Distributed Source Coding Using Continuous-Valued Syndromes

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Abstract—This paper addresses the problem of coding a continuous random source correlated with another source which is only available at the decoder. The proposed approach is based on the extension of the channel coding concept of syndrome from the discrete into the continuous domain. If the correlation between the sources can be described by an additive Gaussian backward channel and capacity-achieving linear codes are employed, it is shown that the performance of the system is asymptotically close to the Wyner-Ziv bound. Even if such an additive channel is not Gaussian, the design procedure can fit the desired correlation and transmission rate. Experiments based on trellis-coded quantization show that the proposed system achieves a performance within 3-4 dB of the theoretical bound in the 0.5-3 bit/sample rate range for any Gaussian correlation, with a reasonable computational complexity.

Index Terms—Distributed source coding, rate-distortion with side information, Wyner-Ziv coding, continuous channels, AWGN channel coding, syndrome-based coding, trellis-coded quantization.

I. INTRODUCTION

DISTRIBUTED SOURCE CODING addresses the problem of coding the outcomes of multiple correlated sources independently, i.e. without allowing the respective encoders to communicate with each other. Theoretical results on this topic appeared in the seventies, and showed that, if the joint distribution characterizing the correlation structure is known, there is no loss in performance with respect to (w.r.t.) the case where the encoders can collaborate with each other. This result goes under the name of the Slepian-Wolf coding theorem [1], [2]. A related problem is coding with side information at the decoder, in which the outcomes of a source have to be encoded and decoded within a given distortion under the condition that a second correlated source is only available at the decoder. Theoretical performance bounds, given by Wyner and Ziv [3], show that in certain cases there is no loss in performance w.r.t. the case where the encoder can access the side information as well.

Encoding methods that approach these theoretical limits are hence highly desirable, and have many practical applications. For example, they can be employed in sensor networks [4], where communication between the nodes not only requires an elaborate intersensor network, but also may be limited by bandwidth constraints. In addition to this natural distributed source coding application, these coding methods can be also used in video coding, not only for independent encoding of multiple correlated video sensors [5], but mostly to reduce the encoding complexity w.r.t. the classical, non-distributed, video coding solutions [6], [7]. Hyperspectral image compression is yet another field where distributed source coding principles can be taken into account [8], [9], [10].

Despite the limits of distributed source coding were theoretically investigated more than thirty years ago, practical solutions to the problem have been proposed only in the last decade. All solutions stemmed from the connection that distributed source coding has to channel coding, as already pointed out by Wyner in its pioneering work [11]. Algorithms based on turbo and low-density parity-check (LDPC) codes [12], [13] have then appeared that approach the theoretical bounds in the discrete binary domain [14], [15].

To solve the coding with side information issue for continuous random sources, current proposals recast the problem into a binary domain or, at least, into a discrete domain, by using some kind of quantization. For example, in the distributed source coding using syndromes (DISCUS) system [16], [17], [18], the random variables are transformed into a discrete domain, in which cosets of some trellis codes are identified on top of the reconstruction codebook used by the quantizer, before actually coding them. The related idea of using nested codes for distributed source coding is discussed by Zamir et al. in [19], [20].

In this work it is proposed to solve the coding with side information problem for continuous random sources entirely in the continuous domain. By operating in this domain, the problem is recast into the traditional source coding problem of a continuous-valued syndrome that is closely related to the actual statistical correlation between the source and the side information. In a certain sense, the paper is an extension of the work in [19], since it is shown that, under some condition, it is not necessary for the codes to be nested for optimum performance, as first proved in [21].

The rest of the paper is organized as follows. In Section II a review on linear block codes and lattices is given, and the concept of syndrome is introduced. Section III discusses the duality between the problems of distributed source coding and of channel coding. The discussion is presented from a perspective that enables us to directly utilize the concepts introduced in Section II to solve the distributed source coding problem. This approach is presented in Section IV and the experimental results are discussed in Section V. Conclusions are drawn in Section VI.

II. LINEAR BLOCK CODES AND LATTICES

In this section, the basic properties of linear block codes and lattices are reviewed. In particular, the focus is on the concept
of syndrome. After reading this section, it should be clear that the definition of such a concept catches some dualities between the two algebraic structures.

A. Linear Block Codes and Syndromes

Consider the Galois field $\mathbb{F} = GF(q)$, with $q = p^m$ for some prime number $p \geq 2$ and some $m \in \{1, 2, \ldots\}$. An $(n, k)$-linear block code $C$ over $\mathbb{F}$ is a $k$-dimensional subspace of the vector space $\mathbb{F}^n$ over $\mathbb{F}$. Equivalently,

$$ C \triangleq \left\{ \sum_{i=1}^{k} a_i g_i : a_i \in \mathbb{F}, \forall i = 1, 2, \ldots, k \right\}, \quad (1) $$

for some set $G = \{g_i\}_{i=1,2,\ldots,k}$ of linearly independent vectors on $\mathbb{F}^n$. It is worth noting that an $(n', k', m)$-convolutional code over $\mathbb{F}$, for a sufficiently large number $L$ of consecutive $n'$-length output codewords, can be as well described by (1), with $n = n'L$ and $k = k'L - m$ [22].

Being a subgroup of the additive group $(\mathbb{F}^n, +)$, $C$ induces a partition of $\mathbb{F}^n$ into cosets. In particular, the set of all cosets is the quotient group $\mathbb{F}^n/C$, and, since the cardinality of $C$ is $|C| = q^k$, there are exactly $|\mathbb{F}^n|/|C| = q^n/q^k = q^{n-k}$ of them. To identify the coset to which an element of $\mathbb{F}^n$ belongs, consider any linear application of $\mathbb{F}^n$ with kernel equal to $C$ into some other $(n - k)$-dimensional vector space over $\mathbb{F}$. Then, such a homomorphism of $(\mathbb{F}^n, +)$ automatically assigns a distinctive vector, called syndrome, to the elements belonging to the same coset.

A simple way to construct such a linear application is as follows. Consider the orthogonal complement $C^\perp$ of $C$ into $\mathbb{F}^n$ (i.e. the dual code of $C$, with the canonical inner product on $\mathbb{F}^n$). Since $C^\perp$ is an $(n - k)$-dimensional subspace of $\mathbb{F}^n$, it is generated by some set $H = \{h_i\}_{i=1,2,\ldots,k}$ of linearly independent vectors on $\mathbb{F}^n$. Associate with each $a \in \mathbb{F}^n$ the vector $s_H(a)$ on $\mathbb{F}^{n-k}$ whose canonical coordinates are the inner products $\langle a, h_i \rangle$, $i = 1, 2, \ldots, n - k$. This vector represents the syndrome of $a$ relative to the $(n - k) \times n$ parity check matrix $H$ (whose rows contain the canonical coordinates of each $h_i$). In addition, it is in principle straightforward to compute $s_H(a)$ since it can be simply obtained by matrix multiplication with $H^T$.

The syndrome plays an important role in minimum-distance decoding (using some distance function such as the Hamming distance) when it can be univocally associated with a minimum-weight element in each coset of $\mathbb{F}^n/C$. In this case the minimum-weight element that by subtraction leads to a codeword of $C$ can be identified from the syndrome of any received codeword. Linear codes with good distance properties permit to associate a unique minimum-weight element with most of the syndromes. When a syndrome without this property occurs, an error is detected but not corrected.

B. Lattices and Continuous-Valued Syndromes

Consider now the additive group of real numbers $(\mathbb{R}, +)$. An $n$-dimensional lattice $\Lambda$ is a discrete subgroup of the Euclidean space $\mathbb{R}^n$ that spans $\mathbb{R}^n$ itself. The group $(\Lambda, +)$ can be equivalently defined as

$$ \Lambda \triangleq \left\{ \sum_{i=1}^{n} a_i v_i : a_i \in \mathbb{Z}, \forall i = 1, 2, \ldots, n \right\}, $$

for some set $\{v_i\}_{i=1,2,\ldots,n}$ of linearly independent vectors on $\mathbb{R}^n$.

Through the quotient group $\mathbb{R}^n/\Lambda$, any lattice induces a partition of $\mathbb{R}^n$ into cosets. For practical reasons, it is useful to identify each coset using one of its elements. Hence, consider an injection $l : \mathbb{R}^n/\Lambda \to \mathbb{R}^n$ such that $l(A) \in A$, $\forall A \subseteq \mathbb{R}^n/\Lambda$, and define labeling function of $\mathbb{R}^n/\Lambda$ any such function. The fundamental region $R_l(\Lambda)$ of $\Lambda$ induced by $l$ is then defined as the image of $l$, i.e. $R_l(\Lambda) \triangleq l(\mathbb{R}^n/\Lambda) \subseteq \mathbb{R}^n$. Clearly, the set of translates $\{R_l(\Lambda) + b : b \in \Lambda\}$ forms a regular tessellation of $\mathbb{R}^n$, and hence the volume of $R_l(\Lambda)$ equals $V(\Lambda)$, the volume of $n$-space per point of $\Lambda$.

Since $\mathbb{R}^n$ is a normed space under the usual $L^2$-norm, it is common to take elements with minimum norm as coset representatives. A labeling function $V$ such that

$$ |V(A)| \leq |a|, \forall A \subseteq \mathbb{R}^n/\Lambda, a \in A $$

defines then a fundamental region $R_V(\Lambda)$ known as fundamental Voronoi region. The corresponding tessellation of $\mathbb{R}^n$ consists of decision regions for a minimum-distance quantizer (or decoder) that uses $\Lambda$ as codebook.

The role of the labeling function becomes evident when the induced group structure on $R_l(\Lambda)$ is considered. The labeling function is in fact invertible in $R_l(\Lambda)$, and hence, upon defining the sum operation in $R_l(\Lambda)$ as

$$ \alpha + \beta \triangleq l(l^{-1}(\alpha) + l^{-1}(\beta)), \forall \alpha, \beta \in R_l(\Lambda), \quad (2) $$

$l$ is an isomorphism. Denote with $\nu$ the natural homomorphism $\nu : \mathbb{R}^n \to \mathbb{R}^n/\Lambda$, and define the function $s_l \triangleq l \circ \nu : \mathbb{R}^n \to R_l(\Lambda)$. Once a labeling function $l$ is defined, this homomorphism identifies indeed the coset to which any element of $\mathbb{R}^n$ belongs. As a consequence, $s_l(a)$ represents the continuous-valued syndrome of $a \in \mathbb{R}^n$, by analogy with the role of the traditional syndrome in linear codes.

The continuous-valued syndrome satisfies the following properties:

$$ s_l(a + \lambda) = s_l(a), \forall a \in \mathbb{R}^n, \lambda \in \Lambda; \quad (3) $$

$$ s_l(a) = a, \forall a \in R_l(\Lambda); \quad (4) $$

$$ s_l(s_l(a)) = s_l(a), \forall a \in \mathbb{R}^n; \quad (5) $$

$$ s_l(a + b) = s_l(a) + s_l(b), \forall a, b \in \mathbb{R}^n. \quad (6) $$

In particular, (3) and (4) follow directly from the definition of $s_l$, and state that $s_l$ is periodic and that the restriction $s_l|_{R_l(\Lambda)} : R_l(\Lambda) \to R_l(\Lambda)$ of $s_l$ is an identity respectively; (5) follows directly from (4), and states that $s_l$ is idempotent, i.e. that $s_l^n = s_l$, $\forall m = 1, 2, \ldots$; (6), in which the sum on the right-hand side is intended as defined in (2), follows from the fact that $s_l$ is a homomorphism. As a remark, $\tilde{s}_l : \mathbb{R}^n \to (\mathbb{R}^n, +)$ is not a homomorphism since $R_l(\Lambda)$

1For the sake of clarity, the continuous-valued syndrome $s_l(\cdot)$, when intended as belonging to $(\mathbb{R}^n, +)$ rather than $(R_l(\Lambda), +)$, will be hereinafter indicated as $\tilde{s}_l(\cdot)$.
is not a subgroup of \((\mathbb{R}^n, +)\). Hence, in general, if both the sums are taken in \((\mathbb{R}^n, +)\), \(\tilde{s}_l(a + b) \neq \tilde{s}_l(a) + \tilde{s}_l(b)\). However, it is straightforward to show that
\[
\tilde{s}_l(a + b) = \tilde{s}_l(\tilde{s}_l(a) + \tilde{s}_l(b)), \quad \forall a, b \in \mathbb{R}^n,
\]
which in turn shows that \(\tilde{s}_l(a + b) = \tilde{s}_l(a) + \tilde{s}_l(b)\) if and only if \(\tilde{s}_l(a) + \tilde{s}_l(b) \in R_l(\Lambda)\).

In order to evaluate the continuous-valued syndrome for a given \(a \in \mathbb{R}^n\), it is immediate to verify that the syndrome \(s_V(a) \in R_V(\Lambda)\) relative to a fundamental Voronoi region can be obtained as quantization error of a minimum-distance quantizer that uses \(\Lambda\) as codebook. In particular, defining
\[
Q_\Lambda(a) \triangleq \lambda \in \Lambda : |\lambda - a| \leq |\gamma - a|, \quad \forall \gamma \in \Lambda
\]
as the closest lattice point to \(a\) (with the further condition \((a - \lambda) \in R_V(\Lambda)\) in case of ambiguity), we have
\[
\tilde{s}_V(a) = a - Q_\Lambda(a).
\]
In this case, as the traditional syndrome, the continuous-valued syndrome of \(a\) identifies the minimum-norm element that, subtracted to \(a\), leads to an element (the closest) of \(\Lambda\).

III. DISTRIBUTED SOURCE CODING AND CHANNEL CODING

After a brief review of the well known concepts of distributed source coding and coding with side information, this section discusses how these problems are intertwined with the more traditional channel coding problem. In particular, this fact is due to the existence of a virtual correlation channel, which can be seen both as forward or backward channel.

A. Distributed Source Coding and Coding with Side Information

The notion of distributed source coding refers to the problem of coding the outcomes of a random vector \(X = [X_i]_{i=1,2,...,N}\) by using \(N\) independent encoders that cannot collaborate with each other, but whose respective \(N\) outputs are jointly fed to a single decoder. Assume that the random variables \(X_i\) have a discrete alphabet. Then, by the source coding theorem \([1]\), perfect reconstruction is asymptotically achieved if each encoder can communicate with a rate greater or equal than the entropy rate \(H_{\infty}(X_i)\) of \(X_i\). However, because of joint decoding, this is no longer a necessary condition. Slepian and Wolf showed that in case of independent and identically distributed outcomes with \(N = 2\) (see Fig. 1), the necessary (and sufficient) conditions are \([2]\)
\[
R_1 \geq H(X_1|X_2), \quad R_2 \geq H(X_2|X_1), \quad R_1 + R_2 \geq H(X_1, X_2),
\]
where \(H(\cdot|\cdot)\) and \(H(\cdot,\cdot)\) denote the conditional and the joint entropy respectively.

\(^2\)For the same reason, while the identity function \(s_l|_{R_l(\Lambda)}\) is obviously a homomorphism \(R_l(\Lambda) \rightarrow (R_l(\Lambda), +)\) (in particular, an automorphism), it is not a homomorphism \(R_l(\Lambda) \rightarrow (\mathbb{R}^n, +)\).

In such a case, denote \(X_1, X_2,\) and \(R_1\) with \(X, Y,\) and \(R\) respectively. If \(R_2 \geq H(Y)\), i.e., if the decoder can perfectly reconstruct the outcomes of \(Y\) upon receiving the output of the second encoder (see Fig. 2), the first encoder just needs to communicate with a rate equal to \(H(X|Y) \leq H(X)\) for perfect reconstruction, since the shaded decoder in Fig. 2 can rely on the side information \(Y\) correlated with \(X\). Given a certain upper bound \(D\) to the distortion between \(\hat{X}\) and \(X\) (that can be discrete or continuous random variables), the general problem of finding the minimum achievable value for \(R\) goes under the name of coding with side information at the decoder. This problem was solved by Wyner and Ziv \([3]\), and it turned out that the obtained rate-distortion function \(R^{*}_{X|Y}(D)\) is in general greater or equal than the rate-distortion function \(R^{*}_{X|Y}(D)\) corresponding to the case where \(Y\) is as well accessible by the encoder.

B. Connections to Channel Coding

Since the ultimate performance limits shown in \([2]\), \([3]\) were obtained with non-constructive proofs, effective, practical solutions have not appeared until recently. Current approaches, that are getting closer and closer to the theoretical limits \([4]\), have stemmed from the connection that distributed source coding has to channel coding \([11]\).

In fact, the statistical dependence between \(X\) and \(Y\) can be exactly characterized in terms of a virtual correlation channel. This channel can be defined in two equivalent ways, as follows.

1) **Forward channel** (FCH): the complete statistical description of the tuple \((X, Y)\) is given by the probability mass function \(p_\cdot\) \(p(x)\) and by the conditional pmf \(p(y|x)\).
   Hence, \(Y\) can be seen as the output of the stochastic channel described by \(p(y|x)\) into which \(X\), distributed according to \(p(x)\), is fed as input.

2) **Backward channel** (BCH): alternatively, the complete statistical description of \((X, Y)\) can be given by the

\(^3\)If the random variables are not discrete, the probability mass functions \(p(\cdot)\) can be replaced with probability density functions (pdf) \(f(\cdot)\).

Fig. 1. Distributed coding of two random sources.

Fig. 2. Coding with side information at the decoder.
pmf \( p(y) \) and by the conditional pmf \( p(x|y) \). In this characterization, \( X \) is seen as the output of the stochastic channel described by \( p(x|y) \) whose input is \( Y \), distributed according to \( p(y) \).

In literature the FCH interpretation is commonly given. In the coding with side information problem, according to this interpretation, a noisy observation \( Y \) of the unknown \( X \) is given to the decoder together with some prior information about \( X \), provided by the encoder. Being aware of the FCH statistics, the encoder must hence provide to the decoder the smallest number of independent constraints which allow for reconstruction within the desired distortion.

In practice, in the discrete case the alphabet of both \( X \) and \( Y \) coincide with a Galois field \( \mathbb{F} \), and the FCH can be described as additive channel. This implies that \( Y = X + N_f \), with \( N_f \) a random variable (noise) independent of \( X \) and distributed according to some pmf \( p(n_f) \) (see Fig. 3). The connection to channel coding then comes from the fact that there may exist a zero-error \((n, k)\)-linear code which achieves the capacity \( C = \log_2 |\mathbb{F}| - H(N_f) \) (bits per channel use) of the FCH, i.e. such that \( k/n \cdot \log_2 |\mathbb{F}| = C \). In this case \( X \) can be perfectly reconstructed once the coset to which each successive \( n \)-tuple of outcomes of \( X \) belongs is signalled to the decoder. This is simply accomplished by decoding \( Y \) into the signalled coset. If \( X \) is uniformly distributed on \( \mathbb{F} \) this communication requires a rate \( R = (n-k)/n \cdot \log_2 |\mathbb{F}| \) (bits per channel use), i.e. it achieves the Slepian-Wolf limit \( H(X|Y) = H(Y|X) = H(N_f) = \log_2 |\mathbb{F}| - C \).

As discussed in Section 4, the syndrome \( s_H(x) \) can be used to inform the decoder about the coset to which the unknown belongs. Details and experimental results regarding this distributed coding strategy called distributed source coding using syndromes (DISCUS) can be found in [4], [16], [17], [18].

Once the decoder has the side information, the remaining information needed for perfect reconstruction should actually regard the noise only. The fact that in the previous approach the communication rate equals exactly \( R = H(N_f) \) seems to confirm this observation. However, within the additive FCH interpretation there is no apparent relation between the noise and the output of the encoder, which is indeed driven by a signal perfectly independent of the noise itself.

According to the BCH interpretation, instead, the input of the encoder can be seen as a direct observation of the unknown noise introduced by the channel, but measured with some offset (that is input into the BCH) known at the decoder only. The encoder, hence, given the BCH statistics, should provide the decoder with the smallest amount of information that allows for reconstruction of the unknown (i.e. of the noise) within the desired distortion. Obviously, it should operate in a way such that any allowable offset does not increase the amount of information needed.

Again, in the discrete case the BCH can be usually described as additive channel over the finite field \( \mathbb{F} \). This implies that \( X = Y + N \) (the term offset used for the side information becomes now clearer), with \( N \) a noise independent of \( Y \) and distributed according to some pmf \( p(n) \) (see Fig. 4). If there exist a zero-error \((n, k)\)-linear code which achieves the capacity \( C = \log_2 |\mathbb{F}| - H(N) \) (bits per channel use) of the BCH (i.e. such that \( k/n \cdot \log_2 |\mathbb{F}| = C \)), \( X \) can be perfectly reconstructed once the coset to which each successive \( n \)-tuple of outcomes of \( X \) belongs is signalled to the decoder. By knowing the offset, in fact, the decoder easily computes first the coset to which the noise belongs, and then the noise itself. Since this communication requires a rate up to \( R = (n-k)/n \cdot \log_2 |\mathbb{F}| \) (bits per channel use), the Slepian-Wolf limit \( H(X|Y) = H(N) = \log_2 |\mathbb{F}| - C \) is exactly achieved.

Apparently, in the discrete case examined above, if both the additive FCH and BCH interpretations are applicable it is not yet clear why one should prefer one over the other. In this paper it is claimed that the BCH interpretation is preferable because of the following.

- The existence of a good linear code for the additive FCH does not guarantee the achievability of the Slepian-Wolf limit when \( X \) is not uniformly distributed. For example, it is always possible to assign non-uniform probabilities \( p(x) \) in a way such that each coset is still equiprobable. Consequently, the transmission rate needed will still equal \( H(N_f) \), even if in this case \( H(Y) > H(X) \) implies \( H(X|Y) = X(Y|X) + H(X) - H(Y|X) < H(Y|X) = H(N_f) \).
- If a good linear code for the additive BCH exists, then the Slepian-Wolf limit \( H(X|Y) = H(N) \) is always achieved, independently of the pmf of \( Y \). Moreover, during the design of the distributed coding system, it is not necessary to know \( p(y) \), but only \( p(n) \), i.e. the BCH statistics.
- Finding a good linear code for the additive BCH is equivalent to looking for a good linear encoder for \( N \).
Within the BCH interpretation, the encoder design is hence directly tailored to the noise $N_f$. Then, since encoding is a linear operation, any offset known by the decoder will not harm the performance of the system. In fact, the decoder can easily remove its effect on the information received by the encoder, which is still uniformly distributed on $\mathbb{F}^{n-k}$, and hence requires the same transmission rate.

It is worth to point out that in the discrete and finite case both forward and backward interpretations give additive channels (with input-noise independence) if and only if $X$ and $Y$ are uniformly distributed, unless $N_f$ (or $N$) is pseudo-aleatory (in which case the channel is deterministic and one-to-one). In particular, if an additive FCH exists and $X$ is uniformly distributed, then $N = -N_f$ is independent of $Y$, which in turn is uniformly distributed. Viceversa, if an additive BCH exists and $Y$ is uniformly distributed, then $N_f = -N$ is independent of $X$, which in turn is uniformly distributed.

In practice, if $N = -N_f$, the two schemes in Fig. 3 and Fig. 4 are equivalent. The point here is that any good linear code for the additive FCH turns out to be equally good for the BCH. In other words, the DISCUS system that is tailored for an additive FCH with a uniformly distributed source is also optimal for the corresponding BCH with any desired side information distribution. Even if in the discrete case it is not necessary to explicitly design codes using the BCH interpretation, this approach will turn out to be more useful for the continuous channels discussed in the following.

C. Continuous Correlation Channels

Assuming that $X$ and $Y$ are continuous random variables that take values on the Euclidean space $\mathbb{R}$, the virtual channel (FCH or BCH) that describes the mutual correlation is a continuous channel. In this case, in general, as $D \to 0^+$, $R_{X|Y}(D) \geq R_{X|Y}(D) \to +\infty$, i.e. in the problem of coding with side information perfect reconstruction is practically not achievable, unless we allow the encoder to communicate at an infinite rate. In the following, assuming for a while that this is possible, the generalization to this domain of the results of the previous sub-section is discussed.

In particular, assume that the FCH is actually an additive channel such that $Y_f = X + N_f$, with $N_f$ independent of $X$ (see Fig. 5). Suppose that a lattice $\Lambda$ exists in $\mathbb{R}^n$ whose intersection with a bounded set $A \subset \mathbb{R}^n$ defines a capacity-achieving code $\Lambda_h$ (subject to some cost constraint) for the FCH. The volume of $n$-space per point of $\Lambda$ approximates the volume of the typical set of $N_f$, because in each coset of $\mathbb{R}^n/\Lambda$ there is a unique noise representative. Perfect reconstruction (with probability close to one) is achieved by channel-decoding $Y_f$ into the coset signalled to the decoder as the coset to which any $n$-tuple of outcomes of $X$ belongs. It would be for example possible to reconstruct $X$ with negligible probability of error by transmitting a continuous-valued syndrome $s_t(x)$ as defined in Section II-B.

However, differently from the discrete case, $Y_f$ and $N_f$ are neither independent nor uncorrelated ($E[Y_f N_f] = E[N_f^2] \neq 0$), not even when the pdf of $X$ achieves channel capacity. Hence, the information brought by $Y_f$ about the noise may relax the need for transmission of as many different messages as the possible realizations of $N_f$. In the example of an additive Gaussian channel with $N_f \in \mathcal{N}(0, \sigma_f^2)$, the volume of $n$-space per point of $\Lambda$ is approximately $V(\Lambda) \approx 2^{n h_C(\sigma_f^2)}$, where $h_C(\cdot)$ and $h_G(\cdot)$ denote the differential entropy and the differential entropy of the Gaussian random variable (i.e. the upper bound of the differential entropy under the specified power constraint) respectively. Invoking a sphere-packing argument, if $X \in \mathcal{N}(0, \sigma_x^2)$ (which achieves the capacity under the constraint $E[X^2] \leq \sigma_x^2$) $Y_f$ is used at the decoder to discriminate, on average, between about $2^{n h_C(\sigma_f^2)}/V(\Lambda) \approx 2^{\frac{n}{2} \log_2(\frac{\sigma_x^2}{\sigma_f^2})}$ different $n$-tuples. But actually the number of distinct messages per $n$ channel uses that could be reliably transmitted through the FCH under the same power constraint is $2^{n C} \approx 2^{\frac{n}{2} \log_2(1+\sigma_x^2/\sigma_f^2)} > 2^{\frac{n}{2} \log_2(\sigma_x^2/\sigma_f^2)}$.

In other words, there must exist a denser code $\Lambda_h$ (which is assumed to be a bounded subset of a lattice $\Lambda' \subset \mathbb{R}^n$) such that the relative coset information is still sufficient for perfect reconstruction. Since at least a coset of $\mathbb{R}^n/\Lambda'$ contains more than one representative of $N_f$, for this code the algorithm sketched above (channel-decoding of $Y_f$ into the coset signalled by the encoder) no longer leads to perfect reconstruction.

As noted in the previous section, a slightly different perspective is possible when the correlation is described in terms of a BCH. In the continuous domain, under the condition that $X$ and $N_f$ are uncorrelated, any additive FCH can be interpreted
as additive BCH as follows. Define

\[ Y \triangleq \alpha Y_f, \quad (9) \]

with

\[ \alpha = \frac{E[X^2]}{E[X^2] + E[N_f^2]} < 1, \]

as the optimum linear predictor of \( X \) from \( Y_f \). Consequently,

\[ N \triangleq X - Y = X - \alpha Y_f = (1 - \alpha)X - \alpha N_f \]

is uncorrelated with \( Y \), and \( X = Y + N \). Hence, if \( Y_f \) is multiplied by \( \alpha \), then the forward channel of Fig. 5 can be interpreted as a backward channel (see Fig. 6), with input, noise and output variances equal to

\[ \begin{align*}
E[Y^2] &= \alpha^2 E[Y_f^2] = \alpha E[X^2] \\
E[N^2] &= E[X^2] - E[Y^2] = (1 - \alpha)E[X^2] = \alpha E[N_f^2] \\
E[X^2] &= \alpha (E[X_f^2] + E[N_f^2]) = \alpha E[Y_f^2].
\end{align*} \]

Moreover, if \( X \) and \( N_f \) are independent and \( X \) is distributed according to the pdf that achieves the FCH capacity, then it is reasonable for \( N \) to be independent of \( Y \). This is certainly true for additive Gaussian channels.

Using this interpretation, and assuming that the pdf of \( X \) is the one that achieves the FCH capacity, a code \( \Lambda'_b \) is found such that signalling the coset of \( n \cdot \Lambda' \) to which any \( n \)-tuple of outcomes of \( X \) belongs is sufficient for perfect reconstruction at the decoder. In the Gaussian case with \( N_f \in \mathcal{N}(0, \sigma_f^2) \), for example, the scaled code \( \Lambda'_b = \alpha^{1/2} \Lambda_b \) can be used. Observe that it represents a capacity-achieving code for the BCH with the power constraint \( E[Y^2] \leq \alpha \sigma_f^2 \) (that \( Y \) as defined in (9) satisfies), which has the same capacity \( C = \frac{1}{2} \log_2(1 + \sigma_f^2/\sigma_y^2) \) of the FCH with the power constraint \( E[X^2] \leq \sigma_x^2 \). Hence, with probability close to one, any other \( n \)-dimensional realization of the noise \( N \) belongs to a different coset of \( \mathbb{R}^n/\Lambda' \), such that in each coset there is a unique noise representative. Being independent of \( N \), \( Y(Y_f) \) does not give any other useful information about \( N \). As in the discrete case, since the information about the noise can be linearly formed at the encoder with an offset known at the decoder, perfect reconstruction is achieved. In particular, this is accomplished by channel-decoding \( Y \) (in place of \( Y_f \)) at the decoder. Now that the volume of \( n \)-space per point of \( \Lambda' \) is approximately \( V(\Lambda') \approx 2^{nhc(\sigma_f^2)} \), it turns out that the number of distinct messages per \( n \) channel uses that could be reliably transmitted through the correlation channel is correctly \( 2^{nhc(\sigma_f^2)/V(\Lambda')} \approx 2^{\log_2(\alpha^{-1}\sigma_f^2/\sigma_y^2)} = 2^n C \).

While in the discrete and finite case any good linear code for the additive FCH turns out to be equally good for the BCH, in the continuous case the coset information sent to the decoder should be relative to a code, tailored to the BCH statistics, which is denser than the former. This approach will be discussed in the next section, where it will be shown that if a good channel code is available for the additive Gaussian BCH, then the rate-distortion function \( R^*_X(Y)(D) \) can be approximately achieved by sending a quantized version of the continuous-valued syndrome, independently of the pdf of \( Y \).

IV. CONTINUOUS-VALUED SYNDROME-BASED DISTRIBUTED SOURCE CODING

In the last fifteen years, the binary codes that have been introduced in the literature, such as the turbo-codes [12] and the low-density parity-check (LDPC) codes [13], come very close to the channel capacity of the binary additive channel with a reasonable computational complexity. But, in many distributed source coding applications, \( X \) and \( Y \) take binary values, and their correlation can be analogously described by a virtual additive channel. Hence, since soon after the discovery of the strong connection to channel coding discussed in Section III-B, several approaches have been successfully proposed for the problem of distributed source coding based on these linear channel codes [14], [15].

Following the success of these coding algorithms in the binary domain, if continuous (or discrete, but non-binary) random variables have to be coded, a transformation into the binary domain is usually applied prior to the actual coding operation, for example by first quantizing the variables and then scanning their values by bit-planes [6].

Otherwise, it is possible to consider non-binary discrete linear codes, such as the trellis codes ([24], [25]), to directly code discrete random variables or quantized versions of continuous random variables. For example, the DISCUS system [17] is actually concerned with the problem of coding with side information a continuous variable \( X \), which is first quantized into a discrete domain, in which cosets of some trellis code [26] are identified. For any \( n \)-dimensional realization \( x \) of \( X \), the encoder sends the syndrome \( s_H(w) \) that identifies the coset to which the quantized version \( w \) of \( x \) belongs (see Fig. 7). Since the version \( \tilde{w} \) of \( w \) reconstructed at the decoder lies in the discrete domain, then a minimum square-error (MSE) estimate \( \hat{x} \), is found, using as well the same side information \( y \) which by channel-decoding led to \( \hat{w} \).

While these coding systems achieve somewhat a good performance, the interplay between the quantizer and the syndrome former [17], [27] does not usually lead to a straightforward optimal design procedure once the correlation and the available transmission rate are known. According to the discussion in Section III-C by using a continuous-valued syndrome it is instead possible to form a syndrome which directly depends on the continuous realization to be coded. If the syndrome formation is exactly tailored to the BCH correlation,
then upon receiving that information perfect reconstruction is possible. Otherwise, if that information cannot be entirely sent due to some transmission rate constrain, it will be still possible to send a quantized version of it. The difference here is that the knowledge of the available transmission rate (responsible for the syndrome formation) and on the knowledge of the available transmission rate (responsible for the syndrome quantization).

The proposed coding system is sketched in Fig. 8. The continuous-valued syndrome $s_V(x)$ is formed in correspondence of any $n$-dimensional realization $x$ of $X$, and an approximation $\hat{s}_V(x)$ is received at the decoder, which estimates $\hat{x}$ by channel-decoding of $y$ (that represents the input of the BCH) and finally reconstructs $\hat{x}_r$ as a MSE estimate of $x$ from $\hat{x}$ and $y$.

In the case that a capacity achieving linear code (i.e. a bounded subset $\Lambda_b$ of a lattice $\Lambda$) exists for the BCH and that the corresponding continuous-valued syndrome $s_V(x)$ is correctly received at the decoder, perfect reconstruction of $x$ is asymptotically achieved when $n \to \infty$. This is shown by the following expressions

\[
x = y + n \\
= y + \hat{s}_V(n) \\
= y + \hat{s}_V(x - y) \\
= y + \hat{s}_V(\hat{s}_V(x) + \hat{s}_V(-y)) ,
\]

where (12) follows from (4) and from the fact that all noise realizations tend to lie uniformly in the fundamental Voronoi region of $\Lambda$, and (13) follows from the property of the sum (7). Even if equation (14) already shows the sufficiency of $s_V(x)$ for perfect reconstruction, from an operative point of view it is useful to note that

\[
x = y + \hat{s}_V(\hat{s}_V(x) - y - Q_\Lambda(-y)) \\
= y + \hat{s}_V(\hat{s}_V(x) - y) \\
= \hat{s}_V(x) - Q_\Lambda(\hat{s}_V(x) - y) ,
\]

where (14) follows from the operative definition of the continuous-valued syndrome $s_V(x)$. (15) follows from the periodicity $\{s_V(x)\}$, and (16), again, from the definition (8). Then, a single quantization operation is sufficient at the decoder for perfect reconstruction.

Assuming now that the channel-decoding algorithm is the same even if the received information from the encoder consist of a quantized (noisy) version of the continuous-valued syndrome, at the decoder the following hold

\[
\hat{x} = y + \hat{s}_V(\hat{s}_V(x) + q + \hat{s}_V(-y)) \\
= y + \hat{s}_V(\hat{s}_V(x) + \hat{s}_V(q) + \hat{s}_V(-y)) \\
= y + \hat{s}_V(x + q - y) \\
= y + \hat{s}_V(n + q) \\
= x + (q - Q_\Lambda(n + q)) = x + (q + q_{ol}) ,
\]

where: (17) is obtained from (13) by substitution of $s_V(x)$ with $\hat{s}_V(x) = \hat{s}_V(x) + q$ (is an $n$-dimensional realization of a random variable $Q$); (18) follows from (13) and from the reasonable assumption that all the realizations of $q$ lie in some region as $n$; (19) and (20) follow, again, from the sum-property (7) and from the definition (8) respectively.

The total reconstruction error $q_{ol} = q + q_{ol}$ is hence the sum of the granular error $q$, and of the overload error $q_{ol} = -Q_\Lambda(n + q)$. If $q$ is negligible w.r.t. $n$, and it is reasonable that this happens at high transmission rates (recall that if $\Lambda_b$ is capacity-achieving $s_V(X)$ is distributed as $N$), then the probability of $q_{ol}$ being not zero (that is called error probability $P_{e}$) is negligible and $q_{ol} \simeq q$. Hence, the same additive error $Q$ would impair both the syndrome and the source at the decoder (as it was shown in a different way in [21]). This in turn shows that the operational rate-distortion function obtained by the proposed coding system $R_{X|Y}(D)$ satisfies at high rates

\[
R_{X|Y}(D) \simeq R_{s_V}(D) ,
\]

where $R_{s_V}(D)$ is the operational rate-distortion function relative to the source encoder used to code the continuous-valued syndrome. Again, since $s_V(X)$ is distributed as $N$, the ultimate performance achievable is then represented by $R_N(D)$, the rate-distortion function relative to the noise added by the BCH.

This is particularly interesting when the noise $N$ (which is the difference of $X$ and $Y$) is Gaussian (and of course independent of $Y$), i.e. $N \in \mathcal{N}(0, \sigma^2)$. In this case, as shown first in [28] and then in [29], $R_{X|Y}(D) = R_N(D)$ independently of the distribution of $Y$ (in particular, without requiring $Y$ to be Gaussian as first assumed in [3]). Consequently,

\[
R_{X|Y}^*(D) \geq R_N(D) = R_{X|Y}^*(D) ,
\]
i.e. in the Gaussian case and at high rates, the ultimate performance of the proposed coding algorithm is the rate-distortion function with side information at the decoder itself. This result is achieved upon utilizing a capacity-achieving channel code $\Lambda_b$ (i.e. upon utilizing a rate-distortion optimal source code [28]) for syndrome formation, and a rate-distortion optimal source code for syndrome encoding.

In case the rate is not sufficiently high for this to hold, $P_e \neq 0$ and an MSE estimate can reduce the global error. In particular, observing that $X = Y + N$ (with $N$ independent of $Y$) and $\hat{X} = X + Q_t$ (with $Q_t$ assumed independent of both $Y$ and $N$ and with variance $\sigma^2_{q_t}$), it is perfectly correct to use

$$\hat{X}_r = \frac{\sigma^2}{\sigma^2_n + \sigma^2_{q_t}} \hat{X} + \frac{\sigma^2_{q_t}}{\sigma^2_n + \sigma^2_{q_t}} Y$$

as optimal linear prediction of $X$ from $\hat{X}$ and $Y$ at the decoder; the residual error variance $\sigma^2_r$ would satisfy then $1/\sigma^2_r = 1/\sigma^2_{q_t} + 1/\sigma^2_n$ [30].

The existence of effective source coders for Gaussian random variables, such as the trellis-coded quantizers [26], suggests the feasibility of their application in this framework, in the syndrome encoder as well as in the channel decoder. This application, with the relative experiments, is discussed in the next section.

V. EXPERIMENTS

In this section, the problem of coding a source when another one is available only at the decoder is targeted with the approach outlined in Section IV. In particular, it is assumed that the correlation is modeled by a backward Gaussian additive channel (Fig. 6) with $N \in N(0, \sigma^2_n)$. The variance $\sigma^2_{n}$ of the Gaussian noise to be generated is chosen in order to guarantee a given backward correlation signal-to-noise ratio (BC-SNR), i.e. a certain ratio between the variance $\sigma^2_{n}$ of the side information $Y$ and $\sigma^2_{n}$. In the experiments, $\sigma^2_{n}$ is fixed (and equals 1), but both the Gaussian and the uniform distribution has been tested for $Y$, obtaining as expected exactly the same results.

As a remark, note that if $Y \in N(0, \sigma^2_n)$, then $X \in N(0, \sigma^2_x)$, with $\sigma^2_x = \sigma^2_n + \sigma^2_{n}$, and the simulated additive BCH can be assumed as derived from a Gaussian additive FCH with Gaussian input (as discussed in Section III.C), which is the problem treated in [17]. For comparison purposes, observe that the correlation signal-to-noise ratio (C-SNR) of the corresponding FCH equals the BC-SNR of the simulated BCH since $\sigma^2_x/\sigma^2_{n} = \sigma^2_n/\sigma^2_{n}$, as given by (10) and (11)

A. Continuous-Valued Syndrome Formation

As trellis-coded quantization (TCQ) based on the partition $aZ/4aZ$ [26] defines a geometrically uniform source code [31], and in particular a lattice $\Lambda$ [24], the encoder computes the continuous-valued syndrome of the source as quantization error after TCQ, as given in (8). The number of samples jointly processed is $n = 1000$, and the tested codes are the ones in [26] relative to 8-, 64-, and 256-state trellises.

The experimental second moment per dimension of the Voronoi region relative to the TCQ codes measured in [26] is used to determine the scaling factor $a$ in a way such that the continuous-valued syndrome variance equals the noise variance $\sigma^2_n$ multiplied by a volumetric factor $K$. This factor takes into account for the non-optimality of the TCQ codes for channel coding and it is numerically estimated. For a capacity-achieving code, $s(y)$ would be distributed as $N$ and hence have the same variance $(K = 1)$; in practice, using sub-optimal codes, the normalized volume (per two dimensions) of the non-spherical Voronoi region must be larger ($K > 1$) for optimal performance.

B. Continuous-Valued Syndrome Coding

Any suitable source coding algorithm can be used to code the $n$-dimensional continuous-valued syndrome $s(y)$ into its approximation $\hat{s}(x)$ at the desired transmission rate. In this paper, since the focus is on practical applications, the target rate $R$ is chosen in the range $0.5 \div 3.0$ bit/sample. Again, TCQ has been preferred over other coding systems because of its simplicity and in order to reutilize the same tool used in syndrome formation. In particular, independently of the TCQ system used to obtain $s(y)$, two TCQ systems have been tested, both based on 8-state trellises.

1) $Z/4Z$-based TCQ. Given the target rate $R$, a regular TCQ based on the partition $bZ/4bZ$ is designed (by dimensioning $b$) such that its normalized volume is $2^{2R}$ times less than the normalized volume relative to the TCQ system used in syndrome formation.

2) Rate-distortion optimized TCQ. Once $R$ is assigned, a finite-alphabet TCQ system achieving the rate $\lceil R \rceil$ (that uses $2^{\lceil R \rceil + 1}$ reconstruction points [26]) is iteratively optimized for a set of 100 syndromes that serve as training sequences. The used algorithm is an adaptation to the trellis structure of the algorithm presented in [32].

In particular, all reconstruction levels are grouped into 2 supersets [33] and the occurrences of any level into each superset at each iteration is used for codeword-length estimation. By varying the Lagrange multiplier $\lambda$, the design can fit not only the rate $R$ but any rate up to about $\lceil R \rceil$.

In both cases, a coding system was not actually designed, but the entropy $H$ of the optimal path into the trellis, estimated using the 2 supersets as contexts, is assumed as achieved rate. On the other hand, it is reasonable that context-based variable-length coding based on the same contexts achieves exactly that rate. Fig. 9(a) and Fig. 9(b) show the average entropy $H_m$ (in the range BC-SNR $\in \{9.0, 9.5, \ldots, 19.0\}$ dB, with 8-state syndrome formation) obtained in the two cases in correspondence of the target rates $R \in \{0.5, 1.0, \ldots, 3.0\}$ bit/sample. Using $Z/4Z$-based TCQ the entropy is somewhat greater than desired, in particular at $R < 2$ bit/sample, due to the impossibility to exactly cover the domain of $s(y)$ with polytopes that are translations of the same fundamental Voronoi region. Instead, with rate-distortion optimized TCQ, it
is possible to achieve exactly the desired rate, paying the price of a slightly more complex encoder that requires non-uniform quantization and rate-distortion aware metric computation.

### C. Finite-Rate Volume Optimization

In practice, by optimizing the value of $K > 1$ it is not only possible to take into account for the non-optimality of the TCQ codes in the case of $s_V(x)$ being correctly received at the decoder, but also possible to take into account for the fact that at low rates $q$ is not negligible w.r.t. $n$, and hence $P_e \neq 0$. In fact, $K$ is responsible for the normalized volume of the lattice and, by increasing its value (at a fixed rate $R$)

- the probability of error $P_e \triangleq P[Q_n(n+q) \neq 0] = P[X - \hat{s}_V(X) \neq X - (\hat{s}_V(X) + Q)]$ reduces;
- the granular error variance $\sigma_q^2$ increases.

When an error happens $q \approx q_{ol}$, and hence the total error variance equals

$$\sigma_{q_t}^2 \approx (1 - P_e)\sigma_q^2 + P_e\sigma_{q_{ol}}^2. \quad (22)$$

In Fig. 10, experimental values of both terms of the right hand side of (22) are shown in function of $K$ as the dot-dashed and the dashed curve respectively. It is clear that there exist an optimum value of the volumetric factor $K$, possibly different at each rate. The solid curve, i.e. the experimental value of the total error variance $\sigma^2 < \sigma_{q_t}^2$ after MSE estimation (with $\sigma^2_q$ in (21) hypothesized to equal $\sigma_{q_{ol}}^2$), shows this effect.

For each different code used in syndrome formation, the best value of $K$ was found by a dichotomic-like search at each BC-SNR and at each target rate $R$. The results relative to the 8-state system, reported in Fig. 11(a), show that as expected $K$ depends only on the target-rate, and is larger at lower rates, where there would be a significant amount of errors if $K$ was too small. Correspondingly, the optimal value of $P_e$ is constant at the various correlations, as shown in Fig. 11(b).

The average values $K_m$ and $P_{em}$ of $K$ and $P_e$ over the various BC-SNR values are plotted against the average entropy achieved by the encoder in Fig. 12(a) and Fig. 12(b) respectively, using the three different TCQ codes for syndrome formation. Since by increasing the number of states the codes get closer to the capacity of the channel, it is expected, as actually obtained (see Fig. 12(a)), that the optimum values of $K$ reduce; however, the same average probability of error seems to be actually required to achieve the best performance (see Fig. 12(b)). It is interesting to note that the optimal probability of error, which increases at low rates, is very close to the probability of error shown in [17] in correspondence of
the C-SNR that gives the optimal performance in the DISCUS system at both 1 and 2 bit/sample.

D. Performance Analysis

The average performance loss w.r.t. the Wyner-Ziv bound $\Delta_m$ is shown in function of the achieved transmission rate in Fig. 13 and in Fig. 14 for $\mathbb{Z}/4\mathbb{Z}$-based syndrome coding and for rate-distortion optimized syndrome coding respectively. These values were obtained by averaging the experimental value of the performance loss $\Delta \triangleq \sigma_r^2/(\sigma_n^22^{-2H_m})$ over the various values of the BC-SNR in correspondence of the optimum value of $K$. Similarly, the error bars show the average 95% confidence interval.\(^3\)

The experiments show that at any correlation and in the rate range $R \in [0.5, 3.0]$ bit/sample, the performance of the system is within $3 \div 4$ dB of the theoretical bound. In particular, at low rates, it is preferable to code the syndrome using rate-distortion optimized TCQ, which allows for a gain of more than 1 dB over the other method.

To conclude the section, comparisons w.r.t. to the DISCUS system are shown in Fig. 15(a) and Fig. 15(b) for the rates $R = 1$ and $R = 2$ bit/sample respectively. The immediate result is that by simply choosing the right scaling factor $a$ the proposed system can adapt to any correlation and give the same performance loss, while the DISCUS system must be redesigned to adapt to different correlations. In addition, once the proposed system is designed for a certain additive BCH, no modifications are needed if the statistics of $Y$ (and consequently the one of $X$) changes. Moreover, while only integer rates can be achieved by the DISCUS system, any rate can be achieved by distributed coding based on continuous-

\(^3\)For each tuple (BC-SNR, $R$, $K$) the 95% confidence interval is estimated after up to 5000 independent simulations.
Fig. 13. Average performance loss (w.r.t. the Wyner-Ziv bound) vs. experimental entropy, varying the number of states in syndrome formation ($\mathbb{Z}/4\mathbb{Z}$-based TCQ).

Fig. 14. Average performance loss (w.r.t. the Wyner-Ziv bound) vs. experimental entropy, varying the number of states in syndrome formation (rate-distortion optimized TCQ). Rate-distortion optimized TCQ allows for better performance at low rates (note that in this case the experimental entropy equals the desired target rate).

valued syndromes. This, again, is simply obtained by choosing the right value of $b$ in case of $\mathbb{Z}/4\mathbb{Z}$-based syndrome coding or, with some increased complexity, by using an ad-hoc rate-distortion optimized TCQ, which finally leads to higher performance at low rates.

While the complexity of the encoding operation is somewhat higher w.r.t. DISCUS, in which the discrete syndrome is computed on-the-fly after quantization, the decoding complexity is essentially the same, and hence it is perfectly feasible to use distributed coding based on continuous-valued syndromes in actual applications.

VI. CONCLUSION AND FUTURE WORK

In this paper a coding system with side information at the decoder has been described that operates entirely in the continuous domain. As a consequence of this approach, the system can be designed exactly according to the correlation between the source and the side information and according to the given transmission rate. Extensive experiments showed that the system achieves a good performance in case the correlation can be modeled by a backward Gaussian additive channel, even at low rates such as for example 0.5 bit/sample.

Since in the proposed system the adaptation to the amount of correlation is simply done by means of a scaling factor, it is very interesting to investigate if on-the-fly adaptation to slightly time-varying correlation is possible, which would be of paramount importance in real applications. Moreover, since the actual syndrome coding operation is performed via traditional source coding, it is in principle possible to use scalable source coding tools to obtain a scalable distributed source coding algorithm, that could be applied in frameworks where the available transmission rate is not constant (such as in...
wireless network-based applications). Both these observations represent a good starting point for future research.

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