Analysis regarding the transport network models. Case study on finding the optimal transport route

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Abstract. Transport networks are studied most of the time from a graph theory perspective, mostly studied in a static way, in order to emphasize their characteristics like: topology, morphology, costs, traffic flows etc. There are many methods used to describe these characteristics at local and global level. Usually when analysing the transport network models, the aim is to achieve minimum capacity transit or minimum cost of operating or investment. Throughout this paper we will get an insight into the many models of the transport network that were presented over the years and we will try to make a short analysis regarding the most important ones. We will make a case study on finding the optimal route by using one of the models presented within this paper.

1. Introduction
Indispensable element for the socio-economic life, through the advantages offered to the company both in respect to travel possibilities and to those specific to the social life (communication, assimilation and understanding of modern civilization), the transport is the final result which is get in pursuance of the activity carried out by a complex system represented by the transport system. Such as any system having in view the realization of a common purpose, the transport system tries to follow the requirements of the involved parties, the elements which represent the base of their realization (means of conveyance, an adequate infrastructure and the necessary equipment), but also the way in which these interact in order to finally get the predetermined performances [1].

Generally, the analysis of transport systems cannot be realized without a detailed analysis of transport networks. Nowadays, in land development, multiple networks interfere, which impose an analysis of their fundamental role within underserved territories, regarding different ways of territorial organization and synchronization, but especially regarding the transport techniques [1]. Transport networks shall be seen not only in terms of flows existing along them. Through the rational use of the area, these ensure connections/relationships between technical capacities and the services that are delivered within the territory.

2. Overview regarding the transport network
A general analysis of a transport network shows that it ensures the freight transport between the shipping point and the destination point, through links (graphically represented under the form of some arches) which connect the network nodes (beneficiaries and terminals – ensuring additional sorting, consolidation, storage or transhipment activities). The networks’ modelling is realized by connecting sources with their related transhipment nodes through an increasing number of links.
As Raicu and Roșca (2006) highlight in their paper [2], the standard definition of a transport network (R) presents it as an undirected graph (a set of nodes and links, at which the crossing direction of the links has no importance) G[R], whose representation is made through its sets of nodes and links:

\[ R = (V, A) \]  \hspace{1cm} (1)

where:

- \( V = \{v_0, v_1, \ldots, v_n\} \) which represents the set of nodes: considering the transport terminal \( v_0 \) and the recipients \( v_1, \ldots, v_n \);

- \( A = \{(v_i, v_j) / i \neq j; \ v_i, v_j \in V\} \) which represents the set of links of the graph.

Ellanti, Gorshe, Raman and Grover (2005) mention in their book [3] that the analysis of networks shall consider both their division depending on the geographical position and on the functions realized by them, but also taking into consideration three fundamental criteria: transport, control and management. In this way, one can easily assert that networks represent basic principles of our society, allowing the study and the optimization of some essential issues for modern society, as are those regarding: external costs, urban expansion, diseases dispersal or development of some regions [4].

Thus, the main elements composing a transport network are [5]:

- the transport node: represented by any location which, in terms of its location, allows the access to a transport network;
- the related arches: transport infrastructure ensuring the connection between two nodes;
- the optimal routes which can exist between two nodes;
- the flow of goods/ passengers existing on the levels of nodes or arches.

3. Transport network modelling

Both the quantitative analysis of a system, in general, and the quantitative analysis of a transport network are realized by means of some conceptual models which shall be similar to the real model, in a great measure. The performance on the level of these models is determined by the help of an objective function or by means of two categories of variables, needed to be considered when a model is realized. The decisional variables are the basic elements which contribute to making a decision from a set of variants, these being fully controllable. In contrast with them, the condition variables are not controllable, they depend on the decisional variables and describe the condition of an element within the model [6].

When it is realized the transport network modelling, there are major difficulties because of multiple criteria taken into consideration in the designing process. Thus, the specialty literature presents a series of syntheses ensuring the modelling and the mathematical formalization of networks by taking into account some optimality criteria. In this way, aiming to get minimum transit capacity of minimum operational or investment cost, the transport network modelling can be classified from the following points of view ([7, 8]):

- topologically, when it is aimed the changing of its structure or the introduction of some new elements in the network;
- of its evolution in time, allowing the differentiation of models depending on the introduction of time parameter between the network attributes;
- of the incertitude degree existing in the network, in relation to the certitude which makes the parameters known;
- of the cost function specific to the network.
The modelling of freight transport networks allows the procurement of some models which, according to the specialty literature, can have the following general structure [9]:

1. determination of the freight volume related to the network;
2. graphical representation of the network by determining both the shipping and the destination places;
3. choosing the transport ways which will be used on the network level;
4. determination of the number of travels done with the means of transport on the level of the network taking into consideration the tons of goods transported and their allocation on the level of the network matrix;
5. calibration and validation of the model.

Ducruet and Lugo (2013) point out in their paper [10] that the analysis of the transport network features shall be realized on global level (by the description of the network as a whole) and on local level (by the analysis of one or more nodes individually). The table below presents the methods used both on global level and on local level for the description of complex transport networks.

| Method adopted at global level | Definition | Formula |
|-------------------------------|------------|---------|
| Average shortest path length  | Average number of stops between two nodes of the transport network | \( l_c = \frac{1}{n(n-1)\sum_{i,j} d(\theta_i, \theta_j)} \) |

| Method adopted at local level | Definition | Formula |
|-------------------------------|------------|---------|
| Degree centrality             | Number of adjacent nodes | \( k_i = c_D(i) = \sum_{j=1}^{N} x_{ij} \) |
| Shimbel Index                 | Sum of the length of all shortest paths connecting all other nodes in the graph | \( A_j = \sum_{i=1}^{N} d_{ij} \) |
| Eccentricity of a node        | Number of links needed to reach the most distant node in the graph | \( e(x) = \max_{y \in X} d(x, y) \) |

Source: [10]

On global level, the structure of transport networks is influenced by physical constraints imposed by the infrastructure, but also by the transport means. In this case, the literature presents more methods which aim the definition of the network characteristics such as: the graphs method, the minimum distance method (determine the route which ensures the connection of all nodes with minimum costs, method which measures the network efficiency) [11], geomorphometry (it analyse the fluvial transport networks) etc.

On local level, it is aimed the determination of the relative position of a node or the identification of a group of nodes within the network. Thus, it can be followed the determination of the eccentricity of a node, of the position towards the centre of a node, the nodality index Shimbel etc.

Over the years several models have been developed in order to increase the general performance of the intermodal transport chain. The designed, analysed and presented models aimed at optimization, simulation and establishing a transport system network [12].
In 1990, Crainic, Florin and Leal [13] have developed a model that sought to minimize the total cost of transport system through optimal allocation of transport flows by mode and different routes. The model is integrated within the STAN software, which is used by many countries for national transport planning. Subsequently, in 2001, Jourquin and Beuthe [14] have developed another transport network model, NODUS system, which aimed to analyze the intermodal transport networks by choosing modes and routes that reduce the total cost of transport.

Regarding the optimization models, the most interesting pure optimization model is considered the one developed by Nozick and Morlok [15], the TOFC model. It is a tactical model for planning the operations that are developed at intermodal transport network by minimizing the intermodal transport cost of a known flow of intermodal transport units-ITU (ensuring compliance with delivery schedules and optimum use of empty semi-trailers and swap-bodies).

Another heuristic model that tried to determine the most efficient operating procedure of an intermodal transport system, was the one conducted by Jensen [16]. The Jensen model shows:
- the transport cost reduction that is obtain by transferring goods from unimodal transport system on intermodal system (road/rail) in order to deliver them door-to-door;
- the increased flow traffic around the terminals.

Later on, in 2007, Flodén [12] developed within is paper the HIT Model (the Heuristics Intermodal Transport Model) that was based on Jensen model (by objectives and features). The HIT model provides information regarding the intermodal transport system on costs, external impact or overall performance. For its implementation are taken into account: the amount of goods that will be transported, the transport connections, the transport routes and the rail transport means used within the transport route.

4. Case study on finding the optimal route

The modelling of a unimodal transport network is usually realized through an equilibrated transport problem. Taking as an example the maritime transport problem, studies have shown that these are modelled through a problem with at least four indices with consider, especially, the economic criteria. The problem is presented below, and it can be more complex when it is transformed in a strict transport problem [6].

It is important to mention that on the level of unimodal maritime transport network, the unitary cost \( C_{mijn} \) differs depending on more attributes such as:
- the type of transported goods \( m \);
- the type of ship used \( n \);
- the loading \( P_i \) and the unloading ports \( P_j \) which determine the distance and the duration of the transport.

In this way, the model follows to get a minimum transport cost, for which one shall determine the quantities of goods \( x_{mijn} \) of type \( m \) transported with a ship of type \( n \), on the route \( i,j \). In this way, it shall be minimalized the amount:

\[
\sum_{m=1}^{p} \sum_{i=1}^{q} \sum_{j=1}^{r} \sum_{n=1}^{s} C_{mijn} x_{mijn}
\]

On the level of this model, the problem is equilibrated if the quantity of goods requested is equal to the quantity to be transported from the loading port.

Using the Bellman Kalaba’s algorithm, one can determine the optimum route in a transport network aiming to get a total minimum cost. In this way, it is considered a maritime transport network (presented in the graphic below), where the ship \( n \), which transports the good \( m \) loaded in the port \( P_1 \) shall reach in the destination port \( P_z \) having at its disposal the ports \( P_2 - P_5 \) met in time.
Applying the Bellman Kalaba’ algorithm, it is formed and filled the matrix of the direct distances in order to determine the optimal route from the transport network, respectively \( P_1 \rightarrow P_2 \rightarrow P_4 \rightarrow P_6 \).

### Table 2. The direct distance matrix.

|       | \( P_1 \) | \( P_2 \) | \( P_3 \) | \( P_4 \) | \( P_5 \) | \( P_6 \) |
|-------|-----------|-----------|-----------|-----------|-----------|-----------|
| \( P_1 \) | 0         | 30        | 60        | 90        | \( \infty \) | \( \infty \) |
| \( P_2 \) | \( \infty \) | 0         | \( \infty \) | 50        | \( \infty \) | \( \infty \) |
| \( P_3 \) | \( \infty \) | \( \infty \) | 0         | \( \infty \) | 40        | 100       |
| \( P_4 \) | \( \infty \) | \( \infty \) | \( \infty \) | \( \infty \) | \( \infty \) | 50        |
| \( P_5 \) | \( \infty \) | \( \infty \) | \( \infty \) | \( \infty \) | 0         | 70        |
| \( P_6 \) | \( \infty \) | \( \infty \) | \( \infty \) | \( \infty \) | \( \infty \) | 0         |
| \( m_1 \) | \( \infty \) | \( \infty \) | 100       | 50        | 70        | 0         |
| \( m_2 \) | 140       | 100       | 100       | 50        | 70        | 0         |
| \( m_3 \) | 130       | 100       | 100       | 50        | 70        | 0         |
| \( m_4 \) | 130       | 100       | 100       | 50        | 70        | 0         |

### 5. Conclusions
The models used both in transports and in logistics for modelling, designing and using the transport networks shall ensure the guidance of flows of travellers or goods along the network of nodes and arches, between the shipping point and the destination point. The literature has always analysed the modelling process of transport networks, through these allowing the efficiency and the productivity, as it could be also noticed in the analysed example. The structure of the transport network is an important element, because on it depend the technical-functional features, the different models analysed and used having an essential role for determining these features.

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