The photo-electric effect in the bi-layer graphite

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August 8, 2008

Abstract

We derive the differential probability of the photoelectric effect realized at the very low temperature of double graphene in the very strong magnetic field. The relation of this effect to the elementary particle physics, nuclear physics and Einstein gravity is mentioned. Our approach is the analogue of the Landau discovery of the diamagnetism, where Landau supposed the parabolic dispersion relations for the model of diamagnetism.

Key words: Mono-layer graphite, bi-layer graphite, Schrödinger equation, photons, photoeffect.

1 Introduction

The photoelectric effect is a quantum electronic phenomenon in which electrons are emitted from matter after the absorption of energy from electromagnetic radiation. Frequency of radiation must be above a threshold frequency, which is specific to the type of surface and material. No electrons are emitted for radiation with a frequency below that of the threshold. These emitted electrons are also known as photoelectrons in this context.
The photoelectric effect was theoretically explained by Einstein who introduced the light quanta. Einstein writes [1]: \textit{In accordance with the assumption to be considered here, the energy of light ray spreading out from point source is not continuously distributed over an increasing space but consists of a finite number of energy quanta which are localized at points in space, which move without dividing, and which can only be produced and absorbed as complete units.}

The effect is also termed as the Hertz effect due to its discovery by Heinrich Rudolf Hertz in 1887.

It is known some prehistory of the photoelectric effect beginning by 1839 when Alexandre Edmond Becquerel observed the photoelectric effect via an current when an electrode was exposed to light. Later in 1873, Willoughby Smith found that selenium is photoconductive.

The linear dependence on the frequency was experimentally determined in 1915 when Robert Andrews Millikan showed that Einstein formula

\[ h\omega = \frac{mv^2}{2} + A \]

was correct. Here \( h\omega \) is the energy of the impinging photon and \( A \) is work function of concrete material. The work function for Aluminium is 4.3 eV, for Beryllium 5.0 eV, for Lead 4.3 eV, for Iron 4.5 eV, and so on [2]. The work function concerns the surface photoelectric effect where the photon is absorbed by an electron in a band. The theoretical determination of the work function is the problem of the solid state physics. On the other hand, there is the so called atomic photoeffect [3], where the ionization energy plays the role of the work function. The system of the ionization energies is involved in the tables of the solid state physics. The work function of graphene, or, work function of the Wigner crystal in graphene was never determined, and it is the one of the prestige problem of the contemporary experimental and theoretical graphene physics and the Wigner crystal physics.

The formula (1) is the law of conservation of energy. The classical analogue of the equation (1) is the motion of the Robins ballistic pendulum in the resistive medium.

The Einstein ballistic principle is not valid inside of the blackbody. The Brownian motion of electrons in this cavity is caused by the repeating Compton process \( \gamma + e \rightarrow \gamma + e \) and not by the ballistic collisions. The diffusion constant for electrons must be calculated from the Compton process and not from the Ballistic process. The same is valid for electrons immersed into the cosmic relic photon sea.

The idea of the existence of the Compton effect is also involved in the Einstein article. He writes [1]: \textit{The possibility should not be excluded, however, that electrons might receive their energy only in part from the light quantum.} However, Einstein was not sure, a priori, that his idea of such process is realistic. Only Compton proved the reality of the Einstein statement.

Eq. (1) represents so called one-photon photoelectric effect, which is valid for very weak electromagnetic waves. At present time of the laser physics, where the strong electromagnetic intensity is possible, we know that so called multiphoton photoelectric effect is possible. Then, instead of equation (1) we can write

\[ h\omega_1 + h\omega_2 + ... + h\omega_n = \frac{mv^2}{2} + A. \]
The time lag between the incidence of radiation and the emission of a photoelectron is very small, less than $10^{-9}$ seconds.

As a analogue of the equation (2), the multiphoton Compton effect is also possible:

$$\gamma_1 + \gamma_2 + ... \gamma_n + e \rightarrow \gamma + e,$$

and two-electron, three-electron,... n-electron photoelectric effect is also possible [3]. To our knowledge the Compton process with the entangled photons was still not discovered and elaborated. On the other hand, there is the deep inelastic Compton effect in the high energy particle physics.

Einstein in his paper [1] introduced the term "light quanta" called "photons" by chemist G. N. Lewis, in 1926. Later Compton, in his famous experiment proved that light quanta have particle properties, or, photons are elementary particles.

At present time the attention of physicists is concentrated to the planar physics at zero temperature and in the strong magnetic field. Namely, in the graphene physics which is probably new revolution in this century physics with assumption that graphene is the silicon of this century.

In 2004, Andre Geim, Kostia Novoselov [4] and co-workers at the University of Manchester in the UK delicately cleaving a sample of graphite with sticky tape produced something that was long considered impossible: a sheet of crystalline carbon just one atom thick, known as graphene. Geims group was able to isolate graphene, and was able to visualize the new crystal using a simple optical microscope. Graphene is the benzene ring ($C_6H_6$) stripped out from their H-atoms. It is allotrope of carbon because carbon can be in the crystalline form of graphite, diamant, fullerene ($C_{60}$) and carbon nanotube.

Graphene unique properties arise from the collective behaviour of electrons. The electrons in graphene are governed by the Dirac equation. The behavior of electrons in graphene was first predicted in 1947 by the Canadian theorist Philip Russell Wallace.

The Dirac fermions in graphene carry one unit of electric charge and so can be manipulated using electromagnetic fields. Strong interactions between the electrons and the honeycomb lattice of carbon atoms mean that the dispersion relation is linear and given by $E = vp$, $v$ is called the Fermi-Dirac velocity, $p$ is momentum of a quasielectron. The energy quantization of the electron in the bi-layer graphene in magnetic field is $E_n \sim \sqrt{n(n - 1)}$ and the dispersion relation is quadratic. The Dirac equation in graphene physics is used for so called quasispin generated by the honeycomb lattice. The parabolic dispersion relation valid for 2-layer graphite means it is possible to use the Schrödinger equation for the calculation of photoeffect in 2-layer graphite.

Our calculation of the photoelectric effect is applicable not only for the 2-layer graphite but also for the Wigner crystals. A Wigner crystal is the crystalline phase of electrons first predicted by Eugene Wigner in 1934 [5], who was probably motivated by the electron quantum states forming the Landau diamagnetism. In other words this is a gas of electrons moving in 2-dimensional or 3-dimensional neutralizing background as a lattice if the electron density is less than a critical value potential energy $E_p$ dominating the kinetic energy $E_k$ at low densities. Or, $E_p > E_k$. To minimize the potential energy, the electrons form a triangular lattice in 2D and body-centered cubic lattice in 3D. A crystalline state of the 2D electron gas can also be realized by applying a sufficiently strong magnetic field. And this is a case of monolayer graphite (graphene) in the magnetic field. It is evident that the 2-layer graphite involves the Wigner crystal too [6].
We derive the differential probability of the photoelectric effect realized at the very low temperature graphene in the very strong magnetic field. The relation of this effect to the elementary particle physics of LHC, nuclear physics and Einstein gravity is mentioned.

## 2 The quantum theory of the photoelectric effect in the 2D films at zero temperature and strong magnetic field

Electrons in graphene respect the linear spectrum and they are described by the Dirac equation. The electron dispersion relations in the 2-layer graphite was proved to be parabolic and we can calculate the photoelectric effect using the Schrödinger equation. In general words the photoelectric effect on graphene can give us information on the state of electrons in general in N-layer graphite.

The photoelectric effect in graphene is presented here 100 years after well know Einstein article [1]. While the experimental investigation of the photoelectrical effect was performed in past many times, the photoelectric effect in graphene is still the missing experiment in the graphene physics, the electronics of the future. Nevertheless the photoeffect is the brilliant method for investigating the graphene and the Wigner crystals.

The quantum mechanical description of the photoeffect is realized as the nonrelativistic, or relativistic and it is described in many textbooks on quantum mechanics. Let us apply the known nonrelativistic elementary quantum theory of the photoeffect to the 2D structures with the 2-layer graphite.

The main idea of the quantum mechanical description of the photoeffect is that it must be described by the appropriate S-matrix element involving the interaction of atom with the impinging photon with the simultaneous generation of the electron, the motion of which can be described approximately by the plane wave

$$\psi_q = \frac{1}{\sqrt{V}} e^{i q \cdot x}, \quad q = \frac{p}{\hbar},$$  

(4)

where $p$ is the momentum of the ejected electron. We suppose that magnetic field is applied locally to the carbon film, so, in a sufficient distance from it the wave function is of the form of the plane wave (4). This situation has an analog in the classical atomic effect discussed in monograph [7]. However, if the photon energy only just exceeds the ionization energy $I$ of atom, then we cannot used the plane wave approximation but the wave function of the continuous spectrum.

The probability of the emission of electron by the electromagnetic wave is of the well-known form [7]:

$$dP = \frac{e^2 p}{8\pi^2 \varepsilon_0 \hbar m \omega} \left| \int e^{i(k-q)\cdot x} (e \cdot \nabla) \psi_0 dx dy dz \right|^2 d\Omega = C |J|^2 d\Omega,$$  

(5)

where the interaction for absorption of the electromagnetic wave is normalized to *one photon in the unit volume*, $e$ is the polarization of the impinging photon, $\varepsilon_0$ is the dielectric constant of vacuum, $\psi_0$ is the basic state of and atom. We have denoted the integral in $||$ by $J$ and the constant before $||$ by $C$. 
In case of the 2D low temperature system in the strong magnetic field the basic function is so called Laughlin function [8], which is of the very sophisticated Jastrow form. We consider the case with electrons in magnetic field as an analog of the Landau diamagnetism. So, we take the basic function $\psi_0$ for one electron in the lowest Landau level, as

$$\psi_0 = \left(\frac{m\omega_c}{2\pi \hbar}\right)^{1/2} \exp\left(-\frac{m\omega_c}{4\hbar}(x^2 + y^2)\right),$$  \hspace{1cm} (6)

which is solution of the Schrödinger equation in the magnetic field with potentials $A = (-Hy/2, -Hx/2, 0)$, $A_0 = 0$ [9]:

$$\left[\frac{p_x^2}{2m} + \frac{p_y^2}{2m} - \frac{m}{2} \left(\frac{\omega_c}{2}\right)^2 (x^2 + y^2)\right] \psi = E\psi.$$  \hspace{1cm} (7)

We have supposed that the motion in the $z$-direction is zero and it means that the wave function $\exp\left[\frac{i}{\hbar}(\hat{p}_z z)\right] = 1$.

So, The main problem is to calculate the integral

$$J = \int e^{i(K \cdot x)}(e \cdot \nabla)\psi_0 dxdydz; \quad K = k - q.$$  \hspace{1cm} (8)

with the basic Landau function $\psi_0$ given by the equation (6).

Operator $(\hbar/i)\nabla$ is Hermitean and it means we can rewrite the last integrals as follows:

$$J = \frac{i}{\hbar} e \cdot K \int e^{-i(K \cdot x)}\psi_0 dxdydz,$$  \hspace{1cm} (9)

which gives

$$J = ie \cdot K \int e^{-i(K \cdot x)}\psi_0 dxdydz,$$  \hspace{1cm} (10)

The integral in eq. (10) can be transformed using the cylindrical coordinates with

$$dxdydz = d\theta d\varphi dz, \quad \varrho^2 = x^2 + y^2$$  \hspace{1cm} (11)

which gives for vector $K$ fixed on the axis $z$ with $K \cdot x = Kz$ and with physical condition $e \cdot k = 0$, expressing the physical situation where polarization is perpendicular to the direction of the wave propagation. So,

$$J = (i)(e \cdot q) \int_0^{\infty} d\varrho \int_{-\infty}^{\infty} dz \int_0^{2\pi} d\varphi e^{-iKz}\psi_0.$$  \hspace{1cm} (12)

Using

$$\psi_0 = A \exp\left(-B\varrho^2\right); \quad A = \left(\frac{m\omega_c}{2\pi \hbar}\right)^{1/2}; \quad B = \frac{m\omega_c}{4\hbar}.$$  \hspace{1cm} (13)

The integral (12) is then

$$J = (-\pi i)\frac{A}{B}(e \cdot q) \int_{-\infty}^{\infty} e^{-iKz}dz = (-\pi i)\frac{A}{B}(e \cdot q)(2\pi)\delta(K).$$  \hspace{1cm} (14)

Then,
\[ dP = C|J|^2d\Omega = 4\pi^4 \frac{A^2}{B^2} C(e \cdot q)^2 \delta^2(K)d\Omega. \]  \hspace{1cm} (15)

Now, let be the angle \( \Theta \) between direction \( \mathbf{k} \) and direction \( \mathbf{q} \), and let be the angle \( \Phi \) between planes \( (\mathbf{k}, \mathbf{q}) \) and \( (\mathbf{e}, \mathbf{k}) \). Then,

\[ (e \cdot q)^2 = q^2 \sin^2 \Theta \cos^2 \Phi. \]  \hspace{1cm} (16)

So, the differential probability of the emission of photons from the 2-layer graphite (double graphene) in the strong magnetic field is as follows:

\[ dP = \frac{4e^2p}{\pi \varepsilon_0 m^2 \omega_c} \left[ q^2 \cos^2 \Theta \sin^2 \Phi \right] \delta^2(K)d\Omega; \quad \omega_c = \frac{|e| H}{mc}. \]  \hspace{1cm} (17)

We can see that our result differs from the result for the original photoelectric effect [7] which involves still the term

\[ \frac{1}{(1 - \frac{\mathbf{v}}{c} \cos \Theta)^4}, \]  \hspace{1cm} (18)

which means that the most intensity of the classical photoeffect is in the direction of the electric vector of the electromagnetic wave \( (\Phi = \pi/2, \Theta = 0) \). While the nonrelativistic solution of the photoeffect in case of the Coulomb potential was performed by Stobbe [10] and the relativistic calculation by Sauter [11], the general magnetic photoeffect (with electrons moving in the magnetic field and forming atom) was not still performed in a such simple form. The delta term \( \delta \cdot \delta \) represents the conservation law \( |\mathbf{k} - \mathbf{q}| = 0 \) in our approximation. While the product \( \delta \cdot \delta \) forms no problem in the quantum field theory, its mathematical meaning is not well defined in the so called theory of the generalized functions.

### 3 Discussion

The article is in a some sense the preamble to the any conferences of ideas related to the photoeffect on graphene, bi-layer graphite, n-layer graphite and on the Wigner crystals which are spontaneously formed in graphite structures, or, in other structures. We have considered the photoeffect in the planar crystal at zero temperature and in the very strong magnetic field. We calculated only the process which can be approximated by the Schrödinger equation for an electron orbiting in magnetic field.

At present time, the most attention in graphene physics is devoted to the conductivity of a graphene with the goal to invent new MOSFETs and new transistors for new computers. However, we do not know, a priory, how many discoveries are involved in the investigation of he photo-electric effects in graphene.

We did not consider the relativistic description in the constant magnetic field. Such description can be realized using the so called Volkov solution of the Dirac equation in the magnetic field instead of the plane wave. The explicit form of such solution was used by Ritus [12], Nikishov [13] and others, and by author [14], [15], [16], in the different situation. For instance, for the description of the electron in the laser field, synchrotron radiation, or, in case of the massive photons leading to the Riccati equation [15].
The Volkov solution for an electron moving in the constant magnetic field is the solution of the Dirac equation with the following four potential

\[ A_\mu = a_\mu \varphi; \quad \varphi = kx; \quad k^2 = 0. \]  \hspace{1cm} (19)

It follows from equation (19) that \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = -a_\mu k_\nu + a_\nu k_\mu = \text{const.} \), which means that electron moves in the constant electromagnetic field with the components \( \mathbf{E} \) and \( \mathbf{H} \). The parameters \( a \) and \( k \) can be chosen in such a way that \( \mathbf{E} = 0 \). So, the motion of electron is performed in the constant magnetic field.

The Volkov solution [17] of the Dirac equation for an electron moving in a field of a plane wave was derived in the form [14],[15],[18]:

\[ \psi_p = \frac{u(p)}{\sqrt{2p_0}} \left[ 1 + e^{\frac{(\gamma k)(\gamma A(\varphi))}{2kp}} \right] \exp \left[ (i/\hbar)S \right] \]  \hspace{1cm} (20)

and \( S \) is an classical action of an electron moving in the potential \( A(\varphi) \) [19].

\[ S = -px - \int_0^{kx} \frac{e}{(kp)} \left[ (pA) - \frac{e}{2}(A)^2 \right] d\varphi. \]  \hspace{1cm} (21)

It was shown that for the potential (19) the Volkov wave function is [19]:

\[ \psi_p = \frac{u(p)}{\sqrt{2p_0}} \left[ 1 + e^{\frac{(\gamma k)(\gamma a)}{2kp}} \varphi \right] \exp \left[ (i/\hbar)S \right] \]  \hspace{1cm} (22)

with

\[ S = -e\frac{ap}{2kp} \varphi^2 + e^2 \frac{a^2}{6kp} \varphi^3 - px. \]  \hspace{1cm} (23)

The basic function is also relativistic and it was derived in the form [20].

\[ \Psi(x, t) = \frac{1}{L} \exp \left\{-\frac{i}{\hbar} \epsilon Et + ik_2y + ik_3z \right\} \psi = \begin{pmatrix} C_1u_{n-1}(\eta) \\ iC_2u_n(\eta) \\ C_3u_{n-1}(\eta) \\ iC_4u_n(\eta) \end{pmatrix}, \hspace{1cm} (24)\]

where \( \epsilon = \pm 1 \) and the spinor components \( u \) and the coefficients \( C_i \) are defined in the Sokolov et al. monograph [20].

\[ u_n(\eta) = \sqrt{\frac{\sqrt{2\gamma}}{2\eta n!}} \sqrt{\pi} e^{-\eta^2/2} H_n(\eta) \]  \hspace{1cm} (25)

with

\[ H_n(\eta) = (-1)^n e^{\eta^2} \left( \frac{d}{d\eta} \right)^n e^{-\eta^2}, \]  \hspace{1cm} (26)

\[ \eta = \sqrt{2\gamma} x + k_2/\sqrt{2\gamma}; \quad \gamma = eH/2\hbar. \]  \hspace{1cm} (27)

The coefficients \( C_i \) are defined in the Sokolov et al. monograph [20]. So, our elementary approach can be generalized.
We have here considered the situation which is an analog of the ionization process in atom. Namely:

\[ \hbar \omega + (e \text{ in atom}) \rightarrow (\text{atom minus } e) + e. \]  

(28)

In other words, we have calculated the process with the following equation:

\[ \hbar \omega + (e \text{ in } G) \rightarrow (G \text{ minus } e) + e, \]  

(29)

where \( G \) is the double graphene and the electron in the crystal is considered as the elementary particle with all attributes of electron as an elementary particle and not the quasielectron which can be also considered in the solid state physics. Quasielectron is the product of the crystalline medium and as such it can move only in the medium. The existence of the quasielectron in vacuum is not possible. However, the process where the photon interacts with the quasielectron and after some time the quasielectron decays in such a way that the integral part of the decay product is also electron, is possible and till present time the theory of such process was not elaborated.

The new experiments are necessary in order to verify the photoelectric equation in graphene. At the same time it will be the dilemma if the process is ballistic, or not. The ballistic interaction of photon with electron is not possible in vacuum for point-like electrons. The experiments in CERN, Hamburg, Orsay, (The L3 collaboration, CELLO collaboration, ALEPH collaboration, ...) and other laboratories confirmed that there are no excited states of electron in vacuum. In other words, the ballistic process in vacuum with electrons was not confirmed. It means that electron in the standard model of elementary particles is a point particle. It seems that the ballistic process is possible only in a medium. We know, that electron accelerated in PASER [21] by the ballistic method is accelerated for instance in CO\(_2\). On the other hand, the photo-desintegration of nuclei involves the ballistic process [22]. The interaction of light with carbon \( C_{60}, C_{70}, C_{80} \ldots \) involves also the ballistic process with photons [23, 24].

The photoelectric effect at zero temperature can be realized only by very short laser pulses, because in case of the continual laser irradiation the zero temperature state is not stable. Only very short pulses can conserve the zero temperature 2D system.

The interaction of light with quasielectron is substantially new process which will be obviously one of the future problem of the nonrelativistic and relativistic graphene physics, or, the graphene quantum optics. Such interaction is the analogue of the interaction of light with phonons (quanta of the elastic waves), magnons (quanta of the magnetic waves), plasmons (quanta of the plasmatic waves), polarons (quanta of the bound states of electrons and their accompanying polarization waves), polaritons (quanta of the polarization waves), dislocons (quanta of the dislocation waves), excitons (quanta of the excitation waves), Cooper pairs (two-electron system of electrons with the opposite spins in superconductor), and so on.

The graphene can be deformed in such a way that the metric of the deformed sheet is the Riemann one. However, the Riemann metric of general relativity is the metric of the deformed 4D sheet as was proposed by Sacharov [25]. So, in other words, there is the analogy between deformation and Einstein gravity. The discussion on this approach was presented also by author [26]. The deformed graphene obviously leads to the modification of the photoeffect in graphene and it can be used as the introduction to the theory of the photoelectric effect influenced by the gravitational field.
The information on the photovoltaic effect in graphene and also the elementary particle interaction with graphene is necessary not only in the solid state physics, but also in the elementary particle physics in the big laboratories where graphene can form the substantial components of the particle detectors. The graphene can be probably used as the appropriate components in the solar elements, the anode and cathode surface in the electron microscope, or, as the medium of the memory hard disks in the computers. While the last century economy growth was based on the Edison-Tesla electricity, the economy growth in this century will be obviously based on the graphene physics. We hope that these perspective ideas will be considered at the universities and in the physical laboratories.

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