Probing the Vacuum Induced Coherence in a Λ-system

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We propose a simple test to demonstrate and detect the presence of vacuum induced coherence in a Λ-system. We show that the probe field absorption is modulated due to the presence of such a coherence which is unobservable in fluorescence. We present analytical and numerical results for the modulated absorption, the cosine and sine components of which display different types of behavior.

I. INTRODUCTION

The vacuum of electromagnetic field is known [1] to give rise to several types of very interesting coherence effects. For example it gives rise to atom-atom correlations and the collective effects. In a single multilevel atom, it also gives rise to coherences among levels. The coherences are especially significant if the relevant levels are near by. In Ref. [1] it was shown that the vacuum of the electromagnetic field can lead to the possibility of a trapped state in a degenerate V-system. The Ref. [1] also analyzed the origin of such a trapped state in terms of suitably defined coupled and uncoupled states. Spontaneous emission produced a coherence between the two excited states of the V-system. Further, it was shown that such a vacuum induced coherence (VIC) can suppress the steady state resonance fluorescence [2], and can substantially modify the emission spectrum [3]. Recently, interest in this subject has been revived due to the various possibilities of manipulating atomic properties using atomic coherence effects [1, 4]. Using this kind of coherence effect, Hegerfeldt and Plenio showed that periodic dark states and quantum beats appear in a near-degenerate V-system [4]. The work of Zhu, Scully and coworkers [5] demonstrate that even spectral line elimination and spontaneous emission cancellation is possible. This was observed experimentally by Xia, Ye and Zhu in sodium dimers [6]. An appealing physical picture to explain spontaneous emission cancellation was provided by Agarwal [7]. The effect of vacuum induced coherence (VIC) on spontaneous emission has been suggested to achieve gain without inversion and sub-natural line-widths [8]. We mention that such a coherence mechanism is also known to occur in quantum well structures [9]. Recent studies have also shown that vacuum induced coherence effects give rise to phase sensitive absorption [10] and emission [11] profiles as well as to obtain phase control of spontaneous emission in V-systems [12].

While much of the work has been in connection with V-systems, other level schemes like Λ-systems [10, 12] and Ξ-systems [14] have also been studied. In case of Λ-systems, the coherence is produced in the ground state. We study in this article the origin of this coherence and the question of a proper probe for such a coherence.

The organization of this paper is as follow. In Sec. II we show the reason behind the origin of VIC. In Sec. III we derive the spontaneous emission spectrum in the presence of VIC, and show that the spectrum is independent of VIC. In Sec. IV we show that absorption of a weak field, as a probe for VIC, will be uniquely modulated due to VIC. Using second order perturbation theory we derive the analytical results which well explains our numerical results. Finally, in Sec. V we present concluding remarks.

II. ORIGIN OF VIC IN Λ-SYSTEMS

Consider a Λ-system as shown in Fig. 1. Let us take the zero of energy at the state $|\beta\rangle$ and let $\hbar\omega_{ij}$ denote the energy difference between the states $|i\rangle$ and $|j\rangle$. The Hamiltonian for this system interacting with the vacuum of radiation field is

$$H = H_0 + H_{AV}. \quad (1)$$

where the unperturbed atom and vacuum field Hamiltonian $H_0$ will be

$$H_0 = \hbar \omega_{1\beta} a_{11} + \hbar \omega_{\alpha\beta} a_{\alpha\alpha} + \sum_{ks} \hbar \omega_k a_{ks}^\dagger a_{ks}, \quad (2)$$

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and the interaction Hamiltonian $H_{AV}$ is

$$H_{AV} = -\sum_{k} \hbar \{ (g_{ks} A_{1\alpha} + f_{ks} A_{1\beta}) a_{ks} + H.c. \}.$$  

(3)

Here the operator $A_{ij} = |i\rangle \langle j|$ is the atomic transition operator for $i \neq j$ and population operator for $i = j$. The field annihilation and creation operators are $a_{ks}$ and $a_{ks}^\dagger$ respectively where the subscript $k$ denotes the $k$th mode of the field with polarization along $\hat{e}_{ks}$. The vacuum coupling strengths are $g_{ks} = i(2\pi ck/hL^3)^{1/2} d_{1\alpha} \cdot \hat{e}_{ks} e^{i\vec{k} \cdot \vec{r}}$ and $f_{ks} = i(2\pi ck/hL^3)^{1/2} d_{1\beta} \cdot \hat{e}_{ks} e^{i\vec{k} \cdot \vec{r}}$, where $d_{1i}$’s ($i = \alpha, \beta$) denote the dipole matrix elements. We have dropped the anti-resonant terms from (3). We work with density matrices and use the standard master equation technique here. A calculation leads to the following master equation for the reduced density matrix $\rho$ of the atomic system

$$\dot{\rho} = -i[\omega_{1\beta} A_{11} + \omega_{\alpha\beta} A_{\alpha\alpha}, \rho] - \gamma_{1\alpha}(A_{11}\rho - 2A_{\alpha\alpha}\rho_{11} + \rho A_{11}) - \gamma_{1\beta}(A_{11}\rho - 2A_{\beta\beta}\rho_{11} + \rho A_{11}) + 2\sqrt{\gamma_{1\alpha}\gamma_{1\beta}} \cos \theta_1 (A_{\alpha\beta}\rho_{11} - A_{\beta\alpha}\rho_{11} + 2\sqrt{\gamma_{1\alpha}\gamma_{1\beta}} \cos \theta_1 A_{\alpha\beta}\rho_{11}).$$  

(4)

Here $2\gamma_{1\alpha} = 4\omega_{1\alpha}^2 |d_{1\alpha}|^2 / 3\hbar \epsilon_c^3$ and $2\gamma_{1\beta} = 4\omega_{1\beta}^2 |d_{1\beta}|^2 / 3\hbar \epsilon_c^3$ denote the spontaneous emission rates from state $|1\rangle$ to states $|\alpha\rangle$ and $|\beta\rangle$ respectively and $\theta_1$ is the angle between the two transition dipole moments $\vec{d}_{1i}$ ($i = \alpha, \beta$). The last two terms in the above equation are the interference term due to coupling of the two atomic transition $|1\rangle \rightarrow |\alpha\rangle$, $|1\rangle \rightarrow |\beta\rangle$ to a common vacuum $|11\rangle$ of the electromagnetic field. The dipole matrix elements should be non-orthogonal for the above interference to occur. The density matrix elements, $\rho_{ij} (\langle i|\rho|j\rangle)$, in the Schrödinger picture obey equations

$$\dot{\rho}_{11} = -2\Gamma_1 \rho_{11}; \quad \dot{\rho}_{1\alpha} = -(\Gamma_1 + i\omega_{1\alpha}) \rho_{1\alpha}; \quad \dot{\rho}_{1\beta} = -(\Gamma_1 + i\omega_{1\beta}) \rho_{1\beta}; \quad \dot{\rho}_{\beta\beta} = 2\gamma_{1\beta}\rho_{11}; \quad \dot{\rho}_{\alpha\beta} = -i\omega_{\alpha\beta}\rho_{1\alpha} + 2\sqrt{\gamma_{1\alpha}\gamma_{1\beta}} \cos \theta_1 \rho_{11},$$

(5)

where $\Gamma_1 = \gamma_{1\alpha} + \gamma_{1\beta}$. Note that the equation for the ground state coherence $\rho_{\alpha\beta}$ is coupled to the population of the excited state. Solving for the coherence $\rho_{\alpha\beta}$ with the initial condition $\rho(0) = A_{11}$ gives,

$$\rho_{\alpha\beta}(t) = \frac{2\sqrt{\gamma_{1\alpha}\gamma_{1\beta}} \cos \theta_1 (e^{-i\omega_{\alpha\beta}t} - e^{-2\Gamma_1 t})}{(2\Gamma_1 - i\omega_{\alpha\beta})}.$$  

(6)

As the equation reads, this coherence is non-zero as a result of interference term. Even in the long time limit ($t \gg 1/\Gamma_1$) this coherence is finite and oscillates with a frequency $\omega_{\alpha\beta}$

$$\rho_{\alpha\beta}(t \rightarrow \infty) = \frac{2\sqrt{\gamma_{1\alpha}\gamma_{1\beta}} \cos \theta_1 e^{-i\omega_{\alpha\beta}t}}{(2\Gamma_1 - i\omega_{\alpha\beta})}.$$  

(7)

The magnitude of this coherence is especially significant only if $\omega_{\alpha\beta} \leq 2\Gamma_1$ [10] and if the dipole matrix elements are parallel. Thus, as mentioned, even vacuum of electromagnetic field can give rise to coherence in systems with near degenerate levels. We next address the questions: (a) what leads to the coherence (6), and (b) how such a coherence can be measured.

In the long time limit the non-zero density matrix in (6) will be

$$\rho_{\alpha\beta} = \gamma_{1\alpha}/\Gamma_1, \quad \rho_{\beta\alpha} = \gamma_{1\beta}/\Gamma_1, \quad \rho_{\alpha\beta} = \sqrt{\rho_{\alpha\alpha}\rho_{\beta\beta}} B, \quad \text{where} \quad B = \frac{2\cos \theta_1}{(2 - i\omega_{\alpha\beta}/\Gamma_1)}.$$  

(8)

The oscillation in $\rho_{\alpha\beta}$ has been removed by writing it in the interaction picture. Thus the density matrix $\rho$ is reduced to an effective matrix $\tilde{\rho}$ where

$$\tilde{\rho} = \begin{bmatrix} \rho_{\alpha\alpha} & \rho_{\alpha\beta} \\ \rho_{\beta\alpha} & \rho_{\beta\beta} \end{bmatrix} \equiv \begin{bmatrix} \rho_{\alpha\alpha} & \sqrt{\rho_{\alpha\alpha}\rho_{\beta\beta}} B \\ \sqrt{\rho_{\alpha\alpha}\rho_{\beta\beta}} B^* & \rho_{\beta\beta} \end{bmatrix}.$$  

(9)

A measure of the purity of the state we calculate $\text{Tr}(\tilde{\rho}^2)$ :

$$\text{Tr}(\tilde{\rho}^2) = \frac{\gamma_{1\alpha}^2 + \gamma_{1\beta}^2}{\Gamma_1^2} + \frac{8\gamma_{1\alpha}\gamma_{1\beta}\cos^2 \theta_1}{(4\Gamma_1^2 + \omega_{\alpha\beta}^2)}.$$  

(10)
For $\omega_{\alpha\beta} = 0$ and $\cos \theta_1 = 1, B = 1$, and we get

$$\text{Tr}(\hat{\rho}^2) = 1,$$

(11)

which means that the atom would be in a pure state. That is when the VIC is maximum. Generally, one would find the system in a mixed state $[\text{Tr}(\hat{\rho}^2) < 1]$ as $|B| \neq 1$. The entropy of the final state depends on the parameter $B$.

The case when the atom is left in a pure state is especially interesting as we can introduce the coupled ($|c\rangle$) and uncoupled ($|uc\rangle$) states given by

$$|c\rangle = \frac{|d_{1\alpha}|\langle \alpha\rangle + |d_{1\beta}|\langle \beta\rangle}{|d|},$$

$$|uc\rangle = \frac{|d_{1\alpha}|\langle \alpha\rangle - |d_{1\beta}|\langle \beta\rangle}{|d|},$$

(12)

where $|d| = \sqrt{|d_{1\alpha}|^2 + |d_{1\beta}|^2}$. The Hamiltonian (11) can be written as

$$H = \hbar \omega_{1\beta}|1\rangle\langle 1| + \sum_{k_\perp} \omega_{k} a^\dagger_{k_\perp} a_{k_\perp} - \sum_{k_\perp} (g'_{k_\perp}|1\rangle\langle a_{k_\perp} + \text{H.c} ,$$

(13)

where $g'_{k_\perp} = i(2\pi c/\hbar L^3)^{1/2}|d| \hat{d} \cdot \hat{e}_{k_\perp} e^{i\hat{k} \cdot \hat{r}}$ is the vacuum coupling between state $|1\rangle$ and $|c\rangle$ and $\hat{d}$ is the unit vector parallel to both $\hat{d}_{1\alpha}$ and $\hat{d}_{1\beta}$. Note that state $|uc\rangle$ is not directly coupled to state $|1\rangle$. Thus $|uc\rangle$ never gets populated if $\rho(0) = A_{11}$. The spontaneous emission from state $|1\rangle$ occurs to the coherent superposition state $|c\rangle$ and not just the individual states $|\alpha\rangle$ and $|\beta\rangle$. Clearly under these conditions the final state will be $|c\rangle$ which agrees with the result (11) for $\omega_{\alpha\beta} = 0, \theta_1 = 0$.

For $\omega_{\alpha\beta} \neq 0$, the proper basis corresponds to the two eigenstates $|\psi_\pm\rangle$ of (11) and the steady state will be an incoherent mixture of $|\psi_+\rangle$ and $|\psi_-\rangle$.

### III. Emission Spectrum

We now come to the question as to how can one probe the existence of VIC in a Λ-system. Thus we naturally think of the spectrum of spontaneous emission. In a V-system the spontaneous emission is significantly affected by the presence of VIC (11). But for a Λ-system, as we show, the emission spectrum is independent of VIC. The emission spectrum corresponds to the normally ordered two time correlation function of electric field amplitudes (11). The radiated fields at spacetime points $\vec{r}, t_1 (l = 1, 2)$ will have a correlation given by

$$\langle E^{(-)}(\vec{r}, t_1) \cdot E^{(+)}(\vec{r}, t_2) \rangle = (r_1 r_2)^{-1} \sum_{i,j=\alpha,\beta} M_{ij} \langle A_{i1}(t_1) A_{j1}(t_2) \rangle, \quad t_1 > t_2,$$

(14)

where

$$M_{ij} = (\frac{\omega_{1\beta} \omega_{1j}}{c^2})^2 [\hat{r}_1 \times (\hat{r}_1 \times \hat{d}_{1i})] \cdot [\hat{r}_2 \times (\hat{r}_2 \times \hat{d}_{1j})],$$

and $r_1$ is much greater than the size of the source. Using quantum regression theorem and equations (11), it can be shown that the two time atomic correlation functions are given by

$$\langle A_{i1}(t_1) A_{i1}(t_2) \rangle = \exp [(i\omega_{1i} - \Gamma_1)(t_1 - t_2)] \exp (-2\Gamma_1 t_2), \quad t_1 > t_2,$$

(15)

and

$$\langle A_{i1}(t_1) A_{j1}(t_2) \rangle = 0 \quad \text{for} \quad i \neq j.$$

(16)

Using (15) in (14) we get the correlation function of the radiated field

$$\langle E^{(-)}(\vec{r}, t_1) \cdot E^{(+)}(\vec{r}, t_2) \rangle = (r_1 r_2)^{-1} \sum_{i=\alpha,\beta} M_{ii} \exp [(i\omega_{1i} - \Gamma_1)(t_1 - t_2)] \exp (-2\Gamma_1 t_2), \quad t_1 > t_2.$$

(17)

This correlation function is the sum of incoherent emissions along the two transitions, $|1\rangle \rightarrow |\alpha\rangle$, $|1\rangle \rightarrow |\beta\rangle$. Thus we conclude that the spontaneous emission spectrum in a Λ-system is not affected by VIC. Therefore one has to consider other types of probes to study VIC in such a system.
IV. MODULATED ABSORPTION AS A PROBE OF VIC

The above result is not surprising because the coherence is created after the spontaneous emission has occurred. An alternative approach to monitor VIC will be to study the absorption of a probe field tuned close to some other transition in the system. In this paper we show that a unique feature in probe absorption appears due to the presence of VIC. The model scheme is as shown in Fig. 3. Here the spontaneous emission from state |1⟩ creates VIC between the two near-degenerate ground levels |α⟩ and |β⟩. We now consider another excited state |2⟩, well separated from |1⟩. A weak coherent field is tuned between state |2⟩ and the two ground states to monitor VIC. The Hamiltonian in the dipole approximation will be

\[ \mathcal{H} = \hbar \omega_{\alpha\beta} A_{\alpha\alpha} + \hbar \omega_{1\beta} A_{11} + \hbar \omega_{2\beta} A_{22} - \{ (\vec{d}_{2\beta} A_{2\beta} + \vec{d}_{\alpha\alpha} A_{2\alpha}) \cdot \vec{E}_2 e^{-i\omega_2 t} + \text{H.c.} \}, \]  

(18)

where the counter rotating terms in the probe field have been dropped. The probe field is treated classically here and has a frequency \( \omega_2 \) and a complex amplitude \( \vec{E}_2 \). We use the master equation method to derive equations for the reduced density matrix of the atomic system. We give the result of such a calculation,

\[ \rho_{11} = -2\Gamma_1 \rho_{11}, \]  

(19a)

\[ \rho_{22} = -2\Gamma_2 \rho_{22} + i(G \rho_{22} + F \rho_{2\beta}) e^{-i\omega_2 t} - i(G^* \rho_{2\beta} + F^* \rho_{2\beta}) e^{i\omega_2 t}, \]  

(19b)

\[ \rho_{\alpha\alpha} = 2\gamma_{1\alpha} \rho_{1\alpha} + 2\gamma_{2\alpha} \rho_{2\alpha} - iG e^{-i\omega_2 t} \rho_{\alpha\alpha} + iG^* e^{i\omega_2 t} \rho_{\alpha\alpha}, \]  

(19c)

\[ \rho_{\alpha\alpha} = -(\Gamma_2 - i\omega_2) \rho_{\beta\beta} - iG^* e^{i\omega_2 t} \rho_{\beta\alpha} + iF^* e^{i\omega_2 t} (2\rho_{2\alpha} + \rho_{11} + \rho_{\alpha\alpha} - 1), \]  

(19d)

\[ \rho_{\beta\beta} = -\rho_{\alpha\alpha}, \]  

(19e)

\[ \rho_{\alpha\beta} = -\rho_{\beta\alpha}, \]  

(19f)

\[ \rho_{\beta\beta} = -\rho_{\alpha\alpha}, \]  

(19g)

\[ \rho_{\alpha\beta} = \eta_1 \rho_{11} + \eta_2 \rho_{22} - i\omega_{\alpha\beta} \rho_{\alpha\beta} - iF e^{-i\omega_2 t} \rho_{\alpha\beta} + iG^* e^{i\omega_2 t} \rho_{\beta\beta}, \]  

(19i)

where we have used the trace condition \( \sum_i \rho_{ii} = 1 \) for (19d). Here

\[ 2\gamma_{2\alpha} = \frac{4\omega_{2\alpha}^3 |d_{1\alpha}|^2}{3\hbar c^3} \quad \text{and} \quad 2\gamma_{2\beta} = \frac{4\omega_{2\beta}^3 |d_{1\beta}|^2}{3\hbar c^3} \]  

(20)

define the spontaneous emission rates from |2⟩ to states |\alpha⟩ and |\beta⟩ respectively and we write \( \Gamma_2 = \gamma_{2\alpha} + \gamma_{2\beta} \). The Rabi frequencies

\[ 2G = 2\vec{E}_2 \cdot \vec{d}_{2\alpha}/\hbar, \quad 2F = 2\vec{E}_2 \cdot \vec{d}_{2\beta}/\hbar \]  

(21)

are for the probe field acting on transitions |1⟩ ↔ |\alpha⟩ and |1⟩ ↔ |\beta⟩ respectively. Further we can write \( G = |G| e^{-i\phi_1} \) and \( F = |F| e^{-i\phi_2} \), where the phase \( \phi = \phi_1 - \phi_2 \) gives the relative phase between the complex dipole matrix elements \( d_{1\alpha} \) and \( d_{1\beta} \). The VIC parameters are

\[ \eta_1 = 2\sqrt{\gamma_{1\alpha} \gamma_{1\beta}} \cos \theta_1, \quad \eta_2 = 2\sqrt{\gamma_{2\alpha} \gamma_{2\beta}} \cos \theta_2. \]  

(22)

We thus include vacuum induced coherence on all possible transitions.

In order to study probe absorption we solve Eqs. (19) perturbatively. We need to know \( \rho_{22}(t) \) to second order in the probe field, assuming that the atom was prepared in the state |1⟩ at \( t = 0 \). Using (19b) we get

\[ \rho_{22}^{(2)}(t) = i \int_0^t d\tau e^{-i\omega_2 \tau} \{ |G| e^{-i\phi_1} \rho_{\alpha\alpha}^{(1)}(\tau) + |F| e^{-i\phi_2} \rho_{\beta\beta}^{(1)}(\tau) \} e^{-2\Gamma_2(t-\tau)} + \text{c.c.} \]  

(23)

The first order contribution is obtained, for example, by integrating (19d)

\[ \rho_{\alpha\alpha}^{(1)}(t) = -i \int_0^t d\tau e^{i\omega_2 \tau} \{ |F| e^{i\phi_2} \rho_{\alpha\alpha}^{(0)}(\tau) - |G| e^{i\phi_1} [\rho_{22}^{(0)}(\tau) - \rho_{\alpha\alpha}^{(0)}(\tau)] \} e^{-(\Gamma_2 - i\omega_2\tau)(t-\tau)}, \]  

(24)

It can be easily show that \( \rho_{22}^{(0)}(t) = 0 \) and the other zeroth order terms are known from Sec. I. The VIC contribution arises from non-zero \( \rho_{\alpha\alpha}^{(0)}(t) \) in (24). Similarly integrating for \( \rho_{\beta\beta}^{(1)}(t) \) and combining with Eqs. (24), (23), and on simplification we find our key results.
\[ \rho_{22}^{(2)} (t \gg \Gamma_1^{-1}, \Gamma_2^{-1}) = \frac{\eta |F|^2 |G|^2 e^{-i(\omega_{\alpha\beta} t + \phi)}}{(2\Gamma_1 - i\omega_{\alpha\beta})(2\Gamma_1 - i\omega_{\alpha\beta})(\Gamma_2 - i(\Delta_2 + \omega_{\alpha\beta}/2))} + \frac{\eta |F|^2 |G|^2 e^{i(\omega_{\alpha\beta} t + \phi)}}{(2\Gamma_2 + i\omega_{\alpha\beta})(2\Gamma_2 + i\omega_{\alpha\beta})(\Gamma_1 - i(\Delta_1 - \omega_{\alpha\beta}/2))} \]

\[ + \frac{|G|^2}{4\Gamma_2[\Gamma_2 - i(\Delta_2 - \omega_{\alpha\beta}/2)]} + \frac{|F|^2}{4\Gamma_2[\Gamma_2 - i(\Delta_2 + \omega_{\alpha\beta}/2)]} + \text{c.c.} \]  

(25)

When \( \eta_1 \to 0 \)

\[ \rho_{22}^{(2)} (t \gg \Gamma_1^{-1}, \Gamma_2^{-1}) = \frac{|G|^2}{4\Gamma_2[\Gamma_2 - i(\Delta_2 - \omega_{\alpha\beta}/2)]} + \frac{|F|^2}{4\Gamma_2[\Gamma_2 - i(\Delta_2 + \omega_{\alpha\beta}/2)]} + \text{c.c.} \]  

(26)

where the last result is the expected result which is the sum of the individual absorptions corresponding to the transitions |\( \alpha \rangle \to |2 \rangle |, |\beta \rangle \to |2 \rangle |. \) The parameter \( \Delta_2 = \omega_{\beta} - \omega_{\alpha\beta}/2 - \omega_2 \) is the probe detuning defined with respect to the center of level |\( \alpha \rangle | \) and |\( \beta \rangle |. \) The modulated term in probe absorption (25) is the result of VIC. This modulation is the signature of the VIC produced by the two paths of spontaneous emission \([1] \to |\alpha \rangle, |1 \rangle \to |\beta \rangle |. \) Note the interesting phase dependence that arises in the probe absorption due to non-zero \( \eta_1 \). This phase dependence is another outcome of the presence of VIC in a system. Since the probe is treated to second order in its amplitude, the result is independent of the coherence parameter \( \eta_2 \) for the transition |\( 2 \rangle \to |\alpha \rangle |, |\alpha \rangle \to |\beta \rangle |. \) Needless to say that the Eqs. (19) can be integrated numerically to obtain the probe absorption for arbitrary times. For this purpose it is useful to remove the optical frequencies by making the transformations \( \tilde{\rho}_{11} \equiv \rho_{11} e^{i\omega_{\alpha\beta}t}, \tilde{\rho}_{21} \equiv \rho_{21} e^{\omega_{\alpha\beta}t} (i = \alpha, \beta) \) and \( \tilde{\rho}_{12} \equiv \rho_{12} e^{i(\omega_{\alpha\beta} - \omega_2)t} \) etc. We solve these using fifth-order Runge-Kutta-Verner method with the initial condition that \( \rho_{11}(0) = 1 \). We take the probe Rabi frequencies \( F, G \) much smaller than \( \Gamma \)'s. The numerical results for excited state population \( \rho_{22}(t) \) as a function of time for both the cases when \( \eta_1 \) is zero and non-zero are plotted in Fig. 3. Figure 3 shows the significant difference that arises due to the presence or absence of VIC. The oscillation in the probe absorption is the reflection of oscillation of in the coherence \( \rho_{\alpha\beta} \) (see (27)) and this confirms the analytical result (25). The numerical result shows a very slow decay of the envelop of the oscillations. This arises from terms which are of higher order in probe strength.

Finally we discuss the changes in absorption spectrum that can arise due to VIC. The modulated component of the population (25) can be written as

\[ \rho_{22}^{(2)} = \frac{2\eta |F||G|}{D} \left[\begin{array}{c}
2\Gamma_1 \Gamma_2^2 + 2\Gamma_1(\Delta_2^2 - \omega_{\alpha\beta}^2/4) - \Gamma_2 \omega_{\alpha\beta}^2 \cos(\omega_{\alpha\beta} t + \phi) \\
\omega_{\alpha\beta} \{2\Gamma_1 \Gamma_2 + \Gamma_2^2 + \Delta_2^2 - \omega_{\alpha\beta}^2/4 \} \sin(\omega_{\alpha\beta} t + \phi)\end{array}\right], \]

(27)

where

\[ D = (4\Gamma_1^2 + \omega_{\alpha\beta}^2)[\Gamma_2^2 + (\Delta_2 + \omega_{\alpha\beta}/2)^2][\Gamma_2^2 + (\Delta_2 - \omega_{\alpha\beta}/2)^2]. \]

Since it is possible to separate the sine and cosine terms by a phase sensitive detection we plot these in Fig. 4 as a function of probe detuning. These two components of the absorption spectrum behave quite differently.

V. CONCLUSIONS

The important criteria for the existence of the vacuum induced coherence between the close lying levels is the nonorthogonality of the dipole matrix elements. In practice the nonorthogonality can be achieved by mixing of the energy levels. The mixing can occur either due to internal fields or due to externally applied fields. For example in the experiment of Xia et al. 11, spin-orbit interaction gives rise to mixing. The VIC has also been studied when the level mixing is produced by using electromagnetic fields [17], dc fields [18], and rf fields [19]. Special configurations involving cavities can also be utilized to study VIC [20].

In conclusion we have found that the VIC in a Λ-system is more difficult to monitor as it does not show up in the fluorescence spectrum. We have however demonstrated that the absorption spectrum carries the information on VIC and that the VIC produces a modulated component in the absorption spectrum.
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FIG. 1. Schematic diagram of a three level Λ system. The two ground states $|\alpha\rangle$ and $|\beta\rangle$ are coupled to the excited state $|1\rangle$ via vacuum field.

FIG. 2. Schematic diagram of a four level model proposed for monitoring vacuum induced coherence. The coherence created after spontaneous emission from $|1\rangle$ can be observed in the probe absorption.
FIG. 3. The excited state population $\rho_{22}$ as a function of scaled time $\gamma t$. Here the probe field is tuned to the center of states $|\alpha\rangle$ and $|\beta\rangle$, and we take $\gamma_{1\alpha} = \gamma_{1\beta} = \gamma_{2\alpha} = \gamma_{2\beta} = \gamma$. The parameters are chosen as $F/\gamma = G/\gamma = 0.1$ and $\omega_{\alpha\beta}/\gamma = 2$, and phase $\phi = 0$. The dashed oscillating curve is in the presence of VIC and the solid curve in the absence of VIC. This observed modulation is consistent with the analytical result (25). Very weak oscillation appears in the solid line because the probe is not exactly tuned to the two transitions.
FIG. 4. The cosine (dashed) and sine (solid) components of the excited state population $\rho^{(2)}_{22} \times 10^2$ as a function of probe detuning. Plot (a) is for $\omega_{\alpha\beta} = 2\gamma$ and (b) is for $\omega_{\alpha\beta} = 5\gamma$. The parameters are as in Fig. (3).