Results on the Gluon Propagator in Lattice Covariant Gauges

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We illustrate preliminary results on the gluon propagator computed in generic covariant lattice gauges in the quenched approximation with the Wilson action. We have applied a recently proposed procedure to fix a generic covariant gauge on the lattice. We make a comparison among results obtained at two different values of the gauge parameter and in the standard Landau’s gauge.

We present preliminary data on the gluon propagator in generalized covariant gauges. The motivation for considering a generalized gauge-fixing procedure stems, just as in the continuum, from the necessity of studying the gauge dependence of the Green’s functions. In the covariant gauges it is possible to change the gauge fixing varying the value of the gauge parameter. In the continuum, the generic covariant gauges satisfy the following condition:

\[ \partial_\mu A_\mu^{G\lambda}(x) = \Lambda_\lambda(x) \]  

where \( A_\lambda(x) \) are matrices belonging to the Lie algebra of the SU(3) group and \( \lambda \) is the gauge parameter which controls the width of the gaussian distribution of the \( \lambda \)'s.

The corresponding gauge fixing procedure on the lattice is implicitly defined by the following expression for the expectation value of a gauge dependent operator \( \mathcal{O} \):

\[ \langle \mathcal{O} \rangle = \frac{1}{Z} \int d\Lambda \int dU \mathcal{O}(U^{G\lambda}) e^{-\frac{1}{\beta} \sum_x \text{Tr}(\Lambda^2)} e^{-\beta S(U)} \]  

(2)

where \( S(U) \) is the Wilson lattice gauge invariant action and \( G_\lambda \) is the gauge transformation that enforces the gauge condition.

After having generated a set of \( \Lambda \) according to a Gaussian distribution, the gauge fixing procedure follows very closely the widely used Landau’s gauge fixing: the gauge transformation \( G(x) \) which rotate the links \( U_\mu(x) \) into the gauge-fixed configuration is obtained by the numerical minimization of a functional \( F[G] \), chosen in such a way that its minimum corresponds to a gauge configuration satisfying the gauge condition. The functional form we adopt in the continuum reads:

\[ F[G] = \int d^4x \text{Tr} \left[ (\partial_\mu A_\mu^{G\lambda} - \Lambda)^2 \right] . \]  

(3)

The discretization of this expression requires a particular care (driven discretization) in order to obtain a functional form suitable to be minimized by the standard gauge-fixing algorithms. More details on this covariant gauge-fixing method can be found in \[1\], \[2\].

The stationary points of the above functional correspond to the following gauge condition in the continuum:

\[ D_\nu \partial_\nu (\partial_\mu A_\mu^{G\lambda} - \Lambda) = 0 \]  

(4)

It should be noted that, while for \( \Lambda = 0 \) eq. (4) reduces to the Landau gauge condition (apart from the presence of possible spurious solutions, i.e. solutions to \( \partial_\mu A_\mu^{G\lambda} - \Lambda = \rho \) with \( D_\nu \partial_\nu \rho = 0 \) which, however, we did not encounter in our study), the same is not true of the lattice construction, where the equations that follow from the minimization of the functional with \( \Lambda = 0 \) differ from the lattice Landau gauge condition by terms \( O(a) \). Thus the
A comparison of the results we will obtain for \( \Lambda = 0 \) with those obtained in the Landau gauge will be a meaningful check on our procedure.

We gauge-fixed 221 SU(3) configurations with \( V = 16^3 \times 32 \) and \( \beta = 6 \) taking the gauge parameter \( \lambda = 0,8 \) where \( \lambda = 0 \) has been considered in order to make a comparison with the Landau gauge. The gauge configurations were retrieved from the repository at the “Gauge Connection” (http://qcd.nersc.gov). The calculation has been performed on Boston University’s Origin2000 and took approximately 19000 CPU hours. We imposed a gauge fixing quality factor \( \theta < 10^{-6} \). With these gauge-fixed configurations we computed the correlator of gluon propagator

\[
D_{\mu\nu}(x - y) = \langle A_{\mu}(x) A_{\nu}(y) \rangle 
\]

(5)

where we adopted for \( A \) the standard expression

\[
A_{\mu}(x) \equiv \left[ U_{\mu}(x) - U_{\mu}^\dagger(x) \right]_{traceless}. 
\]

(6)

Discussions on the effect of different lattice definitions of the gluon field in the context of numerical calculations can be found in refs. [3] and [4]. The general properties of lattice operators were studied in ref. [5].

As usual in gluon propagator calculations [6] we study the quantity

\[
\langle A_0 A_0 \rangle(t) \equiv \frac{1}{V^2} \sum_{x,y} Tr(A_0(x,t)A_0(y,0)) . 
\]

(7)

which is displayed in Fig. 1. This correlator should be a constant in the case of Landau’s gauge-fixing and its behaviour is indicative of the quality of the gauge fixing procedure. Figure 1 shows that even in the case \( \lambda = 8 \) the above correlator is almost a constant. The case \( \lambda = 0 \) is equivalent to the Landau’s one with a different value of the constant.

In Fig. 2 we exhibit the behaviour of the propagator

\[
\langle A_i A_i \rangle(t) \equiv \frac{1}{3V^2} \sum_i \sum_{x,y} Tr(A_i(x,t)A_i(y,0)) 
\]

(8)

as a function of \( t \).

The results of this simulation show clearly that the gauge dependence of the gluon propagator (eq. (8)) on \( \lambda \) cannot be simply understood as a rescaling of the data, as one might have argued of the data on a smaller lattice, reported in the very preliminary analysis of ref. [7]. We also observe that the data for the Landau gauge fixing and for \( \lambda = 0 \) are compatible within errors. We proceed by showing the gluon propagator in Fourier space.

The transformed field \( A_{\mu}(k) \) is given by

\[
A_{\mu}(k) = e^{-i k_{\mu} q_{\mu}} \sum_x e^{-i (k \cdot x)} A_{\mu}(x) 
\]

(9)

where \( k_{\mu} = \frac{2\pi}{L_{\mu}} n_{\mu} \) with \( L_{\mu} = 16 \) or 32 for spatial and temporal indices respectively, \( n_{\mu} \) are integers. Following suggestions from lattice perturbation theory [8], in order to get rid of a well-known lattice artifact we express all the quantities as a function of the variable \( q \)

\[
q_{\mu} = 2\sin\left(\frac{k_{\mu}}{2}\right)
\]
rather than \( k \). This kinematic correction is needed to reduce the anisotropy of the data when displayed as function of \( k \). The Fourier transform of the gluon propagator in eq. (5) is:

\[
D_{\mu\nu}(q) = \frac{1}{V} < A_\mu(q) A_\nu^\dagger(q) >
\]

We analyzed the data in terms of the conventional distinction of transverse and longitudinal parts \( D_T \) and \( D_L \):

\[
D_{\mu\nu}(q) = (\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})D_T(q) + \frac{q_\mu q_\nu}{q^2} D_L(q)
\]

In Fig. 3 we present the behaviour of the transverse part (times \( q^2 \)) of the gluon propagator in covariant gauges, for the two values of the gauge parameter \( \lambda = 0 \), equivalent to the Landau’s gauge, and \( \lambda = 8 \), as a function of \( q = \sqrt{q^2} \). The data have been averaged over the \( \mathbb{Z}_3 \) group and only values corresponding to \( k_0 = \frac{2\pi}{32}(0 \div 12) \) and \( k_i = \frac{2\pi}{16}(0 \div 6) \) are reported. We see that at large \( q^2 \) the data at the two values of the gauge parameter tend to coincide, while the gluon propagator at low \( q^2 \) exhibits a clear gauge dependence. The data with \( \lambda = 8 \) show a lower peak than those with \( \lambda = 0 \), but the peak position stays fixed. In Fig. 4 we illustrate the behaviour of the longitudinal part \( D_L \) as a function of \( q \).

The data at \( \lambda = 0 \) vanish as one expects for Landau’s gauge. At \( \lambda = 8 \) the situation is different and a signal different to zero is observed with a clear increase for increasing \( q \). This differs from the predictions of perturbation theory, where the longitudinal part is a constant proportional to the value of the gauge parameter.

In summary, our gauge fixing procedure can be implemented for medium to large volume simulations; the gauge fixing obtained with our algorithm...
Figure 4. Longitudinal part of the gluon propagator as a function of $q$ at $\lambda = 0$ and $\lambda = 8$.

behaves as one expects in the case of $\lambda = 0$, reproducing the features of Landau’s gauge. The data of the gluon propagator are new and show a clear gauge dependence; the transverse and longitudinal part will be analyzed in more details in a future publication.

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