Thermal Fluctuations and Validity of the 1-loop Effective Potential

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Abstract

We examine the validity of the 1-loop approximation to the effective potential at finite temperatures and present a simple test for its reliability. As an application we study the standard electroweak potential, showing that for a Higgs mass above 70 GeV, and fairly independently of the top mass (with $m_t \geq 90$ GeV), the 1-loop approximation is no longer valid for temperatures in the neighborhood of the critical temperature.
1 Introduction

The effective potential at finite temperatures is an important tool in the study of phase transitions in scalar and gauge field theories [1]. It is equivalent to the homogeneous coarse-grained free-energy density functional of statistical physics, with its minima giving the stable and, when applicable, metastable states of the system. For interacting field theories the effective potential is evaluated perturbatively, with an expansion in loops being equivalent to an expansion in powers of $\hbar$ [2]. The 1-loop approximation is then equivalent to incorporating the first quantum corrections to the classical potential. We start by briefly reviewing the calculation of the 1-loop potential for a self-interacting scalar field theory. The classical action in the presence of an external source $J(x)$ is

$$S[\phi, J] = \int d^4 x \left[ \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + \bar{\hbar} J(x) \phi(x) \right].$$  \hspace{1cm} (1)

The effective action $\Gamma[\phi_c]$ is defined in terms of the connected generating functional $W[J]$ as

$$\Gamma[\phi_c] = W[J] - \int d^4 x J(x) \phi_c(x),$$  \hspace{1cm} (2)

where the classical field $\phi_c(x, t)$ is defined by $\phi_c(x, t) \equiv \delta W[J]/\delta J(x)$, and

$$W[J] = -i\hbar \ln \int D\phi \exp \left[ \frac{i}{\hbar} S[\phi, J] \right].$$  \hspace{1cm} (3)

In order to evaluate $\Gamma[\phi_c]$ perturbatively, one writes the field as $\phi(x, t) \to \phi_0(x, t) + \eta(x, t)$, where $\phi_0(x, t)$ is a field configuration which extremizes the classical action $S[\phi, J]$, $\frac{\delta S[\phi, J]}{\delta \phi}|_{\phi=\phi_0} = 0$, and $\eta(x, t)$ is a small perturbation about that extremum configuration. The action $S[\phi, J]$ can then be expanded about $\phi_0(x, t)$ and, up to quadratic order in $\eta(x, t)$, we can use a saddle-point approximation to the path integral to obtain for the connected generating functional,

$$W[J] = S[\phi_0] + \hbar \int d^4 x \phi_0(x) J(x) + \frac{i\hbar}{2} \text{Tr} \ln \left[ \partial_\mu \partial^\mu + V''(\phi_0) \right].$$  \hspace{1cm} (4)

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In order to obtain the 1-loop expression for $\Gamma[\phi_c]$, we first note that writing $\phi_0 = \phi_c - \eta$ we get to first order in $\bar{h}$, $S[\phi_0] = S[\phi_c] - \bar{h} \int d^4 x \eta(x) J(x) + O(\hbar^2)$. Using this result and Eq. (4) into Eq. (2) we find, as $J \to 0$,

$$\Gamma[\phi_c] = S[\phi_c] + i \frac{\hbar}{2} \text{Tr} \ln \left[ \partial_{\mu} \partial_{\mu} + V''(\phi_c) \right].$$

(5)

The effective action can also be computed as a derivative expansion about $\phi_c(\vec{x}, t)$,

$$\Gamma[\phi_c] = \int d^4 x \left[ -V_{\text{eff}}(\phi_c(x)) + \frac{1}{2} (\partial_{\mu} \phi_c)^2 Z(\phi_c(x)) + \ldots \right].$$

(6)

The function $V_{\text{eff}}(\phi_c)$ is the effective potential. For a constant field configuration $\phi_c(\vec{x}, t) = \phi_c$ we obtain

$$\Gamma[\phi_c] = -\Omega V_{\text{eff}}(\phi_c),$$

(7)

where $\Omega$ is the total volume of space-time. Comparing Eqs. (5) and (7) we obtain for the 1-loop effective potential,

$$V_{\text{eff}}(\phi_c) = V(\phi_c) - i \frac{\hbar}{2} \Omega^{-1} \text{Tr} \ln \left[ \partial_{\mu} \partial_{\mu} + V''(\phi_c) \right].$$

(8)

When working at non-vanishing temperature, the same functional techniques can be used. In this case one is interested in evaluating the generating functional (the partition function) $Z[\beta, J]$ which is given by the path integral

$$Z[\beta, J] = N \int D\phi \exp \left[ - \int_0^\beta d\tau \int d^3 x \left( L_E - J(\phi) \right) \right],$$

(9)

where the integration is restricted to paths periodic in $\tau$ with $\phi(0, \vec{x}) = \phi(\beta, \vec{x})$, $L_E$ is the Euclidean Lagrangian, and $N$ is a normalization constant. Again one expands about an extremum of the Euclidean action and calculates the partition function by a saddle-point evaluation of the path integral. The result for the 1-loop approximation to the effective potential is

$$V_{\text{eff}}(\phi_c, T) = V_{\text{eff}}(\phi_c) + \frac{\hbar}{2\pi^2 \beta^4} \int_0^\infty dx \ x^2 \ln \left\{ 1 - \exp \left[ -\sqrt{x^2 + \beta^2 V''(\phi_c)} \right] \right\}. \quad (10)$$
From the above discussion it is clear that the 1-loop approximation to the effective action, Eq. (5), works best when the classical field does not differ much from the configuration that extremizes the classical action, \( \phi_c = \phi_0 + \eta \sim \phi_0 \), since in this case the saddle-point evaluation to the path integral is adequate. Also, \( \phi_c(\vec{x}, t) \) must be nearly constant so that the effective potential can be obtained from Eq. (7). As \( J \to 0 \), \( \phi_c(\vec{x}, t) \) is identified with \( \langle \phi \rangle \), the vacuum expectation value. How large can be the fluctuation \( \eta(\vec{x}, t) \) without spoiling the validity of the approximation? Clearly, for models that exhibit a second order transition, the approximation worsens as one approaches the critical temperature from above or below, with infrared corrections becoming progressively more important. One way of dealing with this problem is to obtain an improved effective potential where some of the infrared divergences are taken into account, for example by the summation of daisy (or super daisy) diagrams \([4]\). This method has recently been extensively discussed in connection with the standard electroweak potential, in an attempt to include infrared effects from higher gauge loops \([3]\). (For vector boson masses \( m_V \sim g\phi \), the expansion parameter \( g^2T/m_V \sim gT/\phi \) is large for small values of \( \phi \).)

Another approach is to use \( \varepsilon \)-expansion techniques in order to compute corrections to the critical exponents that control the singular behavior of physical quantities near the critical point, as is familiar from the theory of critical phenomena \([6]\). Here, we will not be concerned with improving the 1-loop approximation, but in quantifying its reliability. Our results should be of relevance in particular in the study of weakly first-order transitions, where large fluctuations about equilibrium may be present, invalidating the 1-loop approximation for certain values of the temperature or other relevant physical parameters of \( V_{\text{eff}}(\phi, T) \).

In this paper we propose a simple method to estimate the validity of the 1-loop approximation to the effective potential. We will argue that the statistically dominant ther-
mal fluctuations around the minimum of \( V_{\text{eff}}(\phi, T) \) are spherically symmetric and have roughly a correlation volume, where the correlation length is given by the inverse temperature dependent mass, \( \xi(T) = m^{-1}(T) \). Assuming that these fluctuations are Boltzmann suppressed, we compute the average value for their amplitude. Following Gleiser, Kolb, and Watkins (GKW) we will refer to these fluctuations as sub-critical bubbles. Note, however, that the sub-critical bubbles of GKW had fixed amplitude, while here we will average over all possible fluctuations. Contrary to GKW we are not interested in the dynamics of the transition, but on the validity of the 1-loop potential. (This also explains our emphasis on the effective potential as opposed to the effective action.)

For small enough amplitudes the 1-loop approximation clearly is satisfactory. Otherwise, infrared corrections are important, and the 1-loop approximation is unreliable. In the next Section we obtain the free energy of correlation volume thermal fluctuations. In Section III we discuss the validity of the 1-loop finite temperature effective potential in the presence of sub-critical thermal fluctuations. In Section IV we apply our results to the electroweak potential. Conclusions are presented in Section V.

## 2 Free Energy of Thermal Fluctuations

The idea that the statistically dominant fluctuations around equilibrium can be modelled by sub-critical bubbles of roughly a correlation volume has been discussed by GKW and other recent works. Although the original proposal of GKW, that sub-critical bubbles of the broken-symmetric phase can play an important rôle in the dynamics of weakly first-order transitions is still under debate, their presence in any hot fluctuating system is undisputed. In statistical physics, the coarse-grained free energy functional is built under the assumption that the relevant coarse-graining scale is the correlation length, \( \xi(T) \); if
the coarse-graining scale were to be larger than the correlation length, phase separation would occur within a “grain” and the free energy would become a convex function of the order parameter \cite{10}. Thus, the coarse-graining guarantees that we can identify different phases in our system, so long as the fluctuations about equilibrium are consistent with the coarse-graining scale. The same reasoning applies to the 1-loop approximation to the effective potential. The effective coarse-graining scale is given by the inverse mass of fluctuations about the equilibrium state, the correlation length \( \xi(T) = m^{-1}(T) \). If we want to use the effective potential to describe a first-order phase transition it better be a concave function of the scalar field at, say, the critical temperature. Hence, the value of \( \phi \) inside the correlation volume fluctuations should not differ much from its equilibrium value.

We now briefly estimate the free energy of sub-critical fluctuations. We refer the reader to GKW for details. Although our approach is quite general, it is adequate to perform the calculation for a particular potential which in principle includes interactions of \( \phi \) with itself and other fields. We write the 1-loop effective potential as

\[
V_{\text{eff}}(\phi, T) = \frac{m^2(T)}{2} \phi^2 - \gamma(T)\phi^3 + \frac{\lambda(T)}{4} \phi^4,
\]

where \( \gamma(T) \) and \( \lambda(T) \) are positive functions of \( T \) and \( m^2(T) \) can be negative below a certain temperature \( T_2 < T_c \). \( V_{\text{eff}}(\phi, T) \) has minima at \( \phi = 0 \) (for \( m^2(T) > 0 \)) and at \( \phi_+ = \frac{1}{2\lambda(T)} \left[ 3\gamma(T) + \sqrt{9\gamma^2(T) - 4m^2(T)\lambda(T)} \right] \), for temperatures \( T < T_1 \), with \( T_1 \) given by the solution of \( \gamma^2(T_1) = \frac{4}{9}m^2(T_1)\lambda(T_1) \). At \( T = T_c \), \( V_{\text{eff}}(\phi = 0, T_c) = V_{\text{eff}}(\phi = \phi_+, T_c) \). Below \( T_c \) the minimum at \( \phi = \phi_+ \) becomes the global minimum (the true vacuum) and the minimum at \( \phi = 0 \) becomes metastable (the false vacuum). Note that \( \phi \) could be a real scalar field or the amplitude of the Higgs field. In the latter case, \( V_{\text{eff}} \) is an even function of \( \phi \).
Consider cooling the system described by the above potential from \( T \gg T_c \) down to \( T \approx T_c \). The equilibrium state of the system is at \( \phi = 0 \). The probability of a thermal fluctuation \( \phi_F(\vec{x}) \) about \( \phi = 0 \) is

\[
P[\phi_F] \sim \exp \left[ -\frac{F(\phi_F, T)}{T} \right],
\]

where \( F(\phi_F, T) \) denotes the excess free energy of the thermal fluctuation. Following Ref. \[7\] we write \( F(\phi_F, T) \) as

\[
F(\phi_F, T) = \int dV \left[ \frac{1}{2} (\nabla \phi_F)^2 + V_{\text{eff}}(\phi_F, T) \right].
\]

Note that the free energy is equivalent to the (Euclidean) effective action, as defined in Eq. \[8\], to first order in the derivatives (with \( Z(\phi_c(\vec{x})) = 1 \)), for a static field configuration. One could improve on this approximation by including higher order terms in \( F(\phi_F, T) \), although we refrain from doing so here \[11\]. How can we estimate the free energy of these fluctuations? Since they are not extrema of the classical action (like, e.g., critical bubbles) we must choose an explicit profile for the typical fluctuations. In order to minimize the free energy we choose them to be spherically symmetric. They can then be described by two parameters, their radius and the value of the field \( \phi \) in their interior, \( \phi_A \). (We refer to \( \phi_A \) as the amplitude of the fluctuation.) Following the discussion above we take the radius to be the temperature dependent correlation radius and write

\[
\phi_F(r, T) = \phi_A \exp \left( -\frac{r^2}{\xi(T)^2} \right).
\]

Other choices for \( \phi_F(r, T) \) give larger free energy. The free energy for the correlation volume fluctuations with amplitude \( \phi_A \) becomes

\[
F(\phi_A, T) = \alpha(\phi_A)\xi(T) + \beta(\phi_A, T)\xi(T)^3,
\]
where $\alpha(\phi_A)$ and $\beta(\phi_A, T)$ are given, respectively, by

$$
\alpha(\phi_A) = \frac{3\sqrt{2}}{8} \pi^{\frac{3}{2}} \phi_A^2,
$$

(16)

$$
\beta(\phi_A, T) = \frac{\sqrt{2}}{8} \pi^{\frac{3}{2}} m^2(T) \phi_A^2 - \frac{\sqrt{3}}{9} \pi^{\frac{3}{2}} \gamma(T) \phi_A^2 + \frac{\pi^{\frac{3}{2}}}{32} \lambda(T) \phi_A^4.
$$

3 Validity of the 1-loop Approximation

As we discussed in the Introduction, the 1-loop approximation to the effective action is obtained by expanding the classical action about an extremum configuration, $\phi_0(\vec{x}, t)$, and keeping terms up to second order in the perturbation $\eta(\vec{x}, t)$, with the classical field $\phi_c(\vec{x}, t) = \phi_0(\vec{x}, t) + \eta(\vec{x}, t)$. When $J(x) \to 0$, $\phi_c(\vec{x}, t)$ becomes a constant, the vacuum expectation value $\langle \phi \rangle$, which, by Eq. (7), is a solution of $dV_{\text{eff}}(\phi_c)/d\phi_c|_{\langle \phi \rangle} = 0$. Thus, the 1-loop approximation to the effective potential relies on having fluctuations about $\phi_c = \langle \phi \rangle$ which are small enough that the inhomogeneous terms in the effective action (Eq. (6)) can be neglected. For the models described by the potential of Eq. (11), for $T > T_c$, we are interested in the amplitude of fluctuations about $\phi_c = 0$. For $T < T_1$, $V_{\text{eff}}(\phi, T)$ has an inflexion point closest to $\phi_c = 0$ at

$$
\phi_{\text{inf}}(T) = \frac{\gamma(T)}{\lambda(T)} - \sqrt{\frac{\gamma^2(T)}{\lambda^2(T)} - \frac{m^2(T)}{3\lambda(T)}}.
$$

(17)

Clearly, the rms amplitude of fluctuations, which we write as $\tilde{\phi}(T)$, must be smaller than $\phi_{\text{inf}}(T)$ in order for the 1-loop approximation to be accurate. Thus we can write as a criterion for the validity of the 1-loop approximation,

$$
\tilde{\phi}(T) \leq \phi_{\text{inf}}(T).
$$

(18)
This is a general criterion which can be adapted to different models, including second-order transitions in the neighborhood of the critical point.

What remains is to calculate $\bar{\phi}(T)$ \[^{12}\] Since $\bar{\phi}(T)$ is the rms amplitude of the fluctuations, its definition is simply,

$$\bar{\phi}(T) = \sqrt{\langle \phi^2 \rangle_T - \langle \phi \rangle_T^2},$$

(19)

where the thermal average $\langle \ldots \rangle_T$ is defined in terms of the probability distribution of Eq. \[^{12}\] as

$$\langle \ldots \rangle_T = \frac{\int_{-\infty}^{+\infty} d\phi \ldots P[\phi]}{\int_{-\infty}^{+\infty} d\phi P[\phi]}.$$ 

(20)

Note that with our ansatz of Eq. \[^{14}\] for the thermal fluctuations, the path integrals above become simple integrals over $\phi_A$.

4 Application: The Electroweak 1-loop Potential

As an application we study the 1-loop approximation to the electroweak potential given by \[^{13}\]

$$V_{\text{eff}}(\phi, T) = D(T^2 - T_2^2)\phi^2 - ET\phi^3 + \frac{\lambda}{4}\phi^4,$$

(21)

where $D$ and $E$ are constants given in terms of the $W$ and $Z$ boson masses and of the top quark mass as $D = \frac{1}{2\pi} \left[6 \left(\frac{m_W}{\sigma}\right)^2 + 3\left(\frac{m_Z}{\sigma}\right)^2 + 6\left(\frac{m_t}{\sigma}\right)^2\right]$ and $E = \frac{1}{12\pi} \left[6 \left(\frac{m_W}{\sigma}\right)^3 + 3\left(\frac{m_Z}{\sigma}\right)^3\right] \simeq 10^{-2}$, where $\sigma \simeq 246$ GeV is the vacuum expectation value of the Higgs field. We use $m_W = 80.6$ GeV and $m_Z = 91.2$ GeV. $T_2$ is the spinodal instability temperature, given by

$$T_2 = \sqrt{\frac{m_H^2 - 8B\sigma^2}{4D}},$$

(22)
where \( m_H^2 = \frac{2\lambda + 12B}{\sigma^2} \) is the physical Higgs mass and \( B = \frac{1}{6\pi^2\sigma^4} (6m_W^4 + 3m_Z^4 - 12m_t^4) \).

The temperature dependent Higgs self-coupling \( \lambda_T \) is given by

\[
\lambda_T = \lambda - \frac{1}{16\pi^2} \left[ \sum_b g_b \left( \frac{m_b}{\sigma} \right)^4 \ln \left( \frac{m_b^2}{c_b T^2} \right) - \sum_f g_f \left( \frac{m_f}{\sigma} \right)^4 \ln \left( \frac{m_f^2}{c_f T^2} \right) \right],
\]

where the sums are performed over bosons and fermions, with degrees of freedom \( g_b \) and \( g_f \), respectively. In Eq. (23), \( c_b = 5.41 \) and \( c_f = 2.64 \).

The electroweak potential is equivalent to the potential of Eq. (11), with the identifications

\[
m_2(T) = 2D(T^2 - T_2^2), \quad \gamma(T) = ET, \quad \text{and} \quad \lambda(T) = \lambda_T.
\]

At \( T_c \) the minima at \( \phi = 0 \) and \( \phi_+ \) are degenerate, with

\[
T_c^2 = \frac{T_2^2}{1 - \frac{E^2}{\lambda_T D}}.
\]

For \( T < T_1 \), the nearest inflexion point to the minimum \( \phi = 0 \) is located at

\[
\phi_{\text{inf}}(T) = \frac{ET}{\lambda_T} - \sqrt{\frac{E^2 T^2}{\lambda_T^2} - \frac{2D(T^2 - T_2^2)}{3\lambda_T}}.
\]

It is now simple to obtain the expression for \( \bar{\phi}(T) \). From Eqs. (12), (16), and (20) we obtain (in the electroweak model the potential is left-right symmetric, and \( \langle \phi_A \rangle_T = 0 \))

\[
\left[ \bar{\phi}(T) \right]^2 = \frac{\int_{-\infty}^{\infty} d\phi_A \phi_A^2 e^{-\frac{1}{T} [\alpha(\phi_A)\xi + \beta(\phi_A, T)\xi^3]}}{\int_{-\infty}^{\infty} d\phi_A e^{-\frac{1}{T} [\alpha(\phi_A)\xi + \beta(\phi_A, T)\xi^3]}}.
\]

where \( \xi^{-1}(T) = \sqrt{2D(T^2 - T_2^2)} \). Due to the non-linear terms the integrals above cannot be calculated exactly. However, for the case at hand, the free energy of the fluctuations is dominated by their surface term. We can safely set \( \beta(\phi_A, T) = 0 \) in the integrals above to obtain an approximate analytic expression for \( \bar{\phi}(T) \),

\[
\bar{\phi}(T) \simeq \left[ \frac{4D \frac{1}{2} T(T^2 - T_2^2)^{1/2}}{3\pi^{1/2}} \right]^{1/2}.
\]

This result can be written as \( \bar{\phi}^2(T) \simeq m(T)T/6 \). In Fig. 1 we compare, at the critical temperature, the analytical result above for \( \bar{\phi}(T) \) with the numerical result obtained by
keeping the volume contribution to the free energy. To each pair of curves corresponds a top mass. Within each pair, the top curve is the numerical result, while the bottom curve is the approximation of Eq. (27). It is clear that the approximation is very good, working to within 10% at its worse for all values of the Higgs mass we investigated, being also only weakly dependent on the top mass.

The condition for the validity of the 1-loop approximation, Eq. (18), reads,

$$\left[ \frac{4D^{\frac{1}{2}}T(T^2 - T_2^2)^{\frac{1}{2}}}{3\pi^{\frac{3}{2}}} \right]^2 \leq \frac{ET}{\lambda_T} - \frac{E^2T^2}{\lambda_T^2} - \frac{2D(T^2 - T_2^2)}{3\lambda_T}. \quad (28)$$

This condition can be studied in two ways, assuming the results are fairly independent of the top mass. (Or for a fixed top mass.) We can either fix the Higgs mass and look for the temperature that violates the inequality, or fix the temperature and look for the Higgs mass that violates the inequality. We choose the latter approach and look for the Higgs mass that violates the inequality at the critical temperature. If $V_{eff}(\phi, T)$ is not a good approximation at $T_c$ it should not be trusted for any temperatures $T_c \leq T \leq T_2$.

Solving for $\lambda_T$ we obtain, using Eq. (24),

$$\lambda_T \leq \pi \left[ E \left( 1 - \frac{\sqrt{3}}{2} \right) \right]^{2/3}. \quad (29)$$

In order to express this result in terms of the Higgs mass, note that at $T_c$ we can write [14], (effectively approximating $\lambda_T$ to its tree-level value, $\lambda = m_H^2 / 2\sigma^2$)

$$\lambda_T \simeq 0.08 \left( \frac{m_H}{100 \text{GeV}} \right)^2. \quad (30)$$

Substituting Eq. (30) into Eq. (29), we find that for a Higgs mass $m_H \gtrsim 70$ GeV the 1-loop approximation is no longer valid. (Without the approximation $\lambda_T = \lambda$ we find numerically $m_H \gtrsim 77$ GeV.) In Fig. 2 we compare this approximate analytical result with the numerical result obtained by keeping all terms in the free energy. The results turn out to be quite independent of the top mass, and in very good agreement with each other.
5 Conclusions

We have examined the validity of the 1-loop approximation to the effective potential at finite temperature. By modelling the statistically dominant thermal fluctuations about equilibrium by correlation volume sub-critical bubbles of arbitrary amplitude, we argued that the 1-loop potential is valid so long as the rms amplitude of the fluctuations is smaller than the closest inflexion point.

We applied our results to the electroweak model, showing analytically and numerically that for temperatures at and below the critical temperature the 1-loop approximation breaks down for Higgs masses $m_H \gtrsim 70$ GeV. The results depend only very weakly on the top mass. For smaller Higgs masses, it is possible to trust the 1-loop approximation for temperatures below $T_c$. In a more detailed study, it would be interesting to obtain the maximum value of $m_H$ for which the potential is still valid at the nucleation temperature for critical bubbles. Given that the experimental lower bound on the Higgs mass is $m_H \gtrsim 60$ GeV, we suspect that the 1-loop approximation will be ruled out for all values of $m_H$.

In closing, we mention that the perturbation expansion parameter for scalar loops, $\lambda T/m(T)$, when evaluated at $T_c$, becomes bigger than unity for $m_H \gtrsim 85$ GeV [3,4]. It is reassuring to note that our results are in qualitative agreement with these power counting perturbative arguments, even though they are non-perturbative in nature.

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Figure Captions

Figure 1: Comparison between analytical and numerical results for the rms fluctuation amplitude \( \bar{\varphi}(T) \) at the critical temperature as a function of the Higgs mass. Each pair of curves is for a value of the top mass. Within each pair, the top curve is the numerical result and the bottom curve the approximation of Eq. (26).

Figure 2: Criterion for the validity of the 1-loop approximation for the electroweak potential at the critical temperature as a function of the Higgs mass, for several values of the top mass. The ascending curves are the numerical results for the rms amplitude \( \bar{\varphi}(T) \), while the descending curves give the location of the inflexion point. Values of \( m_H \) to the right of the dots violate the inequality in Eq. (18).

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