Lepton pair production by high-energy neutrino in an external electromagnetic field

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Abstract

The process of the lepton pair production by a neutrino propagating in an external electromagnetic field is investigated in the framework of the Standard Model. Relatively simple exact expression for the probability as the single integral is obtained, which is suitable for a quantitative analysis.

Nowadays, it is well established that a medium makes an active influence on the quantum processes. It stimulates a constantly growing interest in the particle physics in medium, especially in view of possible astrophysical manifestations. It should be noted that a consideration of an intense electromagnetic field as the medium, along with a dense matter, is physically justified indeed. Really, the field strengths inside the astrophysical objects can reach the critical Schwinger value \( B_c = m_e^2/e \simeq 4.41 \times 10^{13} \text{ G} \), and even could exceed it essentially. On the other hand, the situation is possible when

\[ ^1 \text{We use natural units in which } c = \hbar = 1. \]
the so-called field dynamical parameter $\chi$ of the relativistic particle propagating in a relatively weak electromagnetic field, $F < B_e$ ($F = E$ and/or $B$), could appear rather high. In this case the field in the particle rest frame can exceed essentially the critical value and is very close to the crossed field ($\vec{E} \perp \vec{B}$, $\mathcal{E} = B$). Thus, the calculation in a constant crossed field is the relativistic limit of the calculation in an arbitrary relatively weak smooth field. Consequently, the results obtained in a crossed field possesses a great extent of generality, and acquires interest by itself.

It is known that such intense electromagnetic fields allow the processes which are kinematically forbidden in vacuum, such as the creation of the lepton pair by a neutrino, $\nu \rightarrow \nu \ell^- \ell^+$ ($\ell = e, \mu, \tau$). It should be noted, that the $\ell^- \ell^+$ pair can have a sufficiently large space-like total momentum in an electromagnetic field, due to the specific kinematics of a charged particle in this field. Therefore, the process with a relativistic neutrino becomes purely diagonal with respect to the neutrino flavor and is insensitive to its mass and to the mixing in the lepton sector.

The theoretical study of the process of the electron-positron pair production by a neutrino in the crossed field limit has a rather long history [1–5]. A correct type of the dependence of the probability on the dynamical parameter $\chi$ in the leading log approximation, namely, $\sim \chi^2 \ln \chi$, was found in the paper [1], where the numerical coefficient was wrong, however. In the succeeding papers attempts were made to adjust this coefficient and to find the next post-logarithm terms which could appear quite essential when $\ln \chi$ was not very large.

According to the definition of the problem in the crossed field approximation, one should speak about the ultrarelativistic neutrino only, which exists as the left-handed one due to the chiral type of its interaction in the frame of the Standard Model, even if the neutrino mass is non-zero. A lack of understanding of the fact that nonpolarized ultrarelativistic neutrinos did not exist in Nature, often led to the appearance of an erroneous extra factor $1/2$ in expressions for the probabilities of the processes with initial neutrinos, due to an unphysical averaging over the neutrino polarizations, see e.g. the papers [6, 7].

The results for the probability of the process $\nu \rightarrow \nu e^- e^+$ in the crossed

\[ \chi = e(p_\alpha F_{\alpha \beta} F_{\beta \sigma} p_\sigma)^{1/2}/m^3, \]

where $p_\alpha$ is the particle four-momentum, $F_{\alpha \beta}$ is the electromagnetic field tensor.
field, which were obtained in the listed papers, had essential distinctions. This probability for the case of a high-energy neutrino ($\chi \gg 1$) can be presented in the following form

\[ w(\nu \to \nu e^- e^+) = K w_0 \chi^2 \left( \ln \chi - \frac{1}{2} \ln 3 - \gamma_E + \Delta \right), \tag{1} \]

where

\[ w_0 = \frac{G_F^2 (g_V^2 + g_A^2) m_e^6}{27\pi^3 E_\nu}, \tag{2} \]

$\gamma_E = 0.577\ldots$ is the Euler constant, $g_V, g_A$ are the constants in the effective local Lagrangian of the $\nu e e$ interaction, see Eq. (3) below. In the recent paper [7] devoted to the study of the massive neutrino decay $\nu_i \to \nu_j e^- e^+ (m_i > m_j + 2m_e)$ in an external field, a comparison was also made of various results for the process probability. However, the statement which was made in [7] about a mutual agreement of the results was incorrect, in our opinion. Really, the constants $K$ and $\Delta$ introduced in Eq. (1), were obtained by the authors as follows, see Table 1.

Note that in the papers [1, 7] calculations were performed for the case of the electron – neutrino interaction via the $W$-boson only. To compare our Eq. (1) with the results of these papers, one should suppose in this formula $g_V = g_A = 1$ [1], and $g_V = g_A = |U_{ei} U_{e3}|$ [7], correspondingly. Loss of the factor $m_e/E_\nu$ in the resulting expressions for the probability in Ref. [3] was not the numerical mistake but rather the physical one, because it led to the loss of Lorentz invariance of the product of the probability and the neutrino energy.

As it was mentioned above, the formula (1) for the probability described a rather particular case of $\ln \chi \gg 1$. On the other hand, the situation is realized under some physical conditions where the dynamical parameter takes the values which are moderately large, namely, $\chi \gg 1$, but $\ln \chi \sim 1$. The crossed-field approximation is valid in this case, but the condition $\ln \chi \gg 1$ fails, and a consideration is necessary in Eq. (1) of the next terms of expansion over the inverse powers of the large dynamical parameter $\chi$. The expressions for the probability at an arbitrary value of the $\chi$ parameter, presented in some of the listed papers, have a cumbersome form of multiple integrals, which are not suitable for an analysis.

Hence the problem is urgent of obtaining the probability of the lepton pair ($e^- e^+$ or $\mu^- \mu^+$) production by a neutrino propagating in the crossed
Table 1: The constants $K$ and $\Delta$ in Eq. (1), obtained in various papers

|                  | $K$                          | $\Delta$                      |
|------------------|------------------------------|-------------------------------|
| Choban, Ivanov   | $\frac{29}{1024\pi}$        | —                             |
| Borisov et al.   | $1$                          | $-2 \ln 2 - \frac{389}{384} + \frac{9}{128} \frac{g_V^2 - g_A^2}{g_V^2 + g_A^2}$ |
| Knizhnikov et al.| $\frac{9 E_v}{16 m_e}$      | —                             |
| Borisov et al.   | $\frac{1}{2}$                | $\frac{5}{4}$                 |
| Kuznetsov, Mikheev| $1$                          | $-\frac{29}{21}$              |
| Borisov, Zamorin | $\frac{1}{2}$                | $-\frac{29}{21}$              |

field for an arbitrary value of the $\chi$ parameter. In this paper we present our result for the probability of the process $\nu \to \nu \ell^- \ell^+$ which is rather simple and suitable for a quantitative analysis.

We will consider the case of relatively low momentum transfers ($|q^2| \ll m_W^2$). Under this condition, the weak interaction of neutrinos with charged leptons can be considered in the local limit by using the effective Lagrangian

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_\alpha (g_V - g_A \gamma_5) \ell] [\bar{\nu} \gamma^\alpha (1 - \gamma_5) \nu], \quad (3)$$

where $g_V = \pm 1/2 + 2 \sin^2 \theta_W$, $g_A = \pm 1/2$. Here the upper signs correspond to the situation when the neutrino flavor coincides with the lepton $\ell$ flavor ($\nu = \nu_\ell$), in this case both $Z$ and $W$ boson exchange takes part in a process. The lower signs correspond to the case $\nu = \nu_{\ell'}$, $\ell' \neq \ell$, when the $Z$ boson exchange is only presented in the Lagrangian (3).

We omit the details of calculations which can be found e.g. in our paper [7], and present here the result for the probability in a form of the single
integral containing the Airy function:

\[
w(\nu \rightarrow \nu \ell^-) = \frac{G_F^2 (g_V^2 + g_A^2) m_{\ell}^6 \chi_{\ell}^2}{27 \pi^4 E_\nu} \int_0^1 u^2 du z \Phi(z) \left\{ \frac{4}{1 - u^2} \left( 2L(u) - \frac{29}{24} \right) - \frac{15}{2} L(u) - \frac{47}{48} + \frac{1}{8} \left( 1 + (1 - u^2)L(u) \right) \left( 33 - \frac{47}{4} (1 - u^2) \right) \right. \\
+ \left. \frac{9}{16} g_A^2 \left[ 48L(u) + 2 - \left( 1 + (1 - u^2)L(u) \right) (28 - 3(1 - u^2)) \right] \right\}. \tag{4}
\]

Here \( \chi_{\ell} \) is the dynamical parameter of the lepton with the mass \( m_{\ell} \),

\[ \chi_{\ell} = e(p_{\alpha} F_{\alpha \beta} \Phi_{\beta \sigma} p_\sigma) \left( \frac{1}{m_{\ell}} \right)^{1/2} \]

\( \Phi(z) \) is the Airy function

\[ \Phi(z) = \int_0^\infty dt \cos \left( zt + \frac{t^3}{3} \right), \quad z = \left( \frac{4}{\chi_{\ell}(1-u^2)} \right)^{2/3}, \quad L(u) = \frac{1}{2u} \ln \frac{1 + u}{1 - u}. \tag{5} \]

In the case when \( \chi_{\ell} \ll 1 \), one immediately obtains from Eq. (4) the formula for the probability containing the well-known exponential suppression:

\[ w(\chi_{\ell} \ll 1) = \frac{3 \sqrt{3} G_F^2 m_{\ell}^6}{(16\pi)^3 E_\nu} (3g_V^2 + 13g_A^2) \chi_{\ell}^4 \exp \left( -\frac{8}{3\chi_{\ell}} \right), \tag{6} \]

which agrees with corresponding formula of Ref. [4].

In the case when \( \chi_{\ell} \gg 1 \), it is easy to obtain from Eq. (4) the formula (4), where \( K = 1, \Delta = -29/24 \), in agreement with the result of Ref. [5]. It is not difficult also to find from our Eq. (4) the next term of expansion over the inverse powers of the dynamical parameter \( \chi_{\ell} \). One obtains:

\[ w(\chi_{\ell} \gg 1) = \frac{G_F^2 (g_V^2 + g_A^2) m_{\ell}^6 \chi_{\ell}^2}{27 \pi^4 E_\nu} \left\{ \ln \chi_{\ell} - \frac{1}{2} \ln 3 - \gamma_E - \frac{29}{24} \right. \\
- \left. \frac{1}{\chi_{\ell}^{2/3}} \frac{9}{56} \left[ \Gamma \left( \frac{2}{3} \right) \right]^4 \frac{19g_V^2 - 63g_A^2}{g_V^2 + g_A^2} \right\}, \tag{7} \]

where \( \Gamma(x) \) is the gamma function.

It is seen from Eq. (7) that the correction term \( \sim \chi_{\ell}^{-2/3} \) is not universal. It is relatively small and negative in the case when the neutrino flavor coincides with the charged lepton flavor \( \nu_e \rightarrow \nu_e e^+ \), \( \nu_\mu \rightarrow \nu_\mu \mu^- \mu^+ \).
When the flavors of the neutrino and of the charged lepton are different ($\nu_\mu \rightarrow \nu_\mu e^- e^+$, $\nu_e \rightarrow \nu_e \mu^- \mu^+$), the correction term is positive and relatively large.

The dependence of the probability of the process $\nu \rightarrow \nu \ell^- \ell^+$ on the dynamical parameter $\chi_\ell$ in the region where its value is moderately large, is represented in Figs. 1 and 2. One can see that the correction term $\sim \chi_\ell^{-2/3}$ is more likely to worsen than to improve the presentation of the probability in this region. As the analysis shows, this term has a sense just of the correction for the values $\chi_\ell \gtrsim 10^5$ only.

Therefore, our exact formula (4) should be used in a detailed analysis of the probability of the lepton pair production by a neutrino propagating in an external electromagnetic field, when the value of the dynamical parameter $\chi_\ell$ is moderately large.

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Figure captions

Fig. 1
The dependence of the probability of the process $\nu \to \nu \ell^- \ell^+$ on the moderately large dynamical parameter $\chi_\ell$ in the case when the neutrino flavor coincides with the charged lepton flavor ($\nu_e \to \nu_e e^- e^+, \ldots$): a) from the exact formula (4); b) from the approximate formula (1); c) from the formula (7) with the ‘correction’ $\sim \chi_\ell^{-2/3}$.

Fig. 2
The same as in Fig. 1, in the case when the flavors of the neutrino and of the charged lepton are different ($\nu_\mu \to \nu_\mu e^- e^+, \ldots$).
Figure 1: A.V. Kuznetsov et al., “Lepton pair production …”
Figure 2: A.V. Kuznetsov et al., “Lepton pair production …”