Maximal Acceleration Limits on the Mass of the Higgs Boson

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Abstract

Caianiello’s quantum geometrical model with maximal acceleration places the upper limit $\mu \leq 719.5$ GeV on the mass of the Higgs boson. The model also provides an equation linking the mass of the W boson to the Higgs mass $\mu$ and independent symmetry breaking and mass generating mechanisms. These may further restrict the value of $\mu$.

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1 Introduction

In a recent work Kuwata [1] points out that the existence of a maximal acceleration (MA), conjectured on different grounds by several authors [2], [3], [4], would set an upper limit on the mass of the Higgs boson. Kuwata argues that the onset of MA would transform the Higgs–fermion interaction potential, assumed to be of the Yukawa type, into a low order polynomial in $r$ at distances less than a critical value $r_c$. By restricting the polynomial to third order, which requires the MA to occur at $r = 0$, and by demanding the continuity of the two potentials at $r = r_c$ up to second order derivatives, the short range potential can be fully determined. The Higgs Lagrangian is then modified to accomodate the new potential. Standard model relations lead to the upper limit $\mu = M_W \sqrt{1.77 k \sin^2 \theta_W / \alpha}$, where $M_W$ is the mass of the $W$-boson, $\theta_W$ is the Weinberg angle and $\alpha$ the fine structure constant. $k \approx 1$ is a factor that appears in the definition of the maximal acceleration $A = km_r$ ($h = c = 1$) adopted by Kuwata and $m_r$ is the reduced fermion mass.

The expression for $A_m$ used by Kuwata was originally derived by Caianiello [2], [5], and subsequently re-derived by other authors [6], by using quantum mechanics and Landau's theory of fluctuations. It does not require additional assumptions and is, as such, a concept in search of a deeper interpretation. In related work, in fact, Caianiello and collaborators provided a geometrical framework in which the MA of a particle directly affects the spacetime experienced by the particle itself [7], [8]. According to this model, a particle of mass $m$ accelerating along its worldline experiences a spacetime of metric

$$ds^2 = ds^2 \left(1 + \frac{\eta_{\mu\nu} \dddot{x}^{\mu} \dddot{x}^{\nu}}{A_m^2}\right) = \sigma^2(x)ds^2,$$

where $ds^2$ refers to Minkowski space whose metric $\eta_{\mu\nu}$ has signature $-2$ and $\dddot{x}^{\mu} \equiv d^2x^{\mu}/ds^2$ is the four–acceleration of the particle [4]. Unless explicitly stated, we choose $k = 2$ [3].

Generalizations to include background gravitational fields can be obtained by replacing $\eta_{\mu\nu}$ with the corresponding metric tensors. Several important implications of Eq.(1) have already been discussed in the literature [1], [10], [11], [12]. The purpose of this paper is to show that Eq.(1) also places limits on the mass of the Higgs boson. These limits do not require additional
changes to the Higgs Lagrangian, are consistent with those of Ref. [1], when allowance is made for the different values of $k$, and with the experimental limits $50 \text{GeV} \leq \mu \leq 1000 \text{GeV}$ obtained by using radiative corrections [13]. They are derived and discussed in Section 2. Section 3 contains a brief derivation of the Dirac equation for the metric (1) and a discussion of its most noticeable consequences. Eq. (1) also allows the derivation of a formula relating the masses of the standard model fermions and bosons to that of the Higgs boson. The relation between $M_A$ and standard model is further discussed in Section 5 where a mass generating mechanism is also introduced. The conclusions are presented in Section 6.

2 Limits to the Higgs mass

It is possible to establish an upper limit to $\mu$ starting from Eq. (1) directly. An explicit expression for $\sigma(x)$ can be obtained classically by assuming, as in Ref. [1], that the fermion–fermion interaction can be represented by a Yukawa potential

$$ V(r) = \frac{g_H^2}{4\pi} \frac{e^{-\mu r}}{r}. $$

One immediately finds

$$ \sigma(r) = \sqrt{1 - \left( \frac{1}{\pi v^2} \right)^2 \frac{e^{-2\mu r}(1 + \mu r)^2}{k^2 r^4}}, \quad (2) $$

where $v = \sqrt{2} < 0|\phi|0 \geq 246.2185 \text{GeV}$ is the usual standard model parameter.

Since $d\bar{s}$ is real, $\sigma(r)$ must remain real for all values of $\mu$ and $r$. When $|\tilde{x}| = A_m$, $\sigma(r)$ vanishes. On the other hand $\sigma(r) \sim 1$ for $r >> 1/\mu$. At these distances the fermions do not interact appreciably with the Higgs boson. A function $r_c(\mu)$ must therefore exist such that $\sigma(r) \geq 0$ for $r \geq r_c(\mu)$. An expression for $r_c(\mu)$ can be obtained by expanding $\sigma(r)$ to third order in the neighborhood of $1/\mu$, where the acceleration presumably reaches its highest values, and by equating the result to zero. The expansion is given by

$$ \sigma(r) \sim B + \frac{9.32758 \cdot 10^{-12} \mu^5}{B} (r - \frac{1}{\mu})^+ \quad (3) $$

where \( B = (1 - 3.73103 \cdot 10^{-12} \mu^4)^{1/2} \). Expression (3) is real if \( B \) is real, which yields \( \mu \leq 719.52 \text{GeV} \). Of the three solutions for \( r \) obtained by equating (3) to zero, only one is real.

It also follows that \( r \geq 0 \) for \( 316.1 \text{ GeV} \leq \mu \leq 719.5 \text{ GeV} \). These are the limits of validity of the solution. They are not affected greatly by adding additional terms to the expansion for \( \sigma(r) \). At order \((r - 1/\mu)^5\) one finds in fact \( 225.4 \text{ GeV} \leq \mu \leq 719.53 \text{ GeV} \) for \( k = 2 \).

In this model it seems therefore possible to establish a lower limit in addition to an upper one. However the lower limit is only a construct of the approximation, devoid of physical meaning. Eq (2) does in fact diverge at \( r = 0 \). For each value of \( \mu \) there is a value of \( r \) for which \( \sigma = 0 \). For the choice \( k = 1 \) of Kuwata, the corresponding limits at \( O((r - 1/\mu)^3) \) are \( 223.5 \text{ GeV} \leq \mu \leq 508.8 \text{ GeV} \). In both instances the upper limits are well above the energy ranges of recent searches [14]. Contrary to Kuwata’s results, the MA is reached for \( r = 0 \) only for \( \mu \) at the lower limit, where the solution becomes inaccurate. Eq. (4) also affects the fermion–fermion interaction. This is shown in the next section.

### 3 The covariant Dirac equation

With the introduction of the metric tensor \( \tilde{g}_{\mu\nu} = \sigma^2(x) \eta_{\mu\nu} \), the Dirac equation must be generalized to curved space–time and connected to a local Minkowski frame by means of the vierbein field \( e^a_\mu(x) \), where Latin indices refer to the local frame and Greek indices to a generic non–inertial frame. The vierbeins follow from (3) and are given by \( e^a_\mu(x) = \sigma(x) \delta^a_\mu \). The covariant Dirac equation has the form

\[
[i \gamma^\mu(x) D_\mu - V(r) - m] \psi(x) = 0.
\]

(4)
The matrices $\gamma^\mu(x)$ satisfy the anticommutation relations $\{\gamma^\mu(x), \gamma^\nu(x)\} = 2\delta^\mu\nu(x)$. The covariant derivative $D_\mu \equiv \partial_\mu + \omega_\mu(x)$ contains the total connection $\omega_\mu = \sigma^{ab}\omega_{\mu ab}$, where $\sigma^{ab} = [\gamma^a, \gamma^b]/4$, $\omega^a_\mu b = (\Gamma^\lambda_\mu e^a_\lambda - \partial_\mu e^a_\nu)e^\nu_b$ and $\Gamma^\lambda_\mu$ represent the usual Christoffel symbols. For conformally flat metrics $\omega_\mu = \sigma^{ab}\eta_{\mu (\ln \sigma) , b}$. By using the transformations $\gamma^\mu(x) = e^a_\mu \gamma^a = \sigma^{-1}(x)\gamma^\mu$, where $\gamma^\mu$ are the usual position–independent Dirac matrices, Eq. (4) becomes

$$\left[ i\gamma^\mu \partial_\mu + i\frac{3}{2} \gamma^\mu (\ln \sigma) , \mu - m\sigma - V(r)\sigma \right] \psi(x) = 0 . \tag{5}$$

From Eq. (5) one finds the Hamiltonian

$$H = \vec{\alpha} \cdot \vec{\gamma} V(r)\sigma - i\frac{3}{2} \gamma^0 \gamma^\mu (\ln \sigma) , \mu + m\sigma(r)\gamma^0 . \tag{6}$$

The line element (1) therefore induces additional potential terms in the Hamiltonian. They are represented by the last two terms in Eq. (6) and give rise to a combined potential which is in general spin–dependent and highly repulsive for $\sigma(r) < 1$.

In the present calculation $\sigma(r)$, given by Eq. (2), is time–independent, hence all potential terms are conservative [15]. Classically the fermions will therefore accelerate toward the centre of mass while adjusting their speeds to the height of the potential barrier they move over. Once they reach the barrier, they must bounce back. Numerical studies of these terms indicate that when $\mu \simeq 700$ GeV and the total fermion energy is also close to 700 GeV, the bounce back occurs at $r \simeq 0.00184$ GeV$^{-1}$, while $\sigma$ vanishes at $r \simeq 0.0013975$ GeV$^{-1}$. The highest value of the acceleration reached by the fermions at $r \simeq 0.00184$ GeV$^{-1}$ is $\sim 0.49A_m$. For $\mu \sim 500$ GeV and a total fermion energy also of 500 GeV the barrier is hit at $r \simeq 0.002037$ GeV$^{-1}$ where the acceleration is $\sim 0.46A_m$. Only at the lower limit $\mu = 316.1$ GeV$^{-1}$ is $|\vec{x}| = A_m$ at $r = 0$. With the exception of this limiting case, a consequence of the approximations made, Kuwata’s assumption that the MA is reached at $r = 0$ during the interaction cannot be made in the present model. It also follows that the limits determined in Sect. 2, corresponding to accelerations undamped by the repulsive barrier, represent indeed the extreme upper limits derivable from this geometrical model.

Qualitatively one may also conclude that as the two fermions approach the limiting value of the potential and their relative speed becomes non relativistic, the term $\gamma^0 \gamma^\mu (\ln \sigma) , \mu$, that connects the small spinor components,
is switched off. The residual potential then becomes slightly negative allowing the fermions to weakly bind to each other already for $\mu \sim 400$ GeV. No quantitative analysis of the effect of the repulsive barrier has so far been carried out for the quantum scattering problem. The barrier effect on the energy spectrum has however been analyzed in detail in the case of hydrogenic atoms [9].

A study of the curvature invariant

$$R = -\frac{6}{r} \frac{1}{\sigma^3(r)} [2\sigma'(r) + r\sigma''(r)]$$

indicates that $R$ is always positive or null and grows rapidly in the neighborhood of those values of $r$ and $\mu$ for which $\sigma = 0$, as expected.

4 Scale changes

It is interesting to observe that for values of $g_H$ given by the standard model, $\sigma(r)$, represented by (2), is independent of the mass of the accelerated fermions and depends only on the mass of the Higgs boson. The same applies to the accelerated $W$ and $Z$ bosons in the Yukawa potential approximation of Sect. 2. It now follows from Eq. (1) that if the characteristic length of an accelerated particle is $ds \approx m^{-1}$, $ds$ will contract to a length $\tilde{ds} \approx \mu^{-1}$ when the particles are accelerated. The contraction is due to the conformal factor $\sigma$ which provides an immediate, approximate relationship between $m$ and $\mu$. From Eq. (1) and (2) one obtains the formula

$$m = \mu \sqrt{1 - \frac{4\mu^4}{(\pi ev^2)^2 k^2}},$$

where $(\pi ev^2)^2 \sim 3.73103 \cdot 10^{-12}$ GeV$^{-4}$ and $e$ is Neper’s number. Given $\mu$, one may in principle derive the values of $m$ from Eq. (5). In fact this equation has always the real and positive solution

$$\mu \simeq m \quad \text{for} \quad \frac{4m^4}{(\pi ev^2)^2 k^2} \ll 1,$$

corresponding to the uncontracted length, and the solution

$$\mu \simeq \sqrt{\frac{\pi ev^2 k}{2}} \quad \text{for} \quad m^2 \sqrt{\frac{2}{\pi ev^2 k}} \ll 1.$$
The latter solution gives the value of \( \mu \) at which a change in scale occurs. On applying Eq. (8) to scattering processes involving the fermions and bosons of the standard model one obtains the solutions given in Table I. The results for the lower fermion masses are very sensitive to the value of \( \mu \). On using the formula \[ m_t = 180 \pm 7 + 13 \ln \frac{\mu}{300 \text{GeV}}, \]
and the values of \( \mu \) determined from Eq. (8) one finds for \( k = 2 \)
\[ m_t = 180 \pm 7 + 11.372 = \begin{cases} 198.372 \text{GeV} \\ 184.372 \text{GeV} \end{cases} \]
and for \( k = 1 \)
\[ m_t = 180 \pm 7 + 6.867 = \begin{cases} 193.867 \text{GeV} \\ 178.867 \text{GeV}. \end{cases} \]
All these values are consistent with the global fit to all data given in Ref. [16] \[ m_t = 180 \pm 7^{+12}_{-13} \text{GeV} \]
and with the values of \( m_t \) obtained from (8). Finally, the solution \( \mu \approx \sqrt{\pi e v^2 k/2} \) may be written as
\[ \mu = M_W \frac{2}{g} \sqrt{\frac{\pi e k}{2}} = M_W \sqrt{\frac{1.36 k \sin^2 \theta_W}{\alpha}} \] (9)
in good agreement with Kuwata’s formula given in Sect. 1 and the limit of Sect. 2.

5 MA and the Standard Model

The behaviour of the Dirac equation at \( \sigma = 0 \) is pathological. The particle’s behaviour is however well defined for increasing acceleration up to, but excluding, the MA. For decreasing \( \sigma(r) \) the particle’s effective mass, \( m \sigma(r) \), decreases and so does the term \( V(r) \sigma(r) \). These features aptly embody the notion of asymptotic freedom in the model. On the other hand, the repulsive barrier grows logarithmically, which indicates that the MA is classically unattainable.
Eq. (5) remains invariant in form under the transformation $\sigma \rightarrow i|\sigma|$. Fermions with acceleration higher than the MA may therefore be thought of as particles obeying the Dirac equation, but in regions of space–time where the roles of space and time are interchanged relative to the usual ones.

The two regions, $\sigma$ imaginary (I) and $\sigma$ real (II) are disconnected. The detailed behaviour in the neighborhood of $\sigma = 0$ will in general depend on $g_{\mu\nu}$ in the metric $\tilde{g}_{\mu\nu} = \sigma^2 g_{\mu\nu}$. In I the light cone is rotated by $\pi/2$ relative to II. Fermions may possibly travel from I to II and II to I but only as far as $r_c (\sigma = 0)$. The fermions are confined to their world tubes and that of the Higgs boson by the action of $V(r)\sigma(r)$ but are prevented from reaching the MA by the combined action of $m_\sigma(r)$ and the logarithmic potential.

The space–time experienced by the accelerating particle has, however, nonvanishing curvature inside the world tube. This leads to interesting consequences. A Lagrangian term $(\xi/2)R(x)\sigma^2(x)|\phi(x)|^2$, where $\xi$ is a parameter, usually accompanies any field capable of interacting with gravity. Though $R$ is really experienced only by the accelerating particle, its dynamics in the field that produces acceleration must be effectively consistent with the presence of this interaction term. For Higgs fields one must have $\xi \leq 0$ or $\xi \geq 1/6$ [17]. The Higgs potential then becomes

$$V(\phi) = \left(\bar{\mu}^2|\phi|^2 + \frac{1}{2}\xi R|\phi|^2 + \lambda|\phi|^4\right)\sigma^2,$$

where $\bar{\mu}$ is the usual mass parameter that appears in the Higgs mechanism and $\lambda > 0$. Formally, the vacuum has minima at

$$|\phi| = \pm \sqrt{-\bar{\mu}^2 + \frac{(\xi/2)R}{2\lambda}} \equiv v$$

and the mass of the Higgs boson becomes

$$\mu^2 = -2\left(\bar{\mu}^2 + \frac{\xi}{2} R\right)\sigma^2 = 2\lambda v^2 \sigma^2,$$

where, as usual, $\mu^2 = 2\lambda v^2$ for $\sigma^2 = 1$. It follows from (11) and (12) that the MA provides an additional independent symmetry breaking mechanism which is present even when $\bar{\mu} = 0$.

Several possibilities now exist:

i) $\xi > 0$. If in addition $\bar{\mu}^2 > 0$, then $|\phi|$ in Eq. (11) becomes imaginary, the
only minimum of \( V(\phi) \) is at \( |\phi| = 0 \) and the symmetry is not broken. If, on the contrary, \( \bar{\mu}^2 < 0 \) as required by the usual Higgs mechanism, then one can have either \( |\bar{\mu}|^2 > (\xi/2)R \), which leads to spontaneous symmetry breaking and minima at
\[
|\phi| = \pm \sqrt{\frac{|\bar{\mu}|^2 - (\xi/2)R}{2\lambda}},
\]
or \( |\bar{\mu}|^2 < (\xi/2)R \) with the minimum at \( |\phi| = 0 \) as before. The particular value \( |\bar{\mu}|^2 = (\xi/2)R \) restores the original unbroken symmetry and implies \( \mu = 0 \).

ii) \( \xi < 0 \). If one also has \( \bar{\mu}^2 > 0 \), two additional cases are possible: a) \( \bar{\mu}^2 > (|\xi|/2)R \). The only minimum of Eq. (10) is again at \( |\phi| = 0 \); b) \( \bar{\mu}^2 < (|\xi|/2)R \). The new minima occur at
\[
|\phi| = \pm \sqrt{\frac{|\bar{\mu}|^2 - (|\xi|/2)R}{2\lambda}}
\]
and the symmetry is broken. Finally, if \( \bar{\mu}^2 < 0 \), the symmetry is again broken and the vacuum minima are at
\[
|\phi| = \pm \sqrt{\frac{|\bar{\mu}|^2 + (|\xi|/2)R}{2\lambda}}.
\]
The particular value \( \bar{\mu}^2 = (|\xi|/2)R \) also restores the original vacuum value \( |\phi| = 0 \). In all instances the corresponding values of \( \mu \) can be easily derived from Eq. (12).

All possibilities require some sort of mass generating mechanism to be associated with \( \xi R \sigma^2 \). In order to show that this mechanism is present, the choice \( \bar{\mu} = 0 \) in Eq. (11) is particularly appropriate. On averaging \( \sigma^2 R \) over three dimensional space
\[
< \sigma^2 R > = \frac{\int_0^{1/\mu} \sigma^6(r)R(r)r^2dr}{\int_0^{1/\mu} \sigma^4(r)r^2dr}
\]
and expanding the integrands to order \((r - 1/\mu)^5\), one finds that the equation
\[
|\xi| < \sigma^2 R > - \mu^2 = 0
\]
has solutions for \( |\xi| \geq 0 \). For \((1/100) \leq |\xi| \leq 100 \) one gets \( 420 \text{ GeV} \leq \mu \leq 635 \text{ GeV} \). The value \( \mu \sim 555 \text{ GeV} \) occurs at \( |\xi| = 1/6 \). For \( |\xi| = 1 \) one finds
\( \mu \sim 615 \text{ GeV} \) and \( \mu \) does not increase beyond 635 GeV for \( |\xi| \geq 100 \). In fact no value of \( |\xi| \) exists that can move \( \mu \) to the upper limit of Sect. 2. The case \( k = 1 \) is entirely similar. For \((1/100) \leq \xi \leq 100\) one finds 296 GeV \( \leq \mu \leq 450 \text{ GeV} \) with \( \mu \sim 392 \text{ GeV} \) at \( |\xi| = 1/6 \) and increasing from 435 GeV at \( |\xi| = 1 \) to 450 GeV for higher value of \( |\xi| \). Here too no value of \( |\xi| \) yields \( \mu \sim 508 \text{ GeV} \), the upper limit discussed in Sect. 2.

6 Conclusions

Kuwata’s derivation of an upper limit for the value of the Higgs boson mass is based on the maximal value that non-relativistic quantum mechanics places on the acceleration of a particle. It is, as such, perfectly legitimate.

Caianiello’s geometrical model, on the other hand, is more complete. It incorporates the notion of MA, but also contains detailed dynamical prescriptions for the behaviour of an accelerating particle. These are essentially contained in Eq. (1) and its effect on the actual form of the Dirac equation. The reality of \( \sigma(r) \) already is sufficient to place an upper limit on the mass of the Higgs boson. For the sake of comparison with Kuwata’s results estimates have been given for both \( k = 2 \) and \( k = 1 \) (Kuwata’s choice), though present measurements of the Lamb shift in hydrogenic atoms agree with \( k = 2 \), but not with \( k = 1 \).

The result, \( \mu \leq 719.5 \text{ GeV} \), must indeed be considered as an upper limit because the actual value of the MA is not reached by the particle, as derived in Sect. 3 from the covariant Dirac equation. In a non-geometrical approach the Dirac equation takes its usual Minkowski space form and no information is available about the additional potential terms present in Eq. (3). Using Eq. (1) one can also derive Eq. (3) which links the masses of all particles in the standard model to the mass of the Higgs boson. This equation reproduces all values of fermions and bosons well (Table I): they correspond to their original uncontracted scales. A change in scale occurs however for the solution Eq. (4) that agrees well, for \( k = 1 \), with Kuwata’s result and the limit of Sect. 2. Section 5 discusses a number of implications that (1) has for the standard model. In general one may conclude that (3) provides an additional independent symmetry breaking mechanism that may affect the vacuum minima in a variety of ways. A mass generating mechanism can be associated with the average curvature experienced by an accelerating parti-
cle. When the mass parameter $\mu$ of the standard model vanishes, Eq. (12) for the Higgs boson mass has solutions $420 \text{ GeV} \leq \mu \leq 635 \text{ GeV}$ over a wide range of values of the parameter $\xi$. The additional restriction introduced by the mass generation mechanism is perfectly compatible with the upper limit $\mu \leq 719.52 \text{ GeV}$ imposed by the reality of the conformal factor $\sigma(r)$.

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| Interaction | $k = 2$ | $k = 2$ | $k = 1$ | $k = 1$ |
|-------------|---------|---------|---------|---------|
|             | $m$(GeV) | $\mu$(GeV) | $m$(GeV) | $\mu$(GeV) |
| $Hee$       | $0.511 \cdot 10^{-3}$ | 719.52 | $0.511 \cdot 10^{-3}$ | 508.8 |
| $Huu$       | $5 \cdot 10^{-3}$ | 719.52 | $5 \cdot 10^{-3}$ | 508.8 |
| $Hdd$       | $10^{-2}$ | 719.52 | $10^{-2}$ | 508.8 |
| $Hbb$       | 4.3 | 719.51 | 4.3 | 508.8 |
| $Htt$       | 180.716 | 707.54 | 181.8 | 490.7 |
| $Hss$       | 0.34 | 719.52 | 0.34 | 508.8 |
| $Hcc$       | 1.3 | 719.52 | 1.3 | 508.8 |
| $HWW$       | 80.33 | 717.25 | 80.4 | 505.5 |
| $HZZ$       | 91.2 | 716.6 | 91.2 | 504.6 |
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