Reconfigurable Intelligent Surface (RIS)-Enhanced Two-Way OFDM Communications

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Abstract—In this paper, we focus on the reconfigurable intelligent surface (RIS)-enhanced two-way device-to-device (D2D) multi-pair orthogonal-frequency-division-multiplexing (OFDM) communication systems. Specifically, we maximize the minimum bidirectional weighted sum-rate by jointly optimizing the sub-band allocation, the power allocation, and the discrete phase shift (PS) design of the reflecting elements at the RIS. To tackle the main difficulty of the non-convex PS design, we firstly formulate a semi-definite relaxation problem and further devise a low-complexity solution for the PS design by leveraging the projected sub-gradient method. We demonstrate the desirable performance gain for the proposed designs through numerical results, where the number of reflecting elements at the RIS and the deployment location of RIS are shown to play a key role in enhancing the performance gain.

Index Terms—Two-way communications, reconfigurable intelligent surfaces (RISs), OFDM.

I. INTRODUCTION

With the recent advances in electromagnetic (EM) meta-surfaces, the reconfigurable intelligent surfaces (RISs) are foreseen as the cost-effective and energy-efficient substitutes for the relay-based systems [1], [2]. The RIS is a planar array consisting of a large number of reflecting elements, which are implemented with low-cost programmable positive-intrinsic-negative (PIN) diodes or phase shifters (PSTs) [2], [3]. Accordingly, the reflection behaviour of the impinging EM signals can be controlled through an appropriate design of phase shifts (PSs) of the reflecting elements to improve the performance of a wireless network. Compared to the relay-based systems, the deployment of RIS does not involve additional RF chains or the imposition of thermal noise. Furthermore, the RIS can readily be fabricated in small size and low weight, which can be coated on the buildings’ facade, walls, etc [3].

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Due to the above benefits RISs provide, they are envisioned to enhance the performance for various wireless applications, by improving the spectral and energy efficiencies, facilitating the simultaneous wireless information and power transfer, the massive device-to-device (D2D) communications, etc., [2]–[5]. While the majority of current works for the RIS-enhanced communication systems focus on one-way communication, there are only a limited number of works that consider the RIS-enhanced two-way communication systems [6], [7]. The works in [6], [7] are primarily limited to a single pair of full-duplex (FD) nodes, where the benefit of two-way network is conditioned on the proper self-interference (SI) cancellation at the FD nodes. This comes at the cost of high hardware complexity and low energy efficiency [8], which is hence unsuitable for low-cost and power-limited nodes. Furthermore, to the best of our knowledge, the use of orthogonal-frequency-division-multiplexing (OFDM) for the RIS-enhanced two-way communications has not been explored yet.

Motivated by this, we focus on a RIS-enhanced two-way D2D communication system where multiple pairs of transceiver nodes communicate bidirectionally via RIS through the OFDM. Specifically, the available bandwidth is divided into multiple orthogonal sub-bands, where each of the bidirectional communication links across multiple node pairs is allocated a subset of non-overlapping sub-bands. We aim to maximize the minimum bidirectional weighted sum-rate by jointly optimizing the sub-band allocation, the power allocation and the PSs at the RIS, where practical discrete PSs at each of its reflecting elements are considered. To tackle the main difficulty of the non-convex PS design at the RIS, we firstly propose a semi-definite relaxation (SDR) formulation. Subsequently, we devise a low-complexity solution for the PS design by leveraging the projected sub-gradient (PSG) method to achieve a more favorable performance-complexity tradeoff. Numerical results reveal the desirable performance gain for the proposed designs.

Notations: $y$, $y$ and $Y$ denote scalar, vector and matrix, respectively; Conjugate, transpose and conjugate transpose operators are represented by $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$, respectively; $\| \cdot \|_2$ denotes the $l_2$ norm; $\text{Tr}\{ \cdot \}$ denotes the trace operator; $\ast$ denotes the convolution operation; Expectation of a random variable is noted by $E[\cdot]$; $| \cdot |$ and $\angle$ return the absolute value and the argument of a complex number, respectively.

II. SYSTEM MODEL

We consider a two-way D2D communication system with $K$ node pairs, denoted by $\text{Node}_k^1$ and $\text{Node}_k^2, k = \{1, \ldots, K\}$, where each node is equipped with single transmit and receive antennas as shown in Fig. 1.

Figure 1. RIS-enhanced two-way D2D communication system.
A RIS with $R$ reflecting elements is deployed to enhance the communication for the $K$ node pairs. Accordingly, let $\Psi = [\Psi_1, \ldots, \Psi_R]^T \in \mathbb{C}^{R \times 1}$ denote the vector of the reflection coefficients at the RIS, such that each reflection coefficient $\Psi_r$ satisfies $\Psi_r \in \mathbb{R} \triangleq \{ \frac{e^{j2\pi b}}{2^B - 1} | b = 0, \ldots, 2^B - 1 \}$, where $B$ is the number of quantization bits of the PSTDs [6]. Following OFDM, the available bandwidth is divided into $V$ sub-bands, and the sub-bands are occupied to carry out simultaneous communication for the $K$ node pairs, where each node pair further adopts bidirectional communication\(^1\) through non-overlapping sub-bands to avoid SI. Accordingly, the communication within each node pair takes through two links, i.e., the $\text{Node}_k^i$-RIS-$\text{Node}_k^i$ reflected link and the direct $\text{Node}_k^i$-$\text{Node}_k^i$ link, along both the directions.

### A. Channel Model

For the $k$-th node pair, the time-domain $\text{Node}_k^i$-RIS-$\text{Node}_k^i$ reflected channel via each $r$-th reflecting element of the RIS, in the $i$-th direction, is the convolution of the $\text{Node}_k^i$-RIS channel, the RIS reflection coefficient, and the RIS-$\text{Node}_k^i$ channel, i.e., $\xi_{r,k}^i + \psi_r^i \xi_{r,k}^i = \Psi_r^i \xi_{r,k}^i$, where $\xi_{r,k}^i \in \mathbb{C}^{L_k \times 1}$ and $\psi_{r,k}^i \in \mathbb{C}^{L_k \times 1}$ denote the time-domain $\text{Node}_k^i$-RIS and RIS-$\text{Node}_k^i$ channels, respectively, and $L_k$ is the number of the corresponding delay taps [9], [10]. Similarly, let $\xi_{r,k}^i \in \mathbb{C}^{L_k \times 1}$ denote the time-domain direct $\text{Node}_k^i$-$\text{Node}_k^i$ channel, where $L_k$ is the number of delay taps. Furthermore, for each multi-path channel, the channel taps are assumed to follow the exponential power-delay feature characterized by $[\xi_{r,k}^i]_v = \sqrt{\frac{\alpha_v}{\pi}} e^{-\alpha_v/\pi} \xi_{r,k}^i, v = 0, \ldots, L_k - 1, x \in \{ k, k - r, r - k \}$, where $0 < \alpha < 1$, $\alpha_v$ is the path loss, and $\xi_{r,k}^i \sim \mathcal{CN}(0, 1)$ represents the small scale fading [11].

Accordingly, for the $k$-th node pair, the zero-padded concatenated $\text{Node}_k^i$-RIS and RIS-$\text{Node}_k^i$ time-domain channel through each of the $r$-th reflecting element in the $i$-th direction is given by $\bar{h}_{k,v}^i = [(\xi_{r,k}^i)_{0}^T, \ldots, (\xi_{r,k}^i)_{L_k - 1}^T]^T, v = 0, \ldots, V - 1$ in $\mathbb{C}^{V \times 1}$ and $H_{k,v}^i \triangleq [\bar{h}_{k,0}^i, \ldots, \bar{h}_{k,V-1}^i]$ [9]. Thus, the composite $\text{Node}_k^i$-RIS-$\text{Node}_k^i$ reflected channel can be expressed as $H_{k,v}^i \Psi_r^i$. By further denoting $g_{k,v}^i = [\xi_{r,k}^i]_{0}^T, \ldots, [\xi_{r,k}^i]_{L_k - 1}^T]^T \in \mathbb{C}^{V \times 1}$ as the zero-padded time-domain $\text{Node}_k^i$-$\text{Node}_k^i$ direct channel, the effective channel impulse response of the $k$-th node pair in the $i$-th direction is given by $\bar{h}_{k}^i = g_{k,v}^i + H_{k,v}^i \Psi_r^i$. We further assume that the inter-symbol interference is perfectly eliminated through the use of the cyclic prefix as in [9]. Finally, for the $k$-th node pair, the channel frequency response on the $v$-th sub-band in the $i$-th direction is expressed as $\tilde{h}_{k,v}^i = f_{k,v}^i H_{k,v}^i \Psi_r^i = \gamma_{k,v}^i + (h_{k,v}^i)^T \Psi_r^i$, where $f_{k,v}^i$ denotes the $v$-th row of the discrete Fourier transform (DFT) matrix $F \in \mathbb{C}^{V \times V}$, $\gamma_{k,v}^i \triangleq f_{k,v}^i H_{k,v}^i$ and $h_{k,v}^i \triangleq (H_{k,v}^i)^T f_{k,v}^i$. Furthermore, the RIS is considered to be attached with a central controller, which controls the PSSs of its reflecting elements and communicates with the node pairs via dedicated wireless links for coordinating transmission and exchanging information on the channel state information (CSI) [10], [12], where the perfect CSI is assumed to be estimated and known at the nodes through the central controller as in [11], [6], [10]. Based on the CSI, the central controller performs the sub-band and power allocations as well as the PS design at the RIS.

### B. Transmission Model

To avoid the inter-node interference and the self-interference, each sub-band is allocated to only one $\text{Node}_k^i$, $\forall k, i \in \{1, 2\}$, for transmission in at most one direction, which is motivated by the 3rd Generation Partnership Project (3GPP) specification for Long-Term Evolution (LTE) [13]. Accordingly, let $\eta_{k,v}^i$ indicate whether the $v$-th sub-band is allocated to $\text{Node}_k^i$, $i \in \{1, 2\}$ for transmission in the $i$-th direction, i.e., $\eta_{k,v}^i = 1$ if the $v$-th sub-band is assigned to $\text{Node}_k^i$ in the $i$-th direction, and $\eta_{k,v}^i = 0$ otherwise. Accordingly, the received signal on the $v$-th sub-band at $\text{Node}_k^i$, $i \in \{1, 2\}$ in the $i$-th direction, when $\eta_{k,v}^i = 1$, is given by

$$
\gamma_{k,v}^i = \sqrt{p_{k,v}^i h_{k,v}^i s_{k,v}^i + z_{k,v}^i},
$$

where $p_{k,v}^i \geq 0$ denotes the transmit power allocated on the $v$-th sub-band by $\text{Node}_k^i$, $s_{k,v}^i$ is the transmitted signal such that $\mathbb{E}[|s_{k,v}^i|^2] = 1$, and $z_{k,v}^i \sim \mathcal{CN}(0, \sigma_z^2)$ denotes the additive white Gaussian noise (AWGN) in the sub-band. Subsequently, defining $\gamma_{k,v}^i \triangleq \gamma_{k,v}^i / \|h_{k,v}^i\|^2$, the achievable rate in bits per second per Hertz (bps/Hz) for the $k$-th node pair on the $v$-th sub-band in the $i$-th direction is given by [9]

$$
\Gamma_{k,v}^i = \frac{\eta_{k,v}^i}{V} \log_2 (1 + \gamma_{k,v}^i).
$$

### III. Problem Formulation and Proposed Solution

In this work, we aim to maximize the minimum bidirectional weighted sum-rate across all the sub-bands for the $K$ node pairs. Accordingly, we formulate the following max-min optimization problem:

**Problem 1:**

\[
\begin{align*}
    \max_{\{\eta_1, \eta_2, \eta_3, \eta_4\}} \min_{k = 1}^{K} \sum_{v = 1}^{V} \sum_{i = 1}^{2} \rho_0 \Gamma_{k,v}^i
\end{align*}
\]

\[\text{s.t.} \quad C_1: \sum_{v = 1}^{V} \eta_{k,v}^i P_{k,v}^i \leq P_k^i, \forall k, i \in \{1, 2\}, \]

\[C_2: p_{k,v}^i \geq 0, \forall k, v, i \in \{1, 2\}, \]

\[C_3: \sum_{k = 1}^{K} \sum_{i = 1}^{2} \eta_{k,v}^i = 1, \forall v, \]

\[C_4: \eta_{k,v}^i \in \{0, 1\}, \forall k, v, i \in \{1, 2\}, \]

\[C_5: \Psi_r \in \mathbb{R}, \forall r. \]

where $\rho_0$ is the weighting factor of the $k$-th node pair, $\eta_1 \triangleq [\eta_{1,1}, \ldots, \eta_{K,1}]^T$ and $\rho_1 \triangleq [p_{1,1}, \ldots, p_{K,1}]^T, i \in \{1, 2\}$, $C_1$ is the transmit power constraint for Node$_k^i$, where $P_{k,v}^i$ is the maximum power at Node$_k^i, \forall k, i \in \{1, 2\}$, and the constraint $C_2$ ensures the non-negativity of the transmit power at each allocated sub-bands of the nodes. The constraints $C_1$ and $C_4$ assign each sub-band to only one node for transmission in at most one direction. Finally, the constraint $C_3$ restricts the PSSs at the RIS within the feasible set $R$. Note that along with the non-convex constraints $C_4$ and $C_5$, the coupling of $\Gamma_{k,v}^i, \forall k, v, i \in \{1, 2\}$ through $\Psi$ makes $P_1$ difficult to solve. Accordingly, to solve $P_1$, we present a two-stage design, which is described in the subsequent sub-sections.

#### A. First-Stage: Sub-Band Allocation

Firstly, it can be observed from $P_1$ that the sub-band allocation problem, for a given $[\Psi, p_1, \eta_1], i \in \{1, 2\}$, is a non-convex binary integer problem due to $C_4$. Accordingly, to reduce the computational complexity for the sub-band allocation problem, we resort to a sub-optimal algorithm, given in Algorithm 1, similar to that discussed in [14]. Specifically, we first obtain an appropriate initial $\Psi$, denoted by $\tilde{\Psi}$,
Algorithm 1: Sub-Optimal Sub-Band Allocation.

1: Input: Initial $\Psi = \Psi$ according to Appendix A.
2: Initialization: $\Gamma^k_0 = 0$ and $\eta^k_0 = 0, \forall k, v, i \in \{1, 2\}$.
3: for $k = 1$ to $K$ do
4: for $i = 1$ to 2 do
5: Find $v = \arg\max_{v \in \{1, 2\}} \Gamma^k_i(v)$;
6: Update $\Gamma^k_i = \Gamma^k_i + \eta^k_i = 1$ and $\mathcal{V} = \mathcal{V} - \{v\}$;
7: repeat
8: Find $\{k, i\} = \arg\min_{k \in \{1, 2\}, i \in \{1, 2\}} \Gamma^k_i$;
9: Find $v = \arg\max_{v \in \{1, 2\}} \Gamma^k_i$;
10: Update $\Gamma^k_i = \Gamma^k_i + \Gamma^k_i(v)$, $\eta^k_i = 1$ and $\mathcal{V} = \mathcal{V} - \{v\}$;
11: until ($\mathcal{V} \neq \emptyset$)
12: Output: $\eta^k_i, i \in \{1, 2\}$.

which maximizes the minimum bidirectional effective channel gain across all the sub-bands for the $K$ node pairs, as detailed in Appendix A. Consequently, assuming uniform power allocation across the sub-bands, the node with the lowest sum-rate across both the directions is iteratively assigned a sub-band where it achieves the highest rate $\Gamma^k_i$, as described in Algorithm 1, where $\Gamma^k_i \triangleq \log_2(1 + \gamma^k_i |\Psi - \Psi|^2), \forall k, v, i \in \{1, 2\}$ and $\mathcal{V}$ is defined as the set of available sub-bands.

B. Second-Stage: PS Design at the RIS and Power Allocation

With the obtained $\eta^k_i$, $i \in \{1, 2\}$, the goal of maximizing the minimum bidirectional weighted sum-rate across all the sub-bands for the $K$ node pairs is further achieved by refining $\Psi$ and $p_i$, $i \in \{1, 2\}$, through an alternating optimization framework as detailed below.

1) PS Design: With the obtained $\eta^k_i$ and fixed $p_i, i \in \{1, 2\}$, $\Psi$ is obtained by solving $P_1$, which is non-convex with respect to (w.r.t.) $\Psi$ due to $C_5$. Accordingly, we propose the following methods to obtain $\Psi$.

a) Exact Solution: We firstly apply SDR to exactly solve for $\Psi$. Accordingly, by introducing an auxiliary variable $\varsigma$, and defining $\Theta \triangleq \Psi^{1/2}$ and $\Psi \triangleq [\Psi^{1/2}]$, such that $\Theta \succeq 0$ and rank$(\Theta) = 1$, $P_1$ w.r.t. $\Psi$ is transformed into the following convex semidefinite program (SDP) by ignoring the rank-one constraint:

$$
P_2 : \max_{\Theta} \min_{[i \in \{1, 2\}, m = 1]} \sum_{k = 1}^K \sum_{i = 1}^V x^k_i \eta^k_i \log_2 \left(1 + \gamma^k_i (\Theta)\right) \\
\text{s.t.} \quad C_5 : [\Theta](m, m) = 1, \forall m, C_7 : \Theta \succeq 0.
$$

where $\gamma^k_i (\Theta) \triangleq \frac{\phi^k_i (\Theta)}{\sigma^k_i}$ and $H^k_i \triangleq [h^k_0 h^k_1 (\Theta^i_h)^H (h^k_0 (\Theta^i_h)^H)^H]$. $P_2$ can be optimally solved by existing convex optimization solvers [1], which may not lead to a solution satisfying rank$(\Theta) = 1$. Accordingly, for rank$(\Theta) \neq 1$, Gaussian randomization coupled with the projection operation given in (19) in Appendix VII can be leveraged to obtain $\Psi$ as in [1]. The details are omitted for brevity.

b) Low-Complexity Solution: To achieve a lower complexity than the above SDR based solution, let $u^k_i, i \in \{1, 2\}, i \in \{1, 2\}$ be the receive filter such that the estimated signal on the $v$-th sub-band at Node $i$ in the $i$-th direction, when $\eta^k_i = 1$, is given by $s^k_i = u^k_i g^k_i, \forall k, v, i$. Accordingly, the corresponding squared-error (MSE) is given by

$$
\epsilon^k_i = E \left[|s^k_i - \hat{s}^k_i|^2\right] = 1 + 2p^k \Re \left\{\pi^k_i \Psi - u^k_i g^k_i\right\} + p^k \left|\Psi^H \Gamma^k_i \Psi + u^k_i g^k_i\right|^2 + \sigma^2_k |u^k_i|^2,
$$

where $\pi^k_i \Psi \triangleq \sqrt{p^k} (g^k_i)^* |\Psi^H | h^k_i|^H - u^k_i h^k_i |)^H$ and $\Gamma^k_i \triangleq |u^k_i|^2 |h^k_i|^H$. Subsequently, defining $u^k_i = [u^k_0, u^k_1, \ldots, u^k_V]$, we formulate the following problem

$$
P_3 : \min_{[\Psi, w^1, \hat{u}^1, \hat{u}^2]} \max_{[\Theta, u^1, u^2]} \sum_{k = 1}^K \sum_{i = 1}^V x^k_i c^k_i s.t. C_5,
$$

where $c^k_i \triangleq \frac{\rho^k_i (w^k_i) - \log_2(w^k_i) - 1}{w^k_i}$, $w^k_i$ is the weight associated with $\epsilon^k_i$, and $w^k_i \triangleq [w^k_0, w^k_1, \ldots, w^k_V]$. Given $P_3$ is difficult to solve due to the coupling of the variables in $\pi^k_i, v, i \in \{1, 2\}$, we adopt an inner second-phase alternating optimization to solve $P_3$. Accordingly, at the $t$-th iteration of the inner alternating optimization, each element of the optimal receive filter $u^k_i, i \in \{1, 2\}, i \in \{1, 2\}$, for given values of $w^k_i, \Psi^{(t-1)}, i \in \{1, 2\}$, is equivalent to minimizing $\epsilon^k_i$ w.r.t. $(u^k_i)^{(t)}$, which is given by

$$
\left(u^k_i\right)^{(t)} = \sqrt{p^k i} \Psi \left|\Psi^{(t-1)}\right|^2 + \sigma^2_k |u^k_i|^{-1} \Psi^{(t-1)},
$$

where $\Xi^{(t-1)} \triangleq (\Psi^{(t-1)})^H (h^k_i + (g^k_i)^*)$. Subsequently, each element of $w^k_i$, $i \in \{1, 2\}$, for the obtained values of $u^k_i, i \in \{1, 2\}$, and given $\Psi^{(t-1)}$ is computed by minimizing $P_3$ w.r.t. $(u^k_i)^{(t)}$, given by (15)

$$
\left(w^k_i\right)^{(t)} = (\epsilon^k_i)^{(t)}.
$$

Lemma 1: For a given $\Psi^{(t-1)}$, the objective of $P_1$ and $P_3$ are equivalent for the optimal values of $[u^1_i, u^2_i], i \in \{1, 2\}$, given by (7) and (8).

Proof: The lemma can be proved by substituting (7) and (8) into the objective of $P_3$.

Finally, we resort to the PSG method to obtain a low-complexity update for $\Psi$. Accordingly, defining $f_i(\Psi^{(t-1)}) \triangleq \sum_{k = 1}^K \sum_{i = 1}^V x^k_i c^k_i$ and $f(\Psi^{(t-1)}) \triangleq \max_{t \in \{1, 2\}} f_i(\Psi^{(t-1)})$, the sub-differential of the unconstrained $P_3$ for the $t$-th iteration is expressed as $\delta f(\Psi^{(t-1)}) = \text{conv}(U_{\Theta} f_i(\Psi^{(t-1)})) \cap \nabla f_i(\Psi^{(t-1)}) [16]$, which is the convex hull of the union of gradients of $f_i(\Psi^{(t-1)}), i \in \{1, 2\}$, that achieve the maximum at $\Psi^{(t-1)}$, where

$$
\nabla f_i(\Psi^{(t-1)}) \triangleq \sum_{k = 1}^K \sum_{i = 1}^V \eta^k_i u^k_i \left(\frac{h^k_i}{p^k} \left|\Psi^{(t-1)}\right|^2 + \sigma^2_k |u^k_i|^2\right) \times \left[\left(h^k_i (\Theta^{(t-1)} - h^k_i)^H \Psi^{(t-1)} - h^k_i (\Theta^{(t-1)} - h^k_i)^H \Psi^{(t-1)}\right)\left(u^k_i\right)^{(t)}\right].
$$

Let $\delta^k_i \in \nabla f(\Psi^{(t-1)})$ denote any sub-gradient of $f(\Psi^{(t-1)})$ at the $t$-th iteration, where $\delta^k_i$ is uniquely given by $\delta^k_i = \nabla f_i(\Psi^{(t-1)})$ such that $i = \arg\max f_i(\Psi^{(t-1)})$, when $f(\Psi^{(t-1)}) \neq f_i(\Psi^{(t-1)})$

Note that the gradient projection method is used in works like [4], [5] for the PS design at the RIS, where the objective function of the PS design problems is differentiable. However, in our work, the direct adoption of the gradient projection method to obtain the PS design is not feasible due to the non-differentiable nature of the objective of $P_3$. 

Algorithm 2: PS Design With PSG Method.

1: **Input**: \( p_i, i \in \{1, 2\}, \Psi \);
2: Initialize \( \Psi^{(0)} = \Psi \);
3: for \( t = 1 \) to \( T_{\text{max}} \) do
4:  Update \( u_i^{(t)}, i \in \{1, 2\}, \{i\} \) using (7);
5:  Update \( w_i^{(t)}, i \in \{1, 2\} \) using (8);
6:  Update \( \Psi^{(t)} \) using (9);
7:  **Output**: \( \Psi = \Psi^{\text{best}} \).

Algorithm 3: Proposed Two-Way Communication Algorithm.

1: **Input**: \( g_k, h_v, \forall k, v, i \in \{1, 2\} \);
2: Initialize \( \Psi \) according to Appendix A;
3: Obtain \( \eta_i, i \in \{1, 2\} \) using Algorithm 1;
4: repeat
5:  Update \( \Psi \) by solving \( P_2 \) or using Algorithm 2;
6:  Update \( p_i, i \in \{1, 2\} \) using iterative waterfilling;
7: until convergence
8: **Output**: \( \eta_i, p_i, i \in \{1, 2\} \).

and \( \delta^{(t)} = \tau \nabla f_1(\Psi^{(t-1)}) + (1-\tau) \nabla f_2(\Psi^{(t-1)}), \tau \in [0, 1] \), otherwise [16]. Accordingly, \( \Psi \) is updated as following [17]:

\[
\Psi^{(t)} = \text{Proj}_E \left( \Psi^{(t-1)} - \kappa_t \delta^{(t-1)} / \| \delta^{(t-1)} \|_2 \right),
\]

where \( \text{Proj}_E(\cdot) \) is defined as in (19) in Appendix A, \( \kappa_t \equiv 1/t > 0 \) is the diminishing step size [16], [17]. Since the PSG method is generally not a decent method, the best value for \( \Psi \) is given by \( \Psi^{\text{best}} = \arg \min \Psi^{(t)}, \forall t \in \{1, \ldots, T_{\text{max}}\}, f(\Psi^{(t)}) \), where \( T_{\text{max}} \approx 100 \) is sufficient to obtain an adequate performance [16]. Algorithm 2 summarizes the proposed framework to obtain \( \Psi \) through the PSG method, where \( \Psi \) denotes the PS vector obtained at the previous iteration of the outer alternating optimization framework. Note that, considering only the dominant computations, the overall complexity of Algorithm 2 is \( O(2T_{\text{max}}KV^2F) \), which is significantly less compared to \( O((R + 1)^6) \) incurred by solving \( P_2 \), especially for a large \( R \).

2) Power Allocation: For the obtained \( \eta_i, i \in \{1, 2\} \) and \( \Psi, p_i, i \in \{1, 2\} \) is computed by solving \( P_1 \), which has a waterfilling solution, given by

\[
p_{k,v} = \left\{ \begin{array}{ll} \frac{1}{\sum_{\ell=1}^{V} \eta_{\ell,v}} \left( P_{k} + \sum_{\ell=1}^{V} \frac{\eta_{\ell,v}}{\varpi_{k,v}} \right) - \frac{1}{\varpi_{k,v}} & \text{if } \eta_{k,v} = 1, \\ 0 & \text{if } \eta_{k,v} = 0, \end{array} \right.
\]

where \([q]^+ \equiv \max(0, q)\), \( \varpi_{k,v} \equiv \left[ g_{k,v}^2 + (h_{k,v}^2) \right]^{\frac{1}{2}} \). Furthermore, to efficiently utilize the total transmit power of each node, we adopt the iterative waterfilling algorithm, similar to that described in [18]. Specifically, in each iteration, we set \( \eta_{k,v} = 0 \) such that \( \bar{v} = \arg \min \varpi_{k,v}, \) if the obtained \( p^*_{k,v} = 0 \) when \( \eta_{k,v} = 1 \), where the algorithm continues until \( p^*_{k,v} > 0, \forall \eta_{k,v} = 1 \). Note that the details are avoided for brevity. Finally, Algorithm 3 summarizes the overall framework to maximize the minimum bidirectional weighted sum-rate for the proposed two-way communication.

### IV. Numerical Results

In this section, we evaluate the performance of our proposed design via Monte-Carlo simulations. Unless stated otherwise, we assume \( K = 3, V = 16, P_0 = 25 \text{ dBm} \) and \( \sigma_{\text{SNR}}^2 = -110 \text{ dBm} \). Accordingly, we plot the minimum weighted sum-rate among \( \eta \)

\[
\arg \min \eta = \frac{P}{V},
\]

for the obtained \( \eta \). Note that the details are avoided for brevity.

And incurred by solving \( \eta \). Note that the low-complexity 'optPSG' design incurs a marginal performance gain compared to the 'optSDR' design. Furthermore, the higher performance of 'optPSG' design compared to the other designs establishes the merit of the proposed initialization for \( \Psi \) and the proposed update for \( \{p_i, \Psi\}, i \in \{1, 2\} \) as described in Algorithm 3. Subsequently, Fig. 2 (b), (c) and (d) demonstrate the performance of the proposed designs w.r.t. the maximum power of each node, where \( P_k = P, \forall k; i \in \{1, 2\} \), the number of node pairs (K), and the number of sub-bands (V), respectively. The figures show a similar trend to that observed in Fig. 2(a), which verifies the effectiveness of the proposed designs w.r.t. different operating parameters.

Next, we discuss the impact of RIS deployment location assuming \( B = \infty \) by moving the RIS along the x-axis from \( x = -40 \) to \( x = 40 \). Accordingly, we plot the minimum weighted sum-rate among both the directions w.r.t. RIS location at x-axis in Fig. 3. It can be seen that the performance gain increases when the RIS is deployed close to either Node_1 or Node_2, \( \forall k, V \), where deploying the RIS at the central location, i.e., \( x = 0 \), gives the least performance gain. This is because when the RIS is near the nodes, the Node_1 - RIS, \( \forall k, i \in \{1, 2\} \), links attain a stronger channel due to a decrease in the corresponding path-loss, which results in an increase in the performance gain. However, when the RIS moves away from the nodes, the increase in the path-loss for the Node_1 - RIS, \( \forall k, i \in \{1, 2\} \) channels decreases the performance gain. Furthermore, similar to Fig. 2, 'optPSG' design is observed to perform better than the benchmark designs with a marginal performance loss compared to 'optSDR' design.

Finally, in Fig. 4, the minimum bidirectional weighted sum-rate performance for the proposed 'optSDR' and 'optPSG' designs is evaluated w.r.t. the finite value of \( B \), where the proposed designs with \( B = 5 \) are...
at the RIS. Accordingly, the performance of the considered two-way communication system can be significantly enhanced by deploying a large number of lost-complexity low-cost reflecting elements at the RIS. Additionally, the deployment location of RIS is shown to play a crucial part in further improving the performance gain.

APPENDIX A
INITIALIZATION FOR \(\Psi\)

The constant-modulus constraint on \(\Psi\), i.e., \(C_5\), makes \(P_1\) highly non-convex, resulting in multiple local minimum points for \(P_1\). Accordingly, to ensure that the proposed algorithm converges to a near-optimum local point, initialization for \(\Psi\) plays a crucial role. For this purpose, inspired by the work in [1], and defining \(g_i \equiv [g^1_{i\lambda}, \ldots, g^K_{i\lambda}]^T \in \mathbb{C}^{K_v \times 1}\) and \(h_i \equiv [h^1_{i\lambda}, \ldots, h^K_{i\lambda}]^H \in \mathbb{C}^{K_v \times R}\), we initialize \(\Psi\) to the solution of the following optimization problem:

\[
P_4: \max_{\{\Psi\} \in \{1, 2, 3\}} \|g_i + H_i \Psi\|_2^2 \text{s.t. } C_5, \tag{11}
\]

which maximizes the minimum bidirectional effective channel gain across all the sub-bands of the \(K\) node pairs. Accordingly, relaxing \(C_5\) and ignoring the terms independent of \(\Psi\), \(P_4\) can be transformed into the following epigraph form:

\[
P_5: \max_{\{\Psi, \rho\}} \rho \text{ s.t. } C_6: \text{Tr}\left\{\Psi^H H_i \Psi\right\} \geq \rho, i \in \{1, 2\}, \tag{12}
\]

\[
C_6: \|\tilde{\Psi}\|_2^2 \leq R + 1,
\]

where \(\tilde{\Psi} \equiv [\Psi]_{1,1}^T\) and \(\tilde{H}_i \equiv [h^H_{i\lambda} h^H_{i\lambda} h^H_{i\lambda}] \in \mathbb{C}^{K_v \times 1}\). Subsequently, the Lagrangian associated with \(P_5\) is given by

\[
\mathcal{L} = -\rho + \sum_{i=1}^2 \lambda_i \left(\rho - \text{Tr}\left\{\Psi^H H_i \Psi\right\}\right) + \mu \left\|\Psi\right\|_2^2 - R - 1, \tag{13}
\]

and the corresponding the KKT conditions are given by

\[
\frac{\partial \mathcal{L}}{\partial \Psi} = -\sum_{i=1}^2 \lambda_i \tilde{H}_i \tilde{\Psi} + \mu \tilde{\Psi} = 0, \tag{14}
\]

\[
\frac{\partial \mathcal{L}}{\partial \rho} = -1 + \sum_{i=1}^2 \lambda_i = 0, \tag{15}
\]

\[
\lambda_i \left(\rho - \text{tr}\left\{\Psi^H H_i \Psi\right\}\right) = 0, \lambda_i \geq 0, i \in \{1, 2\}, \tag{16}
\]

\[
\mu \left\|\Psi\right\|_2^2 - R - 1 = 0, \mu \geq 0. \tag{17}
\]
where \( \{\lambda_i, \mu\}, i \in \{1, 2\} \) are the Lagrangian multipliers. Note that, from (15), we have \( \sum_{i=1}^2 \lambda_i = 1 \), which implies \( 0 \leq \lambda_i \leq 1, i \in \{1, 2\} \). Accordingly, leveraging (14), we obtain the following condition:

\[
\begin{align*}
\left( \lambda_i \hat{H}_i + (1 - \lambda_i) \tilde{H}_i \right) \Psi = \mu \Psi, \\
\Rightarrow \hat{H}(\lambda_i) \Psi = \mu \Psi,
\end{align*}
\]

where \( \hat{H}(\lambda_i) \equiv (\hat{H}_2 + \lambda_i(\tilde{H}_1 - \tilde{H}_2)) \) is a Hermitian matrix. Subsequently, by expressing the eigen decomposition of \( \hat{H}(\lambda_i) \) as \( \hat{H}(\lambda_i) \equiv U_{\lambda_i} \Sigma_{\lambda_i} U_{\lambda_i}^H \), the solution to \( \mathcal{P}_1 \) as a function of \( \lambda_i \) is given by the principal eigenvector corresponding to the maximum eigenvalue of \( \hat{H}(\lambda_i) \), denoted by \( \hat{u}_{\lambda_i} \). Subsequently, the optimal \( \Psi \) which maximizes the minimum bidirectional effective channel gain across all the sub-bands of the \( K \) node pairs, while satisfying \( C_3 \), is given by

\[
\Psi = \text{Proj}_R(\hat{\Psi}),
\]

where \( \text{Proj}_R(\hat{\Psi}) \equiv \arg\min_{\Psi \in \mathbb{C}} \| \Psi - \hat{\Psi} \|, \quad \hat{\Psi} \equiv [a_{\lambda_i}^{(1, R)}]_{a_{\lambda_i}^{(1, R+1)}} \), \( \lambda_i \equiv \arg\max_{i \in \{1, 2\}} \text{Tr}\left( \hat{u}_{\lambda_i}^H \hat{H} \hat{u}_{\lambda_i} \right) \) and the optimal \( \hat{\lambda}_i \) can be obtained through a linear search over \( 0 \leq \lambda_i \leq 1 \).

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