Hybrid Meson Potentials and the Gluonic van der Waals Force

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The chromoelectric polarizability of mesons governs the strength of the gluonic van der Waals force and therefore of non-quark-exchange processes in hadronic physics. We compute the polarizability of heavy mesons with the aid of lattice gauge theory and the Born–Oppenheimer adiabatic expansion. We find that the operator product expansion breaks down at surprisingly large quark masses due to nonperturbative gluodynamics and that previous conclusions concerning \(J/\psi\)-nuclear matter interactions and \(J/\psi\) dissociation in the quark-gluon plasma must be substantially modified.

I. INTRODUCTION

Although hadronic interactions are a central phenomenon of nuclear and hadronic physics, very little can be said about them from first principles and a microscopic description remains elusive. One of the few attempts at describing hadronic interactions from QCD originated more than twenty years ago with the operator product expansion (OPE) approach of Peskin\(^1\). Peksin argued that the coupling of soft external gluons to small (heavy quark) hadrons may be considered a short distance phenomenon and therefore amenable to Wilson’s operator product formalism. More recently, Luke, Manohar, and Savage\(^2\) have placed Peskin’s argument in the context of effective Lagrangians. Briefly, in the absence of flavour exchange processes, the interactions of hadronic matter are dominated by multigluon exchange processes which may be described with an effective Lagrangian at the compositeness scale, \(\Lambda_Q \sim r_Q^{-1} \sim \alpha_s(\Lambda_Q)m_Q\):\n
\[
L_{\text{eff}}^{(1)} = \sum_{\nu} \frac{1}{\Lambda_Q}(P(v)^\dagger P(v) - V_\nu(v)^\dagger V(v)^\mu)(c_E O_E + c_B O_B).
\]

Here the gluonic operators are \(O_E = -G^{\mu\alpha}G_\nu^\beta v_\nu v_\beta\) and \(O_B = \frac{1}{2}G^{\alpha\beta}G_{\alpha\beta} - G^{\mu\alpha}G_\nu^\beta v_\nu v_\beta\). \(P(v)\) and \(V_\nu(v)\) create pseudoscalar or vector mesons with four velocity \(v^\mu\). In the meson rest frame these operators reduce to \(E^2\) and \(B^2\) respectively. The multipole formalism becomes exact in the large quark mass/small hadron limit and the coupling constants (or Wilson coefficients) \(c_E\) and \(c_B\) may be interpreted as the chromoelectric and magnetic polarizabilities of the heavy meson.

Luke, Manohar, and Savage used this formalism to estimate the binding energy of quarkonium (\(\Upsilon\) and \(\Psi\)) with nuclear matter\(^2\). The gluonic matrix elements were estimated with the aid of the scale anomaly and the experimentally determined gluonic momentum fraction of the nucleon at the scale \(\Lambda_Q\). The final ingredient was Peskin’s original estimate of the chromoelectric polarizability

\[
c_E = \frac{14\pi}{3(N_c^2 - 1)} \quad \text{(1)}
\]

(in the large \(N_c\) limit). We note that theoretical uncertainty is introduced through the choice of the compositeness scale, the strong coupling \(\alpha_s\), and the size of the meson. The final estimates of the binding energies were roughly 3 MeV for the \(\Upsilon\) and 10 MeV for the \(J/\psi\). Subsequently, Brodsky and Miller\(^3\) used this result to obtain a \(J/\psi\)-nuclear matter scattering length of \(a_B = -0.24\) fm and a cross section of roughly 7 mb at threshold. Brodsky and Miller also argued that multiple gluon exchange dominates the \(J/\Psi\)-nuclear matter interaction.

Finally, Kharzeev and Satz\(^4\) have applied Peskin’s results to the interaction of \(J/\Psi\) with comoving matter in heavy ion collisions. They argue that the cross section is small near threshold and that therefore collision-induced dissociation should not confound the use of \(J/\Psi\) suppression as a diagnostic for the formation of the quark-gluon plasma.

The chromoelectric polarizability has appeared in at least one other context. Leutwyler has argued that nonperturbative level shifts in the heavy quarkonium spectrum may be related to the product of the vacuum expectation value of the electric field pair density and the electric polarizability\(^5\) (similar arguments have been made with QCD sum rules\(^5\)). In particular he states that the small size of the heavy meson implies that quarks interact with slowly varying random chromofields. The energy shift is then given by the expectation of the operator

\[
\delta H = -P E \cdot r \frac{1}{H_a - E_\phi} E \cdot r P \quad \text{(2)}
\]

where \(P\) projects onto mesonic states which are orthogonal to the meson, \(E_\phi\) is the mass of the heavy meson, \(E\) is the chromoelectric field, and \(H_a\) is the Hamiltonian which describes the interactions of quarks in the colour octet state,

\[
H_a = 2m_Q + \frac{p^2}{m_Q} + \frac{\alpha_s}{2N_c r} \quad \text{(3)}
\]

The potential in \(H_a\) is the perturbative expression for the interaction of a quark and an anti-quark in the colour adjoint representation. The expectation value of \(\delta H\) is
We have introduced the Coulombic binding energy and the electric polarizability gives the strength of nonperturbative mass shifts (or, equivalently, the strength of the nonlocal nonperturbative potential due to interactions with the gluon condensate).

It is clear that the value of the electric polarizability is crucial to all these conclusions. In the following, we carefully examine Peskin’s computation of \( c_E \) and conclude that its true value is roughly a factor of ten smaller than claimed. More importantly, it will be shown that, in this application, the operator product expansion is never reliable in Nature due to the effects of nonperturbative gluodynamics.

II. CHROMOELECTRIC POLARIZABILITY

Peskin specifies two conditions which permit the application of the operator product expansion. The first is that the meson should be small, \( r_Q^{-1} \gg \Lambda_{QCD} \), which implies \( m_Q \gg \Lambda_{QCD}/\alpha_s(r_Q^{-1}) \). The second constraint arises because gluons coupling to the heavy meson must arrange themselves into colour singlets. Thus the emission of a single gluon – which raises the energy of the meson to that of an octet (or hybrid meson) state – must be followed quickly by a subsequent emission. The correlation time between these events is \( \Delta t \sim 1/(E_a - E_\phi) \) where \( E_a \) is the energy of the intermediate hybrid state. Thus the colour singlet criterion is \( E_a - E_\phi \sim \epsilon_B \gg \Lambda_{QCD} \) which imposes the stronger constraint:

\[
m_Q \gg n^2 \Lambda_{QCD}/\alpha_s^2.
\] (5)

We have introduced the Coulombic binding energy \( \epsilon_B = m_Q C_F^2 \alpha_s^2/4 \) (\( C_F = (N_c^2 - 1)/(2N_c) \)) and the principle quantum number of the heavy meson, \( n \). Eq. 5 implies that the potential felt by the heavy quarks is perturbative and hence that the heavy meson wavefunction is nearly Coulombic. Peskin estimates that the condition of Eq. 5 is met for \( m_Q \gg 25 \text{ GeV} \) for \( n = 1 \). We note that this result is obtained in the large \( N_c \) limit where \( E_a \) tends to \( 2m_Q \) and hence \( \Delta t \sim 1/\epsilon_B \). An updated limit in which this constraint is considerably relaxed will be established in section IV.

Under the conditions specified above, gluon emissions must arrange themselves into small colour singlet clusters which are attached to a small region in spacetime in which the heavy meson is in an octet state. This observation permits the application of the operator product expansion. Peskin applies this idea by exponentiating all possible two-gluon couplings to the heavy meson. The result is a gauge invariant effective interaction of the form

\[
L_{eff} = - \sum_{N=1}^N c_E^{(N)}_{ij} a_0^2 \cdot E_i \cdot D_0^{-2} \cdot E_j
\] (6)

where \( D_0 \) is the temporal component of the covariant derivative. We follow Peskin and introduce the dimensionful parameters \( a_0 = 2/(C_F \alpha_s m_Q) \) (the Coulombic Bohr radius) and \( \epsilon_B \) to make the Wilson coefficients, \( c_E^{(N)} \), dimensionless. We note that the leading term has already been given in covariant form in Eq. 1 and as a model in Eq. 3.

The expression for the Wilson coefficient is

\[
c_E^{(N)}_{ij} = \frac{2\pi \alpha_s^{2N-2}}{N_c a_0^3} \cdot \langle \phi \rangle^{j} \cdot \frac{1}{(H_a - E_\phi)^{2N-1}} \cdot \epsilon_B \cdot \langle \phi \rangle
\] (7)

and \( \phi \) represents the heavy meson of interest. For S-wave states \( c_E^{(N)}_{ij} = \delta^{ij} c_E^{(N)} \). Finally, using 1s Coulombic wavefunctions and neglecting the adjoint potential yields the result [1] for \( c_E^{(1)}(1s) \) given in Eq. 2 (we suppress the superscript from now on). A similar computation gives

\[
c_E(2s) = \frac{502}{7} c(1s).
\] (8)

In general \( r_Q \sim n^2/(m_Q \alpha_s) \), and the energy denominator scales as \( m_Q \alpha_s^2/n^2 \), thus \( c_E(1s) \sim n^6 \) (factors of \( m_Q \) and \( \alpha_s \) cancel against the prefactors in Eq. 7). It is clear that the OPE breaks down very quickly with the principle quantum number.

As we have already remarked, these estimates have been used to compute the strength of hadronic interactions in a variety of applications. However, a number of strong assumptions have been made in deriving them. Certainly, it is not clear that the adjoint potential need be as simple as perturbation theory indicates. Fortunately recent improvements in lattice gauge theory have allowed for an accurate determination of this interaction in the heavy quark regime [7]. It is therefore expedient to confront the assumptions of Refs. 1, 2, 3, 4, 5, 6 with lattice gauge theory in an attempt to establish the validity of Eqs. 2 and 6. Thus we briefly review the current knowledge of hybrid potentials before moving on to a reevaluation of the polarizability and the operator product expansion itself.

III. ADIABATIC HYBRID SPECTRUM

A simple consequence of the fact that glue is confined is that it must manifest itself as a discrete spectrum in the presence of a static colour source and sink. In this case the physical hadrons are heavy hybrid mesons. It is relatively easy to study heavy hybrids by constructing gluonic configurations on the lattice which are analogous to those of a diatomic molecule. Indeed, the gluonic configurations may be described with the same set of quantum numbers as diatomic molecules: \( \Lambda_{Y} \). Here the projection of the total gluonic angular momentum onto the \( QQ \) axis is denoted by \( \Lambda \) which may take on values \( \Sigma, \Pi, \Delta = 0, 1, 2, \) etc. The combined operation of charge and parity conjugation on the gluonic degrees of freedom is denoted by \( \eta = u, g \) and \( Y = \pm \) represents reflection of
the system in a plane containing the $Q\bar{Q}$ axis. As with the diatomic molecule, all systems with $\Lambda$ greater than zero are doubly degenerate in $Y$. Gluonic adiabatic surfaces may be traced by allowing the heavy quark source and sink separation to vary and hybrid mesons containing excited gluonic configurations may be studied in the adiabatic Born-Oppenheimer approximation.

The results of a recent lattice computation are presented in Figure 1. The lowest state is the $\Sigma^+_g$ surface and corresponds to the Wilson loop static interquark potential. The first (second) excited state is the $\Pi_u$ ($\Pi_g$) surface and may be visualized as a gluonic flux tube with the addition of a single ‘phonon’. A similar analogy exists for all of the higher states.

FIG. 1: Low Lying Adiabatic Hybrid Surfaces. Lattice data for the $\Sigma^+_g$ (circles), $\Pi_u$ (diamonds), and $\Pi_g$ (squares) surfaces from Ref. 3. Lines are simple parameterizations of the data. The scale is $r_0 \approx 1/2$ fm. Inset: Potentials at Short Distance. The same data is shown in traditional units. The $\Pi_u$ and $\Pi_g$ lines are those of the main figure with the addition of the perturbative adjoint potential $V_a$.

The inset of the figure shows the lowest hybrid surfaces at scales less than 1.5 femtometres. The dashed and dotted lines are those of the main figure with the addition of the perturbative adjoint potential, $V_a = \alpha_s/(6r_Q)$.[2] The figure demonstrates that perturbative behaviour has not been seen at $r_Q \approx 0.2$ fm (points) and that this is consistent with expectations (dotted and dashed lines).

IV. DISCUSSION

We first note that the nonperturbative gluodynamics shown in Figure 1 indicates that the octet-singlet splitting used in Eq. 3 is not accurately described by the Coulombic binding energy. Rather, the figure indicates that this splitting is roughly 1 GeV at typical hadronic scales. Thus the constraint is not nearly as strong at finite $N_c$ and with reasonable hadron masses. One concludes that gluons are largely correlated in time as required, lending hope to the idea that the application of the OPE to hadronic interactions may be unexpectedly robust. Unfortunately, a new constraint exists, which we now demonstrate.

A central criterion for the validity of the operator product expansion is that the hybrid surfaces shown in Figure 1 approach a universal form at short distances. Figure 1 makes it clear that a universal hybrid potential behaviour (namely $V_a$) does not appear until

$$V_a(r_Q) \gg V_{\Lambda^Y}(r_Q) - V_{\Lambda^Y}(r_Q).$$

Since the typical hybrid surface separation is order $\Lambda_{QCD}$ for small $r_Q$ (and is much larger for the splitting relevant to ground state mesons), one has $m_Q \gg 6\Lambda_{QCD}/\alpha_s^2 \approx 150$ GeV. Thus Eq. 3 is recovered (albeit with an unlucky additional large factor); however, the constraint is now a necessary condition rather than merely sufficient as before. Thus, although the condition which insures the emission of colour singlet states is likely to be satisfied for all quark masses, the hidden assumption in the method, namely that a universal octet potential is relevant, is only true for very heavy quarks.

This conclusion has a simple interpretation in Leutwyler’s random field model: the appearance of a discrete hybrid spectrum makes it clear that the correct representation (here we employ the Born-Oppenheimer approximation) of the matrix element of $\delta H$ of Eq. 3 is as follows

$$\delta E_n = \langle \phi_n | \delta H | \phi_n \rangle$$

$$\rightarrow \langle \phi_n; \Sigma^+_g | \delta H | \phi_n; \Sigma^+_g \rangle$$

$$= \sum_{h, \Lambda, \eta, Y} \frac{|\langle \phi_n; \Sigma^+_g | h ; \Lambda^Y | \eta \rangle|^2}{(E_h(\Lambda^Y) - E_\eta)}.$$  (9)

In this expression $h$ represents all of the nongluonic quantum numbers which describe an intermediate heavy hybrid state. The essence of the operator product expansion is that this expression factorizes (i.e., the Wilson coefficients depend on short range physics only). Factorization requires that the hybrid energies in the denominator do not depend on the gluonic quantum numbers, $\Lambda^Y$. It is only in this circumstance that the expression simplifies:

$$\delta E_n = \sum_h \frac{|\langle \phi_n | h \rangle|^2}{(E_h - E_\eta)} \cdot \langle \Sigma^+_g | E^2 | \Sigma^+_g \rangle.$$  (10)

and the operator product formalism is valid.

Finally, we consider the value of the chromoelectric polarizability in light of the lattice hybrid data of Figure 1. We choose to numerically compute the $\phi$ wavefunction
in the Born-Oppenheimer approximation with the aid of the lattice \( \Sigma^+_g \) surface. The sum over intermediate hybrid states is performed numerically by expanding in the eigenstates of the \( \Pi_g \) surface (this is the lowest surface which couples to a vector heavy hybrid – which is the case we consider in the following).

The results are shown as the open squares in Figure 2. One sees that the Peskin result of Eq. 2 is recovered (arrow) in the very heavy quark mass limit, as expected. However, the value of \( c_E(1s) \) at the \( \Upsilon \) or \( J/\psi \) masses (arrows on the abscissa) is highly suppressed with respect to the asymptotic value. The diamonds are numerically obtained values for the polarizability in the case that the adjoint potential has been included in the \( \Pi_g \) surface parameterization. Again, the analytical result, \( c_E(1s) = 234\pi/425 \), is approached very slowly in quark mass.

![Figure 2: Chromoelectric Polarizability as a Function of Quark Mass in GeV. Points represent \( c_E(1s) \) as computed with the \( \Pi_g \) surface with (diamonds) and without (open squares) the perturbative adjoint potential, \( V_a \). The line is the approximation of Eq. (11). Inset: The Ratio \( c_E(2s)/c_E(1s) \) versus Quark Mass.](image)

It is tempting to speculate that the majority of the finite quark mass correction to Peskin’s result is due to the hybrid mass gap. Allowing for this in Eq. 2 yields the following generalization of Eq. 2:

\[
c_E(1s) = \frac{8\pi}{3(N^2_c - 1)v_0}(256(1 + v)^{3/2} - 256 - 384v - 96v^2 + 16v^3 - 6v^4 + 3v^5)
\]

where \( v = \frac{4V_0}{(c_E(1s)m_Q)} \) and \( V_0 \) is the strength of the relevant hybrid potential at its minimum. The resulting expression is shown as a solid line in Figure 2. Evidently the agreement is quite good and this expression may serve as a useful extrapolation to light quark masses.

Finally we display the ratio \( c_E(2s)/c_E(1s) \) in the inset of Fig. 2. Again the Peskin result (Eq. 2) is approached only very slowly in the heavy quark limit. We note that Eq. 8 leads to the uncomfortable prediction that the \( \Upsilon' \) interacts 5000 times more strongly with nuclear matter than does the \( \Upsilon \). The inset shows, however, that this prediction is substantially moderated (from 5000 to roughly 15) when finite quark mass effects are taken into account.

V. CONCLUSIONS

According to the arguments of Refs [1, 2, 5, 6], the electric polarizability of a small meson controls the strength of its interactions with hadronic matter via the operator product expansion. We have recomputed this strength with the aid of lattice hybrid potentials [11] and find that the large mass gap between the ground state \( (\Sigma^+_g) \) and excited state gluonic configurations leads to a strong suppression of the electric polarizability as the quark mass is reduced. The result is that, if one neglects issues of the applicability of the OPE, previous estimates of interaction strengths are reduced by roughly a factor of 100. Thus the arguments of Khazeev and Satz concerning the utility of \( J/\psi \) suppression as a quark-gluon plasma diagnostic are strengthened. Alternatively non-quark-exchange \( J/\psi \)–nuclear matter interactions are greatly reduced, suggesting that quark exchange mechanisms should be carefully considered in the analysis of the \( J/\psi \)–nuclear matter binding issue.

On a more general level we have argued that the assumptions underlying the operator product expansion description of heavy hadron interactions are violated for all physical states. This situation arises because the sufficient condition on temporal correlations among gluons has been replaced with a necessary condition on the applicability of factorization which is only true for very heavy quarks.

The short length scale required for factorization arises for a number of reasons. Certainly the fact that the strong coupling is not large and that the ratio of fundamental and adjoint Casimir is also small, help to undermine the reliability of the OPE. However, the relatively flat behaviour of the adiabatic hybrid surfaces below 1 fm must be considered the leading cause. One may speculate that this arises due to the robust persistence of string-like field configurations, even at quite small interquark separations. It thus appears that strong nonperturbative gluodynamics conspires to bring about the demise of the operator product formalism in this application.

Although we have said nothing about the utility of the OPE in the very heavy quark limit, the authors of Ref. 8 show that the interaction between very small colour dipoles becomes nonperturbative (it is essentially correlated two pion exchange). It thus appears that the premise of the OPE and any effective field theoretic approach to the interactions of small hadrons is compromised.
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[9] There is a subtlety here: the fitted value of the strong coupling is $C_F\alpha_s = 0.28$. This is very close to the value expected for excitations of string-like degrees of freedom, $\pi/12$, hence one suspects that the value of the strong coupling used here does not represent the perturbative behaviour of QCD, but rather is an intermediate distance effect. Nevertheless, using this value for $\alpha_s$ overestimates the region of validity of the operator product expansion, and the conclusions presented below stand.
[10] We note that the approach to the Coulombic limit is, in part, very slow because of the small value of the strong coupling. For example, it is much faster if typical quark model values for the strong coupling are employed.
[11] The lattice hybrid potentials employed here were obtained in the Born-Oppenheimer approximation, wherein gluonic degrees of freedom respond rapidly to slow quark motion. It may be shown that the requirements for the validity of the Born-Oppenheimer approximation coincide with Eq. 5.