A Predictive Account of Café Wall Illusions Using a Quantitative Model

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Abstract

This paper explores the tilt illusion effect in the Café Wall pattern using a classical Gaussian Receptive Field model. In this illusion, the mortar lines are misperceived as diverging or converging rather than horizontal. We examine the capability of a simple bioplausible filtering model to recognize different degrees of tilt in the Café Wall illusion based on different characteristics of the pattern. Our study employed a Difference of Gaussians model of retinal to cortical “ON” center and/or “OFF” center receptive fields. A wide range of parameters of the stimulus, for example mortar thickness, luminance, tiles contrast, phase of the tile displacement, have been studied for their effects on the inducing tilt in the Café Wall illusion. Our model constructs an edge map representation at multiple scales that reveals tilt cues and clues involved in the illusory perception of the Café Wall pattern. We present here that our model can not only detect the tilt in this pattern, but also can predict the strength of the illusion and quantify the degree of tilt. For the first time quantitative predictions of a model are reported for this stimulus considering different characteristics of the pattern. The results of our simulations are consistent with previous psychophysical findings across the full range of Café Wall variations tested. Our results also suggest that the Difference of Gaussians mechanism is the heart of the effects explained by, and the mechanisms proposed by the Irradiation, Brightness Induction, and Bandpass Filtering models.

Keywords: Visual perception; Biological neural networks; Geometrical illusions; Tilt effects; Café Wall illusion; Difference of Gaussians; Perceptual grouping; Classical Receptive Fields; Retinal processing models; Retinal Ganglion Cells; Lateral Inhibition

1 Introduction

Visual illusions have the potential to give insight into the biology of vision [1, 2, 3]. They further open windows for solving the engineering and application specific problems relevant to image processing tasks including edge detection and feature selection, as we seek to attain human level performance. Many illusions have been used to evaluate such effects, notably Café Wall [4, 5, 6] and Mach Bands [7, 8].

Café Wall is a member of Twisted Cord family of Geometrical Illusions, and has received some fairly convincing explanations [4, 9, 10]. The Munsterberg version of this pattern is a chessboard with dark very thin separator between shifted rows of black and white tiles, giving the illusion of tilted contours dividing the rows. The Café Wall pattern (Fig 1-center) has grey stripes interpreted as mortar lines dividing shifted rows of black and white tiles, inducing a perception of diverging and converging of the mortar lines. Morgan and Moulden suggest the mortar lines are critical for the strength of illu-
sion and that illusion has its highest strength when the luminance of the mortar is intermediate relative to the luminance of the tiles [6]. We consider the whole range of luminance for mortar lines from Black to White as different variations of the Café Wall illusion. Other variations include Hollow Square [9, 11] Fraser Pattern, Spiral Café Wall (Fig 1-left), Spiral Fraser; and even variations of Zollner Illusion (Fig 1-right) [5, 11, 12] where small parallel line segments (inducing horizontal/vertical lines) on parallel lines result in perceiving these lines as being convergent or divergent. The Café Wall illusion seems to originate from the inducing effect of Twisted Cord [13] elements and then integration of these to an extended continuous contour along the whole mortar line [14, 15].

Over the last few decades, many low-level and high-level models have been proposed with the aim of explaining various Geometrical/Tilt Illusions. However, there are many issues unaddressed across the range of the Geometrical Illusions and their underlying percepts. Specially modeling the illusion effect as it is perceived is a challenging task in Computer Vision models. For the Café Wall illusion and its variations, some of the explanations are based on “high-level” models [4, 16], but many rely on “low-level bioplausible” models of simple filtering techniques [6, 10].

A low-level explanatory approach [18-22] employed in this study is a common bioderived model for retinal ganglion cell (RGC) responses to the stimulus. The model presents the simple cell responses using an edge map representation at multiple scales derived from the Differences of Gaussians (DoG). This is an effective filtering technique for identifying the edges [23, 24] in the early stages of visual processing. Symmetrical DoGs at multiple scales have been used for spatial filtering to generate edge maps, modelling ON-center OFF-surround receptive fields (RFs). Our explanations of tilt in Tile Illusions [18, 19, 20-22] connects to the Marr’s theory of primal sketch in general and his speculation of 3D structure above the edge map [25]. Further details about the underlying mechanism of our model arise from the known or theorized mechanisms of early visual processing such as the retinal Point Spread Function (PSF) and lateral inhibition effect of the RFs [26] is presented in Section 2.1. We now concentrate on different explanation models for Café Wall illusion.

One of the first explanations for the Café Wall illusion was the Irradiation Hypothesis [14], which was first introduced by Helmholtz [27], proposing that a compressive transform causes a shift of high contrast boundary towards the dark side of its inflection [6]. The limitation of the Irradiation as a sole explanation of the Café Wall illusion is that the hypothesis does not explain why the illusion is enhanced by the mortar lines and why the pattern doesn’t need to be of a very high contrast as explicitly required by Irradiation Theory. So this explanation is incomplete on its own.

Border Locking is another hypothesis used in high-level explanations for tilt illusions [4]. The appearance of small wedges with local asymmetry is based on the luminance contrast of the light/dark half tiles and their integration along the rows, so that they form long wedges. Gregory and Heard state that the effect depends critically on luminance and that it disappears if the mortar line is either brighter than the light tiles, or dimmer than the dark ones [4]. We will show this in our experimental results.

Brightness Induction (BI) describes a change in the direction of perceived brightness, and is another explanation for Café Wall illusion. The change of direction can be towards the surrounds (Brightness Assimilation) or away from it (Brightness Contrast) [28]. McCourt has shown that a periodic field of a sine or square wave varying brightness would result in inducing brightness on a homogeneous narrow grey field positioned on top of it [5]. He generated a new version of Café Wall based on replacing mortar lines with the patches of light, dark and grey and explained the tilt effect as Bright-
ness Induction, noting that alternately-reversing diagonal line elements can produce the characteristic of illusory convergence in the whole image.

Fraser [13] proposed another hypothesis, connecting the Twisted Cord to the Café Wall illusion without using filtering but his hypothesis is not a complete explanation of the illusion. His idea was based on the fact that the tilt is perceived when two identically colored tiles joined by the mortar lines at their opposite corners create a Twisted Cord element. There are some other proposed explanations such as Fourier-based Models, but it seems that the effect arises from small local wedge shapes rather than global features. Therefore, what is needed is local frequency analysis, which might bring some information out, rather than the global transformation [6].

Bandpass Spatial Filtering is argued to be performed by the retinal ganglion cells of ON-center and OFF-center receptive fields, being model in [6] by a simple discrete Laplacian filter mask viewed as an approximation to Difference of Gaussians (noting that DoG gives the same results), and in [10] by a Difference of Gaussians viewed as “a very close approximation” to a Laplacian of Gaussian (LoG).

There are some more recent explanations for the Café Wall illusion. A similar psychological approach to [4] was proposed by Kitaoka as the Phenomenal Model [16, 29], which is an elemental composition approach to explain a wide variety of Tilt Illusions. Kitaoka and his team created many of these patterns using a heuristic tool [29] for graphical constructions of illusory stimuli from elementary units. Their psychological approach is based on the “contrast polarities of a solid square and its adjacent line segment” to explain the Café Wall illusion. They have suggested that when dark/light squares are adjacent with dark/light line segment, the direction of tilt is to counteract the square angle, otherwise the square angle is going to expand.

Fermuller and Malm proposed a Point and Line Estimation Model [30] for finding the displacement of points and lines as the elementary units in images and used their techniques to explain certain Geometrical Illusions. As applied to Café Wall, this theory is based on categorizing the edges by saying that if mortar lines border both dark tile and bright tile, then the two edges move towards each other. On the other hand, mortar lines between two bright regions or two dark regions cause the edges to move away from each other. Their explanation is similar to [4, 29].

Arai proposed a computational nonlinear model for Contrast Enhancement in early visual processing based on discrete maximal overlap bi-orthogonal wavelet [31]. It is a multiscale model and after decomposition of input signal into four sub bands of approximations, horizontal, vertical, and diagonal details, a mathematical operator, which “enhances small values of detail if the energy of detail is small, and inhibits large values of detail if its energy is large” [31:pp.178] is used. They have investigated the output of their system on Brightness/Lightness Illusions such as Hermann Grid, Chevreul, Mach Bands, Todorovic, and Café Wall.

The Irradiation Effect involving enlargement of bright region to encroach on their dark sides was identified for shifted chessboard pattern by Westheimer [12], who proposed a model including a combination of “light spread”, “compressive nonlinearity”, “center surround organization” and “border locations”, to explain Café Wall illusion. For retinal spatial distribution of excitation, he first calculated the convolution of image with the retinal point spread function, which then passed through a certain nonlinearity equation (Naka-Rushton), and then to the spatial center-surround transformation with the excitatory and inhibitory zones to get the final result.

Elimination of the tilt effect in Café Wall illusion is possible by the enlargement of the black tiles [12: Fig.11]. This results in compensating the border shifts and corner distortions. Similar elimination techniques can be used with additional superimposed dots positioned in the corners (black/white dots on white/black tiles), eliminating the corner effect [30:Fig 5, 29:Figs 5C,D]. The elimination of the
illusion is also possible by replacing the black and white tiles with equiluminant but highly contrasting colour tiles [5, 32].

There are some previous explanations for connecting ‘Brightness Induction’ illusions and ‘Geometrical’ illusions. The Café Wall pattern and its variations are accounted ‘second-order’ tilt patterns [18, 19] involving ‘Brightness Assimilation and Contrast’ [33] as well as ‘Border shifts’ [12, 6]. In a thorough review of lightness, brightness, and transparency (LBT) [34] it has been noted that one of the most promising approaches for modelling brightness coding is multiscale filtering [35] in conjunction with contrast normalization. Lateral inhibition is a common feature of most of the above models such as Irradiation [14], Brightness Induction [5], and Bandpass filtering [6].

In Section 2 we provide a detailed examination of the role of multiscale representation in computational models of vision, with a main focus on evidence of multiscale filtering within the retina (Section 2.1). We then describe the characteristics of a simple classical model based on Differences and Laplacian of Gaussians (DoG, LoG), and utilize this bioplausible model to explain Café Wall illusion qualitatively and quantitatively (Sections 2.2 and 2.3), followed by the details of patterns investigated (Section 2.4). Afterwards, the experimental results on variations of Café Wall illusion are going to be presented in Section 3. These patterns are created by modifying different characteristics of the Café Wall pattern such as mortar luminance and thickness (width), tiles contrast, phase of tile displacement for further investigation of the tilt effect. We then outline a testable framework for the design of psychophysical experiments capable of testing our predictions (Section 3.4). We conclude by highlighting the advantages and disadvantages of the model and proceed to outline a roadmap of our ongoing and future work (Section 4).

2 Material and Method

2.1 Multiscale Representation in Vision and Vision Models

Psychophysical and physiological findings have suggested a multiscale transform model of the processing in the mammalian visual cortex as well as the early stage processing within the retina [36-38]. Kuffler was the pioneer who recorded the activity of the retinal ganglion cells (RGCs) that exit as the optic nerve, and found two types of center surround cells [39]. Hubel and Wiesel performed many pioneering experiments that increased our understanding of cortical visual processing [40]. Daugman showed an approximation of these cortical cells by using Gaussian windows modulated by a sinusoidal wave for this impulse response. A specific spatial orientation tuning of these cells arises from dilation of modulated Gaussian-Gabor functions [41].

The need to extract multiscale information when modelling the visual mechanism in Computer Vision (CV) applications have been established by many researchers in the field [25, 42-44] and some of these early ideas have later been subsumed by the wavelet paradigm. In a multiresolution algorithm [44], the search typically moves from coarse to fine scale, processing low-resolution images first and then zooming selectively into fine scales of the visual data. Mallat showed the impact of wavelets for low-level vision in multiresolution search, multiscale edge detection and texture discrimination [45].

Pyramidal image representations and scale invariant transforms [46] are well matched to human visual encoding and do not need image partitioning like JPEG-DCT [47]. A scale-space analysis is an emergent result of image decomposition by finding the differences between pairs of scaled filters with different parameterizations, notably the Laplacian or Difference of Gaussian filters (LoG/DoG) [48, 49], giving rise to Ricker and Marr wavelets.

Note further that self-organization models of repeated patterns of edge detectors at particular angles are well-established [50]. Higher-level spatial aggregation of regularly spaced spots or edges in turn
automatically gives rise to analogues of DCT and DWT type bases, the latter with localization determined by the higher-level lateral interaction functions or the constraints of an underlying probabilistic connectivity model [51].

In our visual system, light is focused into receptors which transduce it to neural impulses that are further processed in the retina [52]. Then the middle layer of the retina, which is the focus of our study, enhances the neural signals through the process of lateral inhibition [26], causing an activated nerve cell in the middle layer to decrease the ability of its nearby neighbors to become active. This biological convolution with its specific Point Spread Function (PSF) enables feature/texture encoding of the visual scene but also we show, leads to optical illusions. The effect of center-surround processing and lateral inhibition on an indistinctly defined edge with gradual change from dark to light is that it reinforces transition between the light and dark areas to appear more abrupt due to appearance of overshoot and undershoots [52]. These result in sharpening the edges facilitate the visual tasks.

The first layer of the retina has a nonlinear mechanism for retinal gain control, flattening the illumination component, and make it possible for the eye to see under poor light condition [52]. The lateral inhibition in middle layer of the retina thus evidences both a bandpass filtering property and an edge enhancement capability. In the final layer, we find ganglion cells whose axons exit the eye and carry the visual signals to the cortex (inter alia).

The contrast sensitivity of the retinal ganglion cells can be modeled based on Classical Receptive Fields (CRFs), involving circular center and surround antagonism, in which for revealing the edge information it uses the differences and second differences of Gaussians [23, 24] or Laplacian of Gaussian (LoG) [53]. Marr and Hildreth [25] proposed an approximation of LoG with DoG based on a specified ratio of the standard deviations (**σ**) of the Gaussians [23, 24]. Powers showed that DoG-like models can themselves result from a simple biophysical model of ontogenesis and can usefully approximate the interaction functions proposed in a variety of neural models [51].

We hypothesized that visual perception of a scene starts by extracting the multiscale edge map, and a bioplausible implementation of a contrast sensitivity of retinal RFs using DoG filtering produce a stack of multiscale outputs [54]. In retinal encoding, what is sent to the brain is a stack of images or a scale-space, not a single image. One of the first models for foveal retinal vision was proposed by Lindeberg and Florack [55] and our model is most inspired by it. Their model is based on simultaneous sampling of the image at all scales, and the edge map in our model is generated in a similar way. Edge map extraction is an essential and primitive task in most image processing applications, most directly to Marr’s primal sketch for perception of a 3D view of the world [25] and is one of its two main components. There are also possibilities of involvement of higher order derivatives of Gaussians, which can be seen in retinal to cortical visual processing models such as [25, 53, 56, 57] but there is no biological evidence for them.

Our model output has some similarities with Brightness Assimilation and Contrast theory of early model developed by Jameson and Harvich [33] based on DoG filters with multiple spatial scales. They noted the existence of parallel processing that occurs in our visual processing as the result of simultaneous appearance of sharp edges and mixed colour that define delimited regions. They proposed that contrast effect happening when the stimulus components are relatively large in size compared to the center of the filter, and assimilation effect when components of the stimulus are small compared to the filter center.

Recent physiological findings on retinal ganglion cells (RGCs) have dramatically extended our understanding of retinal processing. Previously, it was believed that retinal lateral inhibition could not be a major contributor to Café Wall illusion because the effect is highly directional and arises from both
orientation as well as spatial frequency. Neuro-computational eye models [54-56] have been proposed based on biological findings by considering the size variation of RGCs due to variations of the eccentricity and dendritic field size [58]. Field and Chichilnisky [36] published a sophisticated report about the circuitry and coding of the visual processing inside the retina, by noting the existence of at least 17 distinct retinal ganglion cell types and their specific role in visual encoding.

As mentioned before, some RGCs have been found to have orientation selectivity similar to the cortical cells [59, 60]. Also there is evidence for the existence of other types of retinal cells like horizontal and amacrine cells which have elongated surround beyond the CRF size. This has lead to orientation selectivity for modelling of the eye, as retinal non-CRFs (nCRFs) implementation [61-63].

The underlying mechanics of retinal multiscale processing that encode visual data from fine to coarse scales gets clear indication from all of this evidence from the diversity of intra-retinal circuits, and different types of RGCs [36, 37]. Also there are variations of the size of each individual RGC due to the retinal eccentricity [57]. This indicates a high likelihood of the involvement of retinal/cortical simple cells and their early stages of processing in revealing the tilt cues inside Tile Illusion patterns and in Café Wall in particular.

2.2 Model

The features of our bioplausible approach are intended to model the characteristics of a human’s early visual processing. Based on numerous physiological studies e.g. [36-38] there is a diversity of the receptive field types and sizes inside the retina, resulting in multiscale encoding of the visual scene. This retinal representation is believed to be “scale invariant” in general, and there is an adaptation mechanism for the Receptive Field sizes to some textural elements inside our field of view [54, 64].

Applying Gaussian filter on an image makes a blurred version of the image. The Difference of two Gaussian convolutions generates one scale of the edge map representation in our DoG model. For a 2D signal such as image $I$, the DoG output modelling the retinal GC responses with center surround organization is given by:

$$
DoG_{\sigma, s\sigma}(x,y) = I\times \frac{1}{2\pi \sigma^2} \exp\left[-\frac{x^2+y^2}{2\sigma^2}\right] - I\times \frac{1}{2\pi (s\sigma)^2} \exp\left[-\frac{x^2+y^2}{2(s\sigma)^2}\right]
$$

where $x$ and $y$ are the distance from the origin in the horizontal and vertical axes respectively and $\sigma$ is the standard deviation/scale of the center Gaussian ($\sigma_c$). As shown in Eq (2) $s\sigma$ indicates the standard deviation (or scale) of the surround Gaussian ($\sigma_s=s\sigma$). $s$ is referred to as Surround ratio in our model.

$$
s = \frac{\sigma_s}{\sigma_c}
$$

The Laplacian of Gaussian (LoG) can be estimated as the Differences of two DoGs, estimating the second derivative of Gaussian. For modelling the receptive field of retinal Ganglion Cells, DoG filtering [23, 24] is a good approximation of LoG, if the ratio of dispersion of center to surround is close to 1.6 [10, 25] ($s \approx 1.6 \approx \phi$, the ubiquitous Golden Ratio).

As indicated in Eq (1) by increasing the value of $s$, we reach a wider area of surround suppression, although the height of the surround Gaussian declines. We also tested a broader range of Surround ratio from 1.4 to 8.0 but this made little difference to our results. In addition to the $s$ factor, the filter size is another parameter to be considered in the model. The DoG is only applied within a window in which the value of both Gaussians are insignificant outside the window (less than 5% for the surround Gaussian). A parameter is defined called Window ratio ($h$) to control window size. The size is determined based on this parameter ($h$) and the scale of the center Gaussian ($\sigma_c$) as given in Eq (3):

$$
WindowSize= h\times \sigma_c + 1
$$
Parameter $h$ determines how much of each Gaussian (center and surround) is included inside the DoG filter (+1 as given in Eq (3) guarantees a symmetric filter). For the experimental results, in order to capture both excitation and inhibition effects, the Window ratio ($h$) have been set to 8 in this paper.

A sample DoG edge map of a Tile Illusion that is actually a DoG of the image at multiple scales is shown in Fig 2. The DoG is highly sensitive to spots of matching size, but is also responsive to lines of appropriate thickness and to contrast edges. A crop section (84×84px) of Trampoline pattern [65] was selected as an input image. The edge map has been shown at five different scales, $\sigma_c=0.5, 1.0, 1.5, 2.0$ and 2.5 to capture important information from the image (this is related to the texture/object sizes in the pattern and by applying Eq (3); we can determine a proper range for $\sigma_c$ in the model for any arbitrary pattern). The DoG filters in the figure are shown in jetwhite color map [66].

Scale invariant processing in general is not sensitive to the exact parameter setting, ideally the model’s parameters should be set in a way that at fine scales, they capture high frequency texture details and at coarse scales, the kernel has appropriate size relative to the objects within the scene.

The DoG edge map at multiple scales for a crop section of a Café Wall pattern has been shown in Fig 3, as the output of the model, revealing the Twisted Cord elements along the mortar lines. The appearance of the Twisted Cord elements have been shown as the filtering output of applying either DoG/LoG on a Café Wall image at specific filter sizes (scales) in previous literature [6, 10]. We use a model [18-22] that highlights the perception of divergence and convergence of mortar lines in the “Café Wall” illusion. We will show how the model is capable of revealing the illusory cues in the pattern qualitatively, as well as measuring the strength and orientation of the tilt effect quantitatively in a wide range of Café Wall patterns. We will also show how our explanation of tilt effect can be connected to the Brightness Induction theory.

### 2.3 Processing Pipeline for Tilt Analysis

The DoG transformation, modelling RGC responses, creates an edge map representation at multiple scales for the pattern, consisting of a set of tilted line segments for the Café Wall stimulus [18-22] at its fine scales (noted as Twisted Cord elements in the literature [6, 11, 13]). Then for quantitative measurement of tilt angles in the edge map to compare them with the tilt perceived by a human observer, we embed the DoG model in a processing pipeline using Hough space. We use the houghlines representation for the edges to determine the tilt angles at different scales of the DoG edge map as shown in Fig 3.

**Fig 3.** Flowchart of the model and analytical tilt processing (Reproduced with permission from [72]).

#### 2.3.1 Multiple scale edge map

The most fundamental parameter in the model is the scale of the center Gaussian ($\sigma_c$). Defining a proper range of scales in our model is highly correlated with characteristics of the pattern elements, in particular the mortar lines and tile sizes in the Café Wall pattern. The edge map representation at multiple scales as the output of the MODEL have been shown in Fig 3 for a cropped section of a Café Wall pattern with 200×200px Tiles ($T$) and 8px Mortar ($M$), and in Fig 7 for three variations of Café
Wall pattern presented in the jetwhite color map [66]. Based on the fixed parameters of the Surround and Window ratios, relative to \( \sigma_c \) (\( s=2 \) and \( h=8 \)) and the pattern characteristics, an illustrative range for \( \sigma_c \) to detect both mortar information and tiles is a range of 0.5\( M =4\)px to 3.5\( M =28\)px (refer to Eq (3), at scale 28 the DoG filter has a size of \( 8 \times \sigma_c =8 \times 28=224 \) nearly the same as Tile size =200px. Increasing the scale from this point results in a very distorted edge map, due to the DoG averaging and the filter size). We have selected incremental steps of 0.5\( M \) between scales. This allows for the extraction of both mortar lines (at fine scales) and the Café Wall tiles (at coarse scales) in the edge map, as well as revealing the tilt cues and different perceptual groupings [19] at multiple scales through the gradual increase of the DoG scales. So the output of the model is a DoG edge map at multiple scales for any arbitrary pattern, and we may refer to it as a multiple scale edge map in the text. Now we will explain how to measure the slope of the detected tilted lines in the DoG edge maps.

Since we have used normalized Gaussians in our model, the curve between the center and surround Gaussians in the DoG filter shows the activation response and that the surround Gaussian intersects the center Gaussian at its inflection points (for the 2.0 ratio). Therefore, \( \sigma_c =4 \) (Eq. 3) in our model corresponds to a filter in which the mortar size of 8px lies within one standard deviation (\( \sigma_c \)) away from the mean of the filter, so we are able to detect the mortar lines with high accuracy in the filtered response. Therefore, the detected tilts by the model at \( \sigma_c =4 \) in our results (H) show the prediction of tilt angle in the foveal vision with high acuity in the center of our current gaze direction. We discuss about it in details in Section 3.4.

2.3.2 Analysis with Hough

The quantitative tilt measurement (Fig 3) includes three stages, EDGES, HOUGH and ANALYSIS implemented in MATLAB.

EDGES: At each scale, first the edge map is binarized and then Hough Transform (HT) [67] is applied and allow us to measure the tilt angles in detected line segments in the binary edge map as described in the following stages. HT uses a two-dimensional array called the accumulator (\( H_A \)) to store lines information with quantized values of \( \rho \) and \( \theta \) in its cells. \( \rho \) represents the distance between the line passing through the edge point, and \( \theta \) is the counter-clockwise angle between the normal vector (\( \rho \)) and the x-axis, with the range of [0, \( \pi \)). So based on new parameters of (\( \rho \), \( \theta \)) every edge pixel (\( x \), \( y \)) in the image space corresponds to a sinusoidal curve in the Hough space as given by \( \rho =x \cos \theta +y \sin \theta \).

HOUGH: All possible lines that could pass through every edge point in the edge map, have been accumulated inside the \( H_A \) matrix. We are more interested in the detection of tilt inducing line segments inside the Café Wall pattern. Two MATLAB functions called houghpeaks and houghlines have been employed for this reason. The ‘houghpeaks’ function finds the peaks in the \( H_A \) matrix, which are the dominant line segments. It has parameters of NumPeaks (maximum number of lines to be detected), Threshold (threshold value for searching the \( H_A \) for the peaks), and NhoodSize (neighborhood suppression size which set to zero after the peak is identified). The ‘houghlines’ function, however, extracts line segments associated with a particular bin in the \( H_A \). It has parameters of Fill-Gap (maximum gap allowed between two line segments associated with the same Hough bin), and MinLength (minimum length for merged lines to be kept). Sample outputs of the HOUGH analysis stage have been presented in Fig 3 (for a crop section of a Café Wall pattern) as well as Figs 5, 8, 9, and Figs 12 to 14 for different variations of the Café Wall pattern investigated. Detected houghlines are shown in green, displayed on the edge maps with DoG scales ranges from 4 to 28.

ANALYSIS: For categorizing detected line segments, we have considered four reference orientations of horizontal (\( H \)), vertical (\( V \)), positive diagonal (+45º, \( D1 \)), and negative diagonal (-45º, \( D2 \)). An interval of [-22.5º, 22.5º] around each reference orientation have been chosen to cover the whole
space. The information from HOUGH are saved inside four orientation matrices based on how close they are to one of these reference orientations at this stage for further tilt analysis. The statistical tilt measurements of the detected houghlines in the neighborhood of each reference orientation is the output of this final stage.

2.4 Patterns Investigated

To evaluate the tilts predicted by the model and to find out how close these predictions are to reported psychophysical experiments in the literature, we have generated different variations of the Café Wall pattern similar to the previously tested ones. All the generated patterns have the same configuration (#rows and columns of tiles) as Café Walls of 3×8 tiles with 200×200px tiles.

The patterns investigated include Mortar-Luminance (ML) variations in which the mortar lines have a relative luminance in the range of Black (ML=0) to White (ML=1), and three shades of Grey in between Black and White (0.25, 0.5 and 0.75) and are displayed in Fig 4-top (five patterns).

We investigated also Mortar-Width (MW-thickness) variations, ranging from no mortar lines (MW=0px) through MW= 4, 8, 16, 32 and 64px and are presented in Fig 4-middle (six patterns).

The bottom of Fig 4 show three patterns involving tiles with two shades of Grey (0.25 and 0.75) separated by mortar with one of three levels of luminance (0, 0.5, 1), and below that two variations showing different degrees of displacement (phase shifts of 1/3 and 1/5), and finally a mirrored pattern used to demonstrate that predicted tilts of the pattern reverses as expected and a hollow square pattern (seven variations). These examples are repeated in later figures along with the processed versions with detected Hough lines (eighteen stimuli in total).

3 Experimental Results

The tilt perception in Café Wall pattern is not only affected by foveal and peripheral view to the pattern, which have been investigated in our previous studies [20-22], but also from the patterns characteristics as well, such as mortar luminance, size, phase of tile displacement, tiles contrast and so forth. We have analyzed the effect of these parameters on the magnitude and orientation of the tilt effect in eighteen different variations (Fig 4) of the Café Wall pattern.

3.1 Mortar Luminance variations

3.1.1 Patterns Characteristics

The patterns under investigation are five variations given in Fig 4-top. These patterns are Café Walls of 3×8 tiles with 200×200px tiles (T) and 8px mortar (M) (Café Wall 3×8-T200-M8). To generate the samples we have used a value that can be interpreted as ‘reflectance’ (viewed printed) or ‘relative luminance (viewed on screen) to represent grey levels in the range [0,1]. Together with ambient direct and indirect illumination, these combine to give a subjective effect of relative brightness.
Kingdom [34] defines ‘brightness’ as the perceptual correlate of the perceived luminance. Blakeslee and McCourt [68] explain that ‘luminance’ is the physical intensity of a stimulus and that in achromatic vision, we see patterns of varying luminance, in which the judgement of ‘brightness’, ‘lightness’ and ‘contrast’ would be identical in this case. In this paper we refer to the grey level of mortar lines as simply ‘luminance’, and denote Mortar Luminance with ML for an easier referral to this pattern characteristic in the rest of this paper.

By looking at the Mortar-Luminance variations, we see a very minor tilt effect for the Black (ML=0.0) and White (ML=1.0) mortar patterns, compared to the Grey mortar variations (with three luminance levels: ML=0.25, 0.5, 0.75). Psychophysical experiments reported in the literature have shown that the strongest tilt effect in the Café Wall pattern occurs when the mortar luminance is in the intermediate range between the Black and White tiles [5, 6].

3.1.2 DoG Edge Maps and Quantitative Tilt Results

The binary DoG edge maps at seven different scales for the Mortar-Luminance variations (Fig 4-top) have been presented in Fig 5, in which the edge maps have been overlayed by the detected houghlines displayed in green. Let concentrate on the binary edge map first. The range of DoG scales (Section 2.3.1) starts from $0.5\sigma =4$px and continues to $3.5\sigma =28$px with incremental steps of $0.5\sigma (\sigma =4, 8, 12, 16, 20, 24, 28)$. Note that for the detection of near horizontal tilted line segments, as the perceived mortar lines in the Café Wall illusion, the DoG scale should be in the intermediate range, close to the mortar size, $\sigma _c \sim M=8px$ [20, 22]. We suggest that $\sigma _c =4$ is appropriate for predicting foveal tilt effect and $\sigma _c =8$ for predicting the tilt at the edge of the image in the periphery of the retina. Comparing the edge maps at coarse scales ($\sigma _c =20, 24, 28$) shows nearly similar DoG outputs. This is because at the coarse scales, the scale of the DoG is large enough to capture the tile information. The difference between the DoG edge maps are mainly at fine to medium scales of the DoGs ($\sigma _c =4, 8, 12, 16$). At scale 16 ($\sigma _c =16$), we see a transient state between detecting nearly horizontal tilted line segments connecting tiles with the Twisted Cord elements along the mortar lines, to zigzag vertical grouping of tiles at the coarse scales, in a complete opposite direction. At this scale, we still see some mortar cues left in the edge maps of some of these variations, while in others, those mortar cues are completely disappeared. We will explain later on (Sections 3.2.2 and 3.2.3) how the strength of the illusion (which is different from the magnitude of detected tilt angles) are predictable based on the persistence of the mortar cues at the multiple scales of the DoG edge maps. Now we need to quantify the mean tilts in the edge maps for these patterns using the Hough analysis pipeline in our model (as described in Section 2.3.2).

For quantitative analysis of tilt, same parameters of Hough have been applied for all of these variations, and for every scale of the edge maps to attain reliable tilt results, which are comparable between these variations. The fixed Hough parameters are: NumPeaks=1000, FillGap=40px, and MinLenght=450px, and if the algorithm cannot detect any lines at any specific scale, we simply report it as no detection of lines at that scale. Fig 5 shows the detected houghlines displayed in green on the DoG edge maps for the Mortar-Luminance variations. Blue lines indicate the longest lines detected. The absolute mean tilts and the standard errors of detected tilt range have been provided in Fig 6.
As the left column in Fig 5 shows, for the Black mortar variation (ML=0.0–Munsterberg pattern), there are no detected lines at the finest scale ($\sigma_c = 4$; first row represented by NaN—no detected lines). As the mean tilt results in Fig 6 shows, there is no near horizontal lines detected at any of the DoG scales in the Munsterberg pattern. The detected houghlines at medium scale ($\sigma_c = 12$) are near vertical lines, with then near diagonal lines at larger scales ($\sigma_c = 20$ to 28). As the DoG edge map of the pattern shows the apparent grouping of tiles, is a zigzag vertical pattern. So the edge map leads to a same conclusion as we had in houghlines analysis, with no near horizontal tilts in this pattern. There is no confusion of visual cues in our results that contribute the tilt effect in this pattern, so we state that there is no tilt illusion in the Munsterberg pattern (ML=0.0 – Black mortar).

Fig 6. Mean tilts and the standard error of detected tilt angles of houghlines for the five variations of Mortar-Luminance, from Black (ML 0.0) and White (ML 1.0) mortar on Top, to Grey mortar lines (ML=0.25, 0.5, 0.75) at the Bottom, for DoG edge maps at seven scales ($\sigma_c = 4$, 8, 12, 16, 20, 24, 28) – Hough parameters are kept the same in all experiments for detecting near horizontal ($H$), vertical ($V$) and diagonal ($D1, D2$) tilted lines in the edge maps. NumPeaks=1000, FillGap=40px, and MinLength=450px for all DoG scales of the edge maps. NaN: not a number means no lines detected (Reproduced with permission from [72]).

As the right column in Fig 5 shows, for the White mortar variation (ML=1.0), there is no detected lines at the finest scale ($\sigma_c = 4$) similar to the Munsterberg pattern. But a few lines are detected around 1° of horizontal deviation at the next scale ($\sigma_c = 8$) as with the details given in Fig 6. The detected tilt range is much below the Grey mortar variations, which is approximately more than 6.5° at scale 8. Even at scale 12, the mean tilt is roughly 3° less than the Grey mortar variations. For the variation of the White mortar, we see some cues of mortar lines in the edge map at fine scales, but these are different than the Grey mortar variations. If there is any illusion in the White mortar pattern, the tilts predicted by the model is quite negligible (None at $\sigma_c = 4$ and $\sim 1°$ at $\sigma_c = 8$). Relying on these quantitative results, we can state that similar to the Black mortar, there is no illusion in the White mortar variation as well.

The houghlines of the Grey mortar patterns with three levels of luminance for Dark-, Mid-, and Light-Grey (0.25, 0.5, 0.75), have been shown in Fig 5-center columns. The absolute mean tilts and the standard errors of detected tilt range for these patterns have been shown at the bottom of Fig 6.

For the Dark-Grey (ML=0.25) and Mid-Grey (ML=0.5) mortar, the mean tilts at the finest scale ($\sigma_c = 4$) are $\sim 3.5°$ compared to $\sim 4.5°$ for ML=0.75 (Light-Grey). As Fig 6 shows, we are still able to detect horizontal lines at scale $\sigma_c = 16$ for two patterns of ML=0.5 and ML=0.75, but not for ML=0.25. This gets clear from the DoG edge maps (Fig 5), because the mortar cues are still influential at this scale in the two variations of Mid- and Light-Grey, but not in the Dark-Grey mortar pattern. The near horizontal tilts are $\sim 12°$ at this scale in both of the patterns. Based on the result we conclude that, the range of the detected mean tilts is up to $1°$ more, between $\sim 3.5°$ to $12°$ for ML=0.5 variation, but between $\sim 4.5°$ to $12°$ for ML=0.75 pattern. This could be an indication for a stronger tilt effect in the Mid-Grey pattern compared to the Light-Grey variation. This supports previous psychophysical findings that the highest strength for tilt effect in Cafè Wall illusion is when the luminance of the mortar is in the intermediate luminance of the tiles [5, 6].

At coarse scales after $\sigma_c = 16$, we have the zigzag vertical groupings of tiles. Fig 5 shows that at scales 16 and 20, the vertical lines are fitted in the edge maps with a reliable tilt range. The deviations from the vertical axis are $\sim 6°$ to $9°$ in different variations at coarse scales ($\sigma_c = 16$ to 28). Detected houghlines around the diagonal axes (D1, D2) start at the coarsest scales ($\sigma_c = 24, 28$). The deviations from the diagonal axes are in the range of $\sim 13°$ for the Grey mortar variations, slightly higher $\sim 13.5°$ for the White mortar pattern, and a bit lower $\sim 11.5°$ for the Black mortar (Munsterberg pattern).
3.1.3 Discussion
The perceptual grouping of Café Wall pattern with White mortar seems a bit different from the Munsterberg pattern with Black mortar. In the literature, it has been reported that these variations of Café Wall do not have any illusory tilts [4, 6] or better put, they are not as strong as the intermediate luminance of Grey mortar lines. It has also said that the mortar should be very thin for Munsterberg version to induce the tilt. We have tested one sample of Munsterberg pattern in here with the same mortar size as the others (M=8px). The predicted tilts for the Black and White mortar variations show no to very weak tilts (at $\sigma_c=4, 8$; the finest scales in the model) which support previously reported findings [4, 6].

It was also suggested that the highest strength for the illusion is when the luminance of mortar is in the intermediate luminance of the tiles [5, 6]. Our results on the Grey mortar variations are consistent with this suggestion. We have shown that although both patterns of $ML=0.5$ and $ML=0.75$ (Mid- and Light- Grey mortar) have a highly persistent mortar cues from the finest to medium scale ($\sigma_c=4$ to 16), but mean tilt results for $ML=0.5$ pattern show a higher range of tilt angles compared to $ML=0.75$ variation (1º more - starting from the lower angle at the finest scale with more rapid tilt increase into the medium scale). This indicates a stronger tilt effect for Café Wall illusion having mortar lines with an intermediate luminance. We discuss further in Section 3.4 how the tilts predicted by the model at multiple scales are related to the differences of resolution to the eccentricity.

3.2 Mortar Thickness variations

3.2.1 Patterns Characteristics
The six different variations of Mortar-Width (MW) are shown in Fig 4-middle. It has been reported that by increasing the height (width) of mortar lines, an inverse of the Café Wall illusion is happening [10], which is discussed here as well as in Section 3.2.3. The Mortar Width (Size) is denoted with MW for an easier referral to this pattern characteristic. Note that for these samples, we have used MW to specify patterns that belong to the Mortar-Width category. When we talk about the characteristics of a Café Wall pattern in general, we refer to mortar size simply by using $M$ (Mortar size, thickness, and width all refer to the size of mortar lines).

3.2.2 DoG Edge Maps and Quantitative Tilt Results
Since the mortar sizes are different in these variations while the tile sizes are the same, we have selected a pattern with mid-size mortar thickness, $MW=8px$ as a base, to define the scales of the DoG edge maps for these variations. So considering the mortar size of 8px (the DoG scales vary from $0.5M$ to $3.5M$ with incremental steps of $0.5M$), the appropriate scales are $\sigma_c=4, 8, 12, 16, 20, 24, 28$ which we use for all the Mortar-Width variations (Fig 4-middle). Fig 7 shows the DoG edge maps at seven scales in the jetwhite colour map for the Mortar-Width variations of $MW=16, 32$ and 64px (thick mortar lines). The binary DoG edge maps for all these patterns have been presented in Figs 8 and 9, in which the edge maps have been overlayed by the detected houghlines displayed in green. Let concentrate on the binary edge map first.

Since the tile sizes are the same, by comparing the DoG edge maps in Figs 8 and 9 we see that at the coarsest scale ($\sigma_c=28$), the DoG filter size is large enough to fully capture the tiles ($Window size=8*\sigma_c=224 ~Tile size=200px$ – Section 2.3.1). So for all of the Mortar-Width variations, the coarsest scales ($\sigma_c=24, 28$) show nearly similar DoG outputs. However, we see some substantial changes in the last two variations of very thick mortar lines ($MW=32, 64px$) at these scales.
The cues of mortar lines are still available for $MW=64$px pattern even at scale $\sigma_c=28$. For two variations of the very thick mortar ($MW=32$ and 64px pattern), we see Brightness Induction on the mortar lines, which can be seen more clearly on the jetwhite colour map representations of the edge maps in Fig 7. These brightness artifacts can be seen at scales 8 and 12 for $MW=32$px pattern and at scales 12, 16 and 20 for $MW=64$px pattern.

Fig 7. DoG edge maps at multiple scales ($\sigma_c=4$, 8, 12, 16, 20, 24, 28) for the thick mortar variations displayed in jetwhite colormap. The original patterns are Café Walls of 3×8 tiles with 200×200px tiles and mortar size variations of $MW=16$, 32, and 64px. The other parameters of the model are $s=2$ and $h=8$ for the Surround and Window ratios respectively. The last row shows each variation of the pattern investigated (Reproduced with permission from [72]).

In addition, there is another difference in these two variations versus the rest of the thinner mortar patterns. At the finest scales ($\sigma_c=4$, 8) for $MW=32$px pattern (Fig 7-center column), and at scales 4, 8 and 12 for $MW=64$px pattern (Fig 7-right column), the directions of groupings of identically coloured tiles are not as clear as they were in other patterns with thinner mortar lines. The mortar in these two patterns are quite thick, wide enough to separate the tiles completely, and our only perception when viewing these patterns, are the changes of brightness along the mortar lines. We refer to this effect as Enlarged Twisted Cord. The directions of these brightness changes are similar to what has been detected at their fine DoG scales. However, in these variations, we do not see the tilt effect as clearly as we perceive it on the other thinner mortar variations. These variations have a strong brightness induction, but not a strong tilt effect. The only thing that bridges between the tilt effect to the brightness induction effect in these patterns is the thick mortar sizes. The brightness induction and the direction of Enlarged Twisted Cord elements seem to be more subject dependent in these very thick mortar variations.

In the literature it has been shown that when the diameter of the DoG operator (implementing center-surround operators) is larger than the mortar width, an opposite phase brightness induction appears [69]. This has been reported as a Reverse of Café Wall illusion by Earle and Maskell [10]. Lulich and Stevens also reported for “a reversal of the traditional Café Wall effect that is dependent upon mortar width” [70:pp.428]. The Reversed Twister Cord in thick mortar variations of the pattern, also called Twisted Rope [10] with an opposite direction to the Twisted Cord elements along mortar lines to distinguish these two. The term Enlarged Twisted Cord emphasize the continuity with the Twisted Cord unlike the usage of Twisted Rope in [10]. They note that the effect of Reversed twisted cord occurs for a limited range of spatial frequency that is acting as bandpass filters. Outside this limit, the twisted cord elements breaks into two parallel segments aligned with the mortar lines [10], and thus no twisted edge elements are presented. This breakage of Twisted Cords that are explained in [10] can be observed clearly in the DoG edge maps presented in Fig 9, for example at scale 8 for $MW=64$px pattern and at scale 4 for $MW=32$px variation. Lulich and Stevens [70] note that by increasing mortar width the induced tilt along the mortar are further diminished and disappear when the mortar width is about twice the size of the DoG operator. In our results, the diminishing of the tilt cues can be viewed for $MW=8$px pattern at scale 16 of the edge map, but varies in other samples, with higher range for $MW=4$px, and much lower range for the thicker mortar lines (from the $MW=16$px upwards). The brightness induction observed and explained here is also consistent with Morgan and Moulden’s conclusion [6] that the effect is a consequence of Bandpass filtering. We have shown that this configuration allows us to explore the connection of the edge map representation at multiple scales and the strength of the illusion. We come back to this later on in Section 3.4.
Fig 8. DoG edge maps at seven scales ($\sigma_i=4, 8, 12, 16, 20, 24, 28$) for three variations of Mortar-Width, from no mortar lines ($MW=0px$) to $8px$ mortar, in which the edge maps are overlayed by the detected houghlines displayed in green (Blue lines indicate the longest lines detected) – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines in the edge maps. $NumPeaks=1000$, $FillGap=40px$, and $MinLenght=450px$ for all DoG scales. In all experiments the parameters of the DoG model are kept constant as $s=2$ (Surround ratio), $h=8$ (Window ratio). The last row shows each variation of the pattern investigated (Reproduced with permission from [72]).

For $MW=0px$ pattern (Fig 8-left column), the only grouping of pattern elements that can be seen in the edge map at multiple scale is the zigzag vertical grouping of tiles. We can state based on the DoG edge map that there is no tilt illusion in this pattern. Morgan and Moulden suggest that the mortar line is critical to the strength of illusion [6], and our result supports this suggestion. For $MW=4, 8$ and $16px$ (Figs 8 and 9), based on the defined DoG scales, we can see the mortar cues from the finest scale till the maximum scale of $\sigma_i=20$ for $MW=16px$ pattern. The edge maps show that the mortar cues do not disappear soon after the DoG filter reaches to the mortar size. For instance, in $MW=8px$ pattern, the near horizontal tilt cues along the mortar lines are quite persistent from the finest scale ($\sigma_i=4$) till scale 16 which is twice of the mortar size. From the edge maps in Figs 8 and 9 we see that there is a definite correlation between the width of the mortar lines and the strength of tilt illusion in these variations. The results show that increasing the height of mortar results in decreasing the strength of illusion. For the illusion to appear the mortar size reported in the literature is between $1^\circ$ and $10^\circ$ min of arc [4] based on our eye’s sensitivity and the mechanism of bandpass filtering [6]. We can use the predicted tilts to find the mortar thickness with the highest inducing tilt effect in order to generate tilt illusion patterns with the strongest effects.

The quantitative measurement of detected tilts in the DoG edge maps of the Mortar-Width variations (Fig 4-middle), have been measured based on the same parameters of Hough for all of the patterns, and for every DoG scale of the edge maps ($NumPeaks=1000$, $FillGap=40px$, and $MinLenght=450px$). Figs 8 and 9 show the results of detected houghlines displayed in green on the DoG edge maps at multiple scales for the Mortar-Width variations. The absolute mean tilts and the standard errors of detected tilt range have been tabulated for easier comparison in Fig 10.

Fig 9. DoG edge maps at seven scales ($\sigma_i=4, 8, 12, 16, 20, 24, 28$) for three different variations of Mortar-Width, from mortar size of $16px$ to $64px$, in which the edge maps are overlayed by the detected houghlines displayed in green (Blue lines indicate the longest lines detected) – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines in the edge maps. $NumPeaks=1000$, $FillGap=40px$, and $MinLenght=450px$ for all DoG scales ($\sigma_i=4, 8, 12, 16, 20, 24, 28$). In all experiments the parameters of the DoG model are kept constant as $s=2$ (Surround ratio), $h=8$ (Window ratio). The last row shows each variation of the pattern investigated (Reproduced with permission from [72]).

In $MW=0px$ pattern (Fig 8-left column), there are no detected houghlines around the horizontal orientation at any scales. The results of the mean tilts in Fig 10 confirm this. The only grouping of pattern elements that can be seen in the edge map at multiple scales and also in the detected houghlines, are the zigzag vertical grouping of tiles, with the vertical deviation of $\sim7.5^\circ$-$8^\circ$, and the diagonal mean tilts between $\sim9.4^\circ$-$10.4^\circ$. Therefore based on the edge map and Hough analysis results, we can conclude that there is no tilt illusion in this pattern.

Fig 10. Mean tilts and the standard errors of detected tilt angles of houghlines for six variations of the Mortar-Width, from no mortar ($MW=0px$) to $64px$ mortar and for the multiple scale ($\sigma_i=4, 8, 12, 16, 20, 24, 28$) edge maps – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines in the edge maps ($H, V, D1, D2$). $NumPeaks=1000$, $FillGap=40px$, and $MinLenght=450px$ for all DoG scales. NaN means no detected lines (Reproduced with permission from [72]).
The mean tilts near horizontal at scales 4, 8 and 12 in three patterns of MW=4, 8 and 16px show a range of ~3.5°-9.5°. Fig 10 also indicates that by increasing mortar width from 4px to 32px, there will be an increase of the detected tilt angles from approximately 3.3° in the MW=4px pattern to roughly 5.8° in the MW=32px variation. This supports previous findings that in Café Wall patterns with thick mortar, the inducing bands of Twisted Cords appear at steeper angle to the horizontal compared to thinner mortar [70]. There are no horizontal lines detected at the medium to coarse scales; for MW=4px pattern there is none after $\sigma_c = 16$, for MW=8px there is none after $\sigma_c = 20$, and for MW=16px there is none after $\sigma_c = 24$. The maximum mean tilts of ~14° in the patterns of MW=32 and 64px cannot be detected for the thinner mortar variations (MW≤16px). The vertical and diagonal deviations of the detected houghlines are nearly the same in these variations. The near vertical tilts is about ~7°-8.5°, while the range of diagonal tilts is between ~11.8°-13.8° for MW=4, 8 and 16px at coarse scales ($\sigma_c = 20, 24, 28$).

Now let concentrate on the thickest mortar variation in our samples (MW=64px). Since the mortar size is huge compared to the other variations, the mortar cues still exist at the specified coarsest scale ($\sigma_c = 28$) of the DoG edge map. At scale 28, the DoG captures a complete tile, and houghlines results show that near horizontal lines can be detected until the coarsest scale. There is no horizontal lines detected at the finest scale ($\sigma_c = 4$), and the range of horizontal mean tilts is between ~10.5°-17° along different scales. The mean tilts in this pattern (MW=64px) is more than ~5° larger than the range of horizontal mean tilts in the thinner mortar variations (which is ~3.5°-9.5° in MW=4 and 8px patterns and ~5°-14° in MW=16 and 32px patterns). When we compare the tilt effect in this pattern with the thinner mortar variations such as MW=8px pattern, we see that the tilt effect is very weak here (due to a weak persistence of the mortar cues in relation to the width of the mortar-Fig 11), but the predicted tilts show a strong tilt effect. We will investigate these very thick mortar variations further in the next Section to explain our quantitative tilt results. The diagonal tilts in this pattern get stable around 12.8° at scale $\sigma_c = 28$. The only scale with detected houghlines around the vertical orientation is at $\sigma_c = 12$ with the mean tilt of ~20° which is misleading and again larger than the deviations of the other variations. As Fig 9 shows, the zigzag vertical grouping of tiles did not occur, not even at scale 28 in this pattern.

### 3.2.3 Very Thick mortar variations

In the experiments reported so far in our study, we have assumed the common hypothesis that for detection of near horizontal tilted line segments along mortar lines or the appearance of Twisted Cord elements in the literature, the DoG scale ($\sigma_c$) should be close to mortar size ($\sigma_c \sim M$) [6, 20-22, 70].

We show now that this is not precisely true when mortar size exceeds 16px in our samples (the Café Wall stimuli have tiles of 200×200px). For two patterns of MW=32 and 64px this hypothesis is not valid. We show here that the mortar cues are completely disappeared in the DoG edge maps of these patterns at scales much smaller than the mortar size.

In the previous experiment, we have used a fixed number of scales for the DoGs (seven scales) for all of the Mortar-Width variations (Section 3.2.2). The MW=8px pattern was selected as a base to specify the range of scales as $\sigma_c = 4, 8, 12, 16, 20, 24, 28$ for all these variations to detect both mortar and tiles.

For very thick mortar variations (MW=32 and 64px), we found that in the defined range of scales for the DoGs, the mortar cues still exist at the coarsest scale ($\sigma_c = 28$). In addition, we have detected brightness induction in the DoG edge maps of these variations, much stronger than patterns with thinner mortar lines although the perception of tilt in these variations are very weak. The mean tilts pre-
sented in Fig 10 show overestimated tilt angles with strong tilts in these variations despite of our weakly perceived tilts.

Since the tile sizes are the same for all samples (200×200px), then the coarse scales for detecting them should be similar in size. The DoG edge maps of patterns MW=16, 32 and 64px at seven scales have been shown in the jetwhite colour map in Fig 7 and in binary form in Fig 9. As indicated in Fig 9 for the MW=64px pattern, we have mortar cues at the coarsest scale (σc =28) and a large deviation from the horizontal at this scale along the mortar lines. The zigzag vertical grouping of tiles which appeared clearly at coarse scales of the DoG edge maps for the MW=16 and 32px patterns, are not shown for the MW=64px pattern in the predefined range of DoG scales. So we have examined a few scales above 28 for these patterns, and gathered some of the important results in Fig 11. The figure shows the DoG edge maps at eight different scales (σc =8, 12, 16, 20, 24, 28, 32, 48) for thick mortar patterns. There is a different gap from scale 32 to 48 compared to incremental steps of 4 between the rest of the scales, since we deliberately wanted to show the DoG outputs at scale 48 specifically for the MW=64px pattern. Fig 11 shows that the edge maps of the thick mortar patterns (MW=16, 32 and 64px) are very similar after scale 32, when the tiles are extracted completely. We see a change of groupings of tiles from the near horizontal to the zigzag vertical at scale 24 for the MW=16px pattern, and at scale 32 for the MW=32px pattern.

The other thing worth mentioning is that the mortar lines are detectable at scale 16 (σc=M) for MW=16px pattern, but not at scale 32 (σc=M) for MW=32px pattern (they are detected at the previous scale; σc =24). There are no mortar cues available at scale σc=64=M with MW=64px. In the edge map, the mortar cues exist till scale 32 (σc =32) and then disappear at coarse scales, with a filter size of nearly half of the size of the mortar lines! This might be an indication for a very weak tilt effect, if there is any, for the MW=32 and 64px patterns compared to the MW=16px variation.

If we look back to the MW=8px pattern (Fig 8-right column), we see the mortar cues are not only detectable at scale 8 (σc=M), but also at the following two scales (σc=12, 16). Comparing two variations of MW=8px with MW=16px (Fig 9-left column), we see that for MW=16px pattern, the mortar cues nearly disappear in just one scale after the mortar size (at σc=20) except for very small dots in their place. The persistence of the mortar cues in the edge map of this pattern is not as strong as the MW=8px pattern which last for a few consecutive scales larger than the mortar size (σc=12, 16). For the very thin mortar (MW=4px), the edge map (Fig 8-center column) shows a persistent mortar cues from the finest scale (σc =4=M) till scale 12, similar to the MW=8px pattern. So the tilts predicted by the model for very thin mortar variations (MW=4px and 8px) show the strongest tilt effects.

We argue here that the persistence of mortar cues plays a major role in determining how strongly we perceive the induced tilt in the Café Wall illusion. Our model seems to be a good fit to our biological understanding of the early stages of vision. Quantitative predictions of tilt at multiple scales for a wide range of conditions, and specifically the predictions of the strength of illusion across multiple scales as conditions vary. These quantified results have been shown here for the first time by our predictive model and the DoG edge map representations at multiple scales.

3.2.4 Discussion

The DoG edge maps at multiple scales for the variation of no mortar (MW=0px) indicates the same non-illusory percepts like the previous reports by others [5, 14]. Our DoG edge map representation
supports the previous findings that the strongest tilt effect in the Café Wall illusion occurs with the thin mortar lines ($MW=4$ and $8px$ in our samples). Also the multiple scale edge map nicely unveils the underlying cues involved in thick mortar variations of the Café Wall illusion, and indicates how the tilt effect degrades here while the brightness induction increases in these patterns. The brightness induction that is referred to as *Enlarged Twisted Cord* in this work, previously reported with different names such as *Reversed Twisted Cord* and *Twisted Rope* in the literature [10, 70].

### 3.3 Other Variations of the Pattern

#### 3.3.1 Grey Tiles variations

**Patterns Characteristics**

*Grey-Tiles* are variations of Café Wall patterns with lower contrasted tiles, in which instead of the *Black* and *White* tiles with maximum contrast, the tiles here are two shades of *Grey* (with the relative luminance of tiles equal to 0.25 for *Dark-Grey* and 0.75 for *Light-Grey*). So in these variations the luminance contrast between the tiles are half of the luminance contrast of the original Café Wall pattern with the *Black* and *White* tiles. The mortar lines have one of the three levels of luminance here, either below, between, or above the luminance of both of the Grey tiles selected as $ML=0$ (*Black*), 0.5 (*Mid-Grey*), and 1.0 (*White*), which have been presented in Fig 4-bottom.

#### 3.3.2 DoG Edge Maps and Quantitative Tilt Results

The binary edge maps at seven scales for the three variations of low contrasted tiles (*Grey-Tiles*) have been presented in Fig 12, in which the edge maps have been overlayed by the detected houghlines displayed in green. The scale of the DoGs are similar to our previous investigations in Sections 3.1 and 3.2. For easier comparison of the detected line segments in the edge maps of these variations, the edge map of the original Café Wall with *Black* and *White* tiles and mortar luminance of 0.5 has been provided in the left column of Fig 13.

By comparing the edge map of the intermediate mortar luminance ($ML=0.5$) in the variations of *Grey-Tiles* (Fig 12-center column) with the original Café Wall pattern (Fig 13-left column), we see very similar tilt cues along the mortar lines at multiple scales of the DoGs (However there are some border effects revealed around the variations of the low contrasted tiles, that are not present in the
from the ratio Threshold=...ters are kept the same in all...or diagonal tilting lines. NumPeaks=1000, Threshold=3, FillGap=40, and MinLength=450 for all DoG scales. Other parameters of the DoG model are s=2 (Surround ratio), and h=8 (Window ratio). The last row shows each variation of the pattern investigated (Reproduced with permission from [72]).

Detected houghlines have been shown in Fig 12 displayed in green on the DoG edge maps at multiple scales (σ =4, 8, 12, 16, 20, 24, 28) for the variations of Grey-Tiles. The hough parameters are kept the same as the previous experiments (NumPeaks=1000, Threshold=3, FillGap=40, and MinLength=450 in Sections 3.1 and 3.2). The absolute mean tilts and the standard errors of the low contrasted tile patterns (Grey-Tiles) have been summarized in Fig 15-top. The absolute mean tilts and the standard errors of calculation for the original Café Wall have been provided in the figure for easier comparison of tilt results of other variations with this pattern, at the bottom-left corner of the figure (This is the result from Fig 6 for ML=0.5 or Fig 10 for MW=8px. The original Café Wall has shown the maximum detected tilt range among the patterns investigated in this study).

Comparing the mean tilt results of the Black mortar pattern (ML=0.0) from the variations of Grey-tiles with the original Café Wall pattern in Fig 15 shows that the near horizontal mean tilts are much lower than the original Café Wall, and the rest of the low contrasted tile patterns (ML=0.5, and ML=1.0). As noted in Section 2.3.1 (p. 9) the most appropriate filter size for investigation of the horizontal inducing tilts for mortar size of 8px is the finest scale (σ=4). It is less than 1° for ML=0, 1 (mortar of Black and White) and ~2° for Mid-Grey mortar. The tilts predicted by the model for the Grey-Tiles are weaker than the other variations shown in Fig 15 with high contrasted tiles of Black and White tiles at scales σ =4, 8. At scale 8, it is ~2° for the Black (ML=0.0) and White (ML=1.0) mortar patterns and ~4.5° for the Mid-Grey (ML=0.5) variation. Even for the ML=0.5 version, the mean tilts are nearly 2° smaller than the detected mean tilts in the original Café Wall pattern at scales 8 and 12 (~4.5° to 7.5° compared to ~6.6° to 9.5°). The predicted results indicate for a weak tilt effect in the variations of Grey-Tiles. Gregory and Heard note that the effect depends critically on the luminance and that it disappears if the mortar line is either brighter than the light tiles, or dimmer than the dark ones [4], and our results support these psychophysical findings.

Also at coarse scale (σ =24), our results show the near horizontal tilts with ~2° in the variations of Grey-Tiles compared to none in the original Café Wall pattern. Checking the houghlines in Fig 12 shows that these detected lines are the artefacts of border lines which are out of the interest.

The diagonal mean tilts in the Grey-Tiles are approximately 2° smaller in the ML=0.5 (Mid-Grey) pattern, which is ~11.5°-12° compared to ~12.5°-13.5° in the original Café Wall. So both of the horizontal mean tilts at fine scales and the diagonal mean tilts at coarse scales indicate higher range of detected tilts in the original Café Wall pattern compared to the variations of low contrasted tiles (Grey-Tiles).
3.3.1.3 Discussion

We have shown here that although the illusion persists in the variation of Grey-Tiles with an intermediate mortar luminance between the luminance of both Grey tiles (ML=0.5 pattern), but the illusion strength is not as high as the original Café Wall pattern, which is consistent with previous reports in the literature. It has been given in the literature that the strength of illusion in low contrasted tile variations of the pattern is less than the original Café Wall with high contrasted tiles [4, 6]. Also Café Walls with lower contrasted tiles need thinner mortar lines to have the same degree of apparent convergence/divergence in the illusion percept [4], not yet tested by our model (But for Mortar-Width variations with contrast luminance of one unit for the tiles we have illustrated that patterns with thinner mortar lines produce wider range of tilt angles indicating stronger tilt effect-Section 3.2).

3.3.2 Phase displacement effect on detected tilt

3.3.2.1 Patterns Characteristics

Two more patterns are generated based on two different phase of tile displacement, which is the amount of tile shift between consecutive rows in the Café Wall pattern. One of these patterns has a phase shift of 1/3 (shift of 1/3rd of a tile) and the other one a phase shift of 1/5, and both displayed in Fig 4-bottom.

3.3.2.2 DoG Edge Maps and Quantitative Tilt Results

The results of the binary DoG edge maps at seven scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$) for the two variations of phase shifts of 1/3 and 1/5 (Fig 4-bottom) have been presented in Fig 13, in which the edge maps have been overlayed by the detected houghlines displayed in green. Comparing the edge maps of these patterns with the edge map of the original Café Wall pattern on the left column of Fig 13 shows that the tilt cues are much weaker across multiple scales of the edge maps in the tile shift variations. For the pattern with phase shift of 1/5 (shift=1/5 of a tile), we see the weakest tilt effect and the near horizontal tilts (Fraser Cord elements) reveal only at the finest scales ($\sigma_c=4, 8$). The inducing near horizontal tilts along the mortar lines exist till scale 12 ($\sigma_c=4, 8, 12$) for the phase shift of 1/3 (shift=1/3 of a tile). But, as indicated for the original Café Wall pattern with the shift=1/2 of a tile between the consecutive rows, the mortar cues last till scale 16. The results show again that the persistence of mortar cues in the edge map representation at multiple scales is an indication for the strength of the tilt effect in the Café Wall illusion.

Detected houghlines have been shown in Fig 13 for the DoG edge maps at seven scales for the two variations of phase shift displacement. The hough parameters are the same as the previous experiments (NumPeaks=1000, Threshold=3, FillGap=40, and MinLength=450).

The absolute mean tilts and the standard errors of detected tilts for the two patterns of phase of tile displacement are shown at the bottom of Fig 15. Comparing these results with the mean tilt results of the original Café wall at the bottom-left corner reveals that in small shift of 1/5 of a tile, there is no near horizontal lines detected along the mortar at the fine scales. There are a few near horizontal lines detected at scales 12 and 16 which is much larger than the original Café Wall pattern ($\sim 15.5^\circ-18.9^\circ$ compared to $\sim 9.5^\circ-12^\circ$). The scales of 12 and 16 are much larger than the mortar size ($M=8px$) and the predicted results are not reliable at these scales for the mortar size of 8px (see Section 2.3.1). Fig 12 with the details of detected houghlines also support these. When we compare the phase shift of 1/3 with the original Café Wall pattern, we still see a lower range of mean tilts which is $\sim 4^\circ-9.5^\circ$ compared to $\sim 3.5^\circ-12^\circ$ at the range of fine scales ($\sigma_c=4, 8, 12$).
The vertical mean tilts is much smaller in the pattern with phase shift of 1/5 around 2.5º-3.5º at different scales. It is slightly higher in the phase shift of 1/3 (~4º-6º; recall that for the near vertical and diagonal tilts, we should consider coarse scale DoGs in our model; σ,>16). Checking the edge maps in Fig 13 shows that the vertical grouping of tiles appear at scale 8 in the two variations of tile shift displacement compared to scale 12 and even 16 in the original Café Wall pattern. This is a good indication for a weaker tilt effect along the mortar lines in these variations compared to the original Café Wall because; the tilt cues are more persistent along mortar lines at fine to medium scales in the original Café Wall pattern. In addition, we see sharper vertical lines with less deviation from the vertical in the phase shift of 1/5, highlighting the vertical grouping of tiles that emerges at smaller scales. The near diagonal mean tilts for these two patterns (phase shifts of 1/3 and 1/5) are nearly more than 3º lower than the predicted tilt ranges along the diagonal axes compared to the original Café Wall (~9.5º compared to ~12.5º-13.5º).

### 3.3.2 Discussion

The tilts predicted by the model show higher range of detected mean tilts near the horizontal for the original Café Wall pattern compared to these variations of phase of tile displacement (phase shifts of 1/3 and 1/5). Our results are consistent with previous reports that the tilt effect is maximal with phase shift of half cycle between consecutive rows and minimal or no distortion when it is in a checkerboard phase [5].

### 3.3.3 Tilt effect direction and Hollow Square pattern

#### 3.3.3.1 Patterns Characteristics

The mirrored image of the original Café wall pattern is shown at the bottom of Fig 4 (which is the mirrored image of either ML=0.5 in the Mortar-Luminance variations, or MW=8px in the Mortar-Width variations, shown with * in the figure) results in an opposite direction of inducing tilt. This version may be referred to as Direction Change variation of the Café Wall pattern in the text. The final pattern we consider is the Hollow Square pattern [9, 11], which consist of hollow tiles with the same size of the Café Wall tiles. The outlines of tiles are thinner than the mortar size but the border thickness of these hollow tiles produce roughly similar size to the mortar size of 8px in the Café Wall pattern, since two hollow tiles are adjacent to each other. If the outlines of the hollow squares are thickened in this version, we ultimately reach to a similar pattern to the Café wall without any mortar lines [11]. These two patterns are presented at the bottom of Fig 4.

#### 3.3.3.2 DoG Edge Maps and Quantitative Tilt Results

The results of the binary DoG edge maps at seven scales for two patterns of the mirrored image, inducing opposite direction of tilt, and the Hollow Square pattern have been displayed in Fig 14, in which the edge maps have been overlayed by the detected houghlines displayed in green. The scales of the DoGs are similar to our previous investigations in Sections 3.1 and 3.2. For easier comparison, we have provided again the DoG edge map of the original Café Wall pattern in the figure. The DoG edge map of the Direction Change variation shows exactly the same tilt cues at multiple scales of the DoGs, but with the opposite direction for convergence/divergence along the mortar lines, supporting previous finding that the tilt effect reverses “when alternate rows of tiles are pushed across half a cycle”. [4].

The edge map of the Hollow Square pattern, detect two edges for each side of the hollow tiles in the pattern–inner and outer region. The edge map is completely different from the rest of the Café Wall variations, but some similarities can be found with the MW=0px pattern in Fig 8 and especially
at the finest scales ($\sigma_c=4, 8$) which are the most appropriate scales in the model for predicting the tilts around the horizontal (Section 2.3.1). What we see in the edge map at fine scales are grouping of tiles in vertical-zigzag direction by the high frequency details of the edges. We have seen these vertical grouping of tiles at coarse scales in most of the variations of the Café Wall pattern investigated. There are no near horizontal tilts in the place of mortar (connecting section of rows of hollow tiles) in this pattern at fine scales. Based on the quantitative tilt results we can claim that there is no illusion in this pattern.

Detected houghlines have been shown in Fig 14 for the DoG edge maps at multiple scales for the mirrored image of the Café Wall (Direction Change) and the Hollow Square pattern. The hough parameters are the same as the previous experiments ($\text{NumPeaks}=1000$, $\text{Threshold}=3$, $\text{FillGap}=40$, and $\text{MinLength}=450$).

The mean tilts and the standard errors of detected tilt angles of houghlines have been presented in Fig 16. In the Direction Change pattern, the near horizontal tilt range is quite similar to the original Café Wall pattern ranging from $\sim 3.5^\circ$ to $12^\circ$. The mean tilts along the vertical and diagonal orientations are again very similar to the original Café Wall pattern. Slight changes less than a degree ($<1^\circ$) are in the acceptable mean tilt range due to the standard errors around $0.6^\circ$-$0.7^\circ$ in the original Café Wall (Fig 15), and this error indicates that the results are statistically very close to each other in these two variations. This is what we have expected, and we have shown that the tilt effect has an opposite direction of divergence/convergence tilts in the detected houghlines (Fig 14).

![Fig 15. Top: Mean tilts and the standard errors of detected tilt angles of houghlines for three variations of Grey-Tiles (with relative mortar luminance of Black-Left, Mid-Grey-Center and White-Right) mortar lines for tiles with luminance of 0.25 for Dark Grey, and 0.75 for Light Grey). Bottom: Mean tilts and the standard errors of detected tilt range for three patterns of the original Café Wall pattern on Left, and two variations of phase of tile displacement with the shifts of 1/3 tile in the Center, and 1/5 tile in the Right. The calculations are done for the edge maps at multiple scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$) – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines (H, V, D1, D2) in the edge maps. $\text{NumPeaks}=1000$, $\text{Threshold}=3$, $\text{FillGap}=40$, and $\text{MinLength}=450$ for all DoG scales. NaN means no detected lines (Reproduced with permission from [72]).](image1)

![Fig 16. Left: Mean tilts and the standard errors of detected tilt angles of houghlines for the mirrored image (Direction Change), and Right for the Hollow Square pattern. The calculations are done for the edge maps at multiple scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$) – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines (H, V, D1, D2) in the edge maps. $\text{NumPeaks}=1000$, $\text{Threshold}=3$, $\text{FillGap}=40$, and $\text{MinLength}=450$ for all DoG scales. NaN means no detected lines (Reproduced with permission from [72]).](image2)

The last pattern we consider is the Hollow Square. The mean tilts and the standard errors of detected tilt angles of houghlines have been shown in Fig 16-right. Comparing the near horizontal mean tilts of this pattern with the original Café Wall pattern at the finest scale ($\sigma_c=4$) shows that the tilt angles detected are negligible ($<1^\circ$) here compared to $3.5^\circ$ in the original Café Wall pattern. There is a similar mean tilt $\sim 6^\circ$ (compared to $\sim 6.6^\circ$ at scale 8). But what is important to be considered is that although the detected lines have mean tilts deviation around $6^\circ$, the lines have both positive and negative orientations along each mortar position at $\sigma_c=8$ (connecting of hollow tiles in rows) compared to a single tilt orientation (either positive or negative tilt) for the detected lines along each mortar in the original Café Wall pattern (based on the detected houghlines in Fig 13). In real vision, at this transition resolution, these contradictory tilts tend to cancel each other and result in lower tilt range than the predicted tilt results ($\sim 6^\circ$). Based on the Hough parameters that kept the same for the tilt analysis of the patterns investigated, we found much smaller tilt deviation from the diagonal axes, which is $\sim 3.5^\circ$-$6^\circ$, compared to $13^\circ$ in the original Café Wall pattern. Similar to the diagonal mean tilts, we see again smaller deviation from the vertical axis for the detected lines which is $2^\circ$-$5^\circ$ compared to $6.5^\circ$-$9.5^\circ$ in
the original Café Wall pattern. So the model has not detected any considerable tilt angle in the Hollow Square pattern in.

3.3.3.3 Discussion

We have shown here that lower contrasted tile (Grey-Tiles) patterns compared to the contrast of unity in the original Café Wall pattern have lower range of detected mean tilts, indicating weaker tilt illusion compared to the Black and White tiles in the original Café Wall pattern. Among the three different variations of Grey-Tiles we have shown that the mortar luminance should be in the intermediate range for a strong tilt effect. It has suggested that the decrease of mortar width in low contrasted tile variations of the Café Wall pattern enhances the tilt effect [4]. This has not yet tested by our model. Also our quantitative mean tilt results showed the maximum near horizontal mean tilts with half tile shift (for the original Café Wall) and that the effect is diminished close to a chessboard phase. Our results also show an opposite direction for the illusory tilt in the mirrored image of Café wall variation (half cycle shift of the Café Wall pattern), as well as a very weak tilt effect in the Hollow Square pattern (the variation tested here). It has been reported that the decrease in contrast reduces the apparent tilt of Hollow Square illusion [9] that has not yet tested by our model. In the next section we explain how to verify the tilts predicted by the model in a testable framework connecting the scale of the edge maps to the differences of resolution to the eccentricity and visual angle.

3.4 Outlines of Psychophysical Experiments

In this section we aim to make it clear about what testable predictions the model makes. We show that the tilt prediction by the model for the given stimuli models matching individual foveal and peripheral acuity/eccentricity (at very fine scales) as well as the relative distance/visual angle viewed (for medium to coarse scales). So the model matches a single prediction for each stimulus considering its acuity/eccentricity/distance setting. Also here we outline the critical factors and essential parameters that should be considered in our later psychophysical experiments to validate the predicted results.

The eyes process the scene at different scales at different times, due to the intentional and unintentional eye movements (such as overt saccades and gaze shifts) while we look at the scene/pattern. These result in a rapid scan of the field of view by the fovea for encoding of the high-resolution information. Our perception of illusory tilt in the Café Wall is affected by our fixation on the pattern and the tilt effect is weakened in a region around fixation point, but the peripheral tilts stay unaffected inducing stronger tilts. So in psychophysical experiments we should consider perceptual factors such as foveal/peripheral view to the stimulus to measure the induced tilt in Café Wall illusion.

In the psychophysical experiments we thus need to identify and formalize the effective pixel resolution (how many pixels within a subtended visual angle and the corresponding physical area on a viewer’s retina). Issues related to the design of effective visual experiments include the resolution, visual angle, viewing parameters, the speed of presenting a stimuli, techniques to map data values to their corresponding visual representations and stimulus features and data properties.

The physical measurement of the subtended visual angle (\(\nu\)) can be calculated based on the physical size of an element/object in our field of view (\(S\)) and its viewing distance (\(D\)) as \(\nu = 2 \tan^{-1} \frac{S}{2D}\) or \(\tan \nu = \frac{S}{D}\). Considering the mechanics of the eye, the visual angle can be also calculated based on the size of retinal image (\(R\)) and the distance from the nodal points to the retina (\(n \sim 17\text{mm}\)) as \(\tan \nu = \frac{S}{D} = \frac{R}{n}\), that results in the retinal image of size \(R = 17 \times \frac{S}{D}\text{mm}\). Therefore when 1cm tile is viewed at 0.5m distance then the subtended visual angle of a tile is \(\nu = 0.02\text{rad} = 0.02 \times 57^\circ = 1.14^\circ \sim 1^\circ\).

All the tilt analysis done in our simulations have been considered the resolutions of MATLAB gen-
erated stimuli/patterns, then we should make sure that the pixel sizes in MATLAB that is the image matrix dimensions are displayed with its exact size on the display (monitor) by an equivalent size pixel representation. This will be guaranteed by using truesize function in MATLAB. Note that this is not in general the same as the pixel size of the monitor and is the basis for sizing of image features and thus the definition of pixel that needs to be related to retinal cone density and ocular acuity [73-75].

The limits on visualizations information have been hypothesised by Healey and Sawant nothing that: “a minimum number of pixels ’resolution’ and subtended physical area on the retina ‘visual angle’ are required to perceive different values of a visual feature” [76:pp.2]. They have identified the boundaries for distinguishing different values of luminance, hue, size and orientation. Similar investigations have been reported by other researchers for instance Legge et al. [77, 78] investigate documents thresholds and text legibility for viewer’s readability considering visual features like contrast, size, color and luminance.

The limit of visual acuity for a ‘point acuity’ to resolve two point targets is noted to be 1' and for ‘grating acuity’ to distinguish bright and dark bars in a uniform grey patch is measured in the range of 1-2', while for instance the ‘letter acuity’ to resolve letters in 20/20 vision (5 arc min letters to be seen with 90% accuracy) is 5' and that most visual acuity fall within the 1-2' range corresponding to the center of the fovea [76]. They conclude that: “resolution and visual angle limits depends on feature values being distinguishable from one another, and not on the specific values being displayed” ([76]:pp.14). Rather than the related parameters to the measurement of the visual angle we also need to consider the visual condition of the subjects (for instance subjects with 20/20 visual acuity and above, in a certain age group such as 20-40 years) as well as viewing conditions such as free eye movements for experiments.

A convincing way to demonstrate how our model prediction is going to be tested on real subjects in our future psychophysical experiments is to present the stimulus along with tilts predicted by the model next to each other, with viewers’ task to select the closest possible tilted lines to the perceived tilt in the stimulus. Certainly for a reliable measurement, the presentation framework plays an important role, and in our design we need to make sure we have eliminated any possible side effect of the presentation on the strength of the illusion.

We should note here that in our tilt analysis, we have estimated the mean tilt angles in the DoG edge maps of the stimuli/patterns at seven different scales around four reference orientations of Horizontal, Vertical, and Diagonals (H, V, D; from \( \sigma_c = 0.5M \) to 3.5m with incremental steps of 0.5M; M: mortar size). The edge map at multiple scales which consists of the DoG filtered outputs from fine to coarse scales indicate how our distance from the pattern may affects the tilt or in other words how the perceived tilt is related to the pattern’s visual angle as a whole, and also its elementary parts/figures (mortar and tiles in Café Wall pattern). We believe that the relationship between \( \sigma_c \) in the model and the mortar size is more important than the actual pixel size.

The model uses normalized Gaussians and the curve between the center and surround Gaussians in the DoG filter shows the activation response. The surround Gaussian intersects the center Gaussian at its inflection points (for the 2.0 ratio). Therefore, \( \sigma_c =4 \) in our model corresponds to a filter in which the mortar size of 8px lies within one standard deviation (\( \sigma_c \)) away from the mean of the filter, so we are able to detect the mortar lines with high accuracy in the filtered response as noted previously in Section 2.3.1. Therefore, the tilts predicted by the model at \( \sigma_c =4 \) in our results show the prediction of tilt angles in the foveal vision with high acuity in the center of our current gaze direction. For a fixed size stimulus, the results reported for each scale indicate how the distance affects our tilt perception.
At $\sigma_c=8$ the results show particular distance and so many degree of eccentricity in a local neighbourhood around our fixation points. The mean tilts calculated for larger scales than 8 ($\sigma > 8$; 8 is the Mortar size), do not be considered as predicted tilt angles near the horizontal for the illusion.

Therefore, relying on the tilts predicted by the model presented in Figs 6, 10, 15 and 16, the near horizontal tilts at scales 4, and maximum at scale 8 ($\sigma_c=-4, 8$) are the predicted tilt angles corresponds to the foveal vision of the pattern. These are the appropriate scales to detect the tilt cues in the Café Walls explored. We encounter the disappearance of mortar lines in the pattern as the viewing distance from the stimulus increases (equivalent to decreasing its visual angle). The near vertical and diagonal (V, D) tilts in the predicted results will be then considered for further analysis of tilts (at coarse scales).

All the stimuli used in this study have a same size of 3×8 tiles (with tiles of 200×200px) and an overall size of ~1600×616px (note that for Mortar Width variations the height of the image will have a moderate change). Here we concentrate on the Café Wall of 3×8 with tiles of 200×200px, mortar size of 8px and the intermediate mortar luminance between the luminance of the black and white tiles ($ML=0.5$; Fig 4-top section with *; the resolutions of MATLAB generated pattern) with five copies presented in the left of Fig 17. As indicated before, $\sigma_c=4$ in our model corresponds to the prediction of tilt angles in the foveal vision with high acuity. As indicated in Fig 6, the predicted tilts near the horizontal at scale 4 ($\sigma_c=4$) is $3.5^\circ\pm0.5$ which covers a tilt range between 3-4°.

To let readers compare the predicted tilt results with their own subjective experience of tilt in this stimulus we have presented tilted line segments with actual angles in the range of ±2.5° to ±4.5° with 0.5° difference in between, and positioned them adjacent to the mortar lines of the stimulus in Fig 17. These tilted lines have a very close characteristics to the mortar lines (with a similar size and luminance) and we have overlayed them on top of black backgrounds with blurry outlines at the edges to make the appearance of these line segments as close as possible to the inducing tilt along the mortar lines. The length of the line segments are close to a tile size to prevent any misjudgement of long tilted intersecting lines that may eliminate the inducing tilts in the illusion. In the middle of the figure we have presented the convergent/divergent line segments with magnitudes of 3°, 3.5° and 4° that are the detected range of tilt angles by the model for the stimulus. If you test your perceived tilt on the stimulus presented in Fig 17 with the detected tilt range on the left, you will find that the model prediction are closely matched to our perception of tilt. Psychophysical assessments are required to show this in more details as the future priority of our study.

![Fig 17. Left: five copies of Café Wall of 3×8 tiles with the intermediate mortar luminance between the luminance of the black and white tiles. The resolutions of MATLAB generated pattern is the tile sizes of 200×200px and mortar size of 8px ($ML=0.5$; Fig 4-top section with *). Right: Convergent/divergent line segments in the range of ±2.5° to ±4.5° with 0.5° difference in between. In the middle of the figure the tilted lines indicate the detected range of tilt angles by the model for the stimulus at scale 4 ($\sigma_c=4; 3.5^\circ\pm0.5$) with magnitudes of 3°, 3.5° and 4° around the horizontal (mean tilt table for $ML=0.5$ in Fig. 6).](image-url)

We may perceive the illusory tilt of the mortar lines with some moderate changes among individuals for instance some people may see mortar lines as being more bended/curved than being tilted. The position of the tilted line segments at one end of the mortar lines facilitate the integration of the Fraser Cord elements appeared in our local view to an integrated long continuous contour in our global view.

Note that we have not done any local sampling of the stimuli in this work for tilt investigations. The configurations used as the stimuli in this research (Fig 4) are small Café Wall configurations (Café Walls of 3×8 tiles). The tilt predictions of the model for variations of different sampling sizes for simulating foveal and peripheral vision and investigating the inducing tilt range in these samples have been reported in other research article [71]. We observe that the model predicts tilt angles close to our judgement of tilt across the different configurations of the pattern.
For larger configurations of Café Wall with the same tile size and mortar as the stimulus in Fig 17, the tilt effect appears to be a bit stronger when comparing Fig 17 with Fig 18. A Café Wall of 11×11 tiles (tiles of 200×200px, mortar of 8px, and the same mortar luminance as the stimulus in Fig 17) have been presented in Fig 18. The tilts predicted by the model on different variations of Café Wall pattern with different configurations of tiles in the rows and columns can be found in [71]. The predicted tilt for this stimulus at scale 4 (σc=4) around the horizontal is 4.05°±0.09 ([71]: Fig. 14) that results for tilt angles between 3.96° and 4.14° in which we have rounded them to ~4.0° (rounding to the nearest unit) and have shown ±4° line segments next to the stimulus at the right end points of the mortar lines and adjacent to them for evaluating our/readers perceived tilts with the predicted results (the predicted mean tilt is 3.5°±0.5 at σc=4 for the stimulus in Fig 17 stimulus; from Fig 6). In psychophysical experiments, to validate the results with high accuracy, we might go for presenting only one pair of these lines (convergent/divergent lines) or even one of these lines instead of presenting them all next to the stimulus as shown in Figs 17 and 18. Again if you test your perceived tilt in Fig 18 you find that the predicted mean tilt seems to closely match with our perceived illusory tilt in the stimulus. Now let’s check the effect of visual angle on the perceived tilt.

The effect of the visual angle on the perceived tilt in Café Wall illusion (Café Wall of 11×11 tiles; the resolutions of MATLAB generated pattern is the tile sizes of 200×200px and mortar size of 8px the same as the stimulus in Fig 18). The size of the stimulus is adjusted in these images in a way that the image in the Center has a size reduction of 25% from the image in the Left, and the image in the Right has a size reduction of 50% from the image in the Left.

The effect of the visual angle on the perceived illusory tilt can be checked in Fig 19 for Café Wall of 11×11 tiles. Although the pattern is generated with tiles of 200×200px and mortar of 8px as its original size in MATLAB, but the size of the pattern (fixed aspect ratio of width to height) are adjusted to cover widths of 236px, 177px and 118px in total as shown in the Fig 19 from left to right. The center image has a width reduction of 1/4th from the width (W) of the image in the left (larger image) (W'=3/4×W=3/4×236=177px), and a second width reduction of 1/4th from W has been applied to adjust the size (resolution) of the image in the right (W''=2/4×W=2/4×236=118px). We have used Visio software to generate these copies of Café Wall stimulus considering the exact relative size noted above. This individual images may appear with different resolutions on different monitors/printers or when we change the scale of the displayed image (by resizing it), but the size ratios between these images stay the same and this is what we want to test here (relative visual angle stays constant). The aim is to show that there is a threshold value relative to our eyes sensitivity to the spatial frequency of image/pattern and by reducing the size of the Café Wall stimulus and approaching to that threshold, we encounter with a stronger inducing tilt. Fig 20 demonstrates this with two sample sizes.

In Fig 20 we have shown tilt angles of ±4° as the tilts predicted by the model for this pattern at σc=4 [71: Fig. 14] presented next to the stimulus on the right of the figure (with wider visual angle), and one degree more than the predicted tilts equal to ±5° for a smaller sized image on the left (with narrower visual angle). This can be tested on a digital copy of stimulus on the screen and changing the size of the displayed stimulus to a smaller size. This notified threshold can be further investigated in our later psychophysical experiments in order to find the relationship between the strength of inducing tilt in the illusion, the visual angle of the stimulus as whole, and the visual angle of its elementary
The estimation noted above is consistent with our understanding of tilt based on our foveal/peripheral vision. For larger patterns, when we foveate at the center of the image (facilitated with a red cross in Fig 18), we see a very weak tilt in this vicinity, but in the periphery the induced tilt is much stronger. When the visual angle decreases (such as for smaller pattern on the left of Fig 20), again close to the focal point, the illusory tilt is weak but around the fovea, we are able to capture more tiles in the periphery with more inducing tilt cues on the same retinal area. The strength in our perceived tilt may be affected by cues related to the illusion strength such as activations of more retinal GCs, evident in our model as the persistence of mortar cues across multiple scales in the edge map, or from detecting a larger inducing tilt angle in smaller sized patterns. This should be tested in detail by psychophysical experiments.

To explain how this can be addressed by our model it should be noted that we have considered $\sigma_c = 4$ to evaluate the tilts predicted by the model in this section so far. Based on the predicted results and our understanding of foveal/peripheral vision we expect that for the range of scales tested in our model (related to the pattern characteristics), the mean tilts reported for one scale larger than the finest scale, at scale 8 ($\sigma_c = 8$), corresponds to some neighbourhood region around the foveal area. This seems to approximate the maximum tilt angle around the horizontal as detected for variations of the visual angles and different configurations for diverse eccentricity of retinal GCs that provide us with the peripheral vision. For this stimulus (Fig 18; based on the resolutions of MATLAB generated pattern corresponds to a specific visual angle) we have measured the mean tilts at scale 8 to be $7.5^\circ \pm 0.33$ around the horizontal ([71]: Fig. 14).

4 Conclusion

It is increasingly clear that information in the visual systems is processed at multiple levels of resolution, perhaps simultaneously, perhaps sequentially in some sense. We examined the capability of a bioplausible vision model, simulating retinal/cortical simple cells to address the illusory percept in variations of Café Wall pattern. Exploring the tilt effect in the Café Wall illusion, we have shown that a simple DoG model of lateral inhibition in retinal/cortical simple cells leads to the emergence of tilt in the pattern. Our model generates an intermediate representation at multiple scales that we refer to as an edge map. For the recognition of a line at a particular angle of tilt, further processing by orientation selective cells in the retina and/or cortex is assumed [14, 15] but we have exploited an image processing pipeline for quantitative measurement of tilt angle using Hough transform in our study.

We have shown in this paper that the DoG edge map not only shows the emergence of tilt in Café Wall illusion, but also can explain different degrees of tilt effect in variations of Café Wall illusion based on their characteristics. The qualitative and quantitative tilt results of the Café Wall variations investigated support previous psychophysical and experimental findings [4, 5, 6, 9, 10, 14, 15, 69, 70] on these stimuli.

The edge map has the ability to show some elementary factors that are involved in our local and global view of the pattern (at least in part represented by foveal and peripheral vision). We believe that the tilt effect in the Café Wall illusion arises from two incompatible groupings of pattern ele-
ments (tiles and mortar) that are present simultaneously as a result of multiscale retinal visual encoding of the pattern. At fine scales, there are grouping of two identically coloured tiles with the mortar line in a near horizontal direction (appearance of Fraser’s Twisted Cord elements in focal view). At coarse scales, when the mortar cues disappear from the edge map, another grouping starts to appear in an orthogonal direction, grouping tiles in a zigzag with a broadly vertical orientation. At medium to coarse scales a swapping of the local groupings of the adjacent identically coloured tiles occurs, that contradict with the grouping of the Twisted Cord elements at fine scales. These two incompatible groupings, along with systematic differences relating to the relative size of Gaussian and pattern scales, result in illusory tilt effects that reflect changes in size and density with eccentricity, predicting the change in illusion effects according to distances from the focal point in the pattern versus distance in the retinal image from the fovea into the periphery. For variations on Café Wall involving very thick mortar, the Enlarged Twisted Cord elements result in weak tilt effects plus a different effect, traditionally explained as brightness induction, where an illusory rope like ‘Reversed Twisted Cord-Twisted Rope’ construct along the mortar is perceived and our DoG model picks up a reversed angle of twist.

We have shown that explanatory models and hypothesis for Café Wall illusion such as the Irradiation, Brightness Induction, and Bandpass filtering appear to share the central mechanism of lateral inhibition that ultimately underlies the tilt effect in this illusory pattern. We further expect that these retinal filter models will prove to play an important role in higher-level models of simulating depth and motion processing. This supports the use of Gaussian Filter and their differences or derivatives in Computer Vision. We also have shown empirically that this model, has a high potential in revealing the underlying mechanism connecting low-level filtering approaches to mid- and high-level explanations such as ‘Anchoring theory’ and ‘Perceptual grouping’ [19].

Although we have covered many of the aspects involved in the illusory tilt perceived in variations of the Café Wall pattern by our model in this work (through relying on the available psychophysical reports in the literature), as well as showing examples of how the predicted tilt by the model matches a single prediction for each stimulus considering its acuity/eccentricity/distance setting as a testable framework for the model, but many things are left to be explored. These include psychophysical experiments as a priority in our future study to confirm the predictions implicit in our results, and are expected to lead us to a more precise multiple scale filtering which is adaptable to the patterns characteristics.

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References

[1] Grossberg S, Todorovic D. Neural dynamics of 1-D and 2-D brightness perception: A unified model of classical and recent phenomena. Perception & psychophysics. 1988;43(3): 241-277.
[2] Eagleman DM. Visual illusions and neurobiology. Nature Reviews Neuroscience. 2001;2(12):920-926.
[3] Changizi MA, Hsieh A, Nijhawan R, Kanai R, Shimojo S. Perceiving the present and a systematization of illusions. Cognitive Science. 2008;32(3):459-503.
[4] Gregory RL, Heard P. Border locking and the Café Wall illusion. Perception. 1979;8(4):365-380.
[5] McCourt ME. Brightness induction and the Café Wall illusion. Perception. 1983;12(2):131-142.
[6] Morgan M, Moulden B. The Münsterberg figure and twisted cords. Vision Research. 1986;26(11):1793-1800.
[7] Kingdom F, Moulden B. A multi-channel approach to brightness coding. Vision research. 1992;32(8):1565-1582.
[8] Pessoa L. Mach bands: How many models are possible? Recent experimental findings and modeling attempts. Vision Research. 1996;36(19):3205-3227.

[9] Woodhouse JM, Taylor S. Further studies of the Café Wall and Hollow Squares illusions. Perception. 1987;16(4):467-471.

[10] Earle DC, Maskell SJ, Fraser cords and reversal of the café wall illusion. PERCEPTION-LONDON. 1993; 22: 383-383.

[11] Bressan P. Revisitation of the family tie between Münsterberg and Taylor-Woodhouse illusions. Perception. 1985;14(5):579-585.

[12] Westheimer G. Irradiation, border location, and the shifted-chessboard pattern. Perception. 2007;36(4):483.

[13] Fraser J. A new visual illusion of direction. British Journal of Psychology. 1904-1920. 1908;2(3):307-320.

[14] Moulden B, Renshaw J. The Munsterberg illusion and ‘irradiation’. Perception. 1979;8:275-301.

[15] Grossberg S, Mingolla E. Neural dynamics of form perception: Boundary completion, illusory figures, and neon color spreading. Psychological review. 1985;92(2):173.

[16] Kitaoka A. Tilt illusions after Oyama (1960): A review. Japanese Psychological Research. 2007;49(1):7-19.

[17] Wikipedia. Zollner illusion. https://upload.wikimedia.org/wikipedia/commons/2/2d/Zollner_illusion.svg. 2016.

[18] Nematzadeh N, Lewis TW, Powers DMW. Bioplausible multiscale filtering in retinal to cortical processing as a model of computer vision. ICART2015-International Conference on Agents and Artificial Intelligence, Lisbon, Portugal. SCITEPRESS. 2015.

[19] Nematzadeh N, Powers DMW, Lewis TW. Bioplausible multiscale filtering in retina-cortical processing as a mechanism in perceptual grouping. Brain Informatics, DOI 10.1007/s40708-017-0072-8.

[20] Nematzadeh N, Powers DMW. A quantitative analysis of tilt in the Café Wall illusion: a bioplausible model for foveal and peripheral vision. In Digital Image Computing: Techniques and Applications (DICTA). 2016;1-8. IEEE.

[21] Nematzadeh N, Powers DMW, Trent I.W. Quantitative analysis of a bioplausible model of misperception of slope in the Café Wall illusion. In Workshop on Interpretation and Visualization of Deep Neural Networks (WINVIZNN). ACCV. 2016.

[22] Nematzadeh N, Powers DMW. A Bioplausible Model for Explaining Café Wall Illusion: Foveal vs. Peripheral Resolution. In International Symposium on Visual Computing (ISVC). 2016; 426-438. Springer International Publishing.

[23] Rodieck RW, Stone J. Analysis of receptive fields of cat retinal ganglion cells. Journal of Neurophysiology. 1965;28(5):833-849.

[24] Enroth-Cugel C, Robson JG. The contrast sensitivity of retinal ganglion cells of the cat. The Journal of physiology. 1966;187(3):517-552.

[25] Marr D, Hildreth E. Theory of edge detection, Proceedings of the Royal Society of London. Series B. Biological Sciences. 1980;207 (1167):187-217.

[26] Ratliff F, Knight B, Graham N. On tuning and amplification by lateral inhibition. Proceedings of the National Academy of Sciences. 1969;62(3):733-740.

[27] von Helmholtz H. Handbuch der Physiologischen Optik. 1911; Vol II. In Helmoholc’s Treatise on Physiological. Optics. 1962; Vols I and II (Edited by Southall J. P. C.). Dover, New York.

[28] Penacchio O, Otazu X, Dempere-Marco L. A neurodynamical model of brightness induction in v1. PloS one. 2013;8(5):e64086.

[29] Kitaoka A, Pinna B, Brelstaff G. Contrast polarities determine the direction of Café Wall tilts. PERCEPTION-LONDON. 2004;33(1):11-20.

[30] Fermüller C, Malm H. Uncertainty in visual processes predicts geometrical optical illusions. Vision research. 2004;44(7):727-749.

[31] Arai H. A nonlinear model of visual information processing based on discrete maximal overlap wavelets. Interdisciplinary Information Sciences. 2005;11(2):177-190.

[32] Westheimer G. Illusions in the spatial sense of the eye: Geometrical–optical illusions and the neural representation of space. Vision research. 2008;48(20):2128-2142.

[33] Jameson D, Hurvich LM. Essay concerning color constancy. Annual review of psychology. 1989;40(1):1-24.

[34] Kingdom FA. Lightness, brightness and transparency: A quarter century of new ideas, captivating demonstrations and unrelenting controversy. Vision Research. 2011;51(7):652-673.

[35] Blakeslee B, McCourt ME. A multiscale spatial filtering account of the White effect, simultaneous brightness contrast and grating induction. Vision research. 1999;39(26):4361-4377.

[36] Field G, Chichilnisky E. Information processing in the primate retina: circuitry and coding. Annu. Rev. Neurosci. 2007;30:1-30.

[37] Gauthier JL, Field D, Sher A, Greschner M, Shlens J, Litke AM, Chichilnisky E. Receptive fields in primate retina are coordinated to sample visual space more uniformly. PLoS biology. 2009;7(4):747.

[38] Gollisch T, Meister M. Eye smarter than scientists believed: neural computations in circuits of the retina. Neuron. 2010;65(2):150-164.

[39] Kuffler SW. Neurons in the retina: organization, inhibition and excitation problems. Cold Spring Harbor Symposia on Quantitative Biology. Cold Spring Harbor Laboratory Press. 1952;17:281-292.

[40] Hubel DH, Wiesel TN. Receptive fields, binocular interaction and functional architecture in the cat's visual cortex. The Journal of physiology. 1962;160(1):106-154.

[41] Daugman JG. Two-dimensional spectral analysis of cortical receptive field profiles. Vision research. 1980;20(10):847-856.

[42] Rosenfeld A, Thurston M. Edge and curve detection for visual scene analysis. Computers, IEEE Transactions on. 1971;100(5):562-569.

[43] Marr D, Vision. Vision: A Computational Investigation into the Human Representation and Processing of Visual Information. 1982.

[44] Burt PJ, Adelson EH. The Laplacian pyramid as a compact image code. Communications, IEEE Transactions on. 1983;31(4):532-540.
Blakeslee B, McCourt ME. What visual illusions tell us about underlying neural mechanisms and observer strategies for tackling the inverse problem of achromatic perception. Frontiers in human neuroscience. 2015.

Illingworth J, Kittler J. A survey of the Hough transform. Computer vision, graphics, and image processing. 1988;44(1):87-116.

Foley JM, McCourt ME. Visual grating induction. JOSA A. 1985.

Applied Computational Intelligence and Soft Computing: Special issue of Imaging, Vision, and Pattern Recognition (in press). 2017.

Nematzadeh N, Powers DM. The Café Wall Illusion: Local and Global Perception from multiple scale to multiscale. Journal of cybernetics. 1989;61(6):427-435.

Kitaoka A. Trampoline pattern. http://www.ritsumei.ac.jp/~akitaoka/motion-e.html. 2000.

Lombardo M, Serrao S, Ducoli P, Lombardo G. Eccentricity dependent changes of density, spacing and packing arrangement of parafoveal cones. Ophthalmic and Physiological Optics, 2013;33(4), 516-526.

Nematzadeh N. A Neurophysiological Model for Geometric Visual Illusions. PhD Thesis. Flinders University (in preparation).

Legge GE, Parish DH, Luebker A, Wurm LH. Psychophysics of reading. XI. Comparing color contrast and luminance contrast. JOSA A, 1990;7(10), 2002-2010.
Fig 1. *Left:* Spiral Café Wall illusion [16], *Center:* Café Wall pattern, *Right:* Zollner illusion [17].

Fig 2. DoG edge map of a crop section of Trampoline pattern [REF] as a sample of tile illusion with the size of 84×84px. The scales of DoG filters are $\sigma_c = 0.5, 1.0, 1.5, 2.0, 2.5$ for detecting the important information in the pattern. Other parameters of the model are: $s=2$, and $h=8$ (*Surround and Window ratios* respectively - Reproduced by permission from [72]).
Fig 3. Flowchart of the model and analytical tilt processing (Reproduced by permission from [72]).
Fig 4. Patterns investigated – All the variations are Café Walls of 3×8 tiles with 200×200px tiles. **Top**: Mortar-Luminance (ML) variations from Black (ML=0) to White (ML=1) mortars, and three shades of Grey in between the Black and White (ML=0.25, 0.5, and 0.75) patterns. **Middle**: Mortar thickness (Width–MW) variations, from no mortar lines (MW=0) and MW=4, 8, 16, 32, 64px patterns (*: The original Café Wall with Black and White tiles, the mortar of intermediate luminance between the luminance of the tiles, and the mortar width=8px in our samples). **Bottom**: Other variations investigated from Grey-Tiles with three luminance levels for mortar lines, below, between, or above the luminance of tiles, then phase of tile displacement of shifts of 1/3 and 1/5 of a tile between consecutive rows of the Café Wall, and finally mirrored image inducing opposite direction of tilt as well as Hollow Square pattern (Reproduced by permission from [72]).
Fig 5. DoG edge maps at multiple scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$) for five Mortar-Luminance variations, from Black (Left – $ML\ 0.0$) to White (Right - $ML\ 1.0$) mortars, in which the edge maps are overlayed by the detected houghlines displayed in green (Blue lines indicate the longest lines detected) – Hough parameters are kept the same in all experiments for detecting near Horizontal, Vertical and Diagonal tilted lines in the edge maps. $NumPeaks=1000$, $FillGap=40$px, and $MinLenght=450$px for every DoG scales. The last row shows each variation of the pattern investigated (Reproduced by permission from [72]).
Fig 6. Mean tilts and the standard error of detected tilt angles of houghlines for the five variations of Mortar-Luminance, from Black (ML 0.0) and White (ML 1.0) mortars on Top, to Grey mortar lines (ML=0.25, 0.5, 0.75) at the Bottom, for DoG edge maps at seven scales ($\sigma_c$=4, 8, 12, 16, 20, 24, 28) – Hough parameters are kept the same in all experiments for detecting near horizontal (H), vertical (V) and diagonal (D1, D2) tilted lines in the edge maps. NumPeaks=1000, FillGap=40px, and MinLength=450px for all DoG scales of the edge maps. NaN: not a number means no lines detected (Reproduced by permission from [72]).
Fig 7. DoG edge maps at multiple scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$) for the thick mortar variations displayed in jetwhite colormap. The original patterns are Café Walls of 3×8 tiles with 200×200px tiles and mortar size variations of $MW=16, 32, \text{ and } 64$px. The other parameters of the model are $s=2$ and $h=8$ for the Surround and Window ratios respectively. The last row shows each variation of the pattern investigated (Reproduced by permission from [72]).
Fig 8. DoG edge maps at seven scales ($\sigma_c$=4, 8, 12, 16, 20, 24, 28) for three variations of Mortar-Width, from no mortar lines ($MW\ 0px$) to 8px mortar, in which the edge maps are overlayed by the detected houghlines displayed in green (Blue lines indicate the longest lines detected) – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines in the edge maps. NumPeaks=1000, FillGap=40px, and MinLength=450px for all DoG scales. In all experiments the parameters of the DoG model are kept constant as $s=2$ (Surround ratio), $h=8$ (Window ratio). The last row shows each variation of the pattern investigated (Reproduced by permission from [72]).
Fig 9. DoG edge maps at seven scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$) for three different variations of Mortar-Width, from mortar size of 16px to 64px, in which the edge maps are overlayed by the detected houghlines displayed in green (Blue lines indicate the longest lines detected) – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines in the edge maps. *NumPeaks*=1000, *FillGap*=40px, and *MinLength*=450px for all DoG scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$). In all experiments the parameters of the DoG model are kept constant as $s=2$ *(Surround ratio)*, $h=8$ *(Window ratio)*. The last row shows each variation of the pattern investigated (Reproduced by permission from [72]).
Fig 10. Mean tilts and the standard errors of detected tilt angles of houghlines for six variations of the Mortar-Width, from no mortar (MW 0px) to 64px mortar and for the multiple scale ($\sigma_c$) edge maps – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines in the edge maps (H, V, D1, D2). NumPeaks=1000, FillGap=40px, and MinLength=450px for all DoG scales. NaN means no detected lines (Reproduced by permission from [72]).
Fig 11. Binary DoG edge maps at multiple scales for three thick-mortar variations with mortar lines of width 16px (MW 16px) to 64px for the Café Walls of 3x8 tiles with 200x200px tiles. The edge maps include 8 scales of $\sigma_c=8$, 12, 16, 20, 24, 28, 32, 48 presented in the figure. The other parameters of the DoG model are $s=2$ (Surround ratio) and $h=8$ (Window ratio). The last row shows each variation of the pattern investigated (Reproduced by permission from [72]).
Fig 12. DoG edge maps at seven scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$) for three Grey-Tiles variations (Black-Left, Mid-Grey-Center and White-Right mortar lines for tiles with relative luminance of 0.25-Dark Grey, and 0.75-Light Grey), in which the edge maps are overlayed by the detected houghlines displayed in green (Blue lines indicate the longest lines detected) – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines in the edge maps. $\text{NumPeaks}=1000$, $\text{Threshold}=3$, $\text{FillGap}=40$, and $\text{MinLength}=450$ for all DoG scales. Other parameters of the DoG model are $s=2$ ($\text{Surround ratio}$), and $h=8$ ($\text{Window ratio}$). The last row shows each variation of the pattern investigated (Reproduced by permission from [72]).
Fig 13. DoG edge maps at seven scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$) for three patterns of the original Café Wall-Left column, and two variations of phase of tiles displacement with the shifts of 1/3 and 1/5 of a tile in the Center and Right -columns, in which the edge maps are overlayed by the detected houghlines displayed in green (Blue lines indicate the longest lines detected) – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines in the edge maps. NumPeaks=1000, Threshold=3, FillGap=40, and MinLength=450 for all DoG scales. Other parameters of the DoG model are $s=2$ (Surround ratio) and $h=8$ (Window ratio). The last row shows each variation of the pattern investigated (Reproduced by permission from [72]).
Fig 14. DoG edge maps at seven scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$) for three patterns of the original Café Wall pattern—Left column, mirrored image (Direction Change) in the Center, and Hollow Square in the Right column, in which the edge maps are overlayed by the detected hough lines displayed in green (Blue lines indicate the longest lines detected) – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines. NumPeaks=1000, Threshold=3, FillGap=40, and MinLength=450 for all DoG scales. Other parameters of the DoG model are $s=2$ (Surround ratio), and $h=8$ (Window ratio). The last row shows each variation of the pattern investigated (Reproduced by permission from [72]).
Fig 15. Top: Mean tilts and the standard errors of detected tilt angles of houghlines for three variations of Grey-Tiles (with relative mortar luminance of Black-Left, Mid-Grey-Center and White-Right mortar lines for tiles with luminance of 0.25 for Dark Grey, and 0.75 for Light Grey), Bottom: Mean tilts and the standard errors of detected tilt range for three patterns of the original Café Wall pattern on Left, and two variations of phase of tiles displacement with the shifts of 1/3 tile in the Center, and 1/5 tile in the Right. The calculations are done for the edge maps at multiple scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$) – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines (H, V, D1, D2) in the edge maps. NumPeaks=1000, Threshold=3, FillGap=40, and MinLength=450 for all DoG scales. NaN means no detected lines (Reproduced by permission from [72]).
Fig 16. **Left:** Mean tilts and the standard errors of detected tilt angles of houghlines for the mirrored image (Direction Change), and **Right** for the Hollow Square pattern. The calculations are done for the edge maps at multiple scales ($\sigma_c=4, 8, 12, 16, 20, 24, 28$) – Hough parameters are kept the same in all experiments for detecting near horizontal, vertical and diagonal tilted lines in the edge maps. NumPeaks=1000, Threshold=3, FillGap=40, and MinLength=450 for all DoG scales. NaN means no detected lines (Reproduced by permission from [72]).
Fig 1. Left: five copies of Café Wall of 3×8 tiles with the intermediate mortar luminance between the luminance of the black and white tiles. The resolutions of MATLAB generated pattern is the tile sizes of 200×200px and mortar size of 8px (ML=0.5; Fig 4-top section with *). Right: Convergent/divergent line segments in the range of ±2.5° to ±4.5° with 0.5° difference in between. In the middle of the figure the tilted lines indicate the detected range of tilt angles by the model for the stimulus at scale 4 (σ_c=4; 3.5°±0.5) with magnitudes of 3°, 3.5° and 4° around the horizontal (mean tilt table for ML=0.5 in Fig. 6; Reproduced by permission from [72]).
Fig 2. **Left**: Café Wall of 11×11 tiles (investigated in [71]) with the intermediate mortar luminance between the luminance of the black and white tiles. The resolutions of MATLAB generated pattern is the tile sizes of 200×200px and mortar size of 8px (the same as the stimulus in Fig 17). **Right**: Convergent/divergent line segments at ±4° showing the tilt predicted by the model for the stimulus at scale 4 (σ_c =4; 4.05°±0.09) around the horizontal (mean tilt tables in [71]:Fig. 14).

Fig 3. The effect of the visual angle on the perceived tilt in Café Wall illusion (Café Wall of 11×11 tiles; the resolutions of MATLAB generated pattern is the tile sizes of 200×200px and mortar size of 8px the same as the stimulus in Fig. 18). The size of the stimulus is adjusted in these images in a way that the image in the **Center** has a size reduction of 25% from the image in the **Left**, and the image in the **Right** has a size reduction of 50% from the image in the **Left**.
Fig 4. The Café Wall of 11×11 tiles (the resolutions of MATLAB generated pattern is the tile sizes of 200×200px and mortar size of 8px the same as the stimulus in Figs 18, 19). The image on the Left has a 25% size reduction from the image in the Right. The tilts predicted by the model for the stimulus is reported to be $4.05^\circ \pm 0.09$ at scale 4 ($\sigma_c=4$; $4.05^\circ \pm 0.09$) around the horizontal ([71]:Fig. 14). Left: The Convergent/divergent line segments are shown at $\pm 5^\circ$ that is one degree more than the tilt predicted by the model for the stimulus. Right: The Convergent/divergent line segments are shown at $\pm 4^\circ$ indicating the tilt predicted by the model for the stimulus.