Absorption coefficient regression scheme for splitting radiative heat sources across phases in gas-particulate mixtures

Jian Cai* and Michael F. Modest**

(* Postdoctoral research associate, School of Engineering, University of California, Merced.
(**) Shafer and George Professor of Engineering, University of California, Merced. 5200 North Lake Road, Merced, CA 95343
Corresponding author’s e-mail address: mmodest@ucmerced.edu

Abstract. In a gas-particle mixture particles of different makes and sizes may have different local temperature and are, therefore, modeled with separate energy equations. If the important radiative heat transfer contributions are determined with sophisticated spectral models such as \( k \)-distributions, spectral information is lost during the reordering process, making it difficult to separate the resulting radiative heat source across the various energy equations. It is shown here how radiative sources can be determined accurately with the full-spectrum \( k \)-distribution assembled from a narrowband database and combined with particle properties on a narrowband level. Proper distribution of the radiative source across the various phases is achieved using regression analysis. Numerical examples demonstrate the accuracy of the model.

1. Introduction
Thermal radiation makes major contributions to the heat transfer in fluidized bed and pulverized coal combustors, due to strong particle radiation [1,2]. A high-fidelity radiation model is necessary to improve numerical predictions of the overall heat transfer.

In gas-particle flows, polydisperse particles are grouped into phases by their size and material because of distinguishing characteristics of momentum transfer [3]. Particles of each phase also have individual dynamical responses to heat transfer, which requires a dedicated energy equation and each energy equation requires an individual radiative heat source. On the other hand, the radiation field is described by a single radiative transfer equation (RTE), which determines the total radiative heat source. Therefore, an accurate scheme is needed to split the total heat source across phases in gas-particle mixtures.

Solving the RTE requires spectral models for radiatively participating gases commonly occurring in coal combustion, such as \( CO_2 \), \( CO \) and \( H_2O \). Their absorption coefficients have strong spectral dependency. Common particles in coal combustion include soot, coal, char, ash and limestone [4]. Their radiative properties are not gray even for gray complex refractive indices due to dependency on the size parameter [5].

While the line-by-line (LBL) spectral model [6] provides the most accurate results of radiative heat transfer, its large computational demands prevent its use in practical engineering applications. Recently, \( k \)-distribution methods have been developed for gases [7–9]. A \( k \)-distribution is a spectrally reordered absorption coefficient over a narrow-band or the full spectrum. Using \( k \)-distributions, radiative heat transfer can be solved with excellent accuracy, but at a small fraction of the effort compared to LBL calculations [9].
Although the total heat source can be accurately predicted using \( k \)-distribution methods, the reordering process makes splitting the total heat source across phases challenging because of the loss of spectral information. For example as shown in Fig. 1, for a nongray solid gas-particle mixture, multiple solid absorption coefficients may correspond to one total absorption coefficient value. After reordering, due to the loss of spectral information, the individual solid phase absorption of radiation is indeterminate.

In this paper a regression scheme is introduced to extend the validity of the \( k \)-distribution scheme to multi-phase gas-particle mixtures with multiple emission temperatures, and to accurately distribute the resulting radiative source across the phases. The new method is tested with several examples, demonstrating its validity and accuracy.

2. Theoretical Analysis

Consider a gas-particle mixture with particles grouped into \( M \) different phases according to their material and/or sizes with \( m = 1, 2, \ldots, M \) and in particular let \( m = 0 \) be the gas phase. Particles of different size possess different radiative properties and may have different temperatures due to different reaction rates and responses to heat transfer. The spectral radiative transfer equation (RTE) of radiation intensity \( I_\eta \) at wavenumber \( \eta \) including absorption, scattering and multiple temperature emission for a gas-particle mixture may be stated as [5]

\[
\frac{dI_\eta}{ds} = \sum_{m=0}^{M} \kappa_{m\eta} I_{bg}(T_m) - \sum_{m=0}^{M} \kappa_{m\eta} I_\eta - \sum_{m=0}^{M} \sigma_{s\eta m} I_\eta + \sum_{m=0}^{M} \left[ \sigma_{s\eta m} \frac{1}{4\pi} \int_{4\pi} I_\eta(\hat{\Omega}) \Phi_{m\eta}(\hat{\Omega}, \eta) d\Omega \right]
\]

(1)

where \( \kappa_{m\eta} \) are the absorption coefficient, \( I_{bg} \) the Planck function, \( \sigma_{s\eta m} \) the scattering coefficient and \( \Phi_{m\eta} \) the scattering phase function of phase \( m \). Both absorption and scattering coefficients are functions of state variable arrays \( \phi \) and \( \hat{\phi} \), i.e., \( \phi \) contains the gas properties, such as temperature, pressure and species concentration, and \( \hat{\phi} \) contains information of local particle properties such as particle number density, radius and species.

We now apply reordering according to the total absorption coefficient at reference state \((\kappa_\eta^0)\) over the full spectral range [5, 10]:

\[
\int_{\eta} [\bullet] \delta(k - \kappa_\eta^0) d\eta
\]

(2)

The superscript \( 0 \) indicates a reference state, i.e., \( \kappa_\eta^0 = \kappa_\eta(\phi^0, \hat{\phi}^0) \) indicates the absorption coefficient at a reference state specified by state variable arrays \( \phi^0 \) and \( \hat{\phi}^0 \). Similar to the full spectrum \( k \)-distribution method [5], after performing the integration indicated by Eq. (2), each term in Eq. (1) is transformed (reordered) as follows:

The path derivative (term on the left) becomes

\[
\int_{\eta} \frac{dI_\eta}{ds} \delta(k - \kappa_\eta^0) d\eta = \frac{d}{ds} \int_{\eta} I_\eta \delta(k - \kappa_\eta^0) d\eta = \frac{d}{ds} I_k
\]

(3)

where

\[
I_k = \int_{\eta} I_\eta \delta(k - \kappa_\eta^0) d\eta
\]

(4)

The emission term (first on the right) becomes

\[
\int_{\eta} \sum_{m=0}^{M} \kappa_{m\eta} I_{bg}(T_m) \delta(k - \kappa_\eta^0) d\eta = \sum_{m=0}^{M} \int_{\eta} \kappa_{m\eta} I_{bg}(T_m) \delta(k - \kappa_\eta^0) d\eta
\]
Figure 1. Line-by-line scatter plot of gas phase absorption coefficient ($\kappa_\eta$) vs total absorption coefficient ($\kappa_\eta$). The mixture has 10% CO$_2$ and 90% N$_2$ by volume. The particles are char with a complex refractive index $m = 2.2 - 1.12i$ and a uniform diameter of 400 $\mu$m. The volume fraction of particle is 0.001. The mixture has a temperature of 600K.

$$k_0^* = k_0^*(k) \int_\eta I_{bg}(T_0) \delta(k - \kappa_\eta^0) d\eta + \sum_{m=1}^M \hat{k}_m(k; T_m) \int_\eta I_{bg}(T_m) \delta(k - \kappa_\eta^0) d\eta$$

$$= k_0^* I_{bg}(T_0) f_{FS}(T_0, \phi^0, \phi^0_s, k) + \sum_{m=1}^M [\hat{k}_m(k; T_m) I_{bg}(T_m) f_{FS}(T_m, \phi^0, \phi^0_s, k)]$$

(5)

where the gas absorption coefficients are assumed to be correlated with total absorption coefficients, i.e., there exists a map $K_0$ [10]

$$\kappa_{bg}(\phi, \phi_s, \eta) = K_0(\phi, \phi_s, \kappa_\eta(\phi^0, \phi^0_s, \eta))$$

(6)

which gives

$$k_0^* = K_0(\phi, \phi_s, k; \phi^0, \phi^0_s)$$

(7)

The correlation assumption for a mixture of a gas with nongray particles may be examined by plotting line-by-line gas absorption coefficients against the total absorption coefficients as shown in Fig. 1, for a typical CO$_2$-N$_2$-char mixture with properties given in the figure caption. The gas absorption coefficients cluster around a monotonic curve, suggesting the existence of the map $K_0$ and the validity of correlation assumption. The correlation between gases at varying temperature and concentration has been found to be good in early work on nonluminous reacting flows [9]. Such correlation is not generally available for solid absorption coefficients as shown in Fig. 2 for the same mixture. For a given total absorption coefficient, several different solid absorption coefficients are possible. However, an apparent (or relevant average) solid absorption coefficient may be determined from a regression as

$$\hat{k}_m(k; T) = \frac{\int_\eta \kappa_{mg} I_{bg}(T) \delta(k - \kappa_\eta^0) d\eta}{\int_\eta I_{bg}(T) \delta(k - \kappa_\eta^0) d\eta}$$

(8)
Figure 2. Line-by-line scatter plot of solid phase absorption coefficient ($\kappa_{\eta}$) vs total absorption coefficient ($\kappa_0$). The regression of solid absorption coefficient against total absorption coefficient is shown by red circles. Mixture conditions are the same as Fig. 1.

This allows us to find a proper average value of $\kappa_{\eta}$ that occurs in the presence of a total absorption coefficient $\kappa_0$. The definition of the full spectrum $k$-distribution for a multi-temperature mixture remains the same as its definition for a single-temperature medium:

$$f_{FS}(T; \Phi^0, \Phi_0^0, k) = \frac{1}{I_b(T)} \int_\eta I_{b\eta}(T) \delta(k - \kappa_0^0) d\eta$$

(9)

The absorption term (second on the right) becomes

$$\int_\eta \sum_{m=0}^M k_{\eta m} I_{\eta m}(k - \kappa_0^0) d\eta$$

$$= \int_\eta k_{\eta 0} I_{\eta 0}(k - \kappa_0^0) d\eta + \sum_{m=1}^M \int_\eta k_{\eta m} I_{\eta m}(k - \kappa_0^0) d\eta$$

$$= k_{\eta 0}^* \int_\eta I_{\eta 0}(k - \kappa_0^0) d\eta + \sum_{m=1}^M \left\{ \int_\eta I_{\eta m}(k - \kappa_0^0) d\eta \right\} \int_\eta I_{\eta 0}(k - \kappa_0^0) d\eta$$

$$\approx k_{\eta 0}^* I_k + \left[ \sum_{m=1}^M \tilde{k}_m \right] I_k$$

(10)

where the term in the curly braces is approximated as

$$\frac{\int_\eta k_{\eta m} I_{\eta m}(k - \kappa_0^0) d\eta}{\int_\eta I_{\eta 0}(k - \kappa_0^0) d\eta} \approx \frac{\int_\eta k_{\eta m} I_{b\eta m}(T_m)(k - \kappa_0^0) d\eta}{\int_\eta I_{b\eta m}(T_m)(k - \kappa_0^0) d\eta} = \tilde{k}_m(k; T_m)$$

(11)
This approximation is equivalent to assuming that $I_q \propto I_{b0}(T_m)$ over the spectrum. The proportionality factor may vary with space and direction. For example, if space and spectral dependence are separable

$$I_q(x, \hat{s}, \eta) = p(x, \hat{s})I_{b0}(T_m)$$  \hspace{1cm} (12)

Eq. (11) is still exact when $p$ is not a function of wavenumber. Eq. (12) is valid in an isothermal homogeneous mixture bounded by cold black walls, or for a medium with a scaled absorption coefficient. Deviations may arise near a wall with a different temperature, in the presence of strong temperature gradients and if phase temperatures are very different.

The treatment of scattering will follow Modest and Riazi [9], in which they demonstrated that gray scattering coefficients ($\sigma_{sng} = \sigma_{sm}$) and phase functions ($\Phi_{ng} = \Phi_{sm}$) are valid approximations even for extremely nongray scatterers. By dropping spectral dependence in scattering properties and reordering according to Eq. (2), the scattering terms become

$$\int \left\{ \sum_{m=0}^{M} \sigma_{sng}I_q + \sum_{m=0}^{M} \sigma_{sm} \frac{1}{4\pi} \int_{4\pi} I_q(\hat{s}')\Phi_{sng}(\hat{s}, \hat{s}', \eta) d\Omega \right\} \delta(k - k_0) d\eta$$

$$= \sum_{m=0}^{M} \sigma_{sng}I_q + \sum_{m=0}^{M} \sigma_{sm} \frac{1}{4\pi} \int_{4\pi} I_q \Phi_{sm}(\hat{s}, \hat{s}') I_k(\hat{s}') d\Omega$$  \hspace{1cm} (13)

Assuming gas phase scattering to be negligible, the summation over phases starts from $m = 1$.

Combining the above terms, the reordered RTE becomes

$$\frac{dI_k}{ds} = k_0^0(k)I_0(T_0)f_{FS}(T_0, \Phi^0_0, \Phi^0_0, k) + \sum_{m=1}^{M} \left[ k_m^0(k; T_m)I_0(T_m)f_{FS}(T_m, \Phi^0_0, \Phi^0_0, k) \right]$$

$$- \left[ k_0^0(k) + \sum_{m=1}^{M} k_m^0(k; T_m) \right] I_k - \sum_{m=1}^{M} \sigma_{sng}I_m + \sum_{m=1}^{M} \sigma_{sm} \frac{1}{4\pi} \int_{4\pi} I_k(\hat{s}, \hat{s}') I_k(\hat{s}') d\Omega$$  \hspace{1cm} (14)

Defining the cumulative $k$-distribution as

$$g(k; T^0, \Phi^0_0, \Phi^0_0) = \int_0^d f_{FS}(T^0, \Phi^0_0, \Phi^0_0, k') dk'$$  \hspace{1cm} (15)

and

$$I_q(g(k), \mathbf{r}, \hat{s}) = I_k(k, \mathbf{r}, \hat{s})/f_{FS}(T^0, \Phi^0_0, \Phi^0_0, k)$$  \hspace{1cm} (16)

where $T^0$ is the reference temperature and $T_0$ the gas phase temperature. The new RTE for $I_q$ becomes

$$\frac{dI_q}{ds} = k_0^0(g)a(T_0, T^0, g)I_0(T_0) + \sum_{m=0}^{M} \hat{k}_m(g; T_m)a(T_m, T^0, g)I_0(T_m) - \left[ k_0^0(g) + \sum_{m=1}^{M} \hat{k}_m(g; T_m) \right] I_q$$

$$- \sum_{m=1}^{M} \sigma_{sng}I_q + \sum_{m=1}^{M} \sigma_{sm} \frac{1}{4\pi} \int_{4\pi} I_q(\hat{s}, \hat{s}') I_q(\hat{s}') d\Omega$$  \hspace{1cm} (17)

where the nongray stretching function $a$ is defined as

$$a(T_m, T^0, g) = \frac{f_{FS}(T_m, \Phi^0_0, \Phi^0_0, k(g))}{f_{FS}(T^0, \Phi^0_0, \Phi^0_0, k(g))}$$  \hspace{1cm} (18)
To split the heat source across phases, one may use the same assumption as in the derivation of the absorption term in the RTE for solid phases as given by Eq. (11). The heat source for solid phase \( m \) \((m \geq 1)\) then becomes

\[
S_m = -\nabla \cdot q_m = -\int_\eta \nabla \cdot q_{m\eta} d\eta
\]

\[
= \int_{4\pi} \kappa_{m\eta} I_\eta d\eta d\Omega - 4\pi \int_\eta \kappa_{m\eta} I_{b\eta} d\eta
\]

\[
= \int_{4\pi} \kappa_{m\eta} I_\eta \int_0^\infty \delta(k - k_\eta^0) dk d\eta d\Omega - 4\pi \kappa P_m I_0(T_m)
\]

\[
= \int_{4\pi} \kappa_{m\eta} I_\eta \delta(k - k_\eta^0) d\eta dk d\Omega - 4\pi \kappa P_m I_0(T_m)
\]

\[
\approx \int_{4\pi} \hat{k}_m(k; T_m) I_\eta \delta(k - k_\eta^0) d\eta dk d\Omega - 4\pi \kappa P_m I_0(T_m)
\]

\[
= \int_{4\pi} \hat{k}_m(k; T_m) I_\eta d\Omega dk - 4\pi \kappa P_m I_0(T_m)
\]

\[
= \int_{g=0}^1 \hat{k}_m(k; T_m) G_\eta dk - 4\pi \kappa P_m I_0(T_m)
\]

The gas phase heat source is, assuming the gas to be correlated,

\[
S_0 = -\nabla \cdot q_0 = -\int_\eta \nabla \cdot q_{0\eta} d\eta
\]

\[
= \int_{4\pi} \kappa_{0\eta} I_\eta d\eta d\Omega - 4\pi \int_\eta \kappa_{0\eta} I_{b\eta} d\eta
\]

\[
= \int_{4\pi} \kappa_{0\eta} I_\eta \int_0^\infty \delta(k - k_\eta^0) dk d\eta d\Omega - 4\pi \kappa P_0 I_0(T_0)
\]

\[
= \int_{4\pi} \kappa_{0\eta} I_\eta \delta(k - k_\eta^0) d\eta dk d\Omega - 4\pi \kappa P_0 I_0(T_0)
\]

\[
= \int_{4\pi} \hat{k}_0^0(k) \int_\eta I_\eta d\Omega dk - 4\pi \kappa P_0 I_0(T_0)
\]

\[
= \int_{k=0}^\infty \hat{k}_0^0(k) G_k dk - 4\pi \kappa P_0 I_0(T_0)
\]

\[
= \int_{g=0}^1 \hat{k}_0^0(k; g) G_\eta dg - 4\pi \kappa P_0 I_0(T_0)
\]
because

$$
\sum_{m=0}^{M} S_m = \int_{0}^{1} \left[ k^*_m(k(g)) + \sum_{m=1}^{M} \hat{k}_m(k(g); T_m) \right] G_g dg - \sum_{m=0}^{M} 4\pi \kappa_{pm} I_b(T_m)
$$

$$
= \int_{0}^{1} \left[ k^*_m(g) + \sum_{m=1}^{M} \hat{k}_m(g) \right] G_g dg - 4\pi \int_{0}^{1} k^*_m(g) a_0(g) I_b(T_0) dg - 4\pi \sum_{m=1}^{M} \int_{0}^{1} \hat{k}_m(g) a_m(g) I_b(T_m) dg
$$

$$
= \int_{0}^{1} \left\{ k^*_m(g) + \sum_{m=1}^{M} \hat{k}_m(g) \right\} G_g dg - 4\pi \left\{ k^*_m(g) a_0(g) I_b(T_0) + \sum_{m=1}^{M} \hat{k}_m(g) a_m(g) I_b(T_m) \right\} dg
$$

$$
= -\int_{0}^{1} \nabla \cdot q_g dg = -\nabla \cdot q
$$

(21)

**Regression**

To calculate the full-spectrum $k$-distribution of a gas-particle mixture, we use databased narrowband $k$-distributions [11] of the combustion gases and mix them based on the Modest-Riazzi model [9] to obtain the narrowband $k$-distribution of the gas mixture. Over each narrowband, the solid absorption coefficient of each phase may be approximated by a constant value equal to its average absorption coefficient over the narrowband. Then the narrowband $k$-distribution of the gas-particle mixture is obtained by adding the narrowband solid absorption coefficients of all solid phases to the narrowband gas $k$-distribution over each narrowband [9]. The full-spectrum $k$-distribution of the gas-particle mixture is then compiled from the narrowband $k$-distributions of the gas-particle mixture.

The regression of solid absorption coefficient in Eq. (8) is calculated with narrowband $k$-distributions of the mixture. Suppose that the spectrum is partitioned into $N_{NB}$ narrowbands and the particle absorption coefficient of phase $m$ is approximated by a constant value $k_{m,n}$ over the $n$th narrowband. Over this narrowband, the cumulative $k$-distribution of the gas-particle mixture is $q_{m,n}$ for a sequence of increasing $k_{is}$ ($i = 1, 2, \ldots, N_i$, where $N_i$ is the total number of $k$ values). The regression $\hat{k}_m$ is expressed as a weighted average of $k_{m,n}$ from all narrowbands with the weights proportional to the narrowband $k$-distributions and the narrowband integral Planck function, i.e.,

$$
\hat{k}_m(k_i) = \frac{\int_{\eta} I_{b\eta} k_{m,n} \delta(k_i - \kappa) d\eta}{\int_{\eta} I_{b\eta} \delta(k_i - \kappa) d\eta} \approx \frac{\int_{(k_i, k_{i+1})/2} (k_i, k_{i+1})/2 I_{b\eta} k_{m,n} \delta(k - \kappa) d\eta dk}{\int_{(k_i, k_{i+1})/2} I_{b\eta} \delta(k - \kappa) d\eta dk}
$$

(22)

The numerator may be expressed as

$$
= \frac{\int_{(k_i, k_{i+1})/2} (k_i, k_{i+1})/2 \int_{\eta} I_{b\eta} k_{m,n} \delta(k - \kappa) d\eta dk}{\int_{(k_i, k_{i+1})/2} I_{b\eta} \delta(k - \kappa) d\eta dk}
$$

$$
= \frac{\int_{\eta} \int_{(k_i, k_{i+1})/2} I_{b\eta} k_{m,n} \delta(k - \kappa) dk d\eta}{\int_{\eta} \int_{(k_i, k_{i+1})/2} I_{b\eta} \delta(k - \kappa) dk d\eta}
$$

$$
= \sum_{n=1}^{N_{NB}} \int_{\eta_n}^{\eta_n+\Delta\eta_n} \int_{(k_i, k_{i+1})/2} I_{b\eta_n} k_{m,n} \delta(k - \kappa) dk d\eta
$$

$$
= \sum_{n=1}^{N_{NB}} \int_{\eta_n}^{\eta_n+\Delta\eta_n} \int_{(k_i, k_{i+1})/2} I_{b\eta_n} k_{m,n} \delta(k - \kappa) dk d\eta
$$
where $\kappa_{m,n}$ is the constant solid phase absorption coefficient of phase $m$ over narrowband $n$, $I_{bn} = \int_{\eta_n}^{\eta_n+\Delta\eta_n} I_{bn} d\eta$ is the fraction of Planck function over narrowband $n$, $f_{NB}(k;n)$ is the narrowband $k$-distribution at $k$ over narrowband $n$, $\Delta g_{j,n} = \int_{(k_i+k_{i+1})/2}^{(k_i+k_{i+1})/2} f_{NB}(k;n) dk$ is the change of narrowband $k$-distribution around $k_i$ over narrowband $n$. Similarly, the denominator reads,

$$
\int_{(k_i+k_{i+1})/2}^{(k_i+k_{i+1})/2} \int_{\eta}^{\eta+\Delta\eta} I_{bn}(k)\delta(k-\kappa) d\eta dk = \sum_{n=1}^{N_{NB}} I_{bn,\Delta g_{j,n}} (24)
$$

Combining Eqs. (23) and (24), the regression may be evaluated as

$$
\hat{k}_m(k_i) = k_{m,i} = \frac{\sum_{n=1}^{N_{NB}} I_{bn,\kappa_{m,n}} \Delta g_{n,i}}{\sum_{n=1}^{N_{NB}} I_{bn,\Delta g_{n,i}}} (25)
$$

A sample regression is included in Fig. 2, in which 256 $k$ values are used, with the corresponding $\hat{k}_m$ marked by circles. Note that for small $k$ values, gas absorption is negligible, and the $\hat{k}_m$ takes the same value as $k$. For large $k$ values ($k > 3 \text{cm}^{-1}$), the total absorption coefficient is dominated by gas absorption at the line centers, and $\hat{k}_m$ approaches the solid absorption coefficient at the corresponding wavenumber.

3. Sample calculations

First consider the radiation in a homogeneous gas-particle field bounded by two parallel black cold walls. The gas phase has 10% CO$_2$ and 90% N$_2$. The particles are char with a complex refractive index $m = 2.2 - 1.12i$ [12] and a uniform diameter of 400$\mu$m. The volume fraction of the particles is 0.001. The mixture slab has a temperature of 600K and a thickness of 7cm. The gray scattering coefficient is evaluated at the wavenumber where the Planck function peaks and scattering is assumed to be isotropic. The gas absorption coefficient is calculated from databased narrowband $k$-distributions and the solid absorption coefficient is approximated as constant across each narrowband [9].

Equation (17) is solved with the $P_1$ approximation using 64 quadrature points for $g$. The results of total, gas and solid phase radiative heat sources are compared with line-by-line calculations in Fig. 3. The $k$-distribution method is capable of predicting all of the heat sources accurately (to within 1%).

To demonstrate the accuracy of the regression method in inhomogeneous media, a one-dimensional slab of 10cm width, bounded by two parallel black cold walls is considered. The slab has two homogeneous gas-particle mixture layers with equal thickness. The left layer has a gas temperature of 600K, the composition is 5% CO$_2$ diluted by O$_2$ and N$_2$. The particles have uniform diameter of 200$\mu$m, volume fraction of $1 \times 10^{-3}$ and temperature of 500K. The right layer has a gas temperature of 1200K, the composition is 10% CO$_2$ diluted by O$_2$ and N$_2$. The particles have uniform diameter of 100$\mu$m, volume fraction of $2.5 \times 10^{-4}$ and temperature of 1300K. In both layers the particles are char with a complex refractive index $m = 2.2 - 1.12i$. This configuration approximates both unburned and burning
Figure 3. Heat source calculation of an inhomogeneous slab between line-by-line (solid lines) and full-spectrum $k$-distribution with regression (dots). Line colors are blue (total heat source), red (gas phase heat source) and black (solid phase heat source).

gas-particle environments in a pulverized coal combustor. CO$_2$ is assumed to be the only radiatively participating gas species, which reduces errors from $k$-distributions since only temperature levels (as opposed to concentration) contribute to uncorrelation. The inaccuracy hereby primarily comes from the approximation in Eq.( 11).

Equation (17) is solved with the $P_1$ approximation using 64 quadrature points for $q$. The results of total, gas and solid phase radiative heat sources are compared with line-by-line calculations in Fig. 4. The $k$-distribution method is capable of predicting major trends of total and phase-split heat source terms. In particular, the gas phase heat source is predicted accurately (mostly within 5%) in spite of the fact that it is one order less in magnitude than the total heat source. This is because gas radiation primarily comes from strong bands, and the proposed regression scheme successfully picks the solid absorption coefficients at the corresponding wavenumbers (e.g. large $k$ values in Fig. 2). For the solid phase heat source, the largest deviation occurs near the interface between the two layers. Equation (11) is greatly challenged by the sharp temperature gradient at the interface. In the left (cold) layer, the spectral incident radiation deviates significantly from the emission spectrum at the local temperature because of the emission from the hot layer. Away from the interface, such deviation is reduced and accuracy improves. However, this effect is hardly observed in the hot layer because emission from the cold layer is negligible. In the hot layer, the error is less than 1%, which is due to the temperature difference between gas and solid phases.

4. Conclusion

In a gas-particle mixture particles of different makes and sizes may have different local temperature and, therefore, are modeled with separate energy equations. To model the radiative heat transfer in such a mixture the full-spectrum $k$-distribution model was extended to the case of multi-phase mixtures with individual temperature for each phase. Full spectrum $k$-distributions were assembled from a narrowband database for gases, by assuming particles to be gray across narrowbands, and by using a regression to
determine “effective” absorption coefficients for particles.

It was shown how properly distributed radiative source terms can be determined for the individual energy equations, by using a similar regression analysis. The accuracy of the method was demonstrated with several examples.

References

[1] Hua Y, Flamant G, Lu J and Gauthier D 2005 International Journal of Heat and Mass Transfer 48 1145–1154
[2] Alagoz D E, Kulah G and Selcuk N 2008 Combustion Science and Technology 180 910–926
[3] Syamlal M and O’Brien T 1988 International Journal of Multiphase Flow 14 473–481
[4] Im K H and Ahluwalia R K 1993 International Journal of Heat and Mass Transfer 36 293–302
[5] Modest M F 2003 Radiative Heat Transfer 2nd ed (New York: Academic Press)
[6] Taine J 1983 Journal of Quantitative Spectroscopy and Radiative Transfer 30 371–379
[7] Zhang H and Modest M F 2002 Journal of Quantitative Spectroscopy and Radiative Transfer 73 349–360
[8] Wang L and Modest M F 2005 ASME Journal of Heat Transfer 127 740–748
[9] Modest M F and Riazzi R J 2005 Journal of Quantitative Spectroscopy and Radiative Transfer 90 169–189
[10] Modest M F 2003 Journal of Quantitative Spectroscopy and Radiative Transfer 76 69–83
[11] Wang A and Modest M F 2005 Journal of Quantitative Spectroscopy and Radiative Transfer 93 245–261
[12] Mengüç M P and Viskanta R 1985 Combustion Science and Technology 44 143–159

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Figure 4. Heat source calculation of an inhomogeneous slab between line-by-line (solid lines) and full-spectrum k-distribution with regression (dots). Line colors are blue (total heat source), red (gas phase heat source) and black (solid phase heat source).