Quasi Yukawa Fixed Point due to Decoupling of SUSY Particles

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Abstract

We study effects of SUSY particle decouplings on a quasi fixed point (QFP) of Yukawa coupling. From renormalization group analysis it is shown that if the SUSY breaking scale $M_S$ is large ($\gtrsim 1$TeV), effects of decoupling of Higgsinos and squarks raise the top Yukawa QFP. This tendency is enhanced in most cases of non-universal SUSY breaking. For the case of $M_S \lesssim 1$TeV, the decoupling of gluinos lowers $m_{\tilde{t}}^{QFP}$. We checked some parameter dependencies for the top Yukawa QFP. The bottom-top Yukawa unified case is also studied. When top quark mass is measured more precisely, some patterns of soft mass spectra could be excluded if rather large initial top Yukawa coupling is realized by underlying theory.

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Supersymmetry (SUSY) [1] is now considered as a promising candidate for physics beyond the Standard Model (SM). The minimal supersymmetric extension of the Standard Model (MSSM) shows many successful results, i.e. the gauge coupling unification [2], the radiative symmetry breaking [3], etc. Recently a quasi fixed point (QFP) of the top Yukawa coupling [4] was reconsidered by Lanzagorta and Ross [5] and they pointed out that the QFP is also interesting in supersymmetric models. Such studies show that the QFP under the MSSM can predict the top quark mass closer to the experimental value [6] than the SM. Furthermore it is meaningful to study features of the QFP in the framework of supersymmetric models from various viewpoints, e.g. effects of threshold corrections due to non-universal SUSY particle masses.

In general supergravity theories (SUGRAs) as well as superstring models lead to non-universal soft terms, i.e. non-universal soft scalar masses and gaugino masses [7, 8]. In Refs. [9, 10], it is shown that the effects of non-universal SUSY breaking on the gauge coupling unification are rather sizable. In addition much work has been devoted to phenomenological implications of non-universality of SUSY particles [11, 12]. In this paper we mainly study effects of such non-universal decoupling of SUSY particles on the QFP.

Usually the MSSM has been considered in the framework of $N=1$ minimal SUGRA which leads universal soft SUSY breaking terms. This treatment is simple and has a powerful predictability so that much work has been done under this framework. However, in general, SUGRAs as well as superstring models often yield non-universal soft terms [6]. It seems that the minimal SUGRA is a special case from the viewpoint of superstring theory.

The general forms of Kähler potential $K$ and the superpotential $W$ are written as follows,

$$
K = \kappa^{-2} \hat{K}(\Phi, \bar{\Phi}) + K(\Phi, \bar{\Phi})_{I\bar{J}} Q^I \bar{Q}^J + \left( \frac{1}{2} H(\Phi, \bar{\Phi})_{I\bar{J}} Q^I Q^J + \text{h.c.} \right) + \cdots ,
$$

$$
W = \hat{W}(\Phi) + \frac{1}{2} \tilde{h}(\Phi)_{I\bar{J}} Q^I Q^J + \cdots ,
$$

where $\kappa^2 = 8\pi/M_{Pl}^2$ and $Q^I$ are chiral superfields. The fields $\Phi^I$ belong to the hidden sector contributing the SUSY breaking. The ellipses stand for terms of higher orders in $Q^I$. Using these, one can write down the scalar potential $V$ as follows,

$$
V = \kappa^{-2} e^G |G_\alpha (G^{-1})^{\alpha\beta} G_\beta - 3\kappa^{-2}| + (\text{D-term}),
$$

where $G = K + \kappa^{-2} \log \kappa^6 |W|^2$ and the indices $\alpha$ and $\beta$ denote $Q^I$ as well as $\Phi^m$. If we take the flat limit $M_{Pl} \to \infty$ preserving the gravitino mass $m_{3/2} = \kappa^2 e^{K/2} |\hat{W}|$ fixed in Eq.(3), then the soft scalar masses $m_{I\bar{J}}$ for unnormalized fields $Q_I$ are derived as

$$
m_{I\bar{J}}^2 = m_{3/2}^2 K_{I\bar{J}} - F^m \tilde{F}^\bar{n} \left[ \partial_m \partial_\bar{n} K_{IJ} - (\partial_\bar{n} K \bar{K}) K^{K\bar{L}} (\partial_m K_{IJ}) \right] + \kappa^2 V_0 K_{I\bar{J}},
$$

where $F^m$ are F-terms of $\Phi^m$, $\partial_m$ denote $\partial/\partial \Phi^m$ and $V_0$ is the cosmological constant. The model which has non-minimal kinetic term, i.e. $K_{I\bar{J}} \neq \delta_{I\bar{J}}$ could yield non-universal soft scalar masses. In addition D-term contributions could lead to non-universal soft scalar masses. In Ref. [6], it is pointed out that if the non-universality is large enough, the gauge-coupling unification scale becomes close to the string
scale $M_{st} \sim 0.5 \times 10^{18}$ GeV \cite{3} and there is no need for large string threshold corrections in such a case. Various types of studies have been done about the non-universal soft SUSY breaking\cite{1,2}. It is also shown that such a large non-universality can be realized if we consider the orbifold models with multi-moduli fields\cite{8}.

The canonically normalized gaugino masses $M_a$ are derived through the following equation,

$$M_a = \frac{1}{2} (\text{Re} f_a)^{-1} F_m \partial_m f_a,$$

where $f_a$ is a gauge kinetic function of a gauge group. This shows that in general case the gaugino masses are also non-universal as well as scalar masses. The effects of gaugino mass non-universality enhances the results of Ref.\cite{9} furthermore\cite{10}.

Next we briefly review the QFP\cite{11} following the arguments of Ibáñez and Lopez\cite{4}. For the case with $Y_t \gg Y_b, Y_{\tau}$, renormalization group equations (RGE’s) of gauge and Yukawa couplings are written as,

$$\frac{dg_i^2}{dt} = -\frac{b_i}{(4\pi)^2} g_i^4, \quad \frac{dY_i}{dt} = Y_i \left( \sum_i r_i \tilde{\alpha}_i - sY_i \right),$$

where

$$Y_i = \frac{h_i^2}{(4\pi)^2}, \quad \tilde{\alpha}_i = \frac{g_i^2}{(4\pi)^2}.$$

Here $h_i$ and $g_i$ ($i = 1, 2, 3$) denote the top Yukawa and gauge couplings, respectively. The coefficients $b_i, r_i$ and $s$ are some numerical constants which depend on a model, e.g. $(b_1, b_2, b_3) = (11, 1, -3)$, $(r_1, r_2, r_3) = (13/9, 3, 16/3)$ and $s = 6$ for the MSSM; $(b_1, b_2, b_3) = (41/6, -19/6, -7)$, $(r_1, r_2, r_3) = (17/12, 9/4, 24/3)$ and $s = 9/2$ for the SM. One can solve these equations analytically and obtain the following results;

$$Y_i(t) = \frac{Y_i(0) E_1(t)}{1 + s Y_i(0) F_1(t)},$$

where

$$E_1(t) = \prod_i \left( 1 + \tilde{\alpha}_i(0) t \right)^{\frac{r_i}{s}},$$

$$F_1(t) \equiv \int_0^t E_1(t') dt', \quad t = 2 \log(M_X/Q).$$

Here $M_X$ is the initial scale where we set the initial value of the top Yukawa coupling $Y_t(0)$. This scale can be arbitrary but it seems natural for our purpose to regard $M_X$ as the string scale $M_{st}$. If one takes the limit $Y_t(0) \to \infty$, Eq.(\ref{eq:7}) becomes as follows,

$$Y_t^{QFP}(t) \simeq \frac{E_1(t)}{s F_1(t)}.$$

Note that there is no dependency on an initial value of $Y_t(0)$ in the above formula. This implies that $Y_t^{QFP}$ can be treated as something like a fixed point value as long as $Y_t(0)$ is large enough. This is the

\footnote{Note that this is not the Pendolton-Ross type of fixed points\cite{14}. Their fixed point is exact (not quasi!) if the SU(2) and U(1) gauge couplings and the bottom Yukawa coupling vanish. However, it has been pointed out that the PR fixed point could not be reached since the interval of the energy scales between $M_X$ and $M_W$ is too short to make the Yukawa coupling converge to the PR fixed point value\cite{15}.}
This reason why we call \( Y_t^{\text{QFP}} \) as quasi Yukawa fixed point. It seems necessary to study how large initial value of the top Yukawa coupling is required in order to make the approximation Eq. (10) be realistic. Since we obtain \( F_t(t) \approx 200 \sim 300 \) in Eq. (3), a deviation from the QFP Eq. (4) is less than 1% even in the case with \( Y_t(0) \sim 0.1 \). To show an applicable region of the QFP Eq. (4) more explicitly, we use Eq. (7) with explicit values of \( Y_t(0) \). Even in the case with \( Y_t(0) = 0.1(0.01) \), deviations from the QFP’s are less than 0.3% (2.5%). Therefore the QFP can give a good explanation for the value of the top quark mass when such a initial Yukawa coupling is realized by underlying theories like superstring theory.

One can easily obtain \( m_t^{\text{QFP}} / \sin \beta \equiv (174 \text{GeV}) / 4\pi \sqrt{Y_t^{\text{QFP}}} \sim 205 \text{GeV} \) in the MSSM \((m_t^{\text{QFP}} = 220 \text{GeV} \text{ in the SM}) \) from \( Y_t^{\text{QFP}} \) substituting the following experimentally measured values \([16]\), \( M_Z = 91.187 \text{GeV} \), \( \alpha_3(M_Z) = 0.118 \), \( \alpha(M_Z) = 1/127.9 \) and \( \sin^2 \theta_W(M_Z) = 0.2319 \) into the above formulae. Recently \( m_t \) has been measured at TEVATRON \([3]\):

\[
\begin{align*}
  m_t &= 176 \pm 8 \pm 10 \text{GeV} \quad \text{(CDF)}, \\
  m_t &= 199^{+19}_{-21} \pm 22 \text{GeV} \quad \text{(D0)}.
\end{align*}
\]

It is obvious that \( m_t^{\text{QFP}}(\text{MSSM}) \) is consistent with the D0 result while \( m_t^{\text{QFP}}(\text{SM}) \) exceeds the upper limit slightly. In addition, we can also make \( m_t^{\text{QFP}} \) of the MSSM consistent with the CDF results if we take \( \sin \beta \) to be small enough.

To reduce Eq. (6) for the MSSM, we have assumed that the RGE’s of gauge and Yukawa couplings are exactly supersymmetric from the initial scale to \( M_Z \). This means that the SUSY breaking scale \( M_S \) is just \( M_Z \) although \( M_S \) is, in general, treated as somewhat a higher scale than \( M_Z \). Therefore to discuss these scenario more precisely, one must consider effects of decoupling of SUSY particles. If SUSY breaking is universal, we must evaluate the QFP at \( M_S \) in Eq. (10), input this value of \( Y_t^{\text{QFP}} \) at \( M_S \) into the initial condition of the SM RGE Eq. (10) and then flow \( Y_t \) from \( M_S \) down to \( M_Z \) by the SM RGE. Taking this prescription, we find that \( m_t^{\text{QFP}} \), the top mass predicted by the QFP, is raised slightly if \( M_S \gtrsim 1 \text{ TeV} \). This reason is as follows. We can write down the RGE of the top Yukawa coupling as

\[
\frac{dy_t}{dt} = y_t \left( \frac{16}{3} T_{i3} \delta \alpha_3 + 3 T_{i2} \delta \alpha_2 + \frac{13}{9} T_{i1} \delta \alpha_1 - 6 T_{ii} y_t - T_{i0} y_0 \right). \tag{11}
\]

Here the coefficients \( T' \)'s are obtained as \( T_{i3} = T_{i2} = T_{i1} = T_{tt} = T_{tb} = 1 \) for the MSSM and \( T_{i3} = 3/2, T_{i2} = 3/4, T_{i1} = 51/52, T_{tt} = 3/4 \) and \( T_{tb} = 1/2 \) for the SM. In the above RGE the terms including gauge couplings make \( y_t \) go upward and the Yukawa term plays an opposite role while running from the higher scale. This is caused by the difference of the signs of these terms \([3]\). Note that the SM has a larger value of \( T_{i3} \) and a smaller value of \( T_{tt} \) than the MSSM. Because of both effects of large \( T_{i3} \) and small \( T_{tt} \), the SM top Yukawa RGE has a stronger tendency to push \( Y_t \) upward during the running from \( M_S \) to \( M_Z \) than the MSSM. Therefore if one stops the QFP Eq. (10) at \( M_S \) and runs \( Y_t \) by the SM RGE 

\[\text{A similar situation occurs in the scenario of the radiative symmetry breaking \([3]\). The Higgs squared mass can be negative while running from an initial scale to \( M_Z \) since in the Higgs mass RGE the Yukawa term is dominant against the gauge terms.}\]
from $M_S$ to $M_Z$, one obtains a larger value of $m_t^{\text{QFP}}$ than the usual QFP analysis. We could expect that non-universal decoupling has more complicated effects on the evolution of the top Yukawa coupling.

To discuss such effects on the QFP, we must consider the decoupling in the RGE of the Yukawa coupling. The 1-loop Yukawa RGE's including the effects of decoupling of SUSY particles have been presented by Lahanas and Tamvakis\[17\]. They parameterize the decoupling of SUSY particles by the step-function $\theta_\phi = \theta(Q^2 - m_\phi^2)$. Using this step-function approximation they derive RGE's not only for Yukawa couplings but also gauge couplings, A-parameters, scalar masses and gaugino masses. These RGE's are very useful to analyze features of the decoupling\[3\]. We can investigate the Yukawa coupling QFP in various patterns of SUSY mass spectra using these RGE's.

The non-universality affects $\alpha_i(M_{st})$ and the running of gauge and Yukawa couplings below $M_S$. These are crucial for the determination of the QFP value so that it is important to study effects of non-universal soft SUSY breaking on the QFP. Our procedure is following. Firstly, we determine $\alpha_3(M_Z)$, $\alpha_2(M_Z)$ and $\alpha_1(M_Z)$ by experimental value and let them run from $M_Z$ to $M_{st}$. Up to the SUSY breaking scale $M_S$ we use the corresponding RGE's\[6\] for each non-universal case following Refs.\[9, 10\]. Then we turn on all contributions from SUSY particles at $M_S$, hence the flow of those couplings obeys the usual MSSM RGE's from $M_S$ to $M_{st}$. After evaluating $\alpha_i(M_{st})$'s we input them and $M_S$ to the QFP formula\[10\] to obtain the value of the top Yukawa coupling at the SUSY breaking scale $M_S$. Finally we let gauge and top Yukawa couplings flow down from $M_S$ to $m_t$ by the corresponding RGE's and evaluate $m_t^{\text{QFP}}$ for each non-universal case. Hereafter we consider eight patterns of non-universalities shown in Ref.\[10\]. We review them in Table 1 with $\beta$-coefficients $b_i$, $i = 1, 2, 3$ for each case. Although Ref.\[10\] gives ten cases, we take eight out of ten since Case II and V, Case C and D indicate almost same behavior respectively in the following analysis. For each case the coefficients of the RGE's in Eq.(11) $T$'s are given in Table 2 and 3. We follow the notation of Ref.\[17\]. For the evaluation of $m_t^{\text{QFP}}$, we use the following relation between the running top quark mass and the pole mass\[18\];

$$m_t = \bar{m}_t(m_t) \left[ 1 + \frac{4\bar{\alpha}_3(m_t)}{3\pi} + O(\alpha_3^2) \right],$$

(12)

where $\bar{m}_t(m_t)$ and $\bar{\alpha}_3(m_t)$ denote the running top quark mass and the running SU(3) gauge coupling at $m_t$ respectively.

Firstly we concentrate on the $Y_t \gg Y_b, Y_\tau$ case. We assume $\tan \beta = 2$ in all patterns of non-universalities, for simplicity. If $Y_t \gg Y_b, Y_\tau$, a value of $\tan \beta$ can not be so large. Because a large value of $\tan \beta$ leads to a large value of $Y_b$ in order to realize the bottom quark mass. However, the $\tan \beta$ dependence on $m_t^{\text{QFP}}$ is very large. For example, if $\tan \beta$ is changed from 2 to 1.5, $m_t^{\text{QFP}}$ becomes 168 GeV from 181 GeV. Hereafter we take $\alpha_3 = 0.118$ and $M_X = M_{st} \equiv 5.0 \times 10^{17}$ GeV.

The results are shown in Figures 1-2. Figure 1 shows the $m_t^{\text{QFP}}$ dependencies on $M_S$ for each non-universal case. For the present we assume that all Higgsinos are decoupled at $M_S$. Case 0 is the usual

\[3\] These RGE's correspond to the two-Higgs doublet model when all SUSY particles are decoupled. To get the ordinary SM with one Higgs doublet, we must take into account the mixing of Higgses. However, in the case which contains mixing of fields, the mass-independent renormalization with the $\theta$-function approximation is not applicable and a mass-dependent renormalization scheme should be taken instead of that.
$M_S = M_Z$ prescription taken in the analysis of Ref.1, 2. Case I corresponds to the universal SUSY breaking case which decouples all SUSY particles at $M_S$. All cases raise $m_t^{QFP}$ in the region of $M_S \gtrsim 1$ TeV more than case 0. The cases with the same value of $b_3$ behave similarly. This is because of the fact that the coefficient $T_{t_3}$ is completely identical in all cases. In such cases the difference of $m_t^{QFP}$ mainly depends on $\alpha_3$. The running of $\alpha_3$ is determined by $b_3$, so that the cases with the same $b_3$ value tend to behave similarly.

One can expect from the Yukawa RGE's that the decoupling of Higgsinos is quite effective for this kind of analysis. We also consider the cases with the light Higgsinos and the results are shown in Figure 2. From this figure one can see that in a higher $M_S$ region $m_t^{QFP}$ is rather separated from each other compared with the previous cases with the heavy Higgsinos. As we shown before, this is due to the sameness of $T_{t_3}$. However, the light Higgsinos yield the difference in $T_{t_3}$ and cause such a separation of $m_t^{QFP}$. This effect is significant in Case IV. In this case all squarks are light. This effect appears in the fact that Case IV has the lowest $T_{t_3}$ value as $T_{t_3} = 1$. As shown before, the SU(3) gauge interaction raises the $m_t^{QFP}$ as the renormalization scale is going down. However, for Case IV, such RG effects are not operative because of small $T_{t_3}$. From the above results we conclude that light squarks and Higgsinos are favorable in order to lower $m_t^{QFP}$ for $M_S \gtrsim 1$ TeV region.

When $M_S \lesssim 1$ TeV, the situation becomes different. In this region, Case I and C generate a smaller value of $m_t^{QFP}$ than the usual QFP analysis, Case 0. When $M_S$ is small, the RG effects in Yukawa coupling do not matter because of such a short running interval between $M_S$ and $M_Z$. The common property of Case I and C is the decoupling of gluinos. If gluinos are decoupled, the $\beta$ coefficient for the SU(3) gauge coupling $b_3$ is decreased and $\alpha_3(m_t)$ become small due to RG effects. At the renormalization scale close to $m_t$, the correction term of $O(\alpha_3(m_t))$ in Eq.(12) strongly influences $m_t^{QFP}$. One can read off from Table 1 and Figures 1-2 that $m_t^{QFP}$ of the cases with the same $b_3$ converge to the same points respectively at $M_S \simeq 200$ GeV.

The large ambiguity of the experimental value of $\alpha_3(M_Z)$ is still a big problem for phenomenologists. The QFP is also affected by the value of $\alpha_3(M_Z)$. For $\alpha_3(M_Z) = 0.110(0.130)$ and $M_S = 200$ GeV, Cases 0, II and III with the heavy Higgsinos lead to $m_t^{QFP} = 174(183), 176(185)$ and $177(186)$ GeV, respectively. We also obtain $5 \sim 6$ $\%$ difference for $m_t^{QFP}$ against $\alpha_3(M_Z) = 0.11 \sim 0.13$ in other cases. A smaller value of $\alpha_3^{-1}(M_Z)$ lowers $m_t^{QFP}$.

We also examine the dependence on a starting scale $M_X$. For $M_X = 10^{16}$ GeV $(10^{19}$ GeV) and $M_S = 10$ TeV, Cases 0, II and III with the heavy Higgsinos provide $m_t^{QFP} = 180(176), 182(178)$ and $183(179)$ GeV, respectively. It is obvious that a higher starting scale suppresses the value of $m_t^{QFP}$. Below $M_X$ the top Yukawa coupling tends to decrease monotonically as the renormalization scale is going down. Therefore a large interval of renormalization scale due to the higher $M_X$ lowers the QFP furthermore.

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4 This analysis seems to be unacceptable for the explanation of the separation between Case II and B. In these cases the differences appear in SU(2) coefficients ($T_{t_2}, b_2$). At higher energy scale, the gauge coupling of SU(2) is effective as much as SU(3). Therefore it seems plausible to consider that the separation of the lines in Figure 2 is also triggered by the discrepancy of the SU(2) coefficients.

5 This situation looks like the case of triviality bound of the Higgs mass [34]. According to the argument of triviality, the Higgs mass bound which is determined by the Higgs four-point coupling decreases if cutoff $\Lambda$ increases.
Next we consider the Yukawa-unified case, \( Y_t \simeq Y_b \). In this case, the formula of the top Yukawa QFP is changed into
\[
Y_t(t) \simeq \frac{E_1(t)}{7F_1(t)}. \tag{13}
\]
Similar analysis can be done for the bottom Yukawa and we obtain the QFP for the bottom Yukawa as follows,
\[
Y_b(t) \simeq \frac{E_2(t)}{7F_2(t)}, \tag{14}
\]
where
\[
E_2(t) = (1 + b_3 \tilde{\alpha}_3(0)t)^{16/3b_3}(1 + b_2 \tilde{\alpha}_2(0)t)^{3/3b_2}(1 + b_1 \tilde{\alpha}_1(0)t)^{7/9b_1}, \tag{15}
\]
\[
F_2(t) = \int_0^t E_2(t') dt. \tag{16}
\]
Below \( M_S \) we use the following RGE for bottom Yukawa coupling;
\[
\frac{dY_b}{dt} = Y_b \left( \frac{16}{3} T_{b3} \tilde{\alpha}_3 + 3 T_{b2} \tilde{\alpha}_2 + \frac{7}{9} T_{b1} \tilde{\alpha}_1 - 6 T_{bb} Y_b - T_{bb} Y_t \right). \tag{17}
\]
The coefficients \( T \)'s are shown in Tables 2 and 3. In this case there is a constraint from the bottom quark mass. Here we take the running bottom quark mass \( m_b(m_b) \) as 4.1GeV. Consequently we obtain \( \tan \beta \simeq 65 \). This value is almost common to all of the cases we used. In the interval from \( M_Z \) to \( m_b \) the running of the bottom Yukawa coupling should obey the RGE including only QCD and electromagnetic gauge interaction. Here we simply assume \( m_b(M_Z) = m_b(m_b)/\eta_b \) where \( \eta_b \simeq 1.437 + 0.075[\alpha_3(M_Z) - 0.115]/0.91[\alpha] \).

We investigate the top Yukawa QFP similarly as the \( h_t \gg h_b \) case and the results are shown in Figure 3. The overall tendency of results is similar to the previous cases. It seems strange that even for a quite large \( \tan \beta (\gtrsim 60) \), \( m_t^{QFP} \) is not so large compared to the previous cases. This fact is due to the presence of the contribution from the bottom Yukawa. The top Yukawa QFP becomes smaller than in the previous cases since the factor \( s \) in the denominator of Eq.(11) is larger than in the \( Y_t \gg Y_b \) case. In addition, the top Yukawa RGE below \( M_S \) has an additional term from the bottom Yukawa and this term contributes to lower \( h_t \) further.

Finally we summarize our results. The quasi fixed point of the top Yukawa coupling is investigated from various viewpoints. If one takes into account the universal decoupling of SUSY particles, \( m_t^{QFP} \) is raised slightly when the SUSY breaking scale \( M_S \gtrsim 1 \)TeV. This situation is enhanced in most of the cases with non-universal soft SUSY breaking terms. For the case of \( M_S \lesssim 1 \)TeV, the decoupling of gluinos is crucial for the determination of \( m_t^{QFP} \). The QFP is sensitive about the experimental value of \( \alpha_3 \). In most cases the QFP predicts rather large values of \( m_t \).

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6In the derivation of Eq.(13), we assume \( Y_1 = Y_2 \) in all over the range between \( M_X \) and \( M_Z \). However, this treatment is not correct because RGE coefficients of the top Yukawa is different from those for the bottom Yukawa even in the MSSM, so that the two evolutions are different from each other. Taking into account these effects, we solve RGE's numerically and find that the results which are shown in Figure 3 are entirely raised by 2~3 GeV.

7The ambiguity of the bottom mass is rather large as one of \( \alpha_3 \). However, we check that our results are not so sensitive to the value of \( m_b \) as long as 4.1GeV \( \leq m_b \leq 4.5 \)GeV.
The QFP could provide a good reason for the value of the top quark mass if a high-energy theory like superstring theory or some gauge-Yukawa unified models can give explanation about a suitable initial Yukawa coupling. From our analysis the following prescription may lower $m_t^{QFP}$ enough to reconcile with the CDF results: 1. a lower tan $\beta$, 2. a lower SUSY breaking scale $M_S \lesssim 1$ TeV, 3. heavy gluinos ($m_{\tilde g} \gtrsim M_S$), 4. a lower value of $\alpha_3(M_Z)$, 5. a higher start point ($\sim M_{Pl}$). If $M_S \gtrsim 1$ TeV, the decoupling of squarks and Higgsinos raises the QFP furthermore. In order to lower the QFP, squarks might be light enough. This seems unfavorable for the experimental constraints from electronic dipole moment of neutron (EDMN)\footnote{See e.g. Ref.\cite{23}.}. However it was pointed out that if small $\mu$, tan $\beta$ and $M_2$ are realized simultaneously, the contribution from a dangerous soft CP-violating phase can be suppressed successfully\cite{12} and the EDMN need not to be so large. If the top quark mass as well as $\alpha_3$ is measured more precisely in future, our results become more serious. For example the CDF and D0 results provide $m_t \sim 181$ GeV as the mean value. If this value included a very small error, some cases with smaller $M_S$ could be ruled out.

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Table 1

The patterns of non-universal soft SUSY breaking and corresponding $\beta$ coefficients below $M_S$. The capital letter in the second and third column denotes squark or slepton and $\lambda_i$ express the gaugino. Particles in second column are assumed to be heavy and decouple at $M_S$. The third column is devoted to the light SUSY particles which remain in the scale below $M_S$. The $b'_2$ and $b'_1$ are the SU(2) and U(1) $\beta$ coefficients respectively for the case of light Higgsinos. The decoupling of Higgsinos does not affect $b_3$.

| Case | $\sim M_S$ | $\sim M_Z$ | $b_3$ | $b_2, b_1$ | $b'_2, b'_1$ |
|------|-------------|-------------|-------|------------|--------------|
| I    | $\hat{Q}, \hat{U}, \hat{D}, \hat{L}, \hat{E}, \lambda_3, \lambda_2, \lambda_1$ | (Universal) | −7    | −3, 7      | −7/3, 23/3   |
| II   | $\hat{Q}, \hat{U}, \hat{D}, \hat{L}$ | $\hat{E}, \lambda_3, \lambda_2, \lambda_1$ | −5    | −5/3, 8    | −1, 26/3     |
| III  | $\hat{Q}, \hat{L}$ | $\hat{U}, \hat{D}, \hat{E}, \lambda_3, \lambda_2, \lambda_1$ | −4    | −5/3, 29/3 | −1, 31/3     |
| IV   | $\hat{L}$ | $\hat{Q}, \hat{U}, \hat{D}, \hat{E}, \lambda_3, \lambda_2, \lambda_1$ | −3    | −1/6, 59/6 | 1/2, 21/2    |
| VI   | $\hat{U}, \hat{D}$ | $\hat{Q}, \hat{L}, \hat{E}, \lambda_3, \lambda_2, \lambda_1$ | −4    | 1/3, 26/3  | 1, 28/3      |
| A    | $\hat{Q}, \hat{L}, \lambda_2$ | $\hat{U}, \hat{D}, \hat{E}, \lambda_3, \lambda_1$ | −4    | −3, 29/3   | −7/3, 31/3   |
| B    | $\hat{Q}, \hat{U}, \hat{D}, \hat{L}, \lambda_2$ | $\hat{E}, \lambda_3, \lambda_1$ | −5    | −3, 8      | 26/3, −7/3   |
| C    | $\hat{Q}, \hat{U}, \hat{D}, \lambda_3$ | $\hat{L}, \hat{E}, \lambda_2, \lambda_1$ | −7    | −7/6, 17/2 | −1/2, 55/6   |
## Table 2

The RGE coefficients of top and bottom Yukawa couplings for each non-universal case. These expressions follow the notation of Ref. [17]. In these cases all Higgsinos are assumed to be decoupled.

| Case | $T_{t3}$ | $T_{t2}$ | $T_{t1}$ | $T_{tt}$ | $T_{tb}$ | $T_{b3}$ | $T_{b2}$ | $T_{bl}$ | $T_{bb}$ |
|------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| I    | 3/2      | 3/4      | 51/52    | 3/4      | 1/2      | 3/2      | 3/4      | 15/28    | 1/2      | 3/4      |
| II   | 3/2      | 3/4      | 51/52    | 3/4      | 1/2      | 3/2      | 3/4      | 15/28    | 1/2      | 3/4      |
| III  | 5/4      | 3/4      | 35/52    | 3/4      | 1/2      | 5/4      | 3/4      | 11/28    | 1/2      | 3/4      |
| IV   | 1        | 1/2      | 17/26    | 3/4      | 1/2      | 1        | 1/2      | 5/14     | 1/2      | 3/4      |
| VI   | 5/4      | 1/2      | 25/26    | 3/4      | 1/2      | 5/4      | 1/2      | 1/2      | 1/2      | 3/4      |
| A    | 5/4      | 3/4      | 51/52    | 3/4      | 1/2      | 3/2      | 3/4      | 11/28    | 1/2      | 3/4      |
| B    | 3/2      | 3/4      | 51/52    | 3/4      | 1/2      | 3/2      | 3/4      | 15/28    | 1/2      | 3/4      |
| C    | 3/2      | 3/4      | 51/52    | 3/4      | 1/2      | 3/2      | 3/4      | 15/28    | 1/2      | 3/4      |

## Table 3

The RGE coefficients of top and bottom Yukawa couplings for each non-universal case. In these cases all Higgsinos are assumed to be light.

| Case | $T_{t3}$ | $T_{t2}$ | $T_{t1}$ | $T_{tt}$ | $T_{tb}$ | $T_{b3}$ | $T_{b2}$ | $T_{bl}$ | $T_{bb}$ |
|------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| I    | 3/2      | 3/4      | 51/52    | 3/4      | 1/2      | 3/2      | 3/4      | 15/28    | 1/2      | 3/4      |
| II   | 3/2      | 1/4      | 33/52    | 3/4      | 1/2      | 3/2      | 1/4      | -3/28    | 1/2      | 3/4      |
| III  | 5/4      | 1/4      | 65/52    | 5/6      | 1        | 5/4      | 1/4      | 17/28    | 1        | 5/6      |
| IV   | 1        | 1        | 1        | 1        | 1        | 1        | 1        | 1        | 1        |
| VI   | 5/4      | 1        | 5/13     | 11/12    | 1/2      | 5/4      | 1        | 2/7      | 1/2      | 11/12    |
| A    | 5/4      | 3/4      | 65/52    | 5/6      | 1        | 5/4      | 3/4      | 17/28    | 1        | 5/6      |
| B    | 3/2      | 3/4      | 33/52    | 3/4      | 1/2      | 3/2      | 3/4      | -3/28    | 1/2      | 3/4      |
| C    | 3/2      | 1/4      | 33/52    | 3/4      | 1/2      | 3/2      | 1/4      | -3/28    | 1/2      | 3/4      |
Figure Captions

**Fig. 1** The value of $m_t^{QFP}$ corresponding to each SUSY breaking scale $M_S$. In this case Higgsinos are assumed to be heavy.

**Fig. 2** The value of $m_t^{QFP}$ corresponding to each SUSY breaking scale $M_S$. In this case all Higgsinos are assumed to be light.

**Fig. 3** The value of $m_t^{QFP}$ corresponding to each SUSY breaking scale $M_S$ for the case of Yukawa unification. In this case all Higgsinos are assumed to be light.
Figure 1
Figure 2
Figure 3