Energy Market Risk Management under Uncertainty: A VaR Based on Wavelet Approach

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ABSTRACT
This study contributes to the literature on energy market risk management and portfolio management by examining co-movements between several energy commodities in a portfolio context in light of the impact of several types of uncertainty over time and under high, medium, and low frequencies. Using of wavelet decomposition analysis, we first investigate the lead-lag relationship together with the power of the correlation over time between major renewable and non-renewable energy indexes and uncertainty indexes. Second, we explore the contribution of uncertainty to the energy portfolio. Our procedure reveals that a dependent relationship generally exists between energy returns and changes in uncertainty. The risks of clean energy and crude oil returns are more sensitive to financial uncertainties, whereas investing in GAS markets offers market diversification opportunities during periods of energy uncertainty.

Keywords: VaR Based on Wavelet Approach, Energy Market, Uncertainty
JEL Classifications: C580, G15, E440

1. INTRODUCTION

In recent years, investing in the energy markets has been a notable part of the lifeblood in production process and, thus, in the world’s economic systems and social development. This promising awareness of the role of the energy sector in economic growth, firms’ plans and household expenditure have stimulated investors’ enthusiasm in the capital markets (Liu et al., 2019; Narayan et al., 2017; Aloui et al., 2012). However, the short- and long-term relationship between energy market prices and their economic implications for financial market participants remains a perennial concern. This concern stems from the fact that energy prices cannot be fully explained in the framework of supply and demand because the driving factors behind the energy market are complex and diversified (Ji et al., 2018; Mellios et al., 2016 among others).

Broadly speaking, it is extremely difficult to ignore the existence of extreme risks because to uncertainty has several sources, including turbulent financial markets (Balcilar et al., 2016), climate change, periodical changes in the world economy (Baker et al., 2016) and geopolitical uncertainty (Aloui et al., 2016). Above all, much of the instability in energy investment returns may stem from the heavy capital demand made by energy projects, the long term nature of their production and the long period of cost payback (Balcilar et al., 2017; Bilgin et al., 2015; Kang et al., 2014). These have sharply increased the difficulty of making energy investment decisions.

From this viewpoint, several studies have documented that increased volatility in energy prices due to high volatility at the level of uncertainty greatly hamper the stability of the financial system and may even trigger systemic risk in the global financial markets (Mensi et al., 2017; Joëts, 2014). The complexity of these
volatile energy prices was amplified when the global financial crisis broke in 2008. According to the work of Zhang (2017), among others, the consequences of this crisis not only exerted a significant impact on the energy market situation but also greatly influenced the expectations of energy market investors. These particular data therefore add tremendous challenges to the task for energy investors of identifying an appropriate energy investment scheme.

At this juncture, it must be pointed out that the above situation impelled investors and portfolio managers to seek alternative ways of diversifying their portfolios and reducing risk. Such diversification can be obtained by considering several energy commodities in the same portfolio. In the literature, theoretical models explaining this form of energy portfolio diversification are based on multiple equilibria, endogenous-liquidity shocks causing a portfolio reshuffling and changes in exchange rate regimes, investor psychology, and capital market liquidity. For example, the price fluctuations for fossil energy can exert a powerful influence on the development of the renewable energy sector; they are especially relevant in capital markets to the level of investment in renewable energy and its returns. By contrast, the higher cost of developing forms of renewable energy can seem a sizeable threat when the prices of fossil energy are low. Realigning the weightings of these portfolio assets causes a sell-off of certain asset classes, which in turn lowers asset prices in assets not affected by the initial crisis. Hence, several studies have investigated the effects of diversification on energy portfolios (Francé et al., 2013; Muñoz et al., 2009; Huang and Wu, 2008, to name a few). Of note is the seminal paper of González-Pedraz et al. (2014), who begin by considering oil, gas, coal, and electricity in a portfolio context to evaluate tail risk measures for the portfolio’s profit-and-loss distribution.

To better understand the mechanisms of the dynamic relationship between uncertainties and the movement of energy prices, a number of authors have considered the effect of extreme uncertainty on energy price returns. In this regard, Jurado et al. (2015) argue that the volatility of the indicator rises when the overall economy is slowing down, that is, almost every indicator of uncertainty appears to be countercyclical (Barrero et al., 2017; Baker et al. 2016). Thus, recent investigators (e.g. Lucheroni and Mari, 2017; Ji et al., 2018; Ma et al., 2019) report substantial benefits from including the measurement of uncertainties when considering traditional energy portfolios.

A number of authors have considered the effects of both short-term and long-term components of various uncertainties measurements. As noted by Nalebuff and Scharfstein (1987), the economic cycle is not constant over time among asset returns and can generate asymmetric information. This is further supported in the seminal work of Barrero et al. (2017), where the authors emphasise that investing in the energy market (particularly in fossil fuels) is more sensitive to short-term economic uncertainty, while policy uncertainty is particularly related to long-term uncertainty. Mele et al. (2015) (Adrang et al., 2019, among others) draws our attention to the distinctive nature of financial uncertainties often observed in portfolio risk management. In this regard, Aloui et al. (2016) and Chen and Kettunen (2017) identify that higher economic and financial uncertainty measurement has not always increased oil returns.

This effective co-movement between uncertainty and energy prices in a portfolio setting can be used determine actual diversification investment opportunities, assess optimal hedging strategies, and in the prevention of contagion effects, although little evidence so far has been provided to verify the major co-movement of uncertainty on energy prices. What is clear is the importance of first considering how several energy commodities in a portfolio context co-move with different uncertainties measurements and second, how these co-movements differ in the short and long term despite the volatility and interdependence of the energy markets.

In specific terms, this study contributes to the literature on energy market risk management and portfolio management by examining co-movements between several energy commodities in a portfolio context and considering the impact of several types of uncertainty over time and under high, medium and low frequencies. To this end, the current research addresses the following questions, in turn: (i) Is there any extreme value dependence between energy commodities and different types of uncertainty? If so, (ii) is the dependence symmetric or asymmetric? Finally, (iii) can this dependence contribute to the risk reductions and downside risk reductions of extreme uncertainty movement on energy price returns?

Answering the above questions will contribute to the current literature in three main respects. First, we extend the current research by considering financial market and energy market uncertainty in a comprehensive analysis of the various diffusion channels through which uncertainty influences energy prices. In order to do so, we take two measures of uncertainty, namely, the implied volatility index (the VIX, henceforward) as a proxy for global financial market uncertainty and the crude oil volatility index (OVX) as a proxy for energy market uncertainty. VIX and OVX were chosen in order to compare their distinct influences on both fossil (crude oil and GAS) and clean energy prices.

Second, consensus literature on portfolio management suggests that variations in energy seasonal demands influence the time-varying trend of energy prices and that exposure association over time provides important information on the risk profile of a portfolio over varying horizons (Shao et al., 2015). Accordingly, this paper applies an empirical methodological framework based on a wavelet approach to account for the presence of potential frequency changes over time. The wavelet method encapsulates both short-term speculators and long-term investors whose expectations are time-frequency dependent. The multi-resolution decomposition of the wavelet transform allows us to identify spillovers, contagion and interdependence (Mensi et al., 2018).

Finally, several portfolios (a risk-minimizing portfolio, an equally weighted portfolio, and a hedging portfolio) were considered to assess the risk reductions and downside risk reductions of extreme uncertainty movement on energy price returns. Of more interest, this study provides a new analysis tool for financial investors and risk managers seeking
to control their trading risks during extreme periods by measuring the value-at-risk (VaR) of energy price returns conditional on the VaR of uncertainties at both short-term and long-term frequency.

The remainder of this study is organized as follows. Section 2 presents an overview of the econometric approach. Section 3 presents the data and provides the empirical results. Finally, Section 5 concludes the study.

2. THE ECONOMETRICS APPROACH

Using wavelets is a well-established technique that decomposes a time series into small waves which begin at a finite point in time and end at a later finite point in time. A significant advantage of this approach is that frequency information can be obtained without losing the timescale dimension. Another advantage of wavelet analysis is that it needs no assumptions about the data generating process for the return series under investigation. (Insightful development of the theory and use of wavelets can be found in Percival and Walden, 2000; Gençay et al., 2001).

A discrete signal of a time series \( Y(x) \in L^2 \) on \( T \)-dimension can be written as the sum of a scaling function \( \varnothing(t) \) representing the smooth baseline trend and wavelet function \( \psi(t) \) that together account for all deviations from trends, namely:

\[
f(t) = \sum_k S_k \varnothing_k(t) + \sum_j^\infty a_j \psi_j(t)
\]

From Equation (1), the wavelet function which spans the differences between two adjacent spaces can be given as

\[
\psi_{j,k}(t) = 2^{-j/2} \psi(2^{-j} t - k) 2^{-j/2}
\]

where \( j = 1, \ldots, J \) accounts for the resolution level that can capture the smooth components of the signal and \( k \) represents the applicable scale and translation parameters.

The orthogonal basis functions in Equation (1) are constructed by translating and dilating the wavelet into both time and scale dimensions. The resulting multi-scale decomposition can be simplified as

\[
f(t) = S(x) + D(x)_{i=1} + \cdots + D(x)_{i=6} + D(x)_{i=7} + \cdots + D(x)
\]

where \( D_i \) is the \( i \)th level wavelet and \( S_i \) present the aggregated sum of variations at each detail scale. For the purpose of this study, we used a compact Daubechies function of minimal asymmetry filter of length eight \([LA(8), \text{hereafter}\] to generate uncorrelated coefficients across scales. Following the seminal work in the field, this level of decomposition leads to six levels of wavelet scales \( D_i, \forall i = 1, \ldots, 6 \) representing the variations caused by shocks occurring on a timescale of 2 days. Moreover, \( S_i \) is the residue of the original signal after subtracting \( D_1, D_2, D_3, D_4, D_5 \) and \( D_6 \) in turn.

The co-movement between two-time series can then be examined using the Maximal Overlap Transformation (MOTWT).

Accordingly, for a defined stochastic process \( \tilde{W}_{j_1} = \sum_{t=0} L^{-1} h_{j_1} X_{t-1} \), the time-dependent wavelet variance at scale \( (j) \) of the signal for the obtained series \( \tilde{X} = x, y \) is given by

\[
\sigma^2_{x,y}(\lambda_j) = \text{var}\{\tilde{W}_{j_1}\}
\]

In addition, the wavelet covariance is defined as

\[
\sigma_{x,y}(\lambda_j) = \text{Cov}\{\tilde{W}_{X,j_1}, \tilde{W}_{Y,j_1}\}
\]

the wavelet correlation at scale \( (j) \) can be, then, estimated as

\[
\rho_{x,y}(\lambda_j) = \frac{\sigma_{x,y}(\lambda_j)}{\sigma_x(\lambda_j) \sigma_y(\lambda_j)}
\]

To account for the synchronicity of the series at certain periods and across certain ranges of time, the concepts of cross-wavelet analysis is adopted. This transform of two time series \( (x) \) (with their respective wavelet transforms \( \text{Wave}(x) \)) and \( (y) \) (with its own wavelet transforms \( \text{Wave}(y) \)) decomposes the Fourier co- and quadrature-spectra in the time-scale domain such that

\[
\text{Wave}_{xy} = 1/s \text{Wave}(x) \text{Wave}(y^*)
\]

The so-called phase difference of \( x \) over \( y \) at each localizing time origin and scale can be formulated as

\[
\text{Angle}(\tau,s) = \text{Arg}(\text{Wave}_{xy}(\tau,s))
\]

An absolute value less (greater) than \( \frac{\pi}{2} \) indicates that the two series move in phase (anti-phase) referring to instantaneous time as the time origin and at the frequency (or period) in question, while the sign of the phase difference shows which series is the leading one in this relationship.

3. DATA, EMPIRICAL RESULTS AND DISCUSSION

We consider the daily data for the S and P 500 Global Clean Energy Index (CEX), the crude oil prices (OIL) and the natural gas prices (GAS) covering the sample period from May 10, 2007 to April 13, 2017 (a total of 2591 observations). In order to capture the uncertainty, we consider the CBOE’s Implied Volatility Index (VIX) as a proxy for financial market uncertainty and the CBOE’s Crude Oil Volatility Index (OVX) as a proxy for energy market uncertainty. The return series for the energy indices are computed by taking the logarithm difference of the energy prices and uncertainty changes as measured by the difference in the uncertainty indicators.

3.1. Cross-Wavelet Transform

We use the cross-wavelet transform to investigate the dynamics of co-movement (leads-lags relationships) between energy commodity returns and both energy and financial uncertainties measurements,
with respect to time and frequencies. As shown in Figures 1 and 2, the left-hand horizontal axis is transformed to show the number of days taken for the scale to move from low to high wavelengths.

For ease of interpretation, the phase difference between studied series is indicated by arrows. More precisely, arrows pointing to the right and down (up) signify the leading (lagging) of the uncertainty index. By contrast, arrows to the left and down imply that the uncertainty index is lagging with an anti-phase series and arrows to the left and up track the uncertainty index when it is leading without-phase. It is worth noting that in-phase means that two series are affecting each other cyclically, while out-of-phase or anti-phase indicates that the studied series are affecting each other anti-cyclically.

An analysis of the results obtained from the wavelet coherence in Figure 1 clarifies the lead and lag relationships between the renewable and non-renewable energy commodities under consideration and the energy uncertainty index. Interestingly, the time horizon of interest is quite an important feature when it comes to evaluating the relationship between chosen variables.

During the period 2011 to 2019, the co-movement between the renewable energy index and energy uncertainty was most probably concentrated in the long-term scale band. To be specific, when the time scale is around 8-16 days, the arrows point to the left and up, indicating that the change in energy uncertainty leads a change in the clean energy index without their having an anti-cyclical effect on each other. When it comes to non-renewable energy, the index is surprisingly different: on the one hand, the energy uncertainty lags the change in the crude oil index with an anti-phase at around the 8-16 day period, since the arrows point to the left and down. On the other, the change in energy uncertainty leads the change in the GAS index without any anti-cyclical effect on either side. In periods longer than 32 days, the picture changes. There the arrows point to the left and down, indicating that the energy uncertainty is lagging the change in the clean energy index with anti-phase. Interestingly, the change in energy uncertainty leads the change in the clean energy index.

**Figure 1:** Cross wavelet transform over energy uncertainty. (a) Oil ~ energy uncertainty. (b) Gas ~ energy uncertainty. (c) Cex ~ energy uncertainty.
in the crude oil index when the time scale is longer, whereas the energy uncertainty leads the change in the GAS index with anti-phase.

The sensitivity of the energy commodities market to energy uncertainty makes it highly volatile, between 2020M2 and 2021M1, with arrows pointing left and up at around the 2–4 and 4–8 day time scales, suggesting a leading relationship without phase in the case of the clean energy and GAS indexes and lagging with anti-phase in the case of the crude oil index since the arrows point to the left and down. This lead-lag relation is interesting when the time is greater because the arrows are in the anti-phase, indicating that the clean energy and energy uncertainty are out of phase, while the arrows indicating the oil index and energy uncertainty are in phase. The arrows point right and up, indicating that the energy uncertainty lags the GAS returns.

The evolution over a longer scale from financial uncertainty to commodities resembles that observed for energy uncertainty but is more stable (Figure 2), with the arrows pointing left and down, corresponding to the period 2011 M01–2019 M10 on an 8-16 day scale fluctuation, and indicating that financial uncertainty lags the CEX and GAS indices with a phase shift. However, in the case of crude oil, the arrows point to the left and up, indicating that the financial shock leads to oil returns without phase. The interconnectedness in the system is much stronger in the long-run scale (32-64 and 64-128), with the arrows suggesting that financial shock leads to a change in clean energy prices. By contrast, the arrows point to the left and down, indicating that the financial shock lags both fossil fuels returns.

Turning our attention to the end of period corresponding to 2020 M01–2021 M01, it is apparent from Figure 2 that the behaviour patterns of the energy return are diverse in terms of response to the energy uncertainty index. It is clear that the financial uncertainty leads to a change in the short-term time scale for crude oil (around 2–8 days) since the arrows point to the right and down. Similarly, the shock leads the GAS change but without phase. However, in the case of clean energy, the shocks are lagging with the anti-phase shift. In the long-term scale, at the scale of 32 days and longer, the arrows are in anti-phase, indicating that the financial shock is out of phase with the GAS returns, while the arrows point to right and up suggesting that the shock lags the clean energy prices. The picture is different in the case of the GAS return, in which the arrows are in anti-phase.

In sum, the results from the cross-wavelet transform of returns connectedness seem to complement nicely the comments in the literature. The wavelet results suggest that the energy market returns were influenced strongly during the aggregate demand-side shocks, such as periods of financial turmoil (the COVID-19 crisis; the tussle between Russia and Saudi Arabia1).

3.2. Dynamic Wavelet Correlation

The analysis in the previous sections provides several insights into the leads-lags structure between renewable and non-renewable energy commodities returns and uncertainty indexes. In particular, the analysis in Section 3.1 provides an answer to the question: In what ways are the leads-lags of the x-energy return type conditional on the y-uncertainty index changes? In this section we focus on the question: How does the power of the correlation change over time?

First, to answer the former question, we estimate the time varying correlation between the renewable and non-renewable energy commodities returns and the uncertainty indexes, using the time-localized multiple regression model. These correlation patterns are presented in a time-frequency domain on a scale-by-scale basis. Therefore, in Figures 1 and 2 the correlation coefficients are calculated daily for each pair of energy commodities returns and the uncertainty index. For ease of interpretation, the heat maps indicate the increasing strength of the correlation as they move from blue (lowest correlation) to red (highest correlation).

In Figure 1 there is a clear difference in the correlation patterns with some markets performing better than others. For example, the renewable energy market in some specific periods appears to be less sensitive to energy uncertainty changes over a long horizon than the non-renewable one, since the correlation is roughly between 0.2 for CEX with uncertainty (Figure 1a), while the magnitude of the correlation is around 0.35 (Figure 1a and b). Note, however, that a contagion effect emerges at the pick of the Covid-19 outbreak after January 2020, as highlighted by the red colour in Figure 1, because positive high correlation was exhibited at this time.

Looking now at situations of financial uncertainty, the trajectories seem to display a similar trend in all cases, showing slight differences in magnitude across the markets. In specific, the correlation in the case of crude oil is the greatest of all the three energy return risks, while the correlation in the non-renewable one, since the correlation is roughly between 0.2 for the GAS index when the time scale is longer, whereas the energy uncertainty leads the change in the GAS index with anti-phase.

3.3. Implications for the Energy Portfolio

Having discussed the lead-lag relationship as well as the power of the correlation over time, we are now in a position to answer the question: What are the implications for risk management and portfolio construction strategies?

Following Fernández-Rodríguez et al. (2016) we investigate the impact of uncertainty on the energy portfolio based on Wavelet Value at Risk (WVaR), which is a robust market-based measure of systemic risk across energy markets of differing length

The (1–α)% value at risk (VaR) of an equally weighted energy portfolio of indexes at the -scale components can be given as

\[
VaR_\tau (\alpha) = V_0 \Phi(\alpha) \sqrt{\left( \sigma_\tau^2 \left( \tau_j \right) \right) \left( \sum_{i=1}^{k} \frac{\beta_i(\tau_j)}{k} \right)^2 + \frac{1}{k^2} \sum_{i=1}^{k} \sigma_i^2(\tau_j)}
\]

(9)

1 On 6th March 2020, Russia refused to comply with the decision to cut oil supplies made at the OPEC summit in Vienna on March 5. In response, on 8th March Saudi Arabia announced oil production increases and price discounts ranging from $6 to $8 per barrel for European and Asian customers.

2 To save space, we mention only the useful equations to our analysis. For in-depth details, interested readers may refer to the seminal work of Gençay et al. (2003).
where $\omega$ is a vector of portfolio weights, $V_o$ is the initial value of the portfolio, $V_0 1(\alpha) \equiv \phi^{-1}(1-\alpha)$, and $\phi(\cdot)$ is the cumulative distribution function of the standard normal.

According to Gençay et al. (2003), the wavelet-beta estimator for asset $i$, at scale $j$, can be defined as

$$
\hat{\beta}(\tau_j) = \frac{\hat{\upsilon}_{R_i} (\tau_j)}{\hat{\upsilon}_{R_u} (\tau_j)}
$$

where $\hat{\upsilon}_{R_i} (\tau_j)$ and $\hat{\upsilon}_{R_u} (\tau_j)$ are the wavelet variance and wavelet covariance of the portfolio at scale $j$, respectively.

The contribution of $j$-scale on total value at risk can then be given as

$$
\left( \frac{\sigma^2_m \left( \frac{\sum_{i=1}^{k} \beta_i (\tau_j)}{k} \right)}{\sum_{i=1}^{k} \sigma^2_i (\tau_j)} \right)^2 + \frac{1}{k^2} \sum_{i=1}^{k} \sigma^2_i (\tau_j)
$$

It is worth noting that we test the contribution of renewable energy in different types of portfolio in order to quantify the willingness to hedge against different types of uncertainty risk. Table 1 illustrates a 1-day horizon and a 95% confidence level. In specific, Panel A of Table 1 shows the case of a portfolio containing the non-renewable

**Figure 2:** Cross wavelet transform over financial uncertainty. (a) Oil ~ financial uncertainty. (b) Gas ~ financial. (c) Cex ~ financial uncertainty
indexes (GAS and OIL) along with the VIX financial uncertainty index. As expected, the value at risk generally decreases as the time-scale increases. Second, the contribution to total risk is higher in the lower scales. That is to say, potential portfolio losses are greater when the detailed components of the data are scrutinised. Finally, the contribution to VaR (CVaR, henceforward) suggests that the magnitude of the CVaR for oil is the greatest of all the energy return risks. A possible explanation for this may be that, when financial uncertainty changes are considered, the extreme oil return risks are greater than those of gas returns at a given time; that is, the oil market may be more sensitive to uncertainty changes than the GAS market at certain times.

It is of interest to compare this figure with that in panel B of Table 1 which explains what happens when the renewable energy index is included in the portfolio. In line with the results in Section 4.2, we observe that the VaR and CVaR trajectories display similar trends in all cases, revealing only slight differences in magnitude across the indexes. That is to say, renewable energy returns are less sensitive to extreme uncertainty changes in the financial markets. What is interesting about this result is that even in condition of extreme market distress, investing in renewable energy may play an important role in balancing portfolios.

The VaR represents the potential loss on a 1-day horizon for a 95% confidence level. (2) The VaR and the contribution to VaR at scale j are computed according to Equations (9) and (10), respectively, where scale 1: 2–4 days, scale 2: 4–8 days, scale 3: 8–16 days, scale 4: 16–32 days, scale 5: 32–64 days, and scale 6: 64–128 days.

Looking at the case of energy uncertainty in portfolio diversification, it appears from Table 2 that the time horizon under consideration is quite an important feature when it comes to evaluating the performance of energy indexes as portfolio stabilizers in times of energy market distress. It can be observed that GAS contributes least to the VaR even when investing in renewable energy is considered. This means that renewable energy and oil price returns are more sensitive to energy uncertainty changes in the energy markets, that is, increasing uncertainty tends to have a negative impact on these price returns. Moreover, the correlation patterns change over both the investment horizons and over time.

The VaR represents the potential loss on a 1-day horizon for a 95% confidence level. (2) The VaR and the contribution to VaR at scale j are computed according to Equations (9) and (10), respectively, where scale 1: 2–4 days, scale 2: 4–8 days, scale 3: 8–16 days, scale 4: 16–32 days, scale 5: 32–64 days, and scale 6: 64–128 days.

4. CONCLUSION AND POLICY IMPLICATIONS

The first question in this study was how to determine the value dependence between energy commodities and different types of uncertainty. In this regard, the results of wavelet coherence show that considering the time horizon is important in evaluating the relationship between energy commodities and uncertainty indexes. This importance appears in the changing behaviour of the series under study using different time scales. For instance, in periods of 8–16 days the findings show that the energy index leads the clean energy index with an anti-cyclical effect between them. However, the energy index appears to lag the crude oil index and lead the GAS index with anti-cyclical and cyclical effects respectively.

However, going beyond 32 days, the results show that the renewable energy index leads the change in the energy index with anti-phase. With respect to non-renewable energy indexes, the energy index turns to lead both the crude oil and GAS indexes but with an anti-cyclical effect on the latter.

In the case of financial uncertainty, the results show that financial uncertainty lags both clean energy and GAS indexes without phase. However, it leads the crude oil index without phase. Nonetheless, over a longer period (more than 32 days) this relationship changes: financial uncertainty leads the clean energy index. However, it lags both the crude oil and GAS indexes.

To sum up, the results of the lead-lag structure between the series under study confirm the presence of asymmetry over time. Moreover, the results show that energy market returns are influenced by the crises that occurred during the study period, such as the COVID-19 pandemic.

Moving to the dynamic wavelet correlation, the results of the heat map show that the clean energy index appears to be less sensitive to changes in energy uncertainty than the crude oil and GAS indexes. The case is similar for financial uncertainty with slight differences in the magnitude of correlation. Finally, the dynamic wavelet correlation results display a contagion effect during the COVID-19 pandemic period (starting in January 2020), thus confirming the results of the wavelet coherence analysis.

### Table 1: Optimal portfolio under financial uncertainty

| Portfolio 1 | D1 | D2 | D3 | D4 | D5 | D6 |
|-------------|----|----|----|----|----|----|
| 95% VaR     | 0.31 | 0.29 | 0.19 | 0.11 | 0.07 | 0.05 |
| FU          | 30.03 | 22.90 | 10.40 | 18.02 | 8.81 | 9.64 |
| OIL         | 34.48 | 23.06 | 16.02 | 13.09 | 7.54 | 5.19 |
| GAS         | 29.51 | 23.80 | 16.49 | 10.73 | 9.75 | 8.48 |

### Table 2: Optimal portfolio under energy uncertainty

| Portfolio 3 | D1 | D2 | D3 | D4 | D5 | D6 |
|-------------|----|----|----|----|----|----|
| 95% VaR     | 0.33 | 0.27 | 0.17 | 0.13 | 0.05 | 0.04 |
| EU          | 33.01 | 29.34 | 24.38 | 5.15 | 4.89 | 2.69 |
| OIL         | 34.66 | 33.76 | 16.27 | 6.01 | 4.03 | 3.62 |
| GAS         | 30.44 | 29.82 | 19.96 | 7.91 | 5.07 | 2.88 |

| Portfolio 4 | D1 | D2 | D3 | D4 | D5 | D6 |
|-------------|----|----|----|----|----|----|
| 95% VaR     | 0.23 | 0.18 | 0.06 | 0.05 | 0.02 | 0.01 |
| EU          | 28.78 | 23.86 | 20.16 | 16.39 | 7.40 | 3.06 |
| OIL         | 32.37 | 31.52 | 22.99 | 7.08 | 3.07 | 2.62 |
| GAS         | 27.98 | 24.81 | 19.63 | 13.13 | 6.87 | 6.98 |
| CEX         | 28.74 | 25.21 | 19.73 | 13.27 | 6.99 | 6.74 |
Finally, the findings of contribution to the VaR show that the crude oil index makes a greater contribution to the VaR in a portfolio composed of oil and GAS under financial uncertainty than the GAS index makes. However, including the clean energy index will rebalance such a portfolio because it makes the smallest contribution to the VaR. With regard to energy uncertainty, the results show that the GAS index makes the smallest contribution to the VaR even when the clean energy index is included in the portfolio.

The evidence from this study suggests that portfolio managers should consider investing in the GAS market to hedge against energy shock. However, they should also consider investing in clean energy to hedge against financial shocks.

REFERENCES

Adrangi, B., Chatrath, A., Macri, J., Raffee, K. (2019), Dynamic responses of major equity markets to the US fear index. Journal of Risk and Financial Management, 12(4), 156.

Aloui, C., Nguyen, D.K., Njeh, H. (2012), Assessing the impacts of oil price fluctuations on stock returns in emerging markets. Economic Modelling, 29(6), 2686-2695.

Aloui, R., Gupta, R., Miller, S.M. (2016), Uncertainty and crude oil returns. Energy Economics, 55, 92-100.

Baker, S.R., Bloom, N., Davis, S.J. (2016), Measuring economic policy uncertainty. The Quarterly Journal of Economics, 131(4), 1593-1636.

Balcilar, M., Demirer, R., Hammoudeh, S., Nguyen, D.K. (2016), Risk spillovers across the energy and carbon markets and hedging strategies for carbon risk. Energy Economics, 54, 159-172.

Balcilar, M., Gupta, R., Wohar, M.E. (2017), Common cycles and common trends in the stock and oil markets: Evidence from more than 150 years of data. Energy Economics, 61, 72-86.

Barrero, J.M., Bloom, N., Wright, I. (2017), Short and Long Run Uncertainty (No. w23676). National Bureau of Economic Research.

Bilgin, M.H., Gozgor, G., Karabulut, G. (2015), The impact of world energy price volatility on aggregate economic activity in developing Asian economies. The Singapore Economic Review, 60(1), 1550009.

Fernández-Rodriguez, F., Gómez-Puig, M., Sosvilla-Rivero, S. (2016), Using connectedness analysis to assess financial stress transmission in EMU sovereign bond market volatility. Journal of International Financial Markets, institutions and Money, 43, 126-145.

Francés, G.E., Marín-Quemada, J.M., González, E.S.M. (2013), RES and risk: Renewable energy’s contribution to energy security. A portfolio-based approach. Renewable and Sustainable Energy Reviews, 26, 549-559.

Gençay, R., Selçuk, F., Whitcher, B. (2005), Multiscale systematic risk. Journal of International Money and Finance, 24(1), 55-70.

González-Pedraza, C., Moreno, M., Peña, J.I. (2014), Tail risk in energy portfolios. Energy Economics, 46, 422-434.

Huang, Y.H., Wu, J.H. (2008), A portfolio risk analysis of electricity supply planning. Energy Policy, 36(2), 627-641.

Ji, Q., Liu, B.Y., Nehler, H., Uddin, G.S. (2018), Uncertainties and extreme risk spillover in the energy markets: A time-varying copula-based CoVaR approach. Energy Economics, 76, 115-126.

Joëts, M. (2014), Energy price transmissions during extreme movements. Economic Modelling, 40, 392-399.

Jurado, K., Ludvigson, S.C., Ng, S. (2015), Measuring uncertainty. American Economic Review, 105(3), 1177-1216.

Kang, W., Lee, K., Ratti, R.A. (2014), Economic policy uncertainty and firm-level investment. Journal of Macroeconomics, 39, 42-53.

Liu, B.Y., Ji, Q., Fan, Y. (2017), Dynamic return-volatility dependence and risk measures of CoVaR in the oil market: A time-varying mixed copula model. Energy Economics, 68, 53-65.

Lucheroni, C., Mari, C. (2017), CO2 volatility impact on energy portfolio choice: A fully stochastic LCOE theory analysis. Applied Energy, 190, 278-290.

Ma, Y.R., Zhang, D., Ji, Q., Pan, J. (2019), Spillovers between oil and stock returns in the US energy sector: Does idiosyncratic information matter? Energy Economics, 81, 536-544.

Mele, A., Obayashi, Y., Shalen, C. (2015), Rate fears gauges and the dynamics of fixed income and equity volatilities. Journal of Banking and Finance, 52, 256-265.

Mellios, C., Six, P., Lai, A.N. (2016), Dynamic speculation and hedging in commodity futures markets with a stochastic convenience yield. European Journal of Operational Research, 250(2), 493-504.

Mensi, W., Hammoudah, S., Shahzad, S.J.H., Shahbaz, M. (2017), Modeling systemic risk and dependence structure between oil and stock markets using a variational mode decomposition-based copula method. Journal of Banking and Finance, 75, 258-279.

Mensi, W., Ikiri, B., Al-Yahyae, K.H., Kang, S.H. (2018), Analyzing time-frequency co-movements across gold and oil prices with BRICS stock markets: A VaR based on wavelet approach. International Review of Economics and Finance, 54, 74-102.

Muñoz, J.I., de la Nieta, A.A.S., Contreras, J., Bernal-Agustin, J.L. (2009), Optimal investment portfolio in renewable energy: The Spanish case. Energy Policy, 37(12), 5273-5284.

Nalebuff, B., Scharfstein, D. (1987), Testing in models of asymmetric information. The Review of Economic Studies, 54(2), 265-277.

Narayan, P.K., Ranjenei, K., Bannigidadmath, D. (2017), New evidence of psychological barrier from the oil market. Journal of Behavioral Finance, 18(4), 457-469.

Zhang, D. (2017), Oil shocks and stock markets revisited: Measuring connectedness from a global perspective. Energy Economics, 62, 323-333.