Vortex dynamics, pinning, and critical currents in a Ginzburg-Landau type-II superconductor

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The dynamics of vortices in a type-II superconductor with defects are studied by solving the time-dependent Ginzburg-Landau equations in two and three dimensions. We show that vortex flux tubes are trapped by volume defects up to a critical current density where they begin to jump between pinning sites along static flow channels. We study the dependence of the critical current on the pinning distribution and find for random distributions a maximum critical current three times larger than the vortex line density. Whereas for a regular triangular pinning array, the critical current is significantly larger when the pinning density matches the vortex line density.

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In a type-II superconductor, dissipation is associated with the motion of the vortex lattice. This dissipation is reduced by the presence of defects, which pin the vortex lattice up to a critical current density where depinning occurs. In many applications such as superconducting magnets, one is interested in optimizing the vortex pinning to achieve the maximum critical current. However, the details of the depinning transition are complex involving the non-equilibrium dynamics of an elastic lattice through a disordered medium. Theoretical studies based on molecular dynamics simulations suggest the existence of various dynamical phases of vortex motion including plastic flow, uncoupled static channels and coupled channels. It is also possible to simulate vortex dynamics by solving the time-dependent Ginzburg-Landau equations, where the vortex-vortex interaction is completely characterised by the Ginzburg-Landau parameter, $\kappa$. However, three dimensional Ginzburg-Landau vortex dynamics simulations are computationally intensive, in part because the standard explicit integration methods require very small time-steps. In contrast, semi-implicit methods are second order accurate in time allowing large time-steps. Although semi-implicit methods are widely used to simulate three dimensional vortex dynamics in dilute Bose-Einstein condensates, the Ginzburg-Landau equations, involving coupled time-dependent vector fields, are more complex.

In this paper we develop a semi-implicit method to solve the time-dependent Ginzburg-Landau equations in three dimensions. For intermediate values of $\kappa$, the semi-implicit method is two orders of magnitude faster than explicit methods, making it feasible to study dynamical vortex phases, depinning, and the dependence of the critical current on the density and distribution of pinning sites. Although pinning may arise due to magnetic defects, dislocations, grain boundaries, and correlated disorder such as twin planes in high-$T_c$ superconductors, we restrict the current study to volume defects which exclude the supercurrent.

The time-dependent Ginzburg-Landau equations can be written as

$$\partial_t \psi = (\nabla - iA)^2 \psi - (|\psi|^2 - 1)\psi \tag{1}$$

$$\partial_t A = (\nabla S - A)|\psi|^2 - \kappa^2 \nabla \times \nabla \times A \tag{2}$$

where $\psi$ is the order parameter, $A$ is the vector potential, $S$ is the phase of $\psi$, and $\kappa = \lambda/\xi$, where $\lambda$ and $\xi$ are the penetration depth and coherence length, respectively. In equations (1) and (2), distance is measured in terms of $\xi$, time in terms of relaxation time, $\tau = \xi^2/D$, where $D$ is the diffusion constant, and the magnetic field in terms of the upper critical field, $H_c2$. In addition, the Meissner state critical field is given by $H_c = 1/\sqrt{2}\kappa$, and the depairing current density by $j_0 = 2/3\sqrt{3} = 0.385$. The equations are discretized using a grid of $51 \times 51 \times 51$ points with a grid spacing $h = 0.4$. The gauge invariance of the discretized equations is preserved by introducing link variables of the form $U_{ijk} = \exp(-iA_{ijk} h)$. A current flow along $x$ is induced by imposing a magnetic field difference, $\Delta B_z$, between the upper ($y = 10$) and lower ($y = -10$) boundaries. The supercurrent across the boundary is set to zero. We impose periodic boundary conditions at $x = \pm 10$ and $z = \pm 10$. The average current density is given by, $j = \kappa^2 \Delta B_z/d$, where $d$ is the width of the superconductor. A pinning array is produced by adding a potential term to equation (1) consisting of a random distribution of cubic potential steps with side length $a = 1.2$ and height $V_0 = 5.0$. In agreement with other studies, we find that the pinning strength increases with $a$ for $a < \xi$, and saturates for $a > \xi$. A more sophisticated pinning model would be needed to account for the large pinning forces observed for small defects.

Fig. 1 shows a sequence of images illustrating the motion of the vortex lattice through the pinning array. In frame 1, six flux tubes are visible. By comparing frames 1, 2 and 3, one sees that the central flux tubes are moving whereas the two pairs on either side are pinned. However, between frames 4 and 5 the flux tubes on the left and right jump to the next pinning site. This differential motion between neighbouring planes in the vortex lattice.
plays an important role in the voltage-current characteristic (see below). After frame 6, a similar but not identical sequence recurs. For the simulations presented in Fig. 1, the bending of the vortex lines is increased by the choice of a larger value of $\kappa$ and strong pinning. However, no entangling of vortex lines is observed. For smaller $\kappa$, the vortex lines become more rigid, and the behaviour of the three dimensional system and a two dimensional cross section are qualitatively very similar. For high-$T_c$ superconductors, a comparison between the two and three dimensional dynamics should consider possible effects of the layered structure.

FIG. 1. A sequence of three dimensional images showing the motion of a $\kappa = 5$ vortex lattice through a random pinning array. The axes are shown inset in frame 6. The current flows along $x$, the external magnetic field is along $z$, and the vortices move in the $-y$ direction. Each frame shows a region with dimensions $9 \times 7 \times 20$ coherence lengths containing 12 pinning sites (shown in black, not to scale). The external magnetic field and current are $B_{\text{ext}} = 0.4$ and $j = 5 \times 10^{-3}$, respectively. The grey flux tubes corresponds to surfaces of constant supercurrent density, $|\psi|^2 = 0.05$. The time interval between successive frames is 100.

We use two dimensional simulations to study the effect of pinning on the voltage-current characteristic or $V - I$ curve of a superconductor with $\kappa = 3$, where three dimensional effects are suppressed. In addition, we reduce the size and strength of the pinning sites to $\alpha = 0.8$ and $V_0 = 2.0$, respectively. In Fig. 2 we present contour plots illustrating the vortex lattice in two dimensions. Fig. 2(a) shows the instantaneous vortex distribution for a perfect superconductor (no pinning). The vortex density is proportional to the magnetic field which decreases linearly from the bottom to the top. The vortices move upwards with a speed $v = E/B$, where $B$ is the local magnetic field and the electric field, $E$, is constant throughout the sample. Consequently, the vortex flow obeys a Bernoulli-like equation where the flow is faster in regions of lower density (lower magnetic field) and the dissipation can be thought of as a relaxation of the magnetic flux lattice.

Adding defects transforms the triangular lattice into an irregular vortex glass, Fig. 2(b). For low driving fields,
As the current is increased, individual vortices begin to jump between pinning sites. As in the three dimensional simulations, Fig. 1, this motion begins along channels. The existence of static channels confirms the results of molecular dynamics simulations. In the Ginzburg-Landau model channels can merge or divide at intermediate drive currents, as shown in Fig. 2(c). At larger currents, all the vortices are moving but the channels are still evident, Fig. 2(d).

The on-set of vortex motion coincides with the on-set of dissipation or breakdown of superconductivity. In Fig. 3 we plot the $V - I$ curve for a two-dimensional thin film for different defect densities. The voltage is measured by decreasing the current at a very slow rate of $-1.2 \times 10^{-7}$ in $2.5 \times 10^5$ steps, and the $V - I$ curves is obtained from a 200 point moving average. As our sample size is relatively small, surface effects tends to dominate. The critical current due to the Bean-Livingston barrier for vortices entering and leaving the calculation region is the same order of magnitude as the pinning effect. In order to study pinning only we remove the surface effects from a 200 point moving average. As our sample size is relatively small, surface effects tends to dominate. The critical current due to the Bean-Livingston barrier for vortices entering and leaving the calculation region is the same order of magnitude as the pinning effect. In order to study pinning only we remove the surface effects by adding a boundary layer of width $9 \xi$ on either side of a calculation region with width $30 \xi$. Within the boundary layer, a linear ramp potential reduces the supercurrent density gradually to zero. The current density and the voltage are measured within the calculation region $|y| \leq 15$ only.

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The thin black line corresponds to the $V - I$ curve without pinning, and the dotted line shows the normal resistance, $E = j$. Note that at large currents the slope of the $V - I$ curves is similar to the normal resistance curve.

The shape of the $V - I$ curve is dependent on the details of the vortex dynamics. The characteristic ‘curved foot’ can be explained by the combination of an increase in the number of vortex flow channels and increased flow along each active channel, as illustrated in Fig. 2(c) and (d). The $V - I$ curve becomes linear when all the vortices start to move. The ratio between the $V - I$ curves and the normal resistance (the dotted line in Fig. 3) gives the dimensionless resistivity, which measures the fraction of current carried by normal electrons.

As the current is decreased the voltage becomes zero, i.e., all the vortices become pinned, at some finite current which we define as the critical current density, $j_c$. In the absence of finite temperature induced fluctuations or vortex creep, the value of $j_c$ is well defined. However the critical current is sensitive to the exact distribution of pinning sites, therefore we average over six random distributions with the same density. Fig. 4 shows a plot of the average value of $j_c$ against pinning density. The maximum critical current density is about 2% of the depairing current, $j_D$. For comparison, the optimum critical current density of Nb-Ti alloy is about 3% of $j_D$. The maximum value of $j_c$ occurs at pinning density about three times larger than the vortex line density (indicated by the dotted line in Fig. 4).

![FIG. 3. The $V - I$ curves for a two dimensional section of superconductor with pinning densities (from the right) 0.14, 0.28, 0.39, and 0.56 $\xi^{-2}$ (at these relatively high densities, the critical current decreases with increasing pinning density). The thin black line corresponds to the $V - I$ curve without pinning, and the dotted line shows the normal resistance, $E = j$. Note that at large currents the slope of the $V - I$ curves is similar to the normal resistance curve.](image)

![FIG. 4. The critical current density as a function of the density of pinning sites (in units of $\xi^{-2}$) for both random distributions ($\nabla$) and regular triangular arrays (•). The data points are determined from an average of six random distributions. The error bars (shown for the high density distribution only) indicate the standard deviation. An example illustrating the effect of the distribution on the $V - I$ curves is shown inset. The bold curve is a fit using the function $Ax \exp(-Bx)$, where $A$ and $B$ are fit parameters. The critical current density for a regular triangular array is a maximum when the pinning density is equal to vortex line density (indicated by the dotted line).](image)
follows from the linear dependence of the critical current on the pinning force. The exponential decrease at large pinning densities is due to the competing effect of supercurrent depletion by defects. The shape of the curve and the relatively high optimum pinning density also agree qualitatively with experimental results on silver doped high-$T_c$ superconductors [14].

For certain random distributions one finds persistence static channels which can dramatically reduce the critical current. This is illustrated in Fig. 4(inset), where the curve with lower dissipation at large currents has a significantly lower critical current.

One approach to increase the critical current is to introduce a regular pinning array by nanostructuring [15–17]. In Fig. 4 we show that a regular triangular array increases the critical currents by more than a factor of two, however, the optimum pinning density is sharply peaked around the vortex line density. Consequently, the enhancement is only obtained within a narrow range of the external magnetic field. This agrees with experimental studies where a sharp enhancement peak is obtained at matching magnetic field values [17]. There are two additional critical current peaks, one at one third the vortex line density where every third vortex is trapped, and one at half the vortex line density, which is weaker because the matching only occurs on alternate planes. For small pinning sites ($a = 0.8$ compared to the vortex cores size of 2) the maximum critical current is about 5% of the depairing current, $j_D$. For $a = 2$ we obtain $j_c = 0.074j_D$, which suggests that other pinning mechanisms may be needed to obtain $j_c \sim j_D$.

In summary, we have studied vortex dynamics and pinning in a three dimensional superconductor by solving the time-dependent Ginzburg-Landau equations. We find that above a critical current density vortex flux tubes jump between pinning sites following specific channels. The main features of the dynamics are reproduced by two dimensional simulations. We study the effect of pinning on the voltage-current characteristic of the superconductor, and show that the breakdown of superconductivity is associated with the appearance of channelled vortex flow. The characteristic curved foot in the $V - I$ curve arises due to the combination of the formation of more channels and faster vortex flow along each channel. For a random pinning array we find a maximum critical current equal to 2% of the depairing current occurring at a pinning density of about three times the vortex line density. Finally, we study the critical currents produced by vortex matching pinning arrays. The results suggest that time-dependent Ginzburg-Landau simulations are ideally suited to provide quantitative predictions of critical currents in type-II superconductors.

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