Mathematical modeling of a hydraulic hitched system of gantry tractor with high clearance used in horticulture and viticulture

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Abstract: A mathematical model that allows comprehensive calculation of hydraulic hitched system (HHS) operation of a four-wheel gantry tractor with high clearance was developed in the paper. Analytical dependencies of angles, angular velocities and angular accelerations were determined, as well as the analogs of linear and angular velocities and link accelerations based on the kinematic diagram of the HHS hitch mechanism. A method for complex calculation of the HHS and a computer program using the Pascal algorithmic language were proposed on the basis of developed mathematical model. The changes in values of the driving force and resistance force on the hydraulic cylinder rod and the changes in fluid pressure in the hydraulic cylinder were calculated.

1. Introduction
At present, Uzbekistan is a major exporter of food, primarily fruit and vegetable products; over the past three years, the volume of exported agricultural products has grown more than 3 times. This is largely facilitated by the fact that in the republic the area under orchards and vineyards expands every year. However, it is impossible to obtain high productivity, quality and labor efficiency without ensuring proper mechanization of fruits and grapes cultivation [13].

Unfortunately, at present in the republic there is practically no domestic specialized equipment to use in orchards and vineyards for traditional and intensive agro-cultivation technologies [12]. Traditional tillage in orchards and vineyards often does not give the desired result, since after multiple passages the soil is not only loosened, but also compacted, its structure is destroyed [12].

In such a situation, the absence in the republic of a specialized tractor for horticulture and viticulture, adapted to local conditions, does not make it possible to raise the mechanization level in orchards and vineyards, therefore, it does not allow increasing labor productivity in these sectors [12]. Therefore, the parameters substantiation and calculation of the domestic tractor for orchards and vineyards and its creation is one of the urgent tasks in the process of complex mechanization of fruits and grapes cultivation (Figure 1).
When designing tractors to use in horticulture and viticulture, the reliable substantiation of the tractor HHS parameters plays a special role. These tractors should be able to be aggregated with various hitched units and meet the agro-technical requirements for operation.

A seven-link flat linkage mechanism with rotating kinematic pairs of links is used in four-wheeled gantry tractor with high clearance as an HHS. The HHS is controlled by a separate-aggregate hydraulic drive of the tractor, the main elements of which are: oil tank, pump, distributor, power cylinder and hydraulic lines [6]. The HHS of modern gantry tractor with high clearance should provide a lifting capacity up to 1.8 tons (17.658 kN) and more. The effective operation of the HHS of a tractor substantially depends on the correct calculation of the lifting force, the determination of necessary and sufficient number and types of installed hydraulic cylinders, and substantiation of their main parameters [8-9].

![Figure 1. Diagram of a four-wheel gantry tractor with high clearance for horticulture and viticulture](image)

### 2. Kinematics study

Consider a load lifting using seven-link lever mechanism with rotating kinematic pairs of links, controlled by a power cylinder connected with the tractor hydraulic drive [2, 8].

Consider the kinematics of the HHS (Figure 2) and determine the displacements, velocities, and accelerations of the angles of its links [9].

![Figure 2. Kinematic diagram of the HHS: 2,3,5,6,9,10—the links of the HHS lever mechanism; 1,4,7,8, 11—the artificially introduced links of the HHS lever mechanism; G—the loading weight.](image)
Under the pressure action on the hydraulic cylinder piston, the rod (link $l_3$) is lengthened or shortened, as a result of which link $l_1$ is rotated by an angle $\varphi_2$. Using the well-known technique [3, 8], the vectors closure condition for $\Delta AIB$ is written as (Figure 2):

$$\vec{\ell}_1 + \vec{\ell}_2 = \vec{\ell}_3.$$  

The projection on the $XOV$ coordinate axis gives

$$\begin{cases} 
\ell_1 \cos \varphi_1 + \ell_2 \cos \varphi_2 = \ell_3 \cos \varphi_3 \\
\ell_1 \sin \varphi_1 + \ell_2 \sin \varphi_2 = \ell_3 \sin \varphi_3 
\end{cases} \tag{1}$$

excluding from relations (1), $\varphi_1$ and $\varphi_2$, we obtain

$$\ell_1^2 + \ell_2^2 + 2 \ell_1 \ell_2 \cos (\varphi_1 - \varphi_2) = \ell_3^2, \quad \varphi_2 = \varphi_1 - \arccos \left( \frac{\ell_3^2 - \ell_1^2 - \ell_2^2}{2 \ell_1 \ell_2} \right)$$

$$\ell_3^2 + \ell_1^2 - 2 \ell_1 \ell_3 \cos (\varphi_3 - \varphi_1) = \ell_2^2, \quad \varphi_3 = \varphi_1 + \arccos \left( \frac{\ell_3^2 + \ell_1^2 - \ell_2^2}{2 \ell_1 \ell_3} \right).$$

To determine the angular velocity, the time derivative is taken from expression (1)

$$\begin{cases} 
-\ell_1 \sin \varphi_1 \omega_1 - \ell_2 \sin \varphi_2 \omega_2 + \ell_2 \cos \varphi_2 = -\ell_3 \sin \varphi_3 \omega_3 \\
\ell_1 \cos \varphi_1 \omega_1 + \ell_2 \cos \varphi_2 \omega_2 + \ell_2 \sin \varphi_2 = \ell_3 \cos \varphi_3 \omega_3 
\end{cases} \tag{2}$$

Since $\varphi_1 = \text{const}$, $\omega_1 = 0$. Eliminating $\omega_2$ and $\omega_3$, respectively, from relation (2), we obtain

$$\omega_2 = \frac{l_2}{l_2} \cot g(\varphi_2 - \varphi_3), \quad \omega_3 = \frac{l_2}{l_2} \sin(\varphi_2 - \varphi_3).$$

To determine the angular acceleration, we take the time derivative from (2)

$$\begin{cases} 
-\ell_1 \cos \varphi_2 \omega_2^2 + l_2 \sin \varphi_2 \varepsilon_2 + l_2 \cos \varphi_2 - 2l_2 \sin \varphi_2 \omega_2 \\
+ l_2 \cos \varphi_2 \omega_3^2 + l_2 \sin \varphi_3 \varepsilon_3 = 0 \\
- l_2 \sin \varphi_2 \omega_2^2 + l_2 \cos \varphi_2 \varepsilon_2 + l_2 \sin \varphi_2 + 2l_2 \cos \varphi_2 \omega_2 \\
+ l_2 \sin \varphi_3 \omega_3^2 - l_2 \cos \varphi_3 \varepsilon_3 = 0 
\end{cases} \tag{3}$$

Eliminating $\varepsilon_2$ and $\varepsilon_3$, respectively, from (3), we obtain:

$$\varepsilon_2 = \left( \frac{l_2}{l_2} \omega_2^2 \right) \cot g(\varphi_2 - \varphi_3) - \frac{2l_2}{l_2} \omega_2 + \frac{l_3 \omega_3^2}{l_2 \sin(\varphi_2 - \varphi_3)},$$

$$\varepsilon_3 = -\frac{l_2 \omega_2^2}{l_2 \sin(\varphi_2 - \varphi_3)} - \frac{l_2}{l_2} \sin(\varphi_2 - \varphi_3) - \omega_2^2 \cot g(\varphi_3 - \varphi_2).$$

Introduce the artificial link $O_2C$, whose length is denoted by $l_4$. The value of $l_4$ can be found by calculating the coordinates of points $O_2$ and $C$.

Define the coordinates of point C:

$$\begin{cases} 
X_c = l_{31} \cos \varphi_{31} \\
Y_c = l_{11} + l_{31} \sin \varphi_{31} 
\end{cases}$$

where $\varphi_{31} = \varphi_3 + \text{const}$, $l_{11}$ is the length of segment $O_2A$.

Since $O_2(0,0)$, we determine

$$l_4 = \sqrt{X_c^2 + Y_c^2} = \sqrt{l_{31}^2 \cos^2 \varphi_{31} + (l_{11} + l_{31} \sin \varphi_{31})^2}.$$ 

For $\Delta O_2AC$ the following is written: $\vec{\ell}_{11} + \vec{\ell}_{31} = \vec{\ell}_4$.

Projecting this equation on the $XOV$ coordinate axis, we obtain

$$\begin{cases} 
\ell_{11} \cos \varphi_{11} + \ell_{31} \cos \varphi_{31} = \ell_4 \cos \varphi_4 \\
\ell_{11} \sin \varphi_{11} + \ell_{31} \sin \varphi_{31} = \ell_4 \sin \varphi_4 \tag{4} 
\end{cases}$$
Given that $\varphi_{11} = 270^\circ$, from the first equation (4) we define

$$\varphi_4 = \arccos \left( \frac{\ell_3}{\ell_4} \cos \varphi_{31} \right).$$

To determine $\omega_4$, $\varepsilon_4$, $\dot{l}_4$ and $\ddot{l}_4$, we take the time derivative from (4) and considering that $\omega_{11} = 0$, we get

$$\begin{align*}
-l_{31} \sin \varphi_{31} \omega_{31} &= -l_4 \sin \varphi_4 \omega_4 + \dot{l}_4 \cos \varphi_4 \\
l_{31} \cos \varphi_{31} \omega_{31} &= l_4 \cos \varphi_4 \omega_4 + \dot{l}_4 \sin \varphi_4
\end{align*}$$

Eliminating $\dot{l}_4$ and $\omega_4$, respectively, from (5), we have:

$$\omega_4 = \frac{l_{31} \omega_{11}}{l_4} \cos(\varphi_{31} - \varphi_4), \quad \dot{l}_4 = l_{31} \omega_{31} \cos(\varphi_{31} - \varphi_4)$$

Next, we take the time derivative from (5)

$$\begin{align*}
-l_{31} \cos \varphi_{31} \omega_{31}^2 - l_{31} \sin \varphi_{31} \varepsilon_{31} &= \ddot{l}_4 \cos \varphi_4 - 2l_4 \sin \varphi_4 \omega_4 - l_4 \cos \varphi_4 \omega_4^2 - l_4 \sin \varphi_4 \\
-l_{31} \sin \varphi_{31} \omega_{31}^2 &= l_4 \sin \varphi_4 + 2l_4 \cos \varphi_4 \omega_4 - l_4 \sin \varphi_4 \omega_4^2 + l_4 \sin \varphi_4 \varepsilon_4
\end{align*}$$

Eliminating $\ddot{l}_4$ and $\omega_4$, from (6), respectively, we obtain:

$$\varepsilon_4 = \frac{l_{31}}{l_4} \varepsilon_{31} \cos(\varphi_{31} - \varphi_4) - \frac{l_{31}}{l_4} \omega_4^2 \sin(\varphi_{31} - \varphi_4) - \frac{2l_4}{l_4} \omega_4,$$

$$\ddot{l}_4 = l_4 \omega_4^2 - l_{31} \omega_4^2 \cos(\varphi_{31} - \varphi_4) - l_{31} \varepsilon_{31} \sin(\varphi_{31} - \varphi_4).$$

Considering $\Delta O_2 C K$: $\ddot{l}_4 + \ddot{l}_5 = \ddot{l}_6$.

Similarly to the above sequence, performing corresponding calculations, we obtain

$$\varphi_5 = \varphi_4 - \arccos \left( \frac{\ell_5 - \ell_4}{2} \right), \quad \varphi_6 = \varphi_4 + \arccos \left( \frac{\ell_5 + \ell_4}{2} \right),$$

$$\omega_5 = -\omega_4 \ell_4 \sin(\varphi_5 - \varphi_4), \quad \omega_6 = -\omega_4 \ell_4 \sin(\varphi_6 - \varphi_4)$$

Introduce the artificial links $O_2 O_3$ and $O_2 T$, denote them by numbers 7 and 8, respectively (see Figure 2). The length of the links is determined by the coordinates of the points $O_2$, $O_1$, and $T$.

Coordinates of the points are

$$\begin{align*}
T: \quad &X_T = l_{61} \cos \varphi_6 \\
&Y_T = l_{61} \sin \varphi_6
\end{align*}$$

where $l_{61} = l_6 + l_4$.

Since the coordinates of the points are $O_2 (0,0)$, $O_1 (X_{O1}, Y_{O1})$, the length of the link $O_2 O_1$ is:

$$l_7 = \sqrt{X_{O1}^2 + Y_{O1}^2},$$

and the link:

$$l_6 = \sqrt{(X_T - X_{O1})^2 + (Y_T - Y_{O1})^2}.$$
From $\Delta O_2O_1T$ we obtain: \( \dot{\ell}_7 + \dot{\ell}_8 = \dot{\ell}_{61} \).
Projecting this equation on the XOY coordinate axis, we get
\[
\begin{align*}
\ell_7 \cos \varphi_7 + \ell_8 \cos \varphi_8 &= \ell_{61} \cos \varphi_6 \\
\ell_7 \sin \varphi_7 + \ell_8 \sin \varphi_8 &= \ell_{61} \sin \varphi_6
\end{align*}
\] (7)
Eliminating $\varphi_6$ from relation (7), we obtain
\[
\varphi_8 = \varphi_7 - \arccos \left( \frac{\ell_{61}^2 - \ell_7^2 - \ell_8^2}{2 \ell_7 \cdot \ell_8} \right).
\]
\( \omega_8, \ \epsilon_8, \ \dot{\ell}_8 \) and \( \ddot{\ell}_8 \), similar to the link \( O_6C \)
\[
\begin{align*}
\omega_8 &= \frac{l_6(\omega_6 \cos(\varphi_6 - \varphi_8))}{l_8}, \quad \dot{\ell}_8 = l_6(\omega_6 \cos(\varphi_6 - \varphi_8)). \\
\epsilon_8 &= \frac{l_6 l_8 \cos(\varphi_6 - \varphi_8)}{l_8} = \frac{l_6 l_8^2 \sin(\varphi_6 - \varphi_8)}{l_8} - \frac{2 \dot{\ell}_8}{l_8} \omega_8, \\
\ddot{\ell}_8 &= l_8 \omega_8^2 - l_6 l_8^2 \cos(\varphi_6 - \varphi_8) - l_6 \epsilon_8 \sin(\varphi_6 - \varphi_8).
\end{align*}
\]
Further from $\Delta O_1FT$ we get \( \dot{\ell}_8 + \ddot{\ell}_9 = \ddot{\ell}_{10} \).
Similarly to the above sequence, performing corresponding calculations, we obtain:
\[
\omega_9 = \varphi_9 - \arccos \left( \frac{\ell_{10}^2 - \ell_8^2 - \ell_9^2}{2 \ell_8 \cdot \ell_9} \right),
\]
\[
\begin{align*}
\omega_9 &= -\frac{\alpha_5 l_8 \sin(\varphi_{10} - \varphi_9)}{l_9 \sin(\varphi_{10} - \varphi_9)} - \dot{\ell}_8 \cos(\varphi_{10} - \varphi_9), \\
\omega_{10} &= -\frac{\alpha_5 l_8 \sin(\varphi_9 - \varphi_8)}{l_{10} \sin(\varphi_{10} - \varphi_9)} - \dot{\ell}_9 \cos(\varphi_9 - \varphi_8), \\
\epsilon_9 &= (\alpha_5^2 l_8 - \dot{\epsilon}_9) \cos(\varphi_{10} - \varphi_9) - (\epsilon_9 l_8 + 2 \dot{\ell}_8 \omega_8) \sin(\varphi_{10} - \varphi_9) - \alpha_5^2 l_8 \cos(\varphi_{10} - \varphi_9) - \alpha_5^2 l_{10} / l_9 \sin(\varphi_{10} - \varphi_9), \\
\epsilon_{10} &= (\alpha_5^2 l_8 - \dot{\epsilon}_9) \cos(\varphi_9 - \varphi_8) - (\epsilon_9 l_8 + 2 \dot{\ell}_8 \omega_8) \sin(\varphi_9 - \varphi_8) - \alpha_5^2 l_{10} / l_9 \sin(\varphi_{10} - \varphi_9) + \alpha_5^2 l_9 / l_{10} \sin(\varphi_{10} - \varphi_9).
\end{align*}
\]
3. Methodology for determining analogs of linear and angular velocity and acceleration of the hitched mechanism
In kinematic analysis of the mechanisms by analytical method, the velocity and acceleration of the driven links are convenient to express as a function of the angle of rotation $\varphi$ or of displacement $s$ of the driving link, i.e. as a function of generalized coordinate [3, 11].
So, if the rotation angle $\varphi_i$ of any $i$-th link is given as a function $\varphi_i(\varphi)$ [3], then to determine the analog of angular velocity of link 3 and linear velocity of link 2, we differentiate equations (8) [11] by the generalized coordinate $\varphi_2$
\[
\begin{align*}
-l_1 \frac{d\varphi_1}{d\varphi_2} \sin \varphi_1 - l_2 \sin \varphi_2 + \frac{d l_1}{d \varphi_2} \cos \varphi_2 &= -\frac{d \varphi_3}{d \varphi_2} \ell_3 \sin \varphi_3 \\
l_1 \frac{d \varphi_1}{d \varphi_2} \cos \varphi_1 + l_2 \cos \varphi_2 + \frac{d l_2}{d \varphi_2} \sin \varphi_2 &= \frac{d \varphi_3}{d \varphi_2} \ell_3 \cos \varphi_3
\end{align*}
\] (8)
where, $\varphi_1 = \text{const, } \omega_1 = 0.$
\[
\begin{align*}
-l_2 \sin \varphi_2 + \frac{d l_2}{d \varphi_2} \cos \varphi_2 &= -\frac{d \varphi_3}{d \varphi_2} \ell_3 \sin \varphi_3 \\
l_2 \cos \varphi_2 + \frac{d \varphi_2}{d \varphi_2} \sin \varphi_2 &= \frac{d \varphi_3}{d \varphi_2} \ell_3 \cos \varphi_3
\end{align*}
\] (9)
where, \( \frac{d\varphi_2}{d\varphi_2} = \Omega_3 \) – is the analog of angular velocity of link 3; \( \frac{dl_2}{d\varphi_2} = V_2 \) – is the analog of linear velocity of link 2.

From the first equation of the system of equations (9) we find the value of the analog of angular velocity \( \Omega_3 \) of link 3

\[
\Omega_3 = \frac{\ell_2 \sin \varphi_2 - V_2 \cos \varphi_2}{\ell_3 \sin \varphi_3}
\]

Substituting the value of the analog of the angular velocity \( \Omega_3 \) into the second equation of system (9), we find the value of the analog of linear velocity \( V_2 \) of link 2

\[
V_2 = \frac{\sin(\varphi_2 - \varphi_3)}{\cos(\varphi_2 + \varphi_3)}
\]

To determine the analog of angular acceleration of link 3 and the analog of linear acceleration of link 2, we differentiate equation (9) by the generalized coordinate \( \varphi_2 \)

\[
\begin{align*}
-\ell_2 \cos \varphi_2 - 2V_2 \sin \varphi_2 + \frac{dV_2}{d\varphi_2} \cos \varphi_2 + l_2 \Omega_3^2 \cos \varphi_2 - \frac{d\Omega_3}{d\varphi_2} \sin \varphi_3 & = 0 \\
-\ell_2 \sin \varphi_2 + 2V_2 \cos \varphi_2 - \frac{dV_2}{d\varphi_2} \sin \varphi_2 - l_2 \Omega_3^2 \sin \varphi_3 - \frac{d\Omega_3}{d\varphi_2} \cos \varphi_3 & = 0
\end{align*}
\]

(10)

where \( \frac{d\Omega_3}{d\varphi_2} = E_3 \) is the analog of angular acceleration of link 3; \( \frac{dV_2}{d\varphi_2} = A_2 \) is the analog of linear acceleration of link 2.

From the first equation of the system of equations (10) we find the value of the analog of angular acceleration \( E_3 \) of link 3

\[
E_3 = \frac{-\ell_2 \cos \varphi_2 - 2V_2 \sin \varphi_2 + A_2 \cos \varphi_2 + l_2 \Omega_3^2 \cos \varphi_2}{\sin \varphi_3}
\]

Substituting the value of the analog of angular acceleration \( E_3 \) in the second equation of the system of equations (10), we find the value of the analog of linear acceleration \( A_2 \) of link 2

\[
A_2 = \frac{\cos(\varphi_2 + \varphi_3) + 2V_2 \sin(\varphi_2 + \varphi_3) - l_2 \Omega_3^2}{\cos(\varphi_2 - \varphi_3)}
\]

Thus, the obtained dependences of the change in angular velocities and angular accelerations of the links allow us to use them in the HHS dynamic calculation.

4. Dynamic calculation

When transferring hitched machines from the working position to a transporting one, the dynamic loads arise in the elements of the tractor hydraulic hitched system, which cannot be taken into account in static calculation [2].

The pattern of dynamic processes depends on the parameters of hydraulic actuator, lifting-hitching mechanism and machine, i.e. on the magnitude and mass distribution of the individual units of the system, their moments of inertia, the elasticity of the pipelines, the mass, the resistances of the system elements, and its external and internal disturbances. Soil reaction, resistance forces, cars weight, etc. relate to external disturbances. Switching of control devices due to rupture or restoration of hydraulic connections refer to internal disturbances [5].

We compose a mathematical model to reveal the actual loads acting on the elements of the system. The pressure in the tractor hydraulic system is taken as a load index [2].

According to [4], we make the assumptions:

1) the viscosity, density, temperature of the working fluid and the amount of air undissolved in it do not change during the lifting process;
2) pump performance is constant;
3) there are no leaks of the working fluid in the system;
4) the mode of fluid flow is turbulent;
5) the valve motion occurs instantly;
6) pressure loss along the length of the main line, local and inertial losses of fluid pressure are the functions of the average fluid flow rate in the main line;
7) the mass and elasticity of the links of hitched mechanism are neglected;
8) in the process of lifting the hitched mechanism moves strictly vertically.

The design diagram of the tractor hydraulic system is shown in Figure 3.

\[
\begin{align*}
    \frac{m_{br}}{dt} &= P_0 - k_v v_2 - P_{df} \text{sign} v_2 - P_v - \frac{v_2^2}{2} \frac{d m_{br}}{d l_2} \\
    \frac{d P_1}{dt} &= \frac{E_f (Q_1 - v_2 F_1)}{V_0 + l_2 F_1} \\
    \frac{d P_2}{dt} &= -\frac{E_f (v_2 F_2 - Q_2)}{V_0 + (l_{max} - l_2) F_2}
\end{align*}
\]

where

\[
\begin{align*}
    m_{br} &= \sum_{i=2}^{n} \left( v_i / v_2 \right)^2 + j_i \left( \omega_i / v_2 \right)^2, \\
    P_0 &= P_1 F_1 - P_2 F_2, \\
    P_v &= \sum_{i=2}^{n} P_i v_i \cos \phi_i / v_2 + M_i \omega_i / v_2
\end{align*}
\]
$m_{bi}$ – are the masses of moving parts reduced to the cylinder rod; $l_0$, $v_2$, $\omega_2$ – are the displacement, velocity and angular velocity of hydraulic cylinder; $k_f$ – is the coefficient of viscous friction; $P_{df}$ – is the coefficient of dry friction; $P_d$ – is the driving force acting on the piston of hydraulic cylinder; $P_r$ – are the resistance forces reduced to the hydraulic cylinder rod; $P_1$, $P_2$ – are the pressures in the force and discharge sides of hydraulic cylinder; $F_1$, $F_2$ – are the effective areas in the forcing and discharge sides; $V_{01}$, $V_{02}$ – are the initial volumes of fluid in the force and discharge sides of cylinder; $m_i$ – is the mass of the $i$-th link; $j_i$ is the moment of inertia of the $i$-th link relative to the axis passing through the center of mass; $v_i$ – is the velocity of the center of gravity of the $i$-th link; $\alpha_i$ is the angular velocity of the $i$-th link; $P_s$, $M_i$ – are the magnitude of the active forces and the moments acting on the links; $\phi_i$ – is the angle between the directions of forces $P_i$ – and the velocity $v_i$; $l_i$ is the length of the $i$-th link. 

The coordinates of the center of mass of the mechanism in question can be determined by the formula [1]

$$
\begin{align*}
    x_u &= \sum G_i x_i / \sum G_i \\
    y_u &= \sum G_i y_i / \sum G_i \\
    i &= 2...12
\end{align*}
$$

(12)

The velocities and moments of inertia of the links are determined by the formulas [8]

$$
\begin{align*}
    v_i &= l_i \omega_i / 2, \\
    j_i &= m_i \left[ (x_u - x_{iu})^2 + (y_u - y_{iu})^2 \right], \\
    i &= 2...12
\end{align*}
$$

(13)

where $x_{iu}$, $y_{iu}$ – are the coordinates of the center of gravity of the $i$-th link.

To study the fluid flow in the pipeline, a model was chosen where the fluid is taken as compressible medium concentrated in one or two volumes of small length (a system with concentrated parameters, considering the flexibility of the hydraulic system elements). In this model, there is a possibility to take into account the compressibility of bubbles of undissolved air [4, 8].

$$
\rho \frac{d^2 x}{dt^2} + 27.5 \frac{\ell \mu}{f} \frac{dx}{dt} + \left( 0.443 k \frac{\mu}{\sqrt{f}} + 0.5 \varphi \right) \left( \frac{dx}{dt} \right)^2 \text{sign} \frac{dx}{dt} + p_{out} = p_{in}
$$

(15)

$$
\frac{d \rho}{dt} = \left( \frac{E_\mu \delta_{pp} E_{pp}}{E_{pp} \delta_{pp} + d_{pp} E_\mu} \right) \frac{Q_{in} - Q_{out}}{V_{in}}
$$

(16)

where $\text{sign} \frac{dx}{dt} = \begin{cases} 1 & v>0 \\ -1 & v<0 \end{cases}$, $p_{in}$, $Q_{in}$ are the pressure and fluid flow at the pipeline inlet; $p_{out}$, $Q_{out}$ are the pressure and fluid flow at the pipeline outlet and inlet; $t$ is time; $\rho$ and $E_{\mu}$ are the density and modulus of volume elasticity of fluid; $d_{pp}$, $\delta_{pp}$, $E_{pp}$ are the diameter, wall thickness and elastic modulus of the pipeline material, respectively; $k_\varepsilon$ is the approximation coefficient, the value of which depends on the relative roughness $\varepsilon$ of hydraulic main lines; $\zeta$ is the coefficient of local resistance; $f$ and $l$ are the area and length of the pipeline; $V_{in}$ is the volume of fluid in the pipeline section; $\mu$ is the dynamic viscosity of fluid.

The flow rate through the distributor is determined by the dependence [4, 8]:

$$
Q_p = \eta_p f_p(y) \sqrt{2 |p_{pm} - p_{in}| / \rho}
$$

(17)

where $p_{pm}$ is the pressure created by the pump, $f_p(y)$ is the area of the flow passage, $\eta_p$ is the flow rate coefficient.

The cross-section area of the distributor can be approximated by the following characteristics:
\[ f_p(y) = \begin{cases} 
0, & \text{if } t < \tau \\
 \frac{\pi d_y^2 (t - \tau)}{4(t_k - t)}, & \text{if } \tau \leq t \leq t_k \\
 \frac{\pi d_y^2}{4}, & \text{if } t > t_k 
\end{cases} \]

where \( d_y \) is the conditional passage, \( \tau \) is the delay time, \( t_k \) is the time of complete opening of the passage.

5. Simulation results

Figure 4–6 shows the results of numerical calculation of changes in angles, rad a), angular velocities, rad/s b) and angular accelerations, rad/s\(^2\) c) of the HHS 2nd, 3rd and 6th links. The following values of the links were taken in calculations: \( l_1 = 0.608 \text{ m}, \ l_2 = 0.593 \text{ m}, \ l_3 = 0.14 \text{ m}, \ l_{31} = 0.26 \text{ m}, \ l_4 = 0.585 \text{ m}, \ l_5 = 0.485 \text{ m}, \ l_6 = 0.3 \text{ m}, \ l_{61} = 0.785 \text{ m}, \ l_7 = 0.5 \text{ m}, \ l_8 = 0.56 \text{ m}, \ l_9 = 0.518 \text{ m}, \ l_{10} = 0.638 \text{ m}, \ l_{11} = 1.22 \text{ m}, \ l_{12} = 1.22 \text{ m}, \ X_0 = 0.288 \text{ m}, \ Y_0 = 0.412 \text{ m}, \ \phi_1 = 89^0 = 1.55 \text{ rad}, \ \phi_7 = 56^0 = 0.97 \text{ rad}. \]

![Figure 4](image1)

**Figure 4.** Dependences of change in angles of the HHS 2nd, 3rd and 6th links.

![Figure 5](image2)

**Figure 5.** Dependences of change in angular velocities of the HHS 2nd, 3rd and 6th links.

![Figure 6](image3)

**Figure 6.** Dependences of change in angles a), angular velocities b) and angular accelerations c) of the HHS 2nd, 3rd and 6th links.

The system of equations (11) - (17), together with the initial and boundary conditions, makes up a mathematical model for dynamic calculation of the tractor HHS. To implement the mathematical model, a computer program was developed in the Pascal algorithmic language [13].
Figures 7–9 a), b) and c) present the results of numerical calculation of changes in driving force, resistance force on the hydraulic cylinder rod, and the hydraulic cylinder rod displacement of the HHS when lifting the load (3rd link). The following link values and initial data were taken in calculations: $l_1=0.608$ m, $l_2=0.593$ m, $l_3=0.14$ m, $l_{31}=0.26$ m, $l_4=0.585$ m, $l_{6}=0.03$ m, $l_{61}=0.785$ m, $l_7=0.5$ m, $l_{10}=0.56$ m, $h_{10}=0.518$ m, $l_{11}=0.638$ m, $l_{12}=1.22$ m, $m_2=14.2$ kg, $m_3=6.98$ kg, $m_5=6.84$ kg, $m_6=18.1$ kg, $m_9=1.52$ kg, $m_{10}=2.14$ kg, $m_{12}=1800$ kg, $p_1=142$ N, $p_2=69.8$ N, $p_3=68.4$ N, $p_4=181$ N, $p_5=15.2$, $N$ $P_{10}=21.4$ N, $P_{12}=18289$ N, $t=0$, $x=0.2$, $t_1=2$, $p_n=12$ MPa, $p_{in}=p_{out}=Q_{in}=Q_{out}=l_2=v_2=\omega_2= p_1= p_2 = 0$.

Analysis of the graphs shows that when the distributor is turned on, considering the delay time at $t = 0.2 \ldots 0.65$ seconds, the driving force sharply increases up to about $102$ kN, then it decreases to the values of resistance force. Further, the value of the driving force monotonously increases according to the resistance force till the value of complete opening of the distributor is $t=0.65\ldots 2$ seconds. Upon reaching the value of the complete opening of the distributor, the driving force begins to oscillate and damps over time.

6. Conclusions
A mathematical model of the HHS dynamics was developed based on the method of reducing the mass and resistance forces to a specific link.

The dependences of changes in angles, angular velocities and angular accelerations of the HHS driven links were obtained.
The changes in the driving and resistance forces on the hydraulic cylinder rod were calculated numerically.

It was established that (at $m=1.8t$) when the distributor is turned on considering the delay time ($t=0.2...0.65$ sec.), the driving force sharply increases to $P_d = 102$kN, then it decreases to the values of resistance force $P_r = 72$kN. Further, the values of the driving force monotonously increases according to the resistance force on the hydraulic cylinder rod till the distributor is completely open ($t=0.65...2$ sec.), which shows that results are close to the actual transition process.

To implement mathematical models, a computer program developed in the Pascal algorithmic language was used.

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