Learning Deep Representation with Energy-Based Self-Expressiveness for Subspace Clustering

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Abstract

Deep subspace clustering has attracted increasing attention in recent years. Almost all the existing works are required to load the whole training data into one batch for learning the self-expressive coefficients in the framework of deep learning. Although these methods achieve promising results, such a learning fashion severely prevents from the usage of deeper neural network architectures (e.g., ResNet), leading to the limited representation abilities of the models. In this paper, we propose a new deep subspace clustering framework, motivated by the energy-based models. In contrast to previous approaches taking the weights of a fully connected layer as the self-expressive coefficients, we propose to learn an energy-based network to obtain the self-expressive coefficients by mini-batch training. By this means, it is no longer necessary to load all data into one batch for learning, and it thus becomes a reality that we can utilize deeper neural network models for subspace clustering. Considering the powerful representation ability of the recently popular self-supervised learning, we attempt to leverage self-supervised representation learning to learn the dictionary. Finally, we propose a joint framework to learn both the self-expressive coefficients and dictionary simultaneously, and train the model in an end-to-end manner. The experiments are performed on three publicly available datasets, and extensive experimental results demonstrate our method can significantly outperform the other related approaches. For instance, on the three datasets, our method can averagely achieve 13.8%, 15.4%, 20.8% improvements in terms of Accuracy, NMI, and ARI over SENet which is proposed very recently and obtains the second best results in the experiments.

Introduction

Subspace clustering aims to cluster data into different groups, where the samples in each group are drawn from the same subspace (Li et al. 2021). So far, subspace clustering has been widely studied in the machine learning and computer vision communities (Liu, Lin, and Yu 2010; Lu et al. 2012, 2013; Elhamifar and Vidal 2013; Hu et al. 2014; Li, You, and Vidal 2017). A mainstream strategy among these methods is to take advantage of the self-expressiveness property which seeks to express each data point as a linear combination of other data points. The typical works include low-rank representation(LRR) (Liu, Lin, and Yu 2010), sparse subspace clustering(SSC) (Elhamifar and Vidal 2013), block diagonal representation (Lu et al. 2018), stochastic sparse subspace clustering (Chen, Li, and You 2020), etc. However, these methods usually use linear models to minimize the reconstruction error, which often fails in handling the data with nonlinear structures. To address this issue, several kernel-based subspace clustering methods have been proposed (Patel and Vidal 2014; Patel, Van Nguyen, and Vidal 2015). As we know, it is difficult for kernel based methods to find an optimal kernel function in real-world applications.

In recent years, many deep learning based methods have been proposed (Peng et al. 2016, 2018; Ji et al. 2017; Zhang et al. 2018; Peng et al. 2020), because of the powerful representation ability of deep learning. A basic scheme among these methods is to leverage deep learning to map the original input into a latent space, and introduce a self-representative layer realized by a fully connected layer to learn the affinity graph (Ji et al. 2017). Following this line, more works are successively proposed to boost the performance of subspace clustering, including deep adversarial subspace clustering(DASC) (Zhou, Hou, and Feng 2018), latent distribution preserving(DPSC) (Zhou et al. 2019), multi-scale fusion (Dang et al. 2020), dual self-supervised CNN network (Zhang et al. 2019), learning representations for subspace clustering(LRSC) (Li et al. 2021), etc.

Although these deep subspace clustering methods achieve promising performance, they often suffer from the following limitations in real-world applications: Since these deep methods take advantage of a fully connected layer as the self-expressive layer and take the weights of the fully connected layer as the self-expressive coefficients, it is necessary to once load the whole dataset into one batch for the self-expressive matrix learning. This kind of learning strategy severely prevents from the usage of deeper neural network models (e.g., ResNet (He et al. 2016)), which results in the limited representation abilities of both the dictionary and the self-expressive coefficient matrix. (Zhang et al. 2021) proposes a self-expressive network, called SENet, for subspace clustering, which can avoid putting all data into one batch and thus can handle large-scale data. However, SENet takes the original input matrix as the dictionary, and performs data reconstruction in the original input space, limit-
Proposed Method

In this section, we will elaborate the details of our method. As shown in Figure 1, our method mainly consists of two modules: a dictionary learning module and a self-expressive set of self-expressive layers to handle multi-scale information extracted from different layers, and presents a multi-scale fusion module to obtain a more discriminative self-expressive matrix. Recently, LRSC (Li et al. 2021) leverages meta-learning to transfer knowledge from the external data to improve the representation ability of the model on the target data, such that the performance of subspace clustering can be improved. Because of the introduction of the self-expressive layer, the above methods take the weights of the self-expressive layer as the self-expressive coefficients. Thus, it is required to once load all samples into one batch for training, which limits the usage of deeper neural network models and prevents the models applying large-scale data.

Energy-based Models

The energy based models (LeCun et al. 2006) (EBMs) aim to capture dependencies between variables by computing a scalar energy value. In the learning stage, the goal of the energy based models is to seek an energy function which generates low energies to desired inputs, but high energies to incorrect ones. Due to its effectiveness, energy based models have been successfully applied to the field of machine learning in recent years (Zhai et al. 2016; Nijkamp et al. 2019; Grathwohl et al. 2020b; Arbel, Zhou, and Gretton 2020). (Zhao, Mathieu, and LeCun 2017) utilizes EBMs to enhance the performance of the generative adversarial networks (Goodfellow et al. 2014). (Pang et al. 2020) builds an energy-based prior model that stands on the top-down generator model to effectively capture regularities of data. (Grathwohl et al. 2020a) designs a classifier from the perspective of EBMs to enhance the abilities of calibration, robustness, and out-of-distribution detection. To the best of our knowledge, there is few work to explore energy based models for deep subspace clustering.

Self-Supervised Representation Learning

Self-supervised learning is an active research topic for representation learning in recent years (Caron et al. 2020; Grill et al. 2020; Zbontar et al. 2021; He et al. 2020; Chen and He 2021). These methods usually define two augmentations of one sample as one positive sample pair, and attempt to maximize the similarity between them by various criteria. The representative works include MoCo (He et al. 2020), SimSiam (Chen and He 2021), etc. MoCo (He et al. 2020) attempts to build a dynamic dictionary with a queue and a moving-averaged encoder using a contrastive loss, with the purpose of improving the representation abilities of the encoder. SimSiam (Chen and He 2021) learns a visual representation encoder by adopting a stop-gradient operation to prevent collapsing solutions. Because of not involving label information, self-supervised learning possesses huge potential in representation learning.

Related Work

In this section, we will briefly review several related works, including deep subspace clustering, energy-based models and self-supervised learning.

Deep Subspace Clustering

Many deep subspace clustering methods have been proposed in the past decade (Ji et al. 2017; Zhou, Hou, and Feng 2018; Zhou et al. 2019; Dang et al. 2020). An earlier and prominent work is DSCN proposed in Ji et al. (2017). DSCN proposes to map data into a latent space by leveraging deep learning, and then introduces a self-expressive layer to learn the self-expressive coefficients. Following this work, DASC (Zhou, Hou, and Feng 2018) utilizes GAN (Goodfellow et al. 2014) to enhance the representation ability of the latent space which is well-suited for self-expressive coefficient learning. DPSC (Zhou et al. 2019) introduces a distribution consistency loss to preserve the intrinsic cluster structure of data, so as to improve the performance of subspace clustering. SC-MSFSC (Dang et al. 2020) designs a

The main contributions of this work can be summarized as follows:

- We propose a new subspace clustering framework based on the energy models, which can be end-to-end trainable. To the best of our knowledge, this is the first work to explore energy based models for subspace clustering.
- We introduce an energy based network for learning the self-expressive coefficient matrix, addressing the issue that deeper neural network models can not be adopted to subspace clustering.
- We propose to learn the dictionary in a self-supervised learning mechanism, and present a joint framework to simultaneously learn the dictionary and self-expressive coefficient matrix.
- Empirical studies on three datasets demonstrate the superiority of our method against other state-of-the-art methods. Specially, our method averagely obtains 13.8%, 15.4% and 20.8% improvements in terms of Accuracy, NMI and ARI on the three datasets over SENet which achieves the second best results in the experiments.
Coefficient learning module based on energy based models. The dictionary learning module aims to learn an effective dictionary, which can be obtained by a very deep neural network in a self-supervised learning manner. The self-expressive coefficient learning module is used to learn the self-expressive coefficients, and thus the representation ability of the model is limited. However, such a learning strategy makes it necessary to once load the whole data into one batch for learning, such that the deeper neural network models can not be adopted to subspace clustering, and thus the representation ability of the model is limited.

In (2), the self-expressive matrix $\mathbf{C}$ is realized by a fully connection layer, where the weights of the layer are regarded as the solution. However, such a learning strategy makes it necessary to once load the whole data into one batch for learning, such that the deeper neural network models can not be adopted to subspace clustering, and thus the representation ability of the model is limited.

To address this issue, we propose to learn an energy network to learn the self-expressive coefficient matrix, so as to avoid once putting all data into one batch. In this way, we can learn an effective dictionary in a self-supervised learning strategy using a deeper network architecture, which can significantly improve the performance of subspace clustering. Next, we will introduce them in detail.

Preliminaries

Let $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_n] \in \mathbb{R}^{d \times n}$ be the input matrix, where $d$ is the dimensions of the feature, and $n$ is the number of the sample. For traditional self-expressive based subspace clustering methods, the objective function can be formulated as:

$$
\min_{\mathbf{C}, \mathbf{Y}} ||\mathbf{X} - \mathbf{YC}||^2_F + \lambda \Omega(\mathbf{C})
$$

(1)

where $\lambda$ is a trade-off parameter. $\mathbf{Y} \in \mathbb{R}^{d \times n}$ is a dictionary which can be learned or pre-defined. $\mathbf{C} \in \mathbb{R}^{n \times n}$ is the self-expressive coefficient matrix. $\Omega(\mathbf{C})$ is the regularizer on $\mathbf{C}$ for discovering the subspace structure. The methods in this line aim to seek a linear projection to reconstruct the inputs, which often fails in handling the non-linear data.

In recent years, many works attempt to leverage deep learning to find a latent subspace, where a self-expressive layer is introduced to learn the self-expressive coefficients.

The representative work is DSCN (Ji et al. 2017), whose objective function is as:

$$
\min L = \frac{1}{2} ||\mathbf{X} - \hat{\mathbf{X}}||^2_F + \frac{\lambda_1}{2} ||\mathbf{Z} - \mathbf{ZC}||^2_F + \lambda_2 ||\mathbf{C}||^2_F
$$

(2)

s.t. diag$(\mathbf{C}) = 0$

where $\cdot ||_F$ is the Frobenius norm of a matrix, $||\mathbf{C}||_F$ denotes a certain matrix norm on $\mathbf{C}$. $\mathbf{Z}$ and $\hat{\mathbf{X}}$ are the latent representation and the reconstruction of $\mathbf{X}$, respectively.

In (2), the self-expressive matrix $\mathbf{C}$ is realized by a fully connection layer, where the weights of the layer are regarded as the solution. However, such a learning strategy makes it necessary to once load the whole data into one batch for learning, such that the deeper neural network models can not be adopted to subspace clustering, and thus the representation ability of the model is limited.

To address this issue, we propose to learn an energy network to learn the self-expressive coefficient matrix, so as to avoid once putting all data into one batch. In this way, we can learn an effective dictionary in a self-supervised learning strategy using a deeper network architecture, which can significantly improve the performance of subspace clustering. Next, we will introduce them in detail.

Dictionary Learning Module

Let $\mathbf{X}^1 = [\mathbf{x}_1^1, \ldots, \mathbf{x}_n^1] \in \mathbb{R}^{d \times n}$ and $\mathbf{X}^2 = [\mathbf{x}_1^2, \ldots, \mathbf{x}_n^2] \in \mathbb{R}^{d \times n}$ be two augmentation matrices of $\mathbf{X}$ using two different data augmentation strategies. The Siamese network consists of two encoder networks which are denoted by $f_{\theta_1}$(·) and $f_{\theta_2}$(·). $\mathbf{Z}^1 = [\mathbf{z}_1^1, \ldots, \mathbf{z}_n^1] \in \mathbb{R}^{d \times n}$. $\mathbf{Z}^2 = [\mathbf{z}_1^2, \ldots, \mathbf{z}_n^2] \in \mathbb{R}^{d \times n}$.
Algorithm 1: Compute Self-expressive coefficients using Energy based Network

**Input:** Representations $z_i^x \in \mathbb{R}^{p \times 1}$, a dictionary $Z^B = [z_1^B, \ldots, z_n^B] \in \mathbb{R}^{p \times n}$. Energy based network $E_{\theta_1}$

**Initialization:** initialize an empty collection $E = \phi$

for each $z_i^x \in [z_1^x, \ldots, z_n^x]$

1: Compute $z_{ij} = concatenate(z_i^x, z_j^x)$ and its dimension is $2p$
2: Compute the energy value $e_{ij} = E_{\theta_1}(z_{ij})$, which is a scalar value.
3: Put $e_{ij}$ into collection $E$

end for

**Output:** self-expressive vector $c_i$, where $c_{ij} = \frac{e^{-e_{ij}}}{\sum_{a=1}^{n} e^{-e_{ia}}}$

$\mathbb{R}^{p \times n}$ are the latent representations obtained from $f_{\theta_1}(x^1)$, $f_{\theta_2}(X^B)$, respectively. $p$ is the dimension of the latent representation.

Recall the existing deep subspace clustering methods, one major goal is to find a latent space which is well-suited to perform the self-expressive learning. Hence, the key to the success of deep subspace clustering methods is to find an effective dictionary which can better reconstruct the samples.

Considering the powerful representation abilities of self-supervised learning methods, we attempt to leverage self-supervised learning to enhance the representation ability of the dictionary. Different from DSCN directly taking the latent representations itself as the dictionary, we first utilize the Siamese network to obtain the latent representations of two augmentations of one sample, and then take one representation as the basis vector to form the dictionary, i.e., $Z^B$. And we approximate the representation of the other augmentation using this dictionary. The reconstructed loss can be formulated as:

$$
\min \sum_{i=1}^{n} \| z_i^a - Z^B c_i^T \|^2_2 = \sum_{i=1}^{n} \| z_i^a - \sum_{j=1}^{n} c_{ij} z_j^B \|^2_2
$$

(3)

where $c_i = [c_i^1, \ldots, c_i^n] \in \mathbb{R}^{1 \times n}$ is the self-expressive vector for $z_i^a$, and $c_{ij}$ is a self-expressive coefficient. Considering the fact that C often possesses the group effect \cite{hu2014sparse}, we present another loss function as follows:

$$
\min_{\theta_1, \theta_2} \sum_{i=1}^{n} \| z_i^a - Z^B c_i^T \|^2_2 + \lambda \Omega(C)
$$

(4)

where $\lambda$ is the temperature hyper-parameter. The first term in (3) is the contrastive loss. Here, we can use other self-supervised loss to take place of it. In the experiment, we also use the loss in SimSiam \cite{chen2021exploring} to show our method is compatible with these self-supervised losses.

Our objective function in (4) aims to minimize the reconstructed error and control the complexity of self-expressive coefficients using $\ell_2$-norm to discover the subspace structure of the data.

**Energy based Self-Expressiveness Module**

In this module, we will give the details about how to learn an energy network $E_{\theta_1}$ to learn the self-expressive coefficient matrix $C$ in (3). Firstly, as shown in Figure 2, we utilize the energy based network to measure whether a representation and a basis vector are in a same subspace, i.e.,

$$
e_{ij} = E_{\theta_1}(z_i^a, z_j^B)
$$

(5)

where the $e_{ij}$ is a scalar energy. When they are in the same subspace, the energy value should be smaller, otherwise the opposite. However, since the range of energy values is relatively large, and the physical meaning of energy is contrary to the theory of graph cut \cite{shi2000normalized}, we can’t

$\tau$ is a temperature hyper-parameter. The first term in (5) is the contrastive loss. Here, we can use other self-supervised loss to take place of it. In the experiment, we also use the loss in SimSiam \cite{chen2021exploring} to show our method is compatible with these self-supervised losses.

Algorithm 2: Train all modules together

**Input:** Dataset $X = \{x_1, \ldots, x_n\} \in \mathbb{R}^{p \times n}$, number of iterations $T$, learning rate $\eta$, the Siamese network’s parameters $\theta_1$ and $\theta_2$

**Initialization:** Energy based network parameters $\theta_1$, dictionary $Z^B = \phi$, self-expressive matrix $C$ whose size is $n \times n$, augmentation method $aug_1$, $aug_2$

# Initialize the dictionary
for each $i \in \{1, \ldots, n\}$

1: Sample a data point $x_i$ from X
2: Compute $x_i^a$, $x_i^B = aug_1(x_i), aug_2(x_i)$
3: Compute $z_i^a$, $z_i^B = f_{\theta_1}(x_i^a)$, $f_{\theta_2}(x_i^B)$
4: Put $z_i^B$ into $Z^B$

end for

# Train all modules
foreach $t \in \{1, \ldots, T\}$

1: for each $x_i \in \{x_1, \ldots, x_n\}$ do
2: Compute $x_i^a$, $x_i^B = aug_1(x_i), aug_2(x_i)$
3: # Forward propogation to compute loss
4: Compute $z_i^a$, $z_i^B = f_{\theta_1}(x_i^a)$, $f_{\theta_2}(x_i^B)$
5: Compute $c_i = E_{\theta_1}(z_i^a, z_i^B)$ using Algorithm (1).
6: Compute $z_i = Z^B \cdot c_i^T$
7: Compute loss $L(z_i^a, z_i^B, z_i, c_i; \theta = \{\theta_1, \theta_2, \theta_3\})$
8: # Backward propogation to compute gradient
9: Compute $d\theta = \frac{\partial L(z_i^a, z_i^B, z_i, c_i; \theta = \{\theta_1, \theta_2, \theta_3\})}{\partial \theta}$
10: Set $\theta \leftarrow \theta - \eta \cdot d\theta$
11: Update $Z^B$ where $Z^B[i] = z_i^B$
12: Update $C$ where $C[i] = c_i$
13: end for

end for

**Output:** Self-expressive matrix $C$
Table 1: Clustering results on the three datasets in terms of ACC, NMI, and ARI.

| Dataset      | Metric | SSC-OMP | ENSC | Cluster-GAN | DSCN-L1 | DSCN-L2 | DASC | DPSC | LRSC-L1 | LRSC-L2 | SENet | LDRSC |
|--------------|--------|---------|------|-------------|---------|---------|------|------|---------|---------|-------|-------|
| Fashion-mnist| ACC    | 0.423   | 0.519| 0.581       | 0.608   | 0.585   | 0.647| 0.624| 0.652   | 0.651   | 0.712 | 0.863 |
|              | NMI    | 0.508   | 0.559| 0.597       | 0.640   | 0.626   | 0.647| 0.645| 0.672   | 0.672   | 0.685 | 0.802 |
|              | ARI    | 0.302   | 0.392| 0.446       | 0.470   | 0.447   | 0.483| 0.484| 0.529   | 0.527   | 0.575 | 0.730 |
| notMNIST     | ACC    | 0.277   | 0.267| 0.510       | 0.517   | 0.517   | 0.514| 0.566| 0.587   | 0.608   | 0.694 | 0.841 |
|              | NMI    | 0.223   | 0.226| 0.345       | 0.501   | 0.504   | 0.506| 0.508| 0.517   | 0.523   | 0.594 | 0.780 |
|              | ARI    | 0.076   | 0.026| 0.246       | 0.413   | 0.412   | 0.408| 0.415| 0.4339  | 0.467   | 0.417 | 0.695 |
| CIFAR-10     | ACC    | 0.326   | 0.613| -           | -       | -       | -   | -   | -       | -       | 0.765 | 0.880 |
|              | NMI    | 0.498   | 0.601| -           | -       | -       | -   | -   | -       | -       | 0.655 | 0.813 |
|              | ARI    | 0.196   | 0.430| -           | -       | -       | -   | -   | -       | -       | 0.573 | 0.763 |

Figure 2: illustration of the Energy based network

directly use them as the self-expressive coefficients. Thus, we transform the energy values to obtain the self-expressive coefficients through the Gibbs distribution:

\[ c_{ij} = \frac{e^{-\beta e_{ij}}}{\sum_{k=1}^{n} e^{-\beta e_{ik}}} \]  

where \( \beta \) is an arbitrary positive constant akin to an inverse temperature. For simplicity, we set \( \beta \) to 1. In Eq. 7, when two samples belong to the same subspace, the value of \( c_{ij} \) will be larger. Therefore, \( c_{ij} \) can be used as the self-expressive coefficient. The key steps are summarized in Algorithm 1.

Overall Deep Subspace Clustering Framework

We integrate the dictionary learning module and the energy-based self-expressiveness module into a joint framework. Similar to most DSC methods, we train our method with a two-stage strategy: 1) pre-train dictionary learning module leveraging a self-supervised manner; 2) train all modules by minimizing our objective function (5) with the gradient descent method, as shown in Algorithm 2.

After training, the self-expressive coefficient matrix \( C \) can be obtained. Then, the affinity matrix can be computed by \( \frac{1}{4}(|C| + |C^T|) \). Finally, we apply the spectral clustering algorithm (Shi and Malik 2000) on the affinity matrix to obtain the clustering result.

EXPERIMENTS

In this section, we perform extensive experiments on three publicly available datasets, in order to evaluate the performance of our proposed approach.

**Dataset**

- **Fashion-MNIST** (Xiao, Rasul, and Vollgraf 2017): It contains 70,000 grey-scale images of 10 categories of fashion products. To conduct a fair comparison, we use the same data with (Li et al. 2021), i.e., 1,000 images from each class are randomly selected, leading to a total of 10,000 images in our experiment.

- **notMNIST** (Bulatov 2011): It consists 70,000 grey-scale images of 10 classes with letters A-J taken from different fonts. To compare fairly, we use the same data with (Li et al. 2021), i.e., 1,000 images from each class are randomly selected. Thus, there are totally 10,000 images in our experiment.

- **CIFAR-10** (Krizhevsky, Hinton et al. 2009): It contains 60,000 color images in 10 classes, where each image is of size 32 \( \times \) 32. We use the whole dataset as a large-scale dataset to evaluate the performance of our method in the experiment.

**Experimental Setting**

**Implementation Details.** The backbone of the Siamese network is ResNet-50 (He et al. 2016) and we initialize our energy based network using a MLP with two hidden layers. The data augmentation strategies we used are as follows: a 32 \( \times \) 32-pixel crop is taken from a randomly resized image, and then undergoes random color jittering, random horizontal flip, and random gray-scale conversion. There are two important hyper-parameters in our method, i.e., \( \lambda_1 \) and \( \lambda_2 \). They are used to control the contribution of the reconstructed loss and the complexity of the self-expressive coefficients, respectively. Throughout the experiments, we set \( \lambda_2 \) to 1, and search \( \lambda_1 \) from \{5, 10, 15, 20, 25\}.

**Metrics and Methods.** To evaluate the clustering performance of our method, we use three widely used evaluation measures, including accuracy (ACC), normalized mutual information (NMI) and adjusted rand index (ARI). To further verify our method, we compare it with several related approaches, including seven deep subspace clustering methods: DSCN-L1 (Ji et al. 2017), DSCN-L2 (Ji et al. 2017), DASC (Zhou, Hou, and Feng 2018), DPSC (Zhou et al. 2019), LRSC-L1 (Li et al. 2021), LRSC-L2 (Li et al. 2021), SENet (Zhang et al. 2021). We also compare with two typical linear subspace clustering methods: SSC-OMP (You, 

Table 2: Clustering results on the Fashion-MNIST with different numbers of data points.

| No. Points | Metric | ACC | NMI | ARI |
|------------|--------|-----|-----|-----|
| 1000       | SSC-OMP| 0.343| 0.301| 0.148|
| 5000       | EnSC   | 0.566| 0.510| 0.366|
| 10000      | Cluster-GAN | 0.558| 0.504| 0.354|
|            | DSCN-L1| 0.536| 0.594| 0.406|
|            | DSCN-L2| 0.536| 0.594| 0.406|
|            | DASC   | 0.548| 0.573| 0.400|
|            | DPSC   | 0.560| 0.606| 0.426|
|            | LRSC-L1| 0.588| 0.645| 0.458|
|            | LRSC-L2| 0.607| 0.627| 0.439|
|            | SENet  | 0.719| 0.653| 0.554|
|            | LDRSC  | 0.984| 0.966| 0.965|

Table 3: Clustering results on the notMNIST with different numbers of data points.

| No. Points | Metric | ACC | NMI | ARI |
|------------|--------|-----|-----|-----|
| 1000       | SSC-OMP| 0.424| 0.351| 0.226|
| 5000       | EnSC   | 0.481| 0.372| 0.246|
| 10000      | Cluster-GAN | 0.475| 0.386| 0.230|
|            | DSCN-L1| 0.488| 0.440| 0.318|
|            | DSCN-L2| 0.489| 0.439| 0.316|
|            | DASC   | 0.483| 0.441| 0.312|
|            | DPSC   | 0.471| 0.430| 0.303|
|            | LRSC-L1| 0.574| 0.509| 0.127|
|            | LRSC-L2| 0.578| 0.512| 0.430|
|            | SENet  | 0.614| 0.502| 0.353|
|            | LDRSC  | 0.969| 0.940| 0.933|

Table 4: Clustering results obtained by using the energy based network with different numbers of hidden layers on the Fashion-MNIST dataset.

| No. Points | No. layers | ACC | NMI | ARI |
|------------|------------|-----|-----|-----|
| 1000       | 2          | 0.984| 0.966| 0.965|
|            | 3          | 0.959| 0.928| 0.912|
|            | 4          | 0.963| 0.930| 0.920|
| 5000       | 2          | 0.898| 0.855| 0.803|
|            | 3          | 0.848| 0.799| 0.713|
|            | 4          | 0.887| 0.841| 0.782|
| 10000      | 2          | 0.863| 0.802| 0.730|
|            | 3          | 0.830| 0.774| 0.679|
|            | 4          | 0.822| 0.770| 0.667|

Robinson, and Vidal (2016) and EnSC (You et al. 2016). In addition, we also compare with a deep learning based clustering method, Cluster-GAN (Mukherjee et al. 2019).

Experimental Result

General Performance. The results of our method and the compared methods are listed in Table 1. From the Table 1, we can see that LDRSC significantly outperforms other baselines on the three datasets. Specifically, compared to SENet which obtains the second best results in the experiments, LDRSC achieves 15.1%, 11.7%, 15.5% improvement over SENet in terms of ACC, NMI, and ARI on the Fashion-mnist dataset, respectively. This illustrates that our method is effective by learning the self-expressive coefficients based on an energy network, and simultaneously learning the dictionary in a self-supervised manner. It is worth noting that we can not implement several deep methods on the larger dataset, CIFAR-10, due to their large memory requirements and time costs.

Effect of Different Sizes of Data. To further verify the effectiveness of our method, we implement all methods by varying the size of datasets. The result are listed in Table 2 and Table 3. We can see that our method still significantly and consistently outperforms other methods under all the cases. Specially, when the number of data points is 1000, LDRSC obtains 24.4%, 27.7%, 36.6% improvements over SENet in terms of ACC, NMI, and ARI on the Fashion-MNIST dataset, and achieves 35.5%, 43.8%, 58.0% improvements over SENet in terms of ACC, NMI, and ARI...
Table 5: Clustering results obtained by using the energy based network and the inner product based attention on the Fashion-MNIST and notMNIST datasets, respectively.

| Dataset     | Metric | Energy | Attention |
|-------------|--------|--------|-----------|
| FashionMNIST| ACC    | 0.984  | 0.977     |
|             | NMI    | 0.966  | 0.954     |
|             | ARI    | 0.965  | 0.950     |
| notMNIST    | ACC    | 0.969  | 0.943     |
|             | NMI    | 0.940  | 0.894     |
|             | ARI    | 0.933  | 0.879     |

Table 6: Results by the contrastive loss and the stop-gradient operation on the Fashion-MNIST dataset.

| No.Points | Metric | Method                | Contrastive Learning | Stop-Gradient |
|-----------|--------|-----------------------|----------------------|---------------|
| 1000      | ACC    | 0.984                 | 0.920                |               |
|           | NMI    | 0.966                 | 0.900                |               |
|           | ARI    | 0.965                 | 0.851                |               |
| 5000      | ACC    | 0.898                 | 0.818                |               |
|           | NMI    | 0.855                 | 0.771                |               |
|           | ARI    | 0.803                 | 0.663                |               |
| 10000     | ACC    | 0.863                 | 0.790                |               |
|           | NMI    | 0.802                 | 0.746                |               |
|           | ARI    | 0.730                 | 0.624                |               |

Evaluation on the Architectures of the EBMIs. Since the energy based network is implemented by a MLP, we evaluate the effect of different numbers of hidden layers in MLP on the clustering results. The experiments are performed on the Fashion-Mnist dataset, and the results are reported in Table 4. We can observe when the number of hidden layers is set to 2, our method obtains the best performance. Thus, we set the number of hidden layers to 2 in the energy network throughout the experiment.

Comparison with Self-Attention based Method. In self-expressive coefficient learning module, we propose an energy based network to reach this goal. In order to verify its effectiveness, we use an attention based method as the baseline. For the attention based method, we utilize the inner product of two samples as the self-expressive coefficient, and take place of the energy network in our framework.

The experiments are performed on the FashionMNIST and notMNIST datasets, and the results are listed in Table 5. We can see that our method consistently achieves better performance than the attention based method, which illustrates that our energy based network is effective for the learning self-expressive coefficients.

Evaluation on the Self-Supervised Learning Manner. In order to demonstrate that our method can be compatible with other self-supervised representation learning methods for learning our dictionary, we rewrite the loss function of dictionary learning $L_1$ as a cosine similarity function with stop-gradient operation (Chen and He 2021). The results are shown in Table 6. Although the performance of the stop-gradient operation is a little worse than that of the contrastive loss in our method, it still obtains better performance than other approaches. Thus, our method can be well compatible with various self-supervised learning frameworks.

Visualization. To intuitively demonstrate our method can learn an effective representation for subspace clustering, we visualize self-expressive coefficient matrices obtained by the energy network and attention based method (inner product) on the Fashion-MNIST dataset. The results are shown in Figure 3. We can clearly observe that our method can learn a more effective representation for subspace clustering.

Conclusion

In this paper, we proposed a new deep subspace clustering method, inspired by the energy models. We attempted to learn an energy network to learn the self-expressive coefficients, addressing the issue that deeper neural network can not be involved in subspace clustering. Meanwhile, we jointly learned an effective dictionary in a self-supervised manner using a deeper network architecture, which can significantly boost the performance. Extensive experiments on three datasets verified the effectiveness of our method.

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