CP Violation in the Neutrino Sector

Sandip Pakvasa
University of Hawaii, Honolulu, Hawaii 96822

Abstract. I will summarise the current status of CP violation in the neutrino sector, with a somewhat historical perspective.

1. Some Prehistory:
As everyone knows, the need for a particle like the neutrino was suggested by Pauli [1] in his famous letter in 1930 to account for the continuous energy spectrum of the electrons emitted in nuclear beta-decay. To be fair, a lot of the credit goes to the experimenters, especially, Ellis and Wooster [2] who labored prodigiously during the period between 1921 and 1929 to establish conclusively that the beta decay spectrum was indeed continuous and answered all the criticisms of Meitner. Without their work, Pauli would not have been driven to “the desperate remedy”. The next big step was the 1933 paper by Fermi [3] which laid out a detailed theory of beta decay incorporating Pauli’s neutrino and containing the first fresh application of the quantum field theory (developed by Dirac, Heisenberg and Pauli as recently as 1929) to a new phenomenon. Amazingly, what Fermi wrote down is still almost valid, except needing a factor of $(1 + \gamma_5)/2$ to allow for maximal parity violation! In 1935, Yukawa [4] introduced a meson field with a mass scale of about 100 MeV and coupling strongly to nucleons to account for the strength and range of the nuclear force. Anderson and Neddermeyer[5], while trying to understand the “hard” component in cosmic rays found that it was due to charged particles of intermediate mass. Street and Stevenson[6] independently reached the same conclusion and so did the Nishina group[7] in Tokyo. Its mass seemed to be approximately 100 MeV and so in the range expected of the Yukawa’s meson. But this particle did not behave as a Yukawa meson should! It had the wrong coupling strength to nucleons and incorrect lifetime. The final clincher was the fact that negative mesons were not absorbed by nuclei at the expected rate(Conversi et al.[8]). Sakata and Inoue proposed in 1943(not published till 1946 due to wartime problems[9]) to solve this puzzle by suggesting that the pion of Yukawa is indeed produced in cosmic rays but decays into new particles, now called muon and $\mu$-neutrino and also proposed the the correct decay scheme $\pi \rightarrow \mu \rightarrow e$, they also proposed the correct spins spin 1/2 for both particles (in contrast to the Marshak-Bethe[10] proposal). This scheme was completely confirmed in cosmic ray data as observed by Lattes, Occhialini and Powell(1947)[11]. After more data on the muon capture by nucleons and muon decay became available and the similarity of the various coupling strengths became clear, this observation that the couplings in beta decay, muon decay and muon capture were comparable was elevated to the principle of Universality of Weak Interactions. This observation was made by Klein, by Puppi, by Tiomno and Wheeler and by Lee, Rosenbluth and Yang in the period 1948-9[12]. In the period 1947 onwards the field of particle physics exploded with the discovery of “strange” particles (Kaons, $\Lambda$ hyperons...
etc....). In 1953, Gell-Mann and Nishijima (with Nakano) provided a simple classification scheme [13] with the introduction of strangeness quantum number. In 1954-5 Gell-Mann and Pais[14] analysed the behaviour of $K^0 - \bar{K}^0$ oscillations and the mass-lifetime eigenstates $K_1$ and $K_2$. The decays of the kaons became very puzzling when the analysis by Dalitz(1953)[15] was applied to the three pion decay mode of the $K^0$ particle. It seemed to imply that either parity is not conserved or there is something very strange going on. In 1956 Lee and Yang[16] presented the analysis of several possible experiments to test parity conservation in weak interactions. Very shortly, the experimental results of the Co-60 experiment of Ambler-Wu et al[17], and the $\pi - \mu$ decay experiment[18] of Lederman et al and Friedman-Telegdi appeared proving conclusively maximal parity violation in weak interactions. Within a few months(1957), the Universal V-A interaction was proposed by Sudarshan and Marshak[19] and by Feynman and Gell-Mann[20]:

$$H_W = J^+ J$$

with $J(V - A) = \bar{p}n + e\nu + \bar{\mu}\nu + \bar{\rho}\Lambda$ (2)

This was very successful except for a small problem. According to the data on the beta decay of $\Lambda$ hyperons, the coupling strength for the strangeness changing current was much smaller than required by universality. In 1960, Gell-Mann and Levy[21] proposed a modified version of "Universality" to account for this. They suggested that the form of the hadronic current be modified as:

$$J = (\bar{p}n + e\rho\Lambda) / \left(1 + \epsilon^2 \right)^{1/2} = (\cos \theta \bar{p}n + \sin \theta \rho\Lambda)$$ (3)

2. History

In 1962, an important step was taken by Maki, Nakagawa and Sakata[22] in which they proposed that there is similar mixing in the leptonic sector, and further that the Neutrino flavor states are mixtures of mass eigenstates:

$$\nu_e = \cos \phi \nu_1 + \sin \phi \nu_2$$
$$\nu_\mu = -\sin \phi \nu_1 + \cos \phi \nu_2$$ (4)

They also discussed neutrino mixing and oscillations, estimating the oscillation time in terms of $\delta m^2$ and $L/E$. While the general notion of neutrino oscillations had been raised earlier(1957) by Pontecorvo[23], this was the first time that neutrino flavor oscillations were discussed. A similar analysis was carried out by Gribov and Pontecorvo in 1969[24]. In 1963, Cabibbo[25] analysed with great success semileptonic decays of all hyperons, using the mixing angle $\theta$, and with the baryons as well as the weak current transforming as 8 under Gell-Mann’s flavor SU(3)[26]. 1964 marked the discovery of CP violation in pionic decays of $K_L$ by Christenson, Cronin, Fitch and Turlay[27]. In 1973 there appeared the seminal paper of Kobayashi and Maskawa[28]. They observed that with two families of quarks and leptons it is not possible to have CP non-conservation in the unified gauge theory of electroweak interactions of Glashow, Salam and Weinberg[29]; and proposed several ways to make it possible. One possibility was to add a third family of quarks (t,b). In 1975, this possibility was analysed and shown to be viable and to be able to account for the observed CP violation in Kaon decays by several groups by Pakvasa and Sugawara[30]; by Maiani[31]; and by Ellis, Gaillard and Nanopoulos[32]. In 1975, the tau lepton was discovered by Perl et al at SPEAR[33]. Its decays showed that it was accompanied by its own neutrino. In 1977, there appeared two papers with analyses of a weak current with three lepton families, including a mixing between three neutrinos with a complex (hence CP
non-conserving) unitary 3X3 matrix: Lee, Pakvasa, Shrock and Sugawara; Fritzsch[34]. The 3X3 unitary matrix U is:

\[ U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot V_M \] (5)

where the matrix \( V_M \) is given by

\[ V_M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_1/2} & 0 \\ 0 & 0 & e^{i\alpha_2/2} \end{pmatrix} \] (6)

and defines the Majorana phases[35] \( \alpha_i \) which are not observable in oscillations. With the PDG[36] parametrisation of the unitary matrix U, the Jarlskog invariant[37] for the neutrino sector which is a measure of CP Non-conservation is given by

\[ J = J_{Max} \sin \delta, \text{ with } J_{max} = c_{12}s_{12}c_{23}s_{23}c_{13}s_{13} \] (7)

and is approximately equal to \( 3.10^{-2} \sin(\delta) \) [38] to be compared to the corresponding \( J \) in the quark sector: \( J_{CKM} \sim 3.10^{-5}[36] \).

There are very simple tests of CP conservation that can be easily stated[39]:

\[ P_{\alpha \beta}(t) \neq P_{\alpha \beta}(t) \text{ etc.} \] (8)

More quantitatively, it can be shown that[40]:

\[ \Delta_{\mu e} = P_{\beta \alpha}(L/E) - P_{\bar{\beta} \bar{\alpha}}(L/E) \]

\[ = -16J_{\mu e} \sin(\Delta_{12}) \sin(\Delta_{23}) \sin(\Delta_{31}) \] (9)

where

\[ \Delta_{ij} = \delta m_{ij}^2 L/4E \] (11)

\[ J_{\mu e} = Im(U_{\mu i}U_{\mu j}^*U_{e i}^*U_{e j}) \]

\[ = c_{12}s_{12}c_{23}s_{23}c_{13}s_{13} \] (13)

One can also look for CP violating effects even without access to antineutrino beams. For example, flavor conversion probability over a long time (or large L/E) may contain terms both odd and even in \( t \), to wit

\[ P = A + B \cos(at) + C \sin(at) \] (14)

and if the coefficient C is not zero, then CP is violated[38].

The current knowledge of the neutrino mass/mixing matrix can be summarised (roughly) as follows:

\[ \delta m_{21}^2 \sim 7.5.10^{-5} \text{ eV}^2 \]

\[ \delta m_{31}^2 \sim 2.4.10^{-3} \text{ eV}^2 \]

\[ s_{12}^2 \sim 0.3 \]

\[ s_{23}^2 \sim 0.42 \]

\[ s_{13}^2 \sim 0.22 \] (15)
In addition the current fits show a slight preference for $\cos(\delta) < 0$[38]. The mass hierarchy is unknown, leaving a twofold ambiguity. The normal hierarchy (NH) refers to $m_3 > m_2 > m_1$, and the inverted hierarchy (IH) refers to $m_2 > m_1 > m_3$. That $m_2 > m_1$ is deduced from the fit of the solar neutrino data to the matter effects in the sun.

Everything so far has been in vacuum. However, we know that most neutrino beams in real life travel thru matter for at least some fraction (sometimes quite large) of their path. Since matter effects are not CP invariant, this induces additional "fake" CP violating effects into the oscillation probabilities. For example, the expression for $\Delta_{\mu e}$ now becomes[41]:

$$\Delta_{\mu e}(m) = 2xyg \left[ -\sin \delta \sin (f + \bar{f}) + \cos \delta \cos (f - \bar{f}) \right]$$

(16)

where $\Delta = \Delta_{31}$; and $x, y, g$ and $f, \bar{f}$ are known functions of the parameters and the electron density in the matter.

3. Now

This means that to extract information about the CP phase $\delta$ and to detect "true" CP violation in neutrino oscillation one has to extract them using this expression and study the sensitivity with respect to $\delta$ of various proposed experimental configurations. It is crucial to have knowledge of the mass hierarchy to do this successfully. A number of proposed future facilities will be attempting to do this. The proposed facilities at LBNE, LBNO, T2HK and also ESSnuSB are some examples. The aim is to determine that the phase $\delta$ is not 0 or $\pi$ and find its value to at least 3 $\sigma$. It appears that at LBNE, with a near detector, and a 35 kT LAr far detector, a dedicated run for 4-5 years may be able to yield a measurement to confirm a non-zero $\delta$ over a large range and thus confirm CP non-conservation[42].

As mentioned earlier, these experiments based on neutrino oscillations can only measure the phase $\delta$ but not the Majorana phases, should the neutrino be a Majorana particle. Can those phases be observed and measured? Now if neutrinoless double beta decay is observed, and neutrino mass is directly measured in tritium beta decay, then one can combine these measurements

$$M_{ee} = \sum_i |V_{ei}|^2 m_i$$

$$m_e = |U_{e1}|^2 m_1 + |U_{e2}|^2 m_2 e^{i\alpha_1} + |U_{e3}|^2 m_3 e^{i\alpha_2}.$$  

(17)

In the reasonable approximation that $\delta m^2_{12} << \delta m^2_{13}$, one can show that only one Majorana phase is left, and can be written as:

$$\cos \alpha_1 \cong 1 - \frac{2}{\delta m^2_{12}} \left(1 - \frac{M^2_{ee}}{m_e^2} \right).$$

(18)

This shows that, in principle, it is possible to extract a value for this Majorana phase[43]. However, the uncertainties in the nuclear matrix elements of the neutrinoless double beta decay render this impossible[43]. Some day that these uncertainties will be under better control to make this possible.

My final topic is that of CP violation and the baryon asymmetry in the universe (BAU). The evidence for this is very obvious in the world around us and out to distant galaxies and is quantified as:

$$\eta_B = (n_B - n_{\bar{B}})/n_\gamma \sim 6.10^{-10}$$

(19)

We have known since Sakharov’s ground breaking paper in 1967[44] that to account for this asymmetry there are three essential ingredients needed:
(i) Baryon number violation
(ii) CP violation and C violation
(iii) Departure from thermal equilibrium.

Now, we already have C and CP violation in weak interactions and the departure from thermal equilibrium is supplied by the expansion of the universe. In the attractive scenarios using leptogenesis\[45\], the B violation is to be supplied by the sphalerons of the Standard Model which act at high temperature and conserve B-L but not B and L separately. The basic idea is that the Right Handed neutrinos needed in the see-saw mechanism violate Lepton number and CP in their decays to lighter particles, and generate a lepton asymmetry thereby, which also carries CP violation. After the lepton asymmetry and a net lepton number L has been created at some high scale, as the universe cools and the temperature drops to the electroweak scale, the sphaleron converts the lepton asymmetry into a baryon asymmetry\[46\]. As B-L is conserved, a net non-zero B is left. In general, the CP violation that contributes to create this asymmetry is in the heavy neutral lepton sector at the high scale. But in some specific flavor models, it may be actually related to the phase $\delta$ in the PMNS (Pontecorvo, Maki, Nakagawa, Sakata) matrix\[47\]. In one example, provided $\theta_{13}$ is large enough, and the hierarchy is Normal, a reasonable value of $\delta$ can give rise to the observed BAU\[48\]. But testing these ideas is a long shot. I remark in passing that predicting the phase $\delta$, or for that matter, any of the mass mixing parameters is very far from realisation, to put it mildly.

4. Conclusions/Outlook:
There are some hints from the current best fits that $\delta$ is not equal to zero. Future Long Baseline experiments will eventually be able to detect CP violation in neutrino oscillations, and measure a non-zero value for $\delta$ to a reasonable degree of accuracy. Ongoing searches for neutrinoless double beta decay may find a non-zero rate, and if the error bars on the nuclear matrix elements are reduced sufficiently, we may be able to measure one of the Majorana phases. To test the connection between the baryon asymmetry and the measured value of $\delta$ is a rather long shot. In any case, it should be obvious that the whole question of observable CP violation in neutrino sector is a long term program.

5. Acknowledgments:
I thank the organisers, and especially Nick Solomey for encouragement. I thank Danny Marfatia in help with preparing this talk, and with the selection of the material. This work was supported in part by the U.S. D.O.E. under grant #DE-FG02-04ER-41291 and in part by the Alexander von Humboldt Foundation. I also thank Professor Jose Valle for hospitality during my stay in Valencia where some of this work was done.

6. References
[1] Paul W Letter to a Physicists’ gathering at Tubingem Dec. 4, 1930, Repr in Pauli W, collected scientific papers, Ed. Kroving R and Woisskopf V 1969 Vol. 2 p. 1313, Interscience, NY.
[2] Ellis C D and Wooster W A 1927 Proc. RoySoc A117 109.
[3] Fermi E 1933 La Riviera Scientifica, 12.
[4] Yukawa H Proc. Phys. Math. Soc. Japan 1935 17 48.
[5] Anderson C D and Neddlermeyer S H Phys. Rev. 1937 51, 884.
[6] Street J C and Stevenson E C 1937 Phys. Rev. 51, 1005.
[7] Nishina Y, Takeuchi M and Ichimaya T 1937 Phys. Rev. 52, 1198.
[8] Conversi M, Pancini F and Piccioni O 1945 Phys. Rev. 71 209.
[9] Sakata S and Inoue T 1946 Prog. Theoret. Phys., 1, 143.
[10] Marshak R and Bethe H 1947 Phys. Rev. 72, 506.
[11] Lattes C M G, Occhialini G P S and Powell C F 1947 Nature 160, 453.
[12] Klein O, Nature, 161, 897 (1948); Puppi G, Nuov. Cim. 5, 587 (1948); Tiomno J and Wheeler J 1949 Rev. Mod. Phys., 21, 144; Lee T D, Rosenbluth M N, and Yang C N 1949 Phys. Rev., 75, 905.
[13] M. Gell-Mann M 1952 Nuov. Cim., 4, 52; Nakano T and Nishijima K 1953 Prog. Theoret. Phys., 10, 581.
[14] Gell-Mann M and Pais A 1955 Phys. Rev. 97, 1387.
[15] Dalitz R H Phil. Mag 1953 44 1068.
[16] Lee T D and Yang C N 1949 Phys. Rev. 105, 254.
[17] Wu C S, Ambler E et al. 1957 Phys. Rev. 105, 1415; Friedman J L and Telegdi V L Phys. Rev. 105, 1681.
[18] Garwin R, Lederman L and Weinrich L 1956 Phys. Rev. 104, 254.
[19] Sudarshan E C G and Marshak R E 1957 Proc. Conf. on Mesons and Newly Discovered Particles, Padua-Venice (Conf. held Sept, 1957), ed. Zonichelli N, p. v-14.; Phys. Rev. 109, 1860.
[20] R P Feynman and Gell-Mann 1957 Phys. Rev. 109, 193.
[21] Gell-Mann M and Pais A 1955 Phys. Rev. 97, 1387.
[22] Dalitz R H Phil. Mag. 1953 44 1068.
[23] Lee T D and Yang C N 1949 Phys. Rev. 105, 254.
[24] Wu C S, Ambler E et al. 1957 Phys. Rev. 105, 1415; Friedman J L and Telegdi V L Phys. Rev. 105, 1681.
[25] Garwin R, Lederman L and Weinrich L 1956 Phys. Rev. 104, 254.
[26] Wu C S, Ambler E et al. 1957 Phys. Rev. 105, 1415; Friedman J L and Telegdi V L Phys. Rev. 105, 1681.
[27] Sudarshan E C G and Marshak R E 1957 Proc. Conf. on Mesons and Newly Discovered Particles, Padua-Venice (Conf. held Sept, 1957), ed. Zonichelli N, p. v-14.; Phys. Rev. 109, 1860.
[28] Garwin R, Lederman L and Weinrich L 1956 Phys. Rev. 104, 254.