Hard diffractive Photon-Proton Scattering at large $t$

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Abstract:

We propose to test perturbative QCD in the Regge limit by means of diffractive photon scattering $\gamma p \rightarrow \gamma X$ at large $t$ and very high energies $W^2 \gg |t| \gg \Lambda^2_{QCD}$. The helicity amplitudes of this process were calculated using the Lipatov solution of the BFKL equation for $t \neq 0$. We found that the perturbatively calculated cross section for this process exceeds the cross section for $J/\Psi$ photoproduction assuming similar kinematics.
1 Introduction

In the quest for more and better tests of pQCD in the Regge limit, better known as BFKL [1],
we suggest hard diffractive photon-proton scattering at large momentum transfer $t$. This process
is closely related to the diffractive production of vector mesons at large $t$ [2, 3, 4] where the
vector meson in the final state takes the place of the photon. Although being suppressed by an
extra $\alpha_{em}$ diffractive $\gamma p$-scattering has the great advantage of being completely calculable. No
phenomenological input in terms of a vector meson wave function is needed. The signature of this
process is also very clear. The photoproduced photon is scattered into the backward region of the
detector at a very low angle. The transverse momentum transferred from the photon is balanced
by a jet in the forward region which does not need to be resolved. Important is the very large
rapidity gap between the photon in the backward and the hadronic system in the forward region.
The idea to use this process as test for pQCD was already discussed in [5].

A very precise definition of the cross section is not necessary, since we are interested in a rough
estimate. Indeed, since BFKL enters on the level of the amplitude the resulting enhancement
is very large and therefore the theoretical uncertainty as well. We work with the leading order
BFKL-solution which is by now known to receive strong NLO-corrections [3]. Already a reduction
by factor of 1/2 in the LO-BFKL-kernel leads to an order of magnitude reduction in the elastic
cross section. Another source of uncertainty is the influence of nonperturbative effects despite the
fact that we require a large momentum transfer $t$. On the proton side it has been proven [4, 5]
that the large $t$-Pomeron-exchange factorizes in the sense that it directly couples to partons. The
difference in the coupling to quarks and gluons is only a trivial colour factor and the corresponding
parton distribution can be taken from any conventional LO-pdf. As we already pointed out we are
at the present only interested in a first rough estimate of the pQCD predictions and thus focus
on the elastic photon-quark scattering. For comparison we include in our numerical analysis the
diffractive production of $J/\Psi$ and find that both cross sections are close in magnitude. The elastic
$\gamma q$-cross section, surprisingly, exceeds the cross section for diffractive $J/\Psi$-production. We also
perform a Vector Dominance Model (VDM) inspired estimate for the same cross sections which
shows that with BFKL the $\gamma q$-cross section for large $t$ can hardly be matched with the VDM-result
at low $t$. At this point, of course, we are most curious to see what the corresponding measurement
will give. If the cross section is as high as predicted by pQCD, enough events will be recorded (or
have been recorded) to work out an experimental cross section. We will already learn a lot from
the mere presence or absence of events.

In the technical part of the paper we extend the successful concept of the photon wave function
[12, 13] to include nonzero momentum transfer. We then convolute the corresponding expression
for the photon wave function directly in impact parameter space with the conformal eigenfunction
of the nonforward BFKL-solution [10]. This part is presented in a rather detailed way to illustrate
some of the techniques which might be useful in other related cases, for it seems most appropriate
in dealing with integrals of two dimensional conformal field theories. The impact factor describing
the coupling of two gluons via a quark-box to the photons has effectively been already calculated in
QED in the context of photon-photon transition [9, 11], only colour factors needed to be added in
the case of QCD. Since for our calculation we need a somewhat different form of the impact factor
than found in the literature, we have reconsidered this calculation using slightly different methods.

The paper contains three technical section which deal with the derivation of the $\gamma q$-cross section.
The following section is devoted to the generalization of the photon wave function to include
momentum transfer, in section 3 we explain the convolution of the wave function with the conformal
BFKL-eigenfunctions and in section 4 we collect all pieces to derive the final expression for the cross section. In section 5 we present the numerical results and conclude with section 6.

2 Photon Wave Function

In the standard case of deep inelastic scattering the photon appears only in the initial state and its wave function \[12, 13\] can be formulated in the simplifying photon-proton CMS. In this frame the momentum of the photon has only longitudinal components and no transverse. When we consider, however, photon elastic scattering we have two photons, one in the initial state and the second in the final state of the process. We can still choose a frame in which one of the photons has only longitudinal components, but the second photon receives transverse components due to the momentum transfer \(q\). We therefore have to generalize the photon wave function description to also include transverse components. We use the standard notation for deep inelastic scattering such as \(Q'\) (final state photon) and \(p\) (proton). The light cone vectors \(Q'\) and \(p\) define the CMS we work in. For our discussion it is easier to assume that the outgoing photon which includes the momentum transfer \(q\) lies along the z-axis of our system and the incoming photon with the momentum \(Q\) has components in the transverse plane, i.e. \(Q = Q' + q\). The corresponding polarization vector for the incoming photon reads

\[
\epsilon(\pm) = \epsilon(\pm)_{\perp\mu} - \frac{q_{\perp} \cdot \epsilon(\pm)_{\perp}}{p \cdot Q} p_{\mu}
\]  

\[ (2.1) \]

with two helicity states \((\pm)\) in the transverse plane of our frame:

\[
\epsilon_{\perp}(\pm) = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)
\]  

\[ (2.2) \]

The photon couples to a quark-antiquark pair with the momenta \(k_1\) and \(k_2\), respectively. Using Sudakov decomposition we write the momenta as \((s = 2Q' \cdot p = 4Q_0p_0)\)

\[
q = \beta_q p + q_{\perp}
\]

\[
k_1 = \alpha Q' + \left(\beta_q + \frac{k_2^2}{(1 - \alpha)s}\right) p + k_{1\perp}
\]  

\[ (2.3) \]

\[
k_2 = (1 - \alpha) Q' - \frac{k_2^2}{(1 - \alpha)s} p + k_{2\perp}.
\]

One of these quark momenta is offshell depending on where the t-channel gluon couples. In the case discussed here it is \(k_1\) as indicated in Fig.1. \(k_1\) can be made onshell by adding a component with respect to \(p\). We make use of this trick in order to break up the trace of the quark loop by inserting quark spinors with onshell momenta. This can be done without effecting the final result because the adjacent gluon coupling has as leading component a /\(p\) which cancels our proposed modification in the trace. In the trace we therefore substitute \(k_1\) by \(\tilde{k}_1\):

\[
\tilde{k}_1 = \alpha Q' - \frac{k_1^2}{\alpha s} p + k_{1\perp}
\]  

\[ (2.4) \]

The denominator of our wave function contains of course the original virtuality \(k_1^2\). In the following we will change the transverse components for the momenta from Minkowski space to Euclidean, i.e
$k_\perp \to k$ with $k_\perp^2 = -k^2$, and despite the fact that the photon virtuality is negative we will use the convention that $Q^2 = |Q^2| = \beta_q s + q^2$.

After these preliminaries it is rather straightforward to introduce the photon wave function ($h = \pm$ for the quark helicity, $h_\gamma = \pm$ for the photon helicity):

$$
\Psi_{(h_\gamma, h)}(k_1, k_2, \alpha) = e^{\bar{u}(k_2, h) \gamma \gamma \gamma (h_\gamma)} \frac{u(k_1, h) \sqrt{\alpha(1-\alpha)}}{|k_1 - \alpha q|^2 + \alpha(1-\alpha)Q^2} \frac{1}{(2\pi)^2} \delta^2(q - k_1 - k_2) \quad (2.5)
$$

In the expression above we have made use of the relation $v(k_1, -h) = u(k_1, h)$ and we have already incorporated a factor $\alpha$ from the gluon vertex and a phase space contribution $1/\sqrt{\alpha(1-\alpha)}$. The propagator itself has the form $|k_1^2| = (|k_1 - \alpha q|^2 + \alpha(1-\alpha)Q^2)/(1-\alpha)$. In the chiral representation the spinors are of the following form (in complex notation for $k_1$):

$$
u(k_1, +) = \left( \frac{k_1^\gamma}{\sqrt{2\alpha Q_0}}, \sqrt{2\alpha Q_0'}, 0, 0 \right) \quad (2.6)
$$

$$
u(k_1, -) = \left( 0, 0, \sqrt{2\alpha Q_0'}, \frac{-k_1}{\sqrt{2\alpha Q_0'}} \right)
$$

and similar expressions for $u(k_2, h)$ by substituting $k_1$ by $k_2$ and $\alpha$ by $1-\alpha$. 

Figure 1: Diagrammatical representation of elastic $\gamma q$-scattering with BFKL-ladder exchange.
Eq. (2.5) together with (2.1) and (2.6) yields the following expressions for the wave function (for better readability we present all helicity states explicitly):

\[
\Psi_{(+,+)}(k_1, k_2, \alpha) = -\frac{\sqrt{2} e \alpha [k_2 - (1 - \alpha)q]}{|k_2 - (1 - \alpha)q|^2 + \alpha(1 - \alpha)Q^2} (2\pi)^2 \delta^2(q - k_1 - k_2)
\]

\[
\Psi_{(+,-)}(k_1, k_2, \alpha) = -\frac{\sqrt{2} e (1 - \alpha) [k_1 - \alpha q]}{|k_1 - \alpha q|^2 + \alpha(1 - \alpha)Q^2} (2\pi)^2 \delta^2(q - k_1 - k_2)
\]

\[
\Psi_{(-,+)}(k_1, k_2, \alpha) = -\frac{\sqrt{2} e (1 - \alpha) [k_1 - \alpha q^*]}{|k_1 - \alpha q|^2 + \alpha(1 - \alpha)Q^2} (2\pi)^2 \delta^2(q - k_1 - k_2)
\]

\[
\Psi_{(-,-)}(k_1, k_2, \alpha) = -\frac{\sqrt{2} e \alpha [k_2^* - (1 - \alpha)q^*]}{|k_2^* - (1 - \alpha)q|^2 + \alpha(1 - \alpha)Q^2} (2\pi)^2 \delta^2(q - k_1 - k_2)
\]

One could of course apply momentum conservation \( k_2 = q - k_1 \) and rewrite all equations in terms of \( k_1 \) only. But it is the form presented in (2.7) which emerges first and still exhibits the fact that the same factor \( \alpha(1 - \alpha) \) appears in front of \( q \) in the numerator of all expressions. The latter is a result of the longitudinal contribution \( p_\mu \) in the polarization vector (2.1) which is the same for all quark helicities.

We transform into impact parameter space by taking the Fourier transform with respect to \( k_1 \) and \( k_2 \)

\[
\int \frac{d^2k_1d^2k_2}{(2\pi)^4} \Psi(k_1, k_2, \alpha) e^{ik_1 \cdot r_1} e^{ik_2 \cdot r_2} = \Psi(r_1, r_2, \alpha)
\]

and arrive at

\[
\Psi_{(+,+)}(r_1, r_2, \alpha) = \frac{\sqrt{2} i e}{2\pi} \alpha \sqrt{\alpha(1 - \alpha)Q^2} K_1(\sqrt{\alpha(1 - \alpha)Q^2} |r|) \frac{r}{|r|} e^{i\alpha q \cdot r_1} e^{i(1 - \alpha)q \cdot r_2}
\]

\[
\Psi_{(+,-)}(r_1, r_2, \alpha) = \frac{\sqrt{2} i e}{2\pi} (1 - \alpha) \sqrt{\alpha(1 - \alpha)Q^2} K_1(\sqrt{\alpha(1 - \alpha)Q^2} |r|) \frac{r}{|r|} e^{i\alpha q \cdot r_1} e^{i(1 - \alpha)q \cdot r_2}
\]

\[
\Psi_{(-,+)}(r_1, r_2, \alpha) = \frac{\sqrt{2} i e}{2\pi} (1 - \alpha) \sqrt{\alpha(1 - \alpha)Q^2} K_1(\sqrt{\alpha(1 - \alpha)Q^2} |r|) \frac{r}{|r|} e^{i\alpha q \cdot r_1} e^{i(1 - \alpha)q \cdot r_2}
\]

\[
\Psi_{(-,-)}(r_1, r_2, \alpha) = \frac{\sqrt{2} i e}{2\pi} \alpha \sqrt{\alpha(1 - \alpha)Q^2} K_1(\sqrt{\alpha(1 - \alpha)Q^2} |r|) \frac{r}{|r|} e^{i\alpha q \cdot r_1} e^{i(1 - \alpha)q \cdot r_2}
\]

with \( r = r_1 - r_2 \) and \( K_1 \) being the MacDonald (Bessel) function. The whole \( q \)-dependence is shifted into phase factors, and the rest is the same as for the 'forward' case, i.e. \( q = 0 \).

One could now go ahead and work with the above wave function for virtual photons, i.e study the scattering of virtual photons into virtual photons. At this point we would like to keep the analysis a bit simpler and consider only real photons. To this end we take \( Q^2 \to 0 \) and eq. (2.9) reduces to

\[
\Psi_{(+,+)}(r_1, r_2, \alpha) = \frac{\sqrt{2} i e}{2\pi} \alpha \frac{r}{|r|^2} e^{i\alpha q \cdot r_1} e^{i(1 - \alpha)q \cdot r_2}
\]

\[
\Psi_{(+,-)}(r_1, r_2, \alpha) = \frac{\sqrt{2} i e}{2\pi} (1 - \alpha) \frac{r}{|r|^2} e^{i\alpha q \cdot r_1} e^{i(1 - \alpha)q \cdot r_2}
\]

\[
\Psi_{(-,+)}(r_1, r_2, \alpha) = \frac{\sqrt{2} i e}{2\pi} (1 - \alpha) \frac{r}{|r|^2} e^{i\alpha q \cdot r_1} e^{i(1 - \alpha)q \cdot r_2}
\]

\[
\Psi_{(-,-)}(r_1, r_2, \alpha) = \frac{\sqrt{2} i e}{2\pi} \alpha \frac{r}{|r|^2} e^{i\alpha q \cdot r_1} e^{i(1 - \alpha)q \cdot r_2}
\]
In the next step we have to consider the wave function for the outgoing photon $\Psi^*$. It is basically the complex conjugate of the previous expression except for the phase factors which are absent because the momentum for the outgoing photon is simply $Q$, i.e. $q = 0$:

$$
\Psi_{(+,+)}^*(r_1, r_2, \alpha) = -\frac{\sqrt{2}}{2\pi^2} i e^\alpha \frac{r^*}{|r|^2}
$$

$$
\Psi_{(+,-)}^*(r_1, r_2, \alpha) = -\frac{\sqrt{2}}{2\pi^2} i e^\alpha (1-\alpha) \frac{r^*}{|r|^2}
$$

$$
\Psi_{(-,+)}^*(r_1, r_2, \alpha) = -\frac{\sqrt{2}}{2\pi^2} i e^\alpha (1-\alpha) \frac{r}{|r|^2}
$$

$$
\Psi_{(-,-)}^*(r_1, r_2, \alpha) = -\frac{\sqrt{2}}{2\pi^2} i e^\alpha \frac{r}{|r|^2}.
$$

The impact factor is essentially the product of the two sets of wave functions, since the imaginary part of our amplitude dominates and all intermediate quarks are onshell. With regard to the two t-channel gluons located at $\rho_1$ and $\rho_2$ we have to make sure that each of the gluons couples to each of the quarks (g is the strong coupling constant):

$$
\Phi(h_\gamma, h_\gamma^*)(\rho_1, \rho_2) = g^2 \sum_{h=\pm} \int_0^1 d\alpha \int d^2 r_1 d^2 r_2 \Psi_{(h_\gamma, h_\gamma)}(r_1, r_2, \alpha) \Psi_{(h_\gamma^*, h_\gamma^*)}^*(r_1, r_2, \alpha)
$$

$$
\left[\delta^2(r_1 - \rho_1) - \delta^2(r_2 - \rho_1)\right] \left[\delta^2(r_1 - \rho_2) - \delta^2(r_2 - \rho_2)\right] .
$$

The normalization will be adjusted later in the full expression for the amplitude. Spelling out the previous expression we find:

$$
\Phi_{(+,+)}(\rho_1, \rho_2) =
$$

$$
\Phi_{(+,-)}(\rho_1, \rho_2) = \frac{e^2 g^2}{2\pi^2} \int_0^1 d\alpha \int d^2 r_1 d^2 r_2 \left[\alpha^2 + (1-\alpha)^2\right] \frac{1}{|r|^2} e^{i\alpha q \cdot r_1} e^{i(1-\alpha)q \cdot r_2}
$$

$$
\left[\delta^2(r_1 - \rho_1) - \delta^2(r_2 - \rho_1)\right] \left[\delta^2(r_1 - \rho_2) - \delta^2(r_2 - \rho_2)\right].
$$

$$
\Phi_{(-,+)}(\rho_1, \rho_2) = \frac{e^2 g^2}{\pi^2} \int_0^1 d\alpha \int d^2 r_1 d^2 r_2 (1-\alpha) \frac{r^2}{|r|^4} e^{i\alpha q \cdot r_1} e^{i(1-\alpha)q \cdot r_2}
$$

$$
\left[\delta^2(r_1 - \rho_1) - \delta^2(r_2 - \rho_1)\right] \left[\delta^2(r_1 - \rho_2) - \delta^2(r_2 - \rho_2)\right].
$$

$$
\Phi_{(+,+)}(\rho_1, \rho_2)\Phi_{(+,-)}(\rho_1, \rho_2)\Phi_{(-,+)}(\rho_1, \rho_2)\Phi_{(-,-)}(\rho_1, \rho_2)
$$

are contributions without helicity flip and $\Phi_{(+,-)}$ and $\Phi_{(-,+)}$ are those with flip. It is not apparent from eq.(2.13) that the two helicity flip contributions are the same, but the following calculation will show that they coincide as one may expect on general grounds. In the end it is enough to deal with a single helicity flip and a single helicity non-flip amplitude.

### 3 Projection on Conformal Eigenstates

In this section we exploit the properties of conformal invariance of the BFKL-equation \[4\] which can be solved in terms of the conformal covariant eigenfunctions:

$$
E^{\nu}(\rho_{10}, \rho_{20}) = \left|\frac{\rho_{12}}{\rho_{10}\rho_{20}}\right|^{1+2i\nu}
$$

(3.1)
where $\nu$ is the conformal weight. We have ignored the conformal spin $n$ which is required to form a complete set. In practice, though, any contribution with $n \neq 0$ gives a subleading contribution at high energies. If wished to do so, the following calculation can be generalized to include $n$.

In projecting the impact factor on the eigenfunction we have to perform the following integration

$$
\int d^2\rho_1 d^2\rho_2 \Phi_{(b_1,h_1)}(\rho_1,\rho_2) E^\nu(\rho_{10},\rho_{20}) .
$$

We realize that in two terms of eq.(2.13) the $\delta$-function forces $\rho_1$ and $\rho_2$ into one point which results into a vanishing contribution due to a zero in $E^\nu$ (Re$[1+2i\nu]$ has to be kept positive). The second observation is the symmetry in $\rho_1$ and $\rho_2$ which allows to write the whole amplitude in one term multiplied by 2 (non-flip here):

$$
\frac{e^2g^2}{\pi^2} \int_0^1 d\alpha \left[ \alpha^2 + (1 - \alpha)^2 \right] \int d^2r_1 d^2r_2 \frac{1}{|r|^2} e^{i\alpha q \cdot r_1 + i\bar{\alpha} q \cdot r_2} \frac{r}{|r_1 - \rho_0(r_2 - \rho_0)|^{1+2i\nu}} .
$$

We will shift $r_1$ and $r_2$ by $\rho_0$ generating an overall phase factor and then substitute $r_1$ by $r = r_1 - r_2$:

$$
\frac{e^2g^2}{\pi^2} e^{iq \cdot \rho_0} \int_0^1 d\alpha \left[ \alpha^2 + (1 - \alpha)^2 \right] \int d^2r d^2r \ e^{i\alpha q \cdot r} e^{i\bar{\alpha} q \cdot r_2} |r|^{-1+2i\nu} |(r + r_2) r_2|^{-1-2i\nu} .
$$

The overall phase factor will be ignored for a while and reconsidered at the end of this section. We carry on in our calculation and shift $r_2$ by $-\alpha r$ eliminating the phase factor that depends on $r$

$$
\frac{e^2g^2}{\pi^2} \int_0^1 d\alpha \left[ \alpha^2 + (1 - \alpha)^2 \right] \int d^2r d^2r \ e^{i\alpha q \cdot r} |r|^{-1+2i\nu} |[r_2 + (1 - \alpha) r] [r_2 - \alpha r]|^{-1-2i\nu} .
$$

We then switch from the conventional representation of the transverse vectors to the corresponding complex notation, i.e from $r = (r_1,r_2)$ to $a = r_1 + i r_2$ and $b = r_1 - i r_2$, and make use of the freedom to rotate our system such that $q$ is real:

$$
- \frac{e^2g^2}{4\pi^2} \int_0^1 d\alpha \left[ \alpha^2 + (1 - \alpha)^2 \right] \int da db da_2 db_2 \ e^{i\alpha q / 2(a_2 + b_2)} (a b)^{-1/2+i\nu} (a_2 b_2)^{-1/2-i\nu} .
$$

In what follows we rescale $a$ and $b$ by $a_2$ and $b_2$, respectively, and thus factorize the integration in $a,b$ and $a_2,b_2$:

$$
- \frac{e^2g^2}{4\pi^2} \int_0^1 d\alpha \left[ \alpha^2 + (1 - \alpha)^2 \right] \int da db da_2 db_2 \ e^{i\alpha q / 2(a_2 + b_2)} (a_2 b_2)^{-1/2-i\nu} \int da db (a b)^{-1/2+i\nu} (1 + (1 - \alpha)a) [1 + (1 - \alpha)b] [1 - \alpha a][1 - \alpha b]^{-1/2-i\nu} .
$$

We now rescale $a_2$ and $b_2$ by $2i/q$

$$
\frac{e^2g^2}{4\pi^2} \left( \frac{2}{q} \right)^{1-2i\nu} \int_0^1 d\alpha \left[ \alpha^2 + (1 - \alpha)^2 \right] \int da db da_2 db_2 \ e^{-(a_2 + b_2)} (-a_2 b_2)^{-1/2-i\nu} \int da db (a b)^{-1/2+i\nu} (1 + (1 - \alpha)a) [1 + (1 - \alpha)b] [1 - \alpha a][1 - \alpha b]^{-1/2-i\nu}.
$$

and note the minus sign in the factor $(-a_2 b_2)^{-1/2-i\nu}$.
At this point we have to consider the analytic structure of our integrand and the proper path of integration. We focus on the $a_2$-integration by keeping $b_2$ fixed, quasi as a parameter. With the minus sign there is a cut in the complex $a_2$ plane to right when $b_2 > 0$ or to the left when $b_2 < 0$. Since the exponent also has a minus sign we would like to shift the integration contour to the right. A nonzero contribution only arises, if a singularity is encountered, i.e. if $b_2$ is positive. Closing the contour around the cut means we have to take the discontinuity along the cut and then integrate in $a_2$ over the positive axis:

$$
\int da_2 \,db_2 \, e^{-(a_2+b_2)}(-a_2 \,b_2)^{−1/2−iν} = \frac{−2i}{sin(−1/2−iν)p} \int_0^∞ da_2 \,a_2^{−1/2−iν} \,e^{−a_2} \int_0^∞ db_2 \,b_2^{−1/2−iν} \,e^{−b_2}
$$

(3.9)

The $−2i \,sin([−1/2−iν]p)$ is the result of taking the discontinuity.

In a similar way the integration over $a$ and $b$ can be performed. We again keep $b$ fixed and consider the integration over $a$. The integrand is convergent even without the presence of an exponent. There will be a nonzero contribution only if the integration contour runs between two cuts, those two cuts which emerge on both sides of the complex $a$-plane when $−1/(1−α) < b < 1/α$. We have to face, however, another cut due to the factor $(a \,b)^{−1/2+iν}$ in eq.(3.8). This lies to the right or to the left depending on the sign of $b$. Therefore we have to consider two cases $−1/(1−α) < b < 0$ and $0 < b < 1/α$ and close the contour to that side which has only one cut:

$$
\int da \,db \,(a \,b)^{−1/2+iν} \{(1 + (1−α)a) \,[1 + (1−α)b] \,[1 − α \,a][1 − α \,b]}^{−1/2−iν}
$$

(3.10)

$$
= \frac{2i}{sin(−1/2−iν)p} \int_−1/(1−α)∞ db \,b^{−1/(1−α)} \int_0^1 da \,a^{−1/2+iν} \{(1 − b)[1 + α \,b^{-1}][1 − α \,a][1 − α \,b]}^{−1/2−iν}
$$

$$
= \frac{4i}{sin(−1/2−iν)p} \,\frac{π^2}{sin^2([1/2−iν]p)} \,2F_1 \left(1/2 + iν,1/2 + iν;1; \frac{α}{α−1}\right) \,2F_1 \left(1/2 + iν,1/2 + iν;1; \frac{α−1}{α}\right)
$$

(3.11)

Putting the results in eq.(3.9) and (3.11) back into eq.(3.8) we get:

$$
2 \,e^{2\,g^2} \left(\frac{2}{q}\right)\,1−2iν \,Γ^2(1/2−iν) \int_0^1 da \,\{1 − 2(1−α) + 2(1−α)^2\}
$$

(3.12)

$$
\,2F_1 \left(1/2 − iν,1/2 + iν;1−α\right) \,2F_1 \left(1/2 − iν,1/2 + iν;1−α\right).
$$
For the flip amplitude we have to proceed in a very similar way and find:

\[- 2 e^2 g^2 \left( \frac{2}{q} \right)^{1-2i\nu} \Gamma^2(1/2 - i\nu) \left( 1/4 + \nu^2 \right) \int_0^1 d\alpha \, \alpha(1 - \alpha) \]

\[2F_1(1/2 - i\nu, 1/2 + i\nu; 2; 1 - \alpha) \, 2F_1(1/2 - i\nu, 1/2 + i\nu; 2; \alpha)\]

which is the same for the (+, −)- or the (−, +)-amplitude.

The further calculation is rather tedious but straightforward. The main point is to expand the first Hypergeometric function, perform the \( \alpha \)-integration and then reduce the remaining series. The final result takes on the following form:

\[e^2 g^2 \left( \frac{2}{q} \right)^{1-2i\nu} \frac{\Gamma(1/2 - i\nu)}{\Gamma(1/2 + i\nu)} \frac{\pi^2}{4} \frac{11/4 + 3\nu^2}{1 + \nu^2} \frac{\tanh(\pi\nu)}{\pi\nu}\]

for the non-flip amplitude and

\[e^2 g^2 \left( \frac{2}{q} \right)^{1-2i\nu} \frac{\Gamma(1/2 - i\nu)}{\Gamma(1/2 + i\nu)} \frac{\pi^2}{4} \frac{1/4 + 3\nu^2}{1 + \nu^2} \frac{\tanh(\pi\nu)}{\pi\nu}\]

for the flip amplitude.

We are left with the calculation of the other end of the BFKL-Pomeron, i.e. the coupling to a quark(gluon) in the proton. We follow the consideration in ref.[7] where it was noted that a simple projection of the eigenfunction (3.1) to a single quark line would give zero. In reality the quark is accompanied by a bunch of particles with opposite colour far away in impact parameter space. Let us place the quark in impact parameter space at \( r_1' \) and the opposite colour charge at \( r_2' \). In the limit \( |r_2'| \gg |r_1'| \) the eigenfunction (3.1) reduces to

\[E^\nu(\rho_{10}', \rho_2' \to \infty) \sim \left| \frac{1}{\rho_{10}'} \right|^{1-2i\nu}.\]

(3.16)

The momentum coming from the photon is transferred to the quark but momentum conservation is imposed later on by doing the final \( \rho_0 \)-integration. At the moment we give the quark a general momentum kick \( q' \) which leads to a phase factor \( e^{iq' \cdot r_1'} \). The integration over \( r_2' \) is factored off and only the \( r_1' \)-integration remains:

\[2g^2 \int \frac{d^2 r_1'}{(2\pi)^2} \left| \frac{1}{r_1' - \rho_0} \right|^{1-2i\nu} e^{-iq' \cdot r_1'}\]

\[= g^2 \left( \frac{2}{q} \right)^{1+2i\nu} \frac{\Gamma(1/2 + i\nu)}{\Gamma(1/2 - i\nu)} \frac{1}{2\pi} e^{-iq' \cdot \rho_0}\]

(3.17)

A factor 2 was added to account for the coupling of both gluons.

We have to recall that we dropped the phase factor \( e^{iq \cdot \rho_0} \) in eq.(3.4). In the full amplitude the two phase factors have to be drawn together and the integration over \( \rho_0 \) forces \( q \) and \( q' \) to be equal:

\[\int d^2 \rho_0 e^{i(q - q') \cdot \rho_0} = (2\pi)^2 \delta^2(q - q') .\]

(3.18)

For us important is the extra factor \( (2\pi)^2 \) which is generated in the above equation.
4 The complete Cross Section

In order to write down the cross section we have to find the complete expression for the amplitude. Cross section and amplitude are linked through the relation \( \frac{d\sigma}{dt}(\gamma q \rightarrow \gamma q) = \frac{1}{16\pi s^2} \sigma_{\gamma q \rightarrow \gamma q} \) (4.1)

All missing factors not yet being included are extracted from the 'Born'-diagram: \( 6/9 \) for the light flavour charges, \( 4/6 \) for the colour (in the case of \( \gamma q \)-scattering), \( 4 \) from the coupling to the lower line, \( s/4 \) from the Sudakov decomposition, \( 1/2(2\pi)^3 \) for the onshell quarks (1/2 because we need the imaginary part and not the discontinuity) and \( 1/(2\pi)^8 \) from the phase space integral. All factors related to Fourier transformations have been taken care of, only the factor \( (2\pi)^2 \) from (3.18) needs to be included. Compiling all these factors and add them together with expression (3.14), (3.15) and (3.17) we finally obtain for the amplitudes

\[
A_{(+,+)} = i \frac{6}{9} \alpha_{em} \alpha_s^2 \frac{4\pi}{3} \frac{s}{|t|} \int d\nu \frac{\nu^2}{(1/4 + \nu^2)^2} \frac{11/4 + 3 \nu^2}{1 + \nu^2} \frac{\tanh(\pi \nu)}{\pi \nu} \left( \frac{s}{|t|} \right)^{\omega(\nu)}
\]

and

\[
A_{(+,-)} = i \frac{6}{9} \alpha_{em} \alpha_s^2 \frac{4\pi}{3} \frac{s}{|t|} \int d\nu \frac{\nu^2}{(1/4 + \nu^2)^2} \frac{1/4 + \nu^2}{1 + \nu^2} \frac{\tanh(\pi \nu)}{\pi \nu} \left( \frac{s}{|t|} \right)^{\omega(\nu)}
\]

where

\[
\omega(\nu) = \frac{3\alpha_s}{\pi} \left[ 2 \Psi(1) - \Psi(1/2 + \text{i}\nu) - \Psi(1/2 - \text{i}\nu) \right].
\]

One can derive the contribution for \( \gamma g \)-scattering simply by multiplying the previous expressions by a factor \( 9/4 \) (due to colour).

There is a striking similarity between the results here and those found for the forward jet cross section with azimuthal dependence [14]. The factor \( \frac{11/4 + 3 \nu^2}{1 + \nu^2} \) for the non-flip contribution is identical to the corresponding integrated contribution to the forward jet cross section whereas \( \frac{1/4 + \nu^2}{1 + \nu^2} \), the flip result, is found for the azimuth dependent part.

The saddle point approximation for the amplitudes (4.2) and (4.3) yields

\[
A_{(+,+)} = 11 i \frac{6}{9} \alpha_{em} \alpha_s^2 \frac{s}{|t|} \frac{8}{3} \frac{\pi}{7\zeta(3)} \eta^{3/2} e^{\eta \ln(4)}
\]

and

\[
A_{(+,-)} = i \frac{6}{9} \alpha_{em} \alpha_s^2 \frac{s}{|t|} \frac{8}{3} \frac{\pi}{7\zeta(3)} \eta^{3/2} e^{\eta \ln(4)}
\]

where \( \eta \) is defined as

\[
\eta = \frac{6\alpha_s}{\pi} \ln \left( \frac{s}{|t|} \right)
\]

In this approximation one nicely sees the dominance of the non-flip versus the flip amplitude given by the factor of 11 in eq. (4.5). On the scale we perform our numerical analysis the difference between the exact result and the saddle point approximation is marginally, i.e. less than a factor of 2.
5 Numerical Results

Since we are mainly interested in a rough estimate for the cross section, we will concentrate on elastic photon-quark scattering. In order to get an impression for the size of the cross section we compare with a different process, namely diffractive \( J/\Psi \)-photoproduction. In addition we use the Vector Dominance Model (VDM) to gain an estimate of the quasi elastic \( \gamma q \) scattering at low \( t \), and we perform a comparison between the full BFKL solution and the leading order two-gluon exchange.

The formulae for the \( J/\Psi \)-cross section are taken from refs.\( [2] \) and \( [3] \). The leading order two gluon exchange can be more easily calculated from the momentum representation of the photon wave function (2.7) directly. The amplitudes in this case are given by

\[
A_{(+,+)} = i \frac{s}{|t|} \alpha_{em} \alpha_s^2 \frac{2^6}{3^2} \left( \frac{\pi^2}{3} + 1 \right)
\]

\[
A_{(+,-)} = i \frac{s}{|t|} \alpha_{em} \alpha_s^2 \frac{2^5}{3^2} .
\]

A VDM-estimate can be achieved by employing the optical theorem which relates the elastic with the total \( \gamma p \)-cross section. The total \( \gamma p \)-cross section at the energy of \( W = 200 \text{ GeV} \) is approximately 150 \( \mu b \). This leads together with the \( t \)-slope of 10 \( GeV^{-2} \) to an integrated elastic \( \gamma p \)-cross section of 115 \( nb \). From the diffractive production of \( \rho^0 \)'s one knows that for roughly 11\% of all events the proton dissociates. The resulting value for the cross section divided by 9 (additive quark model) gives an estimate for the \( \gamma q \)-scattering of \( 1.4 \text{ nb} \) at \( W = 200 \text{GeV} \). The diffractive slope in \( t \) is according to the \( \rho^0 \)-data 5.3 \( GeV^{-2} \) for events with proton dissociation. Altogether one finds:

\[
\frac{d\sigma}{dt}(\gamma) \approx 7.5 \text{ nb} \ exp\left(-5.3 \ |t| \text{ GeV}^{-2}\right)
\]

(5.2)

Taking the measured cross section for \( J/\Psi p \) at the same energy and divide it by 9 we find a value of similar size. The main difference is the much smaller slope of 1.6 \( GeV^{-2} \) which is due to the large mass:

\[
\frac{d\sigma}{dt}(J/\Psi) \approx 4.4 \text{ nb} \ exp\left(-1.6 \ |t| \text{ GeV}^{-2}\right)
\]

(5.3)

for \( W = 200 \text{GeV} \).

In Fig.2 we show all discussed options in one plot. For the perturbative result we have assumed a constant value for the strong coupling of \( \alpha_s = 0.2 \) and \( W = 200 \text{ GeV} \). The two solid lines are related to the full BFKL-solution, the upper line denotes the production of a photon and the lower line the production of \( J/\Psi \). In addition to the upper solid line we have plotted the saddle point solution (1.3) and (1.4) for \( \gamma q \)-scattering which lies almost on top of the solid line. The dashed lines show in a similar fashion the cross sections based on the leading order two-gluon exchange. Again the upper line denotes the production of a photon and the lower the production of \( J/\Psi \). The strongly curved, dotted lines represent the VDM-estimates. In this case the line related to the production of \( J/\Psi \) lies above the line for the production of a photon due to the smaller \( t \)-slope.

The VDM-result for \( \gamma q \)-scattering seems to contradict the perturbative result. Moreover, a matching between the low \( t \) nonperturbative and high \( t \) perturbative regime seems to be difficult. For \( J/\Psi \) on the other hand a matching seems feasible. The enhancement due to BFKL is extremely large, in both cases it is a factor of about 100 at \( |t| = 3 \text{ GeV}^2 \) and somewhat smaller (a factor 10) at \( |t| = 100 \text{ GeV}^2 \).
Figure 2: The cross section for diffractive production of photons and $J/\Psi$’s at $W = 200 \text{ GeV}$. The solid line denotes the BFKL-solution (upper line for photons), the dashed line the leading order two-gluon exchange (upper line again for photons) and the dotted line shows the VDM-estimate (in this case the upper line is for $J/\Psi$). In addition the saddle point approximation is given as dash-dotted line.

The fact that the perturbative BFKL-result for $\gamma q$-scattering overshoots the VDM-result so massively makes it hard to believe that the perturbative prediction is close to the true value. There are two main reasons to mention. First, the BFKL-solution is implemented at Leading Order. NLO-corrections are known to reduce the cross section substantially. Second, although we consider large-$t$ the internal integration over the transverse momenta in the virtual loops is performed without any infrared cutoff. The result is still finite, but dominant contributions might come from the infrared region and thus is influenced by confinement. This point needs further investigation by imposing an infrared cutoff on the separation of the quark-antiquark pair. The consequence of all conceivable corrections might be a reduction of the cross section by 1 or 2 orders of magnitude.
6 Conclusions

We have calculated the cross section for $\gamma q$-elastic scattering in the Regge limit ($W^2 \gg |t| \gg \Lambda_{QCD}^2$). The BFKL-solution for nonzero momentum transfer was used leading to a strong enhancement of the cross section. Believing in this prediction a measurement of the process should be feasible. The comparison with a VDM-calculation indicates, however, that the perturbative prediction might be much too high. The simple fact of observing events at HERA or not will already give a hint with regard to the validity of the present pQCD result. We also found that the cross section for elastic $\gamma q$-scattering exceeds the cross section for $\gamma q \rightarrow J/\Psi q$.

Some effort was put in the detailed presentation of the method employed for performing the convolution of the photon wave function with the conformal eigenfunction. The integrals we were faced with are typical for two dimensional conformal field theories. They seem to be solved most efficiently when complex variables are introduced and the integration is factorized in complex times complex-conjugate contributions. A similar technique was used in ref. [1]. We have pointed out that special attention has to be given to the problem of large infrared contributions. One way of tackling this problem is to keep the virtuality $Q^2$ of the initial photon high enough [11]. It will be interesting to see how much the $\gamma^* q$-cross section will decrease when the photon virtuality is increased from 0 to 1 GeV$^2$.

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