Interaction of $K^-$-mesons with light nuclei

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Introduction

Low-energy $KN$ and $KA$ interactions have gained substantial interest during the last two decades. Data on the $K^-p$ scattering length $a(K^-p)$ from KEK [1]  
\[ a(K^-p) = (-0.78 \pm 0.18) + i(0.49 \pm 0.37) \text{ fm}, \]  
and the DEAR experiment at Frascati [3]
\[ a(K^-p) = \begin{array}{c}
-0.468 \pm 0.090 \text{ (stat.)} \pm 0.015 \text{ (syst.)} \\
+ i(0.302 \pm 0.135 \text{ (stat.)} \pm 0.036 \text{ (syst.)}) \end{array} \text{ fm} \]  
show that the energy shift of the $1s$ level of kaonic hydrogen is repulsive, $\text{Re} a(K^-p) < 0$. Nevertheless, it is possible that the actual $K^-p$ interaction is attractive if the isoscalar $\Lambda(1405)$ resonance is a bound state of the $\bar{K}N$ system [4]. A fundamental reason for such a scenario is provided by the leading order term in the chiral expansion for the $K^-N$ amplitude which is attractive.

Furthermore, very recently a strange tribaryon $S^0(3115)$ was detected in the interaction of stopped $K^-$-mesons with $^4\text{He}$ [6]. The width of this state was found to be less than 21 MeV. According to Ref. [7] this state can be interpreted as a candidate of a deeply bound state $(\bar{K}NNN)^{Z=0}$ with $I=1, I_3=-1$. It is clear that further searches for bound kaonic nuclear states as well as new data on the interactions of $\bar{K}$-mesons with lightest nuclei are thus of great importance.

The $K^-d$, $K^-^3\text{He}$, and $K^-\alpha$ scattering lengths and loosely bound $K^-$-nucleus states

Calculations of the $K^-d$ scattering length have recently been performed within the Multiple-Scattering Approach [8] [9] [10] as well as with Faddeev Equations [9] [11]. The results of Refs. [8] and [9] if using the same input are in good agreement. Moreover, the calculations of Barrett and Deloff [7] within the framework of Faddeev equations gave practically the same result as in the Multiple Scattering Approach, with the value for $A(K^-d)$ in the range $-(0.75 \div 0.85) + i(1.10 \div 1.23) \text{ fm}$. 
In Ref. [12] the real and imaginary parts of the $K^0d$ scattering length have been extracted from the data on the $K^0d$ mass spectrum obtained from the reaction $pp\rightarrow dK^0K^+$ measured recently at COSY [13]. Upper limits on the $K^-d$ scattering length have been found, namely $\text{Im} A(K^-d) \leq 1.3\;\text{fm}$ and $|\text{Re} A(K^-d)| \leq 1.3\;\text{fm}$. It has also been shown that the limit for the imaginary part of the $K^-d$ scattering length is strongly supported by data on the total $K^-d$ cross sections. The results for the imaginary part of the $K^-d$ scattering length from Refs. [10] and [11] violate the upper limit found in Ref. [12].

The calculations of the $K^-\alpha$ and $K^-^3\text{He}$ scattering lengths have been performed using five parameter sets for the $\bar{K}N$ lengths shown in Table I. The results from a $K$-matrix fit (Set 1), separable fit (Set 2) and the constant scattering length fit (CSL) denoted as Set 3 were taken from Ref. [14]. We also study the CSL fit from Conboy [15] (Set 4). Recent predictions for $\bar{K}N$ scattering lengths based on the chiral unitary approach of Ref. [16] are denoted as Set 5.

| Set | Ref. | $a_0(\bar{K}N) [\text{fm}]$ | $a_1(\bar{K}N) [\text{fm}]$ | $A(K^-\alpha) [\text{ fm}]$ | $A(K^-^3\text{He}) [\text{ fm}]$ |
|-----|------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1   | [14] | $-1.59+i0.76$               | $0.26+i0.57$               | $-1.80+i0.90$               | $-1.50+i0.83$               |
| 2   | [14] | $-1.61+i0.75$               | $0.32+i0.70$               | $-1.87+i0.95$               | $-1.55+i0.90$               |
| 3   | [14] | $-1.57+i0.78$               | $0.32+i0.75$               | $-1.90+i0.98$               | $-1.58+i0.94$               |
| 4   | [15] | $-1.03+i0.95$               | $0.94+i0.72$               | $-2.24+i1.58$               | $-1.52+i1.80$               |
| 5   | [16] | $-1.31+i1.24$               | $0.26+i0.66$               | $-1.98+i1.08$               | $-1.66+i1.10$               |

Table 1: The $K^-\alpha$ scattering length from Ref. [17] and new results for $K^-^3\text{He}$, calculated for different sets of the elementary $\bar{K}N$ amplitudes $a(\bar{K}N)$ ($I = 0, 1$).

The results of the calculations are listed in the last two columns of Table I. These results are very similar for Sets 1–3. The $K^-\alpha$ and $K^-^3\text{He}$ scattering lengths are in the range $A(K^-\alpha) = -(1.8 \div 1.9) + i(0.9 \div 0.98)\;\text{ fm}$ and $A(K^-^3\text{He}) = -(1.5 \div 1.58) + i(0.83 \div 0.94)\;\text{ fm}$, respectively. The results for Set 4 are quite different: $A(K^-\alpha) = -2.24 + i1.58\;\text{ fm}$ and $A(K^-^3\text{He}) = -1.52 + i1.80\;\text{ fm}$. The calculations with Set 5 are close to the results obtained with Sets 1–3.

Unitarizing the constant scattering length, we can reconstruct the $K^-X$ scattering amplitude within the zero range approximation (ZRA) as

$$f_{K^-X}(k) = \left[A(K^-X)^{-1} - ik\right]^{-1}, \quad (3)$$

where $X = \alpha$ or $^3\text{He}$, $k = k_{K^-X}$ is the relative momentum of the $K^-X$ system. The denominator of the amplitude of Eq. (3) has a zero at the complex energy $E^* = E_R - i\Gamma_R/2 = k^2/(2\mu)$, where $E_R$ and $\Gamma_R$ are the binding energy and width of a possible $K^-X$ resonance, respectively. Here $\mu$ is the reduced mass of the $K^-X$ system.

In case of the $K^-\alpha$ system we find for Sets 1 and 4 a pole at the complex energies $E^* = (-6.7-i18/2)\;\text{MeV}$ and $E^* = (-2.0-i11.3/2)\;\text{MeV}$, respectively. The calculations with Set 5 also result in a loosely bound state, $E^* = (-4.8-i14.9/2)\;\text{MeV}$. Similar results have been obtained for the $K^-^3\text{He}$ system. Note that assuming
a strongly attractive phenomenological $\bar{K}N$ potential, Akaishi and Yamazaki \cite{18} predicted a deeply bound $\bar{K}\alpha$ state at $E^*=(-86-i34/2)$ MeV, which is far from our solutions. This problem can be resolved assuming that the loosely and deeply bound states are different eigenvalues of the $\bar{K}\alpha$ effective Hamiltonian. Our model for the $\bar{K}\alpha$ scattering amplitude is valid only near threshold, i.e. when $kA(\bar{K}\alpha)\ll 1$. The ZRA can not be applied for the description of deeply bound states when the pole of the scattering amplitude is located far away from the threshold. If the same procedure were applied to the $K^-\bar{3}H$ system we would find a similar loosely bound state. This state together with recently discovered deeply bound state, the $S^0(3115)$, can be considered as different eigenvalues of the $K^-\bar{3}H$ effective Hamiltonian. In any case it is very important to measure the $s$-wave $\bar{K}\alpha$ scattering length in order to clarify the situation concerning the existence of bound $\bar{K}\alpha$ states.

The $K^-\alpha$ FSI in the reaction $dd\rightarrow\alpha K^-K^+$

In Refs. \cite{17,19} it was argued that the reaction $dd \rightarrow \alpha K^-K^+$ near threshold is sensitive to the $K^-\alpha$ final state interaction. We calculated the $K^-\alpha$ invariant mass spectrum at excess energy 50 MeV. The result is shown in Fig. 1. The solid line shows the calculations for pure phase space, i.e. for a constant production amplitude and neglecting FSI effects. The dash-dotted and dashed lines show the results obtained for the $K^-\alpha$ FSI calculated with the parameters of Sets 1 and 4, respectively. All lines are normalized to the same total cross section of 1 nb. It is clear that the FSI significantly changes the $K^-\alpha$ mass spectrum.

![Figure 1: The invariant $K^-\alpha$ mass spectrum produced in the $dd\rightarrow\alpha K^+K^-$ reaction at excess energy 50 MeV. The solid line describes the pure phase space distribution, while the dash-dotted and dashed lines show our calculations with $K^-\alpha$ FSI for parameters of Set 1 and 4, respectively. The short-dashed line shows the result obtained using the modified $\bar{K}N$ scattering lengths in nuclear medium.](image)

Akaishi and Yamazaki \cite{18} argued that the $\bar{K}N$ interaction is characterized by a strong $I=0$ attraction, which allows the few-body systems to form dense nuclear objects. The optical potential proposed by Akaishi and Yamazaki for deeply
bound nuclear states contains the following effective $\bar{K}N$ scattering lengths in the medium: $a_{\bar{K}N,\text{med.}}^0 = +2.25 + i0$ fm, $a_{\bar{K}N,\text{med.}}^1 = 0.48 + i0.12$ fm. We used these modified scattering lengths to calculate the enhancement factor for the $K^-\alpha$ FSI in the $dd \rightarrow K^+K^-\alpha$ reaction. The short-dashed line in Fig. [1] demonstrates a very pronounced deformation of the $K^-\alpha$ invariant mass spectrum. Such a strong in-medium modification of the $\bar{K}N$ scattering length apparently can be tested at COSY.

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