Energy conditions in $f(R, L_m)$ gravity

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Abstract

In order to constrain $f(R, L_m)$ gravity from theoretical aspects, its energy conditions are derived in this paper. These energy conditions given by us are quite general and can be degenerated to the well-known energy conditions in general relativity and $f(R)$ theories of gravity with arbitrary coupling, non-minimal coupling and non-coupling between matter and geometry, respectively, as special cases. To exemplify how to use these energy conditions to restrict $f(R, L_m)$ gravity, we consider a special model in the FRW cosmology and give some corresponding results by using astronomical observations.

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1. Introduction

Recent observations of type Ia supernovae (SNe Ia) [1] indicate that the expansion of the Universe is accelerating at the present time. Moreover, from the observational data of the large scale structure (LSS) [2], the cosmic microwave background (CMB) radiation [3] and the baryon acoustic oscillation (BAO), we know that our Universe is spatially flat and the cosmic budget of energy is as follows: usual baryon matter occupies about 4%, dark matter occupies about 23% and dark energy occupies about 73%. It is reasonable to believe that the acceleration of the Universe is probably driven by dark energy, which is an exotic component with negative pressure. Several candidates for dark energy have been extensively investigated [4]. Unfortunately, a satisfactory answer to the questions of what dark energy is and where it comes from has not yet been obtained.

As an alternative to dark energy, the late-time acceleration of the Universe can be explained by modified theories of gravity [5]. There are numerous ways to deviate from Einstein’s theory of general relativity (GR). Among these theories, $f(R)$ theories of gravity (see, for instance, [6] for reviews) are very competitive. Here $f(R)$ is an arbitrary function of the Ricci scalar $R$. One can add any form of $R$ in it, such as $1/R$ [7], $\ln R$ [8], positive and negative powers of $R$ [9], Gauss–Bonnet invariant [10], etc. It is worth stressing that considering some additional conditions, the early-time inflation and late-time acceleration can be unified by different
roles of gravitational terms relevant at small and at large curvatures. Considering the matter–
geometry coupling, models of $f(R)$ gravity with non-minimal coupling [11] and arbitrary
coupling [12] have been proposed. Besides them, a more general model of $f(R)$ gravity, in
which the Lagrangian density takes the form of $f(R, L_m)$, has recently been proposed [13],
and usually called $f(R, L_m)$ gravity. Here $f(R, L_m)$ is an arbitrary function of the Ricci scalar
and of the Lagrangian density corresponding to matter. The astrophysical and cosmological
implications of these mentioned models have been extensively investigated in [14].

Since many models of $f(R)$ theories of gravity have been proposed, it gives rise to the
problem of how to constrain them from theoretical aspects. One possibility is by imposing the
so-called energy conditions [15]. As is well known, these energy conditions are used in different
contexts to derive general results that hold for a variety of situations [16–20]. Considering
these energy conditions, one is allowed not only to establish gravity which remains attractive,
but also to keep the demands that the energy density is positive and cannot flow faster than
light. In order to make $f(R, L_m)$ gravity contain the above-mentioned features, we derive and
discuss its energy conditions in this paper.

This paper is organized as follows. The fundamental elements of $f(R, L_m)$ gravity are
given in section 2. In section 3, the well-known energy conditions, namely the strong energy
condition (SEC), the null energy condition (NEC), the weak energy condition (WEC) and
the dominant energy condition (DEC) are derived in $f(R, L_m)$ gravity. For obtaining the first
two energy conditions, the Raychaudhuri equation which is their physical origin is used. It is
worth stressing that, from the calculation, we find that the equivalent results can be obtained
by taking the transformations $\rho \rightarrow \rho^{\text{ef}}$ and $p \rightarrow p^{\text{ef}}$ into $\rho + 3p \geq 0$ and $\rho + p \geq 0$,
respectively. Thus by extending this approach to $\rho - p \geq 0$ and $\rho \geq 0$, the DEC and WEC
in $f(R, L_m)$ gravity are obtained. In order to obtain some insights into the meaning of these
energy conditions, we apply them to a special model with $f(R, L_m) = \Lambda \exp \left( \frac{1}{\Lambda^2} R + \frac{1}{2} L_m \right)$. In
section 4, by using the parameters of the deceleration ($q$), the jerk ($j$) and the snap ($s$),
the energy conditions of $f(R, L_m)$ gravity are rewritten. Furthermore, by taking the rewritten
WEC and present astronomical observations and considering $L_m = -\rho$ and $L_m = p$ [21],
respectively, we constrain the parameter $\Lambda$ for the same model as in the previous section. The
last section contains our conclusion.

2. $f(R, L_m)$ gravity

A maximal extension of the Hilbert–Einstein action for $f(R)$ gravity has been proposed in
[13]. Its action takes the following form:

$$ S = \int f(R, L_m) \sqrt{-g} \, d^4x, $$

where $f(R, L_m)$ is an arbitrary function of the Ricci scalar $R$ and of the Lagrangian density
corresponding to matter $L_m$. When $f(R, L_m) = \frac{1}{2} f_i(R) + G(L_m) f_2(R)$, the action of $f(R)$
gravity with arbitrary matter–geometry coupling is recovered, where $f_i(R)$ ($i = 1, 2$) and
$G(L_m)$ are arbitrary functions of the Ricci scalar and the Lagrangian density of matter,
respectively. Furthermore, by setting $f_2(R) = 1 + \lambda f_2(R)$, $G(L_m) = L_m$; $f_1(R) = f(R)$,
$f_2(R) = 1$, $G(L_m) = L_m$ and $f_1(R) = R$, $f_2(R) = 1$ and $G(L_m) = L_m$, action (1) can be
reduced to the context of $f(R)$ gravity with non-minimal coupling, non-coupling between
matter and geometry and GR, respectively.

Varying the action (1) with respect to the metric $g^{\mu\nu}$ yields the field equations

$$ f_R(R, L_m) R_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f_R(R, L_m) = \frac{1}{2} [f(R, L_m) - f_{\Lambda m}(R, L_m)] g_{\mu\nu} $$

$$ = \frac{1}{2} f_{\Lambda m}(R, L_m) T_{\mu\nu}, $$

(2)
\[ \Box = g^{\mu \nu} \nabla_\mu \nabla_\nu, \quad f_R(R, L_m) = \frac{\partial f(R, L_m)}{\partial R} \quad \text{and} \quad f_{L_m}(R, L_m) = \frac{\partial f(R, L_m)}{\partial L_m}, \]

respectively. To obtain equation (2), it has been assumed that the Lagrangian density of matter, \( L_m \), only depends on the components of the metric tensor and not on its derivatives. The matter energy–momentum tensor is defined as

\[ T_{\mu \nu} = -\frac{2}{\sqrt{-g}} \delta \left( \sqrt{-gL_m} \right) \delta g_{\mu \nu}. \quad (3) \]

The contraction of equation (2) provides

\[ f_R(R, L_m) R + \frac{3}{2} f_R(R, L_m) - 2 \left[ f(R, L_m) - f_{L_m}(R, L_m) L_m \right] = \frac{1}{2} f_{L_m}(R, L_m) T, \quad (4) \]

where \( T = T^\mu_\mu \).

Taking the covariant divergence of equation (2) and using the mathematical identity [22],

\[ \nabla_\mu \left[ f_R(R, L_m) R_{\mu \nu} - \frac{1}{2} f(R, L_m) g_{\mu \nu} + (g_{\mu \nu} \Box - \nabla_\mu \nabla_\nu) f_R(R, L_m) \right] \equiv 0, \quad (5) \]

one deduces the following generalized covariant conservation equation:

\[ \nabla_\mu T^\mu_\nu = 2 \nabla^\mu \ln \left[ f_{L_m}(R, L_m) \right] \frac{\partial L_m}{\partial g^{\mu \nu}}. \quad (6) \]

It is clear that the non-minimal coupling between curvature and matter yields a non-trivial exchange of energy and momentum between the geometry and matter fields [23]. However, once \( L_m \) is given, by choosing appropriate forms of \( f_{L_m}(R, L_m) \), one can construct, at least in principle, a conservative model in \( f(R, L_m) \) gravity.

3. Energy conditions in \( f(R, L_m) \) gravity

3.1. The Raychaudhuri equation

In order to obtain the energy conditions in \( f(R, L_m) \) gravity, we simply review the Raychaudhuri equation, which is the physical origin of the NEC and the SEC [24].

In the case of a congruence of time-like geodesics defined by the vector field \( u^\mu \), the Raychaudhuri equation is given by

\[ \frac{d\theta}{d\tau} = \frac{1}{3} \theta^2 - \sigma_{\mu \nu} \sigma^{\mu \nu} + \omega_{\mu \nu} \omega^{\mu \nu} - R_{\mu \nu} u^\mu u^\nu, \quad (7) \]

where \( R_{\mu \nu}, \theta, \sigma_{\mu \nu}, \text{and} \omega_{\mu \nu} \) are the Ricci tensor, the expansion parameter, the shear and the rotation associated with the congruence, respectively. While in the case of a congruence of null geodesics defined by the vector field \( k^\mu \), the Raychaudhuri equation is given by

\[ \frac{d\theta}{d\tau} = \frac{1}{2} \theta^2 - \sigma_{\mu \nu} \sigma^{\mu \nu} + \omega_{\mu \nu} \omega^{\mu \nu} - R_{\mu \nu} k^\mu k^\nu. \quad (8) \]

From the above expressions, it is clear that the Raychaudhuri equation is purely geometric and independent of gravity theory. In order to constrain the energy–momentum tensor by the Raychaudhuri equation, one can use the Ricci tensor from the field equations of gravity to make a connection. Namely, through the combination of the field equations of gravity and the Raychaudhuri equation, one can obtain physical conditions for the energy–momentum tensor. Since \( \sigma^2 \equiv \sigma_{\mu \nu} \sigma^{\mu \nu} \geq 0 \) (the shear is a spatial tensor) and \( \omega_{\mu \nu} = 0 \) (hypersurface orthogonal congruence), from equations (7) and (8), the conditions for gravity to remain attractive \((d\theta/d\tau < 0)\) are

\[ R_{\mu \nu} u^\mu u^\nu \geq 0 \quad \text{SEC}, \quad (9) \]
Thus by means of relationship (9) and Einstein’s field equations, one obtains

\[ R_{\mu\nu} u^\mu u^\nu = \left( T_{\mu\nu} - \frac{T}{2} g_{\mu\nu} \right) u^\mu u^\nu \geq 0, \]  

(11)

where \( T_{\mu\nu} \) is the energy–momentum tensor and \( T \) is its trace. If one considers a perfect fluid with energy density \( \rho \) and pressure \( p \),

\[ T_{\mu\nu} = (\rho + p) U_\mu U_\nu - p g_{\mu\nu}, \]

(12)

relationship (11) turns into the well-known SEC of Einstein’s theory, i.e.

\[ \rho + 3p \geq 0. \]  

(13)

Similarly, by using relationship (10) and Einstein’s field equations, one has

\[ T_k^\mu k^\nu \geq 0. \]  

(14)

Then considering equation (12), the familiar NEC of GR can be reproduced as

\[ \rho + p \geq 0. \]  

(15)

### 3.2. Energy conditions in \( f(R, L_m) \) gravity

The Einstein tensor resulting from the field equation (2) is

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}^{\text{eff}}, \]

(16)

where the effective energy–momentum tensor \( T_{\mu\nu}^{\text{eff}} \) is defined as follows:

\[
T_{\mu\nu}^{\text{eff}} = \frac{1}{f_R(R, L_m)} \left\{ \frac{1}{2} g_{\mu\nu} \left[ f(R, L_m) - R f_R(R, L_m) \right] \right.
- (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f_R(R, L_m) + \frac{1}{2} f_{L_m}(R, L_m) L_m g_{\mu\nu}
+ \frac{1}{2} f_{L_m}(R, L_m) T_{\mu\nu} \left\}, \right.
\]

(17)

where \( \Box = g^{\mu\nu} \nabla_\mu \nabla_\nu, \) \( f_R(R, L_m) = \partial f(R, L_m) / \partial R \) and \( f_{L_m}(R, L_m) = \partial f(R, L_m) / \partial L_m, \) respectively. Contracting equation (17), we have

\[
T^{\text{eff}} = \frac{1}{f_R(R, L_m)} \left\{ 2 \left[ f(R, L_m) - R f_R(R, L_m) \right] - \Box f_R(R, L_m) \right.
+ 2 f_{L_m}(R, L_m) L_m + \frac{1}{2} f_{L_m}(R, L_m) T \left\}. \right.
\]

(18)

where \( T = g^{\mu\nu} T_{\mu\nu} \). Thus, we can write the Ricci tensor in terms of an effective stress–energy tensor and its trace, i.e.

\[ R_{\mu\nu} = T_{\mu\nu}^{\text{eff}} - \frac{1}{2} g_{\mu\nu} T^{\text{eff}}. \]

(19)

In order to keep gravity attractive, besides expressions (9) and (10), the following additional condition should be required:

\[ \frac{f_{L_m}(R, L_m)}{f_R(R, L_m)} > 0. \]

(20)

Note that this condition is independent of the ones derived from the Raychaudhuri equation (i.e. expressions (9) and (10)), and only relates to an effective gravitational coupling.
The FRW metric is chosen as
\[ ds^2 = dt^2 - a^2(t) \, dx_i^2, \]
where \( a(t) \) is the scale factor and \( dx_i^2 \) contains the spatial part of the metric. Using this metric, we can obtain \( R = -6(2H^2 + \dot{H}) \), where \( H = \dot{a}(t)/a(t) \) is the Hubble expansion parameter, and \( \Gamma^\mu_{\nu\lambda} = a(t) \delta_{\nu\lambda}/(\mu, \nu \neq 0) \), which are the components of the affine connection.

By using relationship (9) and equation (19), the SEC can be given as
\[ T_{\mu\nu}^{\text{eff}} u^\mu u^\nu - \frac{1}{2} T^{\text{eff}} \geq 0, \]
where we have used the condition \( g_{\mu\nu} u^\mu u^\nu = 1 \). Taking the energy–momentum tensor, \( T_{\mu\nu}^{\text{eff}} \), to be a perfect fluid (i.e. equation (12)) and considering condition (20), equation (22) turns into
\[ \rho + 3p = \frac{2}{f_{L_{m}}(R, L_{m})} [f(R, L_{m}) - R f(R, L_{m})] + \frac{6}{f_{L_{m}}(R, L_{m})} \left[f_{RR}(R, L_{m}) \dot{R}^2 + f_{RR}(R, L_{m}) \dot{R}^2\right] + H f_{RR}(R, L_{m}) \dot{R} - 2L_{m} \geq 0, \]
where the dot denotes differentiation with respect to cosmic time. This is the SEC in \( f(R, L_{m}) \) gravity.

The NEC in \( f(R, L_{m}) \) gravity can be expressed as
\[ T_{\mu\nu}^{\text{eff}} k^\mu k^\nu \geq 0. \]
By the same method as the SEC, the above relationship can be changed into
\[ \rho + p + \frac{2}{f_{L_{m}}(R, L_{m})} \left[f_{RR}(R, L_{m}) \dot{R}^2 + f_{RR}(R, L_{m}) \dot{R}^2\right] \geq 0. \]
From the above discussion, it is worth stressing that by taking \( f(R, L_{m}) = \frac{1}{2}f_1(R) + G(L_{m}) f_2(R) \) in expressions (23) and (25), we can obtain the SEC and the NEC in \( f(R) \) gravity with arbitrary matter–geometry coupling, which are just the results given in [19]. Furthermore, by setting \( f_2(R) = 1 + \lambda f_2(R), G(L_{m}) = L_{m} \) and \( f_1(R) = f(R), f_2(R) = 1 \) and \( G(L_{m}) = L_{m} \), we can obtain the SEC and NEC in \( f(R) \) gravity with non-minimal coupling and non-coupling between matter and geometry, which are just the same as the ones in [18] and [17], respectively. For \( f_1(R) = R, f_2(R) = 1 \) and \( G(L_{m}) = L_{m} \), the SEC and NEC in GR, i.e. \( \rho + 3p \geq 0 \) and \( \rho + p \geq 0 \), can be reproduced.

Note that both the above expressions of the SEC and NEC are directly derived from the Raychaudhuri equation. However, from the calculation, we find that the equivalent results can be obtained by taking the transformations \( \rho \rightarrow \rho^{\text{eff}} \) and \( p \rightarrow p^{\text{eff}} \) into \( \rho + 3p \geq 0 \) and \( \rho + p \geq 0 \), respectively. Thus by extending this approach to \( \rho \rightarrow \rho^{\text{eff}} \) and \( p \rightarrow p^{\text{eff}} \), we can give the DEC and WEC in \( f(R, L_{m}) \) gravity.

By means of equations (17) and (21), the effective energy density and the effective pressure can be derived as
\[ \rho^{\text{eff}} = \frac{1}{f_{R}(R, L_{m})} \left\{ \frac{1}{2} f(R, L_{m}) - R f_{R}(R, L_{m}) \right\} - 3H f_{RR}(R, L_{m}) \dot{R} + \frac{1}{2} f_{L_{m}}(R, L_{m}) L_{m} + \frac{1}{2} f_{L_{m}}(R, L_{m}) \rho, \]
\[ p^{\text{eff}} = \frac{1}{f_{R}(R, L_{m})} \left\{ \frac{1}{2} \left[R f_{R}(R, L_{m}) - f(R, L_{m}) \right] + f_{RR}(R, L_{m}) \dot{R}^2 + f_{RR}(R, L_{m}) \dot{R} + 3H f_{RR}(R, L_{m}) \dot{R} - \frac{1}{2} f_{L_{m}}(R, L_{m}) L_{m} \right\} + \frac{1}{2} f_{L_{m}}(R, L_{m}) p. \]
Then the corresponding DEC and WEC in $f(R, L_m)$ gravity can be respectively given as
\begin{equation}
\rho - p + \frac{2}{f_{,m}(R, L_m)}[f(R, L_m) - Rf_k(R, L_m)] - \frac{2}{f_{,m}(R, L_m)}[f_{,RR}(R, L_m)\dddot{R}^2 + f_{,RR}(R, L_m)\dot{R}]
+ 6H f_{,RR}(R, L_m)\dot{R} + 2L_m \geq 0,
\end{equation}

(28)

\begin{equation}
\rho + \frac{1}{f_{,m}(R, L_m)}[f(R, L_m) - Rf_k(R, L_m)] - \frac{6}{f_{,m}(R, L_m)}H f_{,RR}(R, L_m)\dot{R} + L_m \geq 0.
\end{equation}

(29)

We emphasize that by taking $f(R, L_m) = \frac{1}{2}f_1(R) + G(L_m)f_2(R)$, the above expressions are the DEC and WEC in $f(R)$ gravity with arbitrary matter–geometry coupling, which are just the same as the ones in [19]. Furthermore, by setting $f_2(R) = 1 + \lambda f_2(R)$, $G(L_m) = L_m$ and $f_1(R) = f(R)$, $f_2(R) = 1$ and $G(L_m) = L_m$, the results given by us are the DEC and the WEC in $f(R)$ gravity with non-minimal coupling and non-coupling between matter and geometry, which are consistent with the results given in [18] and [17], respectively. For $f_1(R) = R$, $f_2(R) = 1$ and $G(L_m) = L_m$, the DEC and the WEC in GR, i.e. $\rho - p \geq 0$ and $\rho \geq 0$, can be reproduced.

In order to get some insights into the meaning of these energy conditions, we consider a specific type of model in $f(R, L_m)$ gravity, which takes the form as

\begin{equation}
f(R, L_m) = \Lambda \exp \left(\frac{R}{2\Lambda} + \frac{1}{\Lambda}L_m\right),
\end{equation}

(30)

where $\Lambda > 0$ is an arbitrary constant. In the limit $(1/2\Lambda)R + (1/\Lambda)L_m \ll 1$, we obtain

\begin{equation}
f(R, L_m) \approx \Lambda + \frac{R}{2} + L_m + \cdots,
\end{equation}

(31)

which is the Lagrangian density of GR with a cosmological constant.

In the FRW cosmology, the energy conditions for model of equation (30) can be written as

\begin{equation}
A + B \geq C
\end{equation}

(32)

where $A$, $B$ and $C$ depend on the energy condition under study. For the SEC, one finds

\begin{equation}
A^{SEC} = \rho + 3p,
\end{equation}

(33a)

\begin{equation}
B^{SEC} = R + 3H\dddot{R} + \frac{3(\dddot{R})^2\dddot{R}}{2\Lambda} + L_m \frac{6(\dddot{R})^4}{16\Lambda^3},
\end{equation}

(33b)

\begin{equation}
C^{SEC} = 2(\Lambda + L_m).
\end{equation}

(33c)

For the NEC, one obtains

\begin{equation}
A^{NEC} = \rho + p,
\end{equation}

(34a)

\begin{equation}
B^{NEC} = \frac{\dddot{R}^2\dddot{R}}{16\Lambda^3},
\end{equation}

(34b)

\begin{equation}
C^{NEC} = 0.
\end{equation}

(34c)
For the DEC, one has
\[ A^{\text{DEC}} = \rho - p, \quad (35a) \]
\[ B^{\text{DEC}} = -\left( R + \frac{3H\dot{R}}{\Lambda} + \frac{\hat{R}^2 \hat{R} \exp \left( \frac{\hat{R} + 2L_m}{2\Lambda} \right)}{16\Lambda^3} \right), \quad (35b) \]
\[ C^{\text{DEC}} = -2(\Lambda + L_m). \quad (35c) \]

Finally, for the WEC, one obtains
\[ A^{\text{WEC}} = \rho, \quad (36a) \]
\[ B^{\text{WEC}} = -\frac{1}{2} \left( R + \frac{3H\dot{R}}{\Lambda} \right), \quad (36b) \]
\[ C^{\text{WEC}} = -(\Lambda + L_m). \quad (36c) \]

Given these definitions, the study of all the energy conditions can be performed by satisfying the inequality (32).

4. Constraints on \( f(R, L_m) \) gravity

The Ricci scalar \( R \) and its derivatives can be expressed by the parameters of the deceleration \( q \), the jerk \( j \) and the snap \( s \) [25], namely
\[ R = -6H^2(1 - q), \quad (37a) \]
\[ \dot{R} = -6H^3(j - q - 2), \quad (37b) \]
\[ \ddot{R} = -6H^4(s + q^2 + 8q + 6), \quad (37c) \]

where
\[ q = -\frac{1}{H^2 \ddot{a}}, \quad j = \frac{1}{H^3 \dot{a}} \quad \text{and} \quad s = \frac{1}{H \ddot{a}}. \quad (38) \]

Thus, the energy conditions (23), (25), (28) and (29) can be rewritten as
\[ \rho + 3p - 2 \left( \frac{f(R, L_m) - 6H^2(q - 1)f_R(R, L_m)}{f_{L_m}(R, L_m)} + L_m \right) \]
\[ + \left[ -1 - 36H^6(j - q - 2)(6 + 8q + q^2 + s)f_{RRR}(R, L_m) \right] \]
\[ \times 36H^4(j - q - 2)f_{RR}(R, L_m) \geq 0 \quad \text{(SEC)}, \quad (39a) \]
\[ \rho + p - 432H^8(2 - j + q)^2(6 + 8q + q^2 + s) \]
\[ \times \left( \frac{f_{RR}(R, L_m)f_{RRR}(R, L_m)}{f_{L_m}(R, L_m)} \right) \geq 0 \quad \text{(NEC)}, \quad (39b) \]
\[ \rho - p + 2 \left( \frac{f(R, L_m) - 6H^2(q - 1)f_R(R, L_m)}{f_{L_m}(R, L_m)} + 2L_m \right) \]
\[ - \left[ -1 - 6H^6(j - q - 2)(6 + 8q + q^2 + s)f_{RRR}(R, L_m) \right] \]
\[ \times 72H^4(j - q - 2)f_{RR}(R, L_m) \geq 0 \quad \text{(DEC)}, \quad (39c) \]
In this paper, we have derived the energy conditions (SEC, NEC, DEC, WEC) in $f(R, L_m)$ gravity.

The energy conditions obtained in this paper are quite general, which includes the corresponding results given in $f(R)$ theories of gravity with arbitrary coupling, non-minimal coupling and non-coupling between matter and geometry as well as in GR, respectively, as special cases.

Furthermore, to exemplify how to use these derived energy conditions to constrain $f(R, L_m)$ gravity, we considered a special model with $f(R, L_m) = \Lambda \exp \left( \frac{1}{\Lambda} R + \frac{1}{\Lambda} L_m \right)$. By virtue of the WEC, present astronomical observations and considering $L_m = -\rho$ and $L_m = p$, respectively, we constrained the range of the parameter $\Lambda$.

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