Current status of NLTE analysis of stellar atmospheres

Jiří Kubát

Abstract Various available codes for NLTE modeling and analysis of hot star spectra are reviewed. Generalizations of standard equations of kinetic equilibrium and their consequences are discussed.

1 Importance of radiation

Radiation is not only an information source about stellar atmospheres, it has also the ability to interact with the matter in the atmosphere and to alter its state.

Each photon has a momentum \( h\nu/c \). Consequently, when it is absorbed, the absorbing ion receives this amount of momentum. If it is scattered, the atom may receive from no momentum (if radiation is scattered in the forward direction) to \( 2h\nu/c \), if radiation is scattered backwards. This momentum gain is redistributed to other particles by elastic collisions and, as a consequence, acceleration of matter by radiation occurs.

The energy of a photon \((h\nu)\) can also be transferred to matter. In the case of ionization, part of this energy is used to ionize the atom and the rest goes to kinetic energy of electrons, \( h\nu \rightarrow h\nu_{\text{ion}} + \frac{1}{2}m_e v_e^2 \). In the case of excitation, all photon energy is used for this process, \( h\nu \rightarrow h\nu_{\text{exc}} \). In the case of the free-free absorption the photon energy causes increase of the electron kinetic energy, \( h\nu \rightarrow \frac{1}{2}m_e \Delta v_e^2 \). Reverse processes (recombination, deexcitation, and free-free emission) cause release of a photon and lowering the atomic internal or electron kinetic energy (or both). In the above mentioned cases of bound-free and free-free transitions there is an energy exchange with electron kinetic energy. In this case, subsequent elastic collisions reestablish the equilibrium velocity distribution with a slightly different temperature. Consequently, the thermal energy increases or decreases and we talk about radiative
heating or cooling. If the energy exchange is with atomic internal energy, a change of excitation or ionization balance occurs. If the radiation changes of excitation and ionization states dominate, then we have to take into account these effects explicitly and we have to use the NLTE approximation and solve the kinetic equilibrium equations.

2 NLTE model atmosphere codes

The solution of the kinetic equilibrium equations\(^1\) (see Hubeny and Mihalas, 2014) for given radiative and collisional rates is a relatively simple task, it is just a solution of set of linear equations. The task of NLTE modelling is more difficult. It is the simultaneous solution of the equations of kinetic equilibrium with other equations describing the stellar atmosphere. These equations form a set of nonlinear integro-differential equations.

The basic task is to solve the equations of kinetic equilibrium together with the radiative transfer equation to determine simultaneously the level population numbers \(n_i\) and the radiation field. In this case, temperature \(T(r)\), density \(\rho(r)\), and velocity \(v(r)\) are fixed. This is the standard NLTE task, which may be applied also to trace element NLTE calculations.

The task of calculation of a static NLTE model atmosphere is more complicated. It means adding two equations to the set of simultaneously solved equations, namely the equation of hydrostatic equilibrium and the equation of radiative equilibrium. Consequently, temperature \(T(r)\) and density \(\rho(r)\) are not fixed, but they are consistently calculated. Then the results of solution of this set of nonlinear equations are the NLTE level populations, radiation field, temperature, total density, and electron density. Adding the two structural equations causes significant slowing down the convergence.

If we replace the hydrostatic equilibrium equation by the equation of motion and add the continuity equation, we may solve hydrodynamic NLTE model atmospheres and determine also the velocity field \(v(r)\). However, this full task is too complicated even for the case of 1-D atmospheres, so restricted problems are being usually solved.

Different codes solve different sets of equations using different numerical approaches. Below we list some representative examples of such codes. Development of any NLTE model atmosphere code is a complicated and time consuming task, which usually takes several years or even decades. The description of such code is then usually spread over many publications, if they exist at all. In other cases, the most relevant information can appear only at a www page, which may be even variable in time. Therefore it is sometimes very difficult to pick up only a single clear reference to a particular code, where everything important about the code is described. References in this paper were found with a belief that they give the most relevant information.

\(^1\) These equations are often referred to as the equations of statistical equilibrium. Here we use the term kinetic equilibrium equations as in the textbook [Hubeny and Mihalas, 2014].
appropriate reference to particular codes. The author apologizes if he omitted references, which describe the code better. The url of the www page was added in the footnote for all cases when it was known to the author.

A list of model atmosphere, NLTE, and radiative transfer codes was compiled also by [Hummer and Hubeny](1991) and [Sakhibullin](1996), we refer the interested reader there.

### 2.1 NLTE problem for a given structure

There are many codes which solve the NLTE problem for a given (i.e. fixed) atmospheric structure. Here we list several of them, each of which represents possible method of a solution of the problem.

One of the first codes which were able to solve the NLTE problem started to be developed in mid-60s. It was the code PANDORA (see [Avrett and Loeser](2003)). This code is based on the equivalent two-level atom approach and uses 1-D radiative transfer. The code LINEAR ([Auer et al](1972)) uses the complete linearization method for solution of the multilevel line formation. The code MULTI ([Carlsson](1986)) uses the accelerated lambda iteration method for solution of 1-D NLTE multilevel problems both in static and moving atmospheres in plane-parallel approximation. [Rybicki and Hummel](1991, 1992, 1994) developed a method and a computer code MALI for multilevel radiative transfer using accelerated lambda iteration method. The code described by [Auer et al](1994) and [Fabiani Bendicho et al](1997) solves 2-D multilevel NLTE radiative transfer problem, using again the accelerated lambda iteration method.

Various aspects of solution of the NLTE problem for trace elements together with couple of codes were discussed in detail at the summer school “NLTE Line Formation for Trace Elements in Stellar Atmospheres” ([Monier et al](2010)).

### 2.2 Static NLTE model atmospheres

The pioneering work on the complete linearization method by [Auer and Mihalas](1969) led to a development of a computer code for calculations of static plane parallel NLTE model atmospheres, which was later described in [Mihalas et al](1975). An independent code using the same method was developed by [Kudritzki](1976). The most sophisticated NLTE static plane-parallel model atmosphere code is the code TLUSTY ([Hubeny](1973, 1988)), which was initially developed using the complete linearization method [Hubeny](1973, 1988). Now it combines the latter method with the accel-
erated lambda iteration method (Hubeny and Lanz, 1995) and enables treatment of NLTE line blanketing using the method of superlevels and superlines. This code is accompanied with the code SYNspec, which solves the radiative transfer equation including precalculated NLTE populations for given model atmosphere in detail.

Another code calculating NLTE line blanketed model atmosphere code is the code PRO2 belonging to the Tübingen NLTE Model Atmosphere Package (TMAP, Werner and Dreizler, 1999; Werner et al, 2003), which focuses on modeling the atmospheres of high gravity stars (e.g. white dwarfs) in NLTE. We should also mention a multi purpose model atmosphere code PHOENIX, which started as a combination of two original codes, for cool stars (Allard, 1990), and for novae atmospheres (Hauschildt).[135x626]

Static spherically symmetric model atmospheres were first calculated using a complete linearization method by Mihalas and Hummer (1974). An independent code using the same method was developed by Gruschinske (1978). Another independent computer code ATA uses the accelerated lambda iteration method and combines it with the linearization method to calculate static NLTE model atmospheres of spherically symmetric stellar atmospheres (Kubát, 1994, 1996, 1997, 2001, 2003).

2.3 NLTE wind model codes

There exist several codes, which solve the NLTE model of the wind assuming given density and velocity structure. These codes solve simultaneously the equations of kinetic equilibrium and the radiative transfer equation, which is mostly treated using the Sobolev approximation. In some codes the more exact comoving frame solution of the radiative transfer in spectral lines is used. The continuum radiative transfer may be treated as in the static case due to weak dependence of the opacity on frequency for continuum transitions.

The code CMFGEN (Hillier, 1987, 1990; Hillier and Miller, 1998; Busche and Hillier, 2005) solves the equations of kinetic equilibrium and radiative transfer equation in the comoving frame. The velocity field is assumed to follow the $\beta$-law. In addition, temperature structure of the models can be iteratively determined. This code may be considered as one of the top codes for solution of this task.

The code PoWR (Hamann, 1985; Hamann and Gräfenet, 2004) solves also the equations of kinetic equilibrium and radiative transfer equation in the comoving

---

5 http://nova.astro.umd.edu/Synspec49/synspec.html
6 http://astro.uni-tuebingen.de/~TMAP/
7 http://www.hs.uni-hamburg.de/EN/For,ThA/phoenix/
8 http://perso.ens-lyon.fr/france.allard/
9 http://www.asu.cas.cz/~kubat/ATA/
10 http://kookaburra.phyast.pitt.edu/hillier/web/CMFGEN.htm
11 http://www.astro.physik.uni-potsdam.de/~wrh/PoWR/
frame. The code was primarily designed for Wolf-Rayet stars atmosphere modelling, however, nowadays it is also being applied to other types stars with expanding atmospheres (e.g. Surlan et al. 2013).

Similarly the code FASTWIND (Santolaya-Rey et al. 1997; Puls et al. 2003) solves the NLTE model for given velocity and density structure and has been applied to many studies. On the other hand, the code WM-basic (e.g. Pauldrach 1987; Pauldrach et al. 1986, 2012) solves line blocked and blanketed NLTE model of stellar winds and also their stationary hydrodynamic stratification.

The ISA-WIND code (de Koter et al. 1993) solves the equations of statistical equilibrium and the radiative transfer equation in Sobolev approximation for given velocity, density, and temperature structure.

There are also several codes focused on modelling supernovae, like the code HYDRA (Höflich 2003), which combines radiative hydrodynamics and NLTE radiative transfer. NLTE radiative transfer in supernovae is solved using the Monte Carlo method by Kromer and Sim (2009). The Monte Carlo method was used for NLTE calculations in circumstellar disks in the code HDUST by Carciofi and Bjorkman (2006, 2008).

2.4 Hydrodynamic models with NLTE

The most important task in stellar wind modelling is consistent determination of the radiative force, which drives the wind. Usually the so-called CAK approximation is being used, where the radiative force is expressed with the help of three parameters $k$, $\alpha$, and $\delta$ (Castor et al. 1975; Abbott 1982). However, this approach is far from consistent even if the parameter $k$ is replaced by the more appropriate parameter $Q$ introduced by Gayley (1995). There have been several attempts to improve calculation of the radiative force using detailed list of lines. First such calculations were done by Abbott (1982). Consistent calculation of the radiative force is enabled by the code WM-basic (see Pauldrach et al. 2012, and references therein). Monte Carlo calculations of the radiative force using NLTE radiative transfer with the ISA-WIND code were done by Vink et al. (1999). Recently, Krtiška and Kubát (2004, 2010) developed a code for consistent solution of wind hydrodynamic equations including equations of kinetic equilibrium.

3 Generalized kinetic equilibrium

Currently the most frequently applied mode of NLTE calculations is the simultaneous solution of static (i.e. for macroscopic velocity $v = 0$) and stationary ($\partial / \partial t = 0$) equations of kinetic equilibrium and the radiative transfer equation. This basic set of

---

12 http://www.usm.uni-muenchen.de/people/adi/adi.html
equations may be extended by other constraint equations for determination of temperature, density, and velocity. However, there are many possible generalizations which go beyond this “standard” task. More processes than simple one-electron excitation and ionization may be included, and the assumption of static stationary medium in equilibrium may be relaxed.

### 3.1 Additional processes in the equations of kinetic equilibrium

A standard model atom consists of a number of energy levels, among which collisional and radiative (both allowed and forbidden) transitions occur. Usually, only transitions between levels of a particular ion and ionization transitions from these levels to the ground level of the next higher ion are considered. This set of atomic levels and transitions is usually sufficient for calculations of NLTE model atmospheres of hot stars.

However, there exist more processes which may change the state of the atoms. As an example we may consider Auger ionization, which occurs as a consequence of a strong X-ray radiation.

Strong X-rays (or collisions) may expell an inner-shell electron. This is followed by fluorescence or by another ionization. The equations of kinetic equilibrium obtain additional term \( n_i \sum_{j \neq i} R^\text{Auger}_{ij} \), which describes this process,

\[
-n_i \sum_{j \neq i} (R_{ij} + C_{ij}) - n_i \sum_{j > i} R^\text{Auger}_{ij} + \sum_{j \neq i} n_j (R_{ji} + C_{ji}) = 0
\]  

The Auger ionization is considered as a two-electron ionization process. States with inner-shell vacancy are not explicitly included in the equations of kinetic equilibrium. This form of the kinetic equilibrium equations was included to wind ionization calculations by [Krtička and Kubát (2009)](#).  

### 3.2 Full kinetic equilibrium equations

The full form of the equations of kinetic equilibrium (in the classical limit) reads ([Mihalas 1978, Eq. 5-48]),

\[
\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \nu) = \sum_{j \neq i} (n_j P_{ji} - n_i P_{ij}).
\]

As discussed in [Kubát 2013](#), the left hand side is usually neglected. However, it may become important for dynamical atmospheres. Non-equilibrium hydrogen ionization using the left hand side of the equations of kinetic equilibrium was solved by [Leenaarts et al. 2007](#). The time derivative is important if relaxation timescale is
longer than dynamic timescales. As examples may serve dynamic ionization of the solar chromosphere (Carlsson and Stein, 2002) or rapidly expanding atmospheres of supernovae (Utrobin and Chugai, 2005). This list of examples is far from complete and has to serve as an indication of the importance of the full treatment of the equations of kinetic equilibrium.

3.3 Non-Maxwellian velocity distribution

The presence of non-thermal electrons, which may happen, for example, as a consequence of solar flares, is a reason why collisional transitions do not support the system to evolve towards equilibrium distribution of excitation and ionization states. They rather cause that the populations differ from their equilibrium values. In this case, we may rewrite the equations of kinetic equilibrium as

\[ -n_i \sum_{j \neq i} (R_{ij} + C_{ij} + C_{nt}^{ij}) + \sum_{j \neq i} n_j (R_{ji} + C_{ji}) = 0 \]  

(2)

where the term \( n_i C_{nt}^{ij} \) describes the non-thermal collisional excitation. This type of equations was used for inclusion of electron beams in solar flares (Kašparová and Heinzel, 2002). To include the non-thermal collisional rates consistently, we have to solve the kinetic equation for electrons instead of the assumption of equilibrium distribution of their velocities.

3.4 Polarization

A significantly more general case is the case when we include polarized radiation and polarization of atomic levels. Then the radiation is described using the Stokes vector for intensities

\[ I = (I, Q, U, V)^T. \]  

(3)

The radiative transfer equation becomes

\[ \frac{dI}{ds} = \eta - \kappa I, \]  

(4)

where the emission coefficient \( \eta \) is a vector quantity and the opacity \( \kappa \) is a tensor quantity.

Instead of the equations of kinetic equilibrium we have to solve the more general density matrix evolution equation,

\[ \frac{d\rho^K}{dr} = 0 \]  

(5)
which, in addition to classical occupation numbers, solves also for coherences between them. For more details see Štěpán and Trujillo Bueno (2013) and references therein.

4 Summary

The necessity to consider the equations of kinetic equilibrium (the NLTE approach) is a consequence of radiation-matter interaction. There exist many codes which can handle this relatively complicated task. Not all codes which claim “NLTE” solve the same task, one has always to distinguish between codes for model atmosphere calculations and codes for solution of the restricted NLTE problem (radiative transfer + kinetic equilibrium). The codes which calculate synthetic spectrum are much simpler. They may enable input of NLTE population numbers, but their task is just the formal solution of the radiative transfer equation (solution for given opacity and emissivity). There is also a number of codes which handle results of the previously mentioned codes in a user-friendly manner.

Although NLTE model atmospheres and NLTE line formation problems offer results, which are much closer to reality than those using pure LTE approach, they are not the final step in generalization. Much more has still to be done to include radiation-matter interactions properly.

Acknowledgements The author would like to thank Dr. Ewa Niemczura for inviting him to the Spring School and he would also like to apologize her for the delay in delivering manuscripts. He is also grateful to both referees for their invaluable comments. This work was partly supported by the project 13-10589S of the Grant Agency of the Czech Republic (GA ČR).

References

Abbott DC (1982) The theory of radiatively driven stellar winds II - The line acceleration. Astrophys. J.259:282–301, DOI 10.1086/160166
Allard F (1990) Model atmospheres for M dwarfs. PhD thesis, Centre de Recherche Astrophysique de Lyon
Auer L, Fabiani Bendicho P, Trujillo Bueno J (1994) Multidimensional radiative transfer with multilevel atoms: I. ALI method with preconditioning of the rate equations. Astron. Astrophys.292:599–615
Auer LH, Mihalas D (1969) Non-LTE model atmospheres. III. A complete-linearization method. Astrophys. J.158:641–655, DOI 10.1086/150226
Auer LH, Heasley JN, Milkey RW (1972) A computational program for the solution of non-LTE transfer problems by the complete linearization method. Contributions from the Kitt Peak National Observatory 555
Avrett EH, Loeser R (2003) Solar and stellar atmospheric modeling using the pandora computer program. In: Piskunov N, Weiss WW, Gray DF (eds) Modelling of Stellar Atmospheres, IAU Symposium, vol 210, p 21
Current status of NLTE analysis of stellar atmospheres

Busche JR, Hillier DJ (2005) Spectroscopic effects of rotation in extended stellar atmospheres. Astron. J.129:454–465, DOI 10.1086/426362

Carciofi AC, Bjorkman JE (2006) Non-LTE Monte Carlo radiative transfer. I. The thermal properties of keplerian disks around classical Be stars. Astrophys. J.639:1081–1094, DOI 10.1086/499483

Carciofi AC, Bjorkman JE (2008) Non-LTE Monte Carlo radiative transfer. II. Nonisothermal solutions for viscous keplerian disks. Astrophys. J.684:1374–1383, DOI 10.1086/589875

Carlsson M (1986) A computer program for solving multi-level non-LTE radiative transfer problems in moving or static atmospheres. Uppsala Astronomical Observatory Reports 33

Carlsson M, Stein RF (2002) Dynamic hydrogen ionization. Astrophys. J.572:626–635, DOI 10.1086/340293

Castor JJ, Abbott DC, Klein RI (1975) Radiation-driven winds in of stars. Astrophys. J.195:157–174, DOI 10.1086/153315

Fabiani Bendicho P, Trujillo Bueno J, Auer L (1997) Multidimensional radiative transfer with multilevel atoms. II. The non-linear multigrid method. Astron. Astrophys.324:161–176

Gayley KG (1995) An improved line-strength parameterization in hot-star winds. Astrophys. J.454:410, DOI 10.1086/191187

Grashinske J (1978) Ausgedehnte statische Modell-Photosphaeren heisser Sterne. PhD thesis, Universität Kiel

Hamann WR (1985) Computed He II spectra for Wolf-Rayet stars. Astron. Astrophys.145:443–448

Hamann WR, Gräfener G (2004) Grids of model spectra for WN stars, ready for use. Astron. Astrophys.427:697–704, DOI 10.1051/0004-6361:20040506

Hauschildt P (1991) Modeling the extended atmospheres of WN stars. Astron. J. Suppl. Ser.63:947–964, DOI 10.1086/191187

Hubeny I (1975) Improved complete-linearization method for the solution of the non-LTE line transfer problem. Bulletin of the Astronomical Institutes of Czechoslovakia 26:38–47

Hubeny I (1988) A computer program for calculating non-LTE model stellar atmospheres. Computer Physics Communications 52:103–132, DOI 10.1016/0010-4655(88)90177-4

Hubeny I, Lanz T (1995) Non-LTE line-blanketed model atmospheres of hot stars. 1: Hybrid complete linearization/accelerated lambda iteration method. Astrophys. J.439:875–904, DOI 10.1086/175226

Hubeny I, Mihalas D (2014) Theory of Stellar Atmospheres. Princeton University Press, Princeton and Oxford, in press

Hummer DG, Hubeny I (1991) Computer codes for stellar atmospheric modeling. In: Crivellari L, Hubeny I, Hummer DG (eds) Stellar Atmospheres - Beyond Classical Models, Kluwer Acad. Publ., Dordrecht, NATO ASI C, vol 341, p 119

Kasparová J, Heinzel P (2002) Diagnostics of electron bombardment in solar flares from hydrogen balmer lines. Astron. Astrophys.382:688–698, DOI 10.1051/0004-6361:20011599

de Koter A, Schmutz W, Lamers HJGLM (1993) A fast non-LTE code for expanding atmospheres - a test of the validity of the Sobolev approximation. Astron. Astrophys.277:561

Kromer M, Sim SA (2009) Time-dependent three-dimensional spectrum synthesis for type Ia supernovae. Mon. Not. Roy. Astron. Soc.398:1809–1826, DOI 10.1111/j.1365-2966.2009.15256. x
Krtička J, Kubát J (2004) NLTE models of line-driven stellar winds. I. Method of calculation and first results for O stars. Astron. Astrophys. 417:1003–1016, DOI 10.1051/0004-6361:20034030
Krtička J, Kubát J (2009) NLTE models of line-driven stellar winds - III. Influence of X-ray radiation on wind structure of O stars. Mon. Not. Roy. Astron. Soc. 394:2065–2079, DOI 10.1111/j.1365-2966.2009.14457.x
Krtička J, Kubát J (2010) Comoving frame models of hot star winds. I. Test of the Sobolev approximation in the case of pure line transitions. Astron. Astrophys. 519:A50, DOI 10.1051/0004-6361/201014111
Kubát J (1994) Spherically symmetric model atmospheres using approximate lambda operators I: First results for static NLTE atmospheres. Astron. Astrophys. 287:179–190
Kubát J (1996) Spherically symmetric model atmospheres using approximate lambda operators II. Simple method for calculation of both plane-parallel and spherically symmetric static model atmospheres. Astron. Astrophys. 305:255–264
Kubát J (1997) Spherically symmetric model atmospheres using approximate lambda operators III. The equations of statistical equilibrium with occupation probabilities. Astron. Astrophys. 326:277–286
Kubát J (2001) Spherically symmetric model atmospheres using approximate lambda operators IV. Computational details of the thermal balance method. Astron. Astrophys. 366:210–214, DOI 10.1051/0004-6361:2000102
Kubát J (2003) Calculation of spherically symmetric NLTE model atmospheres using ALI and a thermal balance method. In: Piskunov N, Weiss WW, Gray DF (eds) Modelling of Stellar Atmospheres, IAU Symposium, vol 210, p A8
Kubát J (2013) Basics of the NLTE physics. In: Niemczura et al (2013), these proceedings
Kudritzki RP (1976) Non-LTE model atmospheres of subluminous O-stars. Astron. Astrophys. 52:1–21
Leenaarts J, Carlsson M, Hansteen V, Rutten RJ (2007) Non-equilibrium hydrogen ionization in 2D simulations of the solar atmosphere. Astron. Astrophys. 473:625–632, DOI 10.1051/0004-6361:20078161
Mihalas D (1978) Stellar atmospheres, 2nd edn. W. H. Freeman & Co., San Francisco
Mihalas D, Hummer DG (1974) Theory of extended stellar atmospheres. I- Computational method and first results for static spherical models. Astrophys. J. Suppl. Ser. 28:343–372, DOI 10.1086/190322
Mihalas D, Heasley JN, Auer LH (1975) A non-lte model stellar atmosphere computer program. NASA STI/Recon Technical Report N 76:30,128
Monier R, Smalley B, Wahlgren G, Stee P (eds) (2010) Non-LTE Line Formation for Trace Elements in Stellar Atmospheres, EAS Publications Series, vol 43
Niemczura E, Smalley B, Pych W (eds) (2013) Spring School of Spectroscopic Data Analyses, GeoPlanet: Earth and Planetary Sciences, Springer Verlag, Berlin, these proceedings
Pauldrach A (1987) Radiation driven winds of hot luminous stars. III - Detailed statistical equilibrium calculations for hydrogen to zinc. Astron. Astrophys. 183:295–313
Pauldrach A, Puls J, Kudritzki RP (1986) Radiation-driven winds of hot luminous stars - Improvements of the theory and first results. Astron. Astrophys. 164:86–100
Pauldrach AWA, Vanbeveren D, Hoffmann TL (2012) Radiation-driven winds of hot luminous stars XVI. Expanding atmospheres of massive and very massive stars and the evolution of dense stellar clusters. Astron. Astrophys. 538:A75, DOI 10.1051/0004-6361/201117621
Puls J, Urbanova MA, Venero R, Repolust T, Springmann U, Jokuthy A, Mokiem MR (2005) Atmospheric NLTE-models for the spectroscopic analysis of blue stars with winds. II. Line-blanketed models. Astron. Astrophys. 435:669–698, DOI 10.1051/0004-6361:20042365
Rybicki GB, Hummer DG (1991) An accelerated lambda iteration method for multilevel radiative transfer I - Non-overlapping lines with background continuum. Astron. Astrophys. 245:171–181
Rybicki GB, Hummer DG (1992) An accelerated lambda iteration method for multilevel radiative transfer II - Overlapping transitions with full continuum. Astron. Astrophys. 262:209–215
Current status of NLTE analysis of stellar atmospheres

Rybicki GB, Hummer DG (1994) An accelerated lambda iteration method for multilevel radiative transfer III. Noncoherent electron scattering. Astron. Astrophys. 290: 553–562

Sakhibullin NA (1996) Computer codes for stellar atmospheres analysis used in the CIS. In: Adelman SJ, Kupka F, Weiss WW (eds) Model Atmospheres and Spectrum Synthesis, Astronomical Society of the Pacific Conference Series, vol 108, p 207

Santolaya-Rey AE, Puls J, Herrero A (1997) Atmospheric NLTE-models for the spectroscopic analysis of luminous blue stars with winds. Astron. Astrophys. 323: 488–512

Štěpán J, Trujillo Bueno J (2013) PORTA: A three-dimensional multilevel radiative transfer code for modeling the intensity and polarization of spectral lines with massively parallel computers. Astron. Astrophys. 557: A143, DOI 10.1051/0004-6361/201321742

Šurlan B, Hamann WR, Aret A, Kubát J, Oskinova LM, Torres AF (2013) Macroclumping as solution of the discrepancy between Hα and P V mass loss diagnostics for O-type stars. Astron. Astrophys. 559: A130

Utrobin VP, Chugai NN (2005) Strong effects of time-dependent ionization in early SN 1987A. Astron. Astrophys. 441: 271–281, DOI 10.1051/0004-6361:20042599

Vink JS, de Koter A, Lamers HJGLM (1999) On the nature of the bi-stability jump in the winds of early-type supergiants. Astron. Astrophys. 350: 181–196

Werner K, Dreizler S (1999) The classical stellar atmosphere problem. Journal of Computational and Applied Mathematics 109: 65–93

Werner K, Deetjen JL, Dreizler S, Nagel T, Rauch T, Schuh SL (2003) Model photospheres with accelerated lambda iteration. In: Hubeny I, Mihalas D, Werner K (eds) Stellar Atmosphere Modeling, Astronomical Society of the Pacific Conference Series, vol 288, p 31