Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
Technical Paper

Preventive replacement policies with time of operations, mission durations, minimal repairs and maintenance triggering approaches

Xufeng Zhao \( ^{\text{a},*} \), Jiajia Cai \( ^{\text{a}} \), Satoshi Mizutani \( ^{\text{b}} \), Toshio Nakagawa \( ^{\text{b}} \)

\( ^{\text{a}} \) College of Economics and Management, Nanjing University of Aeronautics and Astronautics, NO.29, Jiangian Avenue, Nanjing 211106, China

\( ^{\text{b}} \) Department of Business Administration, Aichi Institute of Technology, 1247 Yachigusa, Yakusa-cho, Toyota 470-0392, Japan

**A R T I C L E I N F O**

**Keywords:**
Replacement
Minimal repair
Failure rate
Mission duration
Production system

**A B S T R A C T**

When a mission arrives at a random time and lasts for a duration, it becomes an interesting problem to plan replacement policies according to the health condition and repair history of the operating unit, as the reliability is required at mission time and no replacement can be done preventively during the mission duration. From this viewpoint, this paper proposes that effective replacement policies should be collaborative ones gathering data from time of operations, mission durations, minimal repairs and maintenance triggering approaches. We firstly discuss replacement policies with time of operations and random arrival times of mission durations, model the policies and find optimum replacement times and mission durations to minimize the expected replacement cost rates analytically. Secondly, replacement policies with minimal repairs and mission durations are discussed in a similar analytical way. Furthermore, the maintenance triggering approaches, i.e., replacement first and last, are also considered into respective replacement policies. Numerical examples are illustrated when the arrival time of the mission has a gamma distribution and the failure time of the unit has a Weibull distribution. In addition, simple case illustrations of maintaining the production system in glass factories are given based on the assumed data.

1. Introduction

Maintenance policies and their applications have been studied extensively in literatures \([1-7]\). An age replacement model for a finite operational time span, in which the unit operates from installation for a fixed interval due to external factors, and it is replaced at the end of the interval even if no failure occurs, has been firstly given \([1]\). When the above finite time span becomes a random variable such as working cycle, replacement policies with working cycles have been discussed \([8-13]\). In other words, replacement policies can be done more effectively when they are collaborative with the requirements of operations.

For an operating unit, the working cycle, or the mission duration named in this paper, may arrive at a random time rather than starts from the installation of the unit discussed in the above literatures \([8-13]\). In this case, it would be interesting problems to estimate the possible mission durations, meanwhile, to find possible maintenance opportunities according to the health condition and repair history of the operating unit, as a required reliability is needed at mission time and no replacement can be done during the mission duration. That is, planning maintenance policies also should be based on the reliability interval \([14]\) and the history of repairs of the systems \([15]\).

Typical examples often occur in many mission critical systems, such as emergency communication systems, enterprise databases and application development platforms. For example, when the new coronavirus pneumonia first started in Wuhan, China, at the end January, 2020, many pharmaceutical manufacturers temporarily added emergency orders, which are to produce disposable medical mask, disinfected water and other urgently needed items. If machines are stopped for preventive maintenance (it actually may be the maintenance time as it is in holidays), it will cause bad social impact and huge economic losses. Additionally, some database and application development platforms may not be readily available for preventive maintenance when corporate data is used and analyzed by users, including employees, customers, and partners. More extensively, the mission critical problems can also be found for balancing the aircraft during the period of landing \([16]\). So that, it would be interesting to study how to maintain such reliability systems collaborating with the time of operations and mission durations.

Maintaining the mission-oriented systems are becoming important reliability analyses in recent years, which is different from the reliability-oriented maintenance policies \([17]\) and service-oriented
maintenance policies [18]. As far as we know, there was not much research on mission-oriented maintenance policies. For examples, imperfect maintenance policies for the mission-oriented systems subject to degradation and shocks [19], mission success probabilities and mission abort policies for k-out-of-n systems and random environment [20,21], the systems operating missions for selective maintenance optimizations [22], reliability models for semi-Markov missions [23].

In this paper, the maintenance triggering approaches are also collaborated into planning replacement policies, i.e., replacement first and replacement last policies, which are alternatives in points of cost, reliability and maintainability. Replacement first means the unit is replaced preventively at some events such as operating time, number of repairs, or working numbers, etc., whenever takes place first, while replacement last means the unit is replaced preventively at the above events, whichever takes place last. It has been shown that replacement last could let the unit operate as longer as possible while replacement first are more easier to save total maintenance cost [24]. More recent discussions of replacement first and replacement last can be found in [25–29].

It has been indicated that collaborative management are very effective for maintenance policies [31]. In this paper, we will discuss replacement policies with time of operations, mission durations, minimal repairs and maintenance triggering approaches in analytical ways. That is, we will give the descriptions of replacement policies firstly, and then, obtain the models of expected cost rates under the renewal reward theorem [1], and discuss optimum replacement times and mission durations in analytical ways. In addition, numerical examples and simple case illustrations are given when the arrival time of the mission has a gamma distribution and the failure time of the unit has a Weibull distribution.

2. Age replacement

We consider a unit that begins to operate after installation and should be operating at mission time \( T_0 \) \((0 < T_0 < \infty)\) for a duration \([T_0, T_0 + t_0] \) \((0 \leq t_0 < \infty)\), where \( T_0 \) is a random variable and has a general distribution \( Y(t) \equiv Pr[T_0 \leq t] \) with a density function \( y(t) \equiv dY(t)/dt \) and a finite mean \( \gamma = \int_0^\infty t \cdot y(t)dt \). In this paper, \( t_0 \) is considered as a mission duration, during which, the unit provides reliability and no replacement can be done preventively.

However, the unit degrades with time and would be failure finally, so that replacement preventive policies should be planned to make the unit reliable for the mission duration \([T_0, T_0 + t_0] \). We assume that the failure time of the unit has a general distribution \( F(t) \equiv Pr[X \leq t] \) with a density function \( f(t) \equiv dF(t)/dt \) and a finite mean \( \mu = \int_0^\infty t \cdot f(t)dt \). The conditional failure probability [2] is given by:

\[
\lambda(t; x) \equiv \frac{F(t + x) - F(t)}{F(t)} \quad (0 < x < \infty),
\]

(1)

which represents the probability that the unit will fail in \([t, t + x] \), given that it is still survival at time \( t \). Note that \( 0 \leq \lambda(t; x) \leq 1 \). When \( x \to 0 \), \( \lambda(t; x) \) becomes an instant failure rate [1]:

\[
\lambda(t) \equiv \frac{f(t)}{F(t)} = -\frac{1}{F(t)} \frac{dF(t)}{dt}.
\]

(2)

In maintenance modeling, \( h(t) \) should be increasing with \( t \) from \( h(0) \geq 0 \) to \( h(\infty) \equiv \lim_{t \to \infty} h(t) \) that might be infinity, i.e., \( \lambda(t; x) \) increases with \( t \) from \( F(x) \) to 1, that is also assumed in this paper.

2.1. Simple replacement

We plan that the unit is only replaced preventively at time \( T_0 \) + \( t_0 \) when it is still survival at mission time \( T_0 \), so no preventive replacement can be done during the mission duration \([T_0, T_0 + t_0] \), or it is replaced correctly at failure before time \( T_0 \) + \( t_0 \), whichever takes place first. The mean time to replacement is

\[
\int_0^\infty (t + t_0) \cdot F(t + t_0)dt + \int_0^\infty \left[ \int_0^{t_0} u \cdot dF(u) \right] dt = \int_0^\infty \left[ \int_0^{t_0} F(u)du \right] dt.
\]

(3)

Let \( c_0 \) and \( c_1 \) \((c_0 < c_1)\) are the costs of replacement policies done at \( T_0 \) + \( t_0 \) and at failure, respectively. Using the renewal reward theorem [1], we define that the expected cost rate is given by the expected replacement cost divided by the expected time per cycle, where a cycle terminates whenever a renewal takes place, i.e., the unit is replaced upon failure or at time \( T_0 \) + \( t_0 \), whichever takes place first. From (3), the expected cost rate is

\[
C_i(t_0) = \frac{c_0 + (c_1 - c_0) \int_0^\infty F(t + t_0)dt}{\int_0^\infty F(u)du}.
\]

(4)

Clearly, \( \lim_{t_0 \to \infty} C_i(t_0) = c_f/\mu \) and

\[
\lim_{t_0 \to \infty} C_i(t_0) = \frac{c_0 + (c_1 - c_0) \int_0^\infty F(t)dt}{\int_0^\infty F(u)Y(t)du}.
\]

(5)

which becomes the random age replacement model in [8].

We find optimum mission duration \( t_0^* \) to minimize \( C_i(t_0) \) in (4). Differentiating \( C_i(t_0) \) with respect to \( t_0 \) and setting it equal to zero,

\[
h_i(t_0) = \int_0^\infty \left[ \int_0^{t_0} F(u)du \right] dt - \int_0^\infty F(t + t_0)dt = \frac{c_0}{c_f}.
\]

(6)

From Appendix A.1, when \( Y(t) = 1 - e^{-\theta t} \), \( h_i(t_0) \) increases with \( t_0 \) from \( h_i(0) = \int_0^\infty F(t)e^{-\theta t}dt/\int_0^\infty F(t)dt \) to \( h(\infty) \). Then, the left-hand side of (5) increases with \( t_0 \) to \( \infty \) as \( h(\infty) \to \infty \). This case, there exists a finite and unique \( t_0^* \) \((0 \leq t_0^* < \infty)\) which satisfies (5), and the resulting cost rate is

\[
C_i(t_0^*) = (c_f - c_0)h_i(t_0^*).
\]

(7)

When \( y(t) = \theta e^{-\theta t}/\Gamma(k) \) and \( F(t) = 1 - e^{-\theta t} \), Table 1 presents optimum \( t_0^* \) and its cost rate \( C_i(t_0^*) \) for \( k \) and \( c_0 \) when \( \theta = 1, \alpha = 0.1, \beta = 0.1, \) and \( c_f = 100.0 \). Note that \( E(T_0) = k/\theta \) represents the mean arrival time \( T_0 \) of the mission duration. Table 1 shows that optimum mission duration \([T_0, T_0 + t_0^*] \) increases when \( T_0 \) arrives at an early time and \( c_0 \) increases to the cost of \( c_f \).

Taking \( c_0 = 10.0 \) as an example, Fig. 1 represents the curve of the objective function \( C_i(t_0) \) in (4) for given \( k = 1, 2, 3 \) when \( \theta = 1.0, \alpha = 0.1, \beta = 2.0 \) and \( c_f = 100.0 \). It is clear that Fig. 1 agrees with the
discussion of (5), and also shows the approximate locations of the optimum mission durations and their cost rates calculated in Table 1. For example, when \( k = 1 \), optimum \( t_c^* \) falls in (2, 3) and its cost rate is in (6, 6.5), whose exact value is given in Table 1.

2.2. Replacement first

In order to prevent early or late random arrival time \( T_0 \) of the mission duration, the unit is replaced preventively at time \( T \) (0 < \( T \leq \infty \)) or at time \( T_0 + t_k \) (0 ≤ \( t_k < \infty \)), whichever takes place first; however, no replacement policy can be done preventively during the mission duration \([T_0, T_0 + T] \). In this policy, \( t_k \) is constantly given and \( T_0 \) is a random variable with distribution \( Y(t) \). The mean time to replacement is

\[
T(T) = \int_0^T \int_t^T \mathbb{P}(r \mid t) \, dr \, dt + \int_0^T \mathbb{P}(r \mid t) \, dt \int_0^t \mathbb{P}(r \mid u) \, du \, dy(t)
\]

Using the renewal reward theorem [1], the expected cost rate is

\[
C_f(T; t_k) = \frac{c_p + (c_r - c_p)[F(T)\mathbb{P}(T) + \int_t^T F(t + t_k)\, dt]}{\mathbb{P}(T) + \int_0^T \mathbb{P}(r \mid t) \, dt + \int_0^T \mathbb{P}(r \mid u) \, du \, dy(t)}
\]

where \( c_p \) is the cost of replacement policies done at \( T \) and \( T_0 + t_k \).

Note that when \( T \rightarrow \infty \), \( \lim_{T \rightarrow \infty} C_f(T; t_k) = C_f(t_k) \) in (4), and when \( T \rightarrow 0 \), \( \lim_{T \rightarrow 0} C_f(T; t_k) = \infty \). When \( t_k \rightarrow 0 \),

\[
\lim_{t_k \rightarrow 0} C_f(T; t_k) = \frac{c_p + (c_r - c_p)[F(T)\mathbb{P}(T) + \int_t^T F(t + t_k)\, dt]}{\mathbb{P}(T) + \int_0^T \mathbb{P}(r \mid t) \, dt + \int_0^T \mathbb{P}(r \mid u) \, du \, dy(t)}
\]

which becomes the age replacement first model in [8].

We next find optimum replacement time \( T_f \) and mission duration \( t_{f_k} \) to minimize \( C_f(T; t_k) \) in (8), respectively. Differentiating \( C_f(T; t_k) \) with respect to \( T \) for given \( t_k \) and setting it equal to zero,

\[
q_f(T; t_k) = \frac{r(T)\lambda(T; t_k) + h(T)}{r(T)\lambda(T; t_k) + 1} - \frac{r(T)}{\lambda(T; t_k)}
\]

and \( \lambda(t; x) \) is given in (1) and \( h(t) \) given in (2), and

\[
\Lambda(T; t_k) = \frac{F(T + t_k) - F(T)}{r(T)\lambda(T; t_k)}
\]

From Appendix A2, \( \Lambda(T; t_k) \) increases with \( t \) from \( F(t_k) \) to \( F(\infty) \) to \( h(\infty) \).

When \( Y(t) = 1 - e^{-\beta t} \), \( r(t) = \theta \) and

\[
q_f(T; t_k) = \frac{\theta F(T + t_k) - F(T) + f(T)}{\theta \int_0^t F(\alpha) \, d\alpha + \frac{\beta}{2}}
\]

then \( q_f(T; t_k) \) increases strictly with \( T \) to \( \infty \) as \( h(\infty) \rightarrow \infty \), and also increases strictly with \( t_k \) to \( q_f(T; t_k) \). Thus, the left-hand side of (9) increases with \( T \) from 0 to \( \infty \) as \( h(\infty) \rightarrow \infty \). Therefore, there exists a finite and unique \( T_f \) (0 < \( T_f \) < \( \infty \)) which satisfies (9), and the resulting cost rate is

\[
C_f(T_f; t_k) = (c_r - c_p)q_f(T_f; t_k).
\]

In addition, the left-hand side of (9) increases with \( t_k \), then \( T_f \) decreases with \( t_k \) from the following random replacement model [8],

\[
h(T) = \int_0^T e^{-\alpha T} f(t) \, dt - \int_0^T e^{-\beta T} f(t) = \frac{c_p}{c_r - c_p}
\]

Next, differentiating \( C_f(T; t_k) \) with respect to \( t_k \) for given \( T \) and setting it equal to zero,

\[
h_f(T; t_k) = \frac{\int_0^T f(t + t_k) \, dt}{\int_0^T f(t + t_k) \, dt} \leq h(T + t_k)
\]

From Appendix A1, when \( Y(t) = 1 - e^{-\alpha t} \), \( h_f(T; t_k) \) increases with \( t_k \) to \( h(\infty) \). Then, the left-hand side of (11) increases strictly with \( t_k \) from 0 to \( \infty \) as \( h(\infty) \rightarrow \infty \). Therefore, there exists a finite and unique \( t_{f_k} \) (0 < \( t_{f_k} \) < \( \infty \)) which satisfies (11), and the resulting cost rate is

\[
C_f(T_f; t_{f_k}) = (c_r - c_p)h_f(T_f; t_{f_k})
\]

Note that \( t_{f_k} \) decreases with \( T \) as the left-hand side of (8) increases with \( T \).

When \( y(t) = \theta e^{\kappa k} - e^{-\beta t} / \Gamma(k) \) and \( F(t) = 1 - e^{-\alpha t} / \Gamma(t) \), Table 2 presents optimum \( T_f \) and its cost rate \( C_f(T_f; t_k) \) for \( t_k \) and \( c_p \) when \( \theta = 1.0, k = 2, \alpha = 0.1, \beta = 2.0, \) and \( c_r = 100.0, \) and Table 3 presents optimum \( t_{f_k} \) and its cost rate \( C_f(T_f; t_{f_k}) \) for \( T_f \) and \( c_p \) when \( \theta = 1.0, k = 2, \alpha = 0.1, \beta = 2.0, \) and \( c_r = 100.0, \) which agree with the above analytical discussions.

In addition, taking \( c_p = 10.0 \) as an example, Figs. 2 and 3 represent the curve of the objective function \( C_f(T; t_k) \) in (8) for given mission duration and replacement time, respectively, when \( \theta = 1.0, k = 2, \alpha = 0.1, \beta = 2.0, \) and \( c_r = 100.0. \) Clearly, approximate replacement times and mission durations can be found from Figs. 2 and 3, whose exact values are calculated in Tables 2 and 3.
Table 2

| $c_p$ | $t_x = 1.0$ | $t_x = 2.0$ | $t_x = 5.0$ |
|-------|-------------|-------------|-------------|
| $C(T; t_x)$ | $C(T; t_x)$ | $C(T; t_x)$ | $C(T; t_x)$ |
| 10.0  | 3.368       | 6.442       | 2.864       |
| 15.0  | 4.577       | 8.148       | 3.834       |
| 20.0  | 5.888       | 9.769       | 4.864       |
| 25.0  | 7.354       | 11.360      | 6.001       |
| 30.0  | 9.027       | 12.943      | 7.291       |
| 35.0  | 10.961      | 14.524      | 8.776       |
| 40.0  | 13.260      | 16.105      | 10.513      |
| 50.0  | 19.265      | 21.266      | 15.386      |

Table 3

| $c_p$ | $T = 1.0$ | $T = 2.0$ | $T = 5.0$ |
|-------|------------|------------|------------|
| $C_p(T; C_p)$ | $C_p(T; C_p)$ | $C_p(T; C_p)$ | $C_p(T; C_p)$ |
| 10.0  | 3.918      | 8.127      | 2.446      |
| 15.0  | 5.541      | 10.440     | 3.512      |
| 20.0  | 7.135      | 12.374     | 4.543      |
| 25.0  | 8.776      | 14.058     | 5.581      |
| 30.0  | 10.523     | 15.564     | 6.569      |
| 35.0  | 12.442     | 16.943     | 7.811      |
| 40.0  | 14.606     | 18.234     | 9.069      |
| 50.0  | 20.088     | 20.670     | 12.101     |

2.3. Replacement last

In order to make full use of the unit, preventive replacement is planned at time $T$ or at time $T_0 + t_x$, whichever takes place last; however, no replacement policy can be done preventively during the mission duration $[T_0, T_0 + t_x]$. Comparing to replacement first, both policies have an overlapping case when $T$ is planned at $[T_0, T_0 + t_x]$ but the policy is done at $T_0 + t_x$, however, $T$ begins from $t_x$, i.e., $T \geq t_x$, for replacement last. The mean time to replacement is

$$TT(Y) = Y \int_{T-x}^{T} dY(t) + \int_{T-x}^{T} \int_{0}^{Y} f(t) Y(t) dY(t)$$

Using the renewal reward theorem [1], the expected cost rate is

$$C(T; t_x) = c_p + \frac{(c_j - c_p)T}{\int_{T-x}^{T} F(t) dt + \int_{T-x}^{T} \frac{dF(t)}{Y(t)} dt} \left( T \geq t_x \right)$$

Note that when $T \to \infty$, $\lim_{T \to \infty} C(T; t_x) = \frac{c_j}{\mu}$, and when $T \to 0$, $\lim_{T \to 0} C(T; t_x) = \lim_{T \to 0} C(T; t_x)$ that is the random age replacement model in [8]. When $t_x \to \infty$, $\lim_{t_x \to \infty} C(T; t_x) = \frac{c_p}{\mu}$, and when $t_x \to 0$,

$$\lim_{t_x \to 0} C(T; t_x) = \frac{c_p + (c_j - c_p)T}{\int_{T-x}^{T} F(t) dt + \int_{T-x}^{T} \frac{dF(t)}{Y(t)} dt}$$

which becomes the age replacement last model in [8].

We find optimum replacement time $T_1$ and mission duration $t_x$ to minimize $C(T; t_x)$ in (14), respectively. Differentiating $C(T; t_x)$ with respect to $T$ for given $t_x$ and setting it equal to zero,
then $T_x = t_x$.

Differentiating $C_t(T; t_x)$ with respect to $t_x$ for given $T$ and setting it equal to zero,

$$h_t(T; t_x) = \frac{c_f}{c_f - c_p} = \frac{c_f}{c_f - c_p}$$

where

$$h_t(T; t_x) = \frac{\int_t^T f(t + t_x) dY(t)}{\int_t^T f(t + t) dY(t)} > h(T).$$

Letting $L_t(T; t_x)$ be the left-hand side of (17), $dL_t(T; t_x)/dt_x > 0$ if $h_t(T; t_x)$ increases with $t_x$. Therefore, we have the following optimum policies:

1. If $L_t(T, 0) \geq c_f/(c_f - c_p)$, then $t_x^* = 0$.

2. If $L_t(T, 0) < c_f/(c_f - c_p) < L_t(T, T)$, then there exists a finite and unique $t_x^* (0 < t_x^* < T)$ which satisfies (17), and the resulting cost rate is

$$C_t(T; t_x^*) = (c_f - c_p) h_t(T; t_x^*).$$

3. If $L_t(T, T) \leq c_f/(c_f - c_p)$, then $t_x^* = T$.

When $y(t) = \theta t^{k-1} e^{-\alpha t}/Gamma(k)$ and $F(t) = 1 - e^{-(\alpha t)}$, Table 4 presents optimum $T_x$ and its cost rate $C_t(T; t_x)$ for $t_x$ and $c_p$ when $\theta = 1.0, k = 2$, $\alpha = 0.1, \beta = 2.0$, and $c_f = 100.0$, and Table 5 presents optimum $t_x^*$ and its cost rate $C_t(T; t_x^*)$ for $T$ and $c_p$ when $\theta = 1.0, k = 2$, $\alpha = 0.1, \beta = 2.0$, and $c_f = 100.0$.

Comparing Table 4 to Table 2, and Table 5 to Table 3, $C_t(T; t_x) < C_t(T; t_x)$ but $C_t(T; t_x) < C_t(T; t_x^*)$ in most cases, which means we can select the right replacement policies from point of cost and maintainability. From the works [13] and [20], when the bivariate maintenances are planned simultaneously, we have to take actions to prevent early or late maintenance policies that is, the first and last policies should be considered, which have been done in the section.

Fig. 4 and 5 represent the curve of the objective function $C_t(T; t_x)$ in (14) for given mission duration and replacement time, respectively, when $\theta = 1.0, k = 2$, $\alpha = 0.1, \beta = 2.0$, $c_f = 100.0$ and $c_p = 10.0$. It also shows the condition of $T \geq t_x$ in figures.

### 3. Minimal repair

#### 3.1. Simple replacement

We suppose that the unit undergoes minimal repairs at failures and

| $c_p$ | $T_x$ | $c_f(T_x)$ |
|-------|-------|------------|
| 1.0   | 3.465 | 6.237      |
| 1.5   | 4.301 | 7.312      |
| 2.0   | 5.121 | 8.193      |
| 2.5   | 5.944 | 9.816      |
| 3.0   | 6.792 | 9.959      |
| 3.5   | 7.684 | 9.989      |
| 4.0   | 8.646 | 10.376     |
| 5.0   | 10.908| 10.908     |

| $T_x$ | $c_f(T_x)$ |
|-------|------------|
| 1.0   | 3.564      |
| 2.0   | 4.344      |
| 3.0   | 5.139      |
| 4.0   | 5.951      |
| 5.0   | 6.794      |
| 6.0   | 7.685      |
| 7.0   | 8.646      |
| 8.0   | 10.908     |

| $T_x$ | $c_f(T_x)$ |
|-------|------------|
| 1.0   | 6.415      |
| 2.0   | 7.385      |
| 3.0   | 8.223      |
| 4.0   | 9.827      |
| 5.0   | 10.908     |

Table 4 Optimum $T_x$ and its cost rate $C_t(T; t_x)$ when $\theta = 1.0, k = 2$, $\alpha = 0.1, \beta = 2.0$, and $c_f = 100.0$.

Table 5 Optimum $t_x^*$ and its cost rate $C_t(T; t_x^*)$ when $\theta = 1.0, k = 2$, $\alpha = 0.1, \beta = 2.0$, and $c_f = 100.0$.

begins to operate again after repairs, where the time for repairs are negligible and the failure rate remains undisturbed by repairs. In order to prevent an increasing cost of repairs, we plan that the unit is only replaced at time $T_0 + t_x$ $(0 \leq t_x < \infty)$, where $T_0$ is a random variable.
with distribution $Y(t)$. Using the renewal reward theorem [1], the expected cost rate is
\[
C_i(t_i) = c_m \int_0^\infty h(t + t_i) dY(t) + c_p \int_0^\infty dY(t)
\]
where $c_m$ is minimal repair cost at failure, and $c_p$ is given in (4).

Clearly, \( \lim_{t_i \to \infty} C_i(t_i) \to \infty \) and
\[
\lim_{t_i \to \infty} C_i(t_i) = c_m \int_0^\infty h(t + t_i) dY(t) + c_p,
\]
which agrees with the random periodic replacement model in [8].

If there exists an optimum mission duration $t^*_i$ to minimize $C_i(t_i)$ in (19), it satisfies
\[
\int_0^\infty (t + t_i)h(t) dY(t) + \int_0^\infty h(t + t_i) dY(t) = \frac{c_p}{c_m}
\]
where $t_i$ to as $h(\infty) \to \infty$. In this case, the resulting cost rate is
\[
C_i(t^*_i) = c_m \int_0^\infty h(t + t^*_i) dY(t).
\]

When $y(t) = \theta^p e^{-\alpha t}/\Gamma(k)$ and $F(t) = 1 - e^{-\alpha t}$, Table 6 presents optimum $t^*_i$ and its cost rate $C_i(t^*_i)$ for $k$ and $c_m$ when $\theta = 1.0, \alpha = 1.0, \beta = 2.0$, and $c_p = 100.0$. Table 6 shows that optimum mission duration $[T_0, T_0 + t^*_i]$ decreases when $c_m$ increases and $T_0$ arrives at a late time due to the total increasing repair cost.

Fig. 6 represents the curve of the objective function $C_i(t_i)$ in (19) for given $k = 1, 2, 3$ when $\theta = 1.0, \alpha = 1.0, \beta = 2.0$, $c_p = 100$ and $c_m = 10.0$. It is clear that the approximate mission durations in Fig. 6 agree with those in Table 6 when $c_m = 10.0$.

### 3.2. Replacement first

We plan that the unit is replaced at time $T_0$ ($0 < T_0 < \infty$) or at time $T_0 + t^*_i$ ($0 \leq t^*_i < \infty$), whichever takes place first; however, only minimal repairs can be done during the mission duration $[T_0, T_0 + t^*_i]$. Then, the mean time to replacement is
\[
T \bar{T}(T) + \int_0^T (t + t_i)h(t) dY(t) = \int_0^T \bar{Y}(t) dT.
\]

Using the renewal reward theorem [1], the expected cost rate is
\[
C_i(T; t_i) = c_m \left[ H(T) \bar{T}(T) + \int_0^T h(t + t_i) dY(t) \right] + c_p
\]
\[
\int_0^T \bar{Y}(t) dT.
\]

We find optimum replacement time $T^*_i$ and mission duration $t^*_i$ to minimize $C_i(T; t_i)$ in (23). Differentiating $C_i(T; t_i)$ with respect to $T$ and setting it equal to zero,

\[
\begin{array}{c|c|c|c|c|c}
  k & 1 & 2 & 3 \\
  t^*_i & 1.667 & 1.236 & 1.141 \\
  C_i(t^*_i) & 69.167 & 102.559 & 112.152 \\
  c_m & 10.00 & 10.00 & 10.00 \\
\end{array}
\]

which increases with $T$ and $t^*_i$ to $h(\infty)$.

When $y(t) = 1 - e^{-\alpha t}$, $q_i(T; t_i)$ increases with $T$ to $h(\infty)/\theta \alpha$. Therefore, there exists a finite and unique $T^*_i$ ($0 < T^*_i < \infty$) which satisfies (24), and the resulting cost rate is
\[
C_i(T^*_i; t_i) = c_m q_i(T^*_i; t_i).
\]

Next, differentiating $C_i(T; t_i)$ with respect to $t_i$ and setting it equal to zero,
\[
\begin{array}{c|c|c|c|c|c}
  k & 1 & 2 & 3 \\
  t^*_i & 1.667 & 1.236 & 1.141 \\
  C_i(t^*_i) & 69.167 & 102.559 & 112.152 \\
  c_m & 10.00 & 10.00 & 10.00 \\
\end{array}
\]

whose left-hand side increases with $t^*_i$ to as $h(\infty) \to \infty$. Therefore, there exists a finite and unique $t^*_i$ ($0 \leq t^*_i < \infty$) which satisfies (25), and the resulting cost rate is
\[
C_i(T^*_i; t^*_i) = c_m \int_0^T h(t + t^*_i) dY(t) / \bar{Y}(T).
\]

When $y(t) = \theta^p e^{-\alpha t}/\Gamma(k)$ and $F(t) = 1 - e^{-\alpha t}$, Table 7 presents optimum $T^*_i$ and its cost rate $C_i(T^*_i; t_i)$ for $k$ and $c_m$ when $\theta = 1.0, \alpha = 1.0, \beta = 2.0$, and $c_p = 100.0$. Table 8 presents optimum $t^*_i$ and its cost rate $C_i(T^*_i; t^*_i)$ for $k$ and $c_m$ when $\theta = 1.0, \alpha = 1.0, \beta = 2.0$, $T = 1.0$, and $c_p = 100.0$. When $c_m = 10.0$, Figs. 7 and 8 represent the curve of the objective function $C_i(T; t_i)$ in (23) for given $k = 1, 2, 3$, respectively. It is clear that
Table 7
Optimum $T_f^*$ and its cost rate $C_l(T_f^*; t_e)$ when $\theta = 1.0, \alpha = 1.0, \beta = 2.0, t_e = 1.0,$ and $c_m = 100.0.$

| $c_m$ | $k = 1$ | $k = 2$ | $k = 3$ |
|-------|--------|--------|--------|
|       | $T_f^*$ | $C_l(T_f^*; t_e)$ | $T_f^*$ | $C_l(T_f^*; t_e)$ | $T_f^*$ | $C_l(T_f^*; t_e)$ |
| 10.0  | 3.468  | 74.338 | 3.115  | 66.613 | 3.057  | 64.619 |
| 15.0  | 2.586  | 85.137 | 2.451  | 79.764 | 2.451  | 78.299 |
| 20.0  | 2.193  | 95.547 | 2.080  | 91.265 | 2.119  | 90.667 |
| 25.0  | 1.846  | 104.785| 1.846  | 102.121| 1.685  | 101.321|
| 30.0  | 1.631  | 112.852| 1.670  | 111.739| 1.709  | 110.321|
| 35.0  | 1.494  | 122.090| 1.533  | 120.521| 1.592  | 120.077|
| 40.0  | 1.377  | 130.156| 1.416  | 128.062| 1.475  | 127.323|
| 50.0  | 1.201  | 145.117| 1.260  | 143.873| 1.318  | 142.548|

Table 8
Optimum $t_{c}^*$ and its cost rate $C_l(T ; t_{c}^*)$ when $\theta = 1.0, \alpha = 1.0, \beta = 2.0, T = 1.0$ and $c_m = 100.0.$

| $c_m$ | $k = 1$ | $k = 2$ | $k = 3$ |
|-------|--------|--------|--------|
|       | $t_{c}^*$ | $C_l(T ; t_{c}^*)$ | $t_{c}^*$ | $C_l(T ; t_{c}^*)$ | $t_{c}^*$ | $C_l(T ; t_{c}^*)$ |
| 10.0  | 3.096  | 70.275 | 3.564  | 83.445 | 4.189  | 97.977 |
| 15.0  | 2.393  | 84.318 | 2.588  | 95.870 | 2.861  | 107.121|
| 20.0  | 1.982  | 96.018 | 2.041  | 105.952| 2.158  | 114.704|
| 25.0  | 1.709  | 106.350| 1.689  | 114.862| 1.709  | 120.919|
| 30.0  | 1.514  | 115.902| 1.436  | 122.600| 1.416  | 127.524|
| 35.0  | 1.357  | 124.281| 1.240  | 129.362| 1.182  | 132.372|
| 40.0  | 1.221  | 131.098| 1.084  | 135.342| 1.025  | 138.782|
| 50.0  | 1.045  | 146.294| 0.869  | 147.693| 0.791  | 150.040|

3.3. Replacement last

We plan that the unit is replaced at time $T$ or at time $T_0 + t_e,$ whichever takes place last; however, only minimal repairs can be done during the mission duration $[T_0, T_0 + t_e]$. Then, the mean time to replacement is

$$TY(T-t_e) + \int_{T-t_e}^{\infty} (t+t_e)dY(t).$$

Using the renewal reward theorem [1], the expected cost rate is

$$C_l(T ; t_e) = c_m\left[H(T)Y(T-t_e) + \int_{T-t_e}^{\infty} H(t+t_e)dY(t)\right] + c_p \frac{TY(T-t_e) + \int_{T-t_e}^{\infty} (t+t_e)dY(t)}{TY(T-t_e) + \int_{T-t_e}^{\infty} dY(t)} (T \geq t_e).$$

We find optimum replacement time $T_f^*$ and mission duration $t_{c}^*$ to minimize $C_l(T ; t_e)$ in (30). Differentiating $C_l(T ; t_e)$ with respect to $T$ and setting it equal to zero,

$$h(T) \left[TY(T-t_e) + \int_{T-t_e}^{\infty} (t+t_e)dY(t)\right] - \left[H(T)Y(T-t_e) + \int_{T-t_e}^{\infty} H(t+t_e)dY(t)\right] = \frac{c_p}{c_m}$$

whose left-hand side increases with $T$ to $\infty$ as $h(\infty) \to \infty$. Therefore, there exists a finite and unique $T_f^*$ ($t_e \leq T_f^* < \infty$) which satisfies (31), and the resulting cost rate is

$$C_l(T ; t_e) = c_m h(T_f^*).$$

Differentiating $C_l(T ; t_e)$ with respect to $t_e$ and setting it equal to zero,

$$h_l(T ; t_e) \left[TY(T-t_e) + \int_{T-t_e}^{\infty} (t+t_e)dY(t)\right] - \left[H(T)Y(T-t_e) + \int_{T-t_e}^{\infty} H(t+t_e)dY(t)\right] = \frac{c_p}{c_m}$$

where

$$h_l(T ; t_e) \equiv \frac{\int_{T-t_e}^{\infty} h(t+t_e)dY(t)}{Y(T-t_e)} > h(T).$$

Letting $L_l(T ; t_e)$ be the left-hand side of (33), $dL_l(T ; t_e)/dt_e > 0$ if $h_l(T ; t_e)$ increases with $t_e$. Therefore, we have the following optimum policies:

1. If $L_l(T ; 0) \geq c_p/(c_f - c_p),$ then $t_{c}^* = 0.$
2. If $L_l(T ; 0) < c_p/(c_f - c_p) < L_l(T ; T),$ then there exists a finite and unique $t_{c}^* \left(0 < t_{c}^* < T \right)$ which satisfies (33), and the resulting cost rate is

$$C_l(T ; t_{c}^*) = c_m h_l(T ; t_{c}^*).$$

3. If $L_l(T ; T) \leq c_p/(c_f - c_p),$ then $t_{c}^* = T.$
When \(y(t) = \theta t^{\alpha-1} e^{-\theta t}/\Gamma(k)\), and \(F(t) = 1 - e^{-\theta t}\), Table 9 presents optimum \(T_f\) and its cost rate \(C(T_f; t_k)\) for \(t_k\) and \(c_m\) when \(\theta = 1.0\), \(\alpha = 1.0\), \(\beta = 2.0\), \(t_k = 1.0\), and \(c_p = 100.0\), and Table 10 presents optimum \(T_f\) and its cost rate \(C(T_f; t_k)\) for \(k\) and \(c_m\) when \(\theta = 1.0\), \(\alpha = 1.0\), \(\beta = 2.0\), \(T = 1.0\), and \(c_p = 100.0\).

When \(c_m = 10.0\), Figs. 9 and 10 represent the curve of the objective function \(C(T_f; t_k)\) in (30) for given \(k = 1, 2, 3\), respectively. It is clear that the approximate policies in Figs. 9 and 10 agree with those in Tables 9 and 10 when \(c_m = 10.0\).

4. Case illustration

We next illustrate the above discussions in maintaining the production system in glass factories based on the assumed data. It has been well-known that the equipments in glass factories need to be running all the time even if there is no order, as the glass production requires very high temperatures, and it takes a lot of energy, i.e., production cost, to raise the furnace temperature again when the order arrives. On the other hand, it is not easy to stop the equipments for preventive maintenance due to the requirements such as melting procedure, keeping tin liquid, step cooling for tempered glass production, and etc.

For Section 2, we assume that the mean malfunction time of the furnace is about 2.977 years (1087 days), e.g., assigning \(\alpha = 0.3\) and \(\beta = 3.0\) for the above Weibull distribution. In addition, we assume that corrective maintenance cost after malfunction is 1,000,000.00 USD and preventive maintenance costs 100,000.00 USD.

When some emergency order arrives at about 2 years (730 days) or later, e.g., assigning \(\theta = 1.0\) and \(k = 2\) for the above gamma distribution, and lasts 45 days (0.123 years) or longer, it becomes an important problem to plan maintenance policies to preventive possible malfunction of the furnace. From Fig. 11, we know that maintenance first should be planned at 1.304 years (476 days) and costs 124,720.00 USD in average, and maintenance last should be planned at 1.610 years (588 days) and costs 189,420.00 USD in average. In this case, maintenance first is better than maintenance last. Actually, maintenance is done at 476 days as planned for maintenance first, but for maintenance last, it should be done at 775 days after the production of emergency order, which is close to the mean malfunction time.

However, when we assign \(\theta = 1.0\) and \(k = 1\), i.e., the emergency order arrives at an early time, e.g., 1 year (365 days), and lasts for a shorter duration, e.g., 30 days, maintenance last is better than maintenance first. In this case, we will not worry about the production, as the reliability of the furnace is quite high, so that maintenance first done at an early time should not be applied. From Fig. 12, we know that maintenance first should be planned at 1.475 years (538 days) and costs 158,800.00 USD in average, and maintenance last should be planned at 1.402 years (511 days) and costs 143,220.00 USD in average. Actually, maintenance is done at 395 days after the production of emergency order for maintenance first, but for maintenance last, it should be done

| Table 9 | Optimum \(T_f\) and its cost rate \(C(T_f; t_k)\) when \(\theta = 1.0\), \(\alpha = 1.0\), \(\beta = 2.0\), \(t_k = 1.0\), and \(c_p = 100.0\). |
|---------|----------------------------------------------------------------------------|
| \(c_m\) | \(k = 1\) | \(k = 2\) | \(k = 3\) |
| \(T_f\) | \(C(T_f; t_k)\) | \(T_f\) | \(C(T_f; t_k)\) | \(T_f\) | \(C(T_f; t_k)\) |
| 10.0 | 3.195 | 63.906 | 3.320 | 66.406 | 3.525 | 71.084 |
| 15.0 | 2.648 | 79.453 | 2.852 | 85.547 | 3.154 | 94.629 |
| 20.0 | 2.352 | 94.062 | 2.602 | 104.063 | 2.959 | 118.359 |
| 25.0 | 2.148 | 107.422 | 2.461 | 123.047 | 2.822 | 141.113 |
| 30.0 | 2.008 | 120.469 | 2.352 | 141.094 | 2.744 | 164.648 |
| 35.0 | 1.914 | 133.984 | 2.273 | 159.141 | 2.686 | 187.988 |
| 40.0 | 1.836 | 146.875 | 2.227 | 176.125 | 2.646 | 211.719 |
| 45.0 | 1.773 | 159.609 | 2.180 | 196.172 | 2.607 | 234.668 |
| 50.0 | 1.727 | 172.656 | 2.148 | 214.844 | 2.588 | 258.789 |

| Table 10 | Optimum \(T_f\) and its cost rate \(C(T_f; t_k)\) when \(\theta = 1.0\), \(\alpha = 1.0\), \(\beta = 2.0\), \(T = 1.0\) and \(c_p = 100.0\). |
|---------|----------------------------------------------------------------------------|
| \(c_m\) | \(k = 1\) | \(k = 2\) | \(k = 3\) |
| \(T_f\) | \(C(T_f; t_k)\) | \(T_f\) | \(C(T_f; t_k)\) | \(T_f\) | \(C(T_f; t_k)\) |
| 10.0 | 1.0 | 74.987 | 1.0 | 69.210 | 0.592 | 71.635 |
| 15.0 | 1.0 | 87.474 | 0.946 | 88.278 | 0.919 | 91.996 |
| 20.0 | 1.0 | 99.961 | 0.435 | 105.036 | 0.111 | 151.521 |
| 25.0 | 1.0 | 112.448 | 0.0 | 120.208 | 0.131 | 101.047 |
| 30.0 | 1.0 | 124.935 | 0.0 | 134.717 | 0.150 | 157.502 |
| 35.0 | 0 | 136.252 | 0 | 149.226 | 0.170 | 197.098 |
| 40.0 | 0 | 145.270 | 0 | 163.735 | 0.189 | 623.623 |
| 45.0 | 0 | 154.287 | 0 | 178.244 | 0.209 | 149.149 |
| 50.0 | 0 | 163.304 | 0 | 192.753 | 0.228 | 675.675 |

Fig. 9. Replacement time and its cost rate when \(\theta = 1.0\), \(\alpha = 1.0\), \(\beta = 2.0\), \(T = 1.0\), \(c_p = 100.0\) and \(c_m = 10.0\).

Fig. 10. Mission duration and its cost rate when \(\theta = 1.0\), \(\alpha = 1.0\), \(\beta = 2.0\), \(T = 1.0\), \(c_p = 100.0\) and \(c_m = 10.0\).
at 511 days as planned. Maintenance first costs more as it is done at an early time when the furnace is still reliable to run for a longer time.

When minimal repair is considered in Section 3, similar illustrations for maintenance first and last policies can also be shown in Figs. 13 and 14 according to the early or late arrival times of orders. We assign that $\alpha = 0.7$ and $\beta = 3.0$, i.e., the mean numbers of malfunctions are about 0.3 times in 1 year, 1.2 times in 1.5 years, 2.7 times in 2 years, 9.3 times in 3 years, and so on. In other words, the total cost of minimal repairs will increase greatly till 3 years, and maintenance policies should be done at suitable times.

We assume that maintenance cost is 1,000,000.00 USD and the cost of each minimal repair is 100,000.00 USD. When $t_x = 0.0411$, $\theta = 1.0$ and $\kappa = 3$, i.e., the emergency order arrives at about 3 years and lasts for 15 days, it is obviously that maintenance first should be applied to prevent the increasing cost of minimal repairs. In Fig. 13, maintenance first is done at 2.526 years (922 days) and its cost rate is 657,062.00 USD.

However, when $\kappa = 1$, i.e., the emergency order arrives at an early time, i.e., 1 year. In this case, maintenance last should be applied to prevent early maintenance policies. In Fig. 14, maintenance last is done at 2.492 years (910 days) and its cost rate is 639,150.00 USD.

5. Conclusions

In this paper, replacement policies that are collaborative with time of operations, mission durations, minimal repairs and maintenance triggering approaches have been studied. The time $T_o$ and the duration $t_x$ have been considered as a random arrival time of the mission and its mission duration, respectively. We have discussed replacement policies for random arrival times of mission durations. An interesting point would be that no replacement can be done during the mission duration $[T_o, T_o + t_x]$. Optimum replacement times and mission durations have been obtained for the models of replacement first and last, respectively, where they are alternatively from the points of cost and maintainability. Numerical examples have been illustrated when the arrival time of the mission follows a gamma distribution and the failure time of the unit has a Weibull distribution. Simple case illustrations in maintaining the production system in glass factories have been given based one the assumed data, via comparing maintenance first and last policies.
We have known from literatures that mission success probabilities, mission durations and mission abort policies are important reliability analyses in recent years, and we believe that the maintenance models with mission durations should also be very important for the mission-oriented reliability systems. In our future works, we may revisit the number of redundant units that should be prepared for the random arrival times of mission durations to provide system reliability.

**Conflict of interest**

None declared.

**Appendix A**

**Appendix A.1.** When the failure rate $h(t)$ increases strictly with $t$ from $h(0)$ to $h(\infty)$,

$$h_j(T, t_x) \equiv \frac{\int_0^T f(t + t_x) dY(t)}{\int_0^T F(t + t_x) dY(t)}$$

increases strictly with $T$ from $h(t_x)$ to $h(\infty)$, and when $Y(t) = 1 - e^{-\theta t}$, and $h_j(T, t_x)$ increases strictly with $t_x$ from $h(T, 0)$ to $h(\infty)$.

**Proof.** Note that $h(t_x) \leq h_j(T, t_x) \leq h(T + t_x)$. Differentiating $h_j(T, t_x)$ with $T$,

$$f(T + t_x)f(T) \int_0^T \mathcal{F}(T + t_x) dY(t) - \mathcal{F}(T + t_x)f(T) \int_0^T f(t + t_x) dY(t)$$

which follows that $h_j(T, t_x)$ increases strictly with $T$ from $h(t_x)$ to $h(\infty)$.

$$h_j(T, t_x) \equiv \frac{\int_0^{T + t_x} f(\theta) e^{-\theta t} d\theta}{\int_0^{T + t_x} \mathcal{F}(\theta) e^{-\theta t} d\theta}$$

Differentiating $h_j(T, t_x)$ with $t_x$, the proof becomes

$$f(T + t_x)e^{-\theta \theta} - f(t_x)$$

Note that

$$f(T + t_x)e^{-\theta \theta} - f(t_x) + \mathcal{F}(T + t_x)e^{-\theta \theta}h(T + t_x)$$

i.e.,

$$f(T + t_x)e^{-\theta \theta} - f(t_x)$$

which completes the proof as $h(t_x) \leq h_j(T, t_x)$.

**Appendix A.2.** When the failure rate $h(t)$ increases strictly with $t$ from $h(0)$ to $h(\infty)$,

$$\Lambda(t; x) \equiv \frac{F(t + x) - F(t)}{\int_0^t \mathcal{F}(u) du} \quad (0 < x < \infty)$$

increases with $t$ from $F(x)/\int_0^t \mathcal{F}(u) du$ to $h(\infty)$.

**Proof.** Note that

$$\Lambda(t; x) = \frac{\frac{F(t + x) - F(t)}{\int_0^t \mathcal{F}(u) du}}{\int_0^t \mathcal{F}(u) du} = \frac{\dot{\lambda}(t; x)}{\int_0^t \mathcal{F}(u) du}$$

where $\dot{\lambda}(t; x)$ is given in (1). Obviously,

**Acknowledgements**

This research work is supported by National Natural Science Foundation of China (No. 71801126), Natural Science Foundation of Jiangsu Province (No. BK20180412), Aeronautical Science Foundation of China (No. 2018ZGS2080), and Japan Society for the Promotion of Science KAKENHI (No. 18K01713).
Differentiating \( \int_t^{t+x} F(u)du / F(t) \) with \( t \), and noting that
\[
h(t) \int_t^{t+x} \frac{F(u)}{h(t)} du - [F(t+x) - F(t)] \\
\leq h(t) \int_t^{t+x} \frac{F'(u)}{h(t)} du - [F(t+x) - F(t)] = 0.
\]
which shows that \( \int_t^{t+x} F(u)du / F(t) \) decreases with \( t \) from \( \int_0^x F(u)du \) to \( 1/h(\infty) \). Due to \( \lambda(t; x) \) increases with \( t \) from \( F(x) \) to 1, we complete the proof that \( \Lambda(t; x) \) increases with \( t \) from \( F(x) / \int_0^x F(u)du \) to \( h(\infty) \).

References

[1] Barlow RE, Proschan F. Mathematical Theory of Reliability. Wiley; 1965.
[2] Nakagawa T. Maintenance Theory of Reliability. Springer; 2005.
[3] Jiang Y, Chen M, Zhou D. Joint optimization of preventive maintenance and inventory policies for multi-unit systems subject to deteriorating spare part inventory. J Manuf Syst 2015;35:191–205.
[4] Park M, Pham H. Cost models for age replacement policies and block replacement policies under warranty. Appl Math Model 2016;40:5689–702.
[5] Zhao X, Nakagawa T. Advanced Maintenance Policies for Shock and Damage Models. Springer; 2018.
[6] Li Y, Ye Z, Lee C, Yang S, Peng R. A two-phase preventive maintenance policy considering imperfect repair and postponed replacement. Eur J Oper Res 2019;274:966–77.
[7] Młynarski S, Pilch R, Smolnik M, Szybka J, Wiazania G. A model of an adaptive strategy of preventive maintenance of complex technical objects. Eksploatacja i Niezawodność [Maintenance Reliability] 2020;22:35–41.
[8] Nakagawa T. Random Maintenance Policies. Springer; 2014.
[9] Nakagawa T, Zhao X. Maintenance Overtime Policies in Reliability Theory. Springer; 2015.
[10] Shu SH, Liu TH, Zhang ZG, Tsai HN. The generalized age maintenance policies with random working times. Reliab Eng Syst Saf 2018;169:503–14.
[11] Chen M, Zhao X, Nakagawa T. Replacement policies with general models. Ann Oper Res 2019;277:47–61.
[12] Chang CC, Chen YL. Optimization of continuous and discrete scheduled times for a cumulative damage system with age-dependent maintenance. Commun Stat Theory Methods 2019;48:1381–77.
[13] Zhao X, Gao G, Yang Y, Nakagawa T. Approximate calculations of age-based random replacement times. Commun Stat Theory Methods 2020. https://doi.org/10.1080/03610926.2019.1712023. Online.
[14] Chen X, An Y, Zhang Z, Li Y. An approximate non-dominated sorting genetic algorithm to integrate optimization of production scheduling and accurate maintenance based on reliability intervals. J Manuf Syst 2020;54:227–41.
[15] Zhao X, Al-Khalifa KN, Hamouda AMS, Nakagawa T. First and last triggering event approaches for replacement with minimal repairs. IEEE Trans Reliab 2016;65:197–207.
[16] Ba Zuhair M. Balancing an aircraft with symmetrically deflected split elevator and rudder during short landing run. Aviation. 2019;23:23–30.