Dynamic analysis of a high speed motorized spindle for internal grinding of slender holes

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Abstract. The purpose of this paper is to investigate the dynamic characteristics of a high speed motorized spindle for internal grinding of slender holes. The modified transfer matrix method is used to establish the dynamic model of the motorized spindle rotor system including the additional stiffness of the non-contact rotary union. The linear perturbation method is adopted to derive the perturbation equations for the flow passing through the gap in the non-contact rotary union. The finite difference method is used to solve the Reynolds equation and the perturbation equations to obtain the four dimensional stiffness matrix of the non-contact rotary union. Finally, the effect of the non-contact rotary union on the dynamic behaviours of the motorized spindle are studied.

1. Introduction
High speed motorized spindles have been widely used in machine tools due to their advantages of eliminating the need for conventional power transmission devices [1, 2]. The machining productivity and finishing quality of workpieces are mainly determined by the dynamic performance of the motorized spindle. Therefore, it is essential to study the dynamic characteristics of the motorized spindle for reference in design and operation.

For internal grinding of normal holes, the external cooling method [3] is commonly used to supply the coolant into the grinding zone. However, for internal grinding of slender holes, such as the blind hole (ф3mm×45mm) in the nozzle of diesel injector, the inner cooling method is adopted to supply the grinding coolant [4, 5]. Consequently, a non-contact rotary union has to be specially designed for the motorized spindle [1]. When the motorized spindle operates at high speed, the hydrodynamic effect will produce in the gap between the water pipe and the rotating shaft. In other words, the non-contact rotary union will perform like a hydrodynamic bearing, providing additional stiffness to the rotor system. As the dynamic behaviours of the high speed motorized spindle are significantly affected by the supporting stiffness, the effect of the non-contact rotary union on the dynamic performance of the motorized spindle can not be ignored.

By far, considerable researches on the dynamic analysis and design for the motorized spindle have been conducted [1, 2, 6, 7]. Effects of various structural parameters on the dynamic characteristics of the motorized spindle have been studied systematically. However, the effect of the non-contact rotary union has not been investigated yet.

This paper presents a 4-DOF dynamic model for the motorized spindle by using the modified transfer matrix method. The stiffness coefficients of the non-contact rotary union are included. Based on the proposed model, the effect of the non-contact rotary union on the dynamic behaviours of the motorized spindle are studied.
2. Theoretical model

2.1. Dynamic model of the motorized spindle rotor system

Figure 1 illustrates the rotor schematic of the high speed motorized spindle for internal grinding of slender holes. The rotating shaft is supported by a pair of angular contact ball bearings with face-to-face arrangement. The grinding coolant is supplied by a water bump outside. A portion of the coolant passes through the non-contact rotary union to enter the channel in the shaft, and then flows out from the nozzles of the tool bar, and finally sprays into the grinding zone. The other portion of the coolant flows out from the gap between the water pipe and the rotating shaft. The real scene image of the motorized spindle is shown in Fig. 2.

![Figure 1. The rotor schematic of the motorized spindle](image1)

![Figure 2. Real scene image of the motorized spindle](image2)

Figure 3 shows the lumped mass model of the motorized spindle rotor system. The shaft is divided into a series of sections, which are composed of lumped mass, rigid disk, and massless elastic shaft. Both the angular contact ball bearings and the non-contact rotary union are simplified as massless springs possessing 16 stiffness coefficients. By using the transfer matrix method \[8, 9\], the overall transfer matrix can be obtained and the critical speed of the rotor system can be calculated.

![Figure 3. The dynamic model of the motorized spindle rotor system](image3)

2.2. Stiffness coefficients of the non-contact rotary union

Figure 4 illustrates the schematic of the non-contact rotary union. The flow is governed by using the Reynolds equation \[10\]:

\[
\frac{\partial}{R \partial \psi} \left( \frac{h^3}{6 \mu} \frac{\partial P}{\partial \psi} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{6 \mu} \frac{\partial P}{\partial z} \right) = \Omega \frac{\partial h}{\partial \psi}
\]  

(1)
where, $P$ denotes the pressure; $h$ denotes the film thickness; $R$ denotes the radius of the shaft; $\mu$ denotes the viscosity; $\Omega$ denotes the angular velocity; $\psi$ denotes the angular coordinate; and $z$ denotes the axial coordinate.

![Figure 4. Schematic of the non-contact rotary union](image)

From Fig.4, film thickness can be expressed as follows:

$$h = c + e_0 \cos(\psi - \Phi) + z(\theta_0 \cos \psi - \theta_0 \sin \psi)$$

(2)

where, $c$ denotes the radial clearance; $e_0$ denotes the initial eccentricity; $\Phi_0$ denotes the initial attitude angle; $\theta_0$ and $\theta_0$ respectively denote the tilting angles of the rotating shaft around $x$-axis and around $y$-axis.

The linear perturbation method [11] is adopted to derive the perturbation equations, and then the finite difference method [12] is used to solve the steady and perturbed equations to obtain the perturbed pressure. The additional stiffness coefficients provided by the non-contact rotary union are expressed as follows:

$$K' = \int_0^{2\pi} \begin{bmatrix} -\psi \cos \psi & -\sin \psi & z \sin \psi & -z \cos \psi \\ \{ P_x, P_y, P_\theta, P_{\theta_0} \} \\ \{ P_x, P_y, P_\theta, P_{\theta_0} \} \end{bmatrix} d\psi dz$$

(3)

where, $P_\xi (\xi = x, y, \theta_x, \theta_y)$ denotes the perturbed pressure.

3. Results and discussion

3.1. Stiffness coefficients of the non-contact rotary union

The parameters of the non-contact rotary union are summarized in Table 1. Since the grinding force for manufacturing the slender hole is very small, the eccentricity is considered to be zero. The non-zero additional stiffness coefficients of the non-contact rotary union include: the force stiffness coefficients $k_{xx}$, $k_{yy}$, $k_{x\theta_y}$, and $k_{y\theta_x}$; and the moment stiffness coefficients $k_{x\theta_x}$ and $k_{y\theta_y}$. Fig.5 shows the non-zero stiffness coefficients of the non-contact rotary union with respect to the rotary speed. The negative sign represents the direction. It can be seen from Fig.5 that, $k_{xx}$, $k_{yy}$, $k_{x\theta_x}$, and $k_{y\theta_y}$ increase with the rotary speed; but $k_{x\theta_y}$ and $k_{y\theta_x}$ keep constant at all speed. This is due to the fact that, the convergent gap along axial direction is generated when the rotating shaft runs with an angular misalignment; and the axial velocity of the flow slightly changes with the rotary speed. It can be concluded that, the force stiffness coefficients $k_{x\theta_x}$ and $k_{y\theta_y}$ will significantly affected by the tilting angles rather than the rotary speed. Besides, the above coefficients are skew symmetric ($k_{x\theta_x} = k_{y\theta_y}$, $k_{x\theta_y} = -k_{y\theta_x}$, $k_{x\theta_x} = -k_{y\theta_y}$) due to the concentric structure of the water pipe and the rotating shaft when the eccentricity is zero.
Table 1. Parameters of the non-contact rotary union

| Parameter                        | Value |
|----------------------------------|-------|
| Length \( L \) (mm)              | 10    |
| Diameter \( D \) (mm)            | 6     |
| Radial clearance \( c \) (μm)    | 10    |
| Viscosity \( \mu \) (Pa·s)       | 0.001 |
| Supply pressure \( P_s \) (MPa)  | 0.3   |
| Ambient pressure \( P_0 \) (MPa) | 0.1   |
| Rotary speed (rpm)               | 0–20000 |

Figure 5. Non-zero stiffness coefficients of the non-contact rotary union at \( \epsilon_0 = 0 \)

3.2. Effect of the non-contact rotary union on the spindle critical speed

In this section, a comparative study of the rotor dynamic behaviours of the motorized spindle with and without considering the non-contact rotary union is conducted. The parameters of the rotor system are listed in Table 2. The nonlinear stiffness of the angular contact ball bearing can be calculated by using the 5-DOF bearing model firstly established by Jones [13, 14]. In the absence of any external load, the bearing’s non-zero stiffness coefficients include the radial stiffness \( k_{xx} (=k_{yy}) \), the angular stiffness \( k_{\theta x \theta x} (=k_{\theta y \theta y}) \), and the cross stiffness \( k_{x \theta y} (=k_{y \theta x} = k_{\theta y \theta x} = k_{\theta x \theta y}) \). Fig. 6 gives the nonzero stiffness coefficients of the angular contact ball bearing with respect to the rotary speed. It can be seen that, the bearing stiffness soften phenomenon occurs when the rotary speed exceeds 50000rpm. This is caused by the increasing centrifugal force and the gyroscopic moment of the ball. Note that, the cross stiffness of the front bearing must be mapped to the rotor coordinate system [15].

Table 2. Parameters of the rotor system

| Parameter                        | Value |
|----------------------------------|-------|
| Length of the rotating shaft (mm)| 98    |
| Length of the motor rotor (mm)   | 30    |
| Inner diameter of the motor rotor (mm) | 10   |
| Outer diameter of the motor rotor (mm) | 19.4 |
| Bearing type                     | HC708-E-T-P4S |
| Bearing span (mm)                | 65    |
| Axial preload (N)                | 40    |
Figure 6. Non-zero stiffness coefficients of the bearing

Figure 7 shows the Campbell diagram of the motorized spindle rotor system with and without considering the non-contact rotary union. The simulated results show that, the first natural frequency of the rotor system considering the non-contact rotary union is larger than that without considering the non-contact rotary union, indicating that the non-contact rotary union is helpful to enhance the rigidity of the rotor system. The first critical speeds of the rotor system with and without considering the non-contact rotary union are 167474rpm and 158372rpm, respectively. The corresponding first mode shapes are plotted in Fig.8 and Fig.9. It can be seen that, the maximum relative amplitude locates at the middle of the bearing span. The centreline of the rotor is a space curve considering the non-contact rotary union, but it is a plane curve without considering the non-contact rotary union.

Figure 7. Non-zero stiffness coefficients of the bearing

Figure 8. First mode shape of the rotor considering the rotary union

Figure 9. First mode shape of the rotor without considering the rotary union

4. Conclusions

(1) A dynamic model of the motorized spindle for the internal grinding of slender holes is developed including the additional stiffness of the non-contact rotary union, which laid a foundation for the analysis of the effect of the non-contact rotary union on the dynamic behaviours of the motorized spindle rotor system.
(2) The force stiffness coefficients $k_{x0}$ and $k_{y0}$ of the non-contact rotary union are almost constant at all speed, but they are closely related to the tilting angles $\theta_{x0}$ and $\theta_{y0}$;
(3) The additional stiffness coefficients $k_{xy}$, $k_{yx}$, $k_{\theta x\theta y}$, and $k_{\theta y\theta x}$ increase with the rotary speed, while the bearing stiffness keeps constant at a lower speed, but decreases when the rotary speed exceeds a certain value;
(4) The non-contact rotary union is helpful for improving the rigidity and the first critical speed of the motorized spindle rotor system.

5. References
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