Bubble jet impact on a rigid wall of different stand-off parameters

S Li, S P Wang and A M Zhang
College of Shipbuilding, Harbin Engineering University, #515 Chuanhai Building, Nantong Street No.145, Nangang District, Harbin 150001, China
E-mail: zhangaman@hrbeu.edu.cn

Abstract. One of the key features of the dynamics of a bubble near a rigid wall is the development of a high liquid jet, generating highly localized pressure on the wall. In present study, the boundary integral method is employed to simulate this phenomenon, and the vortex ring model is introduced to handle the discontinued potential of the toroidal bubble. Meanwhile, the pressure induced in the whole process is calculated by an auxiliary function. The effect of the stand-off parameter on the bubble dynamics and the pressure on the wall is investigated, and a double-peaked structure occurs in the pressure profile after the jet impact in some cases, which is associated with the jet impact and the high internal pressure inside the bubble.

1. Introduction
Bubble dynamics near a rigid wall is a particularly important subject because of the damage potential that can be caused by the impulsive pressure pulses resulting from the collapse. The growth and collapse of micron-sized bubbles near propeller blades holds the key to understanding the deleterious effects of cavitation [1]. The interaction between underwater explosion bubble and warship has important naval applications [2]. Lauterborn [3] found the velocity of the boundary-induced bubble jet is approximately 120m/s, which would cause severe damage to the structures.

Boundary integral method (BIM) is extensively used to study the dynamic of a non-equilibrium violently oscillating gas bubble for many decades [4~8]. The main features of the bubble could be simulated, such as expansion, collapse, jet and so on. Besides, the velocity and pressure in the fluid domain could also be calculated to analyze the mechanism of these phenomena. The transition of a bubble surface from singly-connected to doubly-connected induces circulation in the flow around the toroidal bubble, results in only a few works on the dynamic of toroidal bubble. Best [9] introduced a vortex sheet that moves with the fluid, rending the domain singly-connected. Wang [5] came up with a simper model consists of placing a vortex ring inside the bubble. The strength of the vortex ring is chosen to be equal to the potential difference of the jet tip and the opposite bubble surface just before the jet impacts.

In this paper, BIM is adopted to study the motion of bubble near a rigid wall, and the vortex ring model is introduced to handle the discontinued potential of the toroidal bubble. The auxiliary function method [10] is employed to calculate the pressure on the rigid wall, which avoid making finite difference of the velocity potential.

1 Corresponding author.
2. Basic Theory
The liquid surrounding the bubble is assumed inviscid and incompressible, and the motion irrotational. The velocity potential $\Phi$ satisfies the Laplace equation:

$$\nabla^2 \Phi = 0$$  \hspace{1cm} (1)

According to Green function, the velocity potential at any point in the domain could be expressed as an integral equation:

$$\int_\delta \left[ G(p,q) \cdot \frac{\partial \Phi(q,t)}{\partial n} - \Phi(q,t) \frac{\partial G(p,q)}{\partial n} \right] dS(q) = \epsilon(p,t) \cdot \Phi(p,t)$$  \hspace{1cm} (2)

where $\epsilon(p,t)$ is the solid angle of a fixed point $p$ on flow boundaries with the integration variable $q$ also situated on boundaries, $\partial/\partial n$ is the normal outward derivative from the boundary $S$. $G(p,q) = 1/|p-q| + 1/|p-q'|$ is the half-space Green function, with $q'$ being the reflected image of $q$ across the wall.

The dynamic boundary condition on the bubble surface can be written as:

$$\frac{D \Phi}{Dt} = \frac{(\nabla \Phi)^2}{2} + \frac{P_\infty}{\rho} - \frac{P}{\rho} - gz$$  \hspace{1cm} (3)

where $\rho$ is the density of the liquid, $P_\infty$ is the ambient pressure of the liquid around the bubble, $P$ is the pressure on the bubble surface, $g$ is the gravity acceleration.

The kinematic boundary condition is:

$$\frac{dr}{dt} = \nabla \Phi$$  \hspace{1cm} (4)

The pressure inside the bubble is assumed to be uniform and consists of a constant vapor pressure and a volume-dependent noncondensable gas pressure [9]. Hence, the bubble pressure $P_b$ as a function of the volume is described as:

$$P_b = P_c + P_{ini}(V_{ini}/V)^{\gamma}$$  \hspace{1cm} (5)

where the subscript $ini$ denotes initial quantities, $\gamma$ is the ratio of the specific heats for the gas, $P_c$ is the vapor pressure.

Consider the jet impacting on the opposite bubble surface, forming a toroidal bubble. To solve this problem, a vortex ring is introduced inside the toroidal bubble [5]. The velocity induced by a circular vortex ring $u_v$ can be calculated from Bio-Savart law:

$$u_v(r,z) = \frac{\Gamma}{4\pi \rho} \int \frac{dl \times r}{r^3}$$  \hspace{1cm} (6)

More details about the vortex ring model can refer to Wang [5]. Here we have scaled length with respect to the maximum radius which the largest bubble under consideration might achieve in an infinite body of fluid $R_m$; pressure by $\Delta P = P_\infty - P_c$; time by $R_m \sqrt{\rho/\Delta P}$. Besides, some dimensionless parameters are given to describe a bubble:

1) Strength parameter: $\varepsilon = P_{ini}/\Delta P$
2) Stand-off parameter: $\gamma = d/R_m$
3) Buoyancy parameter: $\delta = (\rho g R_m/\Delta P)^{0.5}$
3. Results and Discussion

Three cases were calculated to study the dynamic behaviors of a bubble and the pressure induced on a rigid wall. The bubble is initialized at various stand-off parameters $\gamma = 1.3, 1.6, 2.0$, and the strength parameter is set as $\varepsilon = 20$, buoyancy parameter $\delta = 0$. The bubble motion before the jet impact has been discussed extensively before, so we just give the jet tip velocity versus time in Figure 1. Comparing the three cases, it is clear that, the jet velocity at the end of the collapse phase decreases as the bubble is initiated closer to the wall.

Figure 2 shows the toroidal bubble is rebounding for the case $\gamma = 1.3$. Because of the high-speed liquid jet, a protrusion appears at the position of jet impact, and an annular hollow move downward on the bubble surface. It can be seen that the bubble is drawn towards the rigid wall during its rebounding and the jet impacts directly on the rigid wall afterward.

Figure 1. The jet velocity before jet impact versus time for the bubble with $\varepsilon = 20, \delta = 0$ initiated at distances $\gamma = 1.3, 1.6, 2.0$ from a rigid wall.

Figure 2. Evolution of the toroidal bubble with $\varepsilon = 20, \delta = 0, \gamma = 1.3$ at $t = 2.444$ (innermost), 2.478, 2.552, 2.725, 2.873 (outermost).

Figure 3. Evolution of the toroidal bubble with $\varepsilon = 20, \delta = 0, \gamma = 1.6$ at $t = 2.417$ (innermost), 2.454, 2.569, 2.735 (outermost).

Figure 4. Evolution of the toroidal bubble with $\varepsilon = 20, \delta = 0, \gamma = 2.0$ at $t = 2.362$ (innermost), 2.385, 2.426, 2.487, 2.565 (outermost).

Figure 3 concerns the bubble initiated at $\gamma = 1.6$. The centroid of the bubble also moves quickly toward the rigid wall. However, when the liquid jet comes into contact with the wall, the bubble
reaches a large volume, so the inner-bubble pressure drops to a very low level. Meanwhile, the jet becomes very thin, so the bubble might rejoin into a singly connected form, which won’t be discussed in this study. The next case relates the bubble initiated at \( \gamma = 2.0 \), which is shown in figure 4. The bubble rebounds in a toroidal form, moving upward, and it does not contact the rigid wall during the rebounding progress.

Figure 5 shows the kinetic energy and potential energy versus time for the bubble with different stand-off parameters. It is obvious that the bubble cycle increases as the stand-off parameter decreases, which implies the rigid wall slows down the bubble motion. As the stand-off parameter increases, the bubble will achieve greater potential energy and smaller kinetic energy when bubble reaches its minimum volume. In other words, the jet contains more energy when bubble initialised closer to the wall.

![Figure 5](image5.png)

**Figure 5.** The kinetic energy and potential energy versus time for the bubble with \( \varepsilon = 20, \delta = 0 \) initiated at distances \( \gamma = 1.3, 1.6, 2.0 \) from a rigid wall.

![Figure 6](image6.png)

**Figure 6.** The pressure at different points \( r = 0, 0.5, 1.0 \) of the rigid wall versus time for the bubble with \( \varepsilon = 20, \delta = 0 \) initiated at distances \( \gamma = 1.3 \) from a rigid wall.

![Figure 7](image7.png)

**Figure 7.** The pressure at different points \( r = 0, 0.5, 1.0 \) of the rigid wall versus time for the bubble with \( \varepsilon = 20, \delta = 0 \) initiated at distances \( \gamma = 1.6 \) from a rigid wall.

![Figure 8](image8.png)

**Figure 8.** The pressure at different points \( r = 0, 0.5, 1.0 \) of the rigid wall versus time for the bubble with \( \varepsilon = 20, \delta = 0 \) initiated at distances \( \gamma = 2.0 \) from a rigid wall.

The pressures at different position on the wall for the case \( \gamma = 1.3 \) are given in figure 6. At time \( t = 0 \), the pressure inside the bubble is very large, which induce the first maximum pressure on the wall.
However, the pressure caused by the collapsing bubble is much larger than that at $t = 0$. During the collapsing phase, two pressure peaks at the centre point of the wall, with approximately the same value, are observed. The first one occurs when the bubble reach its minimum volume, and the second one occurs when jet impact directly on the wall. As for $r=0.5$ and $r=1.0$, two small peak also turn up on the pressure profiles during the bubble rebounding phase, which are caused by the bubble protrusion pushing the liquid radially outward.

The evolution of the pressures at different position on the wall with time is shown in figure 7 for the case $\gamma = 1.6$. There are also two peaks on the pressure profile during the collapsing phase. The first one occurs when the bubble reach its minimum volume, the second one is not caused by the jet impact because the bubble surface is not contact with the wall at this time. The second peak can be explained that the bubble still maintains high pressure inside while the bubble is moving upward rapidly, so the distance between the bubble and the wall is decreasing. After $t = 2.558$, the pressure at the centre of the wall increases gradually, however, the pressures at $r=0.5$ and $r=1.0$ keeps decreasing, which implies the range of the jet impact is limited.

The evolution of the pressures at different position on the wall with time is shown in figure 8 for the case $\gamma = 2.0$. The maximum pressure during the collapsing phase is little larger than the first maximum pressure at $t = 0$. The effect of the toroidal bubble on the wall is similar to an spherical-oscillated bubble, and the liquid jet can’t be reflected in the pressure profile.

4. Conclusions

Boundary integral method was employed to simulate bubble dynamics near a rigid wall, and the vortex ring model was adopted to handle the double-connected domain problem. Three different cases ($\gamma = 1.3, 1.6, 2.0$) were calculated to discuss the effect of the stand-off parameter on bubble dynamics and the pressure on the wall. As for $\gamma = 1.3$, the pressure caused by jet impact directly on the boundary is approximately the same with the time bubble reach its minimum volume, but the jet impact area is small. As for $\gamma = 1.6$, there were two peaks on the pressure profile during the bubble collapsing phase, the first one is associated with the minimum volume of the bubble, and the second one is caused by the rapidly upward movement of the bubble. As for $\gamma = 2.0$, the effect of the toroidal bubble on the wall is similar to a spherical-oscillated bubble, the jet impact on the wall could be ignored.

References

[1] Choi J, Hsiao C T, Chahine G and Ceccio S 2009 J. Fluid Mech. 624 255
[2] Klaseboer E, Khoo B C and Hung K C 2005 J. Fluid Struct. 21 395
[3] Lauterborn W and Bolle H 1975 J. Fluid Mech. 72 391
[4] Wang Q X, Yeo K S, Khoo B C and Lam K Y 1996 Theoret. Comput. Fluid Dynamics 8 73
[5] Wang Q X, Yeo K S, Khoo B C and Lam K Y 1996 Computers & Fluids 25 607
[6] Blake J R, Tomita Y and Tong R P 1998 Appl. Sci. Res. 58 77
[7] Tong R P, Schiffer W P, Show S J, Blake J R and Emmony D C 1999 J. Fluid. Mech. 380 339
[8] Brujan E A, Keen G S, Vogel A and Blake J R 2002 Phys. Fluids 14 85
[9] Best J P 1993 J. Fluid. Mech. 251 79
[10] Wu G X and Hu Z Z 2004 P Roy. Soc. A 460 2797
[11] Newman J N 1977 Marine Hydrodynamics (London: MIT Press) p 131