Invalidity of the relativity principle and a proposal of the twofold metric principle

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Abstract

In this paper, we first show that all inertial systems are not equivalent, and the Lorentz transformation is not the space-time transformation over two inertial systems moving with relative constant velocity. To do this, we consider imaginary signals travelling over any inertial system $K$ with arbitrarily large velocities. The travelling of an imaginary signal over $K$ is just a time lapse over $K$. Then we present an example to show that all coordinate systems are not equivalent when the related theory is restricted over tensor-based coordinate transformations, i.e., the general relativity principle is not valid.

Instead of the relativity principle, we propose the twofold metric principle which may be roughly stated to assert that the set of equations $H(v)$ describing the motion of a material body with velocity $v > 0$ can be obtained from the corresponding set of equations $H(0)$ for velocity $v = 0$ by replacing, in each differential equation in $H(0)$, each infinitesimal time variable $dt$ with $dt/\beta(v)$, each maximal velocity-critical infinitesimal length variable $dr$ with $\beta(v)dr$, and each zero velocity-critical infinitesimal length variable $dx$ with $dx$, where $\beta(v) = 1/\sqrt{1 - v^2/c^2}$. By depending on the twofold metric principle and the energy-velocity equation, we derive $\beta(v)mc^2$, the travelling distance of a muon with velocity 0.999$c$, the twofold Schwarzshild metric, the centennial procession of planetary orbits and deflection of light. We also present a reason why the Michelson-Morley experiment is observed. Several other topics are also studied.

Key words:
relativity principle, inertial system, Lorentz transformation, time-space metric
1 Introduction

An inertial system is a coordinate system in which the Newton equation of motion holds. The special relativity principle asserts that all inertial systems are equivalent. This principle plays key roles in [3] in which Einstein constructs foundations of special relativity theory. By depending on the relativity principle and the constant light velocity principle, Einstein develops arguments for supporting the validity of the Lorenz transformation over two inertial systems moving with relative constant velocity, observes time dilation and length contraction, and derives transformation formulae of the electromagnetic fields over two inertial systems as above by studying the Maxwell equations from viewpoints of the special relativity principle. Einstein develops general relativity theory in another significant paper[5], which is based on the genareal relativity principle and tensor-based time-space metrics. Since its origin, relativity theory has attracted scientists and other people theoretically and philosophically. Now it seems that there do not exist many people who have any doubts about the validity of the relativity principle. Although there have appeared ”paradoxes” in the literature such as the twin paradox and the garage paradox, it seems to be generally acknowledged that these paradoxes are solved theoretically by being given suitable explanations.

The main purpose of this paper is to show that all inertial systems are not equivalent, present a new principle called the twofold metric principle and show how the new principle works well for solving many problems. We also show that the general relativity principle asserting all coordinate systems are equivalent (w.r.t. tensor-based theory) is not valid by presenting an example showing that when a proper coordinate system $K$ is transformed to another coordinate system $K'$, then the metric is changed so that important principles such as the variational principle cannot be applied over $K'$, and so we conclude that $K$ and $K'$ are not equivalent.

In the Lorentz transformation, the time at any point at rest over an inertial system depends both on the time and the spacial coordinate of the corresponding point at rest over another inertial system. We shall show that the Lorentz transformation is not valid because it does not preserve simultaneity, i.e., if the space-time transformation from a coordinate system $K$ to another coordinate system $K'$ is the nonidentity Lorentz transformation, then $K$ or $K'$ is not synchrinized, i.e., $K$ or $K'$ is not an inertial system. Especially we shall prove the following (see Theorem 1 in Section 4).

Fact A. Let $K$ and $K'$ be two inertial systems moving with a relative constant velocity. The time at any point $P$ at rest on $K$ depends only on the time at the corresponding point $R$ at rest over $K'$, and does not depend on the spacial coordinate of $R$.

One can see easily that if the Lorentz transformation is valid, then the following hold (see Remark 3 in Section 4) : (1) the flying of a small firework using a small amount of energy over an inertial system can correspond to a travelling in an arbitrarily large distance and an arbitrarily large time lapse over another inertial system ; (2) a small distance and the zero time lapse over an inertial system can correspond to an arbitrarily large distance and an arbitrarily large time lapse, respectively, over another inertial system, and hence the (present) age of the universe is infinite, and the whole space is also infinite. We may also observe that if two coordinate systems $K$ and $K'$ are related by the nonidentity Lorentz transformation, then the light velocity is $c$ over both $K$ and $K'$, but the relation of past and future over $K$ is different from the corresponding relation over $K'$ (see Remark 1 in Section 2). Thus in a sense, one may say $K$ and $K'$ are not equivalent. From these observations, we cannot help concluding the Lorentz transformation is not valid. Morewever we shall present a more formal
proof showing invalidity of the Lorentz transformation. To do this, we introduce the notion of an imaginary signal $S(K,u)$ travelling over an inertial system $K$ with velocity $u > 0$, where $u$ can be arbitrarily large. The travelling of $S(K,u)$ from $P$ at time $t$ to point $Q$ both at rest on $K$ is just a time lapse over $K$ from time $t$ at $P$ to time $t + l/u$ at $Q$, where $l$ is the distance between $P$ and $Q$. The travelling of $S(K,u)$ from point $P$ at time $t$ to $Q$ is in a sense "observable" because we can consider by a Gedankenexperiment the corresponding continuous time lapse over the line segment connecting $P$ and $Q$. Thus the notion of imaginary signals are meaningful mathematically and physically, and the travelling of an imaginary signal can be regarded to be a physical phenomenon which really occurs in the universe.

The Lorentz transformations are bijections and of the form of linear combinations of coordinates and compose a group. It seems that the light signals are not sufficiently rich tools for analyzing whether or not the Lorentz transformations are valid since the light velocity is finitely bounded. Consider Achilles-tortoise running race problem. If one does not notice the fact that in certain cases, an infinite sum of time lapses $\sum_{n=0}^{\infty} t_n$ is finite, then one cannot solve this problem. Here we consider an arbitrarily small time lapse $t_n$ for a sufficiently large $n$. In the same way, if we depend only on the travelling of light signals for analyzing the synchronization problem over inertial systems, then we could not show that the Lorentz transformation does not preserve simultaneity. We may need the notion of arbitrarily small time lapses, i.e., the travelling of imaginary signals with arbitrarily large velocities to solve the synchronization problem.

For any inertial system $K$ and any arbitrarily large real $u > 0$, the imaginary signal $S(K,u)$ travels over $K$ with velocity $u$. We develop arguments similar to the well known ones concerning the possibility of the existence of time machines, and establish Time Independence Lemma (in short, TI Lemma, i.e., Lemma 3 in Section 4) and Time Independence Theorem (in short, TI Theorem, i.e., Theorem 1 in Section 4). In the Lorentz transformation, imaginary signals with large velocities over a coordinate system can go into past histories over another coordinate system. TI Lemma and TI Theorem imply that such phenomena never occur over inertial systems and Fact A holds. For any two inertial systems $K$ and $K'$ moving with nonzero relative constant velocity, the travelling of an imaginary signal $S(K,u)$ over $K$ corresponds to a time lapse within which $K$ and $K'$ continues moving each other in the opposite directions. The travelling of $S(K,u)$ corresponds to a physical phenomenon which occurs in our universe. We remark that one usually consider the notion of infinitesimal time lapses, $\lim_{\Delta t \to 0}$, when he analyzes differential equations describing the motions of material bodies or the travelling of electromagnetic waves. Without the notion of $\lim_{\Delta t \to 0}$, one can establish neither any theory of calculus nor any control theory. By depending on TI Theorem, we study properties of inertial systems. Some of the results are the following: (1) the Lorentz transformation is not the space-time transformation over two inertial systems moving with nonzero relative constant velocity; (2) all inertial systems are not equivalent; (3) the Maxwell equations over an inertial system moving with nonzero constant velocity in the vacuum space are not of the same form as those over stationary inertial systems; (4) the velocity of a light signal measured over an inertial system moving with nonzero constant velocity in the vacuum space is not constant but depends on its direction.

It is generally acknowledged that the expression "stationary in the vacuum space" is meaningless since the special relativity principle holds. However, in this paper, we understand that an inertial system $K$ is stationary in the vacuum space (or, equivalently, stationary w.r.t. the universe) if there does not exist any material body of huge mass near $K$ (i.e., with (almost) zero gravitational potential) and the Maxwell equations over $K$ are of the same form as those in "the vacuum space" (the standard ones presented in any textbooks). Thus over
a stationary inertial system, the light velocity is \( c \) independent of its direction. Since a light signal is a wave, its travelling is in a sense independent from the source from which the signal is emitted and depends only on the space. Thus light signals can be regarded to play roles as instruments measuring the velocity of any material body in the space at least locally. In the sequel, we also use often the words ”in the vacuum space” instead of ”over an inertial system stationary in the vacuum space”.

Instead of the relativity principle, we shall propose the twofold metric principle. We also propose the energy-velocity equation. The twofold metric principle may be roughly stated to assert that the set of equations \( H(v) \) describing the motion of a material body with velocity \( v > 0 \) can be obtained from the corresponding set of equations \( H(0) \) for velocity zero by replacing, in each differential equation in \( H(0) \), each infinitesimal time variable \( dt \) with \( dt/\beta(v) \), each maximal velocity-critical infinitesimal length variable \( dr \) with \( \beta(v)dr \), and each zero velocity-critical infinitesimal length variable \( dx \) with \( dx \), where \( \beta(u) = c/\sqrt{c^2-u^2} = 1/\sqrt{1-u^2/c^2} \) for \( 0 \leq u < c \) (for details, see Section 5). This principle implies that the total energy \( E \) of a material body \( M \) plays a key role for determining the motion of \( M \), and it holds the larger \( E \) is, the more slowly \( M \) responds to any outer force, that is, the larger \( E \) is, the more "stubborn" \( M \) becomes. This principle may present a unification of the correct parts of special relativity and general relativity by the term "energy". By depending on the energy-velocity equation and the twofold metric principle, we derive the following: (5) \( E = \beta(v)mc^2 \); (6) the travelling distance of a muon with velocity 0.999\( c \); (7) a modified version of the Schwarzshild metric (called the twofold Schwartzshild metric); (8) the Maxwell equations and the light velocity over the space with a gravitational field; (9) deflection and red shift of light due to gravity; (10) an explanation for the Michelson-Morley experiment; (11) showing the general relativity principle asserting all coordinate systems are "equivalent" (at least w.r.t. the tensor-based laws of physics) is not valid; (12) remarks about Einstein’s (field) equations and cosmology. Our calculated value of procession of planatery orbits depending on the twofold Schwartzshild metric is the same as the corresponding well known value derived from the Schwarzshild metric. However the twofold Schwartzshild metric does not contain the points \( r = 2GM/c^2 \) as its singular ones, and may imply that arguments for the existence of black holes depending on the points \( r = 2GM/c^2 \) in the Schwartzshild metric are wrong. We conclude the Schwarzshild metric is an approximation of the twofold Schwartzshild metric in Section 5. One may observe many two-fold pairs in the universe: (i) the space and the time; (ii) particle and antiparticle; (iii) mass and energy; (iv) hadron and lepton; (v) electricity and magnetism; (vi) plus and minus electricity; (vii) north and south poles in magnetism; (viii) potential and kinematic energy, etc.

In developing arguments in relativity theory, the following two principles are generally acknowledged to be valid (see, e.g., [14]).

**Principle 1**  
Space is isotropic, i.e., all spacial directions are equivalent.

**Principle 2**  
Space and time are homogeneous, i.e., no point of space or time is distinguished from others. The origin of the coordinate system may be chosen arbitrarily, without affecting the measuring devices for space and time.

The author is not certain whether these two principles are valid about the entire universe, but we acknowledge that these principles are valid at least within the scope of the universe which this paper concerns. We also acknowledge the validity of the following principle which is called in this paper the weak constant light velocity principle.
**Principle 3**  A light signal over an inertial system stationary in the vacuum space proceeds into any direction with constant velocity \( c \).

Moreover the following four ”principles” are also acknowledged to hold in relativity theory.

**Strong Constant Light Velocity Principle**  The velocity of a light signal measured over any inertial system is \( c \) independent of its direction.

**Special Relativity Principle**  All inertial systems are equivalent w.r.t. the laws of physics.

**General Relativity Principle 1**  All coordinate systems are equivalent w.r.t. the laws of physics.

**General Relativity Principle 2**  All coordinate systems are equivalent w.r.t. the tensor-based laws of physics.

We shall show that none of these four principles is valid. To do this, we shall show that Strong Constant Light Velocity Principle is not valid by showing Fact A. This implies invalidity of Special Relativity Principle and General Relativity Principle 1. To disprove General Relativity Principle 2, we shall present one coordinate transformation which transforms the Schwartzshild metric into a metric \( g'_{\mu \nu} \) such that from the metric \( g_{\mu \nu} \), one deduce procession of planetary orbits wrongly by applying the variational principle. Thus to apply the variational principle, we need a standard coordinate system and the corresponding standard metric.

We acknowledge the validity of the following proposition due to Principles 1,2.

**Proposition 1**  All points which are at rest on an inertial system \( K \) are equivalent w.r.t. the theory about \( K \), i.e., no point at rest on \( K \) is distinguished from others at rest on \( K \) theoretically. The origin of \( K \) may be chosen arbitrarily without affecting the measuring devices for space and time.

Einstein [3] introduces the notion of synchronization of two clocks \( A \) and \( B \) which are at rest on two stationary points \( P \) and \( Q \), respectively, on an inertial system \( K \), and have the same mechanism as follows. Due to Strong Constant Light Velocity Principle, Einstein admits that the time lapses for light signals to travel from \( P \) to \( Q \) and to travel from \( Q \) to \( P \), respectively, are the same. Now assume that a light signal is emitted at \( P \) at time \( t_1 \) on \( A \), then arrives and is reflected by a mirror at \( Q \) at time \( t_2 \) on \( B \), and finally returns at \( P \) at time \( t_3 \) on \( A \). Then \( A \) and \( B \) are said to be synchronized if it holds \( t_2 - t_1 = t_3 - t_2 \). Einstein asserts that his synchronization relation is an equivalence relation. We note that Einstein’s synchronization relation would be an equivalence relation only when the times at all points at rest on \( K \) proceed with the same rate since all clocks at rest on \( K \) are assumed to be of the same mechanism. This can be noted by the following simple example.

**Example 1**  We say that the mechanisms of two clocks are equivalent if the times shown by them proceed with the same speed (rate) (a more detailed definition will be presented later). Assume that there exist two clocks \( A \) and \( B \) at points \( P \) and \( Q \), respectively, at rest on the earth, and the following hold.

(1) When the Greenwich mean time is 12 o’clock, January 1, 1993, the time shown on \( A \) is 12 o’clock, January 1, 1993 and the time shown on \( B \) is \( 10^{-3} \) seconds, 12 o’clock,
January 1, 1993.

(2) When the Greenwich mean time is 13 o’clock, January 1, 1993, the time shown on \( A \) is 13 o’clock, January 1, 1993 and the time shown on \( B \) is \( 10^{-3} \) seconds, 30 minutes, 12 o’clock, January 1, 1993.

Thus the ratio of the speed (rate) of the time lapse on \( B \) w.r.t. that on \( A \) is 1/2 (so that the mechanisms of \( A \) and \( B \) are not equivalent). Assume that the time lapse measured by the Greenvich mean time within which a light signal travels from \( P \) and \( Q \) (and from \( Q \) to \( P \) also) is \( 2 \times 10^{-3} \) seconds. Assume also that a light signal is emitted from \( P \) at time \( t = 12 \) o’clock, January 1, 1993 on \( A \), arrives and is reflected at \( Q \) at time \( t' \) on \( B \), and finally returns at \( P \) at time \( t'' \) on \( A \). Then it holds \( t' = 2 \times 10^{-3} \) seconds, 12 o’clock, January 1, 1993, \( t'' = 4 \times 10^{-3} \) seconds, 12 o’clock, January 1, 1993, and \( t' - t = t'' - t' \). But if a light signal is emitted from \( P \) toward \( Q \) at time \( t_0 \neq t \) on \( A \), then the above relation clearly does not hold, and it is impossible to make two clocks \( A \) and \( B \) synchronized. For two clocks to be synchronized, it is necessary that their mechanisms are equivalent.

In the sequel, we assume that on each inertial system \( K_0 \), there exists an imaginary clock \( C_0(P) \) to each point \( P \) at rest on \( K_0 \) indicating the time at \( P \) so that the expression "at time \( t \) at \( P' \)" means that the time shown on \( C_0(P) \) is \( t \), and "the speed of the time lapse at \( P' \)" coincides with the speed of the time lapse shown on \( C_0(P) \). Since an inertial system is a coordinate system in which the Newton equation of motion holds, we acknowledge that all clocks \( \{ C_0(P) \mid P \text{ is a point at rest on } K_0 \} \) are synchronized (a new definition of synchronization will be presented later). In this paper, a coordinate system is for use of descriptions of events occurring in the universe by being given corresponding changes of coordinates of events. To each coordinate system \( K_1 \), we assume that to each point \( P \) at rest on \( K_1 \), there exists an imaginary clock \( C_1(P) \) indicating the time at \( P \) as in case of inertial systems. In some cases, all imaginary clocks associated with a coordinate system may not be synchronized. We say that a coordinate system is synchronized if all imaginary clocks associated with the system are synchronized. We shall introduce the notion of clock-coordinate systems as follows. Let \( K \) be a coordinate system. Then a clock-coordinate system (in short, a cc system) of \( K \) is a pair \( H = < K, C > \) such that \( C \) is a bijection from the set \( A = \{ P \mid P \text{ is a point at rest on } K \} \) to a set of imaginary clocks \( B \) such that for each \( P \in A \), \( C(P) \) indicates the time at \( P \). Since all cc systems of \( K \) are equivalent, we call each of them the cc system of \( K \). \( H \) is said to be synchronized if all clocks in \( B \) are synchronized, i.e., if \( K \) is synchronized.

Instead of Strong Constant Light Velocity Principle, we acknowledge the validity of the following proposition since due to Proposition 1, we can choose the origin of \( K \) arbitrarily.

**Proposition 2** The time lapse within which a light signal travels from a point \( P \) to a point \( Q \) both of which are at rest on an inertial system \( K \) is uniquely determined if the distance between \( P \) and \( Q \) and the direction from \( P \) and \( Q \) are fixed.

We note that it is acknowledged that the universe continues expanding, and one might assert that the distance between two points which are at rest on the above \( K \) continues changing as the time passes. But if one acknowledges the validity of this assertion, then one can deduce that the Newton equation of motion does not hold on \( K \) by noting the weak constant light velocity principle. We also note that the distance between any two points which are at rest on the earth coordinate system has been almost unchanged within a very long time
period independent of the expansion of the universe. Moreover in the Lorentz transformation over two inertial systems moving with relative constant velocity, it is acknowledged that the coordinate of a point which is at rest on one of the systems and the distance between two points at rest on the system are constant independent of the time lapse on the system. In this paper, we acknowledge that an inertial system is a coordinate system which will be used for approximate descriptions of events occurring in a rather small local space and a local time period in that space. In Section 6, we shall study the expansion of the universe by depending on Hubble’s law, but it will be left open to present coordinate systems for describing our entire expanding universe.

Synchronization is indispensable for analyzing properties of events occurring on an inertial system as the following two examples indicate.

Example 2  Consider the following situation.

(1) Two persons A and B live at New York and Tokyo, respectively.

(2) A and B have two clocks C and D, respectively, whose mechanisms are equivalent.

(3) When the Greenwich mean time is 12 o’clock, March 1, 1993, the time shown on C is 11 o’clock, April 1, 1991 and the time shown on D is 10 o’clock, May 1, 1992.

(4) An air mail is sent from A at time 9 o’clock, June 1, 1991 on clock C, and arrives at B at time 8 o’clock, July 8, 1992 on clock D.

If they assert that the time lapse in which the mail is carried from A to B is 371 days and 23 hours (= the time difference shown by their clocks), then this assertion is nonsense.

We also need the notion of bias about two clocks. A detailed definition will be given later.

Example 3  Let A be a clock indicating the French standard time, and B be a clock indicating the Japanese standard time. Then their mechanisms are equivalent, but A and B are not synchronized, and the bias of A w.r.t. B is −8 hours in the winter.

We again remark in Example 2 that if the bias of C w.r.t. D is two weeks, then A and B may assert that the time lapse needed for the delivery of the mail is negative. To decide the delivery time of the mail, it may hold that two clocks are synchronized or their mechanisms are equivalent and the bias between them is known.

Since we do not acknowledge the validity of Strong Constant Light Velocity Principle, we shall present our new definition of synchronization which is theoretical rather than feasible as follows. Let K be a coordinate system, P and Q be two points at rest on K, and A and B be two clocks at rest on P and Q, respectively. Let a and l denote the direction from P to Q and the distance between P and Q, respectively. If K is an inertial system, by Proposition 2, it holds that the velocities of a light signal travelling from P to Q and from Q to P, respectively, are uniquely determined if a and l are determined. Let these velocities be r and s, respectively. Now consider time $t_1$ on A, and let a light signal be emitted from P at time $t_1$ on A, arrive and be reflected at Q at time $t_2$ on B, return at P at time $t_3$ on A, and finally arrive at Q at time $t_4$ on B. The time lapse $tl(A, B, t_1)$ of A w.r.t. B at $t_1$ on A is defined by:

$$tl(A, B, t_1) = t_3 - t_1.$$ 

Similarly $tl(B, A, t_2)$ is defined by:

$$tl(B, A, t_2) = t_4 - t_2.$$ 

Clock A (B, respectively) is said to be punctual if for any time $t_{10}$ on A, it holds $tl(A, B, t_{10}) = l/r + l/s$ (for any time $t_{20}$ on B, it holds $tl(B, A, t_{20}) = l/r + l/s$, respectively). The mechanisms of A and B are said to be equivalent if both A and B are punctual. The bias $b(A, B)$ of A w.r.t.
$B$ is defined by: $b(A, B) = l/r - (t_2 - t_1)$. Note that $b(A, B) = l/r - (t_4 - t_3) = t_3 - t_2 - l/s$, and $b(A, B)$ is independent from the choice of $t_1$ when $A$ and $B$ are punctual. Clocks $A$ and $B$ are said to be synchronized if the mechanisms of $A$ and $B$ are equivalent and the bias of $A$ w.r.t. $B$ is zero. In Appendix 1, we present a proof showing the new synchronization relation is an equivalence relation.

Now we shall introduce the contents of the following sections briefly. The paper consists of nine sections and five appendices. Section 2 presents properties of the Lorentz transformation. Section 3 presents an electromagnetic example which seems contradictory to Special Relativity Principle by observing the Maxwell equations about a stationary charged material body and the corresponding moving one. Sections 4-5 are the main parts of this paper. Section 4 presents properties of inertial systems. It introduces the notion of imaginary signals over an inertial system, and presents TI Lemma (Lemma 3) and TI Theorem (Theorem 1). By depending on TI Theorem, we deduce that neither Special Relativity Principle nor Strong Constant Light Velocity Principle is valid, and the Lorentz transformation is not the space-time transformation over two inertial systems moving with nonzero relative constant velocity. In Section 5, we introduce a new principle called the twofold metric principle, and the energy-velocity equation. By depending on the energy-velocity equation and the new principle, we derive the results (5)-(12) explained briefly in the beginning part of this section. We also conclude by depending on the new principle that the existence of black holes can not be predicted from the Schwarzschild metric, and the Schwarzschild metric is an approximation of the twofold Schwarzschild metric. Section 6 presents arguments about the expansion of the universe, which contains the assertion that the (present) age of the universe is greater than the Hubble time $H_0^{-1}$. Section 7 presents a new observation about the dark matter problem. Section 8 presents arguments for concluding that one interpretation of the principle of equivalence in general relativity theory is not sufficiently valid. The final section presents concluding remarks.

2 The Lorentz transformation

By a coordinate system in the sequel of this section, we mean a Cartesian coordinate system $K$ in which the time lapses at all points at rest on $K$ are also described. For each coordinate system $K$, a coordinate on $K$ is a quadruple, $(x, y, z, t)$, such that $x$, $y$ and $z$ are the $x$-axis, $y$-axis and $z$-axis coordinates of some point $P$ on $K$ and $t$ is a time at $P$. $K$ can be used for describing the behavior of any event occurring on $K$ by denoting changes of corresponding coordinates. For example, travellings of electromagnetic waves can be described on $K$. In the sequel, each coordinate system will be denoted often by $K, K', K''$ or $K_0$, and $t, t', t''$ and $t_0$ will be often used for variables denoting times on $K, K', K''$ and $K_0$, respectively. When $K, K', K''$ or $K_0$ is a Cartesian coordinate system, then a-axis, $a'$-axis, $a''$-axis or $a_0$-axis for $a \in \{x, y, z\}$ means the corresponding axis of $K, K', K''$ or $K_0$, respectively, in the standard way. To each coordinate system $K$, we define the clock-coordinate system $H = < K, C >$ (in short, the cc system ) of $K$ as in Section 1. Thus for each point $P$ at rest on $K$, $C(P)$ is an imaginary clock indicating the time at $P$. The set of all coordinates on $K$ will be denoted by $CO(K')$. An inertial system in the sequel means a synchronized Cartesian coordinate system moving with gravitational potential (almost) zero in the vacuum space with constant velocity. Let $K$ and $K'$ be two Cartesian coordinate systems such that their corresponding axises are parallel, the origins of $K$ and $K'$ coincide at the initial time $t = t' = 0$, and the origin of $K'$ moves in the direction of the positive $x$-axis of $K$ with constant velocity $v > 0$. In this situation, we say that $K$ and $K'$ are two parallel Cartesian coordinate systems with relative
constant $x'$-axis velocity $v$. When $K$ and $K'$ are inertial systems, we say in this situation that $K$ and $K'$ are two parallel inertial systems with relative constant $x'$-axis velocity $v$. The space-time transformation from $K$ to $K'$ is a bijection $\mathcal{T}$ from $CO(K)$ to $CO(K')$ such that for each $(x, y, z, t) \in CO(K)$, $\mathcal{T}(x, y, z, t) = (x', y', z', t')$ means that if $(x, y, z)$ is a spacial coordinate at a point $P$ on $K$, then $(x', y', z')$ is the corresponding spacial coordinate at $P$ on $K'$ at time $t$ if $P$ on $K$, and $t'$ is the corresponding time at $P$ on $K'$. $(x', y', z', t')$ is interpreted in a sense as the coordinate corresponding to $(x, y, z, t)$ at $P$ on $K'$ measured by an observer who is at rest on $K'$. $\mathcal{T}$ is called the Lorentz transformation (from $K$ to $K'$) if for any $(x, y, z, t) \in CO(K)$, $(x, y, z, t) = \mathcal{T}(x, y, z, t)$ satisfies the following, where $\beta(v) = c/\sqrt{c^2 - v^2} = 1/\sqrt{1 - v^2/c^2}$.

$$x' = \beta(v)(x - vt), \ t' = \beta(v)(t - vx/c^2), \ y' = y, \ z' = z$$

The inverse $\mathcal{T}^{-1}$ of $\mathcal{T}$ satisfies the following.

$$\mathcal{T}^{-1}(x', y', z', t') = (x, y, z, t)$$

$$x = \beta(v)(x' + vt'), \ t = \beta(v)(t' + vx'/c^2), \ y = y', \ z = z'$$

Einstein[3] asserts that the following statement is true.

**Statement 1** Let $K$ and $K'$ be two parallel inertial systems with relative constant $x'$-axis velocity $v > 0$. Then the Lorentz transformation from $K$ to $K'$ is the space-time transformation from $K$ to $K'$.

In Section 4, we shall conclude that Statement 1 is false by showing that if the space-time transformation over two parallel coordinate systems $K$ and $K'$ with constant $x'$-axis velocity $v > 0$ is the Lorentz transformation, then $K$ or $K'$ is not synchronized, i.e., $K$ or $K'$ is not an inertial system.

The following properties of the Lorentz transformation are well known and can be proved easily (see, e.g., [12,14,15]).

**Fact 1** Let $K$ and $K'$ be two parallel Cartesian coordinate systems with relative constant $x'$-axis velocity $v > 0$ such that the space-time transformation from $K$ to $K'$ is the Lorentz transformation. Then the following hold, where $\beta(v) = c/\sqrt{c^2 - v^2}$.

1. Let $R$ be a point at rest on the $x'$-axis of $K'$ whose $x'$-coordinate is $x'_1$. Consider time $t'_1$ and time $t'_2$ ($t'_1 < t'_2$) at $R$ on $K'$. Let $(x_1, t_1)$ and $(x_2, t_2)$ be the pairs of $x$-coordinate and time on $K$ corresponding to $(x'_1, t'_1)$ and $(x'_2, t'_2)$, respectively. Then it holds $t_2 - t_1 = \beta(v)(t'_2 - t'_1)$.

2. Let $P$ be a point at rest on the $x$-axis of $K$ whose $x$-coordinate is $x_1$. Consider time $t_1$ and time $t_2$ ($t_1 < t_2$) at $P$ on $K$. Let $(x'_1, t'_1)$ and $(x'_2, t'_2)$ be the pairs of $x'$-coordinate and time on $K'$ corresponding to $(x_1, t_1)$ and $(x_2, t_2)$, respectively. Then it holds $t'_2 - t'_1 = \beta(v)(t_2 - t_1)$.

3. Let $R$ and $S$ be two points at rest on the $x'$-axis of $K'$ whose $x'$-coordinates are $x'_1$ and $x'_2$ ($x'_1 < x'_2$), respectively. Consider time $t'_1$ at $R$ and time $t'_2$ at $S$ on $K'$ such that the pairs of $x'$-coordinate and time corresponding to $(x'_1, t'_1)$ and $(x'_2, t'_2)$ are $(x_1, t_1)$ and $(x_2, t_1)$, respectively, on $K$ for some $x_1, x_2, t_1$. Then it holds $x'_2 - x'_1 = \beta(v)(x_2 - x_1)$. 

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(4) Let \( P \) and \( Q \) be two points at rest on the x-axis of \( K \) whose x-coordinates are \( x_1 \) and \( x_2 \) (\( x_1 < x_2 \)), respectively. Consider time \( t_1 \) at \( P \) and time \( t_2 \) at \( Q \) on \( K \) such that the pairs of \( x' \)-coordinate and time corresponding to \( (x_1, t_1) \) and \( (x_2, t_2) \) on \( K' \) are \( (x'_1, t'_1) \) and \( (x'_2, t'_2) \), respectively, for some \( x'_1, x'_2, t'_1 \). Then it holds \( x_2 - x_1 = \beta(v)(x'_2 - x'_1) \).

(5) Let \( R \) be a point at rest on the x'-axis of \( K' \). Then the velocity of \( R \) measured on \( K \) w.r.t. the x-axis of \( K \) is \( v \).

(6) Let \( P \) be a point at rest on the x-axis of \( K \). Then the velocity of \( P \) measured on \( K' \) w.r.t. the x'-axis of \( K' \) is \( -v \).

In Fact 1, (1) and (2) are called time dilations and (3) and (4) are called length contractions. Note that time dilation occurs both in (1) and (2). At this moment, we may acknowledge that this phenomenon is odd since any time lapse over any inertial system should be a net time lapse, and in (1), the speed (rate) of time lapse over \( K' \) is slower than that over \( K \), and in (2), the converse holds. We also acknowledge that Hafele-Keating experiment concerning the differences of time lapses \( \tau_B - \tau_A \) and \( \tau_C - \tau_A \) on clocks carried on jet flights and stationary on the earth, respectively, introduced in Section 8 (see [1]) seems a strong evidence to the invalidity of time dilations (1) and (2) above since (1) and (2) may imply \( \tau_B - \tau_A \) almost equals \( \tau_C - \tau_A \). We shall conclude Hafele-Keating experiment suggests that the speed of the time lapse over a material body moving with high velocity in the universe is smaller than that over a material body moving with low velocity in the universe (see also Example 5 in Section 3 about the life time of muons).

The following remark may be simple, but present a note about the equivalence of coordinate systems.

**Remark 1** Let \( K \) and \( K' \) be two parallel coordinate systems with relative constant \( x' \)-axis velocity \( v > 0 \). Assume that the space-time transformation from \( K \) to \( K' \) is the Lorentz transformation. Let \( P \) and \( Q \) be two points at rest on the x-axis of \( K \) with x-coordinates \( x_1 \) and \( x_2 \), respectively. Consider a time \( t \) over \( K \). Let \( R \) and \( S \) be two points at rest on the x'-axis of \( K' \) which coincide with \( P \) and \( Q \), respectively, at time \( t \) over \( K \). Let \( t' \) be the time shown on \( D(S) \) when \( S \) and \( Q \) coincide. Let \( t'_1 \) be the time shown on \( D(R) \) when \( R \) and \( P \) coincide. Then it holds

\[
t' = \beta(v)(t - vx_2/c^2), \quad t'_1 = \beta(v)(t - vx_1/c^2), \quad t'_1 > t'
\]

Thus over \( K \), at time \( t \), the time at each point at rest on the line segment connecting \( P \) and \( Q \) is \( t \) while the corresponding times at any two distinct points at rest on the line segment connecting \( R \) and \( S \) are distinct (one is a past time and the other is a future time). Thus in a sense, we may say that the past-future relations over \( K \) and \( K' \) are different, and \( K \) and \( K' \) are not equivalent.

In Appendix 2, we present remarks about twin paradox.

### 3 The Maxwell equations

The Maxwell equations in the vacuum space are expressed as follows.

\[
\nabla \cdot D(\vec{r}, t) = \rho(\vec{r}, t), \quad \nabla \cdot B(\vec{r}, t) = 0
\]
\[ \nabla \times H(\vec{r}, t) - \frac{\partial D(\vec{r}, t)}{\partial t} = i(\vec{r}, t), \ \nabla \times E(\vec{r}, t) + \frac{\partial B(\vec{r}, t)}{\partial t} = 0 \]

First we shall present an electromagnetic example which seems to be a strong evidence to the invalidity of Special Relativity Principle.

**Example 4** Let \( K \) and \( K' \) be two parallel inertial systems with relative constant \( x' \)-axis velocity \( v > 0 \), and \( H = \langle K, C \rangle \) and \( J = \langle K', D \rangle \) be the cc systems of \( K \) and \( K' \), respectively. Assume that \( K \) is stationary in the vacuum space (or in the space very close to the surface of the earth for certain small cases). Let an electrically charged material body \( M \) with \( q \) coulombs be at rest on the origin of \( K' \). We consider the origin \( O \) of \( K \). Since the distance between \( M \) and \( O \) changes as \( t \) changes, for each time \( t \ (t > 0) \) on \( C(O) \), it holds at \( O \)

\[ \frac{\partial E_x}{\partial t} \neq 0 \]

Then due to the Maxwell equations, it holds at \( O \)

\[ \frac{1}{c^2} \frac{\partial E_x}{\partial t} = \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \neq 0 \]

We call this case the moving charge example. Conversely we consider the stationary charge example as follows. Let \( M \) be at rest on the origin of \( K \). Then for any point \( P \) at rest on the \( x \)-axis of \( K \) whose \( x \)-coordinate is \( x \ (x \neq 0) \), it holds

\[ \frac{\partial E_x}{\partial t} = 0 \]

Due to the Maxwell equations, \( \frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} = 0 \) at any point, particularly at the origin \( O' \) of \( K' \).

We may also consider the following situations. Consider the moving charge example. In this case, for a sufficiently large integer \( m \geq 1 \) and a real \( l > 0 \), we consider \( 2m + 1 \) electrically charged material bodies \( M_{-m}, M_{-m+1}, \ldots, M_{-1}, M_0, M_1, M_2, \ldots, M_m \), each being with \( q \) coulombs, at rest points on the \( x' \)-axis of \( K' \) whose \( x' \)-coordinates are \( -ml, (-m + 1)l, \ldots, -l, 0, l, 2l, \ldots, ml \), respectively. We also consider a mariner’s compass \( X \) at rest on a point \( P' \) whose \( x' \)-coordinate and \( y' \)-coordinate are zero and whose \( z' \)-coordinate is \( b > 0 \). Then due to Biot-Savart’s law, these \( 2m + 1 \) electrically charged material bodies may produce the nonzero magnetic field over \( K \) and \( K' \), and the needle of \( X \) will be parallel to the \( y' \)-axis of \( K' \). On the other hand, in the stationary charged example, we consider correspondingly \( 2m + 1 \) electrically charged material bodies at rest on the \( x \)-axis of \( K \) and the mariner’s compass \( X \) at rest on the \( z \)-axis of \( K \). Then these \( 2m + 1 \) electrically charged material bodies do not produce any nonzero magnetic field over \( K \) and \( K' \), and the direction of the needle of \( X \) will be arbitrary.

Thus we conclude that \( K \) and \( K' \) may not be equivalent, and these observations may provide a strong evidence suggesting the invalidity of Special Relativity Principle. In Section 5, from Subprinciple 3, we shall develop arguments from which the above phenomena (and the Michelson-Morley experiment) may be explained reasonably.

Einstein[3] asserts that the following statement is true.

**Statement 2** Let \( K \) and \( K' \) be two parallel inertial systems with relative constant \( x' \)-axis velocity \( v > 0 \). Let \( (E_x, E_y, E_z), (B_x, B_y, B_z), (E'_x', E'_y', E'_z'), \) and \( (B'_x', B'_y', B'_z') \) be the
electric field and the magnetic field measured on \( K \), and those measured on \( K' \), respectively. Then the following hold.

\[
\begin{align*}
\frac{1}{c^2} \frac{\partial E_{x'}}{\partial t'} &= \frac{\partial B_{y'}}{\partial y'} - \frac{\partial B_{z'}}{\partial z'}, \\
\frac{1}{c^2} \frac{\partial E_{y'}}{\partial t'} &= \frac{\partial B_{x'}}{\partial x'} - \frac{\partial B_{z'}}{\partial z'}, \\
\frac{1}{c^2} \frac{\partial E_{z'}}{\partial t'} &= \frac{\partial B_{x'}}{\partial x'} - \frac{\partial B_{y'}}{\partial y'} \times \left( \frac{\partial E_{x'}}{\partial y'} - \frac{\partial E_{y'}}{\partial z'} \right)
\end{align*}
\]

\[
E_{x'} = E_x, \quad B_{x'} = B_x, \\
E_{y'} = \beta(v)(E_y - vB_z), \quad B_{y'} = \beta(v)(B_y + vE_z/c^2) \\
E_{z'} = \beta(v)(E_z + vB_y), \quad B_{z'} = \beta(v)(B_z - vE_y/c^2)
\]

In Section 4, we shall conclude that Statement 2 is false.

**Example 5** Let \( K \) and \( K' \) be two parallel inertial systems with relative constant \( x' \)-axis velocity \( v = 0.999c \), and \( K \) be stationary w.r.t. the universe (or w.r.t. the earth since we consider here a small space). It is well known that the mean life time of a muon \( A \) at rest on the earth is \( \tau \approx 2.20 \times 10^{-6} \) s, and a muon \( B \) moving with velocity \( v = 0.999c \) can travel in a distance of about 14.8 km in the space very close to the surface of the earth. Thus the life time \( \tau' \) of muon \( B \) is

\[
\tau' \approx 49.2 \times 10^{-6} \approx \tau \beta(v)
\]

Now let \( K, K' \) and \( v \) be as above. Assume that muon \( A \) is at rest on \( K \) and muon \( B \) is at rest on \( K' \). Then their life times are quite distinct w.r.t. the earth coordinate system, and this fact may suggest the speed of the time lapse over \( B \) is slower than that over \( A \) by factor \( 1/\beta(v) \). Here we acknowledge the time lapses of the life of muon over \( K \) and \( K' \), respectively, should be the net time lapses over \( K \) and \( K' \). Thus we conclude that these observations present a strong evidence suggesting the invalidity of Special Relativity Principle. Note that the time lapses on \( A \) and \( B \) are measured by their own proper clocks, respectively. In Subsection 5.1, we shall present arguments for deriving the above 14.8 km by depending on the twofold metric principle.

### 4 Inertial systems

In this section, we establish Time Independence Lemma (TI Lemma, i.e., Lemma 3) and Time Independence Theorem (TI Theorem, i.e., Theorem 1) and study properties of inertial systems. Throughout this section, an inertial system means a synchronized Cartesian coordinate system which moves in the vacuum space with constant velocity. Over any inertial system \( K \) and any arbitrarily large real \( u > 0 \), we consider an imaginary signal \( S(K, u) \) whose velocity is \( u \) over \( K \). This means that for any two distinct points \( P \) and \( Q \) at rest on \( K \) and any time \( t \) at \( P \), if the signal \( S(k, u) \) is emitted from \( P \) to \( Q \) at time \( t \), then \( S(K, u) \) arrives at \( Q \) at time \( t + l/u \), where \( l \) is the distance between \( P \) and \( Q \). This notion is unambiguously defined since the distance \( l \) is unambiguously defined and \( K \) is assumed
to be synchronized. For other words, the travelling of imaginary signal \( S(K, u) \) from \( P \) to \( Q \) over \( K \) is just a time lapse from a past time \( t \) at \( P \) to a future time \( t + l/u \) at \( Q \). Thus for any points \( P_0 \) and \( Q_0 \) at rest on the line segment connecting \( P \) and \( Q \), we consider the time lapse from time \( t + l_1/u \) at \( P_0 \) to time \( t + l_1/u + l_2/u \) at \( Q_0 \), where \( l_1 \) and \( l_2 \) are the distances between \( P \) and \( P_0 \) and \( P_0 \) and \( Q_0 \), respectively. One may say that the travelling of imaginary signal \( S(K, u) \) from \( P \) at time \( t \) to \( Q \) at time \( t + l/u \) at \( Q \) over \( K \) is in a sense "observable" since by a Gedankenexperiment, one can imagine this travelling occurs from a past time at \( P \) to a future time at \( Q \) over \( K \) continuously. By depending on TI Theorem, we shall show that Strong Constant Light Velocity Principle is not valid, the Lorenz transformation is not the space-time transformation over two inertial systems moving with relative constant velocity, and all inertial systems are not equivalent, i.e., Special Relativity Principle is not valid.

Now we shall proceed to establishing TI Lemma and TI Theorem. We shall recall the definition of synchronization due to Einstein[3]. Let \( K \) be a Cartesian coordinate system, and \( P \) and \( Q \) be two distinct points at rest on \( K \). Let \( H = < K, C > \) be the cc system of \( K \). Assume that a light signal \( L \) is emitted at \( P \) at time \( t_1 \) on \( C(P) \), then arrives and is reflected at \( Q \) at time \( t_2 \) on \( C(Q) \), and finally returns at \( P \) at time \( t_3 \) on \( C(P) \). Then \( C(P) \) and \( C(Q) \) are said to be synchronized due to Einstein[3] if it holds \( t_2 - t_1 = t_3 - t_2 \). Here Einstein acknowledges the validity of Strong Constant Light Velocity Principle, and in this definition, when \( L \) arrives at \( Q \) at time \( t_3 \) on \( C(Q) \), it is acknowledged that the corresponding time on \( C(P) \) is also \( t_2 \). Since we do not acknowledge the validity of Strong Constant Light Velocity Principle, in the above event, we acknowledge only that the time lapse \( t_2 - t_1 \) (the time lapse \( t_3 - t_2 \), respectively) is uniquely determined if the direction from \( P \) to \( Q \) and the distance between \( P \) and \( Q \) (the direction from \( Q \) to \( P \) and the distance between \( P \) and \( Q \), respectively) are fixed. Thus we acknowledge, as in the definition of our new synchronization relation in Section 1, that in the above event, if \( t_2 - t_1 \) and \( t_3 - t_2 \) are the net time lapses over \( K \) in which \( L \) travels from \( P \) to \( Q \) and \( L \) travels from \( Q \) to \( P \), respectively, then, when \( L \) arrives at \( Q \) (when \( L \) finally returns at \( P \), respectively), the corresponding time on \( C(P) \) is \( t_2 \) (the corresponding time on \( C(Q) \) is \( t_3 \), respectively). From all these arguments, we acknowledge that the following fact holds.

**Fact 2** Let \( K \) be an inertial system, and \( H = < K, C > \) be the cc system of \( K \). Let \( P, Q, R, S \) be four points at rest on \( K \). Assume that (i) the direction from \( P \) to \( Q \) is the same as that from \( R \) to \( S \), (ii) a light signal \( L_1 \) is emitted at \( P \) toward \( Q \) at time \( t_1 \) on \( C(P) \) and arrives at \( Q \) at time \( t_2 \) on \( C(Q) \) \((t_2 > t_1)\), and (iii) another light signal \( L_2 \) is emitted at \( R \) toward \( S \) at time \( t_3 \) on \( C(R) \) and arrives at \( S \) at time \( t_4 \) on \( C(S) \) \((t_3 < t_4)\). Then the following hold.

1. When \( L_1 \) arrives at \( Q \) at time \( t_2 \) on \( C(Q) \), the corresponding time on \( C(P) \) is also \( t_2 \), and \( t_2 - t_1 \) is the net time lapse on \( K \) within which \( L_1 \) travels from \( P \) to \( Q \).

2. It holds \( t_2 - t_1 = t_4 - t_3 \) iff the distance between \( P \) and \( Q \) is the same as that between \( R \) and \( S \).

**Remark 2** (1) In Fact 2, we do not acknowledge the validity of Strong Constant Light Velocity Principle, but (1) and (2) in the fact would be, a fortiori, true if Strong Constant Light Velocity Principle would be valid.

(2) To understand Fact 2 more precisely, we shall present the following explanation using imaginary signals. In Fact 2-(1), the sentence "the corresponding time on \( C(P) \) is also \( t_2 \)"
means the following: (i) for any large \( u > 0 \), if the signal \( S(K, u) \) is emitted from \( Q \) at time \( t_2 \) on \( C(Q) \) to \( P \), then \( S(K, u) \) arrives at \( P \) at a time \( t_5 \) on \( C(P) \) with \( t_5 > t_2 \); (ii) for any small \( \epsilon > 0 \), there exists a sufficiently large \( u(\epsilon) > 0 \) such that when the signal \( S(K, u(\epsilon)) \) is emitted from \( Q \) to \( P \) at time \( t_2 \) on \( C(Q) \), then \( S(K, u(\epsilon)) \) arrives at \( P \) at a time \( t_6 \) on \( C(P) \) with \( t_2 < t_6 < t_2 + \epsilon \). If we consider two clocks \( A \) and \( B \) at rest Paris and New York, respectively, whose mechanisms are equivalent and which are synchronized, then the above (i) and (ii) clearly must hold about \( A \) and \( B \).

We note here that the Lorentz transformation from a coordinate system \( K \) onto another coordinate system \( K' \) asserts that if an imaginary signal \( S(K, u) \) with \( u \) being sufficiently large is emitted over \( K \) to the positive x-axis direction of \( K \), then \( S(K, u) \) can reach a past history over \( K' \). TI Lemma presented in the sequel implies that this phenomenon never occurs over inertial systems.

Before presenting TI Lemma and TI Theorem, we shall study properties of the space-time transformation over two inertial systems moving with relative constant velocity (Lemmas 1,2, Proposition 3). We recall that two coordinate (or inertial) systems \( K \) and \( K' \) are said to be two parallel coordinate (or inertial) systems with relative constant x'-axis velocity \( v \geq 0 \) if their corresponding axises are parallel, the origin of \( K \) and \( K' \) coincide at the initial time \( t = t' = 0 \), and the origin of \( K' \) moves in the direction of the positive x-axis of \( K \) with constant velocity \( v \) (see Section 2). In the sequel from Lemma 1 to Proposition 3, let \( K \) and \( K' \) be two parallel inertial systems with relative constant x'-axis velocity \( v > 0 \). Let \( H = < K, C > \) and \( J = < K', D > \) be the cc systems of \( K \) and \( K' \), respectively, and we acknowledge that \( H \) and \( J \) are synchronized, respectively. Moreover we assume that the velocity of a light signal over \( K \) is \( c \) independent of its direction. Let \( T \) be the space-time transformation from \( K \) to \( K' \). For any point \( P \) at rest on \( K \) and time \( t \) on \( C(P) \), we write \( T(P, t) = R \) and \( T(x_1, t) = (x_1', t') \) if \( x_1 \) is the x-coordinate of \( P \), \( R \) is the point at rest on \( K' \) which coincides with \( P \) at time \( t \) on \( C(P) \) and at time \( t' \) on \( D(R) \), and \( x_1' \) is the x'-coordinate of \( R \). Here we note that \( (x_1, t) \) depends only on \( (x_1', t') \) since one can choose the origins of \( K \) and \( K' \) arbitrarily due to Principle 2 (see also the proofs of Lemma 1 and Proposition 3 below). In this situation, we also write \( T^{-1}(R, t') = P \) and \( T^{-1}(x_1', t') = (x_1, t) \).

**Lemma 1** Let \( R \) and \( S \) be two points at rest on \( K' \) whose x'-coordinates are \( x_1' \) and \( x_2' \), respectively. Consider times \( t_1' \) and \( t_2' \) \((t_1' < t_2')\) on \( D(R) \) and \( D(S) \). Let \( P_1, P_2, Q_1 \) and \( Q_2 \) be four points at rest on \( K \) whose x-coordinates are \( x_1, x_2, x_3 \) and \( x_4 \) and let \( t_1, t_2, t_3 \) and \( t_4 \) be times on \( C(P_1), C(P_2), C(Q_1) \) and \( C(Q_2) \), respectively, such that the following hold:

\[
T^{-1}(R, t_1') = P_1, \quad T^{-1}(R, t_2') = P_2, \quad T^{-1}(S, t_1') = Q_1, \quad T^{-1}(S, t_2') = Q_2
\]

\[
T^{-1}(x_1', t_1') = (x_1, t_1), \quad T^{-1}(x_1', t_2') = (x_2, t_2), \quad T^{-1}(x_2', t_1') = (x_3, t_3), \quad T^{-1}(x_2', t_2') = (x_4, t_4)
\]

Then it holds \( x_2 - x_1 = x_4 - x_3 \) and \( t_2 - t_1 = t_4 - t_3 \).

**Proof.** We consider two inertial systems \( K_0' \) and \( K_1' \) both of which are equivalent to \( K' \) and whose origins coincide with \( R \) and \( S \), respectively. Here we acknowledge the validity of Principle 2. Let \( O_0' \) and \( O_1' \) denote the origins of \( K_0' \) and \( K_1' \), respectively, and the time on \( D_0(O_0') \) corresponding to time \( t_1' \) on \( D(R) \) be zero, and in the same way, let the time on \( D_1(O_1') \) corresponding to time \( t_2' \) on \( D(S) \) be zero, where \(< K_0', D_0 > c < K_1', D_1 > c \) are the cc systems of \( K_0' \) and \( K_1' \), respectively. In the same way, we consider two inertial systems \( K_0 \) and \( K_1 \) both of which are equivalent to \( K \), whose cc systems will be denoted by \(< K_0, C_0 > c < K_1, C_1 > \), respectively, and whose origins coincide with \( P_1 \) and \( Q_1 \), respectively. Let \( O_0 \) and \( O_1 \) denote the origins of \( K_0 \) and \( K_1 \), respectively. We also let the time on \( C_0(O_0) \)
corresponding to time $t_1$ on $C(P_1)$ be zero, and the time on $C_1(O_1)$ corresponding to time $t_3$ on $C(Q_1)$ is zero. Since $K', K'_0$ and $K'_1$ are equivalent and are distinct only about their origins, and the same situations hold for $K, K_0$ and $K_1$, we conclude that the assertion in the lemma holds due to Principle 2. □

As in the proof of Lemma 1, we can prove the following lemma by choosing the proper origins.

**Lemma 2** Let $R, S, T$ and $U$ be four points at rest on $K'$ whose $x'$-coordinates are $x'_1, x'_2, x'_3$ and $x'_4$, respectively, and the following hold : $x'_2 - x'_1 = x'_4 - x'_3 > 0$, and the $(y', z')$-coordinate of $R$ (the $(y', z')$-coordinate of $T$, respectively) is the same as that of $S$ (is the same as that of $U$, respectively). Let $t'$ be a time on $K'$. Let $P_1, P_2, Q_1, Q_2, x_1, t_1, x_2, t_2, x_3, t_3, x_4$ and $t_4$ be such that $P_1 = T^{-1}(R, t'), P_2 = T^{-1}(S, t'), Q_1 = T^{-1}(T, t'), Q_2 = T^{-1}(U, t'), (x_1, t_1) = T^{-1}(x'_1, t'), (x_2, t_2) = T^{-1}(x'_2, t'), (x_3, t_3) = T^{-1}(x'_3, t')$ and $(x_4, t_4) = T^{-1}(x'_4, t')$. Then it holds $x_2 - x_1 = x_4 - x_3$ and $t_2 - t_1 = t_4 - t_3$.

We shall prove the following proposition by depending on Lemmas 1.2.

**Proposition 3** There exist positive constants $\alpha_1, \gamma_2, \alpha_3, \gamma_3, \gamma_4$ and constants $\gamma_1, \alpha_2, \alpha_4$ such that the following hold : for any $(x, y, z, t) \in CO(K)$ and $(x', y', z', t') = T(x, y, z, t)$, it holds $x' = \alpha_1 x + \gamma_1 t$, $t' = \alpha_2 x + \gamma_2 t$, $x = \alpha_3 x' + \gamma_3 t'$ and $t = \alpha_4 x' + \gamma_4 t'$.

**Proof.** Let $R$ be a point at rest on $K'$ whose $x'$-coordinate is $x'$. Consider time $t'$ on $D(R)$, and let $P$ and $x$ be such that $P = T^{-1}(R, t')$ and $(x, t) = T^{-1}(x', t')$. We consider three cases.

Case (1) : $x' = 0$. We shall present the proof for the case $t' > 0$. The case $t' = 0$ is trivial and the case $t' < 0$ is symmetric. Consider the set $A$ of points $Q$ at rest on $K$ such that $Q$ coincides with $R$ at some time $t_1$ on $C(Q)$ with $0 \leq t_1 \leq t$. Then from Lemma 1, one can see that there exist positive constants $\gamma_3$ and $\gamma_4$ such that $x = \gamma_3 t'$ and $t = \gamma_4 t'$.

Case (2) : $x' > 0$. We shall present the proof for the case $t \geq 0$. The case $t < 0$ can be handled in the same way. Consider the set $B$ of points $S$ at rest on $K'$ such that the $(y', z')$-coordinates of $S$ is the same as that of $R$ and the $x'$-coordinate $x'_1$ of $S$ satisfies $0 \leq x'_1 \leq x'$. Then due to Lemma 2, one can see that there exists a positive constant $\alpha_3$ to which the following holds : $x = \alpha_3 x' + \gamma_3 t'$. From Case (1), it holds $vt = \gamma_3 t'$. Thus it holds

$$x = \alpha_3 x' + \gamma_3 t'$$

(1)

Now we consider three subcases in order to show $t = \alpha_4 x' + \gamma_4 t'$.

Case (2.1) : $t' = 0$. Consider the set $X$ of points $Q_1$ at rest on $K$ such that the $(y, z)$-coordinate of $Q_1$ is the same as that of $P$, and the $x$-coordinate $x_1$ of $Q_1$ satisfies $0 \leq x_1 \leq x$. Then due to Lemma 2, one can see that there exist a nonzero constant $\delta_1$ and a constant $\alpha_4$ to which the following holds : $x = \delta_1 x'$ and (2) $t = \alpha_4 x'$. From the consistency of $T$, it must hold $\delta_1 = \alpha_3$ from (1) in Case (2).

Case (2.2) : $t' > 0$. Let $P_1$ be the point at rest on $K$ which coincides with $R$ at time zero on $D(R)$, and let $(x_1, t_1)$ be such that $(x_1, t_1) = T^{-1}(x', 0)$. Then from Case (2.1), it holds $t_1 = \alpha_4 x'$. Now consider the set $Y$ of points $Q_2$ at rest on $K$ such that $Q_2$ coincides with $R$ at time $t'_1$ on $D(R)$ with $0 \leq t'_1 \leq t'$. Then due to Lemma 1 and Case (1), it holds that $t = t_1 + \gamma_4 t'_1 = \alpha_4 x' + \gamma_4 t'$. From Case (2.2), it holds $t' = 0$. In this case, let $P_1$ and $(x_1, t_1)$ be as in Case (2.2). Note that $x_1 > 0$ since $x' > 0$. Let $Z$ be the set of points $Q_3$ at rest on $K$ which coincides with $R$ at time $t'_1$
on $D(R)$ with $t' \leq t'_1 \leq 0$. By Lemma 1 and Case (2.2), one can see that $t - t_1 = \gamma_4 t'$. By Case (2.1), it holds $t_1 = \alpha_4 x'$. Thus $t = \alpha_4 x' + \gamma_4 t'$.

Case (3) : $x' < 0$. This case can be handled as Cases (1),(2) by considering the subcases of $t' = 0$, $t' > 0$ and $t' < 0$, and using the result in Case (1).

By changing the roles of $K$ and $K'$, one can see that there exist positive constants $\alpha_1, \gamma_2$ and constants $\gamma_1, \alpha_2$ to which the following hold : $x' = \alpha_1 x + \gamma_1 t, \ t' = \alpha_2 x + \gamma_2 t$. This completes the proof of Proposition 3. □

Before presenting TI Lemma, we note the following two remarks.

**Remark 3** The Lorentz transformation has the following contradiction within itself. Let $K$ and $K'$ be two parallel Cartesian coordinate systems with relative constant $x'$-axis velocity $v > 0$. We assume that $c - v$ is very small and $\beta(v)$ is very large. Assume that a person $B$ travelling in a spaceship with velocity $v$ is at rest on the $x'$-axis of $K'$, and $B$ is spreading his hands on the $x'$-axis so that the $x'$-coordinates at the left and the right hands of $B$, $x'_1$ and $x'_2$, satisfy $x'_2 - x'_1 = 170 \text{cm}$. Assume also that a person $A$ is at rest on $K$ whose $z$-coordinate $z$ satisfies $z = -100 \text{ m}$. Person $A$ is also assumed to spread his hands so that the $x$-coordinates $x_1$ and $x_2$ of the left and the right hands of $A$ satisfy $x_2 - x_1 = 170 \text{ cm}$. Now consider time $t'$ over $K'$, and at this time, $B$ is just over $A$, and let $x_3$ and $x_4$ be the corresponding coordinates to $x'_1$ and $x'_2$ and $t_1$ and $t_2$ be the times corresponding to $t'$, respectively. We acknowledge that this phenomenon would be physically feasible as a Gedankenexperiment if Special Relativity Principle is valid since $A$ and $B$ exist in the same universe. (To visualize the situation, we may imagine that $A$ is at rest on the surface of the moon and $B$ is flying in the space very close to the surface of the moon). If the Lorentz transformation is valid, then the following hold.

$$x_4 - x_3 = \beta(v)(x'_2 - x'_1), \ t_2 - t_1 = \beta(v)v(x'_2 - x'_1)/c^2$$

These equations are clearly wrong since $\beta(v)$ can be arbitrarily large, and $x_3, x_4, t_1$ and $t_2$ are coordinates measured over $K$. If the Lorentz transformation is valid, one could not estimate the (present) age of the universe since the above $t_2 - t_1$ may become arbitrarily large. Thus we have discovered an inconsistency of the Lorentz transformation.

**Remark 4** Let $K$ and $K'$ be two parallel inertial systems with $x'$-axis velocity $v > 0$. Let $B$ be a spear at rest on the $x'$-axis of $K'$. Thus $B$ continues flying over $K$ with velocity $v > 0$. Let $R$ and $S$ denote the end points of $B$ with $x'_1 < x'_2$, where $x'_1$ and $x'_2$ are the $x'$-coordinates of $R$ and $S$. Let $u_1, u_2 > 0$ be reals. Assume that an imaginary signal $S(K, u_1)$ over $K$ is emitted from $R$ to $S$, and another imaginary signal $S(K', u_2)$ over $K'$ is emitted from $S$ to $R$ when $S(K, u_1)$ arrives at $S$. Then we acknowledge that while the travellings of $S(K, u_1)$ and $S(K', u_2)$ over $K$ and $K'$, respectively, $B$ continues flying over $K$ with velocity $v > 0$. Note that we observe this phenomenon by a Gedankenexperiment without using any measuring devices so that we do not need any energy for changing the observation from the travelling of $S(K, u_1)$ to the travelling of $S(K', u_2)$.

The following lemma will be called Time Independence Lemma (in short, TI Lemma).

**Lemma 3** Let $K$ and $K'$ be two parallel inertial systems with relative constant $x'$-axis velocity $v > 0$. Then for any event $E$ occurring over $K$ and $K'$, while $E$ is occurring, the whole $K'$ continues moving into the positive $x'$-axis direction of $K$. Particularly the following hold.
(1) For any real $u > 0$ and two distinct points $P$ and $Q$ at rest on $K$, the whole $K'$ continues moving into the positive $x$-axis direction of $K$ while imaginary signal $S(K, u)$ over $K$ travels from $P$ to $Q$.

(2) For any real $u > 0$ and two distinct points $R$ and $S$ at rest on $K'$, the whole $K'$ continues moving into the positive $x$-axis direction of $K$ while imaginary signal $S(K', u)$ over $K'$ travels from $R$ to $S$.

(3) Let $H = < K, C >$ and $J = < K', D >$ be the cc systems of $K$ and $K'$, respectively. Let $R$ and $S$ be two points at rest on the $x'$-axis of $K'$ whose $x'$-coordinates are $x'_1$ and $x'_2$ ($x'_1 < x'_2$), respectively. Let $u_1, u_2 > 0$ be sufficiently large reals. Let $t_1$ be a time over $K$, and consider the following situation.

(3.1) $R$ coincides with point $P_0$ at rest on the $x$-axis of $K$ at time $t_1$ on $C(P_0)$.

(3.2) Imaginary signal $S(K, u_1)$ is emitted over $K$ from $P_0$ at time $t_1$ on $C(P_0)$ and arrives at $Q$ at time $t_2$ on $C(Q)$, where $Q$ is the point at rest on the $x$-axis of $K$ which coincides with $S$ when $S(K, u_1)$ arrives at $S$.

(3.3) Imaginary signal $S(K', u_2)$ is emitted over $K'$ from $S$ at time $t_2$ on $C(Q)$ and arrives at $R$ at time $t_3$ on $C(P_1)$, where $P_1$ is the point at rest on the $x$-axis of $K$ which coincides with $R$ when $S(K', u_2)$ arrives at $R$.

Let $x_0$ and $x_1$ be the $x$-coordinates of $P_0$ and $P_1$, respectively. Then it holds $x_0 \leq x_1$.

(4) Let $H, J, R, S, x'_1, x'_2$, and $u_1, u_2 > 0$ be as in (3) except $x'_2 < x'_1$. Let $t_1$ be a time over $K$, and consider the following situation.

(4.1) $R$ coincides with point $P_0$ at rest on the $x$-axis of $K$ at time $t_1$ on $C(P_0)$.

(4.2) Imaginary signal $S(K, u_1)$ is emitted over $K$ from $P_0$ at time $t_1$ on $C(P_0)$ and arrives at $Q$ at time $t_2$ on $C(Q)$, where $Q$ is the point at rest on the $x$-axis of $K$ which coincides with $S$ when $S(K, u_1)$ arrives at $S$.

(4.3) Imaginary signal $S(K', u_2)$ is emitted over $K'$ from $S$ at time $t_2$ on $C(Q)$ and arrives at $R$ at time $t_3$ on $C(P_1)$, where $P_1$ is the point at rest on the $x$-axis of $K$ which coincides with $R$ when $S(K', u_2)$ arrives at $R$.

Let $x_0$ and $x_1$ be the $x$-coordinates of $P_0$ and $P_1$, respectively. Then it holds $x_0 \leq x_1$.

**Proof.** (1) and (2) are clear by definition of imaginary signals and the fact that $K$ and $K'$ are synchronized, respectively. (3). Assume that the situations in (3) hold. We shall present three proofs of (3) whose main ideas are almost the same and rely on our observation that the time passes from the past to the future over any inertial system (so that (1) and (2) hold).

The first proof of (3). Assume that $x_1 < x_0$. We note that when imaginary signal $S(K, u_1)$ is emitted from $R$, $R$ coincides with $P_0$, and $x_1 < x_0$ implies that when imaginary signal $S(K', u_2)$ arrives at $R$ and $P_1$, it holds $R$ is to the negative direction from $P_0$ on the $x$-axis of $K$. Then we note that while $S(K, u_1)$ travels form $P_0$ to $Q$, the whole $K'$ has moved into the positive $x$-axis direction of $K$ even when $u_1$ is arbitrarily large. Thus while $S(K', u_2)$ travels from $S$ to $R$, at least a part of $K'$ has moved into the negative $x$-axis direction of $K$. Particularly the time at $R$ over $K'$ passes from a future time to a past time while $S(K', u_2)$ travels from $S$ to $R$. We note that the travelling of $S(K', u_2)$ corresponds to a (positive) time
lapse over $K'$. Thus we conclude that these observations produce a contradiction to (2) since $K'$ is synchronized.

The second proof of (3). Assume that $x_1 < x_0$. As in the first proof, it holds that while $S(K', u_2)$ travels from $S$ to $R$, at least a part of $K'$ containing $R$ has moved into the negative x-axis direction of $K$. Now as a Gedankenexperiment, we consider the travelling of a person instead of imaginary signals. Assume that a person $A$ is at rest at $R$ on $K'$, and another person $B$ travels from $R$ to $S$ over $K$ with velocity $u_1$, arrives at $S$ at time $t_2$ on $C(Q)$, travels from $S$ to $R$ over $K'$ with velocity $u_2$ and arrives at $R$ at time $t_3$ on $C(P_1)$. We note that the travelling of $B$ corresponds to merely a time lapse over $K$ and $K'$, and we observe only successive time lapses over $K$ and $K'$. Assume that from time $t_3$ on $C(P_1)$, $B$ continues his travel with velocity $u_2$ over $K'$ and arrives at a point $T$ at rest on the x'-axis of $K'$ whose x'-coordinate is $x'_0$ with $x'_0 < x'_1$. Assume that $B$ arrives at $T$ at time $t_4$ on $C(P_2)$, where $P_2$ is the point at rest on the x-axis of $K$ with x-coordinate $x_2$ which coincides with $T$ when $S(K', u_2)$ arrives at $T$. Without loss of generality, we may assume that $u_2 > v$ and $x_2 < x_1$. Now assume that $B$ begins his travel from $T$ to $R$ over $K$ with velocity $u_3 > 0$ such that $B$ arrives at $R$ at a time $t_5$ on $C(P_3)$ with $t_5 > t_1$, where $P_3$ is the point at rest on the x-axis of $K$ which coincides with $R$ when $B$ arrives at $R$. Since $R \neq T$ and $x_2 < x_1$, clearly such $u_3$ exists. Thus during the time period while $A$ moves from $P_0$ to $P_3$ over $K$, $B$ travels from $R$ to $S$, $S$ to $R$, $R$ to $T$ and $T$ to $R$. Since $x_1 < x_0$, we conclude that at least a part of the travelling of $B$ occurs in a universe which is different from the universe where $A$ lives since in the A’s universe, during the travelling of $B$, $K'$ continues moving into the positive x-axis direction of $K$. But this is impossible since $S(K, u_1), S(K', u_2)$ and $S(K, u_3)$ travel over $K$ and $K'$ (travellings from past times to future times over $K, K'$ and $K$, respectively), and $K$ and $K'$ exist in the A’universe.

The third proof of (3). To visualize the situation more concretely, we consider the travelings of imaginary signals between Andromeda galaxy and the sun with certain idealization. As a Gedankenexperiment, we assume that Andromeda galaxy is at rest on the x-axis of an inertial system $K$ and the sun is at rest on the x'-axis of another inertial system $K'$ so that $K$ and $K'$ are two parallel inertial systems with relative constant x'-axis velocity $v > 0$. By simplicity, we assume that Andromeda galaxy is at rest at a point $P_0$ on the x-axis of $K$ with x-coordinate $x_0$ and the sun is at rest at a point $S$ on the x'-axis of $K'$ with x'-coordinate $x'_2$. Let $H = \langle K, C \rangle$ and $J = \langle K', D \rangle$ be the cc systems of $K$ and $K'$, respectively. Assume that a light signal $X$ is emitted from $P_0$ at time $t_0$ on $C(P_0)$ and arrives at $S$ at time $t'_0$ on $D(S)$. We assume that $t'_0$ is a time in the year of 1996 over $K'$. Let $u_1$ and $u_2$ be as above. Assume that imaginary signal $S(K, u_1)$ is emitted from $P_0$ at time $t_1$ on $C(P_0)$ and arrives at $Q$ at time $t_2$ on $C(Q)$ and at time $t'_2$ on $D(S)$, where $Q$ is as above. We consider the case where $t'_0 < t'_2$. As above, let $R$ be the point at rest on the x'-axis of $K'$ which coincides with $P_0$ at time $t_1$ on $C(P_0)$. Since $X$ is a light signal and we assume $u_1 > c$, it holds $t_0 < t_1$. Let $L$ denote the line segment connecting $R$ and $S$ over $K'$. For each point $V$ at rest on $L$, Let $C(V, u_1)$ and $D(V, u_1)$ be times shown on $C(V)$ and $D(V)$ when $S(K, u_1)$ arrives at $V$. We acknowledge that the time in the universe proceeds from the past to the future, and for each point $V$ at rest on $L$, after $S(K, u_1)$ arrives at $V$, all times $t$ at $V$ over $K$ with $t < C(V, u_1)$ and all times $t'$ at $V$ over $K'$ with $t' < D(V, u_1)$ disappeared in the universe and only such times $t > C(V, u_1)$ at $V$ over $K$ and such times $t' > D(V, u_1)$ at $V$ over $K'$ can occur at $V$. We admit that the universe continues aging, and $K$ and $K'$ have the common space. Now assume that imaginary signal $S(K', u_2)$ is emitted from $S$ at time $t'_2$ on $D(S)$ and at time $t_2$ on $C(Q)$, and arrives at $R$ at time $t'_3$ on $D(R)$ and at time $t_3$ on $C(P_1)$, where $P_1$ is as above. For each point $V$ at rest over $L$, let $C(V, u_2)$ and $D(V, u_2)$ be
times shown on $C(V)$ and $D(V)$, respectively, when $S(K', u_2)$ arrives at $V$. From the above observation, we acknowledge that for each point $V$ at rest on $L$, it holds $C(V, u_2) > C(V, u_1)$ and $D(V, u_2) > D(V, u_1)$. Particularly it holds $C(R, u_2) > C(R, u_1)$. This implies that it holds $x_1 > x_0$. Moreover we may observe the following. Let $E_1$ and $E_2$ denote the travellings of imaginary signals $S(K, u_1)$ and $S(K', u_2)$ from $P_0$ to $S$ over $K$ and from $S$ to $P_0$ over $K'$, respectively. Let $E_1 + E_2$ denote the composite event in which $E_1$ firstly occurs and $E_2$ successively occurs. By induction, for $n \geq 2$, let $n(E_1 + E_2)$ denote the composite event in which $(n-1)(E_1 + E_2)$ firstly occurs and $E_1 + E_2$ successively occurs. As above, assume it holds $x_1 < x_0$. Then one can see easily that for any sufficiently large $n > 0$, in the event $n(E_1 + E_2)$, the $n$-th travelling of $S(K', u_2)$ could arrive at $P_0$ at a time $t_6$ on $C(P_0)$ with $t_6 < t_0$. This is clearly a contradiction since we admit that Andromeda galaxy at time $t_0$ disappeared in the universe, and does not exist at any place in the universe of the present age. This completes the third proof of (3).

(4) can be proved in the same way. This completes the proof of Time Independence Lemma. \(\square\)

**Remark 5** In special relativity, the possibility of the existence of time machines has been discussed very often. One of the typical arguments presents a conclusion that over an inertial system $K$, one would go back to a past history over $K$ by depending on the existence of time machines whose velocity can be arbitrarily large (see, e.g., [15]). In the above proof of TI Lemma, if (3) does not hold, then imaginary signal $S(K', u_2)$ can go into the past histories over both $K$ and $K'$, which, we conclude, is impossible. We admit the time passes from the past to the future and the “positive” direction of time’s arrow is one of the most fundamental property of the universe [8,10] (see also Appendices 3,4).

The following theorem will be called Time Independence Theorem (in short, TI Theorem).

**Theorem 1** Let $K$ and $K'$ be two parallel inertial systems with relative constant $x'$-axis velocity $v > 0$. Let $T$ be the space-time transformation from $K$ to $K'$ as in Proposition 3. Then it holds $\alpha_2 = \alpha_4 = 0$, and Fact A in Section 1 holds.

**Proof.** Assume that it holds $\alpha_2 \neq 0$ or $\alpha_4 \neq 0$. Since $T$ is a bijection, it holds $\alpha_2\alpha_3 + \alpha_4\gamma_2 = 0$ and $\alpha_2 = 0$ implies $\alpha_4 = 0$. In the same way, $\alpha_4 = 0$ implies $\alpha_2 = 0$. Thus we may assume that $\alpha_2\alpha_4 \neq 0$. We consider the case where $\alpha_2 < 0$, and depend on (3) of TI Lemma. For the case $\alpha_2 > 0$, it suffices to apply (4) of TI Lemma. Let $H = <K, C>$ and $J = <K', D>$ be the cc systems of $K$ and $K'$, respectively. Let $R$ and $S$ be two points at rest on the $x'$-axis of $K'$ whose $x'$-coordinates are $x'_1$ and $x'_2$ with $x'_1 < x'_2$. Put $l = x'_2 - x'_1 > 0$. Let $u_1, u_2 > 0$ be sufficiently large reals. Consider time $t > 0$ over $K$, and assume that the origin $O$ of $K$ coincides with $R$ at time $t$ on $C(O)$, and imaginary signal $S(K, u_1)$ over $K$ is emitted from $O$ to $S$ at time $t$ on $C(O)$, and arrives at $S$ at time $t + a$ on $C(Q)$ with $a$ being very small, where $Q$ is the point at rest on the $x$-axis of $K$ which coincides with $S$ when $S(K, u_1)$ arrives at $S$. Let $t'_1$ and $t'_2$ be times on $D(R)$ and $D(S)$, respectively, such that $O$ and $R$ coincide at time $t'_1$ on $D(R)$, and $Q$ and $S$ coincide at time $t'_2$ on $D(S)$. Assume also that imaginary signal $S(K', u_2)$ is emitted from $S$ at time $t'_2$ on $D(S)$ and arrives at $R$ at time $t'_3$ on $D(R)$. One can see easily that the $x$-axis coordinate $x_2$ of $Q$ satisfies $x_2 = \alpha_3 t' + \gamma_3(t'_2 - t'_1)$ since $0 = \alpha_3 x'_1 + \gamma_3 t'_1$, and the following hold.

\[
\begin{align*}
\alpha_2 & = \alpha_2(\alpha_3) + \gamma_3(t'_2 - t'_1) + \gamma_2(t + a) \\
\gamma_2 & = \alpha_2(\alpha_3 + \gamma_3(t'_2 - t'_1)) + \gamma_2(t + a) \\
(\gamma_2) & = \alpha_2(\alpha_3 + \gamma_3(t'_2 - t'_1)) \\
\gamma_2 & = \alpha_2(\alpha_3 + \gamma_3(t'_2 - t'_1)) \\
(\gamma_2) & = \alpha_2(\alpha_3 + \gamma_3(t'_2 - t'_1)) \\
\gamma_2 & = \alpha_2(\alpha_3 + \gamma_3(t'_2 - t'_1)) \\
(\gamma_2) & = \alpha_2(\alpha_3 + \gamma_3(t'_2 - t'_1)) \end{align*}
\]
Here it holds $\gamma_3 > 0$ due to Proposition 3. Let $P_1$ be the point at rest on the x-axis of $K$ which coincides with $R$ when $S(K', u_2)$ arrives at $R$ at time $t'_3$ on $D(R)$. Let $x_1$ be the x-coordinate of $P_1$. Then the following hold.

$$0 = \alpha_3 x'_1 + \gamma_3 t'_1, \quad x_1 = \alpha_3 x'_1 + \gamma_3 t'_3 = \alpha_3 x'_1 + \gamma_3 (t'_1 + \alpha_2 \alpha_3 l/(1 - \alpha_2 \gamma_3))$$

Thus

$$x_1 - 0 \simeq \alpha_2 \alpha_3 \gamma_3 l/(1 - \alpha_2 \gamma_3) < 0, \quad x_1 < 0$$

This is a contradiction to (3) of TI Lemma. Thus $\alpha_2 = \alpha_4 = 0$. For the case of $\alpha_2 > 0$, we apply (4) of TI Lemma. Here we remark that

$$1 - \alpha_2 \gamma_3 > 0 \text{ since when } u_1 \text{ is sufficiently small and } \gamma_2 a > \alpha_2 \alpha_3 l, \text{ it must hold } t'_2 - t'_1 = (-\alpha_2 \alpha_3 l + \gamma_2 a)/(1 - \alpha_2 \gamma_3) > 0. \text{ Now Fact A is clear. This completes the proof of Theorem 1 (TI Theorem).}$$

From TI Theorem and Proposition 3, the following corollary is clear.

**Corollary 1**

1. Let $H, J, R, S, x'_1, x'_2, u_1, u_2, x_0$ and $x_1$ be as in (3) of TI Lemma. Then it holds $x_0 < x_1$.
2. Let $H, J, R, S, x'_1, x'_2, u_1, u_2, x_0$ and $x_1$ be as in (4) of TI Lemma. Then it holds $x_0 < x_1$.

**Definition 1**

Let $K$ and $K'$ be two parallel inertial systems with relative constant x'-axis velocity $v > 0$. Let $R$ and $S$ be two distinct points at rest on $K'$. Let $u_1, u_2$ be sufficiently large reals. Let $E_1, E_2, E_3$ and $E_4$ denote the travels of imaginary signals $S(K, u_1)$, $S(K, u_1)$, $S(K', u_2)$ and $S(K', u_2)$, respectively, from $R$ to $S$ over $K$, from $S$ to $R$ over $K'$, from $R$ to $S$ over $K'$, and from $S$ to $R$ over $K'$, respectively. For $i_1, i_2 \in \{1, 2, 3, 4\}$, let $E_{i_1} + E_{i_2}$ denote the composite event in which $E_{i_1}$ firstly occurs, and $E_{i_2}$ successively occurs, where we refer to Fact 2-(1). By induction, define $E_{i_1} + \cdots + E_{i_m} = (E_{i_1} + \cdots + E_{i_{m-1}}) + E_{i_m}$ for $i_j \in \{1, 2, 3, 4\}$.

**Corollary 2**

Let $K$ and $K'$ be two parallel inertial systems with relative constant x'-axis velocity $v > 0$. Let $u_1, u_2 > 0$ be sufficiently large, and let $E_1, E_2, E_3$ and $E_4$ be as in Definition 1. For any $i_1, i_2, \cdots, i_m, j_1, j_2, \cdots, j_m \in \{1, 2, 3, 4\}$, if $(j_1, \cdots, j_m)$ is a permutation of $(i_1, \cdots, i_m)$, then it holds $E_{i_1} + \cdots + E_{i_m} \equiv (E_{i_1} + \cdots + E_{i_{m-1}}) + E_{i_m}$, i.e., the time lapse experienced at $R$ while $E_{i_1} + \cdots + E_{i_m}$ occurs is the same as the time lapse experienced at $R$ while $E_{i_1} + \cdots + E_{i_m}$ occurs. Thus the law of superposition holds about the time lapses over $K$ and $K'$.

Let $K$ and $K'$ be two parallel coordinate system with relative constant x'-axis velocity $v > 0$ and the space-time transformation between $K$ and $K'$ be the Lorentz transformation. Then the law of superposition as in Corollary 2 does not hold (see (3) of Appendix 3). In Appendix 4, we present other odd properties of the Lorentz transformation.

The following theorem also holds.

**Theorem 2**

1. The Lorentz transformation is not the space-time transformation over two inertial systems moving with nonzero constant relative velocity.
2. Strong Constant Light Velocity Principle is not valid.
3. All inertial systems are not equivalent, i.e., Special Relativity Principle is not valid.
4. Statement 2 is false.
Proof. (1) is clear from TI Theorem. Let $K$ and $K'$ be two parallel inertial systems with relative constant $x'$-axis velocity $v > 0$. We assume the velocity of a light signal over $K$ is $c$ independent from its direction. Now it suffices to prove (2) since (2) implies the Maxwell equations over $K'$ are not of the same form as those over $K$, and (3)-(4) follow from (2). Let $H = <K, C>$ and $J = <K', D>$ be the cc systems of $K$ and $K'$, respectively. Let $P$ and $R$ denote the origins of $K$ and $K'$, respectively. Let $t > 0$ and $Q$ be the point at rest on the x-axis of $K$ whose x-coordinate is $ct$. Let a light signal $L_1$ be emitted from $P$ at time 0 on $C(P)$ and arrive at $Q$ at time $t$ on $C(Q)$. Let $S$ be the point at rest on the x'-axis of $K'$ which coincides with $Q$ when $L_1$ arrives at $Q$. Let $t'_1$ be the time shown on $D(S)$ when $S$ coincides with $Q$. By TI Theorem, it holds $t'_1 = \gamma_2 t$. Let a light signal $L_2$ be emitted at $Q$ at time $t$ on $C(Q)$, and arrive at $R$ at time $t'_2$ on $D(R)$. Let $P_0$ be the point at rest on the x-axis of $K$ which coincides with $R$ when $L_2$ arrives at $R$. Let $(x_2, t_2)$ be the pair of x-coordinate and time on $K$ at $P_0$ corresponding to $(0, t'_2)$ at $R$. Then it holds $t_2 - t < t$ since $v < c$ and $ct > x_2 > 0$. By $T_1$, it holds $t'_2 - t'_1 = \gamma_2 (t_2 - t) < \gamma_2 t = t'_1$. This implies that the velocities of $L_1$ and $L_2$ over $K'$ are distinct, completing the proof. □

We have shown that TI Lemma and TI Theorem hold. This implies that Strong Constant Light Velocity Principle is not valid as stated in Theorem 2-(2). Thus we must acknowledge that the result discovered in the Michelson-Morley experiment is not due to Strong Constant Light Velocity Principle, but due to other causes. We shall present a subprinciple of the twofold metric principle in Subsection 5.3 which may explain why the Michelson-Morley experiment is observed.

We present the following properties of the Lorentz transformations.

**Theorem 3** Let $K$ and $K'$ be two parallel coordinate systems with relative constant $x'$-axis velocity $v > 0$ such that the space-time transformation from $CO(K)$ onto $CO(K')$ is the Lorentz transformation. Let $H = <K, C>$ and $J = <K', D>$ be the cc systems of $K$ and $K'$, respectively. Let $P$ and $Q$ be two points at rest on the x-axis of $K$ whose x-coordinates are $x$ and $x + ut$, respectively, where $t > 0$ and $u \neq 0$. Let $t_1$ be a time on $C(P)$ and assume that an imaginary signal $S(K, u)$ is emitted from $P$ at time $t_1$ on $C(P)$ and arrives at $Q$ at time $t_2 = t_1 + t$ on $C(Q)$. Here if $u < 0$, then the signal $S(K, u)$ travels into the negative direction of the x-axis of $K$ so that it holds $x + ut < x$. Let $R$ and $S$ be two points at rest on the x'-axis of $K'$ such that $R$ and $S$ coincide with $P$ and $Q$, respectively, at time $t_1$ on $C(P)$ and at time $t_2$ on $C(Q)$, respectively. Let $(x'_1, t'_1)$ and $(x'_2, t'_2)$ be the pairs of x'-coordinates and times at $R$ and $S$, respectively, corresponding to $(x, t)$ and $(x + ut, t_1 + t)$ at $P$ and $Q$, respectively. Let $u'(u)$ denote $(x'_2 - x'_1)/(t'_2 - t'_1)$ when $t'_2 - t'_1 \neq 0$, and $t'(u)$ denote $t'_2 - t'_1$. Then the following hold, where $\beta(v) = c/\sqrt{c^2 - v^2}$.

1. $u'(c) = c, u'(-c) = -c, u'(v) = 0$ and $u'(u) \neq u$ for all $u \notin \{c, -c\}$.
2. $t'(u) = \beta(v)t(1 - uv/c^2)$ and $t'(u) < 0$ if $u > c^2/v$.
3. $u'(u) = c^2(u - v)/(c^2 - uv)$ if $u \neq c^2/v$.
4. $\frac{du'(u)}{du} = \frac{c^2(c^2 - v^2)}{(c^2 - vu)^2} > 0$ if $u \neq c^2/v$.
5. $\frac{dt'(u)}{du} = -\beta(v)vt/c^2 < 0$.

Proof. By the Lorentz transformation, the following hold.
\begin{align}
(6) \quad & x'_1 = \beta(v)(x - vt_1), \quad x'_2 = \beta(v)(x + ut - v(t_1 + t)) \\
(7) \quad & t'_1 = \beta(v)(t_1 - vx/c^2), \quad t'_2 = \beta(v)(t_1 + t - v(x + ut)/c^2) \\
\end{align}

By (6) and (7), the assertions can be proved easily. □

From (1) in the above theorem, one observes any velocity \( u' \) except \( c \) and \(-c \) over the \( x'\)-axis of \( K' \) is observed velocity \( u \) over the \( x \)-axis of \( K \) with \( u \neq u' \).

In the rest of this section, we shall study properties of inertial systems by depending on TI Theorem. Throughout the rest of this section, let \( K \) and \( K' \) be two parallel inertial systems with relative constant \( x'\)-axis velocity \( v > 0 \). Let \( H =< K, C > \) and \( J =< K', D > \) be the \( cc \) systems of \( K \) and \( K' \), respectively. Then \( H \) and \( J \) are synchronized, respectively, by definition. Moreover we assume that the velocity of a light signal over \( K \) is \( c \) independent of its direction. Let \( T \) be the space-time transformation from \( K \) to \( K' \). From Proposition 3, TI Lemma and TI Theorem, the following theorem and corollaries hold.

**Theorem 4**  
Let \( P \) and \( Q \) be two points at rest on \( K \), and consider time \( t_1 \) on \( C(P) \) and \( C(Q) \). Let \( R \) and \( S \) be two points at rest on \( K' \) which coincide with \( P \) and \( Q \), respectively, at time \( t_1 \) on \( C(P) \) and \( C(Q) \). Let \( t'_1 \) and \( t'_2 \) be the corresponding times at \( R \) and \( S \) on \( D(R) \) and \( D(S) \), respectively. Then it holds \( t'_1 = t'_2 \).

**Corollary 3**  
For any point \( P \) at rest on \( K \), if a point \( R \) at rest on \( K' \) coincides with \( P \) at time zero on \( C(P) \), then the corresponding time shown on \( D(R) \) is also zero.

**Corollary 4**  
For any point \( R \) at rest on \( K' \) and for any point \( P \) at rest on \( K \), the ratio of the speed of the time lapse at \( R \) on \( K' \) w.r.t. that at \( P \) on \( K \) is constant.

In Proposition 3, one can see that \( \gamma_1 = -\alpha_1/v \) since the origin \( O' \) of \( K' \) coincides with point \( P \) at rest on the \( x \)-axis of \( K' \) at time \( t \) on \( C(P) \) such that the \( x \)-coordinate of \( P \) is \( vt \). Thus from Propositions 1,3 and TI Theorem, the following theorem holds.

**Theorem 5**  
There exist positive constants \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) such that the space-time transformation \( T \) from \( K \) to \( K' \) satisfies the following.

For any coordinate \( (x, y, z, t) \) on \( K \), the corresponding coordinate \( (x', y', z', t') = T(x, y, z, t) \) on \( K' \) is such that \( x' = \alpha_1(x - vt), y' = \alpha_2y, z' = \alpha_3z \) and \( t' = \alpha_4t \).

Note that the validity of Theorem 5 presents solutions to the twin paradox and the garage paradox. Instead of Strong Constant Light Velocity Principle, the corresponding theorem is presented in Appendix 5.

### 5 The twofold metric principle

In Section 4, we show that all inertial systems are not equivalent, and the Lorentz transformation is not the space-time transformation over two inertial systems moving with relative nonzero constant velocity. We acknowledge that an inertial system \( K \) is stationary in the vacuum space if there does not exist any material body with huge mass near \( K \), and the Maxwell equations of the standard form hold over \( K \). Thus over a stationary inertial system, the light velocity is \( c \) independent of its direction. The velocity of any inertial system \( K' \) may be determined by its relative velocity w.r.t. any stationary inertial system which may coexist over \( K' \). In general relativity, one often asserts that any physical phenomenon can be described by a set of equations over an arbitrarily chosen coordinate system. However
this assertion obviously contains certain misleading feature. For example, let $K$ be a stationary inertial system over which the Maxwell equations of the standard form hold. Let $K'$ be a Cartesian coordinate system such that $t' = t, y' = y, z' = z$ but $x' = 2x$. Clearly the Maxwell equations of the standard form do not hold over $K'$. We may describe any physical phenomenon $E$ in one of the simplest forms of a set of equations only when we choose one of the most appropriate coordinate systems for $E$.

In this section, we shall present the twofold metric principle, which may replace the relativity principle and is described as a combination of four subprinciples, and solve several well known problems appearing in general relativity by depending on the twofold metric principle and a newly introduced equation named the energy-velocity equation. These problems are related with gravity or nonzero velocities w.r.t. the universe. Throughout this section, we often use the equality sign $=$ instead of the approximation sign $\approx$ when the context is clear.

Thus an equation $A = B$ often means $A$ and $B$ are approximately equal. We acknowledge that the time (the most "fundamental" time) at point at rest over a stationary inertial asystem in the universe (or at least in a part of the universe which this paper concerns) proceeds with the same speed (rate). We call this time the global time. We also admit that for any material body $B$ with rest mass $m$, the total energy $E_T$ of $B$ is the same over any coordinate system, and the time at $B$ proceeds more slowly than the global time by factor $1/\beta(v)$, where $E_T = \beta(v)mc^2$, and $\beta(u) = c/\sqrt{c^2 - u^2} = 1/\sqrt{1 - u^2/c^2}$ for $0 \leq u < c$. These observations may be explained more in detail in the sequel.

5.1 Subprinciple 1. In this subsection, we study the motion of a material body $B$ in a space where the gravitational potential can be negligible. The velocity of $B$ can be related with its total energy and its rest mass energy as can be seen in the following. Due to the experimental results, we acknowledge that the total energy of $B$ moving with velocity $v > 0$ and with rest mass $m > 0$ is $\beta(v)mc^2$. Due to this result, we acknowledge the following equation is fundamental.

The energy-velocity equation (in short, the ev-equation):

\[ \frac{d((E_0/E(v))^2)}{d(v^2)} = -\frac{1}{c^2} \quad (1) \]

Here $E_0$ is the rest energy of $B$ with mass $m > 0$ and $E(v)$ is the total energy of $B$ moving with velocity $v \geq 0$. By integration, it holds

\[ \left( \frac{E_0}{E(v)} \right)^2 = \int_0^{v^2} \left( -\frac{1}{c^2} \right) d(u^2) = a - \frac{v^2}{c^2} \quad (2) \]

Since $E(0) = E_0$, it holds $a = 1$. Thus

\[ E(v) = \beta(v)E_0 \quad (3) \]

When $v$ is small, it holds $\beta(v) \approx 1 + v^2/2c^2$ and $mv^2/2 \approx (\beta(v) - 1)E_0 \approx E_0v^2/2c^2$. Here $mv^2/2$ is the kinematic energy of $B$ at velocity $0 \leq v << c$. Hence

\[ E_0 = mc^2 \quad (4) \]

From the ev-equation, we have derived (3) and (4). We shall propose our first subprinciple which presents a new space-time transformation for the case where an outer force $\vec{F} =$
(\vec{F}_x, \vec{F}_y, \vec{F}_z) \) works over \( B \) moving with velocity \( \vec{v} = (v_x, v_y, v_z) \) with mass \( m > 0 \) and with gravitational potential being almost zero. We say that the direction of \( \vec{F} \) is a maximal velocity-critical direction, and the direction of any vector which is perpendicular to \( \vec{F} \) is a zero velocity-critical direction. We remark that if we remove \( \vec{F} \), then \( B \) continues moving in the direction of \( \vec{v} \) with the same velocity. About the travelling of light, we shall admit that any direction is a maximal velocity-critical one. The reason of this acknowledgement will be presented in Subsection 5.3.

**Subprinciple 1** Let \( K \) be a stationary inertial system, and \( B, m, \vec{F} \) and \( \vec{v} \) be as above. Let \( K' \) be a Cartesian coordinate system such that \( x' = \beta(v)x, y' = \beta(v)y, z' = \beta(v)z \) and \( t' = t/\beta(v) \), where \( v = \sqrt{v_x^2 + v_y^2 + v_z^2} \). Then it holds

\[
m \left( \frac{d^2x'}{dt'^2}, \frac{d^2y'}{dt'^2}, \frac{d^2z'}{dt'^2} \right) = (F_x, F_y, F_z) \quad (5)
\]

Thus it holds

\[
m \left( \frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}, \frac{d^2z}{dt^2} \right) = \vec{F}/\beta(v)^3 \quad (6)
\]

Moreover it holds the time at \( B \) with velocity \( \vec{v} \) proceeds more slowly than the global time by factor \( 1/\beta(v) \).

In the above subprinciple, it holds

\[
\frac{d^2x'}{dt'^2} = \frac{d}{dt'} \left( \frac{d'x}{dt} \frac{dx}{dt} \right) = \frac{d}{dt} \left( \frac{dx}{dt} \right) \frac{dt}{dt'} \beta(v)^2 = \beta(v)^3 \frac{d^2x}{dt^2} \quad (7)
\]

, etc. Equations (5) and (6) can be interpreted in such a way that in order to increase the velocity of \( B \) from \( \vec{v} \) to \( \vec{v} + \vec{\alpha} dt \), where \( \vec{\alpha} = (d^2x/dt'^2, d^2y/dt'^2, d^2z/dt'^2) \), it needs an outer force \( \beta(v)^3 \vec{F} \) working over \( B \) during a time period \( dt \), which is greater than the ordinary force (in Newtonian mechanics) \( \vec{F} \) by factor \( \beta(v)^3 \). Note that in (5) and (6), the infinitesimal length variable \((dx',dy',dz')\) into a maximal velocity-critical direction (= the direction of \( \vec{F} \)) is replaced by \( \beta(v)(dx,dy,dz) \).

By depending on Subprinciple 1, we shall derive (3) in a different way. We consider a stationary coordinate system \( K \) and all the values are measured w.r.t. \( K \). Assume that while the velocity of \( B \) increases from zero to \( \vec{v} \), an outer force \( \vec{F}(\vec{u}) \) works over \( B \) when \( B \) is at velocity \( \vec{u} \). We put \( u = |\vec{u}| \) and \( v = |\vec{v}| \), so on. Then \( E_v = (\beta(v) - 1)E_0 \) can be calculated as follows, where \( \vec{u} \) is the velocity w.r.t. \( K \) and is not affected by Subprinciple 1.

\[
E_v = \int_0^v m \vec{u} \cdot \vec{u}' = \int_0^v m \vec{u} \cdot \left( \frac{d^2x'}{dt'^2}, \frac{d^2y'}{dt'^2}, \frac{d^2z'}{dt'^2} \right) dt
\]

\[
= \int_0^v m \vec{u} \cdot \beta(u)^3\vec{u} = m \int_0^v \beta(u)^3\vec{u} \quad (8)
\]

Here we note that the length variable \( \vec{u} \) remains the same as that over a stationary inertial system while the length of \((dx',dy',dz')\) into a maximal velocity-critical direction is multiplied by factor \( \beta(u) \).

We put \( \sqrt{1 - u^2/c^2} = \cos \theta \). Then \( u/c = \sin \theta \) and \( du/d\theta = c \cos \theta \). Thus
\[ E(v) = m \int_0^v \beta(u)^3 u du = m \int_0^\theta(v) \frac{c \sin \theta \cos \theta d\theta}{\cos^3 \theta} = mc^2 \int_0^\theta(v) \frac{\sin \theta d\theta}{\cos^2 \theta} = mc^2 (\beta(v) - 1) \quad (9) \]

This result is consistent with (3) and (4) since it must hold \( E(v) = \beta(v)mc^2 = E(0) + E_v \). We note that in the above calculation, \( du \) may be sometimes negative, and even in this case, the calculation is valid. We note that in order to obtain \( (\beta(v) - 1)mc^2 \) in (9), we need \( \beta(u)^3 \) exactly. We also remark that we can obtain the corresponding space-time transformations over \( K, K', K_0 \) and \( K'_0 \), where \( K_0 \) and \( K'_0 \) are the inertial systems such that \( x_0 = x - v_z t, y_0 = y - v_y t, z_0 = z - v_z t, t_0 = t/\beta(v), x'_0 = \beta(v) x_0, y'_0 = \beta(v) y_0, z'_0 = \beta(v) z_0 \) and \( t'_0 = t/\beta(v) \). Thus the space-time transformation \( \mathcal{T} \) from \( K \) onto the inertial system \( K(u) \) where \( B \) is at rest with velocity \( \vec{u} \), the coordinate \( (t_u, x_u, y_u, z_u) \) of \( \mathcal{T} \) corresponding to coordinate \( (t, x, y, z) \) may be described as \( x_u = x - u_x t, y_u = y - u_y t, z_u = z - u_z t \) (Galileo transformation), \( t_u = t/\beta(u) \) and the equation of motion is changed from the Newton equation of motion to the form as (6).

Subprinciple 1 can be applied to the well known fact of muon’s mean life time. The mean life time of a muon \( A \) at rest on the earth is \( t_0 \approx 2.20 \times 10^{-6} \) s, and a muon \( B \) moving with velocity \( v = 0.999c \) can travel in a distance \( l \) of about 14.8 km in the space very close to the surface of the earth. By depending on Subprinciple 1, the distance \( l \) can be calculated as follows.

\[ l = \int_0^{t_0} v dt' = v \int_0^{t_0} \beta(v) dt = \beta(v) v t_0 \quad (10) \]

Now we shall show that well known arguments in relativistic mechanics for deriving (3) and (4) have defects. Let us trace the arguments briefly (for more details, see, e.g., [14]). We consider a particle \( B \) with mass \( m > 0 \) moving over a stationary inertial system \( K \) with velocity \( \vec{v} \). The well known arguments define the velocity four-vector, the four-acceleration, and the four-momentum, and conclude the following equation holds (see, e.g., [14]).

\[ \frac{d}{dt} (\beta(v)mc^2) = \vec{F} \cdot \vec{v} \quad (11) \]

Here \( \vec{F} \cdot \vec{v} \) means the work done over \( M \) by the force during a unit interval over \( K \). By integration,

\[ E(v) = \beta(v)mc^2 = \int_0^{t(v)} \vec{F}(\vec{u}) \cdot \vec{u} dt + \text{constant} \quad (12) \]

In the well known arguments, it is acknowledged that "constant" in (12) is zero. But this is clearly wrong since when \( v = 0 \), it should hold \( \beta(0)mc^2 = mc^2 = \int_0^0 \vec{F} \cdot \vec{u} dt + \text{constant} = \text{constant} \). Thus it must hold \( \text{constant} = mc^2 \). But under this condition, it must hold \( \int_0^{t(v)} \vec{F} \cdot \vec{u} dt = (\beta(v) - 1)mc^2 \) and this holds only in the case where \( \vec{F} = \vec{K} / \beta(u)^3 \), where \( u \) is the velocity of \( B \) and \( \vec{K} \) is the outer force working over \( B \). Since in the well known arguments, the term \( \vec{K} / \beta(u)^3 \) does not appear, we conclude that the well known arguments are wrong.

5.2 Subprinciple 2. In this subsection, we propose Subprinciple 2 concerning gravity. By depending on this new subprinciple, we shall derive a modified version of the
Schwarzschild metric (called the twofold Schwarzschild metric) which is almost the same, but
the points \( r = 2GM/c^2 \) is not its singular points.

We consider a gravitational field \( \mathcal{F} \) such that at any point \( P \), the (gravitational) potential
at \( P \) is denoted by \( \Phi(P) \), where \( \Phi(P) \) is the work done over a material body \( M_0 \) with unit
mass while \( M_0 \) is carried from infinity to point \( P \) due to the force of \( \mathcal{F} \). Let \( B \) be a material
body which is at point \( P \). We define the gravity-based velocity (in short, the G-velocity) of \( B \),
\( v(\Phi(P)) \), by : \((\beta(v(\Phi(P))) - 1)c^2 = \Phi(P) \). Thus \( v(\Phi(P)) \) depends only on \( \Phi(P) \), independent
from the mass of \( B \). \( v(\Phi(P)) \) will be denoted by \( v_G \) if the context is clear. The (gravitational)
potential energy \( E_{G} \) of \( B \) is \( m\Phi(P) \). We also consider the kinematic energy \( E_{K} \) of \( B \) due to \( \mathcal{F} \).
Thus we conclude that the total energy \( E_{T} \) of \( B \) is \( E_{T} = mc^2 + m\Phi(P) + E_{K} \). We define the
K-velocity \( v_{K} \) and the T-velocity \( v_{T} \) of \( B \) by \((\beta(v_{K}) - 1)mc^2 = E_{K} \) and \( \beta(v_{T})mc^2 = E_{T} \). We
say that \( B \) moves in the semi-eternal mode if it holds \( E_{G} = E_{K} \). We note planetes rounding
the sun move in the semi-eternal mode since its potential and kinematic energies are (almost)
the same so that they keep rounding the sun without falling into the sun or running away
from the sun. If we consider a stone stationary on the surface of the earth, then its potential
energy is greater than its kinematic energy. From our theory, if two stones \( A \) and \( B \) with the same
mass \( m \) are stationary in the sky at height \( h_1 \) and \( h_2 \), respectively, \( 0 < h_1 < h_2 \), then the
G-energy of \( A \) is greater than that of \( B \) since we need positive energy to raise \( A \) to height
\( h_2 \), and to cancel the work done for carrying \( B \) to height \( h_1 \) by gravity. We also note that
(i) at \( h \) being infinity, the effect caused by gravity is zero, (ii) the rounding velocity of the
earth is smaller than that of Mercury, and (iii) the gravitational force works always over \( A \)
and \( B \) so that we need certain forces to make \( A \) and \( B \) be stationary at height \( h_1 \) and \( h_2 \),
respectively. This is a distinction from classical potential theory in which the potential at \( h_1 \)
is smaller than that at height \( h_2 \). Here we admit it holds \( \Phi(P(h_1)) = \Phi(P(h_2)) + E \), where
\( \Phi(P(h_i)) \) is the potential energy of height \( h_i (i = 1, 2) \), and \( E \) is the energy needed to raise
a particle of unit mass from point \( P(h_1) \) to point \( P(h_2) \). We also note that the total energy of
a stone \( C \) at height \( h_1 \), whose mass is \( m \), which is falling freely and whose velocity at height
\( h_1 \) is \( v > 0 \), is greater than the total energy of \( A \) since their potential energies are the same
but the kinematic energy of \( C \) is greater than that of \( A \).

We say that a physical phenomenon or a property of a physical phenomenon is macro if
it is sufficiently large w.r.t. the uncertainty principle.

**Subprinciple 2** Let \( \mathcal{F} \) be a gravitational field and \( \Phi \) be as above. Let \( E \) be any macro
physical phenomenon or a macro property of any physical phenomenon which is described
by a set of equations \( H(0) \) when \( E \) occurs in the space where the gravitational potential
is (almost) zero. Let \( K \) be a coordinate system such that it is stationary over \( \mathcal{F} \), and the
direction of each spacial axis is a maximal velocity-critical one or a zero velocity-critical
one. Then when \( E \) occurs in \( \mathcal{F} \) without any other outer forces, the behavior of \( E \) can be
described by the set of equations \( H(v_{T}(P)) \) which is obtained from \( H(0) \) by replacing, in
each differential equation in \( H(0) \), any infinitesimal time variable \( dt \) with \( dt/\beta(v_{T}(P)) \), any
infinitesimal maximal velocity-critical direction length variable \( dr \) with \( \beta(v_{T}(P))dr \), and any
infinitesimal zero velocity-critical direction length variable \( dr' \) with \( dr' \), where \( v_{T}(P) \) is the
T-velocity at point \( P \) concerning \( E \).

Now we shall study the Schwarzschild metric. Let \( B \) be a material body moving with
constant velocity \( v \geq 0 \) in the space with (gravitational) potential (almost) zero. As explained
in the muon’s example, we acknowledge that the time \( \tau \) at \( B \) proceeds more slowly by factor
\( 1/\beta(v) \) than the time \( t \) at a stationary material body. Thus about \( B \), the following equation
\[ c^2 d\tau^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2) \]  

(13)

Now consider a material body \( A \) of huge mass \( M \) (e.g., the sun), and the space-time near \( A \). Let \( B \) be a material body with mass \( m \ll M \). Assume that an outer force working over \( B \) is only the gravitational force due to \( A \), and \( B \) rounds \( A \) due to the gravity of \( A \) and \( B \) (like the planets). Instead of a Cartesian coordinate system, we consider a polar coordinate system. We choose \( x^0 \) to be a time variable \( t \), and \( x^1, x^2, x^3 \) be the polar coordinates \( r, \theta \) and \( \phi \) in the standard way. The equation corresponding to (13) is

\[ c^2 d\tau^2 = c^2 dt^2 - \left( \frac{dr^2}{d\tau^2} + r^2 \frac{d\theta^2}{d\tau^2} + r^2 \sin^2 \theta \frac{d\phi^2}{d\tau^2} \right) \]  

(14)

Due to Subprinciple 2, the equation over \( F \) corresponding to (15) is

\[ 1 = \frac{dt'}{d\tau^2} - \frac{1}{c^2} \left( \frac{dr'}{d\tau^2} + r^2 \frac{d\theta'}{d\tau^2} + r^2 \sin^2 \theta \frac{d\phi'}{d\tau^2} \right) \]  

(15)

In order to apply Subprinciple 2 to (16), we need the following differential equations (17), where we consider the case \( r \) is sufficiently large (as in the case of planetary orbits). By applying the variational principle to (16), we have

\[ \frac{dt'}{d\tau} = b = \text{constant}, \quad \frac{dr'}{d\tau} \approx d = \text{constant} \]

\[ r^2 \frac{d\theta'}{d\tau} = f = \text{constant}, \quad r^2 \sin^2 \theta \frac{d\phi'}{d\tau} = h = \text{constant} \]  

(17)

where we neglect the term \( dl/dr' \) for obtaining \( dr'/d\tau \approx d = \text{constant} \). Here \( I \) is the right-hand side of (16). By Subprinciple 2, we admit that \( \tau \) is proportional to the proper time at \( B \), and \( dt' = d\tau \) since \( d\tau \) should be determined by the metric. Since the direction of \( r \) is a maximal velocity-critical one and the directions of \( \theta \) and \( \phi \) are zero velocity-critical ones, in (17), we replace \( dt' \) by \( dt/\beta(v_T) \), \( dr' \) by \( \beta(v_T) dr \), \( r d\theta' \) by \( rd\theta \) and \( r \sin^2 \theta d\phi' \) by \( r \sin^2 \theta d\phi \). (Here \( v_T \) is the T-velocity of \( B \).) In order to hold these relations, (16) becomes the following.

\[ c^2 d\tau^2 = c^2 (\frac{dt'}{d\tau})^2 - (\beta(v_T)dr^2 + r^2 d\theta^2 + r^2 \sin^2 \phi d\phi^2) \]  

(18)

The metric (18) will be called the twofold Schwartzshild metric.

We shall calculate \( \beta(v_T) \) in the case \( 0 \leq v_T \ll c \). First we calculate \( \Phi(P) \) as follows. Assume that \( B \) is carried from infinity to a point \( P \) whose distance from \( A \) is \( r \). Then due to Subprinciple 1, during the travelling of \( B \), the equation describing the motion of \( B \) at any point \( Q \) is

\[ m \frac{d^2 w'}{dt^2} = \beta(u)^3 m \frac{dw}{dt^2} = \frac{GmM}{w^2} \]

\[ m \frac{dw}{dt^2} = \frac{1}{\beta(u)^3} \frac{GmM}{w^2} = F(w) \]  

(19)
Here $w$ is the distance between $A$ and $Q$, and $\beta(u)mc^2$ is the total energy of $B$ at $Q$, i.e. it holds $(\beta(u) - 1)mc^2 = E_G + E_K = 2 \int_0^\infty F(x)dx$. We assume that $\beta(u) \approx 1$ and acknowledge that the following holds.

$$\beta(v_G) - 1)mc^2 \approx \int_\alpha^\beta \frac{GmM}{w^2}(-dw) = \frac{GmM}{r}, \beta(v_G) = 1 + GM/c^2r$$

(20)

Here $v_G = v(\Phi(P))$ is the G-velocity of $B$ and when $v_G < c$, it holds $(\beta(v_G) - 1)mc^2 \approx mv^2_c/2$ and $v^2_G \approx 2GM/r$. We also note that the direction of the gravitational force due to $A$ is opposite to that of $r$. By the note above, we admit the total energy of $B$ is $mc^2 + E_G + E_K = mc^2 + 2E_G \approx mc^2 + 2GM/M/r$. Then $(\beta(v_T) - 1)mc^2 \approx 2GM/M/r$. Thus $\beta(v_T) \approx 1 + 2GM/c^2r$. Thus we obtain the following metric which will be called the twofold approximate Schwarzschild metric. When the context is clear, it will be called simply the twofold Schwarzschild metric.

$$c^2d\tau^2 = c^2dt^2/(1 + 2GM/c^2r) - ((1 + 2GM/c^2r)r^2 + r^2d\theta^2 + r^2\sin^2 \theta d\phi^2)$$

(21)

We recall that the well known Schwarzschild metric is of the following form.

$$c^2d\tau^2 = \left(1 - \frac{2GM}{c^2r}\right)c^2dt^2 - \left(\frac{dr^2}{1 - 2GM/c^2r} + r^2d\theta^2 + r^2\sin^2 \theta d\phi^2\right)$$

(22)

In the case $v_T << c$, the twofold approximate Schwarzschild metric and the Schwarzschild metric are almost the same, and the former can be used to solve such problems as centennial procession of planary orbits and light deflect as the latter can be used. Note also that the former does not have the points $r = 2GM/c^2$ as singular points, but the latter does. We note the following Taylor expansion for $x \geq 0$.

$$1/(1 + x) = 1 - x + x^2 - x^3 + \cdots$$

(23)

If we put $x = 2GM/c^2r$, then $g_{00} = 1 - 2GM/c^2r$ in the Schwarzschild metric is an approximation of $g'_{00} = 1/(1 + 2GM/c^2r)$ in the twofold Schwarzschild metric to the first order. Tensors are related with multilinear transformations, and it may be difficult to obtain any approximations correct to a higher order by tensor calculus. Particularly it may be difficult to deduce such a solution $1/(1 + 2GM/c^2r)$ more complicated than $1 - 2GM/c^2r$ by tensor calculus.

One may note the following equation for $x \geq 0$.

$$1/(1 - x) = 1 + x - x^2 + x^3 - \cdots$$

(24)

This equation is not correct for $x = 1$. Thus the assertion $g'_{11} = 1 + 2GM/c^2r$ in the twofold Schwarzschild metric is an approximation of $g_{11} = 1/(1 - 2GM/c^2r)$ in the Schwarzschild metric may be wrong. We note that the Schwarzschild metric is deduced almost purely mathematically except the boundary conditions (1), (2) presented in Subsection 5.7. The boundary conditions seem very normal physically, and it may be incorrect that the solution (= the Schwarzschild metric) has singular points $r = 2GM/c^2$. Moreover in deriving the Schwarzschild metric, $g_{00} = 1 - 2GM/c^2r$ is firstly deduced and $g_{11}$ is deduced by the relation $g_{00}g_{11} = c^2$. It seems very difficult in this tensor-based calculation to deduce $g_{11}$ firstly. Hence we conclude the Schwarzschild metric is an approximation to the first order of the twofold Schwarzschild metric for $2GM/c^2r << 1$, and the latter metric is more right.
than the former metric. These arguments seem to show a limitation of tensor calculus. More comparisons between these metrics will be presented in Subsection 5.7.

In (18), \( cd\tau \) is the distance for light to travel within a time period \( d\tau \). The time variable \( \tau \) may be regarded to denote the time proportional to the proper time lapse over \( B \) (recall (14)). It is admitted the motion of \( B \) is under a world line called a geodesic, and it holds

\[
\tau_{ab} = \int_{\tau_a}^{\tau_b} d\tau \sqrt{t'^2/\beta(v_T)} - (\beta(v_T)r'^2 + r'^2\theta'^2 + r'^2\phi'^2)/c^2 \tag{25}
\]

where \( \beta(v_T) \approx 1 + 2GM/c^2r \), and \( t', r', \theta' \) and \( \phi' \) stand for \( dt/d\tau, dr/d\tau, d\theta/d\tau \) and \( d\phi/d\tau \), respectively. A geodesic takes an extremum path. This law may be associated with the following Fermat’s law, where \( n \) and \( ds \) correspond to \( \sqrt{t'^2/\beta(v_T)} - (\beta(v_T)r'^2 + r'^2\theta'^2 + r'^2\phi'^2)/c^2 \) and \( d\tau \), respectively, in (25).

**Fermat’s law.** When a light signal \( L \) travels from point \( P \) to point \( Q \), then the following value is an extremum value for any other path which \( L \) may take from \( P \) to \( Q \).

\[
W = \int_P^Q nds \tag{26}
\]

Here \( n \) is the refractive index.

Now we shall turn to the twofold Schwrtzshild metric in more general cases. Assume that a material body \( B \) with mass \( m \) is moving over a gravitational field \( F \) with total energy \( \beta(v_T)mc^2 \). Let \( \vec{F} = (F_x, F_y, F_z) \) be the composition of gravitational forces working over \( B \), and \( \Phi(P)m \) be the potential energy of \( B \) due to \( F \). We consider two cases.

Case (1) : \( \vec{F} \neq (0, 0, 0) \). In this case, we choose a polar coordinate \( r, \theta \) and \( \phi \) in such a way that the direction of \( r \) is that of \(-\vec{F}\), and we calculate \( r \) and a mass \( M \) by the following equations.

\[
\frac{GmM}{\beta(v_T)^3r^2} = |\vec{F}|, \quad \int_r^\infty \frac{GmMd\mu}{\beta(v_\mu)^3u^2} = \Phi(P)m \tag{27}
\]

Here \( (\beta(v_u) - 1)mc^2 = \int_u^\infty GmMd\mu/(\beta(v_\mu)^3u^2) \). Then we can regard \( F \) as the gravitational field produced by a material body \( D \) with mass \( M \) and with distance \( r \) from \( B \). Here we assume \( m << M \). Then we may have the following metric.

\[
c^2d\tau^2 = c^2dt^2/\beta(v_T) - (\beta(v_T)dr^2 + r^2d\theta^2 + r^2d\phi^2) \tag{28}
\]

Case (2) : \( \vec{F} = (0, 0, 0) \). In this case, it seems that any direction is a maximal velocity-critical one. Thus due to Subprinciple 2, any infinitesimal length variable \( dx \) should be replaced by \( \beta(v_T)dx \). Thus in this case, we consider a Cartesian coordinate system \((x, y, z)\), and may have the following metric.

\[
c^2d\tau^2 = c^2dt^2/\beta(v_T) - \beta(v_T)(dx^2 + dy^2 + dz^2) \tag{29}
\]

Now we shall study the metric for freely falling material bodies in a gravitational field. We consider the case where a material body \( A \) of huge mass \( M \) exists and a small material body \( B \) with mass \( m \) freely falls into \( A \). From our theory, the metric depends on the total energy of \( B \) (or equivalently on the T-velocity of \( B \)) rather than the acceleration of \( B \). Thus assume that at time \( t < 0 \), \( B \) is at rest over \( A \) at height \( h \) and at time \( t = 0 \), \( B \) begins
falling freely into $A$. At time $t > 0$, we admit the total energy of $B$ is $mc^2 + E_G + E_K = mc^2 + GmM/r + \int_r^\infty GmMdu/u^2 = mc^2 + GmM/r + GmM(1/r - 1/h)$. Here $r$ is the distance between $B$ and $A$ ($0 < r < h$). Now the T-velocity $v_T$ satisfies

$$(\beta (v_T) - 1)c^2 = GM(2/r - 1/h) \quad (30)$$

Thus $v_T$ depends not only on $r$ but also on $h$. Now the corresponding twofold Schwartzshild metric may be as follows.

$$c^2 d\tau^2 = \frac{c^2 dt^2}{\beta(v_T)} - \beta(v_T) dr^2 \quad (31)$$

In general relativity, the corresponding metric is given as follows by putting $\theta = \pi/2$ and $d\phi/dt = 0$ in the Schwartzshild metric.

$$c^2 (1 - 2GM/r) \left( \frac{dt}{d\tau} \right)^2 - (1 - 2GM/r)^{-1} \left( \frac{dr}{d\tau} \right)^2 = c^2 \quad (32)$$

From this metric, one concludes in general relativity that the proper time experienced by the particle at rest at $r = r_0$ and falling freely is given by (where $\tau = 0$ when $r = r_0$)

$$\tau = \frac{1}{\sqrt{2GM}} \int_r^{r_0} \left( \frac{r_0 r}{r_0 - r} \right)^{1/2} dr \quad (33)$$

which is finite (see [6]) as $r \to 2GM/c^2$. On the other hand, one also concludes in general relativity that the coordinate time $t$ for falling from $r_0$ to $r = 2n + \epsilon$ ($n = GM/c^2$, $\epsilon > 0$) is calculated by

$$t_\epsilon = \left( \frac{r_0 - 2n}{2nc^2} \right)^{1/2} \int_{2n+\epsilon}^{r_0} \frac{r^{3/2} dr}{(r - 2n)(r_0 - r)^{1/2}}$$

$$> \left( \frac{r_0 - 2n}{2nc^2} \right)^{1/2} \frac{(2n)^{3/2}}{r_0^{1/2}} - \int_{2n+\epsilon}^{r_0} \frac{dr}{r - 2n} \to \infty \text{ as } \epsilon \to 0 \quad (34)$$

since $\int_{2n+\epsilon}^{r_0} \frac{dr}{r - 2n} = l_n \frac{r_0 - 2n}{\epsilon}$ (see [6]). These calculations in general relativity seem wrong to the author since in (31), $\beta(v_T)$ is always positive and $\beta(v_T) \approx 1/(1 - 2GM/r)$ is an approximation in the case $2GM/r << 1$. It is also very odd that $\tau$ is finite and the corresponding $t$ is infinite. The above relation between $\tau$ and $t$ may be regarded to be a relation like the one between $d = \sum_{n=0}^\infty a_n$ and $e = \sum_{n=0}^\infty b_n$, where $a_i = 1$ and $b_i = 1/2^i$ for all $i$. The infinite time and a finite time are fundamentally different, and this kind of relation between $\tau$ and $t$ may be impossible.

Now we shall show that General Relativuty Principle II is wrong. We shall study the twofold Schwartzshild metric in case $\theta = \pi/2$.

$$c^2 d\tau^2 = \frac{c^2 dt^2}{\beta(v_T)} - (\beta(v_T) dr^2 + r^2 d\phi^2) \quad (35)$$

Here we recall the centennial procession of planatary orbits. This metric precisely predicts the centennial procession of Mercury’s orbit. We consider the following coordinate transformation.
\[ t' = t/\sqrt{\beta(v_T)} , \quad r' = r, \quad \phi' = \phi \quad (36) \]

This transformation is made in the range of \( r \) where the twofold Schwartzshild metric is valid. Then the corresponding metric over the coordinate system \((t', r', \phi')\) is as follows.

\[ c^2 d\tau^2 = c^2 dt'^2 - (\beta(v_T)dr^2 + r^2d\phi^2) \quad (37) \]

One can see this metric predicts the centennial procession of Mercury’s orbit wrongly by applying the variational principle even with the aid of the transformation. Thus under certain coordinate transformations, some principles such as the variational principle cannot be applied over the new transformed coordinate system. Hence we conclude General Relativity Principle II is invalid. Any coordinate transformation \( T \) from a coordinate system \( K \) to another coordinate system \( K' \) should preserve the properties observed over \( K \) so that \( K \) and \( K' \) should be equivalent at least in the following meaning.

1. Any property observed over \( K \) should be observed over \( K' \) with aid of \( T \).
2. Any property observed over \( K' \) should be observed over \( K \) with aid of \( T^{-1} \).

In the above coordinate transformation, the transformed metric is not equivalent to the twofold Schwartzshild metric since in the Schwartzshild metric, the procession of Mercury’s orbit is \( 3GM\pi(1/r_1 + 1/r_2)/c^2 \) (see [6]) which cannot be obtained from the calculated value by (37) with transformation (36). Here \( r_1 \) and \( r_2 \) are the values of \( r \) at aphelion and perihelion, and under transformation (36), \( 3GM\pi(1/r_1 + 1/r_2)/c^2 \) is not changed since \( r' = r \) and \( \phi' = \phi \).

### 5.3 The Maxwell equations over a gravitational field.

Let \( F \) be a gravitational field and \( \Phi \) denote the corresponding (gravitational) potential function as in Subsection 5.2. We shall study the Maxwell equations over \( F \). We admit any direction is a maximal velocity-critical one since we observe the following. Consider the earth and the gravity field over the earth. Let \( O \) denote the center of the earth and the origin of a polar coordinate system \((r, \theta, \phi)\) describing coordinates of points around the earth. Then over the surface of the earth, the \( r \)-direction may be clearly a maximal velocity-critical one. On the other hand, the \( \theta \)-direction and \( \phi \)-direction might be zero velocity-critical ones since into these directions, the gravitational potential does not change. However if we admit this is right, then the light velocity into these directions calculated in the sequel would become \( c \) (as in the space with gravitational potential (almost) zero) w.r.t. the corresponding coordinate system stationary in the universe. Since the earth is not stationary and the Michelson-Morley experiment is observed, we acknowledge that the \( \theta \)-direction and \( \phi \)-direction are also maximal velocity-critical ones, and so are all directions. We admit the light velocity is completely governed by the gravitational field (perhaps except very special cases). We also note if we remove gravity, then the light velocity would become \( c \) changing from \( c/\beta(v_G)^2 \) which will be obtained in the sequel. We shall show the light velocity at point with G-velocity \( v_G \) is \( c/\beta(v_G)^2 \) and if we remove \( F \), then the velocity may become \( c \). On the other hand, in the twofold Schwartzshild metric (18), if we remove the gravity of \( B \), then \( A \) will continue moving with velocity \( \vec{v} \). Thus we admit any direction is a maximal velocity critical one in the Maxwell equations. These observations presents a distinction between the motion of material bodies and the travelling of light (comparing Subprinciples 1,2). Moreover we admit a photon does not have kinematic energy since its mass is zero. Thus we propose the following subprinciple.

**Subprinciple 3.** Let \( F \) and \( \Phi \) be as above, and consider a point \( P \) whose gravitational potential is \( \Phi(P) \). Then the Maxwell equations at \( P \) are of the following form.
\[ \nabla' \cdot D(x,y,z,t) = \rho(x,y,z,t), \quad \nabla' \cdot B(x,y,z,t) = 0 \]
\[ \nabla' \times H(x,y,z,t) - \frac{\partial D(x,y,z,t)}{\partial t'} = i(x,y,t) \]
\[ \nabla' \times E(x,y,z,t) + \frac{\partial B(x,y,z,t)}{\partial t'} = 0 \quad (38) \]

Here \( \nabla' = (\partial/\partial x', \partial/\partial y', \partial/\partial z') \), \( \partial x' = \beta(v_G)\partial x \), \( \partial y' = \beta(v_G)\partial y \), \( \partial z' = \beta(v_G)\partial z \), and \( \partial t' = \partial t/\beta(v_G) \) with G-velocity \( v_G = v(\Phi(P)) \) at \( P \). The coordinate system \((t,x,y,z)\) is a standard coordinate system \( K \) which is stationary over \( F \). This means for each point \( P \) stationary over \( F \), the corresponding spatial coordinate \((x(P),y(P),z(P))\) is constant independent of time, and the time \( t \) at \( K \) proceeds with the same speed (rate) as the global time. Thus if \( F \) continues moving, then \( K \) continues moving together with \( F \) (as in the case of the gravitational field over the earth).

We consider the case where \( \rho(x,y,z,t) = i(x,y,z,t) = 0 \), and calculate the light velocity at \( P \). To do this, we consider the travelling of a plane electromagnetic wave in the case where \( E \) and \( B \) depend only on \( z \) and \( t \). From (38),
\[ \frac{\partial E_z(z,t)}{\partial z'} = \frac{\partial B_z(z,t)}{\partial z'} = \frac{\partial E_z(z,t)}{\partial t'} = \frac{\partial B_z(z,t)}{\partial t'} = 0 \quad (39) \]
For \( x, y \)-components of \( E(z,t) \) and \( B(z,t) \), it holds
\[ -\frac{\partial B_y(z,t)}{\partial z'} - \varepsilon_0\mu_0 \frac{\partial E_x(z,t)}{\partial t'} = 0 \]
\[ \frac{\partial B_x(z,t)}{\partial z'} - \varepsilon_0\mu_0 \frac{\partial E_y(z,t)}{\partial t'} = 0 \]
\[ -\frac{\partial E_y(z,t)}{\partial z'} + \frac{\partial B_x(z,t)}{\partial t'} = 0 \]
\[ \frac{\partial E_x(z,t)}{\partial z'} + \frac{\partial B_y(z,t)}{\partial t'} = 0 \quad (40) \]
From the relations \( \partial z' = \beta(v_G)\partial z \) and \( \partial t' = \partial t/\beta(v_G) \), (40) are of the following form.
\[ -\frac{\partial B_y(z,t)}{\partial z} - \beta(v_G)^2\varepsilon_0\mu_0 \frac{\partial E_x(z,t)}{\partial t} = 0 \]
\[ \frac{\partial B_x(z,t)}{\partial z} - \beta(v_G)^2\varepsilon_0\mu_0 \frac{\partial E_y(z,t)}{\partial t} = 0 \]
\[ -\frac{\partial E_y(z,t)}{\partial z} + \beta(v_G)^2 \frac{\partial B_x(z,t)}{\partial t} = 0 \]
\[ \frac{\partial E_x(z,t)}{\partial z} + \beta(v_G)^2 \frac{\partial B_y(z,t)}{\partial t} = 0 \quad (41) \]
Hence
\[ \frac{\partial^2 E_x(z,t)}{\partial z^2} - \beta(v_G)^4\varepsilon_0\mu_0 \frac{\partial^2 E_x(z,t)}{\partial t^2} = 0 \quad (42) \]
The solution of (42) is
Here $c = 1/\sqrt{G\mu_0}$ is the light velocity at any place where the gravitational potential is (almost) zero. Thus the light velocity at point $P$ with gravitational potential $\Phi(P)$ is $c/\beta(v_G)^2$, where $(\beta(v_G) - 1)c^2 = \Phi(P)$.

Now we shall solve the problem of light deflection due to gravity by depending on $c/\beta(v_G)^2$ and Huygens’s principle in a similar way as Einstein[5] does. Assume that light $L$ passes close to the sun with minimum distance $r_0$ from the sun and with orthogonal angle at time $t = 0$, and eventually is deflected as $L$ recedes from the sun. For time $t \geq 0$, let $\phi(t)$ be the angle which is constructed by the horizontal line $L_1$ on which $L$ passes at $t = 0$ and the center of the sun lies, and the line $L(t)$ on which $L$ passes at $t > 0$ and which crosses through the center of the sun. The angle $\phi(t)$ will be denoted by $\phi$ often.

Let $K$ be a plane which has the $x$-axis and the $y$-axis and whose origin is the center of the sun. We choose the $x$-axis of $K$ as $L_1$ and at time $t = 0$, $L$ is at point $(r_0, 0)$ and at time $t > 0$, $L$ is at point $(x, y)$ with $y > 0$. Consider time $t > 0$, and we shall calculate the deflection angle $\delta \theta$ made within a small change of time from $t$ to $t + \delta t$. As an approximation, we consider this problem over the line $L_2$ such that $L_2$ is orthogonal to $L_1$ and $L_2$ intersects with $L_1$ at point $(r_0, 0)$. Let $L$ be at point $P$ whose angle as above w.r.t. $L$ is $\phi$ and whose distance from the origin is $r$. Then the $(x, y)$-coordinate of $P$ is $x = r_0$, $y = r_0 \tan \phi$ and $r = \sqrt{x^2 + y^2} = x/\cos \phi$. The velocity $\gamma$ of $L$ at $P$ is $c/\beta(v_G)^2$, where from Subsection 5.2, equation (20), $1/\beta(v_G)^2 \approx 1 - 2GM/c^2r$. Thus $\gamma = (1 - 2GM/c^2r)c$. Due to Huygens’s principle, $\delta \theta$ is calculated by

$$\delta \theta = \sin \theta = \frac{\partial \gamma}{\partial x} = \frac{2GMc}{c^2r^2} \frac{12x}{2} = \frac{2GMx}{c^3r^3}$$

(44)

This deflection $\delta \theta$ is the deflection when $L$ travels within the distance $c/\beta(v_G)^2$. Thus when $L$ travels within the distance $dy$, the corresponding deflection $d\theta$ is

$$d\theta = \frac{2GMx}{cr^3} \frac{dy}{c/\beta(v_G)^2} \approx \frac{2GMxdy}{c^2r^3} = \frac{2GMr_0dy}{c^2r_0^3/c^3} \frac{1}{\cos^3 \phi}$$

$$= \frac{2GM \cos^3 \phi}{c^2r_0^3} \frac{r_0d\phi}{\cos^2 \phi} = \frac{2GM \cos \phi d\phi}{c^2r_0}$$

(45)

Then the total deflection is approximately

$$\theta = \int_0^\pi \frac{2GM \cos \phi d\phi}{c^2r_0} = \frac{2GM}{c^2r_0}, \quad 2\theta = \frac{4GM}{c^2r_0}$$

(46)

This value is the same as the well known one which is derived from the Schwartzshild metric. We also note that in solving the problem of radar sounding in general relativity, it is concluded that the light velocity is $c/\beta(v_G)^2 \approx c(1 - 2GM/c^2r)$ (see [6]).

From (43), we can also present a reason why the Michelson-Morley experiment is observed. Consider a small horizontal circle $C$ which is stationary in a space very close and parallel to the surface of the earth. Then the potential energy $\Phi(P)$ at any point $P$ at rest over $C$ is $GM_0/r$, where $r$ is the distance between the center of the earth and $C$, and $M_0$ is the mass of the earth. Due to (43), the velocity of light travelling horizontally over $C$ is $c/\beta(v_G)^2$ independent of its direction, where $(\beta(v_G) - 1)c^2 = GM_0/r$. Thus the Michelson-Morley...
experiment can be observed. The moving charge examples in Example 4 in Section 3 can be explained in the same way. From these observations, one may say that the gravitational force is more fundamental than the electromagnetic force.

Now we shall turn to the red shift problem of light. It is observed that a photon is affected by a gravitational field, which causes a drop in frequency and an increase in wavelength (red shift); traditionally stated in terms of a red shift parameter, \( z = \Delta \nu / \nu \). Consider a material body \( A \) of huge mass \( M \) and the gravitational field \( F \) near \( A \) whose gravitational potential function is denoted by \( \Phi \). By (43), the light velocity \( c(\nu_G) \) at point \( P \) with G-velocity \( \nu_G \) is \( c/\beta(\nu_G)^2 \), where \( \nu_G^2 / c^2 = 2GM/c^2r \) in case \( \nu_G << c \) and \( r \) is the distance between \( A \) and \( P \). Now consider electromagnetic waves \( L \) are continued to be emitted from hydrogen atoms at the surface \( S \) of \( A \). Due to Subprinciple 2, the time at \( S \) proceeds more slowly by factor \( 1/\beta(\nu_0) \) than the time at point with G-velocity (almost) zero, where \( \nu_0^2 / c^2 = 2GM/c^2r_0 \) and \( r_0 \) is the radius of \( A \). Here we admit the kinematic energy of hydrogen can be negligible since due to the nuclear explosion, hydrogen may be regarded to be floating over the earth. Let \( \nu_0 \) be the G-velocity on the surface of the earth. Now let \( \nu_0 \) be the corresponding frequencies of electromagnetic waves emitted from hydrogen atoms over the earth. Let \( \nu_3 \) be the G-velocity on the surface of the earth. Then as above, it holds \( \nu_3 = \nu(0)/\beta(\nu_0) \). We can see the following hold.

\[
\nu(0)\lambda(0) = c, \quad \nu_0 = \frac{\nu(0)}{\beta(\nu_0)} \quad \lambda_0\nu_0 = c/\beta(\nu_0)^2, \quad \lambda_0 = \frac{c}{\beta(\nu_0)^2}\nu(0) = \frac{\lambda(0)}{\beta(\nu_0)} \quad (47)
\]

where \( \lambda_0 \) is the wavelength of \( L \) at \( S \) and \( \lambda(0) \) is the wavelength of \( L \) corresponding to G-velocity zero.

Now consider two points \( P \) and \( Q \) such that \( L \) arrives at \( P \) and \( Q \) and the distance between \( A \) and \( P \) and \( A \) and \( Q \) are \( r_1 \) and \( r_2 \), respectively, with \( r_0 < r_1 < r_2 \). Let \( \nu_1 \) and \( \nu_2 \) be G-velocities at \( P \) and \( Q \), respectively. Then the light velocity \( c(\nu_1) \) corresponding to G-velocity \( \nu_1 \) is \( c/\beta(\nu_1)^2 \) and the corresponding \( c(\nu_2) \) is \( c/\beta(\nu_2)^2 \). The time at \( P \) proceeds more slowly by factor \( 1/\beta(\nu_1) \) than the time corresponding to G-velocity zero (the global time).

We can observe the following. Let \( t_0 \) denote a unit time lapse at G-velocity zero. Then at \( P \) and \( Q \), the frequencies \( \nu_1 \) and \( \nu_2 \) measured during a time lapse \( t_0 \) are still \( \nu_1 = \nu_2 = \nu(0)/\beta(\nu_0) \). Now let \( \nu_0 \) be the corresponding frequencies of electromagnetic waves emitted from hydrogen atoms over the earth. Let \( \nu_3 \) be the G-velocity on the surface of the earth. Then as above, it holds \( \nu_0 = \nu(0)/\beta(\nu_3) \). Thus

\[
\frac{\nu_0 - \nu_0}{\nu_0} = \frac{\beta(\nu_0) - \beta(\nu_3)}{\beta(\nu_0)} \approx \beta(\nu_0) - \beta(\nu_3) \approx \frac{GM}{c^2r_0} - \frac{GM}{c^2r_3} \approx \frac{GM}{c^2r_0} \quad (48)
\]

\[
\frac{\lambda_0 - \lambda_0}{\lambda_0} = \frac{1/\beta(\nu_3) - 1/\beta(\nu_0)}{1/\beta(\nu_3)} \approx \beta(\nu_0) - \beta(\nu_3) \approx \frac{GM}{c^2r_0} \quad (49)
\]

Here \( r_3 \) is the radius of the earth and \( \lambda_0 \) is the wavelength of electromagnetic waves emitted from hydrogen atoms over the earth. This is the red shift of light. Note the acknowledgment the time on the surface of \( A \) proceeds more slowly than the global time by factor \( \beta(\nu_G) = 1 + GM_0/c^2r_0 \) is also done by Einstein [4], and the value \( GM/c^2r_0 \) in (48) is obtained in a similar way by Einstein [4,5]. On the other hand, since the velocities of \( L \) at \( P \) and \( Q \) are \( c/\beta(\nu_1)^2 \) and \( c/\beta(\nu_2)^2 \), respectively, the wavelengths \( \lambda_1 \) and \( \lambda_2 \) at \( P \) and \( Q \) of \( L \) satisfy

\[
\lambda_1\nu_1 = c/\beta(\nu_1)^2, \quad \lambda_1 = \frac{c}{\beta(\nu_1)^2\nu_1} = \frac{c\beta(\nu_0)}{\beta(\nu_1)^2\nu(0)} = \frac{\beta(\nu_0)}{\beta(\nu_1)^2}\lambda(0)
\]
\[
\lambda_2 = \frac{\beta(v_0)}{\beta(v_2)^2} \lambda(0) \quad (50)
\]

But when one measures the frequencies of \( L \) at \( P \) and \( Q \), one uses atomic-clocks at \( P \) and \( Q \) which proceed more slowly than the time at G-velocity zero by factor \( 1/\beta(v_1) \) and \( 1/\beta(v_2) \), respectively. Thus the frequency \( \nu'_1 \) of \( L \) at \( P \) measured during the time lapse \( \beta(v_1)t_0 \) is \( \beta(v_1)\nu(0)/\beta(v_0) \), and the frequency \( \nu'_2 \) of \( L \) at \( Q \) measured during the time lapse \( \beta(v_2)t_0 \) is \( \beta(v_2)\nu(0)/\beta(v_0) \). Thus we have the following equation.

\[
\frac{\nu'_1 - \nu'_2}{\nu'_1} = \frac{\beta(v_1) - \beta(v_2)}{\beta(v_1)} \approx \beta(v_1) - \beta(v_2) \approx \frac{GM}{\epsilon^2 r_1} - \frac{GM}{\epsilon^2 r_2} \quad (51)
\]

About wavelength, due to Subprinciple 2, we may consider that at point \( P \), we choose a coordinate system \( K(v_1) \) such that \( r'_1 = \beta(v_1)r \) and \( t'_1 = t/\beta(v_1) \). Then the length of \( c/\beta(v_1) \) at \( K (= \text{a stationary coordinate system at G-velocity zero with coordinates } r \text{ and } t) \) becomes \( c \) at \( K(v_1) \). Then the corresponding wavelength \( \lambda'_1 \) of \( L \) at \( P \) becomes \( \lambda'_1 = \beta(v_1)\lambda_1 = \beta(v_0)\lambda(0)/\beta(v_1) \). In the same way, if we consider a coordinate system \( K(v_2) \) with \( r'_2 = \beta(v_2)r \) and \( t'_2 = t/\beta(v_2) \), then the corresponding wavelength \( \lambda'_2 \) becomes \( \lambda'_2 = \beta(v_2)\lambda_2 = \beta(v_0)\lambda(0)/\beta(v_2) \). Thus we have

\[
\frac{\lambda'_1 - \lambda'_2}{\lambda_1} = \frac{1/\beta(v_1) - 1/\beta(v_2)}{1/\beta(v_1)} = 1 - \frac{\beta(v_1)}{\beta(v_2)} \approx \beta(v_2) - \beta(v_1) \approx \frac{GM}{\epsilon^2 r_2} - \frac{GM}{\epsilon^2 r_1} \quad (52)
\]

\[
\lambda'_1 \nu'_1 = \lambda'_2 \nu'_2 = c \quad (53)
\]

The relations between \( \lambda'_1 \) and \( \nu'_1 \) and \( \lambda'_2 \) and \( \nu'_2 \), respectively, may be acknowledged to correspond to the constant light velocity principle under the twofold metric principle.

In the above arguments, at any point, the frequency during a unit time lapse of the global time remains \( \nu(0)/\beta(v_0) \) while the corresponding wavelength depends on \( \beta(v_1) \). If we consider the emittance of light from hydrogen atom with K-velocity \( v_0 \) and G-velocity zero, then the corresponding frequency may be \( \nu(0)/\beta(v_0) \), and the wavelength is \( \beta(v_0)\lambda(0) \) since the light velocity is \( c \). We may admit the corresponding energy of these photons is \( h\nu(0)/\beta(v_0) \) (since the gravitational potential is zero) which is proportional to the frequency. Thus about the light emitted from hydrogen with G-velocity \( v_0 \), the energy may be \( h\nu_0 = h\nu(0)/\beta(v_0) \) (not \( hc/\lambda_0 \)) independent of time. Hence we conclude the energy \( E \) of a photon with frequency \( \nu \) and wavelength \( \lambda \) such that \( h\nu = c/\beta(v)^2 \) satisfies \( E = h\nu = hc/\beta(v)^2 \lambda \). Since the mass of a photon is zero, the energy of a photon may be independent from the gravitational field.

**Remark 6** We acknowledge the following:

(*) For any star \( A \) with \( r/M \) at the surface \( S \) of \( A \), the time in \( A \) proceeds more slowly than the global time by factor \( \beta(v_G) \approx 1 + GM/c^2r \sim 1 + 2GM/c^2r \).

About the sun, \( GM_0/c^2r_0 \approx 6.37\times 10^{-6} \). Thus about any star with \( r/M \approx 10^4M_0/r_0 \), the corresponding \( \beta(v_T) \) is about 1.064. Thus the acknowledgment (*) does not seem to produce any significant effects to the well known theory about the life styles of many types of stars.

**Remark 7** If the Big Bang occurred really, then at a very early time \( t \), the corresponding \( \beta(v_T(t)) \) may have been very large. Then \( \nu(0)/\beta(v_T(t)) \) may be very small. If this is true, then the observed microwave background emission may have troubles with relation to the expansion of the universe. To solve this problem, one possible hypothesis may be the following.
Hypothesis A Let $E_T$ denote the total energy of the universe which we assume constant (or the energy-density at the beginning time of the universe, and in this case, the arguments should be modified correspondingly in the following). When some energy $E_0$ is absorbed in the vacuum space, we admit $E_0$ is still contained in $E_T$. In the rest of this remark, let $t$ and $t_0$ denote the time variables which proceed with the same rate as the global time over any stationary inertial system at present age. We assume at $t = 0$, the Big Bang occurred. So $t$ may be used to represent an age of the universe, and may be used for describing any physical phenomenon. We also assume the following (1)-(6).

1. The universe have continued expanding, and at time $t$, the energy density $\rho(t)$ is denoted by $E_T/a(t)^3$, where $a(t)$ may be admitted to represent the size (or radius) of the universe at time $t$.

2. At any early time $t$, $\rho(t)$ is almost constant at any point in the universe.

3. The coefficient $\beta(v_T(t))$ at an early time $t$ is denoted by $1 + k\rho(t)^\alpha$, where $k$ and $\alpha$ may be constants which in this paper, we do not know how to determine.

4. The global time $t_g(t)$ at an early time $t$ proceeds more slowly than $t$ by factor $1/\beta(v_T(t))$.

5. At an early time $t$, any length $l$ w.r.t. the present stationary inertial system plays a role as the length $\beta(v_T(t))l$.

6. Any physical phenomenon at an early time $t$ can be described of the same form (as the form of present age) over the coordinate system $K(t)$ whose coordinates are of the form $(t_g(t), x(t), y(t), z(t))$, where $t_g(t) = t_0/\beta(v_T(t)), x(t) = \beta(v_T(t))x, y(t) = \beta(v_T(t))y, z(t) = \beta(v_T(t))z$, and $(x, y, z)$ is a coordinate variable over any stationary inertial system of present age $t_p$, where we admit $\beta(v_T(t_p)) \approx 1$.

In (4)-(6), at an early time $t$, a unit time lapse of $t_g(t)$ plays a role as a unit time lapse of the present age $t_p$, and a unit length over $K(t)$ plays a role as a unitlength at present time $t_p$. Thus we may admit that at an early time $t$ over $K(t)$, the twofold metric principle effect and the red shift effect may be almost negligible. However we may also admit that in (6), "...can be described of the same form..." are not used for excluding the possibility of certain unifications of four forces (strong, electromagnetic, gravitational and weak forces) which have been discussed often in the literature.

Then we can see that the frequency $\nu(0)$ at an early time $t$ could be still observed to be $\nu(0)$ at present age if the space would not have expanded. We also note that at an early time $t$, $a(t)$ plays a role as $\beta(v_T(t))a(t)$ at present age. Thus in order to accord with Hubble’s law and the microwave background emission, it may hold

$$\frac{\beta(v_T(t_p))a(t_p)}{\beta(v_T(t))a(t)} = \frac{a_0(t_p)}{a_0(t)} \quad (54)$$

where $a_0(t) = \beta(v_T(t))a(t)$ ($a_0(t_p)$ also) is the size (or radius) of the universe at time $t$ so that the frequency $\nu(0)$ at time $t$ is observed as $\nu(0)a_0(t)/a_0(t_p)$ at present age $t_p$. Thus under Hypothesis A, we may be able to develop reasonable arguments, but this problem may be very difficult. More detailed analysis will be left for future studies.

Now we shall study how the acceleration due to the gravity of the earth produce the total energy of a material body $B$ with mass $m$. Let $B$ be at point $P$ with total energy...
\( \beta(v_1)mc^2 \approx mc^2 + GmM/r_1 \) and falls to point \( Q \) with total energy \( \beta(v_2)mc^2 \approx mc^2 + GmM/r_2 \) such that the mass of the earth is \( M \) and the distances between the center of the earth and \( P \) and \( Q \), respectively, are \( r_1 \) and \( r_2 \), respectively, with \( r_1 > r_2 \). Then

\[
(\beta(v_2) - 1)c^2 - (\beta(v_1) - 1)c^2 \approx \frac{GM}{r_2} - \frac{GM}{r_1} \quad (55)
\]

Then Einstein\[4\] asserts that when the total energy of \( B \) is transformed into the corresponding photons' energy, we may observe from (52) that the energy of a photon with frequency \( \nu' \) at \( P \) (or \( \nu'_2 \) at \( Q \), respectively) in (53) has energy \( h\nu'_1 = h\nu' \lambda_1 \) (\( h\nu'_2 = h\nu'/\lambda_2 \), respectively). It seems to the author this assertion is wrong since in the above, we conclude that photons' energy remains constant over any gravitational field. It seems that in the energy \( \beta(v)mc^2 \), only the energy \( mc^2 \) can be transformed into photons, and the energy \( (\beta(v) - 1)mc^2 \) may be absorbed in the vacuum space as the energy of an expanded spring is eventually absorbed in the vacuum space.

We may also remark the following. When \( B \) is at \( P \), the total energy of \( B \) and the earth is \( \beta(v_1)mc^2 + \beta(v_0)Mc^2 \), where \( \beta(v_0)Mc^2 \) is the total energy of the earth. When \( B \) is at \( Q \), the total energy becomes

\[
\beta(v_2)mc^2 + \beta(v_0)Mc^2 > \beta(v_1)mc^2 + \beta(v_0)Mc^2 \quad (56)
\]

Where does the difference energy \( \beta(v_2)mc^2 - \beta(v_1)mc^2 \) come? It seems that this energy comes from the vacuum space so that the vacuum space may be filled with a great amount of energy. When \( B \) crashes over the surface of the earth, the kinematic energy of \( B \) may be absorbed in the vacuum space as the result of disappearing of thermal energy, molecular decomposition energy, etc. Some people assert that the total energy of the universe must be zero so that the total energy consists of the equal (but sign opposite) energy of particles and antiparticles. These opinions seem to assert that our universe was produced from the "naught". However it is true that the universe contains at least the space. The above assertion may imply that there must exist "antispace", and this assertion may be nonsense. Thus it seems that the space is filled with sufficiently large amount of energy, and in order for the universe to be produced, at least there existed the space and the principles and laws governing the property of the universe.

### 5.4 An outer force and a gravitational field

In this subsection, we shall study the motion of a material body \( B \) with mass \( m \) over a gravitational field \( \mathcal{F} \) with potential function \( \Phi \) as in Subsection 5.2. We also assume an outer force \( \vec{F} = (F_x, F_y, F_z) \) works over \( B \). Let \( E_G \) denote the potential energy of \( B \), \( E_{GK} \) denote the kinematic energy due to \( \mathcal{F} \), and \( E_K \) be the kinematic energy which is obtained due to outer forces except the gravity. Thus the total energy \( E_T = mc^2 + E_G + E_{GK} + E_K \). Now let \( v_T \) denote the total-velocity of \( B \) so that \( \beta(v_T)mc^2 = E_T \). We also let \( \vec{F}_G = (F_{Gx}, F_{Gy}, F_{Gz}) \) be the composition of \( \vec{F} \) and the gravitational force. We shall present an equation describing the motion of \( B \) which is similar to the equation (5) in Subsection 5.1.

**Subprinciple 4** Let \( K \) be a stationary Cartesian coordinate system \((t, x, y, z)\) over \( \mathcal{F} \), and \( B, m, \vec{F} \) and \( v_T \) be as above. Let \( K' \) be a Cartesian coordinate system such that \( x' = \beta(v_T)x \), \( y' = \beta(v_T)y \), \( z' = \beta(v_T)z \) and \( t' = t/\beta(v_T) \). Then it holds

\[
m \left( \frac{d^2x'}{dt'^2}, \frac{d^2y'}{dt'^2}, \frac{d^2z'}{dt'^2} \right) = (F_{Gx}, F_{Gy}, F_{Gz}) \quad (57)
\]
Thus it holds
\[ m \left( \frac{d^2 x}{dt^2}, \frac{d^2 y}{dt^2}, \frac{d^2 z}{dt^2} \right) = \vec{F}_G / \beta(v_T)^3 \quad (58) \]

It seems difficult and will be left open to obtain the metric describing the motion of \( B \).
We remark the metric \( A : c^2 d\tau^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2) \) in special relativity is the metric
over an inertial system where no outer force works. Thus when an outer force works over a
material body moving in a space with (almost) zero gravitational potential, the corresponding
metric may be different from the flat metric \( A \) above. This problem will be also left for future
studies. Here we observe the metric may be regarded to specify kinematic motions, rather
than specify the curvature of the space.

5.5 Black holes. We shall discuss the possibility of the existence of black holes. We
recall that the Schwarzschild metric is of the following form:
\[ c^2 dr^2 = (1 - 2GM/c^2 r) c^2 dt^2 - (dr^2/(1 - 2GM/c^2 r) + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \quad (22) \]
The existence of black holes is usually predicted by considering in the Schwarzschild metric
the case \( 1 - 2GM/c^2 r \leq 0 \), i.e., \( r \leq 2GM/c^2 \). But from Subprinciple 2, we have derived
the twofold Schwarzschild metric and the twofold approximate Schwarzschild metric both
of which do not have the points \( r = 2GM/c^2 \) as their singular points since \( \beta(v_T) \) may
become very large, but still remain always finite. Since we acknowledge that it always holds
\( v < c \) and \( \beta(v) > 0 \), it seems that to study the case \( 1 - 2GM/c^2 r \leq 0 \) of the Schwarzschild
metric is meaningless. We may need more reasonable arguments about black holes. We also
acknowledge that the gravitational force working over a material body \( B \) with total energy
\( \beta(v)mc^2 \), \( m \) being mass, due to the gravity of a material body with mass \( M \) is \( GmM/\beta(v)^3 r^2 \),
where \( r \) is the distance between \( A \) and \( B \). This acknowledgment is due to Subprinciples
1,4. From this observation, it seems that we may need new arguments about gravitational
collapses. Here we recall an observation that the nuclear force becomes a repulsive one when
the distance becomes very small.

It seems that the sentence "the space is curved due to gravity" is wrong. In fact, light
is deflected due to the gravity of the sun. However if we consider a spaceship \( B \) having a
very powerful rocket engine \( E \) and consider the travelling of \( B \) near the sun, then \( B \) travels
along an almost "straight" line if \( E \) is sufficiently powerful. If the space is curved, then \( B \)
may travel along a curved line even when \( E \) is very powerful. Moreover we may observe the
following. If the space is curved due to gravity and a point \( P \) moves from a place \( X \) to a place
\( Y \), then another point \( Q \) may occupy the place \( X \). Thus it seems that at least a very large
part of our universe is 3-dimensional and Euclidian, the present theory about the curvature
of the universe is not sufficiently reasonable, and we do not have yet a reasonable model of
our universe. Any reasonable model may have to be at least homeomorphic to our universe
and be consistent with Hubble’s law since the law seems valid.

5.6. The twofold metric principle. Based on the results in Subsections 5.1-5.4, we
propose the following principle.

The twofold metric principle

(1) The time at any stationary point with (gravitational) potential (almost) zero proceeds
with the same rate. This time is called the global time.
(2) The time at a material body with T-velocity \( v_T \) (total velocity) proceeds more slowly than the global time by factor \( 1/\beta(v_T) \). Any infinitesimal change of motion of any macro phenomenon \( E \) with T-velocity \( v_T > 0 \) can be described by the set of equations \( H(v_T) \) which can be obtained as follows. First we choose a coordinate system \( K \) such that the direction of each spacial axis of \( K \) is a maximal velocity-critical one or a zero velocity-critical one. Let \( H(0) \) be the corresponding set of equations for describing the motion of \( E \) with T-velocity zero over \( K \). Then \( H(v_T) \) is obtained from \( H(0) \) by replacing \( \Delta t \) with \( \Delta t/\beta(v_T) \), (ii) each infinitesimal maximal velocity-critical direction length variable \( dr \) with \( \beta(v_T)dr \), and (iii) each infinitesimal zero velocity-critical direction length variable \( dx \) with \( dx \).

(3) Subprinciples 1-4 hold.

Remark 8  In Subsection 5.3, we show that the light velocity over a gravitational field is \( c/\beta(v_G)^2 \). Since we acknowledge that the velocity of any material body cannot exceed the light velocity, this may imply that in New Subprinciples, each occurrence of \( c \) should be replaced by \( c/\beta(v_G)^2 \). About the Maxwell equations in Subsection 5.3, this may be a little confusing. However we acknowledge that in the Maxwell equations, it must hold that \( \beta(v_G) \) in New Subprinciple 3 is actually a value \( \beta'(v_G) \approx c/\sqrt{c^2/\beta(v_G)^4 - v_G^2} \) and from this \( \beta'(v_G) \), the correct light velocity \( c/\beta'(v_G)^2 \) should be derived from (42). These arguments may imply that the correct light velocity over a gravitational field may be a little larger than \( c/\beta(v_G)^2 \), but when \( v_G \) is sufficiently small w.r.t. \( c \), \( c/\beta(v_G)^2 \) can be used as an approximation.

5.7 Einstein’s (field) equations  Einstein’s equations are of the following form.

\[
R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = \kappa T^{\mu\nu} \quad (59)
\]

Here the cosmological term is omitted. The Schwartzshild metric is generally admitted to be derived by Einstein’s equations. However we must remark this metric is derived by depending not only on Einstein’s equations, but also on the following two assumptions.

(1) Schwartzshild postulated the metric should be of the following form.

\[
c^2d\tau^2 = A(r)dt^2 - B(r)dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 \quad (60)
\]

so that \( x^0 = t, x^1 = r, x^2 = \theta, x^3 = \phi, g_{00} = A(r), g_{11} = -B(r), g_{22} = -r^2, g_{33} = -r^2 \sin^2\theta \) etc. and \( A(r) \to 1 \) and \( B(r) \to 1 \) as \( r \to \infty \).

(2) In a weak gravitational field (when \( r \) is sufficiently large), \( A(r) \) should be \( A(r) = \eta_{00} + h_{00} = 1 + h_{00} \) and \( h_{00} = 2V/c^2 = -2GM/c^2r \).

These two assumptions may be regarded to be two boundary conditions. In Einstein’s equations, \( T^{\mu\nu} \) is the energy stress tensor, and \( T^{\mu\nu} = 0 \) at any point where the energy and the mass are zero. We also note the equations are of the symmetric form w.r.t. all \( \mu, \nu \). Thus when one solves Einstein’s equations, one may have to be very careful, especially about boundary conditions. Otherwise one may obtain meaningless solutions.

The problem whether Einstein’s equations are valid or not is beyond the scope of this article. However under the hypothesis that they are valid, it seems that understanding what \( T^{\mu\nu} \) actually should be is important. It seems photons have no mass, and their existence may
not give any effect to any gravitational field. A gravitational field may be governed only by the material bodies with nonzero mass existing over the field. In this situation, mass and energy seem not equivalent. Here we may observe the motion of a spring \( S \) caused by a force. An outer force \( F \) can make \( S \) expanding, and the energy of the expanded \( S \) is eventually absorbed in the vacuum space. But the energy absorbed in the vacuum space cannot produce the force \( F \) by itself. Thus the direction is one-way, and in this situation, the motion of \( S \) and the corresponding kinematic energy may not be equivalent.

Now we shall present the following remarks about the notion of scalar in tensor calculus. The line element \( ds \) satisfies the following.

\[
ds^2 = g_{\mu\nu}dx^\mu dx^\nu \quad (61)
\]

Thus \( ds \) is admitted to be scalar since under a coordinate transformation \( x^i \rightarrow X^i \), \( ds \) satisfies the following.

\[
ds'^2 = g'_{\mu\nu}dX^\mu dX^\nu \quad (62)
\]

However here we must note the equation \( g'_{\mu\nu}dX^\mu dX^\nu = g_{\mu\nu}dx^\mu dx^\nu \) holds because we count the changes \( x^i \rightarrow X^i \). For example, let \( ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \) and consider a transformation \( T = 2t, X = 3x, Y = 4y \) and \( Z = 5z \). Then \( g'_{00} = 1/4, g'_{11} = 1/9, g'_{22} = 1/16 \) and \( g'_{33} = 1/25 \). Thus \( ds^2 = c^2dT^2/4 - dX^2/9 - dY^2/16 - dZ^2/25 \). Thus the metric is changed, and we must be always remembering the relations \( T = 2t \) etc. for developing theory depending on the coordinate \((T, X, Y, Z)\).

In Subsection 5.1, we admit that over two parallel inertial systems \( K \) and \( K' \) with relative constant velocity \( \vec{v} = (v_x, v_y, v_z) \), the transformation \( T \) satisfies the following, where \( K \) is stationary with coordinate \( x^0 = ct, x^1 = x, x^2 = y, x^3 = z \) and \( K' \) has coordinate \( X^0 = ct', X^1 = x', X^2 = y', X^3 = z' \).

\[
X^0 = ct' = ct/\beta(v) = x^0/\beta(v), \quad X^1 = x - v_xt = x_1 - v_xx^0/c
\]

\[
x^1 = X^1 + v_xt = X^1 + \beta(v)v_xtX^0/c, \quad etc. \quad (63)
\]

If we admit the metric over \( K \) is the flat metric \((1, -1, -1, -1)\), then the metric over \( K' \) is of the following form.

\[
g'_{00} = \frac{dx^0}{dX^0} \frac{dx^0}{dX^0} \eta_{00} + \frac{dx^1}{dX^0} \frac{dx^1}{dX^0} \eta_{11} + \frac{dx^2}{dX^0} \frac{dx^2}{dX^0} \eta_{22}
\]

\[
+ \frac{dx^3}{dX^0} \frac{dx^3}{dX^0} \eta_{33} = \beta(v)^2 - \frac{\beta(v)^2v^2}{c^2} = 1
\]

\[
g'_{ii} = \frac{dx^i}{dX^i} \frac{dx^i}{dX^i} \eta_{ii} = -1 \quad (1 \leq i \leq 3) \quad (64)
\]

Thus under the transformation \( T \), the flat metric is preserved.

Thus we conclude that in tensor calculus, we need a standard coordinate system and the corresponding standard metric (if possible) when we begin developing any theory. In this context, we may conclude that in the Schwartzshild metric, the space is 3-dimensional Euclidian, and the procession of planetary orbits is predicted w.r.t. this coordinate system. Thus the metric may be regarded to specify the motion of planets rather than the curvature of the space. It is not the space but the motion of planets which is curved.
Now we turn to the problem of applying Einstein’s equations to cosmology. Generally in cosmology, Einstein’s equations are applied to the universe by assuming the universe consists of macroscopic fluid. It seems that this type of modelling has the following problems.

(3) In a macroscopic fluid (water or gas), two neighboring particles are connected by a molecular force, or collide very often in a very short time. In the universe, two neighboring stars may collide very scarcely, and the distance between them may remain very large and the same in a very long time.

(4) The present radius of the universe may be greater than $10^{10}$ light years, and if the speed of propagation of a gravitational force is that of light, then it may need more than $10^{10}$ years for the propagation of gravitational forces existing very far each other. Consider a galaxy $X$ and a star $B$ in $X$ whose distance from the center of $X$ is $l$, $l$ being very large. If the gravity between $X$ and $B$ is due to exchanging gravitons, then how can each graviton $\alpha$ know the future position of $B$ which may be very far from the point at which $B$ exists when $\alpha$ is emitted from $X$?

(5) From the observational facts, it seems that Hubble’s law plays more significant roles than the gravity in the expansion of the universe. Hubble’s law seems to imply the space itself is expanding. The expansion of the space itself seems to be suggested by observed red shift of galaxies and the microwave background emission which are generally admitted due to the expansion of the universe. If the receding velocity $v$ of a galaxy from the earth exceeds $c$ and the space remains stationary, then $\beta(v)$ is infinity and this is impossible from our acknowledgment about the motions of material bodies. Thus the space itself seems to continue expanding.

From these remarks, it seems that the present universe is too vast to be applied globally by Einstein’s equations. In developing theory, probably we may have to count the effect of propagation speed of gravity if the speed is finite, and also count the effect of Hubble’s law. Or it may be more reasonable to admit that the gravitational force is not due to exchanging gravitons, but the property of the space near $X$ is changed so that the propagation speed of gravity may be very large ($>c$).

In Subsection 5.2, we present Schwartzshild’s method for deriving the metric. In this method, the coordinate are $(t, r, \theta, \phi)$ the dimensions of whose components are not the same, and correspondingly all $\Gamma_{\mu\nu}$ (all $g_{\mu\nu}$) do not have the same dimensions. Components of $\kappa T^{\mu\nu}$ have the dimension $1/(\text{length})^2$ while in the above deduction, for example, $R_{22}$ is dimensionless.

Another method deriving the Schwartzshild metric depends on assuming $ds^2 = A(dx^0)^2 - B \sum_{i=1}^{3}(dx^i)^2$, where $A$ and $B$ should be determined, and are assumed to depend only on $r = [(x^1)^2 + (x^2)^2 + (x^3)^2]^{1/2}$ (see [16]). In this method, the solution is asserted to be the following.

\[
\begin{align*}
    ds^2 &= \frac{(1 - a/r)^2}{(1 + a/r)^2}(dx^0)^2 - \left(\frac{a}{1 + r}\right)^4 \sum_{i=1}^{3}(dx^i)^2 \\
    &= \frac{(1 - a/r)^2}{(1 + a/r)^2}(dx^0)^2 - \left(\frac{a}{1 + r}\right)^4 ((dr)^2 + r^2((d\theta)^2 + \sin^2\theta(d\phi)^2))
\end{align*}
\]  

(65)

where $a = GM/2c^2$. Then the method makes a coordinate transformation $x_0^1 = x_0, \theta' = \theta, \phi' = \phi$ and $r' = d(1 + a/r)^2$, and asserts that the Schwartzshild metric is derived. However in (65), the metric does not have singular points except $r = 0$, and as we note often above, coordinate transformations must preserve the properties of the original coordinate system. Thus this method seems also incomplete.
6 Hubble’s law

Hubble’s law seems to suggest that the space itself continues expanding. (If the space is "stationary" (not expanding) and only the galaxies continue receding each other, then the receding velocities remain the same or decrease due to gravity since no outer forces do not seem to exist). The observed red shifts due to the receding galaxies also seem to support this acknowledgment as shown in the following remark. Let A and B be two galaxies such that the distance \( l(t) \) between A and B is very large for \( t \geq t_0 \), where \( t \) denotes the global time. We assume the velocity \( v_A \) of A (the velocity \( v_B \) of B) w.r.t. a stationary inertial system \( K \) (a stationary inertial system \( K' \)) over which A exists (B exists) does not exceed \( c \) (since \( \beta(v_A), \beta(v_B) < \infty \)). If the receding velocity \( v(t_0) \) of A at time \( t_0 \) from B exceeds \( 2c \), then clearly the light emitted from A at time \( t_0 \) never reaches B. Thus assume that \( v(t_0) < 2c \) and the light \( L(t_0) \) emitted from A at time \( t_0 \) can eventually reaches B. Now assume \( L(t_0) \) arrives at B at time \( t_0 + \alpha, \alpha > 0 \). Since the space is expanding, it holds \( \alpha > l(t_0)/c \). Consider the next light signal \( L(t_0 + \lambda/c) \) is emitted at time \( t_0 + \lambda/c \), where \( \lambda \) is the wavelength of the light. Now let \( P \) denote the "stationary" position at which B existed when \( L(t_0) \) arrives at B, so that the distance \( A \) and \( P \) is always \( \alpha \alpha \). Then \( L(t_0 + \lambda/c) \) arrives at \( P \) at time \( t_0 + \lambda/c + \alpha \). At this global time, due to the expansion of the space, the distance \( l_0 \) between \( B \) and \( P \) can be sufficiently large if \( l(t_0) \) is sufficiently large. Then \( L(t_0 + \lambda/c) \) arrives at \( B \) at time \( t_0 + \lambda/c + \alpha + \beta \), where \( \beta > l_0/c \). Here we assume the receding velocity of \( B \) from \( A \) is less than \( c \) so that \( L(t_0 + \lambda/c) \) can reach \( B \). Since Hubble’s constant is very small, the above \( l_0 \) can be very large, and we may observe a red shift \( \Delta \lambda/\lambda > l_0/\lambda - 1 \) which may be very large.

In the above arguments, we note that in the standard Doppler effect, one considers the case where the space is "stationary", but in Hubble’s law, while the light is travelling, the space itself continues expanding and the receding velocity is also increasing. Thus in Hubble’s law, the receding velocity and the distance should be accumulated in the calculation. The observed background microwave radiation also seems to imply that for any global time \( t \geq t_1 > 0 \), where \( t_1 \) may be very small, the expansion rates of the space were the same at all points in the universe. We also remark that in the above expansion of the space, the space-time transformation from A to B cannot be the Lorentz transformation since if the light \( L \) emitted from A visits \( B \) and returns to \( A \), then the wavelength of \( L \) becomes very large as observed above. In the rest of this subsection, we shall present a remark about a relation possibly lying between the (present) age of the universe and Hubble’s constant under the hypothesis that Hubble’s law is valid. (From observational facts and the above remarks, we may perhaps need a modification of the law, but the modification may be not very serious). In fact, we shall present arguments for supporting the following assertion.

Assertion A: the (present) age of the universe is greater than the Hubble time \( H_0^{-1} \).

In cosmology, the age of the universe is generally convinced to be between \( 2 \times 10^{10} \) and \( 10^{10} \) years, or approximately \( 1.5 \times 10^{10} \) years. Recently large inhomogeneous structures called great walls were discovered whose distances from the earth are estimated to be about \( 10^9 \) light years (see \[1\]), and it has become a problem in dispute to study whether it was possible for such a large inhomogeneity to have been constructed in the estimated age of the universe \( \approx 1.5 \times 10^{10} \) years. It is generally acknowledged that the age of the universe is smaller than the Hubble time \( H_0^{-1} \). Here \( H_0 \) is the Hubble constant to which the following cosmological observation is acknowledged.

Observation A. For any galaxy \( N \) which is not too near nor too far, the receding velocity \( v \) of \( N \) w.r.t. the earth is related to the distance \( d \) between \( N \) and the earth by the following
equation:

\[ v = H_0d \quad (1) \]

From many cosmological observations, the Hubble constant \( H_0 \) is generally acknowledged to be between 50:\( k m s^{-1} M pc^{-1} \) and 100:\( k m s^{-1} M pc^{-1} \). The (present) age \( t_p \) of the universe is assumed to be smaller than \( H_0^{-1} \) by the following argument (see, e.g., [2]).

First one assumes that the Big Bang occurred, and the universe has continued expanding after the Big Bang. Let \( a(t) \) denote the so-called scale factor at time \( t \) so that \( a(t) \) may denote the size or the radius of the universe at time \( t \). Then the history of the expansion of the universe may be described by depicting the curve of \( a(t) \). Depict the figure of \( a(t) \) over the plane where the horizontal axis is the time axis, \( t \)-axis, and the vertical axis is the \( a(t) \)-axis. The origin \( O \) of axes corresponds to the beginning time of the universe, i.e., the time when the Big Bang occurred. Let \( P \) be the point on the \( t \)-axis such that \( OP \) denotes the present time \( t_p \), i.e., the age of the universe. Let \( Q \) denote the point on the curve \( a(t) \) corresponding to \( t_p \) so that \( PQ \) denotes \( a(t_p) \). One assumes that the curve of \( a(t) \) is concaved to the downward, i.e., \( \dot{a}(t) > 0 \) but \( \ddot{a}(t) < 0 \), since \( \dot{a}(t) \) may decrease due to the gravitational forces as \( t \) increases. Let \( \alpha \) be the angle such that the tangent at \( Q \) meets the \( t \)-axis at point \( T \) at the angle \( \alpha \). Then it holds

\[ \tan \alpha = PQ/PT = \dot{a}(t_p) \quad (2) \]

so that

\[ OP < PT = PQ/\dot{a}(t_p) = a(t_p)/\dot{a}(t_p) = H_0^{-1} \quad (3) \]

Note that the relation \( OP < H_0^{-1} \) is possible since one assumes the curve of \( a(t) \) is concaved to the downward. Thus it may hold \( OP > H_0^{-1} \) in case where (i) the curve of \( a(t) \) is concaved to the upward, i.e., it holds \( \dot{a}(t) > 0 \) and \( \ddot{a}(t) > 0 \), and (ii) \( a(t_p)/a(1) \) is sufficiently large, where 1 in \( a(1) \) is a unit global time such that after \( t \geq 1 \), the expansion of the space has been governed by Hubble’s law (here \( H_0 \) may depend on \( t \)). (We shall present a more detailed condition for \( OP > H_0^{-1} \) later). Thus we may conclude the (present) age \( t_p \) of the universe depends not only on \( H_0 \) but also on \( a(t_p) \) (or equivalently \( a(t_p)/a(1) \) or \( a(t_p)/a(0) \)). Indeed, we shall develop arguments for supporting the assertion that it holds \( \dot{a}(t) > 0 \) and \( \ddot{a}(t) > 0 \) by depending on Hubble’s law.

First we consider an explosion of a material body \( M \) of huge mass in the vacuum space at time \( t_2 \). Each broken piece of \( M \) begins moving with initial velocity \( v \). Then after a time lapse \( t \) from \( t_2 \), the maximum distance \( d(t) \) between any two broken pieces may be about \( 2 \times \max \{ vt \mid v \text{ is an initial velocity of a broken piece of } M \} \) when \( t \) is small. Then it may hold \( \dot{d}(t) < 0 \) since \( d(t) \) may decrease as \( t \) increases due to the gravitational forces working over all broken pieces of \( M \). In the Big Bang theory, it seems that the scenario is quite different from the above material explosion case since the expansion of the universe seems to occur homogeneously and isotropically in the space, that is, the space itself continues expanding. Thus the distance between any two distinct spatial points in the universe may increase as the time proceeds.

Now we shall first present an elementary argument for supporting \( \dot{a}(t) > 0 \). Consider two distinct points \( P \) and \( Q \) in the space. We assume that at any point \( R \) in the space, the speed (rate) of time lapse at \( R \) at global time \( t \) proceeds with the same speed since we assume the universe is homogeneous and isotropic. Then let \( t_1 \geq 1 \) be any global time from the Big
Bang, and let $l_1$ be the distance between two distinct points $P$ and $Q$ at time $t_1$. Let $t$ be a small time lapse after time $t_1$, and let $l_2$ be the distance between $P$ and $Q$ at time $t_1 + t$. We call the value $l_2/l_1$ the expansion rate for time lapse $t$ at time $t_1$. Now let $R$ and $Z$ be two points whose distance is $nl_1$ ($n \geq 2$) at time $t_1$. Then since we assume that the space is homogeneous and isotropic and the space itself continues expanding, the distance $l_3$ between $R$ and $Z$ at time $t_1 + t$ may become about $nl_1 + n(l_2 - l_1) = nl_2$. Let $m \geq 1$ be an integer. If $t$ is sufficiently small and the expanding rate is almost constant between time $t_1$ and time $t_1 + mt$, then the distance $l_4$ between $R$ and $Z$ at time $t_1 + mt$ may become approximately $l_4 \approx nl_1(l_2/l_1)^m$.

Then let $e(t_1 + t)$ and $e(t_1 + mt)$ be the expansion rates for time lapse $t$ and time lapse $mt$ at time $t_1$, respectively. Then it may hold

$$e(t_1 + t) = l_2/l_1, \quad e(t_1 + mt) = nl_1(l_2/l_1)^m/nl_1 = (l_2/l_1)^m \quad (4)$$

Then it holds $e(t_1 + mt) > e(t_1 + t)$ if $m > 1$. If the equations (4) are valid, then the figure of the scale factor is concaved to the upward, and in this case, it holds $\ddot{a}(t) > 0$ for $t \geq 1$.

We shall develop a more formal argument for supporting the assertion that for any $1 < t \leq t_p$, the scale factor $a(t)$ is approximately equal to an exponential function of $t$ so that $\ddot{a}(t) > 0$ as follows.

Consider a galaxy $N$ from which a light signal $S$ was emitted at time $t_p - t_1 - t_2$ for $t_1$ being about $10^6 \sim 10^7$ years and $t_2$ being small, and $S$ arrives at the earth at $t_p$ (present time). We assume Hubble’s law holds between $N$ and the earth. We shall consider a sufficiently small time lapse variable $t > 0$ so that it holds $0 < H_0t << 1$ and $e^{H_0t} \approx 1 + H_0t$. We also assume that for $0 < t \leq t_2$, it holds that $0 < H_0t << 1$ and $e^{H_0t} \approx 1 + H_0t$. Then let $L$ be the distance between $N$ and the earth at time $t_p - t_1 - t_2$. We consider the expansion rate $b(t)$ for time lapse $t$ at time $t_p - t_1 - t_2$ as a function of $t$, where for $0 < t \leq t_2$, $b(t)$ denotes the expansion rate of the universe when we set the initial time as $t_p - t_1 - t_2$. Thus it holds $a(t_p - t_1 - t_2 + t) \approx b(t)a(t_p - t_1 - t_2)$.

Now let $0 < t \leq t_2$ and $\Delta t$ be a very small time lapse, and consider times $t_p - t_1 - t_2 + t$ and $t_p - t_1 - t_2 + t + \Delta t$ at $N$. We assume that the distances between $N$ and the earth at time $t_p - t_1 - t_2 + t$ and time $t_p - t_1 - t_2 + t + \Delta t$ are $b(t)L$ and $b(t + \Delta t)L$, respectively. Let $g(b(t)L)$ be the function of $b(t)L$ such that the time lapse $t_3$ within which a light signal $S$ begins its travel from $N$ at time $t_p - t_1 - t_2 + t$ and ends its travel at the earth at time $t_p - t_1 - t_2 + t + t_3$ satisfies the following.

$$c t_3 = g(b(t)L) \quad (5)$$

Then Hubble’s law asserts the following.

$$\frac{g(b(t + \Delta t)L) - g(b(t)L)}{\Delta t} \approx H_0g(b(t)L), \quad (6)$$

where we assume that $H_0g(b(t)L)$ and $H_0g(b(t + \Delta t)L)$ are approximately equal. From (6), we have

$$g(b(t)L) \approx k(L)e^{H_0t} \quad (7)$$

where $k(L)$ is a function of $L$ independent of $t$. Now we want to determine $b(t)$ and $g(u)$ approximately. To do this, from many well known cosmological observations concerning
Hubble’ law, we may assume that $1 < g(u)/u << 2$. Thus we put approximately $g(uL) = (uL)^r$ for a positive number $r$ with $1 < r << 2$. Then from (7), we have

$$k(L) = L^r, \quad b(t)L \approx Le^{H_0 t/r} \quad (8)$$

From (8),

$$b(t) \approx e^{H_0 t/r} \quad (9)$$

Since we assume that the space itself continues expanding, we acknowledge that it holds approximately $b(t + t') \approx b(t)b(t')$ for any $t, t' > 0$ with $t, t'$ being small as above. Note that if $b(t + t') = b(t)b(t')$, then the following holds.

$$\dot{b}(t) = \lim_{h \to 0} \frac{b(t + h) - b(t)}{h} = \lim_{h \to 0} \frac{b(t)(b(h) - 1)}{h} = db(t), \quad b(t) = \alpha e^{dt} \quad (10)$$

where $\lim_{h \to 0} (b(h) - 1)/h = d$, and $\alpha$ is a constant.

If the expansion principle has not been changed so much from the early times of the universe, then $a(t_p)$ may be calculated by accumulating $b(t)$ in the following way.

$$a(t_p) = a(1)e^{A(t_p)}, \quad \text{where } A(t_p) = \int_1^{t_p}(H_0/r)dt \quad (11)$$

The Hubble constant $H$ may have been dependent on the global time, but it seems that the principle has been the same. Thus we consider the case where the Hubble constant has been the same as $H_0$ and $1 \leq r << 2$ is small. Thus we assume $H_0/r \approx H_0$.

From (11), it holds

$$a(t_p) = a(1)e^{H_0 t_p}, \quad t_p = H_0^{-1}\log(a(t_p)/a(1)) \quad (12)$$

Hence $t_p$ depends not only on $H_0^{-1}$ but also on $a(t_p)/a(1)$. Since $a(t_p)/a(1)$ seems very large, $t_p$ may be greater than $H_0^{-1}$ by a large factor. There may exist a possibility such that the Hubble constant is a function $H(t)$ and it holds $\dot{H}(t) < 0$. Then it may hold

$$a(t_p) = a(1)e^{B(t_p)}, \quad B(t_p) = \int_1^{t_p}H(t)dt, \quad \text{and } t_p \text{ may be smaller than } H_0^{-1}\log(a(t_p)/a(1)).$$

The above arguments may have rough approximations, but it seems that the age $t_p$ of the universe is greater than the Hubble time $H_0^{-1}$.

7 The dark matter problem

In cosmology, the dark matter problem is of great concern (see [13]). One of the main reasons the problem concerns is an observational fact that at a galaxy $G$, the rotation speed of a star $S$ in $G$ is about $200 \sim 300$ km/s seemingly independent from the distance between $S$ and the center of $G$. It is generally admitted this fact occurs because there exists a great amount of dark (invisible) matter called invisible halo outside the visible part of $G$. The invisible halo is also admitted to exist in order for $G$ to be stable. We note the sun system is stable because a great part of mass gathers at the sun. Note also if the invisible halo has great mass, then it may not be stable and may round $G$ since otherwise it may fall into the center of $G$, during a very long history of $G$ which seems impossible. Thus if the above invisible halo exists, then $G$ may not be stable a fortiori. We shall present a new remark (although simple but maybe important) about this problem in this section.
We consider a (x,y)-plane such that the sun is at the origin (0,0) and a galaxy G is at point \( P = (0, l) \), where \( l \) is the distance between the sun and G. We assume G is a disc C on the plane whose radius is \( r \). To make the argument simple, we also assume the mass \( M \) of G gathers at point \( P \). Let \( L \) denote the line segment parallel to the x-axis, passes through \( P \) and its endpoints are at the periphery of \( C \). We assume \( G \) rotates at its own axis in the counterclockwise way. Due to the Newtonian mechanics, the rotating speed \( v(u) \) of a star with mass \( m \) whose distance from \( P \) is \( u \) satisfies \( mv(u)^2/u = GmM/u^2 \), i.e., \( v(u) = \sqrt{GM/u} \).

Let \( Q = (-r, l) \) be the left end point of \( L \). Then the rotating speed \( v(r) \) of a star at \( Q \) is \( \sqrt{GM/r} \). This value is observed about \( 200 \sim 300 \text{km/s} \). Now let us consider the rotating speed of a star whose distance from \( P \) is \( u, 0 < u < r \). Let \( R \) be the point \((-u, l)\) on \( L \), and \( L_1 \) be the line segment parallel to the y-axis whose end points are on the periphery of \( C \) and on which \( R \) exists. We note all the light from all the stars on \( L_1 \) is observed to come from \( R \) on the earth. Now consider a star \( S \) at point \( T = (-u, l-b) \) on \( L_1 \), where \( b = d/4 \) and \( d \) is the length of \( L_1 \). Then the rotating speed of \( S \) is \( v(\sqrt{u^2 + b^2}) = \sqrt{GM/\sqrt{u^2 + b^2}} \). Now let \( \alpha \) be the angle between \( L_1 \) and \( L_2 \), where \( L_2 \) is the line segment connecting \( P \) and \( T \). Then \( \tan \alpha = b/u \). We consider the \((−y)\)-direction of the rotating speed \( v(\sqrt{u^2 + b^2}, -y) \) of \( v(\sqrt{u^2 + b^2}) \) at point \( T \). It holds \( v(\sqrt{u^2 + b^2}, -y) = v(\sqrt{u^2 + b^2}) \cos \alpha = \sqrt{GM/\sqrt{u^2 + b^2}u/\sqrt{u^2 + b^2}} < \sqrt{GM/\sqrt{u^2 + b^2}u/\sqrt{u^2 + b^2}/r} = \sqrt{GM/r} \).

Thus due to the Doppler effect, the \((−y)\)-direction of the rotating speed of \( S \) may be observed smaller than \( \sqrt{GM/r} \) on the earth. The \((−y)\)-direction of the rotating speed of a star at point \((-u, l)\) may be \( \sqrt{GM/u} \) greater than \( \sqrt{GM/r} \). But by counting the mean \((−y)\)-direction rotating speed, we may observe the fact presented in the beginning part of this section. There may exist other unnoticed reasons about the dark matter problem, but the above arguments seem worth studying.

\section{The principle of equivalence}

There exist in the literalature many interpretations of the principle of equivalence. One typical interpretation may be the assertion that the gravitational mass is equivalent to the inertial mass. Since it turns out that the relativity principle is not valid, we must be careful about the interpretation of the principle. In this section, we shall present comments about the following interpretation of the principle of equivalence (see e.g. [1]).

A : "In a small laboratory falling freely in a gravitational field, the laws of physics are the same as those observed in a Newtonian inertial system in the absence of gravitational field."

We first remark the following experiment made by Hafele and Keating (see [1]). Consider three clocks \( A, B \) and \( C \) whose mechanisms are equivalent, initially synchronized in setting and rate and at rest on the earth’s surface. Clock \( A \) remains at rest while \( B \) is flown over the earth by an eastward journey at height \( h \) in an aircraft whose speed relative to the ground is \( v \), and \( C \) is flown over the earth by a westward journey at height \( h \) in an aircraft whose speed relative to the ground is also \( v \). After the circumnavigations, \( A, B \) and \( C \) are compared, that is, the proper time \( \tau_A \) experienced by \( A \) is compared with the proper times \( \tau_B \) and \( \tau_C \) experienced by \( B \) and \( C \), respectively.

In 1971, Hafele and Keating made eastward and westward journeys round the earth on commercial jet flights, carrying caesium clocks \( B \) and \( C \) (\( B \) for the eastward journey and \( C \) for the westward journey) which they later compared with clock \( A \) which remained at the US Naval Observatory in Washington (see [1]). Then their experiment showed that \( \tau_B - \tau_A = -59 \pm 10 \) and \( \tau_C - \tau_A = 273 \pm 7 \) (nanoseconds). Now assume that the above principle
of equivalence is valid. Consider a small local space $S$ very close to the ground. Let $K$ be the Cartesian inertial system falling freely in $S$ whose existence is assured by the principle of equivalence. Since $\tau_B < \tau_C$, due to the twofold metric principle, the eastward velocity of $K$ is greater than the westward velocity of $K$ w.r.t. the universe and $K$ is not stationary w.r.t. the eastward and the westward directions. But then as in the proof of Theorem 2, the light velocity into the eastward direction should be smaller than the light velocity into the westward direction, which is a contradiction to the Michelson-Morley experiment and our conclusion in Subsection 5.3. We also recall that over a gravitational field, the light velocity is $c/\beta(v_G)^2$. Thus we acknowledge that any coordinate system over the gravitational field of the earth cannot be identified with any inertial system moving with constant velocity in the vacuum space where the gravitational potential is almost zero. We also remark that in the twofold metric principle, it is not the acceleration but the velocity which plays a key role. The above arguments show that a modified version of the above principle $A$ which is obtained from $A$ by replacing the words "falling freely in a gravitational field" with "falling freely with some velocity in a gravitational field" is also not valid. Hence we conclude that the above version of the principle of equivalence is not valid. From the twofold metric principle, we acknowledge that it holds the kinematic energy of clock $B$ flying eastward is larger than that of clock $C$ flying westward.

9 Concluding remarks

The special relativity principle asserts that all inertial systems are equivalent (and the Lorentz transformation is valid and the light velocity is $c$ over any inertial system). The general relativity principle asserts all coordinate systems are equivalent. In this paper, we show that the special relativity principle is not valid by depending on TI Lemma and TI Theorem (Section 4) and the general relativity principle is not valid even when we concern only tensor-based general relativity theory. We propose a new principle called the twofold metric principle, and show that in many applications, the new principle works well.

In Section 2, we present properties of the Lorentz transformation. Section 3 presents an electromagnetic example which seems to be a strong evidence to the invalidity of Special Relativity Principle by depending on the Maxwell equations. In Section 4, we establish TI Lemma (Lemma 3) and TI Theorem (Theorem 1). By depending on TI Theorem, we study properties of inertial systems. We deduce that neither Special Relativity Principle nor Strong Constant Light Velocity Principle is valid, and the Lorentz transformation is not the space-time transformation over two inertial systems moving with nonzero relative constant velocity. We deduce that Einstein’s transformation formulae of the Maxwell equations are not valid.

In Section 5, we propose the energy-velocity equation and the twofold metric principle. Four Subprinciples are proposed, and by depending on them, we present arguments for solving (1) $E = \beta(v)mc^2$, (2) the travelling distance of a muon with velocity $0.999c$, (3) a modified version of the Schwarzschild metric called the twofold Schwarzschild metric, (4) the Maxwell equations and the light velocity over the space with a gravitational field, (5) deflection and red shift of light due to gravity, and (6) an explanation for the Michelson-Morley experiment. Our calculated value of procession of planetary orbits by depending on the twofold Schwarzschild metric is the same as the corresponding well known value derived from the Schwarzschild metric. But the twofold Schwarzschild metric does not have the points $r = GM/c^2$ as its singular points while the Schwarzschild metric does. In Section 6, we present arguments supporing the assertion that Hubble’s law implies the expansion of the space itself, and the (present) age of the universe is greater than the Hubble time $H_0^{-1}$. In Section 7, we present
a new remark about the dark matter problem. In Section 8, we develop arguments from
which we deduce that one well known interpretation of the principle of equivalence in general
relativity theory is not valid.

We remark the following. Dirac developed relativistic quantum mechanics on electrons
by depending on the Lorentz transformations, and theoretically deduces the existence of
positrons (see [2]). But in Dirac’s arguments, the principle playing a key role may be said to
be the symmetry principle rather than the relativity principle, where the symmetry principle
may be a physical ”philosophy” respecting symmetry in physics. The theoretical effect of
depending on the Lorentz transformation seems time dilation and length contraction, which
may be observed also on the twofold metric principle. The only space-time transformation
$T$ over two coordinate systems $K$ and $K'$ such that $T$ is linear and symmetric w.r.t. $K$ and
$K'$, and time dilation and length contraction can be observed over $T$ may be the Lorentz
transformation. Since time dilation and length contraction may be said to occur in the
twofold metric principle, in a sense, one may say that the Lorentz transformation can be
derived by depending on the twofold metric principle, the ev-equation and the symmetry
principle. In Dirac’s arguments, four-vectors also appear. In any four-vector, the temporal
component may be related with the spacial components and the total energy. Thus four-
vector arguments seem to be related with the twofold metric principle. We also recall that
the well known arguments deriving $\beta(v)mc^2$ by depending on four-vectors have in some part
wrong computation. There may exist some correct results which can be derived by depending
on the Lorentz transformation. Or many laws in physics may respect symmetry, and they
may be ”invalidant” under the Lorentz transformation. But the Lorentz transformation does
not preserve simultaneity, and produces wrong results (such as time machines) sometimes.
Any result obtained by depending on the Lorentz transformation may have to be checked of
its correctness by any other valid principles. We note that sometimes experimental tests may
not be sufficient as in the case of the Schwartzshild metric, where the metric predicts the
orbit of Mercury correctly, but also implies the existence of black holes (probably wrongly).

Inevitably this paper does not present a full analysis of special and general relativity. The
author hopes it contains results and arguments which will contribute to the development
of physics in the new directions. We may need both learning and pondering as an ancient
Asian philosopher Confucious said : ”Learning without pondering may produce darkness,
and pondering without learning may produce crisis”.

10 Appendices

Appendix 1  We shall show that the new synchronization relation introduced in Section 1
is an equivalence relation. Clearly it is reflexive and symmetric. We shall prove its transitivity.
Let $P,Q$ and $R$ be three points at rest on $K$, and $A,B$ and $C$ be three clocks which are at rest
on $P,Q$ and $R$, respectively, and whose mechanisms are equivalent. Assume that $A$ and $C$ and
$B$ and $C$, respectively, are synchronized. As in Section 1, assume that a light signal $L$ travels
between $P$ and $Q$, and let $t_1,t_2$ and $t_3$ be the times on $A$, $B$ and $A$, respectively, and let $l$ and
$r$ be as in Section 1. Now let $t_2 = t_1 + l/r - b$, where $b$ is the bias of $A$ w.r.t. $B$. Since $A$ and
$C$ are synchronized, it holds the bias of $A$ w.r.t. $C$ is zero. The bias of $B$ w.r.t. $C$ is also zero.
Let the distances between $P$ and $R$ and $R$ and $Q$ be $l_1$ and $l_2$, respectively. Let $r_1$ and $r_2$ be the
velocities of light signals for travelling from $P$ to $R$ and from $R$ to $Q$, respectively. Now put
e = $l/r -(l_1/r_1+l_2/r_2)$, and let a light signal $L_2$ be emitted from $P$ at time $t_1 + e$ on $A$, arrive
at $R$ at time $t_1 + e + l_1/r_1$ on $C$, and finally arrive at $Q$ at time $t_1 + e + l_1/r_1 + l_2/r_2 = t_1 + l/r$
on $B$. It holds $b = l/r - (t_2 - t_1), t_2 = t_1 + l/r - (e + l_1/r_1 + l_2/r_2) + l/r = t_1 + l/r$ and $b = 0$
since (i) $A$, $B$ and $C$ are punctual, (ii) $t_1 + l/r$ is the time at $Q$ when $L_2$ arrives at $Q$, (iii) the time lapse for $L_2$ to travel from $P$, via $R$ to $Q$ is $l_1/r_1 + l_2/r_2$, (iv) $L_2$ is emitted from $P$ at time $t_1 + e$, and (v) the time lapse for $L$ to travel from $P$ to $Q$ is $l/r$. Thus the new synchronization relation is an equivalence relation. Remark that for the synchronization of $A$ and $B$, we need to know $r$ and $s$. If the clocks are not synchronized and the biases between them are not known over an inertial system $K_0$, then one cannot even make any experiments for checking whether or not the velocity of a light signal is constant over $K_0$.

Appendix 2. We shall study the twin paradox concerning the Lorentz transformation. The standard argument about the twin paradox proceeds as follows (see, e.g., [14]). Let $K$ and $K'$ be two parallel inertial systems with relative constant $x'$-axis velocity $v > 0$. Assume that there exist twins $A$ and $B$, and let $P$ be a point at rest on the $x$-axis of $K$. Twin $B$ starts his travel from $P$ on the $x$-axis of $K$ with constant velocity $v > 0$ at time $t_1$ on $K$. Then at time $t_2 = t_1 + t$ ($t > 0$) on $K$, he arrives at a point $Q$ at rest on the $x$-axis of $K$. He changes his direction toward $P$ and finally returns at $P$ at time $t_3$ on $K$. Now let $R$ be the point at rest on the $x'$-axis of $K'$ which coincides with $P$ at time $t_1$ on $K$. Then at time $t_2$ on $K$, $R$ coincides with $Q$. Let $t'_1$ and $t'_2$ be the times at $R$ on $K'$ corresponding to $t_1$ and $t_2$, respectively. By time dilation, it holds

$$t_2 - t_1 = \beta(v)(t'_2 - t'_1) > t'_2 - t'_1$$

Let $t'_3$ be the time on $B$ when $B$ returns at $P$. Then as above, one asserts that it holds $t_3 - t_2 = \beta(v)(t'_3 - t'_2)$. Hence $t_3 - t_1 = \beta(v)(t'_3 - t'_1) > t'_3 - t'_1$.

Thus one asserts that if twin $A$ stays at $P$ during the travel of twin $B$, then $B$ is younger than $A$ after his travel. The above case will be called the $(K, K')$ case. By changing the roles of $K$ and $K'$, we can consider the $(K', K)$ case as follows. In this case, $A$ starts his travel at point $R$ at rest on the $x'$-axis of $K'$ at time $t'_1$ on $K'$ with constant velocity $-v$, arrives at point $S$ at rest on the $x'$-axis of $K'$ at time $t'_2$ on $K'$, then changes his direction toward $R$, and finally returns at $R$ at time $t'_3$ on $K'$. Let $P$ and $Q$ be two points at rest on $K$ which coincide with $R$ and $S$, respectively, when $A$ starts his travel at $R$ and returns at $R$, respectively. We assert that while $A$ travels from $R$ to $S$, $A$ is at rest at $P$. Let $t_1$, $t_2$ and $t_3$ be times at $P, P$ and $Q$ on $K$ corresponding to $t'_1$, $t'_2$, $t'_3$, respectively. As above, one asserts it holds

$$t'_3 - t'_1 = \beta(v)(t_3 - t_1) > t_3 - t_1$$

Then the twin paradox asserts that due to the special relativity principle, the $(K', K)$ case and the $(K', K)$ case should be equivalent, and the relation between $(t_3 - t_1)$ and $(t'_3 - t'_1)$ should be the same in both cases, a contradiction.

We note the following two remarks.

(1) In the $(K, K')$ case, the initial clock condition is such that all the clocks on $K$ indicate the same time, and no two distinct clocks on $K'$ indicate the same time, while in the $(K', K)$ case, the initial condition is the converse of the $(K, K')$ case.

(2) In the $(K, K')$ case, when $B$ changes his direction at $Q$ toward $P$ and travels from $Q$ to $P$, the coordinate system in which the time at $B$ is measured is not in fact $K'$, contrary to our assumption above, but the coordinate system which is parallel to $K$ and its origin moves on the $x$-axis of $K$ with velocity $-v$. Thus the times on $B$ are not measured by the same clock while the time on $P$ is measured by the same clock. About the $(K', K)$ case, the same argument holds.
Due to (1) and (2), the \((K, K')\) case and the \((K', K)\) case are two distinct events occurring on \(K\) and \(K'\) as far as we acknowledge that the Lorentz transformation is valid. The Lorentz transformation is a bijection from \(CO(K)\) onto \(CO(K')\). For any coordinate \((x, y, z, t) \in CO(K)\), there exists one and only one coordinate \((x', y', z', t') \in CO(K')\) which is the image of \((x, y, z, t)\) under the Lorentz transformation. Thus for any event \(E\) occurring on \(K\), there exists the unique event \(E'\) occurring on \(K'\) which is the image of \(E\) under the Lorentz transformation, and vice versa. This means that we cannot find any contradictions in the Lorentz transformation as far as we concern only the Lorentz transformation by discarding all physical understandings about the universe. Some people assert that the twin paradox occurs since we do not count the time lapse needed for acceleration. But this explanation is meaningless since \(B\) in the \((K, K')\) case (or \(A\) in the \((K', K)\) case) can travel with constant velocity as long as he wishes so that the effects by the acceleration become negligible. This can be seen as follows. Consider the \((K, K')\)-case. Twin \(B\) begins his travel from \(P\) at time \(t = t' = 0\) and accelerates, and at time \(t'_1\) on his clock, his velocity w.r.t. \(A\) becomes \(v > 0\). Then \(B\) continues his travel with velocity \(v\) until time \(t'_2\) on his clock. By time dilation, on \(A\)'s clock, at least the time lapse \(\beta(v)(t'_2 - t'_1)\) passes. Then \(B\) decelerates and his velocity w.r.t. \(A\) becomes 0 at time \(t'_3\) on his clock. \(B\) changes his direction toward \(A\) and accelerates, and his velocity w.r.t. \(A\) becomes \(-v\) at time \(t'_4\) on his clock. \(B\) continues his travel with velocity \(-v\) until time \(t'_5\) on his clock. Then by time dilation, at least the time lapse \(\beta(v)(t'_5 - t'_4)\) passes on \(A\)'s clock. Then \(B\) decelerates and returns to \(P\) at time \(t'_6\) on his clock. We note that \(t'_1, t'_4 - t'_2\) and \(t'_0 - t'_5\) may depend only on \(v\), and \(B\) can travel as long as he wishes so that \(t'_2 - t'_1 + t'_5 - t'_4 - (t'_1 + t'_0 - t'_2 + t'_4 - t'_5)\) is arbitrarily large. Thus the time lapse at \(A\) during the \(B\) travel is at least \(\beta(v)(t'_5 - t'_4)\) which can become arbitrarily larger than the time lapse \(t'_6\) at \(B\).

From results in Section 4, we conclude that the twin paradox is not a paradox, but theoretically it is wrongly produced due to the invalidity of the Lorentz transformation.

Appendix 3

We shall present the following remarks.

(1) Scientists often talk about the (present) age of the universe. It is not an inertial system that ages, but our universe itself continues aging. Now consider any inertial system \(K'\), and the travelling of an imaginary signal \(S(K', u_2)\) with \(u_2 > 0\) being very large from point \(S\) to point \(R\) both being at rest over \(K'\) as in the proof of TI Theorem. If \(S(K', u_2)\) goes back into a past history over \(K'\), then this means that \(S(K', u_2)\) goes back into a past time \(t_0\) at \(R\) in our universe. But this must be impossible since the time \(t_0\) at \(R\) in our universe already disappeared. In the above travelling of \(S(K', u_2)\), even when \(u_2\) is arbitrary large, \(u_2\) is still finite and \(S(K', u_2)\) reaches a very far point \(a_0\) over \(K'\) within a second, but the time at \(a_0\) in our universe when \(S(K', u_2)\) arrives at \(a_0\) must be greater than the time at \(R\) in our universe when signal \(S(K, u_1)\) is emitted from \(R\) over \(K\). As a Gedankenexperiment, we can imagine the travelling of \(S(K', u_2)\) and conclude that our universe continues aging and consists of the entire only one connected universe, and the travelling of \(S(K', u_2)\) needs a positive time lapse in our universe. We also note that we have not observed any time period in our universe which was repeated.

(2) Consider Andromeda galaxy whose distance from the earth is acknowledged to be about \(2.5 \times 10^6\) light years. We can observe Andromeda galaxy, but actually we receive light signals emitted from Andromeda galaxy \(2.5 \times 10^6\) years ago. (We denote this time by \(t_0\) as the corresponding universe time = the global time in Section 5). This means that Andromeda galaxy at time \(t_0\) already disappeared in our universe, and at the present time of the universe, Andromeda galaxy of the present time \(t_p\) exists there. Time \(t_p\) may be about \(t_0 + 2.5 \times 10^6\).
years. Let $K$ be an inertial system which may be admitted to exist over the solar system. Then if we send imaginary signal $S(K, u_1)$ with $u_1$ being arbitrarily large, signal $S(K, u_1)$ will arrive at the point $P$ in the universe, at which Andromeda existed at time $t_0$, at time $t_0 + \alpha$, $\alpha > 0$, and it is impossible for $S(K, u_1)$ to arrive at $P$ before any time $t_1 \leq t_0$. Signal $S(K, u_1)$ cannot go into any past history in the universe even if $u_1$ is arbitrarily large. The earth of yesterday does not exist over any synchronized coordinate system of today.

(3) Assume that in Proposition 3, it holds $\alpha_2 \alpha_4 \neq 0$. Let $E_1, E_2, E_3$ and $E_4$ be the travelling of imaginary signals $S(K, u_1)$ and $S(K', u_2)$ between $R$ and $S$ as in Definition 1, where $u_1$ and $u_2$ are sufficiently large. Now consider events $k(E_2 + E_3) + k(E_4 + E_1)$ and $k(E_1 + E_4) + k(E_3 + E_2)$ for $k \geq 1$ (recall the third proof of (3) of TI Lemma). Then the more $k, u_1$ and $u_2$ are large, the more vastly imaginary signals can go into a future (can go into a past, respectively) while the event $k(E_2 + E_3) + k(E_4 + E_1)$ occurs (while the event $k(E_1 + E_4) + k(E_3 + E_2)$ occurs, respectively). In the event $k(E_1 + E_4) + k(E_3 + E_2)$ for large $k$, imaginary signals can go into the time when the Big Bang occurred under the assumption that the Big Bang really occurred. This means that the universe in which event $k(E_2 + E_3) + k(E_4 + E_1)$ occurs (or event $k(E_1 + E_4) + k(E_3 + E_2)$ occurs) must be distinct from the universe where we live. Thus if TI Lemma does not hold, then there should exist our infinitely many universes, and for any person $B$, "all the replicas" of $B$ from his birth to his death would always exist somewhere in the universe which we conclude is impossible.

(4) We observe the sun, stars, galaxies and great walls whose distances from the earth range in a very large scope. We acknowledge this implies that we have only one connected universe, this unique universe has continued aging from a very old past to the present and the time proceeds from the past to the future, and not in the backward way. We acknowledge that the travelling of imaginary signals $S(K, u_1)$ and $S(K', u_2)$ above with $u_1$ and $u_2$ being arbitrarily large occur in our unique universe and cannot go into other universes since they occur over inertial systems $K$ and $K'$ which exist in our universe.

Appendix 4. We acknowledge that the proof of TI Lemma is valid. But moreover we can observe the following odd properties of the Lorentz transformations and those transformations presented in Proposition 3 with $\alpha_2 \neq 0$. We first consider the Lorentz transformation case. Recall the proof of TI Theorem for the assumption $\alpha_2 \neq 0$. When $v = 0$, one cannot observe that any imaginary signal $S(K', u_2)$ goes into a past history over $K'$. Thus one may expect that the larger $v$ is, the more deeply imaginary signal $S(K', u_2)$ over $K'$ with $u_2$ being sufficiently large can go into a past history over $K'$. This is exactly true as we can see in the following. Let $K$ and $K'$ be two parallel coordinate systems with relative constant $x'$-axis velocity $v \geq 0$ such that the space-time transformation from $K$ onto $K'$ is the Lorentz transformation. Let $H = <K, C>$ and $J = <K', D>$ be the cc systems of $K$ and $K'$, respectively. Let $R$ and $S$ be two points at rest on the $x'$-axis of $K'$ whose $x'$-coordinates are $x'_1$ and $x'_2$ with $l = x'_2 - x'_1 > 0$. As in the proof of TI Theorem, we consider travelling of imaginary signals from $R$ to $S$ and from $S$ to $R$. Consider time $t > 0$ over $K$, and assume that the origin $O$ of $K$ coincides with $R$ at time $t$ on $C(O)$, and imaginary signal $S(K, u_1)$ over $K$ is emitted from $O$ to $S$ at time $t$ on $C(O)$, and arrives at $S$ at time $t + a$ on $C(Q)$ with $a$ being very small, where $Q$ is the point at rest on the $x$-axis of $K$ which coincides with $S$ when $S(K, u_1)$ arrives at $S$. Let $t'_1$ and $t'_2$ be times on $D(R)$ and $D(S)$, respectively, such that $O$ and $R$ coincides at time $t'_1$ on $D(R)$, and $Q$ and $S$ coincide at time $t'_2$ on $D(S)$. One can see easily that the $x$-coordinate $x_1$ of $Q$ equals $\beta(v)(l + v(t'_2 - t'_1))$. Now assume that imaginary signal $S(K', u_2)$ over $K'$ is emitted from $S$ at time $t'_2$ on $D(S)$ and arrives at $R$ at time $t'_3$ on $D(D)$, where $u_2 > 0$ is sufficiently large w.r.t. $l$. By the Lorentz transformation,
the following hold:

\[ t'_1 = \beta(v)t, \quad t'_2 = \beta(v)(t - v\beta(v)(l + v(t'_2 - t'_1))/c^2), \quad t'_3 = t'_2 + l/u_2 \simeq t'_2, \]
\[ t'_2 - t'_1 \simeq \beta(v)(t - v\beta(v)(l + v(t'_2 - t'_1))/c^2) - \beta(v)t, \]
\[ (t'_2 - t'_1)(1 + \beta(v)^2v^2/c^2) \simeq -\beta(v)^2vl/c^2 \]
\[ t'_3 - t'_1 \simeq t'_2 - t'_1 \simeq -\beta(v)^2vl/(c^2(1 + \beta(v)^2v^2/c^2)) = -vl/c^2 \]

Then one can observe that it holds the larger \( v \) is, the larger \( t'_1 - t'_3 \) is. This observation implies that the larger \( v \) is, the more deeply imaginary signal \( S(K', u_2) \) can go into a past history over \( K' \). Note also that \( \lim_{v \to c} t'_1(v) - t'_3(v) \simeq l/c \). This is contradictory to our acknowledgement that the property of imaginary signal \( S(K', u_2) \) should depend only on \( K' \) and \( u_2 \), and should be independent from \( v \).

Now we consider the situation presented in Proposition 3. Assume that \( \alpha_2 \neq 0 \). We consider the case \( \alpha_2 < 0 \) as in the proof of TI Theorem. The case \( \alpha_2 > 0 \) can be considered similarly. Let \( S, R, t, O, P, a, t'_1, t'_2, t'_3, u_1, u_2, l \) be as above. As in the proof of TI Theorem, the following hold:

\[ t'_1 = \gamma_2t, \quad t'_2 = \alpha_2\alpha_3l + \gamma_2(t'_2 - t'_1) + \gamma_2t, \quad t'_3 = t'_2 + l/u_2 \simeq t'_2, \]
\[ t'_3 - t'_1 \simeq \alpha_2\alpha_3l/(1 - \alpha_2\gamma_3) < 0 \]

We need continuity of \( \alpha_2 \) so that for \( v = 0, \alpha_2(v) \) must be zero, and we conclude that \( \alpha_2 \) is not a constant and a function of \( v \). Then \( t'_2 - t'_1 \) varies as \( v \) varies, and \( t'_3 - t'_1 \) depends on \( l \).

We observe again similar contradictory phenomena as in the above Lorentz transformation case. We also note that in the above cases, \( l \) can be arbitrarily small. For example, for \( l = 1\text{mm} \), the above phenomena can be still observed.

**Appendix 5**

**Theorem 6** Let \( K \) and \( K' \) be two parallel inertial systems with relative constant \( x' \)-axis velocity \( v > 0 \). Assume that the light velocity over \( K \) is \( c \) independent of its direction. Then the velocity of a light signal over \( K' \) is uniquely determined if its direction is fixed.

**Proof.** Let \( P \) and \( Q \) be two points at rest on \( K \). Let \((x_1, y_1, z_1), (x_2, y_2, z_2)\) be the \((x,y,z)\)-coordinates of \( P \) and \( Q \), respectively, and put \( x_2 - x_1 = x, y_2 - y_1 = y, \) and \( z_2 - z_1 = z \). We shall prove the assertion for the case \( x > 0 \). The other cases can be handled in the same way. Consider time \( t_1 \) on \( K \), and let \( R \) and \( S \) be two points at rest on \( K' \) which coincide with \( P \) and \( Q \), respectively, at time \( t_1 \) on \( K \). Assume that a light signal \( L \) is emitted from \( P \) at time \( t_1 \) on \( K \), and arrives at \( Q_1 \) at time \( t_2 = t_1 + t \) \((t > 0)\) on \( K \), where \( Q_1 \) is the point at rest on \( K \) and \( Q_1 \) coincides with \( S \) at time \( t_2 \) on \( K \). Then the direction from \( P \) to \( Q \) can be denoted by \((y/x, z/x) = (a, b)\) and the direction from \( R \) to \( S \) can be denoted by \((a_2y/a_1x, a_3z/a_1x) = (a_4a_2/a_1, b_4a_3/a_1)\), where \( a_1-a_4 \) are as in Theorem 5. Now \( t \) satisfies the following:

\[ c^2t^2 = (x + vt)^2 + y^2 + z^2 = (x + vt)^2 + a^2x^2 + b^2x^2 \]

Thus

\[ t = (xv + x\sqrt{v^2 + (1 + a^2 + b^2)(c^2 - v^2)}/(c^2 - v^2)) \]

Now let \( t' \) be the corresponding time lapse on \( K' \) within which \( L \) travels from \( R \) to \( S \). Then

\[ t' = \alpha_4t = \alpha_4(xv + \sqrt{v^2 + (1 + a^2 + b^2)(c^2 - v^2)})(c^2 - v^2) \]

Now the distance \( l' \) from \( R \) to \( S \) satisfies
\[ l' = \sqrt{\alpha_1^2 x^2 + \alpha_2^2 y^2 + \alpha_3^2 z^2} = x \sqrt{\alpha_1^2 + a^2 \alpha_2^2 + b^2 \alpha_3^2} \]

Thus the velocity \( c' \) of \( L \) for travelling from \( R \) to \( S \) on \( K' \) is

\[
c' = \frac{l'}{t'} = \sqrt{\alpha_1^2 + a^2 \alpha_2^2 + b^2 \alpha_3^2 (c^2 - v^2)}/(\alpha_4(v + \sqrt{v^2 + (1 + a^2 + b^2)(c^2 - v^2)}))
\]

Thus \( c' \) is uniquely determined by \( \alpha_1, \alpha_2, \alpha_3, \alpha_4, v, a \) and \( b \). \( \square \)

Theorem 6 implies that an observer at rest on \( K' \) can determine his velocity w.r.t. the universe (or at least w.r.t. a local space) by observing the velocities of light signals into many directions over \( K' \) if he knows equations for determining constants \( \alpha_i, 1 \leq i \leq 4 \). In Section 5, we present Subprinciples 1-3 from which one may determine the values of \( \alpha_i, 1 \leq i \leq 4 \).

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