Effective Meson Lagrangian with Chiral and Heavy Quark Symmetries from Quark Flavor Dynamics

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Abstract

By bosonization of an extended NJL model we derive an effective meson theory which describes the interplay between chiral symmetry and heavy quark dynamics. This effective theory is worked out in the low-energy regime using the gradient expansion. The resulting effective lagrangian describes strong and weak interactions of heavy $B$ and $D$ mesons with pseudoscalar Goldstone bosons and light vector and axial–vector mesons. Heavy meson weak decay constants, coupling constants and the Isgur–Wise function are predicted in terms of the model parameters partially fixed from the light quark sector. Explicit $SU(3)_F$ symmetry breaking effects are estimated and, if possible, confronted with experiment.

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1 Introduction

In the framework of the Standard Model, Quantum Chromodynamics (QCD) is the theory of strong interaction. However, due to the complicated nature of QCD, hadrons are usually described by means of phenomenological effective lagrangians. It is a big challenge to derive these effective lagrangians describing the dynamics of strongly interacting light and heavy hadrons directly from QCD by using suitable non–perturbative hadronization techniques. Given the complexity of QCD arising from the self–interactions of non–Abelian gauge bosons, such a program evidently requires (possibly crude) approximations on the long way from QCD down to the effective hadron theory. Simplifications arise, however, since we are naturally restricted to the low-energy region. Therefore, from the practical point of view, it is sufficient to find an approximation to QCD which mimics the essential features of low-energy quark flavor dynamics. This could be achieved, at least principally, in two alternative ways: either, by considering quark interactions mediated through non–perturbative gluon propagators or by integrating out the high–energy components of quarks and gluons. Clearly, the possible form of such effective quark lagrangians must be restricted by the underlying symmetries of QCD, which should be viewed as a guide to find tractable models of quark flavor dynamics.

In the sector of light quark flavors \( q = (u, d, s) \), QCD possesses an approximate \( SU(3)_L \times SU(3)_R \) chiral symmetry which is spontaneously broken to \( SU(3)_V \), leading to the emergence of (pseudo)Goldstone bosons \( \pi, K, \eta \), which receive their masses by the explicit breaking of chiral symmetry through current quark masses. As is well known from current algebra and low energy theorems, chiral symmetry alone almost entirely determines the flavor dynamics of light mesons without need of a detailed knowledge of the underlying gluon dynamics. This suggests that a model for quark flavor dynamics which takes into account spontaneous and explicit breaking of chiral symmetry could lead to a realistic effective hadron theory. Of course, writing down such effective quark lagrangians in general would include non–renormalizable operators with dimensionful coupling constants connected with an intrinsic scale \( \Lambda \) above which the model has to be replaced by the full theory. In our case this scale is naturally expected to coincide with the scale of chiral symmetry breaking \( \Lambda_{\chi SB} \), separating the perturbative region of QCD from the non–perturbative domain.

In the past the Nambu–Jona–Lasionio (NJL) model, originally formulated for strongly interacting nucleons [1], has been successfully used to describe the low–energy light flavor dynamics of QCD. Bosonization of this model combined with a gradient expansion leads to an effective meson theory, which gives a surprisingly realistic description of light pseudoscalar, vector and axial–vector mesons [2, 3]. (Related work on bosonization of approximate QCD and effective quark models can also be found in [4].) The success of the NJL model stems mainly from its global chiral invariance (for vanishing current masses) which is, however, spontaneously broken in the ground state. As a consequence of
chiral symmetry the resulting effective meson lagrangian embodies the soft–pion theorems, Goldberger–Treiman and KSFR relations, vector dominance and the integrated chiral anomaly. In addition, the bosonized NJL model also provides a natural explanation of how baryons can emerge as composite quark–diquark states \[ \text{or, in the case of large number of colors } N_c \to \infty, \text{ as chiral solitons} \] (For a recent review on these subjects see \[ \text{and references therein.} \] While the concept of chiral symmetry and its spontaneous breaking has been proven extremely successful in understanding the light quark flavor hadrons, it has to be abandoned for heavy quark flavors which badly break chiral symmetry. Recently new important symmetries have been discovered for heavy quark flavors \[ Q = b, c, \ldots \] which considerably simplify the description of heavy–light \( (Q\bar{q})\)–mesons. These symmetries, which are not manifest in full QCD, arise in the limit of infinite heavy quark masses \( m_Q \to \infty \). This limit \( \Lambda \ll m_Q \) is somehow complementary to the case \( m_q \ll \Lambda \) in the light quark sector. A systematic expansion of QCD in inverse powers of the heavy quark mass \( 1/m_Q \) can be formulated in the framework of Heavy Quark Effective Theory (HQET) \[ \text{.} \] The relevant degrees of freedom for a heavy quark are then given by

\[ Q_v(x) = \frac{1+i}{2} e^{i m_Q v \cdot x} Q(x), \] (1)

where \( Q(x) \) is the heavy quark field in the full theory and \( v_\mu \) is the velocity of the heavy quark with \( v^2 = 1 \). The irrelevant (‘small’ spinor) degrees of freedom are integrated out defining an effective lagrangian for the ‘large’ spinor components \( Q_v \). This effective lagrangian can be expanded into a \( 1/m_Q \)–series of local operators, where the leading term reads

\[ \mathcal{L}^{HQET} = \mathcal{L}_v (i v \cdot D) Q_v + O(1/m_Q), \] (2)

and \( D^\mu \) is the covariant derivative of QCD. The propagator of the field \( Q_v(x) \) in momentum space is simply \( (v \cdot k + i \epsilon)^{-1} \) where \( k_\mu \) describes the residual momentum of the heavy quark such that its total momentum reads \( p^\mu = m_Q v^\mu + k^\mu \). The leading term in the lagrangian \( \mathcal{L}^{HQET} \) manifestly shows two new symmetries. Spin symmetry is due to the fact, that the coupling of the spin to the color–magnetic field is as usual a \( 1/m_Q \)–effect and consequently gluons are blind to the spin of the heavy quark in the limit \( m_Q \to \infty \). This leads to a mass degeneracy for hadrons that differ only in the spin of the heavy quark. For example the masses of \( B(5280) \) and \( B^*(5325) \) agree within 2%, those of \( D(1870) \) and \( D^*(2010) \) within 8%. As the leading term in (2) is independent of the heavy mass \( m_Q \), an additional heavy flavor symmetry arises. Both symmetries relate several form factors of physical matrix elements between QCD bound states containing one heavy quark like \( B, D \) mesons and \( \Lambda_b, \Lambda_c \) baryons. HQET is then the most powerful tool for determining the parameters of the standard model in the heavy quark sector, namely the CKM–matrix elements with heavy
quarks, in a model independent way. Here fruitful information can be obtained from weak semileptonic decays of heavy hadrons.

In transitions between two heavy mesons the most drastic simplification gives rise to a unique form factor, the Isgur–Wise function \( \xi(\omega) \) \[^9\]

\[
\langle H_v | \overline{Q} v' \Gamma Q | H_v \rangle = \xi(v \cdot v') \text{tr} \left[ \overline{H}_v \Gamma H_v \right],
\]

where the \( H_v \) are the matrix representations for the heavy mesons containing one heavy quark with velocity \( v \). Also the normalization in the heavy quark limit is known: \( \xi(v \cdot v' = 1) = 1 \). This is crucial for determining the CKM–matrix element \( V_{cb} \) from semileptonic \( b \rightarrow c \) transitions. The decays of heavy mesons into light mesons are likewise simplified in the heavy quark mass limit, and one can derive new relations between form factors \[^10\].

HQET is still defined in terms of quark and gluon degrees of freedom. For practical applications it would, however, be desirable to reformulate it in terms of hadronic degrees of freedom, which unfortunately cannot exactly be accomplished. Stimulated by the success of the NJL model for light quark flavors, we will extend this model to heavy quark flavors. One might expect that the NJL model cannot be applied to heavy quark flavors, for which the heavy quark mass is larger than the cut–off \( \Lambda \approx 1 \text{ GeV} \), as estimated from the light quark sector. However, in HQET the heavy quark ‘on–shell’ momentum \( m_Q v_\mu \) has already been subtracted such that one is left with the residual momentum \( k_\mu = p_\mu - m_Q v_\mu \). Indeed, the residual momentum of the heavy quark in a hadron containing a single heavy quark arises entirely from its interaction with the light degrees of freedom (light quark flavors and gluons) and is thus of the same order as the momenta of the light quarks. Therefore in an effective quark model \( k_\mu \) should be cut off at the same scale \( \Lambda \). This is the NJL model we are using in the present paper. We will bosonize this model in both the light-light and heavy-light sector. The resulting effective meson theory which describes the interplay between chiral symmetry and heavy quark symmetry, is studied in a low energy (gradient) expansion. Thereby we reproduce basically the effective meson lagrangians with chiral and heavy quark symmetry introduced previously on phenomenological grounds \[^11\], where, however, all the expansion parameters are now microscopically determined by the quark interaction strength, the quark masses and the cut-off.

The organization of the paper is as follows: In section 2 we define the extended NJL model with heavy quark flavor and spin symmetry. In the subsequent section this model is bosonized. The resulting effective lagrangian then describes the interactions of composite light pseudoscalar, vector and axial–vector mesons with heavy mesons organized in \((0^-, 1^-)\) respectively \((0^+, 1^+)\) spin–symmetry doublets. Including electroweak currents in the generating functional further determines the weak decay constants of heavy mesons and the Isgur–Wise function in terms of the model parameters. Section 4 is devoted to a numerical discussion of our results. By determining the heavy–light four–quark coupling
constant from heavy meson mass relations we obtain predictions for weak decay constants \( f_B, f_D \) and axial and vector couplings between light and heavy mesons, including a detailed estimate of \( SU(3)_F \) breaking effects. Finally, the slope of the Isgur–Wise function is estimated and confronted with actual experimental fits. A short summary and some concluding remarks are given in section 5. Furthermore some mathematical details are relegated to appendices.

2 Extended NJL Model

2.1 The quark lagrangian

In the extended NJL model under consideration we add to the free lagrangian

\[
\mathcal{L}_0 = \bar{\psi}(i\partial - \hat{m}_0)q + \mathcal{Q}_v(i\bar{\nu} \cdot \partial)Q_v
\]
a four–quark interaction–term which is motivated by the general quark–current structure of QCD\(^1\)

\[
\mathcal{L}_{\text{int}} = -\frac{\kappa}{2} \left( \bar{\psi} \gamma_\mu \frac{\lambda^a}{2} \psi \right) \left( \bar{\psi} \gamma^\mu \frac{\lambda^a}{2} \psi \right).
\]

Here, light quarks \((q = (u, d, s)^T)\) and heavy quarks \((Q_v = b, c)\) which are combined in \(\psi = (q, Q_v)^T\) are coupled through a universal coupling constant \(\kappa\) of dimension \((\text{mass})^{-2}\), and \(\lambda^a\) are \(SU(N_c)\)–color matrices. Besides a global \(SU(N_c)\) symmetry the lagrangian \(\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}}\) has the chiral \(SU(3)_L \times SU(3)_R\) symmetry of QCD for vanishing light current masses \(\hat{m}_0 = \text{diag}(m_u^0, m_d^0, m_s^0)\).

The symmetries of HQET, \(SU(2)_{\text{spin}} \times SU(2)_{\text{flavor}}\), are as well included in our model, since the interaction is independent of the heavy quark mass and spin (note \(\overline{Q_v} \gamma_\mu Q_v = \overline{Q_v} \gamma_\mu Q_v\)).

The lagrangian \((4)\) separates into a light–light part (denoted by \(\mathcal{L}^{ll}_{\text{int}}\)), a heavy–light one \((\mathcal{L}^{hl}_{\text{int}})\) and a heavy–heavy part \((\mathcal{L}^{hh}_{\text{int}})\)

\[
\mathcal{L}_{\text{int}} = \mathcal{L}^{ll}_{\text{int}} + \mathcal{L}^{hl}_{\text{int}} + \mathcal{L}^{hh}_{\text{int}}.
\]

In the following \(\mathcal{L}^{hh}_{\text{int}}\) is discarded. The interaction \((4)\) acts in the color \((N_c^2 - 1)\)–plet \((\overline{\psi}\psi)\)–channel. For subsequent considerations it is convenient to Fierz–rearrange this interaction into the physical relevant (attractive) color–singlet channel. Defining a coupling constant \(G_1 = \kappa(1 - 1/N_c)/4\) and using \(SU(3)_F\) matrices \(\lambda^a_F\) and \(\lambda^b_F = \sqrt{2/3} \mathbf{1}_F\) with \(\text{tr}_F \left[ \lambda^a_F \lambda^b_F \right] = 2\delta^{ab}\), one obtains

\[
\mathcal{L}^{ll}_{\text{int}} = 2G_1 \left( \frac{\lambda^a}{2} q \right)^2 + \left( \frac{\bar{\psi} \gamma_5 \lambda^a}{2} q \right)^2.
\]

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1 Summation over repeated color, flavor and Lorentz indices is understood.

2 We will not consider bound states of a top quark as corresponding life–times are expected to be too short.
\[-G_1 \left( \left( \overline{\psi} \gamma_\mu \frac{\lambda^a}{2} \psi \right)^2 + \left( \overline{\psi} \gamma^\mu \gamma_5 \frac{\lambda^a}{2} \psi \right)^2 \right) \right], \quad (5)

\[ \mathcal{L}_{int}^{hl} = G_1 \left( (\overline{Q}_v i \gamma_5 q)(\overline{\psi} \gamma_5 Q_v) - (\overline{Q}_v \gamma_\mu q) P^\perp_{\mu \nu} (\overline{\psi} \gamma^\nu Q_v) \right) + G_1 \left( (\overline{Q}_v q)(\overline{\psi} Q_v) - (\overline{Q}_v i \gamma_\mu \gamma_5 q) P^\perp_{\mu \nu} (\overline{\psi} \gamma^\nu Q_v) \right), \quad (6)\]

where additional terms contributing to diquark channels are subleading in $1/N_c$ and have been discarded. In order to obtain expression (4) we have decomposed the interaction terms into longitudinal and transversal parts by means of projection operators

\[ P_{\parallel \mu \nu} = v_\mu v_\nu, \quad P_{\perp \mu \nu} = g_{\mu \nu} - v_\mu v_\nu, \]

leading to the following identities

\[ (\overline{Q}_v \gamma_\mu q)(\overline{\psi} \gamma^\mu Q_v) = (\overline{Q}_v q)(\overline{\psi} Q_v) + (\overline{Q}_v \gamma_\mu q) P^\perp_{\mu \nu} (\overline{\psi} \gamma^\nu Q_v), \quad (7)\]

\[ (\overline{Q}_v i \gamma_\mu \gamma_5 q)(\overline{\psi} \gamma_5 \gamma_\mu Q_v) = (\overline{Q}_v i \gamma_5 q)(\overline{\psi} \gamma_5 Q_v) + (\overline{Q}_v i \gamma_\mu \gamma_5 q) P^\perp_{\mu \nu} (\overline{\psi} \gamma_5 \gamma^\nu Q_v). \quad (8)\]

Note that the longitudinal components of vector respectively axial vector currents can be rewritten by means of $\overline{p} Q_v = Q_v$ in the form of scalar respectively pseudoscalar currents. This has been exploited in eqs. (7), (8) to organize heavy pseudoscalar and vector (respectively scalar and axial vector) interaction channels in symmetry doublets of HQET spin symmetry, occurring with the same interaction strength $G_1$.

### 2.2 Generating functional for Greens functions of quark currents

The generating functional for Greens functions of quark bilinears in terms of our model lagrangian is given by the path integral

\[ Z(\eta) = \int \mathcal{D} \psi \mathcal{D} \overline{\psi} e^{\int d^4x \left( L_0(\psi) + L_{int}^{hl} + L_{source}(\eta) \right)}, \quad (9)\]

where we included a term $L_{source}(\eta)$ containing sources coupled to weak heavy–light and heavy–heavy quark currents of the following type

\[ L_{source}(\eta) = \eta^\dagger (\overline{\psi} \Gamma Q_v) + h.c. \]

\[ + \eta^\dagger_{\mu \nu} (\overline{Q}_v \Gamma Q_v) + h.c., \quad (10)\]

where $\Gamma$ is a suitable combination of Dirac matrices.

Expectation values involving the mesonic bound states are obtained as usual by differentiating with respect to additionally introduced mesonic sources and amputating external meson poles.

Following the standard path integral bosonization procedure \[2, 3, 12\], we introduce color singlet composite $(q\overline{\psi})$– and $(\overline{Q} Q_v)$–meson fields in such a way
that the action in (9) becomes bilinear in the quark fields and the latter can be integrated out.

In the light sector we have scalar \((s = s_a \lambda^a F/2)\), pseudoscalar \((p = p_a \lambda^a F/2)\), vector \((v_\mu = v^a_\mu \lambda^a F/2)\) and axial–vector \((a_\mu = a^a_\mu \lambda^a F/2)\) fields.

In the heavy–light sector (which we will refer to as heavy in the following) the vector and axial vector fields, \(\phi_\mu\) and \(\phi^5_\mu\), satisfy the constraints 
\[ P_\perp \mu\nu \phi_\nu = \phi_\mu, \]
\[ P_\perp \mu\nu \phi^5_\nu = \phi^5_\mu, \]
being equivalent to the transversality condition \(v_\mu \phi_\mu = v_\mu \phi^5_\mu = 0\).

Furthermore, we can collect the pseudoscalar field \(\phi^5\) and the vector field \(\phi_\mu\) into a (super)field \(h\) which represents the \((0^-, 1^-)\)–doublet of spin symmetry.

Analogously the scalar field \(\phi\) and the axial–vector field \(\phi^5_\mu\) are combined in the parity conjugate (super)field \(k\) \[ h = P_+ (i\phi^5 \gamma_5 + \phi^\mu \gamma_\mu) \quad , \] \[ k = P_+ (\phi + i\phi^5 \gamma_\mu \gamma_5) \quad , \] \[ \bar{h} = \gamma_0 h^\dagger \gamma_0 = (i\phi^5 \gamma_5 + \phi^\mu \gamma_\mu) P_+ \quad , \] \[ \bar{k} = \gamma_0 k^\dagger \gamma_0 = (\phi^\dagger + i\phi^5 \gamma_5 \gamma_\mu) P_+ \quad , \]
where the projection operator on the heavy quark velocity is defined through 
\[ P_+ = \frac{(1 + v / \hat{v})}{2}. \]

This is a shorthand notation, as these fields carry light flavor quantum numbers \(h = h_a = (h_u, h_d, h_s), k_a = (k_u, k_d, k_s)\) to form anti–triplets under chiral symmetry and the dependence on the heavy quark velocity \(h = h_v\), etc. has not been quoted explicitly. Due to flavor symmetry of HQET these fields describe both \(B\) or \(D\) mesons. Note that in case of unbroken chiral \(SU(3)_L \times SU(3)_R\) symmetry parity–conjugated heavy mesons \(h\) and \(k\) are degenerated.

For later use let us also introduce left and right combinations 
\[ (h + k)_{L,R} = (h + k) P_{L,R} \]
\[ = P_+ ((\phi \mp i\phi^5) + \gamma_\mu (\phi^\mu \mp i\phi^5 \gamma_\mu)) P_{L,R} \quad , \]
where \(P_{R,L} = (1 \pm \gamma_5)/2\) are the chiral projectors.

Now we can re–express the generating functional as 
\[ Z(\eta) = \mathcal{N} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\mathcal{M} e^{i \int d^4x (L' + L_{\text{source}}(\eta))} \quad , \]
where \(\mathcal{N}\) is an unimportant normalization factor, \(\mathcal{D}\mathcal{M} = \mathcal{D}s \mathcal{D}p \mathcal{D}v \mathcal{D}a \mathcal{D}h \mathcal{D}k\) stands for the differential of the several mesonic fields and the lagrangian is bilinear in the quark fields,

\[ L' = L'^{ll} + L'^{hl} \quad , \]
\[ L'^{ll} = \overline{q}(i\partial - (s + i\gamma_5 p) + (\gamma + \gamma_5)) q - \frac{1}{4G_1} \text{tr}_F \left[(s - \hat{m}_0)^2 + p^2 - 2v_\mu v^\mu - 2a_{\mu} a^\mu\right] \quad , \]

6
\[ L^{hi} = \overline{Q}_v (i v \cdot \partial) Q_v - \overline{Q}_v (h + k) q - \overline{Q}(\bar{h} + \bar{k}) Q_v + \frac{1}{2G_1} \text{Tr} [(\bar{h} + \bar{k})(h - k)] . \]  

Here the trace has to be taken over flavor and Dirac indices, \( \text{Tr} = \text{tr}_F \text{tr}_D \). In the following it will be convenient to use for the light scalar–pseudoscalar fields the chiral representation

\[ s + i p = \xi_L^\dagger \Sigma \xi_R , \]

where \( \Sigma \) is a hermitian matrix and \( \xi_{L,R} \) are unitary matrices. The freedom in the choice of \( \xi_{L,R} \) reflects the local hidden symmetry \( SU(3)_h \) \[13\]. Under \( SU(3)_L \times SU(3)_R \times SU(3)_h \) the fields \( \xi_{L,R}, \Sigma \) transform as

\[ \begin{aligned}
\xi_R(x) &\rightarrow h(x) \xi_R(x) R^\dagger , & \xi_L(x) &\rightarrow h(x) \xi_L(x) L^\dagger , \\
\Sigma &\rightarrow h(x) \Sigma h^\dagger(x) , \\
L &\in SU(3)_L , & R &\in SU(3)_R , & h(x) &\in SU(3)_h .
\end{aligned} \]

The additional degrees of freedom contained in \( \xi_L, \xi_R \) can be gauged away as in usual Higgs mechanism, and all our results will be given later on in unitary gauge where

\[ \xi_R = \xi_L^\dagger = \xi = \exp(i\pi/F) \]

is an element in the coset space \( SU(3)_L \times SU(3)_R / SU(3)_V \). Here \( F \) is the bare decay constant, and \( \pi = \pi^a \lambda^a_F / 2 \) represents the light octet of (pseudo)Goldstone bosons associated to spontaneous breakdown of chiral symmetry through a non–vanishing vacuum expectation value of \( \Sigma \).

It is now convenient to define new ‘chirally rotated’ fields of constituent quarks \( \chi_{L,R} = \xi_{L,R} P_{L,R} q \). Under a chiral rotation of the original quark fields

\[ \begin{aligned}
q_R &\rightarrow R q_R , & q_L &\rightarrow L q_L 
\end{aligned} \]

the constituent quark fields transform according to the hidden gauge symmetry

\[ \chi_{L,R} \rightarrow h(x) \chi_{L,R} . \]

After the chiral rotation the Dirac operator contains the rotated meson fields

\[ \begin{aligned}
V_\mu + A_\mu &= \xi_{L,R}(v_\mu + a_\mu + i \partial_\mu) \xi_{L,R}^\dagger , \\
(H + K) &= (h + k) L \xi_L^\dagger + (h + k) R \xi_R^\dagger ,
\end{aligned} \]

which transform under a left–right transformation according to the hidden symmetry group

\[ \begin{aligned}
V_\mu + A_\mu &\rightarrow h(x)(V_\mu + A_\mu + i \partial_\mu) h^\dagger(x) , \\
(H + K) &\rightarrow (H + K) h^\dagger(x) .
\end{aligned} \]
Source terms have to be rotated appropriately. Note that the light vector field transforms as a gauge field of hidden local symmetry $SU(3)_h$

$$V^\mu \rightarrow h(x)V^\mu h^\dagger(x) + ih(x)\partial^\mu h^\dagger(x) = h(x)iD^\mu h^\dagger(x)$$

defining a covariant derivative

$$D^\mu = \partial^\mu - iV^\mu,$$

while the axial–vector field transforms homogeneously

$$A^\mu \rightarrow h(x)A^\mu h^\dagger(x).$$

The rotated heavy meson fields are still organized in spin–symmetry doublets

$$H = P_+ (i\Phi^5 + \Phi^\mu \gamma^\mu), \quad (31)$$

$$K = P_+ (\Phi + i\Phi^5 \gamma^\mu \gamma^\mu), \quad (32)$$

where the $\Phi$’s are related to the original fields $\phi$ defined by (16) through (27).

The lagrangian is now expressed in terms of rotated quark and meson fields

$$L^{ll} = \chi(i\partial - \Sigma + V + A\gamma_5)\chi$$

$$- \frac{1}{4G_1} \text{tr}_F \left[ \Sigma^2 - \hat{\gamma}_0 (\xi^\dagger_L \Sigma \xi_R + \xi^\dagger_R \Sigma \xi_L) \right]$$

$$+ \frac{1}{4G_2} \text{tr}_F \left[ (V_\mu - V_\mu^\pi)^2 + (A_\mu - A_\mu^\pi)^2 \right], \quad (33)$$

$$L^{hl} = \bar{Q}_v (iv \cdot \partial)Q_v - \bar{Q}_v (H + K)\chi - \bar{\chi} (\bar{H} + \bar{K})Q_v$$

$$+ \frac{1}{2G_3} \text{Tr} \left[ (\bar{H} + \bar{K})(H - K) \right], \quad (34)$$

Here we have defined the vector and axial–vector fields induced by the chiral rotation

$$V_\mu^\pi = \frac{i}{2} (\xi_R \partial_\mu \xi^\dagger_L + \xi_L \partial_\mu \xi^\dagger_R)$$

$$A_\mu^\pi = \frac{i}{2} (\xi_R \partial_\mu \xi^\dagger_R - \xi_L \partial_\mu \xi^\dagger_L)$$

which in unitary gauge can be expanded into powers of the pseudoscalar meson fields $\pi$. Moreover, following [3], we have introduced an independent coupling constant $G_2$ for the light vector and axial–vector channel, to obtain satisfactory results for the $\rho$– and $a_1$–meson masses. (Note that s–p– and v–a–sectors are separately invariant under chiral transformations, so chiral symmetry allows for different coupling constants.) In the same way chiral and heavy quark symmetries admit an independent coupling $G_3$ for the heavy meson sector. As it will turn out, a new coupling $G_3$ is indeed needed in order to get reasonable predictions for heavy meson observables.
The generating functional is then given by

\[ Z(\eta) = N \int \mathcal{D} \chi \mathcal{D} \bar{Q} \mathcal{D} Q' \mathcal{D} \Sigma \mathcal{D} \bar{\Sigma} \mathcal{D} \chi' \mathcal{D} Q \mathcal{D} \Sigma' \mathcal{D} \bar{\Sigma} \mathcal{D} \chi \mathcal{D} Q' \mathcal{D} M' J(\xi) e^{i \int d^4 x (\mathcal{L}^\prime + \mathcal{L}_{\text{source}}(\eta))} , \tag{35} \]

where the differential of rotated meson fields in unitary gauge reads \( \mathcal{D} M' = \mathcal{D} \Sigma \mathcal{D} \bar{\Sigma} \mathcal{D} \bar{V} \mathcal{D} \bar{A} \mathcal{D} \bar{H} \mathcal{D} \bar{K} \) and \( J(\xi) \) is the Jacobian of the chiral rotation, which gives rise to the integrated chiral anomaly [14, 3].

We are now able to integrate out the quark degrees of freedom. In the first step let us integrate over heavy quark fields \( Q_v(x) \) in (35). Since we are neglecting the influence of heavy–heavy mesons, the resulting heavy quark determinant is trivial and will be absorbed into the normalization. Leaving aside source terms which will be separately treated later when needed for applications, we get the lagrangian

\[ \mathcal{L}'' = \chi i \bar{D} \chi - \frac{1}{4G_1} \text{tr}_F \left[ \Sigma^2 - \hat{m}_0 (\xi_L \Sigma \xi_R + \xi_R \Sigma \xi_L) \right] + \frac{1}{4G_2} \text{tr}_F \left[ (V_\mu - V_\mu^\nu)^2 + (A_\mu - A_\mu^\nu)^2 \right] \]

\[ + \frac{1}{2G_3} \text{Tr} \left[ (\bar{H} + \bar{K})(H - K) \right] , \tag{36} \]

\[ \text{where} \]

\[ i \mathcal{D} = i \partial - \Sigma + V + A \gamma_5 - (\bar{H} + \bar{K})(iv \cdot \partial)^{-1}(H + K) \]

\[ \text{is the Dirac operator for the light constituent quarks. Here the last term represents the effect of the heavy mesons. Finally, integrating out the light quarks leads to the quark determinant} \]

\[ \det(i \mathcal{D}) = \exp(N_c \text{Tr} \ln i \mathcal{D}) . \tag{39} \]

To regularize the quark loops arising from (39) we shall use a universal proper–time cut–off \( \Lambda \) which will be fixed from the light meson data.

### 3 The Effective Meson Lagrangian

Expanding the quark determinant (39) in powers of the meson fields leads to the familiar loop expansion given by Feynman diagrams with heavy and light mesons as external lines and heavy and light quarks in internal loops. Combining the loop expansion with the gradient expansion one finds the desired effective meson lagrangian.

#### 3.1 Light sector

Let us summarize the essential physical results that have already been obtained for the light sector by expanding the functional determinant (39) in terms of light
mesonic fields $\Sigma, V, A$ around their vacuum values. This has been performed along traditional diagrammatic quark loop expansion as well as in a heat kernel expansion. Only the scalar field $\Sigma$ develops a non–zero vacuum expectation value $\langle \Sigma \rangle$ that indicates spontaneous breaking of chiral symmetry. It has to be identified with the constituent quark mass $m_i$ and is determined by the Schwinger–Dyson equation

$$\langle \Sigma \rangle = m_i = m_0^i + 8m^i G_1 I_1^i, \quad (40)$$

with $I_1^i$ given in appendix C. Including explicit flavor symmetry breaking through different current quark masses $m_u^0, m_d^0, m_s^0$ triggers the flavor dependence of constituent quark masses and induced coupling constants and masses of light mesons. The physical data of light mesons then fix the parameters of our model $G_1, G_2, \Lambda, \hat{m}_0$.

We will first give the results in the non–strange sector putting $m_u^0 = m_d^0 = m_0$ and afterwards comment on deviations when considering strange mesons.

Neglecting quantum fluctuations of $\Sigma$ around it’s VEV, the effective lagrangian reads

$$L_{\text{light}} = -\frac{1}{2g_V^2} \text{tr}_F \left[ V^2_{\mu\nu} + A^2_{\mu\nu} \right] + \frac{6m_0^2}{g_V^2} \text{tr}_F \left[ A^2_{\mu} \right]$$

$$+ \frac{1}{4G_2} \text{tr}_F \left[ (V_{\mu} - V^\pi_{\mu})^2 + (A_{\mu} - A^\pi_{\mu})^2 \right]$$

$$+ \frac{m_0 m}{4G_1} \text{tr}_F \left[ \xi_L \xi_R^I + \xi_R \xi_L^I \right], \quad (41)$$

with $g_V = (2/3I_2)^{-1/2}$ and $I_2 = I_2^{uu}$ given in appendix C. We redefine the fields in order to get the correctly normalized kinetic terms and remove the mixing between $A_{\mu}$ and $A^\pi_{\mu}$

$$\tilde{\pi} = \frac{F_\pi}{F_\pi} \pi, \quad g_V \tilde{V} = V, \quad g_V \tilde{A} = A - \frac{M_V^2}{M_A} A^\pi. \quad (42)$$

These fields are then to be considered as the fields of physical $\pi, \rho, A_1$ respectively. This yields the following relations for physical meson masses $M$, the weak pion–decay constant $F_\pi = 93$ MeV and the $\rho–\pi–\pi$–coupling $g_{V\pi\pi}$

$$M_\pi^2 = \frac{m_0 m}{G_1 F_\pi^2}, \quad M_V^2 = \frac{g_V^2}{4G_2}, \quad M_A^2 = M_V^2 + 6m^2,$$

$$F_\pi^2 = \frac{1}{4G_2} \left( 1 - \frac{M_V^2}{M_A^2} \right), \quad g_{V\pi\pi} = \frac{g_V}{8G_2 F_\pi^2}. \quad (43)$$

In addition, we recover the usual KSFR–relations

$$g_{V\pi\pi} = \frac{a}{2} g_V, \quad M_V^2 = a F_\pi^2 g_V^2;$$

$^3$In literature one often uses the chiral field $U = \xi_L^I \xi_R$. 

10
if the parameter
\[
a = \left(1 - \frac{M_V^2}{M_A^2}\right)^{-1}
\]
is chosen as \(a = 2\). This choice yields also the Goldberger–Treiman relation
\[
g_\pi = \frac{m}{F_\pi}
\]
with \(g_\pi = (2I_2)^{-1/2}\) being the \(\pi q \bar{q}\)–coupling constant.

Using \(F_\pi = 93\) MeV, \(M_\pi = 140\) MeV, \(m_\rho = 770\) MeV, \(g_\nu\pi\pi = 6\) as input fixes
the model parameters \(G_1, G_2, m_0^u = m_0^d\) and the intrinsic cut–off scale \(\Lambda\) for \(N_c = 3\) to be
\[
G_1 = 5.7 \text{ GeV}^{-2},
\quad G_2 = 13.8 \text{ GeV}^{-2},
\quad m_0^{u,d} = 3 \text{ MeV},
\quad \Lambda = 1.25 \text{ GeV},
\]
(44)
together with \(m_0^{u,d} = 300\) MeV.

For strange mesons, all quantities get flavor dependent \(g_{ij}V \rightarrow g_{ij}V\), \(M_{ij}V \rightarrow M_{ij}V\), \(M_A \rightarrow M_{ij}A\), \(F_\pi \rightarrow F_{ij}\pi\), \(M_\pi \rightarrow M_{ij}\pi\). For details we refer the reader
\[3\]. For our purpose, it is sufficient to include strange quark effects via a
different constituent quark mass \(m_s \approx \frac{M_\phi}{2} = 510\) MeV, as well as to intro-
duce different couplings \(g_{us}V = \left(\frac{1}{6}(I_{uu}^s + I_{ss}^s + 2I_{us}^s)\right)^{-1/2}\) and a different decay
constant \(F_K = F_{us}^\pi \approx 1.2F_\pi\).

### 3.2 Heavy sector

The on–shell condition for a heavy meson \(\Phi \sim (\bar{q}Q)\) is given by
\[
i\partial_\mu \Phi = \Delta M v_\mu \Phi
\]
(45)
with \(\Delta M = M_\Phi - m_Q\) being the mass difference between the heavy meson
(with mass \(M_\Phi\)) and the heavy quark. This is easily seen by substituting \(\Phi' = \exp(-im_Qv \cdot x)\Phi\) in the Klein–Gordon lagrangian for a meson field \(\Phi' \sim (\bar{q}Q)\)
\[
\mathcal{L} = \partial^\mu \Phi' \partial_\mu \Phi' - M_\Phi^2 \Phi' \Phi' = 2M_\Phi \left(\Phi' \left(i v \cdot \partial - \Delta M\right) + O\left(\frac{1}{m_Q}\right)\right).
\]
(46)
In subsequent applications we will use a normalization, where a factor \(\sqrt{M_\Phi}\)
in \[46\] is absorbed into the fields \(\Phi\). In deriving the effective heavy meson
lagrangian \[46\] from the quark determinant \[39\], we shall combine the loop
expansion with a low momentum or gradient expansion.
3.2.1 The free heavy meson lagrangian

The loop expansion of the fermion determinant (39) gives rise to the self-energy diagram for the heavy fields in Figure 1 which yields the following expression

\[ \Pi_{H,K}(v \cdot p) = \Pi_{H,K}(0) + \Pi_{H,K}^{\prime}(0) v \cdot p + O((v \cdot p)^2) \]  

Figure 1: Self–energy diagram for heavy meson fields \( H, K \).

for each single light quark flavor with mass \( m \)

\[ \text{Expanding the self–energy part } \Pi_{H,K}(v \cdot p) \text{ in powers of the external momentum } v \cdot p, \]

\[ \Pi_{H,K}(v \cdot p) = \Pi_{H,K}(0) + \Pi_{H,K}^{\prime}(0) v \cdot p + O((v \cdot p)^2) \]  

yields the quadratic meson lagrangian

\[ \mathcal{L}_{\text{heavy}} = -\text{tr}_D \left[ \overline{H} \left( -\frac{1}{2G_3} + \Pi_{H}(0) + \Pi_{H}^{\prime}(0) v \cdot p \right) H \right] + \text{tr}_D \left[ \overline{K} \left( -\frac{1}{2G_3} + \Pi_{K}(0) + \Pi_{K}^{\prime}(0) v \cdot p \right) K \right] \]  

The here obtained lagrangian takes the standard form when we choose \( \Delta M_{H,K}^{i} \) such that

\[ -\frac{1}{2G_3} + \Pi_{H,K}^{i}(0) + \Pi_{H,K}^{i}(0) \Delta M_{H,K}^{i} = 0 \]  

Finally rescaling the meson fields by \( Z \)–factors, \( Z_{H,K}^{i} \equiv \left( \Pi_{H,K}^{i}(0) \right)^{-1} \),

\[ \tilde{H}^{i} = (Z_{H}^{i})^{-1/2} H^{i} , \]

\[ \tilde{K}^{i} = (Z_{K}^{i})^{-1/2} K^{i} , \]
the effective meson lagrangian acquires in configuration space the desired form

\[ L_{\text{heavy}} = -\text{tr}_D \left[ \hat{H} \left( iv \cdot \partial - \Delta M_H^i \right) \hat{H}^i \right] + \text{tr}_D \left[ \hat{K} \left( iv \cdot \partial - \Delta M_K^i \right) \hat{K}^i \right]. \]  

The explicit expressions for the \( Z_{H,K}^i \) and \( \Delta M_{H,K}^i \) read

\[ Z_{H,K}^i = \left( I_i^3 \pm 2m_i I_i^2 \right)^{-1}, \]

\[ \Delta M_{H,K}^i = Z_{H,K}^i \left( \frac{1}{2G_3} - I_i^1 \mp m_i I_i^3 \right), \]

where the integrals \( I_1^i, I_2^i \) and \( I_3^i \) are given in appendix C. Due to heavy flavor symmetry, mass differences \( \Delta M_{H,K}^i \) do not scale with the heavy quark mass. In addition, we observe a mass–splitting between \( H \) and \( K \) induced through the light constituent quark mass \( m \) which is therefore an effect of spontaneous chiral symmetry breaking. We will discuss numerical results following from (53), (54) for heavy meson masses and decay constants later.

### 3.2.2 Coupling of heavy mesons \( H,K \) to light vector– and axial–vector–mesons

For applications in heavy–flavor decay processes it is necessary to know the strong interaction couplings between heavy mesons and light mesons \( V, A \). The loop expansion of the quark determinant (39) yields such vertex terms through the following expression

\[ -\frac{iN_c}{16\pi^2} \int \frac{d^4k}{(k-p_i)^2-(m_i)^2} \left( k \cdot p_i + m_i \right) (V_{ij} + A_{ij} \gamma_5) (k \cdot p_j + m_j) (\overline{H}^i + K^i) \]

which corresponds to the diagram shown in Figure 2. In the low–momentum expansion around \( v \cdot p_i = v \cdot p_j = 0 \) we get the following contributions to an effective lagrangian in terms of renormalized heavy and light fields

\[ L_{V/A}^{\text{heavy}} = g_{V}^{ij} \lambda_1^j \text{tr}_D \left[ \hat{H}^i \hat{H}^j \right] v \cdot \hat{V}^{ij} - g_{V}^{ij} \lambda_2^j \text{tr}_D \left[ \hat{K}^i \hat{K}^j \right] v \cdot \hat{V}^{ij} \]

\[ + g_{V}^{ij} \lambda_3^j \text{tr}_D \left[ \hat{H}^i \hat{A}^{ij} \gamma_5 \right] + g_{V}^{ij} \lambda_4^j \text{tr}_D \left[ \hat{K}^i \hat{A}^{ij} \gamma_5 \right] \]

\[ - g_{V}^{ij} \lambda_5^j \text{tr}_D \left[ \hat{K}^i \hat{H}^j \hat{V}^{ij} \right] + h.c. \]

\[ + g_{V}^{ij} \lambda_6^j \text{tr}_D \left[ \hat{K}^i \hat{H}^j \hat{A}^{ij} \gamma_5 \right] + h.c. \]

\[ (55) \]
The expressions for the several coupling parameters $\lambda_{ij}^{ij}$ are given in appendix D.

In the flavor symmetry limit $m_i = m_j$ due to $\lambda_1^{ij} = \lambda_2^{jj} = 1$ and $g_V^{ij} = g_V$, we observe that the light vector mesons couple via a covariant derivative of hidden symmetry

$$D_{ij}^\mu = \partial_\mu \delta_{ij} + ig_V \hat{V}_\mu^{ji}. \quad (56)$$

(Note the different sign compared to (30) which is due to the fact that the heavy fields $H,K$ transform as anti–triplets under $SU(3)_{h}$.)

3.2.3 Coupling of heavy mesons $H,K$ to pseudo–Goldstone $\pi$’s

In our approach, the heavy mesons $H,K$ couple to the fields $\pi$ via $\pi - A_1$ mixing due to the coupling between $A^\mu$ and $A_\pi^\mu$ (see eq. (42)). For concreteness, we consider the coupling between two members of the $(0^-,1^-)$–multiplet $\hat{H}^i$ with a pseudoscalar $\hat{\pi}^{ij}$

$$\mathcal{L}_\pi^{\text{heavy}} = g_{HH\pi}^{ij} \text{trD} \left[ \hat{H}^i \hat{H}^j \hat{A}_\pi^{ij} \gamma_5 \right] + \ldots \quad (57)$$

where the coupling is then given by

$$g_{HH\pi}^{ij} = \frac{(M_{ij}^{ij})^2}{(M_A^{ij})^2} \lambda_3^{ij} \quad . \quad (58)$$

The ellipsis denote analogous couplings of $H$ and $K$ where one simply has to replace $\lambda_3$ by $\lambda_4$ or $\lambda_6$ respectively, (cf. eq.(55))

$$g_{KK\pi}^{ij} = \frac{(M_{ij}^{ij})^2}{(M_A^{ij})^2} \lambda_4^{ij} \quad , \quad g_{HK\pi}^{ij} = \frac{(M_{ij}^{ij})^2}{(M_A^{ij})^2} \lambda_6^{ij} \quad . \quad (59)$$

These terms describe direct processes with an odd number of Goldstone bosons. The corresponding decays into an even number of $\pi$’s are given by coupling the vector mesons in (55) to the pion current $\mathcal{V}_\pi$ in (41).
3.2.4 Electroweak decays of heavy mesons $H, K$

Next, we present the results for the electroweak decay constant of heavy mesons $f_{H,K}$ which determines the matrix elements of electroweak heavy–to–light currents between a heavy meson state and an arbitrary number of Goldstone–fields. For this purpose we introduce a source term according to (10)

$$
\eta^{\mu \dagger}_L (\nabla \xi_L \gamma_\mu (1 - \gamma_5) Q_v) + h.c.
$$

(60)

The source can be removed from the Dirac operator by a simple shift in the fields $\Phi$ and $\Phi^\mu$ and appears afterwards in the quadratic term in (34)

$$
\frac{1}{2G_3} \mathrm{Tr} \left[ (\mathcal{P} + \mathcal{K})(H - K) \right] \rightarrow
\frac{1}{2G_3} \mathrm{Tr} \left[ (\mathcal{P} + \mathcal{K} + \eta^{\mu \dagger}_L \xi_L \gamma_\mu (1 - \gamma_5))(H - K + \eta^\mu L \gamma_\mu (1 + \gamma_5) \xi^\dagger_L) \right].
$$

(61)

By variation with respect to $\eta^{\mu \dagger}_L$ and setting sources equal to zero afterwards, we get the following expression for the bosonized current in terms of the rescaled fields of eqs. (51)

$$
\frac{1}{2G_3} \mathrm{Tr} \left[ \xi_L \gamma_\mu (1 - \gamma_5) (\sqrt{Z_H} \hat{H} - \sqrt{Z_K} \hat{K}) \right].
$$

(62)

On the basis of (62) the weak decay constant $f_{H,K}$, defined through

$$
\langle 0 | \bar{q} \gamma_\mu (1 - \gamma_5) Q_v | H_v(0^-) \rangle = i f_H M_H v_\mu,
\langle 0 | \bar{q} \gamma_\mu (1 - \gamma_5) Q_v | K_v(0^+) \rangle = - f_K M_K v_\mu
$$

(63)

is now related to the effective coupling $G_3$ and the renormalization factors of the heavy meson fields by

$$
f_{H,K} \sqrt{M_{H,K}} = \frac{\sqrt{Z_{H,K}}}{G_3}.
$$

(64)

We recover the familiar scaling of the weak decay constant of heavy mesons with the heavy mass in HQET due to heavy spin and flavor symmetry. Note that by inserting the respective masses and $Z_{H,K}$–factors we can account for flavor symmetry breaking effects and parity splitting between the decay constants of heavy meson doublets $H, K$.

Due to spin symmetry of heavy quarks one can generalize (62) to arbitrary Dirac matrices $\Gamma$, considering e.g. the penguin–operator $\nabla \xi_L i \sigma_{\mu\nu} (1 + \gamma_5) Q_v$ that appears in an effective operator basis for rare $b \rightarrow s$ decays. In our approach one can see this explicitly by calculating the diagram in Figure 3.

---

4 Our normalization is $\langle 0 | \Phi^\mu v | H_v(0^-) \rangle = \sqrt{M_H}$, $\langle 0 | \Phi^v v | K_v(0^+) \rangle = \sqrt{M_K}$. The definition of $f_{H,K}$ corresponds to $f_\pi = \sqrt{2F_\pi} = 132$ MeV.
This is completely analogous to the calculation of the mass differences $\Delta M_{H,K}$ and leads to a corresponding piece in the effective lagrangian

$$\frac{1}{2G_3} \text{Tr} \left[ \eta^i \Gamma (\sqrt{Z_H} \hat{H} - \sqrt{Z_K} \hat{K}) \right]$$

(65)

for arbitrary $\Gamma$. The special choice $\eta^i = \eta^{\mu}_L \xi^L \gamma_\mu (1 - \gamma_5)$ then indeed reproduces (62).

In unitary gauge (23) we finally arrive at the following expressions describing electroweak decays of heavy mesons through bosonized currents

$$\mathcal{L}^{\text{weak}} = \eta^i \eta^j J_{\mu L} + \eta^{\mu} J^{\mu \nu} + \text{h.c.},$$

(66)

$$J_{\mu L} = \frac{\sqrt{M_H f_H}}{2} \text{Tr} \left[ \xi^i \gamma_\mu (1 - \gamma_5) \hat{H} \right]$$

$$- \frac{\sqrt{M_K f_K}}{2} \text{Tr} \left[ \xi^i \gamma_\mu (1 - \gamma_5) \hat{K} \right],$$

(67)

$$J^{\mu \nu} = \frac{\sqrt{M_H f_H}}{2} \text{Tr} \left[ \xi^i \sigma^{\mu \nu} (1 + \gamma_5) \hat{H} \right]$$

$$- \frac{\sqrt{M_K f_K}}{2} \text{Tr} \left[ \xi^i \sigma^{\mu \nu} (1 + \gamma_5) \hat{K} \right].$$

(68)

Our result coincides with expressions given on the basis of symmetry arguments in [11]. However a term like $\text{Tr} \left[ \xi^i \gamma_5 H (V_\mu - V_\mu^\pi) \right]$ as considered sometimes in literature is absent. Let us stress that expression (66) describes as well direct decays into multi–$\pi$ states, if one expands $\xi^i$ in unitary gauge. In phenomenological applications these so–called Callan–Treiman contributions play an important role in weak semileptonic decays like $B \rightarrow \pi e \nu$.

### 3.2.5 Isgur–Wise function

In the heavy quark limit the Isgur–Wise function $\xi(v \cdot v')$ describes as a universal form factor the matrix elements of electroweak heavy–to–heavy currents between
two heavy mesons of different velocities $H_v, H_{v'}$. It is defined as

$$\langle H_{v'}(0^-, 1^-) | Q v \Gamma Q v | H_v(0^-, 1^-) \rangle = \xi(v \cdot v') \text{tr}_D \left[ H_{v'} \Gamma H_v \right] , \quad (69)$$

where $\Gamma$ is an arbitrary Dirac matrix and $H_v$ denotes the matrix representation of a heavy pseudoscalar and a heavy vector meson with polarizat ion vector $\varepsilon^\mu$

$$H_v = \frac{1 + \gamma^5}{2} \sqrt{M_H} (i \gamma^5 + \gamma^\mu) . \quad (70)$$

An equivalent relation holds for matrix elements between the parity conjugate heavy mesons $K$.

In our approach it is straightforward to calculate the Isgur–Wise function by simply introducing an appropriate source–term $\eta^\dagger_{vv'} (Q_{v'} \Gamma Q_v) + h.c.$ from the very beginning and differentiating with respect to $\eta^\dagger_{vv'}$. This gives rise to the Feynman diagram shown in Figure 4 and yields for the Isgur–Wise function

\begin{align*}
\xi(v \cdot v') &= Z_H \frac{i N_c}{16 \pi^2} \int^{reg} d^4k \, \frac{\text{tr}_D \left[ (k - p + m) H_{v'} \Gamma H_v \right]}{\text{tr}_D \left[ H_{v'} \Gamma H_v \right]} \\
&\quad \times \frac{1}{((k - p)^2 - m^2)(v \cdot k + i \epsilon)(v' \cdot k' + i \epsilon)} . \quad (71)
\end{align*}

Figure 4: Feynman–diagram for the Isgur–Wise function.

Due to spin symmetry of heavy mesons the result is indeed independent of the particular form of the Dirac matrix $\Gamma$. Performing the same calculational steps \footnote{Recall that $Q = b, c, \ldots$ such that (69) describes both heavy flavor diagonal and non–diagonal transitions.}
as for the determination of $Z_H$ (see section 3.2.1) we arrive at the following expression for $\xi(\omega)$ in terms of the integrals $I_2, I_3, I_5(\omega)$ (see appendix C),

$$\xi(\omega) = Z_H \left( \frac{2}{1 + \omega} I_3 + mI_5(\omega) \right), \quad (72)$$

where the light flavor index has been dropped. The functional dependence of the integral $I_5$ on the momentum transfer $\omega = v \cdot v'$ is given by the function $r(\omega)$,

$$I_5(\omega) = 2I_2r(\omega) \quad, \quad r(\omega) = \frac{\ln(\omega + \sqrt{\omega^2 - 1})}{\sqrt{\omega^2 - 1}} \quad (73)$$

with $I_5(1) = 2I_2$ and $I_5'(1) = -2/3I_2$. We explicitly get the correct normalization $\xi(\omega = 1) = 1$, and the slope of the Isgur–Wise function at zero recoil is obtained as

$$\xi'(\omega = 1) = Z_H \left( -\frac{1}{2}I_3 - \frac{1}{3}2mI_2 \right) \quad . \quad (74)$$

The Isgur–Wise form factor for members of the $(0^+, 1^+)$–multiplet, which we refer to as $\xi_K(\omega)$, is not related to $\xi(\omega)$ by heavy quark symmetries. Here an analogous calculation yields

$$\xi_K(\omega) = Z_K \left( \frac{2}{1 + \omega} I_3 - mI_5(\omega) \right) \quad . \quad (75)$$

It is worth mentioning that this result includes effects of a physical cut–off $\Lambda$ (defining the scale of chiral symmetry breaking) through the integral $I_3$ that would be absent in renormalization schemes like $\overline{MS}$. A numerical discussion of the slope of the Isgur–Wise function at the non–recoil point $v = v'$ will be given in the following section 4.

### 3.2.6 The effective lagrangian

Finally, let us collect all obtained contributions to the effective lagrangian for renormalized light and heavy meson fields, including strong interaction couplings and electroweak currents of heavy mesons

$$\mathcal{L} = \mathcal{L}_{\text{light}} + \mathcal{L}_{\text{heavy}}^{\eta_0} + \mathcal{L}_{\text{heavy}}^{\eta_+} + \mathcal{L}_{\pi}^{\text{heavy}}$$

$$\mathcal{L}_{\text{light}} = -\frac{1}{2} \text{tr}_F \left[ \tilde{V}_{\mu \nu}^2 \right] + aF^2_{\pi} \text{tr}_F \left[ (g_{\mu \nu} \tilde{V}_\mu - V^\pi_\mu)^2 \right]$$

$$+ \text{tr}_F \left[ \tilde{A}_{\mu \nu}^2 \right] + M_\pi^2 \text{tr}_F \left[ (\tilde{A}_\mu)^2 \right]$$

$$+ F^2_{\pi} \text{tr}_F \left[ (A^\pi)^2 \right] + \frac{F^2_{\pi} M_\pi^2}{4} \text{tr}_F \left[ \xi^2 + (\xi^1)^2 \right] \quad , \quad (77)$$
\[ L_{0}^{\text{heavy}} = -\text{tr}_D \left[ \hat{H}^i \left( i v \cdot \nabla - \Delta M_H \right) \hat{H}^i \right] + \text{tr}_D \left[ \hat{K}^i \left( i v \cdot \nabla - \Delta M_K \right) \hat{K}^i \right], \quad (78) \]

\[ L_{V/A}^{\text{heavy}} = +\lambda_{ij}^{ij} g_{ij}^{ij} \text{tr}_D \left[ \hat{H}^i \hat{H}^{\dagger} v \cdot \hat{V}^{ij} - \lambda_{ij}^{ij} g_{ij}^{ij} \text{tr}_D \left[ \hat{K}^j \hat{K}^{\dagger} v \cdot \hat{V}^{ij} \right] \right] + \lambda_{ij}^{ij} g_{ij}^{ij} \text{tr}_D \left[ \hat{K}^j \hat{K}^{\dagger} v \cdot \hat{V}^{ij} \right] \right] + h.c. , \quad (79) \]

\[ L_{\pi}^{\text{heavy}} = +g_{HH\pi}^{ij} \text{tr}_D \left[ \hat{H}^i \hat{H}^{\dagger} A_{\pi}^{ij} v \cdot \hat{V}^{ij} \right] - g_{KK\pi}^{ij} \text{tr}_D \left[ \hat{K}^j \hat{K}^{\dagger} A_{\pi}^{ij} v \cdot \hat{V}^{ij} \right] + \text{h.c.} , \quad (80) \]

\[ J_{\mu L} = \sqrt{M_H} \frac{\bar{H}}{2} \text{tr} \left[ \xi^i \gamma_{\mu} (1 - \gamma_5) \hat{H} \right] - \sqrt{M_K} \frac{\bar{K}}{2} \text{tr} \left[ \xi^i \gamma_{\mu} (1 - \gamma_5) \hat{K} \right], \quad (81) \]

\[ J_{\mu \nu} = \sqrt{M_H} \frac{\bar{H}}{2} \text{tr} \left[ \xi^i \sigma_{\mu \nu} (1 + \gamma_5) \hat{H} \right] - \sqrt{M_K} \frac{\bar{K}}{2} \text{tr} \left[ \xi^i \sigma_{\mu \nu} (1 + \gamma_5) \hat{K} \right], \quad (82) \]

\[ J_{v v'} = \xi (v \cdot v') \text{tr} \left[ \hat{H}_v \Gamma \hat{H}_v \right] - \xi (v \cdot v') \text{tr} \left[ \hat{K}_v \Gamma \hat{K}_v \right]. \quad (83) \]

The above given effective meson lagrangian and currents, which we have obtained from the bosonization of the NJL model, is our main formal result. They describe the low-energy dynamics of light and heavy mesons and in particular the interplay between spontaneous breaking of chiral symmetry and heavy quark symmetry. Such effective meson lagrangians have previously been written down only on phenomenological grounds [11]. Let us emphasize that in the meson lagrangian and bosonized currents obtained here, the heavy meson decay and coupling constants are all expressed in terms of the few parameters of the NJL model, which can be entirely fixed from light meson data (\( G_1, G_2, \hat{m}_0, \Lambda \)) and heavy meson masses (\( G_3 \)).

4 Numerical Discussion

In the previous chapters we have extended the NJL-model of the light quark sector (determining a universal cut-off \( \Lambda = 1.25 \) GeV and light constituent quark masses \( \hat{m} = \text{diag}(300, 300, 510) \) MeV) to the heavy quark sector, intro-
ducing a further coupling constant $G_3$. This new parameter can be fixed from the experimental mass splitting $M_D - M_D = \Delta M_H - \Delta M_H \approx 100$ MeV \[17\]. Varying the coupling $G_3$ in a range of $5 \text{ GeV}^{-2} \leq G_3 \leq 9 \text{ GeV}^{-2}$, we find a favored value $G_3 = 8.7 \text{ GeV}^{-2}$, which then predicts a weak decay constant $f_B = 180$ MeV in perfect agreement with other theoretical approaches, like lattice QCD \[18\] or QCD sum rules \[19\]. We present the results for the range $5 \text{ GeV}^{-2} \leq G_3 \leq 9 \text{ GeV}^{-2}$ in table 1.

| $G_3 [\text{GeV}^{-2}]$ | 5 | 6 | 7 | 8 | 9 |
|------------------------|---|---|---|---|---|
| $\Delta M_H^u [\text{MeV}]$ | 750 | 540 | 390 | 280 | 200 |
| $\Delta M_H^s [\text{MeV}]$ | 990 | 730 | 540 | 400 | 290 |
| $f_B [\text{MeV}]$ | 310 | 260 | 220 | 190 | 170 |
| $f_{B,s} [\text{MeV}]$ | 340 | 280 | 240 | 210 | 190 |

Table 1: Mass differences and decay constants of heavy mesons as functions of the coupling constant $G_3$.

One sees that our fit requires a slightly larger value for $G_3$ compared to $G_1 = 5.7 \text{ GeV}^{-2}$ obtained from the light sector. (Recall the analogous situation in the light meson sector where $G_2$ has to be chosen larger than $G_1$ in order to satisfactorily describe the $\rho$–$a_1$–sector.) Fixing $G_3 = 8.7 \text{ GeV}^{-2}$ we can predict heavy quark masses\[1\] within our model using averaged values $M_B \approx 5.3 \text{ GeV}, M_D \approx 1.9 \text{ GeV} \[17\]$ as input and $\Delta M_H^u = 220$ MeV,

$$m_b = M_B - \Delta M_H^u \approx 5.1 \text{ GeV} \quad , \quad (84)$$
$$m_c = M_D - \Delta M_H^u \approx 1.7 \text{ GeV} \quad , \quad (85)$$

to be compared with the respective masses derived from $b\bar{b}, c\bar{c}$ bound states

$$m_b \approx \frac{M_T}{2} = 4.73 \text{ GeV} \quad ,$$
$$m_c \approx \frac{M_{J/\psi}}{2} = 1.55 \text{ GeV} \quad .$$

In HQET weak decay constants within the same spin–flavor multiplet scale with the heavy meson masses $\sqrt{M_H^u f_H} = \text{const.}, \sqrt{M_K f_K} = \text{const.}$ Observables of different multiplets are, of course, not related by heavy quark symmetries,

---

\[1\] We use the values and notations of the Particle Data Group \[17\] where $D^{(*)}$ stands for $H^{u,d}, D_s^{(*)}$ for $H^s, D_{s1}^{(*)}$ for $K^{u,d}, D_{s1}^{(*)}$ for $K^s$. $B$ mesons are denoted analogously.

\[2\] For heavy flavors we do not distinguish between current quark masses and constituent quark masses.
so one expects $\sqrt{M_H f_H} \neq \sqrt{M_K f_K}$. In our approach this is connected with different renormalization factors $Z_{H,K}$ appearing in (4). In particular, we can account explicitly for the dependence on different light quark masses. The experimental determination of $f_D, f_B$ is still vague. Lattice–calculations [18] and QCD sum rules [19] give values around $f_D \approx 200$ MeV and $f_B \approx 180$ MeV. Both approaches predict large $\Lambda/m_c$ corrections to the HQET scaling law when including effects of non–leading operators in the HQET lagrangian (2).

Nevertheless in the ratio

$$R = \frac{f_H}{f_H} \tag{86}$$

which accounts for light $SU(3)_F$ symmetry breaking effects, these corrections are expected to cancel and our result for this ratio is independent of $G_3$, $R \approx 1.1$. Other approaches based on one–loop calculations in chiral perturbation theory [20], lattice simulations [18] and QCD sum rules [19] yield similar results $R = 1.1 - 1.2$. Note that usual hard gluon QCD–corrections can be taken into account by the scale dependence of the strong coupling constant computed within a leading log approximation in HQET (3). In our approach the required scale matching should be done at the scale $\Lambda$,

$$\sqrt{M_B f_B} = \left( \frac{\alpha_s(\Lambda)}{\alpha_s(M_B)} \right)^{-6/25} \sqrt{M_H f_H} \hspace{1cm},$$

$$\sqrt{M_D f_D} = \left( \frac{\alpha_s(\Lambda)}{\alpha_s(M_D)} \right)^{-6/25} \sqrt{M_H f_H} \hspace{1cm},$$

to give effects up to 10–15%.

Concerning the heavier (0$^+$, 1$^+$) states we observe that $Z_K$ in (53) together with $\Delta M_K$ in (54) get unrealistically large ($\Delta M_K = 2050$ MeV, $\Delta M_K = 6120$ MeV). We expect here essential numerical improvement from on–shell corrections at $v \cdot p = \Delta M_K$ which are not included in the pure gradient expansion around $v \cdot p = 0$.

Let us next focus on the Isgur–Wise function $\xi(\omega)$. The slope parameter at the non–recoil point is defined through

$$\rho^2 = -\xi'(1) \hspace{1cm}.$$

The predictions of our model are

$$\xi''(\omega = 1) = -0.44 \rightarrow \rho = 0.67 \hspace{1cm}, \tag{87}$$

$$\xi'(\omega = 1) = -0.43 \rightarrow \rho = 0.66 \hspace{1cm}. \tag{88}$$

Recent fits on ARGUS and CLEO data prefer a value of $\rho = 1.14 \pm 0.23$ [21] which is in consistency with QCD sum rule estimates of $\rho = 1$ [19].
Notice that naively calculating a triangle quark diagram in a renormalization scheme like $\overline{MS}$ would give $\xi(\omega) = r(\omega)$ together with a lower value $\rho = 1/\sqrt{3} = 0.58$. In contrast, our calculation with composite mesons uses a physical cut–off (related to the scale of chiral symmetry breaking). Following the discussion in connection with analytic bounds on the Isgur–Wise function \[22\], one would expect an increase in $\rho$ if additional effects of bound states consisting of two heavy quarks would be taken into account.

Concerning the several coupling parameters between heavy mesons and light vector or axial vector fields, we focus on the results for $\lambda^{ij}_1$ and $\lambda^{ij}_3$. The values of $\lambda^{ij}_1$ are fixed by $SU(3)_V$ symmetry to $\lambda^{uu}_1 = \lambda^{ss}_1 = 1$ such that the light vector mesons couple through covariant derivatives. Moreover, explicit $SU(3)_F$ symmetry breaking leads to a slight decrease in the parameter $\lambda^{us}_1$,

$$\lambda^{us}_1 = 0.98 \quad \text{(89)}$$

Let us next consider the value for $\lambda^{ij}_3$ which is connected to the pseudoscalar meson coupling $g_{HHA}^{ij}$ in \[58\] and enters into the rate for decays like $B \to \pi e\nu$, $D \to K e\nu$,

$$\begin{align*}
\lambda^{uu}_3 &= -0.33 \quad , \quad g_{HHA}^{uu} = -0.17 \quad , \\
\lambda^{us}_3 &= -0.45 \quad , \quad g_{HHA}^{us} = -0.22 \quad , \\
\lambda^{ss}_3 &= -0.59 \quad , \quad g_{HHA}^{ss} = -0.29 \quad .
\end{align*} \quad \text{(90-92)}$$

From the process $D^{*+} \to D^0 \pi^+$, there exists an upper bound for $g_A = g_{HHA}^{uu}$

$$\Gamma(D^{*+} \to D^0 \pi^+) = \frac{g_A^2 |P_1|^3}{6 \pi F_\pi^2} < 0.131 \text{ MeV} \quad \text{(93)}$$

leading to $g_A^2 < 0.5 \quad \text{(94)}$

which is respected by our result. Recent analyses of CLEO data for $D^*$ branching fractions including electromagnetic interactions using a phenomenological heavy meson chiral lagrangian \[24\] find a larger value of $g_A$ given by $g_A^2 = 0.34 \pm 0.48$ including still a large error.

5 Conclusions

In this work we have studied the properties of composite heavy mesons within the framework of an extended QCD–motivated NJL–model. The light quark flavor dynamics of this model is governed by chiral $SU(3)_L \times SU(3)_R$ symmetry of QCD and its spontaneous and explicit breaking, while the heavy quark sector incorporates the new heavy quark symmetries.

By applying path integral bosonization techniques and performing a gradient expansion, we have derived an effective low–energy lagrangian of light and heavy mesons.
Except for the heavy–light quark interaction strength $G_3$ we have adjusted the parameters of our model (light quark masses, interaction constants $G_1$, $G_2$ and the universal cut–off) by fitting light meson properties. For the $(0^-, 1^-)$ spin–flavour multiplet, heavy meson masses and weak decay constants are then successfully described with a heavy–light four–quark interaction constant $G_3 = 8.7$ GeV$^{-2}$.

The coupling of heavy $(0^-, 1^-)$ mesons to the axial current of Goldstone bosons is calculated as $g_A = -0.17$, lying within experimental bounds $g_A^2 < 0.5$, while light vector mesons couple by covariant derivatives.

Next, by bosonization of electroweak currents of heavy mesons, the weak decay constants have been determined as $f_B = 180$ MeV, $f_D = 300$ MeV. While the value for $f_B$ coincides with recent lattice estimates, the value for $f_D$ is somewhat large and expected to be improved by including $1/m_c$ corrections. The effect of explicit $SU(3)_F$ breaking is estimated through the ratio $R = f_B^H/f_H \approx 1.1$ in consistency with other predictions.

Finally, the Isgur–Wise function is calculated with a slope parameter of $\rho \approx 0.67$ which is somewhat smaller than experimental findings.

Although conceptionally analogous to the $(0^-, 1^-)$ case, our numerical results for the heavy $(0^+, 1^+)$ spin–flavour multiplet are less satisfactory. Work on a possible improvement by performing an on–shell calculation expanding around $v \cdot p = \Delta M$ (instead of $v \cdot p = 0$ ) is in progress.

In summary we have shown that the synthesis of chiral and heavy spin–flavor symmetries within an extended NJL model leads to an effective lagrangian offering a unified description of strong and weak interactions of heavy and light mesons. The above bosonization approach could be further generalized into two directions: First, it would be interesting to study non–local NJL–interactions based on non–perturbative gluon propagators by using the bilocal field approach (see for example [7]). Secondly, to get a complete picture of hadron dynamics, baryons should be included as well.

**Note added**

During completion of this work investigations of Bardeen and Hill [25] and Nowak, Rho and Zahed [26], partially studying similar questions, were brought to our attention. Our work differs, however, in the used model, in important conceptual aspects and naturally also in the numerical results.

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25
A Basic Formulae of HQET

Following [8], we study heavy quarks in the full QCD lagrangian given by

\[ \mathcal{L} = \bar{Q}(iD - m_Q)Q, \]

where the covariant derivative of QCD reads \( D_\mu = \partial_\mu + igA_\mu^a \lambda^a / 2 \).

It is convenient to introduce upper (\( \varphi \)) and lower components (\( \vartheta \)) of the heavy quark spinor \( Q \)

\[ \varphi = P_+ Q, \quad \vartheta = P_- Q \]

with

\[ \hat{\varphi} \varphi = \varphi, \quad \hat{\vartheta} \vartheta = -\vartheta, \]

where \( P_\pm \) are projectors on the heavy quark velocity \( v^\mu (v^2 = 1) \),

\[ P_\pm = \frac{1 \pm \hat{v}}{2}, \]

satisfying the following relations

\[ P_+ \gamma_\mu P_+ = P_+ v_\mu, \quad (A.5) \]
\[ P_- \gamma_\mu P_- = -P_- v_\mu, \quad (A.6) \]
\[ P_+ \gamma_\mu P_- = P_+ (\gamma_\mu - v_\mu), \quad (A.7) \]
\[ P_- \gamma_\mu P_+ = P_- (\gamma_\mu + v_\mu), \quad (A.8) \]

It is further convenient to define a longitudinal and a transversal part of the covariant derivative

\[ D^\parallel = \hat{\varphi}(v \cdot D) + D^\perp, \quad (A.9) \]
\[ D^\perp = \gamma^\mu (g_{\mu\nu} - v_\mu v_\nu)D_\nu, \quad (A.10) \]
\[ \{D^\perp, \hat{\varphi}\} = 0. \quad (A.11) \]
One now has to choose between two kinds of parametrizations, describing particles or anti–particles:

particle: \[ \varphi = e^{-iQ(v \cdot x)} h^+ \quad \bar{\varphi} = e^{-iQ(v \cdot x)} H^+ \] (A.12)

antiparticle: \[ \varphi = e^{+iQ(v \cdot x)} H^- \quad \bar{\varphi} = e^{+iQ(v \cdot x)} h^- \] (A.13)

(The relevant degree of freedom for particles \( h^+ \) is denoted as \( Q_v \) in the text.)

The lagrangian in the particle sector then reads

\[
\mathcal{L} = \bar{h}^+ i(v \cdot D) h^+ + \bar{h}^+ iD^+ H^+ + H^+ iD^+ h^+ ,
\]

where the field \( H^+ \) carries twice the heavy quark mass. The integration over \( H^+ \) in the concerning generating functional of QCD can easily be carried out, defining an effective lagrangian

\[
\mathcal{L}^{HQET} = \bar{h}^+ i(v \cdot D) h^+ + \bar{h}^+ iD^+ H^+ + \mathcal{K}_v + \mathcal{M}_v + O(m_Q^2) ,
\]

where the two operators \( \mathcal{K}_v, \mathcal{M}_v \) of order \( m_Q^{-1} \)

\[
\mathcal{K}_v = -\frac{1}{2m_Q} \bar{h}^+ iD^+ (g_{\mu \nu} - \nu_{v} \nu) D^\nu h^+ ,
\]

\[
\mathcal{M}_v = -\frac{g}{4m_Q} \bar{h}^+ \sigma^{\mu \nu} F_{\mu \nu} h^+ ,
\]

can be identified in the rest–frame \( \vec{v} = 0 \) with the non–relativistic kinetic energy and the chromomagnetic Pauli–term, respectively (\( F_{\mu \nu} \) denotes the gluonic field strength tensor).

The \( SU(2) \) spin–symmetry, that arises in the limit \( m_Q \to \infty \), can be implemented by generalizing the Pauli matrices \( \sigma_i \) in terms of velocity–dependent generators

\[
S_i(v) = \gamma_5 \gamma_i
\]

with \( \varepsilon_i \) being suitable polarization vectors satisfying \( v \cdot \varepsilon_i = 0 \) and \( \varepsilon_i^2 = -1 \).

Their effect on heavy quark spinors \( u_{\uparrow, \downarrow} \) reads

\[
S_3(v) u_{\uparrow}(v) = u_{\uparrow}(v) , \quad S_3(v) u_{\downarrow}(v) = -u_{\downarrow}(v)
\]

(A.17)

B Fierz Transformations

There exist two types of Fierz–rearrangements of the color matrices in the current–current interaction \([\mathcal{L}]\), defined by \([\mathcal{L}]\)

\[
\sum_{\alpha=1}^{N_c^2-1} \frac{1}{2} \left( \frac{(\lambda_\alpha^c)^{ij} (\lambda_\alpha^c)^{kl}}{2} \right) \delta_{il} \delta_{kj} - \frac{1}{N_c} \sum_{\alpha=1}^{N_c^2-1} \frac{1}{2} \frac{(\lambda_\alpha^c)^{ij} (\lambda_\alpha^c)^{kl}}{2} \]

(B.1)
\[
\sum_{\alpha=1}^{N_c^2-1} \left( \frac{\lambda^\alpha_F}{2} \right)_{ij} \left( \frac{\lambda^\alpha_F}{2} \right)_{kl} = \frac{1}{2} \left( 1 - \frac{1}{N_c} \right) \delta_{il} \delta_{kj} + \frac{1}{2N_c} \epsilon_{mik} \epsilon_{mlj} .
\] (B.2)

The first identity transforms the current–current interaction into an attractive color singlet \((q\bar{q})_1\) interaction and a repulsive color \((N_c^2 - 1)\)–plet interaction. For later applications with diquarks within composite baryons, we prefer the second alternative (B.2), where besides of an attractive interaction in the \((q\bar{q})_1\) channel, the second part yields attraction in the color–antisymmetric \((qq)\) channel. In the limit \(N_c \to \infty\) the color singlet terms in (B.1), (B.2) dominate and have equal weight factors.

In the Fierz–transformation of light flavors use has also been made of the completeness relation
\[
\frac{1}{2} \delta_{ij} \delta_{kl} = \sum_{\alpha=0}^{N_f^2-1} \left( \frac{\lambda^\alpha_F}{2} \right)_{il} \left( \frac{\lambda^\alpha_F}{2} \right)_{kj} ; \quad \left( \frac{\lambda^0_F}{2} \right)^2 = \sqrt{\frac{1}{2N_f}} 1_F .
\] (B.3)

Finally, the Fierz–transformations of Dirac matrices read
\[
(\gamma^\mu)_{ij} (\gamma^\nu)_{kl} = \sum_{\alpha=1}^{4} (\hat{O}^\alpha)_{il} (\hat{O}_\alpha)_{kj} = \sum_{\alpha=1}^{4} (\hat{O}^\alpha C)_{ik} (C\hat{O}_\alpha)_{lj} ,
\] (B.4)
\[
\text{where} \quad \hat{O}^\alpha = \begin{cases} 1, i\gamma_5, i\gamma^\mu, i\sqrt{2} \gamma^\mu \gamma_5 \end{cases} , \quad (\alpha = 1, \ldots, 4) \] (B.5)

and \(C = i\gamma^2\gamma^0\) is the charge conjugation matrix.

## C  Feynman Integrals

We present here the values of several integrals needed when calculating the functional determinant (39). In our approach the denominator in Euclidean space of the light quark propagator is regularized with proper–time methods

\[
\int_0^\infty ds e^{-(k^2 + m^2)s} \to \int_{1/\Lambda^2}^\infty ds e^{-(k^2 + m^2)s} ,
\] (C.1)
\[
\int_0^\infty ds\, s\, e^{-(k^2 + m^2)s} \to \int_{1/\Lambda^2}^\infty ds\, s\, e^{-(k^2 + m^2)s} ,
\] (C.2)
whereas the heavy quark propagator is unaffected from regularizations\textsuperscript{8}. One obtains

\begin{align*}
I_i^i &= \frac{iN_c}{16\pi^4} \int_{1/\Lambda^2}^{reg} \frac{d^4k}{k^2 - (m^i)^2} \\
&= \frac{N_c}{16\pi^4} \int_{1/\Lambda^2} dx \int ds \int d^4k e^{-(k^2 + (m^i)^2)s} \\
&= \frac{N_c}{16\pi^2} (m^i)^2 \Gamma(-1, (m^i)^2/\Lambda^2) , \quad (C.3) \\
I_{ij}^j &= -\frac{iN_c}{16\pi^4} \int_{1/\Lambda^2}^{reg} \frac{d^4k}{(k^2 - (m^j)^2)(k^2 - m^j_i)} \\
&= \frac{N_c}{16\pi^4} \int_{1/\Lambda^2} dx \int ds \int d^4k e^{-(k^2 + x(m^i)^2 + (1-x)(m^j)^2)s} \\
&= \frac{N_c}{16\pi^2} \int_{1/\Lambda^2} dx \Gamma(0, (x(m^i)^2 + (1-x)(m^j)^2)/\Lambda^2) , \quad (C.4) \\
I_3^i &= \frac{iN_c}{16\pi^4} \int_{1/\Lambda^2}^{reg} \frac{d^4k}{(k^2 - (m^j)^2)(v \cdot k + i\epsilon)} \\
&= \frac{iN_c}{16\pi^4} \int_{1/\Lambda^2} dx \int ds \int d^4k e^{-(k^2 + (m^j)^2)s} \frac{1}{k_4 + i\epsilon} \\
&= \frac{N_c}{16\pi^3} \int_{1/\Lambda^2} dx \int ds \int d^3k e^{-(k^2 + (m^j)^2)s} \\
&= \frac{N_c}{16\pi^2} \sqrt{\pi} \int_{1/\Lambda^2} dx \Gamma(1/2, (x(m^i)^2 + (1-x)(m^j)^2)/\Lambda^2) , \quad (C.5) \\
I_4^i &= \frac{iN_c}{16\pi^4} \int_{1/\Lambda^2}^{reg} \frac{d^4k}{(k^2 - (m^i)^2)(k^2 - (m^j)^2)(v \cdot k + i\epsilon)} \\
&= \frac{iN_c}{16\pi^4} \int_{1/\Lambda^2} dx \int ds \int d^4k e^{-(k^2 + x(m^i)^2 + (1-x)(m^j)^2)s} \frac{1}{k_4 + i\epsilon} \\
&= \frac{N_c}{16\pi^3} \int_{1/\Lambda^2} dx \int ds \int d^3k e^{-(k^2 + x(m^i)^2 + (1-x)(m^j)^2)s} \\
&= \frac{N_c}{16\pi^2} \sqrt{\pi} \int_{1/\Lambda^2} dx \Gamma(1/2, (x(m^i)^2 + (1-x)(m^j)^2)/\Lambda^2) \sqrt{x(m^i)^2 + (1-x)(m^j)^2} , \quad (C.6) \\
I_5(\omega = v \cdot v') &= \frac{iN_c}{16\pi^4} \int_{1/\Lambda^2}^{reg} \frac{d^4k}{(k^2 - m^2)(v \cdot k + i\epsilon)(v' \cdot k + i\epsilon)} \\
&= -\frac{N_c}{16\pi^4} \int_{1/\Lambda^2} dx \int ds \int d^4k e^{-(k^2 + m^2)s} \frac{1}{(v \cdot k + i\epsilon)(v' \cdot k + i\epsilon)} \\
\end{align*}

\textsuperscript{8}Note that in going to Euclidean space the \textit{i}\textepsilon–prescription in the heavy quark propagator must be treated properly.
\[
\lambda_{ij}^1 = \sqrt{Z_H^i Z_H^j} \left( \frac{1}{2} ((Z_H^i)^{-1} + (Z_H^j)^{-1}) - \frac{1}{2} (m^i - m^j)^2 I_{ij}^4 \right), \quad (D.1)
\]

\[
\lambda_{ij}^2 = \sqrt{Z_K^i Z_K^j} \left( \frac{1}{2} ((Z_K^i)^{-1} + (Z_K^j)^{-1}) - \frac{1}{2} (m^i - m^j)^2 I_{ij}^4 \right), \quad (D.2)
\]

\[
\lambda_{ij}^3 = \sqrt{Z_H^i Z_K^j} \left( \frac{1}{6} (I_3^i + I_3^j) - (m^i + m^j) I_{ij}^2 \right.
\]
\[
\left. - \left( \frac{4}{3} m^i m^j + \frac{1}{6} (m^i - m^j)^2 I_{ij}^4 \right) \right), \quad (D.3)
\]

\[
\lambda_{ij}^4 = \sqrt{Z_K^i Z_K^j} \left( \frac{1}{6} (I_3^i + I_3^j) + (m^i + m^j) I_{ij}^2 \right.
\]
\[
\left. - \left( \frac{4}{3} m^i m^j + \frac{1}{6} (m^i - m^j)^2 I_{ij}^4 \right) \right), \quad (D.4)
\]

\[
\lambda_{ij}^5 = \sqrt{Z_H^i Z_K^j} \left( \frac{1}{2} ((Z_H^i)^{-1} + (Z_K^j)^{-1}) - \frac{1}{2} (m^i - m^j)^2 I_{ij}^4 \right)
\]
\[
+ \left( \frac{2}{3} m^i m^j - \frac{1}{6} (m^i - m^j)^2 I_{ij}^4 \right), \quad (D.5)
\]

\[
\lambda_{ij}^6 = \sqrt{Z_H^i Z_K^j} \left( \frac{1}{2} ((Z_H^i)^{-1} + (Z_K^j)^{-1}) - \frac{1}{2} (m^i + m^j)^2 I_{ij}^4 \right). \quad (D.6)
\]