Critical stable sectional area of downstream surge tank of hydropower plant with sloping ceiling tailrace tunnel

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Abstract
This paper investigates the critical stable sectional area (CSSA) of downstream surge tank (DST) of hydropower plant with sloping ceiling tailrace tunnel (SCTT). Firstly, two types of nonlinear mathematical model of hydropower plant with SCTT during transient process are established. Then, the criterion of operation stability of hydropower plant is obtained based on nonlinear stability theory. Formulae for CSSAs of DST under two types of mathematical model are derived. Finally, the similarities, differences, and relationships of different CSSAs are clarified. The effect mechanisms of the assumption of steady power output of hydro-turbine and the SCTT on CSSA of DST are revealed. The recommended use conditions of different CSSAs are explained. The results indicate that, under the assumption of steady power output of hydro-turbine, the formula for CSSA of DST of hydropower plant with SCTT is analytical and has a simple expression. Under the complete model, the formula for CSSA of DST of hydropower plant with SCTT is not analytical and has a complicated expression. The former formula is suggested to be given the priority of use because of good precision and concise expression. SCTT can improve the stability of hydropower plant and decrease the CSSA of DST.

Keywords
critical stable sectional area, downstream surge tank, hydropower plant, nonlinear stability, sloping ceiling tailrace tunnel

INTRODUCTION

Hydropower energy is a renewable, clean, sustainable, and storable energy.1,2 Hydropower plant produces electricity using water flow. The water flow activates hydro-turbine and the hydro-turbine drives generator that converts the mechanical energy of water to electricity.3 The advantage of hydropower plant is the flexibility of operation. For the hydropower plant, the frequent start and stop of the hydro-turbine unit and

Abbreviations: B, width of SCTT, m; e, wave velocity of free surface flow section, m/s; eg, load self-regulation coefficient; e1, e2, e3, moment transfer coefficients of hydro-turbine; e0, e1, e2, discharge transfer coefficients of hydro-turbine; F, sectional area of DST, m2; f, sectional area of SCTT, m2; g, acceleration of gravity, m/s2; H, hydro-turbine head, m; h0, head loss of penstock, m; hy, head loss of SCTT, m; Ki, integral gain, s1; Kp, proportional gain; M, resisting moment, N·m; Mh, kinetic moment, N·m; N, hydro-turbine unit frequency, Hz; Q, discharge in penstock, that is, hydro-turbine unit discharge, m3/s; Qy, discharge in SCTT, m3/s; t, time, s; Tw, hydro-turbine unit inertia time constant, s; tanα, ceiling slope of SCTT; Tw, flow inertia time constant of penstock, s; Tw, flow inertia time constant of SCTT, s; V, flow velocity of interface of free surface-pressurized flow, m/s; Vx, flow velocity in SCTT, m/s; Y, guide vane opening, mm; Z, change of water level in DST, downward relative to the initial level, m; λ, cross-sectional coefficient of SCTT.

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the variable load operation are required. The operation stability of hydropower plant faces severe challenges. The analysis and control of the operation stability of hydropower plant are an important topic.

Hydropower plant is usually composed of hydro-turbine unit and pipeline. The tailrace system is an important part of pipeline system and usually determines the type of hydropower plant. Sloping ceiling tailrace tunnel (SCTT) is a novel tailrace system and is widely applied in hydropower plants. During transient process, pressurized flow and free surface flow coexist in the SCTT. The SCTT can decrease the pressure rise of water flow during transient process by using its special body type. However, there is a limitation of length of tailrace system for the application of SCTT. If the tailrace system length is greater than 500 m, the pressure rise of water flow during transient process cannot be effectively restricted. For that situation, a downstream surge tank (DST) is necessary to be set on the SCTT. The combined operation of SCTT and DST is an effective measure to regulate and decrease the pressure rise of water flow during transient process.

The stability of hydropower plant is affected by every part of the pipeline and hydro-turbine unit. The variation of water head could change the stability of hydropower plant, and that change is caused by the turbine nonlinearity. The effect of turbine nonlinearity on the stability of hydropower plant is closely related to control modes of governor. The controller of governor has an obvious effect on the stability of hydropower plant. A reasonable controller of governor can improve the stability of hydropower plant significantly. The key for the design of controller of governor is the optimization of governor parameters. By using reasonable optimization algorithm, the optimal governor parameters can be determined to realize a stable hydropower plant. For the grid-connected hydropower plant, the stability of hydropower plant is influenced by the power grid. The hydropower plant and power grid have coupling effect on the stability of the whole system.

When the DST is introduced into hydropower plant, the water level in DST oscillates periodically during transient process. The water level oscillation (WLO) in DST exists the problem of stability. The stability of WLO in DST reflects the operation stability of hydropower plant. If the WLO in DST is not stable, the hydropower plant would not be stable. The stability of WLO in DST is mainly affected by the sectional area of DST. The sectional area of DST that makes the hydropower plant in the critical stable state is the critical stable sectional area (CSSA) of DST. For the design of hydropower plant with DST, the determination of CSSA of DST is the primary issue.

For the hydropower plant with pressurized tailrace tunnel (PTT), the CSSA of DST is easily to be determined. The computational method and formula have been stated in many textbooks and national specifications. But the body type of SCTT is totally different with that of PTT. The flow pattern in SCTT is also different with that in PTT. The combined operation of SCTT and DST introduces the coupling effect of free surface-pressurized flow in SCTT and WLO in DST. Under the coupling effect of the free surface-pressurized flow in SCTT, the phenomena and principle of WLO in DST are obviously different with those of hydropower plant with PTT. Therefore, the formula for CSSA of DST of hydropower plant with PTT cannot be applied to determine the CSSA of DST of hydropower plant with SCTT. The special formula for CSSA of DST of hydropower plant with SCTT is essential.

The research on the CSSA of DST of hydropower plant with SCTT is an important topic. However, there are no valuable researches and achievements on that topic. Until now, the computational method and formula for the CSSA of DST of hydropower plant with SCTT have not been proposed. Related researches mainly focus on the dynamic performance of hydropower plant with SCTT and the coupling effect between SCTT and surge tank. The representative literatures are stated as follows. In Ref., a dynamic equation of penstock with SCTT is proposed and the stability of hydropower plant with SCTT is studied by Hopf bifurcation theory. The stability working principle of SCTT is revealed. In Ref., the nonlinear control methods of dynamic performance of hydropower plant with SCTT are investigated. Both the Hopf bifurcation control and differential geometry theory are adopted, and the corresponding control strategies are designed. The designed control strategies prove to be better than the proportional-integral-derivative (PID) control strategy. In Ref., the operation stability of hydropower plant with SCTT is analyzed by the method of numerical simulation. The effects of governor parameters and ceiling slope of SCTT are studied in detail, and the recommended values are given. In Ref., the operation stability of hydropower plant with SCTT under small load disturbance is analyzed by using numerical simulation method. In that method, the pipeline, hydro-turbine unit, and governor are considered and calculated by a joint algorithm. In the stability of hydropower plant with SCTT is investigated using the generalized Hamiltonian theory. The change law of energy of hydropower plant is clarified to explain the nonlinear dynamic behavior of hydropower plant with SCTT. In a special operating condition, that is, primary frequency regulation, of hydropower plant with SCTT is studied by numerical simulation and field measurement. The governor parameters are optimized to improve the dynamic performance. In the nonlinear model of hydropower plant with upstream surge tank and SCTT is established. The stability and dynamic performance of hydropower plant are analyzed. In the coupling effect of
DST and SCTT is studied. The optimization methods of the design parameters of DST and SCTT are proposed to improve the stability of hydropower plant during transient process.

This paper aims to study the CSSA of DST of hydropower plant with SCTT. The novelty of this paper is to: (1) establish the reasonable mathematical model of hydropower plant with SCTT during transient process for the derivation of CSSA of DST, (2) derive the formula for the CSSA of DST of hydropower plant with SCTT, (3) reveal the similarities, differences, and relationships of different CSSAs of DST, and (4) put forward the recommended use conditions of different CSSAs of DST.

In Section 2, two types of nonlinear mathematical model of hydropower plant with SCTT during transient process, that is, model considering the assumption of steady power output of hydro-turbine and complete model, are established for the derivation of CSSA of DST. State equations of hydropower plant with SCTT under two types of mathematical model are obtained. In Section 3, the criterion of operation stability of hydropower plant is obtained based on model of hydropower plant and stability analysis theory. The algebraic criterion of stability corresponding to the CSSA of DST is determined. Formulae for the CSSAs of DST under two types of mathematical model are derived according to the algebraic criterion of stability. In Section 4, the correctness of the formulae for CSSAs of DST is verified by numerical verification. The similarities, differences, and relationships of different CSSAs of DST are clarified by comparative analysis. The effect mechanisms of the assumption of steady power output of hydro-turbine and the SCTT on the CSSA of DST are revealed. The recommended use conditions of different CSSAs of DST are explained. In Section 5, the conclusions are given.

2 MATHEMATICAL MODEL

The layout of pipeline system of hydropower plant with SCTT and the stability of WLO in DST during transient process are shown in Figure 1.

2.1 Basic equations

The pipeline system of hydropower plant with SCTT in Figure 1 contains penstock, DST, SCTT, hydro-turbine, generator, and governor. For the study of CSSA of surge tank, two types of mathematical model of hydropower plant during transient process are usually adopted. The first type of mathematical model is based on the assumption of steady output of hydro-turbine. In that assumption, the governor is extremely sensitive and the output of hydro-turbine unit can be kept unchanged. The dynamic behavior of penstock, hydro-turbine, generator, and governor can be described by one equation, that is, equation of steady output of hydro-turbine. The second type of mathematical model is not based on the assumption of steady output of hydro-turbine. The power output of hydro-turbine unit changes during transient process. The dynamic behaviors of penstock, hydro-turbine, generator, and governor are described by their own equations. So, the second type of mathematical model is a kind of complete model of hydropower plant.

In this section, both the above two types of mathematical model of hydropower plant with SCTT during transient process are established for the derivation of CSSA of DST. Then, the characteristics and effects of mathematical models on the CSSA of DST can be analyzed and compared. The similarities, differences, and relationships between different CSSAs of DST can be revealed. The first type of mathematical model is denoted as Model A, and the second type of mathematical model is denoted as Model B.

(1) Model A

Equation of steady power output of hydro-turbine:

\[ z_F = \left( 1 - \frac{h_{t0} + 3h_{t0}}{H_0} \right) q_t - \frac{2h_{t0}}{H_0} q_y \] (1)

Continuity equation of DST:

\[ q_y = q_t - \frac{FH_0}{Q_{t0}} \frac{dz_F}{dt} \] (2)

FIGURE 1 Layout of pipeline system of hydropower plant with SCTT and stability of WLO in DST during transient process
Dynamic equation of SCTT: For the SCTT, it is assumed that there is no interspace between the water and the ceiling of tailrace tunnel when the interface of free surface-pressurized flow moves. That assumption is valid when the ceiling slope gradient is less than 5%. The change of flow inertia of SCTT and the water level fluctuation in free surface flow section of SCTT are considered. The dynamic equation of SCTT is shown in Equation (3). Equation (3) is a first-order and nonlinear ordinary differential equation. The nonlinear term is $\frac{\lambda Q_{00} V_s}{g H_0 c B} q_y \frac{dq_y}{dt}$ and introduced by the change of discharge in SCTT.

$$z_F - \left( \frac{2h_0}{H_0} + \frac{\lambda Q_{00}}{H_0 c B} \right) q_c = \frac{\lambda Q_{00} V_s}{g H_0 c B} q_y \frac{dq_y}{dt} + T_{wy} \frac{dq_y}{dt}$$

(3)

(2) Model B

Moment and discharge equations of hydro-turbine: 

$$m_t = e_x h + e_x x + e_y y$$

(4)

$$q_t = e_y h + e_y x + e_q y$$

(5)

Equation of generator:

$$T_a \frac{dx}{dt} = m_t - (m_x + e_x x)$$

(6)

Equation of governor:

$$\frac{dy}{dt} = -K_p \frac{dx}{dt} - K_r x$$

(7)

Dynamic equation of penstock:

$$-h - z_F - \frac{2h_0}{H_0} q_t = T_{wt} \frac{dq_t}{dt}$$

(8)

For Model B, the continuity equation of DST and dynamic equation of SCTT are still Equation (2) and Equation (3), respectively. For Model A, the basic equations are Equations (1)-(3). According to the feature of basic equations and the research purpose of present study, the $q_t$ and $z_F$ are chosen as the state variables. Then, Equations (1)-(3) are converted to the following state equation.

$$\begin{pmatrix}
q_t \\
q_c \\
z_F
\end{pmatrix} = \begin{pmatrix}
\frac{Q_{00} V_s}{g H_0 c B} & \frac{\lambda Q_{00}}{H_0 c B} & 0 \\
\frac{\lambda Q_{00}}{H_0 c B} & \frac{\lambda Q_{00}}{H_0 c B} & 0 \\
\frac{2h_0}{H_0} & \frac{2h_0}{H_0} & \frac{2h_0}{H_0}
\end{pmatrix} \begin{pmatrix}
z_F \\
q_t \\
q_c
\end{pmatrix} + \begin{pmatrix}
T_{wt} \\
T_{wy} \\
0
\end{pmatrix} \frac{dq_t}{dt}$$

(9)

Equation (9) is the state equation of Model A of hydropower plant. Model A is a second-order nonlinear system. When we let $\mathbf{x} = (q_t, z_F)^T$, Equation (9) can be rewritten as the standard form, that is, $\mathbf{x} = M_1 \mathbf{x}$.

For Model B, the basic equations are Equations (2)-(8). According to the feature of basic equations and the research purpose of present study, the $q_t$, $z_F$, $q_c$, $x$ and $y$ are chosen as the state variables. Then Equations (2)-(8) are converted to the following state equation.

$$\begin{pmatrix}
q_t \\
q_c \\
z_F \\
x \\
y
\end{pmatrix} = \begin{pmatrix}
\frac{z_F - (2h_0 + \frac{\lambda Q_{00}}{H_0 c B}) q_t}{z_F - (2h_0 + \frac{\lambda Q_{00}}{H_0 c B})} & \frac{\lambda Q_{00} V_s}{g H_0 c B} & \frac{\lambda Q_{00}}{H_0 c B} & 0 & 0 \\
\frac{\lambda Q_{00}}{H_0 c B} & \frac{\lambda Q_{00}}{H_0 c B} & \frac{\lambda Q_{00}}{H_0 c B} & 0 & 0 \\
\frac{2h_0}{H_0} & \frac{2h_0}{H_0} & \frac{2h_0}{H_0} & \frac{2h_0}{H_0} & \frac{2h_0}{H_0} \\
\frac{\lambda Q_{00}}{H_0 c B} & \frac{\lambda Q_{00}}{H_0 c B} & \frac{\lambda Q_{00}}{H_0 c B} & \frac{\lambda Q_{00}}{H_0 c B} & \frac{\lambda Q_{00}}{H_0 c B} \\
0 & 0 & 0 & 0 & 0
\end{pmatrix} \begin{pmatrix}
z_F \\
q_t \\
q_c \\
x \\
y
\end{pmatrix} + \begin{pmatrix}
T_{wt} \\
T_{wy} \\
0 \\
0 \\
0
\end{pmatrix} \frac{dq_t}{dt}$$

(10)

Equation (10) is the state equation of Model B of hydropower plant. Model B is a fifth-order nonlinear system. When we let $\mathbf{u} = (q_t, z_F, q_c, x, y)^T$, Equation (10) can be rewritten as the standard form, that is, $\mathbf{u} = M_2 \mathbf{u}$. 

For the convenience of the stability analysis of hydropower plant, the state equation is necessary to be obtained firstly. For Model A, the basic equations are Equations (1)-(3). Moreover, the definitions of variables and parameters are shown in Nomenclature. The subscript of 0 refers to the initial value of variable.
3 | DERIVATION OF CSSA OF DST

The stability of WLO in DST reflects the operation stability of hydropower plant. The sectional area of DST that makes the hydropower plant in the critical stable state is the CSSA of DST. So, the stability analysis of hydropower plant is the key for the derivation of CSSA of DST.

In this section, the operation stability of hydropower plant is firstly analyzed. Based on model of hydropower plant and stability analysis theory, the criterion of stability can be obtained. Then, aiming at the critical stable state of hydropower plant, the algebraic criterion of stability corresponding to the CSSA of DST can be obtained. Finally, according to the algebraic criterion of stability, the CSSA of DST can be derived. Moreover, the Lyapunov’s first method is used for the stability analysis of nonlinear dynamic systems in this paper.

In the following paragraphs, the CSSAs of DST under Model A and Model B are derived, respectively.

3.1 | CSSA of DST under Model A

For the state equation of hydropower plant under Model A, that is, $\dot{x} = f(x) = M_1 x$, the equilibrium point $x_E = (q_{x_E}, z_{x_E})^T$ of the dynamic system can be solved from $x = 0$. Based on Equation (9), we can obtain the unique equilibrium point of Model A, that is, $x_E = (0, 0)^T$. The stability analysis of Lyapunov’s first method is based on the coordinate values of state equation at the origin of coordinates. Therefore, the equilibrium point of Model A meets the stability analysis condition of Lyapunov’s first method. For Model A, the necessary and sufficient condition of Lyapunov stability under equilibrium state is that all the characteristic values of $M_1$ have negative real parts.

At the origin of coordinates, the Jacobian matrix of $\dot{x} = f(x) = M_1 x$ is

$$J(x) = \frac{\partial f(x)}{\partial x} = \begin{bmatrix} b_{11} & 1 \\ b_{21} & b_{22} \end{bmatrix}$$

where

$$b_{11} = -\left(\frac{2h_0}{H_0} + \frac{Q_{a0}}{H_0 c_B}\right)$$

$$b_{21} = -\left(\frac{(H_0 - 3h_0 - 3h_0)Q_{a0}}{(H_0 - h_0 - 3h_0)F_{H_0}}\right)$$

$$b_{22} = \frac{Q_{a0}}{(H_0 - h_0 - 3h_0)F}$$

The expansion of $\det [J(x) - \lambda I] = 0$ yields the characteristic equation of Model A as follows.

$$\lambda^2 + b_1 \lambda + b_2 = 0$$

where

$$b_1 = -\left(\frac{b_{11} + b_{22}}{T_{xy}}\right)$$

and

$$b_2 = \frac{b_{11} + b_{22} - \frac{b_{11} b_{22}}{T_{xy}}}{T_{xy}}$$

Equation (12) is a quadratic equation. Based on the Lyapunov stability theory, the criterion of stability of the dynamic system expressed by Equation (12) is that $b_1 > 0$ and $b_2 > 0$ are met simultaneously. In the following paragraphs, $b_1 > 0$ and $b_2 > 0$ are analyzed to derive the CSSA of DST under Model A, respectively.

(a) $b_1 > 0$

By substituting the expression of $b_1$ into $b_1 > 0$ yields

$$\frac{2h_0}{H_0} + \frac{Q_{a0}}{H_0 c_B} > 0.$$  \hspace{1cm} (13)

From $b_1 > 0$ and $b_2 > 0$, we can get $F > \frac{Q_{a0}h_0 c_B}{(H_0 - h_0 - 3h_0)(2h_0 c_B + Q_{a0})}$. Substitution of $T_{xy} = \frac{L_s V_{a0} g}{\sqrt{3}}$ and $Q_{a0} = f_{V_{a0}}$ into $F > \frac{Q_{a0}h_0 c_B}{(H_0 - h_0 - 3h_0)(2h_0 c_B + Q_{a0})}$ gives:

$$F > \frac{L_s f}{2\left(b_{11} + \frac{Q_{a0}}{V_{a0}^2}\right)} - g \left(H_0 - h_0 - 3h_0\right)$$

Equation (13) is an inequality with respect to $F$. When Equation (13) is met, we have $b_1 > 0$.

(b) $b_2 > 0$

By substituting the expression of $b_2$ into $b_2 > 0$ yields

$$\frac{2h_0}{H_0} + \frac{Q_{a0}}{H_0 c_B} < \frac{(H_0 - 3h_0 - 3h_0)Q_{a0}}{(H_0 - h_0 - 3h_0)F_{H_0}} > 0.$$  \hspace{1cm} (14)

The simplification of $\frac{2h_0}{H_0} + \frac{Q_{a0}}{H_0 c_B} > 0$ gives:

$$H_0 > 5h_0 + 3h_0 + \frac{\lambda Q_{a0}}{c_B}$$

Equation (14) is an inequality without $F$. For the actual hydropower plants, Equation (14) can usually be met. So, for $b_2 > 0$, $F$ can be taken any positive value.

Equation (13) is met for partial value of $F$, while Equation (14) can usually be met for any positive value of $F$. Based on the above analysis about $b_1 > 0$ and $b_2 > 0$, Equation (13) is the stability condition of the hydropower plant under Model A. When Equation (13) is taken as equality, the hydropower plant under Model A reaches the critical stable state. Hence, we can obtain the formula for CSSA of DST under Model A as follows.

$$F_{th-A} = \frac{L_s f}{2\left(b_{11} + \frac{Q_{a0}}{V_{a0}^2}\right)} - g \left(H_0 - h_0 - 3h_0\right)$$

where $F_{th-A}$ represents the CSSA of DST under Model A.
### 3.2 CSSA of DST under Model B

For the state equation of hydropower plant under Model B, that is, the equilibrium point $\mathbf{u}_{E} = (q_{E}, z_{FE}, q_{E}, x_{E}, y_{E})^{T}$ of the dynamic system can be solved from $\mathbf{u} = 0$. Based on Equation (10), we can obtain the equilibrium point of Model B as follows.

$$
\begin{align*}
q_{SE} &= q_{IE} = \frac{m_{g}}{e_{qE}} \left( \frac{e_{qE}}{e_{ph}} \right) \left( \frac{2h_{ah} + 2h_{ah}}{H_{ah}} \frac{a_{Qph}}{H_{ph}B} + \frac{1}{e_{ph}} \right) \\
\varepsilon_{FE} &= \frac{e_{qE}}{e_{ph}} \left( \frac{e_{qE}}{e_{ph}} \right) \left( \frac{2h_{ah} + 2h_{ah}}{H_{ah}} \frac{a_{Qph}}{H_{ph}B} + \frac{1}{e_{ph}} \right) \\
y_{E} &= \frac{e_{qE}}{e_{ph}} \left( \frac{e_{qE}}{e_{ph}} \right) \left( \frac{2h_{ah} + 2h_{ah}}{H_{ah}} \frac{a_{Qph}}{H_{ph}B} + \frac{1}{e_{ph}} \right) \\
\end{align*}
$$

Equation (16) indicates that the equilibrium point of Model B is not at the origin of coordinates. The stability analysis of Lyapunov's first method is based on the coordinate values of state equation at the origin of coordinates. Therefore, the coordinate transformation is needed. Based on the expression of Equations (10) and (16), the following coordinate transformation is adopted.

$$
\begin{align*}
A &= q_{E} - q_{SE} \\
B &= z_{FE} - \varepsilon_{FE} \\
C &= q_{IE} - q_{SE} \\
D &= x_{E} - x_{E} \\
E &= y_{E} - y_{E} \\
\end{align*}
$$

Then Equation (10) can be transformed into the following form.

$$
\begin{align*}
\dot{A} &= \frac{a_{11}A + B}{b(A + C) + T_{wy}} \\
\dot{B} &= \frac{-a_{21}}{F} (A - C) \\
\dot{C} &= a_{32}B + a_{33}C + a_{34}D + a_{35}E \\
\dot{D} &= a_{43}C + a_{44}D + a_{45}E \\
\dot{E} &= a_{53}C + a_{54}D + a_{55}E
\end{align*}
$$

Equation (18) is also a fifth-order nonlinear dynamic system. When we let $w = (A, B, C, D, E)^{T}$, Equation (18) can be rewritten as the standard form, that is, $\dot{w} = M_{3}w$. The unique equilibrium point of $\dot{w} = M_{3}w$ is the origin of coordinates, that is, $w_{eq} = (0, 0, 0, 0)^{T}$. Therefore, the equilibrium point of $\dot{w} = M_{3}w$ meets the stability analysis condition of Lyapunov's first method. For $\dot{w} = M_{3}w$, the necessary and sufficient condition of Lyapunov stability under equilibrium state is that all the characteristic values of $M_{3}$ have negative real parts.

At the origin of coordinates, the Jacobian matrix of $\dot{w} = f(w) = M_{3}w$ is

$$
\begin{align*}
J(w) = \frac{\partial f(w)}{\partial w} &= \begin{bmatrix}
\frac{a_{11}}{b(A + C) + T_{wy}} & \frac{a_{21}}{b(A + C) + T_{wy}} & \frac{a_{31}}{b(A + C) + T_{wy}} & 0 & 0 \\
0 & 0 & 0 & 0 & a_{32} \\
0 & 0 & a_{33} a_{34} a_{35} & 0 & 0 \\
0 & 0 & 0 & a_{53} a_{54} a_{55} & 0 \\
\end{bmatrix}
\end{align*}
$$

The expansion of $det \{ J(w) - \gamma I \} = 0$ yields the characteristic equation of $\dot{w} = M_{3}w$ as follows.

$$
a_{0} \gamma^{5} + a_{1} \gamma^{4} + a_{2} \gamma^{3} + a_{3} \gamma^{2} + a_{4} \gamma + a_{5} = 0
$$

where $a_{0} = 1$, $a_{1} = -\left( a_{33} + a_{44} + a_{55} + \frac{a_{11}}{b(A + C) + T_{wy}} \right)$.
Equation (20) is a fifth-order equation. Based on the Lyapunov stability theory, the criteria of stability of the dynamic system expressed by Equation (20) are:

\[ a_i > 0 \quad (i = 0, 1, 2, \ldots, 5) \quad (21) \]

\[ \Delta_2 = a_1a_2 - a_0a_3 > 0 \quad (22) \]

\[ \Delta_3 = \begin{vmatrix} a_1 & a_3 & a_5 & 0 \\ a_0 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & a_5 \\ 0 & a_0 & a_2 & a_4 \end{vmatrix} > 0 \quad (23) \]

When Equations (21)-(23) are met simultaneously, the dynamic system \( \mathbf{w} = M_i \mathbf{w} \) is stable. Then, the hydropower plant under Model B is stable. In the following paragraphs, Equations (21)-(23) are analyzed to derive the CSSA of DST under Model B. According to the working principle of surge tank, the stability of hydropower plant becomes better with the rise of sectional area of surge tank. So, the stability of hydropower plant is monotone and continuous increasing with respect to the rise of sectional area. Equations (21)-(23) are the criteria of stability of hydropower plant. Then, the inequality intervals of \( F \) solved from Equations (21)-(23) must be left open intervals, that is, \( F > F^* \) or \( (F^*, +\infty) \).

The following expressions are defined for the convenience of derivation. It should be noted that both \( Y_i \) (\( i = 0, 1, 2, \ldots, 14 \)) and \( Y_i \) (\( i = 1, 2, 3, \ldots, 6 \)) do not contain \( F \).

\[ X_0 = a_{33} + a_{44} + a_{55} + \frac{a_{11}}{bc + T_{wy}}, \quad X_1 = a_{33}a_{44} - a_{34}a_{43} + a_{33}a_{55} - a_{35}a_{53} + a_{44}a_{55} - a_{45}a_{54}, \quad X_2 = \frac{a_{11} (a_{33} + a_{44} + a_{55})}{bc + T_{wy}}, \]

\[ X_3 = a_{21}a_{32}, \quad X_4 = \frac{a_{21}}{bc + T_{wy}}, \quad X_5 = a_{33}a_{44}a_{54} - a_{34}a_{43}a_{55} + a_{34}a_{43}a_{55} - a_{34}a_{45}a_{53} - a_{35}a_{43}a_{54} + a_{35}a_{44}a_{53}, \]

\[ X_6 = \frac{a_{11} (a_{33}a_{44} - a_{34}a_{43} + a_{33}a_{55} - a_{35}a_{53} + a_{44}a_{55} - a_{45}a_{54})}{bc + T_{wy}}, \quad X_7 = a_{21}a_{32} \left( a_{44} + a_{55} + \frac{a_{11}}{bc + T_{wy}} \right), \]

\[ X_{10} = a_{21}a_{32} \left( a_{44}a_{55} - a_{45}a_{54} \right), \quad X_{11} = \frac{a_{11}a_{21}a_{32}}{bc + T_{wy}} \left( a_{44} + a_{55} \right), \]

\[ X_{12} = \frac{a_{21} (a_{33}a_{44} - a_{34}a_{43} + a_{33}a_{55} - a_{35}a_{53} + a_{44}a_{55} - a_{45}a_{54})}{bc + T_{wy}}, \quad X_{13} = \frac{a_{11}a_{21}a_{32}}{bc + T_{wy}} \left( a_{44}a_{55} - a_{45}a_{54} \right), \]

\[ X_{14} = \frac{a_{21} (a_{33}a_{45}a_{54} - a_{33}a_{44}a_{55} + a_{34}a_{43}a_{55} - a_{34}a_{45}a_{53} - a_{35}a_{43}a_{54} + a_{35}a_{44}a_{53})}{bc + T_{wy}}, \]

\[ Y_1 = X_{10} + X_{11} - X_{12}, \quad Y_2 = X_1 + X_2, \quad Y_3 = X_4 - X_3, \quad Y_4 = X_4, \quad Y_5 = X_7 - X_8, \quad Y_6 = X_{13} + X_{14}. \]

\[ X_5 = \frac{a_{21} (a_{33} + a_{44} + a_{55})}{bc + T_{wy}}, \quad X_9 = \frac{a_{11} (a_{33}a_{45}a_{54} - a_{33}a_{44}a_{55} + a_{34}a_{43}a_{55} - a_{34}a_{45}a_{53} - a_{35}a_{43}a_{54} + a_{35}a_{44}a_{53})}{bc + T_{wy}}. \]
We have $a_0 = 1$. So $a_0 > 0$ can always be met.

(b) $a_1 > 0$ and $a_5 > 0$

By substituting the expressions of $a_1$ and $a_5$ into $a_1 > 0$ and $a_5 > 0$ yields $X_0 < 0$ and $X_{13} + X_{14} > 0$, respectively. $X_0$, $X_{13}$, and $X_{14}$ do not contain $F$. So, the CSSA of DST under Model B does not depend on $a_1 > 0$ or $a_5 > 0$.

(c) $a_2 > 0$, $a_3 > 0$, and $a_4 > 0$

By substituting the expressions of $a_2$, $a_3$, and $a_4$ into $a_2 > 0$, $a_3 > 0$ and $a_4 > 0$ yields $F > \frac{x_2 - x_3}{x_4 + x_5}$, $F > \frac{x_3 - x_4}{x_5 - x_6}$ and $F > \frac{x_{12} - x_{10} - x_{11}}{x_9}$, respectively. If $a_2 > 0$, $a_3 > 0$, and $a_4 > 0$ are met simultaneously, we have.

$$F > \max \left\{ \frac{X_4 - X_3}{X_1 + X_2}, \frac{X_7 - X_8}{X_3 - X_4}, \frac{X_{12} - X_{10} - X_{11}}{X_9} \right\}$$

$$F > \max \left\{ \max \left\{ \frac{X_4 - X_3}{X_1 + X_2}, \frac{X_7 - X_8}{X_3 - X_4}, \frac{X_{12} - X_{10} - X_{11}}{X_9} \right\}, \frac{X_7 - X_8 + X_0 (X_4 - X_3)}{X_0 (X_1 + X_2) + X_5 - X_6}, \max \left\{ F_1, F_2, F_3 \right\} \right\}$$ (28)

(d) $\Delta_2 > 0$

By substituting the expressions of $a_0$, $a_1$, $a_2$, and $a_3$ into $\Delta_2 > 0$ yields

$$-X_0 \left( X_1 + X_2 + \frac{x_2 - x_3}{F} \right) - \left( X_5 - X_6 + \frac{x_3 - x_4}{F} \right) > 0.$$ From

$$F_{th-B} = \max \left\{ \max \left\{ \frac{X_4 - X_3}{X_1 + X_2}, \frac{X_7 - X_8}{X_3 - X_4}, \frac{X_{12} - X_{10} - X_{11}}{X_9} \right\}, \frac{X_7 - X_8 + X_0 (X_4 - X_3)}{X_0 (X_1 + X_2) + X_5 - X_6}, \max \left\{ F_1, F_2, F_3 \right\} \right\}$$ (29)

that inequality, we can get.

$$F > \frac{X_7 - X_8 + X_0 (X_4 - X_3)}{X_0 (X_1 + X_2) + X_5 - X_6}$$ (25)

(e) $\Delta_4 > 0$

By substituting the expressions of $a_0$, $a_1$, $a_2$, $a_3$, $a_4$, and $a_5$ into $\Delta_4 > 0$ yields.

$$n_0 F^3 + n_1 F^2 + n_2 F + n_3 > 0$$ (26)

where $n_0 = X_0 X_9 Y_4 Y_6 - X_0^2 X_9^2$, $n_1 = 2 X_0 X_9 Y_6 + Y_6 Y_2 Y_4 + Y_1 Y_2^2 + 2 X_9 Y_4 Y_5 - 2 X_0 X_9 Y_1 + X_0 Y_2^2 Y_6 + X_9 X_4 Y_2 Y_5 + X_0 X_9 Y_3 Y_4 + X_0 Y_1 Y_2 Y_4$, $n_2 = 2 X_0 Y_1 Y_6 + Y_2 Y_2 Y_6 + Y_4 Y_4 Y_6 + 2 Y_4 Y_4 Y_5 + X_9 Y_2^2 + 2 X_0 Y_2 Y_3 Y_6 + X_9 X_4 Y_3 Y_5 + X_0 Y_1 Y_2 Y_5 + X_0 Y_1 Y_2 Y_4 - X_0^2 Y_1^2 - Y_6^2$.

When Equation (26) is taken as equality, we can obtain three roots, which are denoted as $F_1$, $F_2$, and $F_3$, respectively. The above analysis indicates that the inequality intervals of $F$ solved from the criteria of stability of hydropower plant must be left open intervals and are also continuous. Based on the feature of the solution interval of simple cubic inequality, the inequality interval of $F$ solved from Equation (26) should be.

$$F > \max \{ F_1, F_2, F_3 \}$$ (27)

When Equations (21)-(23) are met simultaneously, the hydropower plant under Model B is stable. According to the above analysis about $a_i > 0 (i = 0, 1, 2, \ldots, 5)$, $\Delta_2 > 0$ and $\Delta_4 > 0$, the inequality interval of $F$ can be obtained as follows.

$$n_3 = Y_3 Y_5 Y_6 + Y_1 Y_2^2 + X_0 Y_3^2 Y_6 + X_0 Y_1 Y_2 Y_5.$$
steady power output of hydro-turbine and the SCCT on the CSSA of DST are also revealed from the comparative analysis. The recommended use conditions of different CSSAs of DST are explained.

4.1 | Numerical verification

A hydropower plant with SCCT is taken as an example for the numerical verification. The basic data of the hydropower plant are as follows: $H_0 = 70.70$ m, $Q_{\theta} = Q_{\theta} = 466.70$ m³/s, $L_v = 2902.38$ m, $f = 230.00$ m², $V_{\nu} = 2.03$ m/s, $h_{\nu} = 4.27$ m, $h_{\nu} = 2.68$ m, $T_{\nu} = 8.50$ s, $T_{\nu} = 3.20$ s, $T_{\nu} = 8.77$ s, $B = 10.00$ m, $\tan \alpha = 0.03$, $\lambda = 3.00$, $H_x = 23.00$ m, $V_x = 2.03$ m/s, $c = 15.01$ m/s, $e_h = 1.50$, $e_x = -1.00$, $e_y = 1.00$, $e_{gh} = 0.50$, $e_{gt} = 0$, $e_{gy} = 0$, $K_p = 2.00$, $K_i = 0.30$ s⁻¹, and $g = 9.81$ m/s².

Equations (15) and (29) are the formulae for the CSSAs of DST under Model A and Model B, respectively. Based on the above basic data of hydropower plant, we can calculate the CSSAs of DST under Model A and Model B by using Equations (15) and (29). The results are shown in Table 1. In order to give a comprehensive analysis of the sectional area of DST on the stability of hydropower plant, more values of sectional area of DST are also selected for the numerical simulation in the following paragraphs.

By substituting the CSSAs and other values of sectional area of DST under Model A and Model B, respectively. Based on the above basic data of hydropower plant, we can calculate the CSSAs of DST under Model A and Model B by using Equations (15) and (29). The results are shown in Table 1. In order to give a comprehensive analysis of the sectional area of DST on the stability of hydropower plant, more values of sectional area of DST are also selected for the numerical simulation in the following paragraphs.

By substituting the CSSAs and other values of sectional area of DST under Model A into Equation (9), we can determine the dynamic processes of WLO in DST. The external disturbance is selected as $d_{\nu} = -0.10$ and the solution of Equation (9) is realized by ode45 in MATLAB. The results of dynamic processes of WLO in DST under Model A are shown in Figure 2(A). By using the same method, the dynamic processes of WLO in DST under Model B can be determined from Equation (18) and the results are shown in Figure 2(B).

Figure 2 shows that:

1. For Model A, the CSSA of DST $F_{th-A}$ is 268.85 m². When the sectional area of DST is taken as $F_{th-A}$, the WLO in DST under Model A is persistent oscillation. When the sectional area of DST is greater than $F_{th-A}$, the WLO in DST under Model A is damped oscillation. And with the rise of sectional area of DST, the attenuation rate of WLO becomes greater. When the sectional area of DST is smaller than $F_{th-A}$, the WLO in DST under Model A is damped oscillation. And with the decrease of sectional area of DST, the divergent rate becomes greater. The above results indicate and verify that $F_{th-A}$ that is, Equation (15), is the CSSA of DST under Model A. The theoretical derivation and analysis about the CSSA of DST under Model A are correct.

2. For Model B, the CSSA of DST $F_{th-B}$ is 275.27 m². When the sectional area of DST is taken as $F_{th-B}$, the WLO in DST under Model B are the same with those under Model A. Therefore, $F_{th-B}$ that is, Equation (29), is the CSSA of DST under Model B. The theoretical derivation and analysis about the CSSA of DST under Model B are correct.

4.2 | Comparative analysis

Equations (15) and (29) are the formulae for the CSSAs of DST under Model A and Model B of hydropower plant with SCCT, respectively. Equation (15) is obtained based on the assumption of steady power output of hydro-turbine, and Equation (29) is obtained from the complete mathematical model of hydropower plant with SCCT. For hydropower plant with PTT, the Thoma CSSA of surge tank is the most widely used formula. For the hydropower plant with PTT, the formula for the Thoma CSSA of DST is

$$F_{th-T} = \frac{L_y f}{2 \frac{h_n}{V_{\nu}}^2 g (H_0 - h_{\nu} - 3h_{\nu})}$$

Equation (30) is also obtained based on the assumption of steady power output of hydro-turbine. In this section, the similarities, differences, and relationships of Equations (15), (29), and (30) are studied. The effect mechanisms of the assumption of steady power output of hydro-turbine and the SCCT on the CSSA of DST are revealed from the comparative analysis. The recommended use conditions of different CSSAs of DST are explained.

The hydropower plant with SCCT in Section 4.1 is taken as an example for the comparative analysis. By substituting the basic data of the hydropower plant into Equation (30) yields $F_{th-T} = 562.44$ m². Then, the values of $F_{th-A}$, $F_{th-B}$, and $F_{th-T}$, for example, can be compared and shown in Table 2. By substituting the values of $F_{th-A}$, $F_{th-B}$, and $F_{th-T}$

| $F/F_{th}$ | 0.90  | 0.95  | 1.00 | 1.05 | 1.10  |
|------------|------|------|------|------|------|
| F (m²)     |      |      |      |      |      |
| Model A    | 241.97 (0.90 $F_{th-A}$) | 255.41 (0.95 $F_{th-A}$) | 268.85 ($F_{th-A}$) | 282.30 (1.05 $F_{th-A}$) | 295.74 (1.10 $F_{th-A}$) |
| Model B    | 206.87 (0.90 $F_{th-B}$) | 218.37 (0.95 $F_{th-B}$) | 229.86 ($F_{th-B}$) | 241.35 (1.05 $F_{th-B}$) | 252.85 (1.10 $F_{th-B}$) |
into Equation (18), we can determine the dynamic processes of WLO in DST using ode45 in MATLAB. The results for the dynamic processes of WLO in DST are shown in Figure 3.

From Equations (15), (29), and (30), Table 2, and Figure 3, we can get that:

(1) The formulae of $F_{th-A}$ and $F_{th-B}$ are analytical, and the values of $F_{th-A}$ and $F_{th-B}$ can be easily solved from Equations (15) and (30). The formula of $F_{th-B}$ is not analytical, and the solution of $F_{th-B}$ from Equation (29) is quite complex. For the example of hydropower plant with SCTT, the value of $F_{th-B}$ is the smallest and the value of $F_{th-T}$ is the greatest. $F_{th-A}$ and $F_{th-T}$ are 1.17 and 2.45 times of $F_{th-B}$, respectively. For Equation (18), the WLO in DST is persistent oscillation under $F_{th-B}$ and damped oscillation under $F_{th-A}$ and $F_{th-T}$.

(2) The comparison of $F_{th-A}$ and $F_{th-B}$ indicates the effect of the assumption of steady power output of hydro-turbine on the CSSA of DST. $F_{th-B}$ is obtained from the complete mathematical model of hydropower plant with SCTT. Therefore, $F_{th-B}$ has the highest precision. If the assumption of steady power output of hydro-turbine is adopted, the value of CSSA of DST becomes greater. Then that assumption makes the stability of hydropower plant with SCTT become worse. The dynamic behavior of penstock, hydro-turbine, generator, and governor is favorable for the stability of WLO in DST. The difference between the values of $F_{th-A}$ and $F_{th-B}$ is not significant, while the formula of $F_{th-B}$ is much more complicated than that of $F_{th-A}$.

(3) The comparison of $F_{th-A}$ and $F_{th-T}$ indicates the effect of SCTT on the CSSA of DST. The expressions of $F_{th-A}$ and $F_{th-T}$ are similar. The difference of formulae of $F_{th-A}$ and $F_{th-T}$ is the term of denominator. The head loss coefficient term is $h_{0}$ in $F_{th-T}$ and becomes $h_{0} + \frac{\Delta Q}{2cB}$ in $F_{th-A}$. The added term is $\frac{\Delta Q}{2cB}$, which is mainly composed of the characteristic parameters of SCTT. The result indicates that the difference of formulae of $F_{th-A}$ and $F_{th-T}$ is caused by the SCTT. The influence of SCTT is...
introduced into the CSSA of DST. Based on the definitions and properties of $A$, $c$, and $B$, the value of $\frac{A}{B}$ is always greater than 0. So $\frac{A}{B}$ can decrease the value of CSSA of DST. Therefore, the stability of hydropower plant with SCTT is better than that of hydropower plant with PTT. The use of SCTT can improve the stability of hydropower plant and decrease the CSSA of DST. The effect degree of SCTT on CSSA of DST depends on the relationship between the values of $b_{0}$ and $\frac{A}{B}$. For the selected example of hydropower plant, $F_{th-T}$ is 2.09 times of $F_{th-A}$, which indicates that the difference of the values $F_{th-A}$ and $F_{th-T}$ is extremely large. Therefore, $F_{th-T}$ cannot be used for the determination of CSSA of DST of hydropower plant with SCTT.

In order to make the above results more convincing, the two models under several different operating conditions are compared. Specifically, the operating conditions with different $H_{0}$ are selected. In Table 2 and Figure 3, $H_{0}$ is taken as 70.70m. In the following analysis, $H_{0}$ is taken as 65.00m and 75.00m, respectively. When $H_{0}$ is taken as 65.00m or 75.00m, $F_{th-A}$, $F_{th-B}$, and $F_{th-T}$ can be determined by using Equations (15), (29), and (30), respectively. The results of $F_{th-A}$, $F_{th-B}$, and $F_{th-T}$ when $H_{0}$ is taken as 65.00m and 75.00m are shown in Table 3 and Table 4, respectively. By substituting the values of $F_{th-A}$, $F_{th-B}$, and $F_{th-T}$ into Equation (18), we can determine the dynamic processes of WLO in DST using ode45 in MATLAB when $H_{0}$ is taken as 65.00 m and 75.00 m. The results are shown in Figures 4 and 5.

The results and rules in Table 3, Table 4, Figures 4 and 5 are the same with those in Table 2 and Figure 3. Therefore, the formulae for the CSSAs of DST under Model A and Model B of hydropower plant with SCTT, that is, Equations (15) and (29), are always applicable under different operating conditions. Under different operating conditions, the rules for the similarities, differences, and relationships of different CSSAs of DST are the same.

To sum up, the stability of hydropower plant with SCTT is obviously different with that of hydropower plant with PTT. SCTT can improve the stability of hydropower plant significantly and decrease the CSSA of DST obviously. For the determination of CSSA of DST of hydropower plant with SCTT, $F_{th-T}$ is not applicable. $F_{th-A}$ and $F_{th-B}$ are the correct and reasonable formulae for the determination of CSSA of DST of hydropower plant with SCTT. $F_{th-B}$ has the highest precision and a complicated expression. $F_{th-A}$ has a good precision and a concise expression. Because the expression of $F_{th-B}$ is much more complex than that of $F_{th-A}$, the calculation time corresponding to $F_{th-B}$ is much longer than that corresponding to $F_{th-A}$ (Table 2). In actual engineering applications, $F_{th-A}$ is suggested to be given the priority of use if both the calculation precision and calculation difficulty are considered. If the calculation difficulty is not unconstrained and the requirement for calculation precision is high, $F_{th-B}$ is suggested to be given the priority of use.

## 5 SUMMARY AND CONCLUSIONS

Two types of nonlinear model of hydropower plant with SCTT during transient process, that is, model considering the assumption of steady power output of hydro-turbine and complete model, are established for the derivation of CSSA of DST. State equations of hydropower plant with SCTT under Model A and Model B are obtained. Based on model of hydropower plant and stability analysis theory, the criterion of operation stability of hydropower plant is obtained. Aiming at the critical stable state of hydropower plant, the algebraic criterion of stability corresponding to the CSSA of DST is determined. Formulae for the CSSAs of DST under Model A and Model B are derived according to the algebraic criterion of stability. The correctness of the formulae for CSSAs of DST is verified by numerical verification. The similarities, differences, and relationships of different CSSAs of DST are clarified by comparative analysis. The effect mechanisms of the assumption of steady power output of hydro-turbine and the SCTT on the CSSA of DST are revealed. The recommended use conditions of different CSSAs of DST are explained.

The main conclusions are as follows:

1. For hydropower plant with SCTT, Model A is based on the assumption of steady power output of hydro-turbine, and the state equation of Model A is a second-order nonlinear dynamic equation. Model B is based on the complete model of hydropower plant with SCTT, and the state equation of Model B is a fifth-order nonlinear dynamic equation.

| TABLE 3 | Comparison of the values of $F_{th-A}$, $F_{th-B}$ and $F_{th-T}$ when $H_{0} = 65.00$ m |
|-----------|---------------------------------|---------------------------------|-----------|
|            | Hydropower plant with SCTT       | Hydropower plant with PTT       |
| Values     | Model A                        | Model B                        | Thoma     |
| CSSA (m²)  | $297.86 (F_{th-A})$            | $253.65 (F_{th-B})$            | $623.19 (F_{th-T})$ |
| Relative CSSA | $1.17 (F_{th,A}/F_{th,B})$       | $1.00 (F_{th,B}/F_{th,B})$      | $2.46 (F_{th,T}/F_{th,B})$ |
Equations (15) and (29) are the formulae for the CSSAs of DST under Model A and Model B of hydropower plant with SCTT, respectively. When the sectional area of DST is taken as $F_{th-A}$ (or $F_{th-B}$), the WLO in DST under Model A (or Model B) is persistent oscillation. The theoretical derivation and analysis about the CSSAs of DST under Model A and Model B are correct.

The formulae of $F_{th-A}$ and $F_{th-T}$ are analytical, while the formula of $F_{th-B}$ is not analytical. For the hydropower plant with SCTT, the value of $F_{th-B}$ is the smallest and the value of $F_{th-T}$ is the greatest. The assumption of steady power output of hydro-turbine makes the stability of hydropower plant with SCTT become worse. The dynamic behavior of penstock, hydro-turbine, generator, and governor is favorable for the stability of WLO in DST. The expressions of $F_{th-A}$ and $F_{th-T}$ are similar and the difference is caused by the SCTT. The stability of hydropower plant with SCTT is better than that of hydropower plant with PTT. The use of SCTT can improve the stability of hydropower plant and decrease the CSSA of DST.

(4) For the determination of CSSA of DST of hydropower plant with SCTT, $F_{th-A}$ and $F_{th-B}$ are the correct and reasonable formulae for the determination of CSSA of DST of hydropower plant with SCTT. $F_{th-B}$ has the highest precision and a complicated expression. $F_{th-A}$ has a good precision and a concise expression. In actual engineering applications, $F_{th-A}$ is suggested to be given the priority of use if both the calculation precision and calculation difficulty are considered.

**TABLE 4** Comparison of the values of $F_{th-A}$, $F_{th-B}$, and $F_{th-T}$ when $H_0=75.00$ m

| Values | Hydropower plant with SCTT | Hydropower plant with PTT |
|--------|----------------------------|---------------------------|
|        | Model A                    | Model B                   | Thoma                     |
| CSSA (m²) | 250.34 ($F_{th-A}$) | 214.29 ($F_{th-B}$) | 523.78 ($F_{th-T}$) |
| Relative CSSA | 1.17 ($F_{th-A}/F_{th-B}$) | 1.00 ($F_{th-B}/F_{th-B}$) | 2.44 ($F_{th-T}/F_{th-B}$) |

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