$D = 3$ $\mathcal{N} = 6$ superconformal symmetry of the AdS$_4 \times \mathbb{CP}^3$ superstring

D V Uvarov

NSC Kharkov Institute of Physics and Technology, 61108 Kharkov, Ukraine
E-mail: d_uvarov@hotmail.com and uvarov@kipt.kharkov.ua

Received 12 February 2011, in final form 20 September 2011
Published 16 November 2011
Online at stacks.iop.org/CQG/28/235010

Abstract

Invariance of the AdS$_4 \times \mathbb{CP}^3$ superstring under $D = 3$ $\mathcal{N} = 6$ superconformal symmetry is discussed in the sector described by the $OSp(4|6)/(SO(1,3) \times U(3))$ supercoset sigma-model action presented in the conformal basis for the $osp(4|6)/(so(1,3) \times u(3))$ Cartan forms. Transformation rules under $D = 3$ $\mathcal{N} = 6$ superconformal symmetry for the $(10|24)$-dimensional ‘reduced’ AdS$_4 \times \mathbb{CP}^3$ superspace coordinates are obtained and used to derive corresponding world-sheet currents.

PACS numbers: 11.25.−w, 11.30.Pb, 11.25.Tq

1. Introduction

The first explicit example of the gauge/string duality [1–3] allowed us to probe analytically the previously unaccessible nonperturbative regime of $\mathcal{N} = 4$ super-Yang–Mills theory via the IIB superstring on the AdS$_5 \times S^5$ background. It is in a sense the simplest instance of the AdS/CFT correspondence due to the maximal supersymmetry described by the $PSU(2,2|4)$ supergroup both of the AdS$_5 \times S^5$ superbackground and $D = 4$ supersymmetric conformal field theory (SCFT) on the boundary of AdS$_5$ space.

Another highly supersymmetric explicit example of the AdS/CFT correspondence that was put forward not long ago by Aharony, Bergman, Jafferis and Maldacena (ABJM) [4] provides dual description of the SCFT in the spacetime of one lower dimension in terms of $M$-theory on the AdS$_4 \times (S^7/Z_4)$ background. In spite of the fact that lower-dimensional theories basically have simpler dynamics compared to 4D ones, the ABJM correspondence appears to be difficult to verify since on both sides of the duality the isometry supergroup $OSp(4|6)$ isomorphic to $D = 3$ $\mathcal{N} = 6$ superconformal symmetry is lower than the maximally allowed one.

Difficulties manifest itself already at the level of constructing the classical action for the IIA superstring on the AdS$_4 \times \mathbb{CP}^3$ superbackground that provides dual description of the ’t Hooft limit of $D = 3$ SCFT proposed by ABJM [4]. Group-theoretic supercoset approach
[5–8] originally elaborated to describe the IIB superstring on the $\text{AdS}_4 \times S^5$ background when applied to the $\text{AdS}_4 \times \mathbb{C}P^3$ superstring gives the partial answer [9, 10] because only the subspace of $\text{AdS}_4 \times \mathbb{C}P^3$ superspace can be realized as the supercoset manifold $OSp(4|6)/(SO(1, 3) \times U(3))$. The $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model action [9, 10] corresponds to fixing half of the gauge freedom related to $\kappa$-symmetry of the complete action [12, 13] that can be obtained via the double-dimensional reduction [14] of the $D = 11$ supermembrane action on the maximally supersymmetric $\text{AdS}_4 \times S^5$ background [15] due to the Hopf fibration realization of the 7-sphere $S^7 = \mathbb{C}P^3 \times S^3$ [16, 17]. Although the $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset sigma-model fails to describe all possible $\text{AdS}_4 \times \mathbb{C}P^3$ superstring configurations [9, 12], it has clear group-theoretical structure and is classically integrable allowing one to utilize for its investigation many of the results obtained for the ‘elder brother’ example of $\text{AdS}_5/\text{CFT}_4$ correspondence relying on the integrable structure exhibited there [18, 19] (for collections of recent reviews see, e.g., [20, 21]).

In [22] we have found the explicit form of the $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset sigma-model action in the conformal basis for $osp(4|6)$ Cartan forms considering the supercoset representative parametrized by Poincaré coordinates for the $\text{AdS}_4$ space with 24 fermionic coordinates split into two sets of 12 related to Poincaré and conformal supersymmetries from the AdS boundary superspace perspective. Such a choice of the supercoset representative allows us to formulate the stringy side of the duality in terms of the variables that contain those parametrizing $D = 3, \mathcal{N} = 6$ boundary superspace, where the ABJM theory could be formulated off-shell [27–29] aiming at getting new insights into the relation between both theories.

In this paper, we establish transformation properties of the $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset coordinates introduced in [22] under $D = 3, \mathcal{N} = 6$ superconformal symmetry and find Noether currents associated with the $D = 3, \mathcal{N} = 6$ superconformal invariance of the $OSp(4|6)/(SO(1, 3) \times U(3))$ superstring action. This hopefully will allow us to further explore the integrable structure of the $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model, in particular higher multi-local currents and the issue of $T$-duality invariance. Previous studies of the $PSU(2, 2|4)/(SO(1, 4) \times SO(5))$ sigma-model describing the $\text{AdS}_5 \times S^5$ superstring showed [30, 31] that the Noether currents of $D = 4, \mathcal{N} = 4$ superconformal symmetry determine the form of higher conserved charges defined by the monodromy of the Lax connection [19]. Moreover, invariance property of the $PSU(2, 2|4)/(SO(1, 4) \times SO(5))$ sigma-model and associated Lax connection under the super-$T$-duality [32, 31] gives a possibility of relating Noether charges in the original formulation to the higher ones in the $T$-dual formulation and vice versa. On the gauge side of $\text{AdS}_3/\text{CFT}_3$ correspondence, this matches the existence of the dual superconformal symmetry [33] (and in fact infinite-dimensional Yangian symmetry [34]) of scattering amplitudes both in the strong coupling region via the scattering amplitudes/Wilson loops duality [35] and in the perturbation theory [36, 37]. The case of $\text{AdS}_3/\text{CFT}_3$ correspondence appears more subtle. In the perturbative regime of ABJM, there has been gained some non-trivial evidence [38] in favor of the dual $D = 3, \mathcal{N} = 6$ superconformal and Yangian symmetry of the scattering amplitudes, as well as of the scattering amplitudes/Wilson loops/correlators duality [39]. However, all attempts to implement super-$T$-duality on the level of $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model [40] and full $\text{AdS}_4 \times \mathbb{C}P^3$ superstring [13] give negative answer so far. In this respect let us note that our realization of

\footnote{An alternative way to construct the $\text{AdS}_4 \times \mathbb{C}P^3$ superstring using the pure spinor approach was followed in [11].}

\footnote{Previously such conformal-type parametrizations were used to examine the string/brane models involved into the higher-dimensional examples of $\text{AdS}/\text{CFT}$ correspondence [23–26].}

\footnote{More recently, another duality was suggested relating correlators of light-like separated local gauge-invariant operators and light-like polygonal Wilson loops in $\mathcal{N} = 4$ super-Yang–Mills theory [37].}
the $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model is well suited to address this issue as it relies on the conformal structure and uses Poincaré coordinates for AdS$_4$ space.

Also the Noether currents of $D = 3, \mathcal{N} = 6$ superconformal symmetry that we derive can be viewed as the zeroth order components of that for the full AdS$_4 \times \mathbb{CP}^3$ superstring when expanded in the Grassmann coordinates related to supersymmetries broken by the AdS$_4 \times \mathbb{CP}^3$ superbackground. It was shown in [41] that such Noether currents play a role in extending integrability beyond the $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model.

Previously Noether currents associated with the $su(2|2) \oplus u(1)$ subalgebra of the $osp(4|6)$ superalgebra that remains the manifest symmetry of the $OSp(4|6)/(SO(1, 3) \times U(3))$ superstring action upon fixing the light-cone gauge corresponding to the choice of light-like directions from both AdS$_4$ and $\mathbb{CP}^3$ parts of the background were examined in [43] in order to calculate the central extension of $su(2|2) \oplus u(1)$ arising in the decompactification limit on the world sheet and relaxing the level-matching condition. Such an extension was first observed for the AdS$_2$/CFT$_1$ correspondence both on the gauge [44] and string theory [45] sides.

Our results for variations of the $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset coordinates under $D = 3, \mathcal{N} = 6$ superconformal symmetry parallel those in [46, 47] for the $D = 4, \mathcal{N} = 4$ superconformal transformations of AdS$_5 \times S^5$ superspace coordinates relevant to the AdS$_5$/CFT$_4$ correspondence.

We start with reviewing the $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model in the conformal basis for Cartan forms, and then examine the action of left $D = 3, \mathcal{N} = 6$ superconformal transformations on the $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset element and proceed to the derivation of the transformation rules for the $osp(4|6)$ Cartan forms and Noether current densities for each of the individual transformations from $D = 3, \mathcal{N} = 6$ superconformal symmetry.

2. $OSp(4|6)/(SO(1, 3) \times U(3))$ superstring in conformal basis

The sigma-model action on the $(10|24)$-dimensional $OSp(4|6)/(SO(1, 3) \times U(3))$ superspace was found in [9, 10] following the general prescription [5–8] for constructing superfield-type actions on supercoset spaces that admit 4-element outer automorphism $\mathbb{Z}_4$ of the underlying isometry superalgebra. It relies on identifying Cartan forms associated with 10 bosonic and 24 fermionic generators of the $osp(4|6)/(so(1, 3) \times u(3))$ supercoset as the $(10|24)$-dimensional supervielbein components. The resulting action is invariant under global $OSp(4|6)$ supersymmetry, as well as gauge $SO(1, 3) \times U(3)$ and $\kappa$-symmetries, describes the requisite number of physical degrees of freedom, has correct bosonic limit and is classically integrable.

In [22], we have considered the $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset element

$$\mathcal{G} = e^{\varphi P_+ + \bar{\varphi} P_- + \bar{\theta}^a \theta_a + 2\eta^m_\mu + \bar{\eta}^m_\mu \bar{\eta}^m_\mu + \eta^m_\mu \bar{\eta}^m_\mu + \eta^m_\mu \eta^m_\mu} e^{\nu a l + \bar{\nu} a l} e^{D}$$

(2.1)

parametrized by $D = 3, \mathcal{N} = 6$ super-Poincaré coordinates $(x^m, \theta^a, \bar{\theta}^a)$, AdS$_4$ radial direction coordinate $\varphi$ related to the boundary-space dilatations, three complex coordinates $(z^a, \bar{z}^a)$ of the $\mathbb{CP}^3$ manifold and 12 fermionic coordinates $(\eta^m_\mu, \bar{\eta}^m_\mu)$ corresponding to $D = 3, \mathcal{N} = 6$ conformal supersymmetry. Here and in what follows, we adhere to the notations of [22], namely small Latin letters from the middle of the alphabet $k, l, m, n = 1, 2, 3$ label vectors of the $SO(1, 2)$ group of rotations in directions parallel to the boundary of AdS$_4$ space, while

---

4 Another option [42] is to take both light-like directions parallel to Minkowski boundary of anti-de Sitter space analogously to the AdS$_5 \times S^5$ superstring case [26]. This, however, requires consideration of the full AdS$_4 \times \mathbb{CP}^3$ superstring [12] as the $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model Lagrangian degenerates for such a configuration [9, 12].
that from the beginning of the alphabet \(a, b, c = 1, 2, 3\) label objects transforming in the (anti)fundamental representation of the \(SU(3)\) subgroup of the \(CP^3\) isometry group \(SU(4)\).

Small Greek letters \(\mu, \nu, \lambda, \gamma\) stand for 2-component spinor indices of \(Spin(1,2)\). Then, the associated current 1-form in the conformal basis reads

\[
\mathcal{C}(d) = \mathcal{F}^{-1}(d) \mathcal{F} = \hat{\omega}^m(d) P_m + \hat{\omega}^a(d) K_m + \Delta(d) D + G^{\alpha n}(d) M_{mn} + \Omega^a_\alpha(d) V^\alpha_\alpha + \Omega^a_\beta(d) V^\beta_\alpha + \Omega^a_\gamma(d) V^\gamma_\alpha + \hat{\omega}^\mu(d) Q^\mu + \hat{\omega}^\mu(d) \tilde{Q}^\mu + \hat{\omega}^\mu(d) S^{\alpha \mu} + \hat{\omega}^\mu(d) \tilde{S}^{\alpha \mu} \tag{2.2}
\]

or manifesting the \(\mathbb{Z}_4\)-grading

\[
\mathcal{C}(d) = \mathcal{C}_0(d) + \mathcal{C}_2(d) + \mathcal{C}_1(d) + \mathcal{C}_3(d), \tag{2.3}
\]

where

\[
\mathcal{C}_0(d) = \frac{1}{2} (\hat{\omega}^m(d) - \hat{\omega}^a(d)) (P_m - K_m) + G^{\alpha n}(d) M_{mn} + \Omega^a_\beta(d) V^\beta_\beta + \Omega^a_\gamma(d) V^\gamma_\beta,
\]

\[
\mathcal{C}_2(d) = \frac{1}{2} (\hat{\omega}^m(d) + \hat{\omega}^a(d)) (P_m + K_m) + \Delta(d) D + \Omega^a_\alpha(d) V^\alpha_\alpha + \Omega^a_\beta(d) V^\beta_\alpha + \Omega^a_\gamma(d) V^\gamma_\alpha,
\]

\[
\mathcal{C}_1(d) = \frac{1}{2} (\hat{\omega}^m(d) + i \hat{\omega}^a(d)) (Q^m + i S^m) + \frac{1}{2} (\hat{\omega}^\mu d^\mu d^\nu + \hat{\omega}^\mu d^\nu d^\mu) (\tilde{Q}^\mu + i \tilde{S}^\mu),
\]

\[
\mathcal{C}_3(d) = \frac{1}{2} (\hat{\omega}^m(d) - i \hat{\omega}^a(d)) (Q^m - i S^m) + \frac{1}{2} (\hat{\omega}^\mu d^\mu d^\nu + \hat{\omega}^\mu d^\nu d^\mu) (Q^\mu + i \tilde{S}^\mu).
\]

Then, the \(\mathbb{Z}_4\)-invariant \(OSp(4|6)/SO(1,3) \times U(3)\) superstring action in the conformal basis for Cartan forms (2.2) acquires the form

\[
S = -\frac{1}{2} \int d^2 \xi \sqrt{-g} \left[ \frac{1}{4} \left( \hat{\omega}^m \hat{\omega}^m + \hat{\omega}^a \hat{\omega}^a \right) - \frac{1}{2} \left( \Omega^a_\alpha \Omega^a_\alpha + \Omega^a_\beta \Omega^a_\beta \right) \right] + S_{WZ} \tag{2.5}
\]

with the Wess–Zumino action given by

\[
S_{WZ} = -\frac{1}{4} \int d^2 \xi \left[ \left( \hat{\omega}^m \hat{\omega}^m + \hat{\omega}^a \hat{\omega}^a \right) - \frac{1}{2} \left( \Omega^a_\alpha \Omega^a_\alpha + \Omega^a_\beta \Omega^a_\beta \right) \right] = -\frac{1}{4} \int d^2 \xi \left( \hat{\omega}^m \hat{\omega}^m + \hat{\omega}^a \hat{\omega}^a \right).
\]

The first two summands in the kinetic part of action (2.5) include Cartan forms associated with the generators \(P_m\) of the spacetime translations on the \(D = 3\) Minkowski boundary of \(AdS_4\) space:

\[
\hat{\omega}^m(d) = e^{-2\nu} \omega^m(d), \quad \hat{\omega}^a(d) = dx^a - i d\theta^\mu d^\alpha \omega^m d^\alpha + i \theta^\alpha d^\beta \omega^m d^\beta, \tag{2.7}
\]

conformal boost generators \(K_m\)

\[
\hat{\omega}^m(d) = e^{2\nu} \omega^m(d), \quad \hat{\omega}^a(d) = -i d\mu d^\alpha \omega^m d^\alpha + i \eta^\alpha d^\mu d^\alpha
\]

\[
+ 2(\tilde{\eta}) \left( \eta^\alpha d^\mu d^\alpha + \left( \hat{\omega}^m(d) \right) - \left( d\theta^\mu + \frac{1}{2} \tilde{\xi}^\mu(d) \right) \right) \eta^\alpha, \quad \tilde{\eta} = \tilde{\eta}^\alpha \eta^\alpha, \tag{2.8}
\]

where

\[
\hat{\xi}^\mu(d) = -\tilde{\sigma}^\mu d^\alpha \omega^m(d) \eta^\alpha = -\hat{\omega}^\mu(d) \eta^\alpha, \quad \hat{\xi}^{\mu \alpha}(d) = -\hat{\sigma}^\mu d^\alpha \omega^m(d) \eta^\alpha = -\hat{\omega}^{\mu \alpha}(d) \eta^\alpha, \tag{2.9}
\]

and the generator of dilatations \(D\)

\[
\Delta(d) = d\varphi + i(d\theta^\mu \tilde{\eta}_\mu + d\tilde{\theta}^{\mu \alpha} \eta^\alpha). \tag{2.10}
\]
Note that the generators \((D, P_\mu + K_m)\) can be identified as the \(so(2, 3)/so(1, 3)\) coset generators\(^5\) and corresponding Cartan forms represent the AdS part of the \((10|24)\) supervielbein bosonic components.

Supervielbein components in the directions tangent to the \(\mathbb{CP}^3\) manifold are identified with the \(su(4)/u(3)\) Cartan forms \((\Omega_a^\mu(d), \Omega_a^4(d))\) that are the off-diagonal components of the traceless Hermitian matrix of \(su(4)\) Cartan forms

\[
\Omega_a^\mu(d) = \begin{pmatrix}
\Omega_a^b & \Omega_a^4 \\
\Omega_a^d & \Omega_a^4
\end{pmatrix}, \quad \Omega_a^4 = -\Omega_a^u. \tag{2.11}
\]

Using the isomorphism \(SU(4) \sim SO(6)\), it is possible to accommodate \(su(4)\) Cartan forms into the \(6 \times 6\) matrix

\[
\Omega_a^\mu(b) = \begin{pmatrix}
\Omega_a^b & -\delta_a^b \Omega_a^c \\
-\delta_a^b \Omega_a^c & -\Omega_b^a + \delta_b^a \Omega_c^c
\end{pmatrix} \tag{2.12}
\]

antisymmetric w.r.t. the metric

\[
H_{ab} = \begin{pmatrix}
0 & \delta_a^b \\
\delta_a^b & 0
\end{pmatrix} \tag{2.13}
\]

following the decomposition of the \(D = 6\) vector representation as \(3 \oplus \bar{3}\) of \(SU(3)\).\(^6\) For the considered choice of the \(OSp(4|6)/(SO(1, 3) \times U(1))\) supercoset element \(su(4)\) Cartan forms are given by the sum of two contributions

\[
\Omega_a^\mu(b) = \Omega_{a \mu}(b) + \Omega_{a \mu}(d) \tag{2.14}
\]

coming from bosons and fermions. The bosonic contribution

\[
\Omega_{a \mu}(d) = i T_a^\mu d \tilde{T}_a^\mu \tag{2.15}
\]

is described by the \(su(4)\) Cartan form matrix associated with the \(SU(4)/U(3)\) coset element

\[
T_a^\mu = \begin{pmatrix}
T_a^\nu & T_a^\sigma \\
T_a^\sigma & T_a^\nu
\end{pmatrix} = \exp \left( \begin{pmatrix}
0 & i \epsilon_{a b c} \\
-i \epsilon_{a b c} & 0
\end{pmatrix} \right). \tag{2.16}
\]

Explicit expressions for the purely bosonic part of \(su(4)\) Cartan forms can be found in [22]. The fermionic contribution to (2.14)

\[
\Omega_{a \nu}(d) = (T \Psi(d) \tilde{T})_{a \nu} \tag{2.17}
\]

is obtained by the \(T\)-transformation of the matrix

\[
\Psi_a^\nu(d) = 2 (\theta_a^\mu \eta_a^\nu - d \theta_a^\mu \eta_a^\nu - \eta_a^\mu \tilde{\theta}_a^\nu (d) \eta_a^\mu), \tag{2.18}
\]

where the Grassmann coordinates have been written as \(D = 6\) vectors in the \(3 \oplus \bar{3}\) basis:

\[
\theta_a^\mu = \begin{pmatrix}
\theta_a^\mu \\
\bar{\theta}_a^\mu
\end{pmatrix}, \quad \eta_a^\mu = \begin{pmatrix}
\eta_a^\mu \\
\bar{\eta}_a^\mu
\end{pmatrix} \tag{2.19}
\]

\(^5\) (Anti)commutation relations of the \(D = 3\) \(N = 6\) superconformal algebra can be found in [22].

\(^6\) The metric \(H_{d\bar{a}}\) is the conventional unit \(D = 6\) metric \(\delta_{d\bar{a}}\) written in the \(3 \oplus \bar{3}\) basis. Both bases are connected by the transformation matrices

\[
M^{d\bar{a}} = \frac{1}{2} (\tilde{\gamma}^{d\bar{a}}, \gamma^{d\bar{a}}), \quad M^{-1} = (\frac{1}{\sqrt{2}} \tilde{\gamma}^{d\bar{a}}), \quad MM^{-1} = I,
\]

where \(\gamma^{d\bar{a}}\) and \(\tilde{\gamma}^{d\bar{a}}\) are \(D = 6\) chiral \(\gamma\)-matrices, such that the components of a \(D = 6\) vector \(O^\mu\) in these bases can be transformed into one another: \(O^\mu = M^{d\bar{a}} O_{d\bar{a}}\) and \(O_{d\bar{a}} = M^{-1} O^\mu\). In particular, for the \(D = 6\) metric, we find that \(\delta_{d\bar{a}} = -2M^{d\bar{a}}H_{d\bar{a}}M_{d\bar{a}}\).
and $a^{\mu} = H^a b^b$, $b^a = H^b b^b$. The $T$-transformed 3 $\bar{3}$ vectors will be endowed with hats: $\hat{\theta}_i^a = T_i^b \hat{\theta}_b^a$, $\hat{\phi}_j^a = H^a \hat{\phi}_j^a$, etc. Using that $H^a (T^f_j H^b \hat{\theta}_j^a = T^f_a \hat{\theta}_a^b$ for the chosen realization of the matrix $T$, the fermionic part of the $su(4)$ Cartan form matrix can be cast into the form
\[
\Omega_4^{(4)}(d) = 2 (\hat{a} \theta_i^a \hat{\tilde{\eta}}_i^a - \hat{\theta}_i^a \hat{\tilde{\eta}}_i^a - \hat{\eta}_i^a \hat{\tilde{G}}_{iij}(d) \hat{\tilde{\eta}}_j^i). 
\]
(2.20)

The WZ term of action (2.5) in the 3 $\bar{3}$ basis can be written in the form
\[
S_{WZ} = -i \frac{1}{8} \xi \tilde{a}^b \int d^2 \xi (\hat{\omega}_i^a \epsilon_{ij} \hat{\omega}_j^i + \hat{\phi}_i^a \epsilon_{ij} \hat{\omega}_j^i - \hat{\eta} a \hat{\tilde{G}}_{iij}(d) \hat{\tilde{\eta}}_j^i). 
\]
(2.21)

It contains the world-sheet projections of fermionic 1-forms
\[
\hat{\omega}_i^a (d) = e^{-\bar{v}} T_i^b \omega_b^a (d), \quad \hat{\omega}_i^a (d) = d \theta_i^a + \xi_i^a (d) 
\]
related to Poincaré supersymmetry generators ($Q_{a}^{\mu}$, $\hat{Q}_{a\mu}$), and
\[
\hat{\phi}_i^a (d) = e^\xi T_i^a \chi^a (d), \quad \chi^a (d) = d \eta a + 2i n_{i} a \eta \eta - i (\tilde{\eta} \eta) a \omega_{i} a (d) 
\]
related to conformal supersymmetry generators ($S_{a}^{\mu}$, $\hat{S}_{a}^{\mu}$).

3. $D = 3$ $\mathcal{N} = 6$ superconformal symmetry of the $OSp(4|6)/SO(1,3) \times U(3)$ superstring: general properties and coordinate transformations

Global $OSp(4|6)$ transformations act on the $OSp(4|6)/SO(1,3) \times U(3)$ coset representative from which the left-invariant Cartan forms (2.2) are constructed in the following way:\:
\[
\mathcal{G} \mathcal{H} = G \mathcal{H}, \quad G \in OSp(4|6), 
\]
(3.1)

with $H$ being the compensating $SO(1,3) \times U(3)$ transformation or passing to infinitesimal parameters
\[
\delta \mathcal{G} = g \mathcal{H} - \mathcal{G} h, \quad g \in oSp(4|6), \quad h \in so(1,3) \oplus u(3). 
\]
(3.2)

Substituting the above relation into (2.2) yields
\[
\mathcal{G} (\delta) = \mathcal{G}^{-1} \delta \mathcal{G} = \mathcal{G}^{-1} g \mathcal{H} - h. 
\]
(3.3)

Consider the $D = 3$ $\mathcal{N} = 6$ superconformal algebra-valued transformation parameter
\[
g = a^{\mu} P_{m} + b_{m} K_{m} + f D + e^{\lambda} M_{m} a^{\mu} + y^{\mu} V_{a} a^{\mu} + \tilde{y}_{a} V_{a} a^{\mu} + w_{a} V_{a} a^{\mu} + w_{a}^{4} V_{a}^{4} + e_4^{\mu} Q_{a}^{\mu} + e_4^{+\mu} \hat{Q}_{a\mu} + e_4^{\mu} S_{a}^{\mu} + e_4^{+\mu} \hat{S}_{a}^{\mu}. 
\]
(3.4)

It includes the parameters of $D = 3$ Minkowski spacetime translations $a^{m}$, conformal boosts $b_{m}$, dilatations $f$ and Lorentz rotations $l^{m}$, as well as anticommuting parameters

\footnote{In the conventional $D = 6$ vector basis, it is given by the expression $\mu^{f} = \frac{1}{4} (\rho_{\mu\nu}^{f} \rho_{\nu\mu}^{f} - \rho_{\nu\mu}^{f} \rho_{\mu\nu}^{f})$. It takes a simple diagonal form when contracted with the 6D rotation generators
\[
J_{\mu}^{\nu} = \frac{1}{4} (\rho_{\mu\nu}^{f} \rho_{\nu\mu}^{f} - \rho_{\nu\mu}^{f} \rho_{\mu\nu}^{f}).
\]

The matrix $J_{\mu}^{\nu}$ can be shown to satisfy the following equation: $J_{\mu}^{c} J_{c\nu} = 4 J_{\mu}^{\nu} - 12 \delta_{\mu}^{\nu} = 0$.}

\footnote{Since the supercoest string action is built out of the Cartan forms, it is exactly invariant under the global symmetry in distinction with the original Green–Schwarz action [48] that is quasi-invariant because its WZ term cannot be presented as a 2-form in supercurrents. For a detailed discussion on that point and the properties of the WZ term on AdS backgrounds, see, e.g., [49].}
of $D = 3 \mathcal{N} = 6$ Poincaré supersymmetry ($e^a_\mu$, $\tilde{e}^{\mu\nu}$) and conformal supersymmetry ($\xi_{\mu\nu}$, $\tilde{\xi}^{\mu
u}$) supplemented by the $SU(4) R$-symmetry parameters ($w^a_\mu$, $y^a_\nu$, $\bar{y}^a_\nu$). Then, the substitution of the $OSp(4|6)/(SO(1,3) \times U(3))$ coset representative (2.1) into (3.3) yields

$$C(\delta) = (\tilde{\omega}^m(\delta) - \hat{b}^m)P_m + (\tilde{\omega}^m(\delta) + \hat{b}^m)K_m + \Delta(\delta)D + (G^{mn}(\delta) + \frac{1}{2} \tilde{\omega}^m)M_{mn} + \Omega^a(\delta)\bar{v}_a^c + \Omega^a(\delta)\bar{v}_a^c + (\Omega_{2}^b(\delta) + \hat{u}_a^b)\bar{v}_a^c + (\tilde{\Omega}_{4}^4(\delta) + \hat{u}_a^4)\bar{v}_a^c + \hat{\omega}_{\mu}^m(\delta)\tilde{Q}_{\mu a} + \hat{\omega}_{\mu}^m(\delta)\tilde{S}_{\mu a} + \hat{\omega}_{\mu}^m(\delta)\tilde{S}_{\mu a}.$$  

(3.5)

The quantities that cannot be accommodated into the individual Cartan form variations like, e.g., $\tilde{\omega}^m(\delta) = i_\delta \tilde{\omega}^m(\delta)$ represent parameters of the compensating transformations. In particular, the vector

$$\hat{b}^m = e^{\nu a} A^{-1} b^m(\theta), \quad A = 1 - e^{\nu b}(\tilde{\eta}_\nu)^2,$$

(3.6)

where

$$b^m(\theta) = b^m - i[(\xi_{\mu\nu}(\theta)\tilde{\eta}^{m\nu}) + (\tilde{\xi}^{\mu\nu}(\theta)\tilde{\eta}^{m\nu})], \quad \xi_{\mu\nu}(\theta) = \xi_{\mu\nu} + b_{\nu\eta}Q^\nu_a,$$

(3.7)

is the parameter of $SO(1,3)/SO(1,2)$ transformations, while the antisymmetric tensor

$$\hat{l}_{mn} = l_{mn}(\theta) + i e^{2\nu}[\eta_{\nu\lambda}(\tilde{b}^{m\lambda} + \tilde{b}^{n\lambda} - \tilde{b}^{m\nu} - \tilde{b}^{n\nu})],$$

(3.8)

with

$$l_{mn}(\theta) = l_{mn} + 2(b^{m\epsilon} b^{\nu} - b^{n\epsilon} b^{\nu}) + i[(\theta_{\nu\lambda} b^{m\lambda} - \tilde{b}^{m\lambda} - \tilde{b}^{m\nu}) + i[(\theta_{\nu\lambda} b^{m\lambda} - \tilde{b}^{m\lambda} - \tilde{b}^{m\nu})]$$

(3.9)

describes compensating $SO(1,2)$ Lorentz rotations. The parameters of compensating $U(3)$ rotations

$$\hat{u}_a^b = \bar{u}_a^b + i[(1 - \cos |z|) (\tilde{z}_a^b - \tilde{z}_a^b) + \frac{1}{2} |z| (1 - \cos |z|) (\tilde{z}_a^b - \tilde{z}_a^b)],$$

(3.10)

have been presented in the form exhibiting explicit dependence on the entries

$$\bar{u}_a^b = u_a^b(\theta) - e^{2\nu_2}[2(\eta_{\nu\lambda}(\tilde{b}^{b\lambda} - \tilde{b}^{b\nu})],$$

(3.11)

$$\gamma^m = \gamma^m(\theta) + e^{2\nu} \varepsilon_{abc}(\eta_{\nu}(\tilde{b}^{b\nu} - \tilde{b}^{b\nu}),$$

where

$$u_a^b(\theta) = u_a^b - 2(\xi_{\nu\lambda}(\tilde{b}^{b\nu} - \tilde{b}^{b\nu}) + \theta_{\nu\lambda}^b(\tilde{\xi}^{b\nu} - \tilde{\xi}^{b\nu}) - 2(\theta_{\nu\lambda} b^{b\nu} + \tilde{\xi}^{b\nu} - \tilde{\xi}^{b\nu})$$

(3.12)

of the $SU(4)$ matrix

$$\tilde{W}_a^b = \left( \begin{array}{cc} \tilde{u}_a^b & -e^{2\nu_2}(\tilde{b}^{b\nu}) \\ -e^{2\nu_2}(\tilde{b}^{b\nu}) & \tilde{u}_a^b \end{array} \right)$$

(3.13)

that, as will be shown below, enters the transformation laws (3.18) of the $SU(4)/U(3)$ coset element (2.16) under the $D = 3 \mathcal{N} = 6$ superconformal symmetry.

$$D = 3 \mathcal{N} = 6$$ superconformal transformations of the $OSp(4|6)/(SO(1,3) \times U(3))$ superspace coordinates that parametrize (2.1) include contributions proportional to the parameters of compensating transformations (3.6), (3.8) and (3.10). In particular, $D = 3 \mathcal{N} = 6$ super-Poincaré coordinates obey the following transformation rules:

$$\delta x^m = a^m + l_{ab}^{m\nu} x^\nu + 2 f x^m + b^m(\tilde{\phi}^m) - 2 a^m b^{\nu} x^\nu$$

$$- i[(\xi_{\mu\nu}(\tilde{b}^{m\nu}) + (\tilde{\xi}^{m\nu}(\tilde{b}^{m\nu})) - i[(\xi_{\mu\nu}(\tilde{b}^{m\nu}) + (\tilde{\xi}^{m\nu}(\tilde{b}^{m\nu}))$$

(3.14)
\[ \delta \theta^a = \epsilon^a + \frac{1}{4} \varepsilon^{a\mu} \delta \sigma_{\mu
u} + f \theta^a + i \omega^b \theta^a - i \omega^b \theta^a - i \omega^b \theta^a + \hat{\theta}^a \]

\[ + \frac{1}{2} \xi^a \xi_{\alpha} - 2 \theta^a (\hat{\theta}_b + \tilde{\theta}^{ab} \theta^b) \eta_{\alpha} + \frac{i}{2} \tilde{\theta}^a \eta_{\alpha} \]

(3.15)

correspondingly for the variation of the SO(1,3)/SO(1,2) rotation parameter \( \hat{b}^m \) in the boundary limit \( \varphi \to -\infty \) so that these coordinates form a closed set under \( D = 3 \). \( \mathcal{N} = 6 \) superconformal symmetry. This allows us to identify them as the boundary superspace coordinates of \( OSp(4|6)/(SO(1,3) \times U(3)) \). Correspondingly for the variation of the \( \varphi \) coordinate related to the \( \text{AdS}_4 \) space bulk direction, we get

\[ \delta \varphi = f(\theta) = f - b_{m\alpha} x^m + i (\xi_{ab} \hat{g}^{ab} + \bar{\xi}_a \theta_a). \]

(3.16)

Transformation properties of the \( CP^3 \) complex coordinates

\[ \delta z^a = i \omega^b \bar{w}^a + i \bar{w}^b \omega^a + \frac{1}{2} \frac{1}{z} \frac{1}{|z|} \left( 1 - \frac{\cos |z| \sin |z|}{|z|^2} \right) (\bar{z} \tilde{z}) \]

\[ + \frac{1}{2 |z|^2} |1 + |z| (\tan |z| - \cot |z|)| (\bar{z} \tilde{z}) \]

(3.17)

can be summarized in the form of the \( SU(4)/U(3) \) coset representative (2.16) transformations

\[ \delta T^a_b = i (\bar{W}^a_b - \bar{W}^a_b), \quad \delta \bar{T}^a_b = -i (\bar{W}^a_b - \bar{W}^a_b), \]

(3.18)

where the \( SU(4) \) matrix \( \bar{W}^a_b \) has been introduced in (3.13) and

\[ \bar{W}^a_b = \begin{pmatrix} \eta^a_b - \delta^a_b \bar{w}^c \bar{w}_c & 0 \\
0 & -\bar{w}^a_b + \delta^a_b \bar{w}_c \bar{w}_c \end{pmatrix} \]

(3.19)

represents the \( U(3) \) compensating rotation matrix. Finally, Grassmann coordinates associated with the conformal supersymmetry generators transform as follows:

\[ \delta n_{\mu a} = \xi_{\mu a} (\varphi) - \frac{1}{2} \varepsilon^{\mu a} \sigma_{\mu \nu} \eta_{\nu a} - f(\theta) \eta_{\mu a} + i \omega^b \theta^a \eta_{\mu a} - i \omega^b \theta^a \eta_{\nu b} - i \omega^b \theta^a \eta_{\nu b} + \bar{\xi}_{a b} \eta_{\mu b} = -2 i \tilde{\theta}^{\nu a} \eta_{\nu b} \]

\[ = 2 i \varepsilon^{\nu a} (\bar{\theta} \eta) \xi^{\lambda a} \tilde{\theta}^{\nu b} \eta_{\nu b} \]

\[ + 2 i \frac{1}{2} \varepsilon^{\nu a} (\bar{\theta} \eta) \xi^{\lambda a} \tilde{\theta}^{\nu b} \eta_{\nu b} \]

(3.20)

corresponding to the stability group generators. The corresponding Cartan forms in their turn transform in a connection-type way. In particular, bosonic 1-forms that are identified with the \( \text{AdS}_4 \) part of the supervielbein in general transform as

\[ \delta \omega^m (d) + \delta \tilde{c}^m (d) = \tilde{b}^m (d), \quad \delta \Delta (d) = -\hat{b}_m (\tilde{a}^m (d) + \tilde{c}^m (d)), \]

(3.21)

while, e.g., \( so(1,2) \) Cartan forms in the spinor realization \( G^{\mu \nu} (d) = \varepsilon^{\mu \nu} \sigma_{\mu \nu}, G^{\mu \nu} (d) \) obey the following rule:

\[ \delta G^{\mu \nu} (d) = \frac{1}{4} (G^{\mu a} (d) \tilde{b}_a + G^{\nu b} (d) \tilde{b}_b + \tilde{b}_a (\tilde{c} (d) - \tilde{c} (d))^{\mu a} + \tilde{b}_b (\tilde{c} (d) - \tilde{c} (d))^{\nu b} - \frac{1}{2} \delta \tilde{b}^{\mu \nu}. \]

(3.22)

For a generic choice of the superverset coordinate transformation is required in order to identify boundary supercoordinates of anti-de Sitter superspace [47].
\( su(4) \) Cartan forms are \( OSp(4|6) \) left-invariant up to the \( U(3) \) gauge transformation:
\[
\delta \Omega_{a}^{b}(d) = i(\Omega_{a}^{b}(d)\hat{W}_{a}^{b} - \hat{W}_{a}^{b}\Omega_{a}^{b}(d)) - d\hat{W}_{a}^{b},
\]
from which we infer that the \( su(4)/u(3) \) 1-forms identified with the \( \mathbb{CP}^3 \) part of the supervielbein transform as
\[
\delta \Omega_{a}^{b}(d) = i(\hat{w}_{b}^{h}\Omega_{a}^{4}(d) - \hat{w}_{b}^{h}\Omega_{a}^{4}(d)), \quad \delta \Omega_{a}^{a}(d) = -i(\hat{w}_{b}^{h}\Omega_{a}^{a}(d) - \Omega_{a}^{h}(d)\hat{w}_{b}^{a})
\]
and \( u(3) \) 1-forms exhibit connection-type transformation properties
\[
\delta \Omega_{a}^{b}(d) = i(\Omega_{a}^{b}(d)\hat{w}_{a}^{b} - \hat{w}_{a}^{b}\Omega_{a}^{b}(d)) - d\hat{w}_{a}^{b}.
\]
Cartan forms that are identified with the supervielbein fermionic components transform in the following way:
\[
\delta \hat{\omega}_{a}^{b}(d) = \frac{1}{2}\hat{\omega}_{a}^{b}(d)\hat{\chi}_{a}^{b} + \hat{b}^{ab}\hat{\chi}_{a}^{b}(d) - i\hat{w}_{a}^{b}\hat{\omega}_{a}^{b}(d)
\]
and
\[
\delta \hat{\chi}_{a}^{b}(d) = -\frac{1}{2}\hat{\chi}_{a}^{b}(d)\hat{\chi}_{a}^{b}(d) + \hat{b}^{ab}\hat{\chi}_{a}^{b}(d) - i\hat{w}_{a}^{b}\hat{\chi}_{a}^{b}(d).
\]
For individual transformations from the \( D = 3 \, \mathcal{N} = 6 \) superconformal symmetry to be discussed below, these expressions simplify by properly restricting the parameters of compensating transformations that will be indicated by the vertical line.

4. \( D = 3 \, \mathcal{N} = 6 \) superconformal symmetry of the \( OSp(4|6)/SO(1, 3) \times U(3) \) superstring: Noether currents

Noether current densities corresponding to the \( D = 3 \, \mathcal{N} = 6 \) superconformal invariance of the superstring action (2.5) can be formally presented as the sum
\[
\mathcal{J}_{\Sigma}^{i}(\tau, \sigma) = \mathcal{J}_{\text{AdS}}^{i} + \mathcal{J}_{\text{CP}}^{i} + \mathcal{J}_{\text{WZ}}^{i},
\]
where \( \Sigma \) is a transformation parameter\(^{10}\) of contributions of the AdS
\[
\mathcal{J}_{\text{AdS}}^{i} = -\sqrt{-g}g^{ij}\left(\frac{1}{4}(\hat{\omega}_{mn} + \hat{\chi}_{mn}) \frac{\partial}{\partial \Sigma} (\hat{\omega}_{m}(\delta_{\Sigma}) + \hat{\chi}_{m}(\delta_{\Sigma})) + \Delta_{i} \frac{\partial}{\partial \Sigma} \Delta(\delta_{\Sigma})\right)
\]
and \( \mathbb{CP}^3 \) parts of the kinetic term
\[
\mathcal{J}_{\text{CP}}^{i} = -\frac{1}{2}\sqrt{-g}g^{ij}\left(\Omega_{j}^{4} \frac{\partial}{\partial \Sigma} \Omega_{a}^{4}(\delta_{\Sigma}) + \Omega_{i}^{4} \frac{\partial}{\partial \Sigma} \Omega_{a}^{4}(\delta_{\Sigma})\right),
\]
as well as that of the Wess–Zumino term
\[
\mathcal{J}_{\text{WZ}}^{i} = \frac{i}{4}g^{ij} \frac{\partial}{\partial \Sigma} \left(\hat{\omega}_{a}^{i} \epsilon_{a}^{ij} \frac{\partial}{\partial \Sigma} \hat{\omega}_{a}^{j}(\delta_{\Sigma}) + \hat{\chi}_{a}^{i} \epsilon_{a}^{ij} \frac{\partial}{\partial \Sigma} \hat{\chi}_{a}^{j}(\delta_{\Sigma})\right).
\]
Below we specialize to the discussion of the individual transformations from the \( D = 3 \, \mathcal{N} = 6 \) superconformal symmetry and present corresponding expressions for the Noether currents.

\(^{10}\) We assume the right derivative for fermions.
4.1. Noether currents associated with $D = 3$ conformal symmetry

4.1.1. Spacetime translations. $osp(4|6)$ Cartan forms are obviously invariant under global translations of the $D = 3$ Minkowski boundary coordinates. Their contributions to the current density are hence related to the coordinate dependence of the transformation parameter. In particular, equation (3.21) representing the variation of the Cartan forms identified with the AdS part of the supervielbein, when restricted to the boundary spacetime translations, acquires the form

$$\delta \hat{\omega}^m(d) + \delta \hat{c}^m(d) = j^m_n \, dd^n, \quad \delta \Delta(d) = 0,$$

(4.5)

where the current contribution tensor equals

$$j^m_n = \frac{\partial (\hat{\omega}^m(\delta_a) + \hat{c}^m(\delta_a))}{\partial d^n} = \epsilon^{-2\phi} A \delta^m_n.$$

(4.6)

$su(4)$ Cartan forms are also invariant under 3D translations

$$\delta \omega^b_a(d) = J^b_m \, da^n,$$

(4.7)

modulo the current contribution matrix

$$J^b_m = \frac{\partial}{\partial d^m} \Omega^b_a(\delta_a) = \left( j^b_m - j^b_m \right) = -2(\hat{\eta}_a \sigma_m \hat{\eta}_b).$$

(4.8)

As a result, the variation of the $CP^3$ components of the supervielbein acquires the form

$$\delta \hat{\omega}^a = \hat{f}^a_m \, dm^n,$$

$$\delta \hat{\omega}_m = \hat{f}_m^a \, dm^n,$$

(4.9)

where we have adopted the following definition of the $SU(3)$ vector dual to a rank 2 tensor that can also carry other indices $\Sigma$:

$$\hat{\omega}_m^a = \frac{1}{2} \varepsilon^{abc} j_{bc} \Sigma, \quad \hat{f}_m^a = \frac{1}{2} \varepsilon_{abc} j_{bc} \Sigma,$$

(4.10)

or simply $\hat{\omega}^a$ and $\hat{f}^a$ if $\Sigma$ does not contain $SU(3)$ indices. The variation of the fermionic supervielbein components associated with the Poincaré supersymmetry under localized boundary spacetime translations reads

$$\delta \hat{\chi}^\mu = j^\mu \, dm^n,$$

(4.11)

with the current contribution

$$j^\mu_m = \frac{\partial \hat{\chi}^\mu(\delta_a)}{\partial d^m} = -\epsilon^{2\phi} \sigma_m \hat{\eta}_a.$$

(4.12)

Then, for the variation of remaining fermionic supervielbein components associated with the conformal supersymmetry, we obtain

$$\delta \hat{x}_{\mu \sigma} = J_{\mu \sigma} \, dm^n,$$

(4.13)

where

$$J_{\mu \sigma} = \frac{\partial \hat{x}_{\mu \sigma}(\delta_a)}{\partial d^m} = -\epsilon^{2\phi} (\hat{\eta}_{\mu} \sigma_{\nu} \hat{\eta}_{\nu}).$$

(4.14)

is the current contribution.

The current density related to the superstring action (2.5) invariance under $D = 3$ Minkowski spacetime translations has the form

$$J_{\tau, \sigma} = J_{\hat{A} \hat{A} m} + J_{CP^3} + J_{WZ}.$$

(4.15)

The AdS part of the current density

$$J_{\hat{A} \hat{A} m} = -\frac{1}{2} \sqrt{-g} \hat{e}^{ij}(\hat{\omega}_{mn} + \hat{c}_{mn}) j^m_n.$$

(4.16)
is determined by the current contribution (4.6), the $\mathbb{CP}^3$ part
\[ \mathcal{J}_{\mathbb{CP}^3} = \frac{1}{2} \sqrt{-g} \frac{i}{4} \left( \omega^{\mu} + \Omega_{\mu} \right) \] (4.17)
is contributed by equation (4.9), and the current contributions (4.12), (4.14) determine the WZ part of the current density
\[ \mathcal{J}_{WZ} = \frac{1}{4} \sqrt{-g} \left( \mathcal{J}^a - \mathcal{J}^\mu \right) \left( \mathcal{J}^\mu - \mathcal{J}^a \right) \] (4.18)

4.1.2. Conformal boosts. Transformation properties of the Cartan forms that enter the AdS part of the $OSp(4|6)/SO(1,3) \times U(3)$ superstring action (2.5) under local conformal boosts follow from expressions (3.21) appropriately restricted:
\[ \delta_b \omega^m(d) + \delta_b \omega^m(d) = \int \Omega^m(d) + \hat{c}(d) + 4(\hat{b} \omega)^m \Delta(d), \]
\[ \delta_b \Delta(d) = \int \Delta^m - \hat{b} \omega^m \Delta(d) + \hat{c}(d) \] (4.19)
modulo the current contributions
\[ \int \omega^m \left[ \frac{\partial}{\partial b^m} + \frac{\partial \Delta b^m}{\partial b^m} \right] = e^{-2\phi} A(\phi^2 + (\bar{\theta} \theta)^2) \eta^m - 2x^m x^m + 4i(\bar{\eta} \sigma^m \hat{\omega}^m) \] (4.20)
where we have introduced the following 3D spin-tensors $\Lambda_{+\mu} = \delta^{\nu}_{\mu} \pm 2i(\bar{\eta} \sigma^m \hat{\omega}^m)$ that will appear to be useful below, and
\[ \int \omega^m \left[ \frac{\partial}{\partial b^m} + \frac{\partial \Delta b^m}{\partial b^m} \right] = -x^m - i(\eta^m \bar{\eta} \sigma^m \hat{\omega}^m). \] (4.21)
The variation of the $su(4)$ Cartan forms is obtained by specializing to the conformal boost parameter dependence in equation (3.23):
\[ \delta_b \Omega^m = \int \omega^m \left( \hat{\omega}^m \right) \] (4.22)
The current contribution matrix
\[ \int \delta_b \left( \hat{\omega}^m \right) = \left( \begin{array}{cc} \frac{\partial}{\partial b^m} \Omega^m \left( \hat{\omega}^m \right) \\ \frac{\partial}{\partial b^m} \hat{\omega}^m \left( \hat{\omega}^m \right) \end{array} \right) \] (4.23)
is obtained by $T$-transforming the matrix
\[ \int \delta_b \left( \hat{\omega}^m \right) = \left( \begin{array}{cc} \frac{\partial}{\partial b} \left( \hat{\omega}^m \right) \\ \frac{\partial}{\partial b} \left( \hat{\omega}^m \right) \end{array} \right) \] (4.24)
where $Z^m_b = \theta^m + i(\bar{\theta} \theta) \eta^m$. The final form of the current contribution matrix (4.23) is
\[ \int \delta_b \left( \hat{\omega}^m \right) = \left( \begin{array}{cc} \frac{\partial}{\partial b} \left( \hat{\omega}^m \right) \\ \frac{\partial}{\partial b} \left( \hat{\omega}^m \right) \end{array} \right) \] (4.25)
Thus, the variation of the supervielbein components tangent to the $\mathbb{CP}^3$ manifold is brought to the form
\[ \delta_b \Omega^m = \left( \begin{array}{cc} \frac{\partial}{\partial b^m} \left( \hat{\omega}^m \right) \\ \frac{\partial}{\partial b^m} \left( \hat{\omega}^m \right) \end{array} \right) \] (4.26)
and c.c. The expressions for the variation of fermionic 1-forms follow from (3.26) and (3.27). Namely, for Cartan forms related to Poincaré supersymmetry, one derives that
\[ \delta \hat{\omega}^b(d) = \int \mu \, db_m + \frac{1}{2} \hat{\omega}^b(d) (\hat{\theta})_\nu^\mu + (\hat{b})_\nu^\mu \hat{\chi}_\nu(d) - i(\hat{W}_{|b})_\nu^\mu \hat{\alpha}_\nu(d), \]
(4.27)
where the current contribution reads
\[ j^m_a = \frac{\partial \hat{\omega}^b(d)}{\partial b_m} = e^{-\left[ \frac{i}{2} \bar{\sigma}^m \sigma^m Z_a^4 (\sigma^2 + (\bar{\theta} \theta)^2) \sigma^m \bar{\eta}^\nu + 2 \sigma^m \bar{\eta}^\nu \eta^\nu + i(\bar{\theta} \theta) \sigma^m \bar{\eta}^\nu \eta^\nu \right]}. \]
(4.28)
Correspondingly for Cartan forms related to conformal supersymmetry, we find
\[ \delta \hat{\chi}_\mu(d) = J_{\mu a}^m \, db_m - \frac{1}{2} (\hat{\theta} \mu)^\nu \hat{\chi}_\nu(d) + (\hat{b} \mu)^{\nu a} \hat{\alpha}_\nu(d) - i(\hat{W}_{|b})_\nu^\mu \hat{\chi}_\nu(d) \]
with the current contribution
\[ J_{\mu a}^m = \frac{\partial \hat{\chi}_\nu(d)}{\partial b_m} = e^v (\Lambda_{\mu}^a - \bar{\sigma}^m \sigma^m \Lambda_{\mu}^a - i e^v (\bar{\eta} \eta) e_{\mu a} J_{\nu}^m). \]
(4.29)

The substitution of equations (4.19), (4.26), (4.27) and (4.29) into the superstring action variation under \( D = 3 \) conformal boosts yields the current density
\[ J^{im}(\tau, \sigma) = J_{\text{AdS}}^{im} + J_{\text{CP}}^{im} + J_{\text{WZ}}^{im}, \]
where
\[ J_{\text{AdS}}^{im} = -\sqrt{-g} g^{ij} \left( \frac{1}{4} (\hat{\omega}_j^m + \hat{\chi}_j^m) + \Delta_j f^m \right), \]
\[ J_{\text{CP}}^{im} = \frac{1}{2} \sqrt{-g} g^{ij} \left( \Omega_{j a}^m \ast f^m + \Omega_{j a}^m \ast f^m \right), \]
\[ J_{\text{WZ}}^{im} = \frac{1}{2} \sqrt{-g} g^{ij} \left( \hat{\omega}_j^m + \hat{\chi}_j^m \right) e_{\mu a} J_{\nu}^m \]
(4.30)

4.1.3. Dilatations. \( \text{osp}(4|6)/(\text{so}(1,3) \times u(3)) \) Cartan forms identified with the \( (10|24) \)-supervielbein components are invariant under the global scale transformations due to the presence of appropriate exponents of the AdS\( 4 \) bulk coordinate \( \varphi \). Hence, their variation under coordinate-dependent scale transformations is determined by the current contributions. In particular, for the components of supervielbein tangent to the AdS\( 4 \) space, we obtain that
\[ \delta f \hat{\omega}^m(d) + \delta f \hat{\chi}^m(d) = f^m \, df, \]
(4.33)
where
\[ f^m = \frac{\partial (\hat{\omega}^m(d) + \hat{\chi}^m(d))}{\partial f} = 2 e^{-2\varphi} A_x^m + 2 \bar{e}^{2\varphi} (\bar{\eta} \sigma^m \bar{\eta}^m), \]
(4.34)
\[ j = \frac{\partial \Delta(f)}{\partial f} = 1 + i(\vartheta^\mu \bar{\eta}^\mu + \bar{\vartheta}^{\mu a} \eta_{\mu a}). \]
(4.35)

\( su(4) \) Cartan forms are also scale-invariant:
\[ \delta f \Omega^a_b = J^a_b \, df, \]
(4.36)
modulo the current contribution matrix
\[ J^a_b = \frac{\partial}{\partial f} \Omega^a_b(d) = \begin{pmatrix} j^a_b & \bar{j}^a_b \\ j^a_b & -\bar{j}^a_b \end{pmatrix} = 2 \left( \hat{\omega}^a_b \hat{\omega}_b^a - \hat{\chi}^a_b \hat{\chi}_b^a \right), \]
(4.37)
where \( \Theta^a_b = \vartheta^a_b - \eta_{a b} \varphi \). So that the variation of \( \mathbb{C}^3 \) part of the supervielbein is governed by the appropriate components of the matrix (4.36):
\[ \delta f \Omega^{(a)}(d) = -\gamma^a_b \, df, \]
\[ \delta f \Omega^{(a)}(d) = -\gamma^a_b \, df. \]
(4.38)
Fermionic supervielbein components related to Poincaré supersymmetry obey the transformation rules
\[ \delta_f \omega^m_n (d) = f^m_n \mathbf{d} f \] (4.38)
with the current contribution
\[ j^m_n = \frac{\partial \omega^m_n (\delta_f)}{\partial f} = e^{-\nu} (\tilde{\Theta}^m_n - 2\tilde{\eta}^m_n \tilde{\eta}^a_n) \] (4.39)
while those related to conformal supersymmetry transform as
\[ \delta_f \tilde{\eta}^m_n (d) = J_{\mu \nu} \mathbf{d} f, \] (4.40)
where the corresponding current contribution is given by
\[ J_{\mu \nu} = \frac{\partial \tilde{\eta}^m_n (\delta_f)}{\partial f} = -e^{\nu} (A_{-\mu \nu} \tilde{\eta}^a_n + i e^{\nu} (\tilde{\eta}^m_n) \delta_{\mu \nu} f^a_n). \] (4.41)

The above-presented current contributions determine the Noether current density related to the scale invariance of superstring action (2.5):
\[ J^i (\tau, \sigma) = J_{\text{AdS}}^i + J_{\text{CP}}^i + J_{\text{WZ}}^i. \] (4.42)
Specifically, the AdS part of the current density
\[ J_{\text{AdS}}^i = -\sqrt{-g} \sqrt{g} \left( \frac{1}{4} (e_{\mu \nu} + \bar{e}_{\mu \nu}) j^i + \Delta_j^i \right) \] (4.43)
is contributed by equation (4.34), the \( \mathbb{C}^3 \) part
\[ J_{\text{CP}}^i = \frac{1}{2} \sqrt{-g} \sqrt{g} \left( \Omega^a_4 j^i + \Omega^a_4 \bar{j}^i \right) \] (4.44)
is determined by the current contributions that enter (4.37), and the WZ part
\[ J_{\text{WZ}}^i = \frac{i}{4} e^{\gamma i} \bar{j}^i \left( e_{\mu \nu} \bar{e}_{\mu \nu} j^i + 2 \bar{f}_{\mu \nu \rho} e^{\mu \nu \rho} j_{\nu \rho} \right). \] (4.45)
receives contributions from (4.39) and (4.41).

4.1.4. Lorentz rotations. Under local SO(1, 2) Lorentz rotations with parameters \( l^m_n \) Cartan forms identified with the supervielbein components tangent to the AdS_4 space exhibit the following transformation properties:
\[ \delta_l \omega^m_n (d) + \delta_l \epsilon^m_n (d) = f^m_n \mathbf{d} l^m_n + l^m_\mu (\omega^m_n (d) + \epsilon^m_n (d)), \]
\[ \delta_l \Delta (d) = j_{\mu \nu} \mathbf{d} l^m_n \] (4.46)
with the current contributions
\[ f^m_n = \frac{\partial (\hat{\omega}^m_n (\delta_l) + \hat{\epsilon}^m_n (\delta_l))}{\partial l^m_n} = \frac{1}{2} e^{-2A} \left( \delta^m_n \xi_n + \delta^m_n \xi_k + i(\bar{\Theta} \theta) e^m_\mu \right) \]
\[ + \frac{1}{2} e^{2A} (\bar{\eta} \eta) \left\{ \delta^m_n \left[ (\eta_m \sigma_a \bar{\theta}^3) + (\bar{\eta}^a \sigma_a \theta) \right] \right\} - (k \leftrightarrow n) + e^m_\kappa [i + (\bar{\eta} \theta) + (\theta \bar{\eta})] \right\}, \]
\[ j_{\mu \nu} = \frac{\partial A (\delta_l)}{\partial l_{\mu \nu}} = \frac{i}{4} e (\theta \sigma_m \bar{\theta}^3) + (\bar{\Theta} \sigma_m \eta_\mu). \] (4.47)
The \( su(4) \) Cartan forms are obviously \( D = 3 \) Lorentz invariant:
\[ \delta_l \Omega^b_\mu (d) = J^b_{\mu mn} \mathbf{d} l^m_n, \] (4.48)
modulo the current contribution matrix
\[ J^b_{\mu mn} = \frac{\partial}{\partial l^m_n} \Omega^b_\mu (\delta_l) = \begin{pmatrix} j^b_{\mu mn} & j^b_{\mu mn} \\ j^b_{mn} & j^b_{mn} \end{pmatrix} = \frac{1}{2} \left[ (\hat{\Theta} \sigma_m \bar{\theta}^3) - (\bar{\Theta} \sigma_m \eta_\mu) \right]. \] (4.49)
Hence, the variation of the supervielbein components tangent to the $\mathbb{CP}^3$ manifold is extracted from (4.49):

$$\delta_i \Omega^{\hat{a}}_a (d) = -j_{\text{AdS}}^m d l^{mn}. \quad \delta_i \Omega^{\hat{a}}_a (d) = -j^{\hat{a}}_m d l^{mn}. \quad (4.50)$$

The variation of the supervielbein fermionic components that are identified with the Cartan forms related to Poincaré supersymmetry follows from the general expression (3.26):

$$\delta_i \hat{\omega}^i (d) = j^\mu_{\text{mn}} d l^{mn} + \frac{1}{2} \hat{\omega}^i (d) \eta^\mu_i. \quad (4.51)$$

where the current contribution reads

$$j^\mu_{\text{mn}} = \frac{\partial \hat{\omega}^i (d)}{\partial l^{mn}} = \frac{1}{4} e^{-v} (\hat{\omega}_a^m \sigma_{mn}^\mu - \hat{\omega}_a^m \sigma_{mn}^\nu \hat{\eta}_{\nu\hat{a}}). \quad (4.52)$$

The transformation properties of the Cartan forms related to conformal supersymmetry identified with another half supervielbein fermionic components follow from equation (3.27):

$$\delta_i \hat{\chi}_a (d) = J_{\text{AdS}}^a d l^{mn} - \frac{1}{4} j_{\text{AdS}}^a \hat{\chi}_a (d). \quad (4.53)$$

with the current contribution

$$J_{\text{AdS}}^a = \frac{\partial \hat{\chi}_a (d)}{\partial l^{mn}} = -e^\nu \left( \frac{1}{4} \Lambda_{-\mu}^a \delta_{mn}^\nu \hat{\eta}_{\nu\hat{a}} + i e^\nu (\eta \bar{\eta}) \epsilon_{\nu\mu a} j_{\text{AdS}}^\mu \right). \quad (4.54)$$

Then, one is able to derive the current density related to the global $SO(1, 2)$ symmetry of the superstring action (2.5):

$$J^{i}_{\text{AdS}} (\tau, \sigma) = J^{i}_{\text{AdS}} + J^{i}_{\text{CP}} + J^{i}_{\text{WZ}}. \quad (4.55)$$

The form of the AdS part of the current density

$$J^{i}_{\text{AdS}} (\tau, \sigma) = \sum\sqrt{-g} (\frac{1}{2} (\hat{\omega}_a^m + \hat{\omega}_a^m) j^\mu_{\text{mn}} + \Delta j_{\text{mn}}) \quad (4.56)$$

is determined by equation (4.47), that of the $\mathbb{CP}^3$ part

$$J^{i}_{\text{CP}} (\tau, \sigma) = \sum\sqrt{-g} (\Omega_{\mu}^a j^{\nu}_{\text{mn}} + \Omega_{\mu}^a j^{\nu}_{\text{mn}}) \quad (4.57)$$

by equation (4.50), and the WZ part of the current density

$$J^{i}_{\text{WZ}} (\tau, \sigma) = \sum\sqrt{-g} (\hat{\chi}_a (\tau, \sigma) \epsilon_{\nu\mu a} j^{\nu}_{\text{mn}} + \hat{\chi}_a (\tau, \sigma) \epsilon_{\nu\mu a} j^{\nu}_{\text{mn}}) \quad (4.58)$$

is contributed by equations (4.52) and (4.54).

4.2. Noether currents associated with SU(4) R-symmetry

4.2.1. $U(3)$ rotations. Global $U(3)$ rotations represent an obvious symmetry of the AdS$_4$ part of the (10|24)-supervielbein; thus, the nontrivial part of its variation under the coordinate-dependent $U(3)$ rotations

$$\delta_w \hat{\omega}^m (d) + \delta_w \tilde{\omega}^m (d) = j^m_{a b} d w_{a b}, \quad \delta_w \Delta = j_{a b} d w_{a b} \quad (4.59)$$

is concentrated in the current contributions

$$j^m_{a b} = \frac{\partial (\tilde{\omega}^m (\delta_w) + \tilde{\omega}^m (\delta_w))}{\partial w_{a b}} = 2 e^{-2v} A \left[ b^m_{a c} \big( \theta_a \sigma_a \tilde{\eta}^m \big) - \big( \theta_a \sigma_a \tilde{\eta}^b \big) \right] + 2 e^{2v} \delta^m_{a b} \left[ (\eta, \tilde{\sigma}^m \tilde{\eta}) \right]$$

$$- i (\tilde{\eta} \eta) \big( \theta_a \sigma_a \tilde{\eta}^m \big) + i (\eta \tilde{\eta}) \big( \theta_a \sigma_a \tilde{\eta}^b \big) \right] - 2 e^{2v} \left[ (\eta, \tilde{\sigma}^m \tilde{\eta}^b) - i (\tilde{\eta} \eta) \big( \theta_a \sigma_a \tilde{\eta}^b \big) \right],$$

$$j_{a b} = \frac{\partial \Delta (\delta_w)}{\partial w_{a b}} = \delta_{a b} \big( \tilde{\eta}^m \eta_{ab} + \tilde{\eta}^m \sigma_{ab} \big) + \theta_{a b} \tilde{\eta}_{ab} + \eta_{aa} \tilde{\eta}^b. \quad (4.60)$$
Transformation properties of the $su(4)$ Cartan forms under $U(3)$ rotations are derived from equation (3.23):
\[ \delta_w \Omega_w^4(d) = J_{e}^{\hat{a}} b d w_b^a + i(\Omega_w^2(d)(\hat{\Omega}^w|_w)^\hat{a} b - (\hat{\Omega}^w|_w)^{\hat{a}} \Omega_w^4(d)) - d(\hat{\Omega}^w|_w)^{\hat{a}}. \] (4.61)

The current contribution matrix
\[ J_{e}^{\hat{a}} b \frac{\partial}{\partial w_b^a} \Omega_w^4(\delta_w) = \begin{pmatrix} \hat{j}_{\hat{a}}^e b & \hat{j}_{\hat{a} a} b \\ \hat{j}_{\hat{a} a} b & -\hat{j}_{\hat{a}}^e b \end{pmatrix} \] (4.62)
is obtained by $T$-transformation of the matrix
\[ J_{e}^{\hat{a}} b = \frac{\partial}{\partial w_b^a} ((\hat{\Omega}^w|_w)^{\hat{a}} b + \Psi_{e}^{\hat{a}} b (\delta_w)) = \delta_e^b \delta_a^b - \delta_e^a \delta_b^a + 4 \eta_{\hat{a} b c}^b (\theta^b \bar{\theta}^b + \theta^b \bar{\theta}^b) \eta_c^a \]
\[ + \delta_a^b \left[ \frac{\partial}{\partial w_b^a} (\bar{\Omega} \theta^{\mu}) \eta_b^a - (\bar{\Omega} \theta^{\mu}) \eta_b^a - 4 \eta_{\hat{a} b c}^b (\theta^b \bar{\theta}^{\mu} + \bar{\theta}^b \theta^{\mu}) \eta_c^a \right] \]
\[ + 2 \left[ \bar{\eta}_{b c}^b (\eta_{\hat{a} b c}^b - \eta_c^b \delta_b^a) + \bar{\theta}_{b a}^b (\theta_{\hat{a} b a}^a - \eta_c^a \delta_b^a) \right], \] (4.63)
where the following objects have been introduced:
\[ \delta_e^b = \begin{pmatrix} \delta_e^b & \delta_e^a \\ 0 & \delta_e^a \end{pmatrix}, \quad \delta_e^a = \begin{pmatrix} 0 & \delta_e^a \\ \delta_e^a & 0 \end{pmatrix}, \quad \delta_a^b = \begin{pmatrix} \delta_a^b & \delta_a^a \\ \delta_a^a & \delta_a^a \end{pmatrix}. \] (4.64)
so that the explicit form of the current contribution matrix is
\[ J_{e}^{\hat{a}} b = (T J_a^b T)_{\hat{e}}^e = T_{\hat{e}}^a T_{\hat{a}}^b - T_{\hat{e}}^a T_{\hat{a}}^b + i \frac{\hat{\delta}_a^b}{2} (T \bar{T})_{\hat{e}}^e. \] (4.65)
The matrix
\[ T_{\hat{a}}^b = \begin{pmatrix} T_a^b & T_{\hat{a}}^b & T_{\hat{a}}^b \end{pmatrix} = T_a^b + 2 i \hat{\eta}_{\hat{a}}^b \theta^b \] (4.66)
will also be used below. From equations (4.61) and (4.65), one derives transformation properties of the $\mathbb{CP}^3$ part of the bosonic supervielbein:
\[ \delta_w \Omega_w^4(d) = -(J_{\hat{c}}^e a)^{\hat{b}} b d w_b^a + i(\hat{w}|_w)^{\hat{a}} \Omega_w^4(d) - i(\hat{w}|_w)^{\hat{a}} \Omega_w^4(d) \] (4.67)
and the c.c. expression.

Fermionic supervielbein components associated with the Poincaré supersymmetry transform under local $U(3)$ rotations as follows:
\[ \delta_w \hat{\Omega}_w^4(d) = J_{\hat{e}}^e a d w_b^a - i(\hat{\Omega}|_w)^{\hat{a}} \hat{\Omega}_w^4(d), \] (4.68)
where
\[ J_{\hat{e}}^e a = \frac{\partial}{\partial w_b^a} (\hat{\Omega}_w^4(\delta_w)) = e^{\phi} \left[ \frac{1}{2} \delta_{\hat{e}}^a (T \bar{T} \theta^\mu) - i \eta_{\hat{a}}^b T_{\hat{e}}^b + i \bar{\theta}_{\hat{a}}^b T_{\hat{e}}^a \right]. \] (4.69)
represents the current contribution. Analogously variation of the fermionic supervielbein components related to conformal supersymmetry is given by the expression
\[ \delta_w \hat{\Omega}_{\hat{e}}^{\mu}(d) = J_{\hat{e}}^{\mu} a d w_b^a - i(\hat{\Omega}|_w)^{\hat{a}} \hat{\Omega}_{\hat{e}}^{\mu}(d) \] (4.70)
with the current contribution
\[ J_{\hat{e}}^{\mu} a = \frac{\partial}{\partial w_b^a} (\delta_w(\hat{\Omega}_w^4(\delta_w))) = e^\phi \left[ \frac{1}{2} \delta_{\hat{e}}^a (T \bar{T} \eta_{\hat{a}}) - i \eta_{\hat{a}}^b T_{\hat{e}}^b + i \bar{\theta}_{\hat{a}}^b T_{\hat{e}}^a + i e^\phi (i \eta) \epsilon_{\hat{e} \hat{a}}^a \right]. \] (4.71)

In summary, the current density associated with the $U(3)$ global invariance of the superstring action (2.5)
\[ J^i a^b (\tau, \sigma) = J_{AB}^i a^b + J_{CP}^i a^b + J_{WZ}^i a^b \] (4.72)
consists of three summands
\[
\mathcal{J}_{\text{AdS}}^{i j} = -\sqrt{-g}g^{ij}\left(\frac{1}{4}(\tilde{\omega}_m + \tilde{\epsilon}_m)J^m_{i j} + \Delta_j I_d b\right),
\]
(4.73)
\[
\mathcal{J}_{\text{CP}}^{i j} = \frac{1}{4}\sqrt{-g}g^{ij}(\bar{\omega}_m^\alpha \Omega_{j}^\alpha (\tilde{\epsilon}_m)_{b} + \Omega_{J}^\epsilon (\tilde{\epsilon}_J)_{a}^b)
\]
(4.74)
and
\[
\mathcal{J}_{\text{WZ}}^{i j} = \frac{i}{4}e^{ij}_a d_m J_{i j}^{m} + \tilde{\chi}_a^m J_{i j}^{m} - b d_i d_j d_u a,
\]
(4.75)
that are determined by the current contributions of the \( \alpha \psi(4/6)/(so(1, 3) \times u(3)) \) Cartan forms \( (4.60), (4.67), (4.69) \) and \( (4.71) \).

4.2.2. \( SU(4)/U(3) \) transformations. As in the case of \( U(3) \) rotations, Cartan forms from the \( \text{AdS}_4 \) sector are invariant under local \( SU(4)/U(3) \) transformations:
\[
\delta_s \bar{\omega}^m(d) + \delta_s \tilde{\epsilon}^m(d) = f^m_{i d} y^a + \tilde{\omega}^m(d) + \tilde{\epsilon}^m(d), \quad \delta_s \Delta(d) = \tilde{\omega}^m(d) + \tilde{\epsilon}^m(d),
\]
(4.76)
modulo the current contributions
\[
f^m_{i d} = \partial (\bar{\omega}^m(d) + \tilde{\epsilon}^m(d)) \]
(4.77)
and c.c. Transformation properties of the \( su(4) \) Cartan forms follow from the general formula \( (3.23) \) by specializing to \( SU(4)/U(3) \) rotations
\[
\delta_s \Omega^{\alpha \beta}_{b c}(d) = f^{\alpha \beta}_{b c} y^a + \tilde{\Omega}^{\alpha \beta}_{b c} d_y a + \bar{\Omega}^{\alpha \beta}_{b c}(d) - \tilde{\Omega}^{\alpha \beta}_{b c}(d) - d(\bar{\Omega}^{\alpha \beta}_{b c}(d)).
\]
(4.78)

The corresponding current contribution matrices equal
\[
J^\alpha_{b c}(d) = \frac{\partial}{\partial y^a} \Omega^{\alpha \beta}_{b c}(d) = \left( j^\alpha_{b a} - j^\mu_{b c a} \right) = -\epsilon^{\alpha \beta \gamma} T^\beta_{\mu \gamma} T^\mu_{\alpha \gamma} \]
(4.79)
and
\[
J^\alpha_{b c}(d) = \frac{\partial}{\partial y^a} \Omega^{\alpha \beta}_{b c}(d) = \left( \tilde{j}^\alpha_{b a} - \tilde{j}^\mu_{b c a} \right) = \epsilon^{\alpha \beta \gamma} T^\beta_{\mu \gamma} T^\mu_{\alpha \gamma},
\]
(4.80)
so that one extracts from the above expressions the transformation rules for the supervielbein bosonic components tangent to the \( \mathbb{CP}^3 \) manifold:
\[
\delta_s \Omega^{\alpha \beta}_{b c}(d) = (\bar{\Omega}^{\alpha \beta}_{b c}(d) - \tilde{\Omega}^{\alpha \beta}_{b c}(d)) - i(\bar{\Omega}^{\alpha \beta}_{b c}(d) - \tilde{\Omega}^{\alpha \beta}_{b c}(d)).
\]
(4.81)
Supervielbein fermionic components associated with the Poincaré supersymmetry have the following properties under \( SU(4)/U(3) \) transformations:
\[
\delta_s \tilde{\omega}_m^\mu(d) = j^\mu_{ba} d_y^a + f^\nu_{ba} d_y^a - i(\bar{\Omega}^{\alpha \beta}_{b c}(d) - \tilde{\Omega}^{\alpha \beta}_{b c}(d)),
\]
(4.82)
with the current contributions
\[
j^\mu_{ba} = \frac{\partial \tilde{\omega}_m^\mu(d)}{\partial y^a} = i e^{-\epsilon^{\alpha \beta \gamma}} T^\beta_{\mu \gamma} T^\mu_{\alpha \gamma}
\]
(4.83)
and c.c. Supervielbein fermionic components related to conformal supersymmetry transform as
\[
\delta_s \tilde{\chi}_{ab}^c(d) = (\bar{\Omega}^{\alpha \beta}_{b c}(d) - \tilde{\Omega}^{\alpha \beta}_{b c}(d)) - i(\bar{\Omega}^{\alpha \beta}_{b c}(d) - \tilde{\Omega}^{\alpha \beta}_{b c}(d)).
\]
(4.84)
where the current contributions can be brought to the form
\[ J_{\mu ba} = \frac{\partial J_{\mu ba}(\delta_\epsilon)}{\partial y^a} = i e^\phi (\epsilon_{a cd} T^c_{\mu} \hat{n}^{d}_{\mu} + e^\phi (\hat{n}\eta) e_{\mu \nu} f_{b \mu a}^\nu) \] (4.85)
and c.c.

The above-derived current contributions of the supervielbein components determine the Noether current density associated with SU(4)/U(3) global invariance of the superstring action
\[ J_a^i(\tau, \sigma) = J_{\text{AdS}}^i a + J_{\text{CP}}^i a + J_{\text{WZ}}^i a \] (4.86)
and the c.c. expression. The summands contributed by equations (4.77), (4.81), (4.83) and (4.85), respectively, take the form
\[ J_{\text{AdS}}^i a = -\frac{1}{\sqrt{-g}} g^{ij} \left( \frac{1}{2} (\hat{\omega}_{jm} + \hat{c}_{jm}) f_{\mu a}^m + \Delta j_{ja} \right), \]
\[ J_{\text{CP}}^i a = \frac{i}{\sqrt{-g}} g^{ij} (\Omega_{zb}^j (\ast \eta_b^i a) - \Omega_{zb}^j (\ast \eta_b^i a)) \]
and
\[ J_{\text{WZ}}^i a = \frac{i}{4} \epsilon^{ij} \left( \hat{\omega}_{jm} e_{\mu \nu} f_{ja}^m + \hat{c}_{jm} e^{\mu \nu} J_{vj a} \right). \]

4.3. Noether currents associated with \( D = 3 \mathcal{N} = 6 \) Poincaré supersymmetry

The superstring action (2.5) is manifestly invariant under Poincaré supersymmetry as \( D = 3 \mathcal{N} = 6 \) supercoordinates \((x^m, \theta^\mu_a, \bar{\theta}^{\bar{\mu}}_{\bar{a}})\) that are the only non-trivially transforming ones enter through the supersymmetric Volkov–Akulov 1-forms [50]. Hence, non-invariance of the AdS part of the supervielbein bosonic components under coordinate-dependent Poincaré supersymmetry transformations
\[ \delta_\epsilon \hat{\omega}^m (d) + \delta_\epsilon \hat{c}^m (d) = f^m_{\mu a} \delta e^\mu_a - f^m_{\mu a} \delta \hat{e}^{\mu a}, \quad \delta_\epsilon \Delta (d) = f^a_{\mu a} \delta \hat{e}^a - \tilde{f}_{\mu a} \delta \hat{e}^{\mu a}, \]
is accounted by the current contributions whose explicit form is
\[ f^m_{\mu a} = \frac{\partial (\hat{\omega}^m (\delta_\epsilon) + \hat{c}^m (\delta_\epsilon))}{\partial e^\mu_a} = 2 \delta_{\mu a} \left[ i e^{-2e_2 A} \hat{\theta}^{\mu a} + e^{\phi} (\hat{n}\eta) \hat{\eta}^{\mu a} \right], \]
\[ f^a_{\mu a} = \frac{\partial \Delta (\delta_\epsilon)}{\partial e^\mu_a} = -\hat{\eta}^{\mu a} \]
and c.c. expressions. The su(4) Cartan forms are also \( D = 3 \mathcal{N} = 6 \) super-Poincaré invariant
\[ \delta_\epsilon \Omega_b^c (d) = J_{b \mu a} \delta e^\mu_a - \tilde{J}_{b \mu a} \delta \hat{e}^{\mu a}, \]
modulo the current contribution matrices
\[ J_{b \mu a} = \frac{\partial \Omega_b^c (\delta_\epsilon)}{\partial e^\mu_a} = \begin{pmatrix} j_{b \mu a} & j_{b \mu a} \\ j_{b \mu a} & j_{b \mu a} \end{pmatrix} = 2 (\tilde{n}_{\mu b} T^{ia} - T_{b}^{ia} \tilde{n}_{\mu a}) \]
and
\[ \tilde{J}_{b \mu a} = -\frac{\partial \Omega_b^c (\delta_\epsilon)}{\partial \hat{e}^{\mu a}} = \begin{pmatrix} \tilde{j}_{b \mu a} & \tilde{j}_{b \mu a} \\ \tilde{j}_{b \mu a} & \tilde{j}_{b \mu a} \end{pmatrix} = 2 (\tilde{T}^{ia} \tilde{n}_{\mu b} - \tilde{n}_{\mu b} T^{ia}) \]
so that the \( \mathbb{C}^3 \) part of the bosonic supervielbein transforms as
\[ \delta_\epsilon \Omega_b^a (d) = (\ast \eta_b^a) \delta e^\mu_a + (\ast \eta_b^a) \delta \hat{e}^{\mu a}, \quad \delta_\epsilon \Omega_b^a (d) = -(\ast \eta_b^a) \delta e^\mu_a - (\ast \eta_b^a) \delta \hat{e}^{\mu a}. \]

17
Cartan forms associated with the Poincaré supersymmetry are manifestly $D = 3$ $\mathcal{N} = 6$ super-Poincaré invariant:

$$\delta_{\xi} \hat{\omega}^\mu (d) = j^{a\mu}_d \delta \xi^a + \tilde{J}^\mu_{a\mu} \delta \xi^a$$

(4.96)

up to the current contributions

$$j^{a\mu}_b = \frac{\partial \hat{\omega}^\mu_b (\delta_{\xi})}{\partial \xi^a} = e^{-\phi} \left( \delta^\mu_a \tau^a_b + 2 i \tilde{\eta}^{ab} \tilde{\chi}^{0a} \right)$$

(4.97)

and c.c., as well as Cartan forms associated with the conformal supersymmetry

$$\delta_{\xi} \hat{X}_{aj}(d) = \hat{J}_{b\mu}^a \delta \xi^a + \hat{J}_{b \mu a} \delta \xi^a$$

(4.98)

with the corresponding current contributions given by

$$J_{b\mu}^a = \frac{\partial \hat{X}_{aj}(\delta_{\xi})}{\partial \xi^a} = \imath e^\phi \left( 2 \tilde{\eta}^{ab} \tilde{\eta}^a - e^{-\phi} (\tilde{\eta}^b)_{\epsilon} \epsilon_{\mu} \hat{J}_{b\mu}^a \right)$$

(4.99)

and c.c.

Putting all together, contributions (4.91), (4.95), (4.97) and (4.99) allows us to determine the current density related to $D = 3$ $\mathcal{N} = 6$ supersymmetry invariance of the superstring action (2.5):

$$J^{ia} (\tau, \sigma) = J_{\text{AdS}}^{ia} + J_{\text{CP}}^{ia} + J_{\text{WZ}}^{ia}$$

(4.100)

and the c.c. one. The individual summands entering the current density (4.100) equal

$$J_{\text{AdS}}^{ia} = \frac{1}{2} \sqrt{-g} g^{ij} \left( \frac{1}{2} (\partial \omega^j_{\mu} + \hat{\epsilon}_{\mu} \hat{\epsilon}^j_a) f^{m a} + \Delta_j f^a_{\mu} \right)$$

(4.101)

$$J_{\text{CP}}^{ia} = \frac{1}{2} \sqrt{-g} g^{ij} \left( \Omega^4_a \hat{\omega}^j_{\mu} + \Omega^4_j \hat{\omega}^a_{\mu} \right)$$

and

$$J_{\text{WZ}}^{ia} = \frac{1}{2} \delta^i_j \delta^j_a (\partial \hat{\omega}^{0a} \hat{J}^j_{b\mu} + \hat{X}^j_{b\mu a})$$

(4.103)

4.4. Noether currents associated with $D = 3$ $\mathcal{N} = 6$ conformal supersymmetry

Transformation properties of the supervielbein bosonic components in the directions tangent to the AdS$_4$ space can be obtained from the general expressions (3.21) by appropriately restricting the parameters of $SO(1, 3)$ compensating rotations:

$$\delta_{\xi} \hat{\omega}^m (d) + \delta_{\xi} \hat{\epsilon}^m (d) = f^{a\mu}_{\nu} \delta \xi^a - \tilde{f}^{a\mu}_{\nu} \delta \xi^a + (\tilde{\Omega}^m_{\nu})(\delta \omega^m (d) + \hat{\epsilon}_m (d)) + (\tilde{\eta}^b_{\mu}) \Delta (d)$$

(4.104)

and adding the current contribution terms

$$f^{a\mu}_{\nu} = \frac{\partial (\hat{\omega}^m (d) + \hat{\epsilon}^m (d))}{\partial \xi^a} = 2 \left[ \imath e^{-2\phi} \hat{A}^{a\mu}_{\nu} \sigma^m_{\nu} \hat{\omega}^m_{\nu} + \imath e^{2\phi} \hat{\eta}^a_{\mu} \hat{\sigma}^{m \nu} \Lambda_{\nu} \right]$$

and

$$j^{a\mu}_{\nu} = \frac{\partial \delta (d)}{\partial \xi^a} = - \imath \left( \hat{\sigma}^{a\mu} \Lambda_{\nu} + \hat{\sigma}^{a\mu} \Lambda_{\nu} \right)$$

(4.105)

c.c.

The variation of the matrix of $su(4)$ Cartan forms under the coordinate-dependent conformal supersymmetry transformations is presented in the following form:

$$\delta_{\xi} \Omega^j_b (d) = \hat{J}^j_{b\mu} \delta \xi^a - \tilde{J}^j_{b\mu} \delta \xi^a + i (\Omega^j_b (d) (W_{\mu} | \xi \right) \hat{\omega}^j_b + (\tilde{W}_{\mu} | \xi \right) \hat{\omega}^j_b - \hat{\omega}^j_b \hat{\omega}^j_b (d) - d (W_{\mu} | \xi \right) \hat{\omega}^j_b$$

(4.106)
where the current contributions

$$\tilde{f}_b^{\tilde{\epsilon} \mu a} = \frac{\partial}{\partial \xi_{\mu a}} \Omega_b^{\tilde{\epsilon}} (\delta \xi) = \left( \begin{array}{c} \tilde{J}_b^{\mu a a} \\ \tilde{J}_b^{\mu a b} \end{array} \right)$$

(4.107)

and

$$\tilde{f}_b^{\tilde{\epsilon} \mu a} = -\frac{\partial}{\partial \xi_{\mu a}} \Omega_b^{\tilde{\epsilon}} (\delta \xi) = \left( \begin{array}{c} -\tilde{J}_b^{\mu a a} \\ -\tilde{J}_b^{\mu a b} \end{array} \right)$$

(4.108)

are obtained by the $T$-transformation of the matrices

$$J_b^{\tilde{\epsilon} \mu a} = \frac{\partial}{\partial \xi_{\mu a}} ((\tilde{W}_{\tilde{\epsilon}})_b^{\tilde{\epsilon}} + \Psi^{\tilde{\epsilon}} (\delta \xi)) = 2(\delta^{\mu}_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b - \delta^\mu_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b) - 4i\tilde{\Theta}^{\mu}_{\nu}(\eta_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b + \Theta^{\mu}_{\tilde{\epsilon}} b \eta_{\nu}),$$

(4.109)

so that the current contribution matrices (4.107) and (4.108) acquire the form

$$\tilde{f}_b^{\tilde{\epsilon} \mu a} = (T J^{\mu a})_b^{\tilde{\epsilon}} = 2(\tilde{T}^{\nu}_{\mu a} \Theta^{\nu}_{\tilde{\epsilon}} b - \tilde{T}^{\mu}_{a b} \Theta^{\nu}_{\tilde{\epsilon}})$$

(4.110)

and

$$\tilde{f}_b^{\tilde{\epsilon} \mu a} = (T \tilde{f}^{\mu a})_b^{\tilde{\epsilon}} = 2(\tilde{T}^{\mu}_{a b} \Theta^{\nu}_{\tilde{\epsilon}} b - \tilde{T}^{\nu}_{\mu a} \Theta^{\nu}_{\tilde{\epsilon}}).$$

(4.111)

Then, the variation under conformal supersymmetry of the $\mathbb{CP}^3$ components of the supervielbein is brought to the form

$$\delta \xi \Omega^4_b (d) = (\tilde{J}_b)_{\mu a} d \xi_{\mu a} + (\tilde{J}_b)_{\nu b} d \xi_{\nu b} + (\tilde{J}_b)_{\nu b} d \xi_{\nu b} + i(\tilde{W}_{\tilde{\epsilon}})_b^{\tilde{\epsilon}} (\delta \xi) = 2(\delta^{\mu}_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b - \delta^\mu_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b) - 4i\tilde{\Theta}^{\mu}_{\nu}(\eta_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b + \Theta^{\mu}_{\tilde{\epsilon}} b \eta_{\nu}).$$

(4.112)

and the c.c. expression.

The variation of Cartan forms associated with the super-Poincaré generators can be extracted from the general expression (3.26):

$$\delta \xi \tilde{\omega}^{\tilde{\epsilon}} (d) = \tilde{J}^{\mu a}_{\tilde{a} b} d \xi_{\mu a} + \tilde{J}^{\nu b}_{a b} d \xi_{\nu b} + \frac{1}{2} \tilde{J}^{\tilde{\epsilon}%}_{b a} (\delta \xi), \tilde{\chi}_{b a} (d) - i(\tilde{W}_{\tilde{\epsilon}})_b^{\tilde{\epsilon}} \tilde{\chi}_{b a} (d) = 2(\delta^{\mu}_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b - \delta^\mu_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b) - 4i\tilde{\Theta}^{\mu}_{\nu}(\eta_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b + \Theta^{\mu}_{\tilde{\epsilon}} b \eta_{\nu}).$$

(4.113)

with the current contributions given by

$$\tilde{J}^{\mu a}_{\tilde{a} b} = \frac{d}{d\xi_{\mu a}} \tilde{\omega}^{\tilde{\epsilon}} (\delta \xi) = e^\nu (\tilde{T}^{\nu}_{a b} \tilde{\Theta}^{\mu}_{\tilde{\epsilon}} b + 2i\tilde{\Theta}^{\mu}_{\nu}(\eta_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b + \Theta^{\mu}_{\tilde{\epsilon}} b \eta_{\nu})).$$

(4.114)

and c.c. Similarly the variation of the Cartan forms associated with the conformal supervielbein generators reads

$$\delta \xi \tilde{\chi}_b (d) = J^{\mu a}_{b} d \xi_{\mu a} + J^{\nu b}_{a b} d \xi_{\nu b} - \frac{1}{2} (\tilde{W}_{\tilde{\epsilon}})_b^{\tilde{\epsilon}} \tilde{\chi}_b (d) = 2(\delta^{\mu}_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b - \delta^\mu_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b) - 4i\tilde{\Theta}^{\mu}_{\nu}(\eta_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b + \Theta^{\mu}_{\tilde{\epsilon}} b \eta_{\nu}).$$

(4.115)

with the corresponding current contributions

$$J^{\mu a}_{b} = \frac{d}{d\xi_{\mu a}} \tilde{\chi}_b (\delta \xi) = e^\nu (\Lambda^{\nu}_{a} \tilde{T}^{\mu}_{b} + 2i\tilde{\Theta}^{\mu}_{\nu}(\eta_{\nu} \Theta^{\mu}_{\tilde{\epsilon}} b + \Theta^{\mu}_{\tilde{\epsilon}} b \eta_{\nu})).$$

(4.116)

and c.c.

As a result, current density associated with the superstring action (2.5) invariance under conformal supersymmetry takes the form

$$J^{\mu a}_{\tilde{a} b} (\tau, \sigma) = J^{\mu a}_{a b} + J^{\nu b}_{a b} + J^{\mu a}_{b}$$

(4.117)

and c.c. one. Current contributions (4.105) enter the AdS part of the current density

$$J^{\mu a}_{a b} = -\sqrt{\frac{8}{9}} \delta^{\mu}_{a} \left( \frac{1}{2} (\tilde{\omega}_{b a} + \tilde{\omega}_{a b}) \right) J^{\mu a}_{b} + \Delta_j j^{\mu a}.$$

(4.118)
and those entering equation (4.112) determine the $\mathbb{CP}^3$ part of the superconformal current density

$$J^{\mu\alpha}_{\mathbb{CP}} = \frac{1}{2} \sqrt{-g} g^{\mu\nu} (\tilde{\Omega}_{\mu}{}^{\lambda})^{\mu\alpha} - \Omega_{\mu}{}^{\nu} (\tilde{J}_{\nu})^{\mu\alpha}. \quad (4.119)$$

The form of the WZ term contribution is obtained by substituting equations (4.114) and (4.116):

$$J^{\mu\alpha}_{WZ} = \frac{i}{4} \epsilon^{i\mu
u} \tilde{\Delta}_{\nu} \left( \tilde{\omega}_{\nu}^{\mu} \delta_{\nu\alpha} J_{\nu}^{\mu\alpha} + \tilde{\chi}_{\nu}^{\mu} \epsilon^{\nu\alpha} J_{\nu}^{\mu\alpha} \right). \quad (4.120)$$

5. Conclusion

The $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset sigma-model [9, 10] describes manifestly classically integrable part of the $AdS_4 \times \mathbb{CP}^3$ superstring action [12]. By virtue of the isomorphism between the $osp(4|6)$ superalgebra and $D = 3, N = 6$ superconformal algebra, it can be presented in the conformal basis for $osp(4|6)/(so(1, 3) \times u(3))$ Cartan forms [22] that are identified with the ‘reduced’ $(10|24)$-dimensional $D = 10, N = 2A$ supervielbein obtained from the full one [12] by setting to zero eight fermionic coordinates related to spacetime supersymmetries broken by the $AdS_4 \times \mathbb{CP}^3$ superbackground. The $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset sigma-model action is by construction invariant under global $OSp(4|6)$ supergroup transformations and hence is also invariant under $D = 3, N = 6$ superconformal symmetry that is the global symmetry of the ABJM gauge theory [51]. In this paper, we have derived explicit expressions for the world-sheet current densities associated with each type of the transformations from $D = 3, N = 6$ superconformal symmetry. Considering the $OSp(4|6)/(SO(1, 3) \times U(3))$ supercoset element parametrized by $D = 3, N = 6$ super-Poincaré coordinates supplemented by the $\mathbb{CP}^3$ coordinates, the bulk coordinate of the $AdS_4$ space and Grassmann coordinates related to the conformal supersymmetry, we have found their full transformations under $D = 3, N = 6$ superconformal symmetry. Then, passing to the canonical formulation, it should be possible to calculate the algebra of associated supercharges. In this respect, our results also constitute the starting point for examining residual symmetry algebras remaining upon fixing the gauges in the $OSp(4|6)/(SO(1, 3) \times U(3))$ sigma-model action like that of [43] and deriving transformation properties of physical variables (see e.g. [52]). Obtained Noether currents can furthermore be used when studying the semiclassical quantization around particular solutions to the $AdS_4 \times \mathbb{CP}^3$ superstring equations of motion [53].

Acknowledgments

It is a pleasure to thank A A Zheltukhin for interesting discussions and the Abdus Salam ICTP, where this work has been finalized, for warm hospitality and support.

References

[1] Maldacena J M 1998 The large $N$ limit of superconformal field theories and supergravity Adv. Theor. Math. Phys. 2 231 (arXiv:hep-th/9711200)
[2] Gubser S S, Klebanov I R and Polyakov A M 1998 Gauge theory correlators from non-critical string theory Phys. Lett. B 428 105 (arXiv:hep-th/9802109)
[3] Witten E 1998 Anti-de Sitter space and holography Adv. Theor. Math. Phys. 2 253 (arXiv:hep-th/9802150)
[4] Aharony O, Bergman O, Jafferis D L and Maldacena J 2008 $N = 6$ superconformal Chern–Simons–matter theories, M2-branes and their gravity duals J. High Energy Phys. JHEP10(2008)091 (arXiv:0806.1218 [hep-th])
[5] Metsaev R R and Tseytlin A A 1998 Type IIB superstring action in $AdS_5 \times S^5$ background. Nucl. Phys. B 533 109 (arXiv:hep-th/9805028)

[6] Kallosh R, Rahmfeld J and Rajaraman A 1998 Near horizon superspace. J. High Energy Phys. JHEP09(1998)002 (arXiv:hep-th/9805217)

[7] Berkovits N, Bershadsky M, Hauer T, Zhukov S and Zwiebach B 2000 Superstring theory on $AdS_2 \times S^2$ as a coset supermanifold. Nucl. Phys. B 567 61 (arXiv:hep-th/9907200)

[8] Roiban R and Siegel W 2000 Superstrings on $AdS_5 \times S^5$ superwistor space. J. High Energy Phys. JHEP11(2000)024 (arXiv:hep-th/0010104)

[9] Arutyunov G and Frolov S 2008 Superstrings on $AdS_4 \times \mathbb{CP}^3$ as a coset sigma-model. J. High Energy Phys. JHEP09(2008)129 (arXiv:0806.4940 [hep-th])

[10] Stefanski B J 2009 Green–Schwarz action for type IIA strings on $AdS_4 \times \mathbb{CP}^3$. Nucl. Phys. B 808 80 (arXiv:0806.4948 [hep-th])

[11] Fre P and Grassi P A 2008 Pure spinor formalism for $OSp(N|4)$ backgrounds. arXiv:0807.0044 [hep-th]

[12] Bonelli G, Grassi P A and Safaii H 2008 Exploring pure spinor string theory on $AdS_4 \times \mathbb{CP}^3$. J. High Energy Phys. JHEP10(2008)085 (arXiv:0808.1051 [hep-th])

[13] D'Auria R, Fre P, Grassi P A and Trigiante M 2009 Supergravity. arXiv:0912.3982 [hep-th]

[14] Gomis J, Sorokin D and Wulff L 2009 The complete $AdS_4$ super-Yang–Mills. J. High Energy Phys. JHEP03(2009)015 (arXiv:0811.1556 [hep-th])

[15] Grassi P A, Sorokin D and Wulff L 2009 Simplifying superstring and $D$-brane actions in $AdS_4 \times \mathbb{CP}^3$. J. High Energy Phys. JHEP08(2009)060 (arXiv:0903.5407 [hep-th])

[16] Duff M J, Howe P S, Inami T and Stelle K S 1987 Superstrings in $AdS_3 \times S^3$ and $AdS_5 \times S^5$. Phys. Rev. D 35 22167 (arXiv:hep-th/98012245)

[17] de Wit B, Peeters K, Plefka J and Sezgin E 2005 The supermembrane revisited. Class. Quantum Grav. 22 2167 (arXiv:hep-th/0412245)

[18] Nilsson B E W and Pope C 1984 Hopf fibration of eleven-dimensional supergravity. Class. Quantum Grav. 1 499

[19] Sorokin D P, Tkach V I and Volkov D V 1984 Kaluza–Klein theories and spontaneous compactification. J. High Energy Phys. JHEP09(1998)002 (arXiv:hep-th/9807200)

[20] Minahan J A and Zarembo K 2003 The Bethe-ansatz for $N = 4$ super-Yang–Mills. J. High Energy Phys. JHEP03(2003)013 (arXiv:hep-th/0212208)

[21] Bena I, Polchinski J and Roiban R 2004 Hidden symmetries of the AdS5 × S5 superstring. Phys. Rev. D 69 046002 (arXiv:hep-th/0305116)

[22] Kristjansen C, Staudacher M and Tseytlin A (eds) 2009 Gauge-string duality and integrability: progress and outlook. J. Phys. A: Math. Theor. 42 N25

[23] Beisert N et al 2010 Review of AdS/CFT integrability: an overview. arXiv:1012.3982 [hep-th]

[24] Uvarov D V 2009 $AdS_4 \times \mathbb{CP}^3$ supergravity and $D = 3 N = 6$ superconformal symmetry. Phys. Rev. D 79 106007 (arXiv:0911.2813 [hep-th])

[25] Dall'Agata G, Fabbrini D, Fraser C, Fre P, Termonia P and Trigiante M 1999 The $OSp(8|4)$ singleton action from the supermembrane. Nucl. Phys. B 542 157 (arXiv:hep-th/9807115)

[26] Kallosh R 1998 Superconformal actions in Killing gauge. arXiv:hep-th/9807206

[27] Pasti P, Sorokin D P and Tonin M 1999 On gauge-fixed superbrane actions in AdS backgrounds. Phys. Lett. B 447 251 (arXiv:hep-th/9809213)

[28] Metsaev R R and Tseytlin A A 2001 Superstring action in $AdS_4 \times S^5$: k-symmetry light cone gauge. Phys. Rev. D 63 046002 (arXiv:hep-th/0007036)

[29] Metsaev R R, Thorng C B and Tseytlin A A 2001 Light-cone superstring in AdS spacetime. Nucl. Phys. B 596 151 (arXiv:hep-th/0009171)

[30] Benna M, Klebanov I, Klose T and Smendt M 2008 Superconformal Chern–Simons theories and $AdS_4/CFT_3$ correspondence. J. High Energy Phys. JHEP09(2008)072 (arXiv:0806.1519 [hep-th])

[31] Cederwall M 2008 Superfield actions for $N = 8$ and $N = 6$ conformal theories in three dimensions. J. High Energy Phys. JHEP10(2008)070 (arXiv:0809.0318 [hep-th])

[32] Buchbinder I L, Ivanov E A, Lechtenfeld O, Pletnev N G, Samsonov I B and Zupnik B M 2009 ABJM models in $N = 3$ harmonic superspace. J. High Energy Phys. JHEP10(2009)096 (arXiv:0811.4774 [hep-th])

[33] Ricci R, Tseytlin A A and Wolf M 2007 On T-duality and integrability for strings on AdS backgrounds. J. High Energy Phys. JHEP12(2007)082 (arXiv:0711.0707 [hep-th])
[31] Beisert N, Ricci R, Tseytlin A A and Wolf M 2008 Dual superconformal symmetry from AdS$_5 \times S^5$ superstring integrability Phys. Rev. D 78 126004 (arXiv:0807.3228 [hep-th])
Beisert N 2009 T-duality, dual conformal symmetry and integrability for strings on AdS$_5 \times S^5$ Fortschr. Phys. 57 329 (arXiv:0903.0699 [hep-th])

[32] Berkovits N and Maldacena J 2008 Fermionic T-duality, dual superconformal symmetry, and the amplitude/Wilson loop connection J. High Energy Phys. JHEP09(2008)062 (arXiv:0807.3196 [hep-th])

[33] Drummond J M, Henn J, Korchemsky G P and Sokatchev E 2010 Dual superconformal symmetry of scattering amplitudes in $\mathcal{N} = 4$ super-Yang–Mills theory Nucl. Phys. B 828 317 (arXiv:0807.1095 [hep-th])

[34] Drummond J M, Henn J M and Plefka J 2009 Yangian symmetry of scattering amplitudes in super-Yang–Mills theory J. High Energy Phys. JHEP05(2009)046 (arXiv:0902.2987 [hep-th])

[35] Alday L F and Maldacena J M 2007 Gluon scattering amplitudes at strong coupling J. High Energy Phys. JHEP06(2007)064 (arXiv:0705.0303 [hep-th])

[36] Alday L F and Maldacena J M 2007 Dual superconformal symmetry, and the amplitude/Wilson loop connection J. High Energy Phys. JHEP09(2008)062 (arXiv:0807.3196 [hep-th])

[37] Alday L F and Maldacena J M 2007 Magic identities for conformal four-point integrals J. High Energy Phys. JHEP01(2007)064 (arXiv:hep-th/0607160)

[38] Brandhuber A, Heslop P and Travaglini G 2008 On dual superconformal symmetry of the $\mathcal{N} = 4$ super-Yang–Mills $S$-matrix Phys. Rev. D 78 125005 (arXiv:0807.4097 [hep-th])

[39] Brandhuber A, Heslop P and Travaglini G 2008 A note on dual superconformal symmetry of the AdS4 light-cone gauge Hamiltonian for AdS4 super-Yang–Mills theory J. High Energy Phys. JHEP11(2010)076 (arXiv:1008.0041 [hep-th])

[40] Beisert N 2008 The superconformal symmetry from AdS5 super-Yang–Mills theory and Wilson loops in Chern–Simons theory Nucl. Phys. B 794 231 (arXiv:0707.1153 [hep-th])

[41] Beisert N 2008 The dual conformal symmetry and integrability for strings on AdS$_5 \times S^5$ Fortschr. Phys. 57 329 (arXiv:0903.0699 [hep-th])

[42] Beisert N, Ricci R, Tseytlin A A and Wolf M 2008 Dual superconformal symmetry from AdS5 Class. Quantum Grav. 25 065001 (arXiv:hep-th/0607160)

[43] Bykov D 2010 The T-duality of the AdS4 super-Yang–Mills theory Adv. Theor. Math. Phys. 12 945 (available at http://www.intpresse.com/ATMP/ATMP-issue_12_5.php) (arXiv:hep-th/0511082)

[44] Beisert N 2008 The su(2|2) dynamic $S$-matrix Adv. Theor. Math. Phys. 12 945 (available at http://www.intpresse.com/ATMP/ATMP-issue_12_5.php) (arXiv:hep-th/0511082)

[45] Arutyunov G, Frolov S, Plefka J and Zamaklar M 2007 The off-shell symmetry algebra of the light-cone AdS$_5 \times S^5$ superstring J. Phys. A: Math. Theor. 40 3583 (arXiv:hep-th/0609157)
[46] Claus P and Kallosh R 1999 Superisometries of the AdS × S superspace J. High Energy Phys. JHEP03(1999)014 (arXiv:hep-th/9812087)

[47] Claus P, Rahmfeld J, Robins H, Tannenhauser J and Zunger Y 2000 Isometries in anti-de Sitter and conformal superspaces J. High Energy Phys. JHEP07(2000)047 (arXiv:hep-th/0007099)

[48] Green M B and Schwarz J H 1984 Covariant description of superstrings Phys. Lett. B 136 367

[49] Hatsuda M and Sakaguchi M 2002 Wess–Zumino term for AdS superstring Phys. Rev. D 66 045020 (arXiv:hep-th/0205092)

[50] Volkov D V and Akulov V P 1972 Possible universal neutrino interaction JETP Lett. 16 438 (available at http://www.jetpletters.ac.ru/ps/1766/article_26864.pdf)

[51] Bandres M, Lipstein A and Schwarz J H 2008 Studies of the ABJM theory in a formulation with manifest SU(4) R-symmetry J. High Energy Phys. JHEP09(2008)027 (arXiv:0807.0880 [hep-th])

[52] Nishimura M and Tanii Y 2009 PSU(2, 2|4) transformations of IIB superstring in AdS5 × S5 J. Phys. A: Math. Theor. 42 095401 (arXiv:hep-th/0609119)

[53] Chen B and Wu J-B 2008 Semi-classical strings in AdS4 × CP3 J. High Energy Phys. JHEP09(2008)096 (arXiv:0807.0802 [hep-th])

McLoughlin T and Roiban R 2008 Spinning strings at one-loop in AdS4 × CP3 J. High Energy Phys. JHEP12(2008)101 (arXiv:0807.3965 [hep-th])

Alday L F, Arutyunov G and Bykov D 2008 Semiclassical quantization of spinning strings in AdS4 × CP3 J. High Energy Phys. JHEP11(2008)089 (arXiv:0807.4400 [hep-th])

Krishnan C 2008 AdS2/CFT1 at one loop J. High Energy Phys. JHEP09(2008)092 (arXiv:0807.4561 [hep-th])

McLoughlin T, Roiban R and Tseytlin A A 2008 Quantum spinning strings in AdS4 × CP3: testing the Bethe ansatz proposal J. High Energy Phys. JHEP11(2008)069 (arXiv:0809.4038 [hep-th])