Inversion of Fracture Parameters in Anisotropic Coal Seam

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Abstract. Inversion of coal seam fracture parameters is of great significance to identify and predict the distribution state of coal seam fractures, and also provides certain theoretical support for the detection and identification of coalbed methane. According to the formula of PP wave reflection coefficient, this paper uses genetic algorithm to carry out inversion of coal seam fracture parameters and Thomsen anisotropy parameters, and analyzes the accuracy and stability of inversion results. The results show that the genetic algorithm has high inversion accuracy for density, shear wave velocity and fracture density of background medium, and has certain reference significance for fracture identification.

1. Introduction

The bedding structure and fracture state of coal seam lead to its anisotropic characteristics. It is of great significance to identify the development and distribution state of fractures in anisotropic coal seam for coalbed methane development. Inversion of fracture parameters using anisotropic theory of coal seam can obtain fracture density, normal weakness and tangential weakness of fractures, which provides certain theoretical support for identification and detection of coalbed methane [1]. For the inversion of fracture parameters, previous scholars first simulated and constructed fractures, and proposed various fracture equivalent model theories, such as Hudson model [2-4], Thomsen model [5], and Schoenberg linear sliding model [6-8]. For the prediction of fracture parameters, Shaw et al. [9] assumed that the overlying medium on the reflection interface is isotropic medium, and estimated the normal and tangential weakness of the fracture from the P-wave AVO response using the least square method. Alhussain et al. [10] solved the reflection coefficient after linear approximation, and used the relationship between the reflection coefficient perpendicular to the fracture direction and the difference between the reflection coefficient parallel to the fracture direction and the fracture weakness to inverse the fracture weakness and density. Chen Huaizhen et al. [11-12] have carried out fracture parameter inversion including fluid factor, i.e. when fracture weakness is 0. On the basis of previous studies, this paper uses anisotropic theory and medium equivalent model theory to build a coal seam model that conforms to the actual formation state, calculates and solves Thomsen anisotropic parameters, elastic parameters and reflection coefficients of the model, then uses reflection coefficients to invert anisotropic parameters and fracture parameters, and compares the inversion results with prior information.

2. Theories

2.1. Model equivalence theory

Many vertical fractures are developed in coal seam, and the fracture density is relatively high. The coal seam with vertical fractures is regarded as HTI coal seam. Hudson [4] proposed that cracks be regarded as closed and parallel coins. Assuming that there is no fluid flow between cracks and the density and
aspect ratio of cracks are small, HTI equivalent medium model is established. Its elastic coefficient matrix C is expressed as:

\[
c = \begin{bmatrix}
c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\
c_{12} & c_{33} & c_{33} - 2c_{44} & 0 & 0 & 0 \\
c_{12} & c_{33} - 2c_{44} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66} \\
0 & 0 & 0 & 0 & 0 & c_{66} \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(1)

As the first-order model proposed by Hudson is only applicable to the case of extremely small fracture density (< 0.05), Liu deduces the elastic constitutive equation of fractured media based on Hudson model [13]. Liu model are more applicable to the case of relatively large fracture density. The deduced elastic parameter expression is:

\[
\begin{align*}
c_{11} &= (\lambda + 2\mu) \left(1 + \frac{(\lambda + 2\mu)eU_{11}}{\mu} \right)^{-1} \\
c_{12} &= \lambda \left(1 + \frac{(\lambda + 2\mu)eU_{11}}{\mu} \right)^{-1} \\
c_{33} &= \lambda + 2\mu \left(1 + \frac{(\lambda + 2\mu)eU_{11}}{\mu} \right)^{-1} \\
c_{44} &= \mu \\
c_{66} &= \mu(1 + eU_{13})^{-1}
\end{align*}
\]

(2)

where \( e \) represents fracture density, \( \lambda \) and \( \mu \) represent Lame constants of background isotropic media, \( U_{11} \) and \( U_{33} \) are jointly determined by fracture density, surrounding rock physical parameters and filler properties. For aerated dry fissures, \( U_{11} \) and \( U_{33} \) are respectively expressed as:

\[
U_{11} = \frac{4}{3} \left(\frac{\lambda + 2\mu}{\lambda + \mu}\right); \quad U_{33} = \frac{16}{3} \left(\frac{\lambda + 2\mu}{\lambda + 4\mu}\right)
\]

(3)

Utilizing equations 2 and 3, the elastic parameters of HTI coal seam can be calculated according to the actual Lame coefficient of coal seam, and then the anisotropic parameters can be further calculated according to Thomson anisotropic medium theory. The calculation formula is as follows:

\[
\begin{align*}
\varepsilon'(v) &= \frac{c_{11} - c_{33}}{2c_{33}} \\
\delta'(v) &= \frac{(c_{12} + c_{66})^2 - (c_{33} - c_{66})^2}{2c_{33}(c_{33} - c_{66})} \\
\gamma'(v) &= \frac{c_{66} - c_{44}}{2c_{44}}
\end{align*}
\]

(4)

in which \( \varepsilon'(v) \), \( \delta'(v) \) and \( \gamma'(v) \) are Thomsen anisotropy parameters of HTI medium, \( \varepsilon'(v) \) represents the degree of longitudinal wave anisotropy, \( \delta'(v) \) represents the degree of how fast longitudinal wave anisotropy changes between transverse and vertical directions, and \( \gamma'(v) \) represents the degree of difference between fast and slow transverse wave velocities.

2.2. HTI coal seam reflection coefficient

Because HTI medium is isotropic in the vertical direction and has a transverse axis of symmetry, PP waves show obvious azimuthal anisotropy with different fracture orientations. Rüger [14-16] derived the formula of pp wave reflection coefficient of HTI medium varying with incident angle and azimuth angle, which has wide applicability:
where $Z = \rho V_p$, which represents vertical longitudinal wave impedance. $G = \rho V_s$, representing shear wave shear modulus. $V_p$ represents longitudinal wave velocity. $V_s$ represents transverse wave velocity. $\theta$ represents the incident angle. $\phi$ represents azimuth angle. Subscripts 1 and 2 represent the upper medium and the lower medium respectively. The upper dash line represents the average value of the upper and lower media parameters. $\Delta$ indicates the difference between the upper and lower medium parameters.

### 2.2.1. Genetic algorithm inversion

As an intelligent optimization algorithm in inversion, genetic algorithm brings a new way to solve nonlinear inversion problems and has the advantages of high precision and stability. The objective function is the criterion to measure and judge the solution, and it has great influence on the optimization algorithm. In the process of coal seam fracture parameter inversion, the objective function is usually selected according to the error criterion and correlation criterion. The essence of inversion of anisotropic parameters and fracture parameters of HTI coal seam by genetic algorithm is to find the appropriate combination of 6 inversion parameters ($V_p, V_s, \rho, \epsilon, \delta, \epsilon$) to fit the determined reflection coefficient with the theoretical reflection coefficient. Therefore, the objective function of PP wave inversion of fracture parameters is as follows:

$$R_{pp}^{HTI}(\theta, \phi) = \frac{\Delta Z}{2Z} + \frac{1}{2} \left( \frac{\Delta V_p}{V_p} \right)^2 \times \frac{\Delta G}{G} \sin^2 \theta$$

$$+ \left( \frac{\delta_{\phi}^{(c)}}{2} + 4 \gamma \left( \frac{V_p}{V_s} \right)^2 \right) \cos^2 \phi \sin^2 \theta + \frac{\Delta V_p}{2V_p} \sin \theta \tan \theta$$

$$+ \frac{\epsilon_{\phi}^{(c)}}{2} \cos^4 \phi \sin^2 \theta \tan \theta + \frac{\delta_{\phi}^{(c)}}{2} - \sin^2 \phi \cos^2 \phi \sin^2 \theta \tan \theta$$

(5)

where $Z = \rho V_p$, which represents vertical longitudinal wave impedance. $G = \rho V_s$, representing shear wave shear modulus. $V_p$ represents longitudinal wave velocity. $V_s$ represents transverse wave velocity. $\theta$ represents the incident angle. $\phi$ represents azimuth angle. Subscripts 1 and 2 represent the upper medium and the lower medium respectively. The upper dash line represents the average value of the upper and lower media parameters. $\Delta$ indicates the difference between the upper and lower medium parameters.

The improved genetic algorithm mainly includes three operators: selection, crossover and mutation. It simulates the phenomena of reproduction, hybridization and mutation in natural selection and genetic process.

The selection process of iterative calculation is the process of survival of the fittest. It mainly sets certain screening conditions according to the objective function, selects individuals with better fitness from the population, and retains their excellent genes for continuous inheritance. A common selection algorithm is roulette.

Cross-link is a randomized recombination operator that generates new individuals after exchanging some parts of different individuals. The crossover operation is that the excellent genes of the excellent individuals are preserved, thus getting closer to the optimal solution. Cross operations are divided into single point cross, multi-point cross and uniform cross, and the cross probability is in the range of 0.4-0.99.

Variation operation generally occurs after crossover operation and is a process of local random optimization. This operation can make the whole iterative process not only have the ability of local random optimization but also keep the diversity of understanding, and further avoid premature and improve the overall optimization level. The types of mutation operations include basic position variation, uniform variation and boundary variation. The probability of variation is usually between 0.0001 and 0.1.
3. Model trial calculation

According to the model equivalent theory, an equivalent medium model with isotropic mudstone in the upper layer and HTI type dry fractured coal seam in the lower layer is established, as shown in Figure 1.

![Isotopic mudstone: \(V_{p1}, V_{s1}, \rho_1\)
HTI coal seam: \(V_{p2}, V_{s2}, \rho_2\)]

**Figure 1.** ISO/HTI coal seam equivalent medium model.

The elastic parameters of gas-bearing dry fractured coal seam calculated by Formula 2 and Formula 3 are shown in Table 1:

| Type        | \(e\) | \(C_{11}\) | \(C_{12}\) | \(C_{33}\) | \(C_{44}\) | \(C_{66}\) |
|-------------|-------|------------|------------|------------|------------|------------|
| Dry fractures | 0.05  | 6.28       | 1.82       | 8.01       | 2.88       | 2.58       |
|             | 0.15  | 4.33       | 1.26       | 7.79       | 2.88       | 2.13       |
|             | 0.25  | 3.30       | 0.96       | 7.70       | 2.88       | 1.82       |

In this model, the longitudinal and transverse wave velocities of roof mudstone were 3.170km/s and 1.585km/s respectively, and the density was 2.360kg/m^3. The reflection coefficient of the calculation model using the above formula and roof elastic parameters is shown in figure 2. From the above figure, it can be seen that the incident angle \(\theta\) in the model varies from 0 to 60 when the P wave is incident, and the azimuth angle \(\phi\) varies from 0 to 180. For convenience of comparison after inversion, the reflection coefficients of different fracture densities when the azimuth angle \(\phi\) is 0 are extracted as shown in Figure 3. According to the reflection coefficient of different fracture density models in figure 3, Thomsen anisotropic parameters and fracture density are calculated by inversion using the objective function, and the results are shown in figure 4. In order to minimize the error, the units of P-wave and S-wave velocity and density are adjusted during calculation to make the order of magnitude difference between elastic parameters, anisotropic parameters and fracture parameters smaller. Figure 4 illustrates the inversion results with different fracture densities.

Comparing the inversion results of fracture densities in figures 4(a), (b) and (c), we find that the fracture density increases and the inversion accuracy increases with the increase of fracture density. With the increase of fracture density, the retrieved Thomsen parameter values also have obvious changes, matching the changes of fracture density. P-wave velocity inversion results have certain errors, and S-wave velocity inversion results are better than P-wave velocity; The density inversion results are stable. At the same time, comparing with figure 4, it can be seen that as the number of iterations increases, the adaptability of the global optimal solution of elastic parameters and anisotropic parameters as well as fracture parameters and model parameters increases, and the inversion results tend to be stable.
In order to make the inversion results more intuitive, the inversion values of figure 4 are listed and compared with the prior values, as shown in table 2, it can be seen that the inversion effect of density, shear wave velocity and fracture density of background medium is better than that of longitudinal wave velocity and Thomsen parameter of background medium. At the same time, with the increase of theoretical fracture density, the sensitivity of reflection coefficient to anisotropic parameters and fracture parameters increases, and the inversion results are more accurate.

4. Conclusions
Based on the analytical solution of P-wave reflection coefficient of HTI coal seam forward model, the inversion objective function is established by using the relationship between anisotropic parameters and fracture density, and the optimal solution is found from nonlinear genetic algorithm. The elastic parameters, anisotropic parameters and fracture parameters of HTI coal seam are inversed. Numerical calculations show that:

Anisotropic parameters mainly affect the higher-order terms of the function when the reflection coefficient is solved, which leads to higher precision requirements of genetic algorithm in inversion. Genetic algorithm performs better in anisotropic parameter inversion in models with higher fracture density.
The inversion results of fracture parameters are constrained by the inversion results of elastic parameters and anisotropic parameters. When there are many inversion parameters, the nonlinear genetic algorithm is more restrictive in calculation and has more application prospects. According to the inversion results of genetic algorithm, it is feasible to analyze the fracture density of HTI coal seam, which is helpful to the exploration and development of coalbed methane.

**Table 2. Inversion Results of Genetic Algorithm for Anisotropic Parameters and Fracture Parameters**

|                  | \(V_p/(\text{km} \cdot \text{s}^{-1})\) | \(V_s/(\text{km} \cdot \text{s}^{-1})\) | \(\rho/(\text{kg} \cdot \text{m}^{-3})\) | \(\varepsilon\) | \(\delta\) | \(\epsilon\) |
|------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------|----------|----------|
| Theoretical value| 2.454                                  | 1.471                                  | 1.330                                  | -0.108         | -0.116   | 0.050    |
| Inversion value  | 2.440                                  | 1.489                                  | 1.318                                  | -0.102         | -0.105   | 0.047    |
| Error (%)        | 5.8                                    | 1.2                                    | 0.9                                    | 5.5            | 9.4      | 6        |
| Theoretical value| 2.420                                  | 1.471                                  | 1.330                                  | -0.222         | -0.233   | 0.150    |
| Inversion value  | 2.431                                  | 1.474                                  | 1.346                                  | -0.231         | -0.231   | 0.167    |
| Error (%)        | 0.45                                   | 0.20                                   | 1.2                                    | 4.05           | 0.86     | 11       |
| Theoretical value| 2.406                                  | 1.471                                  | 1.330                                  | -0.286         | -0.297   | 0.250    |
| Inversion value  | 2.399                                  | 1.471                                  | 1.318                                  | -0.290         | -0.290   | 0.261    |
| Error (%)        | 0.29                                   | 0                                      | 0.9                                    | 1.39           | 2.35     | 4.4      |

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