Slow Decoherence of Superpositions of Macroscopically Distinct States

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I. INTRODUCTION

A Schrödinger cat state is a linear superposition of two quantum states that differ on a macroscopic scale. Whereas superpositions of quantum states are commonplace in the microscopic world, a superposition of macroscopically distinct states is practically never observed. The understanding of why this is so has evolved considerably with the development of the quantum mechanics of dissipative systems. Dissipative systems are systems that are coupled to an environment (also called “heat bath”) with a large number of degrees of freedom. The coupling allows for an exchange of energy between system and bath. Therefore the motion of the system will slow down till only thermal fluctuations in equilibrium with the heat bath are left. This effect is also present in classical mechanics and takes place on a classical time scale, $T_{\text{class}}$. But the coupling to the environment also very rapidly destroys quantum mechanical interference effects, and leads to an effectively classical behavior. The decoherence arises on a time scale $T_{\text{dec}}$ which is typically much shorter than $T_{\text{class}}$. As a general rule, if a separation $D$ in phase space can be assigned to the states involved in the Schrödinger cat state, the two time scales behave as $T_{\text{dec}}/T_{\text{class}} \sim (\hbar/D)^r$ where $r$ is some positive power depending on the details of the coupling and the environment. This separation of time scales has also been termed “accelerated decoherence”. The validity of this picture starts to be confirmed in experiments of Haroche et al., who were able to turn on the coupling to an environment in a controlled way and measure the decay of the coherences.

The basis in which the density matrix becomes diagonal depends on the coupling to the environment, as was pointed out by Zurek. Suppose that the coupling Hamiltonian contains a “coupling agent” $A$, $H_{\text{int}} = Af(\{x, p\})$, where $f(\{x, p\})$ is some function of the environmental coordinates. Accelerated decoherence arises in the basis formed by the eigenstates of $A$, the so-called pointer basis. Thus, after the short time $T_{\text{dec}}$ the reduced density matrix of the system will be diagonal in this basis, but may still contain off–diagonal matrix elements (coherences) in another.

It has recently been shown, however, that even in the pointer basis accelerated decoherence can be absent if the coupling to the environment has a certain symmetry. Indeed, suppose that $A|\psi_1\rangle = a|\psi_1\rangle$ and $A|\psi_2\rangle = a|\psi_2\rangle$, i.e. $a$ is a degenerate eigenvalue of $A$, then the environment cannot distinguish between states $|\psi_1\rangle$ and $|\psi_2\rangle$. A Schrödinger cat state $|\psi\rangle = c_1|\psi\rangle + c_2|\psi\rangle$ is therefore stable against dephasing. It may lose its coherence on the time scale on which the symmetrical arrangement of the state is destroyed, which can be as large as $T_{\text{class}}$.

In this paper we give specific examples of such longlived Schrödinger cat states in the context of superradiance with a well known, experimentally verified damping mechanism. We calculate the decoherence rates and propose a way how the Schrödinger cat states might be realized experimentally.

II. SUPERRADIANCE AND THE SCHRÖDINGER CAT STATES

The system that we consider is well known from the phenomenon of superradiance: A large number of identical two–level atoms in a cavity radiate collectively. We assign formally to each atom a spin–1/2 operator $S^{(i)}$ whose $z$–components tells us whether the atom is in its lower or upper state. If all atoms couple in the same way to a resonant electromagnetic field mode in the cavity, the system as a whole can be described by a collective variable $J = \sum_{i=1}^{N} S^{(i)}$, the so called Bloch vector ($N$ is the number of atoms). It can be thought of formally as an angular momentum. The coupling of the atoms to the resonant electromagnetic field mode (with creation and annihilation operators $a^{\dagger}$ and $a$) be of the form...
\[ H_{\text{int}} = \hbar (J_+ a + J_- a^\dagger), \]  

with some coupling constant \( g \). Dissipation ultimately arises by the cavity’s non–ideal mirrors, i.e. the field mode is itself damped and photons may leak out of the cavity at a rate \( \kappa \). Under the assumption of weak coupling (\( g\sqrt{N}/\kappa \ll 1 \)), low temperatures (\( k_B T \ll \hbar \omega \), the level separation of the atoms), and the Markov assumption (\( t \gg 1/\kappa \)) the master equation

\[ \frac{d}{d\tau} \hat{\rho} (\tau) = \frac{1}{2j} \left( [J_-, \hat{\rho} (\tau) J_+] + [J_+ \hat{\rho} (\tau), J_+] \right) \]  

for the reduced density operator \( \hat{\rho} \) was derived in [1]. We have written the time \( t \) dimensionless in units of the inverse classical damping rate as \( \tau = 2 j g^2 t / \kappa \), i.e. \( \tau \) is already in units of \( T_{\text{class}} \). The classical limit is obtained by \( j \to \infty \) and yields a spherical phase space in which the orientation of the classical Bloch vector can be parametrized by the polar angles \( \theta \) (reckoned against the \( J_z \)-axis) and \( \varphi \) (the azimuth of the equatorial projection reckoned against the \( J_x \)-axis). The variables \( p = J_z / j \equiv \cos \theta \) and \( \varphi \) acquire the role of classical canonical momentum and coordinate [2]. Since the classical phase space contains \( 2j + 1 \) states, we may think of \( \hbar \) as scaling like \( 1/j \). The Bloch vector behaves classically like an overdamped pendulum, \( d\theta / d\tau = \sin \theta \) and \( \varphi = \text{const}. \) Many of the consequences of [3] have been confirmed experimentally [13].

The states that correspond as much as possible to classical states are the so called angular momentum coherent states \( |\gamma \rangle = |\theta, \varphi \rangle \) [20,21]. They correspond to a classical angular momentum pointing in the direction given by the polar angles \( \theta \) and \( \varphi \), with the complex label \( \gamma \) given by \( \gamma = \tan(\theta/2) e^{i\varphi} \). They have minimal uncertainty, \( \Delta p \Delta q \sim 1/j \). In terms of \( |jm \rangle \) states one has the expansion

\[ |\gamma \rangle = (1 + \gamma \gamma^*)^{-j} \sum_{m=-j}^{j} \gamma^{j-m} \sqrt{(2j \choose j-m)} |jm \rangle. \]  

Coherent states may be more familiar from the harmonic oscillator, where they are eigenstates of the annihilation operator. The compactness of Hilbert space prevents the existence of exact eigenstates of \( J_- \), but one can show that the angular momentum coherent states are approximate eigenstates of \( J_- \) in the sense that the angle between \( J_- |\gamma \rangle \) and \( |\gamma \rangle \) is of the order of \( 1/j \). They therefore qualify as pointer states in the limit \( j \to \infty \).

In the following we will consider the fate of a Schrödinger cat state

\[ |\psi \rangle = \mathcal{N}(|\gamma_1 \rangle + |\gamma_2 \rangle), \]

where \( \mathcal{N} \) is the appropriate normalization constant. Note that the two components can indeed be macroscopically distinct if the number of atoms in the superradiance experiment is large. We will show that the damping [3] leads in general to accelerated decoherence. Our central prediction is, however, that \textit{accelerated decoherence is absent for Schrödinger cat states with} \( \gamma_1 \gamma_2^* = 1 \). Such two states correspond to two classical angular momenta arranged symmetrically with respect to the equator \( \theta = \pi/2 \) (i.e. \( \theta_2 = \pi - \theta_1 \)) on a great circle \( \varphi_1 = \varphi_2 = \text{const}.. \) They decohere only on the classical time scale, \( T_{\text{dec}} \sim T_{\text{class}} \).

The initial density matrix corresponding to the state [4] reads \( \hat{\rho}(0) = |\mathcal{N}|^2 (|\gamma_1 \rangle \langle \gamma_1 | + |\gamma_2 \rangle \langle \gamma_2 | + |\gamma_2 \rangle \langle \gamma_1 | + |\gamma_1 \rangle \langle \gamma_2 |) \). Since the evolution equation of \( \hat{\rho}(\tau) \) is linear it suffices to concentrate on one off–diagonal part \( \hat{\rho}(\tau) \) evolving from \( \hat{\rho}(0) = |\gamma_1 \rangle \langle \gamma_2 | \). Its decay is conveniently studied in terms of one of the norms

\[ N_1(\tau) = \text{tr} \hat{\rho} \rho^1, \tag{5} \]

\[ N_2(\tau) = \sum_{m_1, m_2} |\rho_{m_1, m_2}|, \tag{6} \]

where \( \rho_{m_1, m_2} = \langle jm_1 | \hat{\rho} | jm_2 \rangle \). Both norms are obviously zero if coherences in the \( \gamma \) basis are absent. The time on which they decay to zero sets the decoherence time scale \( T_{\text{dec}} \).

Our prediction of slow decoherence, \( T_{\text{dec}} \sim T_{\text{class}} \) for \( \gamma_1 \gamma_2^* = 1 \), is based on three analytical results:

1. The initial time derivative of \( N_1(\tau) \) reads

\[
\frac{dN_1(\tau)}{d\tau} \big|_{\tau=0} = -2j \left( \sin^2 \theta_1 + \sin^2 \theta_2 - 2 \cos(\varphi_2 - \varphi_1) \sin \theta_1 \sin \theta_2 \right) \\
- \left( (1 + \cos \theta_1)^2 (1 + \cos \theta_2)^2 \right). \tag{7}
\]
Thus, for $\varphi_2 = \varphi_1$ and $\sin \theta_1 = \sin \theta_2$, the term proportional to $j$ vanishes. This marks the absence of accelerated decoherence.

In the following we will restrict ourselves to $\varphi_1 = \varphi_2 = 0$ and denote $\gamma_i = |\gamma_i|$ throughout the rest of the paper.

2. In the particular case of the states $|\gamma_1\rangle = |jj\rangle$ and $|\gamma_2\rangle = |j-j\rangle$ (corresponding to $\gamma_1 = 0$ and $\gamma_2 = \infty$), the exact time evolution $N_2(\tau)$ is given by $N_2(\tau) = e^{-\tau}$ for all times! The coherence decays on the classical time scale, even though the two states are macroscopically as distinct as possible.

3. For finite times $\tau$ with $j\tau \ll 1$, a semiclassical evaluation of the norm $N_2(\tau)$ for $\varphi_1 = \varphi_2 = 0$ leads to

$$\frac{N_2(\tau)}{N_2(0)} = \exp \left( -2j \frac{(\gamma_1 - \gamma_2)^2 (1 - \gamma_1 \gamma_2)^2}{(1 + \gamma_1^2)(1 + \gamma_2^2)} \tau \right) \left( 1 + O(1/j) \right).$$

This means accelerated decoherence as long as $\gamma_1 \neq \gamma_2$ and $\gamma_1 \gamma_2 \neq 1$. If, however, $\gamma_1 \gamma_2 = 1$ then the next order in $1/j$ shows that

$$\frac{N_2(\tau)}{N_2(0)} = \exp \left( -\frac{\gamma_1^2 - 1}{\gamma_1^2 + 1} \tau - \frac{3\gamma_1^8 - 3\gamma_1^6 + 4\gamma_1^4 - 3\gamma_1^2 + 3}{2(\gamma_1^2 + 1)^4} \tau^2 \right) \left( 1 + O(1/j) \right).$$

The expression in the exponent is correct up to and including order $(j\tau)^2$. Obviously, accelerated decoherence is absent for $\gamma_1 \gamma_2 = 1$. Indeed, a single coherent state $\gamma_1 = \gamma_2 = \gamma$ leads, in linear order, to almost the same decay,

$$\frac{N_2(\tau)}{N_2(0)} = \exp \left( -\gamma^4 \left( \frac{\gamma^2 - 1}{\gamma^2 + 1} \right)^2 \tau \right).$$

It is clear that the symmetry of the sine function under $\theta \to \pi - \theta$ underlies the longevity. As mentioned the coherent states $|\gamma\rangle$ are approximate eigenstates of the coupling agent $J_\perp$ and therefore qualify as pointer states. The symmetry of the sine function means that the corresponding "approximate eigenvalue", $j \sin \theta e^{-i\phi}$, is doubly degenerate. A linear combination of two vectors of the pertaining subspace is stable against decoherence. We have here the interesting situation that the deviation of order $1/j$ from exact degeneracy is small enough to compensate for the acceleration factor $j$ of the decoherence rates. Similiar but more general statements about the decoherence in the presence of degeneracy breaking coupling agents can be found in [1].

We now describe in some detail the derivation of the three mentioned results. The first, eq. (8) can be obtained straight forwardly by inserting the equation of motion for $\rho(\tau)$ into $\frac{dN_1(\tau)}{N_1(0)} = \text{tr}(\frac{d\rho(\tau)}{dt} \rho(\tau) + \rho(\tau) \frac{d\rho(\tau)}{dt})$.

The second result is an exact solution of the master equation (2) [17,22]. To see this we write (2) in the $|jm\rangle$-basis and introduce mean and relative quantum numbers, $m = \frac{m_1 + m_2}{2}$ and $k = \frac{m_1 - m_2}{2}$. The latter quantum number is conserved and therefore only enters as a parameter, $\rho_{m_1, m_2} = \rho_m(k, \tau)$. The solution can be written as

$$\rho_m(k, \tau) = \sum_{n=-j+j}^j D_{mn}(k, \tau) \rho_{n}(k, 0)$$

in terms of the dissipative propagator $D_{mn}(k, \tau)$. With the help of the abbreviations $Q_{mn} = \frac{(j+n)!(j-m)!}{(j+m)!(j-n)!}$ and $g_l = j(j+1) - l(l-1)$ one has an exact Laplace integral representation for $D_{mn}(k, \tau)$,

$$D_{mn}(k, \tau) = \frac{\sqrt{Q_{m-k,n-k}Q_{m+k,n+k}}}{2\pi i} \int_{b-i\infty}^{b+i\infty} ds e^{rs} \prod_{l=m}^n \frac{1}{s + g_l - k^2},$$

where the real parameter $b$ should be chosen larger than the largest pole in the denominator. Depending on $m$ and $n$, in general a large number of poles contribute to the integral. But for the case at our interest $(m_1 = j, m_2 = -j)$, we have $m = 0 = n$, $k = j$ and therefore only one pole contributes. We immediately obtain $N_2(\tau) = \rho_0(j, \tau) = D_{00}(j, \tau) \rho_0(j, 0) = \exp(-\tau)$.

Starting point for the third result is the short time propagator
\[ D_{mn}(k, \tau) = \sqrt{\frac{Q_{m-k,n-k} Q_{m+k,n+k}}{(n-m)!}} \left( \frac{\tau}{j} \right)^{n-m} e^{-\frac{\tau}{j} \left[j^2 - \left(\frac{s}{\sqrt{n+m}}\right)^2\right]} \]  

(14)

derived in [22]. It is valid up to times \( \frac{|m+n-1|}{j} (n-m) \tau \ll 1 \). Since we are going to sum over \( m \) and \( n \), both \( m+n \) and \( m-n \) can be of order \( j \) and one should therefore have \( j \tau \ll 1 \). The expansion [2] shows that (at \( \varphi = 0 \)) all initial density matrix elements are real and positive. Since also the short time propagator is real and positive, the same is true for the density matrix elements to the later time \( \tau \), such that the norm \( N_2 \) simplifies to

\[ N_2(\tau) = \sum_{k,m} \rho_m(k, \tau) = \sum_{m,n,k} D_{mn}(k, \tau) \rho_n(k, 0). \]  

(15)

Note that in general \( N_2(0) \neq 1 \), such that in order to study the decay of the coherences one should look at the quantity \( n(\tau) \equiv N_2(\tau)/N_2(0) \).

The first step in calculating \( n(\tau) \) is to evaluate the sum over the final states \( m \). This summation does not involve the initial density matrix at all, so we can define \( S(n, k, \tau) \equiv \sum_m D_{mn}(k, \tau) \) and \( N(\tau) = \sum_{n,k} S(n, k, \tau) \rho_n(k, 0) \). The latter two summations will eventually be done by Laplace’s method, that is by an asymptotic expansion in \( 1/j \) for \( j \to \infty \). In order to remain consistent with the limitation \( j \tau \ll 1 \) we will have to rescale time as

\[ \tau = \frac{\tilde{\tau}}{j} \]  

(16)

with \( \tilde{\tau} \ll 1 \) kept fixed for \( j \to \infty \). This is indeed reasonable since we expect decoherence of an ordinary Schrödinger cat state on a time scale \( 1/j \), that is \( N(\tau) \simeq N(0) \exp(-j \tau \tilde{\tau}) \). If we kept \( \tau \) fixed for \( j \to \infty \) we could only recover \( N(\tau) = 0 \), whereas we can study the decay if we keep \( \tilde{\tau} \) fixed. The precise calculation of \( S(n, k, \tau) \) by summing over \( m \) is difficult. However, since we shall be interested only in short times \( (j \tau \ll 1) \) we can reconstruct \( S(n, k, \tau) \) from its initial time derivatives. Let us write the derivatives directly in reduced coordinates that become continuous in the limit \( j \to \infty \), \( \nu = n/j \) and \( \eta = k/j \), as

\[
\frac{\partial S(n, k, \tau = \tilde{\tau})}{\partial \tilde{\tau}} \bigg|_{\tilde{\tau}=0} = w_1 + \eta^2 - 1 + (\nu - \frac{1}{2j})^2 \\
\frac{\partial^2 S(n, k, \tau = \tilde{\tau})}{\partial \tilde{\tau}^2} \bigg|_{\tilde{\tau}=0} = \left( \frac{\partial S(n, k, \tau = \tilde{\tau})}{\partial \tilde{\tau}} \bigg|_{\tilde{\tau}=0} \right)^2 + w_1 (w_2 - w_1) \\
+2w_1 \left( \frac{-\nu}{j} + \frac{3}{4j^2} \right) \\
w_1 = \sqrt{\left( 1 - (\nu - \eta - \frac{1}{2j})^2 \right) \left( 1 - (\nu + \eta - \frac{1}{2j})^2 \right)} \\
w_2 = \sqrt{\left( 1 - (\nu - \eta - \frac{3}{2j})^2 \right) \left( 1 - (\nu + \eta - \frac{3}{2j})^2 \right)}. 
\]

(17)

For the further derivation we will keep \( S \) in the form of a polynomial, \( S(\nu, \eta, \tilde{\tau}) = 1 + a(\nu, \eta, 1/j) \tilde{\tau} + b(\nu, \eta, 1/j) \tilde{\tau}^2 + O(\tilde{\tau}^3) \) with \( a(\nu, \eta, 1/j) = \partial S/\partial \tilde{\tau} \bigg|_{\tilde{\tau}=0} \) and \( b(\nu, \eta, 1/j) = \frac{1}{2} \partial^2 S/\partial \tilde{\tau}^2 \bigg|_{\tilde{\tau}=0} \). Higher terms in \( \tilde{\tau} \) can be included, but already the first two terms account rather well for the initial decay of the coherences.

We now transform the remaining two sums in \( N_2(\tau) \) (eq. (14)) into integrals by Euler–Maclaurin summation and finally integrate via Laplace’s method. To this end we first write the coefficients \( \rho_n(k, 0) \) in the expansion of the initial density matrix as continuous functions of the reduced coordinates \( \eta \) and \( \nu \equiv n/j \). The result can be cast in the form

\[ \rho_n(k, 0) = C \exp^{S_0(\nu, \eta)}, \]  

(18)

where the “action” \( S_0 \) is given by

\[
S_0(\nu, \eta) = (1 - \nu) \ln(\gamma_1 \gamma_2) + \eta \ln \frac{\gamma_2}{\gamma_1} - \frac{1}{2} (p(1 - \nu + \eta) + p(1 + \nu + \eta) + p(1 + \nu - \eta)) \\
p(x) = x \ln x.
\]  

(19)
The action is correct up to lowest order $1/j$. Terms of order $1/j$ have been absorbed in the prefactor $C$ which turns out to be independent of $\nu, \eta$ and therefore will cancel in $n(\tau)$. We thus have to order $\tilde{\tau}^2$

$$N_2(\tau) = 2j^2C \int d\eta \int d\nu \left( 1 + a(\nu, \eta, 1/j)\tilde{\tau} + b(\nu, \eta, 1/j)\tilde{\tau}^2 \right) \exp(jS_0(\nu, \eta)). \quad (22)$$

The prefactor 2 stems from the fact that $k$ and $n$ can be simultaneously integer or semi–integer, but cancels of course as well in $n(\tau)$. Since the slow decoherence is a $1/j$ effect, a careful asymptotic expansion of the double integral in powers of $1/j$ is in order. First of all, we expand the coefficients $a$ and $b$ as $a(\nu, \eta, 1/j) = \sum_{k=0}^{\infty} a_k(\nu, \eta)j^{-k}$ and $b(\nu, \eta, 1/j) = \sum_{k=0}^{\infty} b_k(\nu, \eta)j^{-k}$. The first few terms in the expansion read ($w = \sqrt{(1 - (\nu - \eta)^2)(1 - (\nu + \eta)^2))}$)

$$a_0 = w + \eta^2 - 1 + \nu^2$$
$$a_1 = \frac{1 - \nu^2 + \eta^2 - w}{w}$$
$$b_0 = \frac{1}{2}a_0^2$$
$$b_1 = (a_0a_1 - \nu(\nu^2 - w - \eta^2 - 1))$$
$$b_2 = \frac{1}{2w^2} \left( (2a_0a_2 + a_1^2)\left((\eta^2 - \nu^2)^2 - 2(\eta^2 + \nu^2)ight) - 2 + 4\nu^6 - 2\eta^6 - 10\nu^4\eta^2 + 8\nu^2\eta^4 - 10\nu^4 - 8\nu^2\eta^2 + 2\eta^4 + 8\nu^2 + 2\eta^2 \right) + \frac{1}{4w} \left( 3\eta^4 + 7\nu^4 - 10\nu^2\eta^2 - 10\nu^2 - 6\eta^2 + 3 \right). \quad (27)$$

The expansion to make sense $w$ should be sufficiently far ($\gg 1/j$) away from 0. The coefficient $a_2$ will not be needed. If we insert the expansions in the integral (22) we encounter functionals of the type

$$I[f] = \int d\eta \int d\nu f(\nu, \eta) e^{jS_0(\nu, \eta)}, \quad (28)$$

where $f(\nu, \eta)$ can be 1 or any of the coefficients in (23) to (27). The large parameter $j$ in the exponent suggests to integrate by the two dimensional Laplace method. To obtain correctly the first few orders in $1/j$ for $n(\tau)$ one needs an extension of Laplace’s method to higher orders, i.e. we also have to expand $I[f]$ in powers of $1/j$:

$$I[f] = I^{(0)}[f] + \frac{1}{j}I^{(1)}[f] + \frac{1}{j^2}I^{(2)}[f] + \ldots. \quad (29)$$

The higher orders can be obtained from systematically reexpanding in $1/j$ the following exact representation of $I[f]$ (28).

$$I[f] = e^{jS_0(\nu_0, \eta_0)} \frac{2\pi}{j} \text{det}(\sigma)^{-\frac{1}{2}} \sum_{l=0}^{\infty} \frac{1}{l!(2j)^l} (L_S)^l \left( f(\nu, \eta) e^{jR(\nu, \eta)} \right) |_{\nu_0, \eta_0}, \quad (30)$$

where $(\nu_0, \eta_0)$ denotes the position of the maximum of $S_0$. The matrix $\sigma$ contains the four second derivatives of $S$ at the maximum, $\sigma_{\nu, \eta} = \partial_\nu \partial_\eta S_0 |_{\nu=\nu_0, \eta=\eta_0}$; the new “action” $R(\nu, \eta)$ is the deviation of $S_0$ from its quadratic approximation,

$$R(\nu, \eta) \equiv S_0(\nu, \eta) - S_0(\nu_0, \eta_0) - \frac{1}{2} \begin{pmatrix} \sigma_{\nu, \eta} & (\nu - \nu_0) \end{pmatrix} \begin{pmatrix} (\nu - \nu_0) \end{pmatrix}, \quad (31)$$

and $L_S$ is the second order homogeneous differential operator

$$L_S = \langle -\sigma^{-1} \begin{pmatrix} \partial_\nu \\ \partial_\eta \end{pmatrix}, \begin{pmatrix} \partial_\nu \\ \partial_\eta \end{pmatrix} \rangle. \quad (32)$$

The angular brackets denote a scalar product. It is assumed that the integration range contains exactly one maximum with $\partial_\nu S_0(\nu, \eta) = 0 = \partial_\eta S_0(\nu, \eta)$. Due to the construction of $R$, all derivatives of $R$ of lower than third order vanish and the term $L_S(f e^{jR})$ in (31) is therefore a polynomial in $1/j$ at most of the order $2l/3$. In order to get the first order in $1/j$ beyond the usual Laplace method (the term with $l = 0$) one has therefore to go up to $l = 3$. 

5
Combining the expansions of \(a\) and \(b\) in terms of \(a_k, b_k\) and the expansions of the corresponding functionals \(I[a_k]\) and \(I[b_k]\) according to [30], we find

\[
n(\tau) = 1 + \tilde{\tau} \left[ \frac{I^{(0)}[a_0]}{I^{(0)}[1]} - \frac{1}{j(I^{(0)}[1])^2} (I^{(0)}[a_0]I^{(1)}[1] - (I^{(0)}[a_1] + \mathcal{O}\left(\frac{1}{j^2}\right)) \right] + I^{(1)}[a_0]I^{(0)}[1]) + \mathcal{O}\left(\frac{1}{j^2}\right)\]

\[
+ \tilde{\tau}^2 \left[ \frac{I^{(0)}[b_0]}{I^{(0)}[1]} - \frac{1}{j(I^{(0)}[1])^2} (I^{(0)}[b_0]I^{(1)}[1] - (I^{(0)}[b_1] + I^{(1)}[b_0])I^{(0)}[1]) - \frac{1}{j^2(I^{(0)}[1])^2} \left( - (I^{(0)}[b_2] + I^{(1)}[b_1])I^{(0)}[1] \right)^2 + (I^{(0)}[b_0]I^{(2)}[1]) \right] + I^{(1)}[1]I^{(0)}[b_1] + I^{(1)}[1]I^{(1)}[b_0])I^{(0)}[1] - I^{(0)}[b_0](I^{(1)}[1])^2 \right].
\]

(33)

The point of maximum \(S_0\) is located in our case at

\[
\eta_0 = \frac{\gamma_2^2 - \gamma_1^2}{(1 + \gamma_1^2)(1 + \gamma_2^2)}, \quad \nu_0 = \frac{1 - \gamma_1^2 \gamma_2^2}{(1 + \gamma_1^2)(1 + \gamma_2^2)}.
\]

(34)

To proceed further we distinguish two different cases.

**Case 1:** \(\gamma_1 \neq \gamma_2\) and \(\gamma_1 \gamma_2 \neq 1\)

Here already the ordinary Laplace method leads to a meaningful result,

\[
\frac{I^{(0)}[a_0]}{I^{(0)}[1]} = a_0(\nu_0, \eta_0) = \frac{2(\gamma_1 - \gamma_2)^2(1 - \gamma_1 \gamma_2)^2}{((1 + \gamma_1^2)(1 + \gamma_2^2))^2}
\]

(35)

\[
\frac{I^{(0)}[b_0]}{I^{(0)}[1]} = b_0(\nu_0, \eta_0) = \frac{1}{2}a_0^2(\nu_0, \eta_0).
\]

(36)

We have therefore

\[
n(\tau) = 1 + a_0(\nu_0, \eta_0) \tilde{\tau} + \frac{1}{2}a_0^2(\nu_0, \eta_0) \tilde{\tau}^2,
\]

and thus, correct up to \(\mathcal{O}(\tilde{\tau}^2)\) the announced result (3), where we have resubstituted \(\tilde{\tau}\) in terms of \(j\tau\).

**Case 2:** \(\gamma_1 \gamma_2 = 1\)

The leading terms (35), (36) now vanish due to \(a_0(\nu_0, \eta_0) = 0\). From the prefactor of the term linear in \(\tilde{\tau}\) in (33) only

\[
\frac{1}{j} \frac{I^{(1)}[a_0]}{I^{(0)}[1]} = \frac{1}{j} \left( \frac{\gamma_1^2 - 1}{\gamma_1^2 + 1} \right)^2
\]

(37)

survives, from the quadratic term only

\[
\frac{1}{j^2} \frac{I^{(0)}[b_2]}{I^{(0)}[1]} = \frac{1}{4j^2}(7\eta_0^2 + 1),
\]

(38)

as the reader might verify in a straightforward but lengthy calculation. In particular, the coefficient proportional to \(1/j\) in the \(\tilde{\tau}^2\) term is zero, such that to quadratic order in \(\tilde{\tau} n(\tau)\) depends only on \(\tau\), not on \(j\tau\):

\[
n(\tau) = 1 - \left( \frac{\gamma_1^2 - 1}{\gamma_1^2 + 1} \right)^2 \tau - \frac{1}{4} \left( 7 \left( \frac{\gamma_1^2 - \gamma_2^2}{(1 + \gamma_1^2)(1 + \gamma_2^2)} \right)^2 + 1 \right) \tilde{\tau}^2 + \mathcal{O}(\tilde{\tau}^3)
\]

(39)

The agreement of Eq. (38) with exact numerical results extends well beyond the range \(\tau \ll 1/j\) for which the theory was made initially. This is not surprising since the main contribution to the norm comes from a region where \(n \simeq 0 \simeq m\), so that the limitation on the validity of the short time propagator is much less severe than in the assumed worst case where both \(n - m\) and \(n + m\) are of order \(j\). The agreement with the numerical result becomes even better and leads to rather precise results even for \(\tau \approx 1\) if we rewrite the decay again in exponential form as was done in (3).
III. POSSIBLE EXPERIMENTAL REALIZATION

It has been experimentally verified that (3) describes adequately the radiation by identical atoms resonantly coupled to a leaky resonator mode [19] in a suitable parameter regime (see the discussion in the context of the equation). It should therefore be possible to observe the slow decoherence of the special Schrödinger cat states. We now propose a scheme for their preparation. Starting by all atoms in the ground state and with the field mode in its vacuum state, a resonant laser pulse brings the Bloch vector to a coherent state \(|\theta, \varphi\rangle\). Note that the cavity may be strongly detuned (with respect to the atomic transition frequency) during the whole preparation process (detuning \(\delta \gg \kappa\)). The dissipation mechanism (3) is hereby turned off and the system evolves unitarily with a Hamiltonian containing a non–linear term \(\propto (g^2/\delta)J_+J_-\). The free evolution during a suitable time will split the coherent state \(|\theta, \varphi\rangle\) in a superposition of \(|\theta, \varphi'\rangle\) and \(|\theta, \varphi' + \pi\rangle\) as described in [24]. Finally, a resonant \(\pi/2\) pulse brings the superposition to the desired orientation symmetric to the equator, by rotation through the angle \(\pi/2\) about an axis perpendicular to the plane defined by the directions of the two coherent states produced by the free evolution. At this point the cavity can be turned to resonance, thereby switching on the dissipation mechanism and one can study the decay of coherence.

IV. CONCLUSION

We have shown that a certain symmetry of the coupling to the environment leads in the phenomenon of superradiance to the existence of longlived coherences of superpositions of macroscopically distinct quantum states. Even though the components of the linear superposition are not exact eigenstates of the coupling operator to a degenerate eigenvalue, the deviation from degeneracy is small enough for the coherences to decay on a classical time scale only. We have proposed a preparation scheme with which such Schrödinger cat states might be realizable experimentally.

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