Optimal strategy for resource deployment in performing the ship key operation under conditions of uncertainty

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Abstract. The study considers the problem of determining the rational strategy of ship resources deployment under the conditions when the goal of key operation depends on the parameters, the values of which are known only at the level of nebulous assessment. Thus in a number of operations to ensure safe ships running there arises a problem of available resources deployment aimed at their most efficient using. Formalization of this arrangements causes some difficulty, since the purpose of the operation often depends on the future values of some parameters specified at interval level. The minimax approach is applied, which allows to choose a rational strategy of resources deployment and to ensure the safe running of the vessel when performing key operations. Drawing on the minimax risk criterion, the article shows that it is possible to estimate the reliability of most ship operations, in which only a set of possible values is known for the state vector. Besides, the same criterion could be applied to select the acceptable strategy of resource deployment which is to provide ship’s effectiveness and safety while performing key operations. It is assumed that the search of optimal strategy for resource deployment could be minimized to the search of a pair of values, which is the solution to a radical problem and allows to find the ultimate formula for choosing a rational and effective strategy of resource allocation needed by the ship’s authority, when performing the ship key operations. In conclusion, the paper describes a numerical example of choosing the strategy of the rational resource deployment, illustrating specific but meaningful actions of the fishing vessel’s authority undertaken in the process of implementing a regular commercial job.

1. Introduction

In some operations, to ensure safety of ships running the problem of available resources allocation for the most efficient using can appear. Formalization of such arrangements causes certain difficulties, since the goal of the operation often depends on the future values of some parameters. To eliminate the resulting uncertainty either statistical processing of the accumulated material, or the experts’ assistance is resorted [1].

The first approach is often difficult to apply. It could be explained by the following reasons. First, the number of parameters to be determined is usually large, and the number of available observations is relatively small. Second, even if statistical material is significant it is difficult to select and justify the probabilistic pattern describing the initial experimental data.

The second approach is theoretically less justified, but relatively simple and has the advantage so it could be used in cases where there is no confidence in the ability of the probabilistic description of the available observations. However, the experts’ numerical values of the parameters are usually, allowing only to reduce, but not eliminate the existing uncertainty. In this regard, for the final formalization of
the operation in these conditions it is necessary to choose one of the principles of decision-making under conditions of uncertainty [2], [3].

Further under the conditions of interval prediction of parameter values, the problem of resource deployment, where the decision is chosen on the basis of minimax risk criterion [2] is considered.

2. Objects, materials and objects of research
First, the principle of decision making is explained. For this purpose, we restrict ourselves to the case when the operation pattern for the known parameter values (matrix $A$ and vector $b$, $s$) can be estimated by a validity criterion

$$R_i = \max_{u = b, u \geq 0} f(u, s),$$

where $f(u, s)$ is a given criterion function; $u = (u_1, ..., u_n)$ is the resource allocation strategy in performing a ship operation, $s$ is the company's investment in the ship resource.

It should be noted that this criterion could be used to evaluate the reliability of most ship operations. In this case, the situation when only the set $S$ of its possible values is known for vector $s$, is considered. Under these conditions, each choice of an acceptable strategy can be associated with a metric

$$\max_{A x = b, x \geq 0} f(x, s) - f(u, s),$$

which is called a "regret".

In accordance with the criterion of minimax risk, one of the acceptable strategies capable of realizing a minimum of the maximum "regret" should be chosen as the optimal strategy

$$R_2 = \min_{u \geq 0} \max_{s \in S} \left[ \max_{A x = b, x \geq 0} f(x, s) - f(u, s) \right].$$

(1)

To study the possibility of using this criterion in resource allocation problems, we turn to the specific task of finding a rational strategy for a company to allocate the money for buying a resource that ensures safe running of the vessel during the operation.

Suppose that at the beginning of the ship's operation, the company allocated the money to vessel $M$ for buying the basic resource. When performing a key operation the ship's authority, uses this resource at each stage of the operation for some time to complete the entire key operation $t$. Moreover, at each key stage, the probabilities of possible corrections of monetary investments $p_{i,0}$ for each of the considered key operation stages are known $i = 0, n$.

At the same time, the values of the financial investments for obtaining the resource for each of the considered key operation stages are known $s_{i,0} i = 1, n$ in relation to the money allocated to buy the basic resource at the beginning of a key operation.

Suppose that, in addition to the given values, interval assessment of monetary investments $s_{i,l} i = 1, n$ at the end of the key ship operation are also known

$$s_{i,l} \in [s_{i,1}, s_{i,2}], \quad i = l, n.$$

(2)

In the terms mentioned above, the problem (1) can be viewed as
3. Analysis of the resource allocation pattern using minimax risk criterion

To analyze the ship's resource allocation pattern using minimax risk criterion, we change the numbering in formulas (3), (4) in accordance with the inequation

\[
\lambda_{0}^{2} \geq \lambda_{1}^{2} \geq \ldots \geq \lambda_{n}^{2} \geq \lambda_{0}^{1} \geq \lambda_{1}^{1} \geq \ldots \geq \lambda_{n}^{1} \geq \lambda_{0}^{0} \geq \lambda_{1}^{0} \geq \ldots \geq \lambda_{n}^{0}.
\]  

(5)

Based on the structure of the solution of the linear optimization problem and properties of the operation of maximum obtaining, we transform (3), (4) to the following

\[
R_2 = \min \Psi (u),
\]  

(6)

\[
\sum_{i=0}^{n} u_i = 1, u_i \geq 0,
\]

where

\[
\Psi (u) = \max \{ \lambda_{i}^{2} - (\lambda_{i}^{2} - \lambda_{i}^{1})u_i \} - \sum_{i=0}^{n} \lambda_{i}^{1} u_i.
\]  

(7)
We designate
\[ W_n(u) = \max_{0 \leq i \leq n} \left\{ \lambda_i^{2^*} - (\lambda_i^{2^*} - \lambda_i^{1^*})u_i \right\} \]
\[ r = \min_{0 \leq i \leq n} \{ \text{Arg} \ \max \{ \lambda_i^{1^*} \} \} \]

Thus we can assume the hypothesis that among the optimal solutions to problem (6), taking into account formula (7) and for some \( k \leq n \) value, extreme solutions can be found
\[ \min_u \Psi(u) \]

but only within the conditions written as follows
\[ W_n(u) \geq \lambda_k^{2^*} \]
\[ u_i = 0, \ i = (k+l), \ n, \ i \neq r \ (\text{if } k = n, \text{ doesn't exist}), \]
\[ \lambda_i^{2^*} - (\lambda_i^{2^*} - \lambda_i^{1^*})u_i = W_n(u), \ i = 0, \ k, \ i \neq r, \]
\[ \sum_{i=0}^{n} u_i = 1, \ u_i \geq 0. \]

To confirm the hypothesis formulated above, we assume the opposite statement. Suppose that among the optimal solutions \( u^* = (u_0^*, \ldots, u_n^*) \) of problem (6), (7) there are no strategies implementing (9) under conditions (10). Then it follows from formulas (6), (7), (9) (10) that \( R_2 \) does not exceed the value (9). Further, let \( k \leq n \) be specified that
\[ \lambda_k^{2^*} \geq W_n(u^*) \geq \lambda_{k+1}^{2^*} \]

besides, when \( k = n \), the right inequation disappears. Then if to reduce all the components of the optimal solution \( u^* \) with numbers \( j \geq k + 1, \ j \neq r \) to 0, and \( u_i^*, \) respectively, increase by
\[ \sum_{i=k+1}^{n} u_i^* \]

then the resulting vector \( u^o \) will be valid for formula (6). Therefore, by virtue of formulas (6–11) and the following chain of relations
\[ \Psi(u^o) = W_n(u^o) - \sum_{i=0}^{k} \lambda_i^{1^*} u_i^* - \lambda_i^{1^*} u_i^o \leq W_n(u^*) - \sum_{i=0}^{k} \lambda_i^{1^*} u_i^* - \sum_{i=0}^{k} (\lambda_i^{1^*} - \lambda_i^{1^*})u_i^* \leq \Psi(u^*) = R_2 \]

the vector \( u^o \) will also be optimal.

Further, let \( j_1, \ldots, j_s \) are all numbers that implement \( W_n(u^*) \). If there exists at least one number \( i_0 \leq k, \) for \( i_0 \neq r, \) which does not coincide with any of the numbers \( j_1, \ldots, j_s, \) i. e.
\[ \lambda_{\gamma_0}^{2*} - (\lambda_{\gamma_0}^{2*} - \lambda_{\gamma_0}^{1*}) u_{i_0}^* < W_n(\mathbf{u}^*), \]

we reduce \( u_{i_0}^* \) to \( u_{i_0}^{**} \), that

\[ \lambda_{\gamma_0}^{2*} - (\lambda_{\gamma_0}^{2*} - \lambda_{\gamma_0}^{1*}) u_{i_0}^{**} < W_n(\mathbf{u}^*), \]

and increase the strategy \( u_i^* \) by

\[ \delta = u_{i_0}^* - u_{i_0}^{**} > 0. \]

Here, the new strategy \( u_{i_0}^{**} \) will remain valid, and in virtue to the inequation

\[ \Psi(u^{**}) = W_n(u^{**}) - \sum_{i=0}^{k} \lambda_{i}^{1*} u_i^* - \lambda_{\gamma}^{1*} u_{i_0}^* - \lambda_{\gamma_0}^{1*} u_{i_0}^* \leq 0, \]

the vector \( u_{i_0}^{**} \) will be the optimal strategy.

Repeating the latter operation the required number of times, we find that the optimal solution (6) satisfies conditions (10), contrary to our assumption. Therefore, the resulting contradiction explains the right to use the hypothesis formulated above. In this case the search for the optimal resource distribution strategy in the sense of (6) can be reduced to the search for the pair \((k^*, W_n^*)\), which is a solution to the following problem

\[ \min \min_{0 \leq \omega \leq W \in \Omega_k} \{ A_k W_n + \left[ \sum_{i=0}^{k} \lambda_{i}^{2*} \left( \lambda_{i}^{1*} - \lambda_{\gamma}^{1*} \right) \right] - \lambda_{\gamma}^{1*} \}, \quad (12) \]

where

\[ \Omega_k = \{ W_n : \lambda_{k+1}^{2*} + \max \{0, B/D \} \leq W_n \leq \lambda_{k}^{2*} \} \]

\[ A_k = 1 - \sum_{i=0}^{k} \left( \lambda_{i}^{1*} - \lambda_{\gamma}^{1*} \right) / \left( \lambda_{i}^{2*} - \lambda_{\gamma}^{1*} \right), \]

\[ B_k = \sum_{i=0}^{k} \left( \lambda_{i}^{2*} - \lambda_{k+1}^{2*} \right) / \left( \lambda_{i}^{2*} - \lambda_{\gamma}^{1*} \right) - 1 \]

\[ D_k = \sum_{i=0}^{k} 1 / \left( \lambda_{i}^{2*} - \lambda_{\gamma}^{1*} \right). \]

The solution to problem (12) obtained by sequential comparison of the quantities \( A_k \) and \( B_k \) with zero and the values

\[ \min \{ A_k W_n + \left[ \sum_{i=0}^{k} \lambda_{i}^{2*} \left( \lambda_{i}^{1*} - \lambda_{\gamma}^{1*} \right) / \left( \lambda_{i}^{2*} - \lambda_{\gamma}^{1*} \right) \right] - \lambda_{\gamma}^{1*} \} \]

among themselves, is privided by the ratios

for \( k^* : A_{k^*} > 0, B_{k^* - 1} \leq 0, B_{k^*} > 0 \)

\[ W_n^* = (B_{k^*} + \lambda_{k^* + 1} D_{k^*}) / D_{k^*} \]

(14)
for \( k^*: A_{k^*} > 0, A_{k^* - 1} \leq 0, B_{k^*} < 0 \)

\[
W^*_{\kappa} = \lambda_{k^* + 1}^{2*}.
\]

(15)

Thus, finally, considering formulas (10), (12), (13), we obtain the following finite formulas for determining the strategy \( u^* = (u_0^*, \ldots, u_n^*) \), that fulfills the task (6)

\[
u_i^* = \begin{cases} 
\left( \lambda_i^{2*} - W^*_n \right) / \left( \lambda_i^{2*} - \lambda_i^{1*} \right) & \text{при } i = 0, k^*, \ i \neq r, \\
0 & \text{при } i = k^* + 1, n, \ i \neq r, \\
1 - \sum_{i=0, i \neq r}^{r} u_i^* & \text{при } r > k^*, \\
1 - \sum_{i=0, i \neq r}^{r} u_i^* & \text{при } r \leq k^*
\end{cases}
\]

where \( k^*, W^*_n \) are determined using formulas (14), (15).

4. Discussion of the result obtained

Applying the following hypothetical material we consider solution to the original problem (6) for the distribution of the resource using the criterion of minimax risk in case of a fishing vessel's scheduled task. Suppose that in the framework of regular tasks, the duration of the voyage is equal to \( t = 3 \) months, and the fishing process can be represented by a cyclic graph, in which the stages of the production process are the vertices (I – V), and the arcs represent transitions from one stage of the production process to another.

The stages of the production process making a cyclic graph will be considered: the following steps: preparation for voyage (I) passage to fishing site (II) fishing (III), passage to the port of discharge (IV) the cargo unloading at the port (V).

Assume that the initial amount of money allocated to marine resources to ensure the safety of fishing voyage is equal to the value \( M \). Moreover, the initial data at the beginning of key fishing operations to solve problem (6) are summarized in table 1–3, and the calculation of the data in table was performed using the following formulas

\[
p_i^0 = \beta_i \times t / 12 \times 100, \ i = 0, 4, \ s_j = S^{\gamma_j} j = 1, 4
\]

Table 1. Growth of resources cost for various stages of fishing operation approved as a voyage operation (in % per annum)

| I   | II  | III | IV  | V  |
|-----|-----|-----|-----|----|
| \(\beta_1\) | \(\beta_2\) | \(B_3\) | \(\beta_4\) | \(\beta_5\) |
| 14,6 | 17,1 | 8,9 | 5,9 | 14,4 |

Table 2. Resource cost at various stages in relation to the first stage

| II    | III    | IV    | V     |
|-------|--------|-------|-------|
| \(\gamma_2 = (\beta_2 / \beta_1)\) | \(\gamma_3 = (\beta_3 / \beta_1)\) | \(\gamma_4 = (\beta_4 / \beta_1)\) | \(\gamma_5 = (\beta_5 / \beta_1)\) |
| 0,446 | 1,715  | 1,576 | 4,008 |
Table 3. Resource cost forecast at various stages in relation to the first stage at the end of the voyage

| Φ = (β / β₁) | Φ₂ = (β₂ / β₁) | Φ₃ = (β₃ / β₁) | Φ₄ = (β₄ / β₁) | Φ₅ = (β₅ / β₁) |
|---------------|---------------|---------------|---------------|---------------|
| 1. 0.447 – 0.470 | 1.660 – 1.720 | 1.52 – 1.58 | 3.87 – 4.02 |
| 2. 0.437 – 0.455 | 1.720 – 1.780 | 1.58 – 1.64 | 4.01 – 4.15 |
| 3. 0.439 – 0.459 | 1.660 – 1.720 | 1.513 – 1.573 | 3.93 – 4.258 |
| 4. 0.472 – 0.480 | 1.720 – 1.760 | 1.56 – 1.60 | 4.09 – 4.17 |

Here are some of the results obtained, depending on the forecast options for the various stages of the fishing process. In this case we mean as options vectors θ with the number of components equal to the number of stages from 1 to 4 of the fishing process for which the resource was predicted.

For example, vector θ = (1, 3, 4, 2) means that the resource cost forecast for the first stage was taken for the first version in table 3; the resource cost forecast for the second stage according to the third version in table 3, the resource cost forecast for the third stage was taken for the forth version in table 3, etc.

Table 4. The results of determining the optimal strategies of various options of resource cost forecast

| θ | u₀ | u₁ | u₂ | u₃ | u₄ | (u₀, . . . , u₄) | Π = |
|---|----|----|----|----|----|-----------------|-----|
| 1, 1, 1, 1 | 0,3722 | 0 | 0,1081M | 0 | 0,5197M | 0,9847M | Π = |
| 2, 2, 2, 2 | 0,3415 | 0,6585M | 0 | 0 | 0 | 1,0168M | Π = |
| 2, 2, 2, 2 | 0,9230 | 0,0769M | 0 | 0 | 0 | 1,0342M | Π = |
| 3, 3, 3, 3 | 0,9075 | 0,6706M | 0 | 0,0214M | 0 | 1,0306M | Π = |
| 4, 4, 4, 4 | 1M | 0 | 0 | 0 | 0 | 1,0365M | Π = |

The calculation results for problem (6) are presented in table 4, where Π = (u₀, . . . , u₄) is the amount of money spent on the ship resource received at the end of the key operation for the chosen strategy of the resource deployment. Note that if the cost of the resource is known at every stage of fishing process, in this case the optimal strategy for allocating a resource with a total cost of M is

\[ u^* = (M, 0, 0, 0, 0) \],

and the volume of the predicted resource Π(u*) at the time t will exceed the basic M (fixed in the voyage’s task at the first stage of fishing process) will be represented as

\[ \Pi(u^*) = 1,0365M \]

Thus, the analysis of table 4 shows that the application of the minimax risk criterion in the considered problem most often does not lead to a point distribution. In addition, to obtain positive results (Π > M is a resource with a small deficit) it is sufficient that the forecast given by experts should approximately but correctly reflect only the relations between the parameters λᵢ, i = 0, 4 in formula (4), but not the costs of ship’s resource purchasing.
5. Conclusion

The proposed task of allocating the resources for a fishing company's vessels by the minimax criterion and choosing a rational strategy under the condition of uncertainty can ensure the safe operation of the vessel when performing key operations fixed in the voyage's task.

The practical application of the minimax risk criterion to the problem of resource allocation under conditions of uncertainty most often does not lead to point distribution. In addition, to select the optimal (rational) resource allocation strategy with a positive result ($\Pi > M$ is a resource with a small deficit) it is necessary and sufficient that the forecast given by experts should reflect approximately but correctly the relationships between the pattern parameters, rather than costs of a ship's resource purchasing.

References

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