Gauge Coupling Unification: 
Strings versus SUSY-GUTs

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ABSTRACT

Standard (level one) heterotic string models with standard model gauge group predict the unification of $SU(3)$ and $SU(2)$ gauge couplings whereas the $U(1)$ factor is unified modulo an unknown normalization factor $k_1$. On the other hand the unification mass is known. I argue that this situation is quite analogous (though opposite) to that in SUSY-GUTs in which the $U(1)$ normalization is known ($k_1 = 5/3$) but the unification mass $M_X$ is unknown. I emphasize that $k_1$ should be taken as a free parameter in the string approach (quite in the same way as $M_X$ is taken as a free parameter in SUSY-GUTs). If this is done, the success of the string approach concerning gauge coupling unification is comparable to that in SUSY-GUTs.

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In the last couple of years there has been a revival of interest on the topic of gauge coupling unification at very high energies [1]. More precise experimental data on gauge couplings has allowed for a more detailed check of the predictions of Grand Unified Theories (GUTs) for gauge couplings. In particular, it has been confirmed that there is very good agreement of the results predicted [2] by supersymmetric (SUSY) unified theories with the experimental results [3]. On the other hand, the predictions of non-supersymmetric GUTs are many standard deviations away from these experimental results. Thus, e.g., a non-SUSY version of minimal $SU(5)$ is experimentally ruled out by these measurements.

Given this spectacular success of the SUSY-GUT scenario one is naturally lead to look closer to these theories and see what is their theoretical status. There are two major problems in these theories. The first of them is the notorious doublet-triplet splitting problem of GUTs (and SUSY-GUTs). In all GUT models the Higgs-doublet of the standard model has colour-triplet GUT-partners. These colour triplets have quantum numbers of $d$-quarks and can mediate fast proton decay unless they have masses of the order of the unification scale. Thus we have to split the Higgs multiplet allowing for the usual light Higgs doublets but giving a large mass to the coloured triplet fields. Of course, this can be done by fine-tuning but this would bring us back a new (though milder) form of gauge hierarchy problem. Other mechanisms proposed to perform this splitting without fine-tuning either do not work or require quite baroque and unexpected (huge) new multiplets. Thus it looks like if the Higgs sector refused to be unified, although, on the other hand its presence is essential in getting the appropriate gauge coupling unification!

The other major problem of SUSY-GUTs is the difficulty in unifying this type of theories with gravity in the context of the only consistent known gravity-unified theories, strings. Indeed, it is well known that in order to obtain a usual GUT like $SU(5)$, $SO(10)$ etc. from strings one needs to go beyond the usual compactifications and consider theories with the gauge sector involving higher level Kac-Moody algebras. Only these theories allow for adjoint superfields in the massless level of the string. Although in principle this is not necessarily a problem, these theories are quite cumbersome in practice (see e.g. ref.[4]). Furthermore, once one has succeed in constructing e.g. an $SU(5)$ string, it is very difficult to avoid the presence of extra unwanted adjoints (or other exotic multiplets like 15s, 40s, etc). On top of that, pretending that all the Higgs multiplets in the model have precisely the couplings required to perform the doublet-triplet splitting is just hoping for a miracle. Thus the situation concerning SUSY-GUTs is quite puzzling: they are theoretically in quite a bad shape but give the correct result required for unification!
In the present note I remark that there is another class of well motivated theories which have comparable success concerning gauge coupling unification but are free from the theoretical problems of SUSY-GUTs mentioned above. This class of theories correspond to the assumption that the SUSY standard model is directly unified into a string theory close to the Planck mass, without any GUT intermediate step. In fact gauge coupling unification within this type of scheme has been considered in the recent past reaching apparently a different conclusion [3],[4]. The origin of this difference is a matter of appropriately identifying what are actually the free parameters in string unification. This will be clarified below.

Let us first briefly recall the situation in SUSY-GUTs. Here the normalization of the $U(1)$ factor is known ($k_1 = 5/3$) and the one-loop expressions for the weak angle and $\alpha_s$ yield

$$\sin^2\theta_W(M_Z) = \frac{3}{8} \left( 1 + \frac{5\alpha(M_Z)}{6\pi} (b_2 - \frac{3}{5} b_1) \log\left(\frac{M_X}{M_Z}\right) \right)$$

$$\frac{1}{\alpha_s(M_Z)} = \frac{3}{8} \left( \frac{1}{\alpha(M_Z)} - \frac{1}{2\pi} (b_1 + b_2 - \frac{8}{3} b_3) \log\left(\frac{M_X}{M_Z}\right) \right)$$

where one has $b_1 = 11, b_2 = 1$ and $b_3 = -3$ in the SUSY case. In principle $M_X$ is unknown and the formulae (1) give us a constraint between the values of $\sin\theta_W$ and $\alpha_s$ consistent with unification. This constraint is shown numerically in the figure in which it is represented as a line in the $\sin\theta_W$- $\alpha_s$ plane. Different points in the line correspond to different values for $M_X$ ($\log_{10}M_X$ is shown at various points on the line). We will not attempt here to include a detail treatment of the errors. We have included an error band corresponding to an uncertainty of $\pm 0.01$ in the resulting value for $\alpha_s$. There are different sources of errors coming from the uncertainty in the low-energy and superheavy thresholds, two-loop effects etc. (see e.g., ref. [7] for a detailed discussion of these points). The success of the SUSY-GUT predictions correspond to this line going through the experimental results also depicted in the figure ($\alpha_s$ is taken from jet event shape analysis). On the other hand the lower curve in the figure corresponds to the non-supersymmetric GUT result. It is clear this latter case is ruled out.

Let us go now to the supersymmetric string case. We will consider a situation in which the gauge group is of the form $SU(3) \times SU(2) \times U(1)_Y \times G$, $G$ being some other possible gauge group factors. Furthermore we will assume that the only low-energy particles which are charged under the standard model group are those of the minimal SUSY standard
model. The gauge coupling constants $g_i$ of the three SM interactions are related at the string scale by

$$k_3g_3^2 = k_2g_2^2 = k_1g_1^2$$

(2)

In any standard compactification of the $E_8 \times E_8$ heterotic string one has in fact $k_2 = k_3 = 1$. And hence we have the boundary condition $g_2^2 = g_3^2$ at the string scale. Notice that this boundary condition is present without any GUT type of symmetry relating strong to weak interactions, is just a consequence of direct (level one) string unification. The case of $k_1$ is different and we only know that it is in general a fractional number with $k_1 \geq 1$ (see below for a brief discussion of our limited knowledge about the possible values of $k_1$)). Thus it looks one has less predictivity compared to the GUT case since in the latter one has one more boundary condition ($g_3^2 = g_2^2 = 5/3g_1^2$). In fact the predictivity is the same because in the string case we also know the unification mass $M_X$. It is related in a calculable manner to the Planck mass and one finds $M_{\text{string}} = 5.3g10^{17}$ GeV. Thus the fact that strings are theories which are unified with gravity provide us with an extra constraint not present in GUTs. Now, the one-loop results for the weak angle and $\alpha_s$ yield

$$\sin^2\theta_W(M_Z) = \frac{1}{1 + k_1}(1 + \frac{k_1\alpha(M_Z)}{2\pi}(b_2 - \frac{1}{k_1}b_1) \log(\frac{M_{\text{string}}}{M_Z}))$$

$$\frac{1}{\alpha_s(M_Z)} = \frac{1}{1 + k_1}(\frac{1}{\alpha(M_Z)} - \frac{1}{2\pi}(b_1 + b_2 - (1 + k_1)b_3) \log(\frac{M_{\text{string}}}{M_Z}))$$

(3)

In analogy with the SUSY-GUT case, these two expressions give a constraint between $\sin^2\theta_W$ and $\alpha_s$. Now the free parameter is $k_1$ (instead of $M_X$) and this constraint may be represented as a line in the $\sin^2\theta_W$-$\alpha_s$ plane, the upper line (band) in the figure. Different points in the line correspond to different values for $k_1$ from 1.3 to 1.7 (some sample values for $k_1$ are shown on the line). As in the GUT case we have included an estimated band of error. Some of the error sources (low-energy threshold, two-loop effects) are identical to those present in GUTs. The treatment of the heavy threshold errors is different and can be substantially larger in the string compared to the GUT case, as has been recently emphasized. To give an idea of the uncertainties I just include, as in the previous case, a band of error corresponding to an uncertainty of $\pm 0.01$ in $\alpha_s$ (I briefly come back to the issue of string threshold corrections below). The figure shows that the band corresponding to string unification is quite close to the experimental results and hence one concludes that the data is compatible with direct string unification at the string scale $M_{\text{string}} = 5.3g10^{17}$
GeV. This is the simple fact I want to emphasize in this note. The result is very strongly dependent on the value of $k_1$ and the best agreement is found for values $k_1 \simeq 1.4$. Notice also that the string case seems to prefer higher values of $\alpha_s(M_Z)$ compared to the GUT case. Taking $k_1$ as a free parameter in gauge coupling unification was previously considered in refs. [10], [1] and [11].

There is a forth logical possibility which is considering non-supersymmetric string unification. In this case the results would be incompatible with data for any $k_1$: one obtains e.g. $\alpha_s(M_Z) \simeq 1.0$. This shows that the fact that in the supersymmetric string case agreement is found is a non trivial result.

Let us end this note by adding a couple of comments about threshold effects and a discussion of possible $k_1$ values in string models.

A comment concerning the size of string threshold corrections is in order. The form of these threshold corrections has been computed [9], [12] for a class of 4-dimensional strings (orbifold compactifications) and it has been found that they grow linearly with $R^2$, $R$ being the compactification radius of the orbifold. This can potentially lead to large corrections. En ref. [3] it was found that one can achieve consistent gauge coupling unification even taking $k_1 = 5/3$ for a) sufficiently large $R^2 \simeq 10-16$ b) if the matter fields have appropriate transformation properties. The latter is not always possible and this can lead to interesting constraints in that class of models. In this approach the corrections to $\alpha_s$ can be enormous, e.g., of order $\Delta \alpha_s \simeq 0.07$, as required to get agreement with $k_1 = 5/3$ (see the point $k_1 = 5/3$ in the figure). Concerning the value of $R^2$, such relatively large values would correspond to the compactification scale being slightly below the string scale. The dynamical models of supersymmetry breaking constructed up to now seem to prefer on the other hand relatively low values for $R^2$ in the range $R^2 \simeq 1.3$, as a reflection of the $R \rightarrow 1/R$ duality symmetry of toroidal compactifications. If one takes values $R^2 \simeq 2 - 3$ one finds typical shifts in $\alpha_s$ of order $\pm 0.01$, similar to the error band shown in the figure. Thus the width of the band in the figure just corresponds to assuming modest string threshold corrections of this order of magnitude.

The second comment concerns the possible values of $k_1$ in string models. Not much is known about $k_1$, apart from the fact that it should be a rational number. A massless state which transforms like the representation $(R_3, R_2, Y)$ under the standard model can only be in the massless string spectrum if the following condition is verified (see e.g., [4])

$$\frac{C(R_3)}{k_3 + 3} + \frac{C(R_2)}{k_2 + 2} + \frac{Y^2}{k_1} \leq 1$$  (4)
where \( C(R_i) \) is the quadratic Casimir of the \( R_i \) representation of the group \( SU(i) \) and the usual assignments for the hypercharges has been assumed (e.g., \( Y(Q_L) = 1/6 \)). Now, in order for the \( e_R \) to be in the massless spectrum one necessarily has \( k_1 \geq 1 \). This is the only model independent constraint which is known about \( k_1 \). The hypercharge generator in string models (see e.g. Appendix C in ref. [14]) can be written in terms of the 16 Cartan subalgebra bosonic coordinates \( X_I \) of \( E_8 \times E_8 \) as \( Y = i \sum_I Q_I \partial X_I \). One then finds \( k_1 = 2 \sum_I Q_I^2 \). Since in each model the hypercharge is embedded into \( E_8 \times E_8 \) in a different way (i.e., one has different \( Q_I \)'s), the obtained results for \( k_1 \) is different and can be only computed in a model by model basis. Concerning the actual values of \( k_1 \) in specific models, in the orbifold examples constructed up to now the value of \( k_1 \) is never the canonical 5/3 and has in fact the tendency to be larger. Other models are specifically constructed in order to get a value \( k_1 = 5/3 \). As discussed above, values for \( k_1 \) slightly smaller like 3/2 or 7/5 would be preferable from the point of view of direct standard model unification into a string. It would be very important to look for model independent information about \( k_1 \). In the case of GUTs it is possible to obtain a variety of values for \( k_1 \) which turn out to be always bigger than 5/3 if one embeds the standard model into a bigger simple group. I do not know of any equivalent statement concerning string models.

It would be interesting to find specific string examples with the preferred \( k_1 \) values in the range 1.4-1.5. On the other hand, the equivalent statement for SUSY-GUTs is that it would be interesting to find GUTs in which the natural value of \( M_X \) is of order \( 10^{16} \) GeV. Furthermore, in the GUT case one would also have to find a natural mechanism for doublet-triplet splitting. Which of both alternatives one should take is at the moment a question of taste. However, it is fair to say that leaving \( k_1 \) as free parameter (as one should do till we have a general handle on it) allows for gauge coupling unification (within expected uncertainties) at the string scale.

Figure

Constraints in the \( \sin^2 \theta_W(M_Z) - \alpha_s(M_Z) \) plane coming from unification of gauge coupling constants
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