Inflation wars: a new hope

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Abstract. We explore a class of primordial power spectra that can fit the observed anisotropies in the cosmic microwave background well and that predicts a value for the Hubble parameter consistent with the local measurement of $H_0 = 74$ km/s/Mpc. This class of primordial power spectrum consists of a continuous deformation between the best-fit power law primordial power spectrum and the primordial power spectrum derived from the modified Richardson-Lucy deconvolution algorithm applied to the $C_\ell$s of best-fit power law primordial power spectrum. We find that linear interpolation half-way between the power law and modified Richardson-Lucy power spectra fits the Planck data better than the best-fit ΛCDM by $\Delta \log \mathcal{L} = 2.5$. In effect, this class of deformations of the primordial power spectra offer a new dimension which is correlated with the Hubble parameter. This correlation causes the best-fit value for $H_0$ to shift and the uncertainty to expand to $H_0 = 70.2 \pm 1.2$ km/s/Mpc. When considering the Planck dataset combined with the Cepheid $H_0$ measurement, the best-fit $H_0$ becomes $H_0 = 71.8 \pm 0.9$ km/s/Mpc. We also compute a Bayes factor of $\log K = 5.7$ in favor of the deformation model.

Keywords: CMBR theory, cosmological parameters from CMBR, inflation

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1 Introduction

Much work has been done in recent years to test the assumptions of the ΛCDM model (Λ for a cosmological constant, CDM for cold dark matter). This model, with just six parameters, has successfully explained the anisotropies in the cosmic microwave background (CMB) to a remarkable degree [5]. Two of those parameters, $A_s$ and $n_s$, characterize the amplitude and spectral index of the primordial power spectra (PPS) of the initial Gaussian fluctuations in the early Universe. This parametrization is an explicit assumption about unknown physics. There is, of course, no reason this parametrization must be true. The simplest models of inflation generically predict a power law PPS [3, 4, 7, 29] but more complicated models can have a PPS with large deviations away from a power law, from broad features, to local ones, to oscillations [12, 16]. In order to test whether a power law is a sufficient parametrization for existing CMB data, or whether some alternative is needed, one can iterate over a possibly infinite number of models and check if their evidences show a strong preference for them over ΛCDM, or one can use model independent methods.

Motivation to look for some extension to the base ΛCDM model is found in the “$H_0$ tension”. This tension is a disagreement between the prediction of the Hubble constant $H_0$ from the ΛCDM fit to the Planck CMB data [5], and the local measurement of $H_0$ via Cepheid calibration of supernova [34]. This tension has reached the level of 4.4 σ and, should no potential systematic uncertainty be shown to bias either of the datasets, could point towards new physics beyond ΛCDM [24]. A number of papers have offered a plethora of new physics explanations for this parameter discordance, though they typically involve modifying the expansion history before the surface of last scattering (early dark energy [31], dark radiation, interacting neutrinos [26]), or modifying the expansion history at low redshift (evolving dark energy [24], dark matter interactions with dark energy [11]).

Modifying the PPS is a potential avenue to resolve this tension that has received less attention. Previously, Hazra et al. (2019) [20] have done just this. In their paper, they investigated what sort of PPS would be needed to explain the “$H_0$ tension”. Hazra et al. (2019) investigated possible novel PPS explanations for the $H_0$ tension by fixing the expansion history to be consistent with low-redshift observables such as the Cepheid measurement of $H_0$ [34] and the KiDS-450 weak-lensing measurement of $\Omega_m$ [21]. With the expansion history fixed, they used a modified Richardson-Lucy (MRL) deconvolution algorithm [17–19, 28, 32, 35–37] to find what PPS maps between this fixed expansion history and the $C_\ell$s of the best-fit ΛCDM model. They found a suppression of power at large scales and oscillations at small scales achieves this mapping and offers a potential explanation for the $H_0$ tension.
It is a somewhat generic prediction of slow-roll single-field inflation that the PPS is a nearly scale-invariant power law [1, 10, 14, 30]. However, more complicated models of inflation could predict more complicated forms of the PPS [2, 8, 9, 12, 13, 15, 23]. Indeed, a power law PPS is by no means a necessary prediction of inflation. One common way to generalize the power law PPS is to allow the spectral index to vary with wavenumber, the so-called running of the index. Similar parametrizations for the PPS such as broken power laws and steps have been explored. Similarly, steps in the inflationary potential generically give rise to non-trivial oscillating PPS. A number of papers have explored novel inflationary potentials that give rise to oscillations in the PPS [6, 16]. Thus it is natural to explore PPS beyond the power law parametrization.

In this paper, we generalize the result of Hazra et al. [20] and parametrize a class of PPS that continuously deforms between the best-fit power law and the MRL-reconstructed PPS. In section 2, we elaborate on this parametrization, explaining why it fits the CMB data well, and then show the results of the statistical inference using this class of PPS in section 3. In section 4, we describe additional ways to smooth the MRL-reconstructed PPS and in section 5 we conclude.

2 Deformation model

The primary objective of this paper is to generalize the MRL-reconstructed PPS from Hazra et al. [20], and demonstrate a class of PPS that can fit the CMB well, yet have very different expansion histories.

This sort of degeneracy between the uncertainty in the PPS and the parameters of the transfer function were explored in Kinney (2001) [25], where it was shown that arbitrary deformations in the PPS could mimic the effects of changing the parameters of the background evolution, such as $H_0$, $\omega_b$, or $\omega_c$.

The idea explored by Hazra et al. [20] is that, since the $C_\ell$s are the quantity directly constrained by observations, one can construct an example of a non-power-law PPS that fits the CMB exactly as well as the best-fit power law+$\Lambda$CDM model, yet has a significantly different expansion history. In fact, Hazra et al. [20] fixed the Universe’s expansion history to the best-fit parameters from the Cepheid measurement of $H_0$ from Riess et al. (2018) [33]. Further, they use the $\Lambda$CDM best-fit values for $\Omega_b h^2$ and $\Omega_{\text{CDM}} h^2$, which when combined with the Cepheid measurement of $H_0$, gives a value for $\Omega_m$ consistent with the KiDS-450 measurement from Hildebrandt et al. (2016) [21].

With the $C_\ell$ values from the best-fit $\Lambda$CDM parameters, and a transfer function consistent with low-redshift observables, the MRL algorithm was then employed to deconvolve the transfer function from the $C_\ell$s and produce a novel PPS that predicts a high value for the Hubble parameter while still fitting the CMB well, hence offering another solution to the Hubble tension.

To explain how the MRL algorithm works we reproduce eq. 2.2 from Hazra et al. (2014) [19]

$$
P_{i+1}^{(i)} - P_i^{(i)} = P_k^{(i)} \sum_{\nu} \sum_{\ell=\ell_{\text{min}}^{(\nu)}}^{\ell_{\text{max}}^{(\nu)}} \frac{1}{g_{\nu}(\ell)} \tilde{G}_\ell^{(i)} \left\{ \left( \frac{C_{\nu}^{T(i)} - c_{T(i)}^{\nu}}{c_{\nu}^{T(i)}} \right) \tanh^2 \left[ Q_\ell \left( C_{\nu}^{D(i)} - c_{\nu}^{D(i)} \right) \right] \right\}_{\text{unbinned}} \\
+ \sum_{\ell=\ell_{\text{min}}^{(\nu)}>1900} \frac{1}{g'_{\nu}(\ell)} \tilde{G}'_\ell^{(i)} \left\{ \left( \frac{C_{\nu}^{T(i)} - c_{T(i)}^{\nu}}{\sigma_{\nu}^{D(i)}} \right) \tanh^2 \left[ \frac{C_{\nu}^{D(i)} - c_{\nu}^{D(i)}}{\sigma_{\nu}^{D(i)}} \right] \right\}_{\text{binned}},
$$

(2.1)
where $G'_{\ell k}$ is the transport kernel, $G^D_{\ell}$ is the data, $C^{T(i)}_{\ell}$ is the theoretical angular power spectra corresponding to the PPS at $i$’th iteration. It is important to emphasize that this MRL algorithm is an iterative process, and ensures the positivity of the recovered $P(k$ since $G_{\ell k}$ is a positive definite matrix. The modifications to the original Richardson-Lucy algorithm and the tanh term is used to account for uncertainties in the data and to make modifications where the data and theory $C_{\ell}$s are most different. For more details about how and why the details of the MRL algorithm is implemented, please refer to Hazra et al. (2014) [19].

To understand the effect of this PPS and why it yields a high value for the Hubble parameter, we must first understand what effect changing the Hubble parameter would have when fixing the PPS to the best-fit power law. Just shifting $H_0$ induces a purely geometric modification to the CMB's $C_{\ell}$s by changing the inferred angular diameter distance to the surface of last scattering. This, in turn, generates a phase shift in the $C_{\ell}$s that is consistent across the acoustic peaks. So, in effect, the MRL deconvolution is generating a PPS that de-phase-shifts the $C_{\ell}$s.

The effects, relative to the best-fit ΛCDM, of changing $H_0$, changing the PPS, and changing both, are shown in figure 1. This figure shows plots the $C_{\ell}$s of for a power law

![Figure 1. Ratios of $C_{\ell}$s relative to those from the best-fit power law PPS and background parameters. In blue is shown the ratio of the $C_{\ell}$s for the same PPS but instead $H_0 = 73.5$. In orange is the MRL PPS with $H_0 = 67.8$ and in green is the MRL PPS with $H_0 = 73.5$. Changing the background expansion history to $H_0 = 73.5$ from the one best-fit using a power law PPS effectively induces a phase shift in the acoustic peaks. Including the MRL PPS induces the opposite phase shift, leaving the $C_{\ell}$s mostly unchanged.](image-url)
ΛCDM model with $H_0 = 73.5$ km/s/Mpc, a MRL model with $H_0 = 67.8$ km/s/Mpc, and a MRL model with $H_0 = 73.5$ km/s/Mpc, all relative to the best-fit power law ΛCDM model with $H_0 = 67.8$ km/s/Mpc. The two different modifications induce equal and opposite changes to the $C_\ell$s, which when combined, induce basically no change.

Part of the discussion around this model must include the fact the MRL-reconstructed PPS is non-trivial and hence either potentially over-fit or a priori unlikely. Saying that this PPS is over-fit is similar to saying the MRL deconvolution used to describe was just fitting noise in the Planck 2015 data. However, this PPS survived new additions to the dataset and changes to the modelling of foregrounds and systematics to also explain the 2018 data. That this PPS has a well-defined observable effect on the $C_\ell$s further contradicts the idea that the result is just noise.

Expressed, another way, one might reasonably believe the numerous, non-trivial features in this PPS are a priori unlikely. Such concerns are understandable but such subjective prior belief is nothing to build firm conclusions on. Such a prior preference for a featureless PPS lasts until someone writes down an inflationary potential that predicts the features derived in the deconvolution. We do not seek to rule out ideas solely on a priori arguments.

In any case, the purpose of this paper is to put this MRL-reconstructed PPS on more firm ground, at least phenomenologically, by introducing a parametrized class of PPS that are a continuous deformation between the best-fit power law and the MRL-reconstructed PPS. This parametrization, which we refer to as the “deformation model” from here on, is simply an interpolation between the best-fit power law PPS and the MRL-reconstructed PPS, which we simply use as a template in this model,

$$P(k, f) = P_{\text{MRL}}(k) + f(P_{\text{PL}}(k) - P_{\text{MRL}}(k)).$$

Thus, when $f = 0$ the PPS is the MRL-reconstructed PPS, and when $f = 1$, the PPS is a power law. Figure 2 shows example PPS that span the space of these deformations.

In summary, we seek to perform a Bayesian model selection between the ΛCDM model (with the base six parameters, $\theta_s, \omega_b, \omega_c, A_s, n_s, \tau$) and the deformation model (with those same base six parameters, along with $f$).

3 Results

First, we calculate the posteriors for the parameters of the deformation model using only the “TT” dataset from Planck. This is to show, that independent of the Cepheid $H_0$ measurement, the deformation model can predict $H_0$ values higher than ΛCDM. Then when Cepheid $H_0$ constraint is included, we show that the resulting parameter space is actually a good fit to both datasets, thus resolving the tension.

In figure 3, we show the results for our deformation model. For the “TT” dataset alone, the best-fit parameters of the deformation model are $H_0 = 70.2 \pm 1.2$ km/s/Mpc and $f = 0.64 \pm 0.19$. These best-fit parameters yield a likelihood better than the best-fit ΛCDM model of $\Delta \log L = 2.5$. This is intriguing since even on its own, the deformation model can explain the temperature anisotropies in the CMB better than the ΛCDM model, while also predicting higher values of $H_0$. We calculate the Bayes factor for the two models ($K = Z_{\text{deform}}/Z_{\Lambda \text{CDM}}$, $Z$ is the evidence of that model) with just the Planck TT dataset to be $\log K = 2.2$

When we combine the Planck dataset with the Cepheid $H_0$ measurement from Riess et al. (2019) [34], the best-fit parameters shift to $H_0 = 71.8 \pm 0.9$ km/s/Mpc and
Figure 2. Example primordial power spectra that deform between the best-fit power law PPS and the MRL PPS. The viridis color map varies between purple at the MRL PPS ($f = 0$) and yellow at the best-fit power law PPS ($f = 1$). The top panel shows the wavenumber in log-scale and the bottom panel in linear scale, to emphasize different features of the MRL PPS.
Figure 3. Posteriors for the deformation model from the Planck-TT CMB dataset (blue) and from the Planck-TT+$H_0$ datasets (orange).

$f = 0.39 \pm 0.16$ and the $\Lambda$CDM regime ($f = 1$) is strongly ruled out. Further, we find the Bayes factor to be $\log K = 5.7$, which according to the Jeffreys scale [22], amounts to strong evidence.

Comparing our results to previous studies, a number of groups have used physically agnostic but still parametric methods to generalize the PPS beyond a power law. For instance, Abazajian et al. [1, 10] generalize the power-law PPS via a series of knots whose position and amplitude can change. They find no evidence for a departure from a power law though the specific PPS generated by the MRL algorithm would require more knots than tested to replicate. Similarly, Liu and Huang [27] use a wavelet analysis to look for deviations from a power law. Again, they find no evidence for a deviation from a power law, though their wavelets would likely not be able to reproduce the MRL PPS exactly. MRL is particularly useful in that it can have different features with different frequencies and different amplitudes.
4 Smoothing

In this section, we seek to answer the question of which features in the MRL-reconstructed PPS are primarily driving the preference for parameters values that are different from those inferred from $\Lambda$CDM. One potential way to answer this question is to apply various smoothings, filters, or wavelet transforms to the MRL-reconstructed PPS and check if the resulting PPS can achieve the same likelihood.

Simple techniques like frequency cuts and low-pass filters suppress the features in the range $k \sim 10^{-4}$ to $10^{-2}$ leaving only the sinusoidal oscillations at high wavenumber. To test the effects of the filter, we scanned over the cutoff “frequency” of the low-pass filter and calculated how much the likelihood changed. For larger values of the cutoff frequency, the change in the PPS is smaller. When scanning over the cutoff frequency, we found no preference for any filtering but some amount was still allowed. Applying a cutoff frequency of 0.6 Mpc to the MRL PPS, the likelihood is marginally worse ($\Delta \log \mathcal{L} \sim 0.5$), but the jaggedness in the high-wavenumber part of the MRL PPS is removed. The cost in the likelihood buys a priori subjective belief. Further, all of the low-wavenumber features are also removed.

To more precisely answer the question of which features give rise to the MRL preference for a high $H_0$, we also tested a case where the low-pass filter was applied to only the features above $k > 0.25$. This choice was motivated by the fact that above $k > 0.25$ only meaningfully affect the $C_\ell$ values for $\ell > 2500$, which is a regime that is unprobed by CMB observations. Keeping with these expectations, we find that essentially all of the features above $k > 0.25$ can be filtered away and maintain the same likelihood. We show this “hi-k” filtering case in figure 4 where in green, we show the filtered part of the PPS (the blue MRL and the green filtered curves combine to form the tested PPS). Thus, it is apparent that these mid-wavenumber ($0.01 < k < 0.25$) oscillations are what compensate for the phase shift in the acoustic peaks coming from the different background parameters.

It is not unreasonable that these features could arise from steps or kinks in a physical inflationary potential such as wiggly-whipped inflation [6, 16, 23]. Thus these sort of “deformed” or “filtered” PPS models offer a new hope that inflationary physics beyond the simplest single-field slow-roll inflation might be true.

5 Conclusions

We generalized the results of Hazra et al. (2019) [20] and introduced a class of PPS that can explain the temperature anisotropies in the CMB well, yet also predict high values for the Hubble parameter $H_0$ (i.e. consistent with the Cepheid measurement). This generalization is simply an interpolation between the MRL-reconstructed PPS and the best-fit power law PPS, which we call the deformation model.

We performed a Bayesian analysis to compare the base $\Lambda$CDM model and the deformation model and then calculate the posterior for the parameters of that model. We find that the deformation model correlates $H_0$ with the new degree of freedom in the PPS, thus predicting higher values for $H_0$ from the CMB’s TT dataset than the base $\Lambda$CDM model. This is not just the uncertainties on $H_0$ increasing to alleviate the tension with the Cepheid $H_0$ measurement from Riess et al. (2019) [34], but the best-fit values shift towards higher values ($H_0 = 70.2$ km/s/Mpc), even without the Cepheid constraint. When the CMB’s TT dataset is considered jointly with the Cepheid $H_0$ measurement, we find that the deformation is preferred over $\Lambda$CDM by a Bayes factor of $\log K = 5.7$. Additionally, we have explored the question of which features in the MRL-reconstructed PPS are driving the preference
for different parameter values. Simply put, it is the features at intermediate wavenumber $k \sim 0.01–0.25$ are most important for fitting the acoustic peaks and hence the parameters of the background expansion. The features at high-$k$ ($k > 0.25$) can be replaced with a power law to recover the same $C_{\ell}$s and likelihood of the best-fit $\Lambda$CDM parameters.

The most important conclusion to take away from this work is that there exist unaccounted for degeneracies between the uncertainties in the PPS and the background expansion history. Even beyond the MRL-reconstructed PPS, arbitrary deformations of the PPS can mimic changes arising from different expansion histories. Though whether these classes of deformed PPS are a priori unreasonable is still an open question, we have shown that the deformation model is a posteriori reasonable. Even if one were skeptical that a physically motivated model of inflation could ever generate a PPS with the features in the MRL-reconstructed PPS, this work is still useful to demonstrate how far and in what ways one would have to deform a power law PPS to beat the successes of $\Lambda$CDM. Because we are primarily interested in data driven techniques, we find it intriguing that deformations of the PPS can be correlated with the parameters of the background expansion. This makes inferences from the CMB less certain, and it is important to remain agnostic and open about these ideas especially considering the Hubble tension.

We leave the calculation of the posteriors of the deformation model from the polarization data and additional low-redshift probes for future work.
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