A Not So Little Higgs?

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Abstract

Most recent models assuming the Higgs Boson is a pseudo-Nambu-Goldstone Boson (pNGb) are motivated by the indication from Standard Model fits that its mass is $\leq 200\text{GeV}$. Starting from a modified SM of Forshaw et. al. with a triplet boson added and a heavier Higgs Boson, we consider a pNGb model. This differs in several ways from most little Higgs models: apart from using only one loop, the cutoff scale is reduced to 5 TeV, and consequently a linear sigma model is used to alleviate FCNC effects; no new vector bosons are required, but vector-like isosinglet fermions are needed, but play no part in determining the mass of the Higgs boson. The phenomenology of the isosinglet pNGb that arises from the $SU(3) \times SU(3) \rightarrow SU(3)$ model we use is briefly discussed. Some potential theoretical and phenomenological problems are mentioned briefly.

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1 Introduction

The indication from Standard Model (SM) fits to precision data that the mass of the Higgs Boson is \( \leq 200 \text{GeV} \) has motivated many recent, and often ingenious, models, the Little Higgs Models (LHMs). For reviews see [1]. Typically these models assume a global symmetry group at \( \simeq 10 \text{ TeV} \) which breaks spontaneously to give Nambu-Goldstone bosons amongst which are the Higgs Bosons. These acquire mass from radiative corrections, but the models are constructed so that the one loop quadratic divergences cancel, thereby ensuring a light enough Higgs Boson.

Experimentally, however, there is only a lower bound on the mass of the Higgs Boson. Soon after the precision data appeared several authors [2] considered how the limit on the mass could be raised by modest alterations of the SM. Amongst these was a model due to Forshaw and collaborators [3]. They showed that by adding a real triplet scalar boson with a small vacuum expectation value adequate fits to precision data with a Higgs Boson mass of 500 GeV (and similar mass for the triplet) could be obtained.

This suggests the possibility of a model where the Higgs Boson is a pseudo-Nambu Goldstone boson (pNGb), but the global group is taken at 5 TeV, and, since \( 0.5 \text{ TeV} \simeq \sqrt{\alpha} \times 5 \text{ TeV} \) there may be no need for extra gauge bosons, or fermions to ensure the cancellation of divergences. It transpires that it is possible to eliminate the need for extra gauge bosons, but extra fermions seem necessary, but are not constrained by contributing to the mass of the Higgs boson as in many LHMs.

The model is presented in the next section with a particular emphasis on the need to use a linear, as opposed to the non-linear sigma model generally used in LHMs. The next section gives the Coleman-Weinberg [4] potential of the model, The Coleman-Weinberg potential for the isoscaler partner \( \eta \) of the Higgs is given in the next section, and the phenomenology of the \( \eta \) is discussed briefly. In the final section some open problems which remain to be resolved are discussed, and a conclusion given.

2 The Model

Forshaw et al add a real triplet scalar field to the SM. One must then look for a group whose breaking will produce a triplet \( \phi^i \), a complex doublet \( H^a \) and possibly some singlets as commonly arise in addition in LHMs. Without considering product groups no candidate has appeared, but the group \( SU(3) \times SU(3) \) which breaks to \( SU(3) \) seems well suited to this purpose, and gives just one singlet \( \eta \).
Unlike most LHMs a linear rather than non-linear sigma model is used. There are three reasons for this. First a comparison with Forshaw et al.'s field theoretical analysis would be difficult for the non-linear case as higher powers of fields suppressed only by powers of the breaking scale $f \approx 0.5\, \text{TeV}$ would appear. Secondly recall the old paper of Georgi and Kaplan[5] who used this same group with a non-linear sigma model, but felt dissatisfied as precision tests required $f$ too large, a view strengthened now by $f$ being $\geq 3\, \text{TeV}$[6]. Georgi and Kaplan did not consider a small triplet vev so that one might think that allowing this could improve the situation, but by using their exponential parametrisation one finds that the triplet vev and the 'effective triplet vev' $O(v^2/f^2)$ where $v$ is vev of $H^0$ are out of phase by $\pi/2$, so that the problem is made worse.

A third reason comes from the constraints of FCNC. Chivukula et al.[7] have argued that these constraints require a cutoff scale well above the 10 TeV of LHMs. Clearly if one lowers the scale to 5 TeV this problem becomes more serious. One remedy suggested[8] is to have the LH as a linear sigma model which arises as a little Higgs model from a scale an order of magnitude higher. Such an idea has recently been implemented for the $SU(3) \times U(1)$ LHM[9]. This again suggests the use of a linear sigma model, though it has to be stressed that no UV completion has yet been obtained for the $SU(3) \times SU(3)$ model.

Extra fermions, singlets under $SU(2)$, will now appear to fill triplets along with $t$ and $b$ quarks, as well as along with lighter quark multiplets. The extra singlets can give rise to FCNC problems by mixing with quarks of the first two generations. This has recently been analysed by Deshpande et al.[10] who find the strong constraint $|U_{ds}| \leq 1.2 \times 10^{-5}$ from rare K decays in a model with an extra charge -1/3 quark, where $U_{ds}$ denotes the mixing between $d$ and $s$ induced by the extra quarks. Provided the singlet quarks are heavy, and the decreasing mixing between light and heavy quarks seen in the SM can be extended to new quarks, this constraint may (just) be satisfied.

3 The Coleman-Weinberg Potential for $\phi$ and $H$.

The scalar potential used by Forshaw et al is given, in our notation, by

$$\mu_1^2|H|^2 + \mu_2^2/2|\phi|^2 + \lambda_1|H|^4 + \lambda_2/4|\phi|^4 + \lambda_3/2|H|^2||\phi|^2 + L_3$$

(1)

where

$$L_3 = \lambda_4\bar{\phi}H^\dagger\sigma^3H$$

(2)

One can ask how much of this potential can be produced by a Coleman-Weinberg mechanism. The Coleman-Weinberg potential gives rise to quadratically divergent
coefficients of $\phi^2$ and $H^2$, as well as logarithmic divergences for $\phi H^2$ and terms quartic in $\phi$ and $H$. The $\phi H^2$ term is novel and such a term will not arise in the Coleman Weinberg effective potential generated using only SM gauge bosons and fermion loops. This is because the gauge bosons couple to bilinears in $\phi$, while doublet fermions and right handed singlet fermions do not couple to the isovector $\phi$. As is shown below the terms in $\lambda_1, \lambda_2$, and $\lambda_3$ are also inadequately described by the Coleman-Weinberg potential so that only the terms in $\mu_1$ and $\mu_2$ can be treated, that is the terms which are directly related to the Goldstone origin of $H$ and $\phi$.

The quadratically divergent $\phi^2$ term is given by

$$V(\phi^2) = \frac{3g^2}{32\pi^2}\Lambda^2$$

from gauge bosons. For $\Lambda = 5 TeV$ the (positive) mass squared $= 0.2 TeV^2$ for $\phi$. For $H$ the dominant (negative) mass squared is expected to come from the top quark loop and is of magnitude $\simeq 2 TeV^2$. The positive contribution of gauge bosons is small $\simeq 0.2 TeV^2$, but unlike the case of light $H$ a large positive contribution comes from $H$ loop itself. For $m_H = 0.5 TeV$ this is given by $\frac{\lambda_1\Lambda^2}{8\pi^2}$ where $\lambda_1 \simeq 2$ for $m_H = 0.5 TeV$. This gives a mass squared of $\simeq 0.63 TeV^2$. Furthermore $\lambda_3 \simeq \lambda_1$ typically in the solutions of Forshaw et al with heavy scalars so that this will give a further positive contribution of similar magnitude. Also, as can be seen from the next section, a similar positive contribution can be expected from the $\eta$ loop, though this is more uncertain. Taken together with the contribution of the gauge bosons this could have the disastrous effect of making $\mu_2^2$ positive. This problem can be resolved either by noting that each term is only given up to a constant of $O(1)$ from UV uncertainties or by having $m_H$ somewhat less than 500 GeV when the contributions of the scalar loops are reduced by $(m_H/500 GeV)^2$ so that a negative value of the correct magnitude may be obtained for $\mu_2^2$. The second approach is favoured by the existence of many more solutions for $m_H$ somewhat less than 500 GeV than for $m_H = 500 GeV$, but, because of the uncertainty associated with the first approach, one can hardly regard it as a prediction of $m_H$ of the model. Because of the large $\lambda_3$ there will be a significant positive contribution to $\mu_2^2$. For $m_H = 500 GeV$, $\mu_2^2/2 \simeq 0.5 TeV^2$, much as desired, but the $\lambda_3$ term would give a further positive contribution, which is hard to determine from [9]. Thus there may be a need to invoke the first approach for $\mu_2^2$. Another possibility is to add some bare term, presumably coming from some still higher scale, as for $m_\pi$ in QCD, as done in the SU(3)xU(1) little Higgs model [11].

Overall it appears that fair consistency at least can be achieved with the Coleman-Weinberg mechanism for the quadratic terms in the model, although some uncertainty still remains. The situation is quite different for the quartic terms. As
mentioned in the previous paragraph $\lambda_1$ must be $\approx 2$ to achieve $m_H \approx 0.5\, TeV$ as desired, but the Coleman-Weinberg value $\approx \frac{3\log(\Lambda/m_t)}{8\pi^2}$ dominantly from the box diagram from the top quark, where the fact that the Yukawa coupling of the top quark $\approx 1$ has been used, but with a substantial reduction in magnitude coming from gauge and $H$ boson box diagrams. Even neglecting these, one finds $\lambda \approx 0.12$, far short of 2. Thus it seems impossible to accommodate a heavy Higgs boson purely within the scheme of a radiatively generated Higgs potential. It is clear, however, from the paper of Coleman-Weinberg that quartic tree interaction is allowed of a priori undetermined magnitude, although one may be uneasy that it is an order of magnitude bigger than the radiatively generated one.

A similar problem will arise for the $\lambda_2$ and $\lambda_3$ terms of Forshaw et al’s model. They cannot be much bigger than $\lambda_1$ as obtained by the Coleman-Weinberg mechanism since $g_2^2 \leq \lambda_t^2$ where $\lambda_t$ is the top quark coupling to $H$ and $g_2$ is the coupling of gauge bosons to $\phi$. In any case the necessity of dominant tree contributions is most apparent for $H H^\dagger H H^\dagger$ interactions. Of course, once one invokes large tree terms there is no reason why they should not appear in any term in the potential (beyond the quadratic or there is nothing to discuss).

4 Phenomenology of $\eta$

Recalling that the model has an octet of pNGb’s consisting of the complex doublet Higgs boson, an isotriplet and an isosinglet $\eta$, the phenomenology of the $\eta$ has to be examined to ensure that it causes no problems. The potential for $\eta$ has the form (cf. the potential of Forshaw et al.)

$$\lambda_{2,\eta}/4|\eta|^4 + \lambda_{3,\eta}/2|\eta|^2|H|^2 + L_\eta$$

where

$$L_\eta = \lambda_{4,\eta}\eta H^\dagger H + \lambda_{5,\eta}|\eta|^3 + \lambda_{6,\eta}|\eta|^2|\phi|^2$$

Here $\lambda_{4,\eta}$ is given by d-type SU(3) coupling as $\sqrt{\frac{2}{3}}\lambda_4$. There are no terms such as $\mu_1$ and $\mu_2 \propto \Lambda^2$ from gauge and fermion loops, but mass will be induced from $\lambda_{3,\eta}$, $\lambda_{6,\eta}$, and $\lambda_{2,\eta}$ terms, both $\propto \Lambda^2$ and from the respective vevs.

Because of the term linear in $\eta$ a vev will be induced for $\eta$ in similar fashion to that for the littlest Higgs model\,[13] and here for $\phi$. It is expected to be much

\[1\] A model has been constructed\,[12] introducing vector-like fermions of mass $\simeq 5\, TeV$ in an adaptation of a model due to Popovic\,[14]. While this can reproduce radiatively $L_3$ to an isospin conserving accuracy of a few per cent, there seems little advantage in this complexity, which could be regarded as an attempt to second guess dynamics at the cutoff, once one fails to obtain other terms in the potential in this way.
smaller than \( \langle H \rangle \) as was \( \langle \phi^0 \rangle \). Other than slightly aggravating the already uncontrolled problem of the cosmological constant, it is not clear what consequences such a vev has.

The mass of \( \eta \) is given, as above, by loops of \( H, \phi \) and \( \eta \) itself, as well as from vevs, dominantly of \( H \), although these contribute a small amount relative to the uncertainties from loops. \( \lambda_{3,\eta}, \lambda_{6,\eta} \) and \( \lambda_{2,\eta} \) are not fully fixed by symmetry from \( \lambda \)'s, but it seems likely they will also be O(1) and thereby induce a mass for \( \eta \simeq m_\phi \) or possibly somewhat smaller as the gauge bosons do not contribute.

Because \( \eta \) couples only to Higgs pairs amongst SM particles it seems to require a detailed analysis, beyond the scope of this paper, to give a reliable estimate of its production cross section. However, it seems certain that, involving a Higgs-Higgs collision, and if it weighs several hundred GeV, one can be confident that it would not have been detected in present experiments.

Being neutral its future detection is likely to be strongly dependent on its lifetime as well as its decay modes. If \( m_\eta \simeq 400 \text{ GeV} \), say, its main decay mode should be top, antitop pairs with a Yukawa coupling constant \( \gamma \), which comes from evaluating a loop with \( t \) exchange between Higgses from the \( \lambda_{4,\eta} \) coupling. \( \lambda_{4,\eta} \) can be estimated as follows via constraining \( \lambda_4 \).

\[ \frac{\langle \phi^0 \rangle}{\langle H \rangle} \] is bounded from precision tests by 0.025. From the equation for the minimum of the potential

\[ m_\phi^2 \langle \phi^0 \rangle = \langle H^2 \rangle \lambda_4 \]

one obtains \( \lambda_4 \leq 0.035 \text{ TeV} \) for \( m_\phi = 0.5 \text{ TeV} \). While this is only a bound, one expects \( \lambda_4 \) not to be substantially less than this.

Evaluating the triangle loop, assuming \( m_\eta \) sufficiently heavy for decay to \( t\bar{t} \), gives

\[ \gamma = \frac{\sqrt{(2/3)} \lambda_4 m_t}{4\pi^2 m_H^2 (1 + O(m_t^2/m_H^2))} \]

Taking for illustration \( m_\eta = 400 \text{ GeV} \) one obtains

\[ \Gamma_\eta = \frac{\gamma^2 k}{4\pi} \]

where \( k = 130 \text{ GeV} \) is the momentum of \( t \) in the \( \eta \) rest frame. With \( |\gamma| \leq 6.6.10^{-4} \) from Eq[6] one obtains \( \Gamma_\eta \leq 5000 \text{ eV} \). While this is much less than the width of a SM Higgs boson of the same mass, (and by the same token its production cross section is drastically suppressed compared to that of a Higgs boson) it is too large to give a displaced vertex. If \( 150 \text{ GeV} \leq m_\eta \leq 360 \text{ GeV} \) \( \eta \) will decay to vector boson pairs, and the lifetime will increase by \( O(1/g_2^4) \) or O(10), but still with no displacement. If \( m_\eta \leq 150 \text{ GeV} \) the decay to \( b\bar{b} \) will be suppressed by \( O(m_b^3/m_H^3) \) compared to the case of \( m_\eta = 400 \text{ GeV} \), so that \( \tau_\eta \) could be \( 0(10^{-13}) \)s. Such a
light $\eta$ seems, however, unlikely, and even this $\tau_\eta$ is probably too short to give a detectable displaced vertex. Thus $\eta$ is similar to $H$ in at least its main decay modes, but its production cross section is so small that it will occasion no confusion with $H$. Indeed it is hard to see how it would be produced at any accelerator in the foreseeable future.

5 Conclusion

An approach to resolving the little hierarchy problem using pNGb’s has been presented which does not need new gauge bosons, but at the expense of extra scalar bosons, an isovector and an isovector, though the latter appears very hard to detect. The model gives reasonable masses for the scalars via the Coleman-Weinberg mechanism, though $m_\phi$ tends to be rather large. The interactions of the scalars have to come from tree level, since the interactions generated by the Coleman-Weinberg mechanism are too weak. It is not clear how serious this is. In QCD the $\pi\pi$ interaction is not usually obtained from a Coleman-Weinberg mechanism, though pions are prototypes of pNGb’s, and yet it has a $\sigma$ resonance at $\approx 500$ MeV. It should be noted, however, that the dynamics of the SU(3)$\times$SU(3) employed in this model cannot be similar to that of chiral SU(3)$\times$SU(3) as the interaction of Eq.1 are of non-derivative type.

From a theoretical standpoint there are several issues that remain to be resolved. It is not clear if a Little Higgs model at $\approx 70$ TeV can be constructed so as to give a linear sigma model with SU(3)$\times$SU(3) symmetry as used here. From[9] it appears that this may prove very hard.

Forshaw et al require in their renormalization analysis that the $\lambda$’s do not become too large by 1 TeV scale. One might worry that this scale should be extended to 5 TeV here, which would probably limit further the scalar masses allowed.\textsuperscript{2} A further issue, possibly related to this, is unitarity.\textsuperscript{16} These authors find, in non-linear realisations of pNGbs, that unitarity is violated below $\Lambda$ given by $4\pi f$ when there are many pNGb’s. The implications of this observation for this model remain to be analysed.

Despite these open theoretical issues, it seems worthwhile to present this model, because of its simplicity, economy of new states, and difference in outlook to most current approaches. It is to be hoped that it may stimulate other, and perhaps better, models along similar lines. Finally, and especially if $\Lambda$ is reduced somewhat, following the lines of[16] the intriguing possibility that the LHC could access the UV dynamics might arise.

\textsuperscript{2}A calculation\textsuperscript{15} of the potential in the littlest Higgs model may cast some doubt, however, on the necessity of this.
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