Similarity of gas discharge in low-pressure argon gaps between two plane-parallel electrodes

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Abstract: The similarity of gas discharge in low-pressure argon gaps between two plane-parallel electrodes was investigated by experiments, numerical simulations and theoretical analysis. It was found by the experiments that the breakdown voltages depend not only on the product of gas pressure and gap length, but also on the aspect ratio of the gap, i.e. \( U_b = f(p d, d/r) \). It was theoretically proved that \( U_b = f(pd, d/r) \) is also a special case, the non-uniform electric field between plane-parallel electrodes, of similarity theorem of gas discharge. It was found by the experiments that there exist similar glow discharges only in two gaps with a limited scaled-down factor \( k \). By theoretical analysis, it was explained that the forbidden processes such as the stepwise ionisation and the inelastic collision of the second kind violate the similarity of discharge as \( k \) increases, which was verified by the numerical simulations of the discharges with or without these two forbidden processes taken into account.

1 Introduction

In 1889, Paschen found that the breakdown voltage of a gas gap does not depend individually on the gas pressure \( p \) and the gap length \( d \), but depends on their product \( pd \), i.e. \( U_b = f(pd) \) that was later known as the famous Paschen’s law [1]. Townsend showed that Paschen’s law is a special case, uniform electric field, of a more general similarity theorem which can be applied equally to the breakdowns in non-uniform fields if the breakdown is dependent on the ionisation by the electron collision with neutral particles [2]. The similarity theorem has been fully discussed and summarised by von Engel [3]. He indicated that, under specific conditions, the similar discharges may be produced in the geometrically similar gaps that are same in a geometrical shape, but different in linear dimensions. The similar discharges have not only a same voltage-current characteristic, but also all their physical parameters (the density of the charged particles, the current density and so on) in the right proportions.

Similarity of gas discharge enables us to use the known properties of the discharge in one gap to extrapolate the features of the discharges in another geometrically similar gap in which the experimental studies may not be feasible or even possible. Before this extrapolation could be made, it should be verified that there do exist the similar discharges in these geometrically similar gaps. Recently, the progresses of similar discharge had been made in micro-discharge, huge glow discharge of International Thermonuclear Experimental Reactor (ITER) and picosecond pulse discharge [4–6].

In this paper, the similarity of gas discharge in low-pressure argon gaps between two plane-parallel electrodes was investigated by experiments, numerical simulations and theoretical analysis. It was found by the experiments that the breakdown voltages of these gaps depend not only on the product of gas pressure and gap length, but also on the aspect ratio of the gap, i.e. \( U_b = f(pd, d/r) \). It was theoretically proved that \( U_b = f(pd, d/r) \) is also a special case, the non-uniform electric field between plane-parallel electrodes, of similarity theorem of gas discharge. It was found by the experiments that there exist similar glow discharges only in two gaps with a limited scaled-down factor \( k \). By theoretical analysis, it was explained that the forbidden processes such as the stepwise ionisation and the inelastic collision of the second kind violate the similarity of discharge as \( k \) increases, which was verified by the numerical simulations of the discharges with or without these two forbidden processes taken into account.

2 Conditions for similar discharges

Similar discharges are not readily produced, but produced under specific conditions. There are two necessary conditions for the similarity discharges in two geometrically similar gaps. The first one is the product of \( pd \) for these two gaps should be the same, i.e. \( pd_1 = pd_2 \) which means that the total number of the collisions for one electron to cross the gap is the same. The second one is the reduced field in these two gaps should be the same, i.e. \( E_1/p_1 = E_2/p_2 \) for the uniform electric fields or \( E_1/p_1 = E_2/p_2 \) for the non-uniform fields at the corresponding points where \( p_1x_1 = p_2x_2 \), which means that the averaged energy of the electrons is the same.

In the above addition mentioned necessary conditions, one more condition is that the discharges in these two gaps should be dominated by the physical processes allowable to the similar discharge, the so called allowed processes. There are many physical processes occurring in the gas discharge, including ionisation by single collision, stepwise ionisation, Penning ionisation, photoionisation, diffusion, electron attachment, recombination and so on. In appendix 1 of his book, von Engel described a method how to test whether a process is allowable or forbidden to the similar discharge [3]. For any process, if the change rate of the particle density by this process fulfills \((k)\), it is an allowed process. Otherwise, it is a forbidden process that is not forbidden to the gas discharge, but not allowed to the similar discharge. Whether a discharge is dominated by the allowed processes or by the forbidden processes is not controllable

\[
\left( \frac{\partial N}{\partial t} \right)_{\text{gap1}} = \frac{1}{k^3} \left( \frac{\partial N}{\partial t} \right)_{\text{gap2}}
\]

where \( k \) is the ratio of the linear dimension of gaps 1 to 2 and is called scale-down factor; and \( N \) is the particle density.

3 Similarity in gas breakdown

Townsend reported in 1928 [7] and later confirmed by many others [8–15] that, even with an equal value of \( pd \), \( U_b \) for a longer gap is higher than that for a shorter gap, i.e. \( U_b = f(pd) \neq f(pd) \). As a result, the Paschen’s curves for the gaps with different \( d \) do not
superimpose onto each other. Not being aware that Paschen’s law applies only to the breakdown in the uniform electric fields, some researchers were surprised at their experimental results deviated from Paschen’s law and made incorrect explanations or even modification on Paschen law [16–18].

We investigated this phenomenon by measuring the DC breakdown voltages of low-pressure argon gaps between two plane-parallel electrodes. A DC voltage was applied to the electrodes. The breakdown was determined by the abrupt falling of the voltage across the gap. When determining the breakdown voltage, a slow growth rate of the applied voltage about 1 V/s was used and the breakdown voltage was measured with an accuracy of ±2 V. 

Figs. 1 and 2 are the typical results, where $R$ is the radius of the discharge chamber housing the electrodes. While the Paschen’s curves move upwards and rightwards with the increase of $d/r$ as shown in Fig. 1, they superimpose onto each other with an equal value of $d/r$ as shown in Fig. 2. It could be concluded that the breakdown voltages of these gaps depend not only on the product of gas pressure and gap length, but also on the aspect ratio of the gap, i.e. $U_b = f(pd, d/r)$.

By modelling the electric field using the computer code called Comsol, the distribution of the electric field along the central axis were obtained and shown in Figs. 3 and 4 in which $E_{av} = U/d$ is the averaged field and $U$ is the applied voltage. While the field distributions in two gaps with different $d/r$ are different, they are the same for two gaps with an equal value of $d/r$. Indeed, the distribution of the electric field is a function of $d/r$.

By polynomial fit of the profile of electric field shown in Fig. 3 or Fig. 4, a mathematical expression was obtained

$$E/E_{av} = f(x/d, d/r)$$

or

$$
\frac{E}{p} = \frac{U}{pd} \cdot f \left( \frac{px}{pd}, \frac{d}{r} \right)
$$

As we know, self-sustained condition for Townsend discharge is also the breakdown criterion and can be expressed as [19]

$$
\gamma \left[ \exp \int_0^q \alpha(x) \cdot dx - 1 \right] = 1
$$

or

$$
\int_0^q \alpha(x) \cdot dx = \ln(1 + 1/\gamma)
$$

where $\gamma$ is the coefficient of second electron emission from the cathode by ion bombardment; $\alpha$ is the electron impact ionisation coefficient and is a function of the reduced field $E/p$, i.e.

$$
\alpha = A \cdot p \cdot \exp \left( -\frac{B}{E/p} \right)
$$

where $A$ and $B$ are constants.

Substituting (2) into (4) and then into (3), we obtain

$$A \cdot \int_0^{pd} \exp \left[ -\frac{B \cdot pd}{U_b \cdot f(px/pd, d/r)} \right] \cdot d(px) = \ln(1 + 1/\gamma)$$
The change rates of the charged particles by the stepwise ionization and by the inelastic collision of second kind are expressed, respectively, by the following equations

\[
\frac{dn}{dt}_{gap1} = C_{R1} n_{e1} n_{m1} \tag{6}
\]

\[
\frac{dn}{dt}_{gap1} = C_{R2} n_{e1} n_{m1} \tag{7}
\]

where \(n_{e1}\) and \(n_{m1}\) are, respectively, the electron density and the metastable density in gap 1; and \(C_{R1}\) and \(C_{R2}\) are, respectively, the rate coefficients for R1 and R2.

If the discharges in gaps 1 and 2 are similar, we have

\[
n_{e1} = n_{e2}/k^2 \tag{8}
\]

\[
n_{m1} = n_{m2}/k^2 \tag{9}
\]

where \(n_{e2}\) and \(n_{m2}\) are, respectively, the electron density and the metastable density in gap 2; \(k\) is the scale-down factor.

By substituting (8) and (9) into (6) and (7), we obtain

\[
\frac{dn}{dt}_{gap1} = \frac{C_{R1}}{k} \cdot \frac{n_{e2}}{k^2} \cdot \frac{n_{m2}}{k^2} \neq \frac{C_{R2}}{k} \cdot \frac{n_{m2}}{k^2} \tag{10}
\]

\[
\frac{dn}{dt}_{gap1} = \frac{C_{R2}}{k^2} \cdot \frac{n_{m2}}{k^2} \neq \frac{C_{R2}}{k^2} \cdot \frac{n_{m2}}{k^2} \tag{11}
\]

Obviously, (1) is not satisfied in R1 and R2, therefore both R1 and R2 are forbidden processes. Furthermore, R1 or R2 will make (10) or (11) more deviated from (1) as \(k\) increases. That is why there exist similar glow discharges only in two gaps with a limited \(k\).

In order to verify the correctness of our above explanation, we made the numerical simulations of the discharges with or without R1 and R2 taken into account [22]. While the numerical simulation with R1 and R2 taken into account reproduced the experimental result that there exist similar glow discharges only in two gaps with a limited \(k\), the numerical simulation without R1 and R2 taken into account showed no influences of \(k\) on the similar discharges. In this case, it could be concluded that the similar discharges in two argon gaps are gradually violated as the scale-down factor \(k\) increases.

5 Conclusions

The breakdown voltages of low-pressure gaps between two plane-parallel electrodes can be expressed as \(U_b = f(pd, d/r)\) that is also a special case, the non-uniform electric field between plane-parallel electrodes, of similarity theorem of gas discharge. There exist similar glow discharges only in two argon gaps with a limited scaled-down factor \(k\) since the forbidden processes such as the stepwise ionisation and the inelastic collision of the second kind violate the similarity of discharge as \(k\) increases.

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