Investigations on the deconfining phase transition in QCD

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We investigate the deconfining phase transition in SU(3) pure gauge theory and in full QCD with two flavors of staggered fermions by means of a gauge invariant thermal partition functional. In the pure gauge case our finite size scaling analysis is in agreement with the well known weak first order phase transition. In the case of 2 flavors full QCD we find that the phase transition is consistent with weak first order, contrary to the expectation of a crossover for not too large quark masses.

1. INTRODUCTION

To detect the deconfinement phase transition in pure gauge theories the expectation value of the trace of the Polyakov loop is used as an order parameter. In presence of dynamical fermions the Polyakov loop ceases to be an order parameter since Z(N) symmetry is no longer a symmetry of the action. Alternatively deconfinement could be detected by looking at Abelian monopole condensation which can be detected by means of the vacuum expectation value of a magnetically charged operator whose expectation value is different from zero in the confined phase and goes to zero at the deconfining phase transition.

In a different way, abelian monopole condensation can be signaled by the free energy needed to create an abelian monopole in the vacuum. This free energy can be computed through a gauge invariant lattice effective action which in turn is defined at zero temperature by means of the lattice Schrödinger functional and at finite temperature by means of a thermal partition functional. Since abelian monopole condensation is not related, as the Polyakov loop, to a symmetry of the lattice action, it could still be useful in order to detect a phase transition even if dynamical fermions are included into the action. The lattice thermal partition functional in presence of a static background field is defined as

\[ Z_T \left[ \vec{A}^{\text{ext}} \right] = \int \frac{DU e^{-S_W}}{Z_T[0]}, \]

(1)

where \( S_W \) is the Wilson action and \( M \) is the fermionic matrix. Notice that the fermionic fields

\[ F(\vec{A}^{\text{ext}}) = -\ln \frac{Z_T[\vec{A}^{\text{ext}}]}{Z_T[0]}. \]

(3)

When including dynamical fermions, the thermal partition functional in presence of a static external background gauge field, Eq. (1), becomes:

\[ Z_T \left[ \vec{A}^{\text{ext}} \right] = \int \frac{DU e^{-S_W}}{Z_T[\vec{A}^{\text{ext}}]} \det M, \]

where \( S_W \) is the Wilson action and \( M \) is the fermionic matrix.
are not constrained and the integration constraint is only relative to the gauge fields: this leads, as in the usual QCD partition function, to the appearance of the gauge invariant fermionic determinant after integration on the fermionic fields.  

To detect monopole condensation we consider Eq. (2) the following quantity defined in terms of free energy $F$ needed to create a monopole in the vacuum:

$$e^{-F/T_{\text{phys}}} = \frac{Z_T[A_{\text{ext}}]}{Z_T[0]}.$$  

(4)

In presence of monopole condensation $F$ is finite and $e^{-F/T_{\text{phys}}} \neq 0$. In practice it is easier to compute $F'(\beta)$ the $\beta$-derivative of the free energy (eventually, since $F = 0$ at $\beta = 0$, $F$ can be obtained by numerical integration of $F'(\beta)$).

2. SU(3) Pure Gauge

We consider SU(3) pure gauge in an abelian monopole background field. The gauge potential in the continuum is

$$g^0(\vec{x}) = \delta^{a3} n_{\text{mon}} \frac{\vec{x} \times \vec{n}}{|\vec{x}|(|\vec{x} - \vec{r} \times \vec{n})},$$  

(5)

where $\vec{n}$ is the direction of the Dirac string and $n_{\text{mon}}$ (integer) is the number of monopoles. On the lattice the spatial links exiting from the sites at the boundary of the time slice $x_t = 0$ are constrained to the (lattice version of) the gauge potential Eq. (4). Links at spatial boundaries ($x_t \neq 0$) can be constrained to the monopole background field ("spatial fixed boundary conditions") or can be periodic ("spatial periodic boundary conditions"). These two different choices (for the monopole field which vanishes at infinity) are equivalent in the thermodynamical limit. The simulations were performed on lattices of different spatial sizes ($16^3$, $24^3$, and $32^3$) and fixed the temporal extent ($L_t = 4$), using APEmille/crate in Bari. We find that our data for $F'(\beta, L_s^{\text{eff}})$ can be fitted according to the following scaling law

$$F'(\beta, L_s^{\text{eff}}) = \frac{a_1 (L_s^{\text{eff}})^\gamma}{(L_s^{\text{eff}})^{1/\nu(\beta - \beta_c)} - d_1 \nu},$$  

(6)

where the above scaling relation holds quite well for a large range of the scaling variable $x = (L_s^{\text{eff}})^{1/\nu(\beta - \beta_c)}$, with $\nu = 0.334 \pm 0.021$ consistent with a first order phase (see Fig. 1).
increases as the lattice volume is increased. At fixed spatial volume the critical coupling depends on \( m_q \). All our lattice data can be described by a finite size scaling function where the gauge critical coupling now does depend on the quark mass and is determined by the chiral critical point [6,7,8]

\[
F'(\beta, L_{s\text{eff}}, m_q) = a_1(L_{s\text{eff}})^{\gamma} \left[ (L_{s\text{eff}})^{1/\nu (\beta - \beta_c(0) - cm_q^2)} - d_1 \right]^{\alpha},
\]

the relevant chiral critical exponents being compatible with those of the three-dimensional O(4) symmetric spin models [7]. We find that the critical exponent \( \nu = 0.31 \pm 0.03 \) is consistent with a first order phase transition, even though weaker than in SU(3) pure gauge (analogous indications have been obtained by the Pisa group [9]).

4. CONCLUSIONS

We investigated the phase transition in SU(3) pure gauge theory and in full QCD with two flavors of degenerate staggered fermions. To locate the phase transition we looked at the free energy to create a monopole in the vacuum. In the pure gauge case our finite size scaling analysis is consistent with the well known weak first order phase transition. In the case of QCD we find that the phase transition is compatible with a weak first order phase transition (contrary to the expectation of a crossover for not too large quark masses).

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