Derivation of optical solitons of dimensionless Fokas-Lenells equation with perturbation term using Sardar sub-equation method

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Received: 7 March 2022 / Accepted: 6 May 2022 / Published online: 6 June 2022
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Abstract
This paper presents an investigation of soliton solutions for the perturbed Fokas-Lenells (pFL) equation, which has a vital role in optics, using Sardar sub-equation method. The equation models the propagation of ultrashort light pulses in optical fibers. Using appropriate wave transformation, the pFL equation is reduced to a nonlinear ordinary differential equation (NLODE). The solutions of this NLODE equation are assumed to be in the suggested form by the Sardar sub-equation method. Hence, an algebraic equation system is obtained by substituting the trial solution and their necessary derivatives into the NLODE. After finding the unknowns in the system, the soliton solutions of the perturbed Fokas-Lenells equation are extracted. The method produces various kinds of solitons such as dark, periodic, singular periodic, combined bright-dark. To show physical representations of the solitons, 2D, 3D and contour plots of the solutions are demonstrated via computer algebraic systems. It is expected that derived solutions may be useful for future works in various fields of science, especially optics and so, it may contribute to the optic fiber industry.

Keywords  Fokas-Lenells equation · Optical fiber · Sardar sub-equation method · Solitons
1 Introduction

Nonlinear partial differential equations (NLPDEs) are commonly used to simulate phenomena in a variety of fields, including plasma physics, quantum field theory, nonlinear optics, fluid mechanics, geochemistry, condensed matter, particle physics, meteorology and biophysics (Wazwaz 2022; Saleh et al. 2021a; Sowndarrajan et al. 2020; Cinar et al. 2022a; Saleh et al. 2021b; Ozisik 2022; Polyanin and Zhurov 2020; Esen et al. 2021; Cinar et al. 2021; Owolabi 2017). Researchers in numerous branches of science such as engineering, natural and applied sciences are very interested in studying the exact solutions of NLPDEs to better understand the phenomena in nature. In literature, many powerful methods are available for solving analytically the NLPDEs. For instance, please see the unified algebraic method (Fan 2003), F-expansion method (Yıldırım 2021; Zhou et al. 2003), sine-Gordon method (Yıldırım 2021; Yan 1996; Inc et al. 2018a, b), extended rational sin-cos/sinh-cosh method (Mahak and Akram 2019; Cinar et al. 2022b), new general extended direct algebraic method (Munawar et al. 2021), Kudryashov-expansion method (Alqurani 2021) and modified rational cosine-sine method (Alqurani and Alqawaqneh 2022). Besides, to get more abundant solutions, the researchers try to introduce new methods or modify or extend the existing methods such as modified extended tanh-function method (Darwish et al. 2021; Wazwaz 2007), new Kudryashov method, Kudryashov (2020), extended, modified auxiliary equation method (Akram and Sarfraz 2021; Khater et al. 2019). The considered methods can produce various types of solitons such as dark, bright, cuspon, kink, compacton, bell-shaped, etc. With the rapid advancement in soliton theory, new types of solitons like gap (Mandelik et al. 2004), rotary (Wang et al. 2006) and breather (Dai et al. 2017), were experimentally observed (Triki et al. 2022). Due to their stable features, optical solitons have crucial applications in optical fibers, communication industries, biomedical, fluid dynamics and biology, etc. (Agrawal 2000; Hasegawa 2005).

In this study, we deal with the dimensionless form pFL equation modeling the light propagation inside optical fibers (Fokas 1995; Lenells 2009; Muniyappan et al. 2022):

\[ iu_t + \alpha_1 u_{xx} + \alpha_2 u_{xt} + |u|^2 (bu + i\mu u_x) = i[pu_x + q(|u|^{2\beta}u)_x + \beta(|u|^{2\beta})_x u], \quad i = \sqrt{-1}, \]

where \( u = u(x, t) \) denotes the complex valued wave profile, \( p, q, \beta \) and \( h \) represent intermodal dispersion, effect of the self-steeping perturbation, nonlinear dispersion, and the degree of the full non-linearity, respectively. \( \alpha_1, \alpha_2, b, \mu \) are the coefficients of group velocity dispersion (GVD), spatio-temporal dispersion (STD), self-phase modulation (SPM), nonlinear dispersion (ND), respectively. \( \alpha_1, \alpha_2, b, \mu, p, q \) and \( \beta \) are real constants and \( h \) is an integer.

Recently, pFL equation has been studied by some researchers using improved Adomian decomposition algorithm (Al-Qarni et al. 2022), modified extended tanh function scheme (Muniyappan et al. 2022), sine–Gordon equation approach (Yıldırım et al. 2021), complex envelope ansatz method (Dieu-donne et al. 2020), Kudryashov methods (Dan et al. 2020; Seadawy et al. 2021), Riccati-Bernoulli Sub-ODE method (Shehata et al. 2019), \( \exp(-\phi(\xi)) \)-expansion method (Arshed et al. 2019), semi-inverse variational principle (Biswas 2018), \( G - G' \) expansion and Jacobi’s elliptic function method (Bansal et al. 2018).

In this article, we mainly aim to extract abundant soliton solutions in the explicit form for the pFL equation using Sardar sub-equation method and to determine the constrain conditions for the existence of the solutions.
The following sections are arranged as: The main steps of the Sardar sub-equation method are described in Sect. 2. The considered method is implemented to the pFL equation in Sect. 3. In Sect. 4, graphical representations are given and obtained results are discussed. Last section includes the conclusion of this work.

2 Analysis of the method

Let us deal with a NLPDE in the general form:
\[ \mathcal{J}(g, g_x, g_t, g_{xx}, g_{tt}, \ldots) = 0, \tag{2} \]
and traveling wave transformations as follows:
\[ g(x, t) = e^{i\theta} \mathcal{G}(\eta), \quad \eta = \alpha x - \omega t, \quad \theta = -kx + \lambda t + \phi_0, \tag{3} \]
in which \( \alpha, \omega, k, \lambda \) and \( \phi_0 \) are non-zero reals.

Step 1 Convert the NLPDE in eq. (2) to a NLODE in eq. (4) by substituting the wave transformations in eq. (3) into eq. (2):
\[ \mathcal{P}(\mathcal{G}(\eta), \mathcal{G}'(\eta), \mathcal{G}''(\eta), \ldots) = 0, \tag{4} \]
where \( \mathcal{P} \) is a polynomial in \( \mathcal{G}(\eta) \), the superscripts symbolize the ordinary derivatives of \( \mathcal{G}(\eta) \) with respect to \( \eta \).

Step 2 Assume that NLODE in the eq. (4) has a solution as follows:
\[ \mathcal{G}(\eta) = \sum_{i=0}^{\kappa} \mu_i \Phi^{(i)}(\eta), \quad \mu_0 \neq 0. \tag{5} \]
where \( \mu_0, \mu_1, \ldots, \mu_\kappa \) are real parameters to be found, \( \kappa \in \mathbb{Z}^+ \) is a balancing constant to be determined by applying balancing rule to eq. (4) and \( \Phi(\eta) \) satisfies the following ODE:
\[ \left[ \frac{d\Phi(\eta)}{d\eta} \right]^2 = \rho_1 + \rho_2 \Phi(\eta)^2 + \rho_3 \Phi(\eta)^4. \tag{6} \]
Here, eq. (6) has the following solutions:

- **Case I:** If \( \rho_2 > 0 \) and \( \rho_1 = 0 \),
  \[ \Phi^+_1(\eta) = \pm \sqrt{-rs\rho_2} \sec h_{rs}(\sqrt{\rho_2}\eta), \]
  \[ \Phi^+_2(\eta) = \pm \sqrt{rs\rho_2} \csc h_{rs}(\sqrt{\rho_2}\eta), \tag{7} \]
  where
  \[ \sec h_{rs} = \frac{2}{re^{\eta} + se^{-\eta}}, \quad \csc h_{rs} = \frac{2}{re^{\eta} - se^{-\eta}}. \tag{8} \]
• Case II: If $\rho_2 < 0$ and $\rho_1 = 0$,
\[
\Phi^\pm_3(\eta) = \pm \sqrt{-rs\rho_2} \sec_{rs}(\sqrt{-\rho_2\eta}),
\]
\[
\Phi^\pm_4(\eta) = \pm \sqrt{-rs\rho_2} \csc_{rs}(\sqrt{-\rho_2\eta}),
\]
where
\[
\sec_{rs} = \frac{2}{re^{i\eta} + se^{-i\eta}}, \quad \csc_{rs} = \frac{2i}{re^{i\eta} - se^{-i\eta}},
\]

• Case III: If $\rho_2 < 0$ and $\rho_1 = \rho_2^2/4$,
\[
\Phi^\pm_5(\eta) = \pm \sqrt{-\rho_2/2} \tan_{rs}(\sqrt{-\rho_2/2 \eta}),
\]
\[
\Phi^\pm_6(\eta) = \pm \sqrt{-\rho_2/2} \cot_{rs}(\sqrt{-\rho_2/2 \eta}),
\]
\[
\Phi^\pm_7(\eta) = \pm \sqrt{-\rho_2/2} \left( \tan_{rs}(\sqrt{-2\rho_2\eta}) + i\sqrt{rs} \sec_{rs}(\sqrt{-2\rho_2\eta}) \right),
\]
\[
\Phi^\pm_8(\eta) = \pm \sqrt{-\rho_2/2} \left( \cot_{rs}(\sqrt{-2\rho_2\eta}) + \sqrt{rs} \csc_{rs}(\sqrt{-2\rho_2\eta}) \right),
\]
\[
\Phi^\pm_9(\eta) = \pm \sqrt{-\rho_2/8} \left( \cot_{rs}(\sqrt{-\rho_2/8 \eta}) + \tan_{rs}(\sqrt{-\rho_2/8 \eta}) \right),
\]
where
\[
\tan_{rs} = \frac{re^{-i\eta} - se^{-i\eta}}{re^{-i\eta} + se^{-i\eta}}, \quad \cot_{rs} = \frac{re^{-i\eta} + se^{-i\eta}}{re^{-i\eta} - se^{-i\eta}},
\]

• Case IV: If $\rho_2 > 0$ and $\rho_1 = \rho_2^2/4$,
\[
\Phi^\pm_{10}(\eta) = \pm \sqrt{\rho_2/2} \tan_{rs}(\sqrt{\rho_2/2 \eta}),
\]
\[
\Phi^\pm_{11}(\eta) = \pm \sqrt{\rho_2/2} \cot_{rs}(\sqrt{\rho_2/2 \eta}),
\]
\[
\Phi^\pm_{12}(\eta) = \pm \sqrt{\rho_2/2} \left( \tan_{rs}(\sqrt{2\rho_2\eta}) \pm \sqrt{rs} \sec_{rs}(\sqrt{2\rho_2\eta}) \right),
\]
\[
\Phi^\pm_{13}(\eta) = \pm \sqrt{\rho_2/2} \left( \cot_{rs}(\sqrt{2\rho_2\eta}) + \sqrt{rs} \csc_{rs}(\sqrt{2\rho_2\eta}) \right),
\]
\[
\Phi^\pm_{14}(\eta) = \pm \sqrt{\rho_2/8} \left( -\tan_{rs}(\sqrt{\rho_2/8 \eta}) + \cot_{rs}(\sqrt{\rho_2/8 \eta}) \right),
\]
in which
\[
\tan_{rs} = -i\frac{re^{i\eta} - se^{i\eta}}{re^{i\eta} + se^{i\eta}}, \quad \cot_{rs} = i\frac{re^{i\eta} + se^{i\eta}}{re^{i\eta} - se^{i\eta}}.
\]
where $r, s$ are non-zero real values.

**Step 3** Derive a polynomial in powers of $\Phi(\eta)$ by putting eq. (5) and its required derivatives into eq. (4). An algebraic equation system for $\mu_0, \mu_1, \ldots, \mu_k, \rho_1, \rho_2, \rho_3, \alpha, \omega, k$ and $\lambda$ is obtained by collecting the terms in the polynomial that include the same power of $\Phi(\eta)$ and then setting the coefficient to zero.

**Step 4** Determine the appropriate solution set(s) of the unknown parameters solving the system in the previous step via a computer algebraic system. Insert the solution sets to eq. (5) and $\Phi_i^+(\eta)$, $(i = 1, 2, \ldots, 14)$ in the Step 2 and consider the eq. (3). So, obtain the solution(s) of the NLPDE in eq. (2).

### 3 Application

Inserting the following wave transformation to the pFL equation by considering $h = 1$ in eq. (1):

$$u(x,t) = e^{i\theta} \mathcal{U}(\eta), \quad \eta = x - \omega t, \quad \theta = -kx + \lambda t + \phi_0,$$

where $\mathcal{U}(\eta)$, $\theta$, $\phi_0$, $k$, $\lambda$ and $\omega$ stand for soliton pulse profile, phase component, phase constant, wave number, frequency and soliton velocity, respectively. Then we split up the real and imaginary parts. So, the following NLODEs can be derived from the real and imaginary parts, respectively:

$$\left(\alpha_1 - \alpha_2 \omega\right) \mathcal{U}''(\eta) + \left(\mathcal{U}(\eta)\right)^3(b + k(\sigma - q)) - \mathcal{U}(\eta)\left(\alpha_1 k^2 - \alpha_2 \lambda k + kp + \lambda\right) = 0, \quad (16)$$

$$\mathcal{U}'(\eta)\left(-\alpha_2(\omega k + \lambda) + 2\alpha_1 k + p + \omega + \left(\mathcal{U}(\eta)\right)^2(3q + 2\beta - \sigma)\right) = 0. \quad (17)$$

Integrating the eq. (17) and assuming the integral constant be zero, one may derive:

$$\mathcal{U}(\eta)\left(-\alpha_2(\omega k + \lambda) + 2\alpha_1 k + p + \omega\right) + \frac{1}{3}\left(\mathcal{U}(\eta)\right)^3(3q + 2\beta - \sigma) = 0. \quad (18)$$

The eq. (18) gives the following constraint conditions:

$$\omega = \frac{-\alpha_2 \lambda + 2\alpha_1 k + p}{\alpha_2 k - 1}, \quad \sigma = 3q + 2\beta, \quad \alpha_2 \neq \frac{1}{k}. \quad (19)$$

Substituting eq. (19) into eq. (16), we have:

$$\frac{\mathcal{U}''(\eta)\left(\alpha_1(\alpha_2 k + 1) + \alpha_2(p - \alpha_2 \lambda)\right)}{\alpha_2 k - 1} - \left(\mathcal{U}(\eta)\right)^3(b + 2k(\omega + \beta)) + \mathcal{U}(\eta)\left(\alpha_1 k^2 - \alpha_2 \lambda k + kp + \lambda\right) = 0. \quad (20)$$

Using homogeneous balance principle in the eq. (20), we find $\kappa + 2 = 3\kappa \Rightarrow \kappa = 1$.

So, eq. (5) degenerates to following form:

$$\mathcal{U}(\eta) = \mu_0 + \mu_1 \Phi(\eta), \quad (\mu_1 \neq 0). \quad (21)$$

Using the instructions in Step 3, we get the following system of algebraic equations:
Case II: If where the existence conditions for the solutions are: the following solutions of the pFL equation in eq. (1).

\[
\Phi^0(\eta) : \quad b\mu_0^3 + \alpha_1 k^2 \mu_0 - \alpha_2 k \lambda \mu_0 + kp\mu_0 - 2kq\mu_0^3 - 2k\beta \mu_0^3 + \lambda \mu_0 = 0,
\]

\[
\Phi^1(\eta) : \quad -\frac{\rho_2 \alpha_2^2 \lambda \mu_1}{\alpha_2 k - 1} + \frac{\rho_2 \alpha_2 \alpha_1 \mu_2}{\alpha_2 k - 1} + \frac{\rho_2 \alpha_2 p \mu_1}{\alpha_2 k - 1} - 3b\mu_0^2 \mu_1 + \alpha_1 k^2 \mu_1
\]

\[- \alpha_2 \lambda k \mu_1 + kp \mu_1 - 6kq\mu_0^2 \mu_1 - 6k\beta \mu_0^2 \mu_1 + \lambda \mu_1 = 0,
\]

\[
\Phi^2(\eta) : \quad -3b\mu_0 \mu_1^2 - 6kq \mu_0^2 \mu_1 - 6k\beta \mu_0^2 \mu_1^2 = 0,
\]

\[
\Phi^3(\eta) : \quad -2\rho_2 \alpha_2^2 \lambda \mu_1 + \frac{2\rho_2 \alpha_1 \mu_1}{\alpha_2 k - 1} + \frac{2\rho_2 \alpha_2 p \mu_1}{\alpha_2 k - 1} + \frac{2\rho_3 \alpha_2 p \mu_1}{\alpha_2 k - 1} - b\mu_1^3 - 2kq\mu_1^3 - 2k\beta \mu_1^3 = 0.
\]

Solving the system in eq. (22) via a computer algebraic system, we derive the following sets:

**Set 1, 2.**

\[
\rho_2 = -\frac{(\alpha_2 k - 1)(\alpha_1 k^2 - \alpha_2 \lambda k + kp + \lambda)}{C_1}, \quad \mu_0 = 0, \quad \mu_1 = \pm \sqrt{\frac{2\rho_3 C_1}{C_2}},
\]

\[
C_1 = \alpha_1 (\alpha_2 k - 1) + \alpha_2 (p - \alpha_2 \lambda), \quad C_2 = (\alpha_2 k - 1)(b + 2k(q + \beta)),
\]

where

\[
C_1 > 0, \quad C_2 > 0.
\]

Inserting the sets above into eq. (21) and considering eqs. (7), (9), (11), (13), (15), we find the following solutions of the pFL equation in eq. (1).

**Case I:** If \( \rho_2 > 0 \) and \( \rho_1 = 0, \)

\[
u_+^x(x, t) = \pm \mu_1 \sqrt{-\frac{\rho_2 rs}{\rho_3} e^{i\theta} \sec h_{r_3} \left( \sqrt{\rho_2} \left( x - \frac{t(-\alpha_2 \lambda + 2\alpha_1 k + p)}{\alpha_2 k - 1} \right) \right)}.
\]

where the existence conditions for the solutions are:

\[
rs < 0, \quad \rho_3 > 0.
\]

\[
u_+^x(x, t) = \pm \mu_1 \sqrt{-\frac{\rho_2 rs}{\rho_3} e^{i\theta} \csc h_{r_3} \left( \sqrt{\rho_2} \left( x - \frac{t(-\alpha_2 \lambda + 2\alpha_1 k + p)}{\alpha_2 k - 1} \right) \right)}.
\]

where the existence conditions for the solutions are:

\[
rs > 0, \quad \rho_3 > 0.
\]

**Case II:** If \( \rho_2 < 0 \) and \( \rho_1 = 0, \)

\[
u_+^x(x, t) = \pm \mu_1 \sqrt{-\frac{\rho_2 rs}{\rho_3} e^{i\theta} \sec h_{r_3} \left( \sqrt{-\rho_2} \left( x + \frac{t(-\alpha_2 \lambda + 2\alpha_1 k + p)}{\alpha_2 k - 1} \right) \right)}.
\]

(30)
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The solutions are:

\[
\begin{align*}
\text{Case III: } & \text{If } \rho_2 < 0 \text{ and } \rho_1 = \frac{\rho_2^2}{4}, \\
& u_7^\pm (x, t) = \pm \sqrt{2} \frac{-\rho_2}{\rho_3} \mu_1 e^{i\theta} \tan h_{rs} \left( \sqrt{-\rho_2} \left( x - \frac{t(-a_2k + 2\alpha_kk + p)}{a_kk - 1} \right) \right), \\
& u_8^\pm (x, t) = \pm \sqrt{2} \frac{-\rho_2}{\rho_3} \mu_1 e^{i\theta} \cot h_{rs} \left( \sqrt{-\rho_2} \left( x - \frac{t(-a_2k + 2\alpha_kk + p)}{a_kk - 1} \right) \right), \\
& u_9^\pm (x, t) = \pm \sqrt{2} \frac{-\rho_2}{\rho_3} \mu_1 e^{i\theta} \left( \tan h_{rs} \left( \sqrt{-\rho_2} \left( x - \frac{t(-a_2k + 2\alpha_kk + p)}{a_kk - 1} \right) \right) \right), \\
& u_{10}^\pm (x, t) = \pm \sqrt{2} \frac{-\rho_2}{\rho_3} \mu_1 e^{i\theta} \left( \cot h_{rs} \left( \sqrt{-\rho_2} \left( x - \frac{t(-a_2k + 2\alpha_kk + p)}{a_kk - 1} \right) \right) \right), \\
& u_{11}^\pm (x, t) = \pm \sqrt{2} \frac{-\rho_2}{\rho_3} \mu_1 e^{i\theta} \left( \tan h_{rs} \left( \sqrt{-\rho_2} \left( x - \frac{t(-a_2k + 2\alpha_kk + p)}{a_kk - 1} \right) \right) \right). 
\end{align*}
\]

where the existence condition for the solutions is:

\[
\rho_3 > 0, \quad \rho_3 > 0. 
\]
\[
\begin{align*}
\psi_{12}^{\pm}(x,t) &= \pm \frac{\sqrt{2}}{2} \sqrt{\frac{\rho_2}{\rho_3}} \mu_1 e^{i\theta} \left( \sqrt{\rho_2} \sec \rho_2 \left( \sqrt{2} \sqrt{\rho_2(x - \omega t)} \right) + \tan \rho_2 \left( \sqrt{2} \sqrt{\rho_2(x - \omega t)} \right) \right), \\
\psi_{13}^{\pm}(x,t) &= \pm \frac{\sqrt{2}}{2} \sqrt{\frac{\rho_2}{\rho_3}} \mu_1 e^{i\theta} \left( \sqrt{\rho_2} \csc \rho_2 \left( \sqrt{2} \sqrt{\rho_2(x - \omega t)} \right) + \cot \rho_2 \left( \sqrt{2} \sqrt{\rho_2(x - \omega t)} \right) \right), \\
\psi_{14}^{\pm}(x,t) &= \pm \frac{\sqrt{2}}{4} \sqrt{\frac{\rho_2}{\rho_3}} \mu_1 e^{i\theta} \left( \cot \rho_2 \left( \frac{\sqrt{\rho_2(x - \omega t)}}{2\sqrt{2}} \right) - \tan \rho_2 \left( \frac{\sqrt{\rho_2(x - \omega t)}}{2\sqrt{2}} \right) \right),
\end{align*}
\]

where

\[\rho_3 > 0.\]  

We confirm that all obtained solutions \(\psi_i(x,t)\) for \(i = 1, 2, \ldots, 14\) satisfy the pFL equation in eq. (1) via Mathematica. Now, we will illustrate various plots of the solutions to analyze how the solutions behave under the selection of different parameters.

### 4 Results and discussion

In this research, we successfully extracted plenty of soliton solutions for the pFL equation using Wolfram Mathematica 13 and also constructed constrain conditions for the existence of solutions. We plotted contour, 2D and 3D graphs of some solutions to show the physical behavior of the solitons selecting suitable parameters via Matlab. In addition, for various \(t\) values, we displayed 2D charts to indicate which direction the generated solutions travel. In Fig. 1, we demonstrate some plots of the solution \(u_5(x,t)\) in eq. (33) for \(b = -1, r = -1, s = -2, k = 1, p = -1, q = -1, \beta = -1, \lambda = -1, \rho_3 = 1, \alpha_1 = -1, \alpha_2 = -1\) and \(\phi_0 = -1\). The parameters is chosen in accordance with the constrain conditions for the existence of solutions. 3D and contour plots of \(|u_5(x,t)|^2\) are presented in Fig. 1a, b, respectively. Figure 1c, d illustrate 3D graphs of real and imaginary parts of \(u_5(x,t)\), respectively. While Fig. 1a, b represent a bright soliton solution, Fig. 1c, d illustrate combined bright-dark with different amplitude. In Fig. 1c, blue, red and yellows lines denotes the modulus square, imaginary and real parts of \(u_5(x,t)\) for \(t = 1\), respectively. In Fig. 1d, we present the profile of \(|u_5(x,t_j)|^2\) at \(t_j = 0\) (blue line), \(t_j = 1\) (red line) and \(t_j = 2\) (yellow line), respectively. It can be seen that the soliton moves to the right along the x-axis. In Fig. 1g, h and i, we investigate how the solution behaves when the parameters \(\alpha_1, \beta, \lambda\) change. The Fig. 1g includes the profile of \(|u_5(x,1)|^2\) at \(\alpha_1 = -1\) (blue line), \(\alpha_1 = -1.5\) (red line) and \(\alpha_1 = -2\) (yellow line), respectively. When the parameter \(\alpha_1\) decreases, both the soliton profile of \(|u_5(x,1)|^2\) travels to the right along the x-axis and amplitude of the soliton increases. In Fig. 1h, we present the profile of \(|u_5(x,1)|^2\) at \(\beta = -1\) (blue line), \(\beta = -1.5\) (red line) and \(\beta = -2\) (yellow line), respectively. When the parameter \(\beta\) decreases, no displacement along the x-axis is observed in the wave but the amplitude of \(|u_5(x,1)|^2\) decreases, which means that \(\beta\) has a positive correlation with the amplitude. In Fig. 1i, the profile of \(|u_5(x,1)|^2\) is given at \(\lambda = -1\) (blue line), \(\lambda = -1.5\) (red line) and \(\lambda = -2\) (yellow line), respectively. When the parameter \(\lambda\) decreases, both the soliton profile of \(|u_5(x,1)|^2\) moves to the right along the x-axis and amplitude of the soliton increases.
In Fig. 2, we give the plots of the solution $u_{10}(x, t)$ in eq. (39) by assigning the parameters $b = -1, r = -1, s = -2, k = 1, p = -1, q = -1, \beta = -1, \lambda = -1, \rho_3 = 1, \alpha_1 = -1, \alpha_2 = -2$ and $\phi_0 = -1$. Figure 2a, b show the 3D and contour plots of $|u_{10}(x, t)|^2$, respectively. We supply the 3D plots of real and imaginary parts of $u_{10}(x, t)$ in Fig. 2c, d, respectively. While Fig. 2a, b represent a periodic singular soliton solution, Figs. 2c, d illustrate periodic singular soliton with different amplitude. In Fig. 2d, the colors blue, red and yellow depict the behaviour of the solutions $|u_{10}(x, t_f)|^2$ at $t_f = 0, t_f = 1$ and $t_f = 2$, respectively. The Fig. 2g includes the profile of $|u_{10}(x, 1)|^2$ at $\alpha_1 = -1$ (blue line), $\alpha_1 = -1.1$ (red line) and $\alpha_1 = -1.2$ (yellow line), respectively. When the parameter $\alpha_1$ decreases, the wave has different positions along the x-axis. In Fig. 2h, we present the profile of $|u_{10}(x, 1)|^2$ at $\beta = -1$ (blue line), $\beta = -2$ (red line) and $\beta = -3$ (yellow line), respectively. When the parameter $\beta$ decreases, there is a very small decrease in the horizontal amplitude from this peak without the vertical displacement of the peak point on the x-axis. In Fig. 2i, the profile of $|u_{10}(x, 1)|^2$ is given at $\lambda = -2$ (blue line), $\lambda = -2.25$
When the parameter $\alpha_1$ decreases, the wave has different positions along the x-axis. The Fig. 3 portrays the various plots of $u_{14}(x, t)$ in eq. (43). The parameters are selected as $b = -1, r = -1, s = -1, k = -1, p = 2, q = -1, \beta = -1, \lambda = 2, \alpha_1 = 3, \alpha_2 = -2$ and $\phi_0 = -1$. We illustrate the contour and 3D plots of $|u_{14}(x, t)|^2$ in Fig. 3a, b, respectively. The Fig. 3a, b represent a periodic soliton solution. The 3D plots of real and imaginary parts of $u_{14}(x, t)$ are sketched in Fig. 3c, d, respectively. We depict the modulus square, imaginary and real parts of $u_{14}(x, t)$ using blue, red and yellow line in Fig. 3c. 2D graph of the solution $|u_{14}(x, t_f)|^2$ is drawn at $t_f = 0, t_f = 5$ and $t_f = 10$ in Fig. 3d. The Fig. 3g includes the profile of $|u_{14}(x, 1)|^2$ at $\alpha_1 = 3$ (blue line), $\alpha_1 = 3.5$ (red line) and $\alpha_1 = 4$ (yellow line), respectively. When the parameter $\alpha_1$ increases, the wave amplitude increases very little and the soliton profile of $|u_{14}(x, 1)|^2$ travels to the left along the x-axis. In Fig. 3h, we present the profile of $|u_{14}(x, 1)|^2$ at $\beta = -1$ (blue line), $\beta = -1.1$ (red line) and $\beta = -1.2$ (yellow line), respectively. When the parameter $\beta$ decreases, the wave moves up vertically. In Fig. 3i, the profile of $|u_{14}(x, 1)|^2$ is given (red line) and $\lambda = -2.5$ (yellow line), respectively. When the parameter $\lambda$ decreases, the wave has different positions along the x-axis.
Derivation of optical solitons of dimensionless Fokas-Lenells…

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\(\alpha\) at \(\alpha = 1\) (blue line), \(\alpha = 1.1\) (red line) and \(\alpha = 1.2\) (yellow line), respectively. When the parameter \(\alpha\) increases, the wave moves up vertically.

5 Conclusion

In this article, optical solitons of perturbed Fokas-Lenells equation which characterizes the propagation of ultrashort nonlinear light pulses in optical fibers have been obtained via Sardar sub-equation method. The considered method produces abundant solitons and it can be successfully applied for nonlinear PDEs modeling various phenomena in nature. We have constructed the constrain conditions for the existence of the solutions and have given the various 3D, 2D and contour plots of the derived solutions. Besides, we have presented a detailed graphical analysis such as how the wave moves at different times or how the wave behaves when the parameters of the right hand side of the main equation change. The obtained solutions and the comments may aid in a better understanding of the nature of the optical wave propagation in optical fibers, and so, they may contribute to future studies in optics.

Fig. 3 The graphs of \(u_{14}(x, t)\) in eq. (43) where \(\rho_3 = 2, b = -1, r = -1, s = \frac{1}{10}, k = -\frac{1}{10}, p = -1, q = -1, \beta = -1, \lambda = 2, \alpha_1 = 3, \alpha_2 = \frac{4}{10}\) and \(\phi_0 = \frac{1}{5}\).
Acknowledgements Melih Cinar thanks the Scientific and Technological Research Council of Turkey (TUBITAK) for the financial support named 2211-A Fellowship Program.

Funding The authors have not disclosed any funding.

Declarations

Conflict of interest The authors have not disclosed any conflict of interest.

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