Aerodynamic optimization of the last stage turbine blade for an industrial gas turbine

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Abstract. A multi-objective, aerodynamic optimization of the last stage turbine blade of an 100Hz industrial gas turbine is presented. We aim at maximizing the stage’s efficiency, while at the same time meeting mechanical integrity. The optimization procedure was build, during the optimization process the mass flow rate and stage meridian were kept constant to obtain the required stage loading without changing the operating point. The optimized geometry are chosen taking into account aerodynamic constraints as well as mechanical constrains, such as centrifugal stress and bending stress. The SST turbulence model was used to calculate the flow field of the original and optimized stage, respectively. The geometry of the original and optimized blade was compared. The stage efficiency of the design point and under different operate conditions were compared. Stage and blade efficiency along span was analyzed in details. The blade loading at root mid top span of the original and optimized blade were discussed. Detailed flow field analysis shows that, after optimization, the peak Mach number of the root span profile was decreased from 1.22 to 1.09, which improves the flow field. Profile loss and end wall loss were also reduced. After optimization, maximum centrifugal stress and the maximum air bending stress were decreased which leads to improved blade life and safety. The number of blades and b/h were suggested for the last stage blade.

1. Introduction

Industrial gas turbine are used for power generation and mechanical drive. It requires high efficiency and low emissions to meet economic requirement and environmental friendliness. The last stage turbine blade has complex three- dimensional flow in the flow passage and large centrifugal force. Novak[1] developed streamline curvature computing procedures. Dorman[2] developed a technique for improving turbine aerodynamic performance with application of controlled-vortex aerodynamics to axial flow turbines, which has been made practical by advanced computer technology. It is still widely used in turbine design process nowadays.

The large steam turbine last stage blade flow passage occurs supersonic flow up to Mach number 1.6, which is difficult to deal with. McBean [3]designed three different length of steam turbine last stage blade of 41in,45in and 49in, the design process include aerodynamic design, mechanical design. Xin[4] developed new low pressure stages for 1000MW boiler feed water pump steam turbine which demands good strength and aerodynamic performance. Tip section of the steam turbine last stage blade was analyzed by Sreedharan[5] using 2D simulations, major airfoil design parameters such as un-guided turning and trailing edge angle were studied, a new design feature on the suction side of the LSB which has higher performance benefit was proposed. Static strength criteria based on allowable stress and dynamic strength criteria based on safety factor are currently the main assessment criteria for the blade design and reliability consideration. Blade life evaluation studies such as static stress...
analysis, dynamic stress analysis, high cycle and low cycle fatigue life analysis, the ideas and methods for further studies were summarized by XIE[6][7].

In gas turbine blade design and research area, aerodynamic redesign of a four-stage heavy-duty gas turbine was done by Filippo Rubechini[8], through-flow methods, three-dimensional RANS computations, were applied according to a given hierarchical criterion, the overall redesign procedure relies on a neural-network-based approach aimed at maximizing the turbine’s power output while satisfying geometrical and mechanical constraints. Two dimensional through-flow using an automatic optimization strategy of multistage turbomachinery[9], blade-to-blade viscous solutions for aerodynamic profile design[10][11][12], three dimensional Navier-Stokes with a number of heuristic and gradient based optimisers for a single row simulation [13][14] have been developed in the early years. Bellucci[15] and Cotroneo[16] developed a multi-objective, aerodynamic optimization method for a high-pressure steam turbine stage which contains prismatic vane and blade. The aerodynamic constraints, such as limitation of the pressure recovery in the uncovered part of the suction side, as well as mechanical constraints, such as root tensile stress and dynamic behavior are chosen taking into account. Redesign procedure for a 17-stage steam turbine which contains optimization of stator lean and rotor twist single for a single stage, geometrical transformation between the original reference stage and the optimized one, neural-network-based refinement of the stator and rotor twist of each stage, is presented by Rubechini [17], Hasenjäger [18] and Öksüz [19] developed a multiploid distance based MOGA aerodynamic shape design optimization tool capable of handling surrogate models for a turbine blade. The multi-objective aerodynamic design objectives are maximizing the adiabatic efficiency and torque so as to reduce the weight, size and cost of the gas turbine engine. An automated multi-objective and multidisciplinary design optimization (MDO) of a transonic turbine stage to maximize the isentropic efficiency and minimize the maximum stress of the rotor with constraints on mass flowrate and dynamic frequencies is presented by Song[20][21]. the isentropic efficiency decreased by 0.02%, but obtained a decrease in the profit of the maximum stress by 15.4%.

Supersonic flow in the gas turbine last stage blade passage is not as strong as the large steam turbine last stage blade. Power generated by the last stage accounts for 20% of a four stage turbine, transonic flow always occurs in the turbine last stage, it is a challenge for designer to acquire high blade efficiency. Another important aspect of last stage blade design is the need for a relatively long operational life. The requirement for a reliable blade design, that must operate under difficult conditions in terms of the high temperature (over 850K) without cooling and also under high centrifugal load, poses a great challenge to blade designers. The blade should also be manufactured in a cost effective manner, and be relatively simple to assemble. So that the final product meets all of the requirements in terms of aerodynamic performance and mechanical integrity.

A lot of research has been done for large steam turbine last stage blade. There are relative few research focus on improving the gas turbine last stage efficiency while meeting mechanical integrity. To consider aero thermal performance and mechanical integrity at the same time in the design process is highly demanded due to the fact that an aerodynamic optimum may not satisfy mechanical and aeromechanical criteria. Thus, an aeroelastic optimization might lead to considerable time delay and extra costs. The aim of the present study is to maximize the stage’s efficiency, while meeting the mechanical integrity. Efforts have been made to make the optimization process time cost effectively and exactly. Another purpose of this paper is to find proper number of blades and b/h which can contribute to get high efficiency and good strength for gas turbine last stage blade.

2. Numerical tool

2.1. Optimization procedure

The present optimization is aimed at maximizing the last stage’s total to total efficiency which was selected as the objective function. The optimization of the blade is an iterative complex analysis process to satisfy all the design constraints, related to the different disciplines, such as aerodynamics, structural mechanics.

By previous analysis, the original last stage has good mean line design and 2D(through flow) design, during optimization, 1D and 2D aerodynamic design results which contains velocity triangles
were kept constant. So during the optimization process several constraints were imposed in order to meet the design rules. The mass flow rate was kept constant, to obtain the required stage loading without changing the operating point when replacing the original blade with the optimized one. The stage meridian was also kept constant.

The optimization procedure is shown in figure 1. The blade optimization procedure starts with choosing number of blades which can effect both the efficiency and strength. The following results show that blade with fewer number and longer chord may achieve higher efficiency while improve the blade strength. Seven blade profile was chosen to be optimized, which can represent the blade aerodynamic performance while saving optimization time. In this step reasonable t/b was chosen to reduce the number of iterations. For the blade loading distribution of the airfoil surface especially on the rear part of the suction side, limitation of the diffusion rate was imposed to avoid boundary layer separation. In the root span, the blade airfoil was optimized as aft loaded to reduce secondary loss. 3D blade was optimized to stack 2D profiles and smooth the surface. 3D CFD was used to simulate the aerodynamic performance of the optimized stage to determine if the expected goal were achieved. The main complex problem of such a procedure is that a good aerodynamics design may not satisfy mechanical requirement. In the present optimization, the mechanical integrity checks relied on simplified design rules based on the evaluation of centrifugal stress and bending stress, ensuring the same or better mechanical capability for the optimized geometry as the original one. Simplified design rules play an important role in reducing optimization time.

![Figure 1. Optimization procedure.](image-url)
2.2. Computational method

The computational domain shown in figure 2 consists of the last vane and the last blade for an 100Hz industrial gas turbine which has four stages turbine. The structured grid is generated using Turbogrid. The total nodes of the computational domain is 1.4million, which contains 6800,000 nodes for the vane, 720,000 nodes for the blade. The height of the first layer was kept to ensure y+<1, which satisfies the selected SST turbulence model.

The numerical simulations in the present work were conducted by using the commercial CFD software ANSYS CFX. The steady compressible RANS equations were solved with the SST turbulence model, which combines the advantages of the k-ω model and the k-ε model. The SST turbulence model has good precision when solving the boundary layer flow with the inverse pressure gradient [22]. The g-θ model which is proposed by Menter[23]and Langtry[24] was used as transition model. The g-θ model which has high precision judges the transition process based on local variables, and has strong adaptability to the grid. The convection term is discretized using a high precision differential format.

Boundary conditions are given in table 1. Total pressure and total temperature with radial distribution as shown in figure 3 was given for the inlet which comes from the simulation of the whole four stage turbine. The outlet boundary condition was given by a uniform static pressure distribution. The rotation speed is 6000r/min. The working fluid is real gas which contains CO₂, H₂O, Ar, N₂ and O₂. The walls were simulated as no-slip, adiabatic boundary conditions. Steady simulations were conducted with mixing planes between the stator and rotor.

![Computational domain and grid.](image)

**Table 1. Boundary conditions.**

| Parameters  | Value                  |
|-------------|------------------------|
| p₀          | 2.2 (radial distribution) |
| T₀         | 980 (radial distribution) |
| p₂          | 0.9 (uniform)     |
| ω           | 6000       |
| Material    | real gas      |
3. Aerodynamic results and discussion

3.1. Geometry
The original and optimized blade section geometric features are compared in table 2. The biggest difference between the original and optimized blade is the reduction of the number of blades which are decreased from 101 to 81, followed by the C increased along span and aspect ratio b/h changed from 0.2 to 0.24. But the pitch/chord along blade span remains almost unchanged.

In figure 4 cross section of the last stage is compared between the original and optimized case. Hub and shroud remains unchanged, the vane and distance between the vane and the blade were also kept constant. The optimized blade has longer axial chord.

Table 2. Blade geometric, original vs optimized.

| Parameters              | Original | Optimized |
|-------------------------|----------|-----------|
| Number of blades        | 101      | 81        |
| t/b at root span        | 0.34     | 0.34      |
| C at root span          | 84.6     | 106.5     |
| t/b at mid span         | 0.57     | 0.57      |
| C at mid span           | 60.6     | 72.2      |
| t/b at top span         | 1        | 0.98      |
| C at top span           | 25.2     | 30.6      |
| b/h                     | 0.2      | 0.24      |

Figure 3. Total pressure and total temperature distribution at inlet.

Figure 4. Cross section of the last stage, original vs optimized.
3.2. Turbine efficiency

The total isentropic stage efficiency $\eta_{\text{stage, is, tt}}$ and the total isentropic blade efficiency $\eta_{\text{blade, is, tt}}$ are defined:

$$\eta_{\text{stage, is, tt}} = \frac{m \cdot \Delta h}{m \cdot \Delta h_{\text{is, t}}} = \frac{h_{0, t, \text{abs.}} - h_{2, t, \text{abs.}}}{h_{0, t, \text{is, abs.}} - h_{2, t, \text{is, abs.}}}$$

$$\eta_{\text{blade, is, tt}} = \frac{m \cdot \Delta h}{m \cdot \Delta h_{\text{is, t}}} = \frac{h_{1, \text{t, rel.}} - h_{2, \text{t, rel.}}}{h_{1, \text{is, t, rel.}} - h_{2, \text{is, t, rel.}}}$$

Where $h_{0, t, \text{abs.}}$ is the vane inlet absolute total enthalpy, $h_{2, t, \text{abs.}}$ is the blade outlet absolute total enthalpy, $h_{1, \text{t, rel.}}$ is the blade inlet relative total enthalpy, $h_{2, \text{t, rel.}}$ is the blade outlet relative total enthalpy.

In Table 3 the significant improvement of isentropic stage efficiency is gained by optimization.

| Parameters       | Original | Optimized |
|------------------|----------|-----------|
| Stage efficiency | 93.4%    | 94.3%     |
| Stage reaction   | 0.5      | 0.51      |
| Blade inlet rel. Ma | 0.39   | 0.4       |
| Blade outlet rel. Ma | 0.93   | 0.92      |
| Blade outlet abs. Ma | 0.52   | 0.52      |
| $\Delta h/\alpha^2$ | 1.37    | 1.37      |
| $m$              | 163      | 163       |

In figure 5 the stage efficiency along span is compared between the original and optimized case. The optimized blade has much higher stage efficiency in the first 50% span. From 50% to 100%, with small exception around 85% l/h, the optimized case results in a little higher stage efficiency (almost the same as the original case). In figure 6, the blade efficiency is compared between the original and optimized case. The optimized case has much higher stage efficiency in the first 50% span. From 50% to 100%, the optimized case results in little higher (0.5%) stage efficiency. The results indicate that optimized stage efficiency gains are attributed to 0–50% l/h blade profile optimization. The reason for the optimized case has little higher stage efficiency at 50%–100% l/h compared with the original case is that the original case already has 95%–97.5% blade efficiency at 50%–100% l/h, which has small potential for optimization.

![Figure 5. Stage efficiency along span, original vs optimized.](image-url)
Figure 6. Blade efficiency along span, original vs optimized.

Streamwise distribution of the mass averaged entropy on the blade cross section from inlet to outlet is shown in figure 7. When gas flows in the blade passage, flow loss consists of two main parts: profile loss and secondary flow loss[25]. Flow loss grows very slow before 0.5 axial chord in the blade passage for the original and optimized case, the difference between them is also very small. The reason is that in the suction side and pressure side, flow are under the influence of accelerating pressure gradient, profile loss is very small, and the secondary loss is small because the area of the passage vortex and horse shore vortex is relative small for the long blade. Incidence loss is also small. Based on the above reasons, flow loss grows very slow for the original and optimized blade before 0.5 axial chord.

From 0.5 to 1.0 axial chord, as the fluid velocity increases, the boundary layer thickens and supersonic speed appears in some areas, the loss increases quickly from 0.7 to 1.0 axial chord, adverse pressure gradient occurs at the blade suction surface, boundary layer thickens more, even separation and transition occur. Entropy increases much faster at the end of the blade because of the wake loss. For the optimized case, the lower pressure point position of suction surface moves downstream, adverse pressure gradient decreases, adverse pressure gradient segments shorten, so flow loss is less than that of the original.

Figure 8 provides a comparison of the stage efficiency between the original and optimized blade over the pressure ratio ranging from 1.6 to 2.5 which can cover 20%~110% load of the gas turbine under different back pressure. With the pressure ratio increasing, the increases of stage efficiency become higher and then become lower, resulting from 0.2% to 1.1%, and from 1.1% to 0.8%, which indicates after optimization the stage have better aerodynamic performance over wide range of pressure ratio. Compared with the original blade, the optimized case has much higher efficiency near the design point, away from the design point the increases of the stage efficiency becomes lower which achieves the goal of optimization.
3.3. **Blade Loading**

The last stage blade aerodynamic optimization involves a large number of iterations to reach a design satisfying mechanical integrity and assembly constraints. The original and optimized blade loading and the separation character of the flow are studied.

The hub section of last stage blade typically has to cope with high flow turning in the order of $90^\circ$ and entails transonic inlet and outlet flow conditions. The challenge in the optimization of such sections is to accommodate for the required flow turning whilst minimizing leading, passage, and trailing shock losses. Such high turning sections feature relatively low level of pitch/chord ratio ($t/b=0.34$), and by nature, approaches aerodynamic limit load quite early ($M_2 = 0.93$). The hub section must have appropriate area to meet mechanical integrity. In figure 9 airfoil pressure distribution at root span is compared between the original and optimized case. The original balde is almost uniform loaded, which leads to higher peak Mach number. The optimized blade is highly-aft-loaded, in the first 80% of the axial chord, the pressure on the pressure surface remains almost constant, in the first 60% of the axial chord, the pressure on the suction surface decreases slowly, and then the pressure on the suction surface decreases fast, the blade loading increases rapidly. Compared with the original blade, in the first 50% of the axial chord, the flow in the optimized blade is decelerated resulting in a diminished cross passage pressure gradient. After a short acceleration period, the negative pressure gradient at the rear part of the passage is decreased compared to the original design. This optimization leads to an increased cross passage pressure gradient in the after part and a homogenized static pressure distribution at the throat of the passage.
Figure 9. Airfoil pressure distribution at root span, original vs optimized.

Airfoil pressure distribution at mid span is shown in figure 10. Compared with the original case the optimized blade seems more aft-loaded. From the 50% axial chord to 80% axial chord, pressure in the optimized blade decreases very fast which means the blade loading increases fast. Like the root span section, the optimized mid span section leads to an increased cross passage pressure gradient in the after part and a homogenized static pressure distribution at the throat of the passage.

Figure 10. Airfoil pressure distribution at mid span, original vs optimized.

Last stage blade tip section design is one of the most challenging tasks a gas turbine aero designer faces due to the transonic-supersonic flow. Considerable amount of effort is required during the optimization process to ensure a mechanically robust tip section with low aerodynamic losses. In figure 11 airfoil pressure distribution at top span is compared between the original and optimized case. The blade loading of the optimized blade reaches maximum earlier than the original case. Flow in the optimized blade accelerating fast when goes into the passage, that is different from the root and medium section. Maybe this is contributed to structural integrity considerations. figure 5 and figure 6 show that the original and optimized blade has similar high efficiency at the top span. So the optimized section profile at top span is accepted.
3.4. Blade flow field analysis

Controlled-vortex technique was used to design the last stage. During optimization the blade outlet flow angle $\alpha$ is kept constant. In figure 12 a along blade span is compared between the original and optimized case. The blade outlet flow angle increases from 78° in the hub to 110° in the shroud. The flow angle increasing form hub to shroud is almost linear. But the original case seems more fluctuation than the optimized blade. Due to the application of controlled-vortex design, the streamline in blade outlet is upturned which leads to uniform flow angle, uniform static pressure, uniform velocity. It should be careful to design the exhaust hood to increase the static pressure recovery in order to gain high turbine efficiency, because absolute velocity along the blade span outlet is not axial.

The stage reaction is based on the isentropic enthalpy drop over the blade row and the turbine stage, which can be defined as follows:

$$ \Omega = \frac{h_{s,1} - h_{s,2}}{h_{t,0} - h_{s,2}} $$

Where the stage inlet total enthalpy $h_{t,0} = f(p_{t,0}, T_{t,0})$, the blade inlet static isotropic enthalpy $h_{s,1} = f(p_{1,s0})$, the blade outlet static isotropic enthalpy $h_{s,2} = f(p_{2,s0})$.

In figure 13 the stage reaction distribution of the original and optimized case is compared. The stage reaction along the span was kept very well, 0.46–0.57 from hub to shroud. The stage reaction of the optimized case is similar to the original, except in the hub area where the optimized case has a little higher stage reaction than the original. The optimized stage has more uniform stage reaction along blade which can ensure the stage have high efficiency during variable working condition.

Because of the better stage reaction control, in figure 14 relative Mach number along the blade span outlet is more uniform for the optimized blade. Although during optimization the 2D design results was kept constant, only seven section of profile was chosen for optimization, Mach number along the blade acquired from 3D analysis occurs some difference. If free-vortex aerodynamics method was used, the range of the Mach number would have greater difference between the hub and shroud.

**Figure 11.** Airfoil pressure distribution at top span original vs optimized.

$$ \text{rel. axial chord} [-] $$

![Graph of Airfoil Pressure Distribution](image.png)
Figure 12. $\alpha$ along blade span, original vs optimized.

Figure 13. Stage reaction distribution, original vs optimized.

Figure 14. Relative Mach number distribution along blade span, original vs optimized.

To examine further the loss reduction mechanism, the normalized entropy contours at the exit plane are shown in figure 15. The pictures contain two blade flow passage, the color levels of entropy are the same in both pictures (red is high entropy, blue is low entropy). It is clear that the optimized blade has much less entropy generation in the first 50% span compared with the original case.
Figure 15. Entropy generation contour at the rotor exit. Left: original, right: optimized.

The results of the simulations of the original and optimized blade is compared in figure 16 and figure 17. The span position of 5%–20% l/h, where the data from the line plots is extracted. The color levels of static pressure are the same in both pictures (red is high pressure, blue is low pressure). The peak suction is reduced in the optimized case resulting in weaker diffusion.

The weaker diffusion and the reduced S-shape of the static pressure distribution suggest later reattachment and a smaller separation bubble for the optimized airfoil. Therefore separation likely occur further downstream than in the original.

Figure 16. Suction airfoil surface pressure distribution at root span of original.

Figure 17. Suction airfoil surface pressure distribution at root span of optimization.
To study the state of the suction side flow, the streamline are compared in original and optimized case. In figure 18 on the original airfoil a small separation bubble with downstream reattachment is visible between 15%~35% span (area A). In contrast there is no flow separation on the optimized blade. The feature is located much further downstream than the original one which supports the earlier interpretations of airfoil pressure distributions. Longer distance in laminar flow on the airfoil upstream of the separation and shorter turbulent boundary layer downstream result in lower profile losses, see Denton[25]. This is seen as one indication for the increase of the overall stage efficiency. End wall secondary flows may be responsible for as much as a third of the losses found in a turbine row depending on factors such as aspect ratio [25]. In the last stage long blade that the paper described, the secondary flow effect area B which is below 5% l/h. Long blade secondary flow is not as strong as short blade because of the aspect ratio. After the horseshoe vortex is formed, the pressure side leg of the horseshoe vortex collide and combine with or wrap around the suction side leg of the horseshoe vortex and together climb the suction surface. Compared with the original blade, horseshoe vortex climb the suction surface later, which indicates highly-aft-loaded profile at the optimized blade end wall delayed flow separation. Compared with the original blade, area effected by secondary flow is smaller which means the secondary flow strength is weakened.

Iso-surface of vorticity is compared in figure 19. On the original airfoil suction side, high vorticity is observed between 0%~40% span, while the optimized blade does not have it, which leads to higher efficiency. Compared with the optimized blade, the original blade has much stronger transverse flow at hub which leads to higher vorticity, while the optimized has weak vorticity distribution. High vorticity which represent corner vortex is also found at blade trailing edge outlet, while the optimized blade seems a little weaker.

**Figure 18.** Streamline on blade suction surface.

**Figure 19.** Iso-surface of vorticity.
To study the detailed flow field, Mach number distribution at different span is compared in figure 20 and figure 21. The color levels of Mach number are the same in the same blade height.

At root span, the optimized blade has an increase in flow acceleration ahead of the exit throat, thereby reducing the peak Mach number from 1.22 to 1.09, which indicates the profile loss is reduced. Compared with the original blade at root span section, the optimized blade minimizes the trailing shock strength and can avoid shock induced boundary layer separation. At mid span, compared with the original blade, the optimized blade has a homogenized Mach number distribution after the throat of the passage, which can reduce trailing shock losses. At top span, both the original and the optimized blade have relative low peak Mach number (less than root and mid span). At top span, the optimized blade reaches the peak Mach number earlier than the original case, but the boundary layer of the optimized blade gets reattached before leaving the trailing edge, so the optimized blade has high efficiency as the original case.

4. Mechanical design and analysis
Mechanical design is achieved in figure 22. Tip shroud was redesigned for the optimized blade which can prevent the migration of the flow from pressure to suction side resulting in higher loading of the blade tip section and less leakage. Tip shroud can also add stiffness to the blade which can reduce the risk of buffeting stresses. The same five teeth fir-tree blade root which can support big centrifugal force was applied for the original and optimized case.
Average centrifugal stress along span is shown in figure 23, from 0 to 10\% l/h where the blade has maximum average centrifugal stress, the averaged centrifugal stress of the optimized blade is a little smaller than the original case. From 10\% to 70\% l/h, averaged centrifugal stress of the optimized blade is a little bigger than the original case, the maximum difference is less than 20MPa. Generally, the optimized blade has similar average centrifugal stress distribution to the original case.

In figure 24 bending stress along span of the two blades is compared. With l/h increasing from 0\% to 50\% the bending stress increases, and then it decreases. The maximum bending stress of the optimized blade is decreased by 25\% compared with the original case. The position of the maximum bending stress of the optimized blade is located at 50\% span, which is similar to the original case. After optimization, the blade has lower bending stress which can contribute to extend the blade life.

Detailed finite element mechanical analysis considering temperature distribution and material properties was carried out to predict the LCF life of the blade. The results show that after optimization the blade has more than 100000 equivalent operating hours.
5. Conclusions
The present paper describes the aerodynamic optimization of the last stage blade of an industrial gas turbine. The optimization strategy aimed at maximizing the stage’s efficiency, while meeting the mechanical integrity. During optimization, the original 1D and 2D aerodynamic design results were kept constant, so the mass flow rate and the stage meridian were kept constant. The SST turbulence model was used to calculate the flow field of the original and optimized stage, coupled with mechanical integrity checks by means of simplified design rules based on the evaluation of centrifugal stress and bending stress in order to ensure the optimized blade has same or better mechanical capability of the original one.

1. Numerical results show that the optimized stage efficiency was increased from 93.4% to 94.3% at the design point, while the optimized blade has much higher stage efficiency in the first 50% span. The stage efficiency was increased by 0.3%~1.1% with the stage pressure ratio ranging from 1.6~2.5.

2. Detailed flow field analysis shows that, after optimization, the root span section profile of the optimized blade is highly-aft-loaded, which delays flow separation and reduce the profile loss and secondary loss. The peak Mach number of the root span profile was decreased from 1.22 to 1.09, which improves the flow field. The optimized blade eliminates the separation on the suction surface which occurred on the original blade.

3. The 3D solid model of the optimized blade was built with shroud and fir-tree root. Simplified structural analysis shows that the averaged centrifugal stress of the optimized blade is a little smaller than the original case, the maximum bending stress of the optimized blade was decreased by 25%. The strength performance of the blade is improved.

4. We suggest choosing fewer number of blades with longer chord when designing the last stage blade, which can achieve higher efficiency and good strength.

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