USING NEWTON’S LAW FOR DARK ENERGY

Paul H. Frampton

Department of Physics and Astronomy,
University of North Carolina,
Chapel Hill, NC 27599-3255, USA

and

Centro Universitario Devoto, Buenos Aires, Argentina
(Dated: August 19, 2021)

Abstract

A model is introduced in which Newton’s law is modified between matter and dark energy corpuscles (DECs). The model predicts that the DEC component is presently decelerating in its expansion at 14% of the magnitude of the matter expansion acceleration. In the future, expansion of the DEC universe will continue to decelerate.

a frampton@physics.unc.edu
b Permanent address
The nature of dark energy [1]-[2] is one of the most pressing problems in physics and astronomy. One underlying question is whether it necessitates a modification of general relativity at large cosmological distances greater than $10^{12}$ m, the size of the solar system. If we are content with a cosmological constant term, as introduced by Einstein for an obsolete reason, then general relativity survives. On the other hand, valiant attempts to make a microscopic theory of dark energy remain unconvincing.

Dark energy is a new driving term in the Friedmann equations doctored to fit the observed accelerated expansion. If the equation of state is $w = p/\rho = -1$ it coincides with a cosmological constant. Therefore it is widely assumed that the future evolution will lead to an eternal exponential expansion with acceleration. This assumption will be questioned in this article in which we will consider a model in which the repulsive nature of the associated force is accommodated more directly, and eventual deceleration is inevitable.

The idea is to introduce a constituent particle of dark energy which we shall call a Dark Energy Corpuscle (DEC), and to postulate how it interacts with itself and with matter, by which we include both luminous and dark matter.

We recall that Friedmann’s equations in a flat matter-dominated universe coincide with Newton’s law of gravity. In the FLRW cosmology with vanishing curvature, and including both matter density $\rho_m$ and dark energy $\rho_\Lambda$ they are

$$\frac{\dot{a}}{a} = \frac{8\pi G}{3}(\rho_m + \rho_\Lambda) \quad (1)$$

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_m + \rho_\Lambda + 3P_\Lambda) \quad (2)$$

where $a(t)$ is the scale factor and $P_\Lambda$ is the dark energy pressure.

According to WMAP7 [3], the cosmology is consistent with zero curvature $k = 0$ and critical energy density divided into matter $\Omega_m = 0.28$ and dark energy $\Omega_\Lambda = 0.72$. The outward acceleration of a matter test particle is observed to be

$$a_m(t) = 13.40 \times 10^{-10} \text{ m/s}^2 \quad (3)$$

In order to break down Eq.(3) into the contributions of matter and of DECs, we use for the critical density at the present time $t = t_0$

$$\rho_c(t_0) = 9.46 \times 10^{-27} \text{ kg m}^{-3} \quad (4)$$

so that the matter and dark energy densities are

$$\rho_m(t_0) = \Omega_m \rho_c(t_0) = 2.65 \times 10^{-27} \text{ kg m}^{-3} \quad (5)$$
$$\rho_\Lambda(t_0) = \Omega_\Lambda \rho_c(t_0) = 6.81 \times 10^{-27} \text{ kg m}^{-3} \quad (6)$$

From the same source [3] the accurate comoving radius of the visible universe bounded by the surface of last scatter (SLS) is

$$R_u(t_0) = 4.36 \times 10^{28} \text{ m} \quad (7)$$

The corresponding masses of matter ($m_m(t_0)$) and dark energy ($m_\Lambda(t_0)$) from the densities Eqs. (5) and (6) are

$$m_m(t_0) = 9.10 \times 10^{53} \text{ kg} = 4.55 \times 10^{23} M_\odot \quad (8)$$
$$m_\Lambda(t_0) = 2.37 \times 10^{54} \text{ kg} = 1.18 \times 10^{24} M_\odot \quad (9)$$

using for the solar mass $M_\odot = 2 \times 10^{30}$ kg.
Let us consider a test particle of mass \( m \) type at the SLS. Then, according to Newton’s law its outward acceleration is, due to \( M \) at \( t_0 \),

\[
(a_m(t_0))_m = \frac{GM_m(t_0)}{R^2(t_0)} = -3.22 \times 10^{-10} \text{ m/s}^2
\]

which was the expected prediction before the observation that \( a_m(t_0) \) is given instead by the very different value in Eq. (3) having the opposite sign and much larger magnitude.

In the present DEC-model, we postulate a novel repulsive counterpart of Newton’s law as the appropriate gravitational interaction between DECs and matter

\[
F_{\text{Newton}}^{m-\text{DEC}} = -\frac{2Gm_1m_2}{r^2}
\]

This provides an additional contribution to \( a_m(t_0) \) given by

\[
(a_m(t_0))_\Lambda = \frac{GM_\Lambda(t_0)}{R^2(t_0)} = +16.62 \times 10^{-10} \text{ m/s}^2
\]

The result for \( a_m(t_0) \) is therefore outward by

\[
a_m(t_0) = (a_m(t_0))_m + (a_m(t_0))_\Lambda
\]

\[
= +13.40 \times 10^{-10} \text{ m/s}^2
\]

which agrees exactly with observations in \( \ref{1,2} \) and Eq. (3) above.

To complete our DEC model it is needed only to postulate the interaction between DECs and as the most conservative possibility we assume the same force as between matter particles, namely

\[
F_{\text{DEC-DEC}}^{\text{Newton}} = \frac{Gm_1m_2}{r^2}
\]

Since dark energy has never been directly observed, only indirectly from the expansion of the universe, and indirectly from other cosmological observations, it may seem challenging to confirm Eq. (15)

The DEC model, however, provides a novel view of the future of the universe.

From the Friedmann equation, Eq. (1), and having in mind that the different scaling between \( \rho_m \sim a^{-3} \) and \( \rho_\Lambda \sim \text{constant} \) the distant future evolution of the scale factor is

\[
a(t) = \exp(H_0(t - t_0))
\]

where \( \dot{a}/a \) is the Hubble parameter which is asymptotically constant.

The behavior, Eq. (16), suggests that the expansion of the visible universe continues to accelerate indefinitely.

This conclusion is, however, misleading as follows. To explain perspicuously, it is first necessary to discuss the modest expansion of the visible universe from \( R_u(t_0) \) to \( R_u(t) \), \( t \gg t_0 \), starting from

\[
R_u(t_0) = c \int_{0}^{t_0} \frac{dt}{a(t)} = 4.36 \times 10^{28} \text{ m}
\]

to arrive at

\[
R_u(t_1) = 4.36 \times 10^{28} \text{ m} + c \int_{t_0}^{t_1} \frac{dt}{a(t)}
\]

\[
= 4.36 \times 10^{28} \text{ m} + cH_0^{-1}
\]

\[
= 5.67 \times 10^{28} \text{ m}
\]

for any \( t_1 \gg t_0 \).
As an illustration, let us choose the future redshift as
\[ z_1 = -1 + 10^{-8} \] (21)
so that \( a(t_1) = 10^{-8} \) and the fraction of matter reduces to
\[ \Omega_m(t_1) = 2.8 \times 10^{-25} \] (22)
while the fraction of dark energy is \( \Omega_\Lambda = (1 - \Omega_m) \).

Because of the increase in the scale factor, no other galaxy will be visible \[4\] so that normal matter will not be of interest to cosmologists. Instead, in the then visible universe it will be the DECs which will be the subject of scientific enquiry and detectable by the technology.

For DECs the effect of the normal matter will be negligible and so a DEC test particle at the SLS will have an outward acceleration
\[ a_\Lambda(t_1) = \frac{GM_\Lambda(t_1)}{R_u(t_1)^2} \] (23)
\[ = 10.77 \times 10^{-10} \text{m/s}^2 \] (24)
which will remain essentially constant in this DEC.

Thus, the universe reverts to a decelerating expansion with respect to the overwhelming component of dark energy.

Because the radius of the visible universe remains constant, all of the matter component exit to be rendered invisible \[4\]. All that remains which is of cosmological interest are the dark energy corpuscles which by then we will be able to detect and measure. With regard to these, the cosmic expansion will be decelerating indefinitely into the infinite future even for the case of a constant dark energy equation of state \( w_\Lambda = p_\Lambda/\rho_\Lambda = -1 \).

It is mildly titillating to speculate that future technology may harness the gravitational repulsion in Eq.(11) so that DECs fulfil the antigravity hopes mainly of science fiction writers, although antigravity does appear in some versions of supergravity theory \[5\].

**Acknowledgments:** I am grateful to J. M. Maldacena for a useful discussion and to the Centro Universitario Devoto for providing the luxury of unencumbered time. This work was supported by the U.S. Department of Energy Grant No. DE-FG02-05ER41418.

---

[1] S. Perlmutter et al. [Supernova Cosmology Project Collaboration], Astrophys. J. 517, 565 (1999) [astro-ph/9812133].

[2] A. G. Riess et al. [Supernova Search Team Collaboration], Astron. J. 116, 1009 (1998) [astro-ph/9805201].

[3] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 192, 18 (2011) [arXiv:1001.4538 [astro-ph.CO]].

[4] L. M. Krauss and G. D. Starkman, Astrophys. J. 531, 22 (2000) [astro-ph/9902189].

[5] J. Scherk, Phys. Lett. B 88, 265 (1979).