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Rock Joint Coefficients Derived from the Three-Dimensional Fourier Reliefs

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Abstract. Coefficients of rock joints serve in geotechnical practice for determining shear strength of rock masses. Evaluation of the stability of rocky terrains containing joints is important from the viewpoint of safety. Larger surface irregularities of joints contribute to a better mechanical stability of rock masses. One of the possible ways of evaluating joint rock coefficients is based on the numerical assessment of surface morphology. The comparison of morphologies between the relief of the investigated rock joint and the reliefs of standard specimens whose coefficients are known enables to assign a proper coefficient to the investigated joint. The Fourier digital reliefs are useful auxiliary tools for such a comparative technique. The technique is however accompanied by some rigors that are necessary to know and avoid them. This contribution discuss some problems associated with the comparative technique used for determining joint rock coefficients.

1. Introduction
Joints in rocky massifs usually form a space network that is hidden under the surface of terrains. Although normally invisible, same parts of such networks may emerge when excavation work is running in terrains. Figure 1 shows the network of joints in a limestone quarry. The jointed surfaces are pressed together by the weight of upper layers and due to roughness (asperity roughness) of the opposite sides of the joints, the whole network seems to be mechanically stable in spite of its inclination.

In order to assess the shearing strength $\tau$ of the opposite surfaces of a particular joint, it is necessary to know the normal effective stress $\sigma_n$ acting on the joint, the compressive strength of asperity walls $JCS$, basic friction angle $\phi_r$ (material constant) and the corresponding joint rock coefficient $JRC$ characterizing roughness of the surfaces of the joint

$$\tau = \sigma_n \tan \left[ JRC \cdot \log \left( \frac{JCS}{\sigma_n} \right) + \phi_r \right]$$

Relation (1) was introduced by Barton and Choubey [1, 2] after a thorough research and testing work. Some authors [3-4] tried to develop the model further and find a functional expression for $JRC$. 

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The determination of the joint rock coefficient $JRC$ is the most critical step in calculating shear strength after relationship (1). One of the possible ways of evaluating joint rock coefficients is based on the numerical assessment of surface morphology. The comparison of morphologies between the relief of the investigated rock joint and the reliefs of standard specimens whose coefficients are known enables to assign a proper coefficient to the investigated joint. The Fourier reliefs are useful auxiliary tools for such a comparative technique.

2. Fourier reliefs

In order to create the Fourier replica of a solid surface, first it is necessary to discretely scanned the surface $f(x_i, y_j)$. The scanning may be performed e.g. by the sectional technique [5-7]. As soon as the discretely scanned surface $f(x_i, y_j)$ is available, the Fourier relief may be computed by means of the partial Fourier series $F_N(x_i, y_j)$ as follows.
\[ f(x_i, y_j) \approx F_N(x_i, y_j) = \sum_{k,n=0}^{N-1} \left( a_{kn} \cos \frac{k \pi x_i}{p} \cos \frac{n \pi y_j}{q} + b_{kn} \sin \frac{k \pi x_i}{p} \cos \frac{n \pi y_j}{q} + c_{kn} \cos \frac{k \pi x_i}{p} \sin \frac{n \pi y_j}{q} + d_{kn} \sin \frac{k \pi x_i}{p} \sin \frac{n \pi y_j}{q} \right) \]

where

\[ \Omega = \{ x_i \in (-p, +p), y_j \in (-q, +q) \} \]

\[
\begin{align*}
    a_{kn} &= \frac{\lambda_{kn}}{pq} \int_{\Omega} f(x_i, y_j) \cos \frac{k \pi x_i}{p} \cos \frac{n \pi y_j}{q} \, dx_i \, dy_j \\
b_{kn} &= \frac{\lambda_{kn}}{pq} \int_{\Omega} f(x_i, y_j) \sin \frac{k \pi x_i}{p} \cos \frac{n \pi y_j}{q} \, dx_i \, dy_j \\
c_{kn} &= \frac{\lambda_{kn}}{pq} \int_{\Omega} f(x_i, y_j) \cos \frac{k \pi x_i}{p} \sin \frac{n \pi y_j}{q} \, dx_i \, dy_j \\
d_{kn} &= \frac{\lambda_{kn}}{pq} \int_{\Omega} f(x_i, y_j) \sin \frac{k \pi x_i}{p} \sin \frac{n \pi y_j}{q} \, dx_i \, dy_j
\end{align*}
\]

\[
\lambda_{kn} = \begin{cases} 
    1 & (k > 0, n > 0) \\
    1/2 & k = 0, n > 0 \text{ or } k > 0, n = 0 \\
    1/4 & k = n = 0 
\end{cases}
\]

The computed Fourier expansion coefficients \( a_{kn} \), \( b_{kn} \), \( c_{kn} \), and \( d_{kn} \) may be used for assembling the Fourier matrix \( M(k,n) \)

\[ M_N(k,n) = \sqrt{a_{kn}^2 + b_{kn}^2 + c_{kn}^2 + d_{kn}^2} \]

from which a simple parameter of wavy shape \( S(N) \) may be introduced

\[ S(N) = \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} \left| M_N^{(o)}(k,n) - M_N(k,n) \right| \]

Similarly, another parameter called the profile height \( H(N) \) may be introduced by means of the root-mean-square values \( R(N) \)

\[ R(N) = \sqrt{\frac{1}{K \cdot L} \sum_{i=1}^{K} \sum_{j=1}^{L} \left[ f(x_i, y_j) - F_N^{(low)}(x_i, y_j) \right]^2} \]

\[ H(N) = \left| R^{(o)}(N) - R(N) \right| \]

where \( K \cdot L \) is a pixel resolution of the used digital images created during scanning procedure. It should be add that usually the rock joints have a certain base wavy level \( F_N^{(low)}(x_i, y_j) \) which is not a part of their inherent surface structure and thus should be removed by subtracting it from the scanned
profile, i.e. \( f(x_i,y_j) - F_{N}^{\text{low}}(x_i,y_j) = f^{\text{corr}}(x_i,y_j) \). So that the Fourier coefficients (3) as well as the matrix (4) are computed from the corrected profile \( f^{\text{corr}}(x_i,y_j) \).

Both parameters (5) and (7) have been defined since the computer comparative technique for determining JRC needs some numerical indicators assessing similarity between investigated and standard profiles. The introduced parameter \( S \) and \( H \) in their absolute forms (5) and (7) seem to be convenient for this purpose. In the next paragraphs we will test whether these parameters are convenient to serve as indicators of dynamical conformity between investigated and standard joint three-dimensional (3D) profiles.

At the beginning of our research, we use similar parameters \( S, \) \( H \) and correlation coefficient. All the three parameters were introduced not in absolute forms but as fractions, i.e. in relative forms. Although they show reasonable results in some cases, yet we have to refuse them for further employment since their relative forms suffered from certain artifacts. For example, the relative forms do not permit exchanging the roles of database and tested profiles, namely \( |a - b|/a \neq |b - a|/b \).

They sometimes show singularities (i.e. zero in denominators) or undefined expressions (0/0). The correlation coefficient had to be refused since it indicated the functional proximity rather than shape proximity. For all these reasons we have formulated new absolute forms of the parameters \( S \) and \( H \) as shown in Equations (5) and (7). Absolute forms of these parameters do not suffer from the drawbacks of the relative forms but it is necessary to test their suitability for assessing the dynamical conformity of 3D profiles.

3. Tests of dynamical conformity

We will explore especially two following situations that will assist in testing properties of the parameters \( S \) and \( H \):

- Two 3D identical reliefs rotated owing to the direction of shearing
- Two 3D identical reliefs translated (shifted) in the direction of shearing

The height parameter \( H \) is sensitive neither to rotations nor to translations since it is defined by means of the \( R \) and \( R' \) values that are related to the basic reference surface \( F_{N}^{\text{low}} \) as shown in Eq. (6). So that the parameters \( R \) and \( R' \) are defined as differences between two levels that will be shifted or translated along with the reliefs \( f(x_i,y_j) \). Thus, the tests of rotations and translations will only concern the parameter \( S \).

For testing purposes the four 3D deterministic periodic profiles \( P1, \ P2, \ P3, \ P4 \) are used. They are defined in the domain \( x \times y \in (-\pi, +\pi) \times (-\pi, +\pi) \):

\[
P1(x,y) = \sin(y), \quad P2(x,y) = \sin(x), \quad P3(x,y) = \cos(y), \quad P4(x,y) = \cos(x)
\]

(8)

their corresponding Fourier matrices \( M_{N}(k,n) \) with rank 2 read

\[
\begin{pmatrix}
0 & 1 \\
0 & 0 \\
\end{pmatrix}_{p1}, \quad \begin{pmatrix}
0 & 0 \\
1 & 0 \\
\end{pmatrix}_{p2}, \quad \begin{pmatrix}
0 & 1 \\
0 & 0 \\
\end{pmatrix}_{p3}, \quad \begin{pmatrix}
0 & 0 \\
1 & 0 \\
\end{pmatrix}_{p4}
\]

(9)
**Situation 1:** Two profiles rotated by $\pi/2$ radians owing to the direction of shearing. Shearing directions along y-axes:

- **a)** $P_1(x, y) = \sin(y)$ versus $P_2(x, y) = \sin(x)$  
- **b)** $P_3(x, y) = \cos(y)$ versus $P_4(x, y) = \cos(x)$

The indicators of wavy shapes $S(N)$ for the couples of reliefs a) and b) are easy to calculate from the matrix elements (9)

- **a)** $S(N) = |0 - 0| + |1 - 0| + |0 - 1| + |0 - 0| = 2$  
- **b)** $S(N) = |0 - 0| + |1 - 0| + |0 - 1| + |0 - 0| = 2$

The investigated couples of reliefs $P_1$ versus $P_2$ and $P_3$ versus $P_4$ have been mutually rotated by $\pi/2$ radians. It is clear that a rock joint consisting of two surfaces $P_1$ and another joint consisting of two surfaces $P_2$ and both the joints sheared along the y-axis will manifest different dynamical responses (different friction forces) since they have to overcome different surface irregularities ($P_1$ joint shears against surface humps, $P_2$ joint shears along the surface humps) and thus the result in Equation (12) correctly mirrors the reality (different 'dynamical' responses of the two joints, i.e. the joints are not similar). The same holds for the couple of reliefs (joints) $P_3$ and $P_4$ (case b)).

**Situation 2:** Two profiles translated in the direction of shearing:

- **c)** $P_1(x, y) = \sin(y)$ versus $P_3(x, y) = \cos(y)$ Shearing along the y-axis.
- **d)** $P_2(x, y) = \sin(x)$ versus $P_4(x, y) = \cos(x)$ Shearing along the x-axis.

The indicators of wavy shapes $S(N)$ for the couples of reliefs a) and b) are easy to calculate from the matrix elements (9)

- **c)** $S(N) = |0 - 0| + |1 - 1| + |0 - 0| + |0 - 0| = 0$  
- **d)** $S(N) = |0 - 0| + |0 - 0| + |1 - 1| + |0 - 0| = 0$

Case c) represents a couple of profiles that are identical but mutually shifted by $\pi/2$ radians. It is obvious that the rock joint consisting of two surfaces $P_1$ and another joint consisting of two surfaces $P_3$ and both the joints sheared along the y-axis will manifest the same dynamical responses (identical friction forces) since they have to overcome the same surface irregularities (both the joints shears against surface humps) and thus the result in Equation (16) correctly mirrors the reality (zero difference between 'dynamical' responses of the two joints, i.e. the joints show full similarity). A similar discussion may be added to the couple of reliefs (joints) $P_3$ and $P_4$ (case b)).

Both Situation 1 and 2 are only two theoretical examples that are to illustrate an ideal behavior of the introduced indicator $S(N)$. In practice, the reliable assessment of similarity requires sampling the rock slopes, i.e. taking a set of specimens from various places of the rock slope under investigation and performing averaging, i.e. computing the average values of $\langle R(N) \rangle$ and $\langle M_N(k, n) \rangle$. Next step should be the comparison between the averaged profile parameters $\langle R(N) \rangle$, $\langle M_N(k, n) \rangle$ and their
counterparts from the database $R^{(\omega)}(N), M^*(k,n)$. The resulted indicators $S(N)$ and $H(N)$ will provide us some numerical values that will specify the most similar database profile whose JRC may be ascribed to the investigated rock slope. It might happen that the indicators $S(N)$ and $H(N)$ will identify two different database profiles. Such a situation is quite natural and does not represent a defect of the comparative method. The geometrical indicator $H(N)$ compares the vertical widths of profiles whereas $S(N)$ compares dynamical conformity of profiles on the basis of shape morphologies. Since heights and shapes of profiles are not tightly correlated, they are independent to some extent and thus the indicators $S(N)$ and $H(N)$ may occasionally forecast two bit different results. In such cases it is reasonable to prefer the database relief whose JRC is smaller since such a value is at the side of higher safety.

4. Conclusions
The comparative method has been designed as a fully computerized procedure. The theoretical tests presented in this contribution have indicated prospective features of the two introduced indicators $H(N)$ and $S(N)$. The former compares vertical widths of surface profiles whereas the latter compares dynamical conformity of these profiles. Comparison is carried out between the investigated and database profiles with known JRC values.

Practical computations that employed the aforementioned indicators are in progress. They will incorporate the complete computerized comparative procedure along with extra visual assessment of profiles as an illustration of the older so far used method.

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