Pulsar Kick and Asymmetric Iron Velocity Distribution in SN 1987A

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We have investigated the relation of the direction of the momentum among the matter, neutrino, and proto-neutron star in a collapse-driven supernova in order to discuss the pulsar kick. In particular, we have investigated the effects of the pulsar motion on the explosion, which are neglected in the previous study. As a result, it is suggested that the direction of the total momentum of the matter and neutrino is opposite to that of the momentum of the proto-neutron star in the asymmetric explosion models. This is because the center of the explosion deviates from the center of the progenitor due to the pulsar motion. This picture is common among the asymmetric explosion models. So if we assume that the pulsar motion is caused by an asymmetric supernova explosion, the neutron star born in SN 1987A, which has not been found yet, will be moving in the southern part of the remnant. In other words, if we can find one neutron star in SN 1987A on the south part of the remnant, asymmetric explosion models will be supported by the observation better than the binary models.

§1. Introduction

It is a well known fact that pulsars in our galaxy have velocities much in excess of those of ordinary stars\textsuperscript{6}. It is reported that their transverse speeds range from 0 to \( \sim 1500 \) km s\(^{-1}\) and their mean three-dimensional speed is \( 450 \pm 90 \) km s\(^{-1}\)\textsuperscript{2}.

On the other hand, there are many theoretical models in order to explain the pulsar kick. One is that a neutron star in a binary system can escape from the system with rapid speed due to a supernova explosion of the nascent star\textsuperscript{3}. There are also many models in which effects of asymmetric supernova explosion are taken into consideration\textsuperscript{4,5}. For example, it is reported that a 1\% anisotropic neutrino radiation from the proto-neutron star results in a kick velocity consistent with the observations\textsuperscript{4,5}. However, there are few observations to determine which model is the most promising one.

As for the asymmetric explosion models, only Burrows and Hayes (1996) performed numerical simulations and estimated the speed of the proto-neutron star in the supernova matter. They introduced the anisotropy of the system by artificially decreasing the density of the Chandrasekhar core within 20° of the pole, which may be realized by the convection during the stellar evolution. They also estimated the contributions due to the neutrino emission anisotropy and the ejecta motions. As a result, they reported that the proto-neutron star can be accelerated to \( \sim 530 \) km s\(^{-1}\). They also concluded that the direction of the total momentum of the matter and neutrino is opposite to that of the momentum of the proto-neutron star.
Recently, Nagataki (1999) discussed that the asymmetry of the observed line profiles of Fe[II] in SN 1987A can be also explained by the asymmetry of the explosion. In the case of SN 1987A, more matter has to be conveyed to the north side than to the south side in order to reproduce the observed asymmetric line profiles. As a result, if we believe the result of Burrows and Hayes (1996), the proto-neutron star born in SN 1987A, which has not been found yet, will be moving in the southern part of the remnant.

It is worthwhile discussing whether the conclusion of Burrows and Hayes (1996) on the momentum of the matter, neutrino, and pulsar is common among the asymmetric supernova explosion models. If so, the conclusion on the location of the neutron star born in SN 1987A will be valid as long as the asymmetric explosion models are believed. Moreover, if the neutron star is discovered on the south side as Nagataki (1999) predicted, the asymmetric explosion models will be supported by the observation than the binary models.

In this paper, we investigate whether the conclusion derived by Burrows and Hayes (1996) on the momentum of the matter, neutrino, and pulsar is common among the asymmetric supernova explosion models. In particular, we investigate the effects of the pulsar motion on the explosion, which are neglected in the study of Burrows and Hayes (1996). In section 2, we show analytical estimates for the effects of the neutrino heating by the proto-neutron star. In section 3, numerical estimates of neutrino flux are shown. Discussion and conclusion are presented in section 4.

§2. Analytical Estimates for the effects of the neutrino heating

The mechanism of collapse-driven supernovae has been understood as follows: when the mass of the iron core of the progenitor exceeds the Chandrasekhar mass, the star begins to collapse. The collapse continues until the central density of the collapsing core reaches about (1.5-2) times the nuclear matter density \(\rho = 2.7 \times 10^{14}\text{g cm}^{-3}\), beyond which matter becomes too stiff to be compressed further. A shock wave then forms, propagates outward. At first, the shock wave is not so strong and stall at \(~200\text{ km}\) in the iron core (it is called a stalled shock wave). However, by the continuous neutrino heating (\(~500\text{ ms}\)), the shock wave is revived, begins to propagate outward again, and finally produces the supernova explosion. This phenomenon is called as the delayed explosion, which is the most promising theory for the mechanism of the collapse-driven supernova explosion.

In this section, we investigate the contributions to the shock revival due to the injection of the momentum and the thermal energy by the neutrino heating, respectively.

Behind the stalled shock wave, the equation of state (EOS) is well determined by the radiation pressure, the ideal gas pressure, and the pressure of degenerate electrons at zero temperature. To put it concretely, the pressure and the energy per unit volume are described as follows:

\[
p = a T^4/3 + \rho kT/\mu m_u + p_e(\rho) \quad (2.1)
\]
\[\rho e = a T^4 + \rho k T / (\gamma_g - 1) \mu m_u + 3 \rho e(\rho) \quad (2.2)\]

where \(a\) is the radiation constant, \(k\) Boltzmann constant, \(m_u\) the atomic mass unit, \(\mu\) the mean mass number of the gas, \(\gamma_g\) the adiabatic index of the gas. The pressure of degenerate electrons \(p_e\) is given by

\[p_e = 1.24 \times 10^{15} \text{ dyn cm}^{-2} (Y_e \rho)^{1/3} \quad (2.3)\]

where \(Y_e\) is the electron fraction of the system. The typical values for the \(\rho, T, \mu, \gamma_g,\) and \(Y_e\) behind the stalled shock wave are: \(10^9 \text{ g cm}^{-3}, 1 \text{ MeV}, 1, 1.2, 0.4. \) The density dependence of the pressure is shown in Fig. 1.

![Fig. 1. Density dependence of the pressure. Temperature and electron fraction are set to be 1 MeV and 0.45. Solid line: the radiation pressure. Dotted line: the ideal gas pressure. Short dashed line: the degenerate electron pressure.](image)

At first we consider the contribution of the injection of the thermal energy. From Eq. (2.1) and (2.2), the partial derivative of the pressure by the energy density can be written as follows:

\[\left. \frac{\partial p}{\partial \epsilon} \right|_\rho = \left. \frac{\partial p}{\partial T} \right|_\rho \left. \frac{\partial T}{\partial \epsilon} \right|_\rho = \frac{\rho}{4aT^3} + \frac{\rho k}{(\gamma_g-1)\mu m_u} \left( \frac{4}{3} a T^3 + \frac{\rho k}{\mu m_u} \right). \quad (2.4)\]

At the behind the stalled shock wave, the contribution of the radiation can be neglected (see Fig. 1). So we can rewrite Eq. (2.4) as

\[\left. \frac{\partial p}{\partial \epsilon} \right|_\rho = (\gamma_g - 1) \rho \sim \frac{1}{5} \rho. \quad (2.5)\]
From Eq. (2.5), we can estimate the thermal pressure by the neutrino heating as

\[ p = \frac{\rho \epsilon}{5}. \]  

(2.6)

Next, we consider the contribution of the injection of the momentum. That is, we have to estimate the ram pressure which is the added pressure when the flow is interrupted and the velocity becomes to be zero at the stalled shock front. However, we cannot use the Bernoulli’s equation behind the stalled shock wave. This is because the pressure is determined not only by the electron but also by the ideal gas at \( \rho = 10^{10} \text{ g cm}^{-3} \) (see Fig. 1). In other words, the pressure is not determined only by the density. So we have to give a rough order-estimation for the ram pressure, which will be enough for the discussion in this section. From the dimension analysis, we can estimate that the ram pressure is as follows:

\[ p_{\text{ram}} \sim \rho v^2 \]  

(2.7)

where \( p_{\text{ram}} \) is the obtained ram pressure.

The typical explosion energy of a supernova is \( \sim 10^{51} \text{ ergs} \). The total mass behind the stalled shock wave is estimated to be \( \sim 0.1 M_\odot \) when the average density behind the stalled shock wave and the location of the stalled shock wave are assumed to be \( 10^{10} \text{ g cm}^{-3} \) and 200 km, respectively. So the thermal energy density behind the stalled shock wave is estimated to be \( 5 \times 10^{18} \text{ erg g}^{-1} \). So, using Eq. (2.6), the thermal pressure due to the neutrino heating is estimated to be \( 10^{28} \text{ dyn cm}^{-2} \).

On the other hand, the velocity due to the momentum transfer from neutrinos to the matter is estimated to be \( 2 \times 10^8 \text{ cm s}^{-1} \). Using Eq. (2.7), the ram pressure is estimated to be \( 4 \times 10^{26} \text{ dyn cm}^{-2} \).

In this section, we find that the contribution to the pressure due to the injection of the thermal energy will be greater than that due to the injection of the momentum by the neutrino heating. However, the ratio of the ram pressure relative to the thermal one (\( \sim 1/25 \) in our analysis) is small relative to their absolute value. It will not be strange that their contributions become comparable in a realistic calculation. So we will investigate both of their effects on the explosion in the next section.

§3. Numerical estimates of Neutrino flux

3.1. Angular dependence of the normal component of the neutrino flux

As we noted in introduction, the proto-neutron star might have moved from the center of the collapse due to the kick before the explosion. If the dynamics of the explosion is controlled by the thermal pressure of the matter heated by the emitted neutrino flux, the integrated flux determines the magnitude of the explosion. In this case it is expected that the neutron star (NS) heats the matter behind the soled shock wave anisotropically and the irons would not be blown off spherically. In this subsection we show that there is an anisotropy of the integrated neutrino flux received on the back of the shock front and show how it depends on the azimuthal angle \( \cos \theta \) to the line of the kick.

For the simplification, we assume that the neutron star whose radius is \( r_{ns} \) moves at a constant velocity \( v_{ns} \) along the line of the kick from the center of the
collapse. In addition we assume that the neutron star starts to move as soon as it begins to emit the electron neutrino and it keeps radiating the neutrino at a constant flux $F_0$ (erg cm$^{-2}$ sec$^{-1}$). Here let $t$ the passed time since the neutron star started to move. Then the distance from the center of NS to the shock front is given by $r = l_0 - v_{ns}t$, where $l_0$ is the radius of the shock front. The matter behind the stolen shock wave receives the neutrino flux $\simeq (r_{ns}/r)^2 F_0$ per an unit time. Because NS has the rapid velocity enough to reach near the shock front until the explosion, we consider the integrated flux only for $0 \leq t \leq t_{\text{max}}$, where $t_{\text{max}} = (l_0 - r_{ns})/v_{ns}$.

Here we take the line of the kick as $z$-axis. As we show in Fig. 2, let $r_2$ the distance from NS to an unit area behind the shock front and $\eta$ the angle between the direction to it and $z$-axis. Then the angle $\eta$ is expressed by

$$\cos \eta = \frac{r - l_0(1 - \cos \theta)}{r_2}, \quad (3.1)$$

where $r_2$ is given by $r_2^2 = l_0^2 + (l_0 - r)^2 - 2l_0(l_0 - r) \cos \theta$. If the neutron star keeps radiating the neutrino until it reaches the shock front, a net neutrino flux $\mathcal{F}_\perp(\cos \theta)$ received on an unit area behind the shock front is expressed as the integrated normal component of the neutrino flux for $0 \leq t \leq t_{\text{max}}$. Then it is given by

$$\mathcal{F}_\perp(\cos \theta) = \int_0^{t_{\text{max}}} F_0 \left( \frac{r_{ns}}{r_2} \right)^2 \cos(\eta - \theta) dt \quad \mathcal{F}_0 r_{ns}^2 \left( \frac{1}{l_0 \sqrt{r_{ns}^2 + 2l_0(1 - \cos \theta)(l_0 - r_{ns})}} \right). \quad (3.2)$$

Fig. 2. Definition of coordinates and angles. The filled circle is the moving neutron star (NS). The outer circle represents the shock front.
If the neutron star does not move at all, the normal component of the integrated flux is equal anywhere behind the shock front and it is given by

\[ \bar{F} = \mathcal{F}_0 \left( \frac{r_{ns}}{l_0} \right)^2 \times t_{\text{max}} = \frac{\mathcal{F}_0 (l_0 - r_{ns}) r_{ns}^2}{v_{ns} l_0^2}. \]  

(3.3)

Then the fraction of the received flux is estimated by

\[ \frac{\mathcal{F}_\perp (\cos \theta)}{\mathcal{F}} = \frac{l_0}{\sqrt{r_{ns}^2 + 2l_0 (1 - \cos \theta)(l_0 - r_{ns})}}. \]  

(3.4)

In Fig. 3 we plot \( \mathcal{F}_\perp (\cos \theta)/\mathcal{F} \) as a function of \( \cos \theta \). Here we take the representative values \( r_{ns} = 10 \text{ km}, \ l_0 = 200 \text{ km and } v_{ns} = 450 \text{ km sec}^{-1} \).

Fig. 3. Plot of the integrated normal component of the neutrino flux received on the back of the shock front as a function of the azimuthal angle \( \cos \theta \). The integration of the flux is performed while NS is moving for \( 0 \leq t \leq t_{\text{max}} \).

From Fig. 3 we find that if the explosion is triggered by the thermal energy of the matter heated by the neutrino, it is expected that for \( \theta \geq \pi/3 \) the matter is extremely blown off. Namely, a small amount of the iron is strongly emitted forward because of the large pressure gradient. On the other hand, the majority is mildly pushed backward because the gradient of the thermal pressure is smoother in the backward direction. In addition we can see that the asymmetry of the received flux is much more larger than 1% which is needed for the initial kick of NS. It ensures that we
can neglect the intrinsic anisotropy of the neutrino emission from the proto-neutron star as we assumed.

3.2. Angular dependence of the forward component of the neutrino flux

If the explosion is triggered by ram pressure caused by the momentum transfer from the emitted neutrino to the matter just behind the stalled shock wave, the magnitude of the explosion should be closely related to the forward component of the neutrino flux. In this subsection we show how the integrated forward (z-axis) component of the neutrino flux depends on the azimuthal angle \( \cos \theta \).

If the neutron star keeps radiating the neutrino until it reaches the shock front, the integrated forward component of the neutrino flux received on an unit area is given by

\[
\mathcal{F}(\cos \theta)_z = \int_0^{t_{\text{max}}} \mathcal{F}_0 \left( \frac{r_{\text{ns}}}{r_2} \right)^2 \cos \eta dt
= \mathcal{F}_0 \frac{r_{\text{ns}}^2}{r_{\text{ns}}} \left( \frac{1}{\sqrt{r_{\text{ns}}^2 + 2l_0(1 - \cos \theta)(l_0 - r_{\text{ns}})} - \frac{1}{l_0} \right). \tag{3.5}
\]

On the other hand, if the neutron star does not move at all, the integrated flux is given in Eq. (3.3) as before. Then the fraction of the integrated forward component is estimated by

\[
\frac{\mathcal{F}_z(\cos \theta)}{\mathcal{F}} = \frac{l_0^2}{l_0 - r_{\text{ns}}} \left( \frac{1}{\sqrt{r_{\text{ns}}^2 + 2l_0(1 - \cos \theta)(l_0 - r_{\text{ns}})} - \frac{1}{l_0} \right). \tag{3.6}
\]

In Fig. 4 we plot \( \mathcal{F}_z(\cos \theta)/\mathcal{F} \) as a function of \( \cos \theta \). We find that if the explosion is caused by the momentum transfer from the neutrino flux to the matter behind the shock front, it blows off the matter forward only for \( \theta \geq \pi/3 \) and the majority of the matter is pushed backward, which is consistent with the conclusion derived in the previous subsection. This means that our conclusion on the flow of the matter is valid irrespective to the effects investigated. Also, because the asymmetry of the integrated neutrino flux is much larger than 1% almost everywhere, we should not be worried about the inherent asymmetric emission needed for the origin of kick. This means that the fact that the large amount of the matter is blown off backward does not depend on the mechanism of asymmetric supernova explosion.

§4. Discussion and Conclusion

We have discussed whether the conclusion derived by Burrows and Hayes (1996) on the momentum of the matter, neutrino, and pulsar is common among the asymmetric supernova explosion models. In particular, we have investigated the effects of the pulsar motion on the explosion, which are neglected in their study. As a result, it was suggested by the present discussions that the direction of the total momentum of the matter and neutrino is opposite to that of the momentum of the proto-neutron star in the asymmetric explosion models. This is because the center
of the explosion deviates from the center of the progenitor due to the pulsar motion. This picture is common among the asymmetric explosion models. So if we assume that the pulsar motion is caused by an asymmetric supernova explosion, the neutron star in SN 1987A, which has not been found yet, will be moving in the southern part of the remnant.

Moreover, the discovery of the mystery spot, separated from SN 1987A to the south region by 60 mas\(^[13]\), will support our conclusion. The existence of the bright spot on the south region will suggest that the existence of the strong shock wave on the south region, which is consistent with our conclusion (see Fig. 3 and 4). Here we have to comment on the second bright source which was reported to be detected on the north side of the remnant\(^[13]\). In their paper, it was concluded that the south spot has to be red-shifted and the north side spot has to be blue-shifted. It means that the south part of the ring around SN 1987A is nearer to us than the north part if these bright spots are ejected from the polar regions. This conclusion is opposite to the ones derived by the other many observations on the ring\(^[13,14]\). In this study, we have assumed that the north side of the ring is nearer to us and concluded that the neutron star born in SN 1987A is running in the south region of the remnant. However, if we believe the conclusion derived from the study of the second bright spot, our conclusion on the location of the neutron star is changed oppositely. We also give an additional comment on the mechanism of the jet-induced explosion which

\[ F_x(\cos\theta) / F \]

\[ \cos\theta \]

Fig. 4. Plot of the integrated forward component of the neutrino flux received on the back of the shock front as a function of the azimuthal angle $\cos \theta$. The integration of the flux is performed while NS is moving for $0 \leq t \leq t_{\text{max}}$. 
may explain the existence of the mystery spot. These are the promising ones which may explain the existence of the mystery spot, but there has to be a mechanism which breaks the symmetry with respect to the equatorial plane. This is because the mystery spot seems to exist on only one side as discussed above (at least, the second bright source on the north side is much fainter than the one on the south side). The effects of the pulsar motion on the asymmetric explosion discussed in this study explain such an asymmetry with respect to the equatorial plane naturally.

As mentioned above, the asymmetric explosion models can explain naturally the asymmetry of the iron line profiles and the existence of the mystery spot at the same time. Also it is suggested that the pulsar born in SN 1987A is running in the south part of the remnant. On the other hand, the simple binary models in which the explosion is assumed to be spherically symmetric can not explain these observations and predicts that the pulsar is located at the center of the remnant. Moreover, the only possible candidate as a companion of the progenitor of SN 1987A is a compact object (neutron star or black hole). In this case, two compact objects will be found in the remnant of SN 1987A. One of them is found at the center of the remnant, if we believe the simple binary models. In other words, if we can find one neutron star in SN 1987A on the south part of the remnant and deny the existence of the another compact object at the center of the remnant, asymmetric explosion models will be supported by the observation better than the binary models.

There may be a possibility to detect anisotropy in the young supernova ejecta using VLBA as long as it is located within $\sim 100$ Mpc. There may also be a possibility to find a pulsar in the ejecta. Increase of such observations will make it clear the relation between the asymmetric explosion and pulsar kick.

In the delayed explosion model, about 1% of the neutrino energy is transferred into the energy of the explosion. In this situation we should delicately treat the neutrino transfer, and we should discuss such a subtle problem carefully. However, to qualitatively understand the physical mechanism in the model, we think that the neutrino flux and its integrated value give the adequate informations. Therefore in this study, we have investigated the effects of the pulsar motion on the explosion using simple analysis. It will be necessary to check our conclusion by performing realistic numerical simulations in which the effects of neutrino heating and its back reaction are taken into consideration. In the present circumstances, the effects of the pulsar motion in the iron core have not been taken into account and we have treated the neutrino flux as a simple thermal radiation in the numerical simulations concerning with the collapse-driven supernova explosion. However, we can understand its importance easily because the neutron star can reach to the stalled shock front in $\sim 500$ ms, which is the dynamical timescale of the stalled shock wave, as long as the neutron star moves with the observed mean speed ($450 \text{ km s}^{-1}$). Even if we assume that the pulsar is accelerated constantly and its velocity becomes to be $450 \text{ km s}^{-1}$ in 500 ms, the location of the pulsar at $t = 500$ ms becomes to be $\sim 110$ km from the center. In fact, Burrows and Hayes (1996) reported that the pulsar gets the velocity $\sim 500 \text{ km s}^{-1}$ in 200 ms. We can easily guess that the effects of the pulsar motion on the dynamics of the explosion should be taken into consideration. Such calculations are now underway and we will report the results in the near future.
Acknowledgements

We would like to thank Dr. S. Yamada for useful discussions. This research has been supported in part by a Grant-in-Aid for the Center-of-Excellence (COE) Research (07CE2002) and for the Scientific Research Fund (199804502, 199908802) of the Ministry of Education, Science, Sports and Culture in Japan and by Japan Society for the Promotion of Science Postdoctoral Fellowships for Research Abroad.

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