Z-boson production in association with heavy flavor in the $k_T$-factorization

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Abstract. We present the calculations of associated production of $Z$ bosons and heavy (charm or beauty) quarks at the LHC energies in the framework of $k_T$-factorization QCD approach. Our consideration is mainly based on the $O(\alpha_s^3)$ off-shell gluon-gluon fusion subprocess $g^*g^* \rightarrow ZQ\bar{Q}$, where produced $Z$ boson subsequently decays into a lepton pair. Several subleading contributions from the $O(\alpha_s^2)$ and $O(\alpha_s^3)$ subprocesses are taken into account. Contributions from the double parton scattering mechanism are estimated. The transverse momentum dependent (or unintegrated) gluon densities in a proton are determined using the Catani-Ciafaloni-Fiorani-Marchesini evolution equation. We achieve reasonably good agreement of our predictions and latest experimental data taken by the CMS and ATLAS Collaborations, discuss the theoretical uncertainties of our calculations and demonstrate the importance of subleading quark contributions in description of the LHC data in the whole kinematical region.

Very recently, the CMS Collaboration has presented the measurements of $Z$ boson production in association with charm [1] quarks at the LHC energy $\sqrt{s} = 8$ TeV. Also measurements of angular correlations of $B$-hadrons at $\sqrt{s} = 7$ TeV were presented [2] and the ATLAS Collaboration has reported the experimental data on the total and differential cross sections of $Z$ boson production in association with beauty quark jets at $\sqrt{s} = 7$ TeV [3]. Such processes involve both strong and weak interactions, so are important as global test of the Standard Model (SM). The $Z + b$-jets production is an important background for studies of the associated production of Higgs and $Z$ bosons, where the Higgs boson decays into $b\bar{b}$ pairs [4–6]. Many physics scenarios beyond the Standard Model (SM), for example, new generations of heavy quarks ($b', t'$) decaying into $Z$ bosons and $b$-quarks [7], supersymmetric Higgs bosons produced in association with beauty quarks [8] and some SM extensions with additional $SU(2)$ doublets with enhanced $Zbb$ coupling [9], predict final states with $b$-quarks and $Z$ bosons. In addition, $Z + b/Z + c$ cross sections ratio is highly sensitive to the charm content of the proton [10]. Finally, such processes may serve as potential indicators of the Double Parton Scattering (DPS) mechanism [11–13].

In the present work we analyse recent CMS [1, 2] and ATLAS [3] data using so called $k_T$-factorization QCD approach [14]. This approach is based on the famous Balitsky-Fadin-Kuraev-Lipatov (BFKL) or Ciafaloni-Catani-Fiorani-Marchesini (CCFM) gluon evolution equations and pro-

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vides solid theoretical grounds for the effects of gluon radiation in initial state and intrinsic gluon transverse momentum. The certain advantages are connected with the fact that, even with the LO partonic amplitudes, one can include a large piece of higher-order corrections (namely, part of NLO + NNLO + ... terms containing leading log $1/x$ enhancement of cross sections due to real initial state gluon emissions) taking them into account in the form of CCFM-evolved transverse momentum dependent (TMD) gluon densities. In particular, it gives better agreement with Tevatron data on the associated production of prompt photons and charm or beauty quarks compared to the NLO pQCD predictions [15], that is an additional motivation of our present study.

Let us start from a short review of calculation steps. Our consideration is mainly based on the off-shell gluon-gluon fusion subprocess:

$$g^*(k_1) + g^*(k_2) \rightarrow Z(p) + Q(p_1) + Q(p_2),$$

where $Q$ denotes the produced charm or beauty quark, and four-momenta of the particles are given in parentheses. The corresponding gauge-invariant off-shell amplitude was calculated earlier [16, 17]. To fully reproduce the experimental setup [1–3] we simulate the subsequent decay of the produced $Z$ boson into lepton pair according to the electroweak theory, that has not been made in the previous calculations [16, 17]. So, the off-shell gluon-gluon fusion gives the $O(\alpha s^2)$ contribution to the production cross-section. Additionally, we take into account several subprocesses involving quarks in the initial state, namely

$$q(k_1) + Q(k_2) \rightarrow Z(p) + q(p_1) + Q(p_2),$$

$$q(k_1) + g(k_2) \rightarrow Z(p) + q(p_1) + Q(p_2),$$

$$q(k_1) + \bar{q}(k_2) \rightarrow Z(p) + \bar{q}(p_1) + \bar{Q}(p_2),$$

$$q(k_1) + g(k_2) \rightarrow Z(p) + q(p_1) + Q(p_2) + \bar{Q}(p_3),$$

where the produced $Z$ bosons also decay to lepton pairs. These subprocesses give $O(\alpha s^2)$ (subprocesses (2) and (3)) and $O(\alpha s^3)$ (subprocess (4)) contributions. Subprocess (4) is suppressed by the additional degree of strong coupling QCD constant, however, we include it into the consideration because it can give sizeble contribution to the production cross section due to large gluon flux at the LHC energies, especially at low $\eta - \phi$ distances between the produced heavy quarks. Moreover, while the gluon-gluon fusion (1) contributes mainly at low and moderate transverse momenta, the subprocesses (2) — (4) become important at high transverse momenta, which correspond to the region of relatively large $x$, where the standard Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) parton evolution works perfectly and contribution of the log $1/x$-enhanced terms is negligible. Therefore, the subprocesses (2) — (4) can be safely taken into account in the usual collinear QCD factorization. Thus, we rely on a combination of two techniques with each of them being used where it is most suitable. For the subprocesses (2) and (3) we use here the on-shell limit of formulas obtained earlier [15] supplementing them with the $Z$ boson decays.

According to the collinear QCD factorization theorem, to calculate the contributions of subleading subprocesses (2) — (4) one has to convolute the corresponding partonic cross sections $d\hat{\sigma}_{ab}$ and conventional parton distribution functions $f_a(x_1, \mu^2)$ and $f_b(x_2, \mu^2)$ in a proton:

$$\sigma = \int d x_1 d x_2 d \hat{\sigma}_{ab}(x_1, x_2, \mu^2) f_a(x_1, \mu^2) f_b(x_2, \mu^2),$$

where indices $a$ and $b$ denote quark and/or gluon, $x_1$ and $x_2$ are the fractions of longitudinal momenta of colliding protons and $\mu^2$ is the hard scale. In the case of the leading off-shell gluon-gluon fusion subprocess (1) we employ the $k_T$-factorization approach:

$$\sigma = \int d x_1 d x_2 d k_{1T}^2 d k_{2T}^2 d \hat{\sigma}_{gg}(x_1, x_2, k_{1T}^2, k_{2T}^2, \mu^2) f_g(x_1, k_{1T}^2, \mu^2) f_g(x_2, k_{2T}^2, \mu^2),$$
where \( k_{1T}^2 \) and \( k_{2T}^2 \) are the transverse momenta of the initial off-shell gluons, and \( f_\phi(x_1, k_{T\phi}^2) \) is the TMD distribution in a proton. The essential point of our consideration is that we use the numerical solution of the CCFM evolution equation to determine the distribution. Numerically, we adopt the latest JH’2013 set, where two density functions, namely JH’2013 set 1 and JH’2013 set 2, were released [18]. The input parameters of corresponding initial gluon distributions were fitted from the best description of the precision DIS data on the inclusive \( F_2 \) data (set 1) or both \( F_2 \) and \( F_2^e \) data (set 2). Both these fits are based on the TMD matrix elements (which are directly related with the resummation of DIS coefficient functions) and include two-loop strong coupling constant, kinematic consistency constraint [19, 20] and non-singular terms in the CCFM gluon splitting function [21]. However, the inclusive structure function \( F_2 \) receives significant contributions from quark channels, whereas charm production is dominated by the gluon distribution. Therefore, below we will use JH’2013 set 2 gluon density as a default choice. For the conventional quark and gluon distributions in a proton we apply the LO MSTW’2008 set [22].

To calculate the DPS contributions one commonly makes use of a simple factorization formula (for details see the reviews [11–13] and references therein):

\[
\sigma_{\text{DPS}}(Z + Q + \bar{Q}) = \frac{\sigma_{\text{SPS}}(Z)\sigma_{\text{SPS}}(Q + \bar{Q})}{\sigma_{\text{eff}}},
\]

where \( \sigma_{\text{eff}} = 15 \text{mb} \) is a normalization constant which incorporates all "DPS unknowns" into a single phenomenological parameter. The numerical value of \( \sigma_{\text{eff}} \) was earlier obtained from fits to \( pp \) and \( p\bar{p} \) data. This will be taken as default value throughout the paper. Deriving the formula (7) relies on two simplifying approximations: the double parton distribution functions can be decomposed into longitudinal and transverse components and the longitudinal component reduces to the diagonal product of independent single parton densities. The inclusive SPS cross sections for the individual partonic subprocesses can be derived in a usual way (in the collinear QCD approximation or \( k_T \)-factorization). To calculate them below we strictly follow the approaches described earlier [23–25].

We now are in a position to present our numerical results. First we describe the input parameters and kinematic conditions. The predicted cross sections depend on the renormalization (\( \mu_R \)) and factorization (\( \mu_F \)) scales. As it is often done in the collinear QCD factorization, which we apply for the subprocesses (2) — (4), we set both these scales to be equal to transverse mass of produced Z bosons. In the \( k_T \)-factorization approach, employed for the gluon-gluon fusion subprocess (1), we set \( \mu_F^2 = \hat{s} + Q_T^2 \) with \( \hat{s} \) and \( Q_T^2 \) being the energy of scattering subprocess and transverse momentum of the incoming off-shell gluon pair, respectively. The special choice of \( \mu_F \) is connected with the CCFM evolution [18]. We set charm and beauty masses \( m_c = 1.4 \text{GeV} \) and \( m_b = 4.75 \text{GeV} \), Z boson mass \( m_Z = 91.1876 \text{GeV} \), its total decay width \( \Gamma_Z = 2.4952 \text{GeV} \) and \( \sin^2 \theta_W = 0.23122 \). For the subprocesses (2) — (4) we used LO formula for the strong coupling constant with \( n_f = 4 \) massless quark flavours and \( \Lambda_{QCD} = 200 \text{MeV} \), so that \( \alpha_s(m_Z^2) = 0.1232 \). According to the fit [18], we apply two-loop strong coupling constant for the off-shell gluon-gluon fusion subprocess. The multidimensional integration everywhere was performed by means of a Monte Carlo technique, using the VEGAS routine [26].

Let us consider first the associated production of Z bosons and b-jets [27]. The ATLAS Collaboration collected the data [3] at \( \sqrt{s} = 7 \text{TeV} \). Both leptons originating from the Z boson decay were required to have \( p_T^l > 20 \text{GeV} \) and \( |\eta|^l < 2.4 \), the lepton pair invariant mass lied in the interval \( 76 < M_{ll} < 106 \text{GeV} \), the beauty jets were required to have \( p_T^{jB} > 20 \text{GeV} \) and \( |\eta|^j < 2.4 \). We confront our predictions with the available data in Figs. 1 and 2. To estimate the theoretical uncertainties in the quark-involving subprocesses (2) — (4), calculated using the collinear QCD factorization, we have varied the scales \( \mu_R \) and \( \mu_F \) by a factor of 2 around their default values. In the \( k_T \)-factorization approach, employed for off-shell gluon-gluon fusion subprocess (1), the scale uncertainties have been
Figure 1. Associated $Z + b$ production cross section at $\sqrt{s} = 7$ TeV presented as a function of the $Z$ boson transverse momentum (left panel) or rapidity (right panel). Solid histograms show our predictions at the default scale while shaded bands correspond to scale variations described in the text. The estimated DPS contributions and MCFM [28] predictions (taken from [3]) are shown additionally. The data are from ATLAS [3].

estimated by using the gluon densities JH’2013 set 2+ and JH’2013 set 2- instead of default density JH’2013 set 2. These two sets refer to the varied hard scales in the strong coupling constant $\alpha_s$ in the off-shell amplitude: JH’2013 set 2+ stands for $2\mu_R$, while JH’2013 set 2- refers to $\mu_R/2$ (see [18] for more information). The estimated scale uncertainties are shown as shaded bands. As one can see, we achieve reasonably good agreement with the ATLAS data [3] within the experimental and theoretical uncertainties, although we observe some underestimation of these data at high $p_Z^T$ and slight overestimation at small transverse momenta. The slight overestimation of the data at low $p_Z^T$ can probably be attributed to the TMD gluon density used, since the region $p_Z^T < 100$ GeV is fully dominated by off-shell gluon-gluon fusion, as it is demonstrated in Fig. 2. The rapidity distribution is well described practically everywhere. The NLO pQCD calculations, performed using MCFM routine [28], tend to slightly overestimate our predictions and better describe the data at large transverse momenta.

We find that the quark-initiated subprocesses (2) — (4) become important only at high transverse momenta, where the typical $x$ values are large, and that supports using of the DGLAP quark and gluon dynamics for these subprocesses (see Fig. 2). The subprocesses (2) — (4) are important to achieve an adequate description of the data in the whole $p_T^Z$ region. The estimated DPS contributions are found to be small in the considered kinematic region. Some reasonable variations in $\sigma_{\text{eff}} \approx 15 \pm 5$ mb would affect DPS predictions, though without changing our basic conclusion. We note also that scale uncertainties of the CCFM-based predictions are comparable with the ones of NLO pQCD calculations.

Now we turn to the associated production of $Z$ bosons and two beauty jets [27]. The data provided by the ATLAS Collaboration [3] refer to the same energies and kinematic restrictions as in the previous subsection. The observables shown by the ATLAS Collaboration are the $Z$ boson transverse momentum $p_T^Z$ and rapidity $y^Z$, invariant mass of the $b$-jet pair $M^{bb}$ and angular separation in $\eta - \phi$ plane between the jets $\Delta R^{bb}$. The latter is useful to identify the contributions where scattering amplitudes are dominated by terms involving gluon splitting $g \rightarrow Q + \bar{Q}$. 
The role of off-shell gluon-gluon fusion subprocess is a bit enhanced here compared to the case of $Z + b$ production because the quark-antiquark annihilation subprocess (3) gives a negligible contribution and gluon splitting subprocess (4) populates mainly at low $\eta - \phi$ distances $\Delta R_{bb}$. The estimated DPS contribution is small and can play a role at low $p_T^Z$ only. The NLO pQCD calculations, performed using MCFM program, tend to slightly underestimate the ATLAS data at low $\Delta R_{bb}$ and $M_{bb}$, although provide better description of the data at large transverse momentum $p_T^Z$ and invariant mass $M_{bb}$.

In the measurements reported by CMS Collaboration [2], both $b$-hadrons were identified explicitly by their full decay reconstruction. This data sample allows to study the production properties of a $Zb\bar{b}$ system even in the region of small angular separation between the $b$ quarks (where the usual jet analysis is not possible as the jets would overlap). In a specific subsample, an additional cut on the $Z$ boson transverse momentum is applied, $p_T^Z > 50$ GeV. The CMS Collaboration described the angular configuration of the $Zb\bar{b}$ system in terms of spatial (in $\eta - \phi$ plane) and azimuthal separation between the $b$-hadrons $\Delta R_{bb}$ and $\Delta \phi_{bb}$, spatial separation min $\Delta R_{Zb}$ between the $Z$ boson and closest $b$-hadron and the asymmetry in the $Zb\bar{b}$ system defined as

$$A^{Zbb} = \frac{\max \Delta R_{Zb} - \min \Delta R_{Zb}}{\max \Delta R_{Zb} + \min \Delta R_{Zb}},$$

where $\max \Delta R_{Zb}$ is the distance between the $Z$ boson and remote $b$-hadron. The correlation observables are useful to identify the different production mechanisms (or specific higher-order corrections). For example, low min $\Delta R_{Zb}$ identifies $Z$ bosons in the vicinity of one of the $b$-hadrons ($Z$ bosons promptly radiated from $b$-quarks), small $\Delta \phi_{bb}$ indicates gluon to quark splitting $g \rightarrow Q + \bar{Q}$. Moreover, while the configurations where the two $b$-hadrons are emitted symmetrically with respect to the $Z$ directions leads to a zero value of $A^{Zbb}$ asymmetry, the additional final-state gluon radiation results in a non-zero one, that provides us with the possibility to test the high-order pQCD corrections.
Figure 3. Associated production of a $Z$ boson with two beauty jets at $\sqrt{s} = 7$ TeV calculated as a function of the $Z$ boson transverse momentum, rapidity, invariant mass of the $b$-jet pair and angular separation between the jets. Notation of the histograms is the same as in Fig. 1. The data are from ATLAS [3]. The MCFM [28] predictions are taken from [3]. Our predictions [27] are shown in Figs. 4 and 5 in comparison with the CMS data [2]. As one can see, our results with default $b$-quark fragmentation parameters reasonably well describe the data within the theoretical and experimental uncertainties. To estimate an additional uncertainty coming from the $b$-quark fragmentation, we repeated our calculations with varied shape parameter $\epsilon_b = 0.003$ (not shown), which is often used in NLO pQCD calculations. We find that the predicted cross sections (in the considered $p_T$ region) are larger for smaller $\epsilon_b$ values. However, the typical dependence of numerical predictions on the fragmentation scheme is much smaller than the scale uncertainties of our calculations. The NLO pQCD predictions, obtained using the aMC@NLO [29] event generator, are rather close to our results.
Finally, let us consider measurements of the associated $Z+c$-jet production performed by the CMS Collaboration at $\sqrt{s} = 8$ TeV \cite{ref1}. Similar to $Z+b$ production, leptons originating from the $Z$ boson decay have transverse momenta $p_T > 20$ GeV and pseudo-rapidities $|\eta| < 2.1$. The invariant mass of the lepton pair must lie within the $71 - 111$ GeV interval and $c$-jets are required to have $p_T > 25$ GeV and $|\eta| < 2.5$. The cross sections ratios $\sigma(Z+c)/\sigma(Z+b)$ was also measured as a functions of $Z$ boson and jet transverse momenta. Our predictions are shown in Figs. 6 and 7 in comparison with the CMS data. We have achieved good description of the $Z$ boson transverse momentum distribution, whereas the measured cross section as a function of leading $c$-jet is above the predicted one at $p_T^{c\text{-jet}} < 40$ GeV. Taking into account the DPS contributions only slightly increases the calculated cross sections. The NLO pQCD predictions, obtained using the MCFM program \cite{ref28}, are lower than the data in the first and second $Z$ boson $p_T$ bins and in the first $p_T^{c\text{-jet}}$ bin. The LO pQCD predictions matched with parton...
Figure 5. Associated production of a $Z$ boson and two $b$-hadrons at $\sqrt{s} = 7$ TeV under additional kinematical cut on the $Z$ boson transverse momentum $p_{ZT} > 50$ GeV. Notation of the histograms is the same as in Fig. 1. The data are from CMS [2]. The aMC@NLO [29] predictions are taken from [2].

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showering, obtained with MadGraph event generator, are in better agreement with the data than the NLO ones, although they still slightly underestimate the CMS data. The differences between these predictions and the data are reduced in the $\sigma(Z + c)/\sigma(Z + b)$ ratios. The results of our calculations underestimate these ratios at high $Z$-boson and jet transverse momenta. However, they are rather close to the CMS data within the large experimental uncertainties.

To conclude, we have considered the associated production of $Z$ bosons and charmed or beauty quarks at the LHC energies in the framework of the $k_T$-factorization approach. Our consideration has been based on the $O(\alpha_s^2)$ off-shell gluon-gluon fusion subprocess $g^* g^* \rightarrow ZQ\bar{Q}$, where the produced $Z$ boson subsequently decays into the lepton pair. Several subleading $O(\alpha_s^2)$ and $O(\alpha_s^3)$ contributions from the quark-involved subprocesses, which come into play at high transverse momenta, were
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Figure 6. The associated $Z + c$ production calculated as a function of $Z$ boson and $c$-jet transverse momenta at $\sqrt{s} = 8$ TeV. Notation of histograms is the same as in Fig. 1. The experimental data are from CMS [1].

Figure 7. The ratio $\sigma(Z + c)/\sigma(Z + b)$ calculated as a function of $Z$ boson and $c$-jet transverse momenta at $\sqrt{s} = 8$ TeV. Notation of histograms is the same as in Fig. 1. The experimental data are from CMS [1].
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