Light cone gauge formulation of IIB supergravity in $AdS_5 \times S^5$ background and AdS/CFT correspondence

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Abstract

Light cone gauge manifestly supersymmetric formulations of type IIB 10-dimensional supergravity in $AdS_5 \times S^5$ background and related boundary conformal field theory representations are developed. A precise correspondence between the bulk fields of IIB supergravity and the boundary operators is established. The formulations are given entirely in terms of light cone scalar superfields, allowing us to treat all component fields on an equal footing.

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Introduction. A long-term motivation for our investigation comes from the following potentially important application. Inspired by the conjectured duality between the string theory and $\mathcal{N} = 4$, 4d SYM theory [1] the Green-Schwarz formulation of strings propagating in $AdS_5 \times S^5$ was suggested in [3] (for further developments see [3]-[5]). Despite considerable efforts these strings have not yet been quantized (some related interesting discussions are in [3]). Alternative approaches can be found in [3]. As is well known, quantization of GS superstrings propagating in flat space is straightforward only in the light cone gauge. It is the light cone gauge that removes unphysical degrees of freedom explicitly and reduces the action to quadratic form in string coordinates. The light cone gauge in string theory implies the corresponding light cone formulation for target space fields. The string theories are approximated at low energies by supergravity theories. This suggests that we should first study a light cone gauge formulation of supergravity theory in AdS spacetime. Understanding a light cone description of type IIB supergravity in $AdS_5 \times S^5$ background might help to solve problems of strings in AdS spacetime.

Keeping in mind extremely important applications to string theory, in this paper we develop the light cone gauge formulation of IIB supergravity in $AdS_5 \times S^5$ and the associated boundary CFT and apply our results to the study of AdS/CFT correspondence at the level of state/operator matching. Our method is conceptually very close to the one used in [3] (see also [1]) to find the light cone form of IIB supergravity in flat space and is based essentially on a light cone gauge description of field dynamics developed recently in [10]. A discussion of IIB supergravity at the level of gauge invariant equations of motion and actions can be found in [11] and [12] respectively. As is well known in the case of the massless fields, investigation of AdS/CFT correspondence requires the analysis of some subtleties related to the fact that transformations of massless bulk fields are defined up to local gauge transformations. These complications are absent in the light cone formulation because here we deal only with the physical fields, and this allows us to demonstrate the AdS/CFT correspondence in a rather straightforward way.

Light cone form of $\text{psu}(2,2|4)$ superalgebra and notation. We use the following parametrization of $AdS_5 \times S^5$ space

$$ds^2 = \frac{1}{z^2}( -dt^2 + dx_1^2 + dx_2^2 + dz^2 + dx_4^2 ) + \frac{1}{4}dy_{ij}dy_{ij}^* , \quad z > 0 .$$

Here and below we set the radii of both $AdS_5$ and $S^5$ equal to unity. The boundary at spatial infinity corresponds to $z = 0$. The $S^5$ coordinates $y^{ij}$ are subject to the constraints

$$y^{ik}y_{kj} = \delta^i_j , \quad y_{ij} = \frac{1}{2}\epsilon_{ijkm}y^{kn} , \quad y_{ij} = -y_{ij} , \quad i,j,k,n = 1,2,3,4$$

where $\epsilon_{ijkn} = \pm 1$ is the Levi-Civita tensor of $su(4)$. The coordinates $y_{ij}$ are related to the standard $so(6)$ cartesian coordinates $y^M, M = 1,\ldots,6$, which satisfy the constraint $y^My^M = 1$ through the formula $y_{ij} = \rho_{ij}^M y^M$, where $\rho_{ij}^M$ are the Clebsh Gordan coefficients of $su(4)$ algebra [13]. We use the coordinates $y^{ij}$ instead of $y^M$ as this allows us to avoid using various cumbersome gamma matrix identities. To develop light cone formulation we introduce light cone variables $x^\pm \equiv (x^4 \pm x^0)/\sqrt{2}, x \equiv (x^1 + ix^2)/\sqrt{2}, \bar{x} \equiv x^* $, where $x^0 \equiv t$. In the following we treat $x^+$ as evolution parameter.

Now let us discuss the form of the algebra of isometry transformations of $AdS_5 \times S^5$ superspace, that is $\text{psu}(2,2|4)$, which we are going to use. The even part of this algebra is the algebra $so(4,2)$ which is the isometry algebra of $AdS_5$ and the algebra $so(6)$ which is the isometry algebra of $S^5$. The odd part of the algebra consists of 32 supercharges which are responsible for 32 Killing spinors in $AdS_5 \times S^5$ space. We prefer to use the form of $so(4,2)$ algebra provided by nomenclature of conformal algebra. In this notation we have, as usual, translations $P^a$, conformal boosts...
$K^a$, dilatation $D$ and Lorentz rotations $J^{ab}$ which satisfy the commutation relations

$$[D, P^a] = -P^a, \quad [D, K^a] = K^a, \quad [P^a, P^b] = (\eta^{ab} P^c - \eta^{ac} P^b), \quad [K^a, J^{bc}] = \eta^{ab} K^c - \eta^{ac} K^b,$$

$$[P^a, K^b] = \eta^{ab} D - J^{ab}, \quad [J^{ab}, J^{cd}] = \eta^{bc} J^{ad} + 3 \text{ terms}, \quad a, b, c, d = 0, 1, 2, 4$$

(1)

where $\eta^{ab} = (-, +, +, +)$. Throughout this paper instead of $so(6)$ algebra notation we prefer to use the notation of $su(4)$ algebra. Commutation relation of $su(4)$ algebra are

$$[J^i, J^j_n] = \delta^i_j J^j_n - \delta^j_i J^i_n.$$

In the light cone basis we have: $J^{±x}, J^{±y}, J^{±z}, J^{x²}, P^±, P^x, P^z, K^±, K^x, K^z$. We simplify notation as follows $P \equiv P^x, \hat{P} = P^z, K \equiv K^x, \hat{K} = K^z$. The light cone form of $so(4, 2)$ algebra commutation relations can be obtained from (1) with the light cone metric having the following non vanishing elements $\eta^{++} = \eta^{-+} = 1, \eta^{±x} = \eta^{±z} = 1$.

To describe the odd part of $psu(2, 2|4)$ superalgebra we introduce 32 supercharges $Q^{±i}, Q^i_±, S^{±i}, S^i_±$ which possess $D$, $J^{+±}$ and $J^{x²}$ charges. The commutation relations of supercharges with dilatation $D$

$$[D, Q^{±i}] = -\frac{1}{2} Q^{±i}, \quad [D, Q^i_±] = -\frac{1}{2} Q^i_±, \quad [D, S^{±i}] = \frac{1}{2} S^{±i}, \quad [D, S^i_±] = \frac{1}{2} S^i_±,$$

(2)
tell us that $Q$ are usual supercharges of Poincaré subsuperalgebra, while $S$ are conformal supercharges. The supercharges with superscript $+$ (−) have positive (negative) $J^{+±}$ charge

$$[J^{+±}, Q^{±i}] = \pm \frac{1}{2} Q^{±i}, \quad [J^{+±}, Q^i_±] = \pm \frac{1}{2} Q^i_±, \quad [J^{+±}, S^{±i}] = \pm \frac{1}{2} S^{±i}, \quad [J^{+±}, S^i_±] = \mp \frac{1}{2} S^i_±.$$

The $J^{x²}$ charges are fixed by the commutation relations

$$[J^{x²}, Q^{±i}] = \pm \frac{1}{2} Q^{±i}, \quad [J^{x²}, Q^i_±] = \pm \frac{1}{2} Q^i_±, \quad [J^{x²}, S^{±i}] = \pm \frac{1}{2} S^{±i}, \quad [J^{x²}, S^i_±] = \mp \frac{1}{2} S^i_±.$$

(3)

Transformation properties of supercharges with respect to $su(4)$ algebra are given by

$$[Q^i_±, J^j_k] = \delta^i_j Q^j_k - \frac{1}{4} \delta^j_k Q^i_±, \quad [Q^{±i}, J^j_k] = -\delta^j_i Q^{±j} + \frac{1}{4} \delta^j_k Q^{±i}$$

and the same for supercharges $S$. Anticommutation relations between supercharges are

$$\{Q^{±i}, Q^j_±\} = \pm P^{±δ^i} δ^j_±, \quad \{Q^i_±, Q^j_±\} = P δ^i_± \delta^j_±, \quad \{S^{±i}, S^j_±\} = K δ^i_± \delta^j_±,$$

$$\{Q^{+i}, S^j_+\} = -J^{x²} δ^i_±, \quad \{Q^{-i}, S^j_-\} = -J^{x²} δ^i_±,$$

$$\{Q^{±i}, S^j_±\} = \frac{1}{2} (J^{+±} + J^{x²} + D) δ^i_± \mp J^j_±.$$

Remaining commutation relations between supercharges and even part of superalgebra take the following form

$$[Q^{-i}, J^{+±}] = -Q^{+i}, \quad [S^j_+, J^{+±}] = -S^j_+, \quad [Q^{+i}, J^{−±}] = Q^{-i}, \quad [S^j_−, J^{−±}] = S^j_−,$$

$$[S^{±i}, P^±] = Q^i_±, \quad [S^i_±, P] = Q^i_±, \quad [S^{±i}, P] = -Q^i_±,$$

$$[Q^{±i}, K^±] = S^{±i}, \quad [Q^{+i}, K] = S^{−i}, \quad [Q^{−i}, \hat{K}] = S^{−i}.$$

The above generators are subject to the following hermitian conjugation conditions

$$P^{±} = P^±, \quad P = \bar{P}, \quad (K^±)^\dagger = K^±, \quad K^\dagger = \bar{K}, \quad (Q^{±i})^\dagger = Q^i_±, \quad (S^{±i})^\dagger = S^i_±,$$
describe two Kalb Ramond fields, the graviton and self dual 4 form field. The reality constraint in terms of the superfield $\Phi$ takes the form

$$\Phi = \frac{1}{4!} \epsilon^{ijkn} \lambda_i \lambda_j \lambda_k \lambda_n , \quad (\epsilon \lambda^3)^i = \frac{1}{3!} \epsilon^{ijkn} \lambda_j \lambda_k \lambda_n , \quad (\epsilon \lambda^2)^i j = \frac{1}{2!} \epsilon^{ijkn} \lambda_k \lambda_n ,$$

and the same notation is adapted for $\tau_i$. The field $\phi^{ijkn}$ satisfies the constraint

$$\phi^{ijkn} = \frac{1}{4} \epsilon^{ijkl} e^{knk_{1}n_{1} \phi^{ij \tau k_{1}n_{1} \tau_{1}}}.$$
which we refer to as dynamical generators. The kinematical generators have positive or zero \( J^+ \) charges, while dynamical generators have negative \( J^+ \) charges. For \( x^+ = 0 \) the kinematical generators are quadratic in the physical field \( \Phi \), while the dynamical generators receive corrections in interaction theory. In this paper we deal with free fields. At a quadratic level both kinematical and dynamical generators have the following representation in terms of the physical light cone superfield

\[
\hat{G} = \int dp^+ d^2p d^5d^4\tau p^+ \Phi(-p, z, y, -\lambda, -\tau)G\Phi(p, z, y, \lambda, \tau),
\]

where \( G \) are the differential operators acting on \( \Phi \). Thus we should find representation of \( psu(2,2|4) \) in terms of differential operators acting on light cone scalar superfield \( \Phi \). To simplify expressions let us write down the generators for \( x^+ = 0 \). The kinematical generators are then given by

\[
P = p, \quad \bar{P} = \bar{p}, \quad P^+ = p^+, \quad J^+ = \partial_p p^+, \quad J^{++} = \bar{\partial}_p p^+,
\]

\[
K^+ = \frac{1}{2}(z^2 - 2\partial_p \bar{\partial}_p) p^+, \quad K = K_0 + \frac{1}{2} \partial_p + \theta^i S^i_+,
\]

\[
Q^{+i} = p^+ \theta^i, \quad Q^+ = \lambda_i, \quad S^+_i = \frac{1}{\sqrt{2}} z\tau_i - \lambda_i \partial_p, \quad S^{++}_i = \frac{1}{\sqrt{2}} z p^+ \theta^i + p^+ \theta^i \partial_p,
\]

\[
J^{++} = \partial^- p^+ - \frac{1}{2} \theta \lambda - \frac{1}{2} \chi \tau + 2, \quad J^{x+} = p \partial^- p + \frac{1}{2} \theta \lambda - \frac{1}{2} \chi \tau,
\]

\[
D = -\partial^- p^+ - \partial_p \bar{p} - \bar{\partial}_p p + z\partial_z + \frac{1}{2} \theta \lambda + \frac{1}{2} \chi \tau - \frac{1}{2}, \quad J^{i+}_j = l^i_j + M^i_j,
\]

where we use the notation

\[
K_0 \equiv \frac{1}{2}(z^2 - 2\partial_p \bar{\partial}_p) p - \partial_p(-\partial_p p^+ - \partial_p \bar{p} - \bar{\partial}_p p + z\partial_z),
\]

\[
l^i_j = \frac{1}{2} y^{ik} \nabla_{kj}, \quad M^i_j \equiv \theta^i \lambda_j + \chi^i \tau_j - \frac{1}{4} \delta^i_j (\theta \lambda + \chi \tau), \quad \nabla_{ij} \equiv \rho^{MN}_{ij} (\delta^M N - y^M y^N) \partial_p N,
\]

\[
\partial^\pm_i \equiv \partial / \partial p^\pm, \quad \partial_p \equiv \partial / \partial p, \quad \bar{\partial}_p \equiv \partial / \partial \bar{p}, \quad \theta \lambda \equiv \theta^i \lambda_i, \quad \chi \tau \equiv \chi^i \tau_i.
\]

The orbital part of \( su(4) \) angular momentum \( l^i_j \) satisfies the following important relations

\[
l^i_{ml^j} = \frac{1}{4} l^m_i n^m o^j i + 2 l^i_j, \quad [l^i_j, l^k_n] = \delta^i_j l^k_n - \delta^k_n l^i_j,
\]

frequently used in this paper. Dynamical generators are given by

\[
P^- = -\frac{p\bar{p}}{p^+} + \frac{\partial^2}{2p^+} - \frac{1}{2z^2 p^+} A,
\]

\[
J^{-x} = -\partial^- p + \partial_p P^- + \theta^i Q^- - \frac{2p}{p^+}, \quad J^{-x} = -\partial^- \bar{p} + \bar{\partial}_p P^- - Q^- \lambda_i^{+}_j + (\theta \lambda + \chi \tau - 2) \bar{p} / p^+,
\]

\[
Q^{-i} = \bar{p} \theta^i + \frac{1}{\sqrt{2}} (\chi^i \partial_z + \frac{1}{2z} [\chi^i, A]), \quad Q^- = \frac{p}{p^+} \lambda_i - \frac{1}{\sqrt{2}p^+} (\tau_i \partial_z + \frac{1}{2z} [\tau_i, A]),
\]

where

\[
A \equiv X - \frac{1}{4}, \quad X \equiv l^2_j + 4\tau l \chi + (\chi \tau - 2)^2, \quad l^2_j \equiv l^i_j l^j_i, \quad \tau l \chi \equiv \tau l^i j \chi^i.
\]
For \( \lambda_i, \tau_i \) and \( \tau^i, \chi^i \) we adopt the following anticommutation and hermitean conjugation rules

\[
\{ \theta^i, \lambda_j \} = \delta^i_j, \quad \{ \chi^i, \tau_j \} = \delta^i_j, \quad \lambda^i = p^+ \theta^i, \quad \theta^{i\dagger} = \frac{1}{p^+} \lambda_i, \quad \tau^i = p^+ \chi^i, \quad \chi^{i\dagger} = \frac{1}{p^+} \tau_i.
\]

Remaining generators \( K, S^{-i}, S^{-i}, K^\dagger \) are obtainable from the above generators via commutation relations of \( psu(2, 2|4) \) superalgebra. Because these expressions are not illuminating we do not present them here.

Following [10] we shall call the operator \( A \) the AdS mass operator. Few comments are in order. (i) The operator \( A \) is equal to zero only for massless representations which can be realized as irreducible representations of conformal algebra [14], [10] which for the case of \( \text{AdS}_5 \) space is the \( \text{so}(5, 2) \) algebra. Below we shall demonstrate that operator \( X \) [13] has eigenvalues equal to squared integers in the whole spectrum of compactification of \( IIB \) supergravity on \( \text{AdS}_5 \).

This implies that operator \( A \) [15] is never equal to zero. From this we conclude that the scalar fields [15] as well as all remaining fields of compactification of \( IIB \) supergravity do not satisfy conformally invariant equations of motion. (ii) Generators involve the terms linear and quadratic in Grassmann variables \( \theta^i, \lambda_i \) and terms up to fourth power in \( \chi^i \) and \( \tau_i \). The coordinate \( \theta^i \) (or \( \lambda_i \)) constitutes odd part of light cone superspace appropriate to superfield description of light cone gauge \( N = 4, 4d \) SYM theory. The terms of the third and fourth powers in \( \chi^i \) and \( \tau_i \) are expressible in terms of the operator \( A \) and its commutators with \( \chi^i \) and \( \tau_i \).

Recall that the above representation was given for \( x^+ = 0 \). To study AdS/CFT correspondence we shall need the generators for arbitrary \( x^+ \equiv i \partial^+_p \) which are given by

\[
G_{x^+} = e^{-\partial^+_p P^-} G_{x^+ = 0} e^{\partial^+_p P^-}, \quad G_{x^+} = G_{x^+ = 0} - \partial^+_p [P^-, G_{x^+ = 0}] + \frac{1}{2} \partial^+_p [P^-, [P^-, G_{x^+ = 0}]].
\]

Using these relations we derive the complete expression for conform supergenerators

\[
S^+_i = S^+_i |_{x^+ = 0} + \partial^+_p Q^-_i, \quad S^{++i} = S^{++i} |_{x^+ = 0} - \partial^+_p Q^{-i}.
\]

Below we shall need the complete expression for the following generators

\[
J^{++} = \partial_p p^+ - \partial^+_p p, \quad J^{++} = \partial_p p^+ - \partial^+_p p, \quad J^{+-} = J^{+-} |_{x^+ = 0} - \partial^+_p P^-, \tag{16}
\]

\[
K^+ = \frac{1}{2} (z^2 - 2 \partial_p \partial_p) p^+ - \partial^+_p (-\partial^+_p P^+ - \partial_p \tilde{p} - \partial_p p + z \partial_z + \frac{3}{2}), \quad D = D |_{x^+ = 0} - \partial_p P^-. \tag{17}
\]

Making use of the expression for \( P^- \) [12] we can immediately write down the light cone gauge action

\[
S_{l.c.} = \int d^4 p dz dS^5 d^4 \lambda d^4 \tau p^+ \Phi(-p, z, y, -\lambda, -\tau)(-p^+ + P^-) \Phi(p, z, y, \lambda, \tau).
\]

Since the action is invariant with respect to the symmetries generated by \( psu(2, 2|4) \) superalgebra, the formalism we discuss is sometimes referred to as an off shell light cone formulation [9].

In what follows we shall exploit the following above mentioned important property of the operator \( X \) [13] – that its eigenvalues are squared integers. Let us demonstrate this important fact. First, we expand the scalar superfield [4] in \( S^5 \) coordinates \( y^j \) and Grassmann momentum \( \tau_i \)

\[
\Phi = \sum_{l=0}^{\infty} \Phi_l, \quad \Phi_l = \sum_{a=0}^{4} \Phi_{l,a}, \tag{18}
\]

where ‘spherical’ harmonic superfields \( \Phi_{l,a} \) satisfy the constraints

\[
l^2 \Phi_{l,a} = l(l+4) \Phi_{l,a}, \quad \tau \chi \Phi_{l,a} = a \Phi_{l,a}. \tag{19}
\]
The first constraint tells us that $\Phi_{l,a}$ is an eigenvalue vector of square of $su(4)$ orbital momentum $l^2 j$, while the second constraint tells us that $\Phi_{l,a}$ is a monomial of degree $a$ in $\tau_i$. Note that superfield $\Phi_0$ consists of fields of $N = 8$, $5d$ AdS supergravity, while the superfields $\Phi_{l>0}$ are responsible for Kaluza-Klein modes. Second, we evaluate eigenvalues of $X$ for each superfield $\Phi_{l,a}$ in turn. For the case of $\Phi_{l,0}$, $\Phi_{l,4}$ the eigenvalues of $X$ are easily found to be

$$X\Phi_{l,0} = (l+2)^2\Phi_{l,0}, \quad X\Phi_{l,4} = (l+2)^2\Phi_{l,4}. \quad (20)$$

In deriving the second relation one needs to use the relation like

$$\tau_i(\tau l)_j(\tau l)_k \Phi^{ijk}_l = -\frac{1}{12}l(l+4)\tau_i\tau_j\tau_k\Phi^{ijk}_l, \quad (\tau l)_i \equiv \tau_l^j i, \quad (21)$$

where $\Phi^{ijk}_l$ is totally antisymmetric in $i, j, k$. Next, we analyse the spectrum of $X$ in $\Phi_{l,1}$ and $\Phi_{l,3}$. In contrast to $(20)$ it turns out that these superfields themselves do not diagonalize the operator $X$. Now we decompose the superfields $\Phi_{l,1}$ and $\Phi_{l,3}$ as follows

$$\Phi_{l,1} = \Phi^{(1)}_{l,1} + \Phi^{(2)}_{l,1}, \quad \Phi_{l,3} = \Phi^{(1)}_{l,3} + \Phi^{(2)}_{l,3},$$

where

$$\Phi^{(1)}_{l,1} = (\tau_i - \frac{2}{l+4}(\tau l)_i)\Phi^{(1)}_{l,1}, \quad \Phi^{(2)}_{l,1} = (\tau_i + \frac{2}{l}(\tau l)_i)\Phi^{(1)}_{l,1}, \quad l > 0;$$

$$\Phi^{(1)}_{l,3} = (\tau_i\tau_j\tau_k - \frac{6}{l+4}\tau_i(\tau l)_k\Phi^{ijk}_{l,3}, \quad \Phi^{(2)}_{l,3} = (\tau_i\tau_j\tau_k + \frac{6}{l}\tau_i(\tau l)_k\Phi^{ijk}_{l,3}, \quad l > 0;$$

and superfields $\Phi^{(1)}_{l,1}, \Phi^{(2)}_{l,3}$ do not depend on $\tau_i$. The $\Phi^{ijk}_{l,3}$ is totally antisymmetric in $i, j, k$. Straightforward calculation gives the following eigenvalues of operator $X$

$$X\Phi^{(1)}_{l,1} = (l+1)^2\Phi^{(1)}_{l,1}, \quad X\Phi^{(2)}_{l,1} = (l+3)^2\Phi^{(2)}_{l,1}, \quad X\Phi^{(1)}_{l,3} = (l+1)^2\Phi^{(1)}_{l,3}, \quad X\Phi^{(2)}_{l,3} = (l+3)^2\Phi^{(2)}_{l,3}.$$

Finally, we consider the most challenging case of the superfield $\Phi_{l,2}$. It turns out that this superfield is decomposed into three superfields which are eigenvectors of operator $X$

$$\Phi_{l,2} = \Phi^{(1)}_{l,2} + \Phi^{(2)}_{l,2} + \Phi^{(3)}_{l,2},$$

$$\Phi^{(1)}_{l,2} = (\tau_i\tau_j - \frac{4(l+3)}{(l+2)(l+4)}\tau_i(\tau l)_j + \frac{4}{(l+2)(l+4)}(\tau l)_i(\tau l)_j)\Phi^{ij}_{l,2};$$

$$\Phi^{(2)}_{l,2} = (\tau_i\tau_j + \frac{8}{l(l+4)}\tau_i(\tau l)_j - \frac{4}{l(l+4)}(\tau l)_i(\tau l)_j)\Phi^{ij}_{l,2}, \quad l > 0;$$

$$\Phi^{(3)}_{l,2} = (\tau_i\tau_j + \frac{4(l+1)}{l(l+2)}\tau_i(\tau l)_j + \frac{4}{l(l+2)}(\tau l)_i(\tau l)_j)\Phi^{ij}_{l,2}, \quad l > 0;$$

where the superfield $\Phi^{ij}_{l,2}$ does not depend on $\tau_i$ and is totally antisymmetric in $i, j$. Relatively straightforward calculation gives the following eigenvalues of the operator $X$

$$X\Phi^{(1)}_{l,2} = l^2\Phi^{(1)}_{l,2}, \quad X\Phi^{(2)}_{l,2} = (l+2)^2\Phi^{(2)}_{l,2}, \quad X\Phi^{(3)}_{l,2} = (l+4)^2\Phi^{(3)}_{l,2}.$$

Thus we have demonstrated that the operator $X$ in whole space of superfield $\Phi$ take values which are squares of integers. This implies that the operator $\kappa \equiv \sqrt{X}$ is well defined and possesses integer eigenvalues which are chosen to be positive in what follows.
Light cone form of boundary CFT. The next primary goal of this work is to find a light cone gauge realization of \( \text{psu}(2, 2|4) \) superalgebra at the boundary of \( AdS_5 \times S^5 \) which is \( M^{3,1} \times S^5 \), where \( M^{3,1} \) is a (3+1) Minkowski space time (for a review of CFT see, for instance, [16]). At this boundary the superalgebra acts as the algebra of conformal transformations. Now we have to realize this superalgebra on the space of conformal operators. To this end we introduce boundary light cone superspace which is based on momentum variables \( p^\pm, \bar{p}, \bar{q} \), position \( S^5 \) coordinate \( y^{ij} \) and the Grassmann momentum variables \( \lambda_i, \tau_i \). On this light cone superspace we define a superfield \( \mathcal{O}_{loc} \) which is collection of CFT operators with canonical dimensions (currents) and superfield \( \bar{\mathcal{O}}_{loc} \) which is a collection of shadow operators (sources). These superfields have a similar expansion in \( \lambda_i \) and \( \tau_i \). To be definite, here and below we assume that \( q^2 = p^2 - \bar{q}^2 > 0 \).

\[
O = q^{-\kappa - \frac{1}{2}} \mathcal{O}_{loc}, \quad \bar{O} = q^{\kappa - \frac{1}{2}} \bar{\mathcal{O}}_{loc}, \quad q^2 = -2(p^+ p^- + p\bar{p})
\]  
(22)

then the representations of \( \text{psu}(2, 2|4) \) in \( \mathcal{O} \) and \( \bar{\mathcal{O}} \) coincide. We found the following realization of \( \text{psu}(2, 2|4) \) superalgebra in terms of differential operators acting on CFT superfields \( \mathcal{O}, \bar{\mathcal{O}} \)

\[
P^\pm = p^\pm, \quad P = p, \quad \bar{P} = \bar{p}, \quad J^{+x} = \partial_p p^+ - \partial^+ p, \quad J^{+\bar{x}} = \bar{\partial}_p p^+ - \partial^+ \bar{p},
\]  
(23)

\[
K^+ = K_0^+ - \frac{3}{2} \partial_p^+ + \frac{p^+}{2q^2} A,
\]  
(24)

\[
Q^{+i} = p^+ \theta^i, \quad Q^{-i} = \bar{p} \theta^i + \frac{i}{\sqrt{2}} q \chi^i, \quad Q_i^- = \frac{p}{p^+} \lambda_i - \frac{i}{\sqrt{2} p^+} q \tau_i,
\]  
(25)

\[
S^{+i} = \theta^i p^+ \partial_p + \frac{p^+}{2\sqrt{2} q} [\chi^i, A] - \partial_p^+ Q^{-i}, \quad S_i^+ = -\lambda_i \partial_p + \frac{i}{2\sqrt{2} q} [\tau_i, A] + \partial_p^+ Q_i^-,
\]  
(26)

where the \( AdS \) mass operator \( A \) is given in [13], the \( l^i_j \) and \( M^i_j \) are given in [11], while \( K_0^+ \) is given by

\[
K_0^+ = -\partial^\dagger \bar{p} p^+ - \partial^\dagger p^- (-\partial^\dagger p^+ - \partial^\dagger \bar{p} - \partial^\dagger p).
\]  
(28)

To be definite, here and below we assume that \( q^2 = p^2 - \bar{q}^2 > 0 \). Expressions for \( J^{-x}, J^{-\bar{x}} \) are obtainable from [13] by inserting there the expressions for \( P^- \) and \( Q^- \) given in [23], [24]. The remarkable property of realization we constructed is that the dependence on \( AdS \) mass operator \( A \) in CFT is ‘dual’ to \( AdS \) representations. Namely, on \( AdS \) side the \( A \) appears in \( P^- [13] \) having \( -1 \) \( D^- \) and \( J^{++} \) charges, while on CFT side this operator appears in \( K^+ [24] \) having opposite, i.e., \( +1 \) \( D^+ \) and \( J^{++} \) charges. The same ‘duality’ is the case of \( AdS \) \( Q^- \) generators [14] having \( -1/2 \) \( D^- \) and \( J^{++} \) charges and CFT \( S^+ \) generators [24] having opposite, i.e., \( +1/2 \) \( D^+ \) and \( J^{++} \)-charges. As before, the nonlinear dependence on Grassmann variables \( \chi^i, \tau_i \) is expressible through the operator \( A \). As was said already, the above representation is applicable to both \( \mathcal{O} \) and \( \bar{\mathcal{O}} \). The price for this is that the generators are no longer local with respect to the transverse momenta \( p \) and \( \bar{p} \) included in \( q \). However, these nonlocal terms cancel when we transform from \( \mathcal{O} \) and \( \bar{\mathcal{O}} \) basis into the one of \( \mathcal{O}_{loc} \) and \( \bar{\mathcal{O}}_{loc} \).

AdS/CFT correspondence. After we have derived the light cone formulation for both the bulk superfield \( \Phi \) and the boundary conformal theory operators collected in \( \mathcal{O} \) and \( \bar{\mathcal{O}} \) we are
satisfy the basic relation (30) gives the following nontrivial relation of case of.

It is straightforward to see that if these generators match then the remaining generators \( K^- \) and \( K, K', S^{\pm 1}, S^z \) shall match due to commutation relations of the \( psu(2,2|4) \) superalgebra. We start with a comparison of the kinematical generators (\( K_{ads} \)). As for the generators \( P^+, P, \bar{P}, J^{+-}, J^{++}, J^{+x}, J^{x}, Q^i, Q^{\pm i}, J^{\pm x}, D, J^i \).

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Now let us make a comparison of generators for bulk field \( \Phi \) and boundary operator \( O \). Important technical simplification is that it is sufficient to make comparison only for the part of the algebra spanned by generators

\[ P^\pm, P, \bar{P}, J^{+-}, J^{\pm x}, J^{xx}, K^+, Q^\pm_i, Q^{\pm i}, J^{\pm x}, D, J^i. \]

Here and below we use the notation \( G_{ads} \) and \( G_{cft} \) to indicate the realization of \( psu(2,2|4) \) algebra generators on the bulk field (7)-(10) and conformal operator (23)-(27) respectively.

Now let us consider solutions to light cone equations of motion which, as usual, take the form \( P^- \Phi = p^- \Phi \). Taking into account the expression for \( P^- \) and rewriting these equations as

\[
\left(-\partial_z^2 + \frac{1}{z^2}(\kappa^2 - \frac{1}{4})\right)\Phi = q^2 \Phi, \quad \kappa \equiv \sqrt{X},
\]

we immediately get the following normalizable and non-normalizable solutions

\[
\Phi_{norm} = \sqrt{q}J_{\kappa}(qz)i^\kappa O, \quad \Phi_{non-norm} = \sqrt{q}Y_{\kappa}(qz)i^\kappa \bar{O},
\]

where \( O, \bar{O} \) are scalar superfields not depending on \( z \). The \( J_{\kappa} \) and \( Y_{\kappa} \) are Bessel and Neumann functions respectively. The normalization factor \( i^\kappa \) is included for convenience. In (29) we use the notation \( O, \bar{O} \) since we are going to demonstrate that these are indeed the CFT superfields discussed above. Namely, we are going to prove that AdS transformations for \( \Phi \) lead to conformal theory transformations for \( O \) (and \( \bar{O} \))

\[
G_{ads} \Phi = Z_{\kappa}(qz)i^\kappa G_{cft}O, \quad Z_{\kappa}(z) \equiv \sqrt{q}J_{\kappa}(z).
\]

Now let us consider \( P^-_{ads} \) and \( P^-_{cft} \). Taking into account that solutions to equation of motion (29), by definition, satisfy the relation \( P^-_{ads} \Phi = p^- \Phi \) we get \( P^-_{ads} \Phi = Z_{\kappa}(qz)i^\kappa P^-_{cft}O \). So \( P^-_{ads} \) and \( P^-_{cft} \) also match. Taking this into account it is straightforward to see that the generators \( J^{+-}_{ads}, D_{ads} \) and \( J^{+-}_{cft}, D_{cft} \) satisfy the relation (30). Next we consider kinematical generators \( K^+_{ads} \) and \( K^+_{cft} \). Here we use the following relation

\[
K^+_{ads}Z_{\kappa}(qz) = Z_{\kappa}(qz)\left(K^+_{cft} + \frac{p_+}{2q^2} A - \partial_+ A \left(\frac{3}{2} + z \partial_z\right) + \frac{p_+}{q}\partial_+ Z_{\kappa}(qz)\partial_z\right),
\]

where \( K^+_{ads} \) and \( K^+_{cft} \) are given in (17) and (28) respectively. Using then (29), the above relation and the fact that \( O \) in (29) does not depend \( z \) we get immediately that \( K^+_{ads} \) and \( K^+_{cft} \) satisfy the relation (30).

The last step is to match the generators \( Q^i_{ads}, Q^i_{cft} \). Generalization of our discussion to the case of \( Q^- \) is straightforward. As is seen from (14) and (23) requiring that these supercharges satisfy the basic relation (30) gives the following nontrivial relation

\[
(\chi^i \partial_z + \frac{1}{2z}[\chi^i, X])\sqrt{z}J_{\kappa}(z)i^\kappa = \sqrt{z}J_{\kappa}(z)i^\kappa \chi^i, \quad (31)
\]
which is understood in weak sense, i.e., as an operator relation defined on the space of superfield $O$. This interesting relation is proved in Appendix. The same relation is valid in the case of the Neumann function. Taking into account matching $Q^{-i}, Q_{ij}$ and $P^{-}$ we conclude that $J^{-x}$, $J^{-x}$ \((\ref{7})\) also match. Above analysis is obviously generalized to the case of non-normalizable solutions and shadow operators \((\ref{26})\).

Thus we have demonstrated that boundary operators in $O, \tilde{O}$ in \((\ref{29})\) are indeed the conformal operators. Because of multiplicative factors $J_{\kappa}$ and $Y_{\kappa}$ involving powers of $q$ the operators $O$ and $\tilde{O}$, however, are not boundary values of solutions of the bulk equations of motion \((\ref{29})\). On other hand, by relations \((\ref{22})\) it is easily seen that it is $O_{\text{loc}} (\tilde{O}_{\text{loc}})$ that is the boundary value of $\Phi_{\text{norm}} (\Phi_{\text{non-norm}})$, i.e. for small $z$ one has the local interrelations

$$
\Phi_{\text{norm}} \sim z^{\kappa + \frac{1}{2}} O_{\text{loc}}, \quad \Phi_{\text{non-norm}} \sim z^{-\kappa + \frac{1}{2}} \tilde{O}_{\text{loc}}. \tag{32}
$$

Note that for $\kappa = 0$ the factor $z^{-\kappa}$ in the second relation of \((\ref{32})\) should be replaced by log $z$.

**Conclusions.** We have developed the light cone gauge formulation of IIB supergravity in $AdS_5 \times S^5$ background and applied this formulation to the study of AdS/CFT correspondence (for review see \((\ref{13})\)). Because the formulation is given entirely in terms of the light cone scalar superfield it allows us to treat all fields of IIB supergravity on equal footing and in a manifestly supersymmetric way. Comparison of this formalism with other approaches available in the literature leads us to the conclusion that this is a very efficient formalism indeed. The results presented here should have a number of interesting generalizations, some of which are:

(i) extension of light cone formulation of IIB supergravity in $AdS_5 \times S^5$ background to the level of cubic interaction vertices (see \((\ref{19})\)).

(ii) extension of light cone formulation of conformal field theory to the level of OPE’s and a study of light-cone form of AdS/CFT correspondence to the level of correlation functions.

(iii) generalization of light cone gauge formalism to the study of compactifications of 11-dimensional supergravity on $AdS_4 \times S^7$ and $AdS_7 \times S^4$ \((\ref{20})\). In these cases a formulation in terms of light-cone scalar superfields could also be developed.

In view of previous experiences with massless higher spin fields in $AdS_4$ space it is known that to construct self-consistent interactions for such fields it is necessary to introduce an infinite tower of massless fields \((\ref{21})\). For the case of $AdS_5$ space it is expected that fields of IIB supergravity constitute lower spin multiplet of the infinite tower of $N = 8$ supersymmetric massless higher spin fields theory in $AdS_5$ background. In this perspective, we think that the results of this paper can also get interesting applications to massless higher spin field theory. Note that $AdS_5 \times S^5$ is a unique space where the consistent string and massless higher spin field theories may ‘meet’.

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**Appendix.** In order to prove \((\ref{21})\) we start with

$$
\chi^i_t \equiv e^{tr_t l \chi^i} e^{-tr_t l \chi}, \quad \tau_{i t} \equiv e^{tr_t l \chi} \tau_t e^{-tr_t l \chi}, \quad j^i_{j t} \equiv e^{tr_t l \chi} j_{ij} e^{-tr_t l \chi}.
$$

Making use of the relations

$$
\partial_t \chi^i_t = -(l_t \chi^i_t), \quad \partial_t \tau_{i t} = (\tau_t l_t)_i, \quad \partial_t j^i_{j t} = (\tau_t l_t)_j \chi^i_t - \tau_j (l_t \chi_t)^i,
$$

where $(l \chi)_i^j \equiv l^i_j \chi^j$ (see also \((\ref{21})\)), we get a closed differential equation for $\chi^i_t$ which can be written in the following two equivalent forms

$$
\partial^2_t \chi^i_t = \left(\frac{1}{4} m^2 + \tau l \chi\right) \chi^i_t + (2 - \chi \tau)(\partial_t \chi_t)^i, \quad \partial^2_t \chi^i_t = \chi^i_t \left(\frac{1}{4} m^2 + \tau l \chi\right) + (\partial_t \chi_t)^i (2 - \chi \tau), \tag{33}
$$

10
where the initial conditions are \( \chi^i_{t=0} = \chi^i, \partial_t \chi^i|_{t=0} = -(l \chi)^i \). Solutions to these equations are

\[
\chi^i_t = e^{\frac{1}{2}(2-\chi^i)} \left( \cosh \frac{\kappa}{2} (\chi^i + \frac{\sinh \frac{\kappa}{2} t}{\kappa} ((\chi^i - 2) \chi^i - 2(l \chi)^i)) \right),
\]

\[
\chi^i_t = \left( \chi^i \cosh \frac{\kappa}{2} t + (\chi^i (\chi^i - 2) - 2(l \chi)^i) \frac{\sinh \frac{\kappa}{2} t}{\kappa} \right) e^{\frac{1}{2}(2-\chi^i)}.
\]

Equating these two different forms of the solution we get the following basic formula

\[
\chi^i \cosh \kappa t + (2(l \chi)^i + \chi^i(2 - \chi \tau)) \frac{\sinh \frac{\kappa}{2} t}{\kappa} = e^t \left( \cosh \kappa t \chi^i + \frac{\sinh \frac{\kappa}{2} t}{\kappa} (2(l \chi)^i + (2 - \chi^i) \chi^i) \right).
\]

Inserting \( t = i \varphi \) and taking real and imaginary parts gives

\[
\chi^i \cos \kappa \varphi = \cos \varphi \cos \kappa \varphi \chi^i + \frac{\sin \varphi \sin \frac{\kappa}{2} \varphi}{\kappa} ((\chi^i - 2) \chi^i - 2(l \chi)^i), \tag{34}
\]

\[
(2(l \chi)^i + \chi^i(2 - \chi \tau)) \frac{\cos \varphi \sin \frac{\kappa}{2} \varphi}{\kappa} = \cos \varphi \cos \kappa \varphi \chi^i + \sin \varphi \sin \kappa \varphi \chi^i. \tag{35}
\]

Now we use the integral representation for Bessel function

\[
J_\kappa(z) = \frac{i^{-\kappa}}{\pi} \int_0^\pi d\varphi e^{iz \cos \varphi \cos \kappa \varphi},
\]

which is valid for integer \( \kappa \). Taking into account the relation

\[
(\chi^i \partial_z + \frac{1}{2z} [\chi^i, X]) \sqrt{z} J_\kappa(z) = \sqrt{z} \left( \chi^i \partial_z + \frac{1}{z} (2(l \chi)^i + \chi^i(2 - \chi \tau)) \right) J_\kappa(z)
\]

and the formulas (34), (35) we get

\[
\chi^i \partial_z J_\kappa(z)^i = \frac{1}{\pi} \int_0^\pi d\varphi e^{iz \cos \varphi} i \cos \varphi \left( \cos \varphi \cos \kappa \varphi \chi^i + \frac{\sin \varphi \sin \frac{\kappa}{2} \varphi}{\kappa} ((\chi^i - 2) \chi^i - 2(l \chi)^i) \right),
\]

\[
\frac{1}{z} (2(l \chi)^i + \chi^i(2 - \chi \tau)) J_\kappa(z)^i = \frac{1}{\pi} \int_0^\pi d\varphi e^{iz \cos \varphi} i \cos \varphi \left( \sin \varphi \cos \kappa \varphi \chi^i + \frac{\cos \varphi \sin \frac{\kappa}{2} \varphi}{\kappa} ((2 - \chi^i) \chi^i + 2(l \chi)^i) \right).
\]

To derive the last formula we integrate by parts and since the operator \( \kappa \) takes integer values the boundary terms cancel. Summing these contributions we get the desired relation (31).
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