Revisiting a non-parametric reconstruction of the deceleration parameter from observational data

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ABSTRACT
A non-parametric reconstruction of the deceleration parameter $q$ is carried out. The observational datasets are so chosen that they are model independent as much as possible. The present acceleration and the epoch at which the cosmic acceleration sets in is quite as expected, but beyond a certain redshift ($z \approx 2$), a negative value of $q$ appears to be in the allowed region. A survey of existing literature is given and compared with the results obtained in the present work.

Key words: reconstruction, dark energy, deceleration parameter, cosmology

1 INTRODUCTION
Although observations have established a recent accelerated expansion of the universe (Riess et al. 1998; Perlmutter et al. 1999), the nature of the agent driving this acceleration is yet to be established. For a comprehensive review of various aspects of this accelerated expansion and various tensions between observations, we refer to the work of Wang et al. (2016). A host of models, either in the form of an additional field called dark energy in the matter sector or in the form of modifying the theory of gravity itself can do the trick of explaining the accelerated expansion, yet all of them have the generic problem of not being desperately required by any other branch of physics!

This inspires a reverse way of looking at the evolution, rather than trying to find the evolution from the given matter sector using Einstein field equations, one uses the evolutionary history, that fits with observations, to find out the possible distribution of matter. Normally physical quantities like the equation of state parameter of the dark energy (Saini et al. 2000; Sahni & Starobinsky 2006), the quintessence potential (Piazza 1998; Starobinsky 1998; Huterer & Turner 1999, 2001) occupy the central stage of interest in this game of reconstruction. A recent trend of reconstruction ignores the dynamical evolution and makes an attempt towards finding out the kinematical quantities directly from observations. The Hubble rate of expansion $H$ being an observable, the natural choice as the relevant parameters are the next higher order derivatives like the deceleration parameter $q$ and the jerk parameter $j$. It should be mentioned that the observational quantities also suffer from tensions between different data sets, this is particularly true for the measurement of the present value of the Hubble parameter $H_0$ (see Wang et al. (2016); Mortsell & Dhawan (2018) and references therein). A reconstruction of kinematical quantities might have a say on these tensions different from that of the results given by physical quantities.

Already there are quite a few investigations in this direction. Reconstruction of the deceleration parameter $q$ naturally started quite a long time back (Gong & Wang 2007; Wang et al. 2010; Jesus et al. 2019). As $q$ is evolving, the next higher order derivative, the jerk parameter $j$ has also been reconstructed from observational data (Longo 2005; Rapetti et al. 2007; Zhai et al. 2013; Mukherjee & Banerjee 2016, 2017). These investigations mostly rely on a parametrization of the kinematical quantity and an estimation of the parameters from observational data. This approach normally is a bit biased as the quantities depend on $z$ in a given way depending on the functional form assumed. A more robust form of reconstruction is a non-parametric reconstruction, where the quantity of interest is reconstructed directly from the data without assuming any functional form. For the physical quantities like the equation of state parameter of the dark energy, dark energy potential etc., this practice is already there (Sahlin, Liddle & Parkinson 2005, 2007; Holsclaw et al. 2010a,b, 2011; Crittenden et al. 2012; Nair, Jhingan & Jain 2014; Zhang et al. 2019).

There are also some examples of a non-parametric reconstruction of kinematical quantity like $q$. Bilicki & Seikel (2012) reconstructed $q$ using Union 2.1 (Suzuki et al. 2012) compilation for the Supernova data. Lin, Li & Tang (2019) did a similar reconstruction with the Pantheon (Scolnic et al. 2018) compilation for the Supernova data with various priors for $H_0$. A slightly older similar work in Zhang & Xia (2016) uses Union 2 (Amanullah et al. 2010) and Union 2.1

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2 COSMIC KINEMATICS

The infinitesimal distance element in a spatially homogeneous and isotropic universe is given by the Friedmann-Lemaître-Robertson-Walker (FLRW) metric

\[ ds^2 = -c^2 dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\theta d\phi + r^2 \sin^2 \theta d\phi^2 \right), \]  

where \( a(t) \) is the scale factor and \( k \) is the curvature index.

The Hubble parameter is defined as,

\[ H = \frac{\dot{a}}{a}. \]  

We define the reduced Hubble parameter \( h(z) = \frac{H(z)}{100} \), where the suffix 0 indicates the present value of the quantity. The luminosity distances of luminous objects, like SN-Ia, is given as

\[ d_L(z) = \frac{c}{H_0 \sqrt{\Omega_{\Lambda 0}}} \sin n \left( \sqrt{\Omega_{\Lambda 0}} \int_0^z \frac{dz'}{h(z')} \right), \]  

in which \( z \) is the redshift given by \( 1 + z = \frac{\omega}{a} \) and the \( n \) function is a shorthand for the definition,

\[ \sin n \omega = \begin{cases} \sin x & (\Omega_{\Lambda 0} > 0), \\ x & (\Omega_{\Lambda 0} = 0), \\ \sin x & (\Omega_{\Lambda 0} < 0). \end{cases} \]

The dimensionless parameter \( \Omega_{\Lambda 0} \) is positive, zero or negative corresponding to the spatial curvature \( k = -1, 0, +1 \) respectively. The present work will deal with a zero spatial curvature \( (k = 0) \).

For convenience, we shall define a dimensionless comoving luminosity distance,

\[ D(z) = \frac{H_0}{c(1 + z)} d_L(z). \]  

Combining Eq. (3) and (4), and taking derivative with respect to \( z \), we obtain the Hubble parameter with spatial curvature as,

\[ H(z) = \frac{H_0 \sqrt{1 + \Omega_{\Lambda 0} D^2}}{D'}, \]  

where the prime denotes derivative with respect to \( z \).

3 GAUSSIAN PROCESS METHODOLOGY

Gaussian processes (Williams 1999; MacKay 2003; Rasmussen & Williams 2006) is a model-independent method for reconstructing the target function without limiting to any particular parametrization form. It is a distribution over functions, namely they generalize the idea of a Gaussian distribution for a finite number of quantities to the continuum. Given a set of Gaussian-distributed data points one can use Gaussian processes to reconstruct the most probable underlying continuous function describing the data, and also obtain the associated confidence levels, without assuming a concrete parametrization of the aforesaid function. It requires only a probability on the target function \( D(z) \). Thus, a Gaussian process is a generalization of the Gaussian probability distribution. For a detailed overview one can refer to the Gaussian Process website\(^1\).

In cosmology, it has a wide application in reconstructing dark energy (Holsclaw et al. 2010b; Seikel, Clarkson & Smith 2012) and cosmography (Shafieloo, Kim & Linde 2012), testing standard concordance model (Yahya et al. 2014), distance duality relation (Costa, Busti & Holanda 2015), determining the interaction between dark matter and dark energy (Yang, Guo & Cai 2015), spatial curvature (Cai, Guo & Yang 2010), constraining the dark energy equation of state (Wang & Meng 2017), and many more (Wang & Meng 2019; Zhou et al. 2019; Cai, Khurshudyan & Sari-dakis 2020). In the pedagogical introduction to GP, Seikel, Clarkson & Smith developed the publicly available GaPP (Gaussian Processes in Python) code.

Assuming the observational data, such as the distance data \( D \), obey a Gaussian distribution with mean and variance, the posterior distribution of reconstructed function can be expressed as a joint Gaussian distribution of different data sets involving \( D \). In this process, the key ingredient is the covariance function \( k(z, \tilde{z}) \) which correlates the values of different \( D(z) \) at redshift points \( z \) and \( \tilde{z} \) separated by \( |z - \tilde{z}| \) distance units. The covariance function \( k(z, \tilde{z}) \) depends on a set of hyperparameters (e.g. the characteristic length scale \( l \) and the signal variance \( \sigma_f \)). This approach also provides a robust way to estimate derivatives of the function. The hyperparameter \( l \) corresponds roughly to the distance one needs to move in input space before the function value changes significantly, while \( \sigma_f \) describes typical

\(^1\) http://www.gaussianprocess.org
change in the function value. Different choices for the covariance function may have different effects on the reconstruction, for example the squared exponential covariance function (see Rasmussen & Williams, chapter 4, page 83)

$$k(z, \bar{z}) = \sigma_f^2 \exp \left(-\frac{(z - \bar{z})^2}{2l^2}\right)$$

and the Matérn class covariance function (see Rasmussen & Williams, chapter 4, page 85),

$$k_{\nu=p+\frac{1}{2}}(z, \bar{z}) = \sigma_f^2 \exp \left(-\frac{\sqrt{2p+1}f}{l} \cdot \left|z - \bar{z}\right|\right) \times \frac{pl}{(2p)!} \sum_{i=0}^{p} \frac{(p + i)!(\sqrt{2p+1}f/l)^{p-i}}{i!(p-i)!} \cdot (7)$$

We shall use the squared exponential and the Matérn ($\nu = \frac{1}{2}, p = 4$) covariance function in our analysis. The former is not always a suitable choice, but later one leads to the most reliable and stable results amongst the other significant choices (Seikel & Clarkson 2013) and is given as,

$$k(z, \bar{z}) = \sigma_f^2 \exp \left(-\frac{3\bar{z} - z}{l}\right) \times \left[1 + \frac{3\bar{z} - z}{l}\right] + \frac{27(z - \bar{z})^2}{7l^2} + \frac{18(z - \bar{z})^3}{7l^3} + \frac{27(z - \bar{z})^4}{35l^4}\right].$$

4 OBSERVATIONAL DATASETS

In this work we use the Observational Hubble data (OHD), the Supernova distance modulus data (SNe) and the Cosmic Microwave Background (CMB) Shift parameter data for the reconstruction of the cosmic deceleration parameter $q$ as a function of the redshift $z$. A brief summary of the datasets is given below.

4.1 OHD Data

The Hubble parameter $H(z)$ is usually evaluated as a function of the redshift $z$

$$H(z) = -\frac{1}{1+z} \frac{dz}{dt}.$$  

The Hubble $H(z)$ data are not direct products from a tailored telescope, but can be acquired by two different ways. One is to calculate the differential ages of galaxies (Simon, Verde & Jimenez 2005; Jimenez & Loeb 2008; Stern et al. 2010), usually called cosmic chronometer (CC). Another one is the deduction from the radial BAO peaks in the galaxy power spectrum (Gaztanaga, Cabre & Hui 2009; Moresco et al. 2012) or from the BAO peak using the Ly-α forest of QSOs (Delubac et al. 2013) based on the clustering of galaxies or quasars. In the present paper, we use the compilation of OHD data points collected by Magana et al. (2018) and Geng et al. (2018), including almost all $H(z)$ data reported in various surveys so far. The 31 CC $H(z)$ data points are listed in Table 1 and the 23 $H(z)$ data points obtained from clustering measurements are listed in Table 2. One may find that some of the $H(z)$ data points from clustering measurements are correlated since they either belong to the same analysis or there is an overlap between the galaxy samples. Here in this paper, we mainly take the central value and standard deviation of the OHD data into consideration. Thus, just as in Geng et al., we assume that they are independent measurements. After the preparation of $H(z)$ data, we should normalize them to obtain the dimensionless or reduced Hubble parameter $h(z) = H(z)/H_0$. Considering the error of Hubble constant $\sigma_H$, we can calculate the uncertainty of $h(z)$ as,

$$\sigma_h^2 = \frac{\sigma_H^2}{H_0^2} + \frac{H^2}{H_0^4} \sigma_{H_0}^2$$  

where $\sigma_{H_0}$ is the error associated with $H_0$.

4.2 SNIa Data

For the supernova data, we use the recent Pantheon data sample by Scolnic et al. (2018). The numerical data of the full Pantheon SNIa catalogue is publicly available in the website\(^2\)\(^3\).

Table 1. The latest Hubble parameter measurements $H(z)$ (in units of km s\(^{-1}\) Mpc\(^{-1}\)) and their errors $\sigma_H$ at redshift $z$ obtained from the differential age method (CC).

| Index | $z$  | $H(z)$ | $\sigma_H$ | References |
|-------|------|--------|------------|------------|
| 1     | 0.07 | 69     | 19.6       | Zhang et al. (2014) |
| 2     | 0.12 | 68.6   | 26.2       | Zhang et al. (2014) |
| 3     | 0.2  | 72.9   | 29.6       | Zhang et al. (2014) |
| 4     | 0.28 | 88.8   | 36.6       | Zhang et al. (2014) |
| 5     | 0.1  | 69     | 12         | Stern et al. (2010) |
| 6     | 0.17 | 83     | 8          | Stern et al. (2010) |
| 7     | 0.27 | 77     | 14         | Stern et al. (2010) |
| 8     | 0.4  | 95     | 17         | Stern et al. (2010) |
| 9     | 0.48 | 97     | 60         | Stern et al. (2010) |
| 10    | 0.88 | 90     | 40         | Stern et al. (2010) |
| 11    | 0.9  | 117    | 23         | Stern et al. (2010) |
| 12    | 1.3  | 168    | 17         | Stern et al. (2010) |
| 13    | 1.43 | 177    | 18         | Stern et al. (2010) |
| 14    | 1.53 | 140    | 14         | Stern et al. (2010) |
| 15    | 1.75 | 202    | 40         | Stern et al. (2010) |
| 16    | 0.1797 | 75  | 4          | Moresco et al. (2012) |
| 17    | 0.1993 | 75  | 5          | Moresco et al. (2012) |
| 18    | 0.3519 | 83  | 14         | Moresco et al. (2012) |
| 19    | 0.5929 | 104 | 13         | Moresco et al. (2012) |
| 20    | 0.6797 | 92  | 8          | Moresco et al. (2012) |
| 21    | 0.7812 | 105 | 12         | Moresco et al. (2012) |
| 22    | 0.8754 | 125 | 17         | Moresco et al. (2012) |
| 23    | 1.037 | 154  | 20         | Moresco et al. (2012) |
| 24    | 0.3802 | 83  | 13.5       | Moresco et al. (2016) |
| 25    | 0.4004 | 77  | 10.2       | Moresco et al. (2016) |
| 26    | 0.4247 | 87.1 | 11.2       | Moresco et al. (2016) |
| 27    | 0.4497 | 92.8 | 12.9       | Moresco et al. (2016) |
| 28    | 0.4783 | 80.9 | 9          | Moresco et al. (2016) |
| 29    | 1.363 | 160   | 33.6       | Moresco (2015) |
| 30    | 1.965 | 186.5 | 50.4       | Moresco (2015) |
| 31    | 0.47 | 89     | 34         | Ratsimbazafy et al. (2017) |

\(^2\) http://dx.doi.org/10.17909/T9SPQ4X

\(^3\) https://archive.stsci.edu/prepds/ps1cosmo/index.html
The colour and stretch corrections are no longer required, so we can fix $\alpha = \beta = 0$ and proceed with the following. The absolute magnitude of SN-Ia is degenerated with the Hubble parameter, and we fix it to $MB = -19.35$, the best-fitting value of $\Lambda CDM$. We convert the distance modulus of SN-Ia to the normalized comoving distance through the relation

$$D(z) = \frac{c(1+z)}{H_0} 10^{\frac{\mu}{5} - 5},$$

where $\mu$ is given by the difference between the corrected apparent magnitude $m_H$ and the absolute magnitude $M_B$ in the B-band for SN-Ia.

The statistical uncertainty $C_{stat}$ and systematic uncertainty $C_{sys}$ are also given. The total uncertainty matrix of distance modulus is given by,

$$\Sigma_{\mu} = C_{stat} + C_{sys}. $$

The uncertainty of $D(z)$ is propagated from the uncertainties of $\mu$ and $H_0$ using the standard error propagation formula,

$$\Sigma_D = D_1 \Sigma_\mu D_1^T + \sigma_{H_0}^2 D_2 D_2^T,$$

where $\sigma_{H_0}$ is the uncertainty of Hubble constant, the superscript $'T'$ denotes the transpose of a matrix, $D_1$ and $D_2$ are the Jacobian matrices,

$$D_1 = \text{diag} \left( \frac{\ln 10}{5} D \right),$$

$$D_2 = \text{diag} \left( \frac{1}{H_0} D \right),$$

where $D$ is a vector whose components are the normalized comoving distances of all the SN-Ia.

4.3 CMB Shift Parameter

The so-called shift parameter is related to the position of the first acoustic peak in the power spectrum anisotropies of the cosmic microwave background (CMB). However the shift parameter $R$ is not directly measurable from the cosmic microwave background, and its value is usually derived from data assuming a spatially flat cosmology with dark matter and cosmological constant.

$$R = \sqrt{\Omega_m} \int_0^{z_c} \frac{dz'}{h(z')}$$

where $z_c = 1089$ is the redshift of recombination. We use the CMB shift parameter $R = 1.7488 \pm 0.0074$ and matter density parameter $\Omega_m = 0.308 \pm 0.012$ from the Planck’s release (Ade et al. 2015) as important supplements of SN-Ia data.

5 RECONSTRUCTION OF THE DECELERATION PARAMETER

The deceleration parameter is a dimensionless measure of the cosmic acceleration of the expansion rate. It is defined by,

$$q = \frac{1}{aH^2} \frac{d^2a}{dt^2}. $$
The motivation of the present work is a non-parametric reconstruction of the deceleration parameter from observational data. This can be written as a function the reduced Hubble parameter \(h\) and \(z\) which in turn can be written as function of the comoving luminosity distance \(D\) and their derivatives,

\[
q(z) = \frac{k'}{h} (1 + z)^{-1} - 1 = \frac{\Omega_{k0}DD'' - (1 + \Omega_{k0}D^2)D'\nu}{D'(1 + \Omega_{k0}D^2)} (1 + z)^{-1}.
\]

The uncertainty in \(q(z)\), \(\sigma_q\) is obtained by error propagating Eq. (21).

\[
\left(\frac{\sigma_q}{1 + q}\right)^2 = \left(\frac{\sigma_{k'}}{h}\right)^2 + \left(\frac{\sigma_{h}}{h}\right)^2 - \frac{2\sigma_{k0}\nu}{hh'}
\]

We reconstruct the cosmological deceleration parameter \(q(z)\) using the distance data and their derivatives using the Gaussian Process methodology. In Fig. 1, we plot the reconstructed \(q(z)\) within \(3\sigma\) region using different combinations of datasets for a mean standard value of \(H_0 = 70.35 \pm 2.22\) km s\(^{-1}\) Mpc\(^{-1}\). In all the curves, the thick dark central curve is the best fit curve.

We examine the effect which, the two different strategies for determining value of \(H_0\) has on the reconstruction. Locally, the Hubble parameter has been found to be \(H_0 = 73.52 \pm 1.62\) km s\(^{-1}\) Mpc\(^{-1}\) presented by the Hubble Space Telescope photometry of long-period Milky Way Cepheid and GAIA parallaxes (Riess et al. 2018) (hereafter R18).

Another strategy involves an extrapolation of data on the early Universe from the CMB where, \(H_0 = 67.27 \pm 0.60\) km s\(^{-1}\) Mpc\(^{-1}\) (Aghanim et al. 2018) provided by Planck 2018 power spectra (TT,TE,EE+lowE) measurements by assuming base \(\Lambda\)CDM model (hereafter P18). The effect of these two strategies on the reconstruction has been shown in Fig. 2 and Fig. 3. The shaded regions correspond to the 68%, 95% and 99% confidence levels (CL). The corresponding choice of the hyperparameters for the squared exponential covariance function is given in Table 3, and that for the Matérn 9/2 covariance in Table 4. Throughout the analysis, we have assumed a spatially flat universe, i.e., \(\Omega_k = 0\).

where \(a\) is the scale factor of the universe. Cosmological observations indicate that the universe is undergoing an accelerated expansion in the recent epoch, i.e., \(q < 0\). However, this acceleration must have set in during a recent past and not a permanent feature of the evolution, so that the Big Bang Nucleosynthesis and the formation of structure must have taken place in a perfect ambience.

The motivation of the present work is a non-parametric reconstruction of the deceleration parameter from observational data. This \(q\) can be written as a function the reduced Hubble parameter \(h\) and \(z\) which in turn can be written as function of the comoving luminosity distance \(D\) and their derivatives,

\[
q(z) = \frac{k'}{h} (1 + z)^{-1} - 1 = \frac{\Omega_{k0}DD'' - (1 + \Omega_{k0}D^2)D'\nu}{D'(1 + \Omega_{k0}D^2)} (1 + z)^{-1}.
\]

The uncertainty in \(q(z)\), \(\sigma_q\) is obtained by error propagating Eq. (21).

\[
\left(\frac{\sigma_q}{1 + q}\right)^2 = \left(\frac{\sigma_{k'}}{h}\right)^2 + \left(\frac{\sigma_{h}}{h}\right)^2 - \frac{2\sigma_{k0}\nu}{hh'}
\]

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The present work contains some sophistication in the process, certainly the non-parametric one, which does not assume any functional form of $q$ to start with. As mentioned in the introduction, there are quite a few efforts in this direction. The present work contains some sophistication in the process, but arrives at similar results, at least qualitatively.

We have used the Pantheon compilation for the Supernova data, CMB shift parameter data and the OHD as detailed before. We used the Matérn 9/2 and squared exponential covariance using the both of the conflicting $H_0$ priors, namely the R18 and P18, as well as a mean $H_0$ prior. In all cases, the common feature is that the best-fit curve for $q$ shows that the present acceleration has set in quite recently, for $z > 0.5$ but well below $z = 1$. However, in a recent past the best-fit curve has a dip, indicating another stint of accelerated expansion even in a $1 \sigma$ confidence level in some cases and certainly in a $3 \sigma$ confidence level in all cases. This feature is observed close to $z = 2$ for all cases of the combined datasets, independent of the choice of $H_0$ prior and the covariance function used. However, this dip in the best fit of $q$ in the recent past allows a decelerated expansion as well in $1 \sigma$ except for cases when all the datasets (SNe, CMB, OHD and P18 or R18), this deceleration is allowed only in $3 \sigma$, if Matérn 9/2 covariance is used. It should be noted that from $z = 0$ to close to $z = 0.5$, no deceleration is allowed even in $3 \sigma$ in all the cases.

The existing literature on non-parametric reconstruction of $q$ also indicates this dip in $q$ in the recent past. Bilicki & Seikel (2012) worked with either SNe data (Union 2.1) or OHD and radial Baryon Acoustic Oscillation (BAO) data. A combination of all the data sets was avoided in this work. The present work deals with the combination of various data sets, but avoids the angular BAO data. Lin, Li & Tang (2019) work with the squared exponential covariance solely. With only SNe data (the Pantheon compilation), they find no dip in the best fit of $q$ although such a dip, indicating an accelerated expansion in the recent past beyond a short lived decelerated phase is very much allowed at least in $2 \sigma$. With the OHD data included, the possibility of this dip in $q$ is quite clear in their work. Using Matérn 9/2 covariance, Zhang & Xia (2016), found that with the SNe data alone, a negative $q$ beyond a short lived deceleration is allowed in $2 \sigma$, but all other data sets indicate a dip in $q$ towards a negative value. In this work Zhang & Xia, individual datasets were used, the effect of the combination of data sets were ignored.

The present work is in fact a generalization of all these investigations. We use various datasets, and their combinations, and used both the Matérn 9/2 and squared exponential covariance for the analysis. We qualitatively find the similar astonishing result, perhaps a bit more strongly. The two competing values of $H_0$ can hardly make any difference in this connection. It should be mentioned that we worked out the whole process with the Union 2.1 compilation as well. The data is only up to $z = 1.41$. However, if extrapolated, this also gives very similar result. The inclusion of that in the present paper is avoided for the economy of space.

As a conclusion we can say that not only the nature of dark energy, the evolution history of the universe is yet to be properly ascertained. We agree with Lin, Li & Tang (2019) that we need more data and also perhaps more model independent treatment of the data as well.

6 DISCUSSION OF THE RESULTS

As the universe decelerates at different rates during various phases of the evolution, a reconstruction of $q$ is definitely an important tool in our attempt to understand the history of the universe. An unbiased way of a reconstruction is certainly the non-parametric one, which does not assume any functional form of $q$ to start with. As mentioned in the introduction, there are quite a few efforts in this direction. The present work contains some sophistication in the process, but arrives at similar results, at least qualitatively.

We have used the Pantheon compilation for the Supernova data, CMB shift parameter data and the OHD as detailed before. We used the Matérn 9/2 and squared exponential covariance using the both of the conflicting $H_0$ priors, namely the R18 and P18, as well as a mean $H_0$ prior. In all cases, the common feature is that the best-fit curve for $q$ shows that the present acceleration has set in quite recently, for $z > 0.5$ but well below $z = 1$. However, in a recent past the best-fit curve has a dip, indicating another stint of accelerated expansion even in a $1 \sigma$ confidence level in some cases and certainly in a $3 \sigma$ confidence level in all cases. This feature is observed close to $z = 2$ for all cases of the combined datasets, independent of the choice of $H_0$ prior and the covariance function used. However, this dip in the best fit of $q$ in the recent past allows a decelerated expansion as well in $1 \sigma$ except for cases when all the datasets (SNe, CMB, OHD and P18 or R18), this deceleration is allowed only in $3 \sigma$, if Matérn 9/2 covariance is used. It should be noted that from $z = 0$ to close to $z = 0.5$, no deceleration is allowed even in $3 \sigma$ in all the cases.

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7 DATA AVAILABILITY STATEMENT

The data generated underlying this article are available in Zenodo, at https://doi.org/10.5281/zenodo.3967607. These datasets, derived from sources in the public domain are available in the manuscript, and/or duly cited.
