Cosmic Acceleration and Anisotropic models with Magnetic field

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Abstract

Plane symmetric cosmological models are investigated with or without any dark energy components in the field equations. Keeping an eye on the recent observational constraints concerning the accelerating phase of expansion of the universe, the role of magnetic field is assessed. The presence of magnetic field can favour an accelerating model even if we take a linear relationship between the directional Hubble parameters.

1 Introduction

Observations from distant type Ia Supernovae confirm that presently the universe is undergoing an accelerated phase of expansion [1, 2, 3]. These data led to the development of a lot of novel ideas and solutions. The accelerated expansion is believed to be due to an exotic form of energy, known as dark energy. The host of observational data suggest that the universe is made up of 71.3% dark energy and 27.4% dark matter and baryonic matter. Dark energy provides a strong negative pressure giving rise to an antigravity effect that drives the acceleration (for recent reviews see [4, 5, 6]). The simplest and natural candidate for dark energy is a cosmological constant in classical FRW model or the Λ dominated Cold Dark Matter (ΛCDM) model. But the cosmological constant faces many serious problems like the fine tuning problem, coincidence problem etc. (see [7, 8] for reviews on cosmological constant problem). Many other dark energy models have been proposed in recent times with alternative candidates such as variable cosmological constant, a canonical scalar field like quintessence models [9], a phantom field, a scalar field with negative kinetic term [10], ghost condensate [11] or k-essence [12]. On the other hand, dynamical dark energy is effectively described by a modification of geometrical part of Einstein-Hilbet action or modification of gravity by using functions of curvature scalar \( f(R) \) gravity models [13, 14], of Gauss-Bonnet invariant [15] or higher derivatives of
The standard cosmological model (ΛCDM) based upon the spatial isotropy and flatness of the universe is consistent with the data from precise measurements of the Cosmic Microwave Background (CMB) temperature anisotropy from Wilkinson Microwave Anisotropy Probe (WMAP). However, the ΛCDM model suffers from some anomalous features at large scale such as: (i) observed large scale velocity flows than prediction, (ii) a statistically significant alignment and planarity of the CMB quadrupole and octupole modes and (iii) the observed large scale alignment in the quasar polarization vectors. Recently released Planck data show a slight red-shift of the primordial power spectrum from the exact scale invariance. The anisotropic parameter \( g^* \) in the power spectrum of curvature perturbation \( \varsigma \) with broken statistical isotropy, \( P_\varsigma(k) = P_\varsigma^0(k) \left(1 + g^* \cos^2 \Theta_{k,v}\right) \), describing the deviation from the isotropic behaviour is constrained to be \( g^* = 0.29 \pm 0.031 \) from WMAP data and \( |g^*| \geq 0.5 \) from Planck data. It is clear from the Planck data that, ΛCDM model does not fit well to the temperature power spectrum at low multipoles. Also, precise measurements from WMAP predict asymmetric expansion with one direction expanding differently from the other two transverse direction at equatorial plane which signals a non trivial topology of the large scale geometry of the universe. In order to address the issue of the smallness in the angular power spectrum of the temperature anisotropy plane symmetric models have been proposed in recent times. Plane Symmetric or Locally Rotationally Symmetric Bianchi type I (LRSBI) models have also been studied in different context to address different issues of cosmology. These models are more interesting in the sense that they are more general then FRW models.

The universe contains highly ionized matter and therefore, magnetic field plays an important role in the description of its dynamics and energy distribution. Strong magnetic field may be created due to adiabatic compression in cluster of galaxies. Cosmic anisotropies may also be attributed to the large scale magnetic fields. In anisotropic models they could alter the particle creation rates and can affect the rate of expansion. The problem of the origin and possible amplification of cosmic magnetic field has been discussed by many authors. The origin of cosmic magnetic fields can be attributed to primordial quantum fluctuations. The large scale galactic, intergalactic and super cluster magnetic fields are of the order of \( 10^{-6} \) Gauss to \( 10^{-11} \) Gauss with correlation from 100 Kpc to several Mpc to the extent that they are originated from scalar and possibly gauge field fluctuations after exciting the inflation. Their seeds may be in the range \( 10^{-18} \) to \( 10^{-27} \) Gauss or less. Magnetic field with an amplitude of \( 10^{-8} \) to \( 10^{-9} \) Gauss is believed to leave traces on CMB. In the context of the present observational data concerning the cosmic acceleration and anisotropy in the temperature power spectrum, it is interesting to investigate the role of magnetic field in anisotropic models in getting an accelerated phase. Motivated by this idea, in the present work, we have investigated LRSBI models in the frame work of Barber’s Self Creation Cosmology (BSCC). Barber.
proposed two continuous self creation theories modifying Brans and Dicke [44] theory and general theory of relativity. Brans [45] pointed out that the first theory of Barber violates the equivalence principle and also is in disagreement with experiment and hence the first theory was rejected with gross internal inconsistency. However, the second theory is an interesting cosmological model and passes all experimental tests to date [46]. Barber’s second self-creation theory (BSSC) is a simple modification of general relativity to a variable G-theory. We consider the universe to be filled with an anisotropic pressureless fluid with a cloud of cosmic strings embedded in a magnetic field. The magnetic field is aligned along the direction of the cosmic strings. A linear relationship between the directional Hubble parameters is assumed. In order to assess the role of magnetic field in getting an accelerating model, the deceleration parameter in the presence and in the absence of magnetic field is calculated.

The organisation of the paper is as follows. In Sect-2, dynamics of the LRSBI model is discussed for a cloud of cosmic strings in presence of magnetic field. In order to get a clear picture on the role of magnetic field on the properties of the model, we have also presented the results in the absence of magnetic field. In Sect-3, we have incorporated dark energy components in the form of a time varying cosmological constant in the field equations and investigated the role of magnetic field and dark energy component in getting an accelerated phase. At the end, conclusion of the present work is presented in Sect-4.

2 Anisotropic Cosmological model in presence of Magnetic field

We consider the plane symmetric LRSBI metric in the form
\[ ds^2 = -dt^2 + A^2(t)(dx^2 + dy^2) + B^2(t)dz^2 \]  
where \( A \) and \( B \) are the directional scale factors and are considered as functions of cosmic time \( t \) only. The metric corresponds to considering xy-plane as the symmetry plane. The eccentricity of such a universe is given by \( e = \sqrt{1 - \frac{B^2}{A^2}} \). The average scale factor for this metric is \( a = \left( A^2B \right)^{\frac{1}{2}} \).

The energy momentum tensor for a pressureless cosmic fluid containing one dimensional strings embedded in an electromagnetic field is taken as
\[ T_{ij} = \rho u_iu_j - \lambda x_i x_j + E_{ij} \]  
where \( \rho \) is the rest energy density of the system and \( \lambda \) is the string tension density. \( u^i = \delta^i_0 \) are the four velocity vectors. \( x^i \) is a spacelike vector representing the anisotropic direction of the cosmic strings. \( x^i \) and \( u^i \) satisfy the relations
\[ g_{ij}u^iu^j = -1, \]  
\[ g_{ij}x^ix^j = 1, \]  
\[ u^ix_i = 0. \]

The one dimensional strings are assumed to spread over the surface of cosmic sheet and aligned along the axis of symmetry. The cosmic strings are loaded
with particles with particle energy density \( \rho_p = \rho - \lambda \). \( E_{ij} \) is the part of the energy-momentum tensor corresponding to the electromagnetic field and is given by

\[
E_{ij} = \frac{1}{4\pi} [g^{sp} F_{is} F_{j,sp} - \frac{1}{4} g_{ij} F_{sp} F^{sp}]
\]

where, \( F_{sp} \) is the electromagnetic field tensor. We assume an infinite conductivity of the medium so that only the magnetic components of \( F_{sp} \) will exist and all the electric components will vanish. We consider a parallel alignment of the magnetic field with respect to the cosmic strings. In other words, we will quantize the axis of magnetic field along the axis of symmetry i.e z-axis. From Maxwell’s equations, the only non-vanishing component of electromagnetic field tensor comes out to be a constant quantity i.e.

\[
F_{12} = -F_{21} = \mathcal{H},
\]

where, \( \mathcal{H} \) is a constant representing the presence of magnetic field in and around the cosmic strings. If \( \mathcal{H} \) is zero, the system is free of any magnetic effect.

For the plane symmetric metric considered in eqn (1), the components of the electromagnetic field can be expressed as

\[
E_{11} = E_{22} = \frac{\mathcal{H}^2}{8\pi A^2} = \eta A^2,
\]

\[
E_{33} = -\frac{\mathcal{H}^2 B^2}{8\pi A^2} = -\eta B^2,
\]

\[
E_{44} = \frac{\mathcal{H}^2}{8\pi A^4} = \eta.
\]

The field equations in BSSC

\[
G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = -\frac{8\pi}{\phi} T_{ij}
\]

along with

\[
\Box \phi = \frac{8\pi}{3} \zeta T,
\]

for the metric in eqn(1) can be explicitly expressed as

\[
\left( \frac{\dot{A}}{A} \right)^2 + 2 \frac{\dot{A} \dot{B}}{A B} = \frac{8\pi}{\phi} (\rho + \eta),
\]

\[
\frac{\ddot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{A}}{A} = \frac{8\pi}{\phi} \eta,
\]

\[
2 \frac{\ddot{A}}{A} + \left( \frac{\dot{A}}{A} \right)^2 = \frac{8\pi}{\phi} (\lambda + \eta),
\]

\[
\ddot{\phi} + \left( 2 \frac{\dot{A} \dot{B}}{A B} \right) \dot{\phi} = \frac{8\pi}{3} (\rho + \lambda)
\]
In the above field equations, a dot over a directional scale factor represents a
time derivative. $\zeta$ is a coupling constant usually evaluated from experiment and
$\phi$ is the Barber’s scalar. Here $\phi$ is a function of cosmic time and it encompass
the time varying nature of the Newtonian gravitational constant. In the limit
$\zeta \to 0$, the theory approaches Einstein’s general relativity in every respect. For
an isotropic flat universe, the directional scale factors are the same i.e $A = B = a$
and from eqns (14) and (15), it is evident that, in order to get a Friedman like
equation, we need to have $\lambda = 2\eta$. In otherwords, for a pressureless cosmic
fluid, the contribution of magnetic field and cosmic string comes out as a sort
of psuedo anisotropic pressure along the axis of symmetry and the symmetry
plane. Magnetic field brings about an anisotropy in the cosmic fluid.

The directional Hubble parameters along the axis of symmetry and symmetry
plane are defined as $H_z = \frac{\dot{a}}{a}$ and $H_x = \frac{\dot{b}}{b}$ so that the mean Hubble
parameter becomes $H = \frac{1}{3}(2H_x + H_z)$. The scalar expansion and the shear
scalar for the metric (1) are respectively expressed as

$$\theta = 2H_x + H_z,$$
$$\sigma^2 = \frac{1}{3}(H_x - H_z)^2.$$  

Shear scalar is generally considered to be proportional to the scalar expansion
which envisages a linear relationship between the directional Hubble parameters
i.e.

$$H_x = kH_z$$

which is equivalent to the anisotropic relationship between the directional scale
factors $A$ and $B$ as $A = B^k$. Here $k$ is the anisotropic parameter which should
be a positive constant taking care of the anisotropic nature of the model. If
$k = 1$ the model becomes isotropic otherwise anisotropic. The field equations
(13)-(16) can be expressed in terms of the mean Hubble parameter $H$ as

$$3(k^2 + 3k + 2)\dot{H} + 9(k^2 + k + 1)H^2 = -\frac{8\pi}{\phi} (k + 2)^2 \eta,\quad (20)$$
$$6(k + 2)\dot{H} + 27H^2 = \frac{8\pi}{\phi} (k + 2)^2 (\lambda + \eta),\quad (21)$$
$$9(2k + 1)H^2 = \frac{8\pi}{\phi} (k + 2)^2 (\rho + \eta).\quad (22)$$

From eqs (20)-(22) we get

$$\frac{8\pi}{\phi} \lambda = \frac{1}{(k + 2)^2} \left[ 3(k^2 + 5k + 6)\dot{H} + 9(k^2 + k + 4)H^2 \right]$$

and

$$\frac{8\pi}{\phi} \rho = 3 \left( \frac{k + 1}{k + 2} \right) [\dot{H} + 3H^2].\quad (23)$$

It is interesting to note from eqn (24) that, the functional $\chi(H) = \dot{H} + 3H^2 \neq$
0 unlike that in the general relativistic anisotropic model in Ref. [34]. In
Ref. [34], this quantity is zero giving rise to a positive deceleration parameter $q = -1 - \frac{\dot{H}}{H^2}$ depicting a decelerating universe. In that paper, it has been shown that, a relation of the kind [19] in an LRSBI model in the framework of General Relativity can not predict an accelerating universe as demanded by the host of recent observational data. This is true for what ever matter field taken for the energy momentum tensor excluding electromagnetic contribution. However in the present case i.e, in BSSC, either in the presence or absence of electromagnetic field we will get a non zero value for the functional $\chi(H)$ which shows that there is a possibility of getting an accelerating universe for the ansatz [19] in BSSC. This behaviour of the model may be due to the presence of magnetic field along the direction of the cosmic strings or due to the nature of scalar field. The nature of Barber’s scalar field incorporates the concept of a time varying Newtonian gravitational constant and hence the effect of magnetic field may be more prominent than the presence of scalar field in getting an accelerating model.

For a linear string equation of state $\rho = \gamma \lambda$, the eqs (23) and (24) reduce to

$$-\frac{\dot{H}}{H^2} = 1 + q$$

where the deceleration parameter $q$ is expressed as

$$q = 2 \left[ \frac{(k^2 - k + 3)\gamma - (k^2 + 3k + 2)}{(k^2 + 5k + 6)\gamma - (k^2 + 3k + 2)} \right].$$

(26)

The deceleration parameter comes out to be time independent and depends on the value of the string equation of state and the anisotropic nature of the model. It is worth to mention here that a positive value of the deceleration parameter favours a decelerating universe whereas its negative value signifies a universe with acceleratining phase of expansion. Observations from type Ia Supernovae predict an accelerating universe with deceleration parameter $q = -0.81 \pm 0.14$ in the present time [47]. However, the exact determination of the deceleration parameter requires the observation of objects with redshift greater than one which is a formidable task in the present time and therefore, current observational results on deceleration parameter are not reliable [48]. It can be inferred from (26) that, for different choices of the the string equation of state $\gamma$, the deceleration parameter assumes either positive or negative values or we may get a transition from negative to positive values. Baring very low values of the anisotropic parameter $k$, the deceleration parameter becomes negative in low range in $k$ for $\gamma > 0.36$. In order to get a clear picture about the deceleration parameter, let us now consider two specific choices of the string equation of state i.e the geometric string case with $\gamma = 1$ and vaccum string case with $\gamma = -1$.

For $\gamma = 1$, the deceleration parameter becomes

$$q = \frac{1 - 4k}{2 + k}.$$  

(27)

For an isotropic model, $k = 1$ and the deceleration parameter becomes $q = -1$ implying an accelerating universe. It is clear from (27) that accelerating models can be achieved for all the models with anisotropic parameter greater than 0.25 i.e. $k > 0.25$. Below this critical value we will get decelerating models.
Figure 1: Deceleration parameter as a function of anisotropic parameter for two different choices of the string equation of state in presence of magnetic field. Deceleration parameter in the absence of magnetic field is also shown for comparison.

For the vacuum string case, i.e. $\gamma = -1$, $\rho + \lambda = 0$ i.e. the sum of the rest energy density and string tension density for a cloud of cosmic strings vanishes. Such a situation strips the universe off the string phase and leaves behind an anisotropic cosmic fluid. For this case the deceleration parameter can be expressed as

$$q = \frac{4k^2 + k + 10}{2(k^2 + 4k + 4)}.$$  \hfill (28)

Since $k$ is positive, the decelerating parameter comes out to be positive and hence the vacuum string case for the present model can not predict an accelerating universe. In Fig.1, we have plotted the deceleration parameter as a function of the anisotropic parameter for the two specific choices of the string equation of state. It is clear that, in presence of the magnetic field, the deceleration parameter for $\gamma = 1$ decreases with the increase in $k$. For low values of $k$ below $k = 0.25$, the deceleration parameter remains in the deceleration zone whereas for higher values of $k$ it goes into the acceleration zone. Basing upon the observational value of the deceleration parameter we can constrain the value of the anisotropic parameter for the geometric string case in the range $0.67 < k < 0.95$. For $\gamma = -1$, the deceleration parameter also decreases with the increase in anisotropic parameter but remains completely in the deceleration zone. In other words, for the present anisotropic model, it is unreasonable to
think of a vacuum string case which does not support the host of observational data predicting an accelerating universe.

Integration of Eq. (25) yields

\[ H = \frac{H_0}{(1 + q)H_0(t - t_0) + 1}, \]  

(29)

where \( H_0 \) is the Hubble parameter in the present time \( t_0 \). Consequent upon Eq. (29), we can calculate the scale factor as

\[ a = a_0 \left( \frac{1 - \frac{q}{H_0} \left( (1 + q)H_0(t - t_0) + 1 \right)^{\frac{1}{1 + q}}}{1 + \frac{q}{H_0}} \right), \]  

(30)

where \( a_0 \) is the scale factor in the present epoch. The redshift \( z \) can also be calculated using the fact \( 1 + z = \frac{a}{a_0} \) as

\[ z = -1 + \left[ (1 + q)H_0(t - t_0) + 1 \right]^{\frac{1}{1 + q}}. \]  

(31)

From Eqs. (10) and (30), we can have the expression of the normalized energy density contribution from magnetic field as

\[ \eta = \eta_0 \left[ (1 + q)H_0(t - t_0) + 1 \right]^{\frac{12}{(1 + q)(k^2 + 2)}}, \]  

(32)

where \( \eta_0 = \frac{H_0^2}{8\pi A_0} \) and \( A_0 \) correspond to the values at the present time. The magnetic energy density decreases with the growth of cosmic time and at late time of evolution its effect becomes negligible.

### 2.1 Physical and geometrical properties of the model

The rest energy density (\( \rho \)), the string tension density (\( \lambda \)), the particle density (\( \rho_p \)) for the model are given by

\[ \rho = \rho_0 \left[ (1 + q)H_0(t - t_0) + 1 \right]^{\frac{12}{(1 + q)(k^2 + 2)}}, \]  

(33)

\[ \lambda = \lambda_0 \left[ (1 + q)H_0(t - t_0) + 1 \right]^{\frac{12}{(1 + q)(k^2 + 2)}}, \]  

(34)

\[ \rho_p = \left( \frac{\gamma - 1}{\gamma} \right) \rho_0 \left[ (1 + q)H_0(t - t_0) + 1 \right]^{\frac{12}{(1 + q)(k^2 + 2)}}, \]  

(35)

where

\[ \rho_0 = \frac{(k + 1)(2 - q)(k + 2)\eta_0}{3(k^2 + K + 1) - (1 + q)(k^2 + 3k + 2)}, \]  

(36)

and

\[ \lambda_0 = \frac{(k + 1)(2 - q)(k + 2)\eta_0}{\gamma [(k^2 + K + 1) - (1 + q)(k^2 + 3k + 2)]}, \]  

(37)

are the rest energy density and string tension density in the present epoch \( t_0 \).

The Barber Scalar field can be expresses as

\[ \phi = \phi_0 \left[ (1 + q)H_0(t - t_0) + 1 \right]^{\frac{12}{(1 + q)(k^2 + 2)}}. \]  

(38)

In Eq. (38), \( \phi_0 = \frac{(k + 1)^2}{6(k^2 + K + 1) - 3(1 + q)(k^2 + 3k + 2)} \frac{8\pi \eta_0}{H_0^2} \) is the value of the scalar field in the present time.
The rest energy density and the string tension density decreases with the expansion of the universe. In the beginning of the universe, they have large magnitudes and they roll down to small values at late time of evolution. The increment or decrement of the scalar field with time depends on the choice of the anisotropic parameter and the string equation of state and hence the deceleration parameter. For \((1 + q)(k + 2) < 6\), the scalar field decreases with time whereas for \((1 + q)(k + 2) > 6\), it increases with time. At the critical value \((1 + q)(k + 2) = 6\), the scalar field becomes independent of time. Considering the present observational estimate of deceleration parameter and the plausible constraints on the range of anisotropic parameter \(k\), \((1 + q)(k + 2)\) will always be less than 6 and hence the scalar field decreases with the growth of cosmic time.

The geometrical features of the model are expressed through the shear scalar \(\sigma^2\) and the scalar expansion \(\Theta\). For the present model these quantities are given by

\[
\sigma^2 = 3 \left( \frac{k - 1}{k + 2} \right)^2 \left( \frac{H_0}{(1 + q)H_0(t - t_0) + 1} \right)^2, \tag{39}
\]

\[
\Theta = \frac{3H_0}{(1 + q)H_0(t - t_0) + 1}. \tag{40}
\]

The shear scalar and the scalar expansion decrease from large value at the beginning of cosmic time to small value at late times.

### 2.2 Cosmological model in the absence of magnetic field

In the absence of magnetic field, the field equations in Self Creation Cosmology assume the form

\[
3(k^2 + 3k + 2)\dot{H} + 9(k^2 + k + 1)H^2 = 0, \tag{41}
\]

\[
6(k + 2)\dot{H} + 27H^2 = \frac{8\pi}{\phi} (k + 2)^2 \lambda, \tag{42}
\]

\[
9(2k + 1)H^2 = \frac{8\pi}{\phi} (k + 2)^2 \rho. \tag{43}
\]

From Eq.(41) we can have

\[
-\frac{\dot{H}}{H^2} = 1 + \frac{2k^2 + 1}{k^2 + 3k + 2}, \tag{44}
\]

so that the deceleration parameter \(q = -1 - \frac{\dot{H}}{H^2}\) is given by

\[
q = \frac{2k^2 + 1}{k^2 + 3k + 2}. \tag{45}
\]

It is interesting to note here that, since the anisotropic parameter \(k\) is always positive, the deceleration parameter is also a positive constant quantity independent of cosmic time. It only depends on the anisotropic nature of the model. The deceleration parameter in the absence of magnetic field first decreases then increases with the increase in the anisotropic parameter (see Fig.1). However,
its variation with respect to $k$ is very less. It remains totally in the deceleration zone. It is amply clear from the calculation that in the absence of magnetic field, in the present model, it is not possible to get an accelerated phase of expansion of the universe. A similar conclusion has also been derived in Ref. [34] where it has been shown that in Einstein’s general relativity, a relation of the type [19] in the absence of magnetic field can not predict an accelerating universe which necessitates either the consideration of magnetic field in the field equation or a more evolving relationship among the directional Hubble parameters. Even in the present case of Self Creation Cosmology, the role of magnetic field in getting an accelerating universe is clearly established for a linear relationship among the directional Hubble parameters.

We can get the string equation state from Eqs (42) and (43) as

$$\gamma = \frac{\rho}{\lambda} = 1 + \frac{2k(2k + 1)}{1 + k - 2k^2}. \quad (46)$$

Figure 2: String equation of state as a function of anisotropic parameter in the absence of magnetic field.

In Fig. 2, the string equation of state in the absence of magnetic field is plotted as a function of anisotropic parameter. The string equation of state is independent of time and depends only on the value of the anisotropic parameter $k$. 

\[10\]
However, this expression (46) is not valid for an isotropic model with $k = 1$. The cosmic string in the absence of magnetic field favours a takabayasi string equation of state for all values of $k$ satisfying the relation $1 + k > 2k^2$. In other cases, for all positive values of $k$, the string equation of state favours a relationship $\rho < \lambda$. For $k < 1$, the string equation of state is positive whereas for $k > 1$, $\gamma$ is negative. In other words, we can get a string phase in the universe for anisotropic parameter in the range $k < 1$. In the model with cosmic strings embedded in a parallel magnetic field, we have considered a linear string equation of state. But if we try to figure out the relationship between the rest energy density and the string tension density, we may get a different picture.

In presence of magnetic field, the string equation can be expressed as

$$\gamma = \frac{\rho}{\lambda} = \frac{(k + 1)(k + 2)(\dot{H} + 3H^2)}{\left[(k^2 + 5k + 6)\dot{H} + 3(k^2 + k + 4)H^2\right]}.$$  \hspace{1cm} (47)

Since the functional $\chi(H) = \dot{H} + 3H^2 \neq 0$ for the present model with magnetic field having a positive rest energy density, Eq. (47) clearly indicates an evolving relationship between the rest energy density and the string tension density. In other words, in presence of magnetic field, the relationship among the rest energy density and string tension density evolves all through the expansion history of the universe.

3 Models with Dark Energy components

The presence of exotic matter and/or energy in the universe is believed to provide an acceleration. The driving force is the antigravity affect of the dark energy that generates a strong negative pressure. We consider the presence of components of dark energy in the anisotropic model in the form of a time varying cosmological constant term $\Lambda$ in the field equations. Recent cosmological observations suggest a small but positive cosmological constant with magnitude $\Lambda(G\hbar c^3) = 10^{-123}$.

The field equations in the frame work of Self Creation Cosmology in presence of magnetic field and dark energy components become

$$3(k^2 + 3k + 2)\dot{H} + 9(k^2 + k + 1)H^2 = -\frac{8\pi}{\phi}(k + 2)^2 \eta + \Lambda,$$  \hspace{1cm} (48)

$$6(k + 2)\dot{H} + 27H^2 = \frac{8\pi}{\phi}(k + 2)^2 (\lambda + \eta) + \Lambda,$$  \hspace{1cm} (49)

$$9(2k + 1)H^2 = \frac{8\pi}{\phi}(k + 2)^2 (\rho + \eta) + \Lambda.$$  \hspace{1cm} (50)

From the above field equations it is straightforward to calculate the deceleration parameter for geometric cosmic string as

$$q = \frac{1 - 4k}{2 + k}.$$  \hspace{1cm} (51)
The deceleration parameter depends only on the anisotropic parameter and is time independent. It assumes negative values for the range $k > 0.25$. In other words, the presence of the dark energy component along with the magnetic field favours an accelerating universe in the range $k > 0.25$.

The cosmological constant is now expressed by

$$\Lambda = \Lambda_0 [(1 + q)H_0(t - t_0) + 1]^{-2},$$

(52)

where $\Lambda_0 = 9(2k^2 + k)H_0^2 + (k^2 + 2)^28\pi m_0$ is the value of the cosmological constant in the present epoch. In conformity with the experimental evidences, the cosmological constant decreases with time from large value at an initial epoch to small positive value at late time of evolution.

In the absence of magnetic field, the field equations (48)-(50) predict the same deceleration parameter for geometric cosmic strings spreading the surface of the world sheet as that of (51).

The cosmological constant in the absence of magnetic field becomes

$$\Lambda = \Lambda_0 [(1 + q)H_0(t - t_0) + 1]^{-2},$$

(53)

where $\Lambda_0 = 9(2k^2 + k)H_0^2$. It is interesting to note that even in the absence of magnetic field it is possible to get an accelerating model if we incorporate dark energy component in the field equations. In other words, the role of magnetic field in getting an accelerating model is overshadowed by the dark energy component. Besides having a role in providing an accelerating model, the presence of magnetic field also brings about a change in the magnitude of the cosmological constant in the present epoch. In fact, the cosmological constant in the present epoch is somewhat lowered in absence of magnetic field.

The jerk parameter for this dark energy model is expressed as

$$j = \frac{\ddot{a}}{aH^3} = \frac{28k^2 - 23k + 4}{(k + 2)^2}.$$ 

(54)

The jerk parameter comes out to be a constant quantity and depends on the choice of the anisotropic parameter. In Fig.3, jerk parameter is plotted as a function of anisotropic parameter. It is positive for all values of the anisotropic parameter $k$ except for the range $1/4 < k < 4/7$. In this range $j$ becomes negative. The jerk parameter decreases with the increase in the anisotropic parameter up to $k = 0.41$ and then increases. For an isotropic case with $k = 1$, the jerk parameter becomes $j = 1$ in conformity with the prediction from $\Lambda$CDM model. In the present model, since the anisotropic parameter is constrained to be in the range $k < 1$, $j$ can take values less than one encompassing values from both the positive and negative domain. It is worth to mention here that the exact determination of the jerk parameter requires the observation of Supernovae of redshift greater than one which is presently a difficult task and therefore, current observational data are not able to pin down the value or the sign of the jerk parameter.
4 Conclusion

The universe is not only expanding but the expansion is accelerating. The reason behind the acceleration is not yet known to a satisfactory extent. In the present work, we have investigated the role of magnetic field in presence of cloud of cosmic strings pervading the worldsheet to get accelerating models. For this purpose, we considered plane symmetric cosmological models in the frame work of Barber’s Self Creation Cosmology. In order to get determinate cosmological models we have assumed a linear relationship between the directional Hubble parameters which envisages an anisotropic relationship among the directional scale factors. In an earlier work, it has been shown that a linear relationship among the directional Hubble parameters will not be able to predict the observational facts concerning the accelerating phase of the universe. However, in the present work, we get accelerating models even if we consider such anisotropic relationship. In the absence of magnetic field, the deceleration parameter comes out to be positive for all possible values of anisotropic parameter whereas in presence of magnetic field the deceleration parameter can be negative for a range of anisotropic parameter implying an accelerating universe. It is to be emphasized here that, the role of magnetic field is crucial in getting an accelerating model. Incorporation of dark energy components into the field equations in the form of a time varying cosmological constant overshadows the role of magnetic field.
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