The $\sigma$ and $f_0(980)$ from $K_{e4} \oplus \pi \pi$, $\gamma \gamma$ scatterings, $J/\psi, \phi \rightarrow \gamma \sigma_B$ and $D_s \rightarrow l\nu \sigma_B$

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Abstract

We extract the pole positions, hadronic and $\gamma \gamma$ widths of $\sigma$ and $f_0(980)$, from $\pi \pi$ and $\gamma \gamma$ scattering data using an improved analytic K-matrix model. Our results favour a large gluon component for the $\sigma$ and a $\bar{s}s$ or/and gluon component for the $f_0(980)$ but neither a large four-quark nor a molecule component. Gluonium $\sigma_B$ production from $J/\psi, \phi$, radiative and $D_s$ semi-leptonic decays are also discussed.

Keywords: $\gamma \gamma$ and $\pi \pi$ scatterings, radiative decays, light scalars, gluonia, four-quark states, QCD spectral sum rules.

1. Introduction

The hadronic and $\gamma \gamma$ couplings of light scalar mesons could provide an important information about their nature.

K-matrix model has been used to describe $\pi \pi$ and $\gamma \gamma$ processes \cite{1}. This model is improved by introducing a form factor shape function in a single channel \cite{2}, which is generalized \cite{3-5} to the coupled channels case in order to extract the pole positions and the previous couplings of the $\sigma$ and $f_0(980)$.

In this talk, we review these recent results.

2. Phenomenology of $\pi \pi$ scattering

1 channel $\oplus$ 1 ”bare” resonance

We introduce a real analytic form factor shape function \cite{2}:

$$f_p(s) = \frac{s - s_{AP}}{s - \sigma_{DP}}, \quad P = \pi, K$$

It allows for an Alder zero $s = s_{AP}$ and a pole at $\sigma_{DP} < 0$ to simulate left hand singularities. The unitary $I = 0$ S wave $\pi \pi$ scattering amplitude is then written as:

$$T_{PP} = \frac{g_1^2 f_p(s)}{s - s - g_1^2 f_p(s)} = \frac{g_1^2 f_p(s)}{D_p(s)},$$

where:

$$\text{Im} D = \text{Im}(-g_1^2 f_p) = -(\theta_{PP}) g_1^2 f_p.$$  \hspace{1cm} (3)

and hence:

$$\text{Im}(f_p) = (\theta_{PP}) f_p, \quad \rho_p = \sqrt{1 - 4m_p^2/s}. \hspace{1cm} (4)$$

The real part of $f_p$ is obtained from a dispersion relation with subtraction at $s = 0$,

$$f_p(s) = \frac{2}{\pi} (h_0(s) - h_0(0)),$$  \hspace{1cm} (5)

where $h_0(s)$ has been defined in Ref. \cite{2}.

0 bare resonance $\equiv \Lambda \Phi^4$ model

In this case, one can introduce another shape function $f_2(s)$:

$$T_{PP} = \frac{A f_2(s)}{1 - A f_2(s)}, \quad f_2(s) = \frac{s - s_{AP}}{s - \sigma_{DP}},$$

where $\sigma_{D1} = \sigma_{D2}$, and $f_2(s) = \frac{2}{\pi} (h_2(s) - h_2(0))$. The single channel results are shown in Fig. \[\text{Fig.}\quad \] and Table \[\text{Table}\quad \].

The result of $\Lambda \Phi^4$ model shows that the existence of $\sigma$ is not an artifact of a ”bare” resonance entering into the parametrization of $T_{PP}$.

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2 channels \( \oplus \) 2 "bare" resonances

The generalization to coupled channels is conceptually straightforward. We consider the \( \pi\pi - K\bar{K} \) coupled channels and introduce 2 "bare" resonances labeled \( a \) and \( b \), with bare masses squared \( s_{Ra} \) and \( s_{Rb} \). To leading order in SU(3) breakings, we shall approximately work in the minimal case with only one shape function. The phase shifts and inelasticity \( \eta \) are defined by:

\[
\eta e^{2i\delta} = 1 + 2\alpha \rho_\pi T_{PP} .
\]

We refer to [4] for the explicit expressions of \( T_{PP} \). We analyze 3 cases using different groups of \( \pi\pi - K\bar{K} \) data, which are shown in Table 2. With these choices, we expect to span all possible regions of space of parameters. The fit results are shown in Fig. 2 and Table 3 from which we derive the pole positions and hadronic couplings of the \( \sigma \) and \( J_0(980) \).

### Table 1: Values of the bare parameters in GeV\(^d\)\((d = 1, 2)\).

| Outputs | Set 1 | Set 2 | Set 3 | Average |
|---------|-------|-------|-------|---------|
| \( \sigma_{D_{1}} \) | 6.20(3.2) | 1.41(7) | 1.78(10) | |
| \( \sigma_{D_{2}} \) | 7.6(4.5) | - | - | |
| \( s_{Ra} \) | 108(34) | - | - | |
| \( s_{Rb} \) | 2.54(8) | 10.42(30) | 0.61(31) | |
| \( \Lambda \) | - | - | -0.39(8) | |
| \( \chi^2_{d.o.f} \) | 1244 | 113 | 0.78 | 0.79 |
| \( M_{\sigma} \) | 468(181) | 456(19) | 448(18) | 452(13) |
| \( \Gamma_{\sigma}/2 \) | 261(211) | 265(18) | 260(19) | 259(16) |
| \( |g_{\pi\pi\pi\pi}| \) | 1.291 | 1.016 | 1.016 | 1.016 |
| \( |g_{\pi\pi\pi\pi}| \) | 4.112 | 4.112 | 4.112 | 4.112 |
| \( |g_{\pi\pi\pi\pi}| \) | 0.016 | 0.016 | 0.016 | 0.016 |

### Table 2: Different data used for each set

| Input | Set 1 | Set 2 | Set 3 |
|-------|-------|-------|-------|
| \( \delta_\eta \) | \([6-8]\) | \([6-8]\) | \([6-8]\) |
| \( \eta \) | \([8]\) | \([9]\) | \([9]\) |
| \( \delta_{Ra} = \delta_a + \delta_\eta \) | \([8]\) | \([9]\) | \([9]\) |

### Table 3: Values of the bare parameters in GeV\(^2\)\((d = 1, 2)\).

| Outputs | Set 1 | Set 2 | Set 3 |
|---------|-------|-------|-------|
| \( s_{\Lambda} \) | 0.0161±0.0004 | 0.0132±0.006 | 0.010±0.006 |
| \( s_{\alpha} \) | 0.740±0.097 | 0.909±0.201 | 1.116±0.262 |
| \( s_{Ra} \) | 4.112±0.499 | 2.230±0.271 | 2.447±0.298 |
| \( s_{Rb} \) | -0.557±0.177 | 0.864±0.391 | 0.997±0.516 |
| \( s_{Ra} \) | 3.191±0.499 | 1.458±0.262 | 1.684±0.363 |
| \( s_{Ra} \) | 1.291±0.062 | 1.187±0.094 | 1.354±0.149 |
| \( s_{Rb} \) | -1.562±0.117 | -1.327±0.134 | -1.756±0.183 |
| \( s_{Rb} \) | 0.748±0.062 | 0.999±0.149 | 1.159±0.261 |

\[ \chi^2_{d.o.f} = 70.6/77 = 0.914 \]

\[ \chi^2_{d.o.f} = 48.8/64 = 0.759 \]

\[ \chi^2_{d.o.f} = 44.3/58 = 0.763 \]
duce the direct couplings of only ambiguity comes from the direct term. We introduce to \([5]\) for the expressions of the reduced amplitudes, 

\[ M_f = 989(80) \quad \Gamma_f/2 = 20(32) \quad \Gamma_{f\sigma^*\pi^*} = 1.33(72) \]

\[ |g_{f\pi^*}\pi^*| = 1.17(26) \text{ GeV}, \quad r_{f\pi^*K} = 2.59(1.34). \]

A large value of \( r_{f\pi^*K} \) together with a narrow width disfavor a pure \([u\bar{u} + d\bar{d}]\) assignment of the \( f_0(980) \), whereas its non-negligible width into \( \pi\pi \) indicates that it cannot be a pure \( s\bar{s} \) or \( KK \) molecule.

- A possible gluonium component mixed with a \( q\bar{q} \) state of the \( \sigma \) and \( f_0(980) \) seems to be necessary for evading the previous difficulties.

### 3. \( \gamma\gamma \to \pi\pi \) process

The \( I = 0 \) S-wave amplitude consists of two parts, the Born and unitarized amplitudes shown in Fig. 3 and the direct \( \gamma\gamma \) couplings shown in Fig. 4. The unitarized amplitude can be calculated using chiral lagrangian. The only ambiguity comes from the direct term. We introduce the direct couplings of \( \sigma \) and \( f_0(980) \) to \( \gamma\gamma \) \([11]\):

\[ T_s^\gamma = \sqrt{2} a_\sigma (f_{\gamma\gamma} + s_{\gamma\gamma}) T_{\sigma\pi^+} + (f_{\gamma\gamma} + s_{\gamma\gamma}) T_{f_0\pi^+}. \]

for the S-wave and:

\[ T_s^D = \frac{\alpha}{\sqrt{2}} f_{\gamma\gamma}^{I=0} + s_{\gamma\gamma}^{I=0} T_{f_0\pi^+}. \]

for the D-wave with helicity 0 and 2 respectively. We refer to \([5]\) for the expressions of the reduced amplitudes, \( \tilde{T}_{\sigma\pi}, \tilde{T}_{f_0\pi} \) and \( \tilde{T}_{f_0\pi^+} \),

\[
\begin{align*}
\gamma, V, A &+ \gamma, V, A + \pi, V, A + P, V, A + P, V, A + P
\end{align*}
\]

The previous possible gluonium assignment of the \( \sigma \) can be tested from \( J/\psi, \phi \to \gamma\sigma_B \) and \( D_s \to l\nu\sigma_B \) processes (\( \sigma_B \) is a n unmixed hypothetical gluonium state) as discussed respectively in \([24, 23]\) and \([24]\). We expect to have the branching ratios \( \times 10^3 \):

\[
B(J/\psi \to \sigma_B\gamma) \times B(\sigma_B \to all) \approx (0.4 - 1.0), \quad B(\phi \to \sigma_B\gamma) \approx 0.12.
\]
Figure 5: Fit up to 1.09 GeV, \( \nu^2_{x,\nu} = \frac{\sigma}{\pi} = 0.96. \Sigma = \pi + K. \) Dotted blue (S-wave contribution); dashed green (D-wave contribution); continuous red (S+D contribution); dash-dotted orange (all partial wave contribution).

Table 6: \( \gamma\gamma \) decay widths in unit of keV. \( \Gamma_0 \approx \frac{\sigma_{\gamma\gamma}}{\pi} = f_{\sigma}^2/\sigma_{\gamma\gamma} \)

| Set 2 Set 3 | 12 | 14 | 151 | 16 | 161 | 18 | PDG19 |
|------------|----|----|-----|----|-----|----|-------|
| \( \nu \)  | 1.09 | 1.09 | 0.8 | 1.44 | 0.8 | 1.44 | 1.4 |
| \( r_0 \)  | 0   | 0   | 0.13 | 0.26 | 0.15 |
| \( \Gamma_{\nu}^{f_0} \) | 0.16 | 0.20 | 0.13 | 0.01 |
| \( \Gamma_{\nu}^{f_2} \) | 1.53 | 1.43 | 2.70 |
| \( \Gamma_{\nu}^{S} \) | 3.11 | 3.10 | 3.90 | 1.7 | 3.1 | 2.4 | 2.1 | 1.2 |
| \( \Gamma_{\nu}^{f_0} \) | 0.29 | 0.27 | 0.015 |
| \( \Gamma_{\nu}^{f_2} \) | 0.90 | 0.81 |
| \( \Gamma_{\nu}^{S} \) | 0.17 | 0.15 | 0.42 | 0.10 | 0.13 | 0.29 \( \pm 0.08 \) |

and:

\[
\frac{\Gamma[D \to \sigma(p\bar{p}))\nu]}{\Gamma[D \to S_2(q\bar{q})\nu]} \approx \frac{1}{[f_{\sigma}(0)]^2} \left[ \frac{f_{\bar{q}}}{M_\pi} \right]^2 \approx O(1),
\]

for \( M_\pi \approx 1.5 \text{ GeV}, f_{\bar{q}} \approx 1 \text{ GeV} \), \( [f_{\sigma}(0)] \approx 0.5 \) \( \frac{20}{22} \frac{23}{23} \), where \( f_{\bar{q}} \) is the form factor associated to the \( q\bar{q} \) semileptonic production. The rates of these productions are in fair agreement with existing data supporting again or not excluding a large gluonium component of the \( \sigma \).

5. Conclusions

- We use an improved coupled channel K-matrix model, taking into account Adler zero and left hand singularities. We extract the pole positions and widths, as well as the hadronic and \( \gamma\gamma \) couplings of \( \sigma \) and \( f_{\sigma}(980) \) by fitting experimental data.
- The values of their direct widths favor a large gluon content for the \( \sigma \) meson but are not decisive for explaining the structure of the \( f_{\sigma}(980) \) meson, which can mainly be either a \( \bar{s}s \) or a gluonium.

- The large values of the rescatterings widths, due to meson loops, can be also obtained if they are glueonia states but not necessarily if they are four-quark or molecule states.

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