The Light-Front SU(N) Yang-Mills Theory for the Weyl gauge

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I. INTRODUCTION

The light-front (LF) quantization (at a fixed $x^+$ surfaces) of the gauge field models is almost exclusively performed for the light-cone (LC) gauge condition: $A^a_+ = A^{a+} = 0$. Though this choice leads to a considerable simplifications of the interaction Hamiltonians, then it leads directly to the Cauchy Principal Value (CPV) prescription for the spurious poles of the perturbative gauge field propagators - which is known to be an inconsistent prescription for the higher order computations. The consistent regularization is the causal Mandelstam-Leibbrandt (ML) prescription, which has been first obtained as a mere computational trick and then derived within the canonical equal-time (ET) quantization for the Yang-Mills theories. Since then there were attempts to derive the ML prescription for the LC gauge within the LF canonical quantization, however they were successful only for the free Abelian models with two null surfaces used as the quantization surfaces. Now it becomes clear that in order to have the ML-poles for the gauge propagators in the interacting LF models one has to resort from the LC gauge and choose another null axial gauge - the LF Weyl (LF Weyl) condition: $A^a_+ = A^{a-} = 0$. This gauge condition has been introduced for the LF Quantum Electrodynamics (QED) in within the DLCQ approach. Quite recently it was implemented for the perturbative QED in and , where the ML spurious poles have arised naturally for the gauge field propagator. The fermionic currents in the QED contain no derivatives of fields, thus the only complication which appears in the LF quantization is connected with the presence of nondynamical components of the fermion field $\psi_-$ and $\psi^\dagger_-$. This leads to the nonlocal (in $x^-$) terms in the Hamiltonian, but their consistent perturbation theory is equivalent to the usual ET formulation. For the nonAbelian models things look worse, because the triple gauge-gauge-gauge field couplings contain derivatives of fields and for the LF Weyl gauge there will be also a time derivative $\partial_+ A^a_+$ and the canonical procedure would lead to the Dirac brackets which contain interactions. We notice that a similar phenomenon also happens in the Abelian model of QED for charged scalar fields. For a toy model in 1+1 dimensions the consistent LF quantization has been performed after a proper redefinition of scalar fields which shifted all interaction into nonlocal (in $x^-$) interactions but have left the LF Dirac brackets free of interactions. We may hope that a similar approach can be consistently applied also for the nonAbelian model of SU(N) Yang-Mills fields.

Our present paper is organized as follows. In Section 2 we perform the canonical quantization in three steps. First, we recapitulate the analysis of the Abelian model for the helicity representation of the transverse components $A^a_\perp$. Second, we introduce the scalar representation for the nonAbelian fields $A^a_\parallel$ and choose a suitable form of the LF Lagrangian. Third, the canonical LF quantization is performed with the canonical free form of the LF commutators. In Section 3 we define the generating functional for all Green functions as the canonical phase-space path integral. Within the perturbative theory, this path-integral is shown to be equivalent to the canonical definition of the generating functional, thus proving the consistency of the ML-preservation for the LF Weyl gauge. In Section 4 these results are discussed and further developments are sketched.
II. CANONICAL QUANTIZATION

In this paper we implement the LF Weyl gauge condition $A^a_\nu = 0$ explicitly for the SU(N) Yang-Mills theory, thus, the canonical Lagrangian, written in the LF notation, has the form

$$L_{Y-M}^{Weyl} = \frac{1}{2} \left( \partial_\mu A^a_\mu \right)^2 - \frac{1}{4} \left( \partial_\mu A^a_\mu - D^a_\mu A^a_\mu \right)^2 + \partial_\mu A^a_\mu \left( D^a_\mu A^a_\mu - \partial_\mu A^a_\mu \right) + A^a_\mu j^a_\mu,$$

(2.1)

where $D^a_\mu = \partial_\mu \delta^a_b + g f^{acb} A^c_\mu$, $j^a_\mu$ describe arbitrary external sources and the summation over repeating indices is understood. The transverse components of gauge fields can be parameterized by means of the helicity fields $\phi^a_A$, $\phi^{\dagger}_A$, which allow us to write

$$- \frac{1}{4} \left( \partial_\mu A^a_\mu - D^a_\mu A^a_\mu \right)^2 = - \frac{1}{4} \left( G^a_{AB} - G^a_{AB} \right) \left( G^a_{AB} - G^a_{AB} \right) - \frac{1}{2} \left( G^a_{AB} G^a_{AB} + G^a_{AB} G^a_{AB} \right),$$

(2.2a)

$$\partial_\mu \phi_A^a + \partial_\mu \phi_A^{\dagger} = \partial_\mu \phi_A^a \left( D^a_\mu \phi_A^a - \nabla_A A^a_\mu \right) + \partial_\mu \phi_A^{\dagger} \left( D^a_\mu \phi^{\dagger}_A - \nabla_A A^a_\mu \right).$$

(2.2b)

In this way we reach the expression for the Lagrangian density

$$L_{Y-M}^{Weyl} = \frac{1}{2} \left( \partial_\mu A^a_\mu \right)^2 + \partial_\mu \phi_A^a \left( D^a_\mu \phi_A^a - \nabla_A A^a_\mu \right) + \partial_\mu \phi_A^{\dagger} \left( D^a_\mu \phi^{\dagger}_A - \nabla_A A^a_\mu \right)$$

$$- \frac{1}{4} \left( G^a_{AB} - G^a_{AB} \right) \left( G^a_{AB} - G^a_{AB} \right) - \frac{1}{2} \left( G^a_{AB} G^a_{AB} + G^a_{AB} G^a_{AB} \right)$$

$$+ A^a_\mu j^a_\mu + \phi_A^a s^a_A + s^a_A \phi_A^a,$$

(2.3)

where $s^a_A = \frac{j^{2A}_a - i j^{A+1}_a}{\sqrt{2}}$, $s^{\dagger a}_A = \frac{j^{2A}_a + i j^{A+1}_a}{\sqrt{2}}$. However before starting the canonical quantization procedure we integrate the second and third terms by parts in order to obtain the equivalent Lagrangian density

$$L_{Y-M}^{Weyl} = \frac{1}{2} \left( \partial_\mu A^a_\mu \right)^2 + \partial_\mu \phi_A^a D^a_{\mu} \phi^a_A + \partial_\mu \phi_A^{\dagger} D^a_{\mu} \phi_A^{\dagger} - \partial_\mu A^a_\mu \left( \nabla_A \phi_A^a + \nabla_A \phi_A^{\dagger} \right)$$

$$- \frac{1}{4} \left( G^a_{AB} - G^a_{AB} \right) \left( G^a_{AB} - G^a_{AB} \right) - \frac{1}{2} \left( G^a_{AB} G^a_{AB} + G^a_{AB} G^a_{AB} \right)$$

$$+ A^a_\mu j^a_\mu + \phi_A^a s^a_A + s^a_A \phi_A^a,$$

(2.4)

which further will be taken as the starting point for quantization. We stress here that one could also start with (2.3) and then would reach the same final results, however (2.4) produces least possible complications during the quantization procedure. Also we point out that our formulation is performed within the dimensional regularization $(D = 2\omega)$, which at the stage of canonical quantization allows for the proper definition of independent modes, while at the level of perturbative calculation it covariantly regularizes Feynman integrals.

A. Canonical quantization for the Abelian case

Before analysing the complete non-Abelian model, we will present results for the Abelian case, when all color indices are omitted and we put $g = 0$ in (2.4). Therefore our first starting point is the equivalent Lagrangian density for the Abelian case

$$L_{Abel}^{Weyl} = \frac{1}{2} \left( \partial_\mu A^a_\mu \right)^2 + \partial_\mu \phi_A^a \partial_\mu \phi^a_A + \partial_\mu \phi_A^{\dagger} \partial_\mu \phi_A^{\dagger} - \partial_\mu A^a_\mu \left( \nabla_A \phi_A^a + \nabla_A \phi_A^{\dagger} \right)$$

$$- \frac{1}{4} \left( \nabla_A \phi_B - \nabla_B \phi_A - \nabla_A \phi_B^A + \nabla_B \phi_A^A \right) \left( \nabla_A \phi_A^A - \nabla_B \phi_A^A - \nabla_A \phi_A^A + \nabla_B \phi_A^A \right)$$

$$- \frac{1}{2} \left( \nabla_A \phi_B - \nabla_B \phi_A \right) \left( \nabla_A \phi_B - \nabla_B \phi_A \right) - \frac{1}{2} \left( \nabla_A \phi_B - \nabla_B \phi_A \right) \left( \nabla_A \phi_B - \nabla_B \phi_A \right)$$

$$+ A^a_\mu j^a_\mu + \phi_A^a s^a_A + s^a_A \phi_A^a,$$

(2.5)

3Details of this parameterization and other notations are explained in the Appendix.
and we easily find the relevant canonical momenta

\[ \Pi_\phi = \partial_\phi A_\phi - \nabla_A \phi_A, \quad (2.6a) \]

\[ \Pi_{\phi_A} = \partial_{\phi_A} A_{\phi_A}, \quad (2.6b) \]

\[ \Pi_{\phi_A^\dagger} = \partial_{\phi_A^\dagger} A_{\phi_A^\dagger}, \quad (2.6c) \]

and the canonical Hamiltonian density\footnote{We have integrated by parts the transverse partial derivatives in order to have the most compact notation for the Hamiltonian density. Therefore the sign \( \approx \) means that some boundary terms, completely superfluous in the forthcoming analysis, are omitted.}

\[ H_{can}^{Abel} = \Pi_{\phi_A} \partial_+ A_{\phi_A} + \Pi_{\phi_A^\dagger} \partial_+ A_{\phi_A^\dagger} + \Pi_+ \partial_+ A_\phi - \mathcal{L} \]

\[ \approx \frac{1}{2} \left( \Pi_+ + \nabla_A A_{\phi_A} + \nabla_A A_{\phi_A^\dagger} \right) - \nabla_B A_{\phi_A} \nabla_B A_{\phi_A^\dagger} \]

\[ - \frac{1}{2} \left( - \nabla_A A_{\phi_A} + \nabla_A A_{\phi_A^\dagger} \right)^2 - A_+ j^- - \phi_A^\dagger s_A - s_A^\dagger A_{\phi_A^\dagger}. \quad (2.7) \]

The nonvanishing Dirac brackets are

\[ 2\partial^x \left\{ \phi_A^\dagger (x), \phi(y) B \right\}_{x^+ = y^+} = -\delta_{AB} \delta^{2\omega - 1} (\vec{x} - \vec{y}), \quad (2.8a) \]

\[ \{ A_\phi (x), \Pi_\phi(y) \}_{x^+ = y^+} = \delta^{2\omega - 1} (\vec{x} - \vec{y}), \quad (2.8b) \]

and the Hamilton equations of motion

\[ \partial_+ \Pi_\phi = -j^-, \quad (2.9a) \]

\[ 2(\partial_+ - \nabla_A A_{\phi_A}) \phi_B^\dagger = \nabla_B \Pi_\phi + s_B^\dagger, \quad (2.9b) \]

\[ 2(\partial_+ - \nabla_A A_{\phi_A}) \phi_B = \nabla_B \Pi_\phi + s_B, \quad (2.9c) \]

\[ \partial_+ A_\phi = \Pi_\phi + \nabla_A A_{\phi_A} + \nabla_A A_{\phi_A^\dagger}. \quad (2.9d) \]

are equivalent to the Euler-Lagrange equations which follow from the Lagrangian density \( \mathcal{L} \), thus proving the consistency of the above canonical structure. Next the canonical quantization follows directly; from \( \mathcal{L} \) we obtain the commutation relations

\[ 2\partial^x \left[ \phi_A^\dagger (x), \phi_B (y) \right]_{x^+ = y^+} = -i \delta_{AB} \delta^{2\omega - 1} (\vec{x} - \vec{y}), \quad (2.10a) \]

\[ [A_\phi (x), \Pi_\phi(y)]_{x^+ = y^+} = i \delta^{2\omega - 1} (\vec{x} - \vec{y}), \quad (2.10b) \]

while the quantum Hamiltonian is given exactly by \( \mathcal{L} \). We see that source terms appear only linearly thus the perturbative gauge field propagators will be given just by the chronological products of respective (free) fields. However we will not calculate these propagators here, instead we define the path-integral representation for the Green functions as

\[ Z[j^-, s_A^\dagger, s_A] = \mathcal{N} \int D A_\phi D\Pi_\phi \exp \left\{ i \int d^2\omega x \left( \Pi_\phi \partial_\phi A_\phi + \partial_\phi A_\phi \partial_\phi A_\phi - H_{can}^{Abel} \right) \right\}. \quad (2.11) \]

Because in the Hamiltonian \( \Pi_\phi \) appears utmost quadratically, then we can easily perform the Gaussian integral over \( \Pi_\phi \) and obtain the desired result

\[ Z[j^-, s_A^\dagger, s_A] = \mathcal{N} \int D A_\phi \exp \left\{ i \int d^2\omega x \mathcal{L}_{Abel}^{Weyl} \right\}. \quad (2.12) \]

The remaining integrations are utmost Gaussian, thus one can perform them rather easily and derive the explicit expression for the generating functional

\[ \mathcal{Z} = \mathcal{N} \int D A_\phi \exp \left\{ i \int d^2\omega x \mathcal{L}_{Abel}^{Weyl} \right\}. \]

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\[ \mathcal{Z} = \mathcal{N} \int D A_\phi \exp \left\{ i \int d^2\omega x \mathcal{L}_{Abel}^{Weyl} \right\}. \]
\[ Z[j^-, s_A^1, s_A] = \exp -i \int d^{2\omega} x \ d^{2\omega} y \left[ s_A^1(x) D_F^{2\omega}(x-y) s_A(y) + j^-(x) \partial^X \Delta_{ML}^{2\omega}(x-y) j^-(y) \right] , \]
\[ -s_A^1(x) \nabla_A \Delta_{ML}^{2\omega}(x-y) j^-(y) - s_A(x) \nabla_A \Delta_{ML}^{2\omega}(x-y) j^-(y) \right] , \]

where
\[ D_F^{2\omega}(x) = i \int \frac{d^{2\omega} k}{(2\pi)^4} \frac{e^{-ik \cdot x}}{2k_+ k_- - k_0^2 + i\epsilon} \]  
\[ \Delta_{ML}^{2\omega}(x) = \int_0^{x^+} d\xi \ D_F^{2\omega}(\xi, \bar{x}) = - \int \frac{d^{2\omega} k}{(2\pi)^4} \frac{e^{-ik \cdot x}}{2k_+ k_- - k_0^2 + i\epsilon} \frac{1}{\text{sgn}(k_-)} \]

The canonical quantum field propagators are defined as the chronological products of operators and in the present case one finds the following expressions

\[ \langle 0 \left| T^+ \phi_A(x) \phi_B^\dagger(y) \right| 0 \rangle = \delta_{AB} D_{F}^{2\omega}(x-y), \]
\[ \langle 0 \left| T^+ \phi_A(x) A_-(y) \right| 0 \rangle = \nabla_A \Delta_{ML}^{2\omega}(x-y), \]
\[ \langle 0 \left| T^+ \phi_A^\dagger(x) A_-(y) \right| 0 \rangle = \nabla_A \Delta_{ML}^{2\omega}(x-y), \]
\[ \langle 0 \left| T^+ A_-(x) A_-(y) \right| 0 \rangle = 2 \partial^X \Delta_{ML}^{2\omega}(x-y), \]

therefore establishing the expected equivalence between the canonical and path-integral definition of the generating functional

\[ Z[j^-, s_A^1, s_A] = \left\langle 0 \left| T^+ \exp -i \int d^{2\omega} x \left( A_- j^+ + \phi_A^\dagger s_A + s_A^1 \phi_A \right) \right| 0 \right\rangle . \]

B. Redefinition of transverse components of gauge fields

Now we may start solve the main problem of this problem, namely, the LF quantization of the nonAbelian gauge field system given by the Lagrangian density \( \mathcal{L} \). Because the third and fourth terms in \( \mathcal{L} \) look very similarly to those for the charged scalar fields, therefore we will employ here the same trick [13], [14] which we have used in the scalar QED. This means that we redefine charged complex fields \( \phi_a \) and \( \phi_a^\dagger \) as follows:

\[ \phi_a^\dagger = \chi_a + \sum_{n=1}^{\infty} \left( \frac{-1}{n} \right)^n (\tilde{\alpha})^{n}_{ab} * \chi_b = (1 + \tilde{\alpha})_{ab}^{1/2} * \chi_a^b, \]  
\[ \phi_a^\dagger = \chi_a + \chi^\dagger A_A + \sum_{n=1}^{\infty} \left( \frac{-1}{n} \right)^n (\tilde{\alpha})^{n}_{ab} = \chi^\dagger A_A + (1 + \tilde{\alpha})_{ab}^{-1/2}, \]  
\[ \tilde{\alpha}_{ab} = g f^{abc} A^c_{\alpha}, \quad \tilde{\alpha}_{ab} = g f^{abc} A^c_{\alpha}, \]  
\[ (\tilde{\alpha})_{ab}^{n} = (\tilde{\alpha})_{ac}^{n-1} * \tilde{\alpha}_{cb}, \]

where the integration over \( x^- \) is denoted by \( * \). Next one can calculate the following expressions

\[ D^a_{-} \phi_b^A = (1 + \tilde{\alpha})_{ab}^{1/2} * (\partial^- \chi_b^A), \]  
\[ D^- \phi^a_{ab} = (\partial^- \chi^a_A) * (1 + \tilde{\alpha})_{ab}^{1/2}, \]  
\[ \partial_{\alpha} \phi^a = (1 + \tilde{\alpha})_{ab}^{1/2} * (\partial_{\alpha} \chi^a_A) + L_{ab} [\partial_{\alpha} A_-] * \chi_b^A, \]  
\[ \partial_{\alpha} \phi^a_{ab} = (\partial_{\alpha} \chi^a_A) * (1 + \tilde{\alpha})_{ab}^{-1/2} + \chi^a_A * R_{ba} [\partial_{\alpha} A_-], \]

where we have introduced the notation \( (\partial_{\alpha} = \partial_+, \nabla_A, \nabla_A) \) and
\[ L_{ab}[x^-, y^-; \partial_+ A_-] = \int dz^- \partial_+ A^c_-(z^-) \mathcal{L}_{ab}^c[x^-, y^-, z^-], \]  
\[ R_{ab}[x^-, y^-; \partial_+ A_-] = \int dz^- \partial_+ A^c_-(z^-) \mathcal{R}_{ab}^c[x^-, y^-, z^-, y^-], \]  
\[ \mathcal{L}_{ab}^c[x^-, y^-; z^-] = \frac{\delta}{\delta A^c_-(z^-)} (\mathbb{1} + \hat{a})^{-1/2} [x^-, y^-], \]  
\[ \mathcal{R}_{ab}^c[x^-, y^-, z^-] = \frac{\delta}{\delta A^c_-(z^-)} (\mathbb{1} + \hat{a}^\dagger)^{-1/2} [x^-, y^-]. \] 

The transverse components of the gauge field strength can be expressed in terms of these new fields \( \chi^a_A \), \( \chi^\dagger_A \), and \( A^a_- \) for example

\[ G^a_{AB} = (\mathbb{1} + \hat{a})^{-1/2} * \nabla_A \chi^b_B - (\mathbb{1} + \hat{a})^{-1/2} * \nabla_B \chi^a_A + L_{ab}[\nabla_A A_-] * \chi^b_B \]  
\[ - L_{ab}[\nabla_B A_-] * \chi^a_A - (\mathbb{1} + \hat{a})_{cd}^{1/2} * \chi^d_A * (\mathbb{1} + \hat{a}^\dagger)^{1/2} * \chi^c_B = g^a_{AB}, \]  
\[ G^a_A = (\mathbb{1} + \hat{a})^{-1/2} * \nabla_A \chi^b_B - \nabla_B \chi^a_A + (\mathbb{1} + \hat{a}^\dagger)^{-1/2} + L_{ab}[\nabla_A A_-] * \chi^b_B \]  
\[ - \chi^a_A * R_{ab}[\nabla_B A_-] + g^{abc} \chi^a_A * (\mathbb{1} + \hat{a}^\dagger)_{ab}^{1/2} (\mathbb{1} + \hat{a})_{cd}^{1/2} * \chi^c_B = g^a_{A\bar{A}}, \] 

and the similar expressions for \( g^a_{A\bar{A}} = C^a_{A\bar{A}} \) and \( g^a_{AB} = C^a_{AB} \). All above formulas allow us to write the redefined Lagrangian density as follows

\[ \mathcal{L}_{mod} = \partial_+ \chi^a_A \partial_- \chi^a_A + \partial_+ \chi^\dagger_A \partial_- \chi^\dagger_A + \frac{1}{2} \left( \partial_+ A^a_- \right)^2 + \partial_+ A^a_- \mathcal{J}^a - \mathcal{V} + A^a_- j^- + s^a_A (\mathbb{1} + \hat{a})^{1/2} \chi^a_A + \chi^\dagger_A \left( \mathbb{1} + \hat{a}^\dagger \right)^{1/2} s^a_A, \]  

where for brevity we have introduced the notation

\[ \mathcal{J}^a = \chi^b_A * R_{bc}^a * (\mathbb{1} + \hat{a}^\dagger)_{cd} * \chi^d_A + \chi^\dagger_A * (\mathbb{1} + \hat{a})_{cd}^{1/2} * \mathcal{L}_{cd}^a * \chi^a_A - \mathcal{L}_{a}^c * \chi^c_A - \mathcal{L}_{bc}^a * \nabla_B \chi^b_A \]  
\[ - L_{ab}[\nabla_A A_-] * \partial_- \chi^a_A - (\mathbb{1} + \hat{a})_{ab}^{1/2} \chi^a_A, \]  
\[ \mathcal{V} = \frac{1}{4} \left( g^a_{AB} - g^a_{A\bar{A}} \right) \left( g^a_{AB} - g^a_{A\bar{A}} \right) + \frac{1}{2} \left( g^a_{AB} g^a_{A\bar{A}} + g^a_{AB} g^a_{A\bar{A}} \right) \]  

and also we have used the identities

\[ (\mathbb{1} + \hat{a})_{ab}^{1/2} * (\mathbb{1} + \hat{a}^\dagger)_{bc}^{1/2} = (\mathbb{1} + \hat{a})_{ab}^{1/2} * (\mathbb{1} + \hat{a})_{bc}^{1/2} = \delta_{ac}. \]  

Now we are ready to start the canonical procedure and we find the canonical conjugated momenta to \( \chi^a_A \), \( \chi^\dagger_A \) and \( A^a_- \) respectively

\[ \Pi_{\chi^a_A} = \partial_- \chi^a_A \]  
\[ \Pi_{\chi^\dagger_A} = \partial_- \chi^\dagger_A \]  
\[ \Pi^a_- = \partial_+ A^a_- + \mathcal{J}^a. \]  

Thus the nondynamical momenta conjugated to transverse fields \( \chi^a_A \) and \( \chi^\dagger_A \) have the free form and we expected that they will not generate the interaction dependent Dirac brackets. The price of this success is the more complicated structure of interaction terms but this is not a fundamental difficulty both in the canonical quantization procedure and the later perturbation calculations. One easily can find the canonical Hamiltonian density

\[ \mathcal{H}_{can} = \Pi_{\chi^a_A} \partial_+ \chi^a_A + \Pi_{\chi^\dagger_A} \partial_+ \chi^\dagger_A + \Pi^a_- * \partial_+ A^- - \mathcal{L} \]  
\[ = \frac{1}{2} \left( \Pi^a_- - \mathcal{J}^a \right)^2 + \mathcal{V} - s^a_A (\mathbb{1} + \hat{a})_{ab}^{1/2} \chi^a_A - \chi^\dagger_A * (\mathbb{1} + \hat{a}^\dagger)_{ab}^{1/2} s^a_A - A^a_- j^- , \]  

and then the canonical commutation rules which are direct generalization of those [2.10] in the Abelian case

\[ 2\delta^{ab_{12}} \left[ \phi^a(x), \phi^b(y) \right]_{x^+ = y^+} = -i \delta^{ab} \delta_{AB} \delta^{2\omega-1}(\vec{x} - \vec{y}), \]  
\[ \left[ A^a_-(x), \Pi^b_-(y) \right]_{x^+ = y^+} = i \delta^{ab} \delta^{2\omega-1}(\vec{x} - \vec{y}). \]
One can check that these commutators and the Hamiltonian (2.24) generate Heisenberg equations of motion

\[ 2\partial_+\partial_-\chi_A^a = \partial_+ A^b_+ \frac{\delta J^b}{\delta \chi_A^a} - \frac{\delta V}{\delta \chi_A^a} + (1 + \tilde{a})_{ab}^{-1/2} \ast s_b^a, \]  

(2.26a)

\[ 2\partial_+\partial_-\chi^a_A = \partial_+ A^b_+ \frac{\delta J^b}{\delta \chi_A^a} - \frac{\delta V}{\delta \chi_A^a} + s^b_A \ast (1 + \tilde{a})_{ba}^{-1/2}, \]  

(2.26b)

\[ \partial_+ \Pi_+ = (\Pi_+ - J^b) \ast \frac{\delta J^b}{\delta \Pi_+} - \frac{\delta V}{\delta \Pi_+} + j^a_A \ast L_{bc}^c \ast \chi^b_A + \chi^b_A \ast R_{bc}^a \ast s^a_A, \]  

(2.26c)

\[ \partial_+ A^-_+ = \Pi_+ - J^a, \]  

(2.26d)

which, modulo proper ordering of non-commuting terms, are equivalent to the Euler-Lagrange equations generated from the Lagrangian (2.21).

### III. GENERATING FUNCTIONAL

Having found the canonical structure for the gauge system we can define the generating functional for Green functions as the path-integral over phase space

\[ Z[s_A^a, s^i_A, j_a^i] \equiv \int \mathcal{D}A^a_+ \mathcal{D}\Pi_+ \mathcal{D}\chi_A^a \mathcal{D} \chi^i_A \exp i \int d^2\omega x \left[ 2\partial_-\chi^a_A \partial_+ A^a_+ + 2\partial_+\chi^a_A \partial_- A^a_+ + \Pi_+ \partial_- A^-_+ - H_{\text{can}} \right]. \]  

(3.1)

Next we can perform the Gaussian integration over \( \Pi_+ \) and rewrite the generating functional

\[ Z[s_A^a, s^i_A, j_a^i] \equiv \int \mathcal{D}A^a_- \mathcal{D}\phi_A^i \mathcal{D} \phi^a_A \mathcal{D} t^{1/2} / (1 + \tilde{a}) \mathcal{D} t^{1/2} / (1 + \tilde{a}^\dagger) \exp i \int d^2\omega x L_{Weyl}. \]  

(3.2)

Then we may reinstall linear couplings to the external sources \( s_A^a \) and \( s^i_A \) via the following change of path variables

\[ \chi_A^a = \left( 1 + \tilde{a} \right)_{ab}^{1/2} \phi_A^b, \]  

(3.3a)

\[ \chi^i_A = \phi_A^b \ast \left( 1 + \tilde{a}^\dagger \right)_{ba}^{1/2}, \]  

(3.3b)

which gives the main result of this paper:

\[ Z[s_A^a, s^i_A, j_a^i] \equiv \int \mathcal{D}A^a_- \mathcal{D}\phi_A^i \mathcal{D} \phi^a_A \mathcal{D} t^{1/2} / (1 + \tilde{a}) \mathcal{D} t^{1/2} / (1 + \tilde{a}^\dagger) \exp i \int d^2\omega x L_{Weyl}. \]  

(3.4)

We notice, after \[ \boxed{[\text{cf.}]}, \] that within the dimensional regularization we can safely omit the functional determinants as irrelevant constants. Therefore our LF quantization leads to the same path-integral definition of generating functional as the commonly used ET formulation.

In the Abelian case we have checked that the generating functional can be equivalently defined either as the path-integral or canonically as the chronological product. Below we will check the respectful equivalence for the Yang-Mills case and we start with the canonical definition

\[ Z[j_a^-, s^i_A, s_A^a] = \left( 0 \right| T^+ \exp -i \int d^2\omega x \left( \mathcal{H}_{\text{int}} \left[ A^a_-, \Pi_-, \chi^i_A, \chi_A^a, s^i_A, s_A^a \right] - A^a_+ j_a^a \right) \left| 0 \right). \]  

(3.5)

where

\[ \mathcal{H}_{\text{int}} \left[ A^a_-, \Pi_-, \chi^i_A, \chi_A^a, s^i_A, s_A^a \right] = H_{\text{can}} - H_0 \]

\[ = \frac{1}{2} (\mathcal{J}_0^a)^2 + \mathcal{V}_{\text{int}} - \mathcal{J}_0^a (\Pi_- - \mathcal{J}_0^a) - \chi_A^a \ast \left( 1 + \tilde{a}^\dagger \right)_{ab}^{-1/2} s_b^a, \]  

(3.6a)

\[ H_0 = \frac{1}{2} (\Pi_- - \mathcal{J}_0^a)^2 + \mathcal{V}_0, \]  

(3.6b)

\[ \mathcal{J}_0^a = \nabla_A \chi_A^a + \nabla \chi_A^a, \]  

(3.6c)

\[ \mathcal{V}_0 = \nabla_B \chi^a_B \nabla_A \chi_B^a + \nabla_B \chi^a_B \nabla B \chi_A^a - \frac{1}{2} \left( \nabla_A \chi_A^a + \nabla \chi_A^a \right)^2, \]  

(3.6d)

\[ \mathcal{J}_0^a = \mathcal{J}^a - \mathcal{J}_0^a, \]  

(3.6e)

\[ \mathcal{V}_{\text{int}} = \mathcal{V} - \mathcal{V}_0. \]  

(3.6f)
Introducing auxiliary sources for the canonical fields $\Pi^a_A$, $\chi^a_A$ and $\chi^{\dagger a}_A$ we can take the modified external Hamiltonian density

$$\mathcal{H}_{\text{ext}} = -\Pi^a_k a - \chi^{\dagger a}_A p^a_A - p^{\dagger a}_A \chi^a_A - A^a_j a^-$$

and this allows to factorize the interaction Hamiltonian outside the vacuum expectation value

$$Z[j^a_-, s^{\dagger a}_A, s^a_A] = \exp -i \int d^{2\omega} x \mathcal{H}_{\text{int}} \left[ A^a_-, \Pi^a_-, \chi^{\dagger a}_A, \chi^a_A, s^{\dagger a}_A, s^a_A \right] \left[ 0 \right] T^+ \exp -i \int d^{2\omega} x \mathcal{H}_{\text{ext}} \left. \right|_{k=p=p^+ = 0},$$

where $\tilde{A}^a_- = \frac{i \delta}{\delta j^a_-}$, $\tilde{\Pi}^a_- = \frac{i \delta}{\delta p^a_-}$, $\tilde{\chi}^a_A = \frac{i \delta}{\delta s^{\dagger a}_A}$. First we analyze the free generating functional

$$Z_0[j^a_-, k^a_-, p^{\dagger a}_A, p^a_A] = \left< 0 \right| T^+ \exp -i \int d^{2\omega} x \mathcal{H}_{\text{ext}} \left. \right| 0 \right>$$

$$= \exp -i \int d^{2\omega} x \int d^{2\omega} y \left[ p^a_A(x) D^\omega_F(x-y) p^a_A(y) + p^a_A(x) \partial^\omega \Delta_{\text{ML}}^\omega(x-y) j^a_-(y) 
- p_a^a(x) \chi^a_A \Delta_{\text{ML}}^\omega(x-y) j^a_-(y) - p^{\dagger a}_A(x) \chi^{\dagger a}_A \Delta_{\text{ML}}^\omega(x-y) j^a_-(y) + k^a_A(x) E^\omega(x-y) j^a_-(y) \right],$$

where $D_F(x)$ and $\Delta_{\text{ML}}(x)$ are given by Eqs. (2.14), while $E^\omega(x)$ is defined as

$$E^\omega(x) = \int \frac{d^{2\omega} k}{(2\pi)^{2\omega}} \frac{e^{-i(k\cdot(x-y))}}{\epsilon + \epsilon \text{sgn}(k^-)}.$$

Thus the free generating functional has its path-integral representation

$$Z_0[j^a_-, k^a_-, p^{\dagger a}_A, p^a_A] = N \int \mathcal{D} A^a_- \mathcal{D} \chi^{\dagger a}_A \mathcal{D} \chi^a_A \mathcal{D} \Phi^a_-$$

$$\times \exp i \int d^{2\omega} x \left( \partial_+ \chi^{\dagger a}_A \partial_- \chi^a_A + \partial_+ \chi^a_A \partial_- \chi^{\dagger a}_A + \partial_+ A^a_+ - H_0 - \mathcal{H}_{\text{ext}} \right)$$

In the complete generating functional, the exponential operator can be pushed under the sign of path integration and then easily one obtains the expected result

$$Z[s^a_A, s^{\dagger a}_A, j^a_-] = N' \int \mathcal{D} A^a_- \mathcal{D} \chi^{\dagger a}_A \mathcal{D} \chi^a_A \mathcal{D} \Phi^a_-$$

$$\times \exp i \int d^{2\omega} x \left( \partial_+ \chi^{\dagger a}_A \partial_- \chi^a_A + \partial_+ \chi^a_A \partial_- \chi^{\dagger a}_A + \partial_+ A^a_+ \Pi^a_- - H_{\text{can}} \right).$$

IV. CONCLUSIONS

In this paper we have shown how the nonAbelian couplings can be consistently quantized within the LF approach without spoiling the natural ML-prescription for the spurious poles. While we have used only formal arguments for checking the equivalence of this approach with the usual ET quantization, it will be interesting to check how it works for the explicit calculations. We think that the computation of the gauge invariant quantities like the Wilson loops will be a quite interesting cross-check. Further we expect that also other choices of gauge conditions can be similarly implemented for the LF Yang-Mills theory.

\[^5\text{This would be a generalization of the fermionic QED analysis from \[\text{[3]}.}\]
APPENDIX A: NOTATIONS

In the space-time of 2\(\omega\) dimensions we introduce the light-front notation: 2 longitudinal coordinates \(x^\pm = \frac{x^0 \pm x^{2\omega - 1}}{\sqrt{2}}\) and 2\((\omega - 1)\) transverse coordinates \(x^i\). Similarly we denote components of any vector \(V_\mu\): \(V_\pm = \frac{V_0 \pm V_{2\omega - 1}}{\sqrt{2}}, V_i\). Further, the transverse components of the gauge field potential \(A_i^a\), \(i = 1,\ldots, 2(\omega - 1)\) are parameterized by the pair of Hermitian fields \((\phi_A^a, \phi_i^a, A = 1,\ldots, \omega - 1)\) as follows:

\[
A^a_{2A} = \frac{\phi_A^a + \phi_i^a}{\sqrt{2}},
\]

\[
A^a_{2A+1} = \frac{\phi_A^a - \phi_i^a}{\sqrt{2}}.
\]  

(A1a)

(A1b)

When this notation is implemented for into the components of the gauge field \(F_{ij}^a\) we obtain

\[
F_{2A\ 2B}^a = \partial_{2A}A_{2B}^a - \partial_{2B}A_{2A}^a + gf^{abc}A_{2A}^bA_{2B}^c = \frac{1}{2} (G_{AB}^a + G_{AB}^a + G_{AB}^a),
\]

\[
F_{2A\ 2B+1}^a = \partial_{2A}A_{2B+1}^a - \partial_{2B+1}A_{2A}^a + gf^{abc}A_{2A}^bA_{2B+1}^c = \frac{i}{2} (G_{AB}^a + G_{AB}^a - G_{AB}^a),
\]

\[
F_{2A+1\ 2B+1}^a = \partial_{2A+1}A_{2B+1}^a - \partial_{2B+1}A_{2A+1}^a + gf^{abc}A_{2A+1}^bA_{2B+1}^c = \frac{1}{2} (-G_{AB}^a + G_{AB}^a + G_{AB}^a),
\]

where

\[
G_{AB}^a = \nabla_A \phi_B^a - \nabla_B \phi_A^a + gf^{abc} \phi_B^a \phi_C^b.
\]  

(A3a)

\[
G_{AB}^a = \nabla_A \phi_B^a - \nabla_B \phi_A^a + gf^{abc} \phi_A^a \phi_B^b.
\]  

(A3b)

\[
G_{AB}^a = \nabla_A \phi_B^a - \nabla_B \phi_A^a + gf^{abc} \phi_A^a \phi_B^c = (G_{AB}^a)^\dagger,
\]  

(A3c)

\[
G_{AB}^a = \nabla_A \phi_B^a - \nabla_B \phi_A^a + gf^{abc} \phi_A^a \phi_B^b = (G_{AB}^a)^\dagger,
\]  

(A3d)

\[
\nabla_A = \frac{\partial_{2A} - i \partial_{2A+1}}{\sqrt{2}}, \quad \nabla_A = \frac{\partial_{2A} + i \partial_{2A+1}}{\sqrt{2}}.
\]  

(A3e)

The above notation allows us to write the following useful relations which are used in the main text

\[
\sum_{i,j=1}^{2(\omega - 1)} (F_{ij}^a)^2 = \sum_{A,B=1}^{\omega - 1} \left[ (G_{AB}^a - G_{AB}^a) (G_{AB}^a - G_{AB}^a) + 2 (G_{AB}^a G_{AB}^a + G_{AB}^a G_{AB}^a) \right],
\]

\[
\sum_{i=1}^{2(\omega - 1)} \partial_+ A_i^a \partial_i A_a^a = \sum_{A=1}^{\omega - 1} \left[ \partial_+ \phi_A^a \nabla_A A_a^a + \partial_+ \phi_i^a \nabla_A A_a^a \right],
\]

\[
\sum_{i=1}^{2(\omega - 1)} \partial_i A_i^a D_i^{ab} A_a^b = \sum_{A=1}^{\omega - 1} \left[ \partial_+ \phi_A^a D_i^{ab} \phi_i^a + \partial_+ \phi_i^a D_i^{ab} \phi_A^a \right]
\]

\[
\sum_{i,j=1}^{2(\omega - 1)} \partial_i A_j \partial_j A_i = \sum_{A,B=1}^{\omega - 1} 2 \left[ \nabla_B \phi_A^a \nabla_B \phi_A^a + \nabla_B \phi_i^a \nabla_B \phi_i^a \right],
\]

\[
\sum_{i=1}^{2(\omega - 1)} \partial_i A_i = \sum_{A=1}^{\omega - 1} \left[ \nabla_A \phi_A^a + \nabla_A \phi_i^a \right].
\]

(A4a)

(A4b)

(A4c)

(A4d)

(A4e)

In the main text we tacitly use the convention of summing over repeating indices over all possible values of different indices, thus in the above formulas we omit the sum signs hoping this will lead to no confusion or difficulty.
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