Production and polarization of $\Upsilon$ mesons in the $k_t$-factorization approach in more detail

S. P. Baranov$^{+1}$, N. P. Zotov$^2$

$^{+}$P.N.Lebedev Institute of Physics, 119991 Moscow, Russia
$^*$ D.V.Skobelzyn Institute of Nuclear Physics, 119991 Moscow, Russia

Submitted 20 October 2008

PACS: 12.38.Bx, 13.85.Ni, 14.40.Gx

1. INTRODUCTION

Nowadays, the production of heavy quarkonium states at high energies is under intense theoretical and experimental study [1, 2]. The production mechanism involves the physics of both short and long distances, and so, appeals to both perturbative and nonperturbative methods of QCD. This feature gives rise to two competing theoretical approaches known in the literature as the color-singlet [3, 4] and color-octet [5] models. According to the color-singlet approach, the formation of a colorless final state takes place already at the level of the hard partonic subprocess (which includes the emission of hard gluons when necessary). In the color-octet model, also known as nonrelativistic QCD (NRQCD), the formation of a meson starts from a color-octet $Q\bar{Q}$ pair and proceeds via the emission of soft nonperturbative gluons. The former model has a well defined applicability range and has already demonstrated its predictive power in describing the $J/\psi$ production at HERA, both in the collinear [6] and the $k_t$-factorization [7] approaches. As it was shown in the analysis of recent ZEUS [8] data, there is no need in the color-octet contribution, neither in the collinear nor in the $k_t$-factorization approach. The numerical estimates of the color octet contributions extracted from the analysis of Tevatron data are at odds with the HERA data, especially as far as the inelasticity parameter $z = E_\gamma/E_\psi$ is concerned [9]. In the $k_t$-factorization approach, the values of the color-octet contributions obtained as fits of the Tevatron data appear to be substantially smaller than the ones in the collinear scheme, or even can be neglected at all [10, 11, 12, 13].

Recently, the results of new theoretical calculations of the next-to-leading (NLO) and next-to-next-to-leading (NNLO) order corrections to colour singlet (CS) quarkonium production have been obtained in the framework of standard pQCD [14]. In the region of moderate $p_T$ ($p_T \geq 10 \text{ GeV}$), these corrections enhance the color singlet production rate by one order of magnitude and even larger. These new results are in much better agreement with the $k_t$-factorization predictions than it was seen for leading order collinear calculations.

In the present note we follow the guideline of our previous publication [15] and show a more detailed analysis of the production and polarization of $\Upsilon$ mesons at the Tevatron conditions using the $k_t$-factorization approach.

2. THEORETICAL FRAMEWORK

In the $k_t$-factorization approach, the cross section of a physical process is calculated as a convolution of the off-shell partonic cross section $\hat{\sigma}$ and unintegrated parton distributions $F_g(x, k_T^2, \mu^2)$, which depend on both the longitudinal momentum fraction $x$ and transverse momentum $k_T$:

$$\sigma_{pp} = \int F_g(x_1, k_{1T}^2, \mu^2) F_g(x_2, k_{2T}^2, \mu^2) \times$$

$$\times \hat{\sigma}_{gg}(x_1, x_2, k_{1T}^2, k_{2T}^2, \cdots) dx_1 dx_2 dk_{1T}^2 dk_{2T}^2.$$  (1)

In accordance with the $k_t$-factorization prescriptions [10, 17, 18, 19], the off-shell gluon spin density matrix is taken in the form

$$\hat{\sigma}^{\mu\nu}_{gg} = p_p^\mu p_p^\nu x_g^2 / |k_T|^2 = k_T^\mu k_T^\nu / |k_T|^2.$$  (2)
In all other respects, our calculations follow the standard Feynman rules.

In order to estimate the degree of theoretical uncertainty connected with the choice of unintegrated gluon density, we use two different parametrizations, which are known to show the largest difference with each other, namely, the ones proposed in Refs. [16, 19] and [20].

In the first case [16], the unintegrated gluon density is derived from the ordinary (collinear) density \( G(x, \mu^2) \) by differentiating it with respect to \( \mu^2 \) and setting \( \mu^2 = k_T^2 \). Here we use the leading order Glück-Reya-Vogt (LO GRV) set [21] as the input collinear density. In the following, this will be referred to as dGRV parametrisation. The other unintegrated gluon density [20] is obtained as a solution of leading order Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [19] in the double-logarithm approximation. Technically, it is calculated as a convolution of the ordinary gluon density with some universal weight factor. In the following, this will be referred to as JB parametrisation.

The production of \( \Upsilon(1S) \) mesons in pp collisions can proceed via either direct gluon-gluon fusion or the production of \( P \)-wave states \( \chi_b \) followed by their radiative decays \( \chi_b \rightarrow \Upsilon + \gamma \). The direct mechanism corresponds to the partonic subprocess \( g + g \rightarrow \Upsilon + g \) which includes the emission of an additional hard gluon in the final state. The production of \( P \)-wave mesons is given by \( g + g \rightarrow \chi_b \), and there is no emission of any additional gluons. As we have already argued in our previous publication [15], we see no need in taking the color-octet contributions into consideration.

The polarization state of a vector meson is characterized by the spin alignment parameter \( \alpha \) which is defined as a function of any kinematic variable as

\[
\alpha(P) = (\sigma / dP - 3 \sigma_L / dP) / (\sigma / dP + \sigma_L / dP),
\]

where \( \sigma \) is the reaction cross section and \( \sigma_L \) is the part of cross section corresponding to mesons with longitudinal polarization (zero helicity state). The limiting values \( \alpha = 1 \) and \( \alpha = -1 \) refer to the totally transverse and totally longitudinal polarizations. We will be interested in the behavior of \( \alpha \) as a function of the \( \Upsilon \) transverse momentum: \( P \equiv |p_T| \). The experimental definition of \( \alpha \) is based on measuring the angular distributions of the decay leptons

\[
d\Gamma(\Upsilon \rightarrow \mu^+ \mu^-) / d\cos \theta \sim 1 + \alpha \cos^2 \theta,
\]

where \( \theta \) is the polar angle of the final state muon measured in the decaying meson rest frame.

The definition of helicity and, consequently, the definition of \( \alpha \) is frame-dependent. There are four commonly used different definitions of the helicity frame:

these are the recoil, the target, the Collins-Soper, and the Gottfried-Jackson systems. In our analysis, we will basically use the recoil system (which, at the Tevatron conditions, is the same as the laboratory or proton-proton center-of-mass system), unless a different choice is explicitly declared.

When considering the polarization properties of \( \Upsilon(1S) \) mesons originating from radiative decays of \( P \)-wave states, we rely upon the dominance of electric dipole \( E1 \) transitions\(^3\). The corresponding invariant amplitudes can be written as [22]

\[
iA(\chi_1 \rightarrow \Upsilon \gamma) \propto \varepsilon^{\mu \nu \alpha \beta} \varepsilon_\nu(\chi_1) \varepsilon^{\alpha}_{\beta}(\Upsilon) \varepsilon^{(\gamma)},
\]

\[
iA(\chi_2 \rightarrow \Upsilon \gamma) \propto p^\mu \varepsilon^{\alpha \beta}_{(\chi_2)} \varepsilon^{(\alpha)}_{\beta}[k_\mu \varepsilon^{(\gamma)}_{\beta} - k_\beta \varepsilon^{(\gamma)}_{\mu}],
\]

with \( p \) and \( k \) being the momenta of the decaying meson and the emitted photon; \( \varepsilon^{(\chi_1)} \), \( \varepsilon^{(\chi_2)} \), \( \varepsilon^{(\Upsilon)} \), and \( \varepsilon^{(\gamma)} \) the respective polarization vectors; and \( \varepsilon^{\mu \nu \alpha \beta} \) the antisymmetric Levi-Civita tensor. This leads to the following relations between the production cross sections for different helicity states (see Eq. (14) in [22]):

\[
\sigma(\Upsilon(h=0)) = B(\chi_1 \rightarrow \Upsilon \gamma) \left[ \frac{1}{3} \sigma_{\chi_1(h=|l|=1)} + \frac{1}{3} \sigma_{\chi_2(h=|l|=1)} \right] + B(\chi_2 \rightarrow \Upsilon \gamma) \left[ \frac{1}{3} \sigma_{\chi_1(h=|l|=1)} + \frac{1}{3} \sigma_{\chi_2(h=|l|=1)} \right]
\]

\[
\sigma(\Upsilon(h=|l|=1)) = B(\chi_1 \rightarrow \Upsilon \gamma) \left[ \frac{1}{3} \sigma_{\chi_1(h=|l|=1)} + \frac{1}{3} \sigma_{\chi_2(h=|l|=1)} \right] + B(\chi_2 \rightarrow \Upsilon \gamma) \left[ \frac{1}{3} \sigma_{\chi_1(h=|l|=1)} + \frac{1}{3} \sigma_{\chi_2(h=|l|=1)} \right]
\]

The dominance of electric dipole transitions (at least for the charmonium family) is supported by the recent experimental data collected by the E835 Collaboration [23] at the Fermilab.

All the other essential parameters were taken as in our previous paper: the b-quark mass \( m_b = m_{\Upsilon}/2 = 4.75 \text{ GeV} \); the \( \Upsilon \) meson wave function \( |\Psi_{\Upsilon}(0)|^2 = 0.4 \text{ GeV}^3 \) (known from the leptonic decay width \( \Gamma_{\Upsilon \rightarrow l \nu} \) [24]); the wave function of \( P \)-wave states \( |\Psi_P(0)|^2 = 0.12 \text{ GeV}^3 \) (taken from the potential model [25]); the radiative decay branchings \( Br(\chi_b \rightarrow \Upsilon \gamma) = 0.06, 0.35, 0.22 \) for \( (J = 0, 1, 2) \) [24]; the renormalization and factorization scale \( \mu_R^2 = \mu_F^2 = \mu^2 = m_{\Upsilon}^2 + p_T^2 \).

\(^3\)In our previous paper [15], two somewhat different models were used for this process.
3. NUMERICAL RESULTS

The results of our calculations are presented in Figs. 1-4. Fig. 1 displays the $p_T$ dependence of the differential cross section and spin alignment parameter $\alpha$ for four different intervals of rapidity. Complementary to Fig. 1, Fig. 2 exhibits the rapidity dependence of the cross section and parameter alpha for three different intervals of $p_T$. Everywhere, we separately show the contribution from the direct production mechanism taken solely (thin curves) and after having the $\chi$ decays added (thick curves). When possible, we compare our theoretical predictions with experimental measurements [26]-[28].

First of all, we notice the importance of the feed-down from $\chi_b$ decays, without which the experimental data can hardly be understood. The calculations seem to underestimate the cross section data by approximately a factor of 2. This can be considered as a room for higher order corrections and contributions from other possible subprocesses, such as the associated production of $\Upsilon + b + \bar{b}$ states. The latter was shown to be comparable in size with the ordinary production at high $p_T$ [29]. Any way, the disagreement by a factor of 2 must not be taken too seriously, as it lies within the uncertainty connected with the choice of factorization and renormalization scales 4). The JB gluon density leads to significantly better agreement with the data than the dGRV density.

While the direct and indirect production mechanisms lead to more or less similar $p_T$ and $y$ spectra, the behavior of the polarization is very much different. This is seen in the right parts of Figs. 1 and 2, and is vividly shown in Fig. 3.

Our results for the direct mechanism are also applicable to the production of $\Upsilon (3S)$ states, with the only exception that the overall dimuon rate $BR_{\mu \mu} \sigma (\Upsilon)$ is lower by an approximate factor of 4 because of smaller value of the wave function $|\Psi (0)_{1S}|^2 : |\Psi (0)_{1S}|^2 \propto \Gamma_{t+l-}(3S) : \Gamma_{t+l-}(1S) = 0.44 : 1.34$ and smaller branching fraction (2.18% versus 2.48% [24]). In this case, the absence of the feed-down from $\chi_b$ decays would make the experimental sample cleaner and clearer for theoretical analysis 5).

We also have to draw attention to the fact that the behavior of the spin alignment parameter $\alpha$ is frame dependent, as is demonstrated in Fig. 4. In particular, the sharp dip of $\alpha$ at $y = 0$ is only seen in the recoil system, but not in either of the other three helicity systems. This property has to be not forgotten in order that the comparison between the theoretical and experimental results be fully adequate.

4. CONCLUSIONS

We have considered the production of $\Upsilon$ mesons in high energy $pp$ collisions in the $k_t$-factorization approach and compared the predictions on the differential cross sections and spin alignment parameter $\alpha$ with new D0 and CDF data. We find a more or less reasonable agreement in all cases.

We have argued that measuring the double differential cross sections and, especially, the polarization of quarkonium states in extended $p_T$ and rapidity intervals can provide interesting and important information on their production mechanisms.

The purest probe is provided by the polarization of $\Upsilon (3S)$ mesons. In that case, the polarization is the strongest and the predictions are free from uncertainties coming from radiative $\chi_b$ decays.

ACKNOWLEDGMENTS

This work was supported by the FASI of RF (Grant No. NS-1856.2008.2), the RFBR foundation (Grant No. 08-02-00896-a), and DESY Directorate in the framework of Moscow-DESY project on MC implementation for HERA-LHC.
Differential cross section and spin alignment parameter $\alpha$ as functions of the $\Upsilon(1S)$ transverse momentum $p_T$, integrated over four different rapidity intervals. The panels from top to bottom: $|y| < 0.6$; $0.6 < |y| < 1.2$; $1.2 < |y| < 1.8$; $1.8 < |y|$. Dashed histograms, $d$GRV gluon density; dash-dotted histograms, JB gluon density. Thin lines, the direct contribution only; thick lines, with the feed-down from $\chi_b$ states added. Experimental points: $\bullet$ D0 [26]; $\circ$ CDF [27]; $*$ D0 (preliminary) [28].

Fraction of longitudinally polarised $\Upsilon(1S)$ mesons $d\sigma(\text{helicity}=0)/d\sigma(\text{all helicities})$ as function of the transverse momentum $p_T$ and rapidity $y$. Upper panel, direct subprocess; lower panel, $\chi_b$ decays solely.

Rapidity dependence of the parameter $\alpha$ as seen in the different helicity frames (sole $\chi_b$ contribution with JB gluon densities). Dash-dotted histograms, recoil system; dashed histograms, target system (equivalent to Gottfried-Jackson system); dotted histograms, Collins-Soper system.
