To the optimization model of flexible reinforced concrete elements in monolithic frames of buildings

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Abstract. This article examines a three-span 8-storey frame building with the aim of optimizing the girders based on heuristic algorithms. A comparative analysis of modern heuristic algorithms for optimizing bending elements as applied to reinforced concrete frames of monolithic multi-storey buildings for the possibility of their use in practical engineering calculations. A mathematical model of the adapted algorithm has been formed that meets the requirements of Russian standards. The target function for cost has been determined, including the cost of materials (concrete and reinforcement), formwork, construction with the corresponding rise in price coefficients. In this case, the girder is optimized, since, despite the fact that their number in the building frame is 24, the cost is about 65% of the total cost, including reinforced concrete columns in the amount of 32.5 constant parameters and 19 variable parameters are selected, a set of limitations of the optimization algorithm was developed, including regulatory requirements for the calculation of limit states, design requirements for the reinforcement of the girder and its geometric characteristics, including the concrete cover, the step of the rods, anchoring, etc. The calculation of girders as part of a reinforced concrete monolithic frame of an 8-storey building was carried out using the developed methodology and the comparison of the results obtained with numerical calculations and solutions obtained by other methods. It is shown that the results of numerical calculations have a sufficiently high efficiency of the developed optimization technique. The final cost is after 100,000 iterations. Savings in terms of the main design characteristics range from 3 to 13%, depending on the complexity of the problem and the accepted constraints.

1. Introduction
Design of reinforced concrete elements is a complex task of ensuring the safety and manufacturability of a structure with minimal costs [1]. As a rule, the design of reinforced concrete elements includes two main stages: the assignment of the assumed geometric parameters of the element cross-section and the selection of the required reinforcement for this section. When the criteria for safety or manufacturability in design are met, the economic efficiency of reinforced concrete elements is significantly reduced [2]. When designing any structure, one should not only strive for maximum efficiency of the structure, but also to ensure the strength, rigidity and stability of its elements. The big problem is the number of variable parameters. If it is large, then the number of possible solutions and combinations in the problem becomes so large that it becomes irrational to enumerate them all, the computational cost of the problem grows exponentially. To solve such problems of optimizing design solutions for reinforced concrete elements, modern heuristic algorithms are used that significantly speed up the process of finding the objective function [3].
The bending elements of multi-storey reinforced concrete frames adopted as the object of research are usually considered either as separate structures within various engineering structures, or as a component of the load-bearing frame of multi-storey frame-type buildings [4]. Among the most often used heuristic algorithms for optimizing concrete frames are the simulated annealing (SA), BB-BC, colliding particles (CBO), harmony search (HS), particle swarm (PSO) algorithm [5, 6]. A great contribution to the research of such algorithms for use in the optimization and calculation of reinforced concrete frames was made by the scientists of X. S. Yang and A. Kaveh. Also noteworthy are the studies by A. Akin and M. P. Saka, I. Paya and V. Yepez, Y. C. Toklu, J. Kim, S. Rajeev and C. S. Krishnamoorthy.

2. Optimization model of girders of reinforced concrete frames

The formation of the target function is made necessarily taking into account the following criteria: efficiency, manufacturability and safety [2]. In most works on the study of heuristic algorithms for the optimization of reinforced concrete structures, the function of the element cost is taken as the target optimization function, which includes the totality of costs for concrete and reinforcement, for formwork and the construction of a structure. In general terms, the target function of the design cost of the girder \( C_r \) conventional unit (c.u.) can be represented as

\[
C_r = C_b + C_{rf} + C_f
\]

where \( C_b \) -is the cost of the material to be reinforced (c.u.);

\( C_{rf} \) - cost of the reinforcing material (c.u.);

\( C_f \) - the cost of the formwork and the cost of the work (c.u.).

Here \( C_b \) can be represented as

\[
C_b = b_r \times h_r \times L_r \times C_c
\]

where 
- \( b_r \) - cross-sectional width of the girder (m);
- \( h_r \) - height of the cross-section of the girder (m);
- \( L_r \) - girder span (m);
- \( C_c \) - the cost of 1 m³ of concrete "in business" (c.u. / m³).

The value of reinforcement \( C_{rf} \) is determined as the total value of all reinforcing bars in the girder using the formula

\[
C_{rf} = 0.25\pi C_s \gamma_s \left( \sum_{i=1}^{m} n_i d_i^2 l_i + k d_s^2 l_s \right)
\]

where 
- \( C_s \) - the cost of 1 ton of reinforcing steel "in business", (c.u. / t);
- \( \gamma_s \) - specific weight of reinforcing steel (t / m³);
- \( m \) -is the number of rows of longitudinal reinforcement bars;
- \( n_i \) - the number of longitudinal reinforcement bars of the \( i \)-th row of bars;
- \( d_i \) - diameter of longitudinal reinforcement bars (m);
- \( l_i \) - the length of the bars of the \( i \)-th row of reinforcement (m);
- \( k \) - is the number of transverse reinforcement bars;
- \( d_s \) - diameter of transverse reinforcement bars (m);
- \( l_s \) - length of cross-reinforcement stirrups (m).

The cost of reinforcement \( C_{rf} \) may include the costs of an additional increase in the consumption of reinforcement in the amount of 5 ÷ 10% of the total longitudinal reinforcement volume, which are used for the manufacture of embedded parts, docking devices, etc.

The \( C_f \) indicator is calculated by the formula

\[
C_f = C_w L_r (b_r + 2h_r)
\]
where \( C_w \) is the cost of 1 m\(^2\) of formwork, provided that the upper edge of the girder is open for the production of concrete works (c.u./m\(^2\)). \( C_w \) also includes the costs of construction production: performance of reinforcement, formwork, concreting and auxiliary works (bending, cutting, welding of reinforcing bars, etc. in the amount of \( 5 \div 10\% \) of the main work). Further, this parameter is usually called the "cost of formwork", omitting the additional costs of production.

If necessary, the target function of the cost of the girders \( C_r \) can be multiplied by increasing factors that take into account the rise in the cost of work in the winter, as well as changes in the cost of materials and work due to inflation.

Thus, based on the above, the target function of optimizing the girder as part of the frame can be described by the formula

\[
C_r = b_r h_r L_r C_c + 0.25 \pi C_s \gamma \left( \sum_{i=1}^{m} n_i d_i^2 l_i + kd_i^2 l_i \right) + C_w (b_r + 2h_r)
\]  

(5)

Determination of the main constant and variable parameters of the algorithm

Of the fourteen parameters of the girders optimization objective function proposed above, five are designated as constant and nine - as variables. It should be noted that the varied parameters must necessarily be discrete in order to be able to implement them in real design conditions.

For the remaining nine variable parameters, the range of acceptable values is set. The range of permissible values is determined based on design considerations and the requirements of the manufacturability of the design. In this case, for the geometric dimensions of the section \( b_r \times h_r \), the following restrictions are adopted, reflecting the requirements [7]:

- for section width \( b_r \): values from 250 to 500 mm with a step of 50 mm;
- for section height \( h_r \): values from 400 to 700 mm with a step of 50 mm.

The number of longitudinal reinforcement bars of the bottom row: \( n_1 = 3 \div 5 \); additional bottom row: \( n_2 = 2 \div 4 \); top row: \( n_3 = 2 \div 4 \).

The range of used diameters of reinforcement bars is limited to the range of the most common bars in the design of real structures, regardless of the number of reinforcement: \( d = 12 \div 32 \) mm.

The number of rows of longitudinal reinforcement bars \( m \) is ultimately determined when calculating the actual bearing capacity of the section with the adopted reinforcement option and section dimensions. For the problems under consideration, the values \( m = 2 \) or 3 are taken.

The length of the longitudinal reinforcement bars \( l_i \), generally depends on the arrangement row \( m \), and for the span reinforcement of the lower row \( m = 1 \) the length value \( l_1 = L_r + 40 d_i \), where 40 \( d_i \) is the anchorage length of the bars of the lower row. For the spanning reinforcement of the second row \( l_2 = 0.8 L_r \), for the upper row \( l_2 = L_r + 40 d_i \).

The diameter of the transverse reinforcement \( d_s \) is taken in the range \( d_s = 6 \div 10 \) mm. The number of stirrups of transverse reinforcement \( \tilde{k} \), depending on their pitch in the span and on the supporting sections, is determined by calculation. In order to unify the step of the cross-reinforcement stirrups in the girders near the support sections \( 0.25L_r \) is taken equal to 150 mm, in the span - 300 mm. As a result, the parameter of their number \( \tilde{k} \) can be approximately determined, while rounding the resulting value up

\[
K = \frac{0.5L_r}{0.3m} + 2 \frac{0.25L_r}{0.15m} = \text{ceil}(L_r \times 5 \frac{pcs}{m})
\]

(6)

The length of the stirrups of the transverse reinforcement \( l_s \) directly depends on the geometric parameters of the width and height of the section and is found by the formula

\[
l_s = 2(b_r + h_r) \times 0.9
\]

(7)
The main variable parameters of the girder cost objective function and some design constraints are shown in figure 1 and are summarized in table 1, the penult column of which indicates the range of values of the variable parameters.

**Figure 1.** Main variable parameters of the girder cost objective function (the number and diameter of reinforcement bars are shown conditionally).

**Table 1.** Main variable parameters of the girder cost objective function.

| Denomination                  | Designation | Unit measurements | Value / step | Number of possible values |
|-------------------------------|-------------|-------------------|--------------|---------------------------|
| Section width                 | $br$        | m                 | 250÷500/50   | 6                         |
| Section height                | $hr$        | m                 | 400÷700/50   | 7                         |
| Number of rods, bottom row 1  | $n_1$       | pcs.              | 3÷5/1        | 3                         |
| Number of rods, bottom row 2  | $n_2$       | pcs.              | 2÷4/1        | 3                         |
| Number of rods, top           | $n_3$       | pcs.              | 2÷4/1        | 3                         |
| Number of rows                | $m$         | pcs.              | 2÷3/1        | 2                         |
| Diameter of longitudinal bars | $d_l$       | mm                | 12÷32        | 9                         |
| Stirrups diameter             | $d_s$       | mm                | 6÷10         | 3                         |
| Length of longitudinal bars   | $l_l$       | m                 | depending by $m$ | -                         |
| Stirrups length               | $l_s$       | m                 | by formula 7 | -                         |
| Number of stirrups            | $k$         | pcs.              | by formula 6 | -                         |
As additional variable parameters of the algorithm, it is possible to introduce classes of concrete and reinforcement. In this case, for the class of concrete, values from B25 to B40 are accepted as the most common in practice among structures of this type. The reinforcement classes are limited to those used in non-stressed structures: A400, A500C and A600.

The main characteristics of these reinforcing bars are presented in table 2.

| Table 2. Mechanical properties of reinforcing steels |
|-----------------------------------------------|
| Mechanical properties of reinforcement (MPa) | A400 | A500C | A600 |
| Temporary tensile strength                    | 590  | 600   | 885  |
| Yield point                                    | 390  | 500   | 590  |
| Design tensile strength                       | 365  | 450   | 520  |
| Elongation after rupture \( \delta \), %       | 14   | 14    | 6    |

2.1. Algorithm conditions and limitations

The main limitations of the \( g_i \) algorithm include strict compliance with the requirements of the strength and bearing capacity of elements, structural requirements for reinforcement, including the minimum percentage of reinforcement, requirements for the minimum size of the concrete cover, etc. The main regulatory document for the introduction of all restrictions are the 1\textsuperscript{st} and 2\textsuperscript{nd} groups of limiting states and design restrictions [8]. The performance of these limitations of the objective function ensures the safety and reliability of the structure during its normal operation.

With these restrictions, the minimum concrete cover thickness is assumed to be 20 mm, as well as not less than the diameter of the working rods. The minimum clear distance between the rods is at least 50 mm, between the rows of tensile reinforcement - at least 20 mm and more than the maximum accepted diameter of the reinforcement rods.

2.2. Heuristic optimization algorithm

This paper provides for the use of the heuristic HS algorithm in order to solve the optimization problem. The HS algorithm was chosen as considered due to the fact that it has a sufficiently high convergence rate to the global optimum of the objective function [9], contains a small number of specific parameters for its adjustment, and is also widely used in solving optimization problems of various building structures [10].

For such an algorithm, first of all, the design constants of the objective function, the ranges of variation of the main variable parameters (indicated in table 1), as well as the specific parameters of the algorithm are set.

According to [9, 11] and [10], the variables \( \text{HMCR} = 0.85 \) are written into the specific parameters of the algorithm; \( \text{PAR} = 0.17 \) and \( \text{HMS} = 50 \). Such values of these variables allow achieving good performance when optimizing the objective function according to [11].

The concrete class is B30, the class of reinforcement is A500C.

The initial matrix of variable parameters of the construction \( h, b, n, d, d, m, \) \( m \) is randomly generated and written into the memory of the algorithm, then the conditions and constraints \( g_i \) set for the algorithm are checked, and the objective function \( C_r \) is calculated for each of the possible solutions in the initial matrix.

Next, a new solution is generated by changing the varied parameters in one direction or another with a certain probability (i.e., a new solution is not generated randomly). The new solution is written into the memory of the algorithm over the previous one - the most expensive in comparison with the rest. Then the algorithm is repeated until the moment when the optimization goal is achieved.

The stop criterion of the algorithm (100.000 iterations) is the moment when the maximum difference between the cost of the last five variants of one iteration of the algorithm is less than 2\%.

The stage of generating new solutions includes checks of all design constraints of the \( g_i \) algorithm.

In general, the girder optimization problem can be represented as
Minimize \( f(x_i) = C_r \) provided: \( x_i \in X_i \), \( i=1,2,...,N \)
\[ g_j \leq 0 \quad j=1,2,...,M \]

where
- \( X_i \) - set of design parameters for variable \( i \);
- \( N \) - is the number of variables in the algorithm;
- \( g_j \) - constraints of the objective cost function;
- \( M \) - is the number of constraints on the objective cost function.

The algorithm memory matrix (the so-called HM matrix) is composed of a set of design parameters for the algorithm. The pool of design parameters is initialized at the first step of the algorithm from a discrete set of possible values for each variable parameter. The algorithm memory matrix is written in the following form:

\[
[H] = \begin{pmatrix}
    x_1^1 & x_2^1 & \ldots & x_{N-1}^1 & x_N^1 \\
    x_1^2 & x_2^2 & \ldots & x_{N-1}^2 & x_N^2 \\
    \vdots & \vdots & \ddots & \vdots & \vdots \\
    x_1^{HMS-1} & x_2^{HMS-1} & \ldots & x_{N-1}^{HMS-1} & x_N^{HMS-1} \\
    x_1^{HMS} & x_2^{HMS} & \ldots & x_{N-1}^{HMS} & x_N^{HMS}
\end{pmatrix}
\]  

(9)

Each row in this matrix corresponds to a vector of a possible solution to the problem, i.e. optimal solution candidate and determines a set of variable parameters for its cost function. Moreover, each vector of a possible solution necessarily satisfies all the constraints of the algorithm, and a cost function is calculated for it. The number of columns of the matrix HM corresponds to the number of variable parameters of the algorithm, and the number of rows is limited by the amount of memory of the HMS algorithm. A prerequisite for the third step is to sort the candidates in ascending order of the value of the cost function.

The synthesis of a new solution vector at the fourth step is performed in accordance with the following rules:

1) by setting the probability of accessing the values already written in the memory matrix using the parameter HMCR=0.7÷0.95. The value of the first variable parameter \( x_i \) with a given probability is chosen and the corresponding (first) column of the matrix HM: \( \{x_1^1, x_1^2, x_1^{HMS-1}, x_1^{HMS}\}^T \). The values of the remaining variable parameters are selected in a similar way from the corresponding columns. With a probability of 1-HMCR, the values of the variable parameters can be selected at random from the pool of design solutions for the problem. This rule can be represented as:

\[
[H] = \begin{cases}
    x_i \in \{x_1^1, x_1^2, x_1^{HMS}\}^T \text{ with probability HCMR} \\
    x_i \in X_i \text{ with probability 1-HCMR}
\end{cases}
\]  

(10)

2) each variable of the new vector, if it was selected from the memory matrix of the algorithm, is estimated through the parameter of the probability of additional change in the variable parameter PAR=0.15÷0.20. With a given probability, the selected parameter \( x_i^k \), which is the \( k \)-th element of the pool of values \( X_i \), is replaced with the adjacent parameter \( x_i (k \pm 1) \) from this pool. This rule was introduced into the algorithm in order to prevent stagnation of variable variables and to diversify the memory matrix, which ultimately increases the chance of finding the global optimum of the objective cost function.

As noted above, the vector of the new solution at the fifth step of the algorithm is checked for all the established constraints. The cost of the synthesized solution is calculated. This vector has the right
to overwrite the least economical solution contained in the HM memory only if the cost is lower in comparison with it and all the constraints of the algorithm are checked.

3. Optimization model of girders of reinforced concrete frames

The algorithm for optimizing reinforced concrete structures is proposed to be implemented using the MATLAB software package from The MathWorks. The package uses an interpreted programming language M-language, focused primarily on working with matrices and data vectors, as well as having a wide range of capabilities for implementing various kinds of algorithms, data visualization and interaction with.

As the output of the program, a file of the main properties of the model is generated in the form of coordinates of nodes, geometry, material properties and other initially entered data; it also contains the results of strength analysis - components of the SSS system: mainly, efforts in the rods, displacements of specific nodes, a journal with the history of calculation etc.

In order to make it possible to carry out a comparative analysis, as well as to verify the results obtained, as test problems in this work, it is proposed to use the design scheme of a three-span, eight-story frame (figure 2), described in the works of A Kaveh, O Sabzi [12] and S Gholizadeh, V Aligolizadeh [13].

![Figure 2. Design scheme of a multi-storey frame.](image)

The height of the floor is taken as constant 3.3 m, and the spans are 7.5 m.

In the design model of loads, several load cases are considered in accordance with the requirements of the standards.

The design scheme and design operating conditions also remain constant, not considering possible technogenic loads [14,15,16].

This diagram contains 24 girders, 32 columns. The memory size for this problem is 50 initial solution vectors. The total cost after 100,000 iterations expired was 29,000 (c.u.).
Figure 3 shows the history of the convergence of the crossbar cost objective function to this value. Table 3 compares the main output parameters of the calculation with the results of studies using the BB-BC-algorithm [12].

It should be noted that the constant parameters of the cost of the main structural materials and formwork for all tasks are taken the same and equal:

- the cost of concrete: \( C_c = 105 \, \text{c.u.} / \text{m}^3 \);
- the cost of reinforcing steel: \( C_s = 900 \, \text{c.u.} / \text{t} \);
- the cost of formwork: \( C_w = 92 \, \text{c.u.} / \text{m}^2 \).

![Figure 3. Graph of convergence of the girder cost objective function.](image)

| Elements | Type | Group | Section sizes, mm | The main reinforcement | Section sizes, mm | The main reinforcement |
|----------|------|-------|-------------------|------------------------|-------------------|------------------------|
|          |      |       | Width  | Height | Width | Height | Width | Height |
| Girders  | B1   | 300   | 500    | 5Ø22   | 300   | 500    | 6Ø22  |
|          | B2   | 300   | 500    | 5Ø22   | 300   | 500    | 6Ø22  |
|          | B3   | 300   | 500    | 4Ø22   | 300   | 500    | 5Ø22  |

Cost, c.u. 29000 29730

The final cost difference between the synthesized solution and the one obtained in [12] using the BB-BC heuristic algorithm is 2.52% in favor of the HS-algorithm.

4. Conclusions
1. The spatial frame system made of reinforced concrete is quite complex in mathematical optimization. Therefore, as an object of research, a reinforced concrete girder was adopted as a component of the supporting frame of a three-span, eight-story frame-type frame.
2. The structure of the objective function of a monolithic reinforced concrete girder with 5 constant and 19 variable parameters, including its reinforcement, geometric characteristics, concrete cover, rod pitch, anchoring, etc., as well as an optimization algorithm for given constraints, taking into account the cost criterion.
3. The results of numerical calculations showed a sufficiently high efficiency of the developed optimization technique. Savings on the main design characteristics range from 3 to 13%, depending on the complexity of the task.
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