Reliability of Hull Girder Ultimate Strength of Steel Ships

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Abstract. Hull girder ultimate strength is an evaluation index reflecting the true safety margin or structural redundancy about container ships. Especially, after the hull girder fracture accident of the MOL COMFORT, the 8,000TEU class large container ship, on June 17 2013, larger container ship safety has been paid on much more attention. In this paper, different methods of calculating hull girder ultimate strength are firstly discussed and compared with. The bending ultimate strength can be analyzed by nonlinear finite element method (NFEM) and increment-iterative method, and also the shear ultimate strength can be analyzed by NFEM and simple equations. Then, the probability distribution of hull girder wave loads and still water loads of container ship are summarized. At last, the reliability of hull girder ultimate strength under bending moment and shear forces for three container ships is analyzed by using a first order method. The conclusions can be applied to give guidance for ship design and safety evaluation.

1. Introduction

In the whole life time, container ships are mainly suffered to two kinds of loads, such as wave induced loads and still water loads. Wave loads are always changing and inconstant for different ocean areas. Even for the same ship, the draught, sea-going angle or stern-stem trim can also have effect on the wave induced loads on hull girder. Relatively speaking, the still water loads on hull girder should not have lots of fluctuation owing to fixed hull weight and cargo weight. However, each container weight is not same as the declared weight at the port and even the gap between actual weight and declared weight is more than 7.5%, as pointed out in Class NK report [1]. So, the accurate and reasonable evaluation of wave loads and still water loads on hull girder is very important for the safety of container ships.

The ultimate strength of hull girder may, in principle, be calculated according to any selected computational method. Different analysis methods should be firstly compared with and the potential bias in the calculation should be properly covered in evaluation criteria. In addition, the uncertainties of yielding strength of hull girder members have much influence on ultimate strength. Geometrical dimensions can be modeled as deterministic values, as pointed out in CSR OT background document [2]. So, ultimate strength of hull girder should evaluate the probability distribution and analyze the reliability of actual ships under rules design.

Fang and Das [3] studied the relationship between the risk evaluation and structural reliability, and then reviewed the evolution of structural reliability applied to ship structures. The failure probabilities of damaged ship were obtained based on Monte Carlo simulation technique.
Hussein and GuedesSoares [4] studied the reliability of damaged tankers considering the increase in the still-water bending moment due to damage and the loss in ultimate strength. The sensitivity analysis showed that the ultimate strength and the corresponding uncertainty have the highest importance. The still-water bending moment and the wave-induced bending moment have almost the same importance.

Zhang and Tang [5] studied time-variant reliability analysis of FPSO hull girder considering corrosion based on statistics. The results not only provide the reliability under different corrosion conditions, but also do well for further inspection and maintenance research. The results provide necessary foundation for deciding inspection intervals and maintenance measures, which has practical sense to improve the general safety level of ocean engineering.

Gao [6] summarized common crack types of container ships and analyzed the crack effect on the residual ultimate strength. Based on the numerical results, simple equations are fitted to predict residual ultimate strength of cracked container ship. And also assuming the corrosion additions ruled by classification societies ABS, CCS and DNV as the most serious corrosion, the corrosion effect on residual ultimate strength of container ship was analyzed.

Firstly, this paper discusses different methods of calculating hull girder ultimate strength. The traditional increment-iterative method (hereafter abr. as Smith), firstly proposed by Smith [7], is used to calculate the hull girder ultimate strength of oil tankers. However, container ship has special characteristics, such as large opening in cargo hold and extreme thick plating of higher strength in upper deck and coaming. The correctness of Smith method applied on container ships ultimate strength should be verified by nonlinear finite element method (NFEM).

Secondly, container ships always suffer to large shear forces due to non-uniform weight distribution and therefore it is needed to check the hull girder ultimate strength under shear. A simple equation of hull girder shear ultimate strength, which has considered shear buckling strength of each plating in hull cross section proposed in GL rules [8], is compared with the results by NFEM.

Then, this paper summarizes the probability distribution of hull girder wave loads and still water loads of container ship based on available data. At last, the reliability of hull girder ultimate strength under bending moment and shear forces for three container ships is analyzed by using a first order method.

2. Analysis of bending ultimate strength

2.1. Nonlinear Finite Element Method (NFEM)
NFEM is a powerful tool to solve the problems of complex engineering structures. Hull ultimate strength analysis involves material nonlinearity and large deformation nonlinearity, hull stiffness decreases as external loads increase, so the NFEM in ABAQUS software is chosen to analyze the ultimate strength of container ship and also track the hull collapse modes after the ultimate strength.

The steel in container ships is assumed as a kind of perfect elastic and small plastic harden material, so the lower limit of ultimate strength from material view can be obtained. The elastic modulus of steel is 206GPa and the Poisson’s ratio is 0.3. The middle parallel body of container ship is analyzed and one frame length of hull girder is fabricated in FE model.

In the analysis, the hull structures of container ship can be seemed as thin-walled structures, which are suitable for using the plane stress element, such as S4R element, to build a finite element model. S4R is a kind of 4 nodes shell element, has 6 freedoms at each node, adopts reduced integration to avoid volume self-locking, and also considers finite membrane strain and flexible rotation. The mesh size in the middle frame is about 100mm.

Simply supported conditions are applied on the both ends of FE model. In order to apply boundary conditions, point A of one end and point B of another end are separately created at intersection point of neutral axis. The cross section nodes of one end are constrained by point A, and another end is constrained by point B.
Initial imperfection of first buckling mode of each panel is applied in FE model to trigger the most possible collapse model of hull girder. However, residual welding stress is not considered in this analysis.

Subjected to hogging or sagging moment, part of hull structures are in compression and other part are in tension. As the external loading increases, local buckling may occur in compression panels and then the strain energy in this region will diffuse immediately to neighboring structures. The hull girder ultimate strength under hogging moment is $1.87 \times 10^7$ kN-m, and the hull girder ultimate strength under sagging moment is $2.17 \times 10^7$ kN-m.

2.2. Increment-Iterative Method (Smith)

Using NFEM analyse hull ultimate strength will need plenty of time to fabricate models and do calculations. The most effective method to avoid this problem is to reduce the number of freedom during calculation process, that is to say, decrease the matrix order of finite element stiffness. One kind of method is using large structure element, such as stiffener element, stiffened plating element and hard corner element. The cross section of ship will be divided into a series of separate structure elements, which are considered to act independently and fail in their own damage modes. In order to assure the effectiveness of this method, the structure element should be reasonably divided and their damage mode should be accurately defined.

The moment–curvature curve is obtained by means of an incremental–iterative approach based on the principles of Smith’s method [7]. The bending moment that acts on the hull girder cross section increases due to the imposed curvature $k_i$ for each step of the incremental procedure. This imposed curvature corresponds to an angle of rotation of the hull girder transverse section about its effective horizontal neutral axis, which induces an axial strain $e$ in each hull structural element. In the sagging condition, the structural elements below the neutral axis are lengthened, whilst elements above the neutral axis are shortened. As the external moment applied at two ends of the model increases, the curvature of hull cross section and neutral axis location will be calculated based on given damage mode of each structure element. This procedure will be repeated until the moment-curvature curve reaches the peak, which is corresponding to the hull ultimate strength value.

The software of Mars2000 provided by BV Classification Society adopts the Smith method to analyze the ultimate strength of hull girder between two adjacent frames [6]. The cross section will be divided into two kinds of structure element: stiffener attaching plating element and hard corner element. The former will present buckling or yielding damage mode when in compression; however, it will present only elastic-plastic damage mode when in tension. The latter, constituting plating crossing, collapses mainly according to elastic-plastic failure mode.

The model of target ship is fabricated in Mars2000. Based on the Smith method, the ultimate strength under hogging moment is $1.87 \times 10^7$ kN-m and the ultimate strength under sagging moment is $1.89 \times 10^7$ kN-m.

2.3. Verification of analysis method of bending ultimate strength

The results of hull ultimate strength of the target ship under bending moment, obtained by NFEA and Smith method, are summarized in table 1. The hogging ultimate strength by the two methods shows a good consistence. However, the scatter of sagging ultimate strength is bigger than the hogging. This may be because of different inelastic buckling strength for upper thick plating obtained by various analysis methods. The thickness of upper deck in container ship is 68mm, which is more than 2~3 times common hull plating. The deck has a much larger elastic buckling stress than the yielding stress and will present an inelastic buckling when the hull reaches the sagging ultimate state. Generally, the smith method is conservative in analyzing bending ultimate strength of container ship.
Table 1. Bending ultimate strength of target container ship. [kN·m]

| Load case | NFEA  | Smith | NFEA/Smith |
|-----------|-------|-------|------------|
| Hogging   | 1.87×10^7 | 1.87×10^7 | 1.00       |
| Sagging   | -2.17×10^7 | -1.89×10^7 | 1.15       |

3. Analysis of shear ultimate strength

3.1. Nonlinear Finite Element Method (NFEM)

Based on the FE model described in Sec. 2.1, the shear ultimate strength of the hull girder can be analyzed by NFEM. However, the boundary conditions should be changed to facilitate the application of shear loads and shear deformation on hull girder. One end of the hull girder is constraint all three translation degrees and one rotation degree, UX, UY, UZ and URX. At the other end, point A is created at intersection point of neutral axis, which is used to link the cross section nodes of the end.

As described in Sec. 2.1, initial imperfection has some influence on reducing hull girder ultimate strength. The shear buckling mode along the diagonal line in stiffened panel may bring about the greatest reduction on shear ultimate strength. However, the compression buckling mode, that is a wave pattern along the length direction, may also reduce the shear ultimate strength. Considering the initial imperfection only triggering the most probable failure mode of hull girder, the lateral deformation has a bigger influence. So the compression buckling mode as Sec. 2.1 is used to calculate the shear ultimate strength.

As the external shear load increases, the structures in side shell and inner longitudinal bulkhead of hull structures present shear buckling along diagonal line in each panel. Local buckling only occurs in compression direction and then the strain energy in this region will diffuse immediately to neighboring structures. However, the upper deck and double bottom show low stress level. The hull girder ultimate strength under shear is 5.41×10^5 kN.

3.2. Simple equation

Compared with bending ultimate strength of hull girder, the shear ultimate strength is a much simple issue. Only the vertical plating in hull girder is seemed as effective under shear load, that is, the side shell, inner longitudinal bulkhead, bottom girder and bilge plating. The other structures including stiffener and deck/bottom plating can be excluded in calculating shear ultimate strength.

For each stiffened panel, the shear ultimate strength is equal to the inelastic buckling strength and only the vertical plating is effective for shear load, but the stiffener can always be kept straight and non-deformed. So the boundary conditions of each panel can be kept unchanged during the increase of external hull girder shear load. From this point, each panel can seemed to be independent on resisting shear force. The summation of shear inelastic buckling strength of each panel should be equal to the hull girder shear ultimate strength. The rules requirements provided by GL Classification Society use the same idea to verify hull girder shear ultimate strength of Bulk Carriers, Oil Tankers and Container Ships [8].

It should be noted that the bilge plating in connection of side shell and outer bottom belongs to strengthen longitudinal structures, which should be included in calculating shear ultimate strength by using projected area to the vertical plane. The shear ultimate strength of hull girder can be calculated by (1).

\[ F_U = \frac{1}{\sqrt{3}} \cdot 10^{-3} \cdot \sum_{i=1}^{n} C_i \cdot b_i \cdot t_i \cdot R_{thi}, \text{ kN} \]  \hspace{1cm} (1)

where,

- \( n \) —— plating number for resisting shear load;
- \( C_i \) —— \( i \)th plating reduction factor for shear buckling, referred to CSR rules;
- \( b_i \) —— \( i \)th plating breadth, mm;
3.3. Verification of analysis method of shear ultimate strength

The results of hull ultimate strength of the target ship under shear load, obtained by NFEA and simple equation, are summarized in table 2. The shear ultimate strength by the two methods shows a good consistence. It proves that the simple equation described in Sec. 3.2 can be used to analyze shear ultimate strength of container ship.

Table 2. Shear ultimate strength of target container ship. [kN]

| Method       | NFEA   | Simple equation | NFEA/ Simple equation |
|--------------|--------|-----------------|------------------------|
|              | 5.36×10^5 | 5.41×10^5       | 0.991                  |

4. Reliability analysis of ultimate strength

The limit states are conditions that will cause a hull girder to experience performance failure and ultimate strength is corresponding to the limit state of hull girder. The limit state designs consider various conditions/factors under which the hull girder may fail to function, and account for the uncertainties associated with determining the safety margins. Reliability analysis of ultimate strength is able to evaluate the uncertainties, including still water loads, wave loads and ultimate strength for container ships.

The reliability of hull girder bending ultimate strength of container ship can be calculated by inducing the following limit state equation [4].

\[ g(x) = X_R M_u - [X_{st} X_{nl} M_w + X_{sw} M_{sw}] \]

where,

- \( M_u \) is the bending ultimate strength with a model uncertainty factor \( X_R \);
- \( M_w \) is the wave bending moment with model uncertainty factors;
- \( X_{st} \) is the linear response calculation and \( X_{nl} \) for non-linear effects;
- \( M_{sw} \) is the still-water bending moment with a model uncertainty factor \( X_{sw} \).

Based on technical background of Bulk Carriers and Oil Tankers Common Structure Rules [9], the values of the above mentioned uncertainties are taken as presented in table 3.

Table 3. Distribution of uncertainty factors.

| Uncertainty factor | Distribution | Mean, \( \mu \) | Standard deviation, \( \sigma \) |
|--------------------|--------------|----------------|-------------------------------|
| \( X_R \)          | Normal       | 1.05           | 0.1                           |
| \( X_{st} \)       | Normal       | 1              | 0.1                           |
| \( X_{nl} \)       | Normal       | 1              | 0.1                           |
| \( X_{sw} \)       | Normal       | 1              | 0.1                           |

For hull girder of container ships, the bending moment is an integral of shear forces in each cross section along ship length and the ship length can be seemed as a fixed value. Therefore, the model uncertainty factors for shear forces can be assumed the same as bending moment. The limit state equation of hull girder shear ultimate strength of container ship can be described as the following,

\[ g(x) = X_R F_u - [X_{st} X_{nl} F_w + X_{sw} F_{sw}] \]

where,

- \( F_u \) is the shear ultimate strength with a model uncertainty factor \( X_R \);
- \( F_w \) is the wave shear force with model uncertainty factors;
$X_e$ is the linear response calculation and $X_n$ for non-linear effects;
$F_{sw}$ is the still-water shear force with a model uncertainty factor $X_{sw}$.

### 4.1. Stochastic model of ultimate strength

In general, the following are considered as uncertainties on hull girder ultimate strength, including: Scantlings of structural members, Structural arrangements, Structural details, Yield stresses of steel, Effect of welding residual stress, Effect of initial imperfection, Lateral loads, such as sea pressure and container loads.

Among these factors, yield stress of steel has the greatest influence on the ultimate strength. Therefore, only yield stress is seemed as variables, and the other factors are assumed as definite factors, which are taken in consideration on calculation model to estimate the hull girder ultimate strength of container ships. Based on studies in CSR OT [2], the distribution of yield stress for different grade steel can be approximated as lognormal and their probability characteristics are summarized in table 4.

| Yield stress | Mean, $\mu$ | Standard deviation, $\sigma$ | Coefficient of variation, COV |
|--------------|-------------|------------------------------|-------------------------------|
| ReH=235     | 269         | 21.52                        | 0.08                          |
| ReH=315     | 348         | 20.88                        | 0.06                          |
| ReH=355     | 391         | 21.66                        | 0.06                          |

The distribution of ultimate strength, only the factor of yield stress is seemed as variable, so the ultimate strength can also be roughly assumed as lognormal distribution. The mean value of bending ultimate strength of container ship hull girder is calculated by Smith method as validated in Sec. 2 and the mean value of shear ultimate strength is calculated by (1) as validated in Sec. 3. The COV of ultimate strength are conservatively taken as 0.08 [10], the maximum value of yield stress COV. The stochastic model of all the ultimate strength for the three container ships is summarized in table 5.

|                      | Hogging [kNm] | Sagging [kNm] | Shear [kN] |
|----------------------|--------------|--------------|------------|
|                      | $\mu$        | $\sigma$     | $\mu$      | $\sigma$   | $\mu$      | $\sigma$   |
| Container ship 1     | 1.18E+07     | 9.44E+05     | 1.19E+07   | 9.52E+05   | 3.97E+05   | 3.18E+04   |
| Container ship 2     | 1.57E+07     | 1.26E+06     | 1.56E+07   | 1.25E+06   | 4.21E+05   | 3.37E+04   |
| Container ship 3     | 1.85E+07     | 1.48E+06     | 1.72E+07   | 1.38E+06   | 5.41E+05   | 4.33E+04   |

### 4.2. Stochastic model of wave loads

The wave loads can be based on the use of RAOs for the container ship, and follows the procedure described in the IACS background document [9]. There is another method, which is based on the rule formulae for wave bending moment and wave shear force in IACS unified requirements. In this paper, the latter is chosen. The distribution of the extreme values of the annual wave loads is obtained as a Gumbel law [10].

The mean value and standard deviation of wave loads can be estimated by the following equation [11].

$$
\mu_w = \delta (\ln N + 0.577)
$$

$$
\sigma_w = 1.283\delta
$$

where,

$N$ is the number of wave cycle during ship operation, $10^8$;
\[ \delta = \begin{cases} 
M_{w\text{design}}/19 & \text{for bending moment} \\
F_{w\text{design}}/19 & \text{for shear force} 
\end{cases} \]

\( M_{w\text{design}} \) and \( F_{w\text{design}} \) is the rule formulae value in IACS unified requirements.

The stochastic model of all the wave loads for the three container ships is summarized in table 6.

|                  | \( \mu \) [kNm] | \( \sigma \) [kNm] | \( \mu \) [kN] | \( \sigma \) [kN] |
|------------------|-----------------|-----------------|-----------------|-----------------|
| Container ship 1 | 3.44E+06        | 2.32E+05        | 4.26E+06        | 2.88E+05        |
| Container ship 2 | 5.79E+06        | 3.91E+05        | 6.88E+06        | 4.65E+05        |
| Container ship 3 | 6.68E+06        | 4.51E+05        | 7.85E+06        | 5.30E+05        |

### 4.3. Stochastic Model of Still Water Loads

There are mainly two methods of statistics for still water loads. The first one is assuming that the still water loads can be seemed as normal distribution, and the mean value and standard deviation have some relationship with the allowable still water loads in IACS rule requirements for the target ships. For example, the mean value is equal to 60% of the allowable still water loads and the standard deviation is equal to 40% of the allowable. The second one is assuming that the mean value and standard deviation have some relationship with the maximum still water loads in loading manual of the target ships. For one year period of seagoing, the still water load of each loading condition can be fitted by normal distribution.

In reliability analysis of this paper, the still water moment and shear force is set as normal distribution, the mean value is assumed as the 70% of the maximum value in loading manuals, and the standard deviation is assumed as 20% of the maximum value in loading manuals. The stochastic model of all the still water loads for the three container ships is summarized in table 7.

|                  | \( \mu \) [kNm] | \( \sigma \) [kNm] | \( \mu \) [kN] | \( \sigma \) [kN] |
|------------------|-----------------|-----------------|-----------------|-----------------|
| Container ship 1 | 2.67E+06        | 5.35E+05        | 8.61E+04        | 1.72E+04        |
| Container ship 2 | 3.98E+06        | 7.95E+05        | 4.06E+04        | 8.12E+03        |
| Container ship 3 | 4.68E+06        | 9.37E+05        | 6.30E+04        | 1.26E+04        |

### 4.4. Reliability results

According to the stochastic models presented in tables 5–7 the reliability index of container ships under bending moment and shear force can be calculated by a first order method and the results are presented in table 8. The consistent reliability level for each load condition can be attributed to using the IACS rule value which is already calibrated to achieve certain level of reliability.

Generally, hogging condition is the most dangerous for container ship ultimate strength and shear condition is relatively safer. Compared with the results derived from other researches [12-14], the range of reliability index coincide with the general situation of ship reliability, which indicates the comparability of this study.

|                  | \( \beta \) | \( P(f) \) | \( \beta \) | \( P(f) \) | \( \beta \) | \( P(f) \) |
|------------------|-------------|------------|-------------|------------|-------------|------------|
| Container ship 1 | 3.90        | 4.82E-05   | 3.29        | 4.99E-04   | 6.20        | 2.74E-10   |
| Container ship 2 | 2.93        | 1.69E-03   | 3.99        | 3.34E-05   | 8.04        | 4.62E-16   |
| Container ship 3 | 3.00        | 1.36E-03   | 4.10        | 2.02E-05   | 7.84        | 2.18E-15   |
5. Conclusions
The results of hull ultimate strength of the target ship under bending moment, obtained by NFEA and Smith method. The hogging ultimate strength by the two methods shows a good consistence. Generally, the smith method is conservative in analyzing bending ultimate strength of container ship.

The results of hull ultimate strength of the target ship under shear load, obtained by NFEA and simple equation. It proves that the simple equation described in Sec. 3.2 can be used to analyze shear ultimate strength of container ship.

The stochastic models of ultimate strength, wave bending moment and shear force, and still water bending moment and shear is analyzed in this paper. Then, the reliability index of container ships ultimate strength under bending moment and shear force is calculated by a first order method. The reliability results show that hogging condition is the most dangerous for container ship and shear condition is relatively safer.

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