BERNOULLI POTENTIAL, HALL CONSTANT AND COOPER PAIRS EFFECTIVE MASSES IN DISORDERED BCS SUPERCONDUCTORS

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It is analyzed what fundamental new information for the properties of the superconductors can be obtained by systematic investigation of the Bernoulli effect. It is shown that it is a tool to determine the effective mass of Cooper pairs, the volume density of charge carriers, the temperature dependence of the penetration depth and condensation energy. The theoretical results for disordered and anisotropic gap superconductors are systematized for this aim. For clean-anisotropic-gap superconductors is presented a simple derivation for the temperature dependence of the penetration depth.

Keywords: Ginzburg-Landau theory, effective mass, gap anisotropy, exotic superconductors

1. Introduction

The Landau theory of second-order phase transitions[1] and its realization for superconductors, the Ginzburg-Landau (GL) gauge theory[2] can be classified as belonging to the most illuminating theoretical achievements in XXth-century physics. The basic concepts[3] advanced in these theories often find applications in interdisciplinary research. The microscopic Bardeen-Cooper-Schrieffer (BCS) theory[4] makes it possible to calculate the parameters of the GL theory. Thus the phenomenology of superconductivity can be reliably derived once the parameters of the microscopic Hamiltonian are specified. Such a scheme ensures that there is no missing link between the microscopic theory and the material properties of the superconductors. On the other hand Bernoulli effect is one of the oldest effects known in the physics which in as a result of the energy conservation can be realized strictly speaking only for superfluids. For superconductor the Bernoulli theorem gives a relation between current density and the current induced contact potential difference at constant in

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thermal equilibrium electrochemical potential.

The purpose of the present paper is to systematize the results for the GL theory of disordered and anisotropic gap superconductors and to point out what new information for the fundamental parameters of superconductors can be extracted using the Bernoulli effect. We are describing in short how the experiments can be done and how the effective mass of Cooper pairs, the volume density of charge carriers and London penetration depth can be extracted from investigation of the Bernoulli effect. The proposed experiments require only standard low frequency electronic measurements. In parallel we analyze a simple methodical derivation of the London penetration depth for clean superconductors. We will start our analysis with the GL equation, then we will analyze disorder renormalization and experimental accessibility of the effective Cooper pair mass, we will analyse the Bernoulli effect and temperature dependence of the penetration depth. Finally we will conclude which experimental development we consider as most perspective.

2. Ginzburg-Landau equation

Consider now the gradient terms of the Ginzburg-Landau (GL) theory. A general expression for the tensor of the squared coherence lengths \( \xi_{\alpha\beta}^2 \) has been derived by Pokrovsky and Pokrovsky:

\[
(\xi_{\alpha\beta}^2) = \frac{\hbar^2}{(4\pi k_B T_c)^2} \left( \zeta_{3,0} \langle v_\alpha v_\beta \chi \rangle^2 + 2 x_c \zeta_{3,1} \langle v_\alpha v_\beta \chi \rangle \langle \chi \rangle + x_c^2 \zeta_{3,2} \langle v_\alpha v_\beta \chi \rangle^2 \right),
\]

where

\[
\zeta_{k,l} \equiv \zeta_{k,l}(x_c + 1/2), \quad \zeta_{k,l}(z) = \sum_{n=0}^{\infty} (n + z)^{-k} (n + 1/2)^{-l}, \quad x_c = \frac{\hbar}{2\pi k_B T_c},
\]

\( \tau(T_c) \) is the electron scattering time, \( v_\alpha \) are the components of the velocity, and \( \langle \ldots \rangle \) means averaging on the Fermi surface. In terms of \( \xi^2 \) the GL equation for the space-dependent order parameter \( \Xi(\mathbf{r}) \) can be written as

\[
\left( -D \cdot \xi^2 \cdot D + \epsilon + \frac{b}{a_0} |\Xi|^2 \right) \Xi = 0,
\]

where

\[
D = \frac{\partial}{\partial \mathbf{r}} - \frac{2\pi}{\Phi_0} \mathbf{A}(\mathbf{r}), \quad \Phi_0 = 2\pi\hbar/e^*, \quad |e^*| = 2|e|, \quad \epsilon = \frac{T - T_c}{T_c},
\]

Equation (3) is the extremum (minimum) condition for the GL free energy

\[
F[\Xi, \mathbf{A}] = \int d\mathbf{r} \left\{ -\frac{\hbar^2}{2} [D\Xi(\mathbf{r})]^* \cdot \hat{M}^{-1} \cdot D\Xi(\mathbf{r}) + a(T)|\Xi(\mathbf{r})|^2 + \frac{b}{2} |\Xi(\mathbf{r})|^4 \right\}.
\]
where
\[(\hat{\mathcal{M}}^{-1})_{\alpha\beta} = \frac{2a_0}{\hbar^2} (\tilde{\xi}^2)_{\alpha\beta}, \text{ and } a(T) = \varepsilon a_0. \quad (7)\]

Using the eigenvalues \(\xi_\alpha\) and \(\tilde{M}_\alpha\) of the tensors \(\hat{\xi}^2\) and \(\hat{M}\) we can introduce the temperature-dependent GL coherence lengths and penetration depths, respectively (\(0 < -\varepsilon \ll 1\)):
\[\xi_{\alpha,\text{GL}}(T) \approx \frac{\xi_\alpha}{\sqrt{-\varepsilon}}, \quad \lambda_{\alpha,\text{GL}}(T) \approx \frac{\lambda_\alpha}{\sqrt{-\varepsilon}}, \quad (8)\]
which satisfy the following GL relations
\[\frac{1}{\lambda_\alpha^2} = \frac{(a_0/b)\varepsilon^2}{c^2\varepsilon_0} \frac{1}{M_\alpha} = \frac{2a_0^2\varepsilon^2}{\hbar^2\varepsilon_0c^2b}\xi_\alpha^2 = \frac{8\pi^2\mu_0}{b\Phi_0^2} = \frac{8\pi^2\mu_0}{\Phi_0^2} T_c \Delta C \xi_\alpha^2, \quad (9)\]
where in Gaussian units \(\mu_0 = 4\pi\) and \(\Delta C = a_0^2/T_c b\) is the jump of the specific heat at \(T_c\) per unit volume. The explicit formulae for the GL coefficients \(a_0\) and \(b\) are given in Ref. 5.

3. Isotropic alloys. Disorder renormalization of the Cooper pair mass

Let us illustrate now the operation of the above general expressions on the important for the applications case of dirty isotropic alloys. In this case we have
\[\langle v_\alpha v_\beta \rangle = \frac{1}{3} v_F^2 \delta_{\alpha\beta}, \quad \chi_p = \text{const}, \quad (10)\]

Using the identity
\[\zeta_{3,0} + 2r_c \zeta_{3,1} + x_c^2 \zeta_{3,2} = \zeta_{1,2} \quad (11)\]
one obtains the classical result due to Gor’kov, see the textbook Ref. 8
\[\xi^2 = \frac{1}{3} \left( \frac{\hbar v_F}{4\pi k_B T_c} \right)^2 \zeta_{1,2} = \xi_{\text{clean}}^2 \frac{\hat{M}_{\text{clean}}}{\hat{M}(x)}. \quad (12)\]

Here we wish to recall some notation used in different works on the physics of
Figure 1: Disorder renormalization of the Cooper pair mass $M/M_{\text{clean}} = \frac{\xi^2_{\text{clean}}}{\xi^2} = \frac{\lambda^2}{\lambda_{\text{clean}}^2} \approx H_{\text{cl}} / H_{\text{cl, clean}}$ vs. dimensionless scattering rate $x_c = \hbar / 2 \pi k_B T_c \tau(T_c)$. The exact microscopic result Eq. (13) is shown by the full line. The dashed line is the Padé approximant derived within framework of the Pippard-Landau theory.

superconductivity (see, e.g., Ref. [9]:

$$\xi_{\text{clean}} = \frac{\xi_0}{4 \gamma} \sqrt{\frac{7 \zeta(3)}{12}} = \frac{\xi_0}{4} \frac{2 \Delta(0)/k_B T_c}{\sqrt{\Delta C/C_n(T_c)}} = 0.738 \xi_0 \approx \xi_{0P},$$

$$\xi_0 = \frac{\hbar v_F}{\pi \Delta(0)} = \frac{\gamma}{\pi^2} \frac{\hbar v_F}{k_B T_c} = 0.1805 \frac{\hbar v_F}{k_B T_c},$$

$$2 \Delta(0) = \frac{2 \pi}{\gamma} k_B T_c = 3.53 k_B T_c, \quad \gamma = 1.781 \ldots,$$

$$\frac{\xi^2_{\text{clean}}}{\xi^2} = \frac{\lambda^2_{\text{clean}}}{\lambda^2} = \frac{M_{\text{clean}}}{M(x)} = \frac{\zeta_{1.2} \left(x_c + \frac{1}{2}\right)}{7 \zeta(3)} \approx \frac{1}{1 + \frac{14(3)}{\pi^2} x_c} = \frac{1}{1 + \xi_{0P}^2},$$

$$\xi_{0P} = \frac{7 \zeta(3)}{2 \pi^2} \xi_0 = 0.752 \xi_0 = \frac{7 \zeta(3) \hbar v_F}{2 \pi^3} k_B T_c$$

$$= 0.135 \frac{\hbar v_F}{k_B T_c} = \frac{2 \sqrt{21 \zeta(3)}}{\pi^2} \xi_{\text{clean}} = 1.018 \xi_{\text{clean}},$$

and finally $l = v_F \frac{\sqrt{2}}{2}$. The 2% difference between the length $\xi$, introduced by Ginzburg and Landau, and $\xi_0$ introduced by Pippard, is experimentally inaccessible. So is the several percent difference between the exact microscopic result (13) and its Padé approximant shown in Fig. 1. The Padé approximant for the mass renormalization can be easily derived within framework of the Pippard-Landau theory,
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i.e. using the generalization of the local GL theory and nonlocal Pippard electrodynamics. These are generalized by inserting the Pippard kernel between the two gradients in the GL expression for the free energy:

\[
\int \int d\mathbf{r} d\mathbf{r'} \sum_{\alpha,\beta} [D_{\alpha} \Xi(\mathbf{r})]^* \delta(\mathbf{R}) D_{\beta} \Xi(\mathbf{r'}) \rightarrow \int \int d\mathbf{r} d\mathbf{r'} [D \Xi(\mathbf{r})]^* \cdot \frac{3 \mathbf{R} \otimes \mathbf{R}}{4\pi\xi_0 P R^4} e^{-R/\xi_P} \cdot D \Xi(\mathbf{r'}),
\]

where \( \mathbf{R} = \mathbf{r} - \mathbf{r'} \), \( \xi \approx \xi_P \), and

\[
\frac{1}{\xi_P} = \frac{1}{\xi_0 P} + \frac{1}{\ell}, \quad \xi_{\text{GL}}(T) \approx \frac{\xi_P}{\sqrt{|\epsilon|}}.
\]

4. Experimental accessibility of the effective Cooper pair mass \((T = 0)\)

The tensor introduced in Eq. (7) acquires dimension of mass if we renormalize the GL order parameter so as to have dimension of density. The total charge carrier density \( n_{\text{tot}} \) is a fundamental theoretical notion. Thus it is of principle interests whether this quantity is experimentally accessible through the equilibrium thermodynamic properties of the vortex-free Meissner-Ochsenfeld phase. An approach to this problem has been outlined in Ref. 11 and is based on the current-induced contact-potential difference at the surface of a superconductor (we call it the London-Hall effect):

\[
\Delta \varphi = -R_H \frac{B^2}{2\mu_0}.
\]

For superconductors characterized with local electrodynamics \( \Delta \varphi \) is a manifestation of the energy conservation law and the Bernoulli theorem for charged superfluids in thermodynamic equilibrium:

\[
\Delta \varphi = -\frac{1}{2\varepsilon_0 c^2} R_H \lambda^2(T) j^2.
\]

For \( T = 0 \) this equation reads as one fluid energy conservation law

\[
\frac{1}{2} M_{CP} v^2 + e^* \Delta \varphi = 0,
\]

where \( M_{CP} \) is the effective mass of Cooper pairs. For anisotropic superconductors \( \lambda \) in Eq. (17) corresponds to the direction of the current density \( \mathbf{j} \) at the superconductor surface. All radio frequencies are too small compared to the typical gap parameters of superconductors. Hence \( \Delta \varphi \) can be measured using a lock-in amplifier in conjunction with a suitable low-noise preamplifier. The superconductor surface under investigation, as shown in Fig. 2, plays the role of one of the plates of the capacitor with capacitance \( c \) and resistance \( r_c \).

The condition under which \( \Delta \varphi \) can be directly measured reads

\[
C f_{\text{lock-in}} \gg \frac{1}{R_C} + \frac{1}{R_{\text{lock-in}}},
\]
Figure 2: Cooper pair mass spectroscopy based on the Bernoulli potential (after Ref. 11). (a) Top view (b) Cross section, (c) Equivalent electric scheme. Two electrodes, circle- (1) and ring-shaped electrode (2), should be produced on the insulating layer capping the superconducting film. (3) and (4) denote the contacts of the drive coil with inductance \( L_d \) and resistance \( R_d \); (5)—insulator layer of thickness \( d_{\text{ins}} \); (6)—superconducting film with thickness \( d_{\text{film}} < \lambda_{ab}(0) \); (7)—substrate; \( M_{12} \)—mutual inductance; \( l_1, l_2 \)—variable inductances; \( r \)—load resistor; \( \mathcal{V} \)—voltmeter; \( \mathcal{A} \)—ammeter; \( SW \)—switch; \( C_d \)—capacitor of the drive resonance contour with resonance frequency \( \omega \); \( G \)—Bernoulli voltage generator with doubled frequency \( 2\omega \); \( C_1, C_2 \)—capacitances between the superconducting film and the metal electrodes (1) and (2).

where \( f_{\text{lock-in}} \) and \( R_{\text{lock-in}} \) are, respectively, the operating frequency and the internal resistance of the lock-in amplifier. Typical voltages are \( \sim \text{nV} \) but for high-\( T_c \) superconductors the Bernoulli signals will be considerably stronger. The key technological problem lies in the the properties of the superconductor-insulator interface—its quality should be comparable to that of the samples used for investigation of electric effects in superconductors.

For pure crystals of elemental metals the total charge density is accessible trough Hall-effect measurements in the normal phase, but for dirty superconductors only the London-Hall effect gives such a possibility. Determination of the Hall constant by the current-induced contact-potential difference provides a tool for determination of the superfluid density at zero temperature \( n(0) \), \( e n_{\text{tot}} = e^* n(0) = 1/\mathcal{R}_H \). Thus knowing \( n(0) \) and \( \lambda(0) \) one can determine the effective mass of the Cooper pairs \( M_{\text{CP}} \):

\[
\frac{1}{\lambda^2(0)} = \frac{n(0)e^*}{M_{\text{CP}} c^2 \varepsilon_0}, \quad M_{\text{CP}} = \frac{e^* \lambda^2(0)}{c^2 \varepsilon_0 \mathcal{R}_H}.
\]

All the three important parameters, \( \mathcal{R}_H \), \( M_{\text{CP}} \), and \( \lambda(T) \), can be determined by measuring the contact potential difference in thin \( (d_{\text{film}} \ll \lambda(T)) \) and thick \( (d_{\text{film}} \gg \lambda(T)) \) superconducting films. We should mention that strong enhancement of the effective mass due to disorder is valid for arbitrary band anisotropy if the averaged
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Figure 3: Gedanken set-up proposed in Ref. 15 to determine the vortex charge and the Cooper pair mass. Thin Bi$_2$Sr$_2$CaCu$_2$O$_8$ layer is thread by perpendicular magnetic field $B_z$. The voltage $V_y$ applied through the Ag electrodes in circuit (1) creates a drift of the vortices with mean velocity $v_y$. Due to the Bernoulli effect the superfluid currents around every vortex create a change in the electric potential on the superconducting surface. The Bernoulli potential of the vortex leads to an electric polarization on the normal Au surface. The charge $q_v$, related to the vortex, has the same drift velocity $v_y$. The corresponding current $I_x$ in circuit (2) can be read by a sensitive ammeter. The quality of the SrTiO$_3$ plate should be high enough so as to allow detection of the interface Hall current without being significant perturbed by the leakage currents between circuits (1) and (2).

gap does not vanish:

$$\langle \xi^2_{\text{dirty}} \rangle_{\alpha\beta} \propto \frac{\tau}{T_c} \frac{\langle \chi \rangle^2}{\langle \chi^2 \rangle} \langle v_\alpha v_\beta \rangle, \quad (x \gg 1).$$

(21)

As it was commented earlier, the effective mass of the Cooper pairs was first measured by Fiory et al. using electrostatic doping of YBa$_2$Cu$_3$O$_{7-\delta}$ films. This pioneering experiment confirmed that the effective mass shows weak temperature dependence. Certainly, there are other methods for absolute determination of $M_{CP}$, e.g., from the surface Hall effect. For the layered cuprates the problem of determining the Cooper pair mass is related to problem of the vortex charge. The aim of the model experiment suggested in Ref. 15 was to check whether the Bernoulli potential of the pancake vortices creates an electrical polarization necessary to explain the Hall effect in the vortex creep regime. The supposed experimental setup is sketched in Fig. 3. Up to now the is only one experimental hint concerning the electrostatic polarization around the vortices, the NMR and NQR study by Kumagai, Nozaki and Matsuda.

Finally we wish to mention that electric polarization of a metal-oxide-superconductor
plane capacitor by a sudden light impulse can give a the condensation energy

$$\Delta \varphi = \frac{B^2(T)}{2\mu_0}. \quad (22)$$

We have to take into account that electric polarization is related to work function, workfunction is related to the chemical potential which is the Gibbs free energy per particle and in such a way we have an electric method for measurement of the thermodynamic critical magnetic field $B_c(T)$.

5. What has to be done. A short working program for the experiment

After the analysis of the theory let us describe what has to be done experimentally for creation of Cooper pair mass spectroscopy based on the Bernoulli effect. Among the other electric field effect in superconductors such as surface Hall effect and electrostatic modulation of the kinetic inductance the Bernoulli effect looks simplest and experimental research is better to start with him. The main advantage is that in this case the electric field is very small and there are no restrictions related to the brake-trough voltage of the insulating layer. It is necessary insulator to be good only for small voltages. These investigations can be a by-product of preliminary research of the superconductor insulator interfaces in further trial to make superconducting field effect transistors. The first step will be qualitatively observation of the Bernoulli effect in already existing superconductor structures. A numerical calculation of the current distribution in the framework of London electrodynamics can convert this observation in quantitative measurement. The simplest possible set-up is a superconducting nanobridge in which an appropriate gate can be performed. In this field effect transistor type structure current in the strip can be excited by capacitor connections id source and drain area. In such a way generation of the current harmonics by the contacts can be avoided.

Later on a systematic investigation can be started. The main purpose is to try to use already prepared big number of superconductor films without any destroying patterning to be carried out. The gold Bernoulli electrodes should be evaporated on a thin SrTiO$_3$ substrate. The investigated film should be affixed on this plate. A small coil, or a solenoid with diameter of order of 1 mm, for example, will excite the eddy currents. The detector circuit should be switched between the circular and ring electrodes patterned on the insulator plate, cf. Fig. 2. The cross talk between detector and drive coil should be annulled by a variable mutual inductance. The harmonic current should be applied to the drive coil and the Bernoulli signal will be detected at second harmonics using a low noise preamplifier before the lock-in. A resonance technique additionally can use the high-Q factor to suppres the noise and signal at basic frequency.

In parallel a numerical simulation of the experimental setup will allow the result of measurements to be delivered as material constants. After a systematic investigation of the collected films would be possible to extract fundamental information of the doping dependence of the effective mass of Cooper pairs in cuprate CuO$_2$
superconductors and the almost temperature independent London-Hall coefficient.

Finally the experimental setup could be elaborated as a commercial device (superfluid densitometer) intended for fast contactless investigation of the Bernoulli effect in superconductors. Such a device can find application for the monitoring of the quality of the thin superconducting films used in second-generation superconducting cables, for example. The investigation of other electric field effects in the superconductors will be the next step of the investigation of electric field effect in superconductors.

The measurement of the Bernoulli signal in irradiated thin films can become a routine tool for investigation of these disordered superconductors.

Except for the Cooper pair mass spectroscopy and charge carriers densitometry Bernoulli effect, i.e. measurement of the contact potential difference can become unique tool for determination of the condensation energy especially for type-II superconductors. For sudden heating we can use laser light illumination of the set-up for the Bernoulli measurements.

6. Temperature dependence of the penetration depth and optical mass

Let us attempt now a simple derivation of the temperature dependence of the penetration depth for clean superconductors. Our objective is to show that the result is in agreement with the formula for the coherence length, Eq. (1).

For a superconductor at \( T = 0 \), consider the current response to a small space-homogeneous variation of the vector potential \( \mathbf{A}(t) \). The Fermi surface is shifted in the momentum space as a rigid object; see, e.g., Ref. [6]. In this respect there is no principal difference between a superconductor and a normal metal at optical frequencies. The difference is only that the optical approach, \( \omega \tau \gg 1 \), can be used for the static response of the clean superconductor. The electromagnetic field brings about a small change of the electron momentum \( \mathbf{Q} = -e\mathbf{A}(t) \) and the shift in the momentum space, \( \mathbf{p} \rightarrow \mathbf{p} + \mathbf{Q} \), creates in turn a small shift of the electron kinetic energies:

\[
\epsilon_p \rightarrow \epsilon_{p+Q} \approx \epsilon_p + \mathbf{v}_p \cdot \mathbf{Q} + \frac{1}{2} \mathbf{m}_p^{-1} \cdot \mathbf{Q},
\]

\[
m_p^{-1} = \frac{\partial \mathbf{v}_p}{\partial \mathbf{p}} = \frac{\partial^2 \epsilon_p}{\partial \mathbf{p}^2}.
\]

Thus for the increase of the kinetic energy density we obtain:

\[
w_{\text{kin}} = \frac{1}{2} \mathbf{Q} \cdot \mathbf{m}^{-1} \cdot \mathbf{Q} \mathbf{n}_c = \frac{1}{2} \mathbf{v}_{dr} \cdot \mathbf{m} \cdot \mathbf{v}_{dr} \mathbf{n}_c,
\]

\[
= \frac{1}{2} \epsilon_0 c^2 \sum_{\alpha,\beta=1}^3 A_\alpha (\lambda^{-2}(0))_{\alpha\beta} A_\beta,
\]
where \( j = e n_c v_{dr} \), \( v_{dr} = m^{-1} \cdot Q \), and

\[
n_c = 2 \int_{\varepsilon_p < E_F} \frac{dp}{(2\pi\hbar)^3} m_p^{-1} \theta(E_F - \varepsilon_p) = 2 \int_{\varepsilon_p = E_F} \frac{dS_p}{(2\pi\hbar)^3} v_p \otimes v_p \int_{\varepsilon_p < E_F} \frac{dp}{(2\pi\hbar)^3}.
\]

(26)

Whence the expressions for the penetration depth and the optical mass tensor read

\[
(\lambda^{-2}(0))_{\alpha\beta} = \frac{e^2 n_c}{\varepsilon_0 c^2} (m^{-1})_{\alpha\beta} = \frac{e^2}{\varepsilon_0 c^2} 2\nu_F \langle v_\alpha v_\beta \rangle,
\]

(27)

\[
(m^{-1})_{\alpha\beta} = \frac{2\nu_F}{n_c} \langle v_\alpha v_\beta \rangle,
\]

(28)

\[
\nu_F = \int \frac{dp}{(2\pi\hbar)^3} \delta(E_F - \varepsilon_p).
\]

(29)

The comparison with the Bernoulli effect considered in Sec. 3.1 tells us that the effective mass of the Cooper pairs in the clean limit is just twice the optical mass:

\[
M_{CP} = 2m, \quad n(T = 0) = \frac{1}{2} n_c,
\]

(30)

and neglecting some subtleties, e.g., the appearance of hole pockets, the total number of electrons corresponds to the total number of charge carriers \( n_c \) and \( n_{tot} \).

Formally, the effect of nonzero temperature reduces to introducing an additional multiplier \( r_d \) in the averaging of the velocity-velocity tensor at the Fermi surface, i.e.

\[
(\lambda^{-2}(T))_{\alpha\beta} = \frac{e^2}{\varepsilon_0 c^2} 2\nu_F \left( v_\alpha v_\beta r_d \left( \frac{\Delta_p}{2\pi k_n T} \right) \right),
\]

(31)

where

\[
r_d \left( \frac{\Delta_p}{2k_n T} \right) = \sum_{n=0}^{\infty} \frac{2\pi k_n T \Delta_p^2}{\Delta_p^2 + \varepsilon_n^2} \approx \begin{cases} 1, & \text{for } k_n T \ll |\Delta_p| \\ 7\zeta(3) \left( \frac{\Delta_p}{2\pi k_n T} \right)^2, & \text{for } |\Delta_p| \ll k_n T \\ \end{cases}
\]

where \( \varepsilon_n = (2n + 1)\pi k_n T \). The BCS multiplier \( r_d \) can be easily derived using Matsubara Green function method\(^3\), averaging over the Fermi surface also can be easily derived using this technique. For superconductors with strong coupling corrections we suggest an interpolation formula which can be used before a detailed theory to be developed. Having the solution of the Eliashberg equations \( Z_{n,p} , \tilde{\varepsilon}_{n,p} \) and \( \tilde{\Delta}_{n,p} \) we can perform in Eq. (32) the substitution \( \varepsilon_n \to \tilde{\varepsilon}_{n,p} = Z_{n,p} \varepsilon_n \) and \( \Delta_{n,p} \to \tilde{\Delta}_{n,p} \). The procedure used by Kogan\(^6\) is related to further substitution in Eq. (32) \( \tilde{\varepsilon}_{n,p} \to \tilde{\varepsilon}_n' = \tilde{\varepsilon}_{n,p} + 1/\tau_p \). The function

\[
r_d(z) = z^2 \sum_{n=0}^{\infty} \left[ z^2 + \pi^2 (n + 1/2)^2 \right]^{-3/2} \approx \begin{cases} 7\zeta(3)z^2/\pi^2, & z \ll 1 \\ 1, & z \gg 1, \end{cases}
\]

(32)
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For multiband superconductors, like MgB$_2$, the general formula for the penetration depth reads

\[
(\Delta^{-2}(T))_{\alpha\beta} = \frac{2\nu_{F}}{\varepsilon_{0}c^2} \sum_{b} c_{b} r_{d}(\frac{\Delta_{b}(T)}{2\pi k_{B}T}) \langle v_{\alpha} v_{\beta} \rangle_{b},
\]

\[
= \sum_{b} c_{b}(\hat{\lambda}_{b}^{-2}(0))_{\alpha\beta} r_{d}(\frac{\Delta_{b}(T)}{2\pi k_{B}T}),
\]

where $\lambda_{b}^{-2}(0)$ is the contribution of the $b$th band at $T = 0$. A careful fit of $\lambda(T)$ for MgB$_2$ could make it possible to separate the influence of the $\pi$- and $\sigma$-bands.

Returning to the expression for the kinetic energy density, Eq. (25), let us introduce the momentum of the pairs $\Pi = 2Q$, and the small-gap approximation

\[
r_{d}(p) \approx \frac{7}{3} \zeta(3) |\chi_{p}| \frac{1}{2\pi k_{B}T_{c}} \cdot \langle v \otimes v \chi^{2} \rangle \cdot D\Xi(r),
\]

Then for a clean superconductor one obtains

\[
\hat{w}_{\text{kin}} = \frac{1}{2} \int_{E_{F}}^{E_{F}} \frac{2d\mathbf{p}}{(2\pi\hbar)^{3}} Q \cdot m_{p}^{-1} \cdot Q r_{d}(\mathbf{p})
\]

\[
= \frac{1}{2}(\Pi\Xi)^{*} \cdot \tilde{\mathcal{M}}^{-1} \cdot (\Pi\Xi) = -a_{0}[D\Xi(r)]^{*} \cdot \xi^{2} \cdot D\Xi(r)
\]

\[
= -\frac{7\zeta(3)\hbar^2\nu_{F}}{(4\pi k_{B}T_{c})^2} [D\Xi(r)]^{*} \cdot \langle v \otimes v \chi^{2} \rangle \cdot D\Xi(r),
\]

where we have used the replacement

\[
\Pi\Xi \to -i\hbar D\Xi(r) = \left(-i\hbar \frac{\partial}{\partial r} - e^{*} A(r)\right) \Xi(r).
\]

Equation (34) agrees with Eq. (1) for $x_{c} = 0$ which is perhaps the simplest validation of the gradient terms in the GL expansion. In this way, for temperatures slightly below $T_{c}$ we obtain

\[
\hat{\mathcal{M}}_{\text{clean}}^{-1}(T) \approx \frac{7\zeta(3)\nu_{F}}{8\pi^{2}(k_{B}T_{c})^2} \langle v \otimes v \chi^{2} \rangle,
\]

\[
\hat{\lambda}_{\text{clean}}^{-2}(T) \approx \frac{4e^{2}v_{F}}{\varepsilon_{0}c^2} \frac{\langle \chi^{2} \rangle \langle v \otimes v \chi^{2} \rangle T_{c} - T}{T_{c}}.
\]

For isotropic-gap superconductors, $\chi = \text{const}$, and arbitrary band anisotropy this equation together with Eq. (27) gives

\[
\hat{\lambda}_{\text{clean}}^{-2}(0) = \frac{1}{2} \frac{d}{dT} \hat{\lambda}_{\text{clean}}^{-2}(T) \bigg|_{T_{c}}.
\]
7. Conclusions

We demonstrated that temperature dependence of the penetration depth $\lambda(T)$ in clean superconductors can be presented and an inserting of an additional BCS factor $r_d$ in the formula for far infrared skin depth of the metal. We conclude that London-Hall coefficient is temperature- and disorder independent, and the Bernoulli effect is an adequate method for determination of $R_H$. Given the penetration depth and the London-Hall coefficient at $T = 0$ we can determine the effective mass of the superfluid particles. Measuring the Bernoulli effect for thin and thick films, see Eq. (16) and Eq. (17) we conclude that London penetration depth $\lambda(T)$ also can determined by purely electrostatic measurements. This hydrodynamic mass $M_{CP}$ which parameterizes the relation between $R_H$ and $\lambda(0)$ is in the clean limit the optical mass extrapolated to zero frequency. But for disordered superconductors, irradiated thin films, for example, investigation of the disorder dependence of the Bernoulli signal can give an important information for the disorder renormalization of the effective Cooper pair mass. On Fig. 1 is presented the result for an isotropic gap superconductor but for a different gap anisotropies the curves will be different. In such a way we are coming to the conclusion that the impurity and temperature dependence of the Bernoulli signal is an important property which can be used to check the comparison between the theory and the experiment. For high-$T_c$ superconductors the Bernoulli effect gives the possibility to evaluate the vortex charge. We emphasize that Bernoulli effect and other electric field effects in superconductors, e.g., the surface Hall effect, gives a complete set of parameters describing the low-frequency current response of superconductors. We conclude also that bulk condensation energy of a superconductor can also be determined by electrostatic polarization created by a sudden heating due to Eq. (22).

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