ABSTRACT

A large variety of jet properties can be described perturbatively by evolving the parton cascade down to small scales of order of a few 100 MeV. We discuss two recent applications of this approach: 1. the soft limit of the particle spectrum which is nearly energy independent as expected for the soft gluon bremsstrahlung; further tests of this interpretation in $e^+e^-$ and DIS are presented; 2. the steep decrease of the distribution of rapidity gaps in $e^+e^-$ annihilation can be explained by the Sudakov formfactor for quark jets.

1. Introduction

An important goal in the study of high energy hard collision processes is the detailed understanding of the hadronic final state and its characteristic jet structure. There are two major fields of interest, one concerning the predictions in QCD perturbation theory, mainly on the jet final states, the second concerning the colour confinement phenomena, how the partonic cascade evolves into the final state of hadrons.

In the popular models for particle production in hard collision processes it is assumed that first the primarily produced partons evolve by bremsstrahlung processes according to perturbation theory into jets of partons until a characteristic resolution scale of $Q_0 \sim 1$ GeV is reached. Thereafter, non-perturbative processes take over and the final hadronic particles, often through intermediate resonances, are produced, for example, by a string mechanism or through cluster formation These models have been proven to be very successful in the description of the experimental data over the years. On the other hand, they are only accessible through a Monte Carlo code and involve quite a number of a priori unknown parameters in the description of the hadronization phase.

We consider here another approach, based on the concept of “Local Parton Hadron Duality” (LPHD). The parton cascade is evolved further down to a scale of about $Q_0 \sim 250$ MeV where $Q_0$ is now the cut-off in the transverse momentum. Then one compares directly an observed quantity with the corresponding parton level calculation without any hadronization correction.

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An immediate benefit of this approach are the analytical results in many cases which provide a deeper understanding of how the theory is actually tested by a particular measurement. For example, the scaling properties and the systematics of their violations can only be formulated in a meaningful way using analytical formulae. Some results can be expressed in compact form by simple QCD numbers.

Theoretical calculations can be carried out in the simplest case in the Double Log Approximation (DLA)\cite{4,5}, which provides the high energy limit, or in the Modified Leading Log Approximation (MLLA)\cite{6} which includes finite energy corrections; they are usually essential to obtain quantitative agreement with experiment at present energies. The original success was in the description of the inclusive hadron distributions\cite{3} thereafter it has been applied to a large variety of different observables\cite{7}. In recent years more applications have been worked out and the experimental results from LEP, HERA and TEVATRON gave further support to this approach\cite{8}.

The duality picture is clearly rather bold and it is not clear a priori for which observables and in which kinematic ranges it applies. This has to be clarified ultimately by experiment. It is obvious that this model cannot compete with the standard hadronization models in the description of the various details of the final state like the production of different particle species or resonances. It has so far been applied successfully for suitably averaged inclusive quantities.

The main virtue of this approach is its intrinsic simplicity which involves only two essential parameters, namely the QCD scale Λ and the non-perturbative mass scale Q₀. In some cases the absolute normalization is taken as additional parameter; on the other hand, in a recent calculation of multiplicities beyond MLLA it was suggested that one parton corresponds to one hadron in this duality picture\cite{9}.

The hope is that with increased phenomenological knowledge about successes and limitations of this approach our theoretical understanding of the confinement mechanism will improve. In this presentation we discuss two recent applications, one concerning the soft limit of the energy spectrum\cite{10} and the other one the rate for events with large rapidity gaps\cite{11} i.e. events in a quasi exclusive limit.

2. Main Ingredients of the Analytical Calculations

In this section we give a short overview of the analytical treatment of multiparticle processes. This involves the matrix elements for the primary hard subprocess and the evolution of the primary partons into jets by bremsstrahlung cascades. The gluon bremsstrahlung at small angles \( \vartheta \) with energy \( E \) off the primary parton of type \( A \) (\( A = q, g \)) with jet momentum \( P_j \) is given by

\[
    d\gamma A = \frac{C_A}{N_C} \gamma^2_0(p_\perp) \frac{d\vartheta dE}{\vartheta E}, \quad \gamma^2_0(p_\perp) = \frac{2N_C\alpha_s(p_\perp)}{\pi} = \frac{\beta^2}{\ln(p_\perp/\Lambda)}, \quad p_\perp \geq Q_0
\]
where \( p_\perp \approx E \theta, \beta^2 = 4N_C/b, b \equiv (11N_C - 2n_f)/3 \) with \( N_C, n_f \) the numbers of colours and flavours, also \( C_g = N_C, C_q = 4/3 \). Inside the cascade the soft gluons are coherently produced from all harder partons. For azimuthally averaged quantities the consequences of the coherence effect can be taken into account by the angular ordering prescription\[1\] which requires the angles of subsequent gluon emissions to be in decreasing order.

The multiparticle properties of the jet can be discussed conveniently by using the generating functional \( Z_A(P_j, \Theta; u(p)) \). Here \( P_j \) and \( \Theta \) denote the initial parton momentum and opening angle of the jet, and \( u(p) \) is a profile function for particle momentum \( p \). The functional is constructed from all the exclusive final states. Then the inclusive densities can be obtained by functional differentiation with respect to the profile function \( u(p) \), for example the one particle density by \( \rho^{(1)}(p) = \delta Z\{u\}/\delta u(p) \big|_{u=1} \). The properties of these densities can be derived from the evolution equation for \( Z \) which relates the functional at scales \( P_j, \Theta \) to the one at lower scales according to the “decay” \( A \rightarrow BC \). In MLLA accuracy this evolution equation is given by\[2\]

\[
\frac{d}{d \ln \Theta} Z_A(P_j, \Theta) = \frac{1}{2} \sum_{B,C} \int_0^1 dz \times \frac{\alpha_s(p^2_\perp)}{2\pi} \Phi^{BC}_A(z)[Z_B(zP_j, \Theta) Z_C((1-z)P_j, \Theta) - Z_A(P_j, \Theta)] \tag{2}
\]

where \( \Phi^{BC}_A(z) \) denotes the DGLAP splitting functions. The initial condition for the parton evolution is given by

\[
Z_A(P_j, \Theta; \{u\})|_{P_j \Theta = Q_0} = u_A(p = P_j), \tag{3}
\]

i.e. at threshold there is only the primary parton.

From (2) one can obtain the evolution equations for particle densities by appropriate functional differentiation. Therefore this equation is the basic tool for deriving the multiparticle properties of a jet analytically. A more detailed discussion on the approximations can be found elsewhere\[3\],\[4\],\[7\],\[8\],\[13\],\[14\].

3. The Soft Limit of the Particle Spectrum

3.1. Theoretical Predictions Confronted with Experiment

An essential ingredient of the LPHD approach is the evolution of the parton cascade towards small scales. The normalization of all quantities is given by the initial condition (3). Therefore, one has to assume that the perturbative evolution can be applied at small cms energies as well. Alternatively, one could start the evolution from a larger cms energy whenever the perturbative QCD is considered trustworthy, but then one had to introduce unknown parameters for initialization instead of (3).
and a lot of the predictive power of this approach would disappear. Indeed, applying the MLLA calculations, the moments of the energy spectra are well described down to such low energies as $\sqrt{s} = 3$ GeV using the initial condition \(13\).

Of special interest is the behaviour of the soft end of the particle energy spectrum. The coherence of the soft gluon emission from all harder partons forbids the multiplication of the soft particles and the spectrum in the variable $\xi = \ln(P_j/E)$ obtains a “hump-backed” shape.\(14\)

This problem has been studied recently in more detail.\(10\) The parton density in the variable $\xi = \ln(P_j/E)$ in the DLA is given in lowest order of $\alpha_s$ by

$$\frac{dn_A(E, P_j)}{d\ln E} = \frac{C_A}{N_C} \beta^2 \ln \frac{\ln(E/\Lambda)}{\ln(Q_0/\Lambda)}$$

(4)

This single bremsstrahlung contribution is independent of the jet energy $P_j$; the higher order contributions do depend on $P_j$ but they are negligible in the soft limit $E \to Q_0$. Also the MLLA correction leaves this limit unaltered: the energy conservation effects and large $z$ corrections from the splitting functions which make up the differences between DLA and MLLA can be neglected. If the perturbation theory and LPHD are valid towards such low particle energies $E$ one expects then also that the hadron spectrum becomes independent of the $cms$ energy in the soft limit. To exhibit the energy dependence it is convenient to consider the invariant density $Edn/d^3p$ which remains finite for small momenta.\(10\) Then we expect an energy independent density $I_0$ of hadrons in the limit where the particle momentum $p$, or alternatively rapidity $y$ and transverse momentum $k_{\perp}$, become small:

$$I_0 = \lim_{y \to 0, p_T \to 0} \frac{E}{d^3p} \frac{dn}{d^3p} = \frac{1}{2} \lim_{p \to 0} E \frac{dn}{d^3p}. \quad (5)$$

The factor $\frac{1}{2}$ takes into account that both hemispheres are included in the limit $p \to 0$.

This prediction is a direct consequence of the coherence of the soft gluon emission: the emission rate for the gluon of large wavelength does not depend on the details of the jet evolution at smaller distances; it is essentially determined by the colour charge of the hard initial partons.

In Fig. 1 we show the experimental results on the invariant density of charged particles for $cms$ energies from 3 to 130 GeV in $e^+e^-$ annihilation. An approximate energy independence of the low momentum particle density (within about 20%) is indeed observed; the same is true for identified particles $\pi$, $K$ and $p$.\(10\) The curves in Fig. 1 represent the MLLA results. At very low energies where $E \approx Q_0$ the QCD perturbative results have to be supplemented by a kinematical relation between parton and hadron spectra taking into account their different masses (for more details, see Ref. 10). The theoretical curves show the approach to the scaling limit and

\(^a\)Another alternative is the non-invariant spectrum $dn/d^3p$ which has the advantage of being independent of the particle mass.
describe well the different slopes at larger particle energies. An important role here is played by the running $\alpha_s$ which provides the strong rise towards small energies for $E < 1$ GeV, for fixed $\alpha_s$ this rise would be much weaker.\cite{10}

Recently inclusive spectra from HERA in the current fragmentation region in the Breit frame became available.\cite{18} The data are found to be rather similar in shape to the observation in $e^+e^-$ annihilation at comparable scales. Especially, the approximate energy independence of the soft part of the spectrum is verified in the region $12 < Q^2 < 100$ GeV$^2$. This supports the universality of the low energy phenomena.

### 3.2. Further Tests of the Perturbative Picture

One may suspect that the universal, approximately energy independent soft limit of the invariant spectrum could be some general hadronization phenomenon not related to the coherence properties of perturbative QCD. It is therefore very important to further investigate the perturbative origin of the effect. Whereas there is no parameter free prediction of the absolute size of the density $I_0$, the perturbative approach predicts its dependence on the colour charge of the primary parton as in $|$\cite{4}$|$. Especially, we expect for the ratio of soft particle densities in a $gg$ system to the one in a
The $q\bar{q}$ system

\[
\frac{I^q_0}{I^\bar{q}_0} = \frac{N_C}{C_F} = \frac{9}{4}.
\]  

(6)

Such a ratio is expected asymptotically for the total multiplicities of particles but at present energies there are large corrections. On the other hand, such a large ratio may nevertheless appear in case of the soft particles where the DLA asymptotic behaviour is valid to a good approximation. The preparation of $gg$ final states is not easy; but there are two lines of approach which can test the hypothesis (6), the first at a more qualitative, the second at a quantitative level.

**Study of the central rapidity plateau at small $p_\perp$**

Let’s first consider $e^+e^-$ annihilation. The rate $I_0$ in (3) is defined in the $cms$. If we perform a Lorentz transformation in direction of one primary parton, we arrive at a frame with unequal momenta of $q$ and $\bar{q}$, say at rapidity $y$ from the $cms$. As the soft radiation does not depend on the primary energy, the density $I_0(y)$ in the frame at $y$ has not changed, so $I_0(y)$ should be constant away from the kinematical boundaries. Instead of the density at $p_\perp = 0$ we can also consider the density in an interval at small $p_\perp$. This picture is a bit idealistic, in that the primary parton direction is not known and should be replaced by a suitable jet-axis, then $I_0(y)$ will not be exactly constant.

Next we consider collisions with incoming hadrons or/and photons. The interaction may proceed through quark or gluon exchange where we have to assume some semihard momentum transfer of, say, at least 1 GeV—the soft processes may follow different rules. In case of a semihard quark or gluon exchange the outgoing partons will be either colour triplet or octet sources of radiation respectively, so in the soft limit we expect the ratio as in (6) for both cases.

Let’s now consider specifically the DIS process studied at HERA and construct $I_0(y)$ as above. Analogous results are obtained for $\gamma\gamma$ collisions. In the Breit frame we have a quark in the current direction recoiling against another colour triplet system in proton direction. So in this frame we have $I_0(y_{Breit}) = I^q_0$, the density as in $e^+e^-$ annihilation. However, the same DIS process can also be initiated by other subprocesses, for example, the photon gluon fusion. This process can be viewed best in a frame closer to the proton: a $qq$ pair moves in current direction and the remainder in proton direction. This process is mediated by gluon exchange, so the colour octet sources yield a soft particle rate $I^g_0(y) = \frac{9}{4}I^q_0$ in such a frame. The overall picture then looks as in Fig. 2.

As already mentioned the exchanged parton should be sufficiently hard. The gluon virtuality cannot be simply enforced by the external conditions but it should become larger with increasing $Q^2$. We have no explicit calculation for this effect in the moment, so the conditions are formulated at a qualitative level. However, there
are quantitative asymptotic expectations and predictions which could be falsified. We consider the distribution of particles in rapidity and at small $p_\perp$, at best in the Breit frame. An increase of $Q^2$ should make the small $k_\perp$ current plateau longer but not higher than $I^q_0$. At the same time the plateau in proton direction should be rising and approach for large $Q^2$ and $W^2$ a limit, the “gluon plateau” $I^g_0$, about two times larger than the “quark plateau” $I^q_0$.

**Soft radiation perpendicular to the production plane in multi-jet events**

The collinear parton configurations just discussed correspond to quark or gluon exchanges with the soft radiation expected in the ratio (6). We consider now the configurations not exactly collinear but with moderate transverse momentum, so that the corresponding parton jets can be resolved. In that case we consider the soft particle production in direction perpendicular to the production plane which can be analytically calculated in the soft region taking into account the cut-off $Q_0$.

The simplest case is the process $e^+e^- \rightarrow q\bar{q}g$. The soft radiation in this process has been analysed already some time ago and applied to the case of multiplicity flow in the production plane. The spectrum perpendicular to the plane (say, in a small cone) is conveniently normalized to the corresponding flow in a 2 jet $q\bar{q}$ event by

$$R^a_\perp(p) \equiv \frac{dN^a_\perp/d\Omega dp/d\Omega}{dN^{qq}_\perp/d\Omega dp}$$

for a general process $a$. For the case of $q\bar{q}g$ events one finds for the soft bremsstrahlung

![Fig. 2. Schematic view of particle density at small $p_\perp$ as a function of rapidity $y$ in the cms: in frames at rapidity $y$ with dominant gluon exchange the soft particle density $I_0(y)$ is 9/4 times larger than in the frames with dominant quark exchange (the Breit frame, for example).]
\[ R_{\perp} (p) = \frac{N_C}{4 C_F} \left[ 2 - \cos \Theta_{qg} - \cos \Theta_{\bar{q}g} - \frac{1}{N_C^2} (1 - \cos \Theta_{q\bar{q}}) \right]. \] (8)

where \( \Theta_{ij} \) are the angles between the jets in the cms. This formula yields the two collinear limits \( R_{\perp} = 1 \) for collinear or soft gluons and \( R_{\perp} = \frac{3}{4} \) for antiparallel quarks recoiling against the gluon. In this approximation the shape of the momentum spectrum does not depend on the inter-jet angles but the absolute magnitude does.

The analogous analysis can be carried out for photo-production of dijets in \( \gamma p \) or \( \gamma\gamma \) collisions and also in \( p\bar{p} \) collisions. One can distinguish between direct and resolved production. The former involves the soft radiation from a \( q\bar{q}g \) system, very much like in \( e^+e^- \rightarrow q\bar{q}g \), and for the dominant small angle scattering corresponds to quark exchange, whereas the latter at small angles corresponds to gluon exchange. One expects again the ratio (3) of the soft particle production in both processes at small scattering angles.

It will be interesting to find out whether the particle density varies with the angles as predicted by (3) and occurs in the predicted ratios in different processes even down to low momenta of a few hundred MeV. This would provide a strong argument in favour of the perturbative QCD interpretation of the soft particle production.

4. Large Rapidity Gaps in \( e^+e^- \) Annihilation

In the previous example of inclusive particle spectra the partons and hadrons are typically close in phase space and the hadronization requires only small rearrangements of momenta. If during the evolution of the parton cascade in \( e^+e^- \) annihilation a large rapidity gap occurs, one may expect the colour confinement forces at large distances to fill the gap by hadrons in the hadronization phase. Then there is no close relation between parton and hadron final states. The study of large rapidity gap events can give us therefore further insight into the colour neutralization mechanism.

The interest in such events has been awakened recently by the findings of large rapidity gaps in dijet events at TEVATRON and HERA which are interpreted by the exchange of colour neutral objects.

In \( e^+e^- \) annihilation—if one wants to create a large gap without colour exchange such as to avoid final state rearrangements by confinement forces—one is lead to hard processes of the type \( e^+e^- \rightarrow q\bar{q}q\bar{q} \) or \( q\bar{q}gg \) where the hadrons are produced from colour singlet low mass \( q\bar{q} \) or \( gg \) clusters. As these processes in the perturbative analysis involve a highly virtual intermediate quark or gluon their rate is rather small though. One finds that the rate keeps decreasing with increasing gap size, contrary to the case of \( p\bar{p} \) or \( ep \) collisions.

Indeed, the recent measurement of rapidity gaps by the SLD collaboration has shown an unlimited decrease of the gap rate over five orders of magnitude (such a
A decrease has also been seen at lower energies some time ago (29). However, the gap rate in absolute terms exceeds the expectations from the above calculations by about two orders of magnitude and therefore, this type of hard colour neutralization cannot be the dominant process for the formation of large rapidity gap events.

An alternative approach (11) in line with the previously discussed LPHD, considers a rather soft confinement mechanism, i.e. the hadron formation does not change the parton final state considerably, not even if a large rapidity gap is formed. Clearly, such an application of LPHD cannot be justified a priori. Rather one tries and looks how different the rates of large gaps really are in the parton and hadron cascades.

One has to calculate the probability that no parton is emitted into a certain angular interval, say between $\Theta_1$ and $\Theta_2$ ($\Theta_1 < \Theta_2$) where $\Theta$ is measured with respect to the initial parton direction. The rapidity for a massless parton is then obtained from $y = -\ln \tan \frac{\Theta}{2}$. This probability is given by the so-called Sudakov formfactor. For the actual problem of the angular ordered cascade with transverse momentum cut-off it has been calculated in connection with the multi-jet rates in the Durham/$p_\perp$ algorithm (30).

The probability that no parton be emitted below the angle $\Theta$ in the cascade with cut-off $p_\perp > Q_0$ initiated by the primary parton $A$ of momentum $p_j$ is given by

$$\Delta_A(p_j, \Theta, Q_0) = \exp(-w_A(p_j, \Theta, Q_0))$$

$$w_A(p_j, \Theta, Q_0) = \int d\omega' \int_{k_\perp > Q_0} d\Theta' \varphi_A(\omega', \Theta'),$$

where $\varphi_A(\omega', \Theta') = dn_A/\omega' d\Theta'$ is the emission density of a gluon at energy $\omega'$ and angle $\Theta'$. The probability for a gap between the two angles $\Theta_1 < \Theta_2 < \frac{\pi}{2}$ is then given by the ratio of two Sudakov formfactors

$$f_A(\Theta_1, \Theta_2) = \frac{\Delta_A(\Theta_2)}{\Delta_A(\Theta_1)} = \exp(-w_A(\Theta_2) + w_A(\Theta_1)).$$

In the simplest approximation, the DLA, one finds

$$w_A(Y, \lambda) = \frac{C_A}{N_C} \beta^2 \{(Y + \lambda) \ln \frac{Y + \lambda}{\lambda} - Y\}$$

In this approximation the jets evolve independently in the two hemispheres and the respective gap probabilities factorize. In particular, for the symmetric gap in $e^+e^-$ annihilation one obtains accordingly

$$f_A(\Theta_G) = e^{-2(\omega_A(\frac{\pi}{2}) - \omega_A(\Theta_G))}.$$
As the result of these calculations one finds the probability for the gap in the symmetric rapidity interval $\Delta y \approx -2 \ln \frac{\Theta G}{2}$. It shows an almost exponential decrease very much like the experimental data. The slope is not very different for DLA and MLLA calculations but strongly depends on the parameter $\lambda = \ln(Q_0/\Lambda)$. The SLD data are shown in Fig. 3. They refer to rapidity gaps between charged particles and still include $\tau$-lepton events. The analytical calculations from the DLA are shown for two parameters of $\lambda$ at $\Lambda = 0.244$. An upper limit $\lambda < 0.1$ has been derived from a moment analysis of the spectra. These calculations should rather be compared to data on gaps empty of any particles, not only of charged ones. Their distribution is expected to have a steeper slope than shown by the data in the figure. As an example the prediction for $\lambda = 0.05$ is shown. A recent analysis of particle and jet multiplicities beyond MLLA has given the small value $\lambda = 0.015 \pm 0.005$.

One can conclude that the distribution of gaps in a hadron cascade is well represented by the one in the parton cascade, provided the cut-off $Q_0$ is taken small enough, i.e. very close to the QCD scale $\Lambda$ in the present approximation scheme. If one took the larger cut-off $Q_0 \sim 1$ GeV the gap probability at $\Delta y = 6$ would be larger by 4 orders of magnitude! This suggests that the hadronization phase in a conventional hadronization model with shorter parton cascade has a similar effect as the evolution of the parton cascade from $Q_0 = 1$ GeV down to $Q_0 = 0.25$ GeV. This is not too surprising as it is in this last phase where the running $\alpha_s$ increases most.

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**Fig. 3.** Fraction of rapidity gap events in $e^+e^-$ annihilation as a function of the full width of the symmetric gap. The data points refer to the measurement of gaps between charged particles ($\tau$-lepton events included) by the SLD Collaboration. The dashed histogram shows the expectation from the JETSET Monte Carlo without $\tau$-leptons. Also shown are the DLA predictions for the gaps of a parton cascade for two values of $\lambda = \ln(Q_0/\Lambda)$ at $Q_0 = 0.244$ GeV.
An interesting prediction from this approach concerns the distribution of rapidity gaps in single quark and gluon jets. As the exponent of the Sudakov formfactor is derived from the one-particle-density it is proportional again to the colour factor $C_A$, i.e. the slope of the gap distribution is correspondingly steeper (roughly twice) for gluon jets than for quark jets. This prediction could be checked in high $p_T$ jets at the TEVATRON or at HERA.

5. Conclusions

The description of detailed properties of the hadronic jets in hard processes in terms of a parton cascade assuming a duality between parton and hadron production continues to be successful. It is surprising that this duality works even for observables where a priori there is no good reason to believe in a perturbative approach.

It appears that the evolution of the parton cascade at very low scales below 1 GeV resembles in its main inclusive aspects the hadronization phase of the phenomenological models with all its resonance decays, so one can speak of a dual description of the hadronic phenomena after appropriate averaging (see Fig. 4). As the cut-off $Q_0$ is close to the QCD scale $\Lambda$ within less than 10% the coupling $\alpha_s$ becomes large in the last phase of the perturbative cascade. So it may not be unreasonable that the strong coupling perturbative regime corresponds to the hadronic phase with resonance production.

The QCD expectation of the energy independence of the low momentum particle production from the soft gluon coherence is well met by the hadronic data in the full energy region explored, i.e. in $e^+e^-$ annihilation from ADONE at 1.6 GeV up to LEP-1.5 and in DIS from $Q^2 = 12$ GeV$^2$ to $Q^2 = 100$ GeV$^2$. It will be interesting to test the predictions on the higher production rates of soft particles for other colour
configurations than $q\bar{q}$ which can be obtained in various processes.

The parton cascade with low cut-off $Q_0$ and without hadronization also reproduces the approximately exponential decrease of the rapidity gap distribution in $e^+e^-$-annihilation. This suggests in particular that the confinement of colour in the jet evolution is a soft process.

The correspondence between the parton and hadron final state according to these results is not like the conventional duality between a colour singlet $q\bar{q}$ and a hadronic cluster. Rather it is a correspondence between a coloured parton (typically a gluon) and a colourless hadron. These partons are taken at a particular scale $p_\perp > Q_0 > \Lambda$, so that partons below that scale are not resolved. These soft non-perturbative sea-partons could take the important role in the confinement process without disturbing the momenta of the resolved perturbative partons.

Meanwhile it seems to be a challenge to test further this dual picture of hadron formation at the limit of what is accessible perturbatively.

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