**Performance Analysis of a Low-Interference \(N\)-Continuous OFDM Scheme**

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**Abstract**—This letter analyzes the performance of a low-interference time-domain \(N\)-continuous orthogonal frequency division multiplexing (TD-NC-OFDM) scheme for sidelobe suppression in terms of power spectrum density (PSD) and bit error rate (BER). Firstly, an asymptotic closed-form expression of the low-interference TD-NC-OFDM signal. Then, an asymptotic expansion of BER is achieved for the received signal over the multipath channel. Simulation results confirm the accuracy of our performance analysis.

**Index Terms**—\(N\)-continuous orthogonal frequency division multiplexing (NC-OFDM), sidelobe suppression, time-domain \(N\)-continuous OFDM (TD-NC-OFDM).

I. INTRODUCTION

ORTHOGONAL frequency division multiplexing (OFDM) has been widely adopted in wireless communications \[1\] due to its high-speed data transmission and capability of combating the inter-symbol interference (ISI) in frequency selective fading channels. However, traditional rectangularly pulsed OFDM has high out-of-band power emission, causing severe interference to adjacent channels \[2\]. To solve the problem, many methods have been proposed for sidelobe suppression in OFDM systems \[3\]–\[6\].

Recently, due to excellent sidelobe suppression performance, \(N\)-continuous OFDM (NC-OFDM) has been paid more attention \[7\]–\[17\]. By making the OFDM signal and its first \(N\) derivatives continuous, NC-OFDM removes the discontinuities of the OFDM signal and achieves a steep out-of-band spectrum. However, conventional NC-OFDM \[7\] is computationally inefficient with severe interference, the simplified schemes in \[8\]–\[10\] has a reduction in efficiency, the techniques in \[11\] and \[12\] are with complexity overhead at the transceiver.

To reduce the complexity of NC-OFDM, in \[15\], \[16\], time-domain NC-OFDM (TD-NC-OFDM) is proposed by adding a smooth signal into the conventional OFDM signal in the time domain. The smooth signal is designed by the linear combination of basis signals belonging to a basis set. Considering the high interference of the smooth signal due to its time-domain distribution, Ref. \[17\] proposes a low-interference TD-NC-OFDM with a redesign of the basis signals truncated by a smooth window. The low-interference TD-NC-OFDM can obtain a promising tradeoff among complexity, sidelobe suppression, and bit error rate (BER) performance.

In this work, we examine the potential of the low-interference TD-NC-OFDM over a multipath fading channel, and present a theoretical analysis to evaluate the system performance in terms of sidelobe suppression and bit error rate (BER). Firstly, by considering the effect of the window on the smooth signal, an asymptotic expression of the power spectrum density (PSD) of the low-interference TD-NC-OFDM signal is obtained, with the objective of showing that the proposed scheme can maintain similar sidelobe suppression as opposed to traditional NC-OFDM. Secondly, based on the analyzed signal-to-interference-plus-noise ratio (SINR), an asymptotic expression of BER that the low-interference scheme can achieve a promising system performance close to that of the original OFDM.

II. LOW-INTERERENCE TD-NC-OFDM

In an OFDM system with the symbol length \(M\) and the subcarrier index set \(\mathcal{K} = \{k_0, k_1, \ldots, k_{K-1}\}\), the proposed low-interference scheme \[17\] adds a smooth signal \(w_i(m)\) onto the OFDM signal \(y_i(m)\) in the time domain, given as

\[
y_i(m) = \tilde{y}_i(m) + w_i(m),
\]

where \(m \in \mathcal{M} = \{-M_{cp}, -M_{cp}+1, \ldots, M-1\}\) and \(M_{cp}\) is the cyclic prefix (CP) length. Since the smooth signal is aligned with the beginning of the OFDM symbol and added to the front part of each cyclic-prefixed OFDM symbol, to obtain the \(N\)-continuous OFDM signal, \(w_i(m)\) should satisfy

\[
w_i^{(n)}(m)\bigg|_{m=-M_{cp}} = y_i^{(n)}(m)\bigg|_{m=M} - y_i^{(n)}(m)\bigg|_{m=-M_{cp}},
\]

where \(y_i^{(n)}(m)\) is the \(n\)th-order derivative of \(y_i(m)\) and \(n \in \mathcal{N} = \{0, 1, \ldots, N\}\) with the highest derivative order (HDO) \(N\).

The linear-combination design of \(w_i(m)\) is described as

\[
w_i(m) = \sum_{n=0}^{N} b_{i,n}\tilde{f}_n(m), \quad m \in \mathcal{L},
\]

\[
0, \quad m \in \mathcal{M}\backslash \mathcal{L},
\]

where \(\mathcal{L} \triangleq \{-M_{cp}, -M_{cp}+1, \ldots, -M_{cp}+L-1\}\) indicates the location of \(w_i(m)\) with a support length of \(L\), and the basis signals \(\tilde{f}_n(m)\) belong to the basis set \(\mathcal{Q}\), defined as

\[
\mathcal{Q} \triangleq \{ q_\beta | q_\beta = [\tilde{f}_\beta(-M_{cp}), \ldots, \tilde{f}_\beta(-M_{cp}+L-1)]^T, \beta \in \mathcal{U}_{2N} \},
\]

where \(\mathcal{U}_{2N} = \{0, 1, \ldots, 2N\}\).
On the one hand, in the basis set $Q$, the basis signals $\tilde{f}_n(m)$ is truncated by a window $g(m)$, given as

$$f_n(m) = \begin{cases} f^{(\tilde{n})}(m)g(m)u(m), & m \in \mathcal{L}, \\ 0, & m \in \mathcal{M} \setminus \mathcal{L}, \end{cases}$$

(5)

where $u(m)$ denotes the unit-step function, $g(m)$ is a smooth and zero-edged window function, such as the triangular, Hanning, or Blackman window function, and $f^{(\tilde{n})}(m)$ is calculated by the system configuration [17] as follows

$$f^{(\tilde{n})}(m) = \frac{1}{M} \left( j \frac{2\pi}{M} \right)^{\tilde{n}} \sum_{k_r \in K} k_r e^{-j\varphi_k} e^{j2\pi k m}. \quad (6)$$

On the other hand, from [17], by substituting Eqs. (3)-(6) into Eq. (2), the simplified calculation of coefficients $b_{i,n}$ is given by

$$b_i = P_f^{-1} \begin{bmatrix} y_{i-1}(M) - y_i(-M_L) \\ P_{1x_{i-1}} - P_{2x_i} \end{bmatrix},$$

(7)

where $P_f$ is a $(N+1) \times (N+1)$ symmetric matrix with its $(n+1)$th row and $(\tilde{n}+1)$th column element $f^{(n+\tilde{n})}(-M_L)$ for $\tilde{n} \in \mathcal{U}_N$,

$$P_1 = \frac{1}{M} \begin{bmatrix} j2\pi k_0 / M \cdots j2\pi k_{K-1} / M \\ \vdots \vdots \\ (j2\pi k_0 / M)^N \cdots (j2\pi k_{K-1} / M)^N \end{bmatrix},$$

$$P_2 = P_f \Phi \Phi \triangleq \text{diag}(e^{j2\pi k_1}, e^{j2\pi k_2}, \ldots, e^{j2\pi k_{K-1}}),$$

and $\varphi = -2\pi M_L / M$.

Finally, $w_{i,n}(m)$ is added onto only part of the CP-OFDM symbol to achieve the $N$-continuous symbol $\tilde{y}_i$, as follows

$$\tilde{y}_i = \begin{cases} y_i + \begin{bmatrix} 0_{(M+M_L-L-1) \times 1} \\ Q_f b_i \end{bmatrix}, & 0 \leq i \leq M_\text{s} - 1, \\ 0_{(M+M_L-L) \times 1}, & i = M_\text{s}, \end{cases}$$

(8)

where $Q_f = [q_0 \quad q_1 \quad \ldots \quad q_N]$, $M_\text{s}$ is the number of OFDM symbols, and $x_{i-1}$ is initialized as a zero vector $x_{-1} = 0_{K \times 1}$.

III. THEORETICAL ANALYSIS OF SPECTRUM, SINR, AND COMPLEXITY

A. Spectral Analysis

In a continuous time, the transmit $N$-continuous OFDM signal is

$$s(t) = \sum_{i=0}^{+\infty} \tilde{y}_i(t - iT)$$

(9)

given by

$$\tilde{y}_i(t) = \begin{cases} \sum_{r=0}^{K-1} \bar{x}_{i,k_r} e^{j2\pi k_r \Delta f t} - T_{cp} \leq t < -T_{cp} + T_L, \\ \sum_{r=0}^{K-1} \bar{x}_{i,k_r} e^{j2\pi k_r \Delta f t} - T_{cp} + T_L \leq t < T_s, \end{cases}$$

(10)

with the smoothed symbol $\bar{x}_{i,k_r}$ on the $(r+1)$th subcarrier, the frequency spacing $\Delta f = 1/T_s$, the symbol duration $T_s$, the CP duration $T_{cp}$, and the duration of the smooth signal $T_L$.

Assume that the first $N$ derivatives of the smoothed OFDM signal $\tilde{y}_i(t)$ are continuous, and the $(N+1)$th-order derivative has finite amplitude discontinuity [20]. Thus, according to the definition of the PSD [5] and the relationship between spectral roll-off and continuity in [18], the PSD of the low-interference TD-NC-OFDM signal can be expressed by

$$\Psi(f) = \lim_{T \to +\infty} \frac{1}{2\pi f} \left\{ \sum_{i=-\infty}^{\infty} \mathcal{F} \left\{ \tilde{y}_i^{(N+1)}(t) \right\} e^{j2\pi ft} \right\}^2,$$

(11)

Eq. (11) indicates that the spectrum of the low-interference TD-NC-OFDM signal is related to the expectation of $\tilde{y}_i^{(N+1)}(t)$ multiplied by $f^{-N-1}$. However, in the design of the smooth signal, the truncation window $g(t)$ with zero edges cannot possess infinite derivatives. Thus, when $N$ is large, Eq. (11) may not be available. Thus, in this paper, the symmetric Blackman window function is used as an example for the PSD calculation of the low-interference TD-NC-OFDM signal, where $g(t) = 0.42 - 0.5\cos(\pi t / T_L) + 0.08\cos(2\pi t / T_L)$ with $T_L = (L-1)T_{samp}$ and $T_{samp} = T_s / M$.

According to the first and second derivatives of the Blackman function, $0.5(\pi / T_L)\sin(\pi t / T_L) - 0.16(\pi / T_L)^2\sin(2\pi t / T_L)$ and $0.5(\pi / T_L)^2\cos(\pi t / T_L) - 0.32(\pi / T_L)^2\cos(2\pi t / T_L)$, it can be proved that for $t \in (-\infty, +\infty)$, the former is continuous and the latter is discontinuous with finite amplitudes at the edges of 0 and $2T_L$. As shown in Fig. 1 when $N \geq 1$, the discontinuities may appear at times $-T_{cp}$ and $-T_{cp} + T_L$. Since these two discontinuities at times $-T_{cp}$ and $-T_{cp} + T_L$ may not appear at the same time, three conditions are discussed as follows.

![Smooth signal](image1.png)

- Smooth signal
- Conventional OFDM signal

Fig. 1. Diagram of the discontinuities of high-order derivatives of the low-interference TD-NC-OFDM signal during one baseband OFDM symbol.

Firstly, for the ease expression, Eq. (7) is rewritten by

$$b_i = P_f^{-1} P_{1x_{i-1}} - P_f^{-1} P_1 \Phi x_i,$$

(12)

where

$$P_1 = \begin{bmatrix} 1 \cdots 1 \\ \vdots \vdots \\ (j2\pi k_0 / M)^N \cdots (j2\pi k_{K-1} / M)^N \end{bmatrix}.$$
where $a_{n,r}$ and $b_{n,r}$ are, respectively, the $(n+1)$th row and $(r+1)$th column elements of $P_f^{-1}P_1$ and $P_f^{-1}P_1\Phi$, and $f(n)(t)$ and $g(t)$ consisting of $\tilde{f}_n(t)$ are, respectively, given by

$$f(n)(t) = \frac{1}{M} \sum_{k_r \in K} (j2\pi k_r \Delta f)^n e^{j2\pi k_r \Delta f(t+T_{cp})}, \quad (14)$$

and

$$g(t) = 0.42 - 0.5 \cos(\pi t + T_{cp} + T_L)/T_L + 0.08 \cos(2\pi t + T_{cp} + T_L)/T_L. \quad (15)$$

1) $N = 0$: When $N = 0$, the first derivative of $s(t)$ is discontinuous at time $-T_{cp}$. From (11), the PSD of $s(t)$ can be expressed as

$$\Psi(f) = \lim_{U \to \infty} \frac{1}{2UT} E \left\{ \sum_{i=-U}^{U-1} \left[ \int_{-T_{cp}}^{-T_{cp}+T_L} \frac{g_i(t)}{j2\pi f} e^{-j2\pi ft} dt \right]^2 \right\}. \quad (16)$$

Since the support range of the smooth signal is $\pm T_L$, the Fourier transform $F \{ g_i(t) \}$ can be expressed as

$$F \{ g_i(t) \} = \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} g_i(t)e^{-j2\pi ft} dt + \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} a_i(t)e^{-j2\pi ft} dt. \quad (17)$$

Substituting (10) and (13)-(15) into (17) gives

$$F \{ g_i(t) \} = \sum_{r=0}^{K-1} \left( \frac{j2\pi k_r}{T_s} \right)^2 x_{i,k} e^{j\pi f_s (1-\beta_1)} \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} g_i(t)e^{-j2\pi ft} dt + \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} a_i(t)e^{-j2\pi ft} dt. \quad (18)$$

where $sinc(x) \triangleq \sin(\pi x)/(\pi x)$, $f_s = k_r - T_s f_s$, $\beta_1 = T_{cp}/T_s$.

$$F_0(f) = \frac{1}{M} \sum_{r=0}^{K-1} e^{j2\pi T_{cp} f + 2j\beta_2 f_{cp}} \left( j2\pi k_r \left( 0.42\beta_2 \sin(\beta_2 f) - \cos(\pi \beta_2 f) \right) \right) \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} g_i(t)e^{-j2\pi ft} dt + \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} a_i(t)e^{-j2\pi ft} dt. \quad (19)$$

and $\beta_2 = T_L/T_s$.

By substituting (17)-(19) into (16), we have

$$\Psi(f) = \lim_{U \to \infty} \frac{1}{2UT} E \left\{ \sum_{i=-U}^{U-1} e^{-j2\pi ft} \left[ \sum_{r=0}^{K-1} k_{i,k} e^{j\pi f_s (1-\beta_1)} \sin(f_s (1 + \beta_1)) \right] \right\}^2 \quad (20)$$

which shows that when $f$ is large, the PSD of $s(t)$ with its 0th derivative continuous decays as $f^{-4}$.

2) $N = 1$: When $N = 1$, the second derivative of $s(t)$ has discontinuous at both time $-T_{cp}$ and time $-T_{cp} + T_L$. Thus, according to (11), the PSD of $s(t)$ can be calculated by

$$\Psi(f) = \lim_{U \to \infty} \frac{1}{2UT} E \left\{ \sum_{i=-U}^{U-1} \left[ \int_{-T_{cp}}^{-T_{cp}+T_L} \frac{g_i(t)}{j2\pi f} e^{-j2\pi ft} dt \right]^2 \right\}. \quad (21)$$

where the $\mathcal{F} \{ g_i(t) \}$ is expressed as

$$\mathcal{F} \{ g_i(t) \} = \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} g_i(t)e^{-j2\pi ft} dt + \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} a_i(t)e^{-j2\pi ft} dt. \quad (22)$$

Substituting (10) and (13)-(15) into (22) gives

$$\mathcal{F} \{ g_i(t) \} = \sum_{r=0}^{K-1} \left( \frac{j2\pi k_r}{T_s} \right)^2 x_{i,k} e^{j\pi f_s (1-\beta_1)} \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} g_i(t)e^{-j2\pi ft} dt + \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} a_i(t)e^{-j2\pi ft} dt \quad (23)$$

with

$$F_1(f) = \frac{1}{M} \sum_{r=0}^{K-1} \sum_{n=0}^{2} \left( \frac{2}{n} \right) \left( j2\pi k_r T_s \right)^{n+2} e^{j\pi (2T_{cp} f + \beta_2 f)} \times \left( 0.42(n) T_s \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} g_i(t)e^{-j2\pi ft} dt \right) + \frac{1}{T} \int_{T_{cp}}^{T_{cp}+T_L} a_i(t)e^{-j2\pi ft} dt. \quad (24)$$

where $\left( \frac{2}{n} \right)$ are the binomial coefficients.

According to (22)-(24), Eq. (21) can be expressed as

$$\Psi(f) = \lim_{U \to \infty} \frac{1}{2UT} E \left\{ \left[ \sum_{i=-U}^{U-1} e^{-j2\pi ft} \left[ \sum_{r=0}^{K-1} k_{i,k} e^{j\pi f_s (1-\beta_1)} \sin(f_s (1 + \beta_1)) \right] \right] \right\}^2 \quad (25)$$

which shows that when $s(t)$ and its first derivative continuous, the PSD of $s(t)$ decays as $f^{-6}$.

3) $N \geq 2$: In this case, the discontinuity first appears at time $-T_{cp} + T_L$ for the $u_i^{(2)}(t)$. With the point $u_i(-T_{cp})$, the discontinuity of $u_i^{(N+1)}(t)$ will appear at time $-T_{cp}$. Thus, the PSD of $s(t)$ is written by

$$\Psi(f) = \lim_{U \to \infty} \frac{1}{2UT} E \left\{ \left[ \sum_{i=-U}^{U-1} e^{-j2\pi ft} \left[ \frac{\mathcal{F} \{ u_i^{(N+1)}(t) \}}{j2\pi f} \right] \right] \right\}^2 \quad (26)$$
where

\[ F\{y_i^{(N+1)}(t)\} = \frac{1}{T} \int_{-T_{cp}}^{T_{cp}} (y_i^{(N+1)}(t) + u_i^{(N+1)}(t) \delta(t + T_{cp})) e^{-j2\pi ft} dt \]

be far small, and when \( f \) is large, the PSD decaying of \( s(t) \) approximates to \( f^{-2N-4} \).

Fig. 2 compares the theoretical and simulation results of the low-interference scheme with \( L = M_{cp} = 144 \) and \( M = 2048 \) according to the LTE parameter configuration [19]. It is shown that the simulation results match well with their theoretical counterparts of (20), (25), and (30) with different HDs. When \( N \) is large, the sideloobe suppression performance is improved with the increased length of the smooth signal, for example, \( N = 3 \) and 4.

![Fig. 2. PSD comparison between the analytical and simulation results in low-interference TD-NC-OFDM.](image)

**B. BER Analysis in a Multipath Fading Channel**

In a multipath channel with time-domain coefficients \( h_{i,j} \) on the \( i \)th path, the \( i \)th received OFDM symbol \( r_i(t) \) is given by

\[ r_i(t) = \sum_{l=1}^{L} h_{i,j} y_j(t - \tau_l) + n_i(t), \]  

where \( h_{i,j} \) is a complex valued following independent and identically distributed (i.i.d.) with mean zero and variance \( \sigma_i^2 \), \( \tau_l \) is the time delay along the \( l \)th path, and \( n_i(t) \) is the additive white Gaussian noise (AWGN) noise with mean zero and variance \( \sigma_n^2 \).

Under the assumption of perfect synchronization and long enough CP to combat the channel effect, after removing CP and a \( M \)-point DFT, the received signal on the \((r+1)\)th subcarrier is

\[ R_{i,k_r} = H_{i,k_r} \bar{Y}_{i,k_r} + N_{i,k_r} = H_{i,k_r}(x_{i,k_r} + W_{i,k_r}) + N_{i,k_r}, \]  

where \( R_{i,k_r} \), \( \bar{Y}_{i,k_r} \), and \( N_{i,k_r} \) are the frequency-domain expression of \( r_i(t) \), \( y_j(t) \), and \( n_i(t) \), respectively, \( H_{i,k_r} \) is the frequency-domain channel response, and \( W_{i,k_r} \) is the interference caused by the smooth signal.

Thus, the instantaneous SINR on the subcarrier \( k_r \) can be calculated by

\[ \gamma_{i,k_r} = \frac{|H_{i,k_r}|^2 E\{ |x_{i,k_r}|^2 \}}{|H_{i,k_r}|^2 E\{ |W_{i,k_r}|^2 \} + E\{ |N_{i,k_r}|^2 \}}. \]
Suppose that \(x_{i,k_r}\) is i.i.d. with mean zero and a normalized variance, we have \(E\{x_{i,k_r}^2\} = 1\), \(E\{x_i x_i^H\} = I_K\), and \(E\{x_{i-1} x_i^H\} = 0_{K \times K}\).

\[
E\{|N_{i,k_r}|^2\} = E\{n_i^H F_{k_r} F_{k_r} n_i\} = \text{Tr} \left\{ F_{k_r} E\{n_i n_i^H\} F_{k_r}^H \right\} = M \sigma_n^2.
\]

where \(F_{k_r} = [e^{-j2\pi \frac{f_0}{T}0} \quad e^{-j2\pi \frac{f_1}{T}} \ldots e^{-j2\pi \frac{f_r}{T}(M-1)}] \) and \(n_i = [n_i(0) \quad n_i(1) \ldots n_i(M-1)]^T\).

Fig. 3. A time-domain illustration of the effect of the smooth signal in the multipath channel without power attenuation and Gaussian noise.

To calculate the power of the interference \(W_{i,k_r}\) in the multipath channel, the distribution of the smooth signal should be investigated. As illustrated in Fig. 3 different channel paths with varying time delays lead to varying interferences of \(w_i(l)\). With an increased time delay, the delayed tail of \(w_i(l)\) increases the interference.

Then, we arrive at

\[
E\{|W_{i,k_r}|^2\} = E\left\{ \left| \sum_{l=1}^{\tilde{l}} \tilde{w}_{i,l} \right|^2 \right\} = \text{Tr} \left\{ F_{k_r} E\left\{ \sum_{l=1}^{\tilde{l}} \tilde{w}_{i,l} \right\} \right\}.
\]

where \(\tilde{w}_{i,l}\) denotes the delayed tail of \(w_i(l)\) in the \(\tilde{l}\)th path, expressed by

\[
\tilde{w}_{i,l} = [0_{L-\theta_l} \quad I_{\theta_l}] Q_f b_l,
\]

and \(\theta_l = \text{round}\{\tau_l / T_{\text{ samp}}\} + L - M_{\text{ cp}}\) with the rounding operation \(\text{round}\{\cdot\}\).

Then, Eq. (35) can be further expressed by

\[
E\{|W_{i,k_r}|^2\} = 2 F_{k_r} U Q_f P_f^{-1} \tilde{P}_1 \tilde{P}_1^H (P_f^{-1})^H Q_f^H F_{k_r}^H = 2 \sigma_{w,k_r}^2.
\]

where the matrix \(U\) is expressed by

\[
U = \sum_{l=1}^{\tilde{l}} \left[ \begin{array}{cc} 0_{\theta_l} \times (L - \theta_l) \\ 0_{(M-\theta_l) \times L} \end{array} \right].
\]

Thus, \(\gamma_{i,k_r}\) can be expressed by

\[
\gamma_{i,k_r} = \frac{|H_{i,k_r}|^2}{2 \sigma_{w,k_r}^2 |H_{i,k_r}|^2 + M \sigma_n^2}.
\]

Fig. 4 compares the analyzed average SINR in (38) and simulation results in the 3GPP LTE Extended Vehicular A (EVA) channel model [19], whose parameters will be given in Section IV, where the average SINR is \(\gamma_{\text{SINR}} = \frac{1}{L} \sum_{k_r} E\{\gamma_{i,k_r}\}\). It is shown that the theoretical results match well with the simulation ones. It also revealed that the signal-to-noise ratio (SNR) loss is negligible in low-interference TD-NC-OFDM. Even if the length of \(w_i(l)\) becomes very long, such as \(L = 1000\), its SINR is still much higher than NC-OFDM and TD-NC-OFDM.

Here for concise expressions and without loss of generality, the subscript of subcarrier and time index are removed. Let \(|H_{i,k_r}|^2 = \alpha\) and \(\gamma_{i,k_r} = \gamma\). Then, Eq. (38) is rewritten by

\[
\gamma = \frac{\alpha}{2 \sigma_{w,k_r}^2 \alpha + M \sigma_n^2}.
\]

Eq. (39) shows that \(\gamma\) is a monotonically increasing continuous derivable function with an inverse \(\alpha = f(\gamma) = M \sigma_n^2 / (1 - 2 \sigma_{w,k_r}^2 \gamma)\). Then, according to the property of the composite function, the probability density function (PDF) \(p_\gamma(\gamma)\) of \(\gamma\) can be calculated by

\[
\gamma = \frac{p_\gamma(\gamma)}{\gamma} = \frac{1}{\sigma_\alpha} \exp\left(-\frac{\gamma}{\sigma_\alpha} \right).
\]

with \(\sigma_\alpha = \left\{ \frac{L}{f(\gamma)} \right\} \). Then, we have

\[
p_\gamma(\gamma) = \left\{ \begin{array}{ll} M \sigma_n^2 \exp\left(-\frac{M \sigma_n^2}{L \sigma_{w,k_r}^2 \gamma}\right) & 0 \leq \gamma < \frac{1}{2 \sigma_{w,k_r}^2} \\ 0 & \text{otherwise}. \end{array} \right.
\]

With the interference caused by the smooth signal, the BER expression of an uncoded quadrature amplitude modulation (QAM) based OFDM over Rayleigh fading channel is

\[
p_b(E) = \int_0^{+\infty} p_b(E|\gamma) p_\gamma(\gamma) d\gamma.
\]
where the conditional BER $p_b(E|\gamma)$ of QAM is given by [22]

$$p_b(E|\gamma) = \frac{2}{\sqrt{J \log_2 J}} \sum_{u_1=1}^{\log_2 \sqrt{J}} \sum_{u_2=0}^{\log_2 \sqrt{J} - 1} (-1)^{u_2} \cdot$$

$$\cdot \left(2^{u_1-1} - \left(\frac{2^{u_1-1}}{\sqrt{J}} + \frac{1}{2}\right)Q\left(2^{u_2+1} + \frac{3 \log_2 J}{J - 1} \gamma\right)\right), (43)$$

where $J$ is the size of constellations for the square QAM, $\gamma_b = \gamma / \log_2 J$ denotes the SNR per bit, and $[x]$ denotes the largest integer to $x$.

Thus, according to the Appendix in [22], the asymptotic expression of (42) is derived as

$$p_b(E) = \frac{2}{\sqrt{J \log_2 J}} \sum_{u_1=1}^{\log_2 \sqrt{J}} \sum_{u_2=0}^{\log_2 \sqrt{J} - 1} (-1)^{u_2} \cdot$$

$$\cdot \left(2^{u_1-1} - \left(\frac{2^{u_1-1}}{\sqrt{J}} + \frac{1}{2}\right)\left(\frac{1}{2} - \frac{1}{\sqrt{\pi}} \sum_{v_1=0}^{+\infty} \frac{(-1)^v}{v!} (2v_1 + 1) \right)\right)$$

$$\cdot \left(\frac{3(2^{u_2+1})}{2(J - 1)}\right)^{v_2 + 1} \left(\frac{M_{0,0}^2 v_2 + 1}{v_2! (E\{\alpha\})^2 v_2 + \frac{1}{2}}\right)\cdot$$

$$\cdot \left(\frac{1}{2} - \frac{1}{\sqrt{\pi}} \sum_{v_1=0}^{+\infty} \frac{(-1)^v}{v!} (2v_1 + 1) \right)\right)$$

$$\cdot \left(\frac{1}{2} - \frac{1}{\sqrt{\pi}} \sum_{v_1=0}^{+\infty} \frac{(-1)^v}{v!} (2v_1 + 1) \right)\right)$$

$$\cdot 2F_1(v_2 + 2, v_1 + v_2 + 3/2; v_1 + v_2 + 5/2; 2\sigma^2_{w,k}, \sigma^-), (44)$$

where $2F_1(\cdot; \cdot; \cdot)$ is the hypergeometric function and $\sigma^- = \lim_{\epsilon>0, \epsilon \to 0} \{1 - \sqrt{2\sigma_{w,k} \epsilon}\}$.

Finally, the average BER can be calculated by

$$p(E) = \frac{1}{K} \sum_{k_r \in K} p_{b,k_r}(E) (45)$$

where $p_{b,k_r}(E)$, equal to $p_b(E)$, is the bit error probability on the subcarrier $k_r$.

IV. NUMERICAL RESULTS

Simulations are performed in a baseband-equivalent OFDM system with $K=256$, $M_{cp} = 144$, and 16-QAM. The carrier frequency is 2 GHz, the subcarrier spacing is $\Delta f = 15$ KHz, and the time-domain oversampling factor is 8. The PSD is evaluated by Welch’s averaged periodogram method [21] with a 2048-sample Hanning window and 512-sample overlap. The multipath Rayleigh fading channel using the EVA channel model is considered [19], whose excess tap delay is [0, 30, 150, 310, 370, 710, 1090, 1730, 2510] ns with relative power [0, -1.5, -1.4, -3.6, -0.6, -9.1, -7, -12, -16.9] dB.

Fig. 5 compares the PSDs of NC-OFDM and low-interference TD-NC-OFDM with different HDOs and lengths of the smooth signal. The low-interference scheme can offer a sidelobe suppression performance similar to traditional NC-OFDM, which has the same sidelobe suppression as TD-NC-OFDM. Moreover, As $L$ increases, the sidelobe attenuation of the low-interference scheme is enhanced. For example, $L=1000$, the low-interference scheme can obtain a BER performance close to original OFDM, but TD-NC-OFDM and NC-OFDM result in high BERs. In this case, the BERs in NC-OFDM and TD-NC-OFDM can be well reduced by the signal recovery algorithm [7] at the price of computational complexity. Meanwhile, the increased length of the smooth signal just leads to slight performance degradation for the small SNR loss as analyzed in Section IV-C.

![Fig. 5. PSDs of NC-OFDM and the low-interference scheme with varying HDOs and the smooth signal lengths.](image)

![Fig. 6. BERs of NC-OFDM, TD-NC-OFDM, and its low-interference scheme with varying HDOs and the smooth signal lengths in the multipath Rayleigh fading channel.](image)

V. CONCLUSION

In this letter, the performance of the low-interference TD-NC-OFDM was analyzed in terms of sidelobe suppression and BER. Based on the continuity and sidelobe decaying, a tight asymptotic spectrum expression of the low-interference TD-NC-OFDM signal was derived. We also obtained the closed-form expression of SINR and given the analytical BER. Both analytical and simulation results showed that the low-interference scheme effectively reduces the interference to a negligible level, while maintaining similar sidelobe suppression to traditional NC-OFDM and TD-NC-OFDM.
Substituting (43) into (42) gives

\[
p_w(E) = \frac{2}{\sqrt{J} \log_2 \sqrt{J}} \sum_{u_1=1}^{\log_2 \sqrt{J}} \sum_{u_2=0}^{(1-2^{-u_1})} (-1)^{\left(2^{u_2+1} - 1\right)} \left(\frac{2^{u_2+1} - 1}{\sqrt{J}} + 1/2\right) I_1.
\]  

(46)

where

\[
I_1 = \int_0^\infty Q \left( (2u_2 + 1) \sqrt{\frac{3\gamma}{J - 1}} \right) \frac{M \sigma^2}{E(\alpha)(1 - 2\sigma^2_{w,k,\gamma})^2} \left( e^{\frac{M \sigma^2}{E(\alpha)(1 - 2\sigma^2_{w,k,\gamma})}} \right)^{1/2} d\gamma,
\]

(47)

Similar to (25), by using \( Q(x) = \frac{1}{2} \left( 1 - erf\left( \frac{x}{\sqrt{2}} \right) \right) \) and the error function’s Maclaurin series \( \text{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{v=0}^{\infty} \frac{(-1)^v x^{2v+1}}{(2v+1)!} \), we have

\[
I_1 = \int_0^\infty \left( \frac{1}{2} - \frac{\text{erf}\left( \frac{2u_2 + 1}{\sqrt{J - 1}} \right)}{2} \right) \frac{M \sigma^2}{E(\alpha)(1 - 2\sigma^2_{w,k,\gamma})^2} \left( e^{\frac{M \sigma^2}{E(\alpha)(1 - 2\sigma^2_{w,k,\gamma})}} \right)^{1/2} d\gamma
\]

\[
= \frac{1}{2} - \frac{\text{erf}\left( \frac{2u_2 + 1}{\sqrt{J - 1}} \right)}{2} \frac{M \sigma^2}{E(\alpha)(1 - 2\sigma^2_{w,k,\gamma})^2} \left( e^{\frac{M \sigma^2}{E(\alpha)(1 - 2\sigma^2_{w,k,\gamma})}} \right)^{1/2} d\gamma
\]

\[
= \frac{1}{2} - \frac{\text{erf}\left( \frac{2u_2 + 1}{\sqrt{J - 1}} \right)}{2} \frac{M \sigma^2}{E(\alpha)(1 - 2\sigma^2_{w,k,\gamma})^2} \left( e^{\frac{M \sigma^2}{E(\alpha)(1 - 2\sigma^2_{w,k,\gamma})}} \right)^{1/2} d\gamma
\]

(48)

with

\[
I_2 = \int_0^\infty \frac{\gamma^{v_1 + 1/2}}{(1 - 2\sigma^2_{w,k,\gamma})^2} e^{-\frac{M \sigma^2}{E(\alpha)(1 - 2\sigma^2_{w,k,\gamma})}} d\gamma.
\]

(49)

By using the Taylor series expansion of \( e^x \), Eq. (49) is rewritten as

\[
I_2 = \sum_{v_2=0}^{\infty} \frac{(-M \sigma^2_{w,k,\gamma})^{v_2}}{v_2!(E(\alpha))^{v_2}} \int_0^\infty \frac{\gamma^{v_1 + v_2 + 1/2}}{(1 - 2\sigma^2_{w,k,\gamma})^2} d\gamma.
\]

(50)

According to (24), under the condition of \( |\arg\{1 - 2\sigma^2_{w,k,\gamma} \sigma^-\}| < \pi \) and \( \text{Re}\{v_1 + v_2 + \frac{1}{2}\} > 0 \), we have

\[
\sigma^- \int_0^\infty \frac{\gamma^{v_1 + v_2 + 1/2}}{(1 - 2\sigma^2_{w,k,\gamma})^2} d\gamma = \frac{(\sigma^-)^{v_1 + v_2 + 1/2}}{v_1 + v_2 + \frac{1}{2}} F_1(v_2 + 2, v_1 + v_2 + v_1 + v_2 + \frac{1}{2}; v_1 + v_2 + v_1 + v_2 + \frac{5}{2}; 2\sigma^2_{w,k,\gamma} \sigma^-).
\]

(51)

Finally, by substituting \( I_1 \) and \( I_2 \) into (46), we can obtain the expression in (44).