Scalar form factor of the pion in the Kroll-Lee-Zumino field theory

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The renormalizable Kroll-Lee-Zumino field theory of pions and a neutral rho-meson is used to determine the scalar form factor of the pion in the space-like region at next-to-leading order. Perturbative calculations in this framework are parameter free, as the masses and the rho-pion-pion coupling are known from experiment. Results compare favorably with lattice QCD calculations.

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The scalar form factor of the pion \[1\], and particularly its quadratic radius, plays an important role in chiral perturbation theory (CHPT) \[2\]. This form factor is defined as the pion matrix element of the QCD scalar current \(J_S = m_u \bar{u}u + m_d \bar{d}d\), i.e.

\[F_S(q^2) = \langle \pi(p_2)|J_S|\pi(p_1)\rangle,\]

where \(q^2 = (p_2 - p_1)^2\). The associated quadratic scalar radius is given by

\[F_S(q^2) = F_S(0)\left[1 + \frac{1}{6}(\sigma^2)S q^2 + \ldots\right],\]

where \(F_S(0)\) is the pion sigma term

\[F_S(0) = \sigma_\pi = m_q \frac{\partial M^2_\pi}{\partial m_q}.\]

The scalar radius fixes \(\ell_4\), one of the low energy constants of CHPT, through the relation

\[\langle r^2 \rangle = \frac{3}{8\pi^2 F^2_\pi} \left[\ell_4 - \frac{13}{12} + O(M^4_\pi)\right],\]

where \(F_\pi = 91.9 \pm 0.1\) MeV is the physical pion decay constant \[3\]. The low energy constant \(\ell_4\), in turn, determines the leading contribution in the chiral expansion of the pion decay constant, i.e.

\[\frac{F_\pi}{F} = 1 + \left(\frac{M_\pi}{4\pi F_\pi}\right)^2 \ell_4 + O(M^4_\pi),\]

where \(F\) is the pion decay constant in the chiral limit. This scalar form factor is not accessible experimentally, but it has been determined from lattice QCD (LQCD) \[4-6\], or hadronic models \[7\].

Theoretically, the ideal tool to study this form factor, independently from LQCD, is the Kroll-Lee-Zumino Abelian renormalizable field theory of pions and a neutral \(\rho\)-meson \[8\]. This provides the appropriate field theory platform for the phenomenological Vector Meson Dominance (VMD) model \[9\], allowing for a systematic calculation of higher order quantum corrections \[10\]-\[11\]. Due to the renormalizability of the theory, predictions are parameter free, as the strong \(\rho\pi\pi\) coupling, \(g_{\rho\pi\pi}\), is known from experiment. In spite of this coupling being a strong interaction quantity, perturbative calculations in the \(\overline{MS}\) scheme make sense because the effective expansion parameter turns out to be \((g_{\rho\pi\pi}/4\pi)^2 \simeq 0.2\).

The KLZ theory has been used to compute the next-to-leading order (NLO) correction to the tree level (VMD) electromagnetic form factor of the pion in the space-like region with very good results \[10\]. In fact, it agrees with data up to \(q^2 \simeq -10\) GeV\(^2\) with a chi-squared per degree of freedom \(\chi^2_F = 1.1\), as opposed to VMD which gives \(\chi^2_F = 5.0\). In addition, the mean-squared radius at NLO is \(\langle r^2_{\pi} \rangle = 0.46\) fm\(^2\), compared with the experimental result \(\langle r^2_{\pi} \rangle = 0.45 \pm 0.01\) fm\(^2\), and the VMD value \(\langle r^2_{\pi} \rangle = 0.39\) fm\(^2\).

\[\text{FIG. 1: Leading order (LO) contribution to the scalar form factor of the pion. The cross indicates the coupling of the scalar current to two pions.}\]
The KLZ Lagrangian is given by
\[ \mathcal{L}_{KLZ} = \partial_\mu \phi \partial^\mu \phi^* - M^2_\pi \phi \phi^* - \frac{1}{4} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{1}{2} M^2_\rho \rho_\mu \rho^\mu + g_{\rho\pi\pi} J_\rho^\mu + g_{\rho\pi} \rho_\mu \rho^\mu \phi \phi^*, \]

where \( \rho_\mu \) is a vector field describing the \( \rho^0 \) meson \((\partial_\mu \rho^\mu = 0)\), \( \phi \) is a complex pseudo-scalar field describing the \( \pi^\pm \) mesons, \( \rho_{\mu\nu} \) is the usual field strength tensor: \( \rho_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu \), and \( J_\rho^\mu \) is the \( \pi^\pm \) current: \( J_\rho^\mu = i \partial_\mu \phi^\dagger \). In spite of the explicit presence of the \( \rho^0 \) mass term in the Lagrangian, the theory is renormalizable because the neutral vector meson is coupled to a conserved current \[8\]. Figures 1 and 2 show, respectively, the LO and the NLO diagrams, where the cross indicates the coupling of the current to the two pions. Notice that while the Lagrangian, Eq.(6), contains a \( \rho\rho\pi\pi \) quartic coupling, this term only contributes in this application at NNLO and beyond.

Using the Feynman propagator for the \( \rho \)-meson, and in \( d \) dimensions, the unrenormalized vertex function in Fig.2 in dimensional regularization is given by
\[ G(q^2) = -2 \frac{g^2_{\rho\pi\pi}}{(4\pi)^2} q^2 (2 - \frac{d}{2}) \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \left\{ \frac{2}{\varepsilon} - \ln \left( \frac{\Delta(q^2)}{\mu^2} \right) - \frac{1}{2} - \gamma + \ln(4\pi) \right. \]
\[ + \left. \frac{1}{2\Delta(q^2)} \left[ M^2_\pi (x_1 + x_2 - 2)^2 - q^2 (x_1 x_2 - x_1 - x_2 + 2) \right] + O(\varepsilon) \right\}, \]

where \( \Delta(q^2) \) is defined as
\[ \Delta(q^2) = M^2_\pi (x_1 + x_2)^2 + M^2_\rho (1 - x_1 - x_2) - x_1 x_2 q^2. \]

In the \( \overline{MS} \) scheme, and renormalizing the vertex function at the point \( q^2 = 0 \), the NLO contribution in Fig. 2 is \[11\]
\[ G(q^2) - G(0) = -2 \frac{g^2_{\rho\pi\pi}}{(4\pi)^2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \left( \ln \left( \frac{\Delta(q^2)}{\Delta(0)} \right) + \frac{1}{2} \left[ M^2_\pi (x_1 + x_2 - 2)^2 \left( \frac{1}{\Delta(q^2)} \right) - \frac{1}{\Delta(0)} \right] - \frac{q^2}{\Delta(q^2)} (x_1 x_2 - x_1 - x_2 + 2) \right) \],

with
\[ F_S(q^2) = F_S(0) \left[ 1 + G(q^2) - G(0) \right]. \]

For details on the renormalization procedure for the fields, masses and coupling see \[10\]. The result of a numerical evaluation of Eq.(9), using \( g^2_{\rho\pi\pi} = 36.0 \pm 0.2 \) from the measured width of the \( \rho \)-meson \[4\], is shown in Fig.3. Regarding the scalar radius, defined in Eq.(2), we confirm the NLO result obtained in \[11\]
\[ (r^2_\pi)_S = 0.4 \text{ fm}^2. \]
with a negligible error due to the strong coupling.

This value is smaller than typical values in the literature \[4\]-\[7\]. However, it must be kept in mind that the NLO result is expected to be a lower bound, i.e. with \( |G(q^2) - G(0)| < 0 \) the NNLO would reduce \( F_S(q^2) \), thus increasing the radius. A rough order of magnitude estimate of the size of the NNLO contribution suggests a correction of some 20\% to the NLO term (the NNLO calculation is quite formidable and beyond the scope of this note). This is obtained by estimating a typical two-loop diagram, e.g. the \( \rho \)-meson propagator at NNLO and comparing it with the NLO result. The Feynman integrals in the variables \( x_i \) at NLO and NNLO are of order \( O(1) \) in the \( q^2 \) range explored here. We find the total contribution from this diagram to be over 20\% of the NLO, thus increasing the radius to \( \langle r^2 \rangle_S \simeq 0.5 \text{ fm}^2 \).

A comparison of the KLZ form factor itself at low \( |q^2| < 0.5 \text{ GeV}^2 \) with LQCD results read from figures in \[4\] and \[6\] shows good agreement. It should be mentioned, though, that LQCD results from \[4\] are for light-quark masses in the range from \( m_s/6 \) to \( m_s/2 \), while those from \[6\] are for \( m_s = 325 \text{ MeV} \). These LQCD determinations find values for the scalar radius higher than in this analysis, Eq.(11), i.e. \( \langle r^2 \rangle_S = 0.6 \pm 0.1 \text{ fm}^2 \) from \[4\], and \( \langle r^2 \rangle_S = 0.637 \pm 0.023 \text{ fm}^2 \) from \[6\]. These results for the radius are determined from e.g. chiral extrapolations to the physical pion mass. Our results for the form factor are also in agreement within less than 10\% with a CHPT calculation \[12\] in the range \(-q^2 = 0 - 0.2 \text{ GeV}^2\).

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