An allocation scheme for estimating the reliability of a parallel-series system

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Abstract

We give a hybrid two stage design which can be useful to estimate the reliability of a parallel–series and/or by duality a series–parallel system, when the component reliabilities are unknown as well as the total numbers of units allowed to be tested in each subsystem. When a total sample size is fixed large, asymptotic optimality is proved systematically and validated \textit{via} Monte Carlo simulation.

Keywords: Reliability, Parallel-series, Two stage design, Asymptotic optimality

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1. Introduction

In reliability engineering two crucial objectives are considered: (1) to maximize an estimate of system reliability and (2) to minimize the variance of the reliability estimate. Because system designers and users are risk-averse, they generally prefer the second objective which leads to a system design with a slightly lower reliability estimate but a lower variance of that estimate. In the case of parallel–series and/or by duality series–parallel systems just as was defined in \cite{1}, the variance of the reliability estimate can be lowered by allocation of a fixed sample size, while reliability estimate is obtained by testing components \cite{2}. Allocation schemes for estimation with cost \cite{3} lead

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generally to a discrete optimization problem which can be solved sequentially using adaptive designs in a fixed or a Bayesian framework [4, 5, 6, 2, 7]. In [8] a reliability sequential schemes (R-SS) was applied successfully to a parallel-series systems, when the total number of units to be tested in each subsystem was fixed. Recently in [9] a two stage design for the same purpose was presented and shown to be asymptotically optimal when the subsystems sample sizes are fixed and large at the same order of the total sample size of the full system. The problem considered in this paper is useful to estimate the reliability of a parallel-series and/or by duality a series-parallel system, when the component reliabilities are unknown as well as the total numbers of units allowed to be tested in each subsystem. This work improves the results in [9] by developing a hybrid two stage design to get a dynamic allocation between the sample sizes allowed for subsystems and those allowed for their components.

In section 2, definitions and preliminary results are presented accompanied by the proper two stage design for a parallel subsystem just as was defined in [9] and its asymptotic optimality is proved for a fixed and large sample size. In section 3 the full parallel-series system is considered and it is shown that the variance of its reliability estimate has a lower bound independent of allocation. This leads, in section 4 to the main result of this paper which lies in the hybrid two stage algorithm and its asymptotic optimality for a fixed and large sample size allowed for the full system. In section 5, the results are validated via Monte Carlo simulation and it is shown that our algorithm leads asymptotically to the best allocation scheme to reach the lower bound of the variance of the reliability estimate. The last section is reserved for conclusion and remarks.

2. Preliminary results

Consider a system $S$ of $n$ subsystems $S_1, S_2, \ldots, S_n$ connected in series, each subsystem $S_j$ contains $n_j$ components $S_{1j}, S_{2j}, \ldots, S_{n_j}$ connected in parallel. The system should be referred as parallel-series system (cf. [1]). Assume s-independence within and across populations, then the system reliability is

$$R = \prod_{j=1}^{n} R_j$$  (1)
where

\[ R_j = 1 - \prod_{i=1}^{n_j} (1 - R_{ij}) \]

is the reliability of the parallel subsystem \( S_j \) and \( R_{ij} \) the reliability of component \( S_{ij} \). An estimator of \( R \) is assumed to be the product of sample reliabilities

\[ \hat{R} = \prod_{j=1}^{n} \hat{R}_j \]

where

\[ \hat{R}_j = 1 - \prod_{i=1}^{n_j} (1 - \hat{R}_{ij}) \]

and \( \hat{R}_{ij} \) is the sample mean of functioning units in component \( S_{ij} \),

\[ \hat{R}_{ij} = \frac{\sum_{l=1}^{M_{ij}} X_{ij}^{(l)}}{M_{ij}} \]

\( \hat{R}_{ij} \) is used to estimate \( R_{ij} \) where \( M_{ij} \) is the sample size and \( X_{ij}^{(l)} \) is the binary outcome of the unit \( l \) in component \( S_{ij} \). Hence, for each subsystem \( S_j \), one must allocate

\[ T_j = \sum_{i=1}^{n_j} M_{ij} \]

units such that the estimated reliability of the full system is based on a total sample size

\[ T = \sum_{j=1}^{n} T_j \]

As in the series case, the variance of the estimated reliability \( \hat{R} \) incurred by any allocation scheme can be obtained

\[ Var \left\{ \hat{R} \right\} = \prod_{j=1}^{n} \left( Var \left( \hat{R}_j \right) + R_j^2 \right) - \prod_{j=1}^{n} R_j^2 \]

(2)
where

\[ \text{Var} \left\{ \hat{R}_j \right\} = (1 - R_j)^2 \left[ \prod_{i=1}^{n_j} \left( 1 + \frac{c_{ij}^{-2}}{M_{ij}} \right) - 1 \right] \tag{3} \]

is given as a function of the allocation numbers \( M_{ij} \) and the coefficients of variation of Bernoulli populations

\[ c_{ij} = \sqrt{1/R_{ij} - 1} \]

We have found convenient to work with the equivalent expression of (3)

\[ \text{Var} \left\{ \hat{R}_j \right\} = (1 - R_j)^2 \left[ \sum_{i=1}^{n_j} \frac{c_{ij}^{-2}}{M_{ij}} + F \left( \frac{c_{1j}^{-2}}{M_{1j}}, \ldots, \frac{c_{nj}^{-2}}{M_{nj}} \right) \right] \]

where

\[ F \left( \frac{c_{1j}^{-2}}{M_{1j}}, \ldots, \frac{c_{nj}^{-2}}{M_{nj}} \right) \]

is a sum over all the products of at least two of its arguments.

The problem is to estimate \( \hat{R} \) when components reliabilities are unknowns and a total number of \( T \) units must be tested in the full system. The aim is to minimize the variance of \( \hat{R} \). Hence, the problem can be addressed by developing allocation schemes to select \( M_{ij} \), the numbers of units to be tested in each component \( i \) in the subsystem \( j \), under the constraint

\[ \sum_{j=1}^{n} \sum_{i=1}^{n_j} M_{ij} = T \tag{4} \]

such that the variance of \( \hat{R} \) is as small as possible. Reliability sequential schemes (R-SS) exist for the series, parallel or parallel-series configurations when the sample sizes \( T_j \) of the subsystems are fixed. Therefore, one can fully optimize the variance of \( \hat{R} \) just by applying the (R-SS) to find the best partition \( T_1, T_2, ..., T_n \) of \( T \). Unfortunately, a full sequential design can not be used in practice for large systems since the number of operations will growth dramatically. For this reason, we reasonably propose a hybrid two stage design which is shown to be asymptotically optimal when \( T \) is large.
2.1. Lower bound for the variance of the estimated reliability of the parallel subsystem $S_j$

For the asymptotic optimization of the variance of the estimated reliabilities, we make use of the well-known Lagrange’s identity which can be written in the form:

Let $a_i > 0$, $N_i > 0$, for $i = 1, \ldots, k$ and $N = N_1 + \cdots + N_k$, then the following identity holds.

$$\sum_{i=1}^{k} \frac{a_i}{N_i} = N^{-1} \left( \left( \sum_{i=1}^{k} \sqrt{a_i} \right)^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \frac{(N_i \sqrt{a_j} - N_j \sqrt{a_i})^2}{N_i N_j} \right)$$  \hspace{1cm} (5)

**Proposition 1.** Denote by

$$Q_j = (1 - R_j)^2 T_j^{-1} \left( \sum_{i=1}^{n_j} c_{ij}^{-1} \right)^2$$  \hspace{1cm} (6)

then

$$\text{Var} \left\{ \hat{R}_j \right\} \geq Q_j$$

**Proof.** The proof is a direct consequence of the previous identity (5). Indeed

$$\begin{align*}
\text{Var} \left\{ \hat{R}_j \right\} &= (1 - R_j)^2 T_j^{-1} \left( \sum_{i=1}^{n_j} c_{ij}^{-1} \right)^2 \\
&+ T_j^{-1} (1 - R_j)^2 \sum_{i=1}^{n_j-1} \sum_{k=i+1}^{n_j} \frac{(M_{ij} c_{kj}^{-1} - M_{kj} c_{ij}^{-1})^2}{M_{ij} M_{kj}} \\
&+ (1 - R_j)^2 F \left( \frac{c_{ij}^{-2}}{M_{1j}}, \frac{c_{2j}^{-2}}{M_{2j}}, \ldots, \frac{c_{nj}^{-2}}{M_{nj}} \right)
\end{align*}$$  \hspace{1cm} (7)

2.2. The two stage design for the parallel subsystem $S_j$

Following the expansion (7) and since $F$ contains second order terms (see later), one gives interest to the numbers $M_{ij}$ which minimize the expression

$$T_j^{-1} \sum_{i=1}^{n_j-1} \sum_{k=i+1}^{n_j} \frac{(M_{ij} c_{kj}^{-1} - M_{kj} c_{ij}^{-1})^2}{M_{ij} M_{kj}}$$
Thus $M_{ij}$ must verify for $i = 1, \ldots, n_j$

$$M_{ij} c_{kj}^{-1} = M_{kj} c_{ij}^{-1}$$

which implies that

$$M_{ij} = T_j \frac{c_{ij}^{-1}}{\sum_{k=1}^{n_j} c_{kj}^{-1}}$$

(8)

If one assumes that $T_j$ is fixed then a proper two stage scheme can be used to determine $M_{ij}$, just as was defined in [9], as follows:

Choose $L_j$ as a function of $T_j$ such that:

(i) $L_j$ must be large if $T_j$ is large,
(ii) $L_j \leq \frac{T_j}{n_j}$,
(iii) $\lim_{T_j \rightarrow \infty} \frac{L_j}{T_j} = 0$.

One can take for example $L_j = \lceil \sqrt{T_j} \rceil$, where $[.]$ denotes the integer part.

**Stage 1.** Sample $L_j$ units from each component $i$ in the subsystem $j$, estimate $c_{ij}$ by its maximum likelihood estimator (M.L.E)

$$\hat{c}_{ij} = \sqrt{\frac{L_j}{\sum_{l=1}^{L_j} X_{ij}^{(l)}}} - 1$$

and define the predictor, according to (8),

$$\hat{M}_{ij} = \left[ T_j \frac{\hat{c}_{ij}^{-1}}{\sum_{k=1}^{n_j} \hat{c}_{kj}^{-1}} \right], \ i = 1, \ldots, n_j - 1$$

**Stage 2.** Sample $T_j - n_j L_j$ units for which $M_{ij} - L_j$ are units from component $i$ in the subsystem $j$ where $M_{ij}$ is the corrector of $\hat{M}_{ij}$ defined by

$$M_{ij} = \max \left\{ L_j, \hat{M}_{ij} \right\}, \ i = 1, \ldots, n_j - 1,$$

$$M_{n_j j} = T_j - \sum_{k=1}^{n_j - 1} M_{kj}$$
Theorem 1. Choosing the $M_{ij}$ according to the previous two stage sampling scheme, one obtains

$$\lim_{T_j \to \infty} T_j \left( \text{Var} \left\{ \hat{R}_j \right\} - Q_j \right) = 0$$

Proof. From relation (7), one can write

$$T_j \left( \text{Var} \left\{ \hat{R}_j \right\} - Q_j \right) = (1 - R_j)^2 \sum_{i=1}^{n_j-1} \sum_{k=i+1}^{n_j} \frac{(M_{ij} c^{-1}_{kj} - M_{kj} c^{-1}_{ij})^2}{M_{ij} M_{kj}}$$

$$+ (1 - R_j)^2 T_j F \left( \frac{c_{ij}^{-2}}{M_{ij}}, ..., \frac{c_{n_j j}^{-2}}{M_{n_j j}} \right)$$

(9)

When $T_j$ is large enough, condition (iii) gives $M_{ij} = \hat{M}_{ij}$ for $i = 1, ..., n_j - 1$. So, the strong law of large numbers with the integer part properties give, when $T_j \to \infty$,

$$\frac{M_{ij}}{M_{kj}} \to c_{kj}$$

for $i = 1, ..., n_j$. Hence,

$$\frac{(M_{ij} c^{-1}_{kj} - M_{kj} c^{-1}_{ij})^2}{M_{ij} M_{kj}} = \frac{M_{ij}}{M_{kj}} \left( c_{kj}^{-1} - \frac{M_{kj}}{M_{ij}} c_{ij}^{-1} \right)^2 \to 0, \text{ as } T_j \to \infty,$$

(10)

and on the other hand

$$T_j F \left( \frac{c_{ij}^{-2}}{M_{ij}}, ..., \frac{c_{n_j j}^{-2}}{M_{n_j j}} \right) \to 0, \text{ as } T_j \to \infty,$$

(11)

which achieves the proof.

3. Lower bound for the variance of the estimated reliability of $S$

We consider now the full system $S$. From expression (2), one can write

$$\text{Var} \left\{ \hat{R} \right\} = R^2 \left[ \prod_{j=1}^{n} \left( \frac{\text{Var} \left\{ \hat{R}_j \right\}}{R_j^2} + 1 \right) - 1 \right]$$

The following theorem gives a lower bound for the variance of $\hat{R}$. 

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Theorem 2. Denote by

\[ Q = T^{-1} R^2 \left[ \sum_{j=1}^{n} \frac{1 - R_j}{R_j} \left( \sum_{i=1}^{n_j} c_{ij}^{-1} \right) \right]^2 \]

then

\[ \text{Var} \left\{ \hat{R} \right\} \geq Q \]

Proof. Expanding the right hand side of (2) and using (1), one obtains

\[ \text{Var} \left\{ \hat{R} \right\} = R^2 \left[ \sum_{j=1}^{n} \text{Var} \left( \hat{R}_j \right) \right] + F \left( \frac{\text{Var} \left( \hat{R}_1 \right)}{R_1^2}, \ldots, \frac{\text{Var} \left( \hat{R}_n \right)}{R_n^2} \right), \]

which gives with the help of theorem (1)

\[ \text{Var} \left\{ \hat{R} \right\} \geq R^2 \sum_{j=1}^{n} Q_j \frac{R_j^2}{T_j} = R^2 \sum_{j=1}^{n} \left( \frac{1 - R_j}{R_j} \sum_{i=1}^{n_j} c_{ij}^{-1} \right)^2 \]

(12)

This last expression has the form

\[ R^2 \sum_{j=1}^{n} \frac{a_j}{T_j} \]

which can be expanded, thanks to identity (5), as follows

\[ R^2 T^{-1} \left[ \sum_{j=1}^{n} \frac{1 - R_j}{R_j} \left( \sum_{k=1}^{n_j} c_{kj}^{-1} \right) \right]^2 \]

\[ + R^2 T^{-1} \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left( \frac{T_i}{R_i} \sum_{k=1}^{n_i} c_{ki}^{-1} - \frac{T_j}{R_j} \sum_{k=1}^{n_j} c_{kj}^{-1} \right)^2 \]

(13)

and as a consequence

\[ \text{Var} \left\{ \hat{R} \right\} \geq T^{-1} R^2 \left[ \sum_{j=1}^{n} \frac{1 - R_j}{R_j} \left( \sum_{k=1}^{n_j} c_{kj}^{-1} \right) \right]^2 = Q, \]

which achieves the proof.
4. The hybrid two stage design for the full system $S$

Similarly to the case of a subsystem $S_j$ from expressions (12) and (13), one gives interest to the numbers $T_j$ which minimize the quantity

$$\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} \left( \frac{T_i^{1-R_{i}} \sum_{k=1}^{n_j} c_{kj}^{-1} - T_j^{1-R_{j}} \sum_{k=1}^{n_i} c_{ki}^{-1}}{T_i T_j} \right)^2$$

and obtains the asymptotic optimality criteria

$$\frac{T_i}{T_j} = \frac{\frac{1-R_i}{R_i} \sum_{k=1}^{n_j} c_{ki}}{\frac{1-R_j}{R_j} \sum_{k=1}^{n_i} c_{kj}}$$

for all $i, j \in \{1, 2, ..., n\}$, which gives the rule

$$T_j = T \frac{\frac{1-R_j}{R_j} \sum_{k=1}^{n_j} c_{kj}^{-1}}{\sum_{k=1}^{n} \frac{1-R_k}{R_k} \sum_{i=1}^{n_k} c_{ik}^{-1}}$$  \hspace{1cm} (14)$$

We can now implement a hybrid two stage design for the determination of the numbers $T_j$ as well as $M_{ij}$ as follows:

**Stage 1** choose $L = \left\lceil \sqrt{T} \right\rceil$: one applies the two stage scheme given in sub-section (2.1) for each subsystem $S_j$ with $T_j = L$ and $L_j = \left\lceil \sqrt{T_j} \right\rceil$.

Next, obtain the predictor, according to the rule (14),

$$\hat{T}_j = \left[ \begin{array}{c} \frac{1-R_j}{R_j} \sum_{k=1}^{n_j} \hat{c}_{kj}^{-1} \\ \sum_{k=1}^{n} \frac{1-R_k}{R_k} \sum_{i=1}^{n_k} \hat{c}_{ik}^{-1} \end{array} \right], \ j = 1, \ldots, n-1.$$

**Stage 2** define the corrector

$$T_j = \max \left\{ L, \hat{T}_j \right\}, \ j = 1, \ldots, n-1,$$

$$T_n = T - \sum_{j=1}^{n-1} T_j,$$
and take back the two stage scheme for each subsystem $S_j$ to calculate $M_{ij}$ with the sample size equals $T_j$.

Now, the main result of this paper is given by the following theorem.

**Theorem 3.** Choosing the $T_j$ and $M_{ij}$ according to the hybrid two stage design, one obtains

$$\lim_{T \to \infty} T \left( \text{Var} \left\{ \hat{R} \right\} - Q \right) = 0$$

where $Q$ is defined in theorem (2).

**Proof.** The relation (9) implies that

$$\text{Var} \left\{ \hat{R}_j \right\} = Q_j + T_j^{-1} (1 - R_j)^2 \sum_{i=1}^{n_j-1} \sum_{k=i+1}^{n_j} \frac{(M_{ij} c_{ij}^{-1} - M_{kj} c_{kj}^{-1})^2}{M_{ij} M_{kj}}$$

$$+ (1 - R_j)^2 F \left( \frac{c_{ij}^2}{M_{ij}}, \ldots, \frac{c_{n_j}^2}{M_{n_j}} \right)$$

As a consequence of the hybrid two stage design and the strong law of large numbers, $T_j/T_j$ and $T_j/M_{ij}$ remain bounded for all $i, j$ as $T \to \infty$. It follows that, as $T \to \infty$,

$$F \left( \frac{c_{ij}^2}{M_{ij}}, \ldots, \frac{c_{n_j}^2}{M_{n_j}} \right) = o \left( T^{-1} \right),$$

and

$$T_j^{-1} \sum_{i=1}^{n_j-1} \sum_{k=i+1}^{n_j} \frac{(M_{ij} c_{ij}^{-1} - M_{kj} c_{kj}^{-1})^2}{M_{ij} M_{kj}} = o \left( T^{-1} \right),$$

thanks to (10) and (11). Thus,

$$\text{Var} \left\{ \hat{R}_j \right\} = Q_j + o \left( T^{-1} \right), \text{ as } T \to \infty$$

which implies that

$$\prod_{j=1}^{n} \left( \frac{\text{Var} \left\{ \hat{R}_j \right\}}{R_j^2} + 1 \right) = \prod_{j=1}^{n} \left( \frac{Q_j}{R_j^2} + 1 + o \left( T^{-1} \right) \right)$$

$$= \prod_{j=1}^{n} \left( \frac{Q_j}{R_j^2} + 1 \right) + o \left( T^{-1} \right)$$
As a consequence

$$\lim_{T \to \infty} T \left( \text{Var} \left\{ \hat{R} \right\} - Q \right) = R^2 \lim_{T \to \infty} T \left[ \prod_{j=1}^{n} \left( \frac{Q_j}{R_j^2} + 1 \right) - 1 - Q \right]$$

Now, expanding the product within the limit and applying identity (5), after having replaced $Q_j$ by its expression (6), one obtains

$$\prod_{j=1}^{n} \left( \frac{Q_j}{R_j^2} + 1 \right) - 1 = R^2 \left[ \sum_{j=1}^{n} Q_j R_j^2 + F \left( \frac{Q_1}{R_1^2}, \ldots, \frac{Q_n}{R_n^2} \right) \right]$$

$$= Q + R^2 (A + B)$$

where

$$A = T^{-1} \sum_{i=1}^{n-1} \sum_{k=i+1}^{n} \frac{T_i \left( \frac{1-R_k}{R_k} \right) \left( \sum_{l=1}^{n_k} c_{lk}^{-1} \right) - T_k \left( \frac{1-R_i}{R_i} \right) \left( \sum_{l=1}^{n_k} c_{li}^{-1} \right)^2}{T_i T_k}$$

$$B = F \left( \frac{Q_1}{R_1^2}, \ldots, \frac{Q_n}{R_n^2} \right)$$

Once more, the hybrid two stage allocation scheme and the strong law of large numbers provide

$$\lim_{T \to \infty} T.A = 0$$

and

$$\lim_{T \to \infty} T.B = 0$$

which achieves the proof.

5. Monte Carlo simulation

Let us remark first that the lower bound $Q$ is a first order approximation of the optimal variance of the reliability estimate under the constraint (4) when $T$ is large.

In the first experiment, as in figure (1), we consider a simple parallel-series system of two subsystems each one, with varying reliabilities and a
fixed sample size \( T = 20 \). For each situation A, B, C and D and for each partition sample size \( \{T_1, T - T_1\} \) where \( T_1 \) varies from \( \sqrt{T} \) to \( T - \sqrt{T} \), we have applied the proper two stage design for each parallel subsystem and reported in a bar diagram \( Var(\hat{R}) \) as a function of \( T_1 \), see figure (3). On the other hand, in table (1), we have reported the expected value of \( T_1 = M_{11} + M_{21} \) given by the hybrid two stage design. As expected, our scheme gives the best allocation for each situation.

The second experiment deals with a non trivial parallel-series system just as in [9], where subsystems are composed, respectively, of 2, 3, 4 and 5 components, see figure (2). The partition total numbers \( T_j \) to test in each subsystem are evaluated systematically by the hybrid two stage design while their sum \( T \) is incremented from 100 to 10000 by step of 100. The figure (4) shows the rate of the excess of variance \( T \left( Var(\hat{R}) - Q \right) \) at logarithmic scale as a function of the sample size \( T \). The asymptotic optimality of the hybrid scheme is validated.

\[
\begin{align*}
R_{11} & \quad R_{21} \\
R_{12} & \quad R_{22}
\end{align*}
\]

\( T_1 = M_{11} + M_{21} \)

\( T_2 = M_{12} + M_{22} \)

Figure 1: A simple parallel-series system of two subsystems with two components each one

6. Conclusion

The proof of the first order asymptotic optimality for the proper two stage design for a parallel subsystem as well as for the hybrid two stage design for
Figure 2: A non trivial parallel-series system.

| System | $R_{11} - R_{21} - R_{12} - R_{22}$ | $E(T_1)$ |
|--------|------------------------------------|----------|
| A      | 0.1 - 0.11 - 0.9 - 0.99            | 16       |
| B      | 0.5 - 0.55 - 0.51 - 0.6            | 11       |
| C      | 0.9 - 0.99 - 0.1 - 0.11           | 4        |
| D      | 0.2 - 0.4 - 0.6 - 0.3             | 12       |

Table 1: Expected value of $T_1 = M_{11} + M_{21}$ given by the hybrid two stage design
Figure 3: Bar diagram $\text{Var}(\hat{R})$ as a function of $T_1$ for each case A, B, C, and D. $\hat{\cdot}$ shows the minimum of $\text{Var}(\hat{R})$.
Figure 4: Asymptotic optimality of the hybrid two stage design: the speed of the excess of variance $T(Var(\hat{R}) - Q)$ at logarithmic scale as a function of the sample size $T$

the full system has been obtained mainly through the following steps

- an adequate writing of the variance of the reliability estimate,
- a lower bound for this variance, independent of allocation,
- the allocation defined by the hybrid sampling scheme and the strong law of large numbers.

With a straightforward but tedious adaptation, the above study can be namely extended to deal with complex systems involving a multi-criteria optimization problem under a set of constraints such as risk, system weight, cost, performance and others, in a fixed or in a Bayesian framework.

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