1. Introduction

Fractional differential equations play a key role to describe various problems in different areas of science. Fractional models are more useful than the classical models. Fractional differential equations are used in economics, image processing, physics, and so on. For detailed information on fractional differential equations and their applications, see [1–9].

A. Baleanu and D. Baleanu [10] introduces a nonsingular Caputo and Riemann–Liouville version of the fractional differential operator using the Mittag–Leffler function as its kernel. Bonyah et al. [11] developed a mathematical model for cancer and hepatitis coinfection diseases using the AB-fractional derivative. The AB-fractional derivative of the fractional-order tumor-immune-vitamin model was provided in [12] and discussed the existence, uniqueness, and Hyres–Ulam stability results also. Through the fractional tumor-immune dynamical model with AB-fractional derivative, researchers [13] created a chaotic and comparative work of tumour and effector cells. The fractional AB derivative was used to explore the numerical solution of the fractional immunogenetic tumour model in [14]. Like this, several applications were mentioned in reference [15–19].

In [20], the authors deal with the transmission dynamics of the COVID-19 mathematical model under ABC-fractional-order derivative. With the AB-fractional derivative, Logeswari et al. [21] investigated the mathematical model for COVID-19 infection propagation over the world. They also developed a framework for generating numerical outputs in order to forecast the effect of the illness spreading across India. [22–24] are a few other key publications that sought to address the issue of various diseases modeled as FDEs involving AB-fractional derivative.
In [25], Tidke introduced the following linear evolution equation:
\[
\frac{d^q x}{dt^q} = A(t)x(t) + f(t, x(t)), \quad t \in J = [0, b],
\]
(1)
where \(0 < q < 1\), the unknown \(x(t)\) takes values in the Banach space \(X\); \(f \in C(I \times X, X)\), and \(A(t)\) is a bounded linear operator on a Banach space \(X\), and \(x_0\) is a given element of \(X\). The operator \(D^q\) denotes the Caputo fractional derivative of order \(q\) and he has investigated about the existence, uniqueness, and properties of solutions of fractional semilinear evolution equations in Banach spaces.

In [26], the authors Kucche et al. dealt with the following nonlinear implicit fractional differential equations and investigated the existence and interval of existence of solutions, uniqueness, and properties of continuous dependance.

\[
D^\alpha x(t) = f(t, x(t)),
\]
\(0 = x_0 \in \mathcal{R}, t \in [0, T],\)
(2)
where \(D^\alpha (0 < \alpha < 1)\) denotes the Caputo fractional derivative and \(f \in [0, T] \times \mathcal{R} \times \mathcal{R}\) is a given continuous function.

In [27], Guo et al. discussed the impulsive fractional differential equations with boundary value problems of the form:

\[
C D^\alpha_0 x(t) = f(t, x(t)), \quad t \in J^1,\]
\(J = [0, T], \Delta x(t_k) = u(t_k^+ - t_k^-) = I_k(t_k^+),\)
\(k = 0, 1, 2, \ldots, t, m, \alpha x(0) + bx(T) = c,\)
(3)
where \(C D^\alpha_0\) is the Caputo fractional derivative of order \(\alpha \in (0, 1)\) with the lower limit zero, \(f: J \times \mathcal{R} \rightarrow \mathcal{R}\) is jointly continuous and \(t_k\) satisfy \(0 = t_0 < t_1 < \ldots < t_m < t_{m+1} = T, x(t_k^+) = \lim_{\tau \to t_k^-} x(t_k + \epsilon),\) and \(x(t_k^-) = \lim_{\tau \to t_k^+} x(t_k + \epsilon)\) represent the right and left limits of \(x(t)\) at \(t = t_k, I_k \in C(\mathcal{R}, \mathcal{R}),\) and \(a, b, c\) are real constants with \(a + b + c \neq 0.\)

In [28], Yukunthorn et al. studied the impulsive Hadamard fractional differential equations with boundary value problems of the form:

\[
C D^\alpha_0 x(t) = f(t, x(t)),
\]
\(t \in J_k \subset [t_0, T],\)
\(t = t_k, \Delta x(t_k) = \varphi_k(x(t_k)),\)
\(k = 1, 2, \ldots, t, m, \alpha x(t_0) + \beta x(T) = \sum_{i=0}^m \gamma_i f_i^\alpha x(t_{i+1}),\)
(4)
where \(C D^\alpha_0\) is the Hadamard fractional derivative of order \(0 < \alpha \leq 1\) on intervals \(J_k = (t_k, t_{k+1}], k = 1, 2, \ldots, m,\) with \(J_0 = [t_0, t_1], 0 < t_1 < t_2 < \ldots < t_k < \ldots < t_m < t_{m+1} = T,\) which are the impulsive points, \(J = [t_0, T], f: J \times \mathcal{R} \rightarrow \mathcal{R}\) is a continuous function, and \(\varphi_k \in C(\mathcal{R}, \mathcal{R}), f_i^\alpha\) is the Hadamard fractional integral of order \(q_i > 0, i = 1, 2, \ldots, m.\)

The jump conditions are defined by \(\Delta x(t_k) = x(t_k^-) - x(t_k^+), x(t_0) = \lim_{\tau \to t_k^-} x(t_k + \epsilon), k = 1, 2, \ldots, m.\)

Motivated by the above works, we study multidimensional nonlinear impulsive FDEs including AB-fractional derivative (AB derivative) of the following form:

\[
_0^D_\tau^\alpha f(t, \omega(t), A\omega(t)) = g(t, \omega(t), A\omega(t)), \quad t \in J,
\]
(5)
with integral boundary condition of the form:

\[
_0^\omega(0) = \int_0^T (T - s)^{\alpha - 1} \omega(s)ds,
\]
(6)
where \(J = [0, T], T > 0, 0 < \alpha < 1, D^\alpha_0\) indicates the ABR-fractional differential operator of order \(\alpha\) and \(f \in C(J \times \mathcal{R} \times \mathcal{R} \rightarrow \mathcal{R}), \varphi \in C(J \times \mathcal{R} \rightarrow \mathcal{R})\) are nonlinear functions. Where \(Bw(t) = \int_0^t k(t, s, \omega(s))ds\) and \(k: \alpha \times [0, T] \rightarrow \mathcal{R}, \Delta = \{(\tau, s); 0 \leq s \leq T, 0 \leq \tau \leq T\}, 0 = t_0 < t_1 < \ldots < t_m = 1, \Delta \omega_{t_0} = \omega(t_1^-) - \omega(t_1^+),\) and \(\omega(t_k^-) = \lim_{\tau \to t_k^-} \omega(t_k + h), \omega(t_k^+) = \lim_{\tau \to t_k^+} \omega(t_k + h)\) represent the left and right hand limits of \(t(t)\) at \(t = t_k.\) The fixed point theorem of Krasnoselskii is used to prove the existence of a solution. The Gronwall–Bellman inequality and the properties of the fractional integral operator are used to prove that the solution is unique.

The following is the outline of the paper: the required background for the development of the study is presented in Section 2. Section 3 discusses the existence and uniqueness of impulsive fractional integrodifferential equations. Section 4 describes the examples.

### 2. Preliminaries

The ABC-fractional derivative and the generalized Mittag–Leffler function are defined and discussed in this section.

**Definition 1** (see [29]). For \(p \in [1, \infty)\) and \(\omega \) form an open subset of \(\mathcal{R}^n\) then the Sobolev space \(H^p(\omega)\) can be defined as follows:

\[
H^p(\omega) = \left\{ f \in L^2(\omega); D^\alpha f \in L^2(\omega), \forall |\beta| \leq \omega \right\},
\]
(8)

**Definition 2** (see [10]). We assume \(x \in H^1(0, 1)\) and \(0 < \alpha < 1\). Then, in Riemann–Liouville perspective, the left Atangana–Baleanu fractional derivative of \(\omega\) of order \(\alpha\) (ABR derivative) is defined by

\[
D^\alpha_\tau = \frac{B(\alpha)}{(1 - \alpha)} \frac{d}{d\sigma} \int_0^\sigma \mathbb{E}_1^{\alpha} \left( -\sigma \right)^{\alpha - 1} \omega(\sigma) d\sigma,
\]
(9)
where \(B(\alpha) > 0\) is a normalization function satisfying \(B(0) = B(1) = 1\) and \(E\) is one parameter Mittag–Leffler function.

**Definition 3** (see [10]). Let \(\omega \in H^1(0, 1)\) and \(0 < \alpha < 1\). Then, the left Atangana–Baleanu fractional derivative of \(x\) of order \(\alpha\) in Caputo sense is defined by
\[ D_t^a = \frac{B(a)}{1-a} \int_0^t E_a \left( \frac{-\alpha}{1-a} (t-\sigma)^a \right) \omega(\sigma) d\sigma. \]  

(10)

A normalization function which conforms to \( B(0) = B(1) = 1 \) is \( B(a) > 0 \), and the Mittag–Leffler function is \( E \).

**Definition 4** (see [30, 31]). A Mittag–Leffler generalized function \( [\gamma]_{a,\beta}(z) \) for the complex \( a, \beta \) with \( Re(a) > 0 \) can be defined as follows:

\[ E_{a,\beta}^\gamma(z) = \sum_{k=0}^{\infty} \frac{\gamma_k}{\Gamma(ak + \beta)} z^k, \]

where \( \gamma_k \) is the Pochhammer symbol given by \( \gamma_0 = 1, \gamma_k = \gamma (\gamma + 1 \ldots (\gamma + k - 1)), k \in \mathbb{N}. \)

We note that
\[ E_{a,\beta}^1(z) = E_{a,\beta}(z), E_{a,1}^1(z) = E_{a}(z). \]

The following Laplace transformation results are required.

**Lemma 1** (see [10]). Suppose \( L [ f(\tau); \rho] = F(\rho) \), then \( D_t^a [ f(\tau); \rho] = B(a)/1 - a^p F(\rho) / p^a + a/1 - a. \)

**Lemma 2** (see [32]). \( L \left\{ e^{a \sigma^{\beta - 1}} E_{a,\beta}^{(k)} (\pm a t^a); \rho \right\} = k! p^{a - \beta} (p^a \pm a)^{k+1}, E^{(k)} t = t^{k} / a t^k. \)

**Definition 5** (see [31, 33]). Let \( \rho, \mu, \omega, \gamma \in C(Re(\rho), Re(\mu) > 0), b > a. \) On a class \( \mathcal{L}(a,b) \), the fractional integral operator \( \mathcal{G}^\gamma_{p,a;w;v} \) is defined by

\[ \left( \mathcal{G}^\gamma_{p,a;w;v} \phi \right)(\tau) = \int_a^\tau (\tau - \sigma)^{p - 1} E_{p,\tau}^\gamma [\phi(\tau - \sigma)] d\sigma, \tau \in [a, b]. \]

(14)

**Lemma 3** (see [31, 33]). Let \( \rho, \mu, \omega, \gamma \in C(Re(\rho), Re(\mu) > 0), b > a \), then the operator \( \mathcal{G}^\gamma_{p,a;w;v} \) is bounded on \( \mathcal{C}[a,b] \) such that

\[ \| \mathcal{G}^\gamma_{p,a;w;v} \phi \|_2 \leq C \| \phi \|_2, \]

where

\[ C = (b - a)^{Re(a)} \sum_{k=0}^{\infty} \left| \frac{\gamma_k}{\Gamma(ak + \rho)} \right| \left| \frac{\omega(b - a)^{Re(\rho)}}{\Gamma(\rho)} \right|^k / k!. \]

(16)

**Lemma 4** (see [31, 33]). Let \( \rho, \mu, \omega, \gamma \in C(Re(\rho), Re(\mu) > 0) \), then the operator \( \mathcal{G}^\gamma_{p,a;w;v} \) is invertible in the space \( \mathcal{L}(a,b) \) and for \( f \in \mathcal{L}(a,b) \), its left inversion is given by the relation

\[ \left( \mathcal{G}^\gamma_{p,a;w;v} \right)^{-1} f = (D_a^{\mu + \nu} \mathcal{G}^\gamma_{p,a;w;v} f)(\tau), a < \tau \leq b, \]

where \( \nu \in \mathcal{C}, (Re(\nu) > 0) \) and \( D_a^{\mu + \nu} \) denotes the Riemann–Liouville fractional differential operator of order \( \mu + \nu \) with \( a \).

**Lemma 5** (see [31, 33]). We assume \( \rho, \mu, \omega, \gamma \in C(Re(\rho), Re(\mu) > 0) \). Suppose

\[ \int_{a}^{\tau} (t - \sigma)^{\rho - 1} E_{\rho,\sigma}^\gamma [x(t - \sigma)^{\phi(\sigma)} d\sigma = f(t), a < t \leq b, \]

is solvable in the space \( L(a,b) \), then

\[ \phi(t) = (D_a^{\mu + \nu} \mathcal{G}^\gamma_{p,a;w;v} f)(\tau), a < \tau \leq b, \]

(19)

where \( \nu \in \mathcal{C}, (Re(\nu) > 0) \) and \( D_a^{\mu + \nu} \) denotes the Riemann–Liouville fractional differential operator of order \( \mu + \nu \) with \( a \), which is a unique solution.

**Lemma 6** (Krasnosel’skii’s Fixed Point Theorem [6]). Let \( \omega \) be a Banach space. Let \( \mathcal{S} \) be a bounded, closed, convex subset of \( \omega \) and let \( \mathcal{F}_1, \mathcal{F}_2 \) be maps of \( \mathcal{S} \) into \( \omega \) such that \( \mathcal{F}_1 + \mathcal{F}_2 \) is bounded on every pair \( \omega, v \in \mathcal{S} \). If \( \mathcal{F}_1 \), and \( \mathcal{F}_2 \) are continuously, then the equation

\[ \mathcal{F}_1 \omega + \mathcal{F}_2 \omega = \omega, \]

(20)

has a solution on \( \mathcal{S} \).

**Lemma 7** (Gronwall–Bellman inequality [34]). Let \( u \) and \( f \) be continuous and non-negative functions defined on \( J = [0, T] \) and let \( c \) be a non-negative constant. Then, the inequality

\[ u(\tau) \leq c + \int_{0}^{\tau} f(\sigma) u(\sigma) d(\sigma), \tau \in J, \]

(21)

implies that

\[ u(\tau) \leq c \exp \left( \int_{0}^{\tau} f(\sigma) d(\sigma) \right), \tau \in J. \]

(22)

**Lemma 8.** The function \( \omega \in C(J) \) is a solution of ABR-FDEs, for any function \( h \in C(J) \)

\[ D_t^\alpha \omega(\tau) = h(\tau), \tau \in J, \]

\[ \omega(\tau_k) = \omega(\tau_{k-1}) + y_k, y_k \in \mathbb{R}, \]

(23)

with integral boundary condition of the form

\[ \omega(0) = \int_{0}^{T} \frac{(T - \sigma)^{\alpha - 1}}{\Gamma(\alpha)} \mathcal{A}(\sigma, \omega(\sigma)) d\sigma, \]

(25)

if and only if \( x \) is a solution of fractional integral equation.
Proof. We assume that \( \omega \) satisfies (23) and (25). If \( \tau \in [0, \tau_1] \), then

\[
\omega(\tau) = \int_0^\tau \frac{(T-\sigma)^{\alpha-1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma + B(\alpha) \int_0^{\tau_1} E_\alpha \left( \frac{-\alpha}{(1-\alpha)} (\tau - \sigma)^\alpha \right) \omega'(\sigma) d\sigma + \int_0^{\tau_1} h(\sigma) d\sigma, \tag{26}
\]

\[
\omega(0) = \int_0^T \frac{(T-\sigma)^{\alpha-1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma,
\]

\[
\omega(\tau) = \sum_{i=1}^{m} y_i + \int_0^{\tau} \frac{(T-\sigma)^{\alpha-1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma + B(\alpha) \int_0^{\tau_1} E_\alpha \left( \frac{-\alpha}{(1-\alpha)} (\tau - \sigma)^\alpha \right) \omega'(\sigma) d\sigma + \int_0^{\tau} h(\sigma) d\sigma. \tag{28}
\]

If \( \tau \in (\tau_1, \tau_2) \), then we have

\[
0^+ D_\tau^\alpha \omega(\tau) = h(\tau), \quad \omega(\tau_2) = \omega(\tau_1) + y_k, \quad y_k \in \mathbb{R}, \tag{29}
\]

and so

\[
\omega(\tau) = \omega(\tau_1) - \int_0^{\tau_1} h(\sigma) d\sigma + \int_0^{\tau} \frac{(T-\sigma)^{\alpha-1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma + B(\alpha) \int_0^{\tau_1} E_\alpha \left( \frac{-\alpha}{(1-\alpha)} (\tau - \sigma)^\alpha \right) \omega'(\sigma) d\sigma + \int_0^{\tau} h(\sigma) d\sigma
\]

\[
= \omega(\tau_1) + y_1 - \int_0^{\tau_1} h(\sigma) d\sigma + \int_0^{\tau} \frac{(T-\sigma)^{\alpha-1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma + B(\alpha) \int_0^{\tau_1} E_\alpha \left( \frac{-\alpha}{(1-\alpha)} (\tau - \sigma)^\alpha \right) \omega'(\sigma) d\sigma + \int_0^{\tau} h(\sigma) d\sigma
\]

\[
= y_1 + \int_0^{\tau} \frac{(T-\sigma)^{\alpha-1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma + B(\alpha) \int_0^{\tau_1} E_\alpha \left( \frac{-\alpha}{(1-\alpha)} (\tau - \sigma)^\alpha \right) \omega'(\sigma) d\sigma + \int_0^{\tau} h(\sigma) d\sigma. \tag{30}
\]
If $\tau \in (\tau_2, \tau_3)$, then we find

$$
\omega(\tau) = \omega(\tau_2) - \int_0^{\tau_2} h(\sigma) d\sigma + \int_0^{\tau_2} \frac{(T - \sigma)^{\alpha - 1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma + \frac{B(\alpha)}{1 - \alpha} \int_0^{\tau_2} \mathcal{E}_\alpha \left( \frac{-\alpha}{1 - \alpha} (\tau - \sigma)^{\alpha} \right) \omega'(\sigma) d\sigma + \int_0^{\tau_2} h(\sigma) d\sigma 
$$

$$= \omega(\tau_2) + y_2 - \int_0^{\tau_2} h(\sigma) d\sigma + \int_0^{\tau_2} \frac{(T - \sigma)^{\alpha - 1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma + \frac{B(\alpha)}{1 - \alpha} \int_0^{\tau_2} \mathcal{E}_\alpha \left( \frac{-\alpha}{1 - \alpha} (\tau - \sigma)^{\alpha} \right) \omega'(\sigma) d\sigma + \int_0^{\tau_2} h(\sigma) d\sigma 
$$

$$= y_1 + y_2 + \int_0^{\tau_2} \frac{(T - \sigma)^{\alpha - 1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma + \frac{B(\alpha)}{1 - \alpha} \int_0^{\tau_2} \mathcal{E}_\alpha \left( \frac{-\alpha}{1 - \alpha} (\tau - \sigma)^{\alpha} \right) \omega'(\sigma) d\sigma + \int_0^{\tau_2} h(\sigma) d\sigma. 
$$

(31)

Let us consider the case $\tau \in (\tau_m, T]$. Then, we conclude

$$
\omega(\tau) = \sum_{i=1}^{m} y_i + \int_0^{\tau} \frac{(T - \sigma)^{\alpha - 1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma + \frac{B(\alpha)}{1 - \alpha} \int_0^{\tau} \mathcal{E}_\alpha \left( \frac{-\alpha}{1 - \alpha} (\tau - \sigma)^{\alpha} \right) \omega'(\sigma) d\sigma + \int_0^{\tau} h(\sigma) d\sigma. 
$$

(32)

Conversely, we assume that $\omega$ satisfies the impulsive equation (26). The fractional integral equation (26) to the ABR-FDEs (23) and (25) is $\mathcal{G}^T_{\rho, \mu, \omega, \alpha}$ as given below:

$$
\omega(\tau) = \sum_{i=1}^{m} y_i + \int_0^{\tau} \frac{(T - \sigma)^{\alpha - 1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma + \frac{B(\alpha)}{1 - \alpha} \int_0^{\tau} \mathcal{E}_\alpha \left( \frac{-\alpha}{1 - \alpha} (\tau - \sigma)^{\alpha} \right) \omega'(\sigma) d\sigma + \int_0^{\tau} h(\sigma) d\sigma, \tau \in J. 
$$

(33)

**Theorem 1.** The function $\omega \in C(J)$ is a solution of ABR-FDEs (5) and (7), for any $f \in C(J \times \mathcal{H}, \mathbb{R})$ if and only if $\omega$ is a solution of fractional integral equations

$$
\omega(\tau) = \sum_{i=1}^{m} y_i + \int_0^{\tau} \frac{(T - \sigma)^{\alpha - 1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma)) d\sigma + \frac{B(\alpha)}{1 - \alpha} \int_0^{\tau} \mathcal{E}_\alpha \left( \frac{-\alpha}{1 - \alpha} (\tau - \sigma)^{\alpha} \right) \omega'(\sigma) d\sigma + \int_0^{\tau} h(\sigma) d\sigma, t \in J. 
$$

(34)

**Proof.** Proof follows by taking from Lemma 8, $h(\tau) = f(\tau, \omega(\tau)), \tau \in J$.

The features of fractional integral operator $\mathcal{G}^T_{\rho, \mu, \omega, \alpha}$ are used to prove the following theorem.

**Theorem 2.** We assume $0 < \alpha < 1$. A function $\mathcal{F}$ on $C(J)$ is defined by

$$
(\mathcal{F} \omega)(\tau) = \frac{B(\alpha)}{1 - \alpha} \left( \mathcal{G}^1_{\alpha, 1 - \alpha, 0, 0}(\omega) \right)(\tau), \omega \in C(J), \tau \in J. 
$$

(35)

Then, $\mathcal{F}$ is a $C(J)$ bounded linear operator

(i) Since the integral operator $\mathcal{G}^1_{\alpha, 1 - \alpha, 0, 0}$ is a bounded and linear operator on $C(J)$ by the definition and Lemma 3, the equation becomes

$$
\left\| \mathcal{G}^1_{\alpha, 1 - \alpha, 0, 0}(\omega) \right\| \leq C\|\omega\|, \tau \in J, 
$$

(36)

where we find
\[ \varrho = \sum_{k=0}^{\infty} \frac{(1)_k}{\Gamma(ak + 1)(ak + 1)} \left| -\frac{a}{1 - a}T^a \right|^k = \sum_{k=0}^{\infty} \frac{a/1 - aT^a}{\Gamma(ak + 2)} = E_{\alpha,2}\left(\frac{\alpha}{1 - a}T^{\alpha+1}\right). \tag{37} \]

We have
\[ \mathcal{F} \omega = \left| \frac{B(a)}{1 - \alpha} \| \mathcal{G}_{\alpha,1,-a/1-a,\alpha,\omega} \| \right| \leq \frac{B(a)}{1 - \alpha} \| \omega \|, \text{ for all } \omega \in C(J). \tag{38} \]
As a result, the linear operator \( \mathcal{F} \) on \( C(J) \) is bounded.

(ii) For some \( \omega, \sigma \in C(J) \). Then, for some \( \tau \in J \), we find utilising the linearity of \( \mathcal{F} \) and the boundedness of operator \( \mathcal{G}_{\alpha,1,-a/1-a,\alpha,\omega} \), we have
\[ |\mathcal{F} \omega(\tau) - \mathcal{F} \sigma(\tau)| = |(\mathcal{F} \omega - \mathcal{F} \sigma)(\tau)| = \left| \frac{B(a)}{1 - \alpha} \mathcal{G}_{\alpha,1,-a/1-a,\alpha,\omega - \sigma}(\tau) \right| \leq \frac{B(a)}{1 - \alpha} \| \mathcal{G}_{\alpha,1,-a/1-a,\alpha,\omega - \sigma} \| \leq \frac{B(a)}{1 - \alpha} \| \omega - \sigma \|. \tag{39} \]

This gives
\[ \| \mathcal{F} \omega - \mathcal{F} \sigma \| \leq Q \frac{B(a)}{1 - \alpha} \| \omega - \sigma \|, \omega, \sigma \in C(J). \tag{40} \]
With Lipschitz constant \( \mathcal{F} B(a)/1 - aE_{\alpha,2}(a/1 - a)T^\alpha \), the operator \( \mathcal{F} \) satisfies Lipschitz condition.

(iii) Any closed, bounded subset of \( C(J) \) is \( \mathcal{S} = \omega \in C(J) : \|\omega\| \leq R \). Then, for every \( \omega \in \mathcal{S} \) and any \( \tau_1, \tau_2 \in J \) with \( \tau_1 < \tau_2 \), we get
\[ \| \mathcal{F} \omega(\tau_1) - \mathcal{F} \sigma(\tau_2) \| \leq \frac{B(a)}{1 - \alpha} \| \mathcal{G}_{\alpha,1,-a/1-a,\alpha,\omega}(\tau_1) - \mathcal{G}_{\alpha,1,-a/1-a,\alpha,\omega}(\tau_2) \| \]
\[ = \frac{B(a)}{1 - \alpha} \int_0^\tau \mathcal{E}_\alpha\left(\frac{-\alpha}{1 - \alpha}(\tau_1 - \sigma)^\alpha\right) \omega(\sigma) d\sigma - \int_0^\tau \mathcal{E}_\alpha\left(\frac{-\alpha}{1 - \alpha}(\tau_2 - \sigma)^\alpha\right) \omega(\sigma) d\sigma \]
\[ \leq \frac{B(a)}{1 - \alpha} \| \mathcal{G}_{\alpha,1,-a/1-a,\alpha,\omega} \| \left| \int_0^\tau (\tau_1 - \sigma)^\alpha - (\tau_2 - \sigma)^\alpha \omega(\sigma) d\sigma \right| \]
\[ \leq \frac{B(a)}{1 - \alpha} \sum_{k=0}^{\infty} \left(\frac{-\alpha}{1 - \alpha}\right)^k \frac{1}{\Gamma(ak + 1)} \int_0^\tau (\tau_1 - \sigma)^{ka} - (\tau_2 - \sigma)^{ka} \omega(\sigma) d\sigma \]
\[ \leq \frac{RB(a)}{1 - \alpha} \sum_{k=0}^{\infty} \left(\frac{\alpha}{1 - \alpha}\right)^k \frac{1}{\Gamma(ak + 1)} \int_0^\tau (\tau_2 - \sigma)^{ka} \omega(\sigma) d\sigma \]
\[ \leq \frac{RB(a)}{1 - \alpha} \sum_{k=0}^{\infty} \left(\frac{\alpha}{1 - \alpha}\right)^k \frac{1}{\Gamma(ak + 2)} \left\{ (\tau_2 - \tau_1)^{ka+1} + (\tau_2)^{ka+1} - (\tau_1)^{ka+1} + (\tau_2 - \tau_1)^{ka+1} \right\} \]
\[ \leq \frac{RB(a)}{1 - \alpha} \sum_{k=0}^{\infty} \left(\frac{\alpha}{1 - \alpha}\right)^k \frac{1}{\Gamma(ak + 2)} \left\{ (\tau_2)^{ka+1} - (\tau_1)^{ka+1} \right\}. \tag{41} \]
From above inequality, it follows that, if \(|r_1 - r_2| \to 0\), then \(|\mathcal{F} \omega (r_1) - \mathcal{F} \sigma (r_2)| \to 0\). This establishes the equicontinuous nature of \(\mathcal{F} (S)\) on \(J\).

(iv) For any \(f \in C(J)\), we obtain the below by using Lemmas 4 and 5.

\[
(\mathcal{F}^{-1} f)(\tau) = \left( \frac{Ba}{1 - \alpha} \mathcal{g}^{1}_{a,1-a/1-a,0,f} \right)^{-1}(\tau) = \frac{1 - \alpha}{Ba} \left( \mathcal{g}^{1}_{a,1-a/1-a,0,f} \right)^{-1}(\tau), \tau \in (a, b).
\]

This demonstrates that the operator equation \(\mathcal{F}\) is invertible on \(C(f)\),

\[
(\mathcal{F} \omega)(\tau) = f(\tau), \tau \in J,
\]

has the unique solution

\[
\omega(\tau) = \frac{1 - \alpha}{Ba} \left( \mathcal{g}^{1}_{a,1-a/1-a,0,f} \right)(\tau), \tau \in J,
\]

where \(\beta \in C\) with \(\text{Re}(\beta) > 0\) and \(\int_{0}^{1} f(\sigma, \omega(\sigma))d(\sigma), \tau \in J\).

### 3. Main Results

**Theorem 3.** Suppose \(f \in C(J \times \mathcal{R}, \mathcal{R})\), then the ABR-FDEs \(\mathcal{D}^n f = f(\tau, \omega(\tau)), \tau \in J\) can be solvable on \(C(J)\) and has a solution given by

\[
\omega(\tau) = \frac{1 - \alpha}{Ba} \left( \mathcal{g}^{1}_{a,1-a/1-a,0,f} \right)(\tau), \tau \in J,
\]

where \(\beta \in C\) with \(\text{Re}(\beta) > 0\) and \(\int_{0}^{1} f(\sigma, \omega(\sigma))d(\sigma), \tau \in J\).

### 3. Main Results

**Theorem 4 (Existence theorem).** Suppose \(f \in C(J \times \mathcal{R} \times \mathcal{R}, \mathcal{R})\) and the Lipschitz condition \(\mathcal{H} \in C(J \times \mathcal{R})\) is satisfied.

1. \(|f(\tau, \omega, \kappa_1) - f(\tau, \omega, \kappa_2)| \leq p(\tau)|\omega - \omega| + |\kappa_1 - \kappa_2|\),

2. \(|\mathcal{H}(\tau, \omega) - \mathcal{H}(\tau, \nu)| \leq L_{\mathcal{H}}|\omega - \nu|, \omega, \nu \in C(J)\),

### 3. Main Results

The operator equation for the equivalent fractional integral equation (55) to the ABR–FDEs (5) and (7) is as follows:

\[
(\mathcal{F}_1 \omega)(\tau) = \sum_{i=1}^{m} y_i + \int_{0}^{1} \frac{(T - \sigma)^{\alpha - 1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma))d\sigma + \int_{0}^{1} f(\sigma, \omega(\sigma), Ba(\sigma))d\sigma, \tau \in J,
\]

\[
(\mathcal{F}_2 \omega)(\tau) = \frac{B(a)}{1 - \alpha} \left( \mathcal{g}^{1}_{a,1-a/1-a,0,\omega} \right)(\tau) = (\mathcal{F} \omega)(\tau), \tau \in J.
\]

### 3. Main Results

Step 1. \(\mathcal{F}_1\) is contraction.

For every \(\omega, \nu \in PC(J)\) and \(\tau \in J\), we obtain using the Lipschitz condition on \(f\),

\[
|\mathcal{F}(\tau, \omega(\tau), Ba(\sigma)) - \mathcal{F}(\tau, \nu(\tau), Ba(\nu(\tau))| \leq p(\tau)|\omega - \nu|.
\]

This gives
\[ \| F_1 w - F_1 v \| \leq (L_F + LT) \| w - v \|, \omega, v \in PC(J). \] (53)

**Step 2.** Next, we show that \( F_2 \) is completely continuous.

\[ |(F_1 w + F_2 v)(\tau)| \leq |(F_1 w)(\tau)| + |(F_2 v)(\tau)| \leq \int_0^T \frac{(T - \sigma)^{\alpha - 1}}{\Gamma(\alpha)} H(\sigma, \omega(\sigma))d\sigma + \sum_{i=1}^m y_i + \int_0^T |f(\sigma, \omega(\sigma), B\omega(\sigma))|d\sigma \]

\[ + \frac{B(\alpha)}{1 - \alpha} \left[ \frac{\alpha}{(1 - \alpha)} T^{\alpha+1} \right] \| v \| \]

\[ \leq \frac{T^\alpha}{\Gamma(\alpha + 1)} (C_F \| w \| + M_F) + M^* + L \int_0^T |f(\omega(\sigma), B\omega(\sigma))|d\sigma + M_f \int_0^T d\sigma + \frac{B(\alpha)}{1 - \alpha} \left[ \frac{\alpha}{(1 - \alpha)} T^{\alpha+1} \right] R \]

\[ \leq \frac{T^\alpha}{\Gamma(\alpha + 1)} (C_F R + M_F) + M^* + LRT + M_J T + \frac{B(\alpha)}{1 - \alpha} \left[ \frac{\alpha}{(1 - \alpha)} T^{\alpha+1} \right] R. \] (54)

i.e., by condition (47) with (48), we get

\[ \frac{T^\alpha}{\Gamma(\alpha + 1)} (M_F) + M_J T + M^* = R1 - LT - \frac{T^\alpha}{\Gamma(\alpha + 1)} C_F \]

\[ - \frac{B(\alpha)E_{\alpha,2}(\alpha/\alpha) T^{\alpha+1}}{1 - \alpha}. \] (55)

From inequalities (54) and (55), we have

\[ |(F_1 w + F_2 v)(\tau)| \leq R, \tau \in J. \] (56)

This gives

\[ \|(F_1 w + F_2 v)\| \leq R, \text{for all } w, v \in \mathcal{S}. \] (57)

This shows that \( F_1 w + F_2 v \in \mathcal{S} \), for \( w, v \in \mathcal{S} \).

Then, the operator equation,

\[ \omega = F_1 w + F_2 \omega, \] (58)

has a fixed point in \( S \), which is the solution of ABR–FDEs (5) and (7). This completes the proof.

The uniqueness of solutions to ABR–FDEs (5) and (7) is demonstrated in two methods in the following theorem. We prove the result first using the properties of the fractional integral operator \( \mathcal{S}^{1}_{a,1-\alpha/1-\alpha,0+} \), and then using the Gronwall–Bellman inequality.

**Theorem 5** (Uniqueness result). The ABR-FDEs (5) and (7) has a unique solution in \( C(J) \) under the conditions of Theorem 4.

**Proof.** In operator equation form, the analogous fractional integral equation to ABR-FDEs (5) and (7) is

\[ \left( \mathcal{S}^{1}_{a,1-\alpha/1-\alpha,0+} \right)(\tau) = \tilde{f}(\tau), \tau \in J, \] (59)

where

\[ \tilde{f}(\tau) = \frac{1 - \alpha}{B(\alpha)} \left( \int_0^T \frac{(T - \sigma)^{\alpha - 1}}{\Gamma(\alpha)} H(\sigma, \omega(\sigma))d\sigma - \omega_t \right) \]

\[ + \int_0^T f(\sigma, \omega(\sigma), B\omega(\sigma))d\sigma + \sum_{i=1}^m y_i, \tau \in J. \] (60)

The operator (59) is solvable in \( C(J) \) according to Theorem 4. As a result of applying Lemma 5, the operator equation equation (59) has a unique solution in \( C(J) \), which is the ABR-FDEs (5) and (7).
Proof. Let $\omega$ and $\nu$ be two ABR-FDEs (5) solutions (7). We derive for any $\tau \in J$ using the linearity of the fractional integral operator,

$$
|\omega(\tau) - \nu(\tau)| = \left( \int_{0}^{T} \frac{(T - \sigma)^{\alpha-1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \omega(\sigma))d\sigma - \frac{B(\alpha)}{1 - \alpha} \left( \int_{0}^{1} f(\sigma, \omega(\sigma))d\sigma \right) \right) \\
- \left( \int_{0}^{T} \frac{(T - \sigma)^{\alpha-1}}{\Gamma(\alpha)} \mathcal{H}(\sigma, \nu(\sigma))d\sigma - \frac{B(\alpha)}{1 - \alpha} \left( \int_{0}^{1} f(\sigma, \nu(\sigma), \omega(\sigma))d\sigma \right) \right) \\
\leq \frac{T^{\alpha}}{\Gamma(\alpha + 1)} \left( L_{\mathcal{H}} |\omega(\sigma) - \nu(\sigma)| \right) + \frac{B(\alpha)}{1 - \alpha} \int_{0}^{T} E_{\alpha} \left( \frac{-\alpha}{1 - \alpha} (T - \sigma)^{\alpha} \right) |\omega(\sigma) - \nu(\sigma)|d\sigma \\
+ \int_{0}^{T} p_{1}(\sigma)|\omega(\sigma) - \nu(\sigma)|d\sigma \leq \frac{T^{\alpha}}{\Gamma(\alpha + 1)} \left( L_{\mathcal{H}} |\omega(\sigma) - \nu(\sigma)| \right) + \frac{B(\alpha)}{1 - \alpha} \int_{0}^{T} E_{\alpha} \left( \frac{\alpha}{1 - \alpha} (T)^{\alpha} + p_{1}(\sigma) \right) |\omega(\sigma) - \nu(\sigma)|d\sigma
$$

(61)

Applying Lemma 7, we get

$$
|\omega(\tau) - \nu(\tau)| \leq 0, \tau \in J,
$$

(62)

which shows that $\omega(\tau) = \nu(\tau)$, for every $\tau \in J$. This demonstrates the ABR-FDEs (5) solution is unique (7). □

4. Examples

(1) We consider the following fractional boundary value problem:

$$
D^{1/2}_{0} \omega(t) = \frac{t^{3} + \sin(|\omega(t)|)}{45} \\
+ \frac{1}{10} \int_{0}^{T} t^{2} \sin(s)\omega(s)ds, t \in [0, 1],
$$

(63)

$$
\Delta \omega(t) = \frac{\omega(1/2^-)}{3 + \omega(1/2^-)}
$$

(64)

with integral boundary condition of the form:

$$
\omega(0) = \int_{0}^{1} \frac{(1 - \sigma)^{\alpha-1}}{\Gamma(\alpha)} \frac{1}{25} \exp(-\omega(\sigma))d\sigma,
$$

(65)

where

$$
f(\tau, \omega(\tau), B\omega(\tau)) = \frac{t^{3} + \sin(|\omega(t)|)}{45} + \frac{1}{10} \int_{0}^{T} t^{2} \sin(s)\omega(s)ds,
$$

(66)

As $T = 1, \alpha = 1/2$, let $\omega, \nu \in PC(J)$

$$
|f(\tau, \omega(\tau), B\omega(\tau)) - f(\tau, \nu(\tau), B\nu(\tau))| \leq \frac{t^{3} + \sin(|\omega(t)|)}{45} + \frac{1}{10} \int_{0}^{T} t^{2} \sin(s)\omega(s)ds - \frac{t^{3} + \sin(|\nu(t)|)}{45} \\
- \frac{1}{10} \int_{0}^{T} t^{2} \sin(s)\nu(s)ds \leq \left[ \frac{1}{45} + \frac{1 - \cos(T)}{10} \right] |\omega(t) - \nu(t)|
$$
Thus, we have \( L = \frac{1}{4.5}, C_{\mathcal{F}} = \frac{1}{25}, M_{\mathcal{F}} = \frac{1}{25}, M_{f} = \frac{1}{45}, M^* = \frac{1}{3} \) and consider \( k = 1 \),

\[
\bigg( \frac{T^\alpha_{C_{\mathcal{F}}}}{\Gamma(\alpha + 1)} C_{\mathcal{F}} + \frac{B(\alpha)E_{\alpha,2}(\alpha/1 - \alpha)T^{\alpha + 1}}{1 - \alpha} \bigg) = 0.740 < 1,
\]

\[
R = \frac{T^\alpha_{\Gamma\alpha + 1} \Gamma(\alpha + 1) M_{\mathcal{F}} + M_{f} T + M^*}{1 - LT - T^\alpha_{\Gamma(\alpha + 1) C_{\mathcal{F}} - B(\alpha)E_{\alpha,2}(\alpha/1 - \alpha)T^{\alpha + 1}/1 - \alpha} = 0.401 = 11.13.
\]

Thus, the condition of Theorem 4 is satisfied.

(2) We consider the following example for fractional impulsive integrodifferential equations of the form:

\[
\int_0^t \omega(t)\frac{d}{d\tau}^{1/2} \omega(t)\frac{d}{d\tau}^{1/2} \omega(t)\quad t \in [0, 1],
\]

\[
\Delta \omega(t) = \frac{\omega(1/2^-) - \omega(1/2^+)}{3 + \omega(1/2)}
\]

with integral boundary condition

\[
|f(\tau, \omega(t), B\omega(t)) - f(\tau, \nu(t), B\nu(t))| \leq \frac{\cos(|\omega(t)|)}{35} + \frac{1}{50} \int_0^t (t^2 + s^2)\omega(s)ds - \frac{\cos(|\nu(t)|)}{35} - \frac{1}{50} \int_0^t (t^2 + s^2)\nu(s)ds
\]

\[
\leq \left[ \frac{1}{35} + \frac{2}{75} \right] |\omega(t) - \nu(t)|
\]

\[
|\mathcal{H}(\sigma, \omega(\sigma)) - \mathcal{H}(\sigma, \nu(\sigma))| \leq |\frac{1}{30}\sigma^2\omega(\sigma) - \frac{1}{30}\sigma^2\nu(\sigma)|
\]

\[
\leq \frac{1}{30}|\omega(t) - \nu(t)|
\]

\[
|\mathcal{H}(\sigma, \omega(\sigma))| \leq \left| \frac{1}{30}\sigma^2\omega(\sigma) \right|
\]

\[
\leq \frac{1}{30}|\omega(t)| + \frac{1}{30}.
\]

Thus, we have \( L = 0.054, C_{\mathcal{F}} = \frac{1}{30}, M_{\mathcal{F}} = \frac{1}{30}, M_{f} = \frac{1}{35}, M^* = \frac{1}{3} \) and consider \( k = 1 \),
Thus, the condition of Theorem 4 is satisfied.

5. Conclusion

In this paper, we examined the impulsive fractional integro-differential equations involving ABC derivative with integral boundary conditions. Recently ABC-derivative gained much attention due to the nonsingular property of the kernels. The existence of solution is investigated for the proposed equations by using Krasnoselskii fixed point theorem. The uniqueness of the result is derived with the help of Gronwall–Bellman inequality as well as the properties of fractional integral operator. In future, we extend this work with the delay properties involving Mittag–Leffler function.

Data Availability

There is no data used for this manuscript.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors’ Contributions

All authors have equal contribution and have finalized the manuscript.

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