Devel Finite Element Analysis of Steel Beams Taking Into Account the Development of Plastic Deformation

A A Sventikov and M L Kikaa

1Voronezh state technical University, 20 letiya oktyabrya 84, Voronezh 394006, Russia

E-mail: svarka@vgasu.vrn.ru

Abstract: Two finite element models with different cross-sections are developed in finite element analysis software ABAQUS and investigated for plastic deformation and plastic hinge formation. A simply supported rectangular beam loaded with uniformly distributed load of intensity 2.295 N/mm² is considered for this study. The beam has two sections: rectangular and I sections. The beams were previously analyzed numerically in the book, “finite elements in plasticity: Theory and Practice” written by D.R.J. Owen and E. Hinton. Beams are modeled using a one-dimensional Timoshenko B22 beam element and an ABAQUS S4R shell element. B22 is a three-node quadratic element that takes into account the shear strain and is used for beams subjected to non-uniform bending. While the S4R is a four-node double curvature thin or thick shell, reduced integration, hourglass control, finite membrane strain. Load-displacement diagrams derived from ABAQUS are compared with theoretical results. It is obvious that both results showed the initial loss of stiffness at the ends of the I-section and the final loss of stiffness at the middle section, for rectangular beam. ABAQUS showed the initial loss of stiffness up to the final cross section of the beam at the same load as the theoretical results. Significant plastic deformation developed in both cases after the yield strength. A plastic hinge is formed in the middle of the beam.

1. Introduction

In the design of building structures it is important to take into account the development of plastic deformation. This factor allows us to fully assess the stress-strain state of steel structural elements of buildings and structures. The development of plastic deformations affects the work of beam structures to the greatest extent. Modern research in the field of development of plastic deformations and formation of plastic hinges in many countries of the world confirm the need to investigate this subject area. Note that in the field of development of nonlinear methods of analysis of structural systems, the term design with the development of plastic deformations often has a different name: material nonlinear design of structures.

Ehsan Dehghani, Sajad A. Hamidi, Fariborz M. Tehrani, Aastha Goyal, Rasoul Mirghaderi in their paper titled “new practical approach to plastic analysis of steel structures” proposed a methodology to solve plasticity problems with propagating effect in both section and length. As object of research, they used a variable section in the plastic region of the element. Their findings were compared with
other methods such as plastic hinge or modified plastic hinge methods and apparently, their proposed method was found to be more accurate, easier and more efficient for implementation. Also Hendry, A W in his article «Plastic Analysis and Design of Mild Steel Vierendeel Girders» [8] applied the simple plastic theory to the design of mild steel Vierendeel girders. He described three modes of failure and derived the equations which he used to calculate the collapse load of a girder in each of these modes. On the basis of this, it can be said that at the present time the development of construction science is relevant to the development of building techniques that allow assessing the stress state taking into account the development of plastic deformation.

2. Relevance of the work
One of the most effective design approaches to the design of steel structures is the application of the finite element method (FEM). Development of computational models based on FEM is carried out by using software systems. In this paper we used one of the most effective products of this type ABAQUS complex. ABAQUS is a powerful finite element software that can solve a variety of linear and nonlinear problems, from the simplest linear problem to the most complex nonlinear problem in the field of modern engineering. ABAQUS provides a wide range of possibilities for modeling linear and nonlinear applications. ABAQUS solves nonlinear problems by means of a step-by-step iterative approach based on the Newton-Raphson method.

The aim of this work is to develop two finite element computational models and to substantiate these models by means of numerical testing.

3. Problem statement
As an object of study in the work a simply supported beam (Figure 1 (a)) with two types of cross-sections are used: rectangular (figure 1б) and I-section (figure 1в). General properties of beams: 

\[ E = 210 \text{kN/mm}^2, \quad \nu = 0.3, \quad I_0 = 0.25 \text{kN/mm}^2, \text{beam length 3m.} \]

A uniformly distributed load of intensity \(2.295 \text{N/mm}^2\). Parameters of T-section: flange width 200 mm; flange thickness 20 mm, web height 160 mm; web thickness 10 mm. Parameters of rectangular section: height 300 mm; width 150 mm. The given structural elements were previously used to assess the development of plastic deformation D. R. J. Owen and E. Hinton, in his book "Finite elements plasticity: theory and practice".

4. Modelling, testing and finite element design
For the analysis of a beam of rectangular cross section the beam element design model was used. In this case, the beam was divided along its length into several elements using standard finite elements B22 (universal beam finite element). For the I-beam cross section, a spatial calculation model was developed which was created using standard S4R elements (universal shell finite element) (Figure 2). The resulting computational model consisted of 2880 finite elements. The main feature of the developed computational model is its spatiality and multi-element feature, which allows us to take into account all the laws of the stress-strain state of the beam. The problem was solved taking into account material nonlinearity. The nonlinear calculation was performed using the step by step method. The number of iterations of the calculation was 100.

5. Results analysis and discussion
The finite element analysis of the stress-strain state of the steel beam was carried out and compared with the known data of D. R. J. Owen and E. Hinton [6]. In figure 3 and 4 dependences for change of displacements of a beam of rectangular and I-sections are given. Comparison of the data with the known ones shows that the nature of the changes will preserve the General patterns. This fact allows us to conclude that the proposed calculation models are sufficiently reliable.
Fig. 1. Single span simply supported beam
а – design and loading scheme; б, в – rectangular and I cross sections

Fig. 2. Spatial computational model of the beam
Fig. 3. Graph depicting changes in displacement of rectangular cross section beam.

Fig. 4. Graph depicting changes in displacement of I beam.
The difference in the graphs is observed in the areas of formation of the plastic hinge. This fact can be explained by a finer finite element beam mesh, which makes it possible to take into account the increase in plastic deformations in more specific areas of the studied elements. In figure 5 is given the stress distribution in the beam web for the spatial analysis model.

![Stress distribution in beam web](image)

**Fig. 5.** Stress distribution in beam web

\[ a - q = 0.22 \cdot q_0, \quad b - q = q_0 \quad (q_0 - \text{design load}) \]

The analysis of the results showed that the formation of a plastic hinge (the development of plastic deformations along the entire height of the cross section) occurs in the middle of the beam. This fact also confirms the adequacy of the proposed analysis models. Note that in areas near this hinge in the web of the beam there is an uneven stress state. This points to the need to analyse the steel elements taking into account the development of plastic deformations using spatial analysis models.

6. **Conclusions**

On the basis of the conducted researches it is possible to draw the following conclusions.

1. The results of calculations obtained by the developed finite element models of steel beams in the design complex Abaqus have good comparability with the known data, which indicates the adequacy of the proposed design positions.

2. Material nonlinear analysis of steel beams should be performed using spatial analysis models that take into account the uneven development of plastic deformations in the elements of the beams.

**References**

[1] A.I. Lurie 2005 *Theory of Elasticity*, Springer-Verlag, Berlin, Heidelberg.

[2] Bhaumik A.K., Hanley J.T. 1967 *Elastic-plastic analysis of finite difference plates*, G. Structures. Element div. ASCE p.575.

[3] Chandramani M.F. 2013 Analysis of a layered composite beam using MFE Vol. 2 pp. 198-204

[4] Chao Isaac A, 2003 *Simple geometric model for elastic deformations*, Caltech, TU Munchen Germany.

[5] Cocchetti G. 2003 Maier G. Elastic–plastic and limit-state analyses of frames with softening plastic-hinge models by mathematical programming *International Journal of Solids and Structures* vol. 40 (25).

[6] D. R. J. Owen, E. Hinton 1980 *Finite elements in plasticity: theory and practice* pp. 1-31.
[7] Ehsan Dehghani, Sajad A Hamidi, Fariborz M Tehrani, Aastha Goyal, Rasoul Mirghaderi 2015 *New Practical Approach to Plastic Analysis of Steel Structures* p. 35.

[8] Hendry A.W. 1955 *Plastic Analysis and Design of Mild Steel Vierendeel Girders*.

[9] Jiang X.M., Chen H., Liew J.Y.R. 2002 Spread-of-plasticity analysis of three dimensional steel frames *Journal of Constructional Steel Research* vol. 58 (2).

[10] J.N. Reddy 2004 *Introduction to the nonlinear finite element method*.

[11] Kaliszky S. 1996 Elastoplastic Analysis with Limited Plastic-Deformations and Displacements, *Mechanics of Structures and Machines* vol. 24 (1).

[12] Kaliszky S. 1989 *Plasticity–theory and engineering applications*, Elsevier; Amsterdam.

[13] L.T.M. Blessing 1994 *A process-based approach to computer-supported engineering design*, University of Twente, PhD thesis, Black Bear Press, Cambridge, UK.

[14] Liew J.Y.R., Chen H., Shanmugam N.E., Chen W.F. 20000 Improved nonlinear plastic hinge analysis of space frame structures *Engineering Structures* vol. 22 (10).

[15] Liew J.Y.R, White D.W., Chen W.F. 1993 Second-order refined plastic hinge analysis for frame design: Part I *Journal of Structural Engineering* vol. 119 (11).

[16] Moy S.S.J. 1981 *Plastic methods for steel and concrete structures*, Macmillan, London.

[17] Neal B G 1963 *The Plastic Methods of Structural Analysis*, Chapman & Hall ltd, London.

[18] Parmee I C and Bonham C, 2000 *Towards the support of innovative conceptual design through interactive designer/evolutionary computing strategies*. Artificial Intelligence for Engineering Design, Analysis, and Manufacturing.

[19] Robert D Cook 1974 *Concepts and Applications of Finite Element Analysis*, University of Wisconsin, Madison.

[20] Shahrestany A., Fallah N. 2013 *New composition based on finite volume for elastic-plastic plate analysis*. 