Triple Evolution: An Important Channel in the Formation of Type Ia Supernovae

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Abstract

Type Ia supernovae (SNe Ia) are thought to be the result of thermonuclear explosions in white dwarfs (WDs). Commonly considered formation pathways include two merging WDs (the double-degenerate channel) and a single WD accreting material from a H or He donor (the single-degenerate channel). Since the predicted SN Ia rates from WDs in binaries are thought to be insufficient to explain the observed SN Ia rate, it is important to study similar interactions in higher-order multiple-star systems such as triple systems. We use the evolutionary population synthesis code Multiple Stellar Evolution (MSE) to study the stellar evolution, binary interactions, and gravitational dynamics of the triple-star systems. Also, unlike previous studies, prescriptions are included to simultaneously take into account the single- and double-degenerate channels, and we consider triples across the entire parameter space (including those with tight inner binaries). We explore the impact of typically ignored or uncertain physics such as flybys and common envelope prescription parameters on our results. The majority of systems undergo circular mergers to explode as SNe Ia, while eccentric collisions contribute to 0.4%—4% of SN Ia events. The time-integrated SN Ia rate from the triple channel is found to be $(3.60 \pm 0.04) \times 10^{-4} \ M_{\odot}^{-1}$, which is, surprisingly, similar to that of the isolated binary channel, where the SN Ia rate is $(3.2 \pm 0.1) \times 10^{-4} \ M_{\odot}^{-1}$. This implies that triples, when considering their entire parameter space, yield an important contribution to the overall SN Ia rate.

Unified Astronomy Thesaurus concepts: Type Ia supernovae (1728); White dwarf stars (1799); Multiple stars (1081)

1. Introduction

Type Ia supernovae (SNe Ia) are standard candles that play a key role in distance measurements on cosmological scales. As such, they play an important role in our understanding of the structure and expansion rate of the universe. The SNe Ia are important for our comprehension of the chemical evolution of galaxies and of iron-group element nucleosynthesis. The origin of SNe Ia is thought to be thermonuclear explosions in white dwarfs (WDs), though our understanding of their progenitors and explosion mechanisms is not very clear (Wang & Han 2012; Maoz et al. 2014; Livio & Mazzali 2018; Ruiter 2020).

Nevertheless, there are two commonly suggested binary progenitor channels (Ruiter 2020) to produce SN Ia explosions, which include the double-degenerate (DD) channel (in which both components of the binary system are WDs; Iben & Tutukov 1984; Webbink 1984) and the single-degenerate (SD) channel (in which an accreting Chandrasekhar-mass WD may explode via delayed detonation or an accreting sub-Chandrasekhar-mass WD may explode via double detonation; Whelan & Iben 1973). Although a substantial amount of work (Nomoto 1980; Neunteufel et al. 1982a, 1982b; Yoon & Langer 2004a, 2004b; Neunteufel et al. 2016, 2017, 2019; Bauer et al. 2021) has been done to understand the progenitors through binary evolution, the general consensus is that the rates from the binary channels are too low to explain the observations (Ruiter et al. 2011; Claey et al. 2014). The observed time-integrated rate from Maoz et al. (2012) is $(1.3 \pm 0.2) \times 10^{-3} \ M_{\odot}^{-1}$. Claey et al. (2014) studied the formation channels for SNe Ia through binary population synthesis and estimated the time-integrated overall SN rate to be $4.8 \times 10^{-4} \ M_{\odot}^{-1}$, which could explain only a fraction of the observed rates from Maoz et al. (2012). This motivates the study of other progenitor channels.

A formation channel for SNe Ia that has not been studied as extensively as the binary channel is the triple channel, which involves hierarchical triple-star systems containing WDs. This channel is particularly interesting because of its contribution to producing and disrupting close binaries. For an isolated binary, it is difficult to produce close binaries and mergers within a Hubble time. However, in a triple system, if the initial mutual inclination is sufficiently large, the inner binary can undergo high-amplitude eccentricity oscillations. This, in turn, leads to changes in the mutual inclination of the system. These oscillations are known as von Zeipel–Lidov–Kozai (ZLK) oscillations (von Zeipel 1910; Kozai 1962; Lidov 1962, see Naoz (2016) for a review). Combined with tidal effects, ZLK oscillations can shrink the inner binary, which results in the formation of close binaries (Mazeh & Shaham 1979; Eggleton & Kisseleva-Eggleton 2001; Eggleton & Kisseleva-Eggleton 2006; Fabrycky & Tremaine 2007); cause earlier common envelope (CE) evolution (Hamers & Thompson 2019; Toonen et al. 2020); accelerate mergers (Blaes et al. 2002; Thompson 2011; Toonen et al. 2018); and induce dynamical instability, which, in turn, results in a merger of two WDs and an SN Ia explosion. From studies by Moe & Di Stefano (2017) and Raghavan et al. (2010), we know that about 10% of solar-mass stars are found to be in triples with possible SN Ia progenitors.
Previous studies of triple-star systems considered SN Ia rates from head-on collisions arising from dynamical interactions (Katz & Dong 2012), contributions from WD mergers taking into account stellar evolution and dynamics (Hamers et al. 2013), isolated triples with a circular approximation for mass transfer (Toonen et al. 2018), the postulated progenitor triples from the Gaia DR2 database (Hallakoun & Maoz 2019), and ultrawide WD triples affected by flybys (Hamers & Thompson 2019; Michaely 2021). These studies considered dynamical interactions and/or stellar evolution and binary interactions such as tidal effects. However, in triple systems, their focus was on the DD formation pathway; there are no studies of the contribution of SD channels (in which an accreting Chandrasekhar-mass WD may explode via delayed detonation or an accreting sub-Chandrasekhar-mass WD may explode via double detonation; Whelan & Iben 1973) and in particular, there are no studies that self-consistently take into account stellar and binary evolution (especially mass transfer in eccentric orbits), as well as gravitational dynamics.

In this paper, we present a comprehensive study of candidates for thermonuclear explosions originating from triple-star systems through both the SD and DD channels. We note that not all thermonuclear SNe will result in SNe Ia but may form related transients such as SNe Iax instead. For the purposes of this study, we use SN Ia as a catch-all form for transients resulting from the thermonuclear detonation of a WD. In addition, in our simulations, we take into account the possibility for the tertiary star to transfer mass onto the inner binary system, which, in turn, can produce a triple common envelope (TCE). The paper begins with the methodology in Section 2, followed by the different formation channels for SNe Ia in Section 3. In Section 4, we present our statistical results. We discuss and conclude the results in Sections 5 and 6, respectively.

2. Methodology

2.1. Population Synthesis

2.1.1. MSE

In this work, we use the evolutionary population synthesis code Multiple Stellar Evolution (MSE, version-v0.87; Hamers et al. 2021). The advantage of this code is that it incorporates prescriptions for stellar evolution, binary interactions (tides, mass transfer, etc.), dynamical perturbations from higher-order multiple systems, and flybys. The MSE is a publicly available C/C++ code with a Python interface. It can evolve any number of stars as long as the system is originally hierarchical (later potential dynamical instabilities are modeled self-consistently through N-body methods). In order to tackle the complicated long-term dynamical evolution of multiple-star systems, MSE uses a hybrid approach that switches between the secular approximation (Hamers & Portegies Zwart 2016; Hamers 2018, 2020) and N-body integration (Rantala et al. 2020) during the runtime. Throughout the dynamical evolution, post-Newtonian (PN) terms are taken into account, up to and including 2.5 PN order.

Single-star evolution (SSE) in MSE is based on the SSE algorithms (Hurley et al. 2000) based on stellar evolutionary tracks by Pols et al. (1998). The code uses modified binary star evolution (BSE) prescriptions (Hurley et al. 2002) for binary interactions. Tidal evolution is modeled following the equilibrium tide model (Eggleton et al. 1998). Here the tides are applied to star–star and star–composite systems. The code takes into account eccentric mass transfer, adopting the model of Hamers & Dosopoulou (2019). When mass transfer is deemed unstable, CE is modeled using the energy conservation mechanism, i.e., the $\alpha$–$\lambda$ CE prescription (Paczynski 1976). The outer companion, when massive enough, can transfer mass onto the inner binary components, and the subsequent evolution is modeled following approximate prescriptions (Hamers et al. 2022).

In MSE, the effects of passing stars (flybys) are taken into account as appropriate for low-density ($n_s = 0.1 \, \text{pc}^{-3}$) environments. An exploration of the impact on triples of encounters in high-density environments such as globular clusters, although interesting, is beyond the scope of this paper. The perturber mass is sampled from Kroupa (2001), and encounters are sampled assuming an encounter sphere of radius $R_{\text{enc}} = 10^5$ au with velocities sampled from a Maxwellian distribution of dispersion $\sigma_s = 30 \, \text{km s}^{-1}$. These flybys become significant when the semimajor axis of the orbit exceeds approximately $10^3$ au.

2.1.2. SN Ia Prescription

The initial version of MSE (Hamers et al. 2021) uses BSE (Hurley et al. 2002) prescriptions for SN Ia explosions. Within these prescriptions, an accreting CO WD has accumulated 0.15 $M_\odot$ of helium, and the WD explodes in an SNe Ia. This assumption, however, has been shown to be incomplete since its first implementation. The amount of material required to initiate a helium detonation and subsequent ignition of the CO core has been shown to depend on other parameters of the progenitor binary, most notably the mass of the accretor, the mass transfer rate, and, to some extent, assumptions on rotation, angular momentum transport, and viscose heating (Yoon & Langer 2004a, 2004b, 2005; Woosley & Kasen 2011; Piersanti et al. 2014; Neunteufel et al. 2017). Further, as summarized particularly by Piersanti et al. (2014), depending on the mass transfer rate, outcomes of He accretion onto CO WDs range from possible double detonation ($M \lesssim 7 \times 10^{-8} M_\odot \, \text{yr}^{-1}$) via massive He novae of decreasing intensity ($M \gtrsim 7 \times 10^{-8} M_\odot \, \text{yr}^{-1}$; see Kato & Hachisu 2004) to steady burning and reignition as a He red giant ($M \gtrsim 1 \times 10^{-6} M_\odot \, \text{yr}^{-1}$) in the space of about 2 orders of magnitude. As further shown by Neunteufel et al. (2016), a system may move between these different mass transfer regimes, e.g., with systems first undergoing weak helium flashes and then finally terminating in an SN Ia.

In order to take these different possibilities into account, this study employs a refined prescription, considering the mass of the accretor and the rate of mass accretion, for deciding on the final outcome of helium accretion (Neunteufel et al. 2016). This prescription combines the accretion rate–dependent accretion efficiencies ($\eta$) presented by Kato & Hachisu (2004) at values of mass transfer rates with the occurrence of detonation at low mass transfer rates as presented by Woosley & Kasen (2011). The resulting prescription can be written as

$$
\eta = \begin{cases} 
1, & \text{if } 0 < [\dot{M}] < M_{\text{WK,max}}, \\
0, & \text{if } M_{\text{WK,max}} < \dot{M} < M_{\text{KH,min}}, \\
\eta_{\text{KH}}(M_{\text{WD}}, \dot{M}), & \text{if } M_{\text{KH,min}} < \dot{M}, \\
1, & \text{if } M_{\text{KH,max}} < \dot{M},
\end{cases}
$$

(1)
where $\dot{M}_{\text{WK,max}}$ and $\dot{M}_{\text{KH,min}}$ are the upper and lower limits of the accretion rates studied by Woosley & Kasen (2011) and Kato & Hachisu (2004), respectively. We note that, while Neunteufel et al. (2016) used a time-averaged mass transfer rate in order to exclude ignitions resulting from spurious variations in the mass transfer rate, which are prone to happen in detailed stellar evolution, this approach is unnecessary in the context of population synthesis.

With regard to the DD SNe Ia, a new prescription combining results from various hydrodynamics simulation studies has been incorporated into MSE. Collisions in MSE can happen either via a circular merger, usually following CE evolution, or eccentric collision driven by secular evolution. We assume the outcome will be SNe Ia when there is a circular merger of a He WD and CO WD (irrespective of their masses). We also assume that the coalescence of two CO WDs in which one of them is more massive than 0.9 $M_\odot$ results in an SN Ia explosion (Pakmor et al. 2010, 2013). In the event of an eccentric collision, the collision of two CO WDs, two ONe WDs, or a CO WD and an ONe WD is assumed to lead to SNe Ia.

2.2. Initial Distributions

We adopt a population synthesis method, in which the initial conditions for a large number of triple-star systems are generated based on a Monte Carlo approach. Here we describe the assumptions made in this procedure. The primary mass of the inner binary, $m_1$ (i.e., the mass of the initially most massive star in the inner binary system), is set between 1 and 6.5 $M_\odot$ to ensure the formation of at least one CO WD within a Hubble time in isolation, and it follows Kroupa (2001). The distribution of the secondary mass is modeled after the observational fit functions of Moe & Di Stefano (2017). The initial orbital period and eccentricity distributions (for both inner and outer orbits) are also drawn from the observational fit functions of Moe & Di Stefano (2017). The orbital periods are sampled in the range $0.2 < \log(P/$days$) < 8$. Eccentricities of both orbits are sampled between zero and 1. The initial mutual inclinations are uniformly distributed in $\cos(i)$. The longitudes of the ascending node and arguments of periapsis are sampled from uniform distributions. These assumptions correspond to isotropic orientations of the inner and outer orbits. The systems that do not satisfy the stability criteria of Mardling & Aarseth (2001) are rejected. We also eliminate systems with stars that are filling their Roche lobes at the start of the evolution at periapsis, using the fit of Eggleton (1983), and using the mass–radius relation $R \propto M^{0.7}$ to estimate the initial stellar main-sequence radii.3

The mass ratio distribution involving the tertiary (outer) star in triple systems, specifically, the outer mass ratio $q_{out} \equiv m_3/(m_1 + m_2)$, is not very well constrained. From the Multiple Star Catalogue (MSC; Tokovinin 2018), about 7% of the systems have a tertiary star that is more massive than the total mass of the inner binary. These systems are potentially interesting because they favor channels that involve triple mass transfer, i.e., when the tertiary star fills its Roche lobe around the inner binary (Glanz & Perets 2021; Hamers et al. 2022). In order to allow for the possibility of systems with a massive tertiary star, we have constructed two different models for the initial mass ratio distribution. For the first model (hereafter model 1), we fit a decaying exponential function to the data from the MSC and find the best-fit parameters. The second model (hereafter model 2) is an extrapolation of the mass ratio distribution of Moe & Di Stefano (2017). The first mass ratio model is of the form

$$\frac{dN}{dq_{out}} \propto \exp(-q_{out} \lambda),$$

where $\lambda = 1.05$. The decaying exponential model best fits the current observations. However, the MSC has substantial observational biases for triple (and higher-order) systems, and we especially expect strong observational biases against triples with high-mass tertiaries and low-mass inner binaries (high mass ratio systems). Thus, we take into account both models in our work as a means to explore the current uncertainties in the systems. The initial distribution of the assumed parameter space is shown in Figure 1.

In addition to considering two models for the assumed distribution of $q_{out}$, we vary the physical model parameters (Table 1) in our simulations to investigate the impact of physical uncertainties, as well as effects that are often ignored in the literature. We inspect the effects of flybys and various CE parameters in our work. We also study the SN Ia rate from stars with subsolar metallicity.

2.3. Construction of the Initial Population

Our population pool includes $4 \times 10^5$ triple systems for both model 1 and model 2. Triple systems, which include all of the varying model parameters such as CE, flybys, and metallicity, constitute $4 \times 10^5$ systems. In total, our triple population sample size adds up to be $1.2 \times 10^6$. In addition, in order to investigate the effect of the tertiary star, we rerun our main models without the tertiary star (only inner binary systems). The size of the inner binary population is $8 \times 10^5$. We also study the contribution from isolated binaries, for which we construct a binary population of size $1 \times 10^5$. As explained later in Section 4.4, the latter isolated binary population is significantly different from the triple population with the tertiary star removed. In total, our population pool consists of $2.1 \times 10^6$ systems. The constructed population is evolved for a period of 10 Gyr with an imposed maximum wall time of 5 hr.

3. Evolutionary Pathways

In this section, we summarize the evolutionary pathways for forming SNe Ia in triple systems as found in our population synthesis calculations. We restrict our explanation to formation channels that demand a tertiary to form SNe Ia. In order to select the systems that have an effect from the tertiary, we compare SNe Ia from triple population synthesis with those from inner binary (without tertiary) population synthesis. We provide five unique formation channels for producing SNe Ia only from triple systems. Table 2 quantifies the contribution from these evolutionary pathways. The presented evolutionary pathways are unique to triples and not mutually exclusive. The evolutionary pathways in which the tertiary does not contribute to producing an SN Ia explosion is similar to binary evolution channels and not presented here.

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3. This more approximate method of determining the main-sequence radii is only adopted for sampling purposes.
Figure 1. Initial parameter distributions of the constructed initial populations with two $q_{\text{out}}$ models: the decaying exponential model (model 1; blue filled histogram) and the Moe & Di Stefano (2017) $q$ distribution (model 2; dashed indigo line). The top panels show the mass distributions of the primary, secondary, and tertiary star. The middle panels represent the semimajor axis distributions for the inner and outer orbits. The bottom panels show the eccentricity distributions for the inner and outer orbits.
Table 1

Overview of the Different Models Stating the Assumptions for the Distribution of the Mass Ratio between the Outer Star and the Inner Binary $q_{\text{out}} \equiv m_3/(m_1 + m_2)$, the Choice for the CE Parameter $\alpha_{\text{CE}}$, Whether or Not Flybys Are Accounted for, and the Metallicity

| Models | $q_{\text{out}}$ | $\alpha_{\text{CE}}$ | Flybys | Metallicity |
|--------|-----------------|-----------------|--------|-------------|
| Model 1 | $\exp(-q_{\text{out}})$; $\lambda = 1.05$ | 1 | Included | 0.02 |
| Model 2 | Extrapolating Moe & Di Stefano (2017) | 1 | Included | 0.02 |
| Model 3 | $\exp(-q_{\text{out}})$; $\lambda = 1.05$ | 10 | Included | 0.02 |
| Model 4 | $\exp(-q_{\text{out}})$; $\lambda = 1.05$ | 0.1 | Included | 0.02 |
| Model 5 | $\exp(-q_{\text{out}})$; $\lambda = 1.05$ | 1 | Ignored | 0.02 |
| Model 6 | $\exp(-q_{\text{out}})$; $\lambda = 1.05$ | 1 | Included | 0.001 |

Table 2

Relative Contribution of the Formation Channels for SNe Ia in Triples

| Models | TCE (%) | Double Mergers (%) | Unbound Tertiary (%) | Eccentric Collision (%) | Dynamical Instability (%) |
|--------|---------|--------------------|----------------------|-------------------------|--------------------------|
| Model 1 | 12.0 ± 0.5 | 21.4 ± 0.5 | 28.1 ± 0.7 | 0.8 ± 0.1 | 1.1 ± 0.1 |
| Model 2 | 12.6 ± 0.5 | 20.1 ± 0.5 | 29.9 ± 0.8 | 1.1 ± 0.1 | 1.0 ± 0.1 |
| Model 3 | 3.8 ± 0.6 | 20.1 ± 1.0 | 31.2 ± 1.9 | 4.0 ± 0.6 | 2.3 ± 0.5 |
| Model 4 | 12.6 ± 1.8 | 60.0 ± 3.4 | 14.3 ± 1.9 | 2.3 ± 0.7 | 2.1 ± 0.7 |
| Model 5 | 11.8 ± 0.9 | 20.1 ± 1.0 | 26.5 ± 1.4 | 0.4 ± 0.1 | 0.7 ± 0.2 |
| Model 6 | 11.7 ± 0.9 | 21.0 ± 1.0 | 27.8 ± 1.5 | 0.9 ± 0.2 | 1.1 ± 0.3 |

Note. The channels listed in the table are the ones in which the tertiary plays a role in producing SN Ia events. Note that they are not mutually exclusive. Error bars indicate statistical (Poisson) uncertainties.

3.1. Triple Common Envelope

From our results, TCE is an important channel for producing SNe Ia from triple-star systems, responsible for 4%-13% of all SNe Ia by triples in our set of models. Figure 2 shows a mobile diagram (see Appendix for an explanation of the mobile diagrams presented here) of a triple system undergoing TCE, causing a merger of the inner binary, which then leads to an SN Ia later.

If the tertiary star is relatively close and more massive than the total mass of the inner binary, it can start transferring mass onto the inner binary, forming a TCE around the inner binary. At the end of TCE, if dynamical instability is triggered, one of the inner binary components can get exchanged with the tertiary, forming an exchange triple. Other possibilities include TCE evolution followed by a merger of the inner binary or a merger of an inner binary component with the tertiary. The TCE evolution can disrupt the triple system by unbinding the tertiary or inner binary component, resulting in a binary system with the remaining components. If the CE is assumed to be more efficient ($\alpha_{\text{CE}} = 10$), it induces more inner binary mergers and thereby fewer TCE episodes.

3.2. Double Mergers

We identify two different cases of scenarios leading to SNe Ia and involving double mergers. In the first case, there is an early mass transfer episode during the main sequence, which merges the inner binary components into a rejuvenated main-sequence star. This results in a new binary, with one component being the original tertiary star and the other being the merger remnant of the inner binary. These two stars then evolve, and this later leads to an SN Ia explosion. In the second case, the tertiary star, as a result of secular eccentricity excitation coupled with tides, shrinks the inner binary, which leads to an early CE phase that subsequently merges the inner binary. This forms a new binary with the merger remnant and original tertiary star and produces an SN Ia event later. Figure 3 shows a triple-star system in which the inner binary merges through a CE phase to produce a new star, which then further undergoes two more CE phases and then collides with the tertiary star to produce SNe Ia.

3.3. Unbound Tertiary

In the course of the evolution, the tertiary star can get unbound for different reasons. For example, Figure 4 shows a triple channel in which the tertiary gets unbound when the inner binary undergoes a CE phase. The CE in the inner binary and TCE are responsible for unbinding the tertiary star in about 67% and 7% of SN Ia events through this channel, respectively. When the tertiary star is massive enough, it can collapse into a neutron star, resulting in an SN II. In about 22% of the unbound tertiary channel, the mass loss and/or natal kick during this SN II can unbind the tertiary star, while the inner binary later produces an SN Ia event. The tertiary star can also become unbound when there is a CE episode in the inner binary, which is associated with rapid mass loss in the inner binary. Flybys unbind the tertiary star when the semimajor axis is of the order of $10^3$ au or wider. When the triple system becomes dynamically unstable, one of the stars can get ejected out of the system. Flybys and dynamical instability contribute 2% each to this channel.
3.4. Eccentric Collision

The formation of close binaries in the isolated binary channel is mainly explained by CE phases. These systems are nearly always circularized at the end of the CE phase. But in the case of triple-star systems, there is a possibility that, even after the CE phase, the tertiary can induce eccentricities in the close inner binary through secular evolution. We see that about 1% of systems that form SNe Ia experience eccentric collisions. There is also another possibility of forming an SN Ia only through the dynamical channel. In these types of systems, the

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**Figure 2.** Example of a system undergoing TCE. A triple system with a massive tertiary evolves first to transfer mass on top of the inner binary to form a TCE. The inner binary merges to form a rejuvenated star at the end of TCE, which then interacts with the tertiary star to explode as SNe Ia. Refer to Appendix for more details on the mobile diagrams presented in this paper.
eccentricity of the inner binary oscillates due to the perturbations from the tertiary star, and, after a period of time, the secular approximation breaks down (Antognini et al. 2014; Antonini et al. 2014; Luo et al. 2016) and the inner binary components collide at extremely high eccentricity, leading to an SN Ia explosion. Figure 5 shows an example of a system achieving such high eccentricities ($e \approx 1$) and then causing a collision and hence an SN Ia explosion.

3.5. SNe Ia Driven by Dynamical Instability

In an isolated binary channel, the formation of SNe Ia cannot be explained without undergoing a CE phase. In our simulations,
in addition to mergers during CE, there is also the possibility of physical collisions driven by (chaotic) few-body dynamics following the onset of dynamical instability. For example, Figure 6 shows a pure dynamical channel to produce SNe Ia. During the course of the evolution, if the inner binary becomes dynamically unstable following Mardling & Aarseth (2001), there can be a head-on collision between the binary components, which leads to an SN Ia explosion.

4. Statistical Results

We present the delay time distribution (DTD) and detailed statistical analysis of the contribution from different progenitors in this section. Table 3 gives the overview of the time-integrated rate from triple (various models) and binary channels. Table 4 summarizes the contributions of different progenitors to SN Ia events.

4.1. DTD and SN Ia Rate

We assume a starburst at time \( t = 0 \); the SN Ia rate during a particular time interval \( \Delta T \) is then calculated using

\[
R = \frac{N}{M_* \Delta T}
\]

where \( N \) is the total number of SN Ia explosions during \( \Delta T \), and \( M_* \) is the total mass of the synthesized stellar population. We assume that the synthesized stellar population only constitutes single, binary, and triple stars, and any contribution from higher-order systems is neglected. We use the primary...
mass-dependent multiplicity fraction from Moe & Di Stefano (2017, their Table 13) when calculating the total stellar mass. First, the total number of systems to be sampled is calculated using the expression

\[ N_{\text{tot}} = N_{\text{triple}} + N_{\text{binary}} + N_{\text{single}} \]

where \( N_{\text{calc}} \) is the numerically calculated fraction of triple stars in which the primary mass of the inner binary is in the mass range 1–6.5 \( M_\odot \), and \( N_{\text{calc}} \) is number of triple stars originally sampled (in our case, \( 4 \times 10^5 \) for each \( q_{\text{out}} \) model). Here \( N_{\text{triple},m} \) is the number of triple stars in the particular mass bin, with \( \alpha_{\text{single},m} \), \( \alpha_{\text{binary},m} \), and \( \alpha_{\text{triple},m} \) being the single, binary, and triple fractions in the respective mass bins that we adopt from Moe & Di Stefano (2017). A stellar population is constructed with the calculated number of single, binary, and triple stars in each mass bin. The single-star population is created by assuming the initial mass function (IMF) from Kroupa (2001) between 0.08 and 100 \( M_\odot \). The primary mass of the binary population is constructed similarly to the single-star population. The separations are calculated as a function of primary mass following Moe & Di Stefano (2017). The secondary mass and eccentricities are sampled following the primary mass and period-dependent distribution functions from Moe & Di Stefano (2017). The triple population is constructed similarly to that described in Section 2.3, except that now the

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**Figure 5.** Example of an eccentric collision. The isolated binary channel usually predicts that the binary gets circularized (\( e \approx 0 \)) after a CE episode. Here the figure shows that the inner binary can still achieve high eccentricities from dynamical perturbations, which can then lead to an SN Ia explosion via collision.
primary masses of the inner binary are sampled in the mass range 0.08–10.0 M$_\odot$ to cover the entire mass range. Finally, $M_\ae$ is calculated by adding all of the stellar masses.

The DTDs for the DD and SD pathways are shown in Figures 7 and 8, respectively. The later part of the DD DTD is found to follow a power-law shape, $\propto t^{-1}$. The DTD is calculated in units of number of SNe Ia per 10$^{10}$ M$_\odot$ per century.

The time-integrated SN Ia rates $N_{\text{total}}/M_\ae$ for models 1 and 2 are calculated as $(3.60 \pm 0.04) \times 10^{-4}$ and $(3.50 \pm 0.04) \times 10^{-4}$ M$_\odot^{-1}$, respectively.

4.2. Circular and Eccentric Mergers

In the isolated binary evolution channels, most SN Ia explosions result from the formation of close WD binaries following CE evolution. However, in triple-star systems, in addition to CE evolution, the tertiary star can also aid the formation of SNe Ia. Figure 9 represents the initial semimajor axis distribution of systems that explode as SNe Ia. From the semimajor axis distribution, it is evident that the systems with wide semimajor axes undergo collisions triggered by high-eccentricity oscillations in the inner binary due to the tertiary star. This is because the triples with wide inner binaries have shorter secular timescales (all else being the same), whereas the short-range precession timescales in the inner binaries are longer. Both of these effects contribute to a larger probability for exciting high eccentricities in the inner binary. The ZLK mechanism produces high-amplitude eccentric oscillations in systems with high initial mutual inclinations. Furthermore, from Figure 10, we can see that the systems with high initial mutual inclinations are more likely to undergo eccentric collisions than circular mergers. There is a strong decrease in the number of systems undergoing circular mergers for higher inclinations near 90° (note that Figure 10 uses a log scale). Such a strong dip is not apparent in the systems undergoing eccentric collisions. However, in the case of the opposite sides of the inclination distribution, inner orbits are more likely to be brought closer together by CE evolution, thereby inducing circular mergers. Systems undergoing circular mergers are dominant; their contribution to the total number of SNe Ia is found to be $(99.0 \pm 1.7)$% and $(99 \pm 2)$% for models 1 and 2.
respectively. There are also systems undergoing eccentric collisions but with a smaller contribution. Their fractional contribution is found to be $(0.8 \pm 0.1)\%$ for model 1 and $(1.0 \pm 0.1)\%$ for model 2.

### 4.3. DD and SD SNe Ia

It is interesting to analyze the contribution of DD and SD channels to SNe Ia. From our study, the DD channel surpasses the SD channel in great numbers. The percentage of systems undergoing DD SNe Ia is $(99 \pm 2)\%$ and $(99 \pm 2)\%$ for models 1 and 2, respectively, whereas the SD SNe are $(0.7 \pm 0.1)\%$ for model 1 and $(0.7 \pm 0.1)\%$ for model 2. When we carried out an in-depth analysis of the various subchannels contributing to DD SNe Ia, we found that the majority of them are He and CO WD mergers, and there is also a nonnegligible contribution from CO–CO WD mergers. Figure 11 shows the relative contributions from He–CO and CO–CO WDs. The detailed numbers are displayed in Table 4. The contribution of the SD subchannels is analyzed in the next section.

### 4.4. Chandrasekhar- and Sub-Chandrasekhar-mass SNe Ia

As described in Section 2.1.2, there are two possible scenarios for SD SNe Ia taken into account in our simulations. The first scenario involves the accretor (CO WD) gaining mass by accreting mass from a nondegenerate donor star and exploding as an SN Ia when the WD reaches the Chandrasekhar mass limit $(1.44 M_\odot)$. In the second case, the CO WD undergoes stable accretion via Roche lobe overflow. Here the donor is a H-poor, He-burning stripped star. The WD undergoes double detonation and explodes as an SN Ia well before the Chandrasekhar mass is reached. These two scenarios involve different evolutionary pathways (see, e.g., Ruiter 2020).

From our model 1 simulations of SD SNe Ia, the percentage of systems undergoing Chandrasekhar mass and sub-Chandrasekhar mass are $(9 \pm 5)\%$ and $(90 \pm 20)\%$, respectively. From the model 2 simulations, the percentage of systems undergoing Chandrasekhar mass and sub-Chandrasekhar mass are $(14 \pm 6)\%$ and $(86 \pm 19)\%$, respectively. It is apparent from the statistics that the contribution of sub-Chandrasekhar-mass SNe Ia dominates the fraction of SD SNe Ia over Chandrasekhar-mass SNe Ia.

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Figure 7. The DTD from all DD SNe Ia in our simulations. The solid blue line (decaying exponential $q_{\text{out}}$ model) and dashed green line (extrapolating $q$ from Moe & Di Stefano 2017 $q_{\text{out}}$ model) correspond to the DTD from triple population synthesis. The dotted red line represents the DTD from isolated binary population synthesis, and the black dashed–dotted line shows that the later part of the DTD is found to follow a power-law shape ($\propto t^{-1}$). In addition, observational SN Ia rates from field galaxies (refer to Table 2 of Maoz & Graur 2017) and galaxy clusters (refer to Table 3 of Maoz & Graur 2017) are shown by cyan and magenta points, respectively.

Figure 8. The DTD from SD SNe Ia. The solid blue line ($q_{\text{out}}$ model; decaying exponential $q_{\text{out}}$ model) and dashed green line ($q_{\text{out}}$ model; extrapolating $q$ from Moe & Di Stefano 2017) correspond to the DTD from our simulated triples. The dotted red line represents the DTD from our simulated isolated binaries.
4.5. Isolated Binary Evolution

In order to understand the contribution of the triple evolution channel to the SN Ia rate, it is important to compare to the rate attributed to the binary evolution channel. We constructed our initial binary population to be fully consistent with our sampling of the triple population. Specifically, we assumed the Kroupa (2001) IMF for the primary mass distribution; the secondary mass, eccentricity, and period distributions follow the functional distributions from Moe & Di Stefano (2017). We used the same SN Ia prescriptions as for our triple runs and, for consistency, the same population synthesis code MSE to evolve these binary systems for 10 Gyr. The estimated time-integrated SN Ia rate from our binary system calculations is $(3.2 \pm 0.1) \times 10^{-4} \, M_\odot^{-1}$. The corresponding DTD from the binary evolution channel is shown in Figure 7 with the red dotted line.

4.6. Properties of the Third Star

We examined the nature of the third star at the moment when the inner binary explodes as an SN Ia. We find that, in about 1% of systems, two unbound stars collide during a phase of dynamical instability leading to an SN Ia. 20% of stars are newly formed binaries as a result of the merger of the inner binary and the tertiary component, 30% of systems have an unbound third star, and 49% of systems have a bound third star. The mass distribution of the tertiary star in those cases when it is still bound at the moment of the SN Ia explosion is shown in Figure 12; most of the tertaries have masses less than 1 $M_\odot$. The outer semimajor axis distribution for the cases when the tertiary star is still bound at the time of the SN Ia explosion is shown in Figure 13; the outer semimajor axis is found to be distributed broadly up to more than a million au. In most of the cases, flybys are responsible for these wide orbits, while in a small number of cases, CE evolution in the inner binary can also create these wide orbits. It is also noted that these outer orbits are mostly eccentric and typically expected to be short-lived. In addition, we point out that it would be very difficult to detect these ultrawide orbits.

Table 4

| Models | Channel | DD (%) | SD (%) | SCM (%) | CM (%) | He+CO (%) | CO+CO (%) |
|--------|---------|--------|--------|---------|--------|-----------|-----------|
| Model 1 | Triple  | 99.3 ± 1.7 | 0.7 ± 0.1 | 90.7 ± 20.06 | 9.3 ± 4.9 | 96.6 ± 1.7 | 3.4 ± 0.2 |
|        | Binary  | 99.4 ± 1.9 | 0.6 ± 0.1 | 93.7 ± 23.8 | 6.3 ± 4.5 | 99.3 ± 1.9 | 0.7 ± 0.1 |
| Model 2 | Triple  | 99.1 ± 1.7 | 0.9 ± 0.1 | 86.4 ± 19.1 | 13.6 ± 5.9 | 96.7 ± 1.7 | 3.3 ± 0.2 |
|        | Binary  | 99.3 ± 1.9 | 0.7 ± 0.1 | 90.0 ± 20.7 | 10.0 ± 5.2 | 99.3 ± 1.9 | 0.7 ± 0.1 |
| Model 3 | Triple  | 99.5 ± 4.3 | 0.5 ± 0.2 | 75.0 ± 57.3 | 25.0 ± 28.0 | 97.7 ± 4.2 | 2.3 ± 0.5 |
| Model 4 | Triple  | 100.0 ± 6.8 | 0.0 | 0.0 | 0.0 | 79.7 ± 5.8 | 20.3 ± 2.4 |
| Model 5 | Triple  | 99.7 ± 3.4 | 0.3 ± 0.1 | 80.0 ± 53.7 | 20.0 ± 21.9 | 97.4 ± 3.4 | 2.6 ± 0.4 |
| Model 6 | Triple  | 99.5 ± 3.4 | 0.5 ± 0.1 | 66.7 ± 43.03 | 33.3 ± 27.2 | 96.8 ± 3.4 | 3.2 ± 0.4 |

Note. The “Binary” channel here refers to the inner binaries of the triple population, evolved without the tertiary star. DD—Percentage of DD SNe Ia of the total number of SNe Ia. SD—Percentage of SD SNe Ia of the total number of SNe Ia. SCM—Percentage of sub-Chandrasekhar-mass SD SNe Ia of the total number of SD SNe Ia. CM—Percentage of Chandrasekhar-mass SD SNe Ia of the total number of SD SNe Ia. He+CO—Percentage of mergers/collisions of He and CO WDs of the total number of DD SNe Ia. CO+CO—Percentage of mergers/collisions of CO WDs and CO WDs of the total number of DD SNe Ia. Refer to Table B1 for the rate version of this table.

Figure 9. Distribution of the initial inner semimajor axes of all triple systems (blue solid lines) that explode as SNe Ia via circular mergers (black dashed lines) and eccentric collision (red filled histograms). The two models represent the results from the two $q_{\text{out}}$ models: model 1, decaying exponential fit to $q_{\text{out}}$ observations, and model 2, extrapolating Moe & Di Stefano (2017) $q$ distribution.
5. Discussion

5.1. The Effect of the Tertiary Star

In order to investigate the effect of the tertiary on the formation of SNe Ia, we investigated three different data sets: runs with the hierarchical triple population, runs with the inner binary of the triple population (after removing the tertiary star from the triple system), and runs with the isolated binary population. We find that the time-integrated SN Ia rates from the hierarchical triple population, inner binary of the triple population, and isolated binary population cases are $3.60 \pm 0.04 \times 10^{-4}$, $2.90 \pm 0.04 \times 10^{-4}$, and $(3.2 \pm 0.1) \times 10^{-4} M_{\odot}^{-1}$, respectively. This shows that the hierarchical triple population slightly yields the highest contribution to the SN Ia rate.

The tertiary star is contributing to the SN Ia in different ways. First, the stability configuration of the hierarchical triple population demands the inner binaries of the hierarchical triple population to be in tighter orbits than those of the isolated binary population. We can also see from Figure 14 that these tight inner binaries contribute the most to SN Ia explosions. Second, the tertiary star can assist in shrinking the inner binary orbits and bringing them closer to lead to interactions such as circular merger via tides, stable mass transfer, and/or CE, or an eccentric collision. One can see these two contributions of the tertiary in two different peaks in the filled yellow columns of Figure 14.

Hamers et al. (2013) carried out a similar study by restricting the initial conditions to systems with $a_1(1 - e_1^2) > 12$ au and estimated the SN Ia rates from triples to be on the order of $10^{-6} M_{\odot}^{-1}$. Our rates agree with Hamers et al. (2013) for systems with $a_1(1 - e_1^2) > 12$ au. The isolated binary population rates from our calculations are similar to those of Claeys et al. (2014). Putting our results in perspective, the time-integrated rates from the isolated binary and triple channels in our simulations are $(3.2 \pm 0.1) \times 10^{-4}$ and $(3.60 \pm 0.04) \times 10^{-4} M_{\odot}^{-1}$, respectively. The observed time-integrated rate from
Maoz et al. (2012) is \( \varepsilon M \). The combined rates from the triple and binary channels thus contribute to about \( \sim 52\% \) of the observed rate, of which the largest contribution comes, somewhat surprisingly, from triple systems. The discrepancy between the observed and theoretical rates demands the exploration of other SN Ia progenitors, though it should be noted that there are significant uncertainties in our models, which we address in the next section.

5.2. Uncertainties in the Models

Figure 7 shows the DTDs of DD SNe Ia with a solid blue line, dashed green line, and dotted red line representing the corresponding DTDs from model 1, model 2, and the isolated binary population, respectively. From Figure 7, it is evident that the total rate and DTD are not affected by the underlying \( q_{\text{out}} \) distribution. However, as shown by Figure 15, the CE efficiency parameter \( \alpha_{\text{CE}} \) does strongly affect the number of SNe Ia and hence the rates.

Maoz et al. (2012) is \((1.3 \pm 0.2) \times 10^{-3} M_\odot^{-1}\). The combined rates from the triple and binary channels thus contribute to about \( \sim 52\% \) of the observed rate, of which the largest contribution comes, somewhat surprisingly, from triple systems. The discrepancy between the observed and theoretical rates demands the exploration of other SN Ia progenitors, though it should be noted that there are significant uncertainties in our models, which we address in the next section.

Figure 13. Outer semimajor axis distribution for systems in which the inner binary explodes as an SN Ia.

Figure 14. Initial inner semimajor axis distribution of our triple population (model 1; solid blue line), isolated binary population (dashed green line), and systems that explode as SNe Ia from triple evolution channels (filled yellow columns).

5.2. Uncertainties in the Models

Figure 7 shows the DTDs of DD SNe Ia with a solid blue line, dashed green line, and dotted red line representing the corresponding DTDs from model 1, model 2, and the isolated binary population, respectively. From Figure 7, it is evident that the total rate and DTD are not affected by the underlying \( q_{\text{out}} \) distribution. However, as shown by Figure 15, the CE efficiency parameter \( \alpha_{\text{CE}} \) does strongly affect the number of SNe Ia and hence the rates. A higher-efficiency parameter \( \alpha_{\text{CE}} = 10 \) results in more early mergers and thereby fewer SNe Ia than a lower efficiency \( \alpha_{\text{CE}} = 1 \). Furthermore, a low-efficiency parameter \( \alpha_{\text{CE}} = 0.1 \) results in less transfer of orbital energy during CE evolution, thereby reducing the number of close binaries that could lead to SN Ia explosions. The effects of flybys and lower metallicity are negligible in producing SNe Ia. We have used multiplicity fractions from Moe & Di Stefano (2017) while normalizing the rates for every model. Uncertainties in the multiplicity fractions, as well as the precise values for the upper and lower masses of the synthesized population, propagate into errors in the mass normalization, which we have not considered here for simplicity. We further note as a caveat that all uncertainties that apply to single and binary star evolution (including the criteria for what exactly produces an SN Ia transient) also apply here.

5.3. Predominance of Circular Mergers

As described in Section 4.2, according to our results, SNe Ia attributed to circular mergers via CE are dominant compared to those following eccentric collisions. This is in strong contrast

Figure 15. Impact of different models on the SN Ia rate. See Table 1 for a description of the models.

Figure 16. Wall time for systems that explode as SNe Ia.
to Dong et al. (2015), who suggested that head-on WD collisions in isolated triples are the dominant channel for producing SNe Ia, and agrees with previous works (Hamers et al. 2013; Toonen et al. 2018) that found that the rate of head-on collisions in WD–WD systems is too low to explain the observed SN Ia rate.

About 6% of all evolved triple systems are not complete due to the set restricted wall time of 5 hr. Figure 16 shows the wall time distribution of systems that explode as SNe Ia. It is evident that the majority of the systems that explode as SNe Ia have a wall time within 1 hr; any contribution from systems with a longer wall time is negligible.

Even though the SN Ia rate from the triple evolution channel is found to contribute similarly to that of the binary evolution channel, the combined rate from triples and binaries is still inadequate to explain the complete observed rate. Thus, a detailed study of other possible SN Ia progenitors and the contribution of SNe Ia from higher-order systems should be done in the future.

6. Conclusions

We studied rates of SN Ia explosions in hierarchical triple systems by performing evolutionary population synthesis calculations. The triple populations were constructed following Kroupa (2001) and Moe & Di Stefano (2017), and we only considered initially dynamically stable systems using the stability criterion from Mardling & Aarseth (2001). The systems in which one or more of the stars are filling their Roche lobe at the beginning of the main sequence were ignored. Our sampled triples were evolved using the evolutionary population synthesis code MSE (Hamers et al. 2021) for a period of 10 Gyr, and a statistical analysis was carried out. The results are summarized as follows.

1. We found five unique formation channels to produce SNe Ia.
   (a) Unbound tertiary: a triple evolution channel in which the tertiary gets unbound when it collapses into a neutron star. The other reasons for the tertiary star to get unbound include dynamical instability and CE in the inner binary system.
   (b) Double merger: a triple system in which the inner binary components merge to form a new massive star, which then later interacts with the tertiary star to produce an SN Ia explosion.
   (c) TCE: a triple system in which the massive tertiary transfers mass on top of the inner binary, resulting in exchange, merger, or dynamical instability. The newly formed components then interact to produce an SN Ia explosion at a later time.
   (d) Eccentric collision: a triple system in which higher eccentricities are induced by secular evolution or dynamical instability even after circularising by CE. The components of the inner binary then collide to form an SN Ia.
   (e) Dynamical instability: a purely dynamical channel in which the triple system undergoes dynamical instability to explode as SNe Ia. Unlike, the isolated binary channel, this channel doesn’t involve a CE phase during the course of evolution.

2. Head-on eccentric collisions of WDs contribute only about 1% (model 1) of the total number of SNe Ia, while the rest are circular mergers that involve CE evolution.
3. When an SN Ia occurs in the inner binary, the third star is found to be bound in ~49% of systems. The mass distribution of the bound star peaks around $0.5 \, M_{\odot}$, and the outer orbital semimajor axis is distributed over a broad range of about few million au, peaking at about $10^3$ au.

We estimated the DTDs for SD and DD SNe Ia, which are presented in Figures 7 and 8, respectively. The time-integrated rate of SNe Ia from the triple evolution channel is found to be slightly higher than that of the binary evolution channel, although this conclusion is affected by uncertainties in the models.

5. Previously, Hamers et al. (2013) and Toonen et al. (2018) considered triples with only wide inner binaries and found a comparatively low contribution of triples to the SN Ia rate. However, when the complete set of parameters is included, it is evident that the triple channel is important in producing SN Ia explosions.

6. According to our models, the combined rate from the triple and binary evolution channels contributes about 52% of the observed SN Ia rate.

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Appendix A
Mobile Diagrams

We use mobile diagrams to describe the different evolutionary stages of triple star evolution in MSE. The blue and orange boxes represent the inner and outer orbits of the triple system, respectively. Colors indicate the evolutionary stage following the stellar types defined in Hurley et al. (2000). The legend explains the colors of the different stellar types, and the acronyms are described in Table A1. The red arrows pointing from one star to the other represent strong interactions such as stable mass transfer, unstable mass transfer, and collisions. The orange and red shaded regions around stars represent Roche lobe overflow and CE episodes, respectively. The star symbol in the final panel of every mobile diagram shows an SN Ia explosion. Every SN Ia explosion involves only two stars.

| Acronym       | Stellar Type                      |
|---------------|-----------------------------------|
| Low-mass MS   | Main-sequence star ($M \lesssim 0.7 \, M_{\odot}$) |
| MS            | Main-sequence star ($M \gtrsim 0.7 \, M_{\odot}$) |
| HG            | Hertzsprung gap                   |
| RGB           | Red giant branch                  |
| CheB          | Core He burning                   |
| EAGB          | Early asymptotic giant branch     |
| TPAGB         | Thermally pulsating asymptotic giant branch |
| HeMS          | He main sequence                  |
| HeHG          | Helium Hertzsprung gap           |
| HeGB          | Helium giant branch               |
| HeWD          | He WD                             |
| COWD          | CO WD                             |
| ONEWD         | One WD                            |
| NS            | Neutron star                      |
| BH            | Black hole                        |
Appendix B
Different Progenitors

Table 4 outlines the contributions of different progenitors to SNe Ia events. For readers interested in additional details, Table B1 presents rates from different progenitor channels.

| Models | Channel | DD \(10^{-3}M_\odot^{-1}\) | SD \(10^{-3}M_\odot^{-1}\) | SCM \(10^{-3}M_\odot^{-1}\) | CM \(10^{-3}M_\odot^{-1}\) | He+CO \(10^{-3}M_\odot^{-1}\) | CO+CO \(10^{-3}M_\odot^{-1}\) |
|--------|---------|----------------|----------------|----------------|----------------|----------------|----------------|
| Model 1 | Triple  | 3.57 ± 0.04 | 0.025 ± 0.003 | 0.024 ± 0.004 | 0.0009 ± 0.0007 | 3.45 ± 0.04 | 0.122 ± 0.008 |
| | Binary  | 2.88 ± 0.04 | 0.017 ± 0.003 | 0.017 ± 0.003 | 0.0001 ± 0.0002 | 2.86 ± 0.04 | 0.020 ± 0.003 |
| Model 2 | Triple  | 3.47 ± 0.04 | 0.031 ± 0.004 | 0.030 ± 0.004 | 0.0010 ± 0.0007 | 3.35 ± 0.04 | 0.114 ± 0.008 |
| | Binary  | 2.88 ± 0.04 | 0.020 ± 0.003 | 0.020 ± 0.003 | 0.0001 ± 0.0003 | 2.86 ± 0.04 | 0.020 ± 0.003 |
| Model 3 | Triple  | 2.39 ± 0.07 | 0.013 ± 0.005 | 0.013 ± 0.005 | 0.0003 ± 0.0008 | 2.33 ± 0.071 | 0.054 ± 0.011 |
| | Binary  | 0.90 ± 0.04 | 0.0 | 0.0 | 0.0 | 0.72 ± 0.04 | 0.183 ± 0.020 |
| Model 4 | Triple  | 3.69 ± 0.09 | 0.011 ± 0.005 | 0.010 ± 0.005 | 0.0003 ± 0.0008 | 3.59 ± 0.09 | 0.097 ± 0.014 |
| | Binary  | 3.58 ± 0.09 | 0.017 ± 0.006 | 0.017 ± 0.006 | 0.0006 ± 0.0011 | 3.47 ± 0.09 | 0.114 ± 0.016 |

Note. The “Binary” channel here refers to the inner binaries of the triple population, evolved without the tertiary star. DD—Rate of DD SNe Ia of the total number of SNe Ia. SD—Rate of SD SNe Ia of the total number of SNe Ia. SCM—Rate of sub-Chandrasekhar-mass SD SNe Ia. CM—Rate of Chandrasekhar-mass SD SNe Ia. He+CO—Rate of mergers/collisions of He and CO WDs. CO+CO—Rate of mergers/collisions of CO and CO WDs.
References

Antognini, J. M., Shappee, B. J., Thompson, T. A., & Amaro-Seoane, P. 2014, MNRAS, 439, 1079
Antonini, F., Murray, N., & Mikkola, S. 2014, ApJ, 781, 45
Bauer, E. B., Chandra, V., Shen, K. J., & Hermes, J. J. 2021, ApJL, 923, L34
Blaes, O., Lee, M. H., & Socrates, A. 2002, ApJ, 578, 775
Claeys, J. S. W., Pols, O. R., Izzard, R. G., Vink, J., & Verbunt, F. W. M. 2014, A&A, 563, A83
Dong, S., Katz, B., Kushnir, D., & Prieto, J. L. 2015, MNRAS, 454, L61
Eggleton, P. P. 1983, ApJ, 268, 368
Eggleton, P. P., Kiseleva, L. G., & Hut, P. 1998, ApJ, 509, 1012
Eggleton, P. P., & Kisseleva-Eggleton, L. 2006, Ap&SS, 304, 75
Fabrycky, D., & Tremaine, S. 2007, ApJ, 669, 1298
Glanz, H., & Perets, H. B. 2021, MNRAS, 500, 1921
Hallakoun, N., & Maoz, D. 2019, MNRAS, 490, 657
Hamers, A. S. 2018, MNRAS, 476, 4139
Hamers, A. S. 2020, MNRAS, 494, 5492
Hamers, A. S., & Dossopoulou, F. 2019, ApJ, 872, 119
Hamers, A. S., Glanz, H., & Neunteufel, P. 2022, ApJS, 259, 25
Hamers, A. S., Pols, O. R., Claeys, J. S. W., & Nelemans, G. 2013, MNRAS, 430, 2262
Hamers, A. S., & Portegies Zwart, S. F. 2016, MNRAS, 459, 2827
Hamers, A. S., Rantala, A., Neunteufel, P., Preece, H., & Vynatheya, P. 2021, MNRAS, 502, 4479
Hamers, A. S., & Thompson, T. A. 2019, ApJ, 882, 24
Hurley, J. R., Pols, O. R., & Tout, C. A. 2000, MNRAS, 315, 543
Hurley, J. R., Tout, C. A., & Pols, O. R. 2002, MNRAS, 329, 897
Iben, I. J., & Tutukov, A. V. 1984, ApJS, 54, 335
Kato, M., & Hachisu, I. 2004, ApJL, 613, L119
Katz, B., & Dong, S. 2012, arXiv:1211.4584
Kozai, Y. 1962, AJ, 67, 591
Kroupa, P. 2001, MNRAS, 322, 231

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Lidov, M. L. 1962, P&SS, 9, 719
Livio, M., & Mazzali, P. 2018, PhR, 736, 1
Luo, L., Katz, B., & Dong, S. 2016, MNRAS, 458, 3060
Maoz, D., & Graur, O. 2017, ApJ, 848, 25
Maoz, D., Mannucci, F., & Brandt, T. D. 2012, MNRAS, 426, 3282
Maoz, D., Mannucci, F., & Nelemans, G. 2014, ARA&A, 52, 107
Mardling, R. A., & Aarseth, S. J. 2001, MNRAS, 321, 398
Mazeh, T., & Shaham, J. 1979, A&A, 77, 145
Moe, M., & Di Stefano, R. 2017, ApJS, 230, 15
Maoz, S. 2016, ARA&A, 54, 441
Neunteufel, P., Yoon, S. C., & Langer, N. 1982a, ApJ, 257, 780
Neunteufel, P., Yoon, S. C., & Langer, N. 1982b, ApJ, 253, 798
Neunteufel, P., Yoon, S. C., & Langer, N. 2016, A&A, 589, A43
Neunteufel, P., Yoon, S. C., & Langer, N. 2017, A&A, 602, A55
Neunteufel, P., Yoon, S. C., & Langer, N. 2019, A&A, 627, A14
Nomoto, K. 1980, SSRv, 27, 563
Paczynski, B. 1976, in IAU Symp. 73, Structure and Evolution of Close Binary Systems, ed. P. Eggleton, S. Mitton, & J. Whelan (Dordrecht: Reidel), 75
Pakmor, R., Kromer, M., Röpke, F. K., et al. 2010, Natur, 463, 61
Pakmor, R., Kromer, M., Taubenberger, S., & Springel, V. 2013, ApJL, 770, L8
Piersanti, L., Tornambé, A., & Yungelson, L. R. 2014, MNRAS, 445, 3259
Pols, O. R., Schröder, K.-P., Hurley, J. R., Tout, C. A., & Eggleton, P. P. 1998, MNRAS, 298, 525
Raghavan, D., McAlister, H. A., Henry, T. J., et al. 2010, ApJS, 190, 1
Rantala, A., Pfahajoki, P., Mannerkoski, M., Johansson, P. H., & Naab, T. 2020, MNRAS, 492, 4131
Ruiter, A. J. 2020, IAU, 357, 1
Ruiter, A. J., Belczynski, K., Sim, S. A., et al. 2011, MNRAS, 417, 408
Thompson, T. A. 2011, ApJ, 741, 82
Tokovinin, A. 2018, ApJS, 235, 6
Toonen, S., Perets, H. B., & Hamers, A. S. 2018, A&A, 610, A22
Toonen, S., Portegies Zwart, S., Hamers, A. S., & Bandopadhyay, D. 2020, A&A, 640, A16
von Zeipel, H. 1910, AN, 183, 345
Wang, B., & Han, Z. 2012, NewAR, 56, 122
Webbink, R. F. 1984, ApJ, 277, 355
Whelan, J., & Iben, I., Jr. 1973, ApJ, 186, 1007
Woosley, S. E., & Kasen, D. 2011, ApJ, 734, 38
Yoon, S. C., & Langer, N. 2004a, A&A, 419, 623
Yoon, S. C., & Langer, N. 2004b, A&A, 419, 645
Yoon, S. C., & Langer, N. 2005, A&A, 435, 967