The vibration behaviour of a freight wagon in the presence of irregularities of the track

T Mazilu¹ and V M Dinu²

¹ Department of Railway Vehicles, University Politehnica of Bucharest, 313 Splaiul Independenţei, Bucharest, Romania
² Doctoral School of Transports, University Politehnica of Bucharest, 313 Splaiul Independenţei, Bucharest, Romania

trmazilu@yahoo.com

Abstract. The paper presents an analysis of the behaviour of the vertical vibration of a freight wagon featured with dry friction dampers in suspension. To this end, the wagon model is reduced to an oscillator with two degrees of freedom corresponding to the suspended mass of the wagon and the wheel. The suspension has progressive characteristic in two steps according to the empty/loaded state. Coulomb’s model is considered to model the dry friction damper. The vibration is induced by the track irregularity which is synthetized from an analytic spectrum. Vibration behaviour exhibits two different aspects: at low speed, the suspension is locked, and at high speed, the stick-slip vibration occurs. Acceleration of the suspended mass of the wagon and wheel-rail contact force increase along the speed. Wagon loaded state influences the wheel-rail contact force, but it has no impact upon the acceleration.

1. Introduction

Generally, during running, railway vehicles are subjected to a complex vibration behaviour that develops mainly in the vertical and lateral direction. The sources of these vibrations are very varied, but the main cause is the unevenness of the track. In the case of freight wagons, the vibrations affect the ride quality of the rolling stock, the fatigue resistance of the running gear and the track superstructure, jeopardizing, under certain conditions, the safety of traffic. At the same time, the dynamic loads that appear at the wheel-rail interface due to vibrations can lead to a reduction in the operating time of the track, its exit from the geometric parameters, etc.

From scientific point of view, freight wagons are modelled using non-linear oscillators due to the use of damping based on dry friction. This aspect raises interesting problems from the perspective of modelling the dry friction, related to the simulation of the possibility of the suspension blocking [1-2]. At the same time, the vibration behaviour can exhibit the stick-slip phenomenon, which sometimes has the characteristics of a chaotic motion [3].

In this paper, the vibration behaviour of a freight wagon due to the track irregularity is analysed using a simple model consisting of two-degree of freedom oscillator, representing the wheel and the suspended mass of the wagon. The wagon suspension is modelled with the help of an ideal spring working in parallel with a damper with dry friction. Such simple model has the advantage to allow making the analysis and observations easier.
2. Freight wagon model

Figure 1 shows the Y25 bogie of a Zas tank car. As it is known [4], the Y25 bogie suspension consists of two sets of double springs for each axle box with progressive elastic characteristic. Thus, up to 136 kN axle load, the elasticity of the bogie suspension is 0.3 mm/kN, and from this value, the elasticity becomes 0.1 mm/kN. The total mass of the bogie is about 4400 kg, of which the wheelsets mass is 2600 kg. The total mass of the wagon is 24000 kg for empty state and 80000 kg for loaded state. Each axle box is equipped with a Lenoir damping device.

![Figure 1. Y25 bogie.](image)

Figure 2 presents the proposed model for the study of the vertical vibrations of the tank car. The model is based on the symmetry property of the wagon construction, neglecting the coupling between the two bogies, as well as the coupling between the axles of the same bogie. At the same time, it is observed that the running track has a symmetrical construction with respect to the longitudinal axis. As a result, the wagon model consists of an oscillator with two degrees of freedom in which the mass $M_c$ represents the suspended mass of the wagon that rests on one wheel (1/2 wheelset), and $M_w$ - the mass of the wheel.

![Figure 2. Mechanical model.](image)
The stiffness of the suspension is $k$, and the frictional force developed by the dry friction damper is $F_f$. The track is assumed to be rigid with random irregularity $\eta$. It is assumed that the wheel of the vehicle is perfectly round, without geometrical deviations. The contact between the wheel and the rail is elastic, according to Hertz's non-linear theory of elastic contact between two bodies. The Hertzian contact is characterized by the Hertz $C_H$ constant which depends on the main curvatures of the bodies in contact (wheel and rail) and the stiffness $k_H$. The oscillator moves with constant speed $V$. A similar model, but which considers the wheel-rail contact as rigid and harmonic track irregularity is presented in refs. [5-6].

Considering that the displacement of the suspended mass is $z_c$, and the displacement of the wheel is $z_w$, one can write the equations of motion of the model. Therefore, by applying the second principle of dynamics, we have:

- the equation of motion of the suspended mass

$$M_c \ddot{z}_c = F_e + F_f$$  \hspace{1cm} (1)

- the wheel movement equation

$$M_w \ddot{z}_w = Q_o - Q - F_e - F_f$$,  \hspace{1cm} (2)

wherein $F_e$ is the elastic force in suspension

$$F_e = -k(z_c - z_w)$$,  \hspace{1cm} (3)

$F_f$ is the dry friction force, $Q_o$ is the static load

$$Q_o = (M_c + M_w)g$$,  \hspace{1cm} (4)

where $g$ is the gravitational acceleration and $Q$ is the wheel-rail contact force.

![Figure 3. Friction force.](image)

The friction force depends on the relative speed between the suspended mass and wheel and cannot exceed the static friction force (figure 3)

$$F_f = -\mu_k N \cdot \text{sign}(\dot{z}_c - \dot{z}_w) \quad \text{for} \quad \dot{z}_c - \dot{z}_w \neq 0$$  \hspace{1cm} (5a)

$$|F_f| \leq \mu_k N \quad \text{for} \quad \dot{z}_c - \dot{z}_w = 0$$  \hspace{1cm} (5b)

where $N$ is the normal pressing force in the damping element, $\mu_e$ is the static friction coefficient and $\mu_k$ - the kinematic friction coefficient.
The normal pressing force can be calculated with the relation
\[ N = M_c \cdot g \cdot \tan \alpha, \]  
(6)
where \( \alpha \) is the inclination angle of the Lenoir mechanism; \( M_c g \) represents the static force in the suspension spring and the variation due to the dynamic deflection of the spring is neglected.

The wheel-rail contact force can be calculated based on Hertz's theory
\[ Q = C_H \sqrt{(z_w - \eta)^3} \cdot \sigma(z_w - \eta), \]  
(7)
where \( \sigma(.) \) is the unitary step function of Heaviside.

The use of the unitary step function allows the modelling of the contact loss between the wheel and the rail: if \( z_w - \eta \) is positive, the contact force is positive, but this becomes zero when the condition \( z_w - \eta \leq 0 \) is met - the wheel has broken off the rail (figure 4).

\[ u = z_e - z_w, \]  
(8)
and Eq. (1) becomes
\[ M_c \ddot{u} = F_e + F_f - M_c \ddot{z}_w. \]  
(9)
Wheel acceleration results from Eq. (2)
\[ \ddot{z}_w = \frac{1}{M_w} \left( Q_o - Q - F_e - F_f \right). \]  
(10)
Inserting wheel acceleration in Eq. (9), it results
\[ \ddot{u} = \frac{1}{M_{ech}} \left( F_e + F_f \right), \]  
(11)
where
\[ M_{ech} = \frac{M_e M_w}{M_e + M_w} \]  
(12)
and
\[ F_f = F_e + \frac{M_{ech}}{M_w} (Q - Q_o). \]  
(13)
Equations of motion (10-11) can be solved using a similar numeric procedure as it is presented in reference [6].

It should be noted that, since the wagon model is an oscillator with two degrees of freedom corresponding to the vertical motion of the suspended mass and the wheel, with two elastic levels, one for suspension, and the other for the elasticity of the wheel-rail contact, the vibration behaviour is characterized by two natural frequencies that are calculated with the relation

$$
\nu_{1,2} = \frac{1}{2\pi} \sqrt{\frac{k}{M_e} + \frac{k + k_c}{M_w} \pm \sqrt{\left(\frac{k}{M_e} + \frac{k + k_c}{M_w}\right)^2 - 4\frac{kk_c}{M_e M_w}}} .
$$

On the other hand, the presence of dry friction can lead to the suspension locking, in which case the wheel moves in solidarity with the suspended mass, and the model 'degenerates' into one with a single degree of freedom whose natural frequency is given by the relation

$$
\nu = \frac{1}{2\pi} \sqrt{\frac{k_c}{M_e + M_w}} .
$$

Regarding the track irregularity, described by the function $\eta(x)$, where $x$ is coordinated along the track, this can be determined either experimentally or by calculation based on the spectra obtained by measurements or their analytical approximations.

The longitudinal level can be calculated starting from the power spectral density of the longitudinal level, recommended by the ORE report B176 [7] as representative for the European rail networks,

$$
S(\Omega) = \frac{A\Omega^2}{(\Omega^2 + \Omega_c^2)(\Omega^2 + \Omega_r^2)} ,
$$

where $\Omega$ is the wave number, $\Omega_c = 0.8246$ rad/m, $\Omega_r = 0.0206$ rad/m, and $A$ is a coefficient depending on the quality of the tread, and applying the method recommended in ref. [8].

3. Numerical applications

In this section are presented a series of results obtained by applying the model and method presented above in the MATLAB environment. The parameters of the vehicle model are [4]: $M_e = 2350$ kg for empty wagon and $M_e = 9350$ kg for loaded wagon, $M_w = 650$ kg, $k = 830$ kN/m for axle loads up to 135 kN and $k = 2.5$ MN/m for axle loads greater than 135 kN, $\mu_k = 0.17$, $\mu_s = 0.204$, $N = 9.126$ kN for the empty wagon and $N = 36.699$ kN for the loaded wagon, the inclination angle of the Lenoir mechanism, $\alpha = 21.42^\circ$, $C_H = 118.6$ GNm$^{3/2}$ and $\Delta t = 1/20000$ s.

The wheel load is 30 kN for the empty wagon and 100 kN for the loaded wagon. The wheel-rail contact rigidity is 1.125 GN/m for the empty wagon and 1.681 GN/m for the loaded wagon. The static friction force is 1.882 kN for the empty wagon and 7.486 kN for the loaded wagon, and the kinematic friction force is 1.568 kN for the empty wagon and 6.239 kN for the loaded wagon.

Figure 5 shows the longitudinal level of the track calculated for 1000 m section, including wavelengths between 3 m and 120 m. The peak value reaches 7.4 mm, while the RMS value is 2.38 mm which encloses the section of track in QN2 quality level for speeds between 80 and 120 km/h.

Figure 6 shows the friction force and the elastic force in the loaded wagon suspension at 20 km/h. It is observed that the friction force does not exceed 500 N, a value much lower than the static friction force (7.486 kN). This means that the suspension is locked, and this aspect is also signalled by the fact that the elastic force is zero.
Figure 5. Longitudinal level.

Figure 6. Friction force and elastic force in the loaded wagon suspension at 20 km/h: red line – friction force, black line – elastic force.

Figure 7. Friction force and elastic force in the loaded wagon suspension at 70 km/h: red line – friction force, black line – elastic force.
Figure 8. Friction force and elastic force in the loaded wagon suspension at 90 km/h: red line – friction force, black line – elastic force.

Figure 9. Stick-slip vibration in the loaded wagon suspension at 90 km/h: red line – friction force, black line – elastic force.
Similar diagrams also result in the simulation of the wagon circulation with speeds up to 70 km/h, as shown in figure 7, where the calculation speed is even 70 km/h. The difference is that the friction force is much higher than in the previous example; the peak value reaches 6.414 kN, but this is still lower than the static friction force, which explains why the suspension is locked throughout the simulation.

Figure 8 shows the same forces for the loaded wagon at 90 km/h. This time, the friction force often reaches the static friction force, and the suspension begins to work. This is consistent with the fact that the elastic force is no longer zero. It is interesting what happens when the friction force reaches the static friction threshold and the sliding between the wheel and the suspended mass occurs (see fig. 9). We observe the alternation of stick and slip phases - stick-slip vibrations. During the stick phase, the suspension is locked, the elastic force is constant, and the friction force has a reduced frequency variation, corresponding to the wagon's natural frequency with the locked suspension (approx. 65 Hz) modulated with the frequency induced by the longitudinal level of the track.

During the slip phase, the elastic force varies from one constant value to another, while the friction force has a high frequency variation when it goes from the static friction value to the dynamic friction value. If the slip phase is long enough, then the frictional force stabilizes to the value of the kinematic friction force.

From the point of view of the ride quality, the acceleration of the suspended mass (figure 10) and the wheel-rail contact force (figure 11) are of interest. The peak value of the suspended mass acceleration is 0.931 m/s$^2$, and its RMS value is 0.327 m/s$^2$. The wheel-rail contact force has variations of not more than 10.02 kN around the static load on the wheel (100 kN for the loaded wagon), while the RMS value is 3.28 kN.

The ride quality of the vehicle depends on the state of loading and the running speed. Figures 12 and 13 show the RMS value of the acceleration and the contact force respectively for the speed between 20 and 120 km/h for both empty and loaded wagon. It is observed that in the case of acceleration of the suspended mass, we do not notice differences depending on the condition of the wagon, empty - loaded. RMS acceleration increases with the running speed, reaching 0.5 m/s$^2$ at 120 km/h. The shape of the acceleration curve changes when the suspension starts working, around 70 km/h. The contact force increases with the speed of the wagon and is significantly higher in the case of the loaded wagon. For example, at 120 km/h, the RMS value of the contact force is 5.23 kN for the loaded wagon and only 1.58 kN if the wagon is empty. There is also a change in the allure of the contact force curve when the suspension starts working.

![Figure 10. Acceleration of the suspended mass of the loaded wagon at 90 km/h.](image-url)
**Figure 11.** Wheel-rail contact force at 90 km/h (loaded wagon).

**Figure 12.** Acceleration of the suspended mass of the wagon: red line – empty wagon; black line – loaded wagon.

**Figure 13.** Wheel-rail contact force: red line – empty wagon; black line – loaded wagon.
4. Conclusions

The paper presents a series of characteristics of the vertical vibrations of a freight wagon equipped with suspension with dry friction dampers when circulating on a track in the presence of longitudinal level deviations. For this, the wagon was modelled with the help of an oscillator with two degrees of freedom representing the suspended mass of the wagon and the wheel. The suspension is progressive in two levels for the empty and respectively charged state. For dry friction, Coulomb’s model was considered.

At low speeds up to 70 km/h, the suspension of the wagon is locked, so that, at higher speeds, relative displacements occur between the suspended mass and the wheel in the form of stick-slip vibrations. The stick-slip vibrations have two distinct phases: the stick phase and the slip phase. During the stick phase, the friction force shows variations in the resonance frequency of the vehicle on the elasticity of the wheel-rail contact that are modulated by the longitudinal level deviations; the elastic force is constant, and the suspension does not work. During the slip phase, with the release of the suspension, high frequency vibrations may occur.

From a practical point of view, the acceleration and the wheel-rail contact force are of interest. These quantities increase continuously with the wagon speed after a different allure at the speeds at which the suspension is no longer locked. The results of the numerical simulations showed that the loading state does not influence the RMS acceleration level of the suspended mass. Consequently, the ride quality is not much influenced by the loading which is an advantage offered by the wagon suspension. This is explained by the fact that the wagon's natural frequencies do not differ much from the loading state due to the stiffness of the suspension. The contact force is influenced by the loading condition, being higher in the case of the loaded wagon than in the empty wagon in a ratio approximately equal to that of the static load on the loaded/empty wheel (in the analysed case 3.33: 1).

5. References

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