Enhanced Secondary Frequency Control via Distributed Peer-to-Peer Communication

Chenye Wu, Soummya Kar and Gabriela Hug

Abstract—The prevalent distributed generation resources warrant reconsideration on how their coordination is achieved. In this paper, we particularly focus on how to enhance secondary frequency control by exploiting peer-to-peer communication among the resources. We design a control framework based on a consensus-plus-global-innovation approach. The control signals of distributed resources are updated in response to a global innovation (utilizing the area control error signal), and additional information exchanged via communication among neighboring resources. We show that such a distributed control scheme can be very well approximated by a proportional-integral controller and stabilizes the system. Moreover, since our scheme takes advantage of both the global and neighboring information, simulation results demonstrate that our scheme stabilizes the system significantly faster than the conventional automatic generation control framework. In addition, our control scheme asymptotically achieves the cost effectiveness.

I. INTRODUCTION

The electric power system is facing new challenges but also opportunities as more and more distributed resources such as distributed generation units, storage systems, and demand response participants are integrated into the system. The discrepancy between this increasingly distributed infrastructure and the traditional centralized control structure may lead to unnecessary inefficiencies and reduction in reliability in the electric energy supply. On the other hand, the trend of incorporating more communication technology to the system allows for an enhancement of the control structure by leveraging the capability of direct communication and coordination between distributed resources.

A. Scope of the Work

In modern power system, the sub-5-minute control is dominated by both primary and secondary frequency control [1], [2], without communication among the resources at either level. More precisely, primary frequency control is based on speed droop controllers to balance load fluctuations within seconds. On a slower time scale, but still sub-5-minute, secondary frequency control kicks in to free up the primary control reserves, restore the frequency to 60 Hz, and re-establish the pre-defined tie line flows between control areas. In this paper, we specifically focus on secondary frequency control, where the independent system operators (ISOs) first clear the resources via the ancillary service market and then use the automatic generation control (AGC) framework [3] to dispatch the cleared resources according to the pre-defined participation factors. At this level, the communication only takes place between control center and resources. Thus, we aspire to study how to utilize communications between distributed resources to enhance the control scheme.

To achieve this goal, we utilize the area control error (ACE) signal and propose a secondary frequency control scheme based on a consensus-plus-global-innovation framework [4]. More precisely, in our framework, the participants in real time incrementally update their own control signals in response to the global and via communication exchanged information. A key objective of the proposed method is to maintain the stability of the system, and at the same time adjust the settings of generators, flexible loads and storages such as to achieve a cost effective dispatch, i.e., not just the clearing is optimized but also the dispatch of the cleared resources. We would like to emphasize that, we do not require a completely connected communication network. Instead, the communication among resources can be sparse and limited. Even with limited communication, the proposed control scheme still stabilizes the system significantly faster than the AGC framework.

B. Related Work

Frequency stability has been recognized as one of the most important challenges in power systems since the 1970s [5]. Over the past decade, much effort has been devoted to the design of decentralized robust proportional-integral (PI) controllers to tackle this challenge (see [6] and [7] for two comprehensive surveys). For example, Rerkpreedapong et al. combine $H_{\infty}$ control and genetic algorithm techniques to tune the PI parameters in [8]. Bevrani et al. introduce a sequential decentralized method to obtain a set of low-order robust controllers in [9]. Li et al. propose a modified AGC framework to improve the economic efficiency in [10]. Different from the conventional control perspective without communications among resources, we utilize the consensus-plus-global-innovation approach and develop a control framework, which leverages peer-to-peer communication to improve the control performance.

Our work also belongs to a growing literature that utilizes the consensus-plus-innovations approach to determining the
economic dispatch. For example, Yang et al. propose a consensus based distributed framework to perform economic dispatch in [11]. Zhang et al. introduce a distributed incremental cost consensus algorithm to solve the economic dispatch problem in [12]. The work closest to our approach is proposed by Kar et al. in [13], where the authors employed the consensus-plus-innovations algorithm for distributed economic dispatch. In contrast to previous work, our approach is significantly more complex in that the outcome after every single iteration is used as a control signal which requires that one has to maintain the system stability while following the dynamically changing demand.

C. Our Contributions
Towards exploiting how peer-to-peer communication can enhance secondary frequency control, the principal contributions of this paper are:

- Communication and Control: We propose a control scheme to better utilize the communication between resources. Each regulation resource updates its local control signal based on the ACE signal received from the control center and information obtained from its communication neighbors.
- System Stability Guarantee: The proposed control scheme is proven to be a PI controller. Simulation results illuminate our scheme in Section VI.
- Cost-effectiveness: Under certain assumptions (to be specified in Section IV-C), we prove that the allocations provided by the proposed algorithm are close to the optimally achievable allocations, where $c$ is a positive constant depending only on the cost parameters and the communication topology.

The remainder of this paper is organized as follows. We introduce the single-area system model, including the state space model in Section II. Section III demonstrates the consensus-plus-global-innovation control framework. We analyze its properties in Section IV. In Section V, we generalize our control scheme to the multi-area scenario. Simulation results illuminate our scheme in Section VI. Concluding remarks are given in Section VII.

II. SYSTEM MODEL

A. Notations

- $\Delta f$: Frequency deviation from nominal value.
- $\Delta P_{m}^{i}$: Resource $i$'s regulation contribution.
- $\Delta P_{g}^{i}$: Resource $i$'s governor valve position differential.
- $\Delta P_{L}$: Load deviation from its predicted value.
- $H$: Equivalent inertia constant for the single area.
- $D$: Equivalent damping coefficient for the single area.
- $R_{g}$: Resource $i$'s droop characteristic.
- $\Omega = \{1, \cdots, n\}$: The set of all the frequency regulation resources in the system.
- $A_G$: Adjacency matrix of the communication graph.
- $u_i$: Secondary frequency control action for generator $i$.
- $T_i$: Turbine time constant for resource $i$.
- $T_g^i$: The governor time constant for resource $i$.
- $\Delta f$: Frequency deviation from nominal value.
- $\Delta P^i$: Resource $i$'s regulation contribution.
- $\Delta P_{m}^i$: Resource $i$'s governor valve position differential.
- $\Delta P_{L}$: Load deviation from its predicted value.
- $H$: Equivalent inertia constant for the single area.
- $D$: Equivalent damping coefficient for the single area.
- $R_{g}$: Resource $i$'s droop characteristic.
- $\Omega = \{1, \cdots, n\}$: The set of all the frequency regulation resources in the system.
- $A_G$: Adjacency matrix of the communication graph.

\[ D_G: \text{Degree matrix of the communication graph.} \]
\[ L = D_G - A_G: \text{The Laplacian matrix of the communication graph.} \]
\[ T_g^i: \text{The governor time constant for resource } i. \]
\[ T_i: \text{The turbine time constant for resource } i. \]

B. State Space Model
Consider the single-area secondary frequency control problem. Fig. 1 demonstrates our system’s key difference from the conventional secondary frequency control - the distributed resources can directly communicate with each other and exchange information.

We use the second order model [14] (in Laplace domain) to characterize each frequency regulation resource $i$:

\[
M_i(s) = \frac{1}{1 + T_g^i s} \frac{1}{1 + T_i s}, \forall i \in \Omega.
\] (1)

Thus, the state space model [1] can be described as:

\[
\dot{\Delta f} = -\frac{D}{2H} \Delta f + \frac{1}{2H} \left( \sum_{i \in \Omega} \Delta P_{m}^{i} - \Delta P_{L} \right),
\] (2)
\[
\dot{\Delta P_{m}^{i}} = -\frac{\Delta P_{m}^{i} - \Delta P_{g}^{i}}{T_g^i},
\] (3)
\[
\dot{\Delta P_{g}^{i}} = -\frac{\Delta f}{T_g^i R_g} - \frac{1}{T_g^i} (\Delta P_{g}^{i} - u_i).
\] (4)

C. Time Scales
Next, we clarify various system time scales associated with frequency control. The sub-5-minute control time horizon is divided into time slots, $T = \{1, \cdots, T\}$. Each time slot is of length $\Delta T$. A typical value for $\Delta T$ is 4 seconds. We also make the following assumption so as to formally analyze the performance of our discrete time control scheme using its equivalent continuous time counterpart:

**Assumption 1:** The load deviation $\Delta P_{L}$ from its prediction only changes at the beginning of each time slot, and remains constant for the rest of the time slot.

This assumption merely utilizes the fact that the current AGC framework sends out the control signal every 4 seconds, which implies that the load variation within the 4-second period is minor. However, it allows us to consider the following sequential events. At the beginning of each time

\(^3\)In most major US power systems, the current AGC framework sends out the control signal every 4 seconds.
slot, control center sends out frequency deviation measurement corresponding to load change $\Delta P_L$ to all regulation resources. Subsequently, regulation resources update their own control signals in response to the latest frequency deviation measurement, and also the information received from other resources.

III. COMMUNICATION ENHANCED CONTROL DESIGN

In this section, we first investigate secondary frequency control from an economic dispatch perspective. Motivated by two control objectives (desired equilibrium behavior and cost effective dispatch), we introduce our control scheme to leverage peer-to-peer communication.

A. Economic Dispatch Inspiration

Any equilibrium point of the controlled swing dynamic equations satisfies that $\Delta f$, $\Delta P^m_i$, and $\Delta P^i_j$ are all zero. This yields the following necessary condition to achieve an equilibrium with $\Delta f = 0$:

$$\sum_{i \in \Omega} u_i = \Delta P_L.$$  

(5)

It is readily seen that there could be many control schemes satisfying the necessary condition (5), but we want to design a control scheme that also achieves cost effectiveness in terms of the cost to conduct energy dispatch.

Mathematically, if we consider a quadratic cost function for each regulation resource $i$, from an economic perspective, ideally, we would want, by the end of time slot $t$, $P^m_i(t+1)$'s that minimize the dispatch cost:

$$\begin{align*}
\text{minimize} & \quad \sum_{i \in \Omega} \left( a_i (\Delta P^m_i)^2 + b_i \Delta P^m_i + c_i \right), \\
\text{subject to} & \quad \sum_{i \in \Omega} \Delta P^m_i = \Delta P_L(t + 1).
\end{align*}$$

(6)

Note, in the above we ignore the ramping constraints of the network resources which decouples the temporal constraints, thereby reducing the overall dispatch cost minimization over the control horizon $T$ to solving a collection of temporally decoupled economic dispatch problems (6)-(7) at each time slot $t$. In fact, as long as $|\Delta P_L(t) - \Delta P_L(t - 1)|$ is appropriately bounded throughout $T$, this relaxation is tight. We justify this in supplementary material [15].

Further, note that since the regulation resources are selected by co-optimizing with the energy bids, taking capacity payment (used to compensate lost opportunity cost in energy market [16]) into account, the initial marginal costs (when $\Delta P^m_i = 0$), i.e., $b_i$'s, of all participants to provide secondary frequency control should be almost identical. Therefore, we make the following assumption to simplify analysis:

Assumption 2: All the $b_i$'s are identical.

We know at the equilibrium point, we have $u_i = \Delta P^m_i$, for all $i \in \Omega$. Combining this with (6)-(7), ideally, from a cost of dispatch viewpoint, we would like to design a control scheme, such that $u_i(t+1)$'s are the solution to

$$\begin{align*}
\text{minimize} & \quad \sum_{i \in \Omega} \left( a_i u_i^2 + b_i u_i + c_i \right), \\
\text{subject to} & \quad \sum_{i \in \Omega} u_i = \Delta P_L(t + 1).
\end{align*}$$

(8)

Note that, with identical $b_i$'s, constraint (9) guarantees $\sum_{i \in \Omega} b_i u_i = b \Delta P_L(t + 1)$, which is a constant in the optimization. Hence, problem (8)-(9) can be reduced to

$$\begin{align*}
\text{minimize} & \quad \sum_{i \in \Omega} a_i u_i^2, \\
\text{subject to} & \quad \sum_{i \in \Omega} u_i = \Delta P_L(t + 1).
\end{align*}$$

(10)

Denoting all the control signals $\{u_i(t+1), \forall i \in \Omega\}$ by $u(t + 1)$, and the Lagrangian multiplier associated with (11) by $\lambda_i^{t+1}$, the Lagrangian function $\mathcal{L}(u(t+1), \lambda_i^{t+1})$ for problem (10)-(11) at each time $t$ is given by

$$\mathcal{L}(u(t+1), \lambda_i^{t+1}) = \sum_{i \in \Omega} a_i u_i^2(t+1) - \lambda_i^{t+1} \left( \sum_{i \in \Omega} u_i(t+1) - \Delta P_L(t+1) \right).$$

(12)

The first order optimality conditions are therefore given by

$$\begin{align*}
\frac{\partial \mathcal{L}}{\partial u_i(t+1)} &= 2a_i u_i(t+1) - \lambda_i^{t+1} = 0, \quad \forall i \in \Omega, \\
\frac{\partial \mathcal{L}}{\partial \lambda_i^{t+1}} &= \sum_{i \in \Omega} u_i(t+1) - \Delta P_L(t+1) = 0.
\end{align*}$$

(13)

(14)

If we denote each regulation resource $i$'s marginal cost at time $t$ by $\lambda_i^t$, (13) requires

$$\lambda_i^{t+1} = 2a_i u_i(t+1) + \lambda_i^{t+1}, \quad \forall i \in \Omega.$$  

(15)

Hence, (13)-(15) reflect that the marginal costs for all entities have to be equal in the optimal solution and the resulting provision of power needs to fulfill the power balance.

At a given time $t \in T$ if each resource $i$ has access to the Lagrange multiplier variable $\lambda_i^t$, which may be interpreted as a differential generation price, it may set its control signal according to (13) so that the system achieves the power balance in the most economic way. However, as (14) suggests, the quantity $\lambda^t$ depends on private information such as the cost characteristics of all the entities and global information - the instantaneous system net load deviation $\Delta P_L(t)$. This motivates us to propose the consensus-plus-global-innovation control scheme.

B. Consensus + Global Innovation Control Design

We propose a distributed real-time approach in which participating entities, through neighborhood communication and global information processing, continuously update their control signals to track the optimal power allocation closely. Note that we denote our control scheme as distributed for the following reasons: cost parameters of resource $i$ are only known to resource $i$ and communication is used to find an agreement with neighboring resources. In our control scheme, each resource $i \in \Omega$ maintains and updates a local copy of the variable $\lambda_i^t$. The updates are defined as

$$\begin{align*}
\text{neighborhood consensus} & \quad \lambda_i^{t+1} = \lambda_i^t - 2a_i \beta \sum_{i \in \Omega} (\lambda_i^t - \lambda_j^t) \\
& \quad + 2a_i m^{-1} \left( \Delta P_L(t+1) - \sum_{j \in \Omega} \Delta P^m_j(t) \right), \\
\text{global innovation} & \quad u_i(t+1) = (2a_i)^{-1} \lambda_i^{t+1}, \\
\lambda_i^{t+1} &= 2a_i \Delta P^m_i(t+1),
\end{align*}$$

(16)

(17)

(18)
where $\beta$ is a positive tuning parameter; $\tilde{\lambda}_i^t$ is the estimate of marginal cost $\lambda_i^t$ for resource $i$ at time $t$; $\Omega_i$ denotes the set of participant $i$'s neighbors in the communication network.

Intuitively, note that the neighborhood consensus term in the update rule (16) seeks to enforce an agreement between the marginal price variables $\lambda_i^t$'s so as to optimize the dispatch cost (see (15)); whereas, the innovation term seeks to enforce demand-supply balance which is also necessary to drive $\Delta f$ to zero. This update rule would require that entity $i$ has access to $\Delta P_L(t)$ and all $\Delta P_m^i(t)$'s, which constitutes global as opposed to local information. In order to realize the (global) innovation term in (16) using local information, we use the fact that

$$\Delta P_L = \sum_{i \in \Omega} \Delta P_m^i - D\Delta f - 2H\Delta f,$$  
(19)

which follows from the swing dynamics (2). The discrete time approximation of $\Delta P_L$ therefore is

$$\Delta P_L(t+1) = \sum_{i \in \Omega} \Delta P_m^i(t) - D\Delta f(t) - 2H(T)^{-1}(\Delta f(t+1) - \Delta f(t)).$$  
(20)

Substituting (20) into the global innovation in (16) yields

$$\tilde{\lambda}_i^{t+1} = \lambda_i^t - 2a_i\beta \sum_{j \in \Omega_i}(\lambda_j^t - \lambda_j^t) - 2a_iD\Delta f(t) - \frac{4a_iH}{n \Delta T}(\Delta f(t+1) - \Delta f(t)).$$  
(21)

Note that incorporating this global innovation term requires public information of $H$ and $D$.

### IV. Analytical Performance Evaluation

Given this distributed control scheme, we analyze its key properties - stability, equilibrium behavior, and dispatch cost effectiveness in this section.

#### A. Stability

For each $u_i(t+1)$, we can rewrite the control law as

$$u_i(t+1) = \Delta P_m^i(t) - \beta L^t\Lambda^{-1}\Delta P_m(t) + n^{-1} \Delta P_L(t+1) - 1^T P_m(t),$$  
(22)

where $P_m(t) = [\Delta P_m^1(t), \ldots, \Delta P_m^n(t)]^T$; $L^t$ is the $i$th row of the communication network's Laplacian matrix $L$; and

$$\Lambda = \text{diag}\{(2a_1)^{-1}, \ldots, (2a_n)^{-1}\},$$  
(23)

where $\text{diag}\{\cdot\}$ denotes the diagonal matrix.

Using the Euler forward emulnation and (20), we have the Laplace transform for $u_i$ as

$$u_i(s)(1+Ts) = \Delta P_m^i(s) - \beta L^t\Lambda \Delta P_m(s) + n^{-1} \Delta P_L(s) - \frac{4a_iH}{n \Delta T}(\Delta f(s) + D\Delta f(s)).$$  
(24)

Note that, according to the second order model (1), we can further approximate $\Delta P_m^i(s)$ by

$$\Delta P_m^i(s) \approx u_i(s)(1+T_g^i s)^{-1}(1+T_i^i s)^{-1} \approx u_i(s)(1-(T_g^i + T_i^i) s).$$  
(25)

Combining (25) with (24) yields

$$u_i(s) = \frac{2H}{n T_u} \Delta f(s) - \frac{\beta n L_i^t \Delta P_m(s) + D\Delta f(s)}{n T_u s},$$  
(26)

where $T_u = \Delta T + T_g^i + T_i^i$.

That is, the above approximation reduces to a PI controller, which can be tuned by standard methods [17].

**Remark:** Compared with the conventional AGC framework, our distributed control scheme makes use of more state information (both $\Delta f$ and $\Delta P_m^i$'s) via communication. This grants us additional flexibility in terms of designing the (PI) controller. As demonstrated by the simulation results (see Section VI), the power of communication enables our control scheme to stabilize the system significantly faster than the AGC framework.

#### B. Equilibrium Behavior

**Theorem 1:** The proposed distributed control scheme satisfies the necessary condition (5).

Due to space limit, all the proofs can be found in our supplementary material [15].

#### C. Cost Effectiveness

We analyze the cost effectiveness of the proposed distributed control scheme under the following assumption.

**Assumption 3:** If the time constants $T_g^i$'s and $T_i^i$'s in the second order model (1) are all zero, we have for all $t \in T$

$$\Delta P_m^i(t) = u_i(t), \forall i \in \Omega.$$  
(27)

With increasing storage systems, and demand response resources participating in the secondary frequency control market, more resources enjoy very small time constants. This control scheme can thus guarantee near real time demand-supply balance. We rely on this assumption to prove the cost effectiveness of our control scheme, but for the simulations we still use reasonable time constants for the second order model. This assumption corresponds to guaranteeing that the control settings for the generators are optimal. Specifically, it assumes that the (final) output of the generators, i.e., after the physical delay caused by the governor and the turbine, are equal to the proposed control.

To prove cost effectiveness, we also need the communication graph to be well connected. In particular, the second largest eigenvalue $1 - \rho$ of matrix $I - \beta \Lambda^{-1} L$ need satisfy:

$$(1 - \rho) \max_{i \in N}(a_i)^{-\frac{1}{2}} \max_{i \in N}(a_i)^{-\frac{1}{2}} < 1.$$  
(28)

**Remark:** Condition (28) guarantees sufficient speed to implicitly spread the cost parameters via consensus. The Laplacian matrix has a trivial eigenvalue of zero, corresponding to the largest eigenvalue, 1, of the matrix $I - \beta \Lambda^{-1} L$. The second largest eigenvalue of this matrix measures the connectivity of the communication network. The more connected the network, the smaller the second largest eigenvalue.

**Theorem 2:** If the communication graph is well connected such that (28) holds, $\|\Delta P_L(t+1) - \Delta P_L(t)\| < \epsilon$, $\forall t \in T$, and Assumption 3 holds, then there exists a constant
c > 0 (depending only on the cost parameters and the communication topology) such that control signals (ui’s) are cc-close to their optimal values given by (13)-(14):
\[
\|X_i^t - X^\star\| \leq c, \quad \forall t \in T, i \in \Omega
\]
\[
\|u_i(t) - u_i^\star(t)\| \leq c, \quad \forall t \in T, i \in \Omega
\]
where \(u_i^\star(t)\) denotes the solution to (13)-(14).

V. MULTI-AREA GENERALIZATION

Though our analysis is based on the single area model, where the ACE signal is exactly the frequency deviation, it is not hard to see that by replacing the current control signal \(\Delta f(t)\) with the ACE signal, we can directly apply the proposed distributed secondary frequency control scheme to the multi-area model. More specifically, the frequency swing dynamics in area \(j\) becomes
\[
\Delta f_j = -\frac{1}{2H_j} \left( D_j \Delta f_j + \Delta P_{tie,j} - \sum_{i \in \Omega_j} \Delta P_{m,j}^i + \Delta P_{b,j} \right),
\]
where \(\Delta P_{tie,j}\) denotes the net tie-line flow out of area \(j\). Thus, the global innovation term in the updating rule (16) can be estimated by
\[
\Delta P_{L,j}(t+1) - \sum_{i \in \Omega_j} \Delta P_{m,j}^i(t) = 2H_j \frac{\Delta f_j(t) - \Delta f_j(t+1)}{\Delta T} - D_j \Delta f_j(t) - \Delta P_{tie,j}(t).
\]  (31)

In other words, each area will update its own innovation term in (16) according to (31). Based on this information and the communication with the neighbors in the same control area, the regulation resources update the control signals. Note that in the multi-area model, we assume that the area control center will provide the public information on the tie line flow constraints. The performance analysis is essentially the same as the single-area scenario.

VI. SIMULATION RESULTS

In this section, we first carry out the simulation for the proposed control scheme for a single-area system, and then test our control scheme in the multi-area scenario and compare the performance with the conventional AGC scheme.

A. Single-area Scenario

In the single-area five-participant system, \(H\) is assumed to be 0.0833 pu s, and \(D\) is set to 0.0084 pu Hz. The droop characteristics \(R_i\)’s, \(T_i^p\)’s, \(T_i^q\)’s are generated uniformly at random from [2,3]Hz/pu, [0.05,0.06]s, and [0.3,0.5]s, respectively. The communication network is a 2-nearest neighbor network. In the updating rules, we select the shortest settling time as the tuning objective for the AGC framework. Table I shows the tuned parameters for the AGC framework. For our communication enhanced framework, we choose \(\beta = 0.01\) for all control intervals. In Table I, we also compare the settling times of the two frameworks at different control intervals, when facing a step load increase of 0.1 pu.

In the testing examples, our distributed control framework outperforms the AGC framework in all control intervals with an average of 33% speed up in restoring the frequency back to 60 Hz. The maximal speed up (43%) is achieved when \(\Delta T = 1.6s\). Our framework is also robust in parameter selection: we choose \(\beta = 0.01\) for all the four testing cases while the tuning parameters for AGC framework differ significantly for the various time intervals.

Next, by choosing common \(\Delta T\) as 1.6s, we compare the frequency responses of the two frameworks when the load constantly changes, as demonstrated in Fig. 2(a). Though direct observations of Fig. 2(b) and (c) suggest that both frameworks work reasonably well, statistical features illuminate that our framework outperforms the AGC framework: during the 200-second simulation period, the mean of frequency derivations in our framework is \(-9 \times 10^{-4}\) while that in the AGC framework is \(-0.0021\); the standard deviation in our framework is 0.0113, while that in the AGC framework is 0.0124.

Finally, we study the cost effectiveness of our control scheme in a 100-second time horizon in Fig. 3. Assuming each resource has a quadratic cost function, we know that AGC can perform the cost effective control by setting the participating factor \(\alpha_i\) for resource \(i\) as \(\alpha_i = a_i^{-1}/\sum_{j \in \Omega} a_j^{-1}\).

On the other hand, in Section IV, we show that our control scheme can also asymptotically achieve the cost effectiveness. We use a monotonically increasing load, shown in Fig. 3(a), to better illustrate this property. The relative deviation of the total cost from the optimal dispatch is illustrated in Fig. 3(b): during the 100-second horizon, using \(\Delta T = 1.6s\), our control scheme successfully keeps the relative error below 20% after 20 seconds. The large initial relative error is introduced by the global innovation term in the updating rule as well as the physical delays. The
consensus terms, then, dominate and reduce the relative error over time. Hence, as time goes on, the relative error stays below 12%.

B. Multi-area Scenario

To verify the stability of our framework in the multi-area scenario, we consider a three area model as shown in Fig. 4. Each area consists of five regulation resources. The system parameters are generated similarly as the single area scenario. We again employ the 2-nearest neighbor network as the communication network for each area. We set all the tie-line flow constraints to be zero. In the updating rules, we use $\beta = 0.005$ for all the three areas.

When encountering a step load increase of 0.05 pu in area 2, the frequency responses of the three areas are exemplified in Fig. 5(a)-(b). The larger $\Delta T$ is, the longer our control scheme takes to stabilize the system. Furthermore, as shown in Fig. 5(c)-(d), our control scheme guarantees the tie line flow constraints while stabilizing the frequency for each area. Table II shows the settling times of the frequencies in the three areas and the tie line flows.

VII. CONCLUSIONS AND FUTURE WORK

This paper introduces a cost effective secondary frequency control framework, which leverages peer-to-peer communication to enhance the control performance. We employ a consensus-plus-global-innovation approach to design a distributed control scheme. Theoretical analysis and simulation results further illustrate the stability and cost-effectiveness of this scheme in both single-area and multi-area scenarios. This work can be extended in various directions. For instance, we would like to quantify the impact of communication delay and asynchronizaton issues on the distributed control scheme. We also want to design a fully distributed control scheme, where each resource measures the frequency information locally. This will be substantially challenging in that the local frequency measurements, if not processed carefully, might even lead the resources to perform regulation against each other.

REFERENCES

[1] A. Bergen and V. Vittal, Power systems analysis. Prentice Hall, 1999.

[2] Y. Rebours, D. Kirschen, M. Trottignon, and S. Rossignol, “A survey of frequency and voltage control ancillary services: Part I: Technical features,” IEEE Trans. on Power Syst., vol. 22, no. 1, pp. 350–357, Feb 2007.

[3] R. Bacher and H. P. Van Meeteren, “Real-time optimal power flow in automatic generation control,” IEEE Trans. on Power Syst., vol. 3, no. 4, pp. 1518–1529, Nov 1988.

[4] S. Kar, J. M. F. Moura, and K. Ramanan, “Distributed parameter estimation in sensor networks: Nonlinear observation models and imperfect communication,” IEEE Trans. on Information Theory, vol. 58, no. 6, pp. 3575–3605, 2012.

[5] V. Converiti, D. P. Gelpolos, M. Housley, and G. Steinbrenner, “Long-term stability solution of interconnected power systems,” IEEE Trans. on Power Apparatus and Systems, vol. 95, no. 1, pp. 96–104, 1976.

[6] I. Ibraheem, P. Kumar, and D. Kothari, “Recent philosophies of automatic generation control strategies in power systems,” IEEE Trans. on Power Syst., vol. 20, no. 1, pp. 346–357, 2005.

[7] S. K. Pandey, S. R. Mohanty, and N. Kishor, “A literature survey on load–frequency control for conventional and distribution generation power systems,” Renewable and Sustainable Energy Reviews, vol. 25, pp. 318–334, 2013.

[8] D. Rerkpreedapong, A. Hasanovic, and A. Feliachi, “Robust load frequency control using genetic algorithms and linear matrix inequalities,” IEEE Trans. on Power Syst., vol. 18, no. 2, pp. 855–861, 2003.

[9] H. Bevrani, Y. Mitani, and K. Tsuji, “Sequential design of decentralized load frequency controllers using $\mu$ synthesis and analysis,” Energy conversion and management, vol. 45, no. 6, pp. 865–881, 2004.

[10] N. Li, L. Chen, C. Zhao, and S. H. Low, “Connecting automatic generation control and economic dispatch from an optimization view,” in American Control Conference (ACC), 2014, pp. 735–740.

[11] S. Yang, S. Tan, and J.-X. Xu, “Consensus based approach for economic dispatch problem in a smart grid,” IEEE Trans. on Power Syst., vol. 28, no. 4, pp. 4416–4426, 2013.

[12] Z. Zhang and M.-Y. Chow, “Convergence analysis of the incremental cost consensus algorithm under different communication network topologies in a smart grid,” IEEE Trans. on Power Syst., vol. 27, no. 4, pp. 1761–1768, 2012.

[13] S. Kar and G. Hug, “Distributed robust economic dispatch in power systems: A consensus + innovations approach,” in Proc. of IEEE Power and Energy Society General Meeting, July 2012, pp. 1–8.

[14] E. Camponogara, D. Jia, B. Krogh, and S. Talukdar, “Distributed model predictive control,” IEEE Control Systems, vol. 22, no. 1, pp. 44–52, Feb 2002.

[15] C. Wu, S. Kar, and G. Hug, “Proofs: Supplementary material for paper ‘enhanced secondary frequency control via distributed peer-to-peer communication’,” [Online]. Available: http://goo.gl/Fdrf1UC.

[16] FERC, “Frequency regulation compensation in the organized wholesale power markets,” Order No. 755-A, pp. 1–76, 2011.

[17] Y.-Y. Cao, J. Lam, and Y.-X. Sun, “Static output feedback stabilization: an ilmi approach,” Automatica, vol. 34, no. 12, pp. 1641–1645, 1998.

TABLE II

| $\Delta T$ (s) | $\Delta f_1$ (Hz) | $\Delta f_2$ (Hz) | $\Delta f_3$ (Hz) | $\Delta P_{tie,1-2}$ (pu) | $\Delta P_{tie,2-3}$ (pu) |
|--------------|-----------------|-----------------|-----------------|-----------------------------|-----------------------------|
| Step change   | 1.6             | 6.10            | 6.79            | 6.27                        | 6.35                        | 7.91                        |
| 4.0          | 8.73            | 10.48           | 8.84            | 9.19                        | 10.61                       |