Thermodynamic evidence for two superconducting phases under magnetic field in UTe$_2$

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The recently discovered superconductor UTe$_2$ with a $T_c$ between 1.5 K and 2 K, has attracted much attention due to indications of spin-triplet and topological superconductivity. Its properties under magnetic field are also remarkable, with field-reinforced and field-induced superconducting phases. Here, we report the first complete thermodynamic determination of the phase diagram for fields applied along the three crystallographic directions. Measurements were performed up to 36 T along the hard $b$ axis in order to follow the superconducting transition up to the metamagnetic transition at $H_{\text{sat}} = 34.75$ T. They demonstrate the existence of a phase transition line within the superconducting phase, and drastic differences occurring between these two phases. Detailed analysis supports a different spin state between the two phases, possibly a spin-triplet to spin-singlet transition.

I. INTRODUCTION

The uranium-based heavy-fermion system UTe$_2$, discovered to be superconducting three years ago [1], triggered much experimental and theoretical work [2] as it appeared immediately as a prime candidate for spin-triplet topological superconductivity [1, 3]. It was soon discovered that it belongs to the very select class of unconventional superconductors displaying transitions between different superconducting phases. This is well established under pressure, as all groups converge on similar phase diagrams [4–7], and more controversial at ambient pressure, where a double superconducting transition is observed on some samples at zero field [8]. There is now more and more evidence that this double transition at ambient pressure is not an intrinsic effect [2, 7, 9].

However, a definite identification of the superconducting order parameter symmetry and of the pairing mechanism in UTe$_2$ are still lacking. Microscopic neutron measurements have revealed incommensurate magnetic fluctuations [10–12], and a resonance at finite $Q$ below the superconducting transition temperature $T_c$ [13, 14], but not the expected ferromagnetic fluctuations. It is known that odd parity spin-triplet superconductivity can exist without ferromagnetic fluctuations. For example, UPt$_3$ is a prime example of odd-parity pairing, with evidences for $E_{2u}$ ground state [15, 16]. Nevertheless, neutron scattering studies only detected antiferromagnetic correlations [17], and theories have also predicted the observed $E_{2u}$ state by antiferromagnetic fluctuations [18]. In UTe$_2$, evidence for spin-triplet pairing comes from NMR Knight-shift measurements, recently performed along all crystallographic axes [19], and from the strong violation of the paramagnetic limit by the superconducting upper critical field $H_{c2}$ [1, 5, 20–22]. Several theoretical works have proposed models for the pairing with or without ferromagnetic correlations, and for the different possible symmetry states under fields of pressure [23–28].

Naturally, both questions and notably the symmetry of the superconducting state, are known to be notoriously difficult to answer in heavy-fermion superconductors. Nonetheless, the stunning behaviour under field of UTe$_2$ gives more clues than usual to guide theoretical proposals among the different possible odd-parity states [25, 26, 29]. This is mitigated by the field-reinforced phase observed for magnetic fields applied along the hard $b$ axis, which strongly suggests a field-dependent pairing strength [1, 20]. Indeed, such a field-dependent pairing is difficult to model theoretically, and makes the analysis of $H_{c2}$ much more complex, because it opens other routes for a violation of the paramagnetic limit than spin-triplet pairing [2]. Moreover, even the simplest question of whether or not the field-reinforced phase has a different symmetry than the low-field phase is not settled experimentally. The reason is that the phase diagram has only been determined at high fields by electrical transport measurements, which cannot probe transitions within the superconducting state.

In this work, we have performed both specific-heat and thermal dilatation/magnetostriction measurements up to 36 T, allowing to draw the superconducting phase diagram in the three crystallographic directions of UTe$_2$ by bulk thermodynamic probes. Hence, we can answer part of these questions, and notably reveal a real phase transi-

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tion between a low and high field superconducting phase, for magnetic fields applied along the hard b axis. We uncover also a deep change in the specific-heat anomaly along $H_{c2}$ between the two phases, suggesting that different spin-states could characterise these two phases.

At last, is appears clearly with these bulk probes that $H_{c2}(T)$ is anomalous for all crystallographic directions in UTe$_2$, suggesting also a field dependent pairing strength along the easy a axis and along the c axis.

II. EXPERIMENTAL DETAILS

A. Single crystal growth and samples

Different single crystals of UTe$_2$ from three different batches have been studied by specific-heat measurements and magnetostriction/thermal expansion measurements. All single crystals were prepared by the chemical vapor transport method with iodine as transport medium. A starting ratio of U:Te = 2:3 has been used, and the quartz ampules have been heated slowly up to a final temperature gradient of 1060°C/1000 °C, which was kept for 18 days. The ampules have been slowly cooled down to 300 K during 70 hours. The first sample (#1) has a $T_c$ of 1.45 K, with a mass of 12.3 mg. This sample is from the same batch than those studied in Refs. [20, 30]. It has been studied by specific-heat and magnetostriction. Samples, #2 and #3 with masses of 5.6 mg and 27 µg respectively, are from a different batch with a critical temperature around 1.85 K. In addition, thermal expansion has been measured on a fourth crystal (sample #4). Samples #2, #3 and #4 show almost similar values of the specific-heat in the normal state, and of the specific-heat jumps at $T_c$ ≈ 1.85 K as well as of the residual value $\gamma_0$. (We found $\gamma_0 \approx 0.03 J K^{-2}mol^{-1}$ at 0.1 K, or 0.011 J K$^{-2}$mol$^{-1}$ extrapolated from above 0.3 K.)

B. Specific-heat measurements

The specific-heat of two samples (#1 and #2) has been measured by quasi-adiabatic relaxation method in a dilution refrigerator up to 15 T in a superconducting magnet down to 100 mK. Small heat pulses of maximum 1% of the temperature $T$ (0.5% in the superconducting transition) were applied to the sample. The specific-heat $C$ is extracted from the temperature response of the sample during the whole pulse sequence. Down to the lowest temperatures, only one relaxation time was measured in the exponential decay. The addenda have been measured separately. It represents 8% of the total specific-heat measured with the UTe$_2$ samples at 2 K and 2% at 100 mK. Mainly temperature sweeps were performed, but also field sweeps for the transitions at the lowest temperatures (between 100 mK and 200 mK).

To align the samples in the magnetic field, we used a piezoelectric rotator allowing a rotation over 90° in a plane parallel to the field, and a goniometer allowing a ±3° rotation perpendicular to the plane. Furthermore, the set-up is rigid, so that the torque between magnetic field and the anisotropic magnetisation of the sample could not induce a misalignment.

The third sample (#3) of 27 µg coming from the same batch as sample #2, has been measured with an ac specific-heat technique in a ⁴He refrigerator down to 600 mK, and up to 36 T on the M9 magnet at the high magnetic field laboratory LNCMI in Grenoble. Details of the specific-heat set-up are shown in the Supplemental Materials figures of Ref. [31]. For fields up to 18.5 T, the ac calorimetry has been performed using a 20 T superconducting magnet (M2) in combination with a ⁴He refrigerator down to 400 mK. An attenuator piezorotator allowed for a rotation in the (b,c) plane.

The value of the critical temperature $T_c$ is extracted from the specific-heat transition by a fit to an ideal jump broadened by a Gaussian distribution of critical temperatures: $T_c$ corresponds to the centre of this distribution. This model, which reproduced the data very well, allows to extract directly other parameters like the width and jump of the transition (more details in the Supplemental Material Section V.B.).

C. Linear magnetostriction and thermal expansion measurements

The linear magnetostriction $\Delta L_b/L_b$ of UTe$_2$ has been measured on sample #1. In addition, we measured the linear thermal expansion at constant magnetic field on a second single crystal #4.

These measurements have been performed using a high resolution capacitive dilatometer [32]. The capacitance has been determined using an Andeen Hagerling capacitance bridge AH2550A. High magnetic field experiments have been performed using the 30 T magnet M10 of the high magnetic field laboratory LNCMI Grenoble. Due to the limited diameter of the magnet it has been only possible to measure the length change $\Delta L_b$ of the sample parallel to the magnetic field applied along the b axis of the crystal. The magnetic field has been swept with a maximal rate of 100 G/sec to avoid eddy currents heating. The dilatometer was positioned at the end of a silver cold finger in a ⁴He cryostat, with a base temperature near 370 mK. A RuO$_2$ thermometer and a heater were fixed directly on the dilatometer. Temperature sweeps at fixed magnetic field have been performed with maximal heating rates of 0.1 K/min.

Additional thermal expansion measurements have been performed in a superconducting magnet up to 13 T using a dilution refrigerator in the Pheliks laboratory.
FIG. 1. Temperature dependence of the specific-heat $C/T$ at different magnetic fields $H \parallel b$ from 0 T to 15 T measured on sample #2.

III. RESULTS

A. Magnetic field $H \parallel b$

1. specific-heat

In zero field, all studied samples exhibit a single sharp superconducting transition (width $\Delta T_c \approx 20$ to 38 mK), with a large jump at the superconducting transition (up to $\Delta C/C \approx 1.85$), emphasising the high quality and homogeneity of the samples. Figure 1 shows $C/T$ as a function of temperature measured on sample #2 for several magnetic fields $H \parallel b$ up to 15 T. At zero field the transition is extremely sharp at $T_c = 1.85$ K. Under magnetic field it remains sharp and the jump $\Delta C/T_c$ is pronounced up to 18.5 T (see Fig. 2-a), so the transitions are easily followed under field. The first specific-heat measurements on UTe$_2$ displayed an upturn and a large residual term of $C/T$ at low temperatures [33]. They both became smaller with improved sample quality. Indeed, our measurements on crystal #2 show a small residual term, and an upturn shifted to lower temperatures compared to samples with lower $T_c$ (see also in Supplemental Material [34] Section V.A.). This agrees with more recent work claiming that residual term and upturn are extrinsic to UTe$_2$ [9, 35, 36].

Remarkably, above 15 T for $H \parallel b$, $C/T$ shows two transitions. A second wide transition appears for fields above 15 T (350 mK width at 18 T), which is well detached above $H \gtrsim 17$ T from the sharp transition observed at lower fields (Fig. 2-a). In the following, we will label ”high-field” (HF) and ”low-field” (LF) these two respective transitions. We follow this second wide HF transition (Fig. 2-b) up to the metamagnetic transition at a field $H_m = 34.75$ T [37–39]: its width and height remain roughly constant with field.

These high-field measurements shown on Fig. 2 were performed on crystal #3, which is from the same batch as #2 in Fig. 1. As observed already on resistivity measurements in the field-reinforced superconducting phase, the $T_c$ of this broad anomaly is increasing with field, except very close to $H_m$ where the transition temperature decreases slightly. This may be due to a slight misalignment of the sample in the high-field experiments (no sample rotation), or it could be intrinsic. Above $H_m$, the broad anomaly abruptly disappears. This HF transition is most likely the expected bulk signature of the field-reinforced superconducting phase observed in transport properties for the same field direction [20, 39]. We will see in the next section that there is indeed other experimental evidence that the sample displays bulk superconductivity in this high-field region.
FIG. 3. $C/T$ versus field for $H \parallel b$ on sample #3 at different temperatures between 0.7 K and 1.86 K. The LF superconducting transition and the peak at $H_m$ are clearly visible, but the HF superconducting transition reported on Fig. 2 appears only as a very broad anomaly.

FIG. 4. $C/T$ versus field ($H \parallel b$) at the metamagnetic transition for sample #3 at different temperatures between 0.7 K and 1.86 K. The hysteresis is independent of the field sweep rate. On the left panel, the arrows indicate the direction of the field-sweeps.

Figure 3 displays the field dependence along the $b$-axis of $C/T$ up to 36 T. On approaching the first order metamagnetic transition at $H_m \approx 34.75$ T $C/T$ shows a strong increase, with a large drop (of order 25 %) above $H_m$, and a hysteresis (see Fig. 4), independent of the sweep rate. The drop of the specific-heat is sharpest at lowest temperature with a width of 0.25 T. However, a possible interplay between the superconducting and metamagnetic transition at this temperature may influence the shape of the anomaly. The width of the hysteresis decreases linearly with increasing temperature, starting from 0.17 T at 0.7 K. This behaviour of $C/T$ at $H_m$ agrees qualitatively with previous measurements [40, 41] performed in pulsed magnetic fields (see comparison in Supplemental Material [34] Section IV). As regards the superconducting transitions below $H_m$, the sharp LF transition is well observed on the field sweeps (see Fig. 3). However, the high-field transitions observed on temperature scans appear as a very broad anomaly, and it is noticeable only by comparison with curves at a different temperature. This arises due to the combination of an already large $T_c$ distribution at fixed field, and an almost vertical $H_{c2}$, so that this high-field transition appears extremely broad as a function of field (see Supplemental Material [34] Section V.D.).

2. Linear Magnetostriction

The longitudinal linear magnetostriction $\Delta L_b(H)/L_b$ along the $b$ axis has been measured for $H \parallel b$, and it is shown on Fig. 5 for sample #1. In the normal state ($T = 2$ K), the linear magnetostriction is negative and shows roughly a $H^2$ field-dependence, as usually observed in paramagnetic metals (Fig. 5(a)). This is in agreement with the low field measurements of Ref. [42] and with the very recent measurements in pulsed magnetic fields, which show a strong negative jump of the linear magnetostriction at the metamagnetic transition [43]. Following Maxwell’s relations, the negative sign of $\Delta L_b/L_b$ indicates that under uniaxial stress applied along the $b$ axis, the susceptibility $\chi_b$ along this axis should increase under uniaxial stress along the $b$ axis. We note that under pressure such an increase of $\chi_b$ has been observed [44].

However, in the superconducting state at $T = 0.35$ K, the linear magnetostriction shows a very pronounced hysteretic behaviour. In Fig. 5(b) we display the additional contribution to the linear magnetostriction $\Delta L_b^s$ which appears in the superconducting state. It is obtained from the measured linear magnetostriction at fixed temperature in the superconducting state after subtraction of the paramagnetic contribution measured at 2 K. The linear magnetostriction in the superconducting state is very large and shows a strong hysteresis with a fish-tail-like behaviour both below $\approx 15$ T and above $\approx 20$ T.

This irreversible magnetostriction appears very similar to the behaviour of the magnetisation in the mixed state of type II superconductors with strong vortex pinning.

In the critical state model [45], magnetic flux penetration or expulsion, when increasing (or decreasing) the field, is impeded by vortex pinning. This imposes a field gradient at the sample surface, perpendicular to the applied field, controlled by the critical current den-
sity. Hence, if magnetic flux lines are trapped by the action of pinning forces, equal but opposite forces will act on the lattice. Thus, the length change of the crystal $\Delta L_b/L_b$ is proportional to $H_{irr}^2 E_C^2$, where $\Delta M$ is the non-equilibrium part of the magnetisation, $E$ the Young modulus, and $C$ a constant depending on the Poisson’s ratio [46].

In UTe$_2$, at the lowest temperature, the hysteresis in the linear magnetostriction vanishes above $H_{irr,1} \approx 14.5$ T, and it opens again above $H_{irr,2} \approx 21$ T, being maximal at 30 T, which is the highest field we could reach in these experiments. On increasing the temperature, the lower field $H_{irr,1}$ decreases, while the upper field $H_{irr,2}$ increases and above 0.6 K, it exceeds the achievable field range.

In Fig. 6 we display the irreversibility field $H_{irr}$ of sample #1 ($T_c \approx 1.5$ K) determined from the magnetostriction measurements. We compare this field with $H_{c2}$ determined from specific-heat measurements (only below 15 T on this sample), with thermal conductivity measurements (see section I in the Supplemental Material [34]) and also with resistivity measurements on a sample from the same batch with similar $T_c$ (published in [20]). Obviously, flux pinning in the sample at low magnetic fields is very strong and the irreversibility field follows the upper critical field: $H_{irr} \sim H_{c2}$. In the field-enhanced superconducting phase above 15 T the difference between $H_{irr}$ and $H_{c2}$ gets more pronounced: there is a broader reversible regime, between $H_{c2}$ and $H_{irr}$. However, the observation of the irreversible magnetostriction due to the flux pinning between 20 and 30 T is a first proof of the bulk nature of the field-enhanced superconducting phase of UTe$_2$.

In the Supplemental Material ([34] Section II) we also show the linear thermal expansion $\Delta L_b(T)/L_b$ as a function of temperature measured on sample #4, which has a $T_c = 1.82$ K similar to that of samples #2 and #3 studied by specific-heat. On sample #4, we could observe both $H_{c2}$, which shows up as a kink in the temperature dependence of the sample length (hence a jump of the thermal expansion $\Delta L_b/L_b$), as well as the entrance of the irreversible regime of the magnetisation, controlled by vortex pinning. In the HF phase, as opposed to the LF phase, ([34] Section II), the temperatures for the emergence of $H_{c2}$ and of the irreversible regime are well separated, suggesting a change of the pinning strength in the HF phase.

3. Complete phase diagram

Since samples #2 and #3 come from the same batch and have essentially the same $T_c$ at 0 T (1.847 K and 1.845 K respectively), we use the specific-heat measurements on these two samples to establish the complete superconducting phase diagrams for all field directions from 0 to 36 T. It is shown on Fig. 7. The most important result is that for $H \parallel b$, two superconducting phases are clearly present, with a point around $H = 15$ T and $T = 1$ K where the three transition lines should join.

The low field (LF) superconducting phase corresponds to the sharp LF transition that can be followed from 0 T up to 18.5 T. The high field (HF) phase corresponds to the broad HF transition emerging above 15 T, and followed up to $H_m$ (Figures 1-2). For magnetic fields slightly above 15 T, the two transitions overlap, thus it is difficult to determine unambiguously $T_c$ for the HF phase transition. Knowing that the jump and width of these transitions seem to have a negligible field-dependence (Fig. 11), we fixed them to extract $T_c(H)$ between 15 and 17 T (empty crosses on Fig. 7). (More details on the analysis and the way we determine the transition temperatures are given in the Supplemental Material, [34] Section V.B.) Hence these values need to be taken with some caution, and there is yet no experimental determination of the precise nature of the crossing of the $H_{c2}$ lines of the LF and HF phases: they could merge with
a sharp change of slope, or could be tangential. Moreover, we looked thoroughly but found no sign of a fourth transition line inside the LF phase, either on temperature or field sweeps, as would be expected for a crossing of second order phase transitions \[4, 47\]. This absence of a fourth line however could also arise from a further broadening of the HF transition inside the LF phase (for field below 15 \(T\)).

Nevertheless, the phase diagram clearly demonstrates the existence of at least two different superconducting phases for \(H \parallel b\): comparison with the resistivity measurements, and the observation of vortex pinning in the (HF) phase leave little doubt that it is also a superconducting phase.

**B. \(H_{c2}\) close to \(T_c\), along all directions**

The upper critical field \(H_{c2}\) in UTe\(_2\) present anomalies not only due to the presence of these two superconducting phases: it also has an anomalous temperature dependence close to \(T_c\). It is known that in UTe\(_2\), the initial slope of \(H_{c2}\) at \(T_c\) \(\left(\frac{dH_{c2}}{dT}\right)\) has its largest value for \(H \parallel b\). From our specific-heat measurements, we determine \(\frac{dH_{c2}}{dT} \approx -34 \, T/K\), which is larger than the values obtained from electrical transport measurements. Generally speaking, in electrical transport measurements, filamentary superconductivity easily leads to slightly higher \(T_c\) values. However this filamentary superconductivity is fragile under small fields, hence it is usual to observe a small positive curvature of \(H_{c2}\) determined by resistivity. In UTe\(_2\), it may mask the negative curvature of \(H_{c2}\) clearly visible in Fig. 7 close to \(T_c\).

Figure 7 also shows the upper critical field \(H_{c2}\) of sample #2 along the \(a\) and \(c\) axes. Similar to the previous reports \[1, 20\], \(H_{c2}\) is strongly anisotropic and extrapolates to 9 \(T\) for the \(a\) axis, 15 \(T\) for the \(c\) axis. Along the \(c\) axis, \(H_{c2}\) is almost linear on a large temperature range, and has the lowest slope near \(T_c\), \(\frac{dH_{c2}}{dT_c} \approx -7.5 \, T/K\).

The most striking feature appears along the \(a\) axis: it seems also linear and with a slope at \(T_c\) comparable to that along the \(c\) axis when examined on this broad field and temperature scale. However, a closer look very near \(T_c\) shows that \(H_{c2}\) along this easy axis has a very large initial slope, followed by a very strong negative curvature. Figure 8 displays a zoom on the very low field region \((H < 0.4 \, T)\): it can be seen that the initial slope \(\frac{dH_{c2}}{dT_c}\) is of order -20 \(T/K\) for \(H \parallel a\). It is almost of the same order as that along the \(b\) axis \((-34 \, T/K)\), and much larger than along the \(c\) axis \((-7.5 \, T/K)\). This effect could be easily missed in resistivity measurements, again due to filamentary superconductivity, or because of “too large” field-steps.

In fact, this effect was probably already present in Ref. 48 reporting also specific-heat measurements, but
FIG. 8. Upper critical field $H_{c2}$ as a function of temperature determined from specific-heat measurements on sample #2 for the $a$, $b$ and $c$ axes at very low fields, close to $T_c$. The dashed line is a linear fit following the initial slope of $H_{c2}$ along the easy $a$ axis. It is close to the value along the $b$ axis, however followed by a very strong negative curvature.

It is now well established, notably from the very large specific-heat jump at $T_c$, that UTe$_2$ is in a strong-coupling regime. In such a case, the most natural explanation for the field-reinforced superconducting phase is that the superconducting pairing itself is enhanced under fields along the $b$ axis [1, 20]. As discussed already for the ferromagnetic superconductors, in the strong coupling regime, such a field dependence should be reflected also in the normal state Sommerfeld specific-heat coefficient ($\gamma$) [49, 50]. In UTe$_2$, it is not so easy to determine the Sommerfeld coefficient, because $C/T$ remains strongly temperature dependent almost down to $T_c$, and there is no simple way to analyse this temperature dependence (see Supplemental Material [34] Section III). Hence, as already reported in the literature [38, 40, 41], we show in Fig. 9 $C(H)/T$ normalised to $C/T$ in zero field, at $T = 1.8$ K, which can be considered as a reasonable estimation of the $\gamma$ value, at least not too close from $H_m$.

Figure 9 displays the field dependence of $\gamma/\gamma(H = 0)$ for the three directions up to 15 T on sample #1, and, combined with the ac specific-heat data of sample #3 up to 36 T along the $b$ axis. Along the $b$ axis, $\gamma(H)$ increases under field, and even more strongly on approaching $H_m$. It is similar to already published data [38, 40], with some quantitative differences notably above $H_m$, where the ac technique in static fields probably allows for more precision. By contrast, for the field along the $a$ or $c$ axis, $\gamma(H)$ has a much more complex behaviour. It is known that along the $a$ axis, Lifshitz anomalies appear around 5 T and possibly 9 T [51]. They are indeed visible on our measurements (see Supplemental Material [34] Section III). However, we observe additional anomalies also as pronounced maxima along the $a$ and $c$ axes, respectively at 0.5 T and 1.5 T, whose origin is still unclear. Hence, until better understood, it is difficult to rely on the field dependence of $\gamma$ to discuss the behaviour of the pairing strength with field.

IV. ANALYSIS

A. $H_{c2}$ in the LF phase

Some discussion is first required on the behaviour of $H_{c2}$ close to $T_c$. Indeed, the observed negative curvatures of $H_{c2}$ along the $b$ and $a$ axes might seem to contradict the common belief that UTe$_2$ is a $p$-wave superconductor with a $d$-vector perpendicular to the easy $a$ axis. Indeed, the very strong curvature of $H_{c2}$ along the $a$...
axis may suggest a severe paramagnetic limitation. This is at odds with the value of $H_{c2}(0) \sim 9$ T, which is much larger than the weak-coupling paramagnetic limit of around 3.5 T (for a gyromagnetic factor $g = 2$ with $T_c = 1.85$ K). Actually, the negative curvature is so concentrated close to $T_c$ that it requires a large value of $g \sim 3.2$ (in the weak-coupling limit) to match the initial deviation from linearity of $H_{c2}$ along the $a$ axis, leading to a saturation of $H_{c2}(0)$ at 2.25 T at low temperatures (see also Fig. S13 of the Supplemental Material [34] Section V.F.). In other words, $H_{c2}(T)$ along the easy axis does not follow at all the temperature dependence of an upper critical field solely controlled by paramagnetic and orbital limitations [52]. Adding strong-coupling effects will not change anything to this problem, except for requiring even larger values of $g$ to reproduce the initial negative curvature.

Before discussing further this anomalous temperature dependence of $H_{c2}$ along the $a$ axis, let us examine the new larger value of the initial slope derived from these bulk measurements (see Fig. 7). Previous measurements of the lower critical field [53] had already pointed out an inconsistency between $H_{c1}$ and $H_{c2}$ along the $a$ axis. From the row value of $H_{c1}$, it was deduced ([53]-Supplemental Material) that $dH_{c2}/dT_c$ should be $-16$ T/K (instead of $-6.6$ T/K deduced from resistivity measurements, for a sample with $T_c \sim 1.45$ K), which rescales up to $-20.4$ T for a $T_c$ of 1.85 K. Hence, the observed value of $-20$ T/K is now fully consistent with $H_{c1}$ along the $a$ axis.

As regards the same comparison of $H_{c1}$ with $H_{c2}$ along the $c$ axis, the agreement is also very good. Again, after rescaling the $T_c$, from the values of $dH_{c1}/dT_c$ along the $c$ axis in [53], we expect a value of $dH_{c2}/dT_c$ of $-7.6$ T/K, whereas our experiments yield to $-7.5$ T/K.

This has to be contrasted with the case for $H \parallel b$. As stated already in [53], the anisotropy of $H_{c1}$ between the $b$ and $c$ axes at $T_c$ is very small. Hence $dH_{c2}/dT_c$ should be roughly equal (and of the order of $-8$ T/K) in both directions, whereas $dH_{c2}/dT_c$ is largest along the $b$ axis and of the order of $-34$ T/K (see Table S1 in Supplemental Material [34] Section V.E.). In addition, the temperature dependence of $H_{c2}$ along the $c$ axis and the $b$ axis also cannot be reproduced by any combination of paramagnetic and orbital limitations: for $H \parallel c$, $H_{c2}(T)$ is too linear with even a small positive curvature close to $T_c$, and for $H \parallel b$, the situation is similar to that along the $a$ axis, however with smaller deviations visible mainly above 8 T (Fig. S13 in the Supplemental Material [34] Section V.F.).

The only way we can imagine to understand these anomalies regarding the value of the slope at $T_c$ (for $H \parallel b$) and the temperature dependence of $H_{c2}$ along all directions, is a field dependent pairing strength. This has been observed in ferromagnetic superconductors [49] and been already proposed for UTe$_2$ [1, 20]. If we call $\lambda$ the strong-coupling parameter controlling this pairing strength, it has to be field-dependent in all directions.

We can rely on $H_{c1}$, where the effects of such a field dependence have negligible effects [53] to fix the average Fermi velocities controlling $dH_{c2}/dT_c$ for $H \parallel b$, without the contribution of the field-dependent pairing strength. Along the $a$ and $c$ axes, the agreement between $dH_{c1}/dT_c$ and $dH_{c2}/dT_c$ shows that $\frac{d\lambda(H)}{dT_c} \sim 0$ in zero field (at $T_c$).

For the estimation of $\lambda(H)$ in the different directions, there are general constraints which are model-independent. First of all, along the $b$ axis, the discrepancy between $dH_{c1}/dT_c$ and $dH_{c2}/dT_c$ can only be reconciled with an increase of the pairing strength: increasing $dH_{c2}/dT_c$ requires $\frac{d\lambda(H)}{dT_c}(H = 0) > 0$. Hence, we expect an increase $\lambda(H)$ in the (LF) phase.

For $H \parallel c$, starting with $\frac{d\lambda(H)}{dT_c}(0) = 0$, the small positive curvature and very linear behaviour also requires an increase of $\lambda(H)$, whatever the strong-coupling model and the mechanisms (orbital and/or not paramagnetic limitation).

The situation is not so straightforward for $H \parallel a$. Indeed, the deviation from linearity observed very close to $T_c$ could be explained by a strong suppression of $\lambda(H)$ with an $H_{c2}$ which would be purely orbitally-limited (otherwise, it would be impossible to reach such a ”large" value of $H_{c2}(0)$). But it could also arise from an increase of $\lambda(H)$, starting from an $H_{c2}$ having both an orbital and paramagnetic limitation in order to be smaller than the measured $H_{c2}$. Taking into account the large specific-heat jump at $T_c$ pointing to a developed strong coupling regime, it is reasonable to assume a value of $\lambda(0) \geq 1$. Then, the second scenario would require a gyromagnetic factor $g$ larger than 6.5 (at least twice the weak-coupling value of 3.2 mentioned above). Hence, from $H_{c2}$ (and $H_{c1}$) alone, we can only conclude that for $H \parallel a$, there is either a very strong paramagnetic limitation and a fast increase of $\lambda(H)$, or essentially no paramagnetic limitation and a strong decrease of $\lambda(H)$.

The first hypothesis would be opposite to common belief that UTe$_2$ is a $p$-wave spin-triplet superconductor, and it requires some exchange field mechanism like in paramagnetic Chevrel phases [54] to account for such a large effective gyromagnetic ratio. The second one is well supported by the recent Knight-shift measurements [19], yielding essentially no change of the Knight-shift along the $a$ axis, and only minute changes along the $b$ and $c$ axes. It should be stressed here that NMR measurements are performed at fixed field. They extract the change of electronic spin susceptibility (when there is a mean to estimate the ”reference level” in the normal state) across $T_c$ from the temperature variation of the Knight-shift. Hence, this measurement is not directly influenced by the field dependence of the pairing strength, as opposed to considerations on the violation of the paramagnetic limit on $H_{c2}$.

In the following, we will concentrate on this second hypothesis: spin-triplet superconductivity with an equal-spin pairing state (ESP state) characterised by a $d$-vector perpendicular to the $a$ axis. It is more reasonable as regards values of the gyromagnetic factor, and it is well
FIG. 10. $\lambda(H)$ determined from $H_{c2}$ measured on sample #2. $\lambda$ was set to 1 at 0 T, $g$ to 0 for the LF phase. Red squares are for $H \parallel b$, green triangles for $H \parallel c$ and blue circles for $H \parallel a$. When it comes to the HF phase, red crosses symbolise $\lambda(H)$ determined with $g = 2$, and grey crosses with $g = 0$. A zoom on fields below 1 T is shown in the insert.

FIG. 11. (a) Jump of the transitions $\Delta C/T_c$ as a function of field, for $H \parallel b$, determined on sample #2 and #3 for the LF and HF transitions. (b) Width $\Delta T$ of the transitions as a function of fields. $\Delta T$ is equal to 2.35 times the standard deviation of the Gaussian distribution.

The result for $\lambda(H)$ in the three crystallographic directions is reported on Fig. 10. The increase of $\lambda$ along the $c$ and even $b$ axes is modest in the (LF) phase, at most 10%, whereas a strong suppression (factor 2 between $T_c$ and $T \to 0$) is required for $H \parallel a$. The different parameters of the fit are summarised in Table S2 of the Supplemental Material [34] Section V.F. It is to be noted that the lowest Fermi velocities ($v_F$) are along the $b$ and $c$ axes and highest along $a$ axis. This matches qualitatively the anisotropy found in transport measurements between the different axes [55]. However, $H_{c2}$ depends on an average of the Fermi velocities perpendicular to the applied field direction, weighted by the pairing strength. As a consequence for example, we never succeeded to compare quantitatively anisotropies of $H_{c2}$ in UPt$_3$ or in URu$_2$Si$_2$ with the detailed determination of their respective Fermi surfaces by quantum oscillations [56, 57], even though the order of magnitude of the orbital limitation is consistent with measured effective Fermi velocities. This is probably even more acute if subtle $Q$-dependent pairing is responsible the specific pairing state realised in the system, as could well be the case in UTe$_2$ [24, 26–28].

B. $H_{c2}$ in the HF phase

The field-reinforced HF phase is a central property of UTe$_2$, and from the very start, it has been proposed that it could arise from a symmetry change of the order parameter. The main idea is that for a spin-triplet superconducting state arising from ferromagnetic fluctuations along the easy magnetisation axis, at low fields, the $d$-vector should be perpendicular to this $a$ axis. Hence a $B_{3u}$ (or more generally $B_{3u} + iB_{1u}$ state under field [2]) is favoured at low fields. By contrast, for strong fields along the $b$ axis, a rotation of the $d$-vector is expected toward a $B_{3u}$ state (or $B_{2u} + iA_{2u}$), to minimise the component of the $d$-vector along the $b$ axis. Such a symmetry change would imply a phase transition somewhere between the low and high field regimes, which had not been detected until the present specific-heat measurements. However, this change of $d$-vector orientation alone could only explain that an initial paramagnetic limitation is exceeded thanks to the new orientation of the $d$-vector, it will not explain the positive $dH_{c2}/dT$ observed in the HF phase between 15 and 30 T.

Conversely, empirical explanations of the reinforcement of $H_{c2}$ focused on field-induced enhancement of the pairing (positive $d\lambda/dH$) [1, 20], which is compatible with, but does not require an additional phase transition. Indeed, even rather sharp upturns observed on $H_{c2}$ extracted from electrical transport can be reproduced with a smooth continuous increase of $\lambda(H)$ [2].

However the present experiment shows not only that there is a field-induced phase transition between two different superconducting phases, but also that the nature of these phases is likely different. Indeed, in UPt$_3$ [58]...
or more recently in CeRh$_2$As$_2$ [59, 60], no change is observed on the shape of the specific-heat transition along $H_{c2}$ when switching from the low to the high field superconducting phases. By contrast, in UTe$_2$, the specific-heat anomaly for both phases is markedly different, with a very broad anomaly in the HF phase whereas that of the LF phase remains remarkably sharp (see Fig. 2).

When deconvoluted from broadening effects (thanks to the Gaussian analysis of the temperature dependence of $C/T$), it is observed that the specific-heat jump along $H_{c2}$ in the HF phase is smaller than in the LF phase. The specific-heat jump at the transition from the LF to HF phase displays a marked drop compared to its amplitude for the transition between the normal and the superconducting LF phase (see Fig. 11). Essentially, it goes down to the same level as that of the wide transition of the HF phase, which remains roughly constant up to $H_m$. Hence, as expected, the emergence of the HF transition goes along with a redistribution entropy between both phases.

This does confirm the reality of a double superconducting transition, however with a field induced transition requiring not only a symmetry change, but also a different pairing mechanism leading both to the positive $dH_{c2}/dT$ and the broadened transition. It is again difficult to avoid invoking field-reinforced pairing. A Lebed mechanism [61] would not explain the double transition and does not fit with the overall 3D character of the system [2]. A Jaccarino-Peter mechanism [62] associated to a competing singlet (or at least, strongly paramagnetically limited phase) could lead to such a field-induced phase [63], but it would require a phase with a very large orbital limit hence necessarily with a larger $T_c$ in zero field than the present LF phase.

So again, we need to invoke field-reinforced pairing mechanism to explain the HF phase appearance above 15 T. Most likely, pairing is reinforced on approaching $H_m$. The phase diagram for $H \parallel b$ under pressure [64] shows that the HF phase could well be the same as the pressure-induced higher $T_c$ superconducting phase [4–7]. There are two main theoretical proposals for this “high-$T_c$” pressure-induced phase. The first [25] is that it is a $B_{2a}$ phase, having no component of the d-vector along the $b$ axis, with a pairing mechanism controlled by local ferromagnetic correlations. The second is that it is a spin-singlet ($A_g$) phase [26], induced by antiferromagnetic correlations becoming more dominant under pressure.

The first proposal is the most natural one, explaining the different phases with a single pairing mechanism and transitions imposed by the Zeeman coupling of the $d$-vector with the applied field. In this framework, nothing should be changed to the procedure for the evaluation of the field dependent pairing strength $\lambda(H)$ between the LF and HF phases. The result is reported also on Fig. 10 (grey crosses), displaying a cusp at $\sim 15$ T, but a rather weak increase (30% between zero field and $H_m$) of the pairing strength, far from the factor 1.8 observed on the Sommerfeld coefficient (Fig. 9).

Using the other proposal of a spin-singlet superconducting order parameter for the HF phase seems paradoxical, but yields interesting results. Once again, with a field dependent pairing, the weak-coupling paramagnetic limit is easily exceeded thanks to the effective increase of the “zero-field” $T_c$, and strong coupling effects. Under pressure, this singlet phase can explain the strong paramagnetic limitation observed along the $a$ axis [20, 65]. Here, it leads to a stronger field dependence of the pairing strength (red crosses on Fig. 10), required to overcome the saturation of $H_{c2}$ (at fixed $\lambda$, see Fig. S14 in the Supplemental Material [34] Section V.F.). Estimation of $\lambda(H)$ has been done for a gyromagnetic factor of 2, and (arbitrarily) with the same energy scale $\Omega$ than for the LF phase, considering that both mechanisms should have similar characteristics in order to lead to similar critical temperatures. Using different values of $\Omega$ (but the same $g$-factor) changes little to the following analysis.

Indeed, this spin-singlet scenario has some success to explain the large broadening observed for the superconducting transition in the HF phase. Considering that the field dependence of $\lambda$ might come from the proximity to $H_m$, a simple hypothesis is that $\lambda$ is a function of $H_m$. Then, a dispersion of $H_m$ controlling the broadened specific-heat anomaly reported on Fig. 4 for the metamagnetic transition, will translate into a distribution of $T_c$, hence to a new mechanism for the broadening of the superconducting transition. From the calculation of $H_{c2}$ at fixed $\lambda$ used to extract the field dependence of the pairing, we can as well determine $T_c = \varphi \left( H, \lambda \left( \frac{H}{H_m} \right) \right)$. This allows to determine the effect of the distribution of $H_m$ (deduced from an analysis of the specific-heat at $H_m$), on the specific-heat anomalies of the HF phase, according to the two different determinations of $\lambda(H)$ (see Fig. 12).

With this hypothesis, the dispersion of $H_m$ explains half the width of the superconducting transition in the spin-singlet scenario (it fits well the observed anomaly by

![FIG. 12. HF transition at 24 T, $H \parallel b$, measured by specific-heat. Lines are the transitions calculated for different $g$-factors.](image-url)
FIG. 13. Temperature dependence of $C/T$ measured on sample #3 $H \parallel b$ at 18.5 T for different angles in the (b,c) plane. Dashed lines are the transitions calculated from a $H_m$ distribution, controlled by its angular dependence [5] and a finite mosaicity of 3° in the sample.

V. SUMMARY

A clear result from this work is the requirement of a field-dependent pairing strength along all directions of the applied field, as shown by the anomalous temperature dependence of $H_{c2}$ along the a, b and c axes already close to $T_c$. The (most likely) strong decrease of the pairing strength along the a axis is reminiscent of the results on UCoGe along its easy magnetisation axis, and at first sight, it seems best compatible with a pairing mechanism involving true ferromagnetic fluctuations. Indeed, there are several theoretical proposals showing that spin-triplet pairing is possible with finite momentum magnetic fluctuations [28], or with only local ferromagnetic correlations within a unit cell [25]. It is not clear how such mechanisms could lead to a strong decrease of the pairing strength already at very low fields. However, the Fermi surface instability observed at 6 T along the easy axis [59] could also play a role if $Q$-dependent pairing is important. Hence, the origin of this strong suppression of the pairing along the easy axis in UTe$_2$ requires deeper and more detailed investigations, before strong conclusions can be drawn. A detailed characterisation of the Fermi surface of UTe$_2$ is notably a mandatory step. In any case, hasty comparison with ferromagnetic superconductors could be misleading.

This is even clearer regarding the results along the hard b axis. From this work, we can now claim the existence of two different bulk superconducting phases in UTe$_2$ already at ambient pressure, when a magnetic field is applied along the b axis above 15 T. Analysis of these results has been done looking for the simplest possible explanations. In the LF phase, we confirm that contradiction between $H_{c1}$ and $H_{c2}$ values close to $T_c$ requires a steep increase of the pairing strength at very low fields ($\partial T_c/\partial H > 0$ at $T_c$). In the HF phase, We have found indirect support for a paramagnetically limited $H_{c2}$ along the b axis, as it could explain a large part of the strong broadening of the specific-heat anomaly in this new phase.

However, many open questions remain. A first one concerns the strong difference between the irreversibility line observed on the magnetostriction and the specific-heat anomaly: it suggests a strong increase of the reversibility region in this phase, like the fact that the resistive transition only goes to zero when the bulk transition ends. This is similar to previous observations in UCoGe [66], however, we still have no explanation for this phenomenon. A new generation of crystals with even better homogeneity (at low fields and at the metamagnetic transition) would be key to distinguish intrinsic and extrinsic effects. For example, if broadening of the transition in the HF phase remains unchanged, more exotic explanations (for a 3D, low $T_c$ system) like an enhancement of the superconducting fluctuations would be required.

Another important point concerns the order of the different transition lines and the precise slopes of the lines at the tricritical point. Indeed, as for CeRh$_2$As$_2$ [59], in the case of a spin-triplet to spin-singlet transition, a first
order transition is expected. In our specific-heat measurements, we did not detect hysteresis effects. The only features visible on Fig. 11 are a smaller jump of $C/T$ for the LF to HF transition than along $H_{c2}$ in the LF phase, as well as a slightly smaller width. This slight narrowing leaves open the possibility that the transition from LF to HF phases could be weakly first order, but it requires further experimental confirmation. Unfortunately, magnetostriction measurements could not be performed on sample #1 at low enough temperatures to test volume changes at the LF to HF transition. It is a main target for the next high-field experiment campaign.

If this transition was first order, of course, the question of the tricritical point would be solved. If it is not, it remains an issue to determine if there is an additional transition line within the LF superconducting phase, and whether or not the three transition lines determined in this work join with different slopes or if the $H_{c2}$ line has no change of slope (only a very strong positive curvature) at the tricritical point. Like for the question of the origin of the difference between resistivity and specific-heat in the HF phase, more details on the tricritical point or higher sensitivity to additional transition lines require crystals with even better homogeneity.

Note also that the entrance into the HF phase along $H_{c2}$ cannot be done in a mixed singlet-triplet superconducting phase [67]; it would require, like for the chiral superconducting state [8], a double transition which is not observed.

This scenario, where both ferromagnetic and antiferromagnetic fluctuations lead to (competing) pairing mechanisms, would then be central for the pressure and the field-induced phases of UTe$_2$. At ambient pressure, at the opposite of CeRh$_2$As$_2$, they would lead to a paradoxical spin-singlet (or mixed triplet-spin-singlet) phase at high fields, most likely driven by strong antiferromagnetic correlations on approaching the metamagnetic transition. Under pressure, this high-field phase would become the highest $T_c$ phase with the lowering of the metamagnetic field along the $b$ axis, whereas the pure spin-triplet phase would survive essentially for large enough fields along the easy $a$ axis. UTe$_2$ would then be probably the first superconductor where two competing pairing mechanism of similar strength exist and can be arbitrarily tuned by field and pressure. A promising playground for future experiments and theories.

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Thermal conductivity \( \kappa \) measurements have been performed on a sample from the same batch as sample #1. The temperature sweeps have been measured on a home-made dilution refrigerator with a base temperature of 100 mK and a superconducting magnet with field up to 16 T using a standard “one heater-two thermometers” setup. The temperature dependence of \( \kappa/T \) for different magnetic fields up to 16 T is represented in Fig. S1. \( \kappa/T \) shows a broad maximum at around 3 K. At low field, there is a clear increase in \( \kappa/T \) just below \( T_c \), which is suppressed by increasing the field. Such an increase below \( T_c \) has also been observed in other systems such as CeCoIn\(_5\) or YBCO, and attributed to a suppression of the inelastic scattering of heat carriers (electrons and phonons, respectively) by the opening of the superconducting gap. In the case of UTe\(_2\), the enhanced conductivity below \( T_c \) is likely due to an increase of the electronic mean free path due to the opening of the superconducting gap. At higher field (\( \mu_0 H > 3 \) T), entrance in the superconducting state is marked by a rapid decrease of the thermal conductivity, usually attributed to Andreev scattering on the vortex cores.

**I. THERMAL CONDUCTIVITY \( H \parallel b \)**

FIG. S1. Temperature dependence of \( \kappa/T \) in UTe\(_2\) with \( H \parallel b \) between 0 and 16 T (every Tesla). Traces have been shifted for clarity. The superconducting temperature \( T_c \) is represented by vertical arrows.

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II. THERMAL EXPANSION FOR FIELD \( H \parallel b \)

In addition to the linear magnetostriction measurements discussed in the main text, we performed longitudinal thermal expansion measurements (length change in the field direction) on sample #4 as a function of temperature at fixed magnetic fields up to 29.5 T along the \( b \) axis in the high magnetic field laboratory LNCMI and in addition using a superconducting magnet up to 13 T in the Phelix laboratory.

Figures S2 (a) and (b) show the temperature dependence of the relative length change along the \( b \) axis \( \Delta L_b/L_b \) for different magnetic fields. The data displayed in panel (a) are obtained by cooling from the normal state to the lowest temperature in a field 2 T below the target final field, then increasing the field at the lowest temperature up to the final field, heating at that field above \( T_c \) and cooling again. Increasing the field at low temperature induces a non-equilibrium magnetisation inside the sample due to the flux pinning, while an equilibrium flux distribution occurs in the final-field cooled sweep. \( H_{irr} \) marks the onset of the irreversible magnetisation regime.

The data in panel (b) are measured in a superconducting magnet using a dilution refrigerator. They are obtained from a similar procedure however starting by zero field cooling, then ramping the field up to its target value, heating up to the normal state and measuring the field-cooled length change. Again, the irreversibility line can be clearly determined.

In panel (c) we show the temperature dependence of the upper critical field \( H_{c2} \) and of the irreversibility field \( H_{irr} \) determined from the cycles described above. Data for \( H \leq 13 \) T are from the experiments performed in a superconducting magnet, while for \( H > 13 \) T they stem from the high-field experiments in LNCMI. While in the LF superconducting phase \( H_{irr} \) is lower but very close to \( H_{c2} \), in the HF superconducting phase \( H_{irr} \) is far lower in temperature than \( H_{c2} \), indicating a wide reversible region (with low pinning) behind \( H_{c2} \) in this state.

![Temperature dependence of the linear thermal expansion \( \Delta L_b/L_b \) in UTe\(_2\) with \( H \parallel b \) at different magnetic fields measured in (a) the LNCMI Grenoble, and (b) using a superconducting magnet. The arrows indicate the direction of the temperature sweep, see text for details. The temperature of the irreversibility field \( H_{irr} \) and the upper critical field \( H_{c2} \) are indicated by arrows and vertical bars, respectively. Panel (c) shows the phase diagram obtained from the thermal expansion measurements. While at low field (\( H < 15 \) T) \( H_{irr} \approx H_{c2} \), at high fields the irreversibility line is much lower in temperature.](image-url)
III. SPECIFIC-HEAT: NORMAL PHASE

In general, the leading term of the specific-heat of a heavy-fermion system at low temperatures ($T \ll T_F$, with $T_F$ being the effective Fermi temperature), far from any quantum criticality, is linear in temperature: $C \propto \gamma T$. The Sommerfeld coefficient $\gamma$ is proportional to the density of states at the Fermi level, which is, strongly renormalised compared to the free electron gas. In UTe$_2$, at low temperature, the competition between different natures of magnetic fluctuations (ferromagnetic or antiferromagnetic) as well as the role of valence fluctuations due to the interplay between U$^{3+}$ and U$^{4+}$ configurations may occur. The situation is even more complex in this system, because electronic correlations play the unusual role of driving the system from an insulating toward a metallic state [S1–S4].

As a consequence, even close to $T_c$, $C/T$ is not the sum $\gamma + \beta T^2$ with $\beta T^2$ the phonon contribution far below the Debye temperature. An additional contribution is observed, likely coupled to the "Schottky-like" anomaly detected at $T^* \approx 12$ K [S5]. In Fig. S3(a,b) shows $C/T$ vs $T^2$ for sample #1 and sample #2. The phonon contribution has been calculated following the Debye model with a Debye temperature of $\theta_D = 180$ K [S5]. Clearly, the phonon contribution is low compared to the measured specific-heat and cannot reproduce the strong temperature increase of $C/T$ at zero magnetic field. Under magnetic field, the anomaly at $T^*$ shifts to higher temperatures for $H \parallel a$ and lower temperatures for $H \parallel b$. Accordingly, $C/T$ strongly depends on the magnitude of the applied field and on its direction. Hence the low temperature Sommerfeld coefficient $\gamma$ cannot be determined properly from a direct analysis of the temperature dependence of $C/T$. We tried, as done previously [S6, S7], to follow the field evolution of $C/T$ at the lowest possible temperature, so as to be as close as possible to the value of $\gamma$.

At 15 T for $H \parallel a$, $C/T$ decreases with temperature (down to 0.3 K), which is quite unusual (Fig. S3-c). Thermoelectric power measurements have revealed the presence of several Lifshitz transitions in this field direction [S8]. In the main text, the insert of Figure 9 shows that the field dependence of $C/T$ at 1.8 K has a minimum followed by a maximum close to 5 T and 9 T respectively.

Moreover, field sweeps have been performed up to 31 T for $H \parallel a$, on a sample with a $T_c$ of 1.45 K coming from the same batch as sample #1. Measurements of $C/T$ are displayed in Fig. S4. A minimum (around 6 T) followed by a maximum (around 8 T) are visible for fields above the superconducting transition. At higher fields, a change of slope occurs for fields between 17 T and 22 T, depending on the temperature. We can follow these three anomalies in addition to the superconducting transition, and establish the phase diagram shown in Fig. S5. The temperature dependence and the order of magnitude of the field where they occur are similar to those obtained from thermoelectric

![Graph](image-url)

FIG. S3. Temperature dependence of $C/T$ (sample #1) for fields applied along the three crystallographic directions. (a) $C/T$ as a function of $T^2$, measured on sample #1. No linear behaviour is seen. At 15 T for $H \parallel a$, the temperature dependence is drastically suppressed compared to measurement at 0 T. At 15 T for $H \parallel b$, the temperature dependence is slightly larger compared to zero field. The dotted line corresponds to the phonon contribution calculated from estimation of the Debye temperature by high-temperature measurements on other samples [S5], in addition to a constant Fermi-liquid contribution. (b) Same data for $H \parallel b$ on sample #2. The anomalous magnetic contribution seems more pronounced than for sample #1. (c) $C/T$ at low temperatures at 15 T along the three axis measured on sample #1. The superconducting transition at $\sim 0.5$ K is visible for $H \parallel b$. 
power measurements. The lower transition (at around 5-6 T) was clearly identified as a Lifshitz transition, the origin of the two others is less clear [S8]. Regarding the present specific-heat data, the origin of the pronounced maxima of \( C/T \) observed at \( \sim 1 \) T for \( H \parallel a \) and \( \sim 1.5 \) T for \( H \parallel c \) (Fig. 9) is also not identified.

**FIG. S4.** \( C/T \) measured for field sweeps at different temperatures with \( H \parallel a \). Measurements done on a sample coming from the same batch as sample #1, with a \( T_c \) of 1.5 K.

**FIG. S5.** Phase diagram \( H \parallel a \) up to 31 T. Black triangles represent the superconducting transitions, blue circles are the minima of \( C/T(H) \). Red squares represent the maxima of \( C/T(H) \). Green pentagons represent the inflexion point of \( C/T \) observed on field sweeps. The dashed lines are the corresponding transitions measured by thermoelectric power in ref [S8].
IV. SPECIFIC-HEAT: METAMAGNETIC TRANSITION

As regards the metamagnetic transition, we could measure precisely the field dependence of the anomaly, and check that it did not depend on field sweep rate, varied between ±350 and ±50 Gauss/sec. At this sweep rate, we also did not detect any magnetocaloric effect. Hence, the present continuous field sweep measurement show unambiguously that there is a jump at $H_m$, slightly broadened by a distribution of $H_m$. At 0.7 K, $C/T$ decreases 55 mJK$^{-2}$mol$^{-1}$ and the width of the transition is 0.25 T. This distribution of $H_m$ possibly comes from the strong sensitivity of $H_m$ to pressure [S9, S10] and most likely to stress (crystal defects could then generate this $H_m$ distribution).

Figure S6 shows the comparison of $C/T - C/T(H = 0)$ near $T = 1.8$ K determined from our experiment performed on sample #3 and experiments performed in pulsed field. Ref. S7 reports specific-heat experiments in highly stabilized fields, using the long pulsed fields facility at ISSP. In Ref. S11 the Sommerfeld coefficient $\gamma$ has been determined from the magnetization measurements $M(T)$ under pulsed fields, using Maxwell’s relation for $H \neq H_m$ as $(\partial \gamma / \partial H)_{T} = (\partial^2 M / \partial T^2)_{H}$, and using the Clausius-Clapeyron relation for the first order transition: $\mu_0 dH_m / dT = -\Delta S / \Delta M$ [S11] to get the jump $\Delta \gamma = \Delta S / T$ at $H_m$. The last analysis indicated a discontinuous jump of $\Delta \gamma = -30$ mJK$^{-2}$mol$^{-1}$ at $H_M$ for $H \parallel b$, which is lower than that obtained in the present experiment. However, despite some quantitative differences (e.g. the absolute variation of $C/T - C/T(H = 0)$ is larger in both pulsed field experiments) the general behaviour is similar: an increase with $H$ when approaching $H_m$ and a drop at $H_m$ followed by a strong decrease. A similar field dependence has been observed for the $A$ coefficient of the electrical resistivity, albeit without a clear jump above $H_m$ [S12, S13].

![Figure S6](image1.png)

**FIG. S6.** Cyan triangles: specific-heat measurements done in pulsed fields in ref [S7]. Orange circles: $\gamma(H) - \gamma(H = 0)$ determined from the magnetisation measurements through thermodynamic relations in ref [S14]. Red line: our measurement.

![Figure S7](image2.png)

**FIG. S7.** (a) Jump (drop) of $C/T$ at $H_m$, (b) $H_m$ and (c) width at half-height (2.35$\sigma$) of the transitions as a function of the temperature, for the up and down sweeps.

To extract the position and the width of the specific-heat at the metamagnetic transition, it has been fitted, like a broadened second-order phase transition, with the same Gaussian model as the superconducting transition. Fields replace temperatures, and $H_m$ replaces $T_c$. The parameters extracted are displayed in Figure S7. $H_m$ for the up sweeps are roughly constant at 34.75 T, and for the down sweeps $H_m$ increases with the temperature. This goes along with the trend toward a closing of the hysteresis cycles with increasing temperatures, in agreement with the observations from resistivity measurements [S12, S15]. The width of the transitions increases abruptly from 0.24 T to 0.45 T between 0.7 K and 0.97 K and then stays constant with temperature. The jump of $C/T$ at $H_m$ strongly decreases on cooling from 0.97 K to 0.7 K, otherwise, above 0.97 K, it decreases on warming. This anomaly at 0.7 K might be due to the presence of the HF transition, which is wide enough in field and temperature to influence the drop of $C/T$ at $H_m$. 
V. SPECIFIC-HEAT: SUPERCONDUCTING PHASE

A. Measurements in zero field

All measurements of $C/T$ in UTe$_2$ display an upturn below 0.1K, and an extrapolated (from temperatures above the upturn) residual term at $T = 0$ which was quite large in the first measurements [S16–S18]. More recent studies are claiming that the residual term and the upturn are extrinsic to UTe$_2$ [S19, S20]. Our measurements on different samples in Fig. S8 show diverse behaviours at low temperatures. The upturn is not monotonously correlated to $T_c$, however, it is strongly reduced on our best samples. The residual term seems to be more systematically decreasing with the $T_c$ increase, well in the trend reported in [S20]. In any case, these measurements do agree with an extrinsic nature of these anomalies.

![Graph showing C/T as a function of temperature for different samples.](image)

FIG. S8. $C/T$ as a function of temperature at zero field and low temperatures for different samples. Samples #1 and #2 are the one presented in this article. Sample neutron is a large sample of 241 mg used to perform the neutron diffusion experiments in ref [S21, S22].
B. Gaussian model for the specific-heat anomaly

A simple hypothesis is that broadening of the specific-heat transition is controlled by a Gaussian $T_c$ distribution of the form:

$$ p(T_c) = \frac{1}{\sigma \sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{T_c - T_{c0}}{\sigma} \right)^2 \right) \tag{S1} $$

For the specific-heat, or any additive quantity we can then write that:

$$ \frac{C}{T} = \int_{-\infty}^{\infty} p(T_c) \frac{C}{T}(T, T_c) dT_c \tag{S2} $$

The simplest expression for $\frac{C}{T}(T, T_c)$ is a constant $\gamma$ term above $T_c$, a jump at $T_c$ followed by a constant positive slope below $T_c$. If both the slope and the jump are independent of $T_c$, this amounts to:

$$ \frac{C}{T}(T, T_c) = \gamma + \theta(T_c - T) \left( \frac{\Delta C}{T} + \alpha(T - T_c) \right) \tag{S3} $$

Hence, for the total specific-heat:

$$ \frac{C}{T}(T) = \gamma + \left( \frac{\Delta C}{T} + \alpha(T - T_{c0}) \right) \left[ \frac{1}{2} - \frac{1}{2} \text{erf} \left( \frac{T - T_{c0}}{\sigma} \right) \right] - \alpha \frac{\sigma}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} \left( \frac{T - T_{c0}}{\sigma} \right)^2 \right) \tag{S4} $$

This model is fine for the zero field transition, where $\frac{\Delta C}{T}$ is independent of $T_c$. However, under magnetic field, the broadening of the transition may correspond to a distribution of slopes of $H_{c2}$ (proportional to $T_c$ for clean type II superconductors). We can expect that $\frac{\Delta C}{T}$ will be suppressed with field, with a decrease controlled by $H/H_{c2}(0)$. Hence $\frac{\Delta C}{T}$ will not be constant within the broadened transition. More simply, we can assume that the jump will be suppressed like $T_c(H)/T_c(0)$. The problem is therefore to relate $T_c(H)$ and $T_c(0)$, or more precisely, to get the $T_c(0)$ corresponding to a given $T_c(H)$. Then we could take for a model of the transition that $\frac{\Delta C}{T}$ is proportional, within the transition, to $T_c(H)/T_c(0)$. A simple way to find this relation is to assume a proportionality to the broadening so that:

$$ T_c(H) - T_{c0}(H) = \frac{\sigma}{\sigma_0} (T_c(0) - T_{c0}(0)) $$

$$ \frac{\Delta C}{T}(T_c) = \frac{\Delta C}{T}(T_{c0}) \frac{T_c/T_{c0}}{1 + \frac{\sigma_0}{\sigma} \frac{T_c - T_{c0}}{T_{c0}(0)}} \tag{S5} $$

In the last expression, we wrote $T_c = T_c(H)$ and $T_{c0} = T_{c0}(H)$. As regards the slope, similarly, it should also depend on $T_c$. Indeed, in high fields for example, where the temperature dependence of $C/T$ is close to linear, the slope should depend both on $T_c$ and on $\frac{\Delta C}{T}$. One way to keep some consistency within the transition is to assume that we have the same entropy balance for all the curves at different $T_c$ at a given field. At low field, where $C/T(T)$ has no specific reason to remain close to linear far below $T_c$, there is no peculiar constraint on this entropy balance (the linear behaviour of $C/T$ below $T_c$ is valid only close enough to $T_c$). However, for fields closer to $H_{c2}(0)$, we can expect that this entropy balance should be more or less close to zero. Explicitly, we can enforce that:

$$ \Delta S(T_c) = \int_0^{T_c} \left[ \frac{\Delta C}{T}(T_c) + \alpha(T_c) (T - T_c) \right] dT = \beta T_c \quad \text{with} \quad \beta \text{ independent of } T_c $$

$$ \alpha(T_c) = \frac{2}{T_c} \left[ \frac{\Delta C}{T}(T_c) - \beta \right] \tag{S6} $$

Inserting equations (S5) and (S6) in the equations (S3-S2), we obtain a final expression for $C/T(T)$, easily managed in its integral form by numerical calculations. It depends linearly on the parameters $\gamma$, $\frac{\Delta C}{T}(T_{c0})$ and $\beta$ (close to zero in high fields), and non linearly on $\sigma$ and on $T_{c0}$. It has two additional inputs, taken from the zero field data: $\sigma_0$ and $T_{c0}(0)$. 

C. Measurements of $H_{c2}$ for $H \parallel a$

Three different samples have been measured and their $H_{c2}$ determined from the specific-heat anomaly for $H \parallel a$ including at very low fields. The three samples come from different batches. Sample #1 and 2 have been measured with the same set up, and sample #5 with a different one. They all exhibit a strong negative curvature near $T_c$, proving that this feature is reproducible and intrinsic.

FIG. S9. $H_{c2}$ of 3 different samples for $H \parallel a$: the anomalous strong curvature near $T_c$ is reproducible, even for crystals with very different $T_c$. 

D. Measurements of $H_{c2}$ for $H \parallel b$ - comparison with resistivity.

Resistivity has also been measured on the sample from which we cut off sample #3. The critical field obtained with $R = 0$ as criterion, is compared to $H_{c2}$ determined by specific-heat. For the LF phase, as expected, $R = 0$ is above the specific-heat transition. At low fields, there is a large difference between the initial slopes at $T_c$ for the determination from resistivity or specific-heat anomaly, most likely due to the sensitivity of resistivity measurements to filamentary superconductivity, rapidly suppressed by (small) magnetic fields. For the HF phase, $R = 0$ is below the maximum of the specific-heat transition, which is more unusual. This can be seen on Fig. S11, showing the temperature dependence of $C/T$ and of the resistivity at a fixed field of 18 T.

This discrepancy can arise from extrinsic inhomogeneities, like a continuous gradient of $H_m$ in the sample, or from more intrinsic phenomena like a weaker pinning of vortices in the HF phase, which would induce a smaller critical current and possibly a shift of the resistive transition to lower temperatures. This is well known in organic superconductors [S23] or in High-Tc cuprates [S24], where the resistivity remains non-zero in the vortex liquid state, favoured by the highly 2D anisotropy of their normal and superconducting properties. It has also been observed in iron-based superconductors [S25], and like in the organics or high-Tc cuprates, with much stronger differences on the $T_c$ determination than observed in UTe$_2$. The difficulty for such an explanation in UTe$_2$ is the same as faced for UCoGe [S26]: the systems are 3D rather than 2D, hence superconducting fluctuations should be much less effective. In addition, the discrepancy between resistivity and specific-heat determination of $H_{c2}$ occurs only at very high fields, whereas in the other systems, it arises very fast when entering the mixed state. It could be that the quantitative difference in the effect arises precisely because superconducting fluctuations are much less important in UTe$_2$ or UCoGe than in the quasi 2D systems. However, it remains to be explained why this would happen only in the field-reinforced phase. This point remains a fully open question.

\[ \text{FIG. S10. } H_{c2} \text{ for } H \parallel b \text{ determined by } C/T \text{ measurements (see main article). Gray points correspond to the } R = 0 \text{ determined by resistivity measurements. The shaded region indicates the width of the HF transition. The inset is a zoom for field below 4 T.} \]

\[ \text{FIG. S11. } C/T \text{ as function of temperature compared to } R \text{ as function of temperature, both measured at 18T.} \]
E. Comparison of $H_{c2}$ and $H_{c1}$

Figure S12 shows $H_{c2}$ determined by specific-heat on sample #2 and the $dH_{c2}/dT_c$ at $T_c$ calculated and rescaled from $H_{c1}$ measurements in ref. [S27]. The measurements of $H_{c1}$ have been done on a sample from the same batch as sample #1, and have roughly the same $T_c$ at zero field. Near $T_c$, in the Ginzburg-Landau regime, the Ginzburg-Landau parameter $\kappa$ is determined by $H_{c1} = \frac{H_{c1}}{\sqrt{2}(\ln(\kappa) + 0.49)}$. $dH_{c2}/dT_c$ can then be deduced from $H_{c2} = \sqrt{2\kappa}H_{c1}$, in order to evaluate $dH_{c2}/dT_c$ from $dH_{c1}/dT_c$. The thermodynamic critical field $H_{c}(T)$ is determined by double integration of the specific-heat at 0 T, and we obtain $\frac{dH_{c2}}{dT_c}=-0.056$ T/K. Then, the calculated $dH_{c2}/dT_c$ are rescaled by the ratio of the respective critical temperatures of sample #2 and #1 in order to obtain the values of $dH_{c2}/dT_c$ for each axis. These values are displayed in table S1.

| $H \parallel a$ | $dH_{c1}/dT_c$ (T/K) - ref [S27] | $\kappa$ | $dH_{c2}/dT_c$ (T/K) | $dH_{c2}/dT_c$ rescaled (T/K) | $dH_{c2}/dT_c$ (T/K) - #2 |
|-----------------|-------------------------------|----------|----------------------|-----------------------------|-----------------------------|
| $H \parallel a$ | 0.00113                       | 202.683  | 16.052               | 20.480                      | 20                           |
| $H \parallel b$ | 0.00227                       | 86.482   | 6.849                | 8.738                       | 34.5                         |
| $H \parallel c$ | 0.00252                       | 75.838   | 6.006                | 7.063                       | 7.5                          |

TABLE S1. Table with the values of the slope $dH_{c1}/dT_c$ of $H_{c1}$ at $T_c$ from [S27], the corresponding value of the calculated Ginzburg-Landau parameter $\kappa$, and the predicted value for $dH_{c2}/dT_c$ for the sample of Ref. [S27] (same batch as #1). For sample #2, $dH_{c2}/dT_c$ is rescaled by the ratio of the $T_c$ of these samples. Last column is the initial slope measured on sample #2.
F. Field-dependence of the pairing strength modelled by a strong coupling parameter \( \lambda \)

Figure S13 shows \( H_{c2} \) along the 3 crystallographic directions, calculated with a strong coupling model for the upper critical field already used in ref. [S9, S28], at fixed pairing strength. The measured initial slopes \( dH_{c2}/dT_c \) at \( T_c \) are used to determine \( v_F \). The strong coupling constant \( \lambda \) is set to 1, which seems a reasonable value for UTe\(_2\). The plain lines in Fig. S13 are \( H_{c2}(T) \) calculated with the orbital limit adjusted to match the measured \( dH_{c2}/dT_c \), and the gyromagnetic factor \( g \) adjusted to match the initial negative curvature along the \( a \) and \( b \) axes \((g = 6.5 \text{ along the } a \text{ axis and } g = 0.8 \text{ along the } b \text{ axis})\). Deviations of the measured \( H_{c2} \) to such a usual combined orbital and paramagnetic limitation are observed for all applied directions of the magnetic field.

![Graph](image)

**FIG. S13.** \( H_{c2} \) determined by specific-heat on sample \#2. The lines are the best adjustment of \( H_{c2} \) (orbital and paramagnetic limitations) to match the measured initial slopes and curvatures in each direction. The dashed dotted line corresponds to a pure orbital limitation of \( H_{c2} \) adjusted on its initial slope for \( H \parallel a \), evidencing the very strong negative curvature close to \( T_c \). \( g \) is the gyromagnetic factor and \( \lambda \) the strong coupling constant.

To explain these deviations, a field dependent pairing strength is assumed. It is extracted from the data through a calculation of \( H_{c2}(T) \) at fixed values of \( \lambda \) (see Fig. S14 for the case of \( H \parallel b \)). The typical energy controlling \( T_c \) \((\Omega)\), the Coulomb repulsion parameter \( \mu^* \), and the bare average Fermi velocity for field along the \( i \) axis \((\bar{v}_F^\text{bare, } i)\) controlling the orbital limit are taken independent of \( \lambda \). The effective Fermi velocity controlling the orbital limit and so \( dH_{c2}/dT_c \) (at fixed \( \lambda \)) is renormalised as \( \bar{v}_F = \frac{\bar{v}_F^\text{bare, } i}{1 + \lambda} \). If \( \lambda \) is field dependent, this effective Fermi velocity is also field dependent.

In the table S2, we also calculate the Fermi velocity along each \( i \)-axis: \( v_F^i \), deduced from the effective Fermi velocities through \( v_F^i = \frac{\bar{v}_F^i v_F^k}{\bar{v}_F^k} \), where \( j, k \) are the axis perpendicular to \( i \).

| \( H \parallel a \) | 5400 | 0 | 40 | 14400 | 106 |
| \( H \parallel b \) (LF) | 8600 | 0 | 64 | 5680 | 42 |
| \( H \parallel c \) | 9044 | 0 | 67 | 5130 | 38 |
| \( H \parallel b \) (HF) | 8600 | 2 | 64 | 5680 | 42 |

**TABLE S2.** Parameter values of the fit. We used a strong-coupling parameter \( \lambda(H = 0) = 1 \), with a typical energy (equivalent to the Debye energy) \( \Omega = 28.4 \text{ K} \), \( \mu^* = 0.1 \), pair-breaking impurity scattering rate \( \Gamma = 1.39 \text{ K} \). Values of \( \bar{v}_F \) used in the fit are reported for each field direction. Difference between \( \bar{v}_F \) and \( v_F^i \) is explained in the text. The corresponding coherence length are calculated from and \( \xi_0 = 0.18 \frac{k_F}{\pi T_c} \).
FIG. S14. Crosses and squares are data of $H_{c2}$ for $H || b$. Dashed lines are the $H_{c2}$ calculated for different fixed values of the coupling constant $\lambda$. The green lines correspond to calculation for $g = 0$ by steps of $\Delta \lambda = 0.05$, and the blue lines for $g = 2$ by steps of $\Delta \lambda = 0.2$. 
G. Broadening of the specific-heat transition by a distribution of $H_m$

As explained in section IV of these supplement, we could extract a standard deviation $\sigma = 0.19 \, \text{T}$ for the distribution of $H_m$ (see Fig. S7), hence a relative standard deviation $\frac{\sigma}{H_m} \sim 0.55\%$

As explained in the main text, from the calculation of $H_{c2}$ at fixed values of the pairing strength $\lambda$ (Fig. S14), we can extract also the superconducting critical temperature under field as:

$$T_c = \varphi \left( H, \tilde{\lambda} \left( \frac{H}{H_m} \right) \right)$$

$$\tilde{\lambda} \left( \frac{H}{H_m} \right) = \lambda \left( \frac{H H_{m0}}{H_m} \right)$$

(S7)

Where $H_{m0}$ is the centre of the distribution of metamagnetic fields $H_m$, determined from the specific-heat measurements of the metamagnetic transition. $\lambda(H)$ is the field dependent pairing strength deduced from the different models for $H_{c2}$ and drawn on Fig. 10 of the main text.

From this relation, we can calculate the effect of a Gaussian distribution of $H_m$ on the specific-heat anomaly at constant field of the superconducting transition, using for $C/T$ (instead of Equ. S2):

$$C/T = \int p(H_m) C/T (T, T_c (H, H_m)) \, dH_m$$

(S8)

This is the way we could draw the broadening of the specific-heat anomaly on Fig. 12 and Fig. 13 of the main text, using the two different determinations of $\lambda(H/H_m)$ (with or without paramagnetic limitation of $H_{c2}$).

However, even without a full determination of the shape of the anomaly, requiring a numerical integration of Equ. S8, we can understand why the broadening is larger when there is a paramagnetic limitation of $H_{c2}$. From Equ. S7, we can derive the derivative of $T_c$ with respect to $H_m$ at fixed $H$ and for $H_m = H_{m0}$. It measures the sensitivity of $T_c$ to $H_m$, hence the broadening of the $C/T$ anomaly due to a distribution of $H_m$:

$$\left. \frac{\partial T_c}{\partial H_m} \right|_H = \left. \frac{\partial T_c}{\partial \lambda} \right|_H \left( \frac{H}{H_{m0}} \right) \left( \frac{d\lambda}{dH} \right)$$

(S9)

When comparing both models, it is clear that one has a stronger field dependence of $\lambda$ than the other, but this could be compensated by a different $\left. \frac{\partial T_c}{\partial \lambda} \right|_H$ which has to be computed at finite field (on the $H_{c2}(T)$ line). Indeed, both models share the same $H_{c2}(T)$. We can compute its temperature derivative from Equ. S7:

$$\frac{dT}{dH_{c2}} - \left. \frac{\partial T_c}{\partial H} \right|_{\lambda} = - \left( \frac{H_{m0}}{H} \right) \left. \frac{\partial T_c}{\partial H_m} \right|_H$$

(S10)

The last equation shows that the difference between models for $\left. \frac{\partial T_c}{\partial H_m} \right|_H$ arises not directly from $\left( \frac{d\lambda}{dH} \right)$, but rather from $\left. \frac{\partial T_c}{\partial H} \right|_{\lambda}$; this term is much larger when $H_{c2}(T)$ at fixed $\lambda$ becomes ”flat” due to the paramagnetic limitation arising for singlet pairing, than for a pure orbital limit in case of a spin-triplet ESP state (see Fig. S14).
H. Angular dependence in the \((b,c)\) plane

AC specific-heat measurements have been performed on sample \#2 up to 18.5 T for several angles in the \((b,c)\) plane. Figure S15 shows temperature sweeps for different fields for angles of 0°, 10° and 15° from \(b\) toward the \(c\) axis. As the field is rotated toward the \(c\) axis, the sharp transition of the LF phase is shifted toward lower temperatures. The same behaviour is observed for the wide transition of the HF phase. The corresponding critical temperature for the two transitions at the different angles are reported on the phase diagram of Fig. S16, using the same Gaussian analysis as in the main paper. For \(H_\perp c\) at 15°, it was impossible to extract a reliable value of \(T_c\) for the HF transition.

Similarly, as presented in the main article in Fig. 13 for an applied field of 18.5 T, temperature sweeps have been performed at 16 T for different angles in the \((b,c)\) plane. Results are presented in Fig. S17. The LF and HF transitions overlap and the HF is less defined as at 18.5 T. Nevertheless, we clearly see a shift to the low temperatures of the HF transition as the field is rotated away from \(b\) axis. The HF transition also clearly disappears, shifting to the low temperatures and the jump size decreasing.

**FIG. S15.** \(C/T\) as a function of temperature for several fields between 0 T and 18.5 T, measured on sample \#3. (a) \(C/T\) for \(H \parallel b\), (b) for an angle of 10° toward the \(c\) axis and (c) for an angle of 15° toward the \(c\) axis.

**FIG. S16.** Phase diagram for a field applied along the \(b\) axis (red squares and crosses), at 10° toward \(c\) axis (black triangles and circles), and at 15° (orange diamonds). The empty marker represents fields where the width and jump sizes of the HF transitions need to be fixed to determine \(T_c\) from a Gaussian fit of both anomalies.
FIG. S17. $C/T$ as a function of temperature at 16 T, for different angles in the $(b,c)$ plane.
