Abstract

In this paper we present an efficient integer programming (IP) based data association approach to simultaneous localization and mapping (SLAM). In this approach, the feature-based SLAM data association problem is formulated as a 0–1 IP problem. The IP problem is approached by first solving a relaxed linear programming (LP) problem. Based on the optimal LP solution, a suboptimal solution to the IP problem is then obtained by applying an iterative heuristic greedy rounding (IHGR) procedure. Unlike the traditional nearest-neighbor (NN) algorithm, the proposed algorithm deals with the global matching between existing features and measurements of each scan and is more robust for an environment of high-density features (the feature number is high and the distances between features are often very close) which is usually the case in outdoor applications. Detailed simulation and experimental studies show that the proposed IHGR-based algorithm has moderate computational requirement and offers better performance with higher successful rate of SLAM for complex environments of high density of features than the NN algorithm.

KEY WORDS—simultaneous localization and mapping, data association, integer programming, extended Kalman filtering

1. Introduction

A feature-based approach to simultaneous localization and mapping (SLAM) is to use the information obtained by sensors mounted on a vehicle to build and update a map of the environment and compute the vehicle location in that map (Leonard and Durrant-Whyte 1991; Fox 2001; Zhang, Xie, and Adams 2003). One of the critical problems in obtaining a robust SLAM solution is data association, i.e., relating sensor measurements to features in the map that has been built thus far (Leonard and Durrant-Whyte 1992; Guivant, Nebot, and Durrant-Whyte 2000; Bailey and Nebot, 2001; Castellanos, Neira, and Tardós 2001; Zhang, Xie, and Adams 2004a). Correct correspondences between the sensed feature observations and map landmarks are essential for consistent map construction since any single false matching may invalidate the entire process (Durrant-Whyte et al. 2001). Simple localization may be able to recover from a minor mis-association, because only the vehicle pose estimate is affected, but with SLAM the map is also altered and these inconsistencies tend to be self-propagating, causing divergence. Hence, data association failure is a much more serious problem for SLAM than for simple localization problems.

There have been some approaches to data association. In stochastic mapping, the simplest method is the nearest-neighbor (NN) algorithm which is a classical technique in tracking problems (Leonard and Durrant-Whyte 1992; Guivant, Nebot, and Baiker 2000, 2002; Guivant and Nebot 2001; Leonard and Feder 2001; Adams, Zhang, and Xie 2004). The great advantage of NN is its $O(mn)$ computational complexity in addition to its conceptual simplicity (here $m$ is the number of sensor measurements and $n$ is the number of existing features in the map). It performs satisfactorily when clutter density is low and sensor precision is high. However, during the process of SLAM, especially in complex outdoor environments, clutter level is high and the innovations in matching different observations obtained from the same vehicle position are correlated. In this situation, the NN algorithm may accept a wrong matching, which leads to divergence in the state estimate. In order to improve the robustness of data association, Neira and Tardós (2001) presented an approach using a joint compatibility test based on the branch and bound search with a computational cost which is acceptable in indoor environments. Nieto et al. (2003) give a fast SLAM algorithm for data association by applying the multiple hypotheses tracking method in a variety of outdoor environments. The
experimental complexity estimates show that if the number of features in one scan is large, these algorithms will not be fast enough for real-time implementation. In other approaches, Bailey et al. (2000) considered relative distances and angles between points and lines in two laser scans and used graph theory to find the largest number of compatible pairings between the measurements and existing features. The work of Lim and Leonard (2000) applies a hypotheses test to implement data association of the relocation in SLAM using geometrical constraints. Castellanos et al. (1999) use binary constraints to localize the robot with an a priori map using an interpretation tree. In these methods, geometric constraints among features are used to obtain hypotheses with pairwise compatible pairings. However, pairwise compatibility does not guarantee joint compatibility (Neira and Tardós 2001), and additional validations are required. Other data association algorithms used in multitarget tracking include joint probability data association (JPDA) and multiple hypotheses tracking (MHT; Bar-Shalom and Fortmann 1988). The JPDA associates all the measurements that are falling inside the validation region of a track to itself by a probabilistic weighting procedure and performs fairly well in highly cluttered environments, but the calculation of the weighting probability is difficult in SLAM application. Historically, the MHT algorithm first proposed by Reid (1979) has been considered to be the only approach that can truly provide an optimal data association solution. However, its practicality and feasibility have been hampered since it requires the enumeration of an exponentially increasing number of feasible joint association hypotheses to evaluate probabilities.

In our work, we propose a single-frame 0–1 integer programming (IP) approach to data association. First, we formulate the data association of SLAM as an IP problem. In order to reduce the computational burden, a validation gate is applied to reduce the size of the solution space. An iterative heuristic greedy rounding (IHGR) process, based on linear programming (LP) techniques instead of the traditional Lagrangian relaxation algorithm (Poore 1994; Poore and Robertson 1997), is then proposed to obtain a suboptimal solution to the IP problem. The algorithm has moderate computational requirements. Simulation results show that the proposed method gives a much higher success rate of SLAM for environments of high-density features than the NN algorithm, while the cost of computation is moderately higher than the latter. Further, the experiment in a real outdoor environment shows that the NN algorithm leads to a diverged vehicle pose estimate whereas the proposed algorithm performs satisfactorily. Compared to other existing methods such as the JCBB and MHT algorithms, our approach has a lower computational complexity and provides a good trade-off between accuracy and computational cost. It is also worth noting that the proposed algorithm can be easily extended to multiscan cases with a much lower computational requirement than the existing methods (Neira and Tardós 2001; Nieto et al. 2003).

The paper is organized as follows. Section 2 is devoted to an IP formulation for data association in SLAM. In Section 3 we present an IHGR algorithm for the IP problem. In Section 4 we show the simulation and experimental results. Some conclusions are drawn in Section 5.

2. Problem Formulation

In this section we formulate the data association of SLAM as a 0–1 IP problem similar to Lochana, Wijesoma, and Adams (2003). A mathematical framework of SLAM, which is based on the extended Kalman filter (EKF; Dissanayake et al. 2001), will be applied.

Data association of SLAM is a decision process of associating measurements (observations) with existing features in the stochastic map. It should be noted that the term “measurements” (observations) in this paper refers to the observed features after feature extraction rather than the raw sensor measurements. Generally, the number of measurements obtained in each scan is not equal to the number of features whose positions are estimated by the EKF. Each measurement may either (i) belong to a previously known geometric feature, (ii) be a new geometric feature or (iii) be a spurious measurement (also called a false alarm). On the other hand, there also exist features that do not have associated measurements in the current scan. A dummy element is applied to denote the case of a false alarm or a new feature or a feature that does not have an associated measurement (Poore 1994).

Assume that there are $M$ measurements from the latest scan which are to be assigned to $N$ existing features in the map built based on the previous scans. Typically, $M \neq N$. Define the binary assignment variable

$$x_{nm} = \begin{cases} 1 & \text{if measurement } m \text{ is assigned to feature } n \\ 0 & \text{otherwise.} \end{cases}$$

(1)

Note that $x_{n0} = 1$ implies that the feature $n$ has no associated measurement in the current scan, and $x_{0m} = 1$ implies that measurement $m$ is not assigned to any of the existing $N$ features, but instead assigned to a dummy feature—false alarm or newly initialized feature. In the data association process, we make the reasonable assumption that one measurement originates from at most one feature, and one feature can produce at most one measurement. Therefore, the following constraints can be imposed to the association variables:

$$\sum_{m=0}^{M} x_{nm} = 1, \quad n = 1, 2, \ldots, N \quad (2)$$

$$\sum_{n=0}^{N} x_{nm} = 1, \quad m = 1, 2, \ldots, M. \quad (3)$$
Our goal is to match the sensor’s observations with the features by providing estimates of feature positions relative to the vehicle pose at the time of the current scan. In order to formulate the two-dimensional (2D) assignment problem, a generalized likelihood ratio, which involves feature state estimates for the candidate associations, is used to assign a cost to each association. Similarly to the multitarget tracking problem (Poore 1994), we maximize a likelihood function \( LH \) as follows:

\[
LH = \prod_{\{n,m \in E_{nm}\}} \Lambda(z_m, f_n)
\]

where \( \Lambda(z_m, f_n) = \frac{1}{|2\pi S|^{1/2}} \exp \left\{ -\frac{1}{2} [z_m - \hat{z}_n]^T S^{-1} [z_m - \hat{z}_n] \right\} \)

subject to

\[
\begin{align*}
\sum_{n=0}^{N} x_{nm} &= 1, & n &= 1, 2, \ldots, N \\
\sum_{m=0}^{M} x_{nm} &= 1, & m &= 1, 2, \ldots, M
\end{align*}
\]

where \( x_{nm} \in \{0, 1\} \) and

\[
C_{nm} = \begin{cases} 
0 & \text{if } m = 0 \text{ or } n = 0 \\
-\ln \Lambda(z_m, f_n) & \text{otherwise.}
\end{cases}
\]

In this algorithm, two points should be noted.

- If a measurement does not fall into the \( 3\sigma \) region of any of the existing \( N \) features (see the gating process in Section 3.1), we assume that it is a new feature and add it to the SLAM map directly. It will no longer be considered in the above 2D assignment.

- If an existing feature does not have any measurement that falls into its \( 3\sigma \) region, we consider that this feature is not observed in the current scan. No further matching will be carried out for this feature.

3. Proposed Iterative Heuristic Greedy Rounding Based Data Association Algorithm

In this section, we propose an algorithm to solve the data association problem formulated in the previous section. The method is a combined IHGR and LP algorithm. In order to reduce the computational burden, a validation gate is applied first to reduce the above global association to several local associations.

3.1. Gating

In order to reduce the solution space, gating is first applied. Only measurements that are close enough to the predicted state of an existing feature are considered possible candidates of association with the feature. The criterion of gating is given by

\[
\tau_{ij} = v_i^T S_i^{-1} v_j \leq \epsilon \quad i = 1, 2, \ldots, N; \quad j = 1, 2, \ldots, M
\]

where

\[
v_j(k + 1) = z_j(k + 1) - \hat{z}_i(k + 1 | k)
\]

and \( S_i \) is the covariance of the innovation \( v_j \).

Note that, since \( v_j \) is a Gaussian random variable, \( \tau_{ij} \) is a random variable following the \( \chi^2 \) distribution. Thus, a validation gate, \( \epsilon \), is used to decide whether the measurement \( z_j(k + 1) \) is a close enough match to the predicted feature position. From the \( \chi^2 \) distribution table, we know that \( \tau_{ij} < 6.63 \) with a probability of 0.99 for a variable of two degrees of freedom. Here we set \( \epsilon = 6.63 \).

3.2. Iterative Heuristic Greedy Rounding

3.2.1. Review of Relaxation and Rounding Technique

The 0–1 IP problem that was formulated in the previous section is an NP-hard problem. Many NP-hard combinatorial optimization problems can be attacked with approximation algorithms that yield a feasible solution in polynomial time with cost close enough to the optimal cost (Hochbaum 1997). Such approximation algorithms can be loosely categorized as combinatorial approximation algorithms and LP-based approximation algorithms. Here, we are interested in the latter.
category. In order to solve the IP problem, we often first relax it into an LP problem (Cormen et al. 2001). In general, the relaxation refers to the action of relaxing the integer requirement of a linear IP to turn it into an LP. However, the optimal solution to the LP problem in general does not coincide with the solution to the initial IP problem. One of the basic techniques that is widely exploited to derive an LP-based approximation algorithm is LP-based rounding. It refers to how to construct a feasible solution for the IP from the LP (Parker and Rardin 1988; Cormen et al. 2001). Thus, the LP-based approximation algorithm first relaxes the IP problem to an LP problem, solves the LP problem, and then converts the fractional optimal solution of LP to an integer solution. The heuristic algorithm applied here is based on the relaxation and rounding algorithms as described in Miller and Franz (1996), Storms and Spieksma (2003), and Franz and Miller (1993).

3.2.2. Iterative Heuristic Greedy Rounding Procedure

By changing the integer constraint \( x_{nm} \in \{0, 1\} \) to \( 0 \leq x_{nm} \leq 1 \), the IP problem is relaxed to an LP one. The LP problem can be solved by basic LP algorithms, such as the Simplex algorithm (Vajda 1981). If the optimal solution \( x_{op} \) of the LP relaxation is fully integer-valued (in this case, all decision variables will have the value of either 0 or 1) then the solution \( x_{op} \) is optimal for the 0–1 IP problem in eq. (6) (Parker and Rardin 1988). Otherwise, we apply the IHGR procedure (see, for example, Miller and Franz 1996). Observe that the larger the decision variable \( x_{nm} \), the higher the probability that the \( n \)th measurement associates with the \( n \)th feature. Hence, the algorithm starts by setting the maximum decision variable (with a value close to 1) to 1 and all other entries in the same row and column to zero to meet the constraints (7) and (8). Then, we solve the LP problem for the rest of the assignment matrix and repeat the IHGR procedure to decide the next pairing of measurements and features. The process is continued until all measurements have been assigned. In this manner, a feasible (but not necessarily optimal) solution for the original IP problem is constructed.

In the IHGR procedure, when \( x_{nm} \) is set to 1, all variables in the column and row associated with the specific set in \( E_{nm} \) must be set to 0. Once a variable is forced to a certain value, it is no longer allowed to change. To achieve this, all rounded variables and all implicated variables are discarded from the IHGR procedure. In this way, the IHGR will never set the value of a variable twice. This deletion of variables also applies to the initial LP solution, i.e., all variables with value 1 and all zero-valued variables implicated by them are removed. The IHGR algorithm repeats the actions of selection, rounding, and deletion until there are no variables left. The outcome will then be a feasible solution to eq. (6).

The algorithm described above can be summarized in the following four steps.

**Step 1.** Relax the IP problem to an LP problem and solve the LP problem. If the solution is fully integer valued, then stop. Otherwise, go to step 2.

**Step 2.** Set the maximum decision variable to 1 and other entries in the same row and column to zero.

**Step 3.** Delete the elements that have been decided; go to step 1 for the rest of the assignment matrix.

**Step 4.** Repeat the above relaxation, selection, rounding, and deletion steps until all the elements are assigned.

Observe that IHGR can be implemented efficiently, but it is clear that there is no guarantee that this heuristic procedure yields the optimal solution to the IP problem. However, the simulations and experiments to be discussed in the following section show that IHGR does result in acceptable feature-measurement assignments of which the achieved cost is close to the optimal cost.

3.3. Algorithm Complexity

Due to the application of the gating process that is affected by random factors, we cannot give an exact description of the complexity. However, we know that in any fixed dimension, LP can be solved in strongly polynomial linear time (linear in the input size; Karmarkar 1984). For our case, the input size is \( M \times N \). Thus, we can roughly know the worst-case complexity of the proposed algorithm is \( O(MN+(M-1)(N-1)+...+(M-N+1)) \times 1 \).

Neira and Tardós (2001) presented a data association approach using a joint compatibility test based on the branch and bound search (JCBB). JCBB performs incremental construction and search of an interpretation tree of joint association hypotheses. The gating determines acceptable hypotheses and performs branch and bound pruning of the search space. The discussion in Neira and Tardós (2001) does not provide any theoretical bound, but gives an empirical complexity estimate of \( O(1.53^N) \), where \( N \) is the number of observed features. Therefore, when the observed feature number is large (such as more than 30), our algorithm can work much more efficiently than the JCBB.

4. Simulation and Experimental Results

The algorithm presented was tested in two different environments: an artificial environment and a real outdoor environment. In these two environments, SLAM was implemented by using the data association algorithm proposed in the previous section. The experimental results show that the algorithm is efficient.

4.1. Simulation Environment

The first test environment is established by randomly generating some features and assuming that the vehicle trajectory
is along a circle whose radius is 62 m. The robot moves at a constant speed and the heading angle changes 1° at each sampling instant. The environments involve 105 features which are randomly distributed in a region of $120 \times 120$ m$^2$. When the vehicle moves, some of the features are observed. We assume that the feature positions are unknown, which is the case in SLAM, and we match the features with the observations by using the NN and the IHGR algorithms, respectively.

In order to compare the successful rates of these two data association algorithms, we change feature positions 200 times to implement the SLAM. The feature positions are randomly generated each time. We apply the NN data association algorithm and the IHGR data association method to perform the SLAM processes. The successful SLAM rate when using the IHGR algorithm is 95.5% (191/200) while it is only 78.5% (157/200) for the NN algorithm. Here, a successful SLAM means that the average error of the vehicle position and the average error of the feature position during the SLAM process are both smaller than 0.5 m.

When the features are closely located, the NN algorithm fails. Figures 1 and 2 show the SLAM results where the NN algorithm fails whereas the proposed IHGR algorithm performs well. In this case, the positions of three features in the environment are fixed, and they are located at $(27, 20.5)$, $(26, 19.5)$, and $(26.5, 19)$, respectively. The remaining features are randomly distributed. It can be observed from Figure 1 that the NN algorithm leads to diverged estimates of vehicle pose and feature positions. On the other hand, our IHGR method performs very well, as observed in Figure 2. In fact, the vehicle’s true path is almost overlapped with the estimated one during the SLAM process. The global error between the estimated vehicle’s position and the ground truth can be seen in Figure 3.

A comparison on execution time between the NN algorithm and the IHGR-based data association algorithm versus the number of features observed in one scan is shown in Figure 4 (the algorithms are run on a Pentium IV PC, 1.7 GHz) for the cases when the NN algorithm is able to give successful SLAMs. In the simulation, we extract data at each scan under different feature number and assume that the number of measurements is the same as that of the existing features (this is the most time-consuming case). The result shows that our IHGR-based method has moderate computational requirement and is implementable in real-time applications.

In Table 1, we show that the average global errors of the vehicle’s position estimate versus the number of features observed (observations) in each scan in the same environment mentioned above. We fixed the number of observations in each SLAM process, which lasted a few thousand steps, but changed the number of observations for different SLAM processes. The standard deviations of the laser sensor are $\sigma_r = 0.01$ m and $\sigma_\theta = 0.005$ rad. In the table, the errors are calculated as follows:

$$X\text{-direction error} = \frac{1}{K} \sum_{i=1}^{K} |E_{x_i}|$$

$$Y\text{-direction error} = \frac{1}{K} \sum_{i=1}^{K} |E_{y_i}|$$

where $|E_{x_i}|$ and $|E_{y_i}|$ are the absolute values of the $i$th
Fig. 3. The vehicle’s position errors in the global coordinate between the ground truth (GPS reading) and the estimated position using the IHGR data association algorithm in the SLAM process.

Table 1. The Comparison of the SLAM Performance When We Change the Number of Observations at Each Time-Step (M)

| M   | X-Direction Error | Y-Direction Error |
|-----|-------------------|-------------------|
| 2   | 0.2374            | 0.2518            |
| 5   | 0.1737            | 0.1946            |
| 8   | 0.1278            | 0.1469            |
| 10  | 0.1008            | 0.1348            |
| 13  | 0.0792            | 0.0986            |
| 15  | 0.0771            | 0.0910            |
| 18  | 0.0631            | 0.0874            |
| 20  | 0.0558            | 0.0806            |
| 25  | 0.0501            | 0.0645            |
| 30  | 0.0354            | 0.0439            |
| 35  | 0.0323            | 0.0357            |

Table 2. SLAM Performance Versus the Standard Deviation of the Sensor Range Noise

| σr  | X-Direction Error | Y-Direction Error |
|-----|-------------------|-------------------|
| 0.01| 0.0815            | 0.0979            |
| 0.02| 0.0989            | 0.1135            |
| 0.03| 0.1247            | 0.1389            |
| 0.05| 0.2049            | 0.2209            |
| 0.08| 0.2975            | 0.3108            |
| 0.1 | 0.3564            | 0.4532            |
| 0.15| 0.4753            | 0.6786            |
| 0.2 | 0.6785            | 0.9860            |
| 0.25| 0.9076            | 1.0062            |

Therefore, we only show the results when the number of observations is smaller than 35.

In order to examine the robustness of the IHGR-based algorithm with respect to sensor noise, we also carried out the simulations under various sensor range variances. From Adams (1999), we know that for any range finder, the range noise covariance can vary depending on the received signal amplitude while the angular uncertainty is relatively very small and little changed compared to the range noise. In the simulation, we fix \( \sigma_\theta = 0.005 \text{ rad} \). The sensor used here is a LADAR (laser detection and ranging) sensor; therefore, the standard deviation of the range noise \( \sigma_r \) can be typically from 0.01 to 0.25 m (Adams 1999) for different sensors. Table 2 shows the average performance of 10 SLAM processes for the same environments mentioned earlier with 12 observations.

In Section 3 we have compared the computational burden of our method with that of the JCBB algorithm, which is proposed in Neira and Tardós (2001). Here we compare the accuracy of these two methods. Note that the JCBB algorithm, which uses the branch and bound enumeration algorithm, gives an optimal solution to the data association problem. In this simulation, we use 30 features in the large-scale map because the speed of the JCBB algorithm becomes quite slow when the feature number is too large.

From Table 3 we can see that with the IHGR-based data association method a near optimal solution to the SLAM has been achieved. Observe that when the number of observations is 5, we obtain the same performance as the JCBB algorithm, which gives the optimal solution.

4.2. Real Outdoor Environment

4.2.1. SLAM with Artificial Beacons

In order to implement the IHGR data association algorithm in a SLAM process in a real environment, we first
Table 3. Comparison of the SLAM Performance for the JCBB and IHGR Algorithms Under Different Numbers of Observations

| M   | JCBB method | IHGR method | JCBB method | IHGR method |
|-----|-------------|-------------|-------------|-------------|
| 5   | 0.1802      | 0.1899      | 0.1802      | 0.1899      |
| 8   | 0.1346      | 0.1573      | 0.1389      | 0.1623      |
| 10  | 0.1025      | 0.1389      | 0.1101      | 0.1475      |
| 13  | 0.0834      | 0.1003      | 0.0924      | 0.1120      |
| 15  | 0.0799      | 0.0915      | 0.0856      | 0.1079      |
| 18  | 0.0702      | 0.0832      | 0.0786      | 0.0886      |
| 20  | 0.0628      | 0.0784      | 0.0703      | 0.0826      |
| 25  | 0.0591      | 0.0748      | 0.0651      | 0.0799      |
| 30  | 0.0528      | 0.0537      | 0.0580      | 0.0722      |

Note: The unit for the errors is meters.

use the experimental data set from http://www.acfr.usyd.edu.au/homepages/academic/enebot/dataset.htm obtained by Guivant and Nebot. The testing site is a car park at Sydney University. The vehicle is equipped with a GPS, a laser sensor, and wheel encoders. A kinematic GPS system of 2 cm accuracy was used to evaluate the ground truth. Thus, the true navigation map was available for comparison purposes. Wheel encoders give an odometric measurement of the vehicle location. The dead reckoning sensors and laser range sensor are combined to predict the vehicle’s trajectory using the EKF and to build up the map at the same time. In this experiment, the features used are artificial beacons. The feature detection was performed by using a geometric analysis of the range measurement to obtain the most likely centers of tree trunks using intensity information. The laser scans are processed using the algorithm of Guivant, Nebot, and Durrant-Whyte (2000) to detect tree trunk centers and estimate their radii.

We ran continuous SLAM for more than 5000 time-steps and obtained the map shown in Figure 5 using our proposed IHGR data association during the SLAM process. It can be seen that the IHGR method performs well for the SLAM implementation in real time. Figures 6 and 7 give the measurement (range and angle) innovation and their $2\sigma$ confidence bounds during the SLAM process, respectively. The results show that the IHGR-based algorithm works well during the SLAM process.

In order to check the effectiveness of the IHGR data association, we randomly choose two scans (scans 68 and 87) and show the matching matrix $x_{nm}$ after the IP problem is solved. In scan 68, the laser sensor obtained two measurements and the existing feature number has accumulated to 11. As mentioned, the term “measurement” means the extracted features. As seen in Table 4, measurement 1 is associated with feature 3 and measurement 2 is associated with feature 2. The rest

Fig. 4. Mean execution time of the IHGR and NN algorithms. The mean execution time of IHGR is nearly linear with respect to the observed feature number by repeated experiments.

Fig. 5. SLAM path and the feature map during the SLAM process with IHGR data association.
Table 4. Matching Matrix in Scan 68

| Existed Feature Number and Dummy Element | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 0 |
|-----------------------------------------|---|---|---|---|---|---|---|---|---|----|----|---|
| Scan 68                                 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0 |
| Scan 100                                | 2 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0 |
|                                          | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 0 |

![Fig. 6. Range innovation and its 2σ confidence bounds during the SLAM process with IHGR data association.](image1)

![Fig. 7. Observation angle innovation and its 2σ confidence bounds during the SLAM process with IHGR data association.](image2)

of the features are all undetected in this scan. In Table 5, for scan 87, measurement 1 is matched with a dummy element, which means that this is a new feature or false alarm. Since the probability of false alarm for the laser sensor is low, we regard the measurement as a new feature. The other measurements and features are similar to those in scan 68, which can be seen in Table 4.

REMARK 1. In this environment, the features are sparsely distributed. By checking the experimental data, we know that the number of observations per step ranges from 1 to 5. Therefore, the environment is relatively simple as far as data association is concerned. In this case, the NN algorithm also works well. We ran the whole SLAM with the NN and IHGR algorithms, respectively, and we found that the computational cost is similar although the NN algorithm is slightly faster than our method. For the NN data association, the whole SLAM process (involving more than 5000 time-steps) where the vehicle traveled several hundred meters took 40.9680 s while the IHGR algorithm took 58.1420 s on the same computer (the algorithms were run on a Pentium IV PC, 1.7 GHz, and we calculated the CPU time for each algorithm).

4.2.2. SLAM with Natural Features in the Experimental Environment

In order to verify the effectiveness of the IHGR algorithm, a more complex real environment is explored by our car-like Cycab vehicle (see Figure 8). In this case, the SLAM is processed with natural features other than artificial beacons in the last experiment. The features extracted from the complex campus environment are obtained by using the feature extraction algorithm given in Zhang, Xie, and Adams (2004b). The experimental data used were obtained from a 2D scanning range laser (SICK LMS200) mounted in the front of
Table 5. Matching Matrix in Scan 87

| Existed Feature Number and Dummy Element | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 0 |
|-----------------------------------------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|---|
| Scan 87                                 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 1 |
| 2                                        | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 1 |
| 3                                        | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 1  | 0 |
| 0                                        | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0  | 0  | 0  | 0  | 0  | 0 |

the vehicle. The laser returns a 180° planar sweep of range measurements in 0.5° intervals (i.e., 361 range values in an anticlockwise order) with a range resolution of 50 mm. Similar to the previous experiment, the vehicle is equipped with a GPS, a laser sensor, and encoders. A kinematic GPS system of 2 cm accuracy was used to evaluate the ground truth. Thus, the true navigation map was available for comparison purposes. The testing site is a long walkway around Hall 7 at Nanyang Technological University of Singapore. The real campus map including this road is shown in Figure 9. The dead reckoning sensors and laser range sensor are combined, in order to predict the vehicle’s trajectory using the EKF and to build up the map at the same time. During SLAM, in order to improve our map accuracy, we attempted to detect two types of features, namely the point features and circular features as described in Zhang, Xie, and Adams (2004b), and we used the IHGR algorithm in Section 3 to carry out data association.

Figure 10 shows part of the experimental environment. Figure 11 gives a typical laser scan during the run and shows the main features that are extracted from the scan data. Figure 12 shows the experimental results of SLAM. We can see that the estimated path is close to the GPS reading. In this figure, there is a break in the GPS data. This is because the vehicle is under a building which blocks the GPS signal (the GPS data coordinate is vacant roughly between (−10, −120) and (−30, −115)). That is, no ground truth is available in this segment of road. Figures 13 and 14 indicate the observation innovations and their $2\sigma$ confidence bounds during the whole process, which lasted for 450 s. These figures show that the bounds obtained using all natural features are consistent with the actual errors. It is also important to mention that the average position error of the entire SLAM process, except for the segment where the ground truth is unavailable, is smaller than 0.5 m. On the other hand, we also carried out the SLAM process with the NN data association. Unfortunately, the NN algorithm results in a diverged vehicle path estimation as shown in Figure 15, due to the complex environment with features of high density. In Figure 15, point A is the starting point of the SLAM process as in the previous experiment with the IHGR algorithm.

5. Conclusions

In this paper we have presented a new data association algorithm for SLAM which offers an excellent trade-off between accuracy and computational requirement. We first formulated the data association problem in SLAM as a 0–1 IP problem. In order to obtain a fast solution, the 0–1 IP problem was first relaxed to an LP problem. Then we proposed to use the IHGR procedure in conjunction with basic LP algorithms to obtain a feasible solution of the data association problem. Both the simulation and experimental results have demonstrated that the proposed algorithm is implementable and gives a better performance (higher successful rate of SLAM) than the
Fig. 10. The environment and the vehicle in our experiment.

Fig. 11. A typical laser scan of the experimental environment.

Fig. 12. A comparison between the estimated path and the path by the GPS readings during SLAM.

Fig. 13. The range observation's innovation and its 95% confidence bounds in the SLAM experiment.

Fig. 14. The angular observation's innovation and its 95% confidence bounds in the SLAM experiment.

Fig. 15. The unsuccessful path estimation of this experiment when the NN algorithm is applied. The vehicle started from point A in our experiment; see Figure 9.
commonly used NN algorithm for complex (outdoor) environments with a high density of features. Compared to the optimal JCBB algorithm, the proposed algorithm has lower computational complexity and is more suitable for real-time implementation.

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