Chemically reactive bioconvection flow of tangent hyperbolic nanoliquid with gyrotactic microorganisms and nonlinear thermal radiation

Kamel Al-Khaled\textsuperscript{a,b,*}, Sami Ullah Khan\textsuperscript{b}, Ilyas Khan\textsuperscript{c}

\textsuperscript{a} Department of Mathematics & Statistics, Jordan University of Science and Technology, P.O. Box 3030, Irbid, 22110, Jordan
\textsuperscript{b} Department of Mathematics, COMSATS University Islamabad, Sahiwal, 57000, Pakistan
\textsuperscript{c} College of Engineering, Majmaah University, P.O. Box 66, Majmaah, Saudi Arabia

\section*{A R T I C L E   I N F O}

Keywords:
Computational mathematics
Industrial engineering
Thermodynamics
Theoretical fluid dynamics
Physics methods
Tangent hyperbolic nanofluid
Motile organisms
Variable thermal conductivity
Oscillatory stretching sheet

\section*{A B S T R A C T}

On the account of motivating fabrication of bioconvection phenomenon in various engineering and industrial systems, an attention has been devoted by researchers in current decade. Therefore, this theoretical investigation deals with the utilization of bioconvection phenomenon in flow of tangent hyperbolic nanofluid over an accelerated moving surface. It is assumed that the flow is generated due to periodically motion of the sheet. The energy equation is modified by entertaining the nonlinear thermal radiation features. The chemical reaction effects are elaborated in the concentration equation. Moreover, the significance of present flow problem increases by utilizing the thermophoresis and Brownian motion effects. The governing equations are transmuted into non-dimensional form with utilization of appropriate quantities. The analytical solution is computed by using homotopy analysis method. The implications of promising parameters on velocity profile, temperature profile, nanoparticles volume fraction and microorganisms profile is evaluated graphically. The presence of radiation parameter, thermophoresis and Brownian motion effects are more frequent for enhancement of heat transfer. The reported observations can efficiently use in the improvement of heat transfer devices as well as microbial fuel cells.

1. Introduction

The interest has been developed in recent few years towards the study of non-Newtonian materials by numerous instigators due to their valuable consequences and practical industrial, chemical, biochemical engineering and mechanical applications\cite{1, 2, 3}. Accompanying some prestigious applications include blood, honey, cosmetics, glue, crude oil, asphalts, cream etc. Due to their complex molecular structure, the non-Newtonian fluids accomplish a nonlinear relationship with shear stress and rate of deformation and subsequently assigned in the category of power law model. In contrast to the viscous fluids, such fluids are more complex and the physical properties of such fluids cannot be predicted by using simple relations. In order to overcome this issue, the scientists have suggested a variety of mathematical models regarding non-Newtonian materials in the literature. Among these mathematical models, the tangent hyperbolic model is one which accomplished the shear thinning effects i.e., the viscosity declined by increasing shear rate. Due to such interesting rheological behavior, many authors have used tangent hyperbolic model with distinct flow features. For instance, Nadeem et al.\cite{4} constituted famous boundary layer approximation equations for the flow of tangent hyperbolic fluid past over a stretching surface. Hayat et al.\cite{5} studied a two-dimensional flow of tangent hyperbolic fluid in presence of mass flux conditions and thermophoresis effects. Ullah and Zaman\cite{6} directed the slip flow of tangent hyperbolic liquid over a stretched configuration. The dimensionless analysis for the governing equations has been performed via Lie group technique which was further tackled numerically with utilization of shooting procedure. The flow of tangent hyperbolic liquid in double saturated flow over a stretching cylinder has been signified by Nagendraamma and co-workers\cite{7}. Kumar et al.\cite{8} numerically evaluated the involvement of heat absorption and generation features in tangent nanoparticles flow over a convectively heat surface. The numerical solution of the simulated flow problem was suggested by using shooting technique. Another interesting analytical based approach regarding flow of tangent hyperbolic nanofluid over an oscillatory moving surface was elaborated by Khan et al.\cite{9}. Rehman et al.\cite{10} discovered some interesting thermo-physical properties of tangent hyperbolic fluid over a confined
The peristaltic flow with variable thermal conductivity in tangent hyperbolic fluid was evaluated by Hayat et al. [11].

The transport of heat incorporating the fluid flow is necessitated in large number of nuclear and thermal-hydraulic processes. In order to enhance the heat transportation process, a variety of fluids and operating conditions has been tested. The interaction of such fluid into existing system may be useful in reduction of capital costs, improve the working efficiency and better design of desired system. Among the traditional methods, air is one of the primary methods to cooling the various electronic systems. However, it is often noted that for extremely higher heat fluxes, the role of liquid cooling is more progressive. The role of cooling is quite indispensable in order to sustain the desired thermal performances in various engineering and technological products like computers, motor engines, chemical reactions, laptops and cooling of strips. Nano fluids, with excellent thermo-physical features and relatively slow thermal resistance, are attributed as most attractive attention recently. Recently, the nanotechnology has been considered as most intriguing developments with effective cost and ultra-high output. The nanoparticles are relatively small sized particles and suspended in the liquid. The basic principle and innovation of such suspended particles is to enhance the thermal conductivity of widely used base liquids. The fundamental development on this topic was presented by Choi [12] which was further massively extended by various authors along with addition of some other physical features. For instance, Bhatti and Rashidi [13] observed the Brownian motion and thermodiffusion features in the flow of Williamson nanofluid plunged in porous medium. Hayat et al. [14] provided a mathematical model for melting heat with thermal radiation effects in stagnation-point flow of carbon nanotubes. Kumar et al. [15] exploited the cooling procedure based on involvement of CuO-water based nanoparticles in the semiconductors. Ijaz et al. [16] inspected the entropy generation and activation energy phenomenon in flow of Sisko nanofluid along with interaction of nonlinear thermal radiation. Khan and Shehzad [17] analyzed the third grade fluid over periodically accelerated surface in addition of Joule heating effects.

Microorganisms are unicellular organisms, they live everywhere as in animals, people and in bodies of plants. Microorganisms become cause of bioconvection with the supremacy of microorganisms gathering. Bioconvection phenomenon is presented by oxytactic animals, people and in bodies of plants. Microorganisms become cause of bioconvection phenomenon in following forms:

\[ \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0, \]

\[ \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \left[ (1 - C_m) \rho \beta \gamma \left( f(T - T_w) - (\rho_f - \rho) \gamma C - C_m \right) - (n - n_m) \gamma \left( \rho_{m} - \rho \right) \right], \]

\[ \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \left( \alpha + 16 \sigma n T^3 \right) \frac{\partial^2 T}{\partial \sigma^2} + \tau \left[ D_r \frac{\partial C}{\partial \sigma} + D_t \frac{\partial T}{\partial \sigma} \right] \]

\[ \frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_r \frac{\partial^2 C}{\partial \sigma^2} + D_t \frac{\partial T}{\partial \sigma} - k_s (C - C_m), \]

\[ \frac{\partial n}{\partial t} + u \frac{\partial n}{\partial x} + v \frac{\partial n}{\partial y} = \frac{b_i W_i}{C_m - C} \frac{\partial n}{\partial \sigma} = D_n \frac{\partial^2 n}{\partial \sigma^2}. \]

We assign following boundary conditions for the current problem

\[ u = u_m = \delta T \sin \omega t, \quad v = 0, \quad T = T_w, \quad C = C_m, \quad n = n_m \quad \text{at} \quad \gamma = 0, \quad t > 0, \]

\[ u \rightarrow 0, \quad T \rightarrow T_m, \quad C \rightarrow C_m, \quad n \rightarrow n_m \quad \text{at} \quad \gamma \rightarrow \infty. \]

The initial conditions are

\[ u = 0, \quad T = T_m, \quad C = C_m, \quad n = n_m \quad \text{at} \quad \gamma = 0, \]


\[ u = 0, \ v = 0 \quad t = 0. \]

(8)

where \( \Gamma \) is time constant, \( \rho_f \) represents the fluid density, \( k^* \) denotes the permeability parameter, \( \varphi \) is porous medium, \( \sigma^* \) electrical conductivity, \( \rho_p \) density of nanoparticles, \( \rho_m \) microorganisms particles, \( \beta^* \) is volume expansion coefficient, \( g \) the gravity, \( T \) is temperature, \( \alpha_m \) determine the Stefan Boltzmann constant and \( k^* \) is the absorption constant, \( C \) is concentration, \( D_b \) denotes coefficient of Brownian diffusion, \( D_r \) reports the thermophoretic diffusion constant, \( \tau_1 = (\varphi c_p)_{p}/(\varphi c_p) \) signify the nanoparticles heat capacitance and fluid particles heat capacitance ratio, \( D_{am} \) relates the microorganisms density, \( n \) stands for gyrotactic microorganism density, \( W_s \) is swimming cell speed while \( b_i \) is the chemotaxis constant.

Before compute the desired solution of the flow problem, first we transform the flow problem in dimensionless form by initiating following variables [17].

\[ u = b f_1(y, r), \quad v = -\sqrt{b} f_2(y, r), \quad y = \frac{b}{\sqrt{b}} y, \quad \tau = \tau. \]

(9)

\[ \theta(y, r) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad \phi(y, r) = \frac{C - C_{\infty}}{C_w - C_{\infty}}, \quad \chi(y, r) = \frac{n - n_{\infty}}{n_m - n_{\infty}}. \]

(10)

The substitution of above variables in Eqs. (2), (3), (4), and (5), following dimensionless forms are recovered

\[ f_{yy} - S f_y - f_y^2 + f_{\tau y} - \beta f_{\tau r} + W e f_y f_{\tau y} \phi \theta + \frac{\lambda(1 - \varphi) \theta}{\tau_c} = 0, \]

(11)

\[ \left[ \frac{4}{3} R_d \left( 1 + (\theta_1 - 1) \theta_1^3 \right) \theta_1^3 + R_d \left( \frac{3(\theta_1 - 1)^2}{1 + (\theta_1 - 1) \theta_1^3} + \frac{\lambda}{\theta_1} \right) \theta_1^3 \right] + \frac{\lambda}{\theta_1} \phi \theta + \frac{\lambda}{\theta_1} \phi \theta = 0, \]

(12)

\[ \phi_{yy} - S c_{\phi} \phi - S c_{\phi} \phi + \frac{N_t}{N_b} \phi_{\tau y} - K_r S c_{\phi} \phi = 0, \]

(13)

\[ \chi_{yy} - S L h_{\chi} + L h_{\chi} - Pe \left( \frac{\partial \chi \phi_y}{\partial y} + \frac{\partial \phi \chi_y}{\partial y} + \sigma \right) = 0, \]

(14)

The governing boundary conditions are

\[ f_1(0, \tau) = \sin \tau, \quad f_2(0, \tau) = \theta(0, \tau) = 1, \quad \psi(0, \tau) = \chi(0, \tau) = 1, \]

(15)

\[ f_1(\infty, \tau) \to 0, \quad \theta(\infty, \tau) \to 0, \quad \psi(\infty, \tau) \to 0, \quad \chi(\infty, \tau) \to 0, \]

(16)

where \( W_e = 2\sqrt{\pi} \sqrt{b d_{\infty}/d_V}\) is the local Williamson parameter, \( S = \omega/\beta \) is the ratio of oscillation frequency to stretching rate, \( \beta = \sigma^* B_0^2 / \rho_f b^3 + \varphi \rho_f / k \) is the Hartmann number and porosity parameter (combined parameter), \( \lambda = \beta^* \left( 1 - C_{\infty} \right) \left( T_w - T_{\infty} \right) / \alpha^* \) is the mixed convection parameter, \( R_b = \gamma (n_{\infty} - n_m) (\rho_f - \rho_r) / \beta \rho_f (1 - C_{\infty}) \left( T_w - T_{\infty} \right) \) is the convectored Rayleigh number, \( Pr = \varphi / \eta \) is the Prandtl number, \( R_d = 4 \sigma^* T_{\infty}^3 / 3 k^* \) is the radiation parameter, \( \theta_1 = \theta / \theta_{\infty} \) is the surface heating parameter, \( N_r = (\rho_f - \rho_r) / \beta \rho_f (1 - C_{\infty}) \left( T_w - T_{\infty} \right) \) buoyancy ratio parameter, \( N_t = \tau_1 D_r \left( T_w - T_{\infty} \right) / T_w \) is the thermophoresis parameter, \( S_c = \varphi / D_b \) as the Schmidt number, \( N_h = \tau_1 D_r (C_{\infty} - C_{\infty}) / \eta \) is the Brownian motion parameter, \( L_h = \rho_f / D_b \) is the biocoordinated Lewis number, \( K_r = k / b \) is the chemical reaction parameter, \( P_e = b_i \left( W_s - D_b \right) / D_b \) is the Pelet number and \( \sigma = n_{\infty} \) is the microorganisms concentration difference parameter.

We expressed the physical quantities namely local Nusselt number, local Sherwood number and local motile number in following forms:

\[ N_u = \frac{\tau_{\infty}}{k (T_w - T_{\infty})}, \quad S_h = \frac{\tau_{\infty}}{D_b (C_w - C_{\infty})}, \quad N_s = \frac{\tau_{\infty}}{D_b (n_{\infty} - n_m)}, \quad q_i = -\frac{\partial \theta}{\partial \tau} \tau = \frac{\tau_{\infty}}{D_b (C_w - C_{\infty})}, \quad g_i = -\frac{\partial \phi}{\partial \tau} \tau = \frac{\tau_{\infty}}{D_b (n_{\infty} - n_m)}. \]

(17)

After inserting the dimensionless quantities in above expression, we have

\[ N_u \text{Re}_{\infty}^{-0.5} = -\left( 1 + \frac{4}{3} R_d \theta_1^3 \right) \theta_1(0, \tau), \quad S_h \text{Re}_{\infty}^{-0.5} = -\varphi(0, \tau), \quad N_s \text{Re}_{\infty}^{-0.5} = -\chi(0, \tau). \]

(18)

where \( N_u \) local Nusselt number, \( S_h \) is local Sherwood and \( N_s \) local motile organisms density number.

3. Homotopy analysis method

In various engineering, technological and industrial systems, the resulted differential equations are of highly nonlinear in nature which always poses a challenge for mathematicians and engineers. Such problems are often treated numerically or analytically. Beyond the analytical techniques, homotopy analysis method is one which successfully computes the desired series solution. One of the astonishing aspect of this method is it does not restrict the condition of large or small parameter. The region of convergence associated with this technique can be addressed more conveniently as compared to other techniques. It offers great freedom to develop the desired base functions to compute the solution. The pioneer work on this method was initiated by Liao [28] and later on many investigators use this technique for various nonlinear problems [29, 30, 31, 32]. Following initial guesses are suggested to start the simulations

\[ f_1(0, \tau) = \sin \tau (1 - e^{-\gamma}), \quad \theta(0, \tau) = e^{-\gamma}, \quad \phi(0, \tau) = e^{-\gamma}, \quad \chi(0, \tau) = e^{-\gamma}. \]

(19)

Let us assert the auxiliary linear operators to start the analytical simulations

\[ \tau_{\gamma} = \frac{\partial \gamma}{\partial \tau} \gamma - \varphi - 1, \quad \tau_{\phi} = \frac{\partial \phi}{\partial \tau} \phi - 1, \quad \tau_{\chi} = \frac{\partial \chi}{\partial \tau} \chi - 1, \]

(20)

satisfying

\[ \tau_{\gamma} \left( c_1 + c_2 e^\gamma + c_3 e^{-\gamma} \right) = 0, \]

(21)

\[ \tau_{\phi} \left( c_4 e^\phi + c_5 e^{-\gamma} \right) = 0, \]

(22)

\[ \tau_{\chi} \left( c_6 e^\chi + c_7 e^{-\gamma} \right) = 0, \]

(23)

\[ \tau_{\phi} \left( c_8 e^\phi + c_9 e^{-\gamma} \right) = 0, \]

(24)

where \( c_i \) represents arbitrary constants.

3.1. Convergence analysis

The simulations based on HAM method results a series solution which involves the auxiliary constants \( h_1, h_2, h_3, h_4 \), for which suitable selection of these auxiliary constants are quite necessitated. For this purpose, \( h \) – curves for velocity, temperature, concentration and motile micro-organisms profiles are presented for specified values of emerging parameters in Figure 1(a-d). It is pointed out that more convenient values for the given solution are selected from \(-2.1 \leq h_1 \leq 0, -1.8 \leq h_2 \leq 0.1, -1.7 \leq h_3 \leq 0.2, -1.7 \leq h_4 \leq 0.2 \).

3.2. Validation of results

Before analyse the graphical results, first we verify our solution by comparing it with already reported data. The obtained results are compared as a limiting case with exact solution, suggested by Turkylmazoglu [33] and Hayat et al. [34] in Table 1. Table shows that our
results meet good agreement with these results. Present numerical computations are also compared with Zheng et al. [35] and Abbas et al. [36] in Table 2 for various values of $\tau$:

Table 1. Numerical values of $f'(0, \tau)$ for linear stretching with $We = S = \lambda = \eta = Rb = 0$ and $\tau = \pi/2$.

| $\beta$ | Turkyilmazoglu [33] | Hayat et al. [34] | Present results |
|---------|---------------------|------------------|-----------------|
| 0       | -1.000000           | -1.000000        | -1.000000       |
| 0.5     | -1.224744           | -1.224747        | -1.224747       |
| 1       | -1.414213           | -1.414217        | -1.414217       |
| 1.5     | -1.581138           | -1.581147        | -1.581147       |
| 2.0     | -1.732050           | -1.732057        | -1.732057       |

The detailed physical significance of each parameter is discussed in this section.

4. Discussion

After computing the desired solution, now we examine the rheological behavior of various fluid parameters on velocity profile $f$, temperature profile $\theta$, nanoparticles volume fraction $\varphi$ and motile microorganism profile $\chi$. It is remarked that while varying each flow parameter, the remaining parameters have assigned some constant values like $We = 0.5$, $\beta = 0.5$, $\lambda = 0.2$, $\eta = 0.1$, $Rb = 0.4$, $Rd = 0.2$, $Nt = 0.3$, $Nb = 0.3$, $Pr = 0.71$, $Sc = 0.4$, $Kr = 0.2$, $Pe = 0.5$, $Lb = 0.5$. The detailed physical significance of each parameter is discussed in this section.

4.1. Velocity profile

Since flow is time dependent, we first examine the three-dimensional (3D) illustration of velocity profile $f$, with $y$ and time $\tau$. For this purpose, Figure 2 is plotted for some fixed values of emerging parameters. The

Table 2. Comparison of $f'(0, \tau)$ with [35, 36] when $S = 1$, $\beta = 12$, $\lambda = 0$, $\eta = 0$ and $Rb = 0$.

| $\tau$  | Zheng et al. [35] | Abbas et al. [36] | Present results |
|---------|-------------------|------------------|-----------------|
| $\tau = 1.5\pi$ | 11.678656        | 11.678656        | 11.678656       |
| $\tau = 5.5\pi$ | 11.678706        | 11.678707        | 11.678706       |
| $\tau = 9.5\pi$ | 11.678656        | 11.678656        | 11.678656       |

Figure 1. $h -$ curves for (a) velocity profile, (b) temperature profile, (c) concentration profile and (d) motile micro-organisms profile.

Figure 2. Flow phenomenon of $\xi$ and $\tau$ versus $f$. 
velocity distribution oscillates periodically with time near the surface. Further, a phase shift in the distribution of velocity has also been captured far away from the accelerated surface. Figure 3 portrayed the 3D variation in velocity distribution when all the flow parameters have assigned some constant values. It is noted that velocity distribution gradually varied along \( y \)-direction without oscillation.

### 4.2. Temperature profile

In current analysis we have examined bioconvection of non-Newtonian nanofluid in presence of nonlinear thermal radiation. Now we determined the graphical analysis for temperature distribution \( \theta \). On this end, 3D visualization of temperature \( \theta \) with \( y \) and \( \tau \) is plotted in Figure 4. The temperature distribution varied linearly without any oscillation.

### 4.3. Concentration profile

The role of Williamson parameter \( We \), Schmidt number \( Sc \), mixed convection \( \lambda \), buoyancy ratio \( Nr \), bioconveccted Rayleigh number \( Rh \), thermophoresis \( Nt \) and Brownian motion \( Nb \) on nanoparticles volume fraction \( \varphi \) is visualize in Figures 4 and 5. Again, the graphical analysis has been performed with 3D visualization. It is noted that nanoparticles volume fraction \( \varphi \) linearly varied against \( y \). It is noted that nanoparticles volume fraction \( \varphi \) is not truncated with time efficiently.

### 4.4. Motile microorganism profile

Figure 6 concentrates the 3D illustration of microorganisms distribution \( \chi \) when all the parameters have assigned fixed numerical values. From this figure, we observed that microorganisms distribution \( \chi \) is varied linearly again.

### 4.5. Physical quantities

The variation in the local Nusselt number, local Sherwood number and motile density number for flow parameters are portrayed in Table 3. An increasing variation in these physical quantities is taken out for Prandtl number while decreasing behavior has been observed for Williamson parameter. Similarly, these quantities get lower values of mixed convection parameter and Rayleigh constant.

### 5. Conclusions

This study reports the thermophoresis and Brownian effects in flow of Williamson nanoparticles gyrotactic microorganisms. The flow has been assumed over an accelerated surface. As novelty, nonlinear thermal radiation effects are also utilized. First physical phenomenon is formulated by using boundary layer approximations. The series solution is acquired by using HAM. The solution with excellent accuracy has been obtained and results are compared with already reported continuations. A detailed graphical analysis has been performed by illustrating 3D visualization. It is found that velocity distribution accelerate periodically for flow parameters. The 3D simulations for temperature, concentration and microorganisms distribution does not contains any periodic oscillation. The detected observation can involve theoretical significance in
various engineering processes, bio-fuel cells, solar energy system and enhancement of extrusion systems.

**Declarations**

**Author contribution statement**

Kamel Al-Khaled: Conceived and designed the experiments; Analyzed and interpreted the data.

Sami Ullah Khan: Contributed reagents, materials, analysis tools or data; Wrote the paper.

Ilyas Khan: Performed the experiments; Analyzed and interpreted the data.

**Funding statement**

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

**Competing interest statement**

The authors declare no conflict of interest.

**Additional information**

No additional information is available for this paper.

**References**

[1] G.M. Krolicky, R.W. Maruda, J.B. Krolicky, S. Wojciechowski, M. Mia, P. Nieslony, G. Budzik, Ecological trends in machining as a key factor in sustainable production - a review, J. Clean. Prod. 218 (2019) 601–615. Published: MAY 1 2019.

[2] Mozammel Mia, Munish Kumar Gupta, Jose Adolfo Lozano, Diego Carou, Danil Yu, Pimenov, grzegorz Krolicky, aqib manhood khan, nikhil ranjan dhar, multi-objective optimization and life cycle assessment of eco-friendly cryogenic N-2 assisted turning of Ti-6Al-4V, J. Clean. Prod. 210 (2019) 121–133. Published: FEB 10 2019.

[3] G.M. Krolicky, P. Nieslony, R.W. Maruda, S. Wojciechowski, Dry cutting effect in a duplex stainless steel as a key factor in clean production, J. Clean. Prod. 142 (2017) 3343–3354. Part: 4. Published: JAN 20 2017.

[4] S. Nadem, S.T. Hussain, Changhoon Lee, Flow of a Williamson fluid over a stretching sheet, Brazillian J Chem Eng 30 (01) (2013) 619–625. July - September.

[5] T. Hayat, I. Ullah, A. Alsaeedi, B. Ahmad, Modeling tangent hyperbolic nanoliquid flow with heat and mass flux conditions, Eur. Phys. J. Plus 132 (2017) 112.

[6] Zakir Ullah, G. Zaman, Lie group analysis of magnetohydrodynamic tangent hyperbolic fluid flow towards a stretching sheet with slip conditions, Cite as: Heliyon 3 (2017), e00443.

[7] V. Nagendra, A. Leelarathnam, C.S.K. Raju, S.A. Shehzad, T. Hussain, Doubly stratified MHD tangent hyperbolic nanofluid flow due to permeable stretching cylinder, Results in Physics 9 (June 2018) 23–32.

[8] S. Geethan Kumar, S.V.K. Varma, R.V.M.S.S. Kiran Kumar, C.S.K. Raju, S.A. Shehzad, M.N. Bashir, Three-dimensional hydromagnetic convective flow of chemically reactive Williamson fluid with non-uniform heat absorption and generation, Int. J. Chemical reactor Eng (2019).

[9] Sami Ullah Khan, S.A. Shehzad, N. Ali, Interaction of magneto-nanoparticles in Williamson fluid flow over convective oscillatory moving surface, J. Braz. Soc. Mech. Sci. Eng. 40 (2018) 195.

[10] Khalil Ur Rehman, Ali Saleh Alshomrani, M.Y. Malik, Iffat Zehra, Muhammad Naseer, Thermo-physical aspects in tangent hyperbolic fluid flow regime: a short communication, Case Studies in Thermal Engineering 12 (September 2018) 203–212.

[11] T. Hayat, Asma Riaz, Anum Tanveer, Alsaeedi Ahmed, Peristaltic transport of tangent hyperbolic fluid with variable viscosity, Thermal Science and Engineering Progress 6 (June 2018) 217–225.

[12] S.U.S. Choi, Enhancing thermal conductivity of fluids with nanoparticles, Int. Mech. Eng. Cong. Exp., ASME, FED 231/MD 66 (1995) 99–105.

[13] M.M. Bhatti, M.M. Rashidi, Effects of thermo-diffusion and thermal radiation on Williamson nanofluid over a porous shrinking/stretching sheet, J. Mol. Liq. 212 (2019), 015214.

[14] P.C. Mahesh Kumar, C.M. Arun Kumar, Numerical evaluation of cooling performances of semiconductor using CuO/water nanofluids, Heliyon 5 (2019), e02227.

[15] M. Jia, M. Ayub, H. Khan, Entropy generation and activation energy mechanism in nonlinear radiative flow of Sko iso nanofluid: rotating disk, Heliyon 5 (2019), e01863.

[16] Sami Ullah Khan, Sabir Ali Shehzad, Brownian movement and thermophoretic aspects in third grade nanofluid over oscillatory moving sheet, Phys. Scr. 94 (9) (2019), 095202.

[17] B. Shen, C. Liangzhen, X. Zhang, Zhang Bioconvection heat transfer of a nanofluid over a stretching sheet with velocity slip and temperature jump, Therm. Sci. (6) (2015) 1–12.

[18] H. Xu, L. Pop, Mixed convection flow of a nanofluid over a stretching surface with uniform free stream in the presence of both nanoparticles and gyrotactic microorganisms, Int. J. Heat Mass Transf. 75 (2014) 610–623.

[19] W.A. Khan, O.D. Makinde, MHD nanofluid bioconvection due to 2014 gyrotactic microorganisms over a convectively heat stretching sheet, Int. J. Therm. Sci. 81 (2014) 118–124.

[20] S. Saini, Y.D. Sharma, Analysis of onset of bio-thermal convection in a fluid containing gravitactic microorganisms by the energy method, Chin. J. Phys. 56 (2018) 2031–2038.
T. Chakraborty, K. Das, P.K. Kundu, Framing the impact of external magnetic field on bioconvection of a nanofluid flow containing gyrotactic microorganisms with convective boundary conditions, Alexandria Eng J 57 (2018) 61–71.

W.A. Khan, A.M. Rashad, M.M.M. Abdou, I. Tlili, Natural bioconvection flow of a nanofluid containing gyrotactic microorganisms about a truncated cone, Eur. J. Mech. B Fluid (2019).

Rakesh Kumar, Sheipa Sood, Sabir Ali Shehzad, Mohsen Sheikholeslami, Numerical modeling of time-dependent bio-convective stagnation flow of a nanofluid in slip regime, Results in Physics 7 (2017) 3325–3332.

Fazle Mabood, Waqar A. Khan, I. Ahmad, Md. Ismail, Analytical investigation for free convective flow of non-Newtonian nanofluids flow in porous media with gyrotactic microorganisms, J. Porous Media 18 (7) (2015) 653–663.

W.A. Khan, O.D. Makinde, Z.H. Khan, MHD boundary layer flow of a nanofluid containing gyrotactic microorganisms past a vertical plate with Navier slip, Int. J. Heat Mass Transf. 74 (July 2014) 285–291.

Waqar A. Khan, Mohammed Jashim Uddin, Ahmad I. Md. Ismail, Bioconvective non-Newtonian nanofluid transport over a vertical plate in a porous medium containing microorganisms in a moving free stream, J. Porous Media 18 (4) (2015) 2015.

S.J. Liao, Advance in the Homotopy Analysis Method. 5 Toh Tuck Link, World Scientific Publishing, Singapore, 2014.

M. Turkyilmazoglu, Analytic approximate solutions of rotating disk boundary layer flow subject to a uniform suction or injection, Int. J. Mech. Sci. 52 (2010) 1735–1744.

M. Turkyilmazoglu, Determination of the correct range of physical parameters in the approximate analytical solutions of nonlinear equations using the Adomian decomposition method Mediterr. J. Mat. 13 (2016) 4019–4027.

M. Turkyilmazoglu, Some issues on HPM and HAM methods: a convergence scheme, Math. Comput. Model. 53 (2011) 1929–1936.

Saeed Dinavand, Abbas Abbasi, Reza Hoseini, Ioan Pop, Homotopy analysis method for mixed convective boundary layer flow of a nanofluid over a vertical circular cylinder, Therm. Sci.: Yearbook 19 (2) (2015) 549–561.

M. Turkyilmazoglu, The analytical solution of mixed convection heat transfer and fluid flow of a MHD viscoelastic fluid over a permeable stretching surface, Int. J. Mech. Sci. 77 (2013) 263–268.

T. Hayat, M. Mustafa, I. Pop, Heat and mass transfer for Soret and Dufour’s effect on mixed convection boundary layer flow over a stretching vertical surface in a porous medium filled with a viscoelastic fluid, Commun. Nonlinear Sci. Numer. Simul. 15 (2010) 1183–1196.

Z. Abbas, Y. Wang, T. Hayat, M. Oberlack, Hydromagnetic flow in a viscoelastic fluid due to the oscillatory stretching surface, Int. J. Non-Linear Mech. 43 (2008) 783–797.

L.C. Zheng, X. Jin, X.X. Zhang, J.H. Zhang, Unsteady heat and mass transfer in MHD flow over an oscillatory stretching surface with Soret and Dufour effects, Acta Mech. Sin. 29 (5) (2013) 667–675.