Pion Form Factor and Quark Mass Evolution in a Light-Front Quark Model

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We discuss the soft contribution to the elastic pion form factor with the mass evolution from current to constituent quark being taken into account in a light-front quark model (LFQM).

The pion electromagnetic (EM) form factor is of great interest for the study of Quantum Chromodynamics (QCD). At low momentum transfers ($Q^2$) nonperturbative QCD (NPQCD) dominates, while at large $Q^2$ perturbative QCD (PQCD) can be used to calculate the asymptotic form factor; and the transition from NPQCD to PQCD has long been of interest. The light-front (LF) quantization method\textsuperscript{[1]} may be most useful in connecting the formulations of NPQCD and PQCD since the LF wavefunctions provide the essential link between hadronic phenomena at short distances (perturbative) and at long distances (nonperturbative). Although the relevant minimum momentum scale for the PQCD exclusive processes is still under debate\textsuperscript{[2]}, the LF method has been successfully applied to the constituent quark model and described the hadron properties at low momentum transfer region quite well\textsuperscript{[3,4]}. In many previous quark models\textsuperscript{[3,4]}, a constant constituent quark mass was used in the analysis of the hadron properties especially at $Q^2 < 1$ GeV\textsuperscript{2}. As shown in the literatures\textsuperscript{[3,4]}, such constituent quark model has been quite successful in describing static properties of a hadron such as the form factor, charge radius, and decay constant etc. On the other hand, the approach based on the quantum field theory such as the Dyson Schwinger Equations (DSEs)\textsuperscript{[5]} uses the running mass instead of constant constituent mass and it also gives properties of the pion that are in agreement with the experimental data.

Thus, in this talk, we present the quark mass evolution effect on the pion in a light-front quark model (LFQM)\textsuperscript{[6]}. In the present work we restrict ourselves to the soft NPQCD part with a LFQM, but an essential ingredient is the use of a running quark mass, which is the main subject of this talk.

The form factor of the pion is related to the matrix element of the current by the following equation:

$$\langle J^{\mu}_{e.m.} \rangle \equiv \langle P'|\bar{q}\Gamma^{\mu}q|P \rangle = (P' + P)^{\mu} F_{\pi}(Q^2).$$

In usual LF frame, the form factor of a hadron can be obtained by the sum of valence and nonvalence diagrams. However, if we choose the Drell-Yan-West (DYW) (or $q^+ = 0$) frame with "+"-component of the current, only the valence diagram is needed. Then, the matrix element of the current given by Eq. (1) can be expressed as a convolution integral in terms of LF wave function, $\Psi(x, k_{\perp})$ as follows:

$$\langle J^{\mu}_{e.m.} \rangle = \sum_{\lambda_{q} \lambda'_{q}} \int_{0}^{1} dx \int d^{2}k_{\perp} \Psi_{\lambda_{q} \lambda'_{q}}(x, k_{\perp}) \times \frac{\bar{u}_{\lambda'_{q}}(p'_{q})}{\sqrt{p'_{q}}} \Gamma^{\mu}_{\lambda_{q} \lambda'_{q}} \frac{u_{\lambda_{q}}(p_{q})}{\sqrt{p_{q}}} \Psi_{\lambda_{q} \lambda'_{q}}(x, k_{\perp}),$$

Here, $\bar{u}$ and $u$ are the Dirac spinors of the quark and antiquark, respectively, while $p$ and $p'$ are their fourmomenta, $\lambda_{q}$ and $\lambda'_{q}$ are the color indices of the quark and antiquark, respectively, and $\Gamma^{\mu}_{\lambda_{q} \lambda'_{q}}$ is the matrix element of the current.

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where $p_\perp^2 = p^\perp_\ast^2 = (1 - x) P_\perp^2$ and $k'_\perp = k_\perp - x q_\perp$ in the initial pion rest frame, $P_\perp = 0$. The helicity of the quark(antiquark) is denoted as $\lambda_{\bar{q}(q)}$. In our model calculation of the pion form factor, we use the Gaussian radial wave function as well as the relativistic spin-orbit wave function obtained by the interaction independent Melosh transformation (see Appendix B for more detail).

In the usual light-front constituent quark model, the bare quark-photon vertex, $\Gamma^\mu = \gamma^\mu$, is used. However, when the quark propagator has momentum-dependent dressing, the bare vertex is no longer adequate because it violates the Ward-Takahashi identity (WTI). In general, the solution of the DSE for the renormalized dressed-quark propagator takes the form

$$S(p) = A(p^2) \gamma^\mu - B(p^2)$$

where the quark mass evolution function $m(p^2)$ is defined as $m(p^2) = B(p^2)/A(p^2)$. Also, gauge invariance requires that the quark-photon vertex $\Gamma^\mu$ given by Eq. (2) satisfy the WTI, i.e. $-q^\mu \Gamma^\mu(p;q) = S(p')^{-1} - S(p)^{-1}$ (current conservation) as well as $\Gamma^\mu(p;0) = \partial S(p)^{-1}/\partial p_\mu$ (charge conservation), where $q = p - p'$.

As used in many DSE studies of EM interactions, we take the Ball-Chiu (BC) ansatz for the quark-photon vertex

$$\Gamma^\mu_{BC} = \frac{(p + p')_\mu}{2} (p + p') \frac{A(p^2) - A(p'^2)}{p^2 - p'^2} + \frac{A(p^2) + A(p'^2)}{2} \gamma^\mu - (p + p')^\mu \frac{B(p^2) - B(p'^2)}{p^2 - p'^2}.$$  

(3)

Although the asymptotic behavior of the running mass might require crossing symmetry (under $Q^2 \leftrightarrow -Q^2$) at high momentum transfer, there is no clue yet for the low momentum transfer region. So, we introduce two algebraic parametrizations of the running mass: one satisfying the crossing symmetry (CS) and the other satisfying the crossing asymmetry (CA):

$$m_{CS}(p^2) = m_0 + (m_c - m_0) \frac{1 + e^{-\mu^2/\lambda^2}}{1 + e^{(p^2 - m_0^2)/\lambda^2}},$$

$$m_{CA}(p^2) = m_0 + (m_c - m_0) \frac{1 + e^{-\mu^2/\lambda^2}}{1 + e^{(p^2 - m_0^2)/\lambda^2}},$$  

(4)

where $m_0$ and $m_c$ are the current and constituent quark masses, respectively. The parameters $\mu$ and $\lambda$ are used to adjust the shape of the mass evolution.

For comparison, we use in Fig. 3 two different parameter sets for each mass evolution function. The current and constituent quark masses used are $m_0 = 5$ MeV and $m_c = 220$ MeV, respectively. Simulating the constituent picture at the small momentum region, we have chosen these particular sets of parameters, [Set 1] and [Set 2] for each mass function, to keep the constituent mass up to $(-p^2) \sim 1$ and 0.5 GeV$^2$, respectively, before it drops exponentially.

In order to express the four momentum $p^2$ in terms of LF variables $(x, k_\perp)$, we use the on-mass shell condition, $p^2 = m^2(p^2)$. It implies zero binding energy of a mock meson, i.e. $P^- = p^- + p_\perp^-$ where $P^- = P^0 - P^3$ and $p^- (p^-)$ are the LF energies of the mock meson and the quark(antiquark), respectively. It leads to the following identity for the pion case ($m_q = m_\pi$), $p^2 = x(1 - x)M^2 - k_\perp^2$. For the mock meson mass $M$, we take the average value (so called spin-averaged meson mass) of $\pi(m_\pi)$ and $\rho(m_\rho)$ with appropriate weighting factors from the spin degrees of freedom, i.e. $M = (m_\pi + 3m_\rho)\exp/4 = 612$ MeV.

In our numerical calculations, we use the model parameters $(m_c, \beta) = (0.22, 0.3659)$ [GeV] obtained in Ref. 3 for the linear confining potential model.
Figure 2. Pion EM form factor: (a) Crossing
asymmetry (CA) and (b) Crossing symmetry (CS) mass functions compared with the experimental data [8] as well as the CQM result [4].

In Fig. 2 we show our results of the form factor for the intermediate $Q^2$ region for CA [Fig. 2(a)] and CS [Fig. 2(b)] mass functions compared with the experimental data [8] as well as the CQM result [4]. As one can see from Fig. 2, (1) there are differences between the bare vertex and BC ansatz indicating the breakdown of local gauge invariance from the usage of the bare vertex, (2) the [Set 2] for both CA and CS mass functions show larger deviation from the CQM result than the [Set 1] case for the region of momentum transfer $Q^2 \sim 2$ GeV$^2$ and above, (3) the results with the BC vertex fall off faster (at around $Q^2 = 2$ GeV$^2$) than the CQM result does, and (4) the CA mass evolution function is more sensitive to the variation of the momentum dependence than the CS mass evolution function.

In conclusion, we have reexamined the soft contribution to the pion elastic form factor using LFQM with a running quark mass. The Ball-Chiu ansatz was used for the dressed quark-photon vertex. We were also able to calculate the quark condensate as $-\langle \bar{q}q \rangle = (0.3 \text{ GeV})^3$, which is quite comparable with the value employed in contemporary phenomenological studies: $(0.236 \text{ GeV})^3$. This shows the PCAC relation is reasonably well satisfied in LFQM with our mass evolution function.

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