Ultrahigh energy particle collisions near the black hole horizon in the strong magnetic field

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We consider collision between two charged (or charged and neutral) particles near the black hole horizon in the strong magnetic field $B$. It is shown that there exists a strip near the horizon within which collision of any two such particles leads to ultrahigh energy in the centre of mass frame. The results apply to generic (not necessarily vacuum) black holes.

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I. INTRODUCTION

If two particles move towards a black hole and collide near the horizon, under certain additional conditions their energy $E_{c.m.}$ in the centre of mass can become unbound. There are different scenarios of this kind: a black hole should be rotating [1], electrically charged [2] or immersed in the magnetic field [3]. In the latter case, in the situation considered previously, collision occurs near the innermost stable circular orbit (ISCO) that lies near the horizon for the magnetic field strength $B$ large enough [4]. Formally, $E_{c.m.} \to \infty$ requires $B \to \infty$ in this scenario. In doing so, the individual angular momentum on ISCO also grows unbound.

To realize this scenario, it is sufficient to take the simplest case of the Schwarzschild black hole, so the magnetic field affects motion of particles but not the metric itself [5]. The corresponding approach was generalized to the case of the Kerr metric [5].

The aim of the present letter is to draw attention to one more mechanism. Similarly to

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and \( \text{[5]} \), it requires the strong magnetic field. However, its realization is not connected with ISCO. In the very vicinity of the horizon it becomes a universal phenomenon and works for particles with arbitrary individual energies and angular momenta.

Throughout the paper we use units in which fundamental constants are \( G = c = 1 \).

II. BASIC EQUATIONS

To simplify matter, let us consider the static spherically symmetric metric of the form

\[
 ds^2 = -N^2 dt^2 + \frac{dr^2}{N^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),
\]

where \( N \) depends on \( r \) only. The horizon lies at \( N = 0 \). If \( N^2 = 1 - \frac{2M}{r} \) we return to the case of the Schwarzschild black hole \( \text{[3]} \). In general, we do not specify the form of \( N \) explicitly and even do not require it to be unaffected by the magnetic field. In particular, we do not require \( Br_+ \ll 1 \) (\( r_+ \) is the horizon radius, \( B \) is the effective magnetic field), allowing \( Br_+ \sim 1 \). In eq. \( \text{(1)} \), the metric coefficients satisfy the relation \( g_{rr}g_{00} = -1 \) but this is a weak restriction that somewhat simplifies formulas without the loss of generality for the effect under discussion.

We assume that there is a vector-potential that has the only nonvanishing component

\[
 A^\phi = \frac{B}{2},
\]

where \( B \) is, in general, the function of \( r \). In the vacuum case, the Maxwell equations are satisfied with \( B = \text{const} \). However, we consider a more general case of ”dirty” (surrounded by matter) black holes.

Let a particle with the mass \( m \) and electric charge \( q \), move in the background \( \text{(1)} \) with the vector-potential \( \text{(2)} \). The kinematic momentum \( p_\mu = mu_\mu \) and the generalized one \( P_\mu \) are related according to \( p_\mu = P_\mu - qA_\mu \). Here, \( u^\mu = \frac{dx^\mu}{d\tau} \) is the four-velocity, \( \tau \) is the proper time. The components \( P_t = -E \) and \( P_\phi = L \) are conserved, \( E \) having the meaning of the energy, \( L \) being the angular momentum. Then, equations of motion read

\[
 m \ddot{t} = \frac{E}{N^2},
\]

\[
 m \ddot{\phi} = \frac{L}{r^2} - q B_2,
\]
\[
m \dot{r} = \varepsilon Z, \quad (5)
\]
where
\[
Z = \sqrt{E^2 - m^2 N^2 (1 + \beta^2)}, \quad (6)
\]
\[
\beta = \frac{L}{mr} - b, \quad b = \frac{q Br}{2m}. \quad (7)
\]

We assume that a black hole is electrically neutral.

Let two particle 1 and 2 with masses \(m_1\) and \(m_2\), electric charges \(q_1\) and \(q_2\) and four-velocities \(u_1^\mu\) and \(u_2^\mu\) collide in some point. One can define in the same point the energy \(E_{\text{c.m.}}\) in the centre of mass frame (CM frame) according to
\[
E_{\text{c.m.}}^2 = -(m_1 u_1^\mu + m_2 u_2^\mu)(m_1 u_1^\mu + m_2 u_2^\mu) = m_1^2 + m_2^2 + 2m_1 m_2 \gamma, \quad (8)
\]
where
\[
\gamma = -u_1^\mu u_2^\mu \quad (9)
\]
is the Lorentz factor of relative motion.

Using equations of motion, one can find
\[
m_1 m_2 \gamma = \frac{E_1 E_2 - \varepsilon_1 \varepsilon_2 Z_1 Z_2}{N^2} - m_1 m_2 \beta_1 \beta_2. \quad (10)
\]

As one approaches the horizon, \(N \to 0\). Then, for head-on collision \((\varepsilon_1 \varepsilon_2 = -1)\) the Lorentz factor \(\gamma \to \infty\) for any values of the angular momentum and magnetic field. The rotational analogue of this phenomenon was studied in [8] - [10]. Hereafter, we consider the case \(\varepsilon_1 = \varepsilon_2 = -1\), so both particles move towards the horizon. In general, \(\gamma\) remains finite near the horizon, and the question is whether and how \(\gamma\) can become unbound.

### III. ULTRAHIGH ENERGY COLLISIONS

For any finite values of all relevant quantities, one can calculate the horizon limit \(N \to 0\) of (10) and find that it is finite. More precisely,
\[
\gamma_0 \equiv \lim_{N \to 0} \gamma = \frac{1}{2} \left[ \frac{m_1 E_2 [1 + \beta_{1H}^2]}{m_2 E_1} + \frac{m_2 E_1 [1 + \beta_{2H}^2]}{m_1 E_2} \right] - \beta_{1H} \beta_{2H} = \frac{1}{2} \left( \frac{\alpha_1 \beta_{2H} - \alpha_2 \beta_{1H}}{\alpha_1 \alpha_2} \right)^2 + \frac{\alpha_1}{2 \alpha_2} + \frac{\alpha_2}{2 \alpha_1}, \quad (11)
\]
where subscript "H" means that the corresponding quantity is calculated on the horizon,
\[
\alpha_1 = \frac{E_1}{m_1}, \quad \alpha_2 = \frac{E_2}{m_2}. \quad (12)
\]
However, the situation can change if (i) collisions occur not exactly on the horizon but in its vicinity at \( \rho = r_H \approx r_c \), so \( N(r_c) \equiv N_c \ll 1 \), (ii) the quantity \( \beta_{iH} \gg 1 \), (iii) the factors (i) and (ii) are related to each other in such a way that

\[
N_c \beta_{iH} \equiv s_i \sim 1,
\]  

where \( i = 1, 2 \).

Then, the Lorentz factor

\[
\gamma \approx \frac{F}{N_c^2}, \quad F = \alpha_1 \alpha_2 - \sqrt{\alpha_1^2 - s_1^2} \sqrt{\alpha_2^2 - s_2^2} - s_1 s_2.
\]  

(14)

The numerator is positive, except from for the particular case \( \alpha_1 s_2 = \alpha_2 s_1 \) when \( F \) vanishes. We exclude this case from consideration. Then, \( \gamma \) can become as large as one likes. For \( s_i \ll \alpha_i \), eq. (14) turns into (11) in the main approximation, if finite corrections in (11) are neglected.

As we must have \( Z_i \geq 0 \), this mechanism works in the immediate vicinity of the horizon only, so

\[
0 < N_c \leq \frac{E_i}{m_i \beta_{iH}}
\]  

(15)

or, equivalently, \( \alpha_i \geq s_i \). It is curious that, formally, the restriction (15) on the size of the strip within which \( E_{c.m.} \) is ultra-high, resembles the corresponding restriction (18) in [11] or (18) in [6], for rotating nonextremal black holes without the magnetic field.

Using the algebraic inequality

\[
\frac{1}{2} \left( \frac{\alpha_1 s_2 - \alpha_2 s_1}{\alpha_1 \alpha_2} \right)^2 \leq \alpha_1 \alpha_2 - \sqrt{\alpha_1^2 - s_1^2} \sqrt{\alpha_2^2 - s_2^2} - s_1 s_2,
\]  

(16)

one can easily show that \( \gamma_0 < \gamma(N_c) \) with \( N_c > 0 \). In other words, to gain the maximum possible \( E_{c.m.} \), it is more profitable to arrange collision not on the horizon itself but in its vicinity, notwithstanding the fact that high \( E_{c.m.} \) arise just due to the horizon! This circumstance is similar to the observation made in [13] for rotating nonextremal Kerr black holes and extended in [14] for generic dirty rotating black holes.

In the particular case when particle 2 is neutral, \( s_2 = 0 \). Then, it is seen from (14) that for a given \( \alpha_1, \alpha_2 \), the function \( F \) attains it maximum value if \( s_1 = \alpha_1 \) that corresponds to the turning point (cf. [13], [14]).

There are two ways to achieve large value of \( \beta_1 \). The first one is to increase \( L_1 \). (High energy collisions with indefinitely large \( L_1 \) near the horizon were considered in [12] (case
3) but with additional assumptions that the radial velocity is vanishingly small. However, this imposes rather severe restriction on $L_1$ that represents some problem for the realization of such a scenario. Meanwhile, there is more physical way to achieve large $\beta_{iH}$ since it is possible to take a large magnetic field, so that

$$b_{iH} \gg 1.$$  \hfill (17)

Then, $\beta_{iH} \sim B$, and it follows from (14) that

$$\gamma \sim B^2.$$  \hfill (18)

For the effect to occur, at least one of particle should be electrically charged. If they both are neutral, there is no interaction with a magnetic field, and $B$ does not enter the expression for $\gamma$ (6), (10) at all.

It is important that if (17) is obeyed, the relations (13), (14) are satisfied for particles with any finite values of the energy and momentum. Therefore, the phenomenon under discussion acquires universal character. It turns out that in the vicinity of the horizon in a strong magnetic field, collisions between any two particles with arbitrary energies and angular momenta give rise to ultra-high value of $E_{c.m.}$!

IV. SUMMARY

Thus we considered particle collisions in the strong magnetic field near the black hole horizon. It turned out that in the immediate vicinity of the horizon, $E_{c.m.}$ grows as $B^2$. For comparison, in the Frolov’s process \[3\], $E_{c.m.} \sim B^{1/4}$ (see also eqs. 62, 63 of \[5\]), so the present mechanism is more efficient. In both cases there is restrictions on the location of collision. In our scenario, it should happen near the horizon within the coordinate distance determined by (15). In the Frolov’s case collision occurs near ISCO. It is also worth noting that $L_1 \sim B$ for the particle on ISCO (see eq. 49 of \[3\]) but $L_1$ and $L_2$ are arbitrary finite quantities in our scenario.

The results apply to any dirty static black hole since the condition of spherical symmetry can be relaxed easily without the loss of generality. Also, it admits straightforward extension to rotating black holes. In doing so, there is no need to invoke additional assumptions that the background metric is almost unaffected by the magnetic field since the basic formulas like (14), (15) work anyway.
Now, on the basis of [3], [5] and the present work, we can conclude that there exist at least two main scenarios of ultrahigh energy collisions in the magnetic field near black holes. They include collision (i) in the strong field on ISCO, (ii) in the strong field in the immediate vicinity of the horizon. In the present work, we described scenario (ii). The distinctive feature of this scenario consists in that it does not include dynamic characteristics of particles, so their energy and angular momentum can be arbitrary finite quantities.

Thus once there is a very strong magnetic field near the black hole horizon, one can always find the strip around the horizon where this phenomenon does occur. This lends the property of universality to the scenario under discussion and enlarges hopes to find realization of ultra-high energy collisions in nature.

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