In the Standard Cosmological Model, the matter density of the Universe is dominated by an unknown component, approximately 5 times more abundant than baryons, dubbed Dark Matter (DM). Among the many DM candidates proposed in the literature, Weakly Interacting Massive Particles (WIMPs), i.e. particles with mass $\mathcal{O}(100)$ GeV and weak interactions, appear particularly promising, also in view of their possible connection with well motivated extensions of the Standard Model of particle physics (see Ref. [1] for recent reviews on particle DM, including a discussion of ongoing direct, indirect and accelerator searches). Despite their weak interactions, WIMPs can lead to macroscopic effects in astrophysical objects, provided that they have a sizeable scattering cross section off baryons. In this case, in fact, DM particles traveling through stars can be captured, and sink at the center of the stars. Direct searches and astrophysical arguments, however, severely constrain the strength of DM-baryons interactions (see e.g. Ref. [2] and references therein). Since the capture rate is proportional to the product of the scattering cross section times the local DM density, large effects are thus expected in regions where the DM density is extremely high (this was already noticed in the context of the so called ‘cosmions’ [3]). Recent progress in our understanding of the formation and structure of DM halos has prompted a renewed interest in the consequences of DM capture in stars, in particular the case of White Dwarfs [4], compact objects [5] and main sequence stars [6] at the Galactic center, where the DM density could be extremely high [7].

Alternatively, one may focus on the first stars, which are thought to form from gas collapsing at the center of $10^6 - 10^8 M_\odot$ DM halos at redshift $z \lesssim 10 - 50$. In fact, Spolyar, Freese and Gondolo [8] have shown that the energy released by WIMP annihilations in these minihalos, during the formation of a proto-star (thus even before DM capture becomes efficient), may exceed any cooling mechanism, thus inhibiting or delaying stellar evolution (see also Ref. [9]). The formation of proto-stars with masses between $6M_\odot$ and $600M_\odot$ in DM halos of $10^6 M_\odot$ at $z=20$, can actually be delayed by $\sim 10^3 - 10^4$ yrs [10]. Once the star forms, the scattering of WIMPs off the stellar nuclei becomes more efficient and a large number of WIMPs can be trapped inside the gravitational potential well of the star. The WIMP’s luminosity can overwhelm that from nuclear reactions and therefore strongly modify the star evolution [11, 12], and the core H-burning phase of Pop III stars, in DM halos of density of $10^{11}$ GeV cm$^{-3}$, is substantially prolonged, especially for small mass stars ($M_* < 40 M_\odot$) [10].

In this letter, we perform a detailed study of the impact of DM capture and annihilation on the evolution of Pop. III stars with a suitable modification of the Geneva stellar evolution code [13]. With respect to previous analyses, this already allows us to properly take into account the stellar structure in the calculation of the capture rate, that we compute, following Ref. [14], as

$$C = 4\pi \int_0^{R_\star} dr r^2 \frac{dC(r)}{dV}$$

with

$$\frac{dC(r)}{dV} = \left( \frac{6}{\pi} \right)^{1/2} \frac{\rho_i(r) \rho_N}{M_1} \frac{v_2(r)}{m_\chi} \frac{\bar{v}}{2\eta A^2} \times \left\{ A_+ A_- - \frac{1}{2} \left[ \chi(-\eta, \eta) - \chi(A_-, A_+) \right] + \frac{1}{2} A_+ e^{-A^2} - \frac{1}{2} A_- e^{-A_-^2} - \frac{1}{2} \eta e^{-\eta^2} \right\}$$

$$A^2 = \frac{3v_2^2(r)\mu}{2\bar{v}^2 \mu^2_\pm}, \quad A_\pm = A \pm \eta, \quad \eta^2 = \frac{3v_2^2}{2\bar{v}^2}$$

$$\chi(a, b) = \frac{\sqrt{\pi}}{2} \left[ \text{Erf}(b) - \text{Erf}(a) \right] = \int_a^b dy e^{-y^2}$$

$$\mu_- = (\mu_i - 1)/2, \quad \mu_i = m_\chi/M_i$$

References:
[1] [Ref. 1]
[2] [Ref. 2]
[3] [Ref. 3]
[4] [Ref. 4]
[5] [Ref. 5]
[6] [Ref. 6]
[7] [Ref. 7]
[8] [Ref. 8]
[9] [Ref. 9]
[10] [Ref. 10]
[11] [Ref. 11]
[12] [Ref. 12]
[13] [Ref. 13]
[14] [Ref. 14]
where $\rho_i(r)$ is the mass density profile of a given chemical element in the interior of the star and $M_i$ refers to its atomic mass, while $\rho_\chi$, $m_\chi$ and $\bar{v}$ are respectively the WIMP mass and the WIMP density and velocity dispersion at the star position. The velocity of the star with respect to an observer, labeled as $v_\star$, is assumed to be equal to $\bar{v}$, giving therefore $\eta = \sqrt{3/2}$. The radial escape velocity profile depends on $M(r)$, i.e. the mass enclosed within a radius $r$, $v^2(r) = 2\int_r^\infty GM(r')/r'^2dr'$. The WIMP scattering cross section off nuclei, $\sigma_{\chi,N}$ is constrained by direct detection experiments and for a WIMP mass of 100 GeV the current upper limits are $\sigma_{SI} = 10^{-43}$ cm$^2$ [15] and $\sigma_{SD} = 10^{-38}$ cm$^2$ [16] respectively for spin-independent and spin-dependent WIMP interactions off a proton. We will adopt these reference values throughout the paper, but the capture rate can be easily rescaled for other scattering cross sections by using Eq. 3. The spin-independent interactions with nucleons inside nuclei add up coherently giving an enhancement factor $A^4$ with respect to the interaction with a single nucleon: $\sigma_{\chi,N}^{SI} = A^4\sigma_{\chi,p}^{SI}$, where $A$ is the mass number. There is no such enhancement for the spin-dependent interactions. We consider the contribution to the capture rate from WIMP-hydrogen spin dependent interactions and WIMP-helium $^4$He spin-independent interactions, neglecting the presence of other elements because of their very low abundance. The contribution of Helium is found to be negligible with respect to that from hydrogen.

Once captured, WIMPs get redistributed in the interior of the star reaching, in a characteristic time $\tau_{th}$, a thermal distribution [17]:

$$n_\chi(r) = n_\infty e^{-r^2/\pi} \text{ with } r_\chi = \sqrt{\frac{3kT_\star}{2\pi G\rho_\infty m_\chi}} \quad (3)$$

with $T_\star$ and $\rho_\infty$ referring to the core temperature and density. The distribution results quite concentrated toward the center of the star: e.g. for a 20$M_\odot$ star immersed in a WIMP density of $\rho_\chi = 10^9$ GeV cm$^{-3}$ at the beginning of the core H-burning phase we obtain $r_\chi = 2 \times 10^9$ cm, a value much lower than the radius of the star, $R_\star = 10^{11}$ cm. This consideration underlines the importance of an accurate spatial resolution in the core to properly treat the luminosity produced from WIMPs annihilations. We have also checked that regardless the extremely high concentrations of WIMPs obtained at the center of the stars, the gravity due to WIMPs is completely negligible. The number of scattering events needed for DM particles to thermalize with the nuclei in the star is of order $\approx m_\chi/M_H$, thus an upper limit on the thermalization time can be obtained as $\tau_{th} = (m_\chi/M_H)/\sigma_{SD}\bar{n}_H\bar{v}$ where $\bar{n}_H$ is the average density on the star.

The WIMPs luminosity is simply $L_\chi(r) = 4\pi(\sigma v)m_\chi c^2\bar{n}_\chi^2(r)$. For the annihilation cross section times relative velocity ($\sigma v$), we assume the value $3 \times 10^{-26}$ cm$^2$, as appropriate for a thermal WIMP, but note that the total WIMP luminosity at equilibrium

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**Figure 1:** Evolutionary tracks of a Pop III 20 $M_\odot$ star for different WIMP densities (labels in units of GeV cm$^{-3}$). We have adopted a WIMP model with $m_\chi = 100$ GeV and $\sigma_{SD} = 10^{-38}$ cm$^2$.

**Figure 2:** Temperature of the core as a function of the DM density for the 20 $M_\odot$ model, at different stages of the core H-burning phase. $X_c$ denotes the mass fraction of hydrogen at the centre ($X_c = 0.76$ at the beginning of the core H-burning phase). WIMP parameters as in Fig. 1.
Figure 3: Variation of the core H-burning lifetime as a function of the WIMP densities for the Pop III 20 and 200 $M_{\odot}$ models. WIMP parameters as in Fig. 1.

Figure 4: ZAMS positions of 20 and 200 $M_{\odot}$ Pop. III stars in the g vs. $T_{\text{eff}}$ plane for different DM densities (labels in units of GeV cm$^{-3}$). Big red circles correspond to the critical WIMP density (see text). WIMP parameters as in Fig. 1.

does not depend on this quantity. After a time

$$\tau_\chi = \left( \frac{C(\sigma v)}{\pi^{3/2} r_\chi^3} \right)^{-1/2}$$

an equilibrium between capture and annihilation is established, and this incidentally allows to determine the normalization constant $n_0$ above.

We have checked that the two transients $\tau_\chi$ and $\tau_{\text{th}}$ remain much smaller, during the evolution of the star, than the Kelvin-Helmotz timescale, $\tau_{KH}$ and the timescale needed for the nuclear reactions to burn an hydrogen fraction $\Delta X_c = 0.002$ of the convective core, $\tau_{\text{nucl}}$:

$$\tau_{KH} = \frac{GM^2}{R_s L_s}, \quad \tau_{\text{nucl}} = \frac{q_c \Delta X_c M_s 0.007 c^2}{L_s}$$

where the * labels quantities relative to the the star and $q_c$ is the core convective mass fraction. This argument justifies the assumption of equilibrium between capture and annihilation and the use of the radial distribution in Eq. 3. We assume here an average WIMP velocity $\bar{v} = 10 \text{ Km s}^{-1}$, the virial velocity in an halo of $10^5 - 10^6 M_{\odot}$ at z=20. As for the DM density, semi-analytic computations of the adiabatic contraction of DM halos [12, 13], in agreement with the results extrapolated from simulations of first star formation [14], suggest DM densities of order $10^{12} \text{ GeV cm}^{-3}$ or even higher.

We have implemented the effects of WIMPs annihilation in the Geneva stellar evolution code (see Ref. [13] for details), and followed the evolution of a 20$M_{\odot}$ and 200$M_{\odot}$ stars for different DM densities. We show in Fig.1 the evolutionary tracks for the 20$M_{\odot}$ model, and show for comparison (black line) the case of a standard Pop III star without WIMPs. For DM densities smaller than $10^9 \text{ GeV cm}^{-3}$ the evolutionary tracks closely follow that of a normal star and they are not shown for simplicity. The position of the star at the beginning of the core H-burning phase (zero-age main sequence, or ZAMS) is obtained when, after a short transient, the luminosity produced at the center of the star equals the total luminosity and the star settles down in a stationary regime. For increasing DM densities the WIMPs luminosity produced at the center overwhelms the luminosity from nuclear reactions and makes the star inflate, producing therefore a substantial decrease of the effective temperature and a moderate decrease of the star luminosity at the ZAMS position, with respect to the standard scenario. For $\rho_\chi = 10^{10} \text{ GeV cm}^{-3}$, the energy produced by WIMPs present in the star at a given time, estimated as $E_\chi \sim L_\chi \tau_{KH}$, is, at the ZAMS, $\sim 0.8$ times the gravitational potential energy of the star, and the star therefore starts to contract. In this phase, the core temperature, and consequently also the nuclear reactions, increase. When the latter become comparable with the WIMPs luminosity, the standard situation is recovered and the evolutionary track joins the classical tracks of a star without WIMPs. An important difference from standard evolution is that in the first phase, the nuclear reactions are slowed down and therefore the core H-burning lifetime is prolonged. For Dark Matter densi-
densities $\rho \leq 1.6 \times 10^{10}$ GeV cm$^{-3}$, the picture is qualitatively the same, and for these models we only show in Fig. 1 the first phases of the evolution. In Fig. 2 we show the core temperature as a function of the DM density, at different stages of the core H-burning phase. At high DM densities hydrogen burns at much lower core temperatures than in the usual scenario, till a certain mass fraction is reached, e.g. $X_c = 0.3$ for $\rho_\chi = 10^{10}$ GeV cm$^{-3}$, and the standard evolutionary track is joined. For increasing DM densities the nuclear reaction rate is more and more delayed till the contraction of the star is inhibited, due to the high DM energy accumulated, and the evolution is frozen. In Fig. 1 for $\rho_\chi = 2 \cdot 10^{10}$ GeV cm$^{-3}$ and $\rho_\chi = 3 \cdot 10^{10}$ GeV cm$^{-3}$ the stars remains to remain indefinitely at the ZAMS position. In Fig. 2 we show the core H-burning lifetime as a function of the DM density. In the case of a $20\,M_\odot$ model, for $\rho \leq 10^{10}$ Gev cm$^{-3}$ the core H-burning phase is prolonged by less than 10% but the delay increases rapidly for higher DM densities. Extrapolating the curve we determine a critical density, $\rho_c = 2.5 \cdot 10^{10}$ Gev cm$^{-3}$, beyond which the core H-burning lifetime is longer than the age of the Universe. All the calculations have been repeated for the $200\,M_\odot$ model and we find that both the $20\,M_\odot$ and $200\,M_\odot$ stars evolutions are stopped for DM densities higher than $5.3 \cdot 10^{10} (\sigma_{\text{SD}}/10^{-26} \text{ cm}^3)$ Gev cm$^{-3}$.

We have also verified that the results weakly depend on the WIMP mass, e.g. the core H-burning lifetime is modified by a factor 0.2% and 5% respectively for $m_\chi = 10$ GeV and $m_\chi = 100$ GeV, if $\rho_\chi = 10^{10}$ Gev cm$^{-3}$.

It is remarkable that under these circumstances, frozen Pop III stars can survive until the present epoch, and can be searched for as an anomalous stellar population. In Fig. 3 we show the effective temperature and gravity acceleration at the surface of these frozen Pop III stars, kept in the H-burning phase, for different DM densities. Frozen stars would thus appear much bigger and with much lower surface temperatures with respect to normal stars with the same mass and metallicity. Our results are qualitatively consistent with the preliminary estimates in [11, 12] and the analysis in [10]. However, for a given DM density, we obtain a somewhat longer core H-burning lifetime with respect to [10], possibly due to their use of an approximated expression for the capture rate. We have also followed, for selected models, the evolution during the core He-burning phase. During this evolutionary stage, the Dark Matter luminosity is lower than the nuclear reaction luminosity, therefore the impact of DM annihilations is found to be rather weak. For the $20\,M_\odot$ model and for $\rho_\chi = 1.6 \cdot 10^{10}$ Gev cm$^{-3}$ the H-lifetime is prolonged by a factor 1.2, rather than a factor 37 found for the H-burning phase for the same DM density.

In conclusion, we have adapted a stellar evolution code to the study the evolution of Pop. III stars in presence of WIMPs. We have shown that above a critical DM density, the annihilation of WIMPs captured by Pop. III stars can dramatically alter the evolution of these objects, and prolong their lifetime beyond the age of the Universe. We have determined the properties of these ‘frozen’ stars, and determined the observational properties that may allow to discriminate these objects from ordinary stars.

M.T. thanks the International Doctorate on Astroparticle Physics (IDAPP) for partial support and the Geneva Observatory for the warm hospitality. We thank F. Iocco for useful discussions. During the completion of this work we became aware of a related work done independently by Yoon, Iocco and Akiyama [20]. Their results, obtained with an independent stellar evolution code, appear to be in good agreement with our own.

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