Vibration of high-voltage electric machines with rotors on rolling bearings

H G Shekyan* and A V Gevorgyan
Institute of Mechanics of the National Academy of Sciences of Armenia, Yerevan, Armenia
E-mail: *hamlet@mechins.sci.am

Abstract. The paper presents an investigation of vibrational activity of electric machines due to high-harmonic vibrational loadings. It is shown that the vibrational loadings experienced by bearings may result in the interruption of their normal operation and even take them out of action. Therefore, the values of the vibrational speed-up leading to high harmonics are factors determining the admissible dynamic loading on the bearings. In the paper, an attempt is made to consider the factors which result in origination of high harmonics and to illustrate methods for their smoothing.

1. Introduction
Despite numerous literature sources devoted to calculations of the machine body vibrations, there is still no calculational method which can give a result differing from the experimental data by less than 50%. This can be explained by the fact that, in calculations, it is impossible to take into account all numerous factors influencing the body vibration (nonlinear rigidity, kinematic excitation of rolling bearings, rigid characteristics of the stator shields and beds, etc.). It suffices to say that numerous harmonic components of a machine have vibration spectra, which originate due to the nonlinear rigidity and kinetic perturbation of the bearings and which are to be determined when evaluating the dynamic situation and the working capacity of the bearings of electric machines.

Taking into account these factors would, of course, permit improving the existing methods of calculations [1, 2].

2. Statement of the problem
Let us consider the stationary motion of a rigid rotor on rolling bearings in a rigid shock-proof housing (figure 1).

The motion of the system is described by the differential equations

\[
\begin{align*}
    m_1 \ddot{y}_1 &= P_1 + P_M - F_{y_1} + P(t), \\
    m_2 \ddot{y}_2 &= P_2 + F_{y_1} - F_{y_2} - P_M, \\
    F_{y_1} &= C_1 B_0 (y_1 - y_2 + \lambda_{1st} - \lambda_{2st})^{3/2}, \\
    P_M &= C_M (y_1 - y_2 + \lambda_{1st} - \lambda_{2st}), \\
    F_{y_2} &= C_2 (y_2 + \lambda_{2st}), \\
    P_1 &= C_1 B_0 (\lambda_{1st} - \lambda_{2st})^{3/2} - C_M (\lambda_{1st} - \lambda_{2st}).
\end{align*}
\]  
(1)
For the linear terms for small vibrations, we obtain

\[ P_2 = C_2(y_2 - \lambda_{2st}) - C_1 B(\lambda_{1st} - \lambda_{2st})^{3/2} + C_M(\lambda_{1st} - \lambda_{2st}), \]  
\[ B = B_0 - B_1 \cos(\omega_1 t), \]  
\[ P(t) = m_1 F \sin(\omega_1 t) - m_1 s, \quad F = \omega_1^2 e, \]  

where \( y_1 \) and \( y_2 \) are the respective displacements of the rotor and stator along the axis \( y \); \( F_{y1} \) and \( F_{y2} \) are the respective restoring forces due to the contact elasticity of the bearings and the rigidity of the shock absorbers; \( P_M \) is the force of the one-way magnetic traction, \( C_M \) is the conditional rigidity of the magnetic field; \( P_1 \) is the rotor and stator weight; \( C_1 \) is the Hertz coefficient, \( C_2 \) is the total rigidity of shock absorbers, \( B \) is a periodic function of time which is approximated by an expression [3, 4]; \( P(t) \) is the perturbation force due to the rotor out-of-balance and the kinematic perturbation of the bearings; \( e \) is the rotor eccentricity, and \( s \) is a function of kinematic displacement [3, 4].

Substituting (2)–(7) into (1), we obtain

\[
\begin{align*}
&\left\{
 m_1 \ddot{y}_1 + C_1 [B_0 - B_1 \cos(\omega_1 t)] (y_1 - y_2 + \lambda_{1st} - \lambda_{2st})^{3/2} - C_M (y_1 - y_2 + \lambda_{1st} - \lambda_{2st}) \\
&= C B_0 (\lambda_{1st} - \lambda_{2st})^{3/2} - C_M (\lambda_{1st} - \lambda_{2st}) + P(t), \\
 m_2 \ddot{y}_2 + C_2 (y_2 + \lambda_{2st}) - C [B_0 - B_1 \cos(\omega_1 t)] (y_1 - y_2 + \lambda_{1st} - \lambda_{2st})^{3/2} \\
&+ C_M (y_1 - y_2 + \lambda_{1st} - \lambda_{2st}) = C_2 \lambda_{2st} - C B_0 (\lambda_{1st} - \lambda_{2st}) + C_M (\lambda_{1st} - \lambda_{2st}).
\end{align*}
\]  

Expanding the function \( (y_1 - y_2 + \lambda_{1st} - \lambda_{2st})^{3/2} \) into a Taylor series and preserving only the linear terms for small vibrations, we obtain

\[
\ddot{f} + \ddot{y}_2 + [h^2 - q \cos(\omega_1 t)] \xi = v \cos(\omega_1 t) + P'(t), \\
\ddot{y}_2 + \omega_1^2 y_2 - d [h^2 - q \cos(\omega_1 t)] \xi = -dn \cos(\omega_1 t),
\]  

where

\[
\xi = y_1 - y_2, \quad h^2 = \frac{3 C_1 B_0}{2 m_1} (\lambda_{1st} - \lambda_{2st})^{1/2}, \quad \frac{C_M}{m_1} = \frac{P(t)}{m_1},
\]

\[
q = \frac{3 C_1 B_1}{2 m_1} (\lambda_{1st} - \lambda_{2st})^{1/2}, \quad v = \frac{C_1 B_1}{m_1} (\lambda_{1st} - \lambda_{2st})^{3/2}, \quad d = \frac{m_1}{m_2}, \quad \omega_1^2 = \frac{C_2}{m_2}.
\]

The solution of the system of equations (10) shows that, at certain values of the rigid characteristics of the bearings and shock absorbers, there is a sharp increase in the amplitudes of separate vibrational components of spectrum, and the parametric resonance may occur. In

Figure 1. Rigid rotor on nonlinearly elastic supports in an amortized body.
Figure 2. Dependence of the vibration overload coefficient on the rotor out-or-balance of generator C-75.

In this case, it is necessary to change the rigid characteristics of the machine so as to satisfy the conditions

\[ n^2 \omega_1^2 > \omega_2^2 + (d + 1)h^2 \quad \text{and} \quad [(n - 1)\omega_1 \pm \omega_{ki}]^2 > \omega_2^2 + (d + 1)h^2 \]  

obtained by analyzing the solutions of system (10) by the method of a small parameter [5].

In (11), \( \omega_{ki} \) is the frequency of the \( i \)th harmonic whose speed-up amplitude is the greatest.

With the aim of evaluating the system dynamic state, the vibration overload coefficient is introduced in the form

\[ k_g = \frac{P_y}{G}, \]

where \( P_y \) is the centrifugal force of inertia due to the precession motion of the rotating rotor and \( G \) is the rotor weight. In the case of a high-speed rotor rotating on the rolling bearings, the static equilibrium state is unstable if the rotating rotor is precessing with the angular speed of rotation in a circle whose radius is equal to the bearing deformation.

If the rotor eccentricity is equal to \( e \), then the centrifugal force is

\[ P_y = m_1 \omega_1^2 (e + y_1 - y_2) / g, \]

where \( y_1 - y_2 = [P_y/(C \cdot B)]^{3/2} \).

For \( e = 0 \), we have \( P_y = m_1 \cdot \omega_1^6 / (C \cdot B) \) and \( k_g = m_1^2 \cdot \omega_1^6 / (C \cdot B)^3 \).

As we see, the dynamic factor increases significantly because of the nonlinear rigidity of the bearings.

If the rotor eccentricity is nonzero, then

\[ k_g = \frac{\omega_1^2 (e + y_1 - y_2)}{g}, \]

and the dependence of the vibrational overload coefficient on the rotor out-of-balance is shown in figure 2. The graph is constructed for a bearing of type 160507 in the serial generator C-75 at \( \omega = 800 \text{ rad/s} \).

The graph shows that, up to a certain eccentricity value (\( e = 0.05 \mu m \)), the reaction between the shaft and the bearing is practically constant and only the further increase in the eccentricity leads to a sharp increase in the vibration overload coefficient. In figure 3, the dependence of \( k_g \) on the angular speed of rotation of a rigidly balanced rotor (\( e = 0 \)) is depicted.

Figure 3 shows that, up to some values of the angular speed \( \omega = 300 \text{ rad/s} \), the value of the vibration overload coefficient is practically insignificant. The further increase in the rotor angular speed leads to a sharp increase in the vibration overload coefficient. Therefore, for each type of the bearing carrying a rotor, there is a value of the angular speed at which the
**Figure 3.** Dependence of the vibration overload coefficient on the angular speed of the rotor rotation: 1 – without taking the rotor ductility into account, 2 – with the rotor ductility taken into account, 3 – the rotor on linearly elastic supports.

**Table 1.** Spectral components of bearing harmonics of generator C-75.

| Bearing type | Bearing number | Outer circle | Inner circle |
|--------------|----------------|--------------|--------------|
|              | number | harmonic number | harmonic amplitude, µm | harmonic frequency, Hz | harmonic number | harmonic amplitude, µm | harmonic frequency, Hz |
| 308          | 1      | 1              | 0.11          | 53             | 12           | 0.24         | 960          |
|              |        | 4              | 0.2           | 212            | 21           | 0.13         | 1680         |
|              |        | 10             | 0.16          | 530            | 33           | 0.1          | 2650         |
|              |        | 16             | 0.09          | 850            | 47           | 0.15         | 3760         |
|              |        | 24             | 0.06          | 1270           | 64           | 0.07         | 5150         |
|              |        | 44             | 0.02          | 2420           | 93           | 0.03         | 7450         |
|              |        | 86             | 0.01          | 4600           | 125          | 0.08         | 10000        |
|              |        | 115            | 0.04          | 610            | 130          | 0.01         | 10050        |
| 308          | 2      | 2              | 0.28          | 107            | 15           | 0.18         | 1200         |
|              |        | 5              | 0.13          | 268            | 26           | 0.14         | 2080         |
|              |        | 15             | 0.31          | 810            | 33           | 0.22         | 2640         |
|              |        | 47             | 0.08          | 2500           | 61           | 0.1          | 4860         |
|              |        | 66             | 0.06          | 3520           | 82           | 0.07         | 6550         |
|              |        | 92             | 0.02          | 4900           | 120          | 0.03         | 9600         |
|              |        | 131            | 0.01          | 7000           | 160          | 0.02         | 10500        |

**Comment.** Bearing 1 is the generator bearing on the drive unit, and bearing 2, on the rear shield.

Dynamic reactions begin to increase sharply. The dynamic reaction remains constant until a certain value of eccentricity is attained, and since the further balancing does not decrease the dynamic reaction, it makes no sense to decrease the eccentricity, as in the case \(e = 0\), because the support deformation and the dynamic reaction do not vanish.

Generator C-75-V1 is considered as an example of calculations of the vibration spectrum. The basic parameters of the machine and its elements which are required to calculate the vibrations are given in tables 1 and 2, and the results of calculations the carcass vibrations and the experimental data are given in table 3.

It follows that the data obtained by calculations satisfactorily coincide with the experimental
Table 2. Basic parameters of machine C-75 and its elements.

| Stator weight | Rotor weight | Turnover number | Magnetic field eccentricity, \( h^2 \) | Bearing rigidity, \( q \) | Bearing type, \( \nu \) |
|---------------|--------------|-----------------|-----------------------------|---------------------|-------------|
| 100 kg        | 57 kg        | 8000 turn/min    | 0.84 \times 10^9 \text{ kg} \cdot m^2 | 1.67 \times 10^9 \text{ kg} \cdot m | 2.5 \times 10^4 \text{ kg} \cdot m | 2.6 \times 10^4 \text{ kg} \cdot m |

Table 3. Spectral components of harmonics of the body of machine C-75.

| Harmonic Displacement frequency, \( f \) | Speed Displacement amplitude, \( \mu m \) | Speed amplitude, \( \mu m \) | Speed amplitude, \( \mu m \) | Speed amplitude, \( \mu m \) | Speed amplitude, \( \mu m \) | Speed amplitude, \( \mu m \) |
|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|
| 53 Hz                                    | 0.19 \mu m                               | 0.21 \mu m                               | 0.22 \mu m                               | 0.24 \mu m                               | 100 Hz                                   | 58 \mu m                                | 2.3 \mu m                               | 6 \mu m                                  | 2.5 \mu m                                |
| 100 Hz                                   | 7 \mu m                                  | 2.82 \mu m                               | 7.3 \mu m                                | 3 \mu m                                  | 268 Hz                                   | 0.12 \mu m                              | 0.4 \mu m                               | 0.17 \mu m                               | 0.5 \mu m                                |
| 212 Hz                                   | 0.2 \mu m                                | 0.36 \mu m                               | 0.16 \mu m                               | 0.3 \mu m                                | 810 Hz                                   | 0.25 \mu m                              | 7.4 \mu m                               | 0.3 \mu m                                | 8 \mu m                                  |
| 530 Hz                                   | 0.15 \mu m                               | 3.4 \mu m                                | 0.2 \mu m                                | 4.22 \mu m                               | 1200 Hz                                  | 0.1 \mu m                               | 9 \mu m                                 | 0.15 \mu m                               | 10.5 \mu m                               |
| 850 Hz                                   | 0.075 \mu m                              | 2.3 \mu m                                | 0.07 \mu m                               | 2.4 \mu m                                | 2500 Hz                                  | 0.078 \mu m                             | 20 \mu m                                | 0.09 \mu m                               | 18 \mu m                                 |
| 1270 Hz                                  | 0.1 \mu m                                | 9.4 \mu m                                | 0.18 \mu m                               | 11 \mu m                                 | 2640 Hz                                  | 0.02 \mu m                              | 55.5 \mu m                              | 0.028 \mu m                              | 47 \mu m                                 |
| 2420 Hz                                  | 0.025 \mu m                              | 7 \mu m                                  | 0.04 \mu m                               | 7.6 \mu m                                | 3520 Hz                                  | 0.03 \mu m                              | 20 \mu m                                | 0.04 \mu m                               | 24 \mu m                                 |
| 2650 Hz                                  | 0.03 \mu m                               | 11 \mu m                                 | 0.05 \mu m                               | 16.8 \mu m                               | 4860 Hz                                  | 0.14 \mu m                              | 105 \mu m                               | 0.2 \mu m                                | 110 \mu m                                |
| 3760 Hz                                  | 0.013 \mu m                              | 6.7 \mu m                                | 0.02 \mu m                               | 15 \mu m                                 | 5000 Hz                                  | 0.02 \mu m                              | 22 \mu m                                | 0.028 \mu m                              | 20 \mu m                                 |
| 4600 Hz                                  | 0.01 \mu m                               | 10 \mu m                                 | 0.021 \mu m                              | 20 \mu m                                 | 6550 Hz                                  | 0.025 \mu m                             | 50 \mu m                                | 0.03 \mu m                               | 57 \mu m                                 |
| 6100 Hz                                  | 0.03 \mu m                               | 43.3 \mu m                               | 0.04 \mu m                               | 48 \mu m                                 | 7000 Hz                                  | 0.01 \mu m                              | 19 \mu m                                | 0.015 \mu m                              | 24 \mu m                                 |
| 7450 Hz                                  | 0.02 \mu m                               | 48 \mu m                                 | 0.028 \mu m                              | 60 \mu m                                 | 9600 Hz                                  | 0.034 \mu m                             | 110 \mu m                               | 0.05 \mu m                               | 100 \mu m                                |
| 10050 Hz                                 | 0.07 \mu m                               | 36 \mu m                                 | 0.01 \mu m                               | 39.8 \mu m                               | 10500 Hz                                 | 0.025 \mu m                             | 113 \mu m                               | 0.03 \mu m                               | 98 \mu m                                 |

Data. Besides, the maximum displacement amplitude of the machine carcass takes place at the rotation frequency. As the harmonic number increases, the displacement amplitudes decrease, while the speed-up amplitudes increase. At the frequency 10500 Hz from the gear, the speed-up of the bearing shield is 114 m/s².

Therefore, the dynamic force transmitted from the rotor through the bearings to the carcass is equal to

\[ F_g = m_2 \ddot{y}_2 = 100 \times 11.4 = 1140 \text{ kg}, \]

which is almost 8–9 times greater than the static loading, which means that the state of the considered bearing is unsatisfactory. To improve the state of the bearing, it is necessary to minimize the speed-up of higher harmonics of the carcass either by selecting bearings with more plane characteristics of higher harmonic numbers of the spectrum or by localizing the rotor vibration from the carcass by using an elastic support (bearing), which permits cutting down the higher harmonics.
Conclusions

1. Considering the rotors vibrations of high-speed electric machines with the nonlinear rigidity and kinematic perturbation taken into account, one can explain the reasons of increase in the spectral components of higher harmonics and find ways for their smoothing.

2. To estimate of vibration activity of high-speed electric machines, it is necessary to estimate the vibrational speed-up, which is a force characteristic of the inertial system.

3. Under the technical conditions of high-speed machine operation determined by the durability of the bearings, rotation speed, and rotor mass, the vibration acceleration requirements should be formulated in the whole range of frequencies.

4. To improve the status of the bearings, the acceleration of higher harmonic carcasses should be minimized by selecting the bearings with more smooth characteristics of the spectrum of high harmonics or introducing elastic elements into the bearing construction so as to cut off the higher harmonics.

References

[1] Shekyan H G 2004 Dynamics of Rotor Machines (Yerevan: Izdat. NAN RA “Gitutiun”) p 329 [in Russian]
[2] Shekyan H G 1976 Stability of high-speed rotors, rotating on the bearings of roll Trudy VNIIKE 8 136–47 [Proc. HSII (Engl. Transl.) 8 62–77]
[3] Shekyan H G, Khalatyan R P, and Koshkaryan G N 1980 Dynamics of flexible rotors on bearings of roll Izv. Akad. Nauk ArmSSR. Ser. Tekh. Nauk XXXIII (2) 7–11
[4] Sheftell B T and Shanitsin A A 1973 Vibration of ball bearing with radical gap Mashinoved. No. 4 29–35
[5] Malkin I G 2004 Lyapunov and Poincare Methods in Theory of Nonlinear Oscillations (Moscow: Editorial URSS) p 248