Randomness or stock–flow: which mechanism describes labour market matching in Poland?

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ABSTRACT
I identify which theoretical model (random, stock–flow, or job queuing) best describes the matching mechanism in the labour market in Poland. The purpose of this work is to formulate policy recommendations aimed at increasing the number of matches. I use monthly registered unemployment data for the period January 1999–June 2013 and econometrically correct for temporal aggregation bias in the data. I extend known solutions and apply them directly to a job queuing model. Job seekers (from the pool) seek work among old and new job posts, but only a small fraction of the newly unemployed individuals find work quickly. Vacancies are the driving force in aggregate hiring, but the inflow is more important than the stock. The random model has greater explanatory power, although the results do not negate the non-random model. Hence, better information and higher inflows (especially of job offers) should facilitate matching.

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1. Introduction

In this study, I identify which theoretical model best describes the matching mechanism occurring in the labour market in Poland. I study three frameworks: random, stock–flow, and job queuing. They offer competing explanations of how job seekers and job vacancies are matched. By comparing these matching technologies, I try to identify how stock and inflow variables affect the matching process. With this knowledge, I formulate policy recommendations aimed at improving the effectiveness of a matching process in the labour market.

I use administrative data and focus on public employment intermediation in Poland during the time span January 1999–June 2013. The Polish labour market is undoubtedly an understudied case. Poland has undergone fundamental changes since the beginning of the transition from a centrally planned to a market-oriented economy. Specific goals were partially or fully achieved at the cost of substantial labour market adjustment. Large fractions of workers who could not adapt to new conditions remained unemployed or became inactive (Lehmann, 2012). Over the past 20 years, unemployment rates oscillated...
between less than 7% (in 2008) and more than 20% (in 2004); Münich and Svejnar (2009) indicated that unemployment in Poland originated from insufficient matching.

I analyse the time span January 1999–June 2013. Initial transformation period reforms were implemented before then, but various amendments and small modifications were regularly enforced during the studied period. In terms of a job matching, this time span reflects a relatively homogenous period – a loop around a downward-sloping Beveridge curve. Hence, no major reallocation changes should have occurred.

I use registered unemployment data. The main quality of this dataset is that the data refer to more than two million people registered as unemployed (mean stock value). Their self-reliance in the market is poor (approximately 75% of the unemployment inflow are individuals who have been registered previously). These workers are considered unemployed once they register, fulfil some legal conditions, and periodically renew their eagerness and readiness to work. Unfortunately, public employment offices do not verify active job seeking. Registering is a prerequisite to obtain free health insurance or some social welfare benefits. Hence, not all registered workers actually seek jobs. However, Góra (2005) estimated that in 2002, approximately 85% of the registered workers fulfilled the LFS unemployment definition, meaning that they actively sought jobs. Only registered workers can participate in ALMP programmes.

Registered workers use various job search methods, but job offers announced at public employment offices are offered to them only. When a worker reports a job match, the vacancy does not have to come from public employment intermediation. The outflow rate from unemployment to employment refers to workers, not covered job postings, but it resembles registered vacancy time series. This indicates that registered job postings are the driving force of the matching process of the registered unemployed individuals. Hence, when we realize that registered unemployed workers often do not fend for themselves in the labour market and experience multiple unemployment spells, understanding what helps them collectively form a job match becomes necessary.

In the literature, one can find numerous studies of the aggregate matching function. The seminal review can be found in Petrongolo and Pissarides (2001). They contended that a matching function was a nice but somewhat black box, aggregate function, the existence of which was well documented but lacked microfoundations. Petrongolo and Pissarides (2001) grouped research according to the type of study, methods used, or particular research areas. They referred, among other factors, to worker heterogeneity, mismatch, ranking, and data aggregation. They presented studies on the Beveridge curve and aggregate, sectoral, and micro studies. Petrongolo and Pissarides (2001) showed that particular elasticities differed due to time spans and data properties, but the empirical elasticity on unemployment typically ranged from 0.5 to 0.7. Subsequent analyses focused on certain aspects that could alter the empirical results of the matching function elasticities. Galuščák and Münich (2007) analysed how different worker flows (e.g. to and from inactivity) affected matching function elasticities. Münich and Svejnar (2009) controlled for search endogeneity and spurious scale effects and used panel data to analyse how the matching process affected the unemployment level. Andrews, Bradley, Stott, and Upward (2013) studied the stock–flow matching concept using micro data. Borowczyk-Martins, Jolivet, and Postel-Vinay (2013) referred to search endogeneity bias. Stevens (2002) studied the functional form of the matching function, while Poeschel (2012) analysed the time trend in the matching function.
Previous aggregate studies addressing the Polish labour market yielded more varied results than those summarized in Petrongolo and Pissarides (2001). The empirical elasticity on unemployment ranged from 0.3 to even 0.9 (see, e.g. Kucharski & Tokarski, 2003; Kwiatkowski & Tokarski, 1997; Lehmann, 1995 or Roszkowska, 2009). These papers rarely explicitly defined the type of technology that described the trade process and typically assumed matching between the unemployment stock and vacancy inflow. Often, the aim of such research was to identify the determinants of the efficiency of the matching process from a regional perspective, but spatial interactions were neglected. This goal was primarily achieved by estimating an augmented matching function.

The above arguments convince me that Poland is an interesting but understudied case whose analysis can produce some universal findings. To my knowledge, few papers have previously analysed the matching technology, and even fewer have addressed the bias in the parameter estimates while analysing labour market matching in Poland. Galecka-Burdziak (2012) considered temporal aggregation bias in the data (employing the Gregg and Petrongolo (2005) framework), Antczak, Galecka-Burdziak, and Pater (2016) referred to spatial aggregation bias in the data. Being aware of data limitations, I want to fill this gap in the literature and try to identify how job matchings are being created at the aggregate level.

I contribute to the literature in several ways. I provide a thorough analysis of a matching process in the Polish labour market, specifically concerning public employment intermediation. I use overidentified specifications of selected matching function models to identify which matching technology prevails in the labour market in Poland and which type of agents (stocks or flows) form pairs. I explicitly address the nature of matching technology while econometrically addressing the temporal aggregation bias in the data. I employ two temporal data aggregation solutions (Gregg & Petrongolo 2005; Coles & Petrongolo 2008) to identify matching function elasticities with respect to stocks, flows, and ‘at-risk’ measures. I extend these solutions to directly analyse the job queuing framework.

I find that both stocks and inflows of agents engage in a matching process. Job seekers (from the pool) seek work among old and new job posts, but only a small fraction of newly unemployed individuals enjoy a positive instantaneous re-employment probability. Demand is the driving force in aggregate hiring, but the vacancy inflow is more important than the stock counterpart. The positive elasticity of the vacancy stock proves that not all job offers are covered instantaneously, despite large discrepancies between demand and supply. I find that the random model appears to be more relevant, but the results do not negate the non-random matching mechanism. Random matching suggests that the information in the labour market must be improved to facilitate the matching process. The stock–flow model justifies labour market policy actions intended to create more job offers and encourage new job seekers to register as unemployed. The coexistence of both mechanisms indicates segmentation and heterogeneity of the job seekers and job vacancies, which differentiate the hazard rates of agents leaving the market.

The paper proceeds as follows. In Section 2, I describe labour market matching models and the method I use in the computations. Next, in Section 3, I describe the data used in the empirical verification, which is presented in Section 4. I discuss the results in Section 5 and provide concluding remarks in Section 6.
2. Labour market matching models and the method

The matching function is a well-known tool used to analyse aggregate matching processes. Models differ in their description of the matching mechanism, depending on the assumptions made regarding the impact of stocks and flows. In a random (stock–stock) model (see, e.g. Blanchard & Diamond, 1994), a match occurs once a job seeker is assigned to a particular job. Vacancies and unemployment coexist due to coordination failure among agents, even if demand equals supply. The stock–flow model (see, e.g. Coles & Smith, 1998) assumes perfect information to reflect the fact that agents first consider numerous advertisements before applying for selected job offers, and once an offer has been rejected, reapplication is less likely than a search for new vacancies. Agents who remain in the job market lack a proper partner, as all trade options have been exploited. The job queuing model (see, e.g. Shapiro & Stiglitz, 1984, matching takes place randomly) is formulated to reflect large discrepancies between demand and supply. The short side of the market clears in each period, but an insufficient number of vacancies mean that workers must wait for new job postings.

Each theoretical model, random (stock–stock and job queuing) and stock–flow, offers a different explanation for the matching process. Matching is determined either by stocks or a combination of stocks and flows. Appropriate specifications of the matching function are used to verify theoretical assumptions concerning the matching technologies. Typically, the form is a Cobb–Douglas function, \( M = m(U, u, V, v) \), where \( M \) represents the number of matches, \( U \) is the unemployment stock, \( u \) is the unemployment inflow, \( V \) is the vacancy stock, and \( v \) is the vacancy inflow.

In the empirical analysis, I extended the basic forms and assumed more general specifications of the matching function. I attempted to determine the importance of particular matching mechanisms. Coles and Smith (1998) considered the outflow from unemployment to employment (disaggregated according to search duration) to be dependent on a vacancy stock and an inflow. Gregg and Petrongolo (2005) proposed a test to verify stock–flow and random mechanisms by enabling an unemployment stock to match both the stock and the inflow of job offers, as did Coles and Petrongolo (2008). I extended this test (using overidentified specifications) to determine what types of agents form pairs.

I performed the analysis and econometrically corrected for temporal aggregation bias in the data. Such bias arises when continuous economic processes are described using discrete data. Coles and Smith (1998) presented an example. A job posting is published at the beginning of the month but is not matched until the end of the month. A job seeker arrives in a market at the end of the month. Monthly data present both agents as part of an inflow, although from a job seeker’s perspective, the vacancy is part of a stock. Moreover, the literature suggests that large fractions of newcomers immediately match after entering the market. These agents are not reflected in end-of-period stocks, and hence, the stocks do not properly approximate job seeker or vacancy pools (Gregg & Petrongolo, 2005; Petrongolo & Pissarides, 2001). Temporal aggregation bias in the data leads to understatement of the importance of stocks and overstatement of the importance of inflows.

In this article, I addressed the problem of temporal aggregation bias in the data using the frameworks of Gregg and Petrongolo (2005) – hereafter the GP model – and Coles and Petrongolo (2008) – hereafter CP model – and proposed a slight modification that enables
the direct estimation of a job queuing model. The GP model and the CP model identified
the number of agents available for matching. However, only the second framework fully
accounted for temporal aggregation in the data. Coles and Petrongolo (2008) highlighted
differences between frameworks that mathematically address the temporal aggregation
problem in the data. Studies have adopted different conditions for the hazard rate of
exiting unemployment. In the GP model, this rate depended on beginning-of-period
stocks (in random matching) or on a stock and a relevant inflow (in stock–flow matching).
The CP model conditioned the hazard rate on ‘at-risk’ measures.2 Thus, even in the random
model, the vacancy inflow was operationalized through a vacancy ‘at-risk’ variable. This
indicates that the GP model is biased against the random mechanism because it does
not fully reflect temporal aggregation in the data (Coles & Petrongolo, 2008). However,
the GP model provides coefficients that enable calculations of the mean elasticities of
the dependent variable with respect to stocks and flows. These elasticities are typically
the most important outcome of the empirical analysis. The CP model directly estimates
elasticities of the dependent variable with respect to ‘at-risk’ measures of unemployment
and vacancies as well as the inflows of unemployed individuals and job postings. There-
fore, the quantitative results of the two frameworks are not explicitly comparable, but
they do complement one another.

The GP model conditioned the number of matched unemployed job seekers on unem-
ployment stocks and inflows and corresponding outflow rates. The specific models can be
verified using appropriate specifications of λ (unemployment stock hazard rate) and p
(instantaneous unemployment inflow matching probability). In simple random matching,
for example, λ depends on beginning-of-period stocks, but in full non-random matching, it
depends on the unemployment stock and vacancy inflow, while p depends on the unem-
ployment inflow and vacancy stock. The equation for the outflow from unemployment for
a random model takes the following form:

\[ M_t = U_t \left(1 - e^{-\lambda_t}\right) + u_t \left(1 - \frac{1 - e^{-\lambda_t}}{\lambda_t}\right), \]

where \( \lambda = \lambda(U, V) \). For a stock–flow model, it takes the following form:

\[ M_t = U_t \left(1 - e^{-\lambda_t}\right) + u_t \left(1 - (1 - p_t) \frac{1 - e^{-\lambda_t}}{\lambda_t}\right), \]

where \( \lambda = \lambda(U, v) \), \( p = p(u, V) \) and \( M_t \) – the number of matched unemployed individuals
during month \( t \), \( U_t \) – beginning-of-month \( t \) unemployment stock, \( u_t \) – unemployment
inflow during month \( t \), \( \lambda_t \) – unemployment stock hazard rate, and \( p_t \) – instantaneous
unemployment inflow matching probability.

A straightforward modification produces a job queuing model. We assume a random
matching mechanism and that job seekers only match with vacancy inflow; hence, the
total number of matches equals:

\[ M_t = U_t \left(1 - e^{-\lambda_t}\right) + u_t \left(1 - \frac{1 - e^{-\lambda_t}}{\lambda_t}\right), \]

where \( \lambda = \lambda(U, v) \).
In the CP model, the total number of matched job seeker–vacancy pairs in random matching equals:

$$M_t = \lambda_t \bar{U}_t,$$

while in stock–flow matching, it equals:

$$M_t = \lambda_t \bar{U}_t + p_t u_t,$$

where $\bar{U}_t$ is the unemployment ‘at-risk’ measure.

In a stock-based mechanism, the number of matches depends on the unemployment and vacancy ‘at-risk’ measures ($\bar{V}_t$). The non-random model assumes that members of the unemployment pool match the vacancy inflow, while individuals in the unemployment inflow match members of the vacancy pool.

Under a job queuing framework, we can assume a random mechanism:

$$M_t = \lambda_t \bar{U}_t,$$

where the unemployment pool matches the vacancy inflow:

$$\lambda_t \bar{U}_t = v_t \text{ with } \lambda_t = \lambda_t(\bar{U}_t, v_t).$$

### 3. Data

I analysed the period January 1999–June 2013 (using monthly, seasonally adjusted registered unemployment data). The labour market exhibited an anticlockwise loop around a downward-sloping Beveridge curve. The period 1999–2002 reflected a recession. Since 2002, aggregate economic activity has improved, although the sub-period 2002–2004 was a jobless recovery caused primarily by labour productivity growth (Drozdowicz-Bieć, 2012). In 2008, the UV curve began to reverse.

The values of the labour market tightness indices suggested that on average, job seekers experienced difficulty finding work but that enterprises found workers with relative ease. The best conditions for job seekers existed between 2007 and 2009 (partially prolonged to the end of 2010). The index based on vacancy inflow was constantly above that based on the vacancy stock, confirming the importance of a flow determinant. The former also exhibited greater volatility and short-term variations (Figure 1).

![Figure 1. Labour market tightness indices, January 1999–June 2013. $V/U$ – stock-based index, $v/U$ – inflow-based index. Source: Registered unemployment 1999–2013, seasonally adjusted data, Author’s calculation.](image-url)
Table 1 provides the summary statistics for the selected variables: total outflow from unemployment, outflow from unemployment to employment, unemployment stock, unemployment inflow, vacancy stock, and vacancy inflow. Unit root tests for all variables in first differences rejected the null hypothesis of the existence of a unit root. Higher turnover was observed in vacancies than among the unemployed individuals. The degree of volatility in the monthly inflow/stock ratios was much higher in the case of vacancy variables (from 1.3 to 5.0 for vacancies, versus 0.06 to 0.17 for unemployment). Hence, vacancies were both much more volatile and of lower expected duration than unemployment. All series displayed a high degree of persistence. Monthly autocorrelation coefficients were slightly higher for stock variables than for flow variables.

4. Results

I estimated unemployment outflow equations to analyse matching technologies prevailing in the labour market in Poland. I used the GP model and the CP model, as they produce complementary results. Table 2 presents the results using the GP model for different specifications of \( \lambda \) and \( p \). All of the estimated specifications are reported in Appendix. The analysis was based on seasonally adjusted monthly registered unemployment data, where the outflow from unemployment to employment was an endogenous variable. The estimation was performed using non-linear least squares and including first-order serial correlation in the disturbance term to address autocorrelation. ADF tests indicated that, at the 5% significance level, the null hypothesis of the presence of a unit root in the disturbance term was rejected. Therefore, although the time series were non-stationary, cointegration occurred, and the equations converged to the long-run equilibrium.
Table 2. Estimates of time-aggregated matching models, GP model, January 1999–June 2013.

| Independent variable/statistics | Parameter estimates (student’s t-statistics) | Model I | Model II | Model III | Model IV | Model V |
|--------------------------------|-----------------------------------------------|---------|----------|-----------|----------|---------|
| $\alpha_0$                     | −1.833***                                      | (0.084) | −2.085*** | −2.023*** | −1.917*** | −2.158*** |
| $\alpha_1$                     | 0.093***                                       | (0.025) | –         | 0.076***  | –         | 0.236*** |
| $\alpha_2$                     | 0.271***                                       | (0.09)  | 0.350***  | 0.264***  | 0.388***  | –       |
| $\gamma_0$                     | –                                             | (0.041) | −1.996*** | –         | –         | –       |
| $\gamma_1$                     | –                                             | (0.243) | 0.494***  | –         | –         | –       |
| $\lambda$                      | (.0410)                                       | [0.0364] | [0.0377] | [0.0377]  | [0.0412] |
| $\rho$                         | .004                                          | [.0508]  | –        | –         | –        | –       |
| $R^2$ (adj. $R^2$)             | .881 (0.021)                                  | .877 (.0875) | .878 (.097) | .873 (.007) | .871 (.007) | .856 (.007) |
| ADF test for residuals         | −13.14 (p-value)                              | −13.23 (.00) | −13.22 (.00) | −13.30 (.00) | −14.10 (.00) |
| Log likelihood                 | −1688.21 (p-value)                            | −1690.89 (.00) | −1690.49 (.00) | −1690.49 (.00) | −1704.92 (.00) |
| Mean unemployment duration (in months) | 24.4 | 26.1 | 26.5 | 26.5 | 24.3 |
| Elasticities:                  |                                               |         |         |         |         |         |
| $\partial M / \partial U$      | .593                                          | .544    | .571     | .527     | .722     |
| $\partial M / \partial u$      | .047                                          | .099    | .043     | .043     | .047     |
| $\partial M / \partial V$      | .091                                          | .032    | –        | .232     |
| $\partial M / \partial \lambda$| .264                                          | .306    | .238     | .351     | –        |

Notes: Model I – overidentified stock–flow with $\lambda = \lambda(U, V)$ and $p = \text{const}$, Model II – full stock–flow with $\lambda = \lambda(U, V)$ and $p = p(u, V)$, Model III – overidentified job queuing with $\lambda = \lambda(U, V, v)$, Model IV – job queuing with $\lambda = \lambda(U, V)$, Model V – stock-based with $\lambda = \lambda(U, V)$. Dependent variable: outflow from unemployment to employment. Estimation method: non-linear least squares. Standard errors reported in brackets. Expected duration measured in months and counted as $(1 - p)/\lambda$. The matching elasticities are sample averages. The sample averages of $\lambda$ and $p$ are reported in square brackets.

*Significant at the 10% level,
**Significant at the 5% level,
***Significant at the 1% level.

Source: Registered unemployment 1999–2013, seasonally adjusted data, Author's calculation.

Table 2 includes structural parameter estimates and summary statistics. The hazard rates for leaving the unemployment stock and unemployment inflow (instantaneous match) were estimated or computed on the basis of coefficients (the average values of model predictions are presented in square brackets).

I began the analysis using the most general specifications of $\lambda$ and $p$. Most of the estimations produced statistically insignificant and/or incorrectly signed coefficients. I selected five equations that allowed for qualitative inference. Econometrically, these models were nested, but each one reflected a different economic explanation of how job seekers and vacancies formed pairs. Model I (of Table 2): non-random matching, in which $p$ was assumed to be constant, while the unemployed individuals sought work among old and new vacancies. Model II: full stock–flow specification in which a stock on one side matched an inflow on the other. The random model assumed that $\alpha_1 > 0$ and $p = 0$, while the stock–flow framework assumed that $\alpha_1 = 0$ and $p > 0$. Models III and IV referred to the job queuing concept. Model III: the unemployment stock
matched both vacancy variables. Econometrically, this was a reduced-form version of the specification in column I (except for $p$), but it assumed a random mechanism. Model IV: pure job queuing model. Model V: random stock-based model.

The positive and significant estimate of $\alpha_1$ (column V) confirmed the importance of a vacancy stock in the random model. Equations that included the vacancy inflow provided new insights into the significance of the stock and inflow variables. The stock coefficient remained statistically significant in all specifications, but its magnitude decreased sharply. The elasticities computed with respect to stocks and inflows indicated that the unemployment stock remained the most important variable in generating outflow from unemployment to employment across all specifications. The elasticity of the vacancy stock decreased sharply once its inflow counterpart was incorporated into the analysis.

The exit rate from unemployment to employment had a mean value of 0.043 and fluctuated between 0.025 and 0.066. The mean predicted hazard rate of exiting unemployment was in the range 0.036–0.041. The hazard rate was highest in the random and job queuing models. The small or statistically insignificant values of $p$ did not create substantial variations in $\lambda$ values. Models incorporating vacancy inflow better reflected short-term variations in the exit rate, but they more substantially underestimated the mean value. The goodness-of-fit of the predicted $\lambda$ values deteriorated considerably since 2009 (Figure 2).

Unemployed individuals spent, on average, at least 24 months in unemployment, with relatively comparable results across different specifications. Models that included the vacancy inflow yielded slightly higher values. The instantaneous matching probability for the unemployment inflow was positive and significant at the 5% level only in the full stock–flow equation, implying that only 5% of newcomers obtained suitable job offers immediately after entering the market.

I based the second part of the analysis on the CP model. These specifications (Table 3) were analogous to those presented in Table 2. The full stock–flow model (Model II) yielded

![Figure 2](image-url)
The statistically significant parameter estimates for the stock-based model demonstrated that a random mechanism was operating in the labour market. The vacancy pool had a slightly greater influence on the outflow from unemployment to employment than did unemployment. In the job queuing models, the unemployment pool had a higher elasticity than the vacancy pool. Equations incorporating the vacancy inflow confirmed its role in the matching process. A direct comparison of the vacancy pool and vacancy inflow indicates that the flow had a greater impact on generating matches (not in Model I; however, this stock–flow model also contained an insignificant estimate of $p$). The instantaneous matching probability of the newly unemployed was close to zero and statistically insignificant.

The highest mean re-employment probability was observed in the job queuing model predictions (columns III and IV) – 0.0414. The random model yielded a virtually identical value – 0.0413. Random mechanisms most accurately reflected the mean exit rate from unemployment. The reduced-form stock–flow specification yielded the most severely underestimated $\lambda$ value (apart from the full stock–flow specification). The goodness-of-fit deteriorated slightly, which implied underestimated values of $\lambda$, particularly since 2009 (Figure 3). The mean unemployment duration across the specifications was at least 24 months.

Table 3. Estimates of time-aggregated matching models, CP model, January 1999–June 2013.

| Independent variable/statistics | Parameters estimates (Student’s t-statistics) | Model I | Model II | Model III | Model IV | Model V |
|--------------------------------|------------------------------------------------|---------|----------|-----------|----------|---------|
| Const for $(\bar{U}_t, \bar{V}_t)$ pair | $0.044$ | $0.001$ | $0.230$ | $0.650$ | $0.087$ | $0.069$ |
| $(0.063)$ | $(0.007)$ | $(0.221)$ | $(0.547)$ | $(0.097)$ | $0.230$ |
| $\alpha_1(\bar{U}_t)$ | $0.307**$ | $0.084$ | $-0.428**$ | $-0.487**$ | $-0.353**$ | $0.069$ |
| $(0.078)$ | $(0.619)$ | $(0.051)$ | $(0.043)$ | $(0.056)$ | $(0.069)$ |
| $\alpha_2(\bar{V}_t)$ | $0.224***$ | $0.262$ | $0.149**$ | $-0.396***$ | $0.396***$ | $(0.069)$ |
| $(0.069)$ | $(0.343)$ | $(0.073)$ | $0.087$ | $(0.097)$ | $(0.069)$ |
| $\alpha_3(u_t)$ | $-0.859***$ | $-0.859***$ | $-0.859***$ | $-0.859***$ | $-0.859***$ |
| $\alpha_4(v_t)$ | $0.171**$ | $0.452$ | $0.261***$ | $0.395***$ | $0.395***$ | $0.078$ |
| $(0.078)$ | $(0.301)$ | $(0.072)$ | $(0.031)$ | $(0.031)$ | $(0.078)$ |
| Const for $(u_t, \bar{V}_t)$ pair | $-350.088$ | $-350.088$ | $-350.088$ | $-350.088$ | $-350.088$ |
| $p$ | $0.024$ | $0.024$ | $0.024$ | $0.024$ | $0.024$ |
| $(0.029)$ | $(0.029)$ | $(0.029)$ | $(0.029)$ | $(0.029)$ | $(0.029)$ |
| $R^2$ | $0.881$ | $0.876$ | $0.876$ | $0.873$ | $0.867$ |
| $(adj. R^2)$ | $0.877$ | $0.872$ | $0.873$ | $0.871$ | $0.865$ |
| ADF test for residuals | $-13.11$ | $-13.32$ | $-13.31$ | $-13.31$ | $-13.31$ |
| $(p$-value) | $-13.11$ | $-13.32$ | $-13.31$ | $-13.31$ | $-13.31$ |
| Log likelihood | $-1688.58$ | $-1691.67$ | $-1691.94$ | $-1694.03$ | $-1697.93$ |
| $\lambda$ | $0.0390$ | $0.0258$ | $0.0414$ | $0.0414$ | $0.0413$ |
| Mean unemployment duration (in months) | $25.0$ | $32.7$ | $24.2$ | $24.2$ | $24.2$ |

Notes: Equation describing outflow from unemployment to employment includes AR(1). Model I – overidentified stock–flow $\lambda = \lambda(\bar{U}, \bar{V}, v)$ and $p =$ const, Model III – overidentified job queuing $\lambda = \lambda(\bar{U}, \bar{V}, v)$, Model IV – job queuing $\lambda = \lambda(\bar{U}, v)$, Model V – stock-based $\lambda = \lambda(U, V)$. Estimation method: non-linear least squares. Standard errors reported in brackets. Expected duration measured in months and calculated as $(1 - p)/\lambda$. The sample averages of $\lambda$ and $p$ are reported in square brackets.

*Significant at the 10% level,
**Significant at the 5% level,
***Significant at the 1% level.

Source: Registered unemployment 1999–2013, seasonally adjusted data, Author’s calculation.

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5. Discussion

The estimated elasticities indicated that most of the matches originated from the unemployment stock and vacancy inflow. It appears that many job seekers were situated on the long side of the market and waited for new job postings to arrive. Policy actions should then enhance job posting creation. More vacancies would increase the number of job matches.

Matches between the unemployment inflow and vacancy stock played a lesser role in generating the outflow from unemployment to employment. Few workers enjoyed positive instantaneous re-employment probability. However, the positive elasticity of the vacancy stock also proved that not all vacancies were covered instantaneously (as the vacancy inflow) but remained in the market as a part of the stock. Hence, higher vacancy inflow would likely entail an increase in the magnitude of the vacancy stock. Moreover, newly unemployed individuals would then have more offers to choose from.

Positive elasticities on the vacancy stock and vacancy inflow indicated that job seekers considered both old and new job offers. This finding would imply that random and non-random matching co-existed in the market. Policy actions should then be aimed at increasing the information and decreasing mismatch in the labour market – for example, by increasing the number of diversified available job offers to compensate heterogeneous job seekers.

The random model outweighed the stock–flow model, although the results did not preclude the later matching technology. Superiority of a random mechanism (in either the stock-based or job queuing form) means that labour market policy should be directed at improving the information in the labour market to facilitate matching. The job search process is costly, and once agents have better information about one another’s locations, they should form more matches. Some explanatory power of the stock–flow model proved that heterogeneous agents engaged in search activity. Here, the quality of matches matters, and not all agents can form pairs. In stock–flow matching, all agents are better off when there are more newcomers in the market; thus, the policy recommendation
should be to increase the inflows of job seekers and job vacancies in the market to increase the number of job matches.

All estimates indicated a mean unemployment duration of at least 24 months. These results overestimated the true value of the mean unemployment duration, but the models assumed the outflow from unemployment to employment only, and the outflow from unemployment to employment constituted on average 45% of the total outflow from unemployment during the studied period. The LFS data showed that the mean unemployment duration oscillated between 9 and 19 months during the period 2003–2013, averaging 14 months.

The findings of my study were mostly consistent with findings reported in the literature, but some differences emerged. The elasticity on the unemployment stock fell in the common range, but the vacancy stock played a minor role in the job creation process. Compared to Gregg and Petrongolo (2005), in my study only the stock-based model produced comparable results. In other specifications, the elasticity with respect to the unemployment stock was much higher in Poland. In Gregg and Petrongolo (2005), the newcomers took the lead in creating new matches. In Poland, the unemployment stock matched with the vacancy inflow, regardless of the matching framework offered to explain the findings. I found it more difficult to compare my results with Coles and Petrongolo (2008) or Álvarez de Toledo, Núñez, and Usabiaga (2008), as the full stock–flow model produced insignificant results. However, in Poland, as in Spain, vacancies were the driving force of the new matches. My findings were also comparable to previous results for the Polish labour market. The unemployment stock enjoyed the highest elasticity in most of the results, but demand was the driving force in the job creation process. The exit rate from unemployment to employment appeared to have jointly depended on the stock and inflow of new job offers, whereas certain estimates reflected indices of labour market tightness. However, the point estimates in my paper were more quantitatively robust than those of previous contributions. They differed because I econometrically addressed the temporal aggregation problem in the data and did not adopt the augmented matching function concept, as the matching itself was of primary interest in my study.

Regardless of the findings, I am aware of some limitations of my study. The time series referred to vacancies registered only at public employment offices. This number of job offers often heavily underestimated the number of job offers available in the economy. In 2012, only approximately 16.5% of companies listed job offers at public employment offices (NBP 2012). Of course, some of those who found jobs did so without assistance from public employment offices. Between 1999 and 2005, the outflow from unemployment to employment exceeded the number of job offers available at public employment offices. This indicates the lower bound of the bias on how the number of vacancies was underreported in the quantitative study.

Following the concept of Galuščák and Münich (2007), we could identify the impact (direction) of the underreported number of vacancies on parameters’ estimates. Galuščák and Münich (2007) proved that when we omit on-the-job seekers, the parameter estimates on unemployment are overestimated, and those on vacancies are underestimated, ceteris paribus. We could assume that unemployed individuals were the only workers who could apply for vacancies published at employment offices, but they were competing for not-published-in-employment-offices vacancies with other job seekers. When we omitted not-published-in-employment-offices vacancies, the parameters’ estimates on vacancies...
should then be overestimated, whereas those on unemployment should be under- or
overestimated (depending on the relative strength of the particular bias). The policy rec-
ommendations would then be twofold. On the one hand, if the quality of job matches gen-
erated by public employment intermediation improved, more companies could be
couraged to publish their advertisements. Both job seekers and companies would
then be better off. On the other hand, if employment counselling services were improved,
unemployed individuals would become more self-reliant in the labour market and could
form a job match with not-published-in-employment-offices vacancies.

Quantitatively, I corrected the results for temporal aggregation bias in the data. Burdett,
Coles, and van Ours (1994) indicated that the temporal aggregation bias in monthly data
can be small. However, time-aggregated models more properly reflect particular agent
pools, particularly because they consider an inflow to be a determinant of the magnitude
of the stock. I did not consider other potential sources of bias in the parameter estimates,
which was already tackled in the literature – for example, spatial aggregation, worker
flows, or search endogeneity.5

The time series used in the analysis were non-stationary, but the residuals were station-
ary. Thus, the models converged to the long-run equilibrium. Gałecka-Burdziak (2015) ana-
lysed the aggregate labour market matching in Poland during an analogous period. Using
the Johansen approach, she reported one cointegrating vector in most matching function
models. Thus, I assumed that the results presented in this paper reflect a long-term equili-
brum in the matching process.

6. Concluding remarks

I analysed the technology in the matching process between workers seeking jobs and
companies seeking workers. The comparative macroeconomic analysis referred to the
Polish labour market during the period January 1999–June 2013. I used registered unem-
ployment data. I estimated various model parameters while econometrically addressing
temporal aggregation bias in the data. I extended the known solutions to make them
directly applicable to a job queuing model.

I found that stocks and inflows of agents engaged in a matching process. Job seekers
(from the pool) sought work among old and new job posts. Only a small fraction of the
newly unemployed individuals enjoyed positive instantaneous re-employment prob-
ability. The vacancy inflow was more important than the vacancy stock, but the positive
elasticity of the vacancy stock demonstrated that not all job offers were covered instan-
taneously. The inflows did not match with one another.

The results indicated some superiority of the random matching mechanism, but the
stock–flow explanation was not negated. Thus, the real matching process was found to
be complicated and time consuming. The fact that the random model prevailed indicates
that policy should be aimed at improving the information in the labour market, which
would facilitate the matching process. The stock–flow model showed that agents were
aware of the search and recruitment process and that job search activities were systematic.
Here, heterogeneous agents would be more effective if there were more potential partners
to choose from; thus, the labour market policy should have the goal of increasing the
inflows of job seekers and job vacancies in the labour market. Certain characteristics,
including the exit rate from unemployment, emphasized the need to increase the
number of job offers. Particular model predictions resembled vacancy series properties, indicating that demand was crucial for aggregate job matches of the registered unemployed individuals in Poland. Higher vacancy inflow would increase the number of matches from the unemployment stock. Moreover, as not all vacancies were covered instantaneously, higher vacancy inflow would result in higher vacancy stock in the following period, which in turn would result in a higher matching rate of the unemployment inflow.

Notes
1. Public employment intermediation remains the most common job search method; it is used by approximately 70% of job seekers.
2. An ‘at-risk’ measure of the unemployed individuals (or vacancies) presents a pool of unemployed workers (or vacancies) that are available for matching in every particular moment. This measure includes the respective share of a beginning-of-period stock and an inflow of new agents.
3. For a discussion of the use of both unit root and stationarity tests, see Charemza and Syczewska (1998), Gabriel (2001), or Harris and Sollis (2003).
4. In this article, I used the algorithm presented in Álvarez de Toledo et al. (2008), who based their research on a version of the model presented in Coles and Petrongolo (2003). The job queuing framework equations were estimated assuming a random mechanism and adjusting the code for the stock-based model. I checked the robustness of the results by estimating the parameters on the basis of the unemployment ‘at-risk’ measure obtained from random and reduced-form stock–flow models. The point estimates were virtually identical.
5. Compare, for example, Galuščák and Münich (2007) for the importance of worker flows, Antczak et al. (2016) for spatial aggregation or Borowczyk-Martins et al. (2013), and Münich and Svejnar (2009) for search endogeneity.

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Appendix

Model I: Non-random matching (overidentified stock–flow model), in which $p$ (instantaneous re-employment probability of the unemployment inflow) is assumed constant and the unemployment stock matches the vacancy stock and the vacancy inflow.

\[
M_t = \begin{bmatrix}
& \alpha_0 + \alpha_1 \ln \left( \frac{V_{t-1}}{U_{t-1}} \right) + \alpha_2 \ln \left( \frac{V_t}{U_{t-1}} \right) \\
\end{bmatrix} U_{t-1} \\
\end{bmatrix} \\
\begin{cases}
(1 - p_u) \left[ 1 - e^{-e} \left( \frac{V_{t-1}}{U_{t-1}} \right) + \frac{\alpha_0 + \alpha_1 \ln \left( \frac{V_{t-1}}{U_{t-1}} \right) + \alpha_2 \ln \left( \frac{V_t}{U_{t-1}} \right)}{e} \right] \\
1 - \frac{1 - e^{-e} \left( \frac{V_{t-1}}{U_{t-1}} \right) + \frac{\alpha_0 + \alpha_1 \ln \left( \frac{V_{t-1}}{U_{t-1}} \right) + \alpha_2 \ln \left( \frac{V_t}{U_{t-1}} \right)}{e}} {e} \\
\end{cases} \quad u_t + \epsilon_t.
\]

Model II: Full stock–flow specification, in which the unemployment stock matches the vacancy inflow, and the unemployment inflow matches the vacancy stock.

\[
M_t = \begin{bmatrix}
& \alpha_0 + \alpha_1 \ln \left( \frac{V_t}{U_{t-1}} \right) \\
\end{bmatrix} U_{t-1} + \begin{cases}
1 - e^{-e} \left( \frac{V_{t-1}}{U_{t-1}} \right) \left[ 1 - e^{-e} \left( \frac{V_t}{U_{t-1}} \right) \right] \\
1 - \frac{1 - e^{-e} \left( \frac{V_{t-1}}{U_{t-1}} \right) \left[ 1 - e^{-e} \left( \frac{V_t}{U_{t-1}} \right) \right]} {e^\alpha_0 + \alpha_2 \ln \left( \frac{V_t}{U_{t-1}} \right)} \\
\end{cases} \quad u_t + \epsilon_t.
\]

Models III: Job queuing (overidentified, random matching), in which the unemployment stock
matches the vacancy stock and the vacancy inflow.

\[
M_t = \left[ 1 - e^{-e} a_0 + a_1 \ln \left( \frac{V_t}{U_t} \right) \right] U_{t-1} + \left[ 1 - e^{-e} \left( \frac{V_t}{U_t} \right) \right] u_t + \varepsilon_t.
\]

Model IV: Job queuing (random matching), in which the unemployment stock matches the vacancy inflow.

\[
M_t = \left[ 1 - e^{-e} a_0 + a_1 \ln \left( \frac{V_t}{U_t} \right) \right] U_{t-1} + \left[ 1 - e^{-e} \left( \frac{V_t}{U_t} \right) \right] u_t + \varepsilon_t.
\]

Model V: Random stock-based model, in which the unemployment stock matches the vacancy stock.

\[
M_t = \left[ 1 - e^{-e} a_0 + a_1 \ln \left( \frac{V_t}{U_t} \right) \right] U_{t-1} + \left[ 1 - e^{-e} \left( \frac{V_t}{U_t} \right) \right] u_t + \varepsilon_t.
\]