Spin-Dependent Structure Functions of the Photon

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Abstract

The implications of the positivity constraint on the presently unknown polarized structure function of the photon, \( g_1^\gamma(x, Q^2) \), are studied in detail. In particular the non-trivial consequences of this constraint, \(|g_1^\gamma(x, Q^2)| \leq F_1^\gamma(x, Q^2)\), in the next-to-leading order analysis of \( g_1^\gamma \) are pointed out by employing appropriate (DIS) factorization schemes related to \( g_1^\gamma \) and \( F_1^\gamma \) (rather than to \( F_2^\gamma \)).
The spin dependent structure function $g_1^\gamma(x, Q^2)$ of a longitudinally polarized photon was studied [1, 2] within the framework of the radiative parton model, developed [3] for the presently well measured and known structure function $F_2^\gamma(x, Q^2)$ of an unpolarized photon. In particular the next–to–leading order (NLO) analysis [2] of $g_1^\gamma$ adopted a perturbatively stable DIS, factorization scheme, as advocated in [3], and implemented some boundary conditions [1] at the low input scale $Q^2 = \mu^2 \simeq 0.3$ GeV$^2$ of the radiative parton model. These boundary conditions led, however, to a violation of the positivity constraint

$$|A_1^\gamma(x, Q^2)| \equiv |g_1^\gamma(x, Q^2) / F_1^\gamma(x, Q^2)| \leq 1.$$  

(1)

It was therefore suggested [2] to repeat the analysis [3] of $F_2^\gamma$ in a DIS, factorization scheme related to $F_1^\gamma$ rather than to $F_2^\gamma$ which was the source of the above mentioned violation. Such a reanalysis is obviously rather time consuming and leads, moreover, to a diminished perturbative stability of the resulting parton distributions. In the present letter we propose an alternative solution to the positivity constraint which avoids the need for the above mentioned reanalysis of the data on $F_2^\gamma$.

For this purpose we recall that the DIS, factorization scheme, suggested and adopted in [3] for unpolarized photon structure functions, is related to $F_2^\gamma$ as given in NLO(\overline{\text{MS}}) by

$$\frac{1}{x} F_2^\gamma(x, Q^2) = \sum_{q=u,d,s} e_q^2 \left\{ q^\gamma(x, Q^2) + \bar{q}^\gamma(x, Q^2) \\ + \frac{\alpha_s(Q^2)}{2\pi} [C_{q,2} \otimes (q + \bar{q}) \gamma + 2C_{g,2} \otimes g^\gamma] + 2e_q^2 \frac{\alpha}{2\pi} C_{\gamma,2}(x) \right\}$$  

(2)

where $\otimes$ denotes the usual convolution integral, and $\bar{q}^\gamma(x, Q^2) = q^\gamma(x, Q^2)$ and $g^\gamma(x, Q^2)$ provide the so–called ‘resolved’ contributions of $\gamma$ to $F_2^\gamma$ with the usual \overline{\text{MS}} coefficient
functions

\[ C_{q,2}(x) = C_{q,1}(x) + \frac{4}{3} 2x \]

\[ = \frac{4}{3} \left[ (1 + x^2) \left( \ln(1 - x) \right) + \frac{3}{2} \frac{1}{(1 - x)_+} - \frac{1 + x^2}{1 - x} \ln x \right. \]

\[ \left. + 3 + 2x - \left( \frac{9}{2} + \frac{\pi^2}{3} \right) \delta(1 - x) \right] \]

\[ C_{g,2}(x) = C_{g,1}(x) + \frac{1}{2} 4x(1 - x) \]

\[ = \frac{1}{2} \left\{ [x^2 + (1 - x)^2] \ln \frac{1 - x}{x} + 8x(1 - x) - 1 \right\}, \tag{3} \]

while \( C_{\gamma,2} \) provides the ‘direct’ contribution as calculated according to the ‘box’ diagram \( \gamma^*(Q^2)\gamma \to q\bar{q} \):

\[ C_{\gamma,i}(x) = \frac{3}{(1/2)} C_{g,i}(x) \tag{4} \]

with \( i = 1, 2 \). We have suppressed in (2) the contributions from heavy \((c, b)\) quarks since they are irrelevant for our present considerations. (The \( C_1 \) coefficient functions refer to \( F_1^\gamma \) needed below.) Notice that in unpolarized photon (and proton) DIS it is common to use the ‘mixed’ structure function \( F_2 = 2x F_1 + F_L \), rather than the purely transverse \( F_1 \) structure function, since the measured cross section is, apart from kinematically suppressed contributions, directly proportional to \( F_2 \). In order to avoid the instabilities encountered in NLO(\( \overline{\text{MS}} \)) in the large–\( x \) region due to the \( \ln(1 - x) \) term in \( C_{\gamma} \) in (4), the entire ‘direct’ \( C_{\gamma,2} \) term in (2) is absorbed into the \( \overline{\text{MS}} \) (anti)quark densities \( q^\gamma = \bar{q}^\gamma \) in (2) which defines the so–called DIS\( \gamma \) factorization scheme [3]: Eq.(5)

\[ q^\gamma(x, Q^2)_{\text{DIS}} = q^\gamma(x, Q^2) + e_q^2 \frac{\alpha}{2\pi} C_{\gamma,2}(x) \]

\[ g^\gamma(x, Q^2)_{\text{DIS}} = g^\gamma(x, Q^2). \tag{5} \]

This redefinition of parton distributions implies that the NLO(\( \overline{\text{MS}} \)) splitting functions \( k_{q,g}^{(1)}(x) \) of the photon into quarks and gluons, appearing in the inhomogeneous NLO renormalization group (RG)\( Q^2 \)–evolution equations [3] for \( f^\gamma(x, Q^2) \), have correspondingly to
be transformed according to [3, 4]

\[
\begin{align*}
  k_q^{(1)}(x)_{\text{DIS}_\gamma} &= k_q^{(1)}(x) - e_q^2 P_{qq}^{(0)} \otimes C_{\gamma,2} \\
  k_g^{(1)}(x)_{\text{DIS}_\gamma} &= k_g^{(1)}(x) - 2 \sum_q e_q^2 P_{gq}^{(0)} \otimes C_{\gamma,2}
\end{align*}
\] (6)

where

\[
\begin{align*}
  k_q^{(1)}(x) &= \frac{1}{2} 3e_q^2 \frac{4}{3} \left\{ -(1 - 2x) \ln^2 x - (1 - 4x) \ln x + 4 \ln(1 - x) - 9x + 4 \\
  &+ \left[ x^2 + (1 - x)^2 \right] \left[ 2 \ln^2 x + 2 \ln^2(1 - x) + 4 \ln x - 4 \ln x \ln(1 - x) \
  - 4 \ln(1 - x) + 10 - \frac{2}{3} \pi^2 \right] \right\} \\
  k_g^{(1)}(x) &= 3 \sum_q e_q^2 \frac{4}{3} \left\{ -2(1 + x) \ln^2 x - (6 + 10x) \ln x + \frac{4}{3x} + \frac{20}{3} x^2 + 8x - 16 \right\}
\end{align*}
\] (7)

with \( k_q^{(1)} \) referring to each single (anti)quark flavor. The LO splitting functions are given by \( P_{qq}^{(0)} = \frac{4}{3} \left( \frac{1 + x^2}{1 - x} \right) \) and \( P_{gq}^{(0)} = \frac{4}{3} [1 + (1 - x)^2] / x \). The NLO expression for \( F_{2\gamma}^\gamma \) in the \( \text{DIS}_\gamma \) factorization scheme is thus given by (2) with \( C_{\gamma,2} \) being dropped.

In order to comply with the positivity constraint (1) for the polarized structure function \( g_1^\gamma \) one has to consider a corresponding factorization scheme, \( \text{DIS}_{\gamma,1} \), related to \( F_1^\gamma \), the spin–averaged analogon to \( g_1^\gamma \), which is given in NLO(\( \overline{\text{MS}} \)) by

\[
\begin{align*}
  F_1^\gamma(x, Q^2) &= \frac{1}{2} \sum_{q=u,d,s} e_q^2 \left\{ q^\gamma(x, Q^2) + \bar{q}^\gamma(x, Q^2) \right. \\
  &+ \frac{\alpha_s(Q^2)}{2\pi} \left[ C_{q,1} \otimes (q + \bar{q})^\gamma + 2C_{g,1} \otimes g^\gamma \right] + 2e_q^2 \frac{\alpha}{2\pi} C_{\gamma,1}(x) \left\}
\end{align*}
\] (8)

with the \( C_1 \) coefficient functions being given by eqs. (3) and (4). Absorbing now the entire ‘direct’ \( C_{\gamma,1} \) term into the \( \overline{\text{MS}} \) quark densities \( q^\gamma = \bar{q}^\gamma \) defines the \( \text{DIS}_{\gamma,1} \) factorization scheme:

\[
\begin{align*}
  q^\gamma(x, Q^2)_{\text{DIS}_{\gamma,1}} &= q^\gamma(x, Q^2) + e_q^2 \frac{\alpha}{2\pi} C_{\gamma,1}(x) \\
  g^\gamma(x, Q^2)_{\text{DIS}_{\gamma,1}} &= g^\gamma(x, Q^2)
\end{align*}
\] (9)
with the corresponding change of the NLO(\overline{\text{MS}}) photon splitting functions $k_{q,g}^{(1)}(x)$, appearing in the NLO(\overline{\text{MS}}) RG evolution equations,

\begin{align}
    k_{q}^{(1)}(x)_{\text{DIS}_\gamma,1} &= k_{q}^{(1)}(x) - e_q^2 P_{qq}^{(0)} \otimes C_{\gamma,1} \\
    k_{g}^{(1)}(x)_{\text{DIS}_\gamma,1} &= k_{g}^{(1)}(x) - 2 \sum_q e_q^2 P_{gq}^{(0)} \otimes C_{\gamma,1} \quad (10)
\end{align}

in contrast to eq. (6). From the definitions (5) and (9) one obtains:

\begin{align}
    q^\gamma(x, Q^2)_{\text{DIS}_\gamma,1} &= q^\gamma(x, Q^2)_{\text{DIS}_\gamma} + e_q^2 \frac{\alpha}{2\pi} [C_{\gamma,1}(x) - C_{\gamma,2}(x)] \\
    &= q^\gamma(x, Q^2)_{\text{DIS}_\gamma} - e_q^2 \frac{\alpha}{2\pi} 12x(1 - x) \\
    g^\gamma(x, Q^2)_{\text{DIS}_\gamma,1} &= g^\gamma(x, Q^2)_{\text{DIS}_\gamma} \quad (11)
\end{align}

Thus the NLO expression for $F_{\gamma 1}$ in the DIS$_\gamma,1$ factorization scheme is given by (8) with the $C_{\gamma,1}$ term being dropped. Furthermore, the parton distributions in the DIS$_\gamma,1$ scheme are uniquely determined in terms of the well known DIS$_\gamma$ distributions [5, 6] in eq. (11), $f^\gamma(x, Q^2)_{\text{DIS}_\gamma}$ with $f = u, d, s, g$, related to $F_{\gamma 1}$. Since the perturbative stability has been optimized [5, 6] with respect to the experimentally measured structure function $F_{\gamma 2}$, the stability may obviously be diminished at some other place, e.g. $F_{\gamma 1}$, where the difference between the leading order (LO) predictions and the NLO ones is somewhat more pronounced as compared to $F_{\gamma 2}$. This of course does not affect the reliability of the NLO predictions. (The LO expressions for $F_{\gamma 1,2}$ are obviously obtained from eqs. (2) and (8) by simply setting $C_{q,g,\gamma} = 0$.)

The polarized parton distributions $\Delta f^\gamma$ in the analogous DIS$_\Delta\gamma$ factorization scheme are obtained in a similar way by considering the spin–dependent structure function $g_{1}^\gamma$ in (1) which, for the light $u, d, s$ quarks, is in NLO(\overline{\text{MS}}) given by

\begin{align}
g_{1}^\gamma(x, Q^2) &= \frac{1}{2} \sum_{q=u,d,s} e_q^2 \left\{ \Delta q^\gamma(x, Q^2) + \Delta \bar{q}^\gamma(x, Q^2) \\
    &+ \frac{\alpha_s(Q^2)}{2\pi} [\Delta C_q \otimes \Delta (q + \bar{q})^\gamma + 2\Delta C_g \otimes \Delta g^\gamma] + 2e_q^2 \frac{\alpha}{2\pi} \Delta C_{\gamma}(x) \right\} \quad (12)
\end{align}
with \( \Delta \bar{q}^\gamma = \Delta q^\gamma = q_+^\gamma - q_-^\gamma \) and \( \Delta g^\gamma = g_+^\gamma - g_-^\gamma \) as compared to the spin–averaged \( \bar{q}^\gamma = q^\gamma = q_+^\gamma + q_-^\gamma \) and \( g^\gamma = g_+^\gamma + g_-^\gamma \) in \( F_1^\gamma \) in (8) in terms of the positive and negative helicity densities \( q_\pm^\gamma \) and \( g_\pm^\gamma \). The polarized NLO(\( \overline{\text{MS}} \)) partonic coefficient functions [7, 8] for the ‘resolved’ contributions of a longitudinally polarized photon are given by

\[
\Delta C_q(x) = \frac{4}{3} \left[ (1 + x^2) \left( \frac{\ln(1 - x)}{1 - x} \right)_+ - \frac{3}{2} \frac{1}{(1 - x)_+} - \frac{1 + x^2}{1 - x} \ln x \right] \nonumber \\
+ 2 + x - \left( \frac{9}{2} + \frac{\pi^2}{3} \right) \delta(1 - x) 
\]

\[
\Delta C_g(x) = \frac{1}{2} \left[ (2x - 1) \left( \ln \frac{1 - x}{x} - 1 \right) + 2(1 - x) \right], 
\tag{13}
\]

and the ‘direct’ contribution of the polarized photon follows from

\[
\Delta C_\gamma(x) = \frac{3}{(1/2)} \Delta C_g(x). 
\tag{14}
\]

Absorbing this latter contribution in (12) entirely into the polarized (anti)quark distributions, one obtains, in complete analogy to the DIS\( \gamma,1 \) scheme in (9), the polarized DIS\( \Delta \gamma \) factorization scheme [2].

\[
\Delta q^\gamma(x, Q^2)_{\text{DIS}} = \Delta q^\gamma(x, Q^2) + e_q^2 \frac{\alpha}{2\pi} \Delta C_\gamma(x) \\
\Delta g^\gamma(x, Q^2)_{\text{DIS}} = \Delta g^\gamma(x, Q^2). 
\tag{15}
\]

Correspondingly, the polarized NLO(\( \overline{\text{MS}} \)) splitting functions \( \Delta k^{(1)}_{q,g}(x) \) of the polarized photon into quarks and gluons, appearing in the inhomogeneous NLO RG \( Q^2 \)–evolution equations [2], have to be changed according to

\[
\Delta k^{(1)}_{q}(x)_{\text{DIS}} = \Delta k^{(1)}_{q}(x) - e_q^2 \Delta P^{(0)}_{qq} \otimes \Delta C_\gamma \\
\Delta k^{(1)}_{g}(x)_{\text{DIS}} = \Delta k^{(1)}_{g}(x) - 2 \sum_q e_q^2 \Delta P^{(0)}_{gq} \otimes \Delta C_\gamma 
\tag{16}
\]
where [2]

\[
\Delta k_q^{(1)}(x) = \frac{1}{2} 3 e_q^2 \frac{4}{3} \left\{-9 \ln x + 8(1 - x) \ln(1 - x) + 27 x - 22 \right. \\
+ (2x - 1) \left[\ln^2 x + 2 \ln^2(1 - x) - 4 \ln x \ln(1 - x) - \frac{2}{3} \pi^2\right]\left\}
\]

\[
\Delta k_g^{(1)}(x) = 3 \sum_q e_q^2 \frac{4}{3} \left\{-2(1 + x) \ln^2 x + 2(x - 5) \ln x - 10(1 - x)\right\}
\]

(17)

with \(\Delta k_q^{(1)}\) referring again to each single (anti)quark flavor and \(\Delta P_{qg}^{(0)} = P_{qg}^{(0)}, \Delta P_{gq}^{(0)} = \frac{4}{3}(2 - x)\). The NLO expansion for \(g_1^\gamma\) in the DIS\(\Delta\) scheme is thus given by (12) with the \(\Delta C_{\gamma}\) term being dropped.

Following refs. [1, 2], we shall now study two extreme scenarios:

(i) a ‘maximal’ scenario corresponding to an input

\[
\Delta f_1^\gamma(x, \mu^2)_{\text{DIS}_\Delta} = f_1^\gamma(x, \mu^2)_{\text{DIS}_\gamma,1};
\]

(18)

(ii) a ‘minimal’ scenario corresponding to an input

\[
\Delta q_1^\gamma(x, \mu^2)_{\text{DIS}_\Delta} = e_q^2 \frac{\alpha}{2\pi} [C_{\gamma,1}(x) - C_{\gamma,2}(x)] = -e_q^2 \frac{\alpha}{2\pi} 12x(1 - x)
\]

\[
\Delta g_1^\gamma(x, \mu^2)_{\text{DIS}_\Delta} = 0
\]

(19)

which derives from (11) for the minimal (‘pointlike’) boundary condition \(f_1^\gamma(x, \mu^2)_{\text{DIS}_\gamma} = 0\) of the unpolarized photonic parton distributions in the DIS\(\gamma\) scheme [3, 4]. Notice that (19) differs from the minimal (‘pointlike’) input \(\Delta f_1^\gamma(x, \mu^2)_{\text{DIS}_\Delta} = 0\) considered in [2].

In order to facilitate a direct comparison with the results obtained in [3] we shall also use the older GRV\(\gamma\) results [3] for the unpolarized \(f_1^\gamma(x, \mu^2)_{\text{DIS}_\gamma}\) distributions in the DIS\(\gamma\) factorization scheme, which refer to a NLO input scale \(\mu^2 = 0.3\ \text{GeV}^2\), and which uniquely fix \(f_1^\gamma(x, \mu^2)_{\text{DIS}_\gamma,1}\) in (18) via eq. (11). (Our main conclusions remain unchanged, if we use the more recent unpolarized photonic parton distributions of [3].) In LO the ‘maximal’
input (18) refers just to the common (scheme-independent) LO distributions [3], whereas
the ‘minimal’ input obviously implies, instead of (19), also a vanishing quark–input, i.e.
\[ \Delta f^\gamma(x, \mu^2)_{\text{LO}} = 0 \] which coincides with the ‘pointlike’ solution for \( \Delta f^\gamma(x, Q^2) \) and with
the input of [2] where \( \mu^2_{\text{LO}} = 0.25 \text{ GeV}^2 \).

In fig. 1 we show our maximal and minimal NLO results for \( g^\gamma_1 \) at a typical scale of
\( Q^2 = 10 \text{ GeV}^2 \) as obtained from the maximal and minimal input scenarios in (18) and
(19), respectively, which fall somewhat below the results of [2] as expected. A comparison
with \( F^\gamma_1 \) shows furthermore that the fundamental positivity constraint (1), \( |g^\gamma_1| \leq F^\gamma_1 \), is
fulfilled throughout the entire \( x \)-region (at any \( Q^2 \)), in contrast to the violation of (1)
observed in [2]. The corresponding LO and NLO results for the asymmetry \( A^\gamma_1 = g^\gamma_1/F^\gamma_1 \) is
shown in fig. 2 for two representative values of \( Q^2 \). It should be noticed that at very large
values of \( x (x > 0.9) \) the numerical NLO results in figs. 1 and 2 become unreliable due to
the influence of sizeable spurious \( \mathcal{O}(\alpha_s, \alpha_s^2) \) terms [3, 5] encountered in the convolutions
appearing in eqs. (8) and (12). The ‘maximal’ and ‘minimal’ photonic parton distributions
\( \Delta u^\gamma \) and \( \Delta g^\gamma \) are displayed in fig. 3 in LO and NLO(DIS\( \gamma,1 \)) at \( Q^2 = 10 \text{ GeV}^2 \) and the
corresponding asymmetries \( A^\gamma_J \equiv \Delta f^\gamma/f^\gamma \) are shown in fig. 4, where we have again used
the unpolarized \( f^\gamma \) distributions from [3] in order to facilitate a comparison with [2]. Our
results for \( \Delta u^\gamma \) and \( \Delta g^\gamma \) in fig. 3 are similar to the ones in [2], with a larger difference
between our LO and NLO predictions according to our different NLO inputs (18) and (19)
which refer to the unpolarized DIS\( \gamma,1 \) factorization scheme. This, however, is irrelevant as
discussed above for the reliability of the NLO predictions for the experimentally directly
observable structure functions \( g^\gamma_1 \) and \( F^\gamma_1 \).

In LO QCD, where cross sections (structure functions) are directly related to parton
densities, the positivity constraint (1) for structure functions implies
\[ |\Delta f^\gamma(x, Q^2)| \leq f^\gamma(x, Q^2) \] which is satisfied, \( |A^\gamma_{u,g}| \leq 1 \), as shown in fig. 4 by the dashed curves. At NLO, however,
a simple relation between parton densities and cross sections no longer holds. Parton distributions are renormalization and factorization scheme dependent objects; although universal, they are not physical, i.e. not directly observable. Hence there are NLO contributions which may violate (20) in specific cases \[9\]. Such a curiosity occurs for our photonic parton densities which, for medium to large values of \(x\), are dominated by the photon’s splitting functions \((\Delta)k_{q,g}\) appearing as inhomogeneous terms in the RG evolution equations \[2, 3, 4\]. Up to NLO they are given by

\[(\Delta)k_i(x, Q^2) = \frac{\alpha}{2\pi} (\Delta)k_i^{(0)}(x) + \frac{\alpha \alpha_s(Q^2)}{(2\pi)^2} (\Delta)k_i^{(1)}(x)\]  

(21)

where in LO \((\Delta)k_i^{(0)} = \frac{1}{2} 3e_q^2 x^2 \left[ x^2 \left( 1 - x \right)^2 \right] \), while the NLO two–loops unpolarized splitting functions are given by (6), (7) or (10), and their polarized counterparts by (16) or (17), depending on the choice of the factorization scheme. Our NLO results for \(\Delta u^\gamma\) and \(\Delta d^\gamma\) still satisfy the positivity constraint (20) as demonstrated by the solid curves for \(A^u_\gamma\) in fig. 4 since in LO \(|\Delta k_q^{(0)}| \leq k_q^{(0)} \) despite the fact that the subleading NLO contributions in general violate \(|\Delta k_q^{(1)}/k_q^{(1)}| \leq 1\). The NLO gluon distributions, however, violate (20) since the LO terms in (21) obviously vanish \[2, 3\], \(k_g^{(0)} = \Delta k_g^{(0)} = 0\), and the dominant NLO terms \((\Delta)k_g^{(1)}\) in (21) violate \(|\Delta k_g^{(1)}/k_g^{(1)}| \leq 1\). This violation of the NLO gluon ‘positivity’ is illustrated by the solid curves in fig. 4 for \(A_g^\gamma\) where \(A_g^\gamma > 1\) for \(x \gtrsim 0.6\) and 0.9 for the maximal and minimal scenario, respectively.

To summarize, our approach to the positivity constraint (1) on the polarized structure function of the photon, \(g_1^\gamma\), in NLO was to consider appropriate factorization schemes \(\text{DIS}_{\gamma,1}\) and \(\text{DIS}_{\Delta\gamma}\) which are naturally associated with the structure functions \(F_1^\gamma\) and \(g_1^\gamma\), respectively. Utilizing these factorization schemes we have been able to use the well established \[5, 6\] unpolarized NLO parton distributions of the photon, as given in the \(\text{DIS}_\gamma\) factorization scheme associated with \(F_2^\gamma\), in two different hypothetical ‘maximal’ and ‘minimal’ scenarios for the presently unknown \(g_1^\gamma(x, Q^2)\). We have thus shown that the time consuming NLO reanalysis of the data on \(F_2^\gamma\) in the \(\text{DIS}_{\gamma,1}\) factorization scheme,
as proposed in [2], can in fact be avoided. It turns out that our positivity respecting hypothetical NLO scenarios differ from their corresponding counterparts in [4] thus illustrating the importance of a consistent implementation of the non–trivial NLO positivity constraint on $g_1^\gamma$. 


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**Figure Captions**

**Fig. 1.** NLO predictions of the polarized photon structure function $g_1^\gamma$ for the ‘maximal’ and ‘minimal’ inputs in (18) and (19) with the unpolarized photonic parton distributions $f^{\gamma}_{\text{DIS},1}$ being calculated according to the NLO(DIS$_\gamma$) distributions of ref. [5] (in order to facilitate a direct comparison with the results of ref. [2]). These latter distributions determine also the unpolarized photon structure function $F_1^\gamma$ in (8).

**Fig. 2.** The spin asymmetry $A_1^\gamma \equiv g_1^\gamma/F_1^\gamma$ in LO and NLO for the ‘maximal’ and ‘minimal’ scenarios using the input distributions as in fig. 1 at two representative values of $Q^2$.

**Fig. 3.** Predictions for the NLO polarized photonic parton densities in the DIS$_{\Delta\gamma}$ scheme, using the ‘maximal’ and ‘minimal’ inputs in (18) and (19) referring to the unpolarized DIS$_\gamma,1$ scheme where we use again the NLO(DIS$_\gamma$) distributions of ref. [5] for calculating the $f^{\gamma}_{\text{DIS},1}$ input densities in (18), in order to facilitate a direct comparison with the results of ref. [2]. For comparison we also show the corresponding LO results with the input $f^\gamma(x,\mu^2)_{\text{LO}}$ in (18) being taken from ref. [6] for the ‘maximal’ scenario, and where ‘minimal’ scenario obviously implies, instead of (19), also a vanishing quark input, i.e. $\Delta f^\gamma(x,\mu^2)_{\text{LO}} = 0$.

**Fig. 4.** The parton spin–asymmetries $A_f^\gamma \equiv \Delta f^\gamma/f^\gamma$ in LO and NLO at $Q^2 = 10$ GeV$^2$ for the ‘maximal’ and ‘minimal’ scenarios using the input distributions as in fig. 3.
Fig. 1
$A_1^\gamma = g_1^\gamma / F_1^\gamma$

$Q^2 = 2 \text{ GeV}^2$

$Q^2 = 10 \text{ GeV}^2$

Fig. 2
Fig. 3
\[ A^\gamma_u = \Delta u^\gamma / u^\gamma \]

- NLO
- LO

\[ A^\gamma_g = \Delta g^\gamma / g^\gamma \]

max. scenario
min. scenario

\[ Q^2 = 10 \text{ GeV}^2 \]

Fig. 4