Inductive dressed ring traps for ultracold atoms

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Abstract

We present two novel dressed inductive ring trap geometries, ideal for atom interferometry or studies of superfluidity and well-suited to utilization in atom chip architectures. The design permits ring radii currently only accessible via near-diffraction-limited optical traps, whilst retaining the ultra-smooth magnetic potential afforded by inductive traps. One geometry offers simple parallel implementation of multiple rings, whereas the other geometry permits axial beam-splitting of the torus suitable for whole-ring atom interferometry.

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A toroid is one of the simplest multiply-connected 3D shapes, and the periodic boundary conditions both simplify and enrich experimental and theoretical work. In the limit of small cross-sectional area, annular traps permit approximate access to ideal 1D infinite systems (cf Born–von Karman boundary conditions) using a finite experimental footprint. Ring traps are excellent systems in which to perform atom interferometry [1–3] or study superfluidity [4, 5]. Extremely smooth magnetic ring traps for ultracold atoms can be made via induction [6, 7], which eliminates both wire end effects and roughness from meandering dc currents [9]. Such time-averaged traps are limited to radii \(\geq 5\) mm for the ‘thin-wire’ geometries considered here, however careful optimization of wire thickness can decrease radii to \(\approx 1\) mm [8]. In this communication we show—for the first time—how inductive ring traps can be obtained using RF-dressing, which then permits access to the \(\leq 0.3\) mm ring radius regime, suitable for fabrication on atom chips. Such chips offer the prospect of portable ultracold atomic setups and there have been recent important developments in both chip loading [10] and their application [11, 12]. We envisage prospects for low-decoherence on-chip studies of Sagnac interferometry, superfluidity, atomic SQUIDs [13], ring dark solitons [14] as well as vortices [15] and solitons [16] in low-dimensional systems.

There have been a wide variety of studies of cold matter in toroidal geometries, which we briefly summarize, considering first complementary traps, before concentrating on the purely magnetic techniques relevant to this communication.

The optical dipole force can be used in many ways to confine atom in ring traps: by optically plugging a magnetic trap [17, 18], using static ‘hollow’ Laguerre–Gauss beams [3–5, 19–22] or beams that have intensity profiles spatially-shaped by a spatial light modulator [23]. In addition, one can quickly scan a focused beam [24–26] and trap atoms in the time-averaged dipole potential, opening access to a variety of complex geometries including ring lattices. Static dipole traps using both co-propagating [27] and counter-propagating [28] Laguerre–Gauss mode superpositions also offer the flexibility of extending to more exotic ring geometries.

Magnetic ring-shaped traps have been proposed and realized using either static [29–31] or time-averaged...
Zeeman sub-levels and the resulting interaction Hamiltonian can be written

\[ H(r, t) = \Omega_{RF}(r) F_\omega \cos(\omega t) + \Omega_S(r, t) F_z, \]

(1)

where \( F_\omega \) and \( F_z \) are respectively the projections of the total angular momentum operator along the \( \sigma^+ \) polarization and \( Z \) directions (if we choose a local co-ordinate system with \( Z \) parallel to \( B_0 \)). The parameter \( s = \text{sign}(g_F) \) is the sign of the Landé g-factor \( g_F \) for the total angular momentum \( F \). The Larmor frequency is expressed as \( \Omega_S(r, t) = g_F \mu_B B(r, t)/\hbar \) and \( \Omega_{RF}(r) = g_F \mu_B B_\omega(r)/\hbar \) denotes the Rabi frequency associated with the part of the ac field that contributes to the coupling between the Zeeman sub-levels, where \( \mu_B \) is the Bohr magneton. In this limit, one can simply express the resulting adiabatic potential as

\[ U(r, t) = \hbar m_F \sqrt{\delta^2(r, t) + \Omega_{RF}^2(r)} = \hbar m_F \Omega, \]

(2)

for an atom initially in the \( m_F \) Zeeman sub-level, with \( \delta(r, t) = \Omega_S(r, t) - \omega \) the detuning between the Larmor and drive frequencies. These expressions for \( H(r, t) \) and \( U(r, t) \) stand in the adiabatic regime, i.e. as long as the condition \( |\delta'(r, t)| \ll \Omega(r, t) \) is valid, where \( \theta = \arctan(\Omega_S/\delta) \). Adiabaticity can thus be re-expressed as

\[ A = \left( \frac{\delta^2 + \Omega_{RF}^2}{|\delta \Omega_{RF} - \delta' \Omega_{RF}|} \right)^{3/2} \ll 1, \]

(3)

where \( \gamma \) represents the total time derivative \( \left. \frac{d}{dt} \right|_{\gamma} = \frac{\delta}{\Omega} + \mathbf{v} \cdot \nabla \), allowing for atomic motion at velocity \( \mathbf{v} \), and we refer to \( A \) as the adiabaticity parameter. We will only consider trapping regions in which the rotating wave approximation (RWA) is valid, i.e. \( \eta = |\delta(r, t)|/(\Omega_S(r, t) + \omega) \ll 1 \) and \( \eta_{RF} = \Omega_{RF}/\omega \ll 1 \) [48].

Consider now the experimental apparatus illustrated in figure 1: a set of two Helmholtz coils (orange) provide a spatially homogenous oscillating field \( B_{\parallel} \hat{\mathbf{z}} \text{ext} \) throughout the surface of the conductive ring (red), and parallel to the ring axis (\( \hat{z} \)). Spatial homogeneity of \( B_{\parallel} \) is reasonable if we consider the ring to have much smaller dimensions than the coils, and homogeneity is not essential as long as cylindrical symmetry is not significantly broken. The magnetic flux from \( B_{\parallel} \) induces a current in the ring, which has a complex amplitude calculated using Lenz’s law and characterized using the ring’s electrical resistance \( R \) and inductance \( L \) [6]:

\[ I_{\text{Ring}} = -\frac{\pi r_c^2 B_{\parallel}}{L(1 - i \omega_{\parallel}/\omega)} = |I_{\text{Ring}}(e^{i\phi}). \]

(4)

where \( r_c \) is the radius of the ring. The above formalism using complex currents allows one to drop time dependence. This current, in turn, generates a synchronous magnetic field \( B_{\text{ring}}(r) e^{i\phi} \) which is fully spatially expressed using elliptical integrals [49]. The complete RF field, with time dependence, at position \( r \) is thus:

\[ B_{\text{RF}}(r, t) = (B_{\text{ring}}(r) e^{i\phi} + B_{\text{RF}}(r) e^{i\omega t}). \]

Now we consider a static, or slowly varying, magnetic field \( B_0(\mathbf{r}) \), which we choose as our quantization axis. The RF field has to be expressed in terms of the two components \( B_{\perp}(r) \) and \( B_{\parallel}(r) \) orthogonal to the static field, since the parallel (\( \tau \)) component does not contribute to the coupling. Using Jones’
formalism \[50\], the amplitude of the RF fields that couples to \(\sigma^\pm\) transitions is then given by:

\[
B_{\sigma^\pm}(r) = \frac{1}{\sqrt{2}}[(1 \pm i)B_{1,\perp}(r) + (1 \mp i)B_{1,\parallel}(r)].
\]  

(5)

It is then straightforward to calculate the adiabatic potential using equation (2).

Throughout this communication, unless otherwise stated, we assume a copper ring with a radius of 400 \(\mu\text{m}\) and a wire radius of 28 \(\mu\text{m}\). Helmholtz coils provide an RF field of amplitude \(B_H = 12\text{ G}\) oscillating at \(\omega = 2\pi \times 10\text{ MHz}\). Our simulations include a finite-element solver \[7\] to account for the finite ring size and the skin effect that can significantly change the electrical parameters at high frequencies, as well as for precise calculation of the potential depth. Note, however, that the main results of the communication can still be reproduced to reasonable (\(\approx 10\%\)) accuracy without resorting to finite-element calculations. With the above ring dimensions and driving field, the ring has resistance and inductance of 18 m\(\Omega\) and 1.5 nH, respectively. The current amplitude flowing inside the conductor is 390 mA, dissipating about 1.4 mW that rapidly leads to an equilibrium temperature of the copper of approximatively 230 °C (assuming perfect black body radiation). If required, this temperature rise could be radically reduced using e.g. a diamond substrate as an electrically insulating heatsink. We also consider the specific case of \(^{87}\text{Rb}\) pumped into the \(|F = 2, m_F = 2\rangle\) magnetic sub-level: therefore \(g_F = 1/2\) and only the \(B_{\sigma^+}(r)\) component couples to the atomic spin.

We now focus on the inductive TAAP configuration (figure 1(a)), where the quantization field is spatially homogenous, with a vector direction in the \(xy\) plane of the ring, but rotating around the symmetry axis. For a snapshot in time the cylindrical symmetry of the system is broken by the quantization field \(B_S\)—as the total RF field \(B_{RF}(r)\) is projected into a different ratio of coupling:non-coupling components, depending on the quantization direction. However, as the atoms can’t respond on these time-scales, they experience the cylindrically symmetric time-averaged TAAP potential averaged over one rotation period of the quantization field shown in figure 2. Here the constant magnetic field amplitude of \(B_S(t) = 15.9\text{ G}\) creates a corresponding detuning \(\delta\) which is spatially and temporally constant, i.e. \(2\pi \times 1.15\text{ MHz}\). The time-averaged trap is located slightly below the plane of the ring wire due to gravitational sag, with radial and axial trap frequencies of 400 Hz and 250 Hz, respectively.

There are two factors to consider for the maximum trappable temperature in the inductive TAAP. Firstly, gravity results in a saddle point shown in figure 2, and atoms hotter than 103 \(\mu\text{K}\) will no longer be confined to a ring but have access to a disc. Secondly, one can show that, during the rotation of the quantization field, there are curves of points where the RF coupling totally vanishes (black zones). However, as the detuning is non-zero when the coupling vanishes, the adiabatic potential still exists. One can estimate the loss at these locations using the \(A\) parameter, equation (3). These points would in principle limit the trap depth to 5 \(\mu\text{K}\), however the smallest value of the loss parameter is \(A = 310\) near the instantaneous 3D point of zero-coupling (compared to \(A = 1600\) at the trap minimum) for 100 \(\mu\text{K}\) atoms with a quantization axis rotation rate of 10 kHz. We can therefore expect negligible Landau–Zener loss, so atom temperature and the background pressure will be the only limits to the ring geometry and lifetime, respectively. The RWA is valid at the trap location as \(\eta_R = 0.05\) and \(\eta_E = 0.07\) (cf experimental parameters in figure 2 of \[46\] where \(\eta_R = 0.12\) and \(\eta_E = 0.14\)).

The trap parameters of the inductive TAAP are amenable to scaling arguments and we consider a scenario where the ring radius and wire radius both scale together with the parameter \(\xi\) (i.e. \(r_r \rightarrow \xi r_r\) and \(r_w \rightarrow \xi r_w\)). The natural angular frequency of the ring then transforms as \(\omega_{RL} \rightarrow \xi^{-2} \omega_{RL}\), so if the RF drive angular frequency follows an equal scaling \(\omega \rightarrow \xi^{-2} \omega\), the skin depth of the RF current in the ring scales with \(1/\sqrt{\omega}\) i.e. \(\xi\), and the spatial current distribution in the wire remains the same. Moreover, if the amplitude of the RF field \(\Omega_{RF}\) and the detuning \(\delta\) are kept constant, the TAAP trap potential depth and shape remain the same (if gravitational potential is

\[\text{Figure 1. Magnetic coil schematics. In both geometries, the driving ac-Helmholtz coils (orange) and the ring in which current is magnetically induced (red) oscillate at angular frequency } \omega \text{ and have axes aligned with the } z \text{ direction. The two geometries we consider are: (a) two Helmholtz coil pairs (blue and dark blue) driven in quadrature at an angular frequency } \omega_S < \omega, \text{ (b) a dc anti-Helmholtz coil pair (blue). Black, gray and white arrows indicate current directions at angular frequencies } \omega, \omega_S \text{ and dc, respectively. Co-ordinate pairs are coil current angular frequencies, with associated phases.}\]

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relatively weak), but are scaled to cover a region $\xi$ times the original size—i.e. the radial and axial trap frequencies are $1/\xi$ times larger.

We turn now to the second inductive dressed ring configuration (figure 1(b)), using a static quadrupole quantization field, yielding the cylindrically symmetric potentials depicted in figure 3. Because of the spatially varying magnetic field amplitude, the detuning $\delta$ is spatially dependent. The geometry of the potential is determined by the strength of the magnetic quadrupole. We maintain all other parameters, including $\omega = 2\pi \times 10$ MHz, $B_0 = 12$ G. In the case of a radial quadrupole gradient of 480 G cm$^{-1}$, the trap centre is located at $[r, z] = [300, -10]$ µm, with radial and axial trap frequencies of 750 Hz and 180 Hz, respectively (figure 3(a)). Gravity limits the potential to $\sim 31$ µK. Note the only place where the RF coupling vanishes is along the ring axis, far from the trap minimum. The A parameter is $A = 1100$ near the trap minimum so we can again expect negligible loss. The RWA is valid at the trap location as $\eta_3 = 0.007$ and $\eta_{RF} = 0.07$.

A key advantage of this geometry is that if we change the strength of the quantization quadrupole to a radial gradient of 440 G cm$^{-1}$, one can split the ring into two (figure 3(b)), which could be useful for e.g. atom interferometrical determination of gravity [12]. The simple scaling arguments for the inductive TAAP do not hold for the quadrupole trap, due to the different spatial scaling of $\delta$ and $\Omega_{RF}$. However, with appropriate modifications to the trap parameters, scaling to larger and smaller ring radii is possible.

In terms of loading both kinds of dressed inductive ring trap (TAAP and quadrupole), the relatively shallow depth due to its small dimensions necessitates pre-evaporation of the atomic cloud, after optical molasses, to tens of microKelvin in a magnetic or spin-polarized optical dipole trap. By applying bias fields to a quadrupole/TOP/Ioffe magnetic trap, or focal point scanning of a single-beam dipole trap with an AOM/moving lens, the pre-evaporated cloud could be moved to a localized region of the dressed ring trap location prior to switching it on.

We note that a third dressed inductive geometry is possible, using a dc current-carrying wire along the ring axis (cf [31]). This also has very favourable trapping parameters, however we have omitted detailed description due to the
difficulty of experimentally realizing such a geometry on a chip.

In conclusion we have extended inductive ring traps from the time-averaged case to the dressed domain, suitable for investigations of sub-mm ring traps. Two geometries were considered—an inductive TAAP ring trap, and a splittable inductive quadrupole ring trap—both of which are amenable to implementation on atom chips. Inductive TAAPs have the advantage that they are ideally made with spatially homogeneous inducing and quantization fields, and as such many rings can be implemented in parallel on the same chip for application in e.g. gradiometry. Inductive quadrupole traps have quasistatic fields which must be centred on the ring, however multiple quadrupoles for each ring can be implemented on the same chip using U-wires, making implementation of this geometry feasible as well. The inductive quadrupole trap also enables tunable splitting of the ring into a double ring, permitting measurement of e.g. gravitational acceleration, in addition to the usual use of ring traps as gyroscopes. As both types of inductive ring trap have no reliance on optical dipole forces, their potential should be extremely smooth, even with large (mm) scale diameters, ideal for both interferometry and studies of superfluidity.

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References

[1] Zawadzki M E, Griffin P F, Riis E and Arnold A S 2010 Phys. Rev. A 81 043608
[2] Balkyard P L, Jones M P A and Gardiner S A 2010 Phys. Rev. A 80 013629
[3] Beattie S, Moulder S, Fletcher R J and Hadzibabic Z 2013 Phys. Rev. Lett. 110 025302
[4] Griffin P F, Riis E and Arnold A S 2008 Phys. Rev. A 77 051402
[5] Pritchard J D, Dinkelaker A N, Arnold A S, Griffin P F and Riis E 2012 New J. Phys. 14 103047
[6] Griffin P F 2013 private communication
[7] Trebbia J-B, Garrido Alazar C L, Cornelussen R, Westbrook C K and Bouchoule I 2007 Phys. Rev. Lett. 98 263201
[8] Huet L, Ammar M, Morvan E, Sarazin N, Pocholle J-P, Reichel J, Guerlin C and Schwartz S 2012 Appl. Phys. Lett. 100 124114
[9] Nishi C C, Vangelen M, Cotter J P, Griffin P F, Hinds E A, Ironside C N, See P, Sinclair A G, Riis E and Arnold A S 2013 Nature Nanotechnol. 8 321
[10] Jian B and van Wijnjaarden W A 2013 J. Opt. Soc. Am. B 30 238
[11] Riedel M F, Böhi P, Li Y, Hänsch T W, Sinatra A and Treutlein P 2010 Nature 464 1170
[42] Morizot O, Colombe Y, Lorent V, Perrin H and Garraway B M 2006 Phys. Rev. A 74 023617
[43] Lesanovsky I, Schumm T, Hofferberth S, Andersson L M, Krüger P and Schmiedmayer J 2006 Phys. Rev. A 73 033619
[44] Fernholz T, Gerritsma R, Krüger P and Spreeuw R J C 2007 Phys. Rev. A 75 063406
[45] Lesanovsky I and von Klitzing W 2007 Phys. Rev. Lett. 99 083001
[46] Heathcote W H, Nugent E, Sheard B T and Foot C J 2008 New J. Phys. 10 043012
[47] Sherlock B E, Gildemeister M, Owen E, Nugent E and Foot C J 2011 Phys. Rev. A 83 043408
[48] Hofferberth S, Fischer B, Schumm T, Schmiedmayer J and Lesanovsky I 2007 Phys. Rev. A 76 013401
[49] Bergeman T, Erez G and Metcalf H J 1987 Phys. Rev. A 35 1535
[50] Jones R C 1941 J. Opt. Soc. Am. 31 488