Thermoelectric properties of inversion asymmetric Weyl semimetal-Weyl superconductor hybrid junctions

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We theoretically investigate the thermoelectric properties (electronic contribution) of a hybrid structure comprising of an inversion symmetry broken Weyl semimetal (WSM) proximitized to a bulk s-wave superconductor (WSM-Weyl Superconductor (WSC) junction), employing the Blonder-Tinkham-Klapwijk formulation for non-interacting electrons. Our study unfolds interesting features for various relevant physical quantities such as the thermal conductance, the thermoelectric coefficient and the corresponding figure of merit. We also explore the effects of an interfacial insulating (I) barrier (WSM-I-WSC set-up) on the thermoelectric response in the thin barrier limit. Further, we compute the ratio of the thermal to the electrical conductance in different temperature regimes and find that the Wiedemann-Franz law is violated for small temperatures (below critical temperature \( T_c \)) near the Weyl points while it saturates to the Lorentz number, away from the Weyl points, at all temperatures irrespective of the barrier strength. We compare and contrast this behaviour with other Dirac material heterostructures and provide a detailed analysis of the thermal transport. Our study can facilitate the fabrication of mesoscopic thermoelectric devices based on WSMs.

I. INTRODUCTION

In recent times, Weyl semimetals (WSMs) have been subject to intense theoretical and experimental investigations as they are explicit material realizations of hitherto high energy phenomena such as the Adler-Bell-Jackiw anomaly [1, 2] and the chiral magnetic effect [3]. In terms of their band structure, WSMs exhibit linearly dispersing excitations from non-degenerate band touching points called Weyl nodes accompanying unusual surface projections known as Fermi arcs [4]. Weyl nodes always appear in pairs of opposite chirality and behave as oppositely charged monopoles of Berry flux because the electronic states around the band touching points have non-zero Berry curvature. This gives rise to non-trivial topology in momentum space due to which WSMs exhibit several interesting physical effects such as the quantum anomalous Hall effect, the chiral magnetic effect, negative magneto-resistance, etc. [4–8]. Intense theoretical and experimental research work has been carried out in both time-reversal and inversion symmetry broken WSMs [4, 5, 9].

In principle, the WSM phase can be realized by breaking either the time reversal (TR) or the inversion symmetry (IS) [4, 7, 10, 11]. Breaking of the TR symmetry requires a large external magnetic field, which limits the use and application of these materials. However, recently, a WSM state was confirmed in transition metal monophosphides or mono-arsenides MX materials (M=Nb and Ta; X=P and As) with naturally broken IS due to crystal structure asymmetry [5, 12–16]. In these materials, the Weyl nodes and surface Fermi arcs are detected by using angle-resolved photoemission spectroscopy (ARPES).

Over the past years, superconducting hybrid structures, have attracted a great deal of attention due to the dramatic boosts of thermoelectric effects in them [17–24]. In order to investigate the thermoelectric properties of a material or hybrid junction, it is customary to compute the thermal conductance or thermal current generated by the applied temperature gradient. From the application point of view, it is more desirable to investigate the Seebeck coefficient, known as thermopower. A better way to examine the efficiency of a system as a thermoelectric is to study the thermopower as well as a dimensionless parameter called the figure of merit \( zT \) [25]. Improving this thermoelectric \( zT \) along with enhanced Seebeck coefficient is one of the main challenges in material science [26] as well as mesoscopic hybrid junctions [27]. Along this direction, heat transport has also been investigated in superconducting heterostructures of two-dimensional Dirac systems [28–33].

Although electronic properties of WSMs (both in bulk and hetero-junctions) have been extensively studied in recent times [34–38], far less is known about its thermoelectric properties as far as hybrid junctions are concerned. There have been earlier studies including disorder and interactions both near the Weyl point and with doping away from the Weyl point [12, 39–41]. However, these works only investigate thermoelectric properties in the
bulk material. Heterojunctions which form the foundation of applications in electronics and spintronics, have not yet been explored in the context of its thermal properties involving WSMs and proximity induced superconductivity. Motivated by this fact, in this article, we focus on this gap and explicitly study the thermal properties of a hetero-junction consisting of an inversion symmetry broken WSM (normal region) on one side and with the other side placed in close proximity to a bulk s-wave superconductor, thus tailoring a WSM-Weyl superconductor (WSC) junction. In particular, we obtain the electronic contribution to the thermal conductivity, the Seebeck coefficient and the figure of merit (∼ the ratio of the Seebeck coefficient and the thermal conductivity) for this setup. Note that a similar analysis cannot be carried out for a TR broken WSM-WSC junction, as Andreev reflection is fully suppressed there in the absence of a spin active interface, due to chirality blockade [42].

The remainder of this paper is organized as follows. In Sec. II, we describe the model Hamiltonian of IS broken WSM and the scattering matrix approach to analyse our setup. Sec. III is devoted to the analytical formulae for computing various physical quantities that are required to assess the thermoelectric properties of the system. We discuss the numerical results of the WSM-WSC junction in Sec. IV. Finally, we summarize our findings and discuss some possible outlooks in Sec. V.

II. MODEL AND METHOD

In this section we describe the model Hamiltonian of our setup and discuss the scattering matrix approach to analyse the junction problem.

A. Model Hamiltonian

To begin with, we consider an inversion asymmetric WSM described by the low energy Hamiltonian [37]

\[ H(k) = k_x \sigma_x s_z + k_y \sigma_y s_0 + (k_x^2 - |\vec{k}|^2)\sigma_z s_0 + \beta \sigma_y s_y - \alpha k_y \sigma_x s_y, \]

where, \( \vec{k} \) is a three-dimensional vector, \( \sigma(A,B) \) and \( \sigma(\uparrow, \downarrow) \) represent Pauli matrices for orbital and spin degrees of freedom respectively. Here, \( k_0, \alpha \) and \( \beta \) denote real model parameters. For \( 0 < \beta < k_0 \), the model exhibits four Weyl nodes as shown in Fig. 1(a). We depict schematically the cross-sectional view of the band structure at low energy in Fig. 1(b) where we label the Weyl nodes by \( Q_{1,2,3,4} \). The low energy effective Hamiltonian expanded around the Weyl points can be written as [37, 43]

\[ H_{1,2} = (k_x \mp \beta) s_x + k_y s_y + (k_z \mp k_0) s_z, \]
\[ H_{3,4} = (k_x \mp \beta) s_x + k_y s_y - (k_z \mp k_0) s_z. \]

Note that the Weyl points \( Q_{1,2} (Q_{3,4}) \) carry the positive (negative) chirality and form two time-reversed pairs, namely \( s_y H_1^\dagger(k) s_y = H_2(-k) \) and \( s_y H_3^\dagger(k) s_y = H_4(-k) \). We assume that the system remains metallic with an approximately linear dispersion even in the presence of disorder. This remains a good approximation as long as the Fermi energy is larger than the disorder induced gap in the system [41]. Beyond that, we consider intra-orbital s-wave superconducting pairing which couples Weyl nodes of the same chirality. This is because at low energy, the inter-orbital pairing is known to be suppressed [37]. Consequently, we can consider two decoupled Hamiltonians with opposite chirality as

\[ H_{\pm} = \sum_k \langle \Delta^\dagger_{1(3),\uparrow, k} \sigma_z \Delta_{2(4),\downarrow, -k} + 1(3) \leftrightarrow 2(4) \rangle + \text{h.c.} \]

Here, the pairing potential \( \Delta \) couples electrons only between the time-reversed pair of Weyl nodes \( i.e., \) \( Q_1 \) to \( Q_2 \) and \( Q_3 \) to \( Q_4 \). As a result, the system can be described by two effectively independent subsystems with opposite chirality. We consider below the positive chirality sector for our theoretical analysis.

We analyse the heterostructure comprised of a WSM and a bulk s-wave superconductor, considering the above mentioned model Hamiltonians. Here, we assume that superconductivity has been induced in one part of the WSM via the proximity effect, thus forming a WSM-WSC junction as illustrated in Fig. 2. The thin violet region at the interface (see Fig. 2) indicates an insulating barrier with width \( d \) and height \( V_0 \). We incorporate this to investigate thin barrier effects on thermal transport through the junction.

Considering quasi one-dimensional transport along the \( z \) direction, we write the Bogoliubov-de Gennes (BdG) Hamiltonian for positive chirality as

\[ H_{BdG}^+ = \nu_z \tau_0 \left[-i \partial_z \cdot \vec{s} - \mu(z) s_0 \right] + \Delta(z) e^{isgn(z)2\phi \nu_z/\nu_\tau \tau_0 s_0}, \]

[FIG. 1. (Color online) (a) Band structure of IS broken WSM, modelled by the low energy effective Hamiltonian (Eq. (1)), is shown at \( k_y = 0 \). (b) Four Weyl cones labelled by \( Q_{1,2,3,4} \), are depicted schematically in cross-sectional view of the band structure shown in panel (a).]
Hamiltonians (Eq. (2)) to find the expectation values of for electrons and holes. For this, we use the low energy throughout our analysis. Note that, we fix the Fermi-wavelength in the superconducting region \[44\].

The mean-field theory of superconductivity demands that \[\Delta(\mu)\] is \[\Delta(\mu) = \Delta_0 \tanh (1.74 \sqrt{T_c/T - 1})\]. \(\Theta(z)\) is the Heaviside step function, \(\alpha\) is the phase of the superconducting order parameter and \(T_c\) is the critical temperature of the superconductor. Here, \(\mu_N\) and \(\mu_S\) are the chemical potentials on WSM and WSC sides respectively. The mean-field theory of superconductivity demands that \(\mu_S \gg \Delta_0\), or equivalently, that the superconducting coherence length \(\xi\) is much larger than the Fermi-wavelength in the superconducting region \[44\]. Note that, we fix \(\mu_S = 100\Delta_0\) to satisfy this requirement throughout our analysis.

To understand the normal and Andreev reflection (AR) at the interface between WSM and WSC, it is necessary to analyse the spin texture around the Weyl nodes both for electrons and holes. For this, we use the low energy Hamiltonians (Eq. (2)) to find the expectation values of spin vector for the lower conduction band around each Weyl point and show them in Fig. 3. It is evident from Fig. 3(a) that the conduction electron incident from Weyl point \(Q_1\) is reflected as a normal electron from the opposite Fermi surface at \(Q_3\) to preserve spin. The hole due to retro-AR comes from the time reversed partner which is at \(Q'_1 = -Q_1\) and carries opposite spin texture and momenta compared to the incoming electron. This is schematically shown in Fig. 3(b). Specular AR takes place from the valence band of the hole, and this is depicted in Fig. 3(c).

### B. Scattering matrix approach

Due to the proximity induced superconducting coupling between the time-reversed pairs of Weyl cones with the same chirality \[37\], we can separately compute contributions arising from the positive and negative chirality Weyl cones. Taking the positive chirality Weyl cone in our study, we investigate the thermal transport employing the scattering matrix approach \[23, 45, 46\] for the non-interacting electrons in WSM-WSC setup (see Fig. 2). Let us consider the electronic transport along the perpendicular direction to the interface between normal and superconducting part of the WSM, which means that the momentum parallel to the interface \(k_{\parallel} = (k_x, k_y)\) is conserved. We consider the excitation energy \(E > 0\), such that the incoming electron is above the Fermi level at energy \(\mu_N + E\) while the Andreev reflected hole is at energy \(\mu_N - E\).

Using BdG Hamiltonian for the WSM-WSC junction, the wave-functions in the normal side (WSM) for a given excitation energy \(E\) can be written as

\[
\begin{align*}
\Psi_\uparrow(z) &= (\cos \varphi_e, e^{i\theta_e} \sin \varphi_e, 0, 0)^T e^{ik_e z}, \\
\Psi_\downarrow(z) &= (e^{-i\theta_h} \sin \varphi_e, \cos \varphi_e, 0, 0)^T e^{-ik_e z}, \\
\Psi_h(z) &= (0, 0, -e^{-i\theta_h} \sin \varphi_h, \cos \varphi_h)^T e^{ik_h z}, \\
\Psi_h(z) &= (0, 0, \cos \varphi_h, -e^{i\theta_h} \sin \varphi_h)^T e^{-ik_h z},
\end{align*}
\]

where \(k_{(e,h)} = \sqrt{(E \pm \mu_N)^2 - k_{||}^2}\) are the momenta of the electron and hole respectively, \(\varphi_{(e,h)} = \tan^{-1}(k_{||}/k_{(e,h)})/2\) are the incident angles of the incoming electron and hole respectively and \(\theta_k = \tan^{-1}(k_{||}/k_z)\).

The wave functions in the proximity induced superconducting side (WSC) can be written as

\[
\begin{align*}
\Psi_{\uparrow q}(z) &= (e^{i\beta}\cos \phi_e, e^{i\beta} e^{i\theta_h} \sin \phi_e, e^{-i\beta} \cos \phi_e, e^{-i\beta} e^{i\theta_h} \sin \phi_e)^T e^{iq z}, \\
\Psi_{\downarrow q}(z) &= (e^{i\beta} e^{-i\theta_h} \sin \phi_e, e^{i\beta} \cos \phi_e, e^{-i\beta} e^{-i\theta_h} \sin \phi_e, e^{-i\beta} \cos \phi_e)^T e^{-iq z}, \\
\Psi_h(z) &= (e^{i\beta} \cos \phi_h, e^{i\beta} e^{i\theta_h} \sin \phi_h, e^{-i\beta} \cos \phi_h, e^{-i\beta} e^{i\theta_h} \sin \phi_h)^T e^{iq z}, \\
\Psi_h(z) &= (e^{i\beta} \sin \phi_h, e^{i\beta} e^{i\theta_h} \cos \phi_h, e^{-i\beta} \sin \phi_h, e^{-i\beta} e^{i\theta_h} \cos \phi_h)^T e^{-iq z},
\end{align*}
\]

(7)
where, \( q_{\text{e,h}} = \sqrt{(\mu S \pm \Omega)^2 - k_{||}^2} \) and \( \phi_{\text{e,h}} = \tan^{-1}(k_{||}/q_{\text{e,h}})/2 \) are the momenta and incident angles of the electron and hole like quasiparticles respectively inside the superconducting region. For \( E \leq \Delta \) (sub-gapped regime), \( \beta = \cos^{-1}(E/\Delta) \) and \( \Omega = i\sqrt{\Delta^2 - E^2} \) while for \( E > \Delta \) (above the gap), \( \beta = -i \cosh^{-1}(E/\Delta) \) and \( \Omega = \sqrt{E^2 - \Delta^2} \). Considering the scattering between the same chirality Weyl nodes, we can write the scattering states on both sides of the junction as

\[
\Psi(z < 0) = \psi_{\text{e}}(z) + r_{\text{e}} \psi_{\text{e}}(z) + r_{\text{h}} \psi_{\text{h}}(z),
\]
\[
\Psi(z > 0) = t_{\text{e}} \Psi_{\text{f3}}(z) + t_{\text{h}} \Psi_{\text{f4}}(z),
\]

where, \( r_{\text{e,h}} \) corresponds to the normal and AR scattering coefficients respectively, while \( t_{\text{e,h}} \) denotes the transmission coefficients for the electron and hole like quasiparticles respectively. We obtain the scattering matrix by matching the wave functions (Eq. (8)) of both sides of the junction at \( z = 0 \).

### III. THERMAL TRANSPORT

For the non-interacting electrons, the total transmission function through the junction can be computed using the scattering matrix approach. Integrating over the transverse modes one can write [28, 29]

\[
T(E) = \int \frac{d^2 k_{||}}{4\pi^2} \left( 1 - R_{\text{e}}(E) + \text{Re} \left[ \frac{\cos 2\phi_{\text{h}}}{\cos 2\phi_{\text{e}}} \right] R_{\text{h}}(E) \right),
\]

where, \( k_{||} \) is the momentum parallel to the junction interface, \( R_{\text{e}} \) and \( R_{\text{h}} \) are the normal and AR probability respectively, \( \phi_{\text{e}} \) and \( \phi_{\text{h}} \) are the angle of incidence for electron and hole respectively. Changing the integration variable \( k_{||} \) to \( \varphi_{\text{e}} \) using \( k_{||} = (E + \mu_N) \sin 2\varphi_e \), we can write

\[
T(E) = \frac{1}{2\pi} \int d\varphi_e \left( 1 - R_{\text{e}}(E) + \text{Re} \left[ \frac{\cos 2\phi_{\text{h}}}{\cos 2\phi_{\text{e}}} \right] R_{\text{h}}(E) \right) \frac{(E + \mu_N)^2 \sin 4\varphi_e}{2},
\]

(10)

While the electrical transport can distinguish between the electron and hole contribution by the sign of \( R_{\text{e}} \) and \( R_{\text{h}} \), the thermal transport treats both the electron and the hole as a particle carrying the number density along the same direction (see Appendix A for details). Consequently, the transmission function corresponding to the thermal transport can be computed using [28, 29] as

\[
T_{\text{therm}}(E) = \frac{1}{2\pi} \int d\varphi_e \left( 1 - R_{\text{e}}(E) - \text{Re} \left[ \frac{\cos 2\phi_{\text{h}}}{\cos 2\phi_{\text{e}}} \right] R_{\text{h}}(E) \right) \frac{(E + \mu_N)^2 \sin 4\varphi_e}{4},
\]

(11)

### A. Electrical conductance and Thermoelectric coefficient

We begin with the general expression for total electrical current through the normal-superconductor (NS) junction in the presence of voltage and temperature bi-
the thermoelectric coefﬁcient \([46, 47]\) as
\[
I = \frac{2e}{h} \int_{-\infty}^{\infty} [f_N(E - e\Delta V, T + \Delta T) - f_S(E, T)] \nabla(E) dE .
\] (12)

where, \(f_N\) and \(f_S\) are the Fermi distribution functions for the normal and superconducting reservoirs respectively.

In the linear response regime, one can expand the Fermi functions for very small voltage and temperature differences as
\[
f_N(E - e\Delta V, T + \Delta T) = f_0(E, T) - e\Delta V \frac{\partial f_0}{\partial E} + \Delta T \frac{\partial f_0}{\partial T} ,
\]
\[
f_S(E, T) = f_0(E, T) .
\] (13)

where \(f_0\) is the average Fermi distribution at energy \(E\) and temperature \(T\). Using Eq. (13) we can write the total current as (see Appendix \(\text{A}\) for the complete derivation following Ref. [45])
\[
I = \frac{2e}{h} \int_{-\infty}^{\infty} \left[ -e\Delta V \frac{\partial f_0}{\partial E} + \Delta T \frac{\partial f_0}{\partial T} \right] \nabla(E) dE .
\] (14)

Hence, comparing the total current with the Onsager relation
\[
I = G\Delta V + L_{12}\Delta T ,
\] (15)

we arrive at the expressions for electrical conductance and the thermoelectric coefﬁcient \([46, 47]\) as
\[
G = \frac{2e^2}{h} \int_{-\infty}^{\infty} \left( -\frac{\partial f_0}{\partial E} \right) \nabla(E) dE ,
\]
\[
L_{12} = -\frac{2e}{h} \int_{-\infty}^{\infty} \left( -\frac{\partial f_0}{\partial T} \right) \nabla(E) dE .
\] (16)

B. Thermal conductance, Thermopower and Figure of merit

The normalized thermal conductance through the NS interface can be deﬁned as \([46]\)
\[
\kappa/\kappa_0 = \int_{-\infty}^{\infty} (E - \mu_N) \left( -\frac{\partial f_0}{\partial T} \right) \nabla_{\text{therm}}(E) dE .
\] (17)

where, \(\kappa_0\) is the thermal conductance due to ballistic channels in the bulk at energy \((E + \mu_N)\) and \(T = T_c\). This is given by
\[
\kappa_0 = \int_{-\infty}^{\infty} dE \frac{(E + \mu_N)^2}{4\pi} (E - \mu_N) \left( \frac{\partial f_0}{\partial T} \right)_{T=T_c} .
\] (18)

When a temperature gradient is maintained in a metal and no electric current is allowed to ﬂow, a steady-state electrostatic potential difference builds up between the high and low temperature regions. This is called the thermoelectric effect and is measured by the thermopower (the Seebeck coefﬁcient) which by deﬁnition is given by \([48]\)
\[
S = -\left( \frac{\Delta V}{\Delta T} \right)_{t=0} .
\] (19)

Using Eq. (15) the analytical expression for the thermopower can be written as
\[
S = \frac{L_{12}}{G} .
\] (20)

where, \(S\) is measured in units of \(k_B/e\).

The performance of a thermoelectric device is characterized by a dimensionless quantity called the ﬁgure of merit,
\[
zT = \frac{S^2GT}{\kappa} .
\] (21)

The major objective is to attain higher values of ﬁgure of merit in different materials/hybrid junctions, that would be desirable for their potential applicability in thermoelectric devices. It is clear from Eq. (21) that this can be achieved by improving electrical transport properties and reducing thermal conductivity.

C. Wiedemann-Franz law

The Wiedemann-Franz (WF) law states that the ratio of thermal conductance, \(\kappa\) to electrical conductance, \(G\) is proportional to the absolute temperature
\[
\frac{\kappa}{G} = \mathcal{L}T ,
\] (22)

where the proportionality constant \(\mathcal{L}\) is a universal number called the Lorentz number. For an ideal Fermi gas, we have \(\mathcal{L} = \mathcal{L}_0 = \frac{\pi^2}{3} \left( \frac{k_B}{e} \right)^2\). Violation of the WF law has been reported for semiconductors \([49]\), zero-gap systems \(e.g.,\) graphene \([31, 50]\), quasi one-dimensional Luttinger liquids \([51]\) and heavy-fermion materials \([52]\). We explore the validity of the WF law in our WSM-WSC setup and unveil several interesting features near the Weyl point which we discuss in the next section.

IV. NUMERICAL RESULTS

In this section, we proceed to discuss our numerical results for thermal transport through the WSM-WSC setup. We compute the scattering probabilities employing Eq. (8) and ﬁnd the relevant physical quantities using the analytical expressions discussed in Sec. III.
We compute the normalized thermal conductance using Eq. (17) and show its behavior as a function of temperature in Fig. 4(a). For $\mu_N < \Delta_0$, the thermal conductance remains vanishingly small for lower temperatures and starts increasing exponentially with the enhancement of temperature. The zero value of thermal conductance at $T = 0$ is due to the perfect AR in the subgapped regime, which blocks the thermal transport. When the temperature is very small ($T \ll T_c$), electrons with energy at least $\mu_N$ contribute to the thermal conductance. We show this in Fig. 4(a) for $\mu_N/\Delta_0 = 1$ and $\mu_N/\Delta_0 = 2$. Clearly, the thermal conductance increases as we enhance $\mu_N$, due to the availability of more states. The blockage of thermal transport (at $T = 0$) continues as long as the transport is dominated by the superconducting Cooper pairs. Further, thermal conductance exhibits exponential rise for $\mu_N \leq \Delta_0$ as one approaches $T_c$. We attribute this behavior to the spin singlet s-wave superconductor in Weyl NS junction as has been reported earlier in superconducting hybrid junctions based on Dirac-like materials [23, 28, 29]. When the chemical potential on the normal WSM side becomes much larger than the superconducting gap, the NS junction becomes transparent and behaves like a metal. The metallic behaviour of the thermal conductance is known to be linear with the increase of temperature [53, 54], and we find this characteristic for $\mu_N/\Delta_0 = 100$ as shown in Fig. 4(a). This is because at $\mu_N \gg \Delta_0$, the incoming electrons easily overcome the superconducting gap on the right side which results in a linear increase in thermal conductance.

Further, we discuss the behavior of thermal conductance as a function of chemical potential $\mu_N$ in the normal side, at a fixed temperature. To show the clear dependence on $\mu_N$ at a larger scale ($0.01\Delta_0$ to $100\Delta_0$) we demonstrate this using log scale for $\mu_N$ in Fig. 4(b). It is evident that, as we increase the temperature towards $T_c$, there is a larger enhancement in the thermal conductance as we move away from $\Delta_0$.

We discuss the features of the Weidemann-Franz law in this subsection. We numerically compute the thermal and electrical conductance to find the Lorentz number (in terms of Lorentz number $L_0$ for a metal), as given in Eq. (22). We depict the Lorentz number with the variation of the chemical potential in the WSM side of the junction for three values of temperature $T/T_c = 0.99, 0.3, 0.1$ in Fig. 5. The symmetric behaviour of the Lorentz number with respect to the chemical potential $\mu_N$ is due to the particle-hole symmetry of the system. We generally arrive at the larger Lorentz number when there is a small contribution to the thermal conductance. For moderate temperature (e.g., $T/T_c = 0.3$ in Fig. 5), the Lorentz number is finite, even at $\mu_N = 0$. This is due to the reduction of the superconducting gap with the increase of temperature, and we expect that this is responsible for finite thermal conductance even at $\mu_N < \Delta_0$. Note that, there is always a dip in the Lorentz number at the Weyl point ($\mu_N = 0$) below $T_c$. When $T/T_c = 0.1$, this dip becomes maximum and touches the Weyl point due to the unavailability of density of states. However, at $T/T_c = 0.99$ the superconducting gap becomes vanishingly small and we find the Lorentz number for the system to be 4.1958 at the Weyl point. This large value can be attributed to the quasi-particle states close to the superconducting gap. We analytically compute the Lorentz number for a bulk WSM (see Appendix B for details) fol-

![Fig. 4](image1.png)

**FIG. 4.** (Color online) (a) The behavior of normalized thermal conductance is shown as a function of temperature for $\mu_N/\Delta_0 = 0.001, 1, 2, 100$ represented by dashed, dot-dashed, double dot-dashed and solid line respectively. (b) The normalized thermal conductance with respect to the logarithm of $\mu_N/\Delta_0$ is depicted for $T/T_c = 0.3, 0.5, 0.7$ and 0.99 represented by longer dashed, dashed, dot-dashed and solid lines respectively.

![Fig. 5](image2.png)

**FIG. 5.** (Color online) The variation of Lorentz number, in the WSM-WSC junction, is shown as a function of $\mu_N$ for three different values of temperatures. Red, blue and green colour denote the corresponding behavior for $T/T_c = 0.99, 0.3, 0.1$ respectively.
lowing the semi-classical approach and we find it to be 5.13. This is of the same order of magnitude as the numerical estimate here, where we consider the quantum behaviour of the system.

The physical reason behind this peculiar behaviour (violation of the WF law) of the Lorentz number within the range \(-\Delta_0 < \mu_N < \Delta_0\), can be attributed to the temperature dependence of the superconducting gap in our NS Weyl junction. Such deviations of the WF law from \(L_0\) (close to the Dirac point) have been reported earlier in case of graphene [31, 50]. Beyond this region, the Lorentz number converges to \(\kappa\) (Lorentz number for metals) for low temperatures when \(\mu_N \gg \Delta_0\).

\[\Theta(\mu_N/\Delta_0) = \Theta(0) - \Theta(\Delta_0) + \kappa\]

C. Thermopower and figure of merit

![Graph showing the behavior of thermopower or Seebeck coefficient as a function of \(\mu_N/\Delta_0\) and \(T/T_c\).]

FIG. 6. (Color online) The behavior of thermopower or Seebeck coefficient (in unit of \(k_B/e\)) is depicted as a function of (a) \(\mu_N/\Delta_0\) for \(T/T_c = 0.1, 0.5, 0.99\) represented by red (solid), blue (dash) and green (dot-dash) lines respectively. (b) \(T/T_c\) for fixed \(\mu_N/\Delta_0 = 0.001, 1, 2, 100\) represented by green (dash), red (solid), blue (dot-dash) and purple (double dot-dash) lines respectively.

Here, we analyse the thermopower or Seebeck coefficient that also describes the thermoelectric properties of the system. We have defined this quantity in Sec. III (see Eq. (20)). We present the observed characteristics of thermopower in Fig. 6 as a function of (a) the doping level \(\mu_N\) in the WSM side and (b) the temperature \(T\). Note that the thermopower at \(T \ll T_c\) can be clearly distinguished from the behaviour at \(T < T_c\) for all values of \(\mu_N\).

The efficiency of converting heat into electricity is related to the thermoelectric figure of merit, \(zT\) defined in Sec. III. We compute the dimensionless figure of merit employing \(S^2 GT/\kappa\) (Eq. (21)) and show the corresponding behavior in Fig. 7(a) as a function of \(\mu_N/\Delta_0\) at fixed temperatures. Interestingly, we find larger values of \(zT\) for lower temperature (\(T \ll T_c\)) and doping (\(\mu_N \ll \Delta_0\)), as shown in Fig. 7(a). Quantitatively, such high values of \(zT\) arise due to small values of \(\kappa\) and large \(S\) in this regime. However, we do not fully understand the physical reason behind having such a large value of \(zT\). Nevertheless, we note that the two-dimensional Dirac materials, whose low energy dispersion is effectively described by the massless Dirac Hamiltonian, have been shown to maximize \(zT\) up to 15 [55]. The behaviour of \(zT\) is also shown in Fig. 7(b) as a function of temperature for fixed \(\mu_N\). Note that, high values of \(zT \sim 9 \sim 10\) also appear for low temperatures when \(\mu_N/\Delta_0 \ll 1\).

D. Effect of Insulating Barrier

![Graph showing the effect of insulating barrier on thermoelectric properties.]

FIG. 7. (Color online) The figure of merit \(zT\) is depicted in the log scale as a function of (a) \(\mu_N/\Delta_0\) for \(T/T_c = 0.1, 0.5, 0.99\) represented by red (solid), blue (dash) and green (dot-dash) lines respectively and (b) \(T/T_c\) for \(\mu_N/\Delta_0 = 0.11, 0.5, 0.99, 5\) denoted by red (solid), blue (large dash), green (small dash) and purple (dot-dash) lines respectively.

FIG. 8. (Color online) (a) The normalized thermal conductance is depicted as a function of the barrier strength \(\chi\) at \(\mu_N/\Delta_0 = 0.001, 0.1, 1, 100, T/T_c = 0.5\) (b) the maximum amplitude of the thermal conductance oscillations as a function of \(\mu_N\), is shown in the log scale at \(T/T_c = 0.99, 0.5, 0.1\).

In this subsection, we investigate the changes in thermoelectric properties caused by an insulating barrier introduced at the WSM-WSC interface (see Fig. 2). We assume the barrier to be of length \(d\) and height \(V_0\). This can be modelled as \(U(z) = V_0 \Theta(z)\Theta(d - z)\). We carry out our analysis by matching the wave function at the two interfaces, assuming one to be at \(z = 0\) and the other at
\[ z = d. \] The wave function in the insulating region can be written as Eq. (6) with \( \mu_N \) replaced by \( \mu_N - V_0 \). Here, we consider the thin barrier limit. In this limit, one can define a finite quantity \( \chi = V_0d \) \[29\] with \( V_0 \to \infty \) and \( d \to 0 \), called the strength of the barrier. We present here the distinctive behaviour of the thermoelectric properties with respect to this barrier strength \( \chi \).

To begin with, we show the behavior of thermal conductance with the variation of \( \chi \) in Fig. 8(a). Clearly, \( \pi \)-periodic oscillations in \( \kappa \) are found for larger doping \( (\mu_N/\Delta_0 = 100) \). On the other hand, the amplitude of oscillations becomes negligibly small as one decreases \( \mu_N \). We also find that the maximum amplitude (difference between \( |\kappa(\chi = 0) - \kappa(\chi = \pi/2)| \)) of oscillations increases as we increase the value of \( \mu_N \) at any fixed temperature. We present this behavior in Fig. 8(b). The maximum amplitude of oscillations increases more rapidly as one approaches \( T \to T_c \) for \( \mu_N \gg \Delta_0 \).

Afterwards, we study thermopower \( S \) with the variation of \( T/T_c \) for different values of \( \chi \) and show its corresponding behavior in Fig. 9(a). Note that the thermopower changes sign for \( \chi = \pi/3, \pi/2 \). This characteristic feature of changing sign in thermopower is also clearly visible in Fig. 9(b) where we illustrate its behavior with respect to \( \chi \) for different values of \( \mu_N \ll \Delta_0 \). We expect that this sign change in \( S \) takes place due to the electrons close to the Weyl point.

[FIG. 9. (Color online) (a) Thermopower \( S \) is shown as a function of temperature at \( \chi = 0, \pi/4, \pi/3, \pi/2, \pi \). Here we choose \( \mu_N/\Delta_0 = 0.001 \). (b) \( S \) is depicted with the variation of insulating barrier strength \( \chi \) at \( \mu_N/\Delta_0 = 10^{-5}, 10^{-3}, 10^{-2} \) and \( T/T_c = 0.8 \).]

V. SUMMARY AND DISCUSSION

To summarize, in this work, we have investigated thermal transport through normal-superconductor junctions of inversion symmetry broken Weyl semimetals (WSM-WSC heterostructure). We have discussed the characteristic features of thermal conductance, thermopower and figure of merit under different sets of parameter values. We have shown that the normalized thermal conductance has the conventional exponential dependence on the temperature for lower doping (chemical potential \( \mu_N \) on the WSM side), while it exhibits the typical linear behaviour like in metals for higher doping. We find that the Lorentz number unveils interesting features below the superconducting gap. This includes the violation of the Wiedemann-Franz law for \(-\Delta_0 < \mu_N < \Delta_0 \) and agreement (metal like) for higher doping concentrations (\( |\mu_N| \gg \Delta_0 \)). Further, we have studied the thermopower/Seebeck co-efficient and the figure of merit. Surprisingly, the figure of merit shows a sharp increase for transport near the Weyl point, while it stays close to unity away from it. In addition, we have also investigated the effect of a thin insulating barrier on the thermal conductance and thermopower. Importantly, we find larger oscillations (with period \( \pi \)) in thermal conductance with the variation of barrier strength for larger doping and negligible oscillation amplitude for smaller doping concentrations. Moreover, very close to the Weyl point, the thermopower is shown to change sign from positive to negative, as we increase the barrier strength \( \chi \).

As far as practical realizations of our setup is concerned, proximity induced superconductivity has been reported in Dirac semimetals \[56\] and WSMs \[57, 58\]. Therefore, for a s-wave superconductor (e.g., Nb) with \( \Delta_0 \sim 1.3 \text{ meV} \) \[56\] \( (T_c \sim 10 \text{ K}) \), it may be possible to achieve a high value of \( zT \sim 9 \sim 10 \) at low temperatures \( T \sim 1 \sim 2 \text{ K} \) and chemical potentials of \( \mu_N \sim 0.15 \text{ meV} \).

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Appendix A: Derivation of Eq. (14) i.e., the total current

We follow Ref. [45] to arrive at Eq. (14) (see in main text) for one-dimensional transport. This derivation is potentially important for understanding thermoelectric coefficients \( L_{12} \). This physical quantity plays a significant role in determining the thermoelectric properties of any device, namely thermopower or Seebeck coefficient. Thermoelectric current (determined by thermoelectric coefficient) is conceptually a net charge transfer due to temperature bias. We schematically show the normal-superconductor (NS) junction in Fig. 10 where we assume that the left and right reservoirs are kept at bias voltages \( E - e\Delta V \) and \( E \) respectively as well as at temperatures \( T + \Delta T \) and \( T \) respectively. The flow of electrons from the higher to the lower temperature region would cause a flow of electric current from left to right, as depicted by the electron flow indicated by a red dot in Fig. 10. The flow direction of the Andreev reflected hole is denoted by a green dot.
FIG. 10. (Color online) Cartoon of a NS junction that is attached to left (L) and right (R) reservoirs which are kept at temperatures $T + \Delta T$ and $T$ respectively. The red and green dots represent the flow of electrons and holes respectively due to the applied thermal bias.

The total current through the NS junction can be read from Ref. [45] as

$$I = \frac{2e}{h} \int_{-\infty}^{\infty} [f_{\rightarrow}(E) - f_{\leftarrow}(E)] \, dE \, ,$$

where, $f_{\rightarrow}(E)$ and $f_{\leftarrow}(E)$ are the distribution functions of particles coming from N and S side of the junction respectively.

The distribution functions for finite voltage and temperature bias are given by

$$
\begin{align*}
  f_{\rightarrow}(E) & = f_0(E - e\Delta V, T + \Delta T), \\
  f_{\leftarrow}(E) & = A(E) \left[ 1 - f_{\rightarrow}(-E, T - \Delta T) \right] \\
  & \quad + B(E) f_{\rightarrow}(E, T + \Delta T) \\
  & \quad + [C(E) + D(E)] f_0(E, T) \\
  & = A(E) f_0(E + e\Delta V, T + \Delta T) \\
  & \quad + B(E) f_0(E - e\Delta V, T + \Delta T) \\
  & \quad + [1 - A(E) - B(E)] f_0(E, T) \, ,
\end{align*}
$$

where, we use $1 - f_{\rightarrow}(-E, T - \Delta T) = f_0(E + e\Delta V, T - \Delta T)$ and $A(E) + B(E) + C(E) + D(E) = 1$ to arrive at the last expression. We then Taylor expand the distribution functions for small voltage and temperature biases (linear response regime) as

$$
\begin{align*}
  f_0(E - e\Delta V, T + \Delta T) & = f_0(E, T) - e\Delta V \frac{\partial f_0}{\partial E} + \Delta T \frac{\partial f_0}{\partial T}, \\
  f_0(E + e\Delta V, T - \Delta T) & = f_0(E, T) + e\Delta V \frac{\partial f_0}{\partial E} - \Delta T \frac{\partial f_0}{\partial T} \, .
\end{align*}
$$

Using Eq. (A3), we arrive at the current expression for one-dimensional transport as

$$I = \frac{2e}{h} \int_{-\infty}^{\infty} \left[ -e\Delta V \frac{\partial f_0}{\partial E} + \Delta T \frac{\partial f_0}{\partial T} \right] (1 + A - B) \, dE \, .$$

Therefore, from the Onsager relation (see Eq. (15)), one can arrive at the expression for $L_{12}$ as [46]

$$L_{12} = \frac{2e}{h} \int_{-\infty}^{\infty} \left( -\frac{\partial f_0}{\partial T} \right) (1 + A(E) - B(E)) \, dE \, .$$

**Appendix B: Wiedemann-Franz law for bulk Weyl semimetal close to the Weyl point**

In this appendix, we provide an analytical derivation of WF-law for the three-dimensional (3D) bulk WSM near the Weyl point. The idea is to compare this with the Lorentz number for WSM-WSC junction close to critical temperature ($T \sim T_c$) and $\mu_N \sim 0$ which is discussed in the main text and shown in Fig. 5. We note that following the Blonder-Tinkham-Klapwijk approach [45], we numerically find the Lorentz number to be 4.1958 for $\mu_N = 0$. Here, we follow a semiclassical approach which is adopted in a recent study of WF law in case of bulk graphene [59].

The general expression of thermal conductivity $\kappa$ relating to the specific heat $C$ of the system is given by

$$\kappa = \frac{1}{d} C v_F \langle l \rangle \, ,$$

where $d$, $v_F$, and $\langle l \rangle$ are the dimension, Fermi velocity and average mean free path respectively for the corresponding system.

We compute the specific heat and the mean free path of a bulk 3D WSM considering the low energy effective linear spectrum $\epsilon_k = hv_F |k|$. Using the density of states close to the Weyl point as $D(E) = E^2/(2\pi^2)(hv_F)^3$, the specific heat of the system in terms of the internal energy is

$$C = \frac{dU}{dT} = \frac{d}{dT} \left[ \int_{0}^{\infty} dE D(E) f(E) E \right] \, ,$$

where $f(E)$ is the Fermi distribution function. After a few steps of algebra we arrive at the expression for $C$ in a WSM given by

$$C = \frac{4}{\pi^2} \frac{k_F^3 T^3}{(hv_F)^3} (5.6822) \, .$$

To calculate the mean free path, we use the Drude formula which relates the electrical conductivity to the mean free path of electrons in the system as

$$G = \frac{n e^2 l}{m^* v_F} \, ,$$

where $m^*$ and $n$ denote the effective mass and number density of electrons respectively.

Using the number density $n = k_F^3/6\pi^2$ and considering only the transport along $z$ direction, we find the mean free path as

$$l = G \frac{6 \pi^2 h^3 v_F^3}{E^2 e^2} \, .$$

Here, effective mass of the system in general is defined as

$$m_{ij}^* = \frac{\hbar^2}{\left( \frac{\partial^2 E}{\partial k_i \partial k_j} \right)} \, ,$$
For a spherical Fermi surface, off-diagonal components of the effective mass tensor become zero. Therefore, the remaining terms are \( m_{xx}^* = m_{yy}^* = m_{zz}^* = \frac{\hbar k_F}{v_F} \). For transport along \( z \) direction, we have \( m_{zz}^* = m = \frac{\hbar k_F}{v_F} \).

Hence, the average mean free path can be found as

\[
\langle l \rangle = G \frac{6 \pi^2 \hbar^3 v_F^2}{e^2} \left( \frac{1}{E^2} \right), \quad (B7)
\]

Computing the average value of \( 1/E^2 \) statistically we find the analytical expression of average mean free path as

\[
\langle l \rangle = G \frac{6 \pi^2 \hbar^3 v_F^2}{e^2} \left( \frac{0.384423}{\left( k_B T^2 \right)} \right). \quad (B8)
\]

Using the specific heat in Eq. (B3) and the average mean free path in Eq. (B8) one can compute the thermal conductance employing Eq. (B1). Finally, substituting this quantity in Eq. (22) we arrive at the Lorentz number

\[
L = \frac{\kappa}{GT} = 5.3171 \times L_0, \quad (B9)
\]

where, \( L_0 = \frac{\pi^2}{3} \left( \frac{k_B}{e} \right)^2 \).

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