Modelling Element Abundances in Semi-analytic Models of Galaxy Formation

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ABSTRACT
We update the treatment of chemical evolution in the Munich semi-analytic model, L-GALAXIES. Our new implementation includes delayed enrichment from stellar winds, supernovae type II (SNe-II) and supernovae type Ia (SNe-Ia), as well as metallicity-dependent yields and a reformulation of the associated supernova feedback. Two different sets of SN-II yields and three different SN-Ia delay-time distributions (DTDs) are considered, and eleven heavy elements (including O, Mg and Fe) are self-consistently tracked. We compare the results of this new implementation with data on a) local, star-forming galaxies, b) Milky Way disc G dwarfs, and c) local, elliptical galaxies. We find that the $z = 0$ gas-phase mass-metallicity relation is very well reproduced for all forms of DTD considered, as is the [Fe/H] distribution in the Milky Way disc. The [O/Fe] distribution in the Milky Way disc is best reproduced when using a DTD with $\lesssim 50$ per cent of SNe-Ia exploding within $\sim 400$ Myrs. Positive slopes in the mass-$[\alpha/Fe]$ relations of local ellipticals are also obtained when using a DTD with such a minor ‘prompt’ component. Alternatively, metal-rich winds that drive light $\alpha$ elements directly out into the circumgalactic medium also produce positive slopes for all forms of DTD and SN-II yields considered. Overall, we find that the best model for matching the wide range of observational data considered here should include a power-law SN-Ia DTD, SN-II yields that take account of prior mass loss through stellar winds, and some direct ejection of light $\alpha$ elements out of galaxies.

Key words: Galaxy: abundances – Galaxies: abundances – Galaxies: evolution – Supernovæ: general – Methods: analytical

1 INTRODUCTION
Significant progress has been made in the field of galactic chemical evolution (GCE) since the first postulation of stellar nucleosynthesis by Arthur Eddington in the 1920s (Eddington 1920). The first techniques to determine element abundances in both gas (e.g. Aller 1942) and stars (e.g. Chamberlain & Aller 1951) were developed, and the theory of stellar nucleosynthesis was given a more formal footing by Burbidge et al. (1957). Later, more sophisticated studies of GCE were stimulated by the celebrated review by Beatrice Tinsley (Tinsley 1980). Now, it has been determined that a galaxy’s metallicity is related to its luminosity (e.g. Lequeux et al. 1979), age (e.g. Edvardsson et al. 1993, but see e.g. Frid 1997), and stellar mass (e.g. Tremonti et al. 2004), and that different types of stars contribute to GCE in different ways (e.g. Salaris & Cassisi 2003).

However, many questions relating to the cosmic abundances of heavy elements still remain. For example, it is still unclear what exact role different types of supernovæ (SNe) and stellar winds play in the chemical enrichment of galaxies (e.g. McWilliam 1997), what the shape and universality of the stellar initial mass function (IMF) is (e.g. Bastian, Covey & Meyer 2010), how best to model the metal yields produced in stars (e.g. Romano et al. 2010), and what the progenitors and delay times of SNe-Ia are (e.g. Maoz & Mannucci 2012). These are important questions for us to address, as the chemical evolution of galaxies plays a key part in the evolution of galaxies in general; the presence of metals affects the cooling of gas (e.g. Sutherland & Dopita 1993), the formation of stars (e.g. Walch et al. 2011), stellar evolution (e.g. Salaris & Cassisi 2004), and how different types of stars contribute to GCE in different ways (e.g. Salaris & Cassisi 2003).

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and the yields of newly synthesised metals (e.g. Woosley & Weaver 1993) which are released into the interstellar medium (ISM), circumgalactic medium (CGM) and even the intergalactic medium (IGM).

Aside from the ongoing observational studies into these questions, galaxy evolution models incorporating sophisticated GCE modelling also provide an opportunity to further constrain the chemical evolution of galaxies. Many previous works have focused on reproducing the chemical signatures found in the solar neighbourhood (e.g. Tinsley 1980; Matteucci & Greggio 1986; Matteucci 1986; Thomas, Greggio & Bender 1986; Francois et al. 2004; De Rossi et al. 2004; Calura & Menci 2004; Calura et al. 2010; Romano et al. 2010; Tissera, White & Scannapieco 2012; Pilkington et al. 2012; Calura et al. 2012), chiefly in order to constrain the contributions from different types of SNe and stellar winds. Many others have focused on the chemical properties of local elliptical galaxies (e.g. Matteucci 1994; Thomas & Kauffmann 1999; Thomas, Greggio & Bender 1999; Pipino & Matteucci 2004; Nagashima et al. 2005; Pipino et al. 2009b; Calura & Menci 2009; Arrigoni et al. 2010b; Calura & Menci 2010; Pipino & Matteucci 2011), chiefly in the Munich semi-analytic model and review the basic equations required to model GCE and discuss the SN-Ia DTD. In this work, we investigate if the chemical properties of Milky Way (MW) disc stars and local elliptical galaxies can be simultaneously obtained with a self-consistent model which assumes a large-scale clustering of galaxies, the Tully-Fisher relation, and the optical colours, stellar mass function and gas-phase mass-metallicity relation observed in the local Universe. The model can also reproduce the abundance of galaxies as a function of stellar mass or luminosity out to $z = 3$.

The aim of this work is to address both of these issues, using a new implementation of detailed chemical enrichment in the Munich semi-analytic model of galaxy formation. We investigate if the chemical properties of Milky Way (MW) disc stars and local elliptical galaxies can be simultaneously obtained with a self-consistent model which assumes a $\Lambda$CDM hierarchical merging scenario and varied star formation histories (SFHs). We also compare different SNe-II yield sets and SN-Ia delay-time distributions (DTDs), to see which allow us to best match the observational data considered.

This paper is structured as follows: in §2 we give a general outline of the Munich semi-analytic model, L-Galaxies. In §3 we describe the stellar yields, lifetimes and IMF used as inputs to our model. In §4 we describe the basic equations required to model GCE and discuss the SN-Ia DTD. In §5 we explain how this GCE model is implemented into the larger semi-analytic model and review the key physical processes governing the distribution of metals throughout galaxies. In §6 we discuss our model results for the chemical composition of a) local, star-forming galaxies, b) the G dwarfs of the MW disc, and c) the stellar components of local ellipticals, and compare these results to the latest observations. We conclude our work in §7.

2 THE SEMI-ANALYTIC MODEL

L-Galaxies is currently able to reproduce the mass-metallicity relation observed in the local Universe. The latter is given by the stellar yields, obtained from stellar evolution models.

In order to model the chemical evolution of galaxies, we first need to know the total mass of heavy elements liberated from stars at any given time. To do this, we need to know a) how many stars eject metals at that time, and b) how much of each element they eject. The former is given by the assumed stellar lifetimes, the IMF and the SFHs of galaxies. The latter is given by the stellar yields, obtained from stellar evolution models.

The yields, as well as depending on the initial mass (and metallicity) of the star, also depend on the mode of ejection. We consider three modes in this work: stellar winds from low- and intermediate-mass stars during their run on subhalo trees built from DM N-body simulations such as the Millennium (Springel et al. 2005) and the yields of newly synthesised metals (~e.g. Woosley & Weaver 1993) which are released into the interstellar medium (ISM), circumgalactic medium (CGM) and even the intergalactic medium (IGM).
thermally-pulsing asymptotic giant branch phase (TP-AGB), or simply AGB phase). SNe-Ia from some intermediate-mass binary systems, and the SN-II explosions of massive stars. Each of these three modes releases a different set of heavy elements at different times. Long-lived stars of mass \(0.55 \lesssim M/M_\odot \lesssim 7\) release mainly He, C and N. SNe-Ia produce and eject mainly Fe and other iron-peak elements, whether they originate from single degenerate binaries (Whelan & Iben 1973), double degenerate binaries (Webbink 1984; Iben & Tutukov 1984), or otherwise (e.g. the binary progenitors of double-detonation, sub-Chandrasekhar-mass explosions, see Ruiter et al. 2011). Finally, short-lived stars of mass \(\gtrsim 7M_\odot\) explode as core-collapse SNe-II, ejecting chiefly \(\alpha\) elements (e.g. O, Ne, Mg, Si, S and Ca).

We note here that we only consider eleven chemical elements in our GCE model, namely, H, He, C, N, O, Ne, Mg, Si, S, Ca and Fe, as these elements are included in all of the yield sets we consider.

The following subsections outline in more detail these key ingredients for galactic chemical enrichment. The SFHs of galaxies are tracked self-consistently in our semi-analytic model and are discussed in Section 5.1.

### 3.1 The IMF

The IMF, \(\phi(M)\), is a probability density function, which tells us the fraction of stars in a \(1M_\odot\) simple stellar population (SSP) that are within a given mass range. It is obtained from the observable present day mass function (PDMF) of field stars in the Milky Way, or from PDMF indicators in extragalactic regions. In this work, we assume that the IMF is the same in all regions of space and does not evolve with time. There are, however, currently conflicting conclusions in the literature as to its universality (e.g. Weidner & Kroupa 2004; Elmegreen 2006; Bastian, Covey & Meyer 2010; van Dokkum & Conroy 2010; Gunawardhana et al. 2011; Fumagalli, Da Silva & Krumholz 2011; Conroy & van Dokkum 2012).

The IMF used in this work is taken from Chabrier (2003). This version is commonly used in chemical enrichment models, and is already utilised in L-GALAXIES via the stellar population synthesis models of Bruzual & Charlot (2003; Maraston 2005). It’s use therefore provides both a good comparison to other works and self-consistency within the code. The Chabrier IMF is given analytically as

\[
\phi(M) = \begin{cases} 
A_\phi M^{-1.35} e^{-(\log M - \log M_c)^2/2\sigma^2} & \text{if } M \leq 1M_\odot \\
B_\phi M^{-2.3} & \text{if } M > 1M_\odot
\end{cases}
\]

(1)

where \(M_c = 0.079M_\odot\) and \(\sigma = 0.69\). The values of the coefficients \(A_\phi\) and \(B_\phi\) are determined by requiring that a) the overall function is continuous, and b) the IMF by mass is normalised to \(1M_\odot\) over the full mass range of stars considered;

\[
\int_{M_{\text{min}}}^{M_{\text{max}}} M\phi(M)\,dM = 1M_\odot,
\]

(2)

where \(M_{\text{min}} = 0.1M_\odot\). When assuming \(M_{\text{max}} = 120M_\odot\), as we do in this work, the coefficients in Eqn. 1 are \(A_\phi = 0.842984\) and \(B_\phi = 0.235480\).

Once normalised to the total mass of the SSP, Eqn. 1 can be integrated over a certain mass range to tell us the number density \((n = N/V)\) of stars in that mass range. Assuming that the IMF is the same everywhere, this is equivalent to the number of stars in a \(1M_\odot\) SSP in a given mass range. This integrated, normalised IMF has units of \(1/M_\odot\).

The Chabrier IMF predicts fewer stars of mass \(<1M_\odot\) than the Salpeter (1955) IMF, and does so with a smoother transition than the multi-segment power-law Kroupa (2001) IMF. At masses above \(1M_\odot\), it has the same slope as the Kroupa IMF (an exponent of \(-2.3\) in linear mass units, rather than \(-2.5\) used for the Salpeter IMF).

### 3.2 Stellar lifetimes

We adopt the metallicity-dependent lifetimes tabulated by Portinari, Chiosi & Bressan (1998), kindly provided by R. Wiersma (priv. comm.). These account for stars in the mass range \(0.6 \leq M/M_\odot \leq 120\), and five different initial metallicities, from 0.0004 to 0.05 (where metallicity is the fraction \(Z_\odot/M_\odot\) here). The same study also provided SN-II yield tables, which we also use (see Section 3.3).

The lifetimes for different initial metallicities are plotted as a function of mass in Fig. 1. Within the metallicity range shown, the most massive stars \((\sim 120M_\odot)\) live for up to \(\sim 3.3\) Myrs, depending on their initial metallicity, while the smallest stars that shed material during their lives \((\sim 0.85M_\odot)\) live for \(\sim 10\) to 21 Gyrs. Stars of \(\sim 1M_\odot\) can live from \(\sim 6\) to 10 Gyrs according to these lifetime tables, implying that some G V stars (also known as G dwarfs) would not live for more than a Hubble time. The implications of this are briefly discussed in Section 4.2.

\[\text{Figure 1. The lifetimes of stars as a function of initial mass, for five different initial metallicities, as predicted by Portinari, Chiosi & Bressan (1998).}\]

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1. Note that other authors, such as Lia, Portinari & Carraro (2002) and Arrigoni et al. (2010a), choose to define the IMF as the mass of stars in a \(1M_\odot\) SSP, \(\Phi(M)\). This is related to the IMF defined in this work, \(\phi(M)\), by \(\Phi(M) = M\phi(M)\).
3.3 AGB wind yields

We adopt the metallicity-dependent yield tables of Marigo (2001, hereafter M01) for low- and intermediate mass stars, which eject their metals predominantly through stellar winds during their AGB phase. The SN-II yield tables of P98, which we also use, form a complete set with those of M01 for AGB winds. They are both based on the same Padova evolutionary tracks and do not require a large interpolation between them, as the AGB yields consider stars up to 5\(M_\odot\) and the SN-II yields consider down to 7\(M_\odot\). In this work, we consider the ejecta from AGB winds to occur at the end of a star’s lifetime.

Fig. 2 shows the ejected mass of metals (top left panel), total baryons (top right panel), and individual elements (bottom two panels) from AGB stars as a function of initial mass. This is different from the yield, as it includes both the mass that passes through the stars unprocessed and any newly synthesised material. The element abundances of the Sun from Asplund et al. (2009) are used to scale the amplitudes of the curves in Fig. 2.

We note that no elements heavier than oxygen present in the wind have been synthesised or destroyed in the AGB stars, but have instead been formed in previous generations of stars and pass through the AGB stars unprocessed. We have extrapolated the AGB wind yields from 5\(M_\odot\) to 7\(M_\odot\), so that they meet with the SN-II yields used. The exact position of this interface within the region 5 < \(M/M_\odot\) < 8 does not significantly affect our results.

3.4 SN-Ia yields

As with many other chemical enrichment models, we adopt the spherically symmetric ‘W7’ model for our SN-Ia explosive yields, originally tabulated by Nomoto et al. (1984). We use a more recent iteration, by Thielemann et al. (2003, hereafter T03). These tables provide the synthesised mass of the element ‘yield’ of a star is defined as the mass of that element that is synthesised and ejected (Tinsley 1980). If an element undergoes a net destruction during stellar nucleosynthesis (e.g. hydrogen), then its yield will be negative, whereas the mass of the element ejected will not.

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2 Total yields from the RGB and AGB phases together are included in the M01 tables. For simplicity, we refer to these as ‘AGB wind’ yields hereafter.

3 We note that it is also possible to link the M01 AGB yields to the P98 SN-II yields at 6\(M_\odot\), by including the P98 yields for electron-capture SNe (see P98,§4.2). Doing this makes a negligible difference to the results discussed in this work.

4 The element ‘yield’ of a star is defined as the mass of that element that is synthesised and ejected (Tinsley 1980). If an element undergoes a net destruction during stellar nucleosynthesis (e.g. hydrogen), then its yield will be negative, whereas the mass of the element ejected will not.
The nickel into iron shortly after the SN. \cite{Woosley1995} do so by simply adding II yield tables considered in this work account for the decay of \( ^{56} \text{Ni} \). Unlike the tables of \cite{Woosley1995}, both sets of SN-II yields are based. These corrections halve the yield of C and Fe and double the yield of Mg, relative to the originally tabulated values.

We note that the P98 yields show some sudden drops in the ejecta of certain elements. At low metallicities, the reduction in yield of the heaviest elements above \( \sim 30 \text{M}_\odot \) is due to them being locked in the stellar remnant. Remnant masses increase significantly above \( \sim 30 \text{M}_\odot \) at low metallicities due to low mass-loss efficiency prior to the SN. This effect is less severe for lighter elements, such as oxygen, as ‘pair creation’ SNe are believed to dominate over ‘core collapse’ SNe above \( \sim 60 \text{M}_\odot \), allowing more of these elements to be ejected. At higher metallicities, more efficient mass loss prior to the SN inhibits large remnant formation. Increased mass loss at \( Z_0 \geq 0.02 \) from massive, Wolf-Rayet stars also causes the larger He and C yields in this metallicity range. The removal of these elements in the wind in turn suppresses the explosive \( \alpha \) element yields. For more details, see §5 of P98.

These specific features could have a significant impact on our results. We therefore also test our GCE implementation with an alternative set of SN-II yields, that do not take account of prior mass loss, and therefore appear more stable as a function of initial mass and metallicity. This second set is taken from \cite{Chieffi2004} (hereafter CL04). These account for stars of initial masses from 13 to 35 \text{M}_\odot, and so require both an extrapolation downwards to the upper mass limit for AGB winds (chosen here to be \( 7 \text{M}_\odot \)), and upwards to a more reasonable maximum mass. We choose \( M_{\text{max}} = 120 \text{M}_\odot \) when using the CL04 SN-II yields in order to match the maximum mass considered for the P98 SN-II yields, and because such massive stars are known to exist and contribute to chemical enrichment in the real Universe. However, we caution that this represents a gross extrapolation into a regime well above that constrained by the original yield calculations. For this reason we use the CL04 yields only as a comparison to those of P98, in order to discern what effect prior mass loss might have on our overall results.

### 4 THE GCE EQUATION

In this section, we present the GCE equations required to calculate the mass ejection rate from stars. The implementation of these equations into our semi-analytic model is described in §4.

Following the prescriptions given by \cite{Tinsley1980}, the total rate of mass ejected by an SSP at time \( t \) is given by

\[
e_{M}(t) = \int_{M_{L}}^{M_{U}} \phi(M) \psi(t - \tau_{M}) dM , \tag{3}
\]

where \( M \) is the initial mass of a star, \( \tau_{M} \) is its lifetime, \( \psi(t - \tau_{M}) \) is the delayed SFR, \( \phi(M) \) is the initial mass function, and \( \int_{M_{L}}^{M_{U}} \phi(M) dM = \frac{1}{\tau} \cdot \int_{0}^{\infty} \psi(t) dt \).
Figure 4. Mass released by SNe-II from the Portinari, Chiosi & Bressan (1998) yield tables. Points indicate values from the yield tables. Solid lines indicate the interpolation used between these points. Top left: The mass of metals ejected as a function of mass, for five different initial metallicities. Top right: The total baryonic mass ejected as a function of mass, for five different initial metallicities. Bottom left: The mass of each element ejected as a function of mass, for stars of $Z_0 = 0.004$. Dashed lines indicate the corrected C, Mg and Fe yields (see text). Bottom right: Same as bottom left, for stars of $Z_0 = 0.02$.

$\tau_M$ is the star formation rate when the star was born, $M_r$ is the mass of the stellar remnant, and $\phi(M)$ is the normalised IMF by number, as given by Eqn. 1.

$\psi(t-\tau_M) \cdot \phi(M)$ gives us the birthrate of stars of mass $M$ at time $t-\tau_M$. Multiplying this birthrate by $(M-M_r)$, the mass ejected by one star of mass $M$, then gives us the total mass ejected rate by stars of mass $M$, at time $t$. We can then integrate this quantity over a suitable range of masses ($M_L$ to $M_U$) to obtain $e_M(t)$.

The same equation can be written when only considering the metals ejected by an SSP:

$$e_Z(t) = \int_{M_L}^{M_U} M_Z(M, Z_0) \psi(t-\tau_M) \phi(M) \, dM \quad (4)$$

where $M_Z = y_Z(M, Z_0) + Z_0 \cdot (M-M_r)$ is the mass in metals returned to the gas phase by a star of mass $M$ (as clarified by Maeder 1992, §4.1). This is made up of the mass- and metallicity-dependent yield $y_Z$, plus those metals present at the formation of the star that are later ejected unprocessed, $Z_0 \cdot (M-M_r)$.

The same equation can be written again, when only considering individual chemical elements ejected by an SSP, replacing $M_Z$ with $M_i = y_i(M, Z_0) + (M_i/M)(M-M_r)$, the total mass of element $i$ returned to the gas phase by a star of mass $M$. However, for simplicity, we will proceed by describing the GCE equation in terms of the total metals ejected.

Eqn. 4 can be further split-up into four sub-components, representing the three modes of ejection, AGB winds, SNe-Ia and SNe-II:

$$e_Z(t) = \int_{0.85M_{\odot}}^{7M_{\odot}} M_{\text{AGB}}(M, Z_0) \psi(t-\tau_M) \phi(M) \, dM + A'_{\text{k}} \int_{0.85M_{\odot}}^{7M_{\odot}} M_{\text{Ia}}^{\text{k}}(M, Z_0) \psi(t-\tau) \, DTD(\tau) \, d\tau + (1-A) \int_{7M_{\odot}}^{16M_{\odot}} M_{\text{II}}^\text{H}(M, Z_0) \psi(t-\tau_M) \phi(M) \, dM + \int_{16M_{\odot}}^{M_{\text{max}}} M_{\text{II}}^{\text{H}}(M, Z_0) \psi(t-\tau_M) \phi(M) \, dM \quad (5)$$

The first term in Eqn. 5 represents the contribution to the ejected metals from AGB winds (approximating that the material is shed at the end of the stars’ lives), with the symbols representing the same quantities as in Eqn. 4. As can be seen, the integral extends to masses above the minimum mass of SN-Ia-producing binary systems ($\sim 3M_{\odot}$). There-

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6 We define the metal yield as a mass $y_Z$, rather than the mass fraction $p_Z$ proposed by Tinsley 1984, where $y_Z = M_{\text{Z}}$. 

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fore, we are explicitly accounting for the ejection of metals during the AGB phase of such stars, prior to the SN.

The second term represents the contribution from SNe-Ia, parameterised with an analytic DTD motivated by observed SN-Ia rates (see §4.1). Using a DTD means we do not have to make additional assumptions about the progenitor type of SNe-Ia, the binary mass function $\phi(M_\text{b})$, secondary mass fraction distribution $f(M_\text{s}/M_\text{b})$, or binary lifetimes in our modelling. These uncertain parameters become problematic when using the theoretical SN-Ia rate formalism of Greggio & Renzini (1983). The three SN-Ia DTDs that we consider in this work are described in §4.1.

The coefficient $A$ in the second term of Eqn. 6 gives the fraction of objects from the whole IMF that are SN-Ia progenitors. This is subtly different from $A$ in the third term, which is the fraction of objects only in the mass range 3-16$M_\odot$ that are SN-Ia progenitors. As clarified by Arrigoni et al. (2010a, §3.3), these two coefficients are related by $A = A' \cdot f_{3-16}$, where $f_{3-16}$ is the fraction of all objects in the IMF that have mass between 3 and 16$M_\odot$.

Our chosen value of $A$ is 0.028 (i.e. 2.8 per cent of the stellar systems in the mass range 3 - 16$M_\odot$ are SN-Ia progenitors), as discussed in §5.4. The coefficient $k$ is given by

$$k = \int_{M_{\text{min}}}^{M_{\text{max}}} \phi(M) \, dM,$$

and gives the number of stars in a 1$M_\odot$ SSP. For the Chabrier IMF used here, $f_{3-16} = 0.0385$ and $k = 1.4772$ when assuming $M_{\text{min}} = 0.1M_\odot$ and $M_{\text{max}} = 120M_\odot$.

The third term in Eqn. 6 represents the ejection of metals, via SNe-II explosions, of all objects within the mass range $7.0 \leq M/M_\odot \leq 16.0$ that do not produce SNe-Ia. Hence, the coefficient is $(1 - A)$.8

The fourth term represents the contribution to the ejection of metals from single, massive stars exploding as SNe-II.

We note here that Eqn. 6 can also be rewritten so that all the modes of enrichment are expressed as time integrals, because the stellar lifetimes are a monotonic function of initial mass (e.g. P98, §8.7).

### 4.1 SN-Ia delay-time distribution

There have been many SN-Ia DTDs formulated in the literature. In this work, we consider three, shown in Fig. 5 and compare the results obtained from each.

The first is the power-law DTD with slope $-1.12$ proposed by Maoz, Mannucci & Brandt (2012), formed from a fit to the SN-Ia rate derived from 66,000 galaxies (comprising 132 detected SNe-Ia) from the Sloan Digital Sky Survey II (SDSS-II):

$$\text{DTD}_{\text{PL}} = a(\tau/\text{Gyr})^{-1.12},$$

where $\tau$ is the delay time since the birth of the SN-Ia-producing binary systems, and $a$ is the normalisation constant, taken here to be $a = 0.15242 \text{ Gyr}^{-1}$ (see Eqn. 10). Similar power-law slopes have been suggested by a number of other works (e.g. Totani et al. 2008; Maoz Sharon & Gal-Yam 2010; Maoz & Mannucci 2012).

The second is the narrow, Gaussian DTD proposed by Strolger et al. (2004), based on observations of 56 SNe-Ia in the range $0.2 < z < 1.8$ from the GOODS North and South fields. This form is given by

$$\text{DTD}_{\text{NG}} = \frac{1}{\sqrt{2\pi\sigma_r^2}} e^{-(\tau - \tau_r)^2/2\sigma_r^2},$$

where $\tau_r$ is the characteristic delay time ($\sim 85$ Myr of the birth of the binary, followed by a broader, delayed distribution. The $\text{DTD}_{\text{NG}}$ DTD has been expressed by Matteucci et al. (2008) as

$$\log(\text{DTD}_{\text{BM}}) = \begin{cases} 
1.4 - 50(\log(\tau/\text{Gyr}) - 7.7)^2 & \text{if } \tau < \tau_0 \\
-0.8 - 0.9(\log(\tau/\text{Gyr}) - 8.7)^2 & \text{if } \tau > \tau_0 
\end{cases},$$

where $\tau$ is the delay time, and $\tau_0 = 0.0851 \text{ Gyr}$ is the characteristic lifetime separating the two components.
For all of these DTDs, the normalisation requirement is,
\[
\int_{\tau_{\text{min}}}^{\tau_{\text{max}}} \text{DTD}(\tau) \, d\tau = 1 ,
\]
where \(\tau_{\text{min}} = \tau_{8M_{\odot}}\) and \(\tau_{\text{max}} = \tau_{0.85M_{\odot}}\) are the minimum and maximum assumed lifetimes of a SN-Ia-producing binary in the single-degenerate scenario (i.e. the lifetimes of the largest and smallest possible secondary stars), respectively. Strictly, their values depend on the stellar lifetime tables used, and therefore also on the metallicity of the stars. However, we choose to fix their values for Eqn. 10 to those provided by the P98 lifetime tables for stars of \(Z_0 = 0.02\), namely \(\tau_{\text{min}} = 35\) Myrs and \(\tau_{\text{max}} = 21\) Gyrs. Our chosen value of \(\tau_{\text{min}} = 35\) Myrs is in line with those commonly used in the literature, with assumed values ranging from \(\sim 30\) Myrs (e.g. Matteucci & Greggio 1986; Padovani & Matteucci 1993; Matteucci & Recchi 2001; Matteucci et al. 2009) to \(\sim 40\) Myrs (e.g. Greggio 2005). This choice also means that \(\sim 48\) per cent of SNe-Ia explode within 400 Myrs when using the power-law DTD, which is close to the \(\sim 50\) per cent predicted from observations of the SN-Ia rate by Brandt et al. (2010), and around the lower limit determined from SN remnants in the Small and Large Magellanic Clouds by Maoz & Badenes (2010).

5 IMPLEMENTATION

The GCE equation given by Eqn. 5 has been implemented into our semi-analytic model so that the mass of chemical elements ejected is calculated at each simulation timestep. The key aspects of this implementation are outlined in the following sub-sections.

5.1 SFH, ZH and EH arrays

There are three galaxy-dependent values that are required for us to predict ejection rates from stars: the star formation history (SFH), the total gas-phase metallicity history (ZH) and the gas-phase element abundance history (EH). The SFH is required to identify \(\psi(t - \tau_0)\), the ZH is required to identify \(Z_0\), and the EH is required to calculate the unprocessed ejecta of each individual element within the semi-analytic model. We accommodate these histories into arrays in our code.

The L-GALAXIES time structure is made up of 63 snapshots (when run on the MILLENNIUM simulation), each containing 20 timesteps. As there are nearly 26 million galaxies by \(z = 0\) in the semi-analytic model, it would require a significant amount of memory for us to store the full histories of each galaxy at the resolution of one timestep. Therefore, we instead take a more dynamic approach; each galaxy has SFH, ZH and EH arrays of only 20 array-elements (hereafter, ‘bins’). As time elapses in the simulation, the width of older bins (those storing data from higher redshifts) increases, while new bins are ‘activated’ with a default width of one timestep. Thus, the whole history of each galaxy can be stored with a time resolution that decreases with lookback time. High precision at recent times is especially important when calculating galaxy luminosities as a post-processing step, by ‘stitching together’ the highest-resolution bins from the histories of all output snapshots, rather than just those from \(z = 0\). This procedure is illustrated by the schematic in Fig. 6. We have checked that changing the number of bins in the history arrays does not affect the chemical evolution in the model by testing our model with a range of history bin resolutions including full resolution (i.e. 63 \times 20 bins per galaxy).

By \(z = 0\), the older bins in such histories can be up to \(\sim 3\) Gyrs wide. This is acceptable when calculating the chemical enrichment within the code, as the bins are integrated over more finely at each timestep (see §5.2). However, when plotting relations using only the output \(z = 0\) history bins, the lower resolution at high-\(z\) does not correctly represent the smooth chemical evolution actually occurring in our model. In these cases (for example, the [Fe/H]-[O/Fe] relation in Fig. 11), we construct higher resolution histories as a post-processing step, by ‘stitching together’ the highest-resolution bins from the histories of all output snapshots, rather than just those from \(z = 0\). This procedure is illustrated by the schematic in Fig. 7. In this way, a much smoother evolution can be plotted, which more accurately represents the chemical evolution occurring within the code.

We note that when doing this for the disc components of galaxies, account needs to be taken of stars that move from the disc to the bulge through disc instabilities, by ensuring that the total mass formed in the stitched-together bins does not exceed the mass of the bulge, with lookback time. High precision at recent times is especially important when calculating galaxy luminosities as a post-processing step, by ‘stitching together’ the highest-resolution bins from the histories of all output snapshots, rather than just those from \(z = 0\). This procedure is illustrated by the schematic in Fig. 6. We have checked that changing the number of bins in the history arrays does not affect the chemical evolution in the model by testing our model with a range of history bin resolutions including full resolution (i.e. 63 \times 20 bins per galaxy).

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not exceed the mass formed in the $z = 0$ history bins over the same time span.

5.2 Implementing the GCE equation

In order to model GCE, Eqn. 3 needs to be implemented into L-GALAXIES as an algorithm, involving numerical integration and interpolation between values in a number of look-up tables. All non-model-dependent terms (i.e. everything except the SFHs, ZHs and Ehs) are pre-calculated and stored in look-up tables, in order to speed-up the runtime of the code. This is possible because the time structure of the history arrays is, by construction, the same for all galaxies at any given time. Therefore, we can know a priori the range of masses of stars that will explode in any given timestep.

We can re-write Eqn. 3 as

$$e_Z(t) = \psi(t - \tau) \left[ \int_{M_L}^{M_U} M_Z(M, Z_0) \phi(M) \, dM \right], \quad (11)$$

where $\psi(t - \tau)$ can be put outside the integral if we assume, for each thin strip of the SFH integrated, that all the stars of mass $M_L \leq M \leq M_U$ are born at the same time (i.e. $\tau_{ML} = \tau_{MU} = \tau$). We can then pre-calculate the integral in Eqn. 11 numerically to obtain a value for each initial metallicity, in every history bin of every timestep that the semi-analytic model will run through, and store it in a 3-dimensional look-up table. The true value of $Z_0$ for a given galaxy is then used within the semi-analytic model to interpolate between these pre-calculated results at each timestep, and $\psi(t - \tau)$ is multiplied-in. The total mass in metals ejected is then given by $e_Z(t) \cdot \Delta t$, where $\Delta t$ is the width of the timestep. The same procedure is used to obtain the total mass ejected, and the total amount of each chemical element ejected at each timestep. Once the ejected masses are calculated, we transfer the material to either the galaxy’s ISM or CGM, as described in 5.3.3.

5.3 Infall, Cooling and Outflows

The chemical enrichment recipe outlined above is only part of the relevant physics needed to accurately model the chemical evolution of galaxies. The distribution of these metals between the various components of galaxies, and out into the IGM, as well as the infall and cooling of gas, are also important considerations when looking beyond a simple closed-box model. Treatments of these physical processes are already incorporated into L-GALAXIES, as described by Guo et al. (2011). A brief outline is also given below.

We note here that L-GALAXIES considers three classes of galaxy: those at the centre of a main DM halo, also known as a ‘friends-of-friends (FOF) group’ (type 0 galaxies), those at the centre of their own DM subhalo but not of their associated FOF group (type 1 galaxies), and those galaxies that have lost their DM subhalo through tidal disruption but have not yet merged with a central galaxy or been tidally disrupted themselves (type 2 galaxies). The prescriptions for the physical processes included in the model are then applied to galaxies according to their type. For example, infall of pristine gas is only allowed to occur for type 0 galaxies, whereas stripping of hot gas can only occur for type 1 galaxies (once they are within the virial radius of the central galaxy).

5.3.1 Infall

The mass of pristine gas (assumed to be 75 per cent hydrogen and 25 per cent helium) infalling onto the DM halo is simply determined by the difference between the assumed baryon fraction $f_b$ and the actual baryon fraction $M_b/M_{DM}$ in the DM halo. The assumed baryon fraction is reduced from the cosmic baryon fraction $f_{b,\cos}$ (assumed to be 0.17, as given by WMAP1), due to reionisation, and is parameterised following Gnedin (2000) as

$$f_b(z, M_{vir}) = f_{b,\cos} \left[ 1 + (2^{2/3} - 1) \left( \frac{M_{vir}}{M_c(z)} \right)^{-2} \right]^{-3/2}, \quad (12)$$

where $M_{vir}$ is the virial mass of the DM halo, and $M_c(z)$ is the chosen characteristic halo mass, whose dependence on redshift has been calculated by Okamoto, Gao & Theuns (2008). In this formalism, $f_b$ tends towards $f_{b,\cos}$ as $M_{vir}$
increases. Pre-enriched gas can also be re-accreted onto the DM haloes of central galaxies, in addition to this pristine infall (see §5.3.3).

### 5.3.2 Cooling

Following [White & Frenk (1991)], the cooling of gas from the CGM onto the disc is considered to fall into two regimes; at early times and in low-mass DM haloes, gas is able to cool rapidly in less than the free-fall time, with the cold-flow accretion onto the central galaxy modelled as

\[ M_{\text{cool}} = \frac{M_{\text{acc}}}{t_{\text{dyn,h}}} \]

where \( M_{\text{acc}} \) is the mass of gas accreted onto the DM halo, and the dynamical time of the DM halo is \( t_{\text{dyn,h}} = \frac{R_{\text{vir}}}{V_{\text{vir}}} = 0.1H(z)^{-1} \). At later times and in massive DM haloes, the accretion shock radius is large, leading to the formation of a hot gas atmosphere. In this case, the accretion rate onto the central galaxy is reduced to

\[ M_{\text{cool}} = \frac{r_{\text{cool}} M_{\text{hot}}}{t_{\text{dyn,h}}} \]

where the cooling radius \( r_{\text{cool}} \) is set by the cooling function of [Sutherland & Dopita (1993)], and \( M_{\text{hot}} \) is the mass of shocked gas in the hot gas reservoir (see Guo et al. [2011], §3.2). This accreted gas is then able to form stars, following a simplified form of the Kennicutt-Schmidt law (see Guo et al. [2011], §3.4).

### 5.3.3 SN feedback

SNe explosions can reheat cold gas and also eject it from the DM halo of galaxies. In previous versions of L-Galaxies, the amount of energy released by SNe was assumed to be proportional to the mass of stars formed \( \Delta M_\star \) at that time. Now that we have discarded the instantaneous recycling approximation, it is more appropriate to relate this energy to the mass of material released by stars at that time. The total amount of energy produced by SN feedback is therefore parameterised as

\[ E_{\text{SN}} = \epsilon_h \cdot \frac{1}{2} c_{\text{M}}(t) \Delta t V_{\text{SN}}^2 \]

where \( \epsilon_h \) is the halo-velocity-dependent SN energy efficiency, \( c_{\text{M}}(t) \cdot \Delta t \) is the mass released by stars in one timestep (see Eqn. 9), and \( V_{\text{SN}} \) is the SN ejecta speed, assumed to be fixed at 630 km/s. This differs from the prescription used by Guo et al. [2011] in the use of \( \epsilon_h(t) \cdot \Delta t \) rather than \( \Delta M_\star \). Due to this change, we have doubled the value of \( \epsilon_h \), in order to have the same total SN feedback energy \( E_{\text{SN}} \) as previously used in the model. A thorough investigation into the precise values of model parameters required following our new GCE implementation is reserved for future work.

In our new, default GCE implementation, all stars dying in the stellar disc release material and energy into the ISM, whereas stars dying in the bulge and stellar halo release material and energy into the hot CGM. The energy dumped into the ISM by disc stars can then be used to reheat and possibly eject some (fully mixed) cold gas. Energy dumped into the CGM can also contribute to ejection. The amount of gas ejected from the DM halo into an external reservoir is given by

\[ \Delta M_{\text{ej}} = \frac{E_{\text{SN}} - \frac{1}{2} \epsilon_{\text{disc}} c_{\text{M}}(t) \Delta t V_{\text{SN}}^2}{\frac{1}{2} V_{\text{vir}}^2} \]

where \( \epsilon_{\text{disc}} \cdot c_{\text{M}}(t) \cdot \Delta t \) is the amount of gas that is reheated but does not escape the potential well. The ejected gas is then allowed to return to the DM halo over timescales that are proportional to \( V_{\text{vir}}/t_{\text{dyn,h}} \). This constitutes a second component of gas infall that has been pre-enriched by the galaxy.

We have also implemented an alternative feedback prescription which includes metal-rich winds. These winds dump some material released by disc SNe-II directly into the hot gas. This scheme is discussed in §6.3.3.

#### 5.4 Default set-ups

There can be many free parameters involved when developing a chemical enrichment model. We have limited ourselves to only one new free parameter: the fraction, \( A \), of objects in an SSP in the range \( 3 \leq M/M_\odot \leq 16 \) that are SN-Ia progenitors. \( A \) is specifically ‘tuned’ so that the peak of the [Fe/H] distribution for G dwarfs in our MW-type galaxy sample is around the solar value (see §4.2). An increase in \( A \) corresponds to an increase in [Fe/H], and we find that the best value is \( A \sim 0.028 \) for all three of the DTDs we consider (see §4.1). A single value of \( A \) was also found to be suitable for a range of different SN-Ia DTDs by Matteucci et al. [2009]. All other results discussed in this work are obtained without further tuning. In the following, we label results using the bi-modal, power-law, and narrow Gaussian DTDs with ‘BM’, ‘PL’ and ‘NG’, respectively.

We note that our preferred value of \( A = 0.028 \) is similar to that commonly found in the literature. For example, Greggio [2005] took a value of \( A = 0.001 \) when also using a Chabrier IMF, which equates to a value of \( A = 0.026 \). Similarly, de Plaa et al. [2007] take a preferred value of 0.027 for a Kroupa IMF (assuming a SN-Ia progenitor mass of \( 1.5 \leq M \leq 10M_\odot \)). Arrigoni et al. [2010] allow for a value between 0.015 and 0.05, preferring 0.03 when using a slightly top-heavy Chabrier IMF (i.e. a slope of 2.15 rather than 2.3 for \( M > 1M_\odot \), see Eqn. 1). Other works, which have used IMFs with a smaller fraction of stars above \( 1M_\odot \), have taken slightly higher values. For example, Matteucci & Recchi [2001] and François et al. [2004] prefer \( A = 0.05 \) when using a Scalo [1986] IMF.

Henriques et al. [2013] have found that scaling the reincorporation time to the inverse of the DM halo mass allows the semi-analytic model to better reproduce the evolution of the galaxy stellar mass and luminosity functions with redshift. We will incorporate this improvement with our new GCE model in future work.

We note again that the SN efficiency parameter \( \epsilon_h \) has also been modified to ensure that the total SN feedback energy is unchanged (see §5.3.3). All other model parameters have been kept to the values used by Guo et al. [2011].
The $M_\star-Z_{\text{cold}}$ relation (where $Z_{\text{cold}} = 12 + \log(N_\text{H}/N_\text{H})$) for L-Galaxies with the new GCE implementation and using a power-law SN-Ia DTD (points and black lines). This relation is compared to that of L-Galaxies prior to the new GCE implementation (red lines), and to the observed $M_\star-Z_\phi$ relation for emission-line galaxies from the SDSS-DR7 (orange lines) by Calura & Menci (2009) and Matteucci et al. (2006). This relation is compared to that of L-Galaxies with the new GCE implementation (red lines), the observed relation from the SDSS-DR2 (orange lines) by Gallazzi et al. (2005), a fit to the mass-weighted relation from the SDSS-DR3 (green line) by Panter et al. (2008), and to a set of Local Group dwarf galaxies (blue lines) by Woo, Courteau & Dekel (2008).

6 RESULTS

In the following sub-sections, we compare results from our updated semi-analytic model to observational data for local star-forming galaxies (§6.1), Milky Way disc stars (§6.2) and local elliptical galaxies (§6.3). In doing so, we are attempting both to assess the success of our GCE implementation and to further constrain which of the SN-II yield tables and proton-mass range of 3 \(-\) 8M\(_\odot\) believed to be between 0.03 and 0.1 (Maoz & Mannucci 2012). For the range 3 \(-\) 16M\(_\odot\), this equates to between \(0.024\) and \(0.081\) (for a Chabrier IMF).

6.1 The mass-metallicity relations

One of the key diagnostics used to analyse the chemical evolution of galaxies is the relation between their stellar mass ($M_\star$) and gas-phase metallicity ($Z_\phi$). The large statistical power of the SDSS allowed Tremonti et al. (2004) to determine the $M_\star-Z_\phi$ relation for emission-line galaxies in the local Universe. They found a clear positive correlation below \(10^{10.5}\)M\(_\odot\) with a 1σ scatter of only 0.1 dex. Above this mass, the relation was found to flatten. Here, we compare our $z = 0$ model mass-metallicity relations for gas and stars with those observed. We also have the opportunity to directly compare L-Galaxies results before and after the new GCE implementation – something we are not able to do when discussing individual element ratios in later subsections.

Fig. 8 shows the $M_\star-Z_{\text{cold}}$ relation for L-Galaxies with the new GCE implementation and using a power-law SN-Ia DTD (points and black lines). 95600 model galaxies were selected to compare our model predictions to a set of Local Group dwarf galaxies (blue lines) by Woo, Courteau & Dekel (2008).
little impact on the abundance of oxygen, which is the most abundant heavy element and is produced predominantly by SNe-II.

In Fig. 8 we also plot a fit to the same relation for L-Galaxies prior to the new GCE implementation (red lines), and a fit to the observed $M_\ast-Z_\ast$ relation from the SDSS data release 7 (SDSS-DR7) (orange lines). We can see that there is very good agreement between the observations and our new model at $z = 0$. Both the slope and amplitude of the new model relation are in better agreement with observations than those of the previous model. The increase in amplitude at lower mass is due to a) our new GCE implementation (i.e. the input yields) allowing a different amount of metal into the ISM than the fixed 3 per cent yield assumed before, and b) our new SN feedback scheme allowing more oxygen to stay in the ISM after it is released by stars, rather than being instantly ‘reheated’ into the CGM. This is because the energy input by a population of SNe is now distributed over time, rather than all dumped at once into the ISM straight after star formation, when a lot of oxygen is also released (see Fig. 5.3.3).

The scatter of our new model $M_\ast-Z_\ast$ relation is slightly larger than that seen in the SDSS. Studying the properties of outliers above and below the $M_\ast-Z_\ast$ relation can tell us a lot about the evolution of galaxies (e.g. Dellenbusch, Gallagher & Knezek 2007; Peeples, Pogge & Stanek 2008; Zahid et al. 2012a). We defer a detailed analysis of such galaxies in our model to later work.

We note here that the gas-phase metallicity is now defined as $Z_{\text{cold}} = 12 + \log(N_\text{O}/N_\text{H})$ in our new model, in exactly the same way as in observations, where $N_\text{O}$ and $N_\text{H}$ are the number of atoms of oxygen and hydrogen, respectively. Previously, the approximation $Z_{\text{cold}} = 9.0 + \log(M_{Z_{\text{cold}}}/M_{\text{cold}}/0.02)$ was used, where 9.0 was the assumed solar oxygen abundance and 0.02 the assumed solar metallicity. The difference in the value obtained when using these two methods is only small, with the new formulation estimating a metallicity $\sim 0.04$ dex lower than the old formulation.

Fig. 9 shows the $z = 0$ relation between the stellar mass and stellar metallicity ($Z_\ast$) of our model galaxies (using a power-law DTD), after our new GCE implementation (points and black lines), and prior to it (red lines). In both cases, solar-normalised metallicities are calculated as $Z_\ast = \log(M_{2, Z}/M_\ast/0.02)$, using the same solar metallicity of $Z_\odot = 0.02$ assumed in the stellar population synthesis models that obtained stellar metallicities in the SDSS-DR2 (A. Gallazzi, priv. comm.).

Below $M_\ast = 10^{10.5}M_\odot$, the new model $M_\ast-Z_\ast$ relation is similar in shape to that of the previous model, but with an amplitude $\sim 0.1$ dex higher. This is also higher than observed at low mass (although this is a region where observations are not well constrained). The mass-weighted $M_\ast-Z_\ast$ relation of Panter et al. (2008) (green line) from the SDSS-DR3 probably provides the best comparison with our model, as we also consider mass-weighted metallicities. The Panter et al. (2008) relation also shows good correspondence with observations of Local Group dwarfs by Woo, Courteau & Dekel (2003) (blue lines). We can see that the general trend of decreasing $Z_\ast$ with $M_\ast$ is reproduced in our model, despite low-$M_\ast$, star-forming model galaxies being too metal-rich by $z = 0$.

Henriques & Thomas (2010) have shown that a more realistic treatment of stellar disruption, whereby satellite galaxies have their stellar component gradually stripped, can help steepen the slope of the $M_\ast-Z_\ast$ relation in semi-analytic models. This could bring the low-mass end of our model relation into better agreement with observations. Including such a gradual disruption scheme into L-Galaxies will be the focus of future work.

The model $M_\ast-Z_{\text{cold}}$ and $M_\ast-Z_\ast$ relations when using the CL04 SN-II yields have slightly shallower slopes and are $\sim 0.1$ dex higher than those assuming the P98 yields. They therefore have a higher amplitude than observed. This is because the CL04 yield set allows more oxygen to be produced and ejected from stars when extrapolated to 120$M_\odot$, particularly at low metallicity.

To conclude this section, we can say that our new GCE implementation improves the correspondence between our model and observations of gas-phase metallicities in local, star-forming galaxies. This was by no means a foregone conclusion, considering the significant changes to the chemical evolution modelling we have implemented. However, further improvement to the semi-analytic model is still required in order to better match the observed total stellar metallicities of galaxies at $z = 0$.

6.2 The Milky Way disc

There is now a wealth of data available in the literature on the chemical composition of stars in the MW disc. These data allow us to put firm constraints on the success of our GCE implementation in reproducing realistic MW-type model galaxies. We construct a sample of $\sim 5200$ central (type 0) galaxies at $z = 0$ that are disc dominated (i.e. $M_{\text{bulge}}/(M_{\text{bulge}} + M_{\text{disc}}) < 0.5$), with DM halo masses in

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13 This fit to the SDSS-DR7 is given by $26.6864 - 6.63995 \log(M_\ast) + 0.768653 \log(M_\ast)^2 - 6.0282147 \log(M_\ast)^3$, and is an updated version of the SDSS-DR2 relation from Tremonti et al. (2004), using twice as many galaxies (Yates, Kauffmann & Gil 2012).
Modelling element abundances

13

In this section, the model values are normalised to the solar abundances determined by Anders & Grevesse (1989).

In order to compare with observations, we only consider G dwarfs (0.8 \( \leq M/M_\odot \leq 1.2 \)) still present in the stellar discs of our model MW-type galaxies at \( z = 0 \). When using the P98 stellar lifetimes (§3.2), not all G dwarfs live as long as the age of the MW disc. For example, stars of mass 1.2\( M_\odot \) (the upper mass limit we assume for G dwarfs) live from 3.1 Gyrs at \( Z_{\text{init}} = 0.0004 \) to a maximum of 4.7 Gyrs at \( Z_{\text{init}} = 0.02 \) (see Fig. 1). These timescales are clearly shorter than the typical ages of the oldest SSPs in our MW-type model discs (see Fig. 11). Therefore, we re-weight those SSPs for which some of the G dwarfs would no longer be present at \( z = 0 \) for the plots in this section. This correction removes a very small contribution from the oldest SSPs, reducing very slightly the number of low-[Fe/H], high-[O/Fe] stars. Although this is a more rigorous treatment, the main conclusions drawn from our MW-type sample also hold when assuming that all G dwarfs survive up to \( z = 0 \).

6.2.1 MW-type model galaxies

Fig. 11 shows the [Fe/H]-[O/Fe] relation for the G dwarfs in the stellar discs of our model MW-type galaxies, using the stitched-together histories described in §5.1, for the three DTDs we consider. Care needs to be taken when comparing Fig. 11 to observations. In observational studies of the MW disc, the chemical composition of individual stars of various ages are measured and plotted. In the case of our semi-analytic model, individual stars cannot be resolved, and so we must instead rely on the chemical composition of each population of stars, formed at each timestep during the evolution of a galaxy. Fig. 11 therefore shows the chemical composition of SSPs from \( \sim 5200 \) MW-type galaxies, where one MW-type galaxy contributes many hundreds of points (see Fig. 11). Considering a whole sample of MW-type galaxies provides a statistically significant indication of the typical variation in the chemical composition of MW-type discs in our model. This method of comparison has been used before in semi-analytic models (e.g. Calura & Menci 2009). Note that we therefore weight the SSPs by the mass of stars formed. The evolution of an example, individual MW-type galaxy is also plotted in each of the panels in Fig. 11 (red tracks). This galaxy is discussed in detail in §6.2.2. Each of the panels in Fig. 11 shows a clear decrease in [O/Fe] with increasing [Fe/H] towards the solar composition. There are, however, important differences in the distribution of SSPs for each of the three DTDs we consider. These differences can be seen more clearly in Fig. 12 where we compare the [Fe/H] and [O/Fe] distributions when using our three DTDs (black histograms) with those

\[ \text{the range } 11.5 \leq \log(M_{\text{in}})/M_\odot \leq 12.5, \text{ and recent star formation rates of } 1.0 \leq \text{SFR}/M_\odot/\text{yr}^{-1} \leq 10.0 \text{ over the redshift range } 0.0 \leq z \leq 0.25 (i.e. the last } \sim 3.0 \text{ Gyrs). Our results are not affected by small changes to these criteria. Three example star formation histories (SFHs) from our MW-type model sample are shown in Fig. 10. The chemical evolution of the individual galaxy depicted in red is discussed in §6.2.2. Points on the track denote the chemical composition at discreet times in the past, labelled by the lookback time in Gyrs. The SFH of the same galaxy is plotted in red in Fig. 10.} \]

\[ \text{The smallest G dwarfs considered (0.8}\ M_\odot) \text{ can live from around 14 to 26 Gyrs, and so do survive the age of the disc.} \]
of 16,134 F and G dwarfs from the Geneva-Copenhagen Survey (GCS, orange histograms) [Nordström et al. 2004; Holmberg, Nordström & Andersen 2009] and 293 unique G dwarfs from the Sloan Extension for Galactic Understanding and Exploration (SEGUE, red histograms) survey [Yanny et al. 2009; Bovy et al. 2012a,b].

The difference in [Fe/H] distribution between the two observational samples is likely due to their different depths; the GCS probed strictly the solar neighbourhood ($7.7 \lesssim R_{GC}/kpc \lesssim 8.31$ and $0.0 \lesssim |Z_{GC}|/kpc \lesssim 0.359$), whereas SEGUE covered a wider range of galactic radii but also much higher galactic scale heights ($5 \lesssim R_{GC}/kpc \lesssim 12$ and $0.3 \lesssim |Z_{GC}|/kpc \lesssim 3.0$). This means that the SEGUE sample includes a larger number of metal-poor, 'thick-disc' stars, and so has a [Fe/H] distribution spread to lower iron abundances. Our model, in turn, represents the average chemical composition of stars born at each timestep in the discs of MW-type galaxies, due to the full mixing of material in the various galactic components.

The model [Fe/H] distributions for all three of our set-ups are in reasonable agreement with the GCS data (partly by construction, as we have tuned A to obtain a peak of the [Fe/H] distribution around 0.0), although the NG set-up is skewed slightly more to higher iron abundances. However, there are significant differences in the model [O/Fe] distributions. For example, the high-[O/Fe] tail in our BM set-up is much less extended than seen in the [\(\alpha/Fe\)] distribution from the SEGUE survey. This suggests that stars are being enriched with iron too quickly when using the bi-modal DTD – a conclusion also reached by Matteucci et al. (2009).

\footnote{In Fig. 12 the stars with $|Z_{GC}| < 0.3$ that are missing from the \textit{SEGUE} survey are accounted for via the mass re-weighting of the [Fe/H] distribution described by Bovy et al. (2012a).}

\footnote{Note that Bovy et al. (2012a) and Bovy et al. (2012b) choose [$\alpha/Fe$] to be the average of [Mg/Fe], [Si/Fe], [Ca/Fe] and [Ti/Fe], with no oxygen lines included in the analysis. However, as oxygen is the most abundant $\alpha$ element in galaxies, a comparison between their [$\alpha/Fe$] and our [O/Fe] is still valid here.}
Modelling element abundances

Figure 14. The evolution, from redshift 7 to 0, of the mass (in M⊙), SFR (in M⊙/yr), iron abundance, total metallicity, and heavy element enhancements of four different galaxy components (see legend) in the example MW-type model galaxy shown in Fig. 11, using the power-law DTD.

Interestingly, the extent of the high-[O/Fe] tail in the [O/Fe] distribution increases from left to right in Fig. 12. This is due to the different number of ‘prompt’ SNe-Ia assumed for each of the three DTDs. The smaller the prompt component, the larger the number of low-[Fe/H], high-[O/Fe] stars that can be formed before a significant amount of Fe gets into the star-forming gas. The bi-modal DTD allows ~ 54 per cent of SNe-Ia to explode within 100 Myrs of star formation (~ 58 per cent within 400 Myrs), the power-law DTD allows ~ 23 per cent within 100 Myrs of star formation (~ 48 per cent within 400 Myrs), and the Gaussian has no prompt component at all. Only the Gaussian DTD has a high-[O/Fe] tail as extended as that seen for G dwarfs from SEGUE. However, we reiterate that the lack of any prompt component is in contradiction with recent observations (e.g. Maoz & Mannucci 2012). The smaller high-[O/Fe] tail produced when using the power-law DTD, although not as extended as seen in the SEGUE data, is still promising, especially when considering that a) the SEGUE data contain a large number of α-enhanced, iron-poor stars at high galactic scale heights, and b) our model represents the chemical composition of MW-type stellar discs in a statistical sense, and also assumes full mixing of metals in the stellar disc.

We will also show in § 6.3 that the power-law DTD also produces positive slopes in the $M_*-\alpha$/Fe relations of elliptical galaxies.

In Fig. 13 we show a finer binning of the model [Fe/H] and [O/Fe] distributions for the three SN-Ia DTDs considered (black histograms). Sub-distributions for three distinct age ranges (coloured histograms) are also plotted. All panels show nicely that older SSPs have lower [Fe/H] and higher [O/Fe] than younger SSPs, due to the delayed enrichment of the star-forming gas with iron from SNe-Ia. There is also no sign of an extended tail below [Fe/H] = −1.0 (the ‘G-dwarf problem’) that is common to closed-box models.

The broader plateau present in the [O/Fe] distribution for the PL set-up is due to the shape of the DTD; the power-law DTD assumes a smoother change in SNe-Ia rate with time than the other two DTDs considered (see Fig. 5). This means that the ISM in a typical MW-type galaxy undergoes a fairly constant decrease in [O/Fe] of ~ 0.025 dex/Gyr for the power-law DTD. In contrast, the bi-modal DTD causes a more gradual decrease in [O/Fe] of ~ 0.016 dex/Gyr, after significantly enriching the ISM with iron shortly after the start of star formation. In turn, the Gaussian DTD produces a steep decrease in [O/Fe] of ~ 0.066 dex/Gyr from very high initial values until ~ 1 Gyr after the peak of star formation, with little change thereafter.

The CL04 SN-II yields produce qualitatively similar results to those discussed above, except that the [Fe/H] distribution is shifted to higher values and the [O/Fe] has a decreased high-[O/Fe] tail – in greater contradiction with observations. This is because, when extrapolated to 120M⊙, the CL04 yields predict a higher production of O, Mg and Fe by SNe-II than the P98 yields, particularly at low metallicity.

17 Including a treatment of the radial distribution of metals in galaxies, similar to that done by Fu et al. (2013), will be the focus of future work.

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6.2.2 An individual MW-type model galaxy

In this sub-section, we look more closely at the chemical evolution of an individual MW-type galaxy in our model. This galaxy’s SFH is plotted in red in Fig. [10] and its evolution in the [Fe/H]-[O/Fe] diagram is shown by a red track in each panel of Fig. [11]. Points on the tracks in Fig. [11] denote the chemical composition at discreet times in the past, labelled by the lookback time in Gyrs.

This galaxy nicely demonstrates the fairly smooth evolution that we would expect from a MW-type galaxy. However, it is not necessarily typical of our MW-type model sample as a whole. Some galaxies (such as that shown in blue in Fig. [10]) undergo large infall and star formation events that can cause such a track to double-back on itself and otherwise deviate from a ‘smooth’ path (see also Calura & Menci 2004). However, our chosen galaxy provides a good example of the general chemical evolution undergone by MW-type galaxies in our model.

Fig. [11] shows the evolution from $z = 7$ to 0 of the mass, SFR, iron abundance, total metallicity, and complete set of heavy element enhancements for this example MW-type galaxy, when using the power-law DTD. The different components of the galaxy (stellar disc, cold gas, hot gas, and ejecta reservoir) are coloured according to the legend. Note that Fig. [11] shows the average chemical composition of a whole galaxy component at any given time.

Fig. [11] highlights the dependence of an element’s evolution on the mode of its release, namely SNe-II, SNe-Ia or AGB winds. Those elements that are predominantly produced in massive SNe-II (O, Ne and Mg) show a similar decline in their enhancement with cosmic time, as we would expect for a slowly declining SFR and a delayed enrichment of iron. These light $\alpha$ elements also show lower enhancements in the cold gas than in the stellar disc for this reason. The heavier $\alpha$ elements (Si, S and Ca) are produced mainly in lower-mass SNe-II, and also have a greater contribution from SNe-Ia. They are therefore released into the ISM later than the lighter $\alpha$ elements, showing a gradual increase in enhancement with time (at a decreasing rate), and higher enhancements in the gas than in the stars while gas fractions are high. Nitrogen, an element with a dominant contribution from (delayed) AGB winds at low metallicity, shows a strong increase in [N/Fe] at the onset of AGB wind enrichment (at $z \sim 4$ for this galaxy), followed by a more gradual increase thereafter. Finally, the drop in [C/Fe] between $z \sim 3$ and 4 is due to a decrease in the C/Fe ratio in the ejecta of SNe-II at $Z_{\text{init}} \sim 0.004$ compared to other metallicities. The increase in this ratio at higher metallicities, along with a significant contribution to C from AGB winds, causes the sharp rise in [C/Fe] shortly after. This is a specific property of the P98 SN-II yields. When using the CL04 SN-II yields, the [C/Fe] evolution follows that of the light $\alpha$ elements more closely.

To conclude this section, we can say that our new GCE implementation is able to reproduce the [O/Fe] distribution for G dwarfs in the MW disc if there is only a minor prompt component of SNe-Ia (i.e. $< 50$ per cent within $\sim 400$ Myrs). Our NG set-up (narrow Gaussian DTD, delayed SNe-Ia only) and PL set-up (power-law DTD, $< 48$ per cent of SNe-Ia exploding within $\sim 400$ Myrs) therefore reproduce the observed high-[O/Fe] tail best. However, the power-law DTD achieves this whilst assuming a more realistic fraction of prompt SNe-Ia. Our new GCE implementation also allows us to examine, in detail, the chemical evolution undergone by individual galaxies, which can help us explain the features seen in the sample as a whole.

### 6.3 Elliptical galaxies

The change in various element ratios as a function of velocity dispersion or $M_*$ in ellipticals can also provide insight into the chemical evolution of galaxies. It has been observed that $\alpha$ enhancements increase with $M_*$ (e.g. Graves, Faber & Schiavon 2000; Thomas et al. 2010; Johansson, Thomas & Maraston 2012; Conroy, Graves & van Dokkum 2013). This has been mainly attributed to massive ellipticals undergoing the majority of their star formation at higher redshifts and over shorter timescales. The stars in these galaxies are therefore likely to be deficient in iron, as they were formed before a significant number of SNe-Ia could enrich the star-forming gas. Less massive ellipticals, on the other hand, are believed to form a larger fraction of their stars later, from gas that has had time to be more enriched with iron. These galaxies should therefore have lower stellar $\alpha$ enhancements.

Previous GCE models, working within a hierarchical merging scenario, have found it difficult to reproduce this trend between stellar mass and $\alpha$ enhancement, without invoking either a variable or adapted IMF, morphologically-dependent star formation efficiencies (SFEs), or additional prescriptions to increase star formation at high redshift (e.g. Thomas, Greggio & Bender 1999; Thomas 1999; Nagashima et al. 2005b; Pipino et al. 2009b; Calura & Menci 2009; Arrigoni et al. 2010b; Calura & Menci 2011).

We select $z = 0$ elliptical galaxies by bulge-to-mass ratio.
6.3.1 The mass-age relation

Before discussing element enhancements, we first show the $M_\star$-age relation for our model elliptical sample in Fig. 16. Also shown is a fit to the luminosity-weighted mass-age relation from the Johansson, Thomas & Maraston (2012, hereafter JTM12) sample (solid orange line) and its 1σ scatter (dotted orange lines). The ages of model galaxies are r-band weighted in this plot, in order to make a fairer comparison with the observations. For the observed relation, we have used the stellar masses taken directly from the SDSS-DR7 catalogue. This is also the case for all subsequent plots showing data from the JTM12 sample.

It is known that semi-analytic models tend to produce too many old, red, dwarf galaxies by $z = 0$ compared to observations (e.g. Weinmann et al. 2006, Guo et al. 2011), as can be seen by comparing the model and observations in Fig. 16. This is caused not just by the strong stripping of gas from satellites, but also by the strong SN feedback required to match the observed galaxy stellar mass function. Recent work by Henriques et al. (2013) has improved this problem to some extent, by allowing material ejected from model galaxies at high-$z$ to be reaccreted over longer timescales, allowing them to form more stars at low-$z$, and therefore be younger and bluer at $z = 0$. However, this improvement is not implemented into the L-GALAXIES model presented here. Therefore, in the following sections, we will distinguish between our full elliptical model sample and a ‘mass-age-selected’ sub-sample, which includes only those model galaxies that lie within the 1σ scatter of the observed $M_\star$-age relation. This is not done in order to evade the evident issues still affecting the galaxy formation model, but rather as a means of testing the relation between mass, age and α enhancement in our new GCE implementation.

6.3.2 [$\alpha$/Fe] relations

Fig. 17 shows the $M_\star$-[O/Fe] relation for the bulge and disc stars of our model ellipticals at $z = 0$, for the three SN-Ia DTDs we consider. Light-blue contours represent our full elliptical sample. Dark-blue, dashed, filled contours represent our mass-age-selected sub-sample. The observed relation from the JTM12 sample is given by the solid orange line. The slopes of the linear fits to these three relations are shown in the top left corner of each panel. Model element ratios in this section have been normalised to the solar abundances measured by Grevesse, Noels & Sauval (1996), in accordance with the observations to which we compare.

We note here that estimates of element enhancements from stellar population synthesis (SPS) models, such as those used by JTM12, are found to be fairly good representations of the true global value, and are not as biased by small younger populations as age estimates can be (Serra & Trager 2007). It is therefore reasonable for us to compare our model mass-weighted element enhancements with these observations.

As with the extent of the high-[O/Fe] tail in our MW-type sample (see §6.2.1), we can see from Fig. 17 that the strength of the slope in the model $M_\star$-[O/Fe] relation is inversely proportional to the fraction of prompt SNe-Ia assumed. For the BM set-up (∼ 54 per cent of SNe-Ia explode within 100 Myrs, ∼ 58 per cent within 400 Myrs), the model slope is much flatter than observed. For the PL set-up (∼ 23 per cent of SNe-Ia explode within 100 Myrs, ∼ 48 per cent within 400 Myrs), a positive slope is obtained, although shallower than observed. For the NG set-up (with no prompt component), a strong slope is obtained, with a larger scatter.

19 Available at http://www.mpa-garching.mpg.de/SDSS/DR7

20 The model slopes have been obtained from a linear fit in the range $10.0 \leq \log (M_\star / M_\odot) \leq 12.0$. 

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**Figure 16.** The $M_\star$-age relation for our model elliptical galaxies. Greyscale denotes the number density of galaxies. Model ages are weighted by their r-band luminosity. A linear fit to the same relation for the SDSS-DR4 from JTM12 is given by the solid orange line, with the 1σ spread given by dotted orange lines. Our mass-age-selected sub-sample is made up of model galaxies that lie within one standard deviation (±0.222 dex) of the observed mass-age relation.

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and (g-r) colour, such that $M_{\text{bulge}}/(M_{\text{bulge}} + M_{\text{disc}}) \geq 0.7$ and $(g-r) \geq 0.051 \log (M_\star) + 0.14$, to form a sample of ∼ 8700 galaxies. These cuts are chosen to match the selection criteria used to obtain the sample of SDSS-DR7 ellipticals shown as green points in Fig. 19. The (g-r) colour cut also nicely separates the red sequence from the blue cloud in our model at $z = 0$. Our model sample includes type 0, 1 and 2 galaxies (see §6.3).
The increase in slope is because massive ellipticals have shorter star-formation timescales in the model, and so decreasing the fraction of prompt SNe-Ia has a bigger effect on their final iron abundance, increasing their final $\alpha$ enhancements more than for low-mass ellipticals. The increase in scatter at fixed mass is because older galaxies have shorter star-formation timescales in the model, and so undergo a greater increase in their $\alpha$ enhancements for the same reason. This correlation between mass, age and $\alpha$ enhancement can also be seen in the increasing difference between the slope for the full elliptical sample and the mass-age-selected sub-sample from the top to bottom panel, and also from the age-[O/Fe] relation shown in Fig. 18 for the PL set-up. This result supports the canonical thinking that the slopes in $M_\ast$-[O/Fe] relations are driven by differences in the star-formation timescale. If correct, then our model suggests there should only be a minor fraction of prompt SNe-Ia for any given SSP (i.e. $\leq 50$ per cent within $\sim 400$ Myrs).

Fig. 19 shows the enhancements of all the heavy elements that we track as a function of $M_\ast$ when using the power-law DTD. As in Fig. 17, light-blue contours represent our full elliptical sample. Dark-blue, dashed, filled contours represent our mass-age-selected sub-sample. Fits to observations of ellipticals drawn from the SDSS-DR4 (orange lines, JTM12), the SDSS-DR6 (red line, Graves, Faber & Schiavon 2009), and the SDSS-DR7 (green points, see below) are also shown where possible. The PL set-up is shown here because it provides a positive slope for the $M_\ast$-[O/Fe] relation, while also assuming a more realistic fraction of ‘prompt’ SNe-Ia than the NG set-up. Pleasingly, Fig. 19 shows that positive slopes are obtained for all the $\alpha$ elements when using our PL set-up (except for Mg, as explained below). This is again because of the correlation between mass, age and $\alpha$ enhancement in our model. The same is true for our NG set-up, but not for the BM set-up, which has a large fraction of prompt SNe-Ia. This is an important result, as it has been difficult previously for models to obtain positive slopes without invoking additional physics. It should be noted that the slopes of the different observational data shown in Fig. 19 differ substantially for some element enhancements. This is mainly due to the difference in the SPS modelling techniques used. Therefore, it is more important that our model produces positive slopes at all than reproduces the exact slopes of any particular observational data set.

The methodology of both JTM12 and

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure17.png}
\caption{The $M_\ast$-[O/Fe] relation for the bulge and disc components of our model elliptical sample, when using a bi-modal (top panel), power-law (middle panel), or narrow Gaussian (bottom panel) SN-Ia DTD. Light-blue contours represent our full elliptical sample. Dark-blue, dashed, filled contours represent our mass-age-selected sub-sample (see text and Fig. 16). Contours represent the 68th and 95th percentiles. A linear fit to the observed relation from JTM12 is given by the orange line, with its 1σ spread (dotted orange lines). The slopes of these three relations are given in the top left corner of each panel.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure18.png}
\caption{The relation between mass-weighted age and oxygen enhancement for our model elliptical galaxies, when using the power-law DTD. Points show the full elliptical sample, with the greyscale indicating the stellar mass. Dark blue, dashed contours show the mass-age-selected sub-sample. There is a clear positive correlation between age and [O/Fe] in our model ellipticals, both in general and at fixed mass.}
\end{figure}
Figure 19. Element enhancements as a function of stellar mass for the bulge and disc components of our model elliptical sample for our PL set-up. Light-blue contours represent our full elliptical sample. Dark-blue, dashed, filled contours represent our mass-age-selected sub-sample. For both samples, the contours show the 68th and 95th percentiles. Fits to the observed relations from the JTM12 sample (solid orange lines, with 1σ scatter given by dotted orange lines), Graves et al. (2009) (solid red lines, with 1σ scatter given by dotted red lines) and a newly-selected SDSS-DR7 sample (C. Conroy & G. Graves, priv. comm.) (green points) are also shown for comparison.

Graves, Faber & Schiavon (2009) is based on fitting observed and modelled Lick absorption line indices (e.g. Worthey 1994). JTM12 adopt the SPS models of Thomas, Maraston & Johansson (2011b) and 18 Lick indices, whereas Graves, Faber & Schiavon (2009) adopt the models of Schiavon (2007) and use 7 Lick indices. The fits from JTM12 are based on a sample of visually-classified, early-type galaxies in the redshift range 0.05 < z < 0.06. Graves, Faber & Schiavon (2009) selected red-sequence galaxies, classified by the colour-magnitude diagram, in the redshift range 0.04 < z < 0.08. Both samples exclude star-forming galaxies by applying cuts to certain emission line strengths. Stellar masses are obtained from the MPA-JHU catalogue for the JTM12 data, and from mass-to-light ratios obtained using the Bell et al. (2003) (g-r)-M_∗/L_g relation for the Graves, Faber & Schiavon (2009) data. The additional observational data (green points), kindly provided by C. Conroy & G. Graves (priv. comm.), are drawn from the SDSS-DR7, selecting galaxies in the redshift range 0.025 < z < 0.06 with bulge-to-total light ratios > 0.7 and (g-r) > 0.05 log(M_*) + 0.14. These galaxies are binned by M_⊕ and their stacked spectra are used to obtain element enhancements using the SPS models of Conroy & van Dokkum (2012a); Conroy, Graves & van Dokkum (2013). For a detailed comparison of these different methods, see §6 of Conroy, Graves & van Dokkum (2013).

Looking at the panels in Fig. 19 individually, we can see that the slopes of the model relations for [C/Fe] and [N/Fe] are shallower than observed. This is discussed further in §5.3.4. Although our PL set-up reproduces a slope of the M_∗-[O/Fe] relation for the mass-age-selected sub-sample close to that obtained by JTM12 (as also shown in Fig. 17), the newly-selected SDSS-DR7 data suggests a significantly steeper slope. Steeper slopes are obtained in the model when
either using the Gaussian (delayed only) DTD, or when allowing direct ejection of light $\alpha$ elements out of galaxies via galactic winds, as explained in 6.3.3. An increase in $\tau_{\text{min}}$ (the start time for SN-Ia explosions, see 4.1) also slightly increases the slope. For example, increasing $\tau_{\text{min}}$ from 35 to 45 Myrs increases the slope of the $M_*/[\text{O/Fe}]$ relation by $\sim 0.004$ when using the power-law DTD.

Our PL set-up produces a flat $M_*/[\text{Mg/Fe}]$ relation, with a slope slightly shallower than the well-constrained relation obtained from the SDSS-DR7 data. The SDSS-DR7 $M_*/[\text{Mg/Fe}]$ relation, in turn, is flatter than the other observational data sets we compare to here. In our model, the slope of the $M_*/[\text{Mg/Fe}]$ relation is flatter than the other light $\alpha$ elements because Mg is produced in greater amounts by low-metallicity SNe-II than high-metallicity SNe-II when using the P98 yields (compare the bottom two panels in Fig. 11). This is not the case for the CL04 SN-II yields, which produce a slope for $[\text{Mg/Fe}]$ very similar to that of $[\text{O/Fe}]$, due to the negligible difference in their metallicity dependence.

Strong, positive slopes for the heavier $\alpha$ elements (Si, S and Ca) are obtained for all three of the DTDs considered here when using the P98 SN-II yields. This is because these elements are produced only in small amounts by SSPs at low metallicity, as they are easily locked into the stellar remnants of the most massive, low-metallicity SNe-II (see 6.3.3). Low-mass elliptical galaxies, which never obtain high stellar metallicities, therefore never produce enough Si, S or Ca to obtain high enhancements. As the CL04 SN-II yields do not take account of the prior stellar winds’ effect on remnant composition, the slopes produced for the heavier $\alpha$ elements are very similar to those of the lighter $\alpha$ elements. This means that all slopes are equally sensitive to the choice of DTD when using the CL04 yields, such that positive slopes are only obtained if a minor prompt component of SNe-Ia is assumed. We note that the slightly shallower slopes for $M_*/[\text{Ca/Fe}]$ observed in the real Universe may indicate that a larger proportion of heavy $\alpha$ elements come from SNe-Ia than is the case in our model (see Conroy, Graves & van Dokkum 2012).

6.3.3 Galactic winds

The slopes of the $[\alpha/\text{Fe}]$ relations, for all SN-Ia DTDs and SN-II yields considered, are strengthened by introducing metal-rich winds, which suppress the enhancements in low-mass ellipticals. Currently, L-GALAXIES does not invoke direct ejection of material by SNe, instead always fully mixing SN ejecta with the galaxy’s ISM before reheating a fraction of this enriched gas into the CGM. However, a simple wind model, which allows a fraction of the material and energy ejected by SN-II explosions in the disc to be deposited directly into the hot gas, increases the slopes of the $M_*/[\alpha/\text{Fe}]$ relations. This is shown for our PL set-up in Fig. 20. This figure can be compared to the middle panel of Fig. 17.

A scheme where only SNe-II are expected to form a collimated galactic wind is physically motivated by the fact that metal-rich winds (ubiquitous in local, star-forming galaxies) appear to be oxygen rich, $\alpha$ enhanced, and occur shortly after bouts of star formation (e.g. Martin, Kobulnicky & Heckman 2002; Tumlinson et al. 2011). This scheme also allows the remainder of the mass and energy returned by disc SNe-II to mix with and reheat cold gas.

We set the fraction of material from disc SN-II that is directly ejected via the wind to be inversely proportional to the cold gas surface density of the ISM through which it must pass:

$$f_{\text{wind}} = \min \left[ 1.0, \left( \frac{\Sigma_{\text{cold}}}{10 \, M_\odot \, \text{pc}^{-2}} \right)^{-1} \right].$$  \hspace{1cm} (17)

A similar dependency on the gas surface density has also been used in the smoothed-particle hydrodynamical simulations of Hopkins, Quataert & Murray (2012) and in the GALFORM semi-analytic model by Lagos, Lacey & Baugh (2013) (but see Newman et al. 2012). Interestingly, our preferred characteristic gas surface density of $\sim 10 \, M_\odot \, \text{pc}^{-2}$, below which all SN-II ejecta material is put into the wind, is very close to that below which the SFR surface den-
Fig. 22. The $M_\ast$-[N/O] relation for our model elliptical sample (PL set-up). Contours and lines are as in Fig. 21 plus a fit to the model relation when increasing the yield of nitrogen from high-metallicity SNe-II by a factor of 1.5 (see Sec. 3.3.3) (red line). We find that a simple (although ad hoc) increase in the nitrogen yield is enough to obtain a strong, positive slope in this relation.

This simple wind scheme lowers the stellar $\alpha$ enhancements of low-mass ellipticals more than high-mass ellipticals, because low-mass ellipticals tend to have lower-density ISM, and so can dump their SN-II ejecta more efficiently into the CGM. This does not significantly affect the [Fe/H] distribution in the discs of MW-type model galaxies, although the number of low-[O/Fe] SSPs does increase slightly, as shown in Fig. 21 (compare to the middle panel of Fig. 12). However, this simple wind scheme does cause a significant under-enrichment of the ISM in low-mass star-forming galaxies by $z \sim 0$, which steepens the slope of the model $M_\ast-Z_{\text{cold}}$ relation away from that seen in observations. Therefore, although we show here that metal-rich winds are a way of strengthening positive slopes in the $M_\ast-\alpha$ relations of ellipticals, we do not claim that our simple wind model can solve all the problems of GCE modelling.

6.3.4 Carbon and Nitrogen

The case of C and N is more complicated than that of other heavy elements, not least because N is both a primary and secondary element. Our model produces slopes for C and N that are flatter than for the $\alpha$ elements (see top two panels in Fig. 19). This is to be expected if C and N are predominantly released in AGB winds (as they are at low metallicity in our model), yet observations suggest that these enhancements should also produce positive slopes. To further compound the problem, observations by JTM12 of the $M_\ast$-[C/O] and $M_\ast$-[N/O] relations show that these also have positive slopes.

Fig. 22 shows the $M_\ast$-[N/O] relation for our PL model (without additional galactic winds), along with the fit to observations from JTM12. This model relation is also flatter than observed, and its slope increases with the amount of prompt SNe-Ia in the DTD. We note that there is a scatter of high-mass model galaxies with [N/O] values more similar to those observed. However, these higher-[N/O] galaxies tend to be young for their mass in the model, whereas the opposite is true in the observational sample.

One way to increase the slopes in both the $M_\ast$-[N/Fe] and $M_\ast$-[N/O] relations is to assume a greater amount of N production in high-metallicity massive stars, as suggested by JTM12. Doing so implies a boost in secondary nitrogen production. Given the current uncertainty in the amount of secondary N production in stars, this could be a plausible solution, although this is far from certain. The red line in Fig. 22 is a fit to the full model elliptical sample when arbitrarily increasing the N released by SNe-II of metallicity $\geq 0.02$ by a factor of 1.5. A similar increase is also seen in the slope of the $M_\ast$-[N/Fe] relation. Although this is an ad hoc adjustment made to the stellar yields, it does indicate that such a change is capable of improving the values of both [N/Fe] and [N/O] in our model ellipticals.

When using the CL04 yields, the $M_\ast$-[C/Fe] and $M_\ast$-[N/Fe] relations are even flatter and the $M_\ast$-[N/O] relation has a negative slope, due to the lower production of C and N by SNe-II that these yield tables infer. This, again, is in contradiction with the observational data considered, suggesting that the P98 yields, which take account of prior stellar mass loss from massive stars (and so are more dependent on initial mass and metallicity), produce more realistic results in our GCE model.

To conclude this section, we reiterate that positive slopes in the $M_\ast$-[C/Fe] relations of local elliptical galaxies can be obtained if either a SN-Ia DTD with minor prompt component (i.e. $\leq 50$ per cent within $\sim 400$ Myrs) is used, and/or galactic winds driven by SNe-II are allowed to directly eject metals out of galaxies.

7 CONCLUSIONS

We have implemented a new GCE model into the Munich semi-analytic model, L-GALAXIES, which accounts for the delayed enrichment of a series of heavy elements from SNe-II, SNe-Ia and AGB stars. We have also compared the results of this implementation with the chemical composition of local, star-forming galaxies, the MW stellar disc, and local, elliptical galaxies. Our conclusions are as follows:

- The gas-phase mass-metallicity relation for local, star-forming galaxies (when using the Bayesian, SDSS metallicities of Tremonti et al. 2004) is very well reproduced by our new model. However, we caution that both the slope and amplitude of the observed $M_\ast-Z_{\text{g}}$ relation depend strongly on the metallicity diagnostic chosen (e.g. Kewley & Ellison 2008). The stellar components of low-mass, star-forming galaxies tend to be more metal-rich in our model than observed.
- The [Fe/H] distribution of G dwarfs in the MW disc is reasonably reproduced by our model, for all forms of SN-Ia DTD we consider. However, the high-[O/Fe] tail in the MW [O/Fe] distribution is best reproduced when using a SN-Ia DTD with a minor prompt component (i.e. $\leq 50$ per cent within $\sim 400$ Myrs), such as a Gaussian DTD centered on $\sim 1$ Gyr, or a power-law DTD with slope $\sim -1.12$ and $\tau_{\text{min}} \geq 35$ Myrs.
- Positive slopes in the $M_\ast-\alpha$ relations of local, elliptical galaxies are also obtained when assuming a minor prompt component (i.e. $\leq 50$ per cent within $\sim 400$ Myrs). The strength of the slope is inversely proportional to the
fraction of prompt SNe-Ia. These results are achieved when using the same implementation which produces our star-forming-galaxy and MW results.

- The inclusion of metal-rich galactic winds, driven by SN-II explosions, strengthens the positive slopes in the $M_\text{\alpha}/M_\text{Fe}$ relations of ellipticals for all forms of SN-Ia DTD and SN-II yields considered. However, our simple, ISM-density-dependent wind scheme reduces the gas-phase metallicity of low-mass, star-forming galaxies, and so does not fully solve the problem.
- There is a clear correlation between mass, age and $\alpha$ enhancement in our model. This, along with the above findings, suggests that the chemical compositions of a diverse array of galaxies can be reconciled without requiring a variable IMF. Although an IMF that varied with SFR would likely produce similar results, it is instructive to see that this is not the only solution, given that the true behaviour of the stellar IMF is still uncertain.
- Overall, our results suggest, given the assumptions and limitations discussed, that the best model for matching the wide range of observational data considered here should include a) a power-law SN-Ia DTD, b) SN-II yields that take account of prior mass loss through stellar winds, and c) some direct ejection of light $\alpha$ elements into the CGM.

We conclude by highlighting two unavoidable limitations of this work and GCE models in general. First, the stellar yields used as an input have a strong influence on the results, as shown here by comparing the SN-II yields of P98 and CL04, and also in other studies (e.g. François et al. 2004; Romano et al. 2010). Until the true yields ejected by stars of all masses and metallicities are better understood, the accuracy of GCE models will always be uncertain. Second, we only consider one free GCE parameter in this work, $A$, the fraction of objects in a stellar SSP in the range $3 \leq M/M_\odot \leq 16$ that are SN-Ia progenitors. However, the values assumed for other GCE parameters (such as $\tau_{\text{min}}$, $M_{\text{max}}$, or the high-mass slope of the IMF) could also take on different values in reality. If so, our tuning of $A$ could also be correcting for these other uncertainties, and would not solely represent the efficiency of SN-Ia-progenitor formation. Further testing against additional observational data from Local Group dwarf galaxies, the intracluster medium of galaxy clusters, and outliers from the mass-metallicity relation, should help better constrain such uncertainties. These topics will be the focus of future work.

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