Secrecy Capacity Analysis of an Underlay Cognitive Radio Network in the Presence of Co-channel and Primary Network Interference

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Cite this article as: Bayat E, Aldırmaz Çolak S. Secrecy Capacity Analysis of an Underlay Cognitive Radio Network in the Presence of Co-channel and Primary Network Interference. Electrica, 2021; 21(1): 105-114.

ABSTRACT

This study investigates the impact of primary network and co-channel interference on the secrecy outage probability and strictly positive secrecy capacity of an underlay cognitive decode-and-forward relay network in Rayleigh fading channels. We consider a single eavesdropper node that tries to steal the transmitted signal at each hop. The secrecy outage probability and the strictly positive secrecy capacity are theoretically derived as a function of the interference sources and fading channels, which has not been studied in the literature. The theoretical results are validated with Monte Carlo simulation results. Consequently, the results show that the secrecy outage probability and the strictly positive secrecy capacity drastically degrade when the number of co-channel interference sources and the PN interference power increase.

Keywords: Underlay cognitive radio, secrecy outage probability, co-channel interference, decode-and-forward, cooperative communication

Introduction

With the introduction of fifth-generation communication into our lives, millions of devices are estimated to be added to the existing network with diverse applications in a very wide band from 700 MHz to 100 GHz [1]. Efficient resource allocation techniques will be one of the most crucial functions due to the frequency spectrum scarcity. Network densification was popularized years ago to exploit the frequency resources more efficiently and increase the number of users served. High spectrum utilization is caused by the aggressive employment of reuse strategies of the same frequency channels, which results to a co-channel interference (CCI).

The cognitive radio (CR) network proposed by Mitola in [2] can be considered as another solution to a more efficient usage of the frequency spectrum. In CR, three main spectrum access techniques enable unlicensed users, which are also known as secondary users (SUs), to access the licensed bands of primary users (PUs), namely underlay, overlay, and interweave [3]. This classification is roughly made by specifying under which conditions SUs can access the system. In underlay and overlay systems, SUs can simultaneously access the system with PUs. In an interweave system, SUs can only access the system when PUs do not exist in the frequency band.

The underlay approach is more popular than other spectrum access techniques in CR networks (CRNs) [4-12]. The key point of an underlay CRN (UCRN) is that the SU’s transmission power, which causes interference on the PUs, is limited by a predefined interference level to ensure the desired quality of service (QoS) of PUs. In [4, 5], the outage performance of a cooperative CRN that employs a decode-and-forward (DF) relaying protocol was investigated. In [6-12], performance was analyzed for a dual-hop CRN, where no direct communication exists between the source and destination nodes in the secondary network (SN). Multi relays were employed in the SN investigated in [7, 8]. Consequently, the system performance was found to be improved by using multiple secondary relays.

The system performance is affected by the CCI because CR networks are based on the co-existence of licensed and unlicensed users. It may be caused by adjacent sources using the same
frequency in other networks or in other unlicensed users in the CRN [9]. The impact of the CCI on the CRN performance is widely studied in the literature [9-12]. For example, in [9], the performance of opportunistic scheduling in DF-CRN in the presence of the CCI was investigated in terms of the outage probability, average error probability, and ergodic capacity. Aside from the CCI, the primary network (PN) causes interference on the SN because PUs and SUs can exist in the UCRN at the same time. The PN interference was also considered in [10] and [11] to investigate the outage performance of the UCRN over Rayleigh and Nakagami-m fading channels, respectively. These studies showed that both CCI and PN interferences severely degrade the system’s outage performance. Another study [12] investigated the multi-hop outage performance with various antenna selection methods, showing that employing the multi-hop relay scheme in SN for UCRN enhances the system performance by reducing the effect of the interference power constraint.

The transmitted data can be stolen by illegal users due to the nature of wireless communication systems. To prevent illegal eavesdropping, information security can be ensured by data encryption–decryption algorithms developed at the application layer. However, these algorithms require high computational power, complex architecture, and high implementation cost. The physical layer security (PLS) emerged as an alternative to encryption applications because it has advantages, such as low computational complexity and low power consumption. Unlike encryption approaches, PLS secures privacy by using the time-varying properties of wireless channels, such as fading and noise [13, 14]. More specifically, the basic idea of the PLS is to evaluate whether the achievable rate of the legal transmission channel is higher than that of the wiretap channel (i.e., channel between the source and the eavesdropper).

The PLS investigation in the CRN has attracted the attention of many researchers [15-22]. In [15, 16], the PLS was extensively examined along with various attack scenarios. The secrecy outage probability (SOP) for multiple-input single-output and single-input multiple-output CRN was investigated in [17, 18]. The PLS in the dual-hop UCRN was discussed in [19, 20]. Lei et al. compared the secrecy performances of the dual-hop UCRN over Nakagami-m fading channels by using different relay selection techniques [19]. Meanwhile, [20] showed that SOP worsens as the number of PUs increases. In [21], closed-form expressions of the probability of the strictly positive secrecy capacity (SPSC) and the SOP were derived over generalized Gamma fading channels for a single-input single-output system.

This study presents the importance of considering the impact of the PN interference on the secrecy performance. Yang et al. [22] analyzed the effect of the CCI on the SOP performance. In [9], the outage probability and the bit error rate were investigated for the same system model. However, no analytical and simulation results for the SOP and the SPSC were obtained. Similarly, [22] only performed an SOP analysis. Both [9] and [22] investigated only the CCI effect on the other secondary sources. Inspired by this observation, we merged these two studies and performed SOP and SPSC analyses of a dual-hop UCRN considering the CCI from other SUs and the PN interference.

The contributions of this study are as follows:

- To make the analysis of [22] more realistic, we used the UCRN system model defined in [11], where the PN interference power is considered in the SN performance.
- In addition to the SOP analysis, unlike [22], we also performed an SPSC analysis under the CCI of the SN and the interference from the PN.
- Analytical expressions for the SOP and the SPSC were derived using MATHEMATICA. Their results were validated with Monte Carlo simulation results using MATLAB.

The remainder of this paper is organized as follows: Section 2 describes the UCRN system model; Section 3 presents the theoretical analyses for the SOP and the SPSC; Section 4 discusses the simulation results; and finally, Section 5 concludes the paper.

**Notation:** We use the following notation throughout the paper: probability operator: $P_r(.)$; and cumulative distribution function (CDF) and probability density function (PDF): $F(.)$ and $f(.)$, respectively. Table 1 summarizes the system parameters used.

**System Model**

Figure 1 presents the proposed system model for the UCRN. This system model is originally defined in [11]. We only added an eavesdropper node to the SN. In the figure, $S$, $R$, and $D$ represent the source, relay, and destination nodes in the SN, respectively. Moreover, $E$ denotes an eavesdropper in the SN that attempts to steal information between $S$–$E$ the and $R$–$E$ nodes.

**Table 1. Definition of parameters and symbols**

| Parameter | Definition |
|-----------|------------|
| $h_{ij} \in \{sr,rd,sp,rp,pr,rd,ir,id\}$ | Rayleigh channel coefficient |
| $|h_{ij}|^2 \in \{sr,rd,sp,rp,pr,rd,ir,id\}$ | Rayleigh channel gain |
| $\sigma_{ij}^2 \in \{sr,rd,sp,rp,pr,rd,ir,id\}$ | Rayleigh channel variance |
| $C_i$ | Achievable rate of channel |
| $C_i^{se} i \in \{sr,rd,e\}$ | Secrecy achievable rate |
| $\gamma_{se} i \in \{sr,rd,e\}$ | SINR |
| $P_{max}$ | Maximum allowable transmit power |
| $I_{max}$ | Maximum interference threshold |
| $\gamma_{th}$ | Secrecy threshold |
| $L_{CID}$ | Number of CCI sources |
PUtx and PUrx represent the PN transmitter and receiver, respectively. Each node is equipped with a single antenna. We assume no direct link exists between nodes S and D; thus, relay-assisted communication is employed in the SN, where the relay uses the DF scheme. The secondary transmission is simultaneously realized with the primary transmission; thus, its maximum transmit power is limited by \(P_{max}\). In this manner, the QoS of the PN transmission is guaranteed.

All channels between nodes are assumed to be experienced as independent and non-identical (i.n.i.d) Rayleigh fading. The interference powers of the CCI sources are considered the same; thus, the CCI fading channels are assumed independent and identical (i.i.d) \[9\]. The channel fading coefficients between the S–R and R–D links are written as \(h_{sr}\) and \(h_{rd}\) respectively. The number of CCI sources to the nodes are assumed to be Rayleigh distributed; therefore, the channel fading coefficients between the CCI sources and the nodes are assumed independent and non-identical (i.n.i.d) Rayleigh fading. All channels between nodes are assumed to be experienced as independent and non-identical (i.n.i.d) Rayleigh fading. The interference powers of the CCI sources are considered the same.

The signal-to-interference plus noise ratio (SINR) of the S–R and R–D links is written as

\[
y_{sr}^q = \frac{E_S}{N_0} |h_{sr}|^2 \left(1 + \frac{E_P}{N_0} |h_{pr}|^2 \right)^{-1},
\]

\[
y_{rd}^q = \frac{E_R}{N_0} |h_{rd}|^2 \left(1 + \frac{E_P}{N_0} |h_{pd}|^2 \right)^{-1},
\]

Moreover, the SINR of the eavesdropper links between S–E and R–E can be written as

\[
y_{E}^q = \frac{E_E}{N_0} |h_{E}|^2,
\]

where, \(T \in \{S,R\}\) and \(E \in \{se,re\}\). The achievable rate \(c_i, i \in \{sr,se,rd, re\}\) of all links can be easily obtained as stated in \[23\]:

\[
c_S = \log_2 \left(1 + y_{sr}^q \right).
\]

The secrecy achievable rate is defined as the difference of the achievable rate of the channels between a legal user and an eavesdropper. Thus, the secrecy achievable rate expression per hop is written as in Eqs. (7) and (8).

\[
c_{sr}^q = (\log_2 \left(1 + y_{sr}^q \right) - \log_2 \left(1 + y_{E}^q \right)) = \log_2 \left(1 + \frac{y_{sr}^q}{y_{E}^q} \right).
\]

The SN transmission is performed in two steps considering the usage of relay-assisted communication. In the first step, the data are transmitted from node S to node R. Aside from the desired signals from node S, node R also receives undesired signals, such as an additive white Gaussian noise (AWGN) and an interference due to the CCI from the Ls interfering sources, and PN. In the second step, node D receives the forwarded signal from node R together with the noise and interference signals similar to the first step. At the same time, the eavesdropper is active and tries to steal the transmitted signal from both the S and R nodes. The received signals at the R and S nodes are written as follows:

\[
y_r = \sqrt{E_p} h_{pr} x + \sqrt{E_P} \sum_{j=1}^{l_0} h_{ir} x_j + \sqrt{E_{pr}} h_{id} x_{id} + n_r, \tag{1}
\]

\[
y_d = \sqrt{E_p} h_{rd} x + \sqrt{E_P} \sum_{i=1}^{l_0} h_{id} x_{id} + \sqrt{E_{rd}} h_{pd} x_{pd} + n_d, \tag{2}
\]

where \(E_p\) and \(E_P\) are the transmit powers of the S and R nodes, respectively. Their values are determined by \(E_p = \min \left\{ E_{SP}, E_{SR} \right\} \) and \(E_P = \min \left\{ E_{RP}, E_{RD} \right\} \), where \(P_{max}\) is the maximum transmission power limit at both the S and R nodes. \(E_{pr}\) and \(E_{rd}\) are the energies of the CCI at nodes R and D, respectively. Furthermore, \(E_{pr}\) and \(E_{rd}\) represent the energy of the PN interference signals at nodes R and D, respectively. As in \[11, 22\], we assumed that all the CCI channels have an equal power to simplify the analysis. In (1) and (2), \(n_r\) and \(n_d\) represent the AWGN with variance \(N_r\),
The UCRN system uses the DF scheme; thus, the end-to-end secrecy achievable rate $C_{eq}$ depends on the minimum achievable rate of the $S-R$ and $R-D$ links and can be written as follows:

$$C_{eq} = \log_2 \left( \min \left( C_{eq,SR}, C_{eq,DR} \right) \right). \quad (9)$$

**Secrecy Analysis**

**Secrecy Outage Probability**

The SOP is the key performance metric for evaluating the security of communication systems. A secrecy outage occurs if $C_{eq}$ falls below a predefined secrecy rate ($R_s = \log_2 (y_{th})$). The general expression of the CDF of the SOP is given as

$$F_{Y_{SR}} = \Pr (C_{eq} \leq R_s). \quad (10)$$

The SOP can be expressed as follows by using Eq. (9):

$$F_{Y_{SR}} = \Pr \left( \min \left( \frac{1 + Y_{SR}^{eq}}{C_{eq,SR}}, \frac{1 + Y_{RD}^{eq}}{C_{eq,DR}} \right) \leq y_{th} \right). \quad (11)$$

$$F_{Y_{SR}} = 1 - [1 - F_{Y_{SR}}(y_{th})][1 - F_{Y_{RD}}(y_{th})], \quad (12)$$

where, $y_{th} = 2^{R_s}$, and $F_{Y_{SR}}(y_{th})$, and $F_{Y_{RD}}(y_{th})$ represent the SOP of the first and second hops, respectively.

According to the maximum interference and the power constraints on the CDF of the first hop, $F_{Y_{SR}}(y_{th})$ can be expanded and expressed as

$$F_{Y_{SR}}(y_{th}) = \Pr \left( \frac{I_{hy}^{eq}}{I_{hy,sp}^{eq}} \leq y_{th} \right). \quad (13)$$

$$P_1 = \int_{0}^{\infty} F_{I_{hy}^{eq}}(x) f_{\text{Rayleigh}}(x) \, dx. \quad (14)$$

**Proposition 1:** $P_1$ can be obtained as

$$P_1 = \exp \left( -\frac{I_{hy,sp}^{max} P_{\text{max}}}{\sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} \right) + \frac{1}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} \left( T_1 (-1)^{m} \exp \left( -\frac{I_{hy}^{eq} P_{\text{max}}}{\sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} \right) \right)$$

$$\times \left( \frac{y_{th}}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} - \frac{1}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} \right)$$

$$+ \sum_{n=2}^{\infty} \frac{T_1 (-1)^{m} \Gamma \left( \frac{y_{th}}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} - \frac{1}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} \right)^{n+1}}{\frac{1}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} - \frac{1}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} \frac{T_1 (-1)^{m} \Gamma \left( \frac{y_{th}}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} - \frac{1}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} \right)^{n+1}}{\frac{1}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} - \frac{1}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} \right)} \left( \frac{1}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} - \frac{1}{\alpha_{I_{hy}^{eq}}^{2} \sigma_{I_{hy}^{eq}}^{2} P_{\text{max}}} \right)^{n}.$$
The CDF of the first hop $F_{y_r}(y_{th})$ can be obtained using $P_1$, $P_2$, and $P_3$:

$$F_{y_r}(y_{th}) = P_1 + (P_2 \times P_3)$$  \hfill (19)

The similar derivation steps can be repeated to obtain the second-hop CDF of $F_{y_d}(y_{th})$ by replacing $\sigma_{sr}^2$, $\sigma_{sp}^2$, $\sigma_{pr}^2$, and $L_r$ with $\sigma_{rd}^2$, $\sigma_{dp}^2$, $\sigma_{pd}^2$, and $L_d$, respectively. Lastly, the SOP of a dual-hop UCRN can be obtained by substituting what was given in Eq. (12).

Strictly Positive Secrecy Capacity

The SPSC is defined as the probability of the instantaneous secrecy achievable rate greater than zero. When the threshold value was set to zero, Eq. (11) is rewritten for the SPSC as follows:

$$\text{SPSC}_{\text{sys}} = \Pr \left( \min \left( \frac{\log_2 \left( \frac{1 + y_{y_{th}}}{1 + y_{y_{th}}^{\text{SR}}} \right)}{C_{\text{sys}}}, \frac{\log_2 \left( \frac{1 + y_{y_{th}}}{1 + y_{y_{th}}^{\text{RD}}} \right)}{C_{\text{sys}}} \right) > 0 \right)$$  \hfill (20)

$$= \left[ 1 - F_{\text{sys}}^{\text{SR}}(y_{th}) \right] \left[ 1 - F_{\text{sys}}^{\text{RD}}(y_{th}) \right]$$  \hfill (21)

where, $F_{\text{sys}}^{\text{SR}}(y_{th})$ and $F_{\text{sys}}^{\text{RD}}(y_{th})$ denote the CDF of the first and second hops, respectively.

Proposition 2: The SPSC of the first hop $F_{\text{sys}}^{\text{SR}}(y_{th})$ can be obtained as

$$F_{\text{sys}}^{\text{SR}}(y_{th}) = 1 - \exp \left( \frac{\sigma_{sr}^2 + \sigma_{rd}^2}{\sigma_{sr}^2} \left( \frac{1}{\sigma_{sr}^2} + \frac{1}{\sigma_{rd}^2} \right) \times \frac{\sigma_{sr}^2}{\sigma_{sr}^2} \right) \times \gamma \left( -\log \frac{\sigma_{sr}^2 + \sigma_{rd}^2}{\sigma_{sr}^2} \right)$$  \hfill (22)

Proof: See Appendix B

Using derivation steps similar to $F_{\text{sys}}^{\text{SR}}(y_{th})$, $F_{\text{sys}}^{\text{RD}}(y_{th})$, can be obtained by replacing the parameters.

Simulation Results

This section presents the simulation results of the SOP and the SPSC together with their analytical results. While the analytical derivation was done with MATHEMATICA, all simulations were performed using Monte Carlo with $10^6$ iterations on MATLAB. The simulation results were validated with the analytical results.

The following network parameter values were used in all figures: $P_{\text{max}} = 1 \text{ dB}$, $I_{\text{max}} = 2 \text{ dB}$, and $y_{th} = 1 \text{ dB}$. Lastly, the AWGN variances at all nodes were set to 1. The average SINR of the main link is defined as the ratio between the variance of the SU nodes and the variance of the eavesdropping channel. The eavesdropping channel variances were set as. The analytical results in Figures 2–4 and 5–6 were obtained from Eqs. (19) and (21), respectively. The CCI and PN interference powers were assumed as dB.

Figure 2 depicts the SOP variation versus the average SINR of the main link by taking into account only CCI [22], both CCI and primary network interference effect, and no interference effect.

Figure 3. Effects of the average SINR of the main link for various $\sigma_{sr}^2$ and $\sigma_{rd}^2$. 

Figure 2. SOP vs. the average SINR of the main link by taking into account only CCI [22], both CCI and primary network interference effect, and no interference effect.
The interference to the SN should be considered for achieving a more realistic analysis. The SOP value was at when only the CCI existed. The SOP value was at.

Figure 3 shows the effects of the average main link SINR for various channel variances of $S-R$ and $R-D$. The interference channel variances were selected as $\sigma_{pr}^2 = \sigma_{pd}^2 = \sigma_{ir}^2 = \sigma_{id}^2 = 1$. Furthermore, the numbers of the CCI sources at $R$ and $D$ were selected as $L_R = 2$ and $L_D = 2$, respectively. The system performance evidently improved as the average SINR of the main link increase. The SOP decreased as the quality of the $S-R$ and $R-D$ links was improved, indicating that the high variance improved the SOP. For example, in an SINR of dB, the SOP value for $\sigma_{sr}^2 = \sigma_{rd}^2 = 3$ was better than that for $\sigma_{sr}^2 = \sigma_{rd}^2 = 2$. Moreover, the same SOP curve was obtained for the case of $\sigma_{sr}^2 = 1$, $\sigma_{rd}^2 = 2$ and $\sigma_{sr}^2 = 1$, $\sigma_{rd}^2 = 2$ due to the DF strategy used in the SN.

In Figure 4, the impact of the PN interference channel variance and the number of the CCI sources on the system performance were investigated by linearly increasing them. The main link variances were set to $\sigma_{sr}^2 = \sigma_{rd}^2 = L_R$ and $\sigma_{pr}^2 = \sigma_{pd}^2 = L_D$, respectively. A degradation can be observed as the number of CCI sources and PN interference channel variance increased.

Figure 5 presents the SPSC variation versus the average SINR of the main link for various $\sigma_{sr}^2$ and $\sigma_{rd}^2$ values. The SPSC value approached 1 for all $\sigma_{sr}^2$ and $\sigma_{rd}^2$ values as the SINR of the main link increased. The SPSC performed better when the main link channel powers were $\sigma_{sr}^2 = 3$ and $\sigma_{rd}^2 = 3$. The separation between the curves decreased after $SINR = 10$ dB.

Figure 6 depicts the impact of the PN interference and the CCI on the SPSC. In this analysis, the PN interference channel variances at nodes $R$ and $D$ were selected as $\sigma_{ir}^2 = \sigma_{pr}^2 = L_R$ and $\sigma_{id}^2 = \sigma_{pd}^2 = L_D$, respectively. Moreover, the variances of the $S-R$ and $R-D$ links were selected as $\sigma_{sr}^2 = \sigma_{rd}^2 = 2$. As expected, the SPSC value decreased as the number of CCI sources and PN interference channel variance increased. The SPSC value was 0.4502 when the number of CCI sources was $L_R$ and $L_D = 2$ at $SINR = 10$ dB and 0.2996 at $SINR = 10$ dB when $L_R$ and $L_D = 3$.

Conclusions

This study investigated the SOP of the dual-hop DF UCRN system in the presence of the CCI and the PN interference over slow Rayleigh fading channels. The analytical SOP and SPSC results were derived along with the simulation results. The analytical results evidently matched well with the Monte Carlo simulation results.

The CCI from the undesired sources and the PN interference in the SN degraded the SOP and SPSC performances. This study showed the importance of considering the impact of the PN interference. We also observed that $I_{s_{max}}$, $P_{max}$, and $I_{hsp}$ do not affect the SPSC, as can be seen from Eq. (46).
Appendix A

Proof of Proposition 1

The CDF of $F_x$ can be written as

$$F_X(x) = Pr(X < x), \quad (23)$$

$$= Pr \left( \frac{\left| h_{\text{ray}} \right|^2}{1 + \frac{\left| h_{\text{ray}} \right|^2}{\left| h_{\text{ray}} \right|^2}} < \frac{\left( \left| t_{\text{ray}} - 1 \right| z + \gamma_{\text{ray}} \right)}{\left| h_{\text{ray}} \right|^2} \right)$$

$$= \int_{0}^{\infty} F_G(y) \left( \frac{\left( \left| t_{\text{ray}} - 1 \right| z + \gamma_{\text{ray}} \right)}{\left| h_{\text{ray}} \right|^2} \right) \phi_{h_{\text{ray}}}(y), \quad (25)$$

where, $\epsilon = \left( \frac{\left| h_{\text{ray}} \right|^2}{1 + \frac{\left| h_{\text{ray}} \right|^2}{\left| h_{\text{ray}} \right|^2}} \right)$ and $\phi_{h_{\text{ray}}}(y)$ denote the PDF of the Rayleigh channel gain (i.e., $\phi_{h_{\text{ray}}}(y) = \frac{1}{\sqrt{2\pi y}} \exp \left( - \frac{1}{2y} \right)$).

First, the CDF of $F_x$ should be calculated to obtain $F_x$. $G$ can be expressed as $G = \frac{C}{1 + A + B}$, where $A$, $B$, and $C$ are random variables (RVs) for $\sigma_{\text{ray}}^2$, $\left| h_{\text{ray}} \right|^2$, and $\left| h_{\text{ray}} \right|^2$, respectively. The PDF of $A$, $B$, and $C$ can be written as follows:

$$f_A(a_1) = \left( \frac{1}{\sigma_{\text{ray}}^2} \right) \exp \left( - \frac{a_1}{\sigma_{\text{ray}}^2} \right), \quad (26)$$

$$f_B(b_1) = \left( \frac{1}{\sigma_{\text{ray}}^2} \right) \exp \left( - \frac{b_1}{\sigma_{\text{ray}}^2} \right), \quad (27)$$

$$f_C(c_1) = \left( \frac{1}{\sigma_{\text{ray}}^2} \right) \exp \left( - \frac{c_1}{\sigma_{\text{ray}}^2} \right), \quad (28)$$

The first step of obtaining $F_x$ is finding the PDF of the sum of the two RVs ($U = A + B$). This can be done by using the following formula:

$$f_U(u) = \int_{-\infty}^{u} f_A(a_1)f_B(b_1)(u - a_1)da_1. \quad (29)$$

$f_U(u)$ can be obtained as follows:

$$f_U(u) = \left( \frac{1}{\sigma_{\text{ray}}^2} \right) \exp \left( - \frac{u}{\sigma_{\text{ray}}^2} \right) \left( \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2 - \sigma_{\text{ray}}^2} \right)^{\frac{u}{\sigma_{\text{ray}}^2}} \Gamma \left( \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2 - \sigma_{\text{ray}}^2} \right) \exp \left( - \frac{u}{\sigma_{\text{ray}}^2} \right), \quad (30)$$

where $\gamma(\cdot, \cdot)$ is the upper incomplete Gamma function defined in [24] Eq. (8.2.1). In the next step, the CDF of $\epsilon = \frac{C}{1 + A + B}$ can be obtained using the following formula, which is the CDF of the RV division.

$$F_{\epsilon}(\epsilon) = \int_{0}^{\infty} F_{\epsilon}(\epsilon(u + 1)f_U(u)du. \quad (31)$$

$F_{\epsilon}$ can be directly obtained from Eq. (28), and $F_{\epsilon}(g)$ can be obtained as follows:

$$F_{\epsilon}(g) = 1 - \exp \left( - \frac{g}{\sigma_{\text{ray}}^2} \right) \left( \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2 + g \sigma_{\text{ray}}^2} \right) \left( \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2 + g \sigma_{\text{ray}}^2} \right)^{\frac{g}{\sigma_{\text{ray}}^2}}. \quad (32)$$

The integral in Eq. (25) can be solved by substituting $F_{\epsilon}(g)$ and $f_U(u)$. $F_X(z)$ can then be written as follows:

$$F_X(z) = 1 - \exp \left( \frac{T_2z}{\sigma_{\text{ray}}^2} \right) \left( \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2 + T_2 \sigma_{\text{ray}}^2} \right) \left( \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2 + T_2 \sigma_{\text{ray}}^2} \right)^{\frac{T_2z}{\sigma_{\text{ray}}^2}}. \quad (33)$$

The Gamma function should be simplified according to [55] Eq. (8.4.15) to solve the integral in Eq. (14).

$$\Gamma(1 - \frac{T_2z}{\sigma_{\text{ray}}^2} \right) \left( \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2 + T_2 \sigma_{\text{ray}}^2} \right) \left( \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2 + T_2 \sigma_{\text{ray}}^2} \right)^{\frac{T_2z}{\sigma_{\text{ray}}^2}} \frac{(-1)^{n!}}{T_2 \sigma_{\text{ray}}^2} \left( \frac{T_2 \sigma_{\text{ray}}^2 + T_2 \sigma_{\text{ray}}^2} \right)^{\frac{n!}{T_2 \sigma_{\text{ray}}^2}}. \quad (34)$$

$$= B_1 - B_2. \quad (35)$$

$P_i$ can be rewritten as follows after simplification:

$$P_i = \int_{0}^{\sigma_{\text{ray}}^2} \exp \left( - \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \right) dz \quad \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \left( \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \right)^{\frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2}} \exp \left( - \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \right) dz \quad (36)$$

$$I_1 = \exp \left( - \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \right) \quad \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \left( \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \right)^{\frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2}} \exp \left( - \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \right) dz \quad (37)$$

$I_1$ can be directly derived as follows:

$$I_1 = \exp \left( - \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \right) \quad \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \left( \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \right)^{\frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2}} \exp \left( - \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \right) dz \quad (38)$$

$I_2$ can be rewritten as follows:

$$I_2 = \frac{T_2(-1)^n}{T_2 \sigma_{\text{ray}}^2} \exp \left( - \frac{T_2 \sigma_{\text{ray}}^2}{T_2 \sigma_{\text{ray}}^2} \right) \left( \frac{T_2 \sigma_{\text{ray}}^2}{T_2 \sigma_{\text{ray}}^2} \right)^{\frac{T_2 \sigma_{\text{ray}}^2}{T_2 \sigma_{\text{ray}}^2}} \exp \left( - \frac{T_2 \sigma_{\text{ray}}^2}{T_2 \sigma_{\text{ray}}^2} \right) dz \quad \frac{T_2 \sigma_{\text{ray}}^2}{T_2 \sigma_{\text{ray}}^2} \left( \frac{T_2 \sigma_{\text{ray}}^2}{T_2 \sigma_{\text{ray}}^2} \right)^{\frac{T_2 \sigma_{\text{ray}}^2}{T_2 \sigma_{\text{ray}}^2}} \exp \left( - \frac{T_2 \sigma_{\text{ray}}^2}{T_2 \sigma_{\text{ray}}^2} \right) dz \quad (39)$$

$P$ can be obtained by using the following formula [22]:

$$P = \int_{0}^{\sigma_{\text{ray}}^2} \exp \left( - \frac{\sigma_{\text{ray}}^2}{\sigma_{\text{ray}}^2} \right) dz \quad (31)$$
According to (40), $I_2$ can be obtained as follows:

$$I_2 = \left( \frac{T_1(-1)^{a_1}}{I_0} \sigma_0 \exp \left( -\frac{1}{\sigma_0^2} T_2 \frac{l_{\text{max}}}{P_{\text{max}}} \right) \right) \times \left( \frac{1}{\sigma_0^2} \right) \exp \left( -\frac{a_1}{b_1} E(c_1 - a_1 c_1) \right)$$

where, $a_1 = \frac{1}{\sigma_0^2} - T_2$, $b_1 = T_3 T_4 c_1 + T_3 l_{\text{max}} + T_3 \sigma_0^2 l_{\text{max}}$.

In the next step, $I_3$ can be rewritten as follows:

$$I_3 = \left( \frac{T_4(-1)^{a_2}}{I_0} \sigma_0^2 \right) \times \left( \frac{1}{\sigma_0^2} \right) \exp \left( \left( \frac{1}{\sigma_0^2} T_2 \frac{l_{\text{max}}}{P_{\text{max}}} \right) \left( T_3 T_4 + T_3 \sigma_0^2 l_{\text{max}} \right) \left( T_3 T_4 + T_3 \sigma_0^2 l_{\text{max}} \right) \right) \times \frac{1}{\sigma_0^2} f(y) dy.$$

Now, let $s = T_1 T_3 c_1 + T_3 \sigma_0^2 l_{\text{max}}$. The following expression is obtained:

$$I_3 = G_1 \times \left( \frac{1}{\sigma_0^2} \right) \exp \left( \left( \frac{1}{\sigma_0^2} T_2 \frac{l_{\text{max}}}{P_{\text{max}}} \right) \left( T_3 T_4 + T_3 \sigma_0^2 l_{\text{max}} \right) \right) \times \frac{1}{\sigma_0^2} f(y) dy.$$

According to (25) Eq. (8.350.2), can be obtained as follows:

$$I_3 = G_2 \times \left( \frac{1}{\sigma_0^2} \right) \exp \left( \left( \frac{1}{\sigma_0^2} T_2 \frac{l_{\text{max}}}{P_{\text{max}}} \right) \left( T_3 T_4 + T_3 \sigma_0^2 l_{\text{max}} \right) \right) \times \frac{1}{\sigma_0^2} f(y) dy.$$

Next, the variable in Eq. (43) is changed to $	au = \left( \frac{1}{\sigma_0^2} T_2 \frac{l_{\text{max}}}{P_{\text{max}}} \right) \left( T_3 T_4 + T_3 \sigma_0^2 l_{\text{max}} \right)$.

Appendix B

Proof of Proposition 2

The SPSC of the first hop ($F_{\text{spsc}_{SR}(y)}$) can be defined as follows using the definition in Eq. (20):

$$F_{\text{spsc}_{SR}(y)} = \frac{P_r}{C_{\text{SR}}} > 0$$

$$P_r = \frac{\left| h_y \right|^2}{\left( 1 + \sum_{i=1}^{N} \left| h_{yi} \right|^2 \right) + \left| h_p \right|^2} > \left| h_y \right|^2$$

Eq. (46) can be expressed in an integral form as follows:

$$F_{\text{spsc}_{SR}(y)} = \int_{\text{max}}^{\infty} f_0(y) f_{\text{y}}(y) dy.$$
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