User Subgrouping in Multicast Massive MIMO over Spatially Correlated Rayleigh Fading Channels

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Abstract—Massive multiple-input-multiple-output (MaMIMO) multicasting has received significant attention over the last years. MaMIMO is a key enabler of 5G systems to achieve the extremely demanding data rates of upcoming services. Multicast in the physical layer is an efficient way of serving multiple users, simultaneously demanding the same service and sharing radio resources. This work proposes a subgrouping strategy of multicast users based on their spatial channel characteristics to improve the channel estimation and precoding processes. We employ max-min fairness (MMF) power allocation strategy to maximize the minimum spectral efficiency (SE) of the multicast service. Additionally, we explore the combination of spatial multiplexing with orthogonal (time/frequency) multiple access. By varying the number of antennas at the base station (BS) and users’ spatial distribution, we also provide the optimal subgroup configuration that maximizes the spectral efficiency per subgroup. Finally, we show that serving the multicast users into two orthogonal time/frequency intervals offers better performance than only relying on spatial multiplexing.

Index Terms—Massive MIMO, multicasting, spatial correlation, 5G, digital precoding.

I. INTRODUCTION

Upcoming mobile services demand stringent performance requirements, both in terms of data rates, latency, and the number of connected devices [1]. Massive multiple-input-multiple-output (MaMIMO) plays a key role in fifth-generation (5G) systems to fulfill these requirements [2], [3]. This technology makes use of many antennas at the base station (BS) to jointly and coherently serve multiple users in the same time/frequency resources by yielding array gain, spatial multiplexing gain, and spatial diversity [4], [5]. Importantly, MaMIMO can offer, in most of the propagation environments, two fundamental features that are known as favorable propagation and channel hardening: as the number of BS antennas increases, users’ channels become nearly pairwise orthogonal and deterministic, respectively [6]. All this leads to a significant increase in spectral and energy efficiency.

Contributions: We consider spatially correlated fading channels to study the delivery of multicast service. We propose a user subgrouping method based on the level of the user mutual-orthogonality. Our strategy reduces the pilot contamination making precoding more effective. We also consider optimal max-min fairness (MMF) power control and optimize the number of subgroups that maximizes the spectral efficiency (SE). Finally, we analyze the benefits of splitting the multicast users into two different time/frequency schedules.
II. SYSTEM MODEL

Let us consider a multicast transmission in a single-cell \(\text{MaMIMO}\) system with fully digital precoding. We assume the system has a BS equipped with a uniform linear array (ULA) with \(M\) antennas serving \(K\) single-antenna users. We denote the set of multicast users as \(K\), i.e., \(K = \{1, \ldots, K\}\). Let \(h_{gk} \in \mathbb{C}^{M \times 1}\) be the channel between the BS and user \(k\) included in subgroup \(g\) (we detail the subgrouping model in subsection II-A). The block-fading model is herein assumed.

Although uncorrelated Rayleigh fading channels are widely employed to study \(\text{MaMIMO}\) multicasting performance \([10]\), it is more likely to have spatially correlated fading in practical scenarios. Hence, the corresponding large-scale fading coefficient as loss, shadowing, and spatial correlation fading. We denote the large-scale fading coefficient as

\[
\beta_{gk} = \frac{1}{M} \text{tr} (R_{gk}),
\]

while the small-scale fading follows a Rayleigh distribution.

We characterize the spatially correlated Rayleigh fading channels by the wavelength. Both the nominal angle \(\delta\) where \(\rho_k(n) \in \mathbb{C}\) represents the gain and phase of the \(n\)th physical path for user \(k\) and \(a_k (\varphi_k(n)) \in \mathbb{C}^M\) is the steering vector of the ULA given by

\[
a_k (\varphi_k(n)) = \begin{bmatrix} e^{j2\pi \delta \cos \varphi_k(n)} & \ldots & e^{j2\pi \delta(M-1) \cos \varphi_k(n)} \end{bmatrix}^\top,
\]

where \(\delta\) is the distance between adjacent antennas, normalized by the wavelength. Both the nominal angle \(\Phi_k\) and the \(\text{ASD}\) characterize the spatially correlated Rayleigh fading channels. The BS estimates the spatial correlation matrix of each user on the large-scale fading time-scale (i.e., over several coherence blocks). Therefore, we can reasonably assume \(R_{gk}, \forall k \in K\), to be known at the BS.

A. Multicast \(\text{MaMIMO}\) subgrouping

Multicast subgrouping consists in splitting the \(K\) users, demanding the same multicast service, into \(G\) disjoint subgroups based on their channel similarities. The objective of subgrouping the users is to increase the aggregated SE (ASE).

A specific and unique transmission setting characterizes each multicast subgroup. Let \(K_g\) and \(K_g\) denote the set and the number of multicast users in subgroup \(g\), respectively (i.e., \(|K_g| = K_g\)). Moreover, let \(G\) be the set of subgroups. Hence, we have \(K = \sum_{g=1}^G K_g\).

Fig. 1 illustrates an example of user subgrouping-based multicast transmission in a single-cell \(\text{MaMIMO}\) system. The users are grouped based on the level of orthogonality of their spatial correlation matrices, namely users with similar spatial characteristics belong to the same subgroup.

B. Channel estimation

We assume that the users in the same subgroup are assigned the same pilot sequence, while mutually orthogonal pilots are assigned over different subgroups. As co-pilot users have linearly dependent channel estimates, the BS cannot separate the users of the same subgroup in the spatial domain. Consequently, the BS can effectively construct as many precoding vectors as the number of orthogonal pilots (which must be at least equal to \(G\)), and the same precoding vector is employed to all the users of the same subgroup.

Let \(\Psi = [\psi_1, \ldots, \psi_G] \in \mathbb{C}^{G \times G}\) be the pilot matrix where \(\psi_g\) is the pilot sequence of length \(\tau_p\) symbols assigned to all the users in subgroup \(g\). Without loss of generality, we set \(\tau_p = G\) mutually orthogonal uplink pilots, \(\Psi^H \psi = \tau_p I_G\) (\(G\) is known at the channel estimation stage).

The uplink (UL) pilot signal received at the BS is

\[
Y = \sum_{g \in G} \sum_{k \in K_g} \sqrt{q_{gk}} h_{gk} \psi_g^* + N,
\]

where \(q_{gk}\) is proportional to the UL pilot power per user \(k \in K_g\), and \(N \in \mathbb{C}^{M \times \tau_p}\) is the additive white Gaussian noise (AWGN) with i.i.d. elements distributed as \(CN(0, \sigma^2)\). To estimate the channel of user \(k\) in subgroup \(g\), the BS first correlates the received signal with \(\psi_g^*\), obtaining

\[
y_{gk}^{UL} = \sqrt{q_{gk}} \tau_p h_{gk} + \sum_{k' \in K_g \setminus \{k\}} \sqrt{q_{gk'}} \tau_p h_{gk'} + n_k,
\]

where \(n_k \sim CN(0, \sigma^2 I_d)\) is AWGN. Then, the minimum mean square error (MMSE) channel estimate is
Let \( h_g \) denote the composite channel of subgroup \( g \) given by

\[
h_g = \tau_p \sum_{k=1}^{K_g} \sqrt{\rho_{gk}} R_{gk}.
\]

(8)

The MMSE estimate of the composite channel is

\[
\hat{h}_g = \tau_p \sum_{k=1}^{K_g} \left( \sqrt{\rho_{gk}} R_{gk} \right) \left( \tau_p \rho_{gk} R_{gk} + \sigma^2 I_M \right)^{-1} y_{gk}^H.
\]

(9)

We stack the \( G \) composite channel vectors in a matrix \( \mathcal{C} = [h_1, \ldots, h_G] \in \mathbb{C}^{M \times G} \) and use \( \mathcal{C} \) to formulate the zero-forcing (ZF) precoding vector intended for subgroup \( g \) as

\[
w_g = \frac{v_g}{\|v_g\|_2}.
\]

(10)

where \( v_g \in \mathbb{C}^{M \times 1} \), \( \mathbb{E}[\|w_g\|^2] = 1 \), and \( v_g \) is the \( g \)-th column vector of the matrix \( V = \mathcal{C}^\dagger \mathcal{C}^{-1} \).

C. Downlink data transmission and spectral efficiency

We assume that the BS transmits data to the multicast users by using ZF precoding. Specifically, the same precoding vector, modulation and coding scheme (MCS) are employed for all the users in the same subgroup. Let us denote the data symbols for every user \( k \in \mathcal{K}_g \) as \( x_g \) (unit variance random variables and uncorrelated), with \( p_g \) being the transmit power allocated to the multicast subgroup \( g \). We assume that the users have access only to the statistical channel state information (CSI), which is \( \mathbb{E}[h_{gk}^H w_g] \). Hence, the downlink (DL) data signal received at user \( k \in \mathcal{K}_g \) can be written as

\[
y_{gk} = \sqrt{p_g} \mathbb{E}[h_{gk}^H w_g] x_g + \sqrt{p_g} \left( h_{gk}^H w_g - \mathbb{E}[h_{gk}^H w_g] \right) x_g + \sum_{g' \in \mathcal{G} \setminus \{g\}} \sqrt{p_{g'}} h_{gk}^H w_{g'} x_{g'} + n_k,
\]

(11)

where the first term denotes the desired signal, the second term is interference due to the user’s lack of CSI, the third term denotes the inter-subgroup interference and finally \( n_k \sim \mathcal{CN}(0, \sigma^2) \) is the AWGN. By invoking the capacity-bounding technique in [17, Sec. 2.3.4], which treats the second, third and fourth term of (11) as effective uncorrelated noise, a downlink achievable spectral efficiency is given by

\[
\text{SE}_{gk} = \left( 1 - \frac{\tau_p}{\tau_c} \right) \log_2(1 + \gamma_{gk}),
\]

(12)

where \( \tau_c \) is the coherence block length and \( \gamma_{gk} \) is the effective signal-to-interference-plus-noise ratio (SINR) given by

\[
\gamma_{gk} = \frac{p_g \mathbb{E}[h_{gk}^H w_g]^2}{\sum_{g'=1}^G p_{g'} \mathbb{E}[h_{gk}^H w_{g'}]^2 - p_g \mathbb{E}[h_{gk}^H w_{g}]^2 + \sigma^2},
\]

(13)

where the expectations are w.r.t. the channel realizations.

Recall that each subgroup receives the same service but with a different MCS which must support the user’s SE with the worst channel condition in the subgroup. Thus, all the users \( k \in \mathcal{K}_g \) experience the SE given by

\[
\text{SE}_g = \min_{k \in \mathcal{K}_g} \text{SE}_{gk}.
\]

I. MIMO multicasting

The literature of MaMIMO multicasting [9]–[13] essentially presents two fundamental delivery strategies. The first option consists of serving each user individually by a unicast transmission as in conventional MaMIMO. This approach limits the number of unicast users to the number of orthogonal pilots (assuming no pilot reuse) and requires the number of BS antennas to be larger than the number of users (for the ZF precoding to be performed). Alternatively, multicast transmission may take place over one joint transmission to the whole multicast group. In this case, only one pilot sequence, MCS are used for all the users.

A. User subgrouping based on spatial characteristics

We propose an alternative strategy to deliver a multicast service in a MaMIMO system, which consists of forming disjoint subgroups of multicast users, each served with a specific MCS. The soundness of this approach was shown in single-input-single-output (SISO) and multiple-input-multiple-output (MIMO) systems, using both wideband and subband channel information [18], [19].

The proposed subgrouping criterion relies on the spatial characteristics of the users: users with similar spatial characteristics, and thereby which cause much interference to each other, are grouped together. To this end, we consider the normalized channels \( h_{gk}/\sqrt{\mathbb{E}[\|h_{gk}\|^2]} \) and \( h_{g'k'}/\sqrt{\mathbb{E}[\|h_{g'k'}\|^2]} \) of any pair of multicast users \( k \in \mathcal{K}_g, k' \in \mathcal{K}_{g'} \). The level of orthogonality of the channel directions, quantified by the variance of the inner product of the normalized channels

\[
\mathbb{V} \left[ \frac{h_{gk}^H h_{g'k'}}{\sqrt{\mathbb{E}[\|h_{gk}\|^2] \mathbb{E}[\|h_{g'k'}\|^2]}} \right] = \frac{\text{tr}(R_{gk} R_{g'k'})}{M^2 \beta_{gk} \beta_{g'k'}},
\]

(15)

gives a measure of how much interference the users cause to each other: the larger the value, the smaller the orthogonality level of the channel directions and the higher the mutual interference between the two users.
Before pilot assignment and channel estimation, assuming perfect knowledge of the large-scale fading parameters, the BS forms the user subgroups such that the users in the same subgroup present similar channel characteristics, hence low levels of orthogonality, i.e., those users for which \( (15) \) returns a large value.

The K-means algorithm and its multiple variants provide a simple method to efficiently cluster the multicast users into disjoint subgroups [20]. This algorithm aims at finding a partition of the \( K \) users into \( G \) subgroups, minimizing the mean square error (MSE) according to the selected metric [21].

### B. Max-min fairness power control

Optimal max-min fairness power control is of practical interest and it has been extensively investigated for MmMIMO systems [16], [17]. The MMF optimization problem subject to average power constraints at the BS is formulated as

\[
P_1: \quad \max_{p_g} \min_{g \in \mathcal{G}, k \in \mathcal{K}_g} \text{SE}_{gk} \quad \text{s.t.} \quad \sum_{g \in \mathcal{G}} p_g \leq P_{DL}. \tag{16}
\]

Expression (12) can be rewritten as

\[
\text{SE}_{gk} = \left(1 - \frac{\tau_g}{\tau_c}\right) \log_2 \left(1 + \sum_{g' \in \mathcal{G}} \frac{p_g a_{gk}}{b_{gk}^g + \sigma_k^2} \right), \tag{17}
\]

where \( a_{gk} = \mathbb{E} [h_{gk}^H w_g]^2 \), \( b_{gk} = \mathbb{E} [h_{gk}^H w_{g'}^g]^2 \) for \( g' \neq g \) and \( g \) and \( b_{gk} = \mathbb{E} [h_{gk}^H w_g]^2 - \mathbb{E} [h_{gk}^H w_{g'}^g]^2 \).

Maximizing the minimum SE is equivalent to maximizing the minimum SINR. Therefore, we can rewrite \( P_1 \) in epigraph form as

\[
P_2: \quad \max_{p_g} \Gamma \quad \text{s.t.} \quad \sum_{g' \in \mathcal{G}} p_{g'} b_{g'k} + \sigma_k^2 \geq \Gamma \quad \forall k \in \mathcal{K}_g, g \in \mathcal{G},
\]

\[
\sum_{g \in \mathcal{G}} p_g \leq P_{DL}, \tag{18}
\]

where \( \Gamma \) is an auxiliary variable that must satisfy the constraint \( \gamma_{gk} \geq \Gamma, \forall g \in \mathcal{G}, \) and \( k \in \mathcal{K}_g. \) \( P_2 \) is still non-convex since the SINR constraint is neither convex nor concave with respect to \( p_g. \) To overcome such non-convexity, we use a successive convex approximation (SCA). Note that for a fixed value of \( \Gamma \geq 0, \) the SINR constraint in \( P_2 \) can also be written in a linear form as

\[
p_g a_{gk} \geq \Gamma \left( \sum_{g' \in \mathcal{G}} p_{g'} b_{g'k} + \sigma_k^2 \right).
\]

**Algorithm 1** SCA algorithm for optimal max-min fairness power control with multicast user subgroups

| Constant: \( P_{DL}, \varepsilon \) |
|---|
| Input: \( \{a_{gk}\}, \{b_{gk}^g\} \) |
| Initialization: \( \Gamma_{min} \leftarrow 0 \) |
| \( \Gamma_{max} \leftarrow \min_{g,k} \left( \frac{a_{gk}}{b_{gk}^g + \sigma_k^2} \right) \) |
| \( p_g^0 \leftarrow 0, \forall g \in \mathcal{G} \) |
| do |
| \( \Gamma = \Gamma_{max} + \varepsilon \) |
| Solve (18) |
| if feasible then |
| \( \Gamma_{min} \leftarrow \Gamma \) |
| \( p_g^* \leftarrow p_g, \forall g \in \mathcal{G} \) |
| else |
| \( \Gamma_{max} \leftarrow \Gamma \) |
| end if |
| while \( \Gamma_{max} - \Gamma_{min} > \varepsilon \) |
| Output: \( \Gamma_{min}, p_g^* \) |

If \( \Gamma \) is fixed, \( P_2 \) is a linear feasibility program, and the optimal solution can be efficiently computed by using interior-point methods, for example, with the toolbox CVX [22]. Letting \( \Gamma \) vary over an SINR search range \( \{\Gamma_{min}, \Gamma_{max}\}, \) the optimal solution can be efficiently computed by using the bisection method [23], in each step solving the corresponding linear feasibility problem for a fixed value of \( \Gamma. \)

Algorithm 1 describes the SCA algorithm providing the power allocation that maximizes the minimum SINR among the multicast users. It works for a pre-determined subgrouping configuration (i.e., the formation of the \( \mathcal{K}_g \) sets of users).

### C. Time/frequency schedule in MaMIMO multicasting

MaMIMO multicasting exploits spatial multiplexing through DL precoding to handle the intra-cell interference among different subgroups [24]. Alternatively, multicast subgroups can be served over orthogonal time/frequency resources (time/frequency scheduling) [18], [19].

According to [25], the optimal number of time/frequency multicast subgroups that maximizes the ASE is either one or two in multicast orthogonal frequency division multiplexing (OFDM) systems. Hence, we compare two setups underlying the proposed user subgrouping strategy: i) all the multicast users served in the same time-frequency resource (spatial multiplexing), and ii) users served over two orthogonal time-frequency intervals (time-frequency multiplexing). In the latter, a set of subgroups is served in a fraction of the time-frequency resources denoted by \( 0 < \theta < 1 \), whereas the rest of the subgroups is served in a fraction \( 1 - \theta \) of the resources. This user scheduling takes place by using the K-means algorithm based on the large-scale fading coefficient \( \beta_{gk}. \)

The ASE for the time/frequency schedule is given by
ASE = \left(1 - \frac{\gamma_p}{\gamma_e}\right) \sum_{g \in \hat{G}} \hat{\gamma}_g K_g \log_2 \left(1 + \hat{\gamma}_g\right), \tag{19}

where $\hat{\gamma}_g$ is equal to either 0 or 1 − $\theta$ depending on which interval the subgroup $g$ is scheduled, and

$$\hat{\gamma}_g = \frac{p_g \left| \mathbb{E}[h_{gk}^H w_g]\right|^2}{\sum_{g' \in \mathcal{S}_g} p_{g'} \mathbb{E}[h_{g'k}^H w_{g'}]^2 \left| \mathbb{E}[h_{gk}^H w_g]\right|^2 + \sigma_k^2}, \tag{20}$$

and $\mathcal{S}_g$ denotes the set of the indices of the subgroups scheduled in the same time-frequency interval as subgroup $g$, i.e., the subgroups interfering with subgroup $g$.

### IV. NUMERICAL RESULTS

In this section, we use numerical simulations to assess the performance of our multicast [MaMIMO] sub grouping strategy. We have employed different configurations of multicast users, along with a cell of 200 m. We have placed the users in clusters with a radius of 2 m to assess users’ effect with similar channel characteristics. We have employed spatially correlated fading channels based on each user’s nominal angle to the BS and an [ASE] of 10°. The path-loss (in dB) is given by $32.4 + 20 \log_{10}(f) + 37.6 \log_{10}(d)$, where $f$ is the operating frequency in GHz, and $d$ is the 2D distance between the BS and the user in meters [11]. We have modeled a correlated shadowing with a variance of 10 dB, which presents a low inter-cluster correlation and an extremely high intra-cluster one. We have employed a ULA with $M$ transmit antennas at the BS. The total amount of power available for DL transmission is 2 W, the UL pilot power is 1 W, and the length of the pilot sequence is $G$ (i.e., the number of multicast subgroups).

We consider a noise power spectral density of $-174$ dBm/Hz, a receiver noise figure of 7 dB, and an operating bandwidth of 20 MHz at a carrier frequency of 2 GHz (e.g., applicable in sub-6 GHz non-line-of-sight [NLOS] scenarios). We assumed a channel coherence of 200 time/frequency samples. We have run Monte Carlo simulations for different configurations of multicast users, over 50 random spatial distributions with 50 channel realizations.

First, we analyze the sub grouping based on spatial multiplexing without the time/frequency schedule. Fig. 2 presents the [ASE] of the multicast MaMIMO service, using the sub grouping strategy for different distributions of users in clusters uniformly and randomly placed along with the cell. We compare the utilization of various criteria to select the number of spatial multicast subgroups (only 1 subgroup, the number of geographical clusters of users, unicast transmissions to every user, and the optimal number of multicast subgroups based on matrix $R_{gk}$). We can observe how the [ASE] decreases with the dispersion of the users, i.e., 20 users placed in 2 clusters of 10 users present higher [ASE] than 4 clusters of 5 users and even more than 20 uncorrelated users. When the number of users is low compared to the number of transmit antennas, i.e., 20 users with 64 transmit antennas, 20 and 50 users with 128 transmit antennas, the [ASE] achieved using unicast transmissions is close to the optimal results achieved with our spatial sub grouping proposal. However, when the number of users increases and these users are placed in clusters, the utilization of multicast spatial sub grouping offers notably better performance than unicast transmissions. In any case, the higher the number of users, the lower the [ASE]. Consequently, individual [SE] is significantly reduced. A large number of [MaMIMO] spatially multiplexed transmissions and the utilization of a common precoding vector for many multicast users lead to a low channel gain when a high number of multicast users are served using [MaMIMO] transmissions in the same time/frequency interval.

To overcome this limitation, we test the proposal of combining spatial and time/frequency sub grouping. Fig. 3 shows the [ASE] for the initial configurations employing only optimal spatial sub grouping and the combination of spatial sub grouping with scheduling using two intervals (using one or two intervals provides higher ASE depending on the users’ distribution). Observing the scenario with a more considerable distance between the number of users and transmit antennas, i.e., 20 users and 128 transmit antennas, the option of using only spatial multiplexing is almost always the optimal configuration. Nevertheless, when this relation is diminishing, the design splitting the users suffering from highly different large-scale fading coefficient ($\beta_{gk}$) into two time/frequency scheduling resources is hugely beneficial. Note that 100 uncorrelated users cannot be served by 64 transmit antenna using [MaMIMO] spatial multiplexing. However, splitting these 100 users into two scheduling blocks allows the system to deliver the multicast service with an extraordinary improvement in the [ASE].

Finally, we employ the configuration with 10 clusters of 5 users/cluster to evaluate the impact of the number of BS antennas on the sub grouping strategy’s performance. Fig. 4 illustrates the [ASE] for a variable number of BS antennas. We assess the performance of these options: i) a unique multicast group ($G = 1$) without the time/frequency schedule, ii) the op-
We conclude that the optimal subgroup configuration highly depends on random users’ distribution and the spatial channel correlation. Correlation matrices information allows the system to create multicast subgroups in scenarios where the users are randomly placed in clusters. Nevertheless, the ASE dramatically drops when the number of users increases, mostly where they are not set in clusters. Our proposal of splitting the multicast users into two time/frequency schedule blocks based on their large-scale fading coefficient presents a significant improvement in the ASE. Hence, a resource allocation strategy that first decides using one or two time/frequency schedule blocks and then creates spatial subgroups provides an attractive improvement in the ASE results.

VI. ACKNOWLEDGEMENT

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