Variable Speed of Light Cosmology and Bimetric Gravity: An Alternative to Standard Inflation

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Abstract

A scalar-tensor bimetric gravity model of early universe cosmology is reviewed. The metric frame with a variable speed of light (VSL) and a constant speed of gravitational waves is used to describe a Friedmann-Robertson-Walker universe. The Friedmann equations are solved for a radiation dominated equation of state and the power spectrum is predicted to be scale invariant with a scalar mode spectral index $n_s = 0.97$. The scalar modes are born in a ground state super-horizon and the fluctuation modes are causally connected by the VSL mechanism. The cosmological constant is equated to zero and there is no significant dependence on the scalar field potential energy. A possible way of distinguishing the bimetric gravity model from standard inflationary models is discussed.

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1 Bimetric Gravity Theory

In the scalar-tensor bimetric gravity theory, the metric of spacetime is given by [1, 2, 3, 4]:

\[ \hat{g}_{\mu\nu} = g_{\mu\nu} + B \partial_\mu \phi \partial_\nu \phi, \]  

(1)

where \( B = [\text{length}]^2 \) and \( \phi \) is a scalar field (biscal field). The metric \( \hat{g}_{\mu\nu} \) is called the ‘matter’ metric and \( g_{\mu\nu} \) is the gravitational metric. The biscal field \( \phi \) is a component of the gravitational field.

The model consists of a self-gravitating scalar field coupled to matter through the matter metric \( \hat{g}_{\mu\nu} \) with the action

\[ S = S_{\text{grav}} + S_\phi + \hat{S}_M, \]  

(2)

where

\[ S_{\text{grav}} = -\frac{1}{\kappa} \int d\mu (R[g] + 2\Lambda). \]  

(3)

Here, \( \kappa = 16\pi G/c^4 \), \( \Lambda \) is the cosmological constant, \( d\mu = \sqrt{-g} \, dt \, x \), \( \mu = \sqrt{-\hat{g}} \), \( \hat{\mu} = \sqrt{-\hat{g}} \) and \( c \) denotes the currently measured speed of light. We have the relation \( \mu = \sqrt{K} \hat{\mu} \) where \( K = 1 - B \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \). We use the metric signature \( \eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1) \) where \( \eta_{\mu\nu} \) is the flat Minkowski metric.

The minimally-coupled scalar field action is

\[ S_\phi = \frac{1}{\kappa} \int d\mu \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right]. \]  

(4)

From this action we can derive

\[ \hat{g}^{\mu\nu} \hat{\nabla}_\mu \hat{\nabla}_\nu \phi + K V'(\phi) = 0, \]  

(5)

where

\[ \hat{g}^{\mu\nu} = \hat{g}^{\mu\nu} + \frac{B}{K} \hat{\nabla}^\mu \phi \hat{\nabla}_\nu \phi - \kappa B \sqrt{K} \hat{T}^{\mu\nu}, \]  

(6)

and \( \hat{\nabla}^\mu \) denotes the covariant derivative with respect to \( \hat{g}_{\mu\nu} \).

\(^1\)Bekenstein [5] considered bimetric (disformal) gravity theories in the context of lensing effects in generalized gravity theories, not as variable speed of light cosmological theories.


## 2 Bimetric Gravity Cosmology

We can choose either $\hat{g}_{\mu\nu}$ or $g_{\mu\nu}$ to be comoving metric frames in an FRW universe. If we choose $\hat{g}_{\mu\nu}$ as the comoving metric, then

$$ds^2 \equiv \hat{g}_{\mu\nu}dx^\mu dx^\nu$$

$$= c^2 dt^2 - R^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right),$$

(7)

and

$$ds^2 \equiv \hat{g}_{\mu\nu}dx^\mu dx^\nu$$

$$= c^2(1 + \frac{Bc^2}{c^2 \dot{\phi}^2}) dt^2 - R^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right).$$

(8)

From (8) the speed of gravitational waves (gravitons) is given by $c_g(t) = c(1 - \frac{Bc^2}{c^2 \dot{\phi}^2})^{1/2}$, while the speed of light is constant. Alternatively, if we choose $g_{\mu\nu}$ as the comoving metric, we have

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu$$

$$= c_g^2 dt^2 - R^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right),$$

(9)

and

$$ds^2 \equiv \hat{g}_{\mu\nu}dx^\mu dx^\nu$$

$$= c^2(1 + \frac{Bc^2}{c^2 \dot{\phi}^2}) dt^2 - R^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right).$$

(10)

Now, the speed of light is given by the time dependent quantity $c_\gamma(t) = c(1 + \frac{Bc^2}{c^2 \dot{\phi}^2})^{1/2}$, while the speed of gravitational waves $c_g = c$ is constant.

In general relativity (GR), one can always perform a diffeomorphism transformation which removes the time dependence of the speed of light. In the bimetric gravity theory there are two speeds: one associated with the speed of light (photons) $c_\gamma$ in the VSL metric frame, and another with the speed of gravitational waves (gravitons) $c_g$ in the gravitational metric frame with the dimensionless ratio $\gamma = c_g/c_\gamma$. In contrast to GR, we cannot simultaneoulsy remove the time dependence in both $c_\gamma$ and $c_g$ by a diffeomorphism transformation. Thus, by regauging clocks, we cannot make both $c_\gamma$ and $c_g$
simultaneously constant. This makes the time dependence of either $c_\gamma$ or $c_g$ a non-trivial feature of the theory.

We will first consider the metric frame in which the matter metric $\hat{g}_{\mu\nu}$ is comoving. Then, the Friedmann equation is of the form

$$H^2 + \frac{c_g^2 k}{R^2} = \frac{8\pi G}{3} \rho_M + \frac{1}{3} c_g^2 \Lambda,$$

(11)

Here, the effective gravitational constant $\bar{G}(t) = G \left( 1 - \frac{B}{c^2} \dot{\phi}^2 \right)^{3/2}$ and the effective cosmological constant is

$$\bar{\Lambda} = \Lambda + \frac{1}{2} \left( \frac{1}{2c_g^2} \dot{\phi}^2 + V[\phi] \right).$$

(12)

When the speed of gravitons $c_g(t) \to 0$ in the early universe, then $\bar{G}(t) \to 0$ and the Friedmann equation (11) becomes

$$H^2 = \frac{1}{12} \dot{\phi}^2,$$

(13)

where we have set $\Lambda = V[\phi] = 0$. We can show by solving the wave equation for $\phi$ for a radiation universe with the equation of state, $p = c^2 \rho/3$, that $\dot{\phi}^2 \to$ constant as $t \to 0$, so that we obtain an initially inflationary solution $R \sim \exp(At)$ where $A = \dot{\phi}/\sqrt{12} =$ constant. Note that the derivation of initial inflation does not depend significantly on a choice of potential $V[\phi]$. The initial inflationary phase is produced by the assumption that the speed of gravitons becomes very small in the early universe.

We shall now choose the gravitational metric $g_{\mu\nu}$ as the comoving frame. In the $\hat{g}_{\mu\nu}$ matter metric frame (we call it the VSL metric frame) we have: $K(t) = 1/(1 + \frac{B}{c^2} \dot{\phi}^2)$, so that $c_\gamma(t) = c K^{-1/2}(t)$ and the speed of photons $c_\gamma(t) \to \infty$ as $K(t) \to 0$.

The matter stress-energy tensor (using $\hat{u}^0 = c$) is

$$\hat{T}^{00} = K \rho, \quad \hat{T}^{ij} = \frac{p}{R^2} \gamma^{ij},$$

(14)

and the conservation laws give

$$\dot{\rho} + 3H \left( \rho + \frac{p}{c^2} \right) = 0.$$  

(15)
We adopt the radiation equation of state \( p = \frac{1}{3} c^2 \rho \). It is useful at this point to introduce the following quantities derived from the constant \( B \): \( H_B^2 = \frac{c^2}{12B} \) and \( \rho_B = 1/(2\kappa c^2 B) \), where the latter comes from \( H_B^2 = (1/6)\kappa c^4 \rho_B \).

The Friedmann equation is given by

\[
H^2 + \frac{kc^2}{R^2} = \frac{1}{3} c^2 \Lambda + \frac{1}{6} \left( \frac{1}{2} \dot{\phi}^2 + c^2 V[\phi] \right) + \frac{1}{6} \kappa c^4 \sqrt{K} \rho. \tag{16}
\]

The scalar field equation is

\[
(1 - \kappa c^2 BK^{3/2} \rho) \ddot{\phi} + 3H(1 + \kappa B \sqrt{K} \rho) \dot{\phi} + c^2 V'[\phi] = 0. \tag{17}
\]

We postulate that in the very early universe \( K \) is very small, so that \( B \dot{\phi}^2 \gg c^2 \) and \( c_\gamma(t) \to \infty \). We will set \( V[\phi] = 0 \) and \( \Lambda = 0 \). This gives as \( K \to 0 \) the Friedmann equation

\[
H^2 + \frac{kc^2}{R^2} = \frac{1}{12} \dot{\phi}^2 \tag{18}
\]

and

\[
\ddot{\phi} + 3H \dot{\phi} = 0. \tag{19}
\]

Eq.(19) has the solution \( \dot{\phi} = \sqrt{12H_B} \left( \frac{R_{pt}}{R} \right)^3 \), where we have chosen the arbitrary constant of integration \( R_{pt} \) to parameterize the time at which \( K = 1/2 \). This gives \( K = 1/[1 + (R_{pt}/R)^6] \), so that it indicates the time at which the gravitational and matter metrics are close to coinciding. The subscript \( pt \) indicates that this is the ‘phase transition’, where standard local Lorentz symmetry with a single light cone will be restored—it is also the end of the period that will appear as inflation in the comoving gravitational metric frame.

The Friedmann equation is then

\[
H^2 = H_B^2 \left( \frac{R_{pt}}{R} \right)^6 \tag{20}
\]

and the biscalr field \( \phi \) dominates at early times yielding \( R \sim t^{1/3} \) and \( H \sim 1/3t \).

This is not an inflationary solution, and so would seem to not solve the horizon and flatness problems. We are working in the comoving gravitational
metric frame and the speed of light propagation is much larger than the speed of gravitational waves $c_g = c$. The coordinate distance that light would travel since the initial singularity is

$$d_H = c \int \frac{dt}{\sqrt{K R}} \propto \frac{1}{t^{1/3}},$$

(21)

where we used $R \sim t^{1/3}$. This diverges as $t \to 0$, showing that there is no matter particle horizon in spacetime. This solves the horizon problem and correlates fluctuations ‘born’ superhorizon. The flatness problem can also be solved, because $\Omega_\phi \to 1$ as $t \to 0$ (see ref. [6]) where $\Omega_\phi = \dot{\phi}^2/12H^2$.

### 3 Derivation of Primordial Scalar Power Spectrum

In the comoving gravitational metric $g_{\mu\nu}$, the significant decrease in the radiation density $\rho_r$ due to $K \to 0$ as $t \to 0$ gives a minimally-coupled Einstein-Klein-Gordon field, and so scalar mode fluctuations $\phi = \phi_0(t) + \delta\phi(t, \vec{x})$ about the background cosmological solution can be determined. Ignoring gravitational back-reaction, the equation for the perturbation of the biscal scalar field is

$$\frac{d^2 \delta\phi_{\vec{k}}}{dt^2} + 3H \frac{d\delta\phi_{\vec{k}}}{dt} + \frac{c^2 \vec{k}^2}{R^2} \delta\phi_{\vec{k}} = 0,$$

(22)

where $\delta\phi(\vec{x}, t) = (2\pi)^{-2/3} \int d^3k \exp[-ik \cdot \vec{x}] \delta\phi_{\vec{k}}$. The solution for our spacetime is given by Bessel functions

$$\delta\phi_{\vec{k}} = A(\vec{k}) J_0(\xi) + B(\vec{k}) Y_0(\xi), \quad \xi = \frac{ck}{R_{\text{pl}} H_B} \left( \frac{R}{R_{\text{pl}}} \right)^2.$$

(23)

Since $\xi > 1$ corresponds to the modes passing back inside the horizon at the present time, we see that in the early universe modes of interest satisfy $\xi \ll 1$. Thus, we cannot assume a scenario where the modes are ‘born’ in the quantum vacuum in the early universe. *Instead, we assume that the fluctuation modes are born ‘superhorizon’ in a ground state.*

Hollands and Wald [7] assume that modes are ‘born’ or ‘emerge’ from a fundamental description of spacetime, rather than prior to some time at
scales smaller than a length scale $\ell_0$. When the modes pass through the constant sound horizon: $c_\gamma(t)/H(t) \sim \sqrt{B}\dot{\phi}/H(t) \sim \text{const.}$ ($c_\gamma(t) \sim 1/t$ and $H(t) \sim 1/t$). The modes are described by a normalized plane wave of the flat spacetime wave operator

$$\delta \phi_k(t_k) = \sqrt{\frac{\kappa h c^2}{(2\pi R_k)^3 2\omega_k}} \cos(\omega_k t_k - \vec{k} \cdot \vec{x} + \delta), \quad \omega_k = \frac{ck}{R_k},$$

(24)

where $R_k = R(t_k)$ is the scale at which a mode is born.

The fluctuations born superhorizon are causally correlated by the large speed of light or superluminal VSL mechanism in the early universe.

If one assumes that one should use this as initial data, then we match not only the initial perturbation but also its time derivative. Doing so and keeping only the dominant contribution as $\xi \to 0$ gives

$$\delta \phi_k \approx \sqrt{\frac{9\kappa h c^2}{(2\pi R_k)^3 32\omega_k}} \cos(\omega_k t_k - \vec{k} \cdot \vec{x} + \delta) \ln(\xi_k) J_0(\xi_k),$$

(25)

where $\xi_k$ represents $\xi$ evaluated at $R = R_k$. The new contribution here results from the fact that when matching onto the initial state and its derivative the Bessel function $Y_0(z)$ is logarithmically divergent when $\xi \to 0$. The Hollands and Wald wave function did not have this additional logarithmic term, which will lead to slight deviations from a scale invariant spectrum.

We obtain the spectrum of scalar field fluctuations to be $P_{\delta \phi} = \frac{9}{2(2\pi)^3} \left(\frac{\ell_0}{\ell_p}\right)^2 \ln^2(\xi_k)$, and the curvature power spectrum is then found in the usual way (recall that $\dot{\phi}^2 = 12H^2$ in this spacetime):

$$P_R = \frac{H^2}{\dot{\phi}^2} P_{\delta \phi} = \frac{3}{8(2\pi)^3} \left(\frac{\ell_P}{\ell_0}\right)^2 \ln^2(\xi_k),$$

(26)

where we have used the Hollands-Wald condition $R_k = k\ell_0$ and $\xi_k = \sqrt{12B} \ell_0^3/2R_p^3$.

From this the spectral index is calculated to be $n_s = 1 + \frac{d\ln P_R}{d\ln k} = 1 + \frac{6}{\ln(\xi_k)}$, and the running of the spectral index is calculated from: $\alpha_s = dn_s/d\ln k$, from which we find the relation $\alpha_s = -\frac{1}{2}(1 - n_s)^2$.

In the large scale limit, we therefore have

$$\delta_H = 2\sqrt{\frac{P_R}{3}} \approx 2\sqrt{\frac{3}{8(2\pi)^3} \left(\frac{\ell_P}{\ell_0}\right)^2 |\ln(\xi_k)|}.$$  

(27)
4 Comparison with the Data

Assuming that $\ell_0 \approx \sqrt{2B}$ and evaluating the logarithm at the pivot point, $ck \sim 7R_0H_0$, this simplifies to $\delta_H \approx \ell_P/\ell_0 \approx 10^{-5}$, and so we can match the amplitude of the CMB fluctuations by assuming that $\ell_0 \approx \sqrt{2B} \approx 10^5 \ell_P$. This choice predicts $n_s \approx 0.97$, $\alpha_s \approx -4.5 \times 10^{-4}$. These results agree well with the WMAP data $n_s = 0.99 \pm 0.04$ [8].

We can predict the acoustical waves power spectrum and fit it to the corresponding WMAP data that can be detected at the surface of last scattering. By adopting the ΛCDM data from WMAP and our prediction $n_s = 0.97$, we obtain a fit to the data as good as the fit obtained from an inflationary model using an appropriate slow-roll potential for the inflaton.

An important piece of data obtained from the WMAP observations is the cross polarization E-T temperature power. Since our primordial fluctuations are born ‘superhorizon’ and are causally correlated by the superluminal VSL mechanism early in the universe, we obtain a good fit to the cross correlated polarization data. Again the fit to the data is competitive with inflationary model predictions. This data confirms that a superluminal ‘causal’ mechanism, such as that provided by a generic inflationary model or the VSL bimetric gravity model, is necessary to fit the E-T polarization data obtained by WMAP.

5 Conclusions

We have shown that the VSL mechanism in the bimetric gravity model, choosing the gravitational metric as comoving, predicts a scale invariant power spectrum with a spectral index $n_s = 0.97$, which agrees well with the WMAP data. The mechanism for producing this result is quite different from standard inflationary models, in which a ‘slow rolling’ potential is chosen from a large possible number of potential models [9].

The basic postulate is that $c_\gamma(t)$ becomes very large in the early universe, solving the flatness and horizon problems. It is possible that the VSL mechanism can avoid the initial fine-tuning problems of generic inflationary models.

A possible observational test to distinguish the bimetric gravity model from standard slow-roll inflationary models is to observe the tensor mode
(B-mode) and the ratio $r = T/S$ in the power spectrum. The predictions $r$ in the VSL bimetric gravity theory and generic inflation models may be sufficiently different in magnitude to decide which model agrees better with observations. However, the pure gravitational tensor modes have not been detected by WMAP.

In the VSL metric frame in the bimetric gravity model, we are not required to have a large vacuum energy in the initial phase of the universe, so that no fine tuning is needed to obtain the tiny cosmological constant that is favored by the WMAP and supernovae data. This is in contrast to inflationary models which require a very large vacuum energy in the early universe to generate enough e-folds of inflation.

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