MRT-LBM modelling of the oscillatory gas slide film damping in the transition regime

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Abstract. In this paper, the slide film damping (SFD) of micro-scale oscillatory Couette flow has been researched by an effective multiple relaxation time lattice Boltzmann method (MRT-LBM). The validity and effectiveness of MRT-LBM for solving SFD have been verified through contrasting the velocity distribution between the upper plate and the substrate of MRT-LBM and the direct simulation Monte Carlo (DSMC) model. The impacts of the vibration frequency and gap of plates on the damping are discussed, and the result shows that the nonlinear character of velocity profiles is manifest and SFD increases substantially for a larger vibration frequency in the oscillatory gas flow; SFD reduces obviously with the gap increases. Consequently, the results further confirm the implementation of LBM in analysis of the non-equilibrium micro-scale gas flow.

1. Introduction

Recently, for designing better laterally driven micro-transducers, it is significant to evaluate the slide film damping force on the solid wall resulted by the viscosity of the micro-fluids when the parts of micro-electro-mechanical systems (MEMS) oscillate [1]. However, in the transition regime the existence of the rarefied gas effect causes many difficulties in calculating the slide film damping (SFD), and for micro-scale flow with a large Knudsen number (Kn=λ/L), the Knudsen layer has a remarkable influence on SFD. Here λ is the molecular mean free path, and L is the characteristic length of model.

The micro-scale oscillatory gas flow has been investigated in some of literature [2]. There are some numerical methods such as the direct simulation Monte Carlo (DSMC) and the high-order lattice Boltzmann method (HO-LBM). Some papers study the micro-scale oscillatory flow by DSMC [3,4]. Although all the theoretical results about the oscillatory shear-driven gas flow are validated by direct Monte Carlo simulation, DSMC is usually computationally expensive and has inherent noise in results [3]. Some other papers study the micro-scale oscillatory flow by HO-LBM [5]. However, some works reveal that the standard lattice Boltzmann method (LBM) could not handle the Knudsen boundary layer [6]. Although this deficiency can be overcome by introducing a high-order moment, HO-LBM becomes more complicated and the computational cost has increased dramatically [7]. Guo et al. [8] pointed out that the multiple relaxation time lattice Boltzmann method (MRT-LBM), proposed firstly in [9], contains more degrees of freedom and has certain advantages in dealing with microfluidic problems. On this basis, a corrected effective MRT-LBM has been adopted in which the relaxation time is revised by introducing the Knudsen layer in this paper.

The outline of the paper is that in Section 2, the MRT-LBM is introduced, and the modification of the relaxation time and boundary conditions are determined. In Section 3, the physical description and
the characteristic parameters of the structure are given. In Section 4, the results of the validity and effectiveness of MRT-LBM and the relationship between SFD and the vibration frequency ($\omega$) or the gap of plates are discussed. The conclusion is given in Section 5.

2. Physical model

![Figure 1. The couette flow model.](image)

Figure 1 is the schematic diagram of the plate oscillation gas flow. The upper plate oscillates harmonically in the plane, where $u_0$ is the velocity amplitude of the upper plate, and $\omega$ is the vibration frequency. Besides, the authors choose DSMC as the comparison, and on the basis of Cho’s experiment dates [10], the characteristics of the structure are listed in Table 1.

Different groups of lattice size have been adopted and results are available when simulating this physical model. However, for the reasons of the computation rate, the convergence of results and space limitations, the lattice size employed in the model is 50×8.

| Parameter                   | Value     |
|-----------------------------|-----------|
| Mass $M$ (μg)               | 0.26      |
| Stiffness $K$ (N · μm$^{-1}$)| 3.8×10$^7$|
| Structure thickness $G$ (μm) | 1.8       |
| Top surface of the plate $A$ (μm$^2$) | 2.93×10$^4$ |
| The kinetic viscosity $\nu_k$ (μm$^2$ · s$^{-1}$) | 1.5×10$^3$ |
| The velocity amplitude $u_0$ (μm$^{-1}$) | 11        |

3. Multiple relaxation time lattice boltzmann method

3.1. Governing equation

MRT-LBM can be given as:

$$f(x + c\Delta t, t + \Delta t) - f(x, t) = -Z^{-1} H[z(x, t) - z^eq(x, t)]$$

where $z(x, t)$ and $z^eq(x, t)$ are the moment vectors, $H$ is the relaxation matrix, and $Z$ is the transform matrix.

The D2Q9 is adopted in MRT-LBM, and the lattice velocities are as follows:
\[ c_i = \begin{cases} (0, 0), i = 0 \\ a \left[ \cos(-1)\pi/4, \sin(-1)\pi/4 \right], i = 1, 2, 3, 4 \\ \sqrt{2}a \left[ \cos(i-1)\pi/4, \sin(i-1)\pi/4 \right], i = 5, 6, 7, 8 \end{cases} \] (2)

The relaxation matrix \( H \) is given by:
\[
H = \text{diag}(0, 0, 1.4, 0, d_1, 0, d_2, d_3)
\] (3)
where \( d_1 \) and \( d_2 \) are the relaxation time.

The transform matrix \( Z \) [11] and the moment vector \( z \) are respectively given by:
\[
Z = \begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
-4 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\
4 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & -1 & 0 & 1 & -1 & 1 \\
0 & -2 & 0 & 2 & 0 & 1 & -1 & 1 \\
0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 \\
0 & 0 & -2 & 0 & 2 & 1 & 1 & -1 \\
0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1
\end{pmatrix}
\] (4)

\[
z = (\rho, c, \rho, i_1, t_1, i_2, t_2, h_1, h_2)^{-1}
\] (5)

where the \( i_1 \) and \( i_2 \) are written as:
\[
i_1 = \sum_i f_i c_i
\] (6)
\[
i_2 = \sum_i f_i c_i
\] (7)

The macroscopic density \( \rho \), flow velocity \( u \) and pressure \( p \) can be determined by:
\[
\rho = \sum_i f_i
\] (8)
\[
u = 1/\rho \sum_i c_i f_i
\] (9)
\[
p = \rho c_s^2
\] (10)

where \( f_i \) is the discrete distribution function, \( c_s \) is the lattice sound speed.

3.2. Relaxation time and boundary condition
Guo [12] derived the relaxation time related to the mean free path of the gas molecule in LBM for microscale flow simulation:
\[
d_2 = 0.5 + G/\delta_x \text{ Kn} \sqrt{6/\pi}
\] (11)

where \( G \) is the plates gap height.

However, when the upper plate vibrates in the transition regimes, the viscous force caused by the viscosity of the micro-fluids and the Knudsen layer will form. Therefore, Guo [12] gave the corrected formula of relaxation time:
\[ d_2 = 0.5 + G/\delta_2 \text{Kn} \Psi(y, Kn) \sqrt{6/\pi} \]  

where \( \Psi \) is the modified function:

\[ \Psi(y, Kn) = [S(y/\lambda) + S(G - y/\lambda)]/2 \]

\[ S(\chi) = 1 + (\chi - 1)e^{-\chi} - \chi^2 T_1(\chi) \]  

The relaxation time \( d_l \) is as in:

\[ d_l = [0.5 + \frac{3 + 24s_o^2\tau_o^2(G)B_1}{16\tau_o(G)} + \frac{\tau_o(G)d_D}{16\tau_o(G)}] \]  

Where

\[ s_o = \sqrt{6/\pi}, \tau_o = d_2 - 0.5, \tau_o = d \tau_o/\mathrm{dy} \]

\[ B_1 = (2 - \sigma)/\sigma(1 - 0.1817\sigma) \]

\[ B_2 = 1/\pi + 0.5 \times 1/B_1^2 \]

\[ D = 12 + 30\tau_o(G)s_B \]

In the micro-scale plates oscillatory gas flow, the boundary scheme of bounce-back specular reflection (BSR) is used, and the unknown discrete velocities of the upper boundary are [13]:

\[ f_4 = f_2 \]

\[ f_r = nf_s + (1 - n)f_s + 2n\rho_0c_s\cdot u_q / 2c_s^2 \]

\[ f_s = nf_s + (1 - n)f_s + 2n\rho_0c_s\cdot u_q / 2c_s^2 \]  

where \( u_q \) is the speed of active plate, and the tangential momentum conditioning coefficient \( n \) is given by:

\[ n = [1 + s_B + \tau_o(G)d_D/(8\tau_o^2(G))]^{-1} \]  

The slide film damping (SFD) is given in:

\[ \zeta = v_k u^2 / G \]  

where \( v_k \) is the kinetic viscosity.

4. Numerical results

In this section, the validity and effectiveness of MRT-LBM have been verified by comparing MRT-LBM with DSMC. Figure 2 shows the distribution of velocity standardized by the upper plate velocity amplitude at the mid-section. As the vibration frequency \( \omega_0 \), normalized by the minimum frequency increases, the curvature of velocity profiles is larger. Besides, the max quantization relative error is 23.94% in Figure 2 between MRT-LBM and DSMC, and MRT-LBM is quite consistent with DSMC data [3]. The MRT-LBM with corrected effective relaxation time can capture SFD in the micro-scale oscillatory flow.
Figure 2. Velocity profiles for (a) $Kn=0.1$, $\omega_o=16.0$; (b) $Kn=0.2$, $\omega_o=4.0$; (c) $Kn=0.4$, $\omega_o=1.0$.

Figure 3. The upper plate damping with the $Kn$. 
The dimensionless slide film damping (DSFD, $\zeta / \zeta_{\text{max}}$), standardized by the maximum value of the damping $\zeta_{\text{max}}$, can be acquired through the discrete velocity $f_i$. As a result, DSFD can be computed straightway and then the results will be in comparison with the data given by DSMC. As illustrated in Figure 3, DSFD of the upper plate, acquired by the corrected MRT-LBM, is in excellent agreement with DSMC results [5]. At $Kn=0.1$, as the oscillation frequency decreases, DSFD decreases especially, and in the meanwhile, there is a little difference at $Kn=1.5$. We can acquire the conclusion that the influence of vibration frequency on DSFD is smaller as the Knudsen number increases.

Through the discussion above, the feasibility of the corrected effective relaxation time MRT-LBM about exploring the DSFD of the oscillatory gas has been verified. On this basis, the effects of plates’ gap on DSFD have been probed as shown in Figure 4. At $Kn=0.1$ or 0.4, it can be seen that for the larger the gap, DSFD is smaller and decreases greatly. The value of the gap has an inverse effect on DSFD, and the downward trend is gradually slowing down with the value of the gap increasing. Besides, it also indicates that DSFD decrease with the Knudsen number increases when the value of gap is given.

![Figure 4. Slide film damping on the oscillating plate.](image)

5. Conclusions
In this paper, an effective MRT-LBM for the slide film damping in micro-scale oscillatory Couette gas flow is presented. By the corrected effective relaxation time, the results of the MRT-LBM are in agreement with those of DSMC, and the validity and feasibility of this model for solving the slide film damping have been verified. The authors then discuss the impacts of the vibration frequency ($\omega$) and the gap of plates on the damping. The results show that as the oscillation frequency increases, the velocity profiles between the plates are more curved; at low Knudsen number, such as 0.1, the oscillation frequency plays an important role in the damping, and the effect of vibration frequency on the damping reduces gradually with the Knudsen number increases; as the gap of plates become large, the dimensionless damping value reduces rapidly and the downward trend is gradually slowing down. Compared to the DSMC model, the MRT-LBM with the corrected effective relaxation time has successfully captured the slide film damping of the oscillatory gas flow in the transition regime. The implementation of the LBM in analysis of the non-equilibrium micro-scale gas flow have got further confirmation.

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