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Transition polynomial as a weight system for binary delta-matroids. (English) Zbl 07551760
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Summary: To a singular knot $K$ with $n$ double points, one can associate a chord diagram with $n$ chords. A chord diagram can also be understood as a 4-regular graph endowed with an oriented Euler circuit. L. Traldi introduced a polynomial invariant for such graphs, called a transition polynomial. We specialize this polynomial to a multiplicative weight system, that is, a function on chord diagrams satisfying 4-term relations and determining thus a finite type knot invariant. We prove a similar statement for the transition polynomial of general ribbon graphs and binary delta-matroids defined by R. Brijder and H. J. Hoogeboom, which defines, as a consequence, a finite type invariant of links.

MSC:
05C31 Graph polynomials
05B35 Combinatorial aspects of matroids and geometric lattices
52B40 Matroids in convex geometry (realizations in the context of convex polytopes, convexity in combinatorial structures, etc.)

Keywords:
knot; link; finite type invariant of knots; chord diagram; transition polynomial; delta-matroid

Full Text: Link

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