Constructing paths to avoid multiple obstacles

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Abstract. This article deals with the issues of modeling the trajectory of an object moving on a plane. Its trajectory may intersect with obstacles (circles). The author proposes an algorithm for modifying the initial trajectory of the object taking into account the movement of obstacles. At the beginning, we describe how to divide the initial trajectory into sections. Then we describe how to modify the line on each segment in two cases, depending on the angle between the projection direction and the direction of movement of the obstacle center.

1. Introduction
This article deals with the construction of a trajectory that goes around several obstacles. In pursuit tasks, the pursuer and the target object have to change their trajectory taking into account obstacles, which can be either static or mobile. Previously, we considered the situation when there was only one obstacle. We will now touch on the case of several obstacles.

2. Statement of the problem
Consider the case on the horizontal plane. Let the moving object follow a straight line from the point A to the point B. There are n movable obstacles on the plane. It's assumed that each obstacle can be enclosed in a circle of radius $r_i$ centered at $O_i$ ($i=1..n$). Points $O_i$ move along curves $h_i(u)$. Assume that the circles cannot intersect. Moving obstacles can cross the straight line $AB$. If $AB$ has intersected obstacles, the moving object has to change the initial trajectory on a certain section depending on the obstacle movement direction.

![Figure 1. Initial trajectory](image-url)
This way the task consists of two subtasks: 1) splitting the initial trajectory into sections; 2) modifying trajectory.

3. Computing the new trajectory

Let's analyze the first subtask. The number of split sections will match the number of obstacles. This leads to the question: how to set the boundary points of sections. We suggest the following method. Firstly, we construct projections of points $O_i$ on the line $AB$. The boundary points $T_i$ (i=1..n-1) are midpoints of the segments $O_{i-1}O_i$ (i=2..n). The points $T_0$ and $T_n$ are points $A$ and $B$, respectively. I.e., the boundary points will be found by the formulas:

$$T_i = \frac{\text{proj}_{AB}O_{i-1} + \text{proj}_{AB}O_i}{2}; \quad \text{proj}_{AB}O_i = A + \alpha_i(B - A); \quad \alpha_i = \frac{(B - A)(O_i - A)}{(B - A)^2}$$ (1)

We note that if the circle does not intersect the line $AB$, then the nearest point $T_i$ will be replaced by the projection of its center. This will be useful if the distance between the projections of the circle centers is not enough for the moving object to maneuver. On the one hand, such a replacement will complicate the procedure for splitting a straight line $AB$. On the other hand, it will reduce the number of segments. For example, figure 2 shows there is no need to split a line.

![Figure 2. Splitting and modifying the path (right or obtuse angle)](image-url)
Consider the second subtask. The bypass line will depend on the angle between the vectors $O_i$ and the tangent vector $v_2 = l'(u)$. This angle can be in the interval $\left[ \frac{\pi}{2}; \pi \right]$ (Fig. 2) or $\left[ 0; \frac{\pi}{2} \right]$ (Fig. 3). These cases differ from each other only in the position of one of the key points of the circumference circle, shown in the figures as a dotted line. In both cases, this point is located on a straight-line perpendicular to AB, at a distance from the center of the circle equal to $r_{\text{size}}$ (size – the object’s size). In the first case, the points $O_i$ and the key point will be on opposite sides of the line AB, in the second case - one at a time.

The normal vector is found as follows:

$$
\bar{n}_v = v \cdot \left( \frac{-(B-A)y}{(B-A)_x} \right), \quad v = \begin{cases} 
-1, & \text{if } l'(u)(O'_i - O_i) \leq 0 \\
1, & \text{if } l'(u)(O'_i - O_i) > 0 
\end{cases}, \quad O'_i = \text{projection of point } O_i
$$

Let's briefly describe the formation of the bypass line. First, we find the key points of bypass circle. Based on the key points of the bypass circle and auxiliary constructions, the control points are calculated. The control points define third-order Bezier curves. These Bezier curves are transitive curves for the straight line AB and the bypass circle. Previously, the subroutine was developed for calculating the bypass curve. As a result of its operation, a parametric equation of the curve is obtained, called $\text{Manevr}(t, (O,r), A, B)$, where $t$ is the current value of the parameter, $(O,r)$ is an obstacle, and $A$ and $B$ are the beginning and end of the line, respectively [5].

To form a curve for bypassing several obstacles, we just need to run a ready-made routine with the necessary parameters on each segment. Note that in the Manevr procedure, the parameter value $t=0$ corresponded to point $A$, and $t=1$ to point $B$. Here, the points $T_{i-1}$ and $T_i$ serve as the ends of the segment. These points correspond to the values of the parameter $t=t_{i-1}$ and $t=t_i$. Then the subroutine is executed with the following parameters:

$$
\text{Manevr} \left( \frac{t-t_{i-1}}{t_i-t_{i-1}}, (O_i,r_i), T_{i-1}, T_i \right)
$$

**Conclusion**

When creating the procedure for the task, it was necessary to take into account that the projections of obstacles can change places. Because of this, the splitting procedure is complicated by sorting circles by the value of the $o_i$ parameter. It was also necessary to modify the Manevr procedure so that the obstacle bypass line on the site passed behind it, for which it was necessary to consider two cases.
In the future, the developed procedure will be improved to take into account the cases of overlapping obstacles and the small distance between their projections on a straight line.

References

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