Mode Identification of the Slowly Pulsating F0V Star V398 Aurigae (9 Aur)

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ABSTRACT
We have investigated the F0V star V398 Aurigae (= 9 Aur) under the assumption that it is undergoing non-radial gravity mode oscillations and that the two principal periods given by Krisiunias et al. (1995) are correct. We find that the two periods are manifestations of an ℓ = 3, |m| = 1 spheroidal mode and its toroidal corrections due to the rotation of the star. As far as we know, this is the first detection of toroidal correction terms in a real star. The two modes probably are the result of rotational splitting.

Our analysis provides for the first time a physical explanation of certain characteristics of the observed behavior of the star. The amplitude of the radial part of the pulsation for f1 = 0.795 d−1 is a factor of 4 larger than the one for f2 = 0.346 d−1. Since the photometric variability is determined mostly by temperature variations, which in turn are determined by the radial part of the pulsation, the photometric variability is dominated by the mode with frequency f1. On the other hand, f2 is the more pronounced one in all three spectroscopic moment variations (including the radial velocity), reflecting that the transverse displacement of f2, and not the one of f1, dominates the velocity behavior.

Key words: Stars: pulsation – Stars: variables.

1 INTRODUCTION

Krisiunias & Handler (1995) have compiled a list of 17 stars of similar spectral type and luminosity class which appear to constitute a new class of pulsating variable stars. These stars are typically of spectral type F0 to F2, and they are found on, or just above, the main sequence in the Hertzsprung-Russell Diagram. The best studied examples are γ Doradus (Balona, Krisiunias & Cousins 1994), 9 Aurigae (Krisiunias et al. 1995), HD 224638 and HD 224945 (Mantegazza, Poretti & Zerbi 1994). Given that γ Dor is the brightest member of the list, and that it was the first one found to be variable (Cousins & Warren 1963), it has been suggested that these stars be known as “γ Doradus stars”.

The photometric time scales of the γ Dor stars are 0.5 to 3.5 d – variations an order of magnitude slower than the fundamental radial pulsation period for stars of this density. 9 Aur and γ Dor have shown evidence for radial velocity and spectroscopic line profile variations. In the case of 9 Aur – recently designated V398 Aur in the latest named list of new variable stars (Kazovarets & Samus 1995) – the radial velocity variations appear to follow only one of the two photometric frequencies (f2 = 0.346 d−1) found by Krisiunias et al. (1995).

Given the time scale of variations, the evidence for radial velocity and line profile variations, and the lack of other viable explanations (including star spots), the evidence is strong that γ Dor stars are exhibiting non-radial gravity mode pulsations. This has been a surprising observational development, because until now only much hotter stars have shown evidence non-radial g-modes (Waelkens 1991).

Because the power spectrum of the radial velocities of V398 Aur did not show evidence for the highest peak in the power spectrum of the photometry (f1 = 0.795 d−1), Krisiunias et al. (1995) suggest that f2 arises from an ℓ = 1 or 2 spherical harmonic, while f1 arises from a higher degree harmonic. In this paper we apply the moment method described by Aerts et al. (1992) to the line profiles of V398 Aur in an attempt to identify its pulsational mode(s).

2 CROSS-CORRELATION PROFILES

Krisiunias et al. (1995) present 95 cross-correlation profiles obtained by R. F. Griffin with the Haute Provence Coravel
(Baranne, Mayor & Poncet 1979). While the Coravel is used primarily for the determination of the radial velocities of stars, the cross-correlation profiles give us, effectively, a measure of the mean spectroscopic line depths and a parameter characterizing the width of the lines. The width parameter can be interpreted in terms of the projected rotational velocity $v \sin i$ of the star (Benz & Mayor 1981). The cross-correlation profiles can be thought of as the mean line profiles of the star being studied.

The Coravel data reduction package also gives us a Gaussian fit to the points of the cross-correlation profiles. In order to carry out the identification of the pulsational mode(s) of a star, it is necessary to have a very high signal to noise ratio in spectroscopic parameter space. We found it necessary to use the mathematical fits to the cross-correlation points for our analysis here. (See the sets of curved lines in Fig 10 of Krisciunas et al. 1995.)

3 DETERMINATION OF PULSATIONAL MODE(S)

Mode identification is currently often obtained by means of spectroscopic analyses. The method used here was first introduced by Balona (1986) and was subsequently generalized by Aerts et al. (1992) and by Mathias et al. (1994) in the case of respectively a mono- and a multiperiodic pulsation. It is based on the time variations of the first three moments of a line profile. The periodograms of the three moments can immediately be interpreted in terms of the periods and amplitudes of the non-radial pulsation (NRP) parameters. The observed moment variations are compared with theoretically calculated expressions for these variations in the case of various pulsation modes. Mode identification is obtained by means of a so-called discriminant, which captures the discrepancy between the observed and the calculated amplitudes.

The discriminant used in this paper is a generalization of the one proposed by Aerts et al. (1992) in the sense that it takes into account all seven observed amplitudes of the three moments (instead of only three) and it gives them a weight according to their uncertainty. This discriminant turns out to be more accurate than the one presented by Aerts et al., especially in the odd ones. The presence of the frequency $f_2$ is less clear, but beat- and sum-frequencies of $f_1$ and $f_2$ are present in the second and third moment. We further studied the variations of the moments assuming that the two frequencies $f_1 = 0.795 \, \text{d}^{-1}$ and $f_2 = 0.346 \, \text{d}^{-1}$ are accurate. The solid lines shown in Fig 4 are the theoretical fits to the observations for a model with $f_1$ (right) and $f_2$ (left). The fact that the third moment is centered around $0 \, \text{km s}^{-1}$ reflects that there is no variability on a time scale larger than the time span of our data.

We can determine the ratio of the horizontal to the vertical pulsation velocity amplitude $K$ by means of the boundary conditions as a function of stellar mass and radius if we know the pulsation frequency in the corotating frame. We only know the frequencies in the observer’s inertial frame, we use these to approximate the true $K$-values. The actual mass of 9 Aur A, the primary, is not known. While the B component is an M2 V companion at a distance of $\approx 100$ au (Krisciunas et al. 1993), the orbit of that companion is not well known enough to determine the mass function of the system. Adopting a mass range of $1.5–1.7 \, M_\odot$, which is reasonable for FOV stars (Pappler 1980), and further $R = 1.64 \, R_\odot$ (Mantegazza et al. 1994), we find $K_1 \in [40, 45]$ for $f_1$ and $K_2 \in [211, 240]$ for $f_2$. It is clear from these numbers that we are dealing with high-order $g$-modes: the transverse displacement is dominant over the radial one.

The equivalent width of the profiles varies by $8\%$ for $f_1$ and by $3\%$ for $f_2$. Since equivalent width variations are usually interpreted in terms of temperature variations, these findings are in agreement with the photometric variations in the sense that the most important temperature variation is due to $f_1$.

It is clear from Fig 4 that some of the theoretical amplitudes of the moments are uncertain. For such data the discriminant defined in equation (1) is much better than the one defined by Aerts et al. (1992), because the former is constructed in such a way that the most accurate amplitudes are given the largest weight. The amplitudes of the terms of the second moment varying with frequencies $f_1$ and $f_2$ are at least as large as those varying with frequencies $2f_1$ and $2f_2$. This is a signature of the presence of non-axisymmetric modes (see Aerts et al. 1992).

The amplitudes of the moments are used to calculate the discriminants $\Gamma_2^m(v_p, i, \sigma)$ and their minima $\gamma_1^m$ for each
Figure 1. The first, second, and third moment of the mathematical fits to the cross-correlation profiles of V398 Aur taken between 1993 December 25 and 1994 January 10 UT (see also in Section 2). The dots represent the observations, while the full lines are the fits for a model with $f_2$ (left) and $f_1$ (right).

Table 1. The different minima of the discriminants for the two modes of V398 Aur. $\gamma^m_\ell$, $v_p$, and $\sigma$ are given in km s$^{-1}$. The radial component of the velocity has an amplitude proportional to $v_p$, while the transverse component’s amplitude is proportional to $Kv_p$ (see also Discussion).

| $f_1 = 0.795$ d$^{-1}$ | $f_2 = 0.346$ d$^{-1}$ |
|------------------------|------------------------|
| $\ell_1$ | $m_1$ | $\gamma^{m_1}_{\ell_1}$ | $v_p^1$ | $i$ | $\sigma$ | $\ell_2$ | $m_2$ | $\gamma^{m_2}_{\ell_2}$ | $v_p^2$ | $i$ | $\sigma$ |
| 3 | 1 | 0.92 | 0.097 | 55° | 10.1 | 3 | 1 | 1.23 | 0.023 | 56° | 10.0 |
| 4 | 1 | 0.93 | 0.091 | 32° | 8.4 | 4 | 1 | 1.31 | 0.025 | 66° | 5.0 |
| 3 | 2 | 0.97 | 0.108 | 82° | 10.1 | 1 | 1 | 1.32 | 0.030 | 29° | 4.9 |
| 2 | 1 | 1.01 | 0.093 | 83° | 10.6 | 2 | 1 | 1.36 | 0.008 | 56° | 8.2 |
| 2 | 2 | 1.02 | 0.080 | 33° | 4.8 | 2 | 2 | 1.40 | 0.019 | 32° | 4.9 |
candidate mode \((\ell, m)\). We have used the upper limits for the 
K-values as input numbers for the discriminant. In the case 
of V398 Aur, the solution is independent of this choice 
since the discriminant searches the most likely amplitude 
of the transverse velocity \(K v_i\); a lower input K gives a 
higher \(v_{\sin i}\)-value and vice versa. The adopted K-value is 
important when the radial and transverse velocities are of the 
same order of magnitude. The results of the discriminant 
for \(v_{\sin i} = 17.8 \, \text{km s}^{-1}\) are listed in Table 4. We only give 
the best solutions in parameter space. It is seen from the 
table that the two frequencies give rise to an almost identical 
list of most likely modes. This situation is comparable to 
the results found for the \(\beta\) Cephei star \(\beta\) CMa (Aerts et al. 1994) and are interpreted in terms of the presence of 
two modes with identical degree \(\ell\) but opposite azimuthal 
numbers \(m\). The combination of two \(\ell = 3\) modes, one with 
\(m = +1\) and one with \(m = -1\), seems most likely here, 
since the inclination and intrinsic widths fits nicely for such 
a combination, while this is less the case for the \(\ell = 4\) or \(\ell = 3\) solutions. Nevertheless, we cannot exclude the possibility of 
other combinations of modes given in Table 4 since the error 
on the inclination can be quite large. It does seem fair to 
conclude that an inclination can be quite large. It does seem fair to 
consider this situation to be confirmed by a comparison of the results of the discriminant 
for \(v_{\sin i} = 15 \, \text{km s}^{-1}\). The results are comparable 
to the ones given in Table 4, but with slightly different values of the inclination, the amplitude, and the intrinsic width.

The question then arises if the two modes can be due to 
rotational splitting. Dziembiowski & Goode (1992, equations 
22 and 117) give an expression for the observed frequencies 
\(f\) expected from rotational splitting of a mode \((\ell, m)\) in the 
case of rigid rotation and in the limit of high-order \(g\)-modes:

\[
f = f_0 - m\Omega \left(1 - \frac{1}{\ell(\ell + 1)}\right) - \frac{m^2\Omega^2}{4(\ell + 1)(2\ell(\ell + 1) - 3) - 9\ell\ell(\ell + 1)^2(4\ell(\ell + 1) - 3)}
\]

where \(f_0\) is the frequency in the corotating frame in the case 
of the absence of rotation. This formula is derived under the 
assumption that \(\Omega < f_0\). Uniform spacing occurs if the 
second-order term in \(\Omega\) is negligible. If we assume that this 
is the case, then \(f_0 = 0.571 \, \text{d}^{-1}\). If the \(\ell = 3, m = +1\) solution 
belongs to \(f_2\) and the \(\ell = 3, m = -1\) one to \(f_1\), then we obtain this \(f_0\) from \(f_2\) and from \(f_1\) for \(\Omega = 0.245 \, \text{d}^{-1}\). 
Assuming again that \(v_{\sin i} = 17.8 \, \text{km s}^{-1}\) and a stellar radius 
of 1.64 \(R_\odot\) (Mantegazza et al. 1994), such a rotation frequency 
corresponds to an inclination angle of 61°, a value 
fully in agreement with the ones derived from the two discriminants. 
For \(f_0 = 0.571 \, \text{d}^{-1}\), the second-order term in equation 4 amounts to only 0.008 \, \text{d}^{-1}. None of the other 
combinations of \(\ell_1 = \ell_2, m_1 = -m_2\) listed in Table 4 have an 
inclination that is close to 61° for both modes. We conclude that the 
two modes of V398 Aur can be due to rotational 
splitting of an \(\ell = 3\) mode. Unfortunately, the suggestion of 
Krisiunčius et al. (1995) that the “true” projected rotational 
velocity may be less than the observed mean value cannot 
be confirmed by a comparison of the results of the discriminants 
for different \(v_{\sin i}\)-values, because the uncertainty on the 
inclination is unknown. The true ratios of the horizontal 
to vertical velocity amplitudes for uniform splitting of the 
\(\ell = 3\) mode are \(K_1 \in [83.94], K_2 \in [72.82]\), There is no 
evidence of the presence of other frequency components of the 
uniform rotational splitting belonging to \(\ell = 3\) in the photometry.

Assuming a rotation frequency \(\Omega\) of 0.245 \, \text{d}^{-1} and a 
frequency \(f_0 = 0.571 \, \text{d}^{-1}\) as found in the case of uniform 
rotational splitting of the two \(\ell = 3\) modes, we obtain 
\(\Omega/f_0 = 43\%\). With such a high ratio, the frequency-splitting 
formula given in equation 4 may not be very accurate, but 
it is the best one available. Also, the velocity field of an 
NRP can no longer be described in terms of one \((\ell, m)\)-value, 
because the Coriolis force and the centrifugal forces are not negligible. 
The Coriolis force induces toroidal correction terms described by an \(Y_{3}^{+1}\) and an \(Y_{3}^{-1}\) spherical harmonic 
in the case of an NRP with wavenumbers \((\ell, m)\) in the non-rotating case (see e.g. Aerts & Waelkens 1993). 
It is clear that our results found from the moment method, 
which does not take into account these corrections, have to 
be interpreted with caution. It is possible that some of the 
solutions with \(|m| = 1\) found by the discriminant are mani-
festations of the toroidal corrections since the latter have the 
same azimuthal number as the non-rotation spherical harmonic. 
If this is correct, then it would be, as far as we know, 
the first detection of toroidal corrections in a real star.

In order to have an idea of the line-profile variations 
(LPVs) expected for the combination of the two \(\ell = 3\) modes 
as found by the discriminant, we have generated theoretical 
LPVs with the code presented by Aerts & Waelkens (1993). 
This code takes into account the toroidal correction terms 
due to the Coriolis force by means of a perturbation analysis 
assuming that \(\Omega/f_0 \ll 1\), but not those related to the 
centrifugal forces. As far as we know, no line-profile code 
is available that does take into account the latter effects. 
Because of the large \(\Omega/f_0\)-value, it is questionable if the 
perturbation analysis is accurate. With this in mind, we do 
not expect perfect fits, the more so since our code does not 
include equivalent width variations and assumes an intrin-
sic Gaussian that is time-independent. It is clear that these 
conditions are not exactly fulfilled. Furthermore, the 
effective temperature of the hemisphere of the star facing us 
is not constant. From the variations of \(B-V\) color of the star 
(Table 3 and Fig 6 of Krisiunčius et al. 1995) and the rate 
of change of temperature of mid-main sequence stars as a 
function of \(B-V\) (Allen 1973), we estimate for V398 Aur that 
\(T_{\text{eff}}\) varies by \(\approx 100 \, \text{K}\). Nevertheless, we can be con-
dent that the solution proposed by the discriminant is good 
if the obtained theoretical LPVs are compatible with the 
observed ones.

The width of the intrinsic profile found by the discriminant 
must be an overestimate of the true value, since the 
velocity contributions of the toroidal terms were neglected 
in the non-rotating model. This width can, however, easily 
be found from the observed LPVs once a mode identification 
has been obtained. We have found that \(\sigma = 5 \, \text{km s}^{-1}\) is a 
more realistic value. Also the pulsation amplitudes will be 
overestimated by the discriminant for the same reason. 
On the other hand, the K-values used were also overestimated. 
We have generated theoretical LPVs with all the parame-
ters except the \(v_{\sin i}\)-values fixed (we have again chosen the 
upper limit for both K-values of zeroth-order in \(\Omega\), but as 
explained above, this choice does not matter). In this way, 
a better estimate of the \(v_{\sin i}\)-values could be obtained for the 
two modes. Finally, the phase shift between the two modes

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was determined from the first moment. The results of our theoretically generated LPVs for the best choice of the pulsation amplitudes are shown in Fig. 2, where we compare them with some of the observed profiles. We show 10 arbitrarily chosen observed profiles that are spread in phase according to $f_2$ with respect to an arbitrary reference epoch. The solid lines are the profiles obtained with the code that takes into account the toroidal terms ($\Omega/f_0 = 0.43$), while the dashed profiles are those obtained with a model that neglects the rotation effect (i.e. those with $\Omega/f_0 = 0.0$). As other input parameters, we used:

\[
\begin{align*}
& f_1 = 0.795 \text{ d}^{-1}, \quad \ell_1 = 3, \quad m_1 = -1, \\
& \nu^1_p = 0.100 \text{ km s}^{-1}, \quad K_1 = 94, \quad K_v^1 = 9.4 \text{ km s}^{-1}, \\
& f_2 = 0.346 \text{ d}^{-1}, \quad \ell_2 = 3, \quad m_2 = +1, \\
& \nu^2_p = 0.025 \text{ km s}^{-1}, \quad K_2 = 82, \quad K_v^2 = 2.1 \text{ km s}^{-1}, \\
& v \sin i = 17.8 \text{ km s}^{-1}, \quad \sigma = 5 \text{ km s}^{-1}, \quad i = 60^\circ.
\end{align*}
\]

First of all, the observed and calculated profiles show the same global behavior such that our proposed solution of the velocity parameters listed in (3) remains valid. The variation in line depth is accounted for by some of the theoretical profiles, but not by all of them. The evolution of the two sets of theoretical profiles during the cycle are not too different from each other. It seems that the profiles without the toroidal terms are better during the first part of the cycle, while the others are better during the second part. The effect of the toroidal terms on LPVs depends completely on the kind of modes. It was studied by Aerts & Waelkens (1993) for some monoperiodic $p$-modes but has not yet been studied in the case of $g$-modes or multiperiodicity. A detailed study about this is currently being undertaken (Schrijvers et al., in preparation). The fact that the differences between the two sets of profiles are not very large makes us confident that the results of the discriminant are not too bad. We have also calculated theoretical profiles for the other candidate rotationally splitted modes listed in Table 1, i.e. the $\ell = 4, |m| = 1$ and the $\ell = 2, |m| = 1$ solution. In doing so, we have taken the average of the two listed $r$-values. In both cases, the correspondence between the observed profiles and the theoretical ones is worse compared to the $\ell = 3, |m| = 1$ case described by (4).

Figure 2. A comparison between observed and theoretical LPVs for the parameters listed in (3). The dots are actual observed points from the cross-correlation profiles obtained by R. F. Griffin. The solid lines are profiles based on a theoretical model taking toroidal terms into account. The dashed lines are profiles for a theoretical model that neglects rotational effects.
calculated the velocities in the direction of the observer for both modes, taking into account the toroidal corrections. We indeed find that, although the retrograde mode has a four times smaller spheroidal zero-rotation amplitude, its total velocity component in the line of sight becomes larger due to the toroidal correction terms. The maximum velocity reached by the retrograde mode is a factor 1.8 larger than the one of the prograde mode.

It thus seems possible that the toroidal corrections that appear due to influence of the rotation on a spheroidal mode introduce a different photometric and spectroscopic behavior when both prograde and retrograde modes are involved. To our knowledge, this is the first detected consequence of the presence of such toroidal corrections. Our point of view could be further verified by means of an analysis of a large set of high-resolution, high S/N spectra of a star pulsating in both prograde and retrograde modes and having a large rotation frequency, i.e. having $\Omega/f_0 > 20\%$.

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REFERENCES

Aerts C., De Pauw M., Waelkens C., 1992, A&A, 266, 294
Aerts C., Waelkens C., 1993, A&A, 273, 135
Aerts C., Waelkens C., De Pauw M., 1994, A&A, 286, 136
Allen C. W., 1973, Astrophysical Quantities, 3rd edn. Athlone Press, London, p. 206
Balona L.A., 1986, MNRAS, 219, 111
Balona L. A., Krisciunas K., Cousins A. W. J., 1994, MNRAS, 270, 905
Baranne A., Mayor M., Poncet J. L., 1979, Vistas Astron., 23, 279
Benz W., Mayor M., 1981, A&A, 93, 235
Cousins A. W. J., Warren P. R., 1963, Mon. Notes Astron. Soc. S. Afr., 22, 65
Dziembowski W.A., Goode P.R., 1992, ApJ, 394, 670
Kazovarets E. V., Samus N. N., 1995, Inf. Bull. Variable Stars, No. 4140
Krisciunas K. et al., 1993, MNRAS, 263, 781
Krisciunas K., Griffin R. F., Guinan E. F., Ludeke K. D., McCook G. P., 1995, MNRAS, 273, 662
Krisciunas K., Handler G., 1995, Inf. Bull. Variable Stars, in press
Munteanuza L., Poretti E., Zerbi F. M., 1994, MNRAS, 270, 439
Mathias P., Aerts C., De Pauw M., Gillet D., Waelkens C., 1994, A&A, 283, 813
Pepper D. M., 1980, ARA&A, 18, 115
Stellingwerf R. F., 1978, ApJ, 224, 953
Waelkens C., 1991, A&A, 246, 453
Zerbi F. M. et al., 1996, MNRAS, in preparation

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