Continuous Néel-to-Bloch transition in strips whose thickness increases: Statics and dynamics

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Abstract – For strips of width $2w$ and height $h$, with both exchange and the dipole-dipole interaction, we find that Néel and Bloch domain walls are locally stable. As $h$ is increased to $h_c$, Néel walls evolve continuously to Bloch walls by a second-order transition. It is mediated by a critical mode with $\omega \sim \sqrt{h_c - h}$, corresponding to motion of the domain wall center. A uniform out-of-plane rf-field couples strongly to this critical mode only in the Néel phase. Local, but not global stability relative to a crosstie phase was established.

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Introduction. – Domain walls in ferromagnetic thin films, including exchange, uniaxial anisotropy, and the dipole-dipole interaction, have been extensively studied for over 70 years [1]. For a monodomain of small film thickness $h$, the dipole-dipole interaction causes the magnetization to lie entirely in the plane. Such domains in a thin film, depending on the film thickness, are separated by either Bloch or Néel domain walls. For the Bloch domain wall the magnetization develops an out-of-plane component [2,3]. The system develops surface poles, and the associated total dipole-dipole energy is proportional to $h$. For the Néel domain wall the magnetization lies entirely in the plane of the film [4,5]. The total dipole-dipole energy for a Néel wall comes from the self-interaction of a volume pole density, and thus is proportional to $h^2$. For small $h$ the Néel wall occurs (fig. 1a); for larger $h$ the Bloch wall occurs (fig. 1c).

Recent experimental and micromagnetic studies for relatively thick films show that a Bloch wall has a complex structure [6,7]. Its interior magnetization has an out-of-plane (Bloch-like) component, whereas its near-surface magnetization is in-plane (Néel-like) — the so-called Néel caps — to minimize the surface dipole energy. (Figure 1c shows imperfect Néel caps, since our numerical method employs a lattice, rather than a continuum.) The Néel domain wall, on the other hand, has a narrow central part that resembles the usual “exchange” domain wall, and long logarithmic tails [8,9]. These tails are the consequence of the dipole-dipole interaction.

In addition, the so-called cross-tie domain wall can be an alternative equilibrium configuration [10], and would be included in a more global study of this system.

This work presents numerical calculations for a magnetic system with parameters corresponding to permalloy, periodic in length $L$ ($L_{\max} \sim 1500$ nm), and with large enough half-width $w$ and height $h$ that crosstie solutions sometimes occurred when the magnetization was seeded randomly. Moreover, multiple crosstie solutions, with different numbers of vortex/anti-vortex pairs, sometimes occurred, in part because $L$ did not

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Fig. 1: Néel and asymmetric Bloch domain walls, magnetization distribution in the $(x,z)$-plane. Sample strip (permalloy) is infinite along $y$, has thickness $h$ along $z$ and has width $2w = 200$ nm along $x$. 

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equal an integral multiple of the natural period \( P(h, w) \) of any crosstie solution. Computational limitations prevented thorough exploration of \( P(h, w) \), relevant to the competition between optimized crossties and the Néel-Bloch wall. We find a range of \( h \) and \( w \) where the transition proceeds from the Néel directly to the Bloch wall without an intervening cross-tie phase and that the Néel-Bloch transition is continuous. We cannot rule out a thickness regime where crossties are lower in energy than the Néel-Bloch wall; that would make the transition re-entrant as energy than the Néel-Bloch wall. We find a range of thickness \( h \) and \( w \) where crossties are lower in energy than the Néel-Bloch wall; that would make the Néel-Bloch transition re-entrant as observed experimentally. Both the equilibrium Néel-Bloch walls (with no \( \gamma \)-dependence) and the crossties are locally stable to small changes in the 3d spatial and 3d magnetic configuration. We focus on the surprising result that the Néel-Bloch transition varies continuously with \( h \).

We assume that at the thicknesses of interest there is only one domain wall. For \( h \) less than the critical thickness \( h_c \), this domain wall is of Néel type, whereas for \( h > h_c \) the magnetization in the domain wall itself tips out of plane, forming so-called symmetric and asymmetric Bloch walls [11]. For the thicknesses considered here, Néel walls and asymmetric Bloch walls are the only low-energy states. Hubert’s work indicates that the asymmetric wall has a lower energy than the symmetric wall, and we saw no evidence for the symmetric wall [12].

Despite its fundamental nature, the continuous or discontinuous nature the Néel-to-Bloch transition at \( h_c \) is unsettled [13–15]. A recent and comprehensive magneto-statics study by Kalay and Humphrey [16] indicates that the transition between Néel walls and asymmetric Bloch walls is first order, although the authors were cautious in identifying the actual thickness at the transition.

The present letter studies the nature of this transition (continuous or discontinuous) by application of our work on the statics [9] and dynamics for Néel and Bloch walls. Using methods developed in ref. [17], we numerically study the normal mode spectrum as a function of thickness \( h \) and identify an unstable (critical) mode whose frequency \( \omega \) vanishes at a critical thickness \( h_c \). We numerically show that the ground state of samples with \( h < h_c \) is a Néel domain wall and the ground state of samples with \( h > h_c \) is an asymmetric Bloch wall. Figure 1 indicates the evolution of the domain wall at low thickness (fig. 1a) and at thicknesses just below and just above the transition (figs. 1b and c).

We also show that the transition can be described analytically by a Landau theory of second-order phase transitions with order parameter the characteristic spin amplitude of the unstable mode. We derive the Landau free energy for the transition up to fourth order in the order parameter. In agreement with the numerics, \( \omega \sim \sqrt{h_c - h} \) just below the critical thickness.

**Micromagnetics.** — Consider a ferromagnetic strip of thickness \( h \) along \( z \), width \( 2w \) along \( x \), and infinite length along \( y \) (inset to fig. 1). We choose parameters that make the exchange length \( l_{ex} = \sqrt{A/2\pi M_s^2} \) satisfy \( l_{ex} < w \), and consider samples wide enough to completely enclose the domain wall. The material parameters chosen are appropriate to permalloy: exchange constant \( A = 1.30 \times 10^{-6} \text{erg/cm} \) and saturation magnetization \( M_s = 795 \text{emu/cm}^3 \), which gives \( l_{ex} = 5.72 \text{nm} \); and zero crystalline anisotropy. The magnetization for \( x = 0 \) is taken to be parallel to \( x \)-axis and free boundary conditions are taken on the sides of the strip.

We first worked up from small thicknesses \( h = (1–5 \text{nm}) \) and magnetization \( \vec{m}_0 = \vec{y} M_s \), \( x < 0 \), \( \vec{m}_0 = -\vec{y} M_s \), \( x > 0 \). We equilibrated using a relaxation algorithm and then calculated the normal modes and their coupling to a uniform rf external magnetic field. Using this equilibrium as a starting point, we then gradually increased \( h \) and calculated the new equilibrium configuration, normal modes, and rf couplings. We also worked down from large thicknesses, to check for bi-stable solutions.

The two main results of the micromagnetics calculations are: 1) For small thicknesses the ground state is a Néel domain wall, with three distinct regions—a central region of width \( 2\delta \) and two logarithmic tails [9]. 2) A single normal mode, with frequency \( \omega \sim \sqrt{h_c - h} \) just below the critical thickness \( \omega \) (see fig. 2). This mode approximately corresponds to a \( z \)-dependent oscillation of the domain wall core along \( x \). The oscillations are confined to the central part of the Néel domain wall, whereas the logarithmic tails remain unperturbed. The frequency \( \omega \) depends on both the width \( 2w \) and on the height \( h \). We have also studied the soft mode above the transition, and it varies approximately as \( \sqrt{h - h_c} \), but we do not present our results, which are less extensive than for \( h < h_c \).
To a very good approximation, the (unnormalized) critical mode has in-plane oscillations of the form
\[
m^{(1)}_z(x, z) = \mathcal{R} \left[ \tanh \left( \frac{x}{\delta} \right) \text{sech} \left( \frac{x}{\delta} \right) e^{i\phi(x, z) - i\omega t} \right],
\]
where the \( z \) phase factor \( \phi_z(x, z) \) differs from \( \phi(x, z) \). The amplitude of the out-of-plane oscillations is typically less than 25% of the amplitude of the in-plane oscillations, except for \( h \) near \( h_c \).

For \( w = 100 \) nm, the inset to fig. 2 shows the thickness dependence of \( \omega \) just below \( h_c \) (dots) as well as a fit (line) indicating the \( \sqrt{h_c - h} \) dependence. For \( h > h_c \), if the ground state is taken to be a Néel wall, then the imaginary part of \( \omega \) goes from negative (stable) to positive (unstable). This is the only unstable mode.

Near \( h_c \), conventional methods (conjugate gradient algorithm, Landau-Lifshitz equation with large damping, etc.) failed to find the equilibrium state because among all the modes in the Néel state only one is unstable. Therefore any random perturbation of the initial configuration contains only a small projection on the unstable mode. Moreover, the unstable imaginary part of the eigenfrequency is of order 1000 Hz, which would require an enormous time interval for the instability to be observed by numerical integration. The only minimization method to produce reliable and consistent results was to obtain the normal modes for the unstable initial configuration, and then add a component of the unstable mode eigenvector to the static solution. This significantly decreased the energy and gave an eigenvector close enough to the true local equilibrium that more conventional methods then worked.

For \( w = 100 \) nm the domain wall for \( h > h_c \) (with \( h_c \approx 39 \) nm), is indeed an asymmetric Bloch wall (fig. 1). Figure 3 shows that to numerical accuracy both the total magnetostatic energy and its derivative remain continuous through the transition, although the dipole-dipole and exchange energies individually have discontinuous first derivatives. The second derivative of the magnetostatic energy is slightly discontinuous at the transition, which implies a second-order transition. The difference between this and previous work [15] may be due to the failure of conventional numerical methods at the transition. If those calculations employed a false equilibrium (metastability), then with increasing film thickness more modes become unstable. At some thickness the numerical method would find the true equilibrium, yielding an apparently sudden transition.

We also numerically studied the dependence of \( h_c \) on the exchange length \( l_{ex} = \sqrt{A/2\pi M_s^2} \) and the sample half-width \( w \) (fig. 4). For the range 150 nm \( \leq w \leq 1400 \) nm, a fit accurate to about 5% takes the form
\[
h_c = 5.4 l_{ex}^{0.914} w^{-0.086}.
\]

The \( h_c \)'s of fig. 4, for 100 nm \( < w < 1400 \) nm, can also be represented by a logarithm; \( h_c = 0.8 l_{ex} \ln(205 w/l_{ex}) \) gives a reasonable fit.

**Analytics.** – Our numerical findings permit study of the symmetry of the critical mode. We find that we can relate the half width \( \delta \) of the Néel domain wall center and the thickness of the strip \( h \) at the transition.

For very thin strips \( w \gg h \), below the critical thickness \( h_c \) the stable magnetization configuration \( \vec{M}^0(\vec{r}) \) is a Néel domain wall (fig. 1b) with center at \( x = 0 \) and parallel to the \( (z, y) \)-plane (fig. 1a). Because \( h \) is very small \( M_z^0 \) is negligible, and \( M_x^0 \) and \( M_y^0 \) are nearly independent of \( z \).
(Due to translation invariance $\bar{M}$ does not depend on $y$.) Thus the most general Néel domain wall is
\begin{equation}
\bar{M}^0(\hat{r}) = M_s \hat{n} = M_s \left[ u(x) \hat{i} + v(x) \hat{j} \right],
\end{equation}
where $u^2 + v^2 = 1$, subject to $u(\pm w) = 0$, $v(\pm w) = \mp 1$, with $u(-x) = u(x)$ and $v(-x) = -v(x)$. For $\delta \ll w$, near the center of the domain wall, $u(x) \approx \text{sech}(x/\delta)$ and $v(x) \approx -\tanh(x/\delta)$.

Above the critical thickness $h_c$, the stable configuration is the asymmetric Bloch wall (fig. 1c), whose magnetization $\bar{M}^B(\hat{r})$ lacks the $x$ and $z$ reflection symmetries, but preserves inversion symmetry $(x, z) \to (-x, -z)$, $M_{x,z}^B(-x, -z) = M_{x,z}^B(x, z)$, $M_{x,z}^B(-x, z) = -M_{x,z}^B(x, z)$.

The critical mode responsible for the Néel-to-asymmetric-Bloch-wall transition has magnetization
\begin{equation}
\bar{\mu}(x, z) = \bar{M}^B(x, z) - \bar{M}^0(x, z)
\end{equation}
and is symmetric under reflections of $x$ and $z$, so
\begin{align}
\mu_x(-x, z) &= -\mu_x(x, z), & \mu_{x,y}(x, -z) &= -\mu_{x,y}(x, z), \\
\mu_{y,z}(x, -z) &= \mu_{y,z}(x, z), & \mu_{z,x}(x, z) &= -\mu_{z,x}(x, z).
\end{align}

Since the amplitude of the critical mode is small, its magnetization is orthogonal to the static magnetization and can be written as
\begin{equation}
\bar{\mu} = M_s [\lambda(x, z) v(x) \hat{i} - \lambda(x, z) u(x) \hat{j} + \zeta(x, z) \hat{k}],
\end{equation}
where $\lambda(-x, z) = \lambda(x, z), \lambda(x, -z) = -\lambda(x, z), \zeta(x, z) = -\zeta(x, z)$, and $\zeta(x, -z) = \zeta(x, z)$.

We now expand in small $x$ subject to the above symmetries. With $z = 2x/h_c$, we take
\begin{align}
\lambda(x, z) &= (\lambda_1 z + \lambda_3 z^3) f(x), & \zeta(x, z) &= (\zeta_0 + \zeta_2 z^2) g(x),
\end{align}
where $\lambda_0, \lambda_2, \zeta_1$, and $\zeta_3$ are constants to be determined.

The micromagnetic calculations show that the critical mode is localized near the center of the strip and that it shifts the Néel domain wall core parallel to $x$. Thus $\mu_x(x, z) \sim \frac{A}{w_0} M^0_x(x)$ and $\mu_y(x, z) \sim \frac{A}{w_0} M^0_y(x)$. Comparison with the fits (1-3) to the micromagnetic calculations, we find that a good set of approximations is
\begin{equation}
f(x) = \text{sech}(x/\delta), \quad g(x) = \text{sech}(x/\delta) [2 \text{sech}(x/\delta) - 1].
\end{equation}

The mode energy now takes the form
\begin{equation}
W_{m}[\bar{n}, \bar{\mu}] = W_{ex}[\bar{n}, \bar{\mu}] + W_{dd}[\bar{n}, \bar{\mu}],
\end{equation}
where $W_{ex}$ and $W_{dd}$ are the exchange and dipole-dipole contributions. Although the mode is localized near the center of the domain wall, the exchange contribution to its energy is negligible, so $W_{ex} \ll W_{dd}$, and thus [18]
\begin{equation}
W_{m}[\bar{n}, \bar{\mu}] \approx \int d\bar{r}_1 d\bar{r}_2 (\mu(\bar{r}_1) \tilde{\partial}_{\bar{r}_1})(\mu(\bar{r}_2) \tilde{\partial}_{\bar{r}_2}) \left[ \frac{1}{|\bar{r}_1 - \bar{r}_2|} \right].
\end{equation}

We now expand the free-energy functional using the trial solution (7)–(9). The energy of the mode is calculated to second order in the $\lambda_i, \zeta_i$; because of stability it has no first-order terms. With the row vector $Z = (\lambda_1, \lambda_3, \zeta_0, \zeta_2)$ and its column (transpose) vector $Z^T$, we write
\begin{equation}
W_{m}[\bar{n}, \bar{\mu}] = Z \bar{W}_{\text{mode}} Z^T,
\end{equation}
where $\bar{W}_{\text{mode}}$ is a 4 x 4 matrix whose elements depend on the system parameters.

At $h_c$ the smallest eigenvalue $\lambda_0$ of $\bar{W}_{\text{mode}}$ goes to zero, so the determinant itself goes to zero:
\begin{equation}
\text{Det} \bar{W}_{\text{mode}} = 0.
\end{equation}

Solving (13) yields $h_c \approx h_0 \delta_c$, (to about 10%), where $\delta = \delta_c$ is evaluated at the transition. This is in good agreement with the numerics, where for different $w$ and $h_c$, we find $0.74 < h_c/\delta_c < 0.78$. The unnormalized critical vector is
\begin{equation}
Z_c = (0.19, 0.70, 0.66, 0.17).
\end{equation}

This yields the “soft” spin component $\bar{\mu}_s(x, z)$; its amplitude is determined by a higher-order energy expansion.

The matrix $W_{\text{mode}}$, which determines the statics, has $\lambda_0 \sim (h_c - h)$ for small $h_c - h$. The linearized dynamical response gives both the local value of the “hard” spin component $\bar{\mu}_h(x, z)$ and the eigenfrequency, which varies as $\sqrt{h_c - h}$, in agreement with the inset to fig. 2. We have also studied the soft mode above the transition, and it varies as $\sqrt{h - h_c}$, but we have not performed a theoretical analysis for $h > h_c$.

Note that the exchange energy is much smaller than the dipolar energy because their relative strength is proportional to $(h_c/\delta)^2$, and typical values in the present work are $h_c = 5 \text{ nm}$ and $\delta = 50 \text{ nm}$.

**Discussion and summary.** – Experimental study of the transition by measurement of the magnetization and even the critical mode would be desirable. Because the symmetry under $-z$ to $+z$ of the critical mode changes at the transition, from in-phase to out-of-phase, there are drastic symmetry changes in the absorption. Thus, whereas for $h < h_c$ (Néel wall) the mode strongly couples to a uniform out-of-plane rf field, for $h > h_c$ (asymmetric Bloch wall) it becomes antisymmetric with respect to the $z$-axis and cannot couple to this same rf field.

In summary, we find that the Néel-to-asymmetric-Bloch-wall transition in dipole-dipole coupled thin magnetic strips, infinitely long in one direction, with relatively large width $2w$, is a smooth function of thickness $h$. Normal-mode studies reveal that at the critical thickness $h_c$, where the nature of the domain wall changes, the frequency of a single critical mode goes to zero as $\sqrt{h_c - h}$. To high accuracy, both static and dynamic calculations indicate that this transition is continuous, which is also supported by analytical studies. Although locally stable, global stability relative to a crosstie phase was not established.
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