Conformal anomaly for dilaton coupled electromagnetic field

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ABSTRACT

The derivation of the conformal anomaly for dilaton coupled electromagnetic field in curved space is presented. The models of this sort naturally appear in stringy gravity or after spherical reduction of multidimensional Einstein-Maxwell theory. It is shown that unlike the case of minimal vector in curved space or dilaton coupled scalar the anomaly induced effective action cannot be derived. The reason is the same why anomaly induced effective action cannot be constructed for interacting theories (like QED) in curved space.

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1. Introduction. In the study of string theory the lower energy 4D string effective action maybe presented as following

$$S = \int d^4x \sqrt{-g} \left[ R + 4(\nabla \phi)^2 + F_{\mu\nu}^2 \right] e^{-2\phi}$$

(1)

where $\phi$ is dilaton, $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, $A_\mu$ is electromagnetic field. Investigating of string gravity (1) the quantum effects of electromagnetic field with the lagrangian:

$$L = -\frac{1}{4} f(\phi) F_{\mu\nu} F^{\mu\nu}$$

(2)

maybe dominant in some regions, especially if we consider generalization of above model with N vectors and apply large N expansion. From another point if we start from usual Einstein-Maxwell gravity in $D$-dimensions, we can do spherical reduction to the space $R_4 \times S_{D-4}$ where $R_4$ is an arbitrary 4D curved space. Then the reduced action becomes again of the form (1) (maybe with change of some terms and some coefficients) where the radius of $S_{D-4}$ plays the role of dilatonic function.

Hence, the study of dilaton coupled electromagnetic field with the Lagrangian (1) which describes conformally invariant theory maybe of interest in different respects. In the present letter we calculate the conformal anomaly for dilaton coupled vector (2) and discuss the problems which appear in the attempt to define the correspondent anomaly induced action. Note that recently trace anomaly and anomaly induced action for dilaton coupled matter in two dimensions has been discussed in Refs.[1], [2], [3] and [4].

2. One-loop effective action and conformal anomaly for dilaton coupled electromagnetic field. The study of conformal anomaly [3], and its applications attracts a lot of attention (for a recent review, see [5]). Among the most well-known applications one can list particle creation and Hawking radiation, the construction of non-singular Universe with back-reaction of quantum matter, anti-evaporation of black holes, etc. The conformal anomaly for electromagnetic field has been found in refs.[6, 8]. Our purpose in this section will be the calculation of conformal anomaly for dilaton coupled matter (or electromagnetic) field.

The initial Lagrangian of the theory has the following form;

$$L = -\frac{1}{4} f(\phi) F_{\mu\nu} F^{\mu\nu}$$

(3)
where $A_\mu$ is quantum vector field. We consider quantum theory with action (3) in an external classical gravitational field. Note that electromagnetic field is non-minimally coupled with the external classical dilaton function $f(\phi)$ where $\phi$ is dilaton.

One can show that four dimensional theory with the Lagrangian (3) is conformally invariant. Adding to the Lagrangian (3) gauge-fixing Lagrangian $L_{gf}$:

$$L_{gf} = -\frac{1}{2} f(\phi)(\nabla_\nu A^\nu)^2,$$

one can easily obtain

$$L + L_{gf} = \frac{1}{2} f(\phi) A_\alpha \hat{H}_\nu^\alpha A^\nu.$$  \hspace{1cm} (5)

Here

$$\hat{H}_\nu^\alpha = \delta_\nu^\alpha \Box - R_\nu^\alpha + 2[\hat{h}_\nu^\alpha]_\mu \nabla_\mu,$$

$$[\hat{h}_\nu^\alpha]_\mu = \frac{1}{2f} \{ \delta_\nu^\alpha (\nabla^\mu f) - g^{\alpha\mu}(\nabla_\nu f) + \delta_\mu^\nu (\nabla^\alpha f) \}.$$  \hspace{1cm} (6)

Integrating over quantum vectors and taking into account the ghost contribution which is the same as in theory with $f = 1$, one can get the one-loop effective action (its divergent part):

$$\Gamma^{(1)} = \Gamma_{A_\mu}^{(1)} + \Gamma_{ghost}^{(1)} = -\frac{i}{2} \text{Tr} \ln \hat{H}_\nu^\alpha + i \text{Tr} \ln \Box.$$  \hspace{1cm} (7)

Here second term is ghost contribution. In the dimensional regularization, one obtains

$$\Gamma^{(1)} = \frac{1}{(n-4)} \int d^4x \sqrt{-g} b_4$$

where $b_4$ is $b_4$-coefficient of Schwinger-De Witt expansion. Note that Tr ln $f(\phi)$ does not give the contribution to $\Gamma^{(1)}$ in frames of dimensional regularization. For general algorithm of the calculation (8) one can consult [1]. According to this algorithm, $-\frac{i}{2} \text{Tr} \ln \hat{H}_\nu^\alpha$ can be calculated as

$$\Gamma_{A_\mu}^{(1)} = \frac{1}{(4\pi)^2(n-4)} \int d^4x \sqrt{-g} \text{tr} \left\{ \frac{1}{2} \hat{P}^2 + \frac{1}{12} \hat{S}_{\alpha\beta} \hat{S}^{\alpha\beta} \right. \hspace{1cm} (9)$$

$$\left. + \frac{1}{6} \Box \hat{P} + \frac{1}{180} \{ R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - R_{\mu\nu} R^{\mu\nu} + \Box R \} \right\}.$$
where trace is taken over vector space, i.e., \(\text{tr} \hat{1} = 4\). For the operator \(\hat{H}_\nu^\alpha\) we get

\[
\hat{P}_\nu^\alpha = -R_\nu^\alpha + \frac{\delta_\nu^\alpha}{6} R - \nabla_\mu [\hat{h}_\nu^\alpha]_\mu - [\hat{h}_\mu^\beta]_\mu [\hat{h}_\nu^\beta]_\mu
\]

\[
= -R_\nu^\alpha + \frac{\delta_\nu^\alpha}{6} R - \frac{\delta_\nu^\alpha}{2f} \Box f + \frac{1}{2f^2} [\delta_\nu^\alpha (\nabla_\mu f)(\nabla^\mu f) + (\nabla^\alpha f)(\nabla_\nu f)] .
\]

\[
\hat{S}_{\alpha\beta} \equiv [\hat{S}_{\mu\nu}]_{\alpha\beta}
\]

\[
= (\nabla_\beta \nabla_\alpha - \nabla_\alpha \nabla_\beta) \hat{1} + \nabla_\beta [\hat{h}_{\mu\nu}]_\alpha - \nabla_\alpha [\hat{h}_{\mu\nu}]_\beta + [\hat{h}_{\mu\tau}]_\beta [\hat{h}_{\nu\alpha}]_\tau - [\hat{h}_{\mu\tau}]_\alpha [\hat{h}_{\nu\beta}]_\tau
\]

\[
= R_{\mu\nu\alpha\beta} - \frac{1}{f^2} \left\{ -g_{\alpha\mu}(\nabla_\beta f)(\nabla_\nu f) + g_{\alpha\nu}(\nabla_\beta f)(\nabla_\mu f)
\right.
\]

\[
+ g_{\beta\mu}(\nabla_\alpha f)(\nabla_\nu f) - g_{\beta\nu}(\nabla_\alpha f)(\nabla_\mu f)
\]

\[
+ g_{\mu\beta} g_{\nu\alpha}(\nabla_\tau f)(\nabla^\tau f) - g_{\mu\alpha} g_{\nu\beta}(\nabla_\tau f)(\nabla^\tau f)
\]

\[
+ \frac{1}{2f} \left[ g_{\alpha\nu}(\nabla_\beta \nabla_\mu f) - g_{\alpha\mu}(\nabla_\beta \nabla_\nu f)
\right.
\]

\[
+ g_{\beta\mu}(\nabla_\alpha \nabla_\nu f) - g_{\beta\nu}(\nabla_\alpha \nabla_\mu f) \right\} . \tag{10}
\]

Having explicit expressions \([(10)]\) for the operators \(\hat{S}_{\alpha\beta}, \hat{P}\), one can find:

\[
\frac{1}{2} \text{tr} \hat{P}^2 = \frac{1}{2} R^{\alpha\beta} R_{\alpha\beta} - \frac{1}{9} R^2 + \frac{R}{6f} \Box f + \frac{1}{2f^2} (\Box f)^2
\]

\[
- \frac{R^{\nu\alpha}}{2f^2} (\nabla_\nu f)(\nabla_\alpha f) - \frac{R}{12f^2} (\nabla_\mu f)(\nabla^\mu f)
\]

\[
- \frac{5}{4f^3} (\Box f)(\nabla_\mu f)(\nabla^\mu f)
\]

\[
+ \frac{7}{8f^4} (\nabla_\mu f)(\nabla^\mu f)(\nabla_\alpha f)(\nabla^\alpha f) ,
\]

\[
\frac{1}{6} \text{tr} \Box \hat{P} = -\frac{1}{18} \Box R - \frac{1}{3} \Box \left( \frac{1}{f} \Box f \right) + \frac{5}{12} \Box \left[ \frac{1}{f^2} (\nabla_\alpha f)(\nabla^\alpha f) \right] ,
\]

\[
\frac{1}{12} \hat{S}_{\alpha\beta} \hat{S}_{\beta\alpha} = -\frac{1}{12} R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} + \frac{1}{6f^2} R_{\mu\beta}(\nabla_\nu f)(\nabla^\beta f) + \frac{1}{12f^2} R(\nabla_\mu f)(\nabla^\mu f)
\]

\[
- \frac{1}{3f} R_{\alpha\beta}(\nabla_\mu \nabla^\alpha f) - \frac{5}{16f^2} (\nabla_\beta f)(\nabla_\mu f)(\nabla^\mu f)
\]

\[
+ \frac{1}{6f^3} (\nabla_\mu f)(\nabla^\mu f)(\nabla_\nu f) + \frac{1}{3f^3} (\Box f)(\nabla_\beta f)(\nabla^\beta f)
\]
\[-\frac{1}{6f^2}(\nabla_\alpha \nabla_\beta f)(\nabla^\alpha \nabla^\beta f) - \frac{1}{12f^2}(\Box f)(\Box f)\]  

(11)

On the same time, for the second (ghost) term in Eq.(7), we get:

\[\hat{S}_{\alpha\beta} = 0, \quad \hat{P} = \frac{R}{6}.\]  

(12)

Hence, using Eq.(9) we will find

\[\Gamma^{(1)}_{\text{ghost}} = \frac{1}{(4\pi)^2(n-4)} \int d^4 x \sqrt{-g} \left\{ -\frac{1}{90} R_{\mu\nu\alpha\beta}^2 + \frac{1}{90} R_{\mu\nu}^2 - \frac{R^2}{36} - \frac{1}{15} \Box R \right\} \]

\[= \frac{1}{(n-4)} \int d^4 x \sqrt{-g} b_4^{\text{ghost}}.\]  

(13)

As one can see this is standard expression for the ghost contribution to one-loop effective action, it does not depend on dilaton.

The one-loop effective action due to vectors (9) may be found as

\[\Gamma^{(1)}_{\text{vector}} = \frac{1}{(4\pi)^2(n-4)} \int d^4 x \sqrt{-g} \left\{ -\frac{11}{180} R_{\mu\nu\alpha\beta}^2 + \frac{43}{90} R_{\mu\nu}^2 - \frac{1}{9} R^2 - \frac{1}{30} \Box R - \frac{1}{3} \Box \left( \frac{1}{2} \Box f \right) + \frac{5}{12} \Box \left( \frac{1}{f^2} (\nabla_\alpha f)(\nabla^\alpha f) \right) - \frac{1}{3f^2} R_{\mu\nu}(\nabla^\mu f)(\nabla^\nu f) - \frac{1}{3f} R_{\mu\nu}(\nabla^\mu \nabla^\nu f) + \frac{R}{6f}(\Box f) + \frac{9}{16f^4}(\nabla_\mu f)(\nabla^\mu f)(\nabla_\alpha f)(\nabla^\alpha f) + \frac{1}{6f^3}(\nabla^\beta f)(\nabla^\nu f)(\nabla_\beta \nabla_\nu f) - \frac{11}{12f^3} (\nabla^\nu f)(\nabla_\nu f)(\Box f) - \frac{1}{6f^2} (\nabla_\alpha \nabla_\beta f)(\nabla^\alpha \nabla^\beta f) + \frac{5}{12f^2}(\Box f)(\Box f) \right\} \]

\[= \frac{1}{(n-4)} \int d^4 x \sqrt{-g} b_4^{\text{vector}}.\]  

(14)

Hence, we found the one-loop effective action due to dilaton coupled vectors. The total one-loop effective action \(\Gamma^{(1)}\) is given by sum of (13) and (14).
From the expression for $\Gamma^{(1)}$ one can easily get conformal anomaly:

$$T = b_4 = b_4^{\text{ghost}} + b_4^{\text{vector}}. \quad (15)$$

The first four terms give the well-known conformal anomaly of electromagnetic field \cite{7, 8}. We have to note that using another regularization (like zeta-regularization) may slightly alter the coefficients of total derivative terms in (15), like already happened in this case in the absence of dilaton \cite{7, 8}.

For completeness, we write below the trace anomaly for dilaton coupled conformal scalar with the Lagrangian:

$$L = \varphi f(\phi) \left( \Box - \frac{1}{6} R \right) \varphi. \quad (16)$$

In this case, we get \cite{3}

$$T = \frac{1}{(4\pi)^2} \left\{ \frac{1}{32} \left[ \frac{(\nabla f)(\nabla f)}{f^4} \right]^2 + \frac{1}{24} \Box \left( \frac{(\nabla f)(\nabla f)}{f^2} \right) \right. 
+ \frac{1}{180} \left( R_{\mu \nu \alpha \beta}^2 - R_{\mu \nu}^2 + \Box R \right) \right\} \quad (17)$$

If one considers the system consisting of $n$ dilaton coupled conformal scalars and $m$ dilaton coupled vectors then the total conformal anomaly of the system is given by:

$$T = nT(17) + mT(15) \quad (18)$$

Hence, we found the explicit expression for dilaton coupled vector conformal anomaly. It would be of interest to study the structure of this anomaly extending results of ref.\cite{10}.

3. Anomaly induced effective action. With the help of conformal anomaly one can construct the anomaly induced effective action on the same way as in ref.\cite{10}. First of all, we will rewrite $f$-independent terms of conformal anomaly (15) in a slightly different form:

$$T = \frac{1}{(4\pi)^2} \left\{ b \left( F + \frac{2}{3} \Box R \right) + b' G + b'' \Box R + \cdots \right\} \quad (19)$$

where $b = \frac{1}{10}$, $b' = -\frac{31}{180}$ and in our model $b'' = -\frac{1}{6}$. Note, however, that in principle $b''$ corresponds to arbitrary parameter because $\Box R$ is the variation
of local action: $\sqrt{-g} \Box R = -\frac{1}{6} g^{\mu \nu} \delta \frac{\delta}{\delta g^{\mu \nu}} f \, d^4 x \sqrt{-g} R^2$. Hence, $b''$ may be always changed by the addition of finite counterterms to gravitational effective action.

Let the metric has the form

$$g_{\mu \nu} = e^{2\sigma} \bar{g}_{\mu \nu}.$$  \hspace{1cm} (20)

In this case for Ricci tensor and Ricci scalar we get

$$R = e^{-2\sigma} [\bar{R} - 6 \Box \sigma - 6 (\bar{\nabla}_\mu \sigma)(\bar{\nabla}^\mu \sigma)] \, ,$$
$$R_{\mu \nu} = [\bar{R}_{\mu \nu} - 2 \bar{\nabla}_\mu \bar{\nabla}_\nu \sigma - \bar{g}_{\mu \nu} \Box \sigma$$
$$+ 2 (\bar{\nabla}_\nu \sigma)(\bar{\nabla}_\mu \sigma) - 2 \bar{g}_{\mu \nu} \partial^\tau \sigma \partial_\tau \sigma] \, .$$  \hspace{1cm} (21)

One can write now the conformal anomaly (15), (19) in the following form;

$$T = \frac{1}{\sqrt{-g} \delta \sigma} W[\sigma]$$ \hspace{1cm} (22)

where $W[\sigma]$ is some unknown anomaly induced effective action which should be found after integration of Eq.(22). Substituting conformally transformed curvature tensors and metric to conformal anomaly, we find

$$\sqrt{-g} T = \frac{\sqrt{-g}}{(4\pi)^2} \left\{ b \bar{F} + b' \left( \bar{G} - \frac{2}{3} \Box \bar{R} \right) + 4b' \bar{\Delta} \sigma + \left[ b'' + \frac{2}{3} (b + b') \right] \Box R e^{4\sigma}$$
$$- \frac{1}{3} \left[ - \frac{2(\Box \sigma)}{f} \right] \left( 2(\bar{\nabla}_\mu \sigma)(\bar{\nabla}^\mu f) + \Box f \right)$$
$$- 2(\bar{\nabla}_\mu \sigma) \bar{\nabla}^\mu \left( \frac{1}{f} (2(\bar{\nabla}_\mu \sigma)(\bar{\nabla}^\mu f) + \Box f) \right) + \Box \left( \frac{1}{f} (2(\bar{\nabla}_\mu \sigma)(\bar{\nabla}^\mu f) + \Box f) \right) \right]$$
$$+ \frac{5}{12} \left[ \Box \left( (\bar{\nabla}_\mu f)(\bar{\nabla}^\mu f) \right) - \frac{2(\bar{\nabla}_\mu f)(\bar{\nabla}^\mu f)}{f^2} \right] \Box \sigma$$
$$- 2(\bar{\nabla}^\mu \sigma) \bar{\nabla}_\mu \left[ \frac{(\bar{\nabla}_\mu f)(\bar{\nabla}^\mu f)}{f^2} \right]$$
$$+ \left[ \bar{R}_{\mu \nu} - 2 \bar{\nabla}_\mu \bar{\nabla}_\nu \sigma - \bar{g}_{\mu \nu} \Box \sigma + 2 (\bar{\nabla}_\nu \sigma)(\bar{\nabla}_\mu \sigma) - 2 \bar{g}_{\mu \nu} (\partial^\tau \sigma)(\partial_\tau \sigma) \right]$$
$$\times \left[ - \frac{1}{3 f^2} (\bar{\nabla}^\mu f)(\bar{\nabla}^\nu f) - \frac{1}{3 f} \left( (\bar{\nabla}^\mu \bar{\nabla}^\nu f) - \bar{g}^{\mu \tau} (\bar{\nabla}^\nu \sigma)(\bar{\nabla}_\tau f) \right.$$
$$- \bar{g}^{\nu \tau} (\bar{\nabla}^\mu \sigma)(\bar{\nabla}_\tau f) + \bar{g}^{\mu \nu}(\bar{\nabla}^\tau \sigma)(\bar{\nabla}_\tau f) \right] \right\}$$

7
\[
\begin{align*}
+ \frac{1}{6f} [2(\nabla_\mu \sigma)(\nabla^\mu f) + \Box f] \times [R - 6\Box \sigma - 6(\nabla_\mu \sigma)(\nabla^\mu \sigma)] \\
+ \frac{9}{16f^4} [(\nabla_\mu f)^2]^2 + \frac{1}{6f^3} (\nabla^\beta f)(\nabla_\nu f) \left( \nabla_\beta \nabla_\nu f - (\nabla_\beta \sigma)(\nabla_\nu f) \\
- (\nabla_\beta \sigma)(\nabla_\nu f) + \bar{g}_{\beta \nu}(\nabla^\tau \sigma)(\nabla_\tau f) \right) - \frac{11}{12f^3} (\nabla_\mu f)^2 [2(\nabla_\nu \sigma)(\nabla^\nu f) + \Box f] \\
- \frac{1}{6f^2} \left[ \nabla_\alpha \nabla_\beta f - (\nabla_\alpha f)(\nabla_\beta \sigma) - (\nabla_\alpha \sigma)(\nabla_\beta f) + \bar{g}_{\alpha \beta}(\nabla^\tau \sigma)(\nabla_\tau f) \right] \\
\times [\nabla^\alpha \nabla_\beta f - (\nabla^\alpha f)(\nabla_\beta \sigma) - (\nabla^\alpha \sigma)(\nabla_\beta f) + \bar{g}^{\alpha \beta}(\nabla^\tau \sigma)(\nabla_\tau f)] \\
+ \frac{5}{12f^2} [2(\nabla_\mu \sigma)(\nabla^\mu f) + \Box f]^2 \right].
\end{align*}
\]

Here \( \Delta = \Box^2 + 2 R^{\mu \nu} \nabla_\mu \nabla_\nu - \frac{2}{3} R \Box + \frac{1}{3}(\nabla^\mu R) \nabla_\mu \). It is also easier to keep fourth term in (23) to be non-transformed. Now, using Eq.(21), one can integrate expression (22) with conformal anomaly (23) in order to get the explicit effective action \( W \)

\[
W = b \int d^4x \sqrt{-g} F \sigma + b' \int d^4x \sqrt{-g} \left\{ 2\sigma \Delta \sigma + \left( \bar{G} - \frac{2}{3} \Box \bar{R} \right) \sigma \right\} \\
- \frac{1}{12} \left[ b'' + \frac{2}{3}(b + b') \right] \int d^4x \sqrt{-g} \left[ \bar{R} - 6\Box \sigma - 6(\nabla_\mu \sigma)(\nabla^\mu \sigma) \right]^2 \\
+ \int d^4x \sqrt{-g} \left[ -\frac{1}{3} \left\{ \sigma \Box^2 f + \frac{1}{f}(\nabla_\mu \sigma)(\nabla^\mu \sigma)\Box f \right\} \\
+ \frac{4}{3f} (\nabla_\mu \sigma)(\nabla^\mu \sigma)(\nabla_\mu f)(\nabla^\mu f) \right\} \\
+ \frac{5}{12} \left\{ \sigma \Box \left( (\nabla_\mu f)(\nabla^\mu f) \right) + \frac{1}{f^2}(\nabla_\mu \sigma)(\nabla_\mu \sigma)(\nabla_\mu f)(\nabla^\mu f) \right\} \\
+ \bar{R}_{\mu \nu} \left\{ - \frac{1}{3f^2}(\nabla^\mu f)(\nabla_\nu f) - \frac{1}{3f}(\nabla^\mu \nabla^\mu f) \right\} \sigma \\
+ \frac{1}{6f} \sigma \Box f \bar{R} + \frac{9}{16f^4} \sigma [(\nabla_\mu f)^2]^2 + \frac{1}{6f^3} \sigma (\nabla^\beta f)(\nabla_\nu f)(\nabla^\tau \sigma)(\nabla_\tau f) \\
- \frac{11}{12f^3} \sigma (\nabla_\mu f)^2 \Box f - \frac{1}{6f^2} \sigma \nabla_\alpha \nabla_\beta f \nabla^\alpha \nabla^\beta f + \frac{5}{12f^2} \sigma (\Box f)^2 \right] + \cdots
\]

However, several terms in the trace anomaly (23) cannot be integrated to give the effective action. It is not difficult to write explicitly all such terms
if it is necessary.

The reason why we cannot integrate some terms in conformal anomaly (15) is the same as it was pointed out in the paper by Reigert [11] for $R^2$-ter
m. I.e., the functional derivative of $\sqrt{-g}T(x)$ with respect to $\sigma(y)$ must be symmetric in $y$ and $x$ if anomaly induced action would exist. However it is not difficult to show that functional derivative of (15) is not symmetric one for most of terms. Hence we conclude that anomaly induced action does not exist for dilaton coupled vector, i.e., already in the free theory.

On the contrary, anomaly induced action for free fields non-interacting with dilaton may be easily constructed. Only after taking account of quantum field interaction $R^2$ term with the coefficient proportional to coupling constant appeared. Only after appearance of this term anomaly induced action could not be constructed. However we see for dilaton coupled 4D scalar anomaly induced action does exist [3]:

$$W = b \int d^4x \sqrt{-g} F \sigma + b' \int d^4x \sqrt{-g} \left\{ 2\sigma \Delta \sigma + \left( \bar{G} - \frac{2}{3} \Box R \right) \sigma \right\}$$

$$- \frac{1}{12} \left( b'' + \frac{2}{3} (b + b') \right) \int d^4x \sqrt{-g} \left[ \bar{R} - 6\Box \sigma - 6(\bar{\nabla} \sigma)(\bar{\nabla} \sigma) \right]^2$$

$$+ \int d^4x \sqrt{-g} \left\{ a_1 \left[ (\bar{\nabla} f)(\bar{\nabla} f) \right] f_4^2 \sigma + a_2 \Box \left[ (\bar{\nabla} f)(\bar{\nabla} f) \right] f_2^2 \sigma$$

$$+ a_2 \left[ (\bar{\nabla} f)(\bar{\nabla} f) \right] [(\bar{\nabla} \sigma)(\bar{\nabla} \sigma)] \right\}$$

(25)

where

$$b = \frac{1}{120(4\pi)^2}, \quad b' = -\frac{1}{360(4\pi)^2}, \quad a_1 = \frac{1}{32(4\pi)^2}, \quad a_2 = \frac{1}{24(4\pi)^2}.$$  (26)

One can investigate different cosmological applications of anomaly induced effective action (25), like study of early Universe with back-reaction of dilaton coupled scalar matter, constructing of new versions of IR sector of dilatonic gravity, etc.

4. Conclusion We calculated conformal anomaly for dilaton coupled electromagnetic field. It is shown that it is impossible to construct anomaly induced effective action in this case. Currently we do not understand the physical reason why it is impossible to integrate conformal anomaly. It could be that
only in the case of dilaton coupled conformal supermultiplet the dangerous
terms in (super) conformal anomaly cancel and anomaly induced effective
action can be constructed. This question deserves further study.

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