A Modified Branch and Bound Algorithm for Batch Scheduling in Discrete Manufacture System

A S Indrapriyatna and H Triha
Industrial Engineering Department, Faculty of Engineering, Universitas Andalas, Padang, West Sumatera, Indonesia

ahmadsyaf@ft.unand.ac.id, hadi.06173058@gmail.com

Abstract. Indrapriyatna et al. [1,2] have developed a batch scheduling model. In a model with a discrete batch size, three methods are used to convert batch size to integer. Based on numerical calculations it is found that there is no method that always result in the smallest total cost in every data set. This research is done to try other methods to convert batch size to integer. The approach done is by using the Branch and Bound Algorithm Modification that is adjusted to get scheduling for each batch without changing the number of batch (N) and the number of quantities (q) that are going to be produced. The result is the CSA_Dis_BB Algorithm that gets a discrete (integer) batch size. The implementation of CSA_Dis_BB Algorithm is done on 7 sets of data from [1,2]. This algorithm does work to produce the smallest total cost.

1. Introduction
Scheduling is essential to produce the order with exact allocations of resources, like the machines used, number of working operators, sequencing the work on parts, and the materials needed. [7] defines scheduling as the process of allocating resources over certain time frame in order to finish certain tasks. Problems of scheduling comes up whenever the work need to be done exceeds the resources.

A good scheduling maximizes the effectivity and usability of the available resources, hence scheduling is a critical activity in each planning, in production, and in production supervision. An effective and efficient scheduling reduces waiting time and idle time, which therefore cuts production costs.

[3,4,5,6] discuss the problem of sequencing and deciding on the batch size which means the job size. Thus processing time is a variable of decision. The problem of this is how to decide on the batch size (batching) and the sequencing of the processes on those batches; this is called the batch sequencing. [5] have proposed a batch scheduling model that take into account the storage costs and quality costs in a single machine production system.

[1,2] developed a batch scheduling model that considers the storage costs and quality costs and take into account both storage costs of work-in-process batch and finished-batch. This model (1) differentiates storage costs of work-in-process part and finished part in the in-process batch, (2) varies the sample sizes in proportion to the batch size, and (3) has the batch and sample size in integer.

The research that was done resulted in a batch scheduling model that considered storage costs and quality costs and also calculated storage costs of work-in-process batch and finished-batch, had sample size that was proportional to the batch size, for a discrete manufacture system, and applied to
a flowshop system made up of one machine. But in this model with the discrete batch size there are 3 methods used to change the batch size into integer, and those methods are: (1) Top Decimal Sum (TDS), (2) Bottom Decimal Sum (BDS), and (3) Rounding-Off Method.

Based on numerical analysis, it can be concluded that from the above three conversion methods into integer, there is not a single method that always results in a best solution. In other words, the method of conversion into integer that is being used does not always give dominant results for each set of data being tested. For that reason, a different conversion method is needed to convert batch size into integer. The approach taken here is by using Branch and Bound Algorithm (because it has been proven to solve integer problems well).

Land and Doig’s (1960) mentioned that branch and bound algorithm was developed in the context of finding a solution for linear programs with integer decision variable. The components of this algorithm are [8]: Branching, Bounding, Pruning, and Retracting.

In practice, almost all integer problems (IP) is solved by the branch and bound technic. This technic finds optimal solution of IP by enumerating points within feasible area of the subproblem [9]. Each living node is associated with a cost that sets the bound.

2. Research Methodology

Research Methodology is an essential part of a research. This will describe the steps of the research.

Preliminary Study

The objective of preliminary study is to compile the theories that will be the base of finding good solutions.

Problem Identification and Formulation

This is to explain what need to be solved, and formulating it, and explaining and indentifying the problems within certain limits.

Data Collection

The data that is used in this research is theoretical data from the previous researcher [1,2].

Algorithm Design

The process is by designing Branch and Bound Algorithm to get integer Q[i] and algorithm logic test. Next total cost and schedule of each batch is set based on the existing formulas.

Software Design and Analysis

Based on the algorithm, a software is then designed, and the algorithm is analysed.

Closing

Design and results is then concluded and given suggestions for improvement.

3. Research Results

Data Collection

The data used in this research is the 7 sets of data taken from Indrapriyatna et al. [1,2]. Here the units of time and costs are not specified, because any unit can be used as long as it is appropriate, for example minutes for time unit, and monetary currency for the cost. This is done to show that this model is applicable in general.

Branch and Bound Algorithm Modification Design

This algorithm is called a modification because it does not completely follow the BB algorithm. Optimal tests are only made on combinations that give the smallest total cost to the previous batch size. While the original BB algorithm should try also compared with other possible combinations. CSA Algorithm is the basic reference in implementing Branch and Bound Algorithm in the CSA model. The initial phase of designing is by taking the value Q[i] and the optimal number of batches (N), results of CSA Algorithm where i = 1,2,3,....N. In applying the Branch and Bound Algorithm, some additional variables are needed:
a = Number of iterations done, where a = 1, 2, 3,…, N-1.

$Q_{up}[a]$ = Value of $Q[a]$ rounded up.

$Q_{down}[a]$ = Value of $Q[a]$ rounded down.

$TC_{up}$ = $TC[N, Q]$ when $Q[a]$ is rounded up.

$TC_{down}$ = $TC[N, Q]$ when $Q[a]$ is rounded down.

$q_{awal}$ = Number of queries (q) in CSA Algorithm.

$sisa_{up}$ = result $q$ from $q_{awal} - \sum_{i=1}^{a} Q[i]$, when $Q[a]$ is rounded up.

$sisa_{down}$ = result $q$ from $q_{awal} - \sum_{i=1}^{a} Q[i]$, when $Q[a]$ is rounded down.

$Qbaru[N, a]$ = Values of $Q[i]$ after converted to integer ($Q[i]$’).

$TC_{BB}[N]$ = Total costs of Branch and Bound.

The Figure 1 is the Branch and Bound Algorithm to convert $Q[i]$ to its integer form ($Q[i]$’), called the CSA_Dis_BB Algorithm.

Figure 1. Flowchart of CSA_Dis_BB Algorithm
Testing the CSA_Dis_BB Algorithm
An algorithm is considered correct if it produces the correct outputs for all possibilities of input. Therefore, used to test the algorithm is a set of continuous data with three continuous-sized batches. The test intends to check the logic coherency of the algorithm.

Software Design
After the algorithm is created, the application or software is designed. The programming language used is C# (C Sharp) and the editor used is Visual Studio 2005 and 2008. After obtaining the integer value of batch sizes \((Q'[i])\), Total Cost \((TC[N,Q'])\) is calculated based on the CSA_Dis Model for \(Q'[i]\). After the integer value of sample size is obtained, Total Cost \(TC([N,Q'])\) can be calculated. The following is the total cost of CSA_Dis Method using the integer sample size from CSA_Dis_BB, Rounding Up, TDS, BDS, and rounding-off methods. The recapitulation of total cost is shown in Table 1.

| No. | Method                  | Total Cost       |
|-----|-------------------------|------------------|
| 1   | CSA                     | 79,946,295.95    |
| 2   | CSA_Dis_BB              | 79,946,302.67    |
| 3   | CSA_Dis_RoundingUp      | 79,995,360.68    |
| 4   | CSA_Dis_TDS             | 80,009,882.97    |
| 5   | CSA_Dis_BDS             | 80,011,304.63    |
| 6   | CSA_Dis_Rounding       | 80,000,334.60    |

Table 1. Total Cost Recapitulation

Analysis of CSA_Dis_BB Algorithm Design Result
The CSA_Dis_BB Algorithm design aims to obtain the \(Q[i]\) in integer \((Q[i'])\), Schedule for each batch, and due date for each setup and new parts processing activities \((d')\) and minimum Total Cost \((TC[N,Q'])\). The Schedule of each batch and \(d'\) is shown in Figure 2.

![Figure 2. Schedule for Every Batch](image)

Based on Figure 2, the due date for set up and new parts processing activities \((d')\) is smaller than the initial due date. This shows that the resulting schedule and new due date \((d')\) are within the set parameters.

On the other hand, the Total Costs obtained from the CSA_Dis_BB Algorithm \((TC[N,Q'])\) are smallest than the Total Cost from the CSA Algorithm \(TC[N,Q]\). This is affected by the integer batch sizes. \(TC[N,Q']\) from the CSA_Dis_BB Algorithm are all smaller than the integer conversion methods (TDS, BDS, Rounding-Off, Rounding-Up) used by [2].
Table 2 and 3 show recapitulation of total cost for 7 data sets.

**Table 2.** Calculation Result of CSA, CSA\_Dis\_BB, and CSA\_Dis Models with \( u = 10\% 

| Set     | Model CSA (Rp.) | Model CSA\_Dis (Rp.) | CSA\_Dis\_BB | Top Decimal Sum | Bottom Decimal Sum | Rounding-off | Rounding-up |
|---------|-----------------|----------------------|--------------|----------------|-------------------|--------------|-------------|
| Set 1   | 79,946,395.95   | 79,946,392.67        | 80,039,882.97| 80,011,308.63 | 80,000,334.00    | 79,993,360.08|
| Set 2   | 6,035,997,149.32| 6,037,999,409.60     | 6,037,939,807.26| 6,037,940,808.95| 6,037,942,705.08| 6,037,942,768.08|
| Set 3   | 4,682.16        | 4,683.89             | 4,841.41     | 4,831.55       | 4,836.83        | 4,836.88     |
| Set 4   | 4,024,001,344.59| 4,024,004,959.71     | 4,025,698,186.67| 4,025,700,717.75| 4,025,662,344.82| 4,025,660,400.21|
| Set 5   | 7,945,875,401.08| 7,945,878,610.37     | 7,946,387,171.30| 7,946,389,190.92| NA               | 7,946,370,307.07|
| Set 6   | 1,385,189,734.40| 1,385,190,782.217.74| 1,385,193,097,488.45| 1,385,193,107,318.21| 1,385,193,419,190.73| 1,385,193,498,802.81|
| Set 7   | 2,171,126,412.81| 2,171,126,207,002.80| 2,171,130,933,876.69| 2,171,130,947,604.92| 2,171,130,805,941.50| 2,171,130,804,866.65|

**Table 3.** Calculation Result of CSA, CSA\_Dis\_BB, and CSA\_Dis Models with \( u = 20\% 

| Set     | Model CSA (Rp.) | Model CSA\_Dis (Rp.) | CSA\_Dis\_BB | Top Decimal Sum | Bottom Decimal Sum | Rounding-off | Rounding-up |
|---------|-----------------|----------------------|--------------|----------------|-------------------|--------------|-------------|
| Set 1   | 88,701,045.51   | 88,781,055.16        | 88,781,029.44| 88,781,035.46 | 88,761,962.08    | 88,761,962.08|
| Set 2   | 6,299,363,629.06| 6,299,365,042.70     | 6,299,359,487.20| 6,299,364,137.30| 6,299,823,235.01| 6,299,823,235.01|
| Set 3   | 5,164.72        | 5,167.83             | 5,154.04     | 4,851.55      | NA               | 5,301.45     |
| Set 4   | Not feasible    | Not feasible          | Not feasible | Not feasible | Not feasible     | Not feasible |
| Set 5   | 8,008,561,073.33| 8,008,564,331.18     | 8,009,020,976.71| 8,009,024,453.28| 8,009,018,134.11| 8,009,018,847.50|
| Set 6   | Not feasible    | Not feasible          | Not feasible | Not feasible | Not feasible     | Not feasible |
| Set 7   | 2,176,008,487.542.06| 2,176,008,487,698.73| 2,176,012,679,806.23| 2,176,012,705,088.14| 2,176,012,555,358.62| 2,176,012,540,375.54|

Based on Table 2 and 3 (Rounding-Off column), there are cells highlighted in grey that says NA. This means that for the Total Cost \((TC[N,Q'])\) calculations for data set 5 with sample proportion \( u = 10\% \) and data set 3 with sample proportion \( u = 20\% \) in the Rounding-Off Method were not done because the number of query resulted by the integer conversion did not fulfill the parameter of CSA, where the batch sizes were not equal to the initial query.

The CSA\_Dis\_BB Algorithm obtained discrete batch sizes (integer) without changing the number of batches and queries to be produced. The CSA\_Dis\_BB Algorithm was implemented on 7 data sets from [1,2]. For all 7 data sets and integer conversion methods, the number of queries after the integer conversion process was equal to the initial number of queries, except for the Rounding-Off Method. The Rounding-Off Method resulted in different number of queries, which were data set 5 with sample proportion \( u = 10\% \) and data set 3 with sample proportion \( u = 20\% \). The numbers of queries resulted were 1 unit smaller than the intial numbers of queries. This shows that, in those data sets, that the Rounding-Off Method failed to fulfill one parameter of CSA model, that number of discrete batch sizes must be equal to the number of initial queries.

**4. Conclusion**

We have proposed a modified Branch and Bound algorithm, named The CSA\_Dis\_BB, to obtain an integer batch size. This algorithm does work to produce the smallest total cost.

**5. Recommendations**

In order to improve the research on CSA\_Dis\_BB and CSA\_Dis\_BB Algorithm design, it is recommended to:

1. Implement other models, as this research only referred to one machine model (CSA), while [2] developed scheduling models for 2, 3, and m machines.
2. Use real value data or field observation data with conditions appropriate for the existing model, in order to test the CSA\_Dis\_BB method compatibility with all types of data.
Acknowledgment
The authors would like to thank Prof. Dr.-Ing. Hairul Abral, Jonrinaldi, PhD., and Dr. Rika A. Hadiguna for their help.

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