Adiabatic Tracking of Quantum Many-Body Dynamics

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(Dated: August 4, 2014)

The nonadiabatic dynamics of a many-body system driven through a quantum critical point can be controlled using counterdiabatic driving, where the formation of excitations is suppressed by assisting the dynamics with auxiliary multiple-body nonlocal interactions. We propose an alternative scheme which circumvents practical challenges to realize shortcuts to adiabaticity in mesoscopic systems by tailoring the functional form of the auxiliary counterdiabatic interactions. A driving scheme resorting on few-body short-range interactions is shown to generate an effectively adiabatic dynamics.

PACS numbers: 64.60.Ht, 05.30.Rt, 03.67.Ac, 37.10.Ty

Introduction.—The adiabatic driving of quantum many-body systems is of interest to a wide variety of quantum technologies ranging from quantum simulation to adiabatic quantum computation. However, the implementation of adiabatic driving schemes in the laboratory is often challenging or simply impractical. In recent years a large body of theoretical and experimental progress has been focused on the development of shortcuts to adiabaticity (STA), fast-nonadiabatic protocols that reproduce the preparation of the same final state that would be achieved under slow driving [1]. The feasibility of realizing such shortcuts relies on the control of nonadiabatic excitations away from the ground state manifold of the system of interest. A general technique to achieve this goal is known as counterdiabatic driving (CD) [2–4].

In a nutshell, the adiabatic approximation |Ψ(t)⟩ to the dynamics generated by a slowly-driven Hamiltonian of interest $H_0[λ(t)]$, is found to be the exact solution of the so-called counterdiabatic driving Hamiltonian $H_{CD}$, i.e., $iℏ\partial_t|Ψ(t)⟩ = H_{CD}[Ψ(t)]$, even under fast driving. More precisely, let the instantaneous eigenstates and eigenvalues of $H_0[λ(t)]$ be denoted by \{ε_n[λ(t)]\} and \{ε_n[λ(t)]\}, then the CD Hamiltonian can be always written as the sum $H_{CD} = H_0[λ(t)] + H_{aux}[λ(t)]$ of the system Hamiltonian $H_0[λ(t)]$ and the auxiliary CD term

$$H_{aux} = iℏλ(t) \sum_n [⟨\partial_λ ε_n|ε_n⟩ - ⟨ε_n|\partial_λ ε_n⟩]⟨ε_n|ε_n⟩,$$  \hspace{0.5cm} (1)

where the over-dot denotes the time derivative. This auxiliary term suppresses explicitly excitations away from the adiabatic manifold allowing one to design STA. Conversely, it vanishes as the adiabatic limit is approached [4, 5]. When applied to lattice systems [5], CD is closely related to the notion of quasiadiabatic continuation [6–8] and is equivalent to transitionless quantum driving [9].

To date, CD has been demonstrated in effective two-
tion in digital quantum simulators, e.g. in trapped ion chains [28–30] and Rydberg gases [31].

Counterdiabatic driving of many-body systems.—We consider a quench of a finite transverse Ising chain of \(N\) spins with nearest-neighbor interactions under open boundary condition described by

\[
H_0(t) = -B(t) \sum_{j=1}^{N} \sigma_j^x + J_0 \sum_{j=1}^{N-1} \sigma_j^z \otimes \sigma_{j+1}^z ,
\]

where \(B(t)\) is a time-varying external magnetic field, \(\sigma_j^x, \sigma_j^z\) are usual Pauli sigma operators associated with site \(j\), and \(J_0\) denotes uniform magnetic couplings between adjacent spins assumed to be unity throughout the work. In the thermodynamic limit the system exhibits a quantum phase transition as a function of the transverse field, separating a paramagnetic phase \((|B(t)/J_0| > 1)\) from a doubly-degenerate antiferromagnetic one \((|B(t)/J_0| < 1)\).

The quantum simulation of this model and its variants is the goal of current efforts in ion-trap experiments [32–35], ultracold atoms [36], and NMR experiments [37], to name just a few examples. The achievable system-size with current technology is still tractable by real-space exact diagonalization (ED), that we employ to derive the instantaneous eigenspectrum \(\{ |\varepsilon_n(t)\rangle \}\) and the auxiliary CD term via Eq. (1). \(H_{\text{aux}}\) is found to involve \(R\)-body interactions \((R = 1, \ldots, N)\), in agreement with [5]. We wish to find an alternative CD scheme resorting exclusively in auxiliary control fields associated with a restricted set of operators, assumed to be available in a quantum simulator. The scheme relies on suggesting a new ansatz for the variational construction of the “exact” auxiliary CD term of the form

\[
\hat{H}_{\text{aux}}^{[R, R]}(t) = \sum_{\ell=1}^{R} \sum_{i_1, \ldots, i_R} h_{i_1, \ldots, i_R}^{\alpha_1, \ldots, \alpha_R}(t) \hat{R}_{i_1} \otimes \cdots \otimes \hat{R}_{i_R} ,
\]

where \(\alpha_\ell = \{0, x, y, z\}\) with \(\sigma^0 = 1\) and \(0 < |i_\ell - i_{\ell'}| \leq R\) \(\forall \ell, \ell'\) is understood in the restricted sum over distinct site indices at maximal distance \(R = \{R - 1, \ldots, N - 1\}\) from each other to be identified as the range of the \(R\)-body interaction, \(h_{i_1, \ldots, i_R}^{\alpha_1, \ldots, \alpha_R}\) are the optimal interaction amplitudes to be found via a variational optimization procedure that targets the “exact” auxiliary CD term \(H_{\text{aux}}\) and shall be detailed in the following. This ansatz is general enough to generate an arbitrary unitary. However, we shall see that a \(H_{\text{aux}}^{[R, R]}\) containing exclusively few-body quasi-local terms suffices to induce an effectively adiabatic dynamics. \(H_{\text{aux}}^{[R, R]}\) involves nonlocal \(R\)-body couplings of spins through the chain as Fig. 1(a) illustrates. Considering the driving of an arbitrary eigenstate \(|\Psi(0)\rangle = |\varepsilon_n(0)\rangle\) prepared at \(t = 0\), the optimal amplitudes \(h_{i_1, \ldots, i_R}^{\alpha_1, \ldots, \alpha_R}(t)\) that lead to an optimal representation of \(H_{\text{aux}}\) within the subspace of the ansatz (3) may be obtained by minimizing the quadratic cost function of the form

\[
\min_{\alpha_1, \ldots, \alpha_R} \| (H_{\text{aux}} - H_{\text{aux}}^{[R, R]})|\Psi_n(t)\rangle \|^2 ,
\]

where \(|\Psi_n(t)\rangle \equiv |\varepsilon_n(t)\rangle\) denotes the adiabatic evolution of \(|\Psi(0)\rangle\) generated by \(H_0(t)\). The result of such a minimization can generally be written down in the form

\[
\sum_{\{i_\ell\}} \sum_{\{\alpha_\ell\}} A_{i_1, \ldots, i_R}^{\alpha_1, \ldots, \alpha_R} h_{i_1, \ldots, i_R}^{\alpha_1, \ldots, \alpha_R} = C_{i_1, \ldots, i_R}^{\alpha_1, \ldots, \alpha_R} ,
\]

where

\[
A_{i_1, \ldots, i_R}^{\alpha_1, \ldots, \alpha_R} = \text{Tr} \left[ \rho^t \left\{ \hat{R}_{i_1} \otimes \cdots \otimes \hat{R}_{i_R} \right\}^{\alpha_1, \ldots, \alpha_R} \right] ,
\]

\[
C_{i_1, \ldots, i_R}^{\alpha_1, \ldots, \alpha_R} = \text{Tr} \left[ \rho^t [H_{\text{aux}} \otimes \sigma_{i_1}^{\alpha_1} \otimes \cdots \otimes \sigma_{i_R}^{\alpha_R}] \right] ,
\]

where the anticommutator \(\{A, B\} = AB + BA\) and we have denoted \(\rho^t = |\Psi_n(t)\rangle \langle \Psi_n(t)|\) to stress the flexibility of the formalism in extending to mixed states. At a given time \(t\), the equations above admit a compact form as a single linear tensor equation involving a contraction of the form

\[
\overline{A} h = C ,
\]

with a graphical representation depicted in Fig. 1(b).
The latter tensor equation can be solved by an in situ reshaping of the tensor \( A \) of rank \( 4R \) into a regular matrix \( \bar{A} \) of dimension \((4N)^{2R} \times (4N)^{2R}\) while combining the set of indices \( \{i_r\}\) \( \alpha_r\) and \( \{i'_r\}\) \( \alpha'_r\) into single “superindices” \( I \) and \( I' \), respectively. This amounts to casting the original tensor equation into a matrix inversion problem involving matrix \( \bar{A}_{I,I'} \) and reshaped column vectors \( \bar{C}_I \), given by \( \bar{h} = \bar{A}^{-1} \bar{C} \). Reshaping back properly the column vector \( \bar{h} \) so obtained into the original order yields the optimal amplitudes \( h_{a_1 \ldots a_R}^{\alpha_1 \ldots \alpha_R} \) for approximating the exact auxiliary CD term. A costly part of the implementation of the outlined procedure requires an explicit construction of \( \bar{A} \) and \( \bar{C} \) which in case of the driving of pure states can be economized on by utilizing the following properties and bringing thereby a significant reduction in CPU time: (i) \( A_{I,I'} = A_{I,I'} \); (ii) Simplifying the definitions in Eq. (6) as

\[
\begin{align*}
\bar{A}_{I,I'} &= 2\text{Re}\{\langle \Phi_I | \Phi_I \rangle \} , \\
\bar{C}_I &= 2\text{Re}\{\langle \Phi_I | \Phi_{\text{aux}} \rangle \},
\end{align*}
\]

where \( \langle \Phi_I \rangle \equiv \left( \bigotimes_{r=1}^R \sigma^{\alpha_r}_{i_r} \right) |\Phi_a(t)\rangle \) and \( \langle \Phi_{\text{aux}} \rangle \equiv H_{\text{aux}} |\Psi_a(t)\rangle \).

The ansatz (3) is fairly general and its implementation in a quantum simulator can be expected to be complex. In what follows we show that short-range few-body interactions suffice to generate an effectively adiabatic dynamics. For the sake of illustration, we start discussing the case in which \( \bar{H}_{\text{aux}}^{[R,R]} \) is restricted to two-body interactions and show that it already suffices to reduce the density of excitations (DoE) by orders of magnitude with respect to the \( H_{\text{CD}} = H_0 \) case. After discussing it, we shall include higher-order multiple-body interactions.

As an illustrative case, and without loss of generality, we shall henceforth focus on the driving of the ground state of the initial system Hamiltonian, \( |\Psi(0)\rangle = |\phi_0(0)\rangle \). The quality of the adiabatic tracking scheme can be estimated by means of the time-dependent fidelity

\[
\mathcal{F}(t) \equiv |\langle \Phi_a(t) | \bar{\Psi}(t) \rangle|^2 ,
\]

defined as the overlap between the adiabatic evolution of the starting state and the instantaneous state of the system denoted by \( |\bar{\Psi}(t)\rangle \) under an evolution dictated by \( H_0(t) + \bar{H}_{\text{aux}}^{[R,R]}(t) \) while using the units in which \( \hbar = 1 \). The DoE \( n_{\text{ex}} \) can then be computed as the probability of not ending up in the final ground state, i.e., \( n_{\text{ex}} = 1 - \mathcal{F} \). We consider a quench linear in time of the form \( B(t) = B_0 - vt \) and study the efficiency of \( \bar{H}_{\text{aux}}^{[R,R]} \) in suppressing excitations as a function of the quench rate \( v \) for different system sizes following the paradigm of Kibble-Zurek mechanism [17].

Figure 2 summarizes the results with the simplest possible choice of a subclass of the proposed variational ansatz in Eq. (3) of the form

\[
\hat{H}_{\text{aux}}^{[2,2]}(t) = \sum_{i_1, i_2} h_{i_1i_2}^{[2]}(t) \sigma_{i_1}^y \otimes \sigma_{i_2}^z ,
\]

motivated by our observation that for \( R = 2 \) other permutation of spin components do not contribute to the
suppression of excitations. We take the $H_{\text{CD}} = H_0$ case as reference, where $n_{\text{ex}}$ increases monotonically until reaching saturation due to finite-size effects [38–40]. It is shown that a variational counterdiabatic ansatz restricted to only two-body terms though with an interaction range extending through the whole chain, i.e., $R_{\text{max}} = N - 1$, leads to a successful suppression of $n_{\text{ex}}$ by several orders of magnitude. Here the system is evolved in time from an initial value of the quench parameter $B_0 < B(t_c)$ to a final one deep in the ferromagnetic phase where the value of $n_{\text{ex}}$ in the plot is collected. The results therefore suggest a high-fidelity adiabatic passage through the QCP of the model at $t_c = (B_0 - 1)/v$. The residual excitations can be further suppressed by higher-order multiple-body interactions, as shown in Fig. 2(b).

Shortcuts to adiabatic state preparation. — We demonstrate that the outlined variational counterdiabatic procedure can be used to achieve high-fidelity state preparation. We consider for this purpose a time-dependent modulation of the magnetic field which enforces $H_{\text{CD}} = H_0(\lambda(t))$ to vanish at the beginning and end of the driving scheme, i.e., $t = 0, \tau$. We are thus led to the boundary conditions $B(0) = B_0$, $B(\tau) = B_f$ and $\dot{B}(0) = B(\tau) = 0$. $B(\tau) = 0$ which are satisfied by a polynomial quench of the form $B(s) = B_0 + 3(B_f - B_0)s^2 - 2(B_f - B_0)s^3$ with $s \equiv t/\tau$. The use of the latter quench is further motivated by adiabatic perturbation theory [41]. Figure 3 illustrates the results for high-fidelity preparation of the ground state of the transverse Ising chain in Eq. (2) under such a quench and for various choices of $\mathcal{H}_{\text{aux}}$. We use the fidelity Eq. (9) to assess the quality of the state preparation procedure. With a full-range two-body interaction, the protocol leaves some room for improvement, as shown in Fig. 3(a), indicating the urge to employ higher-body terms for longer chain lengths. However, for higher-body interactions beyond $R = 2$ the computational complexity associated with the number of spins permutations and the dimensions of the matrices to be formed and inverted renders the numerical implementation of the variational procedure intractable. To circumvent such a practical challenge, we suggest to truncate over the range of the $R$-body ansatz by restricting it to manageable values of $R \ll R_{\text{max}}$. The latter truncation strategy is further motivated by the nearly tridiagonal structure of $h_{i_1,i_2}^{y,z}$ evident in the inset of Fig. 3(b) which shows remarkably that the dominant contributions to $\mathcal{H}_{\text{aux}}(y,z)$ consist of only short-range interactions. Figure 3(a) illustrates the success of the truncation scheme by demonstrating significant improvement in fidelity upon employing short-range three-body interactions. We point out the required number of distinct interactions that lead to a high-fidelity state preparation scales with $\frac{1}{2}(N^2)$, which is a polynomial in the size of the system $N$ of the leading order $O(N^2)$.

Conclusions. — We have shown how to engineer an experimentally realizable counterdiabatic control Hamiltonian for the fast driving of many-body spin systems that mimic adiabatic driving. Our approach combines ED with a variational principle to determine the optimal CD scheme with a restricted set of control fields and leads to a suppression of the DoE by several orders of magnitude with respect to the uncontrolled driving dynamics. The computational complexity associated with the implementation of our variational CD scheme grows only polynomial...
mially in spite of the exponential growth of the size of the many-body Hilbert space with system size. Our proposal is ideally suited for digital quantum simulation as well as tailoring the nonequilibrium thermodynamics of many-body systems [42]. It further supplements previous adiabatic tracking schemes aimed at accessing highly excited states [43]. We close by pointing out that the presented framework can be applied as well to continuous variable systems without the restriction to self-similar processes that currently constraints applications of CD in these systems [13–15].

We acknowledge stimulating discussions with Daniel Lidar, Marek M. Rams, and Alexey Gorshkov. H.S. is grateful to Aarhus University for support and hospitality. This work was financed by the European Social Fund and the state budget of the Czech Republic, project CZ.1.07/2.3.00/30.0041. This research is further supported by the U.S. Department of Energy through the LANL/LDRD Program and a LANL J. Robert Oppenheimer fellowship (AD).

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