A Constraint on Planck-scale Modifications to Electrodynamics with CMB polarization data

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Abstract. We show that the Cosmic Microwave Background (CMB) polarization data gathered by the BOOMERanG 2003 flight and WMAP provide an opportunity to investigate in-vacuo birefringence, of a type expected in some quantum pictures of space-time, with a sensitivity that extends even beyond the desired Planck-scale energy. In order to render this constraint more transparent we rely on a well studied phenomenological model of quantum-gravity-induced birefringence, in which one easily establishes that effects introduced at the Planck scale would amount to values of a dimensionless parameter, denoted by $\xi$, with respect to the Planck energy which are roughly of order 1. By combining BOOMERanG and WMAP data we estimate $\xi \simeq -0.110 \pm 0.076$ at the 68% c.l. Moreover, we forecast on the sensitivity to $\xi$ achievable by future CMB polarization experiments (PLANCK, Spider, EPIC), which, in the absence of systematics, will be at the 1-$\sigma$ confidence of $8.5 \times 10^{-4}$ (PLANCK), $6.1 \times 10^{-3}$ (Spider), and $1.0 \times 10^{-5}$ (EPIC) respectively. The cosmic variance-limited sensitivity from CMB is $6.1 \times 10^{-6}$. 

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1. Introduction

The challenge of finding a theory for “quantum gravity” to reconcile quantum mechanics with the General Relativity description of gravitational phenomena has been confronting the theoretical physics community for more than 70 years [1]. A reason why after many decades we understand little on the “quantum-gravity problem” originates from the difficulties encountered in experimentally accessing the realm of quantum corrections to gravity, expected to be originating at the ultrahigh energy scale given by the Planck energy, \( E_p \sim 10^{28}\text{eV} \) (or equivalently the ultrashort Planck length, \( L_p \equiv 1/E_p \sim 10^{-35}\text{m} \)).

Over the last decade the search for experimental manifestations of Planck-scale effects has been reenergized by the realization that certain types of observations in astrophysics do provide indirect access to certain Planck-scale effects [2, 3, 4, 5, 6, 7]. Several authors have already argued that cosmological observations may soon also help in the efforts of searching for hints on the realm of quantum gravity. These expectations originate from the fact that many cosmological observations reflect the properties of the Universe at very early times, when the typical energies of particles were significantly closer to the Planck scale than the energies presently reached in our most advanced particle accelerators. Moreover, the particles studied in cosmology have typically travelled ultra-long (“cosmological”) distances, and therefore even when they are particles of relatively low energies they could be affected by a large accumulation of the effects of the “space-time quantization”, which is one of the most common expectations emerging from quantum-gravity research §.

Over the last few years some authors (see, e.g., Refs. [8, 9, 10, 11]) have indeed started to use ideas coming from quantum-gravity research in cosmology, but the possible connections between cosmological measurements and quantum-gravity research have yet to be fully appreciated in the literature. This is because in the relevant cosmological studies, while one indeed easily sees that the nature of a certain effect is of interest for constraining models of quantum gravity, it is often not easy to establish whether the conjectured magnitude of the change could plausibly be connected with corrections introduced genuinely at the Planck scale. Our main objective here is to establish that in the case of studies of quantum-gravity-induced \( \textit{in-vacuo} \) birefringence, proposed in several quantum-gravity studies (see, e.g., Refs. [3, 12, 13]) with the CMB polarization data we may indeed reach sufficient sensitivity to test effects introduced at the Planck energy scale.

Several studies have already investigated (see, e.g., Refs. [14, 15, 16, 17, 18, 19]) the relevance of CMB polarization data for testing birefringence, and some studies have motivated the analysis also from a broadly-intended quantum-gravity perspective [17, 20], but there is still no dedicated study attempting to establish whether the sensitivity levels provided by CMB observations could provide access to birefringence introduced at the Planck energy scale. Here, we make this connection clear and provide a constraint using existing BOOMERanG and WMAP polarization data. Moreover, we also make predictions for the expectations with future CMB polarization measurements both with space and sub-orbital experiments.

In order to make our case for Planck-scale sensitivity more transparent we model

§ Most approaches to the quantum-gravity problem predict some form of space-time quantization, although different approaches often lead to profoundly different predictions on what space-time quantization should entail.
birefringence in the context of a much-studied formalization of Planck-scale modifications of electrodynamics, a model proposed by Myers and Pospelov \[12\], with Lagrangian density of the form
\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2 E_p} \varepsilon^{jkl} F_{0j} \partial_0 F_{kl}.
\]
(1)

This model was proposed as a field theory formalization of the intuition on Planck-scale modifications of the energy-momentum dispersion relation that had been developed in a series of preceding studies starting from those in Refs. \[2, 3, 21\]. The Lagrangian has been much studied from the quantum-gravity perspective, including several analyses on its observable features (see, e.g. Refs. \[7, 13, 22, 23, 24, 25\]).

We feel that the fact that our suggestion for testing Planck energy scale corrections in CMB polarization data is based on such a well-studied model should render our findings more significant. Moreover, in the case of the Myers-Pospelov field theory it is relatively simple to establish which range of values of the single parameter of the model, \(\xi\), would amount to introducing the effects at the Planck energy scale: the correction term in (1) is of dimension 5, so that the parameter \(\xi\) must be dimensionless, and several studies \[7, 12, 22, 23, 24, 25\] have shown that quantization of space time at the Planck scale would lead to correction terms with \(\xi\) roughly of order 1 (\(|\xi| \sim 1\)) in equation (1). This estimate of course must be handled prudently, since the arguments estimating that the scale of quantum-gravity effects should be given by the Planck scale allow a certain range of uncertainty: for example, most of these arguments rely crucially on the rather optimistic assumption of having no running of the gravitational coupling constant and no role for gravitational effects in the running of other coupling constants, but plausible estimates of how the renormalization-group flow might be affected by gravitational effects show \[24, 26\] that the quantum-gravity scale may well differ by 2 or 3 orders of magnitude from the Planck scale. It is therefore rather significant that the analysis we report here establishes robustly that available CMB polarization data are sensitive to \(|\xi| \sim 1\) and that planned more refined measurements of the CMB polarization will provide access to values of \(|\xi|\) even a few orders of magnitude smaller than 1.

Interestingly, through a combined analysis of BOOMERanG and WMAP data as we describe in detail later we find the following estimate: \(\xi \approx -0.110 \pm 0.076\) at the 68\% c.l.. This constraint very explicitly shows that indeed the relevant CMB polarization data have sensitivity at Planck-scale energies. And it is also intriguing that the analysis leads us to an estimate of \(\xi\) which is nonzero at roughly the 1.5\(\sigma\) level. We shall stress that most previous analyses of birefringence in relation to BOOMERanG and WMAP data assume that the polarization angle rotation, usually denoted by \(\alpha\), is energy independent, \(\alpha = \alpha_0\), whereas within the Myers-Pospelov framework one finds a characteristic energy dependence of \(\alpha\): \(\alpha = \frac{2\pi}{E_p} E^2 T\), where \(E\) is the energy of the radiation and \(T\) is the time of propagation\[1\]. Using the same data that lead to our result \(\xi = -0.110 \pm 0.076\) one would instead find \(\alpha_0 = (-2.4 \pm 1.9)\)\(^\circ\) assuming an energy independent rotation. One could therefore (however weakly) argue that the evidence in favor of birefringence is somewhat more robust for the case of the energy-dependent effect we have considered (a 1.5\(\sigma\) effect versus a 1.3\(\sigma\) effect).

\[\parallel\] This dependence is inverse to the energy dependence expected in polarization rotation associated with Faraday rotation in the presence of a magnetic field with the rotational angle proportional to \(E^{-2}\).
We also comment on previous attempts to constrain $\xi$ using astrophysical data, such as the ones reported in Refs. [7, 22, 23]. These analyses provide bounds on $\xi$ that naively appear to be tighter than the ones we obtained here. However, we find that the frame dependence of the analysis of such symmetry-breaking effects renders those astrophysical analyses inapplicable to the context of CMB studies, performed of course in the reference frame naturally identified by the CMB. A crucial point for this observation is the realization that the space-isotropic form of the Myers-Pospelov Lagrangian density can at best be assumed in a very restricted class of frames of reference: even when the Lagrangian density takes space-isotropic form in one class of frames it will not be in general space-isotropic in other frames. And our reasoning leads us also to conclude that in the hypothetical scenario of future more sensitive CMB polarization data providing more robust evidence of energy-dependent birefringence (say at a 3$\sigma$ level), it would be inappropriate to rush to the conclusion that the space-isotropic Myers-Pospelov framework is finding confirmation. In such a scenario one should then necessarily perform searches of possible evidence of anisotropy.

The paper is organized as follows: In the next Section, we describe the Myers-Pospelov framework, and its implications for birefringence. Then in Section 3 we discuss some aspects of the phenomenology of the Myers-Pospelov framework that are particularly relevant for studies of the CMB. Section 4 contains our key result obtained from CMB polarization observations. In Section 5 we compare our analysis to previous, partly related, analyses, and in particular we discuss the issue of frame-dependent spatial isotropy. We conclude, in section 6, with a brief summary of our results and some observations on the outlook of the studies we are here proposing. We shall work throughout adopting “natural units” ($\hbar = c = 1$), and conventions $\eta^{\mu\nu} = \text{diag}(1, -1, -1, -1)$, for the metric, and $\varepsilon_{123} = 1$, $\varepsilon^{123} = -1$, for the Levi-Civita symbol.

2. Effective field theory for Planck-scale-modified electrodynamics

Motivated by previous studies [2, 3, 21] which had discussed some mechanisms for Planck-scale modifications of the laws of propagation of microscopic particles, in Ref. [12] Myers and Pospelov proposed to describe such effects within the framework of low-energy effective field theory, and observed that, assuming essentially only that the effects are linear in the (inverse of the) Planck scale and are characterized mainly by an external four-vector, one arrives at a single possible correction term for the Lagrangian density of electrodynamics:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2E_p} n^\alpha F_{\alpha\delta} n^\sigma (n_{\beta\varepsilon}^{\beta\delta\gamma\lambda} F_{\gamma\lambda})$$

where the four-vector $n_\alpha$ parameterizes the effect.

The (dimensionless) components of $n_\alpha$ of course take different values in different reference frames, transforming indeed as the components of a four-vector. The quantum-gravity intuition that motivates the study of this model also inspires the choice of the factor $\frac{1}{2E_p}$, and leads one to the expectation that the components of the Myers-Pospelov four-vector should (when nonzero) take values somewhere in the range $10^{-3} < n_\alpha < 10^3$. Values of order 1 would essentially correspond to introducing the effects exactly at the Planck scale. Values as high as $10^3$ would still be plausible, especially in light of some renormalization-group arguments suggesting [26]
that the characteristic scale of quantum-gravity effects might actually be closer to the particle-physics “grand-unification scale” than to the Planck scale. There is no robust argument that would suggest that the characteristic scale of quantum-gravity effects might instead be much higher than the Planck scale, but most authors prudently consider values of parameters such as the $n_\alpha$’s still meaningful even down to $10^{-3}$, because of the desire to be rather cautious before excluding the possibility of a quantum-gravity interpretation.

These arguments provide motivation for developing a phenomenology of the Myers-Pospelov field theory exploring a four-dimensional parameter, $n_\alpha$, keeping in focus the most interesting range of values, $10^{-3} < n_\alpha < 10^3$, and contemplating the characteristic frame dependence of the parameters $n_\alpha$. There is already a rather sizeable literature on this phenomenology (see, e.g., Refs. [23, 24, 25] and references therein) but still fully focused on what turns out to be the simplest possibility for the Myers-Pospelov framework, which is the one of assuming to be in a reference frame where $n_\alpha$ only has a time component, $n_\alpha = (n_0, 0, 0, 0)$. Then, upon introducing the convenient notation $\xi \equiv (n_0)^3$, one can rewrite (2) as

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\xi}{2 E_p} \varepsilon^{jkl} F_{0j} \partial_0 F_{kl},$$

and in particular one can exploit the simplifications provided by spatial isotropy.

We shall also focus here on this case $n_\alpha = (n_0, 0, 0, 0)$, although, unlike previous authors, we shall be rather careful (particularly in Subsection 5.1) in assessing the limitations that this simplifying assumption introduces.

From (3) one easily obtains modified Maxwell equations for the electric and magnetic fields:

$$0 = \partial_j E^j$$
$$0 = -\partial_0 E^k - \partial_j \varepsilon^{jkl} B_l - \frac{2\xi}{E_p} (\partial_0)^2 B^k,$$

and

$$0 = \partial_k \varepsilon^{0kjl} E_l - \partial_0 B^j$$
$$0 = \partial_j B^j.$$

Note that this second set of equations is undeformed with respect to the classical ($\xi = 0$) case, since it follows only from the antisymmetry of the electromagnetic tensor $F_{\mu\nu}$. The resulting equation of motion for the electric field is:

$$0 = -\partial^0 \partial_0 E^k - \partial^l (\partial_l E^k - \partial^k E_l) - \frac{2\xi}{E_p} \varepsilon^{mkn} (\partial_0)^2 \partial_m E_n.$$

For the case of plane waves, $E_m(x) = E_m(k) e^{-ik_\rho x^\rho}$, this equation of motion takes the form:

$$0 = k_0^2 E^3 + k^l (k_1 E^j - k^j E_1) - \frac{i \xi}{E_p} (k_0)^2 (k_m \varepsilon^{mjn} E_n).$$

Since the Lagrangian density of Eq. (3) is still symmetric under space-rotations there is clearly no loss of generality in fixing the direction of propagation of the plane wave in, say, the direction $\hat{z}$, with wave vector $k_\rho = (\omega, 0, 0, p)$. For such a plane wave propagating along the $\hat{z}$ axis the three equations of motion given by Eq. (7) take the form

$$0 = \omega^2 E_3$$
\[ 0 = -\omega^2 E_1 + p^2 E_1 + \frac{2i\xi}{E_p} \omega^2 p E_2 \]
\[ 0 = -\omega^2 E_2 + p^2 E_2 - \frac{2i\xi}{E_p} \omega^2 p E_1. \] (8)

The first of these equations simply states that the longitudinal component of the electric field is still absent (even for \(\xi \neq 0\)), and therefore for a plane wave propagating along the \(\hat{z}\) axis one has \(E_3 = 0\). The remaining two equations can be compactly reorganized as follows:

\[ 0 = (-\omega^2 + p^2) (E_1 \pm iE_2) \pm \frac{2\xi}{E_p} \omega^2 p (E_1 \pm iE_2), \] (9)

meaning that the right- and left- circularly polarized fields \(\vec{E}_\pm \equiv E_1\hat{\epsilon}_x \pm iE_2\hat{\epsilon}_y\) satisfy the equation of motion:

\[ \left(-\omega^2 + p^2 \pm \frac{2\xi}{E_p} \omega^2 p\right) \vec{E}_\pm = 0. \] (10)

So the electric field is subject to birefringence: the left- and right-circular polarized components have different dispersion relations:

\[ \omega_\pm \approx p \left(1 \pm \frac{\xi}{E_p} p\right), \] (11)

where we included only the leading correction in \(\xi/E_p\). The components propagate with different group velocities \(v_{g\pm} = \frac{\partial E_\pm}{\partial p} \approx 1 \pm 2\frac{\xi}{E_p} p\).

Assuming that the electric field is originally linearly polarized, then, due to the different group velocities of its circularly polarized components, if the time of propagation of the radiation is sufficiently long, one might have an observably large difference in the time of arrival of the two circularly-polarized components of the field, and the linear polarization would therefore be lost [22]. If instead the time of propagation is not sufficient to produce a detectable difference in the time of arrival of the two modes with different group velocities, the direction of the original linear polarization is rotated (see [27] and references therein). In particular, if the wave propagates for a time \(T\), the amount of rotation is:

\[ \alpha = (\omega_- - \omega_+)T = 2\frac{\xi}{E_p} p^2 T. \] (12)

3. Effects on CMB polarization and its power spectrum

CMB radiation is known to have been generated with some degree of linear polarization (see e.g. [28] and references therein), and some linear polarization is observed, so our analysis must be performed in the regime governed by Eq. (12). In light of Eq. (12), measurements of the amount of rotation of the CMB polarization can be interpreted as measurements of the parameter \(\xi\). To be more precise, one has to consider the fact that the CMB photons propagate into an expanding universe, so the energy of the photons is redshifted during the propagation. The photon energy redshift dependence is:

\[ \omega(z) = \omega_0(1 + z), \] (13)
where \( \omega_0 \) is the energy of the photon when it arrives on Earth. So the amount of rotation of the polarization plane is, for a CMB photon propagating for a time \( t \) from the last scattering surface \( (t = 0) \):

\[
\alpha(t) = \int_0^t \frac{\xi}{E_p} p(t')^2 dt',
\]

The amount of rotation of the polarization of photons propagating from the last scattering surface toward us \( (t = T) \) can be written in terms of the redshift \( z \) as (on the last scattering surface \( z = Z \), now \( z = 0) \):

\[
\alpha = 2 \frac{\xi}{E_p} p_0^2 \int_0^Z (1 + z) H^{-1} dz.
\]

Since to a first approximation \( H = H_0 \sqrt{\Omega_m(1 + z)^3 + \Omega_\Lambda} \), where \( H_0 \approx 10^{-18} \text{s}^{-1} \) is the Hubble parameter (in terms of the reduced Hubble constant \( H_0 = h \cdot 100(\text{Km/s})/\text{Mpc} \)):

\[
\alpha = 2 \frac{\xi}{E_p} p_0^2 \int_0^Z \frac{(1 + z)}{\sqrt{\Omega_m(1 + z)^3 + \Omega_\Lambda}} dz.
\]

The formal description of the rotation with angular velocity \( \omega \equiv \frac{d\alpha}{d\eta} = \frac{d\alpha}{dt} a_0 \) of the CMB polarization plane due to any physical effect requires a modification of the Boltzmann equation which governs the time evolution of a single Fourier mode of the perturbation of the Stokes parameters \( Q \) and \( U \). For scalar perturbations the Boltzmann equations become [29, 30]:

\[
\dot{\Delta}_Q + ik\mu \Delta_Q = -\tau \left[ \Delta_Q + \frac{1}{2} (1 - P_2(\mu)) S_P \right] + 2\omega \Delta_U
\]

\[
\dot{\Delta}_U + ik\mu \Delta_U = -\tau \Delta_U - 2\omega \Delta_Q
\]

where the dots indicate derivatives with respect to the conformal time \( \eta \), \( \mu \) is the cosine of the angle between the photon direction and the Fourier wave vector, \( \tau \equiv x_e n_e \sigma_T a / a_0 \), with \( x_e \) the ionization fraction, \( n_e \) the electron number density, \( \sigma_T \) the Thomson cross-section and \( a \) the scale factor \( (a_0 \text{ the scale factor today}) \). \( S_P \equiv \Delta_{T^2} + \Delta_{Q^2} - \Delta_{Q^0} \) is the source function, \( \Delta_{T_i} \) and \( \Delta_{Q_i} \) indicate, respectively, the \( i \)-th moment in the expansion of the temperature perturbation \( \Delta_T \) and of the polarization perturbation \( \Delta_Q \) in Legendre polynomials; \( P_2(\mu) \) is the second Legendre polynomial. Note that the equation for \( \Delta_U \) has no generating term, since the \( U \) component of polarization is not generated by scalar perturbations, but by tensors.

These coupled differential equations can be separated taking the sum and the difference of the first equation with \( i \) times the second. Then a formal integration leads to the solution [29]:

\[
(\Delta_Q \pm i\Delta_U)(\eta_0) = \int_0^{\eta_0} d\eta \left[ \tilde{\tau}(\eta)e^{ik\mu(\eta - \eta_0) - \tau(\eta)} S_P(\eta)e^{\pm 2i \int_{\eta_0}^{\eta} d\eta' \omega(\eta')} \right]
\]

where \( \tau(\eta) = \int_0^{\eta_0} d\eta' \tilde{\tau}(\eta') \) and \( \eta_0 \) is the conformal time today.

Since in our case both \( \omega \) and \( \alpha \) are expected to be very small and the visibility function \( \tilde{\tau}(\eta)e^{-\tau(\eta)} \) peaks tightly at the time of decoupling \( \eta = \eta_{dec} \), a very good approximation to the above solution is given by:

\[
\Delta_Q(\eta_0) = \tilde{\Delta}_Q(\eta_0) \cos \left( 2 \int_{\eta_0}^{\eta_{dec}} d\eta' \omega(\eta') \right) = \tilde{\Delta}_Q(\eta_0) \cos (2\alpha)
\]

\[
\Delta_U(\eta_0) = \tilde{\Delta}_Q(\eta_0) \sin \left( 2 \int_{\eta_0}^{\eta_{dec}} d\eta' \omega(\eta') \right) = -\tilde{\Delta}_Q(\eta_0) \sin (2\alpha)
\]
where $\tilde{\Delta}_Q(\eta_0) \equiv \Delta_Q(\eta_0)|_{\omega=0}$ and $\alpha$ is given by eq. (16). Taking into account the tensorial modes of the perturbations, which generate a primordial $U$ mode, it can be shown that the effects of a rotation of the polarization direction are:

$$\Delta_Q(\eta_0) = \tilde{\Delta}_Q(\eta_0) \cos (2\alpha) + \tilde{\Delta}_U(\eta_0) \sin (2\alpha)$$
$$\Delta_U(\eta_0) = - \tilde{\Delta}_Q(\eta_0) \sin (2\alpha) + \tilde{\Delta}_U(\eta_0) \cos (2\alpha).$$

(20)

This leads to the polarization power spectrum $C_{\ell}^{XY} \sim \int dk \ [k^2 \Delta_X(\eta_0) \Delta_Y(\eta_0)]$, $X, Y = T, E, B$:

$$C_{\ell}^{EE} = \tilde{C}_{\ell}^{EE} \cos^2 (2\alpha) + \tilde{C}_{\ell}^{BB} \sin^2 (2\alpha)$$
$$C_{\ell}^{BB} = \tilde{C}_{\ell}^{EE} \sin^2 (2\alpha) + \tilde{C}_{\ell}^{BB} \cos^2 (2\alpha)$$
$$C_{\ell}^{EB} = \frac{1}{2} \left( \tilde{C}_{\ell}^{EE} - \tilde{C}_{\ell}^{BB} \right) \sin (4\alpha)$$
$$C_{\ell}^{TE} = \tilde{C}_{\ell}^{TE} \cos (2\alpha)$$
$$C_{\ell}^{TB} = \tilde{C}_{\ell}^{TE} \sin (2\alpha)$$

(21)

We checked numerically that evaluating the power spectra given by equation (21) instead of the exact formula in equation (18) (and the corresponding one for tensorial modes), leads to an error on each $C_{\ell}^{XY}$ that, for $\int_{\eta_0}^{\eta_{dec}} d\eta' \omega(\eta')$ up to six degrees, is much less than 1%.

4. Data Analysis

In this section we discuss the current constraints on $\alpha$ and on $\xi$ from the most recent CMB polarization datasets coming from the WMAP five-year mission [18, 31] and the BOOMERanG polarization flight in 2003 [32]. We follow the standard approach in the literature by making use of the the publicly available Markov Chain Monte Carlo package cosmomc [33] with a convergence diagnostics done through the Gelman and Rubin statistic.

In our analysis we consider the complete set of power spectra (21), taking into account also $TB$ and $EB$ cross-correlation power spectra, which are usually set to zero in the standard CMB calculations. We sample the following eight-dimensional set of cosmological parameters, adopting flat priors on them: the physical baryon and Cold Dark Matter densities, $\omega_b = \Omega_b h^2$ and $\omega_c = \Omega_c h^2$, the ratio of the sound horizon to the angular diameter distance at decoupling, $\theta_s$, the scalar spectral index $n_s$, the overall normalization of the spectrum $A_s$ at $k = 0.05 \text{ Mpc}^{-1}$, the optical depth to reionization, $\tau$, the tensor to scalar ratio of the primordial spectrum, $r$ and, finally, the rotation of the power spectrum discussed above, $\alpha$. We derive the value of $\xi$ for each Markov chain step using (16), so including the dependence on the geometrical parameters of the universe. Simultaneously we also use a top-hat prior on the age of the Universe as $10 \text{Gyr} < t_0 < 20 \text{ Gyr}$. Furthermore, we consider purely adiabatic initial conditions, we impose flatness, we treat the dark energy component as a cosmological constant, and we include the weak lensing effect in the CMB spectra computation.

In Table 1 we show the current constraints on $\alpha$ and $\xi$ from current experiments. WMAP 5-year data provide the constraint of $\alpha = (-1.6 \pm 2.1)^o$ (consistent with previous analyses [18, 14]), i.e. with no indication for $\alpha$ different from zero, while BOOMERanG data give $\alpha = (-5.2 \pm 4.0)^o$, with a $\sim 1.3\sigma$ hint for $\alpha < 0$. Assuming $\alpha$ energy-independent ($\alpha = \alpha_0$), the
joint estimation from BOOMERanG and WMAP5 data gives $\alpha_0 = (-2.4 \pm 1.9)\degree$, which is also consistent with $[14]$. As regards to the $\xi$ parameter, from WMAP 5-year data we constrain it to be $\xi = -0.09 \pm 0.12$, while with BOOMERanG data we obtain $\xi = -0.123 \pm 0.096$. Since $\xi$ is energy-independent, we can also make a combined analysis of the two datasets, which results in the estimation of $\xi = -0.110 \pm 0.075$.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Experiment & $\alpha \pm \sigma(\alpha)$ & $\xi \pm \sigma(\xi)$ \\
\hline
WMAP (94 GHz) & $-1.6 \pm 2.1$ & $-0.09 \pm 0.12$ \\
BOOMERanG (145 GHz) & $-5.2 \pm 4.0$ & $-0.123 \pm 0.096$ \\
WMAP+BOOMERanG & - & $-0.110 \pm 0.075$ \\
\hline
\end{tabular}
\caption{Mean values and 1$\sigma$ error on $\alpha$ (in degrees) and $\xi$ for WMAP, BOOMERanG and WMAP+BOOMERanG}
\end{table}

We also analyze the ability of future experiments in constraining $\alpha$ and $\xi$. We created several mock datasets with noise properties consistent with the PLANCK satellite $[34]$, the Spider balloon borne experiment $[35]$ and the EPIC satellite $[36]$, assuming, as fiducial model, the best fit of WMAP with $\alpha = 0$ (see Table 2 for the experimental specifications used in the analysis).

We analyze these datasets with the full sky exact likelihood routine $[37]$ in the cosmomc code including the $f_{sky}^2$ factor to reduce the degrees of freedom, ignoring correlations between different multipoles $\ell$. Firstly, we consider the specifications for the PLANCK satellite for the channels at 70, 100, 143 and 217 GHz respectively. Clearly the presence of different channels in the PLANCK experiment has the main goal of foreground removal. However we report the sensitivity on $\alpha$ and $\xi$ for each channel considering the possibility of a frequency dependence of the cosmic signal. As we can see in Table 3 the PLANCK channel with the highest sensitivity on $\alpha$ is the 143 GHz channel, able to constrain $\alpha$ at the level of 0.07$\degree$ and $\xi$ at level of 0.0017. Lower sensitivities on $\alpha$ will be reached by other “high frequency” channels but, because of the energy dependence of $\alpha$, the 217 GHz channel is the most sensitive to $\xi$, since it allows to constrain this parameter at the level of 0.0010. The LFI 70 GHz channel can provide a constraint of the order of 0.64$\degree$ for $\alpha$ and 0.06 for $\xi$. It is interesting to observe that combining the three HFI channels (100, 143, 217 GHz) it is possible to improve the sensitivity on the $\xi$ parameter, up to $8.5 \cdot 10^{-4}$.

Before continuing it is important to bear in mind that any systematic present in the calibration of the polarization angle will be reflected in a spurious $\alpha$ and, as a consequence, a spurious $\xi$. The current HFI and LFI channels are expected to be calibrated at a level of about $\sim 1$° thanks to measurements of polarized emission from the Crab nebula. The expected values reported in the Table are therefore not taking in to account this possible systematic. However, a multi-frequency analysis could permit to distinguish a systematic effect due to miscalibration from a true birefringence effect, since the first is energy independent, while the second has a specific energy dependence.

$\dagger$ For the analysis using the BOOMERanG dataset, we fixed the optical depth $\tau$ to the fiducial value of $\tau = 0.09$. 

\[\]
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| Experiment | Channel | FWHM | ∆T/T |
|------------|---------|------|-------|
| PLANCK    | 70      | 14'  | 4.7   |
|           | 100     | 10’  | 2.5   |
|           | 143     | 7.1’ | 2.2   |
|           | 217     | 5’   | 4.8   |
| Spider    | 145     | 40’  | 0.30  |
| f_{sky} = 0.50 |
| EPIC      | 70      | 12’  | 0.066 |
|           | 100     | 8.4’ | 0.066 |
|           | 150     | 5.6’ | 0.084 |
|           | 220     | 3.8’ | 0.17  |

Table 2. PLANCK, Spider and EPIC experimental specifications. Channel frequency is given in GHz, FWHM in arcminutes and noise per pixel in $10^{-6}$ for the Stokes I parameter; the corresponding sensitivities for the Stokes Q and U parameters are related to this by a factor of $\sqrt{2}$.

| Experiment | Channel | σ(α)  | σ(ξ)  |
|------------|---------|-------|-------|
| PLANCK    | 70      | 0.64  | 6.0 · 10^{-2} |
|           | 100     | 0.14  | 6.5 · 10^{-3} |
|           | 143     | 0.073 | 1.7 · 10^{-3} |
|           | 217     | 0.10  | 1.0 · 10^{-3} |
|           | 100+143+217 | -     | 8.5 · 10^{-4} |
| Spider    | 145     | 0.27  | 6.1 · 10^{-3} |
| EPIC      | 70      | 2.1 · 10^{-3} | 1.9 · 10^{-4} |
|           | 100     | 1.8 · 10^{-3} | 7.8 · 10^{-5} |
|           | 150     | 1.5 · 10^{-3} | 2.9 · 10^{-5} |
|           | 220     | 1.2 · 10^{-3} | 1.1 · 10^{-5} |
|           | 70+100+150+220 | -   | 1.0 · 10^{-5} |
| CVL       | 150     | 6.1 · 10^{-4} | 1.3 · 10^{-5} |
|           | 217     | 6.1 · 10^{-4} | 6.1 · 10^{-6} |

Table 3. Expected 1σ error for PLANCK 70, 100, 143, 217 GHz, Spider 145 GHz, EPIC 70, 100, 150, 220 GHz and two ideal CVL experiment at 150 GHz and 217 GHz on α (in degrees) and ξ.

It is interesting to extend the forecast to other future experiments as Spider and EPIC. Spider, a balloon borne experiment that will fly over Antarctica in 2012, will constrain variation of α at level of 0.27° and of ξ at level of 6.1 · 10^{-3}, with a sensitivity competitive with PLANCK. The EPIC satellite proposal, or an equivalent next generation CMBpol satellite mission, can detect deviations as small as 0.0012° for α and 1.0 · 10^{-5} for ξ, providing a dramatic improvement respect to PLANCK and Spider. Again, the channel at higher frequency provides the best
constrain. Finally we consider an ideal experiment, cosmic variance limited in anisotropy and polarization measurements. This experiment provides a sort of fundamental limit to the precision that can be achieved. We found that in this, ideal, experiment, angles as small as $\alpha = 0.0006^\circ$ could be, in principle, measured, and we observe in Table 3 that this would provide sensitivity to $\xi = 1.3 \cdot 10^{-5}$, for an experiment at 150, or even $\xi = 6.1 \cdot 10^{-6}$, for an experiment at 217 GHz.

5. Comparison with other results

5.1. Comparison with limits previously obtained using observations in astrophysics

While ours is the first explicit work aimed at comparing the predictions of the Myers-Pospelov framework to CMB data, there have been a few other attempts, such as the ones reported in Refs. [22, 7, 23, 25], to place limits on the parameter $\xi$ using certain observations in astrophysics. The outcome of some of these analyses was simply described in terms of limits on the parameter $\xi$, as if they were absolute limits, but we must here stress that, in light of the very explicit frame dependence of the Myers-Pospelov model, one of course cannot establish absolute limits on parameters. From the analysis of experimental data collected in a certain frame one can definitely obtain even very robust limits on the values of some of the components of the four-vector $n_\alpha$ (see Eq. (2)) in that frame, but it is highly nontrivial to then qualify such results from a frame-independent perspective.

All these previous studies have, like the one we are here reporting, focused on the implications of a time component for the Myers-Pospelov symmetry-breaking four-vector $n_\alpha$, a restriction which is introduced very explicitly by assuming $n_\alpha = (n_0, 0, 0, 0)$. But of course the quantum-gravity arguments [2, 3, 21] that motivated the study of the Myers-Pospelov framework do not provide any indication that the four-vector should take this form in any specific frame ($n_\alpha$ may well not even be time-like), and even if it took this form in a certain class of frames it would then definitely have different form in other frames, transforming like a four-vector from frame to frame.

Constraints on all four components of $n_\alpha$ in one frame could be converted, through appropriate Lorentz transformations, into analogous (but possibly very different in magnitude) constraints applicable in another frame. But bounds established exclusively for the time component of $n_\alpha$ are of very limited applicability in other frames.

This is particularly significant for our results since some analyses of data in astrophysics can lead to very stringent bounds on $\xi$. In one recent such study [25] it was even argued that observations of polarized radiation from the Crab Nebula can be used to obtain the impressive bound $|\xi| < 10^{-9}$, which would amount to the constraint $|n_0| < 10^{-3}$ on the time component of the four-vector $n_\alpha$. Clearly these results obtained in astrophysics start to provide us an intuition that large values of $n_0$ are disfavored, but at the same time it should be noticed that it is not easy to convert such results into intelligible constraints applicable in reference frames that are different from the “laboratory frame” where the bound was derived. In principle one could even contemplate the possibility that in such a laboratory frame one has, say, $n_\alpha = (0, 10^3, 10^3, 10^3)$, which would be compatible with any upper bound on $n_0$ and would still be meaningful from a
quantum-gravity perspective (since, as mentioned in Section 2, components of $n_\alpha$ with values of order $10^3$ could be expected in rather popular scenarios estimating the scale of quantum-gravity as roughly given by the grand-unification scale). And if indeed $n_\alpha = (0, 10^3, 10^3, 10^3)$ in some laboratory frame then a boost of unimpressive magnitude, with, say, $\beta \simeq 10^{-3}$, could take us from that laboratory frame to a frame where the time component of $n_\alpha$ is actually of order 1.

In light of the rather significant frame dependence implied by these considerations it is rather clear that for this research program it is a top priority to move on to bounding experimentally all four components of $n_\alpha$. And for works that focus on the time component it appears that it is rather advantageous to focus on studies of the CMB since one can at least rather easily combine different sets of data in a meaningful way, by describing them all in the natural reference frame of the CMB.

Also significant from the perspective of the frame dependence of the Myers-Pospelov framework is the fact that the results of our analysis, while being inspired by this framework, depend most strongly on the assumption of a characteristic (quadratic) energy dependence of $\alpha$, and not very sensitively on the details of the Myers-Pospelov framework with $n_\alpha = (n_0, 0, 0, 0)$. Such an energy dependence is a generic feature of any quantum-gravity-inspired description of birefringence, since it is obtained using only dimensional analysis and the fact that the effects must disappear in the $E_p \to \infty$ limit. One should therefore expect a similar energy dependence also in the Myers-Pospelov framework with generic $n_\alpha$ and in models where the breakdown of Lorentz symmetry is governed by, say, a two-index tensor.

5.2. Comparison with previous CMB limits on energy-dependent birefringence

While ours is the first study to discuss a robust link from the parameters of a model that has been extensively studied in the quantum-gravity literature and CMB polarization data, from a broader phenomenological perspective the possibility of a quadratic energy dependence of $\alpha$, which is indeed the main characteristic of the effects we studied, had already been considered in Ref. [20]. In order to compare of our results to the ones of Ref. [20] we must establish a link between the parameter $\xi$ of the (isotropic) Myers-Pospelov model and the multi-parameter description of birefringence effects for CMB polarization data analyses adopted in Ref. [20]. This can be done by describing the rotation of linear polarization of CMB radiation codified in our Eq. (16) in terms of rotations of the Stokes vector $\vec{s} \equiv (Q, U, V)^T$, since it is at the level of these rotations of the Stokes vector that Ref. [20] introduced its multi-parameter phenomenological picture.

Let us start by noticing that the variation of the Stokes vector due to birefringence (not necessarily between circularly polarized components) can be written as [20]:

$$\frac{d\vec{s}}{dt} = 2\omega \vec{\zeta} \times \vec{s}$$  \hspace{1cm} (22)

where $\vec{\zeta}$ is proportional to the Stokes vector of the faster of the two modes subject to birefringence.

The Stokes vector $\vec{\zeta}$ can be written in the spin-weighted basis, $\vec{\zeta}_{SW} \equiv (\zeta_{(+2)}, \zeta_{(0)}, \zeta_{(-2)})^T \equiv (\zeta^1 - i\zeta^2, \zeta^3, \zeta^1 + i\zeta^2)^T$, and each component of $\vec{\zeta}_{SW}$ can be expanded in spin-weighted spherical
harmonics:
\[
\zeta(\pm 2) = \sum_{lm} (k_{(E)lm} \pm i k_{(B)lm}) \pm 2Y_{lm}, \quad \zeta(0) = \sum_{lm} k_{(V)lm} 0Y_{lm}. \tag{23}
\]

Each coefficient \(k_{(V)lm}\) and, respectively, \(k_{(E,B)lm}\) in Eq. (23) can be expressed as a combination of only odd and, respectively, even powers of \(\omega\). The parameters \(k^{(d)}_{(V)lm}\) and \(k^{(d)}_{(E,B)lm}\) used in [20] are the coefficients of the expansion: \(k_{(V)lm} = \sum_{d \text{ odd}} \omega^{d-4} k^{(d)}_{(V)lm}\), \(k_{(E,B)lm} = \sum_{d \text{ even}} \omega^{d-4} k^{(d)}_{(E,B)lm}\).

In the case of our interest, in which the rotation of the Stokes parameter is due to a birefringence between circularly polarized components of the radiation, taking conventionally as the fastest mode the right-circularly polarized one, one finds that the vector \(\vec{\zeta}\) is proportional to \((0, 1, 0)^T\) in the spin-weighted basis. The variation of the Stokes vector is, in the \((Q, U, V)^T\) basis:
\[
\frac{d\vec{S}}{dt} = \begin{pmatrix} -Q_0 \sin(\alpha) + U_0 \cos(\alpha) \\ -Q_0 \cos(\alpha) - U_0 \sin(\alpha) \\ 0 \end{pmatrix} \cdot \frac{d(\alpha)}{dt}, \tag{24}
\]
where \(\alpha(t)\) is given by Eq. (14):
\[
\frac{d\alpha}{dt}(t) = 2\frac{\xi}{E_p} p(t)^2.
\]
Writing this equation in the spin-weighted basis, evaluating it at time \(t = T\) (today) and comparing with Eq. (22), one finds \(\zeta(0) = -\frac{\xi}{E_p} p_0\). So all the coefficients of the expansion of \(\zeta(\pm 2)\) are null, while for \(\zeta(0)\) the only nonzero coefficient is \(k_{(V)00} = -\frac{\xi}{E_p} p_0 \sqrt{4\pi}\), because \(\zeta(0)\) doesn’t depend on the direction of observation in the sky. Since \(k_{(V)00}\) is linear in the energy of the photon, there is only one nonzero coefficient in the expansion of \(k_{(V)00}\), and this coefficient, which was denoted by \(k^{(5)}_{(V)00}\) in Ref. [20], is therefore the only parameter in the parameterization introduced in Ref. [20] that would reflect the presence of a (time-component-only) Myers-Pospelov term in the Lagrangian density of electrodynamics. The comparison between our results and the ones of Ref. [20] should therefore be based on the relationship between our parameter \(\xi\) and the parameter \(k^{(5)}_{(V)00}\), which is
\[
k^{(5)}_{(V)00} = -\frac{\xi}{E_p} \sqrt{4\pi}. \tag{25}
\]
For this parameter Ref. [20] arrives at an estimate \(k^{(5)}_{(V)00} \simeq (3 \pm 2) \times 10^{-20} GeV^{-1}\), using exclusively BOOMERanG data and without performing a Markov chain analysis, but rather fixing the cosmological model to the WMAP best fit model and neglecting therefore possible correlations between the cosmological parameters.\(^+\) In light of (25) our result \(\xi \simeq -0.110 \pm 0.076\) amounts to \(k^{(5)}_{(V)00} = (3.2 \pm 2.1) \times 10^{-20} GeV^{-1}\). This is fully consistent with the findings of Ref. [20], and we would like to argue that our result should now be viewed as the benchmark.

\(^+\) An effect of this could be the fact that another parameter studied by [20], \(k^{(3)}_{(V)00} \), is estimated also from the BOOMERanG dataset to be \(k^{(3)}_{(V)00} = (12 \pm 7) \times 10^{-43} GeV\), which corresponds to a rotation angle \(\alpha = 12 \pm 7\) degrees, while the estimate of \(k^{(5)}_{(V)00} = (3 \pm 2) \times 10^{-20} GeV^{-1}\) from the same dataset corresponds to a rotation angle \(\alpha = 4 \pm 3\) degrees.
for the limits on $k_{(V)00}^{(5)}$, both because we relied on a more sizeable data sample, combining BOOMERanG and WMAP data, and because of the marginalization over the remaining cosmic parameters.

6. Conclusions and Outlook

The main objective of our analysis was to establish robustly that cosmology and particularly CMB data can provide constraints on effects that are of interest from a quantum-gravity perspective and are introduced at the Planck scale. This possibility has been contemplated in several previous studies but never in a way that would transparently expose sensitivity to effects introduced truly at the Planck scale. We feel that we have fully achieved our goal by relying on the Myers-Pospelov model, and on the rather large quantum-gravity literature on this model, which also provided a clear target for the description of effects introduced at the Planck scale. We therefore hope that our analysis will motivate a more intense program of studies of quantum-gravity effects in cosmology, not necessarily focused on in-vacuo birefringence and/or the Myers-Pospelov model.

It is intriguing that our estimate of the Myers-Pospelov $\xi$ parameter, $\xi = -0.110 \pm 0.076$, provides a faint indication for a nonzero effect. In this respect it is certainly valuable that, as discussed in Section 4, future experiments will be able to measure CMB radiation polarization with higher sensitivities, and we find that this should lead to an improvement of three orders of magnitude in the constraints obtained following our strategy of analysis ($\sigma(\xi) \sim 10^{-5}$).

As stressed in Subsection 5.1, in light of the rather significant frame dependence introduced by the Myers-Pospelov setup, this type of sensitivities obtainable from CMB-data analyses are still rather meaningful, in spite of the apparently ultra stringent constraints that have been obtained on the parameter $\xi$ using astrophysical observations. We also showed that CMB data can be used to determine the sign of $\xi$, a possibility which was never exposed in previous works relying on astrophysics and that may prove challenging in those contexts.

From the methodological aspect, while in the past the birefringence of CMB photons has been widely studied, and several bounds have already been provided on the birefringence angle $\alpha$, here we pointed that in the literature the possible energy dependence of the effect has not been adequately contemplated, leading to possible misleading conclusions when averaging on different photon frequencies. We showed how exploiting the availability of data from different channels of frequency of the photons improves the sensibility on the birefringence parameter $\xi$ and leads to a slightly stronger indication for a nonzero effect, than the results obtained estimating the birefringence angle $\alpha$ with an average on frequencies. This should be taken into account in planning future CMB experiments, and may turn out to be another reason for combining cosmological and astrophysical investigations of the Myers-Pospelov framework.

Here, we have analyzed a uniform, isotropic rotation to the EM polarization vectors. But, as stressed in Subsection 5.1, the quantum-gravity motivation for investigating the Myers-Pospelov framework and the intrinsic structure of the Planck-scale correction term added to the Lagrangian density of electrodynamics provides a strong invitation to contemplate also effects resulting in anisotropic rotation of polarization. In an upcoming paper we plan to return to this issue and address the extent to which CMB polarization measurements can constrain such
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an anisotropic model.

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