Early Universe in the $SU(3)_L \otimes U(1)_X$ electroweak models

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We present status of the 3-3-1 models and their implications to cosmological evolution such as inflation, phase transitions and sphalerons. The models are able to provide quite good agreement with the Standard Cosmology: the inflation happens in the GUT scale, while phase transition has two sequences corresponding two steps of symmetry breaking in the models. Some bounds on the model parameters are obtained.

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I. INTRODUCTION

It is well known that our Universe content is 68.3% of Dark Energy (DE), 26.8% of Dark Matter (DM) and of 4.9% of luminous matter [1]. With the unique fact of accelerated Universe, the core origin of Dark Energy is still under question, while the existence of Dark Matter is unambiguous. According to the Standard Cosmology, in the moment at $10^{-36}$s after the Big Bang (BB), there was inflation, and our Universe has been expanded exponentially. The inflationary scenario solves a number of problems such as the Universe’s flatness, horizon, primordial monopole, etc. It is well known that there is no anti-matter in our Universe, or other word speaking: at present there exists a Baryon Asymmetry of Universe (BAU). The baryon number vanishes ($n_B = 0$) at the BB, and this conflicts with the present BAU. Nowadays, the BAU is one of the greatest challenges in Physics and any physical model has to give an explanation. The BAU is realized if three Sakharov’s conditions are satisfied [2, 3].

1. $B$ violation,
2. $C$ and $CP$ violations,

3. deviation from thermal equilibrium.

Over the half of Century, the Standard Model (SM) of the electromagnetic, weak and strong interactions successfully possesses a great experimental examinations and stands for future development. Despite its great success, the model still contains a number of unresolved problems such as the generation number of quarks and leptons, the neutrino mass and mixing, the electric charge quantization, the existence of about one quarter of DM, etc. The aforementioned problems require that the SM must be extended.

Among the extensions beyond the SM, the models based on $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (3-3-1) gauge group [4, 5] have some interesting features including the ability to explain the generation problem [4, 5] and the electric charge quantization [6]. It is noted that in this scheme the gauge couplings can be unified at the scale of order TeV without supersymmetry [7]. The 3-3-1 models have two interesting properties needed for the mentioned aim, namely: first, the lepton-number violation due to the fact that lepton and anti-lepton are put in the triplet [8]. Second, one generation of quarks transforms differently from other two. This leads to the flavor changing neutral current at the tree level mediated by new $Z'$ gauge boson [9].

The 3-3-1 models have been considered in aspects of collider physics [10–13], muon anomalous magnetic moments [14], neutrino physics [15], DM [16, 17].... In this review I will concentrate on Early Universe aspects of the models.

This paper is organized as follows. In Sec. II we give a brief review of the 3-3-1 models and their modified versions. In Sec. III the cosmological inflation in the supersymmetric economical 3-3-1 model is presented. In Sec. IV we investigate the structure of the electroweak phase transition (EWPT) sequence in the 3-3-1 models with minimal Higgs sector, namely the reduced minimal 3-3-1 model (RM331) and the economical 3-3-1 model (E331), find the parameter ranges where the EWPTs are the strongly first-order to provide B violation necessary for baryogenesis, and show the constraints on the mass of the charged Higgs boson. Section V is devoted for sphalerons in the reduced minimal 3-3-1 model. Finally, in Sec. VI we give conclusion on the possibility to describe cosmological evolution in the framework of the 3-3-1 models.
II. THE MODELS

In the mentioned models, the strong interaction keeps the same as in the SM, while the electroweak part associated with SU(3)$_L$ $\otimes$ U(1)$_X$ has two diagonal generators $T_3$ and $T_8$ from which the electric charge operator is based on

$$Q = T_3 + \beta T_8 + X. \quad (1)$$

The coefficient ($=1$) at the $T_3$ is defined to make the 3-3-1 models embed the SM. The lepton arrangement will define the parameter $\beta$ which distinguishes two main versions: the minimal version with $\beta = \sqrt{3}$ and the version with neutral leptons/neutrinos $\beta = -1/\sqrt{3}$ at the bottom of the triplet.

A. The minimal 3-3-1 model

The minimal version [4] contains lepton triplet in the form

$$f_L = (\nu_l, l, l^c)^T_L \sim (1, 3, 0). \quad (2)$$

Two first quark generations are in anti-triplet and the third one is in triplet:

$$Q_{iL} = (d_{iL}, -u_{iL}, D_{iL})^T \sim (3, \bar{3}, -1/3), \quad (3)$$

$$u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), D_{iR} \sim (3, 1, -4/3), \ i = 1, 2,$$

$$Q_{3L} = (u_{3L}, d_{3L}, T_L)^T \sim (3, 3, 2/3),$$

$$u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_R \sim (3, 1, 5/3).$$

To provide masses for all quarks and lepton, the Higgs sector needs three scalar triplets and one sextet:

$$\chi = (\chi_1^+, \chi_2^-, \chi_3^0)^T \sim (1, 3, -1), \quad (4)$$

$$\eta = (\eta_1^0, \eta_2^-, \eta_3^+) \sim (1, 3, 0),$$

$$\rho = (\rho_1^+, \rho_2^0, \rho_3^{++}) \sim (1, 3, 1),$$

$$S \sim (1, 6, 0).$$

with VEV: $\langle \rho_{2}^0 \rangle = v/\sqrt{2}, \langle \eta_1^0 \rangle = u/\sqrt{2}, \langle \chi_3^0 \rangle = \omega/\sqrt{2}$ and $\langle S_{23}^0 \rangle = v'/\sqrt{2}$. 
The gauge sector of this model contains five new gauge bosons: one neutral $Z'$ and two bileptons carrying lepton number 2: $Y^\pm$ and $X^{\pm\pm}$. In (2), lepton and antilepton lie in the same triplet, and this leads to lepton number violations in the model. Hence, it is better to deal with a new conserved charge $L$ commuting with the gauge symmetry

$$L = \frac{4}{\sqrt{3}} T_8 + \mathcal{L}.$$  

The exotic quarks $T$ and $D_i$ have the electric charges, respectively, $5/3$ and $-4/3$ and carry both baryon and lepton numbers $L = \pm 2$.

The singly charged bilepton is responsible for the wrong muon decay

$$\mu \rightarrow e + \nu_e + \bar{\nu}_\mu,$$

while the doubly charged bilepton with decay

$$X^{--} \rightarrow ll$$

provides four leptons at the final states which is characteristic feature of the model. The model provides an interesting prediction for the Weinberg angle

$$\sin^2 \theta_W(M_{Z'}) \leq \frac{1}{4}.$$  

Besides the complication in the Higgs sector, the model also has one problem that it loses perturbative property at the scale above 5 TeV.

The above Higgs sector is complicated; and recently it is reduced to the minimal with only two Higgs triplets. If the triplet $\rho$ and $\chi$ are used then the model is called reduced minimal 3-3-1 model, while $\rho$ is replaced by $\eta$ then it is called simple 3-3-1 model (S331).

It has been recently shown that due to the $\rho$ parameter and the Landau pole, the minimal and its reduced version should be ruled out. It is noted that the RM331 has nonrenormalizable effective interactions, so situation has to be considered carefully.

**B. The 3-3-1 model with right-handed neutrinos**

Leptons are in triplet:

$$f_L^a = (\nu_L^a, e_L^a, (N_L)^a)^T \sim (1, 3, -1/3), e_R^a \sim (1, 1, -1),$$  

$$\tag{6}$$
where $a = 1, 2, 3$ is a generation index and $N_L$ can be right-handed neutrino or neutral lepton. Two first generations of quarks are in antitriplets, and the third one is in triplet:

$$Q_{iL} = (d_{iL}, -u_{iL}, D_{iL})^T \sim (3, \bar{3}, 0),$$

$$u_{iR} \sim (3, 1, 2/3), d_{iR} \sim (3, 1, -1/3), D_{iR} \sim (3, 1, -1/3), i = 1, 2,$$

$$Q_{3L} = (u_{3L}, d_{3L}, T_L)^T \sim (3, 3, 1/3),$$

$$u_{3R} \sim (3, 1, 2/3), d_{3R} \sim (3, 1, -1/3), T_R \sim (3, 1, 2/3).$$

The model with neutral lepton/neutrino ($\beta = -1/\sqrt{3}$) needs three scalar triplets to provide all fermions masses and the same for spontaneous symmetry breaking (SSB):

$$\chi = \left(\chi^0, \chi^{-}, \chi^0\right)^T \sim (1, 3, -1),$$

$$\rho = \left(\rho^+, \rho^0, \rho^+\right)^T \sim (1, 3, 2),$$

$$\eta = \left(\eta^0, \eta^-, \eta^0\right)^T \sim (1, 3, -1).$$

The exotic quarks $T$ and $D_i$ have electric charges as usual one, i.e., 2/3 and $-1/3$, respectively, and carry both baryon and lepton numbers $L = \pm 2$. The new gauge bosons are: the neutral $Z'$ and two bileptons carrying lepton number 2: $Y^\pm$ and $X^0$. The neutral bilepton $X^0$ is non-Hermitian and is responsible for neutrino oscillation [21].

Note that two Higgs triplets $\eta$ and $\chi$ have the same structure, so ones can reduce number of Higgs triplets from three to two, namely we can use only $\rho$ and $\chi$ to produce masses for quarks and leptons; and resulting model is called economical 3-3-1 model [22]. As in the RM331, the nonrenormalizable interactions, in this case, are needed for production of quark masses [22].

### III. COSMOLOGICAL INFLATION IN THE SUPERSYMMETRIC ECONOMICAL 3-3-1 MODELS

The discovery of the 2.7K microwave background radiation arriving from the farthest reaches of the Universe, gained widespread acceptance, is positive point of the hot-universe theory, where the inflationary scenario [23, 24] plays very important role. Cosmological inflation (CI) can give solutions for above mentioned problems, hence it is a possible theory of the origin of all structures in the Universe, including ourselves!
With above reasons, any beyond standard model has to have the cosmological inflation happened at the interval of $10^{-36} - 10^{-34}$s after the BB. With that moment, the energy scale of CI is about $10^{15}$ GeV. In \[25\], the CI was considered in the framework of the supersymmetric economical 3-3-1 model (SE331), and a reason is the following: the E331 is very simple, but there is no candidate for inflaton - a key element of CI. The SE331 has some advantages such as there are more scalar fields which can play a role of the inflaton, and the Higgs sector is very constrained.

A supersymmetric version of the minimal 3-3-1 model has been constructed in \[26\] and its scalar sector was studied in \[27\]. Lepton masses in the framework of the above-mentioned model were presented in \[28\], while potential discovery of supersymmetric particles was studied in \[29\]. In \[30\], the $R$-parity violating interaction was applied for instability of the proton. A supersymmetric RM331 was presented in \[31\].

The supersymmetric version of the 3-3-1 model with right-handed neutrinos has already been constructed in \[32\]. The scalar sector was considered in \[33\] and neutrino mass was studied in \[34\]. A supersymmetric version of the economical 3-3-1 model has been constructed in \[35\]. Some interesting features such as Higgs bosons with masses equal to that of the gauge bosons: the $W (m^2_{\tilde{e}_1} = m^2_W)$ and the bileptons $X$ and $Y (m^2_{\tilde{\chi}^\pm} = m^2_Y)$, have been pointed out in \[36\]. Sfermions in this model have been considered in \[37\]. In \[38\] it was shown that bino-like neutralino can be a candidate for DM.

In \[25\], the authors have constructed a hybrid inflationary scheme based on a realistic supersymmetric $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model by adding a singlet superfield $\Phi$ which plays the role of the inflaton, namely the inflaton superfield.

We remind that the existence of a $U(1)_Z$ does not belong to the MSSM and it spontaneously breaks down at the scale $M_X$ by Higgs superfield $\phi$, which is singlet under the MSSM. The inflaton superfield couples with this pair of Higgs superfields. Therefore, the additional global supersymmetric renormalizable superpotential for the inflation sector is chosen to be \[39\], \[40\]

$$W_{inf}(\Phi, \chi, \chi') = \alpha \Phi \chi \chi' - \mu^2 \Phi. \quad (9)$$

The superpotential given by \[(9)\] is the most general potential consistent with a continuous $R$ symmetry under which $\phi \rightarrow e^{i\gamma} \phi, \ W \rightarrow e^{i\gamma} W$, while the product $\chi \chi'$ is invariant \[40\], \[41\].

By a suitable redefinition of complex fields $\mu^2, \alpha$ are chosen to be positive real constants.
and the ratio $\frac{\mu}{\sqrt{\alpha}}$ sets the $U(1)_Z$ symmetry breaking scale $M_X$. The most general superpotential consistent with a continuous $R$-symmetry is given by

$$W_{tot} = W_R + W_{inf}(\Phi, \chi, \chi').$$ (10)

With the superpotential given in (9), the Higgs scalar potential takes the form

$$V_{tot} = \sum_i |F_i|^2 + \frac{1}{2} \sum_{\alpha} |D_{\alpha}|^2 + V_{soft},$$

where $i$ runs from 1 to the total number of the chiral superfields in $W_{tot}$, while $V_{soft}$ contains all the soft terms generated by supersymmetry breaking at the low energy.

Hence, the Higgs potential becomes

$$V_{tot} = |\mu + \alpha \Phi|^2|\chi'|^2 + |\mu + \alpha \Phi|^2|\chi|^2 + |\alpha \chi' - \mu|^2 +$$

$$+ |\mu \rho|^2 + |\mu \rho'|^2 + \frac{1}{2} \sum_{\alpha} |D_{\alpha}|^2 + V_{soft}.$$ (11)

The first derivatives $\frac{\partial V_{tot}}{\partial \rho}$, $\frac{\partial V_{tot}}{\partial \rho'}$ are independent of $\chi, \chi', \Phi$, and the fields $\rho, \rho'$ will stay in their minimum independently of what the fields $\chi, \chi', \Phi$ do. If we are mainly interested in what is happening above the electroweak scale, and hence we do not take into account the dimensional Higgs multiplets $\rho, \rho'$. Then, the Higgs scalar potential is given by

$$V_{inf} = |\mu + \alpha \Phi|^2|\chi'|^2 + |\mu + \alpha \Phi|^2|\chi|^2 + |\alpha \chi' - \mu|^2 +$$

$$+ \frac{1}{2} \left( g \sum_a \chi^* T^a \chi \right)^2 + \frac{1}{2} \left( g \sum_a \chi'^* T^a \chi' \right)^2.$$ (11)

Let us denote

$$\mu + \alpha \Phi \equiv \beta S,$$ (12)

where $\beta$ is some constant and $S$ is a new field, the Higgs potential (11) can be rewritten as

$$V_{inf} = \beta^2 |S|^2 \left( |\chi|^2 + |\chi'|^2 \right) + |\alpha \chi' - \mu|^2 +$$

$$+ \frac{1}{2} \left( g \sum_a \chi^* T^a \chi \right)^2 + \frac{1}{2} \left( g \sum_a \chi'^* T^a \chi' \right)^2.$$ (11)

When $D$ term vanishes along its direction, the potential contains only $F$ term and has the form

$$V_{inf} = \beta^2 |S|^2 \left( |\chi|^2 + |\chi'|^2 \right) + |\alpha \chi' - \mu|^2.$$ (13)
From (13), it is clear that $V_{in,f}$ has an unique supersymmetric minimum corresponding to

$$
\langle S \rangle = 0,
$$

$$
M_X \equiv \langle \chi \rangle = \langle \chi' \rangle = \mu \sqrt{\alpha}.
$$

(14)

The ratio $\frac{\mu}{\sqrt{\alpha}}$ sets the $U(1)_Z$ symmetry breaking $M_X$, but Eq. (14) is global minimum, and supersymmetry is not violated [40]. Hence, inflation can take place but supersymmetry is not broken. This is $F$ term inflation [42].

We assume that the initial value for the inflaton field is much greater than its critical value $S_c$. For $|S| > |S_c| \equiv \mu / \sqrt{\alpha}$ the potential is very flat in the $|S|$ direction, and the $\chi, \chi'$ fields settle down to the local minimum of the potential, $\chi = \chi' = 0$, but it does not drive $S$ to its minimum value. The universe is dominated by a nonzero vacuum energy density, $V_0^\frac{1}{4} = \mu$, which can lead to an exponential expanding, inflation starts, and supersymmetry is broken.

By the Coleman-Weinberg formula in [43], at the one-loop level, the effective potential along the inflaton direction is given by

$$
\Delta V = \frac{1}{16\pi^2} \sum_i (-1)^F m_i^4 \ln \left( \frac{m_i^2}{\Lambda^2} \right),
$$

where $F = -1$ for the fermionic fields and $F = 1$ for the bosonic fields. The coefficient $(-1)^F$ shows that bosons and fermions give opposite contributions. The sum runs over each degree of freedom $i$ with mass $m_i$ and $\Lambda$ is a renormalization scale.

The effective potential (along the inflationary trajectory $S > S_c, \chi = \chi' = 0$) is given by

$$
V_{eff}(S) = \mu^4 + \frac{3}{16\pi^2} \left[ 2\beta^2 \frac{\mu^4}{\alpha^2} \ln \frac{\beta^2 |S|^2}{\Lambda^2} + \left( \beta^2 |S|^2 + \alpha \mu^2 \right)^2 \ln \left( 1 + \frac{\alpha \mu^2}{\beta^2 |S|^2} \right) + \left( \beta^2 |S|^2 - \alpha \mu^2 \right)^2 \ln \left( 1 - \frac{\alpha \mu^2}{\beta^2 |S|^2} \right) \right].
$$

(15)

It is to be noted that for $S > S_c$, the universe is dominated by the false vacuum energy $\mu^4$. When $S$ field drops to $S_c$, then the GUT phase transition happens. At the end of inflation, the inflaton field does not need to coincide with the GUT phase transition. The end of inflation can be supposed to be on a region of the potential which satisfies the flatness conditions (see, for example, [44])

$$
\epsilon \ll 1, \eta \ll 1,
$$

(16)
where we have used the conventional notations
\[ \epsilon \equiv \frac{M_P^2}{16 \pi} \left( \frac{V'}{V} \right)^2, \quad \eta \equiv \frac{M_P^2}{8 \pi} \frac{V''}{V}, \]
where primes denote a derivative with respect to $S$.

To compare with observational COBE data, we use the slow-roll approximation with parameters: $\epsilon$ and $\eta$. The first condition in (16): $\epsilon \ll 1$ indicates that the density $\rho$ is close to $V$ and is slowly varying. As a result, the Hubble parameter $H$ is slowly varying, which implies that one can write $a \propto e^{H t}$ at least over a Hubble time or so. The second condition $\eta \ll 1$ is a result of the first condition plus the slow-roll approximation. The conditional phase may end before the GUT transition if the flatness conditions (16) are violated at some point $S > S_c$.

Let us denote a dimensionless variable
\[ y \equiv \frac{\beta |S|}{\alpha S_c}. \]
Imposing the condition $\alpha = \beta$, which means that $|\Phi| \approx |S| \gg \mu$, we get then
\[ \epsilon = \left( \frac{3\alpha^2 M_P}{4\pi^2 M_X} \right)^2 \frac{1}{16 \pi} \left[ y \left( y^2 - 1 \right) \ln \left( \frac{1}{y^2} \right) + y \left( y^2 + 1 \right) \ln \left( \frac{1 + y^2}{y^2} \right) \right]^2, \]
\[ \eta = \left( \frac{\alpha M_P}{4\pi M_X} \right)^2 \frac{3}{2\pi} \left[ (3y^2 + 1) \ln \left( \frac{1 + y^2}{y^2} \right) + (3y^2 - 1) \ln \left( \frac{1}{y^2} \right) \right]. \]

The chaotic inflation driven by the $\phi^3$ is in good agreement with the WMAP data, while for the $\phi^4$ potential, the situation is negative.

The above model cannot resolve the horizon/flatness problems of the BB cosmology and violates the slow-roll conditions $\eta \ll 1$ (the $\eta$ problem. To deal with these problems, we should consider the $F$-term inflation with minimal Kähler potential.

The $F$-term inflation with Kähler potential is defined by
\[ W_{stand}(\Phi, \chi, \chi') = \alpha \tilde{S} \left( \tilde{\chi}^2 - M_X^2 \right). \]
Keeping in mind that $K = \sum_{\alpha} |\phi_\alpha|^2$, we obtain the scalar potential
\[ V_F = 2\alpha^2 S^2 \phi^2 \left[ 1 + \frac{S^2 + 2\phi^2}{m_P^2} + \frac{(S^2 + 2\phi^2)^2}{2m_P^4} \right] + \alpha^2 (\phi^2 - M_X^2)^2 \left[ 1 + \frac{\phi^2}{m_P^2} + \frac{S^4}{2m_P^4} + \frac{2\phi^4}{m_P^4} \right] + \cdots, \]
where we have assumed that $|\phi|^2 = |\phi'|^2$.

Let us consider how does this factor change the result. As we know, the slow-roll parameter is defined as

$$\eta = m_p^2 \left( \frac{V''}{V} \right),$$

where the prime refers to derivative with respect to $S$. The supergravity scalar potential for $S > S_c$ is given by

$$V_o = \alpha^2 M_X^4 + \frac{\alpha^2 M_X^4}{2m_p^2} S^4. \quad (22)$$

From (22), it follows derivative of $V$: $V'' \simeq \frac{\alpha^2 M_X^4}{2m_p^2} S^2$, and $\eta = \frac{1}{2m_p^2} S^2 \ll 1$. Therefore, the $\eta$-problem is overcome.

The potential given in (22) does not contain a term which can drive $S$ to its minimum value, so we have to consider the effective potential. In this case, the spectral index $n$ is given by

$$n = 1 - 6\epsilon + 2\eta$$

$$= 1 - \frac{3\alpha^2}{512\pi^3 \zeta x} \left[ x^2(16 + 9\alpha^2) - 54\alpha^2 \zeta + 6x^8(-40 + 9\alpha^2)\zeta + 16x^4(-5 + 9\alpha^2)\zeta \right], \quad (23)$$

where $\zeta \equiv \frac{M_X^2}{M_p^2}$.

Taking into account the WMAP data, we conclude that the value of e-folding number $N_Q$ must be larger than 45, and get bounds on the values of coupling $\alpha$ and $\zeta$, which are presented in Table II.

| Table I: Bounds on the parameter $\zeta$ and coupling $\alpha$ followed by the WMAP data. |
|-----------------------------------------------|
| $\alpha$ | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ | $10^{-6}$ |
| $\zeta$  | $25 \times 10^{-6}$ | $25 \times 10^{-7}$ | $25 \times 10^{-9}$ | $3 \times 10^{-11}$ |

It is interesting to note that due the inflaton with mass in the GUT scale, the model can provides masses for neutrino different from ones without inflationary scenario. With the help of the lepton-number-violating interactions among the inflaton and right-handed neutrinos, the non-thermal leptogenesis scenario is followed [45].

In recent work [46], the authors have considered the inflationary scenario and leptogenesis in newly proposed 3-3-1-1 model. Here, the scalar field that spontaneously breaks the $U(1)_Y$ symmetry plays as inflaton.
To finish this section, we emphasize that the 3-3-1 models can provide the inflationary scenario or cosmological evolution of the our Universe.

IV. ELECTROWEAK PHASE TRANSITION IN 3-3-1 MODELS

It is known that if baryon number is conserved and is equal to zero, it will equal to zero forever. If baryon number does not satisfy any conservation law, it vanishes in the state of thermal equilibrium. Therefore we need the third Sakharov’s condition. The second condition is appropriate for ensuring a different decay rate for particles and antiparticles [3]. The electroweak phase transition is the transition between symmetric phase to asymmetric phase in order to generate mass for particles. Hence, the phase transition is related to the mass of the Higgs boson [3].

In the basic model of particles, the first and second conditions can be satisfied, but conditions on thermal imbalance is difficult to satisfy. So the analysis of the third condition is the only approach at present in order to explain the baryon asymmetry.

Why is the first order phase transition? For very large temperature, the effective potential has only one minimum at the zero. As temperature drops below the critical temperature ($T_c$), the second minimum appears. If the two minimums are separated by a potential barrier, the phase transition occurs with bubble nucleation. Inside the bubbles, the scalar field acquires a nonzero expectation value. If the bubble nucleation rate exceeds the universe’s expansion rate, the bubbles collide and eventually fill all space. Such a transition is called the first order phase transition. It is very violent and one can expect large deviations from thermal equilibrium [3]. The other possible scenario takes place if the two minimums are never separated by a potential barrier. The phase transition is a smooth transition or the second order phase transition.

A. Phase transition in reduced minimal 3-3-1 model

For the SM, although the EWPT strength is larger than unity at the electroweak scale, it is still too weak for the mass of the Higgs boson to be compatible with current experimental limits [3, 51]; this suggests that electroweak baryogenesis (EWBG) requires new physics beyond the SM at the weak scale [32]. Many extensions such as the two-Higgs-doublet
model or Minimal Supersymmetric Standard Model have a more strongly first-order phase transition and the new sources of CP violation, which are necessary to account for the BAU; triggers for the first-order phase transition in these models are heavy bosons or DM candidates [47–50].

To start, let us consider the high-temperature effective potential

\[ V_{\text{eff}} = D.(T^2 - T'^2)\nu^2 - E.T\nu^3 + \frac{\lambda_T}{4}\nu^4, \]

where \( \nu \) is the VEV of Higgs. In order to have the strongly first-order phase transition, the strength of phase transition has to be larger than 1, i.e., \( \frac{\nu}{\lambda_T} \geq 1 \).

The phase transition has been firstly investigated in the SM. But the difficulty of the SM is that the strength of the first-order electroweak phase transition, which must be larger than 1 at the electroweak scale, appears too weak for the experimentally allowed mass of the SM scalar Higgs boson [3, 51]. Therefore, it seems that EWBG requires a new physics beyond the SM at weak scale [52].

With the discovery of the Higgs boson, the study of phase transitions in the particle models is simplified: only to determine the order of phase transition. This opens a lot of hope for the extended models in examining the electroweak phase transition.

The 3-3-1 models must have at least two Higgs triplets [19, 22]. Therefore, the number of bosons in the 3-3-1 models will many more than in the SM and symmetry breaking structure is different to the SM.

The physical scalar spectrum of the RM331 model is composed by a doubly charged scalar \( h^{++} \) and two neutral scalars \( h_1 \) and \( h_2 \) [19]. These new particles and exotic quarks can be triggers for the first order phase transition.

From the Higgs potential we can obtain \( V_0 \) that depends on VEVs as the following

\[ V_0(\nu_\chi, \nu_\rho) = \mu_1^2\nu_\chi^2 + \mu_2^2\nu_\rho^2 + \lambda_1\nu_\chi^4 + \lambda_2\nu_\rho^4 + (\lambda_3 + \lambda_4)\nu_\chi^2\nu_\rho^2. \]

The effective potential being a function of VEVs and temperature has the form

\[ V = V_0(\nu_\chi, \nu_\rho) + \sum M_{\text{boson}}^2(\nu_\chi, \nu_\rho)W_\mu W_\mu + \sum m_{\text{fermion}}(\nu_\chi, \nu_\rho)\langle f_L^c f_L^c \rangle. \]

Averaging over space, we obtain

\[ V = V_0(\nu_\chi, \nu_\rho) + \sum M_{\text{boson}}^2(\nu_\chi, \nu_\rho)\langle W_\mu W_\mu \rangle + \sum m_{\text{fermion}}(\nu_\chi, \nu_\rho)\langle f_L^c f_L^c \rangle. \]
where $W^\mu$ runs over all gauge fields. The RM331 has the following gauge bosons: Two like the SM bosons $Z_1$, $W^\pm$ and the new heavy neutral boson $Z_2$, the singly and doubly charged boson $U^{\pm\pm}$ and $V^{\pm}$. Two doubly charged Higgs $h^{++}$ and $h^{--}$, one heavy neutral Higgs $h_2$ and one like-SM Higgs $h_1$. Using Bose-Einstein and Fermi-Dirac distributions for bosons and fermions, we can obtain the effective potential in the RM331 as follows

$$V_{\text{eff}}^{\text{RM331}} = V_0(v_\chi, v_\rho) + \frac{3}{64\pi^2} \left( m_{Z_1}^4 \ln \frac{m_{Z_1}^2}{Q^2} + m_{Z_2}^4 \ln \frac{m_{Z_2}^2}{Q^2} + 2m_W^4 \ln \frac{m_W^2}{Q^2} + m_t^4 \ln \frac{m_t^2}{Q^2} - 12m_Q^4 \ln \frac{m_Q^2}{Q^2} \right)$$

$$+ \frac{1}{64\pi^2} \left( m_{h_2}^4 \ln \frac{m_{h_2}^2}{Q^2} + 2m_{h++}^4 \ln \frac{m_{h++}^2}{Q^2} \right)$$

$$+ \frac{3}{64\pi^2} \left( 2m_U^4 \ln \frac{m_U^2}{Q^2} + 2m_V^4 \ln \frac{m_V^2}{Q^2} \right)$$

$$+ \frac{T_4^4}{4\pi^2} \left[ F_- \left( \frac{m_{h_2}}{T} \right) + 2F_- \left( \frac{m_{h++}}{T} \right) \right]$$

$$+ \frac{3T_4^4}{4\pi^2} \left[ 4F_+ \left( \frac{m_t}{T} \right) + 12F_+ \left( \frac{m_Q}{T} \right) \right]$$

$$+ \frac{3T_4^4}{4\pi^2} \left[ F_- \left( \frac{m_{Z_1}}{T} \right) + F_- \left( \frac{m_{Z_2}}{T} \right) + 2F_- \left( \frac{m_W}{T} \right) \right]$$

$$+ 2F_- \left( \frac{m_U}{T} \right) + 2F_- \left( \frac{m_V}{T} \right) \right],$$

where

$$F_+ \left( \frac{m_\phi}{T} \right) = \int_0^{\frac{m_\phi}{T}} \alpha J_+^{(1)}(\alpha, 0) d\alpha$$

$$J_+^{(1)}(\alpha, 0) = 2 \int_0^{\alpha} \frac{(x^2 - \alpha^2)^{1/2}}{e^x + 1} dx.$$

The effective potential can be rewritten as follows

$$V_{\text{eff}} = V_0 + V_{\text{eff}}^{\text{hard}} + V_{\text{eff}}^{\text{light}},$$

where

$$V_{\text{eff}}^{\text{hard}} = \frac{3}{64\pi^2} \left( m_{Z_1}^4 \ln \frac{m_{Z_1}^2}{Q^2} + m_{Z_2}^4 \ln \frac{m_{Z_2}^2}{Q^2} + 2m_W^4 \ln \frac{m_W^2}{Q^2} + m_t^4 \ln \frac{m_t^2}{Q^2} - 12m_Q^4 \ln \frac{m_Q^2}{Q^2} \right)$$

$$+ \frac{3}{64\pi^2} \left( 2m_U^4 \ln \frac{m_U^2}{Q^2} + 2m_V^4 \ln \frac{m_V^2}{Q^2} - 12m_Q^4 \ln \frac{m_Q^2}{Q^2} \right)$$

$$+ \frac{T_4^4}{4\pi^2} \left[ F_- \left( \frac{m_{h_2}}{T} \right) + 2F_- \left( \frac{m_{h++}}{T} \right) \right]$$

$$+ \frac{3T_4^4}{4\pi^2} \left[ F_- \left( \frac{m_{Z_1}}{T} \right) + F_- \left( \frac{m_{Z_2}}{T} \right) + 2F_- \left( \frac{m_W}{T} \right) \right]$$

$$+ 2F_- \left( \frac{m_U}{T} \right) + 2F_- \left( \frac{m_V}{T} \right) + 12F_+ \left( \frac{m_Q}{T} \right) \right] ,$$
and

\[
V_{\text{eff}}^{\text{light}} = \frac{3}{64\pi^2} \left( m_{Z_1}^4 \ln \frac{m_{Z_1}^2}{Q^2} + 2m_W^4 \ln \frac{m_W^2}{Q^2} - 4m_t^4 \ln \frac{m_t^2}{Q^2} \right) \\
+ \frac{3T_1^4}{4\pi^2} \left[ F_-(\frac{m_{Z_1}}{T}) + 2F_-(\frac{m_W}{T}) + 4F_+(\frac{m_t}{T}) \right].
\]

Here \(V_{\text{eff}}^{\text{light}}\) is like the effective potential of the SM, while \(V_{\text{eff}}^{\text{hard}}\) is contributions from heavy particles. We expect that \(V_{\text{eff}}^{\text{hard}}\) contributes heavily in the EWPT.

The symmetry breaking in the RM331 can take place sequentially. Because two scales of symmetry breaking are very different, \(v_{\chi_0} \gg v_{\rho_0} (v_{\chi_0} \sim 4 - 5 \text{ TeV}, v_{\rho_0} = 246 \text{ GeV})\) and because of the accelerating universe, the symmetry breaking \(SU(3) \to SU(2)\) takes place before the symmetry breaking \(SU(2) \to U(1)\). The symmetry breaking \(SU(3) \to SU(2)\) through \(\chi_0\), generates the masses of the heavy gauge bosons such as \(U^{\pm\pm}, V^{\pm}, Z_2\), and exotic quarks.

Through the boson mass formulations in the above sections, we see that boson \(V^\pm\) only involves in the phase transition \(SU(3) \to SU(2)\). \(Z_1, W^\pm\) and \(h_1\) only involve in the phase transition \(SU(2) \to U(1)\). However, \(U^{\pm\pm}, Z_2\) and \(h^{-}\) involve in both two phase transitions.

The first one is the phase transition \(SU(3) \to SU(2)\). This phase transition involves exotic quarks, heavy bosons, without involvement of the SM particles, so \(v_\rho\) is omitted in this phase transition. The effective potential can be rewritten as follows \[53\]

\[
V_{\text{eff}}^{\text{SU}(3)\to SU(2)} = D'(T^2 - T_0^2)v_\chi^2 - E'Tv_\chi^3 + \frac{\lambda_T T^4 v_\chi^4}{4}.
\]

The minimum conditions are

\[
V_{\text{eff}}(\chi_0) = 0; \quad V'_{\text{eff}}(\chi_0) = 0; \quad V''_{\text{eff}}(\chi_0) = m_{h_2}^2,
\]

where

\[
D' = \frac{1}{24v_{\chi_0}^2} \left\{ 6m_U^2 + 3m_{Z_2}^2 + 6m_V^2 + 18m_Q^2 + 2m_{h^\pm}^2 \right\},
\]

\[
T_0^2 = \frac{1}{D} \left\{ \frac{1}{4} m_{h_2}^2 - \frac{1}{32\pi^2 v_{\chi_0}^2} \left( 6m_U^4 + 3m_{Z_2}^4 + 6m_V^4 - 36m_Q^4 + 2m_{h^\pm}^4 \right) \right\},
\]

\[
E' = \frac{1}{12\pi^3 v_{\chi_0}^3} \left( 6m_U^3 + 3m_{Z_2}^3 + 6m_V^3 + 2m_{h^\pm}^3 \right),
\]

\[
\lambda_T = \frac{m_{h_2}^2}{2v_{\chi_0}^2} \left[ 1 - \frac{1}{8\pi^2 v_{\chi_0}^2 m_{h_2}^2} \left( 6m_V^4 \ln \frac{m_V^2}{bT^2} + 3m_{Z_2}^4 \ln \frac{m_{Z_2}^2}{bT^2} \\
+ 6m_U^4 \ln \frac{m_U^2}{bT^2} - 36m_Q^4 \ln \frac{m_Q^2}{bF T^2} + 2m_{h^\pm}^4 \ln \frac{m_{h^\pm}^2}{bT^2} \right) \right].
\]
The critical temperature is determined as follows

\[ T_c' = \frac{T_0'}{\sqrt{1 - E'^2 / D' \lambda T_c'}}. \]  
\[ (24) \]

For simplicity, let us assume \( m_{h_2} = X, m_{h^-} = m_{Z_2} = m_Q = K \). In order to have the first-order phase transition, the phase transition strength must be larger than 1, i.e., \( \frac{\nu_{\chi}}{T_c} \geq 1 \).

If \( X \) is larger than 200 GeV, the heavy particle masses are in range of few TeVs in order to have the first-order phase transition \([53]\). In order to have the first-order phase transition, if the contribution of \( h_2 \) with the mass is smaller than 200 GeV, \( K \) is smaller than 1.5 TeV \([53]\).

The second/last step is the phase transition \( SU(2) \rightarrow U(1) \). This phase transition does not involve the exotic quarks and boson \( V^\pm \). Hence, in this case, \( \nu_{\chi} \) is neglected, and the contribution of \( U^{\mp} \) is equal to \( W^\mp \). Then

\[ V_{SU(2)\rightarrow U(1)}^{\text{eff}} = v_0(v_\rho) \frac{1}{64\pi^2} \left( \frac{m_{h_2}^4}{Q^2} \ln \frac{m_{h_2}^2}{Q^2} + 2m_{h^+}^4 \ln \frac{m_{h^+}^2}{Q^2} \right) \]
\[ + \frac{3}{64\pi^2} \left( 2m_{U}^4 \ln \frac{m_{U}^2}{Q^2} + m_{Z_1}^4 \ln \frac{m_{Z_1}^2}{Q^2} \right) \]
\[ + m_{Z_2}^4 \ln \frac{m_{Z_2}^2}{Q^2} + 2m_{W}^4 \ln \frac{m_{W}^2}{Q^2} - 4m_t^4 \ln \frac{m_t^2}{Q^2} \right) \]
\[ + \frac{T^4}{4\pi^2} \left[ F_\left( \frac{m_{h_2}}{T} \right) + 2F_\left( \frac{m_{h^+}}{T} \right) \right] \]
\[ + \frac{3T^4}{4\pi^2} \left[ 2F_\left( \frac{m_{U}}{T} \right) + F_\left( \frac{m_{Z_1}}{T} \right) \right] \]
\[ + F_\left( \frac{m_{Z_2}}{T} \right) + 4F_\left( \frac{m_{W}}{T} \right) + 4F_\left( \frac{m_t}{T} \right) \]

Denoting \( V_{SU(2)\rightarrow U(1)}^{\text{eff}} \equiv V_{SU(2)\rightarrow U(1)}^{\text{eff}}(v_\rho, T) \), at high-temperature, it becomes

\[ V_{e_{ efficiency}}^{RM331} = D(T^2 - T_0^2), v_\rho^2 - ET|v_\rho|^3 + \frac{\lambda r}{4} v_\rho^4, \]

where

\[ D = \frac{1}{24v_0^2} \left\{ 6m_W^2 + 6m_U^2 + 3m_{Z_1}^2 + 3m_{Z_2}^2 + 6m_t^2 + m_{h_2}^2 + 2m_{h^\pm}^2 \right\}, \]
\[ T_{0}^2 = \frac{1}{D} \left\{ \frac{1}{4} \frac{m_{h_1}^2}{v_0^2} - \frac{1}{32\pi^2 v_0^4} \left( 6m_W^4 + 6m_U^4 + 3m_{Z_1}^4 + 3m_{Z_2}^4 - 12m_t^4 \right) \right\}, \]
\[ E = \frac{1}{12\pi v_0^3} \left( 6m_W^3 + 6m_U^3 + 3m_{Z_1}^3 + 3m_{Z_2}^3 + m_{h_2}^3 + 2m_{h^\pm}^3 \right), \]  
\[ (25) \]
\[ \lambda_T = \frac{m_{h_1}^2}{2v_0^2} \left\{ 1 - \frac{1}{8\pi^2 v^2_0 m_h^2} \left[ 6m_W^4 \ln \frac{m_W^2}{bT^2} + 3m_{Z_1}^4 \ln \frac{m_{Z_1}^2}{bT^2} + 3m_{Z_2}^4 \ln \frac{m_{Z_2}^2}{bT^2} \\ + 6m_U^4 \ln \frac{m_U^2}{bT^2} - 12m_t^4 \ln \frac{m_t^2}{bT^2} + m_{h_2}^4 \ln \frac{m_{h_2}^2}{bT^2} + 2m_{h_1}^4 \ln \frac{m_{h_1}^2}{bT^2} \right] \right\}, \]

where we have assumed \( m_{H_2} = m_{h_{--}} = m_{Z_2} \equiv Y \) with boson \( Z_2 \) and used \( Q \equiv v_{\rho_0} = v_0 = 246 \text{ GeV} \).

In order to have the first-order phase transition, the phase transition strength has to be larger than 1, i.e., \( \frac{v_c}{T_c} \geq 1 \). The critical temperature \( T_c \) is given by

\[ T_c = \sqrt{1 - \frac{E^2}{D \lambda_T}}. \] (26)

To survive the critical temperatures, \( T_c, T_0 \) must be positive, so \( T_0 \) is also positive, from which we can draw on conditions for heavy particles. Therefore, we get

\[ \frac{1}{4} m_{h_1}^2 - \frac{1}{32\pi^2 v_0^2} \left( 6m_W^4 + 6m_U^4 + 3m_{Z_1}^4 + 3m_{Z_2}^4 - 12m_t^4 + m_{h_2}^4 + 2m_{h_1}^4 \right) > 0 \]

With \( m_{h_1} = 125 \text{ GeV} \) and assuming \( m_{Z_2} = m_{h_2} = m_{h_{--}} = Y \), we can obtain \( Y < 344.718 \text{ GeV} \).

When \( \frac{v_c}{T_c} = 1 \), i.e., \( 2E/\lambda_T = 1 \), we can obtain \( Y = 203.825 \text{ GeV} \), and the critical temperature is in range \( 0 < T_c < 111.473 \text{ GeV} \).

The contributions of new particles make of the strongly first-order phase transition that the SM cannot. However, there is one thing special, heavy particles as \( U^{\pm \pm}, h_2, h_{--}, Z_2 \) that contribute only the little part in their mass.

When temperature goes close to \( T_c \), the second minimum slowly formed distinct, i.e., the phase transition nucleation appears.

When temperature goes over \( T_c \), the minimum goes to zero, i.e., the symmetry phase is restored. This was showed that phase transition \( SU(2) \rightarrow U(1) \) is the first-order phase transition.

We find that the effective potential of this model is different from that of the SM, and it has contributions from heavy bosons as triggers for the strongly first-order phase transition with \( m_{h_1} = 125 \text{ GeV} \).

We have got the following constraints on the mass of Higgs in RM331

\[ 285.56 \text{ GeV} < M_{h_2} < 1.746 \text{ TeV}, \quad 3.32 \text{ TeV} < M_{h_{--}} < 5.61 \text{ TeV}. \]
TABLE II: Mass formulations of bosons in the E331 model

| Bosons | $m^2(\omega, v)$ | $m^2(\omega)$ | $m^2(v)$ |
|--------|-----------------|---------------|----------|
| $m_{W^\pm}^2$ | $\frac{g^2}{4} v^2$ | 0 | $80.39^2$ (GeV)$^2$ |
| $m_{Y^\pm}^2$ | $\frac{g^2}{4}(\omega^2 + v^2)$ | $\frac{g^2}{4}\omega^2$ | $80.39^2$ (GeV)$^2$ |
| $m_{X^0}^2$ | $\frac{g^2}{4}\omega^2$ | $\frac{g^2}{4}\omega^2$ | 0 |
| $m_{Z_1}^2 \sim m_{Z_2}^2$ | $\frac{g^2 v^2}{3+4v^2}$ | 0 | $91.68^2$ (GeV)$^2$ |
| $m_{Z_2}^2 \sim m_{Z_2}'$ | $\frac{g^2 v^2}{3-4v^2} \omega^2$ | $\frac{g^2 v^2}{3-4v^2} \omega^2$ | 0 |
| $m_{H^0}^2$ | $(2\lambda_2 - \frac{\lambda_4^2}{2\lambda_1}) v^2$ | 0 | $125^2$ (GeV)$^2$ |
| $m_{H^+}^2$ | $2\lambda_1 \omega^2 + \frac{\lambda_4^2}{2\lambda_1} v^2$ | $2\lambda_1 \omega^2$ | $\frac{\lambda_4^2}{2\lambda_1} v^2$ |
| $m_{H^+}^2$ | $\frac{\lambda_4}{2}(\omega^2 + v^2)$ | $\frac{\lambda_4}{2}\omega^2$ | $\frac{\lambda_4}{2} v^2$ |

Thus we have used the effective potential at finite temperature to study the structure of the EWPT in the RM331 model. This phase transition is split into two phases, namely, the first transition is $SU(3) \rightarrow SU(2)$ or the symmetry breaking in the energy scale $v_{\chi_0}$ in order to generate masses for heavy particles and exotic quarks. The second phase transition is $SU(2) \rightarrow U(1)$ at $v_{\rho_{\chi_0}}$. The EWPT in this model may be the strongly first-order EWPT with $m_{h_1} = 125$ GeV if the heavy bosons masses are some few TeVs.

B. Phase transition in economical 3-3-1 model

In this section, we follow the same approach for E331 model [22], whose lepton sector is more complicated than that of the RM331 model. The E331 model has the right-handed neutrino in the leptonic content, the bileptons (two singly charged gauge bosons $W^\pm$, $Y^\pm$, and a neutral gauge bosons $X^0$), the heavy neutral boson $Z_2$, and the exotic quarks. The masses of particles in the E331 were summarized in Table II

As in the RM331, here EWPT takes place with two transitions: i) $SU(3) \rightarrow SU(2)$ at the scale of $\omega_0$ and the transition $SU(2) \rightarrow U(1)$ at the scale of $v_0$ [54].

The first phase transition $SU(3) \rightarrow SU(2)$ due to $\omega$ provides the bounds on parameters presented in Table II

The new bosons and exotic quarks can be triggers for the EWPT $SU(3) \rightarrow SU(2)$ to
TABLE III: The mass ranges of $H^0_1$ and $H^\pm_2$ for the first-order EWPT $SU(3) \to SU(2)$ and their upper bounds by the condition $m_{boson} < 2.2 \times T'_c$.

| $\omega$ [TeV] | $T'_c$ [GeV] | $m_{H^0_1}$ [GeV] | $m_{H^\pm_2}$ [GeV] | Upper bound [GeV] |
|----------------|--------------|-------------------|-------------------|------------------|
| 1              | 350          | $0 < m_{H^0_1} < 300$ | $0 < m_{H^\pm_2} < 720$ | 770              |
| 2              | 650          | $0 < m_{H^0_1} < 600$ | $0 < m_{H^\pm_2} < 1440$ | 1430             |
| 3              | 950          | $0 < m_{H^0_1} < 900$ | $0 < m_{H^\pm_2} < 2150$ | 2090             |
| 4              | 1300         | $0 < m_{H^0_1} < 1200$ | $0 < m_{H^\pm_2} < 2870$ | 2860             |
| 5              | 1600         | $0 < m_{H^0_1} < 1500$ | $0 < m_{H^\pm_2} < 3590$ | 3520             |

be the first-order. It was shown that the EWPT $SU(2) \to U(1)$ is the first-order phase transition, but it seems quite weak [54].

V. ELECTROWEAK SPHALERONS IN THE REDUCED MINIMAL 3-3-1 MODEL

To be consistent with cosmological evolution, our strategy is the following: the model has to have an inflation or phase transition of the first-order. As a result, the leptogenesis or CP-violation exist. Then sphaleron completes to produce the BAU. Sphaleron is a transition at high temperature where thermal fluctuations can bring the magnitude of the Higgs field from zero VEV over the barrier to nonzero VEV classically without tunneling. In [55], the sphalerons in the RM331 were considered. In the SM, the sphaleron rate is very small, about $10^{-60}$ [56–60]; this rate is much smaller than the rate of BAU and smaller than the cosmological expansion rate.

To study the sphaleron processes, we consider the Lagrangian of the gauge- Higgs system

$$L_{\text{gauge-Higgs}} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \left( D_\mu \chi \right) \dagger \left( D^\mu \chi \right) + \left( D_\mu \rho \right) \dagger \left( D^\mu \rho \right) - V(\chi, \rho).$$  \hspace{1cm} (27)

Assuming the least energy has the pure-gauge configurations ($F^a_{ij} = 0$), we get energy functional in the temporal gauge

$$E = \int d^3 x \left[ \left( D_\mu \chi \right) \dagger \left( D^\mu \chi \right) + \left( D_\mu \rho \right) \dagger \left( D^\mu \rho \right) + V(\chi, \rho) \right],$$ \hspace{1cm} (28)
By the temperature expansion, the energy functional is given by

\[ \mathcal{E} = 4\pi \int_0^\infty d^3x \left[ \frac{1}{2} \left( \nabla^2 v_\chi \right)^2 + \frac{1}{2} \left( \nabla^2 v_\rho \right)^2 + V_{\text{eff}}(v_\chi, v_\rho; T) \right]. \]  

(29)

In the static field approximation, we have two equations of motion for the VEVs in spherical coordinates \[55\] for the VEVs:

\[ \ddot{v}_\chi + \nabla^2 v_\chi - \frac{\partial V_{\text{eff}}(v_\chi, T)}{\partial v_\chi} = 0, \]  

(30)

and

\[ \ddot{v}_\rho + \nabla^2 v_\rho - \frac{\partial V_{\text{eff}}(v_\rho, T)}{\partial v_\rho} = 0. \]  

(31)

Then, the sphaleron energies in the \( SU(3) \rightarrow SU(2) \) and \( SU(2) \rightarrow U(1) \) phase transitions, are given, respectively

\[ \mathcal{E}_{\text{sph.su}(3)} = 4\pi \int \left[ \frac{1}{2} \left( \frac{dv_\chi}{dr} \right)^2 + V_{\text{eff}}(v_\chi, T) \right] r^2 dr, \]  

(32)

and

\[ \mathcal{E}_{\text{sph.su}(2)} = 4\pi \int \left[ \frac{1}{2} \left( \frac{dv_\rho}{dr} \right)^2 + V_{\text{eff}}(v_\rho, T) \right] r^2 dr. \]  

(33)

The sphaleron rate per unit time per unit volume, \( \Gamma/V \), is characterized by a Boltzmann factor, \( \exp (-\mathcal{E}/T) \), as follows \[60-62\]:

\[ \Gamma/V = \alpha^4 T^4 \exp (-\mathcal{E}/T), \]  

(34)

where \( V \) is the volume of the EWPT’s region, \( T \) is the temperature, \( \mathcal{E} \) is the sphaleron energy, and \( \alpha = 1/30 \).

We will compare the sphaleron rate with the Hubble constant, which describes the cosmological expansion rate at the temperature \( T \) \[63-64\]

\[ H^2 = \frac{\pi^2 g T^4}{90 M_{\text{pl}}^2}, \]  

(35)

where \( g = 106.75 \), \( M_{\text{pl}} = 2.43 \times 10^{18} \) GeV.

Assuming that the VEVs of the Higgs fields do not change from point to point in the universe, then we have \( \frac{dv_\chi}{dr} = \frac{dv_\rho}{dr} = 0 \), and

\[ \frac{\partial V_{\text{eff}}(v_\chi)}{\partial v_\chi} = 0, \quad \frac{\partial V_{\text{eff}}(v_\rho)}{\partial v_\rho} = 0. \]  

(36)
Eqs. (36) shows that $v_\chi$ and $v_\rho$ are the extremes of the effective potentials. The sphaleron energies can be rewritten as

$$E_{sph.su(3)} = 4\pi \int V_{eff}(v_\chi, T)r^2 dr = \left. \frac{4\pi r^3}{3} V_{eff}(v_\chi, T) \right|_{v_\chi m},$$ (37)

and

$$E_{sph.su(2)} = 4\pi \int V_{eff}(v_\rho, T)r^2 dr = \left. \frac{4\pi r^3}{3} V_{eff}(v_\rho, T) \right|_{v_\rho m},$$ (38)

where $v_\chi m, v_\rho m$ are the VEVs at the maximum of the effective potentials. From (37) and (38), it follows that the sphaleron energies are equal to the maximum heights of the potential barriers.

The universe’s volume at a temperature $T$ is given by $V = \frac{4\pi r^3}{3} = \frac{1}{T^3}$. Because the whole universe is an identically thermal bath, the sphaleron energies are approximately

$$E_{sph.su(3)} \sim \frac{E'^4 T}{4\lambda'^3 T}; \quad E_{sph.su(2)} \sim \frac{E^4 T}{4\lambda^3 T}.$$ (39)

From the definitions (37) and (38), the sphaleron rates take the form, respectively

$$\Gamma_{su(3)} = \alpha_w^4 T \exp \left(-\frac{E'^4 T}{4\lambda'^3 T}\right),$$ (40)

and

$$\Gamma_{su(2)} = \alpha_w^4 T \exp \left(-\frac{E^4 T}{4\lambda^3 T}\right).$$ (41)

For the heavy particles, $E, \lambda, E'$ and $\lambda'$ are constant, and the sphaleron rates (for the the phase transition $SU(2) \rightarrow U(1)$) in this approximation are the linear functions of temperature \[55\]

Thus, the upper bounds of the sphaleron rates are much larger the Hubble constant \[55\]

$$\Gamma_{su(3)} \sim 10^{-3} \gg H; \quad \Gamma_{su(2)} \sim 10^{-4} \gg H \sim 10^{-13}.$$ (42)

In a thin-wall approximation, sphaleron rates are presented in Tables IV and V

Here $R_{b.su(3)}$ and $\Delta l'$ are respectively the radius and the wall thickness of a bubble which is nucleated in the phase transitions.

We conclude that the sphaleron rates are larger than the cosmological expansion rate at temperatures above the critical temperature and are smaller than the cosmological expansion rate at temperatures below the critical temperature. For each transition, baryon violation rapidly takes place in the symmetric phase regions but it also quickly shuts off in the broken phase regions. This may provide B-violation necessary for baryogenesis, as required by the first of Sakharov’s conditions, in the connection with non-equilibrium physics.
TABLE IV: The sphaleron rate in the EWPT $SU(3) \rightarrow SU(2)$ with $m_q(v_\chi) = m_h(v_\chi) = 1500$ GeV.

| $T$ [GeV] | $R_{b,su}(3)$ $[10^{-6} \times \text{GeV}^{-1}]$ | $R_{b,su}(3)/\Delta l'$ | $\mathcal{E}_{\text{sph.SU}(3)}$ [GeV] | $\Gamma_{SU(3)}$ $[10^{-11} \times \text{GeV}]$ | $H$ $[10^{-12} \times \text{GeV}]$ | $\Gamma_{SU(3)}/H$ |
|-----------|---------------------------------|------------------|-----------------|------------------|-----------------|-----------------|
| 1479.48 ($T'_1$) | 10 | 10 | 6975.17 | 1.63719 $\times$ 10$^6$ | 3.08195 | 5.31 $\times$ 10$^6$ |
| 1450 | 12 | 12 | 12481.3 | 3.2702 $\times$ 10$^4$ | 2.96034 | 1.10 $\times$ 10$^5$ |
| 1400 | 13 | 13 | 17206.3 | 7.94481 $\times$ 10$^2$ | 2.7597 | 2.878 $\times$ 10$^3$ |
| 1390 | 15 | 15 | 23251.7 | 9.3264 | 2.72042 | 3.42 |
| 1388.4556 ($T''_1$) | 16.5 | 16.5 | 28135.1 | 0.2714 | 2.71438 | 1 |
| 1387 | 17 | 17 | 29854.0 | 0.07687 | 2.70869 | 0.28 |
| 1000 | 19 | 19 | 60590.8 | 5.98 $\times$ 10$^{-19}$ | 1.40801 | 4.25 $\times$ 10$^{-18}$ |
| 900 | 22 | 22 | 89250.8 | 9.50 $\times$ 10$^{-36}$ | 1.14049 | 8.33 $\times$ 10$^{-35}$ |
| 865.024 ($T'_0$) | 25 | 25 | 119110.36 | 1.69 $\times$ 10$^{-52}$ | 1.05357 | 1.60 $\times$ 10$^{-51}$ |

TABLE V: The sphaleron rate in the EWPT $SU(2) \rightarrow U(1)$ with $m_{h\pm}(v_p) = 100$ GeV, $m_{h\pm}(v_p) = 350$ GeV.

| $T$ [GeV] | $R_{s,su}(2)$ $[10^{-4} \times \text{GeV}^{-1}]$ | $R_{s,su}(2)/\Delta l'$ | $\mathcal{E}_{\text{sph.SU}(2)}$ [GeV] | $\Gamma_{SU(2)}$ $[10^{-12} \times \text{GeV}]$ | $H$ $[10^{-14} \times \text{GeV}]$ | $\Gamma_{SU(2)}/H$ |
|-----------|---------------------------------|------------------|-----------------|------------------|-----------------|-----------------|
| 141.574 ($T'_1$) | 6 | 10 | 742.838 | 919936.07 | 2.82211 | 3.25 $\times$ 10$^7$ |
| 141.5 | 8 | 10 | 1020.87 | 128525.28 | 2.81916 | 4.55 $\times$ 10$^6$ |
| 141 | 10 | 10 | 1442.75 | 6264.89 | 2.79927 | 2.23 $\times$ 10$^5$ |
| 140 | 12 | 12 | 2342.21 | 9.37289 | 2.7597 | 339.6 |
| 138.562 ($T''_1$) | 13.1 | 13 | 3135.75 | 0.02703 | 2.703 | 1 |
| 137 | 14 | 14 | 3922.29 | 0.0000622 | 2.6427 | 2.357 $\times$ 10$^{-3}$ |
| 130 | 16 | 16 | 6567.08 | 1.847 $\times$ 10$^{-14}$ | 2.379 | 7.76 $\times$ 10$^{-13}$ |
| 120 | 18 | 18 | 10068.2 | 5.403 $\times$ 10$^{-29}$ | 2.02754 | 2.66 $\times$ 10$^{-27}$ |
| 118.42 ($T'_0$) | 20 | 20 | 12656.7 | 5.595 $\times$ 10$^{-39}$ | 6.209 | 9.01 $\times$ 10$^{-38}$ |

VI. CONCLUSIONS

In this review, we have showed that the 3-3-1 models are able to describe the cosmological evolution. The 3-3-1 models contain the hybrid inflationary scenario and the first-order
phase transitions. The inflation happens in the GUT scale, while phase transition has two sequences corresponding two steps of symmetry breaking in the models. The sphaleron rates are much larger than the Hubble constant. They are larger than the cosmological expansion rate at temperatures above the critical temperature and are smaller than the cosmological expansion rate at temperatures below the critical temperature. From these considerations, some bound on model parameters are deduced.

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