Analytical approach to solving dynamic problems on resonators

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Abstract. A sensitive element, its geometric features, and errors affecting the performance characteristics of the product are considered in this article. The analysis was carried out, which showed that the main contribution to the self-care of the angle is made by such errors as elastic-mass defects of the quartz resonator, inaccuracy of the assembly of the structure, errors in the shooting and control system, geometric errors of the resonator. The influence of such errors as the frequency response and the quality difference on the final characteristics of the navigation device is estimated. An urgent task is to improve the control component and increase the accuracy of the entire measuring system of a solid-state wave gyroscope. The use of adaptive automatic control systems provides setting the resonant frequency of the sensitive element, capturing the natural frequency of oscillations, generating signals of the mismatch of the amplitude of oscillations, etc. To improve the efficiency of the automatic control system, to reduce the time to bring the device to the required level of quality, a mathematical model is being developed. This article discusses the model of the circuit of automatic suppression of dissimilarity for the push-pull control system of a solid-state wave gyro. An assumption has been made that it is possible to apply the results of simulation modeling for the further implementation of the circuit of suppressing different-Q-factors in the navigation block.

1. Introduction
One of the leading directions of technical progress is the rapid development of navigation tools. High requirements for sensitive elements of orientation and navigation systems led to the creation of unconventional gyroscopic instruments based on new physical principles (solid-state and micromechanical gyroscopes, satellite navigation systems), and because of their widespread use in many areas of modern society [1].

There are stringent requirements for navigation systems, they must work in harsh environmental conditions, withstand shock loads, have high reliability, long working life, high accuracy, have a small weight, dimensions, and power consumption, go to working mode within 0.1 seconds [2].

One of the most modern and promising representatives of navigation systems is a solid-state wave gyroscope (SWG), whose principle of operation is based on the inert properties of elastic waves excited in a solid. The advantages of tug are high accuracy, low readiness time, absence of moving parts in the structure, low energy consumption, low sensitivity to linear overloads.

The actual direction in gyroscopy is the creation of instruments with high tactical and technical characteristics, the improvement of which is carried out at the stage of design preparation of the product [3].
Despite the differences in design, the control system plays a special role in the development of gyroscopes, to improve the characteristics of which and reduce the processing time, it is important to create a mathematical model of a solid-state wave gyroscope. In the manufacture of tug conducted a large number of measurements. Controlled by various parameters that determine the final characteristics of the device. The main errors that determine the accuracy of the output signal from the device include its own departure (drift) of the standing wave, noise component, nonlinearity, and others. This article discusses issues related to the decrease in drift. Based on the analytical solution of equation 1 and subsequent mathematical modeling. In studies [6, 9, 11] it was noted that the change in the angular orientation of the device is measured by the angular position of the standing wave excited in the resonator. In the process of manufacturing SWG for several reasons, such as asymmetry of the case, uneven distribution of stresses in quartz glass, microcracks, the dependence of the damping factor on the angular position arises.

Drift is the intrinsic rate of departure of a standing wave, the precession in the absence of external rotations. The drift of the SWG is resolved into systematic and random components. Systematic drift depends on various factors: dissimilarity, heterogeneity, ellipticity. The change in the electric angle (standing wave in the resonator) is determined by the following formula [4]:

\[
\dot{\theta} = -K \Omega - L \cos(4(\theta_{\omega})) + \frac{1}{4} \left( \frac{1}{\tau_1} - \frac{1}{\tau_2} \right) \sin(4(\theta - \varphi_{\omega})) + P \sin 2\gamma
\]

The first term of the equation describes the rate of change of the angular position of the device, relative to the rate of change of the standing wave. K is the scale factor of the Brian effect [5]. In a real instrument, this ratio describes the output signal. The challenge is to reduce the influence of terms that distort the Brian effect.

The second term explains the occurrence of systematic drift due to different frequencies. In an ideal resonator, the position of the axes of normal vibrations can be arbitrary, however, due to the asymmetry of the shell, the eigenfrequencies degenerate and two elastic axes appear with eigenfrequencies \(\omega_1\) and \(\omega_2\).

The third term describes the occurrence of a systematic drift associated with a difference in quality. Due to the inhomogeneous dissipation of the energy of free oscillations, two axes arise, characterized by the values and. These values are the smallest and largest damping decrements, respectively.

The fourth term describes the ellipticity of oscillations. Thus, if improperly manufactured, the parametric excitation system will create a drift.

At the last stage of manufacturing the resonator, such technological operations as grinding and polishing are performed, as well as some electrical parameters are adjusted. The occurrence of systematic drift due to different frequencies decreases at the stage of setting up a sensitive element, by performing such technological operation as balancing [6]. To reduce heterogeneity to microhertz, the circuit of automatic adjustment of elastic axes is used. It is believed that the systematic drift caused by different frequencies is almost completely eliminated and does not affect the wave pattern.

The presence in the resonator of asymmetry of the shell and stresses on the surface layer leads to the appearance of two axes of dissipation [12], and as a result, to the appearance of a drift from dissimilarity. Variety is a phenomenon in which the quality factor depends on the angular orientation of the standing wave in the resonator. A confirmation of this will be the display of the error signal in amplitude simultaneously with the drift graph (Figure 1).
Figure 1. Drift and mismatch signal of the amplitude of oscillations of the sensitive element.

It can be seen that in the values of the maximum and minimum of the mismatch signal, the drift value has a minimum value, which means that the standing wave in these positions is in the Q axis. Consequently, the drift estimate can be given from the values of the amplitude mismatch signal. Currently, there is no ready-made implementation of an automatic control system for the variability of different sizes, therefore, the development of such a system is an urgent task.

One of the ways to reduce the drift caused by diversification is to approximate the 4th harmonic output signal. It is necessary and sufficient that the graph of the approximating function coincides with the error signal [13], for convenience, the least squares method is used. The coefficients A and B play an important role in finding a new function, and it will depend on them the difference from the original signal. The task has been reduced to the determination of the coefficients of the approximating function.

2. Modeling

The control system performs the most important functions; to improve it, a mathematical model is needed to investigate the influence of various innovations added to the design and to obtain data on technical parameters.

In the analytical approach, it is proposed to consider the mathematical model of the resonator in the form of a partial oscillator. The mathematical model for measuring the angle of rotation using SWG requires the application of a slowly varying amplitude of oscillations, therefore, for simulation purposes; the equation is used for slowly varying phase variable waves in the cavity. Consider the following model SWG [7]:

\[ X_{\text{obj}} = (G_{\ell} + G_{t} + G_{\omega} + G_{\Delta}) \times X + G_{f} \times F \]

where

\[
G_{\ell} = \frac{1}{\omega t} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},
\]

\[
G_{\omega} = \frac{\Delta_{\omega}}{2\omega} \begin{pmatrix} \cos 4\theta_{\ell} & 0 & \sin 4\theta_{\ell} & 0 \\ 0 & \cos 4\theta_{\ell} & 0 & \sin 4\theta_{\ell} \\ \sin 4\theta_{\ell} & 0 & -\cos 4\theta_{\ell} & 0 \\ 0 & \sin 4\theta_{\ell} & 0 & -\cos 4\theta_{\ell} \end{pmatrix},
\]
\[ G_\Omega = \frac{2K\Omega}{\omega} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}, G_{\Delta \omega} = \frac{\Delta \omega}{2\omega^2} \begin{pmatrix} 0 & -\cos 4\theta_\omega & 0 & -\sin 4\theta_\omega \\ \cos 4\theta_\omega & 0 & \sin 4\theta_\omega & 0 \\ 0 & -\sin 4\theta_\omega & 0 & \cos 4\theta_\omega \\ \sin 4\theta_\omega & 0 & -\cos 4\theta_\omega & 0 \end{pmatrix} \]

\[ G_f = \frac{1}{2\nu\omega^2} \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \]

\( \omega \) - the resonant frequency of the sensitive element, \( \tau \) - the magnitude of the resonator Q factor, \( \Delta_\tau \) - the magnitude of the resonator difference in the Q-factor, \( \Delta_\omega \) - the magnitude of the resonator different frequency, \( \theta_\tau \) - the angular position of the Q axis, \( \theta_\omega \) - the angular position of the frequency axis, \( \nu \) - the control parameter close to unity, characterizes the degree of detuning frequency from resonance.

\[ X = X(t) \] - the vector of phase state variables of the system, of dimension 1 * 4;
\[ F = F(t) \] - a control action limited by a piecewise continuous function with a finite number of discontinuity points;

\( G_\tau \) - the matrix of dissipative forces of the scalar (spherical) type;
\( G_{\text{Qf}} \) - the matrix of forces generating the “Difference in the Q-factor” of the resonator;
\( G_\Omega \) - the matrix of gyroscopic forces;
\( G_{\Delta \omega} \) - the matrix of forces generating the “heterogeneity” of the resonator;
\( G_f \) - the matrix of circular (non-conservative, pseudogyroscopic) forces.

The reliability of calculations is ensured by the choice of parameters so that the model matches and reproduces the characteristics of a real instrument. From the previously conducted numerical and field experiments [8], we take the values equal to:

1) \( Q = 600 \) seconds;
2) Difference in Q-factor = 2%;
3) Scale factor = 0.3;
4) Resonance frequency = 5000 Hz.

After adjustment, the magnitude of the different frequency is reduced to thousandths of Hertz, which is confirmed by numerous field experiments. Also, this defect has practically no effect on the value of the quality factor and, accordingly, of the difference in quality, therefore, in the model under consideration, it does not make sense to take it into account. We accept the values of the matrices and are much less than unity, and at the initial moment of time, we do not take into account the correction signal from the control matrix due to its absence. We get the equation:

\[ (G_\tau + G_{\text{Qf}} + G_\Omega) \cdot X + Bu \quad (3) \]

We consider the high-frequency oscillations of the ring in the second mode of oscillations in the plane, we turn to the calculation of the Difference in the Q-factor, which leads to the appearance of a defect such as drift. The magnitude of the systematic and non-systematic drift depends on the final value of the Q-factor and the Difference in the Q-factor, and as a result, the sensitivity of the device to angular velocities. Systematic drift from the Difference in the Q-factor can be eliminated by creating static hyperbolic-type forces [9].

Since the Q-factor has a finite value, for the operation of the device it is necessary to create and maintain constant undamped oscillations using a control loop that performs energy pumping.

The application of the Q-factor control algorithm is necessary to ensure the perfect rotation of the standing wave, which can be realized in two ways:

1) turning the device around its axis;
2) the creation of forces proportional to the forces of the gyroscopic type, cyclically varying according to the required law.
We use the second method. Of all the errors of the resonator, the most interesting is the value of the
dissimilar parameter, since the final drift of the output signal will depend on it.

Having provided the necessary parameters for the model to work, namely: by specifying forced
oscillations and constant rotation, we proceed to solve equation (3). It is impossible to explicitly find
the internal parameters of the resonator, therefore we will make a qualitative assessment of these
parameters, we will take:

$$\tilde{p} = -\frac{\Delta \tau}{2 \omega}, \tilde{q} = 4 \theta \tau$$

Then the estimated matrix of the forces generating the “Difference in the Q-factor” of the resonator
will look like:

$$\tilde{G}_{\sigma \tau} = \tilde{p} \begin{pmatrix} \cos(\tilde{q}) & 0 & \sin(\tilde{q}) & 0 \\ 0 & \cos(\tilde{q}) & 0 & \sin(\tilde{q}) \\ \sin(\tilde{q}) & 0 & -\cos(\tilde{q}) & 0 \\ 0 & \sin(\tilde{q}) & 0 & -\cos(\tilde{q}) \end{pmatrix}$$

We will carry out the simulation of the algorithm execution in the Matlab: Simulink software
product, and find the desired parameters $\tilde{p}$ and $\tilde{q}$. Figures 2 and 3 show an example of calculating the
required parameters.

![Figure 2](image1.png)  
**Figure 2** Calculation of the $\tilde{p}$ parameter

![Figure 3](image2.png)  
**Figure 3.** Calculation of the $\tilde{q}$ parameter

The graphs show that to determine the values of the estimated parameters accurately, the time of
continuous operation of the device with a constant rotation of at least 5 minutes is necessary. The
initial values are noticeably different; the transient process inside the SWG explains this. For control
coefficients, it is not necessary to take into account the values of the coefficients calculated in the time

interval up to one minute. The control system will operate in two modes: calibration mode and adjustment mode. At initial start-up, the control system operates in a calibration mode; this is necessary to perform a preliminary adjustment of the instrument and determine its position in space. Next, the control system switches to adaptive filtering mode, at which the control coefficients are refined in time, thereby improving the accuracy of the output signal.

Making an estimate of the parameters that affect the value of the Q-factor, we form the control signal inverse to the matrix of forces generating the Difference in the Q-factor. The use of new control coefficients significantly increases the accuracy of the output signal, namely, it reduces such an error as the systematic drift of a solid-state wave gyro caused by the forces generating different resonator resonators. Figure 4 shows a graph of the drift of a solid-state wave gyroscope before and after using adaptive control.

![Figure 4. Drift dependence on the use of adaptive control](image)

The drift from the Difference in the Q-factor is eliminated by the creation of static hyperbolic-type forces [10]. The circuit of the automatic correction of different returns differs from the control circuit of the axes in that at each local time point there is no criterion according to which the dynamics of the resonator can be adjusted. For the control circuit of the axes of stiffness, these criteria are the phase difference of the channels. For the development of the circuit, it is necessary to provide a change in the electric angle $\theta$ in a sufficiently large range, as well as the ability to create forces that generate a Q-factor. We use an adaptive filter, on the basis of which a circuit of difference in the Q-factor suppression has been developed, implemented using a push-pull control system [8]. The proposed control system allows realizing both types of forces: gyroscopic and hyperbolical.

The general algorithm for the operation of the control circuit of difference in the Q-factor is

1. Ensure the condition of immobility of the object, which includes SWG with a push-pull control system;
2. Turn on the device;
3. Start the wave rotation mode [11];
4. Calculate the necessary values to correct the difference in the Q-factor;
5. Form the forces that reduce the difference in the Q-factor;
6. Start the operating mode (angle sensor or angular velocity);

The result of the operation of this algorithm will be a reduction in the forces that cause a difference in the Q-factor and, as a consequence, a decrease in drift.

3. Conclusion

The considered analytical approach and the simulation carried out by the mathematician showed the efficiency of the adaptive circuit with automatic compensation of difference in the Q-factor values and can be implemented as part of an SWG device with a digital signal processing system and a push-pull control system.
The task of further research is to consider issues related to the implementation of the circuit of the suppression of dissimilarity. Develop and implement a software module that is responsible for switching the instrument into the mode of wave rotation, a parameter calculation module for controlling difference in the Q-factor, and a force generation module for suppressing different receptacles. The result of the circuit work will be a reduction in the drift of the SWG.

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