Game Theory to Study Interactions between Mobility Stakeholders

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Abstract—Increasing urbanization and exacerbation of sustainability goals threaten the operational efficiency of current transportation systems and confront cities with complex choices with huge impact on future generations. At the same time, the rise of private, profit-maximizing Mobility Service Providers leveraging public resources, such as ride-hailing companies, entangles current regulation schemes. This calls for tools to study such complex socio-technical problems. In this paper, we provide a game-theoretic framework to study interactions between stakeholders of the mobility ecosystem, modeling regulatory aspects such as taxes and public transport prices, as well as operational matters for Mobility Service Providers such as pricing strategy, fleet sizing, and vehicle design. Our framework is modular and can readily accommodate different types of Mobility Service Providers, actions of municipalities, and low-level models of customers’ choices in the mobility system. Through both an analytical and a numerical case study for the city of Berlin, Germany, we showcase the ability of our framework to compute equilibria of the problem, to study fundamental tradeoffs, and to inform stakeholders and policy makers on the effects of interventions. Among others, we show tradeoffs between customers’ satisfaction, environmental impact, and public revenue, as well as the impact of strategic decisions on these metrics.

I. INTRODUCTION

In past decades, cities worldwide have observed a dramatic urbanization. Today, 55\% of the world’s population resides in urban areas, and by 2050 the proportion is expected to reach 68\% [1]. A direct consequence of the population density growth is the increase of urban travel, and of the externalities it produces [2]. In this rapidly expanding setting, cities have to take important decisions to adapt their transportation system to welcome larger travel demands. This is a very complex task for at least three reasons. First, cities need to accommodate the changing travel needs of the population, by predicting them [3], and by ensuring fairness and equity [4]. Second, designed policies not only concern the population, by predicting them [3], and by ensuring fairness and equity [4]. Third, policies have to be designed while meeting global sustainability goals. It is not surprising that cities are estimated to be responsible for 78\% of the world’s energy consumption and for over 60\% of the global greenhouse emissions (30\% of which is produced by transportation, in US) [10]. Indeed, sustainability is central in policy-making worldwide: NYC plans to increase sustainable trips from 68\% to 80\% [6], and EU plans a 90\% reduction of emissions by 2050 [11].

Taken together, the aforementioned perspectives highlight the complexity of this socio-technical problem, and imperatively call for methods to inform and drive policy makers.

The goal of this paper is to lay the foundations for a framework through which one can model sequential, competitive interactions between stakeholders of the mobility ecosystem, and characterize their equilibria. Specifically, we leverage game theory to frame the problem in a modular fashion, and provide both an analytical and a numerical case study to showcase our methodology.

A. Related Literature

Our work lies at the interface of applications of game-theory in transportation science, and policy making related to future mobility systems. Game theory has been leveraged to solve various mobility-related problems. Main application areas include optimization of pricing strategies for Mobility Service Providers such as ride-hailing companies, micromobility (μM) and, in a near future, Autonomous Mobility-on-Demand (AMoD) systems [5]. Indeed, such services gained a considerable share of the transportation market in recent years; e.g., in NYC, ride-hailing companies have increased their daily trips by 1,000\% from 2012 to 2019 [6]. While offering more choices to travellers, these systems operate benefiting from public resources (such as roads and public spaces), are profit-oriented, and often lead to potentially disruptive consequences for the efficiency of the transportation system and for society at large [7]–[9]. In this avenue, cities gain an important, onerous regulatory role. Third, policies have to be designed while meeting global sustainability goals. It is not surprising that cities are estimated to be responsible for 78% of the world’s energy consumption and for over 60% of the global greenhouse emissions (30% of which is produced by transportation, in US) [10]. Indeed, sustainability is central in policy-making worldwide: NYC plans to increase sustainable trips from 68% to 80% [6], and EU plans a 90% reduction of emissions by 2050 [11].

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There has been research on policy making for future mobility systems not involving game theory [37]–[44]. Strategies to reduce externalities (including tolls, subsidies, electrification) are proposed in [37]–[40], and socially efficient arguments are made in [41], [42]. Finally the economics of ride-hailing, AVs and carpooling companies is studied in [43], [44].

Overall, all these works either focus on specific problems, neglect some mobility stakeholders, or ignore game-theoretic dynamics. So, to the best of our knowledge, there does not exist a comprehensive framework which allows one to formulate and solve mobility problems involving interactions between different stakeholders, at different time-scales, all the way from municipalities to customers, through MSPs.

B. Statement of Contribution

To fill this gap, we study the interactions between a central municipality, MSPs and customers from a game-theoretic perspective. Our contribution is threefold. First, we provide a general game-theoretic framework to model the sequential and simultaneous interactions between a municipality and MSPs. Second, we instantiate our framework with two low-level models of the mobility system: a parallel-arc congestion game, and a game-theoretic model including ride-hailing companies. Third, we present numerical results for the city of Berlin, and derive insights to inform stakeholders of the mobility ecosystem.

C. Organization

The remainder of this paper is as follows. We specify our problem setting and model in Section II. In Section III we detail our case study and present numerical results. We draw conclusions and present an outlook on future research in Section IV. Proofs are relegated to the appendix.

II. MOBILITY INTERACTIONS AS SEQUENTIAL GAMES

As outlined in Section I, urban mobility systems feature a broad variety of complex interactions. We classify such interactions based on the time scale at which they occur. We identify four time scales: a day, a month, a year, and five years. While day-to-day interactions include specific operational conditions such as dynamic pricing and rebalancing policies for MSPs, monthly interactions cover changes in the regions served by a particular MSP. When looking at a horizon of one year, one can consider general price plans for public transport, taxation systems on ride-hailing companies, as well as logistic decisions for MSPs such as fleet sizing and fleet diversification. Finally, a horizon of five years could include infrastructural changes, land use planning, and public contracts for transportation systems. In the following, we detail a model for the yearly horizon. This way we consider long-term horizon interventions (e.g., transportation network topology) as fixed parameters and include short-term horizon dynamics (e.g., mode selection, rebalancing, etc.) in what we call low-level model of the mobility system. Nonetheless, our methodology readily applies to the other settings as well.

A. Game-theoretic Model

We consider a one-stage sequential game between a municipality and \( N \in \mathbb{N} \) also called single-leader multi-follower Stackelberg game. Herein, the municipality first decides on policies such as taxes, public transport prices, and number of released vehicles licenses, to maximize social welfare. The profit-oriented MSPs then selfishly co-design their fleet and their pricing strategies (see Fig. 1). Finally, the outcome of the game results from a low-level model of the mobility system, which includes the dynamics happening on a short-term horizon.

Formally, the game is specified as follows. The municipality chooses its action \( a_0 \) from the (possibly uncountable) set of actions \( \mathcal{U}_0 \). Since the municipality plays first, actions and strategies coincide: \( a_0 = a_0 \) means that the municipality plays action \( a_0 \in \mathcal{U}_0 \); so, the set of strategies \( \Gamma_0 \) and the set of actions \( \mathcal{U}_0 \) coincide. For instance, if the municipality only designs the (flat) price of public transport, then \( \mathcal{U}_0 = \Gamma_0 := \mathbb{R}_{\geq 0} \). MSPs \( j \in \{1, \ldots, N\} \) selects their action \( a_j \) from the (possibly uncountable) set of actions \( \mathcal{U}_j(\gamma_0) \), possibly dependent on the strategy of the municipality. Since MSPs play after the municipality, the strategy \( \gamma_j \) is a map \( \gamma_j : \Gamma_0 \rightarrow \bigcup_{\gamma_0 \in \Gamma_0} \mathcal{U}_j(\gamma_0) \), where \( \gamma_j(\gamma_0) \in \mathcal{U}_j(\gamma_0) \). We denote by \( \Gamma_j \) the set of all such maps. For instance, \( \gamma_j(\gamma_0) \in \mathcal{U}_j \) is the action that MSP \( j \) plays if the municipality played \( \gamma_0 \). For simplicity, we neglect the influence of \( \Gamma_j \) to the set of actions of \( \Gamma_k \) \( k \neq j \). Yet, our framework can be readily extended to accommodate such interactions. Finally, the payoffs of all agents result from a low-level model of the mobility system comprising, among others, day-to-day behavior of the customers, and dynamic pricing of MSPs.

Formally, we associate to the municipality a payoff function

\[
U_j : \Gamma_0 \times \Gamma_1 \times \ldots \times \Gamma_N \rightarrow \mathbb{R}
\]

for each \( j \), and to each \( \mathcal{U}_j \) \( j \in \{1, \ldots, N\} \) a payoff function

\[
U_j : \prod_{i \neq j} \mathcal{U}_i(\gamma_{i,j-1}) \rightarrow \mathbb{R}
\]

In our case studies, we will focus on \( U_j \) being the profit for MSPs and \( U_0 \) being the social welfare for the municipality.

Nonetheless, our framework accommodates players with different interests (e.g., return on investment).

B. Equilibria

To characterize equilibria of our game, we use the classical notion of pure equilibrium in sequential games. Intuitively, a tuple of strategies is an equilibrium of the game if no agent is willing to unilaterally deviate from its strategy. Formally:

**Definition 1 (Equilibrium).** The tuple \( (\gamma_0^*, \ldots, \gamma_N^*) \) is an equilibrium of the game if for all players \( j \in \{0, \ldots, N\} \), \( U_j(\gamma_j^*, \gamma_{-j}^*) \geq U_j(\gamma_j, \gamma_{-j}, \gamma_{-j}) \) holds, where the subscript \( -j \) represents all players but \( j \).
we assume that MSPs can choose arbitrarily large fleet sizes.

Definition 1 emphasizes why we distinguish between actions and strategies. In particular, Definition 1 would fail if expressed in terms of actions, as it would ignore the sequential nature of the game. Conversely, strategies, defined as maps from the “current information” to the set of actions, capture the sequential nature of the game. As well-known in game theory, equilibria need not exist: one may easily come up with examples of sequential games with no equilibrium. Nonetheless, we will see that, when one fixes a low-level model of the mobility system, it may be possible to study sufficient conditions for the existence of equilibria.

C. Discussion

First, we do not a priori fix the low-level model of the mobility system, allowing one to choose the instance which best suits a desired analytical setting. Examples of low-level models include congestion games [30], mobility simulators [45], and approaches based on reaction curves, ubiquitous in economics [46]. Second, we tacitly assumed a market with a fixed number of MSPs. This assumption is realistic for the yearly time scale of our game and can be relaxed for other time scales. Third, we assumed a sequential game with sequence as in Fig. 1. Arguably, one could think about MSPs acting first, making the municipality a reactive player. We believe that the proposed sequence well aligns with the sequential nature of the game. Conversely, strategies, defined if expressed in terms of actions, as it would ignore the actions and strategies. In particular, Definition 1 would fail if expressed in terms of actions, as it would ignore the sequential nature of the game.

Definition 2 (Wardrop equilibrium), \( x^* \in \{0, 1\}^{N+1} \) with \( \sum_{i=0}^{N} x_i = 1 \) is a Wardrop equilibrium if for all \( i, j \in \{0, \ldots, N\} \) one has \( \ell_j(x_i) > \ell_j(x_j) \Rightarrow x_i = 0 \).

In other words, this condition ensures that no agent has an incentive to travel through another arc: at equilibrium an agent has no incentive to travel through another arc.

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D. Specifying Low-level Models

To showcase our framework, we instantiate it with two different low-level models of the mobility system. First, we consider a simplistic mobility system resulting from decoupled congestion games on parallel-arc networks. This allows for a clear exposition of the analyzed dependencies, and a thorough analytical study of equilibria. Second, we consider a mobility system whereby a MSP applies dynamic pricing, and can therefore select its pricing strategy in real-time, and a MSP which strategically decides operational matter, such as fleet sizing and composition. Customers strategically react to minimize their overall travel cost, resulting from fares paid throughout the trip and monetary value of time.

1) Analytical Parallel-arc Congestion Game: We consider one demand (per unit time) between two non-identical nodes, connected by multiple parallel arcs (see Fig. 2). Each arc denotes an homogeneous mode of transportation, which (possibly combined with walking) leads customers from the origin to the destination node. The municipality chooses a non-negative price from the compact set \( U_0 := [0, p_0^{\text{max}}] \), for some \( p_0^{\text{max}} \in \mathbb{R}_{\geq 0} \), while MSPs co-design the price of the ride \( p_j \) and their fleet size \( f_j \), i.e., \( U_j := [0, f_j] \). For simplicity, we assume that MSPs can choose arbitrarily large fleet sizes

and prices. To each MSP \( j \in \{1, \ldots, N\} \) (and therefore to each arc) we assign a non-decreasing smooth delay function

\[ \ell_j(\gamma_j) : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0} \cup \{+\infty\} \]

which captures the total cost of using that arc to reach the destination. We use the notation \( \ell_j(\gamma_j) \) to emphasize the dependency of the delay function of the strategy of provider \( j \). Since customers minimize their individual cost, given as the sum of fares and monetary value of time, we consider delays functions of the form

\[ \ell_j(\gamma_j)(z) = \ell_j(p_j)(z) = p_j + VT \cdot \ell_0(z) \]

\[ \ell_j((p_j, f_j))(z) = \{ p_j + VT \cdot \hat{\ell}_j(z) \text{ if } z \leq f_j \}
\]

\[ \text{else,} \]

where \( VT \in \mathbb{R}_{\geq 0} \) is the customers’ (average) value of time, \( \ell_0 \in \mathbb{R}_{\geq 0} \) is the total time required to reach the destination with public transport, and \( \ell_j : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0} \cup \{+\infty\} \) is a non-negative non-decreasing smooth function accounting for congestion. For instance, in the case of an MSP operating on the road network, one may construct \( \ell_j \) based on the BPR function [47]. In this setting, we model the outcome of the interactions among customers via the well-known notion of Wardrop equilibrium:

Definition 2 (Wardrop equilibrium), \( x^* \in \{0, 1\}^{N+1} \) with \( \sum_{i=0}^{N} x_i = 1 \) is a Wardrop equilibrium if for all \( i, j \in \{0, \ldots, N\} \) one has \( \ell_j(x_i) > \ell_j(x_j) \Rightarrow x_i = 0 \).

In other words, this condition ensures that no agent has an incentive to travel through another arc: at equilibrium an agent has no incentive to travel through another arc.

Finally, if \( \ell_j \) for all \( j \in \{1, \ldots, N\} \) is strictly increasing, then the Wardrop equilibrium is unique.

To formally talk about the equilibrium we make the following assumption:

Assumption 1. The functions \( \ell_j \) are convex and strictly increasing for all \( j \in \{1, \ldots, N\} \).

Assumption 1 encompasses relevant delay functions, such as the BPR function [47]. Strict monotonicity, via Proposition 1 ensures uniqueness of the equilibrium of the congestion game. Hence, we unambiguously denote the Wardrop equilibrium by \( x^*(\gamma_0, \gamma_1, \ldots, \gamma_N) \). Armed with this result, we can establish existence of an equilibrium for our game. To

Formally, \( \ell_j \) is a function from the set of strategies to the set of non-negative smooth functions.
do so, we first introduce utilities for all players. The socially-aware municipality maximizes social welfare, defined in terms of cost for the customers (including fares and monetary value of time), cost of emissions, and public revenue. Let Assumptions 1 and 2 hold and assume that \( \gamma \) is a minimizer of the cost function \( U_0(\gamma, \gamma_0) \) and \( \bar{\gamma} = \Pi_{[0,p_0^{\max}]} \left( \frac{1}{2} + \frac{\gamma_0}{p_0^{\max}} \right) \).

where \( p_0^{\max} \) and \( p_0 \) are strictly positive weights, \( \epsilon_j \) is the marginal cost of emissions of MSP, \( k \), \( j \), and \( k \) are strictly positive weights, and \( \bar{\gamma} \) collects the strategies of all players but \( j \). For ease of exposition, we consider profit-maximizing systems.

Corollary 4. Let Assumptions 1 and 2 hold, and assume that \( \gamma \) is affine in the set \( A \). Theorem 2 (Equilibria of the Sequential Game) shows intuitive features in extreme cases:

1. A municipality prioritizing the cost for customers (i.e., \( k_1 = 1, k_2, k_3 \to 0 \)) will choose \( \gamma_0^* \to 0 \).
2. A municipality prioritizing the environmental impact (i.e., \( k_2 = 1, k_1, k_3 \to 0 \)) will choose \( \gamma_0^* \to 0 \).
3. A municipality prioritizing its revenue (i.e., \( k_3 = 1, k_1, k_2 \to 0 \)) will choose \( \gamma_0^* \to 0 \).

The generalization of this approach to general congestion games is an active field of research [48], and we leave its treatment to future work.

2) Game-theoretic Model of the Mobility System: We adapt the game-theoretic model of the mobility system from [26]. In particular, we consider a mobility system with an AMoD, an [M], and a taxi company, and public transport. We assume that the AMoD operator changes prices dynamically, while all other MSPs and the municipality strategically choose their prices on a longer time horizon. In addition to travelling with MSPs or public transport, customers can also walk from their origin to their destination. Formally, we model the mobility system as a multigraph \( G = (V, A, s, t) \), where \( V \) is the set of vertices, \( A \) is the set of arcs, and \( s: A \to V \) and \( t: A \to V \) map each arc to its origin and destination, respectively. We define arc subsets \( AMoD, [M], AM, APT, ATest, A9y, \) each inducing a subgraph. We model customers demand by a triple \( (o, d, \alpha) \), where \( o \) is the origin of the trip, \( d \) is its destination, and \( \alpha \) is the rate of customers per unit time, and consider \( M \) demands.

Assumption 3 ensures that customers of demand \( d \in \{1, \ldots, M\} \) select their trip via a mobile app, offering them two options. First, the AMoD ride, which takes time \( t_{AMoD} \) and results from the shortest path from the customer’s origin to the customer’s destination on \( AMoD \). Second, the most convenient option between \( [M] \) public transport, taxi, and walking. The most convenient option is defined as the one minimizing the sum of ticket prices and monetary value of time, and takes time \( t_{\text{shortest}} \) (whereby the time is again computed via shortest path). For each demand, we assume a linear reaction curve: the rate of customers choosing the AMoD ride decreases linearly with the price of the AMoD ride. The parameters of the linear curve depend on the customers’ distribution of value of time; see [26], [56]. We consider a transportation system in steady state and suppose that the AMoD operator offers mobility service on her subgraph \( G_{AMoD} \). To prevent imbalances in her fleet, the AMoD operator rebalances her fleet via empty vehicles. Formally, we dictate that the sum of vehicle flows entering a node equals the sum of the vehicle flows exiting it. When taking operational decisions, the AMoD operator has a limited number of AVs and it is subject to two types of taxes imposed by the municipality: a distance-based tax on all AVs and an additional distance-based tax on AVs driving empty, without customers. Then, the AMoD operator selects
prices to maximize profit. Formally, prices result from the quadratic convex optimization problem

$$\max_{x_i \in \Omega(0,1), f_0} \sum_{i=1}^{N} \left[ x_i (m_i x_i + q_i) - \sum_{j \in A} c_j f_{ij} x_i + c_{0,j} f_{0,j} \right]$$

s.t. \( B^T (f_{ij} x_i + f_0) = 0 \) \( \sum_{i} t_{AMoD,ij} x_i + \sum_{j} t_{j} f_{0,j} \leq N_{fleet}, \)

where \( x_i \) denotes the flow of customers served by the AMoD operator and \( f_0 \) are rebalancing vehicle flows. Here, \( m_i \leq 0 \) and \( q_i \) are parameters defining the reaction curve, depending on the value of time, \( \alpha_i \) AMoD, and \( t_{AMoD} \). The parameters \( c_j \) and \( c_{0,j} \) model operational costs, including taxes, corresponding to arc \( j \in A \). The vector \( f_j \in \{0,1\}^{\text{AMoD}} \) denotes the shortest path on \( G_{AMoD} \) from \( o_i \) to \( d_j \). Constraint (5a), with \( B \) being the incidence matrix of \( G_{AMoD} \), ensures vehicle conservation at each node. Constraint (5b) upper-bounds final prices. Finally, result prices correspond from each vehicle’s reaction curve. The profit of other MSFs, defined as excess of revenue over costs, results from the solution of this optimization problem, via the demand function. Finally, social welfare results from the weighted combination of the cost for the customers, CO2 emissions, and public revenue. Specifically, the cost for the customers results from the fares and the monetary value of time. CO2 emissions result from the distance driven by AVs, taxi, and PMV vehicles; here, we neglect emissions of the public transportation systems, as we consider them as fixed and independent from the system’s load, therefore not influencing strategic decisions. Finally, public revenue results from taxes and public transport tickets.

Consistently with the described low-level model of the mobility system, we consider a game among a municipality deciding on public transport prices and taxes (specific values, but also generic taxation strategies) for AMoD vehicles. In line with current public transport prices in many cities, we parameterize fares via a short-distance price (SDP) in \( P_{p,\leq} \subseteq \mathbb{R}_{\geq 0} \), a long-distance price (LDP) in \( P_{l,\leq} \subseteq \mathbb{R}_{\geq 0} \), and a cutoff distance in \( D \subseteq \mathbb{R}_{\geq 0} \). We consider two types of taxes: a distance-based tax on AVs in \( t_{m} \subseteq \mathbb{R}_{\geq 0} \) and an additional distance-based tax on empty AVs in \( t_{e} \subseteq \mathbb{R}_{\geq 0} \), resulting from the scaling of the first tax. So, overall, the strategy space of the municipality is \( \Gamma_0 = P_{p,\leq} \times P_{l,\leq} \times D \times t_{m} \times t_{e} \).

The AMoD operator chooses the propulsion type of AVs, for a set of options \( E \) (e.g., Internal Combustion Engine (ICEV) or Hybrid Electric Vehicle (HEV) or Battery Electric Vehicle (BEV)), the automation level from \( A \) (e.g., standard vehicle (SV), AV), and the fleet size from \( F \subseteq \mathbb{N}_{\geq 0} \).

Our framework can be easily extended to capture different degrees of vehicle automation [42], [57], [58]. As AMoD applies dynamic pricing, decisions on prices do not happen at the level of our game, but are rather embedded in the low-level model of the mobility system. The action space of the AMoD operator is \( U_{1}(\gamma_0) = U_{1} = E \times A \times F \). Hence, \( \Gamma_1 \) consists of all maps from \( \Gamma_0 \) to \( U_1 \). For instance, the action of the AMoD operator if the municipality played 70% is \( \gamma_1(\gamma_0) = (e, a, n) \in E \times A \times F \), where the AMoD operator decides on prices by choosing base and variable, mileage-dependent, prices from \( P_{m,b} \times P_{m,v} \subseteq \mathbb{R}_{\geq 0} \). She also chooses the type of vehicles from \( M \) (e.g., e-scooter (ES) or shared bike (SB)), giving \( U_2(\gamma_0) = U_2 = P_{m,b} \times P_{m,v} \times M \). Finally, the taxi company decides on base and variable prices from \( P_{b,\leq} \times P_{b,v} \subseteq \mathbb{R}_{\geq 0} \), giving \( U_3(\gamma_0) = U_3 = P_{b} \times P_{b,v} \).

To compute equilibrium, we use an average waiting time of 3 min [61]. To compute \( t_{AMoD} \), we use public transport schedules, velocity of PMV vehicles, and an average walking velocity of 3.13 mph. We account for congestion effects by increasing the nominal travel time of each interested arc by 56%, corresponding to congestion levels in the evening peaks in Berlin [62]. In line with [56], we assume the customers’ value of time to be uniformly distributed between 10 USD/h and 17 USD/h. We report the parameters, such as operational costs and emissions in Table I and the action spaces of the players in Table II.

We compute equilibria of the sequential game of Section II via backward induction. First, we look for Nash Equilibria (NE) of the simultaneous game between MSP. We report all of them in Fig. 3, locating them with respect to the three metrics defining social welfare: cost for the customers, emissions, and public revenue. Second, we compute the equilibrium of the sequential game by selecting the NE maximizing social welfare. As this depends on the weight of

| Parameter | Variable | Value | Units | Source |
|-----------|----------|-------|-------|--------|
| Vehicle operational cost | \( c_{ve} \) | 5.78 | USD/mile | [49], [50] |
| Vehicle fixed cost | \( c_{vf} \) | 19,000 | USD/car | [49], [51] |
| Vehicle emissions | \( c_{ve} \) | 0.16 | kg/mile | [49], [51], [52] |
| Vehicle life | \( l_v \) | 186,000 | miles | |

TABLE I: Parameters, variables, numbers, and units for the case studies.
between free public transport and no taxes and too expensive public transport and unsustainable taxes (which also lead to low public revenue). These are three extreme scenarios; for each choice of weights, we can compute the corresponding equilibrium.

Second, while we cannot a priori give an equilibrium, we can identify solutions which are always inconvenient. In general, NE are incomparable: is NE EQ better than NE EQ? It depends on the weights of the metrics in social welfare: NE EQ yields larger public revenue, but also larger cost for customers. Hence, we call NE EQ and NE EQ incomparable. However, some equilibria are objectively better than others. For instance, NE EQ dominates NE EQ, as it outperforms it in all three metrics. We call non-dominated equilibra rational, and depict them in red in Fig. 4. Interestingly, all NE yielding high emissions are dominated, i.e., never rational.

Third, we can study fundamental tradeoffs of the system. For instance, lowering public revenue from 200,000 USD/h to 150,000 USD/h (i.e., 25% reduction) leads to 50% lower emissions and 10% lower cost for customers.

Fourth, we can “zoom-in” and analyze the actions corresponding to each solution, as shown in Fig. 5 for distance-based taxes. As expected, high taxes correlate with high public revenue. However, they also correlate with larger costs for customers, confirming the well-known principle that the tax load partially falls on customers, and not only on sellers.

Fifth, we can evaluate the impact of interventions on other metrics. For instance, Fig. 5 shows the AMoD modal share. As expected, higher emissions correlate with larger AMoD modal share. Interestingly, though, we do not observe a correlation with the cost for customers: there are NE yielding to low costs and low AMoD modal share as well as NE yielding to low costs and large AMoD modal share.

Sixth, we analyzed results from the perspective of a socially-aware municipality. Yet, our framework can be directly used by (profit-oriented) MSFs to reason on strategic decisions.
In this paper, we proposed a game-theoretic framework to study interactions between stakeholders of the mobility ecosystem. Our framework relies on the theory of sequential games, and can modularly accommodate different low-level models of the mobility system. We instantiated our framework in two case studies, a parallel arc congestion game and a game-theoretic model of the transportation system, and study them both analytically and numerically. With our framework, we arm stakeholders of the mobility ecosystem with analytical tools to reason about interventions and tradeoffs in mobility systems. Our work opens the field for various future research streams. First, we would like to instantiate our framework for various classes of low-level models of the mobility system, by explicitly characterizing equilibria and studying algorithms to efficiently compute them. Second, we want to exploit our framework to model and study interactions happening at different time scales. Third, we aim at applying our methodology to study other settings with similar structures, such as energy and global maritime shipping markets.

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[61] The proof follows directly from the KKT conditions of the optimization problem (1) for \( f_j \geq 1 \).

\[ \gamma_j^*(\gamma_0) = (p_j^*(\gamma_0), x_j^*(\gamma_0, \gamma_0)) \] for \( p_j^*(\gamma_0) \in \mathbb{R}_{\geq 0} \).

Proof. It suffices to observe that (i) \( p_j < \bar{c}_j \) and (ii) \( f_j > x_j^*(\gamma_0, \bar{p}_0) \) are always suboptimal.

Proof of Theorem 4. First, by Lemma 6 there no loss of generality in assuming [MSP] only decide on prices. By Lemma 5 \( x_j^* \) only depends on \( \gamma_j \) and \( \gamma_0 \), so the game reduces to \( N \) parallel single leader-single follower Stackelberg games, whereas [MSP] need to select the action maximizing their profit. Here, we can without loss of generality assume \( U_j(\gamma_j) = [\bar{x}_j^j + V_j(\bar{t}_j - \bar{t}_j^j)] \); else, the problem is straightforward. Then, \( U_j(\gamma_j, \gamma_0) = (\gamma_j - \bar{c}_j) \cdot \bar{t}_j^1 - \frac{20}{V_j^T} - \bar{t}_j^1 + \bar{\gamma}_j^0 + \bar{\gamma}_j^0 \).

Since \( \bar{t}_j \) strictly increasing and convex, its inverse is strictly increasing and concave. Thus, \( d^2U_j / d\gamma_j^2 < 0 \), and \( U_j \) is strictly concave. So, its maximizer is unique and continuous in the parameters. In particular, \( \gamma_0 \to \gamma_j^*(\gamma_0) \) is well-defined and continuous. Hence, \( \gamma_0 \to U_j(\gamma_0, \gamma_0) \) is a continuous function, being the composition of continuous functions. Since \( \Gamma_0 \) is compact, Weierstrass’ thereom ensures the existence of a maximizer of \( \gamma_0 \). So, \( \gamma_0^*, \gamma_1^*, \ldots, \gamma_N^* \) is an equilibrium.

Proof of Corollary 3. The proof follows directly from the proof of Theorem 4. Indeed, by strict concavity, \( \gamma_0^* \) is a necessary and sufficient condition for optimality. By the intermediate value theorem, \( \gamma_0^* \) admits at least a solution; uniqueness follows again from strict concavity.

Proof of Corollary 7. With affine delay functions, we have \( \gamma_j^*(\gamma_0) = (\gamma_0 + V_j \cdot \bar{t}_j^1 - \gamma_j - \gamma_j^0) / \beta_j \). Thus, \( \gamma_j^*(\gamma_0) \) reduces to

\[ \gamma_j^*(\gamma_0) = k_3^j \gamma_0 - k_1^j \gamma_0 + V_j \cdot \bar{t}_j^1 - \gamma_j - \gamma_j^0 \]

with \( \gamma_j := (\gamma_0 + V_j \cdot (\bar{t}_j^1 - \gamma_j - \gamma_j^0) - \gamma_j^0) / (2V_j^T \beta_j) \). This is a strongly concave function, whose maximum lies at \( \gamma_j^0 \).