A Novel Conflict Measurement in Decision-Making and Its Application in Fault Diagnosis

Fuyuan Xiao, Member, IEEE, Zehong Cao, Member, IEEE, and Alireza Jolfaei, Senior Member, IEEE

Abstract—Dempster–Shafer evidence (DSE) theory, which allows combining pieces of evidence from different data sources to derive a degree of belief function that is a type of fuzzy measure, is a general framework for reasoning with uncertainty. In this framework, how to optimally manage the conflicts of multiple pieces of evidence in DSE remains an open issue to support decision making. The existing conflict measurement approaches can achieve acceptable outcomes but do not fully consider the optimization at the decision-making level using the novel measurement of conflicts. In this article, we propose a novel evidential correlation coefficient (ECC) for belief functions by measuring the conflict between two pieces of evidence in decision making. Then, we investigate the properties of our proposed evidential correlation and conflict coefficients, which are all proven to satisfy the desirable properties for conflict measurement, including nonnegativity, symmetry, boundedness, extreme consistency, and insensitivity to refinement. We also present several examples and comparisons to demonstrate the superiority of our proposed ECC method. Finally, we apply the proposed ECC in a decision-making application of motor rotor fault diagnosis, which verifies the practicability and effectiveness of our proposed novel measurement.

Index Terms—Basic belief assignments, belief function, conflict management, decision making, Dempster–Shafer evidence theory, evidential correlation coefficient, fault diagnosis, fuzzy measure.

I. INTRODUCTION

Uncertainty is an inherent component in data science and big data, especially in a fuzzy environment [1]–[3]. How to handle and measure the uncertainty to support decisions in various applications [4]–[6], ranging from medicine to engineering has attracted considerable attention in recent decades [7], [8]. Several novel fuzzy techniques and systems have been presented for reasoning with and managing uncertainty, including the extended intuitionistic fuzzy sets [9], rough sets [10], Z numbers [11], evidence theory [12], [13], evidential reasoning [14], D numbers [15], R sets and numbers [16], [17], and other hybrid methods [18]. These theories are applied broadly in various fields, such as image classification [19], medical diagnosis [20], [21], information fusion [22], and decision making [23], [24]. In these fuzziness-related approaches, one of the most useful tools to handle uncertainty is Dempster–Shafer evidence (DSE) theory [25], [26], which has posed several attractive advantages as follows:

1) quantitatively modeling uncertainty by means of a basic probability assignment (BBA) [27];
2) the belief function is a type of fuzzy measure that provides partial information in terms of the appropriate fuzzy measure in relation to an uncertain variable [28], [29];
3) Dempster’s combination rule (DCR) satisfies the commutative and associative laws [30], [31];
4) the results generated by the DCR have the characteristic of fault-tolerance and relieve the uncertainty level by the DCR [32], [33].

Consequently, DSE theory can be of benefit for supporting decision making [34] and has been extensively investigated in extracting the information quality of BBA [35] and evidence reliability evaluation [36], [37]. According to previous studies of evidence theory, considering optimal management of conflicts may improve the accuracy performance at the decision-making level in data science applications [38]–[40]. Therefore, how to measure the conflict of multiple pieces of evidence has attracted considerable research attention in recent years [41]–[43], and many related definitions have been presented [44], which can be used for fuzzy system-based industrial application areas. Although the outcomes of current conflict management methods are acceptable in DSE theory, we assume there still remains room for improving decision-making performance at the decision level in terms of the measure and management of conflicts.

Therefore, in this article, we explored a novel conflict measurement in decision making. Here, we proposed a new evidential correlation coefficient (ECC), inspired by Jiang’s [45] method, to measure the correlation between BBAs in DSE theory, which could be proved, analyzed, and applied in the decision making of data science applications. Specifically, we proposed a new evidential conflict coefficient based on ECC to measure the conflict degree between BBAs. Then, we analyzed and proved
that the newly defined evidential conflict coefficient has the desirable properties for conflict measurement, including nonnegativity, symmetry, boundedness, extreme consistency, and insensitivity to refinement. Furthermore, we compared the proposed evidential conflict coefficient with well-known methods and demonstrated a motor rotor fault diagnosis application devised based on the ECC.

The rest of this article is organized as follows. Section III and IV briefly introduce the preliminaries of evidence theory and some existing conflict measures, respectively. Section V proposes new evidential correlation and conflict coefficients and their properties are analyzed and proved. Section VI compares various conflict measures to demonstrate the superiority of the proposed method. In Section VII, a fault diagnosis algorithm is devised based on the new correlation coefficient measure; then, the algorithm is applied to solve a motor rotor fault diagnosis problem. Finally, Section VIII concludes this article.

II. RELATED WORKS

As we know, the traditional Dempster’s conflict coefficient \( K \) [25] combines the mass allocated to the empty set, accounting for the conflict among focal elements, but it ignores the global consistency between different pieces of evidence.

To overcome this limitation, George and Pal [46], Jousselme et al. [47], and Cheng and Xiao [48] considered the conflict measure from the nonintersecting parts between different pieces of evidence. Another group of researchers quantified the measure of conflict from an alternative perspective. For instance, Liu [49] designed a two-dimensional conflict model by combining Dempster’s conflict coefficient and pignistic probability distance. Daniel [50] considered the plausibility conflict of evidence. Lefevere and Elouedi [51] studied measured conflict by means of the distance between pieces of evidence and the mass of an empty set. Furthermore, some novel strategies, such as divergence measures, have also been leveraged to measure evidential consistency [52]–[54]. For example, Ma and An [52] quantified the divergence of evidence by fuzzy nearness and a correlation coefficient. Xiao [53] measured the divergence of evidence by means of Jensen–Shannon divergence. In addition, some researchers investigated conflict measurement from the perspective of correlation coefficients [45], [55], [56]. For instance, Song et al. [55] defined a correlation coefficient [57] as the cosine of the angle between two vectors of pieces of evidence. Pan and Deng [56] developed a correlation coefficient [58] on the basis of Deng entropy [59]. Jiang [45] discussed the conflict measure by taking into account the nonintersection and the difference among focal elements [60].

In this article, inspired by Jiang’s [45] method, we propose a novel conflict measurement in decision making and apply it in fault diagnosis, which can improve decision-making performance at the decision level.

III. PRELIMINARIES

Many methods handling uncertainty problems have been presented in recent years [61]–[63]. As a useful uncertainty reasoning tool, DSE theory [25], [26] has been widely applied in various areas, such as decision making [64], classification [65], [66], reasoning [67], [68], and industrial alarm systems [69], [70]. The basic concepts and definitions [25], [26] of DSE theory are described as follows.

**Definition 1:** (Frame of discernment) Let \( \Omega \) be a set of mutually exclusive and collective nonempty events defined by [25], [26]

\[
\Omega = \{ F_1, F_2, \ldots, F_i, \ldots, F_n \}
\]

where \( \Omega \) is a frame of discernment (FOD).

The power set of \( \Omega \) is denoted as \( 2^\Omega \)

\[
2^\Omega = \{ \emptyset, \{ F_1 \}, \{ F_2 \}, \ldots, \{ F_n \}, \{ F_1, F_2 \}, \ldots, \{ F_1, F_2, \ldots, F_i \}, \ldots, \Omega \}
\]

where \( \emptyset \) represents an empty set.

If \( A_i \in 2^\Omega \), \( A_i \) is called a hypothesis.

**Definition 2:** (Mass function) A mass function \( m \) in FOD \( \Omega \) can be described as a mapping from \( 2^\Omega \) to \([0, 1] \) [25], [26]

\[
m : 2^\Omega \rightarrow [0, 1]
\]

satisfying

\[
m(\emptyset) = 0, \text{ and } \sum_{A_i \subseteq \Omega} m(A_i) = 1.
\]

In DSE theory, \( m \) is also called a BBA. For \( A_i \subseteq \Omega \), if \( m(A_i) \) is greater than zero, \( A_i \) is called a focal element. Since a BBA can effectively express the uncertainty, various BBA operations have been devised, including negation [71], [72] and an entropy function [73].

**Definition 3:** (Belief function) The belief function of \( A_i \subseteq \Omega \), denoted as \( \text{Bel}(A_i) \), is defined as [25], [26]

\[
\text{Bel}(A_i) = \sum_{A_h \subseteq A_i} m(A_h).
\]

**Definition 4:** (Plausibility function) The plausibility function of \( A_i \subseteq \Omega \), denoted as \( \text{Pl}(A_i) \), is defined as [25], [26]

\[
\text{Pl}(A_i) = \sum_{A_h \cap A_i \neq \emptyset} m(A_h).
\]

\( \text{Bel}(A_i) \) and \( \text{Pl}(A_i) \) represent the lower and upper bound functions of \( A_i \), respectively. An interval-valued belief structure can be used for an uncertainty measure [74], [75].

**Definition 5:** (Dempster’s combination rule) Let \( m_1 \) and \( m_2 \) be two independent BBAs in FOD \( \Omega \). DCR, represented in the form \( m = m_1 \oplus m_2 \), is defined as [25], [26]

\[
m(A_i) = \begin{cases} \frac{1}{K} \sum_{A_h \cap A_k = A_i} m_1(A_h) m_2(A_k), & A_i \neq \emptyset \\ 0, & A_i = \emptyset \end{cases}
\]

with

\[
K = \sum_{A_h \cap A_k = \emptyset} m_1(A_h) m_2(A_k)
\]

where \( A_h, A_k \subseteq \Omega \) and \( K \) is the coefficient of conflict between BBAs \( m_1 \) and \( m_2 \).
IV. EXISTING CONFLICT MEASURES

In this section, some existing conflict measures for belief functions are briefly introduced.

Let \( m_1 \) and \( m_2 \) be two BBAs with hypotheses \( A_i \) and \( A_j \), respectively, on the same FOD \( \Omega = \{ F_1, \ldots, F_i, \ldots, F_n \} \).

**Definition 6:** Jousselme et al.’s distance [47]

\[
d_{\text{JGDB}}(m_1, m_2) = \sqrt{\frac{1}{2} (\vec{m}_1 - \vec{m}_2)^T D(\vec{m}_1 - \vec{m}_2)} \tag{9}
\]

where \( \vec{m}_1 \) and \( \vec{m}_2 \) are the BBAs in vector notation and \( D \) is a \( 2^n \times 2^n \) matrix with elements

\[
D(A_i, A_j) = \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \tag{10}
\]

in which \( | \cdot | \) represents the cardinality function.

**Definition 7:** Lefèvre and Elouedi’s adapted conflict [51]

\[
k_{\text{LE}}(m_1, m_2) = d_{\text{JGDB}}(m_1, m_2) \cdot m_c(\emptyset) \tag{11}
\]

where \( m_c(\emptyset) \) is equal to \( K \) in (8) and \( d_{\text{JGDB}} \) is (9).

**Definition 8:** Song et al.’s correlation coefficient [55]

\[
c_{\text{SW}}(m_1, m_2) = \frac{\langle m_1, m_2 \rangle}{\| m_1 \| \cdot \| m_2 \|} \tag{12}
\]

in which \( m' \) is defined as

\[
\begin{cases}
  m'_1 = m_1 D \\
  m'_2 = m_2 D
\end{cases}
\]

where \( D \) is defined in (10).

Song et al.’s conflict coefficient:

\[
k_{\text{SW}}(m_1, m_2) = 1 - c_{\text{SW}}(m_1, m_2). \tag{13}
\]

**Definition 9:** Jiang’s correlation coefficient [45]

\[
c_f(m_1, m_2) = \frac{c(m_1, m_2)}{\sqrt{c(m_1, m_1)c(m_2, m_2)}} \tag{14}
\]

where \( c(m_1, m_2) \) is defined as

\[
c(m_1, m_2) = \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} m_1(A_i)m_2(A_j)|A_i \cap A_j| |A_i \cup A_j|. \tag{15}
\]

Jiang’s conflict coefficient

\[
k_f(m_1, m_2) = 1 - c_f(m_1, m_2). \tag{16}
\]

**Definition 10:** Cheng and Xiao’s [48] distance

\[
d_{\text{CX}}(m_1, m_2) = \sqrt{\frac{1}{2} (\vec{m}_1 - \vec{m}_2)^T D_o(\vec{m}_1 - \vec{m}_2)} \tag{17}
\]

where \( D_o \) is a \( 2^n \times 2^n \) matrix with elements

\[
D_o(A_i, A_j) = \frac{|A_i \cap A_j|}{|A_i|} \frac{|A_i \cap A_j|}{|A_j|}. \tag{18}
\]

V. NEW EVIDENTIAL CORRELATION AND CONFLICT COEFFICIENTS

For developing an effective conflict measurement, firstly our proposed a new method aims to satisfy the properties of a conflict measurement. Second, we consider determining how conflict identification between BBAs for improving performance. Third, for two arbitrary BBAs \( m_1 \) and \( m_2 \), we explore the conflict from the view of \( m_1 \) to \( m_2 \), as well as the conflict from the view of \( m_2 \) to \( m_1 \). Based on the abovementioned context, inspired by Jiang’s work [45], we design the evidential correlation and conflict coefficients, and specifically address an ECC for measuring the correlation between BBAs. We, then, analyze and prove the properties of ECC. Furthermore, we define an evidential conflict coefficient and discuss desirable properties for conflict management.

**Definition 11:** (ECC measure between BBAs) Let \( m_1 \) and \( m_2 \) be two BBAs on \( \Omega = \{ F_1, \ldots, F_i, \ldots, F_n \} \), where \( A_i \) and \( A_j \) are hypotheses of BBAs. The ECC between BBAs \( m_1 \) and \( m_2 \), denoted as \( \text{ECC}(m_1, m_2) \), is defined as

\[
\text{ECC}(m_1, m_2) = \cos \Theta(\vec{m}_1, \vec{m}_2) \cdot \cos \Theta(\vec{m}_2, \vec{m}_1) \tag{19}
\]

In (20), \( \cos \Theta \) is a cosine angle function between \( \vec{m}_1 \) and \( \vec{m}_2 \)

\[
\cos \Theta(\vec{m}_1, \vec{m}_2) = \frac{\langle \vec{m}_1, \vec{m}_2 \rangle}{\| \vec{m}_1 \| \cdot \| \vec{m}_2 \|} \tag{20}
\]

which has a mathematical formula similar to (12) [55]:

\[
\langle \vec{m}_1, \vec{m}_2 \rangle = \| \vec{m}_1 \| \cdot \| \vec{m}_2 \| = 2^n \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} m_1(A_i)m_2(A_j)|A_i \cap A_j| |A_i \cup A_j|. \tag{21}
\]

and \( \| \vec{m} \| \) is the norm of \( \vec{m} \).

**Theorem 1:** The ECC has the properties of nonnegativity, nondegeneracy, symmetry, and boundedness [45].

**Property 1:** Let \( m_1 \) and \( m_2 \) be two arbitrary BBAs as follows.

P1.1 Nonnegativity: \( \text{ECC}(m_1, m_2) \geq 0 \).

P1.2 Nondegeneracy: \( \text{ECC}(m_1, m_2) = 1 \) if and only if \( m_1 = m_2 \).

P1.3 Symmetry: \( \text{ECC}(m_1, m_2) = \text{ECC}(m_2, m_1) \).

P1.4 Boundedness: \( 0 \leq \text{ECC}(m_1, m_2) \leq 1 \).
Proof: (P1.1) Consider two arbitrary BBAs \( m_a \) and \( m_b \) in FOD \( \Omega \); we have

\[
\text{ECC}(m_a, m_b) = \left( \frac{\langle \overrightarrow{m}_a, \overrightarrow{m}_b \rangle}{\| \overrightarrow{m}_a \| \| \overrightarrow{m}_b \|} \right)^2.
\]

Clearly, \( \text{ECC}(m_a, m_b) \geq 0 \) can be conducted, which proves the property of nonnegativity of the ECC.

(P1.2) Consider two arbitrary BBAs \( m_a = m_b \) in FOD \( \Omega \) with the hypotheses of \( A_i \) and \( A_j \); we have

\[
\text{ECC}(m_a, m_b) = \left( \frac{\langle \overrightarrow{m}_a, \overrightarrow{m}_b \rangle}{\| \overrightarrow{m}_a \| \| \overrightarrow{m}_b \|} \right)^2 = 1.
\]

Conversely, consider \( \text{ECC}(m_a, m_b) = 1 \); we have

\[
\left( \frac{\langle \overrightarrow{m}_a, \overrightarrow{m}_b \rangle}{\| \overrightarrow{m}_a \| \| \overrightarrow{m}_b \|} \right)^2 = 1.
\]

Then, we obtain

\[
\left[ \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} m_a(A_i) m_b(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \right]^2
= 2^n \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} m_a(A_i) m_a(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|}
+ 2^n \sum_{j=1}^{2^n} \sum_{i=1}^{2^n} m_b(A_j) m_b(A_i) \frac{|A_j \cap A_i|}{|A_j \cup A_i|}.
\]

This equation is satisfied only for \( 1 \leq i, j \leq 2^n \)

\[
m_a(A_i) = m_b(A_i) \quad \text{and} \quad m_a(A_j) = m_b(A_j)
\]

such that

\[
m_a = m_b.
\]

Hence, \( \text{ECC}(m_a, m_b) = 1 \iff m_a = m_b \), which proves the property of nondegeneracy of the ECC.

(P1.3) Consider two arbitrary BBAs \( m_a \) and \( m_b \) in FOD \( \Omega \) with the hypotheses of \( A_i \) and \( A_j \).

For \( \langle \overrightarrow{m}_a, \overrightarrow{m}_b \rangle \), we have

\[
\langle \overrightarrow{m}_a, \overrightarrow{m}_b \rangle = \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} m_a(A_i) m_b(A_j) \frac{|A_i \cap A_j|}{|A_i \cup A_j|} \tag{26}
\]

Furthermore, for \( \langle \overrightarrow{m}_b, \overrightarrow{m}_a \rangle \), we have

\[
\langle \overrightarrow{m}_b, \overrightarrow{m}_a \rangle = \sum_{i=1}^{2^n} \sum_{j=1}^{2^n} m_b(A_j) m_a(A_i) \frac{|A_j \cap A_i|}{|A_j \cup A_i|} \tag{27}
\]

From (26) and (27), it is clear that

\[
\langle \overrightarrow{m}_a, \overrightarrow{m}_b \rangle = \langle \overrightarrow{m}_b, \overrightarrow{m}_a \rangle.
\]

Since

\[
\text{ECC}(m_a, m_b) = \left( \frac{\langle \overrightarrow{m}_a, \overrightarrow{m}_b \rangle}{\| \overrightarrow{m}_a \| \| \overrightarrow{m}_b \|} \right)^2
\]

and

\[
\text{ECC}(m_b, m_a) = \left( \frac{\langle \overrightarrow{m}_b, \overrightarrow{m}_a \rangle}{\| \overrightarrow{m}_b \| \| \overrightarrow{m}_a \|} \right)^2
\]

it is easy to conclude that

\[
\text{ECC}(m_a, m_b) = \text{ECC}(m_b, m_a)
\]

which proves the property of symmetry of the ECC.

(P1.4) Consider two arbitrary BBAs \( m_a \) and \( m_b \) in FOD \( \Omega \). Since \( D \) is a Hermitian positive-definite matrix [45], for a \( 2^n \times 2^n \) lower triangular matrix \( G \), we have

\[
D = G^T G.
\]

Thus, from [45], we have

\[
\langle \overrightarrow{m}_a, \overrightarrow{m}_a \rangle = u^T D u = u^T G^T G u
\]

\[
\langle \overrightarrow{m}_a, \overrightarrow{m}_b \rangle = u^T D v = u^T G^T G v
\]

\[
\langle \overrightarrow{m}_b, \overrightarrow{m}_b \rangle = v^T D v = v^T G^T G v.
\]

Thus,

\[
\text{ECC}(m_a, m_b) = \frac{(u^T G^T G v)^2}{(u^T G^T G u)(v^T G^T G v)}.
\]

Because \( u^T G^T G u = (G u)^T (G u) = \| G u \|^2 \), by applying the triangle inequality on the vector 2-norm [45], we obtain

\[
\| G(u + v) \|^2 \leq (\| G u \| + \| G v \|)^2 \implies (u + v)^T G^T G (u + v) \leq (\sqrt{u^T G^T G u} + \sqrt{u^T G^T G v})^2 \implies u^T G^T G u + u^T G^T G v + v^T G^T G u + v^T G^T G v \leq u^T G^T G u + v^T G^T G v + 2 \sqrt{u^T G^T G u v^T G^T G v} \implies u^T G^T G u \leq \sqrt{u^T G^T G u v^T G^T G v} \implies \frac{(u^T G^T G v)^2}{u^T G^T G u v^T G^T G v} \leq 1.
\]

Hence, as proved in (P1.1) that \( \text{ECC}(m_a, m_b) \geq 0 \), we obtain

\[
0 \leq \text{ECC}(m_a, m_b) \leq 1
\]

which proves the property of boundedness of the ECC.

Remark 1: Note that the larger ECC(m_1, m_2) is, the greater the correlation coefficient between the BBAs. Therefore, if ECC(m_1, m_2) = 1, then m_1 and m_2 are completely correlated; if ECC(m_1, m_2) = 0, then m_1 and m_2 are completely uncorrelated.

Next, an example is presented to illustrate the properties of ECC(m_1, m_2).

Example 1: Assume there are two BBAs, m_1 and m_2, in \( \Omega \)

\[
m_1 : \ m_1(\{A_1\}) = \alpha, m_1(\emptyset) = 1 - \alpha
\]

\[
m_2 : \ m_2(\{A_1\}) = 1 - \alpha, m_2(\emptyset) = \alpha.
\]

In Example 1, m_1 and m_2 change according to \( \alpha \) and the subset of \( \emptyset \). Here, \( \alpha \) is set within [0, 1], and the subset \( \emptyset \) is set as \( \{A_2\} \) and \( \{A_1, A_2\} \). The corresponding correlation coefficient measures are shown in Fig. 1.

When \( \alpha = 0.5 \), we have \( m_1(\{A_1\}) = m_2(\{A_1\}) = 0.5 \) and \( m_1(\emptyset) = m_2(\emptyset) = 0.5 \). Regardless of whether the subset \( \emptyset \) is \( \{A_2\} \) or \( \{A_1, A_2\} \), the correlation coefficient measures ECC(m_1, m_2) have the maximum value of 1 since m_1 and m_2
are the same, i.e., completely correlated. Hence, the nondegeneracy property of the ECC is verified. If and only if $m_1 = m_2$, the ECC($m_1, m_2$) has the largest correlation coefficient value of 1.

Furthermore, when $\alpha = 0$, and $\vartheta = \{A_2\}$, we have $m_1(\{A_1\}) = m_2(\{A_2\}) = 0$ and $m_1(\{A_2\}) = m_2(\{A_1\}) = 1$; when $\alpha = 1$, and $\vartheta = \{A_2\}$, we have $m_1(\{A_1\}) = m_2(\{A_2\}) = 1$ and $m_1(\{A_2\}) = m_2(\{A_1\}) = 0$. Under these two cases, the correlation coefficient measures ECC($m_1, m_2$) have the minimum value of 0 since $m_1$ and $m_2$ are completely uncorrelated.

Moreover, when $\alpha = 0$ and $\vartheta = \{A_1, A_2\}$, we have $m_1(\{A_1\}) = m_2(\{A_1, A_2\}) = 0$ and $m_1(\{A_1, A_2\}) = m_2(\{A_1\}) = 1$; when $\alpha = 1$, and $\vartheta = \{A_1, A_2\}$, we have $m_1(\{A_1\}) = m_2(\{A_1, A_2\}) = 1$ and $m_1(\{A_1, A_2\}) = m_2(\{A_1\}) = 0$. Under these two cases, the ECC($m_1, m_2$) measures have a minimal value of 0.25. This result is reasonable since when $\vartheta = \{A_1, A_2\}$, the subsets between $m_1(\{A_1\})$ and $m_2(\{A_1, A_2\})$ and between $m_2(\{A_1\})$ and $m_1(\{A_1, A_2\})$ have an intersection of $\{A_1\}$. Hence, ECC($m_1, m_2$) is equal to 0.25 rather than zero.

As $\alpha$ increases from 0 to 0.5, regardless of the subset $\vartheta = \{A_2\}$ or $\vartheta = \{A_1, A_2\}$, ECC($m_1, m_2$) gradually increases. This satisfies the expected result since $m_1$ and $m_2$ become similar as $\alpha$ increases from 0 to 0.5. On the other hand, as $\alpha$ increases from 0.5 to 1, regardless of the subset $\vartheta = \{A_2\}$ or $\vartheta = \{A_1, A_2\}$, ECC($m_1, m_2$) gradually decreases. This also satisfies the intuitive result, since $m_1$ and $m_2$ become dissimilar when $\alpha$ increases from 0.5 to 1.

Additionally, in this example, the boundedness property of the ECC, in which ECC($m_1, m_2$) is greater than or equal to 0 and less than or equal to 1, is verified. Furthermore, the results shown in Fig. 1 reveal the symmetry property of the ECC.

Based on Definition 11, the evidential conflict coefficient between BBAs is defined as follows.

Definition 12: (The evidential conflict coefficient between BBAs.) The evidential conflict coefficient between BBAs $m_1$ and $m_2$, denoted as $k_{ECC}(m_1, m_2)$, is defined as

$$k_{ECC}(m_1, m_2) = 1 - \text{ECC}(m_1, m_2) = 1 - \left( \frac{\|m_1 - m_2\|}{\|m_1\| \cdot \|m_2\|} \right)^2.$$

Theorem 2: The $k_{ECC}$ has desirable properties for conflict measurement [45], including nonnegativity, symmetry, boundedness, extreme consistency, and insensitivity to refinement.

Property 2: Let $m_1$ and $m_2$ be two arbitrary BBAs as follows.

P2.1 Nonnegativity: $k_{ECC}(m_1, m_2) \geq 0$.

P2.2 Symmetry: $k_{ECC}(m_1, m_2) = k_{ECC}(m_2, m_1)$.

P2.3 Boundedness: $0 \leq k_{ECC}(m_1, m_2) \leq 1$.

P2.4 Extreme consistency: 1) $k_{ECC}(m_1, m_2) = 1$ iff for the focal elements $A_i$ and $A_j$, of $m_1$ and $m_2$, respectively, $(\cap A_i) \cap (\cup A_j) = \emptyset$; 2) $k_{ECC}(m_1, m_2) = 0$ iff $m_1$ is completely equal to $m_2$.

P2.5 Insensitivity to refinement: for $m_1$ and $m_2$ refined from FODs $\Omega$ to $\Omega'$, $k_{ECC}(m_1^\Omega, m_2^\Omega') = k_{ECC}(m_1^\Omega', m_2^\Omega)$.

Proof: The proofs of (P2.1)–(P2.5) are trivial.

Remark 2: Note that the larger $k_{ECC}(m_1, m_2)$ is, the greater the conflict coefficient between the BBAs. If $k_{ECC}(m_1, m_2) = 1$, then $m_1$ and $m_2$ are in complete conflict; if $k_{ECC}(m_1, m_2) = 0$, then $m_1$ and $m_2$ are in no conflict.

Next, an example is presented to illustrate the nonnegativity and boundedness properties of $k_{ECC}$.

Example 2: Assume there are two BBAs $m_1$ and $m_2$ in $\Omega$,

$$m_1 : m_1(\{A_1\}) = \alpha, m_1(\{A_2\}) = 1 - \alpha$$

$$m_2 : m_2(\{A_1\}) = \gamma, m_2(\{A_2\}) = 1 - \gamma.$$

In Example 2, $m_1$ changes according to $\alpha$ and $\beta$, which are set within [0,1] and satisfy $\alpha + \beta \leq 1$, as shown in Fig. 2(a). Then, as $\alpha$ and $\beta$ vary, the corresponding correlation coefficient measures are shown in Fig. 2(b)–(d).

Fig. 2 verifies the nonnegativity and boundedness properties of $k_{ECC}$, where $k_{ECC} \geq 0$ and $0 \leq k_{ECC} \leq 1$.

As shown in Fig. 2(b), when $\alpha = 0.7$ and $\beta = 0.3$, we have $m_1(\{A_1\}) = m_2(\{A_1\}) = 0.7$ and $m_1(\{A_2\}) = m_2(\{A_2\}) = 0.3$. The correlation conflict measure $k_{ECC}(m_1, m_2)$ has the smallest value of 0 since $m_1$ and $m_2$ are exactly the same, that is, completely not in conflict. On the other hand, when $\alpha = \beta = 0$, we have $m_1(\{A_1\}) = m_2(\{A_2\}) = 0$ and $m_1(\{A_3\}) = 1$. In this case, the correlation conflict measure $k_{ECC}(m_1, m_2)$ has the largest value of 1 since $m_1$ and $m_2$ are completely in conflict. Hence, the extreme consistency property of $k_{ECC}$ is verified in this example.

Furthermore, Fig. 2(c) shows the variation of $k_{ECC}$ as $\alpha$ increases from 0 to 1. Clearly, when $\alpha$ increases from 0 to 0.7, since $m_1$ gradually becomes closer to $m_2$, the conflict coefficient $k_{ECC}$ decreases. As $\alpha$ increases from 0.7 to 1, because $m_1$ tends to become dissimilar to $m_2$, the conflict coefficient $k_{ECC}$ increases.

Moreover, Fig. 2(d) shows the variation of $k_{ECC}$ as $\beta$ increases from 0 to 1. Similarly, when $\beta$ increases from 0 to 0.3, since $m_1$ becomes closer to $m_2$, the conflict coefficient $k_{ECC}$ decreases. As $\beta$ increases from 0.3 to 1, because $m_1$ shifts farther from $m_2$, the conflict coefficient $k_{ECC}$ increases.

Next, we present an example to illustrate the symmetry and insensitivity to refinement properties of $k_{ECC}$.

Example 3: Assume there are two BBAs $m_1$ and $m_2$ in $\Omega$, respectively, $\Omega = \{A_1, A_2\}$ and $\Omega' = \{A_1, A_2, A_3\}$,

$$m_1 : m_1(\{A_1\}) = \alpha, m_1(\{A_2\}) = 1 - \alpha$$

$$m_2 : m_2(\{A_1\}) = \gamma, m_2(\{A_2\}) = 1 - \gamma.$$
In Example 3, $m_1$ and $m_2$ change in accordance with the variation in $\alpha$ and $\gamma$, respectively. $\alpha$ is set within [0,1] for $m_1$. For $m_2$, $\gamma$ is set as 0.2, 0.5, 0.7, and 1. Additionally, $m_1$ and $m_2$ have the same subsets and support values under $\Omega$ and $\Omega'$. Then, as $\alpha$ and $\gamma$ vary, the corresponding correlation coefficient measures under $\Omega$ and $\Omega'$ are shown in Fig. 3.

As shown in Fig. 3(a)-(d), $k_{ECC}$ is not impacted by the variation in the FODs from $\Omega$ to $\Omega'$. Even under variation in $\alpha$ and $\gamma$, the value of $k_{ECC}$ under the FOD $\Omega$ is always the same as that under the FOD $\Omega'$. Moreover, when we change the input $k_{ECC}(m_1, m_2)$ to $k_{ECC}(m_2, m_1)$, the results are exactly the same. Consequently, the symmetry and insensitivity to refinement properties of $k_{ECC}$ are verified.

VI. COMPARISON WITH EXISTING METHODS

In this section, several numerical examples are provided to illustrate the characteristics of different conflict measures, namely, the traditional Dempster’s $K$ [25], Jousselme et al.’s [47] $d_{JGB}$, Lefèvre and Elouedi’s [51] $k_{LE}$, Song et al.’s [55] $k_{SW}$, Jiang’s [45] $k_j$, Cheng and Xiao’s [48] $d_{CX}$, and the proposed $k_{ECC}$. In addition, we assess whether these conflict measures satisfy the abovementioned properties, as well as the degrees of conflict.

Example 4: Assume there are two BBAs $m_1$ and $m_2$ in $\Omega = \{A_1, A_2, A_3, A_4\}$

$m_1 : m_1(\{A_1\}) = 0.5, m_1(\{A_2\}) = 0.5$
$m_1(\{A_3\}) = 0.0, m_1(\{A_4\}) = 0.0$

$m_2 : m_2(\{A_1\}) = 0.0, m_2(\{A_2\}) = 0.0$
$m_2(\{A_3\}) = 0.5, m_2(\{A_4\}) = 0.5$.

Table I

| BBA \| $K$ | $d_{JGB}$ | $k_{LE}$ | $k_{SW}$ | $k_j$ | $d_{CX}$ | $k_{ECC}$ |
|-------|-------|--------|--------|--------|-------|--------|--------|
| $(m_1, m_2)$ | 1 | 0.7071 | 0.7071 | 0.6010 | 1 | 0.7071 | 1 |

In Example 4, for the focal elements $A_1$ and $A_2$ of $m_1$ and $m_2$, respectively, we have $(\cup A_1) \cap (\cup A_2) = \emptyset$. Thus, $m_1$ and $m_2$ are in complete conflict, so the conflict grade between $m_1$ and $m_2$ is assumed to be 1.

The results in Table I indicate that the conflict degrees produced by $K$, $k_j$, and $k_{ECC}$ are 1, in accordance with the intuitive result. By contrast, $d_{JGB}$, $k_{LE}$, and $d_{CX}$ generate a conflict value of 0.7071, and $k_{SW}$ has a conflict degree of 0.601. Therefore, $d_{JGB}$, $k_{LE}$, $k_{SW}$, and $d_{CX}$ do not satisfy the extreme consistency property of conflict measures.

Example 5: Assume there are two BBAs $m_1$ and $m_2$ in $\Omega = \{A_1, A_2, A_3, A_4\}$

$m_1 : m_1(\{A_1\}) = 0.25, m_1(\{A_2\}) = 0.25$
$m_1(\{A_3\}) = 0.25, m_1(\{A_4\}) = 0.25$

$m_2 : m_2(\{A_1\}) = 0.25, m_2(\{A_2\}) = 0.25$
$m_2(\{A_3\}) = 0.25, m_2(\{A_4\}) = 0.25$.

In Example 5, $m_1$ is the same as $m_2$, with the same support values for the corresponding subsets. Therefore, $m_1$ and $m_2$ are completely nonconflicting, so the conflict grade between $m_1$ and $m_2$ should be zero.

Table II shows that the conflict degrees calculated by $d_{JGB}$, $k_{LE}$, $k_{SW}$, $k_j$, $d_{CX}$, and $k_{ECC}$ are zero, in agreement with the
expected result. By contrast, $K$ generates a conflict value of 0.75, which does not satisfy the extreme consistency property of conflict measures.

**Example 6:** Assume there are two BBAs $m_1$ and $m_2$ in, respectively, $\Omega = \{A_1, A_2\}$ and $\Omega' = \{A_1, A_2, A_3\}$ have the same BBAs $m_1$ and $m_2$. Intuitively, the conflict grade between $m_1$ and $m_2$ on $\Omega$ and $\Omega'$ should be the same.

As shown in Table III, $K = 0.68$, $d_{JGB} = 0.6$, $k_{LE} = 0.408$, $k_{SW} = 0.5294$, $d_{CX} = 0.6$, and $k_{ECC} = 0.7785$, regardless of FOD $\Omega$ or $\Omega'$. This conforms to the expected result. However, $k_{SW}$ generates a conflict value of 0.3871 on FOD $\Omega$ and conflict of 0.3716 on FOD $\Omega'$, which does not satisfy the conflict measure property of insensitivity to refinement.

In summary, the abovementioned examples demonstrate the disadvantages of the conflict measures derived in related works. $K$ and the proposed $k_{ECC}$ satisfy the properties of conflict measures, especially the last two properties. By contrast, $K$, $d_{JGB}$, $k_{LE}$, $k_{SW}$, and $d_{CX}$ do not satisfy the extreme consistency property, and $k_{SW}$ does not satisfy the property of insensitivity to refinement. To further study the effectiveness of the proposed $k_{ECC}$, we discuss the following examples.

**Example 7:** Assume there exist two BBAs $m_1$ and $m_2$ in $\Omega = \{A_1, A_2, ..., A_{19}\}$

$m_1 : m_1(\{A_1\}) = 0.1, m_1(\{A_2, A_3, A_4\}) = 0.05$

$m_1(\{A_2\}) = 0.05, m_1(\{A_1\}) = 0.8$

$m_2 : m_2(\{A_1\}) = 0.2, m_2(\{A_2\}) = 0.8$.

In Example 6, the different FODs $\Omega = \{A_1, A_2\}$ and $\Omega' = \{A_1, A_2, A_3\}$ have the same BBAs $m_1$ and $m_2$. Intuitively, the conflict grade between $m_1$ and $m_2$ on $\Omega$ and $\Omega'$ should be the same.

As shown in Table III, $K = 0.68$, $d_{JGB} = 0.6$, $k_{LE} = 0.408$, $k_{SW} = 0.5294$, $d_{CX} = 0.6$, and $k_{ECC} = 0.7785$, regardless of FOD $\Omega$ or $\Omega'$. This conforms to the expected result. However, $k_{SW}$ generates a conflict value of 0.3871 on FOD $\Omega$ and conflict of 0.3716 on FOD $\Omega'$, which does not satisfy the conflict measure property of insensitivity to refinement.

In summary, the abovementioned examples demonstrate the disadvantages of the conflict measures derived in related works. $K$ and the proposed $k_{ECC}$ satisfy the properties of conflict measures, especially the last two properties. By contrast, $K$, $d_{JGB}$, $k_{LE}$, $k_{SW}$, and $d_{CX}$ do not satisfy the extreme consistency property, and $k_{SW}$ does not satisfy the property of insensitivity to refinement. To further study the effectiveness of the proposed $k_{ECC}$, we discuss the following examples.

**Example 7:** Assume there exist two BBAs $m_1$ and $m_2$ in $\Omega = \{A_1, A_2, ..., A_{19}\}$

$m_1 : m_1(\{A_1\}) = 0.1, m_1(\{A_2, A_3, A_4\}) = 0.05$

$m_1(\{A_2\}) = 0.05, m_1(\{A_1\}) = 0.8$

$m_2 : m_2(\{A_1\}) = 0.2, m_2(\{A_2\}) = 0.8$.

In Example 7, the subset $\vartheta_i$ of $m_1$ changes from $\{A_1\}$ to $\{A_1, A_2, A_3, A_4, A_5\}$, as shown in Table IV. Note that $m_2$ has one focal element such that $m_2(\{A_1, A_2, A_3, A_4, A_5\}) = 1$. Then, the conflict measures between BBAs $m_1$ and $m_2$ are calculated, as shown in Fig. 4.

When $i = 5$, the support value of subset $\{A_1, A_2, A_3, A_4, A_5\}$ of $m_1$ is 0.8, which is closer to that of $m_2$ with the subset $\{A_1, A_2, A_3, A_4, A_5\}$ than in other cases of $i$. Therefore, the expected conflict grade is assumed to achieve the minimum value. The results in Fig. 4 indicate that...
when $\vartheta_i$ of $m_1$ varies from $\{A_1\}$ to $\{A_1, A_2, A_3, A_4, A_5\}$, the $d_{JGB}, k_{LE}, k_J, d_{CX}$, and $k_{ECC}$ decrease, where $d_{JGB}, k_{LE}, k_J, d_{CX}$, and $k_{ECC}$ achieve minimum values of 0.1315, 0.0066, 0.0094, 0.1315, and 0.0186, respectively. As $\vartheta_i$ increases from $\{A_1, A_2, A_3, A_4, A_5\}$ to $\{A_1, \ldots, A_{19}\}$, the $d_{JGB}, k_{LE}, k_J, d_{CX}$, and $k_{ECC}$ conflict measures increase. Nevertheless, $K$ does not perform well in this example: the conflict measure always maintains the same value of 0.05 regardless of the variation in $\vartheta_i$.

**Example 8:** Assume there exist two BBAs $m_1$ and $m_2$ in $\Omega = \{A_1, A_2, \ldots, A_{19}\}$

$m_1 : m_1(\{A_1\}) = 0.1, m_1(\vartheta_i) = 0.9$

$m_2 : m_2(\{A_1\}) = 0.9, m_2(\vartheta_i) = 0.1$.

In Example 8, the subsets of $m_1$ and $m_2$ change according to the variation in $\vartheta_i$ in Table IV. $m_1$ and $m_2$ have the same subsets, namely, $\{A_1\}$ and $\vartheta_i$, where the subset $\vartheta_i$ increases as $i$ varies from 1 to 19. The conflict degrees between BBAs $m_1$ and $m_2$ produced by different methods are shown in Fig. 5.

In the case that $i = 1$, we have $m_1(\{A_1\}) = m_2(\{A_1\}) = 1$. Hence, all the conflict measures have a value of zero. As $i$ increases from 2 to 19, $d_{JGB}, k_{LE}, k_J, d_{CX}$, and $k_{ECC}$ increase, where it is reasonable and intuitive since as $\vartheta_i$ varies from $\{A_1, A_2\}$ to $\{A_1, \ldots, A_{19}\}$, the uncertainty increases. However, $K$ and $k_{LE}$ remain at zero, regardless of the variation in $\vartheta_i$. On the other hand, as shown by comparison with $d_{JGB}, k_J$, and $d_{CX}$, the proposed $k_{ECC}$ can better distinguish the conflict between $m_1$ and $m_2$ with higher values.

**Example 9:** Assume there exist two BBAs $m_1$ and $m_2$ in $\Omega = \{A_1, A_2, \ldots, A_{19}\}$

$m_1 : m_1(\Omega) = 0.1, m_1(\{A_2, A_3, A_4\}) = 0.05$

$m_1(\{A_7\}) = 0.05, m_1(\delta_i) = 0.8$

$m_2 : m_2(\{A_{10}\}) = 1$.

In Example 9, the subset $\delta_i$ of $m_1$ changes from $\delta_1$ to $\delta_{10}$, as shown in Table VI. When $i = 1, 2, \ldots, 10$, $\delta_i$ is the same as $\vartheta_i$. When $i$ increases from 11 to 19, the subset $\delta_i$ of $m_1$ is pruned from its first element until it becomes $\{A_{10}\}$. Note that $m_2$ has one focal element such that $m_2(\{A_{10}\}) = 1$. The conflict measures between BBAs $m_1$ and $m_2$ are calculated and shown in Fig. 6.

As $i$ increases from 1 to 9, since $m_1$ and $m_2$ are highly dissimilar, the expected conflict measure is assumed to achieve the maximal value. From Fig. 6, we can see that $K$, $d_{JGB}, k_{LE}, k_J, d_{CX}$, and $k_{ECC}$ show the same trend of increasing conflict values. When $i = 10$, $\vartheta_i$ first includes $\{A_{10}\}$. All the conflict measures become smaller than those in the case where $i = 1, \ldots, 9$. As $\vartheta_i$ increases from 11 to 19, while the subset decreases from $\{A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9, A_{10}\}$ to $\{A_{10}\}$, the $d_{JGB}, k_{LE}, k_J, d_{CX}$, and $k_{ECC}$ methods have decreasing conflict measures that reach minimal values of 0.1658, 0.0166, 0.0128, 0.1658, and 0.0255, respectively. By contrast, $K$ remains unchanged at $K = 0.1$, which does not satisfy the intuitive result.

In summary, Examples 4 to 9 clearly show that $k_{ECC}$ is superior to other methods: $k_{ECC}$ not only satisfies all the desired properties but also provides better conflict identification. Consequently, the proposed approach is effective and suitable for measuring conflict between BBAs.

**VII. Algorithm and Application**

Determining how to address decision-making problems has attracted considerable attention in recent years [76], [77]. In this section, a decision-making algorithm for fault diagnosis is devised based on the correlation coefficient measure. Then, a real application of motor rotor fault diagnosis from [45] is used to demonstrate the efficiency of the proposed method.
A. Algorithm

Problem statement: Let \( \{F_1, \ldots, F_i, \ldots, F_n\} \) be a set of fault types for a kind of machine that establishes an FOD \( \Theta \), and let \( \mathcal{M} = \{m_1, m_2, \ldots, m_k\} \) be \( k \) pieces of evidence modeled from the collected data of the sensors. A threshold \( \xi \) can be set in advance for making a decision. The goal of the algorithm is to diagnose, which type of fault occurs according to the given BBAs \( \{m_1, m_2, \ldots, m_k\} \), and threshold \( \xi \).

Step 1: A correlation matrix is constructed by leveraging the ECC:

\[
M_{ECC} = \begin{bmatrix}
ECC(m_1, m_1) & \cdots & ECC(m_1, m_k) \\
\vdots & \ddots & \vdots \\
ECC(m_k, m_1) & \cdots & ECC(m_k, m_k)
\end{bmatrix}.
\] (29)

Step 2: The support degree of \( m_j \) is calculated as

\[
SD(m_j) = \sum_{i=1|i \neq j}^{k} ECC(m_i, m_j).
\] (30)

Step 3: The credibility degree of \( m_j \) is calculated as:

\[
CD(m_j) = \frac{SD(m_j)}{\sum_{j=1}^{k} SD(m_j)}.
\] (31)

Step 4: The weighted average evidence (WAE) is obtained as

\[
WAE(m) = \sum_{j=1}^{k} CD(m_j) \times m_j.
\] (32)

Step 5: The WAE is fused \( k - 1 \) times with the DCR

\[
Fusion(m) = ((m \oplus m_1) \oplus \cdots \oplus m_{k-1}).
\] (33)

Step 6: The \( m(F_o) \) with the highest value is selected

\[
o = \arg \max_{1 \leq i \leq n} \{m(F_i)\}.
\] (34)

Step 7: The fault type is determined as follows:

\[
\begin{cases} 
\text{if } m(F_o) \geq \xi, & F_o \text{ is the fault type} \\
\text{if } m(F_o) < \xi, & \text{Cannot be determined}
\end{cases}
\] (35)

This fault diagnosis based on the ECC is given in Algorithm 1.

B. Application - Fault Diagnosis

In the motor rotor fault diagnosis application [45], three types of evidence are located at different places to collect the acceleration, velocity, and displacement information for a motor rotor. Then, the collected data are modeled as BBAs, as shown in Table VII, where \( m_1, m_2, \) and \( m_3 \) represent three pieces of evidence from the sensors. There are four states for a motor rotor, which establishes an FOD \( \Theta = \{F_1, F_2, F_3, F_4\} \): \( F_1 \) represents “normal operation,” \( F_2 \) represents “unbalance,” \( F_3 \) represents “misalignment,” and \( F_4 \) represents “pedestal looseness.” In this application, the threshold for making a decision is set to 0.7 based on [45]. A decision is difficult to make based solely on the BBAs \( m_1, m_2, \) and \( m_3 \). Specifically, \( m_1 \) has a value of 0.68, which indicates \( F_2 \): “unbalance”; \( m_2 \) has a value of 0.79, which indicates \( F_3 \): “misalignment”; and \( m_3 \) has a value of 0.58, which indicates \( F_2 \): “unbalance”. Since \( m_1(F_2) = 0.68 \) and \( m_3(F_2) = 0.58 \), which are less than the threshold 0.7, a decision cannot be made on the basis of \( m_1 \) and \( m_2 \), whereas according to \( m_3 \), the diagnosis result is \( F_3 \). As a result, conflict exists between \( m_1, m_2, \) and \( m_3 \), so an accurate decision is difficult to make under such circumstances. Thus, a conflict management method is necessary to improve the decision level.

Step 1: The correlation matrix \( M_{ECC} \) is constructed as

\[
M_{ECC} = \begin{bmatrix}
1.0000 & 0.0335 & 0.9516 \\
0.0335 & 1.0000 & 0.1517 \\
0.9516 & 0.1517 & 1.0000
\end{bmatrix}.
\]

Step 2: The support degree of \( m_j \) is calculated as

\[
SD(m_1) = 0.9851; SD(m_2) = 0.1852
\]

\[
SD(m_3) = 1.1033.
\]

Step 3: The credibility degree of \( m_j \) is calculated as

\[
CD(m_1) = 0.4333; CD(m_2) = 0.0815
\]

\[
CD(m_3) = 0.4853.
\]
The WAE is obtained as

\[
m(F_1) = 0.0373; m(F_2) = 0.5761
\]

\[
m(F_3) = 0.1507; m(F_4) = 0.0408
\]

\[
m(\Theta) = 0.1951.
\]

Step 5: The WAE is fused 2 times with the DCR:

\[
m(F_1) = 0.0102; m(F_2) = 0.8964
\]

\[
m(F_3) = 0.0674; m(F_4) = 0.0113
\]

\[
m(\Theta) = 0.0148.
\]

Step 6: The \( m(F_i) \) with the highest value is selected

\[
o = \arg \max_{1 \leq i \leq n} \{ m(F_i) \} = 2.
\]

Step 7: Since \( m(F_2) = 0.8964 \), which is greater than the threshold 0.7, the fault type is \( F_2 \).

### C. Discussion

To demonstrate the effectiveness of the proposed conflict management method, we compared the proposed method with related works, including Dempster’s [25], Murphy’s [39], Deng et al.’s [40], and Jiang’s [45] methods. The results generated by different conflict management methods are shown in Table VIII. Dempster’s and Murphy’s methods cannot determine the fault type because their \( m(F_2) \) values of 0.5230 and 0.6059, respectively, are smaller than the threshold of 0.7. On the other hand, the methods of Deng et al. and Jiang and the proposed method can diagnose the fault type of the motor rotor as “unbalance,” as they obtain \( m(F_2) \) values of 0.7730, 0.8063, and 0.8964, respectively. Moreover, the proposed method has the highest value of 0.8964 and can, thus, diagnose the fault type with a higher rate of identification.

### VIII. Conclusions

In this article, we explored a novel conflict measurement in decision making and its application in fault diagnosis. Here, a new evidential correlation coefficient, called ECC, was proposed for modeling belief functions in evidence theory to support decision making in an uncertain environment. The properties of the ECC were defined and analyzed, and the ECC was confirmed to have the properties of nonnegativity, nondegeneracy, symmetry, and boundedness. Furthermore, on the basis of the ECC, an evidential conflict coefficient was proposed to measure the conflict between two pieces of evidence. The evidential conflict coefficient was proved to have the desired properties for conflict measurement, including nonnegativity, symmetry, boundedness, extreme consistency, and insensitivity to refinement.

We provided several examples to compare our proposed ECC method with the well-known approaches to demonstrate the superiority of this novel conflict measurement. We also applied the ECC in a fault diagnosis application, and the results verified that our proposed conflict measurement is shown to more efficiently handle uncertainty compared with existing approaches.

In summary, our proposed conflict measurement provides a promising way to manage conflict from multiple pieces of evidence and improve the performance of decision making, illustrating a good potential alternative to the analysis of big data from multiple sources. In future work, we intend to further study the properties of ECC as well as its application in more complex environments.

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### TABLE VIII

| Methods          | \( F_1 \)   | \( F_2 \)   | \( F_3 \)   | \( F_4 \)   | \( \Theta \) | Fault type  |
|------------------|------------|------------|------------|------------|-------------|-------------|
| Dempster [25]    | 0.0205     | 0.5230     | 0.3933     | 0.0309     | 0.0324      | Cannot be determined |
| Murphy [39]      | 0.0112     | 0.6059     | 0.3508     | 0.0153     | 0.0168      | Cannot be determined |
| Deng et al. [40] | 0.0111     | 0.7730     | 0.1856     | 0.0139     | 0.0165      | unbalance   |
| Jiang [45]       | 0.0108     | 0.8063     | 0.1534     | 0.0134     | 0.0162      | unbalance   |
| Proposed method  | 0.0102     | 0.8964     | 0.0674     | 0.0113     | 0.0148      | unbalance   |
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L. Fei and Y. Deng, “Multi-criteria decision making in Pythagorean fuzzy environment,” Appl. Intell., vol. 50, no. 2, pp. 537–561, 2020.

Zehong Cao (Member, IEEE) received the B.S. degree from Northeastern University, Shenyang, China, in 2012 and the M.S. degree from The Chinese University of Hong Kong, Hong Kong, in 2013, both in electronic engineering and the Ph.D. degree in information technology from University of Technology Sydney (UTS), Sydney, NSW, Australia, in 2017.

He is currently a Lecturer (a.k.a., Assistant Professor) with the Discipline of Information and Communication Technology (ICT), University of Tasmania (UTAS), Hobart, TAS, Australia, and an Adjunct Fellow with the School of Computer Science, UTS. He has authored or co-authored more than 50 papers published in well-known conferences, such as AAMAS, Fuzzy, IJCNN, and top-tier journals, such as the IEEE TRANSACTIONS ON FUZZY SYSTEMS (IEEE T-FS), IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS, IEEE TRANSACTIONS ON CYBERNETICS, IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS: SYSTEMS, T-BME, and IoT. His research interests include brain–computer interface, computational intelligence, and machine learning. He is currently focusing on the capacity of the “Human-In-The-Loop” machine learning and applications.

Dr. Cao is the Leading Guest Editor of the IEEE T-FS, IEEE TRANSACTIONS ON INTELLIGENT SYSTEMS AND TECHNOLOGIES, and an Associate Editor for Neurocomputing and Scientific Data. He was the recipient of the UTS Centre for Artificial Intelligence Best Student Paper Award, the UTS Faculty of Engineering and IT Ph.D. Publication Award, and the UTS President Ph.D. Scholarship.

Alireza Jolfaei (Senior Member, IEEE) received the Ph.D. degree in applied cryptography from Griffith University, Mount Gravatt QLD, Australia, in 2016. He is a Lecturer (Assistant Professor in North America) and a Program Leader of Cyber Security at Macquarie University, Sydney, Australia. Before this appointment, he was an Assistant Professor with Federation University Australia and Temple University in Philadelphia, USA. He has authored more than 60 peer-reviewed articles on topics related to applied cryptography, computer networks security, and applied linguistics. His current research interests include cyber security, IoT security, human-in-the-loop CPS security, cryptography, A.I., and machine learning for cyber security.

Dr. Jolfaei was the recipient of multiple awards for Academic Excellence, University Contribution, and Inclusion and Diversity Support. He received the prestigious IEEE Australian council award for his research paper published in the IEEE TRANSACTIONS ON INFORMATION FORENSICS AND SECURITY. He received a recognition diploma with a cash award from the IEEE Industrial Electronics Society for his publication at the 2019 IEEE IES International Conference on Industrial Technology. He is a founding member of the Federation University IEEE Student Branch. He was the Chairman of the Computational Intelligence Society in the IEEE Victoria Section and also as the Chairman of Professional and Career Activities for the IEEE Queensland Section. He was the Guest Associate Editor for the IEEE JOURNALS AND TRANSACTIONS, including the IEEE IOT JOURNAL, IEEE SENSORS JOURNAL, IEEE TRANSACTIONS ON INTELLIGENT SYSTEMS AND TECHNOLOGIES, IEEE TRANSACTIONS ON CYBERNETICS, IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS, and IEEE TRANSACTIONS ON EMERGING TOPICS IN COMPUTATIONAL INTELLIGENCE. He has served at more than 10 conferences in leadership capacities, including program Co-Chair, Track Chair, Session Chair, and Technical Program Committee Member, including IEEE TrustCom and IEEE INFOCOM. He is an ACM Distinguished Speaker on the topic of cyber–physical systems security.