Polariton condensation, superradiance and difference combination parametric resonance in mode-locked laser

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Abstract. The generation of the ring mode-locked laser containing resonant absorption medium in the cavity was investigated. It is shown that near the strong resonant absorption lines a condensation of polaritons arises. Intensive radiation looks like as superradiance in a medium without population inversion. We studied theoretically the microscopic mechanism of these phenomena. It was shown that in this system in absorbing medium a strong self-induced difference combination parametric resonance exists. Superradiance on polaritonic modes in the absorbing medium are due to the emergence of light-induced resonant polarization as a result of fast periodic nonadiabatic quantum jumps in the absorber.

1. Introduction

Exciton-polaritons systems in semiconductors placed in microcavity are called polariton laser. These systems pumped by single ultra-short pulse are actively studied so far. For pumping of atomic polariton laser it was proposed for the first time in \cite{1, 2} and further in \cite{3, 4} to use laser radiation with periodical amplitude modulation. It was shown that in the system "field + matter" in the strong light-matter coupling regime when modulation frequency $\Omega_m = 2\omega_c$ there is a difference combination parametric resonance. Here $\Omega_m$ - frequency of amplitude modulation of the pump field, $\omega_c$ is a cooperative (collective) the frequency of the resonant medium (1)

$$\omega_c = \sqrt{2\pi \alpha d_{12}^{-2} \hbar N_0}. \quad (1)$$

Here $N_0$ - atomic density, $\omega_{12}$ - transition frequency of the two-level particles, $d_{12}$ - transition dipole moment.

In this paper, we present a new efficient way of parametric excitation of the polariton laser and superradiance excitation using coherent pump with periodic phase (frequency) modulation. The method is based on the following idea. When system is pumped by this FM field the population difference $N(t)$ can be rapidly changed. Namely, there are periodic surges nonadiabatic quantum energy states between quantum energy states of medium. Furthermore, in this method, energy of all modes of the pump laser can be used, regardless of their position on the frequency axis. Generation of the single mode polariton laser was investigated by the numerical solution of Maxwell-Bloch equations.
Under the condition of strong light-matter coupling self-consistent motion of the Bloch vector is equivalent to the motion of a physical pendulum, oscillating near the equilibrium point with the cooperative frequency $\omega_c$. Nonadiabatic quantum jumps of atoms lead to a periodic change in the population difference $N(t)$ and the parametric excitation of oscillations of a pendulum. Simultaneously with the parametric build up the amplitude of the pendulum increases. The amplitude of the Bloch pendulum oscillations corresponds to radiation of two polaritonic modes - upper polariton and lower polariton.

The condition of the difference combination parametric resonance for the Bloch pendulum is follows: $\Omega_{\text{mod}} = 2\omega_c/m$, $m = 1, 2, 3...$ Here $\Omega_{\text{mod}}$ – frequency of the phase modulation of the pump field. In the numerical simulations, periodic modulation of the phase (frequency) of the pump field was used (FM mode-locking), $\omega(t) = \omega_{012} + d\phi(t)/dt$, $\phi(t) = \phi_0 \cos(\Omega_{\text{mod}} t)$. Here $\omega(t)$ is the frequency detuning between the carrier frequency of the FM pump field and atomic transition frequency.

It is shown that when FM pumping field is applied the efficiency of excitation of atomic polariton laser can be increased by factor $10^6 \div 10^8$ with respect to the case of single pulse pumping (when the energy of FM and non FM pulses are equal). The investigated phenomena opens opportunity to use the laser diodes of small power for the pumping of polariton laser.

2. Polariton condensation and superradiance in a ring mode-locked laser

The effect of polariton condensation occurs on the resonant transition of absorption atomic cell located in the cavity of mode-locked laser was studied in [5-7]. In general case, the spectrum of laser radiation has the form of spectral doublets which contains upper polariton and lower polariton branches. However, discussing the nature of the microscopic mechanism of this phenomenon is still ongoing. In our studies, it was shown that in a multimode laser without mode locking in such case appears the mode-locking regime. The laser spectrum is close to the resonant frequency of the absorber $\omega_{12}$, and it generates $0\pi$-pulses [8, 9].

A superradiance on the resonant lines $D_1$ and $D_2$ of rubidium was observed in [10] in experiments when rubidium vapor was pumped by non-resonant femtosecond pulses of a Ti-sapphire laser. By increasing the density of the resonant particles $N_0$ the strong light-matter coupling was realized. Superradiance on polaritons modes arose and spectrum takes the form of a spectral doublet. As it was for the first time shown in [1, 2] in a single mode cavity model, this effect can be greatly enhanced by using periodically modulated pump laser. This effect increases due to the occurrence of fast periodic nonadiabatic quantum jumps between the energy states of the resonant medium.

In this work, we investigate the mechanism of occurrence of the gain on the polaritonic modes near the absorber resonance $\omega_{12}$ in the case when laser operates in the mode-locking regime. It turns out that a significant increase of this effect is possible by using of non-resonant laser pulses with self-consistent periodic phase (frequency) modulation: $\omega(t) = \omega_{012} + d\phi(t)/dt$. The variable parameter in this parametric resonance is $\omega(t)$.

Generation of the ring mode-locked laser containing intracavity resonant absorption cell was investigated in the frame of the numerical solution of Maxwell-Bloch equations.

This model takes into account a wide class of phenomena: multimode regimes of generation, mode competition and mode-locking effects, chaotic regimes of generation, soliton formation, and many others. This model is based on the system of Maxwell-Bloch equations, which has following form [11]:

\begin{align}
\frac{d}{dt} P_c(t) &= -\frac{P_c}{T_2} - \Delta\omega \cdot P_s(t) - \frac{d_{12}}{2\hbar} \cdot \Delta\rho(t) \cdot B(t), \\
\frac{d}{dt} P_s(t) &= -\frac{P_s}{T_2} + \Delta\omega \cdot P_c(t) + \frac{d_{12}}{2\hbar} \cdot \Delta\rho(t) \cdot A(t), \\
\frac{d}{dt} \Delta\rho &= -\frac{(\Delta\rho - \Delta\rho_0)}{T_1} - \frac{2d_{12}}{\hbar} \cdot (A(t) \cdot P_s(t) - B(t) \cdot P_c(t)),
\end{align}
\[ \frac{\partial A(z,t)}{\partial t} + c \cdot \frac{\partial A(z,t)}{\partial z} = -4 \cdot \pi \cdot \omega \cdot d_{12} \cdot N_0 \cdot P_s(z,t), \]  
(5)

\[ \frac{\partial B(z,t)}{\partial t} + c \cdot \frac{\partial B(z,t)}{\partial z} = 4 \cdot \pi \cdot \omega \cdot d_{12} \cdot N_0 \cdot P_c(z,t). \]  
(6)

Here \( T_1 \) is population difference relaxation time, \( T_2 \) is polarization relaxation time, \( c \) is velocity of light.

Bloch equations (2)-(4) describe the behavior of the slow envelopes of inphase and quadrature components of polarization \( P_c(z,t) \) and \( P_s(z,t) \) and population difference \( \Delta \rho \) between ground and excited states:

\[ P(z,t) = P_c(z,t) \cos(\omega t - kz) + P_s(z,t) \sin(\omega t - kz). \]  
(7)

Similar Bloch equations are solved for the amplifying medium of laser. Maxwell equations (5)-(6) describe the behavior of slow envelopes of the electric field \( A(z,t) \) and \( B(z,t) \) in gain and absorbing laser media in the cavity:

\[ E(z,t) = A(z,t) \cos(\omega t - kz) + B(z,t) \sin(\omega t - kz). \]  
(8)

For numerical simulations the parameters which are realized in the experiment [1, 2, 8] were used. Numerical simulations showed that superradiance and polariton condensation on the polaritonic modes started from the arising of phase modulation of laser pulses from mode-locked pump laser. When the nonresonant pump pulse propagates the frequency deviation of the field \( \Delta \omega(t) \) decreases.

At the initial stage of the lasing process, \( t = 0, \frac{d\phi(t)}{dt} = 0 \) and \( \Delta \omega(0) = \Delta \omega_{12} \). Figure 1 shows the \( d\phi(t)/dt \) of the laser pulses near the threshold of the parametric instability. Resonantly absorbing medium develops a self-consistent field with the phase modulation. As shown in figure 1, the phase modulation has not simple harmonic form. When pump laser frequency \( \omega(t) = \omega_{12} \) nonadiabatic quantum jumps occur, leading to the creation of superposition states in a resonant absorption medium and the generation short pulses of superradiance arises.

**Figure 1.** The upper curve - pumping pulses \( E(t) \), a.u. The lower curve - \( d\phi(t)/dt \) in GHz. \( \omega_c = 6.7 \times 10^{10} \) rad/sec, \( T_1 = T_2 = 10 \) nsec, the laser cavity length - 3 cm. \( \Delta \omega_{12}/2\pi = -115 \) GHz.

Figure 2 has the spectrum of the laser after the build up of parametric resonance. The spectrum of the mode-locked laser becomes asymmetrical and expands to the direction of the resonant absorption.
Figure 2. The laser spectrum at $t = 6$ nsec. Axial cavity modes are visible in the laser spectrum. The arrow shows the spectrum of polariton condensation at the absorption line. $\Delta \omega_{12}/2\pi = -115$ GHz.

Figure 3. Laser spectrum evolution and arising of polariton condensation near the atomic absorption line ($\Delta \omega_{12}/2\pi = -100$ GHz). $N_0 = 10^{12}$ cm$^{-3}$, $\lambda_{12} = 700$ nm, $d_{12} = 5D$, $T_1 = T_2 = 10$ nsec, the laser cavity length - 3 cm.

Figure 4. Laser spectrum evolution and arising of polariton condensation near the atomic absorption line ($\Delta \omega_{12}/2\pi = -100$ GHz). $N_0 = 10^{13}$ cm$^{-3}$. Other parameters are same as in figure 3.
Figure 3 and figure 4 illustrates the influence of the atomic density $N_o$ on the duration of the process of the formation of phase modulation of pulses in mode-locked laser. Increase of atomic density from $N_o = 10^{12}$ cm$^{-3}$ (figure 3) to $N_o = 10^{13}$ cm$^{-3}$ (figure 4) leads to the decrease of its duration from 6 ns to 2 ns. This result is in agreement with the experimental observations [1, 2, 12].

![Figure 3](image1.png)

**Figure 3.** The build up of the steady state of polariton condensate near the atomic transition $(\Delta \omega_{12}/2\pi = -100 \text{ GHz})$ during the time of laser generation, $N_o = 10^{13}$ cm$^{-3}$. Other parameters are same as in figure 3.

Figure 5 shows the stage of the build up of the steady state of amplitude of the polariton condensate near absorber transition frequency $\omega_{12}$ for a long time. The increase of intensity of polaritons radiation near the absorption lines is limited by the saturation effect.

![Figure 5](image2.png)

**Figure 5.** The build up of the steady state of polariton condensate near the atomic transition $(\Delta \omega_{12}/2\pi = -100 \text{ GHz})$ during the time of laser generation, $N_o = 10^{13}$ cm$^{-3}$. Other parameters are same as in figure 3.

3. Conclusion

In conclusion, the new microscopic mechanism of the formation of condensation of polaritons near a strong absorption line of a medium in the ring optical cavity in the mode-locking regime was considered theoretically.

It was shown that this process has two stages. In the first stage, a self-consistent phase modulation of the laser pump pulses occurs. The duration of this process depends on the density of the absorbing atoms $N_o$. The interaction of phase modulated pulses with absorber atoms leads to the appearance of fast nonadiabatic quantum jumps in the atoms in the absorbing cell and to the formation of coherent superposition states of atoms.

In the second stage, the induced polarization of the medium emits the radiation in the form of short pulses of superradiance at the frequencies of the polaritonic modes. The fulfillment of the condition of difference combination parametric resonance leads to a rapid increase in the amplitude of the polaritonic modes. The increment of the parametric instability depends on the frequency detuning $\Delta \omega_{12}$, amplitude and duration of the laser pulses.

The dynamics of the field $E(z,t)$, polarization $P(z,t)$, and population difference $N(z,t)$ in the absorbing and amplifying cells of a ring laser were analyzed via numerical solution of the Maxwell-
Bloch equations. Numerical calculations performed for realistic conditions realized in the experiment showed a qualitative agreement between the results of simulations and experiments.

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