Spectroscopic Variations of the Be-Shell Star EW Lacertae in the V/R Variation Periods

Masahiro MON
Osaka Shoin Women’s University, 958 Sekiya, Kashiwa, Nara 639-0298
mon.masahiro@osaka-shoin.ac.jp
Masakazu SUZUKI
3-219 Onuka, Kamazawa, Ishikawa 921-8147
Yuki MORITANI
Department of Astronomy, Faculty of Science, Kyoto University, Kitashirakawa-Oiwake-cho, Sakyo-ku, Kyoto 606-8502
Hiroshima Astrophysical Science Center, Hiroshima University, 1-3-1 Kagamiyama, Higashi-Hiroshima, Hiroshima 739-8526
and
Tomokazu KOGURE
1-10 Toganoo, Hashimoto, Yawata, Kyoto 614-8322

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Abstract

EW Lac showed remarkable V/R variations in 1976–1986, and a similar V/R variation started again in around 2007, after some quasi-periodic V/R variations. For the first V/R variation event, we analyzed the spectroscopic behaviors of emission lines and shell absorption lines of the Balmer series. The V/R variations of the Hα through Hβ lines are characterized by the different manner of variations of the individual emission line in time lag and in duration of the V/R phases. Weak correlation is also notable between the V/R variations and other variations of the line-profile parameters, such as the peak velocities, emission-line intensities, and peak separations. We analyzed shell absorption lines for higher members of the Balmer series concerning their central depths and radial velocities. The optical depth of the Hα line is in the range of 2000 to 6000, and its long-term variation discloses different behaviors as compared to the V/R variations. Combining the analyses of emission and shell absorption lines, and regarding the V/R variation as wave propagation phenomena, we find for the 1976–1986 event that the V/R variation is of retrograde structure, and that a spiral structure is likely formed inside the disk in the latter half of this event. The weak correlations among physical parameters are suggestive of the disk being truncated at some radius. It is noticed that remarkable stellar brightening occurred in the latter half of the event, accompanying a marked decrease of the emission-line intensities. As for a V/R variation that appeared in around 2007, which looks like a recurrence of a previous event, we found a less developed state of the disk without an appreciable time lag.

Key words: stars: circumstellar disk — stars: emission-line, Be — stars: individual (EW Lac = HD 217050) — stars: V/R variations, long-term variations — techniques: spectroscopic

1. Introduction

Be stars show emission lines in the Balmer lines, singly ionized metal lines, and sometimes neutral helium lines in their optical spectra. It is now widely accepted that Be stars are rapidly rotating stars surrounded by disks or rings in the low-latitude regions, where the emission lines are formed. Be stars are classified into ordinary Be stars and Be-shell stars according to whether sharp absorption lines, so-called shell absorption lines, exist or not in the Balmer and metallic lines. This classification is somehow related to the inclination of the Be disk. Namely, Be-shell stars are considered to have a disk nearly edge-on from the observer. Be stars usually exhibit irregular variations in both photometric and spectral features on various timescales from less than one day to several decades. Phase changes among B, Be, and Be-shell stars are occasionally observed. A general review on Be stars can be seen in Porter and Rivinius (2003). Moujtahid et al. (1998) reviewed long-term variations in various Be stars. Concerning the formation of Balmer emission lines and shell absorption lines, an overview is given in Kogure and Leung (2007).

EW Lac (HD 217050, B4 IIIep) has been known to be a typical Be-shell star since the 1930’s, showing irregular variations in the shell lines (Kogure 1975; Poeckert & Duric 1980; Hubert et al. 1987). Remarkable V/R variations of the Balmer emission lines have been observed in the period from 1976 through 1986, and a similar variation may have started in around 2007.

For the 1976–1986 events, several observations have been carried out. Kogure and Suzuki (1984) examined the V/R ratios for the Hα through the Hβ lines in the period 1960–1983, based on a visual inspection of spectrograms, and qualitatively showed that the V/R variations in 1975–1983 proceeded accompanying some time lag for different Balmer lines. Suzuki and Kogure (1986) showed the V/R variations for the Hα and the Hβ lines in the period from 1970 up to 1985, but without mentioning the time lag. Hubert et al. (1987) divided the spectroscopic history of EW Lac into quiet (1960–1974) and active (1975–1984) phases, and compared the spectroscopic features of these phases. The quiet phase corresponds to the Be-shell
In this paper we consider the spectroscopic behaviors of the Balmer emission lines and shell absorption lines, mostly in the depths, effective disk area, gas motions, and electron density. Circumstellar disk in front of the photosphere, such as optical and can provide information on the internal structure of the numbers (higher Balmer members) are generally optically thin, absorption lines in the Balmer series with higher quantum varying some irregular variation in the period 1961–1972. Shell term variations much longer than one day. Although it is inter-

Kogure (1975) analyzed the shell absorption lines, and showed that the circumstellar disk of this star has a large optical depth of the Hα line, as large as 2000 to 6000, accompanying some irregular variation in the period 1961–1972. Shell absorption lines in the Balmer series with higher quantum numbers (higher Balmer members) are generally optically thin, and can provide information on the internal structure of the circumstellar disk in front of the photosphere, such as optical depths, effective disk area, gas motions, and electron density.

In this paper we consider the spectroscopic behaviors of the Balmer emission lines and shell absorption lines, mostly in the first V/R variation period, 1976–1986. We consider only long-term variations much longer than one day. Although it is interesting to see that EW Lac has shown short-term V/R variations of less than one day (Pavlovski & Schneider 1990; Floquet et al. 2000), such a short-term variability is beyond this work.

In section 2, we present the observational data and reduction procedure. Section 3 is devoted to presenting the spectroscopic variations of emission-line profiles in the Balmer series, including the V/R ratios, peak separations, and central core components. In section 4 we show the new appearance of the V/R variation in around 2007. In sections 5 and 6, we analyze Balmer shell absorption lines for the line intensities and radial velocities, and investigate the internal structure of the disk. We derive some physical parameters in section 7. In section 8, we examine the relationship between photometric variations and V/R variations. The structure of the disk related to the V/R variations is discussed in section 9. We summarize this paper in section 10.

### 2. Observational Data and Measurements of Line-Profile Parameters

#### 2.1. Observational Data and Reduction Procedure

In this section we present observational data obtained in 1966–1986. We measured 195 photographic spectrograms of EW Lac, obtained by the Coudé spectrograph attached to the 188 cm reflector at the Okayama Astrophysical Observatory (OAO). The plate list is partly given in table 1. The list contains the observation journal, plate number, and the names of Balmer lines contained in each plate. The spectral dispersion is 10 Å/mm (violet) and 20 Å/mm (Hα) for C4-plates and 4 Å/mm (violet) and 8 Å/mm (Hα) for C10-plates, respectively.

All of the spectrograms were digitized with the PDS micro-
densitometer at the Kwasan Observatory of Kyoto University. A data analysis was carried out by using a program developed by M. Suzuki, by making use of computers at Kyoto University and Kanazawa Institute of Technology. The results were recorded on magnetic tape in the form of relative intensity traces in wavelength scale corrected for the solar motion. We measured the parameters characterizing the line profiles on these tracings in the way described in the following subsections.

#### 2.2. Subtraction of Emission Lines

We have selected 10 spectrograms with a better S/N ratio among OAO plates, and prepared the average line profiles for the Balmer lines. These profiles were used for the fitting with Kurucz’s model profiles broadened by stellar rotation, under the parameter ranges of $T_{\text{eff}}$ (16000–18000 K), $\log g$ (2.5–3.5), and $V \sin i$ (250–450 km s$^{-1}$) (Kurucz 1979). We thus adopted the following parameter set as the best fitting with the OAO profiles in the wing parts:

$$T_{\text{eff}} = 16000 \text{ K}, \log g = 3.0, \text{ and } V \sin i = 280 \text{ km s}^{-1}.$$  

In addition, we measured the half-width at half-maximum, $\Delta_{1/2}$, of He I $\lambda 4471$ for 10 selected OAO plates. $V \sin i$ was estimated by making use of the correlation between $V \sin i$ and $\Delta_{1/2}$ for several stars that show the parameters nearer to our parameter set. The data were taken from Chauville et al. (2001). The result is that $V \sin i = 279 \pm 15 \text{ km s}^{-1}$, which is in accord with the above parameter set. The derived parameter set is also in accord with the spectral type B4 IIIpe, which is given by Pavlovski et al. (1997), for instance. Notice that our parameter set is used for determining the underlying photospheric profiles, and is not sensitive to the discussions of the present paper. Samples of the profiles of the Balmer lines Hα through Hδ, and the photospheric absorption lines based on this parameter set are shown in figure 1.

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* Full table is given in the electronic version.1

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1 Full table is given in the electronic version (http://pasj.asj.or.jp/v65n4/650077/).
2.3. Measurements of Line-Profile Parameters

In all of the observed Balmer emission-line profiles, we measured the following line-profile parameters, which are defined in figure 2:

1. Heights of emission peaks $I_v (AA')$ and $I_r (BB')$ for the violet and red components, respectively, in units of the nearby continuum; then, the $V/R$ ratio is defined by the ratio $I_v/I_r$.
2. Heliocentric radial velocities of the emission peaks, $V_v$ at $A$, and $V_r$ at $B$ in km s\(^{-1}\).
3. Peak separation, $2\Delta_p$ (velocity difference between $A$ and $B$).
3. Spectroscopic Variations of the Balmer Emission Lines

In this section we present the observed features of the variation in the line profiles of EW Lac. In figure 3 we show the long-term variations of the $V/R$ ratios, heliocentric peak velocities ($V_r$ and $V_c$), and emission equivalent widths ($EW_e$) for the H$_\alpha$ through the H$_\delta$ lines. The main features are described in the following subsections.

3.1. $V/R$ Variations

Although the $V/R$ variations seem to have started in the early 1970's in the H$_\gamma$ and the H$_\delta$ lines with a shallow $V < R$ state ($V/R \sim 0.8$--$0.9$), the $V/R$ variations became notable in 1976, and continued to around 1986, as can be seen in figure 3 (gray region). The $V/R$ variation after 1986 is considered in subsection 4.2. The remarkable $V/R$ variation may be characterized by the following features:

1. The epoch of the $V/R$ maximum or the $V/R$ minimum depends on the lines. That is, the $V/R$ maximum appeared almost at the same epoch within 100 d in the middle of 1979 for all emission lines, whereas each line reached the $V/R$ minimum at different times. The H$_\delta$ and H$_\gamma$ lines reached it earlier, and then the H$_\beta$ and H$_\alpha$ lines followed in this order. The epoch of the $V/R$ minimum in H$_\alpha$ was delayed for approximately 600 d, as compared with that in the H$_\delta$ line (see figure 3 and table 3). Hereafter, we call this delay the time lag of the $V/R$ variation. The time lag became evident after the epoch of the $V/R$ maximum phase in 1979.

2. The durations of the $V/R$ phase ($V > R$ or $V < R$ phase) differ considerably among the lines. The approximate durations are given in table 3. One can see that the total duration gradually declined from H$_\alpha$ toward higher members.

3. The ranges of the $V/R$ variation are moderate for the H$_\gamma$ and the H$_\beta$ lines from $V/R = 0.4$ to 2.1, whereas those for the H$_\gamma$ and the H$_\delta$ lines are large from $V/R = 0.09$ to 2.0. For the latter lines, a small violet emission component is remarkable at the $V/R$ minimum phase.

3.2. Peak Velocities and Emission Line Intensities

Figure 3 also shows the variations of the heliocentric radial velocities at the violet and the red peaks (middle panel) and the emission equivalent widths (bottom panel).

A weak correlation of the peak velocities, $V_r$ and $V_c$, with the $V/R$ variations is apparent. The radial velocities of the red peak slightly shifted red-ward along with the increase of the $V/R$ value, whereas the violet components indicate almost no relationship. As a whole, the correlation is weak. The average velocities and their standard deviations are given in table 4. The standard deviation is less than around 10 km s$^{-1}$.

The equivalent widths of the emission lines seem to show a weak correlation with the $V/R$ variations, though some large scatter is seen. The average equivalent widths in the $V/R$ variation period are as follows: $EW_e$ (Å) = 50.1 (H$_\alpha$), 4.3 (H$_\beta$), 1.0 (H$_\gamma$), and 0.67 (H$_\delta$).

3.3. Peak Separations

The full peak separations, $2\Delta_p$, are measured, and the long-term variations are shown in figure 4. It can be seen that $\Delta_p$ for every line indicates a slight correlation with the $V/R$ variation. The average values and the standard deviations are as follows:

$\Delta_p$ (km s$^{-1}$) = 101.5 ± 6.0 (H$_\alpha$), 110.5 ± 6.1 (H$_\beta$), 112.1 ± 15.3 (H$_\gamma$), and 116.1 ± 11.3 (H$_\delta$).

and B), of the emission line in km s$^{-1}$.
4. Equivalent width of the emission component above the photospheric line $EW_e$ in Å.
5. Heliocentric radial velocities of the deepest point, $V_c$ (at C), in km s$^{-1}$.
6. Depth of the central core absorption measured from the photospheric absorption, $D_c$ (CC').
7. Full width at half-intensity of the central core absorption, $2\Delta_c$ (velocity difference between F and G) in km s$^{-1}$.

The numerical data of these parameters are partly given in table 2, where the mean values of each observational time are given.

| JD 2400000+ | $I_c$ | $I_r$ | $V/R$ | $V_c$ | $V_r$ | $2\Delta_p$ | $EW_e$ | $V_c$ | $D_c$ | $2\Delta_c$ |
|-------------|------|------|-------|------|------|------------|--------|------|------|------------|
| H$\alpha$   |      |      |       |      |      |            |        |      |      |            |
| 40578       | 5.83 | 5.70 | 1.02  | 80.0 | 107.5 | 187.5      | 58.17  | 15.0 | 1.38 |
| 40868       | 5.20 | 5.25 | 0.99  | 112.5| 110.0 | 222.5      | 50.87  | 5.0  | 1.25 |
| 41195       | 3.45 | 3.40 | 1.01  | 130.0| 95.0  | 225.0      | 35.56  | -16.0| 0.25 |
| H$\beta$    |      |      |       |      |      |            |        |      |      |            |
| 39359       | 0.94 | 1.02 | 0.92  | 135.0| 92.5  | 227.5      | 3.61   | -21.5| 0.54 | 64.5       |
| 40580       | 0.82 | 0.77 | 1.07  | 130.0| 90.0  | 220.0      | 4.71   | -20.0| 0.25 | 41.3       |
| 41195       | 0.60 | 0.64 | 0.94  | 135.0| 100.0 | 235.0      | 3.25   | -24.0| 0.25 | 59.0       |
| H$\gamma$   |      |      |       |      |      |            |        |      |      |            |
| 39359       | 0.24 | 0.32 | 0.75  | 125.0| 110.0 | 235.0      | 1.20   | -13.0| 0.60 | 65.5       |
| 40462       | 0.28 | 0.36 | 0.78  | 125.0| 105.0 | 230.0      | 1.10   | -11.5| 0.56 | 58.5       |
| 40578       | 0.26 | 0.28 | 0.93  | 125.0| 101.3 | 226.3      | 1.06   | -16.0| 0.52 | 49.3       |
| H$\delta$   |      |      |       |      |      |            |        |      |      |            |
| 39359       | 0.13 | 0.17 | 0.77  | 122.5| 95.0  | 217.5      | 0.70   | -22.0| 0.58 | 60.5       |
| 40462       | 0.11 | 0.13 | 0.85  | 125.0| 97.5  | 222.5      | 0.62   | -15.5| 0.53 | 57.8       |
| 40578       | 0.15 | 0.16 | 0.94  | 125.0| 101.3 | 226.3      | 0.57   | -12.0| 0.53 | 60.4       |

* Full table is given in the electronic version.1
Fig. 3. Top panel: $V/R$ variations for the H$\alpha$ through the H$\delta$ lines. Middle panel: Radial velocities of the violet and red peaks of emission lines. Bottom panel: Equivalent widths of emission component above the underlying photospheric absorption. The gray region indicates the period of the $V/R$ variation (1976–1986).

Table 3. Durations of the phase of $V/R$ variation in days.

| Phase | $V/R = 1$ | $V > R$ | $V = R_{\text{max}}$ | $V = R_{\text{min}}$ | $V < R$ | Total |
|-------|-----------|---------|----------------------|----------------------|---------|-------|
| H$\alpha$ | 480 | 710 | 680 | 950 | 2820 |
| H$\beta$ | 820 | 370 | 710 | 610 | 2510 |
| H$\gamma$ | 810 | 140 | 580 | 780 | 2310 |
| H$\delta$ | 710 | 120 | 310 | 950 | 2090 |

Table 4. Average radial velocities of the emission peaks and the standard deviations (km s$^{-1}$).

| Line | H$\alpha$ | H$\beta$ | H$\gamma$ | H$\delta$ |
|------|-----------|----------|----------|----------|
| $\langle V_r \rangle$ | $-113.5 \pm 12.3$ | $-127.3 \pm 6.7$ | $-127.4 \pm 5.3$ | $-127.2 \pm 9.9$ |
| $\langle V_v \rangle$ | $90.9 \pm 11.9$ | $94.9 \pm 12.8$ | $100.2 \pm 10.0$ | $102.7 \pm 13.0$ |
Fig. 4. Long-term variations of the full peak separation, $2\Delta p$ (km s$^{-1}$).

Fig. 5. Long-term variations of the central core parameters, defined in figure 2. Top panel: Radial velocities. Middle panel: Depths in units of the residual intensity of the photospheric line center. Bottom panel: Full width at half intensity.
3.4. Central Core Components in the Emission Lines

The central core component is the part of line profile that is formed by combining the emission line and the shell absorption line. We measured the line-profile parameters of this part in the H\(\beta\), H\(\gamma\), and H\(\delta\) lines, such as the radial velocity, \(V_c\), at the deepest point, the line depth below the photospheric absorption, \(D_c = CC'\), and the full width of the central absorption, \(2\Delta_a = FG\), as shown in figure 2. The results are shown in figure 5. We can see the observed features, as follows:

1. The radial velocities, \(V_c\), exhibit different behaviors as compared to the \(V/R\) variations. Although data is lacking for H\(\delta\) during 1980–1981, there seems to be a lag in the time of reaching to the maximum among these lines with different amplitudes. Similarly to the \(V/R\) variation, H\(\delta\) seems to have reached the maximum first, and then the H\(\gamma\) and the H\(\beta\) lines followed. The time lag and the velocity amplitude will be considered in section 6.

2. The depths of the central core showed no particular variations during the \(V/R\) variations. We notice that a marked intensity decrease appeared in 1972–1973, prior to the \(V/R\) variation period. This feature will be considered in subsection 5.2.

3. The full widths of the central core absorption of the H\(\gamma\) and H\(\delta\) lines exhibit a period of highly widened absorption as broad as 80–90 km s\(^{-1}\) during 1978–1981, as compared to that of the H\(\beta\) line, which is in 40–60 km s\(^{-1}\). Since such a large difference is difficult to be explained by the difference of the thermal motion, there should be some velocity structure inside the disk.

In addition, we consider the line asymmetry of the central core absorption. We define the line asymmetry by \(b/a\), where \(a\) denotes the velocity difference between points F and G, and \(b\) the difference between the velocities at point F and at CC’ (see figure 2). Then, \(b/a < 0.5\) indicates the red faded asymmetry and \(b/a > 0.5\) the opposite. The long-term variations of the line asymmetry for the H\(\beta\), H\(\gamma\), and H\(\delta\) are delineated in figure 6. A clear correlation is seen with the \(V/R\) variations. The lines became red faded when \(V > R\) (1980 to 1982), and then violet faded when \(V < R\) (1978 to 1979) for all of the measured lines. Line asymmetry for the H\(\beta\) line has already been shown by Hubert et al. (1987) in the same sense with ours.

Line asymmetry is also shown by Ballereau et al. (1987) in their trace of the \(V/R\) variation in the H\(\beta\) line for HD 184279 (V1294 Aql), where we can see the same sense of asymmetry as shown in figure 6. In both stars, the asymmetry is more prominent when the \(V/R\) ratio is higher.

3.5. Widths of Emission Line Wings

The Balmer emission lines exhibit extended wings over the projected rotational velocity for H\(\alpha\) and higher members. We measured the widths of emission-line wings from the line center for both the violet and red sides, \(W_v\) and \(W_r\), in km s\(^{-1}\) for the H\(\beta\) line (D and E in figure 2). The widths almost remain nearly constant before and during the \(V/R\) variation period, and take average velocities of

\[
W_v (\text{km s}^{-1}) = 311.0 \pm 20.5 \quad \text{and} \quad W_r (\text{km s}^{-1}) = 311.7 \pm 31.0.
\]

These values are larger than the projected rotational velocity of 280 km s\(^{-1}\). The situation is the same for the H\(\gamma\) and the H\(\delta\) lines (see figure 1). These wide wings of emission profiles indicate that the circumstellar disk is not a detached ring, but a plain disk filled with gas.

**Fig. 6.** Line asymmetry of the central core absorption in the H\(\beta\) through the H\(\delta\) lines. The asymmetry \(b/a\) is defined in the text. The large value above 0.5 is blue faded, and below 0.5 is red faded.
4. Recurrence of the \textit{V}/\textit{R} Variation

4.1. Observations

In order to see the state of the \textit{V}/\textit{R} variations at present, an additional observation was made on 2010 October 22 at the Okayama Astrophysical Observatory. By making use of the HIDES (High Dispersion Echelle Spectrograph), attached to the 188 cm reflector, we obtained a spectrum in the range of 4000–7000 Å. Balmer lines were analyzed similarly with the method described in section 2. The results of measurement are summarized in table 5. The definitions of the physical quantities are the same as in figure 2.

4.2. Long-Term \textit{V}/\textit{R} Variations

We traced the long-term \textit{V}/\textit{R} variations of the H\textsc{\textalpha} and H\textsc{\beta} lines up to 2011, using data taken from Ballereau et al. (1987), Grundstrom (2007), Floquet et al. (2000), Rivinius, Stefl, and Baade (2006), and the data sets of spectroscopic observations by Atlas\textsuperscript{2} and BeSS database.\textsuperscript{3} The survey result is shown in figure 7.

It is found that the \textit{V}/\textit{R} variation of 1976–1986 did not cease, but continued thereafter while repeating an irregular form of the \textit{V}/\textit{R} variation. The \textit{V}/\textit{R} maximum appeared around 1979, 1986, 1994, 2000, and 2008, so that the average period is approximately seven years. Among these repeated variations, the one that peaked in 2008 is most remarkable concerning its maximum and minimum \textit{V}/\textit{R} ratios. It seems to be a recurrence of the event of 1976–1986 in its global behavior.

However, the spectral features are different between the 1976–1986 and the 2007–2012 events. Our observation shows that EW Lac was in the state of \textit{V}<\textit{R} in 2010 October. The emission equivalent width of the H\textsc{\textalpha} line (21 Å) is much smaller than that in the previous event (around 50 Å), and the peak separation (228 km s\textsuperscript{-1}) is larger than that in the previous event (203 km s\textsuperscript{-1}). The \textit{V}/\textit{R} ratios are almost the same among all of the emission lines. The implication of these differences are discussed in subsection 9.2.

5. Shell Absorption Lines and Optical Depths

5.1. Method and Fittings

Shell absorption lines are formed in the part of disk lying in front of the stellar photosphere. This part is characterized by two parameters, $\beta$ and $\tau$(H\textsc{\textalpha}), where $\beta$ is the fractional area of this layer relative to the photospheric disk, and $\tau$(H\textsc{\textalpha}) is the optical depth of the H\textsc{\textalpha} line in this layer. In the case of a near edge-on star, this part of the disk is schematically illustrated in figure 8. The formula giving the central depths of the shell absorption lines in higher members of the Balmer series has been derived by Kogure, Hirata, and Asada (1978), and applied to the analysis of Be-shell stars. The same formula can also be

\begin{table}[h]
\centering
\begin{tabular}{|l|ccc|c|c|c|}
\hline
Line & \multicolumn{3}{c|}{Intensity} & \multicolumn{3}{c|}{Emission line parameters on 2010 October 22.} \\
 & $I_v$ & $I_r$ & $V/R$ & $EW_c$ (Å) & $V_c$ & $V_r$ \\
\hline
H\textsc{\textalpha} & 2.38 & 3.40 & 0.70 & 21.37 & $-123.5$ & $104.0$ & $-20.0$ \\
H\textsc{\textbeta} & 0.54 & 0.91 & 0.59 & 2.98 & $-144.0$ & $104.0$ & $-18.0$ \\
H\textsc{\gamma} & 0.29 & 0.38 & 0.76 & 1.46 & $-151.0$ & $106.5$ & $-25.0$ \\
H\textsc{\delta} & 0.24 & 0.34 & 0.71 & 0.94 & $-157.0$ & $110.5$ & $-26.0$ \\
\hline
\end{tabular}
\end{table}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig7.png}
\caption{Long-term \textit{V}/\textit{R} variations in the H\textalpha{} and H\beta{} lines.}
\end{figure}

\textsuperscript{2} http://www.astrosurf.com/buil/us/bestar.htm.

\textsuperscript{3} http://basebe.obspm.fr/basebe/.
derived as follows, based on a simple assumption that the shell lines are formed by pure absorption inside the disk.

Let $r_n$ and $r_s$ be the residual intensities at the centers of the line $\text{H}\alpha$ and that of the photospheric absorption, relative to the nearby continuum, $I_c$, respectively. If we assume that there is no limb darkening on the photosphere, we have a surface intensity of $I_\ast(\lambda)$ for the part of the naked photosphere, whereas we have $I_s(\lambda)e^{-\tau(\lambda)}$ for the part covered by the disk at any wavelength, $\lambda$. We then have $r_n = (I_s/I_c)\beta\exp[-\tau(\text{H}\alpha)]$ and $r_s = (I_s/I_c)(1 - \beta)$, and the central depth, $d_n$, of $\text{H}\alpha$, defined by $d_n = (r_s - r_n)/r_s$, and is written as

$$d_n = \beta[1 - \exp(-\tau(\text{H}\alpha))] = \beta[1 - \exp(-\omega_n \tau(\text{H}\alpha))].$$

(2)

Here, $\tau(\text{H}\alpha)$ denotes the optical depth for the line $\text{H}\alpha$, and

$$\omega_n = \frac{v_{2n} B_{2n}}{v_{23} B_{23}} = \frac{\tau(\text{H}\alpha)}{\tau(\text{H}\alpha)}.$$  

(3)

$B_{2n}$ is the Einstein coefficient for the 2-$\eta$ transition. In deriving equation (2), we assumed that there is no Balmer progression nearby continuum, $I$, equation (2), we assumed that there is no Balmer progression.

To derive the curve. In some cases, we need a combination of two curves of formula (2) having two sets of $\tau(\text{H}\alpha)$, $\beta_1$, and $\tau(\text{H}\alpha)$, $\beta_2$, where $\tau(\text{H}\alpha) > \tau(\text{H}\alpha)$. The decomposition into two layers is made by a simple addition of the same formula (2). Some examples of single-layer and double-layer fittings are shown in figure 9.

The values of $\tau(\text{H}\alpha)$ and $\beta$ measured for EW Lac are partly given in table 6. It contains the date, $S/D$ (sign of adopted single- or double-layer fitting), and the values of $\tau(\text{H}\alpha)$ and $\beta$ for the single and double layers, separately.

It can be seen that the optical depth, $\tau(\text{H}\alpha)$, varies in a range from 2000 to 6000 for an optically thick layer, and from 200 to 800 for an optically thin layer. In contrast, the parameter $\beta$ (single layer) or $\beta_1 + \beta_2$ (double layers) was always larger than 0.6, meaning that the disk is sufficiently extended in front of the photosphere.

5.2. Long-Term Variations

The long-term variations of $\tau(\text{H}\alpha)$ and $\beta$ are shown in figure 10. These parameters evidently exhibit different behaviors as a whole as compared to the $V/R$ variations or other emission-line properties.

As can be seen in figure 10, $\tau(\text{H}\alpha)$ showed a saw-teeth like variation: gradual decline, sudden increase, and gradual decline in the 1970’s to 1980’s. It is to be noticed that the sudden increase in the middle of 1979 nearly corresponds to the epoch of the $V/R$ maximum phase. Moujtahid et al. (1998) also observed a sudden increase of the Balmer discontinuity in the same epoch after a long period of gradual decline. This infers that the disk also became opaque in the Balmer continuum in this epoch.

It is also noticed in figure 10 that, in some epochs, the disk is divided into optically thick and thin layers (double-layer fitting). There seems to exist a tendency that the double layers appear when $\tau(\text{H}\alpha)$ is relatively small, smaller than around 3500. If we assume that EW Lac is seen nearly equator-on, we can suppose that the value of $\beta$ is a measure of the vertical height of the disk. The disk is sufficiently vertically extended when $\tau(\text{H}\alpha)$ is large, whereas, when $\tau(\text{H}\alpha)$ is small, the disk vertically shrinks, and the optically thick layer seems to be

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Fig. 8. Equatorial view of the photosphere and circumstellar disk.

Fig. 9. Fitting samples of central depth analysis for some epochs of EW Lac. Left panel: Case of single-layer fitting. Right panel: Case of double-layer fitting. In both panels, the date of observation and derived parameters are indicated. The arrow in each theoretical curve indicates the position at which $\log(\omega_n)$ is zero on equation (2).
sandwiched between two optically thinner layers.

The value of $\beta$ remained almost constant with its high level of 0.9–1.0 throughout the period of the $V/R$ variation. One may notice that $\beta$ once dropped to around 0.65 in 1972–73, and recovered to the 0.9-level in 1974. It is interesting to see that the central depths of the emission profiles of the H$_\alpha$ through the H$\delta$ lines markedly decreased during this period (subsection 3.4, figure 5). The decrease of $\beta$ indicates the increase of the effective area of the naked photosphere; this provides an explanation why the central depths of the emission lines decreased. It is because additional emission from the photosphere makes the central depths apparently shallow.

5.3. Shell Line Depths Along the Balmer Series

Concerning the central depth analysis, Kupo (1969) plotted the inverse central residual intensities of the Balmer shell lines.
(line depth) against the series number for some epochs in 1965 September through December. His purpose was to determine the electron density of the disk by making use of the Inglis–Teller formula. Kupo determined the value of the maximum number, \( n_m \), by extrapolating the observed line depths to the zero level of the line depth. He noticed that the observed residual intensities are often approximated by two broken lines, for which he interpreted by the existence of two layers with different electron density.

According to our present method, the disappearance of shell absorption lines at the highest members of the Balmer series is caused by the decrease of the optical depths of the lines, which occurs before the lines are mutually blended by the Stark effect. This is apparent when we examine the line profiles of shell lines that are well separated in the highest members around H\( \beta \) in the case of EW Lac. In weaker shell stars, the shell lines disappear in series number much less than 30.

From the view point of our central depth analysis, Kupo’s one straight line indicates the case of one-layer disk with one-value of \( \beta \), whereas the broken two lines correspond to the case of two-layer disk with two values of \( \beta_1 \) and \( \beta_2 \). This is easily shown by producing the series number dependence of the central depths based on our measurements. An example of broken lines is shown in figure 11, in which our values of \( \beta \) and \( \tau(H\alpha) \) for both broken lines are also indicated. In this way we can explain Kupo’s diagram in terms of the existence of single or double layers in the disk.

In Kupo’s observations of 1965, one straight line appeared only in October, whereas two-broken lines were widely observed from September through December. As stated in subsection 5.2, the two-layer state tends to appear when \( \tau(H\alpha) \) is smaller than around 3500. Although we have no data for \( \beta \) and \( \tau(H\alpha) \) during this period, we can guess that the disk in this year was optically thin, \( \tau(H\alpha) \leq 3500 \), but variable in the value of \( \beta \).

### 5.4. Optical Depths of the Disk

Once \( \tau(H\alpha) \) is determined, we can deduce the optical depth for any Balmer lines, \( \tau(Hn) \), through the value of \( \omega_{2n} \). The optical depths of the disk for the H\( \beta \) through the H\( \delta \) lines, relative to H\( \alpha \), are given in table 7 for some cases of \( \tau(H\alpha) \).

In the same way we can obtain the optical depths of the shell lines based on the values of \( \omega_{2n} \). Figure 12 shows the maximum series number, \( n_m \), of the Balmer shell line recognized on the photographic plate, and the series number \( n_1 \) at which \( \tau(Hn) = 1 \). It can be seen that both \( n_m \) and \( n_1 \) are

**Table 7. Optical depth of the disk for the Balmer emission lines.**

| Line    | \( \tau(H\alpha) / \tau(H\alpha) = \omega_{2n} \) | Optical depths for some cases |
|---------|--------------------------------------------------|------------------------------|
| H\( \alpha \) | 1.0                                              | 2000 4000 6000               |
| H\( \beta \) | 0.16                                             | 320 640 960                  |
| H\( \gamma \) | 0.050                                            | 100 200 300                  |
| H\( \delta \) | 0.020                                            | 40 80 120                    |

**Fig. 11.** Shell line depths along the Balmer series number. An example of the double-layer approximation at the date of 1975 August 30 is shown. An optically thick layer is expressed by the regression line 2 \( [\beta_1 = 0.80, \tau_1(H\alpha) = 2450] \), and the optically thinner layer is by the regression line 1 \( [\beta_2 = 0.20, \tau_2(H\alpha) = 720] \).

**Fig. 12.** Maximum series number of recognizable Balmer lines, \( n_m \), and series number, \( n_1 \), at which \( \tau(Hn) = 1 \).
Table 8. Radial velocities of Balmer shell lines.

| Date         | JD 2400000+ | $V'(H6–H15)$ (km s$^{-1}$) | $V'(H16–H25)$ (km s$^{-1}$) | $V(H20)$ (km s$^{-1}$) |
|--------------|-------------|-----------------------------|-----------------------------|------------------------|
| 1966/08/22   | 39359.210   | −17.9                       | −16.4                       | −15.0                  |
| 1968/08/23   | 40091.217   | −12.0                       | −10.5                       | −15.0                  |
| 1969/08/28   | 40462.233   | −14.1                       | −13.6                       | −15.0                  |
| 1969/12/22   | 40577.893   | −12.4                       | −12.2                       | −12.5                  |
| 1969/12/23   | 40578.960   | −10.0                       | −9.2                        | −10.0                  |
| 1970/10/09   | 40868.199   | −17.8                       | −20.0                       | −20.0                  |
| 1970/10/09   | 40869.184   | −12.4                       | −12.2                       | −12.5                  |
| 1970/10/09   | 40869.021   | −15.7                       | −16.3                       |                        |

* Full table is given in the electronic version.

almost unrelated with the $V/R$ variations, and that the values of $n_1$ appear almost in between 16 and 25. This infers that the average radial velocity, $V'(H16–H25)$, given below, is a good measure for indicating the gas motion inside the disk.

6. Radial Velocities of Shell Lines

6.1. Radial Velocities of the Line Centers

We measured the radial velocities of the central deep cores of the line profiles, which correspond to the primary core noticed by Hubert et al. (1987). The average heliocentric velocities of two cases of $V'(H6–H15)$ and $V'(H16–H25)$, along with the value of $V'(H20)$, are partly given in table 8.1 The variation of the radial velocity, $V'(H20)$, measured by Hubert et al. (1987) is in general coincidence with ours.

The long-term variations of the average radial velocities, $V'(H6–H15)$ and $V'(H16–H25)$, are shown in figure 13, along with the data of Granes (1972). Remarkeable is the parallel relationship of the radial velocities with the $V/R$ variations given in figure 3. This relationship infers that the $V/R$ variations are closely connected with the internal gas motion of the disk. It is notable that the radial velocity started to decrease in around 1973, before the commencement of a remarkable $V/R$ variation. This infers that the disk matter in front of the star started to move outward at this early time, then gradually accelerated and turned to the inward motion at around 1978.

Large amplitudes of the velocity curves make it difficult to explain the $V/R$ variation in terms of the contracting–expanding motion of gas elements in a circular disk. This is because, if we calculate the distance of gas elements traveling during the expanding or contracting phase, the resultant distance is well over the whole size of the disk. It is thus evident that the pulsation hypothesis (contraction/expansion) in a circular disk should be ruled out in the case of EW Lac. Close parallelism between $V/R$ and the radial velocity variations is suggestive of the existence of some non-circular disk, reflecting the projected velocity of some Keplerian motion, though the velocity structure of the disk is not clear.

It is also noticed that the radial velocities of the deepest points of the central core for the H$\beta$ through H$\delta$ lines at around 1979, given in figure 5, disclose a close relationship with $V'(H6–H15)$ and $V'(H16–H25)$ concerning the following two points: First, the amplitude of variation is lower for the lower member, and higher for the higher member; secondly, the epoch of the maximum velocity is delayed in the lower
6.2. Balmer Progression

We measured the amount of the Balmer progression, by creating a linear regression line on the radial-velocity distribution along the Balmer series. We simply defined the Balmer progression ($BP$, in km s$^{-1}$) by the formula

$$BP = V(H_30) - V(H_{10}),$$

where $V(H_n)$ denotes the radial velocity of the shell line $H_n$, read out on the linear regression line of the observed progression. The values of $V(H_{10})$ and $BP$ are partly given in table 9.1. Figure 14 illustrates a sample of Balmer progression measured in two epochs.

The long-term variation of $BP$ is shown in figure 15. In most of the observed period, $BP$ is confined within $\pm5$ km s$^{-1}$. The thermal motion inside the disk is around 15 km s$^{-1}$, provided the electron temperature is 10000 K, or so. The small value of $BP$ being less than 15 km s$^{-1}$ provides security for our neglect of the Balmer progression, adopted in subsection 5.1.

Table 9. Balmer progression and radial velocity of the H10 line.*

| Date      | JD 2400000+ | $BP$ (km s$^{-1}$) | $V$(H10) (km s$^{-1}$) |
|-----------|-------------|-------------------|------------------------|
| 1966/08/21| 39359.210   | 1.1               | -17.0                  |
| 1966/08/22| 40091.217   | -3.1              | -15.0                  |
| 1969/08/28| 40462.233   | 1.5               | -12.0                  |
| 1969/12/23| 40578.960   | 4.6               | -11.0                  |
| 1970/10/08| 40868.199   | -1.7              | -18.0                  |
| 1970/10/09| 40869.184   | -0.8              | -17.0                  |
| 1971/09/01| 41195.204   | 0.9               | -20.0                  |
| 1971/12/10| 41296.028   | 1.9               | -21.0                  |

* Full table is given in the electronic version.¹
7. Physical Parameters of the Disk

7.1. Electron Density

By making use of the derived value of \( \tau(\text{H}_\alpha) \), we can roughly estimate the distribution of the electron density in the disk lying in front of the photosphere, provided that the outer radius, \( R_d \), of the disk is known. We shall tentatively adopt the outer radius \( R_d = 7.5 \, R_\star \) given in subsection 3.3, where \( R_\star \) denotes the radius of the photosphere. Then, we have (Kogure & Leung 2007, definitions and references therein),

\[
d \tau(\text{H}_\alpha) = K \frac{N_e^2(r)}{W(r)} dr
\]

and

\[
K = \frac{h v_{2n} A_{c2} B_{2n}}{4 \pi B_{2c} I_{2c}},
\]

where \( K \) is a constant depending on the stellar temperature, and \( K = 3.84 \times 10^{-31} \) for EW Lac (\( T_{\text{eff}} = 15800 \, \text{K} \)). \( W(r) \) is the dilution factor at the distance \( r \) from the star center. We assume that the distribution of electron density, \( N_e(r) \), is given by a power law with index \( \alpha \) as

\[
N_e(r) = N_{e0} \left( \frac{r}{R_\star} \right)^{-\alpha},
\]

where \( N_{e0} \) denotes the electron density at the stellar surface. Integrating equation (5), we have

\[
\tau(\text{H}_\alpha) = K \int_{R_\star}^{R_d} \frac{N_e^2(r)}{W(r)} dr.
\]

Since \( \tau(\text{H}_\alpha) \) is already known, we can derive the distribution of the electron density, \( N_e(r) \), provided that the values of \( R_d \) and the index, \( \alpha \), are known. Thus, a sample of the estimated electron density at the stellar surface is given in table 10. Notice that these values are not sensitive to the adopted outer radius, \( R_d \).

Table 10. Estimated electron density at the stellar surface, \( N_{e0} \) (cm\(^{-3}\)).

| \( \alpha \) | 2000           | 4000           | 6000           |
|-------------|---------------|---------------|---------------|
| 2           | \( 6.91 \times 10^{11} \) | \( 9.78 \times 10^{11} \) | \( 1.20 \times 10^{12} \) |
| 3           | \( 1.17 \times 10^{12} \) | \( 1.65 \times 10^{12} \) | \( 2.03 \times 10^{12} \) |
| 4           | \( 1.56 \times 10^{12} \) | \( 2.20 \times 10^{12} \) | \( 2.70 \times 10^{12} \) |

Table 11. Estimated partial mass of the disk lying in front of the photosphere, \( M_p \) in unit of solar mass.

| \( \alpha \) | 2000           | 4000           | 6000           |
|-------------|---------------|---------------|---------------|
| 2           | \( 6.65 \times 10^{-11} \) | \( 9.41 \times 10^{-11} \) | \( 1.15 \times 10^{-10} \) |
| 3           | \( 6.40 \times 10^{-11} \) | \( 9.05 \times 10^{-11} \) | \( 1.11 \times 10^{-10} \) |
| 4           | \( 5.77 \times 10^{-11} \) | \( 8.16 \times 10^{-11} \) | \( 9.99 \times 10^{-11} \) |

Although the amplitude is small, the \( BP \) variation indicates some parallel correlation with the radial-velocity variation of the shell lines given in figure 13. This may reflect some effects of differential gas motion inside the disk.

8. Photometric Variations and \( V/R \) Variations

EW Lac is known as a remarkable photometric variable, and observations elucidate a large variety from short- to long-term variations. In short-term variations within one day, evidence of multi-periodicity has been widely observed (Pavlovski 1987; Pavlovski et al. 1993), and some correlation with short-term \( V/R \) variations has been pointed out (Pavlovski & Schneider 1990). In order to inspect the photometric behaviors of EW Lac during the \( V/R \) variation, we show in figure 16 the long-term variations of the \( V \) magnitude and colors, based on data of Pavlovski et al. (1997), Jeong, Suh, and Nha (1986), and Stagg et al. (1988). For a comparison, the \( V/R \) variations are also reproduced from figure 3.

It can be seen that the \( V/R \) variation is related in some way to the light variation. The relationship may be given as follows:

1. Remarkable brightening occurred in the latter half period of the \( V/R \) variation, while the darkening of light in
around 1978 appeared just prior to the $V/R$ maximum epoch. In the epoch of the brightening in 1981–1984, marked decrease of the emission-line intensities in the H$\alpha$ through the H$\beta$ lines was observed. As can be seen in figure 3, the equivalent width $EW_\alpha(H\alpha)$ decreased from the 50 Å level down to the 40 Å level in 1982–1985. Ballereau et al. (1987) measured the equivalent widths of the H$\alpha$ line in the epoch from 1983 September to October. Although a marked short-term variation was seen in between 16.43 and 33.85 Å, their average value is considerably lower than the 50 Å level. On a closer inspection of figure 13, we can see that the average radial velocity, $V'(H16–H25)$, reached a deep minimum of around $-35$ km s$^{-1}$ in this epoch, indicating the existence of a strong outward gas motion in the disk. In addition, the optical depth, $\tau(H\alpha)$, showed a small dip, $\Delta\tau(H\alpha) \sim 1000$, in 1982–1983 during its general decline phase, as can be seen in figure 10. These features suggest that the stellar brightening has induced some spectroscopic changes in the disk. One possible scenario may be that the stellar brightening has given rise to strong stellar wind, by which a part of disk mass has blown out, and as a result, the emission-line intensity has decreased markedly.

2. Prior to the $V/R$ maximum phase, a remarkable darkening in the $V$ band appeared accompanying some decrease of the $U - B$ color in 1978. It is also noticed that a marked dip at around 0.05 mag occurred in 1977 (around JD 2443300). The years 1977–1978 correspond to the time when the gas motion in the inner part of the disk in front of the photosphere changed from outward to inward (figure 13). Temporal variations of brightness and colors in some irregular form seem to be connected with the gas motions inside the disk.

3. Stellar brightness seems to have no direct effect on the optical structure of the disk, since the stellar brightening occurred while the optical depth, $\tau(H\alpha)$, was in its declining phase (figure 10).
4. Pavlovski and Ružič (1991) claimed a close correlation between the brightness and radial velocity variations or shell activity for EW Lac. In our observations, no such correlation was confirmed.

9. Discussions

9.1. Retrograde Nature of the \( V/R \) Variation

We suppose that the \( V/R \) variation is caused by the propagation of some density enhancement, not by any change of the global shape (see subsections 3.3 and 6.1). Then, by comparing the long-term variation of the \( V/R \) ratios in figure 3, and of the optical depth, \( \tau(H\alpha) \), in figure 10, we can suppose that the \( V/R \) variation is of retrograde structure.

First of all, notice that the optical depth reached its maximum in late 1978, and had started its nearly monotonous declining for the latter half of the \( V/R \) variation period (figure 10). The \( V/R \) ratio showed its maximum in the middle of 1979. When the \( V/R \) ratio is in its maximum state, the enhanced region lies in the approaching side of the disk. In the case of prograde nature, the enhanced region should move to the front side of the disk, causing an increase of the optical depth. In the case of the retrograde nature, this is opposite: that is, the enhanced region moves to the behind side. A schematic configuration of the enhanced region for EW Lac is illustrated in figure 17 at a phase of nearly the \( V/R \) maximum. After this phase, the optical depth, \( \tau(H\alpha) \), gradually declined, which implies that the \( V/R \) variation is of the retrograde structure in this star.

This retrograde structure of EW Lac seems particular among Be \( V/R \) variables. Telting et al. (1994) found the prograde nature for the case of \( \beta^1 \) Mon. Mennickent, Sterken, and Vogt (1997) carried out photometric observations for six stars (V923 Aql, V1294 Aql, \( \gamma \) Cas, 48 Lib, MX Pup, \( \zeta \) Tau), and found that these stars are compatible with prograde global disk oscillations based on the model of Papaloizou, Savonije, and Henrichs (1992). Ogilvie (2008) calculated 3D eccentric disk models of Be stars, and obtained the prograde modes for all reasonable disk conditions. On the other hand, Okazaki (1991) mentioned in his one-armed oscillation model that the fundamental modes and all of the overtones generally retrograde. He also stated that the prograde density structure found in \( \beta^1 \) Mon is difficult to be explained by his model. EW Lac may belong to this retrograde case.

9.2. Time Lag and Spiral Structure

In the \( V/R \) variations of EW Lac, two behaviors should be noticed. One is the time lag for different Balmer lines, particularly in the latter half of the \( V/R \) variation period. The second is a shortening of the duration of the \( V/R \) variation from \( H\alpha \) toward \( H\beta \). In table 3 we can see that the total durations after the \( V/R \) maximum phase are 2340 (\( H\alpha \)), 1690 (\( H\beta \)), 1500 (\( H\gamma \)), and 1380 (\( H\delta \)) d, respectively. The optical depth is smaller and the peak separation of the Balmer emission is larger for higher members than for lower members. All of this evidence indicates that higher members are formed in deeper layers than lower members. This implies that the angular velocity of wave propagation is higher in deeper layers than in the outer layer. By combining time lag and phase shortening, we can suppose that the spiral structure possibly appeared in this period inside the disk.

In figure 18 we depict a schematic configuration of the density-enhanced regions of the lines \( H\alpha \) through \( H\delta \) at around the phase of the \( V/R \) minimum for \( H\gamma \) and \( H\delta \) in the middle of 1980. After the phase of the \( V/R \) maximum shown in figure 17, the enhanced regions are separated in the sense that higher members propagate faster than lower members, and thus the spiral structure is formed as shown in figure 18. We can trace this spiral structure up to around 1983. In 1984, the enhanced regions successively dispersed before entering the front side of the photosphere, so that the \( V/R \) ratios became unity, and the optical depth decreased down to the value of \( \tau(H\alpha) \approx 2000 \).

Okazaki (1991) mentioned that, in his one-armed oscillation model, when one-armed waves are excited, the most perturbed region is likely to be in the innermost part of the disk, and to propagate outward with a group velocity depending on their frequencies. Carciofi et al. (2009) calculated the 2D global oscillation model in the disk of a binary star, \( \zeta \) Tau, and showed the appearance of a pair of, or single, spiral-like structure. Okazaki (1991) also mentioned that the propagation
of such a perturbed region is merely a transient one. The spiral structure in EW Lac appeared only in the latter half of the $V/R$ variation period, so that the spiral structure of EW Lac may be corresponding to the transient phenomena predicted by Okazaki (1991). Rivinius, Štefl, and Baade (2006) claimed the finding of phase lag and the helical structure of the density waves in the long-term $V/R$ variables, maybe including 48 Lib, though the details are not shown.

In the $V/R$ variation period started in 2007, it may be interesting to see whether a similar spiral structure appeared or not after the $V/R$ maximum phase. Although observational data are quite limited (subsection 4.1), there is one clue to guess the absence of a spiral structure. That is, the $V/R$ ratios have shown similar values for all of emission lines at the epoch of 2010 October, several hundred days after the $V/R$ maximum, suggesting a lacking of time lag as compared with the first event. Time lag seems to be closely related to the formation of a spiral structure.

9.3. Weak Correlation and Possible Binary Effect

As shown in section 3, the $V/R$ variation indicates weak correlations with the peak radial velocities, peak separations of the emission lines, and emission equivalent widths. These weak correlations suggest the structure of the disk truncated at some radius from the star center. One possibility for such a truncation is to assume the existence of an unseen small-mass companion around the primary. According to the stable orbit theory of Hénon and Guyot (1970), gas particles around the primary make some stable orbits, and the maximum stable orbit (MSPO) defines the outer boundary of circumstellar mass around the primary. When the mass ratio is sufficiently small, such as less than 5%, MSPO gives rise to two effects: one is the existence of an outer boundary of the disk; the second is a small eccentricity of the disk. These effects might explain the existence of a weak correlation among the physical parameters when applied to EW Lac. This possibility, however, remains as a future problem.

10. Summary

EW Lac entered the repeating $V/R$ variation phase in 1976 with an approximate period of seven years. Among those, a remarkable $V/R$ variation was observed in 1976–1986, and in the time starting in the late 2000’s.

We mainly investigated the spectroscopic behaviors of EW Lac during the first $V/R$ variation event. We analyzed the emission lines and the shell absorption lines in the Balmer series. In an analysis of the emission lines, we found that the $V/R$ variations of the Hα through the Hδ lines are characterized by time lag and a shortening of the phase duration from lower to higher Balmer members. Time lag is conspicuous after the $V/R$ maximum, and the $V/R$ ratio reached its minimum value in the order from Hδ to Hα. The phase duration is longest in Hα and shortest in Hδ. We also found weak correlations between the $V/R$ variation and the physical parameters, such as the peak velocities, emission equivalent widths, and peak separations.

We then analyzed the shell absorption lines for higher members of the Balmer series, which are formed in the disk lying in front of the photosphere. Our finding is as follows:

1. The disk is sufficiently opaque for the emission lines, and the optical depth is as high as 2000 to 6000 in the Hα line. The optical depth is getting lower, and becomes unity in around H15–H25.
2. The long-term variation of the optical depth of Hα displays a quite different behavior as compared to the $V/R$ variation. However, the epoch of the maximum depth in 1979 nearly coincides with the epoch of the $V/R$ maximum.
3. The effective area of the disk covering the photosphere is high during the $V/R$ variation period, implying that the disk is vertically extended in front of the photosphere.
4. The average radial velocity of the shell absorption lines showed long-term variation parallel with that of the $V/R$ variation. This implies that the $V/R$ variations are associated with the internal velocity structure of the disk.
5. The electron density and the partial mass in front of the photosphere are estimated in some cases. The electron density showed changes around 1.5–1.7 times during the $V/R$ variation.

Based on the analysis of the emission and shell absorption lines, we consider the disk structure to be follows:

1. The $V/R$ variation is found to have a retrograde structure.
2. A spiral structure is likely to be observed inside the disk in the latter half of the $V/R$ variation period.
3. A possible effect of an unseen small-mass companion is suggested to explain the weak correlation between the $V/R$ variation and other physical parameters. A binary effect, however, remains as a future problem.
4. A close relationship between the photometric variation and the $V/R$ variation is pointed out. The remarkable brightening of the star in the later stage of the $V/R$ variation may have induced a mass loss from the disk, causing a marked decrease of the emission-line intensity.

The $V/R$ variation that appeared in 2007 is just like a recurrence of the 1976–1986 event. Our observations in 2010 showed low emission intensities, and similar values of the $V/R$ ratios for Hα through Hδ, several hundred days after the $V/R$ maximum epoch. This suggests the absence of any time lag.

EW Lac may be the first star in which the time lag and variation of the phase duration in the $V/R$ variation have been observed. The present study yields important constraints for the modeling of the $V/R$ variation of this star.

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