Asymmetry at LHC for an $U(1)'$ anomalous extension of MSSM

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Abstract

The measurement of the forward-backward asymmetry at LHC could be an important instrument to pinpoint the features of extra neutral gauge particles obtained by an extension of the gauge symmetry group of the standard model. For definitiveness, in this work we consider an extension of the gauge group of the minimal supersymmetric standard model by an extra anomalous $U(1)$ gauge symmetry. We focus on $pp \rightarrow e^+e^-$ at LHC and use four different definitions of the asymmetry obtained implementing four different cuts on the directions and momenta of the final states of our process of interest. The calculations are performed without imposing constraints on the charges of the extra $Z$’s of our model, since the anomaly is cancelled by a Green-Schwarz type mechanism. Our final result is a fit of our data with a polynomial in the charges from which to extract the values of the charges given the experimental result.

1. Introduction

One of the most motivated extensions, from a theoretical point of view, of the standard model (SM) and minimal supersymmetric standard model (MSSM) of particle physics is obtained by enlarging the gauge group of the theory by admitting extra $U(1)$’s. Such extensions are natural at low energy for models coming from grand unified theories and string theories (see [1] for a recent review). In the string inspired scenarios the anomalies of the extra $U(1)$’s are cancelled by the Green-Schwarz mechanism. To explore such possibility we will use an extension of the MSSM which from now on will be dubbed MiAUMSSM. An alternative version of this model which admits spontaneous supersymmetry breaking was also formulated in [2], but in this work we will use the original formulation of [3]. The phenomenology of the MiAUMSSM has been investigated in different directions. Assuming that the lightest supersymmetric particle (LSP), a candidate for dark matter, comes from the anomalous sector of the model [4, 5], the relic density of such LSP was computed and proved to be compatible with the experimental data of WMAP [6]. Furthermore in [7] the decays of the next to lightest supersymmetric particle (NLSP) into the LSP has been considered, while in [8] the features of a possible signature of the model at LHC has been considered by concentrating on a particular radiative decay of the NLSP. In this paper we will further develop the phenomenology of the MiAUMSSM by computing the forward-backward asymmetry which is induced in the final states of the process $pp \rightarrow e^+e^-$ by keeping into account the new gauge boson, $Z'$, associated to the extra $U(1)$ gauge symmetry. The couplings (charges) of this particle to the others present in our model are not fixed by the requirement of gauge anomaly cancellation and can be determined only by experiment. Our aim is to show that such measurement is feasible and that it can distinguish among the different possible scenarios [9]. Since at LHC the colliding beams are made of the same particle, to generate an asymmetry in the final state, some cuts on the parameter space have to be necessarily performed. Each possible cut leads to a different definition of the asymmetry. In this work we will use four different sets of cuts to show that our results are not dependent from these choices. This work is organized as follow: in sec. 2 we briefly review the main features of the model which we are going to study. In sec. 3 we will discuss the four different definitions of the asymmetry we will use: in sec. 4 we will describe our calculations and collect the results which are finally discussed in the conclusions.

2. Model definition

Our model [3] is an extension of the MSSM with an extra $U(1)$. The charges of the matter fields with respect to the symmetry groups are given in table 1. The gauge invariance of the model implies:

$$
Q_{U^c} = -Q_Q - Q_{H_u} \\
Q_{D^c} = -Q_Q + Q_{H_u} \\
Q_{E^c} = -Q_L + Q_{H_u} \\
Q_{H_d} = -Q_{H_u}
$$

(Table 1)

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Thus, there are only three free charges introduced by the extra symmetry: we can choose $Q_Q$, $Q_L$, and $Q_{H_u}$ without losing generality. The anomalies induced by this extension are cancelled by the GS mechanism: there are no further constraints on the charges.

To evaluate the asymmetry associated to the full process $pp \to e^+e^-$ we have performed the calculation of the cross section of the subprocess $q\bar{q} \to e^+e^-$, which we report in Appendix A. In Appendix B, we give details on the convolution of this differential cross section for the specific definitions of asymmetry we will adopt. We take the mass of our $Z'$ to be 1.5 TeV. There are two main reasons for this choice: on the one hand we wanted a sizeable $Z'$ production (see [3], where there are results for a $Z'$ mass of 1 TeV). On the other hand this mass value allows a comparison with the results in literature [10]. Regardless, our analysis could be repeated for arbitrary value of the $Z'$ mass.

|     | SU(3)$_c$ | SU(2)$_L$ | U(1)$_Y$ | U(1)$_Y'$ |
|-----|-----------|-----------|-----------|-----------|
| $Q_1$ | 3         | 2         | 1/6       | $Q_Q$     |
| $U^c_2$ | 3         | 1         | $-2/3$    | $Q_{U^c}$ |
| $D^c_3$ | 3         | 1         | 1/3       | $Q_{D^c}$ |
| $L_1$ | 1         | 2         | $-1/2$    | $Q_L$     |
| $E^c_1$ | 1         | 1         | 1         | $Q_{E^c}$ |
| $H_u$ | 1         | 2         | 1/2       | $Q_{H_u}$ |
| $H_d$ | 1         | 2         | $-1/2$    | $Q_{H_d}$ |

Table 1: Charge assignment.

3. Asymmetry definition

Because the initial $pp$ state is symmetric, the asymmetry at LHC is zero if we integrate over the whole parameter space. However the partonic subprocess $q\bar{q} \to e^+e^-$ is asymmetric. We can keep this asymmetry by imposing kinematical cuts, which are anyway inevitable because of the limits imposed by the detector. There are many possibilities to perform these cuts and each of them leads to a different definition of the asymmetry. In this work we have used the four definitions of the asymmetry, $A_{RFB}(Y_{cut})$, $A_{OFN}(p_z^\text{cut})$, $A_C(Y_C)$ and $A_E(Y_C)$, which are collected in [10]:

\[
A_{RFB} = \frac{\sigma(|Y_{e^-} > |Y_{e^+}|) - \sigma(|Y_{e^-} < |Y_{e^+}|)}{\sigma(|Y_{e^-} > |Y_{e^+}|) + \sigma(|Y_{e^-} < |Y_{e^+}|)} \left| Y > Y_{cut} \right|
\]

\[
A_{OFN} = \frac{\sigma(|Y_{e^-} > |Y_{e^+}|) - \sigma(|Y_{e^-} < |Y_{e^+}|)}{\sigma(|Y_{e^-} > |Y_{e^+}|) + \sigma(|Y_{e^-} < |Y_{e^+}|)} \left| p_z > p_{z_{cut}} \right|
\]

\[
A_C = \frac{\sigma_{e^-}(|Y_{e^-} < Y_C) - \sigma_{e^+}(|Y_{e^+} < Y_C)}{\sigma_{e^-}(|Y_{e^-} < Y_C) + \sigma_{e^+}(|Y_{e^+} < Y_C)}
\]

\[
A_E = \frac{\sigma_{e^-}(Y_C < |Y_{e^-}|) - \sigma_{e^+}(Y_C < |Y_{e^+}|)}{\sigma_{e^-}(Y_C < |Y_{e^-}|) + \sigma_{e^+}(Y_C < |Y_{e^+}|)}
\]

where $\sigma$ is the total cross section after integrating with the partonic distribution functions (PDFs).

The first two asymmetries are defined in the center of mass (CM) frame. The forward-backward asymmetry $A_{RFB}$ [11, 12, 13, 14] has a cut on the rapidity $Y$ of the $e^-/e^+$ pair

\[
Y = \frac{1}{2} \log \left( \frac{E_{e^-} - p_z}{E_{e^-} + p_z} \right)
\]

The one-side asymmetry $A_{C}$ [15, 16] has a cut on $p_2$, the total momentum associated to the final states ($e^-e^+$) moving longitudinally along the beam direction chosen to be the $z$ axis. In Appendix B this rapidity will be expressed in the CM in terms of the partonic variables $x_1, x_2$. $E_{e^\pm}$ is the sum of the energies associated to the two particles. The other two asymmetries are defined in the laboratory (Labs) frame. The variable $Y_{e^\pm}$ is the pseudo-rapidity associated to the single particle $e^\pm$ and expressed as

\[
Y_{e^\pm} = -\log \left( \tan(\theta_{e^\pm}/2) \right)
\]

with $\theta_{e^\pm}$ the angle of the outgoing fermion with respect to the $z$ axis. In this case the kinematical cut is over the rapidity in the Lab frame which is denoted by $Y_C$ and which will be introduced in Appendix B. The central asymmetry $A_{C}$ [17-21] is calculated integrating in the angular region centered on the axis orthogonal to the beam, while the edge asymmetry $A_{E}$ [22] is defined in the complementary region.

For further details, see Appendix B.

4. Asymmetry calculation

In this work we have calculated the asymmetry in two different ways. First we have used a numerical code that we have written using Mathematica. This code uses the cross-section calculated in Appendix A to numerically compute the integrals discussed in Appendix B. As a second check we have repeated the same computation using the HERWIG package [23, 24], that we have modified to calculate the asymmetry. We have chosen to repeat twice our computation for two main reasons: the first one is that in this way we can have a cross check between our results; the second is that these methods have different peculiarities that we want to use. For example, the numerical integration is less computer time consuming for the Mathematica code, which helps in establishing the dependence of the asymmetry from the free charges of the model. At the same time the HERWIG package permits to study how the cuts influence the rate of production of our final state. For these reasons we have performed the basic calculation (i.e. the asymmetry optimization) using both methods. We remark that all the results that we will show are strongly dependent on the set of PDFs used to
calculate them and that this leads to a systematical error. In the following we do not show results for different sets of PDFs. Where the statistical error is concerned, we have estimated it using the formula \[10\]:

\[
\text{err} \equiv \sqrt{\frac{4N_F N_B}{N^3}} \simeq \frac{1}{\sqrt{\mathcal{L} \sigma}}
\]

where \(N_F, N_B\) are the forward/backward events, \(N\) is the total number of events and \(\mathcal{L}\) is the luminosity. In the following we show the estimated errors for the asymmetry definitions keeping \(\mathcal{L} = 100 \text{ fb}^{-1}\).

We aim to use the asymmetry to distinguish our model from the MSSM or other models which include an extra \(U(1)\). In the following we will perform the asymmetry calculation around the peak region, that is for \(M_{Z'} - 3\Gamma_{Z'} < M_{e^+e^-} < M_{Z'} + 3\Gamma_{Z'}\), where \(\Gamma_{Z'}\) is the total decay rate of the \(Z'\). As we remarked in \[Appendix \ B\] this determines the integration domain, that is \((M_{Z'} - 3\Gamma_{Z'})^2 < s < (M_{Z'} + 3\Gamma_{Z'})^2\). We also compare our results with the ones obtained for the Sequential Standard Model (SSM), in which there is an extra \(Z'\) boson which has the same couplings to fermions such as the SM \(Z\) boson \[11, 25\]. See section \[4.4\] for further details on the corresponding Lagrangian and the values of the quantum numbers.

### 4.1. Optimized asymmetry

As shown in \[10\] the asymmetry magnitude is not a good function to optimize. A better choice is, instead, the statistical significance:

\[
\text{Sig} \equiv A\sqrt{\mathcal{L} \sigma}
\]

where \(A\) can be any of the previously defined asymmetries, \(\mathcal{L}\) is the LHC integrated luminosity, which we take to be 100 \(\text{fb}^{-1}\).

We have found a good agreement between the results obtained by using the Mathematica code and those obtained with the event simulator HERWIG. So we are confident that our results are reliable also when we will use them to calculate other observables, e.g. the dependence from the charges of the asymmetries and the significancies. In figure \[1\] we show the results for the on-peak significance of the MiAUMSSM and SSM for all the definitions of asymmetry that we use. The best cuts are those that maximize the significance. For the SSM we find the same values as in \[10\]. We list the best cuts of the MiAUMSSM in table \[2\]. As in \[10\], we expect that the best cuts are nearly independent from the charges and depend only on the \(Z'\) mass and the partonic distribution functions. Moreover they are also essentially independent from the specific model chosen as it is confirmed by our analysis. As a further check we have performed simulations with the SSM. We have used the same settings of \[10\], obtaining very similar results for all the cuts, confirming the reliability of our numerical codes. We used the SSM not only for having a check of the validity of our calculations, but also to have results that can be compared with those of the MiAUMSSM.

### 4.2. Dependence on the charges

Now we want to use the best cuts previously found to study the asymmetry in function of the free charges of our model. We have studied the value of the four asymmetries keeping alternatively one of the charges fixed to 0 and varying the others two from \(-1\) to \(1\). We choose these...
The asymmetry as a function of the two remaining charges. Those with $Q_L = 0$ or $Q_Q = 0$ are almost symmetric only under the change of sign of both the two unfixed charges. So the asymmetry as a function of the charges must reflect this sort of symmetries in its polynomial dependence on the charges. This implies that if we try to fit the asymmetry with a rational function (which is the best choice, given the definitions (2-5)) we will have constraints on the coefficients of the fit.

4.3. Number of events

We already mentioned that to obtain a non zero asymmetry at LHC we have to impose cuts in the parameter space. Obviously these cuts will diminish the number of events that we can use to measure the asymmetry. It is important to be sure that they do not drastically affect the set of data we have at our disposal. To study the ratio between the number of events obtained applying the cuts and the total number of events expected in our channel of interest ($pp \to e^+e^-$) we have used the HERWIG package. We have studied the ratio $N_i/N_{tot}$, where $N_i$ is the sum of the forward and backward events for the i-th definition of asymmetry and $N_{tot}$ is the number of events that we have generated with HERWIG. We have performed the calculation of $N_i/N_{tot}$ in two cases:

- on peak invariant mass, variable cuts
- variable invariant mass, fixed cuts

Only in the case of fixed invariant mass with fixed cut it is possible to distinguish the behavior of our model from that of the SSM. Therefore we show only the related results in Fig. 5. After the implementation of the cuts we are left with the $65 - 75\%$ of the total number of events for the FB and O asymmetry, while for the C and E asymmetries we are left with the $40 - 50\%$ of the events. In both cases these ratios are good enough to allow the measurement of the observable of interest.

| $A_{RFB}$ | $A_O$ | $A_C$ | $A_E$ |
|-----------|--------|-------|-------|
| $Y_{cut}^{ff} = 0.4$ | $p_{cut}^{eff} = 580$ GeV | $Y_C = 0.8$ | $Y_C = 1.4$ |

Table 2: Best cuts for the on-peak $e^+e^-$ asymmetries.
models. Obviously in the case of the MiAUMSSM we do not have a unique value for the asymmetry, because in the model the charges are not fixed. To show that it is possible to distinguish the MiAUMSSM from the other models we have to estimate the statistical error in this measurement, by using the formula (8). The exact values depend on the cross section which is model dependent. Now, if we fix $Q_H = Q_Q = Q_L = 0.5$, the resulting values for the asymmetries associated to the three models are showed in figure 6. The data plotted in the figure show that it is always possible to discriminate the anomalous model from the non anomalous ones.

Now we want to stress that the three charges of our model are free but the couplings of the fermions to the $Z'$ in the anomalous MiAUMSSM have a peculiar functional form given in table A. As a consequence it is not possible to match the couplings to the extra $Z'$ of the MiAUMSSM with those of other models. But, since the four asymmetries have associated statistical errors we could have a range of values of our three charges where the couplings of the MiAUMSSM (and consequently the asymmetries) could be matched with those of the SSM and LRM models within the considered errors. In reality this does not happen as we can infer from Fig 7, where we consider an error up to 25%, much bigger than the expected experimental error. Observing the amount of points in these figures, it is evident that the SSM is closer to our model than the LRM. This is the reason why throughout this paper we focus our analysis on the comparison with the SSM.

4.5. General case

In this section we want to find the function which describes the asymmetry in terms of the three free charges of our model which can assume values between −1 and 1. From the cross section of the process, that can be found in Appendix A we can see that the amplitude is
proportional to the fourth power of the charges. So, the equations (2)-(5) imply that the asymmetry must be a rational function in which both the numerator and denominator are fourth grade polynomials in the charges:

\[
A = \frac{\sum_{i,j,k=0}^{n} a_{ijk} (Q_{H_u})^i (Q_Q)^j (Q_L)^k}{\sum_{i,j,k=0}^{n} b_{ijk} (Q_{H_u})^i (Q_Q)^j (Q_L)^k} \quad (12)
\]

with \( i + j + k = n \leq 4 \).

The apparent symmetries of the contour plots obtained in subsection 4.2 imply that the terms of odd degree in the charges are suppressed. Then the only relevant terms in subsection 4.2 imply that the terms of odd degree in the numerator and denominator in (12) are those that are linear in the charges. So, the asymmetry is useful for fixing the values of the \( U(1)' \) charges. Moreover, once the values of the three charges are obtained by the previous system, the fourth definition of asymmetry can be used as a check for the validity of the model under scrutiny. In fact its hypothetical experimental value must be recovered by using (12) with the values of the charges already found, within the considered error (we use the mean relative error (MRE) for each asymmetry definition). In the Appendix C we write a table with the coefficients of the four fits. As expected, we have found out that the odd degree polynomials have negligible coefficients, thus confirming the intuitions stemming from the analysis of the contour plots.

The exactness of the fit is evaluated computing the \( R^2 \) and the medium relative error for the polynomial fit of each asymmetry definition. In the Appendix C we show a table with the coefficients of the four fits. As expected, we have found out that the odd degree polynomials have negligible coefficients, thus confirming the intuitions stemming from the analysis of the contour plots. The exactness of the fit is evaluated computing the \( R^2 \) and the medium relative error for the polynomial fit of each asymmetry definition. In the Appendix C we show a table with the coefficients of the four fits. As expected, we have found out that the odd degree polynomials have negligible coefficients, thus confirming the intuitions stemming from the analysis of the contour plots.

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Table 3: Couplings of the SM fermions to the \( Z \)'s for the SSM, LRM and MiAUMSSM models.

| \( f \) | SSM | LRM | MiAUMSSM |
|---|---|---|---|
| \( c, \mu, \tau \) | \( g_Y^c \) | \( g_Y^L \) | \( g_Y^A \) |
| \( u, c, t \) | \( -\frac{1}{2} + 2 \sin^2 \theta_W \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) |
| \( d, s, b \) | \( \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \) | \( \frac{1}{2} \) | \( \frac{1}{2} \) |

Figure 7: Values of the charges that give a MiAUMSSM asymmetry close to the SSM asymmetry within errors of 15 (big red dots), 25 % (left image) and to the LRM within errors of 20 (big red dots), 25 % (right image)
which we can have the product of diagrams where the process Appendix A. Cross section

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Appendix A. Cross section

We have calculated the cross section in the CM for the process $qq \rightarrow e^+e^-$ for a general Drell-Yan interaction in which we can have the product of diagrams where the $\gamma$, $Z_0$ and $Z'$ can be exchanged. Thus we have six possible terms: $\gamma\gamma$, $\gamma Z_0$, $\gamma Z'$, $Z_0 Z_0$, $Z_0 Z'$ and $Z' Z'$. The total amplitude is:

$$|M|^2(q) = \sum_{a,b=\gamma,Z,Z'} g_a^2 g_b^2 M_{ab}$$

where the fractions $\frac{1}{4}$ and $\frac{1}{2}$ come out from the averages over spin and color, $g_0$ is the coupling associated to the $Z'$. We fix its value to 0.1. $M_{ab}$ is the amplitude of each process divided by the couplings:

$$M_{ab} = \frac{64 N_{ab} \left( (s - m^2_q) (s - m^2_b) + (\Gamma_b m_b) \right)}{\left( (s - m^2_q)^2 + (\Gamma_a m_a)^2 \right) \left( (s - m^2_b)^2 + (\Gamma_b m_b)^2 \right)}$$

$$\left\{\begin{array}{l}
2 m_q^2 m_b^2 C_{AAVV}^{ab} C_{AAVV}^{ab} - m^2_a (p_q \cdot p_b) C_{AAVV}^{ab} C_{AAVV}^{ab} \\
+ [(p_q \cdot p_e) (p_b \cdot p_e) - (p_q \cdot p_e) (p_b \cdot p_e)] C_{AAVV}^{ab} C_{AAVV}^{ab} \\
+ [(p_q \cdot p_e) (p_b \cdot p_e) + (p_q \cdot p_e) (p_b \cdot p_e)] C_{AAVV}^{ab} C_{AAVV}^{ab} \\
- m_q^2 (p_e \cdot p_b) C_{AAVV}^{ab} C_{AAVV}^{ab}
\end{array}\right\}$$

where $C_{AAVV}^{ab}$ is a multiplicity factor that is equal to $\frac{1}{2}$ if the exchanged vector bosons are identical and is equal to 1 if they are different. The $C_i$s are simply the vector and axial quantum numbers related to the vector bosons: for the $\gamma$ and the $Z_0$ they are the usual SM quantum numbers that can be found in [25], while the vector and axial couplings related to the $Z'$ have been calculated in [25] and are showed in table A.5.

We remark that in this cross section there are only terms of degree four in the powers of the charges. However, from the equation (A.1) we know that there are different contributions to the total squared amplitude of our process. These terms are divided in three types: the term $Z'Z'$, the terms $\gamma Z'$ and $Z_0 Z'$ and those $\gamma\gamma$, $\gamma Z_0$ and $Z_0 Z_0$ which give contribution of degree four, two and zero in the anomalous charge respectively. Since we are studying the on-peak region, we expect the contribution from the $Z'Z'$ channel to be dominant with respect to the others: this is evident from the table of the coefficients showed in Appendix C. Observing the previous formula we can see that all the combinations of $C_i$s contain two $C_q$ and two $C_a$ because our elementary process involves two leptons and two quarks. Observing that $Q_Q$ and $Q_L$ are related to $C_q$ and $C_a$ respectively, this implies that we cannot have terms of degree larger than two in $Q_Q$ and $Q_L$. This is verified by our fit, where the coefficients related to these terms are suppressed (see Appendix C). The differential cross section can be found multiplying for the usual kinematic prefactor and summing this result over the contribution of the six possible initial quarks:
The formula for \( \frac{\partial^2 \sigma}{\partial s \partial \cos \theta} \) is given by:

\[
\frac{\partial^2 \sigma}{\partial s \partial \cos \theta}_{CM} = \sum_q \frac{p_q}{32 \pi s p_q} |M|^2(q) \quad (A.3)
\]

**Appendix B. Details on the Asymmetry definitions**

The explicit expression of eq. (2) in the CM frame is

\[
A_{RFB} = \frac{\int_C^x dx_1 dx_2 \sum_q f_q(x_1) f_q(x_2) (F - B)}{\int_C^x dx_1 dx_2 \sum_q f_q(x_1) f_q(x_2) (F + B)} \quad (B.1)
\]

where

\[
F = \int_0^1 d \cos \theta \frac{d \sigma(\cos \theta, s)}{d \cos \theta ds} \quad B = \int_{-1}^0 d \cos \theta \frac{d \sigma(\cos \theta, s)}{d \cos \theta ds} \quad (B.2)
\]

are the forward and backward contributions, respectively. The \( f_q/q(x_i) \) are the PDFs of \( q/\bar{q} \). \( C_{cut} \) is the domain of integration, that depends on the type of asymmetry that we want to calculate according to the definitions (14) and (15). The whole domain of variables \( (x_1, x_2) \) and \( (s, Y) \) are not independent. In fact, their definition is \( s = x_1 x_2 \) and \( Y = \frac{1}{2} \log(x_1/x_2) \), where \( S = (14 \text{ TeV})^2 \) is the total squared energy of the accelerator.

Since we have calculated the cross section of the process \( \theta \) in the expression (B.1). The Jacobian of this transformation is \( J = 1/S \). Now we focus on the integral \( \int_{-Y_{cut}}^{Y_{cut}} dY \): if we perform the change of variable \( Y \rightarrow -Y \), because of the \( Y \) definition, this corresponds to the exchange \( x_1 \leftrightarrow x_2 \) and consequently to the exchange of forward with backward \( (F \leftrightarrow B) \). Summarizing

\[
\int_{-Y_{cut}}^{Y_{cut}} dY \sum_q f_q(x_1) f_q(x_2) (F \pm B) =
\int_{Y_{cut}}^{Y_{cut}} dY \sum_q f_q(x_2) f_q(x_1) (\pm F + B) \quad (B.3)
\]

Using this result we obtain the formula that we implemented in the Mathematica evaluation, that is:

\[
A_{RFB} = \frac{\int ds \int_{Y_{cut}}^\infty dY \sum_q f_q^+(x_1(s, Y), x_2(s, Y)) (F - B)}{\int ds \int_{Y_{cut}}^\infty dY \sum_q f_q^-(x_1(s, Y), x_2(s, Y)) (F + B)} \quad (B.4)
\]

with

\[
f_q^\pm(x_1, x_2) = f_q(x_1) f_q(x_2) \pm f_q(x_1) f_q(x_2) \quad (B.5)
\]

The formula for \( A_O \) is very similar and it is obtained by replacing \( Y \) with \( p_z \). This leads to a different cut and a different Jacobian.

The expressions for \( A_C \) and \( A_E \) are also very similar between them and we present them together. In these two cases the cut is performed on the angle \( \theta_{cut} \) and therefore on the limits of integration for \( F \) and \( B \). These asymmetries are defined in the Lab frame, because the Lorentz transformation from the CM frame "squeezes" the final particles [17]. The angles of the outgoing \( e^\pm \) with respect to the z axis, denoted by \( \pm \theta \) respectively in the CM frame, are replaced by \( \theta_{cut} \) and \( \theta_{cut} \) in the Lab frame where the outgoing \( e^\pm \) no longer have the same direction. Therefore in the Lab frame, instead of the definitions (B.2), we have:

\[
(F/B)_C = \int_{-Y_{cut}}^{Y_{cut}} dY \sum_q f_q(x_1) f_q(x_2) \left( F_C - B_C \right) \quad (B.6)
\]

where the limit of integration \( cut \) is defined in terms of \( \theta_{cut} \) as \( cut = \cos \theta_{cut} \). Then (11) and (12) become:

\[
A_{C/E} = \int ds \int_{s/S}^1 dx_1 \sum_q f_q(x_1) f_q(x_2) \left( F_{C/E} - B_{C/E} \right) \quad (B.7)
\]

where \( x_2 = \frac{1}{s/S} \). \( J \) is now the Jacobian of the transformation \( (x_1, x_2) \rightarrow (x_1, s) \). The cut parameter which enters in the definitions (11) and (12) is not directly \( \theta_{cut} \) but the associated pseudorapidity \( \eta_{cut} = -\log \left( \tan \left( \theta_{cut}/2 \right) \right) \). For further details on the calculations sketched in this Appendix, see [26].

**Appendix C. Coefficients of the polynomial fit**

We have performed a numerical calculation of the asymmetries letting the three charges vary in the \( -1 < Q_i < 1 \) range. Then we have fitted the results with the rational function \( 12 \). We have found that only the even grade terms contribute to the results, so we neglect the odd grade terms.

Another point to note is that the formula \( 12 \) implies that all the coefficients \( a_{ijk} \) and \( b_{ijk} \) are defined up to a global multiplicative factor. To permit the comparison among the different types of asymmetries we have fixed \( a_{400} = 1 \) (or \( a_{400} = -1 \) for the C asymmetry that assumes opposite sign with respect to the others). However, if such type of models will be discovered at the LHC, this value will be fixed differently to match the experimental results. The coefficients values for our choice are listed in table \( C \). Note that this table contains only the statistical error and not the systematic error due to the choice of the PDFs.
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| \( A_{RB} \) | \( A_{O} \) | \( A_{C} \) | \( A_{F} \) |
|---------|---------|---------|---------|
| \((-0.52 \pm 0.02) \times 10^{-6}\) | \((-0.31 \pm 0.04) \times 10^{-6}\) | \((1.18 \pm 0.04) \times 10^{-6}\) | \((0.86 \pm 0.18) \times 10^{-6}\) |
| \((82 \pm 3) \times 10^{-6}\) | \((58 \pm 5) \times 10^{-6}\) | \((-17 \pm 5) \times 10^{-6}\) | \((140 \pm 21) \times 10^{-6}\) |
| \((9.9 \pm 1.5) \times 10^{-6}\) | \((9 \pm 2) \times 10^{-6}\) | \((-27 \pm 3) \times 10^{-6}\) | \((12 \pm 11) \times 10^{-6}\) |
| \((5.2 \pm 1.4) \times 10^{-6}\) | \((2 \pm 2) \times 10^{-6}\) | \((11 \pm 2) \times 10^{-6}\) | \((61 \pm 10) \times 10^{-6}\) |
| \((5 \pm 3) \times 10^{-6}\) | \((4 \pm 5) \times 10^{-6}\) | \((19 \pm 6) \times 10^{-6}\) | \((113 \pm 24) \times 10^{-6}\) |
| \((-18 \pm 4) \times 10^{-6}\) | \((-6 \pm 6) \times 10^{-6}\) | \((-148 \pm 7) \times 10^{-6}\) | \((-269 \pm 27) \times 10^{-6}\) |
| \((-80 \pm 3) \times 10^{-6}\) | \((-65 \pm 5) \times 10^{-6}\) | \((34 \pm 5) \times 10^{-6}\) | \((-177 \pm 22) \times 10^{-6}\) |

| \( a_{400} \) | \( a_{401} \) | \( a_{402} \) | \( a_{403} \) |
|---------|---------|---------|---------|
| 0.000009 \pm 0.000035 | 0.00023 \pm 0.00002 | -0.0014 \pm 0.00003 | 0.00018 \pm 0.000015 |

| \( b_{000} \) | \( b_{001} \) | \( b_{002} \) | \( b_{003} \) |
|---------|---------|---------|---------|
| \(-1.19 \pm 0.06) \times 10^{-6}\) | \(-3.19 \pm 0.12) \times 10^{-6}\) | \(-4.54 \pm 0.12) \times 10^{-6}\) | \(-7.0 \pm 0.09) \times 10^{-6}\) |
| \((121 \pm 7) \times 10^{-6}\) | \(-20.5 \pm 0.12) \times 10^{-6}\) | \(-4.54 \pm 0.12) \times 10^{-6}\) | \(-7.0 \pm 0.09) \times 10^{-6}\) |
| \((23 \pm 4) \times 10^{-6}\) | \((18 \pm 12) \times 10^{-6}\) | \((74 \pm 7) \times 10^{-6}\) | \((29 \pm 28) \times 10^{-6}\) |
| \((12 \pm 3) \times 10^{-6}\) | \((5 \pm 5) \times 10^{-6}\) | \((29 \pm 6) \times 10^{-6}\) | \((154 \pm 25) \times 10^{-6}\) |
| \((-58 \pm 26) \times 10^{-6}\) | \((51 \pm 41) \times 10^{-6}\) | \((-282 \pm 52) \times 10^{-6}\) | \((2392 \pm 204) \times 10^{-6}\) |
| \((-116 \pm 13) \times 10^{-6}\) | \((-207 \pm 20) \times 10^{-6}\) | \((408 \pm 27) \times 10^{-6}\) | \((-392 \pm 100) \times 10^{-6}\) |
| \((85 \pm 39) \times 10^{-6}\) | \((-82 \pm 62) \times 10^{-6}\) | \((-104 \pm 79) \times 10^{-6}\) | \((-1120 \pm 306) \times 10^{-6}\) |

| \( c_{000} \) | \( c_{001} \) | \( c_{002} \) | \( c_{003} \) |
|---------|---------|---------|---------|
| 1.90571 \pm 0.00004 | 1.90585 \pm 0.00006 | 2.28541 \pm 0.00008 | 2.1468 \pm 0.0003 |
| 0.036021 \pm 0.000016 | 0.036021 \pm 0.000016 | 0.04190 \pm 0.00002 | 0.03945 \pm 0.00008 |
| 0.002232 \pm 0.000006 | 0.002229 \pm 0.000009 | 0.002670 \pm 0.000012 | 0.00245 \pm 0.00005 |

| \( d_{010} \) | \( d_{011} \) | \( d_{012} \) | \( d_{013} \) |
|---------|---------|---------|---------|
| 1.40007 \pm 0.00018 | 1.3899 \pm 0.0003 | 1.3347 \pm 0.0004 | 1.2650 \pm 0.00015 |
| 3.8269 \pm 0.0003 | 3.823 \pm 0.0004 | 4.5879 \pm 0.0006 | 4.338 \pm 0.002 |
| -0.00386 \pm 0.00018 | -0.0033 \pm 0.0003 | -0.0035 \pm 0.0004 | -0.0103 \pm 0.0014 |
| -3.79461 \pm 0.00014 | -3.7943 \pm 0.0002 | -4.5504 \pm 0.0003 | -4.2761 \pm 0.0011 |
| -2.7984 \pm 0.0004 | -2.7775 \pm 0.0007 | -2.6661 \pm 0.0009 | -2.533 \pm 0.003 |
| -7.5882 \pm 0.0007 | 7.5998 \pm 0.0011 | 9.1030 \pm 0.0014 | -8.592 \pm 0.005 |
| 0.0081 \pm 0.0002 | 0.0061 \pm 0.0003 | 0.0098 \pm 0.0005 | 0.0168 \pm 0.0017 |
| 3.8004 \pm 0.0003 | 3.8021 \pm 0.0004 | 4.5567 \pm 0.0006 | 4.291 \pm 0.002 |
| 2.8005 \pm 0.0005 | 2.7871 \pm 0.0009 | 2.6705 \pm 0.0011 | 2.524 \pm 0.014 |
| 7.6032 \pm 0.0006 | 7.6124 \pm 0.0009 | 9.1181 \pm 0.0012 | 8.599 \pm 0.004 |
| -0.0023 \pm 0.0002 | -0.0026 \pm 0.0004 | -0.0120 \pm 0.0018 | -0.0120 \pm 0.0018 |
| -0.00009 \pm 0.00035 | 0.0002 \pm 0.0006 | -0.0021 \pm 0.0007 | 0.017 \pm 0.003 |

Table C.6: Coefficients of the fits for the four definitions of asymmetry.