Average Rate Analysis of Cooperative NOMA aided Underwater Optical Wireless Systems

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In this paper, we consider a cooperative non-orthogonal multiple access (NOMA) aided underwater optical wireless system in which the source transmits to two users where the near user serves as a relay node to the far user. Our proposed system consists of multiple narrow-angle light-emitting diode (LED)/photodiode (PD) elements at the source, near user, and far user. In order to achieve communication, our system selects a single LED/PD at each node. We propose several low complexity LED/PD selection schemes that aim to maximize the link throughput and in addition consider optimal and random LED/PD selection for benchmarking. In order to characterize the performance of each scheme, bounds and closed-form tight approximations on the average achievable sum rates are presented. The use of multi element nodes and NOMA increase the average sum rate significantly over conventional orthogonal access. Moreover, near-optimal throughput can be achieved using channel gain based and line-of-sight based LED/PD selection schemes in the medium-to-high transmit power regimes. The derived expressions are also useful to investigate the impact of key system and channel parameters such as the source transmit power, power allocation factor, node placement, and the number of elements at each node.

Index Terms—Underwater optical wireless communication, cooperative non-orthogonal multiple access (NOMA), performance analysis, lower bound, achievable sum rate, LED/PD selection.

I. INTRODUCTION

Recent advances in oceanographic research such as observation of marine life, earthquake prediction, water pollution monitoring, and oil/gas exploration have shown the need to develop robust and high data rate underwater communication systems. Optical, and acoustic waves are more commonly used than radio frequency (RF) in underwater applications due to the very high attenuation of RF waves [1]. Optical waves boast higher bandwidths and low latency levels and hence provide a promising solution for short-range high data rate underwater wireless applications [2], [3].

Underwater optical wireless communication (UOWC) systems are subject to performance degradation due to various channel effects. The communication of UOWC systems is limited by absorption, scattering, and turbulence effects [2]. In the literature, several techniques have been studied to combat the channel effects encountered in UOWC systems including the use of pointed transmitters [4], transmit laser selection [5], and relay-based multi-hop systems [6], [7]. A long-distance UOWC system using a single-photon avalanche photodiode (SPAD) has been proposed in [4]. The half-power angle of a light-emitting diode (LED) is narrowed to enhance the optical intensity at the transmitter, and a SPAD is used at the receiver to improve the detection sensitivity. The limitations of such methods are misalignment effects and additional hardware complexity. In addition, transmit laser selection combined with optical spatial modulation is proposed for weak turbulence based UOWC systems to enhance the performance in [5].

A preferred approach to increase the range and reliability of UOWC systems is to employ relays. The end-to-end error performance has been investigated in [6] for multi-hop configurations. In [8], the maximum achievable distance for a multi-hop system and optimum relay placement for decode-and-forward (DF) and amplify-and-forward relaying have been studied. In [9], it was shown that attenuation and blocking effects due to suspended particles degrade the performance of relay-based UOWC systems. In [10], a parallel amplify-and-forward relay system has been designed to mitigate the performance loss due to suspended particles.

More recently, non-orthogonal multiple access (NOMA) has been studied as a promising method to increase the spectral efficiency of UOWC systems [11]. NOMA allows multiple users to use the same frequency and time resources to improve the performance over the traditional orthogonal multiple access concept [12], [13]. In particular, the power domain NOMA uses different power levels to multiplex signals of multiple users. Subsequently, users with better channel gains decode and subtract out messages of the other users before decoding their own messages [13]. Some papers have studied the performance of over air optical wireless communication (OWC) systems. In [14], NOMA is considered in the context of OWC under different channel uncertainty models. Some papers have studied the performance of different NOMA-based OWC systems. In [15], analytical expressions for coverage probability and ergodic sum rate are presented for two cases; quality-of-service guaranteed and opportunistic best-effort service provisioning. In [16], NOMA has been used to enhance the achievable throughput in OWC. Some papers have also applied NOMA for UOWC systems. In [11], the performance of a NOMA aided UOWC system has been studied. In [17], the coverage probability and cell capacity of a NOMA aided UOWC system has been presented and it is shown that NOMA can increase the number of users within a cell. In [17], it has been shown that NOMA can be used to increase the number of users within a cell. A NOMA aided UOWC system with a
There are no papers that have investigated the LED/PD selection for a diver-to-diver UOWC link where the transmitter has multiple laser sources and the receiver has a single PD. However, in the existing body of literature, some studies have experimentally demonstrated the feasibility of underwater NOMA systems. In [21], a NOMA aided high-speed system using green and blue polarization multiplexing has been proposed. The combination of NOMA and relay-based communication is referred to as cooperative NOMA. Compared to the traditional NOMA, cooperative NOMA systems deliver additional gains. In cooperative NOMA, strong users are used as relays to improve the performance of weak users by forwarding decoded messages [22]. In [23], a full-duplex cooperative relay aided underwater NOMA UOWC system has been proposed. In [24], the impact of receiver orientation on a full-duplex relay assisted NOMA aided UOWC system has been studied. Despite recent research on the application of NOMA for UOWC systems, many knowledge gaps remain to be addressed. Some of these include quantifying the NOMA gains under diverse underwater conditions, multi-element designs for improved performance, beamforming, scheduling and resource allocation in multi-user UOWC systems, and accurate analysis of hybrid NOMA underwater/overwater communication systems based on both optical wireless and RF links. Often, these challenges and corresponding solutions are distinct from the widely investigated NOMA aided RF wireless systems presented in the existing literature. Moreover, how emerging machine learning techniques can be applied to better design NOMA aided UOWC systems is an open research question.

In OWC, multi-element deployment is a well-known method to increase the performance [25]–[28]. Several papers have presented multiple LED and/or photodiode (PD) OWC systems. In [25], a multiple LED/PD OWC system with LED grouping to achieve maximum throughput, fairness among users, and quality of service has been presented. In [26], a multi-transceiver spherical free space optics structure has been presented as a basic building block for enabling optical-based ad hoc networking. However, using more LED/PD elements results in a higher cost and complexity in implementation. Fortunately, to reduce the practical burden of operating multiple elements at the same time, LED/PD selection methods can be used [27]. For example in [27], a reduced complexity LED selection scheme has been proposed to study the secrecy probability for an OWC system with spatially distributed eavesdroppers. In [28], a specific LED layout that maximizes the signal-to-noise ratio (SNR) has been used to study the performance of an OWC system. In [29], a sub-optimal LED selection algorithm for distributed multiple-input multiple-output OWC system is presented. In [30], a transmit laser selection for a diver-to-diver UOWC link where the transmitter has multiple laser sources and the receiver has a one PD has been proposed. However, in the existing body of literature, there are no papers that have investigated the LED/PD selection schemes for cooperative NOMA aided UOWC systems.

In UOWC systems laser diodes or narrow-angle LEDs can be used as transmitters to achieve longer transmission distances [4], [5]. However, high turbid conditions and swaying of underwater devices due to water flow can cause misalignment effects in laser based communications. In addition, when the node locations change with time, for example due to wave-induced movement of the water, lasers need to be steered precisely to avoid alignment errors. As an alternative, some papers such as [4], [31], [32] have considered narrow-angle LED transmitters for underwater communication.

In this paper, a cooperative NOMA aided UOWC setup is considered, where an overhead access point (source) located at the surface level transmits to two underwater sensors (users) in weak turbulence conditions. In this system, the near user is anchored above the seabed and assists the far user communication. In order to enhance the communication distance and coverage of the system, multiple narrow-angle LED/PD elements are deployed at each node. In particular, we consider a single LED/PD selection at each node such that NOMA operation can be performed. In general, it is important to stress that element selection schemes for UOWC systems need to consider unique aspects (e.g., directional lightwave travel, blockage due to aquatic life, suspended particles, bubbles, and light color) and requires different design and analysis as compared with widely studied antenna selection schemes in traditional RF wireless systems. The proposed schemes based on channel/lightwave directions are very general since they are applicable to all types of transmitters and receivers. A transceiver of a specific kind used in the design (e.g., narrow angle/wide angle field of view) will only change the numerical values.

Our contributions are summarized as follows:

- We present several new low-complexity LED/PD selection schemes namely, optimal, channel state information (CSI) based, and orientation of LEDs/PDs based methods. We analyze bounds on the average achievable sum rate of the proposed schemes. In particular, we present mathematical expressions and accurate approximations that are useful to obtain design insights.
- We present simulation results applicable to the proposed schemes for different key system parameters which enable a detailed study of the performance trade-offs for different schemes. Our results reveal that near-optimal results can be obtained using both the CSI and orientation of LEDs/PDs based schemes. The proposed schemes are of high practical value and are suitable in order to deliver performance gains in UOWC systems.

The rest of the paper is organized as follows. In Section II system model and weak turbulence propagation model are presented. Section III presents the lower bound of the system and LED/PD selection schemes that can achieve high performance. Analysis of the average achievable sum rate of each LED/PD selection scheme is presented in Section IV. Numerical results for various system and channel parameters of all proposed schemes are presented in Section V. Finally, conclusions are drawn in Section VI.
schemes that will be discussed in Section III. Then the selected PD
respective messages \[33\]. The superimposed signal of
the device constraints.
viiable choices for certain underwater systems depending on
hexagonal, and cylindrical arrangement etc. could also deliver
spherical design. Specifically, LEDs are equally spaced on the
transmitters placed on a hemisphere with equal spacing.
According to the cooperative NOMA principle,
simultaneously transmits two messages \(x_1[n]\) and \(x_2[n]\) at the same
time and frequency by allocating different power levels to the
respective messages \[33\]. The superimposed signal of \(x_1[n]\) and
\(x_2[n]\) is transmitted from a selected LED \(i^*\) from \(S\) in
the first time slot according to a specific LED/PD selection
scheme. The signal transmitted from \(S\) is received at \(U1\) by a
selected PD \(j^*\). Next, \(U1\) decodes and re-transmits the signal
\(x_2[n]\) in the second time slot towards \(U2\) using a selected
transmitter \(k^*\) according to one of the LED/PD selection
schemes that will be discussed in Section III. Then the selected
PD \(l^*\) at \(U2\) receives the signal and \(x_2[n]\) is decoded. In
addition, \(x_1[n]\) is decoded at the selected PD, \(l^*\) at \(U2\) in the first
time slot using the direct signal coming from \(S\).

The source \(S\) transmits a superposition of messages \(x_1[n]\)
and \(x_2[n]\). Hence, the transmitted optical signal is
\[
x_S[n] = a_1 P_S x_1[n] + a_2 P_S x_2[n],
\]
where \(P_S\) is the optical transmit power, and \(a_1\) and \(a_2\) are the
power allocation coefficients for \(x_1[n]\) and \(x_2[n]\), respectively,
such that \(a_1 + a_2 = 1\). The received current signal at \(U1\) in
the first time slot can be written as
\[
y_{11}[n] = R h_{i_j}^{\text{S1}} (a_1 P_S x_1[n] + a_2 P_S x_2[n]) + \eta_1[n],
\]
where \(R\) is the responsivity of the PD, \(h_{i_j}^{\text{S1}}\) is the composite
channel gain between the selected transmitter \(i^*\) of \(S\) and
selected PD \(j^*\) of \(U1\), and \(\eta_1[n]\) is the zero mean additive
white Gaussian noise (AWGN) with variance \(\sigma_1^2\) at the PD at
\(U1\).

The near user \(U1\) performs successive interference cancella-
tion (SIC) according to the NOMA concept and first decodes
\(x_1[n]\) treating \(x_2[n]\) as interference. Next, \(U1\) removes the
decoded message \(x_1[n]\) from the received signal to detect \(x_2[n]\).
After a short processing delay, the near user \(U1\) forwards
the decoded message \(x_2[n]\) towards \(U2\). Hence, the received
current signal at \(U2\) in the second phase of communication in
\(n\)-th frame can be written as
\[
y_{22}[n] = R h_{k_l}^{\text{S2}}, P_l x_2[n] + \eta_2[n],
\]
where \(P_l\) is the optical transmit power at \(U1\), \(h_{k_l}^{\text{S2}}\) is the
composite channel gain between the \(U1\) and \(U2\), and \(\eta_2[n]\)
is zero mean AWGN with variance \(\sigma_2^2\) at the \(U2\) receiver. In
addition, \(x_1[n]\) is decoded at \(U2\) in the first time slot\(^2\).

**B. Channel Model**

In this subsection, we present the underwater light propaga-
tion model which will be used to analyze the performance of
the system in Section IV. The channel gain from transmitter
to receiver \(h\) can be described by the product of three terms as
\[
h = h_g h_l h_t,
\]
where \(h_g\) is the geometric loss, \(h_l\) is the path loss coefficient,
and \(h_t\) is the weak turbulence induced fading \[17, 34, 35\].

1) Geometric Loss

The geometric loss of the line-of-sight (LoS) channel be-
tween the transmitter and receiver is the loss due to the
geometric placement of transmitter and receiver. It can be
determined through \[17, 31\] as
\[
h_g = \begin{cases} (m + 1) A_p \cos^m(\theta) \cos(\psi) T c(\psi), & -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2} \\ 0, & \text{otherwise} \end{cases}
\]
where \(m = -\ln(2)/\ln(\cos(\theta_{1/2}))\) is the Lambertian order of
the transmitter, \(\theta_{1/2}\) is the half power angle of the transmitter,
\(A_p\) is the receiver aperture area, \(d\) is the Euclidean distance
between the transmitter and the receiver, \(\theta\) is the irradiance

\(^2\)In certain maritime applications privacy issues could be important. An
effective solution to guarantee privacy in such applications is to use upper layer
encryption techniques. Privacy-preserving analysis of upper layer encryption
schemes applicable for such cases therefore stands out as an interesting and
worthwhile future direction.

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**II. SYSTEM AND CHANNEL MODELS**

**A. System Model**

As shown in Fig. 1 we consider a cooperative NOMA aided
UOWC system, which consists of a source (\(S\)), a near user
\((U1)\), and a far user \((U2)\). We assume that \(S\), \(U1\), and \(U2\) are
located at the three dimensional coordinates \((x_1, y_1, z_1)\),
\((x_2, y_2, z_2)\) respectively. \(S\) transmits two
message streams; a broadcast message \(x_1[n]\) intended for
both \(U1\) and \(U2\), and a message \(x_2[n]\) intended only for
\(U2\). In addition to direct transmission from \(S\) to \(U2\), \(U1\)
forwards \(x_2[n]\) from \(S\) to \(U2\), thus serving as a DF relay\(^1\).
We consider a situation where \(S\) floats on the water surface at
a height \(z_s\) in from the seabed and is equipped with \(N_l\) optical
transmitters placed on a hemisphere with equal spacing. \(U1\) is
equipped with \(N_f\) PDs to receive from \(S\) and \(N_k\) LEDs for
forwarding signals to \(U2\). \(U2\) has \(N_k\) PDs for data reception.
At each node, the LEDs and PDs are placed according to a
spherical design. Specifically, LEDs are equally spaced on the
lower hemisphere, while PDs are equally spaced on the upper
hemisphere. Other LED/PD placement options such as planar,
hexagonal, and cylindrical arrangement etc. could also deliver
viable choices for certain underwater systems depending on
the device constraints.

According to the cooperative NOMA principle, \(S\) simulta-
aneously transmits two messages \(x_1[n]\) and \(x_2[n]\) at the same
time and frequency by allocating different power levels to the
respective messages \[33\]. The superimposed signal of \(x_1[n]\)
and \(x_2[n]\) is transmitted from a selected LED \(i^*\) from \(S\) in
the first time slot according to a specific LED/PD selection
scheme. The signal transmitted from \(S\) is received at \(U1\) by a
selected PD \(j^*\). Next, \(U1\) decodes and re-transmits the signal
\(x_2[n]\) in the second time slot towards \(U2\) using a selected
transmitter \(k^*\) according to one of the LED/PD selection
schemes that will be discussed in Section III. Then the selected
PD \(l^*\) at \(U2\) receives the signal and \(x_2[n]\) is decoded. In
addition, \(x_1[n]\) is decoded at the selected PD, \(l^*\) at \(U2\) in the first
time slot using the direct signal coming from \(S\).
In our system model, \( \theta \) of the log-normal distributed channel coefficient can be written as:

\[
E \left\{ h_\theta \right\} = \frac{\ln \left( \frac{h_t}{2\pi\sigma_\theta^2} \right)}{8\sigma_\theta^2}
\]

where \( h_t \) denotes the weak turbulence-induced fading coefficient, \( \mu_x \) and \( \sigma_x^2 \) are the mean and variance of the Gaussian distributed log-amplitude factor, \( X = \frac{1}{2} \ln (h_t) \). In order to preserve the energy of the fading coefficient, we normalize the fading amplitude such that \( E\{h_\theta\} = 1 \) where \( E\{\cdot\} \) is the expectation operator giving \( \mu_x = -\sigma_x^2 \) \cite{35}. The variance of the log-amplitude factor \( \sigma_x^2 \) is related to the scintillation index of the propagating signal \( \sigma_x^2 = \frac{1}{4} \ln (\sigma_\theta^2 + 1) \). In a weak turbulence regime, \( \sigma_\theta^2 < 1 \).

Finally, collecting the effects of the geometric loss, path loss and weak turbulence condition, the composite channel gain can be written as

\[
h = G h_t,
\]

where \( G = h_\theta h_t \) is the deterministic part of the channel gain. The distribution of the transformed \( h^2 \) can be obtained with the help of (11) as

\[
f_{h^2}(h^2) = \frac{1}{4h^2\sqrt{2\pi\sigma_x^2}} \exp \left( -\frac{\left( \ln \left( \frac{h^2}{G^2} \right) - 4\mu_x \right)^2}{32\sigma_x^2} \right). \tag{11}
\]

### III. Sum Rate and LED/PD Selection Schemes

In this section we present several LED/PD selection schemes to optimize the performance of the proposed cooperative NOMA aided UOWC system. So far in the literature, the exact capacity of UOWC systems have not been reported. In order to circumvent this difficulty, bounds reported in the literature are used \cite{36} to establish the system’s achievable sum rate.

#### A. Lower Bound on the Average Achievable Sum Rate

First, we present separate expressions for the lower bound on the instantaneous achievable rates for message streams \( x_1[n] \) and \( x_2[n] \) separately. Next, they are averaged and the sum is obtained. The instantaneous achievable rates for decoding the message \( x_1[n] \) at \( U1 \) and \( U2 \) can be lower bounded as \cite{36}

\[
C_1^{S1} \geq \frac{1}{4} \log_2 \left( \frac{2\pi\sigma_1^2 + (RP_2h_{1s}^l)^2}{2\pi\sigma_2^2 + (a_2RP_2h_{1s}^l)^2\epsilon_2} \sum_{i=1}^{S} e_i(\epsilon_i)\sigma_L^2 \right)
\]

and

\[
C_1^{S2} \geq \frac{1}{4} \log_2 \left( \frac{2\pi\sigma_1^2 + (RP_2h_{1s}^l)^2}{2\pi\sigma_2^2 + (a_2RP_2h_{1s}^l)^2\epsilon_2} \sum_{i=1}^{S} e_i(\epsilon_i)\sigma_L^2 \right),
\]

where \( e_i(\epsilon_i) = e^{1+2(\mu_i+\nu_i\epsilon_i)} \), \( \mathbb{E}\{x_i^2\} = \epsilon_i \), \( \mu_i \) and \( \nu_i \) are constants that depends on the input distribution and can be obtained solving equation in \cite{36}, Eq. (12), \( h_{1s}^l \) is the composite channel gain between the selected transmitter \( i^* \) of \( S \) and selected PD \( l^* \) of \( U2 \) respectively. Hence, the lower bound on the instantaneous achievable rate for decoding message stream \( x_1[n] \) can be expressed as \cite{33}

\[
C_1 \geq \min \left\{ C_1^{S1}, C_1^{S2} \right\}.
\tag{12}
\]

Decoding message stream \( x_2[n] \) can be lower bounded as \cite{33}

\[
C_2 \geq \frac{1}{4} \min \left\{ \log_2 \left( 1 + \frac{e_2(\epsilon_2)(a_2RP_2h_{1s}^l)^2}{2\pi\sigma_1^2} \right), \right.
\log_2 \left( 1 + \frac{e_2(\epsilon_2)(RP_2h_{1s}^l)^2}{2\pi\sigma_2^2} \right) \right\}. \tag{13}
\]
The lower bound on the instantaneous achievable sum rate of the proposed system, can be expressed using (12) and (13). Further, by using the expectation operation, we have
\[ \mathbb{E}\{C_{SUM}^{L}\} = \mathbb{E}\{C_{L,1}\} + \mathbb{E}\{C_{L,2}\} \leq \mathbb{E}\{C_{1}\} + \mathbb{E}\{C_{2}\}, \]  
(14)
where \( \mathbb{E}\{C_{L,1}\} \) and \( \mathbb{E}\{C_{L,2}\} \) are the lower bounds on the average achievable rates for message streams \( x_{1}[n] \) and \( x_{2}[n] \) separately, \( \mathbb{E}\{C_{SUM}^{L}\} \) is the average achievable sum rate. Eqs. (12), (13), and (14) are used to find approximate expressions for each LED/PD selection schemes.

B. LED/PD Selection Schemes

In this subsection we propose (1) optimal, (2) max \( S-U1-U2 \) channel gain based, (3) best \( S-U1-U2 \) LoS based, and (4) random LED/PD selection schemes as follows.

1) Optimal LED/PD Selection

The optimal LED/PD selection scheme should be decided to maximize the sum rate in (18). In order to maximize \( \mathbb{E}\{C_{SUM}^{L}\} \) we select the \( i^{\ast}\)-th LED at \( S \) from \( N_{I} \) LEDs, the \( j^{\ast}\)-th PD at \( U1 \) from \( N_{J} \) PDs, the \( k^{\ast}\)-th LED at \( U1 \) from \( N_{K} \) LEDs, and the \( l^{\ast}\)-th PD at \( U2 \) from \( N_{L} \) PDs such that
\[ \{i^{\ast}, j^{\ast}, k^{\ast}, l^{\ast}\} = \arg \max_{i,j,k,l} \{\mathbb{E}\{C_{SUM}^{L}\}\}. \]  
(15)

The optimal LED/PD selection scheme has high implementation complexity as explained in Section IV-F.

2) Max \( S-U1-U2 \) Channel Gain Based Selection

The max \( S-U1-U2 \) channel gain based selection scheme relies on channel state information (CSI) of \( S-U1 \) and \( U1-U2 \) links. First, the \( i^{\ast}\)-th LED at \( S \) and the \( j^{\ast}\)-th PD at \( U1 \) are selected such that \( (h_{ij}^{S1})^{2} \) is maximized which can be expressed as
\[ \{i^{\ast}, j^{\ast}\} = \arg \max_{i,j} \{(h_{ij}^{S1})^{2}\}, \]  
(16)
where \( h_{ij}^{S1} \) is the composite channel gain between the transmitter \( i \) of \( S \) and PD \( j \) of \( U1 \). Similarly, the \( k^{\ast}\)-th LED at \( U1 \) and the \( l^{\ast}\)-th PD at \( U2 \) is selected such that \( (h_{kl}^{12})^{2} \) is maximized which can be expressed as
\[ \{k^{\ast}, l^{\ast}\} = \arg \max_{k,l} \{(h_{kl}^{12})^{2}\}, \]  
(17)
where \( h_{kl}^{12} \) is the composite channel gain between the transmitter \( k \) of \( U1 \) and PD \( l \) of \( U2 \).

3) Best \( S-U1-U2 \) Line-of-Sight Based Selection

In certain underwater implementations, obtaining CSI at all nodes may not be practical. Hence, we present a low-complexity method based on the best LoS links. According to this selection, the LEDs and PDs at \( S, U1, \) and \( U2 \) are selected exploiting position information. First, the \( i^{\ast}\)-th LED at \( S \) is selected such that the \( i^{\ast}\)-th LED is the closest LED to the line connecting \( S \) and \( U1 \) among all LEDs. Hence, \( i^{\ast} \) is given by
\[ \{i^{\ast}\} = \arg \min_{i} |\theta_{S,i} - \alpha|, \]  
(18)
where \( \theta_{S,i} \) is the angle between vertical axis and \( i\)-th LED at \( S \), and \( \alpha \) is the angle between vertical axis and the line connecting \( S \) and \( U1 \). Considering the placement of \( S \) and \( U1, \) \( (18) \) can be explicitly expressed as
\[ \{i^{\ast}\} = \arg \min_{i} \left| \frac{\pi}{2} - \tan^{-1} \left( \frac{\sqrt{(xS-x1)^2 + (yS-y1)^2}}{zS-z1} \right) - \frac{\pi i}{N_{I} + 1} \right| \]  
(19)

Next, the \( j^{\ast}\)-th PD at \( U1 \) is selected such that the \( j^{\ast}\)-th PD is the closest PD to the line connecting \( S \) and \( U1 \) among all PDs.

In this scheme, LED/PD selection for the \( S-U1 \) and \( U1-U2 \) links will be performed at \( S \) and \( U1 \) respectively. In practice, it may not always be feasible to select PDs that are in exact alignment with the respective LED due to imperfect position information. An incorrect selection of PDs will result in LoS misalignment errors. To model such an error at \( U1 \), we consider that a single PD \( j^{1} \) is selected randomly from a set \( J^{1} \) which lies inside a cone around the line connecting \( S \) and \( U1 \) given by
\[ -\Delta \Phi_{1} \leq \alpha - \varphi_{1,j^{1}} \leq \Delta \Phi_{1}, \]  
(20)
where \( \Delta \Phi_{1} \) is the half angle of the cone, \( \varphi_{1,j^{1}} \) is the angle between vertical axis and the \( j^{1}\)-th PD at \( U1 \). Next, \( U1 \) selects the \( k^{\ast}\)-th LED which is closest to the line connecting \( U1 \) and \( U2 \). Hence, selection of \( k^{\ast} \) can be expressed as
\[ \{k^{\ast}\} = \arg \min_{k} |\beta - \theta_{1,k}|, \]  
(21)
where \( \beta \) is the angle between vertical axis and the line connecting \( U1 \) and \( U2, \theta_{1,k} \) is the angle between vertical axis and the \( k\)-th LED. Considering the placement of \( U1 \) and \( U2, \) \( (21) \) can be modified as
\[ \{k^{\ast}\} = \arg \min_{k} \left| \frac{\pi}{2} + \tan^{-1} \left( \frac{\sqrt{(x1-x2)^2 + (y1-y2)^2}}{z1-z2} \right) - \frac{\pi k}{N_{K} + 1} \right| \]  
(22)

Similar to the selection of the PD at \( U1 \), the selection of the \( l^{\ast}\)-th PD at \( U2 \) will also be impaired by PD selection errors. The PD selection error at \( U2 \) is modeled by randomly selecting a single PD \( l^{1} \) from a set \( L^{1} \) which lies inside a cone around line connecting \( U1 \) and \( U2 \) given by
\[ -\Delta \Phi_{2} \leq \beta - \varphi_{2,l^{1}} \leq \Delta \Phi_{2}, \]  
(23)
where \( \Delta \Phi_{2} \) is the half angle of the cone, \( \varphi_{2,l^{1}} \) is the angle between vertical axis and the \( l^{1}\)-th PD at \( U2 \).

4) Random LED/PD Selection

In order to benchmark the channel based and LoS based schemes, a random LED/PD selection approach is also described. Specifically according to this scheme at each node, we arbitrarily select a LED/PD to initiate communication. First, the \( i^{\ast}\)-th LED at \( S \) is selected randomly from the LED set \( \{1, 2, \ldots, N_{I}\} \). The \( j^{\ast}\)-th PD at \( U1 \) is selected randomly from the PD set \( \{1, 2, \ldots, N_{J}\} \). The \( k^{\ast}\)-th LED at \( U1 \) is selected randomly from the PD set \( \{1, 2, \ldots, N_{K}\} \). Similarly, the \( l^{\ast}\)-th PD at \( U2 \) is selected randomly from the PD set \( \{1, 2, \ldots, N_{L}\} \).
IV. PERFORMANCE ANALYSIS

In this section, after establishing a general expression, we study the lower bounds on \( \mathbb{E}\{C_1\} \) and \( \mathbb{E}\{C_2\} \) as applicable to the selection schemes described in Section III above. Moreover, accurate closed-form approximations to complement them are also derived.

With the help of (12), a lower bound on \( C_1 \) can be expressed as

\[
C_1 \geq \frac{1}{4} \log_2 \left( \min \left\{ \frac{2 \pi \sigma^2 + (RP_S h_{i_1 \sigma}^1)^2 \sum_{i=1}^2 e_i (e_i)^2}{2 \pi \sigma^2 + 2 \pi (a_2 RP_S h_{i_1 \sigma}^1)^2 e_2}, \frac{2 \pi \sigma^2 + (RP_S h_{i_2 \sigma}^2)^2 \sum_{i=1}^2 e_i (e_i)^2}{2 \pi \sigma^2 + 2 \pi (a_2 RP_S h_{i_2 \sigma}^2)^2 e_2} \right\} \right),
\]

(24)

Eq. (24) can be re-expressed as

\[
C_1 \geq \frac{1}{4} \log_2 \left( \frac{1 + AX}{1 + BX} \right),
\]

(25)

where \( A = \frac{\sum_{i=1}^2 e_i (e_i)^2}{2 \pi \sigma^2} (RP_S)^2 \) and \( B = \frac{(a_2 RP_S)^2 e_2}{\sigma^2} \). To find the lower bound on \( \mathbb{E}\{C_1\} \), (26) is averaged over the pdf of RV \( X \) as

\[
\mathbb{E}\{C_1\} \geq \frac{1}{4} \int_0^\infty \log_2 \left( \frac{1 + Ax}{1 + Bx} \right) f_X(x) dx,
\]

(27)

where \( f_X(x) \) is the pdf of RV \( X \). To simplify (27), \( f_X(x) \) is expressed using order statistics and with the realistic assumption that squared channel gains \( (h_{i_1 \sigma}^1)^2 \) and \( (h_{i_2 \sigma}^2)^2 \) are RVs that are statistically independent of each other which is valid for all LED/PD selection schemes.

In order to derive a general expression for the lower bound on \( \mathbb{E}\{C_2\} \) we first express (13) as

\[
C_2 \geq \frac{1}{4} \log_2 \left( 1 + \min \left\{ \frac{e_2 (e_2) (a_2 RP_S h_{i_1 \sigma}^1)^2}{2 \pi \sigma^2}, \frac{e_2 (e_2) (RP_S h_{i_2 \sigma}^2)^2}{2 \pi \sigma^2} \right\} \right)
\]

(30)

After simplifying (30) yields

\[
C_2 \geq \frac{1}{4} \log_2 \left( 1 + \frac{e_2 (e_2) R^2 Y}{2 \pi \sigma^2} \right),
\]

(31)

where \( Y = \min \left\{ (a_2 P_S h_{i_1 \sigma}^1)^2, (P_R h_{i_2 \sigma}^2)^2 \right\} \). The statistics of \( Y \) are described below. The lower bound on \( \mathbb{E}\{C_2\} \) can be obtained by averaging (31) over the pdf of RV \( Y \) as

\[
\mathbb{E}\{C_2\} \geq \frac{1}{4} \int_0^\infty \log_2 \left( 1 + \frac{e_2 (e_2) R^2 Y}{2 \pi \sigma^2} \right) f_Y(y) dy,
\]

(32)

The derived general expression for the lower bound on \( \mathbb{E}\{C_2\} \) using (32) and (33) is

\[
\mathbb{E}\{C_2\} \geq \frac{1}{4} \int_0^\infty \log_2 \left( 1 + D Y \right) \left( 1 - F_{a_2 P_S h_{i_1 \sigma}^1}^2 (y) \right) f_Y(y) dy + \frac{1}{4} \int_0^\infty \log_2 \left( 1 + D Y \right) \left( 1 - F_{P_R h_{i_2 \sigma}^2}^2 (y) \right) f_Y(y) dy,
\]

(34)

Using (29), and (34) we derive expressions for the lower bounds on \( \mathbb{E}\{C_1\} \), and \( \mathbb{E}\{C_2\} \) for optimal, max \( S - U_1 - U_2 \) channel gain based, and best \( S - U_1 - U_2 \) LoS based LED/PD selection scheme in the sequel.

A. Optimal LED/PD Selection

In the case of optimal LED/PD selection, the pdf and cdf of the SNR and SINR required to derive the average achievable sum rate is extremely difficult to find. However, in Section V through simulations the corresponding performance of the optimal scheme is shown.

B. Max \( S - U_1 - U_2 \) Channel Gain Based Selection

In order to obtain an exact expression for the lower bound on the average achievable sum rate, first we derive cdfs and pdfs of max \( \left\{ (h_{i_1 \sigma}^1)^2, (h_{i_2 \sigma}^2)^2 \right\} \), max \( \left\{ (a_2 P_S h_{i_1 \sigma}^1)^2 \right\} \), and max \( \left\{ (P_R h_{i_2 \sigma}^2)^2 \right\} \) where \( h_{i_2 \sigma}^2 \) is the composite channel gain between the transmitter \( i \) of \( S \) and PD \( l \) of \( U_2 \). Next, (14), (29), and (34) are used to obtain exact expressions.
Let us focus on deriving expressions for the lower bound on $\mathbb{E}\{C_{1}\}$. The cdf of the distribution of $X = \max \{\{h_{ij}^S\}^2\}$ can be obtained with the help of [34, Eq. (8)] and the use of order statistics as
\[
F_X(x) = \prod_{1 \leq i,j \leq N_f} \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(G_{ij}^S)^2}{x} \right) + 4\mu_x}{4\sqrt{2}\sigma_x} \right), \tag{35}
\]
where $\text{erfc}(x) = 1 - \frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-t^2} dt$ is the complementary error function. By differentiating (35) with respect to (w.r.t.) $x$ the corresponding pdf can be written as
\[
f_X(x) = \sum_{i=1}^{N_t} \sum_{j=1}^{N_f} \frac{1}{4\sqrt{2\pi}\sigma^2} \sum_{1 \leq i \leq N_f, 1 \leq j \leq N_f} \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(G_{ij}^S)^2}{x} \right) + 4\mu_x}{4\sqrt{2}\sigma_x} \right) \times \exp \left( \frac{1}{4\sqrt{2}\sigma^2} \ln \left( \frac{(G_{ij}^S)^2}{x} \right) + 4\mu_x \right). \tag{36}
\]
However, after the selection of $i^*, j^*, k^*$, and $l^*$, the distribution of $(h_{i^*j^*}^S)^2$ can not be obtained straightforwardly. Hence, we obtain the weighting factor of $(h_{i^*j^*}^S)^2$ given by $w_{i^*,l}$ from offline simulations for the use in the analytical expression. The cdf of $X = (h_{i^*j^*}^S)^2$ can be expressed as
\[
F_X(x) = \sum_{i=1}^{N_t} \sum_{j=1}^{N_f} \frac{1}{2} w_{i^*,l} \text{erfc} \left( \frac{\ln \left( \frac{(G_{i^*j^*}^S)^2}{x} \right) + 4\mu_x}{4\sqrt{2}\sigma_x} \right). \tag{37}
\]
By differentiating w.r.t. $x$, the pdf of $(h_{i^*j^*}^S)^2$ is given by
\[
f_X(x) = \sum_{i=1}^{N_t} \sum_{j=1}^{N_f} \frac{1}{4\sqrt{2\pi}\sigma^2} \sum_{1 \leq i \leq N_f, 1 \leq j \leq N_f} \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(G_{i^*j^*}^S)^2}{x} \right) + 4\mu_x}{4\sqrt{2}\sigma_x} \right) \times \exp \left( \frac{1}{4\sqrt{2}\sigma^2} \ln \left( \frac{(G_{i^*j^*}^S)^2}{x} \right) + 4\mu_x \right). \tag{38}
\]
Substituting (35), (36), (37), and (38) into (29), the lower bound on the average achievable sum rate can be established. Similarly, with the help of (40) and (43) yields an approximate expression for the lower bound on the average achievable sum rate.

C. Best $S-U1-U2$ LoS Based Selection

To obtain an expression for the lower bound on the average sum rate in best $S-U1-U2$ LoS based selection, first we obtain cdfs and pdfs for the RVs $(h_{i^*j^*}^S)^2$, $(h_{i^*j}^S)^2$, $(a_2 P_i h_{i^*}^S)^2$, and $(P_i h_{i^*}^S)^2$. Next, using (14), (29), and (34) expressions for lower bounds are obtained.

The cdf of RV $(h_{i^*j^*}^S)^2$ is given by
\[
F_{(h_{i^*j^*}^S)^2}(x) = \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(G_{i^*j^*}^S)^2}{x} \right) + 4\mu_x}{4\sqrt{2}\sigma_x} \right). \tag{46}
\]
Using the cdf given in [34, Eq. (8)] and by differentiating (46) w.r.t. $x$ the pdf is expressed as
\[
f_{(h_{i^*j^*}^S)^2}(x) = \frac{1}{4\sqrt{2\pi}\sigma^2} \exp \left( \frac{1}{4\sqrt{2}\sigma^2} \ln \left( \frac{(G_{i^*j^*}^S)^2}{x} \right) + 4\mu_x \right). \tag{47}
\]
The cdf and pdf of $(h_{i^*j}^S)^2$ can be expressed by replacing $G_{i^*j^*}^S$ with $G_{i^*j}^S$ in (46) and (47). Inserting this pdf and cdf into (29) and averaging over possible combinations of PDs, the lower bound on $\mathbb{E}\{C_{1}\}$ can be established as (44) shown on the page 9, where $J_s$ and $J_e$ are the indices of starting and finishing PDs at $U1$ from the PD set given in (20). Further, with the help of (41), equation (44) can be approximated as (45).

Consider the lower bound on $\mathbb{E}\{C_{2}\}$. The cdf and pdf of RV $(a_2 P_i h_{i^*}^S)^2$ can be found by replacing $G_{i^*j^*}^S$ with $a_2 P_i G_{i^*j}^S$ in (46) and (47) respectively. Similarly, the cdf and pdf of $(P_i h_{i^*}^S)^2$ can be expressed by replacing $G_{i^*j}^S$ with $P_i G_{i^*}^S$ in (46) and (47) respectively. Inserting these pdfs and cdfs into (34) and averaging over possible random selection of PDs at $U2$ given by (23), the expression for the lower bound on $\mathbb{E}\{C_{2}\}$ can be written as (48) where $L_s$ and $L_e$ are indexes of starting and finishing PDs at $U2$ from the PD set given in (23). Further, with the help of (41), (48) can be approximated as (49) shown on the top of the page 10.

Finally, with the help of (44) and (48) in (14) the lower bound on the average achievable sum rate can be established. Similarly, with the help of (45) and (49) yields an approximate
bounds such that possible exact sum rate margins offered by 
exact achievable sum rate. In Section V, numerical results
Establishing an upper bound allows to reaffirm the value of 
rate.
Similarly, with the help of (51) and (53) yields an approximate
bound on the average achievable sum rate can be established.

The lower bound on the average achievable sum rate in
random LED/PD selection method can be found by first
obtaining the lower bounds for $E\{C_1\}$ and $E\{C_2\}$ for a
selected LED/PD set as discussed in previous scheme and then
averaging it over all possible $N_I N_J N_K N_L$ link combinations.

Using a similar approach as in above schemes, the lower
bound on $E\{C_1\}$ can be expressed as (50). Using (41) the
approximate lower bound on $E\{C_1\}$ under random LED/PD
selection can be expressed as (51). Similarly, the expression
for the lower bound on $E\{C_2\}$ can be expressed as (52) shown
on the top of the page 11. The approximate lower bound on
$E\{C_2\}$ can be established with the help of (41) and is given by
(53).

Finally, with the help of (50) and (52) in (14) the lower
bound on the average achievable sum rate can be established.
Similarly, with the help of (51) and (53) yields an approximate
expression for the lower bound on the average achievable sum rate.

D. Random LED/PD Selection

The lower bound on the average achievable sum rate in
random LED/PD selection method can be found by first
obtaining the lower bounds for $E\{C_1\}$ and $E\{C_2\}$ for a
selected LED/PD set as discussed in previous scheme and then
averaging it over all possible $N_I N_J N_K N_L$ link combinations.

We present separate expressions for the upper bound on
the instantaneous achievable rate for message streams $x_1[n]$ and
$x_2[n]$ separately. Next, they are averaged and the sum
is obtained. The instantaneous achievable rates for decoding
message stream $x_1[n]$ at $U1$ and $U2$ are upper bounded by [36]

$$C_{1}^{S1} \leq \frac{1}{4} \log_2 \left( \frac{2 \pi \sigma^2 + 2 \pi (RPS_h^{S1})^2 \sum_{i=1}^{\min} \epsilon_m a^2_m}{2 \pi \sigma^2 + e_2(e_2)(a_2RPS_h^{S1})^2} \right),$$

and

$$C_{1}^{S2} \leq \frac{1}{4} \log_2 \left( \frac{2 \pi \sigma^2 + 2 \pi (RPS_h^{S2})^2 \sum_{i=1}^{\min} \epsilon_m a^2_m}{2 \pi \sigma^2 + e_2(e_2)(a_2RPS_h^{S2})^2} \right).$$

The upper bound on the instantaneous achievable rate for
decoding message stream $x_1[n]$ can be expressed with the help
of results in [33] and after some mathematical manipulation, as

$$C_1 \leq \min \left\{ \frac{1}{4} \log_2 \left( \frac{1 + E(h_{i+j})^2}{1 + F(h_{i+j})^2} \right), \frac{1}{4} \log_2 \left( \frac{1 + E(h_{i+j})^2}{1 + F(h_{i+j})^2} \right) \right\},$$

where $E = 2 \pi (RPS)^2 \sum_{m=1}^{\min} \epsilon_m a^2_m$, and $F = e_2(e_2)(a_2RPS_h^{S1})^2$. Using a similar approach given
in Section IV, the upper bound on the average achievable rate
for message stream $x_1[n]$, $E\{C_U\}$, can be derived using
(29) by replacing $A$ with $E$, and $B$ with $F$.

Now, we present the upper bound on the instantaneous
achievable rates for decoding message stream $x_2[n]$ at $U1$ and
$U2$ is established as [36]

$$C_{2}^{S1} \leq \frac{1}{2} \log_2 \left( 1 + \frac{e_2(e_2)(a_2RPS_h^{S1})^2}{\sigma^2} \right),$$
$$E \{ C_2 \} \geq \frac{1}{16\sqrt{2}\pi \sigma_2^2} \left\{ \sum_{k=1}^{N_K} \sum_{l=1}^{N_L} \int_0^\infty \frac{1}{y} \exp \left( -\left( \frac{\ln \left( \frac{y^{2 \pi \sigma_2^2}}{(P_r G_{kl}^{12})} \right) - 4\mu_y}{32\sigma_y^2} \right)^2 \right) \prod_{1 \leq k_2 \leq N_K, \, 1 \leq l_2 \leq N_L, \, k_2 \neq k \neq j} \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(P_r G_{kl}^{12})^2}{y} \right) + 4\mu_y}{4\sqrt{2}\sigma_y} \right) \right\} \times$$

$$\log_2 (1 + Dy) \left( 1 - \prod_{1 \leq i_1 \leq N_i, \, 1 \leq j_1 \leq N_j} \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(a_2 P_r G_{ij}^{21})^2}{y} \right) + 4\mu_y}{4\sqrt{2}\sigma_y} \right) \right) dy + \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} \int_0^\infty \log_2 (1 + Dy)$$

$$\left( 1 - \prod_{1 \leq k_3 \leq N_K, \, 1 \leq l_3 \leq N_L} \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(P_r G_{kl}^{12})^2}{y} \right) + 4\mu_y}{4\sqrt{2}\sigma_y} \right) \right) \prod_{1 \leq i_4 \leq N_i, \, 1 \leq j_4 \leq N_j, \, i_4 \neq k_3 \neq j} \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(a_2 P_r G_{ij}^{21})^2}{y} \right) + 4\mu_y}{4\sqrt{2}\sigma_y} \right) \right\} \times \frac{1}{y} \exp \left( -\left( \frac{\ln \left( \frac{(a_2 P_r G_{ij}^{21})^2}{y} \right) - 4\mu_y}{32\sigma_y^2} \right)^2 \right) dy \right\}. \quad (42)$$

$$E \{ C_{L,2} \} \approx \frac{1}{6} \sum_{n=-1}^{1} \sum_{k=1}^{N_K} \sum_{l=1}^{N_L} \log_2 \left( 1 + D(P_r G_{kl}^{12})^2 L(n) \right) \prod_{1 \leq k_2 \leq N_K, \, 1 \leq l_2 \leq N_L, \, k_2 \neq k \neq j} \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(P_r G_{kl}^{12})^2}{y} \right) - 4\sqrt{3}\sigma_x}{4\sqrt{2}\sigma_x} \right) \right\} \times$$

$$\left( 1 - \prod_{1 \leq i_1 \leq N_i, \, 1 \leq j_1 \leq N_j} \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(a_2 P_r G_{ij}^{21})^2}{y} \right) - 4\sqrt{3}\sigma_x}{4\sqrt{2}\sigma_x} \right) \right) + \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} \log_2 (1 + D(a_2 P_r G_{ij}^{21})^2 L(n))$$

$$\times \left( 1 - \prod_{1 \leq k_3 \leq N_K, \, 1 \leq l_3 \leq N_L} \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(P_r G_{kl}^{12})^2}{y} \right) - 4\sqrt{3}\sigma_x}{4\sqrt{2}\sigma_x} \right) \right) \prod_{1 \leq i_4 \leq N_i, \, 1 \leq j_4 \leq N_j, \, i_4 \neq k_3 \neq j} \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(a_2 P_r G_{ij}^{21})^2}{y} \right) - 4\sqrt{3}\sigma_x}{4\sqrt{2}\sigma_x} \right) \right\}. \quad (43)$$

$$E \{ C_1 \} \geq \frac{1}{4(J_e - J_s)(L_e - L_s)} \sum_{j=J_s}^{J_e} \sum_{l=J_s}^{L_s} \int_0^\infty \log_2 \left( 1 + Ax \right) \frac{1}{y} \exp \left( -\left( \frac{\ln \left( \frac{\sigma_y}{G_{ij}^{21}} \right) - 4\mu_x}{32\sigma_x^2} \right)^2 \right) dx + \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(P_r G_{kl}^{12})^2}{y} \right) + 4\mu_x}{4\sqrt{2}\sigma_x} \right) \right)$$

$$\times \frac{1}{4x\sqrt{2}\pi \sigma_2^2} \exp \left( -\left( \frac{\ln \left( \frac{\sigma_y}{G_{ij}^{21}} \right) - 4\mu_x}{32\sigma_x^2} \right)^2 \right) dx, \quad (44)$$

$$E \{ C_{L,1} \} \approx \frac{1}{6(J_e - J_s)(L_e - L_s)} \sum_{j=J_s}^{J_e} \sum_{l=J_s}^{L_s} \sum_{n=-1}^{1} \frac{1}{4[m]} \log_2 \left( 1 + A(G_{ij}^{21})^2 L(n) \right)$$

$$\times \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(a_2 P_r G_{ij}^{21})^2}{y} \right) - 4\sqrt{3}\sigma_x}{4\sqrt{2}\sigma_x} \right) \right) \log_2 \left( 1 + A(G_{ij}^{21})^2 L(n) \right)$$

$$\times \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{(a_2 P_r G_{ij}^{21})^2}{y} \right) - 4\sqrt{3}\sigma_x}{4\sqrt{2}\sigma_x} \right) \right) \right\}. \quad (45)$$
The rate of the proposed system can be expressed as \( R/\sigma \) with an upper bound on the average achievable rate for message stream \( \epsilon \) where

\[
\sum_{n=1}^{N_s} \sum_{k=1}^{N_L} \sum_{l=1}^{N_L} \int_{0}^{\infty} \log_2 \left( 1 + \frac{R}{1 + B_{x}} \right) \times \left\{ \left( 1 - \frac{1}{2} \text{erfc} \left( \frac{\ln \left( \frac{G^{(1)}_{x} \sqrt{2}}{\sigma_{x}} \right) + 4 \mu_{x} \sqrt{2} \sigma_{x}}{4 \sqrt{2} \sigma_{x}} \right) \right) \times \frac{1}{4 \sqrt{2} \pi \sigma_{x}^{2}} \exp \left( -\left( \frac{\ln \left( \frac{G^{(1)}_{x} \sqrt{2}}{\sigma_{x}} \right) + 4 \mu_{x} \sqrt{2} \sigma_{x}}{4 \sqrt{2} \sigma_{x}} \right)^{2} \right) \right\} \text{d}x. \tag{50}
\]

The upper bound on the average achievable sum rate of the proposed system can be expressed as

\[
C_{1}^{12} \leq \frac{1}{2} \log_2 \left( 1 + \frac{\epsilon_{2} \left( R P_{R h^{2}_{S}} \right)}{\sigma_{x}^{2}} \right),
\]

Hence the upper bound on the instantaneous achievable rate for decoding message stream \( x_{2}[n] \) can be expressed as [33]

\[
C_{2} \leq \frac{1}{4} \log_2 \left( 1 + G_{1} Y \right), \tag{55}
\]

where \( Y = \min \left\{ \left( a_{2} P_{S} h^{2}_{S_{c}}, \left( R P_{R h^{2}_{S}} \right) \right) \right\} \) and \( G_{1} = \epsilon_{2} R^{2}/\sigma^{2} \). Using a similar approach as in Section IV, the upper bound on the average achievable rate for message stream \( x_{2}[n] \), \( E \{ C_{U,2} \} \), can be derived using (34) by replacing \( U \) with \( R/\sigma \). The upper bound on the average achievable sum rate of the proposed system can be expressed as

\[
E \{ C_{U}^{SUM} \} = E \{ C_{U,1} \} + E \{ C_{U,2} \} \geq E \{ C_{1} \} + E \{ C_{2} \}, \tag{56}
\]

where \( E \{ C_{U,1} \} \) and \( E \{ C_{U,2} \} \) are the upper bounds on the average achievable rates for message streams \( x_{1}[n] \) and \( x_{2}[2] \) separately. Upper bounds on the average achievable rates of optimal, max \( S - U1 - U2 \) channel gain based, and best \( S - U1 - U2 \) LoS based selection schemes can be found by following a similar approach as in Section IV. Due to the limited space we do not proceed further. However, they are presented in Fig. 4(b) for comparison with the lower bound results.

### F. Complexity of LED/PD Selection Schemes

This subsection summarizes the selection and implementation complexity of each LED/PD selection scheme. The optimal LED/PD selection scheme examine all the possible combinations and therefore the highest complexity is equal to \( (N_{1} N_{2} K_{1} N_{L}) \) operations. The proposed max \( S - U1 - U2 \) channel gain based selection scheme selects LEDs/PDs such that highest gains of \( S - U1 \) and \( U1 - U2 \) links are chosen. Such a selection results in a complexity of \( (N_{1} N_{2} + N_{K} N_{L}) \) operations. The best \( S - U1 - U2 \) LoS based selection scheme
selects LEDs/PDs at each node independently, hence a reduced complexity of \((N_I + N_J + N_K + N_L)\) operations is observed.

In terms of implementation complexity, the optimal scheme has the highest complexity since it requires the knowledge of all \(N_I N_J + N_K N_L + N_I N_L\) channel gains and LED/PD selection is performed at \(S\). The proposed “max \(S - U1 - U2\) channel gain based selection” scheme selects LED/PD pairs for transmission and reception at \(S\) and \(U1\) respectively, and \(N_I N_J + N_K N_L\) channel gains are necessary. The best \(S - U1 - U2\) LoS based selection scheme does not require the knowledge of channel gains. Instead, the LED/PD selection is performed at \(S\) and \(U1\) using the position information of \(N_I + N_J + N_K + N\) LED/PD links. Such information can be collected prior to the communication phase and would be remain valid until completion of communication. The random selection scheme uses random numbers for LED/PD selection and does not need the prior knowledge of channel gains or position information.

Typically in communication systems, pilot signals are transmitted for CSI acquisition prior to the data communication phase. These pilots can be used with an estimation scheme such as least squares estimation to acquire the CSI at node level. The nodes can feedback the CSI values or corresponding index of a LED/PD pair in order to compute the LED/PD pairs that should be activated at each node. For example, in the case of the optimal selection scheme the procedure for selection can be explained as follows. Pilot signals are sent from \(S\) to \(U1\), \(S\) to \(U2\), and \(U1\) to \(U2\). Sequential channel estimation is done at \(U1\) and \(U2\). Using low-speed feedback channel from \(U1\) to \(S\) and \(U2\) to \(S\), all CSI are sent to \(S\). The source \(S\) computes LED-PD pairs and the respective indices are notified to \(U1\) and \(U2\).

V. NUMERICAL RESULTS AND DISCUSSION

In this section, numerical results are presented to show the performance of the proposed LED/PD selection schemes and the impact of key system and channel parameters. In particular, we compare the derived expressions and approximations on the average achievable rate of the LED/PD selection schemes with the simulation results to verify the correctness of our analysis. As shown in Fig. 3, a configuration where the \(S\), \(U1\), and \(U2\) are placed at coordinates \((0, 0, 20), (-2, 0, 10),\) and \((2, 0, 0),\) respectively is considered. Unless stated explicitly otherwise in all simulations the employed parameter values are given in Table I. Moreover, the transmission power at \(U1\) is set as \(P_1 = 0.1 P_S\).

To highlight the performance of LED/PD selection schemes with cooperative NOMA, Fig. 4(a) shows the lower bound on the average achievable sum rate versus source transmit power, \(P_S\) using different LED/PD selection schemes for NOMA and OMA. Specifically in the OMA protocol, \(x_1[n]\) is sent from \(S\) to \(U1\) and \(U2\) in time slot 1 while \(x_2[n]\) is sent from \(S\) to \(U1\) and from \(U1\) to \(U2\) in time slot 2 and 3, respectively. The results show that cooperative NOMA aided

| Parameter         | Symbol | Value   |
|-------------------|--------|---------|
| Refractive index  | \(n\)  | 1.25    |
| Absorption coefficient | \(a\)  | 0.114 m\(^{-1}\) |
| Scattering coefficient | \(b\)  | 0.037 m\(^{-1}\) |
| Attenuation coefficient | \(c\)  | 0.151 m\(^{-1}\) |
| Noise variance at receivers | \(\sigma_x^2 = \sigma_y^2\) | 5 \times 10^{-14} |
| Mean of the log amplitude factor | \(\mu_x = \mu_y\) | -0.1 |
| Variance of the log amplitude factor | \(\sigma_x^2 = \sigma_y^2\) | 0.1 |
| Effective area of the PD | \(A_{PD}\) | 10^{-3} m\(^2\) |
| FOV of the concentrator | \(\Psi\) | 90° |
| Divergence angle of the transmitter | \(\theta\) | 10° |
| Power allocation factor for \(x_1[n]\) | \(\alpha_1\) | 0.8 |
| Power allocation factor for \(x_2[n]\) | \(\alpha_2\) | 0.2 |
| Responsivity of PDs | \(R\) | 0.5 A/W |
| Number of LEDs at the source | \(N_I\) | 5 |
| Number of PDs at \(U1\) | \(N_J\) | 5 |
| Number of LEDs at \(U1\) | \(N_K\) | 5 |
| Number of PDs at \(U2\) | \(N_L\) | 5 |
| Half angle of the uncertainty cone | \(\Delta x_1\) | 5° |
UOWC systems outperform conventional OMA in terms of achievable rate. As expected, the optimal solution shows the best lower bound on the achievable rate in the considered setup. Max $S - U1 - U2$ channel gain based selection shows near-optimal performance, while best $S - U1 - U2$ LoS based selection shows better performance than the random LED/PD selection scheme. Since our selection criteria is based on data rates, a scheme that maximizes the channel gains will have a superior performance. Due to the channel fluctuations, there is a finite probability that in LoS based scheme, the best channel is not selected. As such, it yields a inferior performance as compared to the channel gain based scheme. Random LED/PD selection has poorer performance when compared to other schemes. This is expected as random LED/PD selection does not exploit knowledge such as the channel gains or the orientation of elements. However, for applications which demand low-complexity implementations and low data rates, the random LED/PD selection is a possible option. Further, our results show that exact expressions and approximations for the lower bound on the average achievable sum rate closely match each other. Fig. 4(b) shows lower and upper bounds on the average achievable sum rate of the proposed LED/PD selection schemes. The results show that lower and upper bounds provided are fairly tight and are useful to understand the exact achievable sum rate. The rankings of the schemes using the upper bounds is also found to be the same as for the lower bound, confirming that either approach gives a useful indication of system performance. As the transmit power increases, the gap between lower and upper bounds further reduces. In the remainder, curves corresponding to the upper bound and OMA are not shown in the figures to avoid excessive clutter.

Fig. 5 shows the lower bound on the average achievable sum rate versus the source $S$’s transmit power, $P_S$, using best $S - U1 - U2$ LoS based selection scheme for different $\Delta \Phi_1$ values and $x_1$. Results show that when $U1$ is in the vicinity of $S$ and $U2$, higher performance can be obtained from best $S - U1 - U2$ LoS based selection scheme. When $\Delta \Phi_1$ is increased, the performance degrades due to random selection among a higher number of PDs at $U1$ and $U2$. Further, it can be noted that performance degradation due to imperfect position information is higher when $U1$ is not in the vicinity of $S$ and $U2$. Hence, accurate position information and placement of $U1$ in the vicinity of $S$ and $U2$ will lead to better performance.

Results shown in the rest of the figures are for a source transmit power of $P_S = 30$ W. Fig. 6 shows the lower bound on the average achievable sum rate versus power allocation factor $a_2$, for the message stream $x_2[n]$. Max $S - U1 - U2$ channel gain based selection scheme and best $S - U1 - U2$ LoS based selection scheme show near-optimal performance while the performance of random LED/PA selection is inferior. Moreover, the variation of the lower bound on the average achievable sum rate with the power allocation factor is limited. There exists an optimal power allocation factor for each LED/PD selection. This result can be used to select the power allocation factor for different LED/PD selection schemes.

Fig. 7 shows the lower bound on the average achievable sum rate as the placement of $U2$ is varied. The use of multiple LED/PD elements results in a range of rate pairs for $x_1[n]$ and $x_2[n]$. On the other hand, when a single LED/PD is employed, the values of $\mathbb{E}\{C_1\}$ and $\mathbb{E}\{C_2\}$ spans a confined region as seen from Fig. 7(a). The max $S - U1 - U2$ channel gain based selection scheme and best $S - U1 - U2$ LoS based selection scheme show near-optimal performance. The gap between the lower bounds of max $S - U1 - U2$ channel gain based selection scheme and best $S - U1 - U2$ LoS based selection scheme increases when $U1$ is placed away from the line connecting $S$ and $U2$. Further, there is an optimal $U1$ location for each LED/PD selection scheme. Simulation results show that optimal $U1$ placement for optimal, max $S - U1 - U2$ channel gain based, best $S - U1 - U2$ LoS based, and random selection schemes are $(2, 0, 6)$, $(0, 0, 5)$, $(0, 0, 4)$, and $(2, 0, 1)$ respectively. The corresponding lower bounds on the average achievable sum rate are $5.42 \text{bits/Hz/sec}$, $5.23 \text{bits/Hz/sec}$, respectively. Moreover, the maximum value in the case of the single LED/PD configuration is $4.34 \text{bits/Hz/sec}$. Depending on the availability of CSI or orientation of LEDs/PDs an appropriate LED/PD selection scheme can be selected and for each scheme, an optimal placement of $U1$ can be found. Further, our results show that by using the LED/PD selection scheme, better performance can be obtained for a wide range of $U1$ positions when compared to a single LED/PD configuration.

Fig. 8(a) shows the lower bound on the average achievable sum rate versus the number of LEDs/PDs at each node, $N$. When the number of LEDs/PDs is increased, the performance of the system increases. However, the performance improvement beyond six LEDs/PDs is negligible. Hence, $N = 6$ can be considered as a practical design choice for the proposed system. Further, the performance of the random LED/PD selection reduces with the number of LEDs/PDs and saturates due to the
increase of non-LoS LEDs/PDs. The average achievable sum rate fluctuates in configurations where the number of LED/PD elements is low. When the number of LEDs/PDs is increased, the fluctuations reduce since the possibility of finding an LoS LED/PD pair increases. The performance gap between the optimal scheme and the channel gain based scheme shows a fluctuating behavior for the considered parameter values. This behavior can be explained by recalling that our node design is hemispherical. The number of effective LED/PD pairs actively participating in the selection of the highest channel gain most of the time fluctuate with $N$. As the half power angle of the LEDs, $\theta_{1/2}$ increases the gap becomes consistent which is not shown in the figure due to space constraints. This is due to the fact that the effective number of LED/PD pairs actively contributing to the selection pool comes closer to the cardinality of the total selection set when when $\theta_{1/2}$ is increased. In addition, simulations were carried out to observe which node has the highest impact of increasing the number of LEDs/PDs. Our results verified that increasing the number of LEDs at $U_1$ has the highest impact on performance as shown in Fig. 8(b). Also, an extremely poor performance was observed when $U_1$ has only a single LED.

The results above show that the number of LEDs/PDs and the local arrangement of LEDs and PDs within a specific node can affect the performance. Hence when demanded by a specific application requirement, there is scope for optimal design of the number of LEDs/PDs and their best arrangement within the node for additional performance improvement.
Fig. 7. The lower bound on the average achievable sum rate versus position of U1 using different LED/PD selection schemes; (a) single LED/PD configuration; (b) optimal selection; (c) max $S - U1 - U2$ channel gain based selection; (d) best $S - U1 - U2$ LoS based selection; and (e) random selection.

Fig. 8. The lower bound on the average achievable sum rate versus number of LEDs/PDs; (a) $N$ LEDs/PDs at each node; (b) $N_k$ LEDs at U1.

VI. Conclusions

In this paper, we have analyzed a cooperative NOMA aided UOWC system consisting of multiple LED/PD elements at the source, relay, and destination. We have presented several LED/PD selection schemes that exhibit different implementation complexity. The lower bounds and approximate expressions of the average sum rate for maximum channel gain based, best LoS based, and random selection schemes were presented. Our system exhibits significant performance gains over a single LED/PD system as well as orthogonal transmission, specially in the medium-to-high power region. Maximum channel gain based and best LoS based selection schemes achieve results close to the optimum LED/PD selection scheme. Moreover, the performance of random LED/PD selection is significantly inferior to that of optimal selection. This observation clearly highlights the importance of acquiring channel or orientation knowledge to deliver performance gains in low-complexity UOWC systems. For all of the selection schemes, the results reveal that the number of LED/PD elements at the relay and the use of optimal relay placement has a significant impact on the average sum rate.
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