1/N Resolution to Inflationary $\eta$-Problem

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Abstract

We observe that the dominant one loop contribution to the graviton propagator in the theory of $N$ ($N \gg 1$) light scalar fields $\phi_a$ (with masses smaller than $M_{pl}/\sqrt{N}$) minimally coupled to Einstein gravity is proportional to $N$ while that of graviton-scalar-scalar interaction vertex is $N$ independent. We use this to argue that the coefficient of the $R\phi_a^2$ term appearing at one loop level is $1/N$ suppressed. This observation provides a resolution to the $\eta$-problem, that the slow-roll parameter $\eta$ receives order one quantum loop corrections for inflationary models built within the framework of scalar fields minimally coupled to Einstein gravity, for models involving large number of fields. As particular examples, we employ this to argue in favor of the absence of $\eta$-problem in M-flation and N-flation scenarios.
1 Introduction

Recent CMB observations \cite{1} indicate that the early Universe has passed through an inflationary period with Hubble parameter $H \lesssim 10^{-5} M_{\text{pl}}$. The standard theoretical setup for inflationary models generically involve some scalar inflaton fields slowly rolling down their potential. The slow-roll that is needed to ensure a resolution to the problems of standard big bang cosmology, and consistency with the CMB results, demands $\epsilon = -\frac{\dot{H}}{H^2}$ and $\eta = \frac{\ddot{H}}{H \dot{H}}$ to be smaller or of order $10^{-2}$ \cite{1}. In the context of simple single scalar models with potential $V(\phi)$, $\epsilon$ and $\eta$ are a measure of flatness of the potential and, specifically, $\eta \sim m^2/H^2$, where $m$ is the effective mass of the inflaton field. Therefore, the theoretical framework invoked for inflationary model building should also provide mechanisms to protect the potential and its flatness against quantum and/or quantum gravity corrections. In physics models we generically associate smallness and protection of a quantity like $\eta$ with an approximate symmetry, such that when the symmetry is exact this parameter (here the effective mass of inflaton) is zero. Supersymmetry, for example, can be such a symmetry. It turns out that in the presence of gravity, as in our case where we are dealing with inflation models, symmetries protecting $\eta$ are all broken, inducing an inflaton mass term of order the Hubble scale $H$ of the background. We hence end up with an order one $\eta$, i.e., the $\eta$-problem.

From the above discussion it is seen that the $\eta$-problem may appear in two ways: In a top-down approach where we invoke a theory of quantum gravity like string theory for inflationary model building. Or in a bottom-up approach where we take the usual field theory setup of Einstein gravity plus scalar inflaton fields, assuming that this framework is valid up to Planck scale $M_{\text{pl}}$. In the top-down approach the $\eta$-problem appears as a classical (not loop) effect, usually due to the interaction of the inflationary sector with the “moduli stabilization sector”, see, e.g., \cite{2}. In these models it turns out to be easy to have small $\epsilon$ with controlled back-reactions on it, but $\eta$ receives order one corrections. Intuitively, in these top-down setups, the $\eta$-problem can be understood as follows. With a vacuum energy of order $V$ all scalars, including the inflaton, will be endowed with soft masses of order $V/M_{\text{pl}}^2$ because the supersymmetry breaking scale is not below $H$. In this work, however, we will focus on the $\eta$-problem in the bottom-up approach.

In the bottom-up approach the $\eta$-problem arises from quantum loop corrections to the tree-level graviton-scalar-scalar vertex. Despite being non-renormalizable, one can still apply the (Wilsonian) effective field theory techniques to the Einstein gravity theory and consider loop corrections, e.g. see \cite{3, 4, 5}. In the presence of a scalar field $\phi$ minimally coupled to Einstein gravity, as in generic inflationary models, these corrections at one loop level generically involve a $R\phi^2$ term, a term whose presence was noted long ago \cite{7}. As we will review below, such a term is quadratically divergent and in the one loop effective action appears as

$$\zeta \frac{\Lambda^2}{M_{\text{pl}}^2} R \phi^2,$$

where $\Lambda$ is the UV cutoff of the theory and $\zeta$ is an order one coefficient. Assuming a Planckian cutoff scale, $\Lambda \sim M_{\text{pl}}$, in an inflationary background where $R \sim H^2$, this term yields a correction of order $H^2$ to the inflaton mass, causing the $\eta$-problem.
The $\eta$-problem, or the Hubble scale mass term for the effective inflaton field, seems quite
generic and one may put forward the idea of *kinematically* reducing the coefficient $\zeta \Lambda^2 / M_{\text{pl}}^2$. In this Letter we explore this possibility. One obvious possibility is to choose the cutoff $\Lambda$, the scale where quantum gravity effects become important, to be one or two orders of magnitude smaller than $M_{\text{pl}}$ [8]. In this case the coupling constant of gravitons will be reduced like the momentum UV cutoff and the $\eta$ problem persists. Alternatively, one may explore the idea that $\zeta$ is a kinematical factor that for some reason is not of order one, while the bare cutoff $\Lambda$ is $M_{\text{pl}}$. In fact, similar suppressions are already very well known in the context of large $N$ gauge theories [9]: the nonplanar part of a given Feynman diagram comes with powers of $1/N$ suppression compared to the planar part of the same diagram. As we will show similar analysis can be repeated for the theories involving large number of scalar fields minimally coupled to gravity. In particular, if we have $N$ number of “light fields”, lighter than $M_{\text{pl}} / \sqrt{N}$, $\zeta$ turns out to have a $1/N$ suppression factor. In a sense, as if, the diagram leading to (1.1) is a nonplanar diagram. This observation is closely related to the “species dressed gravity cutoff scale” ideas discussed in [10, 11], in light of which the $\zeta \sim 1/N$ result may be interpreted as dealing with a “dressed cutoff” $\Lambda / \sqrt{N}$ while $\Lambda \sim M_{\text{pl}}$.

Inflationary models with many scalar fields have recently got attention in view of their success in providing a natural explanation for the smallness of the inflaton self-couplings (the issue of steepness of the potential) and for the super-Planckian excursion of the effective inflaton in the field space [12]. This idea is not exotic to string theory motivated inflationary settings where it is quite common to have an abundant number of fields/degrees of freedom with masses below the dressed cutoff $M_{\text{pl}} / \sqrt{N}$, see, e.g, [13, 14, 15]. Even though in some of these setups, like N-flation, the individual field excursion is greater than the dressed UV cutoff, some, like Gauged M-flation [16] or multiple M5-branes Inflation [14], remain immune to this “beyond-the-cutoff” problem.

In this work we examine the above proposed $1/N$ resolution to the $\eta$-problem. We assume that there is a hierarchy of scales between $H$, the dressed gravity cutoff $\Lambda_{\text{dressed}} \sim M_{\text{pl}} / \sqrt{N}$ and $M_{\text{pl}}$: $H \ll \Lambda_{\text{dressed}} \ll M_{\text{pl}}$ which is easily achieved by e.g. $N \gtrsim 10^2$. This provides a window where one can safely use the standard techniques of quantum field theory and effectively deal with a system that could be described by Einstein-Hilbert gravity, the inflaton sector, and other heavy remnants of the theory of quantum gravity whose masses $M_a$ falls below the new gravity cutoff, i.e. $M_a \lesssim \Lambda_{\text{dressed}}$.

The outline of this work is as follows. We consider a system of $N$ light scalars minimally
coupled to Einstein gravity and work out basic Feynman rules of the theory and compute the quadratically-divergent part of the one loop contributions to the graviton propagator and graviton-scalar-scalar vertices. We show that one loop graviton two-point function has a linear $N$ parametric dependence while the graviton-scalar-scalar vertex has no $N$ dependence. Therefore, if we (re)normalize the graviton two-point function, the vertex will have a factor of $1/\sqrt{N}$. This latter leads to $\zeta \sim 1/N$ (cf. (1.1)). We discuss how this can resolve the $\eta$-problem in the context of many-field models like N-flation [13] or M-flation [15, 16].

2
2 Loop analysis in multi-field inflationary model

Consider the action of $N$ scalars minimally coupled to gravity

\[ L = -\frac{1}{2}M_{pl}^2 R - \frac{1}{2}\partial_{\mu}\phi_{a}\partial^{\mu}\phi_{a} - \frac{1}{2}M_{a}^2 \phi_{a}^2 - V(\phi_{a}), \]  

(2.1)

where $a = 1, 2, \cdots, N$ is the number of scalar fields, and summation over repeated $a, b$ indices is assumed. One or some of these scalars play the role of inflaton(s), and $V(\phi_{a})$ could be any potential that realizes slow-roll inflation at the classical level, while the rest exhibit possible remnants of the underlying quantum gravity theory. We assume the mass of these remnants to be below our dressed cutoff $\Lambda_{\text{dressed}}$. The action (2.1) once quantized will receive all possible corrections compatible with the symmetries of the system, in particular an $R\phi_{a}^2$ correction which appears at one loop level. As discussed if this term comes with an order one coupling can cause the $\eta$-problem. In what follows we show by carrying out explicit one loop calculations involving gravitons, that this term is suppressed by factors of $1/N$, providing a setting to resolve the $\eta$-problem in the context of multi-field models of inflation. In our analysis in this section and section 3 we ignore the loops involving scalar self-interactions. As we will discuss in the discussion section these diagrams do not change our main result.

2.1 Tree level Feynman diagrams

To perform the one loop analysis, as in any quantum field theory, we need to work out basic tree level Feynman diagrams of propagators and interaction vertices. To do so for the gravity sector, following \[4\], we introduce the tensor densities,

\[ \bar{g}^{\mu\nu} = g^{1/2} g_{\mu\nu} \quad \text{and} \quad \bar{g}_{\mu\nu} = g^{-1/2} g_{\mu\nu}, \]  

(2.2)

to bring the gravitational part of the action to the Goldberg’s form \[17\]

\[ \frac{M_{pl}^2}{16} (2g^{\rho\sigma} \bar{g}_{\lambda\mu} \bar{g}_{\kappa\nu} - \bar{g}^{\rho\sigma} \bar{g}_{\mu\kappa} \bar{g}_{\lambda\nu} - 4\delta_{\kappa}^{\rho} \delta_{\lambda}^{\sigma} \bar{g}_{\mu\nu}) \bar{g}^{\mu\lambda}_{\rho} \bar{g}^{\nu\sigma}_{\lambda}. \]  

(2.3)

We will decompose the density metric to the flat part and the deviation from the flat space part,

\[ \bar{g}_{\mu\nu} = \eta_{\mu\nu} + \hat{h}_{\mu\nu}, \]  

(2.4)

where $\hat{h}_{\mu\nu}$ is defined as

\[ \hat{h}_{\mu\nu} \equiv M_{pl}^{-1} h_{\mu\nu}. \]  

(2.5)

The inverse of $\bar{g}_{\mu\nu}$ is given by

\[ \bar{g}^{\mu\nu} = \eta^{\mu\nu} - \hat{h}^{\mu\nu} + O(\hat{h}^2), \]  

(2.6)

where on the R.H.S. the indices are raised and lowered by the flat Minkowski space metric $\eta_{\mu\nu}$. Perturbing the action (2.1) up to third order in $h_{\mu\nu}$, one obtains

\[ L = -\frac{1}{2} \partial_{\mu} h_{\mu\nu} \partial^{\rho} h^{\rho\nu} + \frac{1}{2} \phi_{a} (\Box - M_{a}^2) \phi_{a} + \frac{1}{2M_{pl}} h_{\mu\nu} T^{\mu\nu} + \frac{1}{M_{pl}} O(h(\partial h)^2) \]  

(2.7)

3
where
\[ T_{\mu\nu} = \partial^\mu \phi_a \partial^\nu \phi_a - \frac{1}{2} \eta^\mu{}^\nu \phi_a \partial_\alpha \phi_a + \eta^\mu{}^\nu \left( \frac{1}{2} M^2_a \phi_a^2 + V(\phi_a) \right). \quad (2.8) \]
Note that $T_{\mu\nu}$ is written to lowest order in $h_{\mu\nu}$ and so it is independent of $h_{\mu\nu}$. From this interaction term, and dropping the last term in (2.8) which is inessential for our purposes, one can read the vertex $\partial^\mu \phi_a \partial^\nu \phi_a$ to be $\frac{1}{M^2_{pl}} \left( p^\mu p^\nu - \frac{1}{2} \eta^\mu{}^\nu (p \cdot p') \right) \delta_{ab}$, where $p^\mu$ and $p'^\mu$ are two external four-momenta on the $\phi_a$ particles. To work out the basic Feynman graphs of the theory we need to gauge-fix the diffeomorphism invariance. This may be done through gauge fixing term
\[ \mathcal{L}_{g.f.} = \frac{M^2_{pl}}{4} \bar{g}^{\mu\nu} g_{\mu\nu}^\alpha \eta_{\alpha\beta}. \quad (2.9) \]
In Figure 1 we have plotted the basic Feynman graphs in this gauge.
2.2 One loop analysis

Having the tree level theory we now proceed to the one loop analysis and revisit the one loop propagator calculations as well as graviton-scalar vertex. Since we are interested in the quadratically divergent term (1.1), it is appropriate to use cutoff regularization; dimensional regularization, which is very well suited in capturing the logarithmic divergences and already used in [5, 6], can not be employed here.

2.2.1 One loop propagators

As depicted in Figure 2, there are five Feynman diagrams contributing to the one loop graviton propagators. The first two are coming from the pure Einstein gravity sector and the other three involve scalar fields running in the loop. Since we are only interested in the $N$ dependence of the diagrams we focus on the ones with scalar fields in the loop. In electrodynamics, the gauge invariance enforces the photon self-energy to be transverse. This reduces the degree of divergence from two to zero. However, in gravity the gauge invariance does not do so. It only relates the three diagrams that involve the scalar field in the loops. Thus the quartic divergence, $\Lambda^4 \delta_{\mu\nu} \delta_{\rho\sigma}$, which corresponds to the cosmological constant term,
\[ \Lambda^4 \sqrt{-g}, \] remains. This difference is due to the fact that \( \Lambda^4 \sqrt{-g} \) is still gauge-invariant whereas \( m^2 A_{\mu} A^\mu \) is not. This is the famous cosmological constant problem which we are not intended to deal with here. Next-to-leading divergent part diverges like \( \Lambda^2 \) and this is the part that renormalizes the graviton propagator. In particular, the diagram that involves two graviton-scalar three-vertices is

\[ I_1 = \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - M_a^2} V_{ab}^{\mu\nu}(-k, k - p; p) \frac{i}{(k - p)^2 - M_a^2} V_{ab}^{\rho\sigma}(k, p - k; -p) \]

\[ \propto N \left( \frac{\Lambda}{M_{pl}} \right)^2 D_{\mu\nu\rho\sigma}, \tag{2.10} \]

as long as \( M_a \ll \Lambda \). The diagram involving the graviton-scalar four vertex is of the form

\[ I_2 = \sum_{a=1}^{N} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - M_a^2} V_4(k, -k; p, -p). \tag{2.11} \]

The leading part of this integral is quartic in the UV cutoff and just renormalizes the cosmological constant. The next-to-leading order is quadratic in the UV cutoff and is proportional to \( N \left( \frac{\Lambda}{M_{pl}} \right)^2 D_{\mu\nu\rho\sigma} \), assuming that the masses \( M_a \) are all much smaller than the “undressed (bare) UV cutoff” \( \Lambda \), which is taken to be \( M_{pl} \). The last diagram in Figure 2 has only a quartic divergence and does not contribute to the renormalization of the graviton propagator at all. Thus we see that the one loop graviton propagator is proportional to number of fields \( N \), as well as \( \left( \frac{\Lambda}{M_{pl}} \right)^2 \).

This term may be viewed as the correction to the Newton constant or \( M_{pl} \). That is, the quantum gravity effects become important when this term becomes of the same order as the classical tree level value. This happens if the cutoff momentum \( \Lambda \) is of order \( M_{pl} \)

\[ \Lambda^2_{\text{dressed}} = \frac{M_{pl}^2}{N} \tag{2.12} \]

which is the species dressed UV cutoff. Besides this “perturbative” argument, the fact that one should use this reduced cutoff instead of \( M_{pl} \) in presence of large number of species has also been backed up by black hole physics and Hawking radiation from black holes in theories with large number of light species \[10, 11\].

One may also compute one loop correction to the scalar propagator. Again there are diagrams involving only scalars and two diagrams involving gravitons. It is immediate to see that the latter two diagrams have no parametric dependence on the number of scalars \( N \).

### 2.2.2 One loop graviton-scalar vertex

As depicted in Figure 3, there are three diagrams contributing to scalar-graviton vertex at one loop level. Our interest in these diagrams are twofold: i) we read the correction to the

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1Analysis of two point function alone is not enough to deduce this result and one should also consider graviton-scalar interaction vertex, which we will have done in the next subsection.
Figure 3: One loop contributions to $h-\phi-\phi$ vertex. These diagrams contribute to the corrections of graviton-scalar tree level interactions (by renormalizing them) as well as generating the $R\phi_a^2$ term at the level of one loop effective action.

tree level graviton-scalar three-vertex depicted in Figure 1 and, ii) compute the coefficient of the $R\phi_a^2$ term which appears at one loop level from these diagrams.

The details of the loop calculations, which are straightforward, are given in the appendix, here we just quote the result. Since we are mainly interested in the $N$ dependence of the loop expressions we only focus on that issue here. There are no $N$ dependence appearing in any of the diagrams in Figure 3 and these diagrams, compared to the tree level results, are proportional to $(\Lambda/M_{pl})^2$. This in particular implies that the coefficient in front of the effective $R\phi_a^2$ term (up to numeric factors of $1/4\pi$) is proportional to $(\Lambda/M_{pl})^2$, and if $\Lambda$ is the species dressed cutoff $\Lambda_{\text{dressed}}$ (2.12) this term is suppressed by the number of light species $N$.

To summarize, the one loop correction to graviton propagator is dressed with a power of $N$, while the graviton-scalar-scalar vertex is $N$ independent. This result is very similar to the well-established ’t Hooft $1/N$ expansion [9], that if we normalize the two point function to one, the interaction term has $1/N$ suppression.

3 $R\phi_a^2$ term in inflationary background and resolution to $\eta$-problem

So far we have shown that in a theory with $N$ number of species with masses lighter than dressed cutoff $\Lambda_{\text{dressed}}$ (2.12), the coefficient of the $R\phi_a^2$ term generated at one loop level is $\xi/N$, where $\xi$ is an order one $c$-number. The above analysis was carried out in a flat space background and should be revisited for inflationary (almost de Sitter) backgrounds. It is readily seen, however, that the basic argument behind the factors of $N$ does not depend on the background geometry. Also, the presence of the new scale $H \ll \Lambda_{\text{dressed}}$ should not affect our argument in any qualitatively important way. We still expect that species lighter than the high energy cutoff $\Lambda_{\text{dressed}}$ in general will contribute to $N$. The only change concerns the very lightest species, with masses roughly below the Hubble scale $H$, where momenta at super-Hubble scales will not contribute, as we will now show.
We recall that the equation of motion of a free massive scalar on an inflationary background is
\[ \ddot{\phi}_a(k; t) + 3H \dot{\phi}_a(k; t) + \left( \frac{k^2}{a(t)^2} + M_a^2 \right) \phi_a(k; t) = 0, \] (3.1)
where dot denotes derivative with respect to the cosmic comoving time and \( a(t) \) is the scale factor. The relevant observation is that the modes contributing to \( N \) are the quantum modes, e.g. those with oscillatory (as opposed to exponentially damping or growing) behavior. To be able to solve the above equation, let us drop the time-dependent piece \( k^2/a(t)^2 \) for the moment and consider the equation
\[ \ddot{\phi}_a(k; t) + 3H \dot{\phi}_a(k; t) + M_a^2 \phi_a(k; t) = 0, \]
whose solution is of the form \( \phi_a(k; t) = \phi_a(0) e^{H\omega t} \) with \( \omega = -3/2 \pm \sqrt{9/4 - M_a^2/H^2} \). To have a quantum mode \( \omega \) should have an imaginary part. This latter implies that \( M_a \geq 3H/2 \). Note that this result is \( k \) independent and that addition of the \( k^2/a(t)^2 \) term will only slightly modify this result, as it is positive definite: All the modes with \( M_a > 3H/2 \), regardless of their \( k \), are always quantum modes, while modes with \( M_a < 3H/2 \) are classical for large wavelengths (i.e. “super-Hubble” physical momenta \( k/a(t) < 3H/2 \)), and quantum mechanical for sub-Hubble momenta. Note also that the “damping coefficient” \( e^{-3Ht/2} \) is removed in the process of canonical quantization as canonical momentum conjugate to \( \phi_a \) is \( \omega Ha(t)^3 \phi_a(0) \). This is of course the standard established result in inflationary cosmic perturbation theory and quantum field theory on curved (de Sitter) space time [18].

Since we are only interested in the UV behavior of the loop integrals, we can instead of integrating over \( k \) all the way from zero to \( \Lambda_{\text{dressed}} \), restrict the integral to go from \( H \) to \( \Lambda_{\text{dressed}} \). In this way we avoid the unnecessary complication with super-Hubble modes. In summary, all the modes lighter than \( \Lambda_{\text{dressed}} \), with both super-Hubble or sub-Hubble masses, contribute to the \( N \) in the loop integral. In other words, as long as \( H \lesssim \Lambda_{\text{dressed}} \), \( N \) is the same for inflationary and flat space and
\[ \Lambda_{\text{dressed}}^2 = \frac{M_{\text{pl}}^2}{N}, \]
where \( N \) is the number of species lighter than the cutoff, \( \Lambda_{\text{dressed}} \). In particular, the coefficient of the \( R\phi_a^2 \) term generated at one loop will become \( \xi/N \), with \( \xi \) of order one.

We are now ready to address the \( \eta \)-problem. To this end, we recall that the one loop corrected action is
\[ L = L_{\text{cl}} + \frac{\xi}{N} R\phi_a^2. \] (3.2)
Hence, the slow-roll parameter \( \eta_{ab} \equiv M_{\text{pl}}^2 V_{ab} / V \), where \( V_{ab} = \frac{\partial^2 V}{\partial \phi_a \partial \phi_b} \), is
\[ \eta_{ab} = \eta_{ab}^{\text{cl}} + \frac{\xi}{N} R / 3H^2 \delta_{ab} \simeq \eta_{ab}^{\text{cl}} + \frac{4\xi}{N} \delta_{ab}. \] (3.3)
To have a successful slow-roll inflationary period we usually demand \( \eta \sim 0.01 \), and if we assume \( \eta_{ab}^{\text{cl}} \sim 0.01 \), quantum corrections to \( \eta \) will be suppressed enough for \( N \gtrsim 100^2 \)

\[ ^2 \text{To complete this discussion we note that in multi-fielded inflationary models, like N-flation [13] or M-flation} \]
4 Discussion

The bottom-up \( \eta \)-problem seems quite generic to all models of inflation that involve a scalar field minimally coupled to gravity. Even if the parameters of the inflaton potential are chosen meticulously at tree-level, the loop corrections that arise from interactions of the graviton with the scalar field create the quadratically divergent conformal mass term which leads to the \( \eta \)-problem, if the UV cutoff of the theory is of order \( M_{\text{pl}} \). This kind of \( \eta \)-problem is of course different from the “top-down” \( \eta \)-problem arising within the string theory setups in which the volume modulus stabilization often resurrects the \( \eta \)-problem. The precision that should be enforced upon the tree-level parameters are often not needed to sustain inflation, but to match the observed density of perturbation [1]. In this letter we examined the possibility of circumventing the \( \eta \)-problem, in the former sense discussed above, in many-field models of inflation that are minimally coupled to gravity. These many-fields whose masses are assumed to be smaller than \( M_{\text{pl}}/\sqrt{N} \), have a natural appearance in effective low energy field theory description of quantum gravity models. As we argued \( N \gtrsim 100 \) will resolve the \( \eta \)-problem. Even though it is not necessary for our argument, the scalar fields should be non-interacting to realize inflation.

One example of such many-field models is N-flation [13] which has the \( O(N) \) symmetric potential \( V = \sum_{i=1}^{N} \frac{1}{2} m^2 \phi_i^2 \). As stated above, a few hundred scalar fields will be enough to circumvent the \( \eta \)-problem. Like its chaotic counterpart, the mass parameter \( m \) has to be \( \sim 10^{-6} M_{\text{pl}} \) which is smaller than \( \Lambda_{\text{dressed}} \) unless one resorts to an unnaturally large number of scalar fields, \( i.e. \ N \sim 10^{12} \). This of course comes at the price of exposing the model to quantum instability of the type discussed in [16]. Namely, the quantum fluctuations of these light fields may dominate over the classical evolution of the inflaton. Assisted model with quartic potential \( V = \sum_{i=1}^{N} \frac{1}{2} \lambda \phi_i^4 \) is another possibility. To have an observationally viable model, the effective coupling must be around \( 10^{-14} \) and thus with \( \lambda \simeq 1 \), one needs around \( 10^{14} \) scalar fields. This scenario also suffers from the above quantum instability with such a large number of massless scalar fields. Another disadvantage of both these two models is that the physical excursion of the fields is larger than \( \Lambda_{\text{dressed}} \) during the required 60 e-folds of inflation.

Another explicit example is M-flation [15] or its gauged version [16] where the inflaton potential is constructed by three \( N \times N \) non-commutative hermitian matrices whose action is invariant under \( U(N) \). The classical dynamics is simplified considerably in the \( SU(2) \) sector where these three scalar fields are proportional to the generators of the \( SU(2) \) algebra. Gauged M-flation, in addition to the above ingredients, has an extra \( U(1) \) field, associated with “center of mass ” \( U(1) \in U(N) \). M-flation in the \( SU(2) \) sector besides the inflaton field contains some number of “spectator fields” which do not contribute to classical inflationary trajectory while can be excited quantum mechanically and appear in the loops. For the gauged M-flation there are \( 2N^2 + 1 \) such scalar modes and \( 3N^2 - 1 \) massive vector modes. [15], there is a certain combination of the fields which plays the role of effective inflaton and the original “physical” field should be rescaled with appropriate powers of number of fields \( N \) so that this effective inflaton field is canonically normalized. The \( R \phi_a^2 \) term, being quadratic in the \( \phi_a \)'s, will not receive any normalization factors due to the \( N \) scaling relating canonically normalized inflaton and the original fields.
These modes have a hierarchical spectrum, i.e. they can be lighter or heavier than the Hubble scale $H$, for the explicit masses see [16]. Not all these modes are light enough to be counted in the dressed cutoff. As discussed in [16], the number of “contributing species” $N_s$ varies between $3 \times 10^5$ and $10^6$, depending on the region of the potential inflation happens. Hence, the species dressed UV cutoff is $10^{-3} M_{pl} \lesssim \Lambda_{dressed} \lesssim 5 \times 10^{-3} M_{pl}$. Consequently the $R\phi^2$ term is suppressed by a factor of $\lesssim 10^{-5}$ and could be safely ignored. Gauged M-flation could be motivated from the branes dynamics in an appropriate flux where the above matrices correspond to three of the perpendicular directions of a stack of $N$ D3-branes which are scalars in the adjoint representation of the $U(N)$. As such, although the “quantum” $\eta$-problem is resolved for M-flation, embedded within string theory, one should still deal with the “stringy $\eta$-problem” (cf. introduction for further references and discussions).

Finally, the term leading to the $\eta$-problem, $R\phi^2$, is a one-loop but marginal operator. One may naturally worry about other loop corrections, that enhancement factors of $N$ will dominate over $\Lambda_{dressed}/M_{pl}$ suppressions. It is straightforward to show that the largest such $N$ enhancement factor appears in the graviton two point function (at higher loops) for which this factor is $(N\Lambda_{dressed}^2/M_{pl}^2)^l$, where $l$ is number of loops. All the other diagrams will be of the form $N^k(\Lambda_{dressed}^2/M_{pl}^2)^l$ with $k < l$. Hence our one loop result seems to be also valid to all orders.

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**A Details of the loop calculations**

Now let us focus on all the diagrams which may generate the $R\phi^2$ term, or equivalently $\frac{1}{M_{pl}}(\partial^2 \tilde{h})\phi^2$, at the one-loop order. The first one, is given by the left diagram in Figure 1 where a graviton exchange between two external scalars modifies the scalar form factor. The diagram is proportional to

$$
\phi(p)\delta h^\mu\nu(q)\phi(p') = \int \frac{d^4k}{(2\pi)^4} \left( \frac{1}{M_{pl}} [p_\alpha k_\beta - (p \cdot k)\eta_{\alpha\beta}] \frac{i(\eta^\alpha\gamma\eta^\beta\kappa + \eta^\alpha\kappa\eta^\beta\gamma - \eta^\alpha\beta\eta^\gamma\kappa)}{(p - k)^2 + i\epsilon} \right)
\frac{1}{k^2 + i\epsilon M_{pl}} \left[p'_\gamma(k + q)_\kappa - \eta_{\gamma\kappa}p'_\mu \cdot (k + q) \right] \frac{1}{(k + q)^2 + i\epsilon}
\frac{1}{M_{pl}} [k_\mu(k + q)_\nu - \eta_{\mu\nu}k \cdot (k + q)] .
$$

(A.1)

What we are interested in is the divergent part of the above integral which multiplies the generated $R\phi^2$ term. There are six momenta in the numerator, only four of which are
internal. Thus the integral will be divergent as

$$\frac{1}{M_{pl}} \left( \frac{\Lambda}{M_{pl}} \right)^2.$$  \hspace{1cm} (A.2)

The other diagram that contributes to the $h - \phi - \phi$ vertex is the middle one in Figure 3. To estimate the leading divergent part of this diagram, one should note that the vertex that involves three $\hat{h}$ is proportional to $1/M_{pl}$. More specifically,

$$W_{3\alpha\beta\mu\nu\gamma\kappa} = \frac{1}{8M_{pl}} \sum_{\text{sym}} \left[ -4\eta_{\mu\nu}\eta_{\kappa\alpha}(p_2.p_3) + 2\eta_{\gamma\kappa}\eta_{\mu\alpha}\eta_{\nu\beta}(p_2.p_3) - \eta_{\gamma\kappa}\eta_{\mu\alpha\nu\beta}p_2\alpha p_3\beta + 2\eta_{\mu\gamma}\eta_{\nu\alpha}\eta_{\nu\beta}p_2\alpha p_3\beta 
+A.3 \right.
\left. + 4\eta_{\gamma\alpha}\eta_{\nu\beta}p_2\mu p_3\eta \right].$$

Sum is over symmetrization on the index pairs $(\alpha, \beta)$, $(\gamma, \kappa)$ and $(\mu, \nu)$ and also the six permutation done over momentum index triplets $\alpha\beta p_1$, $\gamma\kappa p_2$ and $\mu\nu p_3$. Again, this diagram has six momenta in the numerator, two of which are external. Thus the divergent part of diagram behaves as (A.2) times the terms among which $R\phi^2$ exists.

Finally, let us look at the right one-loop diagram in Fig. 3. The vertex $\hat{h}\hat{h}\phi\phi$ is proportional to $1/M_{pl}^2$. In more details:

$$V_{4\alpha\beta\mu\nu} = \frac{1}{M_{pl}^2} \left[ \eta_{\mu\nu}p_1\alpha p_2\beta - 2\eta_{\beta\nu}p_1\alpha p_2\mu + \eta_{\mu\nu}p_1\beta p_2\alpha - 2\eta_{\alpha\nu}p_1\beta p_2\mu + \eta_{\mu\nu}p_1\beta p_2\alpha - 2\eta_{\alpha\nu}p_1\beta p_2\mu 
+A.4 \right. 
\left. - 2\eta_{\alpha\nu}p_1\beta p_2\mu - 2\eta_{\beta\mu}p_1\alpha p_2\nu + \eta_{\alpha\nu}p_1\beta p_2\alpha - 2\eta_{\alpha\nu}p_1\beta p_2\mu \right].$$

The diagram contains two propagators in the denominator and four momenta in the numerator, two of which are external. Thus the leading correction will be, again, of order (A.2) times the terms among which $R\phi^2$ exists. Note that besides the conformal mass term, there are also other higher dimensional operators, whose explicit coefficients could be obtained by exactly calculating the amplitudes. For example, from usual tensorial analysis, terms like $R^{\mu\nu}\partial_\mu \phi \partial_\nu \phi$ are expected to be generated from such one loop diagrams. However, all these terms are suppressed by extra powers of the cutoff and also by slow-roll parameters in an inflationary background.

The other point which is worth mentioning is that the existence of higher order self-interactions for the scalar fields will not disturb our argument. For example inclusion of mass term in the potential for the inflaton, will modify the scalar field propagator and also add corrections proportional to $M_a^2/M_{pl}$ in the $\hat{h} - \phi - \phi$ vertex. Such terms would at most introduce correction of order $\frac{M_a}{\Lambda_{dressed}} \left( \frac{M_a}{M_{pl}} \right)^2 \frac{1}{M_{pl}}$ to the leading contribution (A.2). Thus, as long as $M_a \lesssim \Lambda_{dressed} \ll M_{pl}$, the effect of such terms are very small. Other forms of potential for the scalar field induce vertices that lead to more loops whose effect is more suppressed in comparison with the one-loop diagrams.
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