Experimental and theoretical study of post-tensioned unbonded beams

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Abstract. Paper contains data obtained during a bending test of four post-tensioned unbonded beams as well as description of geometry of test specimens and strength properties of used materials. Midspan deflection, tendon stress increase, cracking moment and ultimate bending moment were measured during the test. Crack propagation pattern was assessed as well. Ultimate bending moment, cracking moment and midspan deflection were then estimated using reinforced concrete analysis methods widespread in Russia. Comparison of the test data and calculation results shows that nowadays there are not domestic analysis methods well enough to fully estimate the unbonded post-tensioned beams.

1. Introduction

First structures with post-tensioning were built in the first half of 20-th century. Approximately in the same time the first experimental studies of post-tensioned constructions were performed. As an example, experimental studies completed at University of Illinois in the beginning of the 1950-es by Feldman [1] and others could be given. Experimental and theoretical data about features of such structures have been accumulated onward: there could be mentioned the works of Burns and Pierce [2], Cooke et all [3] and many others. Obtained theoretical and experimental substantiation together with efficiency and profitability of post-tensioned reinforced concrete [4] have led to wide spreading of this construction technology in many countries in the world.

Despite obvious efficiency and profitability of post-tensioned reinforced concrete, it is used quite seldom in construction of different objects in Russia. Post-tensioned structures are still new type of constructions to the majority of Russian engineers, especially in regions. Among the reasons of low application of post-tensioning in the domestic market there is lack of normative documentation for calculation and design of post-tensioned structures and difficulties in proper understanding of bending behavior and reliability of post-tensioned elements caused by small number of experimental studies performed by domestic researchers (among which [5], [6] and some others), deficiency of experience in construction of real objects as well as language barrier, which makes usage of the worldwide experience much harder.

So, it seems to be necessary to perform additional domestic experimental studies to give exhaustive information about the behavior and the failure pattern of post-tensioned structures.

To reveal behavior features of unbonded post-tensioned beams, the experimental study of several beams was performed, as well as comparison of the test data with the results of theoretical estimation...
of bending capacity, cracking moment and midspan deflection. The latter were obtained from widespread domestic analysis methods of reinforced concrete members.

2. Test specimens
Four simply supported beams with $l/h$ ratio equal to 20 and nominal dimensions of $l \times b \times h = 200 \times 20 \times 10$ cm have been tested. General view of the specimen is shown on figure 1.

![Figure 1. General view of the test specimen (dimensions in mm)](image)

To strengthen the beams against shear on length equal to 42 cm from each end of the beam there were installed reinforcing meshes with spacing 5 cm. To prevent concrete crushing in points where the beam leans on its supports there were mounted steel embedded plates with dimensions of 10×20 cm and 1 cm thick at each end of the beam.

Two unbonded steel wires were installed in the specimens to obtain more uniform prestress distribution. Wires profile was straight along the beams. Distance from the bottom surface of the beams to center of the gravity of the unbonded reinforcement was taken equal to $a_{op} = 2.97$ cm.

2.1. Materials properties
Portland cement M500 was used for preparation of concrete mix. Granite rubble with a maximum size equal to 5-20 mm was used as coarse aggregate, and sand with fineness modulus 1.7 used as fine aggregate. Proportions of the concrete mix components by weight were as follows: cement : water : sand : rubble = 1:0.5:1.87:2.37.

Compressive strength of concrete was determined by testing of standard cubes with the side dimension 10 cm. Then this strength was recounted into standard prismatic compressive strength of concrete which turned out to be equal to $R_b = 2.06$ kN per square cm. Initial modulus of elasticity of the concrete was estimated according to [7], and amounted to $E_o = 3249$ kN per square cm. Tensile strength of the concrete was not measured directly and was calculated by empirical equation proposed in [8]: $R_{bt} = 0.07 \times R_b$.

High-strength reinforcement wire designated as $Vr$-1400 with nominal diameter of 5 mm and nominal tensile stress $R_{s,n} = 140$ kN per square cm was chosen as prestressed steel. Actual cross
section area of the reinforcement wire was amounted to $A_{rp} = 0.202$ square cm. Actual ultimate stress of the reinforcement was determined by tensile test and turned out to be $\sigma_{ru} = 176.5$ kN per square cm.

2.2. Manufacturing of the beams

Water resistant plywood formwork was made to shape the beams. Prestressing reinforcement was mounted into the formwork as a U-shaped unit. At the left end of the beam, as it shown on figure 1, it was turned around the piece of a steel tube with diameter 14 cm welded to a steel plate with cross dimensions 10×20 cm and 1 cm thick. At the right end of the beam free ends of the prestressing wire were fixed by wedge grips. Before mounting into the formwork, the prestressing wires were greased by usual machine oil and enclosed into a plastic shell. Diameter of reinforcing units along the side faces of the plastic shell was equal to 0.66 cm. Required position of the prestressing wires by height and width inside the formwork was provided by plastic clips which were installed with step of about 40 cm.

As placing of all required elements and the reinforcing units into the formwork was finished, casting of concrete was made. Concrete consolidation was performed by 12 mm diameter pin. There were 16 pokes per 100 square cm of concrete surface. Concrete was cured in normal microclimate conditions. Simultaneously with casting of the beams there were made some standard concrete cubes and prisms which were necessary to obtain actual strength and modulus of elasticity of concrete.

Prestressing of the beams B1-B4 was carried out after 60, 70, 78 and 82 days respectively after their manufacturing. Tension of the prestressing wires was performed mechanically using steel plates, bolts and nuts. Prestressing force was applied to the beam’s active end through a steel plate with dimensions 1×13×30 cm. It was made to prevent concrete crushing in points of the prestressing forces transfer. Value of prestressing force in each wire during the prestressing and the test itself was measured by hand-made dynamometers. They were made from short elastic hexagon steel prisms with longitudinal hole for placing the reinforcement. On two opposite faces of the prisms there were mounted electric strain gauges. Before the use, dynamometers were calibrated: some specific compressive force was applied to them and respective indication from the gauges was read. Dynamometers are described in [9] in more details.

After the desirable prestressing force had been applied to the beams, they got upward deflection at midspan approximately equal to 0.03 cm.

3. Experimental study

3.1. Test bench and measuring devices

At one of the ends the beams were rested on the bench construction through a hinge support which was modelled by a hot rolled angle turned up by its outside corner. At the other end the beams were placed on the bench construction through a roller support which was presented by a steel cylinder. Actual spacing between the supports were 192 cm. General view of the test bench with one of the specimens placed into it are shown on figure 2.

Loading of the beams was made by hydraulic jack. Force from it was transferred to the beams through a crossbar which was placed on two roller supports on the top surface of the beams. Distance from the supports of the crossbar to the supports of the beams were 61 cm. To prevent concrete crushing at the points, where forces from the roller supports of the crossbar were applied to the beams, steel plates with dimensions 1×10×20 cm were put. The measure of the force applied to the beams was determined by mechanical dynamometer with the value of one division 0.07 kN.

Dial indicator PAO-6 with the value of division equal to 0.01 mm was used to measure the midspan deflection of the beams. At the distance of 15 cm from each end of the beams steel frames were firmly fixed to the specimens. Between frames thin steel wires were passed. One of their ends was attached to one of the frames, while the other end was passing through a wheel of the dial indicator PAO-6 mounted on the other frame. In the middle part of the beams with the base approximately of 45 cm
long, strains of the top and bottom surfaces of the beams were measured with the use of dial indicators ICh-10 with the division value of 0,01 mm.

As it was mentioned, hand-made calibrated dynamometers were used to control the force in the prestressed unbonded wires during the test.

![Figure 2. Specimen placed into the test bench](image)

3.2. Parameters of the specimens at the day of testing

Testing of the beams B1 and B2 took place 10 and 7 days respectively after they were prestressed. Testing of the specimens B3 and B4 was performed after approximately two months since they were prestressed. During these two months there was a gradually slowing process of pretension losses and the midspan upward deflection increasing. Average loss in prestress after approximately 65 days of observation turned out to be \( \Delta \sigma_{sp} = 12.4 \) kN per square cm, or 9.25% from initial prestress. Midspan upward deflection reached a value of 0.06 cm.

Table 1 shows the geometrical parameters of the specimens, strength of concrete and stresses in the prestressed reinforcement actual to the day of testing.

| №  | Cross section width, \( b \), mm | Cross section height, \( h \), mm | Age at the day of the test, days | Compressive strength of concrete, \( R_b \), kN per square cm | Tensile strength of concrete, \( R_{bt} \), kN per square cm | Stress in prestressed reinforcement due to prestress, \( \sigma_{sp0} \), kN per square cm |
|----|-------------------------------|-------------------------------|----------------|----------------|----------------|-------------------------------|
| B1 | 199.7                         | 100.7                         | 70             | 2.86           | 0.20           | 137.34                        |
| B2 | 199.6                         | 101.8                         | 77             | 2.86           | 0.20           | 136.35                        |
| B3 | 200.8                         | 100.3                         | 145            | 3.00           | 0.21           | 126.78                        |
| B4 | 199.5                         | 99.9                          | 144            | 3.00           | 0.21           | 116.67                        |
3.3. **Test performance and results**

Before formation of a first crack, the load to the beams was applied by 0.5 kN at a step at each support of crossbar. Midspan deflection increase against applied load was almost linear during this stage.

Formation of the first crack in the beams B1 and B2 was accompanied by noticeable growth of the midspan deflection increase against applied load. In case of the beams B3 and B4 some decrease in applied load was observed right after the first crack appearance and then deflection increase against applied load was developing similarly to the beams B1 and B2. Since the significant acceleration of the midspan deflection increase against the applied load was occurred after the crack formation the load increments was estimated rather by value of deflection than the measure of the applied load during this stage of test.

Curves on figure 3 show the midspan deflection of the beams versus the load transferred to them from each support of the crossbar.

![Figure 3. Curves «load–deflection» for the beams](image)

For the beams B1 and B2 the first crack formed at a distance approximately 15–20 cm from the midspan. With increasing of the applied load, two more cracks started consistently in these beams. One of these cracks was located almost in the midspan. Cracks opening pattern at the moment preceding the failure of the beam B1 is shown on figure 4.

![Figure 4. Cracks pattern in the beam B1 at the stage close up to its failure](image)
In case of the beams B3 and B4 there were only one crack at the midspan.
Progressing to the upper faces of the beams, all cracks in all specimens had a tendency to develop into the inclined and even horizontal branches that can be seen on figures 4 and 5.

*Figure 5. Cracks branching in the specimens: a – B3; b – B4*

Failure of all the beams was caused by compressed zone concrete crushing at a small area above the midspan crack. Compressed zone fracture behavior was similar for all the beams. Figure 6 shows fracture of concrete in compressed zone of the beam B4.

*Figure 6. Fracture of the concrete of the compressed zone of the beam B4*

Stress in the reinforcement at the stage before the first crack formation remained approximately on the same level. After the first crack starting with noticeable growth of the midspan deflection, a significant increase in stress was also observed. Curves on figure 7 are showing the value of a current force in the pretensioned reinforcement $P_i$ to the initial force $P_0$ versus applied load for the beams B1, B3 and B4. Prestressing force in the prestressed reinforcement of the specimen B2 changed spasmodically during the test. So, data for this beam are not shown on the figure 7. Moreover, at the failure of this beam some stress decrease in the reinforcement was observed as it is shown in table 2. It also can be noticed, that this beam showed smaller ultimate moment as compared to that of the beam B1. Probably, at compressed zone of the beam B2 there were some concrete defects because of which this beam collapsed early and deflection did not develop enough to reach the stable stress increase in
After the crushing of the compressed concrete this beam had some increase of the deflections which was accompanied by decreasing of applied load. When the maximum midspan deflection was reached, the stress increase in the pretensioned reinforcement turned out to be equal to $\Delta \sigma_{sp} = 15.56 \text{ kN per square cm}$ or 11.41% higher than the initial stress due to prestress alone.

Prestressed steel in all the beams tested remained at an elastic stage during the tests. An exception was observed only in one of the reinforcing wires in the beam B1, where stress at failure of the specimen increased up to 169.04 kN per square cm, or 96% of the actual ultimate tensile stress.

The moment when increase of the applied load to the beams stopped was considered as a loss of bending capacity of the beams.

![Figure 7. $P/P_0$ ratio versus applied load from the initial stage up to the maximum deflection](image)

Experimental data are shown in table 2.

| №   | Cracking moment, $M_{cr}$, kN-cm | Ultimate moment, $M_{ult}$, kN-cm | Stress increase in the prestressed reinforcement at failure, $\Delta \sigma_{sp}$, kN per square cm |
|-----|---------------------------------|-----------------------------------|------------------------------------------------------------------------------------------------|
| B1  | 272.92                          | 365.67                            | 24.86                                                                                           |
| B2  | 288.48                          | 315.40                            | -3.54                                                                                            |
| B3  | 321.69                          | 365.67                            | 19.49                                                                                           |
| B4  | 310.68                          | 354.38                            | 15.08                                                                                           |

After load was removed, all the beams returned nearly into the initial position. Residual midspan deflection turned out to be 0.15 cm approximately and residual cracks opening was about 0.02 cm.

Based on the data from tables 1 and 2 it can be concluded, that the concrete strength as well as the level of prestress influenced the cracking moment and ultimate bending capacity of the beams.

4. Calculation methods and comparison of theoretical and experimental results

Nowadays in Russia there are not normative base dedicated to comprehensive design of post-tensioned unbonded elements. Main normative document dedicated to the calculation and design of concrete and reinforced concrete elements [10] contains information about losses of prestress in unbonded
constructions and some design requirements. In the same time, it does not include formulas and recommendations which can allow to calculate the bearing capacity, crack resistance and deflections of post-tensioned unbonded constructions. In the framework of this study it seems appropriate to bring calculation results obtained by using recommendations and formulas [10] acceptable for bonded constructions to analysis of unbonded post-tensioned beams.

Considering a post-tensioned unbonded beam as an eccentrically compressed element can be quite interesting as well. In this case a bending moment at midspan caused by prestressing force, self-weight of the element and some external loads can be replaced by longitudinal compressive force equal to prestressing force $P_0$ applied with a conditional eccentricity equal to $e=M_{tot}/P_0$. This method in more details described in [11].

Some recommendations on determination of the bearing capacity and deflections of unbonded post-tensioned elements are listed in a domestic normative document [12]. However, this document is appropriate for massive constructions of bridges and contains some specific requirements for such structures. So, expediency of it using to analysis and design of relatively small post-tensioned slabs and beams represented in usual residential and public buildings requires a verification.

Quite detailed recommendations about calculation and design of unbonded post-tensioned constructions can be found in D. V. Portaev’s monography [13].

In addition, methods of the linear mechanics of materials can be applied to analysis of unbonded post-tensioned beams as well. In more details obtained formulas are described in [14].

The beams at the experimental study were tested in two series. In the first two specimens tested there was larger level of prestress and lower concrete compressive strength. In the second two beams prestress level was lower while compressive strength of the concrete was higher. To obtain more representative results it was decided to perform calculations twice: using average parameters of the beams B1 and B2 and then using average parameters of the beams B3 and B4. In all performed calculations, an actual concrete compressive strength and actual stress in reinforcement $\sigma_{sp0}$ were used. Exception was made for method [10] where normative tensile strength of prestressing wire $R_{s,n}$ was used instead actual stresses in reinforcement due to prestress at the moment of the testing $\sigma_{sp0}$.

General view of the considering beam is shown on figure 8. Shear reinforcing meshes at the ends of the beam were neglected in calculations.

![Figure 8. General view of the considering beam (dimensions in mm)](image)

Values of the ultimate bending moment, cracking moment and midspan deflection at ultimate stage obtained by mentioned calculation methods are listed above the line in table 3. Under the line in the table 3, obtained deviations of the calculated results from the experimental data in percent are given.

From the data in the table 3 it can be seen that calculations performed by using of mentioned analysis methods did not allow to obtain results corresponding to experimental results with sufficient accuracy.
Table 3. Calculation results and comparison the theoretical and experimental data

| Parameter and beam number | Method [12] (SP 35.13330, eccentrically compressed elements) | Method [11] (SP 63.13330, elements with bonded reinforcement) | Method [10] (D. V. Portaev’s monograph) | Method [13] (linear mechanics of materials) |
|---------------------------|---------------------------------------------------------------|---------------------------------------------------------------|----------------------------------------|------------------------------------------|
| Ultimate moment, $M_{ult}$ kN-cm | B1, B2 365.84 +6.92% | 313.36 -8.67% | 373.64 +8.86% | 329.34 -3.40% | 366.59 +7.11% |
| Cracking moment, $M_{cr}$ kN-cm | B1, B2 328.99 -9.43% | 288.43 -24.82% | 374.88 +3.96% | 296.18 -21.56% | 330.56 -8.91% |
| Midspan deflection at failure, cm | B1, B2 0.20 – | 1.15 – | – – | 0.17 – |
|                            | B3, B4 0.18 – | – 1.74 – | – – | 0.15 – |

Analysis of the beam according to method presented in [12] allows to get the acceptable value of the ultimate moment but does not allow to evaluate the cracking moment. Deflections calculation in this method based on the linear work of materials in all stages which leads to unacceptably low values of the deflections.

When the post-tensioned unbonded beam was considered as an eccentrically compressed member there were obtained inadmissibly understated values of the ultimate moment and cracking moment of the section. Using of this method does not give an opportunity to estimate deflections of the beam by usual formulas contained in document [10]. Moreover, it is necessary to perform calculations using an iterative process, which has negative effect on practicality of this approach.

Calculation of the unbonded beam like a bonded beam according to recommendations and formulas contained in [10] leads to some overestimation of the value of the ultimate moment. Estimation of the cracking moment by this method shows insufficient accuracy in case of the beams B3 and B4, and quite appropriate results in case of the beams B1 and B2. At the same time, values of the deflections calculated by [10] is in good agreement with experimental data. In this method there are recommendations necessary to calculate curvature of members with cracks. Despite the results obtained by this method is in good agreement with the test data in general, considering of unbonded elements as bonded elements does not match to the nature of their work and may lead to unacceptable results.

Value of the ultimate moment calculated based on the recommendations proposed in the monography [13] turns out to be acceptable in the case of the beams B1 and B2. In the case of the beams B3 and B4 results are significantly worse. Similarly, there is inconsistency in the values of the cracking moment. The mentioned monography does not include analytical formulas for estimation of deflections of post-tensioned unbonded members. Author [13] suggested to use computer programs based on the finite element’s method for this purpose.

Calculation of the beam by formulas obtained by methods of linear mechanics of materials allows to evaluate its bearing capacity with some overestimation in case of the beams of B1 and B2 and with an underestimation in case of the beams B3 and B4. In the case of the beams B1 and B2 there are good agreement between the experimental and theoretical values of the cracking moment. In the same time calculation of the cracking moment with parameters of the beams B3 and B4 gives unacceptable results. Deflection determination in this method performed in suggestion, that materials remain elastic at all stages of the beam work. It leads to obtaining an inadmissible value of the midspan deflection for all specimens.
5. Conclusion
Performed experimental study of the unbonded post-tensioned beams allowed to reveal the main features of their bending behavior. Before the first crack formation, the midspan deflection increased against applied load almost linearly. After the start of the cracking and up to the failure of the specimens the values of the midspan deflection against applied load increased significantly. It was found, that decrease of prestressing level lead to decrease in number of cracks. While cracks are developed to the upper face of the beams, they had tendency to inclined and horizontal branching. Failure of all the beams tested happened due to exhaustion of strength of concrete in compressed zone of the specimens in the small region over the tip of the midspan crack. Prestressed reinforcement at the failure of the beams remained in the elastic stage. Noticeable stress increase in the reinforcement started together with significant growth of the deflections after the crack formation.

Performed calculations and comparison of the test's and theoretical results had shown, that current widespread Russian methods of designing cannot be used to comprehensive analysis of post-tensioned unbonded beams with sufficient accuracy of the obtained results. It is necessary to perform additional experimental studies and develop recommendations, which will allow to do analysis and design of post-tensioned unbonded elements with considering the main features of their behavior.

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