The Sandor-Smarandache function with a prime factor
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ABSTRACT

The Sandor-Smarandache function, denoted by SS(n), is a newly-introduced Smarandache-type arithmetic function. This paper focuses on the functions SS(30p), SS(60p), SS(210p), SS(420p) and SS(840p), where p (≥ 2) is a prime. At the end of the paper, four tables, giving the values of SS(30p), SS(60p), SS(210p) and SS(420p) for the first 200 primes, calculated on a computer, are given.

Introduction

The Sandor-Smarandache function, proposed by Sandor (2001), is denoted by SS(n) and is defined as follows: For n ≥ 7,

$$SS(n) = \max \left\{ k \in \mathbb{N} : 1 \leq k \leq n-2, \ n \text{ divides } \binom{n}{k} \right\}, \quad (1)$$

where by convention,

$$SS(1) = 1, \ SS(2) = 1, \ SS(6) = 1. \quad (2)$$

Let, for 0 ≤ k ≤ n,

$$C(n, k) = \binom{n}{k} = \frac{n(n-1)(n-2) \ldots (n-k+1)}{k!}. \quad (3)$$

Then, the problem of finding SS(n) may be stated as follows: Given an integer n (≥ 7), find the minimum integer k such that k! divides the number \((n-1)(n-2)\ldots(n-k+1)\), where 1 ≤ k ≤ n - 2. With this minimum k, SS(n) = n−k.

Thus, to find SS(8), note that 2! does not divide 7, but 3! = 2 × 3 divides 7 × 6. Hence, the minimum k such that k! divides 7 × 6 is 3; consequently, SS(8) = 8−3= 5.

An extensive study of the function was made by Majumdar (2018). Later, the problem was studied, to some extent, by Majumdar (2019), Islam et al. (2021), Majumdar and Ahmed (2021), Islam et al. (2021), and Islam et al. (2022). The following properties are known about SS(n).

Lemma 1: SS(n) = n − 2 if and only if n (≥ 3) is an odd integer.

Lemma 2: SS(n) = n − 3 if and only if n (≥ 4) is an even integer, not divisible by 3.

From Lemma 1 and Lemma 2, it follows that SS(n) ≤ n − 4 if n is of the form n = 6m, m ≥ 1 being any integer.

This paper considers functions of the forms SS(30p), SS(60p), and SS(210p), where p is a prime. This is done in the next section. In the analysis of the problem, the following results would be needed.

Lemma 3: Let a, b, and c be any three integers. The linear Diophantine equation \(ax + by = c\) has a solution if and only if \(d = \gcd(a, b)\) divides c. Moreover, if \((x_0, y_0)\) is a solution, then the general solution is given parametrically by \(x = x_0 + \left(\frac{b}{d}\right)t\), \(y = y_0 + \left(-\frac{a}{d}\right)t\) for any integer t.

Proof: See, for example, Gioia (2001, Theorem 12.2).

Lemma 4: For any integer m (≥ 1), the product of m consecutive integers is divisible by m!.

Proof: See Hardy and Wright (2002, Theorem 74).
Lemma 3 gives the complete solution of the linear Diophantine equation of the form \( ax + by = c \). Recall that a Diophantine equation involves two or more variables for which positive integer solutions are required.

Main results
First, the following result is proved, which gives an explicit form of \( SS(30p) \).

**Lemma 5:** Let \( p \geq 2 \) be a prime. Then,

\[
SS(30p) = \begin{cases} 
30p - 4, & \text{if } p = 4s + 3, \ s \geq 0 \\
30p - 7, & \text{otherwise}
\end{cases}
\]

**Proof:** Consider the following expression:

\[
C(30p, 4) = 30p \left[ \frac{(30p-1)(15p-1)(10p-1)}{4} \right].
\]

Now, the problem is to find the condition such that the term inside the square bracket is an integer. In other words, the problem is to find the condition on \( p \) such that the term inside the square bracket is an integer. Now, \( p \) may be of one of the two forms, namely, \( p = 4s + 3 \) (for some integer \( s \geq 0 \)), and \( p = 4t + 1 \) (for some integer \( t \geq 1 \)). If \( p = 4s + 3 \), then

\[
15p - 1 = 4(15s + 11),
\]

This shows that 4 divides \( 15p - 1 \), so the term inside the square bracket is an integer.

To complete the proof, consider the case when \( p = 4t + 1 \). Note that, in this case, 4 does not divide \( 15p - 1 \). The expression

\[
C(30p, 5) = 30p \left[ \frac{(30p-1)(15p-1)(10p-1)(15p-2)}{2 \times 5} \right]
\]

shows \( SS(30p) \neq 30p - 5 \) for any prime \( p \geq 2 \). Also, from the expression for \( C(30p, 6) \)

\[
C(30p, 6) = 30p \left[ \frac{(30p-1)(15p-1)(10p-1)(15p-2)(6p-1)}{3 \times 4} \right]
\]

it follows that \( SS(30p) \neq 30p - 6 \) for any prime \( p \geq 3 \).

Now, consider the expression

\[
C(30p, 7) = 30p \left[ \frac{(30p-1)(15p-1)(10p-1)(15p-2)(6p-1)(5p-1)}{2 \times 7} \right].
\]

Here, one of the numbers, \( 15p - 2 \) and \( 15p - 1 \), is even, depending on whether \( p = 2 \) or \( p \) is odd. Also, \( p \neq 7 \) (since by part (1) of the lemma, \( SS(210) = 206 \)). Thus, the term inside the square bracket is an integer by virtue of Lemma 4. All these establish the lemma.

The lemma below finds \( SS(60p) \).

**Lemma 6:** Let \( p \geq 2 \) be a prime. Then,

\[
SS(60p) = \begin{cases} 
30p - 6, & \text{if } p = 6s + 5, \ s \geq 0 \\
60p - 7, & \text{if } p = 6t + 1, \ t \geq 2
\end{cases}
\]

**Proof:** Consider the following expression:

\[
C(60p, 4) = 60p \left[ \frac{(60p-1)(30p-1)(20p-1)}{4} \right].
\]

Clearly, the numerator of the term inside the square bracket is not divisible by 4. Also, the expression

\[
C(60p, 5) = 60p \left[ \frac{(60p-1)(30p-1)(20p-1)(15p-1)}{5} \right]
\]

shows that the term inside the square bracket cannot be an integer. Thus, for any prime \( p \),

\( SS(60p) \neq 60p - 4 \), \( SS(60p) \neq 60p - 5 \). Now, consider the expression:

\[
C(60p, 6) = 60p \left[ \frac{(60p-1)(30p-1)(20p-1)(15p-1)(12p-1)}{6} \right].
\]

Note that, \( p \) is either of the form \( p = 6s + 5 \) (for some integer \( s \geq 0 \)), or it is of the form \( p = 6t + 1 \) (for some integer \( t \geq 0 \)). With \( p = 6s + 5 \),

\[
20p - 1 = 3(40s + 33),
\]

so that \( 20p - 1 \) is divisible by 3; also, with this \( p \), \( 15p - 1 \) is even. Thus, the term inside the square bracket is an integer.

Next, consider the following expression:

\[
C(60p, 7) = 60p \left[ \frac{(60p-1)(30p-1)(20p-1)(15p-1)(12p-1)(10p-1)}{7} \right].
\]

Here, by Lemma 4, the term inside the square bracket is an integer if and only if \( p \neq 7 \). All these complete the proof of the lemma.

It may be mentioned here that, in Lemma 6, \( p \) can be any prime except 7. Thus, Lemma 6 is supplemented by the value \( SS(420) = 412 \).

The next lemma deals with \( SS(210p) \).

**Lemma 7:** Let \( p \geq 2 \) be a prime. Then,
Next, consider the expression below:

$$C(210p,4) = 210p \left[ \frac{(210p-1)(105p-1)(70p-1)}{4} \right]$$. 

Note that, if $p = 4s + 1$, then $105p - 1 = 4(105s + 26)$, this shows that $105p - 1$ is divisible by 4, so the term inside the square bracket is an integer. This establishes part (1) of the lemma.

Next, let $p = 4t + 3$. The expression

$$C(210p,5) = 210p \left[ \frac{(210p-1)(105p-1)(70p-1)(105p-2)}{2\times5} \right]$$

shows that $SS(210p) \neq 210p - 5$ for any prime $p$, from the expression

$$C(210p,6) = 210p \left[ \frac{(210p-1)(105p-1)(70p-1)(105p-2)(42p-1)}{2\times6} \right]$$

(since 4 divides neither $105p - 1$ nor $105p - 2$) it follows that $SS(210p) \neq 210p - 6$ for any prime $p$, and the expression

$$C(210p,7) = 210p \left[ \frac{(210p-1)(105p-1)(70p-1)(105p-2)}{2\times7} \times \frac{(105p-2)(42p-1)(35p-1)}{16} \right]$$

shows that $SS(210p) \neq 210p - 7$ for any prime $p$, since by Lemma 4, the numerator of the term inside the square bracket is not divisible by 7. So, consider

$$C(210p,8) = 210p \left[ \frac{(210p-1)(105p-1)(70p-1)}{16} \times \frac{(105p-2)(42p-1)(35p-1)(30p-1)}{16} \right]$$.

If $p = 8s + 3$, then $105p - 1 = 2(420s + 157), 35p - 1 = 8(35s + 13)$, so that, the term inside the square bracket is an integer if $p = 8s + 3$.

Next, consider the expression:

$$SS(210p) = \begin{cases} 210p - 4, & \text{if } p = 4s + 1, \ s \geq 1 \\ 210p - 8, & \text{if } p = 8t + 3, \ t \geq 0 \\ 210p - 9, & \text{if } p = 72u + 31, \ u \geq 0 \\ 210p - 11, & \text{otherwise} \\ \end{cases}$$

Proof: Consider the expression below:

$$C(210p,9) = 210p \left[ \frac{(210p-1)(105p-1)(70p-1)}{8\times9} \times \frac{(105p-2)(42p-1)(35p-1)(30p-1)(105p-4)}{16} \right]$$.

Now, the problem is to find the condition such that the term inside the square bracket is an integer. Looking at the terms in the numerator, it is clear that, one possibility is that 4 divides $35p - 1$ (in which case, $105p - 1$ is divisible by 2) and 9 divides $70p - 1$.

By inspection, it is found that, when $p = 72s + 31$, then

$$35p - 1 = 4(630s + 271),$$

$$70p - 1 = 9(560s + 241),$$

so that $(105p - 1)(35p - 1)(70p - 1)$ is divisible by 72. The second possibility is that 36 divides $35p - 1$.

With $p = 72s + 71$,

$$35p - 1 = 36(70r + 69),$$

so that $(105p - 1)(35p - 1)$ is divisible by 72. All these prove part (2) of the lemma.

Next, consider the expression below:

$$C(210p,10) = 210p \left[ \frac{(210p-1)(105p-1)(70p-1)(105p-2)}{3\times5\times6} \times \frac{(42p-1)(35p-1)(30p-1)(105p-4)(70p-3)}{16} \right]$$.

Here, in order that the term inside the square bracket is an integer, a necessary condition is that $42p - 1$ must be divisible by 5. This leads to the Diophantine equation $42p - 1 = 5x, \ x \geq 0$ (see Lemma 3). The second condition that must be satisfied is that $35p - 1$ must be divisible by 8.

Since, $35p - 1 = 175s + 104$, it follows that $x = 8$, so that $p = 40s + 3$, which violates part (2) of the lemma.

Finally, consider the following expression for $C(210p,11)$:

$$210p \left[ \frac{(210p-1)(105p-1)(70p-1)(105p-2)(42p-1)}{8\times3\times41} \times \frac{(35p-1)(30p-1)(105p-4)(70p-3)(21p-1)}{16} \right]$$.

Now, by Lemma 4, $(70p - 1)(35p - 1)(70p - 3)$ is divisible by 3. Also, it may easily be verified that $(105p - 1)(35p - 1)(21p - 1)$ is divisible by 8 if $p$ is either of the form $p = 4s + 1$ or of the form $p = 4t + 3$. 


Moreover, \( p \neq 11 \). Hence, the term inside the square bracket is an integer, which was intended to prove.

The lemma below deals with \( SS(420p) \).

**Lemma 8:** Let \( p \geq 2 \) be a prime. Then

\[
420p - 6, \quad \text{if } p = 6s + 5, \ s \geq 0 \\
420p - 8, \quad \text{if } p = 8t + 1, \ t \neq 3x + 2 \\
SS(420p) = \begin{cases} 
420p - 9, & \text{if } p = 18u + 13, \ u \neq 4y + 2 \\
420p - 10, & \text{if } p = 40v + 29, \ v \neq 3a, v \neq 9b + 5 \\
420p - 11, & \text{otherwise}
\end{cases}
\]

**Proof:** The expressions

\[
C(420p, 4) = 420p \left\lfloor \frac{(420p-1)(210p-1)(140p-1)}{4} \right\rfloor \\
C(420p, 5) = 420p \left\lfloor \frac{(420p-1)(210p-1)(140p-1)(105p-1)}{5} \right\rfloor
\]

show that, for any prime \( p \),

\[ SS(420p) \neq 420p - 4, \ SS(420p) \neq 420p - 5. \]

So, consider the expression:

\[
C(420p, 6) = 420p \left\lfloor \frac{(420p-1)(210p-1)(140p-1)(105p-1)(84p-1)}{6} \right\rfloor
\]

Here so that the term inside the square bracket is an integer, \( p \) must be odd, and 3 must divide \( 140p - 1 \). Now, the solution of the Diophantine equation \( 140p - 1 = 3 \alpha \) is \( p = 3x + 2 \). To guarantee that \( p \) is odd, \( x \) must be odd. Therefore, by writing \( x = 2t + 1 \), the desired expression of \( p \) is obtained.

The expression

\[
C(420p, 7) = 420p \left\lfloor \frac{(420p-1)(210p-1)(140p-1)}{7} \right\rfloor \times \\
(105p-1)(84p-1)(70p-1)
\]

shows \( SS(420p) \neq 420p - 7 \) for any prime \( p \). So, consider

\[
C(420p, 8) = 420p \left\lfloor \frac{(420p-1)(210p-1)(140p-1)}{8} \right\rfloor \times \\
(105p-1)(84p-1)(70p-1)(60p-1)
\]

Here, the term inside the square bracket is an integer if and only if 8 divides \( 105p - 1 \). Thus, \( p \) must satisfy the equation \( 105p - 1 = 8 \alpha \), with the solution \( p = 8t + 1 \ (t \geq 2 \text{ being any integer}) \). Now, considering the Diophantine equation \( 8t + 1 = 6a + 5 \), using Lemma 3, the solution is found to be \( t = 3x + 2 \ (x \geq 0 \text{ being any integer}) \).

Next, consider the expression:

\[
C(420p, 9) = 420p \left\lfloor \frac{(420p-1)(210p-1)(140p-1)}{9} \right\rfloor \times \\
(105p-1)(84p-1)(70p-1)(60p-1)(105p-2)
\]

Now, note that, one of 105p - 1 and 105p - 2 is even. Thus, the term inside the square bracket is an integer if and only if 9 divides 70p - 1. This leads to the Diophantine equation \( 70p - 1 = 9 \alpha \), whose solution is \( p = 9x + 4 \). In order to guarantee that \( p \) is odd, \( x \) is replaced by \( 2u + 1 \), to get \( p = 18u + 13 \). To exclude common values, the Diophantine equations \( 18u + 13 = 6a + 5 \), and \( 18u + 13 = 8b + 1 \) are to be considered.

By Lemma 3, the first equation has no solution, while the solution of the second equation is \( u = 4x + 2 \ (x \geq 0 \text{ being any integer}) \).

Now, consider the expression:

\[
C(420p, 10) = 420p \left\lfloor \frac{(420p-1)(210p-1)(140p-1)(105p-1)}{10} \right\rfloor \times \\
(84p-1)(70p-1)(60p-1)(105p-2)(140p-3)
\]

Here, in order that the term inside the square bracket is an integer, the only possibility is that 5 divides \( 84p - 1 \) and 4 divides \( 105p - 1 \). Thus, for some integers \( \alpha \) and \( \beta \),

\[ 84p - 1 = 5 \alpha, \ 105p - 1 = 4 \beta, \]

with the solutions \( p = 5x + 4 \) and \( p = 4y + 1 \) respectively. Now, the combined equation is \( 5x + 4 = 4y + 1 \), whose solution is \( x = 4z + 1 \), so that, finally, \( p = 5(4z + 1) + 4 = 20z + 9 \). Next, the equation \( 20z + 9 = 8b + 1 \). This shows that \( z \) must be even. Therefore, writing \( z = 2v + 1 \), finally, \( p = 20(2v + 1) + 9 = 40v + 29 \). Considering the equations \( 40v + 29 = 6a + 5 \) and \( 40v + 29 = 18c + 13 \), the solutions are found to be \( v = 3a \) and \( v = 9b + 5 \) respectively, \( a \geq 0 \) and \( b \geq 0 \) being any integers.

Finally, consider the expression:

\[
C(420p, 11) = 420p \left\lfloor \frac{(420p-1)(210p-1)(140p-1)(105p-1)}{11} \right\rfloor \times \\
(84p-1)(70p-1)(60p-1)(105p-2)(42p-1)
\]

Here, \( p \neq 11 \). Hence, the term inside the square bracket is an integer.

The lemma below finds \( SS(840p) \).
Lemma 9: Let \( p \geq 2 \) be a prime. Then,
\[
SS(840p) = \begin{cases} 
840p - 9, & \text{if } p = 9s + 1, s \geq 0 \\
840p - 10, & \text{if } p = 10s + 7, s \geq 0 \\
840p - 11, & \text{otherwise}
\end{cases}
\]

Proof: From the expressions of \( C(840, 4), C(840, 5), C(840, 6), C(840, 7), \) and \( C(840, 8), \) it can be seen that, for any prime \( p, \)
\[
SS(840p) \neq 840p - 4, SS(840p) \neq 840p - 5, \\
SS(840p) \neq 840p - 6, SS(840p) \neq 840p - 7, \\
SS(840p) \neq 840p - 8.
\]

So, consider the expression
\[
C(840p, 9) = 840\left[\frac{(840p-1)(420p-1)(280p-1)}{9} \times \\
(210p-1)(168p-1)(140p-1)(120p-1)(105p-1)\right].
\]

Clearly, the term inside the square bracket is an integer if and only if either \( 9 \) divides \( 280p - 1 \) or \( 9 \) divides \( 140p - 1 \). The resulting equations are \( 280p - 1 = 9\alpha \) and \( 140p - 1 = 9\beta \), whose solutions are \( p = 9s + 1 \) and \( p = 9t + 2 \) respectively.

Next, consider
\[
C(840p, 10) = 840\left[\frac{(840p-1)(420p-1)(280p-1)(210p-1)}{2 \times 3 \times 5} \times \\
(168p-1)(140p-1)(120p-1)(105p-1)(280p-3)\right].
\]

Here, so that the term inside the square bracket is an integer, \( 5 \) must divide \( 168p - 1 \); moreover, \( p \) must be odd. Now, the solution of the Diophantine equation \( 168p - 1 = 5\alpha \) is \( p = 5x + 2 \). In order to guarantee that \( p \) is odd, \( x \) is replaced by \( 2s + 1 \) to get the desired result.

Finally, consider
\[
C(840p, 11) = 840\left[\frac{(840p-1)(420p-1)(280p-1)(210p-1)}{3 \times 11} \times \\
(168p-1)(140p-1)(120p-1)(105p-1)(280p-3)(84p-1)\right].
\]

Here, since \( p \neq 11 \), it follows that the term inside the square bracket is an integer.

Conclusions
This paper derives the explicit forms of \( SS(30p), SS(60p), SS(210p) \) and \( SS(420p), \) where \( p \) is a prime. It is found that, surprisingly, \( SS(30p) \) and \( SS(60p) \) behave differently. For example, in \( SS(30p) \), the minimum integer \( k \) such that \( 30p \) divides \( (30p, k) \) can be 4 and 7 only (depending on \( p \)), while the only possible values of the minimum \( k \) in \( SS(60p) \) are 6 and 7. Again, in \( SS(210p) \) (depending on \( p \)), the minimum \( k \) can only be one of the four possible values, namely, 4, 8, 9, and 11, whereas the minimum \( k \) in \( SS(420p) \) is five in number, namely, 6, 8, 9, 10, and 11.

The accompanying tables give the values of \( SS(30p), SS(60p), SS(210p), \) and \( SS(420p) \) for the first 200 primes, calculated on a computer, using the formula (3) for the binomial coefficients.

Conflict of interest
The authors declare that there is no conflict of interest regarding the publication of this article.

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Table 1. Values of $SS(30p), p$ is a prime

| p   | SS(n) | p   | SS(n) | p   | SS(n) | p   | SS(n) |
|-----|-------|-----|-------|-----|-------|-----|-------|
| 1   | 23    | 173 | 5183  | 409 | 12263 | 659 | 19766 | 941 |
| 2   | 53    | 179 | 5366  | 419 | 12566 | 661 | 19823 | 947 |
| 3   | 86    | 181 | 5423  | 421 | 12623 | 673 | 20183 | 953 |
| 5   | 143   | 191 | 5726  | 431 | 12926 | 677 | 20303 | 967 |
| 7   | 206   | 193 | 5783  | 433 | 12983 | 683 | 20486 | 971 |
| 11  | 326   | 197 | 5903  | 439 | 13166 | 691 | 20726 | 977 |
| 13  | 383   | 199 | 5966  | 443 | 13286 | 701 | 21023 | 983 |
| 17  | 503   | 211 | 6326  | 449 | 13463 | 709 | 21263 | 991 |
| 19  | 566   | 223 | 6686  | 457 | 13703 | 719 | 21566 | 997 |
| 23  | 686   | 227 | 6806  | 461 | 13823 | 727 | 21806 | 1009 |
| 29  | 863   | 229 | 6863  | 463 | 13886 | 733 | 21983 | 1013 |
| 31  | 926   | 233 | 6983  | 467 | 14006 | 739 | 22166 | 1019 |
| 37  | 1103  | 239 | 7166  | 479 | 14366 | 743 | 22286 | 1021 |
| 41  | 1223  | 241 | 7223  | 487 | 14606 | 751 | 22526 | 1031 |
| 43  | 1286  | 251 | 7526  | 491 | 14726 | 757 | 22703 | 1033 |
| 47  | 1406  | 257 | 7703  | 499 | 14966 | 761 | 22823 | 1039 |
| 53  | 1583  | 263 | 7886  | 503 | 15086 | 769 | 23063 | 1049 |
| 59  | 1766  | 269 | 8063  | 509 | 15263 | 773 | 23183 | 1051 |
| 61  | 1823  | 271 | 8126  | 521 | 15623 | 787 | 23606 | 1061 |
| 67  | 2006  | 277 | 8303  | 523 | 15686 | 797 | 23903 | 1063 |
| 71  | 2126  | 281 | 8423  | 541 | 16223 | 809 | 24263 | 1069 |
| 73  | 2183  | 283 | 8486  | 547 | 16406 | 811 | 24326 | 1087 |
| 79  | 2366  | 293 | 8783  | 557 | 16703 | 821 | 24623 | 1091 |
| 83  | 2466  | 307 | 9206  | 563 | 16886 | 823 | 24686 | 1093 |
| 89  | 2663  | 311 | 9326  | 569 | 17063 | 827 | 24806 | 1097 |
| 97  | 2903  | 313 | 9383  | 571 | 17126 | 829 | 24863 | 1103 |
| 101 | 3023  | 317 | 9503  | 577 | 17303 | 839 | 25166 | 1109 |
| 103 | 3086  | 331 | 9926  | 587 | 17606 | 853 | 25583 | 1117 |
| 107 | 3206  | 337 | 10103 | 593 | 17783 | 857 | 25703 | 1123 |
| 109 | 3263  | 347 | 10406 | 599 | 17966 | 859 | 25766 | 1129 |
| 113 | 3383  | 349 | 10463 | 601 | 18023 | 863 | 25886 | 1151 |
| 127 | 3806  | 353 | 10583 | 607 | 18206 | 877 | 26303 | 1153 |
| 131 | 3926  | 359 | 10766 | 613 | 18383 | 881 | 26423 | 1163 |
| 137 | 4103  | 367 | 11006 | 617 | 18503 | 883 | 26486 | 1171 |
| 139 | 4166  | 373 | 11183 | 619 | 18566 | 887 | 26606 | 1181 |
| 149 | 4463  | 379 | 11366 | 631 | 18926 | 907 | 27206 | 1187 |
| 151 | 4526  | 383 | 11486 | 641 | 19223 | 911 | 27326 | 1193 |
| 157 | 4703  | 389 | 11663 | 643 | 19286 | 919 | 27566 | 1201 |
| 163 | 4886  | 397 | 11903 | 647 | 19406 | 929 | 27863 | 1213 |
| 167 | 5006  | 401 | 12023 | 653 | 19583 | 937 | 28103 | 1217 |
Table 2. Values of $SS(60p)$, $p$ is a prime

| $p$ | $SS(n)$ | $p$ | $SS(n)$ | $p$ | $SS(n)$ | $p$ | $SS(n)$ |
|-----|---------|-----|---------|-----|---------|-----|---------|
| 1   | 53      | 173 | 10374   | 409 | 24533   | 659 | 39534   | 941 | 56454   |
| 2   | 113     | 179 | 10734   | 419 | 25134   | 661 | 39653   | 947 | 56814   |
| 3   | 173     | 181 | 10853   | 421 | 25253   | 673 | 40373   | 953 | 57174   |
| 5   | 294     | 191 | 11454   | 431 | 25854   | 677 | 40614   | 967 | 58013   |
| 7   | 412     | 193 | 11573   | 433 | 25973   | 683 | 40974   | 971 | 58254   |
| 11  | 654     | 197 | 11814   | 439 | 26333   | 691 | 41453   | 977 | 58614   |
| 13  | 773     | 199 | 11933   | 443 | 26574   | 701 | 42054   | 983 | 58974   |
| 17  | 1014    | 211 | 12653   | 449 | 26934   | 709 | 42533   | 991 | 59453   |
| 19  | 1133    | 223 | 13373   | 457 | 27413   | 719 | 43134   | 997 | 59813   |
| 23  | 1374    | 227 | 13614   | 461 | 27654   | 727 | 43613   | 1009| 60533   |
| 29  | 1734    | 229 | 13733   | 463 | 27773   | 733 | 43973   | 1013| 60774   |
| 31  | 1853    | 233 | 13974   | 467 | 28014   | 739 | 44333   | 1019| 61134   |
| 37  | 2213    | 239 | 14334   | 479 | 28734   | 743 | 44574   | 1021| 61253   |
| 41  | 2454    | 241 | 14453   | 487 | 29213   | 751 | 45053   | 1031| 61854   |
| 43  | 2573    | 251 | 15054   | 491 | 29454   | 757 | 45413   | 1033| 61973   |
| 47  | 2814    | 257 | 15414   | 499 | 29933   | 761 | 45654   | 1039| 62333   |
| 53  | 3174    | 263 | 15774   | 503 | 30174   | 769 | 46133   | 1049| 62934   |
| 59  | 3534    | 269 | 16134   | 509 | 30534   | 773 | 46374   | 1051| 63053   |
| 61  | 3653    | 271 | 16253   | 521 | 31254   | 787 | 47213   | 1061| 63654   |
| 67  | 4013    | 277 | 16613   | 523 | 31373   | 797 | 47814   | 1063| 63773   |
| 71  | 4254    | 281 | 16854   | 541 | 32453   | 809 | 48534   | 1069| 64133   |
| 73  | 4373    | 283 | 16973   | 547 | 32813   | 811 | 48653   | 1087| 65213   |
| 79  | 4733    | 293 | 17574   | 557 | 33414   | 821 | 49254   | 1091| 65454   |
| 83  | 4974    | 307 | 18413   | 563 | 33774   | 823 | 49373   | 1093| 65573   |
| 89  | 5334    | 311 | 18654   | 569 | 34134   | 827 | 49614   | 1097| 65814   |
| 97  | 5813    | 313 | 18773   | 571 | 34253   | 829 | 49733   | 1103| 66174   |
| 101 | 6054    | 317 | 19014   | 577 | 34613   | 839 | 50334   | 1109| 66534   |
| 103 | 6173    | 331 | 19853   | 587 | 35214   | 853 | 51173   | 1117| 67013   |
| 107 | 6414    | 337 | 20213   | 593 | 35574   | 857 | 51414   | 1123| 67373   |
| 109 | 6533    | 347 | 20814   | 599 | 35934   | 859 | 51533   | 1129| 67733   |
| 113 | 6774    | 349 | 20933   | 601 | 36053   | 863 | 51774   | 1151| 69054   |
| 127 | 7613    | 353 | 21174   | 607 | 36413   | 877 | 52613   | 1153| 69173   |
| 131 | 7854    | 359 | 21534   | 613 | 36773   | 881 | 52854   | 1163| 69774   |
| 137 | 8214    | 367 | 22013   | 617 | 37014   | 883 | 52973   | 1171| 70253   |
| 139 | 8333    | 373 | 22373   | 619 | 37133   | 887 | 53214   | 1181| 70854   |
| 149 | 8934    | 379 | 22733   | 631 | 37853   | 907 | 54413   | 1187| 71214   |
| 151 | 9053    | 383 | 22974   | 641 | 38454   | 911 | 54654   | 1193| 71574   |
| 157 | 9413    | 389 | 23334   | 643 | 38573   | 919 | 55133   | 1201| 72053   |
| 163 | 9773    | 397 | 23813   | 647 | 38814   | 929 | 55734   | 1213| 72773   |
| 167 | 10014   | 401 | 24054   | 653 | 39174   | 937 | 56213   | 1217| 73014   |
| p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) |
|----|-------|----|-------|----|-------|----|-------|----|-------|
| 1  | 206   | 173| 36326 | 409| 85886 | 659| 138382 | 941| 197606 |
| 2  | 412   | 179| 37582 | 419| 87982 | 661| 138806 | 947| 198862 |
| 3  | 622   | 181| 38006 | 421| 88406 | 673| 141326 | 953| 200126 |
| 4  | 1046  | 191| 40099 | 431| 90501 | 677| 142166 | 967| 203061 |
| 5  | 1459  | 193| 40526 | 433| 90926 | 683| 143422 | 971| 203902 |
| 6  | 2302  | 197| 41366 | 439| 92179 | 691| 145102 | 977| 205166 |
| 7  | 2726  | 199| 41779 | 443| 93022 | 701| 147206 | 983| 206419 |
| 8  | 3566  | 211| 44302 | 449| 94286 | 709| 148886 | 991| 208099 |
| 9  | 3982  | 223| 46819 | 457| 95966 | 719| 150981 | 997| 209366 |
| 10 | 4819  | 227| 47662 | 461| 96806 | 727| 152659 | 1009| 211886 |
| 11 | 6086  | 229| 48086 | 463| 97221 | 733| 153926 | 1013| 212726 |
| 12 | 6501  | 233| 48296 | 467| 98062 | 739| 155182 | 1019| 213982 |
| 13 | 7766  | 239| 50179 | 479| 100579 | 743| 156019 | 1021| 214406 |
| 14 | 8606  | 241| 50606 | 487| 10259 | 751| 157701 | 1031| 216499 |
| 15 | 9022  | 251| 52702 | 491| 103102 | 757| 158966 | 1033| 216926 |
| 16 | 9859  | 257| 53966 | 499| 104782 | 761| 159806 | 1039| 218181 |
| 17 | 11126 | 263| 55219 | 503| 105621 | 769| 161486 | 1049| 220286 |
| 18 | 12382 | 269| 56486 | 509| 106886 | 773| 162326 | 1051| 220702 |
| 19 | 12806 | 271| 56899 | 521| 109406 | 787| 165262 | 1061| 222806 |
| 20 | 14062 | 277| 58166 | 523| 109822 | 797| 167366 | 1063| 223219 |
| 21 | 14901 | 281| 59006 | 541| 113606 | 809| 169886 | 1069| 224486 |
| 22 | 15326 | 283| 59422 | 547| 114862 | 811| 170302 | 1087| 228259 |
| 23 | 16579 | 293| 61526 | 557| 116966 | 821| 172406 | 1091| 229102 |
| 24 | 17422 | 307| 64462 | 563| 118222 | 823| 172821 | 1093| 229526 |
| 25 | 18686 | 311| 65299 | 569| 119486 | 827| 173662 | 1097| 230366 |
| 26 | 20366 | 313| 65726 | 571| 119902 | 829| 174086 | 1103| 231619 |
| 27 | 21206 | 317| 66566 | 577| 121166 | 839| 176179 | 1109| 232886 |
| 28 | 21621 | 331| 69502 | 587| 123262 | 853| 179126 | 1117| 234566 |
| 29 | 22462 | 337| 70766 | 593| 124526 | 857| 179966 | 1123| 235822 |
| 30 | 22886 | 347| 72862 | 599| 125779 | 859| 180382 | 1129| 237086 |
| 31 | 23726 | 349| 73286 | 601| 126206 | 863| 181221 | 1151| 241701 |
| 32 | 26659 | 353| 74126 | 607| 127461 | 877| 184166 | 1153| 242126 |
| 33 | 27502 | 359| 75381 | 613| 128726 | 881| 185006 | 1163| 244222 |
| 34 | 28766 | 367| 77059 | 617| 129566 | 883| 185422 | 1171| 245902 |
| 35 | 29182 | 373| 78326 | 619| 129982 | 887| 186259 | 1181| 248006 |
| 36 | 31286 | 379| 79582 | 631| 132499 | 907| 190462 | 1187| 249262 |
| 37 | 31699 | 383| 80419 | 641| 134606 | 911| 191299 | 1193| 250526 |
| 38 | 32966 | 389| 81686 | 643| 135022 | 919| 192979 | 1201| 252206 |
| 39 | 34222 | 397| 83366 | 647| 135861 | 929| 195086 | 1213| 254726 |
| 40 | 35059 | 401| 84206 | 653| 137126 | 937| 196766 | 1217| 255566 |
| p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) | p  | SS(n) |
|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|
| 1  | 412   | 173| 72654 | 409| 171772| 659| 276774| 941| 395214| 2  | 831   | 179| 75174 | 419| 175974| 661| 277611| 947| 397734| 3  | 1249  | 181| 76009 | 421| 176809| 673| 282652| 953| 400254| 5  | 2094  | 191| 80214 | 431| 181014| 677| 284334| 967| 406131| 7  | 2929  | 193| 81052 | 433| 181852| 683| 286854| 971| 407814| 11 | 4614  | 197| 82734 | 439| 184369| 691| 290209| 977| 410334| 13 | 5451  | 199| 83569 | 443| 186054| 701| 294414| 983| 412854| 17 | 7134  | 211| 88611 | 449| 188574| 709| 297770| 991| 416209| 19 | 7969  | 223| 93649 | 457| 191932| 719| 301974| 997| 418729| 23 | 9654  | 227| 95334 | 461| 193614| 727| 305329| 1009| 423772| 29 | 12174 | 229| 96171 | 463| 194451| 733| 307851| 1013| 425454| 31 | 13011 | 233| 97854 | 467| 196134| 739| 310369| 1019| 427974| 37 | 15529 | 239| 100374| 479| 201174| 743| 312054| 1021| 428811| 41 | 17214 | 241| 101212| 487| 204529| 751| 315411| 1031| 433014| 43 | 18049 | 251| 105414| 491| 206214| 757| 317929| 1033| 433852| 47 | 19734 | 257| 107934| 499| 209571| 761| 319614| 1039| 436371| 53 | 22254 | 263| 110454| 503| 211254| 769| 322972| 1049| 440574| 59 | 24774 | 269| 112974| 509| 213774| 773| 324654| 1051| 441409| 61 | 25609 | 271| 113809| 521| 218814| 787| 330531| 1061| 445614| 67 | 28131 | 277| 116329| 523| 219649| 797| 334734| 1063| 446449| 71 | 29814 | 281| 118014| 541| 227209| 809| 339774| 1069| 448970| 73 | 30652 | 283| 118851| 547| 229729| 811| 340609| 1087| 456529| 79 | 33169 | 293| 120504| 557| 233934| 821| 344814| 1091| 458214| 83 | 34854 | 307| 128929| 563| 236454| 823| 345651| 1093| 459051| 89 | 37374 | 311| 130614| 569| 238974| 827| 347334| 1097| 460734| 97 | 40732 | 313| 131452| 571| 239811| 829| 348170| 1103| 463254| 101 | 42414 | 317| 133134| 577| 242332| 839| 352374| 1109| 465774| 103 | 43251 | 331| 139009| 587| 246534| 853| 358249| 1117| 469129| 107 | 44934 | 337| 141532| 593| 249054| 857| 359934| 1123| 471649| 109 | 45770 | 347| 145734| 599| 251574| 859| 360771| 1129| 474172| 113 | 47454 | 349| 146570| 601| 252412| 863| 362454| 1151| 483414| 127 | 53329 | 353| 148254| 607| 254931| 877| 368331| 1153| 484252| 131 | 55014 | 359| 150774| 613| 257449| 881| 370014| 1163| 488454| 137 | 57534 | 367| 154129| 617| 259134| 883| 370849| 1171| 491809| 139 | 58371 | 373| 156651| 619| 259969| 887| 372534| 1181| 496014| 149 | 62574 | 379| 159169| 631| 265009| 907| 380929| 1187| 498534| 151 | 63409 | 383| 160854| 641| 269214| 911| 382614| 1193| 501054| 157 | 65931 | 389| 163374| 643| 270051| 919| 385969| 1201| 504412| 163 | 68449 | 397| 166729| 647| 271734| 929| 390174| 1213| 509449| 167 | 70134 | 401| 168414| 653| 274254| 937| 393532| 1217| 511134|