A Quantum Damper

Fumihiro Matsui
Department of Physics, College of Science and Engineering, Ritsumeikan University Noji-higashi 1-1-1, Kusatsu 525-8577, Japan

Hiroaki S. Yamada
Yamada Physics Research Laboratory, Aoyama 5-7-14-205, Niigata 950-2002, Japan

Kensuke S. Ikeda
College of Science and Engineering, Ritsumeikan University Noji-higashi 1-1-1, Kusatsu 525-8577, Japan
(Dated: July 20, 2018)

As an application of the classically decay-able correlation in a quantum chaos system maintained over an extremely long time-scale (Matsui et al.; Europhys.Lett. 113 (2016) 40008), we propose a minimal model of quantum damper composed of a quantum harmonic oscillator (HO) weakly interacting with a bounded quantum chaos system. Although the whole system obeys unitary evolution dynamics of only three quantum degrees of freedom, the mechanical work applied to the HO is stationary converted linearly into the internal energy in time, characterized by an effective temperature in an irreversible way, if the components of the quantum chaos system are mutually entangled enough. A paradoxical dependence of the duration time of the stationary energy conversion on the driving strength is also discussed.

PACS numbers: 05.45.Mt, 05.45.-a, 03.65.-w

I. INTRODUCTION

How irreversible dynamics is self-organized inside of a small closed quantum system? What does the term “thermalize” mean in small closed quantum system? How small the closed thermalizable system can be made? Classical chaos can be the minimal origin of irreversibility [1], because of the presence of mixing, which mean the decay of correlation, in a fully chaotic state. However, its quantum counterpart does not always play the same role.

Consider an assembly of identical quantum chaos systems, which is interacting weakly with each other. Even if the system is unbounded and the associated Hilbert-space dimension $N_{\text{dim}}$ is infinite, persistent quantum interference prevents time correlation from decaying completely, which means microcanonical wandering over the phase space is suppressed, if the number of units (or the dimension of the system) is less than a critical number [2–6]. On the other hand, in the case that $N_{\text{dim}}$ is small, then even the number of units is large enough, the thermalization to equilibrium is often interrupted even though the system is non-integrable, as has been reported for quantum many-body systems on the optical lattices [7, 8]. The decay of correlation is quite delicate problem in the case of quantum system.

Many papers have been published for thermalization of isolated quantum systems composed of many small quantum units (qubits) with a few quantum levels [6, 9]. We, however, would like to explore in the opposite direction, namely, the irreversible relaxation in isolated quantum systems composed of a few large quantum units each of which has many quantum levels and further exhibits chaos in the classical limit [10, 11]. Irreversibility in small classically chaotic quantum system has been examined directly by numerical time-reversal experiments [12–14]. In particular, it has been extensively investigated by many investigators in the context of fidelity [15, 16].

In unbounded quantum chaos systems typically exemplified by the standard map, the absence and presence of diffusion has been of main interest in connection with the Anderson transition problem [3, 5]. The characteristics such as features of eigenspectrum [17], fidelity [13, 16] and so on has been investigated in the context of the Anderson transition. On the other hand, we have been interested in the onset of time-irreversible behavior in
finite bounded chaotic quantum systems, which is not realized as diffusion phenomena and is maintained only a limited time scale because of the finite Hilbert-dimension $10$.

In the previous paper we proposed a simple method which enables to investigate the time-irreversible characteristics of the finite bounded systems. With the method we can map the Fourier spectral features of correlation function to the diffusion characteristics of a fictitious measurement system. Applying the method to coupled quantum chaotic rotors, we found that the time scale on which the decay-able quantum correlation is maintained is proportional to $N_{\text{dim}}^2$, not $N_{\text{dim}}$ if a full entanglement is achieved among the quantum chaos units $18$ $19$. We called this “lifetime” of the decayable correlation. The aim of the present paper is to demonstrate that by applying this extremely long lifetime of decay-able correlation in the quantum kicked rotors in a full entanglement case, the mechanical work into internal energy stored in the HO is parametrically modulated by the chaotic KR perturbs the coherent driving force of mechanical work into internal energy stored in the HO in the classical limit. In Sects. III and IV, we show that the development of entanglement in the coupled KR makes the time scale on which the stationary irreversible energy transport is sustained very long time. However, the actually observed time scale is much shorter than the theoretically predicted one. The origin of such an unexpected result is considered in Sect.V. We give the conclusion in Sect. VI. Appendix A is devoted to exhibiting basic manipulations for exploring energy transfer process in our system.

II. THE MODEL

The main part of our system playing the role of the heat reservoir is a quantum HO driven by externally applied classical periodic force. The external force excites the HO in a usual way, but the coupling of the external force with the HO is parametrically modulated by a chaotic system, which makes the motion of HO completely diffusive, thereby introducing an irreversible nature. Figure 1 illustrates the proposed system in comparison with the classical damper. The total Hamiltonian reads as

$$H_{\text{tot}} = \hat{H}_{\text{HO}}(\hat{q}, \hat{p}) + \hat{H}_{\text{KR}}(\hat{\theta}, \hat{I}, t) + \eta \hat{q} f(\hat{I}) \cos \omega t,$$  \hspace{1cm} (1)

where $\hat{H}_{\text{HO}}(\hat{q}, \hat{p}) = \frac{\hat{p}^2}{2m} + \Omega^2 \hat{q}^2$ is the Hamiltonian of HO with the frequency $\Omega$. The HO plays the role of the energy reservoir, which stores the energy transferred from the mechanical work done by the periodic force $\eta \cos \omega t$ in an “irreversible” manner. $\hat{H}_{\text{KR}}(\hat{\theta}, \hat{I}, t)$ indicate the Hamiltonian of classically chaotic quantum KR composed of two identical units interacting with weak coupling strength $\varepsilon$ as

$$\hat{H}_{\text{KR}}(\hat{\theta}, \hat{I}, t) = \frac{\hat{I}^2}{2T} + \mathcal{V}(\hat{\theta}) \sum_{\ell=-\infty}^{\infty} \delta(t - \ell T)$$

with $$\mathcal{V}(\hat{\theta}) = \sum_{i=1}^{2} KV(\hat{\theta}_i) + \varepsilon \text{V}_{\text{int}}(\hat{\theta}_1, \hat{\theta}_2).$$  \hspace{1cm} (2)

The angle operators $\hat{\theta}_i$ act in the bounded space $[0, 2\pi]$, but the action operators $\hat{I}_i$’s space may be unbounded as is typically exemplified by the standard map. However, the chaotic region of normal dynamical systems is bounded and we are particularly interested in the effect of boundedness of the phase space on the irreversible nature, and we make the system bounded by imposing the periodic boundary conditions with the period $2\pi$ on both the action and the angle representation of wavefunction, respectively. Hence the number of the quantum states is limited because of the finite Hilbert-dimension $\text{dim}$.

In executing the wavepacket propagation of the QD it is convenient to divide the time into the intervals of the kick period $T$ and to construct the time-evolution unitary operator $\hat{U}_\tau$ from just after the $\tau$-th kick ($\tau \in \mathbb{Z}$) to the $(\tau + 1)$-th kick by the product of two unitary evolution operators:

$$\hat{U}_\tau = \hat{U}_{\text{KR}} \hat{U}_{\text{HO}, \tau},$$

with

$$\hat{U}_{\text{HO}, \tau} =$$

$$\mathcal{T} \exp \left\{ -i \frac{\hat{I}}{\hbar} \int_{\tau T}^{(\tau+1)T} dt' \hat{H}_{\text{HO}}(\hat{q}, \hat{p}) + \eta \hat{q} f(\hat{I}) \cos \omega t' \right\},$$

$$\hat{U}_{\text{KR}} = \exp \left\{ -i \frac{\hat{I}^2}{\hbar} \right\} \exp \left\{ -i \frac{\hat{I}}{\hbar} \mathcal{V}(\hat{\theta})\right\} \exp \left\{ -i \frac{\hat{I}^2}{\hbar} \right\}.$$  \hspace{1cm} (3)

The time evolution starts just after the kick $t = \tau T + 0$: first the evolution by the driven HO and KR occurs for the period $T$, i.e., applying $e^{-\frac{i}{\hbar} \hat{H}_{\text{HO}, \tau}} U_{\text{HO}, \tau} = U_{\text{HO}, \tau} e^{-\frac{i}{\hbar} \hat{H}_{\text{HO}, \tau}}$, and next the KRs are kicked at $t = (\tau + 1)T$, i.e., applying $e^{-\frac{i}{\hbar} \mathcal{V}(\hat{\theta})}$. Technically a big merit to use HO is that the evolution by $U_{\text{HO}, \tau}$ can be transformed into a product of three unitary operators each of which contains either $\hat{p}$ or $\hat{q}$ by the Baker-Campbell-Hausdorff expansion, which drastically reduces the computation time.

In the following Sects. III and IV, the coupling strength $\eta$ is taken such a small value that in the classical limit the back-action of the HO to the KRs is negligibly
small ($\eta \simeq O(10^{-4})$) and the chaotic dynamics of the isolated KR is not disturbed by the coupling. In Sect IV the effect of the coupling strength on the quantum dynamics is discussed.

To be concrete, in what follows we take the coupled Arnold cat maps (ACM) as the KR, namely, we choose $V(\hat{\theta}) = -\hat{\theta}^2/2$ and $V_{int}(\hat{\theta}_1, \hat{\theta}_2) = \cos(\hat{\theta}_1 - \hat{\theta}_2)$ in Eq. [2]. Further we couple the HO with only one of the two cats, i.e. $f(\hat{\mathbf{I}}) = \cos \hat{I}_1$, since we would like to restrict the path of influence by chaos. We remark that the results shown in what follows are essentially the same if we choose another type of KR such as the (unbounded) standard map $V(\hat{\theta}) = \cos \hat{\theta}$ when its classical version exhibits strongly chaotic motion.

III. OPERATION OF THE DAMPER

A. Energy transfer process

We consider the time evolution of the absorbed energy by the HO. With using the Heisenberg picture of the dynamics for annihilation and creation operators $\hat{a} \equiv \sqrt{\Omega/2\hbar}(\hat{q} + i\hat{p}/\Omega)$ and $\hat{a}^\dagger$, or their envelope operators $\hat{b} \equiv \hat{a} e^{i\hat{\mu} t}$ and $\hat{b}^\dagger$, one can obtain the following relation as shown in Appendix A

$$
\hat{b}_\tau - \hat{b} \simeq -\frac{\eta(e^{i\nu T} - 1)}{2\nu \sqrt{2\hbar \Omega}} \sum_{k=0}^{\tau-1} e^{ik\nu T} f_k, 
$$

where $\tau \in \mathbb{Z}$ and $\hat{X}_s \equiv \hat{U}_1 \hat{U}_2 \cdots \hat{U}_s \hat{\theta}_s \hat{U}_2 \hat{U}_1$ indicate the Heisenberg operator just after the $s$-th kick. $\nu = \Omega - \omega$ is the difference frequency, and we suppose the nearly resonant condition, namely,

$$
|\Omega - \omega| |\langle 1 \rangle \quad \text{and} \quad |\Omega - \omega| T \sim O(1).
$$

In our model there are two paths by which energy is exchanged with external world. One is the periodic driving force and another is the kick force applied to KR. (Note that the energy of KR is finitely bounded.) The nearly resonant condition mentioned above is very important to make the energy supplied to the HO come from the periodic driving force, and the exchange of energy between KR and HO is negligible compared with the former as is shown in Appendix A

With this approximation the sum frequency term containing the frequency $\Omega + \omega$ is ignored, and the energy stored in the HO is expressed by using the autocorrelation function of the force due to the KR as follows:

$$
E_\tau - E_0 = \sum_{s \leq \tau - 1} A_s \quad \text{with} \quad A_s = \mu \sum_{k=-s}^s C_{rs}(k) e^{-i\omega kT},
$$

where $\mu \equiv \eta^2 |e^{i\nu T} - 1|^2/4k^2$ and $C_{rs}(k)$ is the autocorrelation function of $\hat{f}_k \equiv f(\hat{I}_k)$ defined by $C_{rs}(k) = \langle \hat{f}_s \hat{f}_{s-k} \rangle$ for $k \geq 0$, and $C_{rs}(k) = C_{rs}(-|k|)^*$ for $k < 0$. (see Appendix A) Its statistical property plays the crucial role in the energy transfer process. In the classical limit the quantum average $\langle \cdot \rangle$ over the initial wavepacket is replaced by the average over the ensemble of initial conditions having the same statistical weight in the phase space as the corresponding initial quantum wavepacket.

B. Case of the single kicked rotor ($\eta \neq 0, \varepsilon = 0$)

First, we consider the classical limit in the case of null coupling strength $\varepsilon = 0$, i.e., the HO is coupled with only one cat. The motion of the cat for a single step of the period $T$ is represented by the classical map $\theta_\tau = \theta_{\tau-1} + I_{\tau-1}$, $I_{\tau} = K \theta_{\tau-1} + (K+1)I_{\tau-1}$, where $(\theta, I)$ are the classical canonical variables. We are concerned with the hyperbolic case $K < -4$, $0 < K$ $(K \in \mathbb{Z})$ in which the absolute value of Lyapunov exponent is larger than 1, and the Lyapunov exponent is the same everywhere in the bounded phase space $[0, 2\pi] \times [0, 2\pi]$, where the periodic boundary condition is imposed as mentioned before. The cat map exhibits mixing and so ergodic, and has a uniform invariant measure over the phase space for sufficiently large $K$. Further there remains no correlation between different steps for almost all the initial distributions,

$$
\langle f(I_\tau) \rangle = 0, \quad \langle f(I_k)f(I_j) \rangle = \delta_{k,j} \langle f(I)^2 \rangle, \quad (7)
$$

taking $f(\hat{I}_\tau) = \cos \hat{I}_1$. All the above features provides a typical situation leading to a stationary energy transfer, and energy increases at the constant rate $A = \mu$ as,

$$
\langle E_\tau \rangle - E_0 \simeq A \tau, \quad A = \mu \langle f(I)^2 \rangle, \quad (8)
$$

because $C_{rs}(k) = \langle f(I)^2 \rangle \delta_{k,0}$. We stress that, as is shown in Appendix, the absorbed energy is supplied by the periodic driving force, and not by the stochastic agitation by the KR, if the nearly resonant condition is satisfied.

Moreover, Eq. [4] is the summation of the complex random variable and so the distribution function for $\hat{b}_\tau$ obeys the complexified Gaussian process, which means the complex variables $(a, a^*)$ obey the Gaussian distribution as

$$
P(a^*, a) \propto e^{-|a|^2/(|a|^2)^2}. \quad (9)
$$

Accordingly, the distribution function of energy $E = \hbar \omega |a|^2$ becomes the exponential type $P(E) \propto e^{-E/(E)}$.

Next we consider its quantum version. In the quantum mechanical case we add the cats a perturbation potential $\xi \cos(\hat{\theta}_{2,1} - \hat{\theta}_{1,2})$ with classically negligible strength $\xi$ ($\sim O(\hbar k^2)$), which suffices to break degeneracies caused by the underlying symmetries peculiar to the quantum ACM [29]. The typical value of parameters we took are given in the table. The initial state is set to the direct product state $|\psi_0\rangle = |0\rangle \otimes |I = \hbar N/2\rangle$, where $|0\rangle$ is the ground
as in Fig. 2(a) is about 0.15 for each distribution strength in all cases. The order of saturation levels 0.15, 0.2, 0.25, and 0.3 in the very early stage, but it becomes hardly realized in all cases.

As shown in Fig. 2(a), energy absorbed by HO increases monotonously in the very initial stage but then be-

Increase is suppressed on a short time scale in the single cat case. (b) Absorbed energy \(\langle E_\tau \rangle - E_0\) by HO as a function of time for various \(\varepsilon\)’s with a relatively large \(h = 2\pi/64\) in the twin cats case. (c) Some snapshots of the energy distribution \(P(E_n)\) of HO in the case \(\varepsilon = 1\) and \(h = 2\pi/64\) in the panel (c). All cases are \(\eta = 5 \times 10^{-4}\).

State of the HO, and \(|I = hN/2\rangle\) is the eigenstate of the action operator \(\hat{I}_r\) of the ACM.

TABLE I: The typical value of parameters we used in numerical calculation.

| parameter | value |
|-----------|-------|
| \(K\)     | 10    |
| \(T\)     | 10^2  |
| \(\Omega\)| 1     |
| \(\omega\)| \(1 + \sqrt{2}/T\) |
| \(\xi\)   | 4.25ℏ^2 |
| \(\theta_1\) | \(\sqrt{2}\) |
| \(\theta_2\) | \(-\sqrt{5}\) |
| \(\eta\)  | \(5 \times 10^{-4}\) |
| \(h\)     | \(2\pi/N\) |
| \(N\)     | \(2^5 \sim 2^6\) |
| \(\mu\)   | \(2.64 \times 10^{-4}\) |

In the case of \(\varepsilon = 0\), the time evolutions of absorbed energies by the HO \(\langle E_\tau \rangle - E_0\), calculated numerically under the above conditions are plotted in Fig. 2(a). The solid line in the figure represents theoretical values in the ideal classical limit. Dashed lines represent actual behaviors for \(h = 2\pi/256\), \(h = 2\pi/128\), and \(h = 2\pi/64\) from the top.

As shown in Fig. 2(a), energy absorbed by HO increases monotonously in the very initial stage but the increase is suppressed on a short time \(\tau < O(10^2)\) and turns into fluctuating behaviors around certain saturation levels in all cases. The order of saturation levels in Fig. 2(a) is about \(O(10^{-2})\), which is the same order as \(E_0\), and so we conclude that effective energy transfer was hardly realized in all cases.

As for the distribution function, it takes the Boltzmann type distribution only in the very early stage, but it becomes localized around the ground state whose shape is quite different from the exponential distribution. This is the reflection that the temporal energy absorption rate \(A_\tau\) vanishes very promptly for \(\varepsilon = 0\).

As has been discussed in detail in the previous paper [10], the short saturation time means that in a single KR the number of eigenstates which are significantly connected with an eigenstate by the interaction potential \(\eta qf(I)\cos \omega t\) in Eq. (1) is much less than \(N\). Such is very similar to the localization effect of eigenfunctions in unbounded isolated kicked rotor. Even though \(N\) is increased more and the saturation time surely gets longer, but a recovery of stationary absorption can not be achieved. This is due to the strong quantum interference effect peculiar to low-dimensional quantum systems which has been observed as the “scar” [22]. It corresponds to the Anderson localization phenomenon [2, 3] that has been observed widely in unbounded single KR.

C. Case of the coupled kicked rotors (\(\eta \neq 0\), \(\varepsilon \neq 0\))

Thus a single cat can hardly realize a QD. However, if the two cats are coupled, a remarkable entanglement transition occurs although the increment of the coupling strength \(\varepsilon\) is classically negligible and so very small [19, 21], which is a well-known phenomenon in coupled quantum chaos systems [23].

Then the effective dimension \(N_{\text{dim}}\) of the Hilbert space of the cats increases from \(N\) to \(N^2\). Let us define the time-scale \(\tau_{QD}\) of saturation of the stationary energy absorption caused by the quantum suppression of the classical decay of autocorrelation. It is expected to increase markedly by the coupling between the cats. Indeed, in our previous paper we showed that the finite time Fourier transform of the quantum autocorrelation function

\[
F_{\tau}(z) = \sum_{k=-\tau}^{\tau} C_{r\tau}(k)e^{-izTk},
\]

which is nothing more than the temporal absorption rate \(A_\tau\) for \(z = \nu = (\Omega - \omega)\) in Eq. (10), takes the constant
classical value due to the classical chaotic decay of autocorrelation function if \( \tau \) is less than the “lifetime” \( \tau_L \):

\[
\tau_L \sim \begin{cases} 
CN_{\text{dim}}^2 = CN^4 & \text{for } z \neq 0 \\
CN_{\text{dim}} = CN^2 & \text{for } z = 0,
\end{cases}
\]

(11)

where the coefficient \( C(z) \) is given as

\[
C(z) \sim Cr_{cl}(0)/\sum_{s=-\infty}^{\infty} Cr_{cl}(s)e^{-izTs}
\]

(12)

with the classical correlation function \( Cr_{cl}(s) \). For \( \tau > \tau_L \), \( F_r(z) \) decays to zero due to the multiple-periodic nature of the quantum correlation of the bounded system. The Eq. (11) becomes crucial again in Sect. IV and we will explain the physical origins there.

If we can identify \( \tau_{QD} \) with \( \tau_L \), the absorption rate \( A_r \) keeps the classical rate over an extremely long time-scale of \( \tau_L \), and the stationary and one way conversion of the mechanical energy to the “thermal” energy is almost accomplished.

The fully quantum result of the time-dependent energy absorption by the HO are depicted in Fig. 2(b). Results for different values of \( \varepsilon \) are plotted, where \( h = 2\pi/64 \) and \( E_0 = h\hbar/2 \approx 6.9 \times 10^{-2} \) for all results. The line running close to the bottom is the result of \( \varepsilon = 0 \), while the lines running in the upper sides are those for significantly larger values of \( \varepsilon \). The solid line represents the ideal classical behavior expected by Eq. (10).

It is evident that the quantum behavior approach to the ideal classical one as the value of \( \varepsilon \) increases. The absorbed energy at \( \tau = 2500 \) for a sufficiently large coupling strength \( \varepsilon = 1 \) is about \( 3.4 \times 10^{-1} \), which is 5 times larger than \( E_0 \). This implies that a one-way transfer of the energy from the driving source to the HO occurs if \( \varepsilon \) is taken as a sufficiently large value.

Figure 2(c) depicts the time evolution of energy distributions for a relatively large value of \( \varepsilon \) in the examples in Fig. 2(b), i.e. \( \varepsilon = 1, h = 2\pi/64 \). Exponential distributions, as expected in the ideal case, are certainly realized and are sustained for a long time scale. Namely, it follows very well that the distribution becomes the Maxwell-Boltzmann type one at least up to the attainable time by the numerical simulation. The effective temperature is represented by \( E_r \), which increases almost linearly in time as is shown in Fig. 2(b).

IV. \( h \)-DEPENDENCE OF THE QUANTUM DAMPER

As is expected by Eq. (11), we demonstrate in Fig. 3(a) that the time of the saturation \( \tau_{QD} \) until which the stationary absorption process gets markedly longer than the Planck constant \( h = 2\pi/N \) is reduced by increasing the dimension \( N \). On the other hand, for fixed \( h \), the long-lived stationary absorption process is realized beyond a threshold of the coupling strength \( \varepsilon \). We investigate here the \( h \)-dependence of such a characteristic transition phenomenon.

The change of the absorption process with the increase in \( \varepsilon \) is qualitatively captured by the variation of average absorption rate \( \bar{A} \). We define the transient stationary energy absorption rate \( \bar{A} \) by the least square fit of the data to \( \langle E_r \rangle - E_0 = \bar{A}\tau \) for a fixed interval \( t \in [0, \tau_{av}] \), where \( \tau_{av} = 0.5\tau_{QD} \).

We show in Fig. 3(b) the average energy absorption rate \( \bar{A} \) as a function of \( \varepsilon \) for three values of \( h \) as \( h = 2\pi/2^4, 2\pi/2^5, 2\pi/2^6 \). We set \( \eta = 2.0 \times 10^{-4} \) for all cases. As shown in the figure it seems that the rate suddenly increases as \( \varepsilon \) exceeds a certain threshold value \( \varepsilon = \varepsilon^* \) and beyond it the \( \bar{A} \) reaches to a plateau along which the rate does not change significantly. The plateau height is close to the classical absorption rate for all \( h \) although some significant “quantum fluctuation” is overlapped for relatively large \( h = 2\pi/16 \). We may, therefore, call the plateau as the “classical plateau”. Moreover, it is evident that the plateau and the threshold \( \varepsilon^* \) shifts to the smaller side of \( \varepsilon \) as \( h \) decreases, which becomes more evident by the plots (\( h, \varepsilon^* \)) depicted in Fig. 3(c).

The approach of the average absorption rate to the classical value means the remarkable increase of \( \tau_{QD} \), which should reflect the rapid growth of entanglement between the two cats, as was suggested in the previous section. It means the number of eigenstates connected strongly by the interaction potential \( \eta f(\hat{I}) \cos \omega t \) increases with increase in the coupling strength \( \varepsilon \).

Indeed, the increase in the number of the connected eigenstates via the operator \( f(\hat{I}) = \cos \hat{I}_1 \) contained in the interaction potential is possible only by the development of entanglement between the two cats, which should be directly observed by the entanglement entropy (EE). The approach of the average absorption rate to the classical value means the remarkable increase of \( \tau_{QD} \), which should reflect the rapid growth of entanglement between the two cats, as was suggested in the previous section. It means the number of eigenstates connected strongly by the interaction potential \( \eta f(\hat{I}) \cos \omega t \) increases with increase in the coupling strength \( \varepsilon \).

Indeed, the increase in the number of the connected eigenstates via the operator \( f(\hat{I}) = \cos \hat{I}_1 \) contained in the interaction potential is possible only by the development of entanglement between the two cats, which should be directly observed by the entanglement entropy (EE).

We follow Ref. [19], and introduce the EE defined for every eigenstate \( |n\rangle \) of the evolution operator of the coupled cat \( \hat{U}_{KR} \), i.e.,

\[
S_n = -\text{Tr}|n><n| \log |n\rangle \langle n|
\]

(13)
decreasing

**FIG. 3:** Energy absorption in the case of coupled twin cats. (a) Long-time behavior of the absorbed energy $\langle E_r \rangle - E_0$ for decreasing $h$, where $h = 2\pi/N$ with $N = 2^4, 2^5, 2^6, 2^7$. Here, $\varepsilon = 1$ and $\eta = 2 \times 10^{-4}$. (b) Average absorption rate $\bar{A}$ as a function of $\varepsilon$ for various $h = 2\pi/N$. $\eta = 2 \times 10^{-4}$. The horizontal line shows absorption rate in classical theory. (c) The threshold values $\varepsilon^*$ at some values of $\eta$ and $\varepsilon_S^*$ as a function of $h$. The reference line denotes $\varepsilon^* = 1.3h^{1.65}$, which is fitted in the range $\tau \in [0, \tau_{as}]$.

where trace is taken for either of the twin cats. $S_n$ varies from 0 to $\log N$, and $S_n = 0$ indicates no entanglement happens. Next the mean EE averaged over all the eigenstates, i.e., $S = \sum_n S_n/N^2$ is introduced, and the $\varepsilon$-dependence of $S$ is explored, which draw characteristic curves similar to Fig.3(b) starting with $S = 0$, and steeply increases in the regime close to $\varepsilon \sim \varepsilon^*$. [19]

To analyse $h$-dependence of the threshold of the mean EE, we define the threshold value $\varepsilon_S^*$ satisfying the relationship $S(\varepsilon_S^*) = S(\varepsilon = 1/2)$, and plotted $\varepsilon_S^*$ versus $h$ in Fig.3(c) by rectangle marks. As shown in the figure, the points of $(h, \varepsilon_S^*)$ agree very well with the threshold of absorption $(h, \varepsilon^*)$ and they are aligned very well along the dashed line which indicates $\varepsilon^* \propto h^\alpha$ with $\alpha = 1.65$. This fact is a direct evidence that the enhancement of entanglement happening in the isolated coupled KR is the origin of the increase in the stationary absorption rate of the QD.

V. DURATION TIME OF THE STATIONARY CONVERSION

In this section, we investigate the $\eta$-dependence of the duration time $\tau_{QD}$ and give the interpretation.

A. $\eta$-dependence

Finally, let us focus on the fully entangled regime $\varepsilon \gg \varepsilon^*$ of the two cats and demonstrate how the stationary flow of the energy absorption process by the QD is strengthen with the increase in the Hilbert space dimension $N_{dim} = N^2$ of the coupled cats. As shown in Fig.3(a), it is evident that with increase in $N_{dim}$, the time of saturation of the energy absorption markedly increases. i.e., $\tau_{QD} \sim 10^3$ for $N_{dim} = 16^2$, $\tau_{QD} \sim 5 \times 10^3$ for $N_{dim} = 32^2$, and $\tau_{QD} \sim 2 \times 10^4$ for $N_{dim} = 64^2$. For $N_{dim} > 128^2$ the absorption curve closely follows the ideal curve up to the time numerical simulation is attainable. However the time scale of saturation seems to be much shorter than the prediction of $\tau_L \sim N^4$ in Eq.11 for $z = \nu \neq 0$. The Eq.11 holds under the condition that the coupled cats are completely isolated, i.e. $\eta \rightarrow 0$. That is, the difference between the $\tau_L$ and $\tau_{QD}$ should be due to be the interaction of the coupled cats with HO and the driving force. The increase of $\eta$ changes the condition ($\eta \rightarrow 0$) very significantly. Indeed, in Fig.4 we show how the absorption curve changes as $\eta$ is varied, where $N_{dim} = 32^2$. It is evident that the time of saturation $\tau_{QD}$ is lengthen as $\eta$ is reduced, and there is a threshold $\eta_{th}$ of $\eta$ below which the absorption curves are only parallel downward shifts and thus $\tau_{QD}$ takes the maximum constant value $\tau_{QD} \sim 10^5$. which should $\tau_L$ in Eq.11.
the term having the minimal difference of eigenangle \( \delta = \min_{j,j'} |\delta_{jj'}| \), where \( \delta_{jj'} = \gamma_j - \gamma_{j'} \), becomes oscillatory, namely \( \tau_L \delta \sim O(2\pi) \). Since the strong level repulsion of entangled chaotic cats yields \( \delta \sim 1/N_{\text{dim}} = 1/N^2 \), we conclude that \( \tau_L \sim N_{\text{dim}} = N^2 \), which is \( z = 0 \) case of Eq.(11). This time-scale coincides with the so-called Heisenberg time.

On the other hand, for \( z = \nu \neq 0 \) the additional frequency changes the oscillatory factor as \( e^{-i(\gamma_j \pm zT - \gamma_{j'})} \) and the above rule is modified completely: the minimal difference \( \delta = \min_{j,j'} |\delta_{jj'}| \), where \( \delta_{jj'} \equiv |\gamma_j \pm zT - \gamma_{j'}| \), decides \( \tau_L \). The frequency difference \( \delta_{jj'} \) can be interpreted as the difference in the paired eigenangles of eigenstates in the extended space formed by the direct product of the cats, the HO, and the driving source. The emergence of \( z(= \nu) \) in the frequency difference \( \delta_{jj'} \) makes the choice of the minimal \( |\delta_{jj'}| \) free from the presence of level repulsion among \( \gamma_j \)s, and so \( \delta \) can be much smaller than the case of \( z = 0 \), and the minimal value reaches to \( \delta \sim 1/N_{\text{dim}} = N^{-4} \), which leads to the case of \( z \neq 0 \) of Eq.(11) [18].

C. Entanglement between HO and ACM

However, the growth of the interaction among the coupled cats, external driving force and HO make any closely approaching pair of quasi-energies, say, \( \gamma_j \pm zT \) and \( \gamma_{j'} \) to repulse by introducing the coupling between the paired eigenstates in the extended space mentioned above, and the minimal scale \( \delta \sim 1/N_{\text{dim}} = N^{-4} \) is no longer maintained.

With the above arguments, we can understand the behavior in Fig.4 very roughly as follows: as the coupling strength \( \eta \) becomes greater than the minimal spacing \( \sim 1/N_{\text{dim}}^2 \), then the quasi-degenerate pair of energy levels \( \delta_{jj'} \) repulses with each other to separate in proportion to the perturbation strength \( \eta \), which couples the KR with other degrees of freedom. That is,

\[
\eta_{\text{th}} \sim \frac{1}{N_{\text{dim}}^2}. \quad (14)
\]

In such a limit the saturation level of absorption, which is evaluated by summation over the temporal absorption rate Eq.(11), is dominated by the minimal scale of the energy-level spacing \( \delta_{jj'} \sim \eta \) as

\[
\eta^2 \times \sum_{j,j'} (N\delta_{jj'})^{-2} \propto \eta^2 \times \eta/\eta^2 \sim \eta \quad (15)
\]

because the number of the eigenstates \( j' \) less than \( \delta_{jj'} < \eta \) is roughly given by \( \eta/N_{\text{dim}}^2 = \eta N^2 \) for each \( j \). On the other hand, the absorbed energy classically increases as \( \propto \eta^2 \) according to Eq.(8), and so \( \tau_{QD} \) is given by

\[
\tau_{QD} \propto \eta^{-1}. \quad (16)
\]

Figure 4 indicates that \( \tau_{QD} \) starts to decrease from \( \eta_{\text{th}} \) given by Eq.(14) \( (N = 32) \) for sufficiently small \( \eta \), and
further decreases very according to Eq. (16). The decrease continues with \( \eta \) up to \( \tau_{QD} \sim N_{dim} \), namely the case of \( z = 0 \) in Eq. (11).

It should be noted that the \( N_{dim} \) is no longer \( N^2 \) but \( N^2 \times N_R \), where \( N_R \) being the mean number of the states of HO and the driving source which are entangled with the coupled cats. Detailed analyses for the above rather rough arguments will be presented for more simple standard model proposed in [18].

As long as both HO and the driving source are outsider of the cats group, an extremely long lifetime is promised. But if they are taken into the cats group as the internal degrees of freedom they lose the extremely long lifetime in compensation for the large transfer rate.

VI. CONCLUSION

Based upon the results of previous works, we proposed a very simple fully quantum mechanical model which can transform the mechanical energy into the internal energy in an irreversible way like a classical damper. Our system is composed of only three degrees of freedom: one is a quantum harmonic oscillator (HO) playing the role of the energy storage and another two are quantum kicked rotors (KR) which are classically chaotic. The KRs are defined in bounded phase space and so their Hilbert spaces have finite dimensions. The HO is driven by a coherent periodic force and KRs work as a stochastic source disturbing the phase of the driving force, and one can show an irreversible stationary energy transfer from the driving source to HO is realized in the ideal classical limit. In quantum mechanics, however, the irreversible stationary energy transfer continues up to a finite time scale denoted by \( \tau_{QD} \).

The time scale \( \tau_{QD} \) is expected to be sensitively dependent upon the coupling strength \( \varepsilon \) between the KRs, and it increases very rapidly as \( \varepsilon \) exceeds a very weak threshold decided by the Planck constant. The increment of \( \tau_{QD} \) is closely correlated with the development of entanglement between the two KRs with increase in \( \varepsilon \). However, the maximal \( \tau_{QD} \) realized after the entanglement was much shorter than the one predicted by the previous theory for the isolated chaotic KRs.

The reduction of \( \tau_{QD} \) is caused by the entanglement of the KRs with other systems i.e. HO and the driving source. The entanglement removes the quasi-degeneracy of the eigenstates of the whole system, which was the origin of very large \( \tau_{QD} \). The enhanced entanglement in the coupled KR enhances \( \tau_{QD} \), whereas it makes the threshold \( \eta_{th} \) of the coupling strength \( \eta \) of KR with HO and driving source extremely small. Development of entanglement in the whole system works inversely and reduces the time scale predicted in the ideal limit much shorter. Thus the development of the quantum entanglement plays opposite roles in realizing the time-reversible behavior. A complete understanding of the relation between the time scale of quantum irreversibility \( \tau_{QD} \) and the development of entanglement among the quantum elements is still an open problem, and a further clarification in a more simple and typical situation is strongly desired.

Appendix A: Analytical derivations by balance equation

We summarize here some basic calculations necessary for several equations used in the present paper. We also discuss here the balance between the absorbed energy by the HO and the energy supplied by the periodic driving source.

With using the Heisenberg picture of the dynamics for annihilation and creation operators \( \hat{a} \equiv \sqrt{\Omega/2\hbar}(\hat{q} + i\hat{p}/\Omega) \) and \( \hat{a}^\dagger \), one can immediately obtain

\[
\frac{d\hat{a}}{dt} = -i\Omega\hat{a} - i\eta f(\hat{I}) \cos(\omega t), \tag{A1}
\]

which is easily integrated by introducing the slowly varying envelope operator \( \hat{a} = e^{-i\mu t} \hat{b} \) as,

\[
\Delta \bar{b}(t) := \bar{b}(t) - \bar{b}(T\tau) = -\frac{1}{2\sqrt{2\nu T\hbar}} f(\hat{I}_0) \times \left[ e^{i(\Omega-\omega)\tau} - e^{i(\Omega+\omega)\tau} \right] \frac{\Omega - \omega}{\Omega - \omega} \frac{e^{i(\Omega-\omega)\tau} - e^{i(\Omega+\omega)\tau}}{\Omega - \omega} \tag{A2}
\]

for \( T\tau \leq t \leq T(\tau + 1) \) under the nearly resonant limit such that \( (\Omega-\omega)T = \nu T = C \sim O(1) \). Using this relation iteratively from \( t = 0 \) to \( t = T\tau \), we obtain Eq. (8). By using Eq. (9) and the definition

\[
\hat{q}(t) = \sqrt{2\hbar\Omega}[\hat{b}(t)e^{-i\mu t} + h.c.], \tag{A3}
\]

the energy stored by the HO at the step \( \tau \) is given by

\[
\langle \hat{a}^\dagger \hat{a} \rangle = \mu \sum_{k=0}^{\tau - 1} \sum_{j=0}^{\tau - 1} e^{i(\nu(k-j))} \langle f(\hat{I}_k) f(\hat{I}_j) \rangle, \tag{A4}
\]

which is rewritten as Eq. (10).

Now we suppose that the KR follows chaotically ideal behavior realized in the classical limit, and consider the energy balance between the driving source and HO. We suppose that the coupling strength \( \eta \) among HO, the driving force and KRs is so weak that the correlation characteristics of KR represented by Eq. (12) holds in the classical limit. Then the energy of HO increases linearly obeying Eq. (13) at the classical absorption rate per step

\[
A = \eta^2 \langle f(I)^2 \rangle \frac{1 - \cos(\Omega - \omega)T}{4(\Omega - \omega)^2}. \tag{A5}
\]
Next, we evaluate the energy transfer from the driving source during a single period $\tau T < t \leq (\tau + 1)T$. To this purpose we extend our QD Hamiltonian so as to include the degree of freedom of driving source: let $\omega J$ be the energy of driving source, where $\dot{J} = -i d / d\phi$ is the operator of action variable representing the driving source and $\phi$ is the angle variable describing the phase of the driving source. Then the interaction Hamiltonian $\eta f(\hat{I}) \cos(\omega t) q$ in Eq. (A4) should be replaced as $\eta f(\hat{I}) \cos(\phi) q$, and the extended total Hamiltonian including the driving source is written as,

$$H_{ext} = \hat{H}_{HO}(\hat{q}, \hat{p}) + \hat{H}_{KR}(\hat{\theta}, \hat{I}, t) + \eta f(\hat{I}) \cos(\phi) + \omega \hat{J}.$$  \hspace{2cm} (A6)

The Heisenberg equations of motion for the variables of driving source are the same as the canonical equation of motion

$$\frac{d\phi}{dt} = \frac{i}{\hbar} [H_{ext}, \phi] = \omega, \hspace{2cm} (A7)$$

$$\frac{d\hat{J}}{dt} = \frac{i}{\hbar} [H_{ext}, \hat{J}] = \eta f(\hat{I}) \sin(\phi). \hspace{2cm} (A8)$$

By integrating Eq. (A7) from $(\tau - 1)T$ to $\tau T$, the variation of the source energy $B = \omega (J(T(\tau + 1)) - J(\tau T))$ should be

$$B = \eta \int_{\tau T}^{(\tau + 1)T} (f(\hat{I}) q(t)) \sin(\omega t) dt. \hspace{2cm} (A9)$$

From Eqs. (A3) and (A2), $q(t)$ is the sum of the terms proportional to $\hat{b}(\tau T)$ and to $\Delta \hat{b}(t)$. The contribution from the former to the r.h.s. of Eq. (A9) vanishes because it is the sum of the terms proportional to $f(I_j)$ with $j < \tau$ from Eq. (4). The contribution comes only from $\Delta \hat{b}(t)$, and in the nearly resonant condition $B$, it leads to the result

$$B = \frac{\omega}{\Omega} \int f(\hat{I})^2 \cos((\Omega - \omega)T) - 1}{4(\Omega - \omega)^2}, \hspace{2cm} (A10)$$

which coincides with the absorption rate $A$ of Eq. (A3) in the nearly resonance condition.

Acknowledgments

This work is partly supported by Japanese people’s tax via JPSJ KAKENHI 15H03701, and the authors would like to acknowledge them. They are also very grateful to Kankikai, and Koike memorial house for use of the facilities during this study.

[1] I. Prigogine, *From Being to Becoming: Time and Complexity in the Physical Sciences* (Freeman, San Francisco, 1980).

[2] G. Casati, B. V. Chirikov, F. M. Izraelev, J. Ford, in *Stochastic Behavior in Classical and Quantum Hamiltonian Systems*, edited by G. Casati and J. Ford, Lecture Notes in Physics Vol.93 (Springer-Verlag, Berlin, 1979), p. 334.

[3] S. Fishman, D. R. Grempel, and R. E. Prange, Phys. Rev. Lett., 49, 592 (1982).

[4] G. Casati, I. Guarneri, and D. L. Shepelyansky, Phys. Rev. Lett., 62, 345 (1989).

[5] G. Lemarie, H. Lignier, D. Delande, P. Szriftgiser, and J.-C. Garreau, Phys. Rev. Lett. 105, 090601 (2010); M. Lopez, J. F. Clement, P. Szriftgiser, J. C. Garreau, and D. Delande, Phys. Rev. Lett. 108, 095701 (2012).

[6] R. Nandkishore and D. A. Huse, Ann. Rev. Condensed Matter Phys. 6, 15 (2015).

[7] T. Kinoshita, T. Wenger, and D. S. Weiss, Nature 440, 900 (2006).

[8] I. E. Mazets and J. Schmiedmayer, New. J. Phys., 12, 055027 (2010).

[9] M.A. Cazalilla and M. Rigol, New. J. Phys., 12, 1 (2010).

[10] K. Ikeda, Ann. Phys. 227, 1 (1993).

[11] D. Cohen, Phys. Rev. Lett., 82, 4951 (1999); D. Cohen Ann. Phys. 283, 175, (2000).

[12] K. Ikeda, in *Quantum Chaos*, edited by G. Casati and B. V. Chirikov (Cambridge Univ. Press, 1996), p.145; H. S. Yamada and K. S. Ikeda, Phys. Rev. E 82, 060102 (2010).

[13] L.E. Ballentine and J.P. Zibin, Phys. Rev. A 54, 3813 (1996).

[14] G. Benenti and G. Casati, Phys. Rev. E 79, 025201 (2009).

[15] A. Peres, Phys. Rev. A 30, 1610 (1984).

[16] T. Gorin, T. Prosen, T. H. Seligman, M. Znidaric, Phys. Rep. 435, 33 (2006); Ph. Jacquod and C. Petitjean, Adv. in Phys. 58, 67 (2009).

[17] J. Wang and A.M. Garcia-Garcia, Phys. Rev. E 79, 036206 (2009).

[18] F. Matsui, H.S. Yamada and K.S. Ikeda, Europhys. Lett. 113, 40008 (2016).

[19] F. Matsui, H.S. Yamada and K.S. Ikeda, Europhys. Lett. 114, 60010 (2016).

[20] J.P. Keating and F. Mezzadri, Nonlinearity 13, 747 (1998).

[21] S. Adachi, M. Toda, and K. Ikeda, Phys. Rev. Lett. 61, 659 (1988); B. Gadway, J. Reeves, L. Krinner, and D. Schneble, Phys. Rev. Lett. 110, 190401 (2013).

[22] E.J. Heller, Phys. Rev. Lett. 53, 1515 (1984).

[23] A.Lakshminarayan, Phys. Rev. E 64, 036207 (2001); H. Fujisaki, T. Miyadera, and A. Tanaka, Phys. Rev. E 67, 066201 (2003); R. Demkowicz-Dobrzanski, and M. Kus, Phys. Rev. E 70, 066216 (2004).

[24] The classical absorption process is clearly recovered at least in the the time range $0 \leq \tau < T$ at the classical coupling strength $\varepsilon \sim O(1)$. Then we take $\tau = 0.5 \times T$ as the common time scale on which the least
square fit is done.