CHEAP SPACE-BASED MICROLENS PARALLAXES FOR HIGH-MAGNIFICATION EVENTS

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ABSTRACT

We show that for high-magnification ($A_{\text{max}} \gtrsim 100$) microlensing events, accurate microlens parallaxes can be obtained from three or fewer photometric measurements from a small telescope on a satellite in solar orbit at $O(AU)$ from Earth. This is 1–2 orders of magnitude less observing resources than are required for standard space-based parallaxes. Such microlens parallaxes would yield accurate mass and distance measurements to the lens for all cases in which finite-source effects were observed from the ground over peak. This would include virtually all high-magnification events with detected planets and a substantial fraction of those without. Hence, it would permit accurate estimates of the Galactic distribution of planets.

Key words: gravitational lensing: micro – planetary systems

1. INTRODUCTION

Microlens parallaxes $\pi_E$ can in principle be measured either from the ground (Gould 1992) or by combining space-based and ground-based observations (Refsdal 1966). The magnitude of $\pi_E$ directly yields the ratio of the lens-source relative parallax ($\pi_{\text{rel}} \equiv \text{AU}[D_L^{-1} - D_S^{-1}]$) to the lens mass ($M$).

$$\pi_E^2 = \frac{\pi_{\text{rel}}}{\kappa M} \equiv \frac{4G}{c^2 \text{AU}} \approx 8.1 \frac{\text{mas}}{M_{\odot}}, \quad (1)$$

while the direction is that of lens-source relative proper motion, $\mu_{\text{rel}}$. Moreover, if the angular Einstein radius $\theta_E$ is measured, then the combination of $\theta_E$ and $\pi_E$ yields both the mass and the relative parallax (Gould 1992, 2000):

$$M = \frac{\theta_E}{\kappa \pi_E}, \quad \pi_{\text{rel}} = \pi_E \theta_E. \quad (2)$$

However, both ground-based and space-based microlensing parallaxes face severe challenges. To measure parallax from a single location (e.g., the Earth), the platform must undergo significant deviation from rectilinear motion; otherwise, the platform motion can just be absorbed into $\mu_{\text{rel}}$. Since most microlensing events have Einstein timescales $t_E$ of weeks, the parallax effects are usually too small to make useful measurements.

Space-based measurements work by a substantially different principle. In essence, one measures the basic lens parameters ($t_0, u_0, \theta_0$) from both the Earth and the satellite. Here, $t_0$ is the time of closest approach and $u_0$ is the impact parameter between the source trajectory and the lens. One then determines, in effect, the displacement in the projected Einstein ring between the two observatories,

$$\Delta u = (\Delta \tau, \Delta u_0), \quad \Delta \tau \equiv \frac{t_{0,\text{sat}} - t_{0,\oplus}}{t_E}, \quad \Delta u_0 \equiv \pm (u_{0,\text{sat}} \mp u_{0,\oplus}), \quad (3)$$

and then simply divides by the known separation $D_{\text{sat}}$ to the satellite (projected on the plane of the sky):

$$\pi_E = \frac{\text{AU}}{D_{\text{sat}}} \Delta u. \quad (4)$$

The direction of $\pi_E$ is thus given relative to the Earth–satellite vector. Note that this is conceptually the same as “terrestrial parallax” in which simultaneous observations from different sites on the Earth can yield a parallax measurement (Hardy & Walker 1995; Holz & Wald 1996). However, because of the short baseline, this technique can only be applied in practice to very rare “extreme microlensing events” (Gould 1997; Gould et al. 2009).

Equation (3) makes clear the principal challenges for space-based parallaxes. First, the source must be monitored from the satellite over many epochs to determine the event parameters that enter Equation (3). Since satellite time is expensive, this can be expected to restrict the total number of events measured.

Second, as presented here, the method yields a four-fold degeneracy, which is the product of two two-fold degeneracies: the inner "±" in Equation (3) depends on whether the Earth and satellite lie on same or opposite sides of the lens, and the outer "±" depends on whether the source passes the lens as seen from the Earth on its left or right (see Figures 1 and 2 of Gould 1994). Now, the latter degeneracy affects only the direction of $\pi_E$, not its magnitude, and for many applications the direction is of substantially less interest. But the former degeneracy does affect the magnitude $\pi_E \equiv |\pi_E|$, and can easily be at the factor $\sim 3$ level. Gould (1995) showed that these degeneracies could in principle be resolved by measuring the difference in $t_0$ from the two observatories, but this requires much higher precision and therefore a several-fold increase in telescope time, thus gravely exacerbating the challenge that was discussed in the previous paragraph. Now, it is sometimes possible to break these degeneracies by combining Earth-based and satellite-based parallaxes (Gould 1999; Dong et al. 2007), but only for moderately long events.

Here, we show that for a subset of microlensing events, those that peak at high magnification as seen from Earth, excellent measurements of $|\pi_E|$ can be obtained by combining ground-based observations with a single satellite observation. These high-magnification events are of exceptional interest because they are more sensitive to planetary perturbations (Griest & Safizadeh 1998), more likely to yield measurements of $\theta_0$ (and so, via Equation (2), $M$ and $\pi_{\text{rel}}$), and easier to observe (because they are bright).
Two circles represent the constraints on the position of the source relative to the lens (at the origin) as seen from the Earth, \( u_\oplus \), and the satellite, \( u_{\text{sat}} \). In principle, any line connecting the inner and outer circles is allowed; one example is shown. The unknown angle between the lens–source separation vectors as seen from the satellite and the Earth at the time of the observations is given by \( \phi \). This figure is scaled such that 1.0 equals the Einstein radius.

2. SATELLITE PARALLAXES FOR HIGH-MAGNIFICATION EVENTS

Consider a single observation of a microlensing event taken by a satellite at a time when the event is highly magnified as seen from Earth. In particular, suppose that

\[
u_{\oplus} = \sqrt{u_{\oplus,0}^2 + \left(\frac{t - t_{0,\oplus}}{t_E}\right)^2} \ll 1.
\]

The satellite observation yields a flux measurement \( f_{\text{sat}} \). Using standard techniques (e.g., Gould et al. 2010a), one can infer the source flux in the satellite band \( f_{\text{sat}} \), from the source flux and color in ground-based bands. In many cases, particularly when the satellite measurement is at moderately high magnification and blending is not severe, it will also be possible to adequately estimate satellite blended-light flux \( f_{\text{sat}} \) by photometric transformation of the ground-based images. But even when this is not possible, \( f_{\text{sat}} \) can be determined from a second satellite measurement at a much later date. Then, the satellite-based magnification is just \( A_{\text{sat}} = (f_{\text{sat}} - f_{\text{sat}}) / f_{\text{sat}} \), and so (assuming the lens can be approximated as a point lens), the satellite position in the Einstein ring can be calculated from the inverse of the standard Einstein (1936) formula

\[
u_{\text{sat}} = \sqrt{2\left[1 - A_{\text{sat}}^2\right]^{-1/2} - 1} \to A_{\text{sat}}^{-1},
\]

where the last limit applies when \( A_{\text{max}} \geq 2 \).

By the law of cosines,

\[|\Delta u| = \sqrt{u_{\text{sat}}^2 + u_{\oplus}^2 - 2u_{\text{sat}}u_{\oplus}\cos \phi} \to u_{\text{sat}} - u_{\oplus}\cos \phi,
\]

where \( \phi \) is some unknown angle between the lens–source separation vectors as seen from the Earth and satellite at the time of the observation, and where the last limit applies for \( u_{\text{sat}} \gg u_{\oplus} \). Figure 1 illustrates this geometry.

Let us initially assume that there are no measurement errors. Then, by simply adopting

\[
\pi_E = \frac{\text{AU}}{D_{\text{sat}}}u_{\text{sat}},
\]

one is making a fractional error in the parallax of only \( \delta \pi_E / \pi_E = (u_{\oplus}/u_{\text{sat}})\cos \phi \). The worst errors will occur when \( \pi_E \) is small, i.e., when the lens is in the Galactic bulge (so \( \pi_E \) is small) and has a relatively large mass \( M \). See Equation (1). For example, suppose \( M = 1 M_\odot \) and the distances to lens and source are \( D_L = 7.5 \text{ kpc} \) and \( D_T = 8.5 \text{ kpc} \), and let \( D_{\text{sat}} = 1 \text{ AU} \). Then, \( \pi_E = 0.044 \) so Equation (8) implies that \( u_{\text{sat}} = 0.044 \), and therefore \( |\delta \pi_E / \pi_E| < 0.23(u_{\text{sat}}/0.01) \).

Hence, if the satellite observation is taken when the Earth-based magnification is high, \( A_{\oplus} \gg 100 \) (\( u_{\text{sat}} \ll 0.01 \)), it is possible in principle to make accurate parallax measurements even though the parallax is much smaller than has ever been accurately measured from Earth for events with typical timescales. Another way to say this is that the systematic error (due to unknown relative orientation of the Earth and satellite with respect to the lens geometry) is \( |\delta \pi_E| \ll (\text{AU}/D_{\text{sat}})u_{\text{sat}} \). This means that for a given satellite separation, one can simply choose the events with sufficiently high magnification to achieve the desired precision. Note that this is a hard upper limit on the systematic error, not a 1σ systematic error. We reiterate that these arguments apply under the condition \( u_{\text{sat}} \gg u_{\oplus} \).

We now consider the impact of photometric errors. From the limiting form of Equation (6), one finds that a fractional flux error measurement leads to the same fractional error in \( u_{\text{sat}} \), as shown in Figure 1, assuming \( u_{\text{sat}} \ll 0.5 \). Now, for small \( u_{\text{sat}} \) (where the systematic errors are important), the source is highly magnified, so the photometric errors will generally not compete with the systematic errors. On the other hand, at moderate \( u_{\text{sat}} \sim 0.5 \), the systematic errors will be negligible but the photometric errors could be significant. In particular, if the source is very faint, then it is possible that the measurement will be radically compromised, although even here, one can obtain significant limits just from the fact that the source was not strongly magnified. Finally, if \( u_{\text{sat}} \sim 1 \), then the measurement is likely to be poor because, from Equation (6), a small error in \( A_{\text{sat}} \) leads to a large error in \( u_{\text{sat}} \). However, for satellite separations \( D_{\text{sat}} \ll 1 \text{ AU} \), such large \( \Delta u \) would correspond to a large parallax, which would increase the probability that the parallax could be measured from Earth–orbit parallax or possibly terrestrial parallax (e.g., Gould et al. 2009).

3. APPLICATION TO PLANETS

High-magnification events are an important channel for finding planets because planets that are anywhere in the system give rise to a central caustic near the position of the host star. Hence, if the event is known to be approaching high magnification (i.e., very small projected separation between the lens and source) the probability that the event will probe the central caustic is high, making it advantageous to apply limited observing resources to the brief interval of close passage. Such events are rare, but the specific rate of planet detection is high. For example, Gould et al. (2010b) derived planet frequencies from

\[1 \text{ In their systematic study of microlens parallaxes, Poindexter et al. (2005) measured only one parallax smaller than \( \pi_E < 0.1 \) with accuracy better than 25%. But this event, MACHO-BLG-99-22, had a heliocentric Einstein timescale of \( t_E = 570 \text{ days} \), one of the longest ever observed.}\]
a statistically homogeneous sample of 13 high-magnification events observed over four years, which contained six planets. An important feature of these high-magnification events with planets is that essentially all of them yield measurements of $\theta_E$. This is because the source almost always passes close to or over the central caustic, giving rise to light-curve deviations that depend on $\rho = \theta_*/\theta_E$, the source size in units of the Einstein radius. Since $\theta_*$ can be determined from the dereddened color and magnitude of the source (Yoo et al. 2004), this yields $\theta_E = \theta_*/\rho$.

Now, the basic method outlined in Section 2 assumed a point lens in order to derive $u_{sat}$ from $A_{sat}$. See Equation (6). However, here we are explicitly consider non-point lenses. In general, the magnification pattern for a planet–star lens is very similar to a point lens over most of the source plane, and so the same approach as given above will usually work. Nevertheless, for planetary events, there is some finite probability (which can be explicitly calculated based on the central-caustic perturbation detected from the ground) that the satellite measurement will land on (or very near) the planetary caustic. To be conservative, one should therefore take two measurements separated by a short time (which, again, can be easily calculated based on the ground-based detection of the central caustic). In most cases, these two measurements, combined with the ground-based data, will be sufficient to virtually rule out that the first image was “corrupted” by a planetary caustic. Then, as discussed in Section 2, it may also be necessary to take a third image to determine the blending.

If a large fraction of these events also had measured $\pi_E$, then one would be able to derive the lens mass and lens–source relative parallax using Equation (2). This would greatly increase the value of such high-magnification planet samples. In particular, it would allow one to cleanly distinguish between disk and bulge lenses (and hence planets) and so enable an estimate of the Galactic distribution of planets.

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