Spinor model of a perfect fluid

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Different characteristic of matter influencing the evolution of the Universe has been simulated by means of a nonlinear spinor field. We have considered two cases where the spinor field nonlinearity occurs either as a result of self-action or due to the interaction with a scalar field.

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I. INTRODUCTION

In a series of papers we have studied the evolution of an anisotropic universe where the source is given by a nonlinear spinor field [1, 2, 3, 4, 5]. In those papers it was shown that a suitable choice of nonlinearity (i) accelerates the isotropization process, (ii) gives rise to a singularity-free Universe and (iii) generates late time acceleration. In a recent paper [6] the authors have simulated perfect fluid using spinor field with different nonlinearities. In doing so they used the perfect fluid given by a barotropic equation of state. In this paper beside the barotropic one we consider the Chaplygin gas as well. As far as spinor field is concerned, we have considered two possibilities: (i) the nonlinearity occurs as a result of self action and (ii) the nonlinearity is a induced one, i.e., emerges due to the interaction with a scalar field.

II. SIMULATION OF PERFECT FLUID WITH NONLINEAR SPINOR FIELD

First of all let us note that one of the simplest and popular model of the Universe is a homogeneous and isotropic one filled with a perfect fluid with the energy density $\varepsilon = T_{0}^{0}$ and pressure

$$p = -T_{1}^{1} = -T_{2}^{2} = -T_{3}^{3}$$

obeying the barotropic equation of state

$$p = W \varepsilon,$$

(2.1)

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where $W$ is a constant. Depending on the value of $W$, (2.1) describes perfect fluid from phantom to ekpyrotic matter, namely

\begin{align*}
W &= 0, & \text{(dust),} \\
W &= 1/3, & \text{(radiation),} \\
W &\in (1/3, 1), & \text{(hard Universe),} \\
W &= 1, & \text{(stiff matter),} \\
W &\in (-1/3, -1), & \text{(quintessence),} \\
W &= -1, & \text{(cosmological constant),} \\
W < -1, & \text{(phantom matter),} \\
W > 1, & \text{(ekpyrotic matter).}
\end{align*}

(2.2)

In order to describe the matter given by (2.2) with a spinor field let us now write the corresponding Lagrangian [1]:

\[ L_{sp} = i \frac{1}{2} \left[ \bar{\psi} \gamma^{\mu} \nabla_{\mu} \psi - \nabla_{\mu} \bar{\psi} \gamma^{\mu} \psi \right] - m \bar{\psi} \psi + F, \]

(2.3)

where the nonlinear term $F$ describes the self-interaction of a spinor field and can be presented as some arbitrary functions of invariants generated from the real bilinear forms of a spinor field. For simplicity we consider the case when $F = F(S)$ with $S = \bar{\psi} \psi$. We consider the case when the spinor field depends on $t$ only. In this case for the components of energy-momentum tensor we find

\begin{align*}
T^0_0 &= mS - F, \quad \text{(2.4a)} \\
T^1_1 &= T^2_2 = T^3_3 = \frac{dF}{dS} - F. \quad \text{(2.4b)}
\end{align*}

Inserting $\varepsilon = T^0_0$ and $p = -T^1_1$ into (2.1) we find

\[ S \frac{dF}{dS} - (1 + W)F + mWS = 0, \]

(2.5)

with the solution

\[ F = \lambda S^{1+W} + mS, \]

(2.6)

with $\lambda$ being an integration constant. Inserting (2.6) into (2.4a) we find that

\[ T^0_0 = -\lambda S^{1+W}. \]

(2.7)

Since energy density should be non-negative, we conclude that $\lambda$ is a negative constant, i.e., $\lambda = -\nu$, with $\nu$ being a positive constant. So finally we can write the components of the energy-momentum tensor

\begin{align*}
T^0_0 &= \nu S^{1+W}, \quad \text{(2.8a)} \\
T^1_1 &= T^2_2 = T^3_3 = -\nu WS^{1+W}. \quad \text{(2.8b)}
\end{align*}

(2.8)

As one sees, the energy density $\varepsilon = T^0_0$ is always positive, while the pressure $p = -T^1_1 = -\nu WS^{1+W}$ is positive for $W > 0$, i.e., for usual fluid and negative for $W < 0$, i.e., for dark energy.

In account of it the spinor field Lagrangian now reads

\[ L_{sp} = i \frac{1}{2} \left[ \bar{\psi} \gamma^{\mu} \nabla_{\mu} \psi - \nabla_{\mu} \bar{\psi} \gamma^{\mu} \psi \right] - \nu S^{1+W}, \]

(2.9)
Thus a massless spinor field with the Lagrangian \[ (2.9) \] describes perfect fluid from phantom to ekpyrotic matter. Here the constant of integration \( \nu \) can be viewed as constant of self-coupling. A detailed analysis of this study was given in [6].

Let us now generate a Chaplygin gas by means of a spinor field. A Chaplygin gas is usually described by an equation of state

\[ p = -A/\epsilon^\gamma. \]  

(2.10)

Let us consider the case with \( \gamma = 1 \). Then in case of a massless spinor field for \( F \) one finds

\[ \frac{F dF}{F^2 - A} = 2 \frac{dS}{S}, \]  

(2.11)

with the solution

\[ F = \pm \sqrt{A + \lambda S^2}. \]  

(2.12)

Since, in this case, \( T_0^0 = -F \) should be nonnegative, the expression for \( F \) should be negative. On account of this for the components of energy-momentum tensor we find

\[ T_0^0 = \sqrt{A + \lambda S^2}, \]  

(2.13a)

\[ T_1^1 = T_2^2 = T_3^3 = A/\sqrt{A + \lambda S^2}. \]  

(2.13b)

As was expected, we again get positive energy density and negative pressure.

Thus the spinor field Lagrangian corresponding to a Chaplygin gas reads

\[ L_{sp} = \frac{i}{2} \left[ \psi \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi \right] - \sqrt{A + \lambda S^2}. \]  

(2.14)

Thus we see that a nonlinear spinor field with specific type of nonlinearity can substitute perfect fluid and dark energy, thus give rise to a variety of evolution scenario of the Universe.

### III. SIMULATION OF PERFECT FLUID WITH INTERACTING SPINOR AND SCALAR FIELDS

Now let us consider the system with interacting spinor and scalar fields with the Lagrangian \[ [2]: \]

\[ L_{int} = \frac{i}{2} \left[ \psi \gamma^\mu \nabla_\mu \psi - \nabla_\mu \bar{\psi} \gamma^\mu \psi \right] - m\bar{\psi}\psi + \frac{1}{2} \phi_{,\alpha} \phi^{,\alpha} (1 + \lambda_1 F_1), \]  

(3.1)

where \( F_1 = F_1(S) \). We again consider the case with spinor and scalar fields being the functions of \( t \) only. Denoting \( F_2 = 1 + \lambda_1 F_1 \) in this case for the components of energy-momentum tensor we find

\[ T_0^0 = mS + \frac{1}{2} \frac{S^2}{F_2}, \]  

(3.2a)

\[ T_1^1 = T_2^2 = T_3^3 = \frac{1}{2} \frac{S^2}{F_2^2} \left( F_2 - \frac{dF_2}{dS} \right). \]  

(3.2b)

Here we have taken into account that \( \phi^2 = QS^2/F_2^2 \) [7]. Here \( Q \) is a constant that depends on the concrete cosmological model. Here we also used the fact that \( S = C_0/\tau \) with \( \tau = abc \). For simplicity we set \( Q = 1 \) and \( C_0 = 1 \). These equalities hold for the cosmological models given by [5]

\[ ds^2 = dt^2 - a^2 e^{-2\nu z} dx^2 - b^2 e^{2\nu z} dy^2 - c^2 dz^2. \]  

(3.3)
Here \(a, b, c\) are the functions of \(t\) only and depending on the value of \(m, n\) (3.3) describes Bianchi type VI, V, III, I and FRW cosmological models [5].

Inserting (3.2a) and (3.2b) into (2.1) we find

\[
S^2 \frac{dF_2}{dS} + (W - 1)SF_2 + 2mWF_2^2 = 0. \tag{3.4}
\]

From (3.4) one finds

\[
F_2 = -\frac{1}{2m}S. \tag{3.5}
\]

As one sees, the spinor field nonlinearity in this case does not depend on \(W\). Moreover, inserting this into (3.2a) one finds \(T_0^0 = -mS\). So we can dully neglect this case and move forward with a massless spinor field. In this case from (3.4) we find

\[
F_2 = CS^{1-W}. \tag{3.6}
\]

In this case for the components of the energy momentum tensor we find

\[
T_0^0 = \frac{1}{2C}S^{1+W}, \tag{3.7a}
\]

\[
T_1^1 = T_2^2 = T_3^3 = -\frac{W}{2C}S^{1+W} \tag{3.7b}
\]

which, as was expected coincides with the one given by (2.8a) and (2.8b) with \(\nu = 1/2C\).

As far as Chaplygin gas concerned, inserting (3.2a) and (3.2b) into (2.10) for massless spinor field we find

\[
S\frac{dF_2}{dS} - 4AF_2^3 - F_2 = 0, \tag{3.8}
\]

with the solution

\[
F_2 = \frac{1}{2\sqrt{A}}S^2. \tag{3.9}
\]

In this case for the components of the energy momentum tensor we find

\[
T_0^0 = \sqrt{A}, \tag{3.10a}
\]

\[
T_1^1 = T_2^2 = T_3^3 = \sqrt{A} \tag{3.10b}
\]

which coincides with (2.13a) and (2.13b) when \(\lambda = 0\).

Thus we see that an interacting system of spinor and scalar field can as well describe a perfect fluid and dark energy.

**IV. ANISOTROPIC COSMOLOGICAL MODELS WITH A SPINOR FIELD**

In the previous two sections we showed that the perfect fluid and the dark energy can be simulated by a nonlinear spinor field. In the section II the nonlinearity was the subject to self-action, while in section III the nonlinearity was induced by a scalar field. It was also shown the in our context the results of section III is some special cases those of section II. Taking it into mind we study the evolution an anisotropic Universe filled with a nonlinear spinor field given by the Lagrangian (2.3), with the nonlinear term \(F\) is given by (2.6) of (2.12).
V. CONCLUSION

Within the framework of cosmological gravitational field equivalence between the perfect fluid (and dark energy) and nonlinear spinor field has been established. It is shown that the perfect fluid can be simulated with both self-action and induced nonlinearity of the spinor field. The case with induced nonlinearity can be viewed as partial case that of self-action.

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