A new conception is proposed in Ref. 1, 2, that gravity is one kind of entropic force. In this letter, we try to discuss its applications to the modified gravities by using three different corrections to the area law of entropy which are derived from the quantum effects and extra dimensions. According to the assumption of holographic principle, the number of bits \( N \) which is related to the equipartition law of energy is modified. Then, the modified law of Newton’s gravity and the modified Friedmann equations are obtained by using the new notion. By choosing suitable parameters, the modified area law of entropy leads to de-Sitter solutions which can be used to explain the accelerating expansion of our universe. It suggests that the accelerating phase in our universe may be an emergent phenomenon based on holographic principle and thermodynamics.

**Keywords**: entropic force; modified gravity; thermodynamics; holographic principle.

1. **Introduction**

The gravity was always being regarded as one kind of fundamental force. However, the origin of gravity is still an unaccessible theoretic puzzle. Recently, a new conception of gravity is proposed by Ref. 1, 2. The fundamental basement of these work could be traced back to the profound connections between gravitation and thermodynamics which are suggested by the discovery of the black hole entropy \( S \), the
four laws of classical black hole mechanism \cite{4} and the Hawking radiation \cite{5}. Based on thermodynamics and holographic principle, the new idea, that the gravity can be explained as one kind of entropic force, gives out the Newton’s law. But from a point of view of thermodynamics, is it possible to derive the Einstein equations of gravitational fields? Jacobson first answered this question based on the geometric feature of thermodynamic quantities of black holes whose result is yes \cite{6}. Referred to the new idea of entropic force about gravity, the Friedmann equation which is the time-time component of Einstein equation could be gotten as well \cite{7,8}.

Now, the gravity has the new explanation that it can be treated as one kind of entropic force. In this new theoretic frame, what about the observable effects? Since cosmology covers all energy scales in our universe, our universe could be regarded as a natural lab to detect the observable effect especially for the gravitational force. One of the most important development in cosmology is the proposal of the two accelerations in the early and late times. Inflation, which is an accelerating phase in the early time, was introduced as a way to solve the problems in the standard big-bang theory \cite{9,10}. Meanwhile, the current observable accelerating expansion of our universe was suggested by combining different cosmic probes that primarily involves Supernova data \cite{11,12}. The simplest candidate of the two accelerations is the $\Lambda$CDM model which has the coincidence problem. Nevertheless, the validity of General Relativity on large astrophysical and cosmological scales has never been tested but only assumed, the acceleration in early and late universe might be signals of a breakdown of General Relativity \cite{13,14,15,16}. And, the breakdown of General Relativity is fairly working at solar system and in the weak field regime \cite{17,18}.

Based on the Bekenstein-Hawking entropy in General Relativity \cite{5,19}, the entropy of a Schwarzschild black hole is used in Verlinde’s work,

\[ S = \frac{k_B c^3}{4 G \hbar} A, \quad (1) \]

where $G$ is Newton’s constant, $A$ is the surface area of the horizon; and it is the standard relation between horizon entropy and horizon area. Phenomenologically, one could assume that the horizon entropy is a general function of the horizon area,

\[ S = \frac{k_B}{4} g\left(\frac{c^3 A}{G \hbar}\right), \quad (2) \]

where $g$ is an arbitrary function. At least, after taking quantum correction into account, the form of the entropy in the modified gravity will be changed. However, current observations could not tell us the law of gravity at large or small scales. If we start from the corrected relation between the entropy and the area, can we get the modified Einstein equations? Does this modified gravity make our universe accelerate?

To answer the above questions, in this letter, we try to derive the law of gravity with the modification of area law of entropy by the new notion of gravity which is treated as entropic force. Specifically, we discuss three different corrections for the area law of the entropy. One is the most popular used logarithmic correction
which is derived from quantum effects\cite{20,21,22,23,24,25,26,27,28,29}. One is the power-law correction which is related to the extra dimensions\cite{31,32,33,34,35,36}. The last one is the $f(R)$ correction\cite{37} which may break the energy equipartition law\cite{38,39,40}. Besides the modified gravities, the dark energy model from entropic force has been discussed as well\cite{41,42,43}.

We arrange our paper as follows. In Sec.\ref{sec:newton_law} we give out the approach of getting the Newton’s law by using Verlinde’s method. Next, we introduce the method of getting the Friedmann equations with the energy equipartition law in Sec.\ref{sec:friedmann_equations}. Specifically, in Sec.\ref{sec:logarithmic_correction} we discuss the logarithmic correction which may be related to the quantum effects. Then, we discuss the power-law correction which gives an accelerating universe in Sec.\ref{sec:power-law_correction} and we consider the $f(R)$ correction in Sec.\ref{sec:f(R)_correction}. Finally, the paper is concluded in Sec.\ref{sec:conclusion}.

\section{The Newton law\label{sec:newton_law}}

The essence of Verlinde’s idea is that gravity is not fundamental; it is a kind of entropic force. The process of deriving the Newton’s law mainly depends on thermodynamics and holographic principle. The notion of the entropic force can be expressed as

\begin{equation}
F \Delta x = T \Delta S, \tag{3}
\end{equation}

where $\Delta S$ is the change of entropy of the gravitational system, $\Delta x$ is the displacement of a particle in the gravitational system, and $T$ is the temperature of the system. Motivated by Bekenstein’s original argument on black holes\cite{5}, it is postulated that since the change of entropy $\Delta S$ is associated with the information on the boundary and it equals $2\pi k_B$ when $\Delta x = h/m$, the change of entropy is assumed as the following form

\begin{equation}
\Delta S = 2\pi k_B \frac{mc}{h} \Delta x, \tag{4}
\end{equation}

where $k_B$ is the Boltzmann’s constant, $c$ is the velocity of light, and $h$ is the reduced Planck constant.

The holographic principle assumes that all the information about black holes is encoded in the surface; it could be used in the Schwarzschild black hole and the de-Sitter space. If the number of bits $N$ of the holographic system is proportional to the area of the holographic screen, the following relation will be generalized by the holographic principle

\begin{equation}
N = \frac{4S}{k_B} \tag{5}
\end{equation}

Specially, based on the entropy in general relativity as Eq.\ref{eq:entropy} expressed, the above equation can be rewritten as

\begin{equation}
N = \frac{Ac^3}{Gh} = \frac{4\pi r^2 c^3}{Gh}. \tag{6}
\end{equation}
where the radius of gravitational system is $r$ and the area of the corresponding holographic screen is $A = 4\pi r^2$.

One of the most important assumptions for the notion of gravity of entropic force is that each bit on the holographic screen contributes an energy of $k_B T/2$ to the system. The equipartition law of energy in thermodynamics can be used

$$E = \frac{1}{2}Nk_B T = \frac{2\pi c^3 r^2}{G\hbar} k_B T = Mc^2,$$

(7)

where the energy is defined. Amazingly, combined Eqs. (3), (4) and (7), the Newton’s Law of gravitation is derived,

$$F = G\frac{Mm}{r^2}.$$  

(8)

The above calculation depend on the holographic principle and the energy equipartition law closely. It is interesting to extend Verlinde’s idea from general relativity to the more generalized gravity.

3. The Einstein Equations

The research on Einstein equation is a useful approach to investigate the dynamics of our universe. And, we focus on a $(3 + 1)$-dimensional flat Friedmann-Robertson-Walker (FRW) universe with the metric in double null form,

$$ds^2 = h_{ab}dx^a dx^b + \tilde{r}^2 d\Omega^2_2,$$

(9)

where $a$ is the scale factor, $\tilde{r} = ar$, the 2-dimensional metric is $h_{ab} = \text{diag}(-1, a^2)$ and the unit spherical metric is given by $d\Omega^2_2 = d\theta^2 + \sin^2\theta d\varphi^2$. The energy momentum tensor $T_{\mu\nu}$ of matter in the universe is supposed to have the perfect fluid form

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu + pg_{\mu\nu},$$

(10)

where $\rho$ and $p$ are the energy density of matter and the pressure of matter separately. The evolution of energy density for matter could be assumed as,

$$\dot{\rho} + 3H(\rho + p) = 0,$$

(11)

where $H \equiv \dot{a}/a$ denotes the Hubble parameter. The derivation of the Einstein equations will mainly depend on two different definitions of the energy, one is from the energy equipartition law which is related to the notion of gravity as entropic force, the other one is from the Misner-Sharp mass. In the next sections, we will introduce them separately.

3.1. The Energy Equipartition Law

The notion of horizon plays an important role in cosmology. The apparent horizon, the event horizon and the particle horizon are different both in definitions and physical meanings. In cosmology, the proper chosen horizon is just the apparent
horizon which coincides with the double null metric \(^9\) and it is the boundary surface of anti-trapped region \(^{44}\). The apparent horizon is defined as \(h^{ab}\partial_a\tilde{r}\partial_b\tilde{r} = 0\), it turns out to be
\[
\tilde{r}_A = \frac{c}{H}.
\] (12)

Only in the flat FRW metric, the above form of apparent horizon is the same as the Hubble horizon. From the definition of the apparent horizon, we get
\[
d\tilde{r}_A = -\frac{\tilde{r}_A^3}{c^2}H\tilde{H}dt.
\] (13)

During the time interval \(dt\), the radius of the apparent horizon is supposed to evolve from \(\tilde{r}_A\) to \(\tilde{r}_A + d\tilde{r}_A\), then the change of the area of the apparent horizon is
\[
dA = 8\pi\tilde{r}_Ad\tilde{r}_A.
\]

And, the change of the horizon temperature is
\[
dT = -\frac{c\hbar}{2\pi k_B\tilde{r}_A}d\tilde{r}_A.
\] (15)

The Hawking temperature is determined by geometry itself and it has nothing to do with gravity theory explicitly. Once the geometry is given, the surface gravity and the Hawking temperature can be determined immediately.

Since the apparent horizon with area \(A = 4\pi\tilde{r}_A^2\) carries \(N\) bits of information, from Eq. (5), we could get the change of the number of bits \(dN\). Then, from the energy equipartition rule, we get the changes of the total energy is gotten,
\[
dE = \frac{k_B}{2}NdT + \frac{k_B}{2}TdN.
\] (16)

The expression of \(dE\) is determined by the geometry with variables such as the Hubble parameter \(H\). It is the definition of energy from the view point of the thermodynamics.

### 3.2. The Misner-Sharp Mass

On the other side, for a spherically symmetric space-time with the metric \(^{44}\), using the Misner-Sharp mass \(^{45}\), \(^{46}\), \(^{47}\), \(^{48}\), \(^{49}\)
\[
\mathcal{M} = \tilde{r}(1 - g^{ab}\tilde{r}_a\tilde{r}_b)/2G,
\] (17)

the \(a - b\) components of the Einstein equation give the mass formula\(^{50}\), \(^{51}\)
\[
\mathcal{M}_{,a} = 4\pi\tilde{r}^2(T^b_a - \delta^b_aT)\tilde{r}_b,
\] (18)

where \(T = T^a_a\). At the apparent horizon, the Misner-Sharp mass \(\mathcal{M} = 4\pi\tilde{r}_A^3\rho/3\) can be interpreted as the total energy inside the apparent horizon. To project the
mass formulas, one could use the generator \( k^a = (-1, Hr) \) of the apparent horizon, which is null at the horizon. Since \( k^a \tilde{\gamma}_{,a} = 0 \), using the mass formulae (18), the energy which flows through the apparent horizon could be gotten

\[
dE = k^a \nabla_a M dt = 4\pi \tilde{r}^2 T^b_{\ a} \tilde{\gamma}_{,b} k^a dt = 4\pi \tilde{r}^3 A H (\rho + p) dt.
\]

This is the other definition of the energy which is related to the matter part.

### 3.3. The Equations In Cosmology

Combined with Eqs.(16) and (19), the so-called Raychaudhuri equation is gotten,

\[
\frac{k_B}{2} N dT + \frac{k_B}{2} T dN = 4\pi \tilde{r}^3 A H (\rho + p) dt,
\]

which connects the geometry and the matter. Considering the evolution of matter, after integration, the Friedmann equation which is the time-time component of Einstein equations could be derived easily. And, the other component of Einstein equations can be derived by combining the Raychaudhuri equation and the Friedmann equation.

In a summary, the Misner-Sharp mass related to the Einstein equations decides the right side of Eq.(20). Meanwhile, the left side of Eq.(20) is decided by the equipartition law of energy which is related to the thermal dynamical system. The Misner-Sharp mass which is derived from the Einstein equation is just a variable. To make a correct Raychaudhuri equation, both sides of Eq.(20) must be correct. Because of the validity of the Misner-Sharp mass, we only need to consider the validation of the equipartition law of energy. There is a question that whether we can apply this method to the modified area law of entropy, explicitly speaking, whether the energy equipartition law can be used in modified Einstein gravity. Firstly, the assumption, that the apparent horizon has an entropy proportional to its horizon area, is originated from the black hole thermodynamics which obeys the so-called area formula. The modified theory of gravity may explain the present acceleration without introducing dark energy. If it so happens, Einstein gravity will be only an approximate theory, and the relation of the entropy to the horizon’s area should be changed. Therefore, it is interesting to see thermodynamic properties of the universe may be modified. Secondly, the energy equipartition law only exists in the thermal equilibrium processes. If we can get the correct Raychaudhuri equation (or the correct Friedmann equation), the using of the equipartition law of energy is proper. But if we can not get the correct Friedmann equation, the using of the equipartition law of energy is problematic.

### 4. The Logarithmic Correction

The logarithmic correction to the horizon entropy can arise from the breaking of the conformal invariance of classical massless fields or an anomalous trace for the
modified gravity emerging from thermodynamics and holographic principle

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energy-momentum tensor. As a result, the entropy becomes

\[ S_1 = \frac{k_B c^3}{4G\hbar} A + \frac{\alpha k_B}{4} \ln \frac{A c^3}{G\hbar}, \]  

(21)

where the subscript “1” notifies the logarithmic correction for entropy, \( \alpha \) is a dimensionless parameter whose value is still in debate because there are other physical origins about the logarithmic correction, e.g. such a term appears in the study of black hole entropy of the loop quantum gravity \(^{25,26,27,28}\) and in the correction to black hole entropy of the thermal equilibrium fluctuation or the quantum fluctuation \(^{30}\). Therefore, there are different values of \( \alpha \), \( \alpha = -\frac{3}{2}, \alpha = -\frac{1}{2}, \alpha = 0 \) and \( \alpha > 0 \) are given in Refs. \(^{25,26,27,28}\) separately. Based on the holographic principle, the number of bits which is corresponding the quantum corrected entropy-area relation then becomes

\[ N_1 = \frac{A c^3}{G\hbar} + \alpha \ln \frac{A c^3}{G\hbar}. \]  

(22)

Using the method in Sec2 and Sec3 the Newton’s law and the Friedmann equation of General Relativity could be extended to the gravity with a logarithmic correction.

4.1. The Modified Newton’s Law For Logarithmic Correction

By substituting the modified number of bits \( N_1 \) into the equipartition law of energy \(^7\), the modified energy of the system is gotten. Then combining this modified energy with Eqs.(3) and (4), the Newton’s law could be extended to the gravity with a logarithmic correction,

\[ F_1 = \frac{G M m}{r^2} \left[ 1 + \alpha \ln \frac{4\pi r^2 c^3}{G\hbar} \right] \frac{M m}{r^2} = \frac{G M m}{r^2} \left[ 1 + \alpha \ln \frac{4\pi r^2 c^3}{G\hbar} \right] \frac{M m}{r^2}. \]  

(23)

It is the Newton’s law for the logarithmic correction of gravity. Here, our discussions of the modified Newtonian forces are based on the entropic force which is a quality of thermal statistics. After choosing the proper parameter, it could get back to the Newtonian gravity which satisfies the observations. For example, when \( \alpha \ln \left( \frac{4\pi r^2 c^3}{G\hbar} \right) \ll 1 \), the corresponding leading terms of the corresponding Taylor expansion are

\[ F_1 = \frac{G M m}{r^2} \left[ 1 - \alpha \ln \frac{4\pi r^2 c^3}{G\hbar} \right] \frac{M m}{r^2} \left[ 1 - \alpha \ln \left( \frac{4\pi r^2 c^3}{G\hbar} \right) \right] \left[ 1 - \alpha \ln \left( \frac{4\pi r^2 c^3}{G\hbar} \right) \right]^2. \]  

(24)

In the next section, we try to discuss the modification of Friedmann equation due to the correction of the law of gravity.

4.2. The Friedmann Equation For Logarithmic Correction

For the gravity of the logarithmic correction, the corresponding thermodynamics is changed. Since the apparent horizon with area \( A = 4\pi r_A^2 \) carries \( N_1 \) bits of
information, then from Eq. (22), one could get the change of the number of bits

\[
dN_1 = \frac{8\pi c^3}{G\hbar} \hat{r}_A d\hat{r}_A + 2\alpha \frac{d\hat{r}_A}{\hat{r}_A}. \tag{25}
\]

From the energy equipartition rule, one then obtains the changes of the total energy

\[
dE_1 = \frac{c^4}{G} \left( 1 + \alpha \frac{G\hbar^3}{2\pi c^3 \hat{r}_A^5} - \alpha \ln \left( \frac{4\pi c^3}{G\hbar^3} \right) / \left( \frac{4\pi c^3}{G\hbar^3} \right) \right) d\hat{r}_A. \tag{26}
\]

Combined with the definition of the apparent horizon, the above equation becomes

\[
dE_1 = \frac{c^4}{G} \left( 1 + \alpha \frac{G\hbar^2}{2\pi c^5} - \alpha \ln \left( \frac{4\pi c^5}{G\hbar^2} \right) / \left( \frac{4\pi c^5}{G\hbar^2} \right) \right) d\hat{r}_A. \tag{27}
\]

On the other side, the Misner-Sharp mass

\[
dM_1 = -\frac{4\pi \hat{r}_A^3 (\rho + p)c^2 H dt}{G} \]

is decided by the matter part. Based on the two definitions of energies for the logarithmic correction, the following equation is obtained

\[
\dot{H} \left( 1 + \alpha \frac{G\hbar^2}{2\pi c^5} - \alpha \ln \left( \frac{4\pi c^5}{G\hbar^2} \right) / \left( \frac{4\pi c^5}{G\hbar^2} \right) \right) = -4\pi G (\rho + p). \tag{28}
\]

Using the energy conservation equation of matter (11), one then has the Friedmann equation

\[
H^2 + \frac{3\alpha G\hbar}{16\pi c^3} H^4 + \frac{\alpha G H^4}{8\pi c^5} \ln \left( \frac{G H^2}{4\pi c^5} \right) = \frac{8\pi G}{3} \rho + \Lambda, \tag{29}
\]

where \(\Lambda\) is the integral constant which can be regarded as a cosmological constant. Obviously, a de-Sitter phase can be gotten when \(\frac{8\pi G}{3} \rho \ll \Lambda\). And when \(\alpha < 0\), we can also get de-Sitter solutions as the accelerating phase. The de-Sitter solution with the logarithmic correction is in the quasi-static spacetime where the holographic principle makes sense. Furthermore, if we neglect the term \(\ln \left( \frac{4\pi \hat{r}_A^3 c^3}{G\hbar} \right) / \left( \frac{4\pi \hat{r}_A^3 c^3}{G\hbar} \right)\), one could get

\[
H^2 + \frac{\alpha G\hbar}{4\pi c^3} H^4 \simeq \frac{8\pi G}{3} \rho + \Lambda. \tag{30}
\]

The \(H^4\) term still exists, while the Newton’s law (24) is satisfied at the same time.

In a short summary, for the logarithmic correction of gravity, the field equation has additional terms, e.g. the \(H^4\) term. With proper parameters, the Newton’s law could be recovered. When the parameter \(\alpha\) is negative, the accelerating phase could be gotten.

5. The Power-law Correction

Extra dimensional theories with infinite volume modify the law of gravity in the far infrared energy scale (large distance is called as well). At small distances, its gravitational dynamics is very close to the standard 4-dimensional Einstein gravity. Theories with large extra dimensions are motivated by both the hierarchy and cosmological constant problems. Corresponding to the theories with large
extra dimension(s), we choose the power-law correction to the horizon entropy for extra dimension which could be expressed as

\[ S_2 = k_B \frac{Ac^3}{4G\hbar}(1 - KA^\beta), \]  

(31)

where \( K \) is a constant parameter, the subscript “2” means the power-law correction background. Correspondingly, the number of bits could be written as

\[ N_2 = \frac{Ac^3}{G\hbar}(1 - r_c^{-2\beta}A^\beta), \]  

(32)

where \( r_c^{-2\beta} = 4K/k_B \) has the distance’s dimension. The modified number of bits \( N_2 \) will change the thermodynamics as well.

5.1. The Modified Newton’s Law For The Power-law Correction

With this definition of the number of bits \( N_2 \), using Eqs. (3), (4) and (7), the modified law of gravity could be easily gotten,

\[ F_2 = \frac{G}{1 - (\frac{4\pi r^2}{r_c^2})^\beta} \frac{Mm}{r^2}. \]  

(33)

When \( (\frac{4\pi r^2}{r_c^2})^\beta \ll 1 \), the related leading terms of the Taylor expansion could be obtained

\[ F_2 = \frac{GMm}{r^2} \left(1 + (\frac{4\pi r^2}{r_c^2})^\beta - (\frac{4\pi r^2}{r_c^2})^{2\beta}\right). \]  

(34)

The cutoff parameter \( r_c \) is critical for the dynamics. When \( \beta > 0 \), in small scales where \( r \ll r_c \), the above equation is reduced to Newton’s gravity.

5.2. The Modified Friedmann Equation For The Power-law Correction

For the power-law correction of the entropy, the change of \( N_2 \) is

\[ dN_2 = \frac{8\pi c^3 \tilde{r}_A}{G\hbar} \left(1 - (1 + \beta)(\frac{4\pi \tilde{r}_A^2}{r_c^2})^\beta\right) d\tilde{r}_A. \]  

(35)

Then the change of the energy defined by the energy equipartition rule is

\[ dE_2 = \left(1 - (1 + 2\beta)(\frac{4\pi \tilde{r}_A^2}{r_c^2})^\beta\right) \frac{c^4}{G} d\tilde{r}_A. \]  

(36)

Meanwhile, the Misner-Sharp mass is decided by matter. Therefore, using Eqs. (19) and (36), the Raychaudhuri equation could be obtained

\[ \dot{H} \left(1 - (4\pi)^\beta (1 + 2\beta)(\frac{c}{r_c H})^{2\beta}\right) = -4\pi G(\rho + p). \]  

(37)
Combining the energy conservation equation, one could get the modified Friedmann equation in the power-law correction as well,

\[ H^2 \left( 1 - \frac{1 + 2\beta}{2 - 2\beta} \left( \frac{4\pi c^2}{r_c^2} \right)^\beta H^{-2\beta-2} \right) = \frac{8\pi G}{3} \rho + \Lambda_2, \]  

(38)

where \( \Lambda_2 \) is the cosmological constant. And, a de-Sitter solution exists when \( \beta < 1 \).

The acceleration heavily depends on the cutoff \( r_c \) which is related to the extra dimension closely. Indeed, this is a kind of dark energy model which has been first proposed in Ref. [32]. This model could be also served as an effective holographic dark energy model in Ref. [35] as well. If gravity can be explained by holographic principle and thermodynamics, then, the accelerating phase which is induced by modified gravity has the origin of holographic principle and thermodynamics.

6. The \( f(R) \) Correction

As the simplest case of high-order gravity, the action of the so-called \( f(R) \) gravity is an arbitrary function of curvature scalar \( R \). When \( f(R) = R \), the Einstein’s general relativity is recovered. Here, we try to discuss the \( f(R) \) gravity from the view of entropic force. The area law of the entropy in \( f(R) \) gravity is changed to\[ S_3 = f'(R) \frac{k_B c^3}{4G} A, \]

(39)

where \( R \) is the curvature scalar, the prime means a derivative with respect to \( R \) and the subscript “3” notes the \( f(R) \) correction background. Correspondingly, based on the holographic principle, the number of bits \( N_3 \) is

\[ N_3 = f'(R) \frac{A c^3}{G k_B}. \]

(40)

For \( f(R) \) gravity, however, recently shown in Refs. [38, 39] that in order to derive the corresponding field equations, a treatment with non-equilibrium thermodynamics of spacetime with extra entropy productions is needed in the unified first law.

6.1. The Modified Newton’s Force For The \( f(R) \) Correction

With this definition of the number of bits \( N_3 \), mainly combined with the equipartition law of energy, one could easily get that

\[ F_3 = \frac{G M m}{f'(R) r^2}. \]

(41)

By defining \( \tilde{f}(R) = f(R) - R \), when \( \tilde{f}'(R) \ll 1 \), the leading terms of the related Taylor expansion is obtained,

\[ F_3 = \frac{G M m}{r^2} \left( 1 + \tilde{f}'(R) + \tilde{f}''(R) \right). \]

(42)

If \( f(R) = R \), where \( \tilde{f}'(R) = 0 \), Newton’s gravity is recovered. The constraints from Newton’s law depend on the exact form of \( f(R) \) which require \( \tilde{f}'(R) \ll 1 \) in the small distances.
6.2. The Friedmann Equation For The $f(R)$ Correction

Using the corrected form $N_3$ in $f(R)$ gravity, we get the change of the number of bits,

$$dN_3 = f'(R) \frac{8\pi c^3}{G \hbar} d\tilde{r}_A + f''(R) \frac{4\pi c^3}{G \hbar} \frac{dR}{d\tilde{r}_A} d\tilde{r}_A,$$

where $R = -6(2H^2 + \dot{H})$. Based on the equipartition law of energy, the change of the total energy could be expressed as

$$dE_A = \frac{c^4}{8\pi} f'(R) d\tilde{r}_A - \frac{c^4}{8\pi} f''(R) \frac{H(2AH\dot{H} + 6\dot{H})}{2H} d\tilde{r}_A.$$

Meanwhile, if we treat the Misner-Sharp mass unchanged which is another definition of the energy, by using Eqs. (19) and (44), the Raychaudhuri equation is obtained,

$$\dot{H} \left( f'(R) - f''(R) \frac{H(2AH \dot{H} + 6\dot{H})}{2H} \right) = -4\pi G (\rho + p).$$

When $f(R) = R$, we obtain the standard Friedmann equation by using the energy conservation equation. Unfortunately, Eq. (45) is not the correct Raychaudhuri equation in $f(R)$ gravity. The reason may be traced back to that the $f(R)$ gravity is a non-thermodynamical equilibrium process, so the equipartition law of energy may not be used in this way. The description of the energy in the $f(R)$ correction may be not correct at all.

However, if we use the effective energy density and the pressure in $f(R)$ gravity as below,

$$\tilde{\rho} = \frac{1}{8\pi G f'} \left( -\frac{f - Rf'}{2} - 3H f'' \dot{R} \right) + \frac{\rho}{f'},$$

and keep the “effective” entropy as $\tilde{S} = \frac{k_B c^3}{4\hbar A}$, from the definitions (16) and (19), one could get

$$\dot{H} f'(R) - \frac{H df'(R)}{2dt} - \frac{d^2 f'(R)}{dt^2} = -4\pi G (\rho + p).$$

Combined with the energy density conserved equation of matter, the Friedman equation corresponding to the action could also be obtained

$$H^2 = \frac{8\pi G}{3 f'(R)} \left[ \rho + \frac{R f'(R) - f(R)}{2} - 3H \dot{R} f''(R) \right].$$

In a short summary, in $f(R)$ gravity, the entropic force scenario may not tell the exact thermodynamical process. A thermal equilibrium process exists or not in $f(R)$ gravity is still problem. Here, for the basis of the equipartition law of energy, indeed we treat this process as a thermal equilibrium state and we failed to get the “correct” Friedmann equation with the entropy (39). But if we choose the effective entropy as the Einstein gravity shown, the Friedmann equation corresponding to the action could be gotten.
7. Conclusions

In this letter, by considering three different corrections to the horizon entropy, a trial research on the application of entropic force has been done in three different modified gravities. The modifications of area law of entropy are the logarithmic correction, the power-law correction and the \( f(R) \) correction. Following the new notion of gravity which are assumed as one kind of entropic force, the modified law of Newton’s gravity is obtained which coincides with the existing observations with suitable parameters.

To get the application of the entropic force in cosmology, we try to get the Friedmann equations based on the notion of entropic force. The energies defined by the equipartition law of energy and the Misner Sharp mass were used. According to the form of entropy, we modified the form of the number of bits \( N \), and the energy based on the equipartition law of energy is changed. By matching the energy given by the energy equipartition law and the Misner-Sharp mass, the modified Friedmann equations can be given out with the help of the energy density conserved equation of matter. Specifically, for the logarithmic and power-law corrections to the entropy, the Friedmann equation got an extra term (e.g. \( H^\alpha \) term). The results suggest that the accelerating phase in our universe has the possibility of being explained by entropic force. It may be an emergent phenomenon based on the holographic principle and thermodynamics. During the process of deriving the Friedmann equation, the partition law of energy is critical which should be used in a thermal equilibrium system. For the logarithmic and power-law corrections, we regarded that the energy equipartition law could be used. But, in the \( f(R) \) correction, by using the equipartition law of energy, the choice of the entropy is still a problem.

However, there are still a lot of questions with the new notion of gravity of entropic force, our work is just a pioneer which gives out an attempting study to explain the Friedmann equation in the modified gravity. Through the entropic force is extended to the modified gravity and the validity of the equipartition law of energy is checked in our discussions, the limit of the equipartition law of energy is still worthy of further discussions.

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