From CKM Matrix to MNS Matrix: A Model Based on Supersymmetric $SO(10) \times U(2)_F$ Symmetry

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We construct a realistic model based on SUSY $SO(10)$ with $U(2)$ flavor symmetry. In contrast to the commonly used effective operator approach, 126-dimensional Higgses are used to construct the Yukawa sector. R-parity symmetry is thus preserved at low energies. The Dirac and right-handed Majorana mass matrices in our model have very small mixing, and they combine with the seesaw mechanism resulting in a large leptonic mixing. The symmetric mass textures arising from the left-right symmetry breaking chain of $SO(10)$ give rise to very good predictions: 15 masses (including 3 right-handed Majorana neutrino masses) and 6 mixing angles are predicted by 11 parameters. Both the vacuum oscillation and LOW solutions are favored for the solar neutrino problem.

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The flavor problem with hierarchical fermion masses and mixing has attracted a great deal of attention especially since the advent of the atmospheric neutrino oscillation data from Super-Kamiokande indicating non-zero neutrino masses. The non-zero neutrino masses give support to the idea of grand unification based on $SO(10)$ in which all the 16 fermions (including the right-handed neutrinos) can be accommodated in one single spinor representation. Furthermore, it provides a framework in which seesaw mechanism arises naturally. Models based on $SO(10)$ (and some with $E_6$) combined with a continuous or discrete flavor symmetry group have been constructed to understand the flavor problem. Most of the recent ones have used asymmetric or "lopsided" mass textures to account for the maximal mixing in the neutrino sector. Symmetric mass textures have less parameters and hence could lead to more predictive power. Naively one expects, for symmetric mass textures, six texture zeros in the quark sector. But it has been observed by Ramond, Roberts and Ross that the highest number of zeroes in the quark sector. But it has been observed by Ramond, Roberts and Ross [2] that the highest number of texture zeros has to be five, and using phenomenological analyses, they were able to arrive at five sets of up- and down-quark mass matrices with five texture zeros. Our analysis with recent experimental data and using CP conserving real symmetric matrices indicates that only one set (labeled set (v) in [3]) remains viable (see below). The aim of this paper is to construct a realistic model based on $SO(10)$ combined with $U(2)$ as the flavor group, utilizing this set of symmetric mass textures for charged fermions. We first discuss the viable phenomenology of mass textures followed by the model which accounts for it, and then the implications of the model for neutrino mixing are presented.

Mass Texture Analysis: Throughout this paper we consider CP conserving real mass matrices. We do not lose any generality in our results since the CP violating phases do not have significant contributions to other parameters. A more detailed analysis taking into account CP violating phases will be given elsewhere [3].

We consider the following mass textures at the GUT scale for the up-quark, down-quark and charged lepton sectors [2],

\[
\begin{align*}
M_u &= \begin{pmatrix} 0 & 0 & a \\ 0 & b & c \\ a & c & 1 \end{pmatrix} d, & M_d &= \begin{pmatrix} 0 & e & 0 \\ e & f & 0 \\ 0 & 0 & 1 \end{pmatrix} h \\
M_e &= \begin{pmatrix} 0 & e & 0 \\ e & -3f & 0 \\ 0 & 0 & 1 \end{pmatrix} h
\end{align*}
\]

(1)

with $a \approx b \ll c \ll 1$ and $e \ll f \ll 1$. After diagonalizing $M^T M$, one obtains the following non-negative mass eigenvalues and mass ratios:

\[
\begin{align*}
m_u &\approx \frac{a^2 bd}{\text{max} \{b, c^2\}} \\
m_d &\approx \frac{e^2 fh}{e^2 + f^2} \\
m_s &\approx \frac{(2e^2 f + f^2) h}{e^2 + f^2} \\
m_b &\approx h \\
m_e &\approx \frac{3e^2 fh}{e^2 + 9f^2} \\
m_{\mu} &\approx \frac{6e^2 fh + 27f^2 h}{e^2 + 9f^2}, & m_{\tau} &\approx h(2)
\end{align*}
\]

\[
\begin{align*}
\frac{m_d}{m_c} &\approx \frac{e^2 + 9f^2}{3(e^2 + f^2)} \approx 3 + O\left(\frac{e^2}{f^2}\right) \\
\frac{m_e}{m_{\mu}} &\approx \frac{2e^4 + 19e^2 f^2 + 9f^4}{6e^4 + 33e^2 f^2 + 27f^4} \approx 1 + O\left(\frac{e^2}{f^2}\right) \\
\frac{m_b}{m_{\tau}} &\approx 1
\end{align*}
\]

These analytic expressions are very good approximations to the exact eigenvalues. It can be easily seen that the phenomenologically favored Georgi-Jarlskog relations [1]
are obtained
\[ m_d \simeq 3m_e, \quad m_s \simeq \frac{1}{3}m_\mu, \quad m_b = m_\tau \quad (4) \]

As we will see later, these relations between the down-quark sector and the charged lepton sector can be naturally achieved in SO(10).

In order to explain the smallness of the neutrino masses, we will adopt the type I seesaw mechanism which requires both Dirac and right-handed Majorana mass matrices to be present in the Lagrangian. The right-handed Majorana mass matrix \( M_{\nu RR} \) is at present an unknown sector. The only constraint is that it must be constructed in such a way that it gives a favored low energy Majorana neutrino mass matrix \( M_{\nu LL} \) via the seesaw mechanism. We first consider the low energy (left-handed) Majorana neutrino mass matrix to get some insights into the structure of \( M_{\nu RR} \). We adopt the hierarchical scenario: \( |m_{\nu_3}| > |m_{\nu_2}|, |m_{\nu_1}| \), to accommodate the experimental neutrino oscillation data. One way to achieve a large mixing in the \( \nu_\mu - \nu_\tau \) sector and at the same time a large mass splitting between \( m_{\nu_2} \) and \( m_{\nu_3} \) is to consider
\[
M_{\nu LL} \sim \begin{pmatrix} 0 & 0 & t \\ 0 & 1 & 1 \\ t & 1 & 1 \end{pmatrix} \Lambda \quad (5)
\]

A generic feature of the mass matrix of this type is that it leads to a large mixing in both \( \nu_e - \nu_\mu \) and \( \nu_\mu - \nu_\tau \) sectors (the so-called bimaximal mixing) for a broad range of \( t, 0 \leq t \ll 1 \). In order to have a large mass splitting, we require \( t \ll 1 \). The three eigenvalues of this mass matrix keeping only the dominant orders are given by, in units of \( \Lambda \),
\[
|m_1| \simeq \frac{t}{\sqrt{2}} - \frac{t^2}{8} - \frac{3t^3}{64\sqrt{2}} \\
|m_2| \simeq \frac{t}{\sqrt{2}} + \frac{t^2}{8} - \frac{3t^3}{64\sqrt{2}} \\
|m_3| \simeq 2 + \frac{t^2}{4} \quad (6)
\]

The diagonalization matrix up to order \( O(t^2) \) is given by
\[
U_{\nu LL} = \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
0 & 0 & 1
\end{pmatrix} \frac{1}{\sqrt{2}} \left(\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array}\right)
\]

Note that what the neutrino mixing matrix really means is the mismatch between the charged lepton flavor basis and the neutrino flavor basis analogous to the Cabbibo-Kobayashi-Maskawa quark mixing matrix \( V_{CKM} \), and is the Maki-Nakagawa-Sakata matrix \( U_{MNS} \) defined as,
\[
U_{MNS} \equiv U_{eL}U_{\nu LL}^\dagger = \begin{pmatrix}
U_{\nu_1} & U_{\nu_2} & U_{\nu_3} \\
U_{\mu_1} & U_{\mu_2} & U_{\mu_3} \\
U_{\tau_1} & U_{\tau_2} & U_{\tau_3}
\end{pmatrix} \quad (8)
\]

Since the mixing matrix in the charged lepton sector \( U_{eL} \) is almost diagonal, combining \( U_{\nu LL}^\dagger \) and \( U_{eL} \) results in a nearly bimaximal mixing pattern in the lepton mixing matrix \( U_{MNS} \). The squared mass difference between \( m_{\nu_1}^2 \) and \( m_{\nu_2}^2 \) is of the order of \( O(t^3) \) while the squared mass difference between \( m_{\nu_2}^2 \) and \( m_{\nu_3}^2 \) is of the order \( O(1) \). It is clear that the mass matrix eq.\((4)\) naturally leads to the phenomenologically favored result
\[
|\Delta m_{23}^2| \gg |\Delta m_{12}^2| \quad (9)
\]

Depending on the value of \( t \), both vacuum oscillation (VO) solution and large angle MSW (LAMSW) solution are possible. The VO solution suggests that \( \frac{\Delta m_{23}^2}{\Delta m_{12}^2} \simeq 10^{-7} \). Since \( \frac{\Delta m_{23}^2}{\Delta m_{12}^2} \sim t^3 \), one can see immediately that \( t \sim 10^{-3} \). On the other hand, the LAMSW solution suggests that \( \frac{\Delta m_{23}^2}{\Delta m_{12}^2} \simeq 10^{-2} \), and \( t \) is then required to be \( \sim 10^{-1} \). Since the element \( U_{e\nu_1} \) in \( U_{MNS} \) is proportional to \( t \), an accurate measurement of \( U_{e\nu_1} \) thus could provide some hints to single out one of the solar oscillation solutions if the neutrino mixing pattern is indeed bimaximal.

We assume that the Dirac neutrino mass matrix has the same texture (that is, positions of the zeros) as the up-quark mass matrix
\[
M_{\nu LR} = \begin{pmatrix} 0 & 0 & \alpha \\
0 & \beta & \gamma \\
\alpha & \gamma & 1 \end{pmatrix} \eta \quad (10)
\]

with \( \alpha \simeq \beta \ll \gamma \ll 1 \). We see later that \( M_u \) and \( M_{\nu LR} \) can be in fact identical in SO(10). To achieve \( M_{\nu LL} \) of the form of eq.\((5)\) one needs a right-handed neutrino Majorana mass matrix of the same texture as \( M_{\nu LR} \)
\[
M_{\nu RR} = \begin{pmatrix} 0 & 0 & \delta_1 \\
0 & \delta_2 & \delta_3 \\
\delta_1 & \delta_3 & 1 \end{pmatrix} \quad (11)
\]

with \( \delta_1, \delta_2, \delta_3 \ll 1 \).
\[
\delta_1 \simeq \frac{\alpha^2}{2\alpha - 2\alpha \gamma + \gamma^2 t}, \quad \delta_2 \simeq \frac{\beta^2 t}{2\alpha - 2\alpha \gamma + \gamma^2 t}, \\
\delta_3 \simeq \frac{\alpha(\gamma - \beta) + \beta t}{2\alpha - 2\alpha \gamma + \gamma^2 t}
\]  

(12)

After seesaw mechanism takes place, 
\[M_{\nu LL} = -M_{\nu LR}^T M_{\nu RR}^{-1} M_{\nu LR} \]  

(13)

\[M_{\nu LL} \] of the form eq. (11) results. It is interesting to see that the matrix operation in eq. (13) is form invariant. That is to say, \(M_{\nu LL} \) has the same texture as that of \(M_{\nu LR} \) and \(M_{\nu RR} \). Since \(\delta_1, \delta_2 \) and \(\delta_3 \) are much smaller than 1, the mixing in the right-handed neutrino mass matrix \(M_{\nu RR} \) is generally small. Since the mixings in both \(M_{\nu LR} \) and \(M_{\nu RR} \) are small, our model falls into the category that the large neutrino mixing is purely due to the matrix operations in the seesaw mechanism that charged lepton, Dirac neutrino and right-handed neutrino mixings are small, as classified in Ref. [6]. We note that with the structure of \(M_{\nu LL} \) in eq. (14) we find it hard, though not impossible, to accommodate the small angle MSW solution in our model [3].

We emphasize that, in the neutrino sector, we have been able to get a large mixing and at the same time a large mass splitting by using a symmetric Dirac mass matrix and a hierarchical right-handed mass matrix with very small mixing; the latter gives three superheavy hierarchical right-handed neutrino masses. Asymmetric Dirac mass matrices have been used before to get a large mixing and at the same time \(U(2) \) breaks down in two steps:
\[U(2) \rightarrow \text{nothing} \]  

(15)

with \(\epsilon' \ll \epsilon \ll 1 \) and \(M \) is the UV cut-off of the effective theory mentioned before. These small parameters \(\epsilon \) and \(\epsilon' \) are the ratios of the vacuum expectation values of the flavon fields to the cut-off scale. Note that since
\[\psi_3 \psi_3 \sim 1_S, \quad \psi_3 \psi_a \sim 2, \quad \psi_a \psi_b \sim 2 \otimes 2 = 1_A \oplus 3 \]  

(16)

the only relevant flavon fields are in the \(1_A, 2 \) and 3 dimensional representations of \(U(2) \), namely,
\[A^{ab} \sim 1_A, \quad \phi^a \sim 2, \quad S^{ab} \sim 3 \]  

(17)

Because we are confining ourselves to symmetric mass textures, we use only \(\phi^a \) and \(S^{ab} \). Since all the 16 observed matter fields of each family fall nicely into a 16-dimensional spinor representation of \(SO(10) \), the most general superpotential that generates fermion masses for a \(SO(10) \times U(2) \) model has the following very simple form
\[W = H(\psi_3 \psi_3 + \psi_3 \phi^a M \psi_a + \psi_a S^{ab} M \psi_b) \]  

(18)

In a specific \(U(2) \) basis,
\[\langle \phi \rangle \bigg/ M \sim O \left( \begin{array}{c} \epsilon' \\ \epsilon \end{array} \right), \quad \langle S^{ab} \rangle \bigg/ M \sim O \left( \begin{array}{cc} \epsilon' & \epsilon' \\ \epsilon' & \epsilon \end{array} \right) \]  

(19)

Here we have indicated the VEVs all the flavon fields could acquire for symmetry breaking in eq. (14). The mass matrix would take the following form
\[M \sim O \left( \begin{array}{ccc} \epsilon' & \epsilon' & \epsilon' \\ \epsilon' & \epsilon & \epsilon \\ \epsilon' & \epsilon & 1 \end{array} \right) \]  

(20)

In \(SO(10) \), at the renormalizable level, only three types of Higgs fields can couple to fermions,
\[16 \otimes 16 = 10_S \oplus 120_A \oplus 126_S \]  

(21)

namely, 10, 120, and 126, where the subscripts \(S \) and \(A \) refer to the symmetry property under interchanging two family indices in the Yukawa couplings \(Y_{ab} \). That is,
\[Y_{ab}^{10} = Y_{ba}^{10}, \quad Y_{ab}^{120} = -Y_{ba}^{120}, \quad Y_{ab}^{126} = Y_{ba}^{126} \]  

(22)

\(\phi^a \) and \(S^{ab} \) can couple to only 10 and 120; 126 has no role in giving rise to mass textures. Note that \(SO(10) \) can break down to SM through many different breaking
chains. Different breaking chains give rise to different mass relations among the up-quark, down-quark, charged lepton and neutrino sectors. Since we are interested in symmetric mass textures, a natural choice is the left-right symmetric route, that is,

$$SO(10) \rightarrow SU(4) \times SU(2)_{L} \times SU(2)_{R}$$
$$\rightarrow SU(3) \times SU(2)_{L} \times SU(2)_{R} \times U(1)_{B-L}$$
$$\rightarrow SU(3) \times SU(2)_{L} \times U(1)_{Y}$$
$$\rightarrow SU(3) \times U(1)_{EM}$$

(23)

We have the up-quark sector related to the neutrino sector, and the down-quark sector to the charged lepton sector. A Clebsch-Gordan coefficient ($-3$) appears in the lepton sectors when the $SU(4) \times SU(2)_{L} \times SU(2)_{R}$ components $(15, 2, 2)$ in $126$ are involved in the Yukawa couplings. This factor of $(-3)$ is very crucial for obtaining the Georgi-Jarlskog relations as we have seen in the previous section. The general fermion Dirac mass matrices are thus given schematically by

$$M_{u} \sim Y_{ab}^{10} \langle 10^{+} \rangle + \gamma_{ab}^{126} \langle 126^{+} \rangle$$

and general Majorana mass matrices are given by

$$M_{\nu,RR} \sim \gamma_{ab}^{126} \langle 126^{0} \rangle$$

(25)

$$M_{\nu,LL} \sim \gamma_{ab}^{126} \langle 126^{+} \rangle$$

(26)

where various VEVs are those of the neutral components of $SO(10)$ representations as indicated below (with subscripts referring to the symmetry groups on the r.h.s. of eq. (23); and $+/0/-$ referring to the sign of the hypercharge $Y$).

$$\langle 10^{+} \rangle : (1, 0)_{31} \subset (1, 2, 1)_{321} \subset (1, 2, 2, 0)_{3221} \subset (1, 2, 2)_{422} \subset 10$$
$$\langle 10^{-} \rangle : (1, 0)_{31} \subset (1, 2, -1)_{321} \subset (1, 2, 2, 0)_{3221} \subset (1, 2, 2)_{422} \subset 10$$

$$\langle 126^{+} \rangle : (1, 0)_{31} \subset (1, 2, 1)_{321} \subset (1, 2, 2, 0)_{3221} \subset (15, 2, 2)_{422} \subset 126$$
$$\langle 126^{0} \rangle : (1, 0)_{31} \subset (1, 2, -1)_{321} \subset (1, 2, 2, 0)_{3221} \subset (15, 2, 2)_{422} \subset 126$$
$$\langle 126^{-} \rangle : (1, 0)_{31} \subset (1, 1, 0)_{321} \subset (1, 1, 3, -2)_{3221} \subset (10, 1, 3)_{422} \subset 126$$

(27)

(28)

A remark is in order here. Some models avoid the use of $126$ dimensional Higgses by introducing nonrenormalizable operators of the form $f_{a}f_{b}(16)_{a}(16)_{b}$. Such models appear to be less constrained due to the inclusion of nonrenormalizable operators. Also, a discrete symmetry, the R-parity symmetry, must be imposed by hand to avoid dangerous Baryon number violating terms in the effective potential at low energies which otherwise could lead to fast proton decay rate. Here we use $126$ dimensional representation of Higgses which has the advantage that R-parity symmetry is automatic [14]. The $126$ representation has been used in model building before [3]. It is to be noted that the contribution of the $126$-dimensional representation to the $\beta$-function makes the model nonperturbative (with the onset of the Landau pole) above the unification scale $M_{GUT}$. One could view our model as an effective theory valid below this scale where coupling constants are perturbative.

Other breaking chains of $SO(10)$ have been consid-
Matter fields:

\[ \psi_a \sim (16, 2)^{++} \quad (a = 1, 2) \]
\[ \psi_3 \sim (16, 1)^{+++} \]  \hspace{1cm} (29)

Higgs fields for the mass matrices:

\[ (10, 1) : \quad T_1^{++}, T_2^{++}, T_3^{++} \]
\[ T_4^{--}, T_5^{--} \]
\[ (26', 1) : \quad C^{--}, C_1^{++}, C_2^{++} \]  \hspace{1cm} (30)

Flavon fields:

\[ (1, 2) : \quad \phi_1^{++}, \phi_2^{++}, \Phi^{++} \]
\[ (1, 3) : \quad S_1^{--}, S_2^{--}, \Sigma^{++} \]  \hspace{1cm} (31)

Note that, the entries in the parenthesis indicate the SO(10) and \( U(2) \) representations respectively. The superscript +/- indicates the charges under \( Z_2 \times Z_2 \times Z_2 \) symmetry. Various Higgs fields acquire VEVs in the following directions

\[ T_1 : \quad \langle 10^+_1 \rangle, \quad \langle 10^-_1 \rangle \]
\[ T_2, T_3, T_4 : \quad \langle 10^+_{3,4} \rangle \]
\[ T_5 : \quad \langle 10^- \rangle \]
\[ C : \quad \langle 26' \rangle \]
\[ C_1, C_2 : \quad \langle 26_{1,2}^0 \rangle \]  \hspace{1cm} (32)

and

\[ \langle 10^+_1 \rangle = \langle 10^+_3 \rangle, \quad \langle 10^- \rangle = \langle 10^-_5 \rangle \]
\[ \langle 26'_1 \rangle = \langle 26'_2 \rangle \]  \hspace{1cm} (33)

(Note that, with a \( 26_H \) acquiring VEV, there must be a conjugate \( 26_H \) acquiring VEV to cancel the D-term. Since \( 26_H \) does not couple to \( 16_i \), it has no role in the construction of the Yukawa sector.) The needed flavon VEVs are given by

\[ \langle \phi_1 \rangle = \left( \begin{array}{c} \epsilon' \\ 0 \end{array} \right), \quad \langle \phi_2 \rangle = \left( \begin{array}{c} 0 \\ \epsilon \end{array} \right) \]
\[ \langle S_{1(2)} \rangle = \left( \begin{array}{c} 0 \\ \epsilon' \end{array} \right), \quad \langle S_{2(2)} \rangle = \left( \begin{array}{c} 0 \\ 0 \end{array} \right) \]
\[ \Phi = \left( \begin{array}{c} \delta_1 \\ \delta_2 \end{array} \right), \quad \Sigma = \left( \begin{array}{c} 0 \\ 0 \end{array} \right) \]  \hspace{1cm} (34)

Our \( (Z_2)^3 \) charge assignments give rise to a unique superpotential:

\[ W = W_{\text{Dirac}} + W_{\text{VR}} \]  \hspace{1cm} (35)

\[ W_{\text{Dirac}} = \psi_3 \psi_3 T_1 + \frac{1}{M} \psi_3 \psi_3 \left( T_2 \phi_1 + T_3 \phi_2 \right) \]
\[ + \frac{1}{M} \psi_3 \psi_3 (T_4 + C) S_{1(2)} + \frac{1}{M} \psi_3 \psi_3 T_5 S_{1(2)} \]
\[ W_{\text{VR}} = \psi_3 \psi_3 C_1 + \frac{1}{M} \psi_3 \psi_3 \Phi C_2 + \frac{1}{M} \psi_3 \psi_3 \Sigma C_2 \]  \hspace{1cm} (36)

The mass matrices then can be read from the superpotential to be

\[ M_{\mu,\nu \mu R} = \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & \langle 10^+_1 \rangle \epsilon' & \langle 10^+_1 \rangle \epsilon' \\ 0 & \langle 10^+_1 \rangle \epsilon' & \langle 10^+_1 \rangle \epsilon' \end{array} \right) \]
\[ M_{\nu \mu R} = \left( \begin{array}{ccc} 0 & r_2 \epsilon' & 0 \\ r_2 \epsilon' & 0 & 0 \\ 0 & 0 & 1 \end{array} \right) \]  \hspace{1cm} (37)

\[ M_{d, e} = \left( \begin{array}{ccc} 0 & \langle 10^+_1 \rangle \epsilon' & 0 \\ \langle 10^+_1 \rangle \epsilon'(1, -3) 26' \epsilon & 0 & 0 \\ 0 & 0 & 1 \end{array} \right) \]
\[ M_D = \left( \begin{array}{ccc} 0 & \epsilon'(1, -3) 26' & 0 \\ 0 & 0 & 1 \end{array} \right) \]  \hspace{1cm} (38)

where

\[ M_U \equiv \langle 10^+_1 \rangle, \quad M_D \equiv \langle 10^-_1 \rangle \]  \hspace{1cm} (39)

\[ r_2 \equiv \langle 10^+_2 \rangle / \langle 10^+_1 \rangle, \quad r_4 \equiv \langle 10^+_4 \rangle / \langle 10^+_1 \rangle \]
\[ p \equiv \langle 26' \rangle / \langle 10^-_1 \rangle \]  \hspace{1cm} (40)

The right-handed neutrino mass matrix is

\[ M_{\text{VR}} = \left( \begin{array}{ccc} 0 & 0 & \langle 26'_0 \rangle \delta_1 \\ \langle 26'_2 \rangle \delta_1 & \langle 26'_2 \rangle \delta_2 & \langle 26'_2 \rangle \delta_3 \end{array} \right) \]
\[ = \left( \begin{array}{ccc} 0 & 0 & \delta_1 \\ 0 & \delta_2 & \delta_3 \end{array} \right) M_R \]  \hspace{1cm} (41)

with \( M_R \equiv \langle 26'_1 \rangle \). We have thus arrived at the mass matrices shown in eqs. (32) and \( (33) \).

RGE Analysis and Results: In order to obtain the input parameters at the GUT scale, first we need to know various Yukawa couplings (the diagonal elements) and mixing angles at the GUT scale. We use the expressions derived from 1-loop RGEs given by [13, 14]

\[ m_u = Y_u^0 R_u \eta_h B_3^3 v_u, \quad m_c = Y_c^0 R_u \eta_h B_3^3 v_u \]
\[ m_t = Y_t^0 R_u B_3^3 v_u \]
\[ m_d = Y_d^0 R_d \eta_d v_d, \quad m_s = Y_s^0 R_d \eta_d v_d \]
\[ m_b = Y_b^0 R_d \eta_d B_1^1 v_d \]
\[ m_e = Y_e^0 R_e v_d, \quad m_\mu = Y_\mu^0 R_e v_d \]
\[ m_\tau = Y_\tau^0 R_e v_d \]  \hspace{1cm} (42)

\[ V_{ij} = \{ V_{ij}^0 \} B_i^1, \quad I_{ij} = ud, us, cd, cs, tb \]  \hspace{1cm} (43)
where $V_{ij}$ are CKM matrix elements; quantities with superscript 0 are evaluated at GUT scale, and all the $m_i$ and $V_{ij}$ are the experimental values \[13\]. We will assume $\tan \beta = \frac{u_0}{v_\omega} = 10$ and $v = \sqrt{v_u^2 + v_d^2} = \frac{246}{\sqrt{2}}$ GeV. The running factor $\eta_f$ includes QCD + QED contributions: For $f = b, c$, $\eta_f$ is for the range $m_t$ to $m_t$, and for $f = u, d, s$, $\eta_f$ is for the range 1 GeV to $m_t$;

$$
\eta_u = \eta_d = \eta_s = 2.38^{+0.24}_{-0.19}, \\
\eta_c = 2.05^{+0.13}_{-0.11}, \\
\eta_b = 1.53^{+0.04}_{-0.01}.
$$

$R_{u,d,c}$ are contributions of the gauge-coupling constants running from weak scale $M_z$ to the SUSY breaking scale, taken to be $m_t$, with the SM spectrum, and from $m_t$ to the GUT scale with MSSM spectrum;

$$
R_u = 3.53^{+0.06}_{-0.07}, \quad R_d = 3.43^{+0.07}_{-0.06}, \quad R_c = 1.50.
$$

$B_i$ is the running induced by large top-quark Yukawa coupling defined by

$$
B_i = \exp \left[ \frac{-1}{6\pi^2} \int_{\ln M_{\text{SUSY}}}^{\ln M_{\text{GUT}}} Y_i^2(\mu)d(\ln \mu) \right] \quad (44)
$$

which varies from 0.7 to 0.9 corresponding to the perturbative limit $Y_t^0 \approx 3$ and the lower limit $Y_t^0 \approx 0.5$ imposed by the top-pole mass.

In order to have a good fit to these values, we first obtain the following approximate analytic expressions

$$
a \simeq \sqrt{\frac{Y_u^0 Y_u^0}{Y_t^0 (Y_t^0 + c^2 Y_t^0)}}, \quad b \simeq c^2 + \frac{Y_t^0}{Y_t^0}
$$

$$
d \simeq \frac{Y_t^0}{Y_t^0}
$$

$$
e \simeq \sqrt{\frac{Y_d^0 - Y_c^0}{Y_d^0 - 2Y_t^0 Y_d^0 Y_t^0}}, \quad f \simeq \frac{Y_t^0 - Y_d^0}{Y_t^0}
$$

$$
h = Y_t^0
$$

(45)

With the GUT scale values of $Y_u^0, Y_d^0$ and $Y_t^0$, the three parameters $c, f$, and $h$ in the down-quark and charged lepton sectors are uniquely determined. With the GUT scale values of $Y_u^0, Y_d^0$ and $Y_t^0$, the parameters $c, f$, and $h$ in the down-quark and charged lepton sectors are uniquely determined. Using the GUT scale value of the Cabibbo angle, $V_{us}$, the value of $c$ is determined. At the GUT scale which is taken to be $M_{\text{GUT}} = 2.39 \times 10^{16}$ GeV, with $g_1 = g_2 = g_3 = 0.7530$, our input parameters are chosen to be:

$$
a = \alpha = 0.00226, \quad b = \beta = 0.00381, \quad c = \gamma = 0.0328, \quad d = \eta = 0.572, \quad e = 0.00403, \quad f = 0.0195, \quad h = 0.0678, \quad \delta_1 = 0.00116, \quad \delta_2 = 3.32 \times 10^{-5}, \quad \delta_3 = 0.0152
$$

$$
M_R = 1.32 \times 10^{14}$ GeV
$$
(46)

These parameters are related to $c$ and $c'$ given in eq.\[34\] and Higgs VEVs and their ratios given in eq.\[39\]. $\delta_i$'s could be obtained using $\lambda' = 1 \times 10^{-3}$ in eq.\[12\]; $\lambda$ is expressed in terms of $\frac{m^2}{M^2_R}$ due to eq.\[13\]. The Yukawa couplings in the down-quark and charged lepton sectors are then given by

$$
Y_d^0 = 0.00005441, \quad Y_e^0 = 0.001374, \quad Y_{\mu}^0 = 0.06779
$$

$$
Y_{\tau}^0 = 0.0001880, \quad Y_{\mu}^0 = 0.003979, \quad Y_{\tau}^0 = 0.06779
$$

(47)

and various ratios are given by

$$
\frac{Y_d^0}{Y_e^0} = 2.895, \quad \frac{Y_e^0}{Y_{\mu}^0} = \frac{1}{2.895}, \quad \frac{Y_{\tau}^0}{Y_{\mu}^0} = 1
$$

(48)

which agree with Georgi-Jarlskog relations.

Having determined the GUT scale values of these elements, we then numerically solve the one-loop RGEs for the MSSM spectrum with three right-handed neutrinos \[7\] from GUT scale to the effective right-handed neutrino mass scale, $M_R \simeq 1.32 \times 10^{14}$ GeV. At $M_R$, seesaw mechanism is implemented. We then run the MSSM RGEs \[4\] from $M_R$ down to the SUSY breaking scale $m_t \simeq 176$ GeV, and then the SM RGEs from $m_t$ to $M_z = 91.187$ GeV. The light neutrino RGEs \[7\] are also used from $M_R$ to $M_z$. Predictions obtained at $M_z$ are summarized in Table I, taking into account the SUSY threshold corrections \[7\].

$$
\Delta_a = -0.10, \quad \Delta_b = -0.25
$$

They are to be compared with the values at $M_z$ calculated from the experimental values by the authors of \[20\]. The quark mixing matrix $V_{\text{CKM}}$ at $M_z$ is predicted to be

$$
|V_{\text{CKM,predict}}| = |V_{u_L} V_{d_L}^T|
$$
with $\Delta m^2_{12}$ but have no observable effects on down-quark and
matrix are given by

$$m_{\nu_1} = 2.0052 \times 10^{-4}eV$$
$$m_{\nu_2} = 2.0123 \times 10^{-4}eV$$
$$m_{\nu_3} = 0.05574eV$$

(51)

and the resulting squared mass differences are

$$\Delta m^2_{12} = 2.87 \times 10^{-10}eV^2$$

(52)

The lepton mixing matrix is given by

$$|U_{MNS,predict}| = |U_{eL}U^\dagger_{\nu LL}|$$

(53)

$$= \begin{pmatrix}
0.6710 & 0.7396 & 0.0527 \\
0.5410 & 0.4397 & 0.7169 \\
0.5070 & 0.5096 & 0.6952
\end{pmatrix}$$

This translates into

$$\sin^2 2\theta_{atm} \equiv 4|U_{\nu e_1}|^2(1 - |U_{\nu e_3}|^2) = 0.9992$$
$$\sin^2 2\theta_{\odot} \equiv 4|U_{\nu e_2}|^2(1 - |U_{\nu e_3}|^2) = 0.9912.$$  

(54)

These values agree with the Super-Kamiokande atmospheric neutrino oscillation data \cite{2}, \cite{3}, and the solar VO solution \cite{2}. And the (1, 3) element of $U_{MNS}$ is given by $|U_{\nu e_3}| = 0.0527$ which is far below the bound by the CHOOZ experiment $|U_{\nu e_3}| \lesssim 0.16$ \cite{2}. The three eigenvalues of the right-handed neutrino Majorana mass matrix are given by

$$M_{RR_1} \simeq 2.963 \times 10^7 GeV$$
$$M_{RR_2} \simeq 2.643 \times 10^8 GeV$$
$$M_{RR_3} \simeq 1.319 \times 10^{14} GeV$$

(55)

We can have the LOW solution (a LAMSW solution with $\Delta m^2_{12} \sim 10^{-6} - 10^{-7}eV^2$) with

$$\delta_1 = 0.001147, \quad \delta_2 = 0.0002354, \quad \delta_3 = 0.01675$$

(56)

$$M_R = 1.615 \times 10^{13} GeV$$

These change the predictions of $m_{u,c,t}$ by less than 1% but have no observable effects on down-quark and charged lepton masses, and the CKM matrix remains essentially the same \cite{3}. In the neutrino sector, we get

$$m_{\nu_1} = 0.001626eV$$
$$m_{\nu_2} = 0.001650eV$$
$$m_{\nu_3} = 0.06303eV$$

(57)

and the squared mass differences are

$$\Delta m^2_{23} = 3.973 \times 10^{-3}eV^2$$
$$\Delta m^2_{12} = 1.298 \times 10^{-7}eV^2$$

(58)

The lepton mixing matrix is given by

$$|U_{MNS,predict}| = |U_{eL}U^\dagger_{\nu LL}|$$

(59)

$$= \begin{pmatrix}
0.6665 & 0.7418 & 0.07428 \\
0.5511 & 0.4231 & 0.7192 \\
0.5021 & 0.5202 & 0.6909
\end{pmatrix}$$

The element $|U_{\nu e_3}|$ is predicted to be 0.07428, which is less than the experimental upper bound. The three right-handed neutrino eigenvalues are predicted to be

$$M_{RR_1} \simeq 9.558 \times 10^7 GeV$$
$$M_{RR_2} \simeq 8.453 \times 10^8 GeV$$
$$M_{RR_3} \simeq 1.615 \times 10^{13} GeV$$

(60)

It is also possible to have the LAMSW solution with

$$\delta_1 = 0.001082, \quad \delta_2 = 0.0009870, \quad \delta_3 = 0.02238$$

(61)

$$M_R = 2.415 \times 10^{12} GeV$$

The predictions in the quark and the charged lepton sectors remain the same. In the neutrino sector, we get

$$m_{\nu_1} = 0.01089eV$$
$$m_{\nu_2} = 0.01206eV$$
$$m_{\nu_3} = 0.09999eV$$

(62)

and the squared mass differences are

$$\Delta m^2_{23} = 9.851 \times 10^{-3}eV^2$$
$$\Delta m^2_{12} = 2.752 \times 10^{-7}eV^2$$

(63)
The lepton mixing matrix is given by

\[ |U_{\text{MNS,predict}}| = |U_{eL}^\dagger U_{\nu LL}| \]

\[ = \begin{pmatrix} 0.6439 & 0.7486 & 0.1580 \\ 0.6045 & 0.3712 & 0.7049 \\ 0.4690 & 0.5494 & 0.6915 \end{pmatrix} \]  \quad (64)

The element \( |U_{e23}| \) is predicted to be 0.1580 which is right at the experimental bound \( |U_{e23}| \lesssim 0.16 \). The three right-handed neutrino eigenvalues are given by

\[ M_{RR_1} \approx 5.732 \times 10^6 \text{GeV} \]
\[ M_{RR_2} \approx 1.177 \times 10^9 \text{GeV} \]
\[ M_{RR_3} \approx 2.417 \times 10^{12} \text{GeV} \]  \quad (65)

We note that a \( |U_{e23}| \) value of less than 0.1580 would lead to \( \Delta m_{23}^2 > 10^{-2} \text{eV}^2 \) leading to the elimination of the LAMSW solution in our model. This is a characteristic of the LAMSW solution with \( \Delta m_{23}^2 \gtrsim 10^{-5} \text{eV}^2 \).

**Note added:** The form invariance of eq. (13) \( - M_{\nu LL} \) having the same texture as that of \( M_{\nu LR} \) and \( M_{\nu RR} \) – also occurs in a model of neutrino mixing [24] which uses different symmetric mass matrices.

**Summary:** We have constructed a realistic model based on SUSY \( SO(10) \) combined with \( U(2) \) flavor symmetry. The up-quark sector is related to the Dirac neutrino sector, and the down-quark sector is related to the charged lepton sector via \( SO(10) \) symmetry. The inter-family hierarchy is achieved via \( U(2) \) symmetry. In contrast to the commonly used effective operator approach, we use 126-dim Higgses to construct the Yukawa sector. \( R \)-parity symmetry is thus automatically preserved at low energies. In our model, the Dirac and right-handed Majorana neutrino mass matrices which have very small mixing combine with the seesaw mechanism resulting in a large mixing in the lepton sector. The symmetric mass textures arising from the left-right symmetry breaking chain of \( SO(10) \) which we have considered give rise to very good predictions; 15 masses (including the right-handed neutrino masses) and 6 mixing angles are predicted by 11 parameters. Our model favors the vacuum oscillation and LOW solutions to the solar neutrino problem.

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