Revisiting the four-quark operator matrix elements for the lifetime of $\Lambda_b$

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Heavy quark expansion can nicely explain the lifetime of $\Lambda_b$. However, there still exist sizable uncertainties from the four-quark operator matrix elements in the power of $1/m_b^3$. In this work, the leading order results of the four-quark operator matrix elements $\langle \Lambda_b | (\bar{b}q)_{V-A}(\bar{q}b)_{V-A}| \Lambda_b \rangle$ and $\langle \Lambda_b | (\bar{b}b)_{V-A}(\bar{q}q)_{V-A}| \Lambda_b \rangle$ are obtained using full QCD sum rules. Contributions from dimension-0,3,5 are considered. It turns out that the contributions from dimension-3,5 are proportional to the mass of $u/d$ quark. Stable Borel region can be found, and for this reason, the uncertainties caused by the QCD sum rule parameters are small. The leading logarithmic corrections are also considered, which turn out to be a little destructive. Our results are close to the lower bound of the existing theoretical predictions.

I. INTRODUCTION

Recently, LHCb updated a measurement of the $\Omega_c$ lifetime with $\tau(\Omega_c) = 268 \pm 24 \pm 10 \pm 2$ fs$^1$, which is nearly four times larger than the current world-average value $\tau(\Omega_c) = 69 \pm 12$ fs$^2$. On the one hand, the experimental colleagues are re-examining their measurements; on the other hand, theoretical explanation is highly demanded. Some attempts have been made to solve this puzzle$^3$. However, there is still a lack of more reliable calculation based on QCD for the hadronic matrix elements.

In fact, there was also a conflict between theory and experiment for the lifetime ratio $\tau(\Lambda_b)/\tau(B_d)$ as early as 1996. Taking $\tau(B^0) = (1.519 \pm 0.004)$ ps in PDG2020$^4$ as a benchmark, for $\tau(\Lambda_b) = (1.14 \pm 0.08)$ ps in PDG1996$^5$, one can find the ratio:

$$\tau(\Lambda_b)/\tau(B^0) = 0.75 \pm 0.05.$$ (1)

Theoretically, the ratio deviated from unity at the level of 20% is considered to be too large. Nowadays we know that the low value of $\tau(\Lambda_b)/\tau(B_d)$ or the short $\Lambda_b$ lifetime was a purely experimental issue. The world averages in PDG2020 are

$$\tau(\Lambda_b) = (1.471 \pm 0.009) \text{ ps}, \quad \tau(\Lambda_b^0)/\tau(B^0) = 0.964 \pm 0.007.$$ (2)

The new measurements of the $\Lambda_b$ lifetime are in good agreement with the Heavy Quark Expansion (HQE) result$^6$:

$$\frac{\tau(\Lambda_b)}{\tau(B_d)_{\text{HQE 2014}}} = 1 - (0.8 \pm 0.5)\%_{1/m_b^2} - (4.2 \pm 3.3)\%_{1/m_b^3} - (0.0 \pm 0.5)\%_{1/m_b^4} - (1.6 \pm 1.2)\%_{1/m_b^5}$$

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1. [1]
2. [2]
3. [3]
4. [4]
5. [5]
6. [6]
= 0.935 ± 0.054. \hfill (3)

Heavy quark expansion describes inclusive weak decays of hadrons containing heavy quarks and in particular lifetimes. It is a generalization of the operator product expansion (OPE) in \(1/m_Q\) in the Minkowski space, and nonperturbative effects can be systematically studied.

The starting point of HQE is the following transition operator

\[ T = i \int d^4x \, T[L_W(x)\overline{L_W}(0)], \] \hfill (4)

where \(L_W\) is the effective weak Lagrangian governing the decay \(Q \to X_f\). With the help of the optical theorem the total decay width of \(H_Q\) can be given as

\[ \Gamma(H_Q) = \frac{2 \text{Im} \langle H_Q|T|H_Q \rangle}{2M_H}, \] \hfill (5)

where \(H_Q\) denotes a hadron containing a heavy quark \(Q\), and \(M_H\) is its mass. The right hand side of Eq. \(5\) is then calculated using OPE for the transition operator \(T\) \[3, 6\]

\[ 2 \text{Im}T = \frac{G_m^2 m_Q^5}{192\pi^3} \xi \left( c_3 Q \bar{Q} Q + \frac{c_5 Q}{m_Q^2} \bar{Q} \sigma \cdot G Q + \frac{c_6 Q}{m_Q^2} T_6 + \frac{c_7 Q}{m_Q^2} T_7 + \cdots \right), \] \hfill (6)

where \(\xi\) is the relevant CKM matrix element, \(T_6\) consists of the four-quark operators \((\bar{Q} \Gamma q)(\bar{q} \Gamma Q)\) with \(\Gamma\) representing a combination of the Dirac and color matrices, and a subset of \(T_7\) is the four-quark operators containing derivative insertions.

However, it can also be seen from Eq. \(3\) that, the main uncertainty of the lifetime ratio comes from the \(1/m_Q^3\) corrections from the \(\Lambda_b\) matrix elements. The relevant baryon matrix elements can be parameterized in a model-independent way \[3\]:

\[ \langle \Lambda_b |(\bar{q}b)_{V-A}(\bar{q}b)_{V-A}|\Lambda_b \rangle = f_{B_q} m_{B_q} m_{\Lambda_b} L_1, \]
\[ \langle \Lambda_b |(\bar{q}b)_{S-P}(\bar{q}b)_{S-P}|\Lambda_b \rangle = f_{B_q} m_{B_q} m_{\Lambda_b} L_2, \]
\[ \langle \Lambda_b |(\bar{q}Q)_{V-A}(\bar{q}Q)_{V-A}|\Lambda_b \rangle = f_{B_q} m_{B_q} m_{\Lambda_b} L_3, \]
\[ \langle \Lambda_b |(\bar{b}q^\beta)_{S-P}(\bar{q}^\beta b^\alpha)_{S-P}|\Lambda_b \rangle = f_{B_q} m_{B_q} m_{\Lambda_b} L_4. \] \hfill (7)

where \(V - A\) denotes the weak current and \(S \pm P\) denote \(1 \pm \gamma_5\). These matrix elements are not all independent \[3\], and in this work we will only consider the parameters \(L_1\) and \(L_3\), which can be related by \(\tilde{B}\):

\[ L_3 = -\tilde{B} L_1. \] \hfill (8)

One can see from Table \[1\] that \(L_1\) ranges from \(-0.60\) to \(-0.03\). This article intends to make some efforts in this direction.

In \[12\], we derived the transition form factors from doubly heavy baryons to singly heavy baryons using QCD sum rules for the first time. However, considering there were few theoretical
TABLE I: $L_1$ predicted by different theoretical methods. This table is copied from [8].

| $L_1$      | $B$      | Method                  |
|------------|----------|-------------------------|
| $-0.103(10)$ | 1        | 2014 Spectroscopy update [7] |
| $-0.22(4)$   | 1.21(34) | 1999 Exploratory Lattice [8] |
| $-0.22(5)$   | 1        | 1999 QCDSR v1 [9]       |
| $-0.60(15)$  | 1        | 1999 QCDSR v2 [9]       |
| $-0.033(17)$ | 1        | 1996 QCDSR [10]         |
| $\approx -0.03$ | 1        | 1979 Bag model [11]     |
| $\approx -0.08$ | 1        | 1979 NRQM [11]          |

results and experimental data available to compare with, we then applied our calculation method to the semileptonic decay of $\Lambda_b \to \Lambda_c l \bar{\nu}$ [13]. Our predictions for the form factors and decay widths are comparable with those of HQET and Lattice QCD. Similar to [13], in this work, we will also consider the leading order contributions from the perturbative, quark condensate and quark-gluon mixed condensate diagrams.

The authors of [10] also adopted Cutkosky cutting rules to obtain the spectral density of the correlation function. While the main difference between [10] and this work is that, the former performed the analysis under the framework of HQET, while in this work we will handle the hadronic matrix elements in full QCD. As can be seen in [14, 15], our results can reduce to those of [10] in the heavy quark limit. In addition, we will also consider the leading logarithmic corrections in this work.

The rest of this paper is arranged as follows. In Sec. II, we will show the main steps of calculating the hadronic matrix elements. Numerical analysis will be performed in Sec. III, and some discussions will also be given. We conclude our paper in the last section.

II. QCD SUM RULE CALCULATION

The following interpolating current is adopted for $\Lambda_b$:

$$J = \epsilon_{abc}(u_a^T C \gamma_5 d_b) Q_c,$$

where $Q$ denotes the bottom quark, $a, b, c$ are the color indices and $C$ is the charge conjugate matrix. The correlation function is defined as

$$\Pi(p_1, p_2) = i^2 \int d^4x d^4y \ e^{-ip_1 \cdot x + ip_2 \cdot y} \langle 0 | T \{ J(y) \Gamma_6(0) \bar{J}(x) \} | 0 \rangle$$

with $\Gamma_6$ being one four-quark operator.

Following the standard procedure of QCD sum rules, the correlation function will be calculated at hadronic level and QCD level. At the hadronic level, after inserting the complete set of baryon
In Eq. (12),

\[ \Pi_{\text{had}}(p_1, p_2) = \lambda_H^2 \frac{\langle \not{p}_2 + M \rangle (a + b \gamma_5) (\not{p}_1 + M)}{(p_2^2 - M^2)(p_1^2 - M^2)} + \cdots, \tag{11} \]

where \( \lambda_H = \lambda_{\Lambda_b}, M = m_{\Lambda_b} \) are respectively the pole residue and mass of \( \Lambda_b \), and \( a \) and \( b \) are used to parameterize the hadronic matrix element of interest \( \langle \Lambda_b(q', s') | \Gamma_6 | \Lambda_b(q, s) \rangle = \bar{u}(q', s')(a + b \gamma_5)u(q, s) \), and the ellipsis stands for the contribution from higher resonances and continuum spectra.

It can be seen from Eq. (11) that there are 8 Dirac structures, but only 2 parameters need to be determined. Using the similar prescription as that used in \[ 12, 13, 15, \] that is, by considering the contributions from the negative-parity baryons, Eq. (11) is updated to

\[ \Pi_{\text{had}}(p_1, p_2) = \lambda_+ \lambda_+ \frac{(\not{p}_2 + M_+)(a^{++} + b^{++} \gamma_5)(\not{p}_1 + M_+)}{(p_2^2 - M_+^2)(p_1^2 - M_+^2)} \]

\[ + \lambda_+ \lambda_- \frac{(\not{p}_2 + M_+)(a^{-} + b^{+} \gamma_5)(\not{p}_1 - M_-)}{(p_2^2 - M_+^2)(p_1^2 - M_+^2)} \]

\[ + \lambda_- \lambda_+ \frac{(\not{p}_2 - M_-)(a^{+} + b^{-} \gamma_5)(\not{p}_1 + M_+)}{(p_2^2 - M_-^2)(p_1^2 - M_-^2)} \]

\[ + \lambda_- \lambda_- \frac{(\not{p}_2 - M_-)(a^{-} - b^{-} \gamma_5)(\not{p}_1 - M_-)}{(p_2^2 - M_-^2)(p_1^2 - M_-^2)} \]

\[ + \cdots. \tag{12} \]

In Eq. (12), \( M_{+,(-)} \) and \( \lambda_{+,(-)} \) respectively denote the mass and pole residue of \( \Lambda_b \left( \frac{1}{2}^{+-(-)} \right) \), and \( a^{-} \) is the parameter \( a \) with the negative-parity final state \( \Lambda_b \left( \frac{3}{2}^{-} \right) \), and the positive-parity initial state \( \Lambda_b \left( \frac{1}{2}^{+} \right) \), and so forth. To arrive at Eq. (12), we have also adopted the following definitions:

\[ \langle \Lambda_{b+}(q', s') | \Gamma_6 | \Lambda_{b+}(q, s) \rangle = \bar{u}_+(q', s')(a^{++} + b^{++} \gamma_5)u_+(q, s), \]

\[ \langle \Lambda_{b+}(q', s') | \Gamma_6 | \Lambda_{b-}(q, s) \rangle = \bar{u}_+(q', s')(a^{+} + b^{+} \gamma_5)(i \gamma_5)u_-(q, s), \]

\[ \langle \Lambda_{b-}(q', s') | \Gamma_6 | \Lambda_{b+}(q, s) \rangle = \bar{u}_-(q', s')(i \gamma_5)(a^{++} + b^{++} \gamma_5)u_+(q, s), \]

\[ \langle \Lambda_{b-}(q', s') | \Gamma_6 | \Lambda_{b-}(q, s) \rangle = \bar{u}_-(q', s')(i \gamma_5)(a^{--} + b^{--} \gamma_5)(i \gamma_5)u_-(q, s). \tag{13} \]

In the above equations, it is not necessary to introduce \((i \gamma_5)\), but it is convenient in the calculation. One can then determine \( a^{++} \equiv a \) and \( b^{++} \equiv b \) unambiguously, thereby the following forward scattering matrix element:

\[ \langle \Lambda_b(q, s) | \Gamma_6 | \Lambda_b(q, s) \rangle = \bar{u}(q, s)(a + b \gamma_5)u(q, s) \]

\[ = 2a m_{\Lambda_b}, \tag{14} \]

where we have used \( \bar{u}(q, s)u(q, s) = 2 m_{\Lambda_b} \) and \( \bar{u}(q, s) \gamma_5 u(q, s) = 0 \). It can be seen that to obtain the matrix element, we only need to obtain \( a^{++} \) in Eq. (13).

At the QCD level, the correlation function is written as a double dispersion relation

\[ \Pi_{\text{QCD}}(p_1^2, p_2^2, q^2) = \int_0^\infty ds_1 \int_0^\infty ds_2 \rho_{\text{QCD}}^{(s_1, s_2, q^2)} \frac{\rho_{\text{QCD}}(s_1, s_2, q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}. \tag{15} \]
with $\rho_{\text{QCD}}(s_1, s_2, q^2)$ being the spectral function, which can be obtained by applying Cutkosky cutting rules. The sum rule is given by equating the four pole terms in Eq. (12) to

$$\int_0^{s_0} ds_1 \int_0^{s_2} d\rho_{\text{QCD}}(s_1, s_2, q^2) \frac{(s_1 - p_1^2)(s_2 - p_2^2)}{s_0},$$

where $s_0$ is the continuum threshold parameter. By comparing the coefficients of different Dirac structures on both sides of the equation, one can have 8 equations to solve 8 unknown parameters $a_{\pm\pm}$ and $b_{\pm\pm}$, especially, one can arrive at

$$a_{++} = \frac{\{M_2^2, M_-^2, M_-^2, 1\}}{\lambda_+^2(M_+ + M_-)^2} \exp \left( \frac{2M_+^2}{T^2} \right),$$

where $A_{1,2,3,4}$ are coefficients of $p_2^2p_1^2$, $p_2^2$, $p_1$ and 1 in Eq. (16), $BA_i \equiv B_{T^2, T^2} A_i$ are doubly Borel transformed coefficients, and $T^2$ is the Borel mass parameter. [12, 13] contain more details for obtaining $A_i$.

At the QCD level, we consider the leading order contributions from the perturbative (dimension-0), quark condensate (dimension-3) and quark-gluon mixed condensate (dimension-5) diagrams. For $\langle \Lambda_b | (\bar{b}q)V_{-A}(\bar{q}b)V_{-A} | \Lambda_b \rangle$, we find that the latter two contributions are proportional to the mass of $u/d$ quark. It turns out that only the perturbative contribution survives, as can be seen in Fig. [4]. This situation is the same as those of two-point correlation function of $\Lambda_b$ and three-point correlation function of $\Lambda_b \to \Lambda_c$ transition form factors [13]. Since the only difference between $(\bar{b}q)V_{-A}(\bar{q}b)V_{-A}$ and $(\bar{b}b)V_{-A}((\bar{q}q)V_{-A}$ is in the color space, the same situation occurs for $\langle \Lambda_b | (\bar{b}b)V_{-A}(\bar{q}q)V_{-A} | \Lambda_b \rangle$, except that there is one sign difference between the perturbative contributions of $\langle \Lambda_b | (\bar{b}b)V_{-A}(\bar{q}q)V_{-A} | \Lambda_b \rangle$ and $\langle \Lambda_b | (\bar{b}q)V_{-A}(\bar{q}b)V_{-A} | \Lambda_b \rangle$. Therefore $\langle \Lambda_b | (\bar{b}b)V_{-A}(\bar{q}q)V_{-A} | \Lambda_b \rangle = -\langle \Lambda_b | (\bar{b}q)V_{-A}(\bar{q}b)V_{-A} | \Lambda_b \rangle$ as far as we are concerned. As can be seen in [12], the contributions from gluon condensate are small, therefore we do not consider them in this work.
A. The leading logarithmic corrections

In this work, we will also consider the leading logarithmic (LL) corrections following \[16\]. For convenience, we define

\[
\begin{align*}
\Gamma_6^{(1)} &= (bq)_{V-A}(\bar{q}b)_{V-A}, \\
\Gamma_6^{(2)} &= (\bar{b}b)_{V-A}(\bar{q}q)_{V-A}, \\
\Gamma_6^{(\pm)} &= \frac{1}{2} (\Gamma_6^{(1)} \pm \Gamma_6^{(2)})
\end{align*}
\]

with \( q = u/d \) and denote

\[
\langle \Lambda_b | \Gamma_6 | \Lambda_b \rangle \equiv \langle \Gamma_6 \rangle_{\Lambda_b}
\]

for short. The new basis of \( \Gamma_6^{(\pm)} \) are rescaled without mixing.

The following steps are adopted to obtain \( \langle \Gamma_6^{(1,2)} \rangle_{\Lambda_b}^{\text{LL}} \) - the LL corrected version of \( \langle \Gamma_6^{(1,2)} \rangle_{\Lambda_b} \):

- Calculate \( \langle \Gamma_6^{(1)} \rangle_{\Lambda_b} \) and \( \langle \Gamma_6^{(2)} \rangle_{\Lambda_b} \) to arrive at \( \langle \Gamma_6^{(\pm)} \rangle_{\Lambda_b} \). Since \( \langle \Gamma_6^{(2)} \rangle_{\Lambda_b} = -\langle \Gamma_6^{(1)} \rangle_{\Lambda_b} \) as far as dimension-0,3,5 are concerned, one has \( \langle \Gamma_6^{(+)} \rangle_{\Lambda_b} = 0 \) and \( \langle \Gamma_6^{(-)} \rangle_{\Lambda_b} = \langle \Gamma_6 \rangle_{\Lambda_b} \).

- Multiply \( \langle \Gamma_6^{(-)} \rangle_{\Lambda_b} \) by \( \left( \frac{\log(\mu_0/\Lambda_{\text{QCD}})}{\log(\mu/\Lambda_{\text{QCD}})} \right)^{2\gamma_F+\gamma_6^{(-)}} \) to arrive at the LL corrected \( \langle \Gamma_6^{(-)} \rangle_{\Lambda_b}^{\text{LL}} \) given that only the perturbative contribution survives, where \( \mu_0 \approx 1 \text{ GeV}, \mu \approx m_b, \gamma_F = -1/\beta_0 \), and \( \gamma_6^{(-)} = 4/\beta_0 \).

- \( \langle \Gamma_6^{(1)} \rangle_{\Lambda_b}^{\text{LL}} = \langle \Gamma_6^{(1)} \rangle_{\Lambda_b} + \langle \Gamma_6^{(-)} \rangle_{\Lambda_b}^{\text{LL}} = \langle \Gamma_6^{(-)} \rangle_{\Lambda_b}^{\text{LL}} \) and \( \langle \Gamma_6^{(2)} \rangle_{\Lambda_b}^{\text{LL}} = \langle \Gamma_6^{(2)} \rangle_{\Lambda_b} - \langle \Gamma_6^{(-)} \rangle_{\Lambda_b}^{\text{LL}} = -\langle \Gamma_6^{(-)} \rangle_{\Lambda_b}^{\text{LL}} \).

III. NUMERICAL RESULTS

As can be seen in the end of the last section, \( \langle \Gamma_6^{(+)} \rangle_{\Lambda_b}^{\text{LL}} = 0 \), in this section, we will concentrate on determining \( \langle \Gamma_6^{(-)} \rangle_{\Lambda_b}^{\text{LL}} \). Also because of Eq. \[14\], we only need to determine the \( a^{++} \) of \( \langle \Gamma_6^{(-)} \rangle_{\Lambda_b}^{\text{LL}} \).

The renormalization scale is taken as \( \mu = m_b \), and the bottom quark mass is taken as \[4\]: \( m_b(m_b) = 4.18 \pm 0.03 \text{ GeV}, \lambda_+ = 0.0432 \pm 0.0022 \text{ GeV}^3 \) is taken from \[13\].

The threshold parameter \( s_0 \) and the Borel parameters \( T^2 \) are determined in a standard way. Adjust \( s_0 \) to find the flattest curve of \( a^{++} \) as a function of \( T^2 \), on which, there should exist a stable extremum at some \( T^2 \). The corresponding \( s_0 \) and \( T^2 \) are considered as the optimal choices for these parameters, as shown by the blue curve in Fig. \[2\]. In this figure, we have also plotted the suboptimal choices for error estimation, as shown by the red curve. The corresponding numerical results are listed in Table \[11\].

Our prediction for \( a^{++} \) of \( \langle \Gamma_6^{(-)} \rangle_{\Lambda_b}^{\text{LL}} \) is

\[
a^{++}[\langle \Gamma_6^{(-)} \rangle_{\Lambda_b}^{\text{LL}}] = (-23.8 \pm 1.1 \pm 3.4 \pm 2.2) \times 10^{-4} \text{ GeV}^3,
\]
FIG. 2: $a^{++}$ of $\langle \Gamma_6^{(1)} \rangle_{\Lambda_b}^{LL}$ as a function of the Borel parameters $T^2$. The blue and red curves correspond to optimal and suboptimal choices for $s_0$. The values of $a^{++}$ at the extreme points of these curves are taken as the optimal and suboptimal evaluations. The explicit values for these $s_0$ and $T^2$ can be found in Table II.

| $s_0$/GeV$^2$ | $T^2$/GeV$^2$ | $a^{++}[\langle \Gamma_6^{(-)} \rangle_{\Lambda_b}^{LL}] / (10^{-4}$ GeV$^3$) |
|---------------|---------------|---------------------------------------------------------------------|
| Optimal       | 6.01$^2$      | -23.8                                                              |
| Suboptimal    | 6.02$^2$      | -24.9                                                              |

where the uncertainties respectively come from the QCDSR parameters $s_0$ and $T^2$, the bottom quark mass $m_1$ and the pole residue $\lambda_+$. As can be seen in the last section, $\langle \Gamma_6^{(1)} \rangle_{\Lambda_b}^{LL} = \langle \Gamma_6^{(-)} \rangle_{\Lambda_b}^{LL}$, then our prediction for $L_1$ is given by

$$L_1 = -0.0260 \pm 0.0012 \pm 0.0037 \pm 0.0025,$$

where we have used $m_{B_q} = 5.280$ GeV and the same decay constant $f_{B_q} = 186$ MeV as in [3]. It can be seen from Table II that, our result is close to that of HQET sum rules in [10] and that of the bag model in [11].

Some comments are given in order.

- The spectral density in Eq. (15) also depends on the $q^2 \equiv (p_1 - p_2)^2$. For the forward scattering matrix elements of interest, $q^2$ is taken to be 0.

- The optimal choice for the threshold parameter $s_0 = (6.01$ GeV$)^2$ is close to that of the two-point correlation function of $\Lambda_b$ with $s_0 = (5.95$ GeV$)^2$ [13], as expected.

- The optimal $T^2$ is taken around $T^2 = 20$ GeV$^2$, which is $\sim \mu_b^2$, as expected.

- For $\tilde{B}$ defined in Eq. (8), $\tilde{B} = 1$ is a good approximation: at least for dimension-0,3,5 considered in this work, the equal sign is strictly true.
• It turns out that the LL corrections are destructive. \( \langle \Gamma_6^{(1)} \rangle_{\Lambda_b}^{LL} \) is about 13% smaller than \( \langle \Gamma_6^{(1)} \rangle_{\Lambda_b} \).

IV. CONCLUSIONS

Heavy quark expansion can nicely explain the lifetime of \( \Lambda_b \). However, there still exist sizable uncertainties in the \( 1/m_b^3 \) corrections from the \( \Lambda_b \) matrix elements. In this work, the leading order results of the four-quark operator matrix elements \( \langle \Lambda_b | (\bar{b}q)_{V-A} (\bar{q}b)_{V-A} | \Lambda_b \rangle \) and \( \langle \Lambda_b | (\bar{b}b)_{V-A} (\bar{q}q)_{V-A} | \Lambda_b \rangle \) are obtained using full QCD sum rules. Contributions from dimension-0,3,5 are considered. It turns out that the contributions from dimension-3,5 are proportional to the mass of \( u/d \) quark. Stable Borel region can be found, and for this reason, the uncertainties caused by the threshold parameter \( s_0 \) and the Borel parameter \( T^2 \) are small. We have also considered the leading logarithmic corrections, which turn out to be a little destructive. Our results are close to those of HQET sum rules in [10] and those of the bag model in [11].

In [13], we investigated the semileptonic form factors of \( \Lambda_b \to \Lambda_c \) and our results are comparable to those of HQET and Lattice QCD. Using the same techniques, in this work, we investigate the four-quark operator matrix elements for the lifetime of \( \Lambda_b \). This work can be viewed as the first one in a series of papers towards deciphering the \( \Omega_c \) lifetime puzzle.

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[1] R. Aaij et al. [LHCb], Phys. Rev. Lett. 121, no.9, 092003 (2018) doi:10.1103/PhysRevLett.121.092003 [arXiv:1807.02024 [hep-ex]].
[2] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98, no.3, 030001 (2018) doi:10.1103/PhysRevD.98.030001
[3] H. Y. Cheng, JHEP 11, 014 (2018) doi:10.1007/JHEP11(2018)014 [arXiv:1807.00916 [hep-ph]].
[4] P. A. Zyla et al. [Particle Data Group], Phys. Rev. D 54, 1-720 (1996) doi:10.1103/PhysRevD.54.1
[5] A. Lenz, Int. J. Mod. Phys. A 30, no.10, 1543005 (2015) doi:10.1142/S0217751X15430058 [arXiv:1405.3601 [hep-ph]].
[6] J. L. Rosner, Phys. Lett. B 379, 267-271 (1996) doi:10.1016/0370-2693(96)00352-8 [arXiv:hep-ph/9602265 [hep-ph]].
[7] M. Di Pierro et al. [UKQCD], Phys. Lett. B 468, 143 (1999) [erratum: Phys. Lett. B 525, 360-360 (2002)] doi:10.1016/S0370-2693(99)01166-1 [arXiv:hep-lat/9906031 [hep-lat]].
[9] C. S. Huang, C. Liu and S. L. Zhu, Phys. Rev. D 61, 054004 (2000) doi:10.1103/PhysRevD.61.054004
[arXiv:hep-ph/9906300 [hep-ph]].
[10] P. Colangelo and F. De Fazio, Phys. Lett. B 387, 371-378 (1996) doi:10.1016/0370-2693(96)01049-0
[arXiv:hep-ph/9604425 [hep-ph]].
[11] B. Guberina, S. Nussinov, R. D. Peccei and R. Ruckl, Phys. Lett. B 89, 111-115 (1979)
doi:10.1016/0370-2693(79)90086-8.
[12] Y. J. Shi, W. Wang and Z. X. Zhao, Eur. Phys. J. C 80, no.6, 568 (2020) doi:10.1140/epjc/s10052-020-
8096-2 [arXiv:1902.01092 [hep-ph]].
[13] Z. X. Zhao, R. H. Li, Y. L. Shen, Y. J. Shi and Y. S. Yang, Eur. Phys. J. C 80, no.12, 1181 (2020)
doi:10.1140/epjc/s10052-020-08767-1 [arXiv:2010.07150 [hep-ph]].
[14] E. V. Shuryak, Nucl. Phys. B 198, 83-101 (1982) doi:10.1016/0550-3213(82)90546-6
[15] Z. X. Zhao, R. H. Li, Y. J. Shi and S. H. Zhou, [arXiv:2005.05279 [hep-ph]].
[16] B. L. Ioffe, Nucl. Phys. B 188, 317-341 (1981) [erratum: Nucl. Phys. B 191, 591-592 (1981)]
doi:10.1016/0550-3213(81)90259-5
[17] A. A. Ovchinnikov, A. A. Pivovarov and L. R. Surguladze, Int. J. Mod. Phys. A 6, 2025-2034 (1991)
doi:10.1142/S0217751X91001015
[18] M. E. Peskin and D. V. Schroeder,