Fragmentation of $\omega$ and $\phi$ Mesons in $e^+e^-$ and $pp$ Collisions at NLO

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Abstract

A combined analysis of both $e^+e^-$ (LEP, SLD) and $pp$ (RHIC-PHENIX) hadroproduction processes are done for the first time for the vector meson nonet at the next-to-leading order (NLO) using a model with broken SU(3) symmetry. The scale ($p_T$) and rapidity ($y$) dependence of the differential cross section for $\omega$ and $\phi$ mesons of the $pp$ data are also discussed. The input universal quark (valence and singlet) fragmentation functions at a starting scale of $Q^2_0 = 1.5$ GeV$^2$, after evolution, have values that are consistent with the earlier analysis for $e^+e^-$ at NLO. However, the universal gluon fragmentation function is now well determined from this study with significantly smaller error bars, as the $pp$ hadroproduction cross section is particularly sensitive to the gluon fragmentation since it occurs at the same order as quark fragmentation, in contrast to the $e^+e^-$ hadroproduction process. Additional parameters involved in describing strangeness and sea suppression and octet-singlet mixing are found to be close to earlier analysis; in addition, a new relation between gluon and sea suppression in $K^*$ and $\phi$ hadroproduction has been observed.

Keywords: Vector meson, fragmentation, NLO, strangeness suppression, $pp$, QGP.

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I. INTRODUCTION

A number of analyses are available for fragmentation of pseudoscalar mesons and baryons till date; see, for example, Ref. [1] for π, Refs. [2–9] for K meson, Refs. [10, 11] for proton, and Refs. [3, 12] for η fragmentation with comprehensive reviews in Refs. [4–6, 13] as well. No such considerable interest has been shown towards vector meson production due to the scarcity of the data available so far.

Hadroproduction of φ vector mesons in proton-proton collisions is a good candidate signal for Quark Gluon Plasma (QGP) in heavy nucleus-nucleus collisions. This requires a good understanding of φ hadroproduction in pp collisions, which will serve as a baseline for nucleus–nucleus studies. With this motivation, analyses had been done for vector meson fragmentation in $e^+e^-$ scattering at Leading Order (LO) [14] and Next-to-Leading Order (NLO) [15] and in pp collisions at LO [14] as well. In this paper, for the first time, this has been extended to a combined investigation for the fragmentation of the entire vector meson nonet in $e^+e^-$ and for ω and φ meson production in pp collisions at NLO using the LEP [16–18] and SLD [19] data for $e^+e^-$ and RHIC PHENIX [20] data for pp hadroproduction.

A key feature of the analysis (described earlier in Refs. [14] and [15] and applied to the present study) is the ability to use the entire nonet vector meson hadroproduction data by defining SU(3)-symmetric fragmentation functions common to the entire set of octet mesons. This drastically reduces the number of independent fragmentation functions (from three quark- and one gluon fragmentation function for each member of the octet) to two universal quark- and one gluon fragmentation function. Some additional parameters are subsequently introduced to account for SU(3) symmetry breaking and singlet-octet mixing to allow the study of the entire vector meson nonet. The definition of input fragmentation functions and other parameters relevant to the model [15] remain the same in this study and have been briefly reviewed here for completeness; some differences in the choice of fragmentation functions for analyses are also mentioned below. The hadroproduction in pp collisions at NLO presented here is particularly important in view of the fact that gluons contribute at higher order in $e^+e^-$ collisions but contribute at the same order as quarks in pp processes. This study at NLO therefore enables a more precise determination of the gluon fragmentation functions.

Further studies such as gluon and singlet quark suppression, the dependence of the pp hadroproduction cross sections on transverse momentum $p_T$ as well as rapidity $y$, and inclusion of data on the (branching fraction-weighted) φ and ω cross section ratios helps in a more detailed understanding of the hadroproduction process. With this study we complete the program of vector meson nonet fragmentation using this model.

In section II we list the relevant cross-section formulae for hadroproduction in $e^+e^-$ and pp collisions. In Section III we present highlights of the model used to determine the vector meson fragmentation functions. In Section IV we use the available data to best-fit the parameters involved and show the resultant fits and their quality. We conclude in Section V with some remarks and summary.

II. KINEMATICS AND CROSS SECTIONS

We summarise here for completeness the relevant cross-sections for inclusive hadroproduction in $e^+e^-$ and pp collisions (in the c.m. frame).
A. Hadroproduction in $e^+e^-$ Collisions

The hadronic cross section for inclusive hadroproduction in $e^+e^-$ collisions to NLO is given by [21]:

$$\frac{1}{\sigma_{tot}} \frac{d\sigma_{e^+e^-}}{dx}(x; Q^2) = \frac{1}{\sum_F \lambda_B^F (1 + \alpha_s/\pi)} \left[ f_B^h(x, Q^2) \right],$$

where,

$$f_B^h(x, Q^2) = \int_x^1 \frac{dz}{z} \left[ \sum_F \lambda_B^F \left( \delta(1-z) + \frac{\alpha_s(Q^2)}{2\pi} C_F^{(1)}(z) \right) \left\{ D_{q_f}^h + D_{q_f}^h \right\} \left( \frac{x}{z} \right) + \frac{\alpha_s(Q^2)}{2\pi} \lambda_B^G C_G^{(1)}(z) \left( \frac{x}{z} \right) \right],$$

and $\sigma_{tot} = N_c \sum_F \lambda_B^F \left( \frac{4\pi\alpha^2}{3Q^2} \right) \left( 1 + \frac{\alpha_s}{\pi} \right).$ (2)

Here $x(= 2p_h/\sqrt{s})$ is the fraction of the parent quark momentum carried by the hadron ($h$) having momentum $p_h$. $Q = \sqrt{s}$ is the energy scale at which the analysis is carried out (the data is taken at the Z-pole, with $Q = 91.2$ GeV), functions like $D_{q_f}^h$, $D_{q_f}^h$ and $D_{G}^h$ are the quark, anti-quark and gluon fragmentation functions and $N_c$ refers to the number of colours. Terms like $C_F^{(1)}(z)$, $C_G^{(1)}(z)$, $\lambda_B^F$ and $\lambda_B^G$ are the coefficient functions for quarks (F) and gluons (G), whose expressions are given in detail in Refs. [15] and [21] and where $\alpha_s(Q^2)$ is also defined to NLO.

B. Hadroproduction in $pp$ Collisions

The hadronic cross section for inclusive hadroproduction in $pp$ collisions at NLO is given in terms of the underlying partonic interaction $p_i(x_1)p_j(x_2) \rightarrow p_i(x_3)p_k(x_4)$ as [22],

$$E_3 \frac{d^3\sigma}{d^3k_3} \sim \sum_{i,j,l} \int dx_1dx_2 \frac{dx_3}{x_3} F_{p_i}^{H_1}(x_1, M^2) F_{p_j}^{H_2}(x_2, M^2) D_{p_i}^{H_1}(x_3, M_f^2)$$

$$\times \left[ \frac{1}{v} \left( \frac{d\sigma^0}{dv} \right)_{p_i p_j \rightarrow p_i} (s, v) \delta(1-w) + \frac{\alpha_s(\mu^2)}{2\pi} K_{p_i p_j \rightarrow p_i} (s, v; \mu^2; M^2, M_f^2) \right],$$

where the indices $i, j, l$, sum over all possible flavours of quarks and anti-quarks, and gluons. The term $F_{p_i}^{H_1}(x_1, M^2) (F_{p_j}^{H_2}(x_2, M^2))$ refers to the parton distribution function of parton $p_i(p_j)$ inside hadron $H_1(H_2)$ with a momentum fraction $x_1(x_2)$ and initial factorization scale $M$. Likewise, $D_{p_i}^{H_1}(x_3, M_f^2)$ is the fragmentation function for a parton $p_l$ to fragment into a hadron $H_3$ with a momentum fraction $x_3$ and fragmentation scale $M_f$.

The first term within the bracket, $d\sigma^0$, is the LO Born cross section term for $p_i p_j \rightarrow p_i$ with $s, v$ and $w$ expressed in terms of $x_1, x_2$ and $x_3$ and hadronic momenta $k's$; for example, $s = x_1 x_2 S$ where $S$ is the usual hadronic centre of mass energy (squared).

The second term having $\alpha_s(\mu^2)$ with renormalization scale $\mu$ corresponds to the higher order contribution with its correction factor $K_{p_i p_j \rightarrow p_i} (s, v; \mu^2; M^2, M_f^2)$ for each subprocess. A detailed calculation of the correction factors for various subprocesses is given in Ref. [22]; here we merely note that, unlike in the $e^+e^-$ case, in $pp$ processes the gluon fragmentation function contributes at the LO itself through subprocesses such as $qg \rightarrow qg$ and $gg \rightarrow gg$. 


Hence we expect that inclusion of hadroproduction in \( p p \) processes will significantly improve our knowledge of gluon fragmentation.

Hence from Eqs. 2 and 3 it is very clear that the gluon fragmentation function appears at a higher order of \( \alpha(Q^2) \) as compared to quark fragmentation in \( e^+ e^- \) processes and at the same order in \( p p \) processes.

The LHS of Eq. 3 can be expressed in terms of physical observables, the rapidity \( y \) and the transverse momentum \( p_T \), as

\[
E_3 \frac{d^3\sigma}{d^3k_3} \equiv \frac{1}{p_T} \frac{d^3\sigma}{dp_T dy d\phi} = \frac{1}{\pi} \frac{d^2\sigma}{dp_T^2 dy} ,
\]

where the last simplification occurs because the cross-section is independent of the azimuthal angle \( \phi \). According to the factorization theorem, the cross section for \( p p \) in Eq. 3 is expressed as a convolution over three parts: parton distribution functions, partonic subprocess cross sections and fragmentation functions. For this study, the initial parton distribution functions are taken from GRV 98 NLO code [25], the partonic cross sections for hadroproduction in \( p p \) processes at NLO are taken from Aversa et al. [26], and the fragmentation of the final state parton is obtained using our \( x \)-real space Fortran code based on the broken SU(3) model.

III. THE MODEL

We now briefly describe the broken SU(3) model that is used to describe the input fragmentation functions at NLO in this paper. The details regarding the model were discussed in detail in Refs. [2], [14] and [15] in which the \( e^+ e^- \) data were fitted to the NLO cross sections using this model. In Ref. [14], a study of hadroproduction in \( p p \) processes at LO was also taken up. The present study, which includes consistently an analysis of both \( e^+ e^- \) and \( p p \) hadroproduction to NLO completes this program.

The model uses SU(3) flavour symmetry to express the quark fragmentation functions \( \alpha(x, Q^2), \beta(x, Q^2) \) and \( \gamma(x, Q^2) \) corresponding to the underlying quark fragmentation subprocesses \( q^i \to M^j X^j \), where \( X^j \) is a member of 3-, 6-, or 15-plet respectively. Application of charge conjugation symmetry and isospin invariance significantly reduces the number of unknown fragmentation functions. In addition, fragmentation functions of different mesons are related within this model, and this is what allows for the analysis of the otherwise sparse vector meson data.

The fragmentation functions of all octet vector mesons can be written in terms of three universal functions that are named valence (\( V \)), sea (\( \gamma \)), and gluon (\( D_g \)) fragmentation functions [13] (see Table II). The model defines the fragmentation functions at an initial scale of \( Q_0^2 \), taken to be \( Q_0^2 = 1.5 \text{ GeV}^2 \), for three light quarks \( u, d \) and \( s \), where the charm and bottom flavour contributions are kept zero. The contribution of such heavy flavours are added in at appropriate thresholds during DGLAP evolution. These input fragmentation functions are then evolved to various momentum scales for comparison with available data.

Breaking of SU(3) symmetry due to strangeness suppression is included through an \( x \)-independent strangeness suppression parameter \( \lambda \) at the starting scale. For instance, non-strange quark fragmentation into strange mesons such as \( K^* \) is suppressed by \( \lambda: D_u^{K*} \to \lambda D_u^{K*,\text{pure SU}(3)} \) while strange quark fragmentation is not suppressed: \( D_s^{K*} = D_s^{K*,\text{pure SU}(3)} \) (see Table II for the pure SU(3) expressions). The entire sea quark fragmentation into \( K^* \) is thus suppressed by a factor of \( \lambda \) compared to sea quark fragmentation into \( \rho \) mesons.

The model is extended to include the SU(3) singlet-octet mixing since it is known that the physical \( \omega \) and \( \phi \) mesons are admixtures of the octet and singlet states. An angle \( \theta \) is used
to describe SU(3) singlet–octet mixing. The singlet sector has an additional fragmentation function, $\delta(x, Q^2)$, due to the single subprocess that contributes: $q_i \to M_0 X_i$, where $X_i$ belongs to a 3-plet, which is taken to be proportional to the octet fragmentation function $\alpha$:

$$\frac{\delta(x, Q^2)}{3} = \frac{f_i^1}{3} \alpha(x, Q^2),$$

thus adding only two parameters for $i = \omega, \phi$, viz., $f_{1,\omega}^u$ and $f_{1,\phi}^s = f_{1,\phi}^d = 0$. The former arises from SU(3) and SU(2) symmetry and the latter from the observation that the physical $\phi$ ($\omega$) meson is almost purely an $s\bar{s}$ (non-strange) state since the phenomenological value of the mixing angle $\theta \sim 39^\circ$ is very close to the value $\theta = 35^\circ$ where this is exactly true. Finally the sea suppression factors for $\omega$ and $\phi$ are denoted as $f_{\omega,\text{sea}}^s$ and $f_{\phi,\text{sea}}^u$; they are expected to be of order unity and $\lambda^2$ respectively. Note that no additional singlet fragmentation functions are required.

In toto, we have the fragmentation functions for octet valence, sea and gluon ($V, \gamma$ and $D_g$) with strangeness suppression $\lambda$, the octet-singlet mixing angle $\theta$, and other $x$-independent singlet and suppression factors for the mixed $\omega-\phi$ system such as $f_1^\omega$ ($\phi$), $f_{\omega,\text{sea}}^s$ and $f_{\phi,\text{sea}}^u$. Finally, we have the gluon suppression factors $f_g^{K^*}$, $f_g^\omega$, $f_g^\phi$ [13], where $D_g = f_g^i D_g$, $i = K^*, \omega, \phi$.

There are two modifications in the parameter descriptions compared to the earlier analyses, both leading to better stability during evolution.

1. We have used upto linear terms in $x$ instead of the choice of a quadratic form in the

| fragmenting quark | $K^{*+}$ $(u\bar{s})$ | fragmenting quark | $K^{*0}$ $(d\bar{s})$ |
|-------------------|------------------------|-------------------|------------------------|
| $u, \bar{s}$     | $V + 2\gamma$         | $u, \bar{u}$     | $2\gamma$             |
| $d, \bar{d}$     | $2\gamma$             | $d, \bar{s}$     | $V + 2\gamma$         |
| $s, \bar{s}$     | $2\gamma$             | $s, \bar{d}$     | $2\gamma$             |

Table I: Pure SU(3) quark fragmentation functions for octet mesons in terms of the SU(3) valence ($V(x, Q^2)$) and sea ($\gamma(x, Q^2)$) fragmentation functions. The valence quark content of the mesons is indicated in brackets.
standard polynomial \[ \text{for the parameterization of input quark and gluon fragmentation functions:} \]

\[
F_i(x, Q_0^2) = a_i x^{b_i} (1 - x)^c P_i(x); \quad P_i(x) = (1 + d_i \sqrt{x} + e_i x); \quad (5)
\]

instead of \( P_i = (1 + d_i x + e_i x^2) \) which was the form used in the earlier analysis. This polynomial can have large fluctuations and even go negative during \( Q^2 \) evolution especially in the low-\( x \) region, while the current choice shows smooth behaviour at low and intermediate values of \( x \). Hence, this polynomial choice helps in obtaining a more stable fit at low-\( x \), and a better fit at intermediate-\( x \), particularly for \( pp \) data. Here \( F_i(x) = V(x), \gamma(x) \) and \( D_g(x) \) are the corresponding valence, sea and gluon input fragmentation functions and \( a_i, b_i, c_i, d_i \) and \( e_i \) are the parameters to be determined for these functions at the starting scale \( Q_0^2 \).

2. The sea fragmentation function, \( \gamma(x) \), can be expressed in terms of the valence \( V \) and singlet fragmentation function \( D_0 \) at the starting scale:

\[
\gamma(x, Q_0^2) = (D_0(x, Q_0^2) - 2V(x, Q_0^2))/12 ; \quad D_0(x, Q_0^2) = \sum_f (D_f + D_T) (x, Q_0^2). \quad (6)
\]

Note that \( D_0(x, Q^2) \) is the dominant fragmentation function contributing to the \( e^+ e^- \) hadroproduction cross-section at the Z-pole. The large-\( x \) behaviour of \( D_0 \) and \( V \) is kept the same to avoid rapid fluctuations and hence possible unphysical negative values of \( \gamma \). Hence the exponent \( c \) (power of \((1 - x)\)) is taken to be the same in both, reducing the number of unknown parameters which are to be determined from the fit. Since the available data is either for charge-neutral mesons or sums of charge-conjugate pairs (such as \( \rho^+ + \rho^- \), etc.), it is more convenient to parametrise \((V, D_0)\) rather than \((V, \gamma)\) and indeed this former choice gives a more stable result.

IV. COMBINED ANALYSIS OF \( e^+ e^- \) AND \( pp \) DATA

A. Choice of Data Sets

A combined analysis of both \( e^+ e^- \) and \( pp \) data is done in order to fit the vector meson fragmentation functions. The LEP data \[16, 18\] for \( \rho(\rho^+, \rho^-, \rho^0) \) and \( \omega \) mesons and SLD “pure uds” data \[19\] for \( K^* \) and \( \phi \) are used for \( e^+ e^- \) process at the Z-pole, \( \sqrt{s} = 91.2 \) GeV. The SLD “pure uds” data (three flavours alone) are used in the case of \( K^* \) and \( \phi \) mesons in order to avoid the contamination from heavy flavour meson production such as \( B \) and \( D \) mesons which decay into one of the strange mesons which will contaminate the data on direct hadroproduction into \( K^* \) or \( \phi \) due to large CKM matrix elements \( V_{cs} \) and \( V_{cb} \). In the case of non-strange mesons like \( \rho \) and \( \omega \), the contamination is very small, since heavier \( b \) and \( c \)-mesons will decay mostly (vis \( s \)) to \( \pi \), the lightest non-strange pseudoscalar meson, rather than \( \rho \) or \( \omega \).

Likewise, the 2011 RHIC/PHENIX data \[20\] at centre-of-mass energy, \( \sqrt{s} = 200 \) GeV, with rapidity (to be considered as pseudorapidity throughout the paper), \(|y| \leq 0.35 \) for \( pp \) collisions is used in the analysis for \( \omega \) and \( \phi \) hadroproduction. The data has three types of systematic errors added in quadrature with no statistical errors given in the literature. Effort is taken to add statistical errors from RHIC experimental group paper \[20\] and thesis \[24\] for \( \omega \) and \( \phi \) mesons decaying through various channels. Thus care is taken to include.
both the statistical and systematical errors which are added in quadrature. More recently [31], detailed doubly differential rates in both rapidity $y$ and transverse momentum $p_T$ have been measured by RHIC-PHENIX, in the forward rapidity region $1.2 \leq |y| \leq 2.2$ for $\omega$ and $\phi$ hadroproduction, as well as their weighted events ratio.

In both sets of $pp$ data, the azimuthal angle $\phi$ has an angular acceptance of $\pi$. In order to match the experimental result, therefore, we theoretically divide by a factor of 2 for azimuthal angle in the cross section equation in Eq. 4.

B. Determining the Best-fit Parameters

Using the standard functional form in Eq. 5, the unknown input fragmentation functions for valence, singlet and gluon are parameterized at an initial scale of $Q_0^2 = 1.5$ GeV$^2$. Contributions of the heavy $c$ and $b$ flavours are included at the appropriate thresholds during evolution. The fragmentation functions of all mesons are evolved to various scales, say, $Q^2 = (91.2)^2$ GeV$^2$ for $e^+ e^-$ and $Q^2 = p_T^2$ GeV$^2$ for $pp$ collision, using the DGLAP evolution equations [27–29] for $\rho, K^*, \omega$ and $\phi$.

The best fit to the unknown parameters is found by performing a combined $\chi^2$ minimization with both $e^+ e^-$ LEP [16–18] and SLD [19] data and $pp$ RHIC-PHENIX for both hadroproduction [20] and branching ratio weighted differential cross section [31] data.

The best fit values of the parameters $a, b, c, d$ and $e$ for valence, singlet and gluon input fragmentation functions, with 1$\sigma$ errors are given in Table II. The errors on the quark parameters are about 10%—similar to the earlier studies [15]—while the fits to the gluon parameters are much better determined than earlier.

| parameter | Central Value | Error Bars |
|----------|--------------|------------|
| $V$      |              |            |
| $a$      | 0.40         | 0.04 10    |
| $b$      | 0.81         | 0.20 20    |
| $c$      | 1.42         | 0.07 5     |
| $d$      | -2.22        | 0.58 26    |
| $e$      | 9.01         | 0.79 8     |
| $D_0$    |              |            |
| $a$      | 3.47         | 0.11 4     |
| $b$      | -0.88        | 0.02 3     |
| $c$      | $V_c$        | - -        |
| $d$      | 0.38         | 0.04 10    |
| $e$      | -1.32        | 0.05 4     |
| $D_g$    |              |            |
| $a$      | 1.45         | 0.07 5     |
| $b$      | -0.07        | 0.05 70    |
| $c$      | 2.78         | 0.05 2     |
| $d$      | -4.56        | 0.06 2     |
| $e$      | 6.22         | 0.08 2     |

Table II: Best fit values of the parameters defining the input octet valence and singlet quark and gluon fragmentation functions at the starting scale of $Q_0^2 = 1.5$ GeV$^2$, with their 1-$\sigma$ error bars.
1. Quark fragmentation functions

The small-\(x\) and large-\(x\) behaviour (exponents \(b\) and \(c\) in Eq. 5) of the valence quark fragmentation functions remain the same as before. This is expected since the quark fragmentation functions are well-determined by the \(e^+e^-\) data; the polynomial \(P_i(x)\) is much better determined—with smaller errors, falling from nearly 100% earlier to a few percent in the current analysis. These parameters give the pure octet non-strange fragmentation functions for \(\rho^\pm, \rho^0\) mesons. The corresponding fragmentation functions for \(K^*, \omega\) and \(\rho\) can be determined from these and the best-fit values of the additional (strangeness suppression and singlet-octet mixing) parameters are listed in Table III.

| parameter | Central Value | Error Bars |
|-----------|---------------|------------|
| \(\lambda\) | 0.07 | 0.01 11 |
| \(\theta\) | 39.69 | 1.31 3.5 |
| \(f^\omega_{\text{sea}}\) | 0.99 | 0.09 10 |
| \(f^\phi_{\text{sea}}\) | \(\lambda^2\) const | |
| \(f^1_1(\omega)\) | 0.000 | 0.08 8 |
| \(f^1_1(\phi)\) | 4.77 | 1.47 30 |
| \(f^K_{\gamma}\) | 0.52 | 0.06 12 |
| \(f^\omega_{\gamma}\) | 0.999 | 0.09 10 |
| \(f^\phi_{\gamma}\) | 0.21 | 0.01 5 |

Table III: Best fit values of the strangeness suppression factor \(\lambda\), the singlet–octet mixing angle, \(\theta\), and other suppression factors for \(\omega, \phi\) hadroproduction and gluon suppression factors at the initial scale of \(Q^2_0 = 1.5\ \text{GeV}^2\); note that \(f^\phi_{\text{sea}}\) has been set to \(f^\phi_{\text{sea}} = \lambda^2\). For details see text.

The value of the strangeness suppression factor, \(\lambda = 0.07 \pm 0.01\), is identical to the previous analysis [15] and close to \(\lambda^{\text{pseudo}} = 0.08 \pm 0.02\) for pseudoscalar mesons within error bars. Hence it is very clear from the consistent value of \(\lambda\), that it is a process- and spin- independent global parameter.

The parameters of the singlet fragmentation function can be understood, by doing a meaningful comparison of the sea (\(\gamma\)) fragmentation function determined from Eq. 6 with the one from earlier analysis. The values obtained for parameters \(a = 0.98\) and \(c = 5.6\) are same as before within the error bars and parameter \(b = -0.65\) is close to the earlier result; whereas the polynomial \(P_i(x)\) is quite different (\(d = -1.99\) and \(e = 4.2\)). Thus the current sea fragmentation function is exactly matching with the older one at large \(x\) whereas at small \(x\) it is 10 times larger due to the inclusion of low \(p_T\) \(p\bar{p}\) data. This shows that the sea fragmentation function is consistent with the earlier analysis implying that the singlet fragmentation function parameters are well determined through this analysis.

2. Singlet-octet Mixing and \(\omega-\phi\) fragmentation functions

The best-fit value of the octet-singlet mixing angle, \(\theta\), turns out to be \(\theta = 39.7^\circ \pm 1.31^\circ\) which is close to \(35^\circ\) [30] where \(\omega\) is a pure non-strange physical state and \(\phi\) purely an \(s\bar{s}\)
state. This angle is consistent with the value $\theta = 40.5^\circ$ determined from our earlier $e^+ e^-$ NLO studies [13].

The sea suppression factor, $f^{\omega}_{\text{sea}}$ for $\omega$ came out to be $0.99 \pm 0.09$, the same as earlier, implying that there is no suppression for sea in $\omega$ as it is purely nonstrange. The sea suppression factor for $\phi$, $f^{\phi}_{\text{sea}}$, was kept fixed as before to be equal to the square of the strangeness suppression ($\lambda^2$) since it is dominantly a pure $s\bar{s}$ state.

The singlet proportionality factors are obtained as $f^{u}_i(\omega) = 0.00 \pm 0.08$ and $f^{s}_i(\phi) = 4.77 \pm 1.47$ which are again consistent with the earlier analysis.

3. The gluon fragmentation functions

Through the $pp$ NLO fragmentation study, the gluon fragmentation function and its suppression in strange mesons are better understood compared to the earlier $e^+ e^-$ analysis. The parameter values obtained for input gluon fragmentation functions at the starting scale $Q_{0}^{2}$ (see Table II) have reduced error bars (typically less than 5%) compared to earlier studies [14, 15]. For instance, even the parameters $d$ and $e$ in the polynomial of Eq. 4 have error bars within 5% which implies that the gluon parameters are precisely determined through the analysis. The greatly reduced error bars reflect the higher sensitivity of $pp$ hadroproduction to gluon fragmentation.

Note that the $x$-dependence of the gluon fragmentation function at small-$x$ is still poorly determined, as expected due to both paucity of data, and the pole in the splitting function $P_{gg}$ that cause $D_{g}(x, Q^{2})$ to diverge at low-$x$ on evolution. In particular, the small-$x$ behaviour of the gluon fragmentation function is very different from before—the exponent $b$ in Eq. 5 is driven to nearly zero—due to the inclusion of the low $p_T$ $pp$ data as well as the $pp$ forward rapidity data on $\omega$ and $\phi$ mesons, which we shall discuss in more detail below.

The addition of the $pp$ data allows a more precise determination of the gluon fragmentation functions. Also, we have introduced three new gluon suppression factors, $f^{i}_{g}$, $i = K^{*}, \omega, \phi$ so that $D^{i}_{g} = f^{i}_{g} D^{0}_{g}$. The gluon suppression factor for $\omega$ meson came out to be $f^{\omega}_{g} = 0.99 \pm 0.09$, as expected (and its value is equal to the one obtained in earlier analysis [15]), but with its error reduced by half due to the improved precision.

It turns out that the value of the gluon suppression factor for $K^{*}$ is $f^{K^{*}}_{g} = 0.52 \pm 0.06$ which is quite different from before [15] (1.0 $\pm$ 0.09). Also, the value of gluon suppression factor for $\phi$ meson obtained from the analysis $f^{\phi}_{g} = 0.21 \pm 0.01$ is very stable throughout this analysis; it is lower than the value obtained from $e^+ e^-$ case (0.4 $\pm$ 0.04), but again more precisely determined. Overall, the gluon fragmentation for both $K^{*}$ and $\phi$ have halved compared to the earlier analysis.

It is interesting that the obtained values of $K^{*}$ and $\phi$ gluon suppression parameters can be related as, $f^{i}_{g} \sim (f^{K^{*}}_{g})^2$. This has an analogy with the assumption made in the beginning that the sea suppression factor for $f^{\omega}_{\text{sea}} = \lambda^2$, where $\lambda$ is the strangeness suppression in $K^{*}$. It also explains the relatively large suppression of $\phi$ hadroproduction in general: while non-strange fragmentation into $K^{*}$ is suppressed by $\lambda$ since $K^{*}$ is a $q\bar{q}$ state, $q = u, d$, non-strange fragmentation into $\phi$ is doubly suppressed by $\lambda^2$ since it is dominantly a $s\bar{s}$ state. This appears to hold true for both quark and gluon fragmentation. This may point to the presence of an SU(3) symmetric sea of quarks and gluons.

The input fragmentation functions ($D_{i}(x, Q^{2}); i = \text{valence, sea and gluon}$) at a low energy scale of $Q_{0}^{2} = 1.5$ GeV$^{2}$ for non-strange $\rho$ mesons are plotted as a function of the momentum fraction ($x_{3}$) as shown in the left of Fig. 4 for three light quarks alone. The figure on the right shows the fragmentation functions of the same set for $\rho$ mesons at a sample value of $Q^{2} = 56.3$
GeV$^2$, where the heavy flavours are produced by gluon-initiated process through evolution and not through heavy flavour B- or D- meson hadroproduction and their subsequent decays.

$$D_i(x_3,Q_0^2)$$

Figure 1: Initial fragmentation functions $D_i(x_3,Q^2)$ at the starting scale $Q^2 = Q_0^2 = 1.5$ GeV$^2$ (left) and fragmentation functions at a sample value of $Q^2 = 56.3$ GeV$^2$ (right), $i = V, \gamma, D_g$, for $\rho$ mesons as a function of the momentum fraction $x_3$.

Fig. 2 shows the ratio of the dominant quark fragmentation functions (left) and gluon fragmentation functions (right) for $\omega$ and $\phi$ mesons as a function of $x$. To understand the quark ratio we considered the dominant non-strange quark fragmentation functions ($D^\omega_u$) for $\omega$ mesons and strange quark fragmentation functions ($D^\phi_s$) for $\phi$ mesons. The quark fragmentation ratio came out to be $D^\phi_s/D^\omega_u = \lambda$, that is, equal to the strangeness suppression factor, $\lambda = 0.07$, as expected, independent of $Q^2$. This shows that the model captures strangeness suppression well at all values of $(x, Q^2)$.

![Figure 2](image2.png)

Figure 2: (Dominant) quark fragmentation function ratio (left) and gluon fragmentation function ratio (right) of $\phi$ and $\omega$ mesons as a function of the hadron momentum fraction $x_3$. While the quark fragmentation function ratio is scale (and $x$-) independent, the gluon fragmentation function ratio is shown for three different $Q^2 = p_T^2$ values.

The ratio of the gluon fragmentation functions for the two mesons came out as $D^\phi_g/D^\omega_g = f^\phi_g/f^\omega_g = 0.21$ for large $x$, as expected, whereas in the small-$x$ region $D^\phi_g/D^\omega_g \to 1$, as can be seen from Fig. 2, clearly indicating SU(3) symmetry restoration at small-$x$ for all $Q^2$. As far as we understand, this interesting feature has not been pointed out before.
C. Fits to $e^+ e^-$ hadroproduction data

The best fits to the input parameters obtained in the previous section were used to evolve the fragmentation functions to the $Z$-pole. The resulting cross-sections are shown in the left hand side of Fig. 3 for $\rho^{\pm,0}$ and $\omega^0$ hadroproduction in comparison with the LEP data \cite{16–18} $e^+ e^-$ data and on the right for $K^{\ast,-,0,0}$ and $\phi$ hadroproduction in comparison with the SLD “pure uds” \cite{19} $e^+ e^-$ data.

There is good agreement with data throughout the $x$ range with an overall $\chi^2 \sim 24.0$ for 44 data points and 22 free parameters with 22 degrees of freedom. For individual $\chi^2$ from each meson, see Table IV.

![Figure 3: Cross section behaviour as a function of $x_p$ for vector meson fragmentation in $e^+ e^-$ collisions. The data from LEP \cite{16–18} for $\rho^{\pm,0}$ and $\omega$, and “pure uds data” for $K^*$ and $\phi$ mesons from SLD \cite{19} at $\sqrt{s} = 91.2 GeV$ are shown in comparison with the solid lines which are the best fits resulting from the present model.](image-url)

D. Fits to $p p$ hadroproduction data

The best-fit input parameters for the various fragmentation functions obtained in the earlier section are evolved to different $p_T$ values to obtain the differential hadroproduction cross-section defined in Eq. 4 in the central ($|y| \leq 0.35$) and forward ($1.2 \leq y \leq 2.2$) rapidity regions where data is available. The best-fit (solid central) lines to the cross-section as a function of $p_T$ is shown in Fig. 4 for $\omega$ (left) and $\phi$ (right) in comparison with central rapidity data from RHIC-PHENIX \cite{20}.

The fits reflect the consistency and power of the model which explains two entirely different processes without introduction of any new parameter in the analysis. We have analysed both the momentum ($p_T$) and rapidity ($y$) dependence of differential cross section for $\omega$ and $\phi$ mesons.

1. Scale ($p_T$) dependence

In the $p p$ hadroproduction process, the factorization, renormalization and the fragmentation scales are made equal to the transverse momentum, $M \sim \mu \sim M_f \sim p_T$ and the
| Data Set | No. of data points | $\chi^2$ |
|----------|-------------------|----------|
| Total $e^+ e^-$ | 44 | 24.01 |
| $\rho^0$ | 14 | 7.45 |
| $\rho^{+\mp}$ | 12 | 5.63 |
| K$^*$ | 6 | 7.12 |
| $\omega$ | 6 | 0.67 |
| $\phi$ | 6 | 3.14 |
| Total $pp$ | 46 | 50.51 |
| $\omega$ | 33 | 16.89 |
| $\phi$ | 13 | 33.62 |
| Forward ratio | 5 | 13.86 |
| Total $e^+ e^- + pp +$ forward ratio | 95 | 88.38 |
| Total free parameters | 22 | – |
| Total $e^+ e^- + pp +$ forward ratio | $\chi^2$/dof | 88.38/73 |

Table IV: $\chi^2$ values obtained from best-fits to $\rho$, K$^*$, $\omega$ and $\phi$ hadroproduction from $e^+ e^-$ LEP, SLD data, and $\omega$ and $\phi$ hadroproduction for central rapidity as well as ratio of branching fraction weighted cross sections of $\phi$ and $(\omega + \rho)$ mesons for forward rapidity from $pp$ RHIC-PHENIX data.

Figure 4: Cross section as a function of $p_T$ for $\omega$ (L) and $\phi$ (R) meson hadroproduction in $pp$ collisions at $\sqrt{s} = 200$ GeV and $|y| \leq 0.35$. Bands show the scale uncertainty on changing $Q^2 = p_T^2$ over a range $p_T^2/2 \leq Q^2 \leq 2p_T^2$ (lower curve) for all the three scales. See text for more details.

uncertainty in the scales are determined by changing the value of $Q^2 = p_T^2$ over a range $p_T^2/2 \leq Q^2 \leq 2p_T^2$.

Fig. 4 clearly shows the effect of the scale uncertainty for both the mesons. Keeping all the three scales ($M, \mu, M_f$) equal to $Q$ for convenience and changing $Q^2$ from $p_T^2/2$ to $2p_T^2$ gives an uncertainty band as shown in the figure. The central curve in both left and right side of the Fig. 4 is the actual fit without scaling for original $Q^2 = p_T^2$, whereas the upper curve is for a scale change of $Q^2 = (1/2)p_T^2$ for all the three scales and the lower curve for
\( Q^2 = 2p_T^2 \). For the \( \omega \) meson, the scale uncertainty is seen to be rather small, whereas for the \( \phi \) meson the scale dependence is significant. The reduced scale uncertainty for mesons at NLO compared to the earlier LO analysis [14] shows that the scale dependence decreases with inclusion of higher order terms, as is expected.

2. Fits to the rapidity dependence of \( pp \) data

The RHIC-PHENIX collaboration has also studied [31] the branching ratio (BR)-weighted differential cross section of \((\rho + \omega)\) and \( \phi \) as a function of rapidity over the \( p_T \) range from \( 1 \leq p_T \leq 7 \) GeV/c. The event rates are defined as

\[
(N_\omega + N_\rho) = (BR(\omega \to \mu\mu)\sigma_\omega + BR(\rho \to \mu\mu)\sigma_\rho) ,
\]
\[
N_\phi = BR(\phi \to \mu\mu)\sigma_\phi ,
\]

where the relevant branching ratio to dimuons for \( \rho \) is \((4.55 \pm 0.28) \times 10^{-5}\), for \( \omega \) is \((9.0 \pm 3.1) \times 10^{-5}\) and for \( \phi \) is \((2.87 \pm 0.19) \times 10^{-4}\) [32].

Here \( \sigma_i \) is the integrated cross-section, \( \sigma_i = d\sigma/dy, i = \omega, \rho, \phi \), and the model calculation has been performed by integrating from \( 1.22 \leq p_T(\text{GeV}) \leq 7 \) since the starting scale is \( Q_0^2 = 1.5 \) GeV².

Fig. 5 shows that the cross-section for hadroproduction of non-strange mesons like \( \rho \) and \( \omega \) fall slower with rapidity from central to forward regions and are barely consistent with the data (see Table [V]) while in the case of \( \phi \) hadroproduction, the model fits well with the data. While the fits are still reasonably good, an improvement in the error bars of the data will severely constrain the model parameters, especially that of the gluon fragmentation functions. In fact, reproducing this rapidity dependence was the biggest constraint in determining the model parameter fits.

![Figure 5](image-url)

Figure 5: Solid lines show the model fit to the branching fraction-weighted differential cross sections as a function of rapidity, \( y \), for \((\rho + \omega)\) (L) and \( \phi \) (R) meson hadro-production in \( pp \) collisions at \( \sqrt{s} = 200 \) GeV in comparison with the RHIC data. Both statistical & systematical errors are added in quadrature.
The event ratio is given by,

$$\frac{N_\phi}{(N_\omega + N_\rho)} = \frac{BR(\phi \rightarrow \mu\mu)\sigma_\phi}{(BR(\omega \rightarrow \mu\mu)\sigma_\omega + BR(\rho \rightarrow \mu\mu)\sigma_\rho)},$$

where the ratio is determined for $1.22 \leq p_T \leq 7$ GeV/c, for both central ($|y| \leq 0.35$) and forward rapidity ($1.2 \leq |y| \leq 2.2$) regions.

![Figure 6: Model best-fits to $N_\phi/(N_\omega + N_\rho)$ as a function of $p_T$ over the range $1.22 \leq p_T \leq 7$ GeV/c for $\sqrt{s} = 200$ GeV in comparison with the RHIC data [31]. The statistical and systematical errors are added in quadrature. The upper solid line represents the fit for the central rapidity region ($|y| \leq 0.35$) while the lower one is for the forward rapidity region ($1.2 \leq |y| \leq 2.2$).](image)

The model values for the ratio $N_\phi/(N_\omega + N_\rho)$ are listed in Table VI for central as well as forward rapidity regions. The ratio was determined as 0.43 in the central region, on the average, whereas it was found to be 0.35 in the forward rapidity region. The ratio for both central and forward rapidity regions are in agreement with the data value of $0.390 \pm 0.021$ (stat) $\pm 0.035$ (sys) since the data have large statistical and systematical uncertainties. However, the detailed $p_T$ dependence is not correctly reproduced, especially for forward
rapidities, as can be seen from Fig. 6. This is a reflection of the slower fall with increasing rapidity $|y|$ in $(\rho+\omega)$ as discussed earlier and shown in Fig. 5. With more data, presumably, the fits in this sector can be improved in the future.

| $p_T$ (GeV/c) | $N_\phi/(N_\rho + N_\omega)$ |
|--------------|-----------------------------|
| Central $y$  | Forward $y$ Data for forward $y$ |
| 1.375        | 0.363 0.316 0.33±0.04±0.03 |
| 2.2          | 0.365 0.308 0.44±0.05±0.04 |
| 2.65         | 0.362 0.300 0.43±0.05±0.04 |
| 3.5          | 0.359 0.293 0.40±0.05±0.04 |
| 5.5          | 0.347 0.285 0.45±0.09±0.04 |

Table VI: The ratio $N_\phi/(N_\rho + N_\omega)$ vs. $p_T$ in (GeV/c) for both central ($|y| \leq 0.35$) and forward ($1.2 \leq |y| \leq 2.2$) rapidity regions for $\sqrt{s} = 200$ GeV. For more details, see text.

The $\chi^2$ values obtained from fits to both $\omega$ and $\phi$ mesons in the central rapidity region and their branching fraction-weighted ratios in the forward rapidity region are given in Table [IV]. The model provides reasonable fits to the parameters with reduced error bars with a $\chi^2 = 50.5$ for 22 parameters and 24 degrees of freedom for hadroproduction and $\chi^2 = 13.86$ for 5 degrees of freedom for event ratio of $pp$ data. Apart from individual values, an overall $\chi^2$ of 88.38 is obtained from the combined $e^+ e^-$ and $pp$ (hadroproduction in the central and ratios in the forward rapidity) data with 73 degrees of freedom (95 data points and 22 fit parameters) which is pretty good, and reflects the consistency and efficacy of the model.

An effort was made to understand the LHC/ALICE data [33] for the production of $K^*$ and $\phi$ mesons in $pp$ collisions at $\sqrt{s} = 7$ TeV. The data has $p_T$ values ranges from 1.25 to 5.5 GeV. Therefore, at these low $p_T$, the $x$-values will be of the order of $10^{-4}$. It is well known that DGLAP evolution equations fail at such small $x$-values due to the poles in both the $P_{gq}$ and $P_{gg}$ splitting functions which cause both the singlet quark and gluon fragmentation functions to diverge at small-$x$. Such studies at low $x$-values can be done using modified leading log approximation (MLLA) [2] which yield better (convergent) behaviour of fragmentation functions at small $x$. This is beyond the scope of the present work.

V. CONCLUSION

Vector meson fragmentation has been studied for the first time in both $e^+ e^-$ and $pp$ collisions at NLO with the comparison of LEP, SLD ($e^+ e^-$) and RHIC ($pp$) data using a model with broken SU(3) symmetry.

The model with three light flavours $u, d$ and $s$ uses SU(3) symmetry to describe the unknown fragmentation functions in terms of three independent quark fragmentation functions $\alpha(x, Q^2)$, $\beta(x, Q^2)$ and $\gamma(x, Q^2)$ with their conjugates and a gluon fragmentation function. The model uses further symmetries like isospin invariance and charge conjugation to reduce the functions to two universal functions, the valence $V(x, Q^2)$ and singlet $D_0(x, Q^2)$ quark fragmentation functions and a gluon fragmentation function $D_g(x, Q^2)$.

A strangeness suppression parameter $\lambda$ describes strangeness suppression in $K^*$ mesons. The entire meson nonet (and hence the physical $\omega$ and $\phi$ hadroproduction) is considered by including a singlet–octet mixing parameter, $\theta$. Instead of introducing a (yet another unknown) singlet fragmentation function, this is related to the octet fragmentation function.
\( \alpha(x, Q^2) \) through two proportionality constants, one each for \( \omega \) and \( \phi \) mesons along with a parameter that describes strangeness suppression in \( \omega \) (which turns out to be \( \sim 1 \) since \( \omega \) is dominantly a non-strange meson due to the particular value of the mixing angle). The strangeness suppression factor in \( \phi \) turned out to be close to \( \lambda^2 \), albeit with larger error bars, and was set to be equal to \( \lambda^2 \). No new fragmentation function or additional parameters are introduced in order to explain the \( pp \) hadroproduction data. Finally, individual gluon suppression factors were introduced for \( \omega \), \( K^* \), and \( \phi \), although the first was close to unity.

The best-fit values of the 22 free parameters are given in Tables II and III. The new gluon dependent parameter values are determined more precisely with reduced error bars (within 5\%) compared to the previous analysis with \( e^+ e^- \) data alone [15]. The \( K^* \) and \( \phi \) gluon suppression values are related by \( f_{g}^{\phi} \sim (f_{g}^{K^*})^2 \) similar to the result, \( f_{sea}^{\phi} = (f_{sea}^{K^*})^2 \); this can be used to further reduce the number of fit parameters. This shows the stability of the model and indicates the presence of an SU(3) symmetric sea of quarks and gluons over the entire nonet. Furthermore, the ratio of (dominant) quark fragmentation for \( \phi \) and \( \omega \) mesons came out to be equal to \( \lambda = 0.07 \), the strangeness suppression parameter, which implies that \( \omega \) is dominantly a non-strange meson and \( \phi \) is dominantly a \( ss \) state. In contrast the corresponding gluon ratio tended to unity at low \( x \), indicating restoration of SU(3) symmetry here.

The model explains both the \( e^+ e^- \) and \( pp \) data with good fits and reasonable \( \chi^2 \) as seen from Table IV and Figs. 3 and 4. The \( p_T \) band in Fig. 4 shows the reduced scale dependence for \( pp \) hadroproduction at NLO compared to earlier results [14] at LO, as expected. The branching ratio-weighted differential cross section for \( \rho + \omega \) and \( \phi \) hadroproduction in \( pp \) collisions as a function of rapidity were fitted with the RHIC data [31]. The results show that the rates for \( \rho \) and \( \omega \) mesons fall slower with increase in rapidity away from the central region whereas for \( \phi \) it is comparatively faster; both are consistent with data.

Finally, the ratio of branching fraction-weighted cross-sections for \( \phi \) and \( (\omega + \rho) \) mesons was found to be 0.43 for central rapidity and 0.35 for forward rapidity regions; this agrees with the data while not being fully in agreement with the detailed \( p_T \) dependence of the data. Note that the low \( p_T \) region is particularly intractable because of large changes upon even very small evolution from the starting scale \( Q_0^2 = 1.5 \text{ GeV}^2 \). It may be possible to tune the parameter fits to improve the agreement in this sector with the availability of more data with improved error bars.

This completes the program of describing the fragmentation functions of the entire vector meson nonet at NLO using both \( e^+ e^- \) and \( pp \) hadroproduction data. The fragmentation of vector mesons like \( \omega \) and \( \phi \) meson which have been studied for \( pp \) collisions with RHIC data will be useful to understand, for example, strangeness suppression or \( \phi \) production in nucleus-nucleus collisions as a signal in QGP studies. This was one of the primary motivations for this work.

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