The Earliest Epoch of Reionisation in the Standard ΛCDM Model

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ABSTRACT
We show that the earliest possible reionisation of the Universe is approximately at $z \simeq 13.5$ and the optical depth is $\tau \simeq 0.17$ in the conventionally accepted Λ cold dark matter (ΛCDM) model with adiabatic fluctuations of a flat spectrum normalised to the cosmic microwave background. This is consistent with the reionisation found by WMAP (the apparently earlier reionisation epoch of the WMAP is ascribed to the adoption of the instantaneous reionisation approximation), i.e., the WMAP result is realised only if reionisation of the universe takes place nearly at the maximal efficiency in the ΛCDM model.

Key words: intergalactic medium – cosmology:theory – cosmology:observation – Galaxy:formation

1 INTRODUCTION
The most remarkable discovery from the Wilkinson Microwave Anisotropy Probe (WMAP) Observation is a very early reionisation of the universe at $z \simeq 20$ (Bennett et al. 2003). The optical depth is $\tau = 0.17 \pm 0.04$ (Kogut et al. 2003) indicated by the TE cross power correlation function for low $\ell$ modes, or $\tau = 0.17^{+0.08}_{-0.07}$ from a global fit including both TE and TT power spectra (Spergel et al. 2003).

It is now generally accepted that early reionisation is primarily due to UV light from early OB stars in protogalaxies (Couchman & Rees 1986; Tegmark et al. 1994; Cen & Ostriker 1992; Fukugita & Kawasaki 1994; see Loeb & Barkana 2001 for reviews of the recent progress and further references). Many calculations have been done with increasingly improved physical approximations (see Cen 2003; Razoumov et al. 2002; Ciardi, Ferrara & White 2003 for the most recent calculations). Recent calculations are mostly based on $N$-body simulations to include many physical effects. With the cosmological parameters of ΛCDM models, the reionisation epoch is inferred to be $z \sim 8 - 11$. Although these estimates stand for the ‘best estimates’ with elaborated simulations, there are some subtleties that may depend on specific assumptions, models of physical processes and meshes of numerical simulations. In this paper we study the earliest possible reionisation epoch in the ΛCDM scenario under the standard assumptions concerning star formation. Namely, we consider the case when all energy from baryons that form stars are efficiently injected into the intergalactic medium. For this purpose it is appropriate to take a homogeneous multiphase universe model, where we reduce the detailed physics to a few parameters, which are constrained by empirical knowledge, or otherwise left as free parameters.

The cosmological parameters are now well-constrained by virtue of WMAP itself, together with advancement in optical observations: $H_0 = 73$ km $s^{-1}$Mpc$^{-1}$, $\Omega_m = 0.25$, $\Omega_\Lambda = 0.75$ (i.e., the flat universe), $\Omega_b = 0.043$ (Spergel et al. 2003). The fluctuations show the spectrum close to flat, $n = 0.97 \pm 0.03$, with the amplitude normalisation $\sigma_8 = 0.8 - 0.9$. It seems that there is no much freedom to modify these parameters. The formation of structure is well described by the Press Schechter formalism (Press & Schechter 1974). We assume that a constant fraction of baryons form stars once the perturbation is collapsed and the Jeans and cooling conditions are satisfied. The maximally allowed baryon fraction that goes to stars is constrained by empirical metallicity of the universe. We assume the Salpeter initial mass function (IMF), which shows a rather high weight for high mass stars, relative to Scalo’s IMF (Scalo 1986), although this choice is not essential to us under our empirical constraint from metallicity. We allow the fraction of UV that comes out of OB stars to vary: the extreme case assumes a 100% escaping fraction.

There are many effects that should be taken into account for more realistic reionisation history: inhomogeneity of star forming clouds, feedback from stars and expanding UV shell are obvious examples. The absorption due to galaxies and Lyman alpha clouds are another. These effects all contribute to delay the reionisation epoch, so that the neglect of these effects are justified for the purpose to estimate...
the earliest reionisation epoch. We discuss, however, these physical effects later.

We find that quasars are important only in the late epoch. So we switch off quasars for our calculation of the reionisation epoch.

We follow the formalism given in Fukugita & Kawasaki (1994; hereafter FK). We solve the evolution equation for thermal history with UV from collapsed objects taken into account. We include the effect of heating to the Jeans mass, although it is not very important. A full description of the basic formalism is found in FK. We give a brief description of our calculation and inputs that differ from FK in the next section.

2 CALCULATION

The number of collapsed objects within the mass range \( M \) to \( M + dM \) at redshift \( z \), \( N(z, M) \), is calculated by the Press Schechter formula with the threshold mass density \( \delta_c = 1.68 \). We take a flat \( (n = 1) \) spectrum and the transfer function given by Bardeen et al. (1986) including baryons. We take the high normalisation \( \sigma_8 = 0.9 \), allowed by the WMAP result (Spergel et al. 2003), but later show how the result depends on \( \sigma_8 \). The smearing is made with a top hat window function. We take cosmological parameters from the fit of WMAPext+2dGRS (with constant power spectrum) in Spergel et al. (2003).

We assume that a constant fraction \( f \) of baryons collapsed into bound objects form stars once the Jeans condition

\[
M > M_{\text{J}} = 1.4 \times 10^5 M_\odot \Omega_m^{1/2} h^{-1} \left( \frac{T_e}{\mu T_s} \right)^{3/2},
\]

where \( T_e \) is the electron temperature, \( T_s \) is the temperature of the cosmic background radiation and \( \mu \) is the mean molecular weight, is satisfied, and the molecular cooling time is shorter than the dynamical time (Blumenthal et al. 1984), i.e.,

\[
M > M_{\text{mc}} = 3.7 \times 10^3 (\Omega_m h^2)^{-0.917} (Y_e/10^{-4})^{-0.625} \times (\Omega_b/\Omega_m)^{-2.04} \left( \frac{1 + z}{10} \right)^{-2.75} M_\odot,
\]

where \( Y_e = 10^{-5} \Omega_b^{-1} \Omega_m^{1/2} h^{-1} \) is the fraction of free electrons. \( H_2 \) molecules are fragile, and this condition may have to be replaced with the atomic cooling condition \( T_{\text{vir}} > 10^4 \text{K} \) (\( T_{\text{vir}} \): virial temperature) for a more realistic calculation (Stecher & Williams 1967; Haiman et al. 1997). The atomic cooling condition gives the lower bound on the galactic mass as

\[
M > M_{\text{ac}} = 2.1 \times 10^9 M_\odot (\Omega_m h^2)^{-1/2} (1 + z)^{-3/2},
\]

but the results using this condition differ very little from those we obtained by setting the molecular cooling condition. We take the objects with \max\{\(M_{\text{ac}}, M_{\text{J}}\)\} \( M \) or \max\{\(M_{\text{ac}}, M_{\text{J}}\)\} \( M \) as being collapsed. The cooling time becomes longer than the Hubble time for low redshift \( 1 + z < 7(\Omega_m h^2)^{1/5} \), where Compton cooling is not efficient, and galaxies do not form if

\[
M > 3.0 \times 10^{14} M_\odot (1 + z)^{3/4} (\Omega_b h^2)^{3/2} (\Omega_m h^2)^{-5/4} (Z/0.01)^{3/2}.
\]

\( Z \) being the metallicity (Blumenthal et al. 1984). The constant fraction \( f \) may be estimated by the balance of the local infall rate and the cooling rate, which, however, may depend on the details of calculations (e.g., Cen & Ostriker 1992). Here we simply constrain the \( f \) parameter from the heavy element abundance to avoid uncertainties from the use of specific models. [Note that our \( f \) is the fraction of baryons that form stars against collapsed baryons that satisfy cooling conditions, in contrast to the definition often taken in the literature that a similar number is defined by the ratio of baryons that satisfy the cooling conditions (which equal baryons that form stars) against those in collapsed objects.]

We assume that stars form according to the Salpeter IMF, \( \phi(M) \). The temperature-mass relation (Bond, Carr & Hogan 1986) we adopt is

\[
T_s(M_s) = 6 \times 10^4 \text{K} \min \left[ \left( \frac{M_s}{100 M_\odot} \right)^{0.3}, 1 \right]
\]

for population II stars \( (Z < 0.001) \). The coefficient is replaced with \( 4.3 \times 10^4 \text{K} \) for population I stars \( (Z > 0.01) \). For \( 0.001 < Z < 0.01 \) we consider a mixture of the two populations to adjust the mean metallicity. In principle, somewhat higher temperature is expected for so-called population III stars, i.e., stars from pristine gas, but the local environment of star forming region is quickly enriched once the first stars formed: so it is more appropriate to consider population II stars here\(^1\). The fraction of radiation energy produced, \( \epsilon_s, M_s \), and the time in the main-sequence, \( t_{\text{MS}} \), are, respectively, (Bond, Carr & Hogan 1986)

\[
\epsilon_s = 0.0046 \left( \frac{X}{0.76} \right) \min \left[ \left( \frac{M_s}{100 M_\odot} \right)^{1/2}, 1 \right],
\]

\[
t_{\text{MS}} = 2.3 \times 10^6 \text{yr} \left( \frac{\epsilon_s}{0.0046} \right) \max \left[ \left( \frac{M_s}{100 M_\odot} \right)^{-2}, 1 \right]
\]

where \( X = 0.76 \) is the hydrogen mass fraction. Since the lifetime of massive main-sequence stars with \( M_s > 10 M_\odot \), which are relevant to us, is shorter than the cosmic time for \( z < 50 \), we can assume that ionising photons are produced instantaneously upon the formation of stars. The production rate of UV photons with energy \( \epsilon_\gamma \) is given by

\[
\left[ \frac{d\nu_\gamma}{dz} \right]_{r,\text{star}} = \int dM_s B(\epsilon_\gamma, T_s) \epsilon_s \phi(M_s) \left( \frac{\Omega_b}{\Omega_m} \right) \frac{\Omega_b}{\Omega_m} \int dM \left[ \frac{dN(M,z)}{dz} \right] \left( \frac{\Omega_b}{\Omega_m} \right).
\]

where \( B(\epsilon_\gamma, T_s) \) is the blackbody spectrum, normalised to \( \int d\epsilon B(\epsilon, T_s) = 1 \). For simplicity of calculation we use the

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\footnote{If we take account of population III stars for \( Z < 10^{-4}, \) assuming that star forming regions are not locally enriched after the first stars formed, the reionisation epoch is only slightly delayed (\( \tau_{\text{reion}} = 13, \tau = 0.16, \) which are compared to the numbers given in section 3 below). This results from the fact that the temperature of population III stars is higher (the coefficient of \( \text{footnote} \) being \( 10 \)), but the number of photons is fewer (Carr, Bond & Arnett 1984), and these two effects counteract for reionisation (the latter effect slightly wins); the efficiency of reionisation is more importantly controlled by the energy generation rate, which is fixed by...

\(\text{(end of footnote)}\)
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3 RESULTS AND DISCUSSION

The results of calculations are shown in Figure 1 (for hydrogen) and in Figure 2 (for helium). The figure shows reionisation of hydrogen and excitation of helium I to II at $z_{\text{reion}} = 13.5$, taking 90% ionisation as the reionisation epoch. We define the Thomson optical depth by

$$\tau = \int_0^\infty d\tau \sigma_T \left[ \frac{d\tau}{dz} \right] (n_{\text{HII}} - n_{\text{HII}}(\text{no reionisation})), \quad (10)$$

with $\sigma_T$ the Thomson cross section. Here, we subtract the contribution to $\tau$ from residual ionisation of the standard recombination (the second term in the integrand) to evaluate only the effect of reionisation. We obtain $\tau = 0.172$. We consider that these $z_{\text{reion}}$ and $\tau$ correspond to the earliest epoch of reionisation and the maximum optical depth we can obtain in the $\Lambda$CDM model with the specified cosmological parameters and with flat adiabatic fluctuations normalised to cosmic microwave background observations. The optical depth is consistent with the WMAP result $\tau = 0.17 \pm 0.04$, but the reionisation epoch is later than $z_{\text{reion}} = 17$ the WMAP team quoted, which assumes instantaneous reionisation (Kogut et al. 2003). [We note that $z_{\text{reion}} \approx 20$ often quoted by the WMAP team (Bennett et al. 2003; Kogut et al. 2003) assumes a rather unusual reionisation history.] The difference in $z_{\text{reion}}$ comes from the fact that stars formed earlier than the reionisation epoch contribute to $\tau$. Although the HI fraction drops quite sharply at $z_{\text{reion}}$, there is a non-negligible contribution from early formation of stars at $z > z_{\text{reion}}$. If we integrate (10) to $z=13.5$, we obtain $\tau = 0.13$, i.e. $\Delta \tau = 0.04$ comes from $z > 13.5$.

Note that first stars formed at $z \approx 35$, which is visible as a turn on of the HII component. This epoch is consistent among the authors who calculated reionisation with the CDM model (see e.g., Loeb & Barkana 2001).

In our calculation we have ignored many physical effects, which always contribute to delay the reionisation. We have assumed as a default that all UV photons escape from galaxies. This is not realistic, but this fraction is poorly known observationally, ranging from $>10\%$ (Steidel et al. 2001) to $<2\%$ (Heckman et al. 2001). Bianchi et al. (2001) argued $<20\%$ from the Lyman $\alpha$ cloud abundance. This parameter may also depend on the size, and hence redshift, of objects we consider. We take this as a free parameter. If we assume an escaping fraction of ionising photons $F_{\text{UV}} = 10\%$, the epoch of reionisation is delayed to $z_{\text{reion}} = 11$ and
dependence on epoch and the optical depth upon those parameters. The solid curves) and the Thomson optical depth (dotted curves) on $F_{42}$. M. Fukugita and M. Kawasaki

dependence of the reionisation epoch on $\Omega_b$, for example, arises from the fact that it controls the formation of small objects in early epochs, but on the other hand the decrease of the time interval for a unit redshift as increasing $\Omega_M$ suppresses the increase of $\tau$. The dependence on $\Omega_b$ appears more non-trivial: as baryon density increases, recombination rate becomes faster, delaying reionisation. However, this effect is overcome by the increase of the baryon density, and $\tau$ increases. One may use this figure to compare our calculation with those that use different cosmological parameters.

The other effects that must be taken into account for more realistic calculations are feedback from stars to suppress star formation and the absorption of ionising flux by hydrogen clouds and the envelope of galaxies. The former leads to a milder slope of the luminosity function compared with the dark matter mass function (Dekel & Silk 1986; White & Frenk 1991; Nagamine et al. 2001). We studied the effect by artificially decreasing the net star formation rate for smaller objects so that the collapsed baryonic mass function obeys the form of an empirical Schechter function at $z = 0$, assuming $M/L = $ constant. This suppresses the early star formation and the ionisation does not take place until $z_{\text{reion}} \sim 11$ and $\tau \sim 0.12$. This is probably an extreme case because a sharper faint end slope is expected in the luminosity function at a higher $z$.

The UV absorption by Lyman alpha clouds and galaxies may delay reionisation. We developed a formalism to take account of envelope of galaxies and Lyman $\alpha$ clouds (Fukugita & Kawasaki, in preparation). For an early epoch such as the one we are concerned with, this effect is negligible. The absorption due to galaxies becomes sizable only at low redshifts ($z \lesssim 5$). Adopting a minihalo model for Lyman $\alpha$ clouds (Ikeuchi 1986; Rees 1986), we find that absorption by Lyman $\alpha$ cloud is negligible for the ionisation epoch, because the onset of the ionising radiation decreases rapidly the abundance of Lyman $\alpha$ clouds.

The ionisation by quasars is important only for $z < 3$. Recent Sloan Digital Sky Survey (SDSS) observations showed that early bright quasars form exponentially in $z$ (Fan et al. 2001). We estimate the early quasar population using the quasar luminosity function (Boyle et al. 2000) known at lower redshifts, and put $\left[ d\nu / dz \right]_{\text{quas}}$ with the aid of the empirical spectrum (Vanden Berk et al. 2001) in the evolution equation. We find that quasars alone reionise the universe only at $z = 2.6$ although the effect becomes recognisable at $z \approx 10$. In any case the contribution of quasars is negligible compared to OB stars in the early reionisation epoch. The UV flux from quasars is important to sustain a high degree of ionisation at lower redshifts, as envisaged by the Gunn-Peterson test, when the escaping fraction of UV from galaxies is less than 10%.

In conclusion, the earliest epoch of reionisation we obtained is $z = 13.5$ with the cosmological parameters determined by WMAP (a value of 14 is possible within errors for example with $\Omega_m = 0.3$) and the Thomson optical depth is 0.17. This is consistent with the reionisation WMAP found. The difference in $z$ between our value and WMAP’s is ascribed to different approximations: WMAP assumed that the entire optical depth comes from $z < z_{\text{reion}}$, whereas the contribution earlier than $z = z_{\text{reion}}$ is non-negligible in our calculation. The realistic UV escaping fraction and the feedback of stars may somewhat lower the optical depth and delay the recombination epoch. The uncertainty in IMF that directly controls star formation is effectively absorbed into the $f$ parameter which is constrained by metallicity. We remark that reionisation takes place sharply once the number of ionising photons, which are determined by the thermal balance, reaches some critical number and so it is difficult to sustain a partial ionisation for an extended period, as was assumed in Kogut et al. (2003).

As the final comment our result is consistent with the Salpeter IMF case of Ciardi et al. (2003), who carried out a numerical simulation for the purpose similar to ours. As a check we calculated the case with $\Omega_m = 0.3$, $\sigma_8 = 0.9$ and $F_{UV} = 0.2$, and we obtained the reionisation epoch of $z = 12.8$, which is compared to $z \approx 12$ (see Fig. 3 of Ciardi et al.). This small difference is probably ascribed to feedback effects of stars, as discussed above.

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Figure 3. Dependences of the reionisation epoch (indicated by solid curves) and the Thomson optical depth (dotted curves) on the parameters of our model, (a) $F_{UV}$ (b) $\sigma_8$, (c) $\Omega_m$ and (d) $\Omega_b$. The UV absorption by Lyman alpha clouds and galaxies may delay reionisation. We developed a formalism to take account of envelope of galaxies and Lyman $\alpha$ clouds (Fukugita & Kawasaki, in preparation). For an early epoch such as the one we are concerned with, this effect is negligible. The absorption due to galaxies becomes sizable only at low redshifts ($z \lesssim 5$). Adopting a minihalo model for Lyman $\alpha$ clouds (Ikeuchi 1986; Rees 1986), we find that absorption by Lyman $\alpha$ cloud is negligible for the ionisation epoch, because the onset of the ionising radiation decreases rapidly the abundance of Lyman $\alpha$ clouds.

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