DIMENSIONALLY REDUCED CHERN-SIMONS TERMS AND THEIR SOLITONS

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Abstract. We consider models in which nonrelativistic matter fields interact with gauge fields whose dynamics are governed by the Chern-Simons term. The relevant equations of motion are derived and reduced dimensionally in time or in space. Interesting solitonic equations emerge and their solutions are described. Finally, we consider a Chern-Simons term in three-dimensional Euclidean space, reduced by spherical symmetry, and we discuss its effect on monopole and instanton solutions.

1. Introduction

In (2+1)-dimensions, the possibility of describing gauge theories with a Chern-Simons (C-S) kinetic term, rather than with a Yang-Mills kinetic term, leads to physically and mathematically interesting consequences. The C-S term is given by

\[ W(A) = -\frac{1}{16\pi^2} \int d^3x \epsilon^{\alpha\beta\gamma} \text{Tr}(A_\alpha \partial_\beta A_\gamma + \frac{2}{3} A_\alpha A_\beta A_\gamma) \]  

(1)

where \( \epsilon^{\alpha\beta\gamma} \) is the totally antisymmetric tensor and the gauge field \( A_\mu = A_\mu^a T^a \) takes values in a finite-dimensional representation of a Lie algebra, with generators \( T^a \) satifying \( [T^a, T^b] = f^{abc} T^c \). In the Abelian theory the trilinear term vanishes since \( A_\mu \)'s commute.

\( W(A) \) possesses the important property of being invariant against infinitesimal gauge transformations, while changing under large gauge transformations by the integer winding number \( n \) of the group element \( g \) that effects the transformation:

\[ A_\mu \rightarrow A_\mu'^a = g^{-1} A_\mu g + g^{-1} \partial_\mu g \]  

(2a)

\[ W(A) \rightarrow W(A'^g) = W(A) + n. \]  

(2b)

(We have assumed that no surface terms involving \( A \) contribute.)

\[ n = -\frac{1}{48\pi^2} \int d^3x \epsilon^{\alpha\beta\gamma} \text{Tr}(g^{-1} \partial_\alpha g^{-1} \partial_\beta gg^{-1} \partial_\gamma g) \]  

(2c)

Therefore, when the C-S term \( W(A) \) is used in the action for (2+1)-dimensional gauge fields of a non-Abelian group, the coefficient of \( W(A) \) must be properly quantized in order to ensure gauge

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invariance of the quantum theory. Then the shift in $W(A)$ under a gauge transformation is not seen in the phase exponential of the action. \[1, 2\]

Another important feature of the C-S term is that it describes a topological quantity in the sense that there is no explicit dependence on the space-time metric. Thus when $W(A)$ is used in the action it does not contribute to the energy-momentum tensor, which is obtained by varying the action with respect to the metric tensor.

When the C-S term is coupled to matter fields in a gauge covariant manner we have an action of the following form:

$$I = 8\pi^2 \kappa W(A) - \int d^3 x A_\mu J^\mu + I_{\text{matter}}.$$ (3)

The matter action may describe matter fields with relativistic or nonrelativistic dynamics. The total action is then either Lorentz or Galilean invariant since the C-S term is topological, that is, invariant against all coordinate transformations. Because the variation of $W(A)$ with respect to $A_\mu$ is $\frac{1}{16\pi^2} \epsilon^{\mu\alpha\beta} F_{\alpha\beta}$, the action in Eq. (3) produces a field-current identity

$$\frac{\kappa}{2} \epsilon^{\mu\alpha\beta} F_{\alpha\beta} = J^\mu$$ (4)

which tells us that gauge fields are not dynamical, but they are completely determined by matter variables. Therefore, the resulting matter equations are self-contained and highly nonlinear.

When matter field dynamics are nonrelativistic, we have an interesting class of C-S gauge models described by the action

$$I = 8\pi^2 \kappa W(A) + \int d^3 x \left\{ i\psi^\dagger D_t \psi - \frac{1}{2m} (D\psi)^\dagger (D\psi) - V(\rho) \right\} .$$ (5)

$\psi$ is a scalar multiplet in some definite representation of a non-Abelian gauge group and $(D_t, D)$ are (temporal, spatial) gauge covariant derivatives. $V(\rho)$ is a scalar potential describing matter self-interaction with $\rho^a = -i\psi^\dagger T^a \psi$, where $T^a$ represents anti-Hermitian generators of the gauge algebra in the representation of $\psi$.

When we choose $V(\rho)$ to be

$$V(\rho) = -\frac{1}{2} g \rho^a \rho_a$$ (6)

the matter field equation that emerges from the action (5) is a “gauged” nonlinear Schrödinger equation

$$i\partial_t \psi = -\frac{1}{2} D^2 \psi - iA^0 \psi + ig\rho \psi$$ (7)

where $(A^0, A)$ are given by the field-current identity (4):

$$B^a = -\frac{1}{\kappa} \mu^a \quad \text{(Gauss’ law)}$$ (8a)

$$E^{ia} = \frac{1}{\kappa} \epsilon^{ij} J^a_j$$ (8b)

with the covariantly conserved current density given by

$$J^a_0 = \rho^a$$ (9a)

$$J^a_i = \frac{1}{2} (D^i \rho^a - (D^i \psi)^\dagger T^a \psi)$$ (9b)
In these lectures we discuss models belonging to the general class (3), with both Abelian and non-Abelian gauge groups.

2. Dimensional Reduction of Chern-Simons Models

2.1. Self-dual theories and reduction to Liouville and Toda equations. One obvious reduction for the dynamics of (3) is to reduce in time. When we choose the scalar potential $V(\rho)$ to be quartic as given in (6), with a particular strength $g = \frac{1}{mn}$ and $\kappa > 0$, we obtain a new class of self-dual theories to which one can apply Bogomolny procedures. The second-order static equations of (7) are satisfied by solutions to first-order self-dual equations, and the Gauss law constraint (8a).

$$D_t \psi = i \epsilon_{ij} D_j \psi$$
$$B = -\frac{1}{\kappa} \rho .$$

These coupled equations then can be combined into the completely integrable Liouville equation (Abelian case) or Toda equation (SU(N) case), with well-known soliton solutions. This subject is an old one, well reviewed in the existing literature[3, 4], so we shall not dwell on it.

2.2. Reduction to nonlinear Schrödinger equation. Now we shall describe in detail a reduction to one spatial dimension, which results in an interesting reformulation of the nonlinear Schrödinger equation. On the plane, with coordinates $(x, y)$, we suppress all $y$-dependence and rename $A_y$ as $B$.

Then, in the Abelian case, the action (5) becomes

$$I = \int dt \, dx \left\{ -\kappa BF + i \psi^* D_t \psi - \frac{1}{2m} |D\psi|^2 - \frac{1}{2m} B^2 \rho - V(\rho) \right\} .$$

The “kinetic” gauge field term is the so-called “B-F” expression where $F = \frac{1}{2} \epsilon^{\mu\nu} F_{\mu\nu} = -\dot{a} - A'_0$. [We have renamed $A_1$ as $-a$, and dot/dash refer to differentiation with respect to time/space, that is, $(t/x)$. The covariant derivatives read $D_t \psi = \dot{\psi} + iA_0 \psi, D\psi = \psi' - i\alpha \psi$. Recall that in the Abelian application, the C-S coefficient is not quantized.] Evidently the two-dimensional B-F quantity is a dimensional reduction of the C-S expression.

Because (11) is first-order in time derivatives, the action is already in canonical form, and may be analyzed using the symplectic Hamiltonian procedure. We present (11) as

$$I = \int dt \, dx \left\{ \kappa B\dot{a} + i \psi^* \dot{\psi} - A_0 (\kappa B' + \rho) - \frac{1}{2m} |(\partial_x - i\alpha)\psi|^2 - \frac{1}{2m} B^2 \rho - V(\rho) \right\}$$

$$= \int dt \, dx \left\{ \kappa B\dot{a} + i \psi^* \dot{\psi} - A_0 (\kappa B' + \rho) - \frac{1}{2m} |(\partial_x - i\alpha \pm B)\psi|^2 + \frac{1}{2m} B' \rho - V(\rho) \right\} .$$

(12)

$A_0$ is a Lagrange multiplier, enforcing the Gauss law, which in this theory requires

$$B' = -\frac{1}{\kappa} \rho$$

or equivalently

$$B(x) = -\frac{1}{2\kappa} \int d\tilde{x} \epsilon(x - \tilde{x}) \rho(\tilde{x}) .$$

(13b)
The Green’s function, uniquely determined by parity invariance, is the Heaviside ±1 step. Thus, after we eliminate \( B \), (12) involves a spatially nonlocal Lagrangian.

\[
L = -\frac{1}{2} \int dx \int d\tilde{x} \dot{a}(x) \epsilon(x - \tilde{x}) \rho(\tilde{x}) + \int dx i\psi^* \dot{\psi} - \frac{1}{2m} \int dx \left( \partial_x^2 - \frac{1}{2\kappa} \int d\tilde{x} \epsilon(x - \tilde{x}) \rho(\tilde{x}) \right) \psi(x)^2 + \int dx \left( \pm \frac{1}{2\kappa m} \rho^2 - V(\rho) \right)
\]

(14a)

The \( a \) dependence is removed when \( \psi(x) \) is replaced by \( e^\frac{\pm i}{2} \int d\tilde{x} \epsilon(x - \tilde{x}) a(\tilde{x}) \psi(x) \), leaving

\[
L = \int dx i\psi^* \dot{\psi} - \frac{1}{2m} \int dx \left( \partial_x^2 + \frac{1}{2\kappa} \int d\tilde{x} \epsilon(x - \tilde{x}) \rho(\tilde{x}) \right) \psi(x)^2 + \int dx \left( \pm \frac{1}{2\kappa m} \rho^2 - V(\rho) \right)
\]

(14b)

Finally we choose \( V(\rho) \) to be \( \frac{\pm 1}{2\kappa m} \rho^2 \) [this is the same choice that in (2+1)-dimensions leads to static first-order Bogomolny equations] and our reduced C-S, \( B-F \) theory is governed by the Hamiltonian

\[
H = \frac{1}{2m} \int dx \left( \partial_x^2 + \frac{1}{2\kappa} \int d\tilde{x} \epsilon(x - \tilde{x}) \rho(\tilde{x}) \right) \psi(x)^2
\]

(15)

which implies the first-order Bogomolny equation

\[
\psi'(x) \mp \frac{1}{2\kappa} \int d\tilde{x} \epsilon(x - \tilde{x}) \rho(\tilde{x}) \psi(x) = 0
\]

(16a)

solved by

\[
\psi(x) = \text{phase} \times \frac{\alpha \sqrt{|\kappa|}}{\cosh \alpha x}
\]

(16b)

where \( \mp \kappa \) is taken as positive, and \( \alpha \) is an integration constant.

On the other hand, we can recognize the dynamics described in (15) by expanding the product:

\[
H = \frac{1}{2m} \int dx \left\{ |\psi'|^2 \pm \frac{1}{\kappa} \rho^2 \right\} + \frac{1}{24m\kappa^2} \int dx \int d\tilde{x} \int d\hat{x} \rho(x) \rho(\tilde{x}) \rho(\hat{x}) \left\{ \epsilon(x - \tilde{x}) \epsilon(x - \hat{x}) + \epsilon(\hat{x} - x) \epsilon(x - \tilde{x}) + \epsilon(\tilde{x} - x) \epsilon(x - \hat{x}) \right\}.
\]

(17)

The last term was symmetrized, leading to a sum of step function products, which in fact equals to 1. Consequently the last integral is \( \frac{1}{24m\kappa^2} N^3 \), where \( N = \int dx \rho(x) \), which is conserved in the dynamics implied by (17). Hence this term can be removed by redefining

\[
\psi \to e^{-\frac{N^2}{8m\kappa^2}} \psi.
\]

(18)
What is left is recognized as the Hamiltonian for the nonlinear Schrödinger equation, with equation of motion

\[ i \dot{\psi} = -\frac{1}{2m} \psi'' - \lambda \rho \psi \]

\[ \lambda \equiv \mp \frac{1}{m \kappa} . \]  

The nonlinear Schrödinger equation plays a cycle of interrelated roles in mathematical physics. Viewed as a nonlinear, partial differential equation for the function \( \psi \), it is completely integrable, possessing a complete spectrum of multi-soliton solutions, the simplest of these being the single soliton at rest. This requires \( \lambda > 0 \), which is always achievable in our reduction by adjustment of \( \kappa \):

\[ \psi_{\text{rest}}(t,x) = \pm e^{i \alpha \frac{t^2}{2m}} \frac{1}{\sqrt{\lambda m}} \frac{\alpha}{\cosh \alpha x} . \]  

(20)

Here \( \alpha \) is an integration constant, and the result is consistent with (16b), once the redefinition (18) is taken into account. Because of Galileo invariance, the solution may be boosted with velocity \( v \), yielding

\[ \psi_{\text{moving}}(t,x) = \pm e^{i mvx} e^{i t \left( \frac{\alpha^2}{2m} - \frac{mv^2}{2} \right)} \frac{1}{\sqrt{\lambda m}} \frac{\alpha}{\cosh \alpha (x - vt)} . \]  

(21)

The soliton solutions can be quantized by the well-known methods of soliton quantization. On the other hand, the nonrelativistic field theory can be quantized at fixed \( N \), where it describes \( N \) nonrelativistic point particles with pair-wise \( \delta \)-function interactions. This quantal problem can also be solved exactly, and the results agree with those of soliton quantization. All these properties are well known, and will not be reviewed here.\[6\]

The present development demonstrates that this classical/quantal, completely integrable theory possesses a Bogomolny formulation, which is obtained by using two-dimensional \( B-F \) gauge theory, which in turn descends from three-dimensional C-S dynamics.\[7\]

2.3. Reduction to modified nonlinear Schrödinger equation. While the previous development started with \( B-F \) gauge theory, which descended from a C-S model, and arrived at an interesting (first-order, Bogomolny) formulation for the familiar nonlinear Schrödinger equation, we now further modify the gauge theory and obtain a novel, chiral, nonlinear Schrödinger equation.

Let us observe first that the above dynamics is nontrivial solely because we have chosen \( V \) to be nonvanishing. Indeed with \( V = 0 \) in (11), (12), and (14), the same set of steps (removing \( B \) and \( a \) from the theory) results in a free theory for the \( \psi \) field.

To avoid triviality at \( V = 0 \), we need to make the \( B \) field dynamically active by endowing it with a kinetic term. Such a kinetic term could take the Klein-Gordon form; however we prefer a simpler expression that describes a “chiral” Bose field, propagating in only one direction. A Lagrange density for such a field has been known for some time.\[8\]

\[ L_{\text{chiral}} = \pm \dot{B} B' + v B'' B' . \]  

(22)

Here \( v \) is a velocity, and the consequent equation of motion arising from \( L_{\text{chiral}} \) is solved by \( B = B(x \mp vt) \) (with suitable boundary conditions at infinity), describing propagation in one direction, with velocity \( \pm v \). Note that \( \dot{B} B' \) is not invariant against a Galileo boost, which is a symmetry
of $B'B'$ and of (11), (12), (14): performing a Galileo boost on $\dot{BB}'$ with velocity $\tilde{v}$ gives rise to $\tilde{v}B'B'$, effectively boosting the $v$ parameter in $\mathcal{L}_{\text{chiral}}$ by $\tilde{v}$. Consequently, one may drop the $vB'B'$ contribution, thereby selecting to work in a global “rest frame.” Boosting a solution in this rest frame produces a solution to the theory with a $B'B'$ term.

In view of this discussion, we supplement the previous Lagrange density (11), (12), (14) by

$$L = -\kappa \dot{B} \left( a \mp \frac{1}{\kappa} B' \right) + i \psi^* \dot{\psi} - A_0 (\kappa B' + \rho) - \frac{1}{2m} \left[ (\partial_x - ia) \psi \right]^2 - \frac{1}{2m} B^2 \rho .$$

(23a)

After redefining $a$ as $a \pm \frac{1}{\kappa} B'$, this becomes equivalent to

$$L = \kappa B \dot{a} + i \psi^* \dot{\psi} - A_0 (\kappa B' + \rho) - \frac{1}{2m} \left[ (\partial_x - i a \mp i \frac{1}{\kappa} B') \psi \right]^2 - \frac{1}{2m} B^2 \rho .$$

(23b)

Now we proceed as before: solve Gauss’ law as in (13), remove $a$ by a phase-redefinition of $\psi$, drop the last term in (23b) by a further phase redefinition as in (18). We are then left with

$$L = i \psi^* \dot{\psi} - \frac{1}{2m} \left[ (\partial_x \pm i \frac{1}{\kappa^2} \rho) \psi \right]^2 .$$

(24)

It has been suggested that this theory may be relevant to modeling quantum Hall edge states.[9]

The Euler-Lagrange equation that follows from (24) reads

$$i \dot{\psi} = - \frac{1}{2m} \left( \partial_x \pm i \frac{1}{\kappa^2} \rho \right)^2 \psi \pm \frac{1}{\kappa^2} j \psi$$

(25)

where the current density $j$

$$j = \frac{1}{m} \text{Im} \psi^* \left( \partial_x \pm i \frac{1}{\kappa^2} \rho \right) \psi$$

(26)

is linked to $\rho$ by the continuity equation

$$\dot{\rho} + \partial_x j = 0 .$$

(27)

Next we redefine the $\psi$ field by

$$\psi(t, x) = e^{\mp \int_x^t dy \rho(t, y)} \Psi(t, x)$$

(28)

and see that the equations satisfied by $\Psi$ is

$$i \dot{\Psi}(t, x) \pm \frac{1}{\kappa} \int_x^t dy \dot{\rho}(t, y) \Psi(t, x) = - \frac{1}{2m} \Psi''(t, x) \pm \frac{1}{\kappa} j(t, x) \Psi(t, x) .$$

(29a)

But the integral may be evaluated with the help of (27), so finally we are left with

$$i \dot{\Psi} = - \frac{1}{2m} \Psi'' \pm \frac{2}{\kappa} j \Psi.$$  

(29b)

This is a nonlinear Schrödinger equation similar to (19) but with the current density $j = \frac{1}{m} \text{Im} \Psi^* \Psi'$ replacing the charge density $\rho = \Psi^* \Psi$. The equation is not known to be completely integrable but it does possess an interesting soliton solution, which is readily found by setting the $x$-dependence of the phase of $\Psi$ to be $e^{imvx}$. Then $j = v \rho$, and our new equation (29b) becomes the usual nonlinear Schrödinger equation

$$i \dot{\Psi} = - \frac{1}{2m} \Psi'' \pm \frac{2v}{\kappa^2} \rho \Psi.$$  

(30)
that is, the nonlinear coupling strength of \( \lambda \) is
\[
\lambda = \mp \frac{2v}{\kappa^2}.
\] (31)

The \((\mp)\) sign is inherited from the “chiral” kinetic term, see (22), (23); once a definite choice is made (say +), positive \( \lambda \), which is required for soliton binding, corresponds to definite sign for \( v \) (say positive); that is, the soliton solving (30) moves in only one direction. Explicitly, with the above choice of signs, the one-soliton solution reads
\[
\Psi_s(t, x) = \pm e^{imx} e^{it} \left( \frac{\alpha^2 - \frac{mv^2}{2}}{\kappa \sqrt{2mv}} \right) \frac{\kappa^2}{\cosh \alpha (x - vt)} \right). \] (32)

We see explicitly that \( v \) must be positive; the soliton cannot be brought to rest; Galileo invariance is lost.

The characteristics of the solution are as follows:
\[
N_s = \frac{\alpha \kappa^2}{mv}.
\] (33)

The energy is obtained by integrating the Hamiltonian.
\[
E = \int dx \frac{1}{2m} |\Psi'|^2.
\] (34)
and on the solution (32), takes the value appropriate to a massive, nonrelativistic particle.
\[
E_s = \frac{1}{2} M_s v^2
\] (35)
where
\[
M_s = m N_s \left( 1 + \frac{1}{3\kappa^2 N_s^2} \right).
\] (36)

The conserved field momentum in this theory reads
\[
P = \int dx \left( mj + \frac{1}{\kappa^2 \rho^2} \right)
\] (37)
and on the solution (32) its value again corresponds to that of a massive, nonrelativistic particle
\[
P_s = M_s v
\] (38)
\[
E_s = \frac{p_s^2}{2 M_s}
\] (39)

As already remarked, the model is not Galileo invariant, but one can verify that it is scale invariant. Indeed one can show that the above kinematical relations are a consequence of scale invariance.[10]

The soliton solution (32) can be quantized; also the quantal many-body problem, which is implied by (24), can be analyzed. Because the system does not appear integrable, exact results are unavailable, but one verifies that at weak coupling, the two methods of quantization (soliton, many-body) produce identical results.[10, 11]
3. REDUCING THE CHERN-SIMONS TERM USING A SYMMETRY

Reducing a three-dimensional C-S term in \([1]\) by a symmetry yields another topologically interesting structure. \([3]\) Specifically, reducing by radial symmetry results in a one-dimensional quantum mechanical model, which has recently been used in an analysis of finite-temperature C-S theory. \([13]\) Earlier calculations seem to indicate that the coefficient of the induced C-S term depends on the temperature, \([14]\) contradicting that it must have quantized, discrete values. The puzzle became resolved once it was realized that finite temperature calculations to fixed perturbative order necessarily violate gauge invariance, which is restored only after all orders are summed. \([13, 15]\) (At zero temperature, finite-order calculations suffice to exhibit the complete, induced C-S term. \([16]\))

The all-order summation was first accomplished in a toy quantum mechanical model, which had been introduced a decade earlier for the purpose of exhibiting in a simple setting some of the peculiar topological/geometrical effects of quantized C-S theory. \([17]\) We shall show that this model is not merely a pedagogical toy; in fact, it coincides with the three-dimensional C-S term, reduced by radial symmetry.

Consider the C-S action in three-dimensional space.

\[
NW(A) = -\frac{N}{4\pi} \int d^3x \epsilon^{ijk} \text{Tr}(\partial_i A_j A_k + \frac{2}{3} A_i A_j A_k) \tag{40}
\]

The coefficient of \(W(A)\) is chosen so that the quantization condition is obeyed with integer \(N\). Let us consider SU(2) case and take for \(A^a_i\) the radially symmetric Ansatz, familiar form, from monopole/instanton studies.

\[
A^a_i = (\delta^a_i - \hat{r}^a \hat{r}^i)\frac{1}{r} \psi_1 + \epsilon^{iaj} \hat{r}^j \frac{1}{r} (\psi_2 - 1) + \hat{r}^a \hat{r}^i A \tag{41}
\]

Here \(\psi_m, m = 1, 2 \) and \(A\) are functions just of \(r\). Substituting \((41)\) into \((40)\), performing the angular integral, leaves

\[
NW(A) = N \int_0^\infty dr \left( \epsilon^{mn} \psi_m (D\psi)_n - A \right) \tag{42}
\]

\[
(D\psi)_m = \psi_m - \epsilon_{mn} A \psi_n \tag{43}
\]

Here the dash denotes \(r\)-differentiation and we have dropped an end point contribution, \(\psi_1|_{r=\infty} = 0\). Note that \((42)\) is a \((0+1)\)-dimensional field theory, that is, a quantum mechanical system, except that \(r\) has only half the range of \(t\). The system has local \(U(1)\) gauge symmetry, with a one-dimensional C-S term: \(A\). Under a gauge transformation, the action changes by \(\Delta I = -N \Delta \theta\), where \(\Delta \theta = \int d\theta\); when \(\Delta \theta\) is restricted to an integral multiple of \(2\pi\), gauge invariance of \(e^{iI}\) is assured by the integer \(N\). \([17]\)

The recent analysis of the C-S term at finite temperature \([13, 14]\) was based on the model \((42)\).

Finally, we wish to mention that the radially symmetric C-S term \((42)\) may be inserted into the classical field equation of the Yang-Mills/Higgs model in \((3+1)\) dimensions, for which static ’t Hooft-Polyakov monopole solitons exist, with explicitly known profiles in the Bogomolny-Prasad-Sommerfield limit. These static solutions can also be viewed as instantons of the same theory in \((2+1)\) dimensions, continued to Euclidean 3-space. The C-S term in the \((3+1)\) theory violates Lorentz invariance in an interesting, gauge invariant fashion. \([13]\) On the other hand, when the
C-S term is added to the Euclidean three-dimensional theory, it enters with an imaginary coefficient, which is inherited from the continuation to imaginary time of the (2+1)-dimensional model. Nevertheless, the final equations are real. One can show that the addition of the topological C-S interaction destroys the topological excitations; the equations no longer admit monopole or instanton solutions.

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