ON THE ORIGIN OF PLUTO’S MINOR MOONS, NIX AND HYDRA
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Abstract

How did Pluto’s recently discovered minor moons form? Ward and Canup propose an elegant solution in which Nix and Hydra formed in the collision that produced Charon, then were caught into corotation resonances with Charon, and finally were transported to their current location as Charon migrated outwards. We show with numerical integrations that, if Charon’s eccentricity is judiciously chosen, this scenario works beautifully for either Nix or Hydra. However, it cannot work for both Nix and Hydra simultaneously. To transport Nix, Charon’s eccentricity must satisfy $e_C < 0.024$; otherwise, the second order Lindblad resonance at 4:1 overlaps with the corotation resonance, leading to chaos. To transport Hydra, $e_C > 0.7R_{\text{pluto}}/a_{\text{charon}} > 0.04$; otherwise migration would be faster than libration, and Hydra would slip out of resonance. These two restrictions conflict. Having ruled out this scenario, we suggest an alternative: that many small bodies were captured from the nebular disk, and they were responsible for forming, migrating and damping Nix and Hydra. If this is true, small moons could be common around large Kuiper belt objects.

Subject headings:

1. Introduction

The recent discovery of Pluto’s two minor moons Nix and Hydra (Weaver et al. 2006; Buie et al. 2006) presents interesting puzzles. We summarize current observational data in Table 1.

- The orbits of the moons are nearly circular and nearly coplanar with Charon’s orbit.
- Nix, the inner minor moon, lies just inward of the 4:1 resonance with Charon, while Hydra is close to the 6:1 resonance.

The Pluto-Charon system is doubly synchronized and circularized – it must have gone through significant tidal evolution in the past. In the currently favoured theory for the formation of Charon (Canup 2005; McKinnon 1989), a giant impact chipped off a piece of the proto-Pluto, leaving Charon on an eccentric orbit close to a rapidly spinning Pluto. Subsequent tidal evolution slowed down Pluto’s spin, pushed out Charon to its current position, and damped its orbital eccentricity. This is similar to how Earth’s Moon is thought to have formed and evolved. Tides on Pluto pushed Charon out to its current position in $\sim 2 \times 10^7 (Q_P/100)$ yrs, where $Q_P$ is Pluto’s tidal quality factor.\footnote{ Estimates of various tidal timescales are listed in Appendix B.} Charon’s eccentricity eventually decayed to zero on a comparable timescale, assuming that Charon’s tidal parameter $Q_C$ is not too different from $Q_P$.\footnote{ Its eccentricity would initially have grown if $Q_C/Q_P$ exceeds a number that is of order unity; otherwise, its eccentricity would have decreased monotonically.} By contrast, Nix and Hydra likely cannot evolve tidally at their current positions in the age of the Solar System (Stern et al. 2006; Lithwick & Wu 2008).

2. Forced Resonant Migration (FRM)

2.1. The FRM Scenario

Ward & Canup (2006) propose an elegant scenario to account for Nix and Hydra’s observed orbital properties. In their scenario, the minor moons were formed as byproducts of the collision that formed Charon. Nix was then caught into Charon’s 4:1 corotation resonance, and Hydra into the 6:1 corotation resonance. As Pluto’s tides pushed out Charon, Nix and Hydra remained in resonance and so they too were forced to migrate towards their current orbits. In this scenario, Nix and Hydra must have been caught into the corotation resonance, and not into any of the other sub-resonances at 4:1 and 6:1, because migration in other sub-resonances would have excited the eccentricities of Nix and Hydra to values much larger than are observed.

A corotation resonance can transport a particle only if Charon’s eccentricity $e_C$ is sufficiently large, because the resonant libration time must be shorter than the time for the resonance to migrate a distance of order its width, and the libration time increases with decreasing $e_C$ whereas the width decreases. The more stringent constraint is set by Hydra, which requires $e_C \gtrsim 0.7(R_P/a_C)^{1/5}$, where $a_C$ is Charon’s semimajor axis (Ward & Canup 2006). Therefore when Charon reached its current orbit at $a_C \approx 17R_P$, it must have still had an eccentricity of $e_C \gtrsim 0.04$. As $e_C$ was subsequently damped by tides, the width of the corotation resonances shrank to zero, and Nix and Hydra escaped from resonance. Such a history for Charon’s orbit is plausible, given the uncertainties in the tidal parameters.

Ward & Canup (2006) briefly address the question of how the minor moons were initially trapped into corotation resonances. If Nix and Hydra were produced in a collision, then their free eccentricities were initially large, and it would have been unlikely that they had just the right orbits to end in corotation. But if a lot of debris was produced in the collision, and if this debris was highly collisional, then it is possible that the debris settled into
The forced secular eccentricity is $e_{\text{forced}} = -e_C B_1/2B_2$, and the secular precession frequency is $\omega_{\text{sec}} = 2n_B B_2$. If $e_C$ decays exponentially on timescale $\tau_e$, i.e., $e_C(t) = e_C|t=0|e^{-t/\tau_e}$, and if $Z|t=0 = e_{\text{forced}}$, then the solution of equation (3) is

$$Z = e_{\text{forced}}|t=0\frac{i\omega_{\text{sec}}\epsilon e^{-t/\tau_e} - e^{-i\omega_{\text{sec}} t}}{i\omega_{\text{sec}}\epsilon - 1}$$

Therefore when $t \gg \tau_e$ and $e_C$ has decayed to 0, the particle’s eccentricity will be negligibly small as long as $\omega_{\text{sec}}\epsilon \gg 1$, proving our assertion.

### 2.2. Numerical Simulation of FRM

Figure 1 demonstrates numerically that the FRM scenario works for Hydra when Charon’s migration trajectory is chosen judiciously. We start Charon on an orbit with $a_C = 0.1$, $a_C = 10 R_P$ and force it to migrate outward to $a_C = 17 R_P$ with $\dot{a} = a/10^6$ yrs, keeping the eccentricity constant. Then $e_C$ is forced to decay to zero in $10^6$ yrs. A massless test particle representing Hydra is initially placed in the center of Charon’s 6:1 corotation resonance. (See §3.2 for a description of how we start off a particle in the corotation resonance.) The orbital motions of all bodies are numerically integrated with the SWIFT package (Levison & Duncan 1994), using the hierarchical Jacobi symplectic integrator of Beust (2003). We further modify it to allow semi-major axis and eccentricity evolution due to external forces, following the approach of Lee & Peale (2002).

Figure 1 shows that Charon indeed pushes out Hydra. Hydra remains within the corotation island until Charon’s eccentricity falls to a very small value. Hydra’s eccentricity tracks that of Charon’s along the way (eq. [4]). Fig. 1 also shows that FRM depends extremely sensitively on the initial conditions of the test particle. A second particle started off at exactly the same location as the one above but with a velocity larger by 0.1% (and therefore falling outside the narrow corotation island) is quickly ejected during the migration of Charon. In fact, the width of the corotation island is so narrow that it is comparable to the size of Hydra at the start of our experiment.

### 3. Ruling Out FRM

FRM can migrate either Hydra or Nix to their current orbits. But it cannot simultaneously migrate both

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**TABLE 1**

| orbital period [\(T_C\)] | Pluto | Charon | Nix | Hydra |
|---------------------------|------|--------|-----|-------|
| semi-major axis [\(R_P\)] | 1    | 1.96   | 0.1165 | 5.98 |
| mass [\(M_P\)]            | 41.82| 7.8 \times 10^{-6} | 55.65 |       |
| eccentricity              | 0.0  | 0.0    | 0.002 | 0.005 |
| inclination [\(\text{deg}\)] | 96.14 | 96.14 | 96.18 | 96.36 |

\(a\) Pluto’s radius and mass are \(R_P = 1164\) km (Young & Binzel 1994), and \(M_P = 1.3 \times 10^{23}\) g. Orbital period of Pluto-Charon binary is \(T_C = 6.3872\) days.

\(b\) Charon’s radius and mass from Sicardy et al. (2006).

\(c\) Nix and Hydra’s radii and masses assume a Charon-like albedo of 0.35 (Weaver et al. 2006) and a density of 2g/cm\(^3\) (Charon-like); masses will be \(\sim 20\) times higher (and radii \(\sim 2.7\) times larger) if albedo is 0.04 (comet-like). Orbital parameters are from Buie et al. (2006).

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of them. To transport Hydra, Charon’s eccentricity must be greater than a critical value; otherwise, Hydra would slip out of resonance. To transport Nix, Charon’s eccentricity must be less than a second critical value; otherwise, the 4:1 corotation resonance would be destroyed by resonance overlap. These two constraints conflict. We discuss them in turn.

3.1. Lower Limit on $e_C$ from Hydra’s migration

To be able to transport Hydra in resonance, the migration time across the width of the 6:1 corotation resonance must be longer than the libration period in the resonance,

$$6^{2/3} \dot{a}_C \lesssim \frac{\Delta a_{\text{lib}}}{T_{\text{lib}}} ,$$  \hspace{1cm} (5)

where $\Delta a_{\text{lib}}$ is the resonance width, $T_{\text{lib}}$ is the libration period, and $\dot{a}_C$ is Charon’s tidal migration rate.

We take the expressions for the libration width and libration period from equations (8.58) and (8.47) of Murray & Dermott (1999), and use the Kaula formula (§6.3 of Murray & Dermott 1999) to obtain a resonance strength for the 6:1 corotation resonance of $f_d \approx 0.02$ (as defined in eq. [8.32] of Murray & Dermott 1999). Adopting the tidal timescale for orbital expansion as in equation B6, the above transport condition translates to a lower limit for $e_C$,

$$e_C > 0.04 \left( \frac{k_{2P}}{0.05} \right)^{1/5} \left( \frac{100}{Q_P} \right)^{1/5} \left( \frac{17R_P}{a_C} \right) .$$  \hspace{1cm} (6)

Note the weak dependence on $k_{2P}$ (Pluto’s tidal Love number) and $Q_P$ (Pluto’s tidal quality factor). The above limit has also been given in Ward & Canup (2006). We have performed numerical experiments to confirm the numerical coefficient. We plot this limit in Figure 2.

3.2. Upper Limit on $e_C$ from Resonance Overlap At Nix’s Orbit

In this subsection, we demonstrate with two numerical experiments that Charon’s 4:1 corotation resonance exists only if $e_C < 0.024$. Otherwise, resonance overlap destroys the corotation resonance (as shown in §4). Figure 2 shows that the constraint $e_C < 0.024$, together with the constraint from §3.1, rules out the FRM scenario: it is impossible to satisfy both of these constraints simultaneously.

To demonstrate numerically the destruction of the 4:1 corotation resonance, one first needs a robust method to find it when it exists. This is not entirely trivial because the mass ratio $M_C/M_P$ is not terribly small, and neither are Charon’s and Nix’s eccentricities $e_C$ and $e$. Hence an expansion of the disturbing function will not be very accurate. Instead, our method is based on finding the “coldest” orbits of test particles around the Pluto-Charon binary. A disk of infinitesimal particles that collide inelastically will naturally tend to the coldest orbits, i.e. the orbits that minimize the velocity dispersion within the disk. What are these coldest orbits? Consider first the case that Pluto and Charon orbit each other on circular orbits. Then the coldest orbits are the periodic orbits, which are orbits that, in the rotating reference frame of the binary, close on themselves after a single loop around the binary. The generalization of periodic orbits to the case of non-zero $e_C$ are invariant loops (Ma-
ciejewski & Sparke 1997; Pichardo et al. 2005), which may be understood as follows. If we take a snapshot of the position of an orbiting particle every time Pluto-Charon reach some predefined orbital phase (periapse, say), then in general the snapshots would fill out a two-dimensional region in the $r, \theta$ plane, where $r, \theta$ are the particle’s radius and azimuth. But the coolest orbits—the invariant loops—are those in which the snapshots trace out a one-dimensional closed loop. Far from strong resonances, neighbouring invariant loops do not intersect each other, or themselves. But near strong resonances, they often do intersect.

To find the corotation resonance, we find the set of invariant loops around an eccentric Pluto-Charon binary, largely following the procedure of Pichardo et al. (2005). We express the orbit of the binary in terms of the eccentric anomaly, expanded to fourth order in $e_C$. The motion of the test particle is evolved in the barycentric frame with a 4th-order Runge-Kutta integrator with adaptive step-sizes (Press et al. 1992), with an error tolerance of $10^{-8}$. A test particle is initially launched far from any strong resonances on the periastron axis ($x$-axis) when Charon is at periapse. We assume that its orbit is symmetric with respect to the $x$-axis, so the only unknown is the velocity $v_y$. This is initially guessed using the local Keplerian value. The positions of the test particle at each subsequent periapse passage of Charon are recorded, for a total of $10^4$ binary periods. These are separated into 2,000 angular bins and the radial dispersion within each angular bin is co-added.\footnote{This is different from Pichardo et al. (2005) in that they only use the dispersion near the $x$-axis. We find that our treatment gives a faster and more reliable convergence.} We use the bisection technique (typically within a range 6% of the initial guess) to find the correct $v_y$ that minimizes this dispersion. The resulting 1-D curve $r(\theta)$ is an invariant loop. We then proceed to find the next loop which is closer to the resonance, using the previously obtained $v_y$ as the initial guess. The closer the loops, the better the guess, and the more reliable the convergence.

Fig. 3 depicts the invariant loops we obtain for a test particle near the 4:1 location. When $e_C < 0.024$, 4:1 corotation islands are clearly visible; while when $e_C = 0.024$ and beyond, all islands disappear.

Fig. 4 shows the results from a second experiment that demonstrates the destruction of the 4:1 corotation resonance for large $e_C$. In this experiment, we insert a test particle into the center of the 4:1 corotation resonance when $e_C$ is small, and then slowly raise the value of $e_C$. As $e_C$ rises above 0.024, the motion of a particle initially trapped inside the 4:1 corotation resonance becomes chaotic and the resonance angle circulates.

Similar experiments performed for the 6:1 corotation find the limiting $e_C$ to be 0.24 in that case.

4. RESONANCE OVERLAP DESTROYS THE 4:1 COROTATION RESONANCE

We seek a better understanding of how and why the 4:1 corotation resonance is destroyed when $e_C > 0.024$. We model the evolution of a test particle near the 4:1 resonance of an eccentric Charon-Pluto binary with a truncated disturbing function, keeping only secular and 4:1 resonant terms. The particle’s equations of motion are compactly encoded by the Hamiltonian derived in Appendix A (eq. [A26]), which has two degrees of freedom. The momentum and co-ordinate of the first degree of freedom $\{p_a, q_a\}$ are related to the test particle’s semimajor axis and mean longitude via equations (A13) and (A14). And the momentum and co-ordinate of the second degree of freedom are combined into the complex canonical variable $z$, which is the test particle’s free complex eccentricity (i.e., the complex eccentricity after subtracting the forced secular eccentricity, eqs. [A15] and [A19]). The equations of motion for these two degrees of freedom are just Hamilton’s equations (eq. [A27]), which we shall numerically integrate. Since this is a time-independent Hamiltonian with two degrees of freedom, phase space may be mapped out with Poincaré surfaces of section (e.g., Henon & Heiles 1964)

We first consider the corotation term (the $c_0$ term in Hamiltonian A26) in isolation, discarding the Lindblad terms (the $c_1$-$c_3$ terms), which leaves

$$H = -24p_a^2 - \mu c_0 e_C^3 \cos q_a - \mu B_2 |z|^2 \quad (7)$$

with the two degrees of freedom decoupled from each other; $c_0$ and $B_2$ are order-unity constants whose values are listed in Appendix A. Hamilton’s equations for $q_a, p_a$ yield $\dot{q}_a = -48p_a$ and $\dot{p}_a = -\mu c_0 e_C^3 \sin q_a$, the same as for a pendulum. Hence the angle $q_a$ will either librate around the center of the resonance (where $q_a = p_a = 0$), or, for large enough $|p_a|$, it will circulate (Fig. 5, top...
We then plot the surfaces of section. We place Charon at its current position, and a plane surface of section. We then plot the invariant loops, we first find the fixed point in the $z$-plane surface of section. To see this, recall that an invariant loop is a 1D curve of the test particle’s $z$-plane surface that must be a fixed point in that section; otherwise, there would be an infinite number of different values of $z$ (and hence of $r$) at $\theta = 0$, and the orbit would not trace out a 1D curve in the $r$-$\theta$ plane. In summary, to find the invariant loops, we first find the fixed point in the $z$-plane surface of section. We then plot the $r$ vs. $\theta$ for this orbit, as given by equations (8)-(9). For example, the fixed point marked with an ‘x’ in the upper-right panel of Figure 5 gives rise to one of the curves in the lower panel of that figure. The five other invariant loops shown there were found similarly, but with different values of the energy.

Having described how to find the invariant loops, we turn now to the full 4:1 Hamiltonian (eq. [A26]). Figure 6 shows orbits when $e_C = 0.01$. In the sample surface of section shown, the fixed point is no longer at $z = 0$ because the Lindblad resonances give a non-zero forced eccentricity that is in addition to the corotation secular value. The invariant loops in the right panel of the figure are similar to those with only the corotation resonance (Fig. 5), but are slightly more distorted. When Charon’s eccentricity is increased to $e_C = 0.025$, the invariant loops become highly distorted (Figure 7). By $e_C = 0.03$, many of the librating invariant loops no longer exist—the fixed points break up into a sea of chaos (Fig. 8). At even higher $e_C$, there are no corotation librations left.

In §3.2 we found from direct integration of Newton’s equations that the corotation islands disappear when $e_C > 0.024$. Although the critical value we find in the present section $\sim 0.03$ is similar, we speculate that the

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**Fig. 4.** Maximum $e_C$ above which the 4:1 corotation resonance disappears. We place Charon at its current position, and a test particle at 4:1 corotation resonance. As $e_C$ is increased gradually (solid curve in top panel, $\dot{e}/e = 1/10^7$ years), the barycentric eccentricity of the test particle rises even faster, until $e_C = 0.024$ (marked by the dotted line) after which resonance overlap destroys the corotation resonance—the resonant angle $\phi_{\text{res}} = 4\lambda - \lambda_C - 3\omega_C$ suddenly starts circulating. The particle is released from the resonance and displays chaotic motion. To reduce scatter, we only show test particle data when both bodies have true anomalies $\lambda \sim 0$. The same break-down for the 6:1 resonance occurs at $e_C = 0.24$.

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**Fig. 5.** Orbits arising from the corotation resonance only, from numerical integrations of Hamiltonian (7)′s equations of motion with $e_C = 0.01$. Top two panels show surfaces of section at a fixed value of the energy. Cross-hatched regions are incompatible with the chosen energy and are forbidden. Top left panel shows values of $q_a, p_a$, recorded when $\text{Im}(z) = 0$, for seven orbits; top right panel shows values of $z$ when $q_a = 0$ for the same seven orbits (with two of the orbits overlapping two others, making it appear as if only five orbits are plotted). The fixed point in the $z$-plane marked by an ‘x’ is an invariant loop. When this orbit’s radius is plotted versus its azimuth via equations (8)-(9), it gives a 1D curve, as shown in the bottom panel. (Specifically, it gives the curve in the bottom panel that lies immediately above the librating regions.) Also shown in the bottom panel are five other invariant loops for five other energy values. Compare this plot with Fig. 3 obtained from full numerical integrations.

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**Fig. 6.** Orbits arising from the corotation resonance only, from numerical integrations of Hamiltonian (7)′s equations of motion with $e_C = 0.01$. Top two panels show surfaces of section at a fixed value of the energy. Cross-hatched regions are incompatible with the chosen energy and are forbidden. Top left panel shows values of $q_a, p_a$, recorded when $\text{Im}(z) = 0$, for seven orbits; top right panel shows values of $z$ when $q_a = 0$ for the same seven orbits (with two of the orbits overlapping two others, making it appear as if only five orbits are plotted). The fixed point in the $z$-plane marked by an ‘x’ is an invariant loop. When this orbit’s radius is plotted versus its azimuth via equations (8)-(9), it gives a 1D curve, as shown in the bottom panel. (Specifically, it gives the curve in the bottom panel that lies immediately above the librating regions.) Also shown in the bottom panel are five other invariant loops for five other energy values. Compare this plot with Fig. 3 obtained from full numerical integrations.
The forced eccentricity is infinite when \( e_C \) is given by
\[
e_C = \frac{B_2}{c_2} = 0.031
\]
We note that Ward & Canup (2006) perform a calculation similar to our equation (11) in their Supplementary Notes, although they include only the first Lindblad term.

For our second explanation, we argue that chaos is likely to occur when the center of the second Lindblad resonance (the \( c_2 \) term) overlaps the center of the corotation resonance in the \( q_a, p_a \) plane. To find the center of the second Lindblad resonance, we switch from \( z \) to the canonical variables \( q_e, p_e \), defined via \( \sqrt{2q_e e^{ip_e}} = z \). Then, the Hamiltonian for the second Lindblad resonance becomes
\[
H = -24p_e^2 - 2p_e B_2 - 2\mu_2 c_2 C p_e \cos(q_a + 2q_e).\]
The center of this resonance occurs where \( q_a = 2q_e = 0 \). Employing Hamilton’s equations, we find \( 0 = \dot{q}_a + 2\dot{q}_e = -48p_a - 4\mu B_2 - 4\mu_2 c_2 C \). Therefore the center of this resonance overlaps the center of the corotation resonance (which is at \( p_a = 0 \)) when \( e_C \) is given by \( e_{C*} \).

5. ALTERNATIVE FORMATION SCENARIOS

How did Nix and Hydra form? Ward & Canup (2006) argue that these moons are byproducts of the impact that formed Charon. But here we argue that this is not the case. If Nix and Hydra were byproducts of the impact, one might imagine three possibilities for how they ended up in their current orbits, with very small eccentricities:

- They might have been directly ejected to their current semimajor axes with large eccentricities, which were subsequently damped by tides. However, the timescale for them to damp their eccentricities by...
tidal interaction with Pluto is longer than the age of the Solar System (Stern et al. 2006).\footnote{Nix could have damped its eccentricity if it is a strengthless rubble pile and has both tidal Love number and tidal Q factor of order unity; but Hydra could not have, even in this extreme case.} In addition, they likely could not have damped their eccentricity by exciting Charon’s eccentricity, with Charon in turn tidally damping its own eccentricity (Lithwick & Wu 2008).

- They might have been ejected from the Charon-producing impact to their current semimajor axes along with many small particles. If these small particles formed into a collisional disk, the disk could have damped Nix and Hydra’s eccentricities. However, a post-impact collisional disk probably could not have extended to such a large distance ($\sim 50R_P$) from Pluto: simulations of the Pluto-Charon impact give much more compact disks (Canup 2005).

- As proposed by Ward & Canup (2006), they might have been damped by a collisional disk much closer to Pluto, and then been migrated outward by Charon. However, we have shown in this paper that both Nix and Hydra could not be resonantly migrated outward by Charon in the 4:1 and 6:1 resonances, respectively. Is it possible that Charon was responsible for resonantly migrating only Nix ($e_C < 0.024$), while Hydra was transported outward in Nix’s 3:2 corotation resonance? We have failed to find such an orbit using our algorithm (§3.2) in the presence of an eccentric Charon and a massive Nix. But we have yet to exclude this possibility with more confidence.

Taken together, the above results suggest that Nix and Hydra are not byproducts of the Charon-forming impact. In addition, Nix and Hydra could not have been captured from the Kuiper belt, and subsequently collisionlessly hardened by other passing-by bodies. Although that mechanism might work for other Kuiper belt binaries (Goldreich et al. 2002), it would not be consistent with Nix and Hydra’s small eccentricities and inclinations.

Instead, we argue that Nix and Hydra formed within a collisional Plutocentric disk that was composed of small bodies captured from heliocentric orbits. It is quite plausible that at early times in the history of the Solar System there were many small bodies in heliocentric orbit. Three out of the four largest KBOs (including Pluto) have satellites that have been suggested to form out of impacts (Brown et al. 2006). Charon formed out of an impact between two Pluto-sized objects (Canup 2005). Such events are highly unlikely in the environment of the present day Kuiper belt, where the time before the next impact between two Pluto-sized bodies is $\sim 3 \times 10^{13}$ yr. This implies that in the past the velocity dispersion within the Kuiper belt was much smaller than it is today, in which case gravitational focusing would have enhanced the collision rate between Pluto-sized bodies. The most likely mechanism to cool the population of Plutos is dynamical friction with a large mass of small bodies (Goldreich et al. 2004). Today, these bodies have suffered collisional diminuation and no direct evidence of their past abundance remain. However, they must have been present, at least at some point during the accretion growth period, to form the large KBOs in less than the age of the Solar System (Kenyon & Luu 1998).

The small bodies collide much more frequently and can be collisionally accreted to the big bodies (Sari & Goldreich 2006). Collisions remove their random momenta but cannot remove the net angular momentum. They then form a disk around the big bodies. Moons, formed or captured by such a disk, can be migrated inward, be parked near resonant locations, and have small free eccentricities (Shannon et al., in preparation). The moons are expected to be coplanar with Charon – even if its nascent disk started differently, such a dissipative disk can quickly relax to Charon’s orbital plane.

Hydra’s orbit extends only to $\sim 1\%$ of Pluto’s Hill radius. Searches by Nicholson & Gladman (2006) and Steffl et al. (2006) have excluded the presence of other comparably-sized moons in the Hill sphere. Why do present moons occupy such a small fraction of the available space? We suspect the moon-harboring disk may be limited in size, determined by the net angular momentum of the accreted small bodies. Depending on the velocity dispersion of the small bodies in the circumsolar disk, the size of the accreted circum-Pluto disk might be significantly smaller than Pluto’s Hill radius. Further exploration is underway.

Pluto is not special. Similar accretion disks may also have arisen around other large Kuiper belt objects. Do they also possess small moons? Hopefully the moons of Kuiper belt objects can teach us about the early history of our Solar System, when the planets—and the KBO’s themselves—were being formed.

\begin{equation}
\text{A. HAMILTONIAN NEAR THE 4:1 RESONANCE}
\end{equation}

Consider a massless test particle that is coplanar with the Pluto-Charon binary, near its exterior 4:1 resonance. Pluto and Charon’s mutual orbit has orbital elements \{\(a_C, e_C, \lambda_C, \varpi_C, n_C\)\}, with \(\mathcal{G}(M_P+M_C) = n_C^2 a_C^3\), and \(\lambda_C = \text{const} + n_C t\). We set \(\varpi_C = 0\) without loss of generality. Since \(M_C \ll M_P\), the effect of Charon on the particle may be treated as a perturbation to the effect of Pluto. The particle has Pluto-centric orbital parameters\footnote{Throughout most of this paper, we use Jacobi co-ordinates, where the test particle’s orbital elements are relative to the barycenter of Pluto and Charon. But in this Appendix and in §4, we employ Pluto-centric elements because this is traditional when using a disturbing function (Murray & Dermott 1999)—even though it would be simple to use Jacobi elements instead (Lithwick & Wu 2008).} \{\(a, e, \lambda, \varpi\)\}, and its energy per unit mass is

\[E = -\frac{\mathcal{G}M_P}{2a} - \frac{\mathcal{G}M_C}{a} R\]
where \( \mathcal{R} \) can be expanded as a sum of cosine terms. In Table 2 we list the coefficients and arguments of the leading cosine terms near the 4:1 resonance. To translate our notation to that of Appendix B in Murray & Dermott (1999), from which we extracted the numerical constants in the table, \( \mathcal{R} = \mathcal{R}_D + \alpha^{-2} \mathcal{R}_I \), where \( \alpha \equiv a_C/a \), and \( \{B_1, B_2, C_0, C_1, C_2, C_3\} = \{f_{10}, f_2, f_{82}, f_{83}, f_{84}, f_{85} - 1/(3a^2)\} \). For the numerical constants, we set \( \alpha = a_C/a_{\text{res}} \), where \( a_{\text{res}} \) is the semimajor axis at nominal 4:1 resonance (eq. [A9]).

The Hamiltonian is equal to the energy, after replacing the particle’s orbital elements with canonical variables \( H(\Lambda, \lambda, \Gamma, \gamma) = E \). We adopt Poincaré canonical variables \( \{\Lambda, \lambda, \Gamma, \gamma\} \), where

\[
\Lambda = \frac{(GM\rho a)}{2} \quad (A2)
\]

\[
\Gamma = \frac{(GM\rho a)}{2} \frac{c^2}{2} \quad (A3)
\]

\[
\gamma = -\varpi \quad , \quad (A4)
\]

(Murray & Dermott 1999), dropping terms \( O(e^4) \) from \( \Gamma \).

To simplify the Hamiltonian, we employ a number of variable transformations (Holman & Murray 1996; Murray & Dermott 1999). First, we absorb the time-dependent parameter \( \lambda_C = nct + \text{const} \) into \( \lambda \) and shift \( \Lambda \) so that it vanishes at the nominal 4:1 resonance,

\[
\{\Lambda', \lambda'\} \equiv \left\{ \frac{\Lambda - \Lambda_{\text{res}}}{4}, 4\lambda - \lambda_C \right\} , \quad (A5)
\]

where \( \Lambda_{\text{res}} \) is a constant to be determined. Since the generating function for this transformation is \( F = (\Lambda' + \Lambda_{\text{res}}/4)(4\lambda - \lambda_C) \), the new Hamiltonian is \( H + \partial H/\partial \Lambda' \),

\[
H(\Lambda', \lambda'; \Gamma, \gamma) = -\frac{(GM\rho a)^2}{2(4\Lambda' + \Lambda_{\text{res}})^2} - (\Lambda' + \Lambda_{\text{res}}/4)n_C - \frac{GM_C}{a_{\text{res}}} \mathcal{R} \quad (A6)
\]

setting \( a \) to its value at nominal resonance, \( a = a_{\text{res}} \) (eq. [A9]), everywhere except in the first two terms. To choose \( \Lambda_{\text{res}} \), we require that \( \Lambda' = 0 \) at the nominal 4:1 resonance, which occurs where \( 0 = (d/dt)(4\lambda - \lambda_C) = d\lambda'/dt = \partial H/\partial \Lambda' \).

Since

\[
\frac{\partial H}{\partial \Lambda'} \bigg|_{\Lambda'=0} = \frac{4G^2M_P^3}{\Lambda_{\text{res}}^3} - n_C \to 0 \quad , \quad (A7)
\]

we set

\[
\Lambda_{\text{res}} \equiv \left(\frac{4G^2M_P^3}{n_C}\right)^{1/3} \quad . \quad (A8)
\]

The value of \( a \) at nominal resonance is from equation (A2),

\[
a_{\text{res}} = 4^{2/3}a_C \left(\frac{M_P}{M_C + M_P}\right)^{1/3} \quad . \quad (A9)
\]

Expanding the Hamiltonian to second order in \( \Lambda' \) and dropping the constant term

\[
H(\Lambda', \lambda'; \Gamma, \gamma) = -24 \frac{GM_P}{a_{\text{res}}} \frac{\Lambda'^2}{\Lambda_{\text{res}}^2} - \frac{GM_C}{a_{\text{res}}} \mathcal{R} \quad (A10)
\]

We may rescale the momenta and Hamiltonian by the same constant factor without altering the equations of motion. Rescaling by \( \Lambda_{\text{res}} \), the Hamiltonian becomes

\[
H(p_{\Lambda}, q_{\lambda}; \varpi) = -\frac{n_C}{4} \left(24p_{\lambda}^2 + \mu \mathcal{R}\right) \quad (A11)
\]

\[
\mu \equiv \frac{M_C}{M_P} \quad (A12)
\]

where

\[
p_{\lambda} = \frac{\Lambda'}{\Lambda_{\text{res}}} \approx \frac{1}{8} \frac{a - a_{\text{res}}}{a_{\text{res}}} \quad (A13)
\]

\[
q_{\lambda} = \lambda' \equiv 4\lambda - \lambda_C \quad (A14)
\]
The canonical momenta are $p_a$ and $e^2/2$, and their corresponding conjugate co-ordinates are $q_a$ and $-\omega$.

Next, we transform $\{e^2/2, -\omega\}$ to remove the forced secular eccentricity. It simplifies the algebra to switch to the complex canonical variable (Strocchi 1966; Ogilvie 2007)

$$Z \equiv e e^{-i\omega},$$

which is the usual complex eccentricity. Hamilton’s equations for $\{e^2/2, -\omega\}$ are now expressed as

$$\frac{dZ}{dt} = 2i \frac{\partial H}{\partial Z^*},$$

where $Z^*$ is the complex conjugate of $Z$. The secular part of $R$ in Table 2 is

$$R_{sec} = e_C B_1 \left( \frac{Z + Z^*}{2} + B_2 |Z|^2 \right)$$

$$= B_2 \left[ Z + e_C \frac{B_1}{2B_2} \right]^2,$$

after dropping a constant. Therefore, we transform to the variable

$$z \equiv Z + e_C \frac{B_1}{2B_2},$$

which is the (complex) free eccentricity; the constant offset is the forced eccentricity, $e_{\text{forced}} = -e_C (B_1/2B_2)$. Hamilton’s equation for $z$ is clearly the same as for $Z$ (eq. [A16]), i.e., the transformation is canonical.

Under transformation (A19), the resonant part of $R$ becomes

$$R_{res} = \text{Re} \left( e^{i\eta} \left( C_0 e_C^3 + C_1 e_C^2 Z + C_2 e_C Z^2 + C_3 Z^3 \right) \right)$$

$$= \text{Re} \left( e^{i\eta} \left( C_0 e_C^3 + C_1 e_C^2 z + C_2 e_C z^2 + C_3 z^3 \right) \right)$$

where, defining $\beta \equiv -B_1/2B_2$,

$$c_0 = C_0 + C_1 \beta + C_2 \beta^2 + C_3 \beta^3 = -0.26$$

$$c_1 = C_1 + 2C_2 \beta + 3C_3 \beta^2 = -1.5$$

$$c_2 = C_2 + 3C_3 \beta = -2.9$$

$$c_3 = C_3 = 0.64$$

Collecting results, the Hamiltonian is

$$H(p_a, q_a; z) = -24p_a^2 - \mu B_2 |z|^2 - \mu \text{Re} \left( e^{i\eta} \left( C_0 e_C^3 + C_1 e_C^2 z + C_2 e_C z^2 + C_3 z^3 \right) \right),$$

after dropping the prefactor $n_C/4$, which means that time is now measured in units of $4/n_C$. Hamilton’s equations of motion are

$$\dot{p}_a = -\frac{\partial H}{\partial q_a}; \quad \dot{q}_a = \frac{\partial H}{\partial p_a}; \quad \dot{z} = 2i \frac{\partial H}{\partial Z^*}$$

B. TIDAL DISSIPATION

Assuming orbits and spins are coplanar, tidal dissipation due to tides raised on Charon by Pluto are described by the following evolution equations (Hut 1981)

$$\frac{da}{dt} = -\frac{6k_{2C}}{T_C} q (1 + q) \left( \frac{R_C}{a} \right)^8 \frac{a}{1 - e^2} \left\{ f_1 - (1 - e^2)^{3/2} f_2 \frac{\Omega_C}{n} \right\},$$

$$\frac{de}{dt} = -\frac{27k_{2C}}{T_C} q (1 + q) \left( \frac{R_C}{a} \right)^8 \frac{e}{(1 - e^2)^{3/2}} \left\{ f_3 - \frac{11}{18} (1 - e^2)^{3/2} f_4 \frac{\Omega_C}{n} \right\},$$

$$\frac{d\Omega_C}{dt} = -\frac{3k_{2C} q^2}{T_C r_g^2 (1 - e^2)^6} \left( \frac{R_C}{a} \right)^6 \left\{ f_2 - (1 - e^2)^{3/2} f_5 \frac{\Omega_C}{n} \right\},$$

where $q = M_P/M_C$ is the mass ratio, $\Omega_C$ is Charon’s spin rate, $n = \sqrt{G (M_P + M_C) / a^3}$ is the orbital mean motion, $k_{2C}$ is the tidal Love number of Charon, and $r_g$ is its radius of gyration related to the moment of inertia by $I = r_g^2 m R_C^2$, with $r_g \sim 0.6$ for a uniform density sphere. The values for $f_i$ are all of order unity, $f_i = 1 + O(e^2) + O(e^4)...$, with their exact expressions listed in Hut (1981). In this tidal model, $T_C = R_C^3 / GM_P \tau_C$ where $\tau_C$ is the (assumed) constant tidal lag time. To connect this model to that of Goldreich & Soter (1966) which assumes a constant lag phase (and a constant tidal Q factor), we let the tidal lag time $\tau_C = 1/(2n Q_C)$, with $Q_C$ being Charon’s tidal dissipation quality factor. This choice is somewhat arbitrary: one may also assume, e.g., $\tau_C = 1/[2(\Omega_C - n)Q_C]$. The tide raised by Charon on Pluto causes similar effects but with all subscripts $C$ substituted by $P$ and the mass ratio inverted, $q = M_C/M_P$. The net orbital evolution is given by the sum of both tides.
The Love number for a uniform density sphere is (see, e.g., Dobrovolskis et al. 1997; Murray & Dermott 1999)
\[ k_2 = \frac{3/2}{1 + \mu}, \]  
(B4)
where \( \mu = 19\mu/(2\rho g R) \) is the effective rigidity of the body, \( \mu \) the material strength, \( \rho \) the density and \( g \) the gravitational acceleration. If the body is a rubble pile, \( \mu \ll 1 \) and \( k_2 \sim 3/2 \). However, if the body is a consolidated ice sphere with \( \mu \sim 4 \times 10^{10} \text{dyne/cm}^2 \), \( k_2 \sim C \sim 0.005 \) and \( k_{2P} \sim 0.05 \) (Dobrovolskis et al. 1997).

The following are numerical estimates for the various timescales, \( \tau_a \equiv x/|\dot{x}| \),
\[ \tau_a \sim 10^8 \text{yrs} \]  
\[ \tau_a \sim 2 \times 10^7 \text{yrs} \]  
\[ \tau_c \sim 3 \times 10^7 \text{yrs} \]  
\[ \tau_a \sim 5 \times 10^6 \text{yrs} \]  
\[ \tau_{\Omega_C} \sim 6 \times 10^4 \text{yrs} \]  
\[ \tau_{\Omega_P} \sim 6 \times 10^5 \text{yrs} \]  
(B5)
(B6)
(B7)
(B8)
(B9)
(B10)
where we have scaled \( Q_P \) and \( Q_C \) by values typical of solid bodies and have taken the limit \( \epsilon \ll 1 \).

Charon’s spin is quickly (pseudo-)synchronized with the orbit, and Pluto’s spin synchronization is on-going until the system reaches the final state of double synchronization. So most of the \( a \) evolution is contributed by tides on Pluto (eq. B6).

During the orbital expansion, the tide on Pluto increases \( e \) while the tide on Charon decreases it. With our choice of parameters, the former dominates over the latter. Once Charon reaches its equilibrium location, both tides damp the eccentricity.

If the spin direction of body is misaligned with the orbit normal, it will be tilted to alignment in roughly the spin synchronization time (Hut 1981).

REFERENCES

Beust, H. 2003, A&A, 400, 1129
Brown, M. E., van Dam, M. A., Bouchez, A. H., Le Mignant, D., Campbell, R. D., Chin, J. C. Y., Conrad, A., Hartman, S. K., Johansson, E. M., Lafon, R. E., Rabinowitz, D. L., Stomski, Jr., P. J., Summers, D. M., Trujillo, C. A., & Wizinowich, P. L. 2006, ApJ, 642, L65
Buie, M. W., Grundy, W. M., Young, E. F., Young, L. A., & Stern, S. A. 2006, AJ, 132, 290
Canup, R. M. 2005, Science, 307, 546
Chirikov, B. V. 1979, Phys. Rep., 52, 263
Dobrovolskis, A. R., Peale, S. J., & Harris, A. W. 1997, Dynamics of the Pluto-Charon Binary (Pluto and Charon), 159–+
Goldreich, P., Lithwick, Y., & Sari, R. 2002, Nature, 420, 643
—. 2004, ARA&A, 42, 549
Goldreich, P. & Soter, S. 1966, Icarus, 5, 375
Henon, M. & Heiles, C. 1964, AJ, 69, 89
Henrard, J. 1982, Celestial Mechanics, 27, 3
Holman, M. J. & Murray, N. W. 1996, AJ, 112, 1278
Hut, P. 1981, A&A, 99, 126
Kenyon, S. J. & Luu, J. X. 1998, AJ, 115, 2136
Lee, M. H. & Peale, S. J. 2002, ApJ, 567, 596
Levison, H. F. & Duncan, M. J. 1994, Icarus, 108, 18
Lithwick, Y. & Wu, Y. 2008, ApJ, submitted
Maciejewski, W. & Sparke, L. S. 1997, ApJ, 484, L117+
Mckinnon, W. B. 1989, ApJ, 344, L41
Murray, C. D. & Dermott, S. F. 1999, Solar system dynamics (Solar system dynamics by Murray, C. D., 1999)
Nicholson, P. D. & Gladman, B. 2006, Icarus, 181, 218
Ogilvie, G. I. 2007, MNRAS, 374, 131
Pichardo, B., Sparke, L. S., & Aguilar, L. A. 2005, MNRAS, 359, 521
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical recipes in FORTRAN. The art of scientific computing (Cambridge: University Press, —c1992, 2nd ed.)
Sari, R. & Goldreich, P. 2006, ApJ, 642, L65
Sicardy, B., Bellucci, A., Gundron, E., Lacombe, F., Lacour, S., Lecacheux, J., Lellouch, E., Renner, S., Pau, S., Roques, F., Widemann, T., Colin, F., Vacher, F., Martin, R. V., Aikorges, N., Hainaut, O., Marco, O., Beiser, W., Hummel, E., Feinstein, C., Levato, H., Maury, A., Frappa, E., Gaillard, B., Lavayssiere, M., di Sora, M., Mault, F., Masl, G., Behrend, R., Carrier, F., Mouss, O., Rousset, P., Alvarez-Candal, A., Lazzaro, D., Veiga, C., Andrei, A. H., Assafin, M., da Silva Neto, D. N., Jacques, C., Pimentel, E., Weaver, D., Lecampion, J.-F., Doncel, F., Momiyama, T., & Tancredi, G. 2006, Nature, 439, 52
Steiff, A. J., Mutcher, M. J., Weaver, H. A., Stern, S. A., Durda, D. D., Terrell, D., Merline, W. J., Young, L. A., Young, E. F., Buie, M. W., & Spencer, J. R. 2006, AJ, 132, 614
Stern, S. A., Weaver, H. A., Steiff, A. J., Mutcher, M. J., Merline, W. J., Buie, M. W., Young, E. F., Young, L. A., & Spencer, J. R. 2006, Nature, 439, 946
Strochi, F. 1966, Reviews of Modern Physics, 38, 36
Ward, W. R. & Canup, R. M. 2006, Science, 313, 1107
Weaver, H. A., Stern, S. A., Mutcher, M. J., Steiff, A. J., Buie, M. W., Merline, W. J., Spencer, J. R., Young, E. F., & Young, L. A. 2006, Nature, 439, 943
Young, E. F. & Binzel, R. P. 1994, Icarus, 108, 219