Doubly charmed dibaryon state

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In this work, we study the doubly charmed dibaryon states with the $qqqqcc$ $(q = u, d, s)$ configuration. The mass spectra of doubly charmed dibaryon states are obtained systemically within the chromomagnetic interaction model. In addition to the mass spectrum analysis, we further illustrate their two-body strong decay behaviors. Our results indicate that there may exist narrow states or even stable states, which cannot decay through the strong interaction. Hopefully, our results can provide valuable information for further experimental search for the doubly charmed dibaryon states.

I. INTRODUCTION

With accumulation of experimental hadron matter enter a new stage. In the past two decades, a series of charmoniumlike $XYZ$ states, $P_c$ states, and double charm $T_{cc}^+$ state were reported in experiment, which has stimulated extensive studies of exotic hadron spectroscopy (see review articles $^1$ for more details). Very recently, the LHCb Collaboration announced new observation of the $T_{cc}^+(2900)$ $^{1+0}$ in the processes $B^0 \rightarrow D^0 D^+_c \pi^-$ and $B^- \rightarrow D^- D^+_c \pi^+$ $^2$, $^3$, and a $P_{1/2}^{0}(4338)$ state in $B^- \rightarrow J/\psi \Lambda \bar{p}$ $^8$. In addition, the CMS Collaboration confirmed the observation of the $X(6900)$ and found more structures in $\text{di-}J/\psi$ invariant mass spectrum $^9$, and the ATALAS Collaboration also confirmed the discovery of $P_c$ states and $X(6900)$ $^{10}$. Obviously, we are entering a new era of “Particle Zoo 2.0”, which may deepen our understanding of non-perturbative behavior of strong interaction.

Among different exotic hadrons, the hexaquark or dibaryon state is a special group. The well-known dibaryon state is deuteron discovered in 1932 by Urey, Brickwedde, and Murphy $^{11}$. After proposing the quark model $^{12}$, Dyson and Xuong indicated that SU(6) symmetry provides a multiplet of six non-strange dibaryon states $^{13}$. In 1977, Jaffe predicted the existence of the $H$ dibaryon, a bound state containing $uuddss$ quark component $^{14}$. As a consequence, many studies about dibaryon system have been made $^{15,16,33}$. One can find that a review about the long-standing search for dibaryon system from the early days until recent years $^{34}$. The stability of dibaryon system made of six quarks or antiquarks was studied within a string model $^{35}$, where they discussed the existence of six-quark ($q^6$) states and the three-quark-three-antiquarks ($q^3\bar{q}^3$) states and then extended the investigations to the $(q^4Q^3)$ and $(Q^4q^3)$ systems. Theoretically, the various light flavor baryon-baryon systems have been studied through different methods $^{36,37}$. In fact, exploring the hexaquark with heavy quark(s) is worth to be encouraged $^{38,41}$.

In the framework of the one-boson-exchange (OBE) model $^{26}$, the authors calculated the binding energies and root mean square radii for the possible loosely bound states composed of $\Lambda_a\Lambda_b(\Lambda_c)$, $\Xi_a\Xi_b(\Xi_c)$, $\Sigma_a\Sigma_b(\Sigma_c)$, $\Xi_c^*\Xi_c^*(\Xi_c^*)$, and $\Omega_a\Omega_b(\Omega_c)$. By including the coupled channel effects caused by $\Sigma_a$ and $\Sigma_b$ in the one-pion-exchange potential (OPEP) model, two $\Lambda_c$’s is possibly be bound into a hadronic molecule state $^{27}$. A chiral constituent quark model was used to study the two-baryon system with two units of charm $^{38}$. Here, the results show that there is no charmed $H$-like dibaryon, but it may appear as a resonance above the $\Lambda_c\Lambda_c$ threshold. A variational calculation was presented in the case of double-heavy dibaryons, $qqqqQQ'$ $(q = u, d, or s)$ and $Q$ and $Q'$ denote the different heavy quarks but the mass difference is neglected therein $^{42}$, where no bound or metastable state was found. The lattice QCD simulation is reported for the deuteronlike $(nn$-like) dibaryons with heavy quark flavors in Ref. $^{43}$, and the masses of the ground states $\Omega_c\Omega_c(sscsc), \Omega_b\Omega_b(ssbssb)$, and $\Omega_c\Omega_b(scbcbb)$ lie below their respective two-baryon thresholds.

Especially, inspired by the recent discovery of a narrow structure $T_{cc}^+(cc\bar{u})$ in the LHCb experiment $^{44,45}$, but also based on the similarity of the $cc\bar{u}$ configuration and the $\Xi_{cc}^{++}$ baryon (ccu) $^{44}$, the existence of other open charm multiquark have been studied by the extended chromomagnetic interaction (CMI) model $^{46,48}$, the molecular state model $^{49}$, the quark delocalization color screening model $^{50}$, and the chiral effective theory $^{51}$.

Facing this situation, we are interested in further exploring the doubly charmed dibaryon state. In previous work $^{52}$, we once studied the mass spectrum and decay pattern of hidden charm hexaquark states ($qqqqcc$) in the framework of the chromomagnetic model, which was usually used to calculate the mass spectrum of or-
dinary hadron states or multiquark states [3]. Along this line, the mass spectrum of these doubly charmed dibaryon states will be obtained by the same framework. The present paper aims at discussing the mass spectrum of these doubly charmed dibaryon states, and also the stability of various states briefly. It may provide valuable information to further experimental search for doubly charmed dibaryon states. With the running of High-Luminosity LHC [53], to hunt for this new kind of dibaryon may come true.

This paper is organized as follows. In Sec. II, we introduce the chromomagentic interaction model. According to various permutation symmetry of identical particles, we construct the $F_{\text{flavor}} \otimes \phi_{\text{color}} \otimes \chi_{\text{spin}}$ wave functions for $S$-wave $qqqccc$ dibaryon state in Sec. III. The mass spectrum and decay mode of $qqqccc$ hexaquark state with different constituent quark are discussed in Sec. IV. At last, we make a short summary in Sec. V.

II. THE HAMILTONIAN IN THE CMI MODEL

In the chromomagnetic interaction (CMI) model, the mass of the ground hadron state can be described by the effective Hamiltonian [46][48][54][56]

$$H = \sum_i m_i + H_{CI} + H_{\text{CMI}}$$

$$= \sum_i m_i - \sum_{i<j} A_{ij} \hat{\lambda}_i \cdot \hat{\lambda}_j - \sum_{i<j} v_{ij} \hat{\lambda}_i \cdot \lambda_j \sigma_i \cdot \sigma_j$$

$$= -\frac{3}{4} \sum_{i<j} m_{ij} V_{ij}^C - \sum_{i<j} v_{ij} V_{ij}^{\text{CMI}},$$

where $V_{ij}^C = \hat{\lambda}_i \cdot \hat{\lambda}_j$ and $V_{ij}^{\text{CMI}} = \hat{\lambda}_i \cdot \lambda_j \sigma_i \cdot \sigma_j$ are the color and chromomagnetic interaction between quarks, respectively. The $\sigma_i$ and $\lambda_i$ are the Pauli matrices and the Gell-Mann matrices, respectively. The mass parameter of quark pair $m_{ij} = 1/4(m_i + m_j) + 4/3A_{ij}$ contains the effective quark mass $m_i (m_j)$ and the color interaction strength $A_{ij}$. The $v_{ij}$ is the effective coupling constant between the $i$-th quark and $j$-th quark, determining the mass gaps.

To estimate the mass spectrum of the doubly charmed dibaryon states, we extract the effective coupling parameters $m_{ij}$ and $v_{ij}$ from the conventional baryon masses, which are presented in Table I.

### TABLE I. Relevant coupling parameters for the doubly charmed dibaryon system [57] (in units of MeV).

| Parameter | $m_{nn}$ | $m_{ns}$ | $m_{ss}$ | $m_{nc}$ | $m_{sc}$ | $m_{cc}$ |
|-----------|---------|---------|---------|---------|---------|---------|
| Value     | 181.2   | 226.7   | 262.3   | 520.0   | 545.9   | 792.9   |
| Parameter | $v_{nn}$ | $v_{ns}$ | $v_{ss}$ | $v_{nc}$ | $v_{sc}$ | $v_{cc}$ |
| Value     | 19.1    | 13.3    | 12.2    | 3.9     | 4.4     | 3.5     |

III. THE WAVE FUNCTIONS

To calculate the mass spectrum of the doubly charmed dibaryon states in the CMI model, we need the information on the total wave function, which is composed of space, flavor, color, and spin wave functions, i.e.

$$\Phi_{\text{total}} = \Psi_{\text{space}} \otimes F_{\text{flavor}} \otimes \phi_{\text{color}} \otimes \chi_{\text{spin}}.$$ (2)

Due to the Pauli principle, this wave function should be fully antisymmetric when exchanging identical quarks. Here we only consider the low-lying $S$-wave dibaryon states, so that their spatial wave functions are symmetric and become trivial. Thus, the $F_{\text{flavor}} \otimes \phi_{\text{color}} \otimes \chi_{\text{spin}}$ wave functions of dibaryon states should be fully antisymmetric when exchanging identical quarks. For the doubly charmed dibaryon system, all the possible flavor combinations are $nnnnc$ $(I = 2, 1, 0)$, $nnnnc$ $(I = 3/2, 1/2)$, $nnncc$ $(I = 1, 0)$, $ssncc$, and $ssccc$, respectively. According to symmetry properties, we can divide the $qqqccc$ dibaryon system into three groups and show them in Table II. Here, we need to construct the corresponding $F_{\text{flavor}} \otimes \phi_{\text{color}} \otimes \chi_{\text{spin}}$ wave functions with the $\{123\}\{56\}$, $\{123\}4\{56\}$, and $\{12\}\{34\}\{56\}$ symmetries. Here, we use the notation $\{123...\}$ for the case that the quarks $1, 2, 3$, and so on have antisymmetric property.

### TABLE II. All possible flavor combinations for the $qqqccc$ dibaryon system. Here, $g = n, s$ and $n = u, d$.

| Symmetry | Flavor combinations |
|----------|---------------------|
| $\{123\}\{56\}$ | $nnnnc$ $(I = 2, 1, 0)$, $ssncc$ |
| $\{123\}4\{56\}$ | $nnncc$ $(I = 3/2, 1/2)$, $ssncc$ |
| $\{12\}\{34\}\{56\}$ | $nnncc$ $(I = 1, 0)$ |

Next, we construct the $F_{\text{flavor}} \otimes \phi_{\text{color}} \otimes \chi_{\text{spin}}$ wave functions of the $qqqccc$ dibaryon system group by group.

A. Wave function for $\{123\}\{56\}$ symmetry

Firstly, we construct the wave function of the $nnnnc$ $(I = 2, 1, 0)$ and $ssncc$ dibaryon states. Since $nnnnc$ $(I = 2)$ dibaryon states and $ssncc$ dibaryon states have the absolutely same symmetry requirements, we only need to concentrate on the $nnnnc$ $(I = 2, 1, 0)$ dibaryon states. Here, we introduce the Young tableau to represent the irreducible base of the permutation group $S_n$. It can help us to attach certain symmetry properties to the total wave function of $nnnnc$ dibaryon states. For the $nnnnc$ dibaryon states with isospin $I = 2, 1$, and 0, the flavor states of four $n$-quarks can be represented by partition $[4]$, $[3, 1]$, and $[2, 2]$, respectively, and specific Young tableaux are shown in Table III. Due to the requirement of color confinement, the color part must be a singlet in the SU(3) symmetry. Thus only the partition
TABLE III. The flavor, color, and spin wave functions represented by the partition and the corresponding Young tableaux. Here, the $I/J$ represents the isospin/spin.

| $I/J$ | Partition | The Young Tableau |
|------|-----------|-------------------|
| $I = 2$ | $[4]$ | $F^I_f = (\begin{array}{c} 1 \ 2 \ 3 \ 4 \\ 5 \ 6 \end{array})$ |
| $I = 1$ | $[3,1]$ | $F^I_f = (\begin{array}{c} 1 \ 2 \ 3 \\ 4 \ 5 \end{array})$, $F^I_f = (\begin{array}{c} 1 \ 2 \ 4 \\ 3 \ 5 \end{array})$ |
| $I = 0$ | $[2,2]$ | $F^I_f = (\begin{array}{c} 1 \ 2 \\ 3 \ 4 \\ 5 \ 6 \end{array})$ |

Color part $[2,2,2]$ is satisfied with the color singlet. The spin part can be represented by partition $[6], [5,1], [4,2]$, and $[3,3]$ for total spin $J = 3, 2, 1, 0$, respectively. Based on these, we also show the specific Young tableaux in Table III.

Once we obtain the wave functions represented by the Young tableaux, the next step is adopting an appropriate coupling method to combine these wave functions in different spaces into totally antisymmetric wave functions. Thus we need to introduce the corresponding Clebsch-Gordan (CG) coefficients of the permutation group $S_n$ just like the coupling method used in Refs. [58, 61].

$$|fY⟩ = \sum_{Y''} S([f'Y'][f''Y''][fY][f'Y'][f''Y']) |f''Y''⟩.$$ (3)

Here, $[f], [f'], [f'']$ denotes an irreducible representation of $S_n$, $Y$ denotes a Young tableau and $|fY⟩$ is a basis vector of $S_n$. $S([f'Y'][f''Y''][fY][f'Y'][f''Y'])$ are CG coefficients. As mentioned above, the symmetric of the color wave function is fixed as $[2,2,2]$. Thus we adopt a coupling method that one can first combine the color-SU(3) singlet $[2,2,2]$ with spin-SU(2) representations $[f]_S$ into an SU(6) $[f]_{CS}$, and then combine it with an isospin-SU(2) representations $[f]_I$ to obtain SU(12) representations $[f]_{CSI}$. Thus we can calculate 6-quarks wave function with desired permutation symmetry $[f]$ as a linear combination of $F^{I,J}_{\text{flavor}} \otimes \phi_{\text{color}} \otimes \chi_{\text{spin}}$.

B. Wave function for $\{123\}4\{56\}$ symmetry

Then, we construct the wave function of the nnnscc ($I = 3/2, 1/2$) and ssnscc states. Because nnnscc ($I = 3/2$) states and ssnscc states also have the absolutely same symmetry requirements, we only need to concentrate on the nnnscc ($I = 3/2, 1/2$) states. Note that the mass of $s$ quark is much heavier than those of the $u, d$ quarks, leading to SU(3) flavor symmetry breaking effect. Thus, we include this effect by distinguishing $s$ quark from $u, d$ quarks while still keeping SU(2) isospin symmetry in our calculation. Based on these, it is convenient to use the $|[mn]n\{s(cc)\}|$ basis to construct the total wave functions of these states. We list all the possible flavor, color-singlet, and spin wave functions in Table IV.

In Table IV, S (A) means totally symmetric (antisymmetric), and MS (MA) means that the first two quarks $nn$ or the last two quarks $cc$ are symmetric (antisymmetric) in nnnscc states. In color part, we use the notation $|[123]\{\text{color}_1\{n_3\}\{\text{color}_2\}\{s_4(\text{color}_3\{\text{color}_4\})\}|$ to describe the color-singlet wave functions, where color1, color2, color3, and color4 stand for the color representations of $n_1n_2$, $n_1n_2n_3$, $c_5c_6$, and $s_4c_5c_6$, respectively. Similarly, we use the notation $|[123]\{\text{spin}_1\{n_3\}\{\text{spin}_2\}\{s_4(\text{spin}_3\{\text{spin}_4\})\}\{\text{spin}_5\}|$ to describe the spin wave functions, where spin1, spin2, spin3, spin4, and spin5 represent the spins of $n_1n_2$, $n_1n_2n_3$, $c_5c_6$, $s_4c_5c_6$, and total spin, respectively.
| Flavor | $I = \frac{3}{2}$ | $I = \frac{1}{2}$ |
|--------|-----------------|-----------------|
|        | $F^{S,S}$ | $F^{M,S}$ |
|        | $F^{M,A,S}$ | $F^{A,A,S}$ |
| Color  | $\phi^{M,A,M,A}$ | $\phi^{M,A,M,S}$ | $\phi^{A,A}$ |
|        | $\phi^{M,S,M,A}$ | $\phi^{M,S,MS}$ | $\phi^{A,S,A}$ |

### Spin

| $J = 3$ | $\lambda^{S,S}_1 = \frac{1}{2}$ | $\frac{1}{2}$ |
|        | $\frac{1}{2}$ | $\frac{1}{2}$ |

### $F \otimes \phi \otimes \chi$

| $I = \frac{3}{2}$ | $F^{S,S}$ | $F^{M,A,S}$ |
|-----------------|-----------------|-----------------|
| $F^{S,S}$ | $F^{M,A,S}$ | $F^{A,A,S}$ |
| $F^{S,S}$ | $F^{M,A,S}$ | $F^{A,A,S}$ |

### $nnscc$ states

| Flavor | $I = 1$ | $I = 0$ |
|--------|---------|---------|
|        | $F^{S,S}$ | $F^{A,A}$ |
|        | $F^{A,A}$ | $F^{A,A}$ |

### $F\otimes \phi \otimes \chi$

| $I = \frac{3}{2}$ | $F^{S,S}$ | $F^{A,A}$ |
|-----------------|-----------------|-----------------|
| $F^{S,S}$ | $F^{A,A}$ | $F^{A,A}$ |
| $F^{S,S}$ | $F^{A,A}$ | $F^{A,A}$ |

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**TABLE IV.** All possible flavor, color, and spin wave functions for the $nnscc$ and $nnscc$ states. $F \otimes \phi \otimes \chi$ represents the combination of the wave functions in different spaces. Here, the $I/J$ represents the isospin/spin.
C. Wave function for $\{12\}\{34\}\{56\}$ symmetry

Lastly, we construct the wave function of the $nnsscc$ ($I = 1, 0$) dibaryon states. It’s easy to notice that there are three pairs of identical particles $nn$, $ss$, and $cc$. For these states, it is convenient to use the $|nn\rangle|ss\rangle|cc\rangle$ basis to construct the $nnsscc$ ($I = 1, 0$) dibaryon states. We list corresponding favor, color-singlet, and spin wave functions in Table IV. In color part, we use the notation $|nn\rangle^{\text{color}}|ss\rangle^{\text{color}}|cc\rangle^{\text{color}}$ to describe the color-singlet wave functions, where color1, color2, and color3 stand for the color representations $n_1n_2$, $s_3s_4$, and $c_5c_6$, respectively. Similarly, we use the notation $|[n_1n_2]|s_3|s_4|c_5|c_6\rangle$ to describe the spin wave functions, where spin1, spin2, spin3, spin4, and spin5 represent the spins of $n_1n_2$, $s_3s_4$, $n_1n_2s_3s_4$, $c_5c_6$, and total spin, respectively. In Table IV, $S(A)$ means totally symmetric(antisymmetric) for $nn$, $ss$, and $cc$ of the discussed $nnsscc$ states.

IV. NUMERICAL RESULTS AND DISCUSSION

By constructing all the possible $F_{\text{flavor}} \otimes \phi_{\text{color}} \otimes \chi_{\text{spin}}$ bases satisfied for $\{1234\}\{56\}$, $\{1234\}\{56\}$, and $\{12\}\{34\}\{56\}$ symmetry and introducing the parameters collected in Table I, we show the obtained corresponding mass spectra for the studied doubly charmed dibaryon states in Table V. The masses of ground conventional baryons are adopted for these dibaryon states in Fig. I. Moreover, by rearranging the six constituent quarks, we also plot the possible thresholds of the baryon-baryon decay patterns for the $nnsscc$ states in Fig. I. There are two rearrangement decay types, (single charmed baryon + single charmed baryon) and (light baryon + doubly charmed baryon). Here, all the possible decay channels in the figure are $\Lambda \Lambda$, $\Lambda \Sigma$, $\Lambda \Xi$, $\Sigma \Sigma$, $\Sigma \Xi$, $\Sigma \Omega$, $\Xi \Omega$, and $N \Xi$, $N \Omega$, $\Delta \Xi$, $\Delta \Omega$. The masses of ground conventional baryons are adopted as the thresholds for these decay channels [57]. The $nnsscc$ states can decay to corresponding baryon-baryon final states through $S$-wave or $D$-wave. According to the conservation law in the strong decay process, the initial states and the final states have equal isospin $I$ and spin-parity $J^P$. For convenience, we label the spin (isospin) of the baryon-baryon states with subscript (superscript) in Fig. I. One of these states would be a good $nnsscc$ dibaryon candidate if it were observed in the corresponding decay channels.

The positions of these $nnsscc$ states are shown roughly in Fig. I and their properties may be changed accordingly if the masses of $nnsscc$ states are determined by an observed state. Of course, the mass gaps among the different states are reliable.

Because of the complicated mixing among a large number of color-spin wave functions from the seriously symmetric constraint, the total number of all the possible $nnsscc$ states is restricted to 17. There are 4 isotensor states, 8 isovector states, and 5 isoscalar states.

From Fig. I we can see that for the $nnsscc$ states, the $J^P = 0^+$ particles have the smallest and the largest masses among all quantum numbers. By labeling $nnsscc$ states for $I = 2, 1, 0$, and 0 with green, red, and blue lines, respectively, we can easily find that the $I = 0$ $nnsscc$ hexaquarks generally have smaller masses than the $I = 1$ states which have further smaller masses than the $I = 2$ ones, and thus our results indicate that the states with lower isospin quantum numbers generally form compact structures easily and have lower masses.

From Fig. I there is no state with $I(J^P) = 2(3^+)$ due to the constraint from the Pauli principle. Meanwhile, all states with $I = 2$ are above and beyond possible baryon-
TABLE V. The masses for the $nnncc$, $nnnsc$, $nnscc$, $ssncc$, and $sssscc$ dibaryon states in units of MeV. The $I$, $J$, and $P$ represent the isospin, the angular momentum, and the parity, respectively.

| Dibaryon $I(J^P)$ | Mass | Dibaryon $I(J^P)$ | Mass | Dibaryon $I(J^P)$ | Mass | Dibaryon $I(J^P)$ | Mass |
|-------------------|------|-------------------|------|-------------------|------|-------------------|------|
| 2(2$^+$) | 5098 | $\frac{3}{2}(3^+)$ | 5042 | $\frac{1}{2}(3^+)$ | 5093 | 1(3$^+$) | 5209 |
| 2(1$^+$) | 5140 | $\frac{3}{2}(2^+)$ | 5215 | $\frac{1}{2}(2^+)$ | 5199 | 1(2$^+$) | 5327 |
| 2(0$^+$) | 5374 | 5032 | 4940 | 5002 | 4944 | 5066 | 5025 | 5028 |
| 1(3$^+$) | 4936 | $\frac{3}{2}(1^+)$ | 5272 | $\frac{1}{2}(2^+)$ | 5069 | 1(2$^+$) | 5370 |
| 1(2$^+$) | 4918 | 4945 | 5260 | 5026 | 5022 | 5370 |
| nnncc | (4795) | (4986) | (4383) | (5234) | (5161) | 5350 |
| 1(1$^+$) | 4807 | 5456 | 5063 | 5041 | 5022 | 5203 |
| 1(0$^+$) | 4918 | 5243 | 5041 | 5026 | 5022 | 5350 |
| nnncc | (4986) | (5026) | (5041) | (5022) | (5022) | 5350 |
| 1(2$^+$) | 5002 | 4944 | 5066 | 4986 | 0(3$^+$) | 5209 |
| 1(2$^+$) | 5107 | 5368 | 5011 | 4989 | 4942 |
| 0(0$^+$) | 4824 | $\frac{1}{2}(3^+)$ | 5352 | 4986 | 0(3$^+$) | 5209 |
| 0(2$^+$) | 5045 | 5352 | 4986 | 0(3$^+$) | 5209 |
| 0(1$^+$) | 4683 | 5237 | 4953 | 5224 |
| sssncc | (4893) | (5489) | (4945) | (5097) |
| 0(0$^+$) | 4577 | 5460 | 4908 | 4904* |
| 0(0$^+$) | 4937 | 5307 | 4899 | 5300 |
| sssncc | (4577) | (5307) | (4899) | (5300) |
| 0(2$^+$) | 5557 | 5193 | 5069 | 0(1$^+$) | 5061 |
| 0(1$^+$) | 5604 | 5165 | 5027 | 5025 |
| 0(0$^+$) | 5574 | 5652 | 4973 | 4892 |
| ssssscc | (5574) | (5652) | (4973) | (4892) |
| 0(2$^+$) | 5766 | 5458 | 4944 | 4804* |
| 0(0$^+$) | 5574 | 5269 | 4755 | 4804* |
| ssssscc | (5574) | (5269) | (4755) | (4804*) |

baryon thresholds, and thus we think that there is no stable state and these states should have relatively wide widths for the isotensor sector. In the isovector case, both the heaviest state and the lightest state occur in $J^P = 1^+$. The mass gap between the two states is about 260 MeV. And the $H_{nc}(1, 1^+, 5107)$ can decay through all decay channels via $S$-wave accept $\Lambda\Lambda_c$. Similarly, there also does not exist $I(J^P) = 0(3^+)$ state because of the limitation of symmetry in the isoscalar sectors.

According to Fig. 1, we find that the lowest isoscalar state: $H_{cc}(0, 0^+, 4577)$ state can only decay into $\Lambda\Lambda_c$ and $N\Xi_{cc}$ final states. Moreover, due to the conservation of angular momentum and isospin, the $H_{cc}(2, 0^+, 4683)$ state mainly decay into $N\Xi^*_{cc}$ via $S$-wave, meanwhile, it also can decay into $\Lambda\Lambda_c$ and $N\Xi_{cc}$ via $D$-wave. Moreover, because of the small phase space, this state is expected to be a narrow state.

Next, we investigate the $nnncc$ ($I = 3/2, 1/2$) dibaryon states. Since three $n$ quarks are identical particles, we choose the wave functions in Sec. III as the eigenvector to diagonalize the Hamiltonian matrix. Different from the wave functions of the $nnncc$ state, the color and spin wave functions are constructed in the “baryon ⊗ baryon” configuration. According to the mass spectrum of $nnncc$ states in Table 1, similarly, we plot the relative positions for $nnncc$ states and all the baryon-baryon thresholds of the related rearrangement decay patterns in Fig. 2. The decays may occur through the $S$- or $D$-wave interactions and each $nnncc$ state can decay to these channels from the parity conservation, isospin conversation, and angular momentum conservation. There are three rearrangement decay types:

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(nnn + ccs), (nns + ccn), and (ccn + csn), respectively.

In Fig. 2 we label the nnnscc states for \( I = 3/2 \) and 1/2 with red and blue lines, respectively. We can easily find that the \( I = 1/2 \) states generally have lower masses than that of the \( I = 3/2 \) nnnscc states. Thus, our results indicate that the states with lower isospin quantum numbers generally form compact structures easily and have smaller masses. From Fig. 2 we find that the lowest \( I(J^P) = 1/2(1^+) \) \( H_{cc,s}(4603,1/2,1^+) \) state is below all the rearrangement decay channels and its decay through baryon-baryon channels is kinetically forbidden. So, we consider it a relatively stable state. Meanwhile, we label this stable state with “*” in the Fig. 2 and Table V.

In the nnnscc (\( I = 3/2 \)) case, most states are expected with relatively wide widths, because they all have many different rearrangement decay channels. For example, the lowest \( I(J^P) = 3/2(2^+) \) state: \( H_{cc,s}(3/2,2^+,4940) \) can easily decay to \( \Sigma\Xi^* \) final states through \( S \) wave. It also decays into \( \Sigma\Xi^* \), \( \Sigma\Xi^* \) final states through \( D \) wave.

As for the nnnscc with \( I = 1/2 \), most of them are unstable particles against strong decay. Both the heaviest state and the lightest states occur in \( J^P = 1^+ \). Here, both the \( H_{cc,s}(1/2,0^+,4755) \) and \( H_{cc,s}(1/2,1^+,4771) \) states have the same three quark-rearrangement decay channels \( \Lambda\Xi, \Lambda\Xi, \) and \( N\Omega_{cc} \), respectively. Moreover, the \( H_{cc,s}(1/2,3^+,5093) \) state has only one baryon-baryon decay channel \( \Sigma^*\Xi^* \) via \( S \) wave. Meanwhile, it has many different decay channels via the \( D \) wave.

C. nnssscc

To deal with the permutation symmetry of identical quarks \( n - n, s - s, \) and \( c - c \), respectively, the wave functions have been listed in Sec. III C. The flavor, color, and spin wave functions are constructed in the \(|(nn) \otimes (ss) \otimes (cc)| \) configuration. According to corresponding mass spectrum in Table V we plot the relative positions for the nnsscc dibaryon states in Fig. 3 (a). Meanwhile, all possible baryon-baryon thresholds of the related rearrangement decay patterns are shown in Fig. 3 (a). There are four rearrangement decay types: \( (nss + acc), (nss + ncc), (nsc + nac), \) and \( (nnn + ssc) \), respectively.

By labeling nnsscc states for \( I = 1 \) and 0 with red and blue lines, respectively, we can easily find that the \( I = 0 \) states generally have smaller masses than the \( I = 1 \) ones. Moreover, it is easy to find that the lowest two nnsscc dibaryon states: \( H_{cc,2s}(4827,0,0^+) \) and \( H_{cc,2s}(4804,0,1^+) \), should be stable, which are under all the baryon-baryon thresholds and these two states are probably narrow.

For the isovector nnsscc states, the heaviest state and the lightest state both are the \( J^P = 0^+ \) states, i.e. \( H_{cc,2s}(1,0^+,5550) \) and \( H_{cc,2s}(1,0^+,4942) \). The mass gap between the two states is about 610 MeV. All the states are expected to be broad states, which have many different rearrangement decay channels.

D. sssnc and ssssc

Lastly, we discuss the sssnc and the ssssc dibaryon states. The number of allowed sssnc dibaryon states is the same as the nnnscc (\( I = 3/2 \)) dibaryon states.
because of the same symmetry requirements according to Table V. Similarly, the number of allowed ssssc dibaryon states is the same as the nnncc ($I = 1$) dibaryon states. Further, we also plot the relative positions for the sssnc, ssssc dibaryon states and their all possible rearrangement decay channels in Fig. 3 (b)-(c), respectively.

For sssnc dibaryon states, they have 12 possible rearrangement decay channels, including $\Omega^+_c\Xi^+_c$, $\Omega^+_c\Xi'^_c$, $\Omega^+_c\Xi^0_c$, $\Omega^+_c\Xi^+_c$, $\Omega^+_c\Xi^0_c$, $\Omega^+_c\Xi'^+_c$, $\Omega^+_c\Xi'^0_c$, $\Omega^+_c\Xi'^+_c$, $\Omega^+_c\Xi'^0_c$, $\Omega^+_c\Xi'^+_c$, and $\Omega^+_c\Xi'^0_c$, respectively. For ssssc dibaryon states, they have 5 possible rearrangement decay channels, including $\Omega^+_c\Omega^+_c$, $\Omega^+_c\Omega^0_c$, $\Omega^+_c\Omega^0_c$, $\Omega^+_c\Omega^0_c$, and $\Omega^+_c\Omega^0_c$. From Fig. 3 (c), we notice the ground ssssc dibaryon state with quantum number $I(J^P) = 0(3^+)$ does not exist due to the constraint from the Pauli Principle.

From Fig. 3 (b), all states are above some possible baryon-baryon thresholds in the ssssc dibaryon state. Thus, they should have relatively wide widths. As expected, all of the ssssc dibaryon states are higher than allowed baryon-baryon thresholds from Fig. 3 (c). They can easily decay into the rearrangement channels because of the large phase space. Based on these, all the ssssc dibaryon states are probably broad. It seems not easy to find them experimentally.

V. SUMMARY

In 2021, the LHCb collaboration reported the first doubly charmed tetraquark state $T^{+}_{cc}$ (ccâil). Inspired by this discovery, the existence of double heavy tetraquark states or pentaquark states with different quark configurations
has been studied through various theoretical calculations. Based on the similarity of the ccdd tetraquark state and the \(\Xi^{++}\) baryon (ccu), we would naturally ask if the doubly charmed dibaryon states exist. At the same time, it is an interesting question whether the dibaryon state with double heavy quarks is stable against the strong decay also has a long road.

In this work, we firstly introduce the chromomagnetic interaction (CMI) model and construct the \(F_{\text{flavor}} \otimes \phi_{\text{color}} \otimes \chi_{\text{spin}}\) wave functions for doubly charmed dibaryon system. Here, the biggest challenge is how to apply the Pauli principle to the permutation of the three or four identical particles. Hence we need to introduce the Young tableau to represent the irreducible bases of the permutation group. Then we systematically calculate the corresponding Hamiltonian. Further, we obtain the mass spectra of \(nnncc\), \(nnnsc\), \(nnscc\), \(ssncc\), and \(ssssc\) dibaryon states, which have been listed in Table V. Based on these, the relative positions between dibaryon states and possible baryon-baryon thresholds are also plotted in Figs. 11-13. By comparing with baryon-baryon thresholds, we can analyze the stability of dibaryon states and the decay properties from the possible quark rearrangement decay channels. Moreover, the mass gaps between relevant states are reliable, and if one \(qqqqcc\) dibaryon state would be observed, we can use these mass gaps to predict their corresponding multiples.

According to the obtained mass spectrum, there is no state with \(I(J^P) = 2(3^+)\) in \(nnncc\) and \(ssssc\) states due to the constraint from the Pauli principle. Moreover, our results indicate that the states with lower isospin quantum number are generally form compact structure easily and have smaller masses in the \(nnncc\) \((I = 2, 1, 0)\), \(nnnsc\) \((I = 3/2, 1/2)\), and \(nnnsc\) \((I = 1, 0)\) states. Meanwhile, we find three relative “stable” states: \(H_{cc,s}(1/2, 1^+, 4603)\), \(H_{cc,s}(0, 0^+, 4827)\), and \(H_{cc,s}(0, 1^-, 4804)\), whose decays through baryon-baryon channels are kinetically forbidden. Of course, due to the uncertainty of the CMI model, further dynamical calculations are still needed to clarify their natures. We hope that the present study may inspire relevant experiments to search for these states in the future.

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