Properties of $D_{s2}^*(2573)$ charmed-strange tensor meson

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Abstract

The mass and current coupling constant of the $D_{s2}^*(2573)$ charmed-strange meson is calculated in the framework of two-point QCD sum rule approach. Although the quantum numbers of this meson is not exactly known, its width and decay modes are consistent with $I(J^P) = 0(2^+)$, which we consider to write the interpolating current used in our calculations. Replacing the light strange quark with up or down quark we also compare the results with those of $D_2^*$ charmed tensor meson and estimate the order of SU(3) flavor symmetry violation.

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1 Introduction

During last few years many new particles have been discovered in different experiments. With increased running energies of colliders and improved sensitivity of detectors, more hadrons are expected to be observed. To better understand and analyze the experimental results, parallel theoretical and phenomenological studies on the spectroscopy and decay properties of newly discovered particles are needed. The LHCb Collaboration at CERN reported first observation of the $D_{s2}^*(2573)$ particle through the semileptonic $B_s^0 \rightarrow D_{s2}^{+} X \mu^- \bar{\nu}$ transition in 2011 [1]. This decay has an important contribution to the total branching ratio of the semileptonic $B_s^0$ decays, so its analysis helps us get more information about the semileptonic $B_s^0$ decays which is less known experimentally compared to lighter $B$ mesons.

Although the quantum numbers of the observed $D_{s2}^*(2573)$ particle is not exactly known, however, its width and decay modes favors the quantum number $I(J^P) = 0(2^+)$ [2]. In this article, we calculate the mass and current coupling constant of the $D_{s2}^*(2573)$ in the framework of two-point QCD sum rules considering it as a charmed-strange tensor meson. The interpolating currents of the tensor mesons contain derivatives, so we calculate the two-point correlation function first in coordinate space then transform calculations to the momentum space to apply Borel transformation and continuum subtraction in order to isolate the ground state particle from the higher states and continuum. For some experimental and theoretical works/reviews on the properties, structure and decay channels of charmed-strange mesons, see for instance [3–9] and references therein.

The outline of the article is as follows. Starting from an appreciate two-point correlation function, we derive QCD sum rules for the mass and current coupling constant of the $D_{s2}^*(2573)$ charmed-strange tensor meson in next section. In section 3, we numerically analyze the sum rules obtained in section 2 and obtain working regions for auxiliary Borel parameter and continuum threshold entered to calculations. Making use of the working regions for auxiliary parameters, we obtain the numerical values of the mass and decay constant of the tensor meson under consideration. Replacing the strange quark with the up or down quark we also find the masses and decay constant of the corresponding $\bar{d}(\bar{u})c$ system, by comparison of which we estimate the order of SU(3) flavor symmetry violation in the charmed tensor system.

2 Mass and current coupling of $D_{s2}^*(2573)$ charmed-strange tensor meson

Hadrons are formed in a range of energy very lower than the perturbative or asymptotic region, so to investigate their properties some non-perturbative approaches are required. Among non-perturbative methods the QCD sum rule [10] is one of the most attractive and applicable tools to hadron physics as it is free of any model dependent parameters and is based on QCD Lagrangian. According to the philosophy of this model, to calculate the masses and current coupling constant, we start with a two-point correlation function and calculate it once in terms of hadronic parameters called the physical or phenomenological side, and the other in terms of QCD parameters in deep Euclidean region via operator product expansion (OPE) called the QCD or theoretical side. The QCD sum rules for
the mass and current coupling constant are obtained matching both sides of the two-point correlation function under consideration. To stamp down the contribution of the higher states and continuum we apply Borel transformation to both sides of the acquired sum rules and use the quark-hadron duality assumption.

To derive the QCD sum rules for physical quantities under consideration, we start with the following two-point correlation function:

$$\Pi_{\mu\nu,\alpha\beta} = i \int d^4 x e^{i q (x - y)} \langle 0 \mid T [ j_{\mu\nu}(x) \bar{j}_{\alpha\beta}(y) ] \mid 0 \rangle,$$  \hspace{1cm} (1)

where $T$ is the time-ordering operator and $j_{\mu\nu}$ is the interpolating current of the $D^*_s(2573)$ charmed-strange tensor meson. Considering the quantum numbers of $D^*_s(2573)$ meson, its interpolating current in terms of quark fields can be written as

$$j_{\mu\nu}(x) = \frac{i}{2} \left[ \bar{s}(x) \gamma_\mu \overset{\leftrightarrow}{D}_\nu(x) c(x) + \bar{s}(x) \gamma_\nu \overset{\leftrightarrow}{D}_\mu(x) c(x) \right],$$  \hspace{1cm} (2)

where the two-side covariant derivative $\overset{\leftrightarrow}{D}_\mu(x)$ is defined as

$$\overset{\leftrightarrow}{D}_\mu(x) = \frac{1}{2} \left[ \overset{\rightarrow}{D}_\mu(x) - \overset{\leftarrow}{D}_\mu(x) \right],$$  \hspace{1cm} (3)

and

$$\overset{\rightarrow}{D}_\mu(x) = \overset{}{\nabla}_\mu(x) - ig A^a_\mu(x),$$ $$\overset{\leftarrow}{D}_\mu(x) = \overset{}{\nabla}_\mu(x) + ig A^a_\mu(x).$$  \hspace{1cm} (4)

Here $A^a_\mu(x)$ denote the external gluon fields. In the Fock-Schwinger gauge, where $x^\mu A^a_\mu(0) = 0$, the external gluon fields are expanded in terms of the gluon field strength tensor as

$$A^a_\mu(x) = \int_0^1 d\alpha x^\mu G^a_{\beta\mu}(\alpha x) = \frac{1}{2} x^\beta G^a_{\beta\mu}(0) + \frac{1}{3} x^\eta x^\beta D_\eta G^a_{\beta\mu}(0) + \cdots.$$  \hspace{1cm} (5)

Note that we consider the currents in the aforementioned correlation function at points $x$ and $y$, however, we have only integral over four-$x$. The interpolating current of the tensor meson contains derivatives with respect to the space-time. Hence, after applying derivatives we will set $y = 0$ then perform integral over four-$x$.

In the physical side, the correlation function in Eq.(1) is calculated by saturating it via a complete set of states with the quantum numbers of $D^*_s(2573)$. After isolating the ground state and performing the four-integral we get

$$\Pi_{\mu\nu,\alpha\beta} = \frac{\langle 0 \mid j_{\mu\nu}(0) \mid D^*_s(2573) \rangle \langle D^*_s(2573) \mid \bar{j}_{\alpha\beta}(0) \mid 0 \rangle}{(m_{D^*_s(2573)}^2 - q^2) + \cdots},$$  \hspace{1cm} (6)

where $\cdots$ symbolizes the contribution of higher states and the continuum. To proceed, we need to define the matrix element $\langle 0 \mid j_{\mu\nu}(0) \mid D^*_s(2573) \rangle$ in terms of current coupling constant $f_{D^*_s(2573)}$ and polarization tensor $\varepsilon_{\mu\nu}$:

$$\langle 0 \mid j_{\mu\nu}(0) \mid D^*_s(2573) \rangle = f_{D^*_s(2573)} m_{D^*_s(2573)}^3 \varepsilon_{\mu\nu}. $$  \hspace{1cm} (7)
Using Eq.(7) in Eq.(6) needs performing summation over polarization tensor, which is given as

\[ \varepsilon_{\mu\nu}\varepsilon^*_{\alpha\beta} = \frac{1}{2}\eta_{\mu\alpha}\eta_{\nu\beta} + \frac{1}{2}\eta_{\mu\beta}\eta_{\nu\alpha} - \frac{1}{3}\eta_{\mu\nu}\eta_{\alpha\beta}, \]

(8)

where

\[ \eta_{\mu\nu} = -g_{\mu\nu} + \frac{g_{\mu}q_{\nu} + g_{\nu}q_{\mu}}{m^2_D(2573)}. \]

(9)

As a result, for the final expression of the physical side, we get

\[ \Pi_{\mu\nu,\alpha\beta} = f^2 D^2(2573)m^6 D^2(2573) \left\{ \frac{1}{2}(g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) \right\} + \text{other structures} + \cdots \]

(10)

where we will choose the explicitly written structure to extract the QCD sum rules for the mass and current coupling constant of the tensor meson.

In QCD side, the correlation function in Eq.(1) is calculated in the deep Euclidean region where \( q^2 \ll 0 \), with the help of OPE where the short (perturbative) and long distance (non-perturbative) contributions are separated. The perturbative part is calculated using the perturbation theory, while the non-perturbative part is parameterized in terms of QCD parameters such as quarks masses, quarks and gluon condensates, etc. Therefore, any coefficient of the selected structure in QCD side can be written as a dispersion integral plus a non-perturbative part:

\[ \Pi(q^2) = \int ds \rho^{pert}(s) + \Pi^{non-pert}(q^2), \]

(11)

where the spectral density \( \rho^{pert}(s) \) is obtained from the imaginary of the perturbative contribution, i.e., \( \rho^{pert}(s) = \frac{1}{\pi}Im[\Pi^{pert}(s)] \).

Our main goal in the following is to calculate the spectral density \( \rho^{pert}(s) \) and the non-perturbative part \( \Pi^{non-pert}(q^2) \). Using the tensor current presented in Eq.(2) in the correlation function in Eq.(1) and contracting out all quark pairs via the Wick’s theorem, we obtain

\[ \Pi_{\mu\nu,\alpha\beta} = \frac{i}{4} \int d^4xe^{i(q-x)\cdot y} \left\{ Tr \left[ S_s(y-x)\gamma_\mu \overset{\leftrightarrow}{D}_\nu(x) \overset{\leftrightarrow}{D}_\beta(y)S_c(x-y)\gamma_\alpha \right] + \cdots \right\}. \]

(12)

To proceed, we need to know the expressions of the heavy and light quarks propagators, which are calculated in [11]. By ignoring from the gluon fields which have very small contributions to the mass and current coupling of the tensor meson (see also [12–14]), the explicit expressions of the heavy and light quarks propagators are given by

\[ S^c_{ij}(x-y) = \frac{i}{(2\pi)^4} \int d^4k e^{-ik\cdot(x-y)} \left\{ \frac{(k + m_c)}{(k^2 - m_c^2)} \delta_{ij} + \cdots \right\}, \]

(13)
and
\[
S^{ij}_s(x-y) = \frac{i}{2\pi^2(x-y)^4} \delta_{ij} - \frac{m_s}{4\pi^2(x-y)^2} \delta_{ij} - \frac{(ss)}{12} \left[ 1 - i \frac{m_s}{6} (x-y) \right] \delta_{ij}
\]
\[
- \frac{(x-y)^2}{192} m_0^2 (ss) \left[ 1 - i \frac{m_s}{6} (x-y) \right] \delta_{ij} + \cdots .
\]
(14)

The next step is to use the expressions of the quarks propagators and apply derivatives with respect to \(x\) and \(y\) in Eq.(12). As a result, after setting \(y = 0\), for the QCD side of the correlation function in coordinate space, we get
\[
\Pi_{\mu\nu,\alpha\beta} = \frac{N_c}{16} \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_c^2)} \int d^4x e^{i(q-k)\cdot x} \left\{ \frac{1}{2} \Gamma_{\mu\nu,\alpha\beta} | [\beta \leftrightarrow \alpha] \right. \\
+ \left[ \nu \leftrightarrow \mu \right] + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu] \right\},
\]
(15)

where \(N_c = 3\) is the color factor and the function \(\Gamma_{\mu\nu,\alpha\beta}\) is given by
\[
\Gamma_{\mu\nu,\alpha\beta} = k_{\nu} k_{\beta} \left[ \frac{i}{2\pi^2} \frac{\not{x}}{x^4} + \frac{m_s}{4\pi^2 x^2} + \left( 1 + \frac{im_s}{48} + \frac{x^2 m_0^2}{192} + \frac{ix^2 m_s m_0^2}{1152} \right) (ss) \right] \gamma_{\mu}(k + m_c) \gamma_{\alpha}
+ \left[ \nu \leftrightarrow \mu \right] + [\beta \leftrightarrow \alpha, \nu \leftrightarrow \mu] \right\}.
\]
(16)

After performing all traces in Eq.(15), in order to calculate the integrals, first we transform the terms containing \(\frac{1}{(x-y)^4}\) to the momentum space and replace \(x_{\mu} \rightarrow -i \frac{\partial}{\partial q_{\mu}}\). The integral over four-\(x\) gives us a Dirac Delta function, making use of which we perform the integral over four-\(k\). To perform the final integral over four-\(p\) we use the Feynman parametrization method and the relation
\[
\int d^4p \frac{(p^2)^\beta}{(p^2 + L)^\alpha} = \frac{i \pi^2 (-1)^{\beta-\alpha} \Gamma(\beta + 2) \Gamma(\alpha - \beta - 2)}{\Gamma(2) \Gamma(\alpha) [\Gamma(-L)^{\alpha-\beta-2}]}. \]
(17)

After lengthy calculations for the spectral density, we get
\[
\rho_{\text{pert}}(s) = N_c \frac{(m_c^2 - s)^3 (2m_c^4 + m_s^4 s + 10m_c m_s s - 3s^2)}{960 \pi^2 s^3}.
\]
(18)
For the non-perturbative part, we also obtain
\[
\Pi^{non-pert}(q^2) = m_0^2 \langle \bar{s}s \rangle \frac{(24m_c^2 - m_s^2 - 24m∗q^2 - 5m_s^2)}{1152(m_c^2 - q^2)^2}.
\] (19)

After acquiring the correlation function in both phenomenological and QCD sides, by the procedures mentioned in the beginning of this section, we obtain the following sum rule for the mass and current coupling of the \(D^∗_{s2}(2573)\) tensor meson:
\[
f^2_{D^∗_{s2}(2573)} e^{-m^2_{D^∗_{s2}(2573)}/M^2} = \frac{2}{m^6_{D^∗_{s2}(2573)}} \left( \int_{s_0}^{\infty} \frac{d\rho(s)}{s(M^2 - s)} \frac{e^{-s/M^2}}{M^2} + \hat{\Pi}^{non-pert}(M^2) \right),
\] (20)
where \(s_0\) is the continuum threshold and \(M^2\) is the Borel mass parameter. The function \(\hat{\Pi}^{non-pert}(M^2)\) in Borel scheme is obtained as
\[
\hat{\Pi}^{non-pert}(M^2) = m_0^2 \langle \bar{s}s \rangle \frac{(24M^2m_c + 5M^2m_s - 6m^2_c m_s)}{1152M^2} e^{-m^2_c/M^2}.
\] (21)

The mass of the \(D^∗_{s2}(2573)\) tensor meson alone is obtained from
\[
m^2_{D^∗_{s2}(2573)} = \frac{\int_{s_0}^{\infty} ds \frac{s}{(m_c + m_s)^2} \rho(s) e^{-s/M^2}}{\int_{s_0}^{\infty} d\rho(s) e^{-s/M^2}} + \frac{\partial}{\partial s} \frac{\hat{\Pi}^{non-pert}(M^2)}{\hat{\Pi}^{non-pert}(M^2)}.
\] (22)

3 Numerical results

In this section, we numerically analyze the sum rules obtained for the mass and current coupling constant of the \(D^∗_{s2}(2573)\) tensor meson in the previous section. For this aim we use some input parameters as \(m_c = (1.275 \pm 0.025) GeV\) [2], \(\langle \bar{s}s(1 GeV) \rangle = -0.8(0.24 \pm 0.01)^3 GeV^3\) [15] and \(m_0^2(1 GeV) = (0.8 \pm 0.2) GeV^2\) [16].

The sum rules for above mentioned physical quantities also contain two auxiliary parameters: the Borel parameter \(M^2\) and the continuum threshold \(s_0\) coming from the Borel transformation and the continuum subtraction, respectively. In the following, we shall find working regions of these parameters such that the results of the mass and current coupling show weak dependences on these auxiliary parameters according to the general criteria of the method. The continuum threshold \(s_0\) is not completely capricious, but it is correlated with the energy of the first excited state with the same quantum numbers. As a result, we choose \(s_0 = (10.0 \pm 0.5) GeV^2\) for the continuum threshold.

The working region for the Borel mass parameter is calculated demanding that not only the contributions of the higher state and continuum are stamped down but also the contribution of the higher order operators are very small. The later means that the series of sum rules for physical quantities are convergent and the perturbative part constitutes an important part of the whole contribution. In other words, the upper bound on the Borel parameter is found by demanding that
\[
\frac{\int_{s_{min}}^{s_0} \rho(s) e^{-s/M^2}}{\int_{s_{min}}^{\infty} \rho(s) e^{-s/M^2}} > 1/2,
\] (23)
Figure 1: The dependence of current coupling $f_{D_s^*(2573)}$ on Borel mass parameter $M^2$ at $s_0 = 10 \text{ GeV}^2$.

Figure 2: The dependence of mass of $D_s^*(2573)$ on Borel mass parameter $M^2$ at $s_0 = 10 \text{ GeV}^2$.

which leads to

$$M^2_{\text{max}} = 3 \text{ GeV}^2.$$  \hfill (24)
The lower bound on this parameter is obtained requiring that the contribution of the perturbative part exceeds the nonperturbative contributions. From this restriction we get

\[ M_{\text{min}}^2 = 1.5 \, \text{GeV}^2. \]  

We depict the dependence of the current coupling constant and mass of the tensor meson under consideration on Borel mass parameter at a fixed value of continuum threshold in figures 1 and 2. From these figures we see that the results weakly depend on the Borel mass parameter in its working region. Here, we would like to make the following comment. The above analyses have been done based on, so called, the standard procedure in QCD sum rule technique such that the quark-hadron duality assumption as a systematic error has been used and the continuum threshold has been taken independent of Borel mass parameter. However, as also stated in [17], the continuum threshold can depend on \( M^2 \). Hence the standard procedure does not render the realistic errors and, in fact, the actual errors should be large. Our numerical calculations show that taking the continuum threshold dependent on Borel mass parameter brings an extra systematic error of \( \%15 \), which we will add to our numerical values.

Making use of the working regions for auxiliary parameters and taking into account all systematic uncertainties, we obtain the numerical results of the mass and current coupling constant for \( D_{s2}^*(2573) \) tensor meson as presented in Table 1. We also compare our result on the mass with the existing experimental data which shows a good consistency. The errors quoted in our predictions belong to the uncertainties in determination of working regions for both auxiliary parameters, those existing in other inputs as well as systematic errors. Our result on the current coupling constant of the charmed strange \( D_{s2}^*(2573) \) tensor meson can be checked in future experiments.

\[
\begin{array}{|c|c|c|}
\hline
\text{Parameter} & \text{Present Work} & \text{Experiment [2]} \\
\hline
m_{D_{s2}^*(2573)} & (2549 \pm 440) \, \text{MeV} & (2571.9 \pm 0.8) \, \text{MeV} \\
\bar{f}_{D_{s2}^*(2573)} & 0.023 \pm 0.011 & - \\
\hline
\end{array}
\]

Table 1: Values for the mass and current coupling constant of the \( D_{s2}^*(2573) \) tensor meson.

Our final goal is to replace the strange quark with the up/down quark and estimate the order of SU(3) flavor violation. Our calculations show that this violation is maximally \( \%7 \) in the case of charmed tensor meson.

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