Heavy gravitino in hybrid inflation

Masahiro Kawasaki\textsuperscript{(a,b)}, Naoya Kitajima\textsuperscript{(a)}, Kazunori Nakayama\textsuperscript{(c,b)}
and Tsutomu T. Yanagida\textsuperscript{(b)}

\textsuperscript{a}Institute for Cosmic Ray Research, University of Tokyo, Kashiwa, Chiba 277-8582, Japan
\textsuperscript{b}Kavli Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa, Chiba 277-8568, Japan
\textsuperscript{c}Department of Physics, University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan

Abstract

It is known that supersymmetric hybrid inflation model may require severe tunings on the initial condition for large gravitino mass of order 100 - 1000 TeV due to the constant term in the superpotential. We propose a modified hybrid inflation model, where the constant term is suppressed during inflation and generated after inflation by replacing a constant term with dynamical field. In this modified model, successful inflation consistent with large gravitino mass takes place without severe tunings on the initial condition. Constraint from cosmic strings is also relaxed.
1 Introduction

The current cosmic microwave background (CMB) observations, such as the Wilkinson Microwave Anisotropy Probe (WMAP) observation [1], strongly support the existence of an accelerated expansion era called inflation in the very early stage of the universe. The inflation is driven by some scalar field, called inflaton, whose potential is nearly flat. Unfortunately, such a scalar field does not exist in the framework of the well-established Standard Model of the particle physics, so we must go beyond the Standard Model. One of the most plausible extensions of the Standard Model is the supersymmetry (SUSY) or supergravity. Thus it is well-motivated to consider the inflation model in the framework of SUSY.

Up to now, many SUSY inflation models have been proposed [2]. Among these, the SUSY hybrid inflation is one of the simplest and most plausible models [3]. In this model, the energy scale of the inflation, in terms of the Hubble scale during inflation, is required to be of the order of $10^9–10^{11}$ GeV in order to reproduce the observed density perturbation. The red-tilted power spectrum, $n_S \simeq 0.98$ ($n_S \simeq 0.96$ by relying on the non-minimal Kähler potential [4]), can also be reproduced, which is supported by the CMB observation.

However, there is a drawback in SUSY hybrid inflation model. In supergravity, we must include the constant term in the superpotential: $W_0 = m_{3/2}M_P^2$, where $m_{3/2}$ is the gravitino mass and $M_P$ is the reduced Planck scale, in order to cancel the SUSY breaking vacuum energy. Including such a term, the linear term for the inflaton is induced in the potential and this term may change the dynamics of the inflaton significantly [5, 6, 7]. In particular, severe fine tuning on the initial condition is needed for successful inflation and the allowed parameter region consistent with the WMAP observation shrinks for larger gravitino mass [7]. In addition, the overproduction of thermally-produced gravitinos is also problematic because the reheating temperature tends to be high in the hybrid inflation model. For these reasons, large gravitino mass of $m_{3/2} \gtrsim 100$ TeV is disfavored in the SUSY hybrid inflation model. On the other hand, recent observations of Higgs-boson-like particle at the LHC [8] may indicate a relatively high-scale SUSY [9]. At first sight, therefore, the hybrid inflation model seems to be disfavored in the light of the recent LHC result.\footnote{A variant model of the SUSY hybrid inflation, smooth hybrid inflation, allows large gravitino mass because the inflationary dynamics is less affected by the linear term as shown in [10].}

In this paper we propose a modified model for hybrid inflation, where the problematic constant term in the superpotential is replaced with a dynamical field. It is dynamically set to a small value during hybrid inflation, which avoids the problem with linear term inflaton potential in the original hybrid inflation model, and obtains a large vacuum expectation value (VEV) after inflation yielding a large gravitino mass.\footnote{Another idea to avoid the linear term problem in hybrid inflation was proposed in Ref. [11] based on no-scale supergravity.} The added dynamical term in the superpotential is the same as that for new inflation model [12, 13]. A similar model was proposed in the context of preinflation for solving the severe initial
value problem of new inflation [14], and also in the context of double inflation [15], where a period of new inflation follows after hybrid inflation. In the present purpose, we do not necessarily need a period of new inflation; it only guarantees the successful dynamics of hybrid inflation. Actually we find that in such a setup, heavy gravitino scenario is suitably consistent with hybrid inflation model.

This paper is organized as follows. In Sec. 2, we introduce our inflation model and dynamics after inflation is considered. In Sec. 3, the model parameters are constrained from observations and the initial value problem for the inflaton and the gravitino problem are discussed. Sec. 4 is devoted to the conclusion.

2 Modified supersymmetric hybrid inflation model

2.1 The inflaton potential

First we introduce a modified SUSY hybrid inflation model. The superpotential in our model is given by

\[ W = W_H + W_N. \]

The first term is the superpotential for the hybrid inflation given by

\[ W_H = \kappa S(\Psi \bar{\Psi} - M^2), \]

where \( S \) and \( \Psi (\bar{\Psi}) \) are chiral superfields whose scalar components play roles of the inflaton and the waterfall field, \( \kappa \) is a dimensionless coupling constant and \( M \) gives the VEV of waterfall field. We take the Planck unit, i.e. \( M_P = 1 \), throughout the paper. This model has \( U(1)_R \) symmetry with charge assignments of +2, 0 and 0 for \( S, \Psi \) and \( \bar{\Psi} \) respectively. In addition, there is another \( U(1) \) symmetry whose charge assignments are 0, +1 and −1 for \( S, \Psi \) and \( \bar{\Psi} \) respectively. We assume that this is a gauge symmetry.\(^3\) The second term in (1) is given by

\[ W_N = \Phi \left( v^2 - \frac{g}{n+1} \Phi^n \right), \]

where \( \Phi \) is a chiral superfield, \( v \) gives the energy scale of \( \Phi \) potential, \( g \) is a self coupling constant and \( n \) is an integer larger than 2. A discrete \( R \)-symmetry \( Z_{2n}^R \) under which \( \Phi \) has a charge +2 ensures this form of the superpotential.\(^4\) Note that this superpotential has the same form as the one for the new inflation [12] and it becomes a non-zero constant term at the potential minimum, giving the gravitino mass. The Kähler potential is taken to be

\[ K = K_H + K_N \]

where \( K_H \) and \( K_N \) are respectively given by

\[ K_H = |S|^2 + |\Psi|^2 + |\bar{\Psi}|^2 + |\Phi|^2 + \frac{k_S}{4} |S|^4 + k_1 |S|^2 |\Psi|^2 + k_2 |S|^2 |\bar{\Psi}|^2 + k_{SS} |S|^6 + \ldots, \]

\(^3\)It is possible that this \( U(1) \) is a global symmetry, such as Peccei-Quinn symmetry [16].

\(^4\)It is also understood in the following way: \( \Phi \) has a \( U(1)_R \) charge \( 2/(n+1) \), while it couples to additional chiral matters which condensates at the dynamical SUSY breaking scale yielding \( \Phi v^2 \) term [14].
and

$$K_N = |\Phi|^2 + \frac{c_N}{4} |\Phi|^4 + \ldots, \quad (6)$$

where $k_S$, $k_1$, $k_2$, $k_{SS}$ and $c_N$ are dimensionless coefficients and dots denote higher order Planck suppressed terms. In the following we assume $c_N$ is positive.

The F-term scalar potential is calculated from the formula

$$V_F = e^K [K^{ij} D_i W D_j W^* - 3 |W|^2], \quad (7)$$

where $D_i W = W_i + K_i W$ and the subscript represents derivative with respect to corresponding field and $K^{ij} = K^{-1}_{ij}$. For $|S| > M$, hybrid inflation takes place and the waterfall fields are stabilized at the origin: $\Psi = \bar{\Psi} = 0$. Thus we get the F-term scalar potential during inflation as

$$V = \exp \left( |S|^2 + \frac{k_S}{4} |S|^4 + \frac{k_{SS}}{6} |S|^6 + |\Phi|^2 + \frac{c_N}{4} |\Phi|^4 \right)$$

$$\times \left\{ -\kappa M^2 \left( 1 + |S|^2 + \frac{k_S}{2} |S|^4 \right) + S^* \Phi \left( v^2 - \frac{g}{n+1} \Phi^n \right) \right\}^2$$

$$+ \frac{1}{1 + c_N |\Phi|^2} v^2 \left( 1 + |\Phi|^2 + \frac{c_N}{2} |\Phi|^4 \right) - g \Phi^n \left( 1 + |\Phi|^2 + \frac{c_N |\Phi|^4}{2(n+1)} \right)$$

$$- \kappa M^2 S \Phi^* \left( 1 + \frac{c_N}{2} |\Phi|^2 \right) \left( v^2 + \frac{g}{n+1} \Phi^n \right)^2 \right\} \right\}$$

(8)

The scalar potential is conveniently divided into the following three pieces:

$$V = V_H + V_N + V_{int}. \quad (9)$$

The each term is given by

$$V_H = \kappa^2 M^4 \left( 1 - k_S |S|^2 + \frac{1}{2} \gamma |S|^4 \right) + V_{CW}, \quad (10)$$

$$V_N = |v^2 + g \Phi^n|^2 - c_N v^4 |\Phi|^2, \quad (11)$$

$$V_{int} = \kappa^2 M^4 |\Phi|^2 + \kappa M^2 v^2 (S^* \Phi + \text{c.c.}), \quad (12)$$

where $\gamma = 1 - 7k_S/2 - 3k_{SS} + 2k_S^2$ and we neglected the higher order Planck suppressed terms and assumed $v^2 \gg g \Phi^n$, and $V_{CW}$ is the Coleman-Weinberg effective potential [17] given by

$$V_{CW} = \frac{\kappa^4 M^4}{32\pi^2} \left[ (x^4 + 1) \ln \frac{x^4 - 1}{x^4} + 2x^2 \ln \frac{x^2 + 1}{x^2 - 1} + 2 \ln \frac{\kappa^2 M^2 x^2}{\Lambda^2} - 3 \right] \quad (13)$$

where $x \equiv |S|/M$ and $\Lambda$ is an ultraviolet cut-off scale. In the limit of $|S| \gg M$, this is approximated as

$$V_{CW} \simeq \frac{\kappa^4 M^4}{16\pi^2} \ln \frac{\kappa^2 |S|^2}{\Lambda^2}. \quad (14)$$
In the above calculation, we implicitly assumed that $\Phi$ is a subdominant component during inflation, i.e., $v \ll \sqrt{\kappa M}$. From (12), the minimum of $\Phi$ during inflation is determined by

$$\Phi_{\text{min}} = -\frac{v^2}{\kappa M^2} S,$$

which leads to the effective potential for $S$: $V_{\text{int}}(\Phi_{\text{min}}) = -v^4|S|^2$. Then, defining the inflaton field as $\sigma = \sqrt{2}|S|$, the effective potential of the inflaton is derived as

$$V(\sigma) = \kappa^2 M^4 \left(1 - \frac{1}{2} k_S \sigma^2 + \frac{1}{8} \gamma \sigma^4 \right) - \frac{v^4}{2} \sigma^2 + \frac{\kappa^4 M^4}{16\pi^2} \ln \frac{\kappa^2 \sigma^2}{2\Lambda^2},$$

where we used the approximation for the Coleman-Weinberg potential (14). Note that there is no dangerous linear term in the potential which may spoil the inflaton dynamics. The inflaton potential (16) is illustrated in Fig. 1. The slow-roll parameters are calculated as

$$\epsilon \equiv \frac{1}{2} \left(\frac{V''}{V}\right)^2 = \frac{1}{2} \left[-\left(k_S + \frac{v^4}{\kappa^2 M^4}\right) \sigma + \frac{1}{2} \gamma \sigma^3 + \frac{\kappa^2}{8\pi^2 \sigma} \right]^2$$

and

$$\eta \equiv \frac{V''}{V} = -k_S - \frac{v^4}{\kappa^2 M^4} + \frac{3}{2} \gamma \sigma^2 - \frac{\kappa^2}{8\pi^2 \sigma^2},$$

where the prime denotes the derivative with respect to the inflaton field. The inflation lasts as long as the slow-roll conditions, $\epsilon \ll 1$ and $|\eta| \ll 1$, are satisfied and ends when the inflaton reaches the value where the slow-roll condition breaks down or the waterfall point given by $\sigma_{\text{wf}} = \sqrt{2}M$ where the waterfall fields become tachyonic. Thus the value of $\sigma$ at the end of the hybrid inflation is given by

$$\sigma_f = \max\{\sigma_c, \sqrt{2}M\}, \quad \text{where} \quad \sigma_c \simeq \frac{\kappa}{2\sqrt{2}\pi}.$$  

2.2 Dynamics after inflation

After the end of inflation, the inflaton and waterfall fields starts to oscillate, so the universe is dominated by oscillating scalar fields which behave like matter. On the other hand, $\Phi$ also starts to oscillate around the origin due to the Hubble-induced mass. The initial amplitude of $\Phi$-oscillation is given by the value of $\Phi$ at the end of inflation:

$$\Phi_f \equiv -\frac{v^2 \sigma_f}{\sqrt{2}\kappa M^2}.$$

Because the total energy density is dominated by the oscillating scalar fields, the Hubble-induced mass for $\Phi$ is given by $m^2_{\Phi} = (3/2)H^2$. So the amplitude of $\Phi$-oscillation decreases proportional to $a^{-3/4}$ ($a$ is the scale factor of the cosmic expansion) in this era. Since $V_N \sim v^4$ for $\Phi \sim 0$ while $V_H \propto a^{-3}$, $V_N$ starts to dominate the whole potential at
$H \sim v^2/\sqrt{3}$. Since the mass-squared of $\Phi$ around the origin is given by $\sim -\sqrt{c_N}v^2$, it starts to roll down and oscillate around the true minimum $\Phi_0$ as soon as it comes to dominate the Universe, where

$$|\Phi_0|^n = \frac{v^2}{g}, \quad (21)$$

which corresponds to $D_\Phi W = 0$.\footnote{Since $|\Phi_{\text{min}}|$ is much larger than $H_{\text{inf}} \equiv \kappa M^2/\sqrt{3}$, which is typical scale of the field fluctuation, $\Phi$ can be regarded as a homogeneous field. Thus $\Phi$ rolls down to one of the minima of the potential in whole region of the observable Universe at $H \sim v^2/\sqrt{3}$ and no domain walls are formed.}

Although the second inflation could take place if $c_N \ll 1$, we do not require it. At the minimum, $V_N$ becomes negative and its vacuum energy is given by

$$V_{\text{vac}} = -3e^K|W|^2 \simeq -3\left(\frac{n}{n+1}\right)^2 v^4|\Phi_0|^2. \quad (22)$$

Requiring that $V_{\text{vac}}$ compensates the vacuum energy from the SUSY breaking sector, denoted by $\Lambda_{\text{SUSY}}^4$, we get the gravitino mass as follows:

$$m_{3/2} = \frac{\Lambda_{\text{SUSY}}^4}{\sqrt{3}} = \left(\frac{n}{n+1}\right)v^2\left(\frac{v^2}{g}\right)^{1/n}. \quad (23)$$

The mass of $\Phi$ around the minimum is given by

$$m_{\phi} = ng^{1/n}v^{2-2/n} \simeq ng^{\frac{2}{n+1}}m_{3/2}^{\frac{n+1}{n+1}}. \quad (24)$$

Fig. 2 shows gravitino mass and $\Phi$ mass as a function of $v$ for $n = 4$ (red, solid) and $n = 8$ (green, dashed). Two lines correspond to $g = 10^{-2}$ (upper) and $g = 1$ (lower) in the left panel and $g = 10^{-2}$ (lower) and $g = 1$ (upper) in the right panel. From this figure, it is seen that we should have $v \sim 10^{12} - 10^{13}$ GeV for $m_{3/2} = 100 - 1000$ TeV.

---

Figure 1: The potential of the inflaton is shown. We have taken $M = 10^{15}$ GeV, $\kappa = 0.02$, $v = 4 \times 10^{13}$ GeV, $\Lambda = 10^{16}$ GeV, $k_S = 0$ (solid red) and $k_S = 0.01$ (dashed green).
3 Constraints on the modified hybrid inflation model

3.1 WMAP normalization and spectral index

Now let us constrain our model parameters from the CMB observation. The WMAP observation gives the normalization to the power spectrum of the curvature perturbation as $\mathcal{P}_R \simeq 2.4 \times 10^{-9}$ [1]. The scalar spectral index $n_s$ is also constrained as $n_s = 0.968 \pm 0.012$. These quantities are calculated as

$$\mathcal{P}_R = \frac{V}{24\pi^2 \epsilon},$$

and

$$n_s \simeq 1 - 6\epsilon + 2\eta,$$

where $\epsilon$ and $\eta$ are the ones evaluated when the pivot scale $k_p = 0.002\text{Mpc}^{-1}$ exits the horizon [18]. The allowed parameters satisfying the WMAP normalization lies on the thick contours in Fig. 3. We have taken $n = 4$, $g = 1$ and $k_S = 0$ in Fig. 3(a), $n = 4$, $g = 0.01$ and $k_S = 0$ in Fig. 3(b), $n = 6$, $g = 1$ and $k_S = 0$ in Fig. 3(d), and $n = 4$, $g = 1$ and $k_S = 0.01$ in Fig. 3(d). In each panel, we have taken $m_{3/2} = 10\text{TeV}$ (solid red line), $100\text{TeV}$ (dashed green line), $1000\text{TeV}$ (dotted blue line) and $m_{3/2} = 0$ (small-dotted magenta line). For comparison, we show the case of “traditional” model including the constant term in the superpotential [7] by thin contours. We can see that $M$ can be smaller value down to $M \sim 2 \times 10^{15}\text{GeV}$ even for $m_{3/2} \gtrsim 100\text{TeV}$, which is contrasted to the traditional model. Fig. 4 shows $n_s$ as a function of $M$ with the WMAP normalization is imposed. Parameters are same as the ones used in Fig. 3. We found that $n_s$ can be reduced to the WMAP central value without relying on the non-minimal Kähler potential. This is due to the existence of the negative quadratic term in the potential (16) which comes from $V_{\text{int}}$. For smaller $M$, the condition $v \ll \sqrt{\kappa}M$ becomes violated and the dynamics of hybrid inflation is spoiled by this term.

Figure 2: Gravitino mass (left) and $\Phi$ mass (right) as a function of $v$ for $n = 4$ (red, solid) and $n = 8$ (green, dashed). Two lines correspond to $g = 10^{-2}$ (upper) and $g = 1$ (lower) in the left panel and $g = 10^{-2}$ (lower) and $g = 1$ (upper) in the right panel.
Figure 3: The allowed parameters satisfying the WMAP normalization are shown. They are on the thick contours. We have taken $n = 4$ (Fig. 3(a), 3(c), 3(d)), $n = 6$ (Fig. 3(c)), $k_S = 0$ (Fig. 3(a), 3(b), 3(c)), $k_S = 0.01$ (Fig. 3(d)) and $m_{3/2} = 10$ TeV (solid red lines), $m_{3/2} = 100$ TeV (dashed green lines) and $m_{3/2} = 1000$ TeV (dotted blue lines). Small-dotted magenta lines correspond to $m_{3/2} = 0$ and dashed-and-dotted cyan lines correspond to the upper bound on $\kappa$ from the cosmic string constraint. Thin contours correspond to a “traditional” model including a constant term in the superpotential.
Figure 4: The spectral index $n_s$ as a function of $M$ imposing the WMAP normalization. We have taken $n = 4$ (Fig. 4(a), 4(b), 4(d)), $n = 6$ (Fig. 4(c)), $k_S = 0$ (Fig. 4(a), 4(b), 4(c)), $k_S = 0.01$ (Fig. 4(d)) and $m_{3/2} = 10$ TeV (solid red lines), $m_{3/2} = 100$ TeV (dashed green lines) and $m_{3/2} = 1000$ TeV (dotted blue lines). The small-dotted magenta lines correspond to $m_{3/2} = 0$. 

(a) $n = 4$, $g = 1$ and $k_S = 0$

(b) $n = 4$, $g = 0.01$ and $k_S = 0$

(c) $n = 6$, $g = 1$ and $k_S = 0$

(d) $n = 4$, $g = 1$ and $k_S = 0.01$
3.2 Constraint from cosmic string

Soon after the end of hybrid inflation, the waterfall fields get large vacuum expectation values which spontaneously breaks $U(1)$ symmetry and hence the cosmic strings are inevitably formed [19]. The tension of the cosmic string, denoted as $\mu$, is given by [20, 21]

$$\mu = 2\pi M^2 \epsilon(\beta)$$

(27)

where $\beta = \kappa^2 / 2g^2$, $g^2 = 4\pi / 25$ in grand unification models and $\epsilon(\beta)$ is defined through

$$\epsilon(\beta) = \begin{cases} 
1.04 \beta^{0.195} & \text{for } \beta > 10^{-2} \\
2.4 \ln(2/\beta) & \text{for } \beta < 10^{-2}.
\end{cases}$$

(28)

Cosmic strings can be a source of large scale structure in the Universe in addition to the primordial density perturbation from the inflation. The CMB observations constrain the tension of the cosmic string as $G\mu < (2 - 7) \times 10^{-7}$ [22]. This gives a constraint on the parameter region as shown in Fig. 3, where the allowed parameter region is below the dashed-and-dotted cyan lines.

3.3 Initial condition

Let us consider the initial condition for the inflaton.\(^6\) It is known that the initial condition for inflaton is constrained in the SUSY hybrid inflation model due to the constant term in the superpotential or non-minimal Kähler potential to reproduce the observed spectral index [7].

First, let us see the constraint on initial value for the radial component. In the traditional model the existence of the non-minimal Kähler potential, the $k_S$ term in (5) with $k_S > 0$, gives the local minimum for the inflaton potential, so the initial position of the radial component of the inflaton must be chosen not to be trapped at the local minimum. A similar problem exists in the modified model even if we adopt $k_S = 0$ because of the $-v^4\sigma^2 / 2$ term in the inflaton potential (16). In order not to be trapped at the local minimum, the initial value of the inflaton must be placed smaller than the local maximum, so we require $\sigma_i < \sigma_{\text{max}}$, where

$$\sigma_{\text{max}} \simeq \frac{\kappa}{2\pi \sqrt{2k_S + 2v^4/\kappa^2 M^4}}$$

(29)

Note that the local minimum and maximum do not arise for $V'(\sigma_*) > 0$ where $\sigma_*$ is defined via $V''(\sigma_*) = 0$. Furthermore, the initial value of the inflaton must be chosen so that the inflation last at least 50 e-foldings to solve the horizon problem, hence we impose $\sigma_i > \sigma(N_e)$ with $N_e = 50$, where $\sigma_i$ denotes the initial value of the inflation and $\sigma(N_e)$

\(^6\)Here, we assume that the initial value of the inflaton is placed on the inflationary trajectory since our interest is focused on the dynamics on it. This assumption is reasonable because of the attractor behavior of the inflationary trajectory as shown in [23].
denotes the field value corresponding to the e-folding number $N_e$. Fig. 5 shows the allowed initial values for the inflaton, which are inside the red contours. We have imposed the WMAP normalization on $M$ and $\kappa$ and we have taken $n = 4$, $m_{3/2} = 100$ TeV (Fig. 5(a), 5(c), 5(e)), $m_{3/2} = 1000$ TeV (Fig. 5(b), 5(d), 5(f)), $g = 1$ (Fig. 5(a), 5(b), 5(e), 5(f)), $g = 0.01$ (Fig. 5(c), 5(d)), $k_S = 0$ (Fig. 5(a) - 5(d)) and $k_S = 0.01$ (Fig. 5(c), 5(d)). In the yellow shaded region, $0.95 < n_s < 0.98$ is realised. We have found that, for the minimal Kähler potential, $m_{3/2} \lesssim 100$ TeV is disfavored because the extremely fine tuning for the initial value is necessary to reproduce $n_s < 0.98$. Thus the large gravitino mass is more favored from the initial value problem.

On the other hand, in the traditional model, due to constant term $W_0 = m_{3/2}M_P^2$ in the superpotential, the linear term $\sqrt{2\kappa} M^2 m_{3/2} \sigma \cos \theta_S$ arises in the potential for the inflaton, where $\theta_S$ denotes the phase component of the inflaton $S$. For $\cos \theta_S < 0$, the local minimum can be induced, which may trap the inflaton and inflation cannot end. Hence, the initial phase of the inflaton, $\theta_{S,i}$, must be placed near $\theta_S \sim 0$ so that the angular motion is suppressed and the local minimum does not arise. This constraint becomes more stringent for larger gravitino mass as shown below. In the modified model, however, the potential of the inflaton is independent of $\theta_S$, so the initial phase does not affect the dynamics of the inflaton and it can be chosen freely. To contrast the modified model with the traditional one, we illustrate the allowed region for the initial value for the inflaton in both models in Fig. 6. Fig. 6(a) and 6(b) correspond to the present modified model and Fig. 6(c) and 6(d) correspond to the traditional one with a constant superpotential $W_0$. In these figures, we have not imposed the WMAP normalization and we have taken $m_{3/2} = 100$ TeV, $M = 10^{15}$ GeV and $\kappa = 0.01$ in all figures. Fig. 6(a) and 6(c) correspond to the minimal Kähler potential and Fig. 6(b) and 6(d) correspond to the non-minimal Kähler potential with $k_S = 0.01$. We again emphasize that, compared with the traditional case in which $\theta_{S,i} \lesssim 0.1$ is necessary, the initial phase is not constrained in our present model.

### 3.4 Gravitino problem

Gravitinos are copiously produced from the thermal bath at the reheating [24] and also non-thermally by the decay of inflaton [25]. They are often problematic in cosmology. The abundance of thermally produced gravitinos is sensitive to the reheating temperature and that of non-thermally produced ones depends on the inflaton mass and VEV. The hybrid inflation models often conflict with the gravitino problem since the reheating temperature is expected to be rather high and the inflaton mass and VEV are also large.

Fortunately, in the modified model, the reheating is induced by the decay of $\Phi$, not the inflaton of hybrid inflation. This is because the mass of $\Phi$ is much lighter than that of $S$ or $\Psi$, hence we expect that $\Phi$ dominates the Universe and decays well after the decay of $S$. Therefore we can naturally assume that the reheating temperature is determined by the decay of $\Phi$. All relics from the decay of $S$ are diluted away and become negligible after the decay of $\Phi$. The abundance of the thermally produced gravitino is estimated
Figure 5: Constraints on the initial value of the inflaton are shown. We have imposed the WMAP normalization and the allowed regions for the initial values are inside the red contours. We have taken $n = 4$, $m_{3/2} = 100$ TeV (Fig. 5(a), 5(c), 5(e)), $m_{3/2} = 1000$ TeV (Fig. 5(b), 5(d), 5(f)), $g = 1$ (Fig. 5(a), 5(b), 5(e), 5(f)), $g = 0.01$ (Fig. 5(c), 5(d)), $k_S = 0$ (Fig. 5(a) - 5(d)) and $k_S = 0.01$ (Fig. 5(c), 5(d)). The yellow shaded regions correspond to $0.95 < n_s < 0.98$. 
Figure 6: We show the range of the allowed initial values of the inflaton as the red regions. Fig. 6(a) and 6(b) corresponds to our present model and Fig. 6(c) and 6(d) corresponds to the traditional SUSY hybrid inflation model with the constant superpotential $W_0$. We have taken $M = 10^{15}$ GeV, $\kappa = 0.01$ and $m_{3/2} = 100$ TeV in all figures and $n = 4$, $g = 1$ (Fig. 6(a), 6(b)), $k_S = 0$ (Fig. 6(a), 6(c)), $k_S = 0.01$ (Fig. 6(b), 6(d)). We have imposed that the inflation occurs at least for 50 e-foldings, which places the lower bound on the initial value of the inflaton. The dashed green lines represent the lower or upper bound for $m_{3/2} = 0$. 
as [24]

\[ Y_{3/2}^{(TP)} \simeq 2 \times 10^{-12} \left( 1 + \frac{m_{\tilde{g}}^2}{3m_{3/2}^2} \right) \left( \frac{T_R}{10^{10} \text{GeV}} \right) \]  

where \( T_R \) denotes the reheating temperature and \( m_{\tilde{g}} \) is the gluino mass evaluated at \( T = T_R \). Gravitinos are also produced non-thermally from the direct decay of \( \Phi \) [25]. The decay is induced by, e.g., the following non-renormalizable operator in the Kähler potential:

\[ K \sim |\Phi|^2 z z + \text{h.c.} \]

where \( z \) denotes the SUSY breaking field. The resulting gravitino abundance is estimated as

\[ Y_{3/2}^{(NTP)} \simeq 7 \times 10^{-14} \left( \frac{10^7 \text{GeV}}{T_R} \right) \left( \frac{m_{\phi}}{10^{10} \text{GeV}} \right)^2 \left( \frac{\Phi_0}{10^{16} \text{GeV}} \right)^2. \]

In the dynamical SUSY breaking where \( z \) is charged under some symmetry, such operators are forbidden. The \( \Phi \) decay into hidden sector hadrons are also forbidden since \( \Phi \) is lighter than the SUSY breaking scale. Then the decay rate into gravitinos are suppressed by the factor \( \sim (m_z/m_\phi)^4 \) where \( m_z \) is the mass of SUSY breaking field and constraint from nonthermal gravitino production is significantly relaxed [26].

For \( m_{3/2} \gg 10 \text{TeV} \), gravitinos eventually decay into the lightest SUSY particles (LSPs) well before the big bang nucleosynthesis (BBN). In order for such LSPs not to overclose the Universe, the constraint on the gravitino abundance is given by

\[ m_{3/2} Y_{3/2} \lesssim 4 \times 10^{-10} \text{ GeV} \left( \frac{m_{3/2}}{m_{\text{LSP}}} \right). \]

Assuming the anomaly-mediated SUSY breaking model [27, 28] or pure gravity-mediation model [29], the LSP is the neutral Wino whose mass is given by \( M_2 = (g_2^2/16\pi^2)m_{3/2} \simeq m_{3/2}/400 \) where \( g_2 \) is the weak coupling constant. Using this relation and (32), the reheating temperature is constrained as shown in Fig. 7. The allowed parameters are below the small-dotted magenta lines and above the solid red lines for \( g = 1 \) (above the dashed green lines for \( g = 0.1 \) or dotted blue lines for \( g = 0.01 \)). The situation is much better than the case of traditional hybrid inflation model where the nonthermal gravitino production poses a stringent constraint [7]. Note that, by choosing the parameters properly, the observed dark matter abundance can be explained by the non-thermally-produced Wino-LSP from the gravitino decay as shown in [13] and it may be detected by experiments such as LHC and so on [30, 31, 32].

4 Conclusion

We have revisited the SUSY hybrid inflation model focusing on the large gravitino mass case, motivated by the recent LHC results showing the Higgs mass around \( m_h \simeq 125 \text{ GeV} \) [8]. Instead of the constant term in the superpotential which is required to cancel the vacuum energy, we have replaced it with a dynamical field, which effectively becomes a constant term at the present vacuum. We have shown that the allowed parameters are
Figure 7: Constraints on $T_R - m_{3/2}$ plane from the gravitino abundance are shown. The solid red lines, dashed green lines and dotted blue lines represent lower bounds on reheating temperature from the non-thermally produced gravitinos and small-dotted magenta line represents upper bound from the thermally produced one. We have taken $n = 4$ (Fig. 7(a)), $n = 6$ (Fig. 7(b)) and $g = 1$ (solid red lines), $g = 0.1$ (dashed green line) and $g = 0.01$ (dotted blue lines). We have assumed the anomaly-mediated SUSY breaking model in which $m_{\text{LSP}} \simeq m_{3/2}/400$ is satisfied.

altered from the traditional case and, in particular, the relatively large gravitino mass $m_{3/2} \sim 100 - 1000$ TeV is consistent with hybrid inflation in our model. The constraint from the cosmic string is also relaxed. Furthermore, the observed spectral index can also be reproduced without invoking the non-minimal Kähler potential. Although the initial value of the radial component of the inflaton must be tuned so as not to be trapped at the local minimum, the initial phase component of the inflaton is not constrained, hence the initial value problem becomes milder compared with the traditional model. Since the reheating is induced by the dynamical field $\Phi$, not the inflaton, the cosmological gravitino problem can easily avoided in our model.

Acknowledgment

This work is supported by Grant-in-Aid for Scientific research from the Ministry of Education, Science, Sports, and Culture (MEXT), Japan, No. 22540267 (M.K.), No. 21111006 (M.K. and K.N.), No. 22244030 (K.N.) and also by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. N.K. is supported by the Japan Society for the Promotion of Science (JSPS).
References

[1] G. Hinshaw, D. Larson, E. Komatsu, D. N. Spergel, C. L. Bennett, J. Dunkley, M. R. Nolta and M. Halpern et al., arXiv:1212.5226 [astro-ph.CO].

[2] For a review, see M. Yamaguchi, Class. Quant. Grav. 28, 103001 (2011) [arXiv:1101.2488 [astro-ph.CO]].

[3] E. J. Copeland, A. R. Liddle, D. H. Lyth, E. D. Stewart and D. Wands, Phys. Rev. D 49, 6410 (1994) [astro-ph/9401011]; G. R. Dvali, Q. Shafi and R. K. Schaefer, Phys. Rev. Lett. 73, 1886 (1994) [hep-ph/9406319]; G. Lazarides, R. K. Schaefer and Q. Shafi, Phys. Rev. D 56, 1324 (1997) [hep-ph/9608256]; A. D. Linde and A. Riotto, Phys. Rev. D 56, 1841 (1997) [hep-ph/9703209].

[4] M. Bastero-Gil, S. F. King and Q. Shafi, Phys. Lett. B 651, 345 (2007) [hep-ph/0604198]; M. ur Rehman, V. N. Senoguz and Q. Shafi, Phys. Rev. D 75, 043522 (2007) [hep-ph/0612023].

[5] W. Buchmuller, L. Covi and D. Delepine, Phys. Lett. B 491, 183 (2000) [hep-ph/0006168].

[6] V. N. Senoguz and Q. Shafi, Phys. Rev. D 71, 043514 (2005) [hep-ph/0412102].

[7] K. Nakayama, F. Takahashi and T. T. Yanagida, JCAP 1012, 010 (2010) [arXiv:1007.5152 [hep-ph]].

[8] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716, 1 (2012) [arXiv:1207.7214 [hep-ex]]; S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716, 30 (2012) [arXiv:1207.7235 [hep-ex]].

[9] Y. Okada, M. Yamaguchi and T. Yanagida, Phys. Lett. B 262, 54 (1991).

[10] M. Kawasaki, N. Kitajima and K. Nakayama, Phys. Rev. D 87, 035010 (2013) [arXiv:1211.6516 [hep-ph]].

[11] T. Higaki, K. S. Jeong and F. Takahashi, arXiv:1211.0994 [hep-ph].

[12] K. -I. Izawa and T. Yanagida, Phys. Lett. B 393, 331 (1997) [hep-ph/9608359].

[13] M. Ibe, Y. Shinbara and T. T. Yanagida, Phys. Lett. B 642, 165 (2006) [hep-ph/0608127].

[14] K. I. Izawa, M. Kawasaki and T. Yanagida, Phys. Lett. B 411, 249 (1997) [hep-ph/9707201].
[15] M. Kawasaki, N. Sugiyama and T. Yanagida, Phys. Rev. D 57, 6050 (1998) [hep-ph/9710259]; M. Kawasaki and T. Yanagida, Phys. Rev. D 59, 043512 (1999) [hep-ph/9807544]; T. Kanazawa, M. Kawasaki, N. Sugiyama and T. Yanagida, Phys. Rev. D 61, 023517 (2000) [hep-ph/9908350]; T. Kanazawa, M. Kawasaki and T. Yanagida, Phys. Lett. B 482, 174 (2000) [hep-ph/0002236].

[16] M. Kawasaki, N. Kitajima and K. Nakayama, Phys. Rev. D 82, 123531 (2010) [arXiv:1008.5013 [hep-ph]]; Phys. Rev. D 83, 123521 (2011) [arXiv:1104.1262 [hep-ph]].

[17] S. R. Coleman and E. J. Weinberg, Phys. Rev. D 7, 1888 (1973).

[18] A. R. Liddle and D. H. Lyth, “Cosmological Inflation and Large-scale Structure,” (Cambridge University Press, Cambridge, England, 2000).

[19] R. Jeannerot, J. Rocher and M. Sakellariadou, Phys. Rev. D 68, 103514 (2003) [hep-ph/0308134].

[20] C. T. Hill, H. M. Hodges and M. S. Turner, Phys. Rev. Lett. 59, 2493 (1987).

[21] R. Jeannerot and M. Postma, JHEP 0505, 071 (2005) [hep-ph/0503146].

[22] R. Battye and A. Moss, Phys. Rev. D 82, 023521 (2010) [arXiv:1005.0479 [astro-ph.CO]]; J. Dunkley, R. Hlozek, J. Sievers, V. Acquaviva, P. A. R. Ade, P. Aguirre, M. Amiri and J. W. Appel et al., Astrophys. J. 739, 52 (2011) [arXiv:1009.0866 [astro-ph.CO]]; J. Urrestilla, N. Bevis, M. Hindmarsh and M. Kunz, JCAP 1112, 021 (2011) [arXiv:1108.2730 [astro-ph.CO]]; C. Dvorkin, M. Wyman and W. Hu, Phys. Rev. D 84, 123519 (2011) [arXiv:1109.4947 [astro-ph.CO]].

[23] S. Clesse, C. Ringeval and J. Rocher, Phys. Rev. D 80, 123534 (2009) [arXiv:0909.0402 [astro-ph.CO]].

[24] M. Bolz, A. Brandenburg and W. Buchmuller, Nucl. Phys. B 606, 518 (2001) [Erratum-ibid. B 790, 336 (2008)] [hep-ph/0012052]; J. Pradler and F. D. Steffen, Phys. Rev. D 75, 023509 (2007) [hep-ph/0608344]; Phys. Lett. B 648, 224 (2007) [hep-ph/0612291].

[25] M. Kawasaki, F. Takahashi and T. T. Yanagida, Phys. Lett. B 638, 8 (2006) [arXiv:hep-ph/0603265]; Phys. Rev. D 74, 043519 (2006) [arXiv:hep-ph/0605297]; T. Asaka, S. Nakamura and M. Yamaguchi, Phys. Rev. D 74, 023520 (2006) [arXiv:hep-ph/0604132]; M. Endo, K. Hamaguchi and F. Takahashi, Phys. Rev. D 74, 023531 (2006) [arXiv:hep-ph/0605091]; M. Endo, F. Takahashi and T. T. Yanagida, Phys. Lett. B 658, 236 (2008) [arXiv:hep-ph/0701042]; Phys. Rev. D 76, 083509 (2007) [arXiv:0706.0986 [hep-ph]].

[26] K. Nakayama, F. Takahashi and T. T. Yanagida, Phys. Lett. B 718, 526 (2012) [arXiv:1209.2583 [hep-ph]].
[27] L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999) [hep-th/9810155].

[28] G. F. Giudice, M. A. Luty, H. Murayama and R. Rattazzi, JHEP 9812, 027 (1998) [hep-ph/9810442].

[29] M. Ibe and T. T. Yanagida, Phys. Lett. B 709, 374 (2012) [arXiv:1112.2462 [hep-ph]]; M. Ibe, S. Matsumoto and T. T. Yanagida, Phys. Rev. D 85, 095011 (2012) [arXiv:1202.2253 [hep-ph]].

[30] T. Moroi and K. Nakayama, Phys. Lett. B 710, 159 (2012) [arXiv:1112.3123 [hep-ph]].

[31] B. Bhattacherjee, B. Feldstein, M. Ibe, S. Matsumoto and T. T. Yanagida, arXiv:1207.5453 [hep-ph].

[32] L. J. Hall, Y. Nomura and S. Shirai, arXiv:1210.2395 [hep-ph].