Algorithmic correspondence and canonicity for possibility semantics

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### Possibility semantics
- a variant of standard Kripke semantics for modal logic
- motivation: partial possibilities vs total worlds
- constructive study of classical (modal) logic:
  - intuitionistic-style semantics: refinement relation
  - constructive completeness proofs
  - relation to constructive canonical extension

### Existing works
- Duality: Holliday 2016
- Correspondence: Yamamoto 2016
- Canonicity: Holliday 2016
Possibility semantics

- possibility frame: $\mathcal{F} = (W, R, \sqsubseteq, \text{RO}(W, \sqsubseteq))$
- possibility model: $\mathbb{M} = (\mathcal{F}, V)$ where $V : \text{Prop} \rightarrow \text{RO}(W, \sqsubseteq)$
- refinement relation $\sqsubseteq$: partial order on $W$
- accessibility relation $R$: binary relation on $W$
- $\text{RO}(W, \sqsubseteq)$: set of admissible valuations
- intuition behind $\text{RO}(W, \sqsubseteq)$: subsets equal to their “double negation”
Possibility semantics

Satisfaction relation

1. \( F, V, w \models p \iff w \in V(p); \)
2. \( F, V, w \models \varphi \land \psi \iff F, V, w \models \varphi \text{ and } F, V, w \models \psi; \)
3. \( F, V, w \models \varphi \lor \psi \iff (\forall v \sqsubseteq w)(\exists u \sqsubseteq v)(F, V, u \models \varphi \text{ or } F, V, u \models \psi); \)
4. \( F, V, w \models \varphi \rightarrow \psi \iff (\forall v \sqsubseteq w)(F, V, v \models \varphi \Rightarrow F, V, v \models \psi); \)
5. \( F, V, w \models \neg \varphi \iff (\forall v \sqsubseteq w)(F, V, v \not\models \varphi); \)
6. \( F, V, w \models \Box \varphi \iff \forall v(Rwv \Rightarrow F, V, v \models \varphi). \)
$\mathbb{B} \models \forall \vec{p}(\varphi(\vec{p})) \iff F \models \varphi(\vec{p})$

$\upharpoonleft$

$\mathbb{B} \models \forall \vec{i}\text{Pure}(\varphi(\vec{p})) \iff F \models \text{FO(Pure}(\varphi(\vec{p}))\text{))}$

- In the dual BAO of Kripke frames, nominals are interpreted as atoms.
- How about possibility semantics?
Given $\mathcal{F} = (W, \sqsubseteq, R, \text{RO}(W, \sqsubseteq))$, the regular open dual BAO $\mathcal{B}_{\text{RO}}$

- $\mathcal{B}_{\text{RO}}$ is a complete and completely additive BAO, but not necessarily atomic.
- lack of atomicity: what is the consequence in correspondence theory?
### Nominals and their interpretations

| Algebraic setting                      | Interpretation for nominals | Dually corresponding to                              |
|----------------------------------------|-----------------------------|------------------------------------------------------|
| perfect Boolean algebras               | atoms                       | singletons                                           |
| perfect distributive lattices          | complete join-primes        | $w \uparrow$                                         |
| perfect general lattices               | complete join-irreducibles   | Galois closure of singletons                         |
| constructive canonical extensions      | closed elements              | N.A.                                                |
| complex algebras of possibility frames |                             | regular open closures of singletons                  |
Our results

- Correspondence results for inductive formulas over full possibility frames
- Correspondence results for inductive formulas over filter-descriptive possibility frames
- Constructive canonicity-via-correspondence