The Intrinsic Gluon Component of the Nucleon

Paul Hoyer\textsuperscript{1} and D.P. Roy\textsuperscript{2}

\textsuperscript{1} Nordita, Blegdamsvej 17, DK-2100 Copenhagen, Denmark
\textsuperscript{2} T.I.F.R., Homi Bhabha Road, Mumbai 400 005, India.

Abstract

Using an intrinsic parton model we estimate the rough shape and size of the intrinsic gluon component of the nucleon, corresponding to an energy scale $Q$ of the order $\Lambda_{QCD}$. It is nearly as hard in shape as the valence quark, while its size accounts for a quarter of the nucleon momentum. Both are in qualitative agreement with the input leading twist gluon distribution assumed by Glück, Reya and Vogt at this scale in order to reproduce the observed distributions at $Q^2 \gtrsim 1 \text{ GeV}^2$ via perturbative QCD evolution.
While the $Q^2$ dependence of the parton distribution functions are successfully explained by perturbative QCD, one cannot predict the shape and size of these distribution functions at the starting scale of $Q^2$. The standard practice is to use phenomenological parametrisations of the quark and gluon distributions in the Bjorken $x$ variable [1] at an input scale of

$$Q_0^2 = 1 - 2 \text{ GeV}^2.$$ \hfill (1)

Roughly speaking, in this $Q^2$ region the nucleon momentum seems to be shared about equally between the three valence quarks and an infinite number of soft gluons, with the gluon distribution function

$$g(x) \propto x^{-1} \text{ at low } x.$$ \hfill (2)

Thus to a first approximation

$$\varepsilon_g \equiv \int_0^1 x g(x) \, dx \simeq \frac{1}{2},$$ \hfill (3)

$$\bar{x}_g \equiv \int_0^1 \frac{\varepsilon_g}{g(x)} \, dx = 0,$$ \hfill (4)

where $\varepsilon_g$ is the total fraction of the nucleon momentum which is carried by gluons and $\bar{x}_g$ is the average momentum fraction carried per gluon. There is so far no theoretical understanding of the size or the shape of the input gluon distribution represented by Eqs. (3), (4) even at a qualitative level.

A bold step along this direction was taken by Glück, Reya and Vogt (GRV) [2, 3, 4] by extending the perturbative QCD evolution of the leading twist parton distribution functions down to

$$Q_0^2 \simeq 0.2 \text{ GeV}^2,$$ \hfill (5)

due, to a $Q_0$ of the same order as $\Lambda_{QCD}$. This corresponds to the regime of long distance and time scales, where the input parton distributions can be regarded as intrinsic to the nucleon [5]. An initial attempt to generate the canonical gluon distribution represented by (3) - (4) by starting only with the three valence quarks at the low input scale of (5) did not succeed [2]. However, GRV did reproduce the measured gluon distribution by adding a valence-like gluon component to the valence quarks at the low input scale. While the shape of the input gluon distribution is roughly similar to that of the valence quarks, the gluons carry a smaller momentum fraction

$$\varepsilon_g = 0.25 \text{ and } \varepsilon_q = 0.60 \text{ at } Q_0^2 \simeq 0.2 \text{ GeV}^2,$$ \hfill (6)

in the case of leading order QCD evolution [4]. The remaining 15% of the nucleon momentum is attributed to a sea quark component whose shape is also assumed to be valence like.
In this note we shall provide rough estimates of the shape and size of the intrinsic gluon component, corresponding to the long time scale of Eq. (5) above. For this purpose we shall use the model of Brodsky et al. [5] for the long time scale structure of the nucleon wavefunction. As we shall see below, the predicted intrinsic gluon component has a roughly similar shape and size as the above mentioned inputs of GRV [2-4].

According to the model of [5], the partons in the nucleon Fock states

\[ |qqq\rangle, |qqqg\rangle, |qqq\bar{q}q\rangle, \cdots \] (7)

have similar velocities in order to stay together over a long time scale. More quantitatively, the probability \( P_n \) of an \( n \)-parton Fock state is given by

\[ P_n \propto (\Delta E)^{-2} \propto \left( m_N^2 - \sum_{i=1}^{n} \frac{m_{\perp i}^2}{x_i} \right)^{-2}. \] (8)

Here \( \Delta E \) is the energy difference, in the infinite momentum frame, between the nucleon and the Fock state, and \( m_{\perp i}^2 = m_i^2 + k_{\perp i}^2 \) is the squared transverse mass of parton \( i \). The distribution (8), motivated by old fashioned perturbation theory [5], is relevant for the long time-scale (\( \propto 1/\Delta E \)) structure of the nucleon. The leading twist \( Q^2 \) evolution, on the other hand, reflects the increasing resolution of short-lived ‘extrinsic’ states created through single parton splitting. In the spirit of GRV we thus propose using Eq. (8) to determine the input, low \( Q^2 \) ‘valence’ distribution to which leading twist evolution is applied.

According to Eq. (8) the probability distribution of a given Fock state is peaked at

\[ x_i = \frac{m_{\perp i}}{\sum_i m_{\perp i}}. \] (9)

In particular, if there are heavy partons in a Fock state then they will carry a large fraction of the nucleon momentum. This led to the suggestion of an intrinsic charm component \( |qqq\bar{cc}\rangle \) of the nucleon, where the charm quark pair carries the bulk of the nucleon momentum [5]. The EMC data on muon induced dimuons [6] seems to indicate the presence of such a hard intrinsic charm component in the nucleon [7], but there is no definitive experimental evidence for it so far. For fixed \( m_{\perp i} \), the probability distribution of Eq. (8) implies a power-law fall-off \((1 - x)^n\) for the parton distributions, with \( n = 3 \) and \( 4 \) for the valence quark and gluon respectively [5, 8]. A similar model was used in [9] to predict a hard fragmentation function for the charm quark into a charmed hadron, which is in good agreement with experimental data.

In the present case we are interested in the long time scale structure of the nucleon wavefunction in terms of the multi-gluon Fock states

\[ |qqq\rangle, |qqqg\rangle, |qqq\bar{q}qg\rangle, \cdots \] (10)
Consider the $n$-parton Fock state consisting of the 3 valence quarks and $(n - 3)$ gluons. We take the parton momenta to be distributed according to Eq. (8) with a common transverse mass
\[ m_{\perp i} \simeq \langle k_{\perp i}^2 \rangle^{1/2} \simeq 0.3 - 0.4 \text{ GeV}, \]
\[ ie, \] with a typical intrinsic momentum corresponding to a hadronic scale of $O(1 \text{ fm})$. This implies an equipartition of the nucleon momentum among the $n$ partons, \[ \bar{x}_q = \bar{x}_g = 1/n, \quad \varepsilon_q = 3/n, \quad \varepsilon_g = (n - 3)/n. \]
Thus if a single Fock state were dominant the shape of the gluon distribution would be identical to that of the valence quark. However, in general we have to consider the contribution of all nucleon Fock states
\[ |N\rangle = A_3|qqq\rangle + A_4|qqqg\rangle + A_5|qqqgg\rangle + \cdots. \]
Here
\[ C_n = |A_n|^2 = \int P_n(x_1, \ldots, x_n) \prod_{i=1}^n dx_i \] represents the net probability for the $n$-parton Fock state, and $\sum_{n=3} C_n = 1$. Thus
\[ \varepsilon_q = \sum_{n=3}^{\infty} \frac{3C_n}{n} = 3 \left\langle \frac{1}{n} \right\rangle = 3\bar{x}_q \]
\[ \varepsilon_g = \sum_{n=3}^{\infty} \frac{(n - 3)C_n}{n} = \left\langle \frac{n - 3}{n} \right\rangle, \]
\[ \bar{x}_g = \frac{\varepsilon_g}{\langle n_g \rangle} = \left\langle \frac{n - 3}{n} \right\rangle / (n - 3), \]
\[ ie, \] in general $\bar{x}_g < \bar{x}_q$. As we shall see below, the intrinsic gluon distribution can be soft ($\bar{x}_g \simeq 0$) or nearly as hard as the valence quark ($\bar{x}_g \sim \bar{x}_q$) depending on the nature of the coefficients $C_n$.

In order to proceed further we need to know the $n$-dependence of the probability factors $C_n$ of Eqs. (8) and (14). We shall assume that the $n$-dependence is mainly determined by the energy denominators of Eq. (8) evaluated at their most likely configuration (9), with $m_{\perp i}$ given by Eq. (11). This gives (neglecting the nucleon mass term in Eq. (8))
\[ C_n \propto 1/n^4. \]
Consequently,
\[ \bar{x}_q = \int_3^{\infty} \frac{dn}{n^5} / \int_3^{\infty} \frac{dn}{n^4} = \frac{1}{4}, \quad \varepsilon_q = \frac{3}{4}, \]
Thus the average momentum fractions of gluons and quarks are similar ($\bar{x}_g \simeq 0.7\bar{x}_q$), while their total momentum fractions are in the ratio $\varepsilon_g:\varepsilon_q = 1:3$. Both features are in qualitative agreement with the input gluon distribution of GRV at $Q_0^2 \simeq 0.2$ GeV$^2$ [4], as discussed above.

It is instructive to see how sensitive the results are to the assumed $n$-dependence of the probability factors $C_n$. Let us consider the three other cases

$$C_n \propto 1/n, \quad 1/n^2, \quad \text{and} \quad 1/n^3. \quad (19)$$

The resulting average and total momentum fractions as well as $\langle n_g \rangle$ are shown in Table I along with those of Eqs. (16) - (18). We see that the shape and size of the intrinsic gluon distribution depend sensitively on the distribution of Fock states. While both the quark and the gluon distributions are hard for $C_n \propto 1/n^4$, they are both soft for $C_n \propto 1/n$. Simultaneously the total momentum fraction carried by the gluons ($\varepsilon_g$) increases from 1/4 to 1.

It may also be noticed from Table I that $C_n \propto 1/n^2$ corresponds to hard quark and soft gluon distributions, each carrying half the nucleon momentum fraction, as in the case of the canonical parametrisation at higher $Q^2$ given by Eqs. (1) - (4). However, in this perturbative regime the virtual photon scatters from Fock states having a short life-time of $O(1/Q)$. Hence the dynamics is not determined by the Fock states of lowest energy, as in our intrinsic model based on Eq. (8).

Given that the parton distributions depend sensitively on the Fock state probabilities, we find it significant that the probability distribution (16) of intrinsic states gives roughly the right shape and size of the valence parton distributions, as required for the GRV input at the appropriate scale of $Q_0^2 \simeq 0.2$ GeV$^2$ [2-4]. It would be interesting to compare the Fock state distribution of Eq. (16) with that of solvable field theory models, such as QCD$_{1+1}$ [10] and dimensionally reduced QCD [11].

So far we have neglected the (presumably small) intrinsic sea quark component. Regardless of our specific model, there are at least two reasons to expect that the sea quark component should, like the gluon, have a hard distribution at the low momentum scale of Eq. (5), as is indeed the case in the GRV input [4]. (i) The presence of soft sea quarks ($x_i \rightarrow 0$) in any Fock state will imply the corresponding $\Delta E \rightarrow \infty$, making such states irrelevant for the long time scale structure. (ii) The perturbative evolution of sea quarks in the small $x$ region is driven by the small $x$ behavior of gluons. Thus a soft sea component cannot develop as long as the gluon component remains hard.

It should be emphasized that the hard quark and gluon distributions at the low scale of Eq. (5) represent leading twist parton distributions which are not directly measurable in electron scattering. At such low values of $Q^2$ the physical cross section is in fact dominated by higher twist contributions – the photon
scatters coherently from several quarks. Being of leading twist, the $Q^2$ evolution of the parton distributions are known, however, and they can thus be compared with data at a higher scale, such as that given by (1). This was, of course, how GRV arrived at their parametrization of the parton distributions at low $Q^2$.

In summary, we have estimated the rough shape and size of the intrinsic valence quark and gluon components of the nucleon. At the low scale $Q^2 \simeq 0.2 \text{ GeV}^2$, i.e., for $Q = \mathcal{O}(\Lambda_{\text{QCD}})$, we assume the nucleon Fock state probabilities to be proportional to $(\Delta E)^{-2}$, where $\Delta E$ is the excitation energy of the state. In this approach the average momentum of an intrinsic gluon turns out to be similar to that of a valence quark. The total momentum fractions carried by gluons and quarks are in the ratio 1 : 3. Both features are in qualitative agreement with the shape and size of the input parton distributions found by GRV [2–4] at $Q^2 \simeq 0.2 \text{ GeV}^2$. When evolved to $Q^2 \gtrsim 1 \text{ GeV}^2$ these distributions reproduce the experimental data. Thus our model, taken together with the GRV analysis, provides a theoretical basis for understanding the shape and size of the observed gluon distribution at $Q^2 = 1 - 2 \text{ GeV}^2$.

It is a pleasure to thank Profs. S. Brodsky, R.M. Godbole, J. Kwiecinski, E. Reya, R.G. Roberts and G.G. Ross for illuminating discussions.

REFERENCES

1. See e.g. A.D. Martin, R.G. Roberts and W.J. Stirling, Phys. Lett. B 387 (1996) 419, hep-ph/9606343.

2. M. Glück, E. Reya and A. Vogt, Z. Phys. C 48 (1990) 471.

3. M. Glück, E. Reya and A. Vogt, Z. Phys. C 53 (1992) 127; Phys. Lett. B 306 (1993) 391.

4. M. Glück, E. Reya and A. Vogt, Z. Phys C 67 (1995) 433.

5. S.J. Brodsky, P. Hoyer, C. Peterson and N. Sakai, Phys. Lett. B 93 (1980) 451; S.J. Brodsky, C. Peterson and N. Sakai, Phys. Rev. D 23 (1981) 2745.

6. EMC Collaboration: J.J. Aubert et. al, Nucl. Phys. B 213 (1983) 31.

7. D.P. Roy, On the Indication of Hard Charm in the Latest EMC Dimuon Data, TIFR/TH/83-1 (1983); see also B. Harris, J. Smith and R. Vogt, Nucl. Phys. B 461 (1996) 181, hep-ph/9508403.

8. S.J. Brodsky and I. Schmidt, Phys. Lett. B 234 (1990) 144; S.J. Brodsky, M. Burkhardt and I. Schmidt, Nucl. Phys. B 441 (1995) 197, hep-ph/9401328.
9. M. Suzuki, *Phys. Lett.* B 71 (1977) 139; see also J.D. Bjorken, *Phys. Rev.* D 17 (1978) 171.

10. K. Hornbostel, S. J. Brodsky and H. C. Pauli, *Phys. Rev.* D 41 (1990) 3814.

11. F. Antonuccio and S. Dalley, *Phys. Lett.* B376 (1996) 154, [hep-th/9512106](http://arxiv.org/abs/hep-th/9512106).
Table I: The average and total momentum fractions carried by the valence quarks and the intrinsic gluons are shown along with the average number of gluons for four different types of Fock state distributions.

| $C_n$ | $\epsilon_q$ | $\bar{x}_q$ | $\epsilon_g$ | $\bar{x}_g$ | $< n_g >$ |
|-------|---------------|--------------|---------------|--------------|-----------|
| $1/n$ | 0             | 0            | 1             | 0            | $\infty$ |
| $1/n^2$ | 1/2         | 1/6          | 1/2           | 0            | $\infty$ |
| $1/n^3$ | 2/3         | 2/9          | 1/3           | 1/9          | 3         |
| $1/n^4$ | 3/4         | 1/4          | 1/4           | 1/6          | 3/2       |