THE GRAVITATIONAL FIELD OF STRING MATTER WHEN THE DILATON IS MASSIVE

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Abstract
We study numerically the gravitational field of a star made of massive and neutral string states for the case in which the dilaton is massive. The solution exhibits very simple scaling properties in the dilaton mass. There is no horizon and the singularity is surrounded by a halo (the physical size of which is inversely proportional to the dilaton mass) where the scalar curvature is very large and proportional to the square of the dilaton mass.
1. Neutral and scalar string excitations which are massive already in ten dimensions \cite{1} represent the simplest form of that exotic state of matter in which the constituents are string states not belonging to the massless sector of the string spectrum to which instead ordinary matter, like quarks and leptons, presumably belongs (their masses originating through some still unknown symmetry breaking mechanism).

The physical relevance of such string matter stems from several studies \cite{2} which indicate that, at high energy density, the most probable configuration in string theory is the one in which most of the massive states are excited. According to this view, an electrically neutral collapsing star of sufficiently large mass would start out as a celestial body made of ordinary matter but would eventually evolve into a string star in which most of its mass is now carried by neutral string excitations.

It is therefore of some interest, we believe, to study the gravitational field around such a string star.

The scalar, neutral and massive string excitations couple only to gravitons and dilatons; therefore, the gravitational field around a star made of a large number of such states is described by Einstein’s equations in which the energy-momentum tensor is the one for a scalar field. In the original string scenario, these equations arise as conditions for the vanishing of the one-loop beta functional required by Weyl invariance; the dilaton is massless and the field equations admit an exact solution \cite{3, 4}. This solution, and its relationship to string theory, has been discussed recently in \cite{5}, to which this letter is closely related. The most significant feature of such a solution is the absence of horizon. The relevant elements $g_{rr}$ and $g_{00}$ of the space-time metric are shown in fig.1, where they are compared to the Schwarzschild’s ones. As it is possible to see from the figure, while the two solutions are equivalent at large distances, they are remarkably different closer to the gravitational radius: at the horizon Schwarzschild’s $g_{rr}$ diverges and $-g_{00}$, which is just its inverse, crosses into negative values, whereas the ones corresponding to the string star solution are never either negative or infinite. These results seem to be in agreement with previous work \cite{6} in which the existence of a static and uncharged static solution with a scalar field has been shown to be incompatible with the presence of a non-singular horizon (see, also, \cite{4}).

It is tempting to speculate that also in a realistic scenario, derived from string theory, in which ordinary matter, after obtaining a mass much lower than Planck mass by some yet unknown mechanism, continues to behave with respect to the
dilaton as it were in the massless sector of the string spectrum. According to this view, the dilaton field remains massless as well as decoupled to ordinary matter at large distance. Such a possibility can be entertained without violating any of the present experimental observations. In this case all the results of [5] would hold true.

In this letter we address the question of what happens to the solution with a massless dilaton in the opposite case in which the dilaton does couple to ordinary matter and therefore—because of the current astronomical bounds [6] on Newton’s inverse-square law—it has to become massive. Such an investigation is, for its very nature only preliminary, because of our present ignorance of how the basic ingredients of the superstring, like world-sheet conformal invariance, determine the structure of the effective low-energy theory with a massive dilaton (see [10] for a related discussion in cosmology).

The reader, confronted by a growing literature on the subject of solutions to graviton plus dilaton (and, plus Maxwell fields) gravity in four space-time dimensions, should bear in mind that the general solution for a source with an arbitrary coupling to massless dilaton, graviton and Maxwell fields is contained and discussed in the papers of ref. [8]. Paper [9] discusses the special case in which the coupling to the massless dilaton field is a function of the Maxwell charge and it vanishes as the charge goes to zero. Our solution is the one in which the source is neutral with respect to the the Maxwell field but nonetheless has a non-vanishing coupling to the dilaton field.

2. The field equations are easily written as variations of the action

\[ S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ R - g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - m^2 W(\phi) \right], \]  

(1)

and read

\[ -e^{-\lambda} \left( \frac{\nu'}{r} + \frac{1}{r^2} \right) - \frac{1}{r^2} = -e^{-\lambda} \frac{(\phi')^2}{2} - \frac{m^2}{2} W(\phi) \]

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\[ \phi'' + \left( \frac{2}{r} + \frac{\nu' - \lambda'}{2} \right) \phi' = e^{\lambda} \frac{m^2}{2} \frac{\delta W}{\delta \phi}. \]  

(2)

We have restricted ourselves to the static and spherically symmetric case and therefore used the parametrization

\[ g_{00} = -e^{\nu(r)} \quad g_{rr} = e^{\lambda(r)} \]  

(3)
for the space-time metric, the signature of which is $- + + +$; prime means differentiation with respect to the radial coordinates, $\phi \equiv \kappa \varphi$ is the rescaled dilaton field $\varphi$ and $\kappa^2 = 8\pi G_N$, $G_N$ being Newton’s constant.

In general,

$$W(\phi) = \phi^2 + \alpha \phi^3 + \beta \phi^4 + \ldots,$$

(4)

where the first term gives rise to the mass, and the remaining ones to an effective potential for the dilaton field.

Two questions arise.

The action (1) without the mass term is obtained from the usual one

$$S = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} e^{-2\phi} \left[ R + 4 \nabla \phi \cdot \nabla \phi \right]$$

(5)

by the rescaling $g \to e^{2\phi} g$ and $\phi \to \sqrt{2} \phi/2$, and it represents the low-energy effective theory one derives from string theory. After symmetry breaking, other terms beside the mass $m^2 \phi^2$ may appear and give rise to a potential for the dilaton field as in (4). Consider, for instance, a term like $\beta \phi^4$. Even though the coupling to $\phi^4$ is typically of the order of $G_N m^2$ (that is, $\beta$ of order of one), the net contribution to the equations of motion is of the same order as the ones included in (1), as one can readily see by inspection.

We have explicitly checked our solution against such terms in the potential and verified that its nature, qualitative features as well as most of the numerical values are left unchanged for values of the parameters $\alpha$, $\beta$ up to order of ten. The only case in which there is a substantial change in behavior is when we take, for example, $\alpha > 0$ and $\beta$ and the coefficients of the other even power terms vanishing, thus forcing the potential to become unstable (recall that $\phi$ is negative). Therefore, except for this somewhat pathological case, we are entitled to disregard for the sake of simplicity these extra terms, and take in what follows $W(\phi) = \phi^2$ only.

A second question concerns possible string corrections to (1). These are, for example, of the type $\alpha' R^2$, where $\alpha'$ is the inverse of the string tension. However, in this case, one finds that even for very large values of the scalar curvature (as we are in fact going to have) such terms are suppressed by a factor $\alpha' m^2$ with respect to the term linear in the curvature $\mathcal{R}$, and can accordingly be neglected altogether.

3. Equations (2) do not seem amenable of any exact solution. We study them by numerical methods.
It is convenient to recast (2) as a system of four first-order differential equations:

\[ \begin{align*}
y_1' &= \frac{1}{\rho} y_3 \frac{1}{y_4} \\
y_2' &= \frac{1}{2\rho} y_2 y_3^2 \\
y_3' &= \xi^2 \rho^2 y_1 y_2 \\
y_4' &= y_2 \left( 1 - \frac{1}{2} \xi^2 \rho^2 y_1^2 \right)
\end{align*} \]  

(6)

by means of four dimensionless functions:

\[ \begin{align*}
y_1 &= \phi \\
y_2 &= e^{(\nu + \lambda)/2} \\
y_3 &= \rho e^{(\nu - \lambda)/2} \\
y_4 &= \rho^2 e^{(\nu - \lambda)/2} \phi'.
\end{align*} \]  

(7)

In (6), \( \rho = r/r_g \), where \( r_g = 2G_N M \) is the gravitational radius of the star. The prime is now differentiation with respect to the coordinate \( \rho \).

The parameter \( \xi = m r_g \) is very large for the physically interesting scenario in which the Compton wavelength of the dilaton \( 1/m \) is much shorter than the gravitational radius; we have representatively considered values of \( \xi \) between 10 to \( 10^{10} \).

We set the boundary conditions at large distance for (6) as follows:

\[ \begin{align*}
y_2 &\to 1 \\
y_4 &\to \rho - 1,
\end{align*} \]  

(8)

that is, we require that both \( g_{00} \) and \( g_{rr} \) must go into Schwarzschild metric for sufficiently large distances.

Less clear is the asymptotic value for the dilaton field \( \phi \). The exponential decay

\[ y_1 \to \phi_0 e^{-\xi\rho}/\rho, \]

(9)

that we expect by solving the corresponding equation, leaves the constant \( \phi_0 \) undetermined. While for the massless case this indeterminacy can be resolved by comparison with string perturbation theory [3], the problem seems more complicated here. Luckily, as we shall see, the solution we find is very stable over a large
range of changes in the initial condition for the dilaton field. We therefore fix it by assuming in the region $\rho - 1 \ll 1$ that

$$y_1 \approx K_0 \left(2\xi \sqrt{\rho - 1}\right),$$

(10)

as it is suggested by the study of the dilaton equation in a fixed background, and putting by hand the numerical coefficient in front of the Bessel function. This choice makes the dilaton field behave as in (9) for $\rho - 1 \gg 1$, and as

$$y_1 \approx \exp{-\frac{2\xi \sqrt{\rho - 1}}{2\xi \sqrt{\rho - 1}}}$$

(11)

and

$$y_1 \approx \log \left(2\xi \sqrt{\rho - 1}\right)$$

(12)

for, respectively, $\xi^{-2} \ll \rho - 1 \ll 1$ and $0 < \rho - 1 \ll \xi^{-2}$.

This procedure amounts to placing the position at which the dilaton field begins to be different from zero. Within an acceptable range of initial values, once it has been fixed, the dilaton field rapidly goes into its asymptotic (and universal) behavior beyond the gravitational radius.

4. We can thus solve the system of first order differential equations (8) as an initial-value problem by means of a standard program [11] based on a variable-order, variable-step method implementing the Backward Differential Formulas.

The results of the numerical integration are displayed in figs. 2a-2b for the metric and 3 for the dilaton field, where they are plotted for the special value $\xi = 10$. Although such a small value is not physically relevant, it is useful because it makes the plotting more readable. Moreover, the solution exhibits very simple scaling properties that allow to infer from the plot of figs. 2a-2b and 3 the corresponding ones for higher, and more realistic, values of $\xi$.

As we have already pointed out, we obtain a solution which is surprisingly stable with respect to variations in the dilaton mass, the asymptotic value of $\phi$ and the parameters in $W(\phi)$. The features of the solution we would like to emphasize are that:

- like in the massless case, the horizon is absent;
• there is a very narrow region near the would-be horizon where there is a very sharp transition away from Schwarzschild’s solution;

• outside this region—that is, for \( r > r_g \)—the field \( \phi \) can be neglected and the solution is indistinguishable from the Schwarzschild’s one;

• inside this region—that is, for \( r < r_g \)—the solution becomes stable and depends very little on details.

By varying the input value of \( \xi \) and comparing the outputs so obtained we have verified that the behavior of the space-time metric for \( \rho \rightarrow 0 \) is the following:

\[
g_{rr} \simeq \frac{1}{\xi^2} P_0 \rho^{Q_0^2/2+1} \\
-g_{00} \simeq \frac{1}{\xi} S_0 \rho^{Q_0^2/2-1} \tag{13}
\]

while the dilaton field remains unchanged and:

\[
\phi \simeq Q_0 \log \rho - C_0. \tag{14}
\]

Also the scalar curvature behaves in a simple manner:

\[
\mathcal{R} \simeq \xi^2 \left\{ \frac{Q_0^2}{P_0} \rho^{-(3+Q_0^2/2)} + O(\log \rho) \right\}. \tag{15}
\]

The numerical values for \( Q_0, C_0 \) and \( P_0 \) seem to be universal, independent of the dilaton mass and every other parameter; they turn out to be \( Q_0 \simeq 2.5, C_0 \simeq -1 \) and \( P_0 \simeq 4 \). \( S_0 \) seems to depend more on the input conditions, however being always of order one. Notice that the solution for the massless case \(^5\) was of the same kind with \( Q_0^2 = 2(\sqrt{2} + 1)/(\sqrt{2} - 1) \).

The behaviors (13)-(15) are well satisfied inside the Schwarzschild radius, whereas away from it the solution is more sensitive to the initial conditions and does not follow a simple scaling law in \( \xi \).

5. Let us briefly discuss the solution we have found. It resembles in a qualitative manner the massless case. Away from the would-be horizon, the metric is indistinguishable from Schwarzschild’s. However, when distances of the order of \( 1/\xi^2 \) are reached, the solution departs dramatically from Schwarzschild’s and the horizon disappears. For large values of \( \xi \), this change takes place in a very thin shell
around $r_g$. This can be compared with the massless case in which the departure occurs at distances of the order of $r_g$ instead.

At the same time, the scalar curvature (see fig.4) grows in a very steep manner and produces a high-curvature sphere around the singularity. This is to be expected since in this region $\phi$ changes from being vanishingly small to being of order one, thus acting back on the space-time metric and giving a large contribution to the curvature as well. Fig.4 makes clear how the massive case differs from the massless one only in that the growth of the curvature is more abrupt (again, it takes place in the region of size $1/\xi$ before $r_g$) and steeper. They become equivalent closer to the singularity where the dilaton field becomes effectively massless in any case.

The physical size of the halo of large curvature is

$$\int_0^1 d\rho \sqrt{g_{rr}} \simeq 1/\xi$$

and therefore of order $1/m$ in standard units.

We feel that these results are very natural. The presence of a mass for the dilaton field, by making available a second parameter to the problem, changes the scale at which the departure from Schwarzschild solution takes place and also the magnitude of such departure; it does not however change the nature of it, that remains the one found in the massless case.
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Captions

Fig.1: Schwarzschild’s space-time metric compared to the string star’s for a massless dilaton field.

Fig.2a: Schwarzschild’s $g_{00}$ compared to the string star’s for a massive dilaton field.

Fig.2b: Schwarzschild’s $g_{rr}$ compared to the string star’s for a massive dilaton field.

Fig.3: The massive dilaton field.

Fig.4: The scalar curvature: Massless vs. massive dilaton field.