RARE KAON DECAYS: IL BUONO, IL BRUTTO, IL CATTIVO
G. Redlinger
Brookhaven National Laboratory, Upton, NY, USA 11973

ABSTRACT
I briefly review recent progress in rare kaon decays, where I take “rare” to mean those with $B < \mathcal{O}(10^{-7})$.

1 Introduction
The title of this talk (which by the way borrows from the original title of the famous “spaghetti Western” “The Good, The Bad, and The Ugly”) arose from the convergence of several trains of thought: first, that this conference was being held near Rome, where the filmmaker Sergio Leone was born and lived;
second, various allusions to current world politics, which are best left to the coffee break; lastly, and most importantly, I was inspired by a writeup of a talk by Wilczek [1]:

“Our current, working description of fundamental physics is based on three conceptual systems... it is not inappropriate to call them the Good, the Bad, and the Ugly”

For the purposes of this talk, we concentrate on the “Ugly” which is the flavor sector whose many parameters illustrate our lack of understanding of the Higgs Yukawa couplings. Experiment is a key driving force in making progress here; kaon decays have had a glorious history in elucidating this physics, and continue to serve as sensitive probes.

In the following, I cover rare kaon decay results\(^2\) from approximately 2003 to the present. These can be grouped as follows:

- **The Good.** This includes the study of signatures explicitly beyond the Standard Model (BSM), of which the best known are the lepton flavor violating decays. Here we also include modes sensitive to quark-mixing parameters and CP violation (CPV) with small theoretical uncertainty, thus making them excellent candidates for searches for BSM physics.

- **The Bad.** This include studies of the low-energy behavior of the strong interactions. Obviously this is not “bad” in itself, but is not directly connected to studying the flavor sector.

- **The Ugly.** These are decay modes that potentially probe quark-mixing, CPV and BSM physics but which do not lend themselves to a clean extraction of the fundamental parameters.

I do not cover high-sensitivity experiments that require large numbers of kaons, but where the underlying branching ratios are relatively large, such as \(\epsilon'/\epsilon\) or the search for T violation in \(K^+ \rightarrow \pi^0\mu^+\nu\). Needless to say, in the limited space available I can only give a cursory overview of the field; the interested reader should consult the many reviews available. \(^2\)

\(^2\)All quoted limits are at 90% CL.
2 Lepton flavor violation

The state-of-the-art in lepton flavor violation (LFV) is set by the BNL experiments E871 and E865 which searched for the decays $K_L \rightarrow \mu^\pm e^\mp$ and $K^+ \rightarrow \pi^+ \mu^+ e^-$ respectively. The two decay modes are complementary in that the first probes parity odd couplings and the second parity even. The E871 result on $K_L \rightarrow \mu^\pm e^\mp$ actually predates the time frame of this review, but it is the best limit available: $\text{BR}(K_L \rightarrow \mu^\pm e^\mp) < 4.7 \times 10^{-12}$\,3. Limits on new physics are model dependent; it is typical to derive a limit in a “generic” sense for a heavy particle exchanged at tree level. For the same coupling strength as the electroweak couplings of the quarks, the limit on $K_L \rightarrow \mu^\pm e^\mp$ probes energy scales as high as 150 TeV.\,4 E865 has completed analysis of its 1998 data set for $K^+ \rightarrow \pi^+ \mu^+ e^-.$\,5 The dominant background comes from the overlap of multiple $K^+$ decays, which are estimated from the time sidebands and extrapolated into the signal region from the region of high $K^+$ momentum. Eight events are observed in the signal region, consistent with background expectations; a likelihood analysis is used to obtain the branching ratio limit. Combining this with the E865 results from the 1995 and 1996 runs as well as with the result from the predecessor experiment BNL E777, yields the final E865 limit: $\text{BR}(K^+ \rightarrow \pi^+ \mu^+ e^-) < 1.2 \times 10^{-11}.$\,6 New results from KTeV are also available on $K_L \rightarrow \pi^0 \mu^\pm e^\mp$\,6 and $K_L \rightarrow e^\pm e^\mp \mu^\mp.$\,7

3 Quark mixing and CP violation: $K \rightarrow \pi \nu \bar{\nu}$

The decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ have attracted much attention for their potential (together with the decay $K^+ \rightarrow \pi^0 e^+ \nu$) to completely determine the Unitarity Triangle from kaon decays alone. An inconsistency between the unitarity relation in kaon decays ($s \rightarrow d$ transitions) with that in B decays ($b \rightarrow d$ transitions) would provide clues to the flavor structure of physics beyond the SM. The clean theoretical nature of the $K \rightarrow \pi \nu \bar{\nu}$ decay modes was discussed at this conference by Sehgal; a detailed review can be found in\,8.

First results from BNL E949 on the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ have been published recently.\,9 The signal region is analyzed with a signal-to-noise likelihood ratio technique. An event is seen in the 2002 dataset near the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ kinematic endpoint, albeit with poorer signal-to-noise ratio compared to the previous two candidate events seen by E787; accordingly the new
event has an effective contribution to the branching ratio of about 0.5 events. The best estimate of the branching ratio, combining data from E787 and E949 is $B(K^+ \rightarrow \pi^+ \nu\bar{\nu}) = 1.47^{+1.30}_{-0.89} \times 10^{-10}$, consistent with the SM, although the central value remains about twice the SM value. Further details can be found in the presentation by Sekiguchi at this conference. A result from E787 on $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ from the 1997 dataset in the $\pi^+$ momentum region below 195 MeV/c has also been published recently. 10)

A model-independent bound on the branching ratio for $K_L \rightarrow \pi^0 \nu\bar{\nu}$ can be obtained from the $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ branching ratio. 11) Using the most recent result from E949, this so-called Grossman-Nir bound becomes $B(K_L \rightarrow \pi^0 \nu\bar{\nu}) < 1.4 \times 10^{-9}$, about 400 times lower than the best direct limit from KTeV. 12). As discussed at this conference by Komatsubara, the first experiment (KEK E391a) dedicated to studying this decay mode took its first data this year, hoping to cover the entire region between the KTeV and Grossman-Nir limits.

4 Quark mixing and CP violation: other decay modes

The decays $K_L \rightarrow \pi^0 l^+ l^-$ (where $l = e, \mu$) have also attracted interest for their potential to determine the Wolfenstein parameter $\eta$, responsible for CPV in the SM. However, while $K_L \rightarrow \pi^0 \nu\bar{\nu}$ proceeds almost entirely due to “direct” CPV (sensitive to $\eta$), the $K_L \rightarrow \pi^0 l^+ l^-$ modes have large contributions to the branching ratio from “indirect” CPV ($K_L - K_S$ mixing, followed by CP conserving (CPC) $K_S$ decay) and from the interference between direct and indirect CPV. In the case of $K_L \rightarrow \pi^0 \mu^+ \mu^-$, there is also a large contribution from the CP conserving amplitude. Much theoretical effort has gone into disentangling the various contributions as was covered briefly by Smith, and discussed in detail in 13, 14).

On the experimental side, KTeV has updated the search for $K_L \rightarrow \pi^0 e^+ e^-$, adding the results of their 1999 dataset. 15) One event is seen, consistent with expectations from background, dominated by $K_L \rightarrow e^+ e^- \gamma \gamma$. Combining with the previous result from the 1997 dataset, yields the final KTeV limit: $B(K_L \rightarrow \pi^0 e^+ e^-) < 2.8 \times 10^{-10}$. This is still about a factor of 10 above the SM prediction; to beat down the $K_L \rightarrow e^+ e^- \gamma \gamma$ background further requires higher precision tracking and calorimetry, which may be difficult considering that KTeV is already state-of-the-art. An interference analysis might be a possible way out. 16) $K_L \rightarrow \pi^0 \mu^+ \mu^-$ has a less severe background prob-
lem from $K_L \to \mu^+\mu^-\gamma\gamma$, but the SM branching ratio is smaller by about a factor of two. Results from the 1999 dataset of KTeV are awaited; this data sample contains about a factor 1.3 more K decays compared to the 1997 data.

The measurements of $K_S \to \pi^0\ell^+\ell^-$ are important inputs to the computation of the contributions of indirect CPV (and hence also the magnitude of the interference term, but not the sign) to $K_L \to \pi^0\ell^+\ell^-$. As shown by Ruggiero, NA48/1 has made the first observation of $K_S \to \pi^0\ell^+\ell^-$ and $K_S \to \pi^0\mu^+\mu^-$. $B(K_S \to \pi^0\ell^+\ell^-) = (5.8^{+2.9}_{-2.3}(\text{stat}) \pm 0.8(\text{sys})) \times 10^{-9}$ and $B(K_S \to \pi^0\mu^+\mu^-) = (2.9^{+1.4}_{-1.2} \pm 0.2) \times 10^{-9}$. The precision of these measurements currently set the uncertainty in the SM expectation for $K_L \to \pi^0\ell^+\ell^-$ to around 30%; it is thought that the ultimate theoretical precision could be brought below 10%.

Much effort has also gone into extracting the Wolfenstein parameter $\rho$ from $K_L \to \mu^+\mu^-$ decays. The decay itself is well-measured; the difficulty is that the branching ratio is almost saturated by the two-photon intermediate state, masking the short-distance contribution sensitive to $\rho$. The imaginary part of the two-photon amplitude can be obtained from $B(K_L \to \gamma\gamma)$, as has been known for many years. The real part can be constrained by studies of the form factor for $K_L$ decays to virtual photons, with the final states $e^+e^-\gamma$, $e^+e^-e^+e^-$, $\mu^+\mu^-\gamma$, and $\mu^+\mu^-e^+e^-$. Studies of the $ee\gamma$, $eee\mu\mu$, and $ee\mu\mu$ states have been updated by KTeV, now including their full dataset from the 1997 and 1999 runs. These KTeV form factor measurements are consistent with one another, but the more precise measurements ($ee\gamma$ and $\mu\mu\gamma$) are in disagreement with the previous NA48 measurement from $ee\gamma$. New results are expected from NA48 on $ee\gamma$ and $eee\mu$; a measurement of the $eee\mu$ branching ratio was shown at this conference by Ruggiero. Using the KTeV measurements as input, limits on $\rho$ from $K_L \to \mu^+\mu^-$ have been derived. While these limits are not competitive with other CKM constraints, and are not expected to improve significantly without new theoretical ideas, they do provide constraints on non-standard scenarios.

KLOE is starting to get into the rare decay regime with a new result on the CP-violating decay $K_S \to \pi^0\pi^0\pi^0$, as reported by Martini. The motivation for this decay mode is that the uncertainty on the $K_S \to \pi^0\pi^0\pi^0$ amplitude currently limits the precision on $\text{Im}(\delta)$, where $\delta$ parametrizes the CPT-violating part of the $K_L, K_S$ wavefunctions. The result, $B(K_S \to \pi^0\pi^0\pi^0)$
< 2.1 × 10^{-7}, is about a factor 70 improvement over the current PDG limit, and improves the precision on $Im(\delta)$ by about a factor of 2.5. NA48/1 probes separately the real and imaginary parts of the $K_S \rightarrow \pi^0 \pi^0 \pi^0$ amplitude by looking at $K_L - K_S$ interference, with a similar sensitivity to $Im(\delta)$.\[23]\]

5 Outlook and Conclusion

Concerning LFV, existing data on the most sensitive modes are now fully analyzed. There are no currently running or proposed LFV experiments in the kaon sector; current methods have been estimated to give perhaps another factor of 40 at best.\[24]\ Attention has turned instead to the muon sector where sensitivity gains of 3-4 orders of magnitude are anticipated.\[25]\ SUSY models generally put LFV far out of reach of kaon experiments while large parts of parameter space would be accessible by muon decays.\[26]\ On the other hand, LFV K decays can probe interesting areas of parameter space in ETC models.\[27]\ More generally, LFV K decays involve both quarks and leptons and could provide information complementary to that obtained in the muon sector.

Concerning precision tests of the CKM matrix, future efforts are concentrated on the “golden” modes: $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$. The current experimental situation on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ cries out for completion of the BNL E949 program (only 20% of the proposed running has been completed), but the DOE has halted HEP operations at the AGS; a proposal to NSF to complete E949 has been submitted. In addition, experiments are under consideration at other labs to take the sensitivity one step further to the level of between 50 and 100 events with S/N=10. These include decay-in-flight experiments P940 at Fermilab,\[28]\ NA48/3 at CERN,\[29]\ and a stopped kaon experiment at J-PARC.\[30]\ For $K_L \rightarrow \pi^0 \nu \bar{\nu}$, KEK E391a may have another run in 2005, possibly going below the Grossman-Nir limit. The KOPIO experiment\[31]\ expects to observe 40 events with S/N=2; the project was included in the FY05 President’s Budget for a construction start in 2005. A 5-year construction is envisaged with test runs starting in 2008. There is also a letter of intent to use the E391a technique at J-PARC.\[32]\

In conclusion, rare kaon decays continue to be an active area of study. LFV decays have reached single-event sensitivities at the $(2 - 5) \times 10^{-12}$ level, but further progress requires new measurement techniques. Heroic efforts have been made towards understanding the short distance components of
$K_L \to \pi^0l^+l^-$ and $K_L \to \mu^+\mu^-$. Current experiments for $K_L \to \pi^0l^+l^-$ are within striking distance of the SM by a factor of about 10, but backgrounds are severe. Still, the potential exists for the discovery and study of BSM effects complementary to those in $K \to \pi/\tau\nu$; clever ideas for experiments are needed. The focus of the community is converging on the $K \to \pi/\tau$ decays for precision CKM tests. There are many ideas for experiments at various labs; some even appear to be funded. Together with precision measurements at B factories, these could provide decisive tests of the flavor sector. It is worth recalling that at least in the movie, the “Good” was able to tease out the secret from the “Ugly”.\footnote{But keep in mind the twist at the very end!}

6 Acknowledgements

Thanks to S. Kettell for comments on the draft. This work was supported in part under US Department of Energy contract DE-AC02-98CH10886.

References

1. F. Wilczek, hep-ph/0401126.
2. Start with the review by L. Littenberg and G. Valencia in PDG04, Phys. Lett. B592 (2004) 607.
3. D. Ambrose et al, Phys. Rev. Lett. 81 (1998) 5734.
4. J.L. Ritchie and S.G. Wojcicki, Rev. Mod. Phys. 65 (1993) 1149.
5. A. Sher, Ph.D. Dissertation, Universität Zürich (2004).
6. A. Bellavance, Ph.D Dissertation, Rice University (2002), available from http://kpasa.fnal.gov:8080/public/ktev/theses.html
7. A. Alavi-Harati et al, Phys. Rev. Lett. 90 (2003) 141801.
8. A.J. Buras et al, hep-ph/0405132.
9. V.V. Anisimovsky et al, Phys. Rev. Lett. 93 (2004) 031801.
10. S. Adler et al, hep-ex/0403034. Accepted by Phys. Rev. D.
11. Y. Grossman and Y. Nir, Phys. Lett. B398 (1997) 163.
12. A. Alavi-Harati et al, Phys. Rev. D61 (2000) 072006.
13. G. Buchalla, G. D’Ambrosio and G. Isidori, Nucl. Phys. B672 (2003) 387.
14. G. Isidori, C. Smith and R. Unterdorfer, Eur. Phys. J. C36 (2004) 57.
15. A. Alavi-Harati et al, Phys. Rev. Lett. 93 (2004) 021805.
16. A. Lai et al, Phys. Lett. B576 (2003) 43.
17. J.R. Batley et al, CERN PH-EP Preprint 2004-025. Accepted by Phys. Lett. B.
18. L.M. Sehgal, Phys. Rev. 183 (1969) 1511.
19. J. LaDue, Ph.D. Dissertation, Univ. of Colorado (2003). See [1] for web location.
20. A. Ceccucci, presentation at Lepton-Photon 2003, writeup available at http://conferences.fnal.gov/lp2003/program/.
21. G. Isidori and R. Unterdorfer, J. High Energy Phys. 2004 1 (2004) 9.
22. See the review by P. Bloch in PDG04, Phys. Lett. B592 (2004) 623.
23. N. Cartiglia, hep-ph/0310152.
24. W. Molzon, in Kaon Physics (ed. J.L. Rosner and B.D. Winstein), p.377 (Univ. of Chicago Press, Chicago, 2001).
25. http://meg.psi.ch and http://meco.ps.uci.edu
26. A. Belyaev et al Eur. Phys. J. C22 (2002) 715.
27. T. Applequist, M. Piai, and R. Shrock, Phys. Rev. D69 (2004) 015002.
28. Follow the link to P940 from http://www.fnal.gov/faw/experimentsprojects/
29. http://na48.web.cern.ch/NA48/NA48-3/
30. http://kaon.kek.jp/~kpwg
31. http://pubweb.bnl.gov/users/e926/www
32. http://www-ps.kek.jp/jhf-np/LOIlist/LOIlist.html