An Analysis of Mutual Communication between Qubits by Capacitive Coupling

Takahiro Murakami, Masataka Inuma, Tohru Takahashi, Yutaka Kadoya, and Masamichi Yamanishi

Graduate School of Advanced Sciences of Matter, Hiroshima University, Higashi-Hiroshima, 739-8530, Japan

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Abstract

A behavior of a two qubit system coupled by the electric capacitance has been studied quantum mechanically. We found that the interaction is essentially the same as the one for the dipole-dipole interaction; i.e., qubit-qubit coupling of the NMR quantum gate. Therefore a quantum gate could be constructed by the same operation sequence for the NMR device if the coupling is small enough. The result gives an information to the effort of development of the devices assuming capacitive coupling between qubits.

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*Corresponding Author: tohrut@hiroshima-u.ac.jp
Toward the realization of quantum computers, various type of devices have been studied intensively in systems such as ion traps [1, 2], NMR [3], linear optics [4, 5], cavity QED with atoms [6], quantum dots in optical cavity [7, 8], and Josephson-Junction [9, 10], etc. In terms of the basic physics of the quantum gate, the quantum system must satisfy requirements that; 1) the transition between two levels in each qubit has to be controllable independently for both phase and amplitude, 2) qubits must have suitable mutual interaction to construct a quantum gate. It has to be emphasized that the first requirement means that the two-qubit system has to have a function to switch interaction between qubits during the quantum gate operation.

Recently, an observation of Rabi oscillation in two-qubit system using the Josephson-Junction device operating in the charge regime has been reported [11]. It is an encouraging evidence of the existence of the capacitive interaction between qubits. However, the construction of the universal quantum gate is yet to be demonstrated both theoretically and experimentally, i.e., a way of switching interaction between qubits is necessary to be demonstrated. The switching could be realized by either an embedded mechanism in the device or by a sophisticated operation with proper qubit-qubit interaction. The first one is simple and straightforward to understand, however, it may be difficult in the Josephson-Junction device in the charge regime or the Exciton-Photon device [12, 13] since the device itself has to have a kind of mechanism to decouple two qubits.

For later case, a typical and a widely used way by NMR devices is to utilize the dipole-dipole interaction between qubits which can be described by Hamiltonian as:

\[
H_{\text{dipole}} = \Omega_i \sigma^i_z + \Omega_j \sigma^j_z + \omega_i \sigma^i_x + \omega_j \sigma^j_x + \omega_{ij} \sigma^i_z \sigma^j_z
\]

where \(\Omega_{i(j)}\) and \(\omega_{i(j)}\) are the Rabi oscillation strength and the energy level of the quantum states in the i(j)-th qubit, while the last term describes dipole-dipole coupling between the i-th and j-th qubits with the strength of \(\omega_{ij}\). The Pauli matrix \(\sigma^i_z\) stands for the magnetic or the electric dipole operator depending on the devices being considered.

The possibility of multi-qubits coupling using Josephson-Junction devices with the interaction has been discussed in [9] using a LC-oscillator mode coupling between qubits, however, the Josephson-Junction device described in [11] and the Exciton-Photon device intend to use the capacitive coupling between two qubits rather than the dipole-dipole coupling. In these devices, the excited and the ground state in a qubit are characterized by the
difference of the electric charge rather than the direction of the dipole moment so that the interaction of the two qubits is not described by the same Hamiltonian as (1).

The operation of single qubit by the Josephson-Junction devices in the phase regime has been reported [14] and the quantum mechanical behavior of two-qubit coupling via capacitive coupling in the phase regime has been studied by several authors [15, 16, 17]. These works showed the way to construct the universal quantum gate with the Josephson-Junction devices in the phase regime.

In this letter, the quantum mechanical calculation of the behavior of the two-qubit system operating in the charge regime is reported. We show that the system has the same nature with the dipole-dipole coupling, therefore, the same operation with the NMR devices are applicable to construct quantum gates in weak coupling regime.

In order to see the behavior of the capacitive coupling between two qubits, we analyzed the wave function of a two-qubit system in four dimensional vector space; \( \psi \equiv \varphi_1 \otimes \varphi_2 \), where the basis of the space is defined explicitly as

\[
\begin{pmatrix}
|1\rangle_1 \\
|0\rangle_1
\end{pmatrix} \otimes
\begin{pmatrix}
|1\rangle_2 \\
|0\rangle_2
\end{pmatrix} =
\begin{pmatrix}
|1\rangle |1\rangle \\
|1\rangle |0\rangle \\
|0\rangle |1\rangle \\
|0\rangle |0\rangle
\end{pmatrix}.
\]

(2)

The time evolution of each qubit can be described by the Schrodinger’s equation as \( i\hbar \frac{d\varphi_i}{dt} = H_i \varphi_i \), where \( H_i \) is the Hamiltonian of i-th qubit and its explicit form is, \( H_i = \begin{pmatrix}
\Delta_i & a_i \\
a_i & -\Delta_i
\end{pmatrix} \) with \( \Delta_i \) and \( a_i \) being the energy level and the Rabi oscillation strength of the qubit. It is assumed, as in all proposed devices, that Rabi oscillation in the qubit can be controlled by changing the energy level \( \Delta_i \) via external parameters such as voltages applied to the device. Using these Hamiltonians, the time evolution of the two-qubit system \( \psi \) in four dimensional space is described as \( i\hbar \frac{d\psi}{dt} = (H_1 \otimes I + I \otimes H_2 + H_{12}) \psi \) where \( H_{12} \) stands for the interaction between two qubits. As for the \( H_{12} \), we assume that the electric charge appears only for the excited state so that an additional energy is put only when both qubits are in the excited
state $|1\rangle \langle 1|$ state in (2). The Hamiltonian to describe the system is;

$$H_{\text{cap}} = \begin{pmatrix} \Delta_1 + \Delta_2 & a_2 & a_1 & 0 \\ a_2 & \Delta_1 - \Delta_2 & 0 & a_1 \\ a_1 & 0 & -\Delta_1 + \Delta_2 & a_1 \\ 0 & a_1 & a_1 & -\Delta_1 - \Delta_2 \end{pmatrix} + \begin{pmatrix} \Delta_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$ (3)

where $\Delta_{12}$ in the second term is the coupling energy between two qubits. This assumption contrasts to the dipole-dipole interaction described in Hamiltonian (1). In fact, in four dimensional vector space, Hamiltonian (1) can be expressed as;

$$H_{\text{dipole}} = \begin{pmatrix} \omega_1 + \omega_2 & \Omega_2 & \Omega_1 & 0 \\ \Omega_2 & \omega_1 - \omega_2 & 0 & \Omega_1 \\ \Omega_1 & 0 & -\omega_1 + \omega_2 & \Omega_2 \\ 0 & \Omega_1 & \Omega_2 & -\omega_1 - \omega_2 \end{pmatrix} + \begin{pmatrix} \omega_{12} & 0 & 0 & 0 \\ 0 & -\omega_{12} & 0 & 0 \\ 0 & 0 & -\omega_{12} & 0 \\ 0 & 0 & 0 & \omega_{12} \end{pmatrix}$$ (4)

where the second term is a four dimensional expression of the dipole-dipole interaction. We see the coupling energies are added symmetrically to the diagonal elements of the Hamiltonian and they also change their sign as the direction of a dipole flips. This feature plays an essential role to switch off the interaction between qubits effectively by the refocusing operation.

In order further to see characteristics of the capacitive coupling, it is useful to re-write the Hamiltonian (3) in a two component form as;

$$H'_{\text{cap}} = \frac{\Delta_{12}}{4} I + a_1 \sigma_x^1 + a_2 \sigma_x^2 + \left( \Delta_1 + \frac{\Delta_{12}}{4} \right) \sigma_z^1 + \left( \Delta_2 + \frac{\Delta_{12}}{4} \right) \sigma_z^2 + \frac{\Delta_{12}}{4} \sigma_z^1 \sigma_z^2$$ (5)

A comparison of the Hamiltonian (1) and (5) clearly shows similarities and differences of the two couplings schemes. Both of the two have the same type of the dipole coupling term as seen in the last term of Hamiltonians. On the other hand, in Hamiltonian (5), the coupling energy $\Delta_{12}$ are added to the energy level of each state as shown in the fourth and the fifth term. As a result, the energy level, $E_{1(2)}$, of the quantum state in the 1(2)-th qubit can be expressed as;

$$E_{1(2)} = \Delta_{1(2)} + \frac{\Delta_{12}}{4} \pm \frac{\Delta_{12}}{4}$$ (6)

where the first two terms represent modified but fixed energy level of the quantum state in the qubit while the last term is the contribution from the dipole type coupling in Hamiltonian...
The sign of the last term depends on the relative states of the two qubits. This fact shows that interaction of the capacitive coupling can be described essentially by the same form with the dipole-dipole couplings. Therefore, it may be possible to perform the same quantum gate operations which have been applied on devices of the dipole-dipole coupling such as NMR devices.

To realize the operation discussed above, the most important condition on the parameters is the strength of the qubit-qubit coupling, $\Delta_{12}$. Since the energy level of the quantum state in the qubit, $E_{1(2)}$, depends on the state of the neighboring qubit as expressed by the $\pm$ sign in the equation (6), the quantum state of a qubit affects the condition of Rabi oscillation of neighboring qubit. This fact does not allow the independent control of each qubit. However, if the $\Delta_{12}$ is smaller enough than the $a_{1(2)}$, the condition of Rabi oscillation can be virtually independent of neighboring qubit as typical width of Rabi resonance is its strength $a_{1(2)}$.

It has to be reminded that the situation described above is the same for the interaction with the Hamiltonian (1); i.e., for NMR devices. The quantum operations demonstrated using the NMR devices always have been performed in the weak coupling regime. Since the spin-spin coupling of the NMR devices is so small that the condition of weak coupling has been satisfied without any special treatment.

It is now known that the capacitive coupling interaction also has the dipole-dipole feature as shown in the Hamiltonian (3), so that devices with capacitive coupling could be operative as a universal quantum gate utilizing the same operation sequence on the NMR devices.

In order to see feasibility of the quantum computation with weak capacitive coupling, we performed a numerical calculation of a quantum gate operation using the Hamiltonian (3). As an example of the two-qubit operation, we tried a controlled-NOT operation by the procedure commonly used in NMR devices which is schematically expressed in Fig. 1. It has to be noted that even though it is a single controlled-NOT operation, it consists of all necessary operations for general quantum gate operations. The controlled-NOT operation described in Fig. 1 is expressed in term of a unitary transformation as;

$$\psi_i \Rightarrow \psi_f = \sqrt{-i} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \psi_i$$  \hspace{1cm} (7)$$

after subtracting overall phase factor. In the calculation, the initial state was chosen as $\psi_i = \ldots$
FIG. 1: The controlled-NOT operation for a NMR device described in Ref. [18]. $R_i^{C(T)}(\theta)$ stand for $\theta$ rotation around the $i$ axis on the Control(C) or Target(T) bit. The first and the last $R_y(90)$ stand for Rabi oscillation of 90 degree while a series of operation denoted as $U$ is a phase operation on the qubits, showing that the operation includes all components necessary to construct the general quantum operations.

(1, 0, 0, 0) on the basis shown in Eq. (2) and the expected final state is $\psi_f = (0, e^{-1/4\pi}, 0, 0)$. As the result of the calculation, we plotted, in Fig. 2 the amplitude and the phase of the $|1\rangle |0\rangle$ state as a function of $\Delta_{12}$ normalized to $a_{1(2)}$, where we expect 1.0 and $-1/4\pi$ for the amplitude and the phase respectively. In the calculation, we also have to consider treatment of the Rabi oscillation strength, $a_{1(2)}$, in a qubit. Depending on the devices, we can assume...
that parameter $a_{1(2)}$ exists throughout the operation or appears only when the device is on Rabi oscillation. Since it depends on devices considered, we calculated both cases.

It can be concluded that the result are reasonably close to the expected values and are stable up to $\Delta_{12} \approx 0.1a_{1(2)}$. As for the treatment of $a_{1(2)}$, some deviation from the ideal value is seen if the $a_{1(2)}$ is ON throughout the calculations, particularly for the phase of the state. It is preferable to control $a_{1(2)}$ as is realized in NMR devices, however, the deviation appears to be acceptable level. It is also worthwhile mentioning that $\Delta_{12} \approx 0.1a_{1(2)}$ is much larger than those of typical NMR devices. It indicates that the devices with capacitive coupling potentially have advantage of larger signal than those of NMR devices.

In summary we analyzed the interaction between two charge qubits via capacitive coupling. We conclude that the coupling is essentially the same as the dipole-dipole type interaction and the device can be a operative as a quantum gate if the coupling is smaller enough than the Rabi oscillation strength. This fact gives an important information for the development of those devices such as Josephson-Junction or Exciton-Photon devices.

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