EVOLUTION OF A NETWORK OF COSMIC STRING LOOPS

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Abstract

We discuss and summarise the predictions of a model [1] for the non-equilibrium evolution of a network of cosmic strings initially containing only loops and no infinite strings. The results are of interest given recent work highlighting the problems with structure formation from the standard cosmic string scenario [2, 3].

1 Introduction

Cosmic strings, the analogues of vortices in superfluids, are one of a number of topological defects which may have formed at the GUT phase transition [4]. As they are topologically stable, they may have survived until today, and due to their strong gravitational effects they provide an alternative theory to inflation for explaining the formation of structure in the universe and the temperature fluctuations in the CMB. In the last year, though, the theory seems to have been in some trouble at least in a flat universe with $\Lambda = 0$, as normalisation of the predictions of cosmic strings to COBE was shown to leave the spectrum of density perturbations seriously lacking in power on large scales [2, 3]. However, the cosmic string community is not despondent and there is renewed motivation to understand some very important points which may significantly alter the ‘standard’ picture of cosmic string formation and evolution [4].

In this picture, used to ‘rule out’ cosmic strings [2], the infinite strings lose energy through loop production and reach a scaling solution in which the energy density in strings is a fixed fraction of the energy density of the universe and all scales grow with the horizon. For example, work is underway to understand the effects of the lattice both in the formation of strings [5] and to combat the obvious limitations of numerical simulations on cosmic string network evolution [6]. Attention is also turning to understand whether strings form at all in continuous transitions [7], and if so whether their initial distribution still contains both infinite strings and loops.

Motivation for considering the evolution of a network of cosmic string loops, a very different network to the ‘standard’ one containing both infinite strings and loops, comes from different directions. Firstly, as a result of a lattice-free dynamical simulation of a first order phase transition, Borrill [5] has argued that

1 Talk given at PASCOS 98, Northeastern University, Boston, March 1998.
there is no evidence for infinite strings, but only a population of loops. (This claim has not been disproved even though it has been contested [8].) Loop networks are also produced in other situations [9]. Such a network is expected to evolve very differently from the standard one, and hence have different cosmological consequences; might they be such as to ‘save’ cosmic strings? Secondly, it is interesting to focus on the loop network alone (which, we believe, has not been done). For example, it is often said that the infinite strings are responsible for structure formation; however, could a loop network not survive long enough to perform the same task? Finally, given the complicated nature of phase transitions, we do not believe that an initial distribution containing only loops is ruled out for certain.

Section 2 outlines the main features of our model for the evolution of a cosmic string loop network. Results and discussions are given in section 3.

2 The Model

We describe the network of relativistic cosmic string loops by the distribution function \( n(\ell, t) d\ell \), which is the average number of loops per unit volume with physical length between \( \ell \) and \( \ell + d\ell \) at time \( t \). To obtain an equation for \( \partial n(\ell, t) / \partial t \), length-changing interactions must be specified. Firstly, \( \ell \neq 0 \) since the length of loops changes through gravitational radiation, expansion and redshift. Secondly, loops can interact with each other. Assuming that partners are exchanged when loops intersect, the two dominant length-changing interactions between loops are:

\( \bullet \) A loop of length \( \ell \) can intersect another loop of length \( \ell' \) to give a loop of length \( \ell + \ell' \).

\( \bullet \) A loop of length \( \ell \) can self-intersect to produce two daughter loops of length \( \ell' \) and \( \ell - \ell' \).

In the first case, if the initial lengths of the two loops are in the range \( \ell \rightarrow \ell + d\ell \) and \( \ell' \rightarrow \ell' + d\ell' \) then the number of collisions per unit time per unit volume is given by the number of loops with each of the initial lengths times the probability \( A(\ell + \ell'; \ell, \ell') \) of the collision occurring:

\[
A(\ell + \ell'; \ell, \ell')(n(\ell, t)d\ell)(n(\ell', t)d\ell').
\]

Similarly, for the second process, the number of daughter loops of length \( \ell' \) produced from the self-intersection of a loop of length \( \ell \rightarrow \ell + d\ell \) per unit time per unit volume is given by

\[
B(\ell - \ell', \ell'; \ell)(n(\ell, t)d\ell).
\]

Given (1)-(2) and letting \( H \) be the expansion rate, the equation for \( \partial n / \partial t \) is

\[
\frac{\partial n}{\partial t} = -n(\ell, t) \frac{\partial \ell}{\partial \ell} \frac{\partial n(\ell, t) \partial \ell}{\partial \ell} + \lim_{\ell \rightarrow 0^+} \left[ n(\ell, t) \delta(\ell) \right] - 3Hn(\ell, t)
\]
\[ + \int_0^{\ell/2} A(\ell; \ell', \ell - \ell')n(\ell', t)n(\ell - \ell', t)d\ell' \quad (4) \]

\[ - \int_0^{\infty} A(\ell + \ell'; \ell, \ell')n(\ell', t)d\ell' \quad (5) \]

\[ + \int_\ell^{\infty} B(\ell, \ell' - \ell'; \ell')n(\ell', t)d\ell' - n(\ell, t) \int_0^{\ell/2} B(\ell', t - \ell'; \ell)d\ell'. \quad (6) \]

In line (3), the first two terms can be written as the single differential \(-\partial (n\dot{\ell})/\partial \ell\) which is the net positive flux of loops in \(\ell\) space. The third term in (3) guarantees that \(n(\ell, t) = 0\) for all \(t\) and \(\ell < 0\), as is required for a physical distribution of loops. The four terms in (4)-(6) are scattering integrals. Line (4) gives the rate of production of loops of length \(\ell\) from the intersection of two smaller loops of length \(\ell'\) and \(\ell - \ell'\), and its converse (5) gives the rate at which loops of length \(\ell\) disappear due to intersections with other loops. Loops of length \(\ell\) may be produced as a result of a larger loop self-intersecting — this is the first term of (6). The second term is its counterpart which gives the rate at which loops of length \(\ell\) disappear due to self-intersection.

In a non-expanding universe \(\dot{\ell} = H = 0\) so that \(\partial n/\partial t\) is just given by the scattering terms (4)-(6), and one can verify that the total energy density \(E \propto \int_0^\infty n(\ell, t)\ell d\ell\) is preserved as necessary. In an expanding universe (6) is a non-linear and non-local integro-partial-differential equation. For that reason in existing discussions (6) are ignored leaving a very simple equation to solve. We will not do this below.

The final step on the way to solving (6) given an initial distribution \(n(\ell, t_0)\) is to specify \(A\) and \(B\). This was done by assuming that the loops are Brownian random walks on scales \(\gg \bar{\xi}(t)\), the persistence length, and that they have many small scale kinks on a scale \(\zeta(t)\). Furthermore we assumed \(\bar{\xi}, \zeta\) are constant in a non-expanding universe, and \(\bar{\xi}, \zeta \propto t\) in an expanding one.

### 3 Results and Discussion

#### 3.1 Non-expanding Universe

Here the network should evolve until it reaches a stable equilibrium distribution which has been described by string statistical mechanics (SSM). Analysis of (6) gives results entirely consistent with those of SSM: we find that as long as the constant energy density \(E < E_c\), a critical value determined by parameters of the model (such as \(\bar{\xi}, \zeta\)), then the loops tend to an equilibrium distribution of the form \(n(\ell, t) \sim e^{-\beta I/2} \ell^{-5/2}\) for \(\ell \gg \bar{\xi}\). For \(E > E_c\) we find that \(\beta \to 0\) and non-analytic behaviour of equation (6) together with an extension of the model to include infinite strings leads us to predict that an infinite string is formed at \(\beta = 0\), as in SSM. Though it might perhaps seem surprising that a pure loop distribution can evolve to create infinite strings, this may indeed happen: if the
initial energy density of loops is sufficiently large, we must expect that mutual collisions will lead to a percolating system, i.e., to infinite strings.

### 3.2 Expanding Universe

Here analysis of (6) has led us to conclude [1] that the network evolves in the following ways, depending on the initial distribution of loops:

- For small $E$, the loops disappear both in the radiation and matter eras.
- For larger initial energy densities, $\beta \to 0$ in the radiation era, and we expect the following possibilities:
  1) The point $\beta = 0$ is reached when still in the radiation era or conceivably in the matter era, in which case infinite strings are formed with a scale invariant distribution of loops. Equation (6) then breaks down and the effect of infinite strings need to be included [10].
  2) If $\beta$ does not reach zero, the loops disappear in the matter era.

Thus the only way in which the loop network can evolve so as to stay as a loop network (i.e. not containing infinite strings) is for the loops to disappear in the matter era. At present this is the only scenario which might be compatible with data in a flat universe with $\Lambda = 0$ as it increases the power on large scales.

### Acknowledgments

I would like to thank my collaborators Ed Copeland and Tom Kibble for their part in this long project. This work was supported by P.P.A.R.C.

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