A nonlocal wave-wave interaction among Alfvén waves in an intermediate-β plasma

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A nonlocal coupling mechanism to directly transfer the energy from large-scale Magnetohydrodynamic (MHD) Alfvén waves to small-scale kinetic Alfvén waves is presented. It is shown that the interaction between a MHD Alfvén wave and a reversely propagating kinetic Alfvén wave can generate another kinetic Alfvén wave, and this interaction exists in the plasmas where the thermal to magnetic pressure ratio is larger than the electron to ion mass ratio. The proposed nonlocal interaction may have a potential application to account for the observed electron scale kinetic Alfvén waves in the solar wind and solar corona plasmas.

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I. INTRODUCTION

The large-scale magnetodyndromatic (MHD) Alfvén wave, which is generated by the magnetic stress tensor, is a nondispersive low frequency wave mode and can be directly derived from MHD equations. The dispersive Alfvén wave arises due to the finite-frequency and small-scale modifications. The dispersive Alfvén wave is specially called as the kinetic Alfvén wave (KAW) when its perpendicular wavelength is close to the order of the ion gyroradius, the ion acoustic gyroradius, or the electron inertial length. The KAW can accelerate and heat electrons by its parallel electric field and interact with ions through its perpendicular electric fields. The properties of the KAW have been demonstrated by many experimental investigations in space and laboratory plasmas. In particular, the KAW can play an important role in the acceleration of energetic electrons in Earth’s aurora and solar flares as well as the heating of solar coronal plasma.

Extensive works have been done to discuss the local interaction process of three Alfvén waves in which the magnitudes of three wavenumbers are comparable. A large amplitude dispersive Alfvén wave can bring about the local nonlinear decay among themselves. Local interaction between two counterpropagating Alfvén waves can couple the wave energy to cascade from the energy injection region to the energy dissipation region in the MHD turbulence. Through the local cascade, the KAW can be generated at scales of the order of the ion inertial length or the ion gyroradius in the solar wind turbulence, however, it cannot reach electron scales due to the large electron Landau damping in these small scales.

Recently, some works have found that the nonlinear coupling of Alfvén waves with different scales can occur. Voitenko and Goossens presented a nonlocal interaction among three Alfvén waves and showed that a large-scale MHD Alfvén wave can decay parametrically into two small-scale KAWs. Shukla and Stenflo studied the three-wave interaction involving two KAWs and one field-aligned Alfvén wave with the finite-frequency modification, and they also showed two small-scale KAWs can be nonlocally excited by this large-scale Alfvén wave. Unlike the local energy cascade in the MHD turbulence, the wave energy can be directly transferred from large-scale Alfvén waves to small-scale KAWs in these nonlocal interaction processes.

In this paper, we investigate the nonlocal interaction among one MHD Alfvén wave and two KAWs, and we shall show that the nonlocal coupling, a MHD Alfvén wave + KAW → KAW, may play an important role in generating the KAWs with electron scales. The reminder of this paper is organized in the following fashion. The qualitative and quantitative analyses are given in section 2 and section 3, respectively. Section 4 presents an application in the solar corona. The discussion is set in section 5 and the summary is contained in section 6.

II. QUALITATIVE ANALYSIS

Three waves in the nonlinear coupling process (a MHD Alfvén wave + KAW 1 → KAW 2) must satisfy the resonant relation, which describes the phase relation of these three waves,

\[ \omega_s + \omega_1 = \omega_2 \]
\[ k_s + k_1 = k_2 \] (1)

where \( \omega_s \) and \( k_s \) are the frequency and wave vector of the large-scale MHD Alfvén wave, respectively; \( \omega_{1,2} \) and \( k_{1,2} \) are frequencies and wave vectors of the two KAWs, respectively. In this study, we consider the general oblique propagating MHD Alfvén wave, where wave vector \( \mathbf{k}_s = k_{s,z} \mathbf{z} + k_{s,x} \mathbf{x} \), and \( k_{s,z} \) and \( k_{s,x} \) are the wave vectors perpendicular and parallel to the background magnetic field \( B_0 \mathbf{z} \), respectively.
The linear dispersion relation of the MHD Alfvén wave is \( \omega_A = V_A k_z \), where \( V_A \) is the Alfvén velocity. For the two KAWs in (1), their linear dispersion relations are given as \( \omega_{1,2} = V_A k_{1,2} K_{1,2} \) with \( K_{1,2} = \sqrt{1 + \rho^2 k_{1,2}^2} \) for the intermediate-beta plasmas \( (m_e/m_i \ll \beta \ll 1) \) and \( K_{1,2} = 1/\sqrt{1 + \lambda_e^2 k_{1,2}^2} \) for the low-beta plasmas \( (\beta \ll m_e/m_i \ll 1) \), where \( k_{1,2} \) and \( k_{1,2}^\perp \) are the parallel and perpendicular wavenumbers of the two KAWs, respectively, \( \rho^2 = \rho_i^2 + \rho_s^2 \) (\( \rho_i \) is the ion gyroradius and \( \rho_s \) is the ion acoustic gyroradius), \( \lambda_e \) is the electron inertial length, \( \beta \) is the ratio of the plasma thermal to magnetic pressures, and \( m_e/m_i \) is the electron to ion mass ratio.

Substituting the linear dispersion relations of the MHD Alfvén wave and two KAWs into the frequency relation in (1) and then combining the z-direction wave-vector relation, we can obtain a restricted relation for two wavenumbers of the two KAWs,

\[
k_{1z}(K_1 - s_1) = k_{2z}(K_2 - s_2),
\]

or be written as

\[
(K_1 - s_1)(K_2 - s_2) > 0.
\]

where the MHD Alfvén wave is assumed along the background magnetic field. The subscripts \( s_1 \), \( s_2 \) denote the directions of the two KAWs, for example, \( s_1, s_2 = 1 \) stand for two waves having the same directions as the MHD Alfvén wave and \( s_1, s_2 = -1 \) are two reversely-propagating waves.

There exist three kinds of wave-wave interaction, \( s_1, s_2 = \pm 1 \) and \( s_1 = -s_2 = -1 \), for the coupling of one MHD Alfvén wave and two KAWs. In the first two interaction cases \( (s_1, s_2 = \pm 1) \), the frequency of the KAW 1 is required much larger than the MHD Alfvén wave frequency, and we do not discuss these two cases in this study. The third case \( (s_1 = -s_2 = -1) \), as shown in Fig. 1, describes the parallel-propagating KAW 2 generated in the nonlinear interaction between the parallel-propagating MHD Alfvén wave and the reversely-propagating KAW 1. From the restricted relation Eq. (3), we can see that the third interaction takes place under the condition \( K_2 > 1 \), which means this interaction only exists in the plasmas with \( \beta > m_e/m_i \). Further, we can obtain \( \omega_1 = \omega_s(K_2 - 1)/(K_2/K_1 + 1) \) and \( \omega_2 = \omega_s(K_1 + 1)/(K_1/K_2 + 1) \) from Eqs. (1) and (2).

### III. Two-Fluid Model

Let us consider the homogeneous and collisionless plasmas with two species of particles, electrons and ions. We restrict the plasmas in the intermediate-beta range \( (m_e/m_i \ll \beta \ll 1) \), where the ion finite Larmor radius and ion polarization contributions are both important for the KAWs. The momentum equation is,

\[
\partial_t \mathbf{v}_j + \frac{T_j}{m_j n_j} \nabla n_j - \frac{q_j}{m_j} (\mathbf{E} + \mathbf{v}_j \times \mathbf{B}_0) = -\mathbf{v}_j \cdot \nabla \mathbf{v}_j + \frac{q_j}{m_j} \mathbf{v}_j \times \mathbf{B},
\]

where \( \mathbf{v}_j, m_j, q_j, n_j, \) and \( T_j \) denote the particle velocity, mass, charge, number density, and temperature for species \( j \), respectively; \( \mathbf{E} \) and \( \mathbf{B} \) are the wave electric and magnetic field perturbations, respectively. Note that the isothermal assumption has been used in (4).

The variables can be written as, \( \mathbf{v}_j = \mathbf{v}_{js} + \mathbf{v}_{j1} + \mathbf{v}_{j2}, \) \( n_j = n_0 + n_{j1} + n_{j2}, \) \( \mathbf{E} = \mathbf{E}_s + \mathbf{E}_1 + \mathbf{E}_2 \) and \( \mathbf{B} = \mathbf{B}_s + \mathbf{B}_1 + \mathbf{B}_2 \) (where \( n_0 \) is the background number density). The large-scale MHD Alfvén wave only has the perpendicular velocity and electromagnetic perturbations: \( \mathbf{v}_s = \mathbf{v}_{s\perp}, \mathbf{E}_s = \mathbf{E}_{s\perp} \) and \( \mathbf{B}_s = \mathbf{B}_{s\perp} \). By substituting these expressions into the momentum Eq. (4), we can obtain the fluid velocities of the KAW 2,

\[
\mathbf{v}_{e2\perp} \simeq \frac{1}{B_0} \mathbf{E}_{2\perp} \times \hat{z} - \frac{v_T^2}{\omega_{ce}} \nabla z \frac{n_{e2}}{n_0} \times \hat{z} + \frac{1}{B_0 \omega_{ce}} \partial_z \mathbf{E}_{2\perp},
\]

\[
\mathbf{v}_{c2\perp} \simeq \frac{1}{B_0} \mathbf{E}_{2\perp} \times \hat{z} - \frac{v_T^2}{\omega_{ce}} \nabla z \frac{n_{e2}}{n_0} \times \hat{z} + \frac{1}{B_0} (\mathbf{v}_{e1\perp} \times \mathbf{B}_{s\perp}) \times \hat{z},
\]

and

\[
\partial_t \mathbf{v}_{e2z} \simeq -\frac{e}{m_e} \mathbf{E}_{2z} - \frac{v_T^2}{\omega_{Te}} \partial_z \frac{n_{e2}}{n_0} \mathbf{E}_{2z} - \frac{e}{m_e} [(\mathbf{v}_{e1\perp} + \mathbf{v}_{e1d}) \times \mathbf{B}_{s\perp} + \mathbf{v}_{e2\perp} \times \mathbf{B}_{1\perp}] \cdot \hat{z}.
\]
where $\omega_{ci} = eB_0/m_i$ and $\omega_{ce} = -eB_0/m_e$ are the ion and electron cyclotron frequencies, respectively; $v_{T_i} = (T_i/m_i)^{1/2}$ and $v_{Ti} = (T_{ei}/m_e)^{1/2}$ are the ion and electron thermal velocities, respectively; $v_{i1E} = (1/B_0)E_{1\perp} \times \hat{z}$ and $v_{i1D} = -(v_{T_i}^2/\omega_{ci})V_{\perp}(n_{i1}/n_0) \times \hat{z}$ denote the ion electric field drift velocity and the ion diamagnetic drift velocity of the KAW 1, respectively; $v_{e1z}$, $v_{e1E} = (1/B_0)E_{1\perp} \times \hat{z}$ and $v_{e1D} = -(v_{T_e}^2/\omega_{ce})V_{\perp}(n_{e1}/n_0) \times \hat{z}$ denote the electron parallel velocity, the electric field drift velocity, and the electron diamagnetic drift velocity of the KAW 1, respectively.

Here, the nonlinear terms due to the thermal pressure gradient are neglected because they are smaller than the nonlinear effects in (5) - (7). This study is limited to the weak nonlinear system, where a Fourier analysis can be used. That is, the wave amplitudes are restricted as small amplitudes, and the ratio of energy in the oscillations to the total energy of the plasma is a small parameter (e.g. 0.2). The waves in this system are assumed to be three monochromatic waves, and their amplitudes can be expressed as $A_l = A_k e^{-i \omega_{l}(t + \mathbf{k}_l \cdot \mathbf{r})}$, where $l = (1, 2, s)$.

The ion and electron number density perturbations in equations (5) - (7) can be obtained by using the ion continuity equation and the quasi-neutrality condition $(n_{2i} = n_{2e} \equiv n_2)$,

$$(1 + \rho_i^2 k_{2\perp}^2)n_2/n_0 \simeq -\frac{i e}{m_i \omega_{ci}} k_{2\perp} \cdot E_{2\perp}. \quad (8)$$

The wave electromagnetic fields $E_2$ and $B_2$ in (5) - (8) can be conveniently represented by a scalar potential $\phi_2$ and a z-direction vector potential $A_{2z}$,

$$E_2 = -i k_2 \phi_2 + i \omega_2 (A_{2z} \hat{z}), \quad B_2 = i k_{2\perp} \times (A_{2z} \hat{z}). \quad (9)$$

By using (5) - (9), the current density $J_2 = e n_0 (v_{i2\perp} - v_{e2\perp} - v_{e2z}) \hat{z}$ is written as,

$$k_{2\perp} \cdot J_{2\perp} \simeq -\frac{n_0 e^2}{m_i \omega_{ci}^2} \frac{1 + \rho_i^2 k_{2\perp}^2}{1 + \rho_i^2 k_{2\perp}^2} \omega_2^2 \phi_2 - \frac{n_0 e^2}{\omega_{ci}} k_{2\perp} \cdot \omega_2 A_{2z}$$

$$(v_{i2\perp} \cdot i k_{1\perp} (v_{i1E} + v_{i1D}) + \frac{e}{m_i} v_{e1\perp} \times B_{s\perp}) \cdot \hat{z}, \quad (10)$$

and,

$$\omega_2 J_{2z} \simeq -\frac{n_0 e^2}{m_e} \frac{1 + \rho_i^2 k_{2\perp}^2}{1 + \rho_i^2 k_{2\perp}^2} s_2 k_{2\perp} \phi_2 - \frac{n_0 e^2}{m_e} \omega_2 A_{2z}$$

$$- \frac{n_0 e^2}{m_e} (v_{e1E} + v_{e1D}) \times B_{s\perp} + v_{e2\perp} \times B_{1\perp} \cdot \hat{z}. \quad (11)$$

The current density can also be given in terms of $A_{2z}$ by Ampere’s law $\mu_0 J_2 = \nabla \times B_2$,

$$\mu_0 J_{2\perp} = -k_{2\perp} k_{2z} A_{2z}, \quad \mu_0 J_{2z} = k_{2\perp}^2 A_{2z}. \quad (12)$$

From Eqs. (10) and (12), a nonlinear equation for the KAW 2 is given by,

$$[\omega_2^2 - V_2^2 (1 + \rho_i^2 k_{2\perp}^2) k_{2\perp}^2] \phi_2 = -B_0 (1 + \rho_i^2 k_{2\perp}^2 \omega_2^2) k_{2\perp}^2$$

$$\{ [v_{i\perp} \cdot i k_{1\perp} (v_{i1E} + v_{i1D}) + \frac{e}{m_i} v_{e1\perp} \times B_{s\perp}] \times \hat{z} \}$$

$$+ V_2^2 (1 + \rho_i^2 k_{1\perp}^2) i s_2 k_{2z}$$

$$\{ [v_{i1E} + v_{i1D}] \times B_{s\perp} + v_{e2\perp} \times B_{1\perp} \} \cdot \hat{z}. \quad (13)$$

where the first nonlinear term on the right-hand of the equation comes from the ion convective motion and perpendicular electron nonlinear Lorentz force, and the second nonlinear term arises due to the parallel electron nonlinear Lorentz force. Three nonlinear effects (the ion convective motion, the parallel and perpendicular electron nonlinear Lorentz forces) are all important in the nonlocal coupling of one MHD Alfvén wave and two KAWs, and these three effects are also contained in some other nonlinear models in simplified forms (e.g. 37, 42, 49).

The variables of the KAW 1 in the nonlinear Eq. (13) can be expressed in terms of the scalar potential $\phi_1$,

$$v_{i1E} = v_{i1E} = -\frac{i}{B_0} k_{1\perp} \times \hat{z} \phi_1, \quad v_{i1D} = \frac{i}{B_0} \rho_i^2 k_{1\perp}^2 k_{1\perp} \times \hat{z} \phi_1, \quad v_{e1\perp} = \frac{i}{B_0} \rho_i^2 k_{1\perp}^2 k_{1\perp} \times \hat{z} \phi_1, \quad v_{e1z} = \omega_{ci} k_{1\perp} \times \hat{z} \phi_1, \quad B_{1\perp} = -\omega_{ci} \frac{k_{1\perp}^2}{s_1 k_{12} \omega_1} \phi_1, \quad (14)$$

For the obliquely propagating MHD Alfvén wave, we use a scalar potential $\phi_s$ to express the electric field perturbation $E_s = -\nabla \phi_s$ and other variables in Eq. (13),

$$v_{s\perp} = v_{s\perp} = -\frac{V_A}{B_0} B_{s\perp} = -\frac{i}{B_0} k_{s\perp} \times \hat{z} \phi_s. \quad (15)$$

By using linear relations in Eqs. (14) and (15), we can obtain the nonlinear dispersion relation of the KAW 2,

$$[\omega_2^2 - V_2^2 (1 + \rho_i^2 k_{2\perp}^2) k_{2\perp}^2] \phi_2 = \frac{\omega_2^2 + \rho_i^2 k_{2\perp}^2}{B_0 (1 + \rho_i^2 k_{1\perp}^2)}$$

$$\{ [s_1 k_{1\perp} k_{12}^2 + k_{2\perp}^2 (1 - s_1 k_{1\perp}^2) k_{1\perp}] \} \cdot \phi_1 \quad (16)$$

where $K_{1,2} = \sqrt{1 + \rho_i^2 k_{1,2\perp}^2}$ is the definition in the intermediate-beta plasmas and the relation $k_{1\perp} \approx k_{2\perp} \gg k_{s\perp}$ has been used. The nonlinear dispersion relation of the KAW 1 can be obtained by exchanging the subscripts 1 and 2 and replacing $\phi_1$ by $\phi_s$ in Eq. (16). We here restrict the MHD Alfvén wave as the finite amplitude wave and neglect its nonlinear modification caused by the two
KAWs. This finite amplitude assumption can be permitted for some solar and space plasma environments, such as the solar corona and the solar wind.

Two KAWs frequencies are expressed as \( \omega_1 = \omega_{ke} - i\gamma_1 \) for the KAW 1 and \( \omega_2 = \omega_{ke} + i\gamma_2 \) for the KAW 2, where \( \omega_{ke} \) and \( \gamma_1 \) denote the real parts of two frequencies, \( \gamma_2 \) is the decay rate of the KAW 1 and \( \gamma_2 \) is the growth rate of the KAW 2. To show which parameters affect the nonlocal coupling strength among three Alfvén waves, we assume \( \gamma_1 \simeq \gamma_2 \equiv \gamma \) and \( \gamma \ll \omega_r \). Then, \( \gamma \) can be derived from the nonlinear dispersion relations of the two KAWs,

\[
\gamma^2 = \frac{V_A^2 K_2 k_{2\perp}^2}{4 K_1} (s_1 s_2 + \frac{k_{1\perp}^2}{k_{2\perp}^2})(K_1 - s_1)(K_2 - s_2)\sin^2\theta B_0^2/B_0^2,
\]

(17)

where \( B_{\perp} = k_{\perp}\phi_s/V_A \) and \( \theta \) is the angle between \( k_{\perp} \) and \( k_{1\perp} \). From the equation (17), the restrict relation \( (K_1 - s_1)(K_2 - s_2) > 0 \) can be reobtained. The equation (17) is rewritten as the following form for the interaction case \( s_1 = -s_2 = -1 \),

\[
\gamma^2 = \frac{V_A^2 K_2 k_{2\perp}^2}{4 K_1} (-\frac{K_1}{K_2} + \frac{k_{2\perp}^2}{k_{2\perp}^2})(K_1 + 1)(K_2 - 1)\sin^2\theta B_0^2/B_0^2.
\]

(18)

IV. APPLICATION IN THE SOLAR CORONA

Here we use some typical parameters in the quiet solar corona to discuss the nonlocal interaction proposed in this study: the number density \( n = 10^9 \text{ cm}^{-3} \), the thermal temperature \( T_i = T_e = 10^6 \text{ K} \), and the magnetic field \( B_0 = 10 \text{ G} \). These values give the plasma beta \( \beta = 3.5 \times 10^{-2} \) that is located in the intermediate-beta range. The existence of the MHD Alfvén wave has been demonstrated in the solar corona plasmas by many observations\(^{31-33} \). The frequencies of these MHD Alfvén waves are in the range \( 10^{-5} \text{ Hz} \), \( 10^{-1} \text{ Hz} \)\(^{32,34} \), or the high frequency range \( (0.1 \text{ Hz}, 2.5 \text{ Hz}) \) if these waves are generated by the reconnection in the magnetic networks\(^{35} \). As an example, we consider two MHD Alfvén waves with the same frequencies \( 10^{-1} \text{ Hz} \) but different directions, \( k_{\perp} = k_{sz} (\rho k_{\perp} \sim 10^{-7}) \) and \( k_{\perp} = 10k_{sz} (\rho k_{\perp} \sim 10^{-6}) \). If the two KAWs locate in the frequency range \( (10^{-5} \text{ Hz}, 2.5 \text{ Hz}) \), we can use the frequency relation \( \omega_1 = \omega_s (K_2 - 1)/(K_2/K_1 + 1) \) to estimate the limitation of the perpendicular wavenumber for the KAW 1. It shows that \( \rho k_{\perp} \) is required larger than 0.02, meanwhile, we can also obtain the limitation of \( \rho k_{2\perp} \) because of \( \rho k_{1\perp} \sim \rho k_{2\perp} \).

Figure 2 gives the relation between the maximal coupling strength \( \gamma \) and the perpendicular wavenumber of the KAW 2 \( \rho k_{2\perp} \), where the solid line denotes the MHD Alfvén wave propagating case \( k_{\perp} = k_{sz} \) and the dashed line is for the case \( k_{\perp} = 10k_{sz} \). Figure 2 shows that the coupling strength \( \gamma \) increases with the increment of the \( \rho k_{2\perp} \) and the \( \gamma \) is large for the more obliquely propagating MHD Alfvén wave.

The new generated KAW can suffer the electron Landau damping in the collisionless plasma\(^{35} \), and the expression of the electron Landau damping rate is given as \( \gamma_L/\omega_e = A(m_e/m_i)^{1/2}\beta_e^{1/2} \lambda_e^{5/2}\kappa_L^2 \), where \( \beta_e \) is the electron plasma beta, \( \lambda_e \) is the ion inertial length and \( A \) is a function of \( \beta_e \) and \( T_e/T_i \). For the low beta plasmas with \( \beta_e \ll 1 \), \( A \) is roughly 0.4. We can use the electron Landau damping rate and the expression (18) to give some estimations for the threshold amplitude of the MHD Alfvén wave.

Figure 3 gives the threshold amplitude of the MHD Alfvén wave as a function of the \( \rho k_{2\perp} \). Figure 3 shows that the threshold amplitude is smaller for the more obliquely propagating MHD Alfvén wave and for the KAWs with smaller \( \rho k_{2\perp} \). In the solar low corona, the relative amplitude of the MHD Alfvén wave nearly reaches 0.23, which is larger than all the threshold amplitudes shown in Figure 3. A small threshold amplitude, especially for the more obliquely propagating MHD Alfvén wave, implies that the nonlocal interaction mechanism may happen in the solar corona.

V. DISCUSSION

Our nonlinear wave-wave interaction requires a large-scale MHD Alfvén wave and a small-scale KAW with counterpropagating directions. These two waves can coexist in the solar and space plasmas. The MHD Alfvén wave is a common wave mode and can be easily generated by the shear motion and/or the magnetic reconnection process. The KAW can be produced by many mechanisms, such as the phase mixing and the resonant
absorption of the MHD Alfvén waves, the instability caused by the warm proton beams or by the ion-ion streaming, and the parametric decay by other wave modes. These wave generation mechanisms make it easy to produce the opposite traveling MHD Alfvén wave and KAW, thus causing the nonlocal interaction.

The mechanism proposed in this study is different from the other two kinds of interactions among three Alfvén waves, the local interaction and the nonlocal decay. The proposed wave-wave interaction happens for Alfvén waves with different scales, whereas the local interaction is limited to waves with comparable scales. The nonlocal decay mechanism also relates to three waves with different scales, however, it describes two waves excited by a pump wave. Of course, two nonlocal interactions, either through the nonlocal decay or through our nonlocal interaction, can both directly transport the Alfvén wave energy from the large-scale region to the small-scale region and then dissipate the wave energy there.

Last, let us give a simple discussion for a potential application of our nonlocal mechanism. Recent observations showed there exists the KAW even down to the electron scales (the electron inertial length or the electron gyroradius scale) in the solar wind. This electron scale KAW cannot be explained by the local wave cascade mechanism because the large electron Landau damping can totally dissipate the KAW energy in that small scale region. But our results implies, except for the local wave-wave interaction, the nonlocal wave-wave interaction may also happen in the Alfvén wave turbulence. Furthermore, as shown in figure 2, the nonlocal coupling is stronger for the smaller scale KAWs. So the observed electron scale KAWs may be produced by this nonlocal mechanism. It should be pointed out that the nonlocal turbulence of the KAWs still receives little attention and this problem will be further investigated in our future works.

VI. CONCLUSION

This paper discusses the nonlocal coupling of one MHD Alfvén wave and two KAWs, a MHD Alfvén wave + KAW 1 → KAW 2. The qualitative discussion shows that this wave-wave interaction works in the plasmas with $\beta > m_e/m_i$. The frequency relations among the two KAWs and the MHD Alfvén wave are $\omega_1 = \omega_s(K_2 - 1)/(K_2/K_1 + 1)$ and $\omega_2 = \omega_s(K_1 + 1)/(K_1/K_2 + 1)$. The quantitative discussion shows that the coupling is stronger for the more obliquely propagating MHD Alfvén wave and for the KAWs with larger perpendicular wavenumbers. We also show that the proposed mechanism may play an important role in generating electron scale KAWs in the solar wind.

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