Evolution of Mass and Velocity Field in the Cosmic Web: Comparison Between Baryonic and Dark Matter

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Abstract

We investigate the evolution of the cosmic web since \(z = 5\) in grid-based cosmological hydrodynamical simulations, focusing on the mass and velocity fields of both baryonic and cold dark matter. The tidal tensor of density is used as the main method for web identification, with \(\lambda_{th} = 0.2–1.2\). The evolution trends in baryonic and dark matter are similar, although moderate differences are observed. Sheets appear early, and their large-scale pattern may have been set up by \(z = 3\). In terms of mass, filaments supersedes sheets as the primary collapsing structures from \(z \sim 2–3\). Tenuous filaments assembled with each other to form prominent ones at \(z < 2\). In accordance with the construction of the frame of the sheets, the cosmic divergence velocity, \(v_{div}\), was already well-developed above \(2–3\) Mpc by \(z = 3\). Afterwards, the curl velocity, \(v_{curl}\), grew dramatically along with the rising of filaments, becoming comparable to \(v_{div}\), for \(<2–3\) Mpc at \(z = 0\). The scaling of \(v_{curl}\) can be described by the hierarchical turbulence model. The alignment between the vorticity and the eigenvectors of the shear tensor in the baryonic matter field resembles that in the dark matter field, and is even moderately stronger between \(\omega\) and \(e_1\), and \(\omega\) and \(e_2\). Compared with dark matter, there is slightly less baryonic matter found residing in filaments and clusters, and its vorticity developed more significantly below \(2–3\) Mpc. These differences may be underestimated because of the limited resolution and lack of star formation in our simulation. The impact of the change of dominant structures in overdense regions at \(z \sim 2–3\) on galaxy formation and evolution is shortly discussed.

Key words: cosmology: theory – large-scale structure of universe – methods: numerical

1. Introduction

The spatial distribution of cosmic matter on large scale is called the cosmic web, consisting of voids, sheets/walls, filaments, and clusters/knots, which was first predicted by theoretical study and have been confirmed by galaxy redshift surveys (e.g., de Lapparent et al. 1986; Colless et al. 2003; Tegmark et al. 2004; Mehmet et al. 2014). The formation and evolution of the cosmic web have invoked numerous investigations over the past several decades (see van de Weygaert & Bond 2008). Using a linear Lagrangian model, Zel’dovich (1970) first pointed out that the growth of density perturbation would depend on the deformation tensor, which is connected to the tidal shear field. The Zel’dovich approximation predicted the formation of pancake and filamentary large-scale structures in sequential order, due to the anisotropic gravitational collapse in the linear and mildly nonlinear regimes.

The homogeneous ellipsoidal collapse model (Icke 1973; White & Silk 1979) predicted the appearance and evolution of flattened and elongated structures in the quasilinear regime. Integrated with the effects of the external tidal field (Eisenstein & Loeb 1995; Bond & Myers 1996), the ellipsoidal model has become a useful tool to describe the distribution of virialized objects within the bottom-up structure formation scenario. Based on the peak patch formalism (Bond & Myers 1996), the anisotropic property of gravitational collapse was incorporated within the hierarchical clustering picture, and led to the heuristic cosmic web model given in Bond et al. (1996). According to this scenario, high density peaks in the primordial Gaussian field on a large smoothing scale would evolve first, and then seed the megaparsec-scale tidal shear that drives the formation of prominent filaments connecting clusters, and afterwards, sheets. The proposed evolutionary sequence is in inverse order compared to the Zel’dovich picture, and stresses the dominance of filamentary structures instead of sheets (van de Weygaert & Bond 2008).

Due to their high complexity, the formation and evolution of the cosmic web in the nonlinear regimes have recently been investigated by analyzing cosmological simulation samples (e.g., Aragón-Calvo et al. 2007a; Hahn et al. 2007a; Aragon-Calvo et al. 2010; Cautun et al. 2014, etc.). Various numerical methods have been developed to identify morphological components of the cosmic web in both simulation samples and observed galaxy distribution (Aragón-Calvo et al. 2007a, Hahn et al. 2007a; Forero-Romero et al. 2009; Bond et al. 2010a; Sousbie 2011; Hoffman et al. 2012; Cautun et al. 2013). Of particular interest is the reconstruction of large-scale filamentary structures in observations and simulations, and the study of the statistical properties of their lengths and widths, and alignment with galaxies and halos (Colberg et al. 2005; Zhang et al. 2009, Sousbie 2011; Tempel et al. 2014; Gheller et al. 2015). With the aid of numerical simulations, the mass and volume content, the density distribution, and the spatial extent of the cosmic web components are investigated in detail, as well as their evolution with redshift (for details, see, e.g., Hahn et al. 2007b, Bond et al. 2010a; Cautun et al. 2014, hereafter C14). Filaments are identified as the most prominent structures of the web and contain up to 50% of the cosmic mass, from \(z = 2\). Many of the properties and the evolution of massive filaments in simulation samples are generally in agreement with the prediction of the peak patch model. Nevertheless, clusters become a significant component only at \(z < 0.5\), and tenuous filaments are found to dominate at redshifts \(z > 1\), which cannot be well explained by the peak patch model. A more comprehensive theory for the intricate cosmic web may be needed, in which the evolution of
singular structures in Zel’dovich approximation might be a key factor (Hidding et al. 2013).

On the other hand, the evolution of the cosmic web at redshifts greater than 2 is an important section in building the whole picture, providing direct insights into the properties of each component of the cosmic web at early time and tracking the hierarchical assembly history of structures. A few visual impressions of the filaments at $z > 2$ is presented in Aragon-Calvo (2007) and Bond et al. (2010b). C14 demonstrated that the dominance of filaments holds up to $z = 3.8$ (see their Figure 24), although the margin over sheets decreases toward high redshift. It is reasonable to expect that the sheets might surpass the filaments, i.e., become the primary structure, at epochs earlier than $z > 4$ in their samples. Moreover, the bias effect between baryonic and dark matter in the nonlinear regime should be taken into account when comparing the observed cosmic web with N-body simulations. However, fewer efforts have been made to systematically investigate the evolution of baryonic matter in each large-scale environment during the formation of the cosmic web, except for warm and hot intergalactic medium (WHIM) at low redshifts. Cosmological hydrodynamic simulations predicted that the WHIM resides in moderately overdense filaments and possibly walls, and host $\sim$50% of the total baryons, which are still being searched for by intensive observations (e.g., Cen & Ostriker 1999; Dave et al. 2001; Bregman 2007; Fang et al. 2010; Shull et al. 2012; Tejos et al. 2016).

The velocity fields of cold dark matter and baryonic gas encode crucial information of the cosmic network, especially in the mildly and nonlinear regimes, and can be employed to track the formation history of cosmic structures and mass transport among different large-scale structures. Research on the motions of matter in the cosmic web could provide hints for the following important question: what is the impact of the cosmic environment on galaxy properties, such as stellar mass, morphology, spins, etc? Recently, velocity fields have been investigated in simulations looking for the origin of spins of halos and their alignment with the cosmic web (e.g., Aragon-Calvo et al. 2007b; Codis et al. 2012; Libeskind et al. 2013; Wang et al. 2014a; Dubois et al. 2014), in order to interpret the alignment of galaxy shape and distribution in different cosmic environments revealed by observation and simulation samples (e.g., Lee & Erdogdu 2007; Zhang et al. 2009; Jones et al. 2010; Tempel & Libeskind 2013; Dong et al. 2014; Wang et al. 2014b; Zhang et al. 2015). Nevertheless, investigation using simulations into the evolution of baryonic and cold dark matter flows during the formation of the cosmic web is inadequate, in comparison with the mass content. The velocity on a large scale is usually assumed to be irrotational, just as in the Zel’dovich approximation (Zel’dovich 1970, Shandarin & Zel’dovich 1989), which might become invalid in the mildly nonlinear regimes. The curl velocity would be non-negligible after shell-crossing in multistream regions. The properties of the irrotational and curl velocities of cosmic matter are expected to evolve along the hierarchical clustering of structures, to reflect the anisotropic gravitational collapse of the cosmic web.

Pichon & Bernardadeau (1999) made the pioneering effort to calculate vorticity generation in large-scale structure caustics and demonstrated that the vorticity would be significant at scales above the galaxy clusters. In our previous works (Zhu et al. 2010, 2013; Zhu & Feng 2015, hereafter ZF15), we found that the cosmic shocks appearing in multistream regions will boost the vorticity of baryonic gas because of baroclinicity. The vortical motions of gas are triggered and pumped up in sheets and filaments in simulations. The growth history and statistical properties of the curl velocity is in accordance with the anisotropic collapse process in a bottom-up universe. In order to understand the origin of halo spins beyond the linear tidal torque theory, ambient vortical flow around dark matter halos has been studied over the past few years (e.g., Libeskind et al. 2013). Libeskind et al. (2014) investigated the velocity shear tensor and vorticity, and their relative orientations as a function of scale and redshift, indicating that nonlinear evolution will drive the vorticity tending to be perpendicular to the fastest collapsing axis. Some aspects of the vortical motions of dark matter, including growth history and statistical properties in different environments, remain unclear. The connection between irrotational motion and the evolution of the cosmic web is also not well resolved. In addition, the galaxy spin with respect to the $e_1$ vector (normal vector of sheets) tends to be randomly distributed in observations, which is different from the angular momentum of halos in N-body simulations (Tempel & Libeskind 2013). For a better understanding of the cosmic web, a detailed comparison between baryonic and dark matter with regard to the evolution and alignment of the velocity shear tensor and vorticity is required; this has been partially tackled in Laigle et al. (2015).

We here present a comprehensive comparison study focusing on the evolution of the volume, mass, and irrotational and curl velocities of both baryonic and cold dark matter in the cosmic web since $z = 5$, using fixed-grid cosmological hydrodynamical simulations without star formation and feedback. The connection between matter flows and the evolution of the cosmic web is also explored. This paper is organized as follows. The simulations and numerical methods are described in Section 2. Section 3 studies the volume and mass distribution and their evolution in the cosmic web. The growth history and properties of the irrotational and curl velocities of baryonic and cold dark matter, including magnitude, power spectrum, and scaling relation during the formation and evolution of the cosmic web, are investigated in Section 4. Section 5 compares the alignment of the velocity shear tensor and vorticity of baryonic and dark matter in each class of the cosmic web. Section 6 discusses the growth of vortical motions along with the increase in filamentary structures at $z < 3$, and the possible impact on the formation and evolution of galaxies. We summarize our results in Section 7.

2. Numerical Methodology

2.1. Simulations

Three simulations in periodic cubic boxes of 25, 50, and 100 $h^{-1}$ Mpc were performed in the ΛCDM model with WMAP5 normalization, in which cosmological parameters such as $\Omega_m = 0.274$, $\Omega_\Lambda = 0.726$, $h = 0.705$, $\sigma_8 = 0.812$, $\Omega_b = 0.0456$, and $n_s = 0.96$ (Komatsu et al. 2009) were adopted. We refer to the simulations as L025, L050, and L100, respectively, hereafter, with the numbers standing for the box size. The same random seeds and phases are used in generating the initial conditions of the three simulations. The velocity fields of baryonic matter in L025 and L100 have been investigated in ZF15. The simulations were run with the hybrid N-body/hydrodynamic cosmological code WIGEON,
which employs the positivity-preserving Weighted Essentially Non-Oscillatory finite differences scheme to solve hydrodynamic equations, incorporating the standard particle-mesh method as the gravitational potential solver (Feng et al. 2004; Zhu et al. 2013). All the simulations were evolved from redshift \( z = 99 \) to \( z = 0 \) in a 1024\(^3\) grid with an equal number of dark matter particles. The space resolutions are 24.4 \( h^{-1} \) kpc, 48.8 \( h^{-1} \) kpc, and 97.7 \( h^{-1} \) kpc, respectively. The corresponding particle mass resolutions are 1.30 \( 10^6 \), 1.04 \( 10^5 \), and 8.32 \( 10^4 \) \( M_\odot \). A uniform UV background is switched on at \( z = 11.0 \). The radiative cooling and heating modules are implemented with a primordial composition \((X = 0.76, Y = 0.24)\) following the method in Theuns et al. (1998). The processes of star formation, AGNs, and their feedback are not included.

### 2.2. Density and Velocity Field Resampling and Velocity Decomposition

To reduce the impact of discreteness, especially in underdense regions, the mass and velocity of dark matter particles are first assigned to a 512\(^3\) grid using the cloud-in-cell (CIC) algorithm. The resolution of the CIC grid, \( R_g \), is two grid units of simulations. The resampled density and velocity fields on the CIC grids are then smoothed with Gaussian kernels of one, two, four, and eight CIC grid cells (denoted as smt01, smt02, smt04, and smt08 hereafter) using FFT. For instance, smoothing lengths \( R_s \) of 48.8, 97.6, 195, 390 \( h^{-1} \) kpc are applied to L025. The effective smoothing length is given by

\[
R_{\text{eff}} = \sqrt{R_s^2 + R_g^2}.
\]

The same 512\(^3\) grid is also applied in resampling the density and velocity fields of gas, which are then smoothed with the same kernel as the dark matter. The peculiar velocity fields of both dark matter and baryonic matter are decomposed into three components: the curl-free divergence velocity \( v_{\text{div}} \), the divergence-free curl velocity \( v_{\text{curl}} \), and the uniform velocity \( v_{\text{unif}} \) through the Helmholtz--Hodge decomposition (Sagaut & Cambon 2008),

\[
v = v_{\text{curl}} + v_{\text{div}} + v_{\text{unif}},
\]

where \( \nabla \times v_{\text{div}} = 0 \) and \( \nabla \cdot v_{\text{div}} = \nabla \cdot v \); \( \nabla \cdot v_{\text{curl}} = 0 \) and \( \nabla \times v_{\text{curl}} = \nabla \times v \). The uniform component \( v_{\text{unif}} \) is both curl free and divergence free, which is actually negligible in the simulations and will not be discussed here.

### 2.3. Cosmic Web Identification

The distributions of baryonic and dark matter at the resampled grid cells are classified into four categories of cosmic structures, i.e., voids, sheets/walls, filaments, and clusters/knots, using two identification schemes following Hahn et al. (2007b) and Forero-Romero et al. (2009). The former scheme is based on the density field and the latter is based on the peculiar velocity field of matter. We denote by \( r \) the comoving coordinates and \( x = a(t)r \) the proper coordinates, where \( a(t) \) is the cosmic expansion factor. Furthermore, the comoving peculiar velocity \( v(x, t) = \dot{r} \) is related to the physical velocity \( u \) by \( a(t)v(x, t) = u - \ddot{a}(t)r \). The first method (d-web hereafter) identifies structures on the basis of the eigenvalues \( \lambda_{1,2} \), \( \lambda_{1,3} \), \( \lambda_{2,3} \) of the tidal tensor, which is defined as the Hessian matrix of the rescaled peculiar gravitational potential \( \phi \),

\[
T_{\alpha\beta} = \frac{\partial^2 \phi}{\partial r_\alpha \partial r_\beta},
\]

where \( \alpha, \beta = 1, 2, \) and 3 denotes the components in the three axes. The peculiar gravitational potential is rescaled by \( 4\pi G \hat{\rho}(t) \), and obeys \( \nabla^2 \phi = \delta \), where \( \hat{\rho}(t) \) is the cosmic mean density of matter and \( \delta = (\rho - \bar{\rho})/\bar{\rho} \) is the overdensity field. The mean density and overdensity of baryonic matter, \( \hat{\rho}_b(t), \delta_b(t) \), and of dark matter, \( \hat{\rho}_d(t), \delta_d(t) \), are used separately to identify webs in corresponding matter fields. The second method (v-web hereafter) depends on the eigenvalues \( \lambda_{v,1}, \lambda_{v,2}, \lambda_{v,3} \) of the rescaled velocity shear tensor,

\[
\Sigma_{\alpha\beta} = -\frac{1}{2H(\zeta)} \left( \frac{\partial v_\alpha}{\partial r_\beta} + \frac{\partial v_\beta}{\partial r_\alpha} \right),
\]

where \( H(\zeta) \) is the Hubble parameter and is used as the rescaling factor. The derivatives of the potential and velocity are performed in Fourier space. We count the number of eigenvalues above \( \lambda_{v,1} \) at each CIC grid cell. A cell with a value of 3, 2, 1, or 0 is marked as a cluster, filament, sheet, or void, respectively. The threshold value hence is important for classification results. Forero-Romero et al. (2009) pointed out that \( \lambda_{v,1} \) could be estimated in principle, considering the association of the deformation tensor, collapse timescale, and age of universe as well. A precise estimate of \( \lambda_{v,1} \), however, needs to carefully deal with the anisotropic collapse of structures and is not available thus far. Forero-Romero et al. (2009) relaxed the assumption \( \lambda_{v,1} = 0 \) used in Hahn et al. (2007b), by taking into account that \( \lambda_{v,1} \) is dependent on the collapse timescale, and recommended values of 0.2–0.4 for the d-web. Hoffman et al. (2012) used \( \lambda_{v,1} \sim 0.44 \) for the v-web and a larger value, \( \lambda_{v,1} \sim 0.7 \), for d-web to provide the best match to the visual impression. Here, we investigated the threshold values of 0.2–1.2 for d-web, and \( \lambda_{v,1} \sim 0.44 \) for v-web. \( \lambda_{v,1} \) is kept first constant in time, and then set to vary with redshift.

### 3. The Distribution and Evolution of Mass in the Cosmic Web

#### 3.1. Visual Inspection of the Cosmic Web

Before probing the volume and mass content of the cosmic web, we first provide a direct visual impression of the mass distribution, vorticity, and identified structures in this subsection. Figure 1 displays a projected three-dimensional rendering of the density distribution of baryonic and dark matter, and the vorticity of dark matter in the simulations L025 and L050 at \( z = 0 \). A sharp picture of the cosmic web is visualized in the baryonic density field of L025 (the rendering procedure is improved compared to Figure 1 in ZF15 to feature the structures). In comparison, the large-scale structures are less prominent in L050, probably because lower spatial numerical resolution leads to lower density contrast between the structures in the simulation. It would take more effort to visually identify the large-scale structures in the density field of dark matter, especially in underdense regions. The structures in the vorticity field, \( |\omega| = |\nabla \times v| \), of dark matter, however, are smooth and prominent.

The vorticity of dark matter resembles that of baryonic matter (see Figure 1 in Zhu & Feng 2015) in both spatial configuration and strength distribution. The curl motions have developed significantly in the regions surrounding filaments and clusters, showing more extended morphologies than the
Figure 1. Projected three-dimensional rendering of the density of baryonic matter (top panels) and dark matter (middle panels), and vorticity of dark matter (bottom panels) in simulations L025 (left panels) and L050 (right panels).
Figure 2. Projected three-dimensional rendering of the density field of baryonic matter in the simulation L025 at $z = 5.0$ (top left), 3.0 (top right), 2.0 (middle left), 1.0 (middle right), 0.5 (bottom left), 0.0 (bottom right).
Figure 3. Filaments (left panels) and sheets/walls (right panels) identified by d-web for baryon at $z = 3$ (top panels) and $z = 0$ (middle panels), and for cdm at $z = 0$ (bottom panels).
The evolution of the cosmic web since $z = 5.0$ in L025 is demonstrated in Figure 2. Vague protosheets and sheets appear at early times. Small-scale filaments can also be found visually at $z = 5$, when most of them are tenuous. Prominent filaments emerge late, at a redshift between $z = 2$ and $z = 1$, mainly built from tenuous predecessors. Evident cluster regions form at around $z = 0.5$, in agreement with C14.

Table 1
Mean Density and Mass Fraction of Each Cosmic Web Component Identified by d-web in Three Simulations at $z = 0$ with $\lambda_{th} = 0.6$ and $R_c = R_g$

| Simulation | Mass Fraction | Mean Density |
|------------|---------------|--------------|
|            | Cluster Filament Sheet Void Cluster Filament Sheet Void |
| L100 cdm   | 14.8% 49.1% 23.0% 13.1% 50.91 5.37 0.79 0.21 |
| gas        | 12.8% 47.2% 25.3% 14.6% 51.85 5.62 0.87 0.24 |
| L050 cdm   | 15.3% 54.7% 19.8% 10.1% 72.24 5.67 0.66 0.17 |
| gas        | 13.2% 51.6% 22.9% 12.2% 91.02 5.98 0.77 0.20 |
| L025 cdm   | 14.8% 60.0% 17.1%  8.1% 94.93 5.83 0.56 0.14 |
| gas        | 14.6% 52.0% 21.5% 11.8% 181.7 5.89 0.73 0.19 |

Note. The terms "cdm" and "gas" refer to cold dark matter and baryon, respectively.

Figure 4. Volume and mass fraction of voids, sheets, filaments, and clusters identified by d-web with $\lambda_{th} = 0.2, 0.4, \text{ and } 0.6$, and smf01 in L025 since $z = 5.0$. The top row gives the ratio of the volume fraction in baryonic matter to the volume fraction in dark matter (bottom left), and the ratio of the mass fraction in baryonic matter to the mass fraction in dark matter (bottom right).

density field. The evolution of the cosmic web since $z = 5.0$ in L025 is demonstrated in Figure 2. Vague protosheets and sheets appear at early times. Small-scale filaments can also be found visually at $z = 5$, when most of them are tenuous. Prominent filaments emerge late, at a redshift between $z = 2$ and $z = 1$, mainly built from tenuous predecessors. Evident cluster regions form at around $z = 0.5$, in agreement with C14.

The filaments and sheets identified by d-web at $z = 3.0$ and $z = 0.0$ in L025 with a constant $\lambda_{th} = 0.6$ over redshift and $R_c = R_g$ are presented in Figure 3. d-web is capable of finding filaments with various lengths and radii, as well as sheets. The spatial distribution of filaments at $z = 3$ is significantly different from that at $z = 0$ on a small scale, while it is similar on a large scale to some extent. Longer smoothing length and varying $\lambda_{th}$ at different redshifts may enhance the similarity, as in Bond et al. (2010b). The most prominent filaments are connecting to massive clusters at $z = 0$. However, most of the filaments at high redshifts do not connect to any clusters. They
will coalesce into prominent ones at lower redshifts, e.g., around $z = 1$. The d-web classification has extracted the filaments from the voids and sheets embracing them, leaving empty tubes (Figures 2(b) and (d)). The sheets are fluffy without distinct boundaries at high redshifts, due to low density contrast with respect to the voids. The sheets in baryonic matter evolve to a much smoother and thinner shape at $z = 0$. The distribution of sheets on a larger scale at $z = 3$ is similar to that at $z = 0$ at a certain level; we will revisit this later with the slice view. The discreteness of cold dark matter particles leads to the identified filaments and sheets appearing relatively rough. The sheets suffer more severely from the undersampling in low-density regions, even after smoothing.

### 3.2. Mass Distribution and Its Evolution

Volume and mass content are the most explicit quantities to demonstrate the evolution of the cosmic web (Hahn et al. 2007b; Bond et al. 2010; C14). Table 1 summarizes the mass fraction and mean density of each cosmic web component identified by d-web in our simulations at $z = 0$ with $\lambda_b = 0.6$ and $R_v = R_w$. A lower fraction of baryonic matter is found to residing in clusters and filaments, which is more significant in L025 with the smallest box size. The mass fraction of gas in filaments is less than that of dark matter by 8% in its absolute value, while for both sheets and voids it is $\sim 3.6\%$ higher. It implies that more baryonic matter is likely to inhabit the underdense or mildly overdense region at $z = 0$, which may be attributed to thermal pressure effect and nonthermal turbulent motions. The box size has minor effects on the results on these two global quantities. The mean density of baryon in each environment is usually close to that of dark matter, i.e., $1 + \delta > 50$ in clusters, and around $\sim 6, 0.8, \text{ and } 0.2$ in filaments, sheets, and voids respectively, in agreement with the results in recent simulation studies (e.g., Aragon-Calvo et al. 2010; C14). In L025, however, the mean density of baryon in clusters is nearly twice that of dark matter. In comparison, the excursion model in Shen et al. (2006) gives an approximation 36 and 6 times denser than the critical density for filaments and sheets. The median density of filaments of baryonic matter is also lower than the estimated density of WHIMs, i.e., $\sim 10–30$ (Dave et al. 2001; Fang et al. 2010). It is probably because the mean density in the background, i.e., the voids, is only 0.2 times that of the cosmic mean. Meanwhile, more matter will reside in clusters and filaments when the simulation box gets smaller and the resolution is increased.

In Figure 4, we investigate the evolution of the volume and mass fraction in voids, sheets, filaments, and clusters since $z = 5.0$ in L025 since $z = 5.0$. The Astrophysical Journal, 838:21 (25pp), 2017 March 20 Zhu & Feng
fraction of each component varies slightly with time, while for the evolution of the mass fraction, it is clearly more significant. The volume fraction of clusters is less than 1% throughout time, and hence is not plotted in this figure. The exact volume fractions in voids and sheets, as well as the mass fraction in voids at a given redshift, are sensitive to the value of $\lambda_{th}$. A lower $\lambda_{th}$ would lead to more cells being identified as parts of filaments and sheets, instead of voids. For $\lambda_{th} = 0.4$ and 0.6,
the voids occupy the largest volume share, and then sheets and filaments in turn. However, the volume fraction of sheets is larger than voids at \( z < 5 \) for \( \lambda_{th} = 0.2 \). Forero-Romero et al. (2009) showed that the volume fraction of sheets at \( z = 0 \) would be larger than voids for \( \lambda_{th} < 0.25 \), and a lower \( \lambda_{th} \) close to 0.0 would make the voids too small and isolated.

In terms of the mass fraction, clusters grow gradually with time, while filaments demonstrate rapid growth. On the contrary, the mass content in voids keeps shrinking toward lower redshifts, decreasing more sharply for higher \( \lambda_{th} \). The mass fraction in sheets varies slowly at \( 3 < z < 5 \) and decreases with time from \( z \sim 3 \). At redshift \( z = 5 \), about 33\%–45\% of the mass resides in sheets, while about 18\%–30\% and 2\%–3\% have collapsed into filaments and clusters, respectively. Down to \( z = 0 \), more than half of the mass is found in the filaments.

For threshold values between 0.2 and 0.6, the filaments become the primary collapsed structures, replacing sheets, as measured by the mass fraction at around \( z \sim 2–3 \). The phenomenon that there is less baryonic matter residing in filaments and clusters occurred as early as \( z > 3 \). It is noticeable that the shortage of baryon has been largely remedied in clusters at low redshifts, while the relative difference remains around 10\%–20\% between \( z = 0 \) and \( z = 5 \) in filaments. The shortage of gas is slightly more apparent for higher \( \lambda_{th} \).

The evolution of the volume and mass fractions in L025 with different Gaussian smoothing lengths for \( \lambda_{th} = 0.4 \) is demonstrated in Figure 5. The effect of different values of \( R_s \) is more significant in the mass fraction than in the volume fraction. A larger \( R_s \) will lead to notably more mass in voids, but less mass in filaments. For sheets, however, the dependence of the mass fraction on \( R_s \) turns over before and after \( z \sim 3 \). The redshift where filaments take over sheets as the primary structure decreases moderately for longer smoothing lengths, and is around \( z \sim 2 \) for \( R_s = 195 \ h^{-1} \) kpc. The discrepancy between the mass fraction of baryonic and dark matter in filaments and clusters is narrowed by increasing \( R_s \). For L025, with the space resolution of \( 24.4 \ h^{-1} \) kpc, the relative difference is reduced to \( \sim 5\%–10\% \) while taking \( R_s = 195 \ h^{-1} \) kpc.

Figure 6 presents the evolution of fractions in all three simulations with \( \lambda_{th} = 0.4 \). Results from L025 with \( R_s = 2R_g \), \( 4R_g \) are compared to results from L050 and L100 with \( R_s = R_g \). Both the volume and mass fractions of dark matter in different simulations are very close, once the smoothing lengths are equal. Since we have used the same random seeds and phases to produce the initial conditions, it is obviously not beyond our expectations. In addition, except for...
slight differences, the deficiency of baryonic matter in filaments and clusters can be seen in all simulations. In terms of the mass fraction, baryonic matter is less than dark matter by $\sim 5\% - 15\%$ relatively in filaments and clusters at an effective smoothing scale of $R_{\text{eff}} = 0.29\ h^{-1}\ Mpc$ in L100.

For a short summary, a nice bit of mass has fallen into sheets and protosheets, as well as into filaments, at $z = 5$. In terms of mass, sheets have been dominant over filaments since $z > 2 - 3$ for the d-web web classification scheme with $\lambda_0 = 0.2 - 0.6$. The fast increase of the mass fraction in filaments, with decreasing...
redshifts, is not in conflict with the expectations from the structure growth history in the LCDM universe. Actually, filaments are structures experiencing two-dimensional collapse. The rapid growth of mass in filaments since $z \sim 3$ should come from the further collapsing of sheets and accretion from nearby underdense environments as well.

We further investigate the impact of the web classification scheme, by looking into the results of $v$-web. Figure 7 shows
the mass fraction of dark matter in the cosmic web identified by v-web in L025, and the corresponding ratios of baryon to dark matter. $R_v = 2R_g$ has been used to identify v-web to reduce the noise in undersampled regions. $\lambda_{v,th} = 0.2-0.4$ are used, which are smaller than the $\lambda_{d,th}$ in d-web, similar to Hoffman et al. (2012). Clearly, the overall evolution is consistent with d-web. However, relatively more grid cells are identified as sheets and filaments at low redshifts, rather than the voids and filaments identified in d-web. The mass fraction found in filaments (clusters) is moderately lower (higher) for v-web. Moreover, relative to d-web, the time since the filaments surpass sheets in mass fraction is postponed to around $z \sim 2$. The deficiency in baryonic matter remains at $\sim 5\% - 20\%$, except for filaments at $z = 0$. Our results with the two different classification methods are generally consistent with each other.

Figure 8 gives the probability distributions of the density of baryonic and dark matter in each cosmic web environment. The overtaking of sheets by voids, filaments by sheets, and clusters by filaments by cell number occurs at around $\delta \sim -0.6, 1.0$, and 200.0. Each component covers a large density range. The density range in filaments, especially, almost covers the entire density range in the simulation sample. The cluster has the highest peak density, and then the filament, sheet, and void in a decreasing sequence, which is consistent with theoretical expectations and previous studies. Remarkable changes in time are observed regarding the density distribution function in filaments. In particular, the single-peak distribution at high redshifts gradually changes to a double peak distribution at $z = 0$, while a long tail at the high density end is well-developed at $z = 1$.

Moreover, a plateau appears in the density probability distribution function in sheets, and the covering density range expands progressively with time. The peak value decreases from slightly larger than the cosmic mean ($\delta > 0$) at $z = 3$ to smaller than the cosmic mean, i.e., $\delta < 0$ at $z = 0$. The probability distribution in voids has a single-peak pattern the
entire time, while the width at half peak grows gradually from high redshifts to low redshifts. The density of cells in voids decreases moderately with time; the peak value is $\sim 0.30$ at $z = 3$, and about $\sim 0.06$ at the present epoch. A long tail at higher density has also developed in clusters at $z = 1$, which is where the pdf of the clusters mainly change. The density range of each cosmic morphology environment in L025 is slightly wider than that in L050 and L100, while the probability peak is shallower. As we have argued in the earlier paragraph, the higher resolution in L025 is helpful to develop more refined structures and sharper density contrast in the nonlinear regimes, i.e., resolving higher density peaks and lower density floors.

3.3. Comparison with C14

The mass distribution of dark matter in the cosmic web at $z = 0$ revealed by d-web and v-web in this work is basically in agreement with the results obtained from the dark matter-only simulation in C14. The characteristic density in each environment presented here (see Figure 8) is also consistent with C14. However, our results with the volume and mass fractions, and their evolution in voids and filaments at $z > 0$, show different behaviors in comparison with C14 in several aspects. First, in contrast to C14, the volume fraction in voids/sheets is decreasing/increasing with time in our simulation samples. This is partly because the d-web (also v-web) method tends to identify regions with underdeveloped density and velocity fluctuations as voids. Namely, none of the three eigenvalues is larger than $\lambda_{\text{th}}$, due to lack of significant collapsing activity. For a constant threshold value, the volume and mass fractions in voids would be higher while going to higher redshifts. Second, the mass fraction in filaments changes slowly toward $z = 3.8$ in C14, which is significantly different from our results, even in L100, which has a comparable box size. A larger/smaller share of mass has been found in voids/filaments in our samples, becoming more apparent as the redshift increases. The margin between filaments and sheets in terms of mass fraction at $z > 0.5$ is decreasing going to higher redshifts in C14, albeit in a much slower pace compared to our results. The epoch when sheets surpass filaments in C14 is expected to be much earlier than $z \sim 4$. In addition, the probability distribution functions in our identified web are more widespread.

The key factor leading to these differences is the cosmic web identification method. The NEXUS+ scheme used in C14 is a multiscale morphological analysis tool implemented with filtering of the density field in logarithmic space, and is different from d-web and v-web, which have a single-scale filter. Cautun et al. (2013) have shown that the NEXUS+ scheme was successful in capturing filaments and walls with a wide range of sizes. C14 further showed that the mass fraction is insensitive to the identification tracer, i.e., either density field or tidal tensor, as well as velocity divergence, etc., in their NEXUS scheme. However, it has been justified only at $z = 0$, and its validity is not clear when going to high redshifts. On the other hand, more insightful investigations is needed to set a time-dependent $\lambda_{\text{th}}$ that is physically meaningful for the d-web and v-web schemes. Though d-web with a fixed $\lambda_{\text{th}}$ could identify both filaments and walls with different sizes at low and high redshifts in our simulation samples, it would be interesting to implement a time-varying $\lambda_{\text{th}}$ in our scheme and justify whether it is able to reduce the difference between our statistics and C14.
Figure 9 shows the evolution of the volume and mass fractions of dark matter in L025 with $\lambda(z) = \lambda_0(0)/(1 + z)$. Generally, d-web provides a better match than v-web. Taking a fixed threshold value of $\lambda_0(0) = 1.2$ with $R_s = 2R_g$ in the d-web scheme, the volume fractions of all structures and the mass fraction of filaments and voids at $z = 0$ are found to be almost identical to those in C14. The mass fractions in sheets and clusters, however, still show differences of about 5% when comparing with C14. A time-dependent $\lambda(z)$ is applied, the consistency of the overall evolution trend between d-web and C14 in both mass and volume fractions is somewhat improved, whereas there still exist distinct differences in the absolute value of the volume and mass fractions. The d-web scheme predicts relatively higher mass fractions in sheets and clusters, but lower fractions in filaments at high redshifts, in contrast to C14.

The remaining discrepancy can hardly be reduced by adjusting either $\lambda_{th,0}$ or smoothing length. The numerical results with an alternative monotonic $\lambda_{th}(z)$, $\lambda_0(0) \times H_0/H(z)$, are given in Figure 10 and show slight changes. The differences in volume and mass fractions between d-web and C14 are apparent, especially at high redshifts. It should result from the different kernels we have used in the identification of cosmic web components. In addition, the d-web scheme with time-varying $\lambda_0$ also predicts a turnover redshift of $z \sim 2–3$, at which the universe goes through a transition from being sheet dominated to filament dominated, in agreement with results presented in the last subsection where a fixed $\lambda_0$ was used.

4. Evolution of the Velocity Field in the Cosmic Web

The velocity fields of baryonic and cold dark matter record the anisotropic collapse of the structures and mass transportation among different components in the cosmic web. The flow of cosmic matter is almost curl-free in the linear regime, during which initial vorticity would be diluted by the cosmic expansion. The anisotropic collapse that drives the growth of clustering structures will lead to the appearance of multiflows and hence vorticity in the quasilinear regimes. The curl velocity would be amplified by further anisotropic collapsing and become comparable to the irrotational mode in highly nonlinear regimes. The evolution of the two modes of motions, i.e., irrational and curl, are expected to indicate the variation of density, volume, and mass content presented in the last section. In order to reveal this connection, we investigate the visual and statistical properties of the flows in this section.

4.1. The Divergence and Vorticity Fields

Before studying the properties of velocity in the cosmic web, we first take a look at the divergence and vorticity field of baryonic and dark matter. Figure 11 displays the spatial distributions of divergence and projected vorticity $\omega$ with $R_s = 2R_g$ in a slice at $z = 0$; both have been rescaled to
Figure 15. Mean divergence velocity in each cosmic web environment in L050 at $z = 3.0$, 1.0, 0.0 (top two plots), and in the filaments of all three simulations at $z = 0.0$ (bottom two plots).
strong curved shocks surrounding filaments should be the driving factor for gas (Zhu et al. 2013). The sign of the quadripolar multifold in isolated filaments became more evident at $z = 1.0$ and evolved moderately afterwards, in sync with the growth of filaments. Even more complex patterns of vorticity are found in the cross-sections of prominent filaments. The curl motions in the progenitor of the prominent filaments at the position ($x = 0.50, y = 0.90$) were already more complex than a quadripolar pattern at $z = 1.0$. Matter should have been accreted into filaments along multisheets, as well as directly from nearby voids. The rapid growth of vorticity is in sync with the dramatic rise of filaments at $z < 2–3$.

In Figure 14, we plot the probability distribution function (pdf) of vorticity rescaled by the cosmic time, i.e., $|\omega|/t$ measured in L025 at $z = 0$ with different smoothing lengths. The vorticities of both baryonic and dark matter are found to decrease with increasing smoothing lengths, in agreement with Libeskind et al. (2014). Vortical motions are effectively boosted in the nonlinear regime, but a large smoothing length would reduce the nonlinearity. The vorticity of baryonic matter exhibits a stronger heavy-tailed distribution pattern than dark matter, suggesting that the vorticity of baryonic matter can be developed more effectively in the highly nonlinear regimes. The pdf of the vorticity in baryonic matter at $z = 0$ is likely to be described by a log-normal distribution (Zhu et al. 2010). We also compare the distribution in each cosmic environment in the top right plot of Figure 14. Consistent with ZF15, the vorticities are triggered and pumped up in sheets, and especially in filaments, and then dissipated in clusters for both baryonic and dark matter. The vorticity of dark matter in the voids is relatively higher than that in baryonic matter, as displayed in Figure 11. Actually, it might be due to the undersampling issue of dark matter particles in voids. Moreover, the growth of the vorticity since $z = 3$ can be clearly seen in the bottom left plot of Figure 14. During the epoch between $z = 2$ and $z = 1$, the vortical component of the velocity field experiences the largest gains. Meanwhile, the difference between the vorticities of baryonic and dark matter widened. The bottom right plot in Figure 14 compares the pdf of the vorticity of baryonic matter at $z = 0$ in three simulations. Obviously, as the cosmic vorticity is a typical nonlinear feature emerging from gravitational collapse, higher spatial resolution is capable of resolving higher nonlinearity, and hence the global vorticity of L025-smt01 is the highest. On the other hand, for the same smoothing length, a larger simulation box could host more collapsed structures and have relatively more cells with very large vorticities, i.e., a longer tail in the pdf. For instance, L050-smt01 has more cells with $|\omega|/t > 40$ than L025-smt02.

### 4.2. Mean Divergence and Curl Velocity

The value of velocity can directly characterize the flows corresponding to divergence and vorticity. The mean divergence velocities $v_{\text{div}}$ as a function of density in each web component in L050 since $z = 3.0$ are shown Figure 15. The magnitude of divergence velocity in each structure was set up by $z = 3$ and changed mildly since then, mainly between $z = 3$ and $z = 1$. In Figure 16, we plot the probability distribution function (pdf) of vorticity rescaled by the cosmic time, i.e., $|\omega|/t$ measured in L025 at $z = 0$ with different smoothing lengths. The vorticities of both baryonic and dark matter are found to decrease with increasing smoothing lengths, in agreement with Libeskind et al. (2014). Vortical motions are effectively boosted in the nonlinear regime, but a large smoothing length would reduce the nonlinearity. The vorticity of baryonic matter exhibits a stronger heavy-tailed distribution pattern than dark matter, suggesting that the vorticity of baryonic matter can be developed more effectively in the highly nonlinear regimes. The pdf of the vorticity in baryonic matter at $z = 0$ is likely to be described by a log-normal distribution (Zhu et al. 2010). We also compare the distribution in each cosmic environment in the top right plot of Figure 14. Consistent with ZF15, the vorticities are triggered and pumped up in sheets, and especially in filaments, and then dissipated in clusters for both baryonic and dark matter. The vorticity of dark matter in the voids is relatively higher than that in baryonic matter, as displayed in Figure 11. Actually, it might be due to the undersampling issue of dark matter particles in voids. Moreover, the growth of the vorticity since $z = 3$ can be clearly seen in the bottom left plot of Figure 14. During the epoch between $z = 2$ and $z = 1$, the vortical component of the velocity field experiences the largest gains. Meanwhile, the difference between the vorticities of baryonic and dark matter widened. The bottom right plot in Figure 14 compares the pdf of the vorticity of baryonic matter at $z = 0$ in three simulations. Obviously, as the cosmic vorticity is a typical nonlinear feature emerging from gravitational collapse, higher spatial resolution is capable of resolving higher nonlinearity, and hence the global vorticity of L025-smt01 is the highest. On the other hand, for the same smoothing length, a larger simulation box could host more collapsed structures and have relatively more cells with very large vorticities, i.e., a longer tail in the pdf. For instance, L050-smt01 has more cells with $|\omega|/t > 40$ than L025-smt02.

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It is not beyond our expectation, as the outline of the sheets and voids on a large scale is found, in previous paragraphs, to already be in place by $z = 3.0$. In addition, the divergence velocity of matter evolves slowly with respect to the density in each environment until $\delta \sim 100$, during the formation of cosmic web. At high overdensity regimes, the small number of grid cells leads to significant dispersion. The level of divergence motions in sheets and filaments is marginally higher than in voids and clusters, consistent with the divergence distribution given in Figure 12. For a particular box size, e.g., $L050$, the magnitude is nearly the same in the four morphology environments, around $100 \text{ km s}^{-1}$. Notable differences between baryonic matter and dark matter can only be found in high density regions, $\delta > 30$, where the dispersion due to the limited number of cells becomes evident. The distribution of $v_{\text{div}}$ in the filaments of all three simulations is also compared in Figure 15. The largest mean divergence velocity is found in $L100$, and can be about twice the value in $L025$, because more massive structures can form in a larger simulation box and hence result in a higher bulk velocity.

Similarly, Figure 16 gives the distribution and evolution of the mean curl velocity, $v_{\text{curl}}$, in the cosmic web. The curl motions are effectively developed when the matter flows into the central layer of sheets and into filaments, and then experiences dissipation when flowing into clusters, basically confirming our previous result in ZF15 and the slice view in Figure 12. The resampling and smooth processes in this work may have partly blurred the velocity fields at the boundaries between sheets and filaments for $\delta > 1$, resulting in the fast growth of curl motions in sheets with respect to density, compared to ZF15. On the other hand, Figure 8 shows that most of the cells with $1 < \delta < 100$ belong to filaments. Hence, the curl velocity in the cosmic web is mainly contributed by filaments, which conforms with the visual evolution presented in Figure 13.

The magnitude of $v_{\text{curl}}$ has increased rapidly for $\delta \lesssim 20–30$ since $z = 3$. At $z = 0$, the curl velocity is $\sim 10\%$ of the divergence velocity at $\delta \sim 0$, then increases to $\sim 30\%$ at $\delta \sim 3–5$, and eventually becomes comparable to the divergence velocity at $\delta \sim 20–30$. This evolution pattern is basically consistent with that in Libeskind et al. (2014), which shows that the vorticity of dark matter grows rapidly as $\omega \propto \rho$ for $\rho < 10$, and turns to $\omega \propto \rho^{1/3}$ for $\rho > 10$. In filaments, the curl velocities are pumped up to $\sim 100 \text{ km s}^{-1}$ for $\delta \gtrsim 30$ at $z = 0$. The curl velocity of baryonic matter is moderately higher than that of dark matter in sheets and filaments within the overdensity range $1–30$. The effect of box size on the curl velocity is the same as the effect on divergence velocity. A valley at $\delta \sim 0$ is presented in the sheets and filaments for both modes of motion, which is more evident for $v_{\text{curl}}$ at high redshifts. The valley is observed in all three simulations. A possible explanation is that a fraction of matter with $\delta \sim 0$ in the sheets and filaments have not experienced shell-crossing or curved shocks for dark and baryonic matter, respectively; the fraction decreases with time because of continuous anisotropic gravitational collapse.
4.3. Velocity Power Spectrum

The power spectrum of velocity at different times reflects the spatial correlation of motions associated with the evolving cosmic web. In Figure 17, we plot the compensated power spectrum $k^2 P(k)$ of the total, curl, and divergence velocities. For the sake of clarity, the comoving velocity rather than the proper velocity is used in this figure to separate lines at different redshifts. On large scales, namely, in the linear regime, the power spectrum is dominated by the divergence component, in agreement with theoretical expectation and a previous study (Zheng et al. 2013). The dominance of irrotational velocity can be extended to the mildly nonlinear regime, i.e., around a few megaparsecs. Actually, the power spectrum of the divergence velocity has been well-developed at $>2$ Mpc by $z = 3$. Combined with the exploration on mass and velocity distribution given in the preceding sections, we conclude that the outline of the cosmic web above $\sim 2$–3 Mpc has been formed by $z = 3$, when the sheets served as the frame. The power spectrum of the curl velocity increases rapidly from $z = 3.0$, in accordance with the growth of curl motions that largely developed in filament components. At $z = 0.0$, the contribution from the curl motion reaches 10% at a scale of $\sim 5$ Mpc, and can exceed the divergence velocity at scales smaller than $\sim 2$ Mpc for the baryonic matter. The power spectrum of both modes drops dramatically at a few times the resampling smoothing length.

The relative contribution of curl velocity in dark matter is weaker than baryonic matter, remaining subordinate to irrotational velocity in the highly nonlinear regime. Pichon & Bernardeau (1999) predicted that the curl motion will become significant at scales $\sim 3$–4 Mpc after the shell-crossing in large-scale caustics. The predicted scale is in good agreement with our simulations. Using a set of cold dark-matter-only simulations, Zheng et al. (2013) demonstrated that the power spectrum of curl motions would equal to the divergence motions at $k \sim 10 h$/Mpc, comparable with our result in L100. They also expect that the contribution of curl velocity would be twice that of divergence at sufficiently small scales, which is hard to verify for dark matter due to numerical factors related to the discrete particle, limited resolution, and smooth processes. For the baryonic matter in our simulations, the ratio of the contribution of the curl velocity to divergence velocity reaches a peak value of $\sim 1.5$ at the scales $\sim 1.3$, 0.8, and 0.4 Mpc in L100, L050, and L025, respectively, i.e., close to the expectation in Zheng et al. (2013).

In the nonlinear regime, the power spectrum of velocity approximates a power law within a particular scale range for both baryonic and dark matter. For the baryonic matter, the upper end is about the same, $\sim 3$ Mpc in three simulations. The lower end varies with the resolution, extended to $\sim 0.3$ Mpc in L025. The growth of the upper end of the scale is in sync with the evolution of vorticity in filaments presented in Figure 8. The upper end of the scale range for dark matter is smaller than that for baryonic matter. In the highly nonlinear regime, the anisotropic collapsing of gas would develop numerous strong curved shocks, mostly associated with filaments, and hence a supersonic intermittent velocity field exhibiting $P(k) \propto k^{-2}$ law in the power spectrum (e.g., Porter et al. 1992; Kritsuk et al. 2007; Zhu et al. 2013). After the shell-crossing in caustics, the curl motions of dark matter in filaments may have developed similar intermittent behavior.

![Figure 18. Velocity structure functions of curl motions at $z = 2.0$ and 0.0 for baryonic (top two panels) and cold dark matter (bottom two panels) in L050.](image-url)
4.4. Velocity Structure Functions and Fractal Dimensions

The growth of the magnitude and power spectrum of the curl velocity indicates a close connection between matter flows and cosmic web. Given that the vorticity in filaments may play a very important role in building the spins of halo and galaxy (e.g., Laigle et al. 2015), we carry a further check on the correlations between vorticity and filaments through the study of velocity structure functions and fractal spatial dimensions. The velocity structure functions can give a more insightful view of the coherence of curl motions in sheets and filaments.

In ZF15, we found that the velocity structure functions of curl motions of baryonic matter can be described by the intermittent model proposed by She & Leveque (1994) (see also Dubrulle 1994) in the supersonic-scale range. The velocity structure function reads as

\[ S_p(r) = (\delta v^p) = \langle |v(x) - v(x + r)|^p \rangle \propto r^{Z_p}, \]

where \( p = 1, 2, 3, \ldots \). She & Leveque (1994) and Dubrulle (1994) suggested that the relative scaling exponents \( Z_p \) would follow a universal scaling law, i.e.,

\[ Z_p = \frac{\gamma_p}{\gamma_3} = \left( 1 - \frac{\Delta}{3} \right) \frac{p}{3} + \frac{\Delta}{1 - \beta} (1 - \beta^p / 3), \]

where \( \beta \) and \( \Delta \) are related to the hierarchy properties of intermittent structures (more details can be found in Dubrulle 1994 and ZF15).

We apply the same analysis to the resampled curl velocity fields of both baryonic and cold dark matter in all three simulations. Figure 18 shows the longitudinal and transverse structure functions, \( S_p^L \) and \( S_p^T \), at \( z = 2 \) and \( z = 0 \) in L050. In agreement with ZF15, the structure functions are broken into two distinctive sections, where the breaking occurs at the scale that marks the turnover in the compensated power spectrum of the curl velocity. The breaking scale is almost the same for baryonic and dark matter. In other words, the curl motions show significant coherence below the breaking scale denoted by \( k_b \), strongly confirming the results from the power spectrum study.

The fractal dimensions of the most singular structures, derived from the curl velocity structure functions, are given in Figure 19, where \( d^l \) and \( d^t \) are from \( S_p^L \) and \( S_p^T \), respectively. According to the SL model, the most singular structures of the vortical motions have dimensions of 1.85–2.15 and 1.75–2.05 for baryonic and dark matter, respectively. The \( d^l \) and \( d^t \) in dark matter are smaller than the values in filaments in C14. On the other hand, the fractal dimensions of filaments and sheets can be measured using the box-counting method (Mandelbrot...
Figure 21. Probability distribution $p(\cos \mu | \mu)$ as a function of the angle between the vorticities (shear eigenvectors) of baryonic and dark matter. Top: different smoothing lengths at $z = 0$ in L025; middle: with $R_s = 2R_s$ from $z = 3.0$ to $z = 1.0$ in L025; bottom: with $R_s = R_s$ at $z = 0$ in three simulations.

Figure 22. Probability distribution $p(\cos \mu)$ in different environments between the vorticities of baryonic and dark matter in L025.

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The angle between the vorticities of baryonic and dark matter in L025. A transition of the mass distribution at the scale around $k_p$ is also observed in the filaments. The fractal dimensions of curl motions is close to the fractal dimensions of filaments below $k_p$. In other words, the velocity power spectrum, structure functions, and fractal dimension of mass in filaments show an analogous transition over $k_p$, and the value of the fractal dimensions of curl motions and mass in filaments is similar. Combining the growth history of curl motions revealed in Section 4.2 and in ZF15, we note that the vortical kinetic energy is primarily associated with filaments for both baryonic and dark matter. The statistical properties investigated in our simulations indicate that the curl motions of cosmic matter is mainly driven by the formation of filaments, usually from further collapse within sheets. This result is consistent with that of Laigle et al. (2015), in which they claimed that vorticity in large-scale structures tends to be confined to, and predominantly aligned with, filaments.

5. Relative Orientation of the Shear Tensor and Vorticity

The ambient vortical flow around the dark-matter halo is suggested to play an important role in shaping the halo spin and galaxies, and shows an orientation preference with respect to the velocity shear (Libeskind et al. 2013, 2014). Tentative evidence of the alignment of galaxy spin axes with respect to filaments has been reported in works analyzing observation samples. However, the spin axis of the spiral galaxy is found to be randomly distributed with respect to the $e_1$ vector, which is different from the strong alignment of the angular momentum of dark matter halos with $e_1$ found in N-body simulations (Tempel et al. 2014; Tempel & Libeskind 2014). The limited resolution and lack of star formation in our simulation prohibit us from a direct investigation on the scale of galaxies. Nevertheless, comparison between baryonic and dark matter on the evolution of shear tensor and vorticity over a scale range of tens of kiloparsecs to a couple of megaparsecs is feasible in our samples, which may be helpful in diagnosing the cause of disparity between observations and N-body simulations. Actually, the growth history and statistics including the power spectrum of vorticity reported in the last section already indicate signs of discrepancy between baryonic and dark matter below a couple of megaparsecs.

Figure 21 presents the probability distribution $p(\cos \mu)$ as a function of the angle between the vorticities of baryonic and dark matter, i.e., $\omega_b \cdot \omega_d$. $p(\cos \mu)$ is shown for the shear eigenvectors of $e_{bi} \cdot e_{di}$, where $l = 1, 2, 3$. The velocity shear of the baryonic matter is almost parallel to that of the dark matter; only weak misalignment can be observed for $e_2$. Misalignment of $\omega_b$ with respect to $\omega_d$ is found and became more apparent for a shorter smoothing length, i.e., higher vorticity. The median value of $\cos(\mu)$ decreases from about 0.75 for $R_s = 0.4$ Mpc to 0.38 for $R_s = 0.05$ Mpc in L025. The evolution of $p(\cos \mu)$ since $z = 3.0$ is also demonstrated in Figure 21. The growth history of the misalignment between the vorticity of the two matter components matches the global development of the vorticity displayed in Figure 14. The increased resolution in L025 leads to higher vorticities and hence large misalignments.

As the calculation of the vorticity of dark matter suffers from poor sampling in underdense regions to some extent, the misalignment between $\omega_b$ and $\omega_d$ would likely be amplified in voids by numerical errors. We examine $p(\cos \mu)$ in the four types of large-scale structures identified by d-web applied to the baryonic density field. The result is plotted in Figure 22.
The global misalignment is indeed raised by cells in voids, where vorticity is small. Nevertheless, the misalignment between \( \omega_j \) and \( \omega_f \) is also observable in collapsed structures. The distributions of \( p(|\cos \mu|) \) in filaments, where the vorticity is the highest among the four types of cosmic environment, against different smoothing lengths are shown in Figure 22. The median value of \( |\cos \mu| \) in filaments are about 0.60, 0.78, 0.87, and 0.9 for \( R_s = 1.0, 2.0, 4.0, \) and 8.0 \( R_s \), respectively. Obviously, the misalignment is well-developed mostly below the scale of 0.2 Mpc.

The orientations of vorticity with respect to velocity shear in both baryonic and dark matter are shown in Figure 23. The vorticity prefers to be perpendicular to \( e_1 \) in all three simulations. Meanwhile, \( \omega \) prefers to be aligned with \( e_2 \) and \( e_3 \). These tendencies have been discussed in Libeskind et al. (2013, 2014). In the L025 sample, the median values of \(|\cos \mu|\) between \( \omega \) and \( e_1, e_2, \) and \( e_3 \) are estimated to be 0.21, 0.57, and 0.63 for baryonic matter, and 0.26, 0.59, and 0.52 for dark matter. The latter results for dark matter are actually in good agreement with Libeskind et al. (2013). The preference of \( \omega \) to be perpendicular to \( e_1 \), and \( \omega \) parallel to \( e_3 \) is relatively stronger for baryonic matter, which may partly be due to the vorticity of baryonic matter being more developed than dark matter. The right column of Figure 23 shows the orientation of vorticity filtered with \( R_s = 97.6 \) kpc with respect to the velocity shear eigenvectors, with \( R_s = 97.6, 195.2, 390.4 \) kpc respectively. The vorticity tends to lie in the \( e_2, e_3 \) plane, which is slightly weakened by larger \( R_s \) for dark matter, while for baryonic matter no variation is easily observed. The weakening of the alignment for dark matter became more apparent for \( R_s > 0.5 \) Mpc in Libeskind et al. (2014). The parallel alignment of \( \omega \) with \( e_2 \) is quite similar in the two matter components, and is also getting weaker with larger \( R_s \). The median value of \(|\cos \mu|\) between \( \omega \) and \( e_2 \) of dark matter decreases from 0.65 for \( R_s = 97.6 \) kpc to 0.53 for \( R_s = 390.4 \) kpc in L025, basically in agreement with Libeskind et al. (2014).

Figure 24 presents the alignment between the vorticity eigenvectors of shear tensor in the baryonic and dark matter in different cosmic web environments. Consistent with Libeskind et al. (2013), the strength of the perpendicular alignment between \( \omega \) and \( e_1 \) decreases from sheets to filaments, clusters, and voids. This trend is more evident for dark matter, with the median \(|\cos \mu|\) about 0.17, 0.21, 0.28, and 0.34, respectively. The parallel alignment between \( \omega \) and \( e_2 \), and \( \omega \) and \( e_3 \) in all the environments, except that \( \langle \omega, e_2 \rangle \) in clusters and \( \langle \omega, e_3 \rangle \) in voids for dark matter. The corresponding median \(|\cos \mu|\) is 0.54 and 0.52. As a short summary, the alignment between the vorticity and eigenvectors of shear tensor in the baryonic matter field resembles dark matter, and is even moderately stronger between \( \omega \) and \( e_1, e_3 \) in the highly nonlinear regime. The tension between observation and simulation regarding the spin axis of the spiral galaxy with respect to the \( e_1 \) vector would persist if the vorticity of stellar mass follows ambient gas.

6. Discussion

The formation and evolution of the cosmic web are long-standing questions. The Zel’dovich approximation suggests that the sheets/walls appear first, and then filaments form as a result of subsequent anisotropic collapse. Alternatively, the
peak patch cosmic web scenario (Bond & Myers 1996) offers an inverse sequence, i.e., clusters and their prototypes form first around high density peaks on a large smoothing scale, i.e., from several to tens of megaparsecs, then prominent filaments emerge as cluster–cluster bridges from the quadrupole matter distribution, and sheets/walls arise last. In reality, the evolution of the cosmic web might be more complex. Formation of web components in the nonlinear regime involves hierarchical development (van de Weygaert & Bond 2008). For example, the formation of prominent filaments is likely to result from the assembly of small-scale filaments. Cautun et al. (2014) found that the configuration of prominent filaments and clusters in N-body simulations are basically consistent with the prediction of the peak patch scenario. Meanwhile, the configuration of tenuous filaments and sheets, which are dominant at high redshifts, does not agree with the cluster–filament–cluster picture. Tenuous filaments and sheets of both baryonic and dark matter, identified with the tidal tensor and velocity shear tensor, also appear early in our simulations. Small-scale filaments are embedded in (proto) sheets at high redshifts. The layout of sheets on scales above 2–3 Mpc may have been set up at $z > 3$. Meanwhile, the divergence velocity field can be well-developed over the corresponding scales, and further collapse in secondary axes triggers the emergence of small-scale filaments within sheets. The filaments keep accreting matter from sheets and voids, and surpass the sheets in term of mass fraction at some redshift. With d-web, and taking $\lambda_{th} = 0.2–1.2$, the transition redshift is found to be around $\sim 2–3$ in our simulation samples. The vortical motions are boosted after the transition redshift.

A transition from sheets to filaments as the dominant web component of the universe in terms of mass fraction may have an effect on the cosmic star formation history to some extent. The gas accretion in halos at different redshifts might be affected by the change in the dominant cosmic environment and associated velocity modes. At redshifts larger than $\sim 2$, the overwhelmingly dominant divergence velocity may help the halo acquire gas more rapidly. Afterwards, increasing curl motions may slow down the gas accretion. Moreover, since the accretion shocks surrounding different large-scale structures also have different statistical properties (e.g., Vazza et al. 2009; Zhu et al. 2013), the thermal state of the accreted gas onto halos before and after the transition redshift can be different. This naive scenario requires careful verification by taking account of

Figure 24. Probability distribution $p(\cos \mu | l)$ of the angle between the vorticity and the three eigenvectors of the velocity shear in different web environments in L025. $R_{z} = 2R_{e}$ is used to identify d-web and calculate the vorticity and shear tensor. Left: baryonic matter; right: dark matter.
the formation and evolutionary history of halos. Their connections, however, are worth further investigating in depth, as the impact of the cosmic environment on the star formation in galaxies is not well resolved yet.

The growth of filaments with time is closely related to the role of the vorticity of the ambient flow field in the spins of dark matter halo and galaxies. Recent works suggest that the vorticities of gas and dark matter share the same orientation on large scales, and prefer to be confined and aligned with filaments. Also, the spins of halos and galaxies show alignment with the large-scale vorticity of dark matter and gas, respectively (Libeskind et al. 2013; Dubois et al. 2014; Laigle et al. 2015). Laigle et al. (2015) demonstrated the emergence of vorticity in filaments due to quadrupolar multiform, as predicted by Codis et al. (2012). The primary role of filaments in developing the curl velocity revealed in this work, including the growth and distribution of vorticity, curl velocity, power spectrum, and velocity structure functions, provides solid support for the results in Codis et al. (2012) and Laigle et al. (2015).

Similar to dark matter, the vorticity of baryonic matter is found to be perpendicular to the \( e_1 \) vector, and parallel to \( e_2 \) and \( e_3 \). The alignment between \( \omega \) and \( e_1 \), and \( e_2 \) for baryonic matter is even stronger than that in dark matter, which is more significant with higher resolution. However, the spin axis of spiral galaxies in observations is found to be random with respect to \( e_1 \) (Tempel et al. 2014; Tempel & Libeskind 2013). Hence, the discrepancy between our results on baryonic matter and observations regarding \( \omega \) and \( e_1 \) is likely more serious than that with \( N \)-body simulations. To probe this discrepancy in depth, hybrid \( N \)-body/hydrodynamic simulations with sufficient higher resolutions to track the dynamics of vorticity on galactic scales and implemented with a star formation and feedback module are urged.

7. Conclusion

Based on a set of cosmological hydrodynamical simulations, we comprehensively investigate the mass and velocity evolution of baryonic and cold dark matter from \( z = 5 \) in four types of large-scale environments, i.e., voids, sheets/walls, filaments, and clusters/knots. The web classification methods based on tidal tensor and velocity shear tensor have been used. We found that baryonic and cold dark matter show similar trends regarding the evolution of mass and velocity in the cosmic web, but differences are observed. We summarize our results as follows:

1. Sheets/walls are found to have formed in the early stage of anisotropy collapse processes. The large-scale distribution of sheets had been formed by \( z = 3 \), and served as the frame of the cosmic web. The mass fraction in sheets was up to \( \sim 40\% \) at \( z = 5 \), changed slowly at \( z > 2 \), and then decreased gradually. Filaments also emerged at an earlier time before \( z = 5 \), and became the primary structure as measured by mass fraction after \( z \sim 2 \). The mass fraction in filaments has increased from around 20\% at \( z = 5 \) to \( \sim 50\% \) at \( z = 0 \). Massive clusters/knots became well-developed and evident at around \( z = 0.5 \). The mass fraction in clusters/knots was smaller than 10\% up to \( z = 2.0 \) and is \( \sim 15\% \) at \( z = 0.0 \). The exact numbers of these fractions and redshifts are dependent on the web classification scheme and threshold parameter \( \lambda_{th} \); \( \lambda_{th} = 0.2–1.2 \) have been investigated in this work.

2. In accordance with the formation of the frame of sheets/walls, the cosmic divergence velocity was well-developed above 2–3 Mpc by \( z = 3 \). Vortical motion has grown sharply along with the rapid increase in filamentary structures since \( z = 3.0 \), and became comparable to divergence motion under 2–3 Mpc at \( z = 0 \), due to strong curved shocks and shell-crossing for baryonic and dark matter, respectively, at the boundaries of filaments. The coherent scale of the vortical motion increases with time correspondingly, reaching \( \sim 2 \) Mpc at \( z = 0 \), and is demonstrated by the velocity power spectrum, structure functions, and fractal dimensions. The relative orientation of vorticity with respect to the three eigenvectors of the velocity shear tensor for baryonic matter is similar to dark matter, i.e., vorticity prefers to be perpendicular to \( e_1 \) and parallel to \( e_2 \) and \( e_3 \), which is consistent with the results of \( N \)-body simulations in the literature (e.g., Libeskind et al. 2013, 2014).

3. Differences between baryonic and dark matter arise in the nonlinear regime. Slightly less baryonic matter resides in filaments and clusters than dark matter, at a level of \( \lesssim 20\% \) in relative percentage. The vorticity in baryonic matter is more significant than the vorticity in dark matter. Below a couple of megaparsecs, the velocity power spectrum of the curl velocity can dominate over divergence velocity in baryonic matter, but the divergence mode remains dominant in dark matter. In addition, the alignment between vorticity and \( e_1 \), and \( e_3 \) are moderately stronger in baryonic matter. These differences may be underestimated due to the limited resolution and lack of star formation in our simulation.

In short, our simulations indicate that the rapid growth of vortical motions is in sync with the emergence and increase of filaments within sheets from \( z \sim 3 \). The overdense regions in the universe may have changed from being sheet dominated to filament dominated at about \( z \sim 2 \), which may affect the gas supply for galaxy formation and evolution. The primary role of filaments in developing curl velocity since \( z \sim 3 \) is also expected to have an important impact on the spins of halos and galaxies. Simulations with higher resolution and implemented with star formation and feedback could bring a more vivid view and provide a direct comparison with observation to verify our understanding of the cosmic web.

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