Polarization of Radiation from Planar Channeled Positrons

S. Bellucci

*INFN - Laboratori Nazionali di Frascati, 1-00044, Frascati (Rome), Italy

M. Shatnev

†NSC “Kharkov Institute of Physics and Technology”, 61108, Kharkov, Ukraine

The Stokes parameters have been found in the framework of quantum electrodynamics for the description of polarization of radiation emitted by relativistic positrons channeled between (110) planes in Si crystal. The degree of polarization, which is simply given by the contribution of channeling radiation, has been analyzed. Numerical calculation are presented for the frequencies that are most interesting for the sources of polarized high-energy photons.

Ключевые слова: Positrons Channeling in Crystals, Radiation from Channeled Positrons, Polarization of Radiation.
1. INTRODUCTION

Photon beams of maximum intensity and maximum polarization are needed for performing the nuclear experiments in the intermediate energy range from 10 MeV to 100 MeV. At the same time it is well known that in this range of energy the intensity of the radiation from 5 – 20 GeV channeled positrons and electrons is maximum [1–8]. The radiation is linearly polarized and emission occurs in a narrow cone centered about the relativistic velocity direction with the intensity that can be an order of magnitude larger than bremsstrahlung. Therefore it is interesting to study the polarization of radiation under channeling and to estimate a possibility of the creation of photon beams with high polarization degree which will be sufficient for performing the nuclear experiments in the intermediate energy range, using off-axis collimation of photons under relativistic charged particles channeling in the crystal. In addition to that, radiation emitted from a process at still higher energies - GeV-energy positrons undergoing planar channeling has attracted special attention in the physics community, because the model describing its production is very simple and the radiation itself is almost perfectly monochromatic.

For such radiation of soft photons one can use classical electrodynamics to describe the emission process [9–11]. But when the energy of incoming electrons or positrons rises to levels above a few GeV, the classical description of channeling radiation becomes invalid. We consider a relativistic charged particle directed into a crystal approximately parallel to one of the crystal planes. In our case for positively charged positrons, the channels consist of the ducts in between the crystal planes, and from the quantum mechanical point of view, the channel provides a potential well for the transverse one-dimensional particle motion, in which the particle is in a bound state. Its transitions to lower levels are accompanied by the emission of channeling radiation (CR) with frequencies related to the energy differences of the levels. The polarization properties of the radiation are completely described with a polarization matrix \( \rho_{ij} \) which may be expressed in terms of the Stokes parameters \( \xi_n \) \((n = 1, 2, 3)\):

\[
\rho_{ij} = \frac{1}{2} \begin{pmatrix}
1 + \xi_3 & \xi_1 - i\xi_2 \\
\xi_1 + i\xi_2 & 1 - \xi_3
\end{pmatrix}_{ij}
\]

\(i, j = x, y\).  \(1\)

The degree of linear polarization is \(\sqrt{\xi_1^2 + \xi_3^2}\), while the degree of circular polarization is defined by \(\xi_2\). The angle \(\beta\) defining the direction of the linear polarization in the plane
perpendicular to the direction of photon emission is determined by the following condition
tg(2β) = ξ_1/ξ_3 \[12\].

The aim of this work is to derive the formulae for the Stokes parameters of this radiation. We obtain the formulae for the corresponding Stokes parameters, characterizing the polarization properties of the CR from arbitrary polarized particles as the function of the set which gives the angular dependence of the polarization of the emitted radiation. The calculation of the CR process is carried out by using the rules of quantum electrodynamics. The following analysis utilizes the methods used in \[12, 13\] and is based on the approach which was developed in \[14, 15\]. The numerical results for the CR with the energy of photons ω = 59 MeV emitted by 14 GeV positrons channeled between (110) planes in Si are given.

2. CALCULATION OF PLANAR CR PROCESS

Planar channeling takes place in a one-dimensional potential i.e. a relativistic particle moving in a potential \( U(x) \) periodic in the \( x \) direction, which is normal to the channeling planes, is described by the time-independent Dirac equation. Separation of the wave function \( \Psi(\vec{r}) \), which is the solution of the equation, into large and small components

\[
\Psi(\vec{r}) = \begin{pmatrix} \varphi(\vec{r}) \\ \chi(\vec{r}) \end{pmatrix}
\]

leads to a Pauli-type equation for the large components \( \varphi(\vec{r}) \). Since the potential is independent of \( y \) and \( z \), the solution of this equation can be written in the form

\[
\varphi(\vec{r}) = \sqrt{\frac{E + m}{2E}} \exp(i\vec{p}_||\vec{r}_||)f(x)w,
\]

where \( w^*w = 1 \).

This putting allows us to transform a Pauli-type equation into a one-dimensional, relativistic Schrödinger equation for \( f(x) \) with a relativistic particle mass

\[
\left(-\frac{1}{2\gamma m} \frac{d^2}{dx^2} + U(x)\right)f_n(x) = \varepsilon_n f_n(x),
\]

where \( \gamma = E/m \) is the relativistic factor and \( \varepsilon_n \) is the transverse energy level of the particle. According to quantum electrodynamics, the matrix element for CR is given by

\[
M_{21} = \int \Psi_2^* \vec{\sigma} \vec{e}^* \exp(-i\vec{k}\vec{r}) \Psi_1 d\vec{r} = \int (\varphi_2^* \exp(-i\vec{k}\vec{r})\vec{\sigma} \vec{e}^* \chi_1 + \chi_2^* \exp(-i\vec{k}\vec{r})\vec{\sigma} \vec{e}^* \varphi_1) d\vec{r},
\]

\( \Psi_2 \) and \( \Psi_1 \) are wave functions for initial and final states, respectively.
where $\vec{\alpha}$ and $\vec{\sigma}$ are the Dirac and Pauli matrices respectively, $\vec{k}$ is the photon momentum, $\vec{e}$ is the photon polarization vector. In order to evaluate the matrix element for CR, we solve the wave equation (4) and find for the absolute square of the CR amplitude

$$|M_{21}|^2 = Cw_1^*\vec{e}_1 \left( \vec{A}^* - i \left[ \vec{B}^* \vec{\sigma} \right] \right) w_2 \times w_2^*\vec{e}_2^* \left( \vec{A}^* + i \left[ \vec{B}^* \vec{\sigma} \right] \right) w_1$$

where $C = (2\pi)^4 \frac{(E+m)(E'+m)}{4EE'} \delta^2 \left( \vec{p} - \vec{p}' - \vec{k} \right)$, and $\vec{A}, \vec{B}, I_1, I_2$ are giving by the following expressions

$$A_x = 2I_2 (1 + \frac{\omega^2 E'}{2E^2}), \quad A_y = 0, \quad A_z = 2I_1 (1 + \frac{\omega^2 E}{2E^2}),$$

$$B_x = \frac{\omega E}{E'} (\vec{\theta} \cdot I_1 \cos \varphi - I_2), \quad B_y = \frac{\omega}{E'} \vec{\theta} \cdot I_1 (1 + \frac{\omega E}{2E^2}) \sin \varphi, \quad B_z = \frac{\omega E}{E'} \cdot \frac{m}{E} \cdot I_1,$$

$$I_1 = \int \exp(-ik_xx) \cdot f_2^*(x) \cdot f_1(x) dx, \quad I_2 = -i \frac{\omega}{E} \int \exp(-ik_xx) \cdot f_2^*(x) \frac{d f_1(x)}{dx} dx.$$

### 3. THE STOKES PARAMETERS FOR CHANNELING RADIATION

In our analysis we use the condition $\omega \ll E$, which is correct for this CR case. We also do not take into account here the interaction between the particle’s spin and the potential of planes. For the description of polarization, we use here the set of vectors $\{\vec{e}_1, \vec{e}_2, \vec{n}\}$, which are used in [12] and we give it in terms of the vectors $\vec{p}_1 = \vec{p} - \vec{n}(\vec{n} \cdot \vec{p})$, $\vec{p}'_1 = \vec{p}' - \vec{n}(\vec{n} \cdot \vec{p}')$ ($\vec{p}, \vec{p}'$ are the momentum of the particle before and after radiation respectively, and $\vec{n}$ is the direction in which photon is emitted) in the next form

$$\vec{e}_1 = \frac{\vec{p}_1}{|\vec{p}_1|}, \quad \vec{e}_2 = \frac{\vec{p}_1^2 \vec{p}_1' - \vec{p}_1 (\vec{p}_1 \cdot \vec{p}_1')}{|\vec{p}_1| \sqrt{\vec{p}_1^2 \vec{p}_1'^2 - (\vec{p}_1 \cdot \vec{p}_1')^2}}.$$

The set of vectors $\{\vec{e}_1, \vec{e}_2, \vec{n}\}$ forms an orthogonal basis and the vector $\vec{e}_1$ lies in the radiation plane ($\vec{k}, \vec{p}$). These vectors are related with $\theta, \varphi$ (spherical coordinates of the system in which the spectrum and angular characteristics are calculated; here a relativistic particle moving along $z$ - direction and azimuthal angle $\varphi$ is counted out from $x$ - direction, which is normal to the channeling planes; $\theta \ll 1$; ) by the next relations

$$\vec{e}_1 = (-\cos \varphi, -\sin \varphi, \theta), \quad \vec{e}_2 = (\sin \varphi, -\cos \varphi, 0), \quad \vec{n} = (\theta \cos \varphi, \theta \sin \varphi, \cos \theta).$$
An arbitrary vector \( \vec{R}=(R_x, R_y, R_z) \) in the coordinate system \( \{\vec{e}_1, \vec{e}_2, \vec{n}\} \) is written in the form

\[
R_1 \approx -R_x \cos \varphi - R_y \sin \varphi + R_z \theta, \quad R_2 \approx R_x \sin \varphi - R_y \cos \varphi, \\
R_3 \approx R_x \theta \cos \varphi + R_y \theta \sin \varphi + R_z. \tag{10}
\]

Introducing the density matrices for relativistic particle and photon and after corresponding calculations we find the next general expressions of the Stokes parameters for the outgoing photon:

\[
\begin{align*}
\xi_1 &= (8/\Sigma L^e) (1 + \omega/E')(\theta \cdot Re I_1 \cdot I_2^* - |I_2|^2 \cos \varphi) \cdot \sin \varphi, \\
\xi_2 &= \zeta_3 (8/\Sigma L^e) (2\omega/E') (1 + 2\omega/E') 2\theta \times Re(I_1^* I_2) \cos \varphi \\
&\quad - \zeta_3 (8/\Sigma L^e) (2\omega/E') (1 + 2\omega/E') |I_1|^2 \theta^2 - \zeta_3 (8/\Sigma L^e) (2\omega/E') (1 + 2\omega/E') |I_2|^2 - \\
&\quad - \zeta_3 (8/\Sigma L^e) \left(\omega^2 m^2/E^2 E^2\right) |I_1|^2 - \zeta_2 (8/\Sigma L^e) (2\omega m/E'E) Re(I_1^* I_2) \sin \varphi - \\
&\quad - \zeta_1 (8/\Sigma L^e) (2\omega m/E'E') |I_1|^2 \theta + \zeta_1 (8/\Sigma L^e) (2\omega m/E'E) Re(I_1^* I_2) \cos \varphi, \\
\xi_3 &= (4/\Sigma L^e) (1 + 2\omega/E') |I_1|^2 \theta^2 + (4/\Sigma L^e) (1 + 2\omega/E') |I_1|^2 \cos 2\varphi - \\
&\quad -(4/\Sigma L^e) (1 + 2\omega/E') 2\theta Re(I_1^* I_2) \cos \varphi,
\end{align*}
\]

where \( E \) and \( E' = E - \omega \) are the energy of relativistic particle before and after radiation respectively, \( m \) is the electron mass, \( \omega \) is the energy of emitted photon, \( \zeta_1, \zeta_2, \zeta_3 \) are the components of the unit spin vector \( \vec{\xi} \) of the initial particle given in the coordinate system \( \{\vec{e}_1, \vec{e}_2, \vec{n}\} \). A normalisation factor \( \Sigma L^e \) is found by the formula

\[
\Sigma L^e = 4 \left(1 + \omega/E' + \omega^2/2E'^2\right) \theta^2 |I_1|^2 + 4 \left(1 + \omega/E' + \omega^2/2E'^2\right) |I_2|^2 - \\
8 \left(1 + \omega/E' + \omega^2/2E'^2\right) \theta Re(I_1 \times I_2^*) \cos \varphi + \left(\omega^2 m^2/2E^2 E^2\right) |I_1|^2 
\]

We can obtain from the conservation laws of energy and momentum the next relation between the direction of \( \vec{k} \), the difference of transverse energies before and after radiation \((\varepsilon_n - \varepsilon_{n'})\), and the radiation frequency \( \omega \):

\[
\theta^2 = \frac{2E(E - \omega)(\varepsilon_n - \varepsilon_{n'}) - m^2 \omega}{E\omega(E - \omega \cos^2 \varphi)}. \tag{13}
\]

It may be shown, that in the cases of photons with energy \( \omega \ll E \) there is the following relation for integrals \( I_1, I_2 \):

\[
I_2 = I_1 \left(\frac{\varepsilon_n - \varepsilon_{n'}}{k_x} + \frac{k_x}{2E'}\right). \tag{14}
\]
Then, using (13), the last expression may be written in the form
\[
I_2 = I_1 \frac{\gamma^{-2} + \theta^2(1 - \frac{\omega^2}{E^2} \cos^2 \varphi)}{2\theta(1 - \omega/E) \cos \varphi}.
\] (15)

This allows us to except \(I_1, I_2\) from (12), and the polarization of CR is independent on the planar-continuum potential.

4. CONCLUSION

The degree of circular polarization \(\xi_2\) of Eq. (11) shows that only polarized positrons produce circularly polarized CR and it is proportional to \(\omega/E\). In general, the circular polarization from longitudinally polarized particle is considerably greater than from transversely polarized particle exactly in the same way as it occurs in the case of bremsstrahlung. The linear polarization of CR is not dependent upon the positron spin as it follows from (11), and its degree is given by
\[
P = \sqrt{\xi_1^2 + \xi_3^2}.
\] (16)

If we substitute into (11) the expressions (12) for \(\Sigma L^e\) and (15) for \(I_2\), we can obtain final analytical expressions for the Stokes parameters, and taking (16) into consideration, we can find the degree of linear polarization. These rather complicated formulae are not given here. The results of the numerical calculations under these formulae for \(E = 14 GeV\) and \(\omega = 59 MeV\) are presented in figs. 1 and 2. Fig.1 shows calculated degree of linear polarization \(P(\theta, \varphi)\) of CR as a function of the polar angle \(\theta\) and the azimuthal angle \(\varphi\), with the energy of photons \(\omega = 59 MeV\) from 14 GeV positrons channeled between (110) planes in silicon. Note that in our calculations \(0 \leq \theta \leq 4\gamma^{-1} \ll 1\) at all azimuths i.e. the photons are emitted in the forward direction. As can be seen from Fig.1, the degree of linear polarization of planar CR is almost completely linearly polarized in the direction normal to the crystal planes. Fig.2 illustrates calculated degree of circular polarization \(\xi_2(\theta, \varphi)\), which is \(0.426\% \div 0.429\%\), for various values of \(\theta\) and \(\varphi\), when channeled positrons are completely polarized antiparallel to the direction of the original beam \((\zeta_3 = -1)\).

Thus, we have shown that the channeling radiation emitted in the forward direction by planar channeled positrons is completely linearly polarized. It is important for practical
Fig. 1. Calculated degree of linear polarization $P(\theta, \varphi)$ of CR as a function of the polar angle $\theta$ and the azimuthal angle $\varphi$, with the energy of photons $\omega = 59$ MeV from 14 GeV positrons channeling in Si (110), $T = 293^\circ$ K.

Fig. 2. Calculated degree of circular polarization $\xi_2(\theta, \varphi)$ of CR as a function of the polar angle $\theta$ and the azimuthal angle $\varphi$, with the energy of photons $\omega = 59$ MeV from 14 GeV positrons channeling in Si (110), $T = 293^\circ$ K, 

$$\left(\zeta_1 = \zeta_2 = 0, \zeta_3 = -1\right).$$

Applications of channeling radiation in photonuclear physics, and developed computer simulation method will allow one to get the CR beam parameters which are needed for performing the nuclear experiments in the intermediate energy range from 10 MeV to 100 MeV.

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