The self field effect on the power-law index of superconducting cables

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It is shown that in the absence of inter-strand current redistribution the self-field effect is to always increase the power-law index of the volt-ampere characteristic and to decrease the temperature and magnetic field derivatives of the critical current line. We show that the take-off limit of a strand in a cable made of insulated strands is equal to that of a free strand due to a compensation effect between the increase of the power-law index and the decrease of the magnetic field derivative of the critical current.

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I. THE POWER-LAW INDEX OF A STRAND IN A CABLE

Contrary to the statement, found usually in the text books on superconductivity, that a superconductor below its critical temperature $T_c$ conducts electricity at zero resistance, technical superconductors show a voltage drop (albeit very small) even at temperatures $T < T_c$. The volt-ampere characteristic (VAC) of this ideal superconductor (dashed-line in Fig.1) would have a first range of zero resistance up to a knee point beyond which a voltage develops, linearly with the current. The knee point is identified as the critical current $I_c$. On the contrary, the VAC of a real superconductor is as shown in Fig.1 by the continuous line. The absence of the knee in the VAC in the real case request another definition of the critical current $I_c$. The most popular definition (at least in Europe and the USA) fits the smooth transition of the VAC with a power law with power exponent $n$.

\[ E(I) = E_c \left( \frac{I}{I_c} \right)^n \]

\[ I_c = I_c(B,T,\varepsilon) \] (1)

with $I_c$ -the critical current as given by the scaling law (Sommers [2] in case of Nb$_3$Sn, Bottura [3] for NbTi) and $E_c$ a conveniently chosen voltage criterion, usually $0.1 \mu V/cm$. In Eq.(1) $B$ is the magnetic field, $T$ the temperature and $\varepsilon$ the strain (only for Nb$_3$Sn). The power law index is $n$.

An alternative was proposed in [4] and was suggested by the observed exponential increase of electrical field due to dominating thermally-activated creep over the flux flow

\[ E = J \rho_n \exp \left( \frac{T - T_c}{T_0} + \frac{B}{B_0} + \frac{J}{J_0} \right) \] (2)

where: $J$ is the current density, $T_c$ the critical temperature, $\rho_n$ the normal resistivity and $T_0, B_0, J_0$ are so called grow or increasing fit parameters. They account for the change in VAC as a function of temperature, field and current. The relation and equivalence issues between the exponential form and the power-law VAC were investigated in [5]. Although the next calculations could be performed with the exponential form as well, we will chose to continue our analysis with the power-law functional dependence of VAC.

It is well known, although less used, that the power-law index $n$ is the logarithmic derivative of the VAC. Indeed, if we take the logarithmic derivative of Eq.1 with respect to the current, assuming that $I_c$ is a constant i.e. depends only on temperature and field, we get

\[ \frac{d \log E}{d \log I} = \frac{I}{E \frac{dE}{dI}} = n \] (3)

if the VAC is described by the power-law over the whole range of possible currents. Unfortunately, in most of the cases, the description of the VAC by a power-law is restricted to a limited range, usually $E \in [0.1, 1] \mu V/cm$.
and in this case we speak of an average index in the given range. One can also define a local \( n \) at \( I = I_c \). The corresponding definitions are:

\[
\bar{n} = \frac{I_c}{E_c} \frac{dE}{dT} \bigg|_{E \in [0,1]} \quad \text{or} \quad n_c = \frac{I_c}{E_c} \frac{dE}{dT} \bigg|_{I=I_c} \tag{4}
\]

Now consider the case of a cable exposed to a constant external magnetic field \( B \) in which a total current \( I_t = N_s I \) flows where \( N_s \) is the number of strands in the cable and \( I \) the strand current assumed the same in all strands. The current in the cable generates a magnetic field, the self-field, which adds geometrically to the background magnetic field \( B \). The resulting magnetic field in the cable cross-section becomes non-uniform. In the presence of this self-field, the critical current of a strand in the cable \( I_c \) becomes itself a function of the current in the strand. The dependence is given by

\[
I_c = I_c (B (I), T)
\]

\[
B (I, \varphi) = \sqrt{B^2 + B_s (I) \sin (\varphi)^2 + B_s (I) \cos (\varphi)^2}
\]

\[
B_s (I) = \alpha N_s I
\tag{5}
\]

where \( \alpha \) is the geometrical self field constant depending on the cable radius and the azimuth angle \( \varphi \). For a single round cable i.e. when the return cable is at \( r = \infty \), \( \alpha \) is simply a geometrical constant given by

\[
\alpha = \frac{\mu_0}{2\pi D_c}
\tag{6}
\]

where \( D_c \) is the cable diameter.

Let us consider now for simplicity that only the maximum field counts (peak-field hypothesis). The maximum field corresponds to \( \varphi = \pi / 2 \) and from Eq. \(5\) it is simply

\[
B (I) = B_b + B_s (I) = B_b + \alpha N_s I
\tag{7}
\]

a simple linear form which will be used for convenience throughout this work.

The critical current of one strand in the cable \( I_{c,cable} \) is given by the root \( X \) of the equation

\[
X = I_c (B + \alpha N_s X, T)
\tag{8}
\]

where \( I_c (B, T) \) is the known functional dependence on field and temperature for the superconductor (has different forms for Nb3Sn and NbTi).

Now in order to calculate the index \( n \) of a cable we use Eq. \( 4 \) with this new input. We have for the electric field of a strand in the cable, which is also the electric field of the whole cable if no current transfer takes place

\[
E_{\text{cable}} = E_{\text{strands}} = E_c \left( \frac{I}{I_c (B_b + \alpha N_s I)} \right)^n
\tag{9}
\]

where for convenience we do not write explicitly the temperature dependence in \( I_c \). The cable index from Eq.\( 4 \) is

\[
n_{\text{cable}} = \frac{I}{E_{\text{cable}}} \frac{dE_{\text{cable}}}{dI} = \frac{I}{E_c} n E_c \left( \frac{I}{I_c} \right)^{n-1} \left[ 1 - \frac{I}{I_c^2} \left( \frac{dI_c}{dT} \right) b \right] = \frac{I}{E_c} n E_c \left( \frac{I}{I_c} \right)^{n-1} \left[ 1 - \frac{I}{I_c^2} \left( \frac{dI_c}{dB} \right) \frac{dB}{dT} \right] = \frac{I}{E_c} n \left[ 1 - \frac{I}{I_c} \left( \frac{dI_c}{dB} \right) \alpha N_s \right] \tag{10}
\]

At \( I = I_c \) and taking into account that \( \frac{dI_c}{dB} < 0 \) we get the simple and nice result

\[
n_{\text{cable}} = n \left[ 1 + \alpha N_s \left( \frac{dI_c}{dB} \right) \right] > n \tag{11}
\]

i.e. in the absence of current redistribution the power-law index in a cable is larger than the power-law index of the isolated strands. The enhancement is proportional to the slope of the critical current as a function of magnetic field and proportional to the number of strands in the cable.

\section{II. TEMPERATURE AND MAGNETIC FIELD SLOPES OF CRITICAL CURRENT OF THE CABLE}

The critical current of a strand in a cable is different from the critical current of a free similar strand. According to Eq. \(8\) the critical current of a strand in cable is implicitly defined

\[
I_c = \Omega (B_b + \alpha N_s I_c, T)
\tag{12}
\]

where for simplicity we denote by \( I_c \) the critical current of a strand in cable \( I_c \equiv I_{c,cable} \). In order to avoid confusion we denote by \( \Omega = \Omega (B, T) \equiv \Omega_{c,\text{strand}} \) the scaling relation for the free strand \(8\). Although in implicit form, using the above equation one can get a relation between the temperature slopes of the critical current of a strand in a cable and of a free strand. Remembering that \( I_c \) is a function of field and temperature we have from Eq. \(12\) that

\[
\frac{\partial I_c}{\partial T} = \frac{\partial \Omega}{\partial B} \alpha N_s \frac{\partial I_c}{\partial T} + \frac{\partial \Omega}{\partial T}
\tag{13}
\]

After rearranging the terms and restoring the initial notation \( \Omega = \Omega_{c,\text{strand}} \) we get finally

\[
\frac{\partial I_c}{\partial T} = \frac{\partial \Omega}{\partial B} \alpha N_s \frac{\partial I_c}{\partial T}
\]

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\]
\[
\frac{\partial I_c}{\partial T}_{\text{cable}} = \frac{\partial I_c}{\partial T}_{\text{strand}} \frac{1 - \alpha N_s}{1 - \alpha N_s} = \frac{\partial I_c}{\partial T}_{\text{strand}} \left(1 + \alpha N_s \frac{\partial I_c}{\partial B}_{\text{strand}}\right)
\]

(14)

We will show later by direct calculation of the derivatives that \( \frac{\partial I_c}{\partial B}_{\text{strand}} \) is always negative and therefore the temperature slope of the critical current in the cable is always reduced due to the self field effect.

As shown in [3] the stability limit of a conductor, expressed by the maximum sustainable electric field before a take-off, depends on the first derivative of the critical current with respect to temperature. At the first sight a reduction of the temperature slope should have consequences on the stability limit i.e. it should increase the stability limit. We will show now that this is not the case due to an interesting compensation effect. Indeed, the stability limit in the simplest form i.e. for constant power-law index is [6]

\[
E_q = \frac{\hbar}{n_{\text{cable}}} \left| \frac{\partial I_c}{\partial T}_{\text{cable}} \right|^{-1}
\]

(15)

and we arrive, using Eqs. (14) and (11), at the important relation

\[
E_{q,\text{cable}} = E_{q,\text{strand}}
\]

(16)

i.e. the stability limit is not affected by the self-field effect due to the compensation effect between the increase in the power-law index and decrease in the field slope as stated before.

The field slope is calculated similarly. Differentiating Eq. (12) with respect to \( B \) we get

\[
\frac{\partial I_c}{\partial B} = \frac{\partial \Omega}{\partial B} \left(1 + \alpha N_s \frac{\partial I_c}{\partial B}\right)
\]

(17)

which after some algebra manipulations becomes

\[
\frac{\partial I_c}{\partial B}_{\text{cable}} = \frac{\partial I_c}{\partial B}_{\text{strand}} \frac{1 - \alpha N_s}{1 + \alpha N_s} \frac{\partial I_c}{\partial B}_{\text{strand}}
\]

(18)

a relation similar to that of Eq. (14).

III. COMPARISON BETWEEN Nb3Sn AND NbTi

It is instructive to compare the enhancement of the power-law index for the two typical low temperature superconductors Nb3Sn and NbTi. For this purpose, the temperature and field derivatives of the critical current for the two materials have been calculated for a 0.8mm strand with a copper-non copper ratio of 1.5.

For the critical line of Nb3Sn strand we used the Sommers [2] scaling with a typical, all-purpose set of parameters. Two calculations were done, one at a fixed temperature of 4.5K and a variable magnetic in the range 4-14T and one at a fixed field of 12T and variable temperature in the range 4-12K.

For the NbTi strand, the Bottura [3] scaling was used and the parameterization was in this case a fixed temperature of 4.5K with a magnetic field in the range 4 to 10T and a fixed field of 6T and a variable temperature in the range 4-10K.

The difference in the field ranges for the two superconductors is understandable. One cannot compare Nb3Sn and NbTi at the same field because the shared part of the field ranges of the two superconductors is at fields which are too low for Nb3Sn and too high for NbTi. The "standard" operating fields are: 12T for Nb3Sn and 6T for NbTi.

The derivatives, as calculated from the corresponding scaling relations, are shown in Fig. 2. Please note that in the relation above the modulus of the derivatives appear. It can be seen that the NbTi derivatives are larger than the corresponding derivatives for Nb3Sn which makes NbTi more sensitive to the self field effect. The biggest difference is between the temperature derivatives, around

\[\text{FIG. 2: (Color online) Temperature and magnetic field derivatives of critical current for Nb3Sn and NbTi. In red, } (\partial I_c/\partial T)_B \text{ and in blue, } (\partial I_c/\partial B)_T.\]
110 A/K for NbTi and only 20 A/K for Nb_3Sn. The difference between the field derivatives is also large, 80 A/T for NbTi and 20 A/T for Nb_3Sn. Also noticeable is the difference between the temperature and field derivatives of NbTi: -110 A/K and -80 A/T at 4.5 K and 6 T.

An important point is that the slope lines never cross the zero line i.e. the slopes are always negative and therefore the self-field effect will always increase the power-law index.

IV. CONCLUSIONS

Considering a cable-in-conduit conductor as a collection of insulated strands, relations between the power-law index and the temperature and magnetic field slopes of the critical line of the cable and of the free strands were deduced. The main result is that the power-law index of a strand in the cable is moderately increased with respect to the power-law index of a free strand. The increase is by a factor related to the magnetic field derivative (slope) of the strand critical current, the self field constant and the total number of strands. The temperature and magnetic field derivatives of the critical line in a cabled strand are decreased by the same factor. We have shown also that due to the compensation effect between the increase in the power-law index and the decrease (by the same factor) of the temperature slope of the critical current line, the stability limit of the conductor is the same as that of a free strand.

Measurements on cable-in-conduit conductors show frequently an important reduction of the power-law index as compared to the strand value instead of an increase as predicted here. In view of the facts revealed here it is then clear that the self-field cannot be the cause for this decrease. The self-field effect on the power-law index is moderate and therefore it can be easily removed (covered) by other mechanisms. Mechanisms that act in the opposite direction i.e. to reduce the power-law index are: the current redistribution (both NbTi and Nb_3Sn) and/or conductor degradation by bending and transversal stress at the interstrand contact points (only Nb_3Sn).

In this work we did not take into account the possible dependence of the power-law index on the critical current. In [5] some consequences of this dependence for the stability are investigated. However, the paradigm is that this dependence is a result of current redistribution among the strands an effect which is beyond our goal in the present paper and is probably too complicated to be solved in the frame of an analytical model.

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[7] One must always distinguish carefully between the critical current as a numerical value and as a function. I_c(B, T, ε) is the functional dependence while I_c is a value obtained from I_c(B, T, ε) for given values of B, T and ε.
[8] One could write I_c,strand = I_c(B, T) and I_c,cable = I_c(B_b + αN_sI_c,cable, T) but this would be a little bit confusing.