Testing the chirality of $b$ to $u$ current with $B^0 \rightarrow \rho^- \ell^+ \nu$

Yadong Yang*, Hongjun Hao and Fang Su

Department of Physics, Henan Normal University, Xinxiang, Henan 453007, P.R. China

September 21, 2018

Abstract

We study the effects of a modest V+A admixture strength in $b \rightarrow u$ current in the decay $B \rightarrow \rho \ell \nu$. We have shown that the decay rate, lepton forward-backward distribution asymmetry $A_{FB}$ and polarization ratio are sensitive to the admixture. Future experimental studies of the decay at BaBar and BELLE could clarify the chirality of $b \rightarrow u$ current and might reveal hints for New Physics with right-handed quark currents.

PACS Numbers: 12.60.Cn, 12.15.Ji, 13.20.He

*Corresponding author. E-mail address: yangyd@henannu.edu.cn
With the running of BaBar and Belle B factories, many rare B decays could be well studied and provide tests for the Standard Model (SM). Potentially it will open windows for New Physics beyond the SM, since clear differences between the measurements of observables in B decays and the SM expectations will be indirect evidences for New Physics existence.

In the SM, the weak charge currents are left-handed (LH). Therefore, derivations from pure V-A chirality of weak charge currents would be important probes for New Physics. Years ago, the speculation of right-handed chirality \( b \to c \) current had been put forward by Körner and Schuler \[1\], then by Gronau and Wakaizumi \[2\], Hou and Wyler \[3\]. Since then, the problem has been studied by many authors \[4, 5\]. With the experiment measurements by \[6\] and L3 \[7\], the dominant right-handed \( b \to c \) current was ruled out. However, a modest right-handed admixture in \( b \to c \) current is still allowed. Moreover, as shown by Voloshin \[8\], it may help to resolve the baffling semileptonic branching ratio and the charm-deficit problems in B decays \[9\].

Such a situation was reexamined comprehensively by Rizzo \[10\] later on. Compared to the extensive studies of the chirality of \( b \to c \), the relevant studies about a modest right-handed admixture in \( b \to u \) current are quite few. We note that Wakaizumi \[11\] studied it using the Left-Right Symmetric Model. The main reason for the situation may be due to the fact that \( b \to u \) transitions are much rarer than \( b \to c \). However, many rare decays induced by \( b \to u \) could be well studied now at BaBar and Belle factories. For example, using a sample of \( 3 \times 10^6 \Upsilon(4S) \to B\bar{B} \) events, CLEO \[12\] has given a quite accurate measurement of \( B \to \rho \ell \nu \) decay. It is interesting to note that both BaBar and Belle have accumulated more than 200M \( B\bar{B} \) events by 2004. We can foresee that observables in the decay could be well measured by BaBar and Belle in the near future.

In this letter, we will investigate the sensitivities of lepton angular distribution asymmetry and \( \rho \) meson polarization in \( B^0 \to \rho^- \ell^+ \nu \) decay to a modest V+A admixture in \( b \to u \) current. As shown later, these observables are very sensitive to the admixture.

Following Voloshin \[8\], we introduce a modest V+A admixture in \( b \to u \) coupling, while maintain the leptonic current as purely left-handed to satisfy the tight constraints from \( \mu \) decays \[13\]. So the effective Hamiltonian describing the decay can be written as

\[
H_{\text{eff}} = -4G_F \sqrt{2} V_{ub}[\bar{u}_L \gamma_\mu b_L] + \xi [\bar{u}_R \gamma_\mu b_R] [\bar{\ell}_L \gamma^\mu \nu_L] + h.c.,
\]

where \( G_F \) is the Fermi constant, \( V_{ub} \) is the KM matrix element for the \( b \to u \) transition, and
\( \xi \) denotes the relative strength of the RH admixture to the LH \( b \to u \) coupling. As we do not discuss CP violation, we will treat \( \xi \) as a real parameter.

Neglecting lepton masses, the double differential decay rate in \( q^2 \) and \( \cos \theta \) is given in terms of three \( q^2 \)-dependent helicity amplitudes \( H_{0,\pm} \), where the subscripts refer to the helicity of the \( \rho \)-meson

\[
\frac{d^2\Gamma(B^0 \to \rho^- \ell^+\nu)}{dq^2d\cos \theta} = \frac{G_F^2|V_{ub}|^2}{256\pi^3}K_\rho \frac{q^2}{m_B^2}[(1-\cos \theta)^2|H_+|^2 + (1+\cos \theta)^2|H_-|^2 + 2\sin^2 \theta|H_0|^2]. \tag{2}
\]

Where \( q \) is the momentum of lepton pair, \( \theta \) is the angle of the lepton measured with respect to the \( \rho \)-direction in the \( l\nu \) pair rest frame; \( K_\rho \) is the absolute value of \( \rho \) meson three-momentum in the B rest frame,

\[
K_\rho = \frac{1}{2m_B}[(m_B^2 - m_\rho^2 - q^2)^2 - 4m_\rho^2q^2]^\frac{1}{2}. \tag{3}
\]

The helicity amplitudes \( H_{0,+,\pm} \) are related to the Lorentz-invariant form factors as

\[
H_\pm(q^2) = (m_B + m_\rho)(1-\xi)A_1(q^2) \pm \frac{2m_BK_\rho}{m_B + m_\rho}(1+\xi)V(q^2), \tag{4}
\]

\[
H_0(q^2) = \frac{1-\xi}{2m_\rho\sqrt{q^2}}\left[(m_B^2 - m_\rho^2 - q^2)(m_B + m_\rho)A_1(q^2) - \frac{4m_B^2K_\rho^2}{m_B + m_\rho}A_2(q^2)\right]. \tag{5}
\]

Form factors \( V \) and \( A_{1,2} \) are connected with the meson transition amplitudes, induced by the vector \( V^\mu = \bar{u}\gamma^\mu b \) and axial-vector \( A^\mu = \bar{u}\gamma^\mu\gamma^5 b \) quark transition currents.

All the dynamical content of hadronic current matrix elements is described by the above \( q^2 \)-dependent form factors. The calculation of these form factors requires non-perturbative methods, and are sources of large theoretical uncertainties. Theoretical approaches for calculating these form factors are quark model, QCD sum rule, and lattice QCD. In our calculations, we use both results from light-cone sum rule (LCSR)\(^{15}\) and lattice QCD mode (LQCD)\(^{16}\) for comparison.

Experimentally, the main difficulty in observing signals from \( b \to u\ell\nu \) processes is the very large background due to \( b \to c\ell\nu \). With the lepton-energy requirement \( E_\ell > 2.3\text{GeV} \)\(^{12}\), the \( b \to c\ell\nu \) background can be sufficiently suppressed. It is well known that QCD sum rule is suitable for describing the low \( q^2 \) region of the form factors while lattice QCD for the high \( q^2 \) region. So that, we shall calculate the decay width of \( B^0 \to \rho^- \ell^+\nu \) in the phase space \( E_\ell > 2.3\text{GeV} \) and \( 0 < q^2 < 7\text{GeV}^2 \) using LCSR form factors\(^{15}\), while in the phase space \( 2.3 < E_\ell < 2.6\text{GeV}, 14 < q^2 < 21\text{GeV}^2 \) using lattice QCD form factors\(^{16}\).
Using the upper bound of $q^2$ for a given $E_l$

$$q_{up}^2 = 2E_l \frac{m_B^2 - m_\rho^2 - 2m_B E_l}{m_B - 2E_l}$$

and

$$\cos \theta = -\frac{4m_B E_l - m_B^2 - q^2 + m_\rho^2}{2K_\rho m_B},$$

we can integrate Eq. 2 over the phase spaces of our interests. Our numerical results for partial widths in the phase spaces $0 < \text{leqq}^2 < 7 \text{GeV}^2$ and $14 \text{GeV}^2 < q^2 < 21 \text{GeV}^2$ with the lepton energy requirement $E_l > 2.3 \text{GeV}$ are presented in Fig.1 and Fig.2, respectively. We have used $|V_{ub}| = 4.7 \times 10^{-3}$.

From Fig.1 and Fig.2, we can find that the partial decay widths are sensitive to the V+A admixture strength $\xi$. However, our numerical calculations must use $V_{ub}$ as input, which is poorly known at present. Given accurate measurements of $\Gamma(B \to \rho \ell \nu)$ and $\Gamma(B \to X_u \ell \nu)$ available from BaBar and Belle in the near future, we still may not get a reliable conclusion about the chirality of $b \to u$ current, because the effects of the V+A admixture on the decay widths could be lumped by a redefinition of effective $V_{ub}$. We may have to resort to observables which are $V_{ub}$ independent. To this end, we will study lepton angular asymmetries and polarization ratios in the decay, which are free of our knowledge of $V_{ub}$.
Figure 2: The partial rate $\Gamma$ in the phase-space with $E_l > 2.3\text{GeV}$ and $14 < q^2 < 21\text{GeV}^2$ as a function of $\xi$ (solid curve), calculated using lattice QCD form factors[16]. The horizontal lines are the CLEO data and its $1\sigma$ error bar[12].

The forward-backward asymmetry ($A_{FB}$) of the lepton distribution is defined as[1]

$$A_{FB} = \frac{\int dq^2 \left[ \frac{d\Gamma}{dq^2}(\theta > \frac{\pi}{2}) - \frac{d\Gamma}{dq^2}(\theta < \frac{\pi}{2}) \right]}{\int dq^2 \left[ \frac{d\Gamma}{dq^2}(\theta > \frac{\pi}{2}) + \frac{d\Gamma}{dq^2}(\theta < \frac{\pi}{2}) \right]}.$$  

(8)

Another interesting observable is the polarization ratio $\frac{\Gamma_L}{\Gamma_T}$, where $\Gamma_{L,T}$ are the rates for producing longitudinally and transversely polarized $\rho$ meson in the decay respectively. The polarization ratio can be written as [17]

$$\frac{\Gamma_L}{\Gamma_T} = \frac{\int dq^2 z_0(1 - \frac{z_0^2}{3})K_{\rho q^2}H_0^2}{\int dq^2 z_0(1 + \frac{z_0^2}{3})K_{\rho q^2}(H_+^2 + H_-^2)},$$

(9)

where

$$z_0 = min[1, -\frac{4m_BE_{cut} - m_B^2 - q^2 + m^2}{2K_\rho m_B}].$$

(10)

Since leptons energy cut-off is always required in experiment measurement and the lepton energy $E_\ell$ is a function of the decay angle $\theta$, the measured values of the above observables will be affected by $E_{cut}$. The requirement $E_\ell \geq E_{cut}$ introduces a limit in the experimentally accessible angular range $-1 < \cos \theta \leq z_0$. The range is not symmetric. However we need to use a symmetric cut with respected to $\cos \theta$, i.e., $-z_0 \leq \cos \theta \leq z_0$, in order not to affect the contributions to different helicity amplitudes in a different way[11, 17]. Since CLEO[19] and
Figure 3: The forward-backward asymmetry $A_{FB}$ as a function of $\xi$. The solid(dashed) curve corresponds to calculation with(without) lepton energy cut $E_{cut} = 2.0\text{GeV}$.

BABAR$^{20}$ used $E_{cut} = 2.0\text{GeV}$ in their recent measurement of the rate of $B \rightarrow \rho^- \ell^+ \nu$, we will calculate the asymmetries using the same cutoff. For comparison, we also calculate the asymmetries without the energy cut.

Our numerical results for $A_{FB}$ and $\Gamma_L/\Gamma_T$ are presented in Fig.3 and Fig.4, respectively. The solid lines are the results with energy cut $E_{cut} = 2.0\text{GeV}$ and the dashed lines for the results without energy cut.

As shown in Fig.3 and 4, the $A_{FB}$ and $\Gamma_L/\Gamma_T$ are sensitive to the V+A admixture in $b \rightarrow u$ current. When lepton energy cut required, magnitudes of $A_{FB}$ is reduced about by half, while $\Gamma_L/\Gamma_T$ is reduced not so much. It is interesting to note that LQCD$^{16}$ and LCSR$^{15}$ form factors give very similar numerical results when the lepton energy cut is put. For a modest V+A admixture $\xi = 0.2$, $A_{FB}$ will be enhanced by 17%, and $\Gamma_L/\Gamma_T$ will be reduced by 20%, which might be testable in the near future measurements at BaBar, Belle and LHCb.

In past years, there have been considerable progresses in both theoretical calculations of form-factors$^{15}$ and experimental studies of $B \rightarrow \rho \ell \nu$ $^{19}$ $^{20}$. The progress will surely advance steadily to a higher precision stage of calculating and measurement of the observables studied in this paper. We note that the recent measurement of $Br(B \rightarrow \rho \ell \nu)$ has been preformed by
Figure 4: The polarization ratio $\frac{\Gamma_L}{\Gamma_T}$ as a function of $\xi$. The solid(dashed) curve corresponds to calculation with(without) lepton energy cut $E_{\text{cut}} = 2.0\text{GeV}$.

BaBar\cite{20} with just $55\text{M} \, B\bar{B}$ events. However, BaBar had accumulated more than $250\text{M} \, B\bar{B}$ events by 2004. Moreover, BELLE\cite{21} has recorded data sample as large as $375\text{M}$ recently. So that, refined studies of the decay $B \rightarrow \rho \ell \nu$ could be performed by both BaBar and BELLE to clarify the chirality of $b \rightarrow u$ weak current, which might reveal hints for New Physics.

In summary, we have studies the effects of a modest $V+A$ admixture in $b \rightarrow u$ current in the decay $B \rightarrow \rho^- \ell^+ \nu$. We have shown that the decay rate, lepton forward-backward distribution asymmetry and $\rho$ meson longitudinal to transverse polarizations ratio are quite sensitive to the admixture. The last two observables, which are free of our knowledge of $V_{ub}$, are very suitable for probing New Physics with right-handed current. Further experimental studies of the decay at BaBar and BELLE are urged to clarify the chirality of $b \rightarrow u$ current.

This work is supported by NFSC under contract No.10305003, Henan Provincial Foundation for Prominent Young Scientists under contract No.0312001700 and in part by the project sponsored by SRF for ROCS, SEM.
References

[1] J.G. Körner and G.A. Schuler, Phys. Lett. B226, 185(1989).

[2] M. Gronau and S. Wakaizumi, Phys. Rev. Lett. 68, 1814(1992); Phys. Lett. B280, 79 (1992); Phys. Rev. D47, 1262(1993).

[3] W.S. Hou and D. Wyler, Phys. Lett. B292, 364(1992).

[4] Z. Hioki, Z. Phys. C59, 555(1993); M. Tanaka, Phys. Rev. D47, 4969(1993); R. Mohapatra and S. Nussinov, Phys. Lett. B339, 101(1994); T. Hayashi, Prog.Theor.Phys. 98, 143(1997); D.S. Du, H.Y. Jin and Y.D. Yang, Phys. Lett. B414, 130(1997).

[5] For very recent studies, M. Gronau, D. Pirjol, D. Wyler, Phys. Rev. Lett. 90, 051801(2003); C. H. Chen and C. Q. Geng, arXiv: [hep-ph/0503123] to appear in Phys. Rev. D; Y. D. Yang, G. Lu and R. Wang, Eur. Phys. J. C34, 291 (2004); A.L. Kagan, [hep-ph/0407076]

[6] CLEO Collaboration, J.E. Duboscq et al., Phys. Rev. Lett. 76, 3898(1996).

[7] L3 Collaboration, M. Acciarri et al., Phys. Lett. B351, 375(1995).

[8] M. B. Voloshin, Mod. Phys. Lett. A12, 1823 (1997).

[9] I.I. Bigi, B. Blok, M. A. Shifman, A. I. Vainshtein, Phys. Lett. B323, 408(1994).

[10] T. G. Rizzo, Phys. Rev. D58, 055009 (1998).

[11] S. Wakaizumi, Phys. Lett. B322, 397(1994).

[12] CLEO Collaboration, B.H. Behrens, et al., Phys. Rev. D61, 052001(2000).

[13] W. Fetscher and H.J. Gerber, Muon Decay Parameters, Phys. Lett. B 592, 1 (2004).

[14] M. Bauer, M. Wirbel, Z. Phys. C42, 671(1989).

[15] P. Ball and R. Zwicky, Phys. Rev. D71, 014029, (2005).

[16] L. D. Debbio et al.(UKQCD Collaboration), Phys. Lett. B416, 392 (1998).
[17] M. Neubert, Phys. Rept. 245, 259 (1995).

[18] J. Alexander et al., (Heavy Flavor Averaging Group), hep-ex/0412073.

[19] S. B. Athhar et al. (CLEO Collaboration), Phys. Rev. D68, 072003 (2003) [arXiv: hep-ex/0304019].

[20] B. Aubert et al. (BABAR Collaboration), Phys. Rev. Lett. 90, 181801 (2003) [arXiv: hep-ex/0301001].

[21] For up-to-date Belle Integrated Luminosity, see KEK BELLE homepage http://belle.kek.jp.