Growth and percolation of defect motifs in anisotropic packing problems

Zhaoyu Xie and Timothy J. Atherton

Department of Physics & Astronomy, Tufts University, 574 Boston Ave, Medford MA 02155

We examine the regime between crystalline and amorphous packings of anisotropic objects on surfaces of different genus by continuously varying their size distribution or shape from monodispersed spheres to bidispersed mixtures or monodispersed ellipsoidal particles; we also consider an anisotropic variant of the Thomson problem with a mixture of charges. With increasing anisotropy, we first observe the disruption of translational order with an intermediate orientationally ordered hexatic phase as proposed by Nelson, Rubinstein and Spaepen, and then a transition to amorphous state. By analyzing the structure of the disclination motifs induced, we show that the hexatic-amorphous transition is caused by the growth and connection of disclination grain boundaries, suggesting this transition lies in the percolation universality class in the scenarios considered.

I. INTRODUCTION

Packing problems, where a set of objects are arranged in a specified container to optimize the density, are an important model of many materials including granular media, colloids and amorphous solids[1, 2]. In two-dimensional unbounded Euclidean space, the highest-density packing of disks is the hexagonal lattice where each particle is surrounded by six neighbors. Numerous situations where this highly regular crystalline arrangement becomes disordered have been explored: if the boundaries of the container are not commensurate with the lattice[3, 4], or if the packing occurs on a curved surface so that the lattice is incompatible with the curvature[5–8], or if the particles are no longer circular and equal in size[9–12], then the overall arrangement may lose either translational or orientational order, or both.

Topological defects, deviations from crystalline order that cannot be removed by continuous deformations, are an invaluable concept to understand the resulting packings. The elementary defect in a hexagonal lattice is a disclination, a site that possesses a coordination number $n \neq 6$; these tend to disrupt the orientational order as they promote rotation of the lattice vectors. Interactions between disclinations are analogous to electrostatics, motivating the definition of a topological charge $q = (6 - n)$. Other defect motifs that occur include dislocations, disclination dipoles, scars, chains of disclinations of alternating sign that are induced on curved surfaces to accommodate the curvature[9], pleats that are bound to the edge of an open curved manifold[7] and grain boundaries that separate uncorrelated regions of crystalline order[13].

Consider perturbing the crystalline packing of monodispersed disks of radius $r$ by replacing some fraction $\chi$ with a larger radius $R$ such that $R/r > 1$. We may define a dimensionless parameter, the bidispersity $b = (R - r)/(R + r) \in [0, 1]$, to describe the deviation from monodispersity. As $b$ increases, Nelson, Rubinstein and Spaepen (NRS)[3, 4] predict the following sequence: first dislocations appear introducing stacking faults that disrupt long range translational order. There then exists an intermediate hexatic phase that possesses either long range or power-law orientational order as the lattice vectors of adjacent patches of crystal remain correlated. Further increasing $b$ leads to an amorphous phase that lacks both translational and orientational order. The hexatic phase is a zero-temperature analog of the intermediate hexatic phase that mediates melting in the Kosterlitz-Thouless-Halperin-Nelson-Young theory[14–16].

On a curved surface, such as a sphere, the NRS picture must be modified because defects are required even in the ground state, leading to a regime referred to as spherical crystallography[11, 15, 16], and vector transport properties of the curved surface complicates the measurement of long-range orientational correlations[17, 19]. Isolated disclinations occur for a small number of particles $N$ while for large $N$ these become spatially extended scars trading off the free energy cost of creating additional defects in order to reduce deformation of the lattice[5, 6]. On spheres, these structures are icosahedrally ordered[13], while the distribution for less symmetric surfaces is driven by the distribution of Gaussian curvature[11, 16, 17, 20, 23].

We recently showed[24] for packing bidispersed spheres on a sphere that as the bidispersity is increased from zero, the defect motifs begin to elongate above a critical value of bidispersity $b = 0.08$, continue to grow and eventually form a connected structure; at the same time the orientational order parameter becomes increasingly short range. The hexatic-amorphous transition in this specific system may therefore be equivalently viewed as growth and percolation of the scars, providing a connection between the regimes of spherical crystallography and random close packing and, by leveraging the results of percolation theory[25, 26], successfully predicting the distribution and microstructure of the defects.

In this work, we demonstrate that the percolation mechanism occurs in the original NRS scenario, and for many other kinds of anisotropy that could be present. We examine: bidispersed mixtures on flat surface as considered by NRS, mixtures of identical elongated particles of varying aspect ratio $\lambda$, such as ellipsoids, and packings in curved spaces of different topology. We also consider
a system with long-range interactions, an anisotropic generalization of the Thomson problem [27–29], whereby mixtures of different charge with ratio $\rho = q_2/q_1$ are arranged to minimize the electrostatic energy. Henceforth, we shall unify all these measures of anisotropy by collectively defining a single parameter $a \in [0, 1]$, which depending on the system may be the bidispersity $(R - r) / (R + r)$, shape anisotropy $(\lambda - 1) / (\lambda + 1)$ or charge anisotropy $(q_2 - q_1) / (q_1 + q_2)$.

II. RESULTS & DISCUSSION

To do so, we generate packings of $N = 1000$ particles at $\chi = \frac{1}{2}$ using a Monte Carlo procedure inspired by the Lubachevsky-Stillinger algorithm [30, 31]: particles are initially randomly placed on a large surface, then diffuse both translationally and rotationally by Brownian motion and gradually reduce size of the surface. After reduction moves, gradient descent is performed on an objective function that penalizes overlaps. If overlaps cannot be removed, the algorithm backtracks and reduces the rate of reduction; the algorithm is halted when the...
Some necessary modifications to deal with anisotropic particles suggested in previous work. For the Thomson problem, all charges are initially set equal and a minimum is found by conjugate gradient descent; a fraction $\chi = \frac{1}{2}$ of charges are randomly selected and increased in magnitude; then the energy is minimized.

The packing fraction as a function of anisotropy $a$ for bidispersed spheres on the flat surface and ellipsoids on the surface of a sphere or a torus is displayed in Fig. 1A. For the bidispersed mixture, as $a$ increases, the packing fraction decreases initially due to the introduction of disorder and afterwards slowly increases because the small particles tend to fill in the gap between large particles, consistent with the research on spherical surfaces. On the contrary, for ellipsoids, when $a$ increases, the packing fraction also increases to balance the additional rotational degree of freedom then decreases due to exclusion-volume effects, in agreement with previous results.

Disclinations are identified by the following procedure: We first generate a Voronoi diagram that approximates the navigation map from a cloud of points generated to lie on the boundary of the particles; particles that possess a connected edge in this graph are identified as neighbors. From the resulting neighbor graph, we find the subgraph of defects, i.e. vertices that have connectivity other than 6.

In Fig. 1B we show how the fraction of defects $p$ varies as a function of the relevant anisotropy parameter $a$, showing that though the detailed variation of $p$ depends on the particular scenario considered, these have a similar functional form: As $a \to 0$, $p$ is small and constant. Above a certain value of $a$, $p$ begins to increase rapidly and eventually saturates. The value of anisotropy at which defect clusters begin to form, and the ultimate value of $p$, varies between the scenarios considered; ellipsoidal packings saturate at a significantly lower $p$ than for the isotropically shaped particles; increasing the charge ratios in Thomson problem can achieve a larger $p$ than elongating particles. In Fig. 1C, we also display the growth of a spanning structure indicated by the fraction of scar length of the largest connected defect subgraph among all defects for various extent of anisotropy $a$ in different systems. The fraction quickly rises to 1 as we increase the anisotropy, indicating the percolation transition occurs with the formation of a globally connected cluster. Also note that the formation of a spanning structure is slower by varying the shape of particles than by adding bidispersity.

Representative packings as a function of anisotropy $a$ and their corresponding defect subgraphs are shown in Fig. 1D and E. For monodispersed particles, the packings are crystalline as expected. On flat surface, a few isolated defect motifs are typically present because the lattice may be incommensurate with the periodic boundary conditions. In curved space the scars of spherical crystallography occur together with a number of dislocations. While defects are not topologically required on the torus, because the genus is 1 and the Euler characteristic $\chi = 0$, the higher curvature present locally tends to promote longer scars and star motifs. As the degree of anisotropy is increased, the size of the defect motifs increases for all cases, and, eventually, a system spanning structure emerges.

As the degree of anisotropy and defect motifs increase in size, the system goes from crystalline phase into the amorphous phase, as indicated by various structural order parameters. The translational order can be examined by the pair correlation function.

The packing fraction as a function of anisotropy $a$ for bidispersed packings on flat surfaces and ellipsoidal particles on the sphere. On flat surface, the system goes from crystalline phase into the amorphous phase, as indicated by various structural order parameters. The translational order can be examined by the pair correlation function.

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shape and converges toward \( \theta(p - p_c) \) as the number of particles is increased. The value of \( p_c \) may therefore be extrapolated from a sequence of simulations of different size. An alternative approach is to study the cluster size \( R \) as a function of \( p \), which saturates at \( p_R(N) \) as \( R \) approaches the system size. The saturation point converges on \( p_c \) as \( N \rightarrow \infty \). These different definitions need not necessarily coincide in finite systems

Further, in contrast to the canonical site and bond percolation problems where \( p \) is a parameter that can be directly varied—in these problems \( p \) is the fraction of sites or bonds chosen on a specified lattice—here it is the degree of anisotropy that is varied. The neighbor graph and defect subgraphs are not known ahead of time and must be determined from the packing. We therefore identify \( p \) as the fraction of sites that lie in the defect subgraph.

Now we turn to the growth and structure of the clusters. In each of scenarios considered, we create a zero anisotropy packing—which is of course crystalline—and compute its neighbor graph. We then study the site percolation problem on this graph, where we randomly select \( p \) fraction of sites iteratively and investigate their structure for every trial. The fraction of trials where the selected sites connect into a spanning structure is counted and showed as gray lines in left column of Fig. 3. The fraction becomes nonzero at around \( p_c = 0.5 \) on flat surface and \( p_c = 0.55 \) on spherical or toroidal surface, which is in good agreement with previous literature. We also compute the fraction of sites in the largest connected component, displayed as gray lines in the middle column of Fig. 3. In addition the structures of unselected sites are also computed and the corresponding results are displayed as black lines, which has the mirror symmetry compared with those of the selected sites as expected. The largest cluster radius \( R \) and fractal dimension of the selected sites are shown as a function of \( p \) in right column of Fig. 3. The radius saturates at the percolation transition mentioned above where the fractal dimension is consistent with the universal value of 1.896, marked by the dash lines.

We further compare the structure of the defect subgraphs with this percolation model where the non-hexatic defects are recognized as the selected sites and the hexatic particles are treated as unselected sites. We compute the fraction of packings whose defects forms a global spanning structure, the fraction of sites in the largest defect subgraphs, the largest radius of the defect subgraphs and fractal dimension. The corresponding quantities are overlaid onto the percolation model as points in Fig. 3. The agreement, particularly around the percolation point, is very good indeed, showing that the growth and structure of the clusters are well predicted by percolation theory.

increasing size, and above a critical value \( p_c \), the mean cluster size diverges for infinite lattices. The value of \( p_c \) depends on details of the particular system, but in the vicinity of \( p \rightarrow p_c \), the cluster size distribution and structure exhibit universal behavior: For example, the clusters become fractalline and the cluster radius \( R \),

\[
R^2 = \frac{1}{2} \sum_{i \neq j} \frac{d_{ij}}{n^2}
\]

where \( d_{ij} \) is the distance between pairs of sites \( (i, j) \) and \( n \) is the number of sites, scales with the number of sites like \( n \propto R^D \) where \( D \) is the fractal dimension. In two dimensions this has a value at \( p_c \) of 91/48 = 1.896 independent of the structure of the system. Many disordered systems, both discrete and continuous, lie in this class, including forest fires, distribution of oil inside porous rock, the diffusion of atoms and conductivity of electrical networks.

An important feature of the packing problems considered is that they involve a finite number of particles, either for reasons of tractability or because they occur in compact geometries. In finite systems, the percolation transition becomes second order. For example, the fraction of simulations \( u(p) \) that yield a globally connected cluster as a function of \( p \) is, in an infinite system, the unit step function \( \theta(p - p_c) \) centered on the percolation point. At finite \( N \), \( u(p) \) become sigmoidal in

Figure 2. (A) Pair correlation function \( g(r) \) of different anisotropy for bidispersed packings on flat surface. (B) Bond orientational correlation function \( G_b(r) \) for bidispersed packings on flat surface. (C) Pair correlation function \( g(\theta) \) of different anisotropy for ellipsoidal packings on spherical surface. (D) Bond orientational correlation function \( G_b(\theta) \) of different anisotropy for ellipsoidal packings on spherical surface. The numbers mark the values of anisotropy.
Figure 3. (A) For bidispersed packings on flat surface, fraction of packings with connected spanning structure(left), fraction of particles in the largest connected component(middle), cluster radius of the largest connected component and fractal dimensions(right) as a function of defect fraction. (B)—(D) show corresponding figures for ellipsoid packings on sphere, ellipsoid packings on torus and generalized Thomson problem.

III. CONCLUSIONS

In this work, we have considered the emergence and growth of defect structures as a function of different kinds of anisotropy. Besides bidispersity on flat and curved surfaces that link our work to the well-explored KTHNY transition, we demonstrate that elongating particles, or changing the nature of the interaction have a similar effect: anisotropy induces dislocations that cause the system to successively lose translational and orientational order, during which process the newly generated dislocations gradually form a globally connected cluster. We have further shown that structural features of the clusters, e.g. fractal dimension, are well predicted by the percolation model.

Our results suggest an apparent universality in that the defect structures that emerge when adding anisotropy to a crystalline system appear to be independent of the source of anisotropy. Bidispersity on the flat surface or on the surface of a sphere yield similar results to elongating particles or soft long-range interactions, or to packings on the surface of a torus. Intriguingly, our results for ellipsoidal particles show that the defect fraction only just exceeds the percolation threshold: we speculate that
there may exist kinds of anisotropy that do not yield percolating defects and hence suppress the amorphous phase.

IV. METHODS

Packing algorithm for ellipsoidal particles

Our algorithm to pack spherical particles is as described above and in previous work\cite{21, 52, 33}. The extension to ellipsoidal particles involves modifications to overlap detection and diffusion as follows: The contact distance $\sigma_{ij}$, defined as the distance between the centers of two ellipsoids when they are externally tangent, is given by,

$$\sigma_{ij} = 2b/\sqrt{1 - \frac{\chi}{2} \left( \frac{d_{ij} \cdot \hat{u}_i + d_{ij} \cdot \hat{u}_j}{1 + \chi(\hat{u}_i \cdot \hat{u}_j)} \right)} + \frac{b^2}{1 - \chi(\hat{u}_i \cdot \hat{u}_j)},$$

where $a$ and $b$ are the half-lengths of the major and minor axes, $\chi = (a^2 - b^2)/(a^2 + b^2)$, $\hat{u}$ is the unit vector describing the overall orientation of the particle and $d_{ij}$ is the unit vector pointing from one center to the other. The center-to-center distance $d_{ij}$ is smaller than the contact distance $\sigma_{ij}$, there is overlap. This criterion has been successfully implemented in other work\cite{55, 56}. If two ellipsoidal particles overlap, we exert the Gaussian model potential\cite{59}

$$V(u_i, u_j, d_{ij}) = \epsilon_0 \left[ 1 - \chi^2(u_i \cdot \hat{u}_j)^2 \right]^{-1/2} \exp(-d_{ij}^2/\sigma_{ij}^2),$$

where $\epsilon_0$ is the strength parameter, to remove the overlaps by gradient descent.

Diffusion of particles is another feature of our algorithm. Spheres can diffuse simply by Langevin equation

$$\mathbf{x}_i(t + \delta t) = \mathbf{x}_i(t) + \eta_i \sqrt{2D_0 \delta t},$$

where $\eta_i$ is a random step drawn from Gaussian distribution, $D$ is the diffusion constant such that $\sqrt{2D_0 \delta t}$ determines the variance of the displacement for a timestep $\delta t$. For ellipsoids, we must also account for rotations. First we rotate the director of an ellipsoid by

$$\delta \theta(\delta t) = \eta_0 \sqrt{2D_0 \delta t}.$$

Then in the local coordination system $\bar{x}$ and $\bar{y}$ along the major and minor axes, it is displaced by

$$\delta r(\delta t) = \eta_a \sqrt{2D_a \delta t \bar{x}} + \eta_b \sqrt{2D_b \delta t \bar{y}}.$$

Finally we transform this local displacement into the global coordinate system by multiplying the rotation matrix\cite{60}.

Orientational correlation on spherical surfaces

Here we describe how to find the icosahedron that align with the defects following the method previously reported\cite{13}. A non-trivial icosahedrally symmetric function can be defined as

$$h_6(x) = Y_{6,0}(x) + \sqrt{\frac{\gamma}{11}(Y_{6,-5}(x) - Y_{6,5}(x))},$$

where $Y$ is the spherical harmonics. The positions of local max values align with the vertices of an icosahedron. The defects on the spherical surface can be rotated by the rotational matrix

$$R_6(\theta, \phi) = \begin{pmatrix}
\cos \theta & 0 & \sin \theta \\
0 & 1 & 0 \\
-\sin \theta & 0 & \cos \theta
\end{pmatrix} \begin{pmatrix}
\cos \phi & -\sin \phi & 0 \\
\sin \phi & \cos \phi & 0 \\
0 & 0 & 1
\end{pmatrix}.$$

Then $(\theta, \phi)$ is computed by minimizing $\sum_i \eta_i (R_6(\theta, \phi) \cdot \bar{r}_i)$ where $\bar{r}_i$ represents coordinates of defects. The inverse rotation gives the icosahedron that align with the defects.

CONFLICTS OF INTEREST

There are no conflicts to declare.

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1. M. J. Bowick and L. Giomi, Advances in Physics, 2009, 58, 449–563.
2. S. Torquato and F. H. Stillinger, Reviews of modern physics, 2010, 82, 2633.
3. D. R. Nelson, M. Rubinstein and F. Spaepen, Philosophical Magazine A, 1982, 46, 105–126.
4. M. Rubinstein and D. R. Nelson, Physical Review B, 1982, 26, 6254.
5. A. Bausch, M. Bowick, A. Cacciuto, A. Dinsmore, M. Hsu, D. Nelson, M. Nikolaides, A. Travesset and D. Weitz, Science, 2003, 299, 1716–1718.
6. M. J. Bowick, D. R. Nelson and A. Travesset, Physical Review B, 2000, 62, 8738.
7. W. T. Irvine, V. Vitelli and P. M. Chaikin, Nature, 2010, 468, 947–951.
8. F. L. Jiménez, N. Stoop, R. Lagrange, J. Dunkel and P. M. Reis, Physical review letters, 2016, 116, 104301.
9. S. Williams and A. Philipse, Physical Review E, 2003, 67, 051301.
10. A. Donev, I. Cisse, D. Sachs, E. A. Variano, F. H. Stillinger, R. Connelly, S. Torquato and P. M. Chaikin, Science, 2004, 303, 990–993.
