R-parity violating supersymmetry, $B_s$ mixing, and $D_s \to \ell \nu$

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Recently, it was pointed out that the mixing phase in the $B_s - \Bar{B_s}$ system is large, contrary to the expectations in the Standard Model as well as in minimal flavour violation models. The leptonic decay widths of the $D_s$ meson are also found to be larger than expected. We show how a minimal set of four R-parity violating X-type couplings can explain both these anomalies. We also point out other phenomenological implications of such new physics.

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I. EXPERIMENTAL DATA

A. $B_s - \Bar{B_s}$ mixing

Recently, the UTfit collaboration has claimed that the phase coming from $B_s - \Bar{B_s}$ box diagram, as found on averaging various data, is more than 3$\sigma$ away from the SM expectation [1]. In the Standard Model (SM), $\beta_s$ is defined as

$$\beta_s = \text{arg} \left( -V_{ts} V_{tb}^* / V_{cs} V_{cb}^* \right),$$

which is 0.018 $\pm$ 0.001. If there were no new physics (NP), the angle $\phi_s$ is defined simply as $\phi_s \equiv -\beta_s$. If NP is present, $\phi_s$, the phase coming from the $B_s - \Bar{B_s}$ box, has both SM and NP contributions. UTfit has got two solutions for $\phi_s$, and hence for the NP amplitude:

$$\phi_{s}^{(0)} = -19.9 \pm 5.6 \ [\text{-30.45, -9.29}],$$

$$\phi_{s}^{NP(0)} = -51 \pm 11 \ [\text{-69, -27}],$$

$$\phi_{s}^{NP} = -79 \pm 3 \ [\text{-84, -71}],$$

$$A^{NP} / A^{SM} = 0.73 \pm 0.35 \ [0.24, 1.38],$$

$$= 1.87 \pm 0.06 \ [1.50, 2.47].$$

(2)

In each line, the first number stands for the 68% confidence limit (CL) and the second number stands for the 95% allowed range. The strong phase ambiguity affects the sign of $\cos \phi_s$ and hence $\Re (A^{NP} / A^{SM})$, which can either be $-0.13 \pm 0.31$ or $-1.82 \pm 0.28$ (both at 68% CL), while $\Im (A^{NP} / A^{SM}) = -0.74 \pm 0.26$ in any case. These two solutions are shown separately in eq. (2). Note that while the range of NP contribution for the second solution is more precise, this is more unlikely at the same time as NP amplitude is almost twice that of the SM one. Apart from SM, this result disfavours the minimal flavour violation models too.

However, the situation in the $B_d$ system is markedly different. It has been established that the dominant CP-violation mechanism there is the CKM one, and any NP effect must be subdominant. One can, just to be conservative, discuss the case where there is no effect in the $B_d$ system. We follow such an approach; the NP must be flavour-specific in nature.

B. $D_s \to \ell \nu$

The leptonic decay $D_s \to \ell \nu$, where $\ell = \mu, \tau$, has a branching fraction

$$B = \frac{1}{8 \pi} m_D \tau_D |f_{D_s} |^2 \left| G_F V_{c\ell}^* m_\ell \right| \left( 1 - \frac{m_\ell^2}{m_D^2} \right),$$

(3)

where $\tau_D$ is the lifetime of $D_s$ and the decay constant $f_{D_s}$ is defined through

$$\langle 0 | \bar{s} \gamma_\mu \gamma_5 c | D_s \rangle = i f_{D_s} p_\mu,$$

(4)

where $p_\mu$ is the 4-momentum of $D_s$. While lattice results predict [4]

$$f_{D_s} = 241 \pm 3 \text{ MeV},$$

(5)

the experimental numbers are larger [2, 8]:

$$f_{D_s}(D_s \to \mu \nu) = 273 \pm 11 \text{ MeV},$$

$$f_{D_s}(D_s \to \tau \nu) = 285 \pm 15 \text{ MeV},$$

$$f_{D_s}(D_s \to \ell \nu) = 277 \pm 9 \text{ MeV} \ (\text{average}).$$

(6)

This can be due to an improper estimate of lattice uncertainties. On the other hand, one can also say that $f_{D_s}$ is indeed that of eq. [5] but the discrepancy is due to some NP contribution in the leptonic channels that enhance the branching fractions. The enhancement is about 13 $\pm$ 6% in the $\mu$ channel, 18 $\pm$ 8% in the $\tau$ channel, and 15 $\pm$ 5% on average.

Dobrescu and Kronfeld [8] have attempted an explanation of the $D_s$ lepton anomaly with either charged Higgs bosons or leptoquarks. While they have not talked about the UTfit result, it can hopefully be shown that suitable leptoquark couplings with complex phases can explain both the discrepancies. Two facts, however, are obvious: first, the NP couplings should be large so that they can generate such large effects, and second, as we have just mentioned, these couplings must be flavour-dependent.

In this work, we will try to show that a simultaneous explanation can be found with a minimal set of four R-parity violating supersymmetric couplings.
II. R-PARITY VIOLATION

The discrete symmetry, R-parity, is defined as $(-1)^{3B+L+2S}$ where $B$, $L$ and $S$ are the baryon number, lepton number, and spin of the particle respectively. This is 1 for all particles and $-1$ for all sparticles. While one can demand the conservation of R-parity ad hoc, it is possible to write R-parity violating (RPV) terms in the superpotential. To forbid proton decay, one has to consider either baryon-number or lepton-number violating RPV couplings. For our case, we will consider lepton-number violating $\lambda'$-type couplings, since the interaction involves both quarks and leptons. The Lagrangian, in terms of component fields, is given by

$$\mathcal{L}_{LQD} = \lambda'_{ijk} \left[ \tilde{\nu}_{iL} \tilde{d}_{kR} j_{jL} + \tilde{d}_{jL} \tilde{d}_{kR} \nu_{iL} + (\tilde{d}_{kR})^* \tilde{\nu}_{iR} j_{jL} - \tilde{\nu}_{iL} \tilde{d}_{kR} e_{iL} - (\tilde{d}_{kR})^* \tilde{e}_{iR} u_{jL} \right] + \text{h.c.} \quad (7)$$

Let us mention here that the minimal set is actually three and not four; one must have $\lambda'_{223}$ and $\lambda'_{323}$ to explain $D_s \to \mu \nu$ and $D_s \to \tau \nu$ respectively, and either $\lambda'_{212}$ or $\lambda'_{312}$ which, in conjunction with the $\lambda'$ coupling with the same leptonic index, would contribute to the $B_s - \overline{B_s}$ mixing. However, to keep the couplings symmetric, we will consider both $\lambda'_{212}$ and $\lambda'_{312}$ to be present.

III. EXPLANATION OF $D_s$ BRANCHING RATIO

Let us first consider the $\lambda'_{232}$ couplings. The leptonic index $i$ can be 2 or 3. The relevant four-fermi interaction can be obtained by contracting the $\tilde{b}_R$ field in the third and the sixth terms of eq. (7). Thus, both $\mu \nu_\mu$ and $\mu \nu_\tau$ can occur as final states. Only the former will interfere with the SM amplitude; the second one should be added incoherently. The product carries a minus sign. The $(S - P) \otimes (S + P)$ gives $-\frac{1}{4} (V - A) \otimes (V + A)$ under Fierz reordering. The two charge-conjugated spinors should be replaced by ordinary spinors; that involves another flip of position and the third minus sign (also, $V + A$ changes to $V - A$). Finally, the internal propagator is scalar and not a vector like SM; that brings in the fourth minus sign. Altogether, the SM and the NP come with same sign and the interference is positive, so the branching fraction should increase. However, note that we have to include both neutrino flavours. The product $|\lambda'_{223}|^2$ is always positive; $\lambda'_{232} \lambda'_{323}$ can come with a complex phase, but since this is incoherently added, the phase cancels out in the amplitude squared. The same applies for a $\tau \nu$ final state.

Note that $\lambda'_{212}$ type couplings are highly suppressed from neutrino mass ($\sim 10^{-3}$) [3], and $\lambda'_{21}$ does not resolve the $B_s - \overline{B_s}$ anomaly.

Since the neutrino flavour is not detected, we may replace $|G_F V_{cs}^*|^2$, for $D_s \to \mu \nu$, by

$$\left| G_F V_{cs}^* + \frac{1}{\sqrt{2} m_{\tilde{b}_R}^2} C_{A22}^u \right|^2 + \left| \frac{1}{\sqrt{2} m_{\tilde{b}_R}^2} C_{A23}^u \right|^2, \quad (8)$$
of |sneutrinos even at the GUT scale forces an inequality for large tan $\beta$. We would like to point out that the propagator is a right-handed bottom squark, which may be hardly allowed. When $m_{\tilde{b}_R}$ gets enhanced for three values of $\lambda^i_{23}$, the bound becomes $0.13$. One can easily relax this bound for other choices of the GUT scale input parameters; thus, even with a larger value of $m_{\tilde{b}_R}$ one can reach the 68% CL lower limit of $D_s \to \tau \nu$. This is shown in Fig. 1. It is nevertheless clear that one requires rather large values of $\lambda^i_{23}$ to explain the present data; more precise lattice results are, therefore, eagerly awaited.

### IV. EXPLANATION OF $B_s$ MIXING PHASE

The product $\lambda^i_{23} \lambda_{i12}^{*}$ contributes in the $B_s - \bar{B_s}$ box, with two $i$-type sleptons, a charm, and an up quark flowing in the loop; it can also be leptons and squarks. Let us assume all sleptons degenerate at 100 GeV and all squarks degenerate at 300 GeV (the box amplitude is controlled by the slepton diagram, so the exact value of the squark mass is irrelevant). For simplicity (and without losing any generality), we will assume $\lambda^i_{12} \lambda^i_{23} = \lambda^i_{312} \lambda^i_{323}$, in both magnitude and the weak phase. One can consider the phase to be associated with the $\lambda^i_{12}$ coupling. The relevant formulae can be obtained from [11].

We find that (i) $A_{NP} / A_{SM}$ can at most go up to 38%, above that, the constraint $\Delta M_s = 17.77 \pm 0.12$ ps$^{-1}$ [12] is violated; (ii) the phase coming from the box can lie in the 68% allowed range of UTfit, namely, $[-14.3^\circ, -25.5^\circ]$; (iii) there are two allowed regions where this can happen, viz., $|\lambda^i_{212} \lambda^i_{223}| \in [0.002, 0.004], [0.014, 0.019]$.

Note that we have assumed both $\lambda^i_{212}$ and $\lambda^i_{312}$ to be nonzero (and equal). If only one of them is nonzero, the allowed range would have been enhanced by a factor of four (the two RPV amplitudes add coherently).

One might note that the charged Higgs $H^+$, present in any supersymmetric model, can in principle affect the leptonic branching ratios of $D_s$. However, we would consider the parameter space where such effects are minimal (since the effects go in the opposite direction, it would result in a more serious tension between theory and experiment, and hence one would need larger values of the R-parity violating couplings). This can happen, for example, in the low tan $\beta$ region.

### V. MORE FEATURES

Contracting the slepton index, we get the decay $b \to \text{c}s\bar{u}s$. However, these couplings do not generate $b \to \text{u}c\bar{s}s$. So only $B_s \to D^-_s K^+$ and not $B_s \to D^+_s K^-$ will be affected. Thus, the method for the determination of the angle $\gamma$ of the Unitarity Triangle (UT) based on the simultaneous study of $B_s(\bar{B}_s) \to D^\pm_s K^\mp$ will be affected. The same is true for the $B \to DK$ modes. On the other hand, $\gamma$ determined from channels that are not affected by these RPV couplings will yield the true phase of $V_{ub}$. A signature for this hypothesis would then be to compare the measurements of $\gamma$ from these channels.

The above discussion shows that the $B_s - \bar{B}_s$ mixing box will have an absorptive part. As has been discussed in [14], such new absorptive parts bypass the Grossman theorem [15] of reduction of $\Delta \Gamma$, the width difference of two $B_s$ mass eigenstates, in the presence of new physics. Unfortunately, we find that the effect is too small to be detected over the SM uncertainty in $\Delta \Gamma$, [16], so the result is consistent with the experimental number [17].

If we contract the sneutrino instead of the charged slepton, the decay process is $b \to s d s$. Such $\Delta B = 1, \Delta S = 2$ decays are extremely suppressed in the SM. However, this can now occur with a branching ratio that should be in the range of LHC-B. One can have, for example, the decay $B^+ \to K^{*0} K^+$ and then $K^{*0} \to K^+ \pi^-$. 

![Graph showing the effect of the R-parity violating couplings $\lambda^i_{223}$ and $\lambda^i_{323}$ on $D_s \to \mu(\tau) \nu$. The upper (lower) horizontal line is the 1σ lower (upper) limit for the percentage enhancement of $D_s \to \tau \nu$. The vertical line shows the SPS1a limit of 0.39 (see text). The curves are drawn for different values of $m_{\tilde{b}_R}$, as shown in the plot. We have assumed $\lambda^i_{223} = \lambda^i_{323}$, and both real.](image)
A. Collider signals

It has been noted in [7] that large values of $\lambda'_{123}$ at the GUT scale can generate, through RG evolution, neutrino masses compatible with experiment. The neutralino, in these cases, will decay to $\mu b$ or $\nu_\mu sb$ channel (for $i = 3$, replace $\mu$ by $\tau$). The gaugino signal would be one $b$ jet (plus other jets) and an isolated hard lepton. Thus, an increase in $2j + 2\mu$ (or $2j + 2\tau$) channel would be an encouraging signal for this hypothesis.

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