Charge asymmetry in $e^\pm e^-$, $ep$, $e\gamma$, $\gamma\gamma$ collisions

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Abstract

Study of charge asymmetry within a neutral system $T$ produced in processes $AB \rightarrow T X$ can help solve many problems in particle physics. We collect here some problems, which have been studied well, as well as proposals for future activity with a brief discussion of the main results expected in each case. These are:

1. $e^+e^- \rightarrow TX$. a) $T = \pi^+\pi^-, X = \gamma, \rho f \gamma, \phi f \gamma,...$ vertices.
   b) $T = \pi^+\pi^-, K^+K^-, X = e^+e^-$. Phases of $\pi\pi$ scattering, resonances.
   c) $T = c\bar{c}, bb, X = e^+e^-$. Discovery of C–even $c\bar{c}$ and $bb$ resonances.
   d) $e\gamma \rightarrow e\ell\ell, T = \ell\ell$. Study of possible CP violation in $t$-quark physics.

2. $ep \rightarrow e\pi^+\pi^-X$. a) Possible discovery of the odderon.
   b) Measuring the phases of the forward $\gamma p \rightarrow \rho p$ and $\gamma A \rightarrow \rho A$ amplitudes.
   c) Study of axial current coupling to the Pomeron ($Zp \rightarrow f_2X$).

3. Violation of the quark–hadron duality.

4. Weighted structure functions in DIS.

5. $e\gamma \rightarrow eW^+W^-$. Study of possible strong interaction in the Higgs sector.

6. Polarization charge asymmetry in $\gamma\gamma$ collisions

The greatest part of discussion in the paper is phenomenological with minimization of model dependent details.

1 Introduction

We consider the processes $AB \rightarrow T X$ with production of some truly neutral particle system $T$, which is well separated from other reaction products $X$. The charge asymmetry of the reaction products within the system $T$ — the difference in the distributions of the produced particles and antiparticles — can be used as a powerful tool to study different problems of particle physics. This charge asymmetry can be of different origin:

- CP violation in production or decay.
- Specific charge content of the initial state, for example, quark content of proton.
- Interference of production mechanisms leading to the same final subsystem $T$ via intermediate states with different C–parity.
The well-known example is the forward–backward asymmetry in the process $e^+e^- \rightarrow \mu^+\mu^-$ near $Z$ resonance. The vector current (mainly, the photon in the intermediate state) produces a $C$-odd system, while the axial current in $Z$ boson produces a $C$–even system. Their interference results in the forward–backward asymmetry of the muons. Another example is the observed charge asymmetry in the charm photoproduction in the proton fragmentation region \cite{[1]}, which involves the second and the third mechanisms.

We consider the third mechanism and (in the end) the first mechanism. In the most considered cases the discussed charge asymmetry is determined by the controllable $C$–parity of the intermediate state, for example, by the photon with $C = -1$ or $Z$ with axial $C$–even current. The sign of the asymmetry is defined by the charge sign of the colliding particles $A$ and (or) $B$.

- Hereafter we denote the momenta of colliding particles as $p_1$ and $p_2$, $s = (p_1 + p_2)^2$, $z$–axis is directed along the collision axis, transverse components of the momenta are those orthogonal to both $p_1$ and $p_2$, they are labeled with bold letters. Let the considered neutral system $T$ contain particles $a_i$ with momenta $p_{ia}$ and antiparticles $\bar{a}_j$ with momenta $p_{j\bar{a}}$. The operator of charge conjugation for this system $\hat{C}_T$ acts as:

$$\hat{C}_T M(p_{ia}, p_{j\bar{a}}) = M(p_{j\bar{a}}, p_{ia}).$$

- **Two–particle final states.** We consider in detail the production of two–particle systems $T = P^+P^-$, in the processes $e^+e^- \rightarrow e^+e^- T$, $e\gamma \rightarrow eT$, $\gamma p \rightarrow TX$, $e p \rightarrow eTX$, $\gamma\gamma \rightarrow TX$ with $P = \pi$ or $c$, or $W$, or $\mu$, or ... For such system $\hat{C}_T M(p_+, p_-) = M(p_-, p_+)$. Let momenta of $P^\pm$ and the corresponding light–cone variables $x_\pm, y_\pm$ be

$$p_\pm = (\varepsilon_\pm, p_{\pm\perp}, p_{\pm\perp}), \quad x_\pm = \frac{2p_{\pm\perp}}{s} = \frac{\varepsilon_\pm + p_{\pm\perp}}{2E_1}, \quad y_\pm = \frac{2p_{\pm\perp}}{s};$$

We define

$$\hat{k} = p_+ - p_-, \quad r = p_+ - p_-, \quad M = \sqrt{k^2}, \quad \beta = \sqrt{1 - \frac{4m_P^2}{M^2}};$$

and

$$x = x_+ + x_-, \quad y = y_+ + y_-, \quad t = (p_1 - k)^2 = -\frac{k_\perp^2 + M^2(1 - x)}{x}.$$}

When considering pion pair production ($T = \pi^+\pi^-$), we set $\beta = 1$.

Let $x$ axis be directed along vector $k_\perp$ and $\psi$ be the angle between $x$ axis and some fixed axis. Besides, for vector $r$ we define the angles in the c.m.s. of $T$ system as

$$r_{c.m.s.} = \beta M(0, \sin \theta \cos \phi, \sin \theta \cos \phi, \cos \theta).$$

- **The phenomenon of charge asymmetry is the difference in the distributions of particles and antiparticles.** It is determined by the part of differential cross section that changes its sign with $r^\mu \rightarrow -r^\mu$ change.

Particularly, we describe the forward–backward (FB) asymmetry by variables

$$\xi = \frac{x_+ - x_-}{x} \quad \text{or} \quad \eta = \frac{y_+ - y_-}{y} \quad \text{or} \quad K_\perp = \frac{px - p_{\perp}}{\varepsilon_+ + \varepsilon_-} \bigg|_{c.m.s.}.$$}

Variables $\xi$ and $\eta$ are useful for description of the FB asymmetry for the system $T$ moving approximately along the 3–momentum of $p_1$ and $p_2$ respectively, while $K_\perp$ describes the entire FB asymmetry. For system $T$ moving along $p_1$ (with $x \gg y$) or $p_2$ (with $y \gg x$) we have, respectively, $K_\perp \rightarrow \xi$ or $K_\perp \rightarrow -\eta$. Below we mainly consider the first case.
We also describe the transverse (T) asymmetry by variable
\[ v = \frac{p_{2\perp}^2 - p_{1\perp}^2 - K\cdot k_{\perp}^2}{M|k_{\perp}|} \equiv \frac{(\rho_{\perp} k_{\perp})}{M|k_{\perp}|} \text{ with } \rho_{\perp} = r_{\perp} - K\cdot k_{\perp}. \] (3b)

The charge conjugation operator (1) acts on variables introduced as
\[ \hat{C}_P K_{\perp} = -K_{\perp}, \hat{C}_P k = k \text{ and } \hat{C}_P v = -v. \]

When \(|k_{\perp}| \gg |k_{\perp}|\), simple relations between the angles in c.m.s. take place:
\[ \xi = \beta \cos \theta, \quad v = \beta \sin \theta \cos \phi. \] (3c)

The phase space element for the produced system is
\[ d\Gamma = d^3p_{+} d^3p_{-} = dt dM^2 dx \frac{2dv d\xi}{\sqrt{\beta^2 - v^2 - \xi^2}} d\psi \Rightarrow 4\pi dt dM^2 dx \frac{dv d\xi}{\sqrt{\beta^2 - v^2 - \xi^2}}. \] (4)

The latter form is obtained after integration over \(\psi\).

In respect to eq. (3c), the study of charge asymmetry for two-particle states is similar to the well known partial wave analysis.

- The magnitude of the asymmetry related to some C–odd weight function \(w (\hat{C}_T w = -w)\) is given by integration over some charge symmetric domain \(\mathcal{D} (\hat{C}_T \mathcal{D} = \mathcal{D})\):

\[ \Delta\sigma_w = \int_{\mathcal{D}} \frac{w}{\sqrt{<w^2>}} \, d\sigma \quad \text{with} \quad <w^2> = \int_{\mathcal{D}} \frac{w^2 d\sigma}{\sigma_B}, \quad \sigma_B = \int_{\mathcal{D}} d\sigma. \] (5a)

In numerical estimates below we use the step functions \(w = \epsilon(K_{\perp})\) or \(\epsilon(v)\) for FB or T asymmetries, respectively (with \(\epsilon(x) = \begin{cases} 1 & \text{at } x > 0 \\ -1 & \text{at } x < 0 \end{cases}\) and \(\epsilon^2(x) = 1\)):

\[ \Delta\sigma_K = \int d\sigma(K_{\perp} > 0) - \int d\sigma(K_{\perp} < 0), \quad \Delta\sigma_v = \int d\sigma(v > 0) - \int d\sigma(v < 0). \] (5b)

Value of the given asymmetry is determined by its Statistical Significance defined via the numbers of signal and background events [2]. With the integral luminosity \(\mathcal{L}\):

\[ SS = \frac{\mathcal{L}|\Delta\sigma_w|}{\sqrt{\mathcal{L}\sigma_B^2}}. \] (5c)

In the calculations we use parameters of particles from ref. [3].

- Standard C–even contributions disappear in our signal (as the charge asymmetric part of the cross section disappears in the background). Therefore, to extract the charge asymmetry signal from the data, it is not necessary to know the background with high precision. Only the ratio of signal to statistical fluctuations of the background is essential.

2.1 \(e^+e^- \rightarrow \pi^+\pi^- + \ldots\), etc.

Radiative return studies

There are two main mechanisms of the dipion production in this reaction.

\(\diamond\) Incident \(e^-\) (or \(e^+\)) emits a bremsstrahlung photon (initial state radiation, ISR). Next,
this electron collides with the positron, producing C-odd dipion.

Incident $e^+e^−$ system transforms to dipion (typically, $\rho$, $\omega$ or $\phi$). Next, this system emits a photon, turning the dipion to the C-even state (final state radiation, FSR).

The ratio of cross sections of these processes is of the order of emitting masses ratio, $\sigma_{\text{FSR}}/\sigma_{\text{ISR}} \sim (m_e/m_\pi) \lesssim 0.01$. Nevertheless, better accuracy is necessary in this problem.

To separate ISR and FSR contributions in the data, it was noted that the dipions produced in ISR and FSR processes have opposite C-parity, giving charge asymmetry in the final state. Measuring this charge asymmetry helps separate the effects and extract the cross section $e^+e^− \to \gamma^* \to \pi^+\pi^−$ with high precision [4]. Since FSR effect is small, even rough model of point-like pions (QED) is considered suitable for the analysis of the experimental data [5] to find the $e^+e^− \to \pi^+\pi^−$ cross section with high precision (radiative return method).

Note that in the analysis of these data only FB asymmetry is considered. Accounting the T asymmetry can also be useful (its sign is different for pairs moving along positron and electron directions).

The photons from ISR are concentrated along the directions of incident electron or positron within the angle $\sim 1/\gamma \equiv m/E$. The photons from FSR accompany produced pions and have roughly uniform angular spread. Thus, the interference is $\lesssim 1/\gamma$ (with logarithmic enhancement appearing after the detailed calculation). Its magnitude is about few percent for $\sqrt{s} \sim 1$ GeV [5], and it disappears at higher energies.

One can use this interference to study poorly known vertices $\rho\sigma\gamma$, $\rho f_2\gamma$, $\phi f_0\gamma$, $\phi f_2\gamma$, ... Detailed study of effective mass $M$ dependence in the charge asymmetry effects can help separate effects of these couplings. These results can give also more precise values of $\sigma(e^+e^− \to \gamma^* \to \pi^+\pi^−)$.

2.2 $e^+e^− \to e^+e^− \pi^+\pi^−$, phases of $\pi\pi$ scattering, resonances.

The charge asymmetry of pions in this process appears due to interference between the amplitudes given by diagrams at fig. 1. Open circles in the bremsstrahlung amplitudes describe two QED diagrams for the virtual Compton scattering. Other diagrams contribute negligibly to the cross section.

This asymmetry was considered first in Ref. [6] for small values $k^2_1 \ll m_\pi^2$ (which contribute weakly to the observable effects). The presented equations from Ref. [6] are free from that limitation (similar equations were also obtained in Ref. [5]).

The considered charge asymmetric term in the differential cross section is the sum of terms given by interference between two–photon amplitude and the bremsstrahlung amplitudes with radiation from electron $d\sigma_{2e^-}$ or positron $d\sigma_{2e^+}$:

$$d\sigma_{\text{asym}} = d\sigma_{2e^-} + d\sigma_{2e^+} \quad \text{with} \quad \hat{C}_{\pi}d\sigma_{2e^\pm} = -d\sigma_{2e^\pm}. \quad (6)$$

Figure 1: Two–photon (left) and bremsstrahlung (right) production of pion pairs
The main contribution to the term $d\sigma_{2e^-}$ is given by almost real photon $q_2$. With the logarithmic accuracy (which is about 5% for modern colliders) this contribution is given by the convolution of the equivalent photon $q_2$ spectrum with the charge asymmetric interference for the subprocess $e\gamma \rightarrow e\pi^+\pi^-$. We write it in the helicity basis for the subprocess $\gamma\gamma \rightarrow \pi\pi$

$$
d\sigma_{2e^-} = d\sigma_{e\gamma} \otimes d\sigma_{e\gamma}, \quad d\sigma_{e\gamma} = \frac{\alpha^2}{32\pi^3 q_1^2 M^2 s x \Gamma} \sum_{a=\pm,0} C^{a+} \frac{\text{Re}(F^+_{a}) d\tau}{d\Gamma_{\pi\pi}} \Rightarrow
$$

$$
d\sigma_{2e^-} = \frac{\alpha^3}{8\pi^4} \rho_2^{++} \frac{L_2}{s^2 x [M^2(1-x) + k_{\perp}^2]} \sum_{a=\pm,0} g^{a+} \frac{\text{Re}(F^a_{a}) d\tau}{d\Gamma_{\pi\pi}},
$$

$$
\rho_2^{++} = \frac{2 - 2y_2 + y_2^2}{y_2^3}, \quad L_2 = \ln \left( \frac{|q_2^2|_{\max}(1-y_2)}{m_{\rho}^2 m_{\pi}^2} \right),
$$

$$
y_2 = \frac{2q_2 P_1}{s} = \frac{M^2(1-x) + k_{\perp}^2}{s x (1-x)}, \quad |q_2^2|_{\max} \approx \min \left( \frac{k_{\perp}^2}{1-y_2}, m_{\rho}^2, M^2 \right).
$$

$$
g^{++} = \xi(2-x) + v \left[ (1-x) \frac{M}{k_{\perp}} + \frac{2 - 2x + x^2}{2(1-x)} \frac{k_{\perp}^2}{M} \right],
$$

$$
g^{-+} = -v \left[ 2 - 2x + x^2 - \frac{k_{\perp}^2}{2} \frac{4v^2 - \xi^2 - 3}{1 - \xi^2} \right] \frac{M}{k_{\perp}} - \xi(2-x) \frac{2v^2 + \xi^2 - 1}{1 - \xi^2},
$$

$$
g^{0+} = \sqrt{\frac{1-x}{2(1-x)}} \left[ (2-x)(1-\xi^2) \frac{M}{k_{\perp}} - 4\xi v - \frac{2-x}{1-x}(2v^2 + \xi^2 - 1) \frac{k_{\perp}^2}{M} \right].
$$

The contribution $d\sigma_{2e^+}$ is obtained by changing the variables

$$
d\sigma_{2e^+} = -d\sigma_{2e^-}(p_1 \leftrightarrow p_2, q_1 \leftrightarrow q_2), \quad M_{ab}(q_1, q_2, \Delta) \rightarrow (-1)^{a+b} M_{ba}(q_2, q_1, \Delta).
$$

Note that $\hat{C} \pi M^{\pm+} = M^{\pm+}$, $\hat{C} \pi M^{0+} = -M^{0+}$, $\hat{C} \pi g^{++} = -g^{++}$, $\hat{C} \pi g^{0+} = g^{0+}$.

These equations show that after azimuthal averaging the FB asymmetry (in $\xi$) does not depend on the value of amplitude $M_{+,+}$, while the T asymmetry (in $v$) includes both $M_{+,+}$ and $M_{+,+}$ contributions. Comparing this to the two–photon case, a new term with amplitude $M_{0,0}$ appears.

At small $k_{\perp}$ the considered effects are small, while the background (mainly two-photon production) is strongly peaked. At the same time, the effect only weakly depends on $x$, while two-photon background increases at $x \approx y$, and bremsstrahlung contribution itself is strongly peaked at $x \approx 1$ (center and edges of the rapidity scale). Therefore, cuts on $k_{\perp} \sim 100$ MeV (from below) and on $x$ (from both sides) are useful to increase SS value (\cite{4}). For the contribution $d\sigma_{2e^-}$, the transverse momentum of scattered electron $p_{e\perp} = -k_{\perp}$ with high accuracy.

Here we present some numbers for point–like pions (within QED).

\diamond For DAΦNE with $2E = 1$ GeV for the effective mass interval $M = 300 - 350$ MeV with cuts $k_{\perp} \geq k_0 = 100$ MeV and $0.95 \geq x, y \geq 0.4$ we have $\sigma^B = 14.6$ pb and $\Delta\sigma_K = -1.07 \text{pb}$. At $\mathcal{L} = 500 \text{ pb}^{-1}$ it gives $SS_K \approx 6.3$. The value of SS is increased in the mass interval 350-400 MeV.

\diamond For PEP-II with $\sqrt{s} = 10$ GeV for the effective mass interval $M = 475 - 525$ MeV with cuts $k_{\perp} \geq k_0 = 150$ MeV and $0.95 \geq x, y \geq 0.3$ we have $\sigma^B = 17.2$ pb and $\Delta\sigma_K = -1.62$ pb. At $\mathcal{L} = 30 \text{ fb}^{-1}$ it gives SS=68!
The strong interaction of pions increases both two-photon amplitude and (even more) the form-factor as compare QED. It results in enhancement of SS. Besides, the choice of suitable cut in $k_\perp$ and weight function should be also subject of special studies which should enhance SS.

Physical picture.

At $M < 500$ MeV the Born QED model (point-like pions) describes reasonably the $\gamma\gamma \to \pi^+\pi^-$ amplitudes, while their phases and the phase of the form factor reproduce phase shifts for the $S$ and $P$ waves of the elastic $\pi\pi$ scattering since the unitarity relation is saturated here with two-pion intermediate states. Therefore, the data on the charge asymmetry can give us the energy dependence for the phase shifts $\delta_f^I$ of $\pi\pi$ scattering via the quantity $\cos(\delta_0^I - \delta_1^I)$. One can expect to obtain here the precise values of the scattering lengths. In this energy region non-trivial effects are given by the first term (with $M_{++}$) of eq. (7a) only. For other amplitudes the point-like QED for pions seems to be a good approximation.

![Figure 2: The FB ($\Delta\sigma_K$) and T ($\Delta\sigma_v$) asymmetries and the background, QED for pions – left panel, muons – right panel](image)

Let us present some curves for point-like pions at $\sqrt{s} = 1$ GeV. The fig. 2 (left panel) represents integral effects. At low $M$ the forward-backward asymmetry is higher than the transverse. At the right panel of fig. 2 we show for comparison the same asymmetries for muons in the process $e^+e^- \to e^+e^- \mu^+\mu^-$ (based on equations from Ref. [9]). Comparing contributions of different helicity amplitudes to FB and T asymmetries (fig. 3),

![Figure 3: The contributions of different amplitudes to FB (left panel) and T (right panel) asymmetry](image)

one can see that in the FB asymmetry the amplitude $M_{++}$ dominates. It is useful for the
study of $M \lesssim 1$ GeV and $f_0$'s. In the $T$ asymmetry contributions of $M_{++}$ and $M_{+-}$ almost compensate each other. Strong interaction effects generally break this compensation.

◊ At higher energies the discussed observations should help distinguish between different models for the resonances having two-pion and two-photon decay modes. For example, it can give us a new information about $f_0(980)$ and $f_2(1270)$ mesons, etc.

Figure 4: The charge asymmetries of pions due to ($\rho$, $f_0(980)$) interference.

To get an idea about the magnitude of the charge asymmetry with resonances, we consider a toy model with pion form factor for $P$-wave + $S$-wave given by (i) QED + $f_0(980)$ at $\Gamma_{f_0} = 100$ MeV or (ii) with additional phase shift $\vartheta$ (giving effect of possible $\sigma$ state). In fig. 4 the asymmetries in $K^-$ and $\nu$ (with suitable kinematical cuts) are compared with charge symmetric background (solid lines) and pure QED effects (dotted, etc. lines)

At $M > 1.1$ GeV the main non–QED contribution is given by the term $\text{Re}(F^\ast_\pi M_{++}) \propto \text{Re}(D^\ast_\rho D^0_\vartheta) (\rho - f_2$ interference). The overlap factor which is proportional to this real part is shown in Fig. 7.

2.3 Distinguishing processes

• Two above reactions can be distinguished well, if in addition to pions, the photon (for $e^+e^- \rightarrow \pi^+\pi^-\gamma$) or scattered electron or positron (for $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$) is observed. For the second reaction, the major part of the charge asymmetric effect corresponds to the case when the total transverse momentum of produced pion pair $\gtrsim 100$ MeV. Therefore, the scattered $e^-$ (or $e^+$) is recordable almost without loss of statistics (with scattering angle $\gtrsim 200$ mrad at DAΦNE).

• In the inclusive case, with observation of pions only, these processes can be distinguished with measurements of the missing mass $M_m$,

$$M_m^2 = (p_1 + p_2 - k)^2.$$  

◊ For the process $e^+e^- \rightarrow \pi^+\pi^-\gamma$ it should be zero ($M_m = 0$). To eliminate $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ contribution with $\pi^0 \rightarrow 2\gamma$ decay, one can use cut $M_m < 130$ MeV.

◊ For the process $e^+e^- \rightarrow e^+e^-\pi^+\pi^-$, the missing mass is generally large, $M_m \sim \sqrt{s}$, e.g. one can use cut $M_m > \sqrt{s}/3$. 


2.4 Kaons, heavy quarks in $e^+e^- \rightarrow e^+e^- T, e\gamma \rightarrow eT$

• Eqs. (7) also describe charge asymmetry of kaons in the process $e^+e^- \rightarrow e^+e^- K^+K^-$ (with some corrections due to $\beta \neq 1$). At $M_{KK} \approx 1$ GeV, this asymmetry is given by the phase difference between the amplitudes of $\phi$ meson and still mysterious $f_0(980) + a_0(980)$ mesons production.

• Heavy quarks in $e^+e^- \rightarrow e^+e^- c\bar{c}, e^+e^- \rightarrow e^+e^- b\bar{b}$, $e\gamma \rightarrow e\bar{b}$. These heavy quarks should be seen mainly as D (or B) mesons. Near the threshold these D mesons are produced mainly via resonance states, excitation of $J/\Psi$ (or $\Upsilon$) for C-odd states and yet unobserved mesons with spin 0 or 2 for C–even states. Charge asymmetry in the mentioned processes (e.g., in LEP data) can help discover new C–even (with $J = 0$ or 2) $c\bar{c}$ and $b\bar{b}$ resonances. This problem for the $e\gamma \rightarrow e\bar{b}$ process can be considered as the subject for possible LINX Photon Collider (see [10] for details of project). (The similar analysis of HERA data can also be useful for such discovery.)

• $e\gamma \rightarrow et\bar{t}$. Effects of New Physics are expected to be visible well in the interactions of very heavy $t$–quarks. The analysis of charge asymmetry in the process will be a new effective tool for the study of possible $\mathcal{CP}$ violation effects related to New Physics. The quark-hadron duality works here well since $t$–quark decays before formation of a bound state. Nevertheless, the equation for muons, describing charge asymmetry in QED, cannot be used here since the picture is strongly changed by contribution of the axial current from $Z$ boson exchange. The contribution of the $Z$–bremsstrahlung diagram is essential in the entire phase space, while the contribution from the $t$–channel $Z$ boson exchange becomes substantial with the large transverse momentum of the scattered electron. These effects should be studied in details to distinguish from the effects of pure $\mathcal{CP}$ violation related to the New Physics. The effect of axial $Z\gamma t\bar{t}$ anomaly will also be observed at small transverse momenta of electron [11].

3 Diffractive type process $ep \rightarrow e\pi^+\pi^- p', T = \pi^+\pi^-$

In this section we consider the processes in high energy $ep$ and $eA$ collisions with production of dipion system at $|t| \ll s$ and separated well from other produced hadrons (which can be treated as the products of proton excitation $p'$) — with large rapidity gap. The discussed processes are observable at HERA and similar colliders.

To treat the discussed $ep$ collisions with the notation introduced in Sect. 1, we denote the exchanged photon (or $Z$) momentum as $p_1 = p_e - p'_e$ and its virtuality $Q^2 = -p^2_1$. Besides, we denote, as usual, with $t(\approx k^2_\perp)$ the squared momentum transferred from photon (or $Z$) to dipion (here $k_\perp$ is the transverse momentum of dipion in the $\gamma^*p$ or $Z^*p$ c.m.s.). The different values of virtuality $Q^2$ and dipion transverse momentum $k_\perp$ provide tools to study quite different problems of the hadron physics.

■ The mechanisms of dipion diffractive production. Some features of the case with almost real photons.

• The main source of the C–odd dipions (mainly $\rho$–mesons) is the standard Pomeron exchange ($\mathcal{IP}$) with proton. This amplitude is known well.

◊ The C-odd dipion can be also produced via bremsstrahlung mechanism, like in $e^+e^-$ collisions. However, the amplitude of this production is about $\alpha \sim 10^{-2}$ times less than that via Pomeron. It makes this mechanism negligible and corresponding calculations of ref. [13] meaningless.
• The mechanisms of the C-even dipion production (e.g., in $f_2(1270)$ state) are enumerated in the table,

| mechanism       | reggeon $\rho$, $\omega$ | Primakoff | odderon      | $Z^*\text{IP}$ collision |
|-----------------|---------------------------|-----------|--------------|--------------------------|
| where substantial | $\sqrt{s_{\gamma p}} \lesssim 10$ GeV | $|k_\perp| < 100$ MeV | $|k_\perp| > 200$ MeV, small $Q^2$ | $Q^2 \gtrsim 1000$ GeV^2 |

• The reggeon $\rho$, $\omega$ exchanges. The corresponding cross section is obtained by simple Regge extrapolation from low energy data. In the main HERA energy range this cross section is $\sim 0.15 - 0.3$ nb which is extremely small for observation.

• *The Primakoff effect* (dipion production in collision of incident photon with photon emitted by the proton) is discussed in detail in sect. 3.2. Its cross section is about 8 nb for $f_2$ production. It is concentrated within a narrow interval of $k_\perp$ while at $k_\perp \gtrsim 300$ MeV it is about 0.2 nb, i.e. is negligible.

• *The odderon exchange* is of great interest for the hadron physics. The odderon is yet elusive but necessary element of the QCD motivated hadron physics. The modern status of the odderon in comparison with the Pomeron is discussed in Appendix. It looks reasonable to expect that this contribution is not very low and its transverse momentum dependence is similar to that for other reggeons.

• At large virtuality $Q^2$ in addition to the photoproduction of dipions by virtual photon, one should consider also the $Z$-production of dipions, where virtual $Z$ is emitted from the electron. Via the dominant Pomeron exchange, vector part of $Z$-boson produces C–odd dipions like photon, while its axial part produces C–even dipions.

■ In sect. 3.1-3.3 we consider processes with almost real photons ($Q^2 \approx 0$ — small or negligible electron escape angle) treating them as $\gamma p \rightarrow \pi^+\pi^- p'$ processes. For these studies, recording of scattered electron and proton looks unnecessary. Therefore, in the estimates of the statistical significance SS (5c) for HERA experiments we use integrated effective luminosity and the $\gamma p$ center of mass energy

$$L_{\gamma p} \approx 100 \text{ nb}^{-1} \quad \text{for} \quad \sqrt{s_{\gamma p}} \approx 100 \div 200 \text{ GeV}. \quad (10)$$

Recalculations for other values of luminosity and $\sigma_f$ are evident.

3.1 $Q^2 \approx 0$, $k_\perp \gtrsim 200$ MeV: possible discovery of the odderon

The goal of the discussion in this section (based on ref. [12]) is to show that the odderon effect can be discovered via discussed charge asymmetry. In this respect we prefer to use phenomenological estimates as wide as possible.

The C–odd dipions are produced via Pomeron exchange, and at $k_\perp \gtrsim 200$ MeV at HERA only the odderon exchange can produce C–even dipions with not diminishing rate. The interference between these amplitudes results in charge asymmetry of produced pions. In ref. [12] we propose to record this very charge asymmetry at HERA for the discovery of the odderon.

■ Amplitudes. Assuming that the Pomeron $\text{IP}$ and the odderon $\mathcal{O}$ are Regge poles, their contributions to the scattering amplitude $AB \rightarrow CD$ have the standard form

$$A_H = \zeta(\mathcal{H}) e^{i\pi\alpha_\mathcal{H}/2} G(\mathcal{AHC}) s^{\alpha_\mathcal{H}} G(\mathcal{BHD}) \quad \text{with} \quad \mathcal{H} = \text{IP}, \mathcal{O}, \quad \zeta(\text{IP}) = 1, \quad \zeta(\mathcal{O}) = i. \quad (11)$$
Here the factors $G(AHC)$ and $G(BHD)$ describe couplings $AHC$ and $BHD$ respectively. Additional factor $i$ in the odderon amplitude is related to the opposite signature of the odderon as compared to the Pomeron. At small $|t| \approx k^2$, the dependence of reggeon amplitudes on the difference of helicities in vertexes $G(AHC)$ and $G(BHD)$ is given by factors

$$G(AHC) \propto |t|^{\lambda_A - \lambda_C}/2, \quad G(BRD) \propto |t|^{\lambda_B - \lambda_D}/2. \quad (12)$$

Let us summarize some general features of these amplitudes for the diffractive $\gamma p \to \pi^+\pi^-p'$ process ($A = \gamma$, $B = p$, $C = \pi^+\pi^-$, $D = p'$). In our discussions we have in mind that $\alpha_\gamma \sim \alpha_\Pi \sim 1$ and $\alpha_\Pi - \alpha_\gamma \ll 1$.

- The Pomeron amplitude is studied well at HERA.

1. The main contribution to the cross section is given by amplitudes with production of two pions in the C-odd state ($\rho$-meson + other $\rho$ type resonances at higher effective masses) — (\gamma\Pi\rho vertex). Besides, the $s$-channel helicity conservation (SCHC) takes place at small $t$ — i.e., $\rho$-meson helicity coincides with that of the initial photon.

2. The vertex $p\Pi p'$ is the most significant when $p'$ coincides with proton $p$ (the admixture from proton dissociation to excited states with masses $M' \lesssim 2$ GeV is below 25%). SCHC takes place for this vertex with good accuracy, $\Delta\lambda_p = 0$.

- For the odderon amplitude we can use theoretical estimates only.

1. The vertex $\gamma\mathcal{O}\pi^+\pi^-$ is of main interest to us. We assume that — as it is customary for other phenomena at $M \lesssim 1.5$ GeV — the pion pairs are produced mainly via resonance states ($f_0$ and $f_2$ mesons). At $M \gtrsim 1.1$ GeV, we deal here with the $\gamma\mathcal{O}f_2(1270)$ vertex. below we consider several variants of its helicity structure.

2. The vertex $p\mathcal{O}p'$. In the reggeized 3-gluon exchange quark–diquark model [13] (which is also used — in some variant — in ref. [14]) at small $t$ the properties of the $p\mathcal{O}p'$ vertex are similar roughly to those of $p\Pi p'$ vertex (see Appendix). Therefore, we assume the amplitudes with $p' = p$ and SCHC in this vertex to be either dominant or contributing not less than other amplitudes.

- The conventional approximation for the amplitudes of dipion production in the state with angular momentum $J$ and helicity $\lambda$ (with SCHC in proton vertex) is obtained from eq. (11) by adding factors $D_J(M^2)$ and $\mathcal{E}^{J,\lambda}_{\lambda'}$ which describe mass and angular dependencies for decay $R \to \pi^+\pi^-$ respectively:

$$\mathcal{A}_H = A_H^{\lambda\lambda'} D_J(M^2) \mathcal{E}^{J,\lambda}_{\lambda'} \quad \text{with} \quad H = \Pi, \mathcal{O}. \quad (13a)$$

In our resonant approximation we write $D_J \to D_R$ with $R$ being one of the enumerated resonances. Some of our final equations are written for the region $1.1$ GeV < $M < 1.5$ GeV where C–even dipoles are produced in the $f_2$ meson state while the production of C-odd dipion is described by the $\rho$ meson tail. The $\rho'$ contribution for the Pomeron amplitude can easily be implemented in our equations. At $M \lesssim 1.1$ GeV the $J = 1$ and $J = 0$ interference can also easily be described with the equations written below.

◇ Taking (13a) into account, we specify the first factor, describing the Regge amplitude of production of the resonance $R$ with helicity $\lambda_R$, as

$$A_H^{\lambda_R\lambda'} = \zeta(H) g_R^{\lambda} \sqrt{\sigma_R B_R} e^{i\pi\alpha_R/2} e^{-B_R|t|/2} \frac{(B_R|t|)^{|\lambda_R - \lambda|}/2}{\sqrt{|\lambda_R - \lambda|!}}. \quad (13b)$$

The quantity $|g_R^{\lambda}|^2$ is the fraction of total cross section of the production of resonance $R$ with helicity $\lambda_R$, determined by the initial photon with helicity $1$. Due to $P$–invariance,
this very quantity is related to the transitions of photon with helicity $-1$ to dipion with helicity $-\lambda_R$. The SCHC for Pomeron in the photon vertex means that $g^1_\rho \approx 1 \gg |g^0_\rho|$. Below we neglect the dependence of parameters on $M$ (the energy dependence is included in the quantity $\sigma_R$.)

◊ The second factor in eq. (13a) describes the dependence of the production amplitude for the dipion state with spin $J$ on $\pi^+\pi^-$ invariant mass. We use the standard Breit-Wigner propagation of a resonance with its coupling to pions even at $|M^2 - M_R^2| > M_R\Gamma_R$,

$$D_R(M^2) = \frac{\sqrt{m_R\Gamma_R \mathrm{Br}(R \rightarrow \pi^+\pi^-)}}{\sqrt{\pi}(M^2 - m_R^2 + i m_R\Gamma_R)}.$$  \hspace{1cm} (13c)

◊ The decay factor $E^{J\lambda_R}_{\chi\gamma}$ describes the angular part of the helicity amplitude. Because pions are spinless, it is expressed via the standard angular momentum wave functions $Y_{lm}(\theta, \phi)$ as $E^{J\lambda_R}_{\chi\gamma} = Y_{J\lambda_R}(\theta, \phi)e^{i\lambda\gamma\psi}$.

■ The charge asymmetry effect is given by the interference of the Pomeron and the odderon amplitudes integrated over the redundant phase space variables \[d\sigma_{\text{asym}} = \sum_{J, \lambda_R} 2Re \left( A^\dagger_{\pi I} A^\lambda_{O} \right) d\Gamma.\]  \hspace{1cm} (14)

□ Let us consider the interference of the $\rho$ meson production with the odderon-mediated $f_2$ meson production ($M > 1.1$ GeV). In our approximation its $M$–dependence is given by the helicity-independent overlap function, related to the difference between Pomeron and odderon intercepts $\delta_{\pi O} = (\pi/2)(\alpha_{\pi} - \alpha_{O})$ as

$$I_{\rho f}(M^2) = Re \left[ D_\rho(iD_f)^\dagger e^{i\delta_{\pi O}} \right] = Im \left( \frac{e^{i\delta_{\pi O}}\sqrt{m_\rho m_f \Gamma_\rho \Gamma_f \mathrm{Br}(f_2 \rightarrow \pi^+\pi^-)\mathrm{Br}(\rho \rightarrow \pi^+\pi^-)}}{\pi(M^2 - m_\rho^2 + i m_\rho \Gamma_\rho)(M^2 - m_f^2 - i m_f \Gamma_f)} \right).$$  \hspace{1cm} (15)

This overlap function, shown in Fig. 5, depends on the phase difference $\delta_{\pi O}$ only weakly.

Figure 5: The $\rho - f_2$ overlap function $I_{12}(M^2)$ calculated for $\alpha_{\pi} - \alpha_{O} = 0$ (solid line) and $\alpha_{\pi} - \alpha_{O} = 0.2$ (dashed line).

If the difference between Pomeron and odderon intercepts is small, the overlap function is large ($\sim 1$) when the phase shift between two Breit-Wigner factors is close to $\pi/2$. This
happens in a wide enough region around the resonance peaks, where the $D_{R_1}$ (for one resonance) is almost real while the $D_{R_2}$ (for the other one) is almost imaginary.

\[ \square \] We consider the cross sections averaged over electron scattering angle, i.e. over initial photon spin states. Integration over $\psi$ leaves in the result only terms with identical $\lambda_\gamma$. Besides, due to $P$-invariance, for real photons ($\lambda_\gamma = \pm 1$) the other factors in eq. (13a) depend only on the helicity flip $|\lambda_R - \lambda_\gamma|$, not on the value of helicity itself. Therefore, the interference effects become proportional to sums over opposite initial photon helicities with simultaneous change of sign of final dipion helicities

\[ \mathcal{E}_{\lambda_\gamma}^* \lambda_\rho \mathcal{E}_{-\lambda_\gamma}^* \lambda_R^* \mathcal{E}_{-\lambda_\gamma}^* \lambda_R = \propto \cos[(\lambda_\rho - \lambda_R)\phi]. \]  

(16)

Since $|J_R - J_\rho|$ is odd, this quantity changes sign with $\theta \to \pi - \theta$, $\phi \to \pi + \phi$ (i.e. $p_- \leftrightarrow p_+$).

In particular,

**The terms with odd $\lambda_\rho - \lambda_R$ change sign with $\phi \to \pi + \phi$, i.e. with $v \to -v$. They are responsible for the T asymmetry.**

**The terms with even $\lambda_\rho - \lambda_R$ remain invariant under $\phi \to \pi + \phi$. Therefore, they must change sign with $\theta \to \pi - \theta$, i.e. they are responsible for the FB asymmetry.**

\[ \square \]

Neglecting contributions with higher helicity flips $|\lambda_R - \lambda_\gamma| > 1$ and taking into account explicit forms for spherical harmonics, we obtain final interference ($C$-odd) contribution to the cross section in the form

\[ \frac{d\sigma_{\text{interf}}}{dM^2 d\xi dv} = \frac{3\sqrt{5\mathcal{I}_{\rho f}(M^2)}}{2\pi \sqrt{1 - \xi^2 - \nu^2}} \sqrt{\sigma_\rho \sigma_f B_\rho B_f \exp \left(-\frac{B_\rho + B_f}{2}|t| \right)} \otimes T; \]

\[ T = g_\rho^1 g_f^1 (1 - \xi^2) + \sqrt{|t|} \left\{ v g_\rho^1 \left[ \frac{1}{2} g_f^2 (1 - \xi^2) + \frac{1}{\sqrt{6}} g_f^0 (3 \xi^2 - 1) \right] \right\} \sqrt{B_f} \]

\[ + g_\rho^0 g_f^1 v \xi^2 \sqrt{B_\rho} + \xi g_\rho^0 \left[ \frac{1}{\sqrt{2}} g_f^2 (2v^2 + \xi^2 - 1) + \frac{1}{\sqrt{3}} g_f^0 (3 \xi^2 - 1) \right] \sqrt{B_f B_\rho |t|} \]  

(17)

**The forward–backward asymmetry** is obtained from here by integration over $v$:

\[ \frac{d\sigma_{\text{FB}}}{dM^2 d\xi^2 dv} = \frac{3\sqrt{5\mathcal{I}_{\rho f}(M^2)}}{2} \sqrt{\sigma_\rho \sigma_f B_\rho B_f \exp \left(-\frac{B_\rho + B_f}{2}|t| \right)} \otimes \xi T_\xi, \]

\[ T_\xi = g_\rho^1 g_f^1 (1 - \xi^2) + \frac{1}{\sqrt{3}} g_\rho^0 g_f^0 (3 \xi^2 - 1) \sqrt{B_f B_\rho |t|}. \]  

(18)

The first term is dominant at small $t$. If the SCHC holds for the odderon, then the principal effect would be the FB asymmetry dominated by this first term. If the mechanism of the $f_2$ production significantly violates SCHC, then the first term is dominant only at small $t$. With the growth of $|t|$, the terms with helicity flip both for the Pomeron and odderon become essential, and generally, not small. Note that upon the azimuthal integration over entire region of $v$ variation, the contribution from production of $f_2$ in the state with helicity 2 vanishes because $\int \cos 2\phi d\phi = 0$.

**The transverse asymmetry** is obtained from (17) by integration over $\xi$:

\[ \frac{d\sigma_T}{dM^2 d\xi dv} = \frac{3\sqrt{10\mathcal{I}_{\rho f}(M^2)}}{4} \sqrt{\sigma_\rho \sigma_f B_\rho B_f \exp \left(-\frac{B_\rho + B_f}{2}|t| \right)} \otimes \sqrt{|t|} v T_v, \]

\[ T_v = g_\rho^1 g_f^1 \sqrt{B_f} \left[ \frac{1 + v^2}{2 \sqrt{2}} + \frac{g_\rho^0 g_f^1 \sqrt{B_f}}{2 \sqrt{3}} \frac{1 - 3v^2}{2} + g_\rho^0 g_f^1 \sqrt{B_\rho} (1 - v^2) \right]. \]  

(19)
This asymmetry is dominant in the case of strong s–channel helicity nonconservation (SCHNS) for odderon, for instance, if the $f_2$ meson is produced in the state with maximal helicity $\lambda_f = \pm 2$.

The $T$ asymmetry \cite{19} becomes naturally small at small $t$ where background is high. Therefore, imposing cut from below in $|t|$ improves the signal to background ratio.

- The main background to the discussed Pomeron–odderon charge asymmetry is given by the Pomeron–photon (Pomeron–Primakoff) interference which is predominantly transverse (Primakoff mechanism produces $f_2$ only in the states with helicity 2 or 0). To suppress this background we introduced cuts in $k_\perp$ which are different for the FB and $T$ asymmetries:

\[
|t_{FB}| = \bar{k}_\perp^2 \geq 0.1B_\rho^{-1} \approx 0.01 \text{ GeV}^2 \Rightarrow k_{FB} > 100 \text{ MeV} ;
\]

\[
|t_T| = \bar{k}_\perp^2 \geq B_\rho^{-1} \approx 0.1 \text{ GeV}^2 \Rightarrow k_{T} > 300 \text{ MeV} .
\]

\[\text{Box}\] In the **numerical estimates** we use the following parameters:

\[\text{Box}\] For the $\rho$ meson photoproduction we use the HERA data, $\sigma_\rho \approx 12 \mu b$ (for the diagonal in proton case, $p' = p$), $B_\rho \approx 10 \text{ GeV}^{-2}$, $g_1^\rho \approx 1$, $g_0^\rho \approx 0.1$.

\[\text{Box}\] For the odderon contribution we have no data. The estimates given below and in Appendix show that at HERA the odderon contribution would definitely dominate over the other mechanisms if $\sigma_f \geq 1 \text{ nb}$. Therefore, in order to be able to make as strong conclusions as possible, we take the value $\sigma_f = 1 \text{ nb}$. (We hope that the real value of $\sigma_f$ is significantly higher, e.g. 1 nb is about 5% from both the H1 experimental upper bound \cite{15} and the prediction of \cite{14}.) The slope parameter $B_f$ for the $f_2$–meson photoproduction is also unknown. For definiteness, we assume $B_f = B_\rho$.

\[\text{Box}\] The (charge symmetric) background is the sum of cross sections obliged by Pomeron and odderon. Since the odderon amplitude is considered to be very small, the background can be approximated by the Pomeron $\rho$ contribution even far from the $\rho$ peak, $d\sigma_{\text{bkgd}}/dM^2 \propto |D_1(M^2)|^2$.

\[\text{Box}\] Let us consider values of statistical significance \cite{13} for cross sections averaged over small interval of $M \pm \Delta M$, $SS(M^2)$. According to eqs. \cite{13} and \cite{27}, for all asymmetries,

\[
SS_a(M^2) \propto \left| \frac{I_{12}(M^2)}{|D_1(M^2)|} \right| \equiv \left| \frac{\text{Im}(D_2^*D_1e^{i\delta_{PC}})}{|D_1|} \right| \leq |D_2| .
\]

Therefore the largest values of this $SS(M^2)$ are located near the $f_2$ peak. It is illustrated by Fig. \[\text{Box}\] where local values of these $SS_a(M^2)$ are shown in arbitrary units. Hence, to obtain the best value of $SS$, we consider signals and background integrated over the

![Figure 6: The local statistical significance $SS_a(M^2)$ in arbitrary units.](image)
reasonable mass interval around the $f_2$ peak. A natural choice is

$$M_f - \Gamma_f < M < M_f + \Gamma_f.$$  \hfill (22)

Estimate (21) shows that the influence of nonresonant background as well as tails of other resonances in the Pomeron channel changes our estimates of $SS$ only weakly. Since the overlap function exhibits no strong dependence on phase difference $\delta_{IPO}$ (see Fig. 5), we use for estimates the value of the integral over mass interval (22) at $\delta_{IPO} = 0$:

$$\Delta I = \frac{1}{M_f - \Gamma_f} \int_{M_f + \Gamma_f} |I_\rho f(M^2)|^2 dM^2 \approx 0.095.$$  \hfill (23)

Certainly, such estimate for interference with $S$–wave $\pi^+\pi^-$ final states produced by odderon will show that the corresponding signals are located near $f_0(600)$ and $f_0(980)$ peaks, and these effects are negligible at $M > 1100$ MeV.

We consider two cases of helicity structure of the odderon amplitude. (Numerical factors in front of the integrals below appear due to integration over the region (20)).

\begin{itemize}
  \item The $f_2$ meson is produced in the state with helicity 1 (SCHC takes place also for $f_2$ production), $g_2 \approx 1$. In this case the main asymmetry will be $FB$ (18), and the integration over all variables in the region (20), (22) results in

$$\Delta \sigma_{FB} \approx 0.9 \cdot 3\sqrt{5}/4 \cdot \sqrt{\sigma_\rho \sigma_f} \cdot \Delta I = 15.7 \text{ nb} \Rightarrow SS \approx 7.5.$$  \hfill (25)

\begin{itemize}
  \item The $f_2$ meson is produced via odderon in the state with helicity 2, $g_2 \approx 1$ (SCHNC). In this case the main asymmetry will be the transverse one (19). Performing integration in the region (22), (20), we obtain the same very value as (25)

$$\Delta \sigma_T = 0.507 \cdot \frac{9\sqrt{5}}{16} \cdot \sqrt{\sigma_\rho \sigma_f} \cdot \Delta I = 6.6 \text{ nb} \Rightarrow SS \approx 5.$$  \hfill (26)
\end{itemize}
\end{itemize}

These numbers are still very promising. This offers certain confidence that the odderon signal is indeed within the reach of the current experiments even with very low value for the odderon–induced cross section and relatively low luminosity ($10$).

### 3.1.1 Possible discovery of the hard odderon [17, 18]

\begin{itemize}
  \item $T = c\bar{c}, Q^2 \approx 0$. The idea to discover odderon via the study of charge asymmetry in the process $\gamma p \rightarrow c\bar{c}p$ was proposed first in ref. [17]. Since we deal here with the production of heavy quarks, one can speak here about hard odderon. In the specific calculation in ref. [17], strong interaction of $c$ quarks in the final state was neglected (in the spirit of the quark–hadron duality). In this approximation, the overlap function (15) $I \Rightarrow \sin \delta_{IPO}$ becomes $M$–independent, and the effect is strongly underestimated near
possible $c\bar{c}$ resonances. Therefore the obtained in ref. [17] estimates have chances to be correct only far from the open charm threshold, at $M \gg 2m_c$ (see sect. 4 for other details).

\[ T = \pi^+\pi^-, \quad Q^2 = 2 - 3 \text{ GeV}^2. \] The hard odderon can also be discovered via observation of charge asymmetry (and related azimuthal asymmetry) in the electroproduction of pion pairs by deeply virtual photon [18]. The effect of longitudinal virtual photons becomes essential in this region. When using these results for real analysis, one should be careful.

\( i) \) Their pQCD approach can be valid at $Q^2 \gg \Lambda^2_{QCD}$. Its validity for $Q^2 \sim 2 \div 3 \text{ GeV}^2$ is unclear. For example, even for Pomeron at the considered $Q^2$ the SCHC amplitude for transverse photon is not small. These higher twist contributions can change results strongly.

\( ii) \) The result contains sharp structure at $|t| \approx 0.1 \text{ GeV}^2$. It seems to be an artefact within the pQCD approach, which can describe phenomena only with averaging over interval of momenta wider than $\Lambda^2_{QCD}$.

\( iii) \) The states of proton dissociation can be different for Pomeron and odderon, which will reduce interference.

- At a naive glance, "the main difference of studies of the electroproduction process [18] with respect to refs. [17, 12] is to work in a perturbative framework, which we believe enables us to derive more founded predictions in an accessible kinematical domain" [18]. Unfortunately, as we mention above and in Appendix, in the considered kinematical region the reliability of numerical calculations of papers [17, 18] is low.

\[ \square \] Besides, the discussed cross sections are very small, so that the observation of these effects is hardly possible. Indeed, for the $c\bar{c}$ photoproduction, even the Pomeron mediated $c\bar{c}$ production cross section is small and the efficiency of $c$–quark recording is low. For the electroproduction of dipions by highly virtual photon the observable cross section is also small since in this case (i) the effective $\gamma p$ luminosity is much lower than (10); (ii) the scale of cross section itself is $\sim \alpha/Q^2$ instead of $\alpha/m_\pi^2$ for soft case.

\[ 3.2 \quad 20 \text{ MeV} \lesssim k_\perp \lesssim 100 \text{ MeV}, \quad Q^2 \approx 0. \quad \text{Phases of the forward} \]

\[ \gamma p \to \rho p \text{ and } \gamma A \to \rho A \text{ amplitudes, [19]} \]

\[ \square \] The phase of the forward hadron elastic amplitude $A = |A|e^{i\delta_F}$ at high energy is an important object in hadron physics. In the naive Regge-pole Pomeron model, this phase is given by the Pomeron intercept as $\delta_F = (\pi/2)\alpha_P$ [11]. The object, studied in modern experiments and named as Pomeron, seems to be more complex. Measuring the phase of this object appears useful in order to clarify its nature.

To the moment, phase of this type was measured in the only type of experiments — via study of Coulomb interference in $pp$ or $\bar{p}p$ elastic scattering. These experiments require measurement of extremely low scattering angles, which becomes practically impossible at high enough energies.

\[ \square \] The study of charge asymmetry in the process $\gamma p \to \pi^+\pi^- p$ (or $ep \to e\pi^+\pi^- p$) at very low transverse momenta of the produced dipion provides new method to measure this type of phase [13]. For this purpose, we suggest to study the charge asymmetry in the same mass interval (22) and at very small $k_\perp$,

\[ k_{\text{min}} = 20 \text{ MeV} < k_\perp < k_{\text{max}} \approx 100 \text{ MeV} \Rightarrow |t|_{\text{cut}} = 0.01 \text{ GeV}^2. \quad (27) \]
We speak here about the total transverse momentum of dipion with effective mass $> 1$ GeV. In this case typical transverse momenta of separate pions are $\sim 500$ MeV, i.e. they are measurable well. In the eq. (27) lower limit $k_{\text{min}}$ is determined by accuracy in the measurement of $p_{\perp \pm}$. The upper limit $k_{\text{max}}$ describes the region where the Coulomb (Primakoff) contribution is essential while alternative hadronic contributions are inessential. At $k_{\perp} < k_{\text{max}}$ the contributions of the proton excitations are negligible for both discussed mechanisms. Besides, in this region the form-factor effects are inessential, and one can treat proton as a point-like particle.

In the region (27) the C–even dipion is produced mainly via photon exchange (Primakoff effect), studied well at $e^+e^-$ collisions. It is described with very high precision by the equivalent photon approximation (see for details [21]). Similar to eqs. (13), one can write this amplitude in the form

$$A_{\gamma} = C g_\gamma \frac{|k_{\perp}|}{k_{\perp}^2 + Q_m^2} D_R(M^2) e^{\lambda_R} \text{ with } Q^2_m \approx \left( \frac{m_p M^2}{s} \right)^2$$

(28)

with normalization factor $C$ given by the QED result involving the two photon width $\Gamma_{\gamma\gamma}$

$$d\sigma_f = \frac{8 \pi \alpha \Gamma_{\gamma\gamma}(2J + 1)}{M^4} \cdot \frac{k_{\perp}^2 dK^2}{(k_{\perp}^2 + Q_m^2)^2}.$$  

(29)

The total cross section of the $f_2$ meson production and the cross sections in different kinematical regions for HERA case are

$$\sigma_{\text{tot}} \approx \frac{8 \pi \alpha \Gamma_{\gamma\gamma}(2J + 1)}{M^4} \left( \ln \frac{m_p^2}{Q_m^2} - 1 \right) = 8 \text{ nb},$$

$$\sigma_f(k_{\perp} \leq 100 \text{MeV}) > 7 \text{ nb}, \quad \sigma_f(k_{\perp} \geq 300 \text{ MeV}) \approx 0.2 \text{ nb}.$$  

(30)

Large cross section is concentrated in the narrow region near the forward direction. That is the ground for our choice of the region (27) for the study of the discussed effect.

For the considered collision of almost real photons only two values of dipion helicity are allowed by the conservation laws, 0 and 2, i.e. $g_{\gamma}^{\lambda=1} = 0$. For the $f_2$ meson ($J = 2$) data give $g_{\gamma}^{\lambda=2} \approx 1 \gg |g_{\gamma}^{\lambda=0}|$.

The C-odd dipions are produced via diffractive Pomeron mechanism through the $\rho$–meson like state, mainly in the state with helicity 1 (SCHC), as it was discussed above. This amplitude is described by eqs. (13). At $k_{\perp} < 100$ MeV we have $B_{\rho}|t| < 0.1$. Therefore, one can neglect $t$ dependence of strong interaction ("Pomeron") amplitude.

The background contribution is the sum of Pomeron and Primakoff effects,

$$\sigma_B \equiv \sigma_p + \sigma_f = \left[ 470 B_{\rho}(k_{\text{max}}^2 - k_{\text{min}}^2) + 0.45 \ln \frac{k_{\text{max}}^2}{k_{\text{min}}^2} \right] \text{ nb} \approx 47 + 1.5 = 48.5 \text{ nb}.$$  

(31)

The charge asymmetry is calculated now just as for the odderon case. Certainly, in this case the overlap function $I_{\mathbf{P}}$ is determined by eq. (15) with natural change of $\delta_{\mathbf{P}\mathbf{O}}$ to quantity $\delta_F$. In accordance with discussion in sect. 3.3, the main asymmetry is transverse one, and similarly to (19), but in contrast with the odderon case, asymmetry increases with decreasing of $k_{\perp}$, and (we set $g_\rho^b g_f^2 \approx 1$)

$$\frac{d\sigma_T^{pr}}{dM^2 dK_{\perp}^2 \, dv} = \frac{3\sqrt{5}}{8} C I_{\mathbf{P}}(M^2) \sqrt{\sigma_p B_{\rho}} |k_{\perp}| v(1 + v^2).$$  

(32)
To estimate the opportunity to observe the asymmetry discussed, it is useful to calculate the overall effect, i.e. asymmetry, integrated over $M^2$, $k_{\perp}$ and $v$ in the regions (27), (22). In this estimate for the integral of overlap function one can use quantity (23)

$$\Delta \sigma_T = \frac{9\sqrt{5}}{8} C \sqrt{\sigma_B} (k_{\text{max}} - k_{\text{min}}) \cdot \Delta I \approx 5.6 \text{ nb}. \quad (33)$$

For the $\gamma p$ effective luminosity (10) it results in statistical significance $SS \approx 7.5$. This SS value is sufficient for the observation of an effect. However, to extract the phase under interest, one should study in detail the mass dependence of asymmetry. For this purpose it is useful to have larger effective $\gamma p$ luminosity integral, and this luminosity, necessary for reasonable precision, should be estimated at the stage of planning the experiments.

The simplified model for $\rho$ and $f_2$ amplitudes, used for estimates, seems to be too rough. The contributions of other resonances and nonresonant background should be included. Instead of such calculation (which has many sources of ambiguity), the precise phase of the $C$–even dipion production amplitude can be found via the study of charge asymmetry in the $e^+e^- \to e^+e^-\pi^+\pi^-$ process (sect. 2.2) in the same effective mass region.

### 3.2.1 $eA \to e\pi^+\pi^-A$. The nuclear "Pomeron" phase

The study of charge asymmetry of pions in the $eA \to e\pi^+\pi^-A$ collisions with heavy nuclei (at the future ERHIC and THERA colliders) provides an opportunity to measure the Pomeron phase $\delta_F^A$ for a nuclear target. (Generally $\delta_F^A \neq \delta_F$ for proton target).

When the ultrarelativistic heavy nuclei collide, there is the region of momenta transferred from one of them, which is so small that it does not destroy this nucleus — ultra-peripheral collisions (UPC). In this region the electromagnetic interaction of the nucleus is coherent, its strength is defined not by the fine structure constant $\alpha \ll 1$, but by the quantity $Z\alpha \sim 1$. Therefore, the electromagnetic interactions become comparable with strong interactions (or even become stronger due to Coulomb pole). These interactions are characterized by the nuclear charge $Ze$ and formfactor $F_A(q^2)$ with scale of the $q^2$ dependence $\Lambda^2 \sim 1/R_A^2$. For heavy nuclei considered here $\Lambda \approx 60 \text{ MeV}$.

For the coherence (UPC), one should have $|q^2| < \Lambda^2$. The transferred momentum $q$ is related to its transverse component $q_{\perp}$ as

$$-q^2 = q_m^2 + q_{\perp}^2, \quad q_m^2 = \omega^2/\gamma_A^2, \quad (34)$$

where $\gamma_A$ is the nuclear Lorenz factor and $\omega$ is the energy transferred from the nucleus. Therefore, the transverse momentum end energy transferred to the dipion are limited as

$$k_{\perp} \lesssim \Lambda \approx 60 \text{ MeV}, \quad (35a)$$

$$\omega < \Lambda \gamma_A = \begin{cases} 
6 \text{ GeV for ERHIC } e\text{-Au}, \\
180 \text{ GeV for LHC } e\text{-Pb}.
\end{cases} \quad (35b)$$

For the virtual photon $q_e$, emitted by electron we require only the limitation (35a) for $q_{e\perp}$. It allows to transmit the limitation (35a) to the produced dipion. For this photon we have $-q_e^2 = (Q_m^2 + q_{e\perp}^2)/(1 - \omega/E)$ with $Q_m \approx m_eM^2/s$. Therefore, this photon can be treated as quasireal in respect to the production of hadron system $T$ (in this case $|q_e^2| < \Lambda_T^2 \approx m_\rho^2$), and we treat the UPC $eA \to eT A$ as the $\gamma A \to T A$ processes.
We suggest to look for dipions at the mass interval (22) and in the interval of \( k_\perp \) (instead of (27))

\[
20 \text{ MeV} = k_{\text{min}} < k_\perp < k_{\text{max}} \approx 60 \text{ MeV} \Rightarrow |t|_{\text{cut}} = 0.0036 \text{ GeV}^2.
\]  

(36)

Since the mean number of nuclear collisions per bunch crossing in ERHIC will be less than 1, these UPC can be isolated in events with this kinematical limitation and without other particles in the detector.

Subsequent estimates are similar to those in the previous subsection.

The C–even dipions are produced via photon exchange (Primakoff effect). This production is described by eqs. (28) with additional factor \( Z F_A(k_\perp^2 + Q_m^2) \). For estimates, we write the form-factor in the form \( F_A(Q^2) = 1/(1 + Q^2/A^2) \).

At large enough \( \gamma_A \) energies\(^1\) C-odd dipions are produced via diffractive \( \rho \)-meson–like production, mainly in the state with helicity 1. The approximation for the Pomeron amplitude (13) is valid with additional factor \( A^{2/3} \) and the change \( \sigma_\rho B_\rho \rightarrow \sigma_{\rho}^A B_{\rho}^A \). The values \( \sigma_\rho \) and \( 1/B_\rho^A \) are smaller than the corresponding quantities for proton (21). In the numerical estimates we write \( \sigma_\rho^A B_{\rho}^A \approx k^2 \sigma_\rho B_\rho \) with coefficient \( k \sim 1 \). In any case, in the region (36) the \( t \) dependence of "Pomeron" amplitude is negligible.

For the collision of electron with \( E = 100 \text{ GeV} \) and \( Au \) nuclei with \( \gamma_A = 109 \) we obtain the total cross section of the \( f_2 \) production and the cross sections in the region (36)

\[
\sigma_f^{\text{tot}} \approx Z^2 \frac{8\pi \alpha \Gamma_\gamma (2J + 1)}{M^3} \left( \ln \frac{A^2}{Q_m^2} - 1 \right) \approx 1.5 Z^2 \text{ nb},
\]

\[
\sigma_f(k_\perp \geq 20 \text{ MeV}) \approx 0.6 Z^2 \text{ nb}.
\]

(37)

The background contribution is the sum of the Pomeron and Primakoff effects,

\[
\sigma_B = \left[ 470 A^{1/3} k^2 B_\rho (k_{\text{max}}^2 - k_{\text{min}}^2) + 0.3 Z^2 \ln \frac{k_{\text{max}}^2}{k_{\text{min}}^2} \right] \text{ nb} \approx (16 k^2 A^{1/3} + 0.66 Z^2) \text{ nb}.
\]

(38)

The charge asymmetry is calculated just as for the \( \gamma p \) case with similar overlap function \( \mathcal{I}_{\text{IP}} \). In accordance with discussion in sect. [3.3], the main asymmetry is transverse and, similarly to eq. (32), the asymmetry increases with decreasing \( k_\perp \):

\[
\frac{d\sigma_T^{\perp}}{dM^2 dk_\perp^2 dv} = Z A^{2/3} k \frac{3\sqrt{5}}{8} C \mathcal{I}_{\text{IP}}(M^2) \sqrt{\sigma_\rho B_\rho} |k_\perp| v(1 + v^2).
\]

(39)

\( \square \) The overall asymmetry is an integral over \( M^2, k_\perp \) and \( v \) in the regions of eqs. (36), (22). With the integral of the overlap function given by the quantity (23), we have

\[
\Delta \sigma_T = Z A^{2/3} k \frac{3\sqrt{5}}{8} C \sqrt{\sigma_\rho B_\rho} (k_{\text{max}} - k_{\text{min}}) \cdot \Delta \mathcal{I} \approx 2.8 Z A^{2/3} k \text{ nb}.
\]

(40)

Since \( A^{2/3} \approx Z \), the S/B ratio is almost the same for different nuclei. However, the value of SS (54) increases \( \propto Z \) with \( Z \) growth at fixed luminosity integral \( \mathcal{L} \). Since the Pomeron contribution dominates in the background, the factor \( k A^{2/3} \) disappears from the estimate of SS. For \( Au \) nuclei even at low \( \mathcal{L} \approx 1 \text{ nb}^{-1} \) we have a very good SS value \( \approx 14 \).

\(^1\) At ERHIC — with large enough longitudinal momentum of the dipion directed along the initial electron motion or at THERA.
3.3 Study of double Pomeron exchange in UPC

For pA or AA collisions of ultrarelativistic heavy nuclei A, UPC provides a good tool to study the double Pomeron phenomena. Under the kinematical conditions eq. (35) for the LHC and (35a) for the RHIC and (with bad precision) for HERA-B, the main mechanisms for the dipion production in the process \( A_1 A_2 \rightarrow A_1 A_2 \pi^+ \pi^- \) are

- Pomeron – Pomeron \( (A_1 \mathcal{I} P A_1, A_2 \mathcal{I} P A_2) \otimes \mathcal{I} P \mathcal{I} P \rightarrow \pi^+ \pi^- \),
- Photon – photon \( (A_1 \gamma A_1, A_2 \gamma A_2) \otimes \gamma \gamma \rightarrow \pi^+ \pi^- \),
- Pomeron – photon \( (A_1 \mathcal{I} P A_1, A_2 \gamma A_2) \otimes \mathcal{I} P \gamma \rightarrow \pi^+ \pi^- \).

Similar to the estimates in the previous section, one can expect the Pomeron-Pomeron contribution to dominate, and the main charge asymmetry to appear due to interference of Pomeron–Pomeron and Pomeron–photon amplitudes. After the studies of the Pomeron phase in eA collisions, these investigations in nuclear collisions open the door to a detailed study of the Pomeron-Pomeron amplitude, which is now poorly understood.

3.4 \( Q^2 \gtrsim 1000 \text{ GeV}^2, k_\perp \lesssim 1 \text{ GeV} \). Study of coupling of the axial current to the Pomeron \[22\]

At large electron scattering angles \( (p_\perp e \gtrsim 30 \text{ GeV}) \), the interaction of electron with proton via Z–boson exchange (mainly axial current) becomes essential in addition to the standard photon exchange (vector current). In this region we suggest to consider also dipion final state with large rapidity gap and with no specific final state for proton excitation. In this case the interaction of vector and axial currents with proton (via the Pomeron exchange) produces C–odd and C–even final states respectively. Both amplitudes are described by approximation of form \([13]\). The content of final state \( p' \) is identical in both cases providing as complete interference as possible.

Before a detailed analysis of data it is difficult to say whether any resonant states or nonresonant background dominate in this region. For preliminary estimates at 1.1 GeV \(< M < 1.5 \text{ GeV} \) one can use calculations of \( \rho \) and \( f_2 \) production in pQCD (with massless quarks) in 2-gluon approximation. For \( \rho \) production these amplitudes are written in \[23\]. Calculation of the \( f_2 \) production by axial current can be made in this very manner by using the approach of \[24\] as well. One can expect that this approximation gives correct shape of the charge asymmetry while the value of effect (and background) will be enhanced due to the Pomeron enhancement in comparison with two-gluon approximation.

The interference of these amplitudes results in charge asymmetry effect \( \sim (Q^2/M_\rho^2) \) with overlap factor which is different from that in \[13\]

\[
\mathcal{I}^\rho_{f_2}(M^2) = \text{Re} \left[ D_\rho D_\pi^f \right] = \text{Re} \left( \frac{\sqrt{m_\rho m_f \Gamma_\rho \Gamma_f} Br(f_2 \rightarrow \pi^+ \pi^-) Br(\rho \rightarrow \pi^+ \pi^-)}{\pi (M^2 - m_\rho^2 + im_\rho \Gamma_\rho)(M^2 - m_f^2 - im_f \Gamma_f)} \right).
\]

In particular, near the \( f_2 \) pole the contribution of \( f_2 \) is almost imaginary while the contribution of \( \rho \) is roughly real. Therefore, this overlap function changes its sign at \( M \approx M_f \). The peaks in the mass distribution are disposed at the distance \( \sim \Gamma_f \) from this pole. The idea of estimate of Statistical Significance \[21\] is valid in this case as well.
Finally for the reasonable and careful estimate of SS one should perform mass integration over the region (instead of eq. (22))
\[ D_R : \quad M_{f_2} + \kappa \Gamma_{f_2} < M < M_{f_2} + (1 + \kappa) \Gamma_{f_2}, \quad (\kappa \sim 0.2 \div 0.5). \] (43)

The signal below 1.2 GeV also includes contributions from \( f_0 \) resonances.

## 4 Breaking of quark–hadron duality

It is usually assumed that for heavy quarks the quark–hadron duality (Q-HD) works well (at least in average). However, this Q-HD is violated strongly in the charge asymmetry phenomena due to the final state interaction — FSI.

- Let us remind that the charge asymmetry of muons in the process \( e^+e^- \rightarrow e^+e^- \mu^+\mu^- \) (Fig. 2, right panel) differs strongly from that of pions (Fig. 2, left panel). For muons |\( \Delta \sigma_T^\mu \)\| \( \gg \) |\( \Delta \sigma_{FB}^\mu \)\| while for pions (QED) |\( \Delta \sigma_{FB}^\pi \)\| \( \gg \) |\( \Delta \sigma_T^\pi \)\|. At the first glance, the charge asymmetry for the (point-like) heavy quarks should be mainly transverse as that for muons. In reality, hadronization transforms quarks into mesons with spin 0 and 1, and one can expect that the FB asymmetry will be the largest near the threshold like that for pions. This change of type of the charge asymmetry will be a clear signal of the Q-HD breaking.

- The main source of observed \( D \)-mesons is the decay of \( c\bar{c} \) resonances. These are \( J = 1 \) states of \( c\bar{c} \) system, produced via bremsstrahlung production for \( e^+e^- \) collision or via Pomeron exchange for \( \gamma p \) case. In the same region the \( J = 0 \) and \( J = 2 \) resonance states of this \( c\bar{c} \) system should be produced by two photons or via odderon exchange respectively. All these resonances are not very narrow. Overlapping of these resonances should give essential contribution to the charge asymmetry as it was discussed for pions.

For the \( \gamma p \) collision for point-like quarks the charge asymmetry would be suppressed by a small factor \( \mathcal{I} = \sin \delta_{P\Omega} \) obtained in ref. \([17]\). In reality, due to FSI – resonance production, we expect picture which is similar to pion pair production where this small factor is eliminated due to additional phase shift given by product of two Breit–Wigner factors related to different resonances (see eq. \([15]\) and Fig. \([\ref{fig:bw}]) .
5 Weighted structure functions

Now we consider the deep inelastic scattering of electron with momentum $p_e$ on the proton with momentum $p_p$ (DIS). Let $w$ be some charge asymmetric quantity determined for all observed particles, $\hat{C}w = -w$.

For example, let $x_j$ be standard light cone variable for each produced particle $j$, i.e. $x_j = (E_j - p_{jz})/E_e \equiv 2p_jp_p/s$. We define the weight factor $w_\xi = \sum_{j=+} x_j - \sum_{i=-} x_i \equiv 2(p_+ - p_-)p_p/s$ (cf. (3a)). (The first term here is the sum over all positively charged secondaries, the second one is the sum over all negatively charged secondaries.)

■ The suggested weighted structure functions — WSF — are described via data in the same manner as the usual structure functions for DIS but with weight factor $w$ like $w_\xi$ for each event. (Certainly, the standard polarization analysis of an initial state can be added to this definition.) These weight factors bring the charge asymmetry into standard definitions. In the standard language they are

$$W_{\mu\nu}^C(p, q) = 2\pi^2 \sum_X \int d^4z \langle p|J_\mu(z)||\hat{W}_C^\nu|X\\rangle < X|J_\nu(0)||p > e^{iqz}$$

(44)

with charge odd operators $\hat{W}_C^\mu|X > = w|X >$. It is desirable to have such form of weight $w$, to minimize influence of the target specifics. (This feature works in our example $w_\xi$ since the quantities $x_j$ are small for secondaries $j$, flying along initial proton.)

♦ If the operator $\hat{W}_C^\mu$ acts for hadron and quarks similarly and is measurable explicitly in each event, these WSF will give us new information about quark–gluon structure of matter at small distances.

♦ If the operator $\hat{W}_C^\mu$ acts for hadron and quarks in different ways, the WSF proposed will be sensitive to the details of confinement as well.

■ Useful points for the WSF.

• It is well known that the (multi)gluon exchange effects cannot be seen in the standard $W_3$-like functions. Indeed, for the (multi)gluon colorless exchange with proton, the function (44) corresponds to the interference of C-even (Pomeron-like) exchange and C-odd (odderon-like) exchange, which produces in the collision with photon the final states of the opposite C-parity. Therefore, this contribution disappears in the standard structure functions (see [23] for lower approximation). Respectively, this contribution remains in the weighted object.

• Usually, the contributions of the vector current $J_V$ (mainly from photon exchange) and axial current $J_A$ (from $Z$ exchange) in the structure functions are summed without interference, and the second contribution is small fraction ($\sim (Q^2/M_Z^2)^2$) at $Q^2 < (70 \text{ GeV})^2$. The difference of cross sections for the left-hand and right-hand polarized electrons (like $W_3$ structure function) is

$$d\sigma^L - d\sigma^R \propto Re(J_*^VJ_A).$$

(45)

That is C–odd quantity. Therefore, using some C-odd weight function in WSF makes this interference clean. According to our experience in Higgs physics, this WSF should become large enough at $Q^2 > 1000 \text{ GeV}^2$ ($p_{e\perp} > 30 \text{ GeV}$).
6 \( e\gamma \rightarrow eWW \). Strong interaction in the Higgs sector

The possible strong interaction in the Higgs sector can be seen as that of longitudinal \( W \)'s, in particular, in the process \( \gamma\gamma \rightarrow W_L W_L \). The experience with \( \gamma\gamma \rightarrow \pi^+\pi^- \) makes the following picture very probable: strong interaction modifies weakly the cross section near the threshold in comparison with its QED value but the phase of amplitude reproduces that of strong interacting \( W_L W_L \) scattering and can be not small. This makes strong interaction in the Higgs sector badly observable in the cross sections below expected masses of \( WW \) resonances, about 1.5–2 TeV.

The charge asymmetry in the process \( e\gamma \rightarrow eWW \) is given by interference of two-photon and one-photon production (as in \( e^+e^- \rightarrow e^+e^- \pi^+\pi^- \)) and by interference of photon and \( Z \) boson exchanges. The different interferences dominate in different regions of the final phase space (in dependence on the transverse momentum of the scattered electron and its energy).

The study of charge asymmetry in the process considered can be the key to the discovery of this strong interaction at the relatively low energy of TESLA (0.8 TeV) much below possible resonance production, since it is sensitive to \( W_L W_L \) scattering phase shifts \([26, 27]\).

To distinguish between this charge asymmetry and that for the lepton final states, discussed in the next section, it is necessary to consider quark decays of \( W \)-bosons.

7 Polarization charge asymmetry in \( \gamma\gamma \) collisions

Let us consider the charge asymmetry in \( \gamma\gamma \) collisions, coming from the definite polarization of the initial state in the processes like \( \gamma\gamma \rightarrow WW \rightarrow T\bar{\nu}\bar{\nu} \), \( \gamma\gamma \rightarrow \tau^+\tau^- \rightarrow T\nu\bar{\nu}\nu\bar{\nu} \) or \( \gamma\gamma \rightarrow \chi^+\chi^- \rightarrow T\nu\bar{\nu}\chi_0\bar{\chi}_0 \) with \( T = \mu^+\mu^- \) (or \( T = \mu^+e^-, \mu^-e^+, e^+e^- \)). (Here \( \chi^\pm \) is chargino and \( \chi_0 \) is neutralino – LSP.) These processes will be studied at the Photon Colliders \([28, 29]\) where high energy photons will be prepared mainly in the states with definite helicity \( \lambda_1 \approx \pm 1 \). For discussion, we distinguish the initial states with \( \lambda_1, \lambda_2 = \pm 1 \).

The QED cross sections of pair production \( \gamma\gamma \rightarrow WW \), \( \gamma\gamma \rightarrow \tau^+\tau^- \), etc. depend on the product \( \lambda_1\lambda_2 \) only and exhibit no charge asymmetry (due to P-invariance of electromagnetic interactions). However, the helicity states of the intermediate \( W^\pm, \tau^\pm \) or \( \chi^\pm \) depend separately on photon polarizations. In the subsequent decay of these \( W, \tau \) or \( \chi^\pm \), the P-parity is not conserved. This gives correlations between spin of intermediate \( W, \tau \) or \( \chi^\pm \), etc. and the momentum of the single particle observable in the final state of this decay (in our example — the muon). Due to opposite directions of polarizations of intermediate particles and antiparticles, final distributions of observed \( \mu^+ \) and \( \mu^- \), etc. become different, and the charge asymmetry arises. Note that this asymmetry is absent for massless intermediate particles due to helicity conservation. Therefore, the value of effect increases with the mass of intermediate particle (\( \tau \) or \( W \) or \( \chi^\pm \) in our examples).

Certainly, the observable effect summarizes effects from various intermediate states. So that the detailed study of charge asymmetry, related to different mechanisms, in different regions of final phase space is necessary.

- The initial states \( (\lambda_1, \lambda_2) = ++ \) and \( -- \) choose preferable direction of the collision axis. Therefore, in this case we expect FB asymmetry of final muons. It has opposite signs for \( ++ \) and \( -- \) initial states.
• The initial states \((\lambda_1, \lambda_2) = \-- \text{ and } ++\) choose preferable direction of rotation (left or right, respectively) but the opposite directions of the collision axis are equivalent. Therefore, we expect here T asymmetry of final muons, but not FB asymmetry. It has opposite signs for ++ and -- initial states. Certainly, this FB asymmetry will be smoothed due to nonmonochromaticity of incident photons.

Figure 8: Cross-sections \(d\sigma/dp_{\mu\perp}\) (left) and \(d\sigma/d\cos(\theta_\mu)\) (right) of the process \(\gamma\gamma \to W\mu\bar{\nu}\)

These features are clearly seen in Fig. 8 where muon distributions in the process \(\gamma\gamma \to W^+\mu^-\bar{\nu}\) are shown for different initial photon states. One can see here that the transition from +- to -+ initial states changes transverse distribution of muons, that corresponds to the T asymmetry for \(\gamma\gamma \to \mu^+\mu^-\bar{\nu}\bar{\nu}\) process. The transition from ++ to -- initial states does not change transverse distribution of muons but changes its angular distribution (due to change of distribution in longitudinal momentum), that corresponds to FB asymmetry in \(\gamma\gamma \to \mu^+\mu^-\nu\bar{\nu}\) process.

These asymmetries and the quality of resonance approximation for their description were analyzed in detail in ref. [30] for the simplest EW process \(\gamma\gamma \to WW \to \mu^+\mu^-\nu\bar{\nu}\) within SM together with asymmetries in the process \(\gamma\gamma \to \tau^+\tau^- \to \mu^+\mu^-\nu\bar{\nu}\nu\bar{\nu}\) considered in the resonant approximation. The similar FB asymmetry was considered in Ref. [31] for the \(\gamma\gamma \to \chi^+\chi^- \to e^+e^-\nu\bar{\nu}\chi_0\bar{\chi}_0\) process[2].

The study of this charge asymmetry can be a good tool for investigation of different effects of the New Physics (anomalous triple and quartic interactions of gauge bosons, strong interaction in Higgs sector, SUSY, ...). In this task the mentioned increase of charge asymmetry with the growth of mass of an intermediate particle will be useful.

Unfortunately, the mentioned discussions of \(\gamma\gamma \to \tau^+\tau^- \to \mu^+\mu^-\nu\bar{\nu}\nu\bar{\nu}\) and \(\gamma\gamma \to \chi^+\chi^- \to e^+e^-\nu\bar{\nu}\chi_0\bar{\chi}_0\) are very preliminary since they use resonance approximation for description of final states with 6 particles. This approach neglects a huge number of other diagrams with the same final state (for the process \(\gamma\gamma \to \mu^+\mu^-\nu\bar{\nu}\nu\bar{\nu}\) the resonant approximation deals with 2 diagrams instead of 184 in SM). In the complete form this problem is difficult for computing. (Of course, many omitted diagrams do not contribute to the asymmetry.) The development of corresponding computing algorithms is necessary.

\[^2\text{Note that the discussion of [31] of variations related to change of helicities of initial laser photon and electron at } e \to \gamma \text{ conversion is completely related to variation of initial } \gamma\gamma \text{ state discussed in detail in [28]. Effects discussed there can be obtained by variation of initial beam energy.}\]
Note that in these problems the system $T(=\mu^+\mu^-$ in our example) is organized from two particles of different origin. Therefore, the useful variables for description of these asymmetries become not dimensionless [3], but their dimensional analogs, for example, for the transverse asymmetry $\tilde{v} = p_{+\perp}^2 - p_{-\perp}^2 - K\cdot k_{\perp}^2$.

Appendix. The status of the odderon vs. Pomeron

The Pomeron and odderon are treated as the reggeons, that have vacuum quantum numbers with the only difference: while Pomeron is C–even, the odderon is C-odd, similarly to the photon. Pomeron exchange describes small angle elastic and total cross sections at high energies. Odderon is responsible, e.g., for the difference $\sigma_{pp}^{tot} - \sigma_{p\bar{p}}^{tot}$ at high energies [32, 33]. Assuming the Pomeron and odderon are Regge poles, their contributions to the scattering amplitude $AB \rightarrow CD$ must have the form (11), (12). Because $|\sigma^+ - \sigma^-| < \sigma^+ + \sigma^-$, one should be $\alpha_O \leq \alpha_P$. Since the intercepts $\alpha_P$ and $\alpha_O$ are close to 1, the Pomeron exchange amplitude is predominantly imaginary, while the odderon exchange amplitude is predominantly real.

Within perturbative QCD, the Pomeron and odderon are based on two–gluon and $d$–coupled three–gluon exchanges in $t$–channel respectively [34]. Hence, both the Pomeron and odderon intercepts are close to the gluon spin, $\alpha_O, \alpha_P \sim 1$. The experimental data and BFKL calculations show that the Pomeron intercept $\alpha_P(0) > 1$. The theoretical estimations for odderon intercept vary with the date of preparation of paper ($\alpha_O(0) = 0.94 \rightarrow 0.96 \rightarrow 1 \rightarrow ?$).

The cross section difference $\sigma_{pp} - \sigma_{p\bar{p}}$ is less than the experimental uncertainties. The diffractive photoproduction of $C = +1$ (pseudo) scalar and tensor mesons $M, \gamma p \rightarrow Mp'$ (with $p'$ either proton or its low-mass excitation), seems to be a better signature for the odderon exchange [24, 36]. At asymptotic energies, when the $\rho$ and $\omega$ exchange contributions die out, such processes will be dominated by the odderon exchange. These reactions also have not yet been observed experimentally.

• The available calculations of the odderon amplitude give only the first term — ”Born approximation” — of the reggeization program, which technically is carried out by resummation of logarithms from loop corrections. It is expected that, since the intercept of the odderon is close to 1, the reggeized physical amplitude will be close to the mentioned ”Born” result. However, this approach, used e.g. in refs. [14, 18], requires care.

☐ The proton vertex. In the widely used quark-diquark model, the result depends strongly on the clustering of quarks in the nucleon. With variation of this clustering both the value of amplitude (by a factor $1 \div 4$) and content of final state change strongly [13]. For example, the diagonal transition $p' = p$ is almost forbidden for point-like diquark (this particular case is used in the stochastic vacuum calculation of [14]) and becomes not small (perhaps, dominant) for more realistic finite diquark size.

The similar uncertainty takes place in the description of dipion production via the hard odderon [13]. In the pQCD approach the quark impact–factors for Pomeron and odderon are identical [24]. However, the coupling of Pomeron to the gluon content of proton can be essential, and such coupling is absent for odderon [15]. It can give different final $p'$ states for Pomeron and odderon, reducing charge asymmetric interference.

3 In this Appendix I follow discussion of ref. [12].
The second difficult point is clearly seen in the treatment of $\gamma p \rightarrow f_2p'$ process with its helicity structure. Calculated "Born" term contains both factorizable in helicity terms and non-factorizable terms. Only factorizable terms should be regarded for the estimates of cross sections under interest while non-factorizable terms must be eliminated as having no relation to the Reggeon (in our case — the odderon).

The dominant $\gamma p \rightarrow f_2p'$ amplitude calculated in ref. [14] is exactly non-factorizable: in this amplitude spin flips in the vertices are correlated, $\lambda_\gamma - \lambda_f = -(\lambda_p - \lambda_{p'}) = -1$, and instead of dependence $A \propto t$ following from general property (12), it does not vanish at $t = 0$. This non-factorizable term must be eliminated from the result.

Therefore, two essential conclusions of ref. [14] cannot be related to the odderon: (i) the values of cross sections estimated; (ii) the predictions about nucleon excitations dominance for the proton vertex.

Recently, H1 collaboration tried to observe odderon in the diffractive photoproduction. With event selection including observation of only excited nucleon states (required by [14] results and hardly probable in reality as it was mention above), the signal from odderon was not found and the upper limits for various final states were set. The previous discussion shows that the event selection used in this experiment is misleading and theoretical estimates of cross sections are irrelevant.

The calculations of hard odderon amplitudes were performed in the lowest 3-gluon exchange approximation. The calculations of ref. [24] are valid only for very large $|t|$ where cross sections are very small. The calculations of refs. [37] consider electroproduction at small $t$. The problem of reggeization mentioned above in respect to [14] was not considered in these papers. The other uncertainties in these calculations were discussed in sect. 3.1.1.

Therefore, despite the fact that the odderon is a necessary feature of QCD motivated description of diffractive type processes, up to the moment we have no approach giving reliable estimations for the processes like $\gamma p \rightarrow f_2p'$, etc. at $k_\perp \lesssim 1$ GeV. That is the reason why in ref. [12] and in the text above we use completely phenomenological description and present only calculations for the lowest total odderon cross section exceeding effect of other mechanisms.

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References

[1] E. Cuautle, G. Herrera, J. Magnin, A. Sanchez-Hernandez, hep-ph/0005023

[2] I am thankful to I.P. Ivanov for clarification of this point for me.

[3] Particle Data Group. Eur. Phys. J. C 15 (2000).
[4] S.Binner, J.H. Kühn, K. Melnikov, Phys. Lett. B 459 (1999) 279; for recent references see G.Rodrigo, H.Czyz, J.H. Kuhn, M.Szopa, Eur. Phys. Journ. C 24 (2002) 71; G.Rodrigo, H.Czyz, J.H. Kuhn, hep-ph/0210287; A. Hoefler, J. Gzula, F. Jegerlehner, Eur. Phys. Journ. C 24 (2002) 51.

[5] The KLOE collab., hep-ex/0107023; 0205046; 0210013

[6] V.L. Chernyak, V.G. Serbo: Nucl. Phys. B 67 (1973) 464

[7] I.F. Ginzburg, A. Schiller, V.G. Serbo, Eur. Phys. Journ. C18 (2001) 731

[8] M. Diehl, T. Gosset, B. Pire, Phys. Rev. D 62 073014 (2000).

[9] E.A. Kuraev, L.N. Lipatov, N.P. Merenkov, M.I. Strikman: Sov. Yad. Fiz. 23 (1976) 163

[10] D. Asner. Report at ECFA-DESY Workshop at StMalo (April 12-15, 2002).

[11] I.F. Ginzburg, V.A. Ilyin, in preparation.

[12] I.F. Ginzburg, I.P. Ivanov, N.N. Nikolaev, hep-ph/0207345, submitted to Eur. Phys. J. C; preliminary reports: I.F. Ginzburg, 2-nd THERA meeting materials (DESY, April 2000), I.F. Ginzburg, report at COMPASAS meeting (Dubna, July 2000), I.F. Ginzburg, to be published in Proc. ”Photon2001” Ascona, Switzerland (September, 2001); I.P. Ivanov, N.N. Nikolaev, I.F. Ginzburg, hep-ph/0110181

[13] B.G. Zakharov, Sov.J.Nucl.Phys. 49 (1989) 860

[14] E.R. Berger, A. Donnachie, H.G. Dosch, O. Nachtmann et al., Phys. Rev. D 59 (1999) 014018; Eur. Phys. Journ. C9 (1999) 491; C14 (2000) 673.

[15] T. Golling, for H1 Collab., DIS 2001. IX Int. Workshop on Deep Inelastic Scattering, Bologna, 27 April - 1 May 2001; J. Olsson, for H1 Collab., New Trends in High Energy Physics, Yalta, Crimea, 22–29 September 2001; hep-ex/0112012

[16] M. Galynsky, E.A. Kuraev, P.G. Ratcliffe, B.G. Shaikhatdenov, hep-ph/0003061

[17] S.J. Brodsky, J. Rathsman, C. Merino, Phys. Lett. B461 (1999) 114

[18] Ph. Hagler, B. Pire, L.Szymanowski, O.V. Teryaev, Phys. Lett. B 535 (2002) 117; E: B 540 (2002) 324; hep-ph/0206270, hep-ph/0209242.

[19] I.F. Ginzburg, I.P. Ivanov, in preparation.

[20] V.M. Budnev, I.F. Ginzburg, G.V. Meledin, V.G. Serbo, Phys. Reports 15 C (1975) 181–282.

[21] I am thankful M. Ryskin and B. Kopeliovich who mention me this fact.

[22] I.F. Ginzburg, I.P. Ivanov, M.V. Vychugin, in preparation.

[23] I.F. Ginzburg, D.Yu. Ivanov, Phys. Rev. D 54 (1996) 5523-5535.

[24] I.F. Ginzburg, D.Yu. Ivanov, Nucl. Phys. B 388 (1992) 376-390.
[25] T. Jaroszewicz, J. Kwiecinski, M. Praszałowicz, *Z. Phys.* **C 12** (1982) 167

[26] I.F. Ginzburg, *Proc. 9th Int. Workshop on Photon – Photon Collisions*, San Diego (1992) 474–501, World Sc. Singapore.

[27] D.A. Anipko, I.F. Ginzburg, A.V. Pak, in preparation.

[28] I.F. Ginzburg, G.L. Kotkin, V.G. Serbo, V.I. Telnov, *Nucl. Instrum. Methods* 205 (1983) 47; I.F. Ginzburg, G.L. Kotkin, S.L. Panfil, V.G. Serbo, V.I. Telnov, *Nucl. Instrum. Methods. A* **219** (1984) 5

[29] TESLA Technical Design Report, p. VI DESY 2001-011; [hep-ex/0108012](http://arxiv.org/abs/hep-ex/0108012)

[30] D.A. Anipko, M. Cannoni, I.F. Ginzburg, S. Kolb, O. Panella, A.V. Pak, in preparation.

[31] T. Mayer, C. Blochbinder, F. Franke, H. Fraas, [hep-ph/0108018](http://arxiv.org/abs/hep-ph/0108018)

[32] V.N. Gribov, I.Yu. Kobzarev, V.D. Mur, L.B. Okun, V.S. Popov, *Sov. J. Nucl. Phys.* **12** (1971) 699;

[33] L. Lukaszuk, B. Nicolescu, *Lett. Nuovo Cim.* **8** (1973) 405, D. Joyson, E. Leader, C. Lopez, B. Nicolescu, *Nuovo Cim. A30* (1975) 345.

[34] J. Bartels, *Nucl. Phys. B* **175** (1980) 365; J. Kwiecinski, M. Praszałowicz, *Phys. Lett. B94* (1980) 413; A. Donnachie and P.V. Landshoff, *Nucl. Phys. B231* (1984) 189; *Phys. Lett. B123* (1983) 345

[35] I.F. Ginzburg, *ZhETF Lett.* **59** (1994) 605.

[36] V.V. Barakhovsky, I.R. Zhitnitsky and A.N. Shelkovenko, *Phys. Lett. B267* (1991) 532

[37] J. Czyżewsky, J. Kwiecinski, L. Motyka, M. Sadzikowski, *Phys. Lett. B 398* (1997) 400, E: *B 411* (1997) 402; R. Engel, D.Yu. Ivanov, R. Kirschner, L. Szymanowski, *Eur. Phys. Journ. C* **4** (1998) 93