On Bayesian Estimation of System Reliability in Stress – Strength Model Based on Generalized Inverse Rayleigh Distribution

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ABSTRACT

The parameter and system reliability in stress-strength model are estimated in this paper when the system contains several parallel components that have strengths subject to common stress in case when the stress and strengths follow Generalized Inverse Rayleigh distribution by using different Bayesian estimation methods. Monte Carlo simulation introduced to compare among the proposal methods based on the Mean squared Error criteria.

Keyword: Stress-Strength Reliability, Generalized Inverse Rayleigh distribution, Bayesian methods, Monte Carlo simulation and Mean square error.

1. Introduction

The stress-strength models were useful situations to compute the reliability of a system or component, and when defined by the probability that the strength (random variable) is greater than stress (random variable). While the strength lower than the stress, it makes intuitive sense that a component is deemed to have failed [1]. The parallel stress-strength model occurs when a device under consideration is a combination of k usually independent components with the strengths \( X_1, X_2, \ldots, X_k \) and each component of the system is subject to a common stress \( Y \). If the system is operating successfully whenever at least one of the components survives, it is termed parallel in the analogy with electric circuits [2].

Several researchers assumed and studied various lifetime distributions of the parallel of system reliability in stress-strength model; Hanagal in 1996, studied estimation the system reliability consists two parallel components and the two strength belong to Bivariate Pareto (BVP) distribution subject to common stress or two stresses when the stress follows Pareto distribution [3]. Also, in 1998, Hangal considered the estimation for reliability system in S-S model in two cases (parallel and series) of K components when the strengths subject to common stress follow exponential distributions [4].

In 2013, Sezer and Kinac estimated reliability (R) for a system consists of two parallel components based on masked data by using Exponential distribution, and they used a simulation study to a comparison between Bayes and MLE methods [5]. In 2016, Karam derived and estimated the system reliability for two parallel components which are subjected to common stress, when the stress and strength follows Gompertz distribution with known shape parameters and unknown location. She used
(WLSE, LSE and MLE) to estimate the system reliability and she introduced the numerical simulation study to comparison between the proposed estimators by MSE [6]. On the other hand, Inverse Rayleigh distribution (IRD) considered as significant lifetime distributions. IRD has several applications in the area of reliability and life testing study. Inverse Rayleigh distribution can approximate the distribution of lifetimes of several types of experimental units Voda [7]. However, the generalized inverted scale family distributions were introduced by Potdar et. al. [8]. These newly developed models were formulated by introducing a new shape parameter to the scale family of distributions. These models give major flexibility in modeling complex data and the results drawn from them seem genuine and quite sound.

The Generalized Inverse Rayleigh Distribution (GIRD) is a very useful lifetime model which can be applied for analyzing the lifetime data. It is widely used in operation research, applied statistics, reliability analysis, communication engineering, health and biology approximates the lifetimes of several experimental units Uzam et al.[9]. In addition, GIRD can be used in studying radiations, sounds and wind speed. The generalized model of Inverse Rayleigh distribution was further modified by Bakoban and Abu Baker [10]. They illustrated the different characterizing properties of the generalized Inverse Rayleigh distribution. Then, Reshi et.al [11] estimated the scale parameter of GIRD under the different loss functions. Further, Bakoban [12] considered the optimal design problem and the estimation for GIRD under FSS − PALT using type II censored data. Also, Kawser and Ahmad [13] compared the different informative and non-informative priors for the GIRD under different Loss functions. Therefore, in this paper we derived both the MLE and Bayes estimators of the unknown parameter and for the system reliability of $k^{th}$ parallel components in stress-strength (S-S) model using the assumption of Jeffrey’s Prior Information and Gamma Priors under squared error loss function (SELF) and Linear Exponential (LINEX).

2. Model Description with Mathematical Formulation

Let the random variable $X$ have a GIRD with parameters $\theta, \sigma^2$; $X \sim GIRD (\theta, \sigma^2)$, where $\theta, \sigma^2$ are the shape parameters of the generalized model, then the probability density function (pdf) of $X$ has the form below:

$$f(x) = \frac{2\theta \sigma^2}{x^3} e^{-\frac{\sigma^2 \theta}{x^2}} \text{ for } x > 0, \theta, \sigma^2 > 0$$

(1)

And, the cumulative distribution function (c. d. f) of a r.v. given as:

$$F(x) = e^{-\frac{\sigma^2 \theta}{x}}$$

(2)

The formula of system reliability ($R$) in S-S model which contains $K^{th}$ Parallel components which has $K^{th}$ strength subject to common stress can be derive as below [14]:

$$R = P(Y < \max (X_1X_2, \ldots, X_k))$$

Let $$Z = \max X_1X_2, \ldots, X_k$$

Therefore,

$$R = \int_{0}^{\infty} F_Z(y) f(y) dy$$

$$F_Z(Z) = P(Z < z)$$

$$= P(x_1 < z)P(x_2 < z) \ldots P(x_k < z)$$

$$= e^{-\frac{\sigma^2 \theta_1}{z^2}} e^{-\frac{\sigma^2 \theta_2}{z^2}} \ldots e^{-\frac{\sigma^2 \theta_k}{z^2}}$$

$$= e^{\frac{-\sigma^2 \theta_1}{z^2}} \ldots e^{\frac{-\sigma^2 \theta_k}{z^2}}$$

$$R = \int_{0}^{\infty} (1 - e^{\sum_{i=1}^{k} -\frac{\sigma^2 \theta_i}{z^2}}) f(y) dy$$

Consequently, by simplifications, we get

$$R = 1 - \frac{\theta_{k+1}}{\sum_{i=1}^{k+1} \theta_i}$$
\[ R = \frac{\sum_{i=1}^{k} \theta_i}{\sum_{i=1}^{k+1} \theta_i} \]  

(3)

3. Maximum Likelihood Estimator (MLE)

This subsection concern with system consist \( K^\text{th} \) Parallel components which has \( K^\text{th} \) strength random variables \( X_i \) (\( i = 1, 2, ..., k \)) subject to stress random variable \( Y \). We have to obtain the Maximum Likelihood estimator of \( (R) \). Suppose \( x_{11}, x_{12}, ..., x_{1n_1}, x_{21}, x_{22}, ..., x_{2n_2}, ..., x_{k1}, x_{k2}, ..., x_{kn_k} \) be a random sample from GIRD (\( \theta_i, \sigma^2 \)) respectively, for \( i = 1, 2, ..., k \) and assume \( y_1, y_2, ..., y_m \) be a random sample from GIRD (\( \theta_{k+1}, \sigma^2 \)). Then the likelihood function given by;

\[
L = (x_i, y; \sigma^2, \theta_i, \theta_{k+1}) = \prod_{t=1}^{k} \prod_{i=1}^{n_t} f(x_{ti}) \prod_{j=1}^{m} g(y_j), \quad t = 1, 2, ..., k
\]

\[
\frac{\partial \ln L}{\partial \theta_1} = \frac{n_1}{\theta_1} - \sum_{i=1}^{n_1} \frac{\sigma^2}{X_{1i}^2}, \quad \frac{\partial \ln L}{\partial \theta_1} = 0
\]

\[
\frac{n_1}{\theta_1} = \sum_{i=1}^{n_1} \frac{\sigma^2}{X_{1i}^2}
\]

\[
\theta_1 = \frac{n_1}{\sum_{i=1}^{n_1} \frac{\sigma^2}{X_{1i}^2}}
\]

By the same way find \( \theta_2, ..., \theta_k, \theta_{k+1} \)

Then, the Maximum Likelihood estimator for the parameter \( \beta_i (i = 1, 2, ..., k + 1) \) as bellow:

\[
\hat{\beta}_i = \frac{n_i}{\sum_{i=1}^{n_i} \frac{\sigma^2}{X_{1i}^2}} \quad ; i = 1, 2, ..., k
\]

\[
\hat{\beta}_{k+1} = \frac{m}{\sum_{j=1}^{m} \frac{\sigma^2}{y_j^2}} \quad j = 1, 2, ..., m
\]

4. Bayesian Estimator

In this section, the Bayesian analysis of the parameter is studied using Jeffrey’s Prior Information and Gamma priors under two error loss functions namely; squared error loss function (SELF), and Linear Exponential (LINEX).

4.1 Posterior function of the parameter based on Jeffrey’s Prior Information

To find the Jeffrey’s Prior Information, we must find Fisher information since,

\[
g(\theta) \propto \sqrt{I(\theta)}
\]

\[
g(\theta) = c \sqrt{I(\theta)}
\]

Where \( I(\theta) = -n E \left( \frac{\partial^2 \ln f(x|\theta)}{\partial \theta^2} \right) \)

We need to find second derivative after take \( ln \) to the both side

\[
f(X, \theta, \sigma^2) = 2 \theta \sigma^2 X^{-3} e^{-\theta (\frac{\sigma^2}{\theta})^2}
\]

\[
ln f(x) = ln 2 + ln \theta + 2 ln \sigma - 3 ln X - \theta (\frac{\sigma^2}{\theta})^2
\]

\[
\frac{\partial \ln f(x|\theta)}{\partial \theta} = \frac{1}{\theta} - (\frac{\sigma^2}{\theta})^2
\]

\[
\frac{\partial^2 \ln f(x|\theta)}{\partial \theta^2} = -\frac{1}{\theta^2}
\]

\[
E \left( \frac{\partial^2 \ln f(x|\theta)}{\partial \theta^2} \right) = -\frac{1}{\theta^2}
\]
Hence, \( I(\theta) = \frac{n}{\theta^2} \) then \( g(\theta) = c \sqrt{\frac{n}{\theta^2}} \)

\[
g(\theta) = \frac{c}{\theta} \sqrt{n} \tag{4}
\]

The posterior density function is defined as:

\[
G_1(\theta | x_1, x_2, \ldots, x_n) = \frac{H(x_1, x_2, \ldots, x_n; \theta)}{P(x_1, x_2, \ldots, x_n; \theta)}
\]

When,

\[
H(x_1, x_2, \ldots, x_n; \theta) = L(x_1, x_2, \ldots, x_n; \theta) g(\theta)
\]

Then the marginal probability density function of \((x_1, x_2, \ldots, x_n)\) is given by

\[
P(x_1, x_2, \ldots, x_n; \theta) = \int_0^\infty L(x_1, x_2, \ldots, x_n; \theta) g(\theta) \, d\theta
\]

Therefore,

\[
G_1(\theta | x_1, x_2, \ldots, x_n) = \frac{H(x_1, x_2, \ldots, x_n; \theta)}{\sqrt{n}} \frac{2^n \theta^{n-1} \sigma^{2n} \prod_{i=1}^n X_i^{-3} e^{-\theta w}}{\int_0^\infty c \sqrt{n} 2^n \theta^{n-1} \sigma^{2n} \prod_{i=1}^n X_i^{-3} e^{-\theta w} \, d\theta}
\]

where \( w = \sum_{i=1}^n \frac{\sigma_i^2}{X_i} \) and let \( z = \theta w \), \( \theta = \frac{z}{w} \), \( d\theta = \frac{dz}{w} \).

Hence, the posterior density function for \( \theta \) based on Jeffery’s prior information will be

\[
G_1(\theta | x_1, x_2, \ldots, x_n) = \frac{w^n \theta^{n-1} e^{-z}}{\Gamma(n)}
\]

\[
G_1(\theta | x_1, x_2, \ldots, x_n) = \frac{\sum_{i=1}^n \frac{\sigma_i^2}{X_i}}{\Gamma(n)}
\]

The posterior density in equation can be identified as a density of Gamma distribution, that is

\[
\theta \sim \text{Gamma} \left( n, \frac{1}{w} \right) \quad \text{with} \quad E(\theta) = \frac{n}{w} \quad \text{and} \quad \text{var}(\theta) = \frac{n}{w^2}
\]

\[
\theta_1 \sim \text{Gamma} \left( n_1, \frac{1}{w} \right) \quad \text{with} \quad E(\theta_1) = \frac{n_1}{w} \quad \text{and} \quad \text{var}(\theta_1) = \frac{n_1}{w^2}
\]

By the same way, we find posterior function for \( \theta_2, \ldots, \theta_k, \theta_{k+1} \)

4.2 Posterior function of the parameter based on Gamma prior information

\[
g_2(\theta) = \frac{\beta \delta^\theta e^{-\theta \beta}}{\Gamma(\alpha)} \quad ; \quad \theta > 0, \beta > 0, \sigma > 0
\]

\[
G_2(\theta | x_1, x_2, \ldots, x_n) = \frac{H(x_1, x_2, \ldots, x_n; \theta)}{P(x_1, x_2, \ldots, x_n; \theta)}
\]

\[
= \frac{2^n \theta^n \sigma^{2n} \prod_{i=1}^n X_i^{-3} e^{-\theta w} \beta^\delta \delta^\theta e^{-\theta (w + \beta)}}{\Gamma(\alpha) \int_0^\infty 2^n \sigma^{2n} \prod_{i=1}^n X_i^{-3} \beta^\delta \delta^\theta \Gamma(\alpha) \int_0^\infty g_{n+n+\delta-1} e^{-\theta (w + \beta)} \, d\theta}
\]

Thus

\[
G_2(\theta | x_1, x_2, \ldots, x_n) = \frac{p^{n+\delta} \theta^{n+\delta-1} e^{-\theta \alpha}}{\Gamma(n+\delta)} , \quad \text{where} \quad P = (w + \beta)
\]

It can easily be noted that

\[
\theta \sim \text{Gamma} \left( n + \delta, \frac{1}{w} \right) \quad \text{with} \quad E(\theta) = \frac{n+\delta}{p} \quad \text{and} \quad \text{var}(\theta) = \frac{n+\delta}{p^2}
\]
Therefore,
\[ \theta_1 \sim \text{Gamma} \left( n_1 + \frac{1}{p} \right) \text{ with } E(\theta_1) = \frac{n_1 + \delta}{p} \]  
and \[ \text{var}(\theta_2) = \frac{n_1 + \delta}{p^2} \]
We find posterior density function for \( \theta_2, \ldots, \theta_k, \theta_{k+1} \) based on Gamma prior information by the same way.

5. Bayes estimator under Considered Error Loss Functions

5.1 Bayes estimator under Squared Error Loss Function

Consider the Squared error loss function \( l(\hat{\theta}, \theta) \) as below
\[ l(\hat{\theta}, \theta) = (\hat{\theta} - \theta)^2 \]
Since the mathematical expectation for the loss function is called Risk function is given in the following formula:
\[
R(\hat{\theta}, \theta) = E \left[ l(\hat{\theta}, \theta) \right] = \int_{\theta} l(\hat{\theta}, \theta) G_1(\theta|x_i) \ d\theta
\]
\[= \int_{\theta} (\hat{\theta} - \theta)^2 G_1(\theta|x_i) \ d\theta - 2 \int_{\theta} \hat{\theta} \ G_1(\theta|x_i) \ d\theta + \int_{\theta} \theta^2 \ G_1(\theta|x_i) \ d\theta
\]
\[
R(\hat{\theta}, \theta) = \hat{\theta}^2 - 2 \hat{\theta} \ E(\theta|x_i) + E(\theta^2|x_i)
\]
We differentiate \( R(\hat{\theta}, \theta) \) with respect to \( \hat{\theta} \) and setting the result to zero we get:
\[ 2\hat{\theta} - 2 E(\theta|x_i) = 0 \]
Solving for \( \hat{\theta} \) implies that
\[ \hat{\theta}_S = E(\theta|x_i) \]

5.1.1 The case of Jeffrey's prior information

The estimator that makes the risk function at its minimum limit (the least possible) is the standard Bayes estimator for the parameter \( \theta \).
\[ \hat{\theta}_{1SJ} = E(\theta_1|x_i) \]
And using equation (5) to find \( \theta_1 \);
\[ \hat{\theta}_{1SJ} = \frac{n_1}{w} = \frac{n_1}{\sum_{i=1}^{n_1} \left( \frac{\sigma}{x_{ij}} \right)^2} \]
By the same way find \( \hat{\theta}_2, \ldots, \hat{\theta}_k, \hat{\theta}_{k+1} \)
Thus \[ \hat{R} = \frac{\sum_{i=1}^{k} \hat{\theta}_{iSJ}}{\sum_{i=1}^{n_1} \hat{\theta}_{iSJ}} \]

5.1.2: The case of Gamma prior information.

The estimator that makes the risk function at its minimum limit (the least possible) is the standard Bayes estimate for the parameter \( \theta \).
\[ \hat{\theta} = E(\theta|x_i) \]
And using equation (6) we get
\[ \hat{\theta}_1 = \frac{n_1 + \delta}{p} = \frac{n_1 + \delta}{w + \beta} \]

By the same way find \( \hat{\theta}_2, \ldots, \hat{\theta}_k, \hat{\theta}_{k+1} \)

Thus \( \hat{R} = \frac{\sum_{i=1}^{k} \hat{\theta}_i s_G}{\sum_{i=1}^{k+1} \hat{\theta}_i s_G} \) \hspace{1cm} (8)

5.2 Bayes estimator under Linear Exponential Function

Consider the linear error loss function \( l(\hat{\theta}, \theta) = e^\Delta - \Delta - 1 \)

Where \( \Delta = (\hat{\theta} - \theta) \)

The risk function is given as:

\[ R(\hat{\theta}, \theta) = \int_{\theta} e^{(\hat{\theta} - \theta)} - (\hat{\theta} - \theta) - 1 \] \[ G(\theta|x) \] \[ d\theta \]

\[ = e^{\hat{\theta} \int_{\theta} e^{-\theta} G(\theta|x) d\theta} - \hat{\theta} \int_{\theta} e^{-\theta} G(\theta|x) d\theta + \int_{\theta} \theta G(\theta|x) d\theta - \int_{\theta} G(\theta|x) d\theta \]

Differentiating \( R(\hat{\theta}, \theta) \) with respect to \( \hat{\theta} \) and setting the result to zero, we get

\[ \frac{\partial R(\hat{\theta}, \theta)}{\partial (\hat{\theta})} = e^{\hat{\theta} \int_{\theta} e^{-\theta} G(\theta|x) d\theta} - \int_{\theta} G(\theta|x) d\theta = 0 \]

Which implies that

\[ e^{\hat{\theta} \int_{\theta} e^{-\theta} G(\theta|x) d\theta} = 1 \]

Hence,

\[ e^{\hat{\theta}} = \frac{1}{\int_{\theta} e^{-\theta} G(\theta|x) d\theta} \]

The Bayes estimator of \( \theta \) based on LINEX loss function is

\[ \hat{\theta}_{LEN} = -\ln \int_{\theta} e^{-\theta} G(\theta|x) d\theta \]

5.2.1: The case of Jeffrey’s prior information.

\[ \hat{\theta}_{LENJ} = -\ln \int_{\theta} e^{-\theta} G_1(\theta|x) d\theta \]

\[ = -\ln \int_{0}^{\infty} e^{-\theta} \frac{w^n \theta^{n-1} e^{-\theta w}}{\Gamma(n)} d\theta \]

\[ = -\ln \frac{w^n}{\Gamma(n)} \int_{0}^{\infty} \theta^{n-1} e^{-\theta(1+w)} d\theta \]

\[ = -\ln \frac{w^n}{p^n \Gamma(n)} \]

where \( p = (1 + w) \)

\[ \hat{\theta}_{LENJ} = -\ln \left( \frac{w}{p} \right)^n = -\ln \left( \frac{\sum_{i=1}^{n} \frac{\sigma_i^2}{X_i^2}}{(1+w)} \right)^n \]
Thus, \[ \hat{\theta}_1 = -\ln \left( \frac{n_{1i} \sigma_{1i}^2}{\sum_{i=1}^{n} \sigma_{1i}^2} \right)^{n_1} \]

By the same way find \( \hat{\theta}_2, \ldots, \hat{\theta}_k, \hat{\theta}_{k+1} \)

Thus \[ \hat{R} = \frac{\sum_{i=1}^{k} \hat{\theta}_{iLENJ}}{\sum_{i=1}^{k+1} \hat{\theta}_{iLENJ}} \]

(9)

5.2.2: The case of Gamma prior information.

\[ \hat{\theta}_{LENJ} = -\ln \int_{0}^{\infty} e^{-\theta} G_2(\theta|x) \, d\theta \]

\[ = -\ln \int_{0}^{\infty} e^{-\theta} \frac{p^{n+\delta} \theta^{n+\delta-1} e^{-\theta p}}{\Gamma(n+\delta)} \, d\theta \]

\[ = -\ln \int_{0}^{\infty} \frac{p^{n+\delta} \theta^{n+\delta-1} e^{-\theta (1+p)}}{\Gamma(n+\delta)} \, d\theta \]

\[ = -\ln \frac{p^{n+\delta}}{(1+p)^n \delta} \int_{0}^{\infty} \frac{\theta^{n+\delta-1} e^{-\theta (1+p)}}{\Gamma(n+\delta)} \, d\theta \]

By evaluating the integral we get

\[ \hat{\theta}_{LENJ} = -\ln \left( \frac{p}{(1+p)} \right)^{n+\delta} = -\ln \left( \frac{(w+\beta)}{(1+(w+\beta))} \right)^{n+\delta} \]

Therefore, for \( \hat{\theta}_{1LEN} = -\ln \left( \frac{(w+\beta)}{(1+(w+\beta))} \right)^{n+\delta} \)

By the same way find \( \hat{\theta}_2, \ldots, \hat{\theta}_k, \hat{\theta}_{k+1} \)

Thus \[ \hat{R} = \frac{\sum_{i=1}^{k} \hat{\theta}_{iLENJ}}{\sum_{i=1}^{k+1} \hat{\theta}_{iLENJ}} \]

(10)

6. Simulation Study

In order to verify the performance of the estimation, Monte Carlo simulation was used. The proposed estimation methods in S-S models have been implemented using variety samples (10, 30, and 60). The following steps of Monte Carlo simulation explanation the statistical outcomes for every sample based on Mean Squared Errors criteria with 1000 replicates:

Step1: Generate random samples as \( u_{i1}, u_{i2}, \ldots, u_{in_i} \) and \( w_1, w_2, \ldots, w_m \)for all \( i = 1, 2, \ldots, k = 3 \) respectively, which are follows the continuous uniform distribution defined on the interval (0, 1).

Step2: Transform the above uniform random samples to a random samples follows GIRD using the cumulative distribution function (CDF) as follow;

\[ F(x) = e^{-\alpha^2 x^2}, \quad U_i = e^{-\alpha^2 x^2} \]

\[ x_i = [\alpha^2 \theta_i/\ln(U_i)]^{1/2} \]
And, by the same method, we get
\[ y_i = \left[ a^2 \theta_{k+1} / -\ln(VJ) \right]^2 \]

Step 3: Recall the \( R \) from equation 3.

Step 4: find the \( \hat{R} \) of the squared error loss function, and Linear exponential loss function based on Jeffrey’s Prior Information and gamma priors using equations (7, 8, 9, and 10), respectively.

Step 5: based on \( L=1000 \) replicate, Calculate the MSE as follows:
\[ \text{MSE} = \frac{1}{L} \sum_{i=1}^{L} (\hat{R} - R)^2 \]

7. Results of Simulation

In this section, the simulation results used to determine the best outcome of the conceder estimation methods (\textit{SELF} and \textit{LENX}) of S-S reliability estimator based on one parameters Inverse Rayleigh distribution.

In S-S model of estimate the system reliability \( R = P(\max(X_1, X_2, ..., X_k)) \), the following tables of mean square error showed, at most the orders rank of the estimators as follow \( \hat{R}_{\text{SELF}}, \hat{R}_{\text{SELF}', \text{G}}, \hat{R}_{\text{LENX}, J}, \) and \( \hat{R}_{\text{LENX}, G} \) respectively, that’s mean \( R_{\text{LENJ}} \) was the best than the others estimators. The simulation results was presented as following tables (2 and 4):

| Table 1: Estimation value of \( R = 0.17500000000 \) when \( \sigma = 3, \theta_1 = 2.5, \theta_2 = 3, \theta_3 = 1.5, \theta_4 = 3.5 \) |
|-----------------|----------------|----------------|----------------|----------------|
| (n1,n2,n3,m)    | RSEJ           | RLENJ          | RSEG           | RLENJ          |
| (20,20,20,20)   | 0.174992896482054 | 0.174990823409970 | 0.174991175671877 | 0.174988461142034 |
| (20,20,20,40)   | 0.174979722889881 | 0.174990823407240 | 0.174989873234301 | 0.174985039995665 |
| (40,40,40,40)   | 0.17500481359481 | 0.174990823416745 | 0.174990553795207 | 0.17499593349205 |
| (60,60,60,60)   | 0.174991325778360 | 0.174990823409568 | 0.174991114070636 | 0.174992563242371 |
| (20,40,60,20)   | 0.175043054807755 | 0.174990823425671 | 0.174991568325946 | 0.17502389239817 |
| (40,20,40,60)   | 0.174929231215509 | 0.174990823389533 | 0.174989189461996 | 0.174962911273884 |

| Table (2): MSE value when \( R = 0.17500000000 \) when \( \sigma = 3, \theta_1 = 2.5, \theta_2 = 3, \theta_3 = 1.5, \theta_4 = 3.5 \) |
|-----------------|----------------|----------------|----------------|----------------|
| (n1,n2,n3,m)    | RSEJ           | RSEG           | RLENJ          | RLENJ          |
| (20,20,20,20)   | 0.000000239318743 | 0.00000000084120 | 0.00000001529343 | 0.00000010321630 |
| (20,20,20,40)   | 0.000000252775403 | 0.00000000084120 | 0.00000001640110 | 0.00000006027317 |
| (40,40,40,40)   | 0.000000131739150 | 0.00000000084120 | 0.00000009202664 | 0.000000077852747 |
Table (3): Estimation value of $R = 0.1666666666$ when $\sigma = 3.5, \theta_1 = 1, \theta_2 = 2.5, \theta_3 = 3, \theta_4 = 2$

| $(n_1, n_2, n_3, m)$ | $RSEJ$ | $RLENJ$ | $RSEG$ | $RLENG$ |
|----------------------|--------|--------|--------|--------|
| (20,20,20,20)        | 0.166641601824709 | 0.166638856453412 | 0.166638941412111 | 0.166622583380192 |
| (20,20,20,40)        | 0.166592019225767 | 0.166638856438889 | 0.166636713586408 | 0.166611930398352 |
| (40,40,40,40)        | 0.1667335960655   | 0.166638856452911 | 0.166641513269821 | 0.166652804827086 |
| (60,60,60,60)        | 0.16663043970322  | 0.16668856452911   | 0.16663842313820  | 0.166637005516564 |
| (20,40,60,20)        | 0.166701341021664 | 0.166638856471555 | 0.166640669069385 | 0.166676792289576 |
| (40,20,40,60)        | 0.166635239933565 | 0.16663885645291   | 0.16663829349936  | 0.166638195235606 |

Table (4): MSE value of $R = 0.1666666666$ when $\sigma = 3.5, \theta_1 = 1, \theta_2 = 2.5, \theta_3 = 3, \theta_4 = 2$

| $(n_1, n_2, n_3, m)$ | $R_{SEJ}^\hat{}$ | $R_{LENJ}^\hat{}$ | $R_{SEG}^\hat{}$ | $R_{LENG}^\hat{}$ |
|----------------------|------------------|------------------|------------------|------------------|
| (20,20,20,20)        | 0.00000275740659 | 0.00000000773408 | 0.00000002384350 | 0.00000277878273 |
| (20,20,20,40)        | 0.00000237329976 | 0.00000000773409 | 0.00000002207637 | 0.00000157068099 |
| (40,40,40,40)        | 0.00000108037963 | 0.00000000773408 | 0.00000001270821 | 0.00000034711062 |
| (60,60,60,60)        | 0.000073435793214| 0.000000773407989| 0.00001225307881 | 0.00014828824499 |
| (20,40,60,20)        | 0.00000140642448 | 0.00000000773407 | 0.0000001471647 | 0.00000075711050 |
| (40,20,40,60)        | 0.000000109284432| 0.00000000773408 | 0.00000001414383 | 0.00000034649606 |

**Conclusion**

Parallel system reliability of stress-strength model was introduced when the stress and strength follow the two parameters General Inverse Rayleigh distribution using different two losses functions, such as Squared Error loss function, and Linear Exponential loss function Jeffrey’s Prior Information and gamma priors. The model $R = P(Y < \max(X_1,X_2,...,X_k))$ were used to evaluate and verify the performance of the proposed methods using different samples (20, 40, and 60). After that, the Monte Carlo Simulation has exhibited to analyses and comparison between these methods based criteria of Mean square Error. Based on the results, the performance of Linear Exponential loss function Jeffrey’s Prior Information ($R_{LENG}$) was appropriate behavior and it is an efficient estimator than the others at most.
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