Solving fully fuzzy linear systems by Gauss Jordan Elimination Method

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ABSTRACT—In this paper, we discuss fully fuzzy linear systems with triangular fuzzy numbers. A Gauss Jordan Elimination Method is proposed for solving fully fuzzy linear systems (FFLS). We used elementary row operations augmented matrices of crisp linear system of equation to arrive the row reduced form. The method in detail is discussed and illustrated by a numerical example.

Keywords: Fully fuzzy linear systems, triangular fuzzy numbers, elementary row operations

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1. Introduction

Linear system of equations are vital for contemplating and comprehending an expansive extent of the issues in connected science, ordinarily is numerous applications a portion of the parameters in our issues are spoken to fuzzy numbers Zadeh [17] presented the ideas of fuzzy numbers and fuzzy math, fuzzy numbers number-crunching is connected and valuable in calculation of direct frameworks.

Duba's and Prade [8] proposed two meaning of fuzzy straight arrangement of conditions. Buckley and Qu [4] broadened a few techniques for tackling completely fuzzy direct arrangement of condition. Friedman et al. [10] proposed a general model for comprehending a fuzzy straight framework whose coefficient lattice is fresh and the correct hand side section is a discretionary fuzzy vector. Numerous specialists [2,3,5] presented a few calculations for unraveling completely fuzzy straight frameworks [2,3] proposed arrangement of a fuzzy direct framework by utilizing alterative Gauss Jacobin and Gauss Seidel techniques. Dehghan et al. [5] proposed some new strategies for illuminating completely fuzzy straight frameworks based a the cramers rule Gaussian disposal and LU decay strategy from direct polynomial math and direct programming Dehghan et al. [6] broadened the Adomian Decomposition technique [3] to locate the positive fuzzy vector arrangement of completely fluffy direct framework. Abbas Banday et al. [1] utilized LU disintegration strategy for fathoming fuzzy straight arrangement of condition when the coefficient network is certain positive. Muzzioli et al. [13] grew completely fuzzy straight arrangement of the frame \( A_1x + b_1 = A_2x + b_2 \) where \( A_1, A_2 \) are square grids of fluffy coefficients, \( b_1, b_2 \) are fuzzy numbers.

Mosleh et al. [12] acquainted a technique with discover the arrangement of completely fuzzy direct arrangement of conditions of the frame \( Ax + b = Cx + d \) where \( A, C \) are square frameworks of fuzzy coefficients and \( b, d \) are fuzzy number vectors Nasseri et al. [15] utilized a specific disintegration strategies for the coefficient framework for illuminating completely fuzzy straight arrangement of conditions. Nasseri et al. [16] proposed Huang technique for registering a non negative arrangement of the completely fuzzy straight arrangement of conditions Nasseri et al. [14] presented a direct arrangement of conditions with trapezoidal fuzzy numbers as an expansion of the fuzzy straight framework.

In this paper, our point is to solve \( \hat{A} \otimes \hat{x} = \hat{b} \) where \( \hat{A} \) is a fuzzy lattice and \( \hat{x} \) and \( \hat{b} \) are fuzzy vectors (of triangular fuzzy numbers) with proper sizes.
We sorted out this paper as pursues; In segment 2 we first give some fundamental ideas of fuzzy set hypothesis and some essential definitions on fuzzy numbers are checked on. In section 3, we characterize completely fuzzy direct frameworks of conditions and Gauss Jordan Elimination strategy comprehending completely fuzzy straight frameworks (FFLS). In section 4, numerical precedent is given to analyze a proposed technique. We close in area 5.

2. PRELIMINARIES
In these sections, some basic definitions are reviewed [7, 11].

Definition 2.1
A fuzzy subset $A$ of $R$ is characterized by its participation work: $\mu_A : R \rightarrow [0,1]$ which assigns out a real number $\mu_A$ in the interim $[0,1]$, to every component $x \in R$ where the estimation of $\mu_A$ at $x$ demonstrates the review enrollment of $x$ in $A$.

Definition 2.2
A fuzzy set $A$ characterized on the widespread arrangement of real number $R$, is said to be a fuzzy number if its enrollment work has the accompanying qualities.
(i) $A$ is convex (ie) $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \min\{\mu_A(x_1), \mu_A(x_2)\}$ $\forall x_1, x_2 \in R$, $\forall \lambda \in [0,1]$
(ii) $A$ is normal (ie) there exist $x_0 \in R$ such that $\mu_A(x_0) = 1$
(iii) $\mu_A$ is piecewise continuous.

Definition 2.3
A fuzzy number $A$ is said to be non-negative fuzzy number if and only if $\mu_A(x) = 0$ for all $x < 0$.

Definition 2.4
A fuzzy number $A = (p,q,r)$ is said to be Triangular fuzzy number, if its membership function has the following forms:

$$\mu_A(x) = \begin{cases} 1 - \frac{p-x}{q}, & p - q \leq x \leq p, q > 0 \\ 1 - \frac{x-p}{r}, & p \leq x \leq p + r, r > 0 \\ 0, & \text{otherwise} \end{cases}$$

Definition 2.5
A fuzzy number $A = (p, q, r)$ is said to be Triangular fuzzy number, if its enrollment work has the accompanying structures: (i.e.) $A \geq 0$ if and only if $p - q \geq 0$.

Definition 2.6
A triangular fuzzy number $A = (p, q, r)$ is said to be zero triangular fuzzy number if and only if $p = 0, q = 0, r = 0$. 
Definition 2.7

Two fuzzy numbers $\tilde{A} = (p, q, r)$ and $\tilde{B} = (s, m, n)$ are said to be equal if and only if $p = s$, $q = m$, and $r = n$.

Definition 2.8

Arithmetic operations an triangular fuzzy numbers

Numerous creators have surveyed the essential tasks a triangular fuzzy numbers. In this subsection, we explored the tasks a triangular fuzzy numbers [7].

Let $\tilde{A} = (p, q, r)$ and $\tilde{B} = (s, m, n)$ be two triangular fuzzy numbers.

(i) Addition: $\tilde{A} \oplus \tilde{B} = (p, q, r) \oplus (s, m, n) = (p + s, q + m, r + n)$

(ii) Subtraction: $\tilde{A} \ominus \tilde{B} = (p, q, r) \ominus (s, m, n) = (p - s, q - m, r - n)$

(iii) Multiplication: If $\tilde{A} \geq 0$ and $\tilde{B} \geq 0$ then

$\tilde{A} \Theta \tilde{B} = (p, q, r) \Theta (s, m, n) = (ps, pm + sq, pn + sr)$

(iv) Scalar Multiplication: Let $\lambda$ be scalar then

$\lambda \odot \tilde{A} = \lambda \odot (p, q, r) = \begin{cases} (\lambda p, \lambda q, \lambda r), & \lambda \geq 0 \\ (-\lambda p, -\lambda q, -\lambda r), & \lambda < 0 \end{cases}$

Definition 2.9

A matrix $\tilde{A} = (a_{ij})$ is known as a fuzzy grid if every component of $\tilde{A}$ is a fuzzy number. A fuzzy network $\tilde{A}$ will be certain and signified by $\tilde{A} \succ 0$, if every component of $\tilde{A}$ is sure. We speak to $n \times n$ fuzzy grid $\tilde{A} = (a_{ij})_{n \times n}$ such an extent that $\tilde{a} = (a_{ij}, q_{ij}, r_{ij})$ with the new documentation $\tilde{A} = (A, M, N)$ where $A = (a_{ij})$, $M = (q_{ij})$ and $N = (r_{ij})$ are three $n \times n$ fresh networks.

3. Proposed Method

Consider the $n \times n$ fuzzy linear system of equations

$$(\tilde{a}_{i1} \odot \tilde{x}_1) \oplus (\tilde{a}_{i2} \odot \tilde{x}_2) \oplus \ldots \oplus (\tilde{a}_{in} \odot \tilde{x}_n) = \tilde{b}_1$$

$$(\tilde{a}_{21} \odot \tilde{x}_1) \oplus (\tilde{a}_{22} \odot \tilde{x}_2) \oplus \ldots \oplus (\tilde{a}_{2n} \odot \tilde{x}_n) = \tilde{b}_2$$

$$\vdots$$

$$(\tilde{a}_{ni} \odot \tilde{x}_1) \oplus (\tilde{a}_{nj} \odot \tilde{x}_2) \oplus \ldots \oplus (\tilde{a}_{nn} \odot \tilde{x}_n) = \tilde{b}_n$$

The matrix of the above equation is

$$\tilde{A} \odot \tilde{x} = \tilde{b}$$

where the coefficient matrix $\tilde{A} = (\tilde{a}_{ij})$, $1 \leq i, j \leq n$ is a $n \times n$ fuzzy matrix and $\tilde{x}, \tilde{b} \in F(R)$. This system is called a fully fuzzy linear system (FFLS).

In this paper, we obtain a positive solution of fully fuzzy linear systems $\tilde{A} \odot \tilde{x} = \tilde{b}$.

Consider $\tilde{A} = (A, M, N) \geq 0$, $\tilde{x} = (x, y, z) \geq 0$
and \( \tilde{b} = (b, h, g) \geq 0 \)

The FFLS \( \tilde{A} \otimes \tilde{x} = \tilde{b} \) can be written as

\[
(A, M, N) \otimes (x, y, z) = (b, h, g)
\]

using definition 2.8,

\[
(Ax, Ay + Mx, Az + Nx) = (b, h, g)
\]

using definition 2.7,

\[
\begin{align*}
Ax &= b \\
Ay + Mx &= h \\
Az + Nx &= g
\end{align*}
\] ... (2.1)

Our proposed method is a modification of Gauss Elimination method called Gauss Jordan Method.

In this strategy, the coefficient grid \( A \) of the fresh direct arrangement of condition \( Ax = b \), \( Ay + Mx = h \) and \( Az + Nx = g \) is conveyed to a unit lattice by making the enlarged network upper triangular as well as lower triangular (i.e.) by making all components above and beneath the main corner to corner of expanded framework as zeros.

Step 1: Construct the increased lattices \((A, b),(A, h-Mx)\) and \((A, g-Nx)\) from condition 2.1.

Step 2: Compute the estimations of \( x_i, y_i \) and \( z_i \) by utilizing the basic row tasks.

Step 3: The expanded networks \((A, b),(A, h-Mx)\) and \((A, g-Nx)\) diminished to push decreased frame.

Step 4: The arrangement of fully fluffy straight framework (FFLS) will be spoken to by

\[
\bar{x}_i = (x_i, y_i, z_i) \quad \text{for all } i = 1, 2, ..., n.
\]

4. Numerical Example

Consider the following FFLS (taken from [5]) and solve it by proposed method.

\[
\begin{align*}
(6, 1, 4) \otimes (x_1, y_1, z_1) \oplus (5, 2, 2) \otimes (x_2, y_2, z_2) \oplus (3, 2, 1) \otimes (x_3, y_3, z_3) &= (58, 30, 60) \\
(12, 8, 20) \otimes (x_1, y_1, z_1) \oplus (14, 12, 15) \otimes (x_2, y_2, z_2) \oplus (8, 8, 10) \otimes (x_3, y_3, z_3) &= (142, 139, 257) \\
(24, 10, 34) \otimes (x_1, y_1, z_1) \oplus (32, 30, 30) \otimes (x_2, y_2, z_2) \oplus (20, 19, 24) \otimes (x_3, y_3, z_3) &= (316, 297, 514)
\end{align*}
\]

Solution

The given FFLS may be written as

\[
A = \begin{pmatrix}
6 & 5 & 3 \\
12 & 14 & 8 \\
24 & 32 & 20
\end{pmatrix}, \quad M = \begin{pmatrix}
1 & 2 & 2 \\
8 & 12 & 8 \\
10 & 30 & 19
\end{pmatrix}, \quad N = \begin{pmatrix}
4 & 2 & 1 \\
20 & 15 & 10 \\
34 & 30 & 24
\end{pmatrix}
\]
\[ b = \begin{pmatrix} 58 \\ 142 \\ 316 \end{pmatrix}, h = \begin{pmatrix} 30 \\ 139 \\ 297 \end{pmatrix}, g = \begin{pmatrix} 60 \\ 257 \\ 514 \end{pmatrix} \]

The augmented matrix
\[
(A, b) = \begin{pmatrix}
6 & 5 & 3 & 58 \\
12 & 14 & 8 & 142 \\
24 & 32 & 20 & 316
\end{pmatrix}
\]

Applying elementary row operations on matrix \((A, b)\)

First \(R_1 \rightarrow \frac{R_1}{6}\), we get

\[
\begin{pmatrix}
1 & \frac{5}{6} & \frac{3}{6} & \frac{58}{6} \\
12 & 14 & 8 & 142 \\
24 & 32 & 20 & 316
\end{pmatrix}
\]

Again we apply elementary operations in sequence

\(R_2 \rightarrow R_2 - 12 R_1, R_3 \rightarrow R_3 - 24 R_1, R_2 \rightarrow \frac{R_2}{4}, R_3 \rightarrow R_3 - 12 R_2,\)

\(R_3 \rightarrow \frac{R_3}{2}, R_2 \rightarrow R_2 - \frac{1}{2} R_3, R_1 \rightarrow R_1 - \frac{5}{6} R_2, R_1 \rightarrow R_1 - \frac{1}{2} R_3\)

Finally, we get

\[
\begin{pmatrix}
1 & 0 & 0 & 4 \\
0 & 1 & 0 & 5 \\
0 & 0 & 1 & 3
\end{pmatrix}
\]

From this row reduced form of augmented Matrix \((A, b)\), we have \(x_1 = 4, x_2 = 5, x_3 = 3\).

The augmented matrix
\[
(A, h - Mx) = \begin{pmatrix} 6 & 5 & 3 & 10 \\ 12 & 14 & 8 & 23 \\ 24 & 32 & 20 & 50 \end{pmatrix}
\]

Similarly, applying elementary row operations on Matrix \((A, h - Mx)\) in sequence

\(R_1 \rightarrow \frac{R_1}{6}, R_2 \rightarrow R_2 - 12 R_1, R_3 \rightarrow R_3 - 24 R_1, R_2 \rightarrow \frac{R_2}{4}, R_3 \rightarrow R_3 - 12 R_2,\)

\(R_3 \rightarrow \frac{R_3}{2}, R_2 \rightarrow R_2 - \frac{R_2}{2}, R_1 \rightarrow R_1 - \frac{5}{6} R_2, R_1 \rightarrow R_1 - \frac{R_3}{2}\)
Finally, we get:

\[
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & \frac{1}{2} \\
0 & 0 & 1 & \frac{1}{2}
\end{pmatrix}
\]

From this row reduced form of augmented matrix \((A, h - Mx)\), we have

\[y_1 = 1, \ y_2 = \frac{1}{2}, \ y_3 = \frac{1}{2} \]

Similarly, applying elementary row operations on augmented matrix

\[
(A, g - Nx) = \begin{pmatrix}
6 & 5 & 3 & 31 \\
12 & 14 & 8 & 72 \\
24 & 32 & 20 & 156
\end{pmatrix}
\]

Finally, we get:

\[
\begin{pmatrix}
1 & 0 & 0 & 3 \\
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 1
\end{pmatrix}
\]

From this row reduced form of augmented matrix \((A, g - Nx)\), we have

\[z_1 = 3, \ z_2 = 2, \ z_3 = 1 \]

Substituting the values of \(x_i, y_i, z_i\) where \(i = 1, 2, \ldots, n\) in the FFLS solution \(\tilde{x}_i = (x_i, y_i, z_i)\) for all \(i = 1, 2, \ldots, n\) we get

\[
\tilde{x}_1 = (x_1, y_1, z_1) = (4, 1, 3) \\
\tilde{x}_2 = (x_2, y_2, z_2) = (5, 1/2, 2) \\
\text{and} \quad \tilde{x}_3 = (x_3, y_3, z_3) = (3, 1/2, 1)
\]

We have the same solution with this method as the system given in [5].

5. Conclusion

In this paper, a new methodology is applied to find the solution of fully fuzzy linear system in the form of triangular fuzzy matrices. We used elementary row operations to get row reduced form. We examined are method by solving three fully fuzzy linear systems, which was used in [5]. Our proposed method is easy to determine the solution of FFLS.

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