Stochastic Buffeting Analysis of Uncertain Long-Span Bridge Deck with an Optimized Method

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Abstract: The buffeting analysis of an uncertain long-span bridge deck was carried out in this paper. Due to the effect of strong spatial correlation of wind excitation, it should be assumed as partially coherent multiple excitations. The following includes a theoretical formula for the buffeting analysis of a long-span bridge deck with uncertain parameters, which was achieved mainly by a combination of the stochastic pseudo excitation method (SPEM) and response surface method (RSM). The SPEM-RSM was firstly applied to deal with the complicated spectral density function matrix of wind excitation. The buffeting response of the bridge deck was then calculated and verified by the results from the Monte Carlo simulation (MCS). The efficiency and applicability of the hybrid method for strong spatial correlation was proved. After the comparison, the effect of uncertain structural parameters and wind speed on the buffeting performance of the bridge deck were computed. The results showed that the whole uncertainties essentially affected the buffeting response of the deck. The uncertain wind speed played the most significant role in the vertical and lateral motion of the deck. The joint influences between structural uncertainties and uncertain wind speed further affect the random characteristics of the responses. Finally, the effects of different wind speed and wind angle of attack on the aerodynamic performance of the bridge are examined. The variance of the responses increased with the development of wind speed. The effect of different attack angles on the buffeting responses was significant.

Keywords: buffeting response; structural uncertainties; uncertain wind speed; variance; SPEM; RSM

1. Introduction

Modern structures, such as long-span bridges and slender buildings, tend to be large in scale and have low natural frequencies, which are sensitive to the effects of external loads. In recent decades, random responses of the bridge structures induced by wind loads have received much attention [1–7]. However, several structural parameters, which possess stochastic characteristics, are always regarded as the deterministic values in the classical stochastic wind-induced response analysis, and wind loads are also an uncertain random process [8]. Some studies have investigated the random vibrations of uncertain structures [9–12]. Kareem et al. [13] discussed the uncertain damping in structural systems. The results showed that the uncertain damping significantly affected the response, which increased with the development of variability of the damping ratio. Marano et al. [14] proposed a hybrid method to handle the random vibrations of vibrating structures, which involved both random processes and epistemic variables in calculation. It was found that uncertainties play an important role in random vibrations of structures. Moreover, the wind-induced vibrations of structures with uncertainties have received more and more attention in the last decades.
Huang et al. [15] developed a reliability optimization method for the wind-induced vibrations of tall buildings, where some uncertainties were taken into account. However, the buffeting performances of bridges are different to those of building structures. Ge et al. [16] developed a reliability analysis model and a probability analysis method to study the probability of dynamic response of a bridge subjected to wind loads. Canor et al. [17] selected a 1200 m suspension bridge model to evaluate the efficiency of several numerical approaches on flutter performance, and the advantages and disadvantages of every method were listed in that paper. Zhang and Chen [18] proposed an approach to estimate effects of wind load on various mean recurrence intervals (MRIs) with either directionality or uncertain wind speed. Bruno et al. [19,20] investigated the probabilistic evaluation of the wind-induced performance of a bridge with several uncertainties. Solari et al. [21] estimated a unified model of atmospheric turbulence suited to investigate the 3D gust-excited response of structures. Caracoglia [22] studied the derivation of a closed-form solution for single-mode buffeting vibration with several random parameters. The role of wind power spectral density (PSD), damping, and aeroelastic derivatives were clarified. Zhou et al. [23] proposed a multiplicative dimensional reduction method, which is an effective approach to estimate the complex vortex-induced vibration reliability of long-span bridges. The result showed that the wind attack angle has a significant influence on the performance function. Liu et al. [24] presented a valid method for studying wind-induced index and failure probability; the parameters of wind load and structure were considered in the calculation. Kusano et al. [25] adopted three RBDO methods to carry out the long-span bridge under flutter constraint. It was found that SORA was the most calculation-efficient, while PMA was faster than RIA. Kusano et al. [26] performed RBDO for a long-span suspension bridge; the aerodynamic coefficients were tested by wind tunnel tests and the effects of different deck shapes were considered using CFD simulations. Tor Martin et al. [27], based on the full-scale monitoring of the Hardanger Bridge, carried out a probabilistic model to study the uncertain turbulence parameters. Zhang et al. [28] developed a new procedure involving a nonlinear aeroelastic effect and a nonlinear target. The optimization procedures that can lead to more economical design results were demonstrated. However, most of these studies ignored the effect of structural uncertainties on the buffeting responses. Therefore, the significance of uncertainties on the wind-induced responses of a structure should be pointed out.

In this paper, the buffeting responses of a long-span bridge with uncertain parameters, characterized mainly by an association of SPEM-RSM and finite element approach, are presented. The interaction between uncertain structural parameters and wind speed, which may positively or negatively influence the prediction of the buffeting response of a bridge, is the highlight in this paper. The proposed SPEM-RSM [29,30] is based on the principle of the pseudo excitation method (PEM) [31–33]. SPEM-RSM can assess the classical statistical results, e.g., mean value and variance, similar to the description of PEM. It also can handle the effect of several uncertainties of a structural system and is described in terms of the probability density function (PDF). Since this hybrid method is dependent on PEM, it just handles the dynamic responses of a linear or weak nonlinear structure. The seismic response of the bridge is studied, and the results show that the SPEM-RSM increases the computational efficiency by nearly 42 times [29]. In addition, based on the slender bridge tower, the SPEM-RSM is also applied to analyze the effects of some uncertainties on wind-induced structural vibration [30]. However, the aforementioned results only considered the influences of relatively simple spectral interaction on random vibration. The buffeting load should be assumed as partially coherent multiple excitations, due to its strong spatial correlation. Therefore, the accuracy of the SPEM-RSM to handle the buffeting force and its effect on the uncertain long-scale structure is not verified. How to deal with the spectral density function matrix of the buffeting vibration of the bridge deck is also a major challenge. Therefore, the efficiency and applicability of SPEM-RSM to handle the random buffeting vibration of a long-span bridge deck are not confirmed. The buffeting vibration is now calculated for a long-span bridge with the uncertainties of structure and wind loads.
An initial introduction to the SPEM-RSM for random buffeting analysis of a bridge deck is presented first. Then, the buffeting responses of the bridge deck are calculated and verified by those from Monte Carlo simulation. After the comparison, the effects of uncertain structural parameters and wind speed on the responses of the bridge are computed. Finally, the effects of different wind speed and wind angle of attack are examined.

2. Theoretical Formulas

The following is an initial introduction to the formulation for random analysis [30], which is characterized mainly by an association of finite element method and SPEM-RSM. The advantages of this combined approach are to deal with the varying structural parameters and external excitation, to make good use of the finite element structural models already made for a number of sampling calculations of buffeting response with different parameters, to involve the selected random variables, and to determine buffeting response of the bridge deck with uncertain properties. The equation of motion of the buffeting analysis of the bridge can be written as

$$\mathbf{M} \ddot{\mathbf{Y}}(t) + \mathbf{C} \dot{\mathbf{Y}}(t) + \mathbf{K} \mathbf{Y}(t) = \mathbf{E} \mathbf{P}(t)$$

where $\ddot{\mathbf{Y}}(t), \dot{\mathbf{Y}}(t), \mathbf{Y}(t)$ are the nodal acceleration, velocity, and displacement vector of $N$ dimensions for the bridge deck, respectively. $\mathbf{M}, \mathbf{C}, \mathbf{K}$ are mass, damping, and stiffness matrices of the bridge with $N$ degrees of freedom (DOF). $\mathbf{P}(t)$ is the buffeting loads vector of $m$ dimensions for the bridge deck. $\mathbf{E}$ is the $N \times m$ matrix which can take the $m$-dimensional loading vector account into the $N$-dimensional loading vector.

The $m \times m$ PSD matrix of the buffeting loading $\{x(t)\}$, affecting the bridge deck by stationary wind loads $\{x(t)\}$ $m$ points, and the PSD matrix is expressed as

$$\mathbf{S}_{xx}(\omega) = \begin{bmatrix} S_{11}(\omega) & S_{12}(\omega) & \cdots & S_{1m}(\omega) \\ S_{21}(\omega) & S_{22}(\omega) & \cdots & S_{2m}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ S_{m1}(\omega) & S_{m2}(\omega) & \cdots & S_{mm}(\omega) \end{bmatrix}$$

where $S_{xx}(\omega)$ is the PSD matrix of the excitation $\mathbf{P}(t)$; $S_{mm}(\omega)$ is the cross-PSD matrix of the nodal buffeting load acting as the $m$th and $m$th node.

In order to develop the computational efficiency for investigating buffeting response with random excitation involving uncertain parameters, an SPEM-RSM is adopted to determine PSD and variance for the buffeting response. The SPEM-RSM was proposed by Zhu [29,30], and is based on the PEM [33], to handle the random vibrations of uncertain structures subjected to stochastic external excitation. The SPEM calculates sampling results of random vibrations for uncertain structures, and every obtained sampling can present independent random characteristics of power spectrums of excitations and uncertainties. The RSM, which can further reduce the computational cost, is imported to handle the computed sampling to assess the statistical results for uncertain structures. The advantages of the hybrid method include the more computation efficiency that is required, the maintenance of the whole correlation terms among normal modes, and the consideration of the effect of uncertainties on random vibration. The principle equations involved in buffeting analysis are listed below, and the detailed derivation can be found in Refs. [29,30].

In this paper, the load is assumed as the stationary partial coherent random process, which cannot be multiplied into two vectors. The PSD matrix of the response $\{y\}$ can be listed as

$$\mathbf{S}_{yy}(\omega) = |H(\omega)|^2 \cdot \mathbf{S}_{xx}(\omega) \cdot |H(\omega)|^T$$

where $|H(\omega)|$ is the frequency response function matrix, and $\mathbf{S}_{xx}(\omega)$ is the PSD matrix of wind loads.
Since the PSD matrix of wind loads $S_{xx}(\omega)$ should be a Hermitical matrix, this excitation spectral matrix can be decomposed as

$$[S_{xx}(\omega)] = L^*DL^T = \sum_{k=1}^{m} \lambda_{kk} L_k L_k^T$$

(4)

in which $L$ is the lower triangular matrix; the superscript $*$ and $T$ are complex conjugate and matrix transposition; $D$ is the diagonal matrix; $\lambda_{kk}$ and $L_k$ are eigenvalue and eigenvector. The eigenvalue and eigenvector should satisfy the following relationship:

$$[S_{xx}(\omega)]L_k = \lambda_{kk} L_k$$

(5)

$$L_k L_k^T = \delta_{jj} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

(6)

Therefore, a stochastic pseudo excitation $\{\ddot{x}\}_k$, which makes the input harmonic excitation (original pseudo excitation) possess independent random characteristic, is obtained as

$$\{\ddot{x}\}_k = \alpha L_k \sqrt{\lambda_{kk}} \cdot e^{i\omega t}, (k = 1, 2, \ldots, m)$$

(7)

where $\alpha$ is a random variable, the so-called stochastic amplitude factor [29].

Since the linear structural damping matrix follows orthogonal condition, the equation of dynamic motion of structure can be accessed through modal analysis with essential development in computational efficiency. For $k$th stochastic pseudo excitation, the multiple degree of freedom motion equation can be uncoupled with the first $p$ mode as follows:

$$\ddot{u}_j + 2\xi_j\omega_j \cdot \dot{u}_j + \omega_j^2 \cdot u_j = \alpha \cdot \{\gamma\}_j^T \cdot L_k \cdot \sqrt{\lambda_{kk}} \cdot e^{i\omega t}$$

(8)

where $\xi_j$ and $\omega_j$ are natural damping ratio and frequency for the $j$th $(j = 1 \sim p)$ mode; $u_j$ and $k$ are the $j$th modal response with the $k$th excitation, respectively. In addition,

$$\{\gamma\}_j^T = \{\phi\}_j^T \cdot E$$

(9)

in which $\{\phi\}_j^T$ is the $j$th modal shape.

For every stochastic pseudo excitation $\{\ddot{x}\}_k$ vector, the stochastic pseudo-displacement response vector can be expressed as

$$u_{ijk} = \alpha \cdot H_j(\omega) \cdot \{\gamma\}_j^T \cdot L_k \cdot \sqrt{\lambda_{kk}} \cdot e^{i\omega t}$$

(10)

$$H_j(\omega) = \frac{1}{(\omega_j^2 - \omega^2) + 2\xi_j\omega_j \cdot i}$$

(11)

The corresponding response of the structure with $\{\ddot{x}\}_k$ is

$$\{\ddot{y}\}_k = \sum_{j=1}^{n} \{\phi\}_j \cdot u_{ijk} = \alpha \sum_{j=1}^{n} \{\phi\}_j \cdot H_j(\omega) \cdot \{\gamma\}_j^T \cdot L_k \cdot \sqrt{\lambda_{kk}} \cdot e^{i\omega t}$$

(12)

Substituting the whole computational stationary response,

$$\{\ddot{y}\} = \sum_{k=1}^{m} \{\ddot{y}\}_k \cdot \{\ddot{y}\}_k^T$$

$$= \alpha^2 \sum_{k=1}^{m} \left\{ \sum_{j=1}^{p} \{\phi\}_j \cdot H_j(\omega) \cdot \{\gamma\}_j^T \cdot L_k \cdot \sqrt{\lambda_{kk}} \cdot \{\ddot{y}\}_k \cdot \{\ddot{y}\}_k^T \right\}$$

(13)

Statistical quantity can be written as

$$\langle y \rangle' = \text{Sign}(\alpha) \cdot \sqrt{\int_{-\infty}^{+\infty} \{\ddot{y}\} \cdot \{\ddot{y}\}^T d\omega} = \alpha \cdot \sqrt{\int_{-\infty}^{+\infty} [S_{yy}(\omega)] d\omega}$$

(14)
where \( \text{Sign}(\alpha) \) is a symbolic function of the stochastic amplitude factor.

In general, the random response of a linear structure with Gaussian excitation should follow normal distribution [34]. For the classical random vibration principle, the variance of the structural response can be obtained as

\[
\sigma_y^2 = \int_{-\infty}^{+\infty} S_{yy}(\omega) \, d\omega
\]  

(15)

If the stochastic amplitude factor \( \alpha \sim N(0, 1) \) follows standard normal distribution, the mean value and variance of \([y']\) can be given as

\[
\begin{align*}
E\left[ [y'] \right] &= E \left[ \alpha \cdot \sqrt{\int_{-\infty}^{+\infty} S_{yy}(\omega) \, d\omega} \right] = 0 \\
D\left[ [y'] \right] &= D \left[ \alpha \cdot \sqrt{\int_{-\infty}^{+\infty} S_{yy}(\omega) \, d\omega} \right] = \sigma_y^2
\end{align*}
\]

(16)  

(17)

Since the mean value of the wind-induced structural response is based on the effect of static wind loads, the mean value with buffetting loads should be assumed as zero. The variance of response is same as the original results. It is shown that the stochastic amplitude factor does not affect the original random responses of structure, such as mean value, variance, and distribution.

Unlike the PEM, when the uncertainties are considered in the calculation, the SPEM can handle the significant information for random vibration analysis to compute the required sampling results. The RSM [7,35,36] is selected to deal with those samplings. Therefore, both the preservation of the random characteristics of PDF for external excitation and the effects of uncertainties are presented by SPEM-RSM, whose calculation flow is shown in Figure 1.

**Figure 1.** The computational flow diagram of SPEM-RSM.
3. Features and Parameters for the Case Study

To investigate the application of the SPEM-RSM on the stochastic response of the long-span bridge subjected to buffeting wind loads, a long-span double-tower single-span steel box girder suspension bridge is selected to be computed in this section. The main span is 1196 m, and the heights of two towers are, respectively, 169.688 m and 129.703 m. The finite element model is established in ANSYS software, shown in Figure 2. Table 1 lists the structural natural frequencies and mode features. Since the steel bridge is selected, the uncertainty of mass for the bridge is ignored. The information of all the input uncertainties is presented in Table 2. The mean value of density and the elastic modulus of the bridge are 7.85 × 10³ kg/m³ and 2.10 × 10⁸ MPa, respectively. The mean value of the damping ratio of the structure is selected as 0.02. The mean wind speed is selected as 10 m/s. The major challenges of this paper are investigating the interaction among the different uncertainties, and the effect of them on buffeting responses. The self-excited forces and nonlinear effect will influence the target, so will not be involved in this calculation.

Figure 2. The finite element model of the bridge.

Table 1. Bridge structural natural frequencies and mode features.

| Mode Features                  | Natural Frequencies (Hz) |
|--------------------------------|--------------------------|
| First symmetrical Lateral bending | 0.0580                  |
| First asymmetrical Lateral bending  | 0.1524                  |
| First asymmetrical vertical bending | 0.0950                  |
| First symmetrical vertical bending  | 0.1438                  |
| First symmetrical torsion        | 0.3013                  |
| First asymmetrical torsion       | 0.3569                  |

Table 2. Bridge structural natural frequencies and mode features.

| Parameter         | Distribution            | COV | Refs. |
|-------------------|-------------------------|-----|-------|
| Elastic Modulus   | Normal                  | 0.1 | [37]  |
| Wind Speed        | Type I Extreme Value    | 0.161| [35]  |
| Damping Ratio     | Log-normal              | 0.4 | [36,38]|

Based on the theory of quasi-steady, buffeting excitations affected on deck are expressed as [39]

\[
D_{bu}(x, t) = \frac{1}{2} \rho U^2 B \left[ 2C_D(\alpha_0) \frac{u(x,t)}{U} \gamma_1 + C_D(\alpha_0) \frac{w(x,t)}{U} \gamma_2 \right]
\]

\[
L_{bu}(x, t) = -\frac{1}{2} \rho U^2 B \left[ 2C_L(\alpha_0) \frac{u(x,t)}{U} \gamma_3 + \left( C_L'(\alpha_0) + C_D(\alpha_0) \right) \frac{w(x,t)}{U} \gamma_4 \right]
\]

\[
M_{bu}(x, t) = \frac{1}{2} \rho U^2 B^2 \left[ 2C_M(\alpha_0) \frac{u(x,t)}{U} \gamma_5 + C_M(\alpha_0) \frac{w(x,t)}{U} \gamma_6 \right]
\]

\[
\gamma_1 \sim \gamma_6 \text{ are the aerodynamic admittance functions. The}
\]

\[
D_{bu}, L_{bu}, M_{bu} \text{ are drag, lift, and moment buffeting loads; } \alpha_0 \text{ is the angle between bridge deck and wind flow; } C_M, C_L, C_D \text{ are the slopes of } C_M, C_L, C_D; \ u(x,t) \text{ is the fluctuating wind velocity in the direction of mean wind flow; } w(x,t) \text{ is the fluctuating wind velocity in the deck vertical direction.}
\]
aerodynamic force coefficients of the bridge deck are measured by wind tunnel test, which is shown in Figure 3.

Figure 3. Drag, lift, and moment coefficient curves (body-axes system and wind-axes system).

Since the bridge deck is a flat streamlined box girder section, aerodynamic admittance functions of the bridge deck can be approximated by the simplified flat tablet aerodynamic admittance function. For the passivity cross section, the aerodynamic admittance functions could be approximated as 1.0, which means that the effects of aerodynamic admittance functions are ignored.
According to the wind-resistant design specification, the lateral and vertical spectra are selected as the Kaimal and Panofsky spectrum [38], whose expressions are shown as follows:

\[
\frac{nS(z, n)}{u_s^2} = \frac{200f}{(1 + 50f)^{5/3}}
\]

(19)

\[
\frac{nS(z, n)}{u_s^2} = \frac{6f}{(1 + 4f)^2}
\]

(20)

where \(f\) is the dimensionless frequency \(f = \frac{nU}{U(z)}\), \(U(z)\) is mean value of wind speed at the height of \(z\); \(n\) is frequency in Hz; \(u_s\) is shear velocity of the flow.

In turbulent wind field circumstances, the buffeting wind has a spatial correlation. The special correlation function is written as

\[
Coh_{ij}(f) = \exp\left(-\lambda_{ij} \frac{fr_j}{U_s}\right)
\]

(21)

where \(\lambda_{ij}\) is non-dimensional attenuation factor, and \(\lambda_{ij} = 7\) is adopted in wind-resistant design. \(r_j\) is simulating point. \(j\) is the direction of wind, \((j = u, v, w)\), and \(i\) is the different components of the buffeting wind \((i = u, v, w)\).

4. Comparison of Buffeting Response for Deck

The displacement response of the middle point for the bridge deck is calculated using the SPEM-RSM and is compared with that from the MCS. The mean wind speed is 10 m/s, and several uncertainties are involved in the computation which follows, as listed in Table 2. The total computational times of random buffeting response computed by the SPEM-RSM are 882 realizations. A total of 30,000 samples are adopted in the Monte Carlo method to compare with the results computed by the hybrid method. The errors of buffet responses in the verification are less than 1%. The PDFs of uncertain structure by two different methods are shown in Figure 4.

As shown in Figure 4, the comparison is significantly satisfactory. The efficiency of SPEM-RSM increases by as much as 34.01 times. Therefore, the accuracy and efficiency of the hybrid method is verified by the Monte Carlo method. The hybrid method can perfectly investigate random buffeting responses of bridge deck with uncertainties.
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Figure 4. PSD of middle point for deck with different method.

5. Case Study

5.1. The Interaction between Structural Uncertainties and Uncertain Wind Speed

Four numerical cases are investigated, which are listed in Table 3. There are four uncertainties involved in the calculation, including wind speed, elasticity modulus, damping ratio, and stochastic amplitude factor. The lateral and vertical displacements of the middle point of the bridge deck are performed to investigate the effects of uncertain factors at the wind speeds of 10 m/s, 20 m/s, 30 m/s, and 40 m/s. The corresponding variances of the four cases with the wind speed of 10 m/s are taken as an example to investigate the relationship between two different types of uncertainties and listed in Table 4. As shown in Table 4, the effects of the involved uncertainties on the stochastic response of the bridge deck are significant. The uncertain wind speed is more dominant on the response.
than uncertain structural parameters. The variances in Case 2 of both lateral and vertical responses rise 10% more than those without any uncertainties. The effect of structural uncertainties only increases by 1~2% compared to Case 1 and Case 3. Furthermore, they will not influence each other; an independent relationship exists between them. When the uncertain wind speed associated with the structural uncertainties (Case 4) affects the buffeting response, the variance can further increase compared with Case 2 and Case 3. The variances of responses with two types of uncertain parameters satisfy the superposition principle for both the vertical and lateral responses.

**Table 3.** The detailed information of numerical cases.

| Uncertain Structural Parameters | Uncertain Wind Speed |
|-------------------------------|----------------------|
| Case 1 Involved                | Not involved         |
| Case 2 Not involved            | Involved             |
| Case 3 Involved                | Not involved         |
| Case 4 Involved                | Involved             |

**Table 4.** The variances of four numerical cases (wind speed 10 m/s).

|                  | Lateral Displacement | Vertical Displacement |
|------------------|----------------------|-----------------------|
| Case 1           | 0.03130              | 0.00120               |
| Case 2           | 0.03338              | 0.00130               |
| Case 3 Involved  | 0.03205              | 0.00122               |
| Case 4 Involved  | 0.03414              | 0.00132               |

To investigate the effects of uncertainties on the discretion of responses, the PDFs of the target responses is also plotted in Figures 5 and 6. For the lateral responses with various wind speed, the uncertain wind speed can affect discreteness through comparison with Case 1 and Case 2. The differences between Case 1 and Case 3 are shown, where the obvious asymmetrical distribution is exhibited in the results. The results in Case 2 are similar to those in Case 3, which means the contribution of two types of uncertainties on the buffeting response is almost same. However, the PDF curves of Case 4 have the biggest predictive interval, which show the most discrete distribution. It follows that the association effect between uncertain wind speeds with structural parameters is also positive. Therefore, either the structural uncertainties or uncertain wind speed can impact the lateral responses.

Nevertheless, for the vertical response, the effects of the input uncertainties have several differences to those on lateral responses. The effects of uncertain wind speed are relatively reduced compared to those on lateral responses. The uncertain structural parameters play a dominant role in the vertical buffeting response of the bridge deck. At the wind speed of 10 m/s, the uncertain wind speed inhibits the influences of structural parameters, but the positive interaction between two types of uncertainties also exists in the random buffeting responses for the other cases. In addition, all the input uncertainties can affect the symmetry of PSD results. Therefore, both the uncertain wind speed and also structural uncertainties should be involved in random buffeting analysis.
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The random buffeting responses for the other cases. In addition, all the input uncertainties can affect the symmetry of PS results. Therefore, both the uncertain wind speed and also structural uncertainties should be involved in random buffeting analysis.

Figure 5. The PDF of the lateral stochastic responses of the bridge deck.
5.2. The Effect of Different Wind Speed on Structural Responses

The mean wind speeds are set as 10, 20, 30, and 40 m/s to calculate the effect of wind speed on the stochastic responses of an uncertain bridge. As shown in Figures 7 and 8, the wind speed leads to an increase in the discreteness of buffeting responses for the bridge deck. The response sections also develop with increasing of wind speed. The variances of lateral and vertical displacement at the middle of the bridge deck are shown in Figure 7 for the four numerical cases described above. As shown in Figure 7, the response of the bridge...
increases with the wind speed. The effect of wind speed dominates the joint influence of the total uncertainties. The curve of the variance of Case 1 is similar to that of Case 3 at various wind speeds, and so are Case 2 and Case 4. It is clear that the response is more sensitive to the effect of uncertain wind speed than structural uncertainties. The effect of uncertain wind speed increases with the wind speed, which is influenced by the difference of amplifications between Case 1 and Case 2 or Case 3 and Case 4. However, the effect of structural uncertainties has no disciplinary rule with different wind speed. In addition, the variance of Case 4 is the maximum of all the cases at the various wind speeds. Therefore, the effects of two types of uncertainties cannot be ignored.

![Diagram](Figure 7. The variance of responses of the bridge deck at the different wind speeds.)

### 5.3. The Effect of Different Attack Angle on Structural Responses

The attack angle is one of the most significant influencing factors for the wind-induced vibration of a bridge deck. In this section, the effect of various attack angles, such as $-5^\circ$, $-3^\circ$, $0^\circ$, $+3^\circ$, and $+5^\circ$, on the random buffeting vibration is investigated. The variances of lateral and vertical responses are expressed in Table 5 and Figure 8. For lateral variance, the slopes decrease when the angle changes from $-5^\circ$ to $+5^\circ$. However, there is a little difference for the vertical responses, for which the slope from $-5^\circ$ to $-3^\circ$ is bigger than that of $-3^\circ$ to $0^\circ$. If the attack angle is $+5^\circ$, the variance of the buffeting response reaches the maximum value. Therefore, the random buffeting vibration is significantly sensitive to the varying attack angle. For this case, a positive angle makes the structure appear to be in the most dangerous state.

Table 5. The variances with different attack angle.

| Angle | Lateral Response | Vertical Response |
|-------|------------------|-------------------|
| $-5^\circ$ | 0.88068 | 0.04711 |
| $-3^\circ$ | 0.88068 | 0.04685 |
| $0^\circ$ | 0.82631 | 0.04434 |
| $+3^\circ$ | 0.70252 | 0.04086 |
| $+5^\circ$ | 0.58219 | 0.03939 |
5.3. The Effect of Different Attack Angle on Structural Responses

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![Graph of Lateral Variance](image)

Table 5: Lateral Variance

| Attack angle ($^\circ$) | Lateral Variance |
|------------------------|------------------|
| $-5$                   | 0.5              |
| $-3$                   | 0.6              |
| $0$                    | 0.7              |
| $3$                    | 0.8              |
| $5$                    | 0.9              |

![Graph of Vertical Variance](image)

Table 6: Vertical Variance

| Attack angle ($^\circ$) | Vertical Variance |
|------------------------|-------------------|
| $-5$                   | 0.0400            |
| $-3$                   | 0.0425            |
| $0$                    | 0.0450            |
| $3$                    | 0.0475            |

Figure 8. The variance of responses of the bridge deck with different attack angle.

6. Conclusions

The SPEM-SGI is adopted to investigate the buffeting vibration of a bridge deck with several uncertainties of structure and wind loads in this paper. The stochastic responses of the deck obtained from the suggested method are in great agreement with those calculated by the MCS. The remarkable conclusions are summarized as below:

1. The comparison between SPEM-RSM and MSC is significantly satisfactory. The efficiency of SPEM-RSM increases as much as 34.01 times. The errors between two methods of buffeting analysis responses in the verification are less than 1%.

2. The combined effect of the uncertainties of structural parameters and wind speed cause the slight asymmetrical distribution of the random vibration of the uncertain structure.
(3) The response is more sensitive to the effect of uncertain wind speed than structural uncertainties. The effect of uncertain wind speed increases with the wind speed, but the effect of structural uncertainties has no disciplinary rule with different wind speed.

(4) The random buffeting vibration is significantly sensitive to the varying attack angle. For this case, a positive angle makes the structure appear to be in the most dangerous state.

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