REVIEW OF INDIRECT METHODS USED TO DETERMINE THE $^1S_0$
NEUTRON-NEUTRON SCATTERING LENGTH

C.R. HOWELL
Duke University and Triangle Universities Nuclear Laboratory
Durham, NC USA

Calculations with realistic potential models show that a 1% change in the $^1S_0$ nucleon-nucleon (NN) potential strength changes the scattering length by 30%. It is this high sensitivity that makes the $^1S_0$ NN scattering lengths important in quantifying the amount to which charge symmetry is broken in the strong nuclear force. Of the three $^1S_0$ NN scattering lengths, the value of the neutron-neutron scattering length $a_{nn}$ is the most uncertain. A number of reactions that produce two neutrons with low relative energy have been used to determine $a_{nn}$. However, the two reactions that give $a_{nn}$ with the least theoretical uncertainty are pion-deuteron capture ($\pi^- + d \rightarrow n + n + \gamma$) and neutron-deuteron breakup ($n + d \rightarrow n + n + p$). Curiously, the values obtained using these two reactions are significantly different. In this talk the experimental techniques and theory used to determine $a_{nn}$ for each of these popular reactions will be reviewed. In addition, the results of the two most recent $nd$ breakup experiments will be reported.

1. INTRODUCTION

Most observed charge-symmetry-breaking (CSB) effects can be explained as being due to the differences in the masses (QMD) and electric charges of the $d$ and $u$ quarks [1]. These differences are manifested by the mass splitting in hadronic isospin multiplets and in the values of the $\rho-\omega$ and $\pi-\eta$ mixing amplitudes within meson-exchange potentials [1].

The QMD leads to a difference in the masses of the neutron and proton and many other hadrons, and to a difference between the neutron-neutron ($nn$) and proton-proton ($pp$) $^1S_0$ scattering lengths, $\Delta a = a_{nn} - a_{pp}$ [1]. Therefore, an experimental determination of the scattering length difference, $\Delta a$, gives a direct measure of CSB and can be related to the QMD. This high sensitivity of the scattering lengths to details of the nuclear force at the quark level also is reflected in nucleon-nucleon (NN) potential models that are based on meson-exchange phenomenology. For example, for most realistic NN potential models, a 1% change in the potential strength results in a 30% shift in the value of the calculated scattering length. This enormous sensitivity of the NN scattering length to the potential strength can be understood to first order using effective range phenomenology. For a square well potential, the fractional change in the scattering length due to a small change in the potential depth is given by

$$\frac{\delta a_{nn}}{a_{nn}} = 1.23 \left( \frac{a_{nn}}{r_{nn}} \right) \delta V / V. \quad (1)$$

1This work was supported in part by the U.S. Department of Energy, Office of High Energy and Nuclear Physics, under Grant No. DE-FG02-97-ER41033
Using typical values of $a_{nn} = 18.8 \text{fm}$ and $r_{nn} = 2.8 \text{fm}$, implies that a 1% change in the depth of the square well potential will result in about a 10% shift in the value of $a_{nn}$.

The $a_{pp}$ has been measured directly to high precision (of order 0.01 fm) using two-nucleon scattering. However, there is a relatively large uncertainty of about $\pm 0.4 \text{fm}$ in correcting the measured value of $a_{pp}$ for electromagnetic ($em$) effects, which are sizeable for this parameter. Consequently, the main uncertainty in determining the nuclear part of $a_{pp}$ is due to the theoretical uncertainties in the factors used to relate the measured value of $a_{pp}$, which includes all $em$ effects, to the purely nuclear part of the scattering length.

The situation is quite different for $a_{nn}$. For technical reasons direct measurements of $a_{nn}$ using free neutrons have never been successfully executed. Up to now, all determinations for $a_{nn}$ have been based on studies of reactions with at least three particles in the exit channel [2, 3, 4, 5]. In this paper, the results from measurements of neutron-deuteron ($nd$) breakup and pion-deuteron ($\pi^-d$) capture are reviewed. Studies using these reactions were chosen for review because historically they have provided the most trusted determinations of $a_{nn}$. After the review of two recent kinematically complete $nd$ breakup measurements, some concluding remarks are made and a list of recommended next steps presented.

2. SUMMARY OF $a_{nn}$ RESULTS AND METHODS

In this section the results for $a_{nn}$ are summarized for studies done using $nd$ breakup and $\pi^-d$ capture measurements. This review will cover only experiments reported between 1964 and the present. In all cases $a_{nn}$ was determined by fitting the measured cross section for the $nn$ final-state interaction (FSI), which has a maximum value when the relative momentum of the two interacting neutrons is nearly zero. In the data analysis of these experiments, the relative energy between the two neutrons was typically between 0 and 500 keV. This wide integration range was required to obtain sufficient statistical accuracy in the $a_{nn}$ determination. A graphical summary of the situation is shown in Fig. 1. The graph is divided into three sections. The left most section contains the results from cross-section measurements of kinematically incomplete $nd$ breakup experiments. Five of the experiments reanalyzed by Tornow et al. [6] using modern theory are displayed as open circles. The middle section contains results from kinematically complete $nd$ breakup experiments. The results from $\pi^-d$ capture measurements are in the right section. The solid line represents the statistically weighted average of the data points for each reaction type from 1964 through 1997. The dashed line in the left section is the average of the reanalyzed data.

Some details of the experiments from which the data in Fig. 1 are taken are given in the tables in this section. Each experiment is categorized according to the number of kinematic quantities measured. The two broad types of experiments are kinematically complete (KC) and kinematically incomplete (KI). The experiments are tagged according to the number of measured kinematic parameters, because the level of the kinematic contraints imposed in the experiment can significantly impact the signal-to-background in the measurement and the theory used in the data analysis.

While the lists in the tables below are rather extensive, they are likely not all inclusive. The omission of an experiment of the type being reviewed here is not to be interpreted as the result of a data evaluation exercise but rather as an oversight on the part of the
Figure 1. Summary of indirect determinations of $a_{nn}$ from experiments reported between 1964 and the present. The data points for each reaction type are plotted on the horizontal axis in nearly chronological order and are as listed in Tables 1 and 3. Five of the experiments reanalyzed by Tornow et al. [6] are shown as open circles. The average values given at the tops of the sections for kinematically complete $nd$ breakup measurements and the $\pi^-d$ capture experiments don’t include the results reported since 1997. The second average value in the left section is for the values obtained in the reanalysis [6].

The author apologizes for any such omissions. For each reaction type one or two experiments that reported relatively small error bars for $a_{nn}$ are discussed for the purpose of presenting an overview of the experimental techniques employed and the theory applied in the analysis to determine $a_{nn}$ from the measured cross sections.

2.1. nd breakup

Details of the $nd$ breakup experiments for which the results are plotted in the left and middle sections of Fig. 1 are given in Tables 1 and 2. Typically in the KI experiments the momentum vector of the emitted proton, $\vec{p}_p$, is measured. There are in the general sense two types of KC experiments. In the most common type the deuterium target is an active deuterated scintillator (DS). This type will be referred to as KC1. In these experiments the momentum vectors of both emitted neutrons and the energy of the emitted proton, $p_{n1}$,
Table 1
Survey of \( n d \) breakup cross-section measurements that were used to determine \( a_{nn} \). The survey covers the period between 1964 and the present. The main features of the analyses are given in Table 2.

| ref. | year | \( E_{lab} \) (MeV) | Measured Kinematics | Analysis Details | \( a_{nn} \pm \Delta a_{nn} \) (fm) |
|------|------|----------------------|--------------------|-----------------|-------------------------------|
| 7    | 1964 | 14.4                 | incomplete 3,9     | -21.7 ± 1       |
| 8    | 1965 | 13.9                 | incomplete 4       | -23.6 ± 1.8     |
| 9    | 1968 | 13.9                 | incomplete 2       | -16.7 ± 2.8     |
| 10   | 1967 | 14.0                 | incomplete 2       | -14.3 ± 3       |
| 11   | 1968 | 14.1                 | incomplete 2       | -18.0 ± 1.5     |
| 12   | 1968 | 8-28                 | incomplete 2       | -16.8 ± 1.0     |
| 13   | 1970 | 14.1                 | incomplete 4,9     | -23.2 ± 1.8     |
| 14   | 1972 | 50                   | incomplete 2,9     | -21.7 ± 1.2     |
| 15   | 1973 | 14.1                 | incomplete 2,9     | -19.3 ± 0.8     |
| 16   | 1973 | 14.1                 | incomplete 5,6,7,9 | -18.3 ± 0.2     |
| 17   | 1977 | 14.0                 | incomplete 6,7,9   | -23.2 ± 3.6     |
| 18   | 1986 | 49.6                 | incomplete 6,7,9   | -19.6 ± 1.2     |
| 19   | 1986 | 62.8                 | incomplete 6,7,9   | -18.8 ± 1.0     |
| 20   | 1969 | 14.1                 | complete 3,9,11    | -16.2 ± 2.2     |
| 21   | 1970 | 18.8                 | complete 1,9,11    | -16.4 ± 2.8     |
| 22   | 1973 | 18.4                 | complete 6,7,9,11  | -17.1 ± 0.8     |
| 23   | 1972 | 14.3                 | complete 2,9,11    | -25 ± 3         |
| 24   | 1974 | 18.4                 | complete 6,7,9,11  | -16.3 ± 1.0     |
| 25   | 1975 | 14.1                 | complete 6,7,9,11  | -15.7 ± 2       |
| 26   | 1974 | 14.2                 | complete 6,7,9,11  | -16.0 ± 1.2     |
| 27   | 1978 | 120                  | complete 1,9,11    | -17.5 ± 4       |
| 28   | 1979 | 17-21                | complete 6,7,10,11 | -16.9 ± 0.6     |
| 29   | 1979 | 21-27                | complete 6,7,10,11 | -17.0 ± 0.6     |
| 4    | 1999 | 13.0                 | complete 6,8,10,12 | -18.7 ± 0.6     |
| 5    | 2000 | 25.2                 | complete 6,8,10,12 | -16.3 ± 0.4     |

\( p_n^2 \), and \( E_p \), respectively, are measured. Sizeable experimental effects must be taken into account when comparing theory to data taken in KC1 experiments. The most important effects are: neutron attenuation in the DS, neutron multiple scattering in the DS, the energy dependence of the efficiency of the two neutron detectors and angle and energy averaging over the experimental acceptance. Some early high-precision KC1 experiments were done by B. Zeitnitz et al. [21, 22, 24]. The experimental setup is slightly different in the other type of KC experiment, which will be referred to as KC2. In KC2 \( n d \) breakup experiments the deuterium target is a thin foil so that the momentum of the proton, \( p_p^* \), can be measured. In these experiments both \( p_p^* \) and \( p_n^1 \) are measured for each detected breakup event. As with KC1 experiments, the comparison of data taken in KC2 experiments to theory requires a number of effects to be taken into account: angle and energy averaging over the experimental acceptance, the energy and position dependence.
Table 2
This table gives a short list of the main features of the theory used in the analysis of \( nd \) breakup data in studies designed to obtain a value for \( a_{nn} \) from the \( nn \) FSI cross-section enhancement.

| Detail | Description |
|--------|-------------|
| 1      | Watson-Migdal and effective range theory |
| 2      | Impulse Approximation, Pole Approximation |
| 3      | Born Approximation |
| 4      | Truncated graph method |
| 5      | Hybrid Final-State-Interaction theory |
| 6      | Faddeev theory |
| 7      | Separable nucleon-nucleon potential |
| 8      | Realistic nucleon-nucleon potential |
| 9      | \( \ell = 0 \) only NN partial waves |
| 10     | \( \ell \geq 0 \) NN partial waves |
| 11     | fit shape of \( nn \) FSI enhancement only |
| 12     | fit shape and absolute cross section |

of the efficiency of the neutron detector, and the detection correlation of the two emitted neutrons. An example of an early high-precision KC2 experiment was reported by von Witsch et al. [28, 29].

The results for \( a_{nn} \) from the two most recent KC experiments of González Trotter et al. [4], which was the KC1 type, and of Huhn et al. [5], which was the KC2 type, disagree by more than three standard deviations of the reported experimental uncertainties. This situation is a bit puzzling given that the data taken in both experiments were analyzed with the same theory.

As shown in Fig. 1 there has been a large number of kinematically-incomplete \( nd \) breakup experiments over the last 35 years with the aim of determining \( a_{nn} \). While each experiment has unique features, there are some common characteristics. The main common attributes of these experiments are: (1) the deuterium target is a deuterated polyethylene foil that is thin enough to allow the emitted protons to pass through with sufficient energy for detection, (2) a charged particle detector system is positioned to detect the protons from the breakup reaction that are emitted around \( 0^\circ \), and (3) the value of \( a_{nn} \) is obtained by fitting the enhancement in the proton-energy (\( E_p \)) spectrum due to the \( nn \) FSI. The charged-particle detection system is usually either a magnetic spectrometer or a counter telescope with gas counters to eliminate the direct neutrons from the event trigger. The main technical challenge in these measurements is the reduction of protons from background sources, particularly in the flat part of the \( E_p \) spectrum. Even with more than ten kinematically-incomplete \( nd \) breakup experiments contributing to the effort, the result reported by Shirato et al. [16] dominates the computed average for this
type of measurement due to the small uncertainty in their value relative to that obtained in the experiments. A gas counter telescope was used in the experiment of Shirato et al. \[16\]. The main experimental issue in their measurement was the determination of the breakup protons from the lower energy part of their incident neutron beam. These contaminate protons affected the flat part of the $E_p$ spectrum and had little influence on the $nn$ FSI enhancement, which is at the extreme high end of the spectrum. In their analysis the $E_p$ spectrum was fit with a two-term function by searching on three free parameters, the normalization constant for the term that described the flat part of the spectrum, the normalization constant for the term that described the $nn$ FSI enhancement region of the spectrum, and $a_{nn}$.

The reanalysis of the $E_p$ spectrum from some of the more recent kinematically incomplete experiments by Tornow et al. \[6\] gives very puzzling results. In all cases the magnitude of $a_{nn}$ shifted to a substantially smaller value in the reanalysis with modern theory. On average the shift was about 3 fm out of 19 fm. The shift seems mostly associated with the discrepancy between the cross-section data in the flat part of the proton energy spectrum and the modern calculations. Because this part of the energy distribution is insensitive to the value of $a_{nn}$, the data were normalized to the calculations in this region. At incident neutron energies near 14 MeV the data were typically larger than the calculated cross sections. The consequence of this normalization is that the values of $a_{nn}$ determined in the analysis of Tornow et al. \[6\] are on average lower than the original values.

2.2. $\pi^-d$ capture

The situation for determining $a_{nn}$ from the $\pi^-d$ capture reaction is summarized in Table 3. In these experiments a degraded pion beam is stopped in a liquid deuterium target. As in the case of the $nd$ breakup experiments, the $\pi^-d$ capture measurements also are divided according to the number of kinematic parameters measured.

### Table 3
Survey of $\pi^-d$ capture cross-section measurements that were used to determine $a_{nn}$. The survey covers the period between 1964 and the present.

| ref. | year | Measured Kinematics | Analysis ref. | $a_{nn}$ ± $\Delta a_{ann}$ (fm) |
|------|------|---------------------|---------------|----------------------------------|
| [30] | 1965 | complete            | [31]          | -16.4 ± 1.9                      |
| [32] | 1975 | complete            | [31]          | -16.7 ± 1.3                      |
| [33] | 1979 | incomplete          | [34]          | -18.5 ± 0.4                      |
| [35] | 1987 | complete            | [34]          | -18.7 ± 0.6                      |
| [3]  | 1998 | complete            | [36]          | -18.5 ± 0.5                      |

In KI measurements of the $\pi^- + d \rightarrow 2n + \gamma$ reaction only the energy of the $\gamma$-ray ($E_\gamma$) is measured. The measured $E_\gamma$ spectrum is fit with theory to determine a value of $a_{nn}$. In the KC measurements, the momenta of the $\gamma$-rays and one of the neutrons
are measured for each detected capture event. The time-of-flight (TOF) spectrum of the detected neutron is fit to determine $a_{nn}$. The main experimental effects that must be taken into account when fitting the neutron TOF spectrum are the energy dependence of the neutron detection efficiency, the energy dependence of the attenuation and scattering of neutrons in the liquid deutrium target and the surrounding materials. The results from the two most recent KC experiments are in agreement within the reported uncertainties, which include a $\pm 0.3$ fm due to theoretical uncertainties.

3. SUMMARY and RECOMMENDATIONS

The most popularly accepted value for $a_{nn}$ comes from the $\pi^-d$ capture measurements. The general belief is that these results are more reliable than those obtained from $nd$ breakup or from other reactions in which there are three or more hadrons in the exit channel. This supposition is supported by the observation that the $a_{nn}$ values obtained in the most recent high-precision $\pi^-d$ experiments are in agreement while recent $nd$ breakup experiments and analyses give discrepant results. Therefore, the recommended value for $a_{nn}$ obtained using indirect methods is $-18.6 \pm 0.3$ (experimental) $\pm 0.3$ (theory) fm.

An important next step is to conduct $nd$ breakup experiments for the purpose of resolving the discrepancies between the results from recent KC $nd$ breakup experiments and for determining the cause of the shift in the value of $a_{nn}$ in the reanalysis of the KI $nd$ breakup cross-section data. While it is unclear whether investigating these problems will lead to a better determination of $a_{nn}$, this work will almost certainly strengthen our understanding of three-nucleon reaction dynamics in the kinematic region around the $nn$ FSI.

The next sufficient step in this problem would be a direct measurement of $a_{nn}$. The high thermal neutron flux at the YAGUAR pulsed reactor in Russia opens opportunities for such measurements. The DIANNA collaboration is planning the first direct measurement of $a_{nn}$ that use neutrons from a reactor. Some details of their proposed experiment are given in these proceedings.

REFERENCES
1. G.A. Miller, B.M.K. Nefkens, and I. Slaus, Phys. Reports 194 (1990) 1.
2. I. Šlaus, Y. Akaishi, and H. Tanaka, Phys. Reports 173 (1989) 257.
3. C.R. Howell et al., Phys. Lett. B44 (1998) 252.
4. D.E. González Trotter et al., Phys. Rev. Lett. 83 (1999) 3788.
5. V. Huhn et al., Phys. Rev. Lett. 85 (2000) 1190.
6. W. Tornow, H. Witała and R.T. Braun, Few-Body Systems 21 (1996) 97.
7. M. Cerineo, K. Ilakovac, I. Šlaus, P. Tomaš and V. Valkovič, Phys. Rev. 133 (1964) B948.
8. V.K. Voitovetskii, I.L. Korsunskii and Yu. F. Pazhin, Nucl. Phys. 69 (1965) 513.
9. R.J. Slobodrian, H.E. Conzett and F.G. Resmini, Phys. Lett. 27B (1968) 405.
10. E. Bar-Avraham, R. Fox, Y. Porath, G. Adam and G. Frieder, Nucl. Phys. B1 (1967) 49.
11. S. Shirato and N. Koori, Nucl. Phys. A120 (1968) 387.
12. A. Bond, Nucl. Phys. A120 (1968) 183.
13. A.N. Prokofyev and G.M. Shklyarevsky, Yad. Phys. 11 (1970) 567.
14. A. Stricker et al., Nucl. Phys. A190 (1972) 284.
15. S. Shirato, K. Saitoh and N. Koori, Few Particle Problems in the Nuclear Interaction, p.114, I. Šlaus em et al. (eds). Amsterdam: North-Holland 1973.
16. S. Shirato, K. Saitoh and N. Koori, Nucl. Phys. A215 (1973) 277.
17. R.C. Haight, S.M. Grimes and J.D. Anderson, Phys. Rev. C16 (1977) 97.
18. S. Shirato, Y. Ishibe and K. Shibata, in Book of Contributions for the Conference on Few Body Systems in Particle and Nuclear Physics, p.412, T. Sasakaw et al. (eds.) Tohoku Univ. 1986.
19. N. Koori et al., in Book of Contributions for the Conference on Few Body Systems in Particle and Nuclear Physics, p.406, T. Sasakaw et al. (eds.) Tohoku Univ. 1986.
20. H. Grässler and R. Honecker, Nucl. Phys. A136 (1969) 446.
21. B. Zeitnitz, R. Maschuw and P. Suhr, Nucl. Phys. A149 (1970) 449.
22. B. Zeitnitz et al., Few Particle Problems in the Nuclear Interaction, p.117, I. Šlaus em et al. (eds). Amsterdam: North-Holland 1973.
23. A.I. Saukov et al., Sov. J. Nucl. Phys. 14 (1972) 157.
24. B. Zeitnitz et al., Nucl. Phys. A231 (1974) 13.
25. J. Kecskemeti, T. Csibok and B. Zeitnitz, Nucl. Phys. A254 (1975) 110.
26. W.H. Breunlich, S. Tagesen, W. Bertl and A. Chalupka, Nucl. Phys. A221 (1974) 269.
27. Y. Onel et al., Nucl. Phys. A304 (1978) 51.
28. W. von Witsch et al., Phys. Lett. B80 (1979) 187.
29. W. von Witsch et al., Nucl. Phys. A329 (1979) 141.
30. R.P. Haddock et al., Phys. Rev. Lett. 14 (1965) 318.
31. M. Bander, Phys. Rev. 134 (1964) B1052.
32. R.M. Salter et al., Nucl. Phys. A254 (1975) 241.
33. B. Gabioud et al., Phys. Rev. Lett. 42 (1979) 1508; Phys. Lett. 103B (1981) 9; Nucl. Phys. A420 (1984) 496.
34. G.F. de Téramond, Phys. Rev. C16 (1977) 1976; G.F. de Téramond, J. Paez and C.W. Soto Vargas, Phys. Rev. C21 (1980) 2542.
35. O. Schori et al., Phys. Rev. C35 (1987) 2252.
36. W.R. Gibbs, B.F. Gibson, G.J. Stephenson, Jr., Phys. Rev. C11 (1975) 90; Phys. Rev. C12 (1975) 2130; Phys. Rev. C16 (1977) 327; Phys. Rev. C17 (1978) 856.