A Deep Study of an Intermediate-age Open Cluster SAI 35 (Juchert 20) Using Ground-based Imaging and Gaia EDR3 Astrometry

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Abstract

We present a CCD UBVI photometric study of poorly studied intermediate-age open cluster SAI 35 (Juchert 20) for the first time. To accomplish this study, we also used LAMOST DR5, Two Micron All Sky Survey, and Gaia EDR3 databases. We identified 214 most probable cluster members with membership probability higher than 50%. The mean proper motion of the cluster is found as \( \mu_\alpha \cos \delta = 1.10 \pm 0.01 \) and \( \mu_\delta = -1.66 \pm 0.01 \) mas yr\(^{-1}\). We find the normal interstellar extinction law using the various two-color diagrams. The age, distance, reddening, and \( \log \) values of SAI 35 are derived

1. Introduction

Open clusters (OCs) have long been used for the understanding of stellar physics, to study the star formation scenario and the overall structure of the Milky Way (MW). The OC SAI 35 (Juchert 20; \( \alpha_{2000} = 04^h10^m47^s, \delta_{2000} = 46^\circ52'01''; l = 154^\circ494', b = -3^\circ422' \)) is located in the second Galactic quadrant. This object is listed in the catalog [http://oel.sai.msu.ru/catalog/](http://oel.sai.msu.ru/catalog/).

This OC is listed in the catalog given by Kronberger et al. (2006). Kharchenko et al. (2012, 2013) cataloged the proper motions, distance, reddening, and log(age) value of SAI 35 as \(-2.36, \Delta 6.11 \) mas yr\(^{-1}\), 2812 pc, 0.791 mag, and 8.22, respectively, based on the Two Micron All Sky Survey (2MASS) and the PPMXL catalog. Dias et al. (2014) derived proper-motion values of this object as \(-0.11 \) and \(-0.90 \) mas yr\(^{-1}\) based on the UCAC4 catalog. The integrated \( JHK_s \) magnitudes and luminosity function (LF) have been estimated by Kharchenko et al. (2016). A catalog of cluster membership has been given by Sampedro et al. (2017) based on UCAC4 data. Cantat-Gaudin et al. (2018) have made a catalog for cluster members and obtained fundamental parameters of SAI 35 based on Gaia DR2 data.

OCs generally suffer from the field star contamination toward the fainter ends in the main sequence (MS). Hence, knowledge about the cluster membership status of stars is necessary. The Gaia DR2 catalog was made public on 2018 April 25 (Gaia Collaboration et al. 2018a, 2018b). The (early) Third Gaia Data Release (hereafter EDR3; Gaia Collaboration et al. 2020, in preparation) was made public on 2020 December 3. EDR3 consists of the central coordinates, proper motions in R.A. and decl., and parallax angles \( (\alpha, \delta, \mu_\alpha \cos \delta, \mu_\delta, \pi) \) for around 1.46 billion sources with a limiting magnitude of 3–21 mag in G band. The Gaia EDR3 data are much more precise and accurate in comparison to the second data release.
Comparison with previous photometry is described in Section 4. Section 5 deals with the study of proper motion and determination of the membership probability of stars. The structural properties and derivation of fundamental parameters using the most probable cluster members have been carried out in Sections 6 and 7. The dynamical study of the cluster is discussed in Section 8. The cluster’s orbit is studied in Section 9. Finally, the conclusions are presented in Section 10.

2. Observations and Data Analysis

We carried out CCD $UBVI$ observations of stars in the region of OC SAI 35 on 2010 January 21. We used a 104 cm Sampurnanand reflector telescope ($f/13$) located at Aryabhatta Research Institute of Observational Sciences, Manora Peak, Nainital, India. Images were acquired using a 2K × 2K CCD that has 24 μm square pixel size, resulting in a scale of 0".36 pixel$^{-1}$ and a square field of view of 12/6 size. The CCD gain was 10 e$^-$/ADU, while the readout noise was 5.3 e$^-$. In order to improve the signal-to-noise ratio (S/N), the observations were taken in the $2 \times 2$ pixel$^2$ binned mode. Table 1 lists the date of observations, together with the raw photometric data that consists of bias subtraction, flat-fielding, and cosmic-ray removal. We used DAOPHOT software to estimate the stellar magnitudes. The instrumental magnitudes were derived through point-spread function (PSF) fitting using the DAOPHOT/ALLSTARS (Stetson 1987, 1992) package. To estimate the PSF, we used several well-isolated stars for the entire frame. The Gaussian function was used as an analytical model PSF. The shape of the PSF was made to vary quadratically with the position on the image. Appropriate aperture corrections were determined by using isolated and unsaturated bright stars in the image.

We have cross-identified the stars of different frames and filters using the DAOMATCH/DAOMASTER program available in DAOPHOT II. To determine the transformation coefficients from instrumental to standard magnitudes, CCDLIB and CCDSTD routines have been used. Finally, standard magnitudes and colors of all the stars have been obtained using the routine FINAL.

### Table 1

| Band | Exposure Time | Date       |
|------|---------------|------------|
| $U$  | $1200 \times 2$, $300 \times 1$ | 2010 Jan 21st |
| $B$  | $900 \times 2$, $240 \times 1$  |
| $V$  | $600 \times 3$, $180 \times 2$  |
| $I$  | $300 \times 2$, $60 \times 2$   |

2.1 Photometric Calibration

We have observed the standard field SA 98 (Landolt 1992) for SAI 35 during the observing night for photometric calibration of the CCD system. The 19 standard stars (SA 98-650, 670, 653, 666, 671, 675, 676, 682, 685, 688, 1082, 1087, 1102, 1112, 724, 733, 1124, 1119, 1122) used in the calibrations have brightness and color ranges $9.54 \leq V \leq 15.01$ and $-0.004 < (B - V) < 1.909$, respectively. For the extinction coefficients, we assumed the typical values for the ARIES observational site (Kumar et al. 2000). For translating the instrumental magnitude to the standard magnitude, the calibration equations derived using the least-squares linear regression are as follows:

$$
u = U + Z_U + C_U(U - B) + k_U X$$
$$b = B + Z_B + C_B(B - V) + k_B X$$
$$v = V + Z_V + C_V(B - V) + k_V X$$
$$i = I + Z_I + C_I(V - I) + k_I X,$$

where $u$, $b$, $v$, and $i$ denote the aperture instrumental magnitudes and $U$, $B$, $V$, and $I$ are the standard magnitudes, whereas air mass is denoted by $X$. The color coefficients $(C)$ and zero-points $(Z)$ for different filters are listed in Table 2. The errors in zero-points and color coefficients are ~0.01–0.02 mag. The internal errors derived from DAOPHOT are plotted against $V$ magnitude in Figure 2. This figure shows that the average photometric error is \( \leq 0.06 \) mag for $B$, $V$, and $I$ filters at $V \sim 19$ mag, while it is \( \leq 0.1 \) mag for $U$ filter at $V \sim 18$ mag. Photometric global (DAOPHOT+Calibrations) errors are also calculated, which are listed in Table 3. For the $V$ filter, the errors are 0.06 at $V \sim 17$ mag and 0.08 at $V \sim 20$ mag.

![Figure 1](image.png)

Figure 1. Finding chart of the stars in the field of SAI 35. Filled circles of different sizes represent brightness of the stars. The smallest size denotes stars of $V \sim 20$ mag. The open outer circle represents the cluster size, and the inner circle represents the core region.

### Table 2

| Filter | Color Coeff. $(C)$ | Zero-point $(Z)$ |
|--------|-------------------|-----------------|
| SA 35  |                   |                 |
| $U$    | $-0.01 \pm 0.02$  | $7.80 \pm 0.01$ |
| $B$    | $-0.02 \pm 0.01$  | $5.44 \pm 0.01$ |
| $V$    | $-0.04 \pm 0.01$  | $5.12 \pm 0.01$ |
| $I$    | $-0.05 \pm 0.02$  | $5.53 \pm 0.02$ |

5 IRAF is distributed by the National Optical Astronomical Observatory, which is operated by the Association of Universities for Research in Astronomy, under contract with the National Science Foundation.
To transform the CCD pixel coordinates to celestial coordinates, we used the online digitized European Southern Observatory catalog included in the SKYCAT software as an absolute astrometric reference frame. The CCMAP and CCTRAN routines in IRAF were used to find a transformation equation that gives the celestial coordinates as a function of the pixel coordinates. The resulting celestial coordinates have standard deviations of $\sim 0.01$ in both R.A. and decl.

### 2.2 Archived Data

#### 2.2.1 Gaia EDR3

The Gaia EDR3 (Gaia Collaboration et al. 2020, in preparation) database is used for the astrometric study of SAI 35. These data consist of positions on the sky ($\alpha$, $\delta$), parallaxes, and proper motions ($\mu_{\alpha}, \cos \delta, \mu_{\delta}$), with a limiting magnitude of $G = 21$ mag. The uncertainties in parallax values are $\sim 0.02$--0.03 mas for sources at $G \leq 15$ mag and $\sim 0.07$ mas for sources with $G > 17$ mag. The proper motions with their respective errors are plotted against $G$ magnitude in the right panel of Figure 3. The uncertainties in the corresponding proper-motion components are $\sim 0.01$--0.02 mas yr$^{-1}$ (for $G \leq 15$ mag), $\sim 0.05$ mas yr$^{-1}$ (for $G \sim 17$ mag), $\sim 0.4$ mas yr$^{-1}$ (for $G \sim 20$ mag), and $\sim 1.4$ mas yr$^{-1}$ (for $G \sim 21$ mag). We have compared Gaia EDR3 proper motions and their errors with Gaia DR2 data. We can clearly see in Figure 3 that the Gaia EDR3 database is more precise than Gaia DR2.

#### 2.2.2 2MASS Data

The near-infrared $JHK$ photometric data for SAI 35 were taken from 2MASS. 2MASS consistently scanned the whole sky in three near-IR bands $J$ (1.25 $\mu$m), $H$ (1.65 $\mu$m), and $K$ (2.17 $\mu$m). 2MASS (Skrutskie et al. 2006) used two highly automated 1.3 m aperture, open-tube, equatorial fork-mount telescopes (one at Mt. Hopkins, Arizona, USA, and the other at CTIO, Chile) with a three-channel camera (256 $\times$ 256 array of HgCdTe detectors in each channel). The 2MASS database contains photometry in the near-infrared $J$, $H$, and $K$ bands to a limiting magnitude of 15.8, 15.1, and 14.3, respectively, with an S/N greater than 10.

### 2.3 APASS

The American Association of Variable Star Observers (AAVSO) Photometric All-Sky Survey (APASS) is cataloged in five filters: $B$, $V$ (Landolt), and $g'$, $r'$, $i'$, with $V$-band magnitude ranging from 7 to 17 mag (Henden & Munari 2014). The DR9 catalog covers about 99% of the sky (Henden et al. 2016). To compare the photometry, we have used data in $B$ and $V$ bands for SAI 35.

#### 2.3.1 LAMOST DR5

LAMOST provided 9 million spectra with radial velocities in its fifth data release (DR5). These data also contain 5.3 million spectra with stellar atmospheric parameters (effective temperature, surface gravity, and metallicity). We used these data to obtain the value of mean radial velocity and metallicity toward the region of SAI 35. The mean value of radial velocity has been used to obtain the orbital parameters of the cluster.

### 3. Completeness of the CCD Data

The observational data may be incomplete because of the stellar crowding, saturation of bright stars, poor observing conditions, the inefficiency of CCD data reduction programs, etc. The completeness correction is mandatory to compute the LF of the stars in the cluster. To calculate the completeness level in our photometry for SAI 35, we performed the artificial star (AS) test. We have randomly added only 10%--15% of actually detected stars into the original images so that the crowding characteristics of the original images remain unchanged. The ADDSTAR routine in DAO-PHOT II was used to determine the completeness factor (CF). Detailed information about this experiment is given by Yadav & Sagar (2002) and Sagar & Griffiths (1998). In the present analysis, we have adopted the method given by Sagar & Griffiths (1998). Artificial stars with known magnitude and position were added in the original $V$ frames. These images are re-reduced using a similar method to what was adopted for the original images. The ratio of recovered to added stars in different magnitude bins gives the CF. The CF derived in this way are listed in Table 5 for SAI 35. Figure 4 shows the variation of CF versus $V$ magnitude. The value of CF is found as $\sim 93\%$ at $V = 19$ mag.
4. Comparison with Previous Photometry

To compare the photometry, we cross-matched the present catalog with the APASS catalog. For this matching the maximum difference in the positions of stars is 1″. In this manner, we have found 68 common stars between these two catalogs. A comparison of V magnitudes and (B − V) color between the two catalogs is plotted against V magnitude and shown in Figure 5. The mean difference and standard deviation per magnitude bin are listed in Table 4. The difference indicates that present V and (B − V) measurements are in fair agreement with those given in the APASS catalog.

5. Proper Motions and Field Star Separation

Proper motion is a key parameter to separate field stars from the cluster region to truly understand the MS of clusters. Proper-motion components (μα, μδ, μcosδ) are plotted as vector point diagrams (VPDs) in the bottom panels of Figure 6 after matching our observed UBVJ data with Gaia EDR3. The top panels show the corresponding V versus (B − V) color–magnitude diagrams (CMDs). The left panel shows all the detected stars toward the region of SAI 35, while the middle

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Table 4
Difference in V and (B − V) between the APASS Catalog and the Present Study

| V  | ΔV   | ΔB − V |
|----|------|--------|
| 11–12 | 0.03 (0.03) | 0.02 (0.03) |
| 12–13 | 0.02 (0.05) | 0.02 (0.04) |
| 13–14 | 0.06 (0.08) | 0.03 (0.10) |
| 14–15 | 0.07 (0.08) | 0.05 (0.12) |
| 15–16 | 0.08 (0.10) | 0.04 (0.18) |
| 16–17 | 0.09 (0.14) | 0.07 (0.21) |

Note. The standard deviation in the difference for each magnitude bin is also given in parentheses.

Table 5
Variation of CF in the V, (V − J) Diagram with the MS Brightness

| V mag Range | CF |
|-------------|----|
| 10–11       | 1.00 |
| 11–12       | 1.00 |
| 12–13       | 0.99 |
| 13–14       | 0.99 |
| 14–15       | 0.99 |
| 15–16       | 0.98 |
| 16–17       | 0.97 |
| 17–18       | 0.96 |
| 18–19       | 0.93 |
| 19–20       | 0.79 |
| 20–21       | 0.50 |
and right panels show the probable cluster members and field region stars. A circle of 0.6 mas yr\(^{-1}\) around the distribution of cluster stars in the VPD characterizes our membership criteria. The chosen radius is a compromise between losing member stars with poor proper motions and contamination of field region stars (Sariya & Yadav 2015; Bisht et al. 2020b). We have also used mean parallax for the cluster member selection. We estimated the weighted mean of parallax for stars inside the circle of VPD having \(V\) mag brighter than 20 mag. We obtained the mean value of parallax as \(0.35 \pm 0.02\) mas. We considered a star as the most probable member if it lies within 0.6 mas yr\(^{-1}\) radius in the VPD and has a parallax within 3\(\sigma\) from the mean parallax of SAI 35. The CMD of the most probable cluster members is shown in the top middle panel of Figure 6. In this figure, the MS of the cluster is identified.

To estimate the mean proper motion, we considered probable cluster members selected from the VPD and CMD as shown in Figure 6. Fitting the Gaussian function into the constructed histograms provides the mean proper motion in both the directions of R.A. and decl. as shown in Figure 7. In this way, we found the mean proper motion of SAI 35 to be \(1.10 \pm 0.01\) mas yr\(^{-1}\) and \(-1.66 \pm 0.01\) mas yr\(^{-1}\) in \(\mu_\alpha \cos \delta\) and \(\mu_\delta\), respectively. The estimated value of the mean proper motion for this object is in very good agreement with Cantat-Gaudin et al. (2018). Our derived values of mean proper motion are much more reliable than those of Kharchenko et al. (2012, 2013) and Dias et al. (2014) because the present estimate is based on accurate Gaia EDR3 proper-motion data.

### 5.1 Membership Probabilities

To estimate the membership probabilities of stars toward the region of SAI 35, we adopted the approach given by Balaguer-Núñez et al. (1998) by using proper-motion and parallax data from Gaia EDR3. This membership probability method has been used and described by various authors for a number of clusters (Yadav et al. 2013; Sariya et al. 2017, 2018; Bisht et al. 2020a; Kaur et al. 2020).

To describe the distribution functions for cluster and field stars in the adopted method, we used only good stars that have proper-motion errors better than \(<0.5\) mas yr\(^{-1}\). A clear crowding of stars can be seen at \(\mu_\alpha = 1.10\) mas yr\(^{-1}\), \(\mu_\delta = -1.66\) mas yr\(^{-1}\) and in the circular region having radii of 0.6 mas yr\(^{-1}\). We estimated dispersion \((\sigma_{\mu})\) in proper motions as 0.09 mas yr\(^{-1}\) by fixing cluster distance as 2.9 kpc and the radial velocity dispersion of 1 km s\(^{-1}\) for open star clusters (Girard et al. 1989). For nonmembers, we have estimated \((\mu_{\alpha}\), \(\mu_{\delta}\)) = \((-1.2, 0.2)\) mas yr\(^{-1}\) and \((\sigma_{\mu_{\alpha}}, \sigma_{\mu_{\delta}}) = (2.9, 3.3)\) mas yr\(^{-1}\).

We obtained 214 stars as cluster members after applying completeness to the observational CCD data, along with membership probability being higher than 50% and \(V < 20\) mag. In the left panel of Figure 8, we plotted membership probability versus \(V\) magnitude. In this figure, cluster members and field stars are separated. In the right panel of this figure, we plotted \(V\) magnitude versus parallax of stars. In Figure 9, we plotted the identification chart, proper-motion distribution, and \(V\) versus \(B - V\) CMD using the most probable cluster members. The most probable cluster members with high membership probability (>50%) are shown by filled circles in Figures 8 and 9.

Membership probability has been estimated by Cantat-Gaudin et al. (2018) up to 18.0 mag using the Gaia DR2 catalog for this cluster. To compare the membership probability, we plotted CMDs using our membership catalog and the Cantat-Gaudin et al. (2018) catalog as shown in Figure 10. We used only probable stars with membership probability higher than 50%. The CMDs plotted using selected members of the cluster based on membership probability seem clean. Therefore, both the membership probabilities are comparable.

Blue straggler stars (BSSs) are intriguing objects in diverse environments such as OCs (Johnson & Sandage 1955; Sandage 1962; Ahumada & Lapasset 1995). According to Sandage (1953), in CMDs of OCs BSSs are found along the extension of the MS as the brighter objects than the MS turnoff points in the CMDs of clusters. BSSs are considered crucial objects to study the interplay between stellar evolution and stellar dynamics (Bailyn 1995). In this paper, we found one BSS that is located at a radial distance of \(\sim 2/2\) from the cluster’s center. Our analysis strongly suggests that the BSS is a confirmed cluster member with a membership probability of 99%.

### 6. Structural Analysis: Radial Density Profile

To estimate the center coordinates toward the area of SAI 35, we used the star-count method. The resulting histograms in both the R.A. and decl. directions are shown in Figure 11. The Gaussian curve fitting provides the central coordinates as \(\alpha = 62.70 \pm 0.01\) deg \((4^h 10^m 48^s)\) and \(\delta = 46.86 \pm 0.004\) deg \((46^\circ 51' 56'')\). Our obtained values are very close to the values given by Sampedro et al. (2017) and Cantat-Gaudin et al. (2018).

To obtain structural parameters of SAI 35, we plotted the radial density profile (RDP) as shown in Figure 12. We divided the cluster’s region into many concentric rings and number density \((N_r)\) calculated in each ring by using the formula...
\[ R_i = \frac{N_i}{A_i}, \]
where \( N_i \) is the number of stars and \( A_i \) is the area of the \( i \)th zone. This RDP flattens at \( r \sim 3.9 \) and begins to merge with the background density as shown in the right panel of Figure 12. Therefore, we consider 3.9 as the cluster radius. Our derived value of radius is slightly higher than 3.5 as obtained by Dias et al. (2014). A solid line represents the fitted King (1962) profile:

\[ f(r) = f_{bg} + \frac{f_0}{1 + (r/r_c)^2}, \]

where \( r_c, f_0, \) and \( f_{bg} \) are the core radius, central density, and background density level, respectively. By fitting the King model to the RDP of SAI 35, we obtained the values of central density, background density, and core radius as 25.02 ± 2.5 stars arcmin\(^{-2}\), 9.30 ± 1.5 stars arcmin\(^{-2}\), and 1/20 ± 0.3, respectively. The background density level with errors is also shown by the dotted lines. The cluster limiting radius, \( r_{lim} \), is calculated by using the formula given by Bukowiecki et al. (2011). The estimated value of the limiting radius is found to be 7/6. The concentration parameter is found as 0.8 using the formula given by Peterson & King (1975). Maciejewski & Niedzielski (2007) reported that \( R_{lim} \) may vary for individual clusters from 2\( r_c \) to 7\( r_c \). We found that the value of \( R_{lim} \) (76) for SAI 35 is within the given limit by Maciejewski & Niedzielski (2007).

### 7. Color–Color Diagrams

#### 7.1 Reddening Law

To understand the nature of the extinction law and to find the value of interstellar reddening, we used various color–color diagrams (CCDs) for SAI 35.
the same line of sight (Mathis 1990). The emitted photons from cluster members are scattered and absorbed in the interstellar medium by dust particles. The normal reddening law is not applicable in the line of sight that passes through dust, gas, and molecular clouds (Sneden et al. 1978).

Chini & Wargue (1990) suggested \((V - \lambda)/(B - V)\) CCDs to understand the nature of the reddening law in which \(\lambda\) is any filter, other than \(V\). We plotted various two-color diagrams for SAI 35 as shown in Figure 13 to understand the reddening law. Our obtained values of color excesses with normal values have been listed in Table 6. The estimated values of color excesses are in good agreement with the normal values. Since the stellar color values are found to be linearly dependent on each other, a linear equation is applied to calculate the slope \(m_{\text{cluster}}\) of each CCD. The total-to-selective extinction ratio has been estimated using the relation provided by Neckel & Chini (1981):

\[
R_{\text{cluster}} = \frac{m_{\text{cluster}}}{m_{\text{normal}}} \times R_{\text{normal}},
\]

where \(m_{\text{cluster}}\) is the normal slope value in each CCD and \(R_{\text{normal}}(3.1)\) is the normal value of the total-to-selective extinction ratio. We have estimated \(R_{\text{cluster}}\) in different passbands as \(-2.9\)–\(3.3\), which is close to the normal value. Thus, we found a normal reddening law toward the cluster region of SAI 35.

7.1.2 \((U - B)\) Versus \((B - V)\) Diagram

The knowledge of interstellar reddening is important to obtain the main fundamental parameters (age, distance, etc.) of clusters. In the absence of spectroscopic observations, we can use the \((B - V)\), \((U - B)\) CCD for the reddening estimation of clusters (see Becker & Stock 1954). The resultant \((U - B)\) versus \((B - V)\) plot for SAI 35 is shown in Figure 14 using cluster members with membership probability higher than 50%. Blue circles are the matched stars with the catalog provided by Cantat-Gaudin et al. (2018). We have taken the intrinsic zero-age MS (ZAMS) from Schmid-Kaler (1982). The ZAMS is fitted by the solid curve considering the slope of reddening \(E(U - B)/E(B - V)\) as 0.72. By fitting ZAMS to the MS, we have calculated a mean value of \(E(B - V) = 0.72 \pm 0.05\) mag for SAI 35. The present estimate of reddening is close to the value derived by Kharchenko et al. (2012, 2013) and Sampedro et al. (2017).

7.1.3 Interstellar Extinction in Near-IR

The near-IR photometry is very helpful to understand interstellar extinction (Tapia et al. 1988). Here we have used \(JHK\) photometry from the 2MASS database to study the interstellar extinction law. The \((J - K)\) versus \((V - K)\) diagram for SAI 35 is shown in Figure 15. The ZAMS of solar metallicity is taken from Caldwell et al. (1993) as shown by the solid line. The fit of ZAMS provides \(E(J - K) = 0.33 \pm 0.04\) mag and \(E(V - K) = 1.92 \pm 0.03\) mag. The ratio \(\frac{E(J - K)}{E(V - K)} \sim 0.17 \pm 0.05\) is in good agreement with the normal interstellar extinction value of 0.19 given by Cardelli et al. (1989).

7.2 Age and Distance

Age and distance of OCs are important parameters to trace the galactic structure and to understand the chemical evolution of the Galaxy (Friel & Janes 1993). The main fundamental
parameters (reddening, metallicity, distance modulus, age, etc.) of a cluster can be obtained by fitting the theoretical isochrones to our observed CMDs. We used the theoretical isochrones given by Marigo et al. (2017) for $Z = 0.019$. The $V/(U - B)$, $V/(B - V)$, $V/(V - I)$, $G/(G_{BP} - G)$, $G/(G_{RP} - G_{RP})$, and $G/(G - G_{RP})$ CMDs, along with visually fitted isochrones, are shown in Figures 16 and 17.

We superimpose theoretical isochrones of different ages (log (age) = 8.50, 8.55, and 8.60) in all the plotted CMDs. Based on this, we have found an age of $360 \pm 40$ Myr. Our estimated value of age is the same as that of Sampedro et al. (2017). We obtained distance modulus $(m - M) = 14.60 \pm 0.2$ mag. The estimated distance modulus provides a heliocentric distance as $2.95 \pm 0.3$ kpc. The present distance estimate is in good agreement with that of Cantat-Gaudin et al. (2018). The galactocentric coordinates are $X_e = -2.61$ kpc, $Y_e = 1.25$ kpc, and $Z_e = -0.17$ kpc. The galactocentric distance of the cluster was calculated as $11.20$ kpc. The derived galactocentric coordinates are in good agreement with those of Cantat-Gaudin et al. (2018).

We have also used the parallax of cluster members to find the distance of SAI 35. The resulting histogram is shown in Figure 13.
The mean parallax is estimated as $0.35 \pm 0.02$ mas, which corresponds to a distance of $2.86 \pm 0.17$ kpc. The calculated value of parallax is in good agreement with the value obtained by Cantat-Gaudin et al. (2018). We also estimated distance using the method described by Bailer-Jones et al. (2018). Thus, we obtained a distance of SAI 35 as $2.90 \pm 0.15$ kpc. We find a similar value of distance using the mean parallax and distance modulus of the cluster.

Table 6
Comparison of the Extinction Law in the Direction of the Cluster SAI 35 with a Normal Extinction Law Given by Cardelli et al. (1989)

| SAI 35 | $(V-I)/(B-V)$ | $(V-J)/(B-V)$ | $(V-H)/(B-V)$ | $(V-K)/(B-V)$ |
|--------|----------------|----------------|----------------|----------------|
| Our derived ratios | $1.36 \pm 0.2$ | $2.15 \pm 0.23$ | $2.48 \pm 0.09$ | $2.70 \pm 0.11$ |
| Normal ratios | $1.60$ | $2.22$ | $2.55$ | $2.74$ |

Figure 14. $(U-B)$ vs. $(B-V)$ CCD. The solid curve represents the locus of the Schmid-Kaler (1982) ZAMS for solar metallicity. The arrow indicates the reddening vector.

Figure 15. $(J-K)$ vs. $(V-K)$ CCD. Solid and dotted lines are the ZAMS taken from Caldwell et al. (1993).

Figure 16. CMD of the clusters under study. The curves are the isochrones of log (age) = 8.50, 8.55, and 8.60. These isochrones are taken from Marigo et al. (2017). Black circles are the probable cluster members, while the blue circles represent the matched stars with Cantat-Gaudin et al. (2018).

Figure 17. Same as Figure 16, but using Gaia EDR3 photometric magnitudes.
Several fundamental parameters (center, radius, age, distance, reddening, etc.) for this object have been derived by several authors in the literature. Table 7 presents a comparison of our estimated parameters in this paper with previously published values. All the derived parameter values are comparable with the literature values.

8. Dynamical Study

8.1 Luminosity Function and Mass Function

The LF is the distribution of members of a cluster in different magnitude bins. We considered probable cluster members in the \( V/(V-I) \) CMD to construct the LF for SAI 35. For the construction of the LF, we first converted the apparent \( V \) magnitudes into the absolute magnitudes by using the distance modulus. Then, we plotted the histogram of the LF as shown in Figure 20. The interval of 1.0 mag was picked so that there would be a sufficient number of stars in each bin for statistical usefulness. The LF of SAI 35 rises steadily up to \( M_V = 4.5 \) mag.

The MF is defined as the distribution of masses of cluster stars per unit volume during the time of star formation. The LF can be converted into the MF using a mass–luminosity relation. Since we could not obtain an observational transformation, we must depend on theoretical models. To perform the conversion of LF into MF, we used cluster parameters derived in this paper and theoretical models given by Marigo et al. (2017). The resulting MF is shown in Figure 21. The MF slope can be derived from the linear relation

\[
\log \frac{dN}{dM} = -(1 + x) \times \log(M) + \text{constant.}
\]

In the above relation, \( dN \) represents the number of stars in a mass bin \( dM \) with the central mass \( M \), and \( x \) is the MF slope. The Salpeter (1955) value for the MF slope is \( x = 1.35 \). This form of Salpeter shows that the number of stars in each mass range decreases rapidly with increasing mass. Our derived MF slope value, \( x = 1.49 \pm 0.16 \), is in good agreement with Salpeter’s slope within the uncertainty. Using this value of the MF slope within the mass range of 1.1–3.1 \( M_\odot \), the total mass was obtained as \( \sim 364 M_\odot \).

8.2 Mass Segregation

As a result of the mass segregation, massive stars get concentrated more toward the center than the fainter stars. Many authors have reported the mass segregation phenomenon in clusters (Piatti 2016; Zeidler et al. 2017; Dib & Basu 2018; Bisht et al. 2020b). To study the effect of mass segregation for the clusters, we plot the cumulative radial stellar distribution of stars for different masses in Figure 22. This figure exhibits the mass segregation effect in the clusters under the present study, which means that the massive stars have gradually sunk toward the cluster center as compared to the distribution of the fainter stars. We have divided the MS stars into three mass ranges, \( 3.1 \leq M/M_\odot \leq 2.2, \ 2.2 \leq M/M_\odot \leq 1.5, \text{ and } 1.5 \leq M/M_\odot \leq 1.1 \). To check whether these mass distributions represent the same kind of distribution or not, we have performed the Kolmogorov–Smirnov (K-S) test. This test indicates that the confidence level of the mass segregation effect is 90%.

Further, it is important to know that the effect of mass segregation is due to dynamical evolution or the imprint of star formation or both. In the lifetime of star clusters, encounters between its member stars gradually lead to an increased degree of
energy equipartition throughout the clusters. In this process, the higher-mass cluster members accumulate toward the cluster center and transfer their kinetic energy to the more numerous lower-mass stellar component, thus leading to mass segregation.

8.3 The Relaxation Time

The timescale in which a cluster will lose all traces of its initial conditions is well represented by its relaxation time $T_E$. The relaxation time is the characteristic timescale for a cluster to reach some level of energy equipartition. The relaxation time given by Spitzer & Hart (1971) stated that

$$T_E = \frac{8.9 \times 10^7 N^{3/2} R_h^{1/2}}{(m)^{1/2} \log(0.4N)},$$

where $N$ is the number of cluster members, $R_h$ is the half-mass radius of the cluster, and $(m)$ is the mean mass of the cluster.

A comparison of cluster age with its relaxation time indicates that the relaxation time is smaller than the age. Therefore, we conclude that SAI 35 is a dynamically relaxed cluster.

8.4 Tidal Radius

Tidal radius is the distance from the cluster center where gravitational acceleration caused by the cluster becomes equal to the tidal acceleration due to the parent galaxy (von Hoerner 1957). Tidal interactions play an important role in understanding the initial structure and dynamical evolution of stars. The value of $R_h$ was taken as 1.64 pc, which has been assumed as half of the cluster radius derived by us. Finally, we have estimated the dynamical relaxation time $T_E$ as $\sim 11$ Myr. A comparison of cluster age with its relaxation time indicates that the relaxation time is smaller than the age. Therefore, we conclude that SAI 35 is a dynamically relaxed cluster.

![Figures 20 and 21](https://example.com/figures.png)
Here Note. respectively; and $\phi$ is the position angle relative to the Sun’s direction.

### Figure 22. Cumulative radial distribution of stars in various mass ranges.

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### Table 8
Position and Velocity Components in the Galactocentric Coordinate System

| Cluster   | $R$ (kpc) | $Z$ (kpc) | $U$ (km s$^{-1}$) | $V$ (km s$^{-1}$) | $W$ (km s$^{-1}$) | $\phi$ (rad) |
|-----------|-----------|-----------|-------------------|-------------------|-------------------|-------------|
| SAI 35    | 10.98     | $-0.15$   | 60.17 $\pm$ 5.80  | $-200.86$ $\pm$ 2.90 | $-6.28$ $\pm$ 0.99 | 0.11        |

Note. Here $R$ is the galactocentric distance; $Z$ is the vertical distance from the Galactic disk; $U$, $V$, $W$ are the radial, tangential, and vertical components of velocity, respectively; and $\phi$ is the position angle relative to the Sun’s direction.

clusters (Chumak et al. 2010; Dalessandro et al. 2015). The Galactic mass $M_G$ inside a galactocentric radius $R_G$ is given by Genzel & Townes (1987).

$$M_G = 2 \times 10^8 M_\odot \left( \frac{R_G}{30 \text{ pc}} \right)^{1.2}.$$ 

The estimated value of Galactic mass inside the galactocentric radius (see Section 4.5) is found to be $2.4 \times 10^{11} M_\odot$. Kim et al. (2000) have introduced the formula for the tidal radius $R_t$ of clusters as

$$R_t = \left( \frac{M}{5M_G} \right)^{1/3} R_G,$$

where $R_t$ and $M_c$ indicate the cluster’s tidal radius and total mass (see Section 9), respectively. The estimated value of the tidal radius is $10.22 \pm 0.82$ pc.

### 9. The Orbit of the Cluster

We obtained the Galactic orbit of SAI 35 using the Galactic potential models. We adopted the technique described by Allen & Santillian (1991) for Galactic potentials. Lately, Bajkova & Bobylev (2016) and Bobylev et al. (2017) have refined Galactic potential model parameters with the use of new observational data for the galactocentric distance $R \sim 0–200$ kpc. The equations used for the Galactic potential models are discussed by Rangwal et al. (2019). We have used the main fundamental parameters (cluster center ($\alpha$ and $\delta$), mean proper motions ($\mu_\alpha$ cos $\delta$, $\mu_\delta$), parallax angles, age, and heliocentric distance ($d_c$)) to obtain orbital parameters of SAI 35. Radial velocity is estimated as $-91.62 \pm 6.29$ km s$^{-1}$ from the LAMOST DR5 catalog based on membership probability and the cluster’s VPD.

We have transformed equatorial velocity components into Galactic space velocity components ($U$, $V$, $W$). The Galactic center is taken at (17$^h$45$^m$32$^s$.224, 28$^\circ$56$'$.10$''$), and the north Galactic pole is at (12$^h$5$^m$26$^s$.282, 27$^\circ$7$'$.42$''$) (Reid & Brunthaler 2004). To apply a correction for standard solar motion and motion of the local standard of rest (LSR), we used position coordinates of the Sun of (8.3, 0, 0.02) kpc and space velocity components of (11.1, 12.24, 7.25) km s$^{-1}$ (Schonrich et al. 2010). The transformed parameters in the galactocentric coordinate system are listed in Table 8.

Figure 23 shows the orbits of the cluster SAI 35. In the top left panel of this figure, the motion of the cluster is represented in terms of distance from the Galactic center and Galactic plane, which show a 2D side view of the orbits. In the top right panel, the cluster motion is described in terms of $x$ and $y$ components of galactocentric distance, which shows a top view of orbits. The bottom panel of this figure indicates the motion of SAI 35 in the Galactic disk with time. According to our analysis, SAI 35 follows a boxy pattern. The value of eccentricity is nearly 0, which demonstrates that the OC SAI 35 traces a circular path around the Galactic center. The birth and present-day position of this cluster in the Galaxy are estimated as the filled circle and triangle, respectively, in Figure 23. The orbit is within the solar circle. We also calculated various orbital parameters, which are listed in Table 9.

### 10. Conclusions

We have performed a detailed analysis of the newly discovered OC SAI 35 based on Johnson–Cousins $UBVI$ photometry carried out using the 1.04 m Telescope (ARIES, Nainital, India), 2MASS, the LAMOST DR5 catalog, and the Gaia EDR3 photometric and astrometric database. We have identified 214 member stars with membership probabilities higher than 50%. We investigated the cluster structure, derived the main fundamental parameters, explained the dynamical study, and determined the galactic orbit of SAI 35. The main outcomes of this study can be summarized as follows:

1. The new cluster center is estimated as $\alpha = 62.70 \pm 0.01$ deg ($4^{h}10^{m}48^{s}$) and $\delta = 46.86 \pm 0.004$ deg ($46^{\circ}51^{'}36^{''}$) using cluster members based on VPDs.
2. Using the radial density profile, the cluster radius is obtained as 3.9 (3.3 pc) using the radial density profile.

3. Based on the completeness of CCD data, VPD, and membership probability estimation, we identified 214 most probable cluster members for SAI 35. The mean proper motion is estimated as 1.10 ± 0.01 mas yr⁻¹ and −1.66 ± 0.01 mas yr⁻¹ in both the R.A. and decl. directions, respectively.

4. We detected one BSS toward the region of SAI 35, and it was found to be a confirmed member of the cluster.

5. From the two-color diagram, we have estimated $E(B - V) = 0.72 ± 0.05$ mag. We have plotted various CCDs and obtain the total-to-selective extinction ratio ($R_V$) in the range of 2.9–3.3. Our analysis indicates that the interstellar extinction law is normal toward the direction of SAI 35. From the combined optical and near-infrared data, we obtained $E(J - K) = 0.33 ± 0.04$ mag, while $E(V - K) = 1.92 ± 0.03$ mag.

6. The distance of SAI 35 is determined as 2.9 ± 0.15 kpc. This value is well supported by the distance estimated using the mean parallax of the cluster. Age is determined as 360 ± 40 Myr by comparing the cluster CMD with the solar-metallicity theoretical isochrones given by Marigo et al. (2017).

7. The MF slope is estimated as 1.49 ± 0.16 in the mass range of 1.1–3.1 $M_\odot$, which is in good agreement within uncertainty with the value (1.35) given by Salpeter (1955) for field stars in the solar neighborhood. By using this MF slope, we have estimated the total mass and mean mass as 364 and 1.70 $M_\odot$, respectively.

8. On the basis of the dynamical evolution study of SAI 35, we found a deficiency of low-mass stars in the core. Our study shows a clear mass segregation phenomenon in this cluster. The K-S test indicates 90% confidence level of the mass segregation effect. The dynamical relaxation time is estimated as 10 Myr, which is less than the age of the cluster. Our study indicates that SAI 35 is a dynamically relaxed OC.

9. The Galactic orbits and orbital parameters were estimated using Galactic potential models. We found the value of eccentricity ~0, which concludes that SAI 35 traces a circular path around the center of the Galaxy.

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**Table 9**

| Cluster | $e$ | $R_\alpha$ (kpc) | $R_p$ (kpc) | $Z_{\text{max}}$ (kpc) | $E (100 \text{ km s}^{-1})^2$ | $J_z (100 \text{ kpc km s}^{-1})$ | $T$ (Myr) |
|---------|-----|-----------------|-------------|-----------------|----------------|-----------------|-------|
| SAI 35  | 0.01| 11.56           | 11.81       | 0.18            | −10.12         | −22.06          | 341   |

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**Figure 23.** Galactic orbits of the cluster SAI 35 estimated with the Galactic potential model described in the text in the time interval of the age of the cluster. The top left panel shows the side view and the top right panel shows the top view of the orbit. The bottom panel shows the motion of SAI 35 in the Galactic disk with time. The filled triangle and circle denote the birth and present-day position of the cluster in the Galaxy, respectively.
