Modified Gravitational Theory and the Pioneer 10 and 11 Spacecraft Anomalous Acceleration

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Abstract

The nonsymmetric gravitational theory leads to a modified acceleration law that can at intermediate distance ranges account for the anomalous acceleration experienced by the Pioneer 10 and 11 spacecraft.

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1 Introduction

A gravitational theory explanation of the acceleration of the expansion of the universe [1] [2] [3] and the observed flat rotation curves of galaxies was proposed [4] [5], based on the nonsymmetric gravitational theory (NGT) [6] [7] [8]. The equations of motion of a test particle in a static, spherically symmetric gravitational field modifies Newton’s law of acceleration for weak fields. We shall show that the modified acceleration can account for the anomalous acceleration observed for the Pioneer 10 and 11 spacecraft [9] [10]. Analyses of radio Doppler and ranging data from in the solar system indicate an apparent anomalous acceleration acting on the spacecraft, with a magnitude \(a_P = (8.74 \pm 1.33) \times 10^{-8} \text{cm s}^{-2}\), directed toward the Sun.

A detailed investigation of both external and internal effects have not to-date disclosed a convincing source of the anomalous acceleration. Possible systematic origins of the residuals have not been found. The gravitational source of the spacecraft anomalous acceleration can be linked to the increase in the rotational velocity of a test particle in a circular orbit about the center of a spiral galaxy. They both may share the same gravitational origin in the modified acceleration law in NGT. In the NGT phenomenology, we have suggested that in certain distance regimes gravity increases its strength with the distance from a mass source, corresponding to a
gravitational “confinement” potential. The gravitational constant is “renormalized” by defining the gravitational constant at infinity to be

\[ G \equiv G_\infty = G_0 \left( 1 + \sqrt{\frac{M_0}{M}} \right), \]

(1)

where \( G \) is Newton’s gravitational constant and \( M_0 \) is a positive parameter, which is range-distance dependent, so that as \( M_0 \) increases with distance the gravitational constant \( G \) increases for a fixed value of \( M \). Conversely, for fixed values of \( M_0 \) the gravitational constant \( G \to G_0 \) as \( M \to \infty \).

2 The Equations of Motion of a Particle

We shall consider the equation of motion [5]

\[ \frac{du^\mu}{d\tau} + \left\{ \frac{\mu}{\alpha \beta} \right\} u^\alpha u^\beta = s^{(\mu \sigma)} f_{[\sigma \nu]} u^\nu, \]

(2)

where \( u^\mu = \frac{dx^\mu}{d\tau} \) is the 4-velocity of a particle and \( \tau \) is the proper time along the path of the particle. Moreover, \( s^{(\mu \alpha)} g_{(\nu \alpha)} = \delta^\mu_\nu \) and

\[ \left\{ \frac{\mu}{\alpha \beta} \right\} = \frac{1}{2} s^{(\mu \sigma)} \left( \partial_\beta g_{(\alpha \sigma)} + \partial_\alpha g_{(\beta \sigma)} - \partial_\sigma g_{(\alpha \beta)} \right). \]

(3)

The notation is the same as in refs. [4, 5, 7, 8]. In [5], we obtained the acceleration on a point particle

\[ a(r) = -\frac{G_\infty M}{r^2} + \sigma \frac{\exp(-r/r_0)}{r^2} \left( 1 + \frac{r}{r_0} \right), \]

(4)

where \( r_0 = 1/\mu \) and \( \mu \) is the mass parameter in the NGT action associated with the skew field \( g_{[\mu \nu]} \) [4, 5]. Moreover, \( G_\infty \) defined in Eq. (1) is the effective gravitational constant at infinity where \( G_0 \) is Newton’s gravitational constant, \( G_0 = 6.673 \times 10^{-8} \text{g}^{-1} \text{cm}^3 \text{s}^{-2} \) and \( M_0 \) and \( r_0 \) are two free parameters. The constant \( \sigma \) in (4) is given by [5]

\[ \sigma = \frac{\lambda s G_0^2 M^2}{3c^2 r_0^2}, \]

(5)

where \( \lambda \) and \( s \) denote the strengths of the coupling of a test particle to the force \( f_{[\mu \nu]} \) and of the skew field \( g_{[\mu \nu]} \), respectively.

We set \( \sigma = G_0 \sqrt{M} \sqrt{M_0} \) and write Eq. (4) as

\[ a(r) = -\frac{G_0 M}{r^2} \left\{ 1 + \sqrt{\frac{M_0}{M}} \left[ 1 - \exp(-r/r_0) \left( 1 + \frac{r}{r_0} \right) \right] \right\}. \]

(6)
The form of \( a(r) \) guarantees that it reduces to the Newtonian acceleration \( a_{\text{Newt}} = -G_0 M/r^2 \) at small distances \( r \ll r_0 \).

The two parameters \( M_0 \) and \( r_0 \) in the NGT solution for weak fields are not universal constants. The effective gravitational constant \( G = G_\infty \) given by (1) indicates that the strength of gravity is distance dependent and scales as \( \sqrt{M_0/M} \). Moreover, we postulate that the parameter \( M_0 \) is dependent on the distance range \( r_0 \) such that \( M_0 = M_0(r_0) \) increases from a small value \( M_0 < M_\odot \) to a large galaxy size mass as the range parameter \( r_0 \) increases to larger values. We obtained from a fitting of the galaxy rotation curves \( (r_0)_g = 13.92 \) kpc and \( (M_0)_g = 9.6 \times 10^{11} M_\odot \). For galaxies and clusters of galaxies the range parameter \( r_0 \) was determined by

\[
a_0 = \frac{G_0 M_0}{r_0^2} = cH_0, \tag{7}
\]

where \( H_0 \) is the current value of Hubble’s constant and \( cH_0 = 6.9 \times 10^{-8} \) cm s\(^{-2}\).

The distance-range dependence of the strength of the gravitational force does not directly come from the NGT action or the field equations in the weak field approximation. However, a distance-range dependence could be built into the action by including a scalar field \( \phi = r_0 \), so that \( \mu = \mu(\phi) \). It is hoped that an exact solution of the NGT field equations will reveal the “running” of the gravitational coupling strength with the distance range \( r_0 \). Another possibility is that nonlinear, nonperturbative solutions of the field equations lead to range dependent gravity solutions that deviate significantly from general relativity for small \( g[^{\mu\nu}] \). In the NGT phenomenology, we shall treat the range parameter \( r_0 \) and the parameter \( M_0 \) as constants within their approximate valid ranges.

3 Spacecraft Anomalous Acceleration

Let us expand the exponential in (6) for \( r < r_0 \):

\[
\exp(-r/r_0) \left(1 + \frac{r}{r_0}\right) = 1 - \frac{1}{2} \left(\frac{r}{r_0}\right)^2 + \frac{1}{3} \left(\frac{r}{r_0}\right)^3 - ... \tag{8}
\]

Inserting this into (6) gives for the magnitude of the anomalous acceleration

\[
a_A \equiv a - a_{\text{Newt}} = \frac{G_0 \sqrt{M} \sqrt{M_0}}{2r_0^2}. \tag{9}
\]

For the Pioneer spacecraft we have

\[
a_P = \frac{G_0 \sqrt{M_\odot} \sqrt{(M_0)_{sc}}}{2(r_0)_{sc}^2}. \tag{10}
\]

By adopting the values \( (r_0)_{sc} = 300 \) A.U. = \( 4.49 \times 10^{10} \) km, \( \sqrt{(M_0)_{sc}} = 0.026 \sqrt{M_\odot} \), we obtain the Pioneer spacecraft anomalous acceleration

\[
a_P = 8.7 \times 10^{-8} \) cm s\(^{-2}\). \tag{11}
\]
The anomalous acceleration has not been detected in planetary orbits, particularly for Earth and Mars \cite{11, 12}. A small anomalous acceleration experienced by a planet would cause a perturbed radial difference $\Delta r$ in the planet’s orbit. The anomalous acceleration $a_A$ would be for a circular orbit \cite{10}

$$a_A = \frac{a_{\text{Newt}} \Delta r}{r},$$

where $r$ is the orbital radius. For Earth $\Delta r < -21$ km and for the mean Sun-Earth radial distance $r = 1$ A.U. = $1.496 \times 10^8$ km, we find for Earth that

$$a_A < 8.32 \times 10^{-9} \text{ cm s}^{-2}.$$  

(13)

For the solar planetary system, we adopt the parameter values

$$(r_0)_{pl} \leq 5 \times 10^3 \text{ km}, \quad \sqrt{(M_0)_{pl}} \leq 10^{-19} \sqrt{M_\odot}.$$  

(14)

For planetary distances, the exponential term in Eq. (6) is vanishingly small, because $r_{pl} \gg (r_0)_{pl}$ and $\sqrt{(M_0)_{pl}/M_\odot} \sim 10^{-19}$ whereby $G \equiv G_\infty \sim G_0$. Thus, any anomalous acceleration would satisfy $a_A < 3 \times 10^{-11} \text{ cm s}^{-2}$ and be undetectable for solar system planets.

The perihelion advance of Mercury is given by \cite{5}:

$$\Delta \omega = \frac{6\pi}{c^2 p} (GM_\odot - c^2 K_\odot),$$

(15)

where $p = a(1 - e^2)$ and $a$ and $e$ denote the semimajor axis and eccentricity of Mercury’s orbit, respectively. The parameter $K_\odot$ is

$$K_\odot = \frac{\lambda s G_0^2 M_\odot^2}{3c^4 (r_0)_{pl}^2}.$$  

(16)

We require that $K_\odot < G_0 M_\odot/c^2 = 1.5$ km to agree with the observed perihelion advance of Mercury. This yields the bound

$$\lambda s < \frac{3c^2 (r_0)_{pl}^2}{G_0 M_\odot} = 5.1 \times 10^7 \text{ km}.$$  

(17)

For $(r_0)_{pl} = 10$ km we obtain $\lambda s < 203$ km. The deflection of light grazing the limb of the Sun is given by \cite{5}:

$$\Delta = \frac{4G_0 M_\odot}{c^2 R_\odot}$$  

(18)

in agreement with GR.
4 Conclusions

We have shown that the modified Newtonian acceleration obtained from NGT can with a suitable choice of the two parameters $r_0$ and $M_0$ explain the anomalous acceleration observed for the Pioneer spacecraft 10 and 11. The result depends on the phenomenological premise that the strength of gravity is distance dependent. The range parameter has a growing dependence on distance as does the parameter $M_0$. The running of the gravitational constant $G$ with mass as $\sqrt{M_0/M}$ is a fundamental physical phenomenon that could have its origins in exact solutions of the NGT field equations. This issue will be the subject of a future investigation.

A possible signature of the NGT modified Newtonian acceleration, and the anomalous acceleration for spacecraft moving away from the Sun, is to observe whether there is an eventual increase in the anomalous acceleration as the spacecraft moves to distances from the Sun greater than, say, 200 or 300 A.U., revealing a range dependence on the parameter $r_0$.

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