Electroweak physics in six-fermion final states  

at future $e^+e^-$ colliders

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Abstract

Three studies on six-fermion production processes are presented, in which the production of an intermediate-mass Higgs boson, the top-quark physics and the analysis of possible anomalous quartic gauge couplings are considered. A Monte Carlo event generator has been developed for full electroweak tree-level calculations on six-fermion processes at the Linear Collider. The Monte Carlo procedure has been adapted to deal with a large variety of diagram topologies, so as to keep under control the relevant signals and all the backgrounds of such processes. The effects of initial-state-radiation and beamstrahlung are also taken into account. The relevance of electroweak backgrounds and finite-width effects is discussed and several analyses of final-state distributions are presented, both for the detection of the signals of interest and for the study of properties of the particles involved, thus showing the importance of complete calculations for precision studies at the Linear Collider.

1 Introduction

Many signals of interest for tests of the Standard Model and search for new physics at the Linear Collider will be given by many-particle final states. It is therefore important to develop the calculation techniques and the tools necessary for the physics analysis of these phenomena, taking into account all the background effects and keeping under control all the relevant final-state correlations.

In particular, the six-fermion signatures will be relevant to several subjects, such as intermediate-mass Higgs boson production, top-quark physics and the analysis of anomalous quartic gauge couplings. These topics are addressed in the present contribution. Numerical results are presented and discussed.

The numerical calculations have been performed by means of a computer code that involves the algorithm ALPHA [1], for the automatic calculation of the scattering amplitudes, and a Monte Carlo integration procedure derived from the four-fermion codes HIGGSPV [2] and WWGENPV [3], and developed to deal with six-fermion processes.
2 Intermediate-mass Higgs boson

The search for the Higgs boson, that is carried on presently at LEP and Tevatron, will be also in the physics programme of future high-energy colliders, where the whole range of mass values allowed by the general consistency conditions for the Standard Model, that is up to $\simeq 1 \text{ TeV}$, can be explored.

The current lower bound on the Higgs mass deduced from direct search at LEP is 95.2 GeV at 95 % C.L. $\lbrack 4 \rbrack$, while the upper bound given by fits to the precision data on electroweak observables is 245 GeV at 95 % C.L. $\lbrack 4 \rbrack$.

The Linear Collider will not only be able to discover the Higgs boson, but it will also provide the possibility of making precision studies on its properties. It is then of great interest to make accurate predictions on the processes in which the Higgs boson can be produced at the LC, and to develop the tools for making simulations. In the mass range favoured by the present experimental information, that is between 100 and 250 GeV, the relevant signatures are four-fermion final states if the Higgs mass is below 130-140 GeV, and six-fermion final states if the Higgs mass is greater than 140 GeV. The processes of the first kind have been extensively studied in connection with physics at LEP, while those of the second kind have only recently been addressed $\lbrack 5 \rbrack$–$\lbrack 8 \rbrack$.

In this contribution, complete electroweak tree-level calculations for the processes $e^+e^- \rightarrow q\overline{q}l^+l^-\nu\overline{\nu}$, with $q = u, d, c, s$, $l = e, \mu, \tau$ and $\nu = \nu_e, \nu_\mu, \nu_\tau$ are presented. These processes are characterized by the presence of both charged and neutral currents and of different mechanisms of Higgs production involving Higgs-strahlung and vector boson fusion; moreover, QCD backgrounds are absent.

The total cross-section is shown in fig. 1 as a function of the center-of-mass (c.m.) energy for three values of the Higgs mass, with suitable kinematical cuts, to avoid the soft-pair singularities. The off-shellness effects due to the finite widths of the gauge bosons and of the Higgs boson have also been studied by comparing the result obtained by means of the signal diagrams with the one in the narrow-width approximation. Deviations of the order of $10 \sim 15\%$ have been found $\lbrack 8 \rbrack$.

Various distributions have been studied, after generating samples of unweighted events. The analysis is restricted in this case to the processes with $l = e$ and a luminosity of 500 fb$^{-1}$ is assumed. The invariant masses of different systems of four fermions are plotted in fig. 2, including the effects of initial-state radiation (ISR) $\lbrack 9 \rbrack$ and beamstrahlung (BS) $\lbrack 10 \rbrack$. The different sets of fermions correspond to the Higgs boson in different signal diagrams. It is interesting to observe that at 800 GeV the $qqe^+e^-$ invariant mass gives a clean signal, not affected by ISR and BS, that can be traced back to the $WW$ fusion signal diagram.

Other distributions can be considered in order to reveal the presence of the Higgs boson and to measure its properties $\lbrack 8 \rbrack$. As a conclusion, the processes under consideration turn out to be of interest for the study of intermediate Higgs bosons. Thanks to the sums over quark, charged lepton and neutrino flavours, as well as the combined action of different production mechanisms, assuming a luminosity of 500 fb$^{-1}$/yr and a Higgs mass of, say, 185 GeV, more than 1000 events can be expected at a c.m. energy of 360 GeV and more than 2000 at 800 GeV (see fig. 1). The complete calculation shows the relevance of background and off-shellness effects, and it is possible to exploit the features of the different signal diagrams to find at different energies suitable distributions that are sensitive to the presence and to the properties of the Higgs boson.
Figure 1: Total cross section for the process $e^+e^- \to q\bar{q}l^+l^-\nu\bar{\nu}$ in the Born approximation, as a function of $\sqrt{s}$ for three different values of the Higgs mass $m_H$. The angles $\theta(l^+), \theta(l^-)$ of the charged leptons with the beam axis are in the interval $5^\circ$-$175^\circ$, the $e^+e^-$ and the $q\bar{q}$ invariant masses are larger than 20 GeV.

$m_H = 185 \text{ GeV (Born vs. ISR+BS)}$

Figure 2: Invariant-mass distributions for four-fermion systems in the Born approximation (dashed histograms) and with ISR and beamstrahlung (solid histograms) at $\sqrt{s} = 360$ GeV (upper row) and $\sqrt{s} = 800$ GeV (lower row).
The study of $t\bar{t}$ production both at threshold and above at the Linear Collider will give the opportunity of making significant tests of QCD and to get important information through the determination of the electroweak properties of the top quark.

The production of a $t\bar{t}$ pair gives rise to six fermions in the final state. The $6f$ signatures relevant to the study of the top quark in $e^+e^-$ collisions can be summarized as follows: $b\bar{b}ll\nu\nu$ (leptonic, $\sim 10\%$ of the total rate), $b\bar{b}qq'\nu\nu$ (semi leptonic, $\sim 45\%$), $b\bar{b}+4q$ (hadronic, $\sim 45\%$). Semi leptonic signatures have been considered in refs. [6, 11, 12]. It is then of great interest to carefully evaluate the size of the totally hadronic, six-quark ($6q$) contributions to integrated cross-sections and distributions as well as to determine their phenomenological features.

The $6q$ signatures of the form $b\bar{b}+4q$, where $q = u, d, c, s$ are considered in the present study and the results of complete electroweak tree-level calculations are presented. In particular the rôle of electroweak backgrounds and of ISR and BS are studied and the shape of the events is analysed to the end of isolating the QCD backgrounds.

The integrated cross-section has been studied in the energy range between 350 and 800 GeV for different Higgs masses and has been compared with the signal contribution alone and with the result in the narrow-width-approximation, showing that the background and off-shellness effects are of the order of several per cent [13].

The total electroweak cross-section has also been studied at the threshold for $t\bar{t}$ production as a function of the Higgs mass. Although the dominant effects in this case come from QCD contributions, as is well known [14], the electroweak backgrounds turn out to give a sizeable uncertainty, of the order of $10\%$ of the pure electroweak contribution, in the intermediate range of Higgs masses (see fig. 3), which is related to the fact that the Higgs mass is not known. The topology of the events has been studied by means of various event-shape variables, in order to study the possibility of isolating the top-quark signal from the QCD backgrounds. The pure QCD contributions have been analysed in ref. [15]. In fig. 4 the thrust distribution of the electroweak contribution is shown at a c.m. energy of 500 GeV and with a Higgs mass of 185 GeV in the
Figure 4: Thrust distribution in the Born approximation (dashed histogram) and with initial-state radiation and beamstrahlung (solid histogram), at a c.m. energy of 500 GeV and for a Higgs mass of 185 GeV. A luminosity of 500 fb$^{-1}$ is assumed. The invariant masses of the $b\bar{b}$ pair and of all the pairs of quarks other than $b$ are required to be greater than 10 GeV.

Born approximation (dashed histogram) and with ISR and BS (the solid histogram; in this case, the distribution is calculated after going to the c.m. frame). A luminosity of 500 fb$^{-1}$ is assumed and the invariant masses of the $b\bar{b}$ pair and of all the pairs of quarks other than $b$ are required to be greater than 10 GeV. Remarkable effects due to ISR and BS can be seen in this plot, where the peak in the thrust distribution is strongly reduced with respect to the Born approximation and the events are shifted towards the lower values of $T$, which correspond to spherical events.

As a conclusion, at 500 GeV, in view of the results of ref. [15], the thrust variable is very effective in discriminating pure QCD backgrounds, also in the presence of electroweak backgrounds and of ISR and BS.

4 Anomalous gauge couplings

The situation in which a Higgs boson with mass below 1 TeV is not found can be described by means of the electroweak effective lagrangian, as discussed in ref. [15]. Different models of electroweak symmetry breaking can be parameterized by this effective lagrangian. The contributions at lowest order in the chiral expansion, if the $SU(2)$ custodial symmetry is assumed, are model-independent. At next-to-leading order, dimension-four operators are present, with parameters that depend on the model of symmetry breaking adopted. These operators can give rise to trilinear and quadrilinear couplings of the massive gauge bosons, that modify the standard ones contained in the Yang-Mills lagrangian for the gauge bosons. The dimension-four operators that give only quartic vertices, usually indicated as $\mathcal{L}_4$, $\mathcal{L}_5$, $\mathcal{L}_6$, $\mathcal{L}_7$ and $\mathcal{L}_{10}$, have been implemented in ALPHA. Anomalous $4W$, $WWZZ$ and $4Z$ vertices are provided by these terms. In the following only the two $SU(2)$-custodial symmetry conserving operators $\mathcal{L}_4$ and $\mathcal{L}_5$ are
discussed. Their expressions in the unitary gauge are [16]:

\[ L_4 = \alpha_4 g^4 \left( \frac{1}{2} W^{+\mu} W^{-\nu} W^{-\mu} W^{+\nu} + \frac{1}{2} (W^{+\mu} W^{-\mu})^2 + \frac{1}{c_W^2} W^{+\mu} Z^\nu W^{-\nu} Z^\mu + \frac{1}{4 c_W^4} (Z^\mu Z^\nu)^2 \right) \]

\[ L_5 = \alpha_5 \left( (W^{+\mu} W^{-\mu})^2 + \frac{1}{c_W^2} W^{+\mu} W^{-\nu} Z^\nu Z^\mu + \frac{1}{4 c_W^4} (Z^\mu Z^\nu)^2 \right) \]  

(1)

These anomalous quartic couplings have been studied by several authors at the loop level, where they contribute to radiative corrections to electroweak observables [17], and at tree-level in processes of gauge boson scattering and real gauge boson production [18]. In a more realistic approach, where the gauge bosons are not real, signatures with at least six fermions in the final state have to be considered. For the present study, the processes \( e^+ e^- \rightarrow 2q q' \nu \bar{\nu} e \), with \( q = u, c \) and \( q' = d, s \), have been considered, and a full tree-level calculation has been performed, by using the effective lagrangian containing the dimension-four operators mentioned above.

Through the study of some event samples, several variables that are sensitive to the parameters \( \alpha_4 \) and \( \alpha_5 \) have been found. A set of kinematical cuts has thus been deduced to enhance the effects of anomalous couplings. For example, the cross-section obtained with this set of cuts is shown in fig. 5 as a function of the parameter \( \alpha_5 \) in the range \((-0.01, 0.01)\). The variables involved in the cuts, as indicated in the figure, are the invariant mass of the system of four jets, \( M(WW) \), the angle \( \theta(W) \) of one reconstructed \( W \) (where a simple procedure is used to identify the \( W \) boson from the quarks) and the invariant mass of the pair of jets with lowest transverse momentum. The limits of the 1\( \sigma \) experimental uncertainty around the value at \( \alpha_5 = 0 \) are also shown in fig. 5, by assuming a luminosity of 1000 fb\(^{-1}\). The sensitivity to this parameter with this set of cuts can be seen to be of the order of 10\(^{-2}\). The cross-sections and the variables used in the cuts have been analysed also in the presence of ISR and BS. The above conclusions on the sensitivity to the anomalous couplings should not be modified by the inclusion of such effects as long as variables not involving the missing momentum are considered.

5 Conclusions

The six-fermion final states will be among the most relevant new signatures at future \( e^+ e^- \) linear colliders. In particular they are interesting for Higgs bosons in the intermediate mass range, for \( t\bar{t} \) production and for the study of quartic anomalous gauge couplings. These subjects are addressed in the studies that are presented in this contribution.

A Monte Carlo event generator has been developed for complete tree-level calculations of such processes at the energies of the Linear Collider. This code, that makes use of ALPHA for the calculation of the scattering amplitudes, has been adapted to deal with a large variety of diagram topologies, including both charged and neutral currents, so as to keep under control all the relevant signals of interest as well as the complicated backgrounds that are involved in six-fermion processes where hundreds of diagrams contribute to the tree-level amplitudes. The effects of initial-state-radiation and beamstrahlung are also included.

The studies of Higgs boson production in the intermediate mass range, of \( t\bar{t} \) production and of anomalous gauge couplings have shown the importance of complete calculations to keep under control all the background and finite-width effects, and to obtain simulations of all kinds of final-state distributions such as invariant masses, angular correlations and event-shape variables, that
Figure 5: Integrated cross-section in the Born approximation at a c.m. energy of 1 TeV, as a function of $\alpha_5$ with a set of kinematical cuts studied to enhance the sensitivity to the anomalous couplings. The horizontal lines are the $1\sigma$ bounds around the $\alpha_5 = 0$ value, with the experimental uncertainty corresponding to a luminosity of $1000 \text{ fb}^{-1}$.

are essential both for the detection of the signals of interest and for the analysis of the properties of the particles under study.

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