RESOLUTION OF THE AGE DISCREPANCIES IN PULSAR/SNR ASSOCIATIONS

DAVID MARSDEN\textsuperscript{1}, RICHARD E. LINGENFELTER\textsuperscript{2}, RICHARD E. ROTHSCILD\textsuperscript{2}
\textsuperscript{1} NASA/Goddard Space Flight Center, Greenbelt, MD
\textsuperscript{2} UCSD/Center for Astrophysics and Space Sciences, La Jolla, CA

\textbf{ABSTRACT.} Pulsars associated with supernova remnants (SNRs) are valuable because they provide constraints on the mechanism(s) of pulsar spin-down. Here we discuss two SNR/pulsar associations in which the SNR age is much greater than the age of the pulsar obtained by assuming pure magnetic dipole radiation (MDR) spin-down. The PSR B1757$-$24/SNR G5.4$-$1.2 association has a minimum age of $\sim$40 kyr from proper motion upper limits, yet the MDR timing age of the pulsar is only 16 kyr, and the newly discovered pulsar PSR J1846$-$0258 in the $>2$ kyr old SNR Kes 75 has an MDR timing age of just 0.7 kyr. These and other pulsar/SNR age discrepancies imply that the pulsar spin-down torque is not due to pure MDR, and we discuss a model for the spin-down of the pulsars similar to the ones recently proposed to explain the spin-down of soft gamma-ray repeaters (SGRs) and anomalous x-ray pulsars (AXPs).

1. Introduction

The study of pulsars and their spin-down provides important information on the physics of neutron stars. This information (e.g. magnetic field, moment of inertia, etc.) can be gleaned from the pulsar spin-down by assuming a physical model for the spin-down torque. For isolated pulsars, it is usually assumed that the spin-down torque is due to magnetic dipole radiation (MDR), which produces a timing age $\tau_{MDR} = 0.5P/\dot{P}$ and pulsar braking index $n = \Omega\dot{\Omega}/\dot{\Omega}^2 = 3$, where $P = 2\pi/\Omega$ and $\dot{P} = -2\pi\dot{\Omega}/\dot{\Omega}^2$ are the pulsar spin period and period derivative, respectively (this calculation of $\tau_{MDR}$ assumes that the initial spin period was much smaller than the observed spin period). Assuming MDR spin-down, the surface magnetic field strength of the neutron star is given by the formula $B = 3.2 \times 10^{19}(P\dot{P})^{1/2}\text{G}$, which has been widely used to estimate the magnetic field strengths of isolated pulsars (e.g. Manchester & Taylor 1977).

2. Pulsar/SNR Age Discrepancies?

Perhaps the only way to test the MDR spin-down hypothesis for pulsars is to study the SNRs associated with young pulsars, because the SNRs provide an age constraint independent of the pulsar spin-down. There are at least two pulsars with SNR ages which are inconsistent with the timing ages of the pulsars calculated assuming MDR spin-down: PSR B1757$-$24 and PSR J1846$-$0258. PSR B1757$-$24 is a 0.125 s radio pulsar associated with SNR G5.4$-$1.2. Given the displacement of the pulsar from the center of G5.4$-$1.2, the lack of observed proper motion of the pulsar with respect to its
SNR implies an age $\tau_{\text{psr}} > 39$ kyr (Gaensler & Frail 2000), which is more than a factor of two greater than the MDR timing age of 16 kyr. Because the pulsar/SNR association seems so compelling, Gaensler & Frail (2000) suggested that this age discrepancy raises very serious problems for all pulsar ages based on MDR.

Recently, the 0.324 s x-ray pulsar PSR J1846$-$0258 was discovered (Gotthelf et al. 2000) in the center of the SNR Kes 75. The MDR timing age of the pulsar is $\tau_{\text{MDR}} = 0.7$ kyr, which is much less than the estimated minimum age of Kes 75. A lower limit to the age of the SNR can be estimated by assuming that the remnant is still undergoing free expansion. In this case,

$$M_{\text{ej}} (> v) > \frac{4}{3} \pi R^3 \rho_{\text{ISM}},$$  \hspace{1cm} (1)

where $R$ is the radius of Kes 75 (9.7 pc; Blanton & Helfand 1996), $\rho_{\text{ISM}}$ is the density of the interstellar medium surrounding Kes 75, and $M_{\text{ej}} (> v)$ is the total mass of ejecta with free expansion velocities greater than $v$. Realistic models of the velocity profiles of Type II supernova ejecta in free expansion consist of a broken power law distribution of ejecta velocities, such that the majority of the mass and energy is in the low-velocity ejecta. As a function of the ejecta power law index $q$ (index $n$ in Truelove & McKee 1999),

$$\frac{M_{\text{ej}} (> v)}{M_{\text{tot}}} = 1 - \frac{(v/v_{\text{core}})^{3-q} - q/3}{(v_{\text{core}}/v_{\text{max}})^{q-3} - q/3},$$  \hspace{1cm} (2)

where $M_{\text{tot}}$ is the total ejecta mass, $v_{\text{max}}$ is the maximum ejecta velocity, and $v_{\text{core}}$ is the transition velocity below which the distribution of ejecta velocities (in velocity space) is flat. The parameter $v_{\text{core}}$ can be eliminated by normalizing Equation (2) to a total ejecta kinetic energy $E_{\text{tot}}$, and Equations (1–2) can then be solved for the cutoff velocity $v$ for a given $\rho_{\text{ISM}}$, $R$, $q$, $M_{\text{tot}}$, and $E_{\text{tot}}$ (the result is insensitive to the value of $v_{\text{max}}$). The minimum age in free expansion is then given simply by $T > R/v$. For $q = 9 - 10$ (Chevalier & Fransson 1994), $E_{\text{tot}} = 10^{51}$ ergs, $R = 9.7$ pc, $M_{\text{tot}} = 20 \, \text{M}_\odot$, $v_{\text{max}} = 20,000 \, \text{km} \, \text{s}^{-1}$, and assuming the SNR is in the hot, most tenuous phase of the ISM with $n_{\text{ISM}} = 0.001$, we estimate a minimum age $\tau_{\text{snr}} > 2.0$ kyr for Kes 75. This is a factor of three greater than the pulsar’s MDR timing age of 0.7 kyr.

What conditions must be met for Kes 75 to have the same age as the MDR timing age of PSR J1846$-$0258? The minimum age estimate above is insensitive to $v_{\text{max}}$, but decreases as $E_{\text{tot}}$ increases or $M_{\text{tot}}$ decreases. For $q = 9 - 10$ and $n_{\text{ISM}} = 0.001 \, \text{cm}^{-3}$, and assuming a minimum Type II supernova ejecta mass of $\sim 6.6 \, \text{M}_\odot$ (for an 8 M$_\odot$ progenitor star forming a 1.4 M$_\odot$ neutron star), we find that $\tau_{\text{snr}} = \tau_{\text{MDR}}$ for Kes 75 only if $E > 4 \times 10^{51}$ ergs, which is roughly three times greater than the energy implied by model Type II supernova lightcurves (Shigeyama & Nomoto 1990). Even for extremely high $E_{\text{tot}}$, $\tau_{\text{snr}} = \tau_{\text{MDR}}$ seems unlikely because the ISM density surrounding Kes 75 is probably much greater than 0.001 cm$^{-3}$. This is because an OH maser emission line has been observed from the SNR (Green et al. 1997), which is thought to result from the supernova shock interacting with a dense molecular cloud. Therefore, $\tau_{\text{snr}} > 2$ kyr is probably a very conservative lower limit to the Kes 75 age, since increasing $\rho_{\text{ism}}$ increases the minimum age from the arguments above.
3. Propeller-aided spin-down

The age discrepancy problems outlined above suggest that simple MDR spin-down is incorrect for these pulsars, and that exploration (e.g. Marsen, Lingenfelter, & Rothschild 2001) of more complete spin-down models is warranted. We consider a hybrid spin-down model consisting of MDR torques plus the addition of spin-down torque due to the “propeller effect” (Illarionov & Sunyaev 1975) from material at the pulsar magnetosphere. In the case of PSRs B1757−24 and J1846−0258, this material could be supernova ejecta captured by the neutron star in the form of a fallback disk. Fallback disks may be roughly divided into two categories: “prompt” and “delayed”. Prompt disks may be formed from $\sim 0.001 - 0.1 M_\odot$ (Michel 1991; Lin, Woosley, & Bodenheimer 1991) of ejecta material soon after the initial core collapse in a type II supernova explosion (Woosley & Weaver 1995). Formation of such prompt disks is probably limited to $< 7$ days after the core collapse because of heating of the ejecta by $^{56}$Ni decays (Chevalier 1989). Delayed disks may form years after the explosion from ejecta decelerated by a strong reverse shock (Truelove & McKee 1999) caused by the primary supernova blast wave impinging on dense circumstellar material from the pre-supernova stellar wind. Such models were recently invoked to explain the spin-down of AXPs (Chatterjee, Hernquist & Narayan 2000), and SGRs and AXPs (Marsden et al. 2001).

An artist’s conception of the pulsar and accretion disk system is shown in Figure 1. The total spin-down rate of the neutron star due to the combined disk and MDR torque is given by

$$\dot{\Omega} = \dot{\Omega}_{\text{MDR}} + \dot{\Omega}_A$$

The MDR torque $I_s \dot{\Omega}_{\text{MDR}} = -B_s R_s^6 \Omega^3 / 6 c^3$ (e.g. Manchester & Taylor 1977), where $I_s$ is the neutron star moment of inertia, $R_s$ is the neutron star radius, and the propeller torque $I_s \dot{\Omega}_A = k m \dot{m}_m R_m^2 \Omega_{eq} (1 - \Omega / \Omega_{eq})$ (Menou et al. 1999), where $k$ is a constant of order unity, $\dot{m} = 10^{16} \dot{m}_{16}$ g s$^{-1}$ is the mass infall rate at the magnetosphere, and $R_m$ is...
the magnetospheric radius. Here and elsewhere we assume $R_*=10\text{ km}$, $I_*=1.1\times10^{45}\text{ g cm}^2$, and $B_* = 10^{12}B_{12}\text{ G}$ is the surface magnetic field strength (assumed dipole). The equilibrium angular frequency $\Omega_{eq}$ is defined by the condition $R_m = R_c$, where $R_c = (GM_*/\Omega^2)^{1/3}$ is the Keplerian co-rotation radius for a neutron star of mass $M_*$. The timing age $\tau_{\text{comb}}$ under the action of the combined torque model is then given by
\[
\tau_{\text{comb}} = \int_{\Omega_0}^{\Omega} \frac{d\Omega}{\Omega_{\text{MDR}} + \Omega_{A}},
\]
where $\Omega_0 = 2\pi/P_0$ is the initial angular frequency. The timing ages were calculated for PSR B1757−24 ($P=0.125\text{ s}$) and PSR J1846−0258 ($P=0.324\text{ s}$) for a grid of $B_*$ and $\dot{m}$ values using Equations (3 − 4) and assuming $R_M = 2.4\times10^8B_{12}^{4/7}\dot{m}_{16}^{-2/7}\text{ cm}$ (Chatterjee, Hernquist & Narayan 2000).

### 4. Results

The resulting range of pulsar magnetic fields and magnetospheric accretion rates are shown in Figure 2. For PSR B1757−24 (top panel), we find that values of $9\times10^{10} < B_* < 1.4\times10^{12}\text{ G}$, $7\times10^{13} < \dot{m} < 9\times10^{17}\text{ g s}^{-1}$, and $39 < \tau_{\text{par}} < 80\text{ kyr}$ are consistent with the lower limit on the true age of $39\text{ kyr}$ (Gaensler & Frail 2000) and the present day spin-down rate of the pulsar (heavy solid line). Similarly, for PSR J1846−0258 (right panel) the lower limit on the age ($\tau_{\text{snr}} > 2\text{ kyr}$) gives a range of values $9\times10^{11} < B_* < 1.5\times10^{13}\text{ G}$ and $1.1\times10^{14} < \dot{m} < 8\times10^{18}\text{ g s}^{-1}$. In addition, a more stringent constraint is provided by the x-ray luminosity of the pulsar, since (as seen from the Figure) the magnetic field of the pulsar must be less than $\sim 1.5\times10^{13}\text{ G}$ for consistency with the 2 kyr age lower limit. The MDR luminosity for this magnetic field strength is only $L_{\text{MDR}} \sim 8\times10^{35}\text{ ergs s}^{-1}$ — much less than the observed x-ray luminosity of the pulsar ($L_x \sim 2\times10^{36}\text{ ergs s}^{-1}$; Gotthelf et al. 2000). Therefore the x-ray luminosity may be predominantly powered by the magnetospheric accretion in the context of this model, which constrains $\dot{m}$ for a given x-ray efficiency defined by $L_x = \epsilon\dot{m}c^2$. For $\epsilon < 1$, Figure 2 (bottom) indicates that a solution exists for $7\times10^{11} < B_* < 6\times10^{12}\text{ G}$, $4\times10^{15} < \dot{m} < 8\times10^{18}\text{ g s}^{-1}$, and $2 < \tau_{\text{snr}} < 5\text{ kyr}$. Since most of the accretion to the neutron star surface would be inhibited by the centrifugal barrier in these sources, the value of the x-ray efficiency is probably much less than the usually assumed value of $\epsilon = GM/Rc^2 = 0.2$ appropriate for x-ray binaries.

### 5. Discussion

The discrepancies between the MDR timing ages and supernova remnant ages for pulsars B1757-24 and J1846-0258 can be resolved by using a more complete spin-down model consisting of both MDR and propeller torques. One prediction of this model is the presence of excessive pulsar timing noise which is characteristic of noisy propeller torques. In addition, optical or infrared emission from isolated neutron star accretion disks may be detectable (Perna, Hernquist & Narayan 2000). Since polar cap accretion is suppressed by the centrifugal barrier for $\Omega > \Omega_{eq}$, pulsed radio emission from open
Fig. 2. Contour plots of the PSR B1757−24 (top) and PSR J1846−0258 (bottom) characteristic ages for the combined MDR and propeller torques spin-down model, for various values of the neutron star magnetic field $B_*$ and magnetospheric accretion rate $\dot{m}$. The allowed values of $B_*$ and $\dot{m}$ lie along the heavy solid lines corresponding to the present-day $\dot{P}$, and the shaded areas are excluded by the SNR age constraints or the condition $\dot{P} > 0$. For PSR J1846−0258, the dominant constraint on $B_*$ and $\dot{m}$ are provided by limits on the x-ray efficiency given the observed x-ray luminosity (see text).
field lines above the disk is not precluded by this model (see e.g. Michel 1991, ch. 6). More work is needed, however, to incorporate the effects of time-dependent magnetospheric accretion, as the accretion rate should gradually decrease with time as the disk dissipates (Cannizzo, Lee & Goodman 1990). Propeller-based spin-down models incorporating a time-dependent $\dot{m}$ have been proposed for AXPs (Chatterjee, Hernquist & Narayan 2000), but this model did not include MDR spin-down torque, which may be significant for older pulsars (such as B1757-24) and pulsars with small initial $\dot{m}$. If a significant fraction of pulsars are born with accretion disks, then there may be a population of older pulsars — whose disks have dissipated — with abnormally long MDR timing ages. From just the simple model considered here, spin-down models incorporating propeller torques hold much promise for reconciling the supernova remnant ages and the timing parameters of pulsars.

Acknowledgements

This work was performed while one of the authors (DM) held a National Research Council-GSFC Research Associateship. RER acknowledges support by NASA contract NAS5-30720, and REL support from the Astrophysical Theory Program.

References

Blanton, E.L. & Helfand, D.J.: 1996, Astrophys. J. 470, 961.
Cannizzo, J.K., Lee, H.M. & Goodman, J.: 1990, Astrophys. J. 351, 38.
Chatterjee, P., Hernquist, L. & Narayan, R.: 2000, Astrophys. J. 534, 37.
Chevalier, R. A.: 1989, Astrophys. J. 346, 847.
Chevalier, R.A. & Fransson, : 1994, Astrophys. J. 420, 268.
Gaensler, B.M. & Frail, D.A.: 2000, Nature 406, 158.
Gotthelf, E.V., Vasisht, G., Boylan-Kolchin & Torii, K.: 2000, Astrophys. J. Lett. 542, LL37.
Green, A.J., Frail, D.A., Goss, W.M. & Otrupcek, R.: 1997, Astron. J. 114, 2058.
Illarionov, A.F. & Sunyaev, R.A.: 1975, Astron. Astrophys. 39, 185.
Lin, D. N. C., Woosley, S. E., & Bodenheimer, P. H.: 1991, Nature 353, 827.
Manchester, R.N. & Taylor, J.H.: 1977, Pulsars, (Freeman: San Francisco).
Marsden, D. et al.: 2001, Astrophys. J. in press.
Marsden, D., Lingenfelter, R. E., & Rothschild, R. E.: 2001, Astrophys. J. Lett., L54745.
Menou, K. et al.: 1999, Astrophys. J. 520, 276.
Michel, F.C.: 1988, Nature 333, 644.
Michel, F.C.: 1991, Theory of Neutron Star Magnetospheres, (University of Chicago Press: Chicago).
Perna, R., Hernquist, L. & Narayan, R.: 2000, Astrophys. J. 541, 344.
Shigeyama, T & Nomoto, K.: 1990, Astrophys. J. 360, 242.
Truelove, J.K. & McKee, C.F.: 1999, Astrophys. J. Suppl. 120, 299.
Woosley, S. E., & Weaver, T. A.: 1995, Astrophys. J. Suppl. 101, 181.