the inflaton starts to oscillate, there is a preheating type instability in the gauge field equation of motion, and there can be explosive gauge field particle production. This, at second order in the amplitude of the gauge field perturbations, induces a growing curvature fluctuation mode.

The amplitude of the induced curvature fluctuations is constrained by observations. On one hand, on cosmological scales the amplitude of the induced curvature fluctuations must be smaller than the observed perturbations.\(^2\) This is easy to satisfy if the spectrum of the induced fluctuations is blue, as in our case. If the spectrum is blue then, on the other hand, we must worry about the possible over-production of primordial black holes.

In this paper we will show that both sets of constraints are satisfied. We will first study the preheating of the gauge field fluctuations. Then, we compute the resulting entropy fluctuations and determine the induced curvature fluctuations. We find that the power spectrum of these perturbations is deeply blue, with a spectral index of \(n_s = 4\), which gives

\(^1\)Note, however, that there may be constraints on the scenario coming from string back-reaction effects [3–5] and from the “Weak Gravity Conjecture” [6–12].

\(^2\)They must be strictly smaller since the observed fluctuations are well described by a Gaussian process, whereas the induced fluctuations due to the gauge field perturbations have non-Gaussian statistics.
a leading correction to the curvature power spectrum with index $n_s = 5/2$. Hence, these perturbations have a completely negligible effect on cosmological scales. The amplitude of the perturbations on small scales is influenced both by the tachyonic growth of the modes during inflation, and by the instability during the preheating phase. However, for parameter values used in axion monodromy models, we find that the constraints from possible over-abundance of primordial black holes are easily satisfied.

The Pontryagin term which couples the axion to a gauge field and which is playing the key role in our study has been studied in detail in recent years. The tachyonic instability which the gauge field experiences in the presence of a rolling axion field during inflation has been investigated in [21]. The amplified gauge fields, in turn, lead to axion fluctuations which induce non-Gaussianities in the adiabatic curvature perturbations [23–25]. It was realized that the spectrum of these fluctuations is blue, and there hence are potential constraints on the theory due to possible over-production of primordial black holes. This has been studied e.g. in [26, 27] (see also [28]). The Pontryagin term in the joint action of the axion and gauge field can lead to overdamped motion of the axion in which the axion field value is set by the coupling to the gauge field, and both the acceleration term and the velocity term in the axion equation of motion are negligible [21]. This can lead to inflation on steep potentials for sub-Planckian field values [21, 29, 30]. The same Pontryagin term also leads to an inverse cascade [31, 32] for cosmological magnetic fields, and it can be used to provide a scaling quintessence model for dark energy [33].

2 Review of axion monodromy inflation

Axions are ubiquitous in string theory [34, 35]. They arise in string compactifications by integrating gauge potentials over non-trivial cycles of the compactification. In the absence of branes, the potential for the axions is classically flat, and obtains periodic terms from non-perturbative effects. However, in the presence of branes the periodicity of the axion potential is broken. The axion acquires an infinite field range with a potential which is slowly rising as the absolute value of the field increases. In the original example studied in [1], the potential is linear at large field values.

A ‘realistic’ construction of Axion Monodromy Inflation requires three distinct sectors: (1) the monodromy brane, (2) moduli stabilization, and (3) a realization of the Standard Model. Typically these each are achieved via a brane construction: the DBI action of a D5 brane induces monodromy for the axion associated with the NS two-form (the axion we will focus our attention on), a stack of D7 branes induces gaugino condensation which fixes the radial modulus of the internal space, and a set of intersecting branes realizes the (extension of the) Standard Model. Each of these sectors comes with its own gauge theory: the monodromy brane has a U(1) Super-Yang-Mills (SYM) theory, the stack of D7 branes has a SU(N) SYM, while the intersecting branes have either a GUT group (e.g. SU(5)) or the Standard Model group SU(3)×SU(2)×U(1). The axion of axion monodromy inflation is a bulk field, and thus couples to and can lose energy to each sector. There may be phenomenological issues which arise when the energy loss of the axion into other sectors is considered, but we will not address this issue here.

In this work we will focus on a minimal setup of axion monodromy inflation in which we only consider the monodromy brane and its associated U(1) gauge field. For the case of
the $B_2$ axion $\phi$, this gives the following 4d action\(^3\) (in $(-,+,+,+)$ signature)

$$\mathcal{L} = -(1/2)(\partial \phi)^2 + V(\phi) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{\Lambda} \phi F_{\mu\nu} \tilde{F}^{\mu\nu},$$  \hspace{1cm} (2.2)

where $\Lambda$ is a UV scale, and is different from the axion decay constant. The potential $V(\phi)$ is the monodromy potential:

$$V(\phi) = \mu^3 \sqrt{\phi^2_\epsilon + \phi^2},$$  \hspace{1cm} (2.3)

where $\mu$ is an energy scale whose value can be determined from the observed magnitude of the cosmic microwave background anisotropies, and where $\phi_\epsilon < m_{\text{pl}}$ is a constant, $m_{\text{pl}}$ denoting the Planck mass. The field strength

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$  \hspace{1cm} (2.4)

is that of the abelian gauge field which lives on the brane world-volume, and we have neglected fermions, as preheating into fermions is inefficient. String models of axion monodromy and the resulting values of $\Lambda$ are discussed in appendix A (see also [36–39]). From the point of view of effective field theory, we would expect $\Lambda$ to be given either by the string scale or the Planck scale. More stringent, though indirect, constraints come from models of early universe cosmology based upon this coupling. The gaussianity of the CMB constrains the parameter $\xi$, which we will define shortly, to be $\xi_* \lesssim 2.2$ at the moment when the pivot scale $k_*$ exits the horizon [22], see also [23–25, 41], which can be translated to a bound $\Lambda^{-1} \lesssim 12 m_{\text{pl}}$. Recent results on the validity of perturbation theory during inflation [40] constrain $\xi$ to be $\xi \lesssim 3.5$, which correspond to an even tighter constraint on $\xi_*$ (if the whole inflationary trajectory is to be treated perturbatively). Given these considerations we will take a conservative approach, and work with an upper bound $\Lambda^{-1} \leq O(1)m_{\text{pl}}^{-1}$.

3 Background evolution

We assume that the axion starts out in the large field region $\phi \gg m_{\text{pl}}$ where the slow-roll approximation

$$3H \dot{\phi} = -V'(\phi) \simeq -\mu^3$$  \hspace{1cm} (3.1)

of the equation of motion is self-consistent. The end of inflation occurs at the field value when the slow-roll approximation breaks down, at which point $(1/2)\dot{\phi}^2 = V$. This takes place when

$$|\phi| \equiv \phi_* = \frac{1}{\sqrt{6}} m_{\text{pl}},$$  \hspace{1cm} (3.2)

and the kinetic energy at this point is

$$\frac{1}{2} \dot{\phi}^2_{\phi=\phi_*} = \frac{1}{\sqrt{6}} \mu^3 m_{\text{pl}}.$$  \hspace{1cm} (3.3)

\(^3\)There is additionally a coupling $\phi$ to $F_{\mu\nu} F^{\mu\nu}$, of the form

$$\frac{\beta}{\Sigma} V(\phi) F_{\mu\nu} F^{\mu\nu},$$  \hspace{1cm} (2.1)

which comes from the $(\alpha')^2$ correction to the DBI action. The coupling constant of this term is smaller than the coupling to $\tilde{F}\tilde{F}$ by a factor of $C_0$, where $C_0 \sim 10^2$ in cosmological models based on compactifications which stabilize moduli via gaugino condensation on D7 branes (see e.g. [92]). We will ignore the effect of this term in the current work.

---

3 Background evolution

We assume that the axion starts out in the large field region $\phi \gg m_{\text{pl}}$ where the slow-roll approximation

$$3H \dot{\phi} = -V'(\phi) \simeq -\mu^3$$  \hspace{1cm} (3.1)

of the equation of motion is self-consistent. The end of inflation occurs at the field value when the slow-roll approximation breaks down, at which point $(1/2)\dot{\phi}^2 = V$. This takes place when

$$|\phi| \equiv \phi_* = \frac{1}{\sqrt{6}} m_{\text{pl}},$$  \hspace{1cm} (3.2)

and the kinetic energy at this point is

$$\frac{1}{2} \dot{\phi}^2_{\phi=\phi_*} = \frac{1}{\sqrt{6}} \mu^3 m_{\text{pl}}.$$  \hspace{1cm} (3.3)

There is additionally a coupling $\phi$ to $F_{\mu\nu} F^{\mu\nu}$, of the form

$$\frac{\beta}{\Sigma} V(\phi) F_{\mu\nu} F^{\mu\nu},$$  \hspace{1cm} (2.1)

which comes from the $(\alpha')^2$ correction to the DBI action. The coupling constant of this term is smaller than the coupling to $\tilde{F}\tilde{F}$ by a factor of $C_0$, where $C_0 \sim 10^2$ in cosmological models based on compactifications which stabilize moduli via gaugino condensation on D7 branes (see e.g. [92]). We will ignore the effect of this term in the current work.
The value of the Hubble constant at the end of inflation is \( H = H_e \) with
\[
H_e = 2^{-1/4} 3^{-3/4} m_{\text{pl}}^{-1/2} \mu^{3/2}.
\] (3.4)

After inflation ends \( \phi \) begins anharmonic motion about the ground state \( \phi = 0 \). As long as we can neglect the expansion of space and the loss of energy by particle production, the motion is periodic but anharmonic.

The value of \( \mu \) is set by the observed amplitude of the cosmic microwave background (CMB) anisotropies. A simple application of the usual theory of cosmological perturbations (see e.g. [42] for a detailed review, and [43] for an overview) shows that the power spectrum \( P_\zeta \) of the primordial\(^4\) curvature fluctuation \( \zeta \) has the amplitude
\[
P_\zeta \sim \left( \frac{\mu}{m_{\text{pl}}} \right)^3,
\] (3.5)
from which it follows that
\[
\mu \sim 6 \times 10^{-4} m_{\text{pl}}.
\] (3.6)

4 Preheating of gauge field fluctuations

As first pointed out in [44] and [45], a periodic axion background can lead to explosive particle production for all fields coupled to the axion. This effect is called “preheating” [46–48] (see also [49, 50] for reviews). Here we will consider the resonance of the gauge field fluctuations\(^5\)

The equation of motion for the linear fluctuations of \( A_\mu \) is (see e.g. [21, 23–25, 55])
\[
\frac{d^2 A_{k\pm}}{d\tau^2} + \left( k^2 \pm 2k\xi \frac{\dot{\tau}}{\tau} \right) A_{k\pm} = 0,
\] (4.1)
where \( \pm \) denote the two polarizations of the gauge field, \( \tau \) is conformal time, \( k \) indicates a comoving mode, and \( \xi \) is given by\(^6\)
\[
\xi = \frac{2\dot{\phi}}{\Lambda H},
\] (4.2)
where \( H \) is the Hubble expansion rate and \( \phi \) is the background field, and an overdot denotes the derivative with respect to physical time. As long as the slow-roll approximation is valid, \( \xi \) can be taken to be constant. This is the equation relevant during the inflationary period.

As eq. (4.1) shows, for one of the polarization states there is a tachyonic instability (see e.g. [56] for an initial discussion of tachyonic instabilities in reheating) already during inflation for long wavelength modes, i.e. modes which obey
\[
k - \frac{2\xi}{|\tau|} = k - \frac{4|\dot{\phi}|}{\Lambda H |\tau|} < 0,
\] (4.3)

\(^4\)We add the word “primordial” to make a distinction between the original fluctuations and the induced ones which will be the focus of this paper.

\(^5\)There is also the possibility that there is an efficient self-resonance of the inflaton, leading to oscillons [51–54]. Oscillon formation occurs once the amplitude of \( \phi \) oscillations falls below \( \phi_c \), as defined in equation (2.3). Provided that \( \phi_c \) is small compared to the initial amplitude of oscillations, which is indeed the case in realistic string embeddings, oscillon formation will not occur until preheating in to gauge fields has ceased to be efficient, and will not occur at all if preheating into gauge fields is efficient enough to halt the oscillatory motion of \( \phi \). Given this, we will not consider oscillon formation in this work, although this does deserve further attention.

\(^6\)Our definition of \( \xi \) is equivalent to the definition used in [21, 23–25, 55] with the identification \( \alpha/f = 4/\Lambda \).
where the subscript \( I \) indicates that the time derivative is evaluated during slow-roll inflation. The critical wavelength beyond which there is a tachyonic instability has a fixed value in physical coordinates if we take \( H \) and \( \dot{\phi} \) to be constant in time. The critical wavelength can be called a “gauge horizon” and it plays a similar role as the Hubble radius (Hubble horizon) for cosmological perturbations. The gauge horizon is proportional to the Hubble radius, its physical wavenumber \( k_p \) being given by

\[
k_p = 2\xi H.
\]  

(4.4)

For modes which start in their vacuum state deep inside the horizon, the tachyonic resonance [56] leads to squeezing of the mode function. The Floquet exponent is proportional to \( k \), and hence, among all the modes which become super-horizon (meaning super-gauge horizon) by the end of inflation, the ones which undergo the most squeezing are the ones which exit shortly before the end of inflation, i.e. whose comoving wavenumbers is given by

\[
k = k_* \equiv 2\xi H,
\]  

(4.5)

if we normalize the cosmological scale factor to be \( a(t) = 1 \) at the end of inflation. The value of \( k_* \) is determined by the Hubble rate and the axion field velocity at the end of the period of inflation.

It can be shown [21] that the mode function prepared by inflation is

\[
A_{k^+}^{(0)} = \frac{2^{-1/4}}{\sqrt{2k}} \left( \frac{k}{\xi aH} \right)^{1/4} e^{\pi \xi - 4\xi \sqrt{k/2\xi aH}}
\]

\[
A_{k^-}^{(0)} = 0,
\]  

(4.6)

where \(+/−\) denote the positive/negative chirality mode (the \(-\) mode is not amplified during inflation). This corresponds to a highly blue spectrum of gauge field fluctuations with an ultraviolet cutoff which is set by the gauge horizon; the cutoff comes from the second term in the exponential on the right hand side of (4.6). The major amplification factor \( F_I \) of the amplitude is

\[
F_I = e^{\pi \xi}.
\]  

(4.7)

For the specific potential (2.3) of axion monodromy inflation the values of \( k_* \) and \( \xi \) are (making use of (3.3) and (3.4) )

\[
k_* = 4 \left( \frac{2}{3} \right)^{1/4} m_{\text{pl}}^{1/2} \mu^{3/2} \Lambda^{-1}
\]  

(4.8)

\[
\xi = 2\sqrt{6} m_{\text{pl}} \Lambda.
\]  

(4.9)

This shows that if \( \Lambda \ll m_{\text{pl}} \) there is a large enhancement of the amplitude of \( A_k \) during inflation. On the other hand, if \( \Lambda \gg m_{\text{pl}} \), then the growth is negligible. For small values of \( \Lambda \) (i.e. large values of \( \xi \)), the “gauge horizon” is smaller than the Hubble horizon, whereas for large values of \( \Lambda \) the opposite is true.

As mentioned above, the power spectrum \( P_A \) of gauge field fluctuations is blue. On length scales larger that the gauge horizon we have

\[
P_A(k) \equiv k^3 |A_k|^2 \sim k^{5/2}.
\]  

(4.10)
During reheating the expansion of space can be neglected [44] and the equation (4.1) becomes
\[ \ddot{A}_{k\pm} + \left( k^2 \pm \frac{4 \Lambda}{\phi} \right) A_{k\pm} = 0. \] (4.11)
We immediately see that the tachyonic resonance which was present during the period of inflation persists during the preheating period when $\dot{\phi}$ undergoes damped anharmonic oscillations about $\phi = 0$. While $\dot{\phi}$ is negative, then the same polarization mode gets amplified as during inflation. During the second half cycle, when $\dot{\phi} > 0$, it is the other mode which is amplified while the original mode oscillates.

To obtain an order of magnitude estimate of the amplification of $A_{k}$ during preheating, we focus on the first oscillation period (when the Floquet exponent of the instability is largest). We focus on the first quarter of the oscillation period $T$ when $\phi$ is decreasing from $\phi = \phi_e$ to $\phi = 0$. The velocity during most of this time interval is approximately $\dot{\phi}_e$ (see (3.3)). The amplitude of $A_{k}$ grows exponentially at a rate (for $k/k_* < 1$),
\[ \mu_k = 2 \left( \frac{k}{\Lambda} \right)^{1/2} \sqrt{\dot{\phi}_e} = 2 \left( \frac{2}{3} \right)^{1/8} \left( \frac{k}{\Lambda} \right)^{1/2} m_{\text{pl}}^{1/4} \mu^{3/4}. \] (4.12)

The factor $F_k$ by which the amplitude of $A_{k}$ is amplified is
\[ F_k = e^{X_k}, \] (4.13)
with
\[ X_k = \frac{1}{4} T \mu_k, \] (4.14)
where $T$ is the period. The quarter period is given by
\[ \frac{1}{4} T = \frac{\phi_e}{\dot{\phi}_e}. \] (4.15)

Combining these equations yields
\[ X_k = X_{k_*} \left( \frac{k}{k_*} \right)^{1/2}, \] (4.16)
with
\[ X_{k_*} = 2 \left( \frac{2}{3} \right)^{1/2} m_{\text{pl}}^{1/4} \Lambda^{-1/2}. \] (4.17)

Comparing the amplification factors $F_l$ and $F_k$ (see (4.7) and (4.17) one sees that at the value $k = k_*$ they have similar magnitudes.

The mode function after one period of oscillation of $\phi$ is thus given by
\[ A_{k+} = \frac{2^{-1/4}}{\sqrt{2k}} e^{X_k} \left( \frac{k}{\xi a\bar{H}} \right)^{1/4} e^{\pi \xi - 4\xi \sqrt{k/2\xi a\bar{H}}}. \] (4.18)
As long as the expansion of the universe can be neglected, and before back-reaction shuts off the resonance, the gauge field fluctuations grow by the same factor in each period. Hence, after $N$ periods we obtain
\[ A_{k+} = \frac{2^{-1/4}}{\sqrt{2k}} e^{NX_k} \left( \frac{k}{\xi a\bar{H}} \right)^{1/4} e^{\pi \xi - 4\xi \sqrt{k/2\xi a\bar{H}}}. \] (4.19)
Comparing the expressions for the period $T$ and the Hubble expansion rate $H$ at the end of inflation we see that right at the end of inflation $T \sim H^{-1}$ and hence the expansion of space cannot be neglected. However, once reheating starts, $\phi$ decreases and hence $T$ decreases and the expansion of space becomes negligible. The Floquet exponent can be taken to be approximately constant during half of each period, and vanishing for the other half. Hence, over a period $(0, t)$ of reheating, the increase in the amplitude is

$$F_k \sim e^{\frac{1}{2} \mu t t},$$

and the gauge field amplitude becomes

$$A_{k+} = \frac{2^{-1/4}}{\sqrt{2k}} e^{\frac{1}{2} \mu t} \left( \frac{k}{\xi aH} \right)^{1/4} e^{\pi \xi - 2\sqrt{2k/\xi aH}}.$$  

(4.21)

There is also an amplification for the $(-)$ polarization, $A_{k-}$, but this mode is suppressed during inflation, and enters preheating with a different mode function.

5 Gauge field energy density fluctuations

We have thus far computed the gauge fields produced during preheating. This sources an energy density perturbation, $\delta \rho_A$, which we will now focus on. The gauge field energy density is defined as (in $(-,+,+,+)$ signature)

$$\rho_A(x, t) = -T_{00},$$

(5.1)

where $T_{\mu\nu}$ is given by (again in $(-,+,+,+)$ signature, and assuming a Lagrangian $L = (1/4) F^2$),

$$T_{\mu\nu} = -\frac{1}{4} g_{\mu\nu} F^2 + F_{\mu\lambda} F_{\nu}^{\lambda}.$$  

(5.2)

In terms of the gauge field $A_{\mu}$, and without any gauge fixing, this reduces to

$$\rho_A(x, t) = -\frac{1}{2} \left( \partial^\alpha A^\alpha - \partial^\alpha A^\alpha \right) \left( \partial_\alpha A_\alpha - \partial_\alpha A_\alpha \right) - \frac{1}{4} \left( \partial^\alpha A^\beta - \partial^\alpha A^\beta \right) \left( \partial_\alpha A_\beta - \partial_\alpha A_\beta \right).$$  

(5.3)

We can fix the gauge by setting $A_0 = 0$. The leading term on cosmological scales is given by

$$\rho_A(x, t) \simeq -\frac{1}{2} \partial^\alpha A^\alpha \partial_\alpha A_\alpha.$$  

(5.4)

To find the Fourier modes of $\rho_A(x, t)$, we first expand $A_{\mu}$ in terms of classical oscillators

$$A_{\mu}(x, t) = \sum_{\lambda = +, -} \int \frac{d^3 k}{(2\pi)^3} \left[ \epsilon_{\mu}^{\lambda}(k, t) \alpha_k e^{ikx} + \epsilon_{\mu}^{\lambda*}(k, t) \alpha_k^{\dagger} e^{-ikx} \right],$$

where $\alpha_k$ are classical oscillators drawn from a nearly Gaussian distribution, satisfying

$$\langle \alpha_k \alpha_{k'} \rangle = (2\pi)^3 \delta^3(k + k'),$$

(5.6)

where the angular brackets stand for ensemble averaging. We can expand $\rho$ in a similar fashion

$$\rho_A(x, t) = \int \frac{d^3 k}{(2\pi)^3} \rho_{Ak} \beta_k e^{ikx} + \text{c.c.},$$

(5.7)
where $\beta_k$ are a different set of classical oscillators, whose distribution function can be determined in terms of the $\alpha_k$. The Fourier modes of $\rho(x,t)$ are simply a convolution of Fourier modes of the gauge field $A_\mu$

$$\rho_{Ak}\beta_k = \frac{1}{2}a^{-2}\int \frac{d^3k'}{(2\pi)^3} \hat{A}_{k'} + \hat{A}_{(k-k')} + \alpha_k' \alpha_{k-k'},$$

(5.8)

where the mode function $A_k$ is given by equation (4.21). There is a gradient term $k^2 A_k^2$ which is comparable in magnitude to the time-derivative term, and thus changes $\rho_{Ak}$ by a factor of two.

We can use the above expression to straightforwardly calculate the background energy density in the gauge field and the spectrum of the gauge fluctuations. The homogenous background energy density is simply $\langle \rho_A(x,t) \rangle$, and we define the fluctuations $\delta \rho_A$ about this background as $\delta \rho_A = \rho_A - \langle \rho_A \rangle$, such that $\langle \delta \rho_A \rangle = 0$, and the variance of fluctuations is simply $\langle \delta \rho_A^2 \rangle = (\rho_A^2) - \langle \rho_A \rangle^2$. A simple calculation shows that the background is given by

$$\langle \rho_A(x,t) \rangle = \frac{1}{2}a^{-2}\int d^3k \left| \hat{A}_{k^+} \right|^2.$$

(5.9)

The dominant contribution to the integral comes from the maximally amplified mode $k = k_*$, and we can hence approximate it as

$$\langle \rho_A(x,t) \rangle \sim \sqrt{2}a^{-2} e^{2\mu_* t} \left( \mu_* k_* \right)^2 e^{-2\sqrt{2} \cdot e^{2\pi \xi}},$$

(5.10)

where $\mu_* \equiv \mu_{k*}$. From this we see that the amplitude of $\langle \rho \rangle$ depends inversely on the UV scale $\Lambda$, since a smaller $\Lambda$ means an increased $k_*$. The mode function of fluctuations can be straightforwardly computed using the definition $\delta \rho_A = \rho_A - \langle \rho_A \rangle$ in conjunction with equation (5.8) and the approximation that the $\beta_k$ are drawn from a nearly Gaussian distribution, i.e. $\langle \beta_k \beta_{k'} \rangle = (2\pi)^3 \delta^3(k + k')$. The exact $\beta_k$ are not drawn from a Gaussian distribution, but as we have the modest goal of computing power spectra (i.e. two-point statistics), this is not an important distinction. The dominant term in $\delta \rho_A$ is

$$|\delta \rho_{Ak}|^2 \sim \frac{1}{4} a^{-4} \int d^3q \left| \hat{A}_q \right|^2 \left| \hat{A}_{k-q} \right|^2.$$

(5.11)

For modes in the IR, i.e. $k \ll k_*$, this integral is highly peaked at $q = k_*$ and we can find

$$|\delta \rho_{Ak}|^2 \sim \frac{\langle \rho_A \rangle^2}{k_*^2}.$$

(5.12)

Note, in particular, that the resulting power spectrum of gauge field fluctuations is highly blue. The spectral index is $n_s = 4$.

6 Back-reaction considerations

The exponential increase in the gauge field value cannot continue forever. Eventually, the tachyonic resonance will be shut off by back-reaction effects. Back-reaction in a two field toy model of parametric resonance was considered in [57], where it was concluded that back-reaction does not prevent the exponential production of entropy fluctuations before these perturbations become important. In this subsection we estimate how long the tachyonic resonance in our model will last until back-reaction becomes important.
We will consider the two most important back-reaction effects involving gauge field production. The first is the effect of gauge field production on the axion field dynamics, the dynamics driving the instability. Recall that the axion equation of motion is given by

\[ \ddot{\phi} + 3H \dot{\phi} + V_{,\phi} = \frac{1}{\Lambda} \langle \tilde{F} \tilde{F} \rangle, \]  

(6.1)

where \( \langle \tilde{F} \tilde{F} \rangle \) refers to ensemble or spatial averaging as was done to determine \( \langle \rho_A \rangle \) in the previous section. To obtain an order of magnitude estimate of when back-reaction becomes important, we can compare the term on the right hand side of (6.1) with the force driving the oscillations. The first condition of ‘small backreaction’ comes from demanding that the force term dominates. This translates to

\[ \langle V_{,\phi} \rangle_{\text{rms}} \gg \frac{\langle \tilde{F} \tilde{F} \rangle}{\Lambda_{\text{RMS}}}. \]  

(6.2)

We can estimate the order of magnitude of the right-hand side of the above equation by \( \Lambda^{-1} \rho_A \), and hence the condition (6.2) becomes

\[ V' \gg \frac{1}{\Lambda} \rho_A. \]  

(6.3)

The second back-reaction condition comes from demanding that the energy density is dominated by the scalar field, i.e.

\[ V \gg \rho_A. \]  

(6.4)

In this equation, the value of \( \phi \) appears. We will use the value at the end of inflation.

For the axion monodromy potential we are using, the two conditions differ by a factor \( \Lambda/m_{\text{pl}} \). Inserting the expression (5.10) into the first back-reaction criterium (6.3) yields

\[ 2 \mu_s t = -2\pi \xi + 3 \ln \left( \frac{\Lambda}{\mu} \right) + 2 \ln \left( \frac{\Lambda}{m_{\text{pl}}} \right), \]  

(6.5)

for the time interval \( t \) before back-reaction becomes important, whereas the second condition (6.4) yields

\[ 2 \mu_s t = -2\pi \xi + 3 \ln \left( \frac{\Lambda}{\mu} \right) + 3 \ln \left( \frac{\Lambda}{m_{\text{pl}}} \right), \]  

(6.6)

which is a stronger condition if \( \Lambda < m_{\text{pl}} \) and weaker otherwise.

The amplitude of the gauge field energy density fluctuations when back-reaction becomes important then is bounded from above by

\[ \delta \rho_{Ak} \sim \frac{V}{k^{3/2}} \]  

for \( \Lambda > m_{\text{pl}} \)

\[ \delta \rho_{Ak} \sim \frac{V}{k^{3/2}} \frac{\Lambda}{m_{\text{pl}}} \]  

for \( \Lambda < m_{\text{pl}} \).

Note that there can be back-reaction effects from the production of other fields which may turn off the resonance much earlier. Since we are interested in obtaining upper bounds on the effects generated by gauge field production, we will work with the above upper bounds.
7 Induced curvature perturbations

During reheating purely adiabatic fluctuations on super-Hubble scales cannot be amplified since it can be shown that the curvature fluctuation variable $\zeta$ is conserved. This can be shown in linear cosmological perturbation theory [58–62], but the result holds more generally (see e.g. [63–65]). On the other hand, entropy fluctuations can be parametrically amplified during reheating [66–68] (see also [69–72]). Entropy fluctuations inevitably seed a growing curvature perturbation. Thus, in the presence of entropy modes it is possible to obtain an exponentially growing curvature fluctuation on super-Hubble scales (see e.g. [73–76] for some studies of this question in earlier string-motivated models of inflation).

Consider $\zeta$, the curvature perturbation on uniform density hypersurfaces. This is the variable which determines the amplitude of the CMB anisotropies at late times (see [42] for a detailed overview of the theory of cosmological perturbations). In the absence of entropy fluctuations, this variable is conserved on super-Hubble scales [58–61, 64, 65]. However, in the presence of entropy perturbations, a growing mode of $\zeta$ is induced on super-Hubble scales, as already discussed in the classic review articles on cosmological perturbations [42, 77] and as applied to axion inflation in [78]. For more modern discussions the reader is referred to [79, 80]. The equation of motion for $\zeta_k$ ($k$ denotes the comoving wavenumber) is given by equation (3.29) of [80]

$$
\dot{\zeta}_k = -\frac{H}{p + \rho} \delta P_{nad,k} + \frac{1}{3H a^2} (\Psi_k - \zeta_k) + \frac{k^4}{9HH} \Psi_k , \tag{7.1}
$$

where $\Psi_k$ is the gauge invariant curvature perturbation in longitudinal gauge, $p$ and $\rho$ are the total pressure and energy densities, respectively and $\delta P_{nad,k}$ is the non-adiabatic pressure perturbation. On large length scales the dependence on $\Psi$ disappears and the evolution equation is simply

$$
\dot{\zeta} = -\frac{H}{p + \rho} \delta P_{nad} . \tag{7.2}
$$

Note that $\zeta$ is dimensionless.

The non-adiabatic pressure perturbation $\delta P_{nad}$ is the sum of an intrinsic and a relative perturbation

$$
\delta P_{nad} = \delta P_{int} + \delta P_{rel} . \tag{7.3}
$$

The intrinsic non-adiabatic pressure perturbation is the sum

$$
\delta P_{int} = \sum_\alpha \delta P_{int,\alpha} = \sum_\alpha \left( \delta p_\alpha - c_\alpha^2 \delta \rho_\alpha \right) , \tag{7.4}
$$

while the relative non-adiabatic pressure perturbation is given by

$$
\delta P_{rel} = -\frac{1}{6H\dot{\rho}} \sum_{\alpha\beta} \dot{\rho}_\alpha \dot{\rho}_\beta (c_\alpha^2 - c_\beta^2) S_{\alpha\beta} , \tag{7.5}
$$

where $S_{\alpha\beta}$ is the relative entropy perturbation

$$
S_{\alpha\beta} = -3H \left( \frac{\delta \rho_\alpha}{\rho_\alpha} - \frac{\delta \rho_\beta}{\rho_\beta} \right) . \tag{7.6}
$$

In the above equations, the sum runs over the different components of matter, and $c_\alpha^2$ is the square of the speed of sound of the $\alpha$ component of matter.
The above set of equations can be rewritten in a more compact form (see e.g. [81])

$$\delta P_{\text{nad}} = \dot{p} \left( \frac{\delta p}{\bar{p}} - \frac{\delta \rho}{\bar{\rho}} \right),$$

(7.7)

where in our case the total pressure is the sum of the contributions from the $\phi$ field and from the gauge field, i.e. $p = p_\phi + p_A$, and similarly for $\rho$, and we have set the intrinsic entropy perturbations to zero. For a background that is dominated by $\phi$, and with $\delta \rho_A > \delta \rho_\phi$, the above non-adiabatic pressure perturbation is simply

$$\delta P_{\text{nad}} = \dot{p}_\phi \left( \frac{\delta p_A}{\bar{p}_\phi} - \frac{\delta \rho_A}{\bar{\rho}_\phi} \right),$$

(7.8)

and the evolution equation of $\zeta$ is given by

$$\dot{\zeta} = -\frac{H}{\rho_\phi + \bar{p}_\phi} \left( \frac{1}{3} - c^2_{s\phi} \right) \delta \rho_A.$$

(7.9)

In our case, the gauge field energy density fluctuations $\delta \rho_A$ is increasing exponentially with a Floquet exponent $2\mu_*$ during the preheating phase, as shown in earlier sections. Hence, integrating over time, we get

$$\Delta \zeta_k = -\mu_*^{-1} \frac{H}{\rho_\phi + \bar{p}_\phi} \left( \frac{1}{3} - c^2_{s\phi} \right) \delta \rho_{Ak},$$

(7.10)

where the wavenumber $k$ and the density fluctuation $\delta \rho_A$ are Fourier space quantities. However, since it follows from section 5 that $\delta \rho_A$ is independent of $k$, we find that the power spectrum of the induced fluctuations of $\zeta$ is

$$\mathcal{P}_{\Delta \zeta}(k) \sim k^3,$$

(7.11)

which corresponds to a highly blue tilted spectrum with index $n_s = 4$. Since the spectrum has such a large blue tilt, there are no constraints on our model coming from demanding that the induced curvature fluctuations do not exceed the observational upper bounds.

8 Primordial black hole constraints

Since the power spectrum of induced curvature fluctuations is highly blue, we have to worry about the possible constraints on the model coming from over-production of primordial black holes. Primordial black holes are constrained by a set of cosmological observations, beginning with the original constraints coming from the observational bounds on cosmic rays produced by radiating black holes [82]. Primordial black hole production during reheating has been considered in simple two field inflation models in [83], and in models with spectra with a distinguished scale in [84].

In the context of an inflationary cosmology, primordial black holes of mass $M$ can form when the length scale associated with this mass (i.e. the length $l$ for which the mass inside a sphere of radius $l$ equals $M$) enters the Hubble radius. The number density of black holes of this mass will depend on the amplitude of the primordial power spectrum.\footnote{The are numerous subtleties in computing the precise number density, which tend to suppress the number of primordial black holes formed, see e.g. [85] and references therein. These details will not be important for our analysis.}
Since in our case the power spectrum is highly blue, the tightest constraints will come from the smallest mass for which cosmological constraints exist. These correspond to black holes with a mass such that they evaporate during nucleosynthesis. The extra radiation from these black holes would act as an extra species of radiation, and would destroy the agreement between the theory of nucleosynthesis and observations (see [86–88] for reviews). The smallest length scale (i.e. largest wavenumber $k$) for which constraints exist is [89]

$$k_{\text{max}} \sim 10^{19}\text{Mpc}^{-1},$$

and the approximate bound on the power spectrum is

$$P_\zeta(k_{\text{max}}) < 10^{-1.5}.$$  \hspace{1cm} (8.2)

In fact, the bound for smaller values of $k$ has comparable amplitude.

The power spectrum including the induced curvature perturbations is given by

$$P_\zeta(k) = \frac{k^3}{(2\pi)^2} \left| A_0 k^{-3/2} + \Delta \zeta_k \right|^2,$$ \hspace{1cm} (8.3)

where $A_0 \sim 10^{-10}$ is the amplitude of the power spectrum at the pivot scale $k = k_0 = 0.05\text{Mpc}^{-1}$, and we have approximated the spectrum of curvature perturbations from inflation to be scale invariant. We already computed the value of the induced curvature fluctuations $\Delta \zeta$ in the previous section in eq. (7.10). Inserting the values from (6.7), (4.8) and (4.9) we obtain the following expressions for the leading order correction to the power spectrum of curvature fluctuations

$$\Delta P_\zeta(k) = \mathcal{O}(10^{-3}) \sqrt{A_0} \frac{\Lambda^{5/2}}{m_{\text{pl}}^{7/4} \mu^{9/4}}$$ \hspace{1cm} \text{for } \Lambda > m_{\text{pl}},$$

$$\Delta P_\zeta(k) = \mathcal{O}(10^{-3}) \sqrt{A_0} \frac{\Lambda^{7/2}}{m_{\text{pl}}^{11/4} \mu^{9/4}}$$ \hspace{1cm} \text{for } \Lambda < m_{\text{pl}},$$ \hspace{1cm} (8.4)

corresponding to a spectrum with index $n_s = 5/2$. These expressions hold if the exponential growth of the curvature fluctuations is only limited by the back-reaction effects studied in section 6. Other effects may terminate the growth earlier. Hence, the above equations provide upper bounds on the amplitude of the induced curvature perturbations.

For the largest value of $k$ for which the primordial black hole constraints apply we have

$$\frac{k}{m_{\text{pl}}} \sim 10^{-39}.$$ \hspace{1cm} (8.5)

Inserting this value into (8.4) we find that the primordial black hole constraint (8.2) is trivially satisfied for the realistic range of values of $\Lambda$.

9 Conclusions

In this paper we have considered a minimal axion monodromy model and have calculated the spectrum of the curvature perturbations induced by the entropy mode associated with the gauge field to which the axion couples. We find that the leading correction to the curvature spectrum is blue with spectral index $n_s = 5/2$. Hence, there are no constraints from large
scale cosmological observations. On the other hand, since the spectrum is blue, there is a
danger of overproduction of primordial black holes. We find, however, that the amplitude of
the spectrum is too low even on the smallest scales for which cosmological constraints exist.

Realistic axion monodromy models, on the other hand, typically contain many other
scalar fields which can source entropy modes, and these modes could, in principle, pose cosmo-
logical problems. It is reassuring, however, that the prototypical minimal axion monodromy
model is safe from the constraints studied in this paper.

Acknowledgments

Two of the authors (RB and EM) wish to thank the Institute for Theoretical Studies of the
ETH Zürich for kind hospitality while a portion of this work was completed. RB acknowledges
financial support from Dr. Max Rössler, the Walter Haefner Foundation and the ETH Zurich
Foundation, and from a Simons Foundation fellowship. The research at McGill is supported
in part by funds from NSERC and the Canada Research Chair program. EM is supported
in part by a NSERC PGS D fellowship. HBM is supported in part by a MSRT fellowship
from Iran.

A String theory model building constraints on the coupling of $\phi$ to $F\tilde{F}$

A realistic universe built from axion monodromy necessarilly has three sectors: (1) the
inflation sector, (2) the moduli stabilization sector, and (3) the standard model. However,
without considering all three, there is already interesting couplings purely in the inflation
sector.

In (the standard story of) axion monodromy inflation $[36–39]$, a potential an axion field
is induced by the DBI action of a D5 action. The world volume action of the D5-brane receives
corrections at each order in the string coupling constant $\alpha'$. In particular, the Chern-Simons
part of the action has an $\alpha'^2$ correction:

$$\delta S_{CS} = -\mu_5(2\pi\alpha')^2 \int C_0 B_2 \wedge F \wedge F,$$

where $C_0$ is the RR 0-form potential, $B_2$ is the NSNS two-form, $F$ is the world volume gauge
field (see, for example, equation (2.6) of $[90]$). There are also corrections to the DBI action,
but the coupling of these corrections to the two-form $B_2$ is not known (see for example
equation (2.4) of $[90]$).

From the above action we can derive the 4d interaction,

$$\mathcal{L}_{\text{int}} = \frac{1}{\Lambda} \phi F\tilde{F},$$

where $\Lambda$ is given by

$$\frac{1}{\Lambda} = 2\mu^3 l^3 (2\pi\alpha')^2 C_0,$$

where $\mu^3$ is the coupling constant appearing in the axion potential, and $l^2$ is the size of $\Sigma_2$ (the
two-cycle wrapped by the brane) in string units. The constants in $\Lambda$ are constrained by the
consistency of the model. We can write $\alpha'$ in terms of the string mass scale, $M_s = 1/\sqrt{2\pi\alpha'}$, such that the coupling takes the form

$$\frac{1}{\Lambda} = 2l^3 C_0 \left(\frac{\mu}{M_s}\right)^3 \frac{1}{M_s}.$$
Let us consider the size of the parameters. Firstly, moduli stabilization requires a stack of D7 branes. These branes source $C_0$, and the value of $C_0$ (which is dimensionless) is roughly equal to the number of D7 branes. For more details on the supergravity background in the presence of a stack of D7 branes, see e.g. [91]. In realistic models of 4d physics (see e.g. [92])

$$C_0 \sim 10^2.$$ (A.5)

Secondly, the value of $\mu$ is chosen such that the amplitude of cosmological perturbations arising from axion monodromy inflation matches observations. The potential is

$$V_{\text{int}} = \mu^3 \phi,$$ (A.6)

where $\mu$ is given by

$$\mu = \frac{\mu_5 \alpha'}{f g_s},$$ (A.7)

where $f$ is the axion decay constant. Note that the axion decay constant enters the potential and the interaction term with the same power, and thus it is impossible to change the relative strength of the interaction by fine-tuning the axion decay constant. Consistency with observations requires

$$\mu^3 = (6 \times 10^{-4} m_{\text{pl}})^3.$$ (A.8)

Thirdly, the internal space must be of an ‘intermediate’ size: if the internal space is too small then the supergravity approximation (and the DBI action) ceases to be the correct description of physics, while if the internal space is too large then the 4d Newton’s constant is too small. A reasonable value of the volume of the internal space is $\text{Vol}(X_6) \sim 10^6$ in units of $\alpha'$, which corresponds to a length scale $l_{X_6} \sim 10$.

These constraints determine to a great degree the allowed value of the parameter $\Lambda$. A consistent value is:

$$\frac{1}{\Lambda} \sim 10^{-5} \left(\frac{m_{\text{pl}}}{M_s}\right)^4 \frac{1}{m_{\text{pl}}},$$ (A.9)

where we took $C_0 = 50$, $l = 10$, and $\mu = 6 \times 10^{-4} m_{\text{pl}}$.

References

[1] L. McAllister, E. Silverstein and A. Westphal, *Gravity Waves and Linear Inflation from Axion Monodromy*, *Phys. Rev. D* 82 (2010) 046003 [arXiv:0808.0706] [insPIRE].

[2] E. Silverstein and A. Westphal, *Monodromy in the CMB: Gravity Waves and String Inflation*, *Phys. Rev. D* 78 (2008) 106003 [arXiv:0803.3085] [insPIRE].

[3] D. Baumann and L. McAllister, *Inflation and String Theory*, arXiv:1404.2601 [insPIRE].

[4] J.P. Conlon, *Brane-Antibrane Backreaction in Axion Monodromy Inflation*, *JCAP* 01 (2012) 033 [arXiv:1110.6454] [insPIRE].

[5] A. Hebecker, P. Mangat, F. Rompineve and L.T. Witkowski, *Tuning and Backreaction in F-term Axion Monodromy Inflation*, *Nucl. Phys. B* 894 (2015) 456 [arXiv:1411.2032] [insPIRE].

[6] A. Hebecker, F. Rompineve and A. Westphal, *Axion Monodromy and the Weak Gravity Conjecture*, arXiv:1512.03768 [insPIRE].

[7] L.E. Ibáñez, M. Montero, A. Uranga and I. Valenzuela, *Relaxion Monodromy and the Weak Gravity Conjecture*, *JHEP* 04 (2016) 020 [arXiv:1512.00025] [insPIRE].
[8] B. Heidenreich, M. Reece and T. Rudelius, Weak Gravity Strongly Constrains Large-Field Axion Inflation, *JHEP* **12** (2015) 108 [arXiv:1506.03447] [SPIRE].

[9] J. Brown, W. Cottrell, G. Shiu and P. Soler, On Axionic Field Ranges, Loopholes and the Weak Gravity Conjecture, *JHEP* **04** (2016) 017 [arXiv:1504.00659] [SPIRE].

[10] J. Brown, W. Cottrell, G. Shiu and P. Soler, Fencing in the Swampland: Quantum Gravity Constraints on Large Field Inflation, *JHEP* **10** (2015) 023 [arXiv:1503.04783] [SPIRE].

[11] T. Rudelius, Constraints on Axion Inflation from the Weak Gravity Conjecture, *JCAP* **09** (2015) 020 [arXiv:1503.00795] [SPIRE].

[12] T. Rudelius, On the Possibility of Large Axion Moduli Spaces, *JCAP* **04** (2015) 049 [arXiv:1409.5793] [SPIRE].

[13] R.H. Brandenberger and J.H. Kung, Chaotic Inflation as an Attractor in Initial Condition Space, *Phys. Rev. D* **42** (1990) 1008 [SPIRE].

[14] H.A. Feldman and R.H. Brandenberger, Chaotic Inflation With Metric and Matter Perturbations, *Phys. Lett. B* **227** (1989) 359 [SPIRE].

[15] R.H. Brandenberger and H.A. Feldman, Effects of Gravitational Perturbations on the Evolution of Scalar Fields in the Early Universe, *Phys. Lett. B* **220** (1989) 361 [SPIRE].

[16] R.H. Brandenberger, H. Feldman and J. Kung, Initial conditions for chaotic inflation, *Phys. Scripta T* **36** (1991) 64 [SPIRE].

[17] P. Laguna, H. Kurki-Suonio and R.A. Matzner, Inhomogeneous inflation: The Initial value problem, *Phys. Rev. D* **44** (1991) 3077 [SPIRE].

[18] H. Kurki-Suonio, P. Laguna and R.A. Matzner, Inhomogeneous inflation: Numerical evolution, *Phys. Rev. D* **48** (1993) 3611 [astro-ph/9306009] [SPIRE].

[19] W.E. East, M. Kleban, A. Linde and L. Senatore, Beginning inflation in an inhomogeneous universe, *arXiv:1511.05143* [SPIRE].

[20] R.H. Brandenberger, *Initial Conditions for Inflation — A Short Review*, *arXiv:1601.01918* [SPIRE].

[21] M.M. Anber and L. Sorbo, Naturally inflating on steep potentials through electromagnetic dissipation, *Phys. Rev. D* **81** (2010) 043534 [arXiv:0908.4089] [SPIRE].

[22] PLANCK collaboration, P.A.R. Ade et al., Planck 2015 results. XX. Constraints on inflation, *arXiv:1502.02114* [SPIRE].

[23] N. Barnaby, E. Pajer and M. Peloso, Gauge Field Production in Axion Inflation: Consequences for Monodromy, non-Gaussianity in the CMB and Gravitational Waves at Interferometers, *Phys. Rev. D* **85** (2012) 023525 [arXiv:1110.3327] [SPIRE].

[24] N. Barnaby, R. Namba and M. Peloso, Phenomenology of a Pseudo-Scalar Inflaton: Naturally Large NonGaussianity, *JCAP* **04** (2011) 009 [arXiv:1102.4333] [SPIRE].

[25] N. Barnaby and M. Peloso, Large NonGaussianity in Axion Inflation, *Phys. Rev. Lett.* **106** (2011) 181301 [arXiv:1011.1500] [SPIRE].

[26] E. Bugaev and P. Klimai, Axion inflation with gauge field production and primordial black holes, *Phys. Rev. D* **90** (2014) 023501 [arXiv:1312.7435] [SPIRE].

[27] A. Linde, S. Mooij and E. Pajer, Gauge field production in supergravity inflation: Local non-Gaussianity and primordial black holes, *Phys. Rev. D* **87** (2013) 103506 [arXiv:1212.1693] [SPIRE].

[28] C.-M. Lin and K.-W. Ng, Primordial Black Holes from Passive Density Fluctuations, *Phys. Lett. B* **718** (2013) 1181 [arXiv:1206.1685] [SPIRE].
[29] P. Adshead and M. Wyman, Chromo-Natural Inflation: Natural inflation on a steep potential with classical non-Abelian gauge fields, Phys. Rev. Lett. 108 (2012) 261302 [arXiv:1202.2366] [inSPIRE].
[30] E. Martinec, P. Adshead and M. Wyman, Chern-Simons EM-flation, JHEP 02 (2013) 027 [arXiv:1206.2889] [inSPIRE].
[31] A. Boyarsky, J. Fröhlich and O. Ruchayskiy, Self-consistent evolution of magnetic fields and chiral asymmetry in the early Universe, Phys. Rev. Lett. 108 (2012) 031301 [arXiv:1109.3350] [inSPIRE].
[32] A. Boyarsky, J. Fröhlich and O. Ruchayskiy, Magnetohydrodynamics of Chiral Relativistic Fluids, Phys. Rev. D 92 (2015) 043004 [arXiv:1504.04854] [inSPIRE].
[33] A. Boyarsky, J. Fröhlich and O. Ruchayskiy, Self-consistent evolution of magnetic fields and chiral asymmetry in the early Universe, Phys. Rev. Lett. 108 (2012) 031301 [arXiv:1109.3350] [inSPIRE].
[34] S. Alexander, R.H. Brandenberger and J. Fröhlich, Tracking Dark Energy from Axion-Gauge Field Couplings, arXiv:1601.00057 [inSPIRE].
[35] S. Alexander, R.H. Brandenberger and J. Fröhlich, On axion monodromy inflation in warped throats, JHEP 02 (2015) 017 [arXiv:1309.6916] [inSPIRE].
[36] S. Alexander, R.H. Brandenberger and J. Fröhlich, Particle Production During Out-of-equilibrium Phase Transitions, Phys. Rev. D 42 (1990) 2491 [inSPIRE].
[37] S. Alexander, R.H. Brandenberger and J. Fröhlich, On particle creation by a time dependent scalar field, Sov. J. Nucl. Phys. 51 (1990) 172 [Yad. Fiz. 51 (1990) 273] [inSPIRE].
[38] V.F. Mukhanov, H.A. Feldman and R.H. Brandenberger, Theory of cosmological perturbations. Part 1. Classical perturbations. Part 2. Quantum theory of perturbations. Part 3. Extensions, Phys. Rept. 215 (1992) 295 [inSPIRE].
[39] V.F. Mukhanov, H.A. Feldman and R.H. Brandenberger, Towards the theory of cosmological perturbations, Lect. Notes Phys. 646 (2004) 127 [hep-th/0306071] [inSPIRE].
[49] R. Allahverdi, R.H. Brandenberger, F.Y. Cyr-Racine and A. Mazumdar, Reheating in inflationary cosmology: theory and applications, Ann. Rev. Nucl. Part. Sci. 60 (2010) 27 [arXiv:1001.2600] [inSPIRE].

[50] M.A. Amin, M.P. Hertzberg, D.I. Kaiser and J. Karouby, Nonperturbative Dynamics Of Reheating After Inflation: A Review, Int. J. Mod. Phys. D 24 (2014) 1530003 [arXiv:1410.3808] [IN_SPIRE].

[51] M.A. Amin, Inflaton fragmentation: Emergence of pseudo-stable inflaton lumps (oscillons) after inflation, arXiv:1006.3075 [IN_SPIRE].

[52] M.A. Amin, R. Easther, H. Finkel, Inflaton Fragmentation and Oscillon Formation in Three Dimensions, JCAP 12 (2010) 001 [arXiv:1009.2505] [IN_SPIRE].

[53] M.A. Amin, R. Easther, H. Finkel, R. Flauger and M.P. Hertzberg, Oscillons After Inflation, Phys. Rev. Lett. 108 (2012) 241302 [arXiv:1106.3335] [IN_SPIRE].

[54] S.-Y. Zhou, E.J. Copeland, R. Easther, H. Finkel, Z.-G. Mou and P.M. Saffin, Gravitational Waves from Oscillon Preheating, JHEP 10 (2013) 026 [arXiv:1304.6094] [IN_SPIRE].

[55] J.P. Zibin, R.H. Brandenberger and D. Scott, Back reaction and the parametric resonance of cosmological fluctuations, Phys. Rev. D 63 (2001) 043511 [hep-ph/0007219] [IN_SPIRE].

[56] J.M. Bardeen, Gauge Invariant Cosmological Perturbations, Phys. Rev. D 22 (1980) 1882 [IN_SPIRE].

[57] J.M. Bardeen, P.J. Steinhardt and M.S. Turner, Spontaneous Creation of Almost Scale — Free Density Perturbations in an Inflationary Universe, Phys. Rev. D 28 (1983) 679 [IN_SPIRE].

[58] R.H. Brandenberger and R. Kahn, Cosmological perturbations in inflationary universe models, Phys. Rev. D 29 (1984) 2172 [IN_SPIRE].

[59] D.H. Lyth, Large Scale Energy Density Perturbations and Inflation, Phys. Rev. D 31 (1985) 1792 [IN_SPIRE].

[60] F. Finelli and R.H. Brandenberger, Parametric amplification of gravitational fluctuations during reheating, Phys. Rev. Lett. 82 (1999) 1362 [hep-ph/9809490] [IN_SPIRE].

[61] N. Afshordi and R.H. Brandenberger, Super Hubble nonlinear perturbations during inflation, Phys. Rev. D 63 (2001) 123505 [gr-qc/0011075] [IN_SPIRE].

[62] D. Langlois and F. Vernizzi, Evolution of non-linear cosmological perturbations, Phys. Rev. Lett. 95 (2005) 091303 [astro-ph/0503416] [IN_SPIRE].

[63] D. Langlois and F. Vernizzi, Conserved non-linear quantities in cosmology, Phys. Rev. D 72 (2005) 103501 [astro-ph/0509078] [IN_SPIRE].

[64] B.A. Bassett and F. Viniegra, Massless metric preheating, Phys. Rev. D 62 (2000) 043507 [hep-ph/9909353] [IN_SPIRE].

[65] F. Finelli and R.H. Brandenberger, Parametric amplification of metric fluctuations during reheating in two field models, Phys. Rev. D 62 (2000) 083502 [hep-ph/0003172] [IN_SPIRE].

[66] H.B. Moghaddam, R.H. Brandenberger, Y.-F. Cai and E.G.M. Ferreira, Parametric Resonance of Entropy Perturbations in Massless Preheating, Int. J. Mod. Phys. D 24 (2015) 1550082 [arXiv:1409.1784] [IN_SPIRE].
A. Taruya and Y. Nambu, *Cosmological perturbation with two scalar fields in reheating after inflation*, Phys. Lett. B 428 (1998) 37 [gr-qc/9709036] [inSPIRE].

B.A. Bassett, D.I. Kaiser and R. Maartens, *General relativistic preheating after inflation*, Phys. Lett. B 455 (1999) 84 [hep-ph/9808404] [inSPIRE].

B.A. Bassett, F. Tamburini, D.I. Kaiser and R. Maartens, *Metric preheating and limitations of linearized gravity. 2.*, Nucl. Phys. B 561 (1999) 188 [hep-ph/9901319] [inSPIRE].

B.A. Bassett, C. Gordon, R. Maartens and D.I. Kaiser, *Restoring the sting to metric preheating*, Phys. Rev. D 61 (2000) 061302 [hep-ph/9909482] [inSPIRE].

B.A. Bassett, F. Tamburini, D.I. Kaiser and R. Maartens, *Metric preheating and limitations of linearized gravity. 2.*, Nucl. Phys. B 561 (1999) 188 [hep-ph/9901319] [inSPIRE].

R.H. Brandenberger, A.R. Frey and L.C. Lorenz, *Entropy fluctuations in brane inflation models*, Int. J. Mod. Phys. A 24 (2009) 4327 [arXiv:0712.2178] [inSPIRE].

R.H. Brandenberger, K. Dasgupta and A.-C. Davis, *A Study of Structure Formation and Reheating in the D3/D7 Brane Inflation Model*, Phys. Rev. D 78 (2008) 083502 [hep-ph/0801.3674] [inSPIRE].

H.B. Moghaddam, R.H. Brandenberger, Y.-F. Cai and E.G.M. Ferreira, *Parametric Resonance of Entropy Perturbations in Massless Preheating*, Int. J. Mod. Phys. D 24 (2015) 1550082 [arXiv:1409.1784] [inSPIRE].

H.B. Moghaddam and R.H. Brandenberger, *A First Look at Preheating after Axion Monodromy Inflation*, arXiv:1502.06135 [inSPIRE].

H. Kodama and M. Sasaki, *Cosmological Perturbation Theory*, Prog. Theor. Phys. Suppl. 78 (1984) 1 [inSPIRE].

M. Axenides, R.H. Brandenberger and M.S. Turner, *Development of Axion Perturbations in an Axion Dominated Universe*, Phys. Lett. B 126 (1983) 178 [inSPIRE].

C. Gordon, D. Wands, B.A. Bassett and R. Maartens, *Adiabatic and entropy perturbations from inflation*, Phys. Rev. D 63 (2001) 023506 [astro-ph/0009131] [inSPIRE].

K.A. Malik and D. Wands, *Adiabatic and entropy perturbations with interacting fluids and fields*, JCAP 02 (2005) 007 [astro-ph/0411703] [inSPIRE].

A.R. Liddle, D.H. Lyth, K.A. Malik and D. Wands, *Superhorizon perturbations and preheating*, Phys. Rev. D 61 (2000) 103509 [hep-ph/9912473] [inSPIRE].

D.N. Page and S.W. Hawking, *Gamma rays from primordial black holes*, Astrophys. J. 206 (1976) 1 [inSPIRE].

A.M. Green and K.A. Malik, *Primordial black hole production due to preheating*, Phys. Rev. D 64 (2001) 021301 [hep-ph/0008113] [inSPIRE].

D. Blais, T. Bringmann, C. Kiefer and D. Polarski, *Accurate results for primordial black holes from spectra with a distinguished scale*, Phys. Rev. D 67 (2003) 024024 [astro-ph/0206262] [inSPIRE].

F. Küehnel, C. Rampf and M. Sandstad, *Effects of Critical Collapse on Primordial Black-Hole Mass Spectra*, Eur. Phys. J. C 76 (2016) 93 [arXiv:1512.00488] [inSPIRE].

B.J. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, *New cosmological constraints on primordial black holes*, Phys. Rev. D 81 (2010) 104019 [arXiv:0912.5297] [inSPIRE].

I. Zaballa, A.M. Green, K.A. Malik and M. Sasaki, *Constraints on the primordial curvature perturbation from primordial black holes*, JCAP 03 (2007) 010 [astro-ph/0612379] [inSPIRE].

A.M. Green, A.R. Liddle, K.A. Malik and M. Sasaki, *A New calculation of the mass fraction of primordial black holes*, Phys. Rev. D 70 (2004) 041502 [astro-ph/0403181] [inSPIRE].

A.S. Josan, A.M. Green and K.A. Malik, *Generalised constraints on the curvature perturbation from primordial black holes*, Phys. Rev. D 79 (2009) 103520 [arXiv:0903.3184] [inSPIRE].
[90] D. Junghans and G. Shiu, \textit{Brane curvature corrections to the $\mathcal{N}=1$ type-II/F-theory effective action}, \textit{JHEP} \textbf{03} (2015) 107 [arXiv:1407.0019] [InSPIRE].

[91] P. Ouyang, \textit{Holomorphic D7 branes and flavored $N=1$ gauge theories}, \textit{Nucl. Phys. B} \textbf{699} (2004) 207 [hep-th/0311084] [InSPIRE].

[92] M. Rummel and A. Westphal, \textit{A sufficient condition for de Sitter vacua in type IIB string theory}, \textit{JHEP} \textbf{01} (2012) 020 [arXiv:1107.2115] [InSPIRE].