Electronic Entanglement in the Vicinity of a Superconductor

Gordey B. Lesovik\textsuperscript{a,b,c}, Thierry Martin\textsuperscript{b}, and Gianni Blatter\textsuperscript{c}

\textsuperscript{a}L.D. Landau Institute for Theoretical Physics, Russian Academy of Sciences, Kosygina Str. 2, 117940, Moscow
\textsuperscript{b}Centre de Physique Théorique et Université de la Méditerranée, Case 907, 13288 Marseille, France
\textsuperscript{c}Theoretische Physik, ETH-Hönggerberg, CH-8093 Zürich, Switzerland

(March 21, 2022)

A weakly biased normal-metal–superconductor junction is considered as a potential device injecting entangled pairs of quasi-particles into a normal-metal lead. The two-particle states arise from Cooper pairs decaying into the normal lead and are characterized by entangled spin- and orbital degrees of freedom. The separation of the entangled quasi-particles is achieved with a fork geometry and normal leads containing spin- or energy-selective filters. Measuring the current–current cross-correlator between the two normal leads allows to probe the efficiency of the entanglement apparatus which is proposed here consists of a mesoscopic normal-metal–superconductor (NS) junction with normal leads arranged in a fork geometry (see Fig. 1). Using appropriate spin- or energy-selective filters in the two normal leads the quasi-particle pairs are properly separated and their entanglement can be quantified through a comparison of the intra- and inter-lead noise.

\begin{equation}
\langle \langle I_\sigma I_{-\sigma} \rangle \rangle_{\omega=0} \equiv \int dt \langle \langle I_\sigma(t)I_{-\sigma}(0) \rangle \rangle = \langle \langle I_\sigma^2 \rangle \rangle_{\omega=0},
\end{equation}

FIG. 1. Normal-metal–superconductor (NS) junction with the normal-metal lead arranged in a fork geometry. (a) Without filters, entangled pairs of quasi-particles (Cooper pairs) injected into the lead \(N_3\) propagate into leads \(N_1\) or \(N_2\) either as a whole or one by one. The ferromagnetic filters in setup (b) enforce the separation of the entangled spins, while the Fabry-Perot type interferometers in the setup (c) separate electron- and hole type quasi-particles.

We start by noting that for a single channel NS wire the zero frequency fluctuations of the currents carried by electrons with different spins are completely correlated,
hence $\langle (I_\sigma - I_{-\sigma})^2 \rangle_{\omega=0} = 0$ ($\langle \ldots \rangle$ implies the subtraction of the average currents). This perfect correlation in the (subgap) motion of the quasi-particles with different spins is a consequence of the entanglement of the Cooper pairs injected into the normal wire.

Next, recall that the current noise cross-correlations in a SN-NN fork geometry -- without filters on the normal probes -- are positive when the transmission between the superconductor and the normal leads is low [10]. The unusual sign (for fermions) of these correlations is due to paired electrons penetrating the two normal leads separately, c.f., Fig. 1(a). The positive correlations are further enhanced when the competing channel (with paired electrons entering the leads jointly) is suppressed through the addition of appropriate spin- or energy selective filters to the normal leads, see Figs. 1(b) and (c). The wave function of the entangled states generated with ideal spin/energy filters then is of the type

$$\Psi_{\varepsilon, \sigma} = \alpha \left[ \varepsilon, \sigma; -\varepsilon, -\sigma \right] + \beta \left[ \varepsilon \pm \sigma; \pm \varepsilon \mp \sigma \right],$$

where the first (second) argument in $\Phi_1, \Phi_2$ refers to the quasi-particle state in lead 1 (2) evaluated behind the filters and the upper (lower) signs refer to the setup projecting the spin (energy); the coefficients $\alpha$ and $\beta$ can be tuned by external parameters, e.g., a magnetic field. In such a multi-terminal device, the measurement of the zero frequency noise cross-correlator $\langle (I_\sigma I_{-\sigma}) \rangle_{\omega=0}$ then serves to detect electron entanglement, in analogy with the above single channel NS wire.

A step like dependence of the gap parameter at the NS interface is assumed, subgap transport is specified, while the normal leads are single-channel and ballistic. Using the scattering formulation of NS transport [11], the current operator per spin in normal lead $n$ is defined as

$$I_{\sigma n}(t) = \frac{ie\hbar}{2m} \sum_{\alpha, \sigma'} \int_0^\infty d\varepsilon d\varepsilon' \left[ \{u_{\sigma \alpha}(x) \hat{\partial}_x u_{\sigma' \alpha'}(x) \} \gamma_{\alpha \alpha'}^\dagger \gamma_{\varepsilon \varepsilon'} \right.$$  

$$- \{u_{-\sigma \alpha}(x) \hat{\partial}_x u_{-\sigma' \alpha'}(x) \} \gamma_{\varepsilon \varepsilon'} \gamma_{\alpha \alpha'}^\dagger \} \exp\{i(\varepsilon - \varepsilon')t\},$$

where the operators $\gamma_{\varepsilon \alpha}$ describe electron and hole Bogoliubov quasiparticles (with positive energies $\varepsilon$) on the normal side with $\alpha = \{p, \sigma, n\}$ a multi-index characterizing the ‘charge’ $p (= e, h)$, spin $\sigma (= \pm 1/2)$, and incidence (lead $n$); $f \hat{\partial}_x g = f \partial_x g - g \partial_x f$. The associated wave functions $\{u_{\sigma \alpha}(x)\}$ and $\{v_{\sigma \alpha}(x)\}$ are expressed in terms of the scattering matrix $s_{\alpha, \alpha'}$; e.g., for an electron with spin $\sigma$ incident from lead $n$ and observed in lead $m$ at $x_m$

$$u_{\varepsilon \sigma n}(x_m) \simeq \left[ \delta_{nm} e^{ik_x x_m} + s_{\sigma \varepsilon, \sigma n} e^{-ik_x x_m} \right] / \sqrt{h v_\varepsilon},$$  

$$v_{\varepsilon \sigma n}(x_m) \simeq \left[ s_{\sigma \varepsilon, \sigma n} e^{ik_x x_m} \right] / \sqrt{h v_\varepsilon},$$

with wave numbers $k_\pm = \sqrt{2m(\mu_\pm + \varepsilon)}$ and the quasi-particle velocities $v_\varepsilon = \hbar k_\varepsilon / m$. The difference between the two wave numbers $k_\pm$ will be neglected ($\mu_\pm \gg \Delta$).

Let us now turn to the fork geometry of Fig. 1: the current leads $N_1$ and $N_2$ are connected to $N_3$, which itself is terminated with the NS interface. We discuss two schemes: a) two ferromagnetic metal contacts (with magnetizations in opposite directions) in leads $N_1$ and $N_2$ block the propagation of the opposite spin, see Fig. 1(b), b) two energy filters in $N_1$ and $N_2$ (coherent quantum dots) select the kinetic energies of electrons and holes symmetrically above and below the superconducting chemical potential (Fig. 1(c)). In both proposals, the penetration of a Cooper pair into a given lead is prohibited, while allowing the split pair to pass the filters. E.g., one electron propagates through $N_1$ with spin ‘up’, while simultaneously the other electron (with opposite kinetic energy) propagates through $N_2$ with spin ‘down’.

The scattering matrix $s_{\alpha, \alpha'}$ has to account for all multiple scattering processes: the Andreev reflection at the NS interface can be specified in terms of the transmission and reflection amplitudes $t_{13}$, $t_{23}$, and $r_{ii}$ ($i = 1, 2$) describing the normal-metal part of the device [2]. The latter include the scattering by the ‘beam splitter’ $N_3 \leftrightarrow N_1, N_2$, and account for the presence of spin or energy filters in leads $N_1$ and $N_2$. The corresponding transmission and reflection amplitudes are found iteratively following the scheme sketched in Fig. 2 and accounting for all interference processes in the device.

**FIG. 2.** The scattering matrix $s_{\alpha, \alpha'}$ determining the noise correlator [11] incorporates all the internal scattering features of the fork, the beam splitter ($\rightarrow t^{(2)}, t^{(2)}$), the stubs ($\rightarrow t^{(1)}, r^{(1)}$), and the normal scattering at the NS interface ($\rightarrow t, r$).

**Beam Splitter:** Time reversal invariance is assumed for simplicity. It is then possible to express the transmission probability, say, between leads $N_1$ and $N_2$ in terms of the other two transmissions:

$$T_{12}^{(2)} = T_{13}^{(2)} T_{23}^{(2)} [2 - T_{1\Sigma}^{(2)} + 2(1 - T_{1\Sigma}^{(2)})^{1/2}] / T_{1\Sigma}^{(2)} \cdot T_{2\Sigma}^{(2)},$$

where $T_{1\Sigma}^{(2)} \equiv T_{13}^{(2)} + T_{23}^{(2)}$ and $T_{ij}^{(2)} = |t_{ij}^{(2)}|^2$. For a fully symmetric beam splitter $T_{ij}^{(2)} = 4/9$. Ref. [13] focused on a setup which is symmetric between 1 and 2. The lower
resonances are determined through the sign changes in the transmission amplitudes. These resonances building up within the normal-metal leads. These amplitudes contained in Eq. (9) are the Andreev type resonances which are a consequence of a vanishing transmission probability which enters the (sharp) electron and hole distribution functions produced by the quantum dot will then be decorated by Andreev-type resonances and zeros originating from the NS-fork structure.

For each energy $E$, the amplitudes are obtained by exchanging the lead indices in (8a) and (8b). The main features contained in Eq. (8) are the Andreev type resonances building up within the normal-metal leads. These resonances are determined through the sign changes in Re$(r_{33} + r_{33}^*)$ and their distance $\sim \hbar v_F/L$ is determined via the Fermi velocity $v_F$ and the characteristic size $L$ of the region. In addition, zeros appear in the spectral density which are a consequence of a vanishing transmission for electrons or holes in these three lead geometry.

The above scheme fully specifies the scattering matrix $s_{\alpha,\alpha'}$ for the case with ideal filters. For non-ideal filters the stub should be replaced with a proper description of the lead $N_2$ including its non-ideal filter; in addition, the normal scatterer described through the amplitudes $\{t_{13}, r_{33}, r_{11}\}$ above has to be combined with an additional scatterer describing the non-ideal filter in the lead $N_1$. E.g., an energy filter requires inclusion of a Fabry-Perot interferometer characterized through the scattering amplitudes $t_1, t_2, r_1', r_2$ and the separation $d$ of the double barrier system and producing a transmission $t_{N_2} = t_1 t_2 \exp(\pm ikd)/[1 - r_1 r_2' \exp(2i kd)]$. The resonance spacing should be larger than the applied bias for a proper device operation as a filter. The initial resonance lines produced by the quantum dot will then be decorated by Andreev-type resonances and zeros originating from the NS-fork structure.

The above entangler is essentially a two terminal device where electrons with, say, a given spin from lead 1 are converted into holes with an opposite spin in lead 2. The current correlations between 1 and 2 are positive and can be obtained using the definition of the noise in combination with (3), at $T = 0$.

\[
\langle I_\sigma I_{-\sigma} \rangle = \frac{e^2}{h} \sum_{\alpha,\alpha'} \int_{-eV_1}^{eV_1} d\varepsilon |s_{\alpha,\alpha'}|^2 (1 - |s_{\alpha,\alpha'}|^2),
\]

where a voltage $eV_1$ is applied between the lead $N_1$ and the superconductor while keeping the lead $N_2$ unbiased. For the case of ferromagnetic filters, the chemical potential which enters the (sharp) electron and hole distribution functions depends also on the spin index. The multi-indices $\alpha$ and $\alpha'$ to be summed over in (10) depend on the type of filters in the normal leads $N_1$ and $N_2$: For ferromagnetic filters (SN-FF) with the spin in $F_{1(2)}$ pointing up (down) $\alpha = \{e(h) \uparrow 1\}$ and $\alpha' = \{h(e) \downarrow 2\}$ (the propagation of other states is blocked by the filters). On the other hand, for the setup selecting a definite quasi particle energy via Fabry-Perot type filters we have to sum over spins with $\alpha = \{e \uparrow (\downarrow 1)\}$ and $\alpha' = \{h \downarrow (\uparrow 2)\}$ we assume filters selecting quasi particles and quasi holes in leads $N_1$ and $N_2$, respectively. Applying the same voltage to the lead $N_2$ as well does not change the answer in the normal fork (SN-NN) but renders the result for the ferromagnetic filters (SN-FF) twice larger. The current fluctuations (10) are straightforwardly converted into “counting” correlations as known from quantum optics: with $e n_{\alpha}(t) = \int_0^t dt' I_{\alpha n}(t')$ we find:

\[
\langle I_{\alpha n}(t) n_{-\alpha}(t') \rangle|_{t \rightarrow \infty} \approx \langle t/e^2 \rangle \langle I_{\alpha n} I_{-\alpha} \rangle|_{\omega = 0}.
\]

and hence $(\langle n_{\alpha n} - n_{-\alpha}(t) \rangle^2)/(\langle n_{\alpha n}^2 \rangle) \approx 0$. The result (11) together with the fact that the two currents $I_1$ and $I_{-2}$ are necessarily correlated constitutes the main justification for the proposed entanglement device: Eq. (11)
corresponds precisely to the current noise in lead 1. Ideally, these correlation measurements should be performed using fast electronics in order to generate time resolved voltage pulses for electron injection/detection.

This electronic entanglement apparatus can now be completed with a detection apparatus in order to test non-local correlations (Bell inequalities). For an entangler based on ferro/energy filters, the detection apparatus involves filters of opposite type (energy/ferro). Concentrating on energy filters, a positive energy particle emerging in lead $N_1$ can have either spin orientation, which can be “measured” by connecting this lead to, say, a magnetic contact with known spin orientation. In the opposite arm one should have a similar contact but with a magnetization axis rotated by $\pi/2$ in order to achieve the analog of the spin correlation experiments of Ref. [5]. A detailed discussion of these will be provided later.

The proximity induced entanglement of quasi particles in NS-fork type devices was implicit in Refs. [15] and Ref. [14]. Consider the above SN-NN setup with the lead $N$ in NS-fork type devices was implicit in Refs. [15] and Ref. [14].

While both experiments in Ref. [15] use a magnetic field to separate electron- and hole type quasi particles, the more recent suggestion in Ref. [15] proposes two ferromagnetic needles, a setup similar to our SN-FF device. This Andreev drag effect is quite robust and persists.

Summarizing, we have proposed an electronic entanglement device based on the proximity effect and have shown how to probe the resulting non-local electronic correlations in an emphatic way through a measurement of the current cross-correlator. Using a special fork geometry with, say, Fabry-Perot filters one arrives at a natural source of spin-entangled electron pairs, a device with potential applications in quantum computing architectures based on spintronics [7]. This device presents the advantage – as compared to its ferromagnetic cousin – that it can be realized with nowadays splitters [18] and quantum dot technology, e.g., using semiconductor-superconductor heterostructures [13], and does not require interdot coupling. Moreover, this SN-NN device appears to be more promising regarding potential applications for quantum information processing: the insertion of Fabry-Perot filters destroys only the orbital entanglement of the electrons, while the (most valuable) spin entanglement persists, contrary to the situation in the SN-FF device where the filters project the spin, but where the entanglement of energy degrees of freedom persists nevertheless.

We thank M. Feigelman and M. Reznikov for useful discussions and the Swiss NSF for financial support. GBL acknowledges support from a NWO grant for collaboration with Russia.

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).
[2] A. Ekert, Phys. Rev. Lett. 67, 661 (1991).
[3] C.H. Bennett et al., Phys. Rev. Lett. 70, 1895 (1993).
[4] A. Steane, Rep. Prog. Phys. 61, 117 (1998).
[5] J.F. Glauser and A. Shimony, Rep. Prog. Phys. 41, 1981 (1987), and references therein; A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 47, 460 (1981); see P.G. Kwiat et al., ibid. 75, 4337 (1995) for modern sources of entangled photon pairs.
[6] G. Burkard, D. Loss, and E. Sukhorukov, Phys. Rev. B 61, R16303 (2000).
[7] D. Loss and E.V. Sukhorukov, Phys. Rev. Lett. 84, 1035 (2000); P. Recher, E.V. Sukhorukov, and D. Loss, cond-mat/0009452.
[8] V.T. Petrashov, Phys. Rev. A 30, 1982 (1984); M. Böttiker, Y. Imry, and M. Ya. Azbel, ibid. 60, 11935 (1999).
[9] J. Torrè, and T. Martin, Europhys. J. B 12, 319 (1999).
[10] M.J.M. de Jong and C.W.J. Beenakker, Phys. Rev. B 49, 16070 (1994); B.A. Muzykantskii and D.E. Khmel’nitskii, ibid. 50, 3982 (1994); M.P. Anantram and S. Datta, ibid. 53, 16 390 (1996); G. Lesovik, T. Martin, and J. Torrè, ibid. 60, 11935 (1999).
[11] G. Benfert, G.A. Lauch, and G. Blatter, Phys. Rev. B 55, 3146 (1997); A.L. Fauchère, G.B. Lesovik, and G. Blatter, ibid. 58, 31177 (1998).
[12] Y. Gefen, J. Imry, and R. Landauer, Phys. Rev. Lett. 52, 139 (1984); M. Böttiker, Y. Imry, and M. Ya. Azbel, Phys. Rev. A 30, 1982 (1984).
[13] G. Blatter, N. Chtchelkatchev, G. B. Lesovik and T. Martin (in preparation).
[14] S.I. Bozhko, V.S. Tsoi and S.E. Yakovlev, Pis’ma Zh. Eksp. Teor. Fiz. 36, 123 (1982) [JETP Lett. 36, 153 (1982)]; P.A.M. Benistant, H. van Kampen and P. Wyder, Phys. Rev. Lett. 51, 1817 (1983).
[15] G. Deutscher and D. Feinberg, App. Phys. Lett. 76, 487 (2000).
[16] D. Loss and D.P. di Vincenzo, Phys. Rev. A 57, 120 (1998).
[17] M. Henny et al., Science 284, 296 (1999); W. Oliver et al., ibid., 299 (1999).
[18] H. Takayanagi et al., Phys. Rev. Lett. 75, 3533 (1995).