EXPLORATIONS INTO THE VIABILITY OF COUPLED RADIUS–ORBIT EVOLUTIONARY MODELS FOR INFLATED PLANETS

LAURENT IBGUI1,2, DAVID S. SPIEGEL2, and ADAM BURROWS2

1 LERMA, Observatoire de Paris, CNRS et UPMC, 5 Place J. Janssen, 92195 Meudon, France; laurent.ibgui@obspm.fr
2 Department of Astrophysical Sciences, Peyton Hall, Princeton University, Princeton, NJ 08544, USA; dsp@astro.princeton.edu, burrows@astro.princeton.edu

Received 2009 October 30; accepted 2010 November 18; published 2011 January 5

ABSTRACT

The radii of some transiting extrasolar giant planets are larger than would be expected by the standard theory. We address this puzzle with the model of coupled radius–orbit tidal evolution developed by Ibgui & Burrows. The planetary radius is evolved self-consistently with orbital parameters, under the influence of tidal torques and tidal dissipation in the interior of the planet. A general feature of this model, which we have previously demonstrated in the generic case, is that a possible transient inflation of the planetary radius can temporarily interrupt its standard monotonic shrinking and can lead to the inflated radii that we observe. Importantly, we demonstrate that the use of a constant time lag model for the orbital evolution does not improve the accuracy of the evolutionary calculations. First, though formulated in a closed form by the equations of Hut, it is not valid at large eccentricities, as for the constant phase lag model truncated at the second order in eccentricity that we adopt; ambiguities in tidal theories are perhaps the most significant source of uncertainty in evolutionary calculations. Second, we find evolutionary tracks that fit within the 1σ error bars, the radius, the eccentricity, and the semimajor axis of HD 209458b in its current estimated age range, using the constant time lag model, as we find fitting tracks with the constant phase lag model. Both models show that a bloated planet with a circular orbit may still be inflated, due to thermal inertia. We have modified our constant phase lag model to include an orbital period dependence of the tidal dissipation factor in the star, \( Q_* \propto P^\gamma \), \(-1 < \gamma < 1\). For some inflated planets (WASP-6b and WASP-15b), we find fitting tracks; for another (TrES-4), we do not; and for others (WASP-4b and WASP-12b), we find fitting tracks, but our model might imply that we are observing the planets at a special time. Finally, we stress a 2–3 order-of-magnitude timescale uncertainty of the inspiraling phase of the planet into its host star, arising from uncertainties in \( Q_* \).

Key words: planetary systems – planets and satellites: general

Online-only material: color figures

1. INTRODUCTION

The more than 100 transiting extrasolar giant planets (EGPs) discovered so far offer a unique opportunity to test and improve the models of the structure and evolution of these bodies. The mass and radius of such planets can be inferred from a combination of radial velocity and transit light curve measurements that break the planet mass–inclination angle degeneracy. A large theoretical effort has been undertaken for more than a decade now to model and understand the evolution and the radii of transiting planets (Guillot et al. 1996; Burrows et al. 2000, 2003, 2004, 2007; Bodenheimer et al. 2001, 2003; Baraffe et al. 2003, 2004, 2005, 2006, 2008; Gu et al. 2003; Fortney & Hubbard 2004; Chabrier et al. 2004; Laughlin et al. 2005; Fortney et al. 2007; Marley et al. 2007; Chabrier & Baraffe 2007; Liu et al. 2008; Ibgui & Burrows 2009; Miller et al. 2009; Leconte et al. 2010; Ibgui et al. 2010).

The radius of a gas giant planet depends on many physical effects that are particular to a given planet–star system, including the mass and age of the planet; the stellar irradiation flux and spectrum; the composition—in particular, the heavy-element content—of the atmosphere, the envelope, and the core; the atmospheric circulation that couples the day and the night sides; and any processes that could generate an extra power source in the interior of the planet, such as tidal heating (the subject of this paper and of many references herein) or ohmic dissipation (Batygin & Stevenson 2010). Moreover, the transit radius effect (Burrows et al. 2003; Baraffe et al. 2003) has to be considered in order to infer the transit radius from the planet’s physical radius. Therefore, a custom evolutionary calculation is the most appropriate way to determine a theoretical transit radius, in order to compare it with the observed transit radius.

The objective of the present paper is to test the coupled radius–orbit tidal evolution model developed by Ibgui & Burrows (2009) on some recently discovered inflated planets. We present evolutionary tracks for WASP-4b (Wilson et al. 2008; Southworth et al. 2009; Gillon et al. 2009b; Winn et al. 2009a) and WASP-12b (Hebb et al. 2009). We have also tested the model on TrES-4 (Mandushev et al. 2007; Sozzetti et al. 2009), WASP-6b (Gillon et al. 2009a), and WASP-15b (West et al. 2009). Some other inflated planets have been discovered recently, such as HAT-P-13b (Bakos et al. 2009), WASP-17b (Anderson et al. 2010), and CoRoT-5b (Rauer et al. 2009). We might apply our formalism to them in the future.

The idea of exploring tidal heating as an explanation for the inflated radii was originally formulated by Bodenheimer et al. (2001). They suggested an excitation mechanism to sustain a nonzero eccentricity, for example a planetary companion (Bodenheimer et al. 2003; Mardling 2007). To date, two transiting EGPs are known to be accompanied by a companion, HAT-P-13b (Bakos et al. 2009) and HAT-P-7b (Pál et al. 2008; Winn et al. 2009b). This reinforces the plausibility of such a scenario.

Batygin et al. (2009) have coupled a three-body tidal orbital evolution model with a model of the interior structure of

See J. Schneider’s Extrasolar Planet Encyclopaedia at http://exoplanet.eu, the Geneva Search Programme at http://exoplanets.eu, and the Carnegie/California compilation at http://exoplanets.org.
HAT-P-13b. They found a quasi-stationary solution and possibly consistent core masses, radii, and tidal heating rates. Assuming, as did Liu et al. (2008), that the systems are in a quasi-stationary state, Ibgui et al. (2010) provide, for each of the systems that they have studied (WASP-6b, WASP-12b, WASP-15b, TrES-4, HAT-P-12b) and for each of the associated atmospheric opacities (solar, 10×solar) a relation between the heavy-element core mass $M_{\text{core}}$ and the ratio $\varepsilon^2/Q'_e$, where $\varepsilon$ is the orbital eccentricity and $Q'_e$ is the tidal dissipation factor in the planet. This constraint results from a degeneracy between the dissipation heating rate in the interior of the planet (which increases the radius) and the mass of a possible heavy-element central core (which shrinks the radius).

For close-in EGPs (orbital separation $\lesssim 0.1$–$0.15$ AU), tidal torques are strong enough such that they can result in planetary orbital evolution, and they produce tidal heating (dissipation) inside the planet. Such tidal effects were first suggested for transiting EGPs by Jackson et al. (2008b, 2008c, 2008d). Jackson et al. included the tides raised on the star and the tides raised on the planet. They found tidal rates close to the levels that Burrows et al. (2007) proposed to maintain the observed radii of some transiting EGPs. Ibgui & Burrows (2009) described a model that couples the two consequences of these tidal effects—planetary radius evolution and orbit evolution. They tested their model on HD 209458b and found an explanation for the radius of this planet. Note that they also showed that a supersolar metallicity of the planetary atmosphere, without invoking tides, can explain the radius. Miller et al. (2009) applied a similar method to all the transiting EGPs, albeit with simplified models for the atmospheres of the planets and parent stars, and a restricted range of possibilities for the tidal dissipation factors $Q'_e$. We have chosen a more detailed approach by adopting customized atmospheric models and an extended range for $Q'_e$. Therefore, due to the complexity of the atmospheric calculations, we have selected a couple of planets. We believe that both approaches are complementary. The one followed by Miller et al. (2009) provides a global estimate of the possibilities for matching the observed radii of the transiting EGPs, while our approach, more precise and therefore applied to fewer planetary systems, focuses on more specific issues such as the influence of the atmospheric opacity of the planet, a more detailed model for the tidal dissipation in the star, and a phenomenological study of all the qualitatively possible behaviors of the evolutionary curves (Ibgui & Burrows 2009). The application of our model to a subset of inflated transiting EGPs is the subject of this paper.

The paper outlines, in Section 2, the main assumptions of our coupled radius–orbit tidal evolution model, with a summary of the basic phenomenological results obtained from the previous study by Ibgui & Burrows (2009). The paper shows that the use of the constant time lag model for the orbital evolution modeling (Hut 1981) does not improve the accuracy of the evolutionary calculations, contrary to what has been stated by Leconte et al. (2010), since the equations of Hut, though formulated in a closed form, are not valid at large eccentricities. The paper also explains our upgraded modeling of the tidal dissipation factor in the star. Section 3 demonstrates some additional generic results, such as the effect of $M_{\text{core}}$, $Q'_e$, and the initial semimajor axis. For each of the following planets, TrES-4, WASP-4b, WASP-6b, WASP-12b, and WASP-15b, we search for evolutionary tracks that fall within the observational limits of the radius, the semimajor axis, and the eccentricity of the planet in its current estimated age range. Our results are described in Section 4. We have not found any such track for TrES-4. We have found solutions for WASP-6b and WASP-15b. The cases of the planets WASP-4b and WASP-12b, for which we have coupled models that fit, are more interesting. Therefore, we present evolutionary curves for these planets. In fact, the solutions that we obtain for these two planets are valid only for very short age ranges in comparison with the estimated ages of the planets. This would imply that we are observing both planets at a very special time in their evolution, which would be a priori unlikely. In Section 5, we discuss the plunging timescale of a planet into its host star. Its uncertainty can span 2–3 orders of magnitude. We summarize our results in Section 6.

2. THE COUPLED RADIUS–ORBIT EVOLUTIONARY MODEL AND THE PERIOD DEPENDENCE OF THE TIDAL DISSIPATION FACTOR $Q'_e$

2.1. The Phenomenology of the Coupled Radius–Orbit Evolution

The model that we employ assumes a two-body gravitational and tidal interaction that consistently couples the evolution of the radius with the orbit of the planet. It includes the tides raised on the planet and the tides raised on the star, along with stellar irradiation and detailed model atmospheres. The planetary radius evolves as a result of the competing influences of tidal heating in its convective interior (due to the dissipation of orbital energy) and radiative cooling from its surface. The most interesting phenomenological result that we have obtained is that, for strong enough tides, the planet’s radius can undergo a transient phase of inflation that temporarily interrupts its monotonic shrinking and resets its evolutionary clock (Ibgui & Burrows 2009). Moreover, we have demonstrated that, due to thermal inertia, an earlier episode of tidal heating can result in an inflated radius at the current age of the planet, even though its current orbit has nearly circularized.

2.2. Formalism and Assumptions: A Summary

The formalism, assumptions, and computational techniques have been extensively described in Ibgui & Burrows (2009). We summarize them here.

The planetary structure consists of a gaseous ($H_2$, He) isentropic envelope (helium mass fraction $Y = 0.25$) described by the equation of state of Saumon et al. (1995). It may also contain an inner heavy-element core. The basic effect of the core is to shrink the planetary radius (Burrows et al. 2007; Liu et al. 2008; Ibgui et al. 2010). However, in the context of coupled radius–orbit evolution, its effect is more subtle, as will be discussed in Section 3. We restrict ourselves to a solar atmospheric opacity of the planet. The effects due to a higher opacity, namely an enhanced and accelerated transient phase of radius inflation, have been described in Ibgui & Burrows (2009).

The giant planet radius evolution is modeled with a Henyey code (Burrows et al. 1993, 1997). The radiative cooling from the surface of the planet is linked to boundary conditions (Burrows et al. 2003). The latter incorporate realistic irradiated planetary atmospheres calculated by CoolTLUSTY, a variant of TLUSTY (Hubeny 1988; Hubeny & Lanz 1995). The specific behaviors of such irradiated atmospheres are an active field of research. For example, an extra absorber in the upper atmosphere may produce a temperature inversion (Hubeny et al. 2003; Burrows et al. 2008a, 2008b; Fortney et al. 2008; Knutson et al. 2008; Spiegel et al. 2009). We do not incorporate such an extra absorber in the
models of this paper. High metallicity EGP, such as Neptune-mass planets (Spiegel et al. 2010a), have their own unique characteristics. We calculate a customized host star spectrum by interpolation of the Kurucz (1994) models at the actual effective temperature and gravity of the star. Such a customized approach can only improve the reliability of the model.

The equations governing the tidal evolution of the orbital eccentricity $e$, the semimajor axis $a$, and the tidal heating rate are the ones adopted by Ibgi & Burrows (2009). They are derived from the equilibrium tide model, assuming a constant phase lag, and are truncated at the second order in eccentricity. We rewrite them here for the sake of completeness (Goldreich 1963; Goldreich & Soter 1966; Bodenheimer et al. 2001, 2003; Gu et al. 2004; Mardling 2007; Jackson et al. 2008a, 2008c, 2008d; Ferraz-Mello et al. 2008; Barnes et al. 2009b):

$$\frac{1}{a^{1/2}} \frac{da}{dt} = - \frac{1}{a^{1/2}} \left[ \frac{K_{1p}}{Q_p^{5/2}} R_p^5 + \frac{K_{2p}}{Q_p^{5/2}} R_p^5 \right] + \frac{8}{25} \left( 1 + \frac{57}{4} e^2 \right) R_p^5 Q_p^{3/2},$$

(1)

$$\frac{1}{a^{1/2}} \frac{de}{dt} = - \frac{1}{a^{1/2}} \left[ \frac{K_{1p}}{Q_p^{5/2}} R_p^5 + \frac{K_{2p}}{Q_p^{5/2}} R_p^5 \right],$$

(2)

where $K_{1p}$ and $K_{2p}$ are constants defined by

$$K_{1p} = \frac{63}{4} G^{1/2} \frac{M_s^{3/2}}{M_p},$$

(3)

$$K_{2p} = \frac{225}{16} G^{1/2} \frac{M_s^{3/2}}{M_p},$$

(4)

$$E_{\text{tide}} = \frac{63}{4} G^{1/2} \frac{M_s^{3/2}}{M_p},$$

(5)

In the preceding equations, $G$ is the gravitational constant, $M_p, M_s, R_p, R_s$ are the masses and radii of the planet and star, and $Q_p$ and $Q_s$ are the tidal dissipation factors in the planet and in the star (Goldreich 1963; Goldreich & Soter 1966). The planet radius is time dependent, $R_p(t)$, but the star’s radius is assumed to be constant. The principal assumptions underpinning these equations (more details can be found in Ibgi & Burrows 2009) are as follows. We consider that this two-body interaction starts a few megayears after star formation, precluding any interaction with a protoplanetary disk (Goldreich & Sari 2003) or with other planets (Ford et al. 2003; Juric & Tremaine 2008; Chatterjee et al. 2008; Ford & Rasio 2008; Nagasawa et al. 2008). We also do not consider the Kozai interaction (Wu & Murray 2003; Wu 2003; Wu et al. 2007; Nagasawa et al. 2008). We assume that, after a few megayears, the planet is close enough to its host star that tidal effects can be significant, typically using an “initial” semimajor axis $a_i$ up to 0.1 AU and we consider an “initial” eccentricity $e_i$ that can range from 0.0 to 0.8. We neglect stellar and planetary obliquities. We assume that the planet’s spin is synchronized (tidally locked) with its orbital period, and that the star’s spin rate is small compared with the orbital mean motion. These equations are developed to lowest order in $e$ (Goldreich & Soter 1966). They become less good approximations at higher values of eccentricity and will probably need to be revised when improvements in the modeling of tidal dissipation are available. The major ambiguities come from the tidal dissipation factors, $Q_p$ and $Q_s$, which are inversely proportional to the tidal phase lags, and whose physical description remains poorly understood. Tidal dissipation depends on the nature of the body (fluid or solid), its composition and its equation of state, various tidal forcing frequencies that result from combinations of orbital and spin frequencies. Theoretical research in the modeling of tidal dissipation is an active ongoing effort. Within the framework of the dynamical (wavelike) tide, recent references include Wu et al. 2005a, 2005b; Ogilvie & Lin 2004, 2007; Goodman & Lackner (2009); and Ogilvie (2009). Higher-order terms may be considered for the equilibrium tide model (Hut 1981; Eggleton et al. 1998; Mardling & Lin 2002; Dobbs-Dixon et al. 2004). However, these equations rely on specific assumptions for the response of a body to tidal forcing. For example, one of the assumptions used in some of the aforementioned papers considers unique and constant (frequency-independent) $Q_p$ and $Q_s$. The premise of adopting the same tidal phase lag for all the tidal components was proposed by Goldreich & Soter (1966); the equations that we use in this paper also make it. However, while reasonably legitimate at small eccentricity where tidal forcing frequencies are comparable, this assumption cannot be a priori justified at large eccentricity where a wide range of frequencies appear (Barnes et al. 2008; Greenberg 2009). Only future progress on the physical modeling of the tidal dissipation response of bodies to tidal forcing will help to clarify the adequate modeling at large eccentricities. Considering these uncertainties, and for comparison with previous work, we use the same evolutionary equations with large $e$, as is done by Jackson et al. (2008a, 2008c, 2008d), Barnes et al. (2008, 2009a, 2009b), and Miller et al. (2009), who use them with $e$ reaching values of 0.5 and up to ~0.95. These equations show that $e$ and $a$ can only decrease. A discussion of the effect of the tidal orbit evolution on the evolutionary tracks is detailed in Section 2.3.

Once the properties of both the star and the planet are fixed, the model is controlled by four free parameters $(Q_p, Q_s, e, a_i)$ that we vary in order to fit simultaneously, within the error bars of the age of the planet, the planetary radius, the eccentricity, and the semimajor axis. An important point to bear in mind is that the radius and orbit evolution are strongly nonlinear and depend sensitively on these four free parameters. $e_i$ is varied from 0.00 to 0.80, $a_i$ is varied from its current (measured) value to 0.10 AU, the approximate distance beyond which tidal effects are negligible in our cases. As reminded in the preceding paragraph, tidal dissipation in the planet is ruled by $Q_p$, whose value is poorly constrained. Experimental estimates provide $10^{-5}$–$10^{-6}$ for Jupiter (Goldreich & Soter 1966; Yoder & Peale 1981), while theoretical arguments suggest around $10^{-5}$–$10^{-7}$ (Ogilvie & Lin 2004), and up to $10^{-8}$ for planets with cores (Goodman & Lackner 2009). Consequently, we have considered the range $10^{-5}$–$10^{-8}$ for $Q_p$. A particular model is specified for $Q_s$ as is explained in Section 2.4. It is worthwhile to note that the problem of finding initial conditions that can lead, through tidal and thermal evolution of giant planets, to present-day conditions is poorly constrained, given the large amplitudes of possible values of those free four parameters. In addition, planetary atmospheric opacity is another free parameter.
2.3. On the Applicability of the Tidal Orbital Equations at Large Eccentricity

As stated in the preceding subsection, we adopt tidal equations derived from the equilibrium tide model with constant phase lag, truncated at the second order in eccentricity, and therefore based on the assumption that eccentricities are small (Goldreich 1963; Goldreich & Soter 1966; Jackson et al. 2008c; Ferraz-Mello et al. 2008). It is important to note, once again, that neither equilibrium nor dynamical tidal theory is mature enough to provide equations that are reliable at large eccentricity.

Recently, Leconte et al. (2010) revisited the problem of the tidally coupled radius–orbit evolution of close-in giant planets. Similar to our effort, they work within the framework of the equilibrium tide model; however, they use the weak friction approximation, which assumes a constant time lag (Darwin 1880). They employ the associated tidal orbit evolution equations of Hut (1981). They argue that their approach leads to correct tidal evolution histories, in opposition to the approach that adopts the constant phase lag model, which has been extensively used in the recent papers on tidal evolutions of exoplanets (Jackson et al. 2008a, 2008c, 2008d; Barnes et al. 2008, 2009a; Miller et al. 2009; Ibugi & Burrows 2009).

The major argument that they advocate is that the equations they use are complete, valid to any order in eccentricity. It is true that Hut (1981) provides closed formulae, instead of expansions. However, we emphasize that this formalism relies on the specific assumption of a constant time lag \( \Delta t \) for all tidal driving frequencies, i.e., of a phase lag \( \epsilon \) proportional to the tidal driving frequency \( f_d \): \( \epsilon = \Delta t \times f_d \). As we show below, this assumption is valid only at small eccentricity. One could argue, as Leconte et al. (2010) do, that despite this flaw, the equations of Hut (1981) are mathematically exact for any eccentricity, and, moreover, that the tidal dissipation rate is severely underestimated by the equations that we use. One could object that focusing on the accuracy of the functional form of the tidal dissipation is misplaced, given the uncertainty in the tidal model, and that this uncertainty is adequately covered by the uncertainty (several orders of magnitude) in the \( Q \) parameters. These parameters may be complicated non-monotonic functions of tidal forcing frequency and may also vary dramatically with time. As a result, it is difficult for now to constrain the problem with many falsifiable hypotheses. In any case, we acknowledge that the investigation of the constant time lag model to explore the tidal effects on the radii of close-in EGPs brings an interesting contribution, in the sense that it demonstrates the range of uncertainty in the evolutions of these radii, given the uncertainty in the tidal equations. Still, it is our belief that this approach cannot substitute for an approach backed by a reliable physical rationale, which is not known in the current status of the tidal theory, and that this approach certainly cannot be qualified as “correct.” We detail hereafter our physical arguments.

As mentioned previously, the equations that we use are not applicable at large eccentricity. At the same time, the equations that Leconte et al. (2010) use are not applicable either in this regime, because they are based on the constant time lag assumption that cannot be correct at large eccentricity for at least two reasons, each one related to the multiplication of the tidal driving frequencies and to the uncertainties in the response of the body, planet or star, to this tidal forcing. We detail them hereafter.

1. First, the theory of Darwin (1880) considers the tidal potential acting on a body as a sum of periodic terms with different frequencies. Within the framework of the equilibrium tide, the tidal distortion of the body is the sum of the individual responses to each of the forcing components, each response having its own phase lag caused by dissipation within the body. This is the lag-and-add approach (Ferraz-Mello et al. 2008; Greenberg 2009). The relation between the tidal forcing frequency of each of these components and the value of the phase lag is an open question, because it depends on the physics of the response of a body to the tidal forcing, which is not well understood in the current state of tidal theory. Darwin (1880) assumes that all the tidal components have the same constant time lag, whatever their frequencies, therefore that each phase lag is proportional to the corresponding tidal frequency. This assumption is explored by Hut (1981) to derive equations of orbit evolution and tidal dissipation. These equations are used by Leconte et al. (2010) to justify their calculations at large eccentricity. However, they are entirely based on this premise of linearity between phase lag and tidal frequency, which is not correct at large eccentricity. In fact, tidal dissipation in bodies such as giant planets, or stars with an outer convection zone, is powered by turbulent viscosity in their convection zone. Zahn (2008) (see also Zahn 1966, 1989; Penev et al. 2007, and references therein) demonstrated that turbulent viscosity is not constant with respect to the driving frequency. Now, as the eccentricity increases, there are more and more different driving frequencies, so that the phase lags, which are directly linked to this viscosity (Zahn 2008), cannot be considered as proportional to the driving frequencies: the weak friction approximation (constant time lag) no longer applies.

2. Second, the application of the equilibrium tide model alone, which is used to derive both sets of orbital evolution equations (constant phase lag or constant time lag), is rigorously valid only for a circular orbit, but is a reasonable approach at small eccentricities. As eccentricity gets larger, the dynamical (wavelike) tide has to be considered as a growing additional effect, which coexists with the equilibrium tide. The modeling of these waves and their dissipation, which is the subject of active ongoing research, is neglected by the Leconte et al. (2010) analysis (and by ours). Moreover, recent papers in this field have demonstrated a complex dependence of the response of the body to the tidal forcing frequency, specifically of the tidal dissipation factors \( Q \).

This is a crucial element for fluid bodies, such as EGPs, for which the contribution to the dissipation rate largely depends on this wavelike tide (Ogilvie & Lin 2004, 2007; Goodman & Lackner 2009; Barker & Ogilvie 2010).

For all these reasons, one cannot assert that using the tidal equations of Hut (1981) (which rely on equilibrium tide theory with the constant time lag assumption) results in a valid modeling of the radius evolution of EGPs at any eccentricity.

Now, one can easily show that at small eccentricity, the equations of Hut (1981) and the ones that we use (Equations (1)–(3)) are almost the same ones. More precisely, assuming small eccentricities, a pseudo-synchronization state\(^4\) of the planet in the case of the constant time lag model, and a synchronization state of the planet in the case of the constant phase lag model, one can show analytically that the tidal heating rates coincide.

\(^4\) Pseudo-synchronization involves the planet spinning at a rate spin of \( \omega_p \approx \nu \left(1 + 6e^2 \right) \) (Hut 1981; Levrard et al. 2007; Leconte et al. 2010).
Adding the assumption, made in the constant phase lag model (see Section 2.2), of neglecting the star’s spin rate compared with the orbital mean motion, it is easy to show that the rates of evolution of $a$ and $e$ from both models almost coincide ($\dot{a}$ evolutions due to the tides raised on the planet are identical, so are $\dot{e}$ evolutions; considering the tides raised on the star, $\dot{a}$ is twice as large with the constant time lag model, in comparison with the constant phase lag model, while $\dot{e}$ is 2.88 larger). To summarize, orbital evolution and tidal heating equations are either equal or comparable at small eccentricities, whether the constant time lag or the constant phase lag model is employed, and diverge as eccentricity raises. However, at large eccentricities, none of these equations is correct, for the two reasons mentioned above.

For the sake of completeness, we have nonetheless tested the constant time lag model, by consistently coupling the equations derived by Hut (1981), as recast in the form provided by Leconte et al. (2010), with the Henyey evolutionary code (Burrows et al. 1993, 1997). In order to keep a constant time lag, we have adopted tidal dissipation factors that evolve as $1/(n\Delta t)$ where $\Delta t$ is a constant time lag and $n$ is the orbital mean motion. Then, as explained in Section 2.2, we have searched for sets of the four free parameters, which include the initial values $(e_i, a_i)$, and $(Q_{p_i}^*, Q_{\ast i}^*)$, that allow to fit simultaneously, within the error bars of the age of the planet, the planetary radius, the eccentricity, and the semimajor axis, within 1$\sigma$ of the measurements. Figure 1 shows the results that we have obtained for HD 209458b. Unlike Leconte et al. (2010), we have been able to find solutions. The major difference with their treatment is that we have not restricted the values of $Q_{p_i}^*$ and $Q_{\ast i}^*$, given the large uncertainty on the values of these parameters (Ogilvie & Lin 2004, 2007) (we have considered the range $10^5$–$10^8$, see Section 2.2). Figure 1 displays examples of fitting (within 1$\sigma$ of the measurements) evolutionary curves for HD 209458b at solar atmospheric opacity, assuming coupled evolution with tides of the planetary radius and orbit. The top left, top right, bottom left, and bottom right panels show (versus the age in Gyr) the co-evolution of the planetary radius and orbit. The top left, top right, bottom left, and bottom right panels show (versus the age in Gyr) the co-evolution of the planetary radius and orbit.

![Figure 1. Examples of fitting (within 1σ of the measurements) evolutionary curves for HD 209458b at solar atmospheric opacity, assuming coupled evolution with tides of the planetary radius and orbit. Top left, top right, bottom left, and bottom right panels show (vs. the age in Gyr) the co-evolution of the planetary radius and orbit.](image-url)
\( Q' \approx 10^{7.0}. \) The bunches of red curves represent some examples of fitting curves with the constant time lag model. In this second case, the tidal dissipation factors are set to initial values of \( (Q^p)_i = 10^{7.2} \) and \( (Q'_s)_i = 10^{6.5} \) and evolve as 1/\( (n\Delta t) \), as stated above.

The large peak of the radius evolution, observed with the constant phase lag model, either disappears or is shifted toward very early ages, when the constant time lag model is assumed, since at these early ages the latter model involves larger tidal heating than the truncated constant phase lag model, as the bottom right panel shows. At later ages, from \( \sim 1.0 \) to \( \sim 2.3 \) Gyr, tidal heating from the constant phase lag model prevails, which accounts for the corresponding delayed peak in the radius evolution. The top right panel shows that fitting curves require smaller initial values of eccentricities in the case of the constant time lag model (from 0.67 down to 0.34), in comparison with the large value, 0.77, required with the constant phase lag model. The radius evolutions computed with both models merge at low eccentricities, as shown by top left panel. This can be explained as follows. The free parameters of these models have been selected to fit the radius of the planet within its ranges of uncertainty in age and observed radius. In addition, within this age range, the eccentricity is low, and, more importantly, the tidal heating is already low enough (bottom right panel) to have no longer influence on the inflation of the radii. To sum up, the radii resulting from both models fit at the age of the planet, by construction. Their subsequent evolution is going on under the only radiative cooling regime, the tidal heating being negligible. Incidentally, the curves of both models and the standard evolution curve (no tides: dashed curve) will end up merging at some time, unless the planet falls into its host star prior to that. Finally, the radius remains inflated at the assumed age range of the planet, even though the eccentricity is extremely low, whatever the tidal model. This is due to thermal inertia. This figure shows that we are able to simultaneously fit, within the error bars of the age of the planet HD 209458b, the planetary radius, the eccentricity, and the semimajor axis, within 1\( \sigma \) of the measurements, whatever the tidal model (constant phase lag or constant time lag).

The qualitative result that tidal heating may provide enough energy to inflate the radius of HD 209458b is robust with respect to both models. More precise quantitative results largely depend on the tidal models, and therefore cannot be obtained, given the current stage of the tidal theory, which is an active ongoing research field.

2.4. The Tidal Dissipation Factor in the Star

The tides raised on the star play a specific role in that they are the major if not the only contributor to the planet’s orbital evolution, as soon as the orbital eccentricity \( e \) is small enough or is zero. Moreover, for a zero eccentricity, the evolution of the semimajor axis is described analytically, as was first demonstrated by Goldreich (1963), as can also be directly derived from Equation (2):

\[
a = a_0 \left[ 1 - \frac{117}{4} \frac{G^{1/2}}{a_0^{3/2}} \frac{M_p}{M^*} \frac{R^5}{Q'_s} (t - t_0) \right]^{2/13}, \tag{6}
\]

where \( a_0 \) is the semimajor axis at any time \( t_0 \) after the eccentricity has become null.

The subsequent evolution of the transiting EGPs from their observed current state is a puzzling issue. The stability, but also the timescale, of their evolution is being investigated (Levrard et al. 2009; Hellier et al. 2009; Hamilton 2009; D. S. Spiegel et al. 2010b, in preparation). It is well known that the tides raised on the star ultimately cause the planet to spiral into its host star and probably eventually to be tidally disrupted (Rasio et al. 1996; Levrard et al. 2009; Jackson et al. 2009a, 2009b; Ibgui & Burrows 2009; Miller et al. 2009). The associated timescale is highly dependent on \( Q'_s \).

As an upgrade to the model used for our previous generic study (Ibgui & Burrows 2009), we consider that the tidal dissipation factor in the star, \( Q'_s \), can evolve with the orbital period of the system as follows:

\[
Q'_s = 10^6 \times \left( \frac{P}{P_0} \right)^{\gamma}, \tag{7}
\]

where \( P \) is the orbital period of the system, \( P_0 \) is a reference orbital period (i.e., for which \( Q'_s = 10^6 \)). The exponent \( \gamma \) is between \(-1\) and \(+1\). The theoretical motivation for this range of \( \gamma \) comes from the modeling of tidal dissipation in fluid bodies and the modeling of turbulent kinematic viscosity (Zahn 1966, 1989; Goldreich & Nicholson 1977). The range of \( \gamma \) has been empirically confirmed by D. S. Spiegel et al. (2010b, in preparation) (see also Nordhaus et al. 2010) who show, based on a statistical study of the observed transiting planets’ properties, that only such a range can lead to a stationary rate of plunging planets into their host stars. The range for \( \beta \) is \( 5.0-8.0 \), resulting in \( Q'_s \), within \( 10^5-10^6 \) at \( P = P_0 \). We adopt the same range as for \( Q'_s \). We choose \( P_0 = 10 \) days for the reference orbital period. Indeed, observational data from Meibom & Mathieu (2005) on solar-type binaries in the open cluster M35 suggest a value of \( Q'_s \approx 10^6 \) for a period of 10 days (Ogilvie & Lin 2007).

3. COUPLED EVOLUTION OF THE TIDALLY HEATED RADIUS AND THE ORBIT: THE EFFECTS OF \( M_{core}, Q'_s, \) AND \( a_i \)

Major generic features of the radius and orbital co-evolution of a close-in giant planet are described by Ibgui & Burrows (2009). Here, we present additional results: we evaluate the effect of \( M_{core}, Q'_s, a_i \) on the evolution of the radius and the orbit of a transiting EGP. Our statements can be demonstrated with Equations (1)–(3). They have also been numerically tested on the “generic transiting system” employed by Ibgui & Burrows (2009), namely the HD 209458 system. Here, the tides raised on the star are neglected (\( Q'_s \rightarrow \infty \)) for clarity’s sake. Everything else being equal, increasing the values of either \( M_{core}, Q'_s, \) or \( a_i \) results in delaying the appearance of the radius inflation peak and decreasing its value. It also results in faster circularization of the orbit, and the final semimajor axis \( a_f \) is reached faster. As for the latter, by virtue of conservation of the angular momentum of the system, \( a_f \) is the same whatever (for whatever \( M_{core} \) or \( Q'_s \)), but a larger \( a_i \) results in a larger \( a_f \).

The demonstration of the evolution of the radius inflation peak and the orbit is based on the fact that a higher value of each of the three parameters (\( M_{core}, Q'_s, a_i \)) results in a lower initial tidal heating rate (see Equation (3)). It is straightforward for an enhanced \( Q'_s \) or \( a_i \). For an enhanced \( M_{core} \), everything else being equal, the radius of an EGP (for which an irradiated non-gray atmosphere is modeled) is smaller (Fortney et al. 2007;
Burrows et al. 2007). At the same time, the evolutions of e and a start with a lower rate. The tidal heating being initially lower, the ensuing transient expansion phase is manifest to a lesser degree and later.

The conclusion is that, when tidal heating is coupled with orbital evolution, the addition of a core does not necessarily result in a smaller radius. The result depends on the age of the system. In essence, at earlier age the planet with the larger core has the smaller radius, but it is the opposite at later age.

4. COUPLED EVOLUTION OF THE TIDALLY HEATED RADIUS AND THE ORBIT: APPLICATIONS

The validity of the model can be tested against the available data of the observed inflated transiting EGPs. Our objective is to find evolutionary tracks that fall within the observational limits of the radius, the semimajor axis, and the eccentricity of the planet in its current estimated age range. If we do find such tracks, we say that “we fit the planet.” The first application was presented in Igliu & Burrows (2009) for HD 209458b. We were able to simultaneously fit the radius, the eccentricity, and the semimajor axis of this planet with the set \( \left( \frac{Q'}{Q} \right, e, a) = (10^{6.55}, 10^{7.0}, 0.77, 0.085 \text{ AU}) \), where \( \frac{Q'}{Q} \) is constant, and for a solar opacity. Note that if HD 209458b has 3–10\( \times \) solar opacity, we can explain its radius without invoking the tidal heating argument. In this paper, we present the results of our investigation for other inflated planets, WASP-4b (Section 4.2) and WASP-12b (Section 4.3). We assume a solar opacity, and no central heavy-element core. We adopt a \( \frac{Q'}{Q} \) that varies according to Equation (7) with \( \gamma = -1 \) in order to smooth the ultimate plunging of the planet into its host star. We have also tested our coupled model for TrES-4, WASP-6b, and WASP-15b.

Observational data are listed in Table 1 for the planets’ properties and in Table 2 for the host stars’ characteristics.

For each planet, we have tested a large number of combinations of the parameters \( \left( \frac{Q'}{Q} \right, e, a) \). Recalling the \( Q' \) evolutionary law given by Equation (7) in Section 2.4, the reference orbital period is \( P_0 = 10 \text{ days} \). We took seven values for the \( \beta \) parameter that determines \( \frac{Q'}{Q} \) at \( P = P_0 (\beta = 5.0–8.0 \text{ in intervals of } 0.5) \), 31 values for \( \frac{Q'}{Q} \) \( (\log_{10}(\frac{Q'}{Q}) = 5.0–8.0 \text{ in intervals of } 0.1) \), 81 values for \( e \) (0.00–0.80, in intervals of 0.01), and a certain number (39 for WASP-4b and WASP-12b) of values for \( a \) (the observed value up to 0.10, in intervals of 0.002). This represents \( \sim 680,000 \) evolutionary curves tested for each planet at a given opacity. Given the high nonlinearity of the evolutionary equations, and the sensitive dependence on these parameters (see Section 2.2), this approach represents a fairly exhaustive exploration of all the possible combinations in order to select the ones that fit the observed parameters.

In Section 4.1, we describe results of our models in the cases of WASP-6b, WASP-15b, and TrES-4. Perhaps the most
interesting cases, however, are WASP-4b (Section 4.2 and Figure 2) and WASP-12b (Section 4.3 and Figure 3), which we can fit, assuming a solar opacity.

4.1. WASP-6b, WASP-15b, and TrES-4

WASP-6b and WASP-15b were discovered by the Wide Angle Search for Planets (WASP) survey (Gillon et al. 2009a; West et al. 2009). The radius of WASP-6b is $1.224^{+0.051}_{-0.052} \, R_J$, its mass is $0.503^{+0.019}_{-0.038} \, M_J$, and its age is $11^{+5}_{-3} \, \text{Gyr}$. The radius of WASP-15b is $1.428^{+0.077}_{-0.077} \, R_J$, its mass is $0.542^{+0.050}_{-0.050} \, M_J$, and its age is $3.9^{+2.8}_{-1.3} \, \text{Gyr}$. These two planets can easily be fit with little tidal heating ($\log_{10}(Q_p) > 7.5$) at solar opacity and for large age ranges. Moreover, the opacity effect is sometimes sufficient to explain some radii: WASP-6b can be fit with a $3 \times$ solar opacity, without tidal heating (Ibgui et al. 2010). Evolutionary tracks (not shown) that fit these planets’ observed properties look very similar to those of HD 209458b, presented in Ibgui & Burrows (2009).

TrES-4 was discovered by Sozzetti et al. (2009). Its radius is $1.783^{+0.093}_{-0.086} \, R_J$, its mass is $0.925^{+0.081}_{-0.081} \, M_J$, and its age is $2.9^{+1.5}_{-0.4} \, \text{Gyr}$. This planet is one of the most inflated transiting planets known, and its size is difficult to explain, even with tidal heating, at its estimated age. We were not able to simultaneously fit the radius, eccentricity, and semimajor axis of TrES-4, at any of the opacities that we have tested (solar, $3 \times$ solar, $10 \times$ solar).

4.2. WASP-4b

WASP-4b was discovered by Wilson et al. (2008) and its parameters were further refined by Southworth et al. (2009), Gillon et al. (2009b), and Winn et al. (2009a). Its observed radius is $1.365^{+0.021}_{-0.021} \, R_J$ and its mass is $1.237^{+0.064}_{-0.064} \, M_J$ for an age of $6.5^{+2.3}_{-2.3} \, \text{Gyr}$. Its eccentricity is not well constrained, with an upper limit of $0.096$ (Madhusudhan & Winn 2009).

Examples of evolutionary curves that fit for WASP-4b are portrayed in Figure 2. This figure consists of four panels that depict, versus the age (in Gyr) of the planet, the simultaneous evolution of its radius $R_p(R)$ (top left panel), its eccentricity $e$ (top right), its semimajor axis $a(\text{AU})$ (bottom left), and $\log_{10}(Q_p)$ (solid curves, left $y$-axis) and the orbital period $P(\text{days})$ (dashed curves, right $y$-axis) in the bottom right. Solar opacity is assumed for the planetary atmosphere. The radius evolution without tides and a constant orbit is represented by a black curve in the top left panel. It shows that the standard model prediction is quite far from the measurement. The difference is roughly $0.16 \, R_J$, i.e., $12\%$. Among the tested $Q_p$ at $P = P_0$ ($\log_{10}(Q_p|P_0|) = 5.0$ to $8.0$ in steps of $0.5$), we find fitting solutions for $\log_{10}(Q_p|P_0|) \leq 6.5$. The smaller the $Q_p$, the steeper the plunging slope of the semimajor axis. The results we show in Figure 2 are among the ones that have the smoothest plunging orbits. Moreover, for all the fitting curves, $\log_{10}(Q_p|P_0|)$ is greater than or equal to $6.9$, which is a fairly high value. In the examples in Figure 2, $Q_p = 10^{8.0}$. Also, the initial eccentricity...
is large, $e_i = 0.80$. Solutions with lower $e_i$ exist (the lowest is 0.45), but they are for $Q_e(P_0) = 10^{5.0}$ or $10^{5.5}$, which are the fastest plunging configurations. Three fitting evolutionary curves are plotted, for three different $a_i$: 0.050, 0.052, 0.054 AU. We identify, when $a_i$ increases, the delay of the appearance, and the lower value, of the radius inflation peak, as stated in Section 3. The curves end with thick dots, where the periastron $p = a(1 - e)$ of the orbit might be slightly smaller than the Roche limit. We find $0.0207 \lesssim p \lesssim 0.0229$. See Section 4 for more discussion. (A color version of this figure is available in the online journal.)

Figure 3. Same as Figure 2, but for WASP-12b. Note the very narrow range in age (≈ 50 Myr) for which the fits are simultaneously obtained for $R_p, e$, and $a$. Note also how close the orbit of the planet is to the Roche limit, which is roughly 0.021 AU. Given the uncertainties in the determination of the semimajor axis $a$ and the orbital eccentricity, the periastron $p = a(1 - e)$ of the orbit might be slightly smaller than the Roche limit. We find $0.0207 \lesssim p \lesssim 0.0229$. See Section 4 for more discussion. (A color version of this figure is available in the online journal.)

We identify, when $a_i$ increases, the delay of the appearance, and the lower value, of the radius inflation peak, as stated in Section 3. The curves end with thick dots, where the periastron $p = a(1 - e)$ of the orbit might be slightly smaller than the Roche limit. We find $0.0207 \lesssim p \lesssim 0.0229$. See Section 4 for more discussion. (A color version of this figure is available in the online journal.)

The curves end with thick dots, where the periastron of the orbit reaches the Roche limit represented by a brown horizontal line in the bottom left panel. The ranges of ages for which simultaneous fits are obtained within the plotted 1σ measurement limits for $R_p, e$, and $a$, are represented by two orange vertical segments. These ranges are very narrow ($\lesssim 0.2$ Gyr) in comparison with the estimated age of the planet, 6.5±2.3 Gyr. It is awkward to suggest that we observe this planet at a very special transitional period in its life. However, if we relax the fitting criterion (on $R_p, e$, and $a$) from 1σ to 2σ or 3σ, there would be a wider range of ages for which the evolutionary tracks would fit the observed values. Therefore, our models would no longer imply that the present is such a special time in the life of this planet. The bottom right panel shows the evolution of $Q_e^*$, which increases with time, from $\log_{10}(Q_e^*) \sim 6.8$ to $\sim 7.6$ over the whole evolutionary path. This is because of its inverse dependence on the orbital period $P$, which decreases while the orbit circularizes, from $\sim 4.5$ days to $\sim 0.8$ days at the Roche limit. The fitting ranges are naturally at the measured period $P_\text{measured} \approx 1.3$ days for which $Q_e^* \approx 10^{3.4}$. This increase of $Q_e^*$ corresponds to a decrease in the tidal dissipation in the star and to a smoother evolution of its orbit. In the bottom left panel, we note an apparent jump of the slope in the semimajor axis evolution, previously pointed out and justified by Miller et al. (2009) in their simulation of the evolution of HD 209458b. This jump is respectively at 5, 6, 8 Gyr for $a_i = 0.050, 0.052, 0.054$ AU. It can be explained quasi-analytically using Equation (2) in a more compact form:

$$\dot{a} = \dot{a}_p e^2 + \dot{a}_s \left(1 + \frac{57}{4} e^2\right),$$

where $\dot{a}_p$ and $\dot{a}_s$ are the rates of evolution of $a$ due to the tides, respectively, raised on the planet and on the star.\footnote{Explicitly, the rates are defined by $\dot{a}_p = -2K_1p \frac{R_p^3}{\omega^2 Q_p}$ and $\dot{a}_s = -\frac{K}{\pi} K_2p \frac{R_p^3}{\frac{\pi}{3} Q_p^3}$.} Before this apparent jump, both tides contribute to the decrease of $a$, roughly by a comparable amount according to numerical tests. Then, during the rapid decrease of $e$ as shown in the top right panel of Figure 2, the components of the sum (8) that depend on $e$ fall even more rapidly because of the square dependence on $e$. Finally, after this short transitional phase, $a$ evolves only as $\dot{a}_s$, which is independent of $e$, leading to the analytical evolution described by Equation (6).

Figure 2 shows that, once the orbit has circularized, the radius still remains far above the value reached with no tides, as the top left panel demonstrates at the end of evolution depicted by the thick dots. This specificity, already mentioned by Ibgui & Burrows (2009), is due to thermal inertia. In sum, it is possible that an inflated planet with a circular orbit can be explained with tidal heating.

4.3. WASP-12b

WASP-12b, discovered by Hebb et al. (2009), is unique in many respects. It is the second-largest transiting planet to
date, with a radius of $1.79^{+0.09}_{-0.09} R_J$. Its mass is $1.41^{+0.10}_{-0.10} M_J$. Its estimated eccentricity is $e = 0.049^{+0.015}_{-0.015}$. It is the most heavily irradiated transiting EGP with a flux at the substellar point of $9.098 \times 10^7 \text{erg cm}^{-2} \text{s}^{-1}$. It has one of the shortest orbital periods, $P = 1.09142 \text{days}$. Its orbit is very close to its Roche limit, 0.0221 AU, while its periastron, $p = a(1 - e)$, may be below this limit: $0.0207 \lesssim p \lesssim 0.0229$ (see Table 1). The planet is perhaps on the verge of being tidally disrupted. It may also be losing mass due to Roche lobe overflow (Gu et al. 2003; Li et al. 2010). This phenomenon is not modeled here.

In order to fit the radius, Miller et al. (2009) invoke a floor on the eccentricity and an extremely rapid expansion of the radius when the planet starts to plunge. Our model provides fitting evolutionary curves without imposing a floor on $e$ and without the rapid expansion. Figure 3, similar to Figure 2, shows examples of fitting curves for a solar opacity atmosphere. As is done for WASP-4b, the radius evolution with no tides is drawn in black in the top left panel. The difference between the radii is $\sim 0.50 R_J$, that is 28%, much larger than for WASP-4b. Similar to WASP-4b, we obtain solutions for $\log_{10}[Q_*(P_0)] \leq 6.5$, and high values of $Q_*$ are required, $Q_* \gtrsim 10^{7-8}$. We show in Figure 3 three examples of fitting evolutionary curves with the smoothest plunging orbits ($Q_*(P_0) = 10^{6.5}$), for three different $a_*$: 0.053, 0.055, 0.057 AU, and an initial eccentricity of $e_0 = 0.73$. Lower $e_0$ is possible, down to 0.53, but for lower $Q_*(P_0)$ and, therefore, faster plunging orbits. The delay of the appearance of the radius peak and its lower value when $a_0$ increases are discernable. The bottom left panel shows the evolution of the semimajor axis and how close the planet is to its Roche limit. As for WASP-4b, the thick dots indicate the end of the evolution where the periastron reaches the Roche limit. Despite the $P^{-1}$ dependence of $Q_*$ (bottom right panel) and, therefore, the smoother plunging of the planet when the orbit has circularized (compared with the constant-$Q_*$ case), the eccentricity and the semimajor axis are decreasing extremely fast, especially after 1 Gyr. Thus, the age ranges where the radius of the planet, the eccentricity, and the semimajor axis simultaneously fit the measurements, are even narrower than for WASP-4b. Depicted by vertical orange segments, they are of the order of 50 Myr. Observing the planet in such a short interval of its life, questionable for WASP-4b, is even less likely for WASP-12b. Nevertheless, as we described in Section 4.2, using a $2\sigma$ or $3\sigma$ criterion for fitting the observed properties of the system would similarly alleviate this problem. Finally, as in the case of WASP-4b, the radius when the orbit has circularized is clearly above the radius obtained with no tides.

5. THE PLUNGING TIMESCALE UNCERTAINTY

An intriguing issue about the transiting EGPs is their subsequent evolution. Tidal evolutionary equations show that their fate is tidal disruption as they inscribe into their host star (Rasio et al. 1996; Levillard et al. 2009; Jackson et al. 2009a, 2009b; Ibugi & Burrows 2009; Miller et al. 2009). Once the orbit has circularized, the plunging timescale $\tau$ is described by an analytic expression, directly resulting from the evolution of $a$ (Equation (6)), where $P$ is the orbital period:

$$
\tau = \left( \frac{4}{117} Q_* \right) \left( \frac{a_0^{1/3} M_\star^{1/2}}{G^{1/2} M_p R_\star^2} \right) \left[ \frac{2}{117\pi} Q_* \right] \left( \frac{M_\star}{M_p} \right) \left( \frac{a_0}{R_\star} \right)^5.
$$

We define $\tau$ as the remaining time for the planet to fall into its host star from the present time, assuming that its current eccentricity is zero. We thus choose in this section $t_0$, defined in Section 2.4, to be the present time, and therefore $a_0$ to be the measured semimajor axis.

This formula shows the sensitive dependence of $\tau$ on the current semimajor axis of the system $a_0$. The latter is, however, quite precisely determined by observations (see Table 1). At the same time, Equation (10) exhibits a linear dependence of $\tau$ on $Q_*$, which is a very poorly constrained parameter.

To illustrate our point, we plot in Figure 4 the theoretical evolution, starting from the present time, of the semimajor axes $a(\text{AU})$ of two transiting EGPs, HD 209458b and WASP-12b. The time (in Myr) is represented logarithmically. Our aim is to emphasize the dependence of these evolution on $Q_*$, which follows the generic law described by Equation (7). The reference orbital period $P_0$ is the observed period, $P_0 = P_{\text{measured}}$. We explore three possibilities for $\gamma$, assumed to range from $-1$ to $+1$ (see Section 6): 1 (dotted curves), 0 (solid), and $-1$ (dashed). We also examine three possibilities for the $\beta$ factor: 5 (red curves), 6 (blue), and 7 (green). This enables us to roughly encompass the current observational and theoretical estimates of $Q_*$. The measured eccentricities of these systems are low enough to consider the tides raised on the planet to be negligible and, therefore, to ignore $Q_\varepsilon$. The analytical formula for the evolution of $a$ (Equation (6)) is applicable and so is the above definition of the plunging timescale (Equation (10)). We numerically checked this point by comparison with the integration of the
full evolutionary equations of Section 2.2. The Roche limits of both systems are plotted; they mark the end of the evolutionary curves represented by thick dots.

The linear dependence of $\tau$ on $Q_*$ directly affects the evolutionary curves. For HD 209458b, the order of magnitude of the plunging timescale can be 0.5 Gyr ($\beta = 5$), 5 Gyr ($\beta = 6$), or 50 Gyr ($\beta = 7$). The dependence on the $\gamma$ parameter is weaker, since the ratio $P/P_{\text{measured}}$ remains around 1. The trend is that a positive $\gamma$ results in the decrease of $Q_*$, while the orbital period $P$ of the planet decreases as it spirals into its host star. This results in an increase of the tidal torque exerted on the planet and, therefore, in accelerated plunging in comparison to the case with $\gamma = 0$ ($Q_*$ constant) and a fortiori to the case with $\gamma = 1$ ($Q_* \propto P^{-1}$). Including all these uncertainties, HD 209458b can plunge between 0.5 and 60 Gyr from now, which is a two order-of-magnitude range. Note that Levrard et al. (2009) provide the evolution curve of HD 209458b for one of the cases considered here: $\beta = 6$ and $\gamma = 1$, (blue dotted curve). Their result is consistent with ours. By the same token, we find that WASP-12b can plunge between 0.1 and 100 Myr from now, which is a three order-of-magnitude range. The much shorter timescale, in comparison with that for HD 209458b, is mainly due to the ratio of the semimajor axis ($a = 0.0471$ AU for HD 209458b and $a = 0.0229$ AU for WASP-12b, see Table 1) combined with the power dependence $\tau \propto a^{13/2}$. Note that for WASP-12b, the relative influence of the $\gamma$ parameter compared with that for the $\beta$ parameter is bigger than for HD 209458b.

The evolutionary curves can overlap.

This figure demonstrates that it is difficult to predict the evolution of the orbits of transiting EGPs, given the poor knowledge of the tidal dissipation factors in the host stars. However, the recently discovered WASP-18b (Hellier et al. 2009), which has an orbital period of only 0.94 days, might provide the evolution curve of HD 209458b for one of the cases considered here: $\beta = 6$ and $\gamma = 1$, (blue dotted curve). Their result is consistent with ours. By the same token, we find that WASP-12b can plunge between 0.1 and 100 Myr from now, which is a three order-of-magnitude range. The much shorter timescale, in comparison with that for HD 209458b, is mainly due to the ratio of the semimajor axis ($a = 0.0471$ AU for HD 209458b and $a = 0.0229$ AU for WASP-12b, see Table 1) combined with the power dependence $\tau \propto a^{13/2}$. Note that for WASP-12b, the relative influence of the $\gamma$ parameter compared with that for the $\beta$ parameter is bigger than for HD 209458b.

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6. CONCLUSIONS AND DISCUSSION

We have presented in this paper some new general results of the coupled radius–orbit evolutionary model described in Ibgiu & Burrows (2009), and we have applied the model to the inflated planets WASP-4b and WASP-12b. We assumed a two-body gravitational and tidal interaction between the planet and its host star, coupling the planetary radius and the orbit evolution. We included the tides raised on the planet and the tides raised on its host star, coupling the planetary radius and the orbit evolution. Stellar irradiation and a detailed planetary atmosphere are included. The fundamental result is the transient inflation of the planetary radius that temporarily interrupts its monotonous standard shrinking. An important point is that even though the current orbit of the planet has almost circularized, the radius of the planet can still be inflated due to an earlier episode of tidal heating. This is why we stress that an inflated planet with an observed circular orbit can still have tidal heating as an explanation of its radius. Fixing the planet and star properties, the model is controlled by four free parameters, $(Q_p', Q_*, e_i, a_i)$, that are the tidal dissipation factors in the planet and in the star, and the initial eccentricity and semimajor axis at the beginning of this two-body evolution. We stress the sensitive and nonlinear dependence of the evolutionary curves on these parameters.

We have demonstrated that the use of a constant time lag model for the orbital evolution does not improve the accuracy of the evolutionary calculations. Specifically, the equations of Hut (1981), though formulated in a closed form, are not valid at large eccentricity, as demonstrated by the equilibrium tide theory in a fluid body, which represents an active ongoing research field, along with dynamical tide theory. The latter is neglected in any coupled radius–orbit evolutionary study. Moreover, for the sake of completeness and for comparison with previous studies, we have tested the coupled radius–orbit model with the use of equations of Hut (1981) as recasted by Leconte et al. (2010). Unlike Leconte et al. (2010), we have been able to find evolutionary tracks that fit within the error bars, the radius, the eccentricity, and the semimajor axis of HD 209458b, in its current estimated age range, and for initial eccentricities as low as $\sim 0.3$. This reinforces our statement that tidal heating may explain the inflated radius of this planet. On the other hand, as stated in Ibgiu & Burrows (2009), an increased planet atmospheric opacity with no tidal heating is sufficient to explain the observed radius of HD 209458b. Thus, we provide two plausible explanations for this planet.

We have demonstrated that an increase of either the core mass $M_{\text{core}}$, of $Q_p'$, or $a_i$ results in a lower value of the radius inflation peak and in a delay of its appearance. The final semimajor axis is the same, whatever $M_{\text{core}}$ or $Q_p'$, but is larger when $a_i$ is larger. At an earlier age, the planet with the larger core has the smaller radius, but this is opposite at later ages.

We have enhanced our model by including an orbital period dependence of the tidal dissipation in the star. $Q_* \propto P^\gamma$, $-1 \leq \gamma \leq 1$. $Q_*$ drives the inspiral of the planet into its host star.

Applications of our model to recently detected transiting inflated planets show that

1. WASP-6b and WASP-15b can be fit at solar opacity over Gyr age ranges.
2. We have not found an acceptable fit for TrES-4, at either solar, $3 \times$ solar, or $10 \times$ solar planet atmospheric opacity.
3. WASP-4b can be fit at solar opacity with, for example, the combination $(Q_p', e_i, a_i) = (10^{0.0}, 0.80, 0.050)$ and with $Q_p' = 10^{6.5} \times (P/10 \text{ days})^{-1}$.
4. WASP-12b can be fit at solar opacity with, for example, the combination $(Q_p', e_i, a_i) = (10^{0.3}, 0.73, 0.055)$ and with $Q_p' = 10^{6.5} \times (P/10 \text{ days})^{-1}$.

For WASP-4b and WASP-12b, the ranges of ages that allow simultaneous fits of radius, semimajor axis, and orbital eccentricity, are very narrow, seeming to suggest that, if the two-body coupled evolutionary model described herein is in fact responsible for these planets’ inflated radii, then we are observing them at a special epoch in their evolution. However, relaxing the fit-criterion from $1 \sigma$ to $2 \sigma$ or $3 \sigma$ would alleviate this apparent problem.

Our results (in particular, for TrES-4) suggest that a coupled radius–orbit tidal evolution model might not on its own explain the radii of all the inflated transiting giant planets. An alternative scenario with stationary heating has been proposed (Ibgiu et al.
and applied to all the planets discussed in this paper. Though not providing direct solutions to the inflated radii issue, this scenario constrains the ratio $e^2/Q_p'$ for a given $M_{\text{core}}$. Finally, a combination of these two models could be imagined with a two-body interaction, followed by a quasi-steady low eccentricity phase due to perturbations by a second planet.

The last point we make in this paper is the uncertainty of the plunging timescale during the spiraling of the planet into its host star. This timescale is strongly dependent on the semimajor axis; specifically, it depends on $a$ to the 6.5 power. It also has a linear dependence on $Q_p$, which is a parameter that is uncertain by several orders of magnitude. We have shown that HD 209458b can plunge in between 0.5 and 60 Gyr from now, a two order-of-magnitude range. We have shown that HD 209458b evolutionary tracks (of $R_p$ stars. Improvements in this theory might result in different results, specifically, it depends on $a$, for the boundary conditions. L.I. thanks Chantal Stehlé for useful help on the computing of the atmospheric models related to the sources of change of transit time and duration, and Jason Nordhaus for useful discussions. The authors are pleased to acknowledge that part of the work reported in this paper was substantially performed at the TIGRESS high-performance computer center at Princeton University, which is jointly supported by the Princeton Institute for Computational Science (PICSceI) and Engineering and the Princeton University Office of Information Technology. Another substantial part of the computing work was performed at the DIO, Division Informatique de l’Observatoire de Paris. L.I. is grateful to LERMA, Meudon, France, for providing support to carry on and complete this study. This study was supported by NASA grant NNX07AG80G and under JPL/Spitzer Agreements 1328092, 1348668, and 1312647.

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