Relativistic effects in the search for high density symmetry energy

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Intermediate energy heavy ion collisions open the unique possibility to explore the Equation of State (EOS) of nuclear matter far from saturation, in particular the density dependence of the symmetry energy. Within a relativistic transport model it is shown that the isovector-scalar $\delta$-meson, which affects the high density behavior of the symmetry energy density, influences the dynamics of heavy ion collisions in terms of isospin collective flows. The effect is largely enhanced by a relativistic mechanism related to the covariant nature of the fields contributing to the isovector channel.

The elliptic flows of nucleons and light isobars appear to be quite sensitive to microscopic structure of the symmetry term, in particular for particles with large transverse momenta, since they represent an earlier emission from a compressed source. Thus future more exclusive experiments should be able to set stringent constraints on the density dependence of the symmetry energy far from ground state nuclear matter.

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The high density behaviour of nuclear symmetry energy $E_{\text{sym}}(\rho_B)$ is very important for understanding many interesting astrophysical phenomena, but it is absolutely not constrained by the predictions from several relativistic and non-relativistic [1,2] models of nuclear matter. The results can be roughly classified into two groups, i.e., one where the $E_{\text{sym}}(\rho_B)$ rises, Asy-stiff, and one in which it falls with increasing density, Asy-soft [3]. An increasing $E_{\text{sym}}(\rho_B)$ leads to a more proton-rich neutron star whereas a decreasing one would make it more pure in neutron content. As a consequence the chemical composition and cooling mechanism of protoneutron stars [4,5], mass-radius correlations [8,9], critical densities for kaon condensation in dense stellar matter [6,7] as well as the possibility of a mixed quark-hadron phase [10] in neutrons stars will all be rather different. It has recently been argued by means of simple thermodynamics considerations that the onset of the quark phase has a strong sensitivity to the behaviour of $E_{\text{sym}}(\rho_B)$ even for not very large asymmetries [11].

The search for $E_{\text{sym}}(\rho_B)$ around saturation density has driven a lot of theoretical and experimental efforts. It seems to be rather well established that heavy-ion collisions (HIC) at cyclotron energies can give the possibility to extract some information on the symmetry term of the nuclear Equation of State (EOS) in region below and/or slightly above the normal density [3,12–16]. On the other hand it is quite desirable to get information on the symmetry energy at higher density, where furthermore we cannot have complementary investigations from nuclear structure like in the case of the low density behaviour. Indeed HIC provide a unique way to create asymmetric matter at high density in terrestrial laboratories. Calculations within transport theory show that HICs around 1AGeV allow to reach a transient state of matter with more than twice the normal baryon density. Moreover, although the data are mostly of inclusive type (and the colliding nuclei not very neutron rich), quite clearly a dependence of some observables on charge asymmetry is emerging.

In this paper we show that a relativistic description of the nuclear mean field can account for an enhancement of isospin effects during the dynamics of heavy-ion collision. In particular future experiments with radioactive should be able to provide information on the vector part of isovector mean fields from collective flow analyses.

The isospin dependence of collective flows has been already discussed in a non-relativistic framework [12,17] using very different EOS with opposite be-

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haviours of the symmetry term at high densities, increasing repulsion (*asy-stiff*) vs. increasing attraction (*asy-soft*). The main new result shown here, in a fully relativistic scheme, is the importance at higher energies of the microscopic covariant structure of the effective interaction in the isovector channel: effective forces with very similar symmetry terms can give rise to very different flows in relativistic heavy ion collisions.

We start from the Relativistic Mean Field (*RMF*) picture of the hadronic phase of nuclear matter [18] which has been extensively used to the study of the EOS for symmetric and asymmetric matter. The *RMF* describes an interacting system of nucleons (described as Dirac spinors) and meson classical fields. The most common treatment of the isospin dependent part of the interaction is based on the introduction of an effective $\rho$-meson field (vector-isovector) which can account for the known value of symmetry energy at normal density.

However, a full description in a relativistic framework in principle should rely on the balance between a scalar and a vector field as stressed in some papers within the Hartree approximation [20,19], and as naturally accounted for within the Dirac-Hartree-Fock (*DHF*) [21,22] or the Dirac-Brueckener-Hartree-Fock (*DBHF*) [23] schemes. One could argue that the scalar-isovector $\delta$ meson ($a_0(980)$) is too heavy or that the scalar field in the isoscalar channel may be due to the two pion correlation, while there is no equivalent possibility in the isovector channel. On the other hand we stress that the vector-isovector field in the *RMF* effective picture has not to be viewed as coming only from the exchange of the $\delta$ meson. In fact in the nuclear system contributions to the isovector channel are mostly due to isoscalar mesons, as clearly shown by DHF or DBHF scheme [22–25], through important exchange and correlation terms. Therefore when in the following we will refer to $\delta$ field indeed we mean a $\delta$—*like* field, i.e. the scalar isovector part of the effective “interaction”.

In recent years some efforts have been devoted to the effects of the scalar-isovector channel in finite nuclei, [26–28]. Such investigations have not shown a clear evidence for the $\delta$-field and this can be understood considering that in finite nuclei one can test the interaction properties mainly below the normal density, where the effect of the $\delta$—channel on symmetry energy and on the effective masses is indeed small [19] and eventually could be absorbed into non linear terms of the $\rho$ field. Moreover even studies of the asymmetric nuclear matter by means of the Fermi Liquid Theory [19] and a linear response analysis have concluded that some properties, like the borderline and the dynamical response inside the spinodal instability region, are not affected by the $\delta$ field [29].

Here we show that heavy-ion collisions around 1AGeV with radioactive beams can provide instead a unique opportunity to spot the presence of the scalar isovector channel. In fact, due to the large counterstreaming nuclear currents
Fig. 1. Total (kinetic+potential) symmetry energy as a function of the baryon density. Solid: RMF \(-\rho + \delta\). Dashed: RMF \(-\rho\). Short Dashed: RMF \(-D\rho\). In the insert the density behaviour of the \(\rho\) coupling, \(f_\rho\), for the three models is shown.

one may exploit the different Lorentz nature of a scalar and a vector field.

All models, including RMF, allow to use the parabolic approximation for the description of the EOS of asymmetric nuclear matter:

\[ E(\rho_B, I) = E(\rho_B) + E_{sym}(\rho_B) I^2 \]  

where \(I = \frac{N-Z}{A}\) is the asymmetry parameter. When both \(\rho\)-like and \(\delta\)-like channels are considered \(E_{sym}(\rho_B)\) can be written as [19]:

\[ E_{sym}(\rho_B) = \frac{1}{6} \frac{k_F^2}{E_F^2} + \frac{1}{2} \left[ f_\rho - f_\delta \left( \frac{M^*}{E_F^*} \right)^2 \right] \rho_B \equiv E_{sym}^{kin} + E_{sym}^{pot} \]  

with \(E^* = \sqrt{k_F^2 + M^*^2}\), \(M^*\) the effective Dirac mass and \(f_{\rho,\delta} = (g_{\rho,\delta}/m_{\rho,\delta})^2\) are the coupling constants of the isovector channels.
We see that, when $\delta$ is included, the observed $a_4 = E_{sym}(\rho_0)$ value actually assigns the combination $[f\rho - f\delta(\frac{M}{E})^2]$ of the $(\rho, \delta)$ coupling constants, for further details see ref.[19]. In Fig.1 we report the density dependence of the symmetry energy for three different models: one including only the $\rho$ field ($RMF - \rho$), the other with $(\rho + \delta)$ fields ($RMF - (\rho + \delta)$) and the last, ($RMF - D\rho$), including only $\rho$ field but with a covariant density dependence of $f\rho$, see ref. [26,30]. This is tuned just to give at high density the same $E_{sym}(\rho_B)$ of the the ($RMF - (\rho + \delta)$) case. As shown in the following, the latter is useful for disentagling in the reaction dynamics the effects due to the difference in $E_{sym}(\rho_B)$ from those directly linked to the strenght of the $\rho$ vector field.

Thus these models parametrize the isovector mean field either by only the vector field with $f\rho = 1.1 fm^2$, or with a balance between a vector field with $f\rho = 3.3 fm^2$ and a scalar one with $f\delta = 2.4 fm^2$, or finally by a normal density coupling $f\rho(\rho_0) = 1.1 fm^2$, but with an increasing density dependence as shown in Fig.1 (insert). We stress again that in $RMF - (\rho + \delta)$ the symmetry energy is coming from a balance between a scalar attraction, ($\delta - like$), and a vector repulsion, ($\rho - like$), which is now roughly three times larger than in the $RMF - \rho$ case.

The choice of $f\delta$ is fixed relatively well by DBHF [23] and DHF [22] calculations. Therefore the effect described in the following is not artificially enhanced, but based on a reliable estimate available at the moment.

Collective flows in heavy ion collisions give important information on the dynamic response of excited nuclear matter [31,32]. In particular the proton-neutron differential flow $F_{pn}(y)$ [17] has been found to be a very useful probe of the isovector part of the $EOS$ since it appears rather insensitive to the isoscalar potential and the in medium nuclear cross section. The definition of the $F_{pn}(y)$ is

$$F_{pn}(y) \equiv \frac{1}{N(y)} \sum_{i=1}^{N(y)} p_{x_i} \tau_i$$

where $N(y)$ is the total number of free nucleons at the rapidity $y$, $p_{x_i}$ is the transverse momentum of particle $i$ in the reaction plane, and $\tau_i$ is +1 and -1 for protons and neutrons, respectively.

For the theoretical description of heavy ion collisions we solve the covariant transport equation of the Boltzmann type within the Relativistic Landau Vlasov ($RLV$) method [33] (for the Vlasov part) and applying a Monte-Carlo procedure for the collision term. $RLV$ is a test particle method using covariant Gaussians in phase space for the test particles. The collision term
includes elastic and inelastic processes involving the production/absorption of the $\Delta(1232\text{MeV})$ and $N^*(1440\text{MeV})$ resonances as well as their decays into one- and two-pion channels. Details about the used cross sections for all possible channels can be found in Ref. [34]. An explicit isospin-dependent Pauli blocking term for the fermions is employed. Asymmetry effects are suitably accounted for in a self-consistent way with respect for the RMF models discussed above.

A typical result for the $^{132}\text{Sn} + ^{132}\text{Sn}$ reaction at 1.5AGeV (semicentral collisions) is shown in Fig.2. We notice that the differential flow in case of the $\text{RMF} - (\rho + \delta)$ (full circles and solid line) presents a stiffer behaviour relative to the $\text{RMF} - \rho$ (open circles) model, as expected from the more repulsive symmetry energy $E_{\text{sym}}(\rho_B)$ at high baryon densities, see Fig.1. On the other hand it is quite surprising that a relatively small difference at $2\rho_0$ can result in a such different collective flows. Indeed, we will see below that this is not the whole story.

We have repeated the calculation using the $\text{RMF} - D\rho$ interaction, i.e. with only a $\rho$ contribution but tuned to reproduce the same EOS of the $\text{RMF} - (\rho + \delta)$ case. The results, short-dashed curve of Fig.2, are very similar to the ones of the $\text{RMF} - \rho$ interaction. Therefore we can explain the large flow effect as mainly due to the different strengths of the vector-isovector field between $\text{RMF} - (\rho + \delta)$ and $\text{RMF} - \rho, D\rho$ in the relativistic dynamics. In fact if a source is moving the vector field is enhanced (essentially by the local $\gamma$ Lorentz factor)[36] relative to the scalar one.

In order to get the idea we write down, for an idealized situation, the “force” acting on a particle. From the isovector part of the interaction we get

$$\frac{d\vec{p}_i^*}{d\tau} = \pm f_\rho \frac{E_i^*}{M_i^*} \vec{\nabla}\rho_3 \mp f_\delta \vec{\nabla}\rho_{S3}$$

for protons and (upper signs), respectively neutrons (lower signs). $\vec{p}^*$ is the effective momentum, $\tau$ the particle proper time (averaged) and $\rho_3 = \rho_p - \rho_n$ the isovector baryon density (correspondingly $\rho_{S3}$ the scalar one). Here we have simplified the problem neglecting the contribution from the current gradient in the transverse direction and the current derivative with respect to time. We are interested in the difference between the force acting on a neutron and on a proton, respectively. Oversimplifying the HIC dynamics we consider locally neutrons and protons with the same $\gamma$ factor (i.e. with the same speed). Then Eq.4 can be expressed approximately by the following transparent form ($\rho_{S3} = \frac{M^*}{E^*} \rho_3$):

$$\frac{d\vec{p}_p^*}{d\tau} - \frac{d\vec{p}_n^*}{d\tau} \simeq 2 \left[ \frac{\gamma f_\rho - f_\delta}{\gamma} \right] \vec{\nabla} \rho_3$$

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where γ is the Lorentz factor for the collective motion of a given ideal cell.

Keeping in mind that $RMF - (\rho + \delta)$ has a three times larger ρ field it is clear that dynamically the vector-isovector mean field acting during the HIC is much greater than the one of the $RMF - \rho, D\rho$ cases. Then the isospin effect is mostly caused by the different Lorentz structure of the “interaction” which results in a dynamical breaking of the balance between the ρ vector and δ scalar fields, present in nuclear matter at equilibrium. This effect is analogous to the interplay between the isoscalar vector- and scalar-fields which is seen in the magnitude and energy dependence of the real part of the optical potential, ref. [37].

![Graph](image)

Fig. 2. Proton-neutron differential collective flow in the $^{132}Sn + ^{132}Sn$ reaction at 1.5 AGeV b=6fm for the three different model for the isovector mean fields. Full circles and solid line: $RMF - (\rho + \delta)$. Open circles and dashed line: $RMF - \rho$. Stars and short dashed line : $RMF - D\rho$.

In order to characterize the effect on differential collective flows we have calculated the slope $dF_{pn}(y)/d(y/y_{proj})$ at mid-rapidity. Its value is 46.7 Mev/c for $RMF - (\rho + \delta)$ and 23.4 MeV/c for $RMF - \rho$, i.e. a factor two difference.

We have also performed some calculations at lower beam energies. We have found that up to 500 AMeV there is no valuable difference in the differential flow predictions among the models discussed here. The effect coming from the strength of ρ field starts to become important around 1 AGeV, as expected from the relativistic mechanism.

Another interesting observable is the elliptic flow $v_2(y, p_t)$, which is derived as the second coefficient from a Fourier expansion of the azimuthal distribution.
Fig. 3. Difference between neutron and proton elliptic flow as a function of the transverse momentum in the $^{132}$Sn + $^{132}$Sn reaction at 1.5 AGeV $b=6$fm in the rapidity range $-0.3 \leq y/y_{\text{proj}} \leq 0.3$. Full circles and solid line: $\text{RMF} - (\rho + \delta)$. Open circles and dashed line: $\text{RMF} - \rho$. Stars and short dashed line: $\text{RMF} - D\rho$.

$N(\phi, y, p_t) = v_0 (1 + v_1 \cos(\phi) + 2v_2 \cos(2\phi))$. It can be expressed as

$$v_2 = \langle \frac{p_x^2 - p_y^2}{p_t^2} \rangle$$

where $p_t = \sqrt{p_x^2 + p_y^2}$ is the transverse momentum [35,38].

A negative value of $v_2$ corresponds to the emission of matter perpendicular to the reaction plane, *squeeze-out* flow. The $p_t$-dependence of $v_2$, which has been recently investigated by various groups [38,39,37], is very sensitive to the high density behavior of the EOS since highly energetic particles ($p_t \geq 0.5$) originate from the initial compressed and out-of-equilibrium phase of the collision, see e.g. Ref. [37].

For our purpose we focus on the proton-neutron difference of the elliptic flow. From Fig.3 we see that in the $(\rho + \delta)$ dynamics the high-$p_t$ neutrons show a much larger *squeeze-out*. This is fully consistent with an early emission (more spectator shadowing) due to the larger repulsive $\rho$-field. We can expect this large effect since the relativistic enhancement discussed above is relevant just at the first stage of the collision. The $v_2$ observable, which is a good *chronometer* of the reaction dynamics, appears to be particularly sensitive to the Lorentz structure of the effective interaction.

In conclusion intermediate energy heavy-ion collisions with radioactive beams
can give information on the symmetry energy at high baryon density and on its detailed microscopic structure. We have shown that such experiments provide a unique tool to investigate the strength of the $\delta$ field. The sensitivity is enhanced relative to the static property $E_{\text{sym}}(\rho_B)$ because of the more fundamental covariant nature of the fields involved in $HIC$ dynamics.

Collective flows observables are found to be sensitive to isospin effects. Due to the time selectivity on the emitted particles the elliptic flow measurements appear to be the most appealing, especially for nucleons and light isobars at high transverse momentum.

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