On Noncommutative Black Holes Thermodynamics

Mir Faizal,¹ R. G. G. Amorim,²,³,∗ and S. C. Ulhoa²,⁴,†

¹Department of Physics and Astronomy, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada.

²Instituto de Física, Universidade de Brasília, 70910-900, Brasília, DF, Brazil.

³Faculdade Gama, Universidade de Brasília, Setor Leste (Gama), 72444-240, Brasília-DF, Brazil.

⁴International Center of Condensed Matter Physics, Universidade de Brasília, 70910-900, Brasília, DF, Brazil.

Abstract

In this paper, we will analyze noncommutative deformation of the Schwarzschild black holes and Kerr black holes. We will perform our analysis by relating the commutative and the noncommutative metrics using an Moyal product. We will also analyze the thermodynamics of these noncommutative black hole solutions. We will explicitly derive expression for the corrected entropy and temperature of these black hole solutions.

∗ronniamorim@gmail.com
†sc.ulhoa@gmail.com
‡mirfaizalmir@gmail.com
I. INTRODUCTION

It is important to study black hole thermodynamics as the entropy of the universe be spontaneously reduce by an object crossing the horizon, if we do not associate the entropy with a black hole. This will also violate the second law of thermodynamics [1, 2]. Thus, black holes are proposed to be maximum entropy objects, i.e., black holes have more entropy than any other object of the same volume [3, 4]. The maximum entropy of black holes is proportional to the area of the black hole [5], and this observation has led to the development of the holographic principle [6, 7]. In fact, the precise form of the area-entropy relation for black holes can be written as \( S = A/4 \), where \( S \) is the entropy of the black hole and \( A \) is the area. However, this area-entropy relation is expected to get corrected as the black holes get in size due to Hawking radiation. Thus, the quantum fluctuations are expected to correct the standard this relation area-entropy of a black hole, when they get very small. Thus, at small scales it is expected that the black hole thermodynamics will get modified, and this is also expected to modify the holographic principle [8, 9]. It may be noted that such corrections have been derived using various approaches, such as the non-perturbative quantum general relativity approach [10], the Cardy formula [11, 12], the partition function has been computed for BTZ black holes [13], and string theoretical effects [14–18]. It may be noted that the noncommutativity induces corrections terms for the thermodynamic quantities in the black hole thermodynamic [19–22]. The effect of noncommutativity is usually studied by replacing point-like structures in the original theory by objects which are smeared in spacetime. This is done by replacing the delta function by a Gaussian distribution with minimal width. The width of this Gaussian distribution is fixed by the noncommutative parameter.

The noncommutativity mixes ultraviolet and infrared divergences [23]. It also incorporates non-locality in a controllable way [24]. It has been studied in the context of string theory. This is because it is known that the transverse coordinates of D-branes can be regarded as matrices, and these matrices do not commute [25]. Noncommutativity also occurs in the context of M-theory and [26, 27]. In this analysis the the compactification on the noncommutative torus has been studied. It has been demonstrated that deforming the commutative torus to the noncommutative torus corresponds to have a constant background three form potential. In fact, there are two commutative tori associated with a noncommutative torus, one to its odd and one to its even cohomology, leading to two commuting
actions on the Teichmuller space. Noncommutative geometry has also been studied in the context of open strings ending on the D-branes. In this context gauge theories on noncommutative tori will appear as D-brane world volume theories [28]. The D0-branes in type IIA string theory with a background two form field have also been studied, and it has been observed that the background two form field modifies the replacing ordinary multiplication by a noncommutative product [29]. It has been demonstrated that there is a link between ordinary Yang-Mills theory and ordinary gauge theory [35]. In fact, a relation between the noncommutative instantons and the ordinary instantons for Yang-Mills theory has also been observed [30]. The relation between instantons on branes and the noncommutative Yang-Mills theory has also been observed [31]. In fact, the noncommutative instanton on the torus [32] and the monopole in the noncommutative $U(2)$ Yang-Mills theory [33, 34] have also been studied. It has been demonstrated that the $U(1)$ effective action for branes is the Dirac-Born-Infeld action [36], and the BPS condition of the ordinary Dirac-Born-Infeld action and a noncommutative action are equivalent in a limit $\alpha' \to 0$ [35].

It may be noted that string theory also can give rise to noncommutative gravity [37]. In this analysis, by studing the next to the leading order terms in the Seiberg-Witten limit, it the dynamics of closed strings in the presence of a constant two form field have been studied. Thus, the gravitational action induced by the bosonic string theory on a space-filling D-brane with a constant magnetic field have been studied in the low energy limit. The induced terms for the interaction vertex of three gravitons on the brane have thus been obtained. It has also been observed that the noncommutative deformations of gravity can lead to a complex metric and in this case the tangent space groups is larger than the Lorentz group [38]. Noncommutativity has also been studied by twisting the diffeomorphism invariance of the general relativity [39]. Finally, it may be noted that the deformed algebra corresponding to commutative deformation has been used to construct a covariant tensor calculus for metric, covarient derivatives, curvature and torsion [40]. So, in this paper, we will use the noncommutative gravity to calculate the corrections to the black hole thermodynamics. This analysis will be different from the earlier analysis where the noncommutativity was used to smeared matter in spacetime by the noncommutative parameter [19, 22].
II. NONCOMMUTATIVE GEOMETRY

In this section, we will review the construction of noncommutative gravity \cite{24, 38–40}. In the noncommutative geometry, spacetime coordinates are promoted to a set of noncommutative self-adjoint operators, such that they satisfy

\[ [x^\mu, x^\nu] = i\theta^{\mu\nu}, \]  

(1)

where $\theta^{\mu\nu}$ is a antisymmetric matrix. The product of two fields on this noncommutative spacetime can be replaced by a Moyal product of commutative fields, where the Moyal product is given by \cite{41}

\[ f(x) \star g(x) = \exp \left[ \frac{i}{2} \theta^{\mu\nu} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x^\nu} \right] f(x) g(x + \beta) \big|_{\alpha=\beta=0} \]

\[ = f(x) g(x) + \frac{i}{2} \theta^{\mu\nu} \partial_\mu f(x) \partial_\nu g(x) + \mathcal{O}(\theta^2). \]  

(2)

It may be noted that the noncommutative field theories are nonlocal as the Moyal product of fields involves an infinite number of derivatives. Furthermore, the as the spacetime coordinates do not commute, the noncommutativity gives rise to a minimum length in spacetime

\[ \Delta x^\mu \Delta x^\nu \geq \frac{1}{2} |\theta^{\mu\nu}|. \]  

(3)

We can employ the Weyl quantization procedure \cite{42, 43} to relate the metric tensor on noncommutative spacetime $\hat{g}_{\mu\nu}$ with the metric on commutative spacetime $g_{\mu\nu}$. Now the Fourier transform of the metric is given by

\[ \tilde{g}_{\mu\nu}(k) = \frac{1}{(2\pi)^2} \int d^4x \exp(-ik_\sigma x^\sigma) g_{\mu\nu}(x), \]  

(4)

so, we can write

\[ \hat{g}_{\mu\nu}(\hat{x}) = \frac{1}{(2\pi)^2} \int d^4k \exp(ik_\sigma \hat{x}^\sigma) \tilde{g}_{\mu\nu}(k). \]  

(5)

Thus, the operators $\hat{g}_{\mu\nu}$ and $\hat{x}$ replace the variables $g_{\mu\nu}$ and $x$. Now let us define two tensor fields on this noncommutative spacetime as $\hat{f}_{\mu\nu}$ and $\hat{g}_{\mu\nu}$ as \cite{42},

\[ f(x)_{\lambda\sigma} \star g(x)_{\tau\rho} = \frac{1}{(2\pi)^4} \int d^4kd^4p \exp[i(k_\mu + p_\mu)x^\mu - \frac{i}{2} k_\mu \theta^{\mu\nu} p_\nu] \]

\[ \times \tilde{f}_{\lambda\sigma}(k) \tilde{g}_{\tau\rho}(p) \]

\[ = \exp \left[ \frac{i}{2} \frac{\partial}{\partial x^\mu} \theta^{\mu\nu} \frac{\partial}{\partial y^\nu} \right] f_{\lambda\sigma}(x) g_{\tau\rho}(y) \big|_{y \rightarrow x}. \]  

(6)
where \( \tilde{f}_{\lambda\sigma}(k) \) is the Fourier transform
\[
\tilde{f}_{\lambda\sigma}(k) = \frac{1}{(2\pi)^2} \int d^4 x \exp(-ik_{\sigma}x^\sigma)f_{\lambda\sigma}(x).
\] (7)

In presence of matter the situation with regards to noncommutativity of spacetime coordinates can be imposed on the vierbeins as
\[
g_{\mu\nu} = e^a_{\ (\mu} \epsilon^b_{\ \nu)} \eta_{ab}.
\] (8)

It may be noted that this noncommutative metric preserves the \( SO(3,1) \) symmetry, and induces a deformed diffeomorphism group. There exists a map between this deformed diffeomorphism group and the original diffeomorphism group \[44\].

### III. NONCOMMUTATIVE SCHWARZSCHILD BLACK HOLE

Now we can write the original metric for a Schwarzschild solution as
\[
ds^2 = -\left(1 - \frac{2M}{r}\right)dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),
\] (9)
where \( M \) is the mass of the black hole. Furthermore, we can write the tetrad field as
\[
e^a_{\mu} = \begin{bmatrix}
\sqrt{-g_{00}} & 0 & 0 & 0 \\
0 & \sqrt{g_{11}} \sin \theta \cos \phi & \sqrt{g_{22}} \cos \theta \cos \phi & -\sqrt{g_{33}} \sin \phi \\
0 & \sqrt{g_{11}} \sin \theta \sin \phi & \sqrt{g_{22}} \cos \theta \sin \phi & \sqrt{g_{33}} \cos \phi \\
0 & \sqrt{g_{11}} \cos \phi & -\sqrt{g_{22}} \sin \theta & 0
\end{bmatrix}.
\] (10)

We will now consider the case where the non-vanishing component of \( \theta^{\mu\nu} \) are \( \theta^{23} \) and \( \theta^{32} \). So, we choose the following value for the noncommutative coordinate
\[
\theta^{\mu\nu} = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & \beta \\
0 & -\beta & 0
\end{bmatrix}.
\] (11)
Using this value we can demonstrate that the metric tensor will receive the following corrections,

\[ \tilde{g}_{00} = g_{00}, \]  
\[ \tilde{g}_{11} = g_{11} + \frac{1}{4} \beta^2 g_{11} \cos(2\theta), \]  
\[ \tilde{g}_{22} = g_{22} - \frac{1}{4} \beta^2 g_{22} \cos(2\theta), \]  
\[ \tilde{g}_{33} = g_{33} + \frac{\beta^2}{8} \left( \frac{\partial^2 g_{33}}{\partial \theta^2} \right), \]  
\[ \tilde{g}_{12} = -\left( \frac{\beta^2 \sqrt{g_{11}g_{22}} \sin(2\theta)}{4} \right), \]

We also will obtain the following result,

\[ \sqrt{-\tilde{g}} = \sqrt{-g} + \beta^2 \sqrt{\frac{-g_{00}g_{11}g_{22}}{16g_{33}}} \frac{\partial^2 g_{33}}{\partial \theta^2}. \]  

Now we can calculate the entropy of the noncommutative Schwarzschild black hole and its temperature. The entropy of the noncommutative Schwarzschild black hole is given by as a consequence we have

\[ \hat{S}(r_{+}) = \left( 1 - \frac{\beta^2}{4} \right) S(r_{+}). \]  

where \( S_{+} = A/4 \) is the entropy of the original commutative Schwarzschild black hole, and \( \hat{S}(r_{+}) \) is the entropy of the noncommutative Schwarzschild black hole. The temperature is defined to be

\[ T^{-1} = \frac{\partial \hat{S}}{\partial M}. \]  

Thus, we can write the temperature of the noncommutative Schwarzschild black hole as

\[ T = \frac{1}{8\pi M} \left( 1 + \frac{\beta^2}{4} \right). \]  

As we can see the noncommutativity modifies the thermodynamics of the original Schwarzschild black hole.

**IV. NONCOMMUTATIVE KERR BLACK HOLE**

In this section, we will study the thermodynamics of a noncommutative Kerr black hole. The general form of the line element of a spacetime with axial symmetry can be written as

\[ ds^2 = g_{00}dt^2 + 2g_{03}d\phi dt + g_{11}dr^2 + g_{22}d\theta^2 + g_{33}d\phi^2, \]  

where
where the metric components are functions of $r$ and $\theta$. Now we can write the tetrad field as

$$\boldsymbol{e}^a_\mu = \begin{bmatrix} \sqrt{-g_{00}} & 0 & 0 & -\frac{g_{03}}{\sqrt{-g_{00}}} \\ 0 & \sqrt{g_{11}} \sin \theta \cos \phi & \sqrt{g_{22}} \cos \theta \cos \phi & -\frac{\sqrt{\delta}}{\sqrt{-g_{00}}} \sin \phi \\ 0 & \sqrt{g_{11}} \sin \theta \sin \phi & \sqrt{g_{22}} \cos \theta \sin \phi & \frac{\sqrt{\delta}}{\sqrt{-g_{00}}} \cos \phi \\ 0 & \sqrt{g_{11}} \cos \theta & -\sqrt{g_{22}} \sin \theta & 0 \end{bmatrix}, \quad (22)$$

where $\delta = -g_{33}g_{00} + g_{03}^2$. We will again only consider the case where the non-vanishing component of $\theta^{\mu\nu}$ are $\theta^{23}$ and $\theta^{32}$. Thus, we can write,

$$\theta^{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta \\ 0 & 0 & -\beta & 0 \end{bmatrix}, \quad (23)$$
The non-vanishing components of the corrected metric are given by

\[ \tilde{g}_{00} = g_{00}, \]
\[ \tilde{g}_{03} = g_{03}, \]
\[ \tilde{g}_{11} = g_{11} + \left( \frac{\beta^2}{8} \right) \left[ 4 \sin \theta \cos \theta \left( \frac{\partial g_{11}}{\partial \theta} \right) \right. \]
\[ \left. + \left( \frac{\partial^2 g_{11}}{\partial \theta^2} \right) \sin^2 \theta + 2g_{11} \left( \cos^2 \theta - 1 \right) \right], \]
\[ \tilde{g}_{12} = -\left( \frac{\beta^2}{32g_{11}^{3/2}g_{22}^{3/2}} \right) \left[ -2g_{11}g_{22} \left( \frac{\partial g_{11}}{\partial \theta} \right) \left( \frac{\partial g_{22}}{\partial \theta} \right) \sin \theta \cos \theta \right. \]
\[ \left. -2g_{11}^2 \left( \frac{\partial^2 g_{22}}{\partial \theta^2} \right) \sin \theta \cos \theta + g_{11}^2 \left( \frac{\partial g_{22}}{\partial \theta} \right)^2 \sin \theta \cos \theta \right. \]
\[ \left. -8g_{11}g_{22} \left( \frac{\partial g_{11}}{\partial \theta} \right) \cos^2 \theta + 4g_{11}g_{22} \left( \frac{\partial g_{22}}{\partial \theta} \right) \right. \]
\[ \left. + 16g_{11}g_{22} \sin \theta \cos \theta + 4g_{11}g_{22} \left( \frac{\partial g_{11}}{\partial \theta} \right) \right. \]
\[ \left. + g_{22} \left( \frac{\partial g_{11}}{\partial \theta} \right)^2 \sin \theta \cos \theta \right. \]
\[ \left. - 2g_{22}^2 g_{11} \left( \frac{\partial^2 g_{11}}{\partial \theta^2} \right) \sin \theta \cos \theta - 8g_{22}^2 g_{11} \left( \frac{\partial g_{22}}{\partial \theta} \right) \cos^2 \theta \right], \]
\[ \tilde{g}_{22} = g_{22} - \left( \frac{\beta^2}{8} \right) \left[ 4 \sin \theta \cos \theta \left( \frac{\partial g_{22}}{\partial \theta} \right) - \left( \frac{\partial^2 g_{22}}{\partial \theta^2} \right) \cos^2 \theta \right. \]
\[ \left. + 2g_{22} \left( \cos^2 \theta - 1 \right) \right], \]
\[ \tilde{g}_{33} = g_{33} + \left( \frac{\beta^2}{8g_{00}^3} \right) \left\{ 4g_{00}g_{03} \left( \frac{\partial g_{00}}{\partial \theta} \right) \left( \frac{\partial g_{03}}{\partial \theta} \right) \right. \]
\[ \left. -2g_{00} \left( \frac{\partial g_{03}}{\partial \theta} \right)^2 - 2g_{03} \left( \frac{\partial g_{00}}{\partial \theta} \right)^2 \right. \]
\[ \left. + g_{00}g_{03} \left( \frac{\partial^2 g_{00}}{\partial \theta^2} \right) - g_{03}g_{00} \left( \frac{\partial^2 g_{03}}{\partial \theta^2} \right) + g_{00} \left( \frac{\partial^2 g_{33}}{\partial \theta^2} \right) \right\}. \] (24)

It will be useful to define, \( \Delta g_{\mu \nu} = \tilde{g}_{\mu \nu} - g_{\mu \nu} \). This is because using this definition, we can write

\[ \tilde{S} = \frac{1}{4} \int \int \sqrt{-g} \left[ 1 + \frac{1}{2} \left( \frac{\Delta g_{11}}{g_{11}} + \frac{\Delta g_{22}}{g_{22}} \right) - \frac{g_{00}\Delta g_{33}}{2\beta} \right] d\theta d\phi \] (25)

The metric for the Kerr black hole can be written as

\[ ds^2 = -\frac{\psi^2}{\rho^2} dt^2 - 2\frac{\chi \sin^2 \theta}{\rho^2} dtd\phi + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{\Sigma \sin^2 \theta}{\rho^2} d\phi^2 \] (26)
where
\[
\begin{align*}
\Delta &= r^2 + a^2 - 2mr , \\
\rho^2 &= r^2 + a^2 \cos^2 \theta , \\
\Sigma^2 &= (r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta , \\
\psi^2 &= \Delta - a^2 \sin^2 \theta , \\
\chi &= 2amr .
\end{align*}
\] (27)

Now we can use the standard procedure to calculate the corrections to the entropy of the Kerr black hole from noncommutativity. Thus, if $\tilde{S}(r_+)$ is the entropy of the noncommutative Kerr black hole, then we can write,
\[
\tilde{S}(r_+) = \pi r_+^2 + \frac{\pi \beta^2}{8} \left[ \frac{1}{2} \frac{r_+^2 + a^2}{r_+^2 + a^2 \cos^2 \theta} \int_{\theta_0}^{\pi} \frac{\cos(2\theta)d\theta}{\sin \theta} \right] \\
+ a^4 \int_{\theta_0}^{\pi} \frac{(4 \cos^2 \theta - 1) \cos^2 \theta d\theta}{\sin \theta \left( r_+^2 + a^2 \cos^2 \theta \right)} .
\] (28)

We can use the expressions
\[
\int \frac{\cos(2\theta)d\theta}{\sin \theta \left( r_+^2 + a^2 \cos^2 \theta \right)} = \frac{1}{2(a^2 + r_+^2)} \ln \left( \frac{\cos \theta - 1}{\cos \theta + 1} \right)
\] (29)
and
\[
\int \frac{(4 \cos^2 \theta - 1) \cos^2 \theta d\theta}{\sin \theta \left( r_+^2 + a^2 \cos^2 \theta \right)} = \frac{4 \cos \theta}{a^2} - \frac{r_+^2}{a^2 r_+^2 + 1} \ln \left( \frac{\cos \theta - 1}{\cos \theta + 1} \right)
\] (30)

Thus, we obtain an expression for the entropy of the noncommutative Kerr black hole,
\[
\tilde{S}(r_+) = S(r_+) - \frac{\pi \beta^2}{8} \left\{ 8a^2 + \left[ \frac{a^2 + 2a^2 r_+^2 - 3a^2 r_+^2 + 2a^2}{ar_+^2 (a^2 + r_+^2)} \right] \right\} \times \tan^{-1} \left( \frac{a}{r_+} \right)
\] (31)
where $S(r_+)$ is the entropy for the commutative black hole. Finally, we can write for the noncommutative Kerr black hole

$$\frac{\partial r_+}{\partial M} = \frac{r_+}{r_+ - M}, \quad (32)$$

$$\frac{\partial \tilde{S}}{\partial r_+} = 2\pi r_+ + \frac{\pi \beta^2}{8} \left\{ \frac{(a^6 + 2a^4r_+^2 - 3a^2r_+^4 + 2r_+^6)}{r_+(a^2 + r_+^2)^2} \right\}$$

$$+ \left[ \frac{\Phi}{ar_+^2(a^2 + r_+^2)^2} \right] \tan^{-1} \left( \frac{a}{r_+} \right), \quad (33)$$

where $\Phi = a^6(a^2 + 3r_+^2) - 2a^4r_+^2(a^2 - r_+^2) + 3a^2r_+^4(3a^2 + r_+^2) - 2r_+^6(5a^2 + 3r_+^2)$. So the temperature for the noncommutative Kerr black hole can be written as

$$T^{-1} = \left( \frac{\partial \tilde{S}}{\partial r_+} \right) \left( \frac{\partial r_+}{\partial M} \right),$$

$$= \left[ \frac{r_+}{r_+ - M} \right] \left\{ 2\pi r_+ + \frac{\pi \beta^2}{8} \left\{ \frac{(a^6 + 2a^4r_+^2 - 3a^2r_+^4 + 2r_+^6)}{r_+(a^2 + r_+^2)^2} \right\} \right\}$$

$$+ \left[ \frac{\Phi}{ar_+^2(a^2 + r_+^2)^2} \right] \tan^{-1} \left( \frac{a}{r_+} \right). \quad (34)$$

Thus, we have obtained an expression for the corrections to the thermodynamics of Kerr black holes. It may be noted in the commutative limit these corrections vanish, and we obtain the original case back.

V. CONCLUSION

In this paper, we will studied the noncommutative Schwarzschild black holes and the noncommutative Kerr black holes. This was done by first using Weyl quantization procedure [42, 43] to relate the metric tensor on noncommutative spacetime with the metric on commutative spacetime. The relations thus derived were used specifically to obtain expressions for the corrections to the Schwarzschild metric and the Kerr black metric due to noncommutativity. Furthermore, as we analyzed the thermodynamics of these noncommutative black holes. Thus, explicit expressions for the corrected entropy and temperature of these black hole solutions were obtained. These corrections vanished when the noncommutative parameter is set to zero, and hence, we derive the original commutative results back. It may be noted that this work take a different approach from the earlier works where the effect of noncommutativity has been studied by replacing point-like structures in the original
theory by objects which are smeared in spacetime [19–22]. It will be interesting to analyze other black objects using this approach [45, 46]. In higher dimensions, interesting solutions to the general relativity exist which have interesting topologies. It is possible for black rings and black saturns to exist in higher dimensions. It will be interesting to analyze the thermodynamics of such solutions using noncommutative formalism. It has also been demonstrated that noncommutativity leads to the existence of a minimum length in spacetime. The black hole thermodynamics for minimum length have been analyzed [47–50]. It will be interesting to analyze a possible link between the thermodynamics of black holes with minimum length and noncommutative black holes.

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