Analysis of flow and acoustic radiation in reed instruments by compressible flow simulation

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Abstract: Direct aeroacoustic simulations of flow and sound around an instrument with an oscillating reed were performed on the basis of compressible Navier–Stokes equations along with experiments with an artificial blowing device. The measured reed displacement was utilized as forced vibration in the computations. The predicted sound pressure spectrum shows that the level of the fundamental tone almost agrees with the measured result. The numerical results showed that the lowest acoustic mode of clarinet-type reed instruments (one-quarter wavelength mode) was reproduced. Moreover, the sound generation mechanism was discussed in detail using the predicted gradient of mass flow rate in the instrument. It was found that compression and expansion occur inside the mouthpiece, where the flow separation occurs after the spreading of the air jet from the reed channel exit along the inner wall of the mouthpiece. In addition, vortex ring shedding attributable to the acoustic particle velocity around the open end of the instrument was found to occur, causing an expansion wave from the instrument.

Keywords: Woodwind instruments, Direct aeroacoustic simulation, Reed oscillations, Resonance, Vortex ring

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1. INTRODUCTION

Reed instruments are composed of a single or double reed, a mouthpiece, and a resonator, where the reed acts as a valve that modulates air flow into the mouthpiece [1,2]. The acoustic radiation from the instrument is attributable to the complex aeroelastic–acoustic coupling between the air flow into the instrument, reed oscillations, and the acoustic resonance in the instrument [3]. Therefore, the tone quality depends on various factors, such as reed motion and the shape of the mouthpiece [4].

Backus [5] measured the flow rate into a reed instrument with reed channels of various heights. The flow rate $Q_m$ was found to be related to the pressure difference across the opening $\Delta p$ and the height of the opening $h_{o,1}$. Assumming that inertia and damping can be neglected, the steady Bernoulli equation can be applied to the formulation of the flow rate [3,6]. Moreover, Hirschberg et al. [7] theoretically formulated the behavior of the flow rate by utilizing the quasi-stationary model as:

$$Q_m = \alpha w h_{o,1} \sqrt{\frac{2 \Delta p}{\rho}},$$  \hspace{1cm} (1)

where $\rho$ is the fluid density, $w$ is the width of the opening, and $\alpha$ is the contraction coefficient related to the flow separation around the tip of the reed. In this formulation, the contraction coefficient is the ratio of the actual velocity to the theoretical velocity based on the Bernoulli equation and the assumption of the uniform flow along the opening. The value of $\alpha$ was estimated to be in the range of $0.5 < \alpha < 0.611$ by experiments for a two-dimensional reed channel [7]. This model was validated by measurements performed by van Zon and coworkers [8,9].

Backus [10] also measured simultaneously the reed motion and sound pressure in the mouthpiece. It was clarified that the waveform of the sound pressure was similar to that of the reed oscillations. Furthermore, the waveform of the reed oscillations was found to nearly imitate rectangular and sinusoidal waveforms for intense and small tones, respectively. McIntyre et al. [11] proposed the sound production model, which explains some of the nonlinear behaviors of the reed instruments. Idogawa et al. [12] investigated the variations of mouthpiece pressure and

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air flow velocity at the reed opening along reed motions. The complex transitions of vibratory states caused by the change in mouthpiece pressure were discussed. Moreover, Almeida et al. [13] measured mouth pressure, lip force, and radiated sound pressure using an artificial blowing device for a clarinet, and investigated the effects of mouth pressure and lip force on the radiated sound. Li et al. [14] discussed the effects of the blowing pressure and lip force on the reed vibration. However, the relationship between the flow field and the acoustic radiation has not been sufficiently investigated.

Recently, studies of woodwind instruments with computational fluid dynamics (CFD) and computational aeroacoustics (CAA) have been performed [15–22]. The first Navier–Stokes modeling of a wind instrument was carried out by Skordos on recorders and organ pipes [15,16]. About a decade later, Kühnelt utilized the lattice Boltzmann method to study the recorder, transverse flute, and organ pipe [17]. Miyamoto and coworkers [18,19] and Tateishi et al. [20] discussed the relationship between the jet velocity and the fundamental frequency utilizing large-eddy simulations (LESs) for two-dimensional or three-dimensional flue instruments. Giordano performed direct numerical simulations with artificial viscosity for a two-dimensional or three-dimensional flue instruments. Girlando performed direct numerical simulations with artificial viscosity for a two-dimensional or three-dimensional recorder model [21,22]. Yokoyama et al. [23] conducted direct aeroacoustic simulations for flue instruments, where the flow and acoustic fields in the actual instrument were discussed.

da Silva et al. [24] performed numerical simulations of self-sustained oscillations of flow in reed-mouthpiece models with various channel dimensions based on two-dimensional lattice Boltzmann equations. The contraction coefficient during the reed oscillations was predicted, where the resonator was not included in the model. Richter [25] solved the three-dimensional Navier–Stokes equations for flows inside a bassoon by applying a pressure boundary condition at the inlet of a crook instead of the oscillating reeds. To the best knowledge of the authors, the direct aeroacoustic simulations of flow and sound with the direct reproduction of the reed oscillations around an actual three-dimensional reed instrument with a resonator have not been performed.

In this work, the direct aeroacoustic simulations with the reproduction of reed oscillations were performed for an actual reed instrument (JRS700, KHS Musical Instruments Co., Ltd., Tianjin, China) shown in Fig. 1.

To understand the sound production mechanism in the perspective of the aeroacoustics, the relationship between the flow and acoustic fields is clarified. While the results with the reed oscillations with a sinusoidal wave were investigated in a previous work by Kobayashi et al. [26], the waveform measured by utilizing an artificial blowing device is given in this paper. The acoustic radiation under the conditions with steady oscillations is discussed in this paper. Also, the effects of the vortex ring around an open end on the acoustic radiation are presented.

2. EXPERIMENTAL AND COMPUTATIONAL CONDITIONS

2.1. Reed Instrument

The flow and acoustic fields around the reed instrument shown in Fig. 1 were investigated. This instrument is composed of a clarinet mouthpiece and a recorder-like resonator of the German fingering system with the lowest note of C4. The shape of the mouthpiece is similar to that of CLC40 (YAMAHA Co., Shizuoka, Japan), and a standard ligature with two screws is included in this instrument. The open-end inner edge forms an angle of 90°. Figure 2 shows the experimental setup, where the dimensions of the reed instrument and pressure chamber are described. The length $l$ of the instrument was 307.25 mm and the constant inner diameter $d$ of the resonator was 13.3 mm.

The origin of the coordinate system was set at the spanwise center of the tip of the mouthpiece. The $x$-axis...
was along the axial direction of the reed, the $y$-axis along the direction of the reed oscillations, and the $z$-axis along the spanwise direction.

The reed (Clarinet Traditional, Strength M, Forestone Japan Co., Ltd., Osaka, Japan) whose material is a mixture of polypropylene resin and cellulose wood fiber was used in this study. The length of the reed was 40 mm from the tip to the ligature, and the mouthpiece with the reed was placed in the pressure chamber, where the streamwise extent of the chamber was $b_c = 130$ mm. For the definition of the reed oscillations, the $x_r$-axis along the reed was also utilized with the origin at the tip of the reed. The vibrating length from the reed tip to the imaginary supporting point, $x_r^*$, where the amplitude of the oscillations is approximately zero, was $L_r = 24$ mm and the width was $w/L_r = 0.54$. The streamwise length of the reed channel between the mouthpiece and the reed, which is shown in Fig. 2(b), was $l_c = 0.7$ mm.

### 2.2. Blowing Conditions

To mimic the mouth and teeth of the performer, a rectangular artificial urethane lip (H-0-3K, EXSEAL Co., Ltd., Gifu, Japan) of 3 mm thickness was glued to the reed in the area with $x_r/L_r = 0.35$–0.56. A plate with a length of $0.04L_r$ along the reed was set at $x_r/L_r = 0.46$ as artificial teeth on the artificial lip as shown in Fig. 2(b).

Experiments and computations were performed with all tone holes closed. In this study, the tone hole covers were set along the outer surface of the instrument. Table 1 shows the computational and experimental conditions. The mean gauge pressure in the chamber was adjusted to $P = 8$ kPa, where the experimental and computational mean flow rates were $Q_{avg} = 11$ L/min. These conditions were determined so that a stable tone was obtained with the largest sound level in the preliminary experiments. This chamber pressure is within the region that the clarinet was played (2.9–11.1 kPa) [9]. The stable tonal sound and reed oscillations occurred at $f_r = 265$ Hz in the experiments.

The amplitude of the reed oscillations, which corresponds to the maximum clearance between the tip of the oscillating reed and the mouthpiece, was found to be $A_{pp,1} = 0.49$ mm. The ratio of the maximum clearance to the streamwise reed channel length was $l_c/A_{pp,1} = 1.43$.

### 3. METHODOLOGIES

#### 3.1. Experimental Methodology

As shown in Fig. 2(b), the lip force to the reed was measured using a load cell (eDPU-50N, IMADA CO., LTD., Aichi, Japan), which was found to be $F_{lip} = 0.5$ N under the above-mentioned conditions. The flow rate was measured using a flow meter (FD-A100; KEYENCE, Osaka, Japan).

A pressure sensor was used to measure the average pressure in the chamber (DP101ZA; Panasonic Co., Osaka, Japan) (pressure sensor 1 in Fig. 2). The pressure fluctuations in the chamber, mouthpiece, and resonator were measured using a pressure sensor (Model 8510B-2, Meggitt (Orange County) Inc., California, United States) (pressure sensor 2 in Fig. 2). The sound pressure level was measured at a sampling frequency of 40 kHz in a semi-anechoic room using a 1/2-inch free-field microphone (4191; B&K, Naerum, Denmark) located 100 mm above the end of the instrument ($x/l = 1.0, y/l = -0.32$, and $z/l = 0$), as shown in Fig. 2(a). This position was determined to prevent the effects of the flow from the instrument on the measured sound level.

The displacement of the reed oscillations was measured at different positions within $x_r/L_r = 0.0$–0.71 along the surface of the spanwise center of the reed using a laser displacement meter (LK-800A: KEYENCE, Osaka, Japan) with a spot diameter of 70 μm. The sampling frequency was 50 kHz, which was sufficiently high to capture the oscillations with the fundamental frequency $f_r = 265$ Hz.

#### 3.2. Governing Equations and Finite Difference Formulation for Computations

To simulate the interactions between flow and acoustic fields, the three-dimensional compressible Navier–Stokes equations with mass and energy conservation laws were directly solved using the sixth-order-accurate compact finite difference scheme (the fourth-order-accurate scheme at the boundary) [27]. The time integration was performed using the third-order-accurate Runge–Kutta method, where the time step was set to $\Delta t = 1.45 \times 10^{-8}$ s.

To predict unsteady flow and acoustic behaviors, LESs utilizing spatial filtering were performed for the flow around an actual reed instrument. No explicit Sub-Grid Scale (SGS) model was used. The turbulent energy in the Grid Scale (GS) that should be transferred to SGS eddies was dissipated by the 10th-order spatial filter [28]. This filter also suppressed the numerical instabilities associated with central differencing in the compact scheme [29].

To reproduce the complex shape of the actual reed instrument and the oscillating reed on rectangular grids, the volume penalization method [30,31], one of the immersed-

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### Table 1 Conditions for computations and experiment.

| Chamber pressure $P_m$ [kPa] | Frequency $f_r$ [Hz] | Reed vib. amplitude $A_{pp,1}$ [mm] | Flow rate $Q_{avg}$ [L/min] |
|-----------------------------|----------------------|-----------------------------------|-----------------------------|
| Comp.                       | 8.0                  | 0.49                              | 11                          |
| Exp.                        |                      |                                   |                             |
boundary methods [32], was utilized. The software for the simulation was developed by the authors [23,33,34]. In the previous study [34], the flow and acoustic fields around an oscillating cylinder and plates were correctly predicted.

3.3. Reproduction of Reed Oscillations

As mentioned in Sect. 1, the reed oscillations were imposed as forced vibrations referring to the measured displacement in the simulation. The frequency of forced vibrations was 265 Hz, corresponding to the fundamental frequency of the self-sustained oscillations in thereed instrument with closed tone holes. In the experiment, the ratio of the time intervals with closed and opened channels, $T_c/T_o$, was 0.73–1.39. The ratio was affected by the slight difference in the installation conditions of the artificial lip and teeth in the present experiment. In the present computation, the ratio was set as $T_c/T_o = 0.77$, where the predicted sound pressure spectrum qualitatively agrees with that in the experiment, as shown in Sect. 4.

Figure 3 shows the measured time variation of the clearance between the reed and the mouthpiece at $x_r/L_r = 0, 0.125, 0.30$, and 0.58, where the horizontal axis was normalized by the time interval with the opened reed channel, $T_o$, and the vertical axis was normalized by the peak-to-peak amplitude at each position with the clearance at the timing of the closed channel subtracted, $h_o/A_{pp}$.

As shown in Fig. 3, the nondimensional time variation (waveform) is approximately independent of the position. In the simulation, the waveform of the clearance fitted by the measured displacement at the tip was utilized at each position.

The longitudinal distributions of the amplitude of the reed oscillations were also given with reference to the measured results, as shown in Fig. 4. The ratio of the clearance at each position to the peak-to-peak amplitude of the displacement at the tip, $h_o/A_{pp,t}$, is presented in Eqs. (2)–(6).

$$h_o(x,t)/A_{pp,t} = A(x) \left\{ \frac{C_0}{2} + \sum_{k=1}^{N} [C_{1,k} \cos(2\pi kf_k t) + C_{2,k} \sin(2\pi kf_k t)] \right\},$$

$$A(x) = \begin{cases} \frac{1}{J} & \left[ \cosh \left( \frac{x_l^f - x_l}{x_l^f - x_l^s} \right) - \cosh \left( \frac{x_l^t - x_l}{x_l^f - x_l^s} \right) \right] \\
-K & \left[ \sinh \left( \frac{x_l^f - x_l}{x_l^f - x_l^s} \right) - \sinh \left( \frac{x_l^t - x_l}{x_l^f - x_l^s} \right) \right] \\
0 & \text{if } x_l^s \leq x_l < x_l^{lip}, \\
0.000227(x_l^s - x_l^{lip})x_l^2 - 0.011754(x_l^s - x_l^{lip})x_l \\
+ 0.15152(x_l^s - x_l^{lip}) & \text{if } x_l^{lip} \leq x_l < x_l^t, \\
0 & \text{otherwise}, \end{cases}$$

$$K = \frac{\cosh(\alpha) + \cos(\alpha)}{\sinh(\alpha) + \sin(\alpha)},$$

$$J = \cosh(\alpha) - \cos(\alpha) - K(\sinh(\alpha) - \sin(\alpha)),$$

$$x_l^t = 0.8 L_r,$$

where $C_0$, $C_{1,k}$, and $C_{2,k}$ are Fourier coefficients with $N = 95$ in this simulation, and $\alpha = 1.875$ is the fundamental eigenvalue. From the tip to the lip position ($x_l^t \leq x_l < x_l^{lip}$) shown in Fig. 2, the fundamental mode shape of a cantilever was given, where the imaginary fixed end $x_l^t$
was introduced with reference to the measured distributions. Moreover, the second-order polynomial function was given from the lip position to the supporting position \((x_{lip}^0 \leq x < x_{lip}^0)\). Figures 3 and 4 show that the given fitted displacement of the reed oscillations correctly reproduces the measured displacement.

### 3.4. Computational Grid

Figure 5 shows the computational domain, where the dimensions were nondimensionalized by the fundamental wavelength, \(\lambda_1 = 1.300\) mm. The computational domain was divided into three regions: a flow region, a sound region, and a buffer region with different grid spacings. The grid spacing smoothly changed between the regions to prevent the artificial reflection or damping of the sound.

In the flow region around the reed and the mouthpiece, to capture the air jet from the reed channel between the mouthpiece and the oscillating reed, the \(x\)- and \(y\)-directional grid spacings near the reed were set to \(\Delta_{min} = 0.025\) mm, corresponding to about \(A_{rp}/18\). In the spanwise direction, the grid spacing around the instrument was \(\Delta_{z_{min}} = 0.25\) mm, corresponding to about \(w/52\).

While the grid resolution was generally coarser in the sound region than in the flow region, the resolution was sufficiently fine for the propagation of acoustic waves in the frequency range of \(f/f_1 \leq 1\), where more than 8 points per wavelength were utilized. In particular, the maximum grid resolution in the instrument was \(\Delta = 1.0\) mm, where 1,300 points existed per fundamental wavelength. The pressure fluctuations were sampled at \(x/l = 1.0\), \(y/l = -0.32\), \(z/l = 0\) for comparison with the measured result. Around this point, the grid resolution was finer than \(\lambda_c/260\), and the acoustic propagation of higher harmonics (~the thirtieth harmonic) can be captured using the above-mentioned high-order numerical schemes.

In the buffer region up to \(5\lambda_c\) near the six artificial outlet surfaces, the grid was stretched to \(\Delta/\lambda_c = 0.6\) so as to weaken acoustic waves near the artificial outflow boundary. The total number of grid points was approximately \(1.61 \times 10^8\).

To investigate whether the present grid resolution is sufficient, the present predicted sound pressure spectrum was compared with that predicted by utilizing a finer grid, where the grid resolution near the instrument was twice as fine as the present values. As a result, the sound pressure spectra for the two grids were found to be almost identical, confirming that the present grid resolution is sufficiently fine.

### 3.5. Boundary Conditions

The boundary conditions for the computation are also shown in Fig. 5. The nonreflecting boundary conditions [35–37] were used at the boundaries in the \(x\) and \(y\) directions, and the periodic boundary conditions were used in the \(z\) direction along with buffer regions in all the directions.

The chamber pressure shown in Table 1 was directly given in the inflow region up to \(0.21b_c\) in the chamber. In preliminary experiments, the pressure in this inflow region was found to be almost constant in time. In addition, in this region, the velocity was adjusted by preliminary computations so that the flow rate integrated in the cross section based on the given velocity was confirmed to be consistent with the flow rate into the instrument. As shown in Table 1, the adjusted flow rate was \(Q_{ave} = 11\) L/min, which agreed with the measured flow rate.

### 4. VALIDATION OF COMPUTATIONS

Figure 6 shows the time variations of predicted and measured pressure fluctuations in the mouthpiece \((x/l = 0.076, y/l = 0.012, z/l = 0.013)\) for two periods. The vertical axis was nondimensionalized by the maximum fluctuation pressure, \(p_{ref}/P = 0.7\) (computation) and 0.9 (experiment). The origin of time, \(t = 0\), is defined as the time for the start of the condition of the reed channel to be completely closed. The waveform shows stable fluctuations, indicating that the flow has been sufficiently developed after the initial condition in the computation. The predicted waveform is in good agreement with the measured waveform.

Figure 7 shows the time variations of the volume flow rate predicted in this simulation, which was evaluated by the integration of the velocity along the opening area, and that calculated on the basis of the predicted pressure difference along the reed channel and Eq. (1) [7]. The contraction coefficient in the equation, \(\alpha\), depends on the
geometry of the reed channel inlet. The coefficient was found to be in the range of $0.6 < \alpha < 0.85$ for a clarinet mouthpiece mounted in an artificial mouth in the experiment by Maurin [38].

The value estimated from the results of the present simulation, $\alpha = 0.654$, is reasonably within the above-mentioned range. The difference in the root-mean-square deviation of the fluctuations predicted in the present simulation and those by the theoretical values with this coefficient is 1%. This indicates that the present simulations correctly capture the effects of the reed channel height and the pressure difference across the reed channel on the flow rate in reed instruments. The slight difference is consistent with the results obtained by da Silva et al. [24], where the variation of contraction coefficient during the cycle was reported.

Figure 8 shows the predicted and measured sound pressure spectra in the mouthpiece ($x/l = 0.076$, $y/l = 0.012$, $z/l = 0.013$) and outside of the instrument ($x/l = 1.0$, $y/l = -0.32$, and $z/l = 0$). The dimensional pressure levels in decibels are related to the standard quantity of $2 \times 10^{-5}$ Pa. The frequency resolution for the spectral analysis was $\Delta f = 39$ Hz.

As shown in Fig. 8, the predicted sound pressure level of the fundamental tone agrees with the value obtained experimentally at both positions. Although the sound levels of some harmonics are over- or underestimated, the typical harmonic features of the lowest acoustic mode of clarinet-type instruments are reproduced, where the radiation of the third harmonics is more intense than that of the second harmonics. Therefore, it has been confirmed that this simulation captures the dominant phenomena of the flow and sound in the reed instrument.
5. RESULTS AND DISCUSSION

5.1. Flow Fields in Mouthpiece

Fig. 9 shows the contours of vorticity $\omega_z/(U_{avg}/A_{pp})$ (left), and those of streamwise velocity $u/U_{avg}$ with velocity vectors (right), where $U_{avg} = Q_{avg}/(wA_{pp})$.

At the time of the opening of the reed channel ($t/T_r = 0.4$), the jet is formed from the reed channel and oscillates downstream (left figure). Also, as shown in the right figure, the flow is spread along the inner wall of the mouthpiece until $x/l = 0.076$, where the flow separation leading to the intense pressure fluctuations occurs. At $t/T_r = 0.8$ (left figure), the clockwise and counter-clockwise vortices are formed in the jet from the reed channel.

5.2. Standing Wave in Instrument

Figure 10 shows the time variation of mouthpiece pressure with chamber pressure subtracted along that of the clearance at the tip, $h_{o,z}$, where the chamber pressure is constant as mentioned above. As shown in Fig. 10, the pressure fluctuations in the instrument show periodical behaviors. When the reed is opened, the pressure in the mouthpiece begins to increase. This pressure variation is consistent with the results obtained by Idogawa et al. [12].

To clarify the behavior of standing waves in the instrument, the distributions of the pressure fluctuations along the longitudinal path ($x/l = 0.002$–$1.3$, $z = 0$) shown in Fig. 12 were analyzed. The path for $x/l = 0.002$–$0.13$ is from the inner edge of the tip of the mouthpiece to the axis of the resonator ($y/l = -0.0004$–$0.02$), while that for $x/l = 0.13$–$1.3$ is along the central axis of the resonator ($y/l = 0.02$).

5.3. Phases of Pressure Fluctuations

Figure 13(a) shows the phase distributions of pressure fluctuations, $\psi_p$, with the reference at the center of the resonator ($x/l = 0.5$). As can be observed in the figure, the phase within the resonator for $x/l = 0.014$–$1.0$ is approx-
approximately equal to zero. This means that there are standing waves in the resonator. Also, the phase from $x = l = 1.0$ to $x = l = 1.3$ changes according to the slope of the sound speed.

Figure 13(b) shows the predicted and measured power distributions of the pressure fluctuations, $P_p$, normalized by that at $x/l = 0.05$, $P_{p,\text{ref}}$, along the above-mentioned path.

The power of the fluctuations is almost constant near the reed and gradually decreases from $x/l = 0.3$ to 0.8. Also, the power decreases rapidly near the end of the instrument. As a result, the power at $x/l = 1.07$ ($x = \lambda_r/4$) is far less (approximately 0.002–0.005%) than that of $P_{p,\text{ref}}$. This result also shows that the acoustic resonance with a one-quarter wavelength mode that is a characteristic of general reed instruments, such as clarinets [3], occurs in the present simulation.

5.3. Mechanism of Acoustic Radiation in Mouthpiece

To discuss the acoustic radiation mechanism, the streamwise gradient for the mass flow rate in the instrument is investigated as shown in Fig. 14. Figure 14(a) shows the time variation of the streamwise gradient for the mass flow rate in the instrument, $\partial(\rho Q)/\partial x$, at $x/l = 0.016$, 0.076, 0.13, and 0.2, where the vertical axis is normalized by the mean mass flow rate and the length of the instrument, $(\rho Q)_{\text{avg}}/l$, at each position. In Fig. 14(a),
positive and negative values represent the occurrence of the expansion and compression process of fluids, respectively.

Figure 14(a) indicates that intense expansion occurs in the region near the tip of the mouthpiece \((x/l = 0.016)\) at \(t/T_r = 0.97\), whereas intense compression occurs downstream \((x/l = 0.076)\) at \(t/T_r = 0.56\). This position for the occurrence of intense compression corresponds to that for the flow separation of the spread air jet in the mouthpiece as shown in Fig. 9 (dashed line). The flow separation causes the effective flow passage to become narrow and possibly reinforces the air compression. In addition, the timing of the maximal compression \((t/T_r \approx 0.6)\) at \(x/l = 0.076\) approximately corresponds to that of the maximal volume flow rate into the reed channel, as shown in Fig. 7.

This synchronization indicates that the air is compressed intensely by the increase in volume flow rate into the reed channel.

Figure 14(b) shows the fluctuation pressure at \(x/l = 0.076, y/l = 0.012, z/l = 0\) (red circle in Fig. 11), along the streamwise gradient of the mass flow rate at \(x/l = 0.076\). As shown in Fig. 14(b), the gradient of mass flow rate at \(t/T_r = 0.44-0.94\) is negative. This implies that the air around this region is compressed. This is consistent with the increase in fluctuation pressure. Moreover, in the remaining time, the gradient of the mass flow rate is positive, and the pressure around this region decreases. Thus, the air in the mouthpiece is compressed and expanded, and the standing wave in the resonator is sustained.

The present clarification of the acoustic radiation mechanism contributes to the understanding of the sound source in the model of the sound production mechanism, such as the McIntyre–Schumacher–Woodhouse model [11].

5.4. Acoustic Radiation from Vortex Shedding from Open End

It is known that a vortex ring is formed from the resonator with acoustic excitation [39]. Moreover, the acoustic radiation occurs owing to the generation of the ring [40]. Figure 15 shows the contours of the fluctuation pressure, velocity vectors, and iso-surface of the second invariant of the velocity gradient tensor, \(q/(U_{avg}/A_{pp,i})^2 = 0.001\) near the end of the instrument.

This figure shows that vortex rings 1 and 2 that form the low-pressure regions are observed near the open end of the instrument. These vortex rings are generated owing to the intense acoustic particle velocity around the open end of the instrument as follows. First, vortex ring 1 is generated from the end of the instrument to...
the inside when the acoustic particle velocity to the inside becomes intense. Thereafter, vortex ring 1 is released to the outside of the instrument due to the acoustic particle velocity to the outside. Subsequently, vortex ring 2 is also shed from the end of the instrument.

As shown in Fig. 15, an expansion wave is radiated at $t/T_r = 0.16$ when the vortex rings are shed from the end of the instrument. These results indicate that the acoustic radiation occurs through the shedding of the vortex ring. This relationship between the shedding of the vortex ring and the acoustic radiation is similar to that observed around a rectangular or circular cylinder in a flow [41,42].

Figure 16(a) shows the time variation of the radiated sound pressure in the far-field $(x/l = 1.0$, $y/l = -0.32$, $z/l = 0)$. Considering the retarded time due to acoustic propagation from the instrument to the far-field point, the valleys corresponding to the occurrence of the expansion waves due to standing waves of the resonance and the above-mentioned vortex rings are marked in Fig. 16(a), where the subscripts “sw” and “vor” represent the standing wave and vortex rings, respectively. It is shown that the expansion wave due to the shedding of the vortex ring causes the local minimum in the waveform of the radiated sound.

Figure 16(b) shows the contours of the fluctuation pressure at $t/T_r = 0.3$, when the radiated expansion wave passes the above-mentioned point $(x/l = 1.0, y/l = -0.32, z/l = 0)$. The low-pressure wavefront exists owing to the above-mentioned expansion wave. These results indicate that the shedding of the vortex ring from the open end affects the radiated sound from the instrument.

The radiation impedance, $Z_r = R_r + i X_r$, around the open end evaluated from the predicted velocity and pressure fluctuations is $R_r/(\rho a/(d/2)^2) = 4.6 \times 10^{-5}$ and $X_r/(\rho a/(d/2)^2) = 3.0 \times 10^{-3}$. In this evaluation, the predicted fluctuations within the radial position within 80% of the radius of the resonator, $d/2$, were utilized so that the effects of the fluid-dynamical fluctuations of the vortices near the edge on the impedance are suppressed. The theoretical values for the open end with a baffle plate are $R_r/(\rho a/(d/2)^2) = 5.1 \times 10^{-5}$ and $X_r/(\rho a/(d/2)^2) = 2.7 \times 10^{-3}$ based on the following equation [3,43,44]:

$$Z_r = \frac{\rho a^2}{2\pi a} + i \frac{8\rho a}{3\pi^2(d/2)} \left(\frac{2}{d} - 1\right)^2,$$

where $a$ is the sound speed. The difference between the predicted and theoretical impedances is possibly related to the vortex shedding from the open end, although the detailed clarification of this relationship is a future problem.

To the best knowledge of the authors, the effects of the vortex shedding from the open end on the radiated sound for the actual reed instrument were quantitatively evaluated for the first time. The obtained knowledge contributes to the modification of the sound production model.

6. CONCLUSIONS

Direct aeroacoustical simulations based on the compressible Navier–Stokes equations were performed to clarify the relationship between the flow and acoustic fields around an actual reed instrument. The predicted flow field showed that an air jet is formed from the reed channel into the mouthpiece. The oscillations of the air jet and the shedding of the clockwise and counter-clockwise vortices occur near the tips of the reed and mouthpiece. Also, the air jet is spread along the inner wall of the mouthpiece and the reed, and the flow separation occurs downstream. The streamwise gradient of mass flow rate around this point of the flow separation shows that the intense compression occurs. Moreover, the predicted acoustic field confirmed that the acoustic radiation in the mouthpiece allows for sustained acoustic resonance.

It was found that an expansion wave was radiated by the shedding of the vortex ring near the open end of the instrument. The acoustic radiation due to the vortex ring

![Image 72x624 to 248x792]

![Image 73x471 to 270x605]
causes the local minimum of the waveform of the far-field acoustic pressure.

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