An incomplete model of RRATs and of nulls mode-changes and subpulses

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ABSTRACT
A model for pulsars with polar-cap magnetic flux density $\mathbf{B}$ antiparallel with spin $\Omega$ is described. It recognizes the significance of two elementary processes, proton production in electromagnetic showers and photoelectric transitions in ions accelerated through the blackbody radiation field, which must be present at the polar cap in the $\Omega \cdot \mathbf{B} < 0$ case, but not for pulsars of the opposite spin direction. The two populations are likely to be indistinguishable observationally until curvature radiation pair creation ceases to be possible. The model generates, and provides a physically realistic framework for, the polar-cap potential fluctuations and their time-scales that can produce mode-changes and nulls. The RRATs are then no more than an extreme case of the more commonly observed nulls. The model is also able to support the basic features of subpulse drift and to some extent the null-memory phenomenon that is associated with it. Unfortunately, it appears that the most important neutron-star parameter for quantitative predictive purposes is the whole-surface temperature $T_s$, a quantity which is not readily observable at the neutron-star ages concerned.

Key words: instabilities - plasma - stars:neutron - pulsars:general

1 INTRODUCTION
The discovery within the last decade of further complex pulsar phenomena, particularly the Rotating Radio Transients (RRATs; McLaughlin et al 2006), has extended the problem of understanding these systems by introducing new time-scales. In a number of previous papers (Jones 2010a, 2011, 2012a, 2012b; hereafter Papers I-IV) physical processes at the polar-cap surface in pulsars with spin $\Omega$ and magnetic magnetic flux density $\mathbf{B}$ such that $\Omega \cdot \mathbf{B} < 0$ have been examined to see if they are relevant. The processes that introduce important time-scales are those associated with the formation of electromagnetic showers by reverse-electrons from either electron-positron pair formation or from photoelectric transitions in accelerated ions. They are of no significance in $\Omega \cdot \mathbf{B} > 0$ pulsars with Goldreich-Julian charge density $\rho_{GJ} < 0$ and outward electron acceleration. The work has assumed that the space-charge-limited flow (SCLF) boundary condition $\mathbf{E} \cdot \mathbf{B} = 0$ is satisfied on the polar-cap surface at all times.

The aim of these papers has been to determine the composition and energy distribution of the accelerated particle flux and to look for instabilities that might be relevant to the phenomena which are observed in the radio-frequency spectrum. These properties can then be compared with those needed to produce the observed emission spectra. This has a large bandwidth, of the order of $10^9$ Hz, and efficient conversion of kinetic energy to radio frequencies. For example, the peak power of the brightest pulses in PSR B0656+14 (Weltevrede et al 2006a), is of the order of $7 \times 10^{27}$ erg s$^{-1}$, which is equivalent to approximately $10^{23}$ MeV per unit charge accelerated at the polar caps.

Growth of a collective mode able to transfer energy at this rate to the radiation field constrains the longitudinal effective mass, $m_i \gamma_i^3$, of beam particles with mass $m_i$ and Lorentz factor $\gamma_i$. For a secondary electron-positron pair plasma, it is well known that this requires a low energy, $\gamma_e \sim 100$, which is also of the same order as that needed if coherent curvature radiation, in a dipole-field, were the source of the observed radiation. Papers III and IV showed that, under the SCLF boundary condition, the creation of a reverse flux of electrons by photoelectric transitions in the accelerated ions limits the acceleration potential, analogously with the effect of electron-positron pair creation. Thus the outward particle flux has two principal components: protons formed in the electromagnetic showers and ions with Lorentz factors $\gamma_p \approx 2 \gamma_{A,2}$ which are relativistic, but not ultra-relativistic as they would be in the absence of photoelectric transitions. They can have longitudinal effective masses that allow the rapid growth of the quasi-longitudinal Langmuir mode considered by Asseo, Pelletier & Sol (1990). As noted by Asseo et al, the quasi-longitudinal mode couples
directly with the radiation field and so introduces, in principle, a second source of coherent radio-frequency emission which is not present in $\Omega \cdot B > 0$ pulsars.

Electromagnetic shower theory (see Landau & Rumer 1938, Nordheim & Hebb 1939) makes it possible to calculate the total photon track length per unit interval of photon frequency. It is an almost linear function of primary electron energy in the ultra-relativistic limit. Then known partial cross-sections for the formation and decay of the giant dipole resonance enable $W_p$, the number of protons formed per unit incident electron energy, to be estimated with adequate reliability for the present work. Proton formation is concentrated at shower depths of the order of $10^{13}$ cm. The total mass is poorly known owing to its ion separation energy ($\text{Medin} & \text{Lai} 2006$) is small enough for the mass of the atmosphere at a polar-cap temperature $T_{\text{pc}} \sim 10^6$ K to be significant.

This atmosphere is very compact and in local thermodynamic equilibrium (LTE); its scale height is of the order of $10^{-1}$ cm. The total mass is poorly known owing to its exponential dependence on the ion separation energy, but is possibly in an interval equivalent to $10^{-1}$ to $10^0$ radiation lengths. Thus it may contain the whole or some part of the electromagnetic showers formed by inward accelerated electrons. The extent of a shower is itself uncertain because the Landau-Pomeranchuk-Migdal effect is present at the energies, densities and magnetic fields concerned (see Jones 2010b). At depths immediately below the atmosphere, the state of matter is uncertain and could be either liquid
2.1 Atmospheric fractionation

There is a fractionation of ion charge-to-mass ratio with the largest values at the top of the atmosphere. The relatively small number of protons created in showers are nowhere in equilibrium within the LTE ion atmosphere and, under the influence of the small electric field $E$ present, move outward and are either accelerated or, if their flux exceeds the Goldreich-Julian current density $\rho_{\text{GJ}}$ form an atmosphere above the ions. Its scale height is $2k_BT/m_p\rho_g$, at local temperature $T$ and gravitational acceleration $g$. The chemical potential gradient that causes their motion within the ion sector of the atmosphere is initially mostly an entropy gradient, but at lower densities changes to that derived from the electric field.

This fractionation is of central importance to the model described in this paper, but we have not previously considered the adequacy of our elementary static treatment of the problem. Also, the presence of partial ionization means that the possibility of convective instability has to be considered (for a simple explanation see, for example, Rast 2001). Bearing in mind the functioning of a laboratory high-vacuum diffusion pump, it is also necessary to ask if the upward flux of protons is likely to carry with it sufficient ions to interfere with fractionation.

In order to examine this, we require two transport relaxation times. We employ approximate expressions using lowest-order zero-field perturbation theory, satisfactory here because the proton cyclotron energy quantum is small compared with the polar-capt energy $k_BT$. The first, for upward movement of a proton in the ion atmosphere at ion number density $N_Z$ is,

$$\frac{1}{\tau_p^+} = \frac{2}{3\pi} \left(\frac{1}{2\pi m_p k_B T}\right)^{3/2} N_Z \tilde{Z}^2 e^4 m_p A^{1/2} (A + 1) F_p^+, \quad (1)$$

in which the function $F_p$ is,

$$F_p = \int_0^\infty \frac{d\bar{q}}{(\bar{q}^2 + \kappa_{\text{DP}}^2)^{1/2}} \exp\left(-\frac{\bar{q}^2}{\alpha_p^2}\right) \approx \frac{1}{2} \ln \frac{\alpha_p^2 + \kappa_{\text{DP}}^2}{\kappa_{\text{DP}}^2}. \quad (2)$$

Here, $\kappa_{\text{DP}}$ is the Debye wavenumber for the ions,

$$\kappa_{\text{DP}}^2 = 4\pi e^2/3k_B T N_Z \tilde{Z} (\tilde{Z} + 2) \quad (3)$$

and,

$$\alpha_p^2 = 8m_p k_B T / (A + 1)^2. \quad (4)$$

The equivalent ion relaxation time for movement relative to the proton atmosphere above the ions, if it exists, is also required. At proton number density $N_p$ it is,

$$\frac{1}{\tau_p^+} = \frac{2}{3\pi} \left(\frac{1}{2\pi m_p k_B T}\right)^{3/2} N_p \tilde{Z}^2 e^4 m_p A^{1/2} F_Z \quad (5)$$

in which $\tau_p$ is given by equation (2) with the substitutions,

$$\kappa_{\text{DP}}^2 = \frac{8\pi N_p e^2}{k_B T}, \quad \alpha_p^2 = A \alpha_p^2. \quad (6)$$

It is typically several orders of magnitude larger than $\tau_p^+$. The proton drift time from a depth $z = 0$ with ion number density $N_Z$ to the top of the atmosphere is,

$$\tau_p = \int_0^\infty \frac{dz}{\bar{v}(z)} \approx \frac{l_Z}{\bar{v}(0)}, \quad (7)$$

under the influence of a chemical potential gradient fixed by the ions,

$$m_p g - eE = \left(\frac{\tilde{Z} + 1 - A}{\tilde{Z} + 1}\right) m_p g, \quad (8)$$

equivalent to an upward directed force. The upward velocity is given by,

$$\frac{1}{\bar{v}(z)} = \frac{\tilde{Z} + 1}{A - \tilde{Z} - 1} \frac{1}{gr}\left(\frac{\tilde{Z}}{v}(z)\right), \quad (9)$$

and the scale height of the atmosphere is,

$$l_Z = (\tilde{Z} + 1)k_B T / A m_p g \approx 0.15 \text{ cm}, \quad (10)$$

for $A = 20, \tilde{Z} = 6, g = 2 \times 10^{14} \text{ cm s}^{-2}$ and $T = T_p = 10^6 \text{ K}$. Evaluation for $N_Z = 10^{24} \text{ cm}^{-3}$ gives $1/\tau_p^+ = 2.5 \times 10^{15} \text{ s}^{-1}$ and a drift time of 1.0 s. This should be regarded only as a tentative order of magnitude because equations (1), (3) and (4) are strictly valid only at number densities much lower than $10^{24} \text{ cm}^{-3}$. The position of the change of phase from gas to liquid or solid is also uncertain.

The proton sector number densities are also sufficient for it to form an LTE atmosphere, but during phases of proton emission into the magnetosphere it has the Goldreich-Julian outward flux. Thus as a first approximation, its kinetic distribution function is isotropic in a frame of reference moving outward with a velocity $\rho_{\text{GJ}}/N_p e$ until the proton number density $N_p$ becomes so small that the LTE condition breaks down. If this is not to interfere with fractionation, the force exerted on an ion by the upward flux of protons must be small compared with its chemical potential gradient in the static proton LTE atmosphere, which is,

$$A m_p g - \tilde{Z} eE = \left(\frac{A - \tilde{Z}}{2}\right) m_p g. \quad (10)$$

A proton atmosphere able to maintain a typical Goldreich-Julian flux for one second would have a density at base of $\sim 10^{22} \text{ cm}^{-3}$ at which its ion transport relaxation time would be $1/\tau_p^+ = 5 \times 10^{10} \text{ s}^{-1}$ giving a force several orders of magnitude smaller than equation (10) and of small effect on fractionation.

The existence of convective instability at any density within the atmosphere is unlikely because the lateral motion implicit in a convective cell is strongly suppressed by magnetic fields of the order of $10^{12} \text{ G}$. We refer to Miralles, Urpin & Van Riper (1997) for a full treatment of this problem. But even if convective cells exist, the presumption must be that the electric field $E$, reflecting the internal equilibrium of the adiabatically-moving volume, is still present within it. Thus the velocity of the protons, averaged over many circulations, would remain as calculated above and there would be no interference with fractionation.
2.2 Acceleration potential

The polar-cap radius, as in Papers III and IV, denotes the division between open and closed magnetic flux lines, and is that given by Harding & Muslimov (2001),

\[ u_0(0) = \left( \frac{2\pi R^3}{c^2 f(1)} \right)^{1/2}, \]  

where \( P \) is the rotation period. We assume a neutron-star mass \( 1.4 M_\odot \) and radius \( R = 1.2 \times 10^6 \) cm, for which \( f(1) = 1.368 \). Our approximation for the electrostatic potential \( \Phi \) is based on the Lense-Thirring effect described by Muslimov & Tsygan (1992),

\[ \Phi(u, z) = \pi \left( u_0^2(z) - u^2 \right) \left( \rho(z) - \rho_{CJ}(z) \right), \]

in cylindrical polar coordinates, at altitude \( z \) and radius \( u(z) \), for the specific case of a charge density \( \rho(z) \) that is independent of \( u \) and is only a slowly varying function of \( z \). (It is almost identical with the potential that would be present given a time-independent outward flow of electrons under SCLF boundary conditions in \( \Omega \cdot B \gg 0 \) pulsars.) It also assumes approximate forms, valid at altitudes well within the light-cylinder radius, for the more precise charge densities which were given by Harding & Muslimov (2001). These are independent of \( u \) as required by equation (12) and are,

\[ \rho_{CJ} = -\frac{B}{e}\frac{1 - \kappa}{\eta} \cos \psi, \]

and

\[ \rho = -\frac{B}{e}\frac{1 - \kappa}{\eta} \cos \psi, \]

in which \( \psi \) is the angle between \( \Omega \) and \( B \), \( \kappa = 0.15 \) is the dimensionless Lense-Thirring factor, and \( \eta = (1 + z/R) \).

Equation (12) is certainly a satisfactory approximation to the true potential at altitudes \( z \gg u_0 \), but for lower values of \( z \), we must assume that the H-M potential changes continuously to the one-dimensional potential that exists at \( z < u_0 \), which was originally described by Michel (1974) and then further investigated by Mestel et al. (1985) and by Beloborodov (2008). We have already noted, in Section 1, that the length scale associated with the form of the one-dimensional potential is large, in the \( \Omega \cdot B \ll 0 \) case. Thus equation (12) is a fair representation of the true potential, which would remove any possibility of the backflow described by Beloborodov.

Under the assumption that equation (12) is valid, the condition \( \rho(0) = \rho_{CJ}(0) \) ensures that the SCLF condition \( \mathbf{E} \cdot \mathbf{B} = 0 \) is satisfied at all \( u \) on the polar-cap surface. The maximum potential available for acceleration above the polar cap occurs if \( \rho(z) \) is independent of altitude as would be the case if the particles were ultra-relativistic and there were no charge-separating interactions. Expressed in convenient energy units, it is,

\[ V_{\text{max}}(u, \infty) = \frac{2\pi^2 R^3 eB}{c^2 f(1)P^2} \]

\[ = 1.25 \times 10^3 \left( 1 - \frac{u^2}{u_0^2} \right) \frac{B_{12}}{P^2} \text{ GeV} \]

per unit charge on a flux line with radial coordinate \( u \).

2.3 Photoelectric transitions

The blackbody temperatures responsible for photoelectric transitions are those of the polar cap, \( T_{p\circ} \), and of the whole surface, \( T_s \). The polar cap is not significant at altitudes \( z \gg u_0(0) \) because the photon Lorentz transformation to the ion rest frame becomes unfavourable. There is a little ionization at \( z < h \approx 0.05R \) to a mean charge \( Z_0 \), but the ion Lorentz factors there are small, given the SCLF boundary conditions, as is the contribution to the total reverse-electron energy per ion. We assume a fixed value for this component of \( \epsilon_b = 20 \) GeV. Lorentz transformations of whole-surface photons are much more favourable at higher altitudes and produce the major part of the total reverse-electron energy per ion accelerated, \( \epsilon = \epsilon_b + \epsilon_e \). Ionization may or may not be complete and we define the mean final charge as \( Z_\infty \). We have shown previously that the flux of shower-produced protons reaching the top of the neutron-star atmosphere at any point \( u \) is given by,

\[ J^p(u, t) + \tilde{J}^p(u, t) = \int_{-\infty}^\infty dt' f_p(t - t' \cdot K(u, t')J^s(u, t'), \]

in terms of the ion flux \( J^s \) (see equation (20) of Paper IV). The first component \( J^p \) cannot exceed the Goldreich-Julian flux; the remainder \( \tilde{J}^p \) accumulates at the top of the atmosphere. Paper I assumed the neutron-star atmosphere to be of negligible depth so that \( f_p \) was assumed to be the standard diffusion function. But for the depth considered in Paper IV and here, an expression based on the drift time given by equation (7) is a better approximation. If the proton atmosphere is exhausted, ion emission commences extremely rapidly in order to satisfy the SCLF condition \( \mathbf{E} \cdot \mathbf{B} = 0 \) at the polar-cap surface. In general, the screening of any electric field in the \( \Omega \cdot B < 0 \) case for which a positive charge density is needed occurs preferentially through ion or proton emission in a single relativistic-particle transit time, whereas the process of electron-positron pair multiplication requires many transit times.

Protons accelerated to \( \gamma_p \sim 10^3 \) have only a very small probability of creating electron-positron pairs through interaction with blackbody photons and, with ions of the same energy per unit charge, would have only negligible growth rates for the quasi-longitudinal Langmuir mode. There are three ways in which particle beams capable of giving strong coherent radio-frequency emission might be produced. The two considered here in the first instance are as follows.

(a) \( V_{\text{max}} \) is so large that self-sustaining curvature-radiation (CR) electron-positron pair production occurs. A permanent proton atmosphere exists over much of the polar cap giving a primary current density of protons and positrons which, at least for times long compared with \( u_0/c \), may be in a steady state. A plasma of low-energy secondary electrons and positrons forms.

(b) The potential is so reduced from \( V_{\text{max}} \) by photo-electron backflow that ion and proton Lorentz factors are either in a region that allows rapid growth of the quasi-longitudinal mode or are of magnitude such that downward fluctuations to the necessary values can occur. In this case, the Lorentz factors are such that the mode wave-vector component perpendicular to \( \mathbf{B} \) is unlikely to be negligible so that coherent radio-frequency emission need not be exactly parallel with local flux lines.
The third set of processes are more obscure in the case of $\Omega \cdot B < 0$ pulsars and concern inverse Compton scattering (ICS) of blackbody photons above the polar cap. Pair production by the conversion of outward-moving high-energy ICS photons is known to be significant in $\Omega \cdot B > 0$ pulsars (see Hibschman & Arons 2001, Harding & Muslimov 2002) even if dipole-field geometry is assumed. Also, deviations from such a field, to the extent that they exist, can greatly enhance pair densities (Harding & Muslimov 2011). But in $\Omega \cdot B < 0$ pulsars, high-energy ICS photons are directed inward and the sources of outward-moving positrons to scatter blackbody photons are more obscure. Clearly, a fraction of the photons may have sufficient momentum transverse to $B$ to produce pairs at low altitudes. A further possible source of low-altitude pairs is the conversion of neutron-capture $\gamma$-rays at the top of the atmosphere. Showers produce approximately the same numbers of neutrons and protons, but via these processes would respond to fluctuations in the acceleration potential. However, although the model described in Section 3 has been considered in the context of either low-energy ion-proton beams or CR pair production, it is necessary to bear the more obscure pair-production processes in mind.

3 THE POLAR-CAP MODEL AND ITS PROPERTIES

The previous Section summarized those specific physical properties of the polar cap that are important for the model, but we shall also give a brief description of its framework. This is addressed primarily to the typical radio pulsar with an age of the order of 1 Myr, or older, and not to the small-$P$ case in which continuous pair production is anticipated.

Photoelectric transitions in accelerated ions are a source of electrons which flow inward to the polar-cap surface and partially screen the potential given by equation (12). Data presented in Table 1 of Paper IV, for specific values of $B$ and $T_s$, gave $Z_{\infty}$, the final charge of an accelerated ion, and the energy of its reverse electrons $\epsilon_s$, as functions of the acceleration potential $V(u, \infty)$ experienced by that ion. Then with the inclusion of the small energy $\epsilon_h$ arising from photoelectric transitions at altitudes $z < h$, as described in Section 2.3, the total energy $\epsilon = \epsilon_h + \epsilon_s$ gives the number of protons created in the electron showers. This is the approximately linear function $\epsilon W_p = KZ_{\infty}$ (see Papers I and II). It is convenient here to define $K$ as the number of protons formed per unit ion charge. (In Papers I and II, it was defined as the number of protons per unit nuclear charge of the ion.) The model described here uses an elementary form of equation (16) which describes the temporal distribution of the protons reaching the top of the LTE atmosphere. But the distribution of electron back-flow and the values of $Z_{\infty}$ over the polar-cap also define through solution of Poisson’s equation, at any instant, the actual electrostatic potential which, of course, is not identical with equation (12) because $\rho$ is not independent of $u$. The model then finds by relaxation procedures finite-element approximations to the self-consistent functions $Z_{\infty}(u, t)$ and $V(u, \infty, t)$ for the polar cap. This is circular, with radius given by equation (11), which assumption is not essential but is made for ease of calculation.

3.1 The model

The polar cap is divided into $n_s = n^2$ elements of equal area by first defining $n$ annuli of outer radius $iu_0/n$, where $i = 1, \ldots, n$ and then further dividing each of these into $2\pi - 1$ equal azimuthal elements. Within each annulus, the elements are displaced, by a random azimuthal angle, with respect to the axis $\phi = 0$. It is assumed that the charge density is independent of $u$ within any element. Equation (16) is much simplified by the model assumption $f_p(t) = \delta(t - t' - \tau_p)$ giving,

$$J^p(t) + \bar{J}^p(t) = K(t - \tau_p)J^s(t - \tau_p),$$

within any element, so that the state of the element alternates between proton and ion emission. The total number of protons produced at time $t$ on unit area of surface at the top of the atmosphere following an ion-emission interval of length $\tau_p$ is,

$$q_p = \int_{t-\tau_p}^{t} dt' K(t') J^s(t').$$

(18)

The ion current density is $J^s(t) = N_Z(t)cZ_{\infty}(t)c$ and the Goldreich-Julian charge density is $\rho_{GJ}(h) = N_Z(t)(2Z_h - Z_{\infty}(t))c$. Therefore, if we define the length of the proton phase by $\tilde{K}\tau_p = q_p/\rho_{GJ}c$, then

$$\tilde{K}\tau_p = \int_{t-2\tau_p}^{t-\tau_p} dt' K(t') Z_{\infty}(t') \frac{2Z_h - Z_{\infty}(t')}{2Z_h - Z_{\infty}}.$$  

(19)

An event is defined as a change of state in any element from proton to ion emission or vice-versa. The length of an ion phase is $\tau_p$, which is assumed constant. The length of the following proton phase is $\tilde{K}\tau_p$, and is a function of the state of the whole polar cap within the integration time over the preceding ion phase.

Table 1 of Paper IV gave values of $\epsilon_s$ and of $Z_{\infty}$ as functions of a cut-off potential $V_c$ for different values of the surface magnetic flux density $B_{12}$, in units of $10^{12}$ G, and the whole-surface temperature $T_s$. For reasons which we outline later, we assume a surface nuclear charge $Z_s = 10$. The cut-off potential is a representation of the effect of photoelectric transitions on the maximum possible acceleration potential difference $V_{\max}$ given by equation (15). From Table 1, with some additional values, we have been able to tabulate the functions $\epsilon_s(V)$ and $Z_{\infty}(V)$ for the interval $0 < V(\infty) < V_{\max}$. They are not strongly dependent on $u$ and we can assume that they are functions only of the magnitude of $V(u, \infty)$, the potential at the upper end of the polar-cap acceleration zone which is represented here by a cut-off at $\eta = 4$. In the model, it is assumed to be the potential on the central axis of an element. The excess charge density within an element during the ion phase is,

$$\frac{2(Z_{\infty} - Z_h)}{2Z_h - Z_{\infty}} \rho_{GJ}(h),$$

(20)

and is the source of a potential deviation downward from $V_{\max}(u, \infty)$. In order to calculate this with some economy, we use the approximation on which equation (12) is based, specifically that at an altitude $z > > u_0$, a section of the long
narrow open magnetosphere can be approximated locally by a cylinder of constant radius \(u_0(z)\). Then the Green function satisfying the SCLF boundary condition for a line source at cylindrical polar coordinates \((u, \phi)\) is,

\[
G(u, u') = \ln \left( \frac{u_0^2 + u'^2 - 2u_0 u' \cos(\phi - \phi')}{u_0^2 u'^2 - 2u_0 u' \cos(\phi - \phi')} \right). \tag{21}
\]

The potential on the axis of an element derived from the charge excess within that element is the most important factor and is obtained by numerical integration using the Green function. The potential generated on its axis by any other ion-phase element is found directly from the Green function by using a line approximation for the excess charge. The potential within a proton-phase element is of no interest and is not calculated.

After each event, the \(Z_n\) in all ion-phase elements, also referred to as ion-zones, are recalculated by a relaxation procedure so that there is consistency between the Poisson equation solution for \(V(u, \infty)\) in terms of all the \(Z_n\) and the tabulated function \(Z_{\infty}(V)\) obtained from Table 1 of Paper IV which is based purely on the photoelectric transition rates. This also gives the function \(\varepsilon_n\) and hence \(K\) which, with the values of \(Z_{\infty}\) and their times of duration, then allow calculation of the integral for \(K\) and the duration of the subsequent proton phase.

The initial state of the system has all elements in the proton phase. All \(n_0\) elements, selected in random order, are then assigned sequential times for transition to the ion phase. The interval between any two adjacent times is \(x_{TP}\), where \(x\) is a random number in the interval \(0 < x < x_{max}\).

Our initial choice was \(x_{max} = 0.5\). Throughout the calculation, a list is maintained, in temporal order, of the time of the next event in each of the \(n_0\) elements. This procedure has the advantage that the calculation of every ion phase is entirely self-contained so that cumulative errors are not carried forward. The model has been run for intervals equivalent to real times of the order of ten days.

### 3.2 Model results

For a given random initial state, the subsequent states of the model polar cap are not, of course, random but are chaotic, and variation of \(x_{max}\) appears not to affect their character. The proton production parameter \(W_p\) is a slowly varying function of \(B\) but we have chosen the conservative value \(W_p = 0.2\) GeV\(^{-1}\) and an ion charge \(Z_h = 6\) as in Paper IV. The numbers of elements have been in the interval \(10^2 < n_s < 4 \times 10^3\).

The usual procedure has been to let the model run for a time \(10^7\) \(\tau_p\) before sampling its state at intervals of \(\tau_p\). The presence of the instability described in Paper I has been confirmed. The system has shown no sign of settling down to other than a chaotic state. Its main characteristics are as follows.

(i) Fluctuations in the central potential \(V(0, \infty)\) and in the number of ion-emission elements can be very large and are dependent, principally, on the whole-surface temperature \(T_s\) but also, to a lesser extent, on the parameter \(B_{12}P^{-2}\) which scales \(V_{max}\).

(ii) The elements most likely to be in an ion-emission phase are those near the periphery \(u_0\).

(iii) Peripheral ion-emission elements have a significant autocorrelation function in the angle \(\phi\), unaffected by running time, indicating the formation of clusters.

At this stage, we should note that the model as precisely defined by equations (17) - (19) does not produce, at any point on the polar-cap surface, the mixture of protons and ions which is required for growth of the quasi-longitudinal Langmuir mode. But the growth rate is only a slowly varying function of the proton-ion ratio (see Papers III and IV) and we do not doubt that, given a physically realistic form of the diffusion function \(f_p\) in equation (16), there will be a sufficient interval of time, of the order of \(\tau_p\), within which the current density contains both components.

The potential fluctuations given in the Table are, of course, dependent on \(n_s\). The autocorrelation function in \(\phi\) indicates that a smaller of \(n_s\) should be preferred and, within the limited framework of our model, would be a better approximation to reality. But \(n_s = 100\) is the smallest value for which our approximation for the interaction between two elements can be reasonably adequate. Table 1 therefore underestimates the true scale of fluctuation. The values of \(B_{12}\) and \(P\) have been chosen to be representative of the typical pulsar exhibiting nulls as listed by Wang, Manchester & Johnston (2007). The scale of the downward fluctuations in \(V\) is, as might be anticipated, very strongly dependent on \(T_s\) and hence, presumably on pulsar age. At \(T_s = 10^5\) K, significant downward fluctuations would be present only for very large values of \(B_{12}P^{-2}\), the parameter that scales \(V_{max}\). With increasing rotation period, downward fluctuations also increase.

The most significant feature of the model is the size of the potential fluctuations that occur for larger values of \(T_s\). But this is superimposed on a further instability, with medium time-scale, described in Paper II. Showers reduce the atomic number from \(Z\), possibly \(Z = 26\), to \(Z_s\) at the top of the atmosphere. Provided we ignore capture of shower-produced neutrons and subsequent \(\beta\)-decay of the neutron-rich nuclei formed, this liberates \(Z - Z_s\) protons which diffuse with time-scale \(\tau_p\). At any instant, there is naturally a distribution of \(Z_s\) values, but over a time long.

| \(P\) | \(n_s\) | \(T_s\) | \(V(0)\) | \(p_{cen}\) |
|------|-------|-------|--------|--------|
| 1.0  | 1.0   | 0.000 | 0.004  | 0.008  | 0.21  | 0.967 | 0.000 |
| 2.0  | 0.011 | 0.000 | 0.000  | 0.000  | 0.000 | 0.000 | 0.000 |
| 2.0  | 0.018 | 0.000 | 0.000  | 0.000  | 0.000 | 0.000 | 0.000 |
| 2.0  | 0.040 | 0.000 | 0.000  | 0.000  | 0.000 | 0.000 | 0.000 |
| 3.0  | 0.000 | 0.000 | 0.000  | 0.000  | 0.000 | 0.000 | 0.000 |
| 3.0  | 0.000 | 0.000 | 0.000  | 0.000  | 0.000 | 0.000 | 0.000 |
| 3.0  | 0.077 | 0.000 | 0.000  | 0.000  | 0.000 | 0.000 | 0.000 |
RRATs and pulsar nulls

4 RRATS NULLS AND POTENTIAL FLUCTUATIONS

Our proposal is that the potential fluctuations found in the model provide a basis for understanding why the extent of null lengths and fractions seen in both pulsars and the RRATs occur quite naturally.

4.1 The RRATs

We refer to Burke-Spolaor & Bailes (2010), Keane et al (2011) and Keane & McLaughlin (2011) for reviews of the observational data on these sources. Values of $f_{on}$, the fraction of periods in which a pulse is detected, are broadly in the interval $10^{-4} < f_{on} < 10^{-1}$ within which their distribution is approximately uniform in log $f_{on}$. The bursts of emission are short, usually one but occasionally several periods, and are consistent with being of the same order of magnitude as $\tau_p$. The RRATs listed in Table 3 of Keane et al are all quite distant with the exception of J1840-1419 which also has a period $P = 6.6$ s and a value of the acceleration parameter $B_{12} P^{-2} = 0.15$, that is, below the cut-off value 0.22 which can be found from the ATNF catalogue (Manchester et al 2005).

Thus Weltevrede et al (2006a) proposed that the RRAT pulses are simply analogues of the giant pulses seen in the nearby pulsar B0656+14 and that their emission, apart from the giant pulses, is unobservable owing to distance. However, there is a difference whose significance becomes obvious in view of the model described here. With the exception of J1554-5209, the 14 RRAT listed with surface magnetic fields by Keane et al are well below the threshold for CR secondary pair production as defined by Harding & Muslimov (2002). From Fig.1 of their paper, this condition can be represented approximately in terms of the parameter $X = B_{12} P^{-1.6}$: the threshold is given by the critical value $X_c = 6.5$. PSR 0656+14 lies well above this threshold, which assumes dipole-field flux-line curvature, as does the exceptional RRAT J1554-5209, and it is reasonable to suppose that these pulsars are capable of supporting CR secondary pair creation over at least a fraction of their polar caps.

This means that $Z_s$ and $\bar{K}$ defined by equation (19) cannot be independent variables when considered over medium time-scales of the order of $\tau_{\phi}$. The reason is that large values of $K$ within some interval of time imply reduction of $Z$ at the shower maximum to very small $Z_s$ so producing nuclei able to move to the top of the atmosphere either by diffusion on a time-scale possibly some orders of magnitude larger than $\tau_s$, or by Rayleigh-Taylor instability. In particular, values smaller than $Z_s \sim 5$ have a high probability of being completely ionized in the LTE atmosphere and the consequent absence of a reverse-electron flux stops shower and proton formation. The condition of these surface layers is obviously not well understood, but in Paper II a case was advanced that it would be one of instability rather than a steady-state value of $Z_s$. This is the case for our model assumption of a compromise value $Z_s = 10$ and is also the basis for the observed phenomena, mode-changes and long nulls, with time-scales of the order of $\tau_{\phi}$. The relation between age and proper-frame surface temperature $T_s$ is, unfortunately, obscure in the region assumed in Table 1, which is well within the photon-cooling epoch. Potekhin & Yakovlev (2001; Fig.7) have shown that increasing surface fields accelerates cooling, but an even more important factor would be the presence of low-$Z$ elements in the outer crust, possibly from fall-back, which have the same effect (Potekhin et al 2003).

4.2 Null lengths and surface temperature

With reference to Table 1, it is possible to see qualitatively how nulls change with age during the cooling from high-$T_s$ to low-$T_s$. We consider pulsars which lie below the pair-creation threshold. Initially, values of $\bar{V}$ are usually small enough to give the mode growth rates necessary for emission by process (b) (see Section 2.3). Upward fluctuations of $\bar{V}$ increase the ion and proton Lorentz factors and, owing to the exponential dependence of amplitude growth on
them, can produce short nulls. Upward fluctuations become more frequent as cooling proceeds, giving higher null fractions \(1 - f_{\text{nn}}\). Eventually, cooling reaches the stage at which the mean \(\eta\) approaches unity and downward fluctuations are required to produce the growth rates necessary for observable emission. Null fractions are then near unity and, as the limit is approached, there is no observable emission. We emphasize that these fluctuations are superimposed on medium time-scale fluctuations (see Section 3.2) in the surface atomic number \(Z_s\), which are unlikely to be uniform over the whole polar-cap surface. Values smaller than \(Z_s \sim 5\) produce a negligible reverse-electron energy flux and hence, if present over a substantial area of the polar cap, much reduced deviations of \(\eta\) from unity. It is proposed here that these are responsible for null and burst lengths more nearly of the order of \(\tau_{\text{ad}}\) rather than \(\tau_p\).

Isolated Neutron Stars (INS) are a small group of radio-quiet thermal X-ray emitting positions closed to the RRAT in the \(P - V\) plane (see Keane et al 2011) but at slightly greater periods. The six sources for which timing solutions exist have been listed by Zhu et al (2011). Given their kinematic ages, the observer-frame temperatures are large (\(\sim 10^8\) K) compared with those considered in Table 1 but, with the exception of the anomalous J0420-5022, their \(B_{\text{ej}}P^{-2}\) are typically close to the general cut-off value of 0.22 found from the ATNF catalogue (Manchester et al 2005). If they form part of the \(\Omega \cdot V > 0\) population, they should support pair formation by the ICS mechanism; for \(\Omega \cdot V < 0\), extrapolation from Table 1 shows that small \(V\) would be expected satisfying the condition necessary for rapid growth of the ion-proton beam instability. However, the fact that no radio emission has been observed from this small number of sources is, perhaps, not surprising in view of their anticipated small beaming fractions compared with the 4\(\pi\)-observability of the whole-surface X-rays.

5 SUBPULSE DRIFT AND NULL MEMORY

Subpulse drift, and the null memory which, in some pulsars, is observed with it, is obviously an important diagnostic of polar-cap physics. Unfortunately, our model is incomplete in that it does not have the capacity to spontaneously exhibit this phenomenon following the randomly-constructed initial state described in Section 3. Its finite-element structure and the very elementary approximation represented by equation (17) are the reasons for this. However, the phenomenological model developed by Deshpande & Rankin (1999) from the classic Ruderman & Sutherland (1975) model has proved so useful that any physical polar-cap model must be able to support it.

5.1 Nulls and null memory

It was noted in Section 3 that the model system settles down to a chaotic state, but with ion-emission elements most likely to be near the periphery \(u_0\) and an autocorrelation function in the azimuthal angle \(\phi\) indicating the formation of clusters. This is unsurprising, because ion-emission elements at smaller \(u\) have larger reverse-electron energy fluxes and so lead to longer phases of proton emission. Thus our model generates quite naturally the conal structure that is required for the Deshpande-Rankin carousel model and we can therefore proceed to examine the extent to which it is capable of supporting its organized subpulse motion.

For convenience, we shall consider a circular path of radius \(u\) on the polar cap and associate a moving ion zone with the formation of a subpulse. Then it is possible to envisage organized circular motion in which an ion-emission zone of angular width \(\delta\phi_i(u)\) is followed by a proton-emission zone of \(K\delta\phi_i(u)\). Thus at any instant,

\[
\sum_i \delta\phi_i(u) \left(1 + K_i(u)\right) = 2\pi,
\]

with \(i = 1, \ldots, n\) in which \(n\) is here the number of elements on the carousel and each \(K\) is determined according to equation \((19)\) by the preceding ion-zone. Comparison with measured values of the longitudinal subpulse separation \(P_z\) indicates that the order of magnitude of model values should not be large, \(K_i \sim 3\), but this order of magnitude is associated with values of the acceleration potential, on the ion-zone flux lines, that are unlikely to enable significant electron-positron pair creation. This is then consistent with mechanism (b), based on protons and ions, for the production of coherent radio emission.

It is easy to see that a steady uniform state of equation \((23)\) with \(\delta\phi = \tau_p\phi\) and constant \(\phi\) that is independent of \(u\) does not, in general, exist. However, we have to bear in mind the limitations of our model based on equation \((16)\), which is local in \(u\), and on its very elementary approximation given by equation \((17)\). Thus in a model with a more realistic diffusion function \(f_p\) and without the finite element structure, we would expect that at constant \(\phi\), \(K\) would increase as a function of \(u\), starting from a negligible value at \(u_0\) and reaching a maximum before declining as a proton-emission zone is entered. The presence of this extremum is a possible basis for the existence of short-term quasi-steady-state solutions of equation \((23)\) within a finite interval of \(u\) at \(u_c\), which also coincides with conditions suitable for the growth of the instability giving observable emission. It may be that the tendency to form a cluster, present in our model, indicates that a more realistic model would counteract the dispersion inherent in equation \((23)\) and maintain subpulse shape, but it is not possible to assert that it would be so.

However, it is possible to envisage this motion on a polar cap of any plausible geometrical shape, elliptical or approximately semi-circular. The motion is, of course, independent of \(E \times B\) drift and could be in either direction, as is seen in the survey of Weltevrede, Edwards & Stappers (2006b), or may even be bi-directional at some instant on different areas of the polar cap. There are many ways in which potential fluctuations could disrupt this organized motion. We have seen in the previous Section that a whole-surface temperature \(T_s\), large enough to give low values of \(V\) favours the occurrence of nulls through upward potential fluctuations. The most simple fluctuation is one that increases the potential over much of the polar cap and in particular at \(u = u_c\). The functions \(c_n(V)\) and hence \(K\) increase quite rapidly with \(V\). There are two consequences. Firstly, the exponent in the expression for the mode growth rate (Paper IV, equation \(17)\) is dependent on \(\gamma_{\lambda}^{3/2}\) so that the conditions necessary for observable coherent radio emission are not reached and a null occurs. Secondly, the (unobserved) motion of the ion zones is substantially changed.
It is possible to see the nature of these changes by considering how two variables, the proton density $q(\phi, t)$ at the top of the atmosphere and the ion current density $J^i$ satisfying equations (17) - (19) on a section of the path at $u = u_c$, evolve over a sequence of times. In doing this, we assume that the ion-zone is able to move continuously and abandon the fixed finite-element calculation of the model described in Section 3.1 The ion-zone motion during a null shows how a form of null memory occurs.

It is convenient, for this problem, to represent the density $q$ in units of $\rho_{GJ}(0)c\tau_p$, and $J^i$ in units of $\rho_{GJ}(0)c$. The ion-zones at time $t$ have $J^i \sim 1$, width $\delta \phi^{(1)}$ and move with velocity $\delta \phi^{(1)} = \delta \phi^{(1)}/\tau_p$. Each generates a proton density $q(\phi, t)$ at the top of the atmosphere which is depleted by the proton-zone current density $J^p = 1$. Thus at an instant $t$ and as a function of $\phi$, the proton density following an ion zone rises linearly to a maximum of $K^{(1)} - 1$ in a distance $\delta \phi^{(1)}$ and falls linearly to zero in a further distance $(K^{(1)} - 1)\delta \phi^{(1)}$. The velocity of an ion zone is therefore determined by the gradient $\partial q/\partial \phi$ of the preceding proton zone. This is the quasi-steady-state motion at time $t$, at which instant, an upward potential fluctuation increases proton production to $K^{(2)}$. The increase in ion Lorentz factor reduces the quasi-longitudinal mode growth rate to a sub-critical value and so initiates the null. A new maximum proton density $q = K^{(2)} - 1$ is reached at a later time $t + 2\tau_p$ but the system velocity remains $\delta \phi^{(1)}$ until $t + K^{(1)}\tau_p$. The width of an ion zone is then reduced. In detail, and with reference to equations (17) - (19), this happens because the velocity of its leading edge is reduced whilst that of the rear edge remains at $\delta \phi^{(1)}$ until $t + (K^{(1)} + 1)\tau_p$ at which time the ion-zone width and velocity are both reduced by a factor of $(K^{(2)} - K^{(1)} + 1)$. The ion-zone velocity then tends to its new steady-state value $\delta \phi^{(2)} = \delta \phi^{(1)}/(1 + K^{(1)})/(1 + K^{(2)})$. Hence subpulse drift does not stop in the model, but is much reduced after a null time interval of $K^{(1)}\tau_p$. Thus for short nulls, there is a subpulse memory, but it does not correspond precisely with the conservation of subpulse longitude over a short null that has been observed, for example in PSR B0809+74, by van Leeuwen et al (2002).

We are not unduly disturbed by this, because there are many possible spatial distributions of potential fluctuation over the polar cap other than the uniform case considered above. It is also not clear that the precise form of memory seen in B0809+74 is universally observed. The upward potential fluctuation that caused this could reverse at any time during or after the sequence described in the previous paragraph. In relation to null memory, the question then is, when does observable coherent radio emission recommence? The problem arises owing to the very elementary nature of our model defined by equations (17) - (19) and is that $\delta \phi^{(2)}$ in the model can be very small. The acceleration potential within a very small interval of $\phi$ may be too large to permit adequate growth of the mode and the conditions discussed briefly in Section 4.1 may not be satisfied.

### 5.2 Subpulse drift

Published observations on mode-changes and subpulse drift demonstrate the extremely heterogeneous nature of these phenomena. Some features are quite distinct in a small number of pulsars, but much less distinct or not present at all in others. For this reason, we have used the results of the extensive survey made by Weltevrede et al (2006b) based on observations of 187 pulsars selected only by signal-to-noise ratio, and list some of their conclusions below.

(i) Roughly equal numbers of pulsars have subpulse drift to smaller or greater longitudes, and direction reversal is observed.

(ii) Bi-directional drifting is observed in a small number of pulsars. J0815+09 and B1839-04 have mirrored drift bands.

(iii) The drift rate can be mode-dependent, having one of a number of discrete values of the band separation $P$, and nulls may also be confined to a particular drift-mode.

(iv) Drift bands can be curved or non-linear with pulse longitude-dependent spacing of subpulses: they are often indistinct and can be found only by two-dimensional spectral analysis.

(v) The band separation $P$ is uncorrelated with age $(P/2P)$, or with $P$ and $B$.

Conclusions (i) and (ii) are quite naturally consistent with the model. We emphasize again that $E \times B$ drift is not involved and there is no reason why, given the chaotic state of the polar cap, there should not be reversals or even bi-directionality. The nature of the chaotic state also means that it is unrealistic to suppose that these phenomena can be predictable. Conclusion (iii) is also a natural consequence of medium time-scale instability in the value of the surface atomic number $Z_s$ and hence in the potential as mentioned in Section 4.2. The existence of a polar-cap area emitting nuclei with $Z_s$ too small to produce a significant reverse-electron flux is obviously associated with an upward displacement of $V$ for times very broadly of the order of $\tau_\lambda$. This is quite consistent with the observed time intervals of $10^{3-4}$ s for mode-changes. Non-linear drift bands (iv) are simply a consequence of non-uniformity of $K_1$, values in equation (23). Band separation $P_3$ was discussed in Paper II. It is given by $P_3 = (K + 1)\tau_p$. In this expression, $\tau_p$ is dependent only on the properties of the LTE atmosphere at the polar-cap surface, but $K$ may have some dependence on rotation period and magnetic flux density; also on whole-surface temperature $T_c$ and hence pulsar age. It was noted in Paper II that the distribution of $P_3$ is quite compact, most of the values listed by Weltevrede et al (2006b) being within the interval $1 < P_3 < 10$ s. The fact that the carousel path is restricted to being near the polar-cap periphery $u_0$ may well act as a constraint on the values of $K$ and so $P_3$ that occur.

Finally, Kloumann & Rankin (2010) have observed randomly distributed pseudo-nulls in B1944+17 of length less than $7P$ with weak emission, but well above noise. They interpreted them as a state of no subpulse on that band of polar-cap flux lines from which photons can enter the line of sight. These occur naturally in the model. The final column of Table 1 gives $p_{vn}$, the probability that the whole polar cap is in the proton phase at any instant. There would then be no possible way of producing particle beams satisfying conditions (a) or (b) of Section 2.3 and capable of generating coherent radio emission. This contributes to the total null fraction that is observed. The probability $p_{vn} > p_{vn}$ that there will be no ion-zones merely in the part of the polar cap which is observable along the line of sight is therefore a natural parameter of the model.
6 CONCLUSIONS

Papers I - IV, followed by the present paper, have attempted to predict some of the consequences of there being two populations of isolated pulsars having opposite spin directions. In many important respects, the physical state of the polar-cap has been found to be dependent on the sign of the Goldreich-Julian charge density. In the $\mathbf{\Omega} \cdot \mathbf{B} < 0$ case considered in these papers, SCLF boundary conditions have been assumed rather than the $\mathbf{E} \cdot \mathbf{B} \neq 0$ condition of the original Ruderman & Sutherland (1975) model. Then the state of the polar cap is unstable on time-scales of the order of $\tau_P$ and of the longer polar-cap ablation time $\tau_c$. These are both many orders of magnitude longer than the characteristic polar-cap time-scale of $u_0/c$ and there can be little doubt that the system should be able to allow the movements of charge necessary to maintain the SCLF boundary conditions for them. We therefore need to look at published observational data to see if there is any real evidence for the existence of two populations.

The Weltevrede et al (2006b) survey is an obvious starting point. It lists 187 pulsars selected only by signal-to-noise ratio and examines both subpulse modulation (the wide variations of intensity at a fixed longitude in a sequence of observed pulses) and subpulse drift. Subpulse modulation is an almost universal characteristic and we assume it to be a direct consequence of the plasma turbulence that itself is now widely believed to be the source of the radio emission (see Asseo & Porzio 2006). Subpulse drift with measurable values of both longitudinal separation of successive subpulses $P_3$ and band separation $P_3$ was detected in 72 pulsars and these can be compared with a set of 113 which do not show detectable subpulse drift. The distributions of both sets as a function of age are wide and are broadly similar except that only 7 of the 72 have an age less than 1 Myr as opposed to 31 of the set of 113. The distributions as functions of the parameter $X$ whose critical value $X_c = 6.5$ defines the CR pair creation threshold are also wide but the set of 113 has an excess at $X > X_c$. There are 35 pulsars with $X > X_c$ as opposed to only 7 of the 72. This is consistent with, but does not prove, the existence of two populations which separate as $X$ moves with age to values below $X_c$. We propose that the $\mathbf{\Omega} \cdot \mathbf{B} < 0$ set show the phenomena of mode-changes, nulls and subpulse drift as they age through growth of the ion-proton beam quasi-longitudinal mode (mechanism (b) of Section 2.3 but see also the comments there on more obscure processes of pair creation for pulsars of this spin direction). Pair creation by ICs photons in the $\mathbf{\Omega} \cdot \mathbf{B} > 0$ case has been very fully investigated by Hibschman & Arons (2001) and by Harding & Muslimov (2002, 2011) and our assumption is that this is the source of coherent emission in the population that has no nulls or subpulse drift.

Having divided the Weltevrede et al list into two populations, we should compare these with the extensive survey of nulls made by Wang et al (2007). No pulsar in Table 1 of Wang et al appears in the earlier Weltevrede et al list, presumably because they were not known or did not satisfy the selection criteria. Of the 46 pulsars with measured null fractions given in Table 2 of Wang et al, 18 also do not appear in Weltevrede et al, but 20 are listed with a measured value of $P_3$ and 8 have no detected $P_3$. A smaller but more recent list of nulling pulsars in the paper of Gajjar, Joshi & Kramer (2012) includes a further 8 that were not considered by Wang et al, of which 2 have measured values of $P_3$ but 6 do not appear in the paper of Weltevrede et al. Of the 8 pulsars that have nulls but no detected $P_3$, 4 have null fractions $1 - f_{on} < 0.01$ and B0656+14 is the special case described in Section 4.1. There are only 3 pulsars having substantial null fractions but no detected $P_3$. These are the otherwise unremarkable B1112+50, B2315+21 and B2327-20. But it is not necessarily correct to regard them as anomalous because, as noted by Weltevrede et al, drift bands are often indistinct and detectable only by two-dimensional spectral analysis. We suggest that the results of this comparison are by no means inconsistent with our division of the Weltevrede et al pulsars into two populations.

A different, but perhaps more anomalous set are the three pulsars known to exhibit nulls of very long duration, of the order of 10 d. The first of these to be found (B1931+24; Kramer et al 2006) enabled spin-down rate measurements to be made separately in both on and off states of emission. This was followed by similar measurements for J1832+0029 (Lorimer et al 2012) and J1841-0500 (Camilo et al 2012). In each case, the off-state spin-down rate was about half that of the on state. But these pulsars have quite large values $X = 3.6$, 2.5 and 6.6 respectively, close to the critical value $X_c = 6.5$, and may quite possibly support self-sustaining CR pair creation during the on but not the off state. The consequent difference in the flux and nature of particles passing through the light cylinder appears to be the only physically plausible mechanism for a spin-down torque change of this magnitude. But the $10^6$ s time-scale for both on and off states of emission appears a little too long compared with times of the order of $10^5 \tau_c$ found from equation (22). The question of quasi-periodicity with such time-scales should also be addressed, as in the case of the RRATs (Palliyaguru et al 2011). We have stressed that our model is not random, but deterministic. Quasi-periodicities are therefore not impossible in principle, but remain quite difficult to explain.

Changes in the polar-cap acceleration potential have little direct effect on the spin-down torque. Changes in the proton-ion composition occurring in the process (b) nulls described in section 4.2 would lead to a change in the mean charge to mass ratio of particles crossing the light cylinder, but it must be doubted whether mechanism (b) could explain the large observed difference in spin-down torque. However, it is not yet clear that this is a universal feature of nulls. The only other pulsar for which measurements have been made (PSR B0823+26; Young et al 2012) has a fractional upper limit of 0.06 for the change in spin-down torque. It has a value $X = 2.6$, rather below the putative critical value $X_c = 6.5$ denoting the pair creation threshold, so that the relatively small change in torque could be consistent either with relatively weak pair production or with process (b).

In the absence of electron and positron densities large enough to adjust and so cancel an electric field, as in mechanism (a), we have to remember that acceleration or deceleration remains at higher altitudes beyond $\eta \sim 10$ as a consequence of natural flux-line curvature. But at this stage, growth of the quasi-longitudinal mode has already occurred so that further acceleration would have negligible effect on the coherent radio emission. This is a further distinction between the two populations. In the $\mathbf{\Omega} \cdot \mathbf{B} < 0$ case, the polar...
cap of open magnetic flux lines may well have an approximately semi-circular shape. In this case, the observed drift of subpulses would be along a diameter instead of an arc of a circle.

Although all the above problems remain, the model \( \mathbf{\Omega} \cdot \mathbf{B} < 0 \) polar cap described here has some positive features. It does not require that neutron-star magnetic fields lie in a particular interval. This is important because radio pulsar inferred polar fields can vary by up to six orders of magnitude. The physical processes in electromagnetic shower development or in the photoelectric transitions exist in the zero-field limit and do not change in any qualitative way with increasing field.

The model is deterministic, but chaotic, but is incomplete in that its use of finite elements for the reason stated at the end of Section 3.1 and the very elementary nature of the approximation made in equation (17) appear to preclude the spontaneous appearance of subpulse drift from the random initial state which is used. It is possible to do no more than assert that the model can support subpulse drift in a quasi-stable way.

It is unfortunate that quantitative model predictions depend so much on surface atomic number and particularly on whole-surface temperature \( T_s \), parameters which are not well-known. Cooling calculations (see the review of Yakovlev & Pethick 2004) show that \( T_s \) falls steeply at ages greater than 1 Myr. Bearing in mind that, for a neutron-star with the mass and radius assumed here, the observer-frame temperature is \( T_s^\infty \approx 0.8 T_s \), we can see that the temperatures assumed in Table 1 fall well below the values currently observable. Although cooling in this interval is photon-dominated, the whole-surface temperature must be regarded as very uncertain.

But having made these reservations, the model does represent a physically-realistic framework for understanding the reasons why RRATs and the varied phenomena of mode-changes, nulls and subpulse drift appear during neutron-star aging. It may be that further observations at frequencies below 100 MHz will provide evidence for the existence of a population emitting by process (b) and having spectra biased towards lower frequencies than those for the process (a) population.

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