Storage of polarization-encoded cluster states in an atomic system

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(Dated: May 15, 2009)

We present a scheme for entanglement macroscopic atomic ensembles which are four spatially separate regions of an atomic cloud using cluster-correlated beams. We show that the cluster-type polarization-encoded entanglement could be mapped onto the long-lived collective ground state of the atomic ensembles, and the stored entanglement could be retrieved based on the technique of electromagnetically induced transparency. We also discuss the efficiency of, the lifetime of, and some quantitative restrictions to the proposed quantum memory.

PACS numbers: 03.67.-a, 42.50.Gy, 42.50.Dv

I. INTRODUCTION

Multiparticle graph states, so-called cluster states [1], have attracted much attention for its potential applications as a basic resource in the “one-way” quantum computing scheme [2, 3]. The cluster state encoded by the polarization states of photons has been demonstrated experimentally [4, 5, 6, 7, 8, 9, 10, 11]. Meanwhile, the combination of optical techniques with quantum memory using atoms has shown apparent usefulness to scalable multi-optical quantum computing network [12] and long-distance quantum communication [13]. Several experiments in these aspects have been realized, such as the storage and retrieval of coherent states [14], single-photon wave packets [15, 16], and squeezed vacuum state [17, 18]. With these achievements, it is worth initiating a study on a reversible memory for a cluster state.

As we know, light is a natural carrier of classical and quantum information. Now, the macroscopic atomic system can be efficiently used for its storage due to long lived electronic ground-state as storage units. Based on the light and atomic ensembles, two types of quantum memories have been put forward: one based on the quantum Faraday effect supplemented by measurement and feedback [14], and the other involving electromagnetically induced transparency (EIT) [19, 20] and Raman processes [21, 22]. In addition, photon echo technique has also been proposed for quantum memory [23]. EIT is probably the most actively studied technique to achieve a quantum memory. EIT polaritons (dark state) were first considered theoretically by Mazets and Matisov [24], and later by Fleischhauer and Lukin [20, 25] who suggested storing a probe pulse (stopping the polariton) by adiabatically switching off the control laser. Extending the analysis to a double-Λ system [26, 27, 28], it is possible to simultaneously propagate and store two optical pulses in the medium based on a dark state polariton (DSP) consisting of low-lying atomic excitations and photon states of two frequencies. The existence of the DSP in the double-Λ atomic system studied by Chong et al. [28] required that the fields obey certain conditions for frequency, amplitude, and phase matchings. Quantitative relations in the case of double-Λ type atoms are essentially more complicated than for the standard Λ configuration. If one of the conditions breaks, such as the phase is mismatched, then one of the two pulses will be absorbed and lost [29]. In this sense, the double-Λ system is limited to the storage of two or more optical pulses. Recently, Yoshikawa et al. [30] demonstrated holographic storage of two coherence gratings in a single BEC cloud. Choi et al. [31] demonstrated that the entanglement of two light fields survives quantum storage using two atomic ensembles in a cold cloud, where they realized the coherent conversion of photonic entanglement into and out of a quantum memory. Hence, in principle, it is now achievable to store more than two optical fields in atomic ensembles for different purposes.

In this paper, we propose a scheme that can store a polarization-encoded cluster state reversibly in a cold atomic cloud based on the EIT technique. To our best knowledge, the storage of polarization entangled state is very useful in polarization-encoded quantum computing schemes, such as “one-way” quantum computing and quantum coding. On the other hand, our scheme also presents a natural extension of existing work [31, 32].

Our paper is organized as follows. In Sec. II we describe how the polarization-encoded cluster state can be stored and retrieved, and the method of the measurement and verification of entanglement storage. In Sec. III we analyze and evaluate the efficiency and the fidelity. In Sec. IV we evaluate the memory lifetime. In Sec. V we discuss some restrictions of the proposed quantum memory. Finally, we conclude with a summary of our results.

II. STORAGE OF POLARIZATION-ENCODED CLUSTER STATE

In this section, we show the EIT technique can be used to realize a reversible memory for the polarization encoded cluster state [1]

$$|\phi_{in}\rangle = \frac{1}{2}(|H_1\rangle|H_2\rangle|H_3\rangle|H_4\rangle + |V_1\rangle|V_2\rangle|H_3\rangle|H_4\rangle + |H_1\rangle|H_2\rangle|V_3\rangle|V_4\rangle - |V_1\rangle|V_2\rangle|V_3\rangle|V_4\rangle,$$  (1)
where $|H\rangle$ and $|V\rangle$ stand for the single-photon states with the photon polarized horizontally and vertically, respectively. The four-photon cluster state shown above has been demonstrated experimentally \cite{25, 33, 35}, using different methods. Without loss of generality, here we consider the simple case that the frequencies of all four photons in the state are degenerate. The schematic diagrams of the proposed experimental system are shown in Fig. 1(a) (hereafter noted case 1) and Fig. 2(a) (hereafter noted case 2), where four atomic sub-ensembles (or channels) are used. These four sub-ensembles form four equivalent channels, each of which is used to store the corresponding polarization-encoded single-photon state ($|H\rangle_i$ or $|V\rangle_i$; $i = 1, 2, 3, 4$) in the four-photon cluster state.

![Diagram](image)

**FIG. 1:** (Color online) (a) Schematic diagram of the proposed experiment. The polarization-encoded four-photon cluster state is inputted with a common perpendicularly-propagating control field $E_c$. Four atomic sub-ensembles (or channels) are represented by four spatially separate and symmetric regions in a single cloud of cold atoms. The cold atomic cloud is initially prepared in a magneto-optical trap (MOT) and the MOT fields are turned off during the period of the storage and retrieval process. The quantization axis $z$ is set by the trapping magnetic field $\vec{B}$ in the preparation of the cold atomic cloud. The horizontally and vertically polarized single-photon pulses pass through $\lambda/4$ plates and are converted into circularly left- and right-polarized photons, respectively, i.e., $|H\rangle \rightarrow |\sigma^-\rangle$, and $|V\rangle \rightarrow |\sigma^+\rangle$. (b) The atomic level configuration for the proposal, $E_c$ is the control light with $\pi$-polarization, and $E_{pL}$ and $E_{pR}$ are left and right circularly-polarized lights, respectively. (c) The corresponding decompositions of atom-photon couplings in (b) according to the photon polarizations (I for $\sigma^-$ polarization, II for $\sigma^+$ polarization).

A. Quantum memory for a polarization-encoded single-photon state

![Diagram](image)

**FIG. 2:** (Color online) (a) The figure is the same as Fig. 1(a) except that a common collinearly-propagating control field $E_c$ is used. (b) and (c) are the same as Figs. 1(b) and 1(c), respectively, except the control field $E_c$ is now replaced with right circularly-polarized light in the collinearly-propagating case. The right circularly-polarized light $E_c$ is converted by $\lambda/4$ plates using the vertically polarized light $E_c$.

In order to understand the physics behind the schemes shown in Figs. 1 and 2, first we discuss the quantum memory for a polarization-encoded single-photon state. An atomic ensemble containing $N$ atoms for memory using the $\Lambda$-type atomic level configuration with the excited state $|\alpha\rangle$ and ground states $|\beta\rangle$ and $|\gamma\rangle$ based on EIT was studied in detail by Fleischauer and Lukin \cite{20} and also reviewed by a few authors, such as Petrosyan \cite{33}. Here, we focus on the case of a single-photon probe field with horizontal or vertical polarization and describe the dark state of the system. In the frame rotating with the probe and the driving field frequencies, the interaction Hamiltonian is given by \cite{20, 25, 33}

$$\hat{H} = \hbar \sum_{j=1}^{N} \left[ -g \hat{a}^\dagger_{ab} \hat{\epsilon}(z_j) e^{i k_{pb} z_j} - \Omega_{c,j}(t) \hat{a}^\dagger_{ac} e^{i k_{pz} z_j} + \text{H.c.} \right],$$

where $\hat{a}_{\mu \nu} = |\mu\rangle_j \langle \nu|_j$ is the transition operator of the $j$th atom between states $|\mu\rangle$ and $|\nu\rangle$, and we consider the single- and two-photon resonance cases. $g = \sqrt{\epsilon} / \sqrt{2 \hbar c V}$ is the coupling constant between the atoms and the quantized field mode which for simplicity is assumed to be equal for all atoms. $k_{pb}$ and $k_{pz} = k_{pc} \cdot \hat{\epsilon}_z$ are the wave vectors of the probe field and the control field along the propagation axis $z$, respectively. The traveling-wave quantum field operator $\hat{\epsilon}(z, t) = \sum_q \hat{\epsilon}_q(t) e^{i q z}$ is expressed through the superposition of bosonic operators.
\[ \hat{a}_q(t) \] for the longitudinal field modes \( q \) with wavevectors \( k + q \), where the quantization bandwidth \( \delta q \) \( q \in \{-\delta q/2, \delta q/2\} \) is narrow and is restricted by the width of the EIT window \( \Delta \omega_{tr} \) \( \delta q \leq \Delta \omega_{tr}/c \) [34].

Hamiltonian [2] has a family of dark eigen-states \( |D_1^\alpha\rangle \) with zero eigenvalue \( \hat{H}|D_1^\alpha\rangle = 0 \), which are decoupled from the rapidly decaying excited state \( |a\rangle \). For a single-photon probe field \( n = 1 \), the dark eigen-states \( |D_1^\alpha\rangle \) are given by

\[
|D_1^\alpha\rangle = \cos \theta |1^0\rangle |c(0)\rangle - \sin \theta |0^0\rangle |c(1)\rangle,
\]

where \( \theta = \theta(t) \) is the mixing angle and \( \tan \frac{\theta}{2} = \frac{|\langle n^q\rangle|}{\sqrt{2}} \) and \( \theta \) is independent of the mode \( q \). \( |n^q\rangle \) denotes the state of the quantum field with \( n \) photons in mode \( q \), and \( |c(0)\rangle \) is a symmetric Dicke-type state of the atomic ensemble with \( n \) Raman (spin) excitations, i.e., atoms in state \( |c\rangle \), defined as

\[
|c(0)\rangle = |b_1, ..., b_N\rangle,
\]

\[
|c(1)\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{i(k+q-k)^j} \{ |b_1, b_2, ..., c_j, ..., b_N\rangle \}.
\]

Now, we consider a memory for a single photon with horizontal or vertical polarization. By the \( \lambda/4 \) plate, its polarization is converted into the left circular or right circular polarization. Due to the propagating directions of the probe field and the control field, the corresponding five-level structures are shown in Fig. [1b] for case 1 and Fig. [2b] for case 2 instead of the simple \( \Lambda \)-type. Due to the propagating directions of the probe field and the control field, the corresponding five-level structures are shown in Fig. [1b] for case 1 and Fig. [2b] for case 2 instead of the simple \( \Lambda \)-type. The excited states \( |b_\alpha\rangle \) and \( |c_\alpha\rangle \) correspond to the \( m_F = 0 \) sublevels of the \( F = 1 \), and the states \( |e_+\rangle \) and \( |e_-\rangle \) correspond to the \( m_F = 1 \) and \( m_F = -1 \) sublevels of the \( F = 2 \), respectively. The excited states \( |a_\alpha\rangle \) correspond to the \( m_F = -1 \) and \( m_F = 1 \) sublevels of the \( F = 2 \), respectively. The excited states \( |a_\alpha\rangle \) correspond to the \( m_F = -1 \) and \( m_F = 1 \) sublevels of the \( F = 2 \), respectively. The excited states \( |a_\alpha\rangle \) correspond to the \( m_F = -1 \) and \( m_F = 1 \) sublevels of the \( F = 2 \), respectively.

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or \(|b⟩ \rightarrow |a⁺⟩\), and their group velocities are strongly modified by the control field \(E_c\). The interaction of the different-polarization single-photon probe fields with four atomic sub-ensembles separates into two classes of \(A\)-type EIT, which is illustrated in Figs. 1(c) and 2(c). Each probe field is converted into corresponding single-excitation polariton \(ψ_j (j = 1, 2, 3, 4)\) representing a coupled excitation of the field and atomic coherence, and each polariton \(ψ_j\) is described by polariton \(Ψ_j\) or \(Ψ_{1j}\).

The corresponding transverse section of the four atomic sub-ensembles is shown in Fig. 3. By switching off the control field adiabatically, these coupled excitations are converted into the spin wave excitations with a dominant DSP component, i.e., the cluster state is stored. After a storage period \(τ\), the stored field state can be retrieved by turning on \(E_c\) adiabatically.

Once the four single-photons completely enter the EIT medium, under the adiabatic condition, the state of atomic ensembles \(A, B, C\) and \(D\) will adiabatically follow the specific eigen states of the Hamiltonian (dark states). Then the dark states of the system are the direct products of those corresponding to the subsystems \(A, B, C\) and \(D\), and the system state vector \(|Φ(t)⟩\) is given by

\[
|Φ(t)⟩ = \begin{vmatrix}
|D_{1A}⟩ A |D_{2B}⟩ B |D_{3C}⟩ C |D_{4D}⟩ D \\
\end{vmatrix} \\
= \begin{vmatrix}
\cos \theta_1 |1q_1⟩ |c(0)_A⟩ A - \sin \theta_1 |0q_1⟩ |c(1)_A⟩ A \\
\cos \theta_2 |1q_2⟩ |c(0)_B⟩ B - \sin \theta_2 |0q_2⟩ |c(1)_B⟩ B \\
\cos \theta_3 |1q_3⟩ |c(0)_C⟩ C - \sin \theta_3 |0q_3⟩ |c(1)_C⟩ C \\
\cos \theta_4 |1q_4⟩ |c(0)_D⟩ D - \sin \theta_4 |0q_4⟩ |c(1)_D⟩ D \\
\end{vmatrix} \\
= \cos \theta_1 \cos \theta_2 \cos \theta_3 \cos \theta_4 |1q_1⟩ A |1q_2⟩ B |1q_3⟩ C |1q_4⟩ D + \cdots \cos \theta_1 \cos \theta_2 \cos \theta_3 \sin \theta_4 |0q_1⟩ A |1q_2⟩ B |1q_3⟩ C |1q_4⟩ D \\
+ \sin \theta_1 \sin \theta_2 \sin \theta_3 \sin \theta_4 |1q_1⟩ A |0q_2⟩ B |0q_3⟩ C |1q_4⟩ D \\
+ \cos \theta_1 \sin \theta_2 \cos \theta_3 |0q_1⟩ A |1q_2⟩ B |1q_3⟩ C |1q_4⟩ D \cdot (8)
\]

When the control field \(E_c\) is adiabatically switched off \((θ_i = π/2, i = 1, 2, 3, 4)\), the state of the photonic component of the four pulses is homogeneously coherently mapped onto the collectively atomic excitations

\[
|1q_1⟩ A |1q_2⟩ B |1q_3⟩ C |1q_4⟩ D \rightarrow |c(1)_A⟩ A |c(1)_B⟩ B |c(1)_C⟩ C |c(1)_D⟩ D. (9)
\]

According to the polarizations of the input four single photons, from Eq. 10 we have the following one-to-one mappings:

\[
\begin{align*}
|H_1⟩ & \rightarrow |c(1)_A⟩ A \\
|H_2⟩ & \rightarrow |c(1)_B⟩ B \\
|H_3⟩ & \rightarrow |c(1)_C⟩ C \\
|H_4⟩ & \rightarrow |c(1)_D⟩ D
\end{align*}
\]

\[
\begin{align*}
|V_1⟩ & \rightarrow |c_++⟩ A \\
|V_2⟩ & \rightarrow |c_+⟩ B \\
|V_3⟩ & \rightarrow |c_+⟩ C \\
|V_4⟩ & \rightarrow |c_+⟩ D
\end{align*}
\]

Hence, the state \(|ψ⟩_{atom}\) of four atomic sub-ensembles \((A, B, C\) and \(D\)) will depend on the polarization of the input photons. When the input state is a polarization-encoded cluster state, after adiabatically turning off the control field, the state of the four atomic ensembles is a cluster-type state:

\[
|ψ⟩_{atom} = \frac{1}{2} \left[ |c_−⟩ A |c(1)_B⟩ B |c(1)_C⟩ C |c_−⟩ D + |c_+⟩ A |c(1)_B⟩ B |c(1)_C⟩ C |c_+⟩ D \right] \\
+ \left[ |c_−⟩ C |c(1)_B⟩ B |c(1)_C⟩ C |c_−⟩ D + |c_+⟩ C |c(1)_B⟩ B |c(1)_C⟩ C |c_+⟩ D \right]. (11)
\]

That is to say that the entangled photon state \(|φ_{in}\rangle\) is coherently mapped to the entangled atomic state \(|ψ⟩_{atom}\). At a later time, the entangled photon state can be retrieved on demand from the entangled atomic state by turning on \(E_c\) \((θ_i = 0, i = 1, 2, 3, 4)\). After passing through the \(λ/4\) plates again, the retrieval polarization-encoded cluster state is

\[
|φ_{out}\rangle = \frac{1}{2} \left[ |H_1⟩ A |H_2⟩ B |H_3⟩ C |H_4⟩ D + |V_1⟩ A |V_2⟩ B |V_3⟩ C |V_4⟩ D \right] \\
+ \left[ |H_1⟩ A |H_2⟩ B |V_3⟩ C |V_4⟩ D - |V_1⟩ A |V_2⟩ B |V_3⟩ C |V_4⟩ D \right]. (12)
\]

We describe an ideal transfer of a polarization encoded cluster between light fields and metastable states of atoms [38]. In the ideal case, the retrieved pulses are identical to the input pulses, provided that the same control power is used at the storage and the retrieval stages. However, to realize the ideal storage, two conditions must be met: (1) the whole pulse must spatially compressed into the atomic ensemble, and (2) all spectral components of the pulse must fit inside the EIT transparency window. In Secs. III and IV we consider the realistic parameters of the proposal of realization.

Next, we describe the measurement and verification processes of the retrieval cluster state. Recently, Enk et al. [40] discussed a number of different entanglement-verification protocols: teleportation, violation Bell-Clauser-Horne-Shimony-Holt (CHSH) inequalities, quantum state tomography, entanglement witnesses [41], and direct measurements of entanglement. As for the four-qubit cluster state, the entanglement is verified by measuring the entanglement witness \(W\). The expectation value of \(W\) is positive for any separable state, whereas its negative value detects four-party entanglement close to the cluster state. The theoretically optimal expectation value of \(W\) is \(\text{Tr}(W_{\text{theory}}) = −1\) for the cluster state [41].

III. ANALYSIS OF EFFICIENCY AND FIDELITY

In this section, we analyze the efficiency and the fidelity of the memory. The memory efficiency is defined as the ratio of the number of retrieved photons to the number of incident photons [42, 43, 44, 45]:

\[
η = \int_{T+\tau}^\infty |\hat{E}_{out}(t)|^2 dt / \int_0^T |\hat{E}_{in}(t)|^2 dt, (13)
\]
where $\tau$ is a storage period. Recently, several proposals are presented \cite{42, 43, 45} for the optimal efficiency of light storage and retrieval under the restrictions and limitations of a given system. Based on these proposals, two optimal protocols have been demonstrated experimentally \cite{46, 47}. The first protocol iteratively optimizes the input pulse shape for any given control field \cite{46}, while the second protocol uses optimal control fields calculated for any given input pulse shape \cite{47}. As for the cluster storage situation, it is difficult to shape the input signal pulses. Then the second protocol \cite{44, 47} could be used to improve the efficiency of storage and retrieval of given signal pulses.

Using the method introduced by Gorshkov et al. \cite{43}, we plot the optimal efficiency $\eta$ of any one field of the four input fields due to the four fields are theoretically equivalent. When considering the spin wave decay, one should just multiply the efficiency by $\exp(-2\gamma_s\tau)$ \cite{43} and the efficiency will decrease. Figure 4 shows the optimal efficiency $\eta$ versus the optical depth $d$ under different spin wave decays $\gamma_s$. It is desirable to read out as fast as possible due to the spin wave decay ($\tau \lesssim 1/\gamma_s$).

Next, we analyze the fidelity of memory. For the density matrices $\rho_{in}$ and $\rho_{out}$ of the input and output quantum state, the fidelity is defined as $F$ \cite{48, 49}:

$$F = \langle \rho_{in} | \rho_{exp} | \rho_{in} \rangle^{1/2} = \langle \rho_{out} | \rho_{exp} | \rho_{out} \rangle^{1/2}$$.

(14)

For two pure states, their fidelity coincides with the overlap. This number is equal to one only for the ideal channel transmitting or storing the quantum state perfectly. We describe a method of the projector-based entanglement witness (Tr($\rho_{exp}$)) in Sec. II to verify the cluster state, which is also used to obtain information about the fidelity $F = \langle \phi_{in} | \rho_{exp} | \phi_{in} \rangle$ \cite{50, 51, 52} from the measurement process. The observed fidelity $F > 1/2$ assures that the retrieved state has genuine four-qubit entanglement \cite{11, 51}. The high-fidelity quantum memory is necessary for quantum information and quantum communication. Due to nonsimultaneous retrieval, the influence on the fidelity will be discussed in Sec. IV.

IV. LIFETIME OF QUANTUM MEMORY

In this section, we discuss the lifetime of the proposed memory. Quantum memories for storage and retrieval of quantum information are extremely sensitive to environmental influences, which limits their storage time. In many experiments of cold atoms in the MOT, the quadrupole magnetic field is the main source of the atomic memory decoherence \cite{53, 54}. Using the freely cold rubidium atoms released from MOT \cite{51}, the coherence time will be increased \cite{52} and the longest quantum memory time of the system reported to date is 32 $\mu$s \cite{51} without using the clock states, where the time is limited by dephasing of different Zeeman components in the residual magnetic field. Recently, using magnetic field insensitive states (clock states), the storage time of the quantum memory storing single excitation has improved to 1 ms \cite{53}. Zhao et al. \cite{54} also used the magnetically insensitive clock transition for the quantum memory, but they confined the cold rubidium atomic ensemble in a one-dimensional optical lattice, and the memory lifetime has improved and exceeded 6 ms.

In our scheme, all the light fields responsible for trapping and cooling, as well as the quadrupole magnetic field in the MOT, are shut off during the period of the storage and retrieval process; ballistic expansion of the freely falling gas provides a longer memory time limitation. Assuming one-dimensional case and Maxwell-Boltzmann velocity distribution $f(v) = \sqrt{M/2\pi k_B T} e^{-m v^2/(2k_B T)}$, the atomic mean speed is $\langle v \rangle = \sqrt{k_B T/M}$ where $k_B$ is the Boltzmann constant, $T$ is the temperature, and $M$ is atomic mass. After a storage period $\tau$, the collective state describing the spin wave $|c^{(1)}(t)\rangle$ evolves to

$$|c^{(1)}(t + \tau)\rangle = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} e^{i\Delta k(z_j + v \tau)} |b_1, b_2, ..., c_j, ..., b_N\rangle$$.

(15)

where $\Delta k = k_p - k_f^i + q$ the wavevector of the spin wave. For narrow transparency window $\Delta \omega_T$, $|q| \ll k$, the influence of field mode $q$ on the storage time can be ignored. The stored information due to atomic random motion begins to decrease, and the obtainable information by retrieval process is proportional to

$$R = \left| \langle c^{(1)}(t) | c^{(1)}(t + \tau) \rangle \right|^2 = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i\Delta k v j \tau} \right|^2$$

$$= \left| \int f(v) e^{i\Delta k v \tau} dv \right|^2 \approx \exp(-\frac{\tau^2}{\tau_s^2})$$.

(16)
The dephasing induced by atomic random motion can also be described by the grating wavelength of the spin wave:

\[ \lambda_s = \frac{2\pi}{\Delta k} \]  

(18)

The dephasing induced by atomic random motion can also be described by the grating wavelength of the spin wave.

In case 1 where the control field and the probe fields are propagating in orthogonal direction, \( \Delta k \approx k_p \), the spin wave grating wavelength is about light wavelength and the spin wave would dephase rapidly due to atomic motion, so \( \tau_s \approx 1.5 \mu s \). Considering the efficiency of memory, the storage time is less than 1 \( \mu s \).

In case 2 the Doppler-free configuration propagation, almost no photonic momentum is transferred to the atoms. The atomic coherence is localized in the longitudinal direction, in which oscillations have the small beat frequency \( \Delta \omega \approx \left( k_p - k_{\text{c}} \right) c = 6.8 \text{ GHz} \) between the probe and the control fields, and the calculated spin-wave wavelength by Eq. (18) is \( \lambda_s \approx 27 \text{ cm} \). The dephasing induced by the atomic motion in this localized region is very slowly due to the large spin-wave wavelength \( \lambda_s \), and the computed lifetime \( \tau_s \) is large. However atomic random motion would spread the localized excitation from one ensemble to another ensemble, which will result in the stored information being quickly lost. Considering the atoms flying out of the localized atomic ensemble and the waist of laser beams \( D = 100 \mu m \), the lifetime of the memory can be estimated as \( \tau = D/(2v) \approx 300 \mu s \).

Another factor that influences the memory time is the decoherence of the excited state. The DSP is protected against incoherent decay processes acting on the excited states because of adiabatical elimination of the excited states. Although the collective state \( |c^{(1)} \rangle \) is an entangled state of \( N \) atoms, its decoherence time is not much different from that of the quantum state stored in an individual atom and it is quite stable against one-atom (or few-atom) losses [56]. Due to the memory efficiency, the maximum storage time of our proposal must be far smaller than \( \tau_s \ (\tau < 1/\gamma_s) \), or else the efficiency will be low. From Fig. 4 in order to balance the efficiency and preserve the entanglement, suitable storage times for cases 1 and 2 are less than or equal to 0.15 and 30 \( \mu s \), respectively.

V. DISCUSSION

In this section, we discuss some restrictions. First, we assume that the input the four-photon cluster state \( |\phi_{\text{in}} \rangle \) generated by experiment [4, 5, 6, 7, 8, 9, 10, 11] has high fidelity. The four-photon cluster state should be sent into the atomic cloud simultaneously, which requires that the four incidence points are symmetric about major axis of ellipsoid. If there is a large difference, one single-photon probe field is not synchronously sent into the atomic ensemble with other fields. Then a fraction of single-photon wave packet, captured in the form of a spin wave, is stored for a time period \( \tau \). The efficiency of light storage will decrease, and the shape of the output pulse is different from the initial pulse. This case can be avoided using the perfect cluster state and choosing symmetric atomic ensembles. Second, in the retrieval process, we assume that one stored field is not simultaneously retrieved or is not retrieved even. If the four fields are retrieved non-simultaneously and only one field is retrieved with a little delay, the entanglement is preserved and the entanglement degree decreases [57]. If one field is retrieved with a certain probability, without loss of generality, we also choose field \( E_1 \). After the retrieval, the field \( E_1 \) can be written as \( |0_1 + \beta_1 |1_1 \rangle \rangle/\sqrt{1 + |\beta_1|^2} \), so the fidelity is \( F = |\langle \phi_{\text{in}} | \phi_{\text{out}} \rangle|^2 = |\beta_1|^2/(1 + |\beta_1|^2) \). The retrieval state is a cluster state provided that \( |\beta_1|^2 > 1 \) [11, 41]. In order to retrieve the frequency-entangled state with high fidelity \( F > 95\% \), then the coefficient \( |\beta_1|^2 \) should be necessarily more than 20. That is to say that the four fields would be retrieved nearly simultaneously, or else the fidelity is low. The third factor that limits the performance of the storage is adiabatic condition. Adiabatic following occurs when the population in the excited and bright states is small at all times. For a pulse duration \( T \) and a line-width of the excited state \( \gamma \), the adiabatic condition is \( g^2 N \gg \gamma/T \).

VI. CONCLUSION

In conclusion, we present a scheme for realizing quantum memory for the four-photon polarization encoded cluster state. Our proposal can be realized by current technology [31, 37, 51, 58, 59]. The quantum memory of the cluster is essential for “one-way” quantum computing, and we also expect the ability to store multiple optical modes to be useful for the quantum information and all-optical quantum computing network.

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