Detuning-controlled internal oscillations in an exciton-polariton condensate

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We theoretically analyze exciton-photon oscillatory dynamics within a homogenous polariton condensate in presence of energy detuning between the cavity and quantum well modes. Whereas pure Rabi oscillations consist of the particle exchange between the photon and exciton components in the polariton system without any oscillations of their quantum phases, we demonstrate that any non-zero detuning results in oscillations of the relative phase of the photon and exciton macroscopic wave functions. Different initial conditions reveal a variety of behaviors of the relative phase between the two condensates, and a crossover from Rabi-like to Josephson-like oscillations is predicted.

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Resulting from strong coupling between the photon state in a microcavity and exciton state in a quantum well, exciton-polaritons are the new mixed eigenmodes which inherit properties of both light and matter [1]. Polaritons interact without dephasing due to elastic collisions in their excitonic component, while the photon component provides them with extremely light effective mass allowing for Bose-Einstein condensation (BEC) at high critical temperatures [2, 3]. In the recent years, great wealth of experimental [4–12] and theoretical [13–21] works in the field of polariton BEC demonstrated the coherent effects analogous to those in atomic condensates [22–25], superconductors [26, 27], or liquid Helium [28–30]. At the same time, due to their non-trivial dispersions, which correspond to the condensates of lower and upper polariton branches at the anticrossing, the polariton gas is a half-and-half mixture of 'photon' and 'exciton' subsystems of the condensate. The picture becomes more complicated when the photon and exciton dispersions are shifted with respect to each other: the internal oscillations between exciton and photon components involve oscillations of the relative phase of the two macroscopic wave functions.

In this Letter, we present fully analytical investigation of internal oscillations in the two-component polariton condensate and show the different possible regimes of the dynamics. We consider an idealized polariton system with constant chemical potential and so far neglect all non-equilibrium effects associated with particles gain and dissipation. For this model condensate, we demonstrate that at any non-zero detuning, depending on the initial state of the system, there are different regimes of oscillations possible, from harmonic and anharmonic modifications of Rabi oscillations up to transition to a so-called Josephson regime analogous to internal a.c. Josephson effect in a two-state BEC of $^{87}$Rb atoms [37, 38]. We find the new equilibrium points of the detuned system which correspond to the condensates of lower and upper polaritons where no oscillations take place.

Within the mean field approach [13], temporal evolution of the macroscopic wave functions of cavity photons $\psi_C$ and quantum well excitons $\psi_X$ is described by the coupled Schrödinger and Gross-Pitaevskii equations,

$$i\hbar \partial_t \psi_C = \left[ E_C^0 - \frac{\hbar^2 \nabla^2}{2m_C} \right] \psi_C + \frac{\hbar \Omega_R}{2} \psi_X,$$

$$i\hbar \partial_t \psi_X = \left[ E_X^0 - \frac{\hbar^2 \nabla^2}{2m_X} + g |\psi_X|^2 \right] \psi_X + \frac{\hbar \Omega_R}{2} \psi_C,$$

with $E_{C,X}^0$ the bottoms of energy dispersions and $m_{C,X}$
the effective masses of photons and excitons, respectively. \( \bar{g} > 0 \) is the constant of repulsive exciton-exciton interaction. Particle transfer between the subsystems is described by the coupling term \( \sim \hbar \Omega_R/2 \), and we neglected any external potentials and spin degree of freedom.

When the system is in the strong coupling regime, the polariton state is an eigenstate with an equal (in the absence of interactions) superposition of a photon and an exciton. The positive sign chosen in Eqs. (1), (2) in front of the coupling term \( \hbar \Omega_R/2 \) imposes that the antisymmetric mode \( (\psi_C - \psi_X)/\sqrt{2} \) with the relative quantum phase \( \pi \) is the lower energy level (i.e., corresponds to the condensate of lower polaritons) while the symmetric mode \( (\psi_C + \psi_X)/\sqrt{2} \) with zero relative phase is the upper one. An initial state of the polariton system, being some linear combination of these two modes, results in density oscillations between the photon and exciton subsystems. In the presence of interactions (\( \bar{g} > 0 \)), the effective lower energy level is blueshifted (while the upper energy level appears redshifted), and the eigenmodes are no longer the antisymmetric and the symmetric ones. Still, the relative phase oscillations which will be discussed below go around the time-average values \( \pi \) or 0.

Considering the homogeneous case for simplicity and assuming the momentum equal zero, we neglect the spatial derivatives in Eqs. (1) and (2). After the transformation \( \psi_{C,X}(t) = \sqrt{n_{C,X}(t)}e^{iS_{C,X}(t)} \) we get four nonlinear dynamical equations for photon and exciton densities \( n_{C,X}(t) \) and quantum phases \( S_{C,X}(t) \):

\[
\frac{\partial}{\partial t} n_{C,X} = \mp \sqrt{n_{C,n_X}} \sin (S_C - S_X), \tag{3}
\]

\[
\frac{\partial}{\partial t} S_C = -\epsilon_0^C - \frac{1}{2} \sqrt{n_X/n_C} \cos (S_C - S_X), \tag{4}
\]

\[
\frac{\partial}{\partial t} S_X = -\epsilon_0^X - g n_X - \frac{1}{2} \sqrt{n_C/n_X} \cos (S_C - S_X), \tag{5}
\]

where we have rescaled lengths and energies in terms of \( \sqrt{\hbar/m_C \Omega_R} \) and \( \hbar \Omega_R \), respectively, as time \( t \Omega_R \to t \) and the wavefunctions as \( \psi_{C,X}/\sqrt{\hbar/m_C \Omega_R} \to \psi_{C,X} \).

In order to investigate the dynamics, it is convenient to introduce a new set of variables: relative quantum phase \( S(t) = S_C(t) - S_X(t) \) and population imbalance between the two subsystems \( \rho(t) = (n_C(t) - n_X(t))/n \), where \( n = n_1(t) + n_2(t) \) is total number of polaritons. Variables \( \rho \) and \( S \) obey the coupled evolution equations

\[
\dot{\rho} = -\sqrt{1 - \rho^2} \sin S, \tag{6}
\]

\[
\dot{S} = -e - \frac{g n}{2} \rho + \frac{\rho}{\sqrt{1 - \rho^2}} \cos S, \tag{7}
\]

The dimensionless effective detuning \( e = \delta + gn/2 \) (\( \delta = \epsilon_1^0 - \epsilon_2^0 \)) and the blueshift value \( gn/2 \) are the parameters which determine different regimes of the system behavior. For a system with constant chemical potential the equations (6), (7) are Hamiltonian with the conserved energy

\[
H(S, \rho) = e \rho + \frac{gn}{2} \frac{\rho^2}{2} + \sqrt{1 - \rho^2} \cos S, \tag{8}
\]

where total population \( n \) is constant. Equations (6) and (7) admit analytical solution in terms of quadratures:

\[
\cos S = \frac{H - \frac{gn}{2} \rho^2 - e \rho}{\sqrt{1 - \rho^2}}, \tag{9}
\]

\[
t = \mp \int \frac{d\rho}{\sqrt{1 - \rho^2 - \left( H - \frac{gn}{2} \rho^2 - e \rho \right)^2}}. \tag{10}
\]

After obtaining the formal solution of the evolution equations, it is worth noting that the interaction constant of exciton repulsion \( g \) is of the order of \( 10^{-3} \) (estimated from \( \bar{g} = 0.015 \text{ meV} \cdot \mu \text{m}^2 \) [2]), while for realistic polariton densities, the unscaled \( n \) is of the order of unity (which corresponds to \( \sim 10^{10} \text{ cm}^{-2} \) [2]). Hence, for the closed conservative system with \( n = \text{const} \) the blushift parameter \( gn/2 \) is always of the order of \( 10^{-3} \), and the effect of interactions on the internal oscillations is negligible. This would not be the case if the polariton density changed in time: at the increase of population, interaction would alter the dynamics. For the system we consider, however, neglecting the interactions in the further analysis will result in little loss of accuracy.

In the absence of interactions, the integral in (10) reveals an explicit solution for the population imbalance:

\[
\rho(t) = \frac{he + \sqrt{\omega^2 - h^2}}{\omega^2} \sin(\omega t - \varphi), \tag{11}
\]

where \( h = e \rho(0) + \sqrt{1 + \rho(0)^2} \cos S(0) = \text{const} \) is the energy of the system defined by the detuning \( e \) and the initial conditions, \( \omega = \sqrt{1 + e^2} \) is the renormalized frequency of internal oscillations (in the scaled units, it corresponds to \( \Omega_R \sqrt{1 + e^2} \)), and \( \varphi = \arcsin[he - \rho(0)\omega^2]/\sqrt{\omega^2 - h^2} \).

The phase-plane portrait of the conjugate variables \( (\rho, S) \) is shown in Fig. 1(a)–(c) for three values of the detuning. When effective detuning is zero (or, bare detuning \( \delta \) compensates the blueshift \( gn/2 \)), for any initial conditions the system performs finite motion along the selected trajectory (depending on the energy). This case is displayed in Fig. 1(a) and it corresponds to the Rabi-like oscillations. For this case, if the initial state of the polariton condensate could be prepared in such a way that the exciton and photon populations are equal, i.e., \( \rho(0) = 0 \), the system would stay in the pure Rabi regime of density oscillations between the subsystems with the
The time-average $\langle \rho \rangle$ remaining zero in time and without any change of the relative phase $S = \pi$. Any non-zero $\rho(0)$, however, will result in oscillations in both population imbalance and the relative phase with the Rabi frequency $\Omega_R$. Fig. 1(b) and 1(c) show the phase space trajectories for small and large negative detunings, respectively. In this case, depending on the initial energy, the twocomponent system can evolve in two different dynamical regimes. Closed trajectories representing finite motion at low energies belong to the regime of Rabi-like oscillations similar to the previous case. The difference consists of the renormalization of the oscillation frequency $\omega$, the anharmonicity of the relative phase oscillations (see Fig. 1(f)), and the shift of the time-average population imbalance to a non-zero value $\hbar \omega / \omega^2$. This regime of oscillations represents an interplay between the modified Rabi dynamics and an analog of internal Josephson effect: for small-amplitude oscillations, one may say that the shift of natural frequency corresponds to Josephson "plasma frequency" $\omega_{JP} = e\Omega_R$.

As the energy $h$ of the polariton condensate increases at a fixed detuning (or, equivalently, as the absolute value of the detuning is increased at fixed initial conditions), the phase oscillations grow in amplitude up to $\pi/2$ and acquire strongly anharmonic sawtooth profile shown in Fig. 1(d) while the trajectory on the phase-plane portrait approaches the separatrix line. After crossing the separatrix which is defined by $\cos S = \pm e \sqrt{(1 - \rho)(1 + \rho)}$, the difference between the Rabi-like and the Josephson-like dynamics becomes more dramatic: while the density imbalance oscillates around its new equilibrium value $\hbar e / \sqrt{1 + \rho^2}$, the relative phase between the photon and exciton condensates $S(t)$ becomes monotonously increasing (or decreasing, depending on the sign of $e$) in time like shown in Fig. 1(d). This regime of running phase is analogous to the a.c. Josephson effect in the Josephson junction [29], or to internal Josephson-like oscillations between the populations in a mixture of spin-up and spin-down atoms when external magnetic field is applied [37]. (N.B., all the above explanations imply that the condensate is that of lower polaritons. For upper polaritons, the relative phase would oscillate around zero instead of $\pi$, and the decrease instead of increase in $h$ would bring the system closer to the separatrix and consequently to the Josephson regime).

All the dynamical regimes are finally summarized in the energy-detuning diagram displayed in Fig. 2. Dimensionless energy of the lower polariton system can change in the range $-\omega < h < 0$, while for upper polaritons $0 < h < \omega$. The critical values of $h$ which correspond to the transition between the ‘modified Rabi’ and the ‘internal Josephson’ regimes are defined for each effective detuning as $h = \pm e$. Thus, the diagram $h(e)$ is divided in four regions which correspond to Rabi-like
oscillations and Josephson-like oscillations of lower and upper polaritons (as shown in Fig. 2). The regime of pure Rabi oscillations (density oscillations with constant relative phase) corresponds to the point \( e = 0, \ h = -1 \). (Or, for the hypothetical equilibrium condensate of upper polaritons, \( e = 0, \ h = +1 \).) It can also be seen from this diagram that the larger is the detuning between the modes, the less extra energy is needed for the transition to the Josephson regime to happen.

At last, we analyze how the internal photon-exciton dynamics influence the external condensate features such as quantum phase of the photon field which can be accessed in the experiment. Using the solutions for \( \rho(t) \) and \( S(t) \) given by Eqs. (11) and (9), respectively, we analytically integrate the first-order differential equation (4) for the photon phase, imposing for simplicity zero initial condition \( S_C(0) = 0 \). The solution reads:

\[
S_C = -\frac{e}{2} t + \text{sign}(e - h) \omega \left[ \arctan \left( \frac{\omega^2 - he}{2} - \sqrt{\omega^2 - h^2} \right) \frac{\omega^2 - he}{|e - h|\omega} \right] + \arctan \left( \frac{\omega^2 - he}{2} + \sqrt{\omega^2 - h^2} \right) \frac{\omega^2 - he}{|e - h|\omega}. \tag{12}
\]

The obtained explicit time-dependence of the photon phase allows for Fourier analysis of its periodic part. In Fig. 3, we plot the Fourier spectra and the corresponding photon phase evolutions given by Eq. (12), for a fixed value of negative effective detuning \(-0.5\hbar \Omega_R\) and different initial states of the condensate characterized by the constant energy \( h \). From this analysis, one can clearly see that when the polariton system is deep in the regions ‘R’ or ‘J’ (as marked in Fig. 2), the linear rotation of the photon phase given by the term \(-\frac{et}{2}\) is slightly modified by harmonic beats between \( S_C \) and \( S_X \). On the contrary, when the condensate density is close to the critical lines \( h = \pm e \) (i.e. approaching the transition between the ‘modified Rabi’ and ‘internal Josephson’ regimes), the anharmonicity of the relative phase oscillations results in the ladder-like behavior of the photon phase (see the insets of Fig. 3(b) and (c)). Fourier spectrum of the periodic part of (12) then consists of multiple frequencies.

In conclusion, we fully analytically analyzed the influence of the energy detuning between the photon and exciton modes inside a microcavity on the internal oscillatory dynamics of the polariton condensate. We demonstrated that at any non-zero detuning, the exciton-photon system can, depending on its energy, oscillate around its equilibrium point or transit to the regime of monotonously growing relative phase, which we connect to the so-called internal Josephson oscillations. We also showed that the phase of the photon part of the polariton condensate captured in photoluminescence evolves in time essentially different for the condensates prepared in different initial states.

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