The possibility for superconductor fusion in metallic hydrogen

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Abstract. Hydrogen varies its stable state in accordance with its temperature and density. Though molecular gas is stable in the environment, the monatomic state is stable in high density ranges more than about 1g/cc. Such dense hydrogen has many aspects never seen in the molecular state. For example, in the range over about 10⁸K temperature and over 10³g/cc density, great efforts are taken to realize Inertial Confinement Fusion (ICF). Additionally, in the range of temperature lower than about 10⁵K in temperature and of density more than 10⁴g/cc in density, pycno nuclear fusion is supposed to be occurred. We have derived the expression for the Debye screening length of inter ionic potential and investigated nuclear reaction rates in the superconductive solid metallic hydrogen. It is revealed that the screening length is shortened by correlated electron pairs follow the Bose-Einstein distribution in the superconductive state. The bosonization increases the number of the lower energy states of the electrons to increase the screening effects on the potential with decreases in the temperature, resulting in the significant enhancement of the nuclear reaction rates by more than 10 orders of magnitude.

1. Introduction
Dense hydrogen has various features incident to the temperature change. For example, ICF comes off in the range over about 10⁸K temperature and over 10³g/cc density, and pycno nuclear fusion is said to be occurred in the range of temperature lower than about 10⁵K in temperature and of density more than 10⁴g/cc in density. In low temperature state, monatomic solid metallic hydrogen (MSMH) is expected to format pressures over 400GPa[1, 2, 3], and which is predicted to be a high-temperature elemental superconductor[4]. The metallic hydrogen has beenquested ever since the first prediction of its existence was made in 1935[5]. Due to the lack of orbital electrons surrounding a nuclear, MSMH should be very different to the other alkali metals and the effect of the electronic state on the crystal structure will be much greater than other elements. In the superconductive state, a proportion of the electrons form correlated electron pairs. The mass-center motion of a correlated pair follows Bose-Einstein (BE) statistics, in contrast to normal electrons which follow Fermi-Dirac (FD) statistics. This change of the electronic state would affects inter ionic potential and the reaction rates at sufficiently high density.

2. Derivation of the Debye length in MSMH
Ionic interaction is somewhat weakened by electrons in arbitrary quasineutral states. Generally, ionic Coulomb potential is screened with conductive electrons. Such screened Coulomb potential $\phi(r)$ is represented by $\phi(r) = e \exp(-r/\lambda)/4\pi\varepsilon_0 r$, where $r$ is the distance from the nucleus,
The ideal Fermi sphere, solid metallic hydrogen, the screening effects of the correlated pairs as the Fermi temperature number density. In the same way, in metals the electron temperature is represented by the classical state, electrons follow the Maxwell-Boltzmann distribution with temperature such that $T = \frac{k_B T_F}{3e^2 n_c} \equiv \lambda_e$ [6], where $\lambda_e$ is the Debye length of free electrons in a metal. This quantity $\lambda$ must be modified for the superconductive state. A proportion of the free electrons form correlated electron pairs. The screening effect of the correlated pairs is described by $\lambda_c$. Though a number of the correlated pairs is much smaller than that of total conductive electron, the free electrons and the correlated pairs coexist in the superconductive state. Therefore the total Debye length $\lambda$ is synthesized from $\lambda_e$ and $\lambda_c$. First, we derive an expression for $\lambda$ and then determine the average density of the correlated pairs $n_c$, together with other necessary values.

We treat a correlated pair as a quasi-particle and assume that it follows the BE distribution [7]. The energy of a correlated pair is $\varepsilon = v_F p / 2 - 2e\phi$, where $v_F$ is the Fermi velocity and $p$ is the momentum. To treat the chemical potential $\mu$, we introduce the quantity $z = \exp(\mu/k_B T)$. $z$ is related to the critical temperature $T_c$ and can be approximated by $z = (1 + (T/T_c)^3/\zeta_3(1))^{-1}$, where $\zeta_3(\alpha) = \sum_{n=1}^{\infty} \alpha^n/n^3$ is the Riemann zeta function. By expanding the BE distribution function about $\phi$ to first order, we obtain an expression for the local density of the correlated pairs

$$n_c'(x) = n_c(1 + \frac{g(z)}{k_B T} 2e\phi(x)).$$

Here,

$$g(z) = \frac{1}{1-z} + \frac{1-z}{z} \zeta_2(z) - \frac{\zeta_3(z)}{z}.$$

Since $n_e \gg n_c$, we neglect the change in $n_e$ induced by the creation of correlated pairs. Setting the screening effects of the correlated pairs as $\lambda_c$, the total Debye length in the superconductive solid metallic hydrogen $\lambda_{sc}$ is given as

$$\frac{1}{\lambda_{sc}^2} = \frac{1}{\lambda_e^2} + \frac{1}{\lambda_c^2} = \frac{3e^2 n_e}{2\varepsilon_0 k_B T_F} + \frac{(2e)^2 g(z)n_c}{\varepsilon_0 k_B T},$$

by Poisson’s equation. The second term represents the effect of the correlated pairs. It is not a simple expression for the reason that $\lambda_c$ is a valid parameter at all temperatures, even outside the superconductive region. For example, as $T \to \infty$, the expression shows that $z \to 0$, $g(z) \to 1$ and $\lambda_c \to \sqrt{\varepsilon_0 k_B T/(2e)^2 n_c}$, which is the Debye length of a classical particle. Adversely at $T < T_c$, $g(z) \approx ((T/T_c)^3/\zeta_3(1))^{-1}$. So the Debye length $\lambda_c$ becomes

$$\lambda_c^2 = \frac{\varepsilon_0 k_B T}{(2e)^2 g(z)n_c} \approx \frac{\varepsilon_0 k_B T^4}{\zeta_3(1)(2e)^2 T_c^3 n_c}.$$

By using a correction factor $\gamma$ to express the difference between the actual Fermi surface and the ideal Fermi sphere, $n_c$ is determined by

$$n_c = \frac{3h \omega_D}{4k_B T_F} n_e \gamma,$$

where $h$ is Planck’s constant divided by $2\pi$, and $\omega_D$ is the Debye frequency. By using the relation $(k_B T_c)^3 \zeta_3(1) = \pi^2 h^3 (v_F/2)^3 n_e$, above expression (5) becomes

$$\frac{T_c}{T_F} = \left( \frac{\Theta_D}{4 \zeta_3(1) T_F} \right)^{1/3} \gamma^{1/3},$$

where $\Theta_D$ is the Debye temperature.
where the Debye temperature is \( \Theta_D = \frac{\hbar w_D}{k_B} \). MSMH is predicted to have \( T_c \approx 10^2 \) K and \( \Theta_D \approx 10^3 \) K\[4\]. For the majority of metals \( T_F \approx 10^5 \) K. We assume that the Fermi temperature for MSMH is of the same order, and consequently \( \gamma \approx 10^{-7} \). By using an one dimensional coupled oscillations and the harmonic approximation, we determined \( w_D \) such as

\[
 w_D = \frac{16}{\pi^2} \sqrt{e^2/2\pi \varepsilon_0 ma^2 \exp(-a/2\lambda)} \sqrt{1 + a/\lambda + (a/\lambda)^2/2}
\]

where \( m \) is mass of the hydrogen and \( a \) is a lattice constant. Based on the equations and parameters derived above, we now investigate how the characteristic quantities for MSMH will change. We assume \( \gamma \) does not depend on the density.

Figure 1 shows temperature dependence of the Debye length \( \lambda_{sc} \). At \( T/T_c > 0.1 \), \( \lambda_{sc} \approx \lambda_e \) and at \( T/T_c < 0.1 \), \( \lambda_{sc} \) decreases from \( \lambda_e \) and asymptotically approaches to \( \lambda_c \). \( \lambda_e \) is screening effect of the Fermi-degenerate electron and is constant with respect to the temperature in the region of \( T \ll T_F \). \( \lambda_c \) is the effect of the correlated pairs and \( \lambda_c \propto T^2 \) under the critical temperature as shown in equation (4). When the temperature is higher than 0.1\( T_c \), \( \lambda_c \) in equation (3) is nonsignificant because \( n_e \gg n_c \). On the other hand, \( \lambda_c \) becomes important in lower temperature because \( \lambda_c \propto T^2 \).

![Figure 1. Temperature dependence of the Debye length. \( \lambda_{sc} \) is total Debye length, \( \lambda_e \) is that of the Fermi-degenerate electron and \( \lambda_c \) is that of the correlated electron pairs. The calculations were performed at \( n = 2 \times 10^{31} \) m\(^{-3}\), where \( T_c \) is expected to be 1460 K.](image)

Then we calculate the nuclear reaction rates for superconductive MSMH. Because the nuclei in the crystal do not diffuse from their equilibrium positions due to zero-point oscillation, we estimate the reaction rates using the procedure outlined below rather than using the S-matrix. To simplify the calculation of reaction rates, we use deuterium (a Bose particle) instead of hydrogen (a Fermi particle). Typically, the density \( n \approx n_e \) is nearly equal to \( a^{-3} \). Each nucleus oscillates in an approximately isotropic potential. Because the internuclear distance is much larger than the nuclear radius \( r_a \), the solid angle over which two nuclei can collide is \( \Omega \approx \pi r_a^2/4\pi a^2 = r_a^2/4a^2 \). A reaction can occur when the direction of motion is within the angle \( \Omega \), which implies that the contribution of direction to the reaction rate is \( \Omega/4\pi \). The frequency of the relative motion of a nuclear pair is then regarded as \( w_D \) and the oscillation period is consequently \( t = 2\pi/w_D \). Denoting the penetration probability as \( P \) and the number of nearest-neighbor nuclei as \( N \), the density of a pair of nuclei is \( nN/2 \) and the reaction rate per unit volume and unit time is

\[
 R = \frac{w_D n N \Omega P}{16\pi^2}.
\]
3. Results and discussion

Figure 2 shows temperature dependence of the reaction rates taking into account the correlated electron pairs. $R_e$ and $R_{sc}$ correspond to the Debye length as $\lambda_e$ and $\lambda_{sc}$ respectively. $R_e$ is constant with respect to the temperature as same as $\lambda_e$ in Fig. 1. $R_{sc}$ changes from the temperature about $T/T_c = 0.1$ because $\lambda_{sc}$ changes from same temperature. By the presence of correlated pairs, the reaction rates is enhanced. The temperature dependence of $R_{sc}$ is always $dR_{sc}/dT < 0$. This is because changes of the screening effect is much stronger than decreases in kinetic energy at lower temperature.

![Figure 2. Temperature dependence of the nuclear reaction rates ratio $R_{sc}/R_e$. $R_e$ and $R_{sc}$ are the reaction rates supposing normal conducting and superconductive state respectively. $R_{sc}/R_e$ indicate the enhancement effects and the ratio increases at lower temperature. The calculations were performed at $n = 2 \times 10^{31} \text{ m}^{-3}$, as same as for Fig. 1.](image)

4. Conclusion

In summary, we have inquired into monatomic solid metallic hydrogen in superconductive state taking into account changes in the electronic statistical distribution. We have derived expression for the Debye length including normal electrons and correlated electron pairs. As shown in Fig. 1, effects of the superconductive state appears as shortening of the Debye length in lower temperature. As shown in Fig. 2, the reaction rates is greater in lower temperature. At $T=10\text{K}$, the reaction rates increases about 20-digits. In solid metallic hydrogen, the Debye length is inseparably related to the lattice constant and ionic state. Therefore the result that the reaction rates $R_{sc}$ increases in lower temperature is unique to MSMH in the superconductive state.

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