Clearance optimization of radial bearings for multi-bearing shafting system

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Abstract. There will be a huge difference of fatigue life among the bearings mounted on the shaft for multi-bearing shafting system without designing operating clearance. A unique method for multi-shafting system is presented to optimize operating clearance through selecting proper internal clearance class and designing tolerance of fitting shaft on the basis of thermal conditions and system deflection. An example illustrates that the minimum value of fatigue life calculated by ISO/TR 16281 among radial bearings for multi-bearing shafting system has improved 4.2 times greater than original design through the method mentioned in this paper. It is proved that this method has great advantages on accuracy and convergence, significantly improving the minimum fatigue life among bearings of multi-bearing shafting system.

1. Introduction
Multi-bearing shafting system is a common structure, which is widely used in spindle systems including the precision machine tool, hydro mechanical transmission and other gearboxes[1-3]. A general problem emerges that there is a huge difference among the fatigue life of bearings mounted on the same shaft if those bearings have a same operating clearance. Clearance design plays an important role in improving the minimum fatigue life of bearings in multi-bearing shafting system and it is vital to achieve the equal life among the supporting bearings through optimizing the operating clearance[4].

Changing the operating clearance can alter the stiffness of bearing, and then influence the deflection of the shaft and distribution of loads among supporting bearings. Calculating the deflection of the whole shafting system and obtaining the load distribution of bearings is a basis of optimizing the operating clearance, it is impossible to calculate the deflection of shaft and bearings independently because of the coupling relationship between the deflection of whole system and the stiffness of bearings. Therefore, the only way is to calculate the deflection of whole system with considering coupling relationship between the shaft and bearings, then obtain the loads and predict the fatigue life of bearings[3,5]. There are various studies carrying out to analyze the fatigue life of rolling bearings[6-9], also many papers pay attention to the influence of fatigue life due to misalignment and give some advices about bearing application[7,10]. Early studies have investigated about the methods to predict bearing life[8,11]. Kappaganthu illustrates operating clearance is an important nonlinearity which can cause bifurcations and provides a nonlinear model of rotor-bearing system[12].

However, few research takes notice of huge influence of fatigue life due to operating clearance in multi-bearing shafting system. There are many factors affecting the operating clearance of bearings which are mainly original clearance, working temperature and fitted interference. Hence, this paper
focus on optimizing the bearing clearances through selecting proper internal clearance class of bearings and changing the interference between shaft (or house) and inner ring (or outer ring), meanwhile considering the influence of operating temperature, and the final target is realizing the maximization of the minimum fatigue life calculated by ISO/TR 16281 among supporting bearings.

2. Stiffness of shafting system

2.1. Stepped shaft

It is highly efficient and of reasonable accurate in industrial applications to simplify stepped shaft as beam elements. In order to take into account influence of the deflection due to shear stress especially for a shaft with small length-diameter ratio, this paper chose the Timoshenko beam as calculation element.

The strain energy equation for the Timoshenko beam is shown in Equation (1).

\[ U = U_b + U_s = \frac{1}{2} \int_0^l \left( \frac{d\theta}{dx} \right)^2 EI \left( \frac{d\theta}{dx} \right) dx + \frac{1}{2} \int_0^l \gamma^T \frac{GA}{k} \gamma dx \]  
(1)

Where, \( U_b \) is strain energy of bending, \( U_s \) is strain energy of shear, \( E \) is elasticity modulus, \( I \) is inertia moment, \( G \) is shear elasticity, \( A \) is section surface, \( \theta \) is the bending angle, \( \gamma \) is the shearing angle.

Displacement of element can be divided into two parts, bending displacement \( \delta_b^e \) and shear displacement \( \delta_s^e \), which gives

\[ \begin{bmatrix} \delta_b^e \\ \delta_s^e \end{bmatrix} = \begin{bmatrix} v_{b1} & \theta_1 & v_{b2} & \theta_2 \\ v_{s1} & \theta_1 & v_{s2} & \theta_2 \end{bmatrix} \]  
(2)

Bending displacement \( \delta_b^e \) can be expressed by the equation Hermite interpolation, and the shear displacement \( \delta_s^e \) can be expressed by the linear interpolation method, as shown in Equation (3).

\[ \begin{align*} v_b &= N_1 v_{b1} + N_2 \theta_1 + N_3 v_{b2} + N_4 \theta_2 \\ v_s &= N_5 v_{s1} + N_6 v_{s2} \end{align*} \]  
(3)

where, \( N_1, N_2, N_3, N_4 \); \( N_5, N_6 \).

Substituting equation (3) into equation (1), strain energy of bending \( U_b \) is given by

\[ U_b = \frac{1}{2} \int_0^l \left( \frac{d^2v_b}{dx^2} \right)^2 EI \left( \frac{d^2v_b}{dx^2} \right) dx = \frac{1}{2} \delta_b^e \delta_b^e \left( \int_0^l N_b^T E I N_b dx \right) \delta_b^e = \frac{1}{2} \delta_b^e K_b^e \delta_b^e \]  
(4)

where, \( K_b^e \) is the stiffness matrix of bending for elements.

Also, strain energy of shear \( U_s \) can be expressed as:

\[ U_s = \frac{1}{2} \int_0^l \left( \frac{dv_s}{dx} \right)^2 \frac{GA}{k} \left( \frac{dv_s}{dx} \right) dx = \frac{1}{2} \delta_s^e \delta_s^e \left( \int_0^l N_s^T \frac{GA}{k} N_s dx \right) \delta_s^e = \frac{1}{2} \delta_s^e K_s^e \delta_s^e \]  
(5)

where, \( K_s^e \) is the stiffness matrix of shearing for elements.

2.2. Rolling bearings

Taking the cylindrical rolling bearing as an example, according to ISO/TS 16281, the elastic deformation of a misaligned rolling element can be described by a lamina model, as Figure 1.

It can be seen that deformation of roller can be considered to be composed of three components in Figure 2: Crown drop \( C_\delta \) at lamina \( \lambda \); Deformation \( \delta_i \) due to the radial load at the roller azimuth location \( j \); Deformation \( \delta_{\phi j} \) due to bearing misalignment and roller tilt at the roller azimuth angle \( \phi_j \). \( W \) is the width of the slice; \( l \) is effective length of roller; \( \phi \) is misalignment angle. The actual deformation
of each slice only includes the one due to radial load and misalignment, this leads to the elastic deflection of the slice $\lambda$ as shown in Equation (6).

\[ \delta_{j\lambda} = \delta_j + \delta_{\psi,j\lambda} - c_{j\lambda} \]  

The elastic deflection due to the radial load of the rolling element $j$, $\delta_j$ is:

\[ \delta_j = \delta_j \cos \psi_j - \frac{P_j}{2} \]  

Where, $\delta_j$ is the radial displacement of inner ring; $P_j$ is operating clearance of bearing.

The elastic deflection due to misalignment and tilt of slice $\lambda$ of the rolling element $j$, $\delta_{\psi,j\lambda}$ is:

\[ \delta_{\psi,j\lambda} = \frac{\psi}{2} (\lambda - 0.5) W \cos \psi_j \]  

Total misalignment between inner race and outer race in the plane of rolling element $j$, $\psi_j$ is:

\[ \psi_j = \arctan \left( \tan \psi \cos \varphi_j \right) \]  

Where, $\psi$ is total misalignment between inner race and outer race.

If the radial load applied on the bearing is $F_r$, for static equilibrium to exist, as:

\[ F_r = \frac{c_l}{n_s} \sum_{j=1}^{n_s} \cos \psi_j \sum_{\lambda=1}^{\lambda_{0,9}} \delta_{j\lambda} \]  

here, $n_s$ is the number of slices; CL is spring constant.

For the misaligning moment, the equilibrium condition to be satisfied is

\[ M = \frac{c_l}{n_s} \sum_{j=1}^{n_s} \left( \cos \psi_j \sum_{\lambda=1}^{\lambda_{0,9}} [(\lambda - 0.5) W - 0.5l] \delta_{j\lambda} \right) \]  

In order to obtain the bearing stiffness, it is necessary to solve nonlinear equations (10) and (11). Using the Newton-Raphson method and taking 4 DOF($\delta_{ry}$, $\delta_{rz}$, $\psi_y$ and $\psi_z$) of the inner ring center as unknown quantities, the root can be extracted by the gradient estimation function and independent variable intercept.

3. Clearance of bearings

Bearings are usually mounted on shafts with interference fits, press fitting of inner ring on shaft can cause inner ring to expand slightly, and thus bearing’s clearance tends to decrease. Meanwhile, thermal conditions of bearing operations can also affect operating clearance, which causes operating clearance being different from fitted clearance. Due to differential expansions between inner ring and
outer ring at operating temperature, operating clearance $P_d$ will tend to increase or decrease. It can be given by

$$P_d = P_0 + \Delta_r - \Delta_i - \Delta_o$$  \hspace{1cm} (12)

Where, $P_0$ is original clearance, which can be found in general catalogue from manufacturer; $\Delta_r$ is clearance increase due to thermal expansion; $\Delta_i$ is clearance reduction due to press-fitting of bearing inner ring on shaft; $\Delta_o$ is clearance reduction due to press-fitting of bearing outer ring on house. Generally, $\Delta_o$ can be ignored if there is a transition fit between outer ring and house.

Clearance decrease after mounting can be obtained by using elastic thick ring theory. According to Harris’s research[4], clearance reduction due to press-fitting of inner ring on shaft can be shown in Equation (13).

$$\Delta_i = \frac{2l(D_i/D_s)^2 \left[ \frac{(D_i/D_s)^2 + \xi_i s}{(D_i/D_s)^2 - 1} + \frac{E_i}{E_s} \left[ \frac{(D_i/D_s)^2 + \xi_i s}{(D_i/D_s)^2 - 1} \right] \right]}{\left[ (D_i/D_s)^2 - 1 \right]}$$  \hspace{1cm} (13)

Where, $D_1$ is outside diameter of inner ring; $D_s$ is inside diameter of shaft; $D_i$ is basic fitted diameter of shaft and inner ring; $I$ is interference between inner ring and shaft; $\xi_b$ and $\xi_s$ are respectively Poisson’s ratio of bearing and shaft; $E_b$ and $E_s$ are respectively elasticity modulus of bearing and shaft.

Also, clearance reduction due to fitting of outer ring and house $\Delta_o$ can be calculated using the same method.

Clearance increase due to thermal expansion is given by:

$$\Delta_r = \Gamma_b \left[ d_o (T_o - T_a) - d_i (T_i - T_a) \right]$$  \hspace{1cm} (14)

Where, $\Gamma_b$ is linear expansion coefficient of bearing; $d_i$ and $d_o$ are diameter of bearing inner raceway and outer raceway; $T_i$ and $T_o$ are operating temperature of inner ring and outer ring; $T_a$ is fitting temperature.

4. Arithmetic and block diagram

For a multi-bearing shafting system, the deformable coordination between inner ring and the node of fitting shaft is applicable. Taking bearing as a variable stiffness spring, coupling the DOF of inner ring with shaft and outer ring with house. If the house have a strong stiffness, the displacement of the outer ring can be set to zero.

It is important to choose initial values for the nonlinear equations. Using the default bearing stiffness $K_0$, the initial displacement of the inner ring can be solved through $X_0 = K_0^{-1}F$.

To develop the fifth order nonlinear system combined by Equation (10) and (11), Taylor expand them as:

$$F_i(X + \Delta x) = F_i(X) + \sum_{j=1}^{N} \frac{\partial F_i}{\partial x_j} \Delta x_j + \delta \left( \Delta x^2 \right)$$  \hspace{1cm} (15)

Let $J_i = \frac{\partial F_i}{\partial X_i}$, obtained Jacobian matrix $J_i$ of partial derivative at point $i$.

Using matrix equations to express Equation (16)

$$\{ X_{i+1} \} = \{ X_i \} - [J_i]^{-1} \{ f_i \}$$  \hspace{1cm} (16)

Solve the above equation and obtain modification value $\Delta X$, then take the second calculation, and let $X_2 = X + \Delta X$, here iteration process is generated. Finally, judging whether iteration end or not through setting the allowed error value $\epsilon$.

According to the stiffness matrix of the shafting system, which is combined by stiffness matrices of shaft and bearings, existing the static equilibrium

$$K_s \times \delta = F$$  \hspace{1cm} (17)
Optimal operating clearance can be obtained based on the deflection of system and fatigue life calculation, the block diagram has been investigated as Figure 3.

**Figure 3. Calculation block diagram.**
5. Example

Figure 4 is a typical multi-bearing shafting system. In this structure, the loads distribution of elements in NU2306 can be changed through optimizing the operating clearance, however it is not obvious to change the entire load acted on NU2306 because it accommodates the radial load at one side of the gear by itself, so the method of clearance design mentioned in this paper is applied for two side by side bearings, NU209_N and NU209_F in this structure.

![Geometrical shape](image)

**Figure 4.** Geometrical shape.

Diameters of the shaft and bore of the house fitted with bearings in original design is demonstrated in Table 1.

| Type          | Fitted shaft | Fitted house       |
|---------------|-------------|--------------------|
| NU 2306 EC    | Ø30m6+0.021+0.008 | Ø2H7+0.010 |
| NU209_N       | Ø45m6+0.025+0.009 | Ø5H7+0.015 |
| NU209_F       | Ø45m6+0.025+0.009 | Ø5H7+0.015 |

According to Equation (14), the original and operating clearances before optimization are shown in Table 2.

| Type          | Original clearance $P_o$ (µm) | Clearance after fitting (µm) | Temperature expansion $\Delta T$ (µm) | Operating clearance $P_d$ (µm) |
|---------------|-------------------------------|-------------------------------|----------------------------------------|-------------------------------|
| NU 2306 EC    | Normal 35                     | 21.4                          | 19.6                                   | 40.9                          |
| NU209_N       | Normal 45                     | 32.6                          | 13.3                                   | 45.9                          |
| NU209_F       | Normal 45                     | 32.6                          | 13.3                                   | 45.9                          |

According to ISO/TS 16281, the basic rating life are calculated using the rolling element loads. Elements loads distribution of NU209_N and NU209_F is demonstrated in Figure 5. It can be seen the max element load of NU209_N approximates two times greater than NU209_F.
Figure 5. Elements loads (original design).

Figure 6. Elements loads (after optimization).

The bearing basic rating life is calculated by ISO/TS 16281 and shown in Table 3, which considers the influence factors including clearance during operation.

Table 3. Bearings life (original design).

|                  | NU209_N | NU209_F |
|------------------|---------|---------|
| Operating clearance $P_d$ (mm) | 0.046   | 0.046   |
| $L_{10r}(10^6)$ | 2203    | 19916   |

From Table 3, it reflects a huge difference about bearing rating life that exists between NU209_F and NU209_N because NU209_N accommodates the most load at one side of the gear when NU209_F and NU209_N have the same operating clearances in original design. The basic life of NU209_F is 9 times greater than NU209_N. The minimum bearing life $L_{10r}$ between NU209_F and NU209_N is only 2203 million of revolutions.

Selecting the ranges of the operating clearance from 0 to 0.05mm and calculating the best clearance combination of NU209_F and NU209_N, the best operating clearances of NU209_N and NU209_F are respectively 0.02mm and 0mm. Under this operating clearance combination, elements loads distribution of NU209_N and NU209_F is shown in Figure 6.

The basic fatigue life of NU209_N and NU209_F after optimization is shown in Table 4.

Table 4. Bearings life (after optimization).

|                  | NU209_N | NU209_F |
|------------------|---------|---------|
| Operating clearance $P_d$ (mm) | 0.02    | 0       |
| $L_{10r}(10^6)$ | 10238   | 9320    |

On the basis of Equation (16), clearance increase due to differential expansion is 13.3 μm. The method adopted in this paper is to obtain the desired operating clearance by changing the fitting tolerance of shaft. According to target operating clearance and thermal expansion, the clearance decrease due to fitting can be calculated, the interference can also be calculated by press expansion through Equation (15). The diameter tolerance of fitting shaft can be designed on the basis of interference. Take into account the cost of manufacturing and design intent, the tolerance ranges of the shaft is designed by the 6th stage manufacturing accuracy, the diameter tolerance of shaft for NU209_N (Original clearance class is C2) is $45.009 \pm 0.006$, the diameter tolerance of shaft for NU209_F (Original clearance class is C2) is $45.025 \pm 0.006$. 

6. Conclusion
A unique method of bearing clearance design improving minimum fatigue life of radial bearings in multi-bearing shafting system has been presented, and an example is given to show that the method is feasible. The results reached in this paper demonstrated that:

1. There will be a great difference of basic fatigue life calculated by ISO/TS 16281 among the bearings mounted on multi-bearing shaft if they have the same operating clearances;
2. Clearance design mentioned in this paper is an effective measure to balance the load distribution and equal life design of radial bearings for multi-bearing shafting system;
3. The arithmetic mentioned in this paper shows its great advantages on accuracy and convergence, the best operating clearance can be obtained effectively.

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