Spin-dependent two-photon Bragg scattering in the Kapitza-Dirac effect

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We present the possibility of spin-dependent Kapitza-Dirac scattering based on a two-photon interaction only. The interaction scheme is inspired from a Compton scattering process, for which we explicitly show the mathematical correspondence to the spin-dynamics of an electron diffraction process in a standing light wave. The spin effect has the advantage that it already appears in a Bragg scattering setup with arbitrary low field amplitudes, for which we have estimated the diffraction count rate in a realistic experimental setup at available X-ray free-electron laser facilities.

I. INTRODUCTION

The spin is an intrinsic angular momentum of every elementary particle [1]. While from the theoretical point of view one would identify the spin as a byproduct of the quantization procedure of relativistic wave functions, one might in a classical picture imagine the spin as a tiny spinning sphere. This view might be intuitive, but should be considered as technically incorrect. Nevertheless, one is associating a magnetic moment with a magnetic dipole to the electron, which in the classical imagination of a charged, spinning sphere would be the ‘handle’ to interact with the electron spin. From the historical perspective, it seems that the electron spin was initially rather an implication from the need for a consistent explanation for the atomic structure, as well as from spectroscopic observations [2], with a first explicit experimental indication from the Stern-Gerlach experiment [3–5].

Within the scientific applications of present times, spin-dependent electron interaction appears commonly in photo-emission [6, 7], such that interesting applications like spin- and angle-resolved photoemission spectroscopy (SARPES) [8–10] is possible, with even recording spin-resolved band structure [11–14]. However, these examples have in common that they are bound state systems, in which the electron is not free from interactions with its environment. For isolated electrons, which propagate freely in space, Wolfgang Pauli has already argued in 1932 that an electron interaction with electro-magnetic fields cannot be sensitive to the electron spin in terms of a concept of classical trajectories [15]. The reason is that the Stern-Gerlach experiment has been carried out with electrically neutral silver atoms instead of charged electrons. For charged particles, however, the Lorentz force from the magnetic field in the Stern-Gerlach experiment requires a precise knowledge of the electron’s initial position and momentum, which is in conflict with the Heisenberg uncertainty relation. Therefore, a common assumption is that “it is impossible, by means of a Stern-Gerlach experiment, to determine the magnetic moment of a free electron” [16] and that “Conventional spin filters, the prototype of which is the Stern-Gerlach magnet, do not work with free electrons.” [17]. Nevertheless, proposals for a longitudinal setup of the Stern-Gerlach experiment with electrons exist [18, 19], for a “minimum-spread-longitudinal configuration” [20]. Also, random spin-flips can be induced by radio frequency field injection and thermal radiation at an electron in a Penning trap [21, 22].

Electron diffraction in standing light waves, as first proposed by Kapitza and Dirac [23] (see also [24–29]) could be a way to establish a controlled and explicit spin-dependent interaction of electrons with electro-magnetic fields only. A spin-independent Kapitza-Dirac effect has already been experimentally demonstrated for atoms [30, 31] and also electrons in a strong [32] and weak [33, 34] interaction regime. In the context of electron diffraction in standing light waves we refer to strong interactions when the final diffraction pattern shows many diffraction peaks (diffraction regime) and a weak interaction when the final diffraction pattern only shows a Bragg peak (Bragg regime) [35, 36]. Spin effects [37–45] and also spin-dependent diffraction [46–50] (ie. sorting of electrons according to their spin state) has been discussed theoretically for the Kapitza-Dirac effect. While the original proposal from Kapitza and Dirac considers a two-photon momentum transfer, higher order photon scattering is possible [39, 40, 44–47, 49–51] but might be suppressed for the case of a weak ponderomotive amplitude of the standing wave light field in the Bragg regime. Therefore, possible implementation difficulties of spin-dependent electron diffraction could arise for the case of a higher number of interacting photons or the necessity to wait for larger fractions of a Rabi cycle of the electron quantum state transition, which could hinder the observation of such higher order photon interactions. A further discussion about spin-dependent electron diffraction scenarios in laser fields is carried out in the outlook section V at the end of this article. We also point out other theoretical investigations of electron spin dynamics.

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in strong laser fields \([52–63]\) as well as spin-independent electron diffraction scenarios with a controlled phase-space construction \([64–67]\).

In this article we discuss spin-dependent Kapitza-Dirac diffraction with a two photon interaction which takes place in a Bragg scattering scenario. The approach is inspired by a previous work of one of us, which is investigating spin properties in Compton scattering \([68]\). While in Compton scattering an incoming electron and incoming photon scatter off into an outgoing electron and outgoing photon, one could replace the incoming and outgoing photons by classical electro-magnetic fields of strong laser beams and consider the electron quantum dynamics in terms of the Furry picture \([69–71]\). The mathematical expressions for spin-dynamics in Compton scattering and spin-dependent electron diffraction in strong laser beams are identical, as we discuss by an explicit calculation in this work. Therefore, the situation for a spin interaction of an electron with a photon in Compton scattering can be identified with a spin-dependent laser-electron interaction. The inspiration for the interacting setup which we take from reference \([68]\) is the possibility for two-photon spin-dependent Bragg scattering, which can be achieved by forming a standing light wave from two counterpropagating laser beams, of which one is linearly polarized and the other is circularly polarized. We then predict the existence of a spin-dependent diffraction effect, if a beam of electrons crosses the standing light wave with a momentum of about \(1mc\) along the polarization direction of the linearly polarized laser beam, where \(m\) is the electron rest mass and \(c\) the vacuum speed of light.

This outlines the content of the paper where the article is structured in the following way: Section II discusses the identification of Compton scattering with the process of two-photon electron diffraction in a standing light wave. From this, we take the parameters for the electron and for the photon as in the setup in reference \([68]\) and transfer it to a relativistic quantum simulation which is exhibiting spin-dependent electron diffraction in section III. After having demonstrated the possibility of this type of two-photon spin-dependent electron diffraction in the Bragg regime, we consider the possibility of an experimental implementation of the effect at the Shanghai High Repetition Rate XFEL and Extreme Light Facility (SHINE) in section IV. In the final outlook (section V) we compare our new spin-dependent interaction scheme with other proposals for spin-dependent electron diffraction in the literature.

II. THE THEORETICAL FRAMEWORK

As discussed in the introduction, one can identify the incoming and outgoing photons in Compton scattering with laser beams for the case of electron scattering in strong laser fields. We show this property by an explicit calculation for a perturbative 2-photon interaction, for which we first consider a quantized, interacting electron-photon system. By using perturbation theory (see appendix B) we arrive at an expression which can finally be turned into the Compton tensor. This Compton tensor is encoding the spin and polarization dependent properties of the electron-photon interaction. After that, we also solve the Dirac equation in an external, classical standing light wave perturbatively (see appendix C) for showing that the calculation is equivalent to the previous calculation of the interacting electron-photon system and therefore can also be identified with Compton scattering.

A. Quantized electron-photon system

We start by assuming the initial two particle excitation

\[
\Psi_i = c_{p_i}^{s_+} a_k^{w+} |0\rangle ,
\]

where \(c_{p_i}^{s+}\) is the electron creation operator with spin state \(s\) and initial momentum \(p_i\) and \(a_k^{w+}\) is the photon creation operator with polarization \(w\) and momentum \(k\). The ket \(|0\rangle\) is the quantum vacuum state with a zero number of electron and photon excitations. Note, that we introduce notions and conventions in this article in appendix A in detail and clarify symbols in the main text only where it appears suitable for understanding.

We want to know the perturbative time evolution under the action

\[
\mathcal{L} = \bar{\Psi} (i\gamma_\mu \partial^\mu \! - \! m) \Psi + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \! - \! e\bar{\Psi} \gamma_\mu A^\mu \Psi ,
\]

with electron mass \(m\), electron charge \(e\), Dirac field \(\Psi\), photon field \(A^\mu\) and electro-magnetic field tensor \(F_{\mu\nu}\) (see references \([70, 72–74]\) for introduction). For this we compute the perturbative time evolution from elements of the Dyson series in the Schrödinger picture in the appendix B. In the calculation we focus on a resonant process, for which energy and momentum are conserved in the final state of the lowest order contribution of the perturbative calculation. From the interaction Hamiltonian \((B7)\) one can see that the lowest possible contributions are in the second order perturbation theory, with the four intermediate states

\[
\Psi_0 = c_{p_i + k}^{s_+} |0\rangle ,
\]

\[
\Psi_b = c_{p_i - k}^{s_+} a_k^{w+} |0\rangle,
\]

\[
\Psi_c = c_{p_i}^{s_+} d_{p_i + k}^{s_+} |0\rangle ,
\]

\[
\Psi_d = c_{p_i}^{s_+} d_{-p_i, -k}^{s_+} a_k^{w+} |0\rangle ,
\]

as sketched in Fig. 1. The contributions of interest in the perturbative calculation result in a superposition of final states of the form

\[
\Psi_f = c_{p_f}^{s_+} a_{k'}^{w+} |0\rangle ,
\]

where \(k'\) is the outgoing photon momentum and the final electron momentum \(p_f = p_i + k - k'\) is implied by
momentum conservation. The corresponding spin and polarization dependent quantum state propagation matrix can be brought in the form of the Compton tensor

$$u_{p_i}^{s_i} \left( \frac{1}{2} \frac{p_i + k + m}{k} f^{(w)}(s_i - f^{(w)}(s_i - \frac{m}{k} + \frac{k')}{p_i \cdot k'}) \right) u_{p_i}^{s_i},$$

as shown by the calculation in appendix B 3. The Compton tensor (5) is known in quantum field theory literature \([70, 72]\) and describes the spin and polarization dependent scattering amplitude in Compton scattering. Note, that the results in our previous work \([68]\) were mistakenly based on a Compton tensor expression for linear photon polarizations from reference \([75]\), where the outgoing photon polarization $\epsilon^\text{final} = \epsilon^\text{initial}$ is not complex conjugated. This is corrected in this article (see also appendix A 3 on the helicity of the photon field).

We demonstrate that the propagation scheme with the four different intermediate quantum states in equation (3) is consistent with quantum field theory and the notion of Feynman diagrams, because our calculation in appendix B 3 is resulting in the final expression (5). In this context we would like to point out the following interpretation which is implied by the identity property: The electron propagation with the simultaneous creation of a virtual electron-positron pair as in panel (c) of Figure 1 can be combined into the Feynman graphs (e) and (f). More precisely, panel (e) is composed of the processes in panels (a) and (c) and panel (f) is composed of the processes in panels (b) and (d). The corresponding mathematical identification is carried out in appendix B 3.

B. The Dirac equation in an external field

A similar calculation as discussed in section II A and derived in appendix B 3 can be performed without a quantized photon and electron field, but instead by a per-
turbative solution of the single particle Dirac equation in a classical external potential. The details are carried out in appendix C1. For the external vector potential we use
\[ A_{\mu}(x, t) = \frac{1}{2} \left( a_{\mu} e^{-ik^{i}x} + a_{\mu}^{*} e^{ik^{i}x} + a'_{\mu} e^{-ik'^{i}x} + a'^{*}_{\mu} e^{ik'^{i}x} \right), \]  
which is describing a monochromatic, standing light wave of arbitrary polarization along the x-axis. We have introduced in Eq. (6) the four-vectors of the two counterpropagating laser beams \( k_{l}^{i} = (k_{l}, k_{l}) \), \( k_{l}^{i} = (k_{l}, -k_{l}) \) with wave vector \( k_{l} = k_{l}e_{x} \), laser wave number \( k_{l} \) and the two polarizations \( a_{\mu} \) and \( a'_{\mu} \) of the left and right propagating laser beam. The dot between the four-vectors symbolizes a four-vector contraction \( k_{l} \cdot x = k_{l}^{i}x_{i} \mu \), according to Einstein’s sum convention.

A corresponding approach for the electron wave function
\[ \psi(x, t) = \sum_{n,s} \left( c_{n}^{s}(t)u_{kn}^{s} e^{-i\hat{k}_{n} \cdot x} + d_{n}(t)v_{kn}^{s} e^{-i\hat{k}_{n} \cdot x} \right), \]  
accounts for the transfer of multiple photon momenta \( \hat{k}_{n} = p_{i} + n\hat{k}_{l} \) along the laser propagation direction where \( p_{i} \) is the initial momentum of the electron. The dot between 3 component spacial vectors is denoting the inner product in Euclidean space \( \hat{k}_{n} \cdot x = \sum_{a} k_{n,a} \hat{a}_{a} \). We point out, that the form of the wave function’s plane wave expansion in Eq. (7) is an implication from the standing light wave (6). The choice of a monochromatic, standing light wave in turn has been made for subsequently obtaining the resulting set of differential equations Eq. (C3), which can be solved efficiently in a numeric implementation.

C. The Linkage between Compton scattering and laser electron diffraction

Without showing the explicit expressions in the main text, we make the following statement: The quantum state propagation matrix for the quantized system in Eq. (B20) and the quantum state propagation matrix for the one-particle Dirac equation in Eq. (C11) are equal (note details in footnote [76]). Accordingly, one can say that the spin and polarization dependent interaction of an electron with a photon in the process of Compton scattering is matching to the perturbative description of spin-dependent diffraction dynamics of an electron in an external potential of two plane waves [77]. In other words, the spin/polarization properties in Compton scattering and in electron diffraction of the Kapitza-Dirac effect are of identical form, only the interpretation of the associated process is different, depending on the scenario of consideration (ie. whether the scenario is Compton scattering or electron diffraction).

Having made this statement, we now refer to the Compton scattering scenario in reference [68], for which we want to investigate the electron diffraction analogy later on. Reference [68] is considering a 180° photon back scattering process, in which the photon is initially propagating in the \((1, 0, 0)\) direction and finally propagating in the \((-1, 0, 0)\) direction and the electron has approximately the initial and final momentum [78]
\[ \hat{p}_{i} = -k_{l} + me_{z}, \]  
\[ \hat{p}_{f} = k_{l} + me_{z}. \]  

The spin and polarization properties are determined by matrix entries as in Eq. (C17) for this scattering scenario which, according to our considerations, matches the matrix entries (5) in reference [68]. Correspondingly, the relation in Eq. (19) of Ref. [68] can be expressed in terms of the perturbation expression (B10) of this article by
\[ U(t, t_{0}) c_{\vec{p}_{i}}^{\mu} c_{\vec{k}_{l}}^{\mu} |0⟩ ≈ f(t, t_{0}) c_{\vec{p}_{f}}^{\mu} c_{\vec{k}_{l}}^{\mu} |0⟩ \]  
(9a)
\[ U(t, t_{0}) c_{\vec{p}_{i}}^{\mu} c_{\vec{k}_{l}}^{\mu} |0⟩ ≈ f(t, t_{0}) c_{\vec{p}_{f}}^{\mu} c_{\vec{k}_{l}}^{\mu} |0⟩, \]  
(9b)

with the time dependent prefactor
\[ f(t, t_{0}) = (t - t_{0}) \exp[-i(\vec{z} \cdot \vec{p}_{i} + k)(t - t_{0})], \]  
(10)

where helicity and the tilted spin states are introduced in appendix A3 and appendix A4.

We point out that the polarization of the outgoing photon after scattering depends on the initial spin state of the incoming electron before the interaction (ie. the initial \( c_{\vec{p}_{i}}^{\mu} \) electron state causes the final \( c_{\vec{p}_{f}}^{\mu} c_{\vec{k}_{l}}^{\mu} \) photon state, but the initial \( c_{\vec{p}_{i}}^{\mu} \) electron state causes the final \( c_{\vec{p}_{f}}^{\mu} c_{\vec{k}_{l}}^{\mu} \) photon state). Interesting in this context is that an \( s \leftrightarrow \) electron spin scatters only and exclusively into an L photon polarization as well as the \( s \leftrightarrow \) electron spin scatters only and exclusively into a R photon polarization. In reverse conclusion this means that the matrix elements from an \( s \leftrightarrow \) electron spin in to a L photon polarization and from an \( s \leftrightarrow \) electron spin into an R photon polarization are vanishing. Such processes do not exist for the chosen kinematic configuration.

These statements hold in the context of Compton scattering. Now, if we consider the corresponding diffraction process of an electron in a standing light wave of the form as in Eq. (6) with the polarizations
\[ a = (0, 0, 0, 0)^{T}, \quad a' = (0, 0, 0, 0)^{T}/\sqrt{2}, \]  
the same matrix elements (C17) imply that an electron with spin state \( s \leftrightarrow \) will undergo a diffraction, while an electron with spin state \( s \leftrightarrow \) will not be diffrazed at all. Analogously, for the polarizations
\[ a = (0, 0, 0, 0)^{T}, \quad a' = (0, 0, 0, 0)^{T}/\sqrt{2}, \]  
(11)
\[ a = (0, 0, 0, 0)^{T}, \quad a' = (0, 0, 0, 0)^{T}/\sqrt{2} \]  
(12)
our considerations imply that an electron with spin state \( s \leftrightarrow \) will be diffracted, while an electron with spin state \( s \leftrightarrow \) will not be diffracted.
III. NUMERIC SOLUTION OF THE SPIN-DEPENDENT QUANTUM DYNAMICS

We support the above considerations by performing numeric simulations of the one-particle Dirac equation in momentum space (C3) of an electron in a standing wave of light in Fig. 3. Such a procedure is similar to the numerical simulations shown in references [39–43, 45, 47–49]. In the simulation, the standing wave of light (6) has the polarization (11). The standing wave’s field amplitude is smoothly ramped up and down for the duration of five laser periods at the beginning and the end of the simulation by a sin² temporal envelope, as done in the references [39–43, 45, 47–49]. In this external field, the expansion coefficients $c_n$ and $d_n^*$ of the electron wave function (7) are propagated from the initial time $t = 0$ to the final time $t$ by

$$c_n(t) = \sum_{a,s'} \left[ U^{+,s:s'}_{n,a}(t,0) c^*_a(t) + U^{+,s':-s'}_{n,a}(t,0) d^*_a(t) \right] ,$$

with the quantum state propagation matrix $U^{+,s:s'}_{n,a}(t,0)$ in momentum space.

From the perturbation theory result in appendix C2 we expect that the spin dependent quantum state propagation of the initial spin state $c_0(0)$ with initial momentum $\mathbf{p}_0$ to the final spin state $c_2(t)$ will be proportional to

$$M_s = \frac{1}{\sqrt{8}} \begin{pmatrix} -1 & -1 - \sqrt{2} \\ -1 + \sqrt{2} & 1 \end{pmatrix} = s^\downarrow \cdot s^\uparrow .$$

We take this spin-propagation matrix from the Taylor expansion of the spin propagation matrix $M$ in appendix C2 with the parameters $\alpha_2 = 0$ and $\alpha_3 \approx (\sqrt{2} - 1) k_f^2 / 2$ (see footnote [78]). The right-hand side of Eq. (14) shows an outer product representation of $M_s$ by a pair of two-component spinors $s^\downarrow$ and $s^\uparrow$, from which one immediately obtains

$$\begin{align*}
\langle s^\downarrow | M_s | s^\downarrow \rangle &= 0 \\
\langle s^\uparrow | M_s | s^\downarrow \rangle &= 0 \\
\langle s^\downarrow | M_s | s^\uparrow \rangle &= 1 \\
\langle s^\uparrow | M_s | s^\uparrow \rangle &= 0 ,
\end{align*}$$

as one would expect from the corresponding Compton scattering process (9a) and the discussion in section II.C. For this reason we only show the projection on the diffracted spin state

$$\begin{align*}
\langle s^\downarrow | U^{+,+}_{2,0}(t,0) | s^\uparrow \rangle^2 &\approx 1 \\
\langle s^\uparrow | U^{+,+}_{2,0}(t,0) | s^\uparrow \rangle^2 &\approx 0
\end{align*}$$

and the initial quantum state

$$\begin{align*}
\langle s^\uparrow | U^{+,+}_{0,0}(t,0) | s^\uparrow \rangle^2 &\approx 0 \\
\langle s^\uparrow | U^{+,+}_{2,0}(t,0) | s^\uparrow \rangle^2 &\approx 0
\end{align*}$$

doing so

of the numeric solution of the propagation $U^{+,+}_{a,b}(t,0)$ in Fig. 3. These are the only non-negligible contributions of the time evolution. Equivalently, one can say that both expressions (16) sum up to approximately 1, such that the unitarity of the Dirac equation implies that all other excitations, in particular

$$\begin{align*}
\langle s^\downarrow | U^{+,+}_{0,0}(t,0) | s^\downarrow \rangle^2 &\approx 1 \\
\langle s^\uparrow | U^{+,+}_{0,0}(t,0) | s^\downarrow \rangle^2 &\approx 0 \\
\langle s^\downarrow | U^{+,+}_{2,0}(t,0) | s^\downarrow \rangle^2 &\approx 0 \\
\langle s^\uparrow | U^{+,+}_{2,0}(t,0) | s^\downarrow \rangle^2 &\approx 0 
\end{align*}$$

are negligibly small.

Most importantly, we do not see a dynamical transition from the initial condition with the opposite spin configuration $s^\uparrow$, corresponding to

$$\begin{align*}
\langle s^\downarrow | U^{+,+}_{0,0}(t,0) | s^\downarrow \rangle^2 &\approx 0 \\
\langle s^\uparrow | U^{+,+}_{0,0}(t,0) | s^\downarrow \rangle^2 &\approx 1 \\
\langle s^\downarrow | U^{+,+}_{2,0}(t,0) | s^\downarrow \rangle^2 &\approx 0 \\
\langle s^\uparrow | U^{+,+}_{2,0}(t,0) | s^\downarrow \rangle^2 &\approx 0
\end{align*}$$

Note, that the half period Rabi cycle, which is shown in Fig. 3 lasts for $6.29 \times 10^4$ optical cycles of the laser field, corresponding to the Rabi frequency

$$\Omega_R = 2.02 \times 10^{-7} \text{ m}$$

in the effective Rabi model

$$\langle s^\downarrow | U^{+,+}_{2,0}(t,0) | s^\uparrow \rangle^2 = \sin \left( \frac{\Omega_R t}{2} \right)^2 .$$

This is consistent with the approximate equation for the matrix element

$$\langle s^\downarrow | U^{+,+}_{2,0}(t,0) | s^\downarrow \rangle^2 \approx \frac{\alpha_2 q W / k_f}{8 m^2 \sqrt{2}^2} .$$
of the perturbative solution of the Dirac equation in Eq. (C14). In this context, we assume that the left-hand side of Eq. (21) can be identified with the analytic short time approximation of the Rabi model (20)

$$\left| \langle s^\wedge \mid U_{2,0}^{\pm \mp}(t,0) \mid s^\wedge \rangle \right|^2 \approx \left( \frac{\Omega_R t}{2} \right)^2,$$  

(22)

where we have set the parameters

$$e\mathcal{A}/m = e\mathcal{A}'/m = 4.74 \times 10^{-2}$$  

(23a)

$$k_1/m = 2.54 \times 10^{-2}$$  

(23b)

in our numeric simulation. We chose the photon energy to be 13 keV, corresponding to the value of $k_1$ in Eq. (23b) for the simulation. Similarly, we have set the simulation’s laser field amplitude $\mathcal{A}$ and $\mathcal{A}'$ in (23a), such that the half Rabi period will last exactly 20 fs. This value corresponds to the value of the Rabi frequency (19).

The actual numeric implementation was carried out in the basis of the states $c_n^\uparrow$ and $d_n^\uparrow$ with spin up and spin down $s \in \{\uparrow, \downarrow\}$, where the matrix elements with respect to the spin states $s^\wedge$ and $s^\wedge$ of the numeric propagation $U_{a,b}^{\pm \mp}(t,0)$ are given explicitly in appendix (D). Note that the transition amplitudes $U_{n,0}(t,0)$ of higher momentum states $|n\rangle$ are dropping off exponentially for the chosen parameters in Eq. (23), such that we have truncated the higher modes in the numeric solutions at $|n\rangle = 12$, similar to the procedure in references [39–43, 45, 47–49].

We want to point out that Eqs. (15) demonstrate a spin-dependent diffraction effect: While the initial spin configuration $s^\wedge$ is diffracted into a $s^\wedge$ configuration, an initial spin $s^\wedge$ is not diffracted at all! Thus, electrons are filtered according to their initial spin orientation. Also, the outer product in (14) implies that whatever electron spin is diffracted, the final electron spin will always be $s^\wedge$. This also demonstrates that the electron spin can be polarized by the diffraction mechanism. These two properties (filtering and polarization of the electron spin) are the same characteristics which we have already pointed out in our previous work [48], where a two-photon spin-dependent diffraction is presented which has similar properties as in this work. However, the spin-dependent diffraction effect in reference [48] only appears after multiple Rabi cycles, whereas the spin-dependent diffraction in our current work appears already with the rise of the transition’s oscillation in the form of a Bragg peak, which appears to be more suitable for the experimental implementation.

We also want to point out that the spin-dependent propagation matrix in Eq. (14) is a generalization of our statement in reference [48], in which a spin-dependent quantum state propagation has been identified to be proportional to a projection matrix. In contrast, Eq. (14) demonstrates explicitly that even a projection is not the most general characterization for spin-dependent dynamics. A general and specific criterion for spin-dependent diffraction dynamics might be non-trivial and be a subject of future investigations.

IV. EXPERIMENTAL IMPLEMENTATION CONSIDERATIONS

We want to discuss a possible experimental implementation of the spin-dependent laser electron interaction, according to the setup in Fig. 4. In this example, the source of the X-ray laser beams is assumed to be the Shanghai High Repetition Rate XFEL and Extreme Light Facility (SHINE), which is currently under construction [79]. Within its design parameters, SHINE will provide 100 GW laser pulses at 13 keV photon energy and with a pulse duration of 20 fs. When the beam is focused to 100 nm, the peak intensity reaches $1.2 \times 10^{21} W/cm^2$. A coincident laser pulse overlap at the interaction point is achieved by reflecting the two beams as in the arrangement in Fig. 4. Circular polarization can be converted from the linear polarized laser beam by utilizing a phase retardation setup in X-ray diffraction [80]. In this way, two coincident, counterpropagating, high intensity pulses can be established at the beam focus, with a linearly polarized beam from the left and a circularly polarized beam from the right. By assuming mirror reflectivities of 85%, a phase retarder transmittivity of 55% and a beam splitter design with 34% transmission and 56% reflection [81] one estimates an intensity of $1.2 \times 10^{20} W/cm^2$ for the left and right beam at the laser focus spot. Eq. (21) can be written in terms of SI units as

$$\left| \langle s^\wedge \mid U_{2,0}^{\pm \mp}(t,0) \mid s^\wedge \rangle \right|^2 \approx \left( \frac{\alpha \lambda c}{8\pi \sqrt{2} k_l} \frac{I_1^{1/2} I_2^{3/2} t}{\hbar k_l} \right)^2.$$  

(24)

Here, $\alpha$ is the fine-structure constant and $\lambda_c$ the Compton wavelength and $I_1$ and $I_2$ are the intensities of the
left- and right propagating laser beams. Evaluated with the parameters above, one is expecting a probability of about $1.1 \cdot 10^{-7}$ for an electron with a spin $\uparrow$ orientation to be diffracted in the direction of beam $B$. Since we are discussing a spin-dependent diffraction scheme, electrons with spin $\downarrow$ orientation will not be diffracted into beam $B$. For undergoing spin-dependent diffraction, the electrons have to have the specific momentum of $511 \text{ keV}/c$ along the $z$-axis, corresponding to a kinetic energy $212 \text{ keV}$. When undergoing diffraction, the electron will pick up two longitudinal photon momenta of $13 \text{ keV}/c$ along the $z$-axis. Since the momentum change is longitudinal, one can relate this to a diffraction angle of $\theta = 2.9^\circ$ from the scattering geometry. Spin polarized electron pulses with charges of $10 \text{ fC}$ are available [82] and with the temporal electron bunch width of $10 \text{ ps}$, one expects 124 electrons to cross the beam focal spot in its $20 \text{ fs}$ duration. Therefore, with the SHINE aimed repetition rate of $1 \text{ MHz}$ we estimate a countrate of $13$ electrons per second for the spin-dependent electron diffraction effect. Similar parameters for establishing the considered experimental configuration can also be reached at the LCLS in Stanford [83] and the European X-FEL in Hamburg [84].

V. DISCUSSION AND OUTLOOK

In this article we have discussed a spin-dependent Kapitza-Dirac diffraction effect, which can be implemented in the form of a Bragg scattering setup and which requires only the interaction with two of the standing light wave's photons. Open questions for the effect are the influence of the laser beam focus on the spin-dependent electron dynamics. Within this article, we have treated the laser beam and also the electron wave function as a discrete superposition of a finite number of plane waves, where a Gaussian beam and a Gaussian wave packet would model electron and laser more realistically. In this context the question arises, how a small longitudinal field component [85], which is implied by the laser beam focus, is influencing the spin dynamics. Also, the contribution of spontaneous emission of electromagnetic radiation as compared to the induced emission into the laser beam is of relevance and can be computed [86]. The question on how the quantum state of the laser field is modified by the electron diffraction dynamics is also of relevance, because the Compton scattering version of the effect is known to violate the conservation of intrinsic angular momentum [68].

There are two possible laser frequency regimes for the implementation of the effect, which are realistic in terms of available laser intensity for the experiment: The optical regime and the x-ray regime. The optical regime has the advantage that the classical nonlinearity parameter $\xi = qA/m$ can reach values of $1$ with comparably low effort, such that high photon number Kapitza-Dirac scattering, as for example discussed in references [39, 40, 44–47, 49–51] could be possible. Note, that the short-time diffraction probability and the transition’s Rabi frequency are proportional to the field amplitude $A$ to the power of the number of interacting photons, implying that either $\xi$ should be close to $1$ or the number of contributing photons should be as small as possible. Bi-chromatic setups [47, 49] appear promising for the experiment due to potentially long laser-electron interaction times caused by low initial and final electron momenta. However, one challenge with optical systems would be the control of the transverse electron momentum and the laser polarization such that the effect does not smear out. A look on the matrix (C17) of the polarization dependent spin dynamics for the electron in the laser beam tells that the electron momentum should be under control on the order of the photon momentum $k_l$. Also the laser polarization should be controlled on the accuracy level $k_l/m$, where we have $k_l \approx 10^{-6}m$ in the optical regime.

In the x-ray regime, on the other hand, this need of fine tuning would be only at the percent level. Here, one faces the challenge of providing field amplitudes, such that $\xi$ is close to one, which might be possible for the case of small beam foci. Therefore, for implementing a spin-dependent diffraction setup for x-rays, a lower order photon interaction Kapitza-Dirac effect would be beneficial. Two photon scattering would be the lowest possible configuration for Kapitza-Dirac-like scattering, since a one-photon interaction is not compatible with the conservation of energy and momentum. A two-photon setup from a previous investigation which only depends on a longitudinal electron momentum [43, 48] appears to be promising. However, for this scenario one faces the challenge that the spin oscillations are dependent on simultaneous Rabi oscillations with an enhanced frequency by the factor $m/k_l$, which also would imply the necessity of fine tuning. In contrast, the spin-dependent two photon effect which is discussed within this article is not superimposed to a larger spin-preserving term in the electron spin propagation. Therefore, only the beginning of a Rabi cycle (ie. the Bragg peak) of the diffraction effect would have to be observed for seeing the spin-dependent electron-laser interaction. For this reason, the spin-dependent electron diffraction effect as discussed in section IV appears to be suitable for implementing spin-dependent electron diffraction in standing light waves.

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Appendix A: Notion and Conventions

1. General relations

We work in the Gaussian unit system and set \( h = 1 \) and \( c = 1 \). Correspondingly, the elementary charge evaluates as the square root of the fine structure constant \( q = \sqrt{\alpha} \) in the framework of this convention. As an exception, we use SI units in the experimental section IV.

We define the Dirac gamma matrices as

\[
\gamma_i = \begin{pmatrix} 1 & 0 \\ 0 & \sigma_i \end{pmatrix}, \quad \sigma_i = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},
\]

and Einstein’s sum convention

\[
a_{\mu}b^\mu = \sum_{\mu} a_{\mu}b^{\mu} \quad \text{(A2)}
\]

applies everywhere, where a pair of identical upper and lower indices appears. Co- and contravariant indices may be raised and lowered as usual in covariant notation. A central dot between four-vectors just denotes the Euclidean inner product \( a \cdot b = \sum_n a_n b_n \) over the three vector components.

The Pauli matrices \( \sigma_x = \sigma_1, \sigma_y = \sigma_2 \) and \( \sigma_z = \sigma_3 \) are

\[
\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

We define the Dirac gamma matrices as

\[
\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma^i = \left( -\sigma_i, \sigma_i \right),
\]

where \( \mathbf{1} \) is the \( 2 \times 2 \) identity matrix. We also make use of the standard representation of the Dirac matrices \( \beta = \gamma^0, \alpha^i = \gamma^0 \gamma^i \). We use the symbol \( \ast \) for complex conjugation and the symbol \( T \) for transposition. The combined complex conjugation and transposition is denoted by the dagger symbol \( \dagger \). An adjoint spinor with the \( \gamma^0 \) matrix is abbreviated with a bar on top \( \bar{u} = u^\dagger \gamma^0 \). The Feynman slash notation \( \slash = a_\mu \gamma^\mu \) is used as an abbreviation of the contraction of the Dirac gamma matrices with a four-vector. The dot on top of a time-dependent function \( f(t) \) denotes its time derivative

\[
f(t) = \frac{\partial f}{\partial t}.
\]

The electro-magnetic field tensor in the QED-Lagrangian (2) is

\[
F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu,
\]

with the derivative

\[
\partial^\mu = \frac{\partial}{\partial x^\mu}.
\]

The bi-spinors (four component part of the plane wave eigensolutions of the free Dirac Hamiltonian) are

\[
u_k^\pm = \frac{m}{\mathcal{E}_k} \frac{\pm \mathcal{E}_k + m}{2m} \left( \frac{\sigma_k}{\mathcal{E}_k + m} \chi^s \right), \quad (A8a)
\]

\[
u_k^\mp = \frac{m}{\mathcal{E}_k} \frac{\pm \mathcal{E}_k + m}{2m} \left( \frac{\sigma_k}{\mathcal{E}_k + m} \chi^s \right), \quad (A8b)
\]

where we have absorbed the phase space factor \( m/\mathcal{E}_k \) from the Compton cross section formula into the normalization of the spinor definition.

2. Commutation relations

We assume commutation relations \([\cdot, \cdot]\) for the photon particle operators and anti-commutation \((\cdot, \cdot)\) for the electron particles and anti-particle operators

\[
[a_k^\lambda, a_{k'}^{\eta \dagger}] = (c_k^\lambda, c_{k'}^{\eta \dagger}) = (d_k^{\lambda \dagger}, d_{k'}^{\eta \dagger}) = \delta_{k, k'} \delta_{\lambda, \eta} \quad (A9a)
\]

\[
[a_k^\lambda, a_{k'}^{\eta \dagger}] = (c_k^\lambda, c_{k'}^{\eta \dagger}) = (d_k^{\lambda \dagger}, d_{k'}^{\eta \dagger}) = 0.
\]

We also assume that electron particle and anti-particle operators anti-commute with each other and photon operators commute with electron particle and anti-particle operators.

\[
\{c_k^\lambda, d_{k'}^{\eta \dagger}\} = \{c_k^\lambda, d_{k'}^{\eta \dagger}\} = 0
\]

\[
[a_k^\lambda, a_{k'}^{\eta \dagger}] = \{a_k^\lambda, a_{k'}^{\eta \dagger}\} = 0
\]

3. Helicity

We define the left and right handed photon creation operators

\[
a_{k_1}^{L \dagger} = a_{k_1}^{L \dagger} = a_{k_1}^{L \dagger} - i a_{k_1}^{3 \dagger} \quad (A10a)
\]

\[
a_{k_1}^{R \dagger} = a_{k_1}^{R \dagger} = a_{k_1}^{2 \dagger} + i a_{k_1}^{3 \dagger} \quad (A10b)
\]

\[
a_{k_1}^{L \dagger} = a_{k_1}^{L \dagger} = a_{k_1}^{2 \dagger} + i a_{k_1}^{3 \dagger} \quad (A10c)
\]

\[
a_{-k_1}^{R \dagger} = a_{-k_1}^{R \dagger} = a_{-k_1}^{3 \dagger} - i a_{-k_1}^{3 \dagger} \quad (A10d)
\]

Note, that this definition contains consistent helicities of the photons which are propagating in the \( x \) or \(-x\) direction, in contrast to the left and right circular polarization introduced in reference [68]. Also note that reference [68] is not accounting for the complex conjugation of the outgoing photon polarization in the Compton tensor (5) (see for example [72]). In this work both issues are accounted for.
4. Spin

We denote the spin quantum state as a two component object
\[ s^\sigma = \begin{pmatrix} s_1^\sigma \\ s_2^\sigma \end{pmatrix} \]  
(A11)

with the two components \( s_1^\sigma \) and \( s_2^\sigma \). For spin up and spin down we have
\[ s^\dagger_\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad s^\dagger_\downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \]  
(A12)

As in reference [68] we also define the components of the tilted spin states
\[ s_1^\wedge = \cos(11\pi/8) = N_-(1 - \sqrt{2}) \]  
(A13a)
\[ s_2^\wedge = \sin(11\pi/8) = -N_- \]  
(A13b)
\[ s_1^\hate = \cos(15\pi/8) = N_+(1 + \sqrt{2}) \]  
(A13c)
\[ s_2^\hate = \sin(15\pi/8) = -N_+ \]  
(A13d)

for \( s^\wedge \) and \( s^\hate \), where the specific values 11\( \pi/8 \) and 15\( \pi/8 \) are used to keep consistency to the conventions in reference [68]. The \( N_+ \) and \( N_- \) are the normalization factors
\[ N_+ = \sqrt{\frac{2}{(2 + \sqrt{2})}}^{-1} \]  
(A14a)
\[ N_- = \sqrt{\frac{2}{(2 - \sqrt{2})}}^{-1}. \]  
(A14b)

Correspondingly, with the spin up and spin down electron creation operators \( c_p^\dagger \) and \( c_\uparrow \), we introduce the electron creation operators
\[ c_p^\wedge = s_1^\wedge c_p^\dagger + s_2^\wedge c_\uparrow \]  
(A15a)
\[ c_p^\hate = s_1^\hate c_p^\dagger + s_2^\hate c_\uparrow. \]  
(A15b)

Appendix B: Perturbative electron interaction with a quantized photon field

1. Development of frame-fixed, quantized electron-photon Hamiltonian

The QED Lagrangian density in Eq. (2) implies the free Hamiltonian for electrons and their anti-particles [87]
\[ H_e = \sum_{k,s} \mathcal{E}_k \left( c_{k,s}^\dagger c_k^s + d_{k,s}^\dagger d_k^s \right) \]  
(B1)
as well as the free Hamiltonian for photons [87]
\[ H_p = \sum_{k,r} k a_{k,r}^\dagger a_k^r, \]  
(B2)

where \( \mathcal{E}_k = \sqrt{m^2 + k^2} \) is the relativistic energy momentum relation and \( k = |k| \) the dispersion of light in vacuum. We want to work out the interaction part of the Hamiltonian from the Hamiltonian density
\[ \mathcal{H} = \Pi \bar{\Psi} + \Pi^\mu \mathcal{A}_\mu - \mathcal{L}. \]  
(B3)

\( \Pi \) and \( \Pi^\mu \) are the conjugated momenta of the Dirac field and the photon field, respectively. The interaction part of the Lagrangian density (2) is
\[ \mathcal{L}_{\text{int}} = -e \bar{\Psi} \gamma_\mu A^\mu \Psi, \]  
(B4)

implying the interaction part of the Hamiltonian density
\[ \mathcal{H}_{\text{int}} = e \bar{\Psi} \gamma_\mu A^\mu \Psi. \]  
(B5)

We denote the electron field operators \( \Psi, \bar{\Psi} \) and photon field operator \( A_\mu \) by
\[ \Psi(x,t) = \sum_{k,s} \left( c_{k,s}^\dagger u_k^s e^{-ik \cdot x} + d_{k,s}^\dagger v_k^s e^{ik \cdot x} \right) \]  
(B6a)
\[ \bar{\Psi}(x,t) = \sum_{k,s} \left( c_{k,s}^\dagger u_k^s e^{ik \cdot x} + d_{k,s}^\dagger v_k^s e^{-ik \cdot x} \right) \]  
(B6b)
\[ A_{\mu}(x,t) = \sum_{k,r} \left( \epsilon_{\mu,k}^r e^{-ik \cdot x} + \epsilon_{\mu,k}^r e^{ik \cdot x} \right), \]  
(B6c)

where \( u_k^s \) and \( v_k^s \) are the bi-spinors (A8) and \( \epsilon_{\mu,k}^r \) are the four (r index) four-polarization (\( \mu \) index) vectors of the photon field. Inserting the definitions (B6) in the interaction part of the Hamilton density (B5) results in the interaction Hamiltonian
\[ \mathcal{H}_{\text{int}} = e \int d^3 x \bar{\Psi} \gamma_\mu A^\mu \Psi = e \sum_{k,r,k',r',\mu} \left[ \right] \]  
(B7a)

\[ \left( \bar{u}_{k'}^r \gamma_\mu k u_k^s \right) c_{k'}^\dagger c_k^s \alpha_{\mu,k-k'} \]  
(B7b)
\[ + \left( \bar{u}_{k'}^r \gamma_\mu k' u_k^s \right) c_{k'}^\dagger c_k^s \alpha_{\mu,k-k'} \]  
(B7c)
\[ + \left( \bar{v}_{k'}^r \gamma_\mu k u_k^s \right) d_{k'}^\dagger d_k^s \alpha_{\mu,k-k'} \]  
(B7d)
\[ + \left( \bar{v}_{k'}^r \gamma_\mu k' u_k^s \right) d_{k'}^\dagger d_k^s \alpha_{\mu,k-k'} \]  
(B7e)
\[ + \left( \bar{u}_{k'}^r \gamma_\mu k v_k^s \right) \epsilon_{\mu,k}^r \alpha_{\mu,k-k'} \]  
(B7f)
\[ + \left( \bar{u}_{k'}^r \gamma_\mu k' v_k^s \right) \epsilon_{\mu,k}^r \alpha_{\mu,k-k'} \]  
(B7g)
\[ + \left( \bar{v}_{k'}^r \gamma_\mu k v_k^s \right) \epsilon_{\mu,k}^r \alpha_{\mu,k+k'} \]  
(B7h)
\[ + \left( \bar{v}_{k'}^r \gamma_\mu k' v_k^s \right) \epsilon_{\mu,k}^r \alpha_{\mu,k+k'} \]  
(B7i)

The obtained interaction Hamiltonian (B7), together with the free Hamiltonians (B1) and (B2) determine the time evolution of vacuum excitations in the Schrödinger picture by
\[ i\hbar \frac{d}{dt} \Psi = H \Psi, \]  
(B8)

with
\[ H = H_e + H_p + H_{\text{int}}. \]  
(B9)
2. Perturbative derivation with quantized Hamiltonian

From Eq. (B8) one can establish the time evolution in form of a Dyson series, whose second order interaction term reads \[88\]

\[
U(t, t_0) = \frac{1}{i^2} \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 
\times U_0(t, t_2) H_{\text{int}} U_0(t_2, t_1) H_{\text{int}} U_0(t_1, t_0). \tag{B10}
\]

Here, \(U_0(t, t_0)\) is the free propagation

\[
U_0(t, t_0) = \exp[-i(H_c + H_p)(t - t_0)] \tag{B11}
\]

of electrons, their anti-particles and photons. The first order contributions of the Dyson series are not considered, since they do not contain resonant terms due to energy and momentum conservation. For the initial state \(\Psi_i\) in Eq. (1) we obtain from Eq. (B11)

\[
U_0(t_1, t_0) = \exp[-i(\mathcal{E}_{\psi_i} + k)(t_1 - t_0)] \tag{B12}
\]

for the time interval \([t_0, t_1]\) of the first free quantum state propagation in the second order perturbation (B10). Note, that while Eq. (B11) is an operator equation, the expressions in Eq. (B12) and later in the text also the Eqs. (B14) and (B17) are expressions where the operators have been acting at the operators of the quantum states and turned into ordinary complex numbers by the eigenvalue operations of the operators.

The first action of \(H_{\text{int}}\) on \(\Psi_i\) results in the intermediate states (3), where the interaction term

\[
\begin{align*}
(\bar{u}^{\psi_i}_{f, k'} u^{(r)_w}_{k} u^{(u)_w}_{k'}) \bar{c}_{f, k'} c_{k} a_w \text{ maps to } \Psi_a \tag{B13a} \\
(\bar{u}^{\psi_i}_{f, k'} u^{(f)_w}_{k} u^{(r)_w}_{k'}) \bar{c}_{f, k'} c_{k} a_w \text{ maps to } \Psi_b \tag{B13b} \\
(\bar{u}^{\psi_i}_{f, k'} u^{(f)_w}_{k} u^{(r)_w}_{k'}) \bar{c}_{f, k'} c_{k} a_w \text{ maps to } \Psi_c \tag{B13c} \\
(\bar{u}^{\psi_i}_{f, k'} u^{(f)_w}_{k} u^{(r)_w}_{k'}) \bar{c}_{f, k'} c_{k} a_w \text{ maps to } \Psi_d \tag{B13d}
\end{align*}
\]

as illustrated in Fig. 1. In correspondence, for the time interval \([t_1, t_2]\) of the second free propagation in (B10) one obtains the free propagation

\[
\begin{align*}
U_0(t_2, t_1) &= \exp[-i(\mathcal{E}_{\psi_i} + k)(t_2 - t_1)] \tag{B14a} \\
U_0(t_2, t_1) &= \exp[-i(\mathcal{E}_{\psi_i} + k + k')(t_2 - t_1)] \tag{B14b} \\
U_0(t_2, t_1) &= \exp[-i(\mathcal{E}_{\psi_i} + \mathcal{E}_{\psi_j} + \mathcal{E}_{-\psi_i + k'})(t_2 - t_1)] \tag{B14c} \\
U_0(t_2, t_1) &= \exp[-i(\mathcal{E}_{\psi_i} + \mathcal{E}_{\psi_j} + \mathcal{E}_{-\psi_i - k + k'})(t_2 - t_1)] \tag{B14d}
\end{align*}
\]

for \(\Psi_a, \Psi_b, \Psi_c \text{ and } \Psi_d\), respectively. For the following second interaction \(H_{\text{int}}\) in Eq. (B10) only terms are relevant which fulfill energy conservation, as all other contributions will oscillate in off-resonant Rabi cycles of low amplitude. Particle excitations different than \(\Psi_f\) in Eq. (4) are therefore not possible for asymptotically long times and the final momentum \(p_f\) of the electron and \(k'\) photon must fulfill the two particle energy conservation relation

\[
\mathcal{E}_{\psi_i} + k = \mathcal{E}_{\psi_f} + k'. \tag{B15}
\]

According to the above considerations, the only contributions in the interaction Hamiltonian (B7) that map back to the final state \(\Psi_f\) are

\[
\begin{align*}
\left(\bar{u}^{\psi_i}_{f, k'} u^{(r)_w}_{k} u^{(u)_w}_{k'}\right) c_{f'} c_{k'} a_w & \text{ from } \Psi_a \tag{B16a} \\
\left(\bar{u}^{\psi_i}_{f, k'} u^{(f)_w}_{k} u^{(r)_w}_{k'}\right) c_{f'} c_{k'} a_w & \text{ from } \Psi_b \tag{B16b} \\
\left(\bar{\psi}^{(r)_w}_{-\psi_i - k'} u^{(f)_w}_{k} u^{(r)_w}_{k'}\right) d_{-\psi_i - k'} c_{k'} a_{k'} & \text{ from } \Psi_c \tag{B16c} \\
\left(\bar{\psi}^{(r)_w}_{-\psi_i - k'} u^{(f)_w}_{k} u^{(r)_w}_{k'}\right) d_{-\psi_i - k'} c_{k'} a_{k'} & \text{ from } \Psi_d \tag{B16d}
\end{align*}
\]

The free propagation (B11) of the final state \(\Psi_f\) evaluates to

\[
U_0(t, t_2) = \exp[-i(\mathcal{E}_{\psi_i} + k')(t_2 - t)] \tag{B17a}
\]

\[
= \exp[-i(\mathcal{E}_{\psi_i} + k)(t_2 - t)] \tag{B17b}
\]

and it’s phase oscillates with the same frequency as the free propagation of \(\Psi_i\) in Eq. (B12), due to the energy conservation relation (B15). Consequently, the oscillations of the perturbative contribution of the propagator (B10) with respect to the integration variables \(t_1\) and \(t_2\) oscillate in the exponential with the factor

\[
\begin{align*}
- i(\mathcal{E}_{\psi_i} + k - \mathcal{E}_{\psi_i} - k)(t_2 - t_1) \tag{B18a} \\
- i(\mathcal{E}_{\psi_i} - k' - \mathcal{E}_{\psi_i} + k')(t_2 - t_1) \tag{B18b} \\
- i(\mathcal{E}_{-\psi_i - k'} + \mathcal{E}_{-\psi_i} - k')(t_2 - t_1) \tag{B18c} \\
- i(\mathcal{E}_{-\psi_i - k'} + \mathcal{E}_{-\psi_i}) (t_2 - t_1) \tag{B18d}
\end{align*}
\]

for \(\Psi_a, \Psi_b, \Psi_c \text{ and } \Psi_d\), respectively. Note that Eq. (B15) has been substituted, to arrive at (B18). For the upper integration limit of the integral with respect to \(t_1\) in Eq. (B10), the phase terms (B18) are canceling to zero, such that the integration with respect to \(t_2\) will be independent of \(t_2\), resulting in a solution which is growing linear in time. Such a linear growth results then in Rabi oscillations, if one accounts for higher order terms, as implied by the unitary time evolution (see [25]). In accordance, the integration with respect to \(t_1\) yields the prefactors

\[
\begin{align*}
\mathcal{F}_a &= (\mathcal{E}_{\psi_i} - \mathcal{E}_{\psi_i} + k + k)^{-1} \tag{B19a} \\
\mathcal{F}_b &= (\mathcal{E}_{\psi_i} - \mathcal{E}_{\psi_i} - k' - k')^{-1} \tag{B19b} \\
\mathcal{F}_c &= (\mathcal{E}_{\psi_i} + \mathcal{E}_{\psi_i} - k' - k')^{-1} \tag{B19c} \\
\mathcal{F}_d &= (\mathcal{E}_{\psi_i} + \mathcal{E}_{\psi_i} + k + k)^{-1}. \tag{B19d}
\end{align*}
\]

in Eq. (B10). Here, we accounted for an additional minus sign for Eqs. (B19c) and (B19d) due to the commutation relations (A9) of the additional virtual electron-positron pair and also we multiplied all terms with another factor.
for a perturbative electron-photon interaction. Taking the prefactors (B19), together with the corresponding interaction matrix elements in (B13) and (B16) and substituting them into the propagator (B10) we arrive at the expression

\[
U^{s',s;r,w}(t,t_0) = -i(t-t_0) \exp \left[ -i(E_{p_1} + k)(t-t_0) \right]
\]

\[
\times \sum_{s''} \left[ F_a \left( \bar{u}_{p_1}^{s'}(r) u_{p_1+k}^{s''} \right) \left( \bar{u}_{p_1+k}^{s''}(w) u_{p_1}^{s'} \right) + F_b \left( \bar{u}_{p_1}^{s'}(w) u_{p_1-k}^{s''} \right) \left( \bar{u}_{p_1-k}^{s''}(r) u_{p_1}^{s'} \right) + F_c \left( \bar{u}_{p_1}^{s'}(k) u_{p_1+k}^{s''} \right) \left( \bar{u}_{p_1+k}^{s''}(r) u_{p_1}^{s'} \right) + F_d \left( \bar{u}_{p_1}^{s'}(r) u_{p_1-k}^{s''} \right) \left( \bar{u}_{p_1-k}^{s''}(w) u_{p_1}^{s'} \right) \right].
\]

(B20)

In technical terms, the propagator (B20) should be understood in the following way: The operator \( U(t,t_0) \) in Eq. (B10) is applied at the initial quantum state \( c_{p_1}^{s} a_{k}^{\dagger} |0\rangle \) in Eq. (1) and the propagation matrix (B20) is determined by the amplitude of the final states \( c_{p_1}^{s'} a_{k}^{\dagger} |0\rangle \) of Eq. (4). These final states are the only relevant, resonant states, which are seen as non-vanishing contributions after long times \( t \). In this context we conclude the approximate relation

\[
U(t,t_0) c_{p_1}^{s} a_{k}^{\dagger} |0\rangle \approx \sum_{s',r} U^{s',s;r,w}(t,t_0) c_{p_1}^{s'} a_{k}^{\dagger} |0\rangle
\]

(B21)

for a perturbative electron-photon interaction.

3. Identification with Compton tensor from quantum field theory

The photon polarization dependent electron spin coupling matrix in Eq. (B20) consists of the components

\[
\sum_{s''} F_a \left( \bar{u}_{p_1}^{s'}(r) u_{p_1+k}^{s''} \right) \left( \bar{u}_{p_1+k}^{s''}(w) u_{p_1}^{s'} \right)
\]

(B22a)

\[
\sum_{s''} F_b \left( \bar{u}_{p_1}^{s'}(w) u_{p_1-k}^{s''} \right) \left( \bar{u}_{p_1-k}^{s''}(r) u_{p_1}^{s'} \right)
\]

(B22b)

\[
\sum_{s''} F_c \left( \bar{u}_{p_1}^{s'}(k) u_{p_1+k}^{s''} \right) \left( \bar{u}_{p_1+k}^{s''}(r) u_{p_1}^{s'} \right)
\]

(B22c)

\[
\sum_{s''} F_d \left( \bar{u}_{p_1}^{s'}(r) u_{p_1-k}^{s''} \right) \left( \bar{u}_{p_1-k}^{s''}(w) u_{p_1}^{s'} \right)
\]

(B22d)

where each line corresponds to the intermediate quantum states (3) and the corresponding spin and polarization dependent matrix elements as well as prefactors have been denoted in equations (B13), (B16) and (B19), respectively. These expressions can be further simplified to appear as final \( S \)-matrix expressions in quantum field theory. First we can substitute the identities

\[
\sum_s u_p^s u_p^{s'} = \frac{\bar{p} + m}{2E_p}
\]

(B23a)

\[
\sum_s u_p^s u_p^{s'} = \frac{\bar{p} - m}{2E_p}
\]

(B23b)

into the expressions (B22), resulting in

\[
(F_a - F_d) \bar{u}_{p_1}^{s'}(r) \frac{\bar{p} + \bar{k} + m}{2E_{p_1+k}} u_{p_1}^s + (F_b - F_c) \bar{u}_{p_1}^{s'}(w) \frac{\bar{p} - \bar{k} + m}{2E_{p_1-k'}} u_{p_1}^s.
\]

(B24a)

Eqs. (B22a) and (B22d), as well as Eqs. (B22b) and (B22c) are summed up into Eq. (B24a) and Eq. (B24b), respectively. In Eq. (B24a) we can simplify

\[
\frac{F_a - F_d}{2E_{p_1+k}} = \frac{1}{(E_{p_1} + k)^2 - (E_{p_1} + k)^2} = \frac{1}{2p_l \cdot k}
\]

and similarly in Eq. (B24b) we can simplify

\[
\frac{F_b - F_c}{2E_{p_1-k'}} = \frac{1}{2E_{p_1-k'}^2 - (E_{p_1} - k')^2} = \frac{1}{2p_l \cdot k'}.
\]

(B6)

Summing up also Eqs. (B24a) and (B24b) finally results in the Compton tensor (5).

Appendix C: Perturbative solution of the Dirac equation in an external standing light wave

1. Derivation

The Lagrangian (2) implies the Dirac equation

\[
\imath \dot{\psi}(x,t) = \left( -\imath \nabla - eA(x,t) \right) \cdot \alpha + m \beta + eA^0(x,t) \psi(x,t)
\]

(C1)

from the Euler-Lagrange equations. In context of the one-particle Dirac equation, the quantum field of photons \( A \) in Eq. (B6) changes into the external vector- and scalar potential of a classical standing wave laser field of Eq. (6). When converting into classical variables, the photon creation and annihilation operators \( a_{k}^{\dagger} \) and \( a_{k} \) are substituted by the polarization four vectors \( a_{\mu}^{\dagger} \) and \( a_{\mu} \), where we assume a left propagating field with polarization \( \alpha' \) and wave vector \( k' \), as well as the right propagating field with polarization \( \alpha \) and wave vector \( k_1 \). For the four polarization vectors \( \epsilon_{\mu}^{(r)} \) of the quantized photon field in (B6c) we assume the canonical unit vectors

\[
\epsilon_{0} = \epsilon_{1} = \epsilon_{2} = 1, \quad \epsilon_{3}^{(r)} = 0 \quad \text{else.}
\]

(C2)

We also point out that the vector field (6) allows for longitudinal excitations from Gauge transformations due
to the additional $A^0$ component, in contrast to reference [40]. The zero component is added to ease the identification with the Feynman slash notation for the interaction terms. Though a zero component is added for the vector field, we only consider transverse modes (corresponding to radiation gauge) of the electro-magnetic field in this work.

Similar to the photon field, we replace the quantum field of the electrons $\Psi$ in Eq. (B6a) by the single-particle electron wavefunction (7) in the Dirac equation (C1). Here, the electron annihilation and anti-electron creation field of the electrons $\Psi$ in Eq. (B6a) by the single-particle $c_{n'}^\dagger(t)$ and $d_{n'}^\dagger(t)$ with the discrete photon momenta $k_n = p + n k_l$ and initial momentum $p_l$, where the discrete steps in momentum space are implied by the standing wave field (6).

Inserting the wavefunction (7) and the external potential (6) in the Dirac equation (C1) and projecting on the plane wave eigensolutions of the Dirac equation $u_{k_n}^s e^{-i k_n \cdot x}$ and $v_{-k_n}^s e^{-i k_n \cdot x}$, we obtain the coupled system of differential equations

$$i\dot{c}_{n}^s(t) = \mathcal{E}_{k_n} c_{n}^s(t) + \sum_{n',s'} \left[ V_{n,n'}^{+,s';s'}(t) c_{n'}^s(t) + V_{n,n'}^{-,s';s'}(t) d_{n'}^s(t) \right],$$

$$i\dot{d}_{n}^s(t) = -\mathcal{E}_{k_n} d_{n}^s(t) + \sum_{n',s'} \left[ V_{n,n'}^{-,s';s'}(t) c_{n'}^s(t) + V_{n,n'}^{+,s';s'}(t) d_{n'}^s(t) \right],$$

(C3a, C3b)

similar to references [40] and [48]. Eq. (C3) contains the generalized interaction in momentum space

$$V_{n,n'}^{\gamma,s';s'}(t) = -\frac{e}{2} L_{n,n'}^{\gamma,s';s'}(t) \left[ (a_{\mu} e^{-ik t} + a_{\mu}^* e^{ikt}) \delta_{n',n+1} + (a_{\mu} e^{ikt} + a_{\mu}^* e^{-ikt}) \delta_{n',n-1} \right],$$

(C4)

whose definition is based on the generalized spin- and polarization dependent coupling

$L_{n,n'}^{+,s';s';\mu}$

(C5a)

$L_{n,n'}^{+,s';s';\mu}$

(C5b)

$L_{n,n'}^{-,s';s';\mu}$

(C5c)

$L_{n,n'}^{-,s';s';\mu}$

(C5d)

where the spinors $v_k^s$ (see Eq. (A8b)) have been introduced with opposite momentum $k$.

The second order perturbation (B10) also applies to quantum dynamics in the one-particle Dirac theory. We introduce the free propagation

$$U_{0;n,n'}^{+,s';s'}(t, t_0) = \exp \left(-i\mathcal{E}_{k_n}(t-t_0)\right) \delta_{n,n'} \delta_{s,s'},$$

(C6a)

$$U_{0;n,n'}^{-,s';s'}(t, t_0) = \exp \left(i\mathcal{E}_{k_n}(t-t_0)\right) \delta_{n,n'} \delta_{s,s'},$$

(C6b)

$$U_{0;n,n'}^{+,s';s'}(t, t_0) = U_{n,n'}^{+,s';s'}(t, t_0) = 0$$

(C6c)

of the momentum space expansion coefficients $c_{n}^s$. Then, by applying successively the substitutions

1. $U_{\text{int}} \to V(t_1)$ in between $U_0(t_2, t_1)$ and $U_0(t_1, t_0)$,
2. $U_{\text{int}} \to V(t_2)$ in between $U_0(t_2, t_1)$ and $U_0(t_2, t_1)$,
3. $U_0(t, t_0) \to U(t, t_0)$,
4. $U(t, t_0) \to U(t, t_0)$

in Eq. (B10) results in

$$U(t, t_0) \approx \frac{1}{i2} \int_{t_0}^{t_1} dt_2 \int_{t_0}^{t_1} dt_1 \times U_0(t_2, t_1) V(t_2) U_0(t_2, t_1) V(t_1) U_0(t_1, t_0)$$

(C7)

for the second order perturbative approximation of the quantum state propagation (13) of the electron wave function. Here $U$ and $V$ are matrices with the matrix product

$$[U_0(t_2, t_1) V(t_1)]^{\gamma,s'} = \sum_{n',s'} U_{0;n,n'}^{\gamma,s';s'}(t_2, t_1) V_{n',n'}^{s';s'}(t_1).$$

(C8)

Similar as for the quantized photon-electron case in appendix B, we only account for resonant contributions, which grow linear in time. And as in the rotating wave approximation we neglect the fast oscillating counter-rotating terms $\pm ik_l(t_2 + t_1)$. Hence, for the electron-electron propagation from momentum $k_0$ to $k_2$ we obtain

$$U_{2,0}^{+,s';s}(t, t_0) \approx \frac{q^2 a_{n}^s a_{n'}^{s'}}{4\epsilon_t^2} \sum_{n''} \int_{t_0}^{t} dt_2 \int_{t_0}^{t_2} dt_1 \left\{ L_{1,0}^{+,s';s'}(t_2) \delta_{n,n''} \right\}$$

(C9a)

$$L_{2,1}^{+,s';s',s''} \mathcal{E}_{k_0} \delta_{n,n''} \delta_{s,s''}$$

(C9b)

$$L_{2,1}^{+,s';s',s''} \mathcal{E}_{k_0} \delta_{n,n''} \delta_{s,s''}$$

(C9c)

$$L_{2,1}^{+,s';s',s''} \mathcal{E}_{k_0} \delta_{n,n''} \delta_{s,s''}$$

(C9d)

$$L_{2,1}^{+,s';s',s''} \mathcal{E}_{k_0} \delta_{n,n''} \delta_{s,s''}$$

(C9e)

with the phases

$$\xi_a = \exp \left[-i\mathcal{E}_{k_0} t + i \left(\mathcal{E}_{k_0} - \mathcal{E}_{k_0} + k_l\right)(t_2 - t_1) + i\mathcal{E}_{k_0} t_0\right]$$

(C9f)

$$\xi_b = \exp \left[-i\mathcal{E}_{k_0} t + i \left(\mathcal{E}_{k_0} - \mathcal{E}_{k_0} - k_l\right)(t_2 - t_1) + i\mathcal{E}_{k_0} t_0\right]$$

(C9g)

$$\xi_c = \exp \left[-i\mathcal{E}_{k_0} t + i \left(\mathcal{E}_{k_0} + \mathcal{E}_{k_0} - k_l\right)(t_2 - t_1) + i\mathcal{E}_{k_0} t_0\right]$$

(C9h)

$$\xi_d = \exp \left[-i\mathcal{E}_{k_0} t + i \left(\mathcal{E}_{k_0} + \mathcal{E}_{k_0} + k_l\right)(t_2 - t_1) + i\mathcal{E}_{k_0} t_0\right],$$

(C9i)
where the argument \((t, t_2, t_1, t_0)\) is left away at the left-hand side of Eqs. (C10). We also assumed \(E_{k_0} = E_{k_{00}}\) in the calculation which is implied by energy conservation. In Eqs. (C10) one can identify similar phase terms as in Eqs. (B18). Therefore, after integration over \(t_1\) and \(t_2\) one obtains

\[
U_{2,0}^{+,'++}(t, t_0) \approx -i \frac{q^2 a_{2s}^s}{4} (t - t_0) \exp \left[ -i E_{k_0} (t - t_0) \right] \sum_{s''} \left( F_a L_{2,1}^{s;+,s';+;s''} + F_b L_{2,1}^{s;+,s';+;s''} + F_c L_{2,1}^{s;--;s';+;s''} + F_d L_{2,1}^{s;--;s';+;s''} \right),
\]

with the prefactors

\[
F_a = (E_{k_0} - E_{k_1} + k_t)^{-1},
F_b = (E_{k_0} - E_{k_1} - k_t)^{-1},
F_c = (E_{k_0} + E_{k_1} - k_t)^{-1},
F_d = (E_{k_0} + E_{k_1} + k_t)^{-1}.
\]

Note, that the lower integration limit of the \(t_1\) integral in Eq. (C9) is only contributing non-resonant terms, which are neglected in Eq. (C11). Also, we point out that the prefactors \(F\) in Eq. (C12) are similar to the prefactors in Eq. (B19). In fact, \(F\) in Eq. (C12) could also be obtained from \(F\) in Eq. (B19) by imposing the substitutions \(E_{k_0}, E_{k_1}, k_t \rightarrow E_{k_0}, E_{k_1}, k_t\), \(k \rightarrow k_t\) and \(k' \rightarrow k_l\).

\section{Second order Taylor expansion of the Compton tensor}

For the terms in the last four lines in Eq. (C11) we define the expression

\[
M^{s',s';\mu\nu} = m \sqrt{\frac{E_{k_2}}{m}} \sqrt{\frac{E_{k_0}}{m}} \sum_{s'} \left( F_a L_{2,1}^{s',+,s';+;s''\mu\nu} + F_b L_{2,1}^{s',+,s';+;s''\mu\nu} + F_c L_{2,1}^{s',-,s';+;s''\mu\nu} + F_d L_{2,1}^{s',-,s';+;s''\mu\nu} \right),
\]

such that Eq. (C11) can be written as

\[
U_{2,0}^{+,'++}(t, t_0) \approx -i \sqrt{\frac{m}{E_{k_2}}} \sqrt{\frac{m}{E_{k_0}}} \frac{q^2 a_{2s}^s}{4m} M^{s',s';\mu\nu} (t - t_0) \exp \left[ -i E_{k_0} (t - t_0) \right].
\]

The matrix elements \(M^{s',s';\mu\nu}\) in (C13) are functions of the photon momentum \(k_t\) and the two transverse photon momenta \(k_2\) and \(k_3\). For the following calculation, we introduce the scaled parameters

\[
q_t = \frac{k_t}{m}, \quad q_2 = \frac{k_2}{m}, \quad q_3 = \frac{k_3}{m}
\]

and

\[
\tilde{q}_3 = \frac{k_3 - m}{m} = q_3 - 1.
\]

The Taylor expansion of \(M^{s',s';\mu\nu}\) with respect to the three parameters (C15) is

\[
M^{22} = \left( 1 + \frac{\sqrt{2} - 1}{2} q_t^2 - \frac{q_t^2}{2} \right) \mathbf{1} - i \frac{\sqrt{2} - 1}{\sqrt{2}} q_t \sigma_y - i \frac{3 - 2 \sqrt{2}}{\sqrt{2} q_3} q_t q_2 \sigma_z
\]

\[
M^{23} = -q_t \mathbf{1} + \left[ \frac{i}{2} \sigma_x + \frac{i}{2} \tilde{q}_3 \sigma_x + \frac{i}{2} q_2 \sigma_y - \frac{i}{2} \sigma_z \right] q_t
\]

\[
M^{32} = -q_t \mathbf{1} + \left[ \frac{i}{2} \sigma_x - \frac{i}{2} \tilde{q}_3 \sigma_x + \frac{i}{2} q_2 \sigma_y - \frac{i}{2} \sigma_z \right] q_t
\]

\[
M^{33} = -\tilde{q}_3 \left[ q_t + \frac{1}{2} q_2 + \frac{\sqrt{2} - 1}{2} q_t^2 + \frac{q_t^2}{2} \right] \mathbf{1} - i \frac{\sqrt{2} - 1}{\sqrt{2} q_3} q_t \sigma_y + i \frac{\sqrt{2} - 1}{\sqrt{2}} q_2 q_t \sigma_z.
\]

Here, we have accounted for all contributions up to the quadratic order in the expansion parameters \(q_t, q_2\) and \(q_3\) and their mixed orders. Note, that the Taylor expansion with respect to \(q_t\) and \(q_2\) is performed around their zero value \(k_t = 0\) and \(k_2 = 0\), while the Taylor expansion with respect to \(q_3\) is performed around the value \(k_3 = m\), to get an approximate expression in the vicinity around the initial and final momenta \(\tilde{p}_1\) and \(\tilde{p}_f\) (as defined in...
Eq. (8), about which the whole article is about. For this reason we have rewritten the electron momentum $q_3$ into the shifted momentum $\tilde{q}_3$ in Eq. (C16), where the Taylor expansion around the value $k_3 = m$ corresponds to a Taylor expansion around $\tilde{q}_3 = 0$. We point out that the Taylor expanded matrix (C17) shows the same matrix entries as the matrix (5) in reference [68], with the addition that Eq. (C17) also shows the second order terms of the Taylor expansion.

### Appendix D: Spin-projection of propagator

Assume, that $U$ is a complex $2 \times 2$ matrix, which is representing the propagation of a two-component spinor. Then the following matrix elements can be written as

\[
\begin{align}
\langle s \mid U_{a,b}^{+,+} \mid s \rangle &= \frac{1}{\sqrt{8}} \left( (1 - \sqrt{2}) U_{a,b}^{+,+,+,+} - U_{a,b}^{+,+,+,+} - (1 + \sqrt{2}) U_{a,b}^{+,+,+,+} \right) \\
\langle s \mid U_{a,b}^{+,+} \mid s \rangle &= \frac{1}{\sqrt{8}} \left( - U_{a,b}^{+,+,+,+} - (1 - \sqrt{2}) U_{a,b}^{+,+,+,+} - (1 + \sqrt{2}) U_{a,b}^{+,+,+,+} \right) \\
\langle s \mid U_{a,b}^{+,+} \mid s \rangle &= \frac{1}{\sqrt{8}} \left( - U_{a,b}^{+,+,+,+} - (1 + \sqrt{2}) U_{a,b}^{+,+,+,+} - (1 - \sqrt{2}) U_{a,b}^{+,+,+,+} \right) \\
\langle s \mid U_{a,b}^{+,+} \mid s \rangle &= \frac{1}{\sqrt{8}} \left( - (1 + \sqrt{2}) U_{a,b}^{+,+,+,+} - U_{a,b}^{+,+,+,+} - U_{a,b}^{+,+,+,+} - (1 - \sqrt{2}) U_{a,b}^{+,+,+,+} \right). 
\end{align}
\]
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