Experimental study on gas-liquid flow characteristics of submerged air jets

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Abstract. The gas-liquid flow structure and interfacial behavior of submerged air jets were investigated experimentally using high speed digital video camera and image processing techniques. The jet pressure ratio varied from 1.8 to 4.8 in the experiment. And results from different jet nozzles were processed and compared. Statistical characteristics of the jet diameters along the axial distance were obtained and analyzed. Time series analysis was implemented to study the interface unsteadiness by calculating the gas-liquid interface deviation. The results showed that the jet diameters increase first linearly then nonlinearly and its growth rate decreases along the axial distance. The reason for the divergence between the result of this experiment and those done by other researchers was analyzed. Comparing the results of different pressure ratios and nozzle diameters, we found that larger jet pressure ratios have larger jet diameters and nozzle diameters nearly have no bearing on the distribution of dimensionless jet diameters. The interface unsteadiness in low and high pressure ratios exhibited totally distinct properties. And a minimum unsteady value was found along the axis of the air jets.

1. Introduction

Air jets submerged in liquids are quite common in many industrial applications. In chemical and metallurgical industry, such as aeration, fluidization, liquid metal stirring and gas-metal reactions, gas injections have been widely used for enhancing reaction efficiency, gas-liquid mixing, heat and mass transfer and etc. For underwater equipment applications, such as the conceptual new generation missile launching platform for submarines proposed by Yagla[1], the submerged gaseous region structure and motion are crucial for this unique design. Hence, understanding the flow structure and interface motion of submerged gas jets is relatively important[2].

Numerous previous studies[3-5] have been trying to gain a better understanding of the gas-liquid flow structure and motion at low and moderate injection pressure. Up to now, the characteristics of flow structure and interfacial behavior of submerged gas jets at low or moderate jet pressure have been widely researched and well understood. However, our understanding to the properties of submerged gas jets at high jet pressure is still obscure, because the flow structure and gas-liquid interface motion

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are quite complicated owing to the extreme turbulence and vortex. Until recent decades, increasing investigations have been conducted at high pressure submerged gas jets. Mori et al. (1982)[6] experimentally studied the bubbling/jetting transition point for nitrogen injected into a mercury bath. And the transonic flow was identified as the transition point to distinguish bubbling and jetting flow. McNallan and King (1982)[7] investigated the processes of several gas injected into several liquids. They also found that the transition point is near the sonic point. Then, using the measurements of pressure probe, $\gamma$-ray absorption and laser-Doppler anemometer, Loth and Faeth (1989)[8, 9] studied the structure and mixing properties of round gas jets at various under-expanded jetting conditions. Mean properties such as static pressure distributions, void-fraction distributions and entrainment rates were compiled and analyzed experimentally. Prediction models based on experimentally results were established and compared. Interestingly, flow widths based on void fractions exhibited unusually large flow widths, compared to typical single-phase jets. And they argued that the main reason was the multiphase mixing thick layer around. Besides, they also discovered that using intrusive probe to detect the properties of submerged gas jets may cause inaccuracy.

Recently, with the development of photographic and image processing techniques, method called flow visualization was applied by the researchers like Christopher[10] and Harby[11] to investigated the jet penetration distance, pinch-off events and the unsteadiness of the gas-liquid jet interface. Compared to the results of probe measurements, these non-invade measurement results can be more precise. Christopher considered that RT and KH instability mechanisms are both essential for driving the instability and mixing along the jet interface. Harby found a linear relationship between the dimensionless jetting half-width and axial distance. A function was delivered to predict this relationship using different coefficient for different nozzles. And the Froude number in their view has little effect on this relationship for a constant nozzle.

In this paper, compressed air with different pressure ratios injected from a vertical nozzle in stagnant water were investigated using high speed digital video camera and image processing techniques. Statistical characteristics of jet diameters along the axis of the nozzle were quantified. The influence of nozzle diameters and pressure ratios were analyzed. And the interface unsteadiness based on the calculated boundary of each frame was compared between low and high pressure jets.

2. Experimental method

2.1. Experimental apparatus

An experimental setup was applied to study the characteristics of two-phase flow from an underwater air jet. Figure 1 shows the schematic of the experimental Apparatus. Experiment was carried out in a water-filled tank (3000mm×1500mm×2000mm) with transparent windows on the sidewalls, which allows for visualization and optical supplement. The air jet was generated from a vertically upward nozzle installed on the bottom of the tank. The nozzle was located near the center of the tank and the exit plane of nozzle was kept 450 mm below the liquid surface, in order to ensure that the sidewalls had minimal effects on the flow and the camera can directly record the flow through the window. Two round nozzles made from stainless steel with inner diameters of 6 and 8.7mm were used in this investigation. All the nozzles had a constant length of 90mm and a uniform thickness of 1.5mm. The compressed air was delivered from an air line. Prior to each test, air was compressed into a gas tank (0.8m$^3$) to a high pressure (0.7Mpa), which could be regarded as a stable gas source during the test. Air volume flow rate was monitored by a pressure regulator valve and metered by a target flowmeter (SBL-BQ50CCZTPX). And the pressure and temperature of compressed air were recorded by transducers synchronously. Experimental parameters consisted stagnation pressure ($P_S$) inside the injector, hydrostatic pressure ($P_H$) in the nozzle exit plane, air temperature ($T_0$), pressure ratio ($R_P$) of stagnation pressure to hydrostatic pressure and mass flow ($Q_M$) are showed in Table 1. Here the hydrostatic pressure ($P_H$) was defined by equation (1). Pa and h refer to the barometric pressure and the constant depth of water.
\[ P_l = P_s + \rho gh = 1.01 \times 10^5 + 1000 \times 9.8 \times 0.215 \text{Pa} = 0.105 \text{MPa} \quad (1) \]

A high-speed digital video camera (NAC Memrecam HX-3) was employed in this experiment. Flow images were taken by this camera at the frame rate of 1000 fps in conjunction with a Nikon AF Nikkor 60mm f 1.8d camera lens. And the exposure time was set automatic mode to obtain distinct images. Two 80w halogen lamps were put on the same side of the camera for illuminating the jet, while the surrounding stagnant water produced a dark background. Figure 2 (a) shows a flow image taken by high-speed digital camera. It is clear that the gas flow and the stagnant water can be distinguished significantly. All the images had a resolution of 768×960 pixels, and the pixels of per millimeter could be calculated from the nozzle’s pixels on the image. Over 10 seconds of the gas-liquid flow were recorded under a steady flow rate, among which 2 seconds with 2000 images were taken to ensure a typical analysis.

![Figure 1. Schematic diagram of experimental set-up.](image)

**Table 1. Summary of experimental parameters.**

| D / mm | \( T_0 / K \) | \( P_S / \text{MPa} \) | \( P_H / \text{MPa} \) | \( R_p \) | \( Q_M / \text{g/s} \) | \( \rho / \text{kg/m}^3 \) |
|-------|--------------|----------------|----------------|-------|----------------|----------------|
| 289   | 0.185        | 0.105          | 1.8            | 147.4 | 2.24           |                |
| 289   | 0.215        | 0.105          | 2.0            | 212.2 | 2.61           |                |
| 289   | 0.295        | 0.105          | 2.8            | 368.0 | 3.58           |                |
| 289   | 0.365        | 0.105          | 3.5            | 552.7 | 4.43           |                |
| 6     |              |                |                |        |                |                |
| 289   | 0.425        | 0.105          | 4.0            | 760.7 | 5.16           |                |
| 289   | 0.485        | 0.105          | 4.6            | 983.7 | 5.89           |                |
| 289   | 0.505        | 0.105          | 4.8            | 1130.3| 6.13           |                |
| 288   | 0.190        | 0.105          | 1.8            | 360.6 | 2.61           |                |
| 288   | 0.215        | 0.105          | 2.0            | 545.8 | 3.22           |                |
| 8.7   |              |                |                |        |                |                |
| 288   | 0.305        | 0.105          | 2.9            | 882.0 | 4.07           |                |
| 288   | 0.370        | 0.105          | 3.5            | 1243.0| 4.80           |                |
| 288   | 0.425        | 0.105          | 4.0            | 1732.7| 5.65           |                |
2.2. Image processing method

As the main purpose of this work was to study the characteristics of the gas-liquid flow, images taken by camera were processed in MATLAB using the image processing module, just as the method mentioned by other researchers[12, 13] to obtain the interface of gas and liquid. Figure 2 shows the principal steps to detect the jet interface. Firstly, the image was changed into a gray image and then converted into a binary image using a threshold of pixel intensity. An appropriate threshold was selected to obtain a precise jet (white) and the background (black) area. Then, a median filter and “imfill” function were applied to the images, aiming to eliminate the noise caused by non-uniform illumination and separated gas bubbles. After that, two morphological functions called dilate and erode were used to smooth the boundary of the air jet and background. Finally, edge detection function detected the boundary and stored the coordinates in an array for further analysis. As saw in the final image of figure 2, the detected boundary (red line) agrees quite well with the original image. Namely, the interface of gas and liquid from a submerged air jet is precisely extracted.

![Image processing steps](image)

**Figure 2.** Key steps used to detect the jet boundary, (a) Raw image, (b) Gray image, (c) Binary image, (d) Median filter and “imfill”, (e) Dilate and erode, (f) Detected boundary.
3. Result and discussion

3.1. Jet diameter

Jet diameters along the axis with pressure ratio from $R_p=1.8$ to $R_p=4.8$ are shown in figure 3. Results of nozzle diameter 6 and 8.7mm are showed, respectively. Axial distance ($Y/L_q$), or can be referred to as the downstream distance to the nozzle, plots against jet diameters ($D/L_q$) defined in this paper using the 80% contour of added images. Axial distance and jet diameters are both normalized by $L_q$, so that results of different nozzle diameters can be compared. Here, $L_q$ is a geometric length scale associated with nozzle structure which for a round jet is the square root of the nozzle area [14]. Resulting from the fact that image sizes taken in experiments with different diameters are alike, the normalized axis distances in nozzle diameter $d=6$ and 8.7mm are normally unlike. The smaller nozzle diameter has a wider normalized axial distance. It can be obviously seen in figure 3 that normalized axial distance ranges of $d=6$ and 8.7mm are 0–30 and 0–52 respectively.

It is known that gas-liquid interface of submerged air jets is extremely unsteady, so instantaneous jet diameter detected from single images are meaningless [15, 16]. Therefore, statistical jet diameters are calculated by summing all the images in 2s. In the binary images, region occupied by gas is given a value of 0 and all other positions occupied by water are given a value of 1. So in the binary image, we can see that the gaseous region is black and the surrounding water is white. Before all the images are summed, the value of each pixel must be reversed. Next, values of all the pixels over time are summed and averaged, which produces an added image with the value of each pixel between 0 and 1. Thus, a gray image is generated and a color bar is added for a clearly presentation, just like the example showed in figure 4 (a) and (b). The added image generates an image with a spatially varying intensity and the spatial distribution can be distinctly identified with different colors. According to the statistical results above, statistical characteristics of jet diameter under different pressure ratios and different nozzle diameters can be analyzed.

As can be seen from figure 3, the relation that jet diameters increase with increasing axial distance is consistent for all cases. However, the growth rate of jet diameter decreases along the axis distance. Near the nozzle exit, jet diameter grows fast and the relationship of jet diameter and axis distance is almost linear. On the downstream, it increases slowly and even begins to decrease at some positions ($d=6mm$, $Y/L_q>47$). The relationship of jet diameter and axial distance described above is contradictory to the work of Tross [4] who obtained the flow widths along the axis by detecting the time-averaged void fraction using electrical conductivity probe. This difference mainly because of the fact that jet diameters reported here and the flow widths defined by Tross are only nominally similar, but they are fundamentally different. The flow widths depending on time-averaged void fraction are the radius of the gas-containing region without the bubble cloud around it. However, jet diameters calculated from added images in this paper contain both the gas-containing region and the bubble cloud around it. In the near nozzle region, the discharged flow is predominantly governed by the jet momentum force. Jet flow in this region is a continuous gas with fast jet speed and high internal pressure and its boundary layer thickness which mainly consists of bubbles and drops is relatively thin. Therefore, in this region jet diameters calculated from flow images and the flow width detected by probes are normally alike. With the development of air jet, it disintegrates at the position far from the nozzle exit, where the jet breaks up into a column of rising bubbles. This region is named for the plume regime region by other researchers [17, 18]. In this region, bubbles and drops dominated by gravitational or buoyancy effects form a thick boundary layer. This dense layer can be exhibited on flow images, but can't be detected by the probe. The difference that we contain the boundary layer in jet diameters while Tross didn’t, leads to a divergent conclusion.

The second observation is that the higher pressure ratio has a wider jet diameter, particularly in the downstream region. And low pressure ratio air jets ($R_p<3$) are more susceptible to pressure ratio than high pressure ratio air jets ($R_p>3$). As can be seen in figure 3, curves of low pressure ratio air jets generally have a wide distribution, and jet diameters at the same position obviously increase with increasing pressure ratio. However, curves of high pressure ratio air jets are relatively close. Therefore,
we can presume that the jet diameters increase with pressure ratio nonlinearly. Initially, it grows fast, while across the pressure ratio $R_p=3$, it slows down. Both in the experiment of 6 and 8.7 mm, jet diameters of $R_p=3$ do not obey the regularity above. And the reason for this discrepancy is not yet clear.

![Figure 3. Jet diameters along the axis under different pressure ratio, (a) d=6 mm, (b) d=8.7 mm.](image)

To investigate the influence of nozzle diameters on the relation of jet diameter and axial distance, normalized jet diameter versus axial distance of nozzle diameters d=6 and 8.7 mm are shown together. Seven graphs are exhibited in figure 5 (a-g) with pressure ratio varying from 1.8 to 4.8. It can be observed in the graphs in figure 5 that nozzle diameters do not have impact on the relation of dimensionless jet diameter and axial distance. Both the curves of d=6 and 8.7 mm under certain pressure ratio in the graph nearly grow in a similar path. First, the dimensionless jet diameter (D/Lq) increases linearly with the dimensionless axial distance (Y/Lq). Then, from certain position downstream, its growth becomes nonlinearity and the growth rate decreases. It is possible to suppose that a definite growth function may exist between the dimensionless jet diameter and axial distance.

![Figure 4. Spatial distribution of an added image in 2s (d=8.7 mm, R_p=4.8), (a) gray distribution image, (b) RGB distribution image with color bar.](image)
even though under different nozzle diameters. Further work is required to explore this growth function. The results reported above corroborate the findings of the previous work done by Harby et al.[11]. A linear relationship of normalized jet half widths and centerline path is also obtained from horizontal gas jets, but what they didn’t report was the nonlinear relationship downstream. A probable explanation for this might be that the downstream flow which chiefly controlled by the buoyancy force and characterized by the production of bubbles, rising vertically, which deviate from the horizontal centerline. Therefore, only the rectilinear relationship demonstrated. As to vertical gas jets, the bubbles controlled by the buoyancy force still rise and expand along the axial direction only the growth rate of jet diameter becomes moderate.
3.2. Characteristics of interface unsteady

The flow structure and the process of a submerged air jets are essentially unsteady and turbulent. The unsteadiness of the submerged air jets is accompanied by appreciable pressure fluctuations in the flow passage and beyond the jet interface. By detecting the pressure fluctuations with pressure probes, prior studies have confirmed the innate unsteadiness of gas-liquid jet interface which is not normally found in typical single-phase jets[8].

It has been established in section 2.2 that the gas-liquid interface of each frame is detected and positioned for 2 seconds. Hence, a time series analysis of the right interface unsteadiness is displayed in figure 6 with nozzle diameter d=8.7mm. As showed in figure 6 and figure 7, normalized deviations at different position along the axis are illustrated for Rp= 1.8, 2.0, 4.6 and 4.8. The normalized deviation is calculated by equation (2). In this equation, Xi is the jet boundary position, X is the mean jet boundary position and D refers to the mean jet diameter. The deviation of relative deviation (RD) which is commonly used is divided by the mean value. But the deviation of normalized deviation (ND) defined in this paper is divided by mean jet diameter not the right mean boundary. It is explained by the fact that that the deviations between different pressure ratio can be compared by using the normalized deviation (ND). Thus, all the deviations at a certain position under a certain pressure ratio are normalized by the mean jet diameters. The normalized deviation \( \delta_i(y) \) denotes unsteadiness, or fluctuations of the jet boundary for the chosen axial positions. Eight typical positions are chosen for analysis under a certain pressure ratio, from Y/Lq=3.5 to 28.1.

\[
\delta_i(y) = \frac{(X_i - X)}{D} \tag{2}
\]

Time series interface deviations of low pressure ratio \((R_P=1.8 \text{ and } 2.0)\) and high pressure ratio \((R_P=4.6 \text{ and } 4.8)\) vertical air jets are shown in figure 6 and figure 7. The results exhibited here are the deviations of right boundary. As can be observed in figure 6 (a) and (b), normalized deviations are relatively large. The deviation gradually increases along the axis from the nozzle exit to downstream, while it decreases as the pressure ratio being larger. It means that the near nozzle region is more stable than the downstream in low pressure ratio air jets and larger pressure ratio yields steadier jets. And this phenomenon is more clearly in figure 8 (a). The mean absolute deviation (MAD) is plotted against the positions. It is because that the momentum force is gradually weakened by mixing drops which have eroded the flow pattern. Losing the power force of momentum, the interface of the jet can be readily influenced by the waves propagating from upstream or surface. In addition, positions where deviations are extremely large indicate the so called pinch-off phenomenon[10,19], such as the certain positions at 271ms, 698ms, 986ms and 1630ms in figure 6 (a). It is evident in the graphs that higher pressure ratio have less pinch-off in low pressure ratio jets. And high pressure ratio jets, no pinch-off is observed.
Figure 6. Time series normalized deviations along certain axis positions of low pressure ratio jet, (a) \( R_P = 1.8 \), (b) \( R_P = 2.0 \).

Figure 7. The mean absolute deviation along axis positions, (a) low pressure ratio jet, (b) high pressure ratio jets.

Dislike the features exhibited in low pressure ratio jet, the maximum deviation of high pressure ratio jet shown in figure 7 (a) and (b) is close to the nozzle, and then it decreases along the axis and later increases downstream. Between \( Y/L_q = 14.0 \) and 17.5, the deviation reaches to a minimum value. And the position of this minimum value is somewhat related to the pressure ratio. When \( R_P = 4.6 \) and
4.8, as can be seen from the graph in figure 8 (b), the minimum position is Y/Lq=14.0 and 17.5 pixels respectively. As to the reason why the maximum deviation of a supersonic jet is near the nozzle may be the existence of the shock cell in under-expanded jets[9, 20]. When the gas jet pressure is high near the nozzle, pressure equalization is achieved through a complex external expansion region containing repeating shock cells, which have both compression and expansion waves[20]. The compression and expansion waves cause huge unsteadiness of the boundary. Thus, the deviation near the nozzle is incredibly large. However, when the jet pressure near the nozzle exit is low, pressure disturbances can propagate upstream of the passage to equilibrate the exit pressure with the lower ambient pressure. So deviations of low pressure ratio jets are naturally smaller than the high pressure ratio jets. Now, we can explain the reason why position of minimum deviation appears between Y/Lq=14.0 and 17.5, namely, in the intermediate region of the flow. It is due to the combined effect of two factors: the existence of the shock cell near nozzle and the gradually diminished momentum force along the stream wise.

**Figure 8.** Time series normalized deviations along certain axis positions of high pressure ratio jet, (a) R_P =4.6, (b) R_P =4.8.

4. conclusion
An experimental study on submerged air jets was performed to investigate jet diameters along axis and unsteadiness of gas-liquid interface. The tests included low pressure air jets with pressure ratio, ranging from R_P=1.8 to R_P=3, and high pressure air jets with pressure ratio, ranging from R_P=3.5 to R_P=4.8. And two different diameter nozzles, d=6 and 8.7mm, were used and compared. Flow images were recorded by high speed digital video camera and processed using image processing method. The main conclusions of this work are:
1. Jet diameter increases with increasing axial distance, but its growth rate decreases along the axis distance. Near the nozzle exit, jet diameter grows fast and the relationship of jet diameter and axis distance is almost linear. On the downstream, it increases slowly and even starts to decrease at some positions. The relationship of jet diameter and axis distance detected by flow visualization in this paper is different from the result obtained by Tross using probes. The difference that we contain the boundary layer in jet diameters while Tross didn’t, leads to this divergence.

2. A larger pressure ratio air jet generally has a wider jet diameter. Jet diameters of low pressure ratio air jets (R_p <3) are more sensitive to pressure ratio than those in high pressure ratio air jets (R_p>3).

3. From the result of this experiment, we can see that relations of dimensionless jet diameter and axial distance of different nozzles under certain pressure ratio nearly grow in a similar path. And a growth function of certain pressure ratio was assumed existence without the influence of nozzle diameters.

4. The unsteadiness of gas-liquid interface is exhibited by calculated deviation. In low pressure ratio air jets, the minimum deviation is close to the nozzle exit. And the deviation which means the interface unsteadiness gradually increases along the axis from the nozzle exit to downstream. Pinch-off phenomenon reported by other researchers also can be clearly seen in the time series interface. The minimum deviation in a high pressure ratio air jet is in the middle region of the jet. And this position is a bit concerned with pressure ratio. This mainly because of the combined effect of these two factors: the existence of the shock cell near nozzle and the gradually diminished momentum force along the stream wise. The difference of minimum position between low and high pressure ratio air jets mainly due to the complex interaction of shock cell, momentum force and buoyancy force.

5. The shock cell and pinch-off mentioned in some literatures is illustrated in the time series interface deviation, which has extremely large deviation in the near nozzle region in high pressure ratio air jets and downstream in the low pressure air jets, respectively.

5. References

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