RECENT DEVELOPMENT ON COLLECTIVE NEUTRINO INTERACTIONS

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Quantum Field Theory is applied to study an electron plasma under an intense neutrino flux. The dispersion relation of the longitudinal waves is derived and the damping rate is calculated. It is shown that in the case of Supernova emission the neutrinos are not collimated enough to cause plasma instabilities associated to a strong neutrino resonance effect.

1 Introduction

Bingham et. al. \cite{1} have studied the behavior of an electron plasma under a neutrino flux and concluded that neutrino fluxes as intense as the ones produced in Supernovae cause plasma instabilities. If true this could provide the physical mechanism of energy transfer from the neutrinos to the medium that would explain the Supernovae explosion. The weak interaction between the neutrinos and the plasma was described with the concept of a ponderomotive force acting on the electrons,

$$ F_{\text{pond}}^\nu = -\sqrt{2} G_F c'_V \vec{\nabla} n_\nu , \quad (1) $$

proportional to the gradient of the neutrino number density \((c'_V = \pm 1/2 + 2\sin^2\theta_W, (+) \text{ for } \nu_e \text{ and } (-) \text{ for } \nu_\mu, \nu_\tau)\). The neutrino wave function was assumed to obey a naive Klein-Gordon equation modified to include matter effects. However, that equation does not account for the neutrino spin degrees of freedom and those works were subject to justified controversy. \cite{2,3}

More recently, a classic kinetic theory was applied to describe the electron and neutrino dynamics. \cite{4} In that work the neutrinos too are subject to a ponderomotive force analogous to Eq. \((1)\) with \(n_\nu\) replaced by the electron number density \(n_e\). Again the lepton spin and chiral structure of the weak interactions were ignored. In addition, the dispersion relation derived for the plasma waves is substantially different from the one previously obtained with

\cite{a}Talk given at "Second Meeting on New Worlds in Astroparticle Physics", Faro, Portugal, 1998.
the Klein-Gordon equation. Nevertheless, the authors reiterated the claim that the neutrinos produce resonant effects for certain modes of plasma oscillation which cause instabilities characterized by large growth rates proportional not to $G_F^2 n_\nu$ but to a smaller power of that number.

The aim of the present work was to apply consistent Quantum Field Theory techniques to obtain a kinetic description consistent with the quantum mechanics and spin content of particles and interactions. First, we derive the dispersion relation of the longitudinal waves of an electron plasma in the presence of a neutrino flux and afterwards, analyze the possibility of neutrino induced plasma instabilities.

2 Classic Kinetic Theory

It proves useful to go first to a classic description before going to Quantum Field Theory. The system of interest is a large number of electrons, neutrinos and anti-neutrinos. Classically they are described by distribution functions, $f_e$, $f_\nu$ and $f_{\bar{\nu}}$, on the respective position and momentum phase spaces. In low dense plasmas the collisions are less important than the collective interactions and the evolution of the distribution functions $f(t, \vec{x},\vec{p})$ reduces to the Vlasov equations:

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \frac{\partial f}{\partial \vec{x}} + \vec{F} \cdot \frac{\partial f}{\partial \vec{p}} = 0 .$$

The dynamics is defined by specifying the velocity $\vec{v}$ and force $\vec{F}$ as functions of $(t, \vec{x},\vec{p})$ for each of the particles species. In the electron case $\vec{v} = 0/(\vec{p}^2 + m_e^2)^{1/2}$, but the neutrino mass will be neglected.

At low energies the electron interactions are described by the Standard Model effective Lagrangian:

$$L_{\text{int}} = e A_\mu \bar{\nu}_L \gamma^\mu \nu_L - \sqrt{2} G_F \langle \bar{\nu}_L \gamma_\mu \nu_L \rangle \bar{\nu}_L \gamma^\mu (c'_V - c'_A \gamma_5) e ,$$

where $e$ is the positron charge, $G_F$ the Fermi constant, and $c'_V = c'_A + 2 \sin^2 \theta_W$ with $c'_A = +1/2$ for $\nu_e$ and $c'_A = -1/2$ for $\nu_\mu$, $\nu_\tau$. By analogy with electrodynamics one immediately infers the Lagrangian of a classic spinless electron as

$$L_e = \left( e A_\mu - \sqrt{2} G_F c'_V j^\mu \right) \dot{x}_\mu .$$

The average current density $j^\mu = \langle \bar{\nu}_L \gamma^\mu \nu_L \rangle$, equal to the difference between the neutrino and anti-neutrino current densities, $j^\mu = j^\mu_\nu - j^\mu_{\bar{\nu}}$, acts in the same way as the electromagnetic potential $A_\mu$. Hence, the electro-weak force applied to an electron is a straightforward generalization of the Lorentz force,

$$F'_e = -e(\vec{E} + \vec{v}_e \wedge \vec{B}) + \sqrt{2} G_F c'_V (\vec{E}_w e + \vec{v}_e \wedge \vec{B}_w e) ,$$

where $\vec{E}_w e$ and $\vec{B}_w e$ denote the electromagnetic fields for the electron.
with weak-electric and weak-magnetic fields given by

\[ \vec{E}_{we} = -\vec{\nabla} J^0, \quad \vec{B}_{we} = \vec{\nabla} \times \vec{J} . \]  

(6)

In a similar fashion, the weak forces applied to massless neutrinos is

\[ \vec{F}_\nu = \sqrt{2} G_F (\vec{E}_{\nu e} + \vec{v}_\nu \times \vec{B}_{\nu e}) \]  

(7)

for neutrinos (\( \nu_L \)) and \(-\vec{F}_\nu\) for anti-neutrinos (\( \bar{\nu}_R \)), where

\[ \vec{E}_{\nu e} = -\vec{\nabla} J^0, \quad \vec{B}_{\nu e} = \vec{\nabla} \times \vec{J} . \]  

(8)

are obtained from the weak electron current,

\[ J^\mu_{we} = \langle \bar{e} \gamma^\mu (c' V - c' A \gamma_5) e \rangle = c'_V J^\mu_e - c'_A J^\mu_{5e} . \]  

(9)

It contains an axial current as well, that is different from zero if the electrons are polarized: for an electron density \( n_e \) and average polarization \( \langle \vec{\sigma} \rangle \),

\[ \langle \bar{e} \vec{\gamma} \gamma_5 e \rangle = \langle \vec{\sigma} \rangle n_e . \]

The first point to note is that the weak forces contain other terms than the ones identified before, namely, those with the vector current densities \( J^\nu e \) and \( \vec{J}_{we} \). That is true even in the limit of non-relativistic and unpolarized electrons because the neutrinos are ultra-relativistic. Then, the following approximations apply:

\[ \vec{F}_e \approx -e \vec{E} - \sqrt{2} G_F c'_V \left( \vec{\nabla} n_\nu - \vec{\nabla} n_{\bar{\nu}} + \frac{\partial \vec{j}_{\nu e}}{\partial t} - \frac{\partial \vec{j}_{\bar{\nu} e}}{\partial t} \right) , \]  

(10)

\[ \vec{F}_\nu = -\vec{F}_{\bar{\nu}} \approx -\sqrt{2} G_F c'_V \left( \vec{\nabla} n_e + \frac{\partial \vec{j}_{e}}{\partial t} \right) . \]  

(11)

The electron current can be neglected, as presented in the conference, except for extreme wavelengths, comparable to the inverse plasma frequency times the velocity of light. For longitudinal waves \( \vec{\nabla} \times \vec{j}_e \) is still zero and does not contribute to the neutrino force.

The proper modes of a system are usually investigated by expanding the distribution functions around certain zero order functions \( f^0 \). If the background is static and uniform, \( f^0 \) only depend on the momenta and

\[ f(t, \vec{x}, \vec{p}) = f^0(\vec{p}) + \delta f(t, \vec{x}, \vec{p}) . \]  

(12)
The Vlasov equations \( (3) \) are linearized by neglecting the quadratic terms in \( \delta f \) and the Fourier analysis is applied afterwards. Whenever the forces vanish at zero order, as in the case of the electro-weak forces, the Fourier transforms obey the equations

\[
(\omega - \vec{k} \cdot \vec{v}) \delta f(k, \vec{p}) + i \vec{F}(k, \vec{p}) \cdot \frac{\partial f^0}{\partial \vec{p}} = 0 ,
\]

where \( \omega = k^0 \) is the plasma frequency and \( \vec{k} \) the wave vector. The problem of deriving the dispersion relation is facilitated by the fact that the forces \( (10), (11) \) depend linearly on the current densities

\[
j^\mu(t, \vec{x}) = \int d^3p f(t, \vec{x}, \vec{p}) v^\mu(13)
\]

\((v^0 = 1)\). Then, the fluctuations

\[
\delta j^\mu(k) = \int d^3p \frac{-1}{\omega - \vec{k} \cdot \vec{v}} \frac{\partial f^0}{\partial \vec{p}} \cdot i \vec{F}(k, \vec{p}) v^\mu(\vec{p})(14)
\]

form a closed system of linear equations whose secular equation determines the dispersion relation.

The Langmuir waves \( 5, 6 \) are electron density waves associated with an electrostatic field \( (\vec{B} = \vec{0}, \text{rot} \vec{E} = \vec{0}) \) whose potentials are

\[
A^0(k) = -\frac{e}{k^2} \delta n_e(k) , \quad \vec{A}(k) = \vec{0} . \quad (15)
\]

They are also called longitudinal electromagnetic waves and sometimes plasmons in the Particle Physics literature \( 8 \). When the electro-weak forces \( (10), (11) \) are substituted in \( (14) \), one obtains \( \delta n_e \) as a linear function of \( \delta n_e, \delta n_\nu - \delta n_{\bar{\nu}}, \) and \( \delta j_\nu - \delta j_{\bar{\nu}} \) and, in turn, \( \delta n_\nu, \delta j_\nu, \delta n_{\bar{\nu}}, \delta j_{\bar{\nu}} \) proportional to \( \delta n_e \) \( (\delta j_\nu = \omega \vec{k} \delta n_e / k^2) \). It yields an equation of the type \( \delta n_e = -\chi \delta n_e \), where the susceptibility \( \chi \) has, in addition to the purely electromagnetic \( \chi_{EM} \), a weak interaction term \( \chi_w \). The plasma dispersion relation is

\[
1 + \chi_{EM}(\omega, \vec{k}) + \chi_w(\omega, \vec{k}) = 0 . \quad (16)
\]

\( \chi_{EM} \) only depends on the electron charge and distribution function. It is shown in many books \( 9, 10 \) that \( (e^2 = 4\pi \alpha) \)

\[
\chi_{EM} = \frac{e^2}{k^2} \int d^3p \frac{1}{\omega - \vec{k} \cdot \vec{v}_e} \frac{\partial f^0}{\partial \vec{p}} - \vec{k} . \quad (17)
\]
In the presence of a neutrino beam, the weak correction is given by

$$\chi_w = -2G^2 \frac{e^2 \chi_{EM} c^2}{\epsilon} \left(1 - \frac{\omega^2}{k^2}\right) \int d^3 p \frac{\vec{k} \cdot (\vec{k} - \omega \vec{v}_\nu)}{\omega - k \cdot \vec{v}_\nu} \left(\frac{\partial f^0}{\partial \vec{p}} + \frac{\partial f^0}{\partial \vec{p}}\right) \cdot \vec{k}.$$  \hspace{1cm} (18)

Notice that despite the relative sign in the expression of the electron force, Eq. (10), the $\nu$ and $\bar{\nu}$ beams contribute in the same way to the plasma susceptibility. The reason is the $\nu$ and $\bar{\nu}$ density fluctuations get also opposite signs from the forces $\vec{F}_\nu$ and $\vec{F}_{\bar{\nu}}$. That expression of $\chi_w$ differs from the result obtained in $^4$ by the term $-\omega \vec{v}_\nu$ under the integral and the factor $1 - \omega^2/k^2$, which come from the time derivative of $\vec{J}_\nu$ and $\vec{j}_e$ in the forces (10) and (11) respectively.

3 Quantum Field Theory

A basic property of the weak interactions is their chiral structure i.e., the differentiation between the left and right-handed components of the fermions. This aspect is ignored in the classic theory because it does not include the dynamics of the spin degrees of freedom. At the fundamental level the basic entities are the quantum fields but in the situation of interest they interact within a bath of plasma and neutrinos. For a medium in thermal equilibrium there are well established Finite Temperature Field Theory techniques $^3$, however, the passing neutrinos are not in thermal equilibrium and the plasma waves represent themselves deviations from equilibrium. The matter content is represented by a density of states $\rho$. In the Heisenberg picture, $\rho$ is constant and the fields evolve in time. In the Standard Model they are subject to local, relatively simple interactions and we can take advantage of that to solve the field equations of motion perturbatively $^4$.

In matter, the one-particle propagators are different from the propagators in vacuum. I define the distribution functions as a difference between the expectation values in matter and in vacuum of the time-order products of the fields in the Heisenberg picture i.e.,

$$\langle T \psi^\alpha(x) \overline{\psi}^\beta(y) \rangle_{\rho} = \langle 0 | T \psi^\alpha(x) \overline{\psi}^\beta(y) | 0 \rangle - f^{\alpha\beta}(x,y).$$  \hspace{1cm} (19)

That $f^{\alpha\beta}(x,y)$ represent one-particle distribution functions can been seen by evaluating observables like the current density

$$J^\mu(x) = \langle T \bar{\psi}(x) \gamma^\mu \psi(x) \rangle_{\rho} - \langle 0 | \bar{\psi}(x) \gamma^\mu \psi(x) | 0 \rangle$$  \hspace{1cm} (20)

$$= \text{tr} \{ f(x,x) \gamma^\mu \}.$$
where the trace is over the spinor indices.

In absence of interactions one knows the exact expressions of the quantum fields and exact distribution functions can be easily specified. The function

\[
f_\varepsilon^0(x, y) = \int \frac{d^3 \vec{p}}{4E_\varepsilon} f_\varepsilon^0(\vec{p}) (\vec{p}_\varepsilon + m_\varepsilon) e^{-i \vec{p}_\varepsilon \cdot (x-y)} \tag{21}
\]
describes a homogeneous (invariant under translations) distribution of unpolarized electrons with momentum distribution \(f_\varepsilon^0(\vec{p})\), as shows the expression of the current density,

\[
J_\varepsilon^{\mu 0}(x) = \text{tr} \left\{ f_\varepsilon^0(x, x) \gamma^\mu \right\} = \int d^3 p f_\varepsilon^0(\vec{p}) \frac{p_\mu}{E_\varepsilon} .
\]

Likewise, uniform beams of \(massless\) neutrinos have a distribution function (associated to the field \(\nu_L\))

\[
f_\nu^0(x, y) = \frac{1 - \gamma_5}{2} \int \frac{d^3 \vec{p}}{2E_\nu} \left( f_\nu^0(\vec{p}) \vec{p}_\nu e^{-i \vec{p}_\nu \cdot (x-y)} - f_\nu^0(\vec{p}) \vec{p}_\nu e^{i \vec{p}_\nu \cdot (x-y)} \right), \tag{22}
\]

where \(f_\nu^0\) and \(f_\bar{\nu}^0\) represent the (left-handed) neutrino and (right-handed) anti-neutrino momentum distributions. \(f_\varepsilon^0(x, y)\) and \(f_\nu^0(x, y)\) will be used as background distribution functions which implies a plasma with no positrons and therefore temperatures below the electron mass.

The functions \(f_\varepsilon^0(x, y)\) and \(f_\nu^0(x, y)\) stop to be consistent with the definition (19) when the field equations of motion include interactions. The deviations

\[
\delta f(x, y) = f(x, y) - f_\varepsilon^0(x, y) \tag{23}
\]
can be calculated perturbatively as part of higher order corrections to the one-particle Green functions along with other radiative corrections such as vacuum self-energies and matter induced 'potential' energies. \(\delta f_\varepsilon\) and \(\delta f_\nu\)
are the corrections associated to the very matter fluctuations $\delta f(x,y)$ and, in first approximation, they are given by the diagrams in Figs. 1 and 2.

At one-loop, the Fourier transforms of $\delta f(x,x)$, $\delta f_e(k)$ and $\delta f_\nu(k)$, are just linear combinations of the leptonic current fluctuations namely, $\delta J_\mu^e = \text{tr}\{\delta f_e\gamma^\mu\}$, $\delta J_\mu^\nu = \text{tr}\{\delta f_\nu\gamma^\mu\}$ and $\delta J_{5e}^\mu = \text{tr}\{\delta f_e\gamma^\mu\gamma^5\}$. As a result, $\delta J_\mu^e(k)$, $\delta J_{5e}^\mu(k)$ and $\delta J_\mu^\nu(k)$ form a closed system of coupled equations, $\delta J = A \delta J$ in a matrix notation. In principle one could derive the dispersion relation $\omega(\vec{k})$ from the condition $\det(1-A) = 0$ but that is not easy. In particular, the Lorentz structure is non-trivial because the plasma rest frame, wave vector $\vec{k}$ and neutrino fluxes define privileged directions in space-time. The first possible simplification is to take the macroscopic limit by neglecting $\omega$ and $\vec{k}$ with respect to the one-particle energies $E_e$, $E_\nu$, making in particular $(p_e \pm k)^2 - m_e^2 \approx \pm 2k \cdot p_e$ and $(p_\nu \pm k)^2 \approx \pm 2k \cdot p_\nu$ in the electron and neutrino propagators. It turns out that in this limit the field theory results for the currents $\delta J_\mu^e$ and $\delta J_\mu^\nu$ are equivalent to the classic expressions (14) of $\delta j_\mu^e$ and $\delta j_\mu^\nu - \delta j_{5e}^\mu$ respectively, with forces $\vec{F}_e(k,\vec{p})$ and $\vec{F}_\nu(k,\vec{p})$ given exactly by the Fourier transforms of the forces (5) and (7), derived from the appropriate classic theory.

Of course, the axial current $\delta J_{5e}^\mu$ can only be calculated within field theory. One expects from the Standard Model Lagrangian (3) that the neutrino vector
current polarizes the electron spin like a magnetic field does on a magnetic moment. The density \( \delta J_0 \) is suppressed for non-relativistic electrons but the axial current \( \delta \vec{J} \) gets polarized along the component of the neutrino velocity orthogonal to the wavevector \( \vec{k} \). It is also accompanied by a transverse electric current both of them proportional to \( G_F^2 \). However, their contribution to the dispersion relation only comes at the \( G_F^4 \) order and can so be discarded. The classic dispersion relation (18) is recovered for the non-relativistic plasma.

4 Plasma oscillation. Damping.

Let \( \omega_{pl}(\vec{k}) \) denote the plasma frequency as a function of \( \vec{k} \) in the absence of neutrinos. For wavelengths much larger than the Debye radius \( \vec{k} \), the electromagnetic susceptibility is given by \( \chi_{EM}(\omega, \vec{k}) = -\frac{\omega^2_{pl}(\vec{k})}{\omega^2} \). After applying this relation in the Eqs. (16) and (18) the dispersion relation of the non-relativistic plasma in the presence of neutrinos reads as

\[
\omega^2 - \omega^2_{pl}(\vec{k}) = -\frac{G_F^2 c_V^2}{2\pi\alpha} \frac{\omega^2_{pl}(\vec{k})}{k^2} (k^2 - \omega^2) \int d^3p \frac{\vec{k} \cdot (\vec{k} - \omega \vec{v}_\nu)}{\omega - \vec{k} \cdot \vec{v}_\nu} \left( \frac{\partial f_0^\nu}{\partial \vec{p}} + \frac{\partial \bar{f}_0^\nu}{\partial \vec{p}} \right) \cdot \vec{k},
\]

or, after integrating by parts,

\[
\omega^2 - \omega^2_{pl}(\vec{k}) = \frac{G_F^2 c_V^2}{2\pi\alpha} \frac{\omega^2_{pl}(\vec{k})}{k^2} (k^2 - \omega^2) \int d^3p \frac{f_0^\nu + \bar{f}_0^\nu}{E_\nu} \frac{\vec{k}^2}{(\omega - \vec{k} \cdot \vec{v}_\nu)^2}.\]

Again, as a consequence of the discrepancies in the weak forces, these results differ from (18), namely, by the factor \((1 - \omega^2 / k^2)^2\) in the second equation.

The constant \( c_V^2 \) is much smaller for \( \nu_\mu, \nu_\tau \) \((c_V^2 \approx -0.04)\) than for \( \nu_e \) \((c_V^2 \approx 0.96)\), so only \( \nu_e \) and \( \bar{\nu}_e \) deserve to be taken in consideration. The expressions above provide a good perspective of the real impact of the neutrinos. Keeping only the main factors,

\[
\omega^2 - \omega^2_{pl} \propto \omega^2_{pl} \frac{G_F n_\nu}{E_\nu} G_F k^2
\]
clearly indicates that the neutrino effect is severely suppressed by \( G_F^2 \). The only potential exception are the oscillation modes that catch the neutrinos in the resonance \( \omega = \vec{k} \cdot \vec{v}_\nu \).

If the neutrino spectrum goes from one side of the pole to the other the situation is analogous to the electron Landau damping (18). The integral in Eq. (24) separates into a negligible principal part and a complex quantity that is determined by the so-called Landau prescription, here \( \omega - \vec{k} \cdot \vec{v}_\nu + i\varepsilon, \varepsilon \to \)}
Denoting the imaginary part of $\omega$ as $-i\gamma$, the neutrino contribution to the decay rate $\gamma$ is then

$$\gamma_{\nu} = \frac{G_F^2 \alpha^2}{4\alpha} \frac{\omega}{k^2} \left( \tilde{k}^2 - \omega^2 \right)^2 \int d^3 p \, \delta (\omega - \tilde{k} \cdot \tilde{v}_\nu) \left( \frac{\partial f^0_\nu}{\partial p} + \frac{\partial f^0_{\bar{\nu}}}{\partial p} \right) \cdot \tilde{k} .$$

(26)

Hardy and Melrose obtained this result from the calculation of the decay rate of neutrinos into longitudinal photons. However, by the very nature of such calculation the full neutrino contribution to the dispersion relation is not derived and second, cannot be utilized to investigate possible hydrodynamic instabilities.

$\gamma_{\nu}$ goes as $G_F^2$ and is exceedingly small. If however all the neutrinos were on the top of the resonance they could generate an hydrodynamic instability.

The claim was that this occurs in the conditions of Supernova neutrino emission causing much larger growth rates. The question I rise is, in the end one has to check whether or not the entire neutrino flux lies in the resonance i.e., whether $|\omega - \tilde{k} \cdot \tilde{v}_\nu|$ stays within the calculated resonance width $|\gamma|$. The neutrino velocities only spread in direction. At radii, $r$, much larger than the neutrinosphere radius, $R_\nu$, they move essentially in the radial direction yet, the angular spread $\theta_\nu \approx 2R_\nu/r$ is finite. Under the resonance condition, $\tilde{k} \cdot \tilde{v}_\nu \approx \omega_{pl}$, the interval of variation of $\tilde{k} \cdot \tilde{v}_\nu = |\tilde{k}| \cos \theta_{\nu k}$ lies in the range

$$\frac{\omega_{pl}^2}{2} \leq \Delta \tilde{k} \cdot \tilde{v}_\nu \approx \theta_{\nu k} \omega_{pl} \leq \omega_{pl} .$$

(27)

The left-hand side holds for a vector $\tilde{k}$ with radial direction whereas the right-hand bound is necessary to exclude the situation of Landau damping and the result (25).

The largest values of $|\gamma|$ are obtained by assuming that $\tilde{k} \cdot \tilde{v}_\nu$ is approximately constant over the neutrino spectrum and can be factorized out of the integral in Eq. (25). One obtains growth rates of the order of

$$\gamma_{\text{optimal}} = \omega_{pl} \left\{ \frac{G_F n_\nu}{E_\nu} \frac{G_F n_e}{E_e} \right\}^{1/3} \left( \frac{\tilde{k}^2 - \omega^2_{pl}}{\omega^2_{pl}} \right)^{2/3}$$

for the resonant modes ($\tilde{k} \cdot \tilde{v}_\nu \approx \omega_{pl}$). $\gamma_{\text{opt}}$ varies with $\tan^{4/3} \theta_{\nu k}$ and $\Delta \tilde{k} \cdot \tilde{v}_\nu$ with $\tan \theta_{\nu k}$, so the most favorable case for a hydrodynamic instability is a vector $\tilde{k}$ orthogonal to the radial direction i.e., $\Delta \tilde{k} \cdot \tilde{v}_\nu \approx \omega_{pl}$ and $\tan \theta_{\nu k} \approx 1/\theta_{\nu}$. Then, the condition that the neutrinos be inside the resonance requires

$$\tilde{\gamma} = \left\{ \frac{G_F n_\nu}{E_\nu} \frac{G_F n_e}{E_e} \right\}^{1/3} \theta_{\nu}^{-4/3} \leq \gamma_{\text{optimal}} / \Delta \tilde{k} \cdot \tilde{v}_\nu \geq 1 .$$

9
Knowing how small the energies $G_F n_\nu$ and $G_F n_e$ are, that looks quite unlikely. For numerical estimates I used the values $L_{\nu_e} = 10^{53}$ ergs/s, $E_\nu = 10$ MeV, $n_e = 10^{33}$ cm$^{-3}$ and $\theta_\nu = 100$ km/r. For $\theta_\nu = 0.1$, $\hat{\gamma}$ is of the order of $10^{-7}$, quite small indeed. Furthermore, since the neutrino density varies as $1/r^2$, $\gamma_{\text{opt.}}/\omega_{\text{pl}}$ is proportional to $(n_e r^2)^{1/3}$ and necessarily drops to zero for large enough radius. One may also fix the value of the electron density. It turns out that the parameter $\hat{\gamma}$ approaches the unity only at a radius equal to $10^{13}$ km for $n_e$ as large as $10^{33}$ cm$^{-3}$! That is clearly absurd.

It should be also taken in consideration that for wavelengths smaller than the Debye distance ($k > k_D$), the plasma waves are electromagnetically Landau damped. That puts a further upper limit on the interesting values of $\tan \theta_{\nu k}$ for the resonant neutrinos ($< k_D/\omega_{\text{pl}}$) in disfavour of the hydrodynamic instabilities.

5 Conclusions

We have shown that Quantum Field Theory yields results that are compatible with the classic kinetic theory provided that one uses the right specification for the forces. Simple considerations show that the weak forces between electrons and neutrinos are of the same type as the Lorentz electromagnetic force. In particular, the force done by a neutrino flux on a non-relativistic electron is

$$\vec{F}_e = -\sqrt{2} G_F c'_V \left( \vec{\nabla} n_\nu + \frac{\partial j_\nu}{\partial t} \right).$$  \hspace{1cm} (28)

In Quantum Field Theory the spin degrees of freedom are also taken in account. Under a neutrino flux the electrons get polarized at $G_F^2$ order but the final effect on the plasma dispersion relation is of higher order and can be neglected.

In what concerns possible plasma instabilities caused by the neutrino flux in a Supernova envelope star we have shown that the neutrinos are never collimated enough to produce strong resonance effects. As a consequence, the neutrino induced growth rates are of the Landau damping type, suppressed by $G_F^2$.

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