Deconfining Phase Boundary of Rapidly Rotating Hot and Dense Matter and Analysis of Moment of Inertia

Yuki Fujimoto
The University of Tokyo

Reference:
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Rotating quark-gluon matter

Non-central heavy-ion collisions:

Rotating matter is created with angular momentum \( L \sim 10^6 \hbar \)

**Theory:**
\( \omega \sim 20 \text{ MeV} \)  
Jiang, Lin, Liao (2016)

**Experiment (global \( \Lambda \) polarization):**
\( \omega \sim 6 \text{ MeV} \)  
STAR collaboration (2017)
Chiral transition of rotating matter

NJL model analysis shows...

\[ \frac{\partial M}{\partial \omega} = \frac{1}{\omega} \left( \frac{\partial M}{\partial \omega} \right)_0 + \frac{1}{\omega^2} \left( \frac{\partial^2 M}{\partial \omega^2} \right)_0 \]

\[ T = \begin{cases} 90 \text{ MeV} & \text{for } r = 0.1 \text{ GeV}^{-1} \\ 30 \text{ MeV} & \text{for } r = 10 \text{ MeV} \\ 150 \text{ MeV} & \text{for } r = 150 \text{ MeV} \\ 210 \text{ MeV} & \text{for } r = 210 \text{ MeV} \end{cases} \]

\[ T_{\text{CEP}} \]

\[ T_{\text{Crossover}} \]

Rotation suppresses the chiral condensate

More or less accepted consensus:

**Critical temperature** \( T_c \) **drops** with increasing \( \omega \)

Other studies: Ebihara,Fukushima,Mameda (2016); Chernodub,Gongyo (2016); Wang,Wei,Li,Huang (2018); Zhang,Hou,Liao (2018); …
Deconfinement of rotating matter

Lattice formulation of imaginary rotation: Yamamoto,Hirono (2013)
Braguta,Kotov,Kuznedelev,Roenko (2020,21)

Lattice result of the Polyakov loop in pure QCD under imaginary rotation $\Omega_I = -i\omega$:

Deconfinement temperature $T_c$ rises with increasing $\omega$

At odds with chiral transition!? 

\[
\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C\Omega_I^2
\]

\[
= 1 + C\omega^2
\]
Deconfinement of rotating matter

Our result based on the hadron resonance gas model:

Deconfinement temperature $T_c$ drops with increasing $\omega$.

See also: Holography approach: Chen, Zhang, Li, Hou, Huang (2020)
Compact QED approach: Cherdodub (2020)
these works also give the same behavior as ours.
Our phenomenological approach

Hadron resonance gas (HRG) model

Total pressure: \( p(T, \mu) = \sum_i p_i^{\text{ideal}} \)

Only control parameter \((T, \mu)\); Parameter free (fixed by experiments)

Each particle’s contribution is very small, but in total, it becomes big

\[
p_i^{\text{ideal}} = \pm \frac{T}{8\pi^2} \int dk_r^2 \int dk_z (2S_i + 1) \times \log \left\{ 1 \pm \exp\left[ -\frac{E_{k,i} - \mu_i}{T} \right] \right\}
\]

\( i \): particle specie (e.g., \( \pi, K, p, n, \ldots \)); \( E_{k,i} = \sqrt{k^2 + m_i^2} \)
Rotating reference frame

**General coordinate transformation:**

\( \bar{\chi}^\mu: \text{non-rotating} \rightarrow \chi^\mu: \text{rotating} \)

\[
\begin{align*}
\bar{x} & \rightarrow x = + \bar{x} \cos \omega t + \bar{y} \sin \omega t \\
\bar{y} & \rightarrow y = - \bar{x} \sin \omega t + \bar{y} \cos \omega t
\end{align*}
\]

\[
g_{\mu\nu} = \eta_{ab} \frac{\partial \bar{x}^a}{\partial x^\mu} \frac{\partial \bar{x}^b}{\partial x^\nu} = \begin{pmatrix}
1 - (x^2 + y^2)\omega^2 & y\omega & -x\omega & 0 \\
y\omega & -1 & 0 & 0 \\
-x\omega & 0 & -1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

**Energy spectrum:**

\( \varepsilon \rightarrow \varepsilon - (\ell + s)\omega \)

\( (s = -S, -S + 1, \ldots, S - 1, S \text{ for spin-S particles}) \)
Rotating hadron resonance gas model

\[
p(T, \mu, \omega) = \sum_i p_i^{\text{rot}}
\]

\[
p_i^{\text{rot}} = \pm \frac{T}{8\pi^2} \int dk_r^2 \int dk_z \sum_{\ell=-\infty}^{\infty} \sum_{v=\ell}^{\ell+2S_i} J_v^2(k_r r) \times \log \left\{ 1 \pm \exp \left[ -\frac{E_{k,i} - (\ell + S_i)\omega - \mu_i}{T} \right] \right\}
\]

Compare with non rotating expression:

\[
p_i^{\text{ideal}} = \pm \frac{T}{8\pi^2} \int dk_r^2 \int dk_z (2S_i + 1) \times \log \left\{ 1 \pm \exp \left[ -\frac{E_{k,i} - \mu_i}{T} \right] \right\}
\]

HRG model is purely hadronic model, but how can it capture the deconfinement of quarks?
Deconfinement in hadron resonance gas

HRG blow up → Signal for deconfinement

\[ P/T^4 \]

\[ T \sim T_c \quad \sim 2-3T_c \]

Taken from: Baym, Hatsuda, Kojo, Powell, Song, Takatsuka (2017)
Deconfinement in hadron resonance gas

\[ Z = N \int dm \rho(m) e^{-m/T}, \quad \rho(m) \propto e^{m/T_H} \]

\( T_H \): Hagedorn’s limiting temperature

hadron mass spectrum:

\[ \rho(m) \text{ [GeV}^{-1}] \]

\[ m \text{ [GeV]} \]

\( a = 3 \)
Our criterion of deconfinement

For each given \((\mu, \omega)\), we identify \(T\) that satisfies the following condition as \(T_c\):

\[
\frac{p}{p_{SB}}(T = T_c, \mu, \omega) = 0.18
\]

\[
p_{SB} \equiv (N_c^2 - 1)p_{\text{gluon}} + N_cN_f(p_{\text{quark}} + p_{\text{antiquark}})
\]

- \(p_{\text{gluon}}\)
- \(p_{\text{quark}}\)
- \(p_{\text{antiquark}}\)

\(p/p_{SB}\) vs. \(T\)

- \(\omega > 0\)
- \(\omega = 0\)

\(T_c(\omega > 0)\) vs. \(T_c(\omega = 0) = 154\) MeV
Deconfinement boundary

Deconfinement temperature $T_c$ drops with increasing $\omega$

Fujimoto, Fukushima, Hidaka (2021)
\[ \langle j \rangle (r) = \frac{\partial p(r)}{\partial \omega} \]

\[ \langle j \rangle (r) \ dV \simeq dI(r) \omega \]

moment of inertia \( \propto r^2 \)

\[ p(r) = p(0) + \Delta p(r) \]

\[ \Delta p(r) \simeq \frac{\sigma}{2} T^4 r^2 \omega^2 \]

\[ p_i^{\text{rot}} = \pm \frac{T}{8 \pi^2} \int dk_r^2 \int dk_z \sum_{\ell = -\infty}^{\infty} \sum_{\nu = \ell}^{\ell + 2S_i} J_{\nu}^2(k_r r) \] 

\[ \times \log \left\{ 1 \pm \exp \left[ -\frac{E_{k,i} - (\ell + S_i) \omega - \mu_i}{T} \right] \right\} \]

\( r \)-dependence originates here
Summary

- Estimated the rotation effect on the deconfinement transition in QCD:
  the critical temperature $T_c$ drops with increasing rotation

- We used the Hadron Resonance Gas model: a phenomenological and parameter-free approach

- Still there is a tension between our and the lattice result; the lattice result only includes gluon. We are looking for the thermodynamics at finite rotation on lattice.

- Radial dependent pressure may be interesting to see in the future analysis.