Comparison of approximate to exact next-to-next-to leading order corrections for Higgs and pseudoscalar Higgs boson production

A.P. Contogouris\textsuperscript{a,b,*} and P.K. Papachristou\textsuperscript{b,†}

(a) Department of Physics, McGill University
Montreal, Quebec, H3A 2T8, CANADA
(b) Nuclear and Particle Physics, University of Athens
Panepistimiopolis, Athens 15771, GREECE

Abstract

Recently obtained NNLO exact corrections for Higgs and Pseudoscalar Higgs boson production in hadron colliders are compared with approximate ones. As shown before, it is found that there is a range of a proper variable where these corrections differ little.

\textsuperscript{*}e-mail: apcont@physics.mcgill.ca, acontog@cc.uoa.gr

\textsuperscript{†}e-mail: ppapachr@cc.uoa.gr
Some time ago it was argued that for processes involving structure functions and/or fragmentation functions, over a range of a proper kinematic variable $w$, there is a part that dominates the next- to leading order (NLO) correction and that this part contains the distributions $\delta(1-w)$ and $[\ln^n(1-w)/1-w]+ n = 0, 1$. Subsequently this argument was extended to the then existing next- to- next- to leading order (NNLO) calculations, namely Drell- Yan (D-Y) production of lepton pairs ($q + \bar{q} \to \gamma^*$) and deep inelastic structure (DIS) functions ($q + \gamma^* \to q$).

In the meantime two more processes have been calculated in NNLO: Higgs boson production in hadron- hadron collisions ($g + g \to H$) and neutral pseudoscalar Higgs boson production in hadron- hadron collisions ($g + g \to A$). Clearly, it would be important to see whether the procedures developed in the NNLO calculations apply also to Higgs and pseudoscalar Higgs boson production as well.

In the calculation of Higgs boson production in hadron- hadron collisions to leading order (LO) and to NLO no approximations of the Higgs- two gluon vertex are necessary. This vertex is dominated by the top quark, which is known to have a mass $m_t$ much greater than that of the other quarks. However, the NNLO calculation was possible only in the limit of the Higgs mass $m_H$

$$m_H << 2m_t.$$  \hspace{1cm} (1)

In this limit the top-quark loops are replaced by point-like vertices and the corresponding effective Lagrangian is known to provide a satisfactory description of the cross section for a Higgs boson at NLO.

In the calculation of the pseudoscalar Higgs boson production the situation is more complicated. The Higgs boson sector of the Minimal Supersymmetric Standard Model consists of two complex Higgs doublets. Thus, apart from the mass of the neutral pseudoscalar Higgs boson $m_A$, the ratio of the vacuum expectation values of the two Higgs doublets $v_1/v_2 \equiv \tan \beta$ also enters. The calculation of is valid for small and moderate values of $\tan \beta$; only then the $gg \to A$ is dominated by a top-quark loop. Then, for

$$m_A << 2m_t$$  \hspace{1cm} (2)

the interaction of the pseudoscalar Higgs boson can be described by an effective Lagrangian.

In our approach the proper variable is proportional to $\tau = m_H^2/S$ (or $\tau = m_A^2/S$), where $\sqrt{S}$ is the total c.m. energy of the initial hadrons. Our approach requires inclusion of the region $\tau$ large (inclusion of $\tau$ near 1). Since experiment excludes values of $m_H$ (or $m_A$) $\leq 100$ GeV, we have to consider $\sqrt{S}$ well exceeding this value. On the other hand, inclusion of $\sqrt{S} \geq 2$ TeV would require $m_H$ (or $m_A$) well exceeding 1 TeV, which would

---

*This paper, in preliminary form, was presented in "QCD 2000", Montpellier, France, July 2000, and published in Nucl. Phys. B, (Proceed. Suppl.) 96 (2001) 94 (hep-ph/0109070).

†To NLO this is due to an accidental cancellation of the dependence on the top quark mass between real and virtual corrections. See the last paper of Ref. [6].
render questionable the field-theoretic approach. We then have considered a nominal energy of $\sqrt{S} = 520$ GeV. Clearly, for $m_H$ (or $m_A$) $\geq 200$ GeV the inequality (1) (or (2)) is violated and the whole results of [3] and [4], which we use, should be considered as just providing a mathematical model, where the approach of [2] can be tested.

We begin with $pp \to H + X$ (or $p\overline{p} \to H + X$) mediated via the subprocess $gg \to H^*$ and we consider the cross-section

$$\sigma_{h_1+h_2\to H+X(m_H^2, S)} = \int_0^1 dx_1 dx_2 \overline{f}_{g/p}(x_1)\overline{f}_{g/p}(x_2)\sigma_{gg\to H}(m_H^2, x_1 x_2 S)$$

(3)

where $h_1, h_2$ denote $p, p$ (or $p, \overline{p}$) and $\overline{f}_{g/p}(x)$ is the standard distribution of gluons inside the $p$ (or $\overline{p}$). Using dimensional analysis we write the partonic cross-section in terms of the dimensionless variable

$$z = \frac{m_H^2}{x_1 x_2 S} = \frac{\tau}{x_1 x_2}$$

(4)

and after factoring the collinear singularities (usually in the $\overline{MS}$ scheme) we end up with the following expression [3]

$$\sigma_{h_1+h_2\to H+X(\tau, S)} = \tau f_{g/p} \otimes f_{g/p} \otimes (\sigma_{gg}(z)/z)(\tau)$$

(5)

where $\otimes$ denotes the standard convolution defined as

$$[f_1 \otimes f_2](\tau) = \int_0^1 dx_1 dx_2 f_1(x_1) f_2(x_2) \delta(\tau - x_1 x_2).$$

(6)

The partonic cross-section $\sigma_{gg}(z)$ is given by the following perturbation expansion

$$\sigma_{gg}(z) = \sigma_0 \left[ \eta_{gg}^{(0)}(z) + \frac{\alpha_s}{\pi} \eta_{gg}^{(1)}(z) + \left( \frac{\alpha_s}{\pi} \right)^2 \eta_{gg}^{(2)}(z) + O(\alpha_s^2) \right]$$

(7)

where the functions $\eta_{gg}^{(k)}$, $k = 0, 1, 2$, are given in Eqs. (44), (45), (47), (48) and (49) of [3], and

$$\sigma_0 = \frac{\pi}{576 \upsilon^2} \left( \frac{\alpha_s}{\pi} \right)^2$$

(8)

with $\upsilon \approx 246$ GeV the Higgs vacuum expectation value.

Subsequently we proceed as in [2]. We write, for simplicity, $\sigma_{h_1+h_2\to H+X(\tau, S)} \equiv \sigma_H(\tau, S)$ and denote by $\sigma_H^{(k)}(\tau, S)$, $k = 0, 1, 2$, the $O(\alpha_s^k)$ part of $\sigma_H(\tau, S)$, by $\sigma^{(k)}_H$ the part of $\sigma_H^{(k)}$

---

*We remind that in our approach [1] the various perturbation orders (LO, NLO, NNLO) should refer to the same subprocess.

†Although known since long ago (see [1]), as in [2], we present also results for $k = 1$. 

2
arising from distributions $\delta(1-z)$ and $[\ln^n(1-z)/(1-z)]_+$, here $n=0,1,2,3$ (virtual, collinear and soft gluons) and by $\sigma_{hh}^{(k)}$ the rest. We also define

$$L_{H}^{(k)}(\tau, S) = \frac{\sigma_{hh}^{(k)}(\tau, S)}{\sigma_{H}^{(k)}(\tau, S)}. \quad (9)$$

In the subsequent calculations we use $n_f=5$ flavors and fix the renormalization and factorization scales at $\mu = M = m_H$. For the gluon distributions we use the updated \(\bar{MS} \) CTEQ5M1 set of \cite{10}.

Fig. 1, upper part, shows $L_{H}^{(k)}$, $k = 1, 2$, as functions of $\sqrt{\tau}$. For $L_{H}^{(1)}$, while for relatively small $\sqrt{\tau}$ is significant, for $\sqrt{\tau} \geq .63$ is below 30%. As for $L_{H}^{(2)}$, for $\sqrt{\tau} \geq .43$ is smaller than 20%. Moreover, both $L_{H}^{(k)}$ decrease fast as $\sqrt{\tau}$ increases towards 1.

As in \cite{2}, it is of interest to see the percentage of $\sigma_{hh}^{(k)}$ of the total cross section determined up to $O(\alpha_s^k)$. Fig. 1, upper part, also shows the ratios $\sigma_{hh}^{(1)}/(\sigma_{H}^{(0)} + \sigma_{H}^{(1)})$ and $\sigma_{hh}^{(2)}/(\sigma_{H}^{(0)} + \sigma_{H}^{(1)} + \sigma_{H}^{(2)})$. The former is below 31% and the latter below 15% for all $\sqrt{\tau}$. Again, both ratios decrease fast as $\sqrt{\tau}$ increase.

Now we turn to the calculation of the pseudoscalar Higgs boson production and consider $pp \rightarrow A+X$ (or $p\overline{p} \rightarrow A+X$) mediated via the subprocess $gg \rightarrow A$. As before, the partonic cross-sections $\sigma_{gg}(z)$ have an expansion similar to (7)

$$\sigma_{gg}(z) = \sigma_0 \left[ \phi_{gg}^{(0)}(z) + \frac{\alpha_s}{\pi} \phi_{gg}^{(1)}(z) + \left( \frac{\alpha_s}{\pi} \right)^2 \phi_{gg}^{(2)}(z) + o(\alpha_s^3) \right]. \quad (10)$$

where $z$ is given by (4) with $\tau = m_A^2/S$, $\phi_{gg}^{(k)}(z)$ are given in Eqs. (8)-(11) of \cite{4} (together with the expressions of $\eta_{gg}^{(k)}(z)$), and here

$$\sigma_0 = \frac{\pi}{256\pi^2 \tan^2 \beta} \left( \frac{\alpha_s}{\pi} \right)^2. \quad (11)$$

Now we write $\tan^2 \beta \sigma_{h_1+h_2\rightarrow A+X} \equiv \sigma_A(\tau, S)$ and, as before, denote by $\sigma_A^{(k)}(\tau, S)$ the $O(\alpha_s^k)$ part of $\sigma_A(\tau, S)$, by $\sigma_A^{(k)}$ the part of $\sigma_A^{(k)}$ arising from distributions and by $\sigma_A^{(k)}$ the rest. We define

$$L_A^{(k)}(\tau, S) = \frac{\sigma_A^{(k)}(\tau, S)}{\sigma_A(\tau, S)}. \quad (12)$$

and fix the renormalization and factorization scales at $\mu = M = m_A$. Again, for the gluon distributions we use the set CTEQ5M1 set of \cite{10}.

Fig. 1, lower part, shows $L_A^{(k)}$, $k = 1, 2$, as functions of $\sqrt{\tau}$. All the results are similar as for $L_H^{(k)}$. Similar are also the results for the ratios $\sigma_A^{(1)}/(\sigma_A^{(0)} + \sigma_A^{(1)})$ and $\sigma_A^{(2)}/(\sigma_A^{(0)} + \sigma_A^{(1)} + \sigma_A^{(2)})$.

*This can also be seen in the first paper of Ref.[11].
We note the following*: suppose that in $\sigma^{(k)}_{Hs}$ and $\sigma^{(k)}_{As}$, apart from the terms arising from the distributions $\delta(1-z)$ and $[\ln^n(1-z)/(1-z)]_+$ we include also the terms $\ln^m(1-z)$, $m = 1, 2, 3$. Defining as $\sigma^{(k)}_{Hs}$ and $\sigma^{(k)}_{Ah}$ the rest, we find that the ratios $\sigma^{(1)}_{Hh}/\sigma^{(0)}_{H}$ and $\sigma^{(1)}_{Ah}/\sigma^{(0)}_{A}$ decrease significantly in magnitude over the entire range of $\sqrt{\tau}$. Of course, the same holds for the ratios $\sigma^{(1)}_{Hh}/(\sigma^{(0)}_{H} + \sigma^{(1)}_{H})$, $\sigma^{(2)}_{Hh}/(\sigma^{(0)}_{H} + \sigma^{(1)}_{H} + \sigma^{(2)}_{H})$ and the corresponding ratios with $H$ replaced by $A$.

Note also that Ref.[11] has obtained numerical results very similar to [3] and [4] by expanding the phase-space integrals around the kinematic point $z = \tau/x_1x_2 = 1$, where $\tau = m^2_H/\sqrt{S}$ or $\tau = m^2_A/\sqrt{S}$, and keeping a number of terms. Although the first paper of [11] was published before [3], we prefer the methods of [3] as they avoid expansions.$\dagger$

Finally, in Fig. 2, upper part, we present the total cross-sections $\sigma^{(0)}_{H} + \sigma^{(1)}_{H} + \sigma^{(2)}_{H}$ (dashed line) and $\sigma^{(0)}_{H} + \sigma^{(1)}_{H} + \sigma^{(2)}_{H}$ (solid line). What is important is that as $\sqrt{\tau}$ increases towards 1 both cross-sections approach each other, and for $\tau \geq 0.8$ practically coincide. The same is observed in Fig. 2, lower part, which shows the quantities $\sigma^{(0)}_{A} + \sigma^{(1)}_{A} + \sigma^{(2)}_{A}$ (dashed) and $\sigma^{(0)}_{A} + \sigma^{(1)}_{As} + \sigma^{(2)}_{As}$ (solid).

In conclusion, under the assumptions discussed at the beginning, we have shown that not only in D-Y production and DIS [2], but also in Higgs and pseudoscalar Higgs boson production ($gg \to H$ and $gg \to A$) there is a part containing the distributions $\delta(1-z)$ and $[\ln^n(1-z)/(1-z)]_+$, here $n = 0, 1, 2, 3$ (virtual, soft and collinear part) that for $\sqrt{\tau} = m_H/\sqrt{S}$ or $m_A/\sqrt{S}$ not too small dominates the NLO and NNLO correction. This part is determined much easier than the NLO and in particular the NNLO correction. Of course, as it was stressed in [2], this part should not be restricted to too small a region near 1, for threshold resummation [12] becomes very important.

NOTE ADDED

After the completion of this article, the paper by V. Ravindran, J. Smith and W. van Neerven, [hep-ph/0302335] appeared, confirming the results of [3], [4] and [11] by a different method.

Acknowledgments

We are much indebted to C. Anastasiou and K. Melnikov for very useful private communications and correspondence. A correspondence by R. Harlander and W. Kilgore is also gratefully acknowledged. The work was also supported by the Natural Sciences and

* A similar remark regarding resummations was first made by M. Kramer, E. Laenen and M. Spira, Nucl. Phys. B 511 (1998) 523.

† In the first of Ref.[11] an error was found in the calculation of T. Matsuura et al., Nucl. Phys. B 319 (1989) 570 on D-Y production. We have repeated the relevant calculations of [2] and found no significant change.
Engineering Research Council of Canada, by the Research Committee of the University of Athens and by the Greek State Scholarships foundation (IKY).

References

[1] A. P. Contogouris, N. Merbaki and S. Papadopoulos, Intern. J. Mod. Phys. A 5 (1990) 1951 and A. P. Contogouris and S. Papadopoulos, Mod. Phys. Lett. A5 (1990) 901.

[2] A. P. Contogouris and Z. Merebashvili, Intern. J. Mod. Phys. A 18 (2003) 957 (hep-ph/0205236).

[3] C. Anastasiou and K. Melnikov, Nucl. Phys. B 646 (2002) 220 (hep-ph/0207004).

[4] C. Anastasiou and K. Melnikov, hep-ph/0208115.

[5] J. Ellis et al., Phys. Lett. B 83 (1979) 339; H. Georgi et al., Phys. Rev. Lett. 40 (1978) 692.

[6] A. Djouadi, M. Spira and P. Zerwas, Phys. Lett. B 264 (1991) 440; D. Graudenz, M. Spira and P. Zerwas, Phys. Rev. Lett. 70 (1993) 1372; M. Spira, A. Djouadi, D. Graudenz and P. Zerwas, Nucl. Phys. B 453 (1995) 17.

[7] J. Ellis, M. Gaillard and D. Nanopoulos, Nucl. Phys. B 106 (1976) 292.

[8] M. Voloshin, Yad. Fiz. 44 (1986) 738; M. Shifman, Usp. Fiz. Nauk 157 (1989) 561.

[9] M. Spira et. al., Phys. Lett. B 318 (1993) 347, R. Kauffman and W. Shaffer, Phys. Rev. D 49 (1994) 551, K. Chetyrkin, B. Kniehl and M. Steinhauser, Phys. Rev. Lett. 79 (1997) 353 and Nucl. Phys. B 510 (1998) 61.

[10] H. Lai et al., (CTEQ collaboration), Eur. Phys. J. C 12 (2000) 375.

[11] R. Harlander and W. Kilgore, Phys. Rev. Lett. 88 (2002) 201801 and JHEP 0210 (2002) 017.

[12] N. Kidonakis, Phys. Rev. D 64 (2001) 014009; E. Laenen et al., ibid, D 63 (2001), 114018; A. Vogt, Phys. Lett. B 497 (2001) 228.
Figure 1: Upper part: The ratios $L_{H}^{(1)}$ and $\sigma_{HH}^{(1)}/(\sigma_{H}^{(0)} + \sigma_{H}^{(1)})$ (dashed lines) and the ratios $L_{H}^{(2)}$ and $\sigma_{HH}^{(2)}/(\sigma_{H}^{(0)} + \sigma_{H}^{(1)} + \sigma_{H}^{(2)})$ (solid lines) versus $\sqrt{\tau} = m_{H}/\sqrt{S}$. Lower part: The quantities $L_{A}^{(1)}$ and $\sigma_{Ah}^{(1)}/(\sigma_{A}^{(0)} + \sigma_{A}^{(1)})$ (dashed) and the quantities $L_{A}^{(2)}$ and $\sigma_{Ah}^{(2)}/(\sigma_{A}^{(0)} + \sigma_{A}^{(1)} + \sigma_{A}^{(2)})$ (solid) versus $\sqrt{\tau} = m_{A}/\sqrt{S}$. 
Figure 2: Upper part: The cross-sections $\sigma_{H}^{(0)} + \sigma_{H}^{(1)} + \sigma_{H}^{(2)}$ (dashed line) and $\sigma_{H}^{(0)} + \sigma_{H}^{(1)} + \sigma_{H}^{(2)}$ (solid line) versus $\sqrt{\tau} = m_H/\sqrt{S}$. Lower part: The quantities $\sigma_{A}^{(0)} + \sigma_{A}^{(1)} + \sigma_{A}^{(2)}$ (dashed) and $\sigma_{A}^{(0)} + \sigma_{As}^{(1)} + \sigma_{As}^{(2)}$ (solid) versus $\sqrt{\tau} = m_A/\sqrt{S}$.