Y$^2$ Isochrones with an Improved Core Overshoot Treatment

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ABSTRACT

Convective core overshoot affects stellar evolution rates and the dating of stellar populations. In this paper, we provide a patch to the Y$^2$ isochrones with an improved treatment of convective core overshoot. The new tracks cover the transition mass range from no convective core to a fully developed convective core. We compare the improved isochrones to CMDs of a few well observed open star clusters in the Galaxy and the Large Magellanic Cloud. Finally we discuss future prospects for improving the treatment of core overshoot with the help of asteroseismology.

Subject headings: convection — stars: interiors — stars: evolution

1. Introduction

Understanding the physics of convective core overshoot is important in interpreting the color-magnitude diagrams (CMDs) and the luminosity functions of open star clusters, and in dating young and intermediate age stellar populations. Core overshoot has several effects on stellar evolution (see e.g. Stothers 1991). Among the most notable are the effects on the shape of the main sequence turnoff, on the rate of evolution in the main sequence and subgiant phases, and on the ratio of total lifetimes spent in the core hydrogen burning phase and the shell hydrogen burning stage. The implications of convective core overshoot for deriving a chronology of open star clusters in the Galaxy have long been recognized (Maeder
& Mermilliod 1981). Detailed comparisons of theoretical isochrones with star cluster CMDs suggest the need to include convective overshoot in stellar evolution calculations. Gaps in the stellar distribution near the main sequence, which are identified with the hydrogen exhaustion phase, have been observed in the CMDs of open star clusters. The location and size of these gaps yield better agreement with those theoretical isochrones that admit some amount of core overshoot (Stothers & Chin 1991; Carraro et al. 1993; Daniel et al. 1994; Demarque, Sarajedini & Guo 1994; Kozhurina-Platais et al. 1997; Nordström, Andersen & Andersen 1997).

The purpose of this paper is to improve on the treatment of convective core overshoot in the $Y^2$ isochrones (Yi et al. 2001, hereafter Paper I; Kim et al. 2003, hereafter Paper II). We focus on the mass range where convective cores begin to appear in stellar models near the main sequence. This transition region affects sensitively the morphology of isochrones and the luminosity function near the main sequence turnoff. As a result, it affects the ages derived for stellar populations based on the CMDs of star clusters. It also modifies the predicted integrated spectral energy distributions of stellar populations and their derived spectral ages in studies of distant galaxies (Yi et al. 2000).

In view of the importance of core overshoot for intermediate age stellar populations, we have calculated a patch to the original $Y^2$ isochrones to prevent the troublesome distortions that occurred in some instances near the turnoff in the original isochrones. Most of the interest in core overshoot in the grids of evolutionary tracks available in the literature has concentrated on massive stars with well developed convective cores (Maeder & Meynet 1988; Bertelli et al. 1990; Stothers & Chin 1991; Meynet et al. 1994). Even in the Geneva grids, in which intermediate mass evolutionary tracks have been included, no mention is made on the problems posed for isochrone construction by the narrow transition from the absence to the presence of a convective core as the mass increases (Schaller et al. 1992; Charbonnel et al. 1993). In a more recent study based on the Padova approach to core overshoot, Girardi et al. (2000) describe a treatment of the transition region in which the chosen critical transition mass, taken to be $1.0M_\odot$, is independent of chemical composition.

It is however well-known that the mass at which a convective core first appears along the main sequence is a sensitive function of chemical composition. This result must be taken into account in any detailed comparison with observation. Already, the original isochrones of Papers I and II addressed this chemical composition dependence by introducing a critical mass $M^\text{conv}_{\text{crit}}$, defined as the mass above which stars continue to have a substantial convective core even after the pre-MS phase of evolution is ended.

In addition to taking into account the dependence of $M^\text{conv}_{\text{crit}}$ on chemical composition, our new evolutionary tracks also include several improvements which are physically more
realistic, as detailed in §4. These changes are useful for chemical compositions that differ markedly from solar and for applications which require interpolation between the isochrones.

§2 summarizes the input physics and parameters used in the previous $Y^2$ isochrone papers. In §3, we discuss the physics of convective core overshoot and the procedures used to model its structural and evolutionary effects in stars. Semi-empirical estimates of the extent of core overshoot based principally on the CMDs of star clusters are given in §4. In §5, we then describe the new overshoot approach adopted in the critical mass transition region. Some comparisons between observed open cluster CMD’s are presented in §6. Finally, future prospects for a better physical understanding of convective core overshoot are briefly summarized in §7.

2. Input physics and parameters of the $Y^2$ isochrones

The evolutionary tracks presented in this paper were constructed using the Yale Stellar Evolution Code YREC with the same input physics and parameters as in Paper I and Paper II. A more complete discussion of the choice of parameters can be found in these two papers. A useful assessment of validity of the $Y^2$ stellar models and other models including a detailed comparison with observational data for stars in the pre-main sequence and early post-main sequence phases of evolution has recently been published by Hillenbrand & White (2004).

All evolutionary tracks were started at the pre-main sequence birthline. In order to preserve internal consistency with the rest of the $Y^2$ database, the same set of chemical compositions were used. The solar mixture of Grevesse, Noels & Sauval (1996) was adopted for the heavy elements abundances, which yields a metal-to-hydrogen ratio $(Z/X)_\odot = 0.0244$. The mixing length parameter in the convective envelopes, calibrated on the Sun, was also kept unchanged, i.e. $l/H_p = 1.7432$. Consistent with the solar calibration and the initial chemical composition $(Y, Z)_0 = (0.23, 0.00)$, we adopted the chemical enrichment formula $Y = 0.23 + 2Z$. Following Paper II, we constructed evolutionary tracks for $\alpha$-enhanced mixtures corresponding to $[\alpha/Fe] = 0.0, 0.3$ and 0.6, respectively. Table 1 lists the chemical compositions of the evolutionary tracks constructed for this paper. A more complete discussion of the input parameters can be found in Papers I and II.

2.1. Microscopic physics

In the interior, the OPAL radiative opacities (Rogers & Iglesias 1995; Iglesias & Rogers 1996) for the appropriate mixtures were used. At low temperatures, the opacities from
Alexander & Ferguson (1994) were used for the scaled solar mixtures. For the $\alpha$-enhanced mixtures, the low temperature opacity tables specifically calculated for Paper II by Alexander were adopted. The conductive opacities from Hubbard & Lampe (1969) were adopted for $\log \rho \leq 6.0$ and from Canuto (1970) in the relativistic regime where $\log \rho > 6.0$. The OPAL equation of state was taken from the work of Rogers, Swenson & Iglesias (1996). Helium diffusion was included in the evolutionary calculations with the help of the Bahcall-Loeb formula (Bahcall & Loeb 1990; Thoul, Bahcall & Loeb 1994). As before, the energy generation routines of Bahcall & Pinsonneault (1992) and the cross-sections listed in Bahcall (1989) have been used.

2.2. Color transformations

Theoretical properties ($[Fe/H], L, T_{\text{eff}}$) have been transformed into colors and magnitudes using the semi-empirical color transformation tables of Lejeune, Cuisinier & Buser (1998; hereafter LCB) and of Green, Demarque & King (1987; hereafter GDK). The tables have been normalized so that the absolute visual magnitude of the Sun ($M_v^\odot$) becomes 4.82 (Livingston 2000), regardless of the table used. The color transformation is for the filter systems of $(UBV)_{\text{Johnson}}(RI)_{\text{Cousins}}$ in the GDK table and of $(UBV)_{\text{Johnson}}(RI)_{\text{Cousins}}(JHKL)_{\text{ESO}}$ in the LCB table. A fuller discussion can be found in Paper I.

3. Convective core overshoot

3.1. The physical process

Convective core overshoot is understood here as the presence of material motions and/or mixing beyond the canonical boundary for convection defined by the classic Schwarzschild criterion (1906). Early investigations by Roxburgh (1965), who used the mixing length theory, and by Saslaw & Schwarzschild (1965), who based their argument on thermodynamic grounds (i.e. the edge of the convective core in massive stars is sharply defined in an entropy diagram), suggested that little overshoot takes place at the edge of convective cores. But Shaviv & Salpeter (1973) pointed out subsequently that if one takes into account the presence of hydrodynamic motions and turbulence, one might expect some non-negligible amount of core overshoot.

More recently, Zahn (1991) has discussed the complex physical interaction between convective and radiative transfer in the overshoot region. It turns out that the nature of the overshoot depends sensitively on the details of the local physics at the convective core
edge. The local Péclet number, which characterizes the relative importance of radiative and
turbulent diffusivity, determines whether the temperature gradient in the overshoot region
is adiabatic (the situation called “penetration” by Zahn), or whether the local radiative
transfer dominates the energy transport. In the latter case, overshoot does not modify the
stable temperature gradient beyond the Schwarzschild limit, and its main effect is to cause
mixing into the radiatively stable layers just outside the convective core edge. This kind of
overshoot has been described as “overmixing”.

It follows from the above discussion that a proper treatment of convective core overshoot
requires a radiative hydrodynamic treatment near the convective boundary. Kuhfuß (1986)
has proposed a non-local treatment based on the anelastic approximation for convection and
the diffusion approximation for radiation. An application of the Kuhfuß theory to describe
overshoot from convective stars of the upper main sequence has been made by Straka et al.
(2004), who proposed some seismic tests to test the theory.

3.2. Core overshoot in stellar models

In the absence of a general theory of convective overshoot which would enable us to
calculate the amount of core overshoot for a star of a given mass and chemical composition,
a semi-empirical phenomenological approach must be used in calculations of stellar evolution.
Several computational schemes of various degrees of sophistication in treating the physics of
overshoot have been discussed in the literature (Prather & Demarque 1974; Maeder 1975;
Maeder & Meynet 1988; Bressan, Bertelli & Chiosi 1981; Bertelli et al. 1990; Straka et
al. 2004). The most common parameterization of core overshoot, and the one that was
adopted in the $Y^2$ evolutionary tracks, is to evaluate the overshoot length in terms of the
local pressure scale height $H_p$ at or across the formal edge of the convective core, where the
core edge is defined by the Schwarzschild (1906) criterion. The amount of core overshoot is
then given by the product $\Lambda_{OS} H_p$, where $\Lambda_{OS}$ is a constant parameter less than unity. In the
$Y^2$ models, the temperature gradient in the overshoot region was assumed to be unaffected
by the overshoot, and the main result was an extension of the mixed region beyond the edge
of the convectively unstable core.

The present paper is primarily concerned with the calculation of core overshoot for stars
with masses in the vicinity of $M_{\text{crit}}^{\text{conv}}$, the critical mass at which a convective core makes its
appearance on the main sequence. This simple parameterization of core overshoot in terms
of the pressure scale height breaks down as the convective core vanishes because $H_p \to \infty$
as $r \to 0$ (see e.g. Wuchterl & Feuchtinger 1998).
Roxburgh’s integral constraint is another way to quantify the amount of core overshoot since it provides an upper limit to the radial extent of convective penetration (Roxburgh 1989,1998; Zahn 1991; Canuto 1997). Because it only provides an upper limit, the value of Roxburgh’s integral must in practice be multiplied by an adjustable parameter to achieve good agreement with observations of cluster CMDs. This free parameter, which is analogous to the \( \Lambda_{\text{OS}} \) parameter discussed above, was evaluated for the open cluster NGC 6819 to be about 0.5 (Rosvick & VandenBerg 1998).

It is often desirable to combine the theoretical limit to core overshoot set by Roxburgh’s integral constraint and the convenience of the pressure scale height approach in stellar evolution calculations. Recent examples can be found in the work of Woo & Demarque (2001) and Di Mauro et al. (2003). The Woo-Demarque approach, however, requires a much finer stellar mass grid of evolutionary tracks, than is available for the \( Y^2 \) isochrones. This is because the transition in the Woo-Demarque treatment takes place in a mass range narrower than the mass grid size available in the \( Y^2 \) database (0.1\( M_\odot \)).

4. Empirical estimates of the extent of core overshoot

Observations of star cluster CMDs provide a useful guide in evaluating \( \Lambda_{\text{OS}} \) for a large grid of stellar evolutionary tracks. The semi-empirical approach of comparing synthetic CMDs based on theoretical isochrones to the CMDs and luminosity functions of observed star clusters is adopted. Studies of star cluster CMDs indicate that \( \Lambda_{\text{OS}} \approx 0.2 H_p \) for clusters with ages in the range 1-3 Gyr, with turnoff masses \( M_{\text{TO}} \) in the range 1.4-2.0 \( M_\odot \), and that \( \text{OS} \approx 0.0-0.1 H_p \) for older clusters (4-6 Gyr and \( M_{\text{TO}} = 1.1-1.3 \) \( M_\odot \)) (Stothers 1991; Demarque, Sarajedini & Guo 1994; Kozhurina-Platais et al. 1997). This statement is compatible with the conclusion discussed in the previous section that if the radial extent of core overshoot is expressed in terms of the local pressure height at the core edge, the effective value of the parameter \( \Lambda_{\text{OS}} \) must decrease as the core radius decreases.

There are other independent ways to determine \( \Lambda_{\text{OS}} \). Detached eclipsing binaries in which the components have convective cores are consistent with some core overshoot, of the order of 0.2\( H_p \) for stars with fully developed convective cores (Ribas, Jordi, & Giménez 2000). It is important to remember, however, that such overshoot estimates depend on all other features of the standard stellar models being correct. There are several uncertainties in stellar models that may affect our core overshoot estimates. The boundaries of convective regions in stellar models can be sensitive to the adopted composition parameters, such as helium abundance, \([\text{Fe/H}]\) and \([\alpha/\text{Fe}]\), all of which entail uncertainties. And even if the composition were perfectly well-known, Stothers & Chin (1991) have pointed out the high
sensitivity of core overshoot to the adopted opacities. For example, increases in radiative opacities from the OPAL group (Iglesias & Rogers 1996) over the previous generation Los Alamos Opacity Library (Huebner et al. 1977) decrease the need for overshoot in comparison with observational data.

Internal rotation may also play a role in core overshoot. Deupree (1998, 2000) has considered the combined effects of rotation and convective overshoot in massive stars, which have large convective cores, based on the two-dimensional hydrodynamic simulations. Rotation could be significant in the case of less massive stars as well (in the range $1.5 - 2.0M_\odot$), with shear-driven turbulence near the edge of the convective core mixing material from the helium-enriched core into the envelope (e.g. see rotating star models by Pinsonneault et al. 1991). Rotationally induced mixing could thus in effect enlarge the size of the mixed convective core.

By providing a powerful independent way of probing the stellar interior, stellar seismology will provide new tools to measure the extent of convective regions and overshooting. The promise of seismology will be briefly discussed in the last section of this paper.

5. Isochrone construction

5.1. Original $Y^2$ treatment

Guided by the observational studies mentioned above, the published $Y^2$ isochrones were constructed assuming $\Lambda_{OS} = 0.2$ in stars with well developed convective cores. For convenience, and also based on the available observational data, the isochrones were divided into two classes, i.e. young isochrones ($\leq 2$ Gyr), in which full core overshoot was taken into account, and old isochrones ($\geq 3$ Gyr) in which no overshoot (i.e. $\Lambda_{OS} = 0.0$) was included regardless of the stellar mass. By inspection of the no overshoot evolutionary tracks, we first found the critical mass $M_{conv}^{crit}$ above which stars continue to have a substantial convective core even after the pre-MS phase is ended. The critical mass $M_{conv}^{crit}$ is listed in Table 1 for each chemical composition. Evolutionary tracks were constructed including core overshoot for stars with masses in excess of $M_{conv}^{crit}$. The young isochrones were then constructed using the evolutionary tracks with overshoot for $M \geq M_{conv}^{crit}$, and evolutionary tracks with no overshoot for lower masses. Note that the mass interval between our adjacent models in the grid of evolutionary tracks being 0.1 $M_\odot$, the value of $M_{conv}^{crit}$ effectively used in the interpolation is between the listed value of $M_{conv}^{crit}$ and ($M_{conv}^{crit} - 0.1$). For isochrones older than 3 Gyr, we used stellar models with no overshoot regardless of mass. This procedure is satisfactory for stars with metallicities close to that of the Sun. For example, for solar composition $M_{conv}^{crit}$
is near 1.2 $M_\odot$, which means that the procedure seems acceptable when $M_{TO} \geq (M_{crit}^{conv} + 0.2)$.

Subsequent interpolations of the $Y^2$ isochrones in a variety of research applications revealed that this step function approach could create locally unphysical distortions of the isochrone and luminosity function. The assumptions made in the original $Y^2$ isochrones can be invalid for chemical compositions very different from the solar abundance. In addition, setting the transition age at 3 Gyr, which was selected on the basis of the available cluster CMD’s, is unrealistic for high metallicity isochrones. The “transition age” $t_{trans}$ (i.e. the age at which convective cores begin to affect the turnoff morphology) increases sharply with increasing metallicity. For $Z = 0.0001$, $t_{trans}$ is 1.2 Gyr, whereas for $Z = 0.04$, $t_{trans}$ reaches 7 Gyr.

5.2. Improved core overshoot treatment

The improvement in our overshoot treatment is two folds. First, we take into account the metallicity dependence of $M_{crit}^{conv}$. As a result, the transition age $t_{trans}$ defined in §5.1 is no longer fixed at 3 Gyr but changes as a function of metallicity. Second, we include the condition that the extent of core overshoot should decrease to zero as the convective core radius approaches zero. As noted above, this change is required for the most metal-rich mixtures, in which small convective cores may still be present near the turnoff for ages well above 3 Gyr. The step function has now been replaced by a smoother function for the overshoot parameter in the evolutionary calculations. The precise dependence and the numerical procedure are chosen for compatibility with the rest of the $Y^2$ evolutionary tracks, within the simple pressure scale height description of overshoot used in the models. A grid of evolutionary tracks has been constructed in the transition mass region, and a revised set of isochrones was then calculated which incorporates the improved evolutionary tracks.

For the purpose of this study, we have adopted the following prescription for ramping between $M_{crit}^{conv}$ and $(M_{crit}^{conv} + 0.2)$:

\begin{align*}
\Lambda_{OS} &= 0.0 \quad \text{for} \quad M < M_{crit}^{conv} \\
\Lambda_{OS} &= 0.05 \quad \text{for} \quad M = M_{crit}^{conv} \\
\Lambda_{OS} &= 0.1 \quad \text{for} \quad M = M_{crit}^{conv} + 0.1 \\
\Lambda_{OS} &= 0.15 \quad \text{for} \quad M = M_{crit}^{conv} + 0.2 \\
\Lambda_{OS} &= 0.2 \quad \text{for} \quad M > M_{crit}^{conv} + 0.2
\end{align*}
The values of $M_{\text{crit}}^{\text{conv}}$ used in the evolutionary tracks are given in Table 1, as function of chemical composition. They were then combined with the existing evolutionary tracks calculated with $\Lambda_{\text{OS}} = 0.2$ for $M > M_{\text{crit}}^{\text{conv}} + 0.2$, and with $\Lambda_{\text{OS}} = 0.0$ for $M < M_{\text{crit}}^{\text{conv}}$. New isochrones were constructed using the same isochrone generation codes as in Papers I and II only after minor alterations.

The four panels of Figure 1 compare the transition region for isochrones before and after applying the patch, for $Z = 0.0001$, 0.001, 0.004 and 0.02, respectively. A major usefulness of the overshoot-ramping is that it allows a much smoother age interpolation between isochrones when the population’s turnoff mass is near $M_{\text{crit}}^{\text{conv}}$, that is, near the transition age. Figure 2 illustrates the improvement. The old and new overshoot treatments make little or no changes for ages larger than the transition age because such old isochrones are constructed solely based on the stellar models without a convective core. In the case of $Z = 0.001$, the transition age is roughly 1.9–2.0 Gyr. As expected, little change has been made to the 4 Gyr isochrone. The problems in the previous isochrones are however clearly visible in the interpolated isochrones for smaller ages. The two intermediate-age isochrones (1.8 and 2.5 Gyr) were produced using the interpolation routine provided in Paper 1. Three features, marked as (a), (b) and (c), summarize the problems with the Paper 1 isochrones. Feature (a) in dotted line shows a second dip below the major main-sequence dip. This is simply an artifact caused by the previous delta function treatment on the overshoot parameter. Feature (b) clearly shows how the interpolation fails for the age near the transition age. Lastly, feature (c) shows a failure in the temperature interpolation on the red giant branch. All these problems have now been solved by the introduction of the ramping treatment on the overshoot. In the rest of this paper, we shall refer to the patched isochrones as $Y^2 OS$ isochrones, which can be found at any of the three $Y^2$ isochrone websites:

http://csaweb.yonsei.ac.kr/ kim/yyiso.html
http://www-astro.physics.ox.ac.uk/ yi/yyiso.html
http://www.astro.yale.edu/demarque/yyiso.html

6. Comparisons with intermediate-age star clusters

Identifiable gaps in the stellar distribution, which can be used to evaluate the extent of core overshoot, are best observed in the CMDs of intermediate age clusters, which are generally more populous than the young star clusters. Such gaps have been observed in open clusters in the Galactic disk, and more recently in intermediate age clusters of the Large Magellanic Clouds. In this section, we compare the $Y^2 OS$ isochrones to a few well observed open cluster CMDs to illustrate the improvement of $Y^2 OS$ isochrones. We emphasize that
these comparisons merely show the compatibility of the OS isochrones with some of the available observational data, which cover only a small portion of parameter space. Indeed, the glitches encountered with the original $Y^2$ isochrone interpolations mostly occurred for compositions and ages for which we do not have observed CMDs in the Galaxy or the LMC, but which are important in the systematic modeling of stellar populations.

6.1. Open star clusters in the Galaxy

Field star contamination is a problem in establishing open cluster membership (Kozhurina-Platais et al. 1995; Platais et al. 2003). In addition, the presence of binary stars modifies the stellar distribution near the main sequence turnoff. We present CMD fits for selected Galactic open clusters for which proper motion membership and binary membership have been established by the WOCS (WIYN open cluster studies) collaboration (Mathieu 2000; Sarajedini, Mathieu & Platais 2003). We have selected the following WOCS clusters: NGC 3680 (Khozurina-Platais et al. 1997), NGC 2420 (Demarque et al. 1994), M67 (Girard et al. 1989; Sandquist 2004), and NGC 6791 (Kaluzny & Udalski 1992), in order of increasing age. NGC 3680 and M67 are close to solar in [Fe/H]. NGC 6791 is more metal rich than the Sun (Peterson & Green 1998), while the thick disk cluster NGC 2420 is metal poor (Grocholski & Sarajedini 2003).

In order to fit isochrones to observed CMDs of star clusters, we first chose [Fe/H] for each cluster and determined the best-fitting age by adjusting the cluster distance modulus and reddening. The adopted parameters for each cluster are listed in Table 2. Although it is not straightforward to compare our adopted values of the distance modulus and reddening with literature values, due to the uncertainties of metallicity and metallicity dependence of the models, our values agree in general with literature values, e.g., $(m-M)_V=10.20$ and $E(B-V)_o=0.06$ of NGC 3680 (Anthony-Twarog & Twrog 2004), $(m-M)_V=12.10$ and $E(B-V)_o=0.04$ of NGC 2420 (von Hippel & Gilmore 2000), $(m-M)_V=9.65$ and $E(B-V)_o=0.038$ of M67 (Sandage et al. 2003), and $(m-M)_V=13.42$ and $E(B-V)_o=0.17$ of NGC 6791 (Chaboyer et al. 1999a; Kaluzny & Rucinski 1995). Within the uncertainties, the ages are in each case within the range of recent age determinations based on theoretical isochrones. Since the total metallicity $Z$ includes both [Fe/H] and [$\alpha$/Fe], we determined ages for each [Fe/H] for several values of the $\alpha$ enrichment parameter.

Figure 3 shows three isochrones, depending on the $\alpha$ parameter, fitted to the observed CMD for each cluster. Within the uncertainties which are still very large (see e.g. the recent compilation by Salaris et al. 2004), the ages listed in Table 2 are compatible with the ages derived in other investigations using independent isochrone calculations. For NGC 3680, we
assign an age in the range 1.3–1.7 Gyr, to be compared with 1.3–1.7 Gyr from Kozhurina-Platais et al. (1997). For NGC 2420, we find 1.4–2.0 Gyr, using [Fe/H]=−0.27, which is compatible with the age of 2.4±0.2 Gyr previously derived by Demarque et al. (1994) using [Fe/H]=−0.7 (and more recently by Grocholski & Sarajedini 2003). For M67, 3.6 Gyr (for solar abundance) is in good agreement with the recently derived age of 3.7 Gyr, based on a model including detailed diffusion effects (Michaud et al. 2004). Finally, in the case of NGC 6791, the only good fit was found for the solar mixture. Our age estimate for NGC 6791 is 8.0 Gyr, to be compared to 8.0±0.5 Gyr derived by Chaboyer et al. (1999a).

It is clear from inspection of Figure 1 and Figure 2 that as satisfactory fits as in Figure 3 could not be achieved with the old-version isochrones in the transition age region. We must emphasize, however, that the original approach was tailored to fitting isochrones to CMD’s specifically for the ages and compositions of nearby stars clusters in the Galaxy. The need to improve the ramping and interpolation procedure became more evident when the $Y^2$ isochrones became increasingly utilized for more systematic studies of population synthesis covering more complete samplings of the parameter space.

6.2. Three clusters in the Large Magellanic Cloud

The CMDs of intermediate age open clusters in the Large Magellanic Cloud (LMC) are of particular interest because they are more metal poor than in the Galaxy. Because of their large distances from us, it is not possible to test membership through proper motion studies. Few radial velocity measurements are available. Although the membership problem is not as severe in the LMC as in the Galaxy because the clusters are more populated and field star contamination is less important, estimates of convective core overshoot from their CMDs cannot be made as securely as in Galactic open clusters. In addition, the binary star population is significant in these clusters (Woo et al. 2003). Figure 4 shows a first-cut comparison (ignoring binary stars) of our $Y^2$ OS isochrones to three CMDs of LMC clusters obtained with the Very Large Telescope at the European Southern Observatory. Isochrone fits to the CMDs for NGC 2173, SL 556 and NGC 2155 are displayed, assuming the metallicity $Z = 0.004$ (Gallart et al. 2003). More detailed analyses of the three CMDs, which provide independent age estimates based on the Yale and Padova isochrones, have been published by Woo et al. (2003) and Bertelli et al. (2003), respectively. Once [Fe/H] is fixed, it is not possible to use the same distance modulus and reddening for each different value of $\alpha$. Since the purpose of the CMD fitting is to demonstrate the improved fit near the turnoff of $Y^2$ OS isochrones, rather than to determine exact cluster parameters, we used a fixed metallicity $Z$, instead of a fixed value of [Fe/H], and the same age for each $\alpha$ parameter.
As in the case of Galactic open clusters, the new isochrones show a good fit to the observed CMDs.

7. Future prospects: observations and modeling

On the observational side, two space asteroseismology missions, WIRE (Buzasi 2000) and MOST (Matthews et al. 2000) have begun to provide data on a few selected bright star targets. The analysis of these data holds great promise for testing the internal structure of these stars and the chemical composition profile in their interiors (Basu et al. 2004; Guenther & Brown 2004). In particular, seismology is expected to reveal the presence and extent of convective cores and envelopes in stellar interiors (Audard et al. 1995; Chaboyer et al. 1999b).

On the side of theoretical convection modeling, it is expected that future progress in core overshoot understanding will result from a combination of analytical and numerical hydrodynamical approaches. As an example, the implementation of the Kühnfluss theory in the YREC stellar evolution code has already yielded a more physically realistic description of the overshoot process (Straka et al. 2004). Because core overshoot reaches its full development in a narrow mass interval above \( M_{\text{conv crit}} \), a fine mass grid of evolutionary tracks, i.e. with a mass increment as small as 0.001 \( M_\odot \), will be required for future isochrone construction in the transition region. Already, extremely fine mass grids have been found to be necessary to fully exploit the information contained in high quality asteroseismic data (Guenther & Brown 2004). The confrontation of these models with seismic observations from the WIRE and MOST space missions will in the next few years provide the foundation for a physical theory of convective core overshoot on which to base the next generation of isochrones.

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Table 1: Adopted values of $M_{\text{crit}}^{\text{conv}}$

| Composition | $[\alpha/\text{Fe}]$ | 0.0 | 0.3 | 0.6 |
|-------------|----------------------|-----|-----|-----|
| x76997z00001 | 2.1 | 2.1 | 1.9 |
| x7697z0001  | 1.6 | 1.6 | 1.5 |
| x7688z0004  | 1.4 | 1.5 | 1.3 |
| x767z001    | 1.3 | 1.4 | 1.2 |
| x758z004    | 1.2 | 1.3 | 1.2 |
| x749z007    | 1.2 | 1.2 | 1.2 |
| x74z01      | 1.2 | 1.2 | 1.2 |
| x71z02      | 1.2 | 1.2 | 1.2 |
| x65z04      | 1.1 | 1.1 | 1.0 |
| x59z06      | 1.1 | 1.1 | 1.0 |
| x53z08      | 1.0 | 1.0 | 0.9 |

Table 2: Selected parameters for each cluster

| Name         | [Fe/H] | $(m - M_V)$ | E(B-V) | $[\alpha/\text{Fe}]$ | 0.0 | 0.3 | 0.6 | age   |
|--------------|--------|-------------|--------|----------------------|-----|-----|-----|-------|
| NGC 3680     | 0.0    | 10.4        | 0.04   | 1.7                  | 1.4 | 1.2 |
| NGC 2420     | -0.27  | 12.2        | 0.05   | 2.0                  | 1.7 | 1.4 |
| M67          | 0.0    | 9.9         | 0.03   | 3.6                  | 3.4 | 2.8 |
| NGC 6791     | 0.3    | 13.5        | 0.17   | 8.0                  | -   | -   |
Fig. 1.— Comparison between old (dotted) and revised (continuous) isochrones in the convective core transition region. Note that the transition occurs at a different age range for different chemical composition. The change due to the improvement in the overshoot treatment in this study is only visible near the “transition age”.
Fig. 2.— The problems with our previous isochrones due to their delta-function prescription for the overshoot are illustrated as (a), (b), and (c). See the text for details. The problems, apparent for the two interpolated isochrones (1.8 and 2.5 Gyr) near the transition age (1.9–2.0 Gyr), are all resolved in the new isochrones via our overshoot-ramping prescription.
Fig. 3.— The best-fitting isochrones with observed CMDs of four Galactic open clusters. Thick line: $Y^2$ OS isochrone with $[\alpha/\text{Fe}]=0.0$, dashed line: $[\alpha/\text{Fe}]=0.3$, thin line: $[\alpha/\text{Fe}]=0.6$. The smoothed turn-off shape of the new isochrones represents the observed CMD very well. Note that the isochrone ages for the same $[\text{Fe/H}]$ are different due to different $Z$ values. For NGC 6791, we could not fit the CMD with $[\alpha/\text{Fe}]=0.6$ isochrone. Even with the $[\alpha/\text{Fe}]=0.3$ isochrone, the fit is not acceptable, especially on the giant branch.
Fig. 4.— The best-fitting isochrones with observed CMDs of three LMC clusters. Thick line: Y^2 OS isochrone with [\alpha/Fe]=0.0, dashed line: [\alpha/Fe]=0.3, thin line: [\alpha/Fe]=0.6. Isochrones are compared with the part of CMDs where the single stars are located. In constrast to the Galactic open clusters, these LCM clusters are well populated and many unresolved binary stars are present. (For details on the effect of binaries on the synthetic CMD fitting, see Woo et al. 2003). The new isochrones represent the turn-off region of the observed CMD very well. For each cluster, (m – M_V) = 18.9, 18.6, 18.6 and E(V-R)=0.08,0.06,0.03 are used for NGC 2173, SL 556 and NGC 2155 respectively.