A CRITERION FOR ERGODICITY OF NON-UNIFORMLY HYPERBOLIC DIFFEOMORPHISMS

F. RODRIGUEZ HERTZ, M. A. RODRIGUEZ HERTZ, A. TAHZIBI, AND R. URES

Abstract. In this work we exhibit a new criteria for ergodicity of diffeomorphisms involving conditions on Lyapunov exponents and general position of some invariant manifolds. On one hand we derive uniqueness of SRB-measures for transitive surface diffeomorphisms. On the other hand, using recent results on the existence of blenders we give a positive answer, in the $C^1$ topology, to a conjecture of Pugh-Shub in the context of partially hyperbolic conservative diffeomorphisms with two dimensional center bundle.

1. Introduction

In this note we announce certain criteria to prove ergodicity, and their application in two different settings: stable ergodicity and uniqueness of SRB-measures.

1.1. Stable ergodicity. Let $f : M \to M$ be a volume preserving diffeomorphism of a compact Riemannian manifold. We will denote by $m$ the probability Lebesgue measure that is induced by a volume form $\omega$. A challenging problem in smooth ergodic theory is to prove the ergodicity of the Lebesgue measure. Eberhard Hopf [14] provided the first and still the only argument of wide use to establish ergodicity and proved the ergodicity of the geodesic flow in the case of negatively curved surfaces. Another interesting problem is the quest of abundance and stability of ergodicity in the setting of volume preserving diffeomorphisms. The celebrated Kolmogrov-Arnold-Moser

2000 Mathematics Subject Classification. Primary: 37D25. Secondary: 37D30, 37D35.

This work was done in Universidad de la República- Uruguay and ICMC-USP, São Carlos, Brazil. A. Tahzibi would like to thank FAPESP for financial support and Universidad de la República for warm hospitality and financial support. F. Rodriguez Hertz and R. Ures also acknowledge warm hospitality of ICMC-USP and financial support of CNPq and FAPESP respectively. F. Rodriguez Hertz, M. A. Rodriguez Hertz, and Raúl Ures were partially supported by PDT 54/18 and 63/204 grants and A. Tahzibi was also partially supported by CNPq (Projeto Universal).
result, showing the presence of elliptic dynamics, is an obvious ob-
struction to obtain density of ergodicity. Pugh and Shub proposed
a program to prove abundance (density) of stable ergodicity among
partially hyperbolic dynamical systems.

By Pugh-Shub conjecture we refer the density of stable ergod-
icity among partially hyperbolic volume preserving diffeomor-
phisms. Partial hyperbolicity is a weak form of hyperbolicity and partially hy-
perbolic systems are far from elliptic dynamics. A diffeomorphism
$f : M \to M$ is partially hyperbolic if $TM$ admits a non trivial $Df$-
invariant splitting $TM = E^s \oplus E^c \oplus E^u$, such that all unit vectors
$v^\sigma \in E^\sigma_x$ ($\sigma = s, c, u$) with $x \in M$ verify:

$$
||D_x f v^s|| < ||D_x f v^c|| < ||D_x f v^u||
$$

for some suitable Riemannian metric. It is also required that $||Df|| < 1$ and $||Df^{-1}|| < 1$. We denote the subset of partially hyperbolic $C^{1+}$-diffeomorphisms of $M$ by $\mathcal{PH}(M)$ and $\mathcal{PH}_m(M)$ repre-
scents the partially hyperbolic diffeomorphisms preserving the vol-
ume Lebesgue measure. By a $C^{1+}$ diffeomorphism we mean one with
Hölder continuous first order derivatives.

It is a known fact that there are foliations $\mathcal{W}^\sigma$ tangent to the distri-
butions $E^\sigma$ for $\sigma = s, u$ (see for instance [9]). As a part of their
program, Pugh and Shub conjectured that transitivity of the pair of
stable and unstable foliations, called accessibility, should be enough
to prove ergodicity. That is, a diffeomorphism is accessible if the
unique nonempty set containing the stable and unstable manifold
of every point is the whole $M$. By essential accessibility we mean
that sets saturated by stable and unstable manifolds have full or null
measure. Recently, Burns-Wilkinson [11] proved that essential ac-
cessibility and a bunching condition in the center direction imply
ergodicity and consequently gave a positive answer to one of the
Pugh-Shub’s conjectures. Center bunching is a technical condition
that informally speaking means quasi-conformality in the central
direction. R. Hertz-R. Hertz-Ures [19] proved the same result but
for central direction of dimension one (where the center bunching
condition is obviously verified)

We remark that the first examples of stably ergodic diffeomor-
phisms which are not partially hyperbolic were given in [21].

In the partially hyperbolic context we apply our criterion to obtain
stable ergodicity under some conditions on central Lyapunov expo-
nents and the existence of some topological blenders. The novelty of
this work is the use of topological instruments to prove ergodicity.
We remark that an antecedence of taking advantage of the interplay
between nonuniform and partial hyperbolicity is the following theorem of Burns-Dolgopyat-Pesin [10].

**Theorem 1.1** ([10]). Let $f \in \mathcal{PH}_m(M)$. Assume that $f$ is essentially accessible and has negative central exponents on an invariant set $A$ of positive measure. Then $A$ has full measure and $f$ is stably ergodic.

A natural question arises: what happens when the exponents are different from zero but the signs may vary? In this paper we consider this mixed case when the central dimension is two.

Finally, we show how to apply our results to prove Pugh-Shub conjecture (in $C^1$ setting) for partially hyperbolic dynamical systems with two dimensional central bundle.

### 1.2. SRB-measures

Another application of our criteria is the case of certain observable measures, in the sense that typical trajectories have positive Lebesgue measure. Let $f$ be a diffeomorphism of a surface $M$. We will say that an $f$-invariant ergodic measure $\mu$ is a Sinai-Ruelle-Bowen measure (SRB-measure for short) if the largest Lyapunov exponent is strictly positive and the Pesin’s formula holds that is $\lambda^+(\mu) = h_\mu$ (where $h_\mu$ is the entropy of the measure $\mu$).

SRB-measures are important objects of study when Lebesgue measure is not preserved. Since SRB-measures has a positive Lyapunov exponent, unstable manifolds form measurable partitions of their support. Conditional measures along these partitions are absolutely continuous with respect to the Lebesgue measure of the unstable manifolds. See, for instance, the survey [22] or [3] for presentations of the subject.

An adaptation of our criterion (Theorem A) to this case allows us to show that transitive surface diffeomorphisms have at most one SRB-measure (Theorem D). This contrasts with known examples of transitive endomorphisms of surfaces having intermingled basins of SRB-measures and with diffeomorphisms having this property in greater dimensions (see [16] and [8, Ch. 11]).

### 2. A Criterion for Ergodicity

Given a hyperbolic periodic point $p$ we define the unstable manifold of the orbit of $p$ as $W^u(O(p)) = \bigcup_{k=0}^{n(p)-1} W^u(f^k(p))$, similarly for the stable manifold. Given a regular point, we define its Pesin’s stable manifold as usual, i.e.

$$W^s_p(x) = \{y : \limsup_{n \to +\infty} \frac{1}{n} d(f^n(x), f^n(y)) < 0\}$$
similarly for the unstable manifold.

Given a hyperbolic periodic point \( p \), let us define the following sets:

\[
B^s(p) = \{ x : W^s_p(x) \cap W^u(O(p)) \neq \emptyset \} \\
B^u(p) = \{ x : W^u_p(x) \cap W^s(O(p)) \neq \emptyset \}
\]

where \( \cap \) means that the intersection is transversal. \( B^\sigma(p), \sigma = s, u, \) is clearly \( f \)-invariant. By the above definition any \( x \in B^s(p) \) has at least \( \dim W^s(p) \) negative Lyapunov exponents and similarly for \( x \in B^u(p) \).

**Theorem A.** If \( m(B^\sigma(p)) > 0 \), for both \( \sigma = s \) and \( \sigma = u \) then,

\[
B(p) := B^u(p) \cap B^s(p) = B^u(p) \circ B^s(p),
\]

where the two last equalities are \( m \)-almost sure. Moreover, \( f|_{B(p)} \) is ergodic and non-uniformly hyperbolic and for \( x \in B(p) \), \( \dim W^s_p(x) = \dim W^s(p) \) and \( \dim W^u_p(x) = \dim W^u(p) \).

Here, no partial hyperbolicity is required. Observe also that in the above theorem we do not require all the exponents to be nonzero, although some of them should be. Also we do not require a priori that \( \dim W^s_p(x) = \dim W^s(p) \). Let us state an immediate consequence of the \( \lambda \)-lemma.

**Lemma 2.1.** If \( W^u(p) \cap W^s(q) \neq \emptyset \) then \( B^u(p) \subset B^u(q) \) and \( B^s(q) \subset B^s(p) \).

So we have:

**Corollary 2.2.** If \( W^u(p) \cap W^s(q) \neq \emptyset \), \( m(B^u(p)) > 0 \) and \( m(B^s(q)) > 0 \), then \( B(p) \subset B(q) \). If in addition their union has full measure then \( f \) is ergodic and non-uniformly hyperbolic.

We will apply the above criteria for the proof of the stable ergodicity of Lebesgue measure for a special class of partially hyperbolic systems. The novelty here is the use of blenders (robust topological objects) to obtain stable ergodicity.

**Theorem B.** Let \( f \in PH_m(M) \) satisfy the following properties:

1. \( f \) satisfies the accessibility property,
2. there is a dominated splitting \( E^c = E^- \oplus E^+ \) of the central bundle into one-dimensional subbundles,
3. \( \int_M \lambda^- dm < 0 \) and \( \int_M \lambda^+ dm > 0 \),
4. \( f \) admits a cs--blender of stable dimension \( s + 1 \) and a cu--blender of unstable dimension \( u + 1 \) and their periodic points are homoclinically related.

then \( f \) is stably ergodic.
Let us define $\mathcal{P}H_n(M, 2)$ the subset of partially hyperbolic conservative diffeomorphisms with two dimensional central bundle.

**Theorem C** (Pugh-Shub Conjecture). There is a $C^1$-dense subset $\mathcal{D} \subset \mathcal{P}H_n(M, 2)$ such that any $f \in \mathcal{D}$ is stably ergodic.

### 3. Blenders and Proof of theorem \[B\]

In this section we briefly state the definition and the main properties of blenders that we need. We refer the reader to [8] for more details and references.

In order to give a definition of blender we adopt the operational point of view of [8].

**Definition 3.1** ([8]). Let $f : M \to M$ be a diffeomorphism and $Q$ be a hyperbolic periodic point of unstable dimension $u + 1$. We say that $f$ has a cu-blender associated to $Q$ if there is a $C^1$-neighborhood $\mathcal{U}$ of $f$ and a $C^1$-open set $\mathcal{D}$ of embeddings of an $u$-dimensional disk $D^u$ into $M$, such that for every $g \in \mathcal{U}$ every disk $D \in \mathcal{D}$ intersects the closure of $W^s(Q_g)$, where $Q_g$ is the analytic continuation of $Q$ for $g$. Define cs-blender in an analogous way interchanging $u$ and $s$ and unstable by stable.

In fact, $(u+1)$-dimensional strips containing a disk of $\mathcal{D}$ and whose tangent spaces does not contain stable directions intersect $W^s(Q_g)$. These strips are called vertical strips (for details see [8, Ch. 6.2]).

This property of blenders is the key point of our proof of ergodicity. In fact by means of this geometrical property, we obtain transversal intersection between stable and unstable manifolds even if we do not have control on their size and shape.

**Remark 3.2.** Suppose that we have a cu-blender of a partially hyperbolic diffeomorphism associated to a periodic point $Q$ of unstable dimension $u + 1$. The construction of blenders imply that there is an open ball of size $\varepsilon_0$ such that any disk of dimension $u + 1$ through a point in this ball, whose tangent space is inside a cone around $E^u \oplus E^s$ and containing a (large enough) strong unstable disk must intersect $W^s(Q_g)$.

#### 3.1. Proof of Theorem \[B\]

Let $f$ be a diffeomorphism as in Theorem \[B\]. Observe that conditions 2), 3) in Theorem \[B\] are stable under perturbations, hence there is a neighborhood $\mathcal{U}$ of $f$ in the $C^1$ topology such that every $g$ in $\mathcal{U}$ satisfies these conditions. Observe also that by condition 4), we may assume that the boxes appearing in Remark 3.2 contain balls of radius $\varepsilon_0$ for every $g$ in this $\mathcal{U}$.

As in the proof of [10, Theorem 2], we have that since $f$ is accessible, there is a $C^1$ neighborhood $\mathcal{V}$ of $f$ such that for every $g$ in this
neighborhood, every $g$-orbit is $\varepsilon_0$ dense. We shall see that every $C^1$ diffeomorphism in $V \cap U$ is ergodic.

Let $g$ be in $U \cap V$ and let us see that we are in the hypothesis of Corollary 2.2. So, we have $cs$- and $cu$- blenders with boxes containing $\varepsilon_0$ balls, $B_{cs}$ and $B_{cu}$ respectively. Let $p$ be the periodic point associated to $B_{cs}$ and $q$ be the periodic point associated to $B_{cu}$ (see Definition 3.1 and the discussion after it). We also have that $W^u(O(p))$ intersects $W^u(O(q))$ transversally.

Let us see that for a.e. $x \in M$, if $\lambda^-(x) < 0$ then $x \in B^u(p)$. Since $B^u(p)$ is invariant it is enough to see that an iterate $x'$ of $x$ is in $B^u(p)$. The $\varepsilon_0$-density of a.e. orbit implies that we can take $x'$ in $B_{cs}$. Moreover, since $\lambda^-$ is invariant we have that $\lambda^-(x') < 0$. So we have that $W^u_p(x')$ contains a disk of dimension $(u+1)$, tangent to $E^u \oplus E^-$ and containing a large strip in the direction of $E^u$. Hence, by the main property of the blenders we have that $W^u_p(x')$ intersects transversally $W^s(p)$ and hence $x' \in B^u(p)$. Similarly we prove that for a.e. $x \in M$, if $\lambda^+(x) > 0$ then $x \in B^s(q)$.

Since the splitting $E^- \oplus E^+$ is dominated we have that $\lambda^-(x) < \lambda^+(x)$ for a.e. $x \in M$, and hence either $\lambda^-(x) < 0$ or $\lambda^+(x) > 0$. So we obtain that either $x \in B^u(p)$ or $x \in B^s(q)$ for a.e. $x \in M$. In other words $B^u(p) \cup B^s(q) \supseteq M$. On the other hand, since $\int_M \lambda^-(x) dm < 0$ we have that there is a set of positive measure where $\lambda^-(x) < 0$ for every $x$ in this set and hence $m(B^u(p)) > 0$. Similarly $\int_M \lambda^+(x) dm > 0$ implies that $m(B^s(q)) > 0$. Then we are in the hypotheses of Corollary 2.2 concluding that the system is ergodic and non-uniformly hyperbolic.

Finally observe, that any iterate of $g$ satisfies also the same properties of $g$ and hence is also ergodic, so $g$ is Bernoulli.

4. Pugh-Shub Conjecture

In this section, we explain how to apply Theorem 3 to prove Pugh-Shub Conjecture in $C^1$ topology. Let $f \in \mathcal{PH}_m$. We shall perform a finite number of arbitrarily small $C^1$-perturbations, among $C^{1+}$ volume preserving partially hyperbolic diffeomorphisms, so that we obtain a stably ergodic diffeomorphism arbitrarily close to $f$.

Due to [12], we loose no generality in assuming that $f$ is stably accessible.

Let us call $\lambda^{-}_f(x) \leq \lambda^{+}_f(x)$ the central Lyapunov exponents of $x$ with respect to $f$, and recall that

$$\int_M (\lambda^{-}_f(x) + \lambda^{+}_f(x)) dm(x) = \int_M \log \text{Jac}(Df(x)|E^c_\epsilon) dm(x)$$
Observe that this amount depends continuously on $f$, due to continuity of $E_c^c(f)$. We have

**Theorem 4.1** (Baraviera-Bonatti [2]). *Let $f$ be a $C^1$ partially hyperbolic diffeomorphism, then there are arbitrarily small $C^1$-perturbations $g$ of $f$ such that*

$$
\int_M \log \text{Jac}(Dg(x)|E^-_c)dm(x) > \int_M \log \text{Jac}(Dg(x)|E^+_c)dm(x).
$$

This theorem allows us to choose a $C^1$-perturbation $g \in \mathcal{P}H_m(M)$ arbitrarily near to $f$ such that

$$
\int_M (\lambda^+_g(x) + \lambda^-_g(x))dm(x) > 0. 
$$

(4.1)

or

$$
\int_M (\lambda^+_g(x) + \lambda^-_g(x))dm(x) < 0. 
$$

(4.2)

We deal with the first case and the second case requires a similar argument. Observe that the condition (4.1) is verified for any diffeomorphism $C^1$-close enough to $g$. We consider the two following cases:

1. $E^c$ does not admit a dominated splitting,
2. There is a dominated splitting of $E^c = E^- \oplus E^+.$

If the first case occurs, we use a result of Bochi-Viana [4] to obtain a new diffeomorphism $h$ $C^1$-close to $g$ such that $\int_M \lambda^-_h(x)dm(x) > 0$. This implies that there exists a subset $A$ of $M$ with positive Lebesgue measure such that $\lambda^-_h(x) > 0$ for all $x \in A$. Consequently $\lambda^+_h(x) > 0$ for $x \in A$ and we are in the setting of Theorem [1.1].

So, let us deal with the second case: $E^c$ admits a dominated splitting. Note that (4.1) implies that $\int_M \lambda^+_g(x)dm(x) > 0$. Theorem 4.1 above implies that either $\int_M \lambda^-_g(x)dm(x) > 0$ and Theorem [1.1] applies, or else $\int_M \lambda^-_g(x)dm(x) < 0$. In this last case, we want to produce a perturbation in such a way that we are in the hypotheses of Theorem [3] that is, we want to find $h \in \mathcal{P}H_m(M)$ $C^1$-arbitrarily near $g$ admitting a $cs$-blender of stable dimension $(s + 1)$ and a $cu$-blender of unstable dimension $(u - 1)$ which are homoclinically related. We begin by stating the following lemma:

**Lemma 4.2.** *Let $f \in \mathcal{P}H_m(M)$ be a stably accessible diffeomorphisms such that $\text{dim } E^c = 2$. Then, either*

1. $f$ is $C^1$-approximated by stably ergodic diffeomorphisms, or
(2) \( f \) is \( C^1 \)-approximated by \( g \in \mathcal{PH}_m(M) \) having three hyperbolic periodic points with stable dimension \( s \), \( (s + 1) \) and \( (s + 2) \), respectively, where \( s = \dim E^s \).

To prove the above lemma we apply conservative versions of Mañé’s Ergodic Closing Lemma [1] and Frank’s Lemma [7].

To finish the proof we will need the following version of the Connecting Lemma. A proof of more general results can be found in [1] or [5].

**Theorem 4.3** (Connecting Lemma). Let \( p, q \) be hyperbolic periodic points of a \( C^1 \) conservative transitive diffeomorphism \( f \). Then, there exists a \( C^1 \) conservative diffeomorphism \( g \) \( C^1 \)-close to \( f \) such that \( W^s(p) \cap W^u(q) \neq \emptyset \).

We are almost done. Since the diffeomorphism is transitive, Connecting Lemma implies, by making a perturbation, the existence of cycles in the conditions of the following proposition. This proposition is a conservative version of results in [6].

**Proposition 4.4.** Let \( f \) be a \( C^1 \)-conservative diffeomorphism having a co-index one cycle with real central eigenvalues associated to saddles. Then \( f \) can be approximated (in the \( C^1 \)-topology) by \( C^1 \)-conservative diffeomorphisms which robustly (\( C^1 \)-topology) admit blenders.

Here, a co-index one cycle is a cycle where the difference between the stable dimensions of the saddles is one.

After this we obtain the desired blenders and by applying the Connecting Lemma again, we have that their periodic points are homoclinically related. Finally, Theorem B implies stable ergodicity.

5. **Uniqueness of SRB-measures**

As we have said in the introduction we can also apply our criteria to show uniqueness of SRB-measures of surface diffeomorphisms.

**Theorem D.** Let \( f \) be a \( C^1 \) transitive diffeomorphism of a surface. Then, \( f \) has at most one SRB-measure.

As a corollary we also obtain that

**Corollary 5.1.** Let \( f \) be a non-uniformly hyperbolic \( C^1 \) conservative transitive diffeomorphism of a surface. Then, \( f \) is ergodic.

Let us say a few words about the proof of Theorem D. First of all let us consider the easier case where \( f \) satisfies the Kupka-Smale (KS) condition. Suppose that \( p_i, i = 1, 2 \), are two (typical) periodic points (as the ones obtained in [15]) associated to SRB-measures \( \mu_i, i = 1, 2 \).
We can take “rectangles” $R_i$, $i = 1, 2$, with sides in $W^{\sigma}(p_i)$, $i = 1, 2$, $\sigma = s, u$. Transitivity of $f$ implies that $f^k(R_1)$ intersects $R_2$ for some large enough $k$. If the rectangles $R_i$ have been chosen small enough the intersection between $f^k(R_1)$ and $R_2$ implies that an iterate of $W^u(p_1)$ intersects (transversally, since $f$ verifies KS condition) $W^s(p_2)$. Now, analyzing the stable “foliations” as in our criterion, the equivalence of the measures $\mu_i$ to the Lebesgue measure along unstable manifolds will imply that $\mu_1 = \mu_2$.

In case $f$ does not verify the KS condition the strategy is to show that many transversal intersections exist in spite of the possible existence of tangencies. This is obtained by a subtle argument using Sards Theorem.

References

[1] M.C. Arnaud, Création de connexions en topologie $C^1$, Erg. Th. & Dyn. Systems 21, (2001) 339–381.
[2] A. Baraviera, C. Bonatti, Removing zero Lyapunov exponents, Erg. Th. & Dyn. Systems 23, (2003) 1655–1670.
[3] L. Barreira, Y. Pesin, Nonuniform Hyperbolicity: Dynamics of Systems with Nonzero Lyapunov Exponents, Encyclopedia of Mathematics and its Applications, Cambridge University Press, 2007.
[4] J. Bochi, M. Viana, The Lyapunov exponents of generic volume-preserving and symplectic maps, Ann. Math. 161, (2005) 1423–1485.
[5] C. Bonatti, S. Crovisier, Récurrence et généricité, Inv. Math. 158, (2004) 33–104.
[6] C. Bonatti, L. Díaz, Robust heterodimensional cycles and $C^1$-generic dynamics, Preprint 2006.
[7] C. Bonatti, L. Díaz, E. Pujals, A $C^1$–generic dichotomy Weak forms of hyperbolicity or infinitely many sinks or sources, Ann. Math. 158, (2003) 355–418.
[8] C. Bonatti, L. Díaz, and M. Viana, Dynamics beyond uniform hyperbolicity: A global geometric and probabilistic perspective, Encyclopaedia of Mathematical Sciences, Vol 102, Mathematical Physics, III, Springer-Verlag, Berlin, 2005.
[9] M. Brin, Ya. Pesin, Partially hyperbolic dynamical systems, Math. USSR Izv. 8, (1974) 177–218.
[10] K. Burns, D. Dolgopyat, Ya. Pesin, Partial hyperbolicity, Lyapunov exponents and stable ergodicity, J. Stat. Phys. 108, (2002) 927–942.
[11] K. Burns, A. Wilkinson, On the ergodicity of partially hyperbolic diffeomorphisms, to appear in Ann. Math.
[12] D. Dolgopyat, A. Wilkinson, Stable accessibility is $C^1$ dense, Astérisque 287, (2003) 33–60.
[13] M. Grayson, C. Pugh, M. Shub, Stably ergodic diffeomorphisms, Ann. Math. 140, (1994) 295–329.
[14] E. Hopf, Statistik der geodätischen Linien in Mannigfaltigkeiten negativer Krümmung, Ber. Verh. Sächs. Akad. Wiss. Leipzig 91, (1939) 261-304.
[15] A. Katok, Lyapunov exponents, entropy and periodic orbits for diffeomorphisms, *IHES Publ. Math* **51**, (1980) 137–173.

[16] I. Kan, Open sets of diffeomorphisms having two attractors, each with an everywhere dense basin, *Bull. AMS* **31**, (1994) 68–74.

[17] C. Pugh, M. Shub, Stable ergodicity and partial hyperbolicity, Ledrappier, F. (ed.) et al., 1st International Conference on Dynamical Systems, Montevideo, Uruguay, 1995 - A tribute to Ricardo Mañé. Proceedings. Harlow: Longman. Pitman Res. Notes Math. Ser. **362**, (1996) 182–187.

[18] C. Pugh, M. Shub, Stable ergodicity and julienne quasiconformality, *J. EMS* **2**, (2000) 1–52.

[19] F. Rodríguez Hertz, M. Rodríguez Hertz, R. Ures, Accessibility and stable ergodicity for partially hyperbolic diffeomorphisms with 1d-center bundle, *Preprint* 2006.

[20] F. Rodríguez Hertz, M. Rodríguez Hertz, R. Ures, A survey of partially hyperbolic dynamics, to appear in *Fields Inst. Comm*.

[21] A. Tahzibi, Stably ergodic systems which are not partially hyperbolic, *Israel J. Math.* **142**, (2004) 315–344.

[22] L.-S. Young, What are SRB measures, and which dynamical systems have them?, *J. Stat. Phys.* **108**, (2002) 733–754.

IMERL-Facultad de Ingeniería, Universidad de la República, CC 30 Montevideo, Uruguay.

E-mail address: frhertz@fing.edu.uy

URL: [http://www.fing.edu.uy/~frhertz](http://www.fing.edu.uy/~frhertz)

IMERL-Facultad de Ingeniería, Universidad de la República, CC 30 Montevideo, Uruguay.

E-mail address: jana@fing.edu.uy

URL: [http://www.fing.edu.uy/~jana](http://www.fing.edu.uy/~jana)

Departamento de Matemática, ICMC-USP São Carlos, Caixa Postal 668, 13560-970 São Carlos-SP, Brazil.

URL: [http://www.icmc.sc.usp.br/~tahzibi](http://www.icmc.sc.usp.br/~tahzibi)

E-mail address: tahzibi@icmc.sc.usp.br

IMERL-Facultad de Ingeniería, Universidad de la República, CC 30 Montevideo, Uruguay.

E-mail address: ures@fing.edu.uy

URL: [http://www.fing.edu.uy/~ures](http://www.fing.edu.uy/~ures)