Special feature of a screw vacuum-compressor ‘chamber’
model development

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Abstract. Wildly using of screw vacuum pumps and vacuum-compressors (which discharge pressure is higher than atmosphere pressure) can be explained by their easy service, reliability, excellent energetic and capacity characteristic and excellent vacuum, they produce. Their future optimization can be reached by using of the working process mathematic modelling, which can takes the place of expensive experimental research (can reduce the count of it). In an engineer point of view, chamber models of the positive displacement machine are still popular and wildly used due to simple of the initial data entering, small calculation time and simple of the results’ explanation. Therefore, their development and updating are still actual goal. The chamber model accuracy are determined generally by the accuracy of leakage flow calculation through the gaps and ports and by the accuracy of heat exchange calculation inside the working chamber. This is especially important for the vacuum machines, which minimal vacuum pressure are determined by the internal leakages. The vacuum inside the working chamber can lead to not only viscosity flow, but also to molecular and mixed flows. Therefore, using only viscosity flow calculation methodises leads to the significant errors. The vacuum also leads to change the heat exchange conditions inside the working chamber. Therefore, those facts should be taken into account. This paper is dedicated to the presenting the way of the solving the problems described above.

Keywords: vacuum compressor, ‘chamber’ model, compressor characteristics

1. Introduction
The object of the study is a screw vacuum compressor, the structural diagram of which is given in figure 1.

The screw vacuum compressor [1] falls within the class of machines of the positive displacement principle of operation – that is, machines in which the processes of suction, transfer, compression and displacement of the aspirated medium take place in chambers with a time-variable volume. The main working bodies are rotors 1 and 2, which are coarse pitch screws with the pitch variable along the length. Working cavities are formed in their depressions and are limited by their surfaces as well as by the surface of the pump housing. Suction is performed through nozzle 3 and the suction port located on the inner part of the pump housing. The gas is released through the discharge port, also located on the inner part of the housing, through check valve 4. This valve prevents backflow of the medium into the pumped out volume. The pump rotors are in constant engagement with each other, while the torque
transfer from drive rotor 1 to driven rotor 2 is carried out by means of synchronising gears 6, ensuring the absence of direct contact between the rotors during the pump operation. The pump is driven by electric motor 5, the internal windings of which are directly mounted at the end of drive rotor 1.

![Figure 1. Structural diagram of the screw vacuum pump:](image)

1 – main rotor, 2 – driven rotor, 3 – suction nozzle, 4 – discharge nozzle with check valve, 5 – electric motor, 6 – synchronising gears, 7 – water channels

One of the features of vacuum pumps and positive displacement vacuum compressors in comparison with similar compressors is the extremely large values of the compression ratios at low pressure drops in the cavities. This leads to significant heating of the pumped-out medium, the heat from which is partially removed with the cooling water flowing in channels 7 of the compressor housing, which washes over it from all four sides.

2. Mathematical model of the working process

For the theoretical study of the vacuum compressor under consideration, a 'chamber' model has been selected based on the law of conservation of energy and the equation of state and presented in the form of two differential equations of the first order [2, 3]:

\[
\frac{dP}{d\varphi} = \frac{(k-1)}{V \cdot \omega} \left( \frac{dQ}{d\tau} + \sum_{i=1}^{n} (h_{m,i} \cdot m_{m,i}) - \sum_{j=1}^{m} (h_{c,i} \cdot m_{c,i}) - \frac{\omega \cdot k \cdot P}{(k-1)} \cdot \frac{dV}{d\varphi} \right)
\]

\[
\frac{dT}{d\varphi} = \frac{(k-1) \cdot T}{P \cdot V \cdot \omega} \left( \frac{dQ}{d\tau} + \sum_{i=1}^{n} (h_{m,i} - h) \cdot m_{m,i} + h \cdot \left( \frac{k-1}{k} \right) \left( \sum_{i=1}^{n} m_{m,i} - \sum_{j=1}^{m} m_{c,i} \right) - P \cdot \omega \cdot \frac{dV}{d\varphi} \right),
\]

where \( \varphi \) is the angle of rotation of the drive element (drive rotor); \( P \) and \( T \) are the pressure and the temperature in the working chamber, respectively; \( V \) is the current volume of the working chamber; \( \omega \) is the angular speed of the drive element (drive rotor); \( h = c_p \cdot T \) is the enthalpy of the compressed medium; \( c_p \) is the specific heat capacity at constant pressure of the compressed medium; \( c_v \) is the specific heat capacity at constant volume of the compressed medium; \( k = \frac{c_p}{c_v} \) is the adiabatic index of
the compressed medium; \( \frac{dQ}{d\tau} \) is the heat gain supplied or removed from the compressed medium due to heat exchange with the walls of the housing and the surface of the rotors; \( \dot{m} \) is the mass flow rate of inflow or outflow gas from the working cavity; without the index, the parameters refer to parameters in the working cavity; \( n \) and \( m \) are the numbers of slots and cavities through which inflow and outflow occur; the index ‘\( \text{in.} \)’ relates to the parameters of the inflow medium; ‘\( \text{ex.} \)’ relates to the parameters of the outflow medium.

The system (1) is a mathematical model of the process in the working cavity of ‘dry’ machines with the positive displacement principle of operation, based on the following assumptions:

- The medium in the volume under consideration is homogeneous and continuous.
- The parameters of the medium vary throughout the volume simultaneously and in the same way.
- The change in the kinetic energy and energy of the medium associated with its position in the volume under consideration is disregarded.
- The medium in the reference volume is considered an ideal gas obeying the Mendeleev–Clapeyron equation.

The dependences of the volume of the working chamber, the geometric parameters of the ports of the vacuum compressor as well as the dimensions of the slots formed in the working chamber as a function of the angles of rotation of the drive element are determined based on the analysis of the design, and the dimensions, configuration and profile of the rotor screws. In fact, these dependencies are determined initially and are the initial data along with the parameters of the investigated operating mode for the calculation of the operating mode. They can only be refined iteratively, taking into account the thermal strains of the main parts of the vacuum compressor and calculated based on the parameters of the working process in a given operating mode.

The mathematical model describes the phenomena and processes of a real object approximately, which is mainly governed by the assumptions made during the derivation of each of the equations used, the assumptions of the mathematical model development and the errors of the numerical values used.

The development of the mathematical model of the screw vacuum compressor at this stage has been performed subject to the following assumptions:

1. The pressure in the suction and discharge nozzles is assumed to be constant – that is, on suction and discharge, the pump is connected to the cavities of infinitely large volume, and the discrete gas supply does not affect the pressure change in them.
2. The working fluid within the cavity under consideration is assumed to be homogeneous – that is, its parameters are the same for any point, and mixing of the incoming gas and the gas in the working cavity occurs instantly.
3. Since the processes in the working cavities occur in the same way and are displaced along the angle of rotation of the rotor, only one working cavity is under consideration. This creates the need for an iterative calculation using the indicator and temperature diagram of the previous approximation. The parameters in adjacent cavities in the calculations of each approximation are taken to be the same as in the given cavity at the respective angle of rotation in the previous approximation.

The following are taken as initial conditions:

1. The pressure and the temperature at the end of the suction process are taken as the initial pressure and temperature of the working process.
2. The temperature of the working fluid in the discharge nozzle is initially set in sufficient degree arbitrarily; therefore, the discharge temperature in the first approximation is selected based on the adiabatic compression condition. At the moment of equalisation of pressures in the working cavity and the discharge nozzle, the discharge temperature is redistributed and taken as equal to that in the working cavity. This predetermines the need to calculate the parameters of the gas upon discharge in an iterative way.
3. Indicator and temperature diagrams of the first approximation are set from the condition of constant pressure and temperature in the suction and discharge processes and their instant change when the working cavity is connected to the discharge port, the compression and expansion processes are taken as polytropic.

We shall highlight the following stages of the working process, which differ both in terms of the nature of the phenomena occurring and in terms of the pattern of leaks: suction; gas compression; discharge.

The processes of suction, compression and discharge are themselves subdivided into processes that differ in the pattern of leakage through the slot. There are several cavities communicating with each other through the slotted gaps in the machine simultaneously, in which various stages of the working process occur. They are displaced relative to each other along the angle of rotation of the rotor by the angles that are multiples of periodisation. It is enough to conduct the simulation for one working cavity since the rest represent the same cavity at other angles of rotation of the drive rotor. Therefore, the parameters in adjacent cavities are determined from the previous iteration. The calculation is carried out until the convergence of the iterative values of temperature and pressure is less than 2%.

The volume delivery of the compressor is determined by integrating the indicator and temperature diagrams:

\[
S = \frac{1}{\rho_k} \cdot \frac{m_w}{\omega} \cdot d \phi 
\]

(2)

where \( \rho_k \) is the density of the medium in the suction nozzle; \( m_w \) is the instantaneous mass flow through the port; \( \phi_1 \) is the discharge start angle; \( \phi_2 \) is the discharge start angle; \( z \) is the number of cavities involved in the discharge process per one rotor revolution; \( n \) is the rotation frequency.

The work done on the gas within one cycle is determined on the basis of the indicator and temperature diagrams of the working cavity, obtained by integrating the differential equations of the working process of the vacuum pump, by integrating the differential equation:

\[
\frac{dL}{d \phi} = \frac{(k-1)}{\omega} \left( \frac{dQ}{d \tau} + \sum_{i=1}^{n} (h_{in,i} \cdot \dot{m}_{in,i}) - \sum_{i=1}^{n} (h_{ex,i} \cdot \dot{m}_{ex,i}) \right) - \frac{\omega \cdot k \cdot P}{(k-1)} \cdot \frac{dV}{d \phi},
\]

(3)

The indicator power is determined as:

\[
N_{ind} = Z_1 \cdot L \cdot \frac{\omega}{2 \cdot \pi}
\]

(4)

where \( Z_1 \) is the number of cavities (teeth) of the drive rotor.

3. Calculation of gas leaks in the working chamber of the vacuum compressor

To solve equation (1), it is required to determine the gas flow rate through the slots formed in the working chamber. The calculation method from \([4, 5, 6]\) was taken as a basis. The mass flow is calculated using the following formula:

\[
\dot{m} = \frac{U(P_1 - P_2)}{RT},
\]

(5)

where \( P_1 \) and \( P_2 \) are the pressure values upstream and downstream of the channel, \( T \) is the temperature upstream of the channel, \( R \) is the universal gas constant, and \( U \) is the channel conductivity. The operating pressure range of non-contact pumps can cover more than eight orders of magnitude (from 10 to \( 10^5 \) Pa). Accordingly, the flow regime in the channels can be either molecular, transient or viscous. The geometry of the slotted channels of the rotor mechanism of non-contact
vacuum pumps and vacuum compressors is complicated and depends on the type of profile, the size of the gaps and the relative position of the rotors. Of greatest interest are the four types of slots shown in figure 2 since almost all channels can be reduced to one of these types. The following formula was used to calculate the conductivity:

\[
U = \begin{cases} 
U_V, & \text{if } Kn \leq 0.01 \\
U_V + zU_M, & \text{if } 0.1 > Kn > 0.01, \\
U_M, & \text{if } Kn \geq 0.1 
\end{cases}
\]  

where \( Kn \) is the Knudsen number, \( z \) is the empirical coefficient, \( U_M \) is the conductivity of the element in the molecular flow regime; \( U_V \) is the conductivity of the element in a viscous mode with a flow without sliding on the walls.

![Figure 2. Geometry of channels](image)

For channels 1–3, the following equation has been used

\[
U_V = \frac{\sqrt{2RT}\xi(1+\tau)\delta L}{9\pi\left(1+\sqrt{1+\alpha(1-\tau^2)}\xi^2\right)},
\]  

where \( \alpha = c_0 + c_1x_k + c_2x_k^2 + c_3x_k^3 - 0.00129r^{0.397}\xi \), \( x_k = \frac{\ln(\tau)}{\ln(\tau)-1} \), \( \xi = \frac{\delta^2P_i}{\eta\sqrt{RTl_{EQ}}} \), \( \tau = \frac{P_i}{P_f} \), \( \eta \) is the coefficient of dynamic viscosity, \( l_{EQ} = \sqrt{\frac{\delta R R}{R_1 + R_2}} \) is the effective channel length,

\[
c_0 = \frac{0.0008}{1 + 0.00013\xi^2 + 10^6 + 0.202\xi^3} + 10^6 + 0.562\xi^2, \quad c_1 = -0.00153 + \frac{0.0103\xi^2}{2544 + \xi^2 + 0.0000275\xi^2}, \quad c_2 = \frac{284.6}{9086 + \xi^2} + \frac{0.0105\xi^2}{58153 + \xi^2} + \frac{0.0137\xi^3}{1.11 \cdot 10^6 + \xi^2}, \quad c_3 = \left(\frac{0.00805\xi^2}{36345 + \xi^2} + \frac{39823}{1,646 \cdot 10^6 + \xi^3} + \frac{0.0129\xi^3}{6.93 \cdot 10^6 + \xi^3}\right) \cdot 
\]

When determining the effective channel length, the plus sign is selected for channel 1; the minus sign, for channel 2; and \( l_{EQ} = \sqrt{\delta R_2} \) is adopted for channel 3. For slot 4:

\[
U_V = \frac{\xi^2LP_i}{12\eta \left(1 + \frac{\alpha(1-\tau^2)P_i^2\delta^4}{6\delta^2 RT\eta^2}\right)},
\]
where \( \alpha = c_0 + c_1/(\tau + 0.03) + c_2\ln^2(\tau + 0.03), c_0 = 0.0687 - 0.00581 \ln(1/\xi) - 0.001 \ln^2(1/\xi), \)
\( c_1 = -0.264 - 0.0677 \ln(1/\xi) - 0.00452 \ln^2(1/\xi), \)
\( c_2 = -0.0711 - 0.0163 \ln(1/\xi) - 0.000142 \ln^2(1/\xi), \)
\( \xi = 0.0687 - 0.00581 \ln(1/\xi) - 0.001 \ln^2(1/\xi), \)
\( \xi = 0.0687 - 0.00581 \ln(1/\xi) - 0.001 \ln^2(1/\xi), \)
\( \frac{\alpha^2}{\| \hat{P} \| \sqrt{RT}}. \)

Analysis of the results of numerical calculation of the mass flow rate of gas has made it possible to obtain an expression for determining the critical pressure ratio, which for a flat slot is written in the following form

\[ \tau_c = 0.528 \left( \frac{1}{2} + \frac{1}{\pi} \arctg \frac{0.65(\ln \xi - 1.94) + 0.012(\ln \xi - 1.94)^3}{\pm \ln \sqrt{RT}} \right)^2. \]  
(9)

For calculation in the molecular mode \( U_M, \) it is proposed to use the well-known Clausing expression, presented in the following form:

\[ U_M = \frac{C}{4} L \delta K, \]
(10)

where \( C \) is the arithmetic mean velocity of gas molecules; \( K \) is the coefficient of conductivity reduced to the minimum gap; \( \delta \) is the size of the minimum gap; and \( L \) is the slot height. For channels 1–3, it is recommended to use the following expressions:

\[ K_1' = \exp \left[ 0.7884 + 0.4443x + \frac{3.27}{x} + \frac{4.807}{x^2} + \frac{2.883}{x^3} + 0.0238 \arctg \frac{8.95 R_2}{R_1} \right], \]
\[ K_3'' = K_1' \exp \left[ -\exp[2y] \left( 0.057 y + 0.0304 \sqrt{y} + 0.0067 y^{0.16} \right) \right], \]

where \( x = \ln \left( \delta \left( \frac{1}{R_2} \pm \frac{1}{R_1} \right) \right), \) “+” sign for channel 1, “-” sign for channel 2, \( y = R_2 / |R_1|. \)

Finally, we have:

\[ K_3 = \begin{cases} 
K_1', & \text{for channel 1} \\
K_3'', & \text{for channel 2} \\
\frac{\delta}{L} \ln \left( \frac{L}{\delta} \right), & \text{for channel 4}.
\end{cases} \]
(11)

Coefficient \( z \) for the variable cross-section channels 1–3 can be calculated as:

\[ z = \frac{1 + \delta P_M \sqrt{M(\tau - 1)^{-1} \eta^{-1}}}{1 + (1.24 + 0.5(1-\tau^3)) \delta P_M \sqrt{M(\tau - 1)^{-1} \eta^{-1}}}, \]
(12)

where \( M \) is the molar mass of the compressible medium, and \( P_M = \frac{P_1 + P_2}{2} \) is the average pressure.

Coefficient \( z \) for the variable cross-section channel 4 can be calculated as:

\[ z = \frac{1 + d P_M \sqrt{M(\tau - 1)^{-1} \eta^{-1}}}{1 + 1.24d P_M \sqrt{M(\tau - 1)^{-1} \eta^{-1}}}, \]
(13)
where \( d = \frac{4F}{\pi} \) is the hydraulic diameter of the channel, and \( F \) is the cross-sectional area of the channel.

The mass exchange of the working cavity through the ports with the cavities of the suction and discharge nozzles as well as through the ‘triangular’ slots is determined by the formula

\[
m_w = \mu F_w W_s \rho_2, \tag{14}
\]

where the port discharge coefficient \( \mu \), which takes into account the difference between real and theoretical flow conditions, is determined in the first approximation as for an orifice flowmeter; \( F_w \) is the passage area. The rate of adiabatic outflow of gas \( W_s \) is determined by the Saint-Venant–Wantzel formula:

\[
W_s = \sqrt{\frac{2RT_2}{k+1} \left[ 1 - \frac{p_1}{p_2} \right]^{\frac{k-1}{k}}}, \quad \text{if} \quad \frac{p_1}{p_2} \geq \beta = \left( \frac{2}{k+1} \right)^{\frac{1}{k-1}}
\]

\[
W_s = \sqrt{\frac{2RT_2}{k+1}}, \quad \text{if} \quad \frac{p_1}{p_2} < \beta = \left( \frac{2}{k+1} \right)^{\frac{1}{k-1}}.
\tag{15}
\]

The equation (14) has been used for the viscous mode of the flow. For the molecular and mixed flow regime, the equations (5), (6), (10) and (13) have been used. It should be noted, that those kinds of the vacuum compressor gaps have short length in the flow direction, therefore their coefficient of conductivity can be taken as \( K_3 = 1 \).

4. **Heat exchange calculation**

The heat gain supplied or removed from the compressed medium due to heat exchange with the walls of the housing and the surface of the rotors is calculated using the Newton–Richmann formula:

\[
\frac{dQ}{d\tau} = \alpha F_w (T_w - T), \tag{16}
\]

where \( \alpha = \frac{Nu \cdot \lambda}{d_{eq}} \) is the heat transfer coefficient; \( F_w \) is the area of the surface that limits the working chamber; \( T_w \) is the average temperature of the surface that limits the working chamber; \( Nu \) is the Nusselt number; \( \lambda \) is the coefficient of thermal conductivity of the compressible medium; \( d_{eq} = \frac{4F}{P} \) is the equivalent diameter; \( F \) is the cross-sectional area of the channel; \( P \) is the channel perimeter.

When considering the heat transfer processes in compressors, it is assumed that the gas is a continuum. However, at low absolute pressures, the phenomenon of heat transfer can be explained only if the molecular structure of the substance is taken into account. The flow of rarefied gases is characterised by the speed and temperature jumps on the surface. Quantitatively, these phenomena are characterised using slip and accommodation coefficients. To calculate the heat transfer of a rarefied gas, the following formula was used:

\[
Nu = \frac{Nu_0}{1 + Nu_0 \cdot Kn \cdot c}, \tag{17}
\]

where \( Nu_0 \) is the Nusselt number for the heat transfer of non-rarefied gas under the same heat transfer conditions; \( c \) is the auxiliary coefficient.
\[
c = \frac{2 - a}{a} \cdot \frac{2 \cdot k}{k + 1} \cdot \frac{1}{\text{Pr}}.
\]  

(18)

where \( \text{Pr} \) is the Prandtl number; \( a \) is the accommodation coefficient introduced by Knudsen, which characterises the proportion of true energy exchange in comparison with the maximum possible one. For air and processed metals, the range of the accommodation coefficient is 0.95 to 0.97. However, in calculations, the accommodation coefficient is most commonly taken as equal to 1.

To calculate the heat transfer in channels in a turbulent mode, the following formula is proposed in the paper [2]:

\[
Nu_0 = 0.023 \cdot \text{Re}^{0.8} \cdot \text{Pr}^{0.4} \left( 1 + 1.77 \frac{d_{\text{eq}}}{R_g} \right),
\]  

(19)

where \( \text{Re} \) is the Reynolds number, \( R_g \) is the rotor cavity radius. The average speed of the contact points of the rotors was chosen as the determining speed. To take into account the influence of rarefaction, a correction to the value of the dynamic viscosity of the gas was also introduced:

\[
\eta = \frac{\eta_0}{1 + \beta \cdot \text{Kn}},
\]  

(20)

where \( \eta_0 \) is the dynamic viscosity of the gas at atmospheric pressure, and the value \( \beta \) is determined from the solution of the system of equations:

\[
\begin{aligned}
Kn &= z - \frac{A}{(2 - c_1)(3 - c_2)} z^{2 - c_2} \\
\beta &= c_1 z^{c_2} + \frac{A}{c_1 c_2 (2 - c_1)(3 - c_2)} z
\end{aligned}
\]  

(21)

where \( z \) is the parametric variable; \( A = 0.15; c_1 = 1.479952; c_2 = 0.1551753. \)

5. Simulation results

As a result of mathematical simulation, the calculated pump characteristics of the vacuum compressor under study were obtained. They are given below (figure 3) for the operating mode in the vacuum pump mode since the preliminary test data are available for this mode.

The qualitative analysis of the obtained characteristics evidences that the design mode corresponding to the internal compression ratio in the screw vacuum compressor occurs at a residual pressure of slightly less than 10 kPa (upon discharge into the atmosphere). In modes with a residual pressure below 10 kPa, a characteristic pattern of 'external compression' of the evacuated air is observed; and in modes with a residual pressure of 10 kPa and above, a pattern of 'internal additional compression' of the evacuated air (at 10 kPa, the pattern of 'internal additional compression' is already present, but not notably).
6. Conclusion

The obtained characteristics have the greatest discrepancy with the experimental data in the area of low pressures and minimum productivity (no more than 15%) – that is, in those areas where internal leaks have a dominant influence, while a molecular flow regime is observed. At the same time, it should be noted that the mobility of the walls has a significant effect on leaks in it and requires further correction of the above methodology. However, it can already be recommended for preliminary calculations in the presented form, and the method for calculating leaks in the viscous mode can be extended to a wide range of positive displacement compressor machines without coolant injection.

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