RR Lyrae Distance Scale: Theory and Observations

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Abstract. The RR Lyrae distance scale is reviewed. In particular, we discuss theoretical and empirical methods currently adopted in the literature. Moreover, we also outline pros and cons of optical and near-infrared mean magnitudes to overcome some of the problems currently affecting RR Lyrae distances. The importance of the K-band Period-Luminosity-Metallicity \((PLZ_K)\) relation for RR Lyrae is also discussed, together with the absolute calibration of the zero-point. We also mention some preliminary results based on NIR \((J,K)\) time series data of the LMC cluster Reticulum. This cluster hosts a sizable sample of RR Lyrae and its distance is found to be \(18.45 \pm 0.04\) mag using the predicted \(PLZ_K\) relation and \(18.51 \pm 0.06\) using the \(PLZ_J\) relation. We briefly discuss the evolutionary status of Anomalous Cepheids and their possible use as distance indicators. Finally, we point out some possible improvements to improve the intrinsic accuracy of theory and observations.

1 Introduction

During the last half a century RR Lyrae stars have been the crossroad of paramount theoretical and observational efforts. The reasons are manifold. From a theoretical point of view RR Lyrae play a crucial role because they are a fundamental laboratory not only to test the accuracy of evolutionary (Casalli et al. [34]; Brown et al. [21]; VandenBerg & Bell [91]) and pulsation (Bono & Stellingwerf [19]; Bono et al. [10]; Feuchtner [52]) models but also to constrain fundamental physics problems such as the neutrino magnetic moment (Castellani & Degl’Innocenti [41]).

From an observational point of view RR Lyrae are even more important since they are the most popular primary distance indicators for old, low-mass stars (see e.g. Smith [84]; Caputo [24]; Walker [95, 96]; Carretta et al. [32]; Walker, these proceedings; Cacciari & Clementini these proceedings). Dating back to Baade [2], RR Lyrae stars have been also adopted to trace the old stellar component in the Galaxy (Suntzeff et al. [88]; Layden [64]) and in nearby galaxies (Mateo [68]; Monelli et al. [72]). The use of RR Lyrae as stellar tracers received during the last few years a new spin. On the basis of time-series data recent photometric surveys identified a local overdensity of RR Lyrae stars in the Galactic halo (Vivas et al. [92]). Current empirical (Yanny et al. [96]; Ibata et al. [68]; Martinez-Delgado et al. [66]) and theoretical (Helmi [56], and references therein) evidence suggests that such a clump is the northern tidal stream left over by the Sagittarius dwarf spheroidal (dSph).
This notwithstanding, several empirical phenomena connected with the evolutionary and pulsation properties of RR Lyrae stars have not been settled yet. We still do not know the physical mechanisms that govern the occurrence of the **Blazhko effect** (Kolenberg et al. [61]; Smith et al. [85]) as well as of the mixed-mode behavior (Bono et al. [9]; Feuchtinger [51]). The same outcome applies for the formation and propagation of the shock front along the pulsation cycle (Bono et al. [15]; Chadid et al. [42]).

However, the lack of a detailed knowledge of the physical phenomena that take place in the interior, in the envelope, and in the atmosphere of RR Lyrae stars only partially hampers the use of these objects as standard candles. In the following we discuss pros and cons in using different theoretical and empirical relations to derive RR Lyrae distances. In particular, we will focus our attention on optical and near-infrared (NIR) data for field and cluster RR Lyrae. Finally, we briefly outline the current status of RR Lyrae and classical Cepheid distance scales.

### 2 Theoretical and Empirical Circumstantial Evidence

At present, the most popular approach to estimate the RR Lyrae distances is the $M_V$-[Fe/H] relation. This relation is widely adopted because it only requires two observables, namely the apparent visual magnitude and the metallicity. From a theoretical point of view it is also well-defined, because the RR Lyrae instability strip is located in a region of the Horizontal Branch (HB) that is quite flat. In spite of these straightforward positive features the absolute calibration of the $M_V$-[Fe/H] relation is still an open problem. Current theoretical and empirical calibrations provide difference in absolute distances that range from 0.1 to 0.25 mag (Bono et al. [12]). Oddly enough, the internal errors are quite often of the order of a few hundredths of magnitude. This indicates that current methods might be affected by deceptive systematic errors. The main problems affecting the $M_V$-[Fe/H] relation are the following:

1. **Evolutionary effects.** The use of the $M_V$-[Fe/H] relation relies on the assumption that RR Lyrae stars are on the Zero-Age-Horizontal-Branch (ZAHB). This is on average a plausible but thorny assumption, since field and cluster RR Lyrae do show a spread in luminosity. Moreover, theoretical (Bono et al. [16]; Cassisi & Salaris [35]) and empirical (Carney, Storm, & Jones [29]; Sandage [82]) evidence suggest that the intrinsic width in luminosity of the ZAHB becomes larger when moving from metal-poor to metal-rich Galactic Globular Clusters (GGCs). As a consequence, RR Lyrae samples at different metal contents are differentially affected by off-ZAHB evolution as well as by the HB morphology (Caputo [24], and references therein). Moreover, a spread in luminosity of the order of $\pm 0.1$ dex causes a spread in the visual magnitude of the order of $\pm 0.25$ mag (see Fig. 1 in Bono et al. [12]). To investigate in more detail this effect we estimated, using the atmosphere models provided by Castelli, Gratton,
Kurucz\textsuperscript{38,39}, the bolometric correction in the V-band for two different ZAHBs that cover the metallicity range typical of Galactic RR Lyrae. According to current evolutionary predictions we adopted log \(L/L_\odot = 1.75\), \(M/M_\odot = 0.75\) for \(Z=0.0001\) and log \(L/L_\odot = 1.51\), \(M/M_\odot = 0.55\) for \(Z=0.02\). Data plotted in the top panel of Fig. 1 show that spread in luminosity is mainly due to the change in the bolometric correction. When moving from the hot (blue) to the cool (red) edge of the instability strip \(BC_V\) undergoes a change of the order of 0.1 mag\textsuperscript{2}. Therefore, RR Lyrae stars with exactly the same stellar mass and luminosity but different effective temperatures (pulsation periods) present an intrinsic spread in the visual magnitude of approximately a tenth of a magnitude. Data plotted in the top panel are also suggesting that the ZAHB luminosity, at fixed bolometric magnitude, tilts when moving toward hotter effective temperatures (Brocato et al.\textsuperscript{20}).

This trend presents a substantial change when moving toward longer wavelengths, and indeed the bolometric correction in the I-band provides for the same ZAHBs a change of the order of 0.3-0.4 mag. This means that GGCs characterized by well-populated instability strips start to display a slope when moving from hotter to cooler objects. Therefore, cluster RR Lyrae stars with the same stellar mass and luminosity become systematically brighter when moving from shorter to longer periods. It turns out that RR Lyrae in the I-band start to obey a Period-Luminosity (PL) relation. This effect become more and more evident once we move from the I to the K-band, and indeed the bolometric correction increases by more than one magnitude when moving from the blue to the red edge of the instability strip. Therefore, RR Lyrae stars should show a well-defined PL relation in the K-band. This result strongly supports the seminal empirical finding brought forward by Longmore et al.\textsuperscript{65} concerning the occurrence of the K-band PL relation among cluster RR Lyrae. Moreover, the \(PL_K\) should also be marginally affected by the intrinsic spread in luminosity, since a mild decrease in the effective temperature (increase in the period) causes a strict increase in brightness, and in turn a decrease in \(M_K\). It is worth mentioning that in performing this test we adopted the same evolutionary predictions (bolometric magnitudes and effective temperatures) and that the change from the V to the K-band is mainly due to the bolometric corrections and the color-temperature relations predicted by atmosphere models. Finally we note that RR Lyrae \(M_K\) magnitudes, in contrast with \(M_V\) magnitudes, are also marginally affected by a spread in stellar mass inside the instability strip (see Fig. 2 in Bono et al.\textsuperscript{12}).

\textit{ii) Linearity.} Recent theoretical and empirical evidence indicates that the \(M_V-[\text{Fe/H}]\) relation is not linear when moving from metal-poor to metal-rich RR Lyrae (Castellani, Chieffi, Pulone\textsuperscript{10}; Caputo et al.\textsuperscript{25}; Layden\textsuperscript{59}). In particular, the slope appears to be quite shallow in the metal-poor regime (0.18

\textsuperscript{1} The models labeled \textit{NOVER} were constructed by adopting a canonical treatment for the mixing-length, i.e. the convective overshooting into the convective stable regions is neglected.

\textsuperscript{2} Note that we assumed a range in temperature of more than 3,000 K to account for the temperature variation along the pulsation cycle.
for \([Fe/H] \leq -1.6\) while it is quite steep in the metal-rich regime (0.35 for \([Fe/H] > -1.6\)). However, recent photometric and spectroscopic measurements of RR Lyrae stars in the Large Magellanic Cloud (LMC) do not show evidence of a change in the slope at \([Fe/H] \approx -1.5\) (Clementini et al.\cite{Ref}). The change in the slope, if confirmed by new and independent estimates, means that metal abundance might also introduce a systematic uncertainty when moving from metal-poor to metal-rich RR Lyrae. In fact, an uncertainty of ±0.2 dex in metallicity implies an uncertainty in visual magnitude that ranges from 0.04 to 0.07 mag. Note that such an uncertainty would affect not only the slope but also the
zero-point of the $M_V$-[Fe/H] relation. On the other hand, data plotted in the bottom panel of Fig. 1 and in Fig. 3 by Bono et al. [12], together with K-band observational data (Longmore et al. [65]) suggest that the $P_{L_K}$ presents a linear dependence on the metal content. Note that

iii) Reddening. It is well-known that an uncertainty in the reddening, E(B-V), of 0.01 mag implies an uncertainty of the order of 0.03 mag in the visual magnitude. The same error in the reddening causes an uncertainty that is a factor of two smaller in the I-band, and negligible in the K-band (Cardelli et al. [27]). Moreover, new empirical optical (B-V, V-I) relations based on cluster (Kovacs & Walker [62]) and field (Piersimoni, Bono, & Ripepi [75]) fundamental mode RR Lyrae stars might supply accurate individual reddening estimates. Interestingly enough, these relations rely on reddening free parameters, such as period, luminosity amplitude, and metallicity and present a small intrinsic dispersion.

iv) Mean Magnitudes. The mean magnitude of RR Lyrae stars is estimated as time average either in magnitude or in intensity along the pulsation cycle. However, current theoretical (Bono, Caputo, & Stellingwerf [14]; Marconi et al. [67]) and empirical (Corwin & Carney [45]) evidence suggest that the two mean magnitudes present a systematic difference with the mean "static" magnitude of equivalent nonpulsating stars. The discrepancy for fundamental RR Lyrae stars (RRab) increases from a few hundredths of a magnitude close to the red edge to $\approx 0.1$ mag close to the blue edge. This discrepancy becomes marginal in the K-band, since the luminosity amplitude becomes a factor of $\approx 3$ smaller than in the V-band.

v) Metallicity. Recent spectroscopic investigations based on high-resolution spectra collected with 8m-class telescopes disclosed that hot HB stars present a quite complicate pattern of both helium and heavy element abundances (Behr et al. [4]; Moehler et al. [70]). These peculiarities have also been identified as jumps along the ZAHB in the near ultraviolet bands (Grundhal et al. [54]; Momanhy et al. [71]). According to current beliefs these peculiar abundances are the balance between two competing effects, namely gravitational settling and radiative levitation (Michaud, Vauclair, & Vauclair [69]). Up to now only for a few field RR Lyrae are available high resolution spectra (Clementini et al. [43]), and therefore we do not know whether cluster RR Lyrae present the same chemical peculiarities. Moreover, the metallicity of cluster RR Lyrae is generally estimated using the $\Delta S$ method or the $hk$ index (Anthony-Twarog et al. [1]; Rey et al. [80]) but both of them are based on the Ca abundance that is an $\alpha$-element. The empirical scenario was further jazzed up by the evidence that the stellar rotation shows a bimodal distribution in the hot region of the HB (Recio-Blanco et al. [70]). On the other hand, current empirical evidence indicate that RR Lyrae stars do not rotate rapidly enough for the rotation to be detected (Peterson, Carney, & Latham [74]).

Together with this is the indisputable fact we are still facing the problem of the metallicity scale. In fact, the Zinn & West [97] and the Carretta & Gratton [31] scales show in the intermediate metallicity range a difference of the order of
0.2 dex (Rutledge, Hesser, & Stetson[78]; Kraft & Ivans[63]). Note that the zero-point of the \(M_V-[Fe/H]\) relation is generally estimated at \([Fe/H]=-1.5\), and therefore current uncertainties on the metallicity scale might introduce an error of the order of 0.04 mag(!). Finally, we mention that we still lack a detailed knowledge of \(\alpha-\)element abundances among cluster RR Lyrae stars. This parameter is crucial to estimate the global metallicity, i.e. the metallicity currently adopted in constructing both evolutionary and pulsation models (Salaris, Chieffi, & Straniero[81]; Zoccali et al.[98]). According to current empirical evidence the overabundance of \(\alpha-\)elements should decrease when moving from metal-poor to metal-rich stars (Carney[28]), but current spectroscopic data for RR Lyrae stars are scanty.

Obviously the abundance of \(\alpha-\)elements also affects atmosphere models, and in turn the BCs and the color-temperature (CT) relations. We performed a test using the \(\alpha-\)enhanced atmosphere models recently constructed by Castelli et al. (2003) assuming an \(\alpha\) over iron enhancement of \([\alpha/Fe]\approx0.4\). We found that at fixed iron abundance \((-2.0\leq [Fe/H] \leq -1)\) the difference in BCs and in CTs, for surface gravities \((\log g = 2.5 - 3.0)\) and effective temperatures \((5300 \leq T_e \leq 8300 \text{ K})\) typical of RR Lyrae stars, between solar-scaled and \(\alpha-\)enhanced models is small and of the order of a few hundredths of magnitude.

\textit{vi)} Microturbulent velocity. Theoretical and empirical evidence suggests that the microturbulent velocity \((\xi)\) in the atmosphere of RR Lyrae stars ranges from a few \(\text{km/s}\) to more than 10 \(\text{km/s}\) along the pulsation cycle and peaks close to the phases of maximum compression (Benz & Stellingwerf[6]; Cacciari et al.[22]; Fokin, Gillet, & Chadid[53]). The sample of RR Lyrae for which this information are available is quite limited; however, current data seem to indicate that the microturbulent velocity is larger than 5 \(\text{km/s}\) for a substantial fraction of the pulsation period.

On the other hand, current atmosphere models are constructed by adopting a microturbulent velocity of 2 \(\text{km/s}\), since this is a typical value for static stars. As a consequence, evolutionary and pulsation predictions when transformed into the observational plane might be affected by a systematic uncertainty. Therefore we decided to investigate the dependence of both BCs and CTs on this fundamental parameter. Fig. 2 shows the variation of BCs, at fixed surface gravity \((\log g = 2.5)\), for two sets of \(\alpha-\)enhanced atmosphere models constructed by adopting different iron abundances and microturbulent velocities (see labeled values). Data plotted in the top panel show quite clearly that \(BC_V\) values of metal-poor models are marginally affected by this parameter inside the instability strip, while the metal-rich ones present a difference of the order of a few hundredths of magnitude. The same outcome applies for the \(BC_I\)s. Once again the BC in the

\footnote{These models as well as the Castelli et al.[38] models are available at the following web site \texttt{http://kurucz.harvard.edu}\footnote{Current models are also available in the Kurucz web site. Note that for \([Fe/H]=-2.0\) we adopted \(\xi = 1\) and 4 \(\text{km/s}\), because \(\alpha-\)enhanced atmosphere models for \(\xi = 0\) are not available yet.}}
Fig. 2. Top: predicted bolometric correction in the visual band as a function of the effective temperature. The solid and the dashed lines display the change of $BC_V$ at fixed gravity ($\log g = 2.5$) for metal-poor ([Fe/H]=-2.0) and metal-rich ([Fe/H]=0.0) RR Lyrae stars. The atmosphere models (C03) adopted to estimate the $BC_V$ values were constructed by adopting an overabundance of $\alpha$-elements of $[\alpha/Fe] = 0.4$ and different assumptions for the microturbulent velocity (see labeled values). Middle: same as the top, but the bolometric corrections refer to the I-band. Bottom: same as the top, but the bolometric corrections refer to the K-band.

K band shows marginal changes both in the metal-poor and in the metal-rich regime.

Let us now investigate the dependence of the CT relations on the microturbulent velocity. The top panel of Fig. 3 shows the B-V color as a function of the effective temperature for the same grid of atmosphere models adopted in Fig. 2. Metal-poor B-V colors present a marginal dependence on $\xi$ inside the instability strip. On the other hand, the models at solar chemical composition show that the difference between the models with $\xi = 0$ and $\xi = 4$ km/s strictly increases when...
moving from hotter to cooler effective temperatures. The difference, close to the red edge of the instability strip, becomes of the order of a tenth of magnitude (!). Interestingly enough, the V-I colors (middle panel) do not show any dependence at all on the metal abundance as well as on the microturbulent velocity, thus suggesting that in the V and the I-band the two effects cancel out. Finally, the V-K colors (bottom panel) show a mild reversed (Bono et al. [12]) dependence on metal abundance and a marginal dependence on the microturbulent velocity. In this context it is worth mentioning that Cacciari et al. [23] have recently revised the zero-point of the Baade-Wesselink (BW) method using a new set of atmosphere models that partially overlaps with the atmosphere models currently adopted. Using the entire set of photometric and spectroscopic data available in the literature for two fields RR Lyrae they found that the absolute magnitude of RR Cet ([Fe/H] = −1.45) is ≈ 0.12 mag brighter than previously estimated.
However, they did not find any significant change between old and new estimates for SW And ([Fe/H]=−0.24).

In this section we discussed some possible uncertainties affecting the RR Lyrae distance scale. It is worth mentioning that several of them might affect both theory and observations, therefore it is quite difficult to estimate the global error budget on a quantitative basis. However, it turns out that sets of atmosphere models, constructed by adopting different physical assumptions, predict BCs that might differ by a few hundredths of magnitude. The impact on the B-V colors is larger and of the order of 0.1 mag.

3 New Theoretical Approach

The circumstantial evidence discussed in the previous section suggested that the K-band PL relation of RR Lyrae should present several advantages when compared with the other methods currently adopted in the literature. Moreover, and even more importantly, Longmore et al.\[65\] demonstrated, on the basis of K-band photometry for a good sample of GGCs, that cluster RR Lyrae do obey to a well-defined PL relation. Therefore, we decided to investigate whether non-linear, time-dependent convective models of RR Lyrae (Bono & Stellingwerf\[19\]) support this empirical scenario. To cover the metal abundances typical of Galactic RR Lyrae we computed several sequences of models ranging from Y=0.24, Z=0.0001 to Y=0.28, Z=0.02. For each given chemical composition we adopted a single mass-value and 2-3 different luminosity levels to account for off-ZAHB evolution and for possible uncertainties on the ZAHB luminosity predicted by current evolutionary models. The main advantage in adopting this approach is that the edges of the instability region can be consistently estimated. Even though current predictions depend on the adopted mixing-length parameter, they do not rely on ad hoc assumptions concerning the position of the red edge (Bono et al.\[18\]).

We found that RR Lyrae models do obey to a well-defined $PLZ_K$ relation:

$$M_K = 0.139 - 2.071(\log P + 0.30) + 0.167 \log Z$$ \hspace{1cm} (1)$$

with an intrinsic dispersion of 0.037 mag. The symbols have their usual meaning. On the basis of this relation and of K-band data for RR Lyrae stars in M3 collected by Longmore et al.\[65\] we found for this cluster a true distance modulus of 15.07 ± 0.07 mag. This estimate is in very good agreement with the distance provided by Longmore et al., i.e. $DM = 15.00 ± 0.04 ± 0.15$ mag, where the former error refers to uncertainties in the zero-point, while the latter in the slope of the PL relation. It is noteworthy that the quoted distances are also in good agreement with the M3 distance based on the First Overtone Blue Edge (FOBE) method developed by Caputo et al.\[25\]. This method is based on the comparison between the predicted first overtone blue edge and the location of $RRc$ variables in the $\log P$ vs $M_V$ plane. The accuracy of this method depends on the number of $RRc$ variables present in a given stellar system and seems to
provide accurate distances for GCs characterized by well-populated instability strips. In the case of M3 they found $DM = 15.00 \pm 0.07$ mag.

Although the $PLZ_K$ relation for RR Lyrae presents several indisputable advantages, when compared with other methods available in the literature, we still lack accurate measurements of mean K-band magnitude for cluster RR Lyrae. Therefore, the comparison with empirical PL relations did not allow us to constrain the intrinsic accuracy of our predictions. Fortunately enough, Benedict et al. [5] provided an accurate estimate of the trigonometric parallax of RR Lyr itself using FGS3, the interferometer on board of the Hubble Space Telescope (HST). Note that the new estimate, $\pi_{\text{abs}} = 3.82 \pm 0.20$ mas, is approximately a factor of three more accurate than the previous evaluation provided by Hipparcos, i.e. $\pi_{\text{abs}} = 4.38 \pm 0.59$ mas. Therefore, we investigated whether the theoretical framework we developed accounts for this accurate absolute distance. By adopting for RR Lyr a mean interstellar extinction of $<A_V> = 0.12 \pm 0.1$ (Benedict et al. [5]), an iron abundance of $[\text{Fe/H}]=-1.39$ ($Z \approx 0.0008$, Fernley et al. [49]; Clementini et al. [43]), a mean K magnitude $K = 6.54 \pm 0.04$ mag (Fernley, Skillen, & Burki [50]), and a period of $\log P = -0.2466$ (Hardie [55]) we found a pulsation parallax of $\pi_{\text{abs}} = 3.858 \pm 0.131$ mas. The absolute distance we obtained agrees quite well with the new parallax for RR Lyr provided by HST. This result, once confirmed by new and accurate geometrical distances, emphasizes the potential of the $PLZ_K$ in view of a new NIR RR Lyrae distance scale.

4 New Observational Approach

We already mentioned that accurate mean K-band magnitudes are only available for a limited sample of cluster RR Lyrae (Liu & Janes [60]; Longmore et al. [65]; Storm et al. [86][87]). These data are not very accurate, since NIR photometry with small format detectors was partially hampered by crowding. A few K-band measurements have also been collected by Carney et al. [30] for RR Lyrae stars in the Galactic bulge. However, field RR Lyrae whose distances were estimated using the BW method (26 $RRab$ plus 3 RR Lyrae pulsating in the first overtones, $RRc$) have mean K-magnitudes with an accuracy of the order of a few hundredths of magnitude. A preliminary comparison between distances based on the BW method and on the $PLZ_K$ relation discloses a systematic difference that decreases when moving from metal-poor to metal-rich objects (Bono et al. [11]). It is worth mentioning that this discrepancy between the two different methods is substantially reduced once we adopt the new calibration of the BW method provided by Cacciari et al. [29].

It goes without saying that new and accurate mean K-band magnitudes for cluster RR Lyrae are mandatory to improve current theoretical and empirical scenarios. Therefore, we decided to start a new observational project aimed at collecting J and K band data in a dozen Galactic and Magellanic Cloud clusters. Figures 4 and 5 show the K,V-K and the J,V-J Color-Magnitude Diagram of the LMC cluster Reticulum. We selected this cluster because it contains a sizable
sample of RR Lyrae (32) and it is characterized by a very low central density. Moreover, accurate periods for the entire sample are available in the literature (Walker94).

Fig. 4. Left: Color magnitude diagram of the LMC cluster Reticulum in $K, V-K$. Data were collected over three different observing runs with SOFI@NTT and reduced using DAOPHOT/ALLFRAME. A glance at the data shows that RR Lyrae stars in this cluster present a well-defined slope. Right: intrinsic photometric error. The strategy adopted to perform the photometry allowed us to reach a $K$-band limiting magnitude of 19.5 with an accuracy better than 0.05 mag.

Optical (UBVI) data were collected using SUSI2 at ESO/NTT, the NIR ones with SOFI at ESO/NTT and cover a time interval of three years. In summary, we collected approximately 170 phase points in the $K$-band and at roughly 50 phase points in the $J$-band. The individual exposure times range from 1 to 2
minutes in the K-band and from 20 s to 1 minute in the J-band. To improve the accuracy of individual measurements we adopted a new reduction strategy, i.e. we performed with DAOPHOT/ALLFRAME the photometry over the entire set of J and K individual exposures.

![Figure 5](image)

**Fig. 5.** Left: same as Fig. 4, but the CMD is J,V-J. Right: intrinsic photometric error. Note that RR Lyrae stars also show a slope in the J-band but flatter than in the K-band. The strategy adopted to perform the photometry allowed us to reach a J-band limiting magnitude of 20.5 with an accuracy better than 0.05 mag.

A glance at the data plotted in Figures 4 and 5, and in particular the small color dispersion along the HB and the Red Giant Branch (RGB) show that photometry is very accurate down to limiting magnitudes of $K \approx 19.5$ and $J \approx 20.5$. Moreover and even more importantly RR Lyrae stars show a well-defined slope both in the J and in the K-band.
Using the mean K magnitudes provided by ALLFRAME, a mean metallicity of [Fe/H]=−1.71 based on spectroscopic data (Suntzeff et al. [89]), a mean reddening of E(B-V)=0.02 (Walker [94]), the Cardelli et al. [27] relation, and the $PLZ_K$ relation discussed in section 3, we found a true distance modulus of $18.45 \pm 0.04$ mag, where the uncertainty only accounts for internal photometric errors. Interestingly enough, by adopting the mean J-band magnitude, the same assumptions concerning metallicity and reddening, and a new $PLZ_J$ relation, we found a distance modulus of $18.51 \pm 0.06$ mag, where the uncertainty only accounts for internal photometric errors.

5 Anomalous Cepheids

Anomalous Cepheids are an interesting group of variable stars, since they are brighter than RR Lyrae stars and have periods that range from 0.5 days to a few days. They have been identified both in GGCs and in LG dwarf galaxies (Nemec, Nemec, & Lutz [73]). Dating back to Demarque & Hirshfeld [46] and to Hirshfeld [57] the common belief concerning the evolutionary status of these objects is that they are metal-poor, intermediate-mass stars with an age of the order of 1 Gyr. This hypothesis was confirmed by more recent evolutionary (Castellani & Degl’Innocenti [37]; Caputo & Degl’Innocenti [26]) and pulsational (Bono et al. [13]) investigations. However, the region of the HR diagram roughly located at $\log L/L_\odot = 2$ presents several intrinsic features worth being discussed in some detail. The top panel of Fig. 6 shows the HR diagram for metal-poor, intermediate-mass stars ranging from 2.2 to 3.5 $M/\odot$. It is worth mentioning that the minimum mass that performs the blue loop for this composition is $2.2 M/\odot$. This mass value is smaller than the corresponding minimum mass for the chemical compositions typical of the Small (3.25 $M/\odot$, $Z=0.004$) and of the Large (4.25 $M/\odot$, $Z=0.01$) Magellanic Cloud (Bono et al. [8]). This means that metal-poor stellar systems such as IC1613 should produce a substantial fraction of short-period classical Cepheids. This suggestion is supported by current empirical evidence (Udalski et al. [90]). Moreover and even more importantly evolutionary tracks plotted in this panel show that the blue loop takes place at hotter effective temperatures when moving from 2.2 to 3.5 $M/\odot$. The occurrence of this behavior was explained by Cassisi & Castellani [33] as the consequence of the fact that metal-poor intermediate-mass models do not reach the Hayashi track before central helium ignition. Therefore, these models do not undergo the canonical dredge-up phase. We also note that for evolutionary models more massive than 3.5 $M/\odot$ the amount of time spent inside the instability strip is substantially shorter (Pietrinferni et al. 2003, in preparation) when compared to more metal-rich models.

Data plotted in the bottom panel of Fig. 6 show quite clearly that the structures less massive than 2.2 $M/\odot$ show a substantially different behavior, and indeed they start to burn helium in the center at effective temperatures of the order of $\log T_e = 3.9 - 4.0$. The temperature range moves to lower effective temperatures and crosses the instability strip for structures with mass values
of the order of $1.4-1.8 \, M/M_\odot$. These structures spend a substantial amount of He-burning phases inside the instability strip and should produce Anomalous Cepheids. The central He-burning phases of less-massive structures perform a "hook", i.e. they move at first toward lower effective temperatures ($1.0-1.2 \, M/M_\odot$) and then toward higher effective temperatures ($0.9-1.0 \, M/M_\odot$). Structures with mass values smaller than the latter limit produce RR Lyrae stars. This is a very qualitative scenario and a more detailed analysis can be found in Castellani & Degl'Innocenti[37]. A few caveats concerning the previous observational scenario: i The evolutionary tracks plotted in Fig. 6 have been computed by adopting a Reimers mass-loss rate with $\eta = 0.4$. This means that mass values that cross the instability strip slightly depend on this assumption. ii Stellar structures producing Anomalous Cepheids and classical Cepheids show a substantial difference in the age range covered by main sequence stars. In fact, evolutionary models with stellar mass $\approx 1.6 M/M_\odot$ spend on the main sequence a lifetime of roughly 0.7 Gyr, while models of $3.0 M/M_\odot$ leave the main sequence after approximately 0.2 Gyr. This means that stellar systems producing classical Cepheids should also show a well-populated blue main sequence region when compared with stellar systems producing Anomalous Cepheids. iii Current evolutionary scenario for Anomalous Cepheids relies on the assumption that they are the aftermath of single star evolution. However, the occurrence of a few Anomalous Cepheids in GGCs suggest that a fraction of them might be the progeny of binary collisions or of binary mergings (Renzini[77]; Nemec et al.[73]; Bono et al.[13]).

The observational scenario concerning Anomalous Cepheids has been substantially improved during the last few years (Siegel & Majewski[83]; Bersier & Wood[7]; Dolphin et al.[47]; Pritzl et al.[76]). Fig. 7 shows the distribution of both RR Lyrae and Anomalous Cepheids detected by Dall’Ora et al.[48] in the Carina dwarf galaxy. The comparison between theory and observations supports the evolutionary scenario we discussed in this section. In fact, Dall’Ora et al.[48] and Monelli et al.[72] found, on the basis of pulsational and evolutionary arguments that these objects are approximately a factor of two more massive than RR Lyrae stars present in the same galaxy. Note that the metal abundance adopted for this stellar system is $Z=0.0004$. Interestingly enough, evolutionary predictions also suggest that more metal-rich structures should not produce Anomalous Cepheids, since the so-called "hook" of intermediate-mass helium burning structures do not cross the instability strip.

Theory and observations suggest that Anomalous Cepheids pulsate both in the fundamental and in the first overtone (Nemec, et al.[73]; Bono et al.[13]). However, more data are required to constrain on a quantitative basis the accuracy of distance determinations based on these objects (Pritzl et al.[76]). Obviously LG dwarf galaxies are crucial systems to investigate this problem, since several of them host both RR Lyrae, Anomalous Cepheids, and large samples of red clump stars.
6 Conclusions

The results concerning the RR Lyrae distance scale discussed in this investigation can be summarized along two different paths:

Theoretical path. a) Theoretical predictions based on pulsation models, namely the \( PLZ_K \) relation and the FOBE method supply, within current uncertainties, similar absolute distances. The distance to M3 provided by the former method is in very good agreement with the empirical calibration provided by Longmore et al.\cite{65}. Moreover, the pulsation parallax obtained for RR Lyr itself is in remarkable agreement with the trigonometric parallax recently obtained by Benedict et al.\cite{5}. These findings, once confirmed by independent investigations, together with plain physical arguments concerning the dependence of the bolometric correction and of the color-temperature relation on input physics indicate that the \( PLZ_K \) might be less affected by deceptive uncertainties affecting the other methods. This approach can be further improved. Up to now theoretical and empirical \( PLZ_K \) relations were derived by simultaneously accounting for RR\( ab \) and RR\( c \) variables. First overtones (FOs) are "fundamentalized" by adding 0.13 to the logarithm of the period. However, preliminary theoretical results suggest that FOs also obey a well-defined \( PLZ_K \) relation. The main advantage in using FOs is that the instability region of these objects is narrower when compared with fundamental mode RR Lyrae. Therefore the FO \( PLZ_K \) relation presents a smaller intrinsic dispersion.

b) Theoretical predictions based on evolutionary models have been widely discussed in the literature (Bono, Castellani, & Marconi\cite{17}; Cassisi et al.\cite{34}; Caputo et al.\cite{25}). The main outcome of these investigations is that current HB models seem to predict HB luminosities that are \approx 0.1 \) mag brighter than estimated using the pulsational approach. However, different sets of HB models constructed by adopting different assumptions on input physics present a spread in HB luminosities of the order of 0.15 mag. This means that in the near future new observational constraints based either on geometrical distances or on robust distance indicators might supply the unique opportunity to nail down the intrinsic accuracy of the ingredients currently adopted in evolutionary and pulsation models.

c) In section 2, we mentioned that we still lack homogeneous sets of atmosphere models that cover a broad range of microturbulent velocities. New models are strongly required to check on a quantitative basis the impact that such a parameter has on the transformation of theoretical predictions into the observational plane. The new models might also play a crucial role in understanding the plausibility of the physical assumptions currently adopted by the BW method.

Observational path. a) Theoretical and empirical evidence suggests that the \( PLZ_K \) relation for RR Lyrae presents several advantages when compared with other methods available in the literature. This notwithstanding we still lack of accurate K-band measurements for both field and cluster RR Lyrae. The use of current generation NIR detectors at 4m class telescopes and careful reduction strategies seem to suggest that accurate mean K-band magnitudes can be obtained down to \( K \approx 18.5 - 19.0 \). This means that we should be able to supply
an accurate distance scale for Galactic and Large Magellanic cloud clusters. In the near future the use of NIR detectors at 8m class telescopes should allow us to detect and measure RR Lyrae stars in several Local Group (LG) galaxies (see the ARAUCARIA project, Pietrinzski et al. in these proceedings). This is a fundamental step to improve the global accuracy of cosmic distances, because LG galaxies display complex star formation histories, and often host not only RR Lyrae but also intermediate-mass distance indicators, such as Red Clump stars, Anomalous Cepheids, and classical Cepheids.

b) We focused our attention on the mean K-band magnitude of RR Lyrae stars. However, theory and observations suggest that RR Lyrae do obey a PL relation also in the J and the H-band. Once again the amount of data available in the literature for these bands is quite limited.

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Fig. 6. Top: HR diagram for intermediate-mass stars at fixed chemical composition (Y=0.23, Z=0.0001). Note that the blue loop when moving from 2.2 $M_\odot$ evolutionary models to 3.5 $M_\odot$ takes place at hotter effective temperatures. The vertical line marks the center of the Cepheid instability strip. The width in temperature of the instability strip is typically ±0.05 dex. Bottom: same as the top panel but for low and intermediate-mass stars. Filled circles mark the beginning of central He-burning phases for models ranging from 1.0 $M_\odot$ to 2.1 $M_\odot$. Data plotted in this figure illustrate that central He-burning phases take place inside the instability strip for evolutionary models more massive than 1.4 $M_\odot$ and less massive than 1.0 $M_\odot$. 
Fig. 7. Comparison between predicted He-burning structures at fixed chemical composition ($Z=0.0004$, $Y=0.23$) and bright stars in the Carina dwarf galaxy (Dall’Ora et al. [48]; Monelli et al. [72]). Data plotted in this figure show static (small dots) and variable stars: circles RR Lyrae stars, triangles, ACs. Crosses mark variables that present poor-phase coverage. Solid, dashed, and dotted-dashed lines display predicted Zero Age He-burning structures for different progenitor ages ranging from 12 ($M = 0.8 M_\odot$) to 0.6 ($M = 2.2 M_\odot$) Gyr. The dotted lines show the He-burning evolution for three intermediate-age structures of 1.8 (redder), 2.0, and 2.2 (bluer) $M/M_\odot$. 

$ZAHB$ 12 Gyr $Y=0.23$ $Z=0.0004$

" 1 Gyr $DM=20.24$

" 0.6 Gyr $E(B-V)=0.03$