Composite Slab Made from Precast, Pre-Tensioned Concrete Planks and Lightweight Concrete

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Abstract. New architecture trends consisting of designing large, space-free spaces, force structural engineers to develop new solutions that allow them to bridge large spans with relatively small thicknesses. Therefore, structural engineers are looking for new solutions and improvements to existing solutions both in the field of post-tensioned slabs as well as pre-tensioned, precast elements. In particular, precast concrete manufacturers offer many solutions of partially or fully precast slabs, each of which performs better in some situations and worse in others. The advantages of precast structures are increasingly noticed by all participants of the construction process, including producers who want to meet the requirements of the construction market for developing newer, better and more hybrid solutions, so that competition with traditional concrete technology is possible. In the field of the optimisation of existing precast slab solutions in Poland, work has been undertaken in recent years on the use of pre-tensioned concrete planks proposed a few decades ago as shuttering elements, which include a tension reinforcement zone in the slab. The author of this work proposes making a slab made of pre-tensioned concrete elements of low thickness for use as shuttering for reinforced concrete made of lightweight concrete, which is lighter than conventional, traditional concrete by around 30%. The resulting composite slab of precast elements would be relatively cheap with regard to transportation and assembly, which makes them competitive for the popular ribbed floor structures usually used in houses, whilst at the same time having the advantages of solid slabs, such as good mechanical and acoustic properties. In addition, they also compete with each other with regard to pricing, which is why they could be a good alternative for investors to choose. The work presents the concept of such a slab and an example of analysis.

1. Introduction
The technological advancements of recent decades have increased the importance of precast concrete structures in relation to in-situ structures. Investors, designers and contractors are increasingly aware of the many advantages of using precast concrete. Development in civil engineering is mainly the result of growing demands of society with regard to shortening the time and reducing the cost of building construction, while maintaining a high level of use. Floor slabs remain the subject of attempts to implement new solutions, which satisfy each of the members of the construction process. In addition, in modern architecture, a trend can still be observed to shape slabs of a small structural thickness and a large span, while meeting all functional requirements. Prestressed structures make it possible to meet these requirements – in this respect, the existing solutions in post-tension concrete [1, 2] and pre-tension concrete solutions [3, 4] are improved.
At present, many solutions are available on the construction market; each of them has certain advantages and disadvantages – one may be better in one regard, another may be better in a different manner. Among the precast or semi-precast slabs, two groups of products can be distinguished: beam-and-block slabs and large-size element slabs. The first group is a range of products consisting of beams (ribs) at spacing not exceeding 60 cm – the free spaces are filled with airbricks from various materials depending on the system (ceramics, derivatives of concrete or wood). Slabs of this type, although they are relatively easy to install, are time-consuming to erect, especially with complex shapes of buildings. In addition, research carried out at Cracow University of Technology [5] has shown that slabs are not rigid enough for long spans; this applies to slabs with a significant structural thickness of 30 cm for a span not exceeding 9.00 m. These slabs have a relatively high bearing capacity; however, cracking occurs with only a small load. This causes the long-term cost of exploitation to increase due to the reduced durability. An important disadvantage having a negative impact on the use of slabs in housing construction is inadequate acoustic properties.

The second group consists of large size elements, such as hollow-core or ribbed plates (TT). With the use of this type of slab, assembly is very fast, often it is not necessary to design topping and their durability due to the use of high quality materials is very high. Significant transportation and assembly costs are often a key factor for investors choosing other solutions.

A hybrid solution combining the advantages of both solutions may prove to be pre-tension concrete planks with a lightweight topping layer. In Poland, attempts have been made in recent years to optimise the solutions proposed a few decades ago for the use of small pre-stressed concrete elements as lost formwork and lower tensile reinforcement [6]. The low weight and dimensions of the element will significantly reduce transportation costs (up to 180 m² of floor in one transport operation) and assembly (no need to mobilize heavy equipment for lifting – each plank weighs around 450 kg).

The term ‘pre-stressed concrete plank’ is known from literature. It was first used in the 1950s by W. Grzegorzewski [7], who developed an element resembling the shape of a board used as lost formwork and lower reinforcement. After a period of laboratory testing and experiments, some full-scale bridge structures with spans of up to 10 m as tanks for liquids, floor slabs and retaining walls were made. In the context of using pre-tensioned boards for concrete floors, the most similar solution is the version of the prestressed planks and prestressed concrete elements acting as secondary reinforcement. Unfortunately, due to the fact that in those days, both the material technology and the elements themselves, as well as the inaccuracy of computational models, were not sufficiently developed; thus, the production of elements was abandoned. Nevertheless, the need from the market for the application of innovative and optimal solutions may in the near future be a reason to reactivate a new composite floor system.

2. The proposed new type of slab

The composite slab which is the subject of this analysis, works on the basis of the simplest known static scheme, namely, the simply supported beam. This fact underlines the additional advantage of such a solution – the simplicity of analytical calculations without the need to use advanced software. Precast elements fulfil a dual role – they act as formworks for the in-situ topping layer and they function as bottom reinforcement in transient and persistent design situations. The calculation assumptions adopted in the analysis are presented below:

- span length: 8.00 m
- cross section: h = 0.08 m, b = 0.30 m
- precast concrete: C50 / 60 on basalt aggregate with aggregate diameter 4/8 mm, cement of strength class and type – CEM I 42.5R
- concrete of the casting layer on the construction site: lightweight concrete LC30 / 33 class based on Pollytag aggregate and 32.5N cement class, 18% aggregate humidity, aggregate diameter 6/12 mm, density class – 1.8
- prestressing steel: 7Φ4 Y1860 strands
- fire resistance: REI60
category for imposed loads: domestic, residential areas

Figure 1. The proposed composite slab

Figure 1 presents the concept of the designed slab. Planks with a width of 30 cm are proposed, prestressed with two strands $7Φ4$ – it seems reasonable that this is the optimum width for the purposes of relatively quick assembly and low element weight. Nevertheless, the production of elements with a smaller width (15 cm) or larger width (45 cm) can be considered as an alternative solution depending on the area necessary for covering and the possibility of mobilising the heavy equipment used for vertical lifting. The depth of the plank should be not less than 5 cm due to the fire resistance analysis. Axial prestress would be the most beneficial. The introduction of the prestress eccentricity results in a slightly beneficial effect.

In order to reduce self-weight of the slab, the proposed topping was a lightweight structural concrete (density class 1.8) – this constitutes a reduction of dead load by about 30% while maintaining the same strength parameters as normal concrete. In order to protect against excessive shrinkage, the topping layer should be reinforced with $Φ8$ bars and with spacing in both directions not exceeding 30 cm (bottom and top).

2.1. Fire resistance

Residential buildings should be designed to meet the fire safety regulations, usually it is REI60 class. The European standard [8] used for designing concrete structures in an accidental situation defines a base concrete cover value of 20 mm for the analysed fire resistance. This value should be increased by a $Δa$ correction coefficient that equals 15 mm for prestressed structures. By introducing the prestressing force axially, the smallest possible thickness of the plank would have to be 70 mm – 40% more than initially assumed. A detailed analysis of limiting the use of steel in fire conditions can reduce deviation. The maximum stresses in prestressing steel can be calculated in accordance with formula 5.2 of the standard [8]:

$$\sigma_{p,fi} = \eta_β \cdot \frac{f_{pk} \cdot (20^\circ \text{C}) \cdot A_{p,req}}{\gamma_p \cdot A_{p,prov}}$$

(1)

where:
- $\eta_β$ – reduction coefficient of load level design in fire conditions, equal to 0.56, according to 2.5 [8]
$f_{pk}$ (20°C) – yield strength of prestressing steel equal to 1860 MPa

$\gamma_p$ – safety coefficient of prestressing steel, in fire conditions equal to 1.0

$\frac{A_{p,\text{req}}}{A_{p,\text{prov}}}$ – the ratio of the area of prestressing steel that is required to the area of prestressing steel that is provided is 0.60, which corresponds to the ratio of maximum stresses in prestressing steel in design situation $\sigma_p$, and characteristic prestressing steel strength $f_{pk}$. The value of maximum stresses in prestressing steel $\sigma_{p,\beta}$ of 543.4 MPa was obtained.

Based on the value of $\frac{\sigma_{p,\beta}}{f_{pk} (20^\circ C)}$ the critical temperature $\theta_{cr}$ was assumed to be 450°C. This assumption allows the determination of the value of $\Delta a$ according to the equation (2):

$$\Delta a = 0.1 \cdot (500 - \theta_{cr}) = 5\text{mm},$$

Detailed calculations allowed a reduction in the distance of the prestressing strand from the edge of the precast plank to 25 mm, which in effect made it possible to meet the initial assumption of the thickness of the precast element at 50 mm. It should be emphasised that such a procedure is possible in the situation of limiting the final stresses in prestressing steel to 0.60$f_{pk}$.

2.2. The benefits of using lightweight concrete

In typical design considerations, it is not usual to consider anything other than normal concrete types with strength classes between C12/15 and C50/60. The development of technology has made alternative types of concrete more popular in the world. Standard [9] allows the application of lightweight aggregate concrete in structures. As shown by calculations and tests [10], lightweight concrete is as good as normal concrete.

The lightweight concrete is classified by the standard as a concrete with a closed structure and a density of not more than 2200 kg/m$^3$ consisting of or containing a proportion of artificial or natural lightweight aggregates having a particle density of less than 2000 kg/m$^3$. With the use of mineral aggregate, it is possible to meet the requirement of a minimum compressive strength of 15 MPa. Due to the development of production technology and production of aggregates, the price of this type of material is becoming more attractive. The result of this phenomenon is an increase in the number of buildings erected in the world in which normal concrete has been replaced with a lightweight version. The benefits of using lightweight concrete include:

- geometric reduction of the cross-section;
- increasing the span length of the structure;
- good resistance to fatigue and dynamic loads;
- the possibility of increasing the imposed loads;
- better thermal insulation.

The analysis assumes concrete with a density class of 1.8, which corresponds to a weight of 1800 kg/m$^3$. Thus, the self-weight of topping was reduced by 30% compared to normal concrete. The key parameters in the analysis are the tensile strength, which for the adopted class LC30/33 is 3.6 MPa, and the modulus of elasticity which is 21.4 MPa. It has been computed that there will be no cracking at the joint between concrete cast at different times and the final sag will not be exceeded in any phase of construction, which ultimately confirms the correct use of lightweight structural concrete as an alternative to normal concretes.
3. Results and discussions

3.1. Stress limitation

The ceiling subjected to the computational analysis was designed on the basis of the following five phases of construction/usage:

- transmission of the prestressing force;
- assembly phase;
- composite cross section;
- beginning usage;
- design working life of 50 years.

In the assembly phase, the load was assumed as the weight of the plank (0.40 kN/m) resting on the temporary supports spaced at 1/3 and 2/3 of the total slab length and the weight of wet concrete (0.8 kN/m). After the concrete reaches its full capacity, it is time to remove the temporary supports – when this is done, the structure is able to function as a simply supported slab. When usage of building starts, additional loads from the weight of the finishing layers (0.20 kN/m) and the imposed load (0.6 kN/m) should be considered. The values above are given as the characteristic load on the width of one board (30 cm). Figure 2 shows the individual static schemes together with the points of stress verification (cross sections A-A, B-B).

![Static scheme of prestressed plank](image1)

**Figure 2.** Static scheme of prestressed plank: a) assembly phase, b) composite slab

Figure 3 presents stress increments in the precast element in subsequent phases of the life of the structure in the most reliable cross sections due to the stress limitation (A-A and B-B). The calculations also take into account loss due to relaxation, shrink and creep – this is equal to 2.8 MPa over the fifty-year period of use.
The initial prestressing force applied to the cross section is \( P_0 = 275 \text{ kN} \). Immediate and time dependent losses are 11.2% (\( P_{m0} = 244.2 \text{ kN} \)) and 16% (\( P_{mt} = 200.1 \text{ kN} \)), respectively. As a result of compression in the cross-section, normal stresses are 16.2 MPa.

As a result of the difference of shrinkage between concrete cast at different times periods, the following values of shrinkage were calculated:
- prestressed plank at the time of assembly (after 30 days) – 0.026%
- prestressed plank at the end (after 50 years) – 0.062%
- topping at the end (after 50 years) – 0.064%

The different shrinkage causes an additional bending moment and normal force which in the fifty year-period of use causes additional stresses in the bottom and top edge of the topping (0.9 MPa and -1.3 MPa, respectively) and bottom and top edge of plank (1.9 MPa and -0.1 MPa, respectively).

It is possible to determine the mean increase in compressive stresses – this is 6.5 MPa. Therefore, after taking into account the modified cross-section properties through the ratio of modulus of elasticity, \( \frac{E_p}{E_{cm}} = 4.6 \). As a result, the total stresses in the steel are:

\[
\sigma_p = \frac{P_{mt}}{A_{p,prov}} + \Delta \sigma_p = 1075.8\text{MPa} + 4.6 \cdot 6.5\text{MPa} = 1105.7\text{MPa}
\]

The use of prestressing steel in a final usable situation is therefore \( \frac{\sigma_p}{f_{pk}} = 0.59 \) – it is considered that the work condition of the element under fire conditions requires limitation of steel stress to a maximum level of 60%. The assumed level of steel stress was not exceeded.

After analysing the stress diagrams shown in Figure 3, it can be observed that there is no risk of cracks in any of the cross sections. The maximum tensile stresses occurring in a topping of 2.8 MPa are lower than the tensile capacity of lightweight concrete. In addition, in the most intense section (centre of the board, section A-A) they reach an extreme value and decrease in the support. However,
the most vulnerable section of the plank (centre of the span, Figure 2) is permanently compressed. The value of stresses in the bottom fibre of the precast element is 1.7 MPa.

3.2. Deflection control
In this type of relatively slender composite slab, deflection control is the second important limit state. The limit of the deflection is $L/250$, which is equal to a maximum sag of the slab of 32 mm. Deflection was calculated with the following assumptions:
- the cross section is uncracked;
- the values of the cross section have been modified;
- the plank creep coefficient is $\Phi = 2.2$ and the topping creep coefficient is $\Phi = 2.5$;
- there is elastic deflection and creep deflection for a quasi-permanent combination of interactions for each working phase.

After a detailed calculation, including different phases of the building life, the total long-term deflection for the analysed situation is 31 mm and it is less than $L/250$; therefore, the deflection limit state is not exceeded.

3.3. Shear at the interface between concrete cast at different times
The key assumption increasing the cost-effectiveness of planks in relation to other types of slabs is the lack of reinforcement between concrete cast at different times. For the calculations, the most unfavourable situation with the smooth surface was taken according to standard [9] and according to formula 6.24 [9], the maximum shear stress in the contact surface is:

$$V_{Edi} = \beta \cdot \frac{V_{Ed}}{z \cdot b_z}$$

where:
- $\beta$ – the ratio of the longitudinal force in the new concrete area and the total longitudinal force either in the compression or tension zone, both calculated for the section considered (1.0)
- $V_{Ed}$ – the transverse shear force (10.3 kN)
- $z$ – the lever arm of the composite section (140 mm)
- $b_z$ – the width of the interface (300 mm)

The bearing capacity without reinforcement is affected only by the type of surface and the tensile capacity of concrete, which is 0.20 for the smooth surface and 1.5 MPa for the topping, respectively. The maximum shear stress near the support is 0.24 MPa. The effort of the cross section is at a level of 80%. The result shows that the initial assumption was correct – there is no need for joint reinforcement.

4. Conclusions
The work presents, as a response to the need to develop new solutions that increase competitiveness to the existing solutions, the concept of a semi-prefabricated, solid light slab. The computational analysis shows that it is possible to make this type of structure. It should be emphasised that this is an innovative solution and requires significant refinement. All benefits for each participant in the construction process are achievable through the efficient and optimal management of every stage of the project.

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