How are the process of abstraction of the division of fraction numbers by elementary school students?

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Abstract: The division of fractions is one of the difficult materials for elementary students. That is a concept that they must understand. Therefore, the abstraction process is needed to achieve it. The purpose of this study is to describe the abstraction process of dividing fractions by elementary school students. The subjects of this study were elementary school students in Bengkulu City, Indonesia. The subjects of this study were chosen 3 out of 85 elementary school students in Bengkulu City, Indonesia. There are two story questions submitted to research subjects about the distribution of mineral water after playing soccer. Subjects were interviewed in depth. That is a task-based interview. Data were analyzed through genetic decomposition. The result is students can be more meaningful in preparing propositions about fraction operations. Students can state that the division of fractions is a repetitive reduction. By providing contextual problems students can complete the process of abstraction, and can reach the concept of fraction division appropriately.

1. Introduction
Fraction is one of the most difficult topics for learning mathematics [1]. Findings in mathematics learning, there are students who misconceptions about the concept of fractions. Such misconceptions are found, such as 6/3 are fractions. But when asked "are 2 fractions?", he answered not fractions. This shows the inconsistency of the existing conception in the cognitive process, when mentioning 6/3 fractions, and when asked about 2 which is equal to 6/3, he answers not fractions. The student has the conception that every number that can be written in a / b with a, b integers and b ≠ 0 is a fraction. The concept is that every rational number is a fraction [2]. That is not understanding the true boundaries of fractions. Fraction is an integer in the form of a / b, where a, b is an integer, b ≠ 0, and a ≠ kb, for a k integer [3, 4]. Fractions can also be more than 1, such as 5/3, and 3/2. Therefore, a fraction consists of two integers separated by a bar in between. The numbers located below are called denominators. That is the total number of parts divided equally in one whole. The integers located above are called numerators. The bar represents division operations. It is a different concept from the concepts students have previously learned. Usually students must make accommodations to achieve this concept.

One concept of fractions is a part of the same size as a whole or from a particular unit. It is a fraction concept that is easily understood by students. In life, we often find types of numbers that are not integers, such as measurement, length, height, width, capacity of an object that is between two integers. Also, situations where quantity is shared often require numbers other than integers [5]. Fractions can be part of
the whole; place in the number line; the answer to the calculation of division; or how to compare two sets or sizes [4].

To develop basic concepts of rational numbers, it is necessary to start earlier and maintain continuity for the entire duration of teaching activities [6]. Fractional surgery is a difficult learning topic, one reason for the difficulty of learning fractions is that there are many different properties with integers [1]. In the process of developing students' knowledge about the concept of numbers, arithmetic and algebra are used, although actually the concept of numbers was formed since preschool. It is necessary to develop a more efficient study of the concept of numbers. That is done by linking this process with subtraction and division operations with the set of integers [6]. This technique makes it easy for students to do the process of abstraction about fractions and their operations.

Abstraction ability is one of the abilities students must achieve when learning mathematics. Bruner states that the process of abstraction through three stages of enactive-iconic-symbolic. The results of Widada et al. [7] that students who are at the abstract level are able to use all the statements given to solve problems. He can explain the relationship of the statement given with the argument in solving the problem. Also, explain the statements compiled as a result of existing statements using good arguments and draw conclusions that have been made on paper and pencil. Students are able to reach the abstract level if they go through a learning process that is close to their culture [8, 9]. These students achieve it through learning with an ethnomathematics approach [10, 11].

Learning mathematics through an ethnomathematics approach makes it easy for students to do mathematical representations. At present, very different perspectives on teaching mathematical concepts have been developed and emphasized. This approach states that structuring mathematical knowledge and relating it to real life situations is very important [10]. Students can do a double representation of mathematical relationships as a different entity. These representations contribute to the development of understanding mathematical concepts for students [11]. Learning with ethnomathematics approach provides support for cognitive processes in solving problems and mathematical representation of students is more meaningful. The study showed that students who use visual representations are often more successful in problem solving and that they have positive beliefs about using visual representations in solving verbal mathematical problems [10]. Thus, the local cultural approach as a student's initial model for visual representation can help students carry out mathematical processes and mathematical problem solving. Extra-mathematical connections can reduce the separation between mathematics and reality to give meaning to learning. It provides an understanding of the world through a mathematical perspective and fosters a symbiotic relationship between these two worlds [12], mathematics learning that connects the real world with mathematics [13]. Thus, the focus of the study is to explore the ability of students to understand the division of fraction based on the habit of playing football on the beach.

2. Method

This research is a qualitative research with ethnographic approach. It is part of the research development of a grounded mathematics learning model in Bengkulu, Indonesia [14]. The focus of this research is how the process of abstraction of elementary school students in understanding the division of fractions through local cultural activities. The subjects of this study were selected 3 out of 85 elementary students in Bengkulu City. The 85 students were given an assignment sheet to complete the questions: First: "When finished playing football, Joko bought 1500 ml mineral water. Because he wanted to share mineral water with his friends, Joko poured his mineral water into a 250 ml glass in full. Question: Can you pour 250 ml of glass, 1500 ml of mineral water? Then students are asked to solve the second problem. "When finished playing football, Joko bought 1½ liter mineral water. Because he wanted to share mineral water with his friends, Joko poured his mineral water into ¼-liter glass in full. Question: Can you pour into ¼ liter glass size, 1½ liter mineral water?" We choose the right student answers and describe the achievement of the ability of abstraction about the division of fractions. In this study, there were 3 (three) students who fulfilled these characteristics. The three research subjects are Tg, Jh, and Kt. Selected students were interviewed in depth
by the research team. That is to find out exactly about the actual abstraction process that occurs in the student's thinking system. Data were analyzed qualitatively through the stages of qualitative data analysis. It is a technique of analysis through genetic decomposition of research subjects. Genetic decomposition (or cognitive models) is a structured collection of mental and physical activities that construct blocks (categories) to describe how concepts or principles can be developed in an individual's mind [15].

3. Results and Discussion
Learning about fractions makes it a challenge for elementary students. They find it difficult to understand. Some constraints are about fractions worth and fraction operations, such as 2/3 = 4/6. Analysis of genetic decomposition is done through a collection of mental and physical activities, namely action-process-object-schema [16]. Students perform actions through a reaction to external stimuli received, in the form of expressions in detail the stages that must be carried out. That is the area of the first 2/3 part rectangle, and the second 4/6 part. Students continue with the process of activities from an action to an activity carried out internally (imagined in his mind). This is interiorization in the form of representation, such as Figure 1.

![Figure 1. Representation of fractions of 2/3 = 4/6](image)

Look at Figure 1, visually students can represent that fraction 2/3 is equal to 4/6. That means that 2/3 = 4/6, as an object. By understanding fractions of value students can perform addition operations. One of the things that teachers teach elementary students is to utilize visual representations as can be seen in Figure 2a. In the picture, 2/3 and ½ representations are given.

![Figure 2a. Representation of 2/3 + ½](image)

Students perform actions and processes, that is through their visual representation abilities can state that ½ = 3/6 (see Figure 2b). That is the way students equate the denominator of the two fractions that are operated. In this case students do mental transformation (in the form of cognitive coordination) of a process on a cognitive object, namely ½ = 3/6.

Based on Figure 2b, students carry out additional operations by adding 3/6 to 2/3 as shown in Figure 3a. It seems that part 2/3 has been added to 3/6, which is purple plus blue. That is the process of forming objects to produce a scheme about equalizing the denominator. Students state that the fraction addition operation can be carried out if the denominator is the same.
The representation in Figure 3a can be completed by making 2/3 to 4/6. This process produces all blue representations and 2/3 + ½ = 7/6. It is a construction that links separate actions, processes, or objects to a particular object so as to produce a scheme about adding fraction operations. Thus, students have a coherent system of actions, processes, objects, and other schemes that have been built before. It is coordinated (synthesized) by students in the form of structures used to deal with situations related to fraction addition. The scheme is stored in memory to be recalled when needed. (see Figure 3b).

Through the representation of Figures 2-5, it is easy for students to learn fraction operations visually. But to know quantitatively the level of students' initial ability about fractions is done the initial test. This test is given before students get fraction learning using visual representation, even though they have learned fractions without it.

Based on the results of the initial test about fractions of 85 elementary students in Bengkulu City, Indonesia, most students made mistakes about fractions. These results can be shown in Figure 4.

Based on the test on fractions, Figure 4 shows that 18% of students make mistakes in facts (such as errors about symbols relating to fractions and operations), 21% make concept errors (such as definitions of fractions), 35% make mistakes in principle (such as theorems relating to fractions), there are 18% of students making fractional operations errors and 8% of students making random errors. This shows that the most mistakes about fractions are mistakes in principle, of course this will be related to fraction operations. Like ½ + 1/3 =? There are students who answer 2/5 with the reason that ½ + 1/3 = (1 + 1) / (2 + 3) = 2/5. The following are excerpts of interviews with research subjects (Kt).

Q: How do you complete the addition of ½ + 1/3?
Kt: Yes sir ... the top with the top and the bottom with the bottom ...
Q: What are the results?
Note: This is 2/5
Looking to the information about students' abilities about fractional operations, we have the initiative to improve it. One of its efforts is to make concrete use of spatial representations and activities. Researchers provide learning fraction operations through a visual representation approach. Then give a story problem and ask them to demonstrate it. The question was "When finished playing football, Joko bought 1500 ml mineral water. Because he wanted to share mineral water with his friends, Joko poured his mineral water into a 250 ml glass in full. Question: Can you pour into 250 ml of glass, 1,500 ml of mineral water?" See Figure 5, Joko and his friends playing ball at Pantai Panjang (the long beach). It is a very long beach located in the Bengkulu region.

![Figure 5. Children playing ball at the Pantai Panjang](http://aprilendy.blogspot.com/2015/05/foto-essay-aktivitas-di-pantai-panjang.html)

To answer the question of the story, Tg (a research subject) has prepared a bottle of 1,500 milliliters (ml) mineral water and a 250 ml container. Note that figure 6a and 6b are the intended containers of mineral water.

![Figure 6a. 1.500 ml Mineral Water Bottle](image)

![Figure 6b. Glass of Mineral Water 250 ml](image)

Based on the questions and the water available in the two containers, Tg began distributing water from a 1,500 ml bottle into a 250 ml small container. It turns out that Tg can distribute all the water from a bottle into six 250 ml small containers. He stated that the water in a 1,500 ml bottle could be distributed in six 250 ml sized small containers. In their genetic decomposition, students perform actions, and processes so as to reach an object about the division operation. He is able to do thematization in the form of a scheme that is the division of integers is a repeatable reduction. The genetic decomposition was described in our interview with Tg. Consider the following interview excerpt.

Q: What is the process of sharing from large bottles to small containers?
Tg: I distributed every glass full from the bottle ...
Tg: ... there are 6 cups measuring 250 ml that I gave to 5 of my friends and for me one glass of mineral water ...
Q: What next?
Tg: .... means 1,500-250-250-250-250-250 = 0 and this means 1,500: 250 = 6 ... six glasses ...
Q: What can you conclude?
Tg: ... it is a division as a repetitive reduction ...
Based on the interview excerpt above, the research subjects were able to carry out their duties correctly. He is able to complete each stage correctly. Concretely he can state the concept of division through the distribution of mineral water into small containers. Tg is able to make the conclusion that division is a recurring reduction.

By utilizing the demonstration conducted by Tg, the researcher then triggers students through the next story matter. The problem is a modification of the first problem, which is to replace 1,500 ml with 1½ liters. Please see Figure 4a and 4b. We have given the word problem: "When finished playing football, Joko bought a 1½ liter mineral water. Because he wanted to share mineral water with his friends, Joko poured his mineral water into ¼-liter glass in full. Question: Can you pour into ¼ liter glass size, 1½ liter mineral water?"

Based on the matter of the story, a research subject (Jh) gave a very positive response. He was able to demonstrate it with the analogy that Tg. Jh divide 1½ liters of mineral water into ¼ liter small containers. He produced six small containers that could be filled completely and the water in the large containers was used up. Consider the following interview excerpt.

Q: What can you explain from this activity?
Jh: yes sir ... I can distribute 1½ liters of water into six ¼ liter sized containers ...
Q: how do you conclude ..
Jh: That means that 1½ - ¼ - ¼ - ¼ - ¼ - ¼ - ¼ - ¼ = 0 ... this means that 1½: ¼ = 6 and this is that the division of fractions is a repeatable reduction.

Jh has good analogy skills. That is a representation of genetic decomposition of Tg. A complete activity in the process of achieving the concept of division operations of fraction. Through concrete activities, he was able to show that the operation of dividing fractions is a repeatable reduction. Note that Jh states 1½ - ¼ - ¼ - ¼ - ¼ - ¼ - ¼ - ¼ = 0, so 1½: ¼ = 6. Jh able to reach the concept precisely.

This description provides evidence that the culture-based mathematics learning of students in elementary schools can provide a good cognitive process. Students are able to build knowledge correctly. It makes easier for students to do the mathematical process vertically, so that formal mathematics is obtained. Therefore, learning mathematics with local culture context can increase students' mathematical activities appropriately. The results of the study indicate that ethnomathematics is useful in teaching and learning mathematics [8, 19-20]. The problem solving ability of mathematics students who study with ethnomathematics-oriented YouTube media is higher than students who without ethnomathematics-oriented [18]. Learning mathematics through local cultural approaches can improve students' genetic decomposition [17, 22-23], and overcome the mistakes of mathematical concepts of junior high school students [19]. Ethnomathematics-based mathematics learning also has a positive impact on cognitive processes, mathematical abilities and other mathematical activities [26-29]. Thus, it is recommended to utilize the culture around students as starting points for learning mathematics. One proof is that elementary students are able to reach the concept of fraction sharing through learning based on local culture.

4. Conclusions
Culture is one of the local wisdoms which is the nation's heritage. It becomes something that is always close to the minds of students, therefore utilizing local culture as a starting-point of learning mathematics can make it easier for students to do the process of horizontal mathematical toward vertical mathematical [30-32]. The results of the study concluded that students could be more meaningful in preparing propositions about fractional operations. Students can state that the division of fractions is a repeat reduction. Through giving contextual problems students can solve abstractions starting from symbolic-active-iconic, and can reach the concept of dividing fractions appropriately.

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