Parameter dependence of the neutral Higgs boson production and decay in the two Higgs doublet model

Majid Hashemi* and Neda Nobakht†

Physics Department, College of Sciences, Shiraz University, Shiraz, 71946-84795, Iran

In this work, we present a study of the neutral Higgs bosons in the two Higgs doublet model (2HDM) in terms of their production processes and decay channels as a function of the model parameters. The analysis is performed for all four types of the 2HDM and the most promising processes and decay channels are identified for each type of the model. Several Higgs boson mass scenarios below and above the threshold of decay to gauge boson pair are introduced and the corresponding categories of final states are analyzed. It is shown that future lepton colliders have the potential to explore regions in the 2HDM parameter space which are out of the reach of LHC. Final results are presented in terms of the signal cross sections in different final states at relevant center of mass energies of $\sqrt{s} = 365$ GeV at the Future Circular Collider in $e^+e^-$ collision mode (FCC-ee) and $\sqrt{s} = 500$ GeV at the Compact Linear Collider (CLIC) or International Linear Collider (ILC).

INTRODUCTION

One of the main achievements in high energy physics in the last decade is the observation of a new boson at the Large Hadron Collider (LHC) by the two collaborations ATLAS and CMS [1, 2].

The observed particle is the candidate for the missing key element of the standard model, i.e., the Higgs boson, $h_{\text{SM}}$ [3] and its properties are in reasonable agreement with SM predictions as verified by various analyses at the LHC [9–16].

Within the uncertainty of these measurements, there is still possibility to consider beyond Standard Model (BSM) such as the two Higgs doublet model (2HDM) [17–19] which introduces SM-like Higgs boson candidate together with extra neutral and charged Higgs bosons.

Although 2HDM provides the Higgs sector for supersymmetry in the minimal form (MSSM) [20–22], it is still attractive as a standalone model due to the possibility of better agreement with experimental data [23].

The structure of 2HDM and its parameters provide the possibility to coincide the lightest Higgs boson properties to those of the SM Higgs boson [24]. The heavy neutral CP-even(CP-odd) Higgs bosons $H(A)$ and the two charged Higgs bosons $H^\pm$ are considered as extra Higgs bosons to be observed or excluded in the current or future experiments.

After the discovery of the light Higgs boson candidate, one of the main goals of the ATLAS and CMS collaborations has been the search for the extra Higgs bosons.

The ATLAS collaboration has reported an analysis of \(pp \rightarrow A \rightarrow Zh\) [25] where the CP-odd Higgs boson, $A$, decays to $Z$ boson and 125 GeV Higgs boson. They cover four types of 2HDM based on the Higgs-fermion couplings and results are presented in terms of exclusion contours in the parameter space. These results are confirmed by the CMS collaboration [26]. The heavy Higgs conversion, i.e., $A \rightarrow ZH$ has been analyzed by the two collaborations CMS [27] and ATLAS [28, 29]. We will discuss about these results in the next sections.

While collision data is taken by the two LHC collaborations CMS and ATLAS, there are ongoing analyses focusing on the possibility of observing extra Higgs bosons at the LHC luminosity upgrade [30, 31] and also future lepton colliders such as CLIC [32, 33], ILC [34], FCC [35] and CEPC [36].

In a number of recent works, we analyzed charged [37–39] and neutral [40–42] Higgs boson production and decay at lepton colliders and provided prospects for their observation in different scenarios using benchmark points in the parameter space. The above analyses were based on the alignment limit [43–45] which is defined as the scenario in which the properties of one of the neutral CP-even Higgs mass eigenstates coincide with those of the SM Higgs boson.

The alignment limit is naturally achieved in the so-called decoupling limit where the masses of other scalar states are large and decouple from the SM-like Higgs boson [46]. However, it is possible to achieve the alignment limit even without decoupling [43–45] which has been the case in our previous studies.

In the current work, we consider the possibility of migrating from the alignment limit and we perform a general scan of the parameter space to analyze the neutral Higgs boson branching ratio of decays. The analysis is not limited to a specific type of the 2HDM and all types are analyzed and compared to reach a conclusion on the choice of the most relevant production process and decay channel for the neutral Higgs bosons in each part of the parameter space.

In what follows, a brief theoretical description of the 2HDM and the software setup used for the analysis are presented. Next, we discuss about the signal processes adopted by LHC collaborations and then present our detailed study of the neutral Higgs boson decay channels in different mass scenarios, theoretical constraints and their
relevance in each type of the model. The final conclusion for each type of the 2HDM is presented at the end.

I. THE TWO HIGGS DOUBLET MODEL

The SM Higgs Lagrangian is written in the form

$$\mathcal{L} = (\partial_{\mu} \phi)^{(\partial_{\mu} \phi)} - V(\phi)$$

(1)

where $V$ is the Higgs potential based on only one Higgs doublet $\Phi$:

$$V = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

(2)

With this form of the potential, the condition to have a non-zero vacuum expectation value for the Higgs field is $\mu^2 < 0$.

The two Higgs doublet model is made as an extension of the SM Higgs sector by introducing two Higgs doublets $\Phi_1$ and $\Phi_2$.

Writing all possible Lagrangian terms requires additional degrees of freedom. The SM Higgs potential $\mu^2$ term is extended to include three parameters $m_{h_1}^2$, $m_{h_2}^2$ and $m_{A_2}^2$ and the $\lambda$ term is extended to seven terms containing $\tan \beta$ [44, 48]. In such a general Higgs potential, Higgs-boson-associated FCNC interactions exist. It has been shown that such FCNC terms are avoided at tree level by imposing discrete Z2 symmetry ($\Phi_1 \rightarrow \Phi_1$ and $\Phi_2 \rightarrow -\Phi_2$) [18]. The 2HDM Higgs potential under softly broken Z2 symmetry (allowing $m_{12} \neq 0$) reduces to the following form [49]:

$$V = m_{h_1}^2 \Phi_1^\dagger \Phi_1 + m_{h_2}^2 \Phi_2^\dagger \Phi_2 - m_{A_2}^2 \Phi_1^\dagger \Phi_2 + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \frac{1}{2} \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)
+ \frac{1}{2} \lambda_5 \left[ (\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right]$$

The condition corresponding to the SM $\mu^2 < 0$ is that the Higgs mass matrix made of $m_{ij}^2$ has at least one negative eigenvalue. If this is the case, the two doublets can be written in terms of their vacuum expectation values:

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

(3)

where the ratio of the two vevs is a free parameter of the model denoted by $\tan \beta = v_2/v_1$ with $v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2$.

The other parameter is the mixing angle $\alpha$ used to diagonalize the CP-even Higgs mass-squared matrix. The two parameters $\alpha$ and $\beta$ appear in the Higgs-fermion and Higgs-gauge couplings [49]. The Higgs-fermion Yukawa interactions, keeping only the neutral Higgs interactions, take the following form

$$\mathcal{L}_Y = \sum_{f=u,d,\ell} \frac{m_f}{v} \left( \xi_h f f h + \xi_h^f f H - i \xi_h^f \bar{f} \gamma_5 f A \right)$$

(4)

where the couplings are expressed in terms of the corresponding SM value, $m_f/v$, times the type dependent factors $\xi_{h/H/A}$ presented in Tab. 1. The CP-even Higgs coupling terms are sometimes written in terms of $\sin(\beta-\alpha)$ or $\cos(\beta-\alpha)$ using trigonometric relations [46]:

$$\frac{\sin \alpha}{\sin \beta} = \cos(\beta - \alpha) - \cot \beta \sin(\beta - \alpha)$$

$$\frac{\cos \alpha}{\cos \beta} = \cos(\beta - \alpha) + \tan \beta \sin(\beta - \alpha)$$

$$\frac{-\sin \alpha}{\cos \beta} = \sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha)$$

$$\frac{\cos \alpha}{\sin \beta} = \sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha)$$

(5)

The Higgs boson couplings to gauge bosons are model independent and, normalized to their corresponding SM values, are

$$g_{hVV} = \sin(\beta - \alpha), \quad g_{hVV} = \cos(\beta - \alpha).$$

(6)

There is no tree level coupling of CP-odd Higgs boson $A$ to vector bosons.

Since $\xi_{h,d,l}^{u,d,l}$ is either $\sin(\beta - \alpha)$ or $-\sin(\beta - \alpha)$, it is obvious through Eq. [5] and [6] that both $h$-fermion and $h$-gauge couplings align to their corresponding SM values if $\sin(\beta - \alpha) = 1$. One of the consequences of this alignment is that the heavier CP-even Higgs coupling to gauge bosons vanishes and its couplings to fermions is expressed in terms of $\tan \beta$ or $\cot \beta$.

The above simplified scheme of Higgs-fermion/gauge couplings has been analyzed in various analyses. The extra Higgs bosons ($H$ and $A$) are gaugeophobic and their couplings to fermions, normalized to the corresponding SM couplings, depend on $\beta$ while $\alpha$ is fixed through $\sin(\beta - \alpha) = 1$.

In this work, we do not restrict ourselves to the above requirement and instead, we take $\sin(\beta - \alpha)$ and $\tan \beta$ as input to evaluate the couplings of Tab. 1
table[7] with the use of 2HDMC 1.8 [61, 62]. The full combination of experimental limits are also obtained from the LHC 13 TeV run analyses using 

II. THE LHC SEARCH CHANNEL FOR 2HDM NEUTRAL HIGGS BOSONS

Before proceeding to our detailed analysis, we discuss about the LHC (ATLAS and CMS) search channel for the 2HDM neutral Higgs bosons presented in [25, 29].
The above analyses are based on the single CP-odd Higgs boson production followed by the subsequent decay $A \rightarrow Zh$ or $A \rightarrow Zb$. The CMS collaboration also considers the $m_H > m_A$ possibility through $H \rightarrow Za$ decay [27]. In our study, we assume $m_A > m_H$ while analysis of the opposite case can be performed in a similar way.

The LHC searches for the neutral Higgs bosons are divided into two categories of Higgs boson conversion, i.e., $A \rightarrow Zh_{SM}$ and $A \rightarrow ZH$ where the final state contains SM-like Higgs boson or the CP-even heavy Higgs boson.

The decay chains for the two processes are slightly different. The case of $A \rightarrow Zh$ involves $\cos(\beta - \alpha)$ as the coupling factor and vanishes at the alignment limit which is defined as $\cos(\beta - \alpha) = 0$. While the $A \rightarrow Zh$ coupling is type independent, $h_{SM} \rightarrow b\bar{b}$ depends on the type of the 2HDM which results in different patterns for the four types of the 2HDM in two dimensional $\tan \beta$ vs $\cos(\beta - \alpha)$ space.

Figure 1 shows $BR(A \rightarrow Zh) \times BR(h \rightarrow b\bar{b})$ for the four types as a function of $\tan \beta$ and $\cos(\beta - \alpha)$. Except for the lepton-specific type which is essentially designed for $h \rightarrow \ell\ell$, any suppression of the product of branching ratios in $A \rightarrow Zh \rightarrow Zb\bar{b}$ which occurs at $\cos(\beta - \alpha) \neq 0$ is due to the suppression of $h \rightarrow b\bar{b}$, another symmetric pattern around the vertical line of $\cos(\beta - \alpha) = 0$ would have been obtained.

The area inside the red line is allowed and the points outside are excluded by the most recent LHC analyses. However, ATLAS and CMS analyses which used $A \rightarrow Zh$ followed by $h \rightarrow b\bar{b}$ [25, 26] left the low BR (blue) regions especially the central vertical line of $\cos(\beta - \alpha)$ and excluded the rest.

The case of heavy Higgs boson conversion through $A \rightarrow Zh$ analyzed in [27, 29] is suitable at the alignment limit as the coupling is $\sin(\beta - \alpha)$. Therefore, for a given $\tan \beta$, the two production processes, i.e., $A \rightarrow Zh$ and $A \rightarrow Zh$ are complementary along the $\cos(\beta - \alpha)$ axis.

One of the reasons for using $A \rightarrow Zh$ by the LHC collaborations is less number of free parameters in the signal due to the fixed value of $h_{SM}$ mass. Moreover, $A \rightarrow Zh$ can be tested for $A$ masses as low as $m_h + m_Z$ while such masses are not allowed in $A \rightarrow ZH$ due to $m_H > m_h$ assumption. However, as long as the alignment limit and its nearby area is concerned, $A \rightarrow ZH$ provides a higher sensitivity near $\cos(\beta - \alpha) = 0$.

There are also differences in the CP-even Higgs boson decays to fermions as well as gauge bosons. Since $h \rightarrow b\bar{b}$ has been analyzed by LHC, we will discuss about $H \rightarrow b\bar{b}$ in the next sections. Concerning the heavy Higgs boson decay to gauge bosons, it was mentioned that the coupling, normalized to the corresponding SM value, is $\cos(\beta - \alpha)$ (Eq. 6). Therefore combining $A \rightarrow ZH$ and $H \rightarrow VV$ ($V = W$ or $Z$) may not be a reasonable idea as the two coupling factors compensate each other and the higher the production cross section, the lower the $H \rightarrow VV$ decay rate. This is not the case for $A \rightarrow Zh$ followed by $h \rightarrow VV$ as both involve $\cos(\beta - \alpha)$ factors. However, it is essentially a production chain most suitable far from the alignment area. On the other hand, the higher final state particle multiplicity due to gauge boson decays leads to no superiority over fermionic final states.

Therefore the conclusion for both processes ($A \rightarrow ZH$ and $A \rightarrow Zh$) is to preferably use $H/h \rightarrow f\bar{f}$. Here, we denote the decay final state as $f\bar{f}$ to remember that in lepton-specific type, the $\tau\tau$ final state has to be used while in other types $bb$ is the most suitable final state of the light or heavy CP-even Higgs boson.

Another possibility, which is currently missing among the list of LHC analyses, is to use single neutral gauge boson production leading to the Higgs boson pair production, i.e., $pp$ or $\ell^+\ell^- \rightarrow ZZ \rightarrow AH$. Here we also include lepton collisions at future colliders. The final state can be set by $A \rightarrow bb$, $H \rightarrow bb$ or $A \rightarrow bb$, $H \rightarrow VV$.

| Type | Type I | Type II | Type III | Type IV |
|------|--------|--------|----------|--------|
| $\xi^e_A$ | $\cos\alpha/\sin\beta$ | $\cos\alpha/\sin\beta$ | $\cos\alpha/\sin\beta$ | $\cos\alpha/\sin\beta$ |
| $\xi^d_A$ | $\cos\alpha/\sin\beta$ | $-\sin\alpha/\cos\beta$ | $-\sin\alpha/\cos\beta$ | $\cos\alpha/\sin\beta$ |
| $\xi^L_A$ | $\cos\alpha/\sin\beta$ | $\cos\alpha/\sin\beta$ | $\sin\alpha/\cos\beta$ | $\cos\alpha/\sin\beta$ |
| $\xi_f^+ A$ | $\cos\alpha/\sin\beta$ | $\sin\alpha/\sin\beta$ | $\sin\alpha/\sin\beta$ | $\sin\alpha/\sin\beta$ |
| $\xi_f^0 A$ | $\sin\alpha/\sin\beta$ | $\cos\alpha/\cos\beta$ | $\sin\alpha/\sin\beta$ | $\cos\alpha/\cos\beta$ |
| $\xi_f^- A$ | $\sin\alpha/\sin\beta$ | $\cos\alpha/\cos\beta$ | $\sin\alpha/\sin\beta$ | $\cos\alpha/\cos\beta$ |
| $\xi^e_A$ | $\cot\beta$ | $\tan\beta$ | $\tan\beta$ | $-\cot\beta$ |
| $\xi^d_A$ | $-\cot\beta$ | $\tan\beta$ | $\tan\beta$ | $-\cot\beta$ |

Table I. Yukawa couplings of the up-type ($u$) and down-type ($d$) quarks and leptons ($\ell$) to the neutral Higgs bosons $h/H/A$ in different types of 2HDM. There are also other names for types III and IV: flipped and lepton-specific [24].
with $b$ replaced by $\tau$ for the lepton-specific type. The $A \to VV$ can not be considered due to vanishing CP-odd Higgs-gauge coupling. Figures 2 and 3 show the Feynman diagrams related to the Higgs boson pair production in the two final states discussed above. These are example diagrams for lepton pair collision and $V = W$ while for the case of LHC, the same signal is initiated through quark anti-quark annihilation.

Since the signal is proposed to be analyzed in the four $b$- or $\tau$-jet final state, reasonable control of the QCD background at the LHC event environment is crucial. However, we have shown in a number of analyses that the signal of this process can well be observed at future lepton colliders (the most recent results are found in [64]).

In the following sections, taking the Higgs boson pair production as the golden channel for extra Higgs boson studies, we discuss about the branching ratio of CP-even and CP-odd Higgs boson decays and reach the final conclusion by analyzing all main combinations of decays.

III. CP-EVEN HEAVY HIGGS BOSON DECAY

The Higgs boson pair production should be analyzed in a specific final state. The decay of the CP-even Higgs boson can occur in fermionic mode if the Higgs boson mass is below the threshold of the lightest gauge boson pair production, i.e., if $m_H < 2m_W$. With $m_H > 2m_W$, decay to $WW$ and then $ZZ$ (if $m_H > 2m_Z$) are kinematically allowed. However, one should be aware of possible Higgs boson conversion, i.e., $H \to hh$, which turns on if $m_H > 2m_h$. Therefore we discuss about the three regions as follows.

A. $m_H < 2m_W$

In this region the Higgs boson decays to fermions, i.e., $b$ quarks in types I to III, and $\tau$ leptons in type IV (lepton-specific) unless decay to gauge bosons is enhanced by migrating from the alignment limit.

As seen in Fig. 4 in type I, $H \to bb$ is dominant near the alignment limit where $H \to VV$ is suppressed. However, increasing $|\cos(\beta - \alpha)|$ enhances $H \to VV$ in off-shell mode resulting in reduction of $H \to bb$ down to 0.2 or lower. However, the lower the Higgs boson mass, the higher the suppression of $H \to VV$. The two similar types II and III allow $H \to bb$ to be dominant in a wider area of the parameter space due to the tan $\beta$ factor in the $H \to bb$ coupling (Eq. 3). The type IV behaves similar to type I with $b$ replaced by $\tau$. However, contrary to type I in which $H \to bb$ is almost tan $\beta$ independent, in type IV, $H \to \tau\tau$ is enhanced at high tan $\beta$.

In order to compare the two fermionic and bosonic decay modes, we plot $H \to WW$ in the same parameter space as shown in Fig. 5. The two complementary plots shown in Figs. 4 and 5 show how the two decay modes $H \to ff$ and $H \to WW$ compete. These two plots assume $m_H = 150$ GeV.

Here we verify that the region of parameter space shown in Figs. 4 and 5 is theoretically accessible within the requirements of unitarity, stability and perturbativity.

In order to do so, for each point in the parameter space, we search for a range of $m_{12}^2$ values which satisfy theoretical constraints. Results are type independent and are shown for the four chosen values of tan $\beta = 1$, 5, 10 and 20 in Fig. 6. As is seen, increasing tan $\beta$ shrinks the available $m_{12}^2$ range for a given point defined by the values of $\cos(\beta - \alpha)$ and tan $\beta$.

For the mass scenario adopted in this section, BR($H \to ff$) and BR($H \to VV$) are independent of $m_{12}^2$ and any value of $m_{12}^2$ can be picked up from the range shown in Fig. 6. However, plots shown in Fig. 6 confirm that there is at least one such $m_{12}^2$ value for each point in the parameter space in the range of tan $\beta$ and $\cos(\beta - \alpha)$ under study.

FIG. 2. Higgs boson pair production in the four fermion final state.

FIG. 3. Higgs boson pair production in $WWff$ channel. Due to the large particle multiplicity in the final state, this channel may not provide better signal significance compared to the four fermion final state.
tan β = 1

BR(H → bb) in types I to III and ττ in type IV. The Higgs boson mass is set to 150 GeV which is near the threshold of on-shell H → WW. The lower masses lead to H → ff dominance in a wider region on both sides of cos(β − α) = 0. The area outside the red line is excluded.

FIG. 4. Branching ratio of Higgs boson decay to fermions (bb final state in types I to III and ττ in type IV). The Higgs boson mass is set to 150 GeV which is near the threshold of on-shell H → WW. The lower masses lead to H → ff dominance in a wider region on both sides of cos(β − α) = 0. The area outside the red line is excluded.

FIG. 5. Branching ratio of Higgs boson decay to W boson pair assuming m_H = 150 GeV. The area outside the red line is excluded.

2m_W < m_H < 2m_h

In this region, H → WW starts to occur in on-shell mode and if m_H > 2m_Z, H → ZZ will also be present. The relevant domain of H → VV is limited to Higgs boson masses below the threshold of SM-like Higgs boson pair production, i.e., 2m_W ≤ m_H ≤ 2m_h.

As for illustration, we show BR(H → ff) and BR(H → WW) in Figs. 7 and 8 respectively. The Higgs boson mass is set to m_H = 200 GeV. The Higgs boson decay to gauge boson pair is dominant when |cos(β − α)| approaches unity unless H → ff is enhanced at high tan β values in types II to IV. Inverse colors in the two plots shown in Figs. 7 and 8 show that there is no other relevant decay mode for such Higgs boson masses in the range 2m_W ≤ m_H ≤ 2m_h.

The red hashed region in Figs. 7 and 8 are excluded by theoretical constraints. The approach is the same as what was discussed in the previous section. For a given tan β value, a range of m_{12}^2 is obtained under theoretical constraints. Fig. 9 shows results for the four values of tan β. In this case, at high tan β, for some values of negative cos(β − α) there is no m_{12}^2 value respecting theoretical constraints.

C. m_H > 2m_h

If m_H > 2m_h, i.e., with a Higgs boson mass above 250 GeV, there is possibility of H → hh with a type independent coupling which depends on m_{12}^2. The presence of this decay mode causes suppression of BR(H → VV). One needs to search for a range of m_{12}^2 which satisfy theoretical constraints. On the other hand, m_{12}^2 dependence of the H → hh coupling leads to dependence of branching ratio of all other decay modes especially H → VV on m_{12}^2 as the sum of all BRs has to be unity.

The only safe area in this mass region is the vertical line of cos(β − α) = 0 and nearby where both H → VV and H → hh are suppressed and there is always a range of m_{12}^2 which respects theoretical constraints. Moreover, due to smallness of the above decay modes in the central
FIG. 7. Branching ratio of Higgs boson decay to fermion pair assuming $m_H = 200$ GeV. The red hashed region is theoretically inaccessible and the area outside the red line is experimentally excluded. In type 2, the whole region under study is excluded for this mass.

FIG. 8. Branching ratio of Higgs boson decay to $W$ boson pair assuming $m_H = 200$ GeV. The red hashed region is theoretically inaccessible and the area outside the red line is experimentally excluded. In type 2, the whole region under study is excluded for this mass.

region, BR($H \rightarrow ff$) is effectively independent of $m_{T2}^2$.

Let us show BR($H \rightarrow ff$) in Fig. 10 which features dominance over other decay modes as well as $m_{T2}^2$ independence at the alignment limit. The $m_{T2}^2$ value has been set to 1000 GeV in Fig. 10. However, $m_{T2}^2$ concerns rise when migrating from the alignment limit where BR($H \rightarrow ff$) is essentially small.

The BR($H \rightarrow hh$) has been shown in Fig. 11 with fixed value of $m_{T2}^2 = 1000$ GeV, which shows that $|\cos(\beta - \alpha)| > 0$ area is under control of this decay mode except for the very low $\tan \beta$ values.

It is notable that there is a larger theoretically excluded area at this mass compared to the lower mass of 200 GeV. The excluded area at negative $\cos(\beta - \alpha)$ is larger and also extends to positive $\cos(\beta - \alpha)$ values at high $\tan \beta$. The dominant decay mode at $|\cos(\beta - \alpha)| > 0.3$ is $H \rightarrow hh$ with BR($H \rightarrow hh$) > 0.5. At the central region of $\cos(\beta - \alpha) \simeq 0$, $H \rightarrow bb$ is still dominant as both $H \rightarrow hh$ and $H \rightarrow VV$ are suppressed when approaching this area.

There is a point in Fig. 11 which should be cautious about. We plotted BR($H \rightarrow hh$) for a fixed value of $m_{T2}^2$ to show the relevant domain of this decay mode in the parameter space. However, theoretical constraints rule out some parts of the plot in Fig. 11 because for them the chosen $m_{T2}^2$ may not be in the allowed range. Therefore for each point in the parameter space of $\tan \beta$ vs $\cos(\beta - \alpha)$ a value of $m_{T2}^2$ should be picked up from the specified range to respect the theoretical constraints. The complexity is thus due to $m_{T2}^2$ dependence of BR($H \rightarrow hh$) which provides a range of theoretically allowed branching ratios (not a single value) for each point in Fig. 11.

To conclude, study of $H \rightarrow VV$ at high masses faces difficulties due to theoretical considerations as well as the presence of other decay modes. However, the alignment limit can still be analyzed by $H \rightarrow ff$ at high masses.
FIG. 10. Branching ratio of Higgs boson decay to fermion pair assuming $m_H = 260$ GeV. The red hashed region is theoretically inaccessible and the area outside the red line is experimentally excluded.

FIG. 11. Branching ratio of Higgs boson decay to SM-like Higgs boson pair assuming $m_H = 260$ GeV. This is a type independent decay. However, the allowed regions are different for each type of the model and are shown by the red lines in Fig. 10.

IV. CP-ODD HEAVY HIGGS BOSON DECAY

The situation with CP-odd Higgs boson decays is simpler as $A \rightarrow VV$ vanishes and $A \rightarrow ff$ depends only on $\beta$ through $\tan \beta$ or $\cot \beta$. Therefore $\text{BR}(A \rightarrow ff)$ can be plotted as a function of $\tan \beta$ as shown in Figs. 12 and 13 for the two masses $m_A = 150$ and 200 GeV. The main difference between the two masses is observed in type 1, where $A \rightarrow bb$ is more suppressed by $A \rightarrow gg$ at $m_A = 200$ GeV. The other types essentially prefer $A \rightarrow ff$ at $\tan \beta > 5$.

FIG. 12. Branching ratio of CP-odd Higgs boson decay to $bb$ in types 1 to 3 and $\tau \tau$ in type 4. The Higgs boson mass is set to 150 GeV.

FIG. 13. Branching ratio of CP-odd Higgs boson decay to $bb$ in types 1 to 3 and $\tau \tau$ in type 4. The Higgs boson mass is set to 200 GeV.

V. CROSS SECTION OF THE HIGGS BOSON PAIR PRODUCTION

The only missing element for analyzing the Higgs boson pair production in the final states shown in Figs. 2 and 3 is now the total cross section of $HA$ production which should then be multiplied by branching ratios of Higgs boson decays.

The cross section of these events depends on $\sin(\beta - \alpha)$ through the second vertex and for a fixed value of $\sin(\beta - \alpha)$ is independent of $\tan \beta$. The cross section of the first mass scenario ($m_H = m_A = 150$ GeV) can be calculated for FCC-ee center of mass energy of 365 GeV [70] as well as ILC [71, 72] and CLIC stage 1 [73] operating at center of mass energy of 500 GeV. The second mass scenario above the vector boson pair production threshold can be realized at the same operation scenario of ILC and CLIC.

As previously mentioned, the signal cross section prefers the central region of the alignment due to the $\sin(\beta - \alpha)$ factor thus preferring $H \rightarrow ff$ over $H \rightarrow VV$ because at the alignment limit, in all mass scenarios mentioned before, $H \rightarrow ff$ is dominant.

The final results showing the product of cross sections and branching ratio of CP-even and CP-odd Higgs bosons are presented in Figs. 15 and 16 for FCC-ee center of
mass energy $\sqrt{s} = 365$ GeV and Figs. 17 and 18 for
CLIC/ILC center of mass energy $\sqrt{s} = 500$ GeV for the
two final states $H/A \rightarrow ff$ and $H \rightarrow WW, A \rightarrow ff$. The
color palette obviously shows that relevant final state for
the central region is $H/A \rightarrow ff$ while regions far from the
alignment limit can be probed by $H \rightarrow WW, A \rightarrow ff$
with its own difficulties.

FIG. 14. The signal cross section as a function of $\sin(\beta - \alpha)$
for the two center of mass energies $\sqrt{s} = 365$ GeV (FCC-ee)
and 500 GeV (ILC or CLIC). The two scenarios of $m_H = m_A = 150$ GeV
and 200 GeV are shown. The signal cross section has a quadratic dependence on $\sin(\beta - \alpha)$ and reaches
its maximum at $\sin(\beta - \alpha) = 1$ for each mass scenario.

VI. CONCLUSIONS

The two Higgs double model was studied in this work
as the theoretical framework for study of extra neutral
Higgs bosons in terms of their production processes and
decay channels. The analysis was performed for all four
types of the CP-conserving 2HDM and parameter space
scans were presented including relevant parameters which
determine the production cross sections and branching
ratio of decays.

The final results were divided into two domains of the
Higgs boson mass, i.e., below and above the threshold
doing decay to gauge boson pair. It was shown that the
$\cos(\beta - \alpha) = 0$ limit known as the alignment limit can
well be verified at lepton colliders if $e^+e^- \rightarrow HA \rightarrow 4$
fermion final state is analyzed. The reason is due to the
dominance of the cross section as well as $\text{BR}(H/A \rightarrow ff)$
in this region.

We also included experimental limits from the latest

FIG. 15. The signal cross section in the four fermion final
state for the mass scenario $m_H = m_A = 150$ GeV at $\sqrt{s} = 365$
GeV.

FIG. 16. The signal cross section in the $WWff$ final state for
the mass scenario $m_H = m_A = 150$ GeV at $\sqrt{s} = 365$ GeV.

FIG. 17. The signal cross section in the four fermion final
state for the mass scenario $m_H = m_A = 200$ GeV at $\sqrt{s} = 500$
GeV.
LHC results for each Higgs boson mass scenario. Figures 15 and 17 show that the allowed regions, which are not yet excluded by LHC, focus on regions where the product of the cross section of the proposed process and BR(H/A → ff) is high compared to other points in the parameter space. This is contrary to the case of H → VV shown in Figs. 16 and 18 where the event rate is low at the experimentally allowed region.

This work does not include event analysis due to the missing beam spectra available in a public repository for FCC-ee. A detailed analysis, when missing software elements are available, will be performed in a future work.

The cross sections at FCC-ee center of mass energy of 365 GeV are of the order of 10 fb for the low mass scenario m_H = m_A = 150 GeV.

The high mass scenario m_H = m_A = 200 GeV is realized at a higher center of mass energy of \( \sqrt{s} = 500 \) GeV at CLIC or ILC with cross sections of the order of few femtobarns. The other final state, i.e., \( e^+e^- \rightarrow HA \rightarrow VVff \), reaches maximum cross section of 6 fb at \( \cos(\beta - \alpha) \simeq 0.5 \) and low tan \( \beta \) values while one should be aware of the theoretical constraints which rule out parts of the parameter space at Higgs boson masses above 200 GeV.

The study presented in this work is not limited to lepton colliders and can be verified at LHC as well. However, the analysis of \( pp \rightarrow HA \rightarrow 4f \) in both cases of \( \tau \)-jet as well as \( b \)-jet final states suffers from the hadronic environment of the LHC events and the large cross section of QCD multi-jet events. Therefore a reasonable control of the background events is crucial for a concrete conclusion at LHC.

At lepton colliders, the 2HDM type 1 has in general lower cross sections compared to other types. We have shown in [10] that signal cross section of about 4 fb in 2HDM type 1 (although with slightly different final state from what we propose here) can be observed at \( \sqrt{s} = 500 \) GeV. We have also analyzed the four \( b \)-jet final state in 2HDM type 1 at \( \sqrt{s} = 1000 \) GeV in [69] and reasonable results have been obtained for the low mass scenarios. Therefore, we expect the parameter space of \( \tan\beta \) vs \( \cos(\beta - \alpha) \) can be probed using the proposed channel at lepton colliders thus confirming LHC results and also extending to the alignment limit in the full range of \( \tan\beta \) values.

[1] G. Aad et al. (ATLAS), Phys. Lett. B716, 1 (2012), 1207.7214.
[2] S. Chatrchyan et al. (CMS), Phys. Lett. B716, 30 (2012), 1207.7235.
[3] P. W. Higgs, Phys. Rev. Lett. 13, 508 (1964).
[4] J. W. Giddings, Phys. Rev. Lett. 12, 132 (1964).
[5] P. W. Higgs, Phys. Rev. 145, 1156 (1966).
[6] F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964).
[7] G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, Phys. Rev. Lett. 13, 585 (1964).
[8] T. W. B. Kibble, Phys. Rev. 155, 1554 (1967).
[9] G. Aad et al. (ATLAS), Phys. Rev. D 101, 012002 (2020), 1909.02845.
[10] G. Aad et al. (ATLAS), Phys. Rev. Lett. 125, 061801 (2020), 061801 (2020), 2003.10866.
[11] G. Aad et al. (ATLAS, CMS), JHEP 08, 045 (2016), 1606.02266.
[12] T. D. Lee, Phys. Rev. D8, 1226 (1973).
[13] S. L. Glashow and S. Weinberg, Phys. Rev. D15, 1958 (1977).
[14] G. C. Branco, Phys. Rev. D49, 2901 (1980).
[15] I. J. R. Aitchison (2005), hep-ph/0505105.
[16] E. Ma and D. Ng, Phys. Rev. D49, 6164 (1994), hep-ph/9305230.
[17] A. Djouadi, Phys. Rept. 459, 1 (2008), hep-ph/0503173.
[18] F. Mahmoudi and O. Stal, Phys. Rev. D81, 035016 (2010), 0907.1791.
[19] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, Phys. Rept. 516, 1 (2012), 1106.0034.
[20] A. Djouadi, Phys. Rept. 516, 1 (2012), 1106.0034.
[21] A. Djouadi, Phys. Rept. 516, 1 (2012), 1106.0034.
[22] A. Djouadi, Phys. Rept. 459, 1 (2008), hep-ph/0503173.
[23] F. Mahmoudi and O. Stal, Phys. Rev. D81, 035016 (2010), 0907.1791.
[24] G. C. Branco, P. M. Ferreira, L. Lavoura, M. N. Rebelo, M. Sher, and J. P. Silva, Phys. Rept. 516, 1 (2012), 1106.0034.
[25] A. M. Sirunyan et al. (CMS), Phys. Lett. B792, 369 (2019), 1812.06504.
[26] A. M. Sirunyan et al. (CMS), Eur. Phys. J. C 79, 564 (2019), 1903.00941.
