Constraining the Milky Way’s Ultraviolet to Infrared SED with Gaussian Process Regression

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ABSTRACT

Improving our knowledge of global Milky Way (MW) properties is critical for connecting the detailed measurements only possible from within our Galaxy to our understanding of the broader galaxy population. We train Gaussian Process Regression (GPR) models on SDSS galaxies to map from galaxy properties (stellar mass, apparent axis ratio, star formation rate, bulge-to-total ratio, disk scale length, and bar vote fraction) to UV (GALEX \( FUV/NUV \)), optical (SDSS \( ugriz \)) and IR (2MASS \( JHK_s \) and WISE \( W1/W2/W3/W4 \)) fluxes and uncertainties. With these models we estimate the photometric properties of the MW, resulting in a full UV-to-IR spectral energy distribution (SED) as it would be measured externally, viewed face-on. We confirm that the Milky Way lies in the green valley in optical diagnostic diagrams, but show for the first time that the MW is in the star-forming region in standard UV and IR diagnostics—characteristic of the population of red spiral galaxies. Although our GPR method predicts one band at a time, the resulting MW UV–IR SED is consistent with SEDs of local spirals with characteristics broadly similar to the MW, suggesting that these independent predictions can be combined reliably. Our UV–IR SED will be invaluable for reconstructing the MW’s star formation history using the same tools employed for external galaxies, allowing comparisons of results from \textit{in situ} measurements to those from the methods used for extra-galactic objects.

Key words: Galaxy:general – Galaxy:fundamental parameters – Galaxy:structure

1 INTRODUCTION

Within the Milky Way we have a unique opportunity to study the nuances of galactic properties, allowing us to test galaxy formation and evolution models at an unrivalled level of detail. For example, chemical abundances for hundreds of thousands of stars have been obtained from spectroscopic surveys (Majewski et al. 2017; Martell et al. 2017), and stellar surveys that catalogue distance and dynamical measurements on millions of stars have been performed (Gaia Collaboration et al. 2018). This exhaustive stellar information has helped constrain the Milky Way’s evolutionary history. In turn, high-resolution dynamical simulations have been able to produce galaxies of increased similarity to the Milky Way (Guedes et al. 2011; Sawala et al. 2016; Wetzel et al. 2016), matching fundamental galaxy properties such as dwarf satellite populations and reproducing characteristics of our Galaxy’s gas, dust, and stellar components. Comparisons between Milky Way stellar data and high-resolution hydrodynamical simulations of Milky Way-like galaxies are an increasingly useful way to improve our understanding of galaxy formation. However, the marriage between observations and models is delicate: incorrect assumptions on one side can propagate into the other. Simulators must make choices about how to implement crucial parameters that affect the galaxy evolution process, such as the gas density threshold for star formation to occur and the efficiency with which it proceeds; this is sometimes done by attempting to match observed properties of the Milky Way. But without knowing how our Galaxy fits in amongst the broader galaxy population, it is difficult to determine whether simulations match the Milky Way because they have the correct physics or because they have incorrectly tuned parameters that match by coincidence or design.

This is complicated by the exceptional difficulty of obtaining a global picture of the Milky Way, given our location in the disk and the obscuration caused by interstellar dust. As a result, there are properties that we can easily measure in external galaxies that are impossible to measure directly within our own, making it difficult to determine where we fit within the broader galaxy population.

Creating an outside-in picture of the Milky Way that spans a multitude of broad-band wavelengths will enable simulators to more accurately tune their physics assumptions, as it will then be possible to test whether quantities that can only be determined from large-scale stellar surveys and those that can be measured directly only for extragalactic objects are reproduced. The most basic, easiest-to-

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measure intrinsic quantities we can use to study galaxies are their luminosities and colours; once redshift is known, these can be inferred from broad-band photometry. Hence the focus of this paper will be in determining these properties for the Milky Way. This will enable our Galaxy to be placed on standard colour-magnitude and colour-colour diagrams and result in a multi-wavelength spectral energy distribution (SED) for the Milky Way.

Astronomers have found that galaxies in the local Universe predominantly fall into two populations: passively-evolving red galaxies with older stellar populations; or blue galaxies that are still forming stars. In the optical colour-magnitude diagram (CMD), these two galaxy populations are commonly referred to as the “red sequence” and the “blue cloud”. The colour bimodality of galaxies has been observed at both low redshift, \((z \sim 0.1\text{ e.g., } \text{Strateva et al. 2001; Baldry et al. 2004})\) and up to redshift of \(z \sim 1\) (e.g., Bell et al. 2004; Weiner et al. 2005). The region of the CMD between these two distinct galaxy populations is often referred to as the “green valley”.

This locus is thought to contain a transitional population of galaxies that are “passively” evolving in the sense that no new star formation is occurring (e.g., Bell et al. 2004; Faber et al. 2007), though they still can contain some younger stars. The increase in the fraction of red galaxies over time has led many astronomers to conclude that a galaxy first lives in the star forming blue cloud and then transitions into the green valley and ultimately into the red sequence in complete quiescence, with the galaxy growing more and more red over time due to the ageing stellar population. Green valley galaxies are presumed to be undergoing some form of quenching of their star formation (Salim et al. 2007; Schawinski et al. 2014; Smethurst et al. 2015) - either late-type galaxies that are gradually running out of their cold gas reservoir or having their star formation suppressed, or early-type galaxies that had their gas reservoirs rapidly destroyed. Objects found within the green valley may also simply be in the tails of the blue cloud or red sequence, rather than being in a transitional state (Taylor et al. 2015). While the precise details of quenching processes and the origin of the galaxy colour bimodality have yet to be determined, a galaxy’s location on the CMD remains a very useful tool for determining how a galaxy fits into the broader population.

The radiation emitted by a galaxy is characterised by its spectral energy distribution (SED), or flux as a function of wavelength. Galaxy SEDs contain the imprints of the physical processes occurring within - the stellar population’s ages and abundances (i.e., the star formation history and metallicity of the galaxy), the dust and gas content and the chemistry and physical state of the interstellar medium (ISM). Because different sources dominate the emission at different wavelengths, long-wavelength-baseline SEDs allow one to disentangle the contributing effects. This makes SEDs one of the best direct probes for studying galaxy formation and evolution from both an observational and theoretical modelling perspective.

However, comparing colours and luminosities of the Milky Way to external galaxies is not trivial, regardless of whether we compare to observed galaxies or to mock images from high-resolution hydrodynamical simulations (such as Eris Guedes et al. 2011, APOSTLE Sawala et al. 2016, and Latte Wetzel et al. 2016). Much of our view of the Galaxy is obscured by interstellar dust, especially at UV and optical wavelengths (e.g., Cardelli et al. 1989; Schlegel et al. 1998). Stars outside of the local solar region are reddened as a result of the dust obscuration. Determining the integrated light of stellar populations in the Milky Way is challenging due to the spread of stars over large and varying distances, with correspondingly large and varying dust extinction along lines of sight to the Earth. This makes the study of any portions of the Galactic disk beyond the solar neighbourhood exceptionally difficult, and results in a fragmented picture of the Milky Way. Integrated properties that are relatively painless to obtain in external galaxies (though dust obscuration can affect these observations as well, see e.g., Masters et al. 2003, 2010a), are impossible to obtain directly within our own Galaxy (e.g., Much et al. 2011). As a result, simulators often must resort to comparing their simulated Milky Ways to very general galaxy populations (such as sets of Sbc or late type galaxies) that, while superficially resembling the Milky Way, have a wide range of other global properties (e.g., Guedes et al. 2011).

In an effort to circumvent our limited view of the Milky Way, we can study galaxies that mimic the properties of our Galaxy but can be observed from outside, which we label Milky Way Analogues (MWAs). This method hinges on the Copernican assumption that the Milky Way should not be extraordinary among a galaxy population that shares some key properties with it. These comparisons are enabled by working within volume-limited subsets of large surveys, which ensure that the observed objects constitute a representative population. Previous work suggests that galaxies with similar stellar mass and star formation rates are also similar in other properties, as the observed galaxy population is well-matched by models that parameterise galactic star formation histories with a limited collection of curves (Behroozi et al. 2013; Gladders et al. 2013; Abramson et al. 2014; Kelson et al. 2014). Even further, Bell & de Jong (2001) showed mass and star formation rate are strongly correlated with the photometric properties of a galaxy. Therefore we can exploit the fact that two galaxies of identical mass and star formation rate should have similar luminosities and colours, with some scatter given the range of galaxy photometric properties at fixed physical parameters. Licquia et al. (2015), hereafter LNB15, utilised this to constrain the Milky Way’s optical colours and magnitudes based on the range of observed properties of MWAs that were matched in stellar mass and star formation rate. MWAs also allow direct comparison of properties of our Galaxy to its closest peers (e.g., Licquia & Newman 2016; Licquia et al. 2016; Fraser-McKelvie et al. 2019; Boardman et al. 2020a; Krishnarao et al. 2020) and have been a successful tool for improving our understanding of the Milky Way in an extra-galactic context.

The Milky Way, however, has some characteristics that are atypical (at the \(<2\sigma\text{ level}\) amongst its peers - e.g., the Milky Way has an unusually compact disk (i.e., a small disk scale-length) (Bovy & Rix 2013; Bland-Hawthorn & Gerhard 2016; Licquia & Newman 2016), and an unusually quiescent merger history (from observation Unavane et al. 1996; Ruchti et al. 2015), (and simulation, e.g., Fielder et al. 2019; Carlesi et al. 2020). The deviations of the Milky Way from the average suggest that we should consider parameters beyond just stellar mass and star formation rate in order to identify samples of objects that more closely resemble the Milky Way.

Galaxy morphological characteristics such as disk scale length \(R_d\) and bulge-to-total ratio \((B/T)\) are tied to a galaxy’s evolutionary history and therefore should connect to its photometric properties (Cappellari 2016; Saha & Cortesi 2018) as well as to the ways in which the Milky Way is atypical. We would therefore wish to incorporate these properties in addition to stellar mass and star formation rate in defining an MWA. However, as the number of parameters required to match the Milky Way increases, the number of MWAs correspondingly reduces dramatically. For example Fraser-McKelvie et al. (2019) only found 179 analogues when selecting on stellar mass, bulge-to-total mass ratio, and morphology; Boardman et al. (2020b) and Boardman et al. (2020a) found no MWAs within 1\(\sigma\) of the MW when selecting on stellar mass, star formation rate, bulge-to-total ratio, and disk scale length in either the SDSS-IV MaNGA survey (Bundy et al. 2015) or a larger photometric sample.
LNB15 found that the colour of the Milky Way is consistent with the green valley region of the CMD as it has been defined using purely optical passbands. Characterising the UV and IR colours of the Milky Way can provide more sensitive probes of whether it would be classified as in the process of quenching if seen from outside, allowing us to better understand what type of population the Milky Way may belong to. LNB15 speculated that the Milky Way might belong to the population of massive "red spiral" galaxies, which are characterised by their red optical colours despite ongoing star formation (Masters et al. 2010b; Cortese 2012). Galaxies within this population may be moving into the green valley due to slow quenching (cf. Schawinski et al. (2014)). This conjecture can only be fully tested by examining wavelengths outside of the optical range; in $g - r$, the colours of massive spiral galaxies on the star-forming main sequence (a population that should include the Galaxy) overlap with both the red sequence and the blue cloud (Cortese 2012; Salim 2014). However, samples of MWAs that have high-quality photometry over a broader wavelength range will have reduced numbers due to the limited coverage of sufficiently deep photometry in GALEX.

To address the lack of analogues when multi-dimensional parameter spaces are used, and the smaller overall sample size resulting from the increase in wavelength coverage, we introduce a Gaussian process regression (GPR) approach in this work. GPR is an emergent tool in astrophysics. For example, Bocquet et al. (2020) employed GPR to emulate results of cosmological simulations, while Gordon et al. (2020) used GPR to detect and classify exoplanets. We can use GPR to leverage information from a wider variety of galaxies, instead of just the closest Milky Way analogues, in order to extract information from large-scale trends between galaxy physical and photometric properties. Thanks to the probabilistic framework that underlies GPR, we obtain uncertainty estimates for all predicted quantities for free. The primary result from this paper will be an ultraviolet to infrared SED of the Milky Way as viewed face-on, determined via GPR based on star formation history and structural parameters (i.e., galaxy physical parameters) that have been measured well for both the Milky Way and galaxies from the Sloan Digital Sky Survey (Aihara et al. 2011).

The paper is organised in the following manner. In Section 2 we describe the observational data used, including the external galaxy data in Section 2.1 and Section 2.2, and estimated properties of the Milky Way in Section 2.3. Section 3 details our new Gaussian process regression-based methodology. In Section 4 we compare the luminosity and colours of the Milky Way at multiple wavelengths and the Milky Way’s predicted SED to properties of other galaxies. Finally we summarise our results, and discuss implications and future work in Section 5. Appendix A provides a summary of the galaxy parameters and tables that list predicted photometry for the Milky Way. Appendix B describes tests of the accuracy of the GPR procedures used here, and Appendix C describes how we address the systematic corrections needed for $k$-corrections and Eddington bias.

In this paper all magnitudes are reported in the AB system, except for the Johnson-Cousin $UBVRI$ magnitudes which are presented in the Vega system. Absolute magnitudes are derived using a Hubble constant $H_0 = 100$ km s$^{-1}$Mpc$^{-1}$, so they are equivalent to $M_B - 5 \log h$ (where $M_B$ is the $B$-band absolute magnitude and $h = H_0/100$) for other values of $h$. For other properties in which measurements for the Milky Way are compared to extra-galactic galaxy measurements we assume $H_0 = 70$ km s$^{-1}$Mpc$^{-1}$ ($h = 0.7$) in accordance with Licquia et al. (2015), for a standard flat $\Lambda$CDM cosmology with $\Omega_m = 0.3$. Parameters such as log stellar mass and log star formation rate can be modified for different $h$ values by subtracting $2 \log h/0.7$. We do this to avoid confusion and to allow for potential updates to future $h$ measurements.

2 OBSERVATIONAL DATA

In this section we describe the many galaxy catalogues utilised in this work. We break this up by photometry (Section 2.1) and inferred galaxy properties (Section 2.2), with the Milky Way measurements included in the final subsection (Section 2.3).

2.1 Photometry

2.1.1 SDSS Galaxies

The sample of galaxies that we use as a starting point originates from the eighth data release (DR8; Aihara et al. (2011)) of the Sloan Digital Sky Survey III (SDSS-III; York et al. (2000)). DR8 provides both images and photometry of thousands (almost $10^5$) of local galaxies. The optical broadband passbands, $u, g, r, i, z$ and $z$ were the subjects of previous Milky Way analogue work by LNB15 and are used in this study in addition to bands outside of the optical range.

We make use of both the "mode1" and "cmode1" magnitudes from SDSS. The former refers to magnitudes derived from the better of either a de Vaucouleurs or an exponential profile fit to the galaxy surface brightness distribution. These types of magnitudes are expected to produce the highest signal-to-noise estimate of galaxy colours; thus when we refer to galaxy colours derived from SDSS we will be using mode1 magnitudes for the calculations. Alternatively, cmode1 magnitudes are derived from the best fit to a linear combination of de Vaucouleurs and exponential profile. These magnitudes provide the best estimate of the total flux of a galaxy in each passband. When we refer to galaxy absolute magnitudes for SDSS bands we will use "cmode1" magnitudes.

$k$-corrections on these passbands to rest-frame $z = 0$ were calculated via the kcorrect v4.2 software (Blanton & Roweis 2007), as described in LNB15. This provided AB absolute magnitudes for the SDSS $ugriz$ photometry. Additionally, kcorrect was used to convert the SDSS $ugriz$ photometry to restframe Johnsons-Cousins $UVBRi$ Vega magnitudes in order to make easy comparisons to literature values. Results are presented with the adoption of the Blanton & Roweis (2007) and LNB15 notation, where an absolute magnitude of passband $y$ at redshift $z$ is denoted as $^yM_y$.

Our main galaxy sample is derived from the volume-limited sample presented in LNB15. A volume-limited sample is required for accurate results from Milky Way analogues in order to alleviate a radial selection effect known as Malmquist bias, i.e., the preferential inclusion of intrinsically bright galaxies. At higher redshifts within the main SDSS sample (Strauss et al. 2002), only the most luminous galaxies will be brighter than the sample magnitude limit and followed-up spectroscopically. By using a volume-limited sample we ensure that galaxies within the range of the Milky Way’s parameters are included equally at all distances considered. LNB15 determined the limits for their volume-limited sample from an initial draw of Milky Way analogues from the full SDSS DR8 parent catalogue without any redshift cuts. Then in $^0(g - r)$ vs. $^0M_r$ (i.e., restframe $g-r$ colour derived using $z = 0$ passbands versus $r$-band absolute magnitude, again evaluated with the $z = 0$ passband) colour-magnitude space a maximum redshift was chosen such that all objects as low in luminosity as the faintest Milky Way analogues would still be included at that $z$. A minimum redshift was...
also applied to limit the impact of the finite SDSS fiber aperture on measured galaxy properties. The resulting volume-limited sample contains a total of 124,232 target galaxies within the redshift range of \(0.03 < z < 0.09\). Some initial cuts on SDSS quality flags were also employed; for further details on the construction of this volume-limited sample refer to LNB15, Section 3.1. All cross-matches from SDSS to other catalogues presented here were constructed only using the volume-limited sample. Both the SDSS sample used here and the cross-matched catalogues presented in this paper are available at our catalogue GitHub repository\(^1\).

### 2.1.2 GALEX–SDSS–WISE Legacy Catalogue

Photometry in ultraviolet and infrared wavelengths used in this work comes from the GSWLC (Salim et al. 2016, 2018). We use GSWLC-M2, the medium-deep catalogue of GSWLC-2, which covers 49\% of the SDSS DR10 footprint. While this reduces the number of targets for study, the improved signal-to-noise in the UV-imaging over the shallow catalogue enables tighter results. SDSS Photometry between DR7/DR8 and DR10 is the same, so no issue arises between cross matches of the SDSS volume-limited sample and the GSWLC-M2 sample. In order to account for any differences in astrometry we consider matches to be separations within 1.5 arcseconds.

The ultraviolet sample in the GSWLC catalogue originates from the GALEX survey (\(FUV, NUV\)), and has been corrected for galactic reddening and calibration errors (Martin & GALEX Team 2005). UV detections are available for 74\% of SDSS targets in the GSWLC-M2 catalogue. The infrared sample in the GSWLC catalogue originated from the 2MASS and WISE surveys. The 2MASS photometry (\(JHK_s\)) is available for 48\% of SDSS targets and WISE photometry (\(W1, W2, W3, W4\)) is available for 41\% of SDSS targets.

### 2.1.3 DESI Legacy Imaging Surveys

Data release 8 of the DESI (Dark Energy Spectroscopic Instrument) Legacy imaging surveys includes \(g, r, z\), and WISE photometry of sources detected in Decam or BASS/MzLS imaging (Dey et al. 2019). We use these catalogues for all WISE-band photometry presented here, as the matched-model measurements from the Legacy Surveys Tractor catalogues go substantially deeper and have lower errors than other public WISE data products. We also calculate \(k\) corrections for the WISE bands using this photometry; we discuss our procedures for this, which allow accurate \(k\) corrections in the IR without making any assumptions about SED templates at those wavelengths, briefly in Section C1 and more extensively in a forthcoming paper (Fielder, Newman, & Andrews, in prep.).

DESI Legacy and SDSS contain photometry that comes from differing filter sets and detectors. DESI Legacy North uses BASS and MOSAIC filters while DESI Legacy South used Decam filters (Dey et al. 2019). Our corrections for offsets between the DESI Legacy Survey and SDSS photometry is described in Section C2.

### 2.2 SDSS-based Properties

Only one of the galaxy properties which we use to create SEDs is recorded in the standard SDSS photometric catalogues and required no further analysis; specifically, the best fit axis ratio (\(b/a\)) of a galaxy. We use the \(b/a\) value determined from an exponential fit to the galaxy’s surface brightness density in the \(r\)-band throughout this work.

Axis ratio is used here as a proxy for galaxy inclination. Disk galaxies with very small axis ratios are likely heavily tilted from our perspective, in contrast to face-on disks which would typically have axis ratios of \(b/a \sim 0.9\) (Cho & Park 2009; Maller et al. 2009). Highly-inclined star forming galaxies suffer greatly from dust obscuration which can effect their colours and magnitudes by making them appear redder and fainter than their intrinsic properties (e.g., Conselice et al. 2010; Masters et al. 2010a; Morselli et al. 2016; Kourkchi et al. 2019). This effect is important to consider when we are making predictions for a disk galaxy like the Milky Way.

#### 2.2.1 MPA-JHU Masses and Star Formation Rates

The MPA-JHU galaxy property catalogue provides total mass and star formation rate estimates for galaxies within SDSS DR7. LNB15 developed an updated version of this catalogue using the SDSS DR8 photometry. To summarise, the masses are calculated by fitting stellar population synthesis model to the galaxy’s photometry (instead of spectral features, as in Kauffmann et al. (2003); Gallazzi et al. (2005)), similar to Salim et al. (2007). The star formation rates are calculated by emission-line modelling based upon Brinchmann et al. (2004) with some updates. In order to account for aperture bias LNB15 follows the method of Salim et al. (2007) in calculating photometry for the light that falls outside of the fibre and fitting stochastic stellar population synthesis models to it. Thus each galaxy SFR measurement consists of a combination of the SFR measured from inside and outside of the SDSS fibre. For more details on these calculations refer to LNB15, Section 2.2.2 and Brinchmann et al. (2004). In both cases a Kroupa initial mass function (broken power law) is assumed (Kroupa & Weidner 2003).

Each galaxy in our volume-limited sample is assigned a posterior probability distribution function (PDF) of log stellar mass and log star formation rate, as well as a corresponding cumulative distribution function (CDF) determined by the posterior. Nominal values used for the \(\log M_\star\) and log SFR of each object are taken to be the mean value of the posterior. The standard deviation (\(\sigma\)) on these values are calculated from percentiles (\(P\)) in the CDF provided in the MPA-JHU measurements; we take \(\sigma = (P_{84} - P_{16})/2\), i.e., half the difference between the 16\% and 84\% percentile values. This gives us estimates of the stellar mass and star formation rates as well as their associated errors for each galaxy in the sample.

The GSWLC-2 catalogues which we use for photometry also provide computations of stellar masses and star formation rates. We chose to use the MPA-JHU results instead in order to avoid any systematic effects that may result from using the same measurements for our galaxy properties that were used to determine these derived quantities. We have tested how our results change if we use GSWLC-2 masses and star formation rates and find minimal differences, as summarised in Section 4.2.4.

#### 2.2.2 Simard et al. Bulge and Disk Decompositions

In order to characterise the light-weighted bulge-to-total ratios (\(B/T\)) and disk scale lengths (\(R_d\)) for SDSS galaxies, we use the catalogue of Simard et al. (2011). This work performed galaxy image decompositions for objects within the Legacy area in SDSS via the GIM2D software package (Simard et al. 2002).

Specifically, we use the catalogue of fits based on composites of a Sersic \(n = 4\) bulge and a pure exponential disk (Sersic \(n = 1\)). For

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\(^1\) https://github.com/cfielder/Catalogs
our analysis we use the $B/T$ computed in $r$ band, as it is expected to be more stable than $g$-band measurements. As a check we have performed our analyses using $B/T_e$ instead of $B/T$, and found minimal differences (cf. Section 4.2.4).

It is worth noting that Kruk et al. (2018) finds that these bulge+disk decompositions can be somewhat inaccurate when applied to strongly-barred galaxies, which may lead to some biases within our galaxy sample. However, fits optimised for barred galaxies have been performed for only samples of a few thousand objects, inadequate for our purposes, so we rely on the Simard et al. (2011) results here. The Simard et al. (2002) bulge and disk decompositions are derived using a $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$.

2.2.3 Galaxy Zoo 2 Bar Presence

The presence of bars have been speculated to be correlated to the star formation history within a galaxy. However, the sense of the effect is unknown; a bar may be related to an increase in star formation (e.g., Hawarden et al. 1986, 1996; Devereux 1987; Hummel et al. 1990), no effect (Pompea & Rieke 1990; Martinet & Friedli 1997; Chapelon et al. 1999), or decreased star formation (e.g., Vera et al. 2016; Díaz-García et al. 2020; Fraser-McKelvie et al. 2020), along with other potential effects on colour and metallicity (see e.g., Masters et al. 2011). Regardless of the source of any correlations, we wish to incorporate any possible differences having a bar may cause when we determine the Milky Way’s SED, since our Galaxy does exhibit clear evidence of a bar (e.g., Blitz & Spergel 1991). The Galaxy Zoo 2 (GZ2) catalogue (Willett et al. 2013; Hart et al. 2016) contains identifications of detailed morphological features in disk galaxies, such as spiral arms and bar presence. The galaxies classified in Galaxy Zoo 2 are a sub-sample of the brightest and largest galaxies in the SDSS Main Galaxy Sample.

The Galaxy Zoo projects are open public projects in which members of the community identify whether they found a variety of galaxy features in the images provided. In the Galaxy Zoo 2 catalogue the number of raw votes for each morphological feature is weighted (to account for user consistency) and adjusted to mitigate the impact of redshift-dependent biases, yielding a corrected fraction of volunteers who identified a given morphological feature for each galaxy. In our case we focus on the debiased fraction of volunteers who identified a galaxy as having a bar, which we denote by $p_{bar}$ (following Galaxy Zoo labelling conventions; however, this is a fraction of votes, not a probability). Above some threshold in $p_{bar}$ (after cuts in related parameters) we can be confident that a given galaxy indeed hosts a bar. Willett et al. (2013) developed the initial version of Galaxy Zoo 2 bar thresholds; more conservative thresholds were later defined by Galloway et al. (2015).

Often, when one uses Galaxy Zoo results it is necessary to consider responses to previous questions that influence whether the question of interest is even presented to the volunteers. For example, as described in Willett et al. (2013), a voter is only asked “Is there a sign of a bar feature through the centre of the galaxy?” if they first selected that the object has “features or disk” when asked “Is the galaxy simply smooth and rounded, with no sign of a disk?”, and then responded in the negative when asked “Could this be a disk viewed edge-on?”. One would then identify galaxies that have received a large fraction of “yes” votes as containing bars. It is worth noting that Willett et al. (2013) states galaxies that receive fewer than 10 net votes for a given question may not have a reliable classification. Therefore to construct a bar sample using the minimum allowances of Willett et al. (2013), using the debiased vote fractions from Hart et al. (2016) (which was an improvement to the debiasing of Willett et al. (2013)), one would use the cuts in the second and third row of the third column of Table 3 in Willett et al. (2013) with $p_{bar} \geq 0.3$ as recommended by Galloway et al. (2015) and additional voting number thresholds $N_{not\ edgeon} \geq 10$, and $N_{bar} \geq 10$.

However, here we do not want to consider only information on the relationship between galaxy star formation history, structural properties, and photometric properties that comes from those galaxies that most definitively host bars. Rather, it is desirable for the training sample for our Gaussian process regression model to include objects spanning a broad range of parameter space: this yielded smaller net prediction errors in Milky Way photometric parameters than when small, restricted samples are used for training. For additionally discussion on this choice refer to paragraph 4.2.4.3.

To summarise, the primary set of data that we employ in this paper consists of a cross-match between the original SDSS DR8 volume-limited sample reported in LNB15, an updated version of the MPA-JHU catalogue (Brinchmann et al. 2004) of masses and star formation rates in SDSS, the Simard et al. (2011) morphological bulge-disk decomposition catalogue, the GSWLC-2 medium-deep photometry catalogue (Salim et al. 2016, 2018), the Galaxy Zoo 2 catalogue (Willett et al. 2013; Hart et al. 2016), and the DESI Legacy survey DR8 catalogue (Dey et al. 2019). These cross-matches were performed using the astropy coordinates package. We required that objects be separated by less than 1.5 arcseconds from a counterpart in the SDSS volume-limited sample to be considered a match, and discard any galaxies that are not included in all catalogues considered.

Our sample is smaller than the original volume-limited sample as a result. The MPA-JHU catalogue contains all objects from the volume-limited sample, so we begin with the same set of 124,232 galaxies utilised by LNB15. After matching to Simard et al. (2011), 123,167 galaxies remain; some objects are lost due to minor differences between the DR7 and DR8 SDSS catalogues. When we require GSWLC-M2 measurements, we are left with only 60,857 galaxies, as roughly half of the SDSS footprint is covered by medium-deep GALEX, 2MASS, and WISE photometry (see Section 2.1.2). After matching to GZ2 29,836 galaxies remain, a consequence of the brighter magnitude limit used to select GZ2 objects compared to the original SDSS Main Galaxy Sample (see Section 2.2.3). We note that due to the brighter magnitude limit of GZ2, our final sample is no longer volume-limited. This would yield biased results if we were measuring aggregate properties of Milky Way analogues. However, for our GPR methodology this only modulates the density of our training set within parameter space, causing larger prediction errors due to the sparser sampling, but not leading to a bias.

Finally, matching to DESI Legacy leads to only a minor reduction in galaxy number as it covers a super-set of the SDSS area with deeper photometry (but different reduction algorithms). After these cross matches we then remove objects with photometric values of “NaN”, infinity, or $-99$, which all indicate missing photometry in one catalogue or another. This is only done when necessary for the evaluation of the GPR. For our WISE $k$-correction calculations (see Appendix C1) we will exclude objects with large WISE photometry errors in a given band, which we do not propagate into our main galaxy sample.

The final data sample consists of 29,588 galaxies in total from redshift $0.03 < z < 0.09$, which is publicly available at our catalogue GitHub. The parameters which we will use to predict photometric properties are stellar mass ($M_*$), star formation rate (SFR), galaxy axis ratio as a proxy for inclination ($b/a$), bulge-to-total ratio ($B/T$), disk scale length ($R_d$), and corrected bar vote fraction ($p_{bar}$). Covariances amongst these parameters are minimal, as discussed further in
Appendix A1: we show joint distributions for these parameters in Fig. A1.

2.3 Milky Way Properties

A number of the properties of the Milky Way used in this study have been derived using the Bayesian mixture model meta-analysis method first presented in Licquia & Newman (2015). This technique combines information from multiple measurements in order to obtain aggregate constraints on the Milky Way’s properties, taking into account the possibility that individual measurements could be incorrect or have their errors miss-estimated. This Bayesian method is combined with Monte Carlo simulations in order to account for uncertainties in the Sun’s measured Galactocentric radius, $R_0$; the Galactic exponential disk scale length; and uncertainties in the local surface density of stellar mass. The inferred mass of the Milky Way’s stellar disk depends upon all three of these parameters.

Gravity Collaboration et al. (2019) recently obtained a greatly improved geometric measurement of our distance from the centre of the Milky Way, $R_0 = 8.178 \pm 0.026$ kpc (Licquia & Newman 2015) used the value $8.3 \pm 0.35$ kpc from Gillessen et al. (2009)). We have rerun the Bayesian mixture model inference from Licquia et al. (2015) and Licquia & Newman (2016) with this updated measurement of the Galactocentric distance to obtain updated measurements of the Milky Way’s mass, bulge-to-total mass ratio, and disk scale length; see those papers for all details of the data sets used and the calculations (the star formation rate estimate from Licquia & Newman (2015) is not affected by the value of $R_0$, so we adopt it unchanged). The resulting Milky Way parameters used in this study are as follows:

- Bulge Mass $M_B^D = 0.90 \pm 0.06 \times 10^{10} \, M_\odot$
- Disk Mass $M_\ast^D = 4.58_{-0.94}^{+1.18} \times 10^{10} \, M_\odot$
- Total Stellar Mass $M_\ast = 5.48_{-0.94}^{+1.18} \times 10^{10} \, M_\odot$
- Star Formation Rate SFR= $1.65 \pm 0.19 \, M_\odot$ yr$^{-1}$ (Licquia & Newman 2015)
- Log Specific Star Formation Rate log $\frac{\text{SFR}}{M_\ast}$ = $-10.52 \pm 0.10$
- Bulge-to-total Mass Ratio $B/T = 0.16 \pm 0.03$
- Disk Scale Length $R_d = 2.48_{-0.15}^{+0.14}$ kpc

The revised stellar mass estimate for the Milky Way is smaller than the previous estimate, but by an amount that is significantly smaller than the previously-estimated uncertainties.

By design these physical parameters are constructed such that they can be directly compared to the extragalactic catalogues used to predict photometric properties. For example, the stellar mass and star-formation rate estimates for the Milky Way are constructed assuming a Kroupa initial mass function (Kroupa & Weidner 2003) and an exponential disk model, which is the same way in which mass and star formation rates were calculated in the MPA-JHU catalogue (Brinchmann et al. 2004) that we use in this analysis. The only parameter where this is not fully the case is the bulge-to-total ratio, $B/T$. For the Milky Way we can securely estimate only a mass-weighted value for this quantity. In external galaxies, however, the mass-weighted $B/T$ is much more difficult to obtain, and light-weighted measurements tend to be more reliable. However, our predicted Milky Way properties do not change significantly when we switch from $r$-band-based to $g$-band-based $B/T$ values, even though mass-to-light ratios differ significantly between these bands, suggesting that this is not a major issue.

Our analysis also depends on two additional parameters that are determined independently of any Bayesian mixture model meta-analysis. These quantities have values assigned to them based on their meaning in their respective catalogues and our understanding of the Milky Way:

- Axis ratio (inclination proxy) $b/a = 0.9 \pm 0.1$
- Bar vote fraction $p_{\text{bar}} = 0.45 \pm 0.15$

Galaxy inclination has a strong effect on colour and luminosity measurements for disk galaxies. As mentioned in Section 2.2, dust alters the observed colours and magnitudes of star-forming galaxies that are highly inclined or edge-on. Our perspective within the Milky Way makes it somewhat equivalent to being edge-on to us (though our position within the Galaxy, rather than outside, does cause some differences). However, photometric properties are most cleanly determined for those objects which are observed face-on. Therefore, we predict the SED that would be observed for the Milky Way for axis ratio values drawn from a uniform distribution spanning from $b/a = 0.8$ to $1.0$, consistent with the intrinsic axis ratios of spiral galaxy disks as described by Maller et al. (2009) and in Section 2.2. Our results should therefore correspond to the properties of our Galaxy if it were observed face-on. While axis ratio is a good proxy for inclination, it is not a perfect substitute. For example, van de Sande et al. (2018) found that disk galaxies that are rounder ($b/a \sim 1$) tend to be older and therefore intrinsically redder. This means that small biases could result from treating the Milky Way as having a face-on axis ratio. However, there is no straightforward way to avoid this, and the effect should be small compared to other sources of error.

The Milky Way exhibits clear evidence that it contains a bar (e.g., Blitz & Spergel 1991; Shen & Zheng 2020). However, very few Galaxy Zoo 2 galaxies have $p_{\text{bar}} = 1.0$, and those galaxies with the highest bar vote fractions are expected to have very strong bars, which may not match our Galaxy. Therefore we assume that in Galaxy Zoo 2 the Milky Way would have a vote fraction above the threshold for defining a bar, but not a value higher than the bulk of barred galaxies. Galloway et al. (2015) and Willett et al. (2013) find that $p_{\text{bar}} \geq 0.3$ serves as a reliable threshold between bar presence and lack thereof. Galaxies with $0.3 \leq p_{\text{bar}} < 0.5$ likely have weaker bars while galaxies with $p_{\text{bar}} > 0.5$ likely have stronger bars. Because the bar strength of the Milky Way’s bar as it would be determined from outside our Galaxy is not well-constrained, we treat the bar vote fraction for the Milky Way as uniformly distributed between $p_{\text{bar}} = 0.3$ and $p_{\text{bar}} = 0.6$. Choosing a larger mean vote fraction has small effect on our results, given the large range of fractions considered (compared to the distribution of $p_{\text{bar}}$ in GZ2).

3 GAUSSIAN PROCESS REGRESSION FOR PREDICTING MILKY WAY PHOTOMETRY

In this subsection we describe how Gaussian process regression can be used to estimate photometric properties for the Milky Way. First we explain the need to transition from Milky Way analogue-based methods to GPR when we consider higher-dimensionality parameter spaces in Section 3.1. In Section 3.2 we explain the basic concepts behind GPR, and in Section 3.2.3 we highlight the fundamental differences between GPR and analogue galaxy methods. Section 3.2.1 describes the kernel used to set up our GPR, which guides how information is propagated from training objects to predictions. We briefly describe the computational limitations of the GPR implementation we are using in Section 3.2.2. Lastly, Section 3.2.4 investigates the contributions of various sources of uncertainty to our GPR predictions.
3.1 Limitations of Using Analogue Galaxies

Using Milky Way analogues to predict the photometric properties of the Milky Way, as was done in Licquia et al. (2015), has been a very useful methodology but also has limitations. Of particular concern is the dramatic reduction in MWA sample size that occurs as the number of parameters that must be matched increases; we would like to move from the two parameters considered by Licquia et al. (2015) (M* and SFR), to a total of six, adding b/a, B/T, R_d, and p_bar. Requiring that analogues be Milky Way-like in more ways should reduce the spread in photometric properties of the resulting sample, potentially enabling stronger constraints. There are only limited correlations between these six parameters (cf. Fig. A1 in Appendix A1), so degeneracies between them are minimal: they each add new information. For instance, galaxies with stellar mass and star formation rate matching the Milky Way exhibit bulge-to-total ratios ranging from zero to one: structural and star-formation history parameters carry distinct information. However, while matching on additional galaxy parameters produces a population of analogues that must each be closer in properties to the Milky Way, the resulting MWA sample becomes much smaller (with as few as ~5 analogues in the sample that are within 3σ of the Milky Way in all of the properties considered, and none within 2σ for every aspect).

The reduction in the size of analogue galaxy samples as more parameters are considered is illustrated in Fig. 1. We plot the total number of galaxies that are within a given number of σ for every Milky Way parameter considered (where σ represents the uncertainty in the Milky Way value for a given property) as a function of the number of σ used as a threshold. The lightest shade of blue denotes the number of analogues within a given threshold when only considering stellar mass (M*) and axis ratio (b/a). The consecutive additions indicated in the legend represent the inclusion of the listed parameter in addition to all previous ones; i.e., we consecutively incorporate star formation rate, disk scale length, bulge-to-total ratio, and finally bar vote fraction. Hence, the darkest purple line shows the number of analogues when using all six parameters (which we have used in order of decreasing constraining power on Milky Way colours). The σ tolerances used for each parameter are the same Milky Way measurement errors defined in Section 2.3, except for p_bar. The significant uncertainty we fiducially ascribe to bar vote fraction would cause the 5 parameter and 6 parameter lines to be degenerate with one another; to avoid confusion we use σ_p_bar = 0.05 instead of 0.15 when constructing this plot.

The previous work by LNB15 would most closely correspond to the +SFR (3 parameters), medium blue line. If one were to restrict to objects within ±2σ of the Milky Way value for all parameters employed, there would be a total of zero Milky Way analogue galaxies when using five or more parameters. In the six-dimensional space that we employ below, there are only ~200 galaxies that are within even ±6σ of the Milky Way value in all six properties; at that extreme, analogue samples would be selecting objects that are not very close to the Milky Way at all. The lack of close analogues in high-dimensional parameter spaces makes constraints on Milky Way properties from the MWA method weak in that limit, with correspondingly large uncertainties.

3.2 Gaussian Process Regression: A Powerful Method for Interpolation and Prediction

To address the lack of Milky Way analogue galaxies in our multi-dimensional parameter space, we have developed alternative methods for predicting Milky Way properties based upon Gaussian process regression. In this sub-section we summarise the basic properties of GPR relevant for this work. For in-depth discussion, we refer the reader to Rasmussen & Williams (2006) and Görstler et al. (2019).

Gaussian process regression (sometimes called kriging) is effectively a method of interpolation where information from training data is accounted for by a smooth and continuous weighting function, called a “kernel” or covariance function. The joint probability distribution of the values of a Gaussian process at any finite set of points in parameter space will be a multi-variate Gaussian (with a number of dimensions set by the number of points in the set); the kernel specifies how the covariance between points depends upon their separation. The kernel should be a smooth and continuous function, with a length scale (which governs how far information propagates from a given point) that is optimised by training on the observed data. We can then predict what the value and the uncertainty of the desired quantity would be at any arbitrary point in space by applying this kernel to the training data. This is in contrast to other supervised learning algorithms which typically make single-valued, “point” predictions rather than predicting PDFs. It can be shown that GPR yields the minimum variance out of any unbiased interpolation method that depends only linearly on the training data; this makes GPR an optimal interpolation algorithm.

For our application of GPR, the galaxy sample described in Section 2 will serve as the training set. The six parameters we defined in Section 2.2 (stellar mass, star formation rate, etc.) serve as the features we will use for prediction. Our goal is to determine an optimised mapping from these physical parameters to a single output photometric parameter in our catalogue, e.g., the r-band absolute magnitude, 0_M_r.

Once our training data is selected we then go into the model-selection phase of GPR, during which the mean function and covari-
ance function (or kernel) used for GPR are selected and tuned. We
detail our selection of the covariance function in Section 3.2.1. Effect-
ively, the kernel determines how information from a given training
point will be propagated to make predictions at other points in pa-
rameter space. Hyper-parameters describing the kernel are tuned at
this step to maximise the log-marginal likelihood of the training data.
After this step we consider the model to be “fit”.

Finally we enter the inference phase of GPR. At any point in our
six-dimensional parameter space, we can now determine the posterior
probability distribution for the parameter of interest by applying
the kernel to the training data. When evaluated at a single point in
parameter space, a Gaussian process corresponds to a 1-D Gaussian;
we thus obtain both a predicted mean for the property of interest
(e.g., \( \dot{M}_r \)) and the standard deviation of the Gaussian describing
its uncertainty. In our example we would pass in a set of physical
parameters measured for the Milky Way and obtain a predicted value
for the \( \dot{M}_r \) of the Milky Way, as well as the uncertainty in that value.

In reality we do not only query the GPR at the mean measured
Milky Way properties presented in Section 2.3; rather, we perform
random draws from the PDFs describing M*, SFR, b/a, B/T, R_d,
and \( p_{\text{par}} \) in order to incorporate the uncertainties in the Milky Way’s
measured properties into our analysis. For log M*, log SFR, B/T,
and R_d we assume a normal distribution. For b/a and \( p_{\text{par}} \) we draw
from uniform distributions, as described in Section 2.3.

We perform these random draws 1000 times (so that we have 1000
full sets of Milky Way parameters). We then evaluate the GPR predic-
tions for Milky Way photometric properties at each of these points in
parameter space. This gives us a prediction and error estimate corre-
sponding to each draw from the PDFs of Milky Way characteristics.
Thus we end up with 1,000 total predictions for each Milky Way
photometric property. Our mean prediction for the Milky Way in a
given photometric band corresponds to the arithmetic mean of all
these predictions.

In this work all Gaussian Process regression calculations have been
done with the Python scikit-learn Gaussian Process module,
sklearn.gaussian_process (Pedregosa et al. 2011). For details
on the implementation of this module, refer to Pedregosa et al. (2011)
and the scikit-learn documentation.

The following sub-sections provide more details on some aspects
of our GPR methods and their advantages.

### 3.2.1 Choice of Kernel for GPR

In this work we use a combination of two kernels for Gaussian
Process Regression: a Radial Basis Function kernel and a white
noise kernel. The Radial Basis Function (RBF) kernel decreases
proportionally to \( \exp(-\gamma D^2) \), where \( \gamma \) is a free parameter and \( D \)
is the Euclidean distance between points; this kernel will cause the
covariance between the predicted values from GPR at different points
in parameter space to decrease as a Gaussian in distance as the
separation between those points increases.

However, there is also scatter in galaxy photometric properties
even for objects measured to have the same physical properties. In
order to capture that, we also incorporate a white noise kernel, which
models the spread in values for the predicted property at a fixed point
in parameter space with normally distributed noise (Rasmussen &
Williams 2006). The net covariance used for the Gaussian process
regression is then the sum of the distance-dependent covariance from
the RBF kernel and the (diagonal) covariance matrix corresponding
to the white noise kernel.

For a given set of training data, there is a nearly endless number
of functions that can fit the given data points, each one a realisation
of the Gaussian process. The kernel creates a prior on the GP to con-
strain which functions from that set are most likely to describe the
parameter space. The posterior is then determined using the training
data values. Due to this probabilistic approach, the Gaussian pro-
cess provides both predicted values and uncertainties at any points
within the parameter space. Uncertainties due to the finite training
sample size and its distribution in parameter space and those corre-
spending to intrinsic variation between training objects that have the
same physical parameters are both captured. In regions of parameter
space that are poorly constrained by the training data, the prediction
uncertainties are correspondingly larger.

The kernels we use in this work are available in the
sklearn.gaussian_process.kernels base class. For the white
noise kernel (WhiteKernel in scikit-learn) we initialise the
noise level to be 1; similarly, we initialise the length scale for the
RBF kernel to be 1 for each parameter. We opt to normalize the
output photometric property to have mean zero and variance one
across the training set, which helps to ensure that these initial guesses
will have the right order of magnitude. The noise level and length
scales are then optimised and the regression model is built via the
sklearn.gaussian_process.GaussianProcessRegressor
class. In our model we allow the optimiser to restart 10 times in
order to find the kernel parameters that maximise the likelihood
without being trapped in a false maximum.

### 3.2.2 Optimizing Training Samples

The computation time and required memory for the scikit-learn
implementation of GPR scales as the number of data points used
to train the model squared and cubed, respectively. As a result, we find
that the maximum training sample size we can use without running
out of memory on the computers used for this work is \( \sim 6,000 \); it
is infeasible to train from our entire catalogue when using this GPR
implementation.

We have therefore tested the effects of either restricting to objects
with physical parameters within some tolerance of the MW fiducial
values or randomly selecting a subset of objects in order to reduce
the training set size. We have focused on the root mean squared
error (RMSE) of predicted Milky Way photometry for the \( \text{NUV, } r, \text{ and } J \) bands for this optimisation. We use five-fold cross-validation
for all the tests; i.e., we always train with 80% of the data and test
with 20%, but rotate what objects are used for training and testing
through the whole dataset, and only retain the values for an object
when it was in the test set. This provides unbiased estimates of the
RMSE for a training set 80% as large as the one we actually have.
We find that the combination that offered the lowest RMSE
across all bands while keeping computational time manageable was
to randomly select 2,000 galaxies out of the set of objects that are
within \( 12\sigma \) of the Milky Way for every parameters of interest. We
therefore adopted this training strategy for all results below.

### 3.2.3 Comparison to Results from Analogue Samples

GPR can provide more accurate predictions than many other tech-
niques thanks to its ability to leverage information from both nearby
objects in the training set as well as from more distant objects that
characterise larger-scale trends. In our application, this allows the
GPR to map from the Milky Way’s physical properties to its pho-
tometric properties much more accurately than if we had only used
the few objects that are similar to the Milky Way in all respects (i.e.,
those which would be classified as MWAs) to inform the mapping.
The orange ellipses depict the region which is shaded in purple. When we have many analogues, the sample of galaxies used to train the Milky Way analogues, the sample of galaxies used to train the GPR (where additions are all cumulative, so entries further to the right incorporate more parameters). Since independent errors will add in quadrature, the contribution to the net variance from each factor is proportional to the height of its bar. The variance due to the scatter at fixed properties is shown in a lighter violet shade, while that resulting from the uncertainty in Milky Way properties is shown in dark purple. The scatter at fixed properties contributes to the majority of the error in the Gaussian process regression for \(0(g - r)\) colour.

We have also evaluated the contribution to uncertainties resulting from the finite size of the training sets used. This scatter is isolated by varying the randomly-selected training sample 100 times. For each training sample we evaluate the mean predicted value from GPR parameter varies. The left panel of Fig. 3 shows one of the physical parameters being regressed from, star formation rate, on the x axis and the target value, \(0(g - r)\) colour, on the y axis. We chose this pair as galaxy star formation rate is expected to correlate with galaxy colour well at fixed stellar mass.

First we isolate the scatter in colour at fixed properties; this corresponds to the contribution of the white noise kernel to the covariance function of the GPR. To determine the magnitude of this scatter we query the GPR at the Milky Way’s fiducial physical properties to obtain a predicted PDF of \(0(g - r)\) at this point in parameter space from which we can draw samples. The standard deviation of the colour of these samples corresponds to the scatter encoded in the white noise kernel.

In the panel at the right of Fig. 3 we plot a histogram of 10,000 possible \(0(g - r)\) colours drawn from the GP’s predicted PDF, evaluating it at the fiducial value of the Milky Way’s SFR. In the left panel the lavender star denotes the mean predicted \(0(g - r)\) for the Milky Way with this model, and the error bar corresponds to the standard deviation of the sample values. This error bar therefore corresponds to the \(1\sigma\) scatter in colour at fixed properties. The grey shaded area corresponds to the \(\pm 1\sigma\) band for the GPR prediction of colour for a range of \(\log(\text{SFR})\) values. By construction the half-width of the band must match the standard deviation of samples from the PDF at fixed properties.

To instead isolate the contribution to errors resulting from the uncertainty in Milky Way properties, we determine the distribution of mean predicted colours evaluated at varying values of SFR drawn from the PDF for the Milky Way. We perform 1000 draws from the fiducial MW \(\log\text{SFR}\) PDF in total and evaluate the GPR mean predicted \(0(g - r)\) for each. In Fig. 3 the dark purple points in the left panel shows the resulting predictions, which all fall on a continuous curve by construction. The panel to the right shows the histogram of this set of predictions in purple. We quantify the scatter in colour attributable to uncertainties in the Milky Way’s measurements via the standard deviation of the \(0(g - r)\) values at these 1,000 points.

To illustrate the full range of values obtained via GPR we plot ten samples from the distribution of predictions for each of the thousand Milky Way SFR draws as faint blue points in Fig. 3. These samples vary in colour due to both the scatter in colour at fixed properties and due to the uncertainty in Milky Way properties.

We can extend these same ideas in 1-D to evaluate the relative contributions of uncertainties in Milky Way properties and of the scatter in properties at fixed colour to the error in Milky Way \(0(g - r)\) colour, for any GPR model of interest. The key difference from Fig. 3 is that, in order to quantify the full scatter due to uncertainties in Milky Way properties, we allow all the parameters to vary, not only SFR. We present the results of this analysis for GPR models based on two to six physical parameters in Fig. 4. We use a stacked bar plot to display the contribution to the variance from each error source, where the x-axis is labelled according to the physical parameters used to train the GPR (where additions are all cumulative, so entries further to the right incorporate more parameters). Since independent errors will add in quadrature, the contribution to the net variance from each factor is proportional to the height of its bar. The variance due to the scatter at fixed properties is shown in a lighter violet shade, while that resulting from the uncertainty in Milky Way properties is shown in dark purple. The scatter at fixed properties contributes to the majority of the error in the Gaussian process regression for \(0(g - r)\) colour.

We can quantify the contributions of different sources of uncertainty to our GPR estimates by changing how we perform the regression. We illustrate our methods by evaluating how the prediction from a six-parameter Gaussian process regression fit changes as a single

Fig. 2 illustrates the fundamental difference between how properties are constrained by the Milky Way analogue selection method versus Gaussian process regression. For simplicity’s sake we perform this comparison based on only three parameters (stellar mass, star formation rate, and axis ratio, the same ones utilised in LNB15), as the analogue method starts to break down when more parameters are included. We also do not correct for Eddington bias in either measurement (q.v. Appendix D) for simplicity. Objects included in a set of MWAs based on 5,000 samples from the distribution of possible Milky Way properties via methods equivalent to those from Licquia et al. (2015) are depicted by the orange points. The analogues fall within a narrow range of stellar mass, limiting the set of objects that contribute information. The orange star represents the mean prediction for the Milky Way’s \(0(g - r)\) colour \(\langle 0(g - r) = 0.682 \rangle\) resulting from this set (derived via the Hodges–Lehmann robust estimator, Hodges & Lehmann (1963)). The orange ellipse depicts the 1σ confidence region for \(0(g - r)\) colour and stellar mass. In contrast to the small window that the Milky Way analogues lie in, the GPR utilises a wider variety of galaxies to capture larger-scale trends.
at the Milky Way’s fiducial physical parameters. The contribution to uncertainties from the finite size of training samples is obtained by calculating the variance of the GPR predictions across all of the training sets. These values are minute: the variances are an order of magnitude smaller than error attributed to the scatter due to Milky Way measurements. Thus any contribution to errors resulting from the finite training sample size is negligible.

We have performed the same error budget test for colours in the UV, near-IR, and mid-IR. The results mirror those presented in Fig. 4: the errors are dominated by scatter at fixed properties, followed by scatter from the Milky Way measurements. In all cases errors attributable to finite training set size are negligible compared to other sources. While the cumulative variance decreases for every parameter added when we predict colours, this is not the case for absolute magnitudes. In that case, the cumulative variance decreases as we add parameters until the sixth parameter, $p_{\text{bar}}$, is incorporated. At that point, the variance increases and the scatter due to finite training becomes more important. For this reason all absolute magnitude predictions within this paper are performed using only 5 parameters, excluding $p_{\text{bar}}$.

While uncertainties in Milky Way characteristics will contribute to the random errors in the derived photometric properties of our Galaxy, uncertainties in the physical parameters of the training galaxies can cause systematic errors. If the density of objects in parameter space varies quickly (with non-negligible second or higher derivatives), objects will more often scatter from well-populated regions of parameter space into sparser regions than vice versa. The resulting systematic shift in the measured distribution of parameters compared to the underlying distribution with no scatter is known as Eddington bias.

In the context of this work, Eddington bias will lead to shifts in the colour and luminosity predicted for the Milky Way. We derive corrections for Eddington bias using methods similar to those of LNB15; we detail our procedures in Appendix D. The estimated Eddington bias is subtracted off from the GPR-predicted colours and luminosities for the Milky Way to produce our final estimates for the Galaxy’s photometric properties and likewise the uncertainty on the Eddington bias calculations is propagated into our final error estimates. In general Eddington bias has small but nonzero effects on our results ($< 1 \sigma$ for almost all parameters, as listed in Appendix A).

In the following sections, the errors on GPR results presented include the contributions from scatter at fixed properties, uncertainties in Milky Way properties, and uncertainty from Eddington bias.

3.2.5 Summary of the GPR Algorithm for Determining Milky Way Photometric Properties

Here we summarise the steps taken to predict Milky Way photometric properties via GPR. Our method proceeds as follows:

(i) Construct the training sample by restricting to objects within $12\sigma$ of the Milky Way in all physical parameters considered and then randomly down-sampling to 2,000 objects.

(ii) Adopt the combination of a Radial Basis Function (RBF) kernel and a white noise kernel as the covariance function to be used for GPR.

(iii) Train the GPR using a single photometric property (normalised to have mean zero and variance one) as the output or “y” value and the physical galaxy parameters as the “x” values. This training will tune the hyperparameters of the kernel.

(iv) Perform 1,000 random draws from the PDFs that describe the fiducial Milky Way’s properties. This will allow us to incorporate uncertainties in the Milky Way measurements into our results.

(v) Use GPR to apply the optimised kernel to the training set and predict the photometric property of interest. For each randomly-drawn set of physical properties for the Milky Way, we obtain the mean prediction, predicted variance, and a set of 1,000 values drawn from the GPR-predicted PDF corresponding to that position in physical parameter space (which we refer to as a set of samples).

(vi) The mean photometric prediction for the Milky Way is then calculated as the mean of the set of GPR output means at the position of each MW draw. The error on the prediction is calculated as the standard deviation of the values from the complete set of samples generated, allowing us to incorporate both uncertainties associated with the scatter at fixed properties and errors resulting from the uncertainties in MW properties.

![Figure 3](image-url). A breakdown of uncertainty due to scatter at fixed properties and scatter due to Milky Way measurements uncertainty. We plot results from a six parameter Gaussian process regression trained to predict $(g - r)$ colour. For this example we vary only one of the input parameters, the star formation rate (SFR). The light violet colour corresponds to results when we use a fixed training set and evaluate the GPR at fixed MW properties; this isolates the variation attributable to the white noise kernel, corresponding to the scatter in colour at fixed galaxy properties. The dark purple colour corresponds to results when the training set remains fixed but the SFR value is drawn randomly from the fiducial MW PDF, so that the contribution to the colour error attributable to the uncertainty in the star formation rate of the Milky Way can be evaluated (the other five parameters used to train the GPR are held constant for simplicity for this example). In the left panel the shaded region depicts the $\pm 1\sigma$ range predicted by the GPR fit, plotted out to $\pm 3\sigma$ of the Milky Way’s SFR. The predicted $0_\text{MW}^\text{(g - r)}$ colour and $1\sigma$ error for the Milky Way’s fiducial SFR (and hence only including error at fixed properties) is shown by the star-shaped point and error bar in light purple. The predicted $0_\text{MW}^\text{(g - r)}$ colour from each of 5,000 draws from the SFR PDF is shown by the purple points, which accounts for scatter due to the SFR measurement. These predictions perfectly trace the GPR fit and tend to fall within $\sim 3\sigma$ of the Milky Way’s SFR, by construction. For reference we also show 10 samples from the GPR-predicted PDF for each of the 5,000 random SFR values as faint blue points. The distribution of these points reflects both the scatter of galaxy colours at fixed properties and the uncertainty in the MW measurements. Histograms of the colours for each sample, whose distributions correspond to the scatter at fixed properties and the scatter due to Milky Way SFR measurement uncertainties, are shown in the right panel. It is evident that the spread in GPR predictions at fixed properties is much larger than the scatter that results from uncertainties in the Milky Way’s SFR.
Figure 4. Contributions to the variance in restframe $^0(g - r)$ colour for Gaussian process regression employing varying sets of galaxy physical parameters. The x-axis shows the set of parameters used to predict colour; they increase cumulatively as we go from left to right, in order from the most constraining to the least constraining parameter. The variance decreases monotonically as the number of parameters increases. In every case the scatter in colour at fixed properties dominates errors; uncertainties in the physical properties of the Milky Way are sub-dominant. Contributions to uncertainties due to having a finite training set are small enough to be considered negligible, enough so that they would not be visible on this plot if we included them.

The code used to construct the GPR is provided on our GP GitHub page for public use here. At this site we provide sample code for determining photometry estimates, addressing systematics, and constructing an SED.

4 RESULTS

Via GPR predictions for Milky Way photometric properties across the spectrum, we can produce a comprehensive outside-in portrait of the Milky Way SED, allowing comparisons to the colors and luminosities of other galaxies. In this section we apply a variety of diagnostics from the literature, such as colour-luminosity, colour-mass, and colour-colour diagrams, in order to assess how the Milky Way compares to the broader population. We also construct a multi-wavelength SED for the Milky Way and compare our results to templates from the literature.

4.1 The Milky Way Compared to the Broader Galaxy Population

As discussed in Section 3, we have predicted the Milky Way colours and luminosities based upon the six parameters of stellar mass ($M_*$), star formation rate (SFR), axis ratio ($b/a$), bulge-to-total ratio ($B/T$), disk scale length ($R_d$), and bar vote fraction ($p_{bar}$). In the following colour diagrams, all magnitudes and colours are presented as rest-frame AB magnitudes (evaluating all passbands at redshift zero).

Our quantitative results are summarised in Table A1-Table A3.

The values provided correspond to the mean rest-frame predictions based upon the Gaussian process regression derived via the methods presented in Section 3, and have been corrected for Eddington bias as described in Appendix D. Colours and magnitudes are all calculated independently of one another. For example, we use GPR to predict $^0(g - r)$ galaxy colour directly, as opposed to deriving this value by subtracting the predicted $^0M_r$ from the predicted $^0M_g$. For SDSS photometry, our derived colours are based upon model magnitudes, as these yield the most accurate colour estimates for SDSS galaxies; however, the absolute magnitudes provided are based upon model magnitudes, as those most accurately represent the total brightness of an object.

Log-spaced density contours corresponding to the cross-matched galaxy sample of 29,836 galaxies described in Section 2.1 and Section 2.2 are plotted in grey-scale on all of the following colour-based diagrams. We also overlay red and blue ellipses which denote the rough locus of the red sequence and blue cloud, respectively, in each plot. These shadings are intended to guide the eye and should not be interpreted in a quantitative manner. In a corner of each plot we provide error bars that correspond to the mean uncertainties in each galaxy property being plotted for the training set.

In each diagram we also show the locations of the 36 red spiral galaxies selected in Masters et al. (2010b) that overlap with our cross-matched sample (out of 294 in the original catalogue). This sample of objects was selected based upon their colour, presence of spiral features, and shape/structural parameters from SDSS. They are required to have colour $^0(g - r) > 0.63 - 0.02(0M_r + 20)$, overlapping the blue edge of the red sequence. They are also selected to have a spiral likelihood $p_{spiral} \geq 0.8$ in the prescription of Bamford et al. (2009), and are required to have visible arms in Galaxy Zoo 1 $p_{CW} > 0.8$ or $p_{ACW} > 0.8$ (Lintott et al. 2011) in order to ensure they have spiral morphology. These objects are also selected to be approximately face-on (equivalent to an axis ratio requirement $b/a > 0.63$), as dust reddening is expected to have a substantial impact on the apparent colours of spirals (Masters et al. 2010a). However, in that paper the axis ratio values were calculated via r-band isophotal measurements, while ours are determined from an exponential profile fit. Therefore we apply a profile-fit-based cut of $b/a > 0.6$ to this sample to enable a more direct comparison to our face-on results for the Milky Way. Finally, Masters et al. (2010b) requires that the red spiral sample contain galaxies with an SDSS $f_{dV} \leq 0.5$, where $f_{dV}$ is defined as the weight of the de Vaucouleurs profile in the best-fit linear combination with the exponential profile matched to the object’s image. This ensures that S0 galaxies do not contaminate the sample, although they are already only a small percentage of the GZ1 sample.

The resulting red spiral sample is represented by red points in our plot. We overlay the positions of these objects in each parameter space to help assess the consistency of the inferred properties of the Milky Way with this population. Two objects whose $^0(g - r)$ colours in the cross-matched catalogue differed by $> 0.1$ mag from the photometry used in Masters et al. (2010b) due to changes in SDSS pipelines were excluded.

4.1.1 Optical Colours

We first present results at optical wavelengths, as they allow us to compare directly to previous work done with Milky Way analogues in LNB15. We focus on the SDSS ugriz bands (cf. Section 2.1.1). Fig. 5 presents predictions for Milky Way optical optical colours as a function of stellar mass ($M_*$) in solar mass units.

The upper panel shows $^0(u - r)$ colour and the lower panel shows
The green valley (and by extension the galaxy colour-bimodality) has been used as a basic tool to distinguish transitional galaxies from the general galaxy population. Transitional galaxies have lower specific star formation rates \( \text{sSFR} = \text{SFR}/M_* \) than a star-forming galaxy of the same mass; specific star formation rate can be used as a proxy for the evolutionary state of a galaxy and its star forming history. Salim (2014) defines the transitional region in sSFR space to be below the sSFR of massive Sbc galaxies, as these Sbc’s are the earliest galaxy type expected to proceed with regular star formation rather than being star forming in the UV. As described in that work, this region between the two lines correspond to the “green valley” population. For \( 0 \text{mag} \) we use the divisions of Schawinski et al. (2014) and in \( 0 \text{mag} \) we follow Mendel et al. (2013) (cf. Section 4.1.1). The results and \( 1 \sigma \) confidence region from LNB15 are marked in orange. Our results from applying GPR to all six galaxy physical parameters considered are marked in purple; the 1\( \sigma \) region is determined by the covariance between Gaussian process samples. For comparison, in lighter red we show the six-parameter GPR result obtained when setting the axis ratio of the Milky Way to \( b/a = 0.3 \pm 0.1 \) rather than \( 0.9 \). The stellar masses differ between our prediction and LNB15 due to the uncertainties in Milky Way properties and scatter in colours at fixed properties (cf. Section 3.2.4). In the lower right corner of each panel we use the divisions of Schawinski et al. (2014) and in the literature. We expect “red sequence” galaxies to be in the upper portion of each plot, with the “blue cloud” corresponding to the bluest colours. The region between the two lines corresponds to the “green valley” population. For \( 0 \text{mag} \) we use the divisions of Schawinski et al. (2014) and in \( 0 \text{mag} \) we follow Mendel et al. (2013) (cf. Section 4.1.1). The results and \( 1 \sigma \) confidence region from LNB15 are marked in orange. Our results from applying GPR to all six galaxy physical parameters considered are marked in purple; the 1\( \sigma \) region is determined by the covariance between Gaussian process samples. For comparison, in lighter red we show the six-parameter GPR result obtained when setting the axis ratio of the Milky Way to \( b/a = 0.3 \pm 0.1 \) rather than \( 0.9 \). The stellar masses differ between our prediction and LNB15 due to the updates to the mass estimate for the Milky Way described in Section 2.3. Red points correspond to members of the red spiral galaxy sample of Masters et al. (2010b). In the lower right corner of each panel we depict error bars representative of the mean uncertainties for galaxies in the comparison sample. Our results are consistent with LNB15 and indicate that at optical wavelengths the Milky Way is redder than the typical star-forming spiral galaxy, in addition to being more massive.

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Figure 5. Rest-frame optical colour as a function of stellar mass. The grey-scale, log-spaced contours depict the density of \( 0.03 < z < 0.09 \) galaxies in our cross-matched sample, without any limiting magnitude applied. The dashed grey lines correspond to divisions of the galaxy population used in the literature. We expect “red sequence” galaxies to be in the upper portion of each plot, with the “blue cloud” corresponding to the bluest colours. The region between the two lines corresponds to the “green valley” population. For \( 0 \text{mag} \) we use the divisions of Schawinski et al. (2014) and in \( 0 \text{mag} \) we follow Mendel et al. (2013) (cf. Section 4.1.1). The results and \( 1 \sigma \) confidence region from LNB15 are marked in orange. Our results from applying GPR to all six galaxy physical parameters considered are marked in purple; the 1\( \sigma \) region is determined by the covariance between Gaussian process samples. For comparison, in lighter red we show the six-parameter GPR result obtained when setting the axis ratio of the Milky Way to \( b/a = 0.3 \pm 0.1 \) rather than \( 0.9 \). The stellar masses differ between our prediction and LNB15 due to the updates to the mass estimate for the Milky Way described in Section 2.3. Red points correspond to members of the red spiral galaxy sample of Masters et al. (2010b). In the lower right corner of each panel we depict error bars representative of the mean uncertainties for galaxies in the comparison sample. Our results are consistent with LNB15 and indicate that at optical wavelengths the Milky Way is redder than the typical star-forming spiral galaxy, in addition to being more massive.
Salim (2014). The region with log sSFR above -10.8 corresponds to
galaxies that are actively forming stars while objects with log
specific star formation rate below -11.8 are quiescent; transitional
objects are between them. In the lower panel the green horizontal
lines correspond to the green valley definition of Lackner & Gunn
(2012) evaluated with the predicted r band absolute magnitude for
the Milky Way ($M_r = -20.65$). Galaxies residing within this range in
$0(g − r)$ are expected to reside within the green valley. Galaxies
above this designation are expected to lie in or near the red sequence,
and galaxies below are expected to lie in or near the blue cloud.

Based on its specific star formation rate, the Milky Way must
lie within the star-forming population, rather than in the transitional
range. While one might expect an object that meets optical definitions
of the green valley to have a transitional sSFR this is not necessarily
true, as galaxies of different evolutionary states can share the same
optical colour (see e.g., Cortese 2012; Salim 2014).

In $0(g − r)$ if we take the green valley to be 0.1 in width, as defined
by Mendez et al. (2011), the Galaxy would either fall within or be
redder than the green valley, consistent with the results shown in
Fig. 5. Despite ongoing star formation, more massive spiral galaxies
tend to be redder in the optical than their lower-mass counterparts
(e.g., Masters et al. 2010b). However, our estimated properties for the
Milky Way lie in the middle of the distributions of colours, masses,
and sSFRs of the red spiral sample from Masters et al. (2010b); these
objects are redder in the optical than is typical for even the most
massive spirals.

As these plots exemplify, differentiating between galaxy populations
based only on optical photometry is challenging. For instance,
in the lower panel in Fig. 6 we can see that red sequence, transitional,
and star-forming objects can all have colours of $0(g − r) − 0.7$. In
Fig. 5 the blue cloud becomes difficult to distinguish from the red
sequence at high masses as the most massive spirals have lower sSFRs
and, therefore, redder colours. In the following subsections we
investigate constraints on the colour of the Milky Way at UV and IR
wavelengths where galaxy populations may separate more clearly.

4.1.2 UV Colours

We utilise far-ultraviolet (FUV) and near-ultraviolet (NUV) photom-
etry from GALEX provided in the GSWLC-M2 catalogue (Martin &
GALEX Team 2005; Salim et al. 2016, 2018), as discussed in Sec-
tion 2.1.2. Thermal emission from massive stars with lifetimes < 100
Myr peaks at ultraviolet wavelengths, while lower-mass, longer-lived
stars play a larger role at optical wavelengths (Salim 2014; Tuttle
& Tonnesen 2020). Because UV radiation is produced by short-lived
but high-luminosity stars it provides a sensitive indicator of recent
star formation. As a result, UV photometry can more clearly differen-
tiate star-forming from quiescent galaxies than optical measurements
can.

Much as in Section 4.1.1, we can use our GPR results to place the
Milky Way on UV-based diagnostic diagrams from the literature. In
Fig. 7 we plot $0(FUV − r)$ and $0(NUV − r)$ UV-optical colours versus
specific star formation rate. The contours and vertical reference lines
shown are defined in the same way as in Fig. 6. For $0(NUV − r)$ we
show horizontal lines corresponding to the “green valley” definition
of Salim (2014), bounded at 4 < $0(NUV − r) < 5$. As before, our
face-on MW prediction and the corresponding 1σ confidence region
are plotted in purple and the inclined MW prediction is plotted in red;
unlike in the optical, there are no previous estimates of Milky Way
properties in this space that we could plot. Much as in the optical,
uncertainties in the UV photometry for individual objects account

**Figure 6.** Rest-frame optical colour as a function of log specific star formation rate (log (SFR/M$_\odot$)) in units of yr$^{-1}$. As before the grey-scale log contours depict the density of the parent cross-matched sample, red points correspond to the red spiral sample of Masters et al. (2010b), the orange star and ellipse correspond to the results of Licquia et al. (2015), and the purple star and ellipse are the results of our GPR analysis. The vertical dashed grey lines designate divisions of galaxy populations according to their specific star formation rates, following the definitions of Salim (2014): quiescent galaxies are at left, transitional objects are in the middle, and star-forming objects are at right. The Milky Way lies on the star-forming side of these divisions. The green horizontal lines in the bottom panel correspond to the green valley definition of Lackner & Gunn (2012), evaluated at the r-band absolute magnitude of the Milky Way. According to the prescription by Mendez et al. (2011) the green valley has a width of 0.1 in $0(g − r)$, leading to the limits shown here here. The Milky Way mean value falls above this “green valley” region. While the optical colour of the Milky Way is redder than most star-forming galaxies in the local Universe, based on its specific star formation rate the Milky Way would not be considered a transitional galaxy.
for roughly half of the total scatter ascribed to our Milky Way UV predictions.

Compared to typical star-forming galaxies in the local Universe, the Milky Way has redder than average UV colours and lower than average sSFR. The Milky Way appears to lie on the blue side of the $0(NUV - r)$ green valley border, in contrast to its location in the optical (cf. Fig. 5 and Fig. 6). This reflects the limited discriminating power of optical colour; the green valley is only 0.1 mag wide in $0(g - r)$, allowing objects to easily scatter over its borders due to even small photometric errors or inclination effects, but it spans an entire magnitude in $0(NUV - r)$. A more inclined Milky Way is predicted to be notably more red in the UV than in the optical, so much so that it could be consistent with the UV green valley in colour.

As in the optical, our estimates for the Milky Way in the UV-sSFR plane lie in the middle of the Masters et al. (2010b) red spiral population. Red spiral galaxies tend to lie outside of the UV green valley as they have star formation rates comparable to typical blue spirals of the same mass (Cortese 2012), and UV colour is more sensitive to recent star formation rate than the optical is.

### 4.1.3 Infrared/WISE Colours

The infrared data for our galaxy sample originates from the 2MASS and WISE surveys, as included in the GSWLC-M2 (Salim et al. 2016, 2018) and DESI Legacy catalogues (Dey et al. 2019), respectively (cf. Section 2.1). Similar to in the ultraviolet, the infrared brightness of a galaxy is sensitive to recent star formation due to re-emission of UV photons absorbed by dust. The infrared colours of galaxies also exhibit a bi-modal nature, with star-forming galaxies exhibiting redder IR colours than the passively-evolving population, rather than bluer. Instead of the “green valley,” the region between the star-forming and quiescent populations in the IR is commonly referred to as the infrared transition zone (IRTZ), following Alatalo et al. (2014).

In Fig. 8a and Fig. 8b we plot WISE colour-colour diagrams for the cross-matched galaxy sample in addition to the GPR prediction for the Milky Way. The Milky Way colour is poorly constrained in some WISE bands due to the lower signal-to-noise of these detections. If we compare our covariance ellipse in the $0(W1 - W2)$ direction to the average errors in the photometry (lower right error bar) the photometric errors account for a modest fraction of the total uncertainties in our GPR prediction. However, in $0(W2 - W3)$ and $0(W3 - W4)$ errors in the photometry for individual objects dominate the estimated uncertainties in the Milky Way GPR predictions. The vertical lines in these plots designate the IRTZ from Alatalo et al. (2014); objects with $0(W2 - W3) > 0.565$ in AB magnitudes correspond to late-type galaxies, while those with $0(W2 - W3) < -1.035$ are early-type galaxies, and in between lies the IRTZ. Magnitudes were converted from Vega to AB magnitudes via the prescription of Jarrett et al. (2011).

As before, we show predictions for the Milky Way if it were approximately face-on or more steeply inclined by evaluating the GPR at different axis ratios. An inclined Milky Way appears the most notably different in the W3 band, which traces prominent dust emission features, particularly those associated with polycyclic aromatic hydrocarbons (Wright et al. 2010). It appears that an inclined spiral galaxy would be measured to be more IR bright compared to a face-on counterpart matching it in all other ways; this may represent a systematic effect related to data processing, since the dust emission would be expected to be optically thin. We find similar results in Section 4.2.3.

In both diagrams the prediction for the Milky Way lies on the star-forming side of the infrared transition zone. If we compare Fig. 8a to the classification scheme in Figure 12 of Wright et al. (2010) (note that this requires converting our AB magnitudes to Vega magnitudes), the Milky Way lies within the region of colour space they label as typical for spiral galaxies, as would be expected. Similarly, the classification scheme of Figure 11.b of Jarrett et al. (2017) would place the Milky Way in the intermediate disk region, consistent with the expectation for a massive spiral galaxy. Intermediate disk objects are thought to be in transition towards being quenched due to star formation rates that are decreasing with time; our estimate for the

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**Figure 7.** As Fig. 6, but for UV-optical colours. The vertical reference lines come from Salim (2014) where objects with a log sSFR $> -10.8$ are actively forming stars, objects with log sSFR $<-11.8$ are quiescent, and objects in between are considered transitional. In $0(NUV - r)$ we also plot the bounds of the UV-optical “green valley” as defined in Salim (2014). Our predictions for the Milky Way show that it lies on the star forming side of the transitional regions, and is most likely on the blue side of the UV-optical green valley border. Our Galaxy lies in a similar region of these colour spaces to the red spiral sample of Masters et al. (2010b).
Figure 8. WISE colour-colour diagrams for both our parent sample (log density contours) and the predicted results for the Milky Way from Gaussian process regression. Reference lines from Alatalo et al. (2014) designate the infrared transition zone (1.035 < (W2 – W3) < 0.565; IRTZ). The Milky Way appears to lie on the star forming side of the IRTZ, much closer to the median colours of typical spiral galaxies, in contrast to the UV and IR. Again, our Galaxy lies in a similar region of these colour spaces to the red spiral sample of Masters et al. (2010). The infrared results mirror what we find from the UV: the Milky Way is still forming enough stars to appear bright in the IR due to re-emission from dust. Galaxies are expected to transition in the optical before they do in the infrared (Alatalo et al. 2014; Tuttle & Tonnesen 2020). If we follow the narrative of Alatalo et al. (2014) and Smethurst et al. (2015), the Milky Way may be in the early transition phase from star-forming to quiescent. It is brighter and redder in the optical compared to the typical star-forming galaxy. In the UV the Milky Way is on the blue side of the green valley but near it. In the IR the Milky Way’s inferred colour is more typical for a star-forming galaxy, though uncertainties are substantial. This would track with the expectation that a galaxy transitions in the optical before it does in the IR.

As before, the inferred colours of the Milky Way in the mid-IR are consistent with the range of values for the Masters et al. (2010b) red spiral sample, though some WISE bands are not well constraining due to the low signal-to-noise of the underlying measurements used for prediction. Based on this, it remains plausible that the Milky Way is a part of the red spiral population. We discuss how the Milky Way’s colours compare to other galaxy populations further in Section 5.2.

4.2 The Multiwavelength Spectral Energy Distribution of the Milky Way

Thus far, we have focused on predictions for a single Milky Way colour at a time. However, we can assemble colour information across all passbands to construct a spectral energy distribution (SED) for the Galaxy. SEDs, which quantify the total energy of emitted photons as a function of wavelength or frequency, are valuable tools in the study of galaxies. Many physical characteristics of galaxies can alter their SEDs - the age of their stellar population, stellar abundances, gas and dust content, inter-stellar medium (ISM) chemistry, details of star formation history, and the presence of an AGN can all leave distinct signposts that give observers insight into the formation and evolution of a given galaxy (see e.g., Silva et al. 2011). Because these effects each tend to alter the SED at specific portions of the spectral range, with broad enough wavelength coverage and detailed enough spectral information one can disentangle the dominant processes in a given galaxy.

Detailed modelling of the Milky Way’s SED will be the focus of a follow-up paper. In this work, we will present a proof-of-concept for a GPR-constructed SED for the Milky Way and provide an initial analysis of its properties. In the following sub-section we outline our GPR-based methods for determining the SED of the Galaxy before presenting quantitative results and assessing the effects the galaxy physical parameters used for prediction each has on the SED.

4.2.1 Algorithm for Calculating the SED for the Milky Way

We work in frequency (ν) space instead of wavelength (λ) space when calculating the SED of the Milky Way, as spectral energy distributions are most typically presented in units of energy per unit frequency. Our algorithm for calculating the SED proceeds as follows:

(i) Estimate \( M_r \) and colours for the Milky Way – Our SED is calculated in reference to the r-band. Therefore using the GPR (described in Section 3) we predict the r-band AB absolute magnitude \( M_r \) and all colours with restframe r as the reference band; i.e., \( (FUV - r) \) where x spans from FUV to W4 (e.g., \( (FUV - r), ..., (W4 - r) \)). Eddington bias is subtracted off separately from our predicted colours and \( M_r \) before we combine them. Similarly, the uncertainty in the Eddington bias is added in quadrature to the uncertainty in the GP calculations (which incorporates both scatter at fixed properties and errors due to the Milky Way property uncertainties; cf. Section 3.2.4). For details on the Eddington bias calculations refer to Appendix D.
(ii) Calculate flux ratios –
We calculate the flux in each band $f_{\nu,x}$ relative to the flux in the $r$-band $f_{\nu,r}$ via the relation

$$\log \frac{f_{\nu,x}}{f_{\nu,r}} = \frac{0(x-r)}{-2.5},$$

(1)

where $0(x-r)$ is the Eddington bias-corrected colour in the $x$ band compared to the $r$ band, which we have predicted via GPR.

(iii) Calculate luminosity –
From the $r$-band absolute magnitude combined with the flux ratios it is straightforward to convert to luminosity. We first calculate the $r$ band luminosity as

$$\log (L_{r,r}) = \frac{(0M_r - 34.04)}{-2.5} \left[ \log \left( \frac{W}{Hz} \right) \right],$$

(2)

where $0M_r$ is the Eddington bias-corrected $r$-band absolute magnitude for the Milky Way obtained via GPR. This formula is derived via the relation for converting flux to AB magnitudes in combination with the area of a 10 pc radius sphere to convert flux to luminosity. The luminosity in any other band can then be calculated via the relation

$$\log (L_{r,x}) = \log \left( \frac{f_{\nu,x}}{f_{\nu,r}} \right) + \log (L_{r,r}).$$

(3)

We can then add log frequency ($\log \nu$) to obtain $\log \nu L_{\nu,x}$.

(iv) Calculate errors –
To convert uncertainties in magnitudes and colours to uncertainties in $\log L_{\nu,x}$, we calculate the partial derivative of $\log L_{\nu,x}$ with respect to $0M_r$. This yields

$$\sigma_{\log L_{\nu,x}} = 0.4 \sigma_{0M_r}.$$  

(4)

Errors in the other bands are calculated in a similar manner, but they depend on both the error in colour and the error in $0M_r$:

$$\sigma_{\log L_{\nu,x}} = 0.4 \sqrt{\sigma_{0(x-r)}^2 + \sigma_{0M_r}^2}.$$  

(5)

In the plots that follow we do not plot the contribution to errors from $\sigma_{0M_r}$ as it is fully covariant across all bands; as a result, when templates are normalised to match the observed SED, any error in $0M_r$ would simply change the normalisation. We do provide its value for reference. Thus the error bars presented in the Milky Way SED plots are equivalent to $0.4 \sigma_{0(x-r)}$.

The colour predictions used to derive the luminosities used for our SED are provided for reference in Table A1 and the value of $0M_r$ is provided in the absolute magnitude table Table A3. We present our estimated luminosities and associated uncertainties, incorporating Eddington bias corrections in each case, in Table 1.

4.2.2 Interpreting the SED of the Milky Way

In Fig. 9 we present our full predicted spectral energy distribution for the Milky Way, along with a variety of empirical template galaxy spectra from the literature. We plot $\log \nu L_{\nu}$ (the power emitted per log interval in frequency) on the vertical axis, in units of log Watts. The horizontal axis corresponds to restframe wavelength in units of $\mu$m; each photometric band used is labelled at its effective wavelength along the top of the plot. The black open circles represent the estimates for the Milky Way’s luminosity along with their associated errors, calculated as described in Section 4.2.1. We also show the error bar corresponding to the uncertainty in $0M_r$ near the bottom of the Fig. 9a for reference. The numerical values for the Milky Way SED corresponding to the plotted points are provided in Table 1.

While detailed fitting of the Milky Way’s SED using physical models lies beyond the scope of this paper, we will compare to observed SEDs of individual galaxies and composite galaxy templates from the literature as a sanity check on the realism of our results. Using photometry for extragalactic samples to constrain the SED of the Milky Way, and then comparing the results to observed galaxy photometry (albeit for different objects) is somewhat circular. However, given that our analysis has treated every band completely independently, there were no guarantees that we should get a sensible SED when combining GPR results across the spectrum.

In Fig. 9a we compare our predicted Milky Way SED to templates from Benítez et al. (2004), which are refinements to the templates from Coleman et al. (1980) and (for starburst galaxies) Kinney et al. (1996). The Coleman et al. (1980) templates were based upon averaging the observed SEDs of relatively blue galaxies of a given morphological type. Given the broad range of observed SEDs for objects with similar morphological classification, these should not be considered universally applicable for all galaxies of a given type; however, we use the same labelling as Benítez et al. (2004) for consistency with the literature.

In this and successive plots, we have normalised all templates to match the estimated Milky Way SED in the $r$ band. We normalise in an optical band as those bands have the smallest errors in the predicted SED; which particular optical band we choose has minimal effect on our comparisons. It is apparent that the Milky Way SED is generally consistent with the (Benítez et al. 2004) “Sbc” galaxy template. The galaxies used to construct this template, M51 (NGC 5194) and NGC 2903 are undergoing moderate amounts of star formation, with log specific star formation rates of -10.1 and -10.4 (versus -10.5 for the Milky Way) (Muñoz-Mateos et al. 2007). The Milky Way is generally expected to have an SBbc morphological type (see, e.g., Bland-Hawthorn & Gerhard 2016) for a recent review on Milky Way structure, as well as Hodge (1983); Kennicutt (2001); Efremov (2011), though given how it was constructed, we should not read too much into the agreement of our SED with the Benítez et al. (2004) “Sbc” template in particular. It is clear, however, that the

| Passband | $\lambda_{ef}$ [\mu m] | $\log \nu L_{\nu}$ [log W] | $\sigma_{\log \nu L_{\nu}}$ [log W] |
|----------|----------------------|--------------------------|---------------------------------|
| FUV      | 0.155                | 35.53                    | 0.20                            |
| NUV      | 0.2275               | 35.63                    | 0.20                            |
| u        | 0.354                | 36.61                    | 0.06                            |
| g        | 0.4750               | 36.44                    | 0.02                            |
| r        | 0.622                | 36.62                    | 0.09                            |
| i        | 0.763                | 36.66                    | 0.01                            |
| z        | 0.905                | 36.70                    | 0.02                            |
| J        | 1.25                 | 36.67                    | 0.04                            |
| H        | 1.65                 | 36.54                    | 0.05                            |
| Ks       | 2.15                 | 36.39                    | 0.06                            |
| W1       | 3.368                | 35.59                    | 0.08                            |
| W2       | 4.618                | 35.58                    | 0.10                            |
| W3       | 12.082               | 35.61                    | 0.20                            |
| W4       | 22.194               | 35.49                    | 0.30                            |

Table 1. The passbands and corresponding power and uncertainties for the predicted SED of the Milky Way, as plotted in Fig. 9. These values have already had Eddington bias subtracted out.
Figure 9. The predicted SED for the Milky Way from a six-parameter Gaussian process regression, depicted by black open circles in all panels; the method of calculation is described in Section 4.2.1. (a) We compare to empirical galaxy templates from Benítez et al. (2004) (re-calibrated from Coleman et al. (1980)) normalised to match the Milky Way in the $r$-band. The Milky Way SED is consistent with the 'Sbc' template from this set, which is labelled as such because it was originally based on the average of the SEDs of two blue galaxies of morphological type Sbc. (b) We compare the SED of the Milky Way to the most closely-matched templates from Brown et al. (2014), which are based on spectra and model fits to bright nearby galaxies. We show all templates whose $\chi^2$ values below the 95\% upper limit have their photometry plotted as round points, which are been offset in the wavelength direction for clarity. Their accompanying spectra from the SED atlas are also plotted, without an offset. We also show as fainter curves the SEDs for the remaining two galaxies with potentially matching SEDs, which have $\chi^2$ values below the 68\% limit. Images of the four best-fitting galaxies are provided in Fig. 10. Portions of the spectra from the SED atlas that are based on models are depicted using dotted lines, while those that are directly based on observations are shown using solid lines. (c) The optical and near-IR portion of the Milky Way SED, with Brown et al. (2014) templates, as in (b). (d) The mid-IR portion of (b). The GPR method produces a Milky Way SED that is consistent in shape with both composite SED templates and individual observed SEDs.

The data tables from Brown et al. (2014) provide extinction-corrected photometry as well as a variety of summary values such as luminosity distance. Because the magnitudes are presented in the AB system we can use the relation:

$$\log f_\nu = \frac{m_{AB} - 8.9}{-2.5},$$

(6)
to calculated flux, where $f_\nu$ is flux in units of Jansky and $m_{AB}$ is the observed magnitude in each band. We neglect $k$-corrections as these galaxies are very nearby ($3.1 < D_L < 249.2\text{Mpc}$ at the most extreme), so the corrections are generally negligible. We then can use the flux-luminosity relation $L_\nu = 4\pi D_L^2 f_\nu$ to calculate the...
SED for a given band, the uncertainties in the Brown et al. (2014) 
$\chi^2$ of its SED to the Galactic one; we present them in order from smallest to $\chi^2$ compared to the estimated SED of the Milky Way. At the top of each image

The optical through near-IR regime is depicted in Fig. 9c, while the near- to mid-IR SED is depicted in Fig. 9d. Portions of the spectra that are based on observations are plotted with solid lines, while modellised portions are plotted with dotted lines.

We also provide images for the four Brown et al. (2014) atlas galaxies that fall below the 68% limit in Fig. 10. At the top of each postage stamp image we list the galaxy NGC number, as well as the $\chi^2$ value for comparing photometry for that galaxy to our Milky Way SED. The tiles are presented in order from smallest to largest $\chi^2$ value. At the bottom left of each tile is a letter marking (A) or (B) which refers to the source for each given image. (A) images are from ESA/Hubble (2021) and (B) images are from SDSS (2021). The borders that surround each image match the colour coding in Fig. 9b.

It is clear from this comparison that our GPR method produces results whose spectral shape is comparable to observed galaxy SEDs. The small total number of SEDs within the Brown et al. (2014) atlas that are consistent with the Milky Way is not surprising; after cutting on inclination we are reduced to only 70 objects, the majority of which are early-type galaxies, leaving a limited number of examples to cover the full range of star-forming galaxies. We emphasise that there remains a need for more careful investigation and modelling of the Milky Way SED we have obtained; this will be the topic of a follow-up paper.

However, we will briefly comment on the galaxies from the Brown et al. (2014) atlas whose SEDs are most consistent with the Milky Way. NGC 4138 is an Hubble type Sa(r)0 galaxy which contains an AGN and a star-forming ring. Using estimates for mass from Jure et al. (1996) ($2.92 \times 10^{10} M_\odot$) and Kassin (2004) ($6.23 \times 10^{10} M_\odot$) and the star formation rate estimates from Wiegert & English (2014) ($0.14 M_\odot$yr$^{-1}$) and Brown et al. (2017) ($0.2 M_\odot$yr$^{-1}$) yield a log sSFR of $\sim -11.3$ to $\sim -11.5$, significantly smaller than the Milky Way value ($-10.52$). NGC 4138 has $A_g = 73$ of colour of $0.73 \pm 0.05$ Brown et al. (2014), matching the Milky Way value.

In contrast, NGC 3351/M95 is a galaxy of Hubble type SB(r)b, versus SBb or Sbc for the Galaxy; it contains a pseudo-bulge (Sandage & Bedke 1994; Fisher et al. 2009; Brown et al. 2014), much as the luminosity in each band. Via propagation of errors the uncertainty in log $\nu L_\nu$ for these templates is equivalent to $\sigma_{\log \nu L_\nu} = 0.4 \sigma_{m_{AB}}$.

We take a few further steps before comparing the Brown et al. (2014) SEDs to our Galaxy’s. First, we have obtained the axis ratios (as a proxy for inclination) for 89 out of the 129 galaxies in the Brown et al. (2014) sample from the Siena Galaxy Atlas, which have been distributed as part of DESI Legacy imaging surveys Data Release 9. All galaxies with $b/a$ below 0.5 or unknown axis ratios were excluded from comparisons. Lower axis ratios should correspond to highly-inclined galaxies for which reddening will strongly affect the SED (e.g., Unterborn & Ryden 2008; Maller et al. 2009), making them inappropriate comparisons to our face-on SED for the Milky Way. We then normalise the observed SEDs to match the Galactic SED in the $r$-band, as was done for the Benítez et al. (2004) templates before.

Finally, we calculate the $\chi^2$ difference between our predicted SED for the Milky Way and each of the galaxy SEDs presented in the Brown et al. (2014) atlas. We emphasise that no fitting is performed in this comparison other than matching in the $r$ band. We then calculate $\chi^2$ using log quantities, as that is the space in which we perform our predictions and for which errors are (by construction) symmetric:

$$\chi^2 = \sum \left( \frac{\log \nu L_\nu^{\text{atlas}} - \log \nu L_\nu^{\text{MW}}}{\sigma_{\log \nu L_\nu}} \right)^2,$$

where $\sigma_{\log \nu L_\nu}$ combines in quadrature the total error in the MW SED for a given band, the uncertainties in the Brown et al. (2014) photometry, and $\log_{10}(1.1)$, which corresponds to a 10% error in $\nu L_\nu$. This extra error is added to account for systematic uncertainties in the photometry for a given band relative to others; if this were not included, optical bands would dominate the $\chi^2$ value due to their small nominal uncertainties. We calculate $\chi^2$ using the 14 bandpasses in which we have measured the Milky Way’s predicted SED, which yields 13 total degrees of freedom (one is lost due to the $r$-band normalisation performed).

Fig. 9b over-plots the Brown et al. (2014) SEDs for galaxies whose $\chi^2$ values fall within the 68% upper limits for a $\chi^2$ distribution with 13 degrees of freedom (corresponding to $\chi^2 = 14.8$). We also examine galaxies that fall below the 95% limit (with $\chi^2 < 22.4$), but exclude them from plotting for brevity. Objects below the 68% upper limit (four in total) are clearly consistent with the SED of the Milky Way, while those between the 68 and 95 percent limits (comprising three objects) are in some tension with the SED of the Milky Way, but could still be a match. While we calculate $\chi^2$ using the broadband photometry for each galaxy, we also plot the full SED from Brown et al. (2014) for each of these galaxies for reference.

In Fig. 9b the two galaxies with the smallest $\chi^2$ are labelled and plotted with the highest opacity in teal and gold. For these two galaxies we also plot the observed photometry as points offset in the wavelength direction so they are easier to compare to the Milky Way values. The higher $\chi^2$ objects are plotted with low opacity in pale blue. We also provide more detailed plots of two separate ranges of the SEDs. The optical through near-IR regime is depicted in Fig. 9c, while the near- to mid-IR SED is depicted in Fig. 9d. Portions of the spectra that are based on observations are plotted with solid lines, while modellised portions are plotted with dotted lines.
Milky Way is conjectured to possess (see Bland-Hawthorn & Gerhard 2016, and references therein). According to measurements provided by Leroy et al. (2008); George et al. (2019) NGC 3351 has a log specific star formation rate of $-10.43$ per year (SFR = $0.940 \ M_\odot yr^{-1}$), which is comparable to the log sSFR of the Milky Way of $-10.52$.

According to the measurements compiled by Brown et al. (2014), NGC 3351 has a $0.9 (g - r)$ colour of $0.74 \pm 0.05$, very similar to that of the Milky Way's $(0.73 \pm 0.05$, see Table A1). It matches the Milky Way SED equally well as NGC 4138 at most wavelengths, with the exception of the longest-wavelength W3 and W4 bands. However, we note that the photometry for NGC 3351 from Dale et al. (2017) does not show the strong red slope in the mid-IR seen in the SED atlas spectrum. If the Dale et al. (2017) measurements were used in the mid-IR, the agreement with the Milky Way SED would be significantly better.

Two other galaxies in the atlas have SEDs for which the $\chi^2$ value when compared to the Milky Way SED are below the 68% significance level. NGC 5055 is a galaxy of Hubble type SAbc (Brown et al. 2014) with log sSFR of $-10.53$ as tabulated in Kennicutt et al. (2011). NGC 3265 is of Hubble type SA(rs)0 pec (Ann et al. 2015) with log sSFR of $-9.12$ (Kennicutt et al. 2011). The set of objects whose SEDs are consistent with the Milky Way's span a diverse range of visual morphologies.

4.2.3 Impact of Physical Parameters on the Estimated SED of the Milky Way

Thus far we present results based upon a set of fiducial values (with uncertainties) for the Milky Way's stellar mass, star formation rate, axis ratio, disk scale length, bulge-to-total mass ratio, and bar presence (presented in Section 2.3). Because of the generality of the Gaussian process regression fit, we can vary each of these parameters one at a time and test its impact on the inferred SED.

In our previous analyses we randomly drew values from the distributions of Milky Way properties and made predictions based on each of those draws, which were then combined, as described in Section 3. This allowed us to incorporate the uncertainties in the Milky Way's physical properties into our results. In this subsection, however, we will neglect these uncertainties in order to isolate the effect of changing the central values for each parameter. We have performed a similar analysis to the one presented below by sampling from Milky Way uncertainties as a cross-check; the results are very similar so we do not present them here for brevity.

In this analysis we keep the values for the five Milky Way parameters not being studied at their fiducial mean. We then choose a discrete set of values for the sixth parameter at which to evaluate the SED via GPR. In the case of star formation rate, bulge-to-total mass ratio, and bar vote fraction we select 8 values that lie evenly spaced between $\pm 2 \sigma$ of the fiducial mean value for the Milky Way (inclusive), in addition to evaluating at the nominal value. For axis ratio we step through a wide range of possible galaxy axis ratios instead of focusing around 0.9 in order to capture the full effects of inclination on the Galaxy SED. We have excluded mass from this exercise, as changing mass would radically alter the normalisation of the predicted SED.

For each of the values that we step through for the given test parameter, the GPR is evaluated as before. We apply a $12 \sigma$ cutoff on the training sample for all parameters except the one being varied. By focusing on one parameter at a time we can explore the impact each has on the predicted SED for the Milky Way and can assess whether the GP is able to capture the expected correlations between galaxy properties.

The results of this analysis are presented in Fig. 11. In the colour bars for each panel the lighter shades correspond to smaller values for the parameter being varied and darker shades correspond to larger values. In all panels the fiducial value for the Milky Way is marked by a horizontal white line on the colour bar.

The upper left panel shows the GPR-predicted SED for each axis ratio value considered, with axes similar to Fig. 9. The SED evaluated with $b/a = 0.9$, the fiducial axis ratio value used for the Milky Way, is marked by open points. In this panel one can see the effects of inclination reddening first hand, an effect that has been seen repeatedly in analyses in spiral galaxies (e.g., Shao et al. 2007; Unterborn & Ryden 2008; Maller et al. 2009; Masters et al. 2010b). The cross-section for dust extinction and scattering generally increases with decreasing wavelength, causing reddening effects to be the strongest at the shortest wavelengths. Thus we expect the SEDs of galaxies to appear redder the more inclined they are (Xiao et al. 2012). The increased attenuation at higher inclinations for galaxies in the training sample causes the GPR to predict a redder SED as the inclination increases (lower $b/a$). We find a decreased brightness in the UV/optical and an increased brightness in the IR as disks are viewed more edge-on, an effect also observed by SED modellers (see e.g., Noll et al. 2009).

In the remainder of the panels we plot SEDs divided by the SED evaluated at the fiducial Milky Way values, as effects are more subtle and would be difficult to discern in an un-normalised plot. We include a plot of this type based on varying axis ratio in the lower left panel, though we use a larger y-range than the other normalised plots due to the large dynamic range spanned.

An SED which matches the prediction for the fiducial MW values exactly would fall along the horizontal line at $\log (V_L_r) - \log (V_L_MW) = 0$. The photometric predictions for the Milky Way nominal parameters hence correspond to the open circles along this line.

Star formation rate has the clearest effect on the SED, as seen in the top middle panel. The amount of UV flux is a sensitive indicator of star formation as it is dominated by hot, massive, short-lived stars. These stars contribute to the flux at optical and near-IR wavelengths, but are subdominant there; but in the mid-IR the SED responds strongly to star formation due to light from hot stars that is reprocessed by dust. The GPR predictions reflect all of these phenomena.

The upper right panel depicts the effect of disk scale length on the predicted SED. We note that the predictions become unstable at shorter scale lengths due to the small number of Milky Way-mass galaxies with radius smaller than our Galaxy in the training samples (cf. Licquia et al. (2016)), so results for disk scale lengths more than $1 \sigma$ below the Milky Way value may not be robust. Outside of that regime, it is clear that Milky Way-like galaxies with shorter scale lengths exhibit significantly less flux in the UV than those with longer scale lengths when $R_d$, SFR, etc. are all held fixed, along with smaller effects at optical-IR wavelengths.

A smaller disk, with other properties held fixed, could imply that the gas within the disk is denser and dust columns are correspondingly greater. This would in turn cause the SED to look fainter in the UV relative to the IR compared to if the disk were more extended. We have tested the effect of varying $b/a$ and $R_d$ simultaneously and find that we can compensate for a change in one with a change in the other almost perfectly. Given the long-standing scenario that inclination effects on the SEDs of disc galaxies are driven by dust (Maller et al. 2009, e.g.), it is reasonable to hypothesize that the effects of varying $R_d$ must relate to varying dust impact as well.

The bottom middle panel shows the effect of varying only the bulge-to-total ratio. Overall, the impact on the SED is small. We see that if the Milky Way were to have a more massive bulge, it
Figure 11. Isolating the contributions of each physical parameter to the SED. Each panels shows the effect of varying one parameter while fixing the other five parameters to the fiducial Milky Way values (see Section 2.3). We vary most parameters over a range of ±2σ from the Milky Way’s fiducial values. In the case of axis ratio, we explore a wider range of values to convey better the impact of inclination on the observed SED. The upper-left panel depicts the different SEDs that would be inferred for different galaxy axis ratios, with the expected results that an inclined Milky Way would look much fainter than face-on. In the remainder of the panels we depict the log of the predicted SED divided by the SED evaluated for fiducial Milky Way parameters in order to make differences more clear; the plotted quantity is thus ∆(log (νLₜ)) = log (νLₜ) – log (νLₜ_%MW). The predicted SED for the Milky Way value is plotted with a dashed black line and open circles at the passbands. On the colour bars the Milky Way’s value is marked by a horizontal white line. In b/a the GP captures the effects of dust reddening at higher inclinations. If the Milky Way were to have a higher SFR, the SED would be brighter in both the UV and IR. If the Milky Way’s disk were more extended we would observe an increased UV brightness and decreased mid-IR brightness. We caution the reader that predictions for disk scale lengths more than 1σ below the Milky Way value may not be reliable. Relatively few galaxies of the mass of Milky Way have sizes smaller than the Milky Way, causing the GPR to be poorly trained for small R_d values (Licquia et al. 2016). Changes in the Milky Way’s B/T on the SED would be minimal. As we decrease the intensity of the Milky Way’s bar it seems to have a minimal effect, with a slightly increased UV brightness. In general these follow our expectations of galaxy evolution which we discuss further in Section 4.2.3.

would appear slightly fainter in the UV. This could reflect the fact that bulges have older stellar populations than spiral disks; the effect may be subtle here as to first order the effect is captured by variation in specific star formation rate. Bulges are also susceptible to dust reddening (Tuffs et al. 2004; Driver et al. 2007), so a larger bulge may also suffer from greater reddening effects at short wavelengths.

The lower right panel shows the variation in SEDs as we change the bar vote fraction. As this quantity increases we see a decrease in UV brightness and an increase in mid-IR brightness. In general we speculate that most effects caused by a bar may have been captured by affecting the ability of gas to cool and form molecular clouds; having more dust (at fixed M*, SFR, and inclination) should cause a reduction of flux in the UV and an increase in flux in the mid-IR, as observed here.

4.2.4 Exploration of Other Sources of Physical Parameter Measurements

As described in Section 2.2.1 and Section 2.2.2, there exist multiple options for the values used for the stellar mass, star formation rate, and bulge-to-total ratio for SDSS galaxies. In this subsection we discuss the impact on our results of using different measurements of galaxy parameters or different methods of defining our training samples.

4.2.4.1 Stellar Mass and Star Formation Rate

The GSWLC-M2 catalogue (Salim et al. 2016, 2018) includes estimates of stellar masses and star formation rates computed based on the photometry within the catalogue. While the M* values in the GSWLC-M2 catalogue closely match those from the MPA-JHU catalogue (Brinchmann et al. 2004) used for our main results, the SFRs presented in the GSWLC-M2 catalogue are far less bimodal than those presented in the MPA-JHU catalogue; a significant number of galaxies would be classified as star-forming in GSWLC that would be considered quiescent based on the MPA-JHU catalogue.

We have produced a Milky Way SED using the GSWLC-M2 stellar masses and star formation rates as features in place of the MPA-JHU values, but otherwise following the same methods used to produce the results in Section 4.1 and Section 4.2. The impact on predictions in the optical and IR is small. The effect is more notable in the UV, where the Milky Way is predicted to be closer to the mean colour of star-forming galaxies with the same sSFR (corresponding to smaller
values of \( \sigma^0(FUV - r) \) and \( \sigma^0(NUV - r) \) in Fig. 7. Even so, the predicted UV colors are still within 0.5\( \sigma \) of those resulting from using MPA-JHU M_\text{star} and SFRs. In addition to being brighter in the NUV and FUV, the predicted SED is also marginally fainter in W3 and W4 compared to the results presented in Fig. 9, with shifts that are again well below 1\( \sigma \) in each band.

Overall we find that using the stellar masses and star formation rates derived from the GSWLC-M2 catalogue instead of the MPA-JHU catalogue has little effect on our predictions for the Milky Way, and is subdominant to other sources of uncertainty.

4.2.4.2 Bulge-to-Total Ratio There are also multiple options for which band to measure bulge-to-total ratios; the Simard et al. (2011) catalogue contains bulge and disk decompositions performed both in the \( g \) and \( r \) bands. For our main results we use the \( r \)-band value, \( B/T_r \), but have also tested the impact of instead using \( B/T_g \) on our results presented in Section 4.1 and Section 4.2.

Overall we find only very mild effects on predicted colours from changing to \( B/T_g \). All results are well within a few hundreds of a magnitude of the previous values, except in the case of \( (u - r) \). For that value, the predicted value for the Milky Way from the GPR trained on \( B/T_g \) is larger than when we use \( B/T_r \) (corresponding to redder color; cf. Fig. 6), though still within 0.5\( \sigma \) of our primary result. As a consequence, the predicted SED for the Milky Way using \( B/T_g \) instead of \( B/T_r \) is almost identical to before.

4.2.4.3 Treatment of Galaxy Zoo 2 Votes As discussed in Section 2.2.3, using citizen science votes from Galaxy Zoo 2 requires consideration of responses to previous questions that influence whether the question of interest is even asked. If we wished to select a pure sample of barred galaxies, it would be necessary to ensure not only that a high fraction of people who were asked whether a bar is present voted in the affirmative, but also that a substantial total number of people voted on each of the preceding questions in order to minimise errors in vote fractions (see, e.g., Willett et al. 2013). However, with GPR we may gain more information about trends that influence photometric properties by including a broader set of objects, so we do not necessarily wish to exclude non-barred or non-spiral galaxies from the training sample. We explore here how changing the treatment of GZ2 votes influences our GPR predictions.

We have tested what impact restricting the training set to only barred, face-on spirals would have by testing how predicted colours for the Milky Way vary when using training samples with a variety of different constraints: (1) a control sample without any restrictions based on Galaxy Zoo 2 vote results; (2) a sample where if the number of votes on whether or not a galaxy has a bar, \( N_{\text{bar}} \), is less than 10 we set \( p_{\text{bar}} = 0 \); (3) a sample using the bar selection cuts from Willett et al. (2013) Table 3, column 3, rows 2 and 3 in addition to the vote count thresholds mentioned in Section 2.2.3; or (4) a sample using the same cuts as Willett et al. (2013) except setting \( p_{\text{bar}} = 0 \) when \( N_{\text{bar}} < 10 \), rather than rejecting objects with low \( N_{\text{bar}} \) from the set entirely.

Applying all of the Willett et al. (2013) cuts reduces the size of the training sample by roughly an order of magnitude, degrading the ability of GPR to predict colours and increasing net errors. Furthermore, requiring \( N_{\text{bar}} \geq 10 \) not only shrinks the size of the sample but also greatly biases the luminosity distribution of the training sample compared to a volume-limited sample, which may result in biases in inferred photometry. We have explored how the GPR-predicted Milky Way colours change for each of these four training sample definitions (but otherwise using the methodologies described in Section 3). The results from (2) compared to our fiducial case, (1), are nearly identical, so the particular values of \( p_{\text{bar}} \) assigned to objects with poorly-constrained vote counts cannot have had a large systematic impact on our predicted Milky Way photometry. We also find that restricting to training sample (3) or (4) yields much \((\geq 2x)\) larger errors on all predictions. Results from (3) and (4) are still within 1\( \sigma \) of those from (1) and (2), however. Therefore we conclude that with GPR we get better predictions when we include more objects (including some with noisier vote fractions) than when we instead restrict training to just the best-constrained objects. Therefore we perform no GZ2-based cuts on the galaxy sample used for training, and instead include objects spanning the full range of bar, face-on, and features vote fractions in the training set.

5 SUMMARY AND CONCLUSIONS

5.1 Summary

In this work we have set out to estimate a full SED for the Milky Way, spanning wavelengths from the UV to the IR. Our central motivation is twofold: (1) to improve our understanding of how the Milky Way compares to the general galaxy population and by doing so (2) guide the tuning of parameters in simulations in order to create more realistic galaxies.

The previous work by Licquia et al. (2015) constrained the optical colours and luminosity of the Milky Way using Milky Way analogue galaxies selected based on their stellar mass and star formation rate, obtaining the best constraints on the Milky Way’s photometric properties available previous to this work. Here, we have been able to reduce the uncertainties on these constraints further by incorporating information from additional parameters such as disk scale length and bulge-to-total ratio, that also connect to a galaxy’s evolutionary history (Cappellari 2016; Saha & Cortesi 2018), and have for the first time developed predictions for Milky Way photometry at wavelengths beyond the optical.

We have shown that the Milky Way analogue method breaks down when we attempt to match the Galaxy in many physical parameters; the number of Milky Way analogues rapidly approaches zero in higher-dimensional spaces (cf. Fig. 1). Expanding to a wider wavelength range requires information from datasets that do not cover the full SDSS footprint, making the problem worse. We instead have predicted the photometric properties of the Milky Way using Gaussian process regression, which provides an optimal means of interpolating information from a limited training set. We have performed a series of tests throughout this paper that have demonstrated that GPR is able to produce realistic and reliable photometric predictions.

We have compared predictions for the Milky Way to the broader local galaxy population in colour-mass, colour-specific star formation rate, and colour-colour diagrams. As exemplified by Fig. 5, we obtain similar results in the optical to those reported by Mutch et al. (2011) and LNB15, though with reduced errors, further confirming the Milky Way has optical colors consistent with the green valley population. For the first time we have also predicted UV (Fig. 7) and IR colours Fig. 8 for the Milky Way, which provide more sensitive diagnostics of the evolutionary status of a galaxy. We find that in both these regimes the Milky Way appears to lie on the star-forming side of the green valley.

In this work we have determined the luminosity and colours of the Milky Way for GALEX \( FUV \) and \( NUV \), SDSS \( ugriz \), 2MASS \( JHKs \), and WISE \( W1 - W4 \) bands in an entirely self-consistent way, giving us unprecedented constraints on its spectral energy distribution. We have constructed the first multi-wavelength SED for
the Milky Way. This SED has a shape consistent with both composite galaxy templates (Fig. 9a) and observed SEDs of individual galaxies (Fig. 9b). The GPR method produces a realistic SED with errors and captures previously known galaxy property correlations, such as those between reddening in spiral galaxies and viewing angle or between star-formation rate and UV and IR flux (Fig. 11). High-resolution hydrodynamical simulators (e.g., Guedes et al. 2011; Sawala et al. 2016; Wetzel et al. 2016) no longer have to compare their mocks of the Milky Way blindly to photometric constraints from broad galaxy populations that span a wide range of properties. Rather, it should now be possible to tune the treatment of star formation efficiency, threshold gas density for star formation, and dust properties to produce galaxies which match the photometric properties of the Milky Way directly, while simultaneously exploiting those properties that we can measure well from inside the Galaxy.

5.2 Discussion: The Milky Way as a Red Spiral

As previously suggested in a variety of works (e.g., Salim (2014), Schawinski et al. (2014), and Licquia et al. (2015)), definitions of the green valley that rely only on optical bands may lead to misleading conclusions. The Milky Way has a specific star formation rate that is higher than the canonical values for green valley galaxies, log sSFR = −10.52 as compared to ~−11.8 < log sSFR < −10.8 for transitioning galaxies from Salim (2014), even though it has red optical colours for a star-forming object. However, at UV and IR wavelengths the colours of the Milky Way more clearly place it amongst the star-forming population. This combination of red optical colours when viewed face-on with significant star formation evident at UV and IR wavelengths is characteristic of the previously-identified population of red spiral galaxies.

A population of red spiral galaxies in clusters was first identified by van den Bergh (1976). Since then, these “passive spirals” have been identified at a range of redshifts and in multiple datasets. As noted by to Cortese (2012), for galaxies with a stellar mass above $10^{10} M_\odot$ like the Milky Way ($M_* = 5.48 \pm 1.18 \times 10^{10} M_\odot$), the blue cloud and red sequence overlap in their optical colours (this is also consistent with findings by Salim 2014). This makes optical photometry a poor choice for constraining the star formation activity for galaxies like our own. However, these massive objects still exhibit a distinct colour bi-modality in the UV, as shown by Wyder et al. (2007); Salim (2014) and is evident from comparing Fig. 6 and Fig. 7. In comparison to their lower-mass counterparts, massive galaxies produced the great majority of their stars at earlier epochs (e.g., Boselli et al. 2001). This causes the optical colours of massive galaxies to be dominated by relatively old stellar populations as opposed to probing recent star formation activity (Wyder et al. 2007; Chilingarian & Zolotukhin 2012; Cortese 2012). Hao et al. (2019) and Zhou et al. (2020) provide evidence that this is the case for red spirals. Direct or re-radiated light from young stars still dominates the red spiral SEDs at UV and IR wavelengths, however.

Reflecting that, Cortese (2012) and Smethurst et al. (2015) both find that red spiral galaxies tend to be UV bright; this can be driven by a relatively small amount of total star formation. In order to facilitate comparison of the Milky Way to the red spiral galaxy population, we have over-plotted the red spirals from the Masters et al. (2010b) catalogue (based on a Galaxy Zoo 2 and optical colour selection) that are also part of our cross-matched galaxy catalogue on all colour-mass, colour-sSFR, and colour-colour diagrams presented here. In each diagram the Milky Way falls near the middle of the red spiral population.

Cortese (2012) notes that 85 – 90% of objects in their red sample maintain SFRs of $\sim 1 M_\odot \text{yr}^{-1}$; Masters et al. (2010b) found that red spiral galaxies selected from Galaxy Zoo 2 typically had lower rates of ongoing star formation than blue spirals of the same mass, but still non-negligible. For comparison the Milky Way has a SFR of $1.65 \pm 0.19 M_\odot \text{yr}^{-1}$ (Licquia & Newman 2015), while the average star formation rate of galaxies of approximately the same mass as the Milky Way ($\pm 0.3$ in log stellar mass) with $B/T < 0.75$ (to exclude ellipticals) within our cross-matched galaxy sample is $1.69 M_\odot \text{yr}^{-1}$; the Galaxy is very close to average in this respect.

Masters et al. (2010b) also finds that red spiral galaxies have a significantly higher bar fraction compared to blue spirals of the same mass; 70% versus 27%. This matches with the clear evidence that the Milky Way possesses a bar (e.g., Blitz & Spergel 1991; Bland-Hawthorn & Gerhard 2016; Shen & Zheng 2020). Masters et al. (2010b) notes that one possible evolutionary scenario for red spirals is bar-driven gas inflows. This removes gas from the outer disk and funnels it into central star formation, which in turn causes the disk to appear more and more red over time (Masters et al. 2012; Saintonge et al. 2012; Cheung et al. 2013; Fraser-McKelvie et al. 2020).

Masters et al. (2011) and Fraser-McKelvie et al. (2020) find that barred spirals tend to have redder colours than their unbarred counterparts. It thus may be the case that the bar has played a role in the colours that we observe for the Milky Way in this work. For example, bar quenching may play a role in the development of red spirals, as Bamford et al. (2009) finds many high-stellar-mass red spirals in the field and Smethurst et al. (2015) notes that field red spirals most likely evolve primarily via secular evolution due to the lack of nearby galaxies. In the case of the Milky Way, work by Haywood et al. (2016) and Khoperskov et al. (2018) find that the bar may have played a substantial role in the star formation history of the Milky Way (leading to a significant decrease in star formation 9-10 Gyr ago, and thereby causing the observed pattern of chemical abundances in the disk). Although the effect of bar vote fraction in Fig. 11 is small, it may be that the effects of a bar are primarily captured by other parameters (e.g., SFR).

We note that Evans et al. (2018) studied a population somewhat similar to red spirals, which they labelled “red misfits”. Evans et al. (2018) define this population as corresponding to objects with log(sSFR) $> -10.8$ and restframe $g - r > 0.67$ (i.e., specific star formation rate measured to be above the value for the saddle point in the bimodal distribution and colour redder than the saddle point in the colour bimodality). Based on these divisions, the Milky Way almost certainly meets this definition (which is less stringent than most red spiral classifications).

5.3 Outlook

As seen in Fig. 10, there is a significant diversity in the set of galaxies that have SEDs consistent with the Milky Way, given the measurement uncertainties in both our results and the Brown SED atlas. The goal of this paper has been to construct the Milky Way’s UV-to-IR SED to enable comparisons to samples of external galaxies and to improve the tuning of simulations. However, in a follow-up paper we will fit the estimated SED of the Milky Way using population synthesis models to obtain more detailed constraints on how the star formation history, dust reddening properties, and metallicity of the Galaxy would be interpreted from outside (see e.g., Conroy 2013). This will require proper treatment of covariances between different photometric bands; we will address this by employing multi-output Gaussian process regression in this future work.

The longest-wavelength WISE W3 and W4 bands could have substantial discriminating power on what SEDs are consistent with the
Milky Way’s, if they only had smaller errors, as is evident in Fig. 9. However, currently these bands are poorly measured compared to the optical or near-IR; for most objects used in training the Milky Way SED, the signal-to-noise ratio in these bands is below one. Given the low effects of dust extinction in these bands, investigation of the flux ratio (or colour) in these bands across the all-sky WISE imaging, potentially combining modelling of smooth components of the Milky Way with mapping of the contributions from dust, may provide an alternative method to constrain the colour of the Milky Way at the longest wavelengths. If luminosities in the W3 and W4 bands can be measured relative to the luminosity in W2, long-wavelength measurements could be effectively anchored well to the SED presented here; measuring such relative quantities should be affected less by modelling uncertainties than absolute measurements would be.

The SED presented in the paper (or future improved versions) can be used to identify multiwavelength Milky Way analogue galaxies by matching in unresolved photometric properties. If we do not need to require detailed morphological measurements or citizen science inspection of images it would greatly increase the size of the parent catalogues that could be used to identify MWAs, which could be useful for a variety of follow-up studies such as determining gas masses for the Milky Way or studying environments of Milky Way-like galaxies.

The Milky Way appears to be atypical in its satellite population and mass assembly history. For example, Evans et al. (2020) finds that the assembly history of the Milky Way is only reproduced in 0.65% of Milky Way-mass EAGLE galaxies. This assembly history should be closely related to the local environment surrounding our Galaxy, and environment has been found to play a key role in the formation and evolution of galaxies (e.g., van den Bosch et al. 2008; Peng et al. 2012; Bluck et al. 2016).

In future work we plan to explore how incorporating measures of galaxy environment (e.g., measures of the local overdensity of galaxies) within a GPR model affect the predicted SED of the Milky Way. The noisiness of environment measures (Hogg et al. 2004) and the impact of SDSS fiber collisions on Local Group-like systems (as typically only one galaxy out of two close neighbours would be observed, causing analogues of a Milky Way-M31 pair to be missed) may limit the information that may be gained from this, however. While we anticipate the environment to have a small impact on the Galactic SED compared to the dominant effects of stellar mass and star formation rate on galaxy colours (e.g., Grützbauch et al. 2011), assessing the local environments of the most Milky Way-like galaxies may allow us to explore and to what extent our Galaxy’s environment has shaped its exhibited characteristics.

Gaussian process regression can be useful for a variety of studies beyond the photometric estimates for the Milky Way considered here. For this reason the authors have provided their analysis code on our project GitHub for full public access for adaption to any other project, under a CC BY-SA 4.0 license.

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DATA AVAILABILITY

The data used in this article is provided publicly on our catalogue GitHub. We include both the original volume-limited sample of LNB15 and the cross-matched final sample used in this work. The ReadMe also provides a detailed description of the columns within these tables, which is also provided by each respective catalogue that our data originates from.

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APPENDIX A: DATA SUMMARY

A1 Galaxy Properties

In this subsection we summarise the distribution of properties of the galaxy sample used for training the GPR (which is described in Section 2). This sample consists of cross-match pairs between the SDSS DR8 volume-limited sample reported in LNB15, the MPA-JHU catalogue of galaxy stellar masses and star formation rates (Brinchmann et al. 2004), the Simard et al. (2011) morphological catalogue, the Salim et al. (2016, 2018) GSWLC-M2 photometric catalogue, the DESI Legacy imaging survey DR8 (Dey et al. 2019), and the Galaxy Zoo 2 catalogue (Willett et al. 2013; Hart et al. 2016).

In Fig. A1 we present a corner plot for the distributions of the various galaxy physical properties used to train the GPR: mass ($M_*$), star formation rate (SFR), axis ratio ($b/a$), bulge-to-total ratio ($B/T$), disk scale length ($R_d$), and bar fraction (p_bar). In each contour plot the fiducial value for the Milky Way (from Section 2.3) is marked by a black star, while in the histograms the fiducial value is marked by a vertical dashed black line. Overall we can conclude that covariances are fairly weak for most parameter pairs, with the exceptions of the well-known $M_*-R_d$ (or mass–size) relation, the bimodal distribution of objects in the $M_*-$SFR plane (corresponding to the red sequence/blue cloud division), and weaker correlations between galaxy bulge-to-total ratio or bar vote fraction and star formation rate.

A2 Tabulated Photometric Predictions for the Milky Way

In this section we present the estimates of the colours and luminosities for the Milky Way which we have obtained via Gaussian process regression. Table A1, Table A2, and Table A3 update Table 1 and 3 from in Licquia et al. (2015), and also incorporate results for the non-SDSS bands used in this work. In addition, Table A1 tabulates the colours used to derive the luminosities presented in Table 1, a necessary ingredient for our SED calculation.

In Table A3 all absolute magnitudes are calculated assuming a Hubble constant of $H_0 = 100$ km s$^{-1}$ Mpc$^{-1}$. Results for FUV, NUV, ugriz, JHKs, and W1 – W4 are all presented in the AB magnitude system, as in the body of the paper. The Johnson-Cousins UBVRI values are provided in the Vega magnitude system. These were converted from the ugriz measurements via the scorrect v4.2 software on an object-by-object basis, as in LNB15. We remind the reader that absolute magnitudes are computer with 5 physical parameters instead of 6, due to the increased cumulative variance when $p_{bar}$ is incorporated in the GPR for them. Therefore we do not include derivatives with respect to $p_{bar}$ for them, as bar vote fraction was not used in the predictions of these quantities.

All tabulated results are presented for rest-frame $z = 0$ passbands. We note that each table row is calculated independently from all others; for example, $(0(FUV - r))$ is not calculated from subtracting the predicted $0M_r$ from the predicted $0M_{FUV}$, but rather from a Gaussian process regression that predicts $(0(FUV - r))$ directly. This will in general yield smaller errors for colour calculations, as some of the errors in absolute magnitudes from different bands will be covariant. We derive colour predictions from model1 magnitudes while absolute magnitudes are determined from cmodel1 magnitudes, again to minimise errors.

In these tables, the “Corrected Value” column corresponds to the predicted colour or absolute magnitude from the GPR, derived as the mean predicted value from 1,000 draws from the fiducial distributions of Milky Way physical properties (as described in Section 3), after Eddington bias has been subtracted. The amount subtracted is tabulated for reference in the “Bias Removed” column. The errors in the “Corrected Value” columns have had the uncertainty in the Eddington bias added in quadrature and represent 1$\sigma$ errors.

We also tabulate the derivatives of each photometric property with respect to every galaxy physical property used for the prediction, allowing the photometry presented here to be updated for values of Milky Way parameters that differ from the fiducial values used in this work (or, conversely, to correct for a re-calibration of extragalactic values). The methods applied are directly adapted from those presented in Section 5 of LNB15, but we provide a brief summary here.

To calculate the derivatives we offset each of the Galaxy’s measured properties by $\pm 0.1$ multiplied by the fiducial error in that property ($\sigma$). We then evaluate the GPR using these “offset” properties, rather than performing random draws from the Milky Way distributions. For example, in order to calculate $\partial(0M_r)/\partial(\log SFR)$ we would query the GPR for predictions of the Milky Way values at log SFR – $\sigma_{SFR}/10$, the nominal log SFR, and log SFR + $\sigma_{SFR}/10$, while all other parameters (stellar mass, disk scale length, etc.) are held at the fiducial Milky Way values. This yields three total predictions, two with offset SFR values and one at the nominal SFR. We then use these three values to calculate the derivative using the three-point Lagrangian interpolation method. Because these derivatives are small, they are sensitive to the limited training sample size required for the sci-kit – learn implementation of GPR (refer to Section 3.2.2 for details). In order to mitigate this effect we repeat this process for ten different training sets, re-evaluating the GPR prediction at the offset physical parameter and nominal value each time and calculating the derivative again. The derivatives presented here for each parameter are then computed as the average of the ten derivative values.

APPENDIX B: TESTING THE ACCURACY OF OUR GAUSSIAN PROCESS REGRESSION METHODS

In order to have confidence in the results we obtain from the Gaussian process regression, it is necessary to test the accuracy of the predicted photometry and error estimates. As in any supervised machine learning problem, we want to ensure that the given inputs map correctly to the given outputs. In order to enable an unbiased evaluation of our model we must split our data into separate training and test samples, as we do not want to evaluate a model with information that
Figure A1. Distribution of the various galaxy physical properties used to predict the Milky Way SED, viewed as both two-dimensional projections and one-dimensional histograms. In the contour plots the fiducial values for the Milky Way are designated by a black star, while in the histograms the fiducial value is indicated by a vertical black dashed line. The covariances are weak in almost all parameter combinations, except for the well-known $M_\star - R_d$ (mass–size) relation, the bimodal distribution of galaxies in the $M_\star$-SFR plane, and weaker correlations of bulge-to-total ratio and bar vote fraction with star formation rate.

was used to train it. To do this we use the scikit-learn function `sklearn.model_selection.test_train_split`. We use a split of 75% of our data for training and 25% of the data for testing. We do not construct a separate validation set as we do not have many hyper parameters to tune (the kernel defaults work well for our case, as discussed in Section 3.2.1).

For each photometric property that we study, we train the GPR using only the physical and photometric properties of the galaxies within the training sample. With that trained model we then make predictions for a given photometric property using the testing sample’s physical properties. This gives us predicted absolute magnitudes and colours for every galaxy in our testing sample, which we can then compare to their actual observed values.

Fig. B1 depicts the results of this training and testing analysis for all of the bands relevant to constructing the SED. We present histograms based on the differences between the intrinsic value of a photometric property and the predicted value from the GPR, $\Delta$, for each galaxy in the testing sample. We then divide these $\Delta$ values by the standard deviation predicted by the GPR for that galaxy, $\sigma$, to obtain the normalised residual, $\frac{\Delta}{\sigma}$. By focusing on this quantity we can assess whether a bias in a given galaxy is present and how large it is compared to the prediction uncertainties, and can additionally evaluate whether the error estimates from the GPR are too small (too optimistic) or too large (too conservative) compared to actual deviations.

In each histogram, we overlay a Gaussian with a mean of zero and a variance of one in grey. One would expect the distribution of $\frac{\Delta}{\sigma}$ to follow this distribution if there is no bias in the predictions and the error values are all accurate. We also list the means and medians of $\frac{\Delta}{\sigma}$ for each band on the corresponding histogram. Some quantities exhibit a small skew, but invariably the mean offsets are $< 0.05\sigma$ and the median offsets are $< 0.2\sigma$, so any bias is far subdominant to other sources of errors. In general the distribution of $\frac{\Delta}{\sigma}$ is tighter than the expected Gaussian, indicating that the $\sigma$ estimates from GPR are slightly too large (i.e., overly conservative).

In order to assess what may cause the small offsets, we have eval-
Section D). Colours presented here are in the AB magnitude system. Have been determined by the methods described in Section 3. The corrected value column refers to our final estimate after accounting for Eddington bias (cf. Table A1).

When we observe extragalactic objects their spectrum is redshifted into the “observed” reference frame. In order to convert from observed flux to rest-frame values, astronomers utilise k-corrections, which account for the shifting of restframe light into different passbands as redshift increases. While most of the photometry that we have used in this work (as described in Section 2.1) has readily available k-corrections into restframe bands, measurements in the WISE bands (Wright et al. 2010) do not.

Typically, k-corrections for a given band are calculated by calculating the effect on observed colours of redshifting a theoretical or empirical spectral energy distribution (SED) template; existing software packages such as kcorrect Blanton & Roweis (2007) have been designed to perform this task. However, this approach is not feasible at wavelengths where SED templates are not available or poorly constrained.

For this reason, in the WISE bands we adopt a new, almost purely empirical approach which builds on the template-based methods presented in Beare et al. (2014). Our k-correction techniques will be the subject of an upcoming paper, (Fielder, Newman, & Andrews, in prep). While we do not go into detail here, we summarise the strategy below.

We rely on the assumption that the k-correction needed for a given band is a simple (polynomial) function of a galaxy’s restframe colour determined in some pair of bands; i.e., that galaxy SEDs

| Property | Corrected Value | Bias Removed | $\partial \delta / \partial (\text{log M}_*)$ | $\partial \delta / \partial (\text{log SFR})$ | $\partial \delta / \partial (b/a)$ | $\partial \delta / \partial R_d$ | $\partial \delta / \partial (B/T)$ | $\partial \delta / \partial p_{bar}$ |
|----------|----------------|--------------|---------------------------------|---------------------------------|----------------------------|----------------|----------------|----------------|
| $^{0}(FUV \, - \, r)$ | 4.24 ± 0.59 | 0.16 ± 0.05 | 1.70 | -1.03 | -1.50 | -0.15 | 0.51 | 0.14 |
| $^{0}(NUV \, - \, r)$ | 3.60 ± 0.38 | 0.16 ± 0.05 | 1.34 | -0.85 | -1.13 | -0.12 | 0.76 | 0.09 |
| $^{0}(u \, - \, r)$ | 2.14 ± 0.14 | 0.06 ± 0.01 | 2.02 | -0.96 | -0.58 | -0.29 | -0.19 | -0.05 |
| $^{0}(g \, - \, r)$ | 0.73 ± 0.04 | 0.05 ± 0.01 | 0.13 | -0.13 | -0.09 | 0.00 | 0.07 | 0.01 |
| $^{0}(i \, - \, r)$ | -0.33 ± 0.03 | 0.03 ± 0.00 | -0.08 | 0.05 | 0.06 | 0.01 | -0.03 | 0.00 |
| $^{0}(z \, - \, r)$ | -0.61 ± 0.06 | 0.04 ± 0.01 | -0.16 | 0.09 | 0.14 | 0.01 | -0.03 | -0.02 |
| $^{0}(J \, - \, r)$ | -0.89 ± 0.11 | 0.02 ± 0.01 | -0.24 | 0.05 | 0.16 | 0.03 | 0.03 | -0.01 |
| $^{0}(H \, - \, r)$ | -1.11 ± 0.14 | 0.03 ± 0.01 | -0.28 | 0.05 | 0.09 | 0.03 | 0.02 | -0.03 |
| $^{0}(Ks \, - \, r)$ | -0.78 ± 0.15 | 0.02 ± 0.02 | -0.29 | 0.05 | 0.20 | 0.03 | 0.03 | -0.02 |
| $^{0}(W1 \, - \, r)$ | -0.17 ± 0.19 | 0.02 ± 0.01 | -0.30 | -0.12 | 0.44 | 0.09 | -0.09 | -0.03 |
| $^{0}(W2 \, - \, r)$ | 0.41 ± 0.26 | 0.00 ± 0.01 | -0.25 | -0.19 | 0.67 | 0.11 | -0.11 | -0.03 |
| $^{0}(W3 \, - \, r)$ | -0.30 ± 0.48 | 0.42 ± 0.04 | 0.40 | -1.18 | 0.54 | 0.12 | 0.06 | 0.07 |
| $^{0}(W4 \, - \, r)$ | -1.01 ± 0.70 | 0.06 ± 0.03 | 1.08 | -1.54 | 0.84 | 0.23 | -0.19 | -0.38 |

Table A1. X – r colour estimates for each photometric band X used in this work; these values are used to calculate the luminosities presented in Table 1, and have been determined by the methods described in Section 3. The corrected value column refers to our final estimate after accounting for Eddington bias (cf. Section D). Colours presented here are in the AB magnitude system.

Table A2. Additional colour estimates for the Milky Way, updating values from Table 1 and Table 3 of LNB15 in addition to values for WISE colours (as plotted in Fig. 8). The SDSS ugriz and WISE W1 – W4 values are presented in AB magnitudes. The Johnsons-Cousins U BV R I bands have been converted from the ugriz measurements via kcorrect on an object-by-object basis, as in LNB15, and are presented in the Vega magnitude system.

The histograms of $\frac{\delta \delta}{\delta p}$ separately for face-on galaxies (which we take to be those with axis ratio $b/a > 0.5$) and more inclined objects ($b/a \leq 0.5$). We find that most of the slight skewing that is observable in Fig. B1 is driven by the galaxies with higher inclinations. In this regime galaxy colour can change quickly and non-linearly with $b/a$, which can explain why the GPR struggles to accurately predict colours for these galaxies.

Overall, we find that the GPR performs well at mapping from galaxy physical parameter inputs to photometric properties, with biases that are much smaller than the predicted uncertainties, and error estimates that err on the conservative side. We therefore can use the method with confidence to make predictions for the photometric properties of the Milky Way.

APPENDIX C: K-CORRECTIONS FOR WISE BANDS

C1 Calculating the k-correction

When we observe extragalactic objects their spectrum is redshifted into the “observed” reference frame. In order to convert from observed flux to rest-frame values, astronomers utilise k-corrections, which account for the shifting of restframe light into different passbands as redshift increases. While most of the photometry that we have used in this work (as described in Section 2.1) has readily available k-corrections into restframe bands, measurements in the WISE bands (Wright et al. 2010) do not.
To the observed photometry. To do this, we first fit for the value of this function in two steps. First, we determine polynomial coefficients as functions of $a_1(0(g - r))$, $a_2(0(g - r))$, and $a_2^0(0(g - r))$ via a linear regression algorithm which is robust to outliers, specifically the Huber regression method implemented by scikit-learn (sklearn.linear_model.HuberRegressor). We assume a linear dependence between each coefficient and the restframe reference colour ($0(g - r)$ here).

The result of this process is a function that predicts the observed colour in a new pair of bands, as a function of the restframe colour in some other pair of bands and the redshift. At $z = 0$, the observed colour and restframe colour must agree, by definition. Hence, the difference between the prediction for the observed colour for an object at a given redshift, and the prediction for the observed colour at $z = 0$ (which corresponds to the value of $a_0(0(g - r))$, must be identical to the $k$-correction for the target colour. If one subtracts off the $k$-corrected reference band (which again can be an optical band where $k$-corrections are already well-determined), one is left with the $k$-corrected unknown band on its own.

We apply this (small) correction to the observed photometry to map to the restframe equivalent (i.e., we do not simply adopt the colour in the new bands predicted from the restframe colour in known bands – which could entail large shifts from the observed photometry for some objects – but rather we only make use of the small offset between observed and restframe colours that was fit across the sample). To describe the process symbolically, we use the restframe $0(g - r)$ colour for an object to obtain the $a_1$ and $a_2$ values for the mapping of colour in some band $X$ relative to $r$, $X - r$. Then we can obtain the restframe colour in the new band $0(X - r) = (X - r)_\text{obs} - a_1z - a_2z^2$. Finally, we can determine the restframe absolute magnitude in band $X$ via the expression $0M_X = 0M_r + 0(r - r)$.

The result of this process is a simple but effective empirical method for obtaining $k$-corrections for bands where SED templates are not well known, which can be applied so long as $k$-corrections have already been determined in another set of better-characterised bands. In this work, we use $0(g - r)$ colour to determine the $k$-correction constituting a one-parameter family. This colour can be determined in the optical where more conventional $k$-correction methods are effective; we just need some way to sort SEDs into a one-parameter family. The assumption that one parameter is sufficient is not perfect, as there is some diversity in SEDs at fixed restframe colour, but it is good to first order as variations in dust, specific star formation rate, and metallicity all have coarsely similar effects on the spectrum of a galaxy. Because we are focused on $k$ corrections over a limited redshift range near $z = 0$, getting things right to this level is sufficient to make $k$-correction errors far subdominant to other sources of uncertainty in the WISE bands.

Our goal is to produce a function that takes as inputs the apparent magnitude in some band, the redshift, and the restframe colour in some pair of bands (which can be determined by conventional $k$-correction methods, we will use $0(g - r)$ for this example), and returns the restframe absolute magnitude in the desired band. We determine this function in two steps. First, we determine polynomial relationships between the observed colour in a pair of bands (one of which serves as the anchor, and one of which is the desired target band) and redshift, in bins of the restframe colour in another (e.g., optical) pair of bands which is provided as one of the inputs. This relationship is determined via a second order polynomial fit; i.e., we parameterize the observed colour ($X - r)_\text{obs} = f(z) = a_0 + a_1z + a_2z^2$. For each bin in restframe colour, we determine the fit coefficients separately. This is similar to the technique applied in Brown et al. (2014); see, e.g., Fig. 2 of that work.

We then fit for the linear dependence of each of the fit coefficients on the mean restframe colour of a bin; i.e., we now treat the coefficients as functions of $a_1(0(g - r))$ and $a_2(0(g - r))$. We ignore the value of $a_0$ as it corresponds to the predicted mean restframe colour at $z = 0$; rather than assuming this mean colour is appropriate for all objects, we will instead only make small adjustments to the observed photometry. To do this, we first fit for $a_1(0(g - r))$ and $a_2^0(0(g - r))$ via a linear regression algorithm which is robust to outliers, specifically the Huber regression method implemented by scikit-learn (sklearn.linear_model.HuberRegressor). We assume a linear dependence between each coefficient and the restframe reference colour ($0(g - r)$ here).
mappings as a function of restframe colour and redshift, and WISE-\(r\) colours as the target in each case. We have selected these bands as \(g\) and \(r\) both have small photometric uncertainties and well-characterised \(k\)-corrections. We will present more details of our procedures and tests of their effectiveness in Fielder et al. (2021).

When evaluating the fits for the \(k\)-corrections we exclude objects that have large WISE photometric errors in the \(W1\) and \(W2\) channels. We do not place requirements on \(W3\) or \(W4\) errors as they are invariably large. Specifically, in \(W1\) we perform the \(k\)-correction fits restricting to galaxies with errors \(\sigma_{MW1} < 0.125\), while in \(W2\) we require errors \(\sigma_{MW2} < 0.25\). Objects with larger errors are still included when performing our GPR analyses.

### C2 Photometric Offsets Between SDSS and DESI Legacy Survey Imaging

As described in Section 2.1.3, we rely on Legacy Survey catalogues for WISE photometry, as the Tractor-based measurements have lower uncertainties than other public catalogues. Because of the use of a matched object model across all passbands, colours in WISE bands relative to optical bands will be determined with greater accuracy than total magnitudes in a single band. However, because the filters and instruments used for the Legacy Surveys have different transmission and response curves and photometry was performed with differing analysis pipelines (BASS \(g\) and \(r\) and MOSAIC-3 \(z\) filters were used in the northern portion of the Legacy Survey footprint, while in the south DECam was used for \(g\), \(r\), and \(z\)), the rest-frame \(r\) band absolute magnitudes for a given object should differ between the SDSS and Legacy data.

To calculate the WISE minus SDSS \(r\) band colours used to construct the Milky Way SED, we therefore must take the WISE minus Legacy Survey \(r\) band colour (which should be measured self-consistently due to the use of a common model) and apply a small correction to compensate for the differences between Legacy \(r\) and SDSS \(r\). To do this we proceed as follows:

(i) **Calculate absolute magnitudes for DESI Legacy bands** – We calculate restframe absolute magnitudes for all objects in the cross-matched catalogue in the BASS and DECam \(r\) and SDSS \(g\) and \(r\) bands, using the kcorrect (Blanton & Roweis 2007) software. We base these calculations only on the SDSS \(ugriz\) photometry for our galaxy sample.

(ii) **Calculate the filter offsets in \(r\)** – We now wish to determine the offsets between restframe \(r\) absolute magnitude in Legacy Survey and SDSS filters, as a function of restframe colour: \(\Delta_r = M_{r,\text{BASS}} - M_{r,\text{SDSS}}\) for the North, or \(\Delta_r = M_{r,\text{DECam}} - M_{r,\text{SDSS}}\) for the South. In Fig. C1 we plot \(\Delta_r\) as a function of SDSS \(0(g-r)\) for our sample. Almost all objects fall along a linear relationship between the \(r\)-band offset and colour.

(iii) **Fit for offsets as a function of \(0(g-r)\)** – We perform a robust least-squares fit using the scikit – learn (Pedregosa et al. 2011) Huber regression function with \(0(g-r)\) as the independent variable.
and \( \Delta_r \) as the dependent variable. In Fig. C1 we plot these fits with black dashed lines.

Using the coefficients resulting from this fit, we can convert a restframe colour referenced to Legacy Survey \( r \) (e.g., \( 0(W1 - r_{\text{DECam}}) \)) to one referenced to SDSS \( r \) \( 0(W1 - r_{\text{SDSS}}) \), for this example) by evaluating the fit line at the restframe \( 0(g - r) \) colour for a given object. Our GPR predictions for WISE absolute magnitudes and WISE- \( r \) colours require use of this correction; however, for WISE colours such as \( (W1 - W2) \) the dependence on the offset cancels out.

We apply this correction in tandem with calculating the \( k \)-corrected WISE colours, before training the Gaussian process regression. In the previous sub-section we expressed our calculation of absolute magnitudes as \( 0M_X = 0M_r + 0(X - r) \). Re-written to explicitly use colour relative to the SDSS \( r \) band, this expression becomes

\[
0M_X = 0M_{SDSS} + 0(X - r_{SDSS}) = 0M_{r_{SDSS}} + (0M_X - 0M_{r_{SDSS}}). \quad (C1)
\]

However, we have measured the \( X - r_{\text{Legacy}} \) colour, not \( X - r_{SDSS} \). In order to account for the offset between Legacy and SDSS photometry we therefore require the correction derived above:

\[
0M_X = 0M_{r_{SDSS}} + (0M_X - 0M_{r_{Legacy}}) + (0M_{r_{Legacy}} - 0M_{r_{SDSS}}), \quad (C2)
\]

where \( (0M_{r_{Legacy}} - 0M_{r_{SDSS}}) \) is the offset between restframe SDSS \( r \) and DESI Legacy Survey \( r \), or \( \Delta_r \) above. Applying this correction before performing further analysis allows us to train the GPR based upon the combined North and South DESI Legacy catalogues rather than training with each separately.

The magnitude of the offsets applied to the WISE photometric band estimates are small, spanning from \(-0.14\) to \(0.05\) in magnitude. Compared to the photometric errors of these bands in Table A1 and Table A3, the errors on the photometry far outshine any error attributed to the DESI Legacy - SDSS offset (especially for W3 and W4).

**APPENDIX D: EDDINGTON BIAS CORRECTIONS**

Uncertainties in stellar mass, star formation rate, disc scale length, etc. can lead to biases in the inferred photometric properties of the Milky Way. When the distribution of objects in parameter space varies non-linearly, scatter from errors will preferentially move objects from more sparsely populated regions to rarer locations compared to the opposite situation. This can lead to an “Eddington” bias in inferred properties. For example, because of the rarity of massive galaxies, a galaxy with a large stellar mass measurement is more likely to have an intrinsic stellar mass that is smaller than the measured value than one that is larger, as there is a much larger number of objects that can up-scatter compared to the number that down-scatter. As a result, the stellar mass measurements used when training the Gaussian process regression would then be biased high. Similar effects can occur with star formation rate, disk scale length, etc.

We quantify this bias through an empirical approach based upon the methods presented in LNB15. We perturb the measured values of the galaxy sample used to train the GPR with Gaussian noise sampled from the measured errors. Specifically this means we repeatedly add to each galaxy’s measured \( M_* \), SFR, \( B/T \), etc. a value randomly drawn from a Gaussian distribution centred at zero with a standard deviation of that object’s error in the given quantity (e.g., \( \sigma_{\log M_*} \)), and determine the effects that adding errors to each quantity has. We make the simplifying assumption that the bar vote fraction should not significantly contribute to the Eddington bias (as uncertainties in that quantity are difficult to characterise, but also should have limited effects on photometry).

To obtain predicted photometry for a given noise level, we run the GPR in a similar manner to the methods used in Section 3, except we now train the regression on the perturbed sample before predicting photometric quantities. We then predict the photometric quantity of interest by evaluating each model assuming the fiducial value for each Milky Way physical property (i.e., we use a fixed set of values for all evaluations rather than sampling from the MW distributions). In our application, we perform this procedure on samples with Gaussian noise applied to each physical property from one to four times successively.

In the original cross-matched training set, the values for each physical property have been perturbed from their true, intrinsic values due to measurement errors; we define this case as corresponding to having noise applied \( n = 1 \) times. However, we wish to evaluate what the measurement would be if there were no noise; i.e., for the case where \( n = 0 \). We therefore wish to characterise the difference in the property of interest between the case where noise has been applied...
n or \( n - 1 \) times, and evaluate for \( n = 1 \) to determine the correction needed to remove Eddington bias.

To determine this value we perform a least squares quadratic fit to the set of differences between GPR predictions as we add more noise; i.e., we fit a relation of the form \( P_n - P_{n-1} = A n^2 + B n + C \), where \( P_n \) is the prediction from GPR when noise has been applied a total of \( n \) times and \( A, B, \) and \( C \) are coefficients of the fit. We use the resulting fit to extrapolate to \( P_1 - P_0 \), which should correspond to the effect of Eddington bias. The mean Eddington bias at \( n = 1 \) is then quantified as the sum of the coefficients \( A + B + C \) from the quadratic fit of the set of differences \( P_n - P_{n-1} \). In practice, we perform the entire analysis (adding random noise repeatedly, training the GPR, fitting for \( P_n - P_{n-1} \), and adding the coefficients) 25 times. The effect of Eddington bias for a given band is then calculated as the mean of the set of 25 values. We then subtract this value from the GPR-predicted mean for a given photometric property of the Milky Way to obtain a corrected value. The bias removed in each of our predictions is documented in the third column of Table A1-Table A3.

The uncertainty in each of the 25 Eddington bias estimates (i.e., in the quadratic function evaluated at the point \( n = 1 \)) can be calculated via the square root of the sum of the covariance matrix for the coefficients of the least-squares quadratic fit (as when \( n = 1 \) the quadratic result is simply \( A + B + C \)). The uncertainty in the Eddington bias in each band is therefore equivalent to \( \sigma_{\text{bias}} = \frac{\sqrt{\sum_{i=1}^{n} \sigma_i^2}}{n} \), where \( \sigma_i \) is the estimated uncertainty from the \( i \)'th sample and here \( n = 25 \). As a check we have also computed errors via the standard error on the mean Eddington bias and find comparable (though somewhat more optimistic) results. The Eddington bias uncertainty is combined in quadrature with the uncertainty estimated from sampling the GPR to produce the errors on the corrected values in the tables in Section A.

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