The relationship between fracture toughness and compressive strength of self-compacting lightweight concrete

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Abstract. This study evaluates the relationship between fracture toughness and compressive strength of self-compacting lightweight concrete (SCLC). For this purpose, three mix designs with different w/c ratios were used, and three-point bending tests were done on 36 notched beams. The maximum loads of the beams were achieved, and size effect method was applied for analysing the results. The results of analyses displayed that the fracture toughness and mechanical properties were significantly increased as w/c ratio decreased. In fact, as the w/c ratio decreased from 0.47 to 0.37, the fracture toughness and compressive strength increased by 56% and 52%, respectively. Fracture toughness of mode I was compared with the compressive strength of SCLC. According to the experimental results, an acceptable relation is presented between the fracture toughness of mode I and compressive strength of SCLC in this paper.

1. Introduction

Concrete is one of the most frequently utilized materials in the universe. It is used in the forms of normal, high and ultra-high strength concrete, and the kind of concrete depends on the required design strength and the significance of the building. It should be noted that achievements in modern concrete technology have contributed to the invention of self-compacting lightweight concrete (SCLC). In SCLC, lightweight coarse aggregates are incorporated into the highly flowable paste [1]. Weight reduction, elimination of noise emission, simplicity of placement, higher speed of construction, assurance of compaction in the structure are some main SCLC advantages [2], which can be attractive in construction industries.

When cracks develop in a structure, its load bearing capacity decreases considerably. Provided that the cracks grow in the major structural elements such as beams, columns, and shear walls, the entire structure may collapse within several minutes. Since inherent flaws exist in concrete, and it can be activated for any load combination, it is significant to study the behavior of concrete with inherent flaws under static and dynamic loads. Fracture toughness test of concrete is essential for this reason. It should be noted that the fracture parameters of concrete are quite related to the properties of the materials used in the mix compositions. In addition, the residual load carrying capacity of an element in the presence of cracks is a function of the fracture toughness of materials. Generally, cracks are present in concrete...
structures before their collapse. The strength of cracked concrete is determined by using fracture mechanics concepts [3]. In fact, the crack propagates in concrete structures as the crack tip stress intensity factor reaches the fracture toughness of concrete ($K_{IC}$). Many researchers have studied the fracture toughness of normal strength concrete with various mixing designs [3]. Peterson [4] studied the fracture of cement mortar and concrete to measure $K_{IC}$ and $G_I$. He mentioned that linear elastic fracture mechanics (LEFM) can be applied here. Hamoush and Abdel-Fattah [5] examined three-point bending tests for determining the fracture toughness of cracked beams. Moreover, Ince and Alyamac [6] stated that by increasing w/c ratio, the critical crack-tip opening displacement ($CTOD_c$) and fracture toughness reduced. Petersson [7] showed that the fracture parameters were enormously affected by w/c ratio and the quality of the aggregate. In fact, by decreasing w/c ratio and consuming stronger aggregate, the fracture energy of concrete increased. Wittmann et al. [8] executed an experimental research on the effects of w/c ratios on the fracture energy. The result was that by increasing w/c ratio, the fracture energy of concrete decreased. Nonetheless, some researchers have presented that there are no regular relations between w/c ratio and fracture energy. For instance, Carpinteri and Brighenti [9] said that the maximum fracture energy has achieved in a middle w/c ratio. The reason for it was that the cracks often propagate in cement paste, and an optimal w/c ratio can produce cement paste with the highest strength value. Phillips and Binnsheng [10] believe that fracture parameters are reliant on the strength of concrete, and they increased linearly and quadratically with compressive and tensile strength, respectively.

Self-compacting concrete (SCC) is a kind of concrete that has high fluidity under its own weight; therefore, it can easily fill the molds without mechanical vibration, and it can be pumped to long distances [11]. Some investigators have reported that as the w/c ratio reduced, the fracture toughness of SCC increased [12]. Alyhya et al. [13] indicated the concrete toughness and $CTOD_c$ increased with decreasing w/c ratio and increasing coarse aggregate volume. Cifuentes and Karihaloo [14] said the fracture energy and tensile strength of SCC were lower, and its modulus of elasticity was higher compared to normal concrete (NC) mixes with the same strength. Another study indicated that growing the size and content of coarse aggregate enlarged the fracture toughness of SCC [15].

As a lightweight concrete, SCLC is a respectable candidate to be used in long-span bridges and reinforced concrete structures. Karamloo et al. [16, 17] studied the effects of w/c ratio and maximum aggregate size on fracture behavior of SCLC. They concluded that the fracture energy increased by reducing the w/c ratio and improving the maximum aggregate size.

Lowering coarse aggregate content and improving the amount of ultra-fine particles raise the risk of cracking in SCLC compared to NC [18]. Moreover, using weak lightweight aggregate instead of normal aggregate may cause differences in fracture behavior of concrete. Consequently, evaluating the initiation of crack and fracture behavior of SCLC is quite important.

The problem of using SCLC is that flaws inherent in the material can grow under loading to intolerable lengths. Therefore, essential steps should be taken to improve the current understanding of concrete in order to account for the possible failure mechanisms that may happen reliably. Most of the previous studies on fracture energy of concrete have used normal aggregate to prepare the concrete specimens.

A literature review on concrete shows that there is very little important experimental data about the relationship between fracture toughness, critical crack-tip opening displacement ($CTOD_c$), and compressive strength of SCLC. Consequently, this study has considered $CTOD_c$ and fracture toughness of SCLC specimens made of LECA, cement, sand, water and superplasticizer.
2. Size effect method (SEM)

In the recent decades, extensive studies have been conducted on crack propagation and fracture behavior of concrete. Due to the existence of the sizeable fracture process zone in ahead of the crack tip, often the concept of linear elastic fracture mechanic (LEFM) cannot be applied to quasi-brittle materials such as concrete. Hence, several procedures are suggested for the determination of fracture parameters [19]. One of these methods is size effect method (SEM) that was recommended by Bazant and Pfeiffer [19]. In SEM, the key fracture parameters are quantified using three-point bending test on dissimilar sizes of notched samples.

Bazant and Pfeiffer [19] proposed a nonlinear relation between nominal failure stress ($\sigma_N$) and characteristic dimension of the sample as follows:

$$\sigma_N = \frac{Bf'_t}{\sqrt{(1 + \beta)^2}}$$

(1)

where $\sigma_N$ is the nominal failure stress of the specimen, $f'_t$ tensile strength and B is an empirical constant, and $\beta$ is the brittleness number of the specimen. For determining the fracture behavior of concrete, the following parameters should be calculated for each sample [20]:

$$Y_i = A X_i + C$$

(2)

Figure 1 shows the arrangement of the specimens. $Y_i$ and $X_i$ are parameters that used to calculate the coefficients of the equation 2. Moreover, $A$ and $C$ are the slope and intercept of the regression line.

The fracture toughness of mode I ($K_{ic}$) and critical effective crack-tip opening displacements ($CTOD_c$) are the main fracture parameters as follow [19]:

$$K_{ic} = a_0^{0.5} B \sqrt{g(\alpha)}$$

(3)

$$CTOD_c = \frac{5.66 K_{ic}}{E} \times \sqrt{\frac{C_T}{\pi}}$$

(4)

in which $E$ is modulus of elasticity, $g(\alpha)$ is non-dimensional energy release rate, $a_0$ is the notch length and $C_T$ is the effective size of the process zone.

Figure 1. Formation of specimens.
3. Experimental work

3.1. Materials and mix proportions
For analyzing the fracture toughness of SCLC, three different w/c ratios were considered. The studied w/c ratios were 0.37, 0.42 and 0.47. In each mix, the coarse aggregate was lightweight expanded clay aggregate (LECA), and the maximum size of it was 12.5 mm. The LECA particles were pre-soaked for 24 hours before adding to the mix for turning coarse aggregate to saturated surface dry (SSD) condition. The fine aggregate was river sand with fineness modulus of three. The specific gravity of the sand was 2.66. Type I-425 Portland cement was used in this research, and cement properties are presented in Table 1. A poly-carboxylate based superplasticizer (SP) was used to achieve satisfactory workability. In addition, Ultra-fine limestone powder was used to improve the viscosity of the mixtures.

| Fineness: Blaine $Cm^{2}/g$ | Autoclave Expansion (%) | 28 days (MPa) | 28 days (MPa) | Initial (min) | Final (hr.) |
|-----------------------------|--------------------------|---------------|---------------|---------------|-------------|
| 3307                        | 0.14                     | 21.9          | 48.4          | 140           | 3:10        |

As presented in Table 2, the most mix proportions of SCLC were constant to make the evaluation possible. To achieve a better homogeneity in all the mixtures, they were mixed for 6 minutes. At first the solid materials, including LECA, fine aggregate, limestone powder and cement were mixed. Then the combination of water and superplasticizer was added to the mix. After mixing, the rheology of the fresh concrete was evaluated by slump flow and J-ring tests based on EFNARC recommendation [2].

| Materials | Weight (kg/m³) |
|-----------|----------------|
| S01       | 395 Cement 146.2 water 6.7 Superplasticizer 210 Limestone Powder 780 Sand 230 LECA 0.37 |
| S02       | 360 Cement 151.2 water 4.8 Superplasticizer 210 Limestone Powder 780 Sand 230 LECA 0.42 |
| S03       | 335 Cement 157.4 water 3.7 Superplasticizer 210 Limestone Powder 780 Sand 230 LECA 0.47 |

3.2. Test procedure
For evaluating the relationship between fracture toughness and compressive strength of SCLC, SEM method was used. For receiving acceptable results, RILEM FMT-89 [20] has proposed that the beam depth to the maximum aggregate size ratio ($d/da$) should be chosen 4, 8 and 16. In fact, the bigger size ranges lead to more precise results. In this investigation, the selected $d/da$ ratios were 3, 6, 12 and 24. In this part of the research, 12 notched beams with the dimensions of $38.1 \times 38.1 \times 102$, $38.1 \times 76.2 \times 203$, $38.1 \times 152.4 \times 407$ and $38.1 \times 304.8 \times 814$ mm³ were used. All of the samples were geometrically similar in two dimension ratios and all of them had the persistent width of 38.1 mm. The length-to-depth (L/d), span-to-depth (l/d), and notch-to-beam depth ratios ($α = a_n/d$) in all the samples were 2.67, 2.5 and 0.2, respectively. Figure 2 shows the configuration of the test setup.

Besides the twelve notched beams, six 150×300 mm standard cylinders were cast to determine the splitting tensile strength ($f_t$) and the modulus of elasticity (E) according to ASTM-C496 [21] and ASTM-C469 [22], respectively. To measure the compressive strength ($f_c$) of each mix, three standard cubes
(100 mm) were cast and tested according to BS-EN-12390 [23]. All of the specimens were cured in lime-saturated water until the testing time at the age of 28 days.

As mentioned earlier, in this study, the size effect method has been used to determine the fracture parameters such as fracture toughness, nominal failure stress and CTOD. This method is independent of specimen shapes and dimensions and uses from equations 1 to 4 to determine these parameters.

![Test setup configuration for fracture toughness determination.](image)

Figure 2. Test setup configuration for fracture toughness determination.

4. Results and discussion

4.1. Tests on fresh and hardened state of SCLC

The rheology of the fresh SCLC mixtures was evaluated by measuring the slump flow, and J-ring tests based on EFNARC reference [2]. The properties of the fresh SCLC mixes can be seen in figure 3. The slump flow test was organized for assessing the ability of the mixes to fill the mold under their own weights. The slump flow test results showed that there was a good resistance to coarse aggregate segregation and bleeding in all the mixtures. It is worth noting that the distribution of the paste and coarse aggregate were uniform in all the broken tensile specimens. Moreover, the diameters of the slump flow tests were between 661 to 695 mm; it means, all the mixes were satisfied according to the recommendations of EFNARC [2].
Based on ACI318-14 [24], in lightweight aggregate concrete (LWAC), shear strength, friction properties, development requirements and bond between concrete and reinforcement, and splitting resistance are not the same as the ones in NC with similar compressive strength. Consequently, the relation between SCLC compressive and tensile strengths may not follow the same relations in NC and LWAC. In fact, by using high dosages of superplasticizer and ultra-fine powders in SCLC, the paste volume of concrete increases compared to LWAC; therefore, the cracking risk improves. Figure 4 shows an evaluation between the mechanical properties of SCLC made with three different w/c ratios. It is clear that the compressive strength and modulus of elasticity improved by 52% and 23% respectively as the w/c ratio reduced from 0.47 to 0.37. This finding is in agreement with those stated in earlier studies [12, 17]. As declared previously, this is due to the fact that in concrete mixes with lower w/c ratios, ITZ becomes tougher and the cracks cannot pass around the aggregate particles [25]. The first author of this article has considerable investigations in this field [26-34].

![Figure 3. Results of slump flow and J-ring test.](image)

![Figure 4. Variation of the compressive strength and modulus of elasticity of SCLC.](image)
4.2. Calculation of fracture parameters

According to SEM, the peak loads of geometrically similar beams define the initial fracture energy. The peak loads should be adjusted considering the effect of the specimen weight. According to RILEM TC89-FMC [20], the following equation is used to achieve the adjusted maximum loads:

\[ F_C^e = F_I + \frac{2I_i - L_i}{2l_i} g \times m_i \]  \hspace{1cm} (5)

where \( F_C^e \) is the corrected maximum load, \( F_I \) is the maximum load prepared by experimental test, \( l \) and \( L \) are span and length of specimen as shown in figure 1, \( g \) is acceleration of gravity, \( m \) is the specimen mass, and \( i \) is the number of executed tests. All the corrected maximum loads are presented in figure 5.

Key fracture parameters for each mixture were achieved using linear regression analysis based on RILEM-TC 89 [20]. Figure 6 shows the results of linear regression analyses for SCLC with w/c ratio of 0.47. It is clear that a linear regression analysis for mix S03 resulted in slope and intercept coefficients of \( A=0.0204 \) (MPa m^{-1}) and \( C=1.5166 \) (MPa^{-2}) respectively. Moreover, the coefficients of variation of the intercept and slope of the regression line and the width of scatter band were 12.8%, 4.6% and 10.6% respectively. For increasing the exactness of the analysis, regression line intercept variation coefficient (\( \omega_c \)), slop variation coefficient (\( \omega_A \)) and width of scatter band (\( m \)) must be less than 20, 10 and 20 % respectively [20]. The critical crack-tip opening displacement (CTOD\(_c\)) and fracture toughness (\( K_{IC} \)) in all the mixes are presented in Table 3.

![Figure 5](image.png)

**Figure 5.** Corrected peak loads of the samples.

| Series | S01 | S02 | S03 |
|--------|-----|-----|-----|
| \( f_I \) (MPa) | 2.88 | 2.58 | 2.02 |
| \( K_{IC} \) (MPa\( m^{0.5} \)) | 27.20 | 23.36 | 17.43 |
| CTOD\(_c\) (mm) | 0.0171 | 0.0157 | 0.0107 |
Figure 7 shows the effect of three different w/c ratios on $K_{ic}$ and CTOD$_c$ of SCLC. As can be perceived, by reducing the w/c ratios, $K_{ic}$ improved. The reason for it is that as the w/c ratio improves, ITZ becomes weaker and micro cracks can be extended in this zone extremely. Accordingly, the peak load and the fracture toughness of SCLC decline.

For specimens with the w/c ratios of 0.47 to 0.37, $K_{ic}$ increased from 17.43 to 27.2 MPa mm$^{0.5}$, respectively. Reduction of fracture toughness by growing w/c ratios was described previously by Karamloo et al. [17], Beygi et al. [12] and Ince and Alyamac [6] for SCLC, SCC and NC, respectively. All of the authors above have presented some equations to estimate the fracture toughness of concrete.

Since there is a significant correlation between $K_{ic}$ and $f_c$, different investigators have projected various power equations between them in NC and SCC. In this research, based on the limited experimental data, the following relation between $K_{ic}$ and $f_c$ is achieved:

$$K_{ic} = 0.74 f_c - 0.21 \quad R^2 = 0.954$$

(6)

where $K_{ic}$ and $f_c$ are in MPa mm$^{0.5}$ and MPa, respectively. Moreover, by decreasing w/c ratios from 0.47 to 0.37, the critical crack-tip opening displacement (CTOD$_c$) increased from 0.0107 to 0.0171 mm. By conducting the linear regression analysis, the following equation is obtained between CTOD$_c$ and $f_c$.

$$CTOD_c = 0.015 \ln(f_c) - 0.037 \quad R^2 = 0.889$$

(7)
It is worth noting that by means of equations 6 and 7, it is possible to achieve the fracture parameters of SCLC such as $K_{IC}$ and $CTOD_c$ easily and only with compressive strength of concrete.

5. Conclusions
The conclusions from this study can be summarized as follows:
1. Water/cement (w/c) ratio influenced the fracture toughness ($K_{IC}$) of self-compacting lightweight concrete (SCLC), and as the w/c ratio reduced from 0.47 to 0.37, the $K_{IC}$ of SCLC increased by 56%.
2. The achieved results publicized that the critical crack-tip opening displacement ($CTOD_c$) improved by 60% as w/c ratio reduced from 0.47 to 0.37 in SCLC.
3. There was a significant correlation between fracture parameters and compressive strength ($f_c$) of SCLC. In other words, it is possible to achieve $K_{IC}$ and $CTOD_c$ according to the compressive strength of SCLC.

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