QCD sum rules for $\rho$, $\omega$, $\phi$ meson-nucleon scattering lengths and the mass shifts in nuclear medium

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Abstract

A new QCD sum rule analysis on the spin-isospin averaged $\rho$, $\omega$ and $\phi$ meson-nucleon scattering lengths is presented. By introducing the constraint relation on the low energy limit of the vector-current nucleon forward scattering amplitude (low energy theorem), we get $a_\rho = -0.47 \pm 0.05$ fm, $a_\omega = -0.41 \pm 0.05$ fm and $a_\phi = -0.15 \pm 0.02$ fm, which suggests that these $V-N$ interactions are attractive. It is also proved that the previous studies on the mass shift of these vector mesons in the nuclear medium are essentially the ones obtained from these scattering lengths in the linear density approximation.
1 Introduction

Modifications of hadron properties in nuclear medium is of great interest in connection with the ongoing experimental plans at CEBAF and RHIC etc. Especially, the mass shift of vector mesons is directly accessible by inspecting the change of the lepton pair spectra in the electro- or photo- production experiments of the vector mesons from the nuclear targets. To study this issue, Hatsuda-Lee (HL) applied the QCD sum rule (QSR) method to the vector mesons in the nuclear medium, and got 10-20% decrease of the masses of the ρ and ω mesons at the nuclear matter density [1]. Later one of the present authors [2] reexamined the analysis of [1] based on the observation that their density effect in the vector current correlator comes from the current-nucleon forward scattering amplitude, and accordingly the effect should be interpretable in terms of the physical effect in the forward amplitude [3]. This analysis showed slight increase of the ρ, ω meson masses in contradiction to [1]. Subsequently, the analysis in [2] was criticized by Hatsuda-Lee-Shiomi[4]. This paper is prepared as a reexamination and a more expanded discussion of [2]. We present a new analysis on the ρ, ω and φ- nucleon scattering lengths. By introducing a constraint relation among the parameters in the spectral function, we eventually got a decreasing mass similar to [1], although the interpretation presented in [2] essentially persists. We also provide informative comments and replies to [4], and clarify the misunderstanding in the literature on the interpretation of the mass shift [1, 4, 18].

We first wish to give a brief sketch of the debate. The information about the spectrum of a vector meson in the nuclear medium with the nucleon density $\rho_N$ can be extracted from the correlation function

$$\Pi_{\mu\nu}^{NM}(q) = i \int d^4xe^{iq\cdot x} \langle T J_{\mu}(x) J_{\nu}^\dagger(0) \rangle_{\rho_N},$$

(1.1)

where $q = (\omega, q)$ is the four momentum and $J_{\mu}$ denotes the vector current for the vector mesons in our interest:

$$J_\mu^\rho(x) = \frac{1}{2} (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)(x), \quad J_\mu^\omega(x) = \frac{1}{2} (\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d)(x), \quad J_\mu^\phi(x) = \bar{s}\gamma_\mu s(x).$$

(1.2)

Following a common wisdom in the QSR method [5], Hatsuda-Lee applied an operator product expansion (OPE) to this correlator at large $Q^2 = -q^2 > 0$. The basic assumption employed in this procedure is that the $\rho_N$-dependence of the correlator is wholly ascribed to the $\rho_N$ dependence in the condensates $\langle O_i \rangle_{\rho_N}$:

$$\Pi^{NM}(q^2 \to -\infty) = \sum_i C_i(q^2, \mu^2) \langle O_i(\mu^2) \rangle_{\rho_N},$$

(1.3)

where $C_i$ is the Wilson coefficient for the operator $O_i$ and we suppressed all the Lorentz indices for simplicity. A new feature in the finite density sum rule is that both Lorentz scalar and nonscalar operators survive as the condensates $\langle O_i \rangle_{\rho_N}$. An assumption of the Fermi gas model for the nuclear medium was introduced to estimate the $\rho_N$-dependence of the correlator is wholly ascribed to the $\rho_N$ dependence in the condensates $\langle O_i \rangle_{\rho_N}$:

$$\langle O_i \rangle_{\rho_N} = \langle O_i \rangle_0 + \frac{\rho_N}{2M_N} \langle O_i \rangle_{N} + o(\rho_N),$$

(1.4)
where \( \langle \cdot \rangle_0 \) represents the vacuum expectation value, \( |ps\rangle \) denotes the nucleon state with momentum \( p \) and the spin \( s \) normalized covariantly as \( \langle ps|p's'\rangle = (2\pi)^32p^0\delta_{ss'}\delta^{(3)}(p - p') \), and \( \langle \cdot \rangle_N \) denotes the expectation value with respect to the nucleon state with \( p = 0 \). The effect of \( p \neq 0 \) introduces \( O(\rho_{N}^{5/3}) \) correction to (1.4). This way the \( \rho_N \)-dependence of the condensates can be incorporated through the nucleon matrix elements in the linear density approximation. By inserting (1.4) in (1.3), one can easily see that the approximation to the condensate, (1.4), is equivalent to the following approximation to the correlation function itself:

\[
\Pi_{\mu\nu}^{NM}(q) = \Pi_{\mu\nu}^0(q) + \sum_{\text{spin, isospin}} \int_{p_f}^{p_i} \frac{d^3p}{(2\pi)^32p^0} T_{\mu\nu}(p, q),
\]

where \( \Pi_{\mu\nu}^0(q) \) is the vector current correlator in the vacuum,

\[
\Pi_{\mu\nu}^0(q) = i \int d^4x e^{iq\cdot x} \langle T J_\mu(x) J_\nu^\dagger(0) \rangle_0,
\]

and \( T_{\mu\nu}(p, q) \) is the current-nucleon forward amplitude defined as

\[
T_{\mu\nu}(p, q) = i \int d^4x e^{iq\cdot x} \langle ps| T J_\mu(x) J_\nu^\dagger(0) |ps\rangle.
\]

Since [1] adopted (1.4), one should be able to interpret the result in [1] from the point of view of the current-nucleon forward amplitude. What was the essential ingredient in \( T_{\mu\nu} \) which led to the decreasing mass in [1]? What kind of approximation in the analysis of \( T_{\mu\nu}(p, q) \) corresponds to the analysis of \( \Pi_{\mu\nu}^{NM} \) in [1]?

To answer these questions we first note that the linear density approximation (1.4) to the condensates becomes better at smaller \( \rho_N \) or equivalently smaller \( p_f \). As long as the OPE side is concerned, the effect of the nucleon Fermi motion can be included in \( \langle O \rangle_0 \rho_N \) as is discussed in [4]. It turned out, however, that its effect is negligible. Therefore what is relevant in the mass shift in the QSR approach is the structure of \( T_{\mu\nu} \) in the \( p = 0 \) limit. We observe that in this limit, \( T_{\mu\nu} \) is reduced to the vector meson-nucleon scattering length \( a_V \) at \( q = (\omega = m_V, q = 0) \) \( (m_V \) is the mass of the vector meson). If one knows \( a_V \), the mass shift of the vector meson becomes

\[
\delta m_V = 2\pi \frac{M_N + m_V}{M_N m_V} a_V \rho_N
\]

in the linear density approximation. In the following discussion we argue that what was observed in [1] as a decreasing mass shift is essentially the one in (1.8). Of course, whether the approximation (1.4), (1.3) to \( \Pi_{\mu\nu}^{NM} \) is a good one or not at the nuclear matter density is a different issue. What we wish to stress is that the approximation adopted in [1] is certainly interpretable in terms of the vector meson-nucleon \( (V - N) \) scattering lengths unlike the argument in [4].

To motivate our idea from a purely mathematical point of view, let’s forget about the \( V - N \) scattering lengths for the moment, and translate what was observed in [1] into the language of \( T_{\mu\nu} \). HL analyzed \( \Pi_1^{NM}(\omega^2) \equiv \Pi_{\mu\nu}^{NM}(q)/(-3\omega^2) \) at \( q = 0 \) in QSR. At
\( \rho_N = 0 \), namely in the vacuum, \( \Pi_{1}^{\text{NM}}(\omega^2) \) is reduced to \( \Pi_1(q^2) \) defined by the relation \( \Pi_{\mu\nu}^{0}(q) = (q_{\mu}q_{\nu} - g_{\mu\nu}q^2)\Pi_1(q^2) \). HL obtained a QSR relation for \( \Pi_{1}^{\text{NM}} \) as

\[
\frac{1}{8\pi^2} \ln \left( \frac{s_0^*-q^2}{-q^2} \right) + \frac{A^*}{q^4} + \frac{B^*}{q^6} = \frac{F_{\nu}^*}{m_V^2-q^2} + \frac{\rho_{sc}}{q^2}, \quad (1.9)
\]

where \( A^* \) and \( B^* \) are the in-medium condensates with \( \text{dim.}=4 \) and \( \text{dim.}=6 \), respectively, and \( m_V^2, F^* \) and \( s_0^* \) are the in-medium values of the (squared) vector meson mass, pole residue and the continuum threshold, which are to be determined by fitting the above equation. \( \rho_{sc} \) is the so called Landau damping term which is purely a medium effect and is thus \( O(\rho_N) \).

Actual values are \( \rho_{sc} = -\frac{\rho_{\omega}}{4M_N} \) for the \( \rho, \omega \) mesons and \( \rho_{sc} = 0 \) for \( \phi \) meson \([1, 7]\). At \( \rho_N = 0 \), \( \Pi_{1}^{\text{NM}} \) is simply the well known sum rule in the vacuum \([3]\):

\[
\frac{1}{8\pi^2} \ln \left( \frac{s_0-q^2}{-q^2} \right) + \frac{A}{q^4} + \frac{B}{q^6} = \frac{F'}{m^2_V - q^2}. \quad (1.10)
\]

Since HL included the linear density correction \([1, 4]\) in \( A^* \) and \( B^* \), they got the change in \( m_V^2, F^* \) and \( s_0^* \) to \( O(\rho_N) \) accuracy. Indeed, HL got a clear linear change in these quantities. We write \( A^* = A + \frac{\rho_{\omega}}{2M_N} \delta A \) and similarly for \( B^* \) corresponding to \([1, 4]\), where \( \delta A \) and \( \delta B \) are the nucleon matrix elements of the same operators as \( A \) and \( B \) respectively. Correspondingly it is legitimate to write \( m_V^{2*} = m_V^2 + \frac{\rho_{\omega}}{2M_N} \delta m_V^2, F_{\nu}^* = F' + \frac{\rho_{\omega}}{2M_N} \delta F' \) and \( s_0^* = s_0 + \frac{\rho_{\omega}}{2M_N} \delta s_0 \).

Expand \((1.9)\) to \( O(\rho_N) \) and subtract \((1.11)\) from it. Then one gets

\[
\frac{\delta A}{q^4} + \frac{\delta B}{q^6} = \frac{\delta F'}{m_V^2-q^2} + \frac{\delta s_0/(8\pi^2)}{s_0-q^2} + \frac{\delta \rho_{sc}}{q^2}. \quad (1.11)
\]

The left hand side of this equation is precisely the OPE expression for \( T_{\mu}^\rho(p, q)/(-3\omega^2) \) at \( p = q = 0 \), and thus \((1.11)\) is the QSR for the same quantity which is equivalent to the QSR for \( \Pi_{1}^{\text{NM}}(\omega^2) \) assumed in \([1]\). Regardless of what HL intended in their sum rule analysis for the the vector mesons in the medium, \((1.11)\) is the equivalent sum rule relation for \( T_{\mu\nu} \) in their analysis. What is the physical content of this sum rule for \( T_{\mu\nu} \)? In this paper we shall show that our analysis on the vector meson nucleon scattering lengths precisely leads to the sum rule \((1.11)\).

This paper is organized as follows. In section 2, we present a new analysis for the \( \rho, \omega \) and \( \phi \) meson-nucleon spin-isospin averaged scattering lengths in the framework of QSR. The difference from the previous analysis \([2]\) is emphasized. The contents of this section should be taken as independent from the issue of the mass shift of these vector mesons in the nuclear medium. In section 3, we discuss the relation between the scattering lengths obtained in section 2 and the mass shift of \([1]\). In section 4, we shall give detailed answers and comments to the criticisms raised in \([3]\). Section 5 is devoted to summary and conclusion. Some of the formula will be discussed in the appendix.

## 2 \( \rho, \omega, \phi \)-nucleon scattering lengths

In this section we analyze the vector current-nucleon forward scattering amplitude \([1, 7]\) at \( p = 0 \) in the framework of the QCD sum rule, and present a new estimate for the \( \rho, \omega \) and
\[ T_{\mu\nu}(\omega, q) = i \int d^4 x e^{iqx} \langle ps | T J_\mu(x) J_\nu^\dagger(0) | ps \rangle, \]  

(2.1)

suppressing the explicit dependence on the four momentum of the nucleon \( p = (M_N, 0) \). As was noticed in the introduction, we are interested in the structure of \( T_{\mu\nu}(\omega, q = 0) \) around \( \omega = m_V \) which affects the pole structure of the vector current correlator in the medium. Near the pole position of the vector meson, \( T_{\mu\nu} \) can be associated with the \( T \) matrix for the forward \( V - N \) \((V = \rho, \omega, \phi)\) scattering amplitude \( T_{hH,h'H'} \) by the following relation

\[ \epsilon_{(h)}^\mu(q) T_{\mu\nu}(\omega, q) \epsilon_{(h')}^\nu(q) \simeq \frac{-f_V^2 m_V^4}{(q^2 - m_V^2 + i\varepsilon)^2} T_{hH,h'H'}(\omega, q), \]

(2.2)

where \( h(h') \) denotes the helicities for the initial (final) vector meson, and similarly \( H(H') \) for the nucleon. In (2.2) the coupling \( f_V \) is introduced by the relation \( \langle 0 | J_\mu^V | V^{(h)}(q) \rangle = f_V m_V^2 \epsilon_{(h)}^\mu(q) \) with the polarization vector \( \epsilon_{(h)}^\mu(q) \) normalized as \( \sum_h \epsilon_{(h)}^{\mu*}(q) \epsilon_{(h)}^\nu(q) = -g_{\mu\nu} + q_\mu q_\nu / q^2 \). \( T_{\mu\nu} \) can be decomposed into the four scalar functions respecting the invariance under parity and time reversal and the current conservation. Taking the spin average on both sides of (2.2) (see appendix A), \( T_{\mu\nu}(\omega, q) \) is projected onto \( T(\omega, q) = T_{\mu\nu}^\mu / (-3) \) \([8]\) and \( T_{hH,h'H'} \) is projected onto the spin averaged \( V - N \) \( T \) matrix, \( T(\omega, q) \). At \( q = (m_V, 0) \) and \( p = (M_N, 0) \), \( T \) is connected to the spin averaged \( V - N \) scattering length \( a_V \) as \( T(m_V, 0) = 8\pi (M_N + m_V) a_V \) \([8]\) with \( a_V = \frac{1}{2}(2a_{3/2} + a_{1/2}) \) where \( a_{3/2} \) and \( a_{1/2} \) are the \( V - N \) scattering lengths in the spin-3/2 and 1/2 channels respectively. We also remind that the \( \rho^0 \) \( - N \) scattering length corresponds to the isospin-averaged scattering length owing to the isospin symmetry.

The retarded correlation function defined by

\[ T_{\mu\nu}^R(\omega, q) = i \int d^4 x e^{iqx} \langle N | \theta(x^0) [J_\mu(x), J_\nu^\dagger(0)] | N \rangle \]

(2.3)

satisfies the following dispersion relation

\[ T_{\mu\nu}^R(\omega, q) = \frac{1}{\pi} \int_{-\infty}^\infty du \frac{\text{Im} T_{\mu\nu}^R(u, q)}{u - \omega - i\varepsilon}. \]

(2.4)

We recall that for nonreal values of \( \omega \), \( T_{\mu\nu}^R(\omega, q) \) becomes identical to \( T_{\mu\nu}(\omega, q) \). Applying the same spin-averaging procedure to both sides of (2.4) as above, we get the following dispersion relation for \( \omega^2 \neq \) positive real number:

\[ T(\omega, 0) = \int_{-\infty}^\infty du \frac{\rho(u, 0)}{u - \omega - i\varepsilon} = \int_0^\infty du \frac{\rho(u, 0)}{u^2 - \omega^2}, \]

(2.5)

where we introduced the spin-averaged spectral function \( \rho(\omega, q) \) constructed from \( \frac{1}{\pi} \text{Im} T_{\mu\nu}^R(\omega, q) \). The second equality in (2.4) comes from the relation \( \rho(-\omega, -q) = -\rho(\omega, q) \). Using (2.2), \( \rho(u, 0) \) can be expressed in terms of the spin-averaged \( V - N \) forward \( T \)-matrix \( T \) as

\[ \rho(u > 0, q = 0) \]
\[
= \frac{1}{\pi} \Im \left[ \frac{-f^2_{V} m_{V}^4}{(u^2 - m_{V}^2 + i \varepsilon)^2} T(u, 0) \right] + \cdots
\]
\[
= -\frac{f^2_{V} m_{V}^4}{\pi} \Im \left[ \frac{1}{(u^2 - m_{V}^2 + i \varepsilon)^2} \Re T(u, 0) + \frac{1}{(u^2 - m_{V}^2 + i \varepsilon)^2} \Im T(u, 0) \right] + \cdots
(2.6)
\]
\[
\equiv a \delta'(u^2 - m_{V}^2) + b \delta(u^2 - m_{V}^2) + c \delta(u^2 - s_0),
(2.7)
\]
where
\[
a = -f^2_{V} m_{V}^4 \Re T(u, 0)|_{u = m_{V}} = -8\pi f^2_{V} m_{V}^4 (M_N + m_{V}) a_V,
(2.8)
\]
\[
b = -f^2_{V} m_{V}^4 \frac{d}{du^2} \Re T(u, 0)|_{u = m_{V}},
(2.9)
\]
and \(\cdots\) in (2.6) represents the continuum contribution which is not associated with the \(\rho - N\) scattering. The first two terms in (2.7) come from the first term in (2.6) when (2.6) is substituted into the dispersion integral (2.5). The b-term (simple pole term) in (2.7) represents the off-shell effect in the \(T\) matrix of the forward \(VN \rightarrow VN\) scattering. We note that no other higher derivatives of \(\Re T(u, 0)\) appear here. The third term in (2.7) corresponds to \(\cdots\) in (2.6) and represents the scattering contribution in the continuum part of \(J_V\) which starts at the threshold \(s_0\). The value of \(s_0\) is fixed as \(s_0 = 1.75\) GeV\(^2\) for the \(\rho\) and \(\omega\) mesons and \(s_0 = 2.0\) GeV\(^2\) for the \(\phi\) meson, since these values are known to reproduce the masses of these mesons\(^{[3]}\). What is not included in the ansatz (2.7) is the second term in \([\cdots]\) of (2.6) which represents inelastic (continuum) contribution such as \(\rho N \rightarrow \pi N, \pi \Delta\) for the \(\rho\) meson and \(\phi N \rightarrow K \Lambda, K \Sigma\) for the \(\phi\) meson etc. The strength of these contributions could be sizable, so we should take the following analysis with caution. (See discussion below.)

The OPE expression for \(T(q^2 = \omega^2) = T(\omega, 0)\) in (2.3) is given in Eq. (6) of [2] (and Eq. (2.13) of [3]) for the \(\rho\) and \(\omega\)-mesons, and it is not repeated here. It takes the following form including the operators with dimension up to 6:

\[
T^{\text{OPE}}(q^2) = \frac{\alpha}{q^2} + \frac{\beta}{q^4},
(2.10)
\]
where \(\alpha\) is the sum of the nucleon matrix elements of the dim.=4 operators and \(\beta\) for the dim.=6 operators. In our analysis, we adopt the same values for these matrix elements as [2]: \(\alpha = 0.39\) GeV\(^2\) for the \(\rho\) and \(\omega\) mesons and \(\beta = -0.23 \pm 0.07 (-0.16 \pm 0.10)\) GeV\(^4\) for the \(\rho\) (\(\omega\)) mesons. The difference in \(\beta\) between \(\rho\) and \(\omega\) originates from the twist-4 matrix elements for which we adopted the parameterization used in [10]. For the \(\phi\) meson, \(\alpha = 0.24\) GeV\(^2\) and \(\beta = -0.12\) GeV\(^4\). See [2] for the detail.

Up to now our procedure for analyzing \(T_{\mu \nu}\) is completely the same as [3]. Here we start to deviate from [3] and introduce a constraint relation among \(a\), \(b\) and \(c\) which is imposed by the low energy theorem for the vector current-nucleon forward scattering amplitude. In the low energy limit, \(p \rightarrow (M_N, 0)\) and \(q = (\omega, q) \rightarrow (0, 0)\), \(T_{\mu \nu}(\omega, q)\) is determined by the Born diagram contribution (Fig.1) as in the case of the Compton scattering [11]. Since we are considering the case \(q = 0\), we first put \(q = 0\) and then take the limit \(\omega \rightarrow 0\) (See appendix

\[\text{[In (2.2), } T\text{ is defined only around } \omega = m_V\text{ and thus we introduced the contribution } \cdots\text{ in (2.6).}\]
B): 

\[ T^{\text{Born}}(\omega^2) \equiv T^{\text{Born}}(\omega, 0) = \begin{cases} \frac{-2M_N^2}{4M_N^2 - \omega^2} & \omega \to 0 \\ \frac{\omega^2}{\phi} & \end{cases} \]

At \( q_\mu \neq 0 \), the Born term is not the total contribution and there remains an ambiguity in dealing with \( T^{\text{Born}} \). We thus assume two forms of the parameterization for the phenomenological side of the sum rules for \( \rho \) and \( \omega \) mesons:

(i) With explicit Born term:

\[ T^{\text{ph}}(q^2) = T^{\text{Born}}(q^2) + \frac{a}{(m^2_V - q^2)^2} + \frac{b}{m^2_V - q^2} + \frac{c}{s_0 - q^2} \]

with the condition

\[ \frac{a}{m^4_V} + \frac{b}{m^2_V} + \frac{c}{s_0} = 0. \]

(ii) Without explicit Born term:

\[ T^{\text{ph}}(q^2) = \frac{a}{(m^2_V - q^2)^2} + \frac{b}{m^2_V - q^2} + \frac{c}{s_0 - q^2} \]

with the condition

\[ \frac{a}{m^4_V} + \frac{b}{m^2_V} + \frac{c}{s_0} = T^{\text{Born}}(0). \]

With the phenomenological sides of the sum rules ((2.12) or (2.14)) and the OPE side (2.10), the QSR is given by the relation

\[ T^{\text{OPE}}(q^2) = T^{\text{ph}}(q^2). \]

Several comments are in order here.

1. Because of the conditions (2.13) and (2.15), \( T^{\text{ph}}(q^2) \) satisfies \( T^{\text{ph}}(0) = T^{\text{Born}}(0) \) and has two independent parameters to be determined in either case. This part is the essential difference from the previous study in [2]. In [2], \( a, b \) and \( c \) were treated as independent parameters which were determined in the Borel sum rule (BSR). In the following, we eliminate \( c \) by these relations and regard \( T^{\text{ph}} \) as a functions of \( a \) and \( b \).

2. The leading behavior of \( T^{\text{ph}}(q^2) \) at large \( -q^2 > 0 \) is consistent with \( T^{\text{OPE}}(q^2) \): Both sides start with the \( \frac{1}{q^4} \) term, which supports the form of the spectral function in (2.7).

3. Inclusion of \( T^{\text{Born}}(q^2) \) in (2.12) has a similar effect as the inclusion of the “second continuum” contribution with the threshold \( 4M_N^2 \). In the QSR analysis for the lowest resonance contribution, the result is more reliable if it does not depend on the details of the higher energy part. We shall see this is indeed the case in the following Borel sum rule method.
By expanding $T^{\text{ph}}(q^2)$ with respect to $1/(-q^2)$ and comparing the coefficients of $1/q^2$ and $1/q^4$ in $T^{\text{ph}}(q^2)$ with those in $T^{\text{OPE}}(q^2)$, one gets the finite energy sum rules (FESR). These relations are solved to give

$$a = \frac{1}{1 - \frac{s_0}{m_V^2}} \left[ m_V^2 \left( 1 + \frac{s_0}{m_V^2} \right) \left( -\alpha + 2M_N^2 + \left( \beta - 8M_N^4 \right) \right) \right],$$

$$b = \frac{1}{\left( 1 - \frac{s_0}{m_V^2} \right)^2} \left[ \left( 1 + \frac{s_0^2}{m_V^2} \right) \left( -\alpha + 2M_N^2 + \frac{s_0}{m_V^2} \left( \beta - 8M_N^4 \right) \right) \right],$$

for the case (i) and

$$a = \frac{1}{1 - \frac{s_0}{m_V^2}} \left[ m_V^2 \left( 1 + \frac{s_0}{m_V^2} \right) \left( -\alpha + \frac{1}{2}s_0 + \left( \beta - \frac{1}{2}s_0^2 \right) \right) \right],$$

$$b = \frac{1}{\left( 1 - \frac{s_0}{m_V^2} \right)^2} \left[ \left( 1 + \frac{s_0^2}{m_V^2} \right) \left( -\alpha + \frac{1}{2}s_0 + \frac{s_0}{m_V^2} \left( \beta - \frac{1}{2}s_0^2 \right) \right) \right],$$

for the case (ii). These FESR relations give $a_\rho = -0.68$ fm, $a_\omega = -0.66$ fm for the case (i) and $a_\rho = -0.13$ fm, $a_\omega = -0.11$ fm for the case (ii). For the $\phi$ meson, $a_\phi = -0.06$ fm. Two ways of dealing with the Born term give quite different results. This is not surprising. Since the leading order contribution in $T^{\text{ph}}$ comes from the continuum contribution, the results in FESR strongly depends on the treatment of this part. These small negative numbers, however, suggest that the $V - N$ interaction is weakly attractive.

In order to give more quantitative prediction, we proceed to the Borel sum rule (BSR) analysis. In this method, the higher energy contribution in the spectral function is suppressed compared to the $V - N$ scattering contribution. We thus have an advantage that the ambiguity in dealing with the Born term becomes less important in BSR. We shall try the following two methods in BSR:

1. Derivative Borel Sum Rule (DBSR): After the Borel transform of (2.16) with respect to $Q^2 = -q^2 > 0$, take the derivative of both sides with respect to the Borel mass $M^2$, and use these two equations to get $a$ and $b$ by taking the average in a Borel window, $M^2_{\text{min}} < M^2 < M^2_{\text{max}}$.

2. Fitting Borel Sum Rule (FBSR): Determine $a$ and $b$ in order to make the following quantity minimum in a Borel window $M^2_{\text{min}} < M^2 < M^2_{\text{max}}$:

$$F(a, b) = \int_{M^2_{\text{min}}}^{M^2_{\text{max}}} dM^2 [T^{\text{OPE}}(M^2) - T^{\text{ph}}(M^2; a, b)]^2$$

where $T^{\text{ph}}(M^2; a, b)$ is the Borel transform of $T^{\text{ph}}(q^2)$ which is a functional of $a$ and $b$.

After getting $a$ and $b$ by these methods, we determine $a_V$ from the relation (2.8) using the experimental values of $M_N$, $m_V$ and $f_V$. The numbers we adopted are $M_N = 940$ MeV, $m_{\rho,\omega} = 770$ MeV, $f_{\rho,\omega} = 0.18$, $m_\phi = 1020$ MeV and $f_\phi = 0.25$. Borel curves for $a_V$ ($V = \rho$, $


ω, φ) in the DBSR are shown in Figs. 2 and 3. Stability of these curves is reasonably good around $M^2 = 1$ GeV$^2$ for the ρ and ω mesons and around $M^2 = 1.5$ GeV$^2$ for the φ meson. We take the average over the window $0.8$ GeV$^2 < M^2 < 1.3$ GeV$^2$ for ρ, ω mesons and $1.3$ GeV$^2 < M^2 < 1.8$ GeV$^2$ for the φ meson. These windows are typical for the analysis of these vector meson masses, and the experimental values are well reproduced with the continuum threshold $s_0 = 1.75$ GeV$^2$ for ρ, ω and $s_0 = 2.0$ GeV$^2$ for φ [5]. The obtained values are $a_ρ = -0.5 (-0.4)$ fm and $a_ω = -0.45 (-0.35)$ fm for the case (i) (ii), and $a_φ = -0.15$ fm. In the FBSR method with the same window, we get close numbers $a_ρ = -0.52 (-0.42)$ fm and $a_ω = -0.46 (-0.36)$ fm for (i) (ii), and $a_φ = -0.15$ fm. We tried FBSR for various Borel windows within $0.6$ GeV$^2 < M^2 < 1.8$ GeV$^2$ ($0.9$ GeV$^2 < M^2 < 2.0$ GeV$^2$) for the ρ, ω (φ) mesons and found that the results change within 20% level. From these analyses, we get

$$a_ρ = -0.47 ± 0.05 \text{ fm},$$

$$a_ω = -0.41 ± 0.05 \text{ fm},$$

$$a_φ = -0.15 ± 0.02 \text{ fm},$$

(2.22)

where the assigned error bars are due to the uncertainty in the Borel analysis.

We first note that the magnitudes of these scattering lengths are quite small, i.e., smaller than the typical hadronic size of 1 fm. For πN and Kν systems, the scattering lengths are known to be small due to the chiral symmetry. The above numbers are not so different from $a_πN$ and $a_KN$. Small negative values suggest that these $V - N$ interactions are weakly attractive. The ansatz (2.7) for the spectral function ignores various inelastic contributions as was noted below (2.3). So we should take the above numbers as a rough estimate of the order of magnitude.

Recently Kondo-Morimatsu-Nishino calculated the πN and Kν scattering lengths by applying the same QSR method to the correlator of the axial vector current [12]. The results with the lowest dimensional operators in the OPE side is the same as the current algebra calculation. QSR supplies the correction due to the nucleon matrix elements of the higher dimensional operator. Since there is no algebraic technique (such as current algebra) to calculate the scattering lengths in the vector channels, it is interesting to see that OPE provides a possibility to estimate the strengths of the $VN$ interactions.

3 Mass shift of the vector mesons in the nuclear medium

In the previous section, we have identified the pole structure of $T_{μν}(ω, 0)$ around $ω^2 = m_V^2$ as

$$T_{μν}(ω, 0) = \left( \frac{q_μq_ν}{ω^2} - g_μν \right) \left( \frac{a}{(m_V^2 - ω^2)^2} + \frac{b}{m_V^2 - ω^2} + \ldots \right).$$

(3.1)

By combining this piece with the vacuum piece $Π_0^{NM}(ω, 0)$ in (1.6), the vector current correlation function in the nuclear medium take the following form around $ω^2 = m_V^2$:

$$Π_{μν}^{NM}(ω, 0) \simeq \left( \frac{q_μq_ν}{ω^2} - g_μν \right) \left( Π(ω^2) + \frac{ρ_N}{2M_N} T(ω, 0) \right)$$
\[
\begin{align*}
&\propto \frac{F}{m_V^2 - \omega^2} + \frac{\rho_N}{2M_N} \left\{ \frac{a}{(m_V^2 - \omega^2)^2} + \frac{b}{m_V^2 - \omega^2} \right\} \ldots \\
&\simeq \frac{F + \delta F}{(m_V^2 + \Delta m_V^2) - \omega^2} + \ldots, \quad (3.2)
\end{align*}
\]

where \( \Pi(q^2) \) is defined as \( \Pi^0_{\mu\nu}(q) = \left( \frac{q_{\mu}q_{\nu} - g_{\mu\nu}}{q^2} \right) \Pi(q^2) \) and the pole residue \( F \) in \( \Pi^0_{\mu\nu} \) is related to \( f_V \) and \( m_V \) by the relation \( F = f_V^2 m_V^4 \) and \( \delta F = \frac{\rho_N^2}{2M_N} b \). The quantity

\[
\Delta m_V^2 = -\frac{\rho_N a}{2M_N} = \frac{\rho_N}{2M_N} 8\pi (M_N + m_V) a_V
\]

is regarded as the shift of the squared vector meson mass in nuclear matter. We thus have the mass shift \( \delta m_V \) as shown in (1.8) from the relation

\[
m^*_V = m_V + \delta m_V = \sqrt{m_V^2 + \Delta m_V^2}. \quad (3.4)
\]

Using the scattering lengths obtained in the previous section, we plotted the vector meson masses in Fig. 4 as a function of the density \( \rho_N \) based on the linear density approximation.

At the nuclear matter density \( \rho_N = 0.17 \text{ fm}^{-3} \) as

\[
\begin{align*}
\delta m_\rho &= -45 \sim -55 \text{ MeV (6 \sim 7\%)}, \\
\delta m_\omega &= -40 \sim -50 \text{ MeV (5 \sim 6\%)}, \\
\delta m_\phi &= -10 \sim -20 \text{ MeV (1 \sim 2\%)}. \quad (3.5)
\end{align*}
\]

In order to clarify the relation between the above mass shifts and the approach by Hatsuda-Lee, we briefly recall QSR for the vector meson mass in the vacuum. The correlation function in the vacuum defined in (1.6) has the structure

\[
\Pi^0_{\mu\nu}(q^2) = \left( \frac{q_{\mu}q_{\nu} - g_{\mu\nu}}{q^2} \right) \Pi_1(q^2). \quad (3.6)
\]

In QSR one starts with the dispersion relation for \( \Pi_1(q^2) \) (See Appendix C):

\[
\Pi_1(q^2) = \frac{q^2}{\pi} \int_{0^+}^{\infty} ds \frac{\text{Im}\Pi_1(s)}{s(s - q^2)} + \Pi_1(0), \quad (3.7)
\]

where we introduced one subtraction to avoid the logarithmic divergence. In the deep Euclidean region \( q^2 \to -\infty \), \( \Pi_1(q^2) \) in the the left hand side of (3.7) has the OPE expression including the operators up to dim.=6 as

\[
\Pi_1^{\text{OPE}}(q^2) = -\frac{1}{8\pi^2} \ln(-q^2) + \frac{A}{q^4} + \frac{B}{q^6}, \quad (3.8)
\]

where \( A \) and \( B \) are respectively the sums of dim.=4 and dim.=6 condensates, and the perturbative correction factor \( 1 + \frac{q^2}{\pi} \) to the first term is omitted for simplicity. We also suppressed the scale dependence in each term in (3.8). The spectral function in (3.7) is often modeled by the sum of the pole contribution from the vector meson and the continuum contribution:

\[
\frac{1}{\pi} \text{Im}\Pi_1(s) = F'\delta(s - m_V^2) + \frac{1}{8\pi^2} \theta(s - s_0), \quad (3.9)
\]
where \( F' = f_2^2 m_V^2 \). With this form in (3.7) together with (3.8), one gets the sum rule relation (See Appendix C) as

\[
\frac{1}{8\pi^2} \ln \left( \frac{s_0 - q^2}{-q^2} \right) + \frac{A}{q^4} + \frac{B}{q^6} = \frac{F'}{m_V^2 - q^2}.
\] (3.10)

Hatsuda-Lee considered the sum rule for \( \Pi_{NM}^{\omega^2} = \Pi_{NM}^{\rho N}(\omega, q = 0)/(-3\omega^2) \). The QSR for \( \Pi_{NM}^{\rho N}(q^2) \) is reduced to (3.10) at \( \rho_N \to 0 \) limit. At \( q = 0 \), \( \Pi_{NM}^{\rho N}(q^2) \) becomes

\[
\Pi_{NM}^{\rho N}(q^2) = \Pi_{1}(q^2) + \frac{\rho_N}{2M_N} \frac{T(q^2)}{q^2} + O(\rho_N^{5/3}).
\] (3.11)

Thus one has to analyze \( T(q^2)/q^2 \) to understand the density dependence in \( \Pi_{NM}^{\rho N}(\omega^2) \). We write the dispersion relation for \( T(q^2)/q^2 \):

\[
\frac{T(q^2)}{q^2} = \int_0^\infty ds \frac{\rho(s)}{s(s - q^2)} + \frac{T(0)}{q^2},
\] (3.12)

where the pole contribution at \( q^2 = 0 \) is explicitly taken care of by \( T(0) \). Substituting the spectral function (2.7) in this equation and equating it to the OPE side, one gets the QSR relation for the case (ii) as

\[
\frac{\alpha}{q^4} + \frac{\beta}{q^6} = \frac{a'}{(m_V^2 - q^2)^2} + \frac{b'}{m_V^2 - q^2} + \frac{(T^{\text{Born}}(0) - b')}{s_0 - q^2} + \frac{T^{\text{Born}}(0)}{q^2},
\] (3.13)

where

\[
a' = \frac{a}{m_V^2}, \quad b' = \frac{a}{m_V^2} + \frac{b}{m_V^2},
\] (3.14)

and the relation \( T(0) = T^{\text{Born}}(0) \) is used in the last term of (3.13). We note that (3.13) is nothing but the relation obtained by dividing both sides of (2.16) by \( q^2 \) for the case (ii), which guarantees the absence of the \( \frac{1}{q^2} \) term in the right hand side. (Note that the condition \( T(0) = T^{\text{Born}}(0) \) itself is not required to guarantee this consistency condition.) Using (3.10) and (3.13) in (3.11), we can construct the QSR for \( \Pi_{NM}^{\rho N}(q) \) in the linear density approximation:

\[
\frac{1}{8\pi^2} \ln \left( \frac{s_0^* - q^2}{-q^2} \right) + \frac{A + \tilde{\alpha}}{q^4} + \frac{B + \tilde{\beta}}{q^6} = \frac{F'^* + \tilde{b}'}{m_V^2 - q^2} + \frac{\tilde{a}'}{(m_V^2 - q^2)^2} + \frac{T^{\text{Born}}(0) - \tilde{b}'}{s_0 - q^2} + \frac{T^{\text{Born}}(0)}{q^2},
\] (3.15)

where \( \tilde{\alpha} = \frac{\rho_N}{2M_N} \alpha, \tilde{\beta} = \frac{\rho_N}{2M_N} \beta, \) etc. To \( O(\rho_N) \) accuracy (3.15) can be rewritten as

\[
\frac{1}{8\pi^2} \ln \left( \frac{s_0^* - q^2}{-q^2} \right) + \frac{A^*}{q^4} + \frac{B^*}{q^6} = \frac{F'^*}{m_V^2 - q^2} + \frac{T^{\text{Born}}(0)}{q^2},
\] (3.16)

with

\[
A^* = A + \tilde{\alpha}, \quad B^* = B + \tilde{\beta},
\] (3.17)
\[ F'_{\pi} = F' + \tilde{b}, \quad m_{\pi}^2 = m_{\pi}^2 - \frac{q'^2}{F'}, \quad s_0^* = s_0 - 8\pi^2(\widetilde{T}^{\text{Born}}(0) - \tilde{b}'). \] (3.18)

From the above demonstration, it is now clear that our analysis of \( T_{\mu\nu} \) in the previous section (case (ii)) leads to (1.9) for \( \Pi_{1}^{\text{NM}} \) by the identification \( \rho_{sc} = \widetilde{T}^{\text{Born}}(0) \). In fact \( \rho_{sc} = \frac{2\pi^2}{M_N^2} \) for the \( \rho, \omega \) mesons and \( \rho_{sc} = 0 \) for the \( \phi \) meson in [1], which is consistent with (2.11). The mass shift in (3.18) is obviously the same as given in (3.3). We should emphasize that it is our constraint relation \( T(0) = T^{\text{Born}}(0) \) in the analysis of the scattering lengths which leads to the same sum rule for \( \Pi_{1}^{\text{NM}}(q^2) \) as in [1]. Our use of low energy theorem is in parallel with the calculation of the Landau damping term \( \rho_{sc} \) from the Born diagram in [1]. If one did not have such information on \( T(0) \), one would have to use the approach in [2] with the matrix elements of the dim.=8 or higher operators.

We point out, however, a small difference from [1]. From the first and the third relation in (3.18), one obtains

\[ F'_{\pi} - F' = \frac{1}{8\pi^2} (s_0^* - s_0) + \widetilde{T}^{\text{Born}}(0), \] (3.19)

which is the same as the first FESR relation obtained from \( \Pi_{1}^{\text{NM}} \). (In FESR our present analysis is completely equivalent to [1].) Namely the shift of \( F' \) is determined by that of \( s_0 \). In the Borel sum rule in [1], \( F'_{\pi} \) and \( s_0^* \) are regarded as independent fitting parameters. But if one recognizes that the QSR for \( T_{\mu\nu} \) is independent from that for \( \Pi_{1}^{\text{NM}} \), it is easy to see that this condition has to be also satisfied in the approach of [1]. In fact, in (1.11) which was derived purely mathematically from the sum rule in [1], absence of \( 1/q^2 \) term in the left hand side of (1.11) imposes the consistency requirement in the right hand side of (1.11), which is exactly (3.19). Since HL took the view that \( \rho_{sc} \) is calculable (owing to the low energy theorem), they could have eliminated \( \delta F' \) or \( \delta s_0 \) from the outset. In our BSR for \( T_{\mu\nu} \), we were lead to use the condition (3.19) explicitly, which is imposed by the low energy theorem \( T(0) = T^{\text{Born}}(0) \). In our opinion, this is more natural because the QSR for \( T_{\mu\nu} \) is completely independent from the one for \( \Pi_{1}^{\text{NM}} \), i.e., the density \( \rho_N \) is simply an external parameter which connects these quantities in the sum rule for \( \Pi_{1}^{\text{NM}} \).

Although the mass shifts discussed in this section are essentially the same as those in [1], the numerical values in (3.5) are approximately factor two smaller than those in [1], especially for the \( \rho \) and \( \omega \) mesons. This is mainly because their calculation is done at the chiral limit (they ignored a correction due to the condensate \( m_q \langle \bar{\psi}\psi \rangle \)), and correspondingly their value for the continuum threshold \( s_0 \) is different from ours. They used \( s_0 = 1.43 \) GeV\(^2\) for \( \rho, \omega \) in the vacuum. Another reason is that their QSR was for the total sum of \( \Pi_{1}^{\rho} \) and \( T_{\mu\nu} \), the latter being small \( (O(\rho_N)) \) correction to the former as noted above, while our QSR is for the latter. These differences eventually leads to factor-two difference in the mass shifts at around nuclear matter density. Hatsuda claims [18] that, although \( m_{\pi}^2/m_{\pi} \) at \( \rho_N = 0 \) in [1, 13] is consistent with our scattering lengths, the mass shift in [1, 13] at higher \( \rho_N \) becomes bigger than expected from the scattering length, with the reasoning that the scattering length can be used only at very close to zero density and the prediction in [1] contains more than that. This deviation, however, should not be regarded as a meaningful one, since the OPE side includes only \( O(\rho_N) \) density effect and therefore only the \( O(\rho_N) \) effect represented by the scattering length is a valid physical prediction.

11
It is probably useful to add a brief comment on the calculation of \( \rho_{sc} \) in [1]. Using the general relation

\[
\lim_{q \to 0} \Pi_{1}^{NM}(\omega, q) = \lim_{q \to 0} \frac{\Pi_{00}^{NM}(\omega, q)}{|q|^2},
\]

they calculated \( \rho_{sc} \) from the spectral function of \( \Pi_{00}^{NM}(\omega, q)/|q|^2 \) which corresponds to \( T_{00}(\omega, q)/|q|^2 \) in our method. They included the pole contributions which appear at \( \omega = \pm 0 \) and ignored the contributions from \( \omega = \pm 2M_N \). But this treatment suffices as long as one needs the value of \( T_{\text{Born}}(0) \). The residue at \( \omega = \pm 0 \) (\( = -1/2 \) for \( \rho \), \( \omega \) meson) of \( \lim_{q \to 0} T_{00}(\omega, q)/|q|^2 \) precisely gives \( T_{\text{Born}}(0) \). (See Appendix B.) As was noticed below (2.16), their neglect of the poles at \( \omega = \pm 2M_N \) in \( \lim_{q \to 0} \Pi_{00}^{NM}(\omega, q)/|q|^2 \) corresponds to the assumption that those contributions are taken care of in the continuum part of (2.12) (\( 1/(s_0 - q^2) \) term) in our language.

4 Comments and Replies to Hatsuda-Lee-Shiomi (HLS)

It is by now clear that our present QSR analysis on the current-nucleon forward scattering amplitude is essentially equivalent to the medium QSR for the vector mesons in [1]. Namely their result is certainly interpretable in terms of the \( V-N \) scattering lengths. Although this already resolves the essential controversy between [2] and [1], we summarize in the following our replies and comments to HLS [4].

(1) HLS claims that \( V-N \) scattering lengths are not calculable in QSR without including dim.=8 matrix elements. This is because phenomenological side contains three unknown parameters but finite energy sum rules (FESR) provide only two relations. (Sec. III.B of [4])

Reply:

In our present analysis, we eliminated one parameter by the constraint relation at \( q^\mu = 0 \) and thus have two unknown parameters to be determined by QSR. This constraint relation due to the low energy theorem renders our analysis equivalent to HL in the FESR.

(2) HLS claims \( \Pi^{NM}(\omega^2) = \omega^2 \Pi_{1}^{NM}(\omega^2) \) is not usable to predict the mass of the vector mesons either in medium or in the vacuum. (Sec. III.C of [4])

Reply:

This argumentation is based on the number of available FESRs’ and the Borel stability of the sum rules. As is shown in Appendix C, the QSR itself (before Borel transform) is the same for \( \Pi^{NM}(\omega^2) \) and \( \Pi_{1}^{NM}(\omega^2) \) as long as one starts with the same consistent assumption for these quantities. The QSR for \( \Pi^{NM} \) is simply the one obtained by multiplying \( \omega^2 \) to \( \Pi_{1}^{NM} \). Accordingly the FESRs’ are the same. Whether one applies Borel transform before or after multiplying \( \omega^2 \) to \( \Pi_{1}^{NM}(\omega^2) \) causes numerical difference, especially because one loses
information from the polynomial terms in the BSR method. We agree that applying Borel transform to $\Pi_{1}^{nm}$ leads to more stable Borel curve than applying to $\Pi^{nm}$. We, however, note that the reason HL obtained the stable Borel curve for $\Pi_{1}$ in the vacuum is very stable and the curve for $\frac{\rho_{N}}{2M_{N}} T(q^{2})$ (see (3.11)) is only an $O(\rho_{N})$ correction to the former. In our case, what is plotted in Figs. 2 and 3 are the Borel curves for $T(q^{2})$ itself.

The authors of [13] raised a similar criticism against [2] and claimed that they have clarified the origin of discrepancy between [1] and [2]. But this does not solve the problem.

In [18], it was advertised that the Borel curves for $m_{V}^{*}$ in [1] are more stable than those for our scattering lengths shown in Figs. 2 and 3. But the reason for this is obvious. The stability of the Borel curve for $m_{V}$ itself. The authors of [13] raised a similar criticism against [2] and claimed that they have clarified the origin of discrepancy between [1] and [2]. But this does not solve the problem.

In [18], it was advertised that the Borel curves for $m_{V}^{*}$ in [1] are more stable than those for our scattering lengths shown in Figs. 2 and 3. But the reason for this is obvious. The stability of the Borel curve for $m_{V}$ itself.

(3) HLS claims that the $V - N$ scattering lengths and the mass shift of the vector mesons in the nuclear matter have no direct relation due to the momentum dependence of the $V - N$ forward scattering amplitude. They also claim that the analysis in [1] did not use this relation. (Sec. III.A of [4])

Reply: As is noted in the introduction, the analysis in [1] is mathematically equivalent to the QSR for $T_{\mu \nu}$ shown in (1.11). The right hand side of (1.11) is precisely reproduced by the spectral function shown in (2.7) and the Born contribution to $T_{\mu \nu}$. Thus the physical effect which caused the mass shift in [1] is essentially the same as the one based on the $V - N$ scattering lengths.

HLS stressed the importance of the momentum dependence of $T_{\mu \nu}$. However, it is not conspicuous in the OPE side. So one can not claim its importance in the phenomenological side from the QSR analysis itself. In fact the effect of the fermi motion of the nucleon can be included in the OPE side, but they are at least $O(\rho_{N}^{5/3})$ and they can be neglected as was shown in sec. IV of [4]. How come one can claim the importance of the effect which is negligible in the OPE side? Since the common starting point of our analysis was the linear density approximation to the OPE side shown in (1.4) the negligible effect in (1.4) should be taken as the effect which is either negligible in the phenomenological side or beyond the resolution of the analysis.

The phenomenological basis on which HLS emphasize the effect of Fermi motion of the nucleons is as follows: Nucleon’s fermi momentum is $p_{f} = 270$ MeV in Nuclear matter and thus one should take into account the $\rho - N$ scattering from $\sqrt{s} = m_{\rho} + M_{N} = 1709$ MeV through $\sqrt{s} = [(m_{\rho} + \sqrt{M_{N}^{2} + \vec{p}_{f}^{2}})^{2} - \vec{p}_{f}^{2}]^{1/2} = 1726$ MeV. In this interval there are some $s$-channel resonances such as $N(1710)$ and $N(1720)$ which couple to $\rho - N$ channel, thus $T_{\mu \nu}$ should change rapidly in this interval. However, these resonances together with the other near resonances ($N(1700), \Delta(1700)$) have broad widths of over 100 MeV and that the whole interval $0 < |\vec{p}| < p_{f}$ is buried under these broad resonance regions. In this situation, it is unlikely that the $V - N$ phase shift changes rapidly in this interval. However, these resonances together with the other near resonances ($N(1700), \Delta(1700)$) have broad widths of over 100 MeV and that the whole interval $0 < |\vec{p}| < p_{f}$ is buried under these broad resonance regions. In this situation, it is unlikely that the $V - N$ phase shift changes rapidly in this interval. It might be a good approximation to take the $T$-matrix at $\vec{p} = 0$ as a representative value of it. How
about $\phi$ meson? The $\phi - N$ scattering occurs from $\sqrt{s} = m_\phi + M_N = 1960$ MeV through $\sqrt{s} = [(m_\phi + \sqrt{M_N^2 + p_f^2})^2 - p_f^2]^{1/2} = 1980$ MeV. In this interval there is no resonance which could couple to $\phi - N$ system. The situation is better.

In [14, 15], there is a debate on the interpretation of the nucleon sum rule in the nuclear medium. We agree with the interpretation of [15]. A difference between the sum rules for the $V - N$ and $N - N$ interactions is the smallness of the obtained $V - N$ scattering lengths, which, together with the argument above, may justify the use of (1.8) to predict the mass shift in the linear density approximation.

One can organize the finite temperature ($T$) QSR in a similar way, replacing the Fermi gas of nucleons by the ideal gas of pions [9, 16]. In this formalism, the $T$-dependence of correlation functions comes from the current-pion forward amplitude. Since the pion-hadron scattering lengths are zero in the chiral limit, there is no $O(T^2)$ mass shift [16, 17]. This is in parallel with our present analysis that $O(p_N)$-dependence of the mass is determined by the scattering length.

5 Summary and Conclusions

In this paper, we have presented a new analysis on the $\rho$, $\omega$ and $\phi$ meson-nucleon spin-isospin averaged scattering lengths $a_V$ ($V = \rho, \omega, \phi$) in the framework of the QCD sum rule. Essential difference from the previous calculation in [2] is that the parameters in the spectral function of the vector current-nucleon forward amplitude is constrained by the relation at $q^\mu = 0$ (low energy theorem for the vector-current nucleon scattering amplitude). We obtained small negative values for $a_V$ as

\begin{align*}
a_\rho &= -0.47 \pm 0.05 \text{ fm}, \\
a_\omega &= -0.41 \pm 0.05 \text{ fm}, \\
a_\phi &= -0.15 \pm 0.02 \text{ fm}.
\end{align*}

(5.1)

This suggests that these $V - N$ interactions are weakly attractive in contrast to the previous study [2]. Since the form of the spectral function is greatly simplified, these numbers should be taken as a rough estimate of the order of magnitude. In the axial vector channel, the method works as a tool to introduce a correction to the current algebra calculation due to the higher dimensional operators. Present application to the vector channel in which current algebra technique does not work is suggestive in that the QSR provides us with a possibility to express the $V - N$ scattering lengths in terms of various nucleon matrix elements.

If one applies above $a_V$'s to the vector meson masses in the nuclear medium in the linear density approximation, one gets for the mass shifts as

\begin{align*}
\delta m_\rho &= -45 \sim -55 \text{ MeV (6 \sim 7%)}, \\
\delta m_\omega &= -40 \sim -50 \text{ MeV (5 \sim 6%)}, \\
\delta m_\phi &= -10 \sim -20 \text{ MeV (1 \sim 2%)},
\end{align*}

(5.2)

at the nuclear matter density. We have shown that the physical content of the mass shifts discussed in [1] are essentially the one due to the scattering lengths shown above and have resolved the discrepancy between [1] and [2].
One might naturally ask whether the previous QSR [2] for the scattering lengths is wrong or not. Compared with [2], the present analysis utilizes more available information, i.e. the constraint from the low energy theorem. In this sense, one may say that the present analysis is a more sound one. If one did not have such information on $T(0)$, one would have to use the approach in [2] with the inclusion of the matrix elements of the dim.$\geq 8$ or higher operators. In this sense, the way of constructing sum rule itself in [2] is also correct.

Another point we wish to emphasize is that regardless of the availability of the information on $T(0)$ (such as the low energy theorem), the sum rule for $m^*_V$ in (1.3) [1] in the linear density approximation is automatically equivalent to the mass shift due to the scattering lengths as is shown in (1.11).

Finally, we wish to make some comments on the interpretation in the literature about the mass shifts of the vector mesons in the nuclear medium. Several effective theories for the vector mesons ($\rho$, $\omega$) [19] predicts decreasing masses in the nuclear medium, and the magnitude of the mass shifts is quite similar to the QSR analysis in [1]. Accordingly, the “similarity” and “consistency” between QSR in medium and the effective theories has been erroneously advertized in the literature [18, 4]. The essential ingredient of the mass shifts predicted by those effective theories is the polarization in the Dirac sea of the nuclear medium, which leads to a smaller effective mass of the nucleon in the nuclear medium. If one switch off this effect, the vector meson propagators receives only the effects of the Fermi sea of the nucleons, which leads to small positive mass shifts of those vector mesons [20]. One has to recognize that the physical effect which QSR for the vector mesons in medium is enjoying is simply the scattering with this Fermi sea of the nucleons (through the forward scattering amplitude with the nucleon) which has the same mass as in the vacuum, and accordingly the QSR in medium does not pick up any effect of the polarization of the Dirac sea of the nucleons. Similarity in prediction on the mass shift between the medium QSR [1] and the effective theories [19] looks fortuitous and rather causes new problems. As has been clarified in this work, the medium QSR presented by [1] should be interpreted as a QCD sum rule analysis on the vector current-nucleon forward amplitude, and should not be interpreted as a method which picks up an effect of the vacuum polarization due the finite baryon number density. It is misleading to celebrate the medium QSR in [1] as a tool to incorporate the effect of “change of QCD vacuum” due to the finite baryon density.

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APPENDIX

A. Spin-average of $T_{\mu\nu}$

$T_{\mu\nu}$ in (2.1) can be decomposed as

$$T_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}\right)T_1 + \frac{1}{M_N^2} \left(p_{\mu} - \frac{p \cdot q}{q^2} q_{\mu}\right) \left(p_{\nu} - \frac{p \cdot q}{q^2} q_{\nu}\right) T_2 + \cdots,$$

(A.1)

where $\cdots$ denotes the pieces which depend on the nucleon spin and vanish after averaging over the nucleon spin. Averaging over the helicities of the vector current can be done by the following procedure:

$$T(\omega, q) \equiv \frac{1}{3} \sum_{\epsilon=0} \epsilon^{(h)}_{\mu}(q) T_{\mu\nu} \epsilon^*_{\nu}(q) = \frac{1}{3} \left(-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2}\right) T^{\mu\nu} = \frac{1}{3} T^{\mu}_{\mu} = T_1 - \frac{1}{3} \left(1 - \frac{(p \cdot q)^2}{M_N^2 q^2}\right) T_2,$$

(A.2)

where the use has been made of the relation $\sum_{\epsilon=0} \epsilon^{(h)}_{\mu} \epsilon^*_{\nu} = -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2}$. By this spin-averaging $T$-matrix is projected to the combination,

$$\sum_{J_z=\pm 1/2, \pm 3/2} \mathcal{T}(J = 3/2, J_z) + \sum_{J_z=\pm 1/2} \mathcal{T}(J = 1/2, J_z)$$

$$\omega \to m_N q \to 0 \quad \frac{1}{6} 8\pi (m_V + M_N) (4a_{3/2} + 2a_{1/2})$$

$$= 8\pi (m_V + M_N) a_V$$

(A.3)

with the $V - N$ spin-averaged scattering length $a_V = \frac{1}{3}(2a_{3/2} + a_{1/2})$.

B. Born diagram contribution to $T_{\mu\nu}$

Here we summarize the Born diagram contribution (Fig. 1) to $T_{\mu\nu}$. This is the only contribution to the vector current-nucleon forward amplitude in the $q^\mu \to 0$ limit. For $J^\rho_\mu$, it becomes after averaging over the nucleon spin,

$$T_{\mu\nu}(p, q) = \frac{-1}{(p + q)^2 - M_N^2 + i\varepsilon} \frac{1}{42} \text{Tr} \left[\gamma_\mu (\not{q} + M_N) \gamma_\nu (\not{q} + M_N)\right]$$

$$+ \frac{-1}{(p - q)^2 - M_N^2 + i\varepsilon} \frac{1}{42} \text{Tr} \left[\gamma_\mu (\not{q} - M_N) \gamma_\nu (\not{q} + M_N)\right].$$

(B.1)

In the following we consider the two quantities, $T = T^\mu_{\mu}/(-3)$ and $T_{00}$. A straightforward calculation gives

$$T(p, q) = \frac{1}{3} \left[\frac{M_N^2 - p \cdot q}{q^2 + 2p \cdot q + i\varepsilon} + \frac{M_N^2 + p \cdot q}{q^2 - 2p \cdot q + i\varepsilon}\right].$$

(B.2)
At \( p = 0 \) and \( q = 0 \), \( T \) becomes
\[
T(\omega, q = 0) = \frac{2M_N^2}{\omega^2 - 4M_N^2}. \tag{B.3}
\]
On the other hand, \( T_{00} \) at \( p = 0 \) is calculated to be
\[
T_{00}(\omega, q) = \frac{2M_N^2 q^2}{(q^2)^2 - 4M_N^2 \omega^2}. \tag{B.4}
\]
From (B.3) and (B.4), one sees
\[
\lim_{|q| \to 0} T_{00}(\omega, q) |q|^2 = \lim_{|q| \to 0} \frac{T_{\mu\mu}(\omega, q)}{-3\omega^2} = \frac{2M_N^2}{\omega^2(\omega^2 - 4M_N^2)}. \tag{B.5}
\]
To understand the relation between the Landau damping term \( \rho_{sc} \) in [1] and \( T^{\text{Born}}(0) \), we consider the spectral function
\[
\rho_{00}(\omega, q) = \frac{1}{\pi} (\theta(\omega) - \theta(-\omega)) \Im T_{00}(\omega, q)
\equiv \rho_{00}^{(0)}(\omega, q) + \rho_{00}^{(1)}(\omega, q), \tag{B.6}
\]
where
\[
\begin{align*}
\rho_{00}^{(0)}(\omega, q) &= \frac{M_N(2M_N + \omega)}{\sqrt{M_N^2 + q^2}} \delta(\omega + M_N - \sqrt{M_N^2 + q^2}) \\
&\quad - \frac{M_N(2M_N - \omega)}{\sqrt{M_N^2 + q^2}} \delta(\omega - M_N + \sqrt{M_N^2 + q^2}), \tag{B.8}
\end{align*}
\]
\[
\begin{align*}
\rho_{00}^{(1)}(\omega, q) &= -\frac{M_N(2M_N + \omega)}{\sqrt{M_N^2 + q^2}} \delta(\omega + M_N + \sqrt{M_N^2 + q^2}) \\
&\quad + \frac{M_N(2M_N - \omega)}{\sqrt{M_N^2 + q^2}} \delta(\omega - M_N - \sqrt{M_N^2 + q^2}). \tag{B.9}
\end{align*}
\]
At \( q \to 0 \), \( \rho_{00}^{(0)}(\omega, q) \) has a pole at \( \omega = \pm 0 \) and \( \rho_{00}^{(1)}(\omega, q) \) has poles at \( \omega = \pm 2M_N \). If we define \( T^{(0,1)}_{00}(\omega, q) \) by the dispersion integral
\[
T^{(0,1)}_{00}(\omega, q) = \int_{-\infty}^{\infty} du \rho^{(0,1)}_{00}(u, q) \frac{\rho_{00}^{(0,1)}(u, q)}{u - \omega - i\varepsilon}, \tag{B.10}
\]
then we get
\[
\lim_{q \to 0} \frac{T^{(0)}_{00}(\omega, q)}{|q|^2} = -\frac{1}{2\omega^2}, \tag{B.11}
\]
\[
\lim_{q \to 0} \frac{T^{(1)}_{00}(\omega, q)}{|q|^2} = \frac{1}{2(\omega^2 - 4M_N^2)}. \tag{B.12}
\]
Thus \( \lim_{q \to 0} T^{\mu\mu}/(-3\omega^2) \), \( \lim_{q \to 0} T^{(0)}_{00}(\omega, q)/|q|^2 \) and \( \lim_{q \to 0} T_{00}(\omega, q)/|q|^2 \) have a pole at \( \omega = 0 \) with the same residue \(-1/2\). In the QSR in the medium [1], HL essentially calculated \( \lim_{q \to 0} T^{(0)}_{00}(\omega, q)/|q|^2 \) starting from the spectral function for the Landau damping term.
C. Dispersion relations

Here we summarize the basics of the dispersion relations, since they are essential in carrying out the QSR analyses. The contents in this appendix is trivial, as long as one is careful enough. We dare to add this appendix, since some of the criticisms raised in [1] appear to originate from the misunderstanding of the dispersion relation.

In the text, scalar functions $\Pi(q^2)$ and $\Pi_1(q^2)$ are defined from the vector current correlator in the vacuum as

$$\Pi_0^{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2) \Pi_1(q^2) = \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \Pi(q^2). \tag{C.1}$$

In the medium, $\Pi^{\text{NM}}(q^2)$ and $\Pi_1^{\text{NM}}(q^2)$ are defined at $q = 0$ as

$$\Pi^{\text{NM}}(\omega^2) = \Pi^{\text{NM}}_{\mu\mu}(\omega, 0)/(−3), \quad \Pi_1^{\text{NM}}(\omega^2) = \Pi^{\text{NM}}(\omega^2)/\omega^2. \tag{C.2}$$

$\Pi^{\text{NM}} \to \Pi$ and $\Pi_1^{\text{NM}} \to \Pi_1$ as $\rho_N \to 0$. In the following, we discuss the dispersion relations for $\Pi^{\text{NM}}_1$ and $\Pi^{\text{NM}}$. (Those for $\Pi_1$ and $\Pi$ are parallel.) $\Pi^{\text{NM}}_1(\omega^2)$ has an isolated pole of first order at $\omega^2 = 0$ and has a Laurent expansion as $\Pi^{\text{NM}}_1(\omega^2) = c_{−1}^1 + c_0 + c_1 \omega^2 + \cdots$. Anticipating a logarithmic divergence in the dispersion integral, we introduce one subtraction for $\Pi^{\text{NM}}_1$.

Applying the Cauchy’s theorem, one gets

$$\frac{1}{2\pi i} \oint_{C_1} ds \frac{\Pi^{\text{NM}}_1(s)}{s(s - \omega^2)} = \frac{\Pi^{\text{NM}}_1(\omega^2)}{\omega^2} + \lim_{s \to 0} \frac{d}{ds} \left[ \frac{s\Pi^{\text{NM}}_1(s)}{s - \omega^2} \right] = \frac{\Pi_1(\omega^2)}{\omega^2} - \frac{c_0}{\omega^2} - \frac{c_{−1}}{\omega^4}, \tag{C.3}$$

where the closed path $C_1$ is taken as shown in Fig. 5. Equation (C.3) can be written as

$$\Pi^{\text{NM}}_1(\omega^2) = \frac{c_{−1}}{\omega^2} + c_0 + \frac{\omega^2}{\pi} \int_{0^+}^{\infty} ds \frac{\text{Im}\Pi^{\text{NM}}_1(s)}{(s - i\varepsilon)(s - \omega^2 - i\varepsilon)}. \tag{C.4}$$

In (C.4) the pole contribution at $\omega^2 = 0$ is explicitly taken into account separately and thus the integral along the positive real axis excludes $s = 0$. This is also the same for (C.6) below. For $\Pi^{\text{NM}}$, we need additional subtraction. We thus consider

$$\frac{1}{2\pi i} \oint_{C_1} ds \frac{\Pi^{\text{NM}}(s^2)}{s^2(s - \omega^2)}. \tag{C.5}$$

By repeating the same step as above, we get

$$\Pi^{\text{NM}}(\omega^2) = c_{−1} + c_0 \omega^2 + \frac{\omega^4}{\pi} \int_{0^+}^{\infty} ds \frac{\text{Im}\Pi^{\text{NM}}(s)}{(s - i\varepsilon)^2(s - \omega^2 - i\varepsilon)}. \tag{C.6}$$

Since $\text{Im}\Pi^{\text{NM}}(s) = \text{Im} s \Pi_1(s)$, (C.6) is nothing but the relation obtained by multiplying $\omega^2$ to both sides of (C.3). Therefore it is trivially correct that, regardless of which sum rules we start with, $\Pi^{\text{NM}}_1$ or $\Pi^{\text{NM}}$, we get the same FESR. This also applies to $\Pi_1$ and $\Pi$ in the vacuum, in which case $c_{−1} = 0$ and $c_0 = \Pi_1(0)$. 

18
Next we explicitly demonstrate how (3.10) and the FESR for $\Pi_1$ and $\Pi$ are obtained in the vacuum, making clear the implicit assumptions behind. As is shown in (C.4), the QSR for $\Pi_1(q^2)$ is

$$\Pi_1^{\text{OPE}}(q^2) = \Pi_1(0) + \frac{q^2}{\pi} \int_{0^+}^{\infty} ds \frac{\text{Im}\Pi_1(s)}{s(s-q^2)}. \quad (C.7)$$

Using the explicit forms in (3.8) and (3.9), we get

$$-\frac{1}{8\pi^2}\ln\frac{q^2}{\Lambda^2} + \frac{A}{q^4} + \frac{B}{q^6} = \frac{F'}{m^2_V - q^2} - \frac{F'}{m^2_V} - \frac{1}{8\pi^2}\ln\frac{s_0 - q^2}{-q^2} + \Pi_1(0) \quad (C.8)$$

where we explicitly introduced a finite scale $\Lambda^2$ which specifies a renormalization scheme in the OPE side, and the integral in the right hand side converges due to the subtraction introduced in the dispersion relation. We rewrite (C.9) as

$$\frac{1}{8\pi^2}\ln\frac{s_0 - q^2}{-q^2} + \frac{A}{q^4} + \frac{B}{q^6} = \frac{F'}{m^2_V - q^2} + \left\{-\frac{F'}{m^2_V} - \frac{1}{8\pi^2}\ln\frac{\Lambda^2}{s_0} + \Pi_1(0)\right\}. \quad (C.9)$$

By comparing the large $-q^2$-behavior in both sides of (C.9), one sees that $\{\cdots\}$ piece (constant terms) in the right hand side should vanish, which is the consistency requirement for this sum rule. This is the implicit assumption which leads to the sum rule in (3.10). In the Borel sum rule analysis, this mentioning is irrelevant, since $\{\cdots\}$ piece simply disappears after the Borel transform. Under this condition we get three FESRs' for the three unknowns $F$, $m_V$ and $s_0$, by expanding both sides of (C.9) with respect to $1/q^2$ and comparing the coefficients of $1/q^2$, $1/q^4$ and $1/q^6$.

If one start with $\Pi(q^2)$ using the dispersion relation (C.6) what we get is simply the relation which is obtained by multiplying $q^2$ to both sides of (C.9). We then get the same FESRs'.

As is mentioned in the text, whether one applies Borel transform to $\Pi_1$ or $\Pi$ causes numerical difference, especially because the polynomial terms disappear by the Borel transform. But the FESR gives the same result regardless of which one we start with.
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Figure captions

**Fig. 1** Born diagram contribution to $T_{\mu\nu}$.

**Fig. 2** Borel curves for the $V - N (V = \rho, \omega)$ spin-isospin averaged scattering lengths in the DBSR. The results with and without explicit Born term (cases (i) and (ii)) are shown.

**Fig. 3** Borel curve for the $\phi - N$ spin-averaged scattering length in the DBSR.

**Fig. 4** Vector meson masses in the nuclear medium ($m_V^*$) normalized by their vacuum values ($m_V$) as a function of the nucleon density $\rho_N$ obtained from the scattering lengths $a_V$ in the linear density approximation.

**Fig. 5** Closed path $C_1$ for the dispersion integral in (C.3).
Fig. 1
Fig. 2
Scattering Length (fm)

Borel Mass $M^2$ (GeV$^2$)

$S_0 = 2.0$

Fig. 3
Fig. 4
Fig. 5