Spatiotemporal Differentiators Generating Optical Vortices with Transverse Orbital Angular Momentum and Detecting Sharp Change of Pulse Envelope

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As a new degree of freedom for optical manipulation, recently, spatiotemporal optical vortices (STOVs) carrying transverse orbital angular momentums have been experimentally demonstrated with pulse shapers. Here, a spatiotemporal differentiator is proposed to generate STOVs with transverse orbital angular momentum. In order to create phase singularity in the spatiotemporal domain, the spatiotemporal differentiator is designed by breaking spatial mirror symmetry. In contrast to pulse shapers, the device proposed here is a simple one-dimensional periodic nanostructure and thus it is much more compact. For a normal incident pulse, the differentiator generates a transmitted STOV pulse with transverse orbital angular momentum. Furthermore, the interference of the generated STOVs can be used to detect the sharp changes of pulse envelopes, in both spatial and temporal dimensions.

1. Introduction

Optical vortex (OV) is a particular type of optical beam that carries orbital angular momentum (OAM) of photons.[1–3] Typically, by creating phase singularities in 2D transverse electromagnetic fields, the generated OV beams can carry OAM along the longitudinal direction. With such a degree of freedom, longitudinal OAM enables various advanced applications in broadband optical communication,[4–6] quantum informatics,[7–9] optical tweezers,[10,11] super-resolution imaging,[12,13] and quantum key distribution.[14]

Beyond the spatial domain, recently intense interest has been attracted to explore the OV in the spatiotemporal domain.[15–18] Remarkably, as an optical pulse, the OAM of spatiotemporal optical vortices (STOVs) can be tilted with respect to the propagation direction and exhibits the transverse components. Recently, such STOVs with transverse OAM have been theoretically proposed and experimentally demonstrated, based on the spatiotemporal control methods with adjustable resolution and applications.[19–26] As a new degree of freedom for optical manipulation, transverse OAM unique in the spatiotemporal domain strongly inspires potential advances to generate STOVs in a novel way.

In this paper, we propose to generate STOVs with an optical spatiotemporal differentiator. Recently, optical analog computation of mathematical differentiation has attracted particular attention because of its advantages of high throughput and real-time processing.[27] We note that previous optical differentiators separately operate in either the spatial[28–44] or temporal[45–54] domain. Here, we design the differentiator to couple both the spatial and temporal computation in the spatiotemporal domain. Moreover, we show that in order to create phase singularities and generate OVs, it is necessary for the spatiotemporal differentiator to break spatial mirror symmetry. In contrast to the pulse shapers,[19–21] the device we propose here is a simple 1D periodic nanostructure directly implemented to the pulse without spatial or temporal Fourier transform, and thus, it is much more compact. We demonstrate that for a Gaussian envelope pulse normally incident on the structure, the transmitted pulse is a STOV carrying transverse OAM. Furthermore, the interference of the generated STOVs can be used to detect the sharp changes of pulse envelopes, in both spatial and temporal domains. Our results show the spatial and temporal detection resolutions of 18 μm and 182 fs, respectively, with great benefits of high-resolution and ultra-fast detection.

2. Principle and Design

Figure 1 schematically shows the spatial-symmetry analysis of spatiotemporal differentiator to generate STOVs. Without loss of generality, we first consider a 1D periodic grating structure with mirror symmetry about the plane \( x = 0 \) (indicated by the dashed line) (Figure 1a). Suppose that a pulse with both Gaussian envelopes in the spatial and temporal domains impinges the structure at normal incidence. Due to the mirror symmetry, the phase distribution of the transmitted pulse must also be symmetric about the mirror plane, and thus there is no phase singularity. Therefore, in order to create a phase singularity and generate
STOVs for any incident pulse at normal incidence, it is necessary to break the mirror symmetry of the structure; that is, the mirror symmetry cannot exist for any incident plane. Below we show that by mirror symmetry breaking, a spatiotemporal differentiator can generate STOVs with transverse OAM (Figure 1b).

To demonstrate the generation of STOVs, we propose a spatiotemporal differentiator for optical pulses with a center wavelength \( \lambda_0 = 1196 \text{ nm} \), by considering practical experimental demonstration with a femtosecond light source with optical parameters. Here, in order to break the mirror symmetry, we design a spatiotemporal differentiator where the two silicon rods in each period are etched on a silicon substrate with different sizes (Figure 1b). By calculating the transmission spectrum function and inspecting the winding number in the phase diagram, the geometry parameters of the two rods are determined as \( h_1 = 388 \text{ nm}, h_2 = 160 \text{ nm}, w_1 = 160 \text{ nm}, \) and \( w_2 = 432 \text{ nm} \), and the gap between them is \( g = 64 \text{ nm} \). The thickness of substrate and the structure period are \( t = 20 \text{ nm} \) and \( p = 1000 \text{ nm} \), respectively. We note that there are much more degrees of freedom for the parameters to generate STOVs for the normal incident light, by considering the spatial mirror symmetry breaking. Such a device with these suitable geometric parameters can be fabricated through standard lithography and etching processing, with exposing different doses of hydrogen silsesquioxane photoresist.[55,56]

We consider an incident pulse with the s-polarization whose electric field is only along the y direction. In order to specifically depict the transformation between the input and the transmitted pulse envelopes, we decompose the incident (transmitted) field into a series of plane waves by Fourier transform as \( s_{\text{in(tra)}}(x,t) = \int s_{\text{in(tra)}}(k_x,\Omega) \exp(ik_x x - i\Omega t) dk_x d\Omega \), where \( s_{\text{in(tra)}} \) are the envelope amplitude and the corresponding envelope spectrum of the incident (transmitted) field, respectively. \( k_x \) is the wavevector component parallel to the structure interface. \( \Omega \) is the sideband angular frequency from the center one \( \omega_0 \), that is, \( \Omega = \omega - \omega_0 \). Then, the pulse envelope transformation is determined by the transmission spectrum function \( H(k_x,\Omega) \equiv \tilde{s}_{\text{tra}}(k_x,\Omega)/\tilde{s}_{\text{ina}}(k_x,\Omega) \).

With the periodicity, our STOV device can only diffract normal incident light in the zeroth order due to \( 2\pi p > k_0 \), where \( k_0 \) is the wavenumber of light in vacuum at the center wavelength \( \lambda_0 \). We numerically calculate the transmission spectrum function \( H \) by the finite-element method using the commercial software package COMSOL with frequency domain module and check the convergence of results. In the simulation, the optical constants for silicon are referred to the experimental data in ref. [57].

Figure 2a,b shows the amplitude and phase distributions of the transmission spectrum function with respect to \( k_x \) and \( \Omega \), respectively. Furthermore, the blue circles and orange rhombi in Figure 2c,d correspond to the amplitudes and phases of \( H \) along \( \Omega = 0 \) and \( k_x = 0 \), respectively. They show that \( H \) exhibits good linear dependence on \( k_x \) and \( \Omega \) and the figure shows the distribution of the structure enables the first-order differentiation in both the spatial and temporal domains. Around \( k_x = 0 \) and \( \Omega = 0 \), the transmission spectrum function \( H \) has the form

\[
H = C_4 k_x + C_3 \Omega \tag{1}
\]

where \( C_4 \) and \( C_3 \) are two complex numbers. We note that the parameter \( C_3 \) has a phase shift from \( C_4 \), which leads to the phase singularity in the spectrum domain shown as Figure 2b.

The blue and orange lines in Figure 2c,d, respectively, show the fitting results of amplitudes and phases with Equation (1), where \( C_4 = 1.92 \exp(-0.31i)/k_0 \) and \( C_3 = 1.39 \exp(-1.88i)/\omega_0 \). Here, \( \omega_0 \) is the frequency at the center wavelength \( \lambda_0 \). We note that since \( C_4/C_3 = 1.38c \times \exp(1.57i) \), where \( c \) is the velocity of light.
light in vacuum, the winding number of the phase singularity is equal to 1, which means that there must be a zero amplitude in an enclosed loop around \( k_x = 0 \) and \( \Omega = 0 \) shown as Figure 2a. Importantly, such a phase singularity leads to the asymmetry phase modulations for the plane waves with \( k_x < 0 \) and with the same \( \Omega \), which can contribute to generating STOVs in the spatiotemporal domain.

We note that for the input signal with \( E_{\text{in}}(x, t) = s_{\text{in}}(x, t) e^{-i\omega_0 t} \), after propagating through the spatiotemporal differentiator, the output signal is expressed as

\[
E_{\text{tran}}(x, t) = s_{\text{tran}}(x, t) e^{-i\omega_0 t}
\]

\[
= \left( -iC_x \frac{\partial s_{\text{in}}}{\partial x} + iC_t \frac{\partial s_{\text{in}}}{\partial t} \right) e^{-i\omega_0 t}
\]

(2)

Here, the output envelope \( s_{\text{tran}} \) is the first order differentiation of the input envelope \( s_{\text{in}} \) in both spatial and temporal domains. Therefore, the STOV generator can be applied to detect sharp changes of pulse envelopes, which is similar to the image processing of edge detection by the spatial differentiators in the real space.[33,39,40] [44]

3. Generation of Spatiotemporal Optical Vortices

To demonstrate the generation of STOVs with transverse OAM, we first simulate an incident pulse with Gaussian envelopes in both spatial and temporal domains. Figure 3a shows the field amplitude distribution of the incident pulse envelope in \((x, t)\) coordinate, where the beam waist and the pulse width are 586 μm and 2046 fs, respectively, ensuring a spectrum bandwidth within the range shown in Figure 2. The incident pulse has a constant phase distribution shown as Figure 3b. After passing through the structure, we simulate the transmitted pulse by the Fourier transform method, with the transmission spectrum function in Figure 2. The amplitude and phase distributions of the transmitted pulse envelope are depicted in Figures 3c and 3d, respectively.

We note that the transmitted pulse has a phase singularity at the pulse center (Figure 3d), where a STOV exhibits the zero amplitude of the pulse (Figure 3c). In comparison with the symmetrical phase distribution of the incident pulse (Figure 3b), the asymmetrical phase of the transmitted pulse envelope originates from the mirror symmetry breaking of the structure. Moreover, as expected, the field amplitude profile of the transmitted pulse in Figure 3c indeed shows a typical first-order Hermite-Gaussian profile along each direction, which has a zero value amplitude at the pulse center. Importantly, the vortex phase distribution in Figure 3d carries a singularity, corresponding to the central zero amplitude in Figure 3c. Since the optical vortex exists only in \((x, t)\) coordinate, it carries a transverse orbital angular momentum.

The spatiotemporal differentiator can generate STOVs for arbitrary amplitude modulated spatiotemporal pulses because the
Figure 3. Generation of a STOV by the spatiotemporal differentiator for a Gaussian-enveloped incident pulse. a) Amplitude and b) phase distributions of the incident pulse envelope. c) Amplitude and d) phase distributions of the transmitted one. The transmitted pulse with transverse OAM has a phase singularity, leading to the zero amplitude at the pulse center.

Figure 4. Generation of STOVs for an arbitrary amplitude modulated spatiotemporal pulse. a) Amplitude and b) phase distributions of an incident pulse envelope as the Zhejiang University logo. The phases of the incident pulse envelope are binary with only 0 or $\pi$, without phase singularities. c) Amplitude and d) phase distributions of the transmitted pulse envelope.

Phase singularity in the transmission spectrum function is non-local, akin to the topological spatial differentiator. To further illustrate the STOV generations, here we consider the incident pulse with the amplitude modulation as the Zhejiang University logo in the spatiotemporal domain (Figure 4a). Correspondingly, the phases of the incident pulse envelope are binary with only 0 or $\pi$ (Figure 4b), and thus without phase singularities. We simulate the pulse transmitted through the spatiotemporal differentiator. Figures 4c and 4d correspond to the amplitude and phase distributions of the transmitted pulse envelope, respectively. Remarkably, Figure 4d shows the generation of large numbers of adjacent STOVs in the spatiotemporal domain.

Moreover, as the generated STOVs interfere with each other, the amplitude distribution of Figure 4c shows that the constructive interference occurs at the sharp changes of the incident pulse envelope in both the spatial and temporal domains, while the destructive one strongly takes place where the amplitudes have slight variations. As shown in Equation (2), this highlighting-sharpness effect is contributed by the differentiation computing of the STOV generator in the spatiotemporal domain.
4. Resolution of Detecting Sharp Change

Since the differentiation computing of the STOV generator works within narrow spatiotemporal bandwidths of the transmission spectrum function, we investigate the resolution of detecting sharp changes. We first estimate the spatial resolution by considering rectangular enveloped pulses with a relatively long duration in order to ensure the temporal spectrum within the bandwidth of the STOV generator. Figure 5a shows the incident pulsed fields with different widths of 2058, 1038, 346, 91, 55, and 18 μm, respectively, while the signal durations are fixed to 24.305 ps. Figure 5b depicts the amplitude distributions of the transmitted pulses. Indeed, the sharp changes of the pulses are clearly highlighted in both the spatial and temporal domains. However, the left and right spatial edges become blurred, and it is difficult to be separated when the envelope width reduces. From the narrowest width, where the two edges can be distinguished, the spatial resolution of the STOV generator is about 18 μm.

Figure 5c,d shows the estimation of the temporal detection resolution for the STOV generator. The incident pulses are also with rectangular envelopes in both spatial and temporal domains (Figure 5c). In order to estimate the temporal resolution, the incident pulses have different durations as 6866, 3464, 1155, 304, 182, and 61 fs. Meanwhile, they have a relatively large width of 7286 μm so as to reduce the impact of the spatial bandwidth. Figure 5d exhibits the amplitude distributions of transmitted pulses. We note that the edges of the four long durations in the spatiotemporal domain are clearly detected. In contrast, it is difficult to distinguish the short duration, and thus the temporal detection resolution of the STOV generator is about 182 fs.

5. Conclusion and Discussion

In summary, we have proposed a spatiotemporal differentiator, which generates STOVs with transverse orbital angular momentum and can be used to detect sharp changes in pulse envelopes. The spatiotemporal differentiator is designed by breaking the mirror symmetry, and the nonlocal effect enables the generation of STOVs for arbitrary modulated spatiotemporal pulses. We note that breaking the mirror symmetry of the device is necessary for the normal incident light, but not sufficient to generate STOVs regarding the winding phase. Furthermore, we show that the interference of the generated STOVs can be used to detect the sharp changes of pulse envelopes, for both spatial and temporal dimensions, with great benefits of high-resolution and ultrafast optical detection. Practically, the input spatiotemporal modulated pulse could be generated by ultrafast moving objects, and a
long duration is preferred in order to alleviate the spatiotemporal diffraction effect.

Also, we can generate an opposite topological charge of the phase singularity, by flipping the structure about the plane $x = 0$. Moreover, by breaking the mirror symmetry, transverse OAM with high-order topological charges can be generated based on the high-order spatiotemporal differentiation. We note that the designed spatiotemporal differentiator is much more compact than the pulse shapers to generate STOVs. Therefore, it paves the way forward generating and modulating the transverse OAM with integrated devices, which could be important in the applications of STOVs.

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Conflict of Interest

Z.R., J.H., J.Z., and T.Z. are named inventors on a number of patent applications related to this work.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Keywords

optical analog computation, optical spatiotemporal differentiators, orbital angular momentums, spatiotemporal optical vortices

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