Construction Scheme of Proof Based on Assimilation and Accommodation Processes: Theorem of Number Theory

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Abstract. Number theory is a branch of mathematics that deals with the properties of natural numbers, aimed at finding interesting and unexpected relationships of various types of numbers, then proving that the relationship is true. Problem solutions and proof theorems of number theory often require a strong mathematical background. Weak background, such as: giving examples only applies to definitions not to theorems and lemmas, understanding of the definitions and proven theorems, ways of looking at new theorems built on definitions, and previous theorems, and knowledge of when a definitions or lemma-theorems can be used; enough to affect the ability to solve the problem of number theory. So it is necessary to explore the possibility of other problems in the activity of constructing proof, which in this study uses the framework of the process of assimilation and accommodation, followed by interviews based on student responses. The results obtained are 3 groups of construction schemes, namely: 1) schemes using definitions and theorems, 2) schemes using theorems, and 3) schemes using definitions. Within the three groups there are variations of the problem found that the diantranya occurs is dis-equilibrium in the construction scheme group with theorems and definitions; and group construction schemes by definition only. His advice is to reflect on the ability to understand definitions, theorems and lemmas; and the development of lecture designs that can make the activity of constructing proof into routine activities.

1. Introduction

The proof of a mathematical theorem is a logical argument given in accordance with the rules of a deductive system, and as a justification for the truth of a theorem's statement, and is a fundamental part of the mathematical thinking process [1], [2], [3], and [4]. The need to understand and especially write proof in mathematics lectures is very important [5], [6], [7]. Proof involves reasoning, conviction and communication and helps meaningful learning. Proofs can be used to show students that understanding and performing mathematics means more than just learning to execute certain procedures [6]. According to [8], proving is “a complex mathematical activity with logical, conceptual, social and problem-solving dimensions.”

Even [9] and [10] revealed that the proof aims to help in deciding whether and why our answers are logical, developing the habit of giving arguments, and making investigating activities an integral part of every solution and is a process that can enhance understanding of concepts. Meanwhile, [11] explains that the purpose of proof is to: (1) explain, (2) systemization, (3) communication, (4) discovering new results, (5) consideration of a definition, (6) developing intuition, and (7) provides autonomy. However, difficulties in learning and learning proof are well recognized internationally [12]. Many studies state that proof and proof are very difficult concepts for students [13], [14]. Difficulties of students in
constructing proof are found in almost all subjects that require proof (deductive thinking). According to [15], three causes of students difficulties in constructing proof, namely: (1) Students do not construct based on a framework of proof, (2) students are not able to dismantle or break down conclusions, (3) students are not right in using definitions. While [7], said that the difficulty of constructing proof is partly due to the interpretation of logical structures and the use of previous definitions and theorems in proof.

Research findings made by (eg [16]; [17]; [18]; [19] and [20]) points out that the study findings show “many school and university students and even mathematics teachers only have superficial ideas about the nature of proof”. Furthermore, based on observations on student assignments and lecture activities, we see that the difficulty of Mathematics Education students in semester 3 of the Faculty of Teacher Training and Science Education, Universitas Mataram, is because: (1) giving examples of definitions also apply to theorems and lemmas, (2) understanding definitions and lemmas that have previously proven, (3) views of new theorems built on definitions, and previous lemmas, and (4) knowledge of where theorems are important and when they should be used.

The ability to use methods of proof, axioms, definitions, lemmas, and theorems to show the truth of a statement in mathematics is part of the process of constructing proof [21]. One of the subjects that demands the ability to construct proof is the Number Theory. Number theory is the study of the properties of numbers, where the “numbers” in question are integers and, more specifically, positive integers. An interesting characteristic of number theory is that although many of the results can be stated in simple and elegant terms, the proof is sometimes long and complicated [2], [22], and [23]. Number theory aims to find interesting and unexpected relationships between various types of numbers and then prove that the relationship is true [24] and [25]. In contrast to other branches of mathematics, many problems and theorems of number theory can be understood, although solutions to problems and proofs of theorems often require a strong mathematical background [25]. Many theorems in number theory are suitable for problem solving approaches. Also, the nature of the problem of number theory is such that if students do not try to use suitable problem-solving heuristics at all, they will not be able to make progress in proving theorems [26].

Number theory is presented both as an important mathematical subject area and a valuable, ‘friendly’ field for examining and promoting general mathematical skills like conjecturing, generalizing, proving and refuting mathematical statements [27]. This study wants to investigate the construction schemes of proof in proving the theorem of the divisibility of integers in number theory. Some previous studies have used the basic concepts of number theory as a mathematical context to investigate various problems; for example, [28] use the idea of division in research about pre-service teachers’ understanding of mathematical proof. And [29] conducts research to (a) explore pre-service teacher understanding of basic concepts in number theory, with emphasis given to concepts that involve the division and multiplicative structure of natural numbers; (c) to analyze and describe cognitive strategies used in solving foreign problems that involve and incorporate these concepts in this context; (c) to adapt the constructivist-oriented theoretical framework to the analysis and interpretation of these strategies and to model the cognitive structures that support them. Several previous studies have used the basic concepts of number theory as a mathematical context to investigate various problems; for example, [28] in [29] use the idea of division in research on pre-service teachers’ understanding of mathematical proof.

Still in number theory, this study wants to investigate the scheme of constructing student proof in proving the theorem of the division of integers. Piaget states that the construction activities are divided into two, namely assimilation and accommodation. If someone wants to construct (build) a new knowledge/information, it means he wants to link the new information to the scheme in his mind then there are two possibilities, namely (1) the first possibility is if the new information structure is in accordance with the existing structure in scheme so that the information can be linked to and integrated into the scheme, a construction process called assimilation takes place; and (2) the second possibility is if the new information structure is not in accordance with the structure of the scheme so that there is disequilibrium (imbalance) in the mind which causes a strong urge on the person to change the structure of the scheme so that the new information can be linked (assimilated), then the equilibrium (balance)
returns again, then the second process is called accommodation [30], [31], [32] and [13]. According to [31], our cognitive structures are schemata, which are collections of schemes (structures). An individual can remember, understand, and respond to stimuli because of the operation of this scheme. So that the ability to construct proof related to the divisibility of integers in this study will be represented in the form of construction schemes of proof based on the process of assimilation and accommodation from Piaget.

2. Research Method

This research is a qualitative research that aims to describe the construction scheme of proof based on the process of assimilation and accommodation. The construction of the proof referred to here is a theorem in number theory. The subjects in this study were 37 third semester students of undergraduate Mathematics Education, the Faculty of Teacher Training and Science Education, Universitas Mataram, who answered one of the five questions on proof of number theory courses. Based on the proof sheet answer, 3 answer sheets were chosen randomly with consideration that they have notes related to interviews. Interviews and documentation also aim to confirm the

Based on (***) and (****), it is obtained that \( k|ad \) and \( k|bc \), then according to theorem b1: \( k|ad - bc \)

**Figure 1.** Construction scheme of proof

Confirmation of the construction of proof carried out using interviews and documentation (small notes related to interviews). Interviews and documentation also aim to confirm the proof that has been constructed, so that it becomes supporting data to interpret the ability to construct proof from research subjects. The interview is based on the response given by the subject. The interview process will be recorded with a recording device that is confidential (the subject does not know that it is being recorded), the next recorded data will be made transcript of the conversation to support the interpretation or matching of written data on the proof sheet answer.

3. Result and Discussion

Based on the results of the data analysis of the construction of proof, proof obtained by the construction of 3 characters, each character consists of several subjects, as shown in Chart 1 below:
Chart 1 above shows that the number of subjects with construction scheme using the definitions and theorems by 2 subjects, with a construction scheme using the theorem only a total of 18 subjects, and the construction scheme using the definition as much as 17 subjects. The three groups of this scheme, each of which will be taken 1 subject as a representative. The three subjects in question are:

3.1. Construction Scheme with Theorem and Definition

Ulmi subject (a pseudonym) is one of two subjects with the ability to construct proof using theorems and definitions. But in constructing proof of subjects experienced difficulties in concluding the results of the proof construction. The results of the construction of proof are based on the intended proof sheet are as follows:

If the results of the construction of the proof above are illustrated in the proof construction scheme, then the scheme of the construction of proof carried out by the Ulmi subject is as follows:
Figure 4. Scheme of construction Ulmi subject

Based on picture 2 above, Ulmi subject, that is, a subject with a construction scheme using theorems and definitions, has an mistake writing the statement, assuming that $-b$ is a positive integer, and concludes the results of the construction of the proof, namely: First: mistake in writing the statement if $k|a - b$, then $k|a$ and $k|b$; second: the mistake in thinking about $-b$ is positive integer, but in the end does not use it; and third: an mistake in concluding the results of the construction of $k|ad - bc$, which should be based on the description of the argument is $k| - ad - bc$. This is caused by an error in declaring $-ad = (kn)(km)$ and not manipulating the result becomes $ad = k(-nm)$ with $(-nm) \in \mathbb{Z}$.

From these results, it is necessary to conduct an interview to ask again to declare the results obtained, and concludes the results of the construction of the proof, namely: First: mistake in writing the statement if $k|a - b$, then $k|a$ and $k|b$; second: the mistake in thinking about $-b$ is positive integer, but in the end does not use it; and third: an mistake in concluding the results of the construction obtained is $k| - ad - bc$. This is caused by an error in declaring $-ad = (kn)(km)$ and not manipulating the result becomes $ad = k(-nm)$ with $(-nm) \in \mathbb{Z}$.

From these results, it is necessary to conduct an interview to ask again to show why using theorems and definitions; at the same time asking for a series of reasons derived from the stages of construction of proof. This interview activity, as well as being part of helping the subject in accommodation activities to produce construction results in accordance with the picture of construction schemes of proof in Figure 1. The results of the interview are then transcribed with the following results:

Table 1. Ulmi Subject Interview Transcript

| Researcher/ Subject | Stimulus or response                                                                 |
|---------------------|--------------------------------------------------------------------------------------|
| Researcher          | From the results of the construction of the proof you are working on. We found a mistake. First, you make a mistake in writing the statement, assuming that $-b$ is a positive integer, and concludes the results of the construction of the proof, namely: First: mistake in writing the statement if $k|a - b$, then $k|a$ and $k|b$; second: the mistake in thinking about $-b$ is positive integer, but in the end does not use it; and third: an mistake in concluding the results of the construction of $k|ad - bc$, which should be based on the description of the argument is $k| - ad - bc$. This is caused by an error in declaring $-ad = (kn)(km)$ and not manipulating the result becomes $ad = k(-nm)$ with $(-nm) \in \mathbb{Z}$. From these results, it is necessary to conduct an interview to ask again to show why using theorems and definitions; at the same time asking for a series of reasons derived from the stages of construction of proof. This interview activity, as well as being part of helping the subject in accommodation activities to produce construction results in accordance with the picture of construction schemes of proof in Figure 1. The results of the interview are then transcribed with the following results: |
| Ulmi                | Okay right. Later the proof, make it your duty. Next, why do you suppose that $-b$ is a positive integer? Where do you use these examples? |
| Ulmi                | No sir, Sir. Yes it should be if $k|a, b \in \mathbb{Z}, k|a$ and $k|b$, then $k|a - b$ |
| Researcher          | Next time we have to be more careful in using the separator. Next your mistake is to conclude the results of construction $k|ad - bc$, which should be the result of the construction obtained is $k| - ad - bc$. How did you come to this conclusion? |
| Ulmi                | Based on the definition of b1; and the theorem b1 and b2 the construction solution that I have done I think is right. But why at the conclusion the results are not the same as they should. Is there a little help that can justify the results of my construction, sir? |
| Researcher          | Try to re-analyze this construction argument: Multiply equation 1 and ii), then: $-ad = (kn)(km)$, and according to the definition of b1 we get $k| - ad$ What needs to be done, so that your conclusions are proven? |
| Ulmi                | What, sir? Isn’t that right? |
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Researcher/Subject Stimulus or response

Researcher: It is true. But what can manipulation of \(-ad = k(nm)\)?
Ulmi: I wonder how? Eeh. \(ad = -k(nm)\) \(\Rightarrow\) but \(k\) is positive
or this one:
\(ad = k(-nm)\), with \((-nm) \in \mathbb{Z}\)

And according to the definition of \(b1\), we get \(k|ad\)
Researcher: Very nice. Now continue construction.
Ulmi: From equation multiplication 2) and i), we get \(-bc = k(mn)\), and according to the definition of \(b1\), we get \(k| -bc\). So according to the theorem b2, if \(k|ad\) and \(k| -bc\), then \(k|ad + (-bc)\) or \(k|ad - bc\).
Proven sir.

Researcher: Well. Your construction is right, practice hard.
Ulmi: Yes, Sir.

From table 1 above, subjects with construction schemes use theorems and definitions, make a mistake first in writing the theorem if \(k|a - b\), then \(k|a\) and \(k| -b\), what should be true is if \(k, a, b \in \mathbb{Z}\), \(k|a\) and \(k| -b\). The second mistake is to suppose that \(-b\) is a positive integer, but in the end does not use it; and finally the third mistake is to conclude the results of the construction of \(k|ad - bc\), which should be based on the definition of the argument is \(k| -ad - bc\). This is caused by an error in declaring \(-ad = (kn)(km)\) and not manipulating the result becomes \(ad = (-nm)\) with \((-nm) \in \mathbb{Z}\). The first assumption of an error in concluding the construction results is that the Ulmi subject seems to be intentional, because it is impossible to conclude \(k| -ad - bc\), while \(k| -ad\) and \(k| -bc\) have been obtained. The second assumption is that the Ulmi subject is in a hurry to work on the proof, bearing in mind that the proof given to prove it is insufficient.

These mistakes are then confirmed through an interview process to ask again to show why they made a mistake; at the same time asking for a series of reasons derived from the stage of construction of the proof carried out. This interview activity, as well as being part of helping the subject in accommodation activities to produce construction results in accordance with the description of the construction of proof schemes in Figure 1. Interview of researchers with subjects doing construction using theorems and definitions, showing that the structure of new information/new theorem is not appropriate with the structure of the existing schema so that there is dis-equilibrium (imbalance) in the mind which causes a strong urge on the subject to change the structure of the scheme so that the new theorem can be linked (assimilated), then the equilibrium (balance) returns to do construction correct proof (accommodation).

3.2. Construction Scheme with Theorem

Hanah subject (a pseudonym) is one of the subjects with a proof construction scheme using only theorems and the results of the construction of the proof are correct. The construction of proof by Hanah subject is as follows:

![Figure 5](image)

**Figure 5.** The results of the construction of the Hannah subject are based on the intended proof sheet
If the results of the construction of proof with the theorem above are illustrated in the form of a construction scheme of proof. Then the construction scheme of proof by Hanah subject is as follows.

**Figure 6. Schema of construction Hanah subject**

Based on picture 5 above, it is possible to conduct interviews to find out why in the process of constructing proof only uses theorems; at the same time asking for a series of reasons derived from the stages of construction of proof. The results of the interview are then transcribed with the following results:

**Table 2. Hanah Subject Interview Transcript**

| Stimulus atau respon | Hanah Subject Interview Transcript |
|----------------------|-----------------------------------|
| researcher:          | From the results of the construction of the proof you do. Your proof is correct and you only use a few theorems to prove it. Are you having trouble using the definition? |
| Hanah:               | Actually not really sir. However, due to limited time, only 90 minutes to work on 4 proving questions and 1 question that requires accuracy (looking for FPB using the Algorithm Algorithm). So using a theorem that has been proven before is very helpful in concluding the results of proof. |
| researcher:          | Oke. Now try to convince me that you can prove theorems using definitions, which are: |
| Hanah:               | If k ≠ 0, a, b ∈ Z, k|a| and k|b|, then k|a − b|, |
| researcher:          | Yes, sir. Proof |
| Hanah:               | Subtract a = kn and b = km, then: |
| Hanah:               | It is obtained k|a − b| (Q.E.D) |
| researcher:          | Well. Your construction is right, practice hard. |
| Hanah:               | Yes, sir. |

From table 2 above, it is certain that under the subject of Hanah, he has been able to construct proof using either theorems and definitions, or only theorems. Or in other words, the interview activity with the subject of Hanah is to confirm ideas/concepts that have not been confirmed in the results of the
construction provided, so that the construction of the proof is in accordance with the process of assimilation and accommodation or the construction scheme is in accordance with the construction scheme of proof in picture 1. Subjects with a construction scheme using theorems have constructed the correct or appropriate proof. This condition states that the subject is familiar with the theorem he is dealing with, in other word, the pattern of theorem structure already exists in the subject’s schema so that the subject easily interprets the theorem it faces through the process of assimilation. This means that the structure of the new theorem to be proven is already in accordance with the existing structure in the scheme so that the new theorem can be linked to the cognitive structure of the subject and merges with the scheme so that the construction process (assimilation). The next interview is part of the suspicion whether the subject is also able to construct proof using theorems and definition?; at the same time asking for a series of reasons derived from the stage of construction of the written proof. Interview results show that subjects with construction schemes using theorem, are also able to construct proof using theorems and definitions. It means that there is an assimilation process that shows that the new information structure or new theorem is in accordance with the existing cognitive structure in the subject's scheme so that the new theorem that wants to be proven can be linked to the scheme and integrated with the subject's scheme.

3.3. Construction Scheme with Definition

Umah subject (a pseudonym) is the subject of proof construction using only a definition. In the process of constructing proof, the subject writes an error in writing the symbol of formal logic to “such that” and operations \((ad - bc) + (ad - bc)\) after moving segments. So this error does not make the results obtained wrong or not proven. The results of the construction carried out by the subjects Umah are as follows:

If it is depicted in the form of a construction scheme, then the process of constructing the proof carried out by Umah can be described with the following construction scheme is as follows:
Prove that if \( k|a - b \) and \( k|c - d \), then \( k|ad - bc \)

Definition b1: Let \( a, b \in \mathbb{Z} \) and \( a|b \) \( \iff \exists k \in \mathbb{Z} \) (single), \( \exists b = ak \)

Theorem b1: If \( k|a - b \), then \( k|a \) and \( k|b \)

Theorem b2: If \( k, a, b \in \mathbb{Z}, k|a \) and \( k|b \), then \( k|a + b \)

Theorem b3: If \( k, a, b \in \mathbb{Z}, k|a \) and \( k|b \), then \( k|ad \)

According to definition b1: If \( k|a - b \), then \( k|a - b = mk \)

Because of \( a - b = mk \), then \( a = mk + b \) or \( b = a - mk \) ...(*)

Because of \( c - d = nk \), then \( c = nk + d \) or \( d = c - nk \) ...(**)

Substitution of (*) and (**) to:
\[
\begin{align*}
ad - bc &= (mk + b)(c - nk) - (a - mk)(nk + d) \\
&= (mck - mnk^2 + bc) - (an) + ad - mnk^2 - mdk \\
&= mck + mdk - bnk - ank + bc - ad \\
ad - bc &= k(mc + md - bn - an), \text{ with } mc + md - bn - an \in \mathbb{Z}
\end{align*}
\]

Because of \( ad - bc = k(mc + md - bn - an) \), then according to definition b1 is obtained \( k|ad - bc \).

Q.E.D (Quod Erat Demonstrandum)

Figure 8. Schema of construction Umah subject.

Based on diagram 4 above, there are some ideas/concepts that are not written down so that it causes difficulties in the process of accommodation (disequilibrium), so it is necessary to conduct an interview to ask for an explanation of why to use only the definition, as well as to ask for an explanation related to the mistakes made. The interview results obtained are then transcribed with the following results:

Table 3. Transcript of Subject Umah Interviews.

| Researcher/Subject | Stimulus atau response |
|--------------------|-----------------------|
| Researcher :      | Before we get to the topic of conversation. We do the question and answer first: Is it \( 3 - 1 = -3 + 1 + 4 \)? |
| Umah :            | Right sir             |
| Researcher :      | Is it \( 3 - 1 = -3 + 1 + 2.2 \)? |
| Umah :            | Right sir             |
| Researcher :      | Is it \( 3-1 = 2.2 \)? |
| Umah :            | No sir.               |
| Researcher :      | Then how is the right one? |
| Umah :            | \( 3 - 1 + 1 + 3 - 1 = 2.2 \) |
| Researcher :      | State the operation on the right side as a multiplication operation 2 |
| Umah :            | \( 2.3 - 2.1 = 2.2 \) |
| Researcher :      | \( 2(3 - 1) = 2.2 \) |
| Umah :            | Right sir             |
| Researcher :      | What is \( 2|3 - 1 ; 2|2(3 - 1) \) |
| Umah :            | Right sir             |
| Researcher :      | How did it happen?    |
| Umah :            | Using theorem 3.1: If \( a|b \) then \( a|bz \), for each \( z \in \mathbb{Z} \). So if \( 2|3 - 1 \) (right), then \( 2|2(3 - 1) \) (right) |
| Peneliti :        | Oke, the question and answer operation is complete integer. We continue the problem of the construction of proof. From the results of the construction of the proof you do. We found there were errors of argument, even though the conclusions were correct, including: |
|                  | 1) Formal logic, about the symbol “such that” |
|                  | 2) \( ad - bc = mck + mdk - bnk - ank + bc - ad \) becomes \( ad - bc = mck + mdk - bnk - ank \) |
| Peneliti :        | Now try to write and explain the error we meant. |
| Umah :            | Look sir, related to the symbol “so that” it turns out I was WRONG, it should be \( \exists \). |
|                  | Next to: |
|                  | \( ad - bc = mck + mdk - bnk - ank + bc - ad \) |
|                  | Based on the question and answer operations above, then the right solution is: |
Based on table 3 above, subjects with construction schemes of proof using only definitions make mistakes in writing the symbol of formal logic to “such that” and operations $(ad - bc) + (ad - bc)$ after moving segments. This error does not make the results obtained wrong or not proven. In other words, the premises used to infer correct arguments turn out to be wrong. Some ideas/concepts that are not written that cause difficulties in the process of accommodation (disequilibrium), it is necessary to do an interview to ask for an explanation why only use the definition, as well as asking for an explanation related to mistakes made.

Just as subjects constructing proof using theorems and definitions, subjects constructing proof using definitions alone make interviewing a part of helping the subject in accommodation activities to produce construction results that are in accordance with the description of the proof construction scheme in figure 1. The interviews show that the new information structure/new theorem does not fit into the structure of the existing schema so that dis-equilibrium (imbalance) occurs in the mind which causes a strong urge on the subject to change the structure of the scheme so that the new theorem can be linked (assimilated), then occurs the equilibrium (balance) returns to construct the correct proof (accommodation).

Summary of the Construction Scheme

To provide an overview of each construction scheme, the following table can be used as a summary of the scheme in question.

| Construction Schemes | Construction of Process |
|----------------------|-------------------------|
| Construction Scheme with Theorem and Definition | – Identify and translate information from the proposed theorem,  
– Choose and use previous definitions and theorems in each step of the argument,  
– Using formal mathematical symbols and notations in carrying out the stages of proof,  
– Requires reflection on arguments that are still wrong,  
– Draw conclusions correctly based on the reflection given. |
| Construction Scheme with Theorem | – Identify and translate information from the proposed theorem,  
– Choose and use previous theorems in each step of the argument,  
– Using formal mathematical symbols and notations in carrying out the proof stages,  
– The steps of each argument are based on the previous theorem,  
– Draw conclusions correctly based on the arguments of each statement,  
– Able to use the definition seen from the ability to prove the theorem used. |
| Construction Scheme with Definition | – Identify and translate information from theorems,  
– Select and use definitions in each step of the argument,  
– Using formal mathematical symbols and notations, even if there are errors,  
– There was an error in constructing (moving segments) and was able to be fixed after reflection,  
– Draw conclusions based on unrelated arguments to answer the hypotheses of each statement. |

4. Conclusion

The ability of students in constructing proof is illustrated through 3 construction schemes, namely: 1) construction scheme of proof using definitions and theorems (2 people), 2) construction schemes of proof using theorems (18 people), and 3) construction schemes of proof using definitions (17 people). Subjects with a construction scheme using theorems have constructed the correct proof. It means that
there is an assimilation process that shows that the new information structure or new theorem is in accordance with the existing cognitive structure in the subject’s scheme so that the new theorem that wants to be proven can be linked to the scheme and integrated with the subject’s scheme and in accordance with the construction scheme of proof described in figure 1. Furthermore, the subject with the construction scheme uses theorems and definitions, making mistakes especially in writing the theorem, assuming a number, but in the end it seems not to be used, and the conclusion of the construction of proof is not right. Likewise, subjects with constructing schemes of proof using only definitions make mistakes in writing the symbol of formal logic to “such that” and operations after moving segments. This error does not make the results obtained wrong or not proven. But the premises used to infer correct arguments turn out to be wrong. Errors in the process of constructing proof (subjects using theorems and definitions, subjects using only definitions) have shown that the new information structure / new theorem does not fit into the existing schema structure so that there is a dis-equilibrium (imbalance) in the mind that causes there is a strong urge on the subject to change the structure of the scheme so that the new theorem can be linked (assimilated), then the equilibrium (balance) returns to construct the correct proof (accommodation).

From these findings, it concludes that the difficulty in constructing proof is influenced by: 1) students' understanding of previous definitions and theorems, 2) ability to see that new theorems are built on definitions, and previous lemmas, and (3) ability to determine when the theorem must be used. So the suggestions in this study are: 1) Reflection on students' ability to understand definitions, theorems and lemmas; 2) Study on how to measure students' understanding in constructing proof, 3) exploration of thinking as a result of understanding of definitions, theorems and lemmas, and 4) developing learning designs that can make the activities of constructing proof into routine activities, not non-routine.

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