“IT FROM BIT”

How does information shape the structures in the universe?

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Based on a synthesis of three main ingredients: (i) the Shannon information in nonequilibrium systems, (ii) the semiclassical-energy-time quantization rule, and (iii) the quasistatic information-energy correspondence, a new general rule for the quantization of quasistatic information states supported by an environment away from equilibrium is introduced if the history of the environment is known as a function of time in terms of its thermodynamic potential for information $T(t)\Delta S(t)$ that is a free energy measuring the distance from equilibrium $\Delta S(t)$, and $T(t)$ is the mean temperature of the environment at time $t$. This all new quasistatic information-time quantization rule is applied to the expanding universe using a phenomenological thermodynamic potential for information in the matter dominated era in order to find the eigen-informations of the persistent structures that are supported by the universe (or the local environments therein) at any given epoch, thus providing an information-theoretic foundation for formation of structures and rise of complexity with time that embodies the cosmic evolution as epitomized by the late Wheeler’s famous conjecture “it from bit”. This theoretical procedure must also open new avenues for further research into the quantum theory of information and complexity in nonequilibrium thermodynamics.

I. INTRODUCTION

The most intriguing and fundamental problem in science is probably the discovery of the very basic laws that govern the emergence and gradual evolution of structures of increasing complexity since the inception of time. The term “cosmic evolution” is often used to describe the formation of persistent structures of increasing complexity or information content from the beginning primordial radiation, called the big-bang, to date including the elementary particles, nuclei, light atoms, galaxies, stars, planets, and life itself in the order of higher complexity. The primitive radiation era gave way to the more complex matter era 380,000 years following the big-bang, and now after 13.7 billion years we are well within the life era characterized by highly complex life forms (including us the Homo sapiens) coming to existence and evolving with the passage of time [1].

The above established facts at first may seem to contradict the second-law of thermodynamics or the law of increasing entropy that made physicists in the late nineteenth century believe that the universe must approach a final state of heat-death. The key to resolving this apparent paradox is to realize that the universe has never been static but always expanding. To be more precise, in any isolated system the total energy content or the internal energy $U$ is a constant by the first-law of thermodynamics $\Delta U = Q_{in} + W_{on}$, where $Q_{in}$ is the net heat intake and $W_{on}$ is the net work done on the system. Although by the first-law the total energy content $U$ of an isolated system is constant because no energy in the form of heat or work can either enter or leave the system, $Q_{in} = W_{on} = 0$, the second-law of thermodynamics stipulates that there is a natural tendency for the high-quality energy available for doing work or the free energy $F$ to run down in an isolated system such that in the final state of equilibrium free energy minimizes to a value consistent with the external constraints: $\Delta F = 0$, and $\Delta^2 F > 0$. Clearly in equilibrium the free energy is at the rock bottom and all thermodynamic processes cease to continue. Because free energy $F = U - TS$ is minimized by maximizing the entropy $S$, the state of equilibrium is also characterized by the maximum entropy: $\Delta S = 0$, and $\Delta^2 S < 0$ [2]. Indeed entropy is a measure of macroscopic uniformity or lack of persistent structures within the system that may epitomize order. So entropy is a measure of disorder and reaches its maximum attainable value in equilibrium consistent with the external constraints imposed on the system: $S = S_{\text{max}}$ in equilibrium. In a static universe, therefore, it is natural to assume that no matter how far in time, there comes a time when the universe equilibrates and all natural processes including life will stop. This was the idea held by the nineteenth century physicists such as Helmholtz and Clausius who of course were not aware of the fact that the universe is not static but expanding. The discovery of cosmic expansion in 1920s by Hubble and Lemaître among others, had far reaching implications not only for modern cosmology but also for negating the cosmic heat-death scenario if the expansion rate of the universe exceeds the rates of reactions and processes responsible for equilibration. The cosmic expansion is succinctly summarized in Hubble’s law $v = H d$, which states that the recession speed of a galaxy $v$ is proportional to the distance from observer $d$. The proportionality parameter $H$ called the Hubble parameter is of the order of the inverse of the age of the universe $H \sim t^{-1}$.

The nonequilibrium nature of the expanding universe is not only responsible for the formation of persistent living organisms in the later stages of cosmic evolution
but also for the birth of the tiniest of structures as fundamental as elementary particles in the early universe. Considering the birth of the elementary particles, nucleons, light nuclei, and light atoms in the early universe as entities independent of the primordial radiation, it is the generally accepted view among astroparticle physicists that

As the universe expanded and cooled, the cooling rate was proportional to the Hubble parameter, that is, of the order of the inverse of the age of the universe at that point in expansion. This led to a sequence of important events when different particles and interactions fell out of equilibrium with the gas of blackbody photons [primordial radiation]. The neutrino and neutron-proton conversion reactions [weak interaction] fell out of equilibrium at $t \approx 1$ second. Nuclear reactions [strong interaction] that formed light nuclei fell out of equilibrium at $t \approx 3$ minutes. Neutral atoms fell out of equilibrium at $t \approx 380,000$ years. All these degrees of freedom froze out when the reaction rates that had kept them in equilibrium with the blackbody photons fell far below the cooling rate of the expanding universe. Clearly, a necessary driving force behind cosmic evolution is the universal expansion that not only averts the heat-death of a static universe but also creates the nonequilibrium conditions required for formation of structures and rise of complexity within the universe and the local environments therein [1,4].

In fact every nonequilibrium environment has an inherent thermodynamic information content, which according to Shannon’s definition of information is rigorously quantified by the distance of the environment from the state of thermodynamic equilibrium or maximum entropy [4]:

$$\Delta S(t) \equiv S_{\text{max}}(t) - S(t). \quad (1)$$

$S(t)$ is the actual entropy of the environment be it the universe or a local environment therein, and $S_{\text{max}}(t)$ is the maximum attainable or equilibrium entropy of the environment consistent with the external constraints at the same time $t$. The details of this quantification of information content in terms of the deviation from the state of thermodynamic equilibrium, is presented in the next section following the line of argument given in Ref. [1]. It must be emphasized that Eq. (1) is in essence the Shannon’s measure of the information content of a message received about the thermodynamic system, $\Delta I = \log_2 \Omega_i - \log_2 \Omega_i$, that has the effect of reducing the number of possible microscopic configurations of the system from $\Omega_i$ to $\Omega_i$. Here the logarithm of base two is chosen to give the information content in units of binary bits. In Eq. (1) the equilibrium state forms the vacuum of information as it cannot support any persistent structures but transient equilibrium fluctuations. Eq. (1) also embodies the Layzer’s definition of order as the lack of disorder such that every time a thermodynamic system deviates from the state of maximum entropy (disorder), it must inherently hold a quantifiable amount of Shannon information (order) [3]. Hence, although in general the entropic measure of disorder $S(t)$ is rising by the second-law of thermodynamics, the informational measure of order $\Delta S(t)$ can also increase if $S_{\text{max}}(t)$ is rising at a rate faster than $S(t)$ (see, e.g., Fig. 1). This is believed to be the basis for increasing order, information content, or complexity that so vividly characterizes our expanding universe [1,4].

Therefore, there are already clear signs backed by empirical data as well as theoretical physics that the information content (bit) inherent to the nonequilibrium universe or local environments therein such as ours, must play a fundamental role in the formation and evolution of persistent structures (it), or to use the catch phrase prevalent among physicists due to Wheeler “it from bit” [6]. Now a question that immediately arises is how to develop a quantitative theory based on information that employs this presumably fundamental quantity presented by Shannon information, $\Delta S(t)$, to account for the cosmic evolution at a quantitative level and calculate the persistent information states supported by the nonequilibrium universe at different epochs that are so vividly reflected in both animate and inanimate physical species with discrete levels of complexity. In a serious attempt to end we present a phenomenological top-down approach to the problem of cosmic evolution based on information that is of course independent of the microscopic bottom-up views such as the inflationary theories [7], and the more recent rival theories based on cyclic cosmology [8], in that here we treat information as the most fundamental quantity by employing (i) the thermodynamic information content of Eq. (1) in the appropriate form of an information free energy $T(t)\Delta S(t)$ (see, also, Fig. 1) where $T(t)$ is the mean temperature of the environment at time $t$, (ii) the energy-time quantization rule rigorously derived from Wentzel-Kramers-Brillouin (WKB) semi-classical approximation of quantum mechanics for bound systems, and (iii) the quasistatic information-energy correspondence that is inferred from Landauer’s quasistatic limit [9], together with Szilard’s model of a quasistatic information heat engine [10], both of which have found experimental validity in the recent years [11,12].

The rest of this paper is organized as follows. In Sec. II the semi-empirical theory leading to the quasistatic information-time quantization rule is developed based on the above three ingredients. The method is then applied to the expanding universe in Sec. III and the resulting quasistatic information states are compared with the general phenomenology of the cosmic evolution at a quantitative level. The paper is concluded with a summary, concluding remarks and directions for further research in Sec. IV.

**II. QUASISTATIC INFORMATION-TIME QUANTIZATION RULE**

In this section we expand on the three main ingredients referred to in the introduction in order to derive the quasistatic information-time quantization rule from their
A. Shannon information in nonequilibrium systems

A fundamental quantity in the definition of Shannon information is the surprisal $s$, which quantifies the degree of surprise or amazement in observing a random event. The surprisal of an event with probability $p$ is proportional to the logarithm of the inverse probability

$$s = K \log \frac{1}{p} = K \log \Omega$$

such that the surprisal associated with a probable event ($p \to 1$) is small while that of an improbable event ($p \to 0$) is large. The logarithmic nature of surprisal is due to its additive property when applied to the compound probabilities of multiple independent events $\mathbb{F}$. $K$ is a proportionality constant that determines the units or alternatively the units can be set by choosing a particular base for the logarithmic function. For equally likely events the probability of any one event is $p = 1/\Omega$, where $\Omega$ is the number of all possible outcomes. It is worth noting that the surprisal for equally likely events is already beginning to look like the Boltzmann entropy for the microcanonical statistical ensemble $S = k_B \ln \Omega$. Now according to Shannon the information content of a message received that reduces the total number of possible initial outcomes $\Omega_i$ to a final number $\Omega_f (< \Omega_i)$, is given by the difference between initial and final surprisal:

$$\Delta I = K \log \Omega_i - K \log \Omega_f = K \log \frac{\Omega_i}{\Omega_f}.$$  \hspace{1cm} (3)

For example in tossing a fair die the initial surprisal is $s_i = K \log 6$ (in arbitrary units) because all six outcomes are equally likely. Now if we receive a message that reduces the number of possible outcomes to four by informing us that the outcome of the trial is a number less than five, the final surprisal reduces to $s_f = K \log 4$ (in arbitrary units). Therefore in this example the information content of the message received according to Shannon is $\Delta I = K \log 6 - K \log 4 = K \log 1.5$. Equivalently if the initial probability distribution $\{p_i\}$ is reduced to a final distribution $\{p_f\}$ as a result of the message received, the information content of the message is measured in terms of the difference between the average surprisal of initial and final distribution $\mathbb{F}$:

$$\Delta I = -K \langle \log p_i \rangle - (-K \langle \log p_f \rangle)$$

$$= -K \sum p_i \log p_i - (-K \sum p_f \log p_f).$$  \hspace{1cm} (4)

The above definition of Shannon information manifestly is the Gibbs entropy of the initial probability distribution minus that of the final reduced distribution as a result of the message received. This reduction in entropy or missing information quantifies the amount of Shannon information made available.

Similarly one can measure the information content of any thermodynamic system in terms of the difference between two entropies: one of the maximum attainable entropy consistent with the external constraints or the entropy of equilibrium state characterized by the probability distribution of microstates $\{p_{\max}\}$, and the other of the actual entropy of the system with a distribution $\{p\}$:

$$\Delta S = -k_B \sum p_{\max} \ln p_{\max} - (-k_B \sum p \ln p)$$

$$= S_{\max} - S,$$

where $k_B$ is the Boltzmann constant. This is indeed Eq. (1) first proposed by David Layzer \(^4\) and elaborated further by Eric Chaisson \(^1\). Clearly, information has the effect of reducing the entropy from its maximum attainable or equilibrium value. As a way of demonstration we imagine entering a messy and disordered room that is high in entropy or missing information. It is difficult to know where things are therefore low in information content. On the other hand a tidy and ordered room is high in information content because we know where things are–cloths are in the closet, books are on the shelves, etc., thanks to the sentient agent that tidied up the room at the first place. So this definition of information content in terms of distance from equilibrium is indeed consistent with our intuitive notions of order and information available. Furthermore, as pointed out in the introduction, Eq. (1) identifies the equilibrium state with the vacuum of information $\Delta S = 0$. Indeed the state of thermodynamic equilibrium is the simplest state imaginable because it is macroscopically uniform and devoid of any structures, which means that it can be simply described by a handful of thermodynamic degrees of freedom such as a homogeneous temperature, a homogeneous pressure, and homogeneous chemical potentials of constituent particles. But the description of a nonequilibrium system tends to be more complex for it involves the spatial gradients and higher-order derivatives of the thermodynamic variables as well as their time evolutions. So the equilibrium state as the vacuum of information, $\Delta S = 0$, is consistent with our intuitive notion of simplicity while nonequilibrium states with their inherent information contents, $\Delta S > 0$, tend to exhibit more complexity.

B. Semiclassical energy-time quantization rule

In quantum mechanics the incompatible observables whose products have the physical dimensions of action and satisfy commutation relations of the form $[\hat{p}, \hat{q}] = -i\hbar$, where $\hat{p}$ is the momentum operator conjugate with the coordinate $\hat{q}$, not only satisfy uncertainty relations of the form $\Delta p \Delta q \geq \hbar/2$, but also constituted heuristic quantization relations of the form $\oint pdq = nh$ for bound systems in the spirit of Bohr-Wilson-Sommerfeld old quantum theory, until a mathematically rigorous synthesis.
derivation of such quantization relations were obtained from Schrodinger wave equation through the WKB semiclassical approximation \[16\]:

\[
\int_{x_1}^{x_2} p(x)dx = (n + \delta)\pi \hbar. \tag{5}
\]

In Eq. \[15\] \( p(x) \equiv \sqrt{2m(E - V(x))} \) is the magnitude of momentum, \( V(x) \) is the potential energy function, \( x_1 \) and \( x_2 \) are the classical turning points of the bound system such that \( E = V(x_1) = V(x_2) \), \( n = 1, 2, \ldots \) is a positive integer, \( \delta \) is a system-dependent fractional constant in the range \(-1 < \delta < 1\), and \( \hbar = h/2\pi \) with \( h \) the Planck’s constant. Therefore, it is natural to conclude that there must be a semiclassical energy-time quantization rule consistent with the WKB approximation Eq. \[5\], which is of considerable interest to our analysis. In fact by simply applying a change of variables from position to time in Eq. \[5\]

\[
\int_{x_1}^{x_2} p(x)dx = \int_{t_1}^{t_2} p(x(t))\dot{x}dt, \tag{6}
\]

and using \( p \equiv \sqrt{2m(E - V(x(t)))} \equiv m\dot{x} \) for the momentum together with \( E - V(x(t)) = m\dot{x}^2/2 \) for the instantaneous kinetic energy, we readily find the right semiclassical energy-time quantization rule:

\[
\int_{t_1}^{t_2} [E_n - V(x(t))] dt = \frac{1}{2}(n + \delta)\pi \hbar. \tag{7}
\]

In the above semiclassical energy-time quantization rule, Eq. \[7\], \( E_n \)'s are the semiclassical energy eigen-values, and \( V(x(t)) \) is the potential energy function expressed as a function of time. Of all possible trajectories, \( x(t) \) represents the classical or least-action trajectory of the mechanical system in question, and the time limits \( t_1 \) and \( t_2 \) are the classical turning times when \( E_n = V(x(t_1)) = V(x(t_2)) \). Indeed the application of Eq. \[7\] together with the virial theorem to the system of harmonic oscillator and particle in a box with \( \delta = -1/2 \) and \( \delta = 0 \), respectively, results in the exact energy eigen-values being recovered. Eq. \[7\] also is validated by testing it for other potential energy wells to obtain the semiclassical energy eigen-values that are born out of Eq. \[6\]. It is worth noting that recently a similar semiclassical energy-time quantization rule of the form \[ \oint Edt = 2\pi \hbar(n + \beta) \] has been derived for extreme relativistic particles characterized by linear energy-momentum relation \( E = pc \), where \( c \) is the speed of light (in relativistic units \( c = 1 \) \[17\]. We note that exactly the same result follows from WKB semiclassical approximation Eq. \[5\] with our procedure of changing variables from position to time \( dx = \dot{x}dt \) and further noting that \( \dot{x} = c = 1 \) (in relativistic units). Finally it must be pointed out that Eq. \[7\] is subject to the same limitations on the WKB semiclassical approximation Eq. \[5\] from which it derives. However when applied judiciously to bounded systems they can greatly simplify the eigen-energy problem albeit at the cost of some accuracy. The WKB approximation has also been used to handle scattering problems \[18\].

As a way of demonstrating the utility of semiclassical energy-time quantization rule, Eq. \[7\], here we consider the canonical example of the harmonic oscillator potential energy well \( V(x) = \frac{1}{2}m\omega^2 x^2 \). Considering the classical or least-action trajectory \( x(t) = x_0 \cos(\omega t) \), the potential energy as a function of time becomes \( V(x(t)) = \frac{1}{2}m\omega^2 x_0^2 \cos^2(\omega t) \), which on substitution in Eq. \[7\] gives

\[
\frac{1}{2}(n + \delta)\pi \hbar = \int_0^T \left[ E_n - \frac{1}{2}m\omega^2 x_0^2 \cos^2(\omega t) \right] dt
\]

\[
= E_n \frac{T}{2} - \frac{1}{2}m\omega^2 x_0^2 \frac{T}{4}. \tag{8}
\]

On solving Eq. \[8\] for quantized energies \( E_n \), and noting that the time period \( T = 2\pi/\omega \), we get

\[
E_n = (n + \delta)\frac{\hbar \omega}{2} + \frac{1}{4}m\omega^2 x_0^2 = \langle \text{K.E.} \rangle + \langle \text{P.E.} \rangle. \tag{9}
\]

In Eq. \[9\] the second term on the right hand side is manifestly the average potential energy, and therefore the first term must be the average kinetic energy. By virial theorem therefore \( \langle \text{P.E.} \rangle = \langle \text{K.E.} \rangle = (n + \delta)\hbar \omega/2 \), which immediately gives the exact eigen-energies \( E_n = (n + \delta)\hbar \omega \) for harmonic oscillator with \( n = 1, 2, \ldots \) and \( \delta = -1/2 \).

C. Quasistatic information-energy correspondence

At this point we invoke the quasistatic information-energy correspondence that is based on empirically proven facts embodied in both Landauer’s quasistatic limit that specifies the minimum amount of heat generated by erasing a binary bit of information in the limit of long erase cycles,

\[
Q(1 \text{ bit}) = k_B T \ln 2 \tag{10}
\]

\((T \text{ being the ambient temperature}) \[19\], and a well-studied model of quasistatic information heat engine due to Szilárd that implies a bit of information can be converted to work or free energy equivalent of the Landauer’s limit without contradicting the second law of thermodynamics \[10\]:

\[
W(1 \text{ bit}) = k_B T \ln 2. \tag{11}
\]

Indeed more recent experiments conducted on double-well potential systems as binary bits of information have unequivocally proven the validity of Eq. \[10\] for quasistatic erase cycles \[11\] \[13\]. Similarly recent state-of-the-art observations of microscopic particles in stairwell potentials have given definitive proof of Eq. \[11\] \[14\] \[13\].

Although in microscopic systems the quantities such as heat, work, energy, etc., fluctuate and stochastic violations of the second-law have been recorded \[19\], but
on average the second-law of thermodynamics holds well such that according to the Jarzynski equality \[20\]
\[
(\Delta F - W) \leq 0,
\]
where \(F\) and \(W\) are the stochastic free energy and mechanical work, respectively, and the angular brackets denote the ensemble average such that all thermodynamic quantities are obtained from ensemble averaging. According to this statement of the second-law of thermodynamics, Eq. (12), the mechanical work done on the system by an external force tends to increase the free energy of the system. The less than sign means that some of the energy transferred to the system in the form of work is lost by entropy production. The equality therefore only applies to quasistatic processes where all of the external work adds up to the free energy of the system.

It is a matter of considerable interest that the information obtained about a microscopic system through measurements can also be used as a resource for free energy such that the system can gain free energy larger than the thermodynamic mechanical work \(W_{\text{on}} \equiv \langle W \rangle\) applied to it. Thus the generalized second-law that includes the effect of information must read \[21\]
\[
(\Delta F) - W_{\text{on}} \leq k_B T \ln 2
\]
where, \(I\) represents the bits of information obtained by the measurements carried out on the system via a feedback control mechanism, and the equal sign only holds for the quasistatic information heat engines. The generalized second-law, Eq. (13), asserts that in addition to mechanical work, the information obtained by observing the system can be used to increase its free energy through a feedback control mechanism. As an example of how information can be used to raise the free energy in the absence of any mechanical work, consider a microscopic particle in a potential energy staircase at ambient temperature. Due to the finite temperature the particle sometimes jumps up the steps as a result of the random thermal fluctuations although the general tendency is to run down the staircase in order to minimize its free energy consistent with the external constraints. Now if there is a sentient being (such as a fairy, if not a demon!) that blocks the downward path of the particle every time it observes an upward jump, the free energy of the particle increases just through the information obtained in a feedback control procedure, without any mechanical work actually done on the system.

Now because a bit of information can be quasistatically converted to a well-defined amount of heat \(Q\) and work \(W\) given by Eq. (10) and Eq. (11), respectively, it must find correspondence with the energy \(E\) in the limit of quasistatically slow processes through the first-law of thermodynamics:
\[
(\Delta E) = Q_{\text{in}} + W_{\text{on}}
\]
where, \(\langle \Delta E \rangle\) is the change in internal energy, and \(Q_{\text{in}} \equiv \langle Q \rangle\) the thermodynamic heat injected into the system. Hence, a bit of information can raise the energy \(E\) by a well-specified amount through either quasistatic heat or work. It must be emphasized that internal energy \(U = \langle E_n \rangle\) is nothing but the ensemble average of the eigen-energies, which are obtained at a semiclassical level from the energy-time quantization rule Eq. (7). It is worth noting that a similar correspondence between digital information and energy has been proposed in Ref. \[22\] under the title of mass-energy-information equivalence principle.

This empirical quasistatic information-energy correspondence therefore justifies the generalization of semiclassical energy-time quantization rule, Eq. (7), to an all new quasistatic information-time quantization rule
\[
\int_{t_n}^{t_0} [I_n - V(t)] dt = \frac{1}{2} (n + \delta) \pi \hbar,
\]
where \(|I_n|\)'s are the information eigen-values in units of energy, and \(V(t)\) is given by the thermodynamic potential for information \(T(t) \Delta S(t)\), which sets an upper bound (or maximum) for the information content at any time \(t\) during the history of the environment:
\[
V(t) = -T(t) \Delta S(t) .
\]
It must be noted that \(T(t) \Delta S(t)\) is a genuine thermodynamic potential for information because not only it has the dimensions of free energy but also on minimization it gives the state of thermodynamic equilibrium characterized by \(\Delta S = \Delta S_{\text{max}} = -S = 0\) \[22\]. The above Eq. (15), together with Eq. (16), constitute an all new quasistatic information-time quantization rule generally applicable to any slowly varying nonequilibrium thermodynamic system whose thermodynamic potential for information can be determined accurately as a function of time in order to obtain the eigen-informations of the quasistatically evolving physical objects supported by the medium. Hence a significant advantage of Eq. (7) over Eq. (6) is that it can be adapted for application to situations beyond simple mechanical systems by employing a time-dependent thermodynamic potential for information in nonequilibrium environments, Eq. (15), with the right free energy dimensions, and the interpretation of eigen-informations \(|I_n|\)'s as the quantized information contents of the persistent physical structures that are supported by the out-of-equilibrium environment.

With application to the expanding universe in mind, in Eq. (15) \(t_0\) is the present epoch or the age of the universe, and \(t_n\) is given by \(I_n = V(t_n)\) that is comparable with the time that the information state \(I_n\) first appears. By picking a different epoch in place of \(t_0\), Eq. (15) can be applied to different stages of the cosmic evolution including the future ones if the thermodynamic potential for information in Eq. (16) can be predicted for the foreseeable future. It must be noted that \(|I_n|\)'s have the dimensions of (informational) energy and that the information eigen-values in units of bits are given by \(|I_n|/k_B T \ln 2\), where \(k_B T \ln 2\) is the quasistatic energy equivalent of one bit of
information (the Landauer’s limit), and \( \bar{T} \) is the average temperature of the environment that can be estimated from \( \bar{T} \approx \int_{t_0}^{t} T(t) dt / (t_0 - t) \). \( t_0 - t \) is a good upper bound for the age of the information state. It must be emphasized that Eq. (15) sets the maximum informational energy that a system may have at any given time through the thermodynamic potential for information \( T(t) \Delta S(t) \), and plays the role of a bound for quantization in Eq. (15) similar to the minimum mechanical energy set by the potential energy function \( V(x(t)) \) in Eq. (7).

It must also be pointed out that the semiclassical information-time quantization rule Eq. (15), must be regarded as a first step towards a quantum mechanical theory of information in nonequilibrium thermodynamics as it predicts eigen-informations belonging to an as yet to be determined information operator, which may well be linked to the density operator in the form of \(-\ln \rho \). Its ensemble average gives the von Neumann entropy:

\[
S = -\ln \rho = -\sum \lambda_i \ln \lambda_i.
\]

Furthermore, a quantum phase processing algorithm has been recently introduced that also extracts the eigen-information of the quantum systems by measuring the ancilla qubit [23]. Finally, we must also point out that another definition of information states has been recently introduced in Ref. [24] that although instructive and very useful in its own right, does not directly relate to the information eigen-values introduced here.

FIG. 1: The deviation of the universe (or a local environment therein such as ours) from the state of equilibrium \( \Delta S(t_0)/k_B = 2 \ln 2 \times 10^{16} (t/t_0)^2 \) is shown for the matter dominated era with the parameters determined semi-empirically, and highlighted in blue. The growth of the thermodynamic potential for information \( T(t) \Delta S(t) = 5.74 \times 10^{-5} (t/t_0)^{1/3} \) J also is shown and highlighted in red that acts as a bound for the quantization of quasistatic information states (see text for details).

### III. APPLICATION TO EXPANDING UNIVERSE

In order to apply the quasistatic information-time quantization rule, Eq. (15), one needs to know the history of the environment in terms of the thermodynamic potential for information in Eq. (15). So we begin by deriving a phenomenological expression for the thermodynamic potential for information in matter dominated era by employing the holographic principle and taking note of the fact that the most complex phenomenon of the present epoch is the human brain.

The phenomenologically derived expression for the deviation from equilibrium \( \Delta S(t) \) (which became significant following the end of the radiation era), is pictorially depicted in Fig. 1 for the matter dominated era that began 380,000 years following the big-bang:

\[
\frac{\Delta S(t)}{k_B} = 2 \ln 2 \times 10^{16} \left( \frac{t}{t_0} \right)^2.
\]

To arrive at this expression, the holographic principle of the quantum gravity [25], and the upper bound for the equilibrium entropy imprinted on the boundary of the observable universe given by Bekenstein-Hawking formula [26, 27], are used to determine the time-variation of the maximum attainable entropy

\[
\frac{S_{\text{max}}}{k_B} = A / 4 l_p^2 \sim t^2.
\]

In Eq. (18), \( A = 4 \pi \bar{r}^2 \) is the area of the apparent horizon with \( \bar{r} = c / H \) the cosmic radius of the observable flat universe, and \( l_p \equiv \sqrt{\hbar G / c^3} \) is the Planck’s length. The Hubble parameter is given by \( H \equiv a / a = 2 \pi / t \) in the matter dominated era because the cosmic scale factor varies with the time as \( a(t) \sim t^{-2} \). Hence, the maximum attainable or equilibrium entropy of the universe is expected to vary with the second power of time in the matter dominated era as given by Eq. (18). The actual entropy \( S(t) \), and hence \( \Delta S(t) \) shown in Fig. 1, also are taken to have a similar dependence on time but with lower proportionality constants that are chosen phenomenologically by requiring that at the present epoch \( \Delta S(t_0)/k_B \approx 2 \times 10^{16} \ln 2 \), which corresponds to the information content of the most complex phenomenon of our present time, namely, the human brain (more of which later) [28]. Furthermore, the cosmic temperature in matter dominated era varies like \( T(t) \sim a^{-1}(t) \sim t^{-7/2} \). Hence the time-variation of the thermodynamic potential for information in the matter dominated era becomes

\[
T(t) \Delta S(t) \sim t^{7/3}
\]

with a power-law exponent of four-thirds that is also depicted in Fig. 1. It must be noted that determining the precise expression for the actual entropy \( S(t) \) from first principles, and therefore that of the distance from equilibrium \( \Delta S(t) \), is almost impossible because the exact nature of much of the matter and the energy in the
universe in the form of dark matter and dark energy is unknown. So one has to resort to reasonable assumptions and phenomenological numerical parameters to determine the thermodynamic potential for information as accurately as possible.

In view of the power-law nature of the thermodynamic potential for information in matter dominated era, Eq. (19), we begin by considering a generic information well of the power-law form

\[ V(t) = -V_0 \left( \frac{t}{t_0} \right)^\alpha \quad (\alpha > 0). \]  

(20)

\( V_0 \) is a phenomenological parameter denoting the information well-depth at the present epoch the value of which must be determined by requiring that on substitution in Eq. (19) the highest information eigen-value \( |I_1| \) matches the most complex phenomenon of the present epoch, which is almost always taken to be the human brain. The complexity of a species, rather similar to its information content \( [22] \), is usually attributed to the number of accessible states in its environmental response or cognitive bandwidth \( [24] \). For humans this is often quantified in terms of the brain information content, which in more recent estimates is the equivalent of 2.5 million gigabytes or \( 2 \times 10^{16} \) bits of digital memory \( [25] \), although other estimates differ by few orders of magnitude ranging from \( 10^{14} - 10^{18} \) bits. For practical purposes, therefore, the information well-depth at the present epoch is given by

\[ V_0 \approx 2 \times 10^{16} k_B T \ln 2 = 5.74 \times 10^{-5} \text{ J}, \]  

(21)

where \( T = 300 \text{ K} \) is the ambient temperature of the environment within which the brain operates. This is the information well-depth at the present epoch, also shown in Fig. 1.

A. Early stages

Before application to the present epoch, Eq. (15) is solved for the information eigen-values in a range of times that the universe has just begun to deviate from equilibrium in order to demonstrate the sequential birth of the quasistatic information states and their subsequent dynamical evolution. To obtain the eigen-values for an arbitrary epoch \( \tau \), the upper limit \( t_0 \) in Eq. (15) must be replaced by \( \tau \) and the integral performed with \( V(t) \) given by Eq. (20). The lower limit \( t_n \) satisfies \( I_n = V(t_n) \), which solves to give \( t_n = t_0/(|I_n|/V_0)^{1/\alpha} \). On carrying out the integral, the following algebraic equation for the information eigen-values \( x_n \equiv -I_n = |I_n| \) is obtained:

\[ \frac{\alpha}{\alpha + 1} \left( \frac{x_n}{V_0} \right)^{\frac{\alpha + 1}{\alpha + 1}} - \frac{\tau}{t_0} \left( \frac{x_n}{V_0} \right) + \frac{1}{\alpha + 1} \left( \frac{\tau}{t_0} \right)^{\frac{\alpha + 1}{\alpha + 1}} - \frac{\tilde{n}}{t_0 V_0} = 0 \]  

(22)

where \( \tilde{n} \equiv \frac{1}{2}(\alpha + 1)\pi \hbar \). In general Eq. (22) must be solved numerically for the eigen-informations \( x_n(\tau) \) at different epochs. But it solves exactly for the case \( \alpha = 1 \) or the linear information well, which we consider for the sake of illustrating the dynamics of quasistatic information states, as it reduces to a second-order algebraic equation with the solutions

\[ x_n(\tau) = V_0 \tau/t_0 \pm \sqrt{V_0(n + \delta)\pi \hbar/t_0}. \]  

(23)

In Eq. (23) the negative sign gives the physical solutions because the allowed eigen-informations cannot exceed the information well-depth at the epoch \( \tau \) being considered: \( 0 < x_n(\tau) < |V(\tau)| \). It must be noted that the first term in Eq. (23) is nothing but the information well-depth, \( |V(\tau)| = V_0(\tau/t_0)^\alpha \), with \( \alpha = 1 \) for the linear information well. The maximum number of eigen-informations supported by the universe as a function of epoch, \( n_{\text{max}}(\tau) \), is obtained by letting \( x_n = 0 \) in Eq. (22) for the generic power-law information well with the result

\[ n_{\text{max}}(\tau) = \frac{2 |V(\tau)|}{\pi \hbar(1 + \alpha)} - \delta \]  

(24)

(or the integer part thereof). Clearly, in the case of linear information well, \( \alpha = 1 \), the number of quasistatic information states increases with the second power of \( \tau \), while their information contents or complexities increase linearly with \( \tau \). When does the \( n^{\text{th}} \) state first appear in the age of the universe? \( \tau_n \) is obtained by substituting \( n \) for \( n_{\text{max}} \) and \( \tau_n \) for \( \tau \) in Eq. (24), which solves to give

\[ \tau_n = t_0 \left[ \frac{\pi \hbar(1 + \alpha)}{2 t_0 V_0} (n + \delta) \right]^{1/(\alpha + 1)}. \]  

(25)

For \( \alpha = 1 \) or the linear information well, Eq. (25) becomes \( \tau_n = \sqrt{(n + \delta)(\hbar/2V_0) / V_0} \approx \sqrt{n} (1.58 \text{ ms}) \), where the numerical substitutions \( \hbar = 1.055 \times 10^{-34} \text{ J s}, t_0 = 13.7 \times 10^9 \text{ yrs} = 4.32 \times 10^{17} \text{ s}, V_0 \approx 5.74 \times 10^{-5} \text{ J}, \) and \( \delta = 0 \) have been made. This means that the first quasistatic information state approximately appears within the first microsecond of the environment deviating from equilibrium, and so on, as they are marked on the abscissa of Fig. 2. The physical solutions in Eq. (25) with the above numerical substitutions, take the (positive only) values \( x_n(1.33 \times 10^{-28} J) = \tau (\text{ ms}) = 1.58\sqrt{n} \), where the eigen-information \( x_n \) is given in units of \( 1.33 \times 10^{-28} \text{ J} \) when \( \tau \) is measured in microseconds. Furthermore, the minimum information well-depth required to support the \( n^{\text{th}} \) state is obtained by substituting \( \tau_n \) in \( |V(\tau)| \), i.e. \( |V(\tau_n)| = \sqrt{(n + \delta)\pi \hbar V_0/t_0} \approx \sqrt{n} (2.1 \times 10^{-28} \text{ J}) \), which are marked on the ordinate of Fig. 2. In Fig. 2 the dynamics of eigen-informations is plotted for the exactly solvable linear information well. Clearly, the first information state is formed at \( \tau_1 \), the second at \( \tau_2 \), and so on. This is indeed reminiscent of particles (as physical realizations of information states) one by one falling out of equilibrium from the primordial radiation in the early universe whose phenomenology was discussed in the introduction. Although qualitatively consistent with the cosmic evolution phenomenology in the early universe,
that for the particular case of \( \alpha > \tau \) this speed limit is a constant independent of \( \tau \). For \( \alpha < 1 \) this speed limit is expected to decrease while for \( \alpha > 1 \) increase with the time \( \tau \). This means that for the particular case of \( \alpha = \frac{4}{3} \) in the matter dominated era, the evolution changes to the information states are expected to speed up as time passes.

It must be emphasized that the order of the eigen-informations is such that at any epoch \( n = 1 \) identifies the deepest information state with the highest eigenvalue \( x_1(\tau) \). It is the most complex state of its time with the highest information content. On the other hand, the smallest eigen-information corresponds to \( n_{\text{max}} \), which identifies the most primitive of the elementary particles with the lowest information content.

![Diagram](image)

**FIG. 2:** The eigen-informations \( x_n(\tau) \equiv |I_n(\tau)| \) as a function of time in the early stages of the deviation from equilibrium for the exactly solvable linear information well are shown to illustrate qualitatively the emergence and gradual evolution of eigen-informations in early universe. Clearly new information states (shown in blue, red, green, etc.) begin to emerge one by one at \( \tau_n \approx \tau_n(1.58 \mu s) \) when the information well-depth (shown in black) reaches \( |V(\tau_n)| \approx \sqrt{n}(2.1 \times 10^{-28} \text{ J}) \).

**B. Present epoch**

In the matter dominated era the power-law exponent for the information well of Eq. (20) is \( \alpha = \frac{4}{3} \) as shown in Fig. 1. In cosmological applications at the present epoch it is often convenient to work with the red-shift parameter \( z \) instead of the time \( t \). Using \( 1 + z = a^{-1}(t) \), \( dz = -(a/a') dt \), and the Hubble parameter \( H(z) = \alpha/a \), for the change of variables from \( t \) to \( z \) one gets \( dt = -dz/(H(z)(1+z)) \). Eq. (15) with \( z \) as integration variable therefore becomes

\[
\int_{z_n}^{0} [I_n - V(z)] \left( -\frac{dz}{H(z)(1+z)} \right) = \frac{1}{2} (n+\delta)\pi h \tag{26}
\]

where, the upper limit \( z = 0 \) corresponds to the present epoch, the lower limit \( z_n \) is given by \( I_n = V(z_n) \), and the dynamics of the universe is contained in Hubble parameter \( H(z) \). From Friedmann equation \( H(z) = H_0(\Omega_m,0(1+z)^3 + \Omega_{\Lambda,0})^{1/2} \), where \( \Omega_{r,0} \approx 0 \) and \( \Omega_{k,0} \approx 0 \) have been neglected. With \( \Omega_{m,0} + \Omega_{\Lambda,0} = 1 \) and considering *matter-only* Friedmann model with \( \Omega_{\Lambda,0} = 0 \), the Hubble parameter finally becomes \( H(z) = H_0(1+z)^{3/2} \), where \( H_0 = 70 \text{ km/s/Mpc} = 2.27 \times 10^{-18} \text{ s}^{-1} \) is the Hubble constant. On substituting for \( H(z) \) and \( z = -V_0 a^{-1}/(H_0/H_0)^2 = -V_0(1+z)^2 \) in Eq. (26), the following algebraic equation for the information eigen-values \( x_n = -I_n = |I_n| \) at the present epoch is obtained:

\[
\frac{8}{21} \left( \frac{x_n}{V_0} \right)^{7/4} - \frac{2}{3} \left( \frac{x_n}{V_0} \right) + \frac{2}{7} - \frac{\pi h H_0}{2 V_0} (n+\delta) = 0. \tag{27}
\]

It is worth noting that essentially the same result can be obtained from Eq. (22) with \( \alpha = \frac{4}{3} \), \( \tau = t_0 \), and \( H_0 \sim t_0^{-1} \). Eq. (27) gives a *static* picture of the quasistatic information states at the present epoch as shown compactly in Fig. 3. The total number of quasistatic information states supported by the universe at this epoch is obtained from Eq. (27) with \( x_n = 0 \), which gives \( n_{\text{max}} = 4.36 \times 10^{46} \) for the present epoch. Although very large, the total number of eigen-informations supported by the universe at the present epoch is *finite and countable*. It must be noted that the ratio \( V_0/h H_0 = 2.4 \times 10^{47} \) in Eq. (27) represents the information well-depth \( V_0 \) in units of the smallest quantum of energy at the present epoch \( h H_0 = 2.4 \times 10^{-52} \text{ J} \) [32]. As expected, and much unlike the early stages shown in Fig. 2, the spectrum of eigen-informations at the present epoch is *quasi-continuous*.

Figure 3 shows the normalized information potential well \( V(z)/V_0 \) at the present epoch with some \( I_n \)'s shown at representative intervals \( n = 10^{46}, 10^{45}, 10^{44}, \ldots \) from top to bottom. The inset shows a semi-log plot of the normalized quasi-continuous eigen-informations \( x_n/V_0 \), which at about \( n = 10^{45} \) makes a sharp turn towards a plateau that converges to unity. This is interpreted as a turn of events towards a radically more complex era within the matter dominated era that occurs at a time...
given by $V(t_{1045}) = I_{1045} = -0.84V_0$, which solves to give $t_{1045} = (0.84)^{\frac{3}{4}} t_0 = 0.88t_0$. It is a matter of considerable interest to note that this time corresponds to 1.7 billion years ago when for the first time multi-cellular life forms appeared on the earth and the life era or biological evolution began in its earnest. Evidently, evolution can be more predictable than previously thought \cite{33}, and that biological evolution can be regarded as a special case of a more general cosmic evolutionary scenario \cite{1}. Furthermore, the transition from non-living to living species as depicted in the inset of Fig. 3 is best characterized by a gradual cross-over rather than an abrupt phase transition, which means that there is no life force separating the living from the non-living but their difference is primarily due to their vastly different information contents or eigen-informations. Finally, it must be noted that the exponent $\alpha = \frac{4}{3} (> 1)$ in the matter dominated era indicates that, unlike the case of linear information well shown in Fig. 2, the rates of change of the quasistatic eigen-informations must increase with the time in matter dominated era and should be at its highest ever at the present epoch, which is consistent with the most recent studies finding accelerated changes in all studied animal genomes at the present time \cite{34}.

**FIG. 3:** The normalized information well at the present epoch $V(z)/V_0 = - (1+z)^{-2}$ as a function of red-shift parameter $z$. The spectrum of eigen-informations is quasi-continuous with some representative $I_n$’s drawn explicitly as horizontal lines for $n = 10^{46}, 10^{45}, 10^{44}, \ldots$ from top to bottom. Inset shows the semi-log plot of the normalized eigen-informations $x_n/V_0$ as a function of log $n$ with a characteristic sharp turn to a plateau of complexity at about $n = 10^{45}$, which corresponds to the commencement of the life era.

**IV. SUMMARY AND FINAL REMARKS**

To summarize, therefore, an all new information-time quantization rule for quasistatic information states in nonequilibrium thermodynamics based on (i) Shannon information in nonequilibrium systems, (ii) semiclassical energy-time quantization rule, and (iii) quasistatic information-energy correspondence, is presented that must be generally applicable to any slowly varying nonequilibrium environment where the distance from equilibrium and the mean temperature are known as a function of time. This all new quasistatic information-time quantization rule is applied to cosmic evolution in the matter dominated era (albeit at a mean-field level) by employing a global thermodynamic potential for information with phenomenologically determined parameters. It is a matter of considerable interest that the results obtained from this procedure are in agreement with the general phenomenology of the cosmic evolution at a quantitative level. In particular a turn of events to the life era or biological evolution is detected by the theory starting about 1.7 billion years ago that is in fact when the multi-cellular life forms appeared on the earth of which we are only one of the descendants. As a result biological evolution must be regarded as a special case of a more general cosmic evolutionary scenario that encompasses both animate and inanimate species as material realizations of the quasistatic information states in nonequilibrium environments \cite{1}. In our theory, therefore, the persistent structures (it) are just the material realizations of the quasistatic information states (bit) supported by the universe, which is best epitomized by the late Wheeler’s famous catch phrase “it from bit”, or to use his precise words \cite{2}

“Otherwise stated, every physical quantity, every it, derives its ultimate significance from bits, binary yes-or-no indications, a conclusion which we epitomize in the phrase, it from bit.”

Furthermore, the semiclassical information-time quantization rule introduced here, opens new avenues for further research. Indeed a more comprehensive application to cosmic evolution must also involve the inclusion of the radiation dominated era of the early universe, which can be included in the calculations through its appropriate thermodynamic potential for information. This must lead to a quantitatively accurate time line for the birth of elementary particles, nuclei, light atoms, etc., a qualitative description of which can be already seen in Fig. 2. Such a calculation may also detect other elementary information states that are candidates for dark matter, in addition to the better known baryonic matter. Another significant direction for further research arises from the fact that past history plays a significant role in future evolution through the thermodynamic potential for information in the quasistatic information-time quantization rule. As a result of this thermodynamic memory, long after the environment reaches the state of thermodynamic equilibrium the quantized information states can
continue to exist within the environment, which is to say that quantum processes can continue for a long time after equilibration due to the past deviations from equilibrium registered in the memory. This also is consistent with the recently proposed so-called second-law of quantum complexity [35], which asserts that quantum processes do persist in a system for a long time after equilibration until they too die out. Finally it must be noted that the quasistatic information-time quantization rule presented here constitutes the first step towards a quantum mechanical theory of information in nonequilibrium thermodynamics, which may entail time-evolution operators acting on information states, information operator and information-time uncertainty relation, etc.

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