A Unified Approach for Beam-Split Mitigation in Terahertz Wideband Hybrid Beamforming

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Abstract—The sixth generation (6G) wireless networks envision the deployment of terahertz (THz) band as one of the key enabling property thanks to its abundant bandwidth. However, the ultra-wide bandwidth in THz causes beam-split phenomenon due to the use of a single analog beamformer (AB). Specifically, beam-split makes different subcarriers to observe distinct directions since the same AB is adopted for all subcarriers. Previous works mostly employ additional hardware components, e.g., time-delay networks to mitigate beam-split by realizing virtual subcarrier-dependent ABs. This article introduces an efficient and unified approach, called beam-split-aware (BSA) hybrid beamforming. In particular, instead of virtually generating subcarrier-dependent ABs, a single AB is used and the effect of beam-split is computed and passed into the digital beamformers, which are subcarrier-dependent while maximizing spectral efficiency. Hence, the proposed BSA approach effectively mitigates the impact of beam-split and it can be applied to any hybrid beamforming architecture. Manifold optimization and orthogonal matching pursuit techniques are considered for the evaluation of the proposed approach in multi-user scenario. Numerical simulations show that significant performance improvement can be achieved as compared to the conventional techniques.

Index Terms—Terahertz, beam split, beam-squint hybrid beamforming, multi-user, massive MIMO.

I. INTRODUCTION

The sixth generation (6G) wireless networks envision to demonstrate revolutionary enhancement of the data rate (> 100Gb/s), extremely low latency (< 1 ms) and ultra reliability (99.999%) [1], [2]. Therefore, terahertz (THz) band (0.1–10 GHz) is one of the key components of 6 G thanks to abundant available bandwidth. Although demonstrating the aforementioned advantages, THz band faces several challenges (see the full list in [3], [4]) that should be taken into account accordingly. For instance, path loss is more severe than that of millimeter-wave (mm-Wave) bands (0.03–0.3 THz) due to spreading loss and molecular absorption. In order to overcome path loss, massive number of antennas are employed in multiple-input multiple output (MIMO) configuration for beamforming gain. Nevertheless, massive MIMO architectures require dedicated radio frequency (RF) chains for each antenna element. However this is costly. As a result, hybrid beamforming architectures involving a few digital beamformers and large number of analog beamformers (ABs) are employed [5], [6], [7].

The wideband massive MIMO systems employ subcarrier-dependent (SD) digital beamformers while the ABs are subcarrier-independent (SI) [8]. The single AB assumption causes the generated beams at different subcarriers look at different directions, which is called beam-split phenomenon since the same AB is adopted for all subcarriers. While beam-split does not have notable impact at mm-Wave, it causes significant performance degradation in THz band, at which the bandwidth is relatively wider (see [9] for field experiments). Meanwhile, beam-squint is the term commonly used for the same effect in mm-Wave, where the generated beams are squinted [10], [11]. In comparison, in case of beam-split, the main lobes of the array gain corresponding to the lowest and highest subcarriers totally split and do not overlap while the squinted beam can still cover the entire bandwidth [10], [12]. For instance, the angular deviation due to beam-split is approximately 6° (0.4°) for 0.3 THz with 30 GHz (60 GHz with 1 GHz) bandwidth, respectively for a broadband target (see [12, Fig. 11] for illustration). As a result, beam-split should be handled properly for reliable system performance.

Previous works to mitigate beam-split mostly rely on additional hardware components such as time-delay (TD) networks. Specifically, these approaches employ several TD components between the RF chains and the phase shifters to virtually realize SD ABs. For instance, the authors in [13] devised a wideband channel estimation and combining method based on orthogonal matching pursuit (OMP), wherein the SD dictionaries are used. In [14], a delay-phase precoding (DPP) approach was proposed for hybrid beamforming, which involves time-delay components. Also in [15], the unconstrained beamformers were optimized for all subcarriers, then hybrid beamformers are computed for each subcarrier, which requires SD ABs. The usage of TD components has high hardware cost and power consumption, especially at THz band. For instance, a single TD consumes approximately 100 mW, which is more than that of a phase shifter (40 mW) in THz [3]. A beam-broadening approach was proposed in [16], wherein a subarrayed configuration of the phase shifters across all subcarriers is employed to generate a beam with evenly distributed array gain across the whole frequency band. Hence, beamforming performance is limited since the generated beam is broadened due to the use of subarrayed phase shifters. By exploiting the full instantaneous channel state information (CSI), authors in [17] projected all the subcarriers to the central subcarrier, and constructed a common ABs for all subcarriers based on the channel covariance matrix. However, the use of covariance leads performance loss compared to instantaneous CSI-based beamforming. As a result, the existing solutions either have limited performance or necessary additional hardware components.

This article introduces a unified approach for beam-split mitigation, called beam-split-aware (BSA) hybrid beamforming. Different from the existing works realizing the SD AB via TD networks, a single SI AB is used in the proposed approach. The key idea is that the effect of beam-split is handled by passing it into the digital beamformers which are SD, and the beam-split is effectively corrected without using TD components. To that end, the effect of beam-split is computed for each AB. The phase deviations in the AB weights due to beam-split are computed. Then, the beam-split-corrected beamformer phases are obtained and used to construct a virtual SD beamformer, which is then, employed to obtain BSA digital beamformer. The proposed BSA hybrid beamforming technique suggests a unified approach and it is applicable to any architecture involving hybrid analog/digital beamforming. In this work, the proposed BSA technique is applied to state-of-the-art hybrid beamforming algorithms, i.e., OMP [6] and manifold optimization (MO) [18]. Through numerical simulations, we show that the proposed BSA approach achieves significant improvement in terms of spectral efficiency and effectively mitigates the impact of beam-split.

Notation: Throughout the article, (·)∗, (·)T and (·)H denote the conjugate, transpose and conjugate transpose operations, respectively.
For a matrix $A$, $A_{ij}$ and $A_{ik}$ correspond to the $(i, j)$-th entry and $k$-th column, while $A^H$ denotes the Moore-Penrose pseudo-inverse of $A$. A unit matrix of size $N$ is represented by $I_N$. $\Sigma(a) = \sin \frac{\pi a N}{N - 1}$ is the Dirichlet sinc function. $\ominus$ and $\otimes$ stand for the Khatri-Rao and Kronecker products, respectively. $P(A)$ computes the unwrapped phases of constant-modulus $a \in \mathbb{C}^N$ as $\mathcal{L}$ and $P^{-1}(P(a)) = \frac{1}{\sqrt{N}} a$. Finally, $\mathbb{E}\{\cdot\}$ represents the expectation operation.

II. SIGNAL MODEL

Consider a multi-user wideband THz system with massive MIMO configuration with hybrid analog/digital beamforming over $M$ subcarriers. The base station (BS) employs $N_T$ antennas and $N_R$ RF chains to serve $K$ $N_R$-antenna users to convey $N_S$ data streams. By taking into account a cheaper hardware at each user and low power consumption, it is assumed that the users employ only analog precoder, hence a single data stream, i.e., $N_S = 1$, is received in a single transmission block. In the downlink, the BS first applies SD baseband precoder $F_{BB}[m] = [f_{BB,1}[m], \ldots, f_{BB,K}[m]] \in \mathbb{C}^{N_R \times K}$ $(m \in M = \{1, \ldots, M\})$ to transmit the signal vector $s[m] = [s_1[m], \ldots, s_K[m]] \in \mathbb{C}^K$, where $\mathbb{E}\{s[m]s^H[m]\} = \frac{1}{\pi K}$ for average power $P$ by assuming equal power allocation among the users. Then, the SI analog precoder $F_{RF} \in \mathbb{C}^{N_S \times N_R}$ is used to steer the beams toward $K$ users. We assume that each user is served by a single analog precoder vector, hence we have $N_{RF} = K < N_R$. Since the analog precoders are realized with phase-shifters, they have constant-modulus constraint, i.e., $\|F_{RF}\| = \frac{1}{\sqrt{N_R}}$ as $i = 1, \ldots, N_R$ and $j = 1, \ldots, N_T$. We have also total power constraint as $\sum_{m=1}^{M} \|F_{RF}F_{BB}[m]\|^2_2 = K \leq P$. Then, the transmitted signal, i.e., $F_{RF}F_{BB}[m][s[m]]$, is received at the user as

$$y_k[m] = \mathbf{w}_{RF,k}^H \mathbf{H}_k[m] \sum_{i=1}^{K} F_{RF}F_{BB}[m][s[m]] + n_k[m],$$

where $n_k[m] \in \mathbb{C}^{N_S}$ is the complex additive white Gaussian noise (AWGN) vector with $n_k[m] \sim \mathcal{C}\mathcal{N}(0, \sigma_0^2 I_{N_S})$. In (1), $\mathbf{w}_{RF,k} \in \mathbb{C}^{N_R}$ represents the SI analog combiner of the $k$th user with $\|\mathbf{w}_{RF,k}\| = \sqrt{\frac{P}{N_R}}$ for $i = 1, \ldots, N_R$.

A. Channel Model

The THz channel can modeled as LoS-dominant NLoS paths due to limited reflected NLoS paths, which are about 10 dB weaker as compared to the LoS paths in THz transmission [12], [19], [20]. In addition, multipath channel models are also widely used, especially for indoor applications, wherein lower antenna gains can be tolerated [4], [21]. Nevertheless, the THz channel is sparser than the mmWave channel [4]. That is to say about 5 NLoS path components survive at 0.3 THz scenario [1]. Hence, we consider a general scenario, wherein the $N_{RX} \times N_{TX}$ channel matrix for the $k$th user at the $m$th subcarrier is represented by the combination of $L$ paths as

$$H_k[m] = \zeta \sum_{l=1}^{L} \alpha_{k,m,l} a_{RL}(\theta_{k,m,l}) a_{H}^H(\varphi_{k,m,l}) e^{-j2\pi \tau_{k,l}f_m},$$

where $\zeta = \sqrt{\frac{N_{TX} N_{RX}}{L}}$ and $\tau_{k,l}$ represents the time delay of the $l$th path corresponding to the array origin. $\alpha_{k,m,l} \in \mathbb{C}$ denotes the complex path gain and the expected value of its magnitude for the indoor THz multipath model is given by $\mathbb{E}\{|\alpha_{k,m,l}|\}^2 = \left(\frac{c_0}{4\pi f_d} \right)^2 e^{-k_0} f_m d$, where $c_0$ is speed of light, $f_m$ is the $m$th subcarrier frequency and $d$ denotes the transmission distance and $k_{abs}(f_m)$ is the frequency-dependent medium absorption coefficient [4], [19], [20]. Furthermore, $f_m = f_c + \frac{f_d}{2} \left(1 - \frac{M - 1}{M} \right)$ and $B$ is the bandwidth. The steering vector $a_{RL}(\theta_{k,m,l}) \in \mathbb{C}^{N_T}$ $(\theta_{k,m,l} \in \mathbb{C}^N)$ corresponds to the SD spatial direction-of-departure (DOD) (direction-of-arrival (DOA)) $\theta_{k,m,l}(\theta_{k,m,l})$, respectively. For a uniform linear array (ULA), the $n$th entry of $a_{RL}(\theta_{k,m,l})$ is

$$a_{RL}(\theta_{k,m,l})_n = \frac{1}{\sqrt{N_T}} \exp \{ -2\pi d n \},$$

where $\lambda_m = \frac{\lambda}{\sin \theta_m}$ represents the wavelength corresponding to $f_m$, $c_0$ is speed of light, and $d$ is inter-element distance, which is selected as half-wavelength, i.e., $d = \frac{\lambda}{2}$. The SD spatial DOA/DODs angles denote the directions that can be observed in spatial domain. Therefore, the spatial directions $\theta_{k,m,l}$ and $\varphi_{k,m,l}$ differentiate from the physical directions as $\phi_{k,l}$ and $\phi_{k,l}$ for DOA and DODs, respectively, where $\phi_{k,l} = \sin \theta_{k,l}$ and $\phi_{k,l} = \sin \varphi_{k,l}$. On the other hand, the beam-split-free channel is $\mathbf{H}_k[m] = \zeta \sum_{l=1}^{L} \alpha_{k,m,l} a_{RL}(\theta_{k,m,l}) a_{H}^H(\varphi_{k,l}) e^{-j2\pi \tau_{k,l}f_m}$, where we have

$$a_{RL}(\varphi_{k,l})_n = \frac{1}{\sqrt{N_T}} \exp \{ -j\pi n \}.$$
where \( \mu_m = \frac{\frac{1}{2}(f_m \phi - f_c \bar{\phi})}{\phi} \). The array gain in (8) implies that most of the power is focused only on a small portion of the beamspace due to the power-focusing capability of \( \Sigma(\alpha) \), which substantially reduces across the subcarriers as \( |f_m - f_c| \) increases. Furthermore, \( |\Sigma(\mu_m)|^2 \) gives a peak when \( \mu_m = 0 \), i.e., \( f_m \phi - f_c \bar{\phi} = 0 \), at \( \phi = \eta_m \phi \), which is the deviated direction in the beamspace.

### C. Problem Formulation

Our aim in this work is to efficiently mitigate the beam-splitt by using SI AB without any additional hardware components. The hybrid beamforming design problem maximizes the sum-rate of the multi-user system, which is defined as

\[
R = \sum_{m=1}^{M} \sum_{k=1}^{K} \log_2(1 + \gamma_k[m]),
\]

where

\[
\gamma_k[m] = \frac{\frac{1}{2} \mathbf{w}_{RF,k}^H \mathbf{H}_k[m] \mathbf{F}_{RF} \mathbf{F}_{BB,k}[m]^2}{\frac{1}{M} \sum_{m \neq k} \mathbf{w}_{RF,k}^H \mathbf{H}_k[m] \mathbf{F}_{RF} \mathbf{F}_{BB,k}[m]^2 + \sigma_n^2}.
\]

Then, the hybrid beamformer design problem can be formulated as

\[
\max_{\mathbf{F}_{RF}} \{ R \}
\]

subject to:

\[
||\mathbf{W}_{RF}||_2 = 1
\]

\[
\sum_{m=1}^{M} ||\mathbf{F}_{RF}\mathbf{F}_{BB}[m]||_F^2 = MK,
\]

where \( \mathbf{W}_{RF} = [\mathbf{w}_{RF,1}, \ldots, \mathbf{w}_{RF,K}] \in \mathbb{C}^{N_T \times K} \) includes the combiners of all users. Various methods have been proposed to solve (11) [7] such as OMP [6] and MO [8]. However, all of these algorithms fail to take into account the impact of beam-splat due to the usage of SI AB. In what follows, we introduce our BSA hybrid beamforming approach to efficiently mitigate beam-splitt by using SI AB without any additional hardware components.

### III. Proposed Method

The ABs in the problem formulated in (11) are SI while the aforementioned analysis on beam-splat implies that the ABs should be SD so that beam-splat can be eliminated. The problem in (11) with SD ABs can be recast as

\[
\max_{\{\mathbf{f}_{RF}[m]\}_{m \in M}, \{\mathbf{f}_{BB}[m]\}_{m \in M}, \{\mathbf{w}_{RF}[m]\}_{m \in M}} \tilde{R}
\]

subject to:

\[
||\mathbf{W}_{RF}[m]||_2 = 1
\]

\[
\sum_{m=1}^{M} ||\mathbf{F}_{RF}[m]\mathbf{F}_{BB}[m]||_F^2 = MK,
\]

where \( \mathbf{F}_{RF}[m] \in \mathbb{C}^{N_T \times N_{RF}} \) and \( \mathbf{W}_{RF}[m] = [\mathbf{w}_{RF,1}[m], \ldots, \mathbf{w}_{RF,K}[m]] \in \mathbb{C}^{N_T \times N_{RF}} \) are SD ABs. Also, for \( m_1, m_2 \in M \), (19) allows us to obtain the SD ABs from the SI AB \( \mathbf{F}_{RF} \) without solving (12) \( \forall m \in M \).

\[
R = \sum_{m=1}^{M} \sum_{k=1}^{K} \log_2(1 + \gamma_k[m])
\]

\[
\gamma_k[m] = \frac{\frac{1}{2} \mathbf{w}_{RF,k}^H \mathbf{H}_k[m] \mathbf{F}_{RF} \mathbf{F}_{BB,k}[m]^2}{\frac{1}{M} \sum_{m \neq k} \mathbf{w}_{RF,k}^H \mathbf{H}_k[m] \mathbf{F}_{RF} \mathbf{F}_{BB,k}[m]^2 + \sigma_n^2}.
\]

While the problem in (12) eliminates the beam-splat since SD ABs are used, it requires \((M - 1)N_T N_{RF}\) more phase-shifters (each of which consumes approximately 40 mW in THz [3]), hence it is cost-ineffective. Instead, we propose to use SI AB while the beam-splat problem is handled in the baseband beamformers, which are SD. In this regard, we define \( \mathbf{F}_{BB}[m] \in \mathbb{C}^{N_T \times N_{RF}} \) as the BSA digital beamformer in order to achieve SD beamforming performance that can be obtained in (12). Hence, we aim to match the proposed BSA hybrid beamformer \( \mathbf{F}_{BB}[m] \) with the SD hybrid beamformer \( \mathbf{F}_{RF}\mathbf{F}_{BB}[m] \) as

\[
\min_{\mathbf{F}_{BB}[m]} \| \mathbf{F}_{RF} \mathbf{F}_{BB}[m] - \mathbf{F}_{BB}[m] \|_F^2,
\]

for which \( \mathbf{F}_{BB}[m] \) can be obtained as

\[
\mathbf{F}_{BB}[m] = \mathbf{F}_{RF} \mathbf{F}_{RF}[m] \mathbf{F}_{BB}[m].
\]

As a result, the proposed BSA hybrid beamformer \( \mathbf{F}_{RF} \mathbf{F}_{BB}[m] \) can yield the performance of the SD hybrid beamformer \( \mathbf{F}_{RF}[m] \). While it may seem (15) still requires the solution of SD problem in (12) for \( m \in M \), in what follows, we show that \( \mathbf{F}_{BB}[m] \) can be easily constructed from \( \mathbf{F}_{RF} \) without solving (12), \( \forall m \in M \). To this end, we define the function \( \mathcal{P}(-) \) to compute the unwrapped angles of a vector quantity. For example, from (3), we have

\[
\mathcal{P}(\mathbf{a}_T(\varphi_{k,m}))) = \begin{bmatrix} 0, -j\xi_m \varphi_{k,m,1}, \ldots, -j\xi_m (N_T - 1) \varphi_{k,m,1} \end{bmatrix}^T,
\]

where \( \xi_m = 2\pi \frac{1}{\xi_m} \). Then, one can show that \( \mathbf{a}_T(\varphi_{k,m}) \) can be obtained from \( \mathbf{a}_T(\varphi_{k,m}) \) for any \( m \in M \) as

\[
\mathbf{a}_T(\varphi_{k,m}) = \mathcal{P}\left( \mathbf{a}_T(\varphi_{k,m}) \right) = \begin{bmatrix} 0, -j\xi_m \varphi_{k,m,1}, \ldots, -j\xi_m (N_T - 1) \varphi_{k,m,1} \end{bmatrix}^T
\]

which utilizes the fact that \( \varphi_{k,m,1} = \varphi_{k,m,1} \) from (5). Using (17), we can formulate the following useful expression for the relationship between the beamformers \( \mathbf{f}_{m_1} \) and \( \mathbf{f}_{m_2} \) as

\[
\mathbf{T}[m_1] = \frac{1}{N_T} \mathcal{P}^{-1}\left( \mathcal{P}(\mathbf{f}[m_1]) \right) \mathbf{f}[m_2],
\]

for \( m_1, m_2 \in M \). Then, the SD beamformer \( \mathbf{F}_{RF}[m] \) is

\[
\mathbf{F}_{RF}[m] = \frac{1}{N_T} \mathcal{P}^{-1}\left( \mathcal{P}(\mathbf{f}[m]) \mathbf{f}[m] \right).
\]
Algorithm 1: OMP-Based Hybrid Beamforming

Input: $D_P$, $D_W$, $F_{\text{opt}}[m]$, $W_{\text{opt}}[m]$, $\eta_m$, $m \in \mathcal{M}$
Output: $F_{\text{RF}}, F_{\text{BB}}[m]$.
1: $F_{\text{RF}} = \text{Empty}, F_{\text{res}}[m] = F_{\text{opt}}[m], W_{\text{RF}}[m] = W_{\text{opt}}[m]$.
2: $D_P[m] = \mathcal{P}(\mathcal{P}^{-1}(D_P))\eta_m, D_W[m] = \mathcal{P}(\mathcal{P}^{-1}(D_W))\eta_m$.
3: for $k = 1, \ldots, N_{\text{RF}}$ do
4: $\mathbf{g}_k[m] = f_{\text{res},k}[m] \otimes w_{\text{res},k}[m], m \in \mathcal{M}$.
5: $\{p^*, q^*\} = \argmax_{p,q \in \mathbb{N}^1} \sum_{m=1}^{M} |d_{p,q}^m[m]| |\mathbf{g}_k[m]|$,
where
6: $d_{p,q}^m[m] = [D_P[m]]_{p} \otimes [D_W[m]]_{q}$.
7: end for
8: $[H_{\text{eff}}[m]]_k = \mathbf{w}_{\text{RF,k}}^H \mathbf{H}_k[m] \mathbf{F}_{\text{RF}}, k \in \mathcal{K}$.
9: $F_{\text{BB}}[m] = [H_{\text{eff}}[m]], m \in \mathcal{M}$.
10: $[F_{\text{BB}}[m]]_k = \frac{F_{\text{RF}} F_{\text{BB}}[m]}{\|F_{\text{RF}} F_{\text{BB}}[m]\|_F}, k \in \mathcal{K}$.

Algorithm 2: BSA Hybrid Beamforming

Input: $F_{\text{RF}}, F_{\text{BB}}[m], \eta_m$, $m \in \mathcal{M}$.
Output: $F_{\text{BB}}[m]$.
1: $\mathbf{F}_{\text{RF}}[m] = \frac{1}{\sqrt{N_T}} \mathcal{P}(\eta_m \mathcal{P}^{-1}(\mathbf{F}_{\text{RF}}))$.
2: $F_{\text{BB}}[m] = F_{\text{RF}} F_{\text{BB}}[m] F_{\text{RF}}[m]$.
3: $[F_{\text{BB}}[m]]_k = \frac{F_{\text{RF}} F_{\text{BB}}[m]}{\|F_{\text{RF}} F_{\text{BB}}[m]\|_F}, k \in \mathcal{K}$.

A. BSA Hybrid Beamforming

We propose an OMP based approach, wherein the analog precoder $F_{\text{RF}}$ and combiner $W_{\text{RF}}$ are selected, respectively, from the dictionary matrices

\[
D_P = [\mathbf{a}_T(\phi_1), \ldots, \mathbf{a}_T(\phi_{N_T})] \in \mathbb{C}^{N_T \times N_T},
\]

and

\[
D_W = [\mathbf{a}_R(\phi_1), \ldots, \mathbf{a}_R(\phi_{N_W})] \in \mathbb{C}^{N_R \times N_W},
\]

for which the SD dictionaries can be defined as

\[
D_P = D_P^* \otimes D_W^*, \quad D_W = D_P^* \otimes D_W^*.
\]

Then, the analog precoder/combiner pairs for the $k$th user can be found as $[D_P^*]_{p^*, q^*}$ and $[D_W^*]_{p^*, q^*}$ via

\[
\{p^*, q^*\} = \arg\max_{p,q} \sum_{m=1}^{M} |d_{p,q}^m[m]| |\mathbf{g}_k[m]|,
\]

where

\[
d_{p,q}^m[m] = [D_P[m]]_{p} \otimes [D_W[m]]_{q},
\]

and

\[
\mathbf{g}_k[m] = f_{\text{res},k}[m] \otimes w_{\text{res},k}[m], m \in \mathcal{M},
\]

for which $f_{\text{res},k}[m]$ and $w_{\text{res},k}[m]$ are the $k$th column of the unconstrained precoder/combiners $F_{\text{opt}}[m]$ and $W_{\text{opt}}[m]$, respectively. In particular, the $k$th column of $F_{\text{opt}}[m]$ is $f_{\text{opt},k}[m]$, which can be obtained from the singular value decomposition (SVD) of $H_k[m]$ [6], [22]. Similarly, the unconstrained combiner $w_{\text{opt},k}[m]$ is defined as

\[
w_{\text{opt},k}[m] = \frac{1}{\mathcal{P}} \left( \mathbf{f}_{\text{opt},k}[m] \mathbf{H}_k[m] \mathbf{H}_k[m] \mathbf{f}_{\text{opt},k}[m] \right) + \frac{\sigma_n^2}{\mathcal{P}} \mathbf{f}_{\text{opt},k}[m] \mathbf{H}_k[m].
\]

B. Limitations and Required Conditions

The proposed BSA approach is applicable for any array size and geometry while it requires the knowledge of the beam-split ratio $\eta_m$ as well as the SI analog and SD digital beamformers as input in Algorithm 2. Due to the reduce dimension of the baseband beamformer (i.e., $N_{\text{RF}} < N_T$), the BSA approach does not completely mitigate beam-split. In other words, the beam-split can be fully mitigated only if $F_{\text{RF}} F_{\text{RF}}^H = I_{N_T}$, which requires $N_{\text{RF}} = N_T$. Nevertheless, the proposed approach provides satisfactory SE performance for a wide range of bandwidth (see Fig. 2). Furthermore, the proposed approach is only applicable to the hybrid analog/digital systems. Therefore, it cannot be applied to analog-only systems.

C. Computational Complexity

The complexity of the OMP-based hybrid beamforming approach in Algorithm 1 is similar to the conventional OMP techniques [8] and it is mainly due to the matrix multiplications in the steps 4 and 5 with the time complexity order of $O(M N_T N_{\text{RF}})$, $O(N_D N_W M N_T^2 N_{\text{RF}})$, respectively. Hence, the total complexity order is $O(K M N_T N_{\text{RF}} (1 + N_T N_{\text{RF}}))$. Similarly, the computational complexity order of the proposed BSA hybrid beamforming technique in Algorithm 2 is $O(N_T N_{\text{RF}} (N_T + N_{\text{RF}}) + N_T^2)$.

IV. NUMERICAL EXPERIMENTS

In this part, the performance of the proposed BSA hybrid beamforming approach is evaluated. Throughout the simulations, the THz system model is realized with $f_c = 300$ GHz, $B = 30$ GHz, $M = 128$, $N_T = 128$, $N_R = 8$, $N_{\text{RF}} = K = 8$, $L = 3$ [4], [19], [23]. The DOA/DOD angles are selected uniformly random from the interval $[-\pi, \pi]$, and 100 Monte Carlo experiments are conducted. We assume the channel matrix $H_k[m]$ is obtained via estimation techniques [10], [13], [24] prior to the beamforming stage.

Fig. 1 shows the hybrid beamforming performance of the proposed BSA approach in terms of spectral efficiency (SE) with respect to signal-to-noise ratio (SNR). In order to demonstrate the effectiveness
Fig. 1. Spectral efficiency versus SNR.

Fig. 2. Spectral efficiency versus system bandwidth, SNR = 0 dB, K = 8.

Fig. 3. Spectral efficiency versus number of users, SNR = 0 dB, B = 30 GHz.

We evaluate the SE performance of the competing algorithms in Fig. 3 with respect to number of users, K. We can see that the performance of all algorithms are degraded as K increases while the fully-digital beamformer provides an interference-free performance since it is computed per user [6]. The proposed BSA approach provides a superior performance than both DPP which relies on virtual SD ABs, and others (MO and OMP), which do not include beam-split-correction stage.

V. CONCLUSION

In this work, we introduced a unified hybrid beamforming technique to effectively compensate the impact of beam-split. The SI analog beamformer was first obtained via an OMP-based approach. Then, the SD analog beamformers were incorporated in order to convey the effect of beam-split into the baseband beamformers. The proposed BSA-OMP approach also performs better than DPP-based hybrid beamforming [14] which employs TD network to mitigate beam-split. In contrast, our BSA approach does not require TD network, hence, it is more hardware-efficient. Furthermore, it can be applied to any hybrid beamforming algorithm. The effectiveness of the proposed BSA approach is due to the fact that it can accurately compensate the impact of beam-split. This is done by conveying the beam-split effect from the SI AB to the SD digital beamformers. Hence, it can yield the performance of SD beamformer without the necessity of additional hardware components.

The proposed BSA hybrid beamforming technique is applicable to any architecture involving hybrid analog/digital beamforming, e.g., RIS-assisted systems [25], near-field hybrid beamforming [26], and integrated sensing and communications [3], [12]. In addition, for a massive-user scenario, wherein N_{RF} < K, the proposed approach can be applied to the symbol-by-symbol beamforming techniques [27].

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