NEUTRALINO INDUCED MAJORANA NEUTRINO TRANSITION MAGNETIC MOMENTS

MAREK GÓŻDŻ and WIESŁAW A. KAMIŃSKI

Department of Informatics, Maria Curie-Skłodowska University,
pl. Marii Curie-Skłodowskiej 5, 20-031 Lublin, Poland
mgozdz@kft.umcs.lublin.pl, kaminski@neuron.umcs.lublin.pl

We calculate the effect of neutrino-neutralino mixing on the neutrino magnetic moment and compare it with the contribution of pure particle-sparticle loop. We have found that the dominated mechanism is still the bare loop, and that the bilinear insertions on the external neutrino lines contribute at least one order of magnitude weaker.

1. Bilinear R-parity breaking in MSSM and neutrino magnetic moment from RpV loops

Supersymmetry is believed to be the necessary ingredient in the high energy regime of particle physics, and that it should be included in all models of grand unified theories (GUT). Despite the fact that it has been formulated long before the string theory, it soon turned out that the latter needs supersymmetry to include not only interactions, mediated by bosonic strings, but also matter in the form of fermionic strings. Since the string and M-theories suffer from great complexity and infinite number of solutions, simpler phenomenological models are used, from which one of the most popular is the minimal supersymmetric standard model (MSSM).

In its basic formulation the MSSM exhibits the same feature as the ordinary standard model regarding the conservation of the lepton and baryon numbers. This feature is realized by an artificial introduction of the so-called R-parity defined as

\[ R = (-1)^{3B + L + 2S}, \]

where \( B \), \( L \), and \( S \) are the baryon, lepton, and spin numbers, respectively. In its more general form, however, the MSSM does not preserve the R-parity either through a spontaneous symmetry breaking or by explicit retaining the previously rejected R-parity violating (RpV) terms in the superpotential. In such a case the basic superpotential

\[ W = \epsilon_{ab} \left[ (Y_E)_{ij} \hat{L}^a_i \hat{H}^b_j \hat{E}^c_j + (Y_D)_{ij} \hat{Q}^{ax}_i \hat{H}^b_d \hat{D}^c_j + (Y_U)_{ij} \hat{Q}^{ax}_i \hat{H}^b_u \hat{U}^c_j + \mu \hat{H}_d \hat{H}_u \right] \]

is extended by

\[ W_R = \epsilon_{ab} \left[ \lambda_{ijk} \hat{L}^a_i \hat{L}^b_j \hat{E}^c_k + \lambda'_{ijk} \hat{L}^a_i \hat{Q}^{ax}_j \hat{D}^c_k + \lambda''_{ijk} \hat{U}^a_i \hat{E}^c_j + \epsilon_{abc} \hat{L}^a_i \hat{H}^b_j \hat{H}_u \right], \]

where \( \epsilon_{abc} \) is the Levi-Civita symbol.
where the superfields are denoted by a tilde. The quark and lepton left-handed doublets are $Q$ and $L$, and the right-handed singlets are $U$, $D$, and $E$, respectively.

The $\lambda$, $\lambda'$, and $\kappa_i$ coupling constants mediate lepton number violating interactions. The $\lambda''$ breaks the baryon number and should be treated carefully because of the very stringent limits on the proton life-time; it is therefore often set to zero to avoid unnecessary problems.

For completeness the $\mathcal{R}$MSSM Lagrangian is supplied with mass terms and the mechanism of supergravity (SUGRA), in which the supersymmetry is broken at the Planck scale by explicit soft terms. (See Refs. 1, 5 for details.)

In this paper we neglect all the trilinear couplings and concentrate on the bilinear $\kappa$'s. The most important implication of the existence of the bilinear RpV terms is mixing between different types of particles: neutrinos with neutralinos ($\nu_1,2,3, \tilde{H}_u^0, \tilde{H}_d^0, B^0, W^3$), as well as charged leptons with charginos, and neutral Higgs bosons with sneutrinos.

The $\mathcal{R}$MSSM provides an elegant mechanism of generating Majorana neutrino mass terms. It is done by considering RpV particle-sparticle loops (mostly lepton-slepton or quark-squark) without the need of introducing a heavy right-handed singlet neutrino, i.e. it provides an attractive alternative to the popular see-saw mechanism. By introducing an interaction vertex between one of the virtual particles and an external photon, a Majorana neutrino transition magnetic moment is generated. These processes have been described already [6] but the previous calculations were based on many simplifying assumptions. The present work is a continuation of a series of papers [7] in which we present exact analytic formulas, and perform the numerical calculations using GUT constrained $\mathcal{R}$MSSM. The GUT conditions give the starting point for the renormalization group equations (RGE) from which the low energy spectrum of the model is obtained. This approach has been recognized as the most exact by now. In this conference contribution we present calculations of the transition magnetic moment generated in a mechanism in which the external neutrino lines contain bilinear insertions, causing them to transform into neutralinos. The relevant Feynman diagram is depicted on Fig. 1. There is eleven possible configurations, presented in Tab. 1 which all contribute to the magnetic moment. The contribution to the Majorana neutrino magnetic moment from
the discussed diagrams is given by (in Bohr magnetons $\mu_B$)

$$
\mu_{ab} = (1 - \delta_{ab}) \frac{m_{\nu_1}}{4\pi^2} \left( C_{1a} \frac{C_2 C_3}{m_{\chi_1} m_{\chi_{11}}} C_{4b} \right) \sum_{jk} \left[ \frac{w_{jk}^{(q)}}{m_{q'}} Q_{q'} + \frac{w_{jk}^{(l)}}{m_{l'}} Q_{l'} \right] \mu_B. \tag{3}
$$

Here we have denoted the sneutrinos’ vacuum expectation values by $\omega$, the electric charge of a particle (in units of $e$) by $Q$. The dimensionless loop functions $w$ take the forms

$$
w_{jk}^{(q)} = \frac{1}{2} \sin(2\theta^k) \left[ \frac{x^{jk}_2 \log(x^{jk}_2) - x^{jk}_2 + 1}{(1 - x^{jk}_2)^2} - (x_2 \rightarrow x_1) \right], \tag{4}
$$

where $\theta$ is the squarks’ mixing angle and $x^{jk}_i = (m_{q_i}/m_{\tilde{q}_j})^2$. A similar expression holds for $w_{jk}^{(l)}$ with (s)quarks replaced by (s)leptons. The sum over $j$ and $k$ in Eq. (3) accounts for all the possible quark-squark and lepton-slepton configurations for given neutralinos. The factor 3 in front of $w^{(q)}$ counts the three quark colors.

2. Numerical calculations

In order to reduce the great number of free parameters of the model we assume the GUT scenario at high energies. The initial values of the $\kappa$’s are drawn randomly and evolved down using RGE equations. Then the neutrino mass matrix is calculated and compared with the mass matrix obtained from experimental data. If in agreement, the magnetic moment is calculated.

The phenomenological neutrino mass matrices can be calculated when one knows the three neutrino mixing angles and the two differences of masses squared. The best fit values of these parameters are

$$
|m_1^2 - m_2^2| = 7.1 \times 10^{-5} \text{eV}^2, \quad |m_2^2 - m_3^2| = 2.1 \times 10^{-3} \text{eV}^2, \quad \sin^2(\theta_{12}) = 0.2857, \quad \sin^2(\theta_{23}) = 0.5, \quad \sin^2(\theta_{13}) = 0. \tag{5}
$$

Table 1. All possible loops with neutrino-neutralino mixing on the external lines. The names of the columns correspond to the denotations on Fig. 1.

| $I$ | $II$ | $III$ | $C_1$ | $C_2$ | $C_3$ | $C_4$
|-----|-----|-----|------|------|------|------
| $H_u$ | $u \tilde{u}$ | $H_u$ | $\kappa_a$ | $\sqrt{2} m_u / v_u$ | $\sqrt{2} m_u / v_u$ | $\kappa_b$
| $\tilde{H}_u$ | $u \tilde{u}$ | $B \tilde{B}$ | $\kappa_a$ | $\sqrt{2} m_u / v_u$ | $-g'_y / (3 \sqrt{2})$ | $g'_y \omega_b$
| $\tilde{H}_u$ | $u \tilde{u}$ | $\tilde{W}_3$ | $\kappa_a$ | $\sqrt{2} m_u / v_u$ | $-g / \sqrt{7}$ | $g \omega_b$
| $\tilde{B}$ | $q \tilde{q}$ | $\tilde{B}$ | $g' \omega_a$ | $-g'_y / (3 \sqrt{2})$ | $-g'_y / (3 \sqrt{2})$ | $g'_y \omega_b$
| $B$ | $\tilde{l} \tilde{l}$ | $\tilde{B}$ | $g' \omega_a$ | $-g' / \sqrt{7}$ | $-g' / \sqrt{7}$ | $g' \omega_b$
| $\tilde{W}_3$ | $u \tilde{u}$ | $\tilde{W}_3$ | $g \omega_a$ | $-g / \sqrt{7}$ | $g / \sqrt{7}$ | $g \omega_b$
| $\tilde{W}_3$ | $d \tilde{d}$ | $\tilde{W}_3$ | $g' \omega_a$ | $g / \sqrt{7}$ | $g / \sqrt{7}$ | $g \omega_b$
| $\tilde{B}$ | $u \tilde{u}$ | $\tilde{W}_3$ | $g' \omega_a$ | $-g' / (3 \sqrt{2})$ | $-g / \sqrt{7}$ | $g \omega_b$
| $\tilde{B}$ | $d \tilde{d}$ | $\tilde{W}_3$ | $g' \omega_a$ | $-g' / (3 \sqrt{2})$ | $g / \sqrt{7}$ | $g \omega_b$
| $\tilde{B}$ | $\tilde{l} \tilde{l}$ | $\tilde{W}_3$ | $g' \omega_a$ | $g' / \sqrt{7}$ | $g / \sqrt{7}$ | $g \omega_b$
Additionally, it is necessary to know at least one element of the mass matrix. For example one can assume a certain scenario of the hierarchy of masses and set the lowest mass to zero. The normal hierarchy aligns the masses as \( m_1 \ll m_2 \ll m_3 \) whence in the inverted hierarchy \( m_3 \ll m_1 \ll m_2 \). A separate problem arises from the unknown CP phases which enter the Majorana neutrino mass matrix. We cope with this problem twofold. The simpler case is to assume that the CP symmetry is conserved and neglect the phases. The more complicated approach requires to check all possible values of the phase factors for each of the mass matrix elements separately and for each element pick the combination which leads to its highest value. In such a case the obtained matrix do not correspond to any physically allowed situation, but instead presents an upper bound on all its elements. It is therefore useful in discussion of the allowed parameter space of the model. We call the such obtained matrices ‘maximal’. The results are presented for two GUT

| Scenario | \( A_0 = 100, m_0 = m_{1/2} = 150 \text{ GeV}, \tan \beta = 19 \) | \( A_0 = 500, m_0 = m_{1/2} = 1000 \text{ GeV}, \tan \beta = 19 \) |
|----------|---------------------------------|---------------------------------|
| IH-CP    | \( 3.0 \times 10^{-21} \)   | \( 3.7 \times 10^{-22} \)   |
| IH-max   | \( 3.7 \times 10^{-19} \)   | \( 4.6 \times 10^{-20} \)   |
| NH-CP    | \( 3.2 \times 10^{-20} \)   | \( 4.0 \times 10^{-21} \)   |
| NH-max   | \( 1.2 \times 10^{-19} \)   | \( 1.4 \times 10^{-20} \)   |

scenarios in Tab. 2. The last column shows the upper bounds coming from pure trilinear RpV coupling constants, i.e. from quark-squark and lepton-slepton loops without bilinear mixing on the external lines. The conclusion is clear, that the discussed contribution to the main process is at best of the same order of magnitude, in most cases being at least an order of magnitude weaker.

Acknowledgements. The first author (MG) greatly acknowledges financial support from the Polish State Committee for Scientific Research under grant no. N N202 0764 33.

References
1. H. L. Haber, G. L. Kane, *Phys. Rep.* **117** (1985) 75.
2. C. Aulakh, R. Mohapatra, *Phys. Lett.* **B119** (1982) 136; G. G. Ross, J. W. F. Valle, *Phys. Lett.* **B151** (1985) 375; J. Ellis et al., *Phys. Lett.* **B150** (1985) 142; A. Santa-
neutralino induced majorana neutrino transition magnetic moments 5

Maria, J. W. F. Valle, Phys. Rev. D39 (1989) 1780; A. Masiero, J. W. F. Valle, Phys. Lett. B251 (1990) 273.

3. M. A. Diaz, J. C. Romao, J. W. F. Valle, Nucl. Phys. B524, 23 (1998); A. Akeroyd et al., Nucl. Phys. B529, 3 (1998); A. S. Joshipura, M. Nowakowski, Phys. Rev. D51 (1995) 2421, ibid. 5271; M. Nowakowski, A. Pilaftsis, Nucl. Phys. B461 (1996) 19.

4. L. J. Hall, M. Suzuki, Nucl. Phys. B231 (1984) 419; R. Barbieri, D. E. Brahm, L. J. Hall, S. D. Hsu, Phys. Lett. B238 (1990) 86; H. Dreiner, G. G. Ross, Nucl. Phys. B410 (1993) 188; G. Bhattacharyya, D. Choudhury, K. Sridhar, Phys. Lett. B355 (1995) 193; G. Bhattacharyya, A. Raychaudhuri Phys. Lett. B374 (1996) 93; A. Y. Smirnov, F. Vissani Phys. Lett. B380 (1996) 317.

5. M. Gózdz, W. A. Kamiński, Phys. Rev. D 78 (2008) 075021.

6. C. Liu, Mod. Phys. Lett. A 12 (1997) 329; R. Barbieri, M. M. Guzzo, A. Masiero, D. Tommasini, Phys. Lett. B 252 (1990) 251; O. Haug, J. D. Vergados, A. Faessler, S. Kovalenko, Nucl. Phys. B 565 (2000) 38; G. Bhattacharyya, H. V. Klapdor-Kleingrothaus, H. Päs, Phys. Lett. B 463 (1999) 77; A. Abada, M. Losada, Phys. Lett. B 492 (2000) 310; Nucl. Phys. B 585 (2000) 45.

7. M. Gózdz, W. A. Kamiński, F. Simkovic, Phys. Rev. D 70 (2004) 095005; Int. J. Mod. Phys. E 15 (2006) 441; Acta Phys. Pol. B 37 (2006) 2203; M. Gózdz, W. A. Kamiński, F. Simkovic, A. Faessler, Phys. Rev. D 74 (2006) 055007; M. Gózdz, W. A. Kamiński, Int. J. Mod. Phys. E 17 (2008) 276.

8. J. D. Vergados, private communication.

9. G. Altarelli and F. Feruglio, New J.Phys. 6 (2004) 106 and references therein.