A moving wave generating-absorbing boundary based on OpenFOAM

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Abstract. This paper develops a moving wave generating-absorbing boundary based on the CFD toolbox OpenFOAM. The surface elevation of the moving boundary gauged in every time step is as the feedback signal of active absorption system. Three different equations of correction are considered in the present model to absorb the secondary reflection from the structure in the wave tank. The effect of this boundary under the conditions of regular and irregular waves is investigated and discussed. The stability factor method is used to analyze the absorption effect of regular waves, and a time-domain method is employed to separate the incident and reflected waves of the irregular waves. Results show that this moving boundary has good capability to not only generate regular/irregular waves but also absorb the secondary reflection satisfactorily.

1 Introduction
Active absorption technology originated in the physical model test whose purpose is to eliminate the secondary reflection in the wave tank. The secondary reflected wave in the wave tank refers to the wave that occurs when the wave is reflected on the structure and is reflected again at the wave paddle. As the model test progresses, the secondary reflected waves in the wave tank will cause multiple reflections between the piston paddle and the structure, resulting in deviations of the model test results. In the laboratory, there are three main methods to reduce the impact of the secondary reflection in the wave tank [1]. First, widen the wave tank, so that the structure of the width of the proportion of small enough; Second, increase the length of the wave tank, so that the second reflected wave will not arrive the structure before the completion of the model test; Third, take the initiative to active absorbing technology to eliminate the secondary reflected wave

Most of the theoretical research of active absorption technology are concentrated in the 1990s [2-4], Schäffer [5] put forward a correction velocity of the wave paddle based on shallow water equation, which is the easiest active absorption technology so far. In the 21st century, with the rapid development of computational fluid dynamics and computer hardware technology, a large number of numerical wave tanks based OpenFOAM have emerged. Many experts and scholars have based on different problems to establish their own numerical wave tanks and join the active absorption function: Jacobsen [6] develops a numerical toolbox named wave2foam, which links with the relaxation zone to eliminate the secondary reflection in the wave tank. Pable [7] develops another toolbox with the method of [5] to add the active absorption system. Wang [8] considers the attenuation of standing waves before the wave paddle and establishes the active absorption system, but considering the attenuation of the standing wave will increase the amount of calculation and lead to "slow drift" phenomenon, so there is room for optimization. In summary, it is meaningful to develop a numerical
program with good capability to absorb the secondary reflection. In this paper, the basic theory of the active absorption of the regular wave and irregular wave is given. Then the principle is simplified and finally the active absorption model is established. The stability coefficient method [9] and frequency domain analysis is used to quantitatively analyze the results.

2 Numerical Model

2.1. Governing Equations

The governing equations of the wave field are the combination of the continuity equation and Reynolds-averaged Navier-Stokes equations (RANS) as well as the VOF equation:

\[ \nabla \cdot \mathbf{u} = 0, \quad (1) \]

\[ \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) = -\nabla p + \rho \mathbf{g} + \nabla \cdot \tau + S^*, \quad (2) \]

\[ \frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{u}) = 0, \quad (3) \]

Where \( \mathbf{u} \) is the velocity vector; \( \alpha \) stands for the volume of fluid in every cell, i.e., 0 for air and 1 for water, with values between 0 and 1 indicating cells containing a mixture of the two fluids; \( \tau \) is the specific Reynolds stress tensor; \( S^* \) is the source term for wave absorption in the damping zone and will be discussed in the next section; and \( \rho \) and \( \mu \) are the weighting density and dynamic viscosity of two phases, respectively, calculated by

\[ \rho = \alpha \rho_1 + (1 - \alpha) \rho_2, \quad (4) \]

\[ \mu = \alpha \mu_1 + (1 - \alpha) \mu_2. \quad (5) \]

\( C \kappa \nabla \alpha \) in Eq. 2 is the surface tension term, in which \( C \) is the surface tension coefficient and \( \kappa \) is the curvature of the interface. \( G \) is the acceleration of gravity. \( X \) is the position vector. \( P_{gh} \) is the pseudo-dynamic pressure:

\[ P_{gh} = P - \rho g X, \quad (6) \]

Which is without any physical meaning but is used in the application of a convenient numerical technique. Eq. 6 represents the dynamic pressure only if the free surface is located at \( Z = 0 \). The third term on the left-hand side of Eq. 3 is the artificial compression term, which is only effective at the free surface. \( U_a \) is the artificial compressive velocity:

\[ U_a = \min \{ c_a \min \{ |U| , \max \{ |U| \} \} \}. \quad (7) \]

Where \( c_a \) is a factor with a value of 1 by default and its value can be specified by users to enhance or eliminate \((c_a = 0)\) the compression of the interface.

2.2 Active Absorption Theory

Because of the existence of the wave-maker, the secondary reflection, which always occurs at the wave paddle, has been a major problem in both the physical model test and the wave tank. Lengthening or widening the wave flume is one widely used traditional way to reduce the influence of secondary reflection. However, in a numerical model, this traditional approach may require a large number of calculations. Therefore, the capability of active absorption is often required to provide a better result with a lower cost. In the present model, the moving boundary for wave generation is controlled to absorb waves impinging on it. If \( u_{corr} \) denotes the correct velocity of the active absorbing wave paddle, the corresponding displacement in every time step \( \Delta t \) is

\[ x_{corr} = u_{corr} \cdot \Delta t. \quad (8) \]

The sum of these displacements constitutes the final correction of the active absorbing wave-maker. Thus, the final displacement of the active absorbing wave paddle is the superposition of the normal displacement and the correction part:

\[ X(t) = \frac{S}{2} \sin(\omega t) + \int u_{corr} \cdot dt. \quad (9) \]
For regular waves, three types of $u_{corr}$ are applied in the present model and users can select any one of them to obtain a good result. The first, by Schäffer [9], is the simplest method:

$$u_{corr} = -\sqrt{\frac{g}{d}}(\eta_m - \eta_0).$$

(10)

Here, $g$ is the gravitational acceleration, with the value of 9.81 m/s$^2$. $d$ is the water depth. $\eta_m$ and $\eta_0$ are the measured and theoretical surface elevations at the wave paddle, respectively.

$u_{corr}$ of the second method, which is the most widely used, can be expressed as

$$u_{corr} = -\frac{\omega}{H_s}(\eta_m - \eta_0).$$

(11)

Here, $\omega$, as noted above, is the angular frequency of the target wave. $H_s$ is the linear relationship between the wave height and the paddle stroke. Other variables are the same as in Eq. 10.

The last type of $u_{corr}$ has the following form:

$$u_{corr} = -\frac{\omega}{H_s}\sum_{n=1}^{\infty} H_{s_n}(\eta_m - \eta_0).$$

(12)

In Eq. 12, $x$ denotes the displacement of the wave paddle; thus, the iteration in every time step is needed to solve this equation. $H_{s_n}$ is the transfer function of the evanescent wave and is calculated as

$$H_{s_n} = \frac{4\sin^2(k_n d)}{2kd + \sin 2k_n d} n = 1, 2, 3, \ldots$$

(13)

Where $k_n$ is the wave number of the evanescent modes, which are due to the mismatch between the shape of the progressive wave velocity profile and the type of wave paddle motion. $k_n$ is real, and $k_n > 0$. The wave number is calculated by the dispersion relation for the evanescent modes:

$$\omega = -gk_n\tan k_n d n = 1, 2, 3, \ldots$$

(14)

Clearly, there are an infinite number of solutions $k_n$ to this dispersion relation. Returning to Eq. 12, a definite value of $n$ is needed to make this numerical model effective. Zhang [10] gives the relationship between the transfer function and $n$ (Fig. 1). In the figure, the abscissa axis is the non-dimensional frequency ($\omega \sqrt{d/g}$) and the ordinate axis is the value of the transfer function.

\[\sum_{n=1}^{\infty} H_{s_n}\]

is abbreviated as $R$, and the number at the right side of the black curves is the value of $n$. In the figure, the evanescent transfer function $R$ increases continuously with increasing frequency and it appears that the value of $R$ at $n = 20$ matches well with that at $n = 200$. Thus, in the present model, $n$ is set to 20 by default. It is worth pointing out that if the non-dimensional frequency is larger than 3.4, the sum of the evanescent wave transfer function is larger than that of the linear one, which means the local disturbance at the wave paddle exceeds the progressive wave.

During the calculation, the third term on the right-hand side of Eq. 58 always results in “slow drift” of the wave paddle. Slow drift is caused by a linear growth of deviations in every period of the wave paddle, and it will cause saturation, i.e., the paddle will end up in an extreme position outside of the
stroke. Wang [8] analyses this problem and uses two modifications, i.e., linear compensation and PI control, to minimize the unnecessary motion of the paddle. The results seem very good, but the universal values of the parameters in the two modifications are not obtained, which means that if the wave condition changes, the parameters may need to be redefined. Therefore, in the present model, for convenience as well as accuracy, a limiter is added to this model to make sure the displacement of the paddle is within a reasonable range. The corrected velocity is changed to

$$u_{corr} = \begin{cases} \frac{\omega}{H_s} [2 \cdot \eta_0 - \eta_m + \sum_{i=1}^{n} H_{S_i} \cdot x] & x < \mu S \\ \frac{\omega}{H_s} [2 \cdot \eta_0 - \eta_m] & x > \mu S \end{cases}.$$  

(15)

Obviously, after adding this limiter, the method becomes the combination of the second and the third methods. $S$ is the stroke of the paddle, and $\mu$ is the limiting factor, which is equal to 0.7 by default. The validation and discussion of this method will be presented in the next section.

Because the irregular wave is considered as the sum of several regular waves, the active absorption is similar to regular situations. Thus, the final $u$ of the wave paddle is directly given as

$$u = \sum_{i=1}^{N} \frac{\omega_i}{H_{S_i}} (2 \cdot \eta_{0i} - \lambda_i \eta_{m}).$$  

(16)

Here, $\omega_i$, $H_{S_i}$, and $\eta_{0i}$ are the angular frequency, the linear transfer function and the theoretical surface elevation at the paddle of component $i$, respectively. $\eta_m$ is the measured surface elevation at the wave paddle. $\lambda_i$ is a weight coefficient [9] calculated by

$$\lambda_i = \frac{S(\omega_i)}{\sum_{i=1}^{N} S(\omega_i)}.$$  

(17)

In the present model, as noted above, $N$ is equal to 100 and every component has the same energy; therefore, $\lambda_i$ is equal to 0.01.

3 Investigation and Discussions

3.1 Regular Waves

The active absorption system of regular waves is tested in this section. A two-dimensional wave tank, with a vertical wall at the end of it, is set up to simulate the generation of regular waves as well as their absorption. The initial water depth $d$ is 2.0 m, and the wave height $H$ is 0.1 m. The period of the target wave is 2.0 s. The wavelength $L$, according to the linear theory, is 6 m. The length of the wave tank is 12 m, which is twice as long as the wavelength. Three distinct types of corrected velocities, named methods #1-#3 and defined by Eqs. 10-12, are applied and calculated. The displacements of wave paddles are shown in Fig. 2. From the figure, we find that the displacement of total reflection is a sinusoidal motion. Methods #1 and #2 have very similar displacements. It is worth noting that because of the existence of the limiter, method #3 has a different displacement. If the displacement of method #3 is not modified by the limiter, it will continue to decrease from $t = 10$ s, which means the system will be out of operation.
The results of these three methods, as well as the result in the case of total reflection at the wave paddle, are contrasted in Fig. 3. The x-axis and y-axis stand for time and non-dimensional surface elevation, respectively. Elevations calculated by different active absorption systems seem to show almost no difference. The final stable amplitude at x = 0.5 m is 1.5 times that of the target wave and 2.0 times that of the target wave at x = 9.0 m. Stable standing waves can be observed in comparison with an increasing wave obtained by total reflection. This means the secondary reflection at the wave paddle has been absorbed by the active absorption system.

For quantitative analysis of this stability, a method called “stability factor” [9] is added. In the time range [t-Δt, t+Δt], a stability factor is defined as

\[ r(t) = \frac{\eta_{max} - \eta_{min}}{H_m}. \]  

Here, \( \eta_{max} \) and \( \eta_{min} \) are the maximum and minimum surface elevations in the time range, respectively. \( H_m \) is the average wave height of this wave profile. \( r(t) \) should be approximately 1 and does not change with time acutely if the wave field in the wave tank is stable. Stability factors of those four cases at x = 9.0 m are shown in Fig. 4 with \( \Delta t = 10 \) s. Obviously, the stability factor of total reflection is always increasing and stability factors of other cases are approximately 1 or approaching 1 as time passes. This condition indicates that there are stable wave fields in those wave tanks. In other words, the active absorption system in the present model is effective in eliminating the secondary reflection.
3.2 Irregular Waves

Verification of the active absorption system of irregular waves is performed as follows. A two-dimensional wave tank, same as section 3.1, is set up to simulate the generation of irregular waves as well as their absorption. The initial water depth is set to 0.5 m. The significant wave height is 0.06 m, and the peak period is 1.5 s, which means the peak frequency is 0.667 Hz. The wavelength of the significant wave is approximately 2.8 m. Two wave gauges, whose sampling interval is 0.02 s, are set in the wave tank at positions $x = 1.0$ m and $x = 2.0$ m. Two cases, one with active absorption and another without it, are calculated. In contrast to regular waves, there is no stable standing wave in the irregular NWF. Therefore, it is essential to separate the incident and reflected waves. A time-domain method [1] is employed to separate the incident and reflected waves. Because of the method, there is one more wave gauge located at $x = 1.5$ m for separation. Fig. 5 shows the time histories of surface elevations when the active absorption system is on/off. Fig. 6 is the separated spectrum calculated by the present model. From those two figures, we can see that lower surface elevations are present with the active absorption. Moreover, the separated spectrum agrees well with the target spectrum, which means this active absorption system has a satisfactory effect.

![Fig. 4 Stability factors of the four cases at $x = 9.0$ m.](image)

![Fig. 5 The time histories of surface elevations with the active absorption system on/off.](image)
4 Conclusion

This paper presents and discusses a moving wave generating-absorbing boundary based on OpenFOAM. The theory of active absorption system is elaborated and discussed. The 2-D numerical wave tank is set up to investigate the effect of active absorption system under the conditions of regular/irregular waves. Results show that this new boundary owns a good capability to not only generate waves but also absorb the secondary reflection satisfactorily at the same time.

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