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Local ontology for a dual-rail qubit

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Abstract. We show that quantum predictions for the dual-rail realisation of a qubit can be faithfully simulated with classical stochastic gates and particles which interact entirely in a local manner. In the presented model ‘non-locality’ appears only on the epistemic level of description.

We consider single quantum particle in a circuit which consists of two spatially separated paths \(i = 0, 1\) and a sequence of gates which include phase shifters \(P(\omega)\) and detectors \(D_i\) attached to each path separately, and beam splitters \(B(\xi)\) on which the paths cross; cf. Fig. 1 (on the left). Quantum description of such a dual-rail system boils down to a qubit. In this note we report construction of a classical local stochastic model which faithfully reconstructs quantum predictions for the dual-rail qubit [1]. It shows that this kind of framework is not enough to establish genuine quantum non-locality, since for a dual-rail qubit alleged ‘non-locality’ can be explained as an epistemic effect due to restricted means of investigating the system. In the following, we sketch out the main concepts of the model.

Ontology of the model. We postulate existence of two kinds of local particles carrying inner degrees of freedom which propagate along the paths: real ones described by a unit vector \(\vec{n}\) in 3D and ghosts carrying a phase \(\varphi\). The crucial difference between the particles is that detectors register only real particles and remain blind to ghosts. For the purpose at hand, we restrict our attention to situations in which there is a single real particle present in the system with a ghost in the other path or the other path being empty; see Fig. 1 (on the left). Hence, at each time the system is fully characterised by a triple \((i, \vec{n}, \varphi)\) in the ontic state space:

\[
\Lambda \equiv \{0, 1\} \times S^2 \times S^1^*,
\]

where \(S^1^* \equiv S^1 \cup \{\emptyset\}\). The first component \(i = 0, 1\) specifies position of the real particle carrying vector \(\vec{n}\) and the other path featuring a ghost with phase \(\varphi\) or being empty \(\varphi = \emptyset\). In general, the system is described by a probability distribution over the ontic states, i.e. a point in \(\mathcal{P}(\Lambda)\).

Building blocks of the model. To complete description of the model we need to specify the stochastic counterparts of the interferometric gates by defining action on the ontic states \(\Lambda \rightarrow \mathcal{P}(\Lambda)\) which does not violate the locality principle. Here are the definitions:

- Detector \(D_j\) placed in the \(j\)-th path is a deterministic gate:

\[
(i, \vec{n}, \varphi) \xrightarrow{\, D_j\,} \begin{cases} \delta_i \delta_{\vec{n}} \delta_{\varphi} & \text{for } i = j \text{ (detector 'CLICKS')} , \\ \delta_i \delta_{\vec{n}} \delta_{\emptyset} & \text{for } i \neq j \text{ (detector doesn’t 'CLICK').} \end{cases}
\]

Note that in either case the detector affects only the particle in the \(j\)-th path, i.e. real particle is set to point north \(\vec{n} \rightarrow \hat{z}\) and the ghost is removed from the path \(\varphi \rightarrow \emptyset\).
Figure 1. On the left: Possible configurations of real (●) and ghost (○) particles in the model which traverse the circuit built from phase shifters $\mathbb{P}_j(\omega)$, detectors $\mathbb{D}_j$ and beam splitters $\mathbb{B}(\xi)$ defined in Eqs. (2)-(4). On the right: Agent investigating the system explores only a restricted set of distributions in $\mathcal{P}(\Lambda)$. This set can be coarse-grained into classes $[[\mathbb{R}]]$ indistinguishable to the agent and their geometry is shown to be equivalent to the Bloch ball representation.

- **Phase shifter** $\mathbb{P}_j(\omega)$ placed in the $j$-th path is a deterministic gate:
  $$
  (i, \vec{n}, \varphi) \xrightarrow{\mathbb{P}_j(\omega)} \begin{cases} 
  \delta_i \delta_{i''} \delta_{\varphi} & \text{for } i = j, \\
  \delta_i \delta_{i''} \delta_{\varphi'} & \text{for } i \neq j,
  \end{cases}
  \tag{3}
  $$
  where $\vec{n}' = R_2((-)^j \vec{n}) \vec{n}$ and $\varphi' = \varphi + (-)^j \omega$ are obtained by appropriate rotation by angle $\pm \omega$ about the $\hat{z}$-axis, and in the case of empty path $\varphi = \emptyset$ we put $\varphi' = \emptyset$.

- **Beam splitter** $\mathbb{B}(\xi)$ is a stochastic which acts as follows:
  $$
  (i, \vec{n}, \varphi) \xrightarrow{\mathbb{B}(\xi)} \cos^2 \left( \frac{\xi}{2} \right) \delta_i \delta_{\varphi} + \sin^2 \left( \frac{\xi}{2} \right) \delta_i \delta_{-\varphi} \delta_0,
  \tag{4}
  $$
  which means that particles remain in their respective paths ($i \rightarrow i$) or get swapped ($i \rightarrow i' \equiv 1-i$) with respective probabilities $\cos^2 \left( \frac{\xi}{2} \right)$ and $\sin^2 \left( \frac{\xi}{2} \right)$. The vector $\vec{n}' = (\theta', \varphi') = R_\xi(\xi) R_z(-\varphi) \vec{n}$ obtains by appropriate rotation about $\hat{z}$ and $\hat{x}$-axes, and in the case of empty path $\varphi = \emptyset$ we put $\vec{n}' = (\theta', \varphi') = R_\xi(\xi) \hat{z}$. In either case, notice presence of the ghost $\varphi = 0$ after transformation.

**Operational description of the model.** Imagine an agent without any prior notion of the model trying to make sense of how it works only on the basis of experiments she performs. Clearly, the agent acts under *epistemic constraints* which confine her perception of the system under study – she has a limited choice of gates for building circuits and her detectors ‘see’ only real particles; cf. Eqs. (2)-(4). That being the case the agent is legitimate to be concerned only with situations within her reach. In other words, for all practical purposes it is sufficient to account for a limited set of preparation, transformation and measurement procedures that arise in any experimental circuit that she can build according to the rules of the model.

Following this reasoning, we have shown in [1] that such an agent in all her actions remains confined within a certain non-trivial subset of distributions in $\mathcal{P}(\Lambda)$. This set has an interesting geometry which can be analysed by coarse-graining distributions into classes indistinguishable to the agent. It can be shown that these classes have the geometry of the Bloch ball, i.e. they are in one-to-one correspondence with points in the Bloch ball, transform congruently under the action of stochastic gates defined above, and these transformations are the same as for the quantum interferometric gates, see Fig. 1 (on the right). This means that from the agent’s perspective predictions for the quantum case of a dual-rail qubit and classical circuits built from the stochastic counterparts defined in the model are the same, and thus for all practical purposes both systems are equivalent. We emphasise that the model is local on the ontological level, and alleged ‘non-locality’ is only an epistemic effect caused by restrictions on gaining knowledge.

**References**

[1] Blasiak P 2015 Local model of a qubit in the interferometric setup *New. J. Phys.* **17** 113043