Fermionic current-carrying cosmic strings: zero-temperature limit and equation of state

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The equation of state for a superconducting cosmic string whose current is due to fermionic zero modes is derived analytically in the case where the back-reaction of the fermions to the background is neglected. It is first shown that the zero mode fermions follow a zero temperature distribution because of their interactions (or lack thereof) with the string-forming Higgs and gauge fields. It is then found that the energy per unit length \( U \) and the tension \( T \) are related to the background string mass \( m \) through the simple relation \( U + T = 2m^2 \). Cosmological consequences are briefly discussed.

I. INTRODUCTION

Topological defects [1] have been considered in various physical situations, e.g., in the context of condensed matter and cosmology [2,3]. In many cases of interest in cosmology [4,5], they can be approximated as structureless, the relevant dynamics being often assumed not to depend on any specific choice of their internal content. For cosmic defects, this internal content would correspond to the particles that couple to the string-forming Higgs field [6]. However, in the latter example of cosmic strings, it was shown that such a structure might lead to drastic modifications not only of these object dynamics [7,8], which could be seen as a mere academic situation given our present ignorance on their very existence, but also, because of the appearance of new accessible equilibrium states, of the cosmological setting, leading in some instances to actual catastrophes [9,10]. To make a long story short, let us just say that currents imply a breakdown of the Lorentz-boost invariance along the string worldsheet, thereby allowing loop configurations to rotate, the centrifugal force hereby induced having the ability to sustain the loop tendency to shrink because of the tension. The resulting states, called vortons, might be stable even over cosmological timescales, scaling as matter and thus rapidly coming to dominate the Universe evolution, in contradiction with the observations. This leads to constraints on the particle physics theories that predict them at energy scales that are believed to be unreachable experimentally (in accelerators say) in the foreseeable future.

Unfortunately, it appears that the string structure, contrary to their counterparts as fundamental objects [11], is not determined by any consistency relation, and is therefore somehow arbitrary, at least at the effective description level [8]. This means in practice that in order to be able to tell anything relevant to (cosmic) string cosmology, one needs to set up a complete underlying microscopic model, arising say, from ones favorite Grand Unified Theory (GUT) [12] or some low-energy approximation of some superstring-inspired model [13].

Some generic constructions can however be arranged, as was shown to be the case whence a bosonic condensate gets frozen in the string core [7,8]. In such a situation, the boson field phase \( \varphi \), thanks to a random Kibble-like mechanism, may wind along the string itself, thereby producing a current that turns out to be essentially a function of a single state parameter \( w \), thus expressible as a phase gradient as

\[
w \equiv \kappa_0 \gamma^{ab} \partial_a \varphi \partial_b \varphi,
\]

with indices \( a, b, \ldots \) varying within the string worldsheet coordinates defined by the relations

\[
x^\mu = X^\mu_a(\xi^a), \quad \xi^a \in \{ \tau, \sigma \},
\]

and \( \gamma^{ab} \) the inverse of the induced metric defined with the background metric \( g_{\mu\nu} \) as

\[
\gamma^{ab} = g_{\mu\nu} \partial_{\xi^a} X^\mu \partial_{\xi^b} X^\nu.
\]

A straightforward generalization of the Nambu-Goto action is then provided by the \( w \)-weighted measure as

\[
S = -m^2 \int d^2 \xi \sqrt{-\gamma} L\{w\},
\]

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\end{itemize}
with $m$ the typical mass scale of symmetry breaking leading to string formation and $\gamma$ the determinant of the induced metric \cite{1}. Reasonable microscopic models \cite{14} then yield approximate forms for the Lagrangian function $L\{u\}$, out of which the dynamical properties of the corresponding strings can be derived \cite{13,17}.

Such a description remains however essentially classical even though an alternative formalism, also proposed by Carter \cite{15}, in terms of a dilatonic model, appears more suitable for quantization. This last formalism however, being fully two-dimensional, cannot be used to derive interesting quantities such as the relevant cross-sections for trapped excitations to leave the string worldsheet. This is unfortunate since this is precisely the information one would need for cosmological applications \cite{12}.

It would therefore seem that by considering fermionic current carriers instead of bosonic ones, one would, because of the intrinsically quantum nature of fermions, obtain a more appropriate description \cite{12,19}. Besides, fermions are trapped in topological defects because of Yukawa couplings with the string forming Higgs field in the form of zero modes \cite{20}, so that their dynamics is described by simple (although coupled) Dirac equations, which are linear. In the bosonic case, the non-linear (quartic) term is essential in order to ensure the dynamical stability of the condensate so that a solitonic treatment \cite{21,22} seems the only way to deal with the underlying quantum physics. This fact dramatically complicates matters and as a result, a complete description yet fails to exist.

However, the fermionic case is not that simple either as here, one faces another technical difficulty for the classical description: it can be shown that there doesn’t exist a simple state parameter \cite{19}. An arbitrary spacelike or timelike current can only be built out of at least two opposite chirality spinor fields and will be given by the knowledge of four occupation numbers per unit length. As this is true in particular at least in the zero-temperature limit, we shall be concerned here first with this limit whose validity was assumed to depend on the particular model under consideration \cite{12}. In the following section, we show that setting the temperature to zero is always a good approximation because of the couplings between the fermionic fields and the string-forming Higgs and gauge fields. These results would seem to imply that the previously derived macroscopic formalism is irrelevant to the fermionic situation (see Ref. \cite{23} for a many-parameter formalism). In practice however, as a simple relationship between the energy per unit length and the tension can be found for fermionic currents, the single state parameter formalism can be used that permits to draw some cosmological consequences.

II. THE ZERO-TEMPERATURE LIMIT

In order to have an arbitrary current built upon fermionic fields, one needs at least two Dirac fermions $\Psi$ and $\chi$ say, coupled to the string-forming Higgs field $\Phi$ through Yukawa terms as well as to the associated gauge field $B_\mu$, the later acquiring a mass from the vacuum expectation value (VEV) of the Higgs field. Fermions may condense in the string core in the form of zero modes, and by filling up the accessible states, one forms a current which can be timelike, spacelike, or lightlike. If one wants to give a classical description of such a current-carrying string, one must be in a configuration for which quantum effects are negligible. Such quantum effects, as for instance tunneling outside the vortex, will indeed be negligible provided most of the fermions are on energy levels whose excitation energy is much below their vacuum mass, the latter thus playing the role of a Fermi energy.

As a result, if a temperature may be defined for the fermion ensemble along the vortex, quantum effects will be negligible if the temperature is small compared to the vacuum mass of the fermions, which essentially imply a zero temperature state. Note that the situation we are having in mind is reached only whenever the background temperature is low compared to that at which the string formed for otherwise interactions with the surrounding plasma could populate the high energy levels. In practice, this is what will happen at the time of string formation, and if the fermions did not interact at all, or only through time reversible interactions, one would be left with a frozen distribution corresponding to a high temperature state \cite{25}.

That this is not the case can be seen through an exhaustive list of all the possible fermion interactions. For that purpose, it must be emphasized that fermionic condensates arise in the string core in the form of zero modes, i.e., chiral states \cite{17}. One has essentially two coupling possibilities, namely a coupling of the fermion with the Higgs field or with the gauge field, illustrated on the figure. The first case (diagram \(a\)) is seen to vanish identically in the case of a chiral mode \cite{19} so we shall not consider it. The second case is more interesting and comes from the second diagram (\(b\)) of the figure. In this case, any trapped fermionic zero mode is seen to be able to radiate a gauge vector boson.

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\(^1\)Otherwise, the temperature itself could be taken as a state parameter, so that the usual formalism would be applicable \cite{24}.

\(^2\)We do not consider here the possible massive bound states as those are expected \cite{12} to interact with each other because of diagram (\(c\)) of the figure and therefore move rapidly away from the string core.
These terms do not vanish identically, and in fact can be seen to be the source for some backreacted components of the gauge field: as all the vectors emitted this way will eventually condense into a classical field, this diagram contributes to a small (and indeed usually negligible \[19\]) contribution in the energy per unit length and tension.

Finally the third term, (c), of the figure, represents a would-be interaction term between the fermions that could be responsible for an equilibrium configuration. When one considers only zero modes in a usual model, such a term actually vanishes because both fermions must have opposite chiralities in order for the theory to be well defined. As a result, interactions between fermions turn out to be negligible.

We are now in a position to understand the microphysics of what might happen inside a fermionic current-carrying cosmic string. First, when the fermions become trapped in the string core, they do so on arbitrary high energy levels inside the string. Then, they have the possibility to radiate most of their energy away in the form of the vector field, thereby creating the backreacted component. Note that the vector field itself can also interact with the Higgs field thereby producing an effective lack of symmetry between diagram (b) and its time reversed counterpart. This implies that the overall effect is indeed a radiative decay and not an equilibrium. Thus, all the populated states end up being the lowest reachable states. In practice, that means that the effective temperature of the fermion gas is vanishing. Moreover, such a configuration in turn is stable as no interaction between the various fermion field can be present.

III. FERMIONIC STRING EQUATION OF STATE

As was discussed above, fermions that are coupled to the Higgs field may be bound to cosmic strings in the form of zero modes \[6,19,20\] and therefore the current they generate arises from lightlike components. We recall briefly the formalism necessary to handle this case and then move on to derive the resulting stress-energy tensor as well as a simple relationship relating its eigenvalues. This leads to a plausible classical description in terms of a state parameter whose validity is discussed.

A. Stress-energy tensor and equation of state

Chiral currents have special properties and need be studied on their own. As was shown earlier, a phase gradient formalism hold for them, similar to the $\sigma$–model \[18\] with vanishing potential, namely \[20\]

\[
S_\sigma = - \int d^2 \xi \sqrt{-\gamma} \left( m^2 + \frac{1}{2} \psi^2 \gamma^{ab} \partial_\nu \varphi \partial_\psi \varphi \right),
\]

where the normalization constant $\kappa_0$ of Eq. (1) in the $w$–formalism has now been promoted to a dynamical field whose variations lead to an ever-lightlike current.

Considering now a system of fermionic zero modes, and neglecting back-reaction, one may arrive at the conclusion that the relevant action describing a general fermionic current-carrier cosmic string will be given by

\[
S_F = \int d^2 \xi \sqrt{-\gamma} \mathcal{L}
\]

where the Lagrangian function $\mathcal{L}$ is
\[ \mathcal{L} = -m^2 - \frac{1}{2} \sum_{i}^{N} \psi_{(i)}^2 \gamma^{ab} \partial_a \varphi_{(i)} \partial_b \varphi_{(i)}, \tag{7} \]

\( N \) being the number of fermionic degrees of freedom (at least four \([19]\) if the model is to describe arbitrary spacelike as well as timelike and chiral currents). Having obtained the action, it is now a simple matter to derive the corresponding dynamics by varying it with respect to the various fields involved. However, as we show below, it turns out not to be strictly necessary as many consequences, in particular in cosmology, stem directly from the energy momentum tensor eigenvalues \( U \) and \( T \), i.e., respectively the energy per unit length and tension, for which a very simple relationship is now derived.

First of all, as all the fields \( \psi_{(i)} \) are independent, it is evident that variations of (7) lead to the chirality condition on the various currents induced by the phase gradients, namely

\[ \delta \mathcal{S}_F \delta \psi_{(i)} = 0 \quad \Rightarrow \quad \gamma^{ab} \partial_a \varphi_{(i)} \partial_b \varphi_{(i)} = 0 \quad \forall i \in [1, N], \tag{8} \]

while the currents are obtained through variations with respect to the phases themselves

\[ \delta \mathcal{S}_F \delta \varphi_{(i)} = 0 \quad \Rightarrow \quad \nabla_a \left( \psi_{(i)}^2 \gamma^{ab} \partial_b \varphi_{(i)} \right) = \nabla_a j_{(i)}^a = 0, \quad \forall i \in [1, N]. \tag{9} \]

Eq. (8) can be seen to imply, in our two-dimensional case, that each function \( \varphi_{(i)}(\xi^a) \) separately is harmonic, i.e.

\[ \gamma^{ab} \nabla_a \nabla_b \varphi_{(i)} = 0 \quad \forall i \in [1, N], \tag{10} \]

so that the current conservation equations (9) can be cast in the form

\[ \gamma^{ab} \nabla_a \psi_{(i)} \nabla_b \varphi_{(i)} = 0 \quad \forall i \in [1, N], \tag{11} \]

which means that every \( \psi_{(i)} \) is a function of \( \varphi_{(i)}(\xi^a) \) only, for any fixed value of \( i \). As a result, the formalism really describes only \( N \) degrees of freedom, and not \( 2N \), and one may interpret the phase gradients as occupation numbers per unit length in a given underlying fermionic current-carrying model.

The stress energy tensor is now obtained by the standard procedure of variation with respect to the metric, i.e.,

\[ T^{\mu\nu} = 2 \frac{\delta \mathcal{L}}{\delta g_{\mu\nu}} + \mathcal{L} \eta^{\mu\nu}, \tag{12} \]

where the first fundamental tensor of the worldsheet \([7]\)

\[ \eta^{\mu\nu} = \gamma^{ab} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b} \tag{13} \]

is definable in terms of the eigenvectors \( u^\mu \) and \( v^\mu \), respectively timelike and spacelike \( (u_\mu u^\mu = v_\mu v^\mu = -1) \), of the stress energy tensor for a non-chiral current:

\[ \eta^{\mu\nu} = u^\mu v^\nu - u^\nu v^\mu, \tag{14} \]

and

\[ T^{\mu\nu} = U u^\mu u^\nu - T v^\mu v^\nu. \tag{15} \]

The stress-energy tensor now reads

\[ T^{\mu\nu} = -m^2 \eta^{\mu\nu} + \frac{1}{2} \sum_{i}^{N} \psi_{(i)}^2 \left( X_{(i)}^\mu \partial_i X_{(i)}^\nu - \eta^{\mu\nu} \varphi_{(i)} \partial_i \varphi_{(i)} \right) \tag{16} \]

with a comma denoting partial differentiation with respect to a worldsheet coordinate. The last term identically vanishes because of the on-shell relation (8), and we can now compute the eigenvalues by projecting on the eigenvectors as
\[ U = T^{\mu\nu} u_\mu u_\nu = m^2 + \frac{1}{2} \left( \sum_i \psi_i^2 \varphi_i^a \varphi_i^b \right) X_{s,a} X_{s,b}^\nu u_\mu u_\nu, \]  
(17)

and

\[ T = -T^{\mu\nu} v_\mu v_\nu = m^2 - \frac{1}{2} \left( \sum_i \psi_i^2 \varphi_i^a \varphi_i^b \right) X_{s,a} X_{s,b}^\nu v_\mu v_\nu. \]  
(18)

By adding these two equations up, one gets that the last term is proportional to the first fundamental tensor \( \eta^{\mu\nu} \) projected onto the worldsheet coordinates, i.e. a term proportional to the induced metric \( \gamma^{ab} \). As each part of the current is made of chiral fields, this last term eventually cancels out and one is left with [a relation obtainable directly by taking the trace of the stress tensor (16)]

\[ U + T = 2m^2, \]  
(19)

which will be our final equation of state for a fermionic current carrying cosmic string in the zero temperature limit. Note that this relation holds in the specific case of the model discussed in Ref. [19] whenever one neglects the fermion backreaction on the string fields.

**B. A macroscopic model**

Let us now discuss the various implications of this result. The most important point related with cosmological models involving current-carrying strings concerns vorton stability. As such a model is exclusively classical in nature, we shall not examine the quantum stability here, especially since this was already discussed in Ref. [12]. Before however turning to this physical point, we should like to stress a simple technical detail concerning the equation of state itself.

As we have said, a fermionic current-carrying cosmic string does not in general admit a classical description in terms of a single state parameter. However, in the case where a functional relationship exists between the energy per unit length and the tension, as is indeed what happens in the situation under consideration here, a state parameter can easily be derived as follows.

Let us consider again the \( w \)-formalism. Performing the Legendre transform

\[ \Lambda = \mathcal{L} - 2w \frac{d\mathcal{L}}{dw}, \]  
(20)

it can be shown that [7], depending on the timelike or spacelike character of the current, the energy per unit length and tension can be identified, up to a sign, with \( \mathcal{L} \) and \( \Lambda \). As a result, the knowledge of \( \mathcal{L} \) as a function of \( \Lambda \) or, in other words that of \( U(T) \), permits to integrate Eq. (20) to yield the state parameter through

\[ \ln \left( \frac{w}{w_0} \right) = \int \frac{d\mathcal{L}}{2(\mathcal{L} - \Lambda)}, \]  
(21)

whose inversion, in turn, gives the functional form of the Lagrangian \( \mathcal{L}\{w\} \), up to a normalization factor. Applied to our case, Eq. (21) implies immediately

\[ \mathcal{L}\{w\} = -m^2 - \frac{w}{2}, \]  
(22)

so we see that an arbitrary current formed with many lightlike currents can be described by means of a single state parameter with almost the simplest possible model; in Eq. (3), it suffices to replace the sum over the many chiral models by the standard form of \( w \), i.e. Eq. (1), which can be viewed as the auxiliary field \( \psi \) acquiring a fixed value "on shell". This is just the action of Eq. (5) with \( \psi^2 = \kappa_0 \).

The model described by Eq. (22) was however ruled out as a valid description of a realistic Witten-like current-carrying string in Ref. [14], so one may wonder how it can be re-introduced here. There are two answers to that question. First, it can be argued that most of the statements in this reference applied to bosonic current-carriers, and have therefore no reason to be true in the fermionic case, except that bosons and fermions are known to be equivalent in two dimensions [6]. As a result, a classical description of a vortex must somehow take into account the finite thickness effects before averaging over the transverse degrees of freedom, so that the string keeps a track of its 3 + 1-dimensional nature.
The second, perhaps more important reason, why the model given by Eq. (22) was not considered seriously as a candidate to describe a current-carrying string is the saturation effect. There must indeed be a maximum current flowing along a string as individual particles making the current are limited in energy because they are bound states. In the case of bosons, thanks to Bose condensate, all the particles are essentially in the same state and the saturation effect stems from the non-linear (interaction) term between them. As it turns out, even the interaction terms can be adequately treated through a mean field approximation, so that a classical field description is valid in this case. For fermions however, this effect finds its origin in a completely different mechanism, related to the exclusion principle: it is necessary, in order to increase the value of the current, to add more particles on higher and higher energy levels, up to the point where it becomes energetically favorable for them to leave the worldsheet as massive modes. This is therefore a purely quantum effect which cannot, of course, be properly taken into account in the classical description developed here whose range of validity is thus limited to small currents. Moreover, contrary to the bosonic situation in which the boson mass enters explicitly as a relevant dynamical parameter, fermionic zero modes exist independently of the vacuum fermion mass, so there is no mass scale that could determine the saturation regime in such a classical description.

The conclusion of the previous discussion is that the model described by Eq. (22) is indeed an accurate representation for fermionic current-carrying cosmic strings provided the current is far from the saturation regime. It should be emphasized that it will be the case for most of the evolution of a network of such strings, so that one is entitled, for cosmological application purposes (e.g. numerical simulation), to derive the string dynamics with the linear model.

IV. CONSEQUENCES

Let us now move to the consequences of such an equation of state. We shall assume for now on that the macroscopic formalism with the Lagrangian given by Eq. (22) is valid to describe a fermionic carrier cosmic string, provided the string never leaves the elastic regime. In other words, we shall assume that the string, whatever its shape, has a curvature radius everywhere much larger than its thickness and that the Fermi level is below the vacuum mass of the fermion so that the quantum effects are negligible.

Given a functional relationship between the energy per unit length $U$ and the tension $T$, one can calculate the perturbation velocities respectively as

$$c_T^2 = \frac{T}{U}$$

(23)

for the transverse perturbations, and

$$c_L^2 = -\frac{dT}{dU}$$

(24)

for the longitudinal ones. In the case at hand (19), this gives

$$c_L^2 = 1, \quad c_T^2 = \frac{2m^2}{U} - 1 < 1$$

(25)

since $m^2 < U < 2m^2$ by construction. On a plot $c_T^2$ versus $c_L^2$, such an equation of state would therefore just be the line $c_L^2 = 1$. For cosmological considerations, one may also consider for instance the back-reaction of the fermions on the background vortex fields, or even the electromagnetic back reaction for charged carriers. This means in practice, if one suppose that these will indeed lead to corrections which in principle could not be properly placed on such a diagram, that the corrected equation of state would be a curve somewhere near the $c_L^2 = 1$ line.

The situation is exactly the opposite of what happens for a boson field [27] for which it had been found that the equation of state in this plot is a curve close to the $c_L^2 = 1$ line. One can understand this result as a kind of duality between fermion and boson condensates, the corresponding equations of state being roughly symmetrical with respect to the line $c_L^2 = c_T^2$. This adds further insight on the fact that a purely 2-dimensional description is not valid before the full field theory has been solved. It may be conjectured at this point that a string carrying a current generated by both fermions and bosons with an underlying supersymmetric model [29] could produce an equation of state exactly lying on the line $c_L^2 = c_T^2$, i.e. the so-called fixed determinant model (arising also from a Kaluza-Klein projection [29] or as a smoothed average description of the large scale behavior of a simple Nambu-Goto model over the small scale wiggles [30,31]) for which $UT = m^4$. The advantage of this model, if the conjecture turned out to be a reasonable approximation of a more realistic equation of state, lies in its complete integrability [31] in the case of a flat background. Such a feature might be useful in network simulations.
The last point that needs to be mentioned here concerns vorton stability. It was shown under rather general conditions that circular loops reaching an equilibrium state thanks to a current may suffer from classical instabilities, the fate of which presumably leading to quantum effects \cite{32}, provided the equation of state is in the region above the $c_2^2 = c_r^2$ line \cite{33}. Inclusion of the electromagnetic corrections has also been achieved, showing that these can reduce the number of vortons that can form during the loops evolution \cite{17}, but that once they are formed, they are, classically, more stable \cite{34}. In our case, if the corrections do not change drastically the form of the equation of state, the vortons would exist comfortably below the critical line. Therefore, we expect them to be much more stable with respect to classical perturbations. In fact, it is very hard to imagine anything, except quantum background interaction \cite{12}, that could destabilize a vorton whose dynamics stems from the Lagrangian (22).

V. CONCLUSION

Fermionic zero modes trapped in cosmic strings are shown to follow a vanishing temperature Fermi-Dirac distribution. This is so because the chirality of the zero modes involved are such that the only possible interaction of the fermions is through gauge boson radiation, leading to an effective loss of energy (on average). As a result, as strings are formed and fermions get condensed along them in the form of zero modes, populating arbitrary high energy levels, they have the possibility to decay radiatively until they reach a zero temperature distribution. Then, as all other interaction terms are identically vanishing, they remain in this state which thus happens to be stable.

Assuming therefore such a vanishing temperature fermionic current-carrying cosmic string, it turns out that the equation of state relating the energy per unit length $U$ and the tension $T$ is of the self-dual \cite{14} fixed trace kind, namely $U + T = 2m^2$, with $m$ the characteristic string-forming Higgs mass. Although fermionic carriers imply the need of more than one state parameter, this implies that the simplest linear Lagrangian (22) provides a good approximation for a classical description of such a vortex. This could in fact have been anticipated as this is the only available equation of state that does not involve any new dimensionfull constant.

Vortons formed with such currents are completely stable, at least at the classical level (see however Ref. \cite{12} for quantum excitations). Assuming backreaction and electromagnetic corrections to be small, one finds that the vorton excess problem \cite{9} is therefore seriously enhanced for fermionic current-carrier cosmic strings.

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