The rare decays of $D$ mesons

S. Fajfer$^{ab*}$, A. Prapotnik$^b$, S. Prelovšek$^{ab}$, P. Singer$^c$, and J. Zupan$^b$

$^a$Physics Department, University of Ljubljana, SI-1001 Ljubljana, Slovenia

$^b$J. Stefan Institute, SI-1000 Ljubljana, Slovenia

$^c$Department of Physics, Technion – Israel Institute of Technology, Haifa IL-32000, Israel

The flavor changing transitions in the $c \to u\gamma$, $c \to u\gamma\gamma$ and $c \to ul^+l^-$ offer the possibility to search for new physics in the charm sector. We investigate dominant decay mechanisms in the radiative decays $D \to V\gamma$, $D \to P(V)l^+l^-$, $D \to \gamma\gamma$ and we discuss chances to see physics beyond the standard model in these decays. In addition, we analyze Cabibbo allowed $D \to K\pi\gamma$ decays with nonresonant $K\pi$, and we probe the role of light vector mesons in these decays.

The search for physics beyond the standard model has been focused on the down-like quark sector, while the up-like quark sector has been less researched. The rare $D$ decays offer an opportunity to investigate the FCNC effects in the charm sector. To date, no radiative or dilepton weak decay of $D$ mesons has been detected. Only upper bounds have been established so far for a sizable number of the radiative or dilepton weak decay of $D$ mesons. The radiative decays $D^0 \to \pi^0, \omega, \phi, \bar{K}^*0 + \gamma$ were recently bounded to branching ratios in the $10^{-4}$ range, which is approaching the standard model expectations (see, e.g. where additional previous works are listed). The dilepton decays $D \to Pl^+l^-$, $D \to Vl^+l^-$ are the subject of intensive searches at CLEO and Fermilab. Here again, with upper bounds of $\lesssim 10^{-5}$ for branching ratios of the various modes one approaches the expectations of the standard model. The situation should improve in the future, due to new possibilities for observation of charm meson decays at BELLE, BABAR and Tevatron. Recently, upper limits in the $10^{-5} - 10^{-4}$ range were established also for $D^0$ dilepton decays with two nonresonant pseudoscalar mesons in the final state $D^0 \to (\pi^+\pi^-, K^-\pi^+, K^+K^-) \mu^+\mu^-$, though no comparable results are available yet for similar photonic decays.

The inclusive $c \to u\gamma$ transition is strongly GIM suppressed at one loop electroweak order giving the branching ratio of the order $10^{-18}$. In our approach we include the $c \to u\gamma$ short distance contribution by using the Lagrangian

$$\mathcal{L} = -A C_{7\gamma} \frac{\mu}{4\pi^2} F_{\mu\nu} \left[ \bar{u}d_{\mu\nu} \cdot (1 + \gamma_5)c \right], \quad (1)$$

where $m_c$ is a charm quark mass and $A = (G_F/\sqrt{2}) V_{us} V_{cs}^*$.

We follow and we take $C_{7\gamma} = (-0.7 + 2i) \times 10^{-2}$. The branching ratio induced by this QCD corrected effective Lagrangian is $\text{BR}(c \to u\gamma) \simeq 3 \times 10^{-8}$. A variety of models beyond the standard model were investigated and it was found that the gluino exchange diagrams within general MSSM give the largest enhancement

$$\frac{\text{BR}(c \to u\gamma)_{\text{MSSM}}}{\text{BR}(c \to u\gamma)_{\text{SM}}} \simeq 10^2. \quad (2)$$

Unfortunately, the long distance physics screens such effects since it usually dominates the decay amplitude. The long distance contribution is induced by the effective nonleptonic $|\Delta c| = 1$ weak Lagrangian

$$\mathcal{L} = -A_{ij} \left[ a_1 \bar{u}\gamma^\mu(1 - \gamma_5)q_i \bar{q}_j \gamma_\mu(1 - \gamma_5)c + a_2 \bar{q}_j \gamma_\mu(1 - \gamma_5)q_i \bar{u}\gamma^\mu(1 - \gamma_5)c \right], \quad (3)$$

with $A_{ij} = (G_F/\sqrt{2}) V_{eq}^* V_{aq}$, accompanied by the emission of the virtual photon. Here $q_{i,j}$ denote the $d$ or $s$ quark fields. The effective Wilson
coefficients are $a_1 = 1.2$ and $a_2 = -0.5$. In our calculations of the long distance effects we use the theoretical framework of heavy meson chiral Lagrangian. In the treatment of the $D$ mesons rare decays we use the factorization approximation for the calculation of weak transition elements. We consider the use of this approach to be justified by the "near" success of the approach for the nonleptonic amplitudes. In Table 1 we present the branching ratios of $D \to V\gamma$ decays. The uncertainty is due to relative unknown phases of various contributions. Although the branching ratios are dominated by the long distance contributions, the size of the short distance contribution can be obtained from the difference of the decay widths $\Gamma(D^0 \to \rho^0\gamma)$ and $\Gamma(D^0 \to \omega\gamma)$ [13]. Namely, the long distance mechanism $c\bar{u} \to d\bar{d}\gamma$ overshadows the $c\bar{u} \to u\bar{u}\gamma$ transition in $D^0 \to \rho^0\gamma$ and $D^0 \to \omega\gamma$, the $\rho^0$ and $\omega$ mesons being mixture of $u\bar{u}$ and $d\bar{d}$. However, the LD contributions are mostly canceled in the ratio

$$R = \frac{\text{BR}(D^0 \to \rho^0\gamma) - \text{BR}(D^0 \to \omega\gamma)}{\Gamma(D^0 \to \omega\gamma)} \propto \text{Re} \frac{A(D^0 \to u\bar{u}\gamma)}{A(D^0 \to d\bar{d}\gamma)},$$

which is proportional to the SD amplitude $A(D^0 \to u\bar{u}\gamma)$ driven by $c \to u\gamma$. The ratio $R$ is $6 \pm 15\%$ in the standard model [3], and can be enhanced up to $O(1)$ in the MSSM. In addition to the $c \to u\gamma$ searches in the charm meson decays, we have suggested to search for this transition in $B_c \to B_s^*\gamma$ decay [14], where the long distance contribution is much smaller.

The $c \to u\ell^+\ell^-$ amplitude is given by the $\gamma$ and $Z$ penguin diagrams and $W$ box diagram at one-loop electroweak order in the standard model, and is dominated by the light quark contributions in the loop. In Table 2 we give the branching ratios of $c \to u\ell^+\ell^-$ branching ratios calculated in the standard model and MSSM [3]. The amplitudes for exclusive decays $D \to V\ell^+\ell^-$ and $D \to P\ell^+\ell^-$ are dominated by the long distance contributions. The branching ratios for these decays, as obtained in [3], are given in Tables 3 and 4. In these Tables the first column represents the SD contributions, while in the second column the rates coming from the LD contributions are given. The rates for $D \to V\ell^+\ell^-$ are comparable to those in Table 3 and can be found in [3]. There is a

| Table 1 |
|---------|
| The branching ratios for $D \to V\gamma$ decays. |
| $D \to V\gamma$ | BR |
| $D^0 \to K^{*0}\gamma$ | $[6 - 36] \times 10^{-5}$ |
| $D^0 \to K^{*+}\gamma$ | $[20 - 80] \times 10^{-5}$ |
| $D^0 \to \rho^0\gamma$ | $[0.1 - 1] \times 10^{-5}$ |
| $D^0 \to \rho^+\gamma$ | $[0.1 - 0.9] \times 10^{-5}$ |
| $D^0 \to \phi\gamma$ | $[0.4 - 1.9] \times 10^{-5}$ |
| $D^+ \to \rho^+\gamma$ | $[0.4 - 6.3] \times 10^{-5}$ |
| $D^+ \to K^{*+}\gamma$ | $[1.2 - 5.1] \times 10^{-5}$ |
| $D^+ \to K^{*0}\gamma$ | $[0.3 - 4.4] \times 10^{-6}$ |
| $D^0 \to K^{*0}\gamma$ | $[0.3 - 2.0] \times 10^{-6}$ |

| Table 2 |
|---------|
| The $c \to u\ell^+\ell^-$ branching ratios. |
| $c \to u\ell^+\ell^-$ | BR SD | BR MSSM |
| $c \to u\ell^+\ell^-$ | $[6 \pm 1] \times 10^{-9}$ | $6.0 \times 10^{-8}$ |
| $c \to u\ell^+\ell^-$ | $[6 \pm 1] \times 10^{-9}$ | $2.0 \times 10^{-8}$ |

| Table 3 |
|---------|
| The $D \to V\mu^+\mu^-$ branching ratios. |
| $D \to V\mu^+\mu^-$ | BR SD | BR LD |
| $D^0 \to K^{*0}\mu^+\mu^-$ | $[1.6 - 1.9] \times 10^{-6}$ |
| $D^+ \to \rho^+\mu^+\mu^-$ | $[3.0 - 3.3] \times 10^{-5}$ |
| $D^0 \to \rho^0\mu^+\mu^-$ | $9.7 \times 10^{-10}$ | $[3.5 - 4.7] \times 10^{-7}$ |
| $D^0 \to \omega\mu^+\mu^-$ | $9.1 \times 10^{-10}$ | $[3.3 - 4.5] \times 10^{-7}$ |
| $D^0 \to \phi\mu^+\mu^-$ | $6.5 \times 10^{-9}$ | $[6.5 - 9.0] \times 10^{-8}$ |
| $D^+ \to \rho^+\mu^+\mu^-$ | $4.8 \times 10^{-9}$ | $[1.5 - 1.8] \times 10^{-6}$ |
| $D^+ \to K^{*+}\mu^+\mu^-$ | $1.6 \times 10^{-9}$ | $[5.0 - 7.0] \times 10^{-7}$ |
| $D^0 \to K^{*0}\mu^+\mu^-$ | $[3.1 - 3.7] \times 10^{-8}$ |
| $D^0 \to K^{*0}\mu^+\mu^-$ | $[4.4 - 5.1] \times 10^{-9}$ |
The short distance contribution is rather flat. Decays in terms of $m$ decay is $0.1 \times 10^{-6}$. The difference in their rates due to the kinematic consider them separately.

The allowed kinematic region for the dilepton mass $m_{ll}$ in the $D \rightarrow P l^+l^-$ decay is $m_{ll} = [2m_l, m_D - m_P]$. The long distance contribution has resonant shape with poles at $m_{ll} = m_{\rho}, m_\omega, m_\phi$. There is no pole at $m_{ll} = 0$ since the decay $D \rightarrow P \gamma$ is forbidden. The short distance contribution is rather flat. The spectra of $D \rightarrow P e^+e^-$ and $D \rightarrow P \mu^+\mu^-$ decays in terms of $m_{ll}$ are practically identical. The difference in their rates due to the kinematic region $m_{ll} = [2m_\epsilon, 2m_\mu]$ is small and we do not consider them separately.

The differential distribution for $D^{+,0} \rightarrow \pi^{+0}l^+l^-$ indicates that the high mass dilepton region might give an opportunity for detecting $c \rightarrow ul^+l^-$. Before making a definite statement on such possibility, we should examine this kinematical region of high dilepton mass in $D \rightarrow \pi^+l^-l^-$ decays more closely. For instance, in this region the excited states of the vector mesons $\rho, \omega$ and $\phi$ may become important. Even with the presence of these states we found that the only viable channel for the $c \rightarrow ul^+l^-$ transition is $D \rightarrow \pi l^+l^-$, in the kinematic region of the dilepton invariant mass $m_{ll}$ above the resonance $\phi$, where the long distance contribution is reduced. The kinematics of the processes $D \rightarrow V l^+l^-$ would be more favorable to probe the possible supersymmetric enhancement at small $m_{ll}$, but the long distance contributions in these channels are even more disturbing.

Motivated by the experimental efforts to observe rare $D$ meson decays $^{13}$, and noticing that $B_s \rightarrow \gamma \gamma$ offers possibility to observe physics beyond the SM, we undertook an investigation of the $D^0 \rightarrow \gamma \gamma$ decay $^{10}$. The short distance contribution is expected to be rather small, as already encountered in the one photon decays $^{4}$, hence the main contribution would come from long distance interactions. The total amplitude is dominated by terms proportional to $a_2$ that contribute only through loops with Goldstone bosons. Loop contributions proportional to $a_2$ vanish at this order. We remark that the contribution of the order $O(p)$ does not exist in the $D^0 \rightarrow \gamma \gamma$ decay, and the amplitude starts with contribution of the order $O(p^2)$. The chiral loops of order $O(p^2)$ are finite, as they are in the similar case of $K \rightarrow \gamma \gamma$ decays. At this order the amplitude receives also an annihilation type contribution proportional to the $a_2$ Wilson coefficient, given by the Wess-Zumino anomalous term coupling light pseudoscalars to two photons. Terms which contain the anomalous electromagnetic coupling of the heavy quark Lagrangian are suppressed compared to the leading loop effects $^{14}$. The invariant amplitude for $D^0 \rightarrow \gamma \gamma$ decay can be written using gauge and Lorentz invariance in the following form,

$$M = \left[i M^{(-)}(g^{\mu\nu} - k^{\mu}_{1}k^{\nu}_{1}) + M^{(+)}(5)\epsilon_{\mu1}\epsilon_{\nu2}\right]$$

where $M^{(-)}$ is a parity violating and $M^{(+)}$ a parity conserving part of the amplitude, while $k_{1(2)}$, $\epsilon_{1(2)}$ are respectively the four momenta and the polarization vectors of the outgoing photons. We give in Table 5 the numerical results for the amplitudes originating from the different contributions. Within this framework, the leading contributions are found to arise from the charged $\pi$ and $K$ mesons running in the chiral loops and our calculation predicts that the $D \rightarrow 2\gamma$ decay is mostly a parity violating transition $^{14}$. We estimate that the total uncertainty is not larger than 50% $^{16}$, including possible effects from light res-

| $D \rightarrow P l^+l^-$ | $Br_{SD}$ | $Br_{LD}$ |
|--------------------------|----------|----------|
| $D^0 \rightarrow K^0 l^+l^-$ | 0 | $4.3 \times 10^{-7}$ |
| $D^0 \rightarrow \pi^+ l^+l^-$ | 0 | $6.1 \times 10^{-6}$ |
| $D^0 \rightarrow \eta^0 l^+l^-$ | 1.9 $\times 10^{-9}$ | 2.1 $\times 10^{-7}$ |
| $D^0 \rightarrow \eta l^+l^-$ | 2.5 $\times 10^{-10}$ | 4.9 $\times 10^{-8}$ |
| $D^0 \rightarrow \phi l^+l^-$ | 9.7 $\times 10^{-12}$ | 2.4 $\times 10^{-8}$ |
| $D^0 \rightarrow \pi^0 l^+l^-$ | 9.4 $\times 10^{-9}$ | 1.0 $\times 10^{-6}$ |
| $D^0 \rightarrow K^0 l^+l^-$ | 9.0 $\times 10^{-10}$ | 4.3 $\times 10^{-8}$ |
| $D^0 \rightarrow K^+ l^+l^-$ | 0 | 7.1 $\times 10^{-9}$ |
| $D^0 \rightarrow K^0 l^+l^-$ | 0 | 1.1 $\times 10^{-9}$ |
The nonvanishing amplitudes in $D^0 \to \gamma \gamma$. The amplitudes coming from the anomalous and short distance $C_7^{\text{eff}}$ Lagrangians are given in first two lines. The finite and gauge invariant sums of one-loop amplitudes are listed in the next three lines ($M_i^{(\pm)} = \sum_j M_{i,j}^{(\pm)}$). The numbers 1, 2, 3 denote the row of diagrams on the Fig. 2 in [16]. In the last line the sum of all amplitudes is given. All amplitudes are given in the units $10^{-10}$ GeV.

| $M_i^{(-)}$ | $M_i^{(+)}$ |
|------------|------------|
| A          | 0          | -0.53     |
| SD         | -0.27      | -0.81i    | -0.16 | -0.47i |
| 1          | 3.55       | +9.36i    | 0     |
| 2          | 1.67       | 0         |       |
| 3          | -0.54      | +2.84i    | 0     |
| $\sum_i M_i^{(\pm)}$ | 4.41 | +11.39i | -0.69 | -0.47i |

Such a calculation gives a rather good result for the $D^+ \to \bar{K}^0 \pi^+$ channel but is less successful for the $D^0 \to K^+ \pi^-$ decay. In order to overcome this deficiency and to be able to present accurately the bremsstrahlung component of the radiative transition, we shall use an alternative approach for its derivation. It means, we take the experimental values for the $D \to K \pi$ amplitudes, in the calculation of the bremsstrahlung component. In order to accommodate this, we write the decay amplitude as

$$
\mathcal{M} = -\frac{G_F}{\sqrt{2}} V_{cs} V_{da}^* \left\{ F_0 \left[ \frac{q \cdot \varepsilon}{q \cdot k} - \frac{p \cdot \varepsilon}{p \cdot k} \right] + F_1 \left[ (q \cdot \varepsilon) (p \cdot k) - (p \cdot \varepsilon) (q \cdot k) \right] + F_2 \varepsilon^{\alpha \beta \gamma} \varepsilon_\alpha v_\beta k_\gamma \right\},
$$

(7)

where $F_0$ is the experimentally determined $D \to K \pi$ amplitude and $F_1, F_2$ are the form factors of the electric and magnetic direct transitions which we calculate with our model. When intermediate states appear to be on the mass shell, we use Breit Wigner formula. Thus, we get for the branching ratios of the electric transitions including bremsstrahlung, with $|F_0|$ determined experimentally and taking the photon energy cut $E_\gamma \geq 100$ MeV

$$
\text{BR}(D^0 \to \bar{K}^0 \pi^+) = (2.3 - 2.5) \times 10^{-4}.
$$

(8)

For the $D^0$ radiative decay we get

$$
\text{BR}(D^0 \to K^- \pi^+) = (4.3 - 6.0) \times 10^{-4}.
$$

(9)

The uncertainty in the $F_0/F_1$ phase is less of a problem in $D^+ \to \bar{K}^0 \pi^+$ than in $D^0 \to K^- \pi^+$. If we take the bremsstrahlung amplitude alone as determined from the knowledge of $|F_0|$, disregarding the direct electric $F_1$ term, the above numbers are replaced by $2.3 \times 10^{-4}$ for $D^+$ decay and $5.5 \times 10^{-4}$ for the $D^0$ decay. The contribution of the direct parity violating term (putting $F_0 = 0$), is $\text{BR}(D^+ \to \bar{K}^0 \pi^+)_{\text{dir, PV}} = 1.0 \times 10^{-5}$ and $\text{BR}(D^0 \to K^- \pi^+)_{\text{dir, PV}} = 1.64 \times 10^{-4}$. For the parity conserving direct magnetic transition

Table 5
The nonvanishing amplitudes in $D^0 \to \gamma \gamma$. The amplitudes coming from the anomalous and short distance $C_7^{\text{eff}}$ Lagrangians are given in first two lines. The finite and gauge invariant sums of one-loop amplitudes are listed in the next three lines ($M_i^{(\pm)} = \sum_j M_{i,j}^{(\pm)}$). The numbers 1, 2, 3 denote the row of diagrams on the Fig. 2 in [16]. In the last line the sum of all amplitudes is given. All amplitudes are given in the units $10^{-10}$ GeV.

| $M_i^{(-)}$ | $M_i^{(+)}$ |
|------------|------------|
| A          | 0          | -0.53     |
| SD         | -0.27      | -0.81i    | -0.16 | -0.47i |
| 1          | 3.55       | +9.36i    | 0     |
| 2          | 1.67       | 0         |       |
| 3          | -0.54      | +2.84i    | 0     |
| $\sum_i M_i^{(\pm)}$ | 4.41 | +11.39i | -0.69 | -0.47i |

Onances like $\rho, K^*, a_0(980), f_0(975)$. Accordingly, we conclude that the predicted branching ratio is

$$
\text{BR}(D^0 \to \gamma \gamma) = (1.0 \pm 0.5) \times 10^{-8}.
$$

(6)

We look forward to experimental attempts of detecting this decay. Our result suggests that the observation of $D \to 2\gamma$ at a rate which is an order of magnitude larger than $[3]$ could be a signal for the type of "new physics" [3].

In addition to the $D$ meson rare decays in which the FCNC transitions might occur, we undertook a study of the Cabibbo allowed radiative decays $D^+ \to \bar{K}^0 \pi^+ \gamma$ and $D^0 \to K^- \pi^+ \gamma$ with nonresonant $K \pi$, which we consider to be the most likely candidates for early detection. Here we used the heavy quark chiral Lagrangian supplemented by light vector mesons, as the theoretical framework.

These decays are the charm sector counterpart of the $K \to \pi \pi \gamma$ decays, which have provided a wealth of information on meson dynamics. Using the factorization approximation for the calculation of weak transition elements we use the information obtained from semileptonic decays [7][8]. The nonleptonic $D \to K \pi$ amplitude cannot be calculated accurately in the factorization approximation from the diagrams provided by our model.
we get

\[ \text{BR}(D^+ \rightarrow \bar{K}^0 \pi^+ \gamma)_{\text{PC}} = 2.0 \times 10^{-5}, \]
\[ \text{BR}(D^0 \rightarrow K^- \pi^+ \gamma)_{\text{PC}} = 1.4 \times 10^{-4}. \]

Hence, the two direct transitions are predicted to be of about the same strength.

If we disregard the contribution of vector mesons to the direct part of the radiative decays, the parity-conserving part of the amplitude is considerably decreased, by one order of magnitude in the rate in \( D^+ \rightarrow \bar{K}^0 \pi^+ \gamma \) decay and by two orders of magnitude in \( D^0 \rightarrow K^- \pi^+ \gamma \). On the other hand, their contribution is not felt in a significant way in the parity-violating part of the amplitudes. In any case, the detection of the direct part of these decays at the predicted rates, will constitute a proof of the important role of the light vector mesons.

We conclude by expressing the hope that the interesting features which these decays provide and were analyzed in [19], will bring to an experimental search in the near future.

REFERENCES

1. D.M. Asner et al., CLEO Collaboration, Phys. Rev. D 58 (1998) 092001.
2. S. Fajfer, S. Prelovšek, and P. Singer, Eur. Phys. J. C 6 (1999) 471; 6 (1999) 751(E).
3. A. Freyberger et al., CLEO collaboration, Phys. Rev. Lett. 76 (1996) 3065; E.M. Aitala et al., E 791 Collaboration, Phys. Rev. Lett. 76 (1996) 364; Phys. Lett. B 462 (1999) 401; D.A. Sanders, Mod. Phys. Lett. A 15 (2000) 1399.
4. W.E. Jhons, FOCUS Collaboration, hep-ex/0207013.
5. E.M. Aitala et al., E 791 Collaboration, Phys. Rev. Lett. 86 (2001) 3969; D.A. Sanders et al., E791 Collaboration, hep-ph/0105028. D.J. Summers, Int. J. Mod. Phys. A 16 Suppl. 1B (2001) 536.
6. S. Fajfer, S. Prelovšek, and P. Singer, Phys. Rev. D 58 (1998) 094038.
7. G. Burdman, E. Golowich, J.L. Hewett, and S. Pakvasa, Phys. Rev. D 52 (1995) 1399.
8. G. Burdman, E. Golowich, J.L. Hewett, and S. Pakvasa, hep-ph/0112239.
9. S. Fajfer, S. Prelovšek, and P. Singer, Phys. Rev. D 64 (2001) 114009.
10. C. Greub, T. Hurth, M. Misiak, and D. Wyler, Phys. Lett. B 382 (1996) 415; Q. Ho-Kim and X.-Y. Pham, Phys. Rev. D 61 (2000) 013008.
11. S. Prelovšek and D. Wyler, Phys. Lett. B 500 (2001) 304.
12. M.B. Wise, Phys. Rev. D 45 (1992) 2188; G. Burdman and J. Donoghue, Phys. Lett. B 280 (1992) 287.
13. S. Fajfer, S. Prelovšek, P. Singer, and D. Wyler, Phys. Lett. B 487 (2000) 81.
14. S. Fajfer, S. Prelovšek, P. Singer, Phys. Rev. D 59 (1999) 114003.
15. M. Selen, Workshop on Prospects for CLEO/CESR with \( 3 < E_{cm} < 5 \) GeV, Cornell University, May 5-7, 2001; I. Bediaga, private communication.
16. S. Fajfer, P. Singer, and J. Zupan, Phys. Rev. D 64 (2001) 074008.
17. R. Casalbuoni et al., Phys. Lett. B 299 (1993) 139; Phys. Rep. 281 (1997) 145.
18. B. Bajc, S. Fajfer, and R.J. Oakes, Phys. Rev. D 53, 4957 (1996).
19. S. Fajfer, A. Prapotnik, and P. Singer, hep-ph/0204306 (to appear in Phys. Rev. D).