Abstract—Distributed orthogonal space–time block codes (DOSTBCs) achieving full-diversity order and single-symbol maximum-likelihood (ML) decodability have been introduced recently by Yi and Kim for cooperative networks, and an upper bound on the maximal rate of such codes along with code constructions has been presented. In this paper, a new class of single-symbol ML decodable precoded distributed space–time block codes (SSD-PDSTBCs) called semiorthogonal SSD-PDSTBCs (semi-SSD-PDSTBCs) is introduced wherein, the source performs linear precoding of information symbols appropriately before transmitting it to all the relays. It is shown that DOSTBCs are a special case of semi-SSD-PDSTBCs. A special class of semi-SSD-PDSTBCs having diagonal covariance matrix at the destination is studied and an upper bound on the maximal rate of such codes is derived. The bounds obtained are approximately twice larger than that of the DOSTBCs. A systematic construction of semi-SSD-PDSTBCs is presented when the number of relays \( K \geq 4 \). The constructed codes are shown to achieve the upper bound on the rate when \( K \) is of the form 0 or 3 modulo 4. For the rest of the values of \( K \), the constructed codes are shown to have rates higher than that of DOSTBCs. It is shown that semi-SSD-PDSTBCs cannot be constructed with any form of linear processing at the relays when the source does not perform linear precoding of the information symbols.

Index Terms—Cooperative diversity, distributed space–time coding, precoding, single-symbol maximum-likelihood (ML) decoding.

I. INTRODUCTION AND PRELIMINARIES

Cooperative communication is a promising means of achieving spatial diversity without the need of multiple antennas at the individual nodes in a wireless network. The idea is based on the relay channel model as shown in Fig. 1, where a set of distributed antennas belonging to multiple users in the network cooperate to encode the signal transmitted from the source, and forward it to the destination so that a required diversity order is achieved [1]–[4]. Spatial diversity obtained from such a cooperation is referred to as cooperative diversity. In [5], the idea of space–time coding devised for point to point colocated multiple-antenna systems is applied for a wireless relay network and is referred to as distributed space–time coding. The technique involves a two phase protocol where, in the first phase, the source broadcasts the information to the relays and in the second phase, the relays linearly process the signals received from the source and forward them to the destination such that the signal at the destination appears as a space–time block code (STBC). Such a class of STBCs is called distributed space–time block codes (DSTBCs).

In a colocated multiple-input–multiple-output (MIMO) channel, an STBC is said to be single-symbol maximum-likelihood (SSD) if the ML decoding metric splits as a sum of several terms, with each term being a function of only one of the information symbols. Since the work of [1]–[5], much efforts have been made to generalize various aspects of space–time coding, which were originally proposed for colocated multiple-antenna systems to the cooperative framework. One such important aspect is the design of low-complexity ML decodable DSTBCs—in particular, the design of SSD DSTBCs. A DSTBC is said to be SSD if the STBC seen by the destination from the set of relays is SSD. For a background on SSD STBCs for colocated MIMO channels, we refer the reader to [6]–[12]. Apart from SSD STBCs, multigroup decodable STBCs have also been studied for colocated MIMO channels [13]. In this paper, we discuss the issues related to the design of SSD DSTBCs only. Throughout this paper, we assume that an ML decoder is used at the destination for decoding the symbols of the source.

Recently, in [14], distributed orthogonal space–time codes (DOSTBCs) achieving single-symbol decodability have been introduced for cooperative networks. The authors considered a special class of DOSTBCs, which make the covariance matrix of the additive noise vector at the destination, a diagonal
one, and such a class of codes has been referred as row-monomial DOSTBCs. Upper bounds on the maximum symbol rate (in complex symbols per channel use in the second phase) of row-monomial DOSTBCs have been derived and a systematic construction of such codes has also been proposed. The constructed codes were shown to meet the upper bound for even number of relays. In [15], the same authors have also derived an upper bound on the symbol rate of DOSTBCs when the additive noise at the destination is correlated and have shown that the improvement in the rate is not significant when compared to the case when the noise at the destination is uncorrelated [14].

In [15] and [16], SSD DSTBCs have been proposed when every relay node is assumed to have the perfect knowledge of the phase component of the channel from the source to the relay. An upper bound on the symbol rate for such a set up is shown to be $\frac{1}{2}$, which is independent of the number of relays.

In [14]–[16], the source node transmits the information symbols to all the relays without any processing. Using the framework proposed in [14], in this paper, we propose SSD DSTBCs aided by linear precoding of the information vector at the source. In our setup, we assume that the relay nodes do not have the knowledge of the channel from the source to itself. In particular, it is shown that linear precoding of information symbols at the source along with the appropriate choice of relay matrices SSD DSTBCs with maximal rates higher than that of DOSTBCs can be constructed. The contributions and the organization of this paper can be summarized as follows.

- A new class of DSTBCs called precoded DSTBCs (PDSTBCs) (Definition 1) is introduced where the source performs linear precoding of information symbols appropriately before transmitting it to all the relays. Within this class, we identify codes that are SSD and refer to them as precoded distributed single symbol decodable STBCs (SSD-PDSTBCs) (Definition 2). The well-known DOSTBCs studied in [14] are shown to be a special case of SSD-PDSTBCs (see Section II).
- A set of necessary and sufficient conditions on the relay matrices for the existence of SSD-PDSTBCs is proved (see Theorem 1 in Section II).
- Within the set of SSD-PDSTBCs, a class of semiorthogonal SSD-PDSTBCs (semi-SSD-PDSTBC) (Definition 5) is defined. The known DOSTBCs are shown to belong to the class of semi-SSD-PDSTBCs. On the similar lines of [14], a special class of semi-SSD-PDSTBCs having a diagonal covariance matrix at the destination is studied and are referred to as row-monomial semi-SSD-PDSTBCs. An upper bound on the maximal symbol rate of row-monomial semi-SSD-PDSTBCs is derived. It is shown that the symbol rate of such codes is upper bounded by $\frac{1}{2}$ and $\frac{2}{2+1}$, when the number of relays $K$ is of the form $2l$ and $2l+1$, respectively, where $l$ is any natural number (see Section III).
- Construction of row-monomial semi-SSD-PDSTBCs is presented when $K \geq 4$. The proposed construction provides codes achieving the upper bound on the symbol rate when $K$ is 0 or 3 modulo 4. For other values of $K$, the constructed codes do not achieve the upper bound, but are higher than those of DOSTBCs (see Section IV).

- Precoding of information symbols at the source has resulted in the construction of high-rate semi-SSD-PDSTBCs. In this setup, the relays do not perform coordinate interleaving of the received symbols. It is shown that, when the source transmits information symbols to all the relays without any precoding, and if the relays are allowed to perform any form of linear processing of the received vector, semi-SSD-PDSTBCs other than DOSTBCs cannot be constructed, thereby necessitating the source to perform information symbols in order to construct high-rate semi-SSD-PDSTBCs (see Section V).

The discussions on the full diversity of row-monomial semi-SSD-PDSTBCs is presented in Section VI, whereas concluding remarks and possible directions for further work constitute Section VII.

**Notations:** Throughout this paper, boldface letters and capital boldface letters are used to represent vectors and matrices, respectively. For a complex matrix $X$, the matrices $X^*$, $X^T$, $X^H$, $[X]$, $\text{Re} X$, and $\text{Im} X$ denote, respectively, the conjugate, transpose, conjugate transpose, determinant, real part, and imaginary part of $X$. The element in the $r_1$th row and the $r_2$th column of the matrix $X$ is denoted by $[X]_{r_1,r_2}$. The diagonal matrix $\text{diag} \{[X]_{1,1},[X]_{2,2}, \ldots ,[X]_{T,T}\}$ constructed from the diagonal elements of a $T \times T$ matrix $X$ is denoted by $\text{diag}[X]$. For complex matrices $X$ and $Y$, $X \otimes Y$ denotes the tensor product of $X$ and $Y$. The tensor product of the matrix $X$ with itself $r$ times where $r$ is any positive integer is represented by $X^{\otimes r}$. The $T \times T$ identity matrix and the $T \times T$ zero matrix are, respectively, denoted by $I_T$ and $0_T$. The magnitude of a complex number $x$ is denoted by $|x|$ and $E[x]$ is used to denote the expectation of the random variable $x$. A circularly symmetric complex Gaussian random vector $\mathbf{x}$ with mean $\mu$ and covariance matrix $\Gamma$ is denoted by $\mathbf{x} \sim \mathcal{C}\mathcal{S}\mathcal{G}(\mu, \Gamma)$. The set of all integers, the real numbers, and the complex numbers are, respectively, denoted by $\mathbb{Z}$, $\mathbb{R}$, and $\mathbb{C}$ and $j$ is used to represent $\sqrt{-1}$. The set of all $T \times T$ complex diagonal matrices is denoted by $\mathcal{D}_T$ and a subset of $\mathcal{D}_T$ with strictly positive diagonal elements is denoted by $\mathcal{D}_T^+$.

II. PRECODED DISTRIBUTED SPACE–TIME CODING

A. Signal Model

The wireless network considered as shown in Fig. 1 consists of $K + 2$ nodes, each having a single antenna. There is one source node and one destination node. All the other $K$ nodes are relays. We denote the channel from the source node to the $k$th relay as $h_k$, and the channel from the $k$th relay to the destination node as $g_k$ for $k = 1, 2, \ldots , K$. The following assumptions are made in our model.

- All the nodes are half duplex constrained.
- Fading coefficients $h_k$ and $g_k$ are independent identically distributed (i.i.d.) $\mathcal{C}\mathcal{S}\mathcal{G}(0, 1)$ with a coherence time interval of at least $N$ and $T$ channel uses, respectively.
- All the nodes are synchronized at the symbol level.
- Relay nodes do not have the knowledge of the fade coefficients $h_k$.
- Destination knows all the fade coefficients $g_k, h_k$ for $k = 1, 2, \ldots , K$. 

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The source is equipped with a codebook \( S = \{x_1, x_2, x_3, \ldots, x_L\} \) consisting of information vectors \( x_l \in \mathbb{C}^{1 \times N} \) such that \( E[x_l x_l^H] = 1 \). The source is also equipped with a pair of \( N \times N \) matrices \( P \) and \( Q \) called precoding matrices. Every transmission from the source to the destination comprises two phases. When the source needs to transmit an information vector \( x \in S \) to the destination, it generates a new vector \( \tilde{x} \) as

\[
\tilde{x} = xP + x^Q
\]

(1)

where the precoding matrices satisfy the condition \( E[\tilde{x} \tilde{x}^H] = 1 \) and broadcast the vector \( \tilde{x} \) to all the \( K \) relays (but not to the destination). The received vector at the \( k \)th relay is given by

\[
r_k = \sqrt{P_2/N} h_k x_k + n_k, \quad \text{for all } k = 1, 2, \ldots, K
\]

where \( n_k \sim \mathcal{C}\mathcal{S}\mathcal{G}(0, I_T) \) is the additive noise at the \( k \)th relay and \( P_1 \) is the total power used at the source node for every channel use. In the second phase, all the relay nodes are scheduled to transmit \( T \) length vectors to the destination simultaneously. Each relay is equipped with a fixed pair of \( N \times T \) rectangular matrices \( A_k \) and \( B_k \) and is allowed to linearly process the received vector. The \( k \)th relay is scheduled to transmit

\[
t_k = \sqrt{P_2 T/(1 + P_1)} \{r_k A_k + r_k^* B_k\}
\]

where \( P_2 \) is the total power used at each relay for every channel use in the second phase. The vector received at the destination is given by

\[
y = \sum_{k=1}^K g_k t_k + w
\]

where \( w \sim \mathcal{C}\mathcal{S}\mathcal{G}(0, I_T) \) is the additive noise at the destination. Substituting for \( t_k \), \( y \) can be written as

\[
y = \sqrt{P_1 P_2 T/(1 + P_1) N} g X(\mathbf{x}) + n
\]

where

- \( n = \sqrt{P_2 T/(1 + P_1) N} \sum_{k=1}^K g_k \{n_k A_k + n_k^* B_k\} + w \)
- the equivalent channel \( g \) is given by \( [g_1 \ g_2 \ \cdots \ g_K] \in \mathbb{C}^{1 \times K} \)
- every codeword \( X(\mathbf{x}) \in \mathbb{C}^{K \times T} \), which is of the form (2), shown at the bottom of the page, is a function of the information vector \( x \) through \( \tilde{x} \).

The covariance matrix \( R \in \mathbb{C}^{T \times T} \) of the noise vector \( n \) is given by

\[
R = \frac{P_2 T}{(1 + P_1) N} \left[ \sum_{k=1}^K |g_k|^2 \{A_k^H A_k + B_k^H B_k\} \right] + I_T. \tag{3}
\]

The ML decoder decodes for a vector \( \hat{x} \) where \( \hat{x} \) is given in (4), shown at the bottom of the page.

**Definition 1:** The collection \( C \) of \( K \times T \) codeword matrices

\[
C = \{X(x) \mid x \in S\}
\]

is called a PDSTBC, which is determined by the sets \( \{P, Q, A_k, B_k\} \) and \( S \).

**Remark 1:** From (5), every codeword of a PDSTBC includes random variables \( h_k \) for all \( k = 1, 2, \ldots, K \). Even though \( h_k \) can take any complex value, since the destination knows the channel set \( \{h_1, h_2, \ldots, h_K\} \) for every codeword use, the cardinality of \( C \) is equal to the cardinality of \( S \). The properties of a PDSTBC will depend on the sets \( \{P, Q, A_k, B_k\} \) and \( S \) alone but not on the realization of the channels \( h_k \)'s. In this paper, on the similar lines of [14], we derive conditions on the set \( \{P, Q, A_k, B_k\} \) such that the PDSTBC in (5) is SSD for any realization of \( \{h_1, h_2, \ldots, h_K\} \). In other words, the derived conditions are such that irrespective of the realization of \( h_k \)'s, the PDSTBC in (5) is SSD.

Note that a PDSTBC can be generated from a design \( X \) (which is completely determined by the set \( \{P, Q, A_k, B_k\} \) in the real variables \( x_1, x_2, \ldots, x_N \)) by making the complex variables \( x_1, x_2, \ldots, x_N \) take values from a 2-D signal set. It is to be observed that, excluding the scaling factors and constant terms, the ML decoding metric in (4) is a function of the following two terms: i) \( gXR^{-1}y^H \) and ii) \( gXR^{-1}X^H g^H \) where \( R \) is a function of the set \( \{A_k, B_k\} \). If the set \( \{P, Q, A_k, B_k\} \) that makes the design \( X \) is such that the terms in i) and ii) can be written as a sum of \( N \) terms where each term is strictly a function of only one of the complex variables, then every complex variable can be decoded independent of the other. With this property, the design \( X \) is said to be single complex symbol ML decodable. In this paper, we propose a class of PDSTBCs with single complex symbol ML decodable property called SSD-PDSTBCs from a new class of complex designs called precoded distributed separable designs. Precoded distributed separable designs are defined as follows.

\[
\begin{align*}
X(x) &= \begin{bmatrix} h_1 \tilde{x} A_1 + h_2^* \tilde{x}^* B_1^T \end{bmatrix}^T \begin{bmatrix} h_2 \tilde{x} A_2 + h_2^* \tilde{x}^* B_2^T \end{bmatrix}^T \cdots \begin{bmatrix} h_K \tilde{x} A_K + h_K^* \tilde{x}^* B_K^T \end{bmatrix}^T.
\tag{2}
\end{align*}
\]

\[
\hat{x} = \arg \min_{x \in S} \left[ -2 \text{Re} \left( \sqrt{P_1 P_2 T/(1 + P_1) N} g X^{-1} y^H \right) + \frac{P_1 P_2 T}{(1 + P_1) N} g X^{-1} X^H g^H \right]. \tag{4}
\]
Definition 2: A matrix \( \mathbf{X} \) in variables \( x_{11}, x_{1Q}, \ldots, x_{NJ}, x_{NQ} \) is called a precoded distributed separable design (PDSD), if it satisfies the following conditions:

- The entries of the \( k \)th row of \( \mathbf{X} \) are \( 0, \pm h_k \bar{x}_n, \pm h_k^* \bar{x}_n^* \) or multiples of these by \( j \) where \( j = \sqrt{-1} \) for any complex variable \( h_k \). The complex variables \( \bar{x}_n \) for \( 1 \leq n \leq N \) are the components of the transmitted vector \( \bar{\mathbf{x}} \) where \( \bar{\mathbf{x}} = [\bar{x}_1 \bar{x}_2 \cdots \bar{x}_N] \).
- The matrix \( \mathbf{X} \) satisfies the equality

\[
\mathbf{XR}^{-1}\mathbf{X}^H = \sum_{i=1}^{N} \mathbf{W}_i \text{ with }
\mathbf{W}_i[k,k] = |h_k|^2 (v^{(1)}_{ik}|v_{iI}|^2 + v^{(2)}_{ik}|v_{iQ}|^2)
\]

where each \( \mathbf{W}_i \) is a \( K \times K \) matrix that is not necessarily diagonal and its nonzero entries are functions of only \( x_{iI}, x_{iQ} \) (but not \( x_{jI}, x_{jQ} \) for all \( j \neq i \)) and \( h_k \) for all \( k = 1,2,\ldots,K \) and \( (1)^{(1)}_{ik}, v^{(2)}_{ik} \in \mathbb{R} \).

Throughout this paper, we use the terms PDSD and SSD-PDSTBC interchangeably in the appropriate context. We study the properties of the relay matrices \( \mathbf{A}_k, \mathbf{B}_k \) and the precoding matrices \( \mathbf{P} \) and \( \mathbf{Q} \) such that the vectors transmitted simultaneously from all the relays appear as a codeword of a SSD-PDSTBC at the destination. Some of the properties of the relay matrices have already been studied in the context of DOSTBCs in [14]. We recall some of the definitions and properties used in [14] so as to study the properties of the relay matrices of a PDSD.

Definition 3 [14]: A matrix is said to be column (row) monomial, if there is at most one nonzero entry in every column (row) of it.

Lemma 1: The relay matrices \( \mathbf{A}_k \) and \( \mathbf{B}_k \) of a PDSD satisfy the following conditions.

i) The entries of \( \mathbf{A}_k \) and \( \mathbf{B}_k \) are 0, \pm 1, \pm j.

ii) \( \mathbf{A}_k \) and \( \mathbf{B}_k \) cannot have nonzeros at the same position.

iii) \( \mathbf{A}_k \mathbf{B}_k \), \( \mathbf{A}_k^* \mathbf{B}_k^* \), and \( \mathbf{A}_k + \mathbf{B}_k \) are column monomial matrices.

Proof: The proof is on the similar lines of the proof for Lemma 1 in [14].

Lemma 2: Let \( \mathbf{A}, \mathbf{C}, \mathbf{D} \in \mathbb{C}^{N \times N} \). There exist functions \( f_1, f_2, \ldots, f_N \) from \( \mathbb{C} \) to itself with the property that for every vector \( \mathbf{x} = [x_1, x_2, \ldots, x_N] \), we have

\[
\mathbf{x} \mathbf{A} \mathbf{x}^H + \mathbf{x} \mathbf{C} \mathbf{x}^T + \mathbf{x}^* \mathbf{D} \mathbf{x}^H = \sum_{i=1}^{N} f_i (x_{iI}, x_{iQ})
\]

if and only if \( \mathbf{A} + \mathbf{D}^* = \mathbf{C} = \mathbf{C}^T \), \( \mathbf{D} + \mathbf{D}^T \in \mathcal{D}_N \).

Proof: See Appendix I.

Using the results of Lemma 2, in the following theorem, we provide a set of necessary and sufficient conditions on the matrix set \( \{\mathbf{P}, \mathbf{Q}, \mathbf{A}_k, \mathbf{B}_k\} \) such that a design \( \mathbf{X} \) with the above matrix set is a PDSD.

Theorem 1: A design \( \mathbf{X} \) is a PDSD if and only if the relay matrices \( \mathbf{A}_k, \mathbf{B}_k \) satisfy the following conditions.

i) For \( 1 \leq k \neq k' \leq K \)

\[
\mathbf{Y}_1 \mathbf{A}_k \mathbf{R}^{-1} \mathbf{A}_k^H \mathbf{Y}_2^H + \mathbf{Y}_1 \mathbf{A}_k^* \mathbf{R}^{-1} \mathbf{A}_k^H \mathbf{Y}_2^H \in \mathcal{D}_N, \\
\mathbf{P}_2 = \mathbf{P} \quad \text{and} \quad \mathbf{P}_1 = \mathbf{Q}
\]

for \( \mathbf{Y}_1 = \mathbf{Y}_2 = \mathbf{P} \) and \( \mathbf{P}_1 = \mathbf{P} \)

ii) For \( 1 \leq k, k' \leq K \)

\[
\mathbf{Y}_1 \mathbf{B}_k \mathbf{R}^{-1} \mathbf{B}_k^H \mathbf{Y}_2^H + \mathbf{Y}_1 \mathbf{B}_k^* \mathbf{R}^{-1} \mathbf{B}_k^H \mathbf{Y}_2^H \in \mathcal{D}_N, \\
\mathbf{P}_2 = \mathbf{P} \quad \text{and} \quad \mathbf{P}_1 = \mathbf{Q}
\]

for \( \mathbf{Y}_1 = \mathbf{Y}_2 = \mathbf{P} \) and \( \mathbf{P}_1 = \mathbf{P} \)

iii) For \( 1 \leq k \leq K \)

\[
\mathbf{A}_k \mathbf{R}^{-1} \mathbf{A}_k^H + \mathbf{B}_k \mathbf{R}^{-1} \mathbf{B}_k^H = \text{diag} [D_{1k}, D_{2k}, \ldots, D_{Nk}]
\]

where \( D_{nk} \in \mathbb{R} \) for all \( n = 1,2,\ldots,N \).

Proof: See Appendix II.

It can be verified that DOSTBCs studied in [14] are a special class of SSD-PDSTBCs since the relay matrices of DOSTBCs [14, Lemma 1] satisfy the conditions of Theorem 1. In particular, the necessary and sufficient conditions on the relay matrices of DOSTBCs as shown in Lemma 1 of [14] can be obtained from the necessary and sufficient conditions of PDSD by making \( \mathbf{P} = \mathbf{I}_N \), \( \mathbf{Q} = \mathbf{0}_N \), and \( \mathcal{D}_N = \mathbf{0}_N \) in (8)–(10).

A PDSD \( \mathbf{X} \) in variables \( x_{1I}, x_{1Q}, \ldots, x_{NJ}, x_{NQ} \) can be written in the form of a linear dispersion code [17] as \( \mathbf{X} = \sum_{i=1}^{N} x_{iI} \Phi_{iI} + x_{iQ} \Phi_{iQ} \) where \( \Phi_{iI}, \Phi_{iQ} \in \mathbb{C}^{K \times T} \) are called the weight matrices of \( \mathbf{X} \). Within the class of PDSDs, we study a special set consisting of unitary PDSDs defined as follows.

Definition 4: A PDSD \( \mathbf{X} \) is called a unitary PDSD, if the weight matrices of \( \mathbf{X} \) satisfy the following conditions: \( \Phi_{iI} \Phi_{iI}^H, \Phi_{iQ} \Phi_{iQ}^H \in \mathcal{D}_k^U \) for all \( i = 1,2,\ldots,N \).

Various classes of single-symbol decodable STBCs for cooperative networks are captured in Fig. 2, which is partitioned into two sets depending on whether the codes are obtained from unitary or nonunitary designs (Definition 4). In the rest of this paper, we consider SSD-PDSTBCs from unitary PDSDs only. More details on nonunitary SSD-PDSTBCs and classification of SSD codes for cooperative networks can be found in [18, Sec. 2].

III. SEMIORTHOGONAL SSD-PDSTBC

From the definition of a PDSD (Definition 2), for any \( k \neq k' \), \( [\mathbf{XR}^{-1}\mathbf{X}^H]_{kk'} \) can take a nonzero value, i.e., the \( k \)th and the \( k' \)th row of a PDSD \( \mathbf{X} \), need not satisfy the equality \( [\mathbf{XR}^{-1}\mathbf{X}^H]_{kk'} = 0 \); instead \( [\mathbf{XR}^{-1}\mathbf{X}^H]_{kk'} \) can be a complex linear combination of several terms with each term being a function of only a single complex variable. Throughout this paper, the \( k \)th and the \( k' \)th row of a PDSD are referred
to as \( \mathbf{R} \)-orthogonal if \( \left[ \mathbf{X} \mathbf{R}^{-1} \mathbf{X}^H \right]_{k,k'} = 0 \). Similarly, the \( k \)th and the \( k' \)th row are referred to as \( \mathbf{R} \)-nonorthogonal if \( \left[ \mathbf{X} \mathbf{R}^{-1} \mathbf{X}^H \right]_{k,k'} \neq 0 \). In this paper, we identify a special class of PDSDs where every row of \( \mathbf{X} \) is \( \mathbf{R} \)-nonorthogonal to at most one of its rows. Such a class of PDSDs is formally defined as follows.

**Definition 5:** A PDSD is said to be a semi-orthogonal PDSD (semi-PDSD) if every row of the PDSD is \( \mathbf{R} \)-nonorthogonal to at most one of its rows.

From the above definition, it can be observed that DOSTBCs are a proper subclass of the DSTBCs obtained from semi-PDSDs since every row of a DOSTBC is \( \mathbf{R} \)-orthogonal to every other row. DSTBCs which are obtained from semi-PDSDs are called semi-SSD-PDSTBCs. The definition of a semi-PDSD implies that the set of \( K \) rows can be partitioned into at least \( \left\lceil \frac{k}{2} \right\rceil \) groups such that every group has at most two rows and two rows coming from different groups are \( \mathbf{R} \)-orthogonal.

Note that the covariance matrix \( \mathbf{R} \) in (3) is a realization of the channels from the relays to the destination and ii) the relay matrices \( \mathbf{A}_k, \mathbf{B}_k \). In general, \( \mathbf{R} \) may not be diagonal in which case the construction of semi-PDSDs is not straightforward. On the similar lines of [14], we consider a subset of semi-PDSDs wherein the covariance matrix is diagonal and refer to such a subset as row-monomial semi-PDSDs. It can be proved that the covariance matrix \( \mathbf{R} \) is diagonal if and only if the relay matrices of a semi-PDSDs are row monomial (refer to [14, Th. 1]). The row-monomial property of the relay matrices implies that every row of a row-monomial semi-PDSD contains the variables \( \pm h_k \hat{x}_n \) and \( \pm h_k \hat{r}_n \) at most once for all \( n \) such that \( 1 \leq n \leq N \). In the rest of this paper, we consider constructing DSTBCs from row-monomial semi-PDSDs only. Henceforth, we continue to refer row-monomial semi-PDSD as semi-PDSDs.

### A. An Upper Bound on the Symbol Rate of Semi-PDSDs

In this subsection, we derive an upper bound on the rate of semi-PDSDs in complex symbols per channel use, i.e., an upper bound on \( \frac{N}{T} \). Towards that end, the properties of the relay matrices \( \mathbf{A}_k, \mathbf{A}_{k'}, \mathbf{B}_k, \) and \( \mathbf{B}_{k'} \) of a semi-PDSD are studied when the rows corresponding to the indices \( k \) and \( k' \) are i) \( \mathbf{R} \)-orthogonal and ii) \( \mathbf{R} \)-nonorthogonal. For the former case, the properties of \( \mathbf{A}_k, \mathbf{A}_{k'}, \mathbf{B}_k, \) and \( \mathbf{B}_{k'} \) have been studied in [14]. If \( k \) and \( k' \) are the indices of the rows of a semi-PDSD that are \( \mathbf{R} \)-orthogonal, then the corresponding relay matrices \( \mathbf{A}_k, \mathbf{A}_{k'}, \mathbf{B}_k \) and \( \mathbf{B}_{k'} \) satisfy the following conditions [14]:

\[
\mathbf{A}_k \mathbf{A}_{k'}^H = 0_N \quad \text{and} \quad \mathbf{B}_k \mathbf{B}_{k'}^T = 0_N. \tag{12}
\]

For the latter case, the properties of the relay matrices are given in Lemma 3.

**Lemma 3:** Let \( k \) and \( k' \) represent the indices of two distinct rows of a semi-PDSD that are \( \mathbf{R} \)-nonorthogonal, then the corresponding relay matrices \( \mathbf{A}_k, \mathbf{B}_k, \mathbf{A}_{k'}, \) and \( \mathbf{B}_{k'} \) satisfy the following conditions:

i) \( \left[ \mathbf{A}_k \mathbf{A}_{k'}^H \right]_{i,i} = \left[ \mathbf{B}_k \mathbf{B}_{k'}^T \right]_{i,j} = 0 \) for all \( i = 1,2 \cdots N \);

ii) \( \mathbf{A}_k \mathbf{A}_{k'}^H \) and \( \mathbf{B}_k \mathbf{B}_{k'}^T \) are column and row-monomial matrices;

iii) \( \mathbf{A}_k \mathbf{A}_{k'}^H + \mathbf{B}_k \mathbf{B}_{k'}^T \) is a column and row-monomial matrix;

iv) the number of nonzero entries in \( \mathbf{A}_k \mathbf{A}_{k'}^H + \mathbf{B}_k \mathbf{B}_{k'}^T \) is even;

v) the matrices \( \mathbf{A}_{k,k} \) and \( \mathbf{B}_{k,k'} \) given by \( \mathbf{A}_{k,k'} = \left[ \mathbf{A}^T \mathbf{A}_{k'}^T \right]^T \) and \( \mathbf{B}_{k,k'} = \left[ \mathbf{B}^T \mathbf{B}_{k'}^T \right]^T \) satisfy the following inequality:

\[
\text{Rank} \left[ \mathbf{A}_{k,k'} \mathbf{A}_{k,k'}^H + \mathbf{B}_{k,k'} \mathbf{B}_{k,k'}^T \right] \geq 2 \left\lceil \frac{N}{2} \right\rceil. \tag{13}
\]

**Proof:** See Appendix III.

To derive an upper bound on the symbol rate of PDSDs, we also need to study the properties of the relay matrices when \( k = k' \). From Definition 2, nonzero entries of the \( k \)th row contains variables of the form \( \pm h_k \hat{x}_n, \pm h_k \hat{r}_n \), or multiples of these by \( j \). Therefore

\[
[\mathbf{X} \mathbf{X}^H]_{k,k} = |h_k|^2 \left[ \hat{x} \mathbf{A}_k \mathbf{A}_k^H \hat{x}^H + \hat{x}^* \mathbf{B}_k \mathbf{B}_k^H \hat{x}^T \right] + h_k h_k^* \left[ \hat{x} \mathbf{A}_k \mathbf{B}_k^H \hat{r}^T \right] + h_k h_k^* \left[ \hat{x}^* \mathbf{B}_k \mathbf{A}_k^H \hat{x}^H \right] = \sum_{i=1}^N |h_k|^2 \left( \omega_{i,k}^1 |\hat{x}_i| \hat{x}_i^H + \omega_{i,k}^2 |\hat{x}_i| \hat{r}_i^H \right)
\]

where \( \omega_{i,k}^1, \omega_{i,k}^2 \in \mathbb{R}^+ \) for all \( k = 1,2,\ldots,K \). From the results of [10, Lemma 1], we have

\[
\mathbf{A}_k \mathbf{A}_k^H + \mathbf{B}_k \mathbf{B}_k^T = \text{diag} [E_{1,k}, E_{2,k}, \ldots, E_{N,k}] \tag{14}
\]

where \( E_{n,k} \) are strictly positive real numbers.

Using the properties of relay matrices \( \mathbf{A}_k, \mathbf{A}_{k'}, \mathbf{B}_k, \) and \( \mathbf{B}_{k'} \) of a semi-PDSD corresponding to two different rows that are i) \( \mathbf{R} \)-orthogonal and ii) \( \mathbf{R} \)-nonorthogonal, an upper bound on the maximum rate, \( \frac{N}{T} \) is derived in the following theorem.
Theorem 2: The symbol rate of a semi-PDSD satisfies the inequality
\[ \frac{N}{T} \leq \frac{N}{\left\lceil \frac{Q_N}{2} \right\rceil}. \] (15)

Proof: See Appendix IV. \qed

IV. CONSTRUCTION OF SEMI-PDSDS

In this section, we construct semi-PDSDs when \( K \geq 4 \) and \( N \geq 4 \). The construction provides semi-PDSDs achieving the upper bound in (15) only when i) \( N \) and \( K \) are multiples of four and ii) \( N \) is a multiple of four and \( K \) is 3 modulo 4. For the rest of the values of \( N \) and \( K \), designs meeting the upper bound are not known. However, for such values of \( N \) and \( K \), it is shown that the codes from PDSDs have rates higher than that of DOSTBCs. We first provide the construction of the precoding matrices \( P \) and \( Q \) and then present the construction of semi-PDSDs.

A. Construction of Precoding Matrices \( P \) and \( Q \)

Let \( \Gamma, \Omega \in \mathbb{C}^{4 \times 4} \) be given by
\[
\Gamma = \frac{1}{2} \begin{bmatrix}
1 & 0 & -j & 0 \\
0 & 1 & 0 & -j \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix} \quad \text{and} \quad \Omega = \frac{1}{2} \begin{bmatrix}
1 & 0 & 0 & j \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

Let \( N = 4y + a \), where \( a \) can take values of 0, 1, 2, and 3 and \( y \) is any positive integer. For a given value of \( a \) and \( y \), the precoding matrices \( P \) and \( Q \) at the source are constructed as
\[
P = \left( \Gamma \otimes I_y \right)_{a \times 4y} \text{ and } Q = \left( \Omega \otimes I_y \right)_{a \times 4y}.
\]

Example 1: For \( N = 6 \), we have \( y = 1 \) and \( a = 2 \). Following the above construction method, precoding matrices \( P \) and \( Q \) are given by
\[
P = \frac{1}{2} \begin{bmatrix}
1 & 0 & -j & 0 & 0 & 0 \\
0 & 1 & 0 & -j & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]
\[
Q = \frac{1}{2} \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & j & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & j \\
0 & 0 & 0 & 0 & 0 & j
\end{bmatrix}.
\]

B. Construction of Semi-PDSDs

Throughout this subsection, we denote a semi-PDSD for \( K \) relays in \( N \) variables as \( X(N,K) \). Construction of semi-PDSDs is divided into three cases depending on the values of \( N \) and \( K \).

Case 1: \( N = 4y \) and \( K = 4z \): In this case, we construct semi-PDSDs in the following four steps.

i) Let \( U_{x_1, x_2} \) be a \( 2 \times 2 \) Alamouti design in complex variables \( x_1, x_2 \) as given in the following:
\[
U_{x_1, x_2} = \begin{bmatrix}
x_1 & -x_2 \\
x_2 & x_1
\end{bmatrix}.
\]

Using the design in (16), construct a \( 4 \times 4 \) design \( \Omega_m \) in four complex variables \( x_{4m+1}, x_{4m+2}, x_{4m+3}, x_{4m+4} \) as shown below for all \( m = 0, 1, \ldots, y-1 \):
\[
\Omega_m = \begin{bmatrix}
U_{x_{4m+1} x_{4m+2}} & U_{x_{4m+3} x_{4m+4}} \\
U_{x_{4m+3} x_{4m+1}} & U_{x_{4m+1} x_{4m+2}}
\end{bmatrix} = \begin{bmatrix}
x_{4m+1} & x_{4m+2} & x_{4m+3} & x_{4m+4} \\
x_{4m+2} & x_{4m+1} & x_{4m+4} & x_{4m+3} \\
x_{4m+3} & x_{4m+4} & x_{4m+1} & x_{4m+2} \\
x_{4m+4} & x_{4m+3} & x_{4m+2} & x_{4m+1}
\end{bmatrix}
\]

where \( x_{4m+1} = x_{(4m+1)Q} + jx_{(4m+1)Q} \), \( x_{4m+2} = x_{(4m+2)Q} + jx_{(4m+2)Q} \), \( x_{4m+3} = x_{(4m+3)Q} + jx_{(4m+3)Q} \), and \( x_{4m+4} = x_{(4m+4)Q} + jx_{(4m+4)Q} \).

ii) Let \( H, \Delta \) and \( \Theta \) be diagonal matrices \( \Delta \in \mathbb{C}^{K \times K} \) given by \( H = \text{diag} \left\{ h_1, h_2, \ldots, h_K \right\} \), \( \Delta = \text{diag} \left\{ 0, 1, 0, \ldots, 0 \right\} \), and \( \Theta = \text{diag} \left\{ 0, 1, 0, \ldots, 1 \right\} \), where \( h_1, h_2, \ldots, h_K \) are complex variables and \( \Delta, \Theta \) are such that \( \Delta + \Theta = I_K \).

Using \( H, \Delta \) and \( \Theta \), construct a diagonal matrix \( G \) as \( G = H \Delta + H \Theta \).

iii) Using \( \Omega_m \), construct a \( 4 \times 4 \) matrix \( X_m \) given by \( \Omega_m \otimes I_2 \) for each \( m = 0, 1, \ldots, y-1 \). Notice that once \( C \) is fixed, number of rows in \( X_m \) is also fixed equal to \( 4z \).

iv) A semi-PDSD \( X(N,K) \) is constructed using \( X_m \) and \( G \) as \( X(N,K) = G [X_0 | X_1 | \cdots | X_{y-1}] \). Since \( G \) is a diagonal matrix containing variables \( h_1, h_2, \ldots, h_K \), after multiplying \( G \) with \( [X_0 | X_1 | \cdots | X_{y-1}] \), it is to be observed that the elements of the matrix \( X(N,K) \) are in the required form as given by the definition of a PDSD.

Example 2: For \( N = 4 \) and \( K = 4 \), we have \( x = y = 1 \). Following steps i)–iv) in the above construction, we have \( G = \text{diag} \left\{ h_1, h_2, h_3, h_4 \right\} \) and \( X_0 = \Omega_0 \). Hence, \( X(4,4) \) is given by
\[
X(4,4) = \begin{bmatrix}
h_1 \bar{x}_1 & h_1 \bar{x}_2 & h_2 \bar{x}_3 & h_3 \bar{x}_4 \\
h_2 \bar{x}_2 & h_2 \bar{x}_1 & h_3 \bar{x}_4 & h_4 \bar{x}_3 \\
h_3 \bar{x}_3 & h_3 \bar{x}_4 & h_4 \bar{x}_1 & h_1 \bar{x}_2 \\
h_4 \bar{x}_4 & h_4 \bar{x}_3 & h_1 \bar{x}_2 & h_2 \bar{x}_1
\end{bmatrix}
\]

where \( \bar{x}_1 = x_{1Q} + jx_{1Q} \), \( \bar{x}_2 = x_{2Q} + jx_{2Q} \), \( \bar{x}_3 = x_{3Q} + jx_{3Q} \), and \( \bar{x}_4 = x_{4Q} + jx_{4Q} \). The variables \( \bar{x}_1, \bar{x}_2, \ldots, \bar{x}_4 \) are obtained using the precoding matrices \( P \) and \( Q \) as given in (1). The precoding matrices \( P \) and \( Q \) are constructed as in Section IV-A.
The relay specific matrices $A_k, B_k$ for the semi-PDSD in (17) are

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}.$$

and

$$B_1 = A_2 = B_3 = A_4 = 0_4.$$

Case 2: $N = 4y$ and $K = 4x + a$ for $a = 1, 2,$ and $3$: In this case, a semi-SSD-PDSTBC is constructed in two steps.

i) Construct a semi-PDSD for parameters $N = 4y$ and $K = 4(x + 1)$ as given in Case 1.

ii) Drop the last 4 − a rows of the semi-PDSD constructed in step i).

Example 3: When $N = 4$ and $K = 7$, the parameters $a$, $x$, and $y$ are 3, 1, and 1, respectively. As given in Case 2, a semi-PDSD for $N = 4$ and $K = 8$ is constructed and the last row of the design is dropped.

Case 3: $N = 4y + b$ and $K = 4x + a$ where $b = 1, 2, 3$ and $a = 0, 1, 2, 3$: In this case, semi-PDSDs are constructed in the following three steps.

i) Construct a semi-PDSD $X(4y, 4x + a)$ for parameters $N = 4y$ and $K = 4x + a$ as in Case 2 using the first $4y$ variables.

ii) Construct a design for DOSTBC $X'(b, 4x + a)$ with parameters $N = b$ and $K = 4x + a$ using the last $b$ variables as in [14].

iii) The semi-PDSD $X(N, K)$ is given by juxtaposing $X(4y, 4x + a)$ and $X'(b, 4x + a)$

$$X(N, K) = \left[ X(4y, 4x + a) \quad X'(b, 4x + a) \right].$$

V. ON THE CONSTRUCTION OF SEMI-SSD-PDSTBCS WITHOUT PRECODING AT THE SOURCE

The existence of high-rate semi-SSD-PDSTBCs has been shown in the preceding sections, when the source performs linear precoding of information symbols before broadcasting it to all the relays. One obvious question that arises is whether linear processing of the received symbols at the relays alone is sufficient to construct semi-SSD-PDSTBCs when the source does not perform precoding of information symbols. In other words, is precoding of the information symbols at the source necessary to construct semi-SSD-PDSTBCs? The answer is yes.

In the rest of this section, we show that semi-SSD-PDSTBCs cannot be constructed by linear processing of the received symbols at the relays when the source transmits the information symbols to all the relays without precoding. Towards that end, let the $k$th relay be equipped with a pair of matrices $A_k$ and $B_k \in \mathbb{C}^{N \times T}$, which perform linear processing on the received vector. Excluding the additive noise component, the received vector at the $k$th relay is $h_k x = [h_k x_1, h_k x_2, \ldots, h_k x_N]$, where $x_i$’s are information symbols and $h_k$ is any complex number. The matrices $A_k$ and $B_k \in \mathbb{C}^{N \times T}$ act on the vector $h_k x$ to generate a vector of the form

$$h_k x A_k + h_k^* x^* B_k.$$  \hspace{1cm} (18)

From (18), the nonzero entries of $h_k x A_k + h_k^* x^* B_k$ contain complex variables of the form $\pm x_i, \pm x^*_i$ or multiples of these by $j$ where $j = \sqrt{-1}$ and

$$\text{Re}(x), \text{Im}(x) \in \{\text{Re}(h_k x_n), \text{Im}(h_k x_n) \mid 1 \leq n \leq N\}.$$  \hspace{1cm} (19)

To be precise, $\text{Re}(h_k x_n)$ and $\text{Im}(h_k x_n)$ are given by $h_k x_n I + h_k^* x^*_n I - h_k Q x_n Q$ and $h_k x_n I + h_k^* x^*_n I + h_k Q x_n I$, respectively. The above vector can also contain linear combination of the complex variables specified above.

From Definition 2, nonzero entries of the $k$th row of a SSD-PDSTBC are of the form $\pm x_n, \pm x^*_n$, where $\pm x_n$ and $\pm x^*_n$ can be in-phase and quadrature components of two different information symbols. Since $h_k$ is any complex variable, from (19), linear processing of the received symbols at the relays alone cannot contribute variables of the form $\pm x_n, \pm x^*_n$. Therefore, semi-SSD-PDSTBCs cannot be constructed by linear processing of the received symbols at the relays alone when the source transmits the information symbols to all the relays without precoding.

Remark 2: If $h_k$’s are real variables, then $\text{Re}(x), \text{Im}(x) \in \{h_k \text{Re}(x_n), h_k \text{Im}(x_n) \mid 1 \leq n \leq N\}$, in which case, the nonzero entries of the $k$th row can be of the form $\pm h_k x_n, \pm h_k^* x^*_n$, where $\pm x_n$ and $\pm x^*_n$ can be in-phase and quadrature components of two different information variables for any real variable $h_k$. This aspect has been well studied in [15], [16], and [19], where the relays are assumed to have the knowledge of the phase component [partial channel state information (CSI)] of their corresponding channels thereby making $h_k$, a real variable. Hence, with the knowledge of partial CSI at the relays, high-rate distributed SSD codes can be constructed by linear processing at the relays alone, i.e., with the knowledge of partial CSI at the relays, the source need not perform precoding of information symbols before transmitting to all the relays in the first phase.

VI. ON THE FULL DIVERSITY OF DSTBCS FROM SEMI-PDSDS

In this section, we consider the problem of designing a 2-D signal set $A$ such that a DSTBC obtained from a semi-PDSD is fully diverse when the variables $x_1, x_2, \ldots, x_N$ take values from $A$. Since every codeword of a semi-SSD-PDSTBC (Definition 2) contains complex variables $h_k$’s, pairwise error probability (PEP) analysis of semi-SSD-PDSTBCs is not straightforward. The authors do not have an analytical result regarding the conditions on the choice of a complex signal set for a semi-SSD-PDSTBC to be fully diverse. However, we make the following conjecture.

Conjecture: A DSTBC obtained from a semi-PDSD in variables $x_1, x_2, \ldots, x_N$ is fully diverse if the variables take values...
from a complex signal set say $\Lambda$ such that the difference signal set $\Delta \Lambda$ given by

$$\Delta \Lambda = \{a - b | a, b \in \Lambda\}$$

does not have any point on the lines that are $\pm 45^\circ$ in the complex plane apart from the origin.

In the rest of this section, we provide simulation results on the performance comparison of a semi-SSD-PDSTBC $X(4, 4)$ (given in (17)) and a row-monomial DOSTBC $X'(4, 4)$ (given in (20), shown at the bottom of the page) in terms of bit error rate (BER). The BER comparison is provided in Fig. 3 when the total power per channel use $P$ used for both the designs is the same. Since the design in (17) double the symbol rate compared to the design in (20), for a fair comparison, 16-quadrature-amplitude modulation (16-QAM) and a rotated quadrature phase-shift keying (QPSK) are used as signal sets for $X'(4, 4)$ and $X(4, 4)$, respectively, to maintain a common rate of 2 b/s/Hz. Note that DOSTBCs are shown to be fully diverse in [14]. From Fig. 3, it is observed that $X(4, 4)$ provides full diversity, since at large values of $P$, the BER curve for $X(4, 4)$ moves parallel to that of $X'(4, 4)$. It can be noticed from Fig. 3 that the design $X(4, 4)$ performs better than $X'(4, 4)$ by close to 1.5–2 dB for large values of $P$.

VII. DISCUSSION

We have studied the problem of designing high-rate, single-symbol decodable DSTBCs when the source is allowed to perform linear precoding of information symbols before transmitting it to all the relays. We introduced PDSDs (Definition 2) and showed that DOSTBCs are a special case of DSTBCs from PDSDs.

A special class of PDSDs having semiorthogonal property was defined (Definition 5). A subset of semi-PDSDs called row-monomial semi-PDSDs is studied and an upper bound on the maximal rate of such designs is derived. A systematic construction of row-monomial semi-PDSDs is presented for the case when the number of relays $K \geq 4$. Codes achieving the bound are found when $K$ is of the form 0 or 3 modulo 4. For the rest of the choices of $K$, semi-PDSDs meeting the above bound on the rate are not known. The constructed codes are shown to have rate higher than that of row-monomial DOSTBCs. Some of the possible directions for future work are as follows.

- In this paper, we studied a special class of PDSDs called unitary PDSDs (see Definition 4). The design of high-rate nonunitary PDSDs is an interesting direction for future work.
- The authors are not aware of semi-PDSDs achieving the bound on the maximum rate other than the case when $K$ is 0 or 3 modulo 4. The upper bounds on the maximum rate for rest of the values of $K$ can be possibly tightened.
- A class of semi-PDSDs was defined, by making every row of the PDSD $R$-nonorthogonal to at most one of its rows. It will be interesting whether the bounds on the maximal rate of PDSDs can be increased further by making a row $R$-nonorthogonal to more than one of its rows.
- In this paper, we have studied the class of SSD precoded DSTBCs only. Study of multigroup decodable precoded DSTBCs is also an interesting direction for future work.

APPENDIX I

PROOF OF LEMMA 2

Let $A' = A - \text{diag}[A]$, $C' = C - \text{diag}[C]$ and $D' = D - \text{diag}[D]$. To prove the “only if” part of the lemma, if

$$x A^H x^T + x^* D x^H = \sum_{i=1}^{N} f_i (x_{i1}, x_{iQ})$$

then

$$x A' x^H + x C' x^T + x^* D' x^H + x \text{diag}[A] x^H + x \text{diag}[A] x^H = \sum_{i=1}^{N} f_i (x_{i1}, x_{iQ})$$

which implies that

$$x A' x^H + x C' x^T + x^* D' x^H = 0.$$
Therefore
\[ A, \ C + C^T, \ D + D^T \in \mathcal{D}_N. \]
To prove the “if” part, suppose \( A, \ C + C^T, \ D + D^T \in \mathcal{D}_N \), then
\[
A' = C' + C'^T = D' + D^T = 0_N
\]
and from the results of [10, Lemma 1], we have
\[
x'A'x^H + x'Cx^T + x'Dx^H = 0
\]
which implies \( x'Ax^H + x'Cx^T + x'Dx^H = 0 \), where
\[
\sum_{i=1}^{N} f_i(x_iI, x_iQ).
\]
This completes the proof.

**APPENDIX II**

**PROOF OF THEOREM 1**

The “if” part can be proved by direct substitution of the conditions in (8)–(11) in \( XR^{-1}X^H \), which is straightforward. Hence, we prove the “only if” part of the theorem. From the structure of a PDSTBC in (5), \( [XR^{-1}X^H]_{k,k'} \) for some \( k \neq k' \) is given by
\[
h_{kk'} \sum_{i=1}^{N} f_i(x_iI, x_iQ).
\]
Therefore, \( A_k, A_{k'} \) for \( k \neq k' \) satisfies the conditions in (8).

Similarly, applying Lemma 2 on the rest of the equalities in (23), conditions in (9) and (10) can be proved.

We have proved the necessary conditions for the case when \( k \neq k' \). In the rest of the proof, we consider the case when \( k = k' \). The term \( [XR^{-1}X^H]_{k,k} \) is given by
\[
h_{kk} \sum_{i=1}^{N} f_i(x_iI, x_iQ).
\]
Applying the results of [10, Lemma 1] on (22), we have
\[
A_kR^{-1}A_k^H = 0_N
\]
\[
B_kR^{-1}B_k^H = 0_N
\]
\[
A_kR^{-1}B_k^H + B_k^*R^{-1}A_k^H = 0_N
\]
\[
B_kR^{-1}A_k^H + A_k^*R^{-1}B_k^H = 0_N.
\]
It can be verified that using the above matrices, conditions in (8)–(10) are satisfied where the diagonal matrices become zero matrices.

If \( [XR^{-1}X^H]_{k,k} \neq 0 \), since \( X \) is a PDSF, we have
\[
[XR^{-1}X^H]_{k,k} = \sum_{i=1}^{N} f_i(x_iI, x_iQ, h_k, h_k)
\]
for any complex variables \( h_k, h_k \), where \( f_i(x_iI, x_iQ, h_k, h_k) \) is a complex-valued function of information variables \( x_iI, x_iQ \) alone. Out of the four terms in (21), some of them can be zeros and some can be nonzeros or every term can be nonzero.

Without loss of generality, we consider the case when all the four terms in (21) are nonzeros. Therefore
\[
x'A_kR^{-1}A_k^Hx^H = \sum_{i=1}^{N} f_i(x_iI, x_iQ)
\]
\[
x'A_kR^{-1}B_k^Hx^T = \sum_{i=1}^{N} f_{2i}(x_iI, x_iQ)
\]
\[
x'B_kR^{-1}A_k^Hx^T = \sum_{i=1}^{N} f_{3i}(x_iI, x_iQ)
\]
\[
x'B_kR^{-1}B_k^Hx^T = \sum_{i=1}^{N} f_{4i}(x_iI, x_iQ),
\]
where \( f_{1i}(x_iI, x_iQ), f_{2i}(x_iI, x_iQ), f_{3i}(x_iI, x_iQ), \) and \( f_{4i}(x_iI, x_iQ) \) are some complex-valued functions of information variables \( x_iI, x_iQ \) alone. Using (1), the term \( x'A_kR^{-1}A_k^Hx^H \) on the left-hand side of the first expression in (23) is written as
\[
xPA_kR^{-1}A_k^HQHx^H + x'QA_kR^{-1}A_k^HQ^Hx^T
\]
\[
+xPA_kR^{-1}A_k^HQ^Hx^T + x'QA_kR^{-1}A_k^HP^HX^H.
\]
Applying the results of Lemma 2 on the first equality of (23) by using the above expression, we have
\[
PA_kR^{-1}A_k^HP^H + Q^*A_k^*R^{-1}A_k^TQ^T \in \mathcal{D}_N
\]
\[
PA_kR^{-1}A_k^HQ^H + Q^*A_k^*R^{-1}A_k^TP^T \in \mathcal{D}_N
\]
\[
QA_kR^{-1}A_k^HP^H + P^*A_k^*R^{-1}A_k^TQ^T \in \mathcal{D}_N.
\]
Therefore, \( A_k, A_{k'} \) for \( k \neq k' \) satisfies the conditions in (8). Similarly, applying Lemma 2 on the rest of the equalities in (23), conditions in (9) and (10) can be proved.

We have proved the necessary conditions for the case when \( k \neq k' \). In the rest of the proof, we consider the case when \( k = k' \). The term \( [XR^{-1}X^H]_{k,k} \) is given by
\[
h_{kk} \sum_{i=1}^{N} f_i(x_iI, x_iQ, h_k, h_k)
\]
Applying the results of [10, Lemma 1] on (22), we have
\[
A_kR^{-1}A_k^H = 0_N
\]
\[
B_kR^{-1}B_k^H = 0_N
\]
\[
A_kR^{-1}B_k^H + B_k^*R^{-1}A_k^H = 0_N
\]
\[
B_kR^{-1}A_k^H + A_k^*R^{-1}B_k^H = 0_N.
\]
It can be verified that using the above matrices, conditions in (8)–(10) are satisfied where the diagonal matrices become zero matrices.

If \( [XR^{-1}X^H]_{k,k} \neq 0 \), since \( X \) is a PDSF, we have
\[
[XR^{-1}X^H]_{k,k} = \sum_{j=1}^{N} \left| h_k \right|^2 \left| v_{j,k}^{(1)} \right|^2 + \left| v_{j,k}^{(2)} \right|^2 \left| x_iQ \right|^2.
\]
Using (25) and (24), for any complex variable \( h_k \), we have
\[
h_{kk} \sum_{i=1}^{N} f_i(x_iI, x_iQ, h_k, h_k)
\]
\[
[XR^{-1}X^H]_{k,k} = \sum_{j=1}^{N} \left| h_k \right|^2 \left| v_{j,k}^{(1)} \right|^2 + \left| v_{j,k}^{(2)} \right|^2 \left| x_iQ \right|^2.
\]
Since (26) is true for all complex variables \( h_k \), invoking the results of [10, Lemma 1] on (26), we have
\[
A_kR^{-1}B_k^H + B_k^*R^{-1}A_k^H = 0_N
\]
\[
B_kR^{-1}A_k^H + A_k^*R^{-1}B_k^H = 0_N
\]
which in turn satisfies conditions in (10) trivially. Using (26) in (24), we have
\[
[XR^{-1}X^H]_{k,k} = \left| h_k \right|^2 \left| v_{j,k}^{(1)} \right|^2 + \left| v_{j,k}^{(2)} \right|^2 \left| x_iQ \right|^2.
\]
This implies
\[
\hat{x} [A_k R^{-1} A_k^H + B_k R^{-1} B_k^H] \hat{x}^H
= \hat{x} \text{diag} [D_{1,k}, D_{2,k}, \ldots, D_{N,k}] \hat{x}^H
\]
and hence (11) holds.

This completes the proof.

**APPENDIX III**

**PROOF OF LEMMA 3**

See equations (27) and (28) at the bottom of the page.

i) If \( A_k A_k^H = 0_N \), then the result follows. Hence, we consider the case when \( A_k A_k^H \neq 0_N \). The term \( [A_k A_k^H]_{i,i} \) corresponds to the inner product of the \( i \)-th row of \( A_k \) and the \( i \)-th row of \( A_k^H \). We prove the result by contradiction. Suppose if \( [A_k A_k^H]_{i,i} \) is nonzero, then at least for some \( t \), \( [A_k]_{i,t} \) and \( [A_k^H]_{t,i} \) are nonzero entries. Since \( A_k \) and \( B_k \) do not take values at the same position, a nonzero entry at \( [A_k]_{i,t} \) implies that \( [X]_{i,k'} \) has an entry of the form \( h_k \tilde{x}_{i} \) or its scaled version by \( \pm 1 \) or \( \pm j \). Similarly, \( [X]_{k',t} \) has an entry of the form \( h_{k'} \tilde{x}_{t} \) or its scaled version by \( \pm 1 \) or \( \pm j \). With this, the \( t \)-th column of the weight matrices \( \Phi_J \) and \( \Phi_Q \) corresponding to \( \tilde{x}_{i} \) or \( \tilde{x}_{Q} \), respectively, have nonzero entries in the \( i \)-th row and the \( i \)-th row, hence, for \( k \neq k' \), \( [\Phi_J \Phi_Q^H]_{i,k} \neq 0 \) and \( [\Phi_Q \Phi_Q^H]_{k,k} \neq 0 \). This is a contradiction since \( X \) is a unitary SSD-PDSTBC.

On the similar lines, it can be proved that \( [B_k B_k^H]_{i,i} = 0 \).

ii) If \( A_k A_k^H = 0_N \), then the result follows. If \( A_k A_k^H \neq 0_N \), then the result is proved by contradiction. We first prove the row-nonmonomial property. Assume that for \( m \neq j \), \( m \neq i \), and \( i \neq j \), \( [A_k A_k^H]_{i,m} \) and \( [A_k A_k^H]_{j,m} \) are nonzero elements in the \( i \)-th row of \( A_k A_k^H \). This implies that for some \( t \), \( [X]_{k,t} \) and \( [X]_{k',t} \) have entries of the form \( h_k \tilde{x}_{i} \) or \( h_{k'} \tilde{x}_{m} \), respectively, or their scaled versions. In [18], Lemma 5 provides conditions on the real variables \( \tilde{x}_{i} \), \( \tilde{x}_{Q} \), \( \tilde{x}_{m} \) and \( \tilde{x}_{m_Q} \) such that \( [XX]^H_{k,k'} \) is written as the sum of several terms with each term being a function of real and quadrature component of a single information variable. We refer the reader to [18] for more details on the above result. Applying the results of Lemma 5 in [18], \( \tilde{x}_{i} \) and \( \tilde{x}_{m} \) satisfy the following conditions:

\[
\tilde{x}_{i} \in \tilde{x}_{iQ}, \tilde{x}_{m} \in \{x_{mI}, x_{mQ}, x_{m'I}, x_{m'Q}\}
\]
with \( \tilde{x}_{i} \neq \tilde{x}_{iQ} \) and \( \tilde{x}_{m} \neq \tilde{x}_{m_Q} \)

and (29)

for some \( n \neq n' \) where \( 1 \leq n, n' \leq N \) and the subscript \( \square \) represents either \( I \) or \( Q \). Since \( [A_k A_k^H]_{i,j} \neq 0 \), for some \( t \neq t' \), \( [X]_{i,j} \) and \( [X]_{i',j} \) have entries of the form \( h_k \tilde{x}_{i} \) and \( h_{k'} \tilde{x}_{j} \), respectively, or their scaled versions. Therefore, from the results of Lemma 5 in [18], \( \tilde{x}_{i} \) and \( \tilde{x}_{j} \) must also satisfy the conditions in (29) for \( m = j \). This is a contradiction, since \( \tilde{x}_{j} \neq \tilde{x}_{m} \). Therefore, \( A_k A_k^H \) is row non-monomial. On the similar lines, it can be proved that \( A_k A_k^H \) and \( B_k B_k^H \) are both column and row-nonmonomial matrices.

iii) If \( A_k A_k^H + B_k B_k^H = 0_N \), then the result is true and hence we consider the case when \( A_k A_k^H + B_k B_k^H \neq 0_N \). We first prove the row-nonmonomial property of \( A_k A_k^H + B_k B_k^H \). Consider the \( i \)-th row of \( A_k A_k^H + B_k B_k^H \). Nonzero entries of the \( i \)-th row are contributed by the nonzero entries in the \( i \)-th row of \( A_k A_k^H \) and \( B_k B_k^H \). Since both matrices are row nonmonomials, each matrix can at most contribute one nonzero element at some column in the \( i \)-th row. If both matrices have nonzero entries at the same column in the \( i \)-th column, then the result follows. Otherwise, we prove by contradiction that both matrices cannot contribute nonzero entries in different columns of the \( i \)-th row. Towards that end, \( j \neq m \), \( j \neq i \), and \( i \neq m \), assume that \( (i,m) \)th and \( (i,j) \)th entries of \( A_k A_k^H + B_k B_k^H \) are nonzero contributed from \( A_k A_k^H \) and \( B_k B_k^H \), respectively. This implies that \( (k,t) \)th and \( (k',t) \)th entries of \( X \) are \( h_k \tilde{x}_{i} \) and \( h_{k'} \tilde{x}_{m} \), respectively, or their scaled versions. From the results of [18, Lemma 5], \( \tilde{x}_{i} \) and \( \tilde{x}_{m} \) must satisfy the conditions in (29). Since \( (i,j) \)th entry is also nonzero and contributed by \( B_k B_k^H \), for some \( t \neq t', (k,t) \)th and \( (k',t) \)th entries of \( X \) are \( h_k \tilde{x}_{i} \) and \( h_{k'} \tilde{x}_{m} \), respectively. Therefore, \( \tilde{x}_{i} \) and \( \tilde{x}_{m} \) must also satisfy the conditions in (29) for \( m = j \) [18, Lemma 6]. This is a contradiction, since \( \tilde{x}_{j} \neq \tilde{x}_{m} \). Hence, there cannot be more than one nonzero entry in any row of \( A_k A_k^H + B_k B_k^H \). Similarly, it can be proved that there cannot be more than one nonzero entry in any column also. Thus, \( A_k A_k^H + B_k B_k^H \) is a column and row-nonmonomial matrix.

iv) If \( A_k A_k^H + B_k B_k^H = 0_N \), then the number of nonzero entries is trivially even. If \( A_k A_k^H + B_k B_k^H \neq 0_N \), then from the results of [18, Corollary 1], the number of nonzero entries is even.

V) With \( \hat{A}_{k,k'} = [A_k A_k^H]^T \) and \( \hat{B}_{k,k'} = [B_k B_k^H]^T \),
\[
\hat{A}_{k,k'} \hat{A}^H_{k,k'} + \hat{B}_{k,k'} \hat{B}^H_{k,k'} \in C^{2N \times 2N}
\]
is given by
\[
\hat{A}_{k,k'} \hat{A}^H_{k,k'} + \hat{B}_{k,k'} \hat{B}^H_{k,k'} = \left[ A_k A_k^H + B_k B_k^H \right]^T \left[ A_k A_k^H + B_k B_k^H \right].
\]

\[
M_1 = \text{diag} \left( \hat{A}_{1,2} \hat{A}_{1,2}^H + \hat{B}_{1,2} \hat{B}_{1,2}^H, \hat{A}_{3,4} \hat{A}_{3,4}^H + \hat{B}_{3,4} \hat{B}_{3,4}^H, \ldots, \hat{A}_{2t-1,2t-1} \hat{A}_{2t-1,2t-1}^H + \hat{B}_{2t-1,2t-1} \hat{B}_{2t-1,2t-1}^H \right),
\]
\[
M_2 = \text{diag} \left( A_{2t+1} A_{2t+1}^H + B_{2t+1} B_{2t+1}^H, A_{2t+2} A_{2t+2}^H + B_{2t+2} B_{2t+2}^H, \ldots, A_{2N+1} A_{2N+1}^H + B_{2N+1} B_{2N+1}^H \right).
\]
From (14), $A_kA_k^H + B_k^*B_k^T = \text{diag}[E_{1,k}, E_{2,k}, \ldots, E_{N,k}]$ for $1 \leq k \leq K$, where every $E_{n,k}$ is a strictly positive real number. Therefore, the above matrix can be written as

$$
\begin{bmatrix}
\text{diag}[E_{1,k}, E_{2,k}, \ldots, E_{N,k}] & A_kA_k^H + B_k^*B_k^T \\
A_k^H + B_k^*B_k^T & \text{diag}[E_{1,k}, E_{2,k}, \ldots, E_{N,k}']
\end{bmatrix}.
$$

(30)

Since $A_kA_k^H + B_k^*B_k^T$ and $A_{k'}A_{k'}^H + B_{k'}^*B_{k'}^T$ are column and row-monomial matrices, every column and row of (30) has at least one and at most two nonzero entries. This implies that every column of (30) is orthogonal to at least $2N - 2$ of its columns. If $A_kA_k^H + B_k^*B_k^T$ and $A_{k'}A_{k'}^H + B_{k'}^*B_{k'}^T$ are zero matrices, then the rank of $A_kA_k^H + B_k^*B_k^T$ is $2N$ and hence the result follows. We show that nonzero entries in $A_kA_k^H + B_k^*B_k^T$ can possibly result in the reduction of rank of $A_kA_k^H + B_k^*B_k^T$, which proves the lower bound. From [18, Corollary 1], a nonzero entry at $[A_kA_k^H + B_k^*B_k^T]_{im}$ for some $i \neq m$ implies that $[A_kA_k^H + B_k^*B_k^T]_{im}$, $[A_kA_k^H + B_k^*B_k^T]_{mi}$, and $[A_kA_k^H + B_k^*B_k^T]_{mi}$ are also nonzero entries. This implies that $(i, N + m)$th, $(m, N + i)$th, $(N + m, i)$th, and $(N + i, m)$th entries of $A_kA_k^H + B_k^*B_k^T$ are nonzero entries.

Since the diagonal entries of $A_kA_k^H + B_k^*B_k^T$ are strictly positive, the $i$th and $(N + m)$th column have nonzero entries in the same rows. Similarly, the $m$th and the $(N + i)$th columns have nonzero entries in the same rows. With this, the $m$th and the $(N + i)$th columns can be proportional to each other. Similarly, the $i$th and the $(N + m)$th columns can also be proportional to each other in which case, the rank of the matrix is at least $2N - 2$. Therefore, with a pair of nonzero entries in $A_kA_k^H + B_k^*B_k^T$, the rank of the matrix in (30) can possibly reduce by two. Since the number of nonzero entries in $A_kA_k^H + B_k^*B_k^T$ is always even, irrespective of whether $N = 2m$ or $2m + 1$, for any positive integer $m$, there can be at most $2m$ nonzero entries in $A_kA_k^H + B_k^*B_k^T$. Extending the same argument as before, the rank of (30) can possibly reduce by at most $2m$ in which case the rank of $A_kA_k^H + B_k^*B_k^T$ is at least $2m$ when $N = 2m$ or $2m + 2$ when $N = 2m + 1$. Hence, we have the bound

$$\text{Rank}[A_kA_k^H + B_k^*B_k^T] \geq 2\left\lceil \frac{N}{2} \right\rceil.$$

This completes the proof.

**APPENDIX IV**

**PROOF OF THEOREM 2**

Let $A = [A_1^T, A_2^T, \ldots, A_K^T]^T$ and $B = [B_1^T, B_2^T, \ldots, B_K^T]^T$ be $NK \times T$ matrices constructed using the relay matrices $A_k$.

$$
\begin{bmatrix}
A_1A_1^H + B_1^*B_1^T & A_1A_2^H + B_1^*B_2^T & \cdots & A_1A_K^H + B_1^*B_K^T \\
A_2A_1^H + B_2^*B_1^T & A_2A_2^H + B_2^*B_2^T & \cdots & A_2A_K^H + B_2^*B_K^T \\
\vdots & \vdots & \ddots & \vdots \\
A_KA_1^H + B_K^*B_1^T & A_KA_2^H + B_K^*B_2^T & \cdots & A_KA_K^H + B_K^*B_K^T
\end{bmatrix}.
$$

(31)
Since $\text{Rank} \ [A]$ and $\text{Rank} \ [B]$ are upper bounded by $T$
\[ T \geq a \left[ \frac{N}{2} \right] + \left[ \frac{N}{2} \right] b, \]
If $K$ is even, the right-hand side of the above inequality is minimized for $a = \frac{K}{2}$ and $b = 0$, and hence
\[ T \geq \frac{K}{2} \left[ \frac{N}{2} \right]. \]
If $K$ is odd, the inequality is minimized for $a = \left[ \frac{K}{2} \right]$ and $b = 1$, and hence
\[ T \geq \left[ \frac{K}{2} \right] \left[ \frac{N}{2} \right] + \left[ \frac{N}{2} \right]. \]
Combining the cases of $K$ being even and odd, we have $T \geq \left[ \frac{K}{2} \right] \left[ \frac{N}{2} \right]$ and hence
\[ \frac{N}{T} \leq \left[ \frac{N}{2} \right]. \]
This completes the proof.

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