Cumulative particle production as a rare event

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Abstract
The generalization of the Glauber formula for cumulative production events is derived. On its basis the multiplicity distribution in such events is related to the one in the minimum bias events. As compared to the rare events of type $C$, the formula involves a shift in the arguments determined by the multiplicity from a collision with a cluster of several nucleons.
1 Introduction.

In [1,2] a simple formula was derived which relates the multiplicity distribution $P_C(n)$ in a rare event of the so-called type $C$ to the minimum bias (MB) multiplicity distribution $P(n)$:

$$P_C(M) = \frac{MP(M)}{\langle M \rangle}$$

Remarkably the right-hand side of (1) does not depend on a particular nature of the rare process. So (1) gives a universal probability for rare events, independent of the nature of the trigger process. This universal formula was tested in several different rare processes for $h$ and $hA$ collisions like $W$ and Drell-Yan dilepton pair production.

In this paper we would like to discuss the possibility of applying (1) to cumulative events in $hA$ collisions. In deriving (1) it was assumed that the geometric properties of the $hA$ collision in the rare event were the same as for MB events. From this point of view production of cumulative particles, that is, in the kinematical region outside the one allowed for the scattering on a nucleon at rest, is different, since it involves collisions with two or more nucleons at once. Thus it is not evident that (1) is applicable to cumulative events, although they are certainly rare, experimentally.

Our results show that under certain reasonable approximations one obtains an expression for the multiplicity distribution in a cumulative event quite similar to (1), except for a certain shift in the arguments. The shift is due to a possible change of the multiplicity in a collision with a cluster of several nucleons. In the colour string approach, clustering of nucleons leads to fusion of strings and their percolation, which tends to diminish the multiplicity [3,4] The mentioned shift in the arguments can be observed experimentally and thus can serve as a signature of string fusion, once the multiplicity distributions in cumulative and MB events are compared.

2 The Gribov-Glauber formula for cumulative events

To study the multiplicity distribution in a cumulative event, one needs a suitable generalization of the standard Glauber formula for $hA$ collisions. The derivation of the standard Glauber $hA$ amplitude for high energies made by V.Gribov [5], expresses the $hA$ amplitude via $hN$ amplitudes with the nucleon at rest. The corresponding kinematical region then evidently coincides with the one for the scattering on a nucleon at rest. Cumulative events, in contrast, involve collisions with several nucleons at once. Such multinucleon collisions were considered earlier in relation to the loop contribution in nuclear-nuclear collisions [6] or string fusion [7]. It is remarkable, however, that one can naturally introduce them strictly following the Gribov approach, provided one takes into account a more general structure for the high-energy part of the $hA$ amplitude. Here we only present the main steps and the final formula. One can find some details in the Appendix.

The basic novelty in deriving the Glauber amplitude for cumulative events is taking into account that the high energy part $H$ of the $hA$ scattering amplitude (see Figure), apart from the usual terms which depend on all momenta transferred to the nucleus $q_i$ separately, may include other terms which depend on their partial sums, say, $q_1 + q_2$, $q_3 + q_4 + q_5$ etc. As we shall see, such terms describe simultaneous interaction of the projectile with clusters of two, three etc. nucleons. Correspondingly, we shall call these sums of the transferred momenta clusters for brevity. In the general case, clustering implies a subdivision of all transferred momenta $q_1, ... q_n$ into various groups (clusters) of one, two or more momenta. The high-energy part can be split into terms which depend only on the total momentum of each cluster. Some of the clusters may contain equal number of nucleons, say, $q_1 + q_2$ and $q_3 + q_4$ with...
two nucleons each. The order of such clusters has no influence on the resulting contribution to the amplitude. For this reason it is convenient to characterize the clustering structure by occupation numbers $\nu_k$, $k = 1, 2, \ldots$, telling the number of clusters containing $k$ nucleons (that is, $k$ transferred momenta $q_j$). Evidently for the $n$-fold interaction with the nucleons one should have

$$\sum_k k\nu_k = n$$

(2)

and, of course, all $\nu_k$ with $k > n$ are zero. The standard situation when the high-energy part depends on all $q_j$ separately corresponds to $\nu_1 = n$ and $\nu_k = 0$ for $k > 1$.

Taking the possibility of clustering into account, we present the high-energy part as a sum of contribution from all possible clusters:

$$H(q_1, \ldots q_n) = \prod_k \sum_{\nu_k} S_{\nu_k} H_{\nu_k}(q_1, \ldots q_n)$$

(3)

Here $H_{\nu_k}(q_1, \ldots q_n) = H_{\nu_k}(\kappa_1, \kappa_2, \ldots, \kappa_\nu)$ is supposed to depend only on the total cluster momenta $\kappa_j$, $j = 1, 2, \ldots, \nu$ where

$$\nu = \sum_k \nu_k$$

is the total number of clusters. We also assume that the initial momenta $q_j$ are distributed among the clusters in some fixed way, say, in the order of the growing cluster size. Various redistributions are then taken into account by the evident symmetry factor

$$S_{\nu_k} = \frac{n!}{\prod_k (k!)^{\nu_k} \nu_k!}$$

(4)

Eqs. (3) and (4) present the essence of our generalization of the Gribov derivation. Then, following the standard procedure of integrating over the longitudinal momentum transfers by means of closing the contour around the right-hand singularities of the high-energy part and retaining the lowest mass singularities, one readily finds a contribution to the $hA$ amplitude from a given clustering $\nu_k$

$$iA_{\nu_k} = S_{\nu_k} \int d^2r \prod_k [i a_k T_k(r)]^{\nu_k}$$

(5)

Here $a_k$ are the scattering amplitudes for a collision of the projectile with a cluster of $k$ nucleons at rest. Their normalization is chosen to include an extra factor $(2s)^{-1}(2m)^{1-k}$ as compared to the standard relativistic one, where $m$ is the nucleon mass and $s$ the c.m. energy squared. With this normalization for $k = 1$ (and for the total $hA$ amplitude) twice the imaginary part directly gives the total cross-section. The generalized profile functions $T_k$ are defined as

$$T_k(r) = \int dz \rho^k(r, z)dz$$

(6)

where $\rho(r, z)$ is the 3-dimensional nuclear density. As expected, they give the probability to find $k$ nucleons at the same point in the transverse plane.

To find the total $hA$ amplitude we have to sum (5) over all cluster structures for fixed $n$ and then sum over all $n$ with the standard factor $C_n^A$. This reduces to summing over all $\nu_k$ with the only restriction

$$\sum_k k\nu_k \leq A$$

(7)

In this way we obtain the $hA$ amplitude in the form

$$A = \int d^2r A(r)$$

(8)
where the hA amplitude for fixed impact parameter is given by

$$iA(r) = \sum_{\nu_k} \frac{A!}{(A - \sum_k k\nu_k)!} \prod_k \nu_k! \prod_k [ia_k T_k(r)/k!]^{\nu_k} \tag{9}$$

and the summations are restricted by condition (7).

The standard Glauber formula is obtained if all $\nu_k = 0$ for $k > 1$. In the general case we cannot do the summations explicitly for arbitrary $A$. However they are easily done for a very heavy nucleus with $A >> 1$. In this limit we can drop condition (7) and approximately take

$$\frac{A!}{(A - \sum_k k\nu_k)!} \approx \prod_k A^{k\nu_k}$$

Then the summation over each $\nu_k$ leads to an exponential factor and we obtain

$$iA(r) = \exp \left( \sum_k ia_k A^{kT_k(r)/k!} \right) - 1 \tag{10}$$

The subtracted unity corresponds to a term with all $\nu_k = 0$ and thus without any interaction. Evidently, neglecting all amplitudes with $k > 1$ one obtains the standard Glauber expression for a heavy nucleus.

Note that $T_k \sim A^{1/3-k}$ and so $A^k T_k \sim A^{1/3}$. As a result all terms in the exponent in (10) have the same order $A^{1/3}$ independent of the number of nucleons in the cluster $k$. This means that the relative weight of the multinucleon interactions is essentially independent of $A$. Their smallness is determined not by the large nuclear volume as a whole but by the large internucleon distance $R_0$ in the nucleus as compared to the strong interaction radius $R_s$, which determines the magnitude of $a_k$. Indeed $A^k T_k \sim A^{1/3} R_0^{-3k}$. So the relative weight of the interaction with $k$ nucleons at once is of the order of the dimensionless parameter $a_k R_0^{-3k} \sim (R_s/R_0)^{3k-1}$. Taking $R_s \sim 1/m_\rho \sim 0.27 f$ one finds that raising the number of nucleons in the cluster by unity leads to a damping factor of the order $(1/4)^3 = 1/64$

To calculate the contribution of multinucleon interactions one has to know the multinucleon amplitudes $a_k$. They only weakly depend on the energy $s$. Although being on-mass-shell quantities, they do not seem to be directly measurable. An obvious way is to determine them from simpler processes, like the scattering on the deuteron, triton etc, and then use the found values for studying the cumulative kinematical region in interactions with other nuclei.

### 3 The AGK rules, cross-sections and multiplicities

To simplify our notations we define $t_k(r) = T_k(r)/k!$ Then at fixed $r$

$$iA(r) = \sum_{\nu_k} \frac{A!}{(A - \sum_k k\nu_k)!} \prod_k \nu_k! \prod_k [ia_k t_k(r)]^{\nu_k} \tag{11}$$

The twice imaginary part of this amplitude gives the total cross-section on the nucleus at fixed $r$

$$\sigma_A^{\text{tot}}(r) = -iA - (iA)^* = \sum_{\nu_k} \frac{A!}{(A - \sum_k k\nu_k)!} \prod_k \nu_k! \prod_k t_k(r)^{\nu_k} [- (ia_k)^{\nu_k} - (ia_k)^*^{\nu_k}] \tag{12}$$

To obtain the AGK rules we have to split this into terms corresponding to a given number of inelastic collision with the nuclear clusters. We use a trivial identity

$$ia_k + (ia_k)^* + 2 \text{Im} a_k = 0$$
It demonstrates that for a given discontinuity of the diagram (cutting of the diagram) each interaction may be either cut or taken uncut on both sides of the cut. We present in (12)

\[- (ia_k)^{\nu_k} - (ia_k)^{t_{\nu_k}} = (ia_k + (ia_k)^* + 2\text{Im} a_k)^{\nu_k} - (ia_k)^{t_{\nu_k}}\]

(13)

Terms proportional to \((2\text{Im} a_k)_{\xi_k}\) on the right-hand side correspond to \(\xi_k\)-fold inelastic collision with clusters \(k\). They are

\[C_{\nu_k}^{\xi_k}(2\text{Im} a_k)^{\xi_k}(ia_k + (ia_k)^*)^{\nu_k - \xi_k} = C_{\nu_k}^{\xi_k} a_k^{\nu_k} (-1)^{\nu_k - \xi_k}\]

where we denoted

\[\sigma_k = 2\text{Im} a_k\]

Terms without \(2\text{Im} a_k\) represent the diffractive part of the cross-section

\[(ia_k + (ia_k)^*)^{\nu_k} - (ia_k)^{t_{\nu_k}} - (ia_k)^{t_{\nu_k}}\]

Summing over all sets of clusters we get the cross-section at a given \(r\) corresponding to \(\xi_k\) inelastic interactions with cluster \(k\) in the form

\[
\sigma_A^{(\xi_k)}(r) = \sum_{\nu_k} \frac{A!}{(A - \sum_k k\nu_k)! \prod_k \nu_k!} \prod_k C_{\nu_k}^{\xi_k} \sigma_k^{\nu_k} (r)^{\nu_k - \xi_k} \]

(14)

where of course \(\nu_k \geq \xi_k\). Changing the summation variable \(\nu_k \rightarrow \nu_k - \xi_k\) we rewrite this as

\[
\sigma_A^{(\xi_k)}(r) = \frac{A!}{(A - \sum_k k\xi_k)! \prod_k \xi_k! \prod_k (\sigma_k t_k(r))^{\xi_k} \sum_{\nu_k} \frac{(A - \sum_k k\xi_k)!}{(A - \sum_k k\xi_k - \sum_k k\nu_k)! \prod_k \nu_k!} (-\sigma_k t_k(r))^{t_{\nu_k}}}
\]

(15)

This formula expresses the AGK rules in the general case. It can easily be simplified for a heavy nucleus when in the same way as in deriving (10) we easily obtain

\[
\sigma_A^{(\xi_k)}(r) = \prod_k \left(\frac{\sigma_k A^k t_k(r)}{\xi_k!}\right)^{\xi_k} \exp \left( - \sum_k \sigma_k A^k t_k(r) \right)\]

(16)

in an obvious generalization of the well-known formula for collisions with separate nucleons inside the nucleus.

The multiplicity \(\mu_k\) observed in an inelastic collision with a cluster of \(k\) nucleons is not known \textit{apriori}. Its magnitude depends on the dynamics of the interaction with the cluster. In a model in which this interaction is realized via creation and decay of colour strings, the multiplicity \(\mu_k\) depends on the interaction between the strings. With no interaction, the nucleons, although located at the same longitudinal point, produce strings independently so that \(\mu_k = k\mu_1\). In the opposite case all strings fuse into a pair and \(\mu_k = \mu_1\).

To find the multiplicity in the event with \(\xi_k\) inelastic interactions with cluster \(k\) one has first to find the corresponding inclusive cross-section. This corresponds to substituting in (15) one of the cross-sections \(\sigma_k\) by \(\mu_k \sigma_k\) and then summing over all such substitutions. The multiplicity is obtained by dividing the result by the cross-section (15) itself. This gives a simple formula

\[
\mu^{(\xi_k)} = \sum_k \xi_k \mu_k\]

(17)
4 Rare events.

As discussed, the probability for the nucleons to form a cluster is rather small. So in the cumulative event, in which at least one cluster with a given \( k > 1 \) interacts with the projectile, in the first approximation one can neglect all configurations with two or more clusters with \( k > 1 \). We shall assume that the nucleus has a sharp edge and a constant density inside, so that

\[
\rho(r, z) = \left( \frac{1}{V_A} \right) \theta\left( R_A^2 - r^2 - z^2 \right)
\]

where \( R_A = A^{1/3} R_0 \) is the nuclear radius and \( V_A = AV_0 \) is the nuclear volume. Then we find

\[
t_k(r) = \frac{T(r)}{k! V_A^{k-1}}
\]

with \( T(r) \equiv T_1(r) \) the standard nuclear profile function.

With these simplifications and neglecting the contribution of clusters in the sum over \( \nu_k \) in (15) we find the cross-section for a single inelastic interaction with a cluster \( k \) and \( m - 1 \) interactions with separate nucleons at a given impact parameter \( r \) as

\[
\sigma^{(m,k)}_A = \frac{A!}{(A-m-k+1)! (m-1)!} \frac{\sigma_k}{k! V_A^{k-1}} (\sigma T(r))^m (1 - \sigma T(r))^{A-m-k+1}
\]

For a heavy nucleus this transforms into

\[
\sigma^{(m,k)}_A = \frac{1}{(m-1)!} \frac{\sigma_k}{k! V_0^{k-1}} (\sigma AT(r))^m e^{-\sigma AT(r)}
\]

Let us compare this cross-section with the one for \( m \) inelastic interactions with isolated nucleons (without clustering)

\[
\sigma^{(m)}_A = \frac{A!}{(A-m)! m!} (\sigma T(r))^m (1 - \sigma T(r))^{A-m}
\]

which for a heavy nucleus goes into

\[
\sigma^{(m)}_A = \frac{1}{m!} (\sigma AT(r))^m e^{-\sigma AT(r)}
\]

The comparison of (21) and (23) gives a relation

\[
\sigma^{(m,k)}_A = \alpha_k m \sigma^{(m)}_A
\]

where we have defined

\[
\alpha_k = \frac{\sigma_k}{k! V_0^{k-1}}
\]

Eq.(24) means that the probability \( P_k(m) \) to find apart from a single interacting cluster \( k \) also \( m - 1 \) interacting separate nucleons, \( m = 1, 2, \ldots \), is related to the probability \( P(m) \) to find \( m \) interacting separate nucleons and no clusters at all according to

\[
P_k(m) = c_k \alpha_k m P(m)
\]

Since the total probability to have any number of separate interactions with the nucleons apart from an interaction with the cluster is unity, the sum of (26) over all \( m \) should give unity, which determines the constant \( c_k \). We then obtain

\[
P_k(m) = \frac{m P(m)}{\sum_n n P(n)} = \frac{m P(m)}{\langle m \rangle}
\]
This relation has the same form (1) as obtained in [1,2] for rare events of the so-called C-type. It is remarkable that the relation is independent of the cluster characteristics \((k \text{ and } \alpha_k)\), the only requirement being that the interaction with it should be rare.

However the resulting observable consequences will be different. The point is that passing to observable multiplicities, we have to introduce the multiplicity for the interacting cluster \(\mu_k\). Then the total multiplicity \(M\), observed in the event in which, apart from the cluster, \(m-1\) isolated nucleons interact will be

\[
M = \mu_k + (m-1)\mu = m\mu + \mu_k - \mu \equiv m\mu + \Delta_k
\]  

(28)

where \(\mu \equiv \mu_1\) is the multiplicity for the interaction with an isolated nucleon. So, in terms of the multiplicities, the relation (27) implies

\[
P_k(M) = \frac{(M - \Delta_k)P(M - \Delta_k)}{\langle M - \Delta_k \rangle}
\]  

(29)

This is our final result. It shows that, applied to rare cluster interactions, the approach of [1,2] leads to a very similar relation, in which however the argument is shifted on the right-hand side, the shift depending on the multiplicity for the given cluster. As mentioned, the latter depends on the assumed dynamics of colour strings in the cluster. Without any interaction between them \(\mu_k - \mu = (k-1)\mu\) and the shift is maximal. In the opposite case when the cluster behaves in the same manner as a single nucleon \(\mu_k = \mu\) and there is no shift at all. Thus experimental studies of the relation (29) can shed light on the strength of the interaction between colour strings and the probability of their fusion and percolation.

In terms of averages \(\langle \rangle\) for a cumulative event involving a cluster with \(k\) nucleons and \(\langle \rangle\) for MB events, one gets from (29) for the average multiplicity

\[
\langle M \rangle_k = \frac{\langle M^2 \rangle}{\langle M \rangle} + \Delta_k
\]  

(30)

and for its dispersion squared

\[
D^2_k = \langle M^2 \rangle_k - \langle M \rangle^2_k = \frac{\langle M^3 \rangle}{\langle M \rangle} - \frac{\langle M^2 \rangle^2}{\langle M \rangle^2}
\]  

(31)

One observes that the dispersion of the multiplicity does not depend on the the cluster properties and is the same as for rare events of the type \(C\) found in [1,2]. As discussed there it is smaller than for MB events, so that production of cumulative particles triggers sharpening of the multiplicity distribution. On the other hand, the average multiplicity in the cumulative event generally results still greater than for rare events of the type \(C\) (given by the first term in (30)) due to the extra term \(\Delta_k\). Only in the limiting case when \(\Delta_k = 0\), which in the string language corresponds to a very strong fusion, the two multiplicities coincide, remaining greater than for MB events.

## 5 Conclusions

The standard Glauber-Gribov derivation of the \(hA\)-amplitude in terms of \(hn\) amplitudes has been naturally extended to include cumulative effects. In this way the \(hA\) amplitude is obtained in terms of the amplitudes for the scattering of the projectile hadron off clusters of \(k\) nucleons, \(k = 1, 2, \ldots\). The resulting formula has the same structure as the original Glauber formula. By means of the AGK cutting rules cross-sections for a given number of inelastic collisions with clusters of \(k\) nucleons are found. Assuming a collision with a cluster of more
than one nucleon to be a rare event and thus neglecting contributions from two and more
clusters with such clusters, the associated multiplicity distribution is expressed in terms of
the MB multiplicity distribution. The obtained formula has the same universal form as (1),
except for a shift $\Delta_k$ in the arguments. The shift is determined by the difference between
the multiplicities in the scattering with a cluster of $k$ nucleons and with a single nucleon.
Observation of this shift potentially gives a possibility to study the multiplicity coming from a
cluster of nucleons. Although this does not seem to be easy experimentally, the shift could be
studied in the forthcoming experiments at RHIC and LHC, using, for instance, the BRAHMS
detector.

6 Acknowledgements

This study was supported by the NATO grant CRG. 971461. C.P. is also thankful to CICYT
(Spain) for the financial support under the contract AEN 96-1673.

7 Appendix. The hN amplitude with multinucleon interactions

The hA amplitude with several $(n)$ rescatterings can be represented by a generic diagram
shown in Figure. It divides into a part related to the structure of the nucleus and the high-
energy part represented by the blob $H$. Separation of the nuclear part is, in fact, standard
and follows the original approach of V.Gribov [5]. We briefly repeat it for self-consistency
and to introduce the necessary notations. The latter are as follows. We denote the nucleus
momentum as $Ap$ and take the nucleus at rest: $\mathbf{p} = 0$. The nucleon momenta before (after)
the interaction are denoted $k_i$ ($k_i' = k_i + q_i$). The spectators correspond to $i = n + 1,...A$. For them $k_i = k_i'$ and $q_i = 0$. Also $\sum q_i = 0$. The projectile momentum is denoted $l$.

The expression for the amplitude corresponding to the diagram in the Figure is

$$iA = \int \prod_{j=2}^{n} \frac{d^4 k_j}{(2\pi)^4} \frac{d^4 k_j'}{(2\pi)^4} P(k_j)P(k_j') \prod_{j=n+1}^{A} \frac{d^4 k_j}{(2\pi)^4} P(k_j) i\Gamma(k_i)i\Gamma(k_i') iH(l, k_j, k_j') \tag{32}$$

In this expression $H$ is the above mentioned high-energy part and $P(k)$ is the propagator
of the nucleon with momentum $k$:

$$P(k) = \frac{-i}{m^2 - k^2 - i0} \approx \frac{-i}{\alpha^2 + k^2 - 2mk_0 - i0} \tag{33}$$

where $m$ is the nucleon mass and $A\alpha^2/(2m)$ is the nucleus binding energy. The vertex $\Gamma$
describes the transition of the nucleus into $A$ nucleons. We have chosen the momenta of the
first nucleon $k_1$ and $k_1'$ as dependent variables, although in future we shall use a different
choice also. Evidently $k_1 = -\sum_{i=2}^{A} k_i$ and similarly $k_1' = -\sum_{i=2}^{A} k_i'$.

Standardly one starts by the integration over the zero components of the momenta. Since
the poles coming from the propagators of the first nucleon all lie in the upper half-plane,
we can integrate over $k_{0i}$ or $k_{0i}'$, $i = 2,...A$, just taking the residue at the pole of the corresponding
propagator $P(k_i)$ or $P(k_i')$. Each such integration provides a factor $2\pi/(2m)$. The two
propagators of the active nucleon in the initial and final state together with the factors
$i\Gamma(k_i)i\Gamma(k_i')$ combine into a product of two nuclear wave functions

$$(2m(2\pi)^3)^{A-1} \phi(k_i)\phi(k_i')$$

so that the expression for the amplitude (32) becomes

$$iA = (2m(2\pi)^3)^{A-1} \int \prod_{j=2}^{n} \frac{d^3 k_j}{2m(2\pi)^3} \frac{d^3 k_j'}{2m(2\pi)^3} \prod_{j=n+1}^{A} \frac{d^3 k_j}{2m(2\pi)^3} \phi(k_i)\phi(k_i') iH(l, k_j, k_j') \tag{34}$$
The next step is to pass to the coordinate space to simplify the dependence on the wave functions. We present

$$\phi(k_i) = \int \prod_{i} \frac{d^3r_i}{(2\pi)^{3/2}} \psi(r_i) \exp(-i \sum k_jr_j)$$

and then integrate over the transverse momenta. The high-energy part $H$ does not depend on the transverse momenta of the nucleons in the high-energy limit. So the integration over them is trivial, since all the relevant dependence is concentrated in the exponentials. We readily obtain

$$iA = \int \prod \frac{dk_{zj}}{2\pi} \prod_{j=2}^{n} dq_{zj} \frac{d^2r_j \psi(r_1 = r_2 = r_3 + ... r_n, r_j; z_i) \psi(r_1 = r_2 = r_3 + ... r_n, r_j; z'_i)}{2m(2\pi)}$$

$$iH(l_z, k_{jz}, q_{jz}) \prod dz_i \exp(-i \sum k_{zj}(z_j - z'_j)) \prod dz'_i \exp(i \sum q_{zj}(z'_j - z'_i))$$

$$d^2r_1 \prod_{j=n+1}^{A-1} d^2r_j \psi(r_1 = r_2 = r_3 + ... r_n, r_j; z_i) \psi(r_1 = r_2 = r_3 + ... r_n, r_j; z'_i)$$

(36)

As expected on the physical grounds, the $n$ nucleons which take part in the rescattering have to be taken at the same impact parameter.

Now we have to integrate over the longitudinal momenta. Evidently the integrand does not depend on the $k_z$ of the spectators. So these interactions are done trivially and add a factor

$$\prod_{i=n+1}^{A-1} 2\pi \delta(z_i - z'_i)$$

Apart from the factor $(2\pi)^{A-n-1}$, these $\delta$ functions convert the double integration over $z$ into a single one for the spectators. Together with the integration over their transverse coordinates this turns the product of the wave functions into the nuclear $\rho$-matrix for $n+1$ nucleons taking part in the interaction.

The high-energy part $H$ does not depend on $k_{jz}$ but only on $q_{jz}$. This property follows from the relativistic invariance, due to which $H$ depends on products like $lk_j$ and $lq_j$. With small spatial componets of $k_j$, the products of the first type all are approximately equal to $lp$ and only the second are real variables. So one can also trivially integrate over $k_{jz}$, $j = 1, ..., n$, which gives a factor $(2\pi)^n$ and puts $z_j = z'_j$, $j = 1, ..., n$. We get

$$iA = \int \prod \frac{dq_{zj}}{2m(2\pi)} iH(l_z, q_{jz}) \prod dz_j \exp(i \sum q_{zj}(z_j - z_1)) d^2r \rho(rz_1, rz_2, ..., rz_n | rz_1, rz_2, ..., rz_n)$$

(37)

At this point all nuclear effects are taken into account by the nuclear $\rho$-matrix taken at interaction points $(r, z_j)$. The final task is to integrate over the transferred longitudinal momenta $q_{jz}$. To do this we take into account the generalized structure of the high-energy part as a function of the transferred momenta, discussed in Sec. 3 and presented in Eqs (3) and (4).

Let us find the contribution to the hA amplitude with $n$ interactions (7) from a term in (3) with given $\nu_k$. All integrations over $q_j$ can be divided into $\nu - 1$ integrations over the total cluster momenta $\kappa_2, ..., \kappa_\nu$ ($\kappa_1 = -\sum_{j=2}^{\nu} \kappa_j$) and the rest $q_j$, without, say, the first $q$ from each cluster. The integration over the latters is trivial, since, the high-energy part does not depend on them. It will make all longitudinal points $z_j$ equal within each cluster, in accordance with the intuitive definition of a cluster or simultaneous interaction with several nucleons.
To write down the obtained expression in an understandable way, at this point we make the usual simplifying assumption about the structure of the nuclear $\rho$ matrix: that of absence of correlations and consequently of factorization

$$\rho(rz_1, rz_2, ..., rz_n | rz_1, rz_2, ... rz_n) = \prod_j \rho(rz_j)$$

(38)

where $\rho(rz)$ is the usual nuclear density (normalized to unity). Then the contribution to the amplitude from $H_{\nu k}$ will be given by

$$iA = S_{\nu k} (2m)^{-n+1} \int \prod_j \frac{d\kappa_j}{2\pi} iH_{\nu k} (\kappa_j) \prod z_i \exp(i \sum z_j (z_j - z_1)) d^2r \prod \rho^{k_j} (rz_j)$$

(39)

where $k_j$ is the number of nucleons in the cluster at point $z_j$.

This expression can now be treated in the same way as in the original Gribov approach. Instead of $\kappa_i$ we introduce $\nu - 1$ cumulative longitudinal momentum transfers $t_i = \sum_{j=1}^{i} \kappa_j$. This gives a factor $(2l)^{1-\nu}$. The Feynman integration contour over each of $t_i$ can be closed around the right-hand side singularities. In the spirit of the Glauber approximation, from these singularities we retain one the pole singularity at $t_i = 0$ corresponding to the single nucleon intermediate state. Taking the residue at this point makes the exponential factor in (12) equal to unity. The residue of $iH_{\nu k}$ itself is a product of $\nu$ on-mass-shell forward connected amplitudes $ia_k$ for the interaction of the projectile with $k$ nucleons at rest, where $k$ is the number of nucleons in a cluster. After that the final integration over the cluster points $z_j$ becomes trivial and leads to a standard profile function and its evident generalization. We obtain

$$iA_{\nu k} = S_{\nu k} (4ml)^{1-\nu}(2m)^{\nu-n} \int d^2r \prod_k (ia_k T_k(r))^{\nu_k}$$

(40)

where

$$T_k(r) = \int dz \rho^k (r, z) dz$$

(41)

is an obvious generalization of the profile function. Note that $4ml = s$ where $s$ is the standard c.m. energy squared. It is convenient to pass to reduced amplitudes

$$a_k \rightarrow \frac{a_k}{2s(2m)^{k-1}}, \text{ dim } a_k = 1 - 3k$$

(42)

and similarly for $A$. After that one obtains Eq. (5).

8 References

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9 Figure caption

The hA scattering amplitude. Double lines show the nucleus target.
Figure