Natural Convection MHD Effect on Heat Transfer in Vertical Cylinder

Madan Lal, Swati Agarwal

Abstract: In this paper, natural convection MHD effect on heat transfer in vertical cylinder is analyzed. Firstly heat transfer equation and convert it in dimensionless form by introducing dimensionless variables has considered. By use of finite Hankel and inverse Hankel transforms in dimensionless heat transfer equation, the values of temperature and Nusselt number is obtained. Finally, graphical representation is used to study the effect of Prandtl number Pr on temperature and Nusselt number Nu. It is observed that the fluid at the center region of the cylinder is not heated for value of Prandtl number Pr greater than 1.7 for small values of t.

Keywords: Free convection, Heat transfer, Liquid metal, Mixed convection, MHD

I. INTRODUCTION

The phenomenon of heat transfer due to natural convection and mixed convection has been analyzed by many researchers. R.U.Haq, S. Nadeem, Z.H.Khan and N.F.M.Noor [6] examined the thermal conductivity within the base fluids in presence of Carbon Nanotubes. Tao Fan, Hang Xu and I. Pop [10] computed the influence of parameters such as Brownian motion, the thermophoresis and Lewis number on the temperature and nanoparticle concentration distributions. They analyzed that the nanoparticles enhance the heat transfer attributes of the flow in the horizontal channel. E.V.Murphree [2] proposed a first phenomenology between fluid friction and heat transfer. In an alternate phenomenology, H.T.Lin [4] obtained the analogies of Reynolds, Prandtl, Karman and Coiburn for Prandtl number with uniform wall temperature. S. Cuevas and B.F.Picoligloiu [7], [8] emphasized to specify turbulent velocity profiles to solve the heat transfer equation. Dawid Taler [1] proposed turbulent heat transfer in the tubes and calculated Nusselt numbers to find a relevant heat transfer correlation. He also compared this heat transfer correlation with experimental data. Ilyas Khan, Nehad Ali Shah, Asifa Tassaddiq, Norzieha Mustapha and Seripah Awang Kechil [5] computed the heat transfer from the surface of the cylinder to the fluid. Hulin Huang and Ying Fang [3] investigated heat transfer by applying the MHD “k- ε” model and the induced magnetic field equation. Srinathuni Lavanya and D. Chenna Kesavaiah [9] focused on effects of velocity for dusty gas and dust particle, magnetic field, temperature and concentration. Ya. Listratov, D. Ognerubov, E. Sviridov [11] determined the threshold value of the Grashof number at which buoyancy begins to affect heat transfer.

In present paper natural convection MHD effect on heat transfer in vertical cylinder has been obtained by using Hankel transformation.

II. MATHEMATICAL FORMULATION OF A PROBLEM

Consider a vertical cylinder of radius r_0 and axis of the cylinder is in upward direction. At time t, cylinder starts to oscillate along its axis then cylinder temperature raised to T_c as shown in Fig. 1.

Assuming that the temperature T which is a function of r and t. First consider the PDE for heat transfer turbulent flow is governed by

\[
\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r,t)}{\partial r} = \frac{1}{\alpha} \frac{\partial T(r,t)}{\partial t} ; 0 < r < r_0 , t > 0
\]

With initial condition and mixed boundary condition

\[
T(r,0) = T_c ; 0 \leq r \leq r_0 , t > 0
\]

\[
\frac{\partial T(r_0,t)}{\partial r} = -hT
\]

Where h is a positive constant.

Let us assume the following dimensionless variables as

\[
\Theta = \frac{T + h}{T_c + h}, \quad r' = \frac{r}{r_0}, \quad t' = \frac{t}{\delta r_0^2}
\]

Dropping the star notation, the equations (1) to (3) reduced to

\[
Pr \frac{\partial \Theta(r,t)}{\partial t} = \frac{\partial^2 \Theta(r,t)}{\partial r^2} + \frac{1}{r} \frac{\partial \Theta(r,t)}{\partial r}
\]

\[
\Theta (r,0) = 1 ; 0 \leq r \leq 1, t > 0
\]

\[
\frac{\partial \Theta(1,t)}{\partial r} = -h \Theta(1,t) + g
\]

Revised Manuscript Received on January 22, 2020.

Madan Lal, Department of Applied Mathematics, M.J.P. Rohilkhand University, Bareilly, India.madan.mupri@gmail.com

Swati Agarwal, Department of Applied Mathematics, M.J.P. Rohilkhand University,Bareilly.India.swattagar85@gmail.com

Retrieved Number: D1318029420/202006BEIESP
DOI: 10.35940/ijitee.D1318.029420

International Journal of Innovative Technology and Exploring Engineering (IJITEE) ISSN: 2278-3075, Volume-9 Issue-4, February 2020

Published By: Blue Eyes Intelligence Engineering & Sciences Publication
Natural Convection MHD Effect on Heat Transfer in Vertical Cylinder

Where, \( g = \frac{h^2}{\tau + b} \), \( \frac{\theta}{a} \),

Using the finite Hankel transform of order zero, the equation (4) reduced to

\[
\begin{align*}
\frac{d^2 \Theta_H(s_n, t)}{dt^2} - \frac{s_n^2}{Pr} \Theta_H(s_n, t) &= 0 \\
\frac{d \Theta_H}{dx} + \frac{s_n^2 t}{Pr} \Theta_H &= 0. 
\end{align*}
\]

(7)

Where, \( \Theta_H(s_n, t) = \int_0^1 r \Theta(r, t) J_0(s_n r) dr \), is finite Hankel transform of \( \Theta(r, t) \) and \( s_n, n = 0, 1, 2, 3 \ldots \) are the roots of equation \( J_0(x) = 0 \) where \( J_0(x) \) represents the Bessel functions of first kind of order zero.

So that \( \Theta_H(s_n, 0) = \int_0^1 r J_0(s_n r) dr = \frac{l_1(s_n)}{s_n} \) \( \tag{8} \)

Using initial condition (8), the solution of equation (7) is given by:

\[
\Theta_H(s_n, t) = \frac{l_1(s_n)}{s_n} e^{-\frac{s_n^2 t}{Pr}} \quad \tag{9}
\]

Taking inverse Hankel transform, expression for temperature is given by

\[
\Theta(r, t) = \frac{1}{2 \pi} \sum_{n=1}^{\infty} \left( s_n^2 J_0(s_n r) \right) e^{\frac{-s_n^2 t}{Pr}}. 
\]

(10)

The Nusselt number is defined as:

\[
Nu = \frac{\frac{\partial \Theta(r, t)}{\partial r}}{\Theta(r, t)} = \frac{1}{2 \pi} \sum_{n=1}^{\infty} e^{-\frac{s_n^2 t}{Pr}}. \quad \tag{11}
\]

III. RESULTS AND DISCUSSIONS

Representative results for temperature as a function of radial coordinate for distinct values of Prandtl number at time \( t \) are obtained. Additionally we have computed the Nusselt number \( Nu \). To observe the effect of the Prandtl number \( Pr \) on fluid temperature \( \Theta(r, t) \) and Nusselt number \( Nu \), results are shown graphically. In Table I, Table II and Table III, as the value of \( r \) increases the temperature decreases. Figures 2(a), 2(b) and 2(c) show the temperature, for distinct values of the Prandtl number \( Pr \) at time \( t \). Temperature was obtained for three different values of time 0.1, 0.2 and 0.3 corresponding to the distinct values of Prandtl number. In Table IV, it is shown that Nusselt number decreases with increase in value of \( t \) or decrease in value of \( Pr \) and vice-versa. The equation (10) shows that the temperature tends to zero for small values of the Prandtl number or large values of time \( t \). Finally, it is observed that for small values of the Prandtl number, the heat transfer from the cylinder surface to the fluid is meaningful because for the values of the Prandtl number \( Pr \) larger than 1.7, the fluid at center region of the cylinder is not heated.

Fig. 2(a). Temperature for distinct values of the Prandtl number at time \( t = 0.1 \).

Fig. 2(b). Temperature for distinct values of the Prandtl number at time \( t = 0.2 \).

Fig. 2(c). Temperature for distinct values of the Prandtl number at time \( t = 0.3 \).

Fig. 3. The Nusselt number for distinct values of the Prandtl number at time \( t \).
Table I: Temperature for distinct values of Prandtl number at time t=0.1

| Pr  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|-----|-----|-----|-----|-----|-----|-----|
| Pr  | 0.4 | 0.6 | 0.8 | 1.1 | 1.5 | 1.7 |
| R   | 0.00105 | 0.00156 | 0.02113 | 0.02576 | 0.02964 | 0.03101 | 0.03161 | 0.03124 | 0.03264 | 0.03308 |

Table II: Temperature for distinct values of Prandtl number at time t=0.2

| Pr  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|-----|-----|-----|-----|-----|-----|-----|
| Pr  | 0.4 | 0.6 | 0.8 | 1.1 | 1.5 | 1.7 |
| R   | 0.000241 | 0.000646 | 0.001027 | 0.001526 | 0.002016 | 0.002207 | 0.002292 | 0.002371 | 0.002444 | 0.002512 |

Table III: Temperature for distinct values of Prandtl number at time t=0.3

| Pr  | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
|-----|-----|-----|-----|-----|-----|-----|
| Pr  | 0.4 | 0.6 | 0.8 | 1.1 | 1.5 | 1.7 |
| R   | 0.000057 | 0.000242 | 0.000498 | 0.0009 | 0.01371 | 0.0157 | 0.01662 | 0.01749 | 0.0183 | 0.01908 |

Table IV: The Nusselt number Nu for distinct values of Prandtl number Pr at time t

| T   | 0.02 | 0.05 | 0.07 | 0.1 | 0.12 | 0.14 |
|-----|-----|-----|-----|-----|-----|-----|
| Nu  | 1.49778 | 1.64932 | 1.75078 | 1.80384 | 1.85158 | 1.86444 | 1.87952 | 1.88188 | 1.88762 | 1.89282 |

IV. CONCLUSION

Natural convection MHD effect on heat transfer in vertical cylinder has investigated. The fluid temperature and Nusselt number has obtained for distinct values of the Prandtl number Pr by using finite Hankel and inverse Hankel represents that, when the value of time t increases, the temperature decreases i.e., for large values of time t, heat transfer is most influential. Fig.3 shows the effect of the Prandtl number Pr on the Nusselt number Nu. Equation (11) represents that when the values of the Prandtl number Pr increases, the Nusselt number Nu increases, as shown in Fig.3.

REFERENCES

1. Dawid Taler, “Heat transfer in turbulent tube flow of liquid metals,” Proc. Engng., Vol. 157, 2016, pp. 148-157.
2. E.V. Murphree, “Relation between heat transfer and fluid friction,” Ind. & Engng. Chem., Vol. 24, 1932, pp. 726-736.
3. Hulin Huang and Ying Fang, “MHD effect on heat transfer in liquid metal free surface flow around a cylinder,” Engng. Appl. of Com. Fluid Mech., Vol. 1, 2007, pp. 88-95.
4. H.T. Lin, “The analogy between fluid friction and heat transfer of laminar forced convection on a flat plate,” Warmer- und Stoffubertragung Vol.29, 1994, pp. 181-184.
5. Ilyas Khan,Nehad Ali Khan, Asifa Tassaddiq, Norzieha Mustapha, Seripah Awang Kechil, “Natural convection heat transfer in an oscillating vertical cylinder,” PLOS ONE, Vol. 13, 2018, pp. 1-14.
6. R.U. Haq, S. Nadeem, Z.H. Khan , “Convective heat transfer in MHD slip flow over a stretching surface in the presence of carbon nanotubes,” Phy. B, Vol. 457, 2015, pp. 40-47.
7. S. Cuevas and B.F. Picologlou, “Liquid-metal MHD flow in rectangular ducts with thin conducting or insulating walls: laminar and turbulent solutions,” Int. J. Engng. Sci., Vol.35, No. 5,1997, pp. 485-503.
8. S. Cuevas and B.F. Picologlou, “Heat transfer in laminar and turbulent liquid metal MHD flows in square ducts with thin conducting or insulating walls,” Int. J. Engng. Sci., Vol.35, 1997, pp. 505-514.
9. S. Lavanya and D. Chenna Kesavaiah, “Heat transfer to MHD free convection flow of a viscoelastic dusty gas through a porous medium with chemical reaction,” Int. J. Pure and App. Researches, Vol.3, No.1, 2017, pp. 43-56.
10. Tao Fan, Hang Xu and I. Pop, “Mixed convection heat transfer in horizontal channel filled with nanofluids,” Int. J. spring. Plus, Vol. 34, 2013, pp. 339-350.
11. Ya. Listratov, D. Ognerubov, E. Sviridov, O. Zikanov, V. Sviridov, “Direct numerical simulations of heat transfer and convection in MHD liquid metal flow in a pipe,” MHDs, Vol. 49, 2013, pp. 87-99.

AUTHORS PROFILE

Dr. Madan Lal has completed M.Sc. degree in Mathematics in the year 1997 from M.J.P.Rohilkhand University, Bareilly (U.P.) India. and cleared NET (CSIR JRF) examination in december 2001. Ph.D. degree was awarded to him in 2003 (Mathematics) on the topic “Some Problems on Hydrodynamics and MHD flows” and his more than 15 research papers on “Magnetohydrodynamics” has been published in reputed National/International journals . He has participated more than 10 National /International conferences /workshops/seminars etc. Dr. Lal also chaired the session in various National and International conferences. He is also the life member of Indian Science Congress Association, Indian Mathematical Society etc. Currently he is working as Head of Applied Mathematics Department, M.J.P.Rohilkhand University Bareilly (U.P.) India.

Swati Agarwal has completed M.Sc. degree in Mathematics in the year 2008 from M.J.P.Rohilkhand University, Bareilly (U.P.) India. and cleared NET examination in 2010. Author has joined for Ph.D. degree in Mathematics on the topic “Study on Magnetohydrodynamic Turbulence.” in the year 2014 and three research papers has been published in National/International journals . She has also participated National /International conferences /workshops/seminars. She is the life member of Indian Science Congress Association and Indian Mathematical Society.