Efficient Numerical Methods for Secrecy Capacity of Gaussian MIMO Wiretap Channel

Anshu Mukherjee∗, Björn Ottersten†, and Le Nam Tran∗

∗School of Electrical and Electronic Engineering, University College Dublin, Ireland
Email: anshu.mukherjee@ucdconnect.ie; nam.tran@ucd.ie
†Interdisciplinary Centre for Security, Reliability and Trust, University of Luxembourg, Luxembourg
Email: bjorn.ottersten@uni.lu

Abstract—This paper presents two different low-complexity methods for obtaining the secrecy capacity of multiple-input multiple-output (MIMO) wiretap channel subject to a sum power constraint (SPC). The challenges in deriving computationally efficient solutions to the secrecy capacity problem are due to the fact that the secrecy rate is a difference of convex functions (DC) of the transmit covariance matrix, for which its convexity is only known for the degraded case. In the first method, we capitalize on the accelerated DC algorithm, which requires solving a sequence of convex subproblems. In particular, we show that each subproblem indeed admits a water-filling solution. In the second method, based on the equivalent convex-concave reformulation of the secrecy capacity problem, we develop a so-called partial best response algorithm (PBRA). Each iteration of the PBRA is also done in closed form. Simulation results are provided to demonstrate the superior performance of the proposed methods.

Index Terms - MIMO, secrecy capacity, sum power constraint, convex-concave, closed form solution.

I. INTRODUCTION:

Wireless communication is an integral part of our modern life. Due to its broadcasting nature, information sent over wireless channels is vulnerable to security breach. Significant measures and techniques have been developed by both industry and academia to address this critical issue. Particularly, cryptography is a conventional method to ensure data security in wireless networks. In recent years, physical layer security has received growing interest as a promising alternative to addressing wireless security. While cryptographic methods are based on computational complexity and implemented in high network layers, physical layer security is concerned with exploiting distinguishing properties of wireless channels to achieve secure communication.

The wiretap channel (WTC), in which an eavesdropper aims to decode to the message exchanged by a pair of legitimate transceivers, represents a fundamental information-theoretic model for physical layer security. The secrecy capacity of the WTC was first studied by Wyner in [1]. Since Wyner’s seminal paper, the WTC has been extended, covering various scenarios. In particular, the secrecy capacity of the Gaussian WTC was studied [2]. The use of multiple antennas at transceivers in contemporary wireless communications systems naturally gives rise to the so-called multiple-input multiple-output (MIMO) Gaussian WTC. The secrecy capacity of Gaussian MIMO WTC has received significant interests since the late 2000s. In this regard, there have been many significant results in the literature, which are discussed as follows.

The analytical solution for the Gaussian multiple-input single-out (MISO) WTC where both the eavesdropper and the legitimate receiver have a single antenna was proposed in [3]. When the channel state information is perfectly known, the secrecy capacity of MIMO WTC was characterized in [4], [5], [6]. Particularly, explicit expressions for optimal signaling for Gaussian MIMO WTC are possible under some special cases [7], [8], [9]. Power minimization and secrecy rate maximization for MIMO WTC was studied in [10] using a difference of convex functions algorithm (DCA). More recently, a low-complex solution for Gaussian MIMO WTC was proposed in [11] using the equivalent convex-concave reformulation of the secrecy capacity problem.

In this paper we consider the problem of finding the secrecy capacity-achieving input covariance for Gaussian MIMO WTC subject to a sum power constraint (SPC). The case of general linear transmit covariance constraints such as per-antenna power constraint is studied in [12]. In particular, we develop two low-complexity methods to solve the secrecy-capacity problem for the Gaussian MIMO WTC. The first method is an accelerated difference of convex functions algorithm (ADCA) [13] where each subproblem is found in closed form. In the second method, we propose an efficient iterative method to calculate the secrecy capacity, which is based on the equivalent concave-convex reformulation of the secrecy capacity problem. We refer to this proposed method as the partial best response algorithm (PBRA). The idea of PBRA is to find a saddle point of the concave-convex problem, for which efficient numerical methods are also derived. We remark that the method presented in [11] is a double-loop iterative algorithm, while the proposed PBRA in this paper only requires a single loop.

Notation: We use bold uppercase and lowercase letters to denote matrices and vectors, respectively. \(\mathbb{C}^{M \times N}\) denotes the space of \(M \times N\) complex matrices. To lighten the notation, \(\mathbf{I}\) and \(\mathbf{0}\) define identity and zero matrices respectively, of which the size can be easily inferred from the context. \(\mathbf{H}^\dagger\) and \(\mathbf{H}^\top\) are Hermitian and ordinary transpose of \(\mathbf{H}\), respectively; \(\mathbf{H}_{i,j}\) is the \((i,j)\)-entry of \(\mathbf{H}\); \(|\mathbf{H}|\) is the determinant of \(\mathbf{H}\); Furthermore, we denote the expected value of a random variable by \(E[.\] = \(\mathbb{E}\). For \(\mathbf{x} \in \mathbb{R}^N\), \(x_+ = \)
\[ \max(x_1, 0), \, \max(x_2, 0), \, \cdots \, \max(x_N, 0) \]. The index vector (i.e., its \(i\)th entry is equal to one and all other entries are zero) is denoted by \(e_i\). The notation \(A \succeq (\succ) B\) means \(A - B\) is positive semidefinite (definite). \(\text{diag}(x)\) creates a diagonal matrix whose diagonal elements are taken from \(x\). \(\|H\|\) denotes the Frobenius norm of \(H\).

II. SYSTEM MODEL

A. MIMO Wiretap Channel Model

We consider a MIMO WTC, where Alice wants to transmit information to the legitimate receiver Bob in presence of Eve, the eavesdropper. The number of antennas at Bob, Alice and Eve is denoted by \(N_i, N_r,\) and \(N_e,\) respectively. \(H_{B} \in \mathbb{C}^{N_r \times N_i}\) is the channel matrix between Alice and Bob. The channel matrix between Bob and Eve is denoted by \(H_{E} \in \mathbb{C}^{N_r \times N_e}\). The received signals at Bob and Eve are respectively expressed as

\[
y_b = H_{B}x + z_b \quad (1a)
\]

\[
y_e = H_{E}x + z_e \quad (1b)
\]

where, \(x \in \mathbb{C}^{N_r \times 1}\) represents the transmitted signal ; \(z_b \in \mathbb{C}^{N_r \times 1} \sim CN(0, I)\) and \(z_e \in \mathbb{C}^{N_e \times 1} \sim CN(0, I)\) are the additive white Gaussian noise at Bob and Eve, respectively. In this paper \(H_{B}\) and \(H_{E}\) are assumed to be quasi-static and perfectly known at Alice and Bob. For a given input covariance matrix \(X = E\{xx^\dagger\} \succeq 0\), where \(E\{\cdot\}\) is the statistical expectation, the maximum secrecy rate (in nat/s/Hz) between Alice and Bob is given by [6]

\[
C_s(x) = \ln \left( \frac{\|I + H_{B}XH_{B}^\dagger\|}{\|I + H_{E}XH_{E}^\dagger\|} \right). \quad (2)
\]

The secrecy capacity under the sum power constraint (SPC) is written as

\[
\text{maximize} \quad C_s(X) \quad (3a)
\]

subject to \(\text{tr}(X) \leq P_0\) \quad (3b)

where \(P_0\) is the total transmit power. We remark that the problem (3) is non-convex in general, and thus, it is very difficult to find a globally optimal solution. However, if the channel is degraded, i.e., \(H_{B}^\dagger H_{E} \succeq H_{E}^\dagger H_{E}\), (3) becomes convex but off-the-shelf solvers cannot be used to solve it. In this regard, the equivalent minimax reformulation of (3) presented in the next subsection is more numerically useful.

B. Minimax Reformulation

It is interesting to note that the secrecy capacity of MIMO WTC in (2) is equivalent to the following minimax optimization problem

\[
C_s = \min_{Q \in \mathcal{Q}} \max_{X \in \mathcal{X}} f(Q, X) \triangleq \log \left( \frac{\|I + Q^{-1}H_{B}XH_{B}^\dagger\|}{\|I + H_{E}XH_{E}^\dagger\|} \right) \quad (4)
\]

where \(H = [H_{B}^\dagger, H_{E}^\dagger]^\dagger\) and \(Q \in \mathbb{C}^{N_r \times N_e}\). The sets \(\mathcal{Q}\) and \(\mathcal{X}\) are defined as

\[
\mathcal{X} = \{X | X \succeq 0; \text{tr}(X) = P_0\} \quad (5)
\]

and

\[
\mathcal{Q} = \{Q | Q \succeq 0; Q = \begin{bmatrix} I & Q \\ Q^\dagger & I \end{bmatrix} \} \quad (6)
\]

Compared to (3), (4) is more tractable since the objective of (4) is concave with \(X\) for a given \(Q\) and convex with \(Q\) for a given \(X\). In particular, we can compute the secrecy capacity and the optimal signaling by finding the saddle point of \(f(Q, X)\).

III. PROPOSED ALGORITHMS

In this section we present two low-complexity methods for finding the secrecy capacity of the MIMO WTC. The first method is a result of applying an ADCA to (3) and the second one is based on finding a saddle point of (4).

A. ADCA for Solving (3)

To solve (3), we propose a simple but efficient method derived based on the obvious observation that \(C_s(X)\) is a DC function, which naturally motivates the use of DCA. In this work we apply the ADCA presented in [13]. The idea is that from the current and previous iterates, denoted by \(X_n\) and \(X_{n-1}\) respectively, we compute an extrapolated point \(Z_n\) using the Nesterov’s acceleration technique: \(X_n + (t_k - 1)/t_{k+1} (X_n - X_{n-1})\), where \(t\) is the acceleration parameter. We remark that the specific \(t\)-update in Line 4 is a condition to guarantee the convergence of the iterative process as described in [14]. Since \(C_s(X)\) is possibly non-convex for a general MIMO WTC, \(Z_n\) can be a bad extrapolation and a monitor is required. Specifically, if \(Z_n\) is better than one of the last \(q\) iterates, then \(Z_n\) is considered a good extrapolation and thus will be used instead of \(X_n\) to generate the next iterate. Thus, the ADCA is generally non-monotone. The algorithmic description of ADCA for solving (3) is outlined in Algorithm 1. Note that the subproblem in (7) is achieved by linearizing \(f_s(X)\) around \(V_n\) and by omitting the associated constants that do not affect the optimization. In Algorithm 1, \(q\) is any non-negative integer and \(\gamma_n\) is the minimum of the secrecy rate of the last \(q\) iterates. We remark that the case when \(q = 0\) reduces to the conventional DCA, which is exactly the same as the AO method in [15].

Algorithm 1 ADCA for solving (3)

1: Initialization: \(W_0 = X_0 \in \mathcal{X}, \, t = \frac{1+\sqrt{1+4t}}{2}, \, q: \text{integer.} \)
2: for \(n = 1, 2, \ldots\) do
3: Update:
4: \(X_n = \arg \max_{X \in \mathcal{X}} f_s(X) - tr(\nabla f_s(W_{n-1})X) \quad (7)\)
5: \(Z_n = X_n + \frac{t_n-1}{t_{n+1}} (X_n - X_{n-1})\)
6: \(\gamma_n = \min(C_s(X_{n+1}), C_s(X_{n-1}), \ldots, C(X_{[n-q]+}))\)
7: \(W_n = \begin{cases} Z_n & \text{if } C_s(Z_n) \geq \gamma_n \text{ and } Z_n \text{ is feasible} \\ X_n & \text{otherwise} \end{cases} \)
8: end for
9: Output: \(X_n\)
To implement Algorithm 1, we need to efficiently solve (7), which is explicitly written as
\[
\text{maximize } \ln \left| I + H_0 X H_0^\dagger \right| - \text{tr}(\Phi_{n-1} X) \quad (8a)
\]
subject to \( \text{tr}(X) = P_0 \) \quad (8b)
where \( \Phi_{n-1} = H_0^\dagger (I + H_0 W_{n-1} H_0^\dagger)^{-1} H_0 \). We now show that (8) admits a water-filling solution. To begin with, let us form the partial Lagrangian function associated with (7) as
\[
\mathcal{L}(X, \mu) = \ln \left| I + H_0 X H_0^\dagger \right| - \text{tr}(\Phi_{n-1} X) + \mu \text{tr}(X) \quad (9)
\]
where \( \Phi_{n-1} = \Phi_{n-1} + \mu I_N \), and \( \mu \geq 0 \) is the Lagrangian multiplier. Let \( \bar{X} \triangleq \Phi_{n-1}^{1/2} X \Phi_{n-1}^{1/2} \) and rewrite the Lagrangian function as a function of \( \bar{X} \) as
\[
\mathcal{L}(X, \mu) = \ln \left| I + H_0 \bar{X} H_0^\dagger \right| - \text{tr}(\Phi_{n-1} \bar{X}) \quad (10)
\]
To derive the dual function, the following lemma is ordered in [11].

**Lemma 1.** For a given \( \mu \geq 0 \), let \( \bar{X} \) be the eigen value decomposition (EVD) of \( \Phi_{n-1}^{-1/2} H_0 H_0^\dagger \Phi_{n-1}^{-1/2} = V \Sigma V^\dagger \) be the eigenvalue decomposition (EVD) of \( \Phi_{n-1}^{-1/2} H_0 H_0^\dagger \Phi_{n-1}^{-1/2} \), where \( V \in \mathbb{C}^{N_r \times N_t} \) is unitary, \( \Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_r, 0, \ldots, 0) \) and \( r \) is the rank of \( \Phi_{n-1}^{-1/2} H_0 \).

Then the solution to the problem \( \max \mathcal{L}(X, \mu) \) is given by
\[
X = \Phi_{n-1}^{-1/2} V \Sigma V^\dagger (\Phi_{n-1}^{-1/2}) \quad (11)
\]
where \( \Sigma = \text{diag}([1 - \frac{1}{\sigma_1}]_+, \ldots, [1 - \frac{1}{\sigma_r}]_+, 0, \ldots, 0) \).

Next, to solve (8) we need to find the optimal value of \( \mu \) which can be done by a bisection search. We skip the details here for the sake of brevity. The convergence proof of Algorithm 1 is provided in [12]. The idea is to show that the sequence \( \{\gamma_n\} \) increasing and the feasible set \( \mathcal{X} \) is compact and convex. Thus, there exists a convergent subsequence, the accumulation point of which is then proved to be a stationary point.

**B. Partial Best Response Method for Solving (4)**

The second proposed method is an iterative algorithm to find the saddle point of the concave-convex problem in (4). Suppose \((X_{n-1}, Q_{n-1})\) has been computed at the \(n\)-th iteration. Then \(X_n\) is found as
\[
X_n = \arg \max_{X \in \mathcal{X}} f(Q_n, X)
\]
\[
= \arg \max_{X \in \mathcal{X}} \log |Q_n + H X H^\dagger| - \log |I + H_0 X H_0^\dagger|, \quad (12)
\]
In words, \(X_k\) is the best response to \(Q_{n-1}\) as usual. Now given \(X_n\), due to the concavity of the term \(\log |Q + H X H^\dagger|\), the following inequality holds
\[
f(Q_n, X_n) \leq \log |Q_n + H X_n H^\dagger| + \text{tr}(\Psi_n (Q_n - Q_{n-1})) - \log |Q| - \log |I + H_0 X_n H_0^\dagger|, \quad \forall Q \in \mathcal{Q}.
\]
\[
\triangleq \tilde{f}(Q, X_n). \quad (13)
\]
where \(\Psi_n = (Q_n + H X_n H^\dagger)^{-1}\). Note that the above inequality is tight when \(Q = Q_{n-1}\). Next, \(Q_n\) is obtained as
\[
Q_n = \arg \min_{Q \in \mathcal{Q}} \tilde{f}(Q, X_n) = \arg \min_{Q \in \mathcal{Q}} \text{tr}(\Psi_n Q) - \log |Q|. \quad (14)
\]
That is to say, \(Q_n\) is found be the best response to \(X_n\) using an upper bound of the objective. The proposed solution for finding the secrecy capacity is summarized in Algorithm 2.

**Algorithm 2 PBRA for solving (4)**

1: Input: \(Q_1 \in \mathcal{Q}, \epsilon_1 > 0\)
2: for \(n = 1, 2, \ldots\) do
3: Update \(X_n\) according to (12)
4: Update \(Q_{n+1}\) according to (14)
5: end for
6: Output: \(X_n\)

We remark that the iterative method presented in [11] also aims to find the saddle-point of (4). However, it is a double-loop algorithm where a lower bound of \(f(Q_n, X)\) is used for the \(X\)-update. In contrast, Algorithm 2 is a single-loop one where the \(X\)-update is exact. The efficient methods for the \(X\)-update and \(Q\)-update are detailed in the following subsections.

1) \(X\)-update: To compute \(X_n\), as in Line 3 of Algorithm 2, we need to solve (12) which is a convex problem. Since the projection onto \(\mathcal{X}\) can be done in closed form, we can apply an accelerated projected gradient method (APGM) [16] to solve it efficiently, which is described as follows. To avoid confusion we use the superscript to denote the iteration count of the APGM. Suppose \(Y^{(k)}\), the extrapolated point at iteration \(k\), is available. The next iterate \(X^{(k)}\) is found as
\[
X^{(k)} = p_X \left( Y^{(k)} + \frac{1}{\beta} \nabla f(Q_{n-1}, Y^{(k)}) \right) \quad (15)
\]
where \(\frac{1}{\beta}\) is a step size and \(p_X(X)\) denotes the projection of \(X\) onto \(\mathcal{X}\). The gradient of \(f(Q_{n-1}, X)\) is given by
\[
\nabla f(Q_{n-1}, X) = (H^\dagger (Q_{n-1} + H X H^\dagger)^{-1} H) - (H_e^\dagger (I + H_e X H_e^\dagger)^{-1} H_e). \quad (16)
\]
For a given point \( \bar{X} \), the projection \( p_X(\bar{X}) \) is mathematically stated as
\[
\max \{ \|X - \bar{X}\| \ | \text{tr}(X) = P_0 \} \quad (17)
\]
which admits a closed-form solution as
\[
X = U \text{diag}(\left[ \frac{n_{\tau}}{E} - \tau \right]_+) U^\dagger \quad (18)
\]
where \(X = U \text{diag}(\sigma) U^\dagger\) be the eigenvalue decomposition of \(X\), \( \sigma = [\sigma_1, \sigma_2, \ldots, \sigma_r] \) where \(r\) is the rank of \(X\), and \(\tau\) is the unique number such that \(\sum_{i=1}^{r} \max(\left[ \frac{n_{\tau}}{E} \right]_+, 0) = P_0\).

The APGM for solving (12) is outlined in Algorithm 3. Note that a proper step size can be found by a backtracking line search as done in Lines (4)-(7). Starting from the step size of the previous iteration, the idea of the backtracking line search is to reduce it by a factor of \(\theta\) until (7) is met. That is, we try to find a lower quadratic approximation of the objective at the current iterate. It is shown in [16] that 3 achieves the optimal convergence rate of \(O(1/k^2)\).

2) \(Q\)-update: A closed-form solution is also possible for the \(Q\)-update. Specifically, we can partition \(\Psi_n\) into
\[
\Psi_n = \begin{bmatrix} \Psi_{n,1} & \Psi_{n,2} \\ \Psi_{n,1}^H & \Psi_{n,2}^H \end{bmatrix}. \quad (19)
\]
To lighten the notation, we will drop the subscript \(n\) onwards. Now, let \(\Psi_{12} \Psi_{12}^H = U_{\Psi} \Sigma_{\Psi} U_{\Psi}^H\) be the eigenvalue decom-
Algorithm 3 Accelerated projected gradient method for solving (12)

1: Input: $\mathbf{Y}^{(1)} = \mathbf{X}^{(0)} = \mathbf{X}_{n-1}$, $\eta_0 > 0$, $\theta > 1$, $\xi_1 = 1$.
2: for $k = 1, 2, \ldots$ do
3: \hspace{1em} $\beta = \eta_{k-1}/\theta$
4: \hspace{1em} repeat
5: \hspace{2em} $\beta \leftarrow \theta \beta$
6: \hspace{2em} $\mathbf{X}^{(k)} = \mathbf{p}_X(\mathbf{Y}^{(k)} + \frac{1}{\beta} \nabla f(\mathbf{Q}_{n-1}, \mathbf{Y}^{(k)}))$
7: \hspace{2em} until $f(\mathbf{Q}_{n-1}, \mathbf{X}^{(k)}) \geq f(\mathbf{Q}_{n-1}, \mathbf{Y}^{(k)}) + \langle \nabla f(\mathbf{Q}_{n-1}, \mathbf{Y}^{(k)}), (\mathbf{X}^{(k)} - \mathbf{Y}^{(k)}) \rangle - \frac{\alpha}{2} \|\mathbf{X}^{(k)} - \mathbf{Y}^{(k)}\|^2$
8: \hspace{1em} $\xi_{k+1} = 0.5 (1 + \sqrt{1 + 4\xi_k^2})$; $\eta_k = \beta$
9: \hspace{1em} $\mathbf{Y}^{(k+1)} = \mathbf{X}^{(k)} + \frac{\xi_k - 1}{\xi_{k+1}} (\mathbf{X}^{(k)} - \mathbf{X}^{(k-1)})$
10: end for

The main idea behind the convergence proof of Algorithm 2 is that the monotonic decrease of the objective sequence $f(\mathbf{Q}_n, \mathbf{X}_n)$, which is due to the fact that the term $\log |\mathbf{Q} + \mathbf{H}_e \mathbf{X}\mathbf{H}^H_1| - \log |\mathbf{I} + \mathbf{H}_e \mathbf{X}\mathbf{H}^H_1|$. We refer the reader to [12] for the proof of (20).

The main idea behind the convergence proof of Algorithm 2 is that the monotonic decrease of the objective sequence $f(\mathbf{Q}_n, \mathbf{X}_n)$, which is due to the fact that the term $\log |\mathbf{Q} + \mathbf{H}_e \mathbf{X}\mathbf{H}^H_1| - \log |\mathbf{I} + \mathbf{H}_e \mathbf{X}\mathbf{H}^H_1|$. We refer the reader to [12] for the proof of (20).

The main idea behind the convergence proof of Algorithm 2 is that the monotonic decrease of the objective sequence $f(\mathbf{Q}_n, \mathbf{X}_n)$, which is due to the fact that the term $\log |\mathbf{Q} + \mathbf{H}_e \mathbf{X}\mathbf{H}^H_1| - \log |\mathbf{I} + \mathbf{H}_e \mathbf{X}\mathbf{H}^H_1|$. We refer the reader to [12] for the proof of (20).

We refer the reader to [12] for the proof of (20).
Simultaneously, we also observe that the secrecy capacity is reduced when the number of antennas at Eve increases. In particular, Eve can significantly decrease the secrecy capacity when $N_e$ is much larger than $N_t$. This is because the null space of $H_b$ will increasingly intersect with the space spanned by $H_e$.

Finally, Figure 3 plots the average secrecy capacity as a function of SNR for different numbers of antennas at Eve. The purpose is to understand the gap between the true secrecy capacity and the asymptotic capacity obtained in [5]. As expected, the true secrecy capacity converges to the asymptotic capacity when the SNR is sufficiently high. Again, we can observe the secrecy capacity decreases when the number of receive antennas at Eve increases.

V. Conclusion

In this paper, we have proposed two efficient numerical methods for computing the secrecy capacity and the optimal signaling of MIMO WTC. In the first method, the secrecy capacity problem is viewed as a DC program and we have applied an accelerated version of the celebrated DCA, referred to as the ADCA. In the second method, we have drawn on the convex-concave reformulation of the secrecy capacity problem and developed the PBRA in which each iteration is done in closed form. Numerical results have been provided to demonstrate that the proposed solutions can reduce the run time of a known solution by 5 times for the considered scenarios. Moreover, through extensive numerical experiments, we have observed that the ADCA, albeit inherently a local optimization method, always achieve the optimal solution. Our conjecture is that the proposed ADCA is indeed a global optimization method, the proof of which is left for future work.

ACKNOWLEDGMENT

This publication has emanated from research supported by a Grant from Science Foundation Ireland under Grant number 17/CDA/4786.

REFERENCES

[1] A. D. Wyner, “The wire-tap channel,” Bell System Technical Journal, vol. 54, no. 8, pp. 1355–1387, 1975.
[2] S. Leung-Yan-Cheong and M. Hellman, “The Gaussian wire-tap channel,” IEEE Trans. Inf. Theory, vol. 24, no. 4, pp. 451–456, Jul. 1978.
[3] Z. Li, W. Trappe, and R. Yates, “Secret communication via multi-antenna transmission,” in 41st Annual Conference on Information Sciences and Systems 2007, Mar. 2007, pp. 905–910.
[4] A. Khisti and G. W. Wornell, “Secure transmission with multiple antennas I: The MISOME wiretap channel,” IEEE Trans. Inf. Theory, vol. 56, no. 7, pp. 3088–3104, Jun. 2010.
[5] ——, “Secure transmission with multiple antennas part II: The MIMO wiretap channel,” IEEE Trans. Inf. Theory, vol. 56, no. 11, pp. 5515–5532, Oct. 2010.
[6] F. Oggier and B. Hassibi, “The secrecy capacity of the MIMO wiretap channel,” IEEE Trans. Inf. Theory, vol. 57, no. 8, pp. 4961–4972, Aug. 2011.
[7] J. Li and A. Petropulu, “Optimal input covariance for achieving secrecy capacity in Gaussian MIMO wiretap channels,” in Proc. IEEE ICASSP 2010, Mar. 2010, pp. 3362–3365.
[8] S. Fakoorian and A. L. Swindlehurst, “Full rank solutions for the MIMO Gaussian wiretap channel with an average power constraint,” IEEE Trans. Signal Process., vol. 61, no. 10, pp. 2620–2631, Mar. 2013.
[9] S. Loyka and C. D. Charalambous, “Optimal signaling for secure communications over Gaussian MIMO wiretap channels,” in IEEE Trans. Inf. Theory, vol. 62, no. 12, Dec. 2016, pp. 7207–7215.
[10] K. Cumanan et al., “Secrecy rate optimizations for a MIMO secrecy channel with a multiple-antenna eavesdropper,” IEEE Trans. Veh. Technol., vol. 63, no. 4, pp. 1678–1690, May 2014.
[11] T. V. Nguyen et al., “A low-complexity algorithm for achieving secrecy capacity in MIMO wiretap channels,” in Proc. IEEE ICC 2020, Jun. 2020.
[12] A. Mukherjee et al., “On the MIMO secrecy capacity of MIMO wiretap channels: Convex reformulation and efficient numerical methods,” submitted to IEEE Trans. Commun., 2020. [Online]. Available: https://arxiv.org/abs/2012.05667
[13] D. N. Phan et al., “Accelerated difference of convex functions algorithm and its application to sparse binary logistic regression,” in Proc. Twenty-Seventh Int. It. Conf. Artif. Intell., Jul. 2018, pp. 1369–1375.
[14] Y. Nesterov, “A method for solving the convex programming problem with convergence rate of $O(1/k^2)$,” Proceedings of the USSR Academy of Sciences, vol. 269, pp. 543–547, 1983.
[15] Q. Li et al., “An alternating optimization algorithm for the MIMO secrecy capacity problem under sum power and per-antenna power constraints,” in Proc. IEEE ICASSP 2013, May 2013, pp. 4359–4363.
[16] A. Beck and M. Teboulle, “A fast iterative shrinkage-thresholding algorithm for linear inverse problems,” SIAM J. Imaging Sci., vol. 2, pp. 183–202, 2009.
[17] Chen-Nee Chuah et al., “Capacity scaling in MIMO wireless systems under correlated fading,” IEEE Trans. Inf. Theory, vol. 48, no. 3, pp. 637–650, Mar. 2002.