Chiral perturbation theory
in the environment with chiral imbalance

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/based on different published and unpublished papers/
Outline of the talk

1. Introducing chiral imbalance

2. Effective meson theory in chiral imbalanced hadron medium

3. QCD inspired effective meson lagrangian (SU(2) case)

4. Extended chiral lagrangian in the chiral imbalance background

5. Status of couplings of Gasser-Leutwyler lagrangian of order $p^4$

6. Comparison of predictions for chiral imbalance corrections from the sigma model and chiral lagrangian in the leading order
The exact law in QCD, the partial conservation of axial current (broken by gluon anomaly)

$$\partial_\mu J_5^\mu - 2i m_q J_5 = \frac{N_f}{2\pi^2} \partial_\mu K^\mu$$

predicts the induced axial charge (for small quark masses $m_q \approx 0$)

$$\frac{d}{dt} (Q_5^q - 2N_f T_5) \approx 0, \quad Q_5^q = \int_{\text{vol.}} d^3x \bar{q} \gamma_0 \gamma_5 q = \langle N_L - N_R \rangle$$

to be conserved during $\tau_{\text{fireball}}$. 

**Chiral imbalance**
\[ < T_5 = \frac{1}{8\pi^2} \int_{\text{vol.}} d^3x \varepsilon_{jkl} \text{Tr} \left( G^j \partial^k G^l - i \frac{2}{3} G^j G^k G^l \right) > \]

it may survive for a sizeable lifetime in a heavy-ion fireball

\[ \langle \Delta T_5 \rangle \neq 0 \quad \text{for} \quad \Delta t \simeq \tau_{\text{fireball}} \simeq 5 \div 10 \text{ fm/c}; \]
Sizeable chiral imbalance

| Initial Stage | Hadronization: |
|---------------|----------------|
| After HI collision | Hadron vacuum with chiral imbalance inherited from quark vacuum |

< 1 fm/c  
1 fm/c < 7-10 fm/c

**Quark-hadron continuity** during hadronization through crossover (Fukushima et al)
Axial baryon charge and axial chemical potential

The characteristic left-right oscillation time is governed by inverse quark masses.

- For $u, d$ quarks $1/m_q \sim 1/5 \text{ MeV}^{-1} \sim 40 \text{ fm} \gg \tau_{\text{fireball}}$ and the left-right quark mixing can be neglected.

- For $s$ quark $1/m_s \sim 1/150 \text{ MeV}^{-1} \sim 1 \text{ fm} \ll \tau_{\text{fireball}}$ and $\langle Q_5^s \rangle \approx 0$ due to left-right oscillations.

For $u, d$ quarks QCD with a topological charge $\langle \Delta T_5 \rangle \neq 0$ can be equally described at the Lagrangian level by topological chemical potential $\mu_\theta$ or by axial chemical potential $\mu_5$

$$\langle \Delta T_5 \rangle \approx \frac{1}{2N_f} \langle Q_5^q \rangle \iff \mu_5 \approx \frac{1}{2N_f} \mu_\theta,$$

$$\Delta \mathcal{L}_{\text{top}} = \mu_\theta \Delta T_5 \iff \Delta \mathcal{L}_q = \mu_5 Q_5^q$$
CME disappears in central collisions but chiral imbalance NOT!
The characteristic left-right oscillation time is governed by inverse quark masses.

- For $u, d$ quarks $1/m_q \sim 1/5 \text{ MeV}^{-1} \sim 40 \text{ fm} \gg \tau_{\text{fireball}}$ and the left-right quark mixing can be neglected.

  \[ 1/m_s \sim 1/150 \text{ MeV}^{-1} \sim 1 \text{ fm} \ll \tau_{\text{fireball}} \text{ and } \langle Q_5^s \rangle \simeq 0 \]

  due to left-right oscillations.

For $u, d$ quarks QCD with a topological charge $\langle \Delta T_5 \rangle \neq 0$ can be equally described at the Lagrangian level by topological chemical potential $\mu_\theta$ or by axial chemical potential $\mu_5$

\[ \langle \Delta T_5 \rangle \simeq \frac{1}{2N_f} \langle Q_5^q \rangle \iff \mu_5 \simeq \frac{1}{2N_f} \mu_\theta, \]

\[ \Delta L_{\text{top}} = \mu_\theta \Delta T_5 \iff \Delta L_q = \mu_5 Q_5^q \]
Effective meson theory in a medium with LPB

- Scalar (and pseudoscalar) mesons
  The scalar sector can be estimated by using the spurion technique in the chiral Lagrangian
  \[
  D_{\nu} \rightarrow D_{\nu} - i\{\mu_5 \delta_{0\nu}, \cdot\}
  \]

- Vector mesons
  Low energy QCD can be described by Vector Meson Dominance. In this framework, the following term appears
  \[
  \Delta \mathcal{L} \sim \varepsilon^{\mu \nu \rho \sigma} \text{Tr} \left[ \hat{\xi}_\mu V_\nu V_{\rho \sigma} \right]
  \]
  with \( \hat{\xi}_\mu = \hat{\xi} \delta_\mu^0 \) for a spatially homogeneous and isotropic background (\( \hat{\cdot} \equiv \) isospin content) and \( \xi \propto \mu_5 \).

Two different cases of isospin structure for \( \mu_5 \):
- Isosinglet pseudoscalar background (\( T \gg \mu \)) [RHIC, LHC]
- Pion-like (isotriplet) background (not considered) (\( \mu \gg T \)) [FAIR, NICA]
QCD inspired effective meson lagrangian (SU(2) case)

\[ L = \frac{1}{4} Tr (D_\mu H (D^\mu H)^\dagger) + \frac{b}{2} Tr [m(H + H^\dagger)] + \frac{M^2}{2} Tr (H H^\dagger) \]

\[ - \frac{\lambda_1}{2} Tr [(H H^\dagger)^2] - \frac{\lambda_2}{4} [Tr (H H^\dagger)]^2 + \frac{c}{2} (\det H + \det H^\dagger) \]

\[ U = \xi \xi = \exp \left( i \frac{\bar{\pi} \pi}{f_\pi} \right) \]

\[ \bar{\pi} \pi = \begin{bmatrix} \pi^0 & \sqrt{2} \pi^+ \\ \sqrt{2} \pi^- & -\pi^0 \end{bmatrix} \]

\[ \Sigma = \begin{bmatrix} \nu + \sigma + a_0^0 & \sqrt{2} a^+ \\ \sqrt{2} a^-_0 & \nu + \sigma - a_0^0 \end{bmatrix} \]

\[ D_\mu H = \partial_\mu H - i \mathcal{L}_\mu H + i H \mathcal{R}_\mu \]

\[ \mathcal{R}_\mu = e Q_{em} A_\mu - \mu_5 \delta_{\mu,0} \cdot 1_{2 \times 2}; \quad \mathcal{L}_\mu = e Q_{em} A_\mu + \mu_5 \delta_{\mu,0} \cdot 1_{2 \times 2} \]

\[ Q_{em} = \frac{1}{2} \tau_3 + \frac{1}{6} 1_{2 \times 2}. \]

Current quark mass

Pions

Scalar mesons (isosingles + isotriplets)

Chiral chemical potential = time-component of axial field
Mass spectrum in vacuum

Take

\[ \mu_5 = 0, \ M = 300 \text{ MeV}, \ v = 92 \text{ MeV} \]

Obtain

\[ m_\pi = 139 \text{ MeV}, \ m_\rho = 980 \text{ MeV}, \ m_\sigma = 500 \text{ MeV}, \ m = 5.5 \text{ MeV}, \]

for

\[ \lambda_1 = 16.4850, \ \lambda_2 = -13.1313, \ c = -4.46874 \times 10^4 \text{ MeV}^2, \ b = 1.61594 \times 10^5 \text{ MeV}^2 \]
Mass spectrum

\[ \sigma \text{ meson,} \]

\[
\frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m^2_\sigma \sigma^2
\]

Neutral meson sector,

\[
\frac{1}{2} \partial_\mu a_0^0 \partial^\mu a_0^0 + \frac{1}{2} \partial_\mu \pi^0 \partial^\mu \pi^0 - \frac{1}{2} m^2_\pi (a_0^0)^2 - \frac{1}{2} m^2_\pi (\pi^0)^2 - 4 \mu_5 \pi^0 a_0^0
\]

Charged meson sector,

\[
\partial_\mu a_0^- \partial^\mu a_0^+ + \partial_\mu \pi^- \partial^\mu \pi^+ - m^2_\pi a_0^- a_0^+ - m^2_\pi \pi^- \pi^+ - 4 \mu_5 \pi^+ a_0^- - 4 \mu_5 \pi^- a_0^+
\]

\[
\begin{aligned}
    m^2_\sigma &= -2 \left( M^2 - 6 (\lambda_1 + \lambda_2) v^2 + c + 2 \mu^2_5 \right) \\
    m^2_a &= -2 \left( M^2 - 2 (3\lambda_1 + \lambda_2) v^2 - c + 2 \mu^2_5 \right) \\
    m^2_\pi &= \frac{2 b m}{v} \\
    v(\mu_5) &= \sqrt{\frac{M^2 + 2 \mu^2_5 + c}{2(\lambda_1 + \lambda_2)}} + \frac{b}{2(M^2 + 2 \mu^2_5 + c)} m
\end{aligned}
\]

\[
F^2_\pi(\mu_5) \approx \frac{M^2 + c}{2(\lambda_1 + \lambda_2)} + \frac{\mu^2_5}{(\lambda_1 + \lambda_2)}
\]
In the above model quark condensate is governed by the decay constant $\nu$.
Masses with chiral imbalance

\[ m_{\text{eff}}^- = \frac{1}{2} \left( 16 \mu_5^2 + m_a^2 + m_\pi^2 - \sqrt{(16 \mu_5^2 + m_a^2 + m_\pi^2)^2 - 4 \left( m_a^2 m_\pi^2 - 16 \mu_5^2 |k|^2 \right)} \right) \]

\[ m_{\text{eff}}^+ = \frac{1}{2} \left( 16 \mu_5^2 + m_a^2 + m_\pi^2 + \sqrt{(16 \mu_5^2 + m_a^2 + m_\pi^2)^2 - 4 \left( m_a^2 m_\pi^2 - 16 \mu_5^2 |k|^2 \right)} \right) \]

When \[ |k| > m_1 m_2 / (4 \mu_5) \equiv k_\pi \] \[ m_{\text{eff}}^- < 0 \] but no instability!

“tachyon” in flight
Mass spectrum in chiral imbalanced medium

\[ \pi^\pm \rightarrow \mu^\pm \bar{\nu} \]
Extended chiral lagrangian in the chiral imbalance background

\[ \mathcal{L}_2 = \frac{F^2}{4} \text{tr} \left[ d_\mu U^\dagger d_\mu U + \chi^\dagger U + \chi U^\dagger \right] + C \text{tr} \left[ Q_R U Q_L U^\dagger \right] \]

\[ d_\mu U = \partial_\mu U - i(v_\mu + Q_R A_\mu + a_\mu)U + iU(v_\mu + Q_L A_\mu - a_\mu) \]

External e.m.charges breaking isospin symmetry (R.Urech)

\[ \mathcal{L}_{p^+} = \frac{l_1}{4} \left< d_\mu U^+ d_\mu U \right>^2 + \frac{l_2}{4} \left< d_\mu U^+ d^\nu U \right> \left< d_\mu U^+ d_\nu U \right> \]

\[ + \frac{l_3}{16} \left< \chi^+ U + U^+ \chi \right>^2 + \frac{l_4}{4} \left< d_\mu U^+ d_\mu \chi + d^\mu \chi^+ d_\mu U \right> \]

\[ + l_5 \left< G_{\mu\nu} R U G_{\mu\nu} U^+ \right> + i \frac{l_6}{2} \left< G_{\mu\nu} R d_\mu U d^\nu U^+ + G_{\mu\nu} L d_\mu U^+ d^\nu U \right> \]

\[ - \frac{l_7}{16} \left< \chi^+ U - U^+ \chi \right>^2 + \frac{1}{4} (h_1 + h_3) \left< \chi^+ \chi \right> \]

\[ + \frac{1}{2} (h_1 - h_3) \text{Re} \left( \text{det} \chi \right) - h_2 \left< G_{\mu\nu} R G_{\mu\nu} R + G_{\mu\nu} L G_{\mu\nu} L \right>. \]
One loop renormalization of chiral lagrangian of G-L

\[ l_i = l_i^r + \gamma_i \lambda, \quad i = 1, \ldots, 7 \]
\[ h_i = h_i^r + \delta_i \lambda, \quad i = 1, 2, 3 \]

\[ \lambda = (4\pi)^{-2} \mu^{d-4} \left\{ \frac{1}{d-4} - \frac{1}{2} \left( \ln 4\pi + \Gamma'(1) + 1 \right) \right\} \]

\[ \gamma_1 = \frac{1}{3}, \quad \gamma_2 = \frac{2}{3}, \quad \gamma_3 = -\frac{1}{2}, \quad \gamma_4 = 2, \quad \gamma_5 = -\frac{1}{6}, \quad \gamma_6 = -\frac{1}{3}, \quad \gamma_7 = 0 \]
\[ \delta_1 = 2, \quad \delta_2 = \frac{1}{12}, \quad \delta_3 = 0. \]

\[ \bar{l}_i = \frac{32\pi^2}{\gamma_i} l_i^r(\mu) - \ln \frac{M^2}{\mu^2}. \]

For the subtraction scale \( \mu = 0.77 \text{ GeV}, \)

\[ -\ln \frac{M^2}{\mu^2} = 3.42 \]
Fits of constants

G. Colangelo, J. Gasser, H. Leutwyler

\[ \bar{\ell}_1 = -0.4 \pm 0.6, \quad \bar{\ell}_2 = 4.3 \pm 0.1, \quad \bar{\ell}_4 = 4.4 \pm 0.2. \]

Aoki S. et al FLAG working group [arXiv:1607.00299 [hep-lat]] give similar results for \( \bar{\ell}_4 \).

From resonance saturation

G.~Ecker, J.~Gasser, A.~Pich and E.~de Rafael, Nucl.\ Phys.\ B {\bf 321}, 311 (1989).

\[ \bar{\ell}_1 \simeq -0.7, \quad \bar{\ell}_2 \simeq 5.0, \quad \bar{\ell}_3 \simeq 1.9, \quad \bar{\ell}_4 \simeq 3.7, \]
Chiral imbalance contribution:
leading order in chiral chemical potential

\[ L_4 \rightarrow \frac{l_1}{4} \text{tr}^2 \left[ (\partial^\mu - 2i\mu_5\delta^\mu_0) U^\dagger (\partial_\mu + 2i\mu_5\delta_\mu_0) U \right] \]
\[ + \frac{l_2}{4} \text{tr} \left[ (\partial^\mu - 2i\mu_5\delta^\mu_0) U^\dagger (\partial^\nu + 2i\mu_5\delta^\nu_0) U \right] \text{tr} \left[ (\partial_\mu - 2i\mu_5\delta_\mu_0) U^\dagger (\partial_\nu + 2i\mu_5\delta_\nu_0) U \right] \]
\[ + \frac{l_3}{8} \text{tr} \left[ (\partial^\mu - 2i\mu_5\delta^\mu_0) U^\dagger (\partial_\mu + 2i\mu_5\delta_\mu_0) U \right] \text{tr} \left[ (\chi^\dagger U + \chi U^\dagger) \right] \]

\[ L_4 \rightarrow L_4 + \mu_5^2 \left[ 4l_1 \text{tr} (\partial_\mu U^\dagger \partial^\mu U) + 4l_2 \text{tr} (\partial_0 U^\dagger \partial^0 U) + l_3 \text{tr} (\chi^\dagger U + \chi U^\dagger) \right] + 4\mu_5^4 (l_1 + l_2) \]

Comparison of the ChPT and sigma model

In the rest frame of pion

\[ F_\pi^2(\mu_5) \approx F_0^2 + 4\mu_5^2 \left[ 4l_1 + 4l_2 \right] \]

whereas in the sigma model with chiral chemical potential

\[ F_\pi^2(\mu_5) \approx \frac{M^2 + c}{2(\lambda_1 + \lambda_2)} + \frac{\mu_5^2}{(\lambda_1 + \lambda_2)} \]

Numerically they have the same sign but differ in magnitude.
One loop renormalization of chiral lagrangian

\[ \mathcal{L}_2 = l_1 (\nabla^\mu U^T \nabla_\mu U)^2 + l_2 (\nabla^\mu U^T \nabla^\nu U)(\nabla_\mu U^T \nabla_\nu U) \\
+ l_3 (\chi^T U)^2 + l_4 (\nabla^\mu \chi^T \nabla_\mu U) + l_5 (U^T F^{\mu \nu} F_{\mu \nu} U) \\
+ l_6 (\nabla^\mu U^T F_{\mu \nu} \nabla^\nu U) + l_7 (\tilde{\chi}^T U)^2 + h_1 \chi^T \chi + h_2 \text{tr } F_{\mu \nu} F^{\mu \nu} \\
+ h_3 \tilde{\chi}^T \tilde{\chi} \]

\[ l_i = l_i^0 + \gamma_i \lambda, \quad i = 1, ..., 7 \]

\[ h_i = h_i^0 + \delta_i \lambda, \quad i = 1, 2, 3 \]

\[ \lambda = (4\pi)^{-2} \mu^{d-4} \left\{ \frac{1}{d-4} - \frac{1}{2} (\ln 4\pi + \Gamma'(1) + 1) \right\} \]

\[ \gamma_1 = \frac{1}{3}, \quad \gamma_2 = \frac{2}{3}, \quad \gamma_3 = -\frac{1}{2}, \quad \gamma_4 = 2, \quad \gamma_5 = -\frac{1}{6}, \quad \gamma_6 = -\frac{1}{3}, \quad \gamma_7 = 0 \]

\[ \delta_1 = 2, \quad \delta_2 = \frac{1}{12}, \quad \delta_3 = 0. \] (9.6)

\[ \bar{I}_i = \frac{32\pi^2}{\gamma_i} I_i^0 (\mu) - \ln \left( \frac{M^2}{\mu^2} \right) \]

For the subtraction scale \( \mu = 0.77 \text{ GeV} \),

\[ -\ln \left( \frac{M^2}{\mu^2} \right) = 3.42 \]
\[
\bar{\ell}_1 = -0.4 \pm 0.6, \quad \bar{\ell}_2 = 4.3 \pm 0.1, \quad \bar{\ell}_4 = 4.4 \pm 0.2.
\]

From resonance saturation
\[
\bar{\ell}_1 \simeq -0.7, \quad \bar{\ell}_2 \simeq 5.0, \quad \bar{\ell}_3 \simeq 1.9, \quad \bar{\ell}_4 \simeq 3.7,
\]
lattice results concerning low-energy particle physics

| Quantity   | $N_f = 2 + 1$ | $f_{K^\pm}/f_{\pi^\pm}$ | $f_{\pi^\pm}$ [MeV] | $f_{K^\pm}$ [MeV] | $\Sigma^{1/3}$ [MeV] | $F_{\pi}/F$ | $\bar{\ell}_3$ | $\bar{\ell}_4$ | $\bar{\ell}_6$ |
|------------|---------------|--------------------------|----------------------|-------------------|----------------------|--------------|---------------|---------------|---------------|
| $m_s$ [MeV]| 92.0(2.1)     | 1.192(5)                 | 130.2(1.4)           | 155.9(9)          | 274(3)               | 1.064(7)     | 2.81(64)      | 4.10(45)      | 15.1(1.2)     |
| $m_{ud}$ [MeV]| 3.373(80)     |                         |                      |                   |                      |              |               |               |               |
| $m_s/m_{ud}$| 27.43(31)     |                         |                      |                   |                      |              |               |               |               |
| $m_u$ [MeV] | 2.16(9)(7)    |                         |                      |                   |                      |              |               |               |               |
| $m_d$ [MeV] | 4.68(14)(7)   |                         |                      |                   |                      |              |               |               |               |
| $m_u/m_d$   | 0.46(2)(2)    |                         |                      |                   |                      |              |               |               |               |
Wess-Zumino-Witten action

Describing of anomalous decay of strong interaction $\pi \to \gamma \gamma$
and other interaction: $\gamma \pi^- \to \pi^0 \pi^-$ and $\gamma \to \pi \pi \pi$

\begin{align*}
- \frac{e^2 N_c}{24 \pi^2 f_\pi} e^{\nu \lambda \rho} \partial_\sigma A_{\lambda} \partial_{\nu} A_{\rho} \pi^0 & \quad (1) \\
- \frac{\text{i} e \mu_5 N_c}{6 \pi^2 f_\pi^2} e^{\sigma \lambda \rho} A_{\rho} \partial_\sigma \pi^+ \partial_\lambda \pi^- & \quad (2)
\end{align*}

M. Kawaguchi, M. Harada, S. Matsuzaki, R. Ouyang, PHYS. REV. C 95, 065204 (2017)
Our prediction: Scalar resonance enhancement!

Processes are parity conjugate:

\[
\pi^\pm(\bar{p}) + \gamma(\bar{q}) \rightarrow \pi^\pm(\bar{l}) + \gamma_+(\bar{k}), \\
\pi^\pm(-\bar{p}) + \gamma(-\bar{q}) \rightarrow \pi^\pm(-\bar{l}) + \gamma_-(\bar{k}),
\]

where ± attached on photons in the final state denote photon helicities.

Asymmetry \((A)\) can be evaluated as

\[
A = \left| \frac{N_+ - \mathcal{P}[N_+]}{\sum_\lambda (N_\lambda + \mathcal{P}[N_\lambda])} \right|
\]

where \(N_\lambda\) stands for the number of events per the phase space, \(dE_\gamma \, d\cos \theta \, d\phi\), for the parity conjugate processes with the helicity \(\lambda\) and the photon energy \(E_\gamma\) in the final state. The symbol \(\mathcal{P}\) acts as the parity conjugation projection. The denominator represents the total number of the \(\pi^\pm\gamma\) emission events with unpolarized photons per the phase space.

\[
A^{s\text{-channel}}_{\text{max}} = \frac{\mu_5 E_{\pi} N_c}{6\pi^2 f^2_{\pi}} \simeq 0.2 \times \left( \frac{\mu_5}{200 \text{ MeV}} \right) \left( \frac{E_{\pi}}{1 \text{ GeV}} \right)
\]
Asymmetry in photon polarizations

\[ \pi^+ \gamma \rightarrow a_0^{++} \rightarrow \pi^+ \gamma \]

Asymmetry, \( \mu_5 = 200 \text{ MeV} \), \( E_{\pi_2} = 1 \text{ GeV} \)
Conclusions and outlook

1. Topological charge fluctuations transmit their influence from QGP to hadron physics via chiral chemical potential: in this way local parity breaking (LPB) occurs in hadron sector

2. LPB enhances dynamical chiral symmetry breaking in QCD: chiral condensates are increasing with chiral chemical potential

3. The constants of Chiral Perturbation Theory may enhance predictivity of low-energy pion dynamics in the chiral imbalanced medium, but precision is not satisfactory. There are perspectives to include two-loop and e.m. corrections

4. LPB modifies dispersion laws for scalar and vector mesons: lightest “pseudoscalar” mesons tend to massless states in flight, vector meson polarizations split with different in-flight masses.
   There exist observables unambiguously indicating LPB (STAR, ALICE LHC?): suppression of charged pion decays into leptons, exotic scalar/pseudoscalar meson decays, asymmetry in pion polarizabilities, mass splitting of vector mesons and quarks/nucleons etc.
Effective meson theory in a medium with LPB

Vector mesons

Low energy QCD can be described with the help of Vector Meson Dominance

\[ \mathcal{L}_{\text{int}} = \bar{q} \gamma_{\mu} \hat{V}^\mu q; \quad \hat{V}_\mu \equiv -e A_\mu Q + \frac{1}{2} g_\omega \omega_\mu + \frac{1}{2} g_\rho \rho_\mu \tau_3, \]

\[ (V_{\mu,a}) \equiv (A_\mu, \omega_\mu, \rho_\mu^0) \]

where \( Q = \frac{\tau_3}{2} + \frac{1}{6}, \) \( g_\omega \approx g_\rho \approx g \approx 6. \)

In this framework, the following term is generated in the effective lagrangian for vector mesons

\[ \Delta \mathcal{L} \approx \varepsilon^{\mu\nu\rho\sigma} \text{Tr} \left[ \hat{\zeta}_\mu V_\nu V_{\rho\sigma} \right] \]

with \( \hat{\zeta}_\mu = \hat{\zeta} \delta_{\mu0} \) for a spatially homogeneous and isotropic background (\( ^\hat{\text{}} \equiv \text{isospin content} \)) and \( \zeta \approx \mu_5. \)
Vector Meson spectrum in PB medium

After diagonalization of mass matrix

\[ m_{V,\epsilon}^2 = m_V^2 - \epsilon \zeta |\vec{k}| \implies |\zeta|, \]

where \( \epsilon = 0, \pm 1 \) is the meson polarization.

The photon itself happens to be unaffected by a singlet \( \zeta \).

The position of the poles for \( \pm \) polarized mesons is changing with wave vector \( |\vec{k}| \).

Massive vector mesons split into three polarizations with masses

\[ m_{V,+}^2 < m_{V,L}^2 < m_{V,-}^2. \]

This splitting unambiguously signifies LPB. Can it be measured?

→ dilepton production in HIC from the decays \( \rho, \omega \rightarrow e^+e^- \)

More details in
A.A., V.A. Andrianov’s, D. Espriu and X. Planells, Phys. Lett. B 684 (2010) 101;
B 710 (2012) 230,...
Unfortunately the splitting is strongly contaminated by thermal effects.
Chiral magnetic effect

Topological Charge + Magnetic field = Chirality + Polarization =

Q < -1: Positively charged particles move parallel to magnetic field, negatively charged antiparallel

... = Electromagnetic Current

P- and CP-odd effect --> Chiral Magnetic Effect:

D.Kharzeev, L.McLerran, K.Fukushima, H.Warringa,...
Topological number fluctuations in QCD vacuum

ITEP Lattice Group

P. Buividovich, M. Chernodub, E. Luschevskaya, M. Polikarpov
Extended chiral lagrangian with virtual photon loops

\[ \mathcal{L} = \mathcal{L}^0_{\text{QCD}} + \mathcal{L}^0_\gamma + \bar{q} \gamma^\mu [v_\mu(x) + \gamma_5 a_\mu(x)] q - \bar{q} [s(x) - i \gamma_5 p(x)] q \\
+ A_\mu \bar{q} \gamma^\mu \left\{ Q_L(x) \left( \frac{1 - \gamma_5}{2} \right) + Q_R(x) \left( \frac{1 + \gamma_5}{2} \right) \right\} q. \]
\[ \mathcal{L}_{e^2 p^2} = F^2 \{ k_1 \langle d^\mu U^+ d_\mu U \rangle \langle Q^2 \rangle \\
+ k_2 \langle d^\mu U^+ d_\mu U \rangle \langle QUQU^+ \rangle \\
+ k_3 (\langle d^\mu U^+ QU \rangle \langle d_\mu U^+ QU \rangle + \langle d^\mu UQU^+ \rangle \langle d_\mu UQU^+ \rangle) \\
+ k_4 \langle d^\mu U^+ QU \rangle \langle d_\mu UQU^+ \rangle \\
+ k_5 \langle \chi^+ U + U^+ \chi \rangle \langle Q^2 \rangle \\
+ k_6 \langle \chi^+ U + U^+ \chi \rangle \langle QUQU^+ \rangle \\
+ k_7 \langle (\chi U^+ + U \chi^+) Q + (\chi^+ U + U^+ \chi) Q \rangle \langle Q \rangle \\
+ k_8 \langle (\chi U^+ - U \chi^+) QUQU^+ + (\chi^+ U - U^+ \chi) QU^+ QU \rangle \\
+ k_9 \langle d_\mu U^+ [(c_R^\mu Q), Q] U + d_\mu U [(c_L^\mu Q), Q] U^+ \rangle \\
+ k_{10} \langle (c_R^\mu Q) U (c_{L\mu} Q) U^+ \rangle \\
+ k_{11} \langle (c_R^\mu Q) \cdot (c_R Q) + (c_L Q) \cdot (c_L Q) \rangle \}, \]
| Type:          | $V$   | $A$   | $S$  | $S_1$ | Total    |
|---------------|-------|-------|------|-------|----------|
| $Z^R(M_\rho)$ | $-0.88$ | $1.79$ | $0$  | $0$   | $0.91(\ast)$ |
| $K_1^R(M_\rho)$ | $-4.6$ | $-2.2$ | $0.4$ | $0$   | $-6.4$   |
| $K_2^R(M_\rho)$ | $-4.9$ | $2.2$  | $-0.4$ | $0$   | $-3.1$   |
| $K_3^R(M_\rho)$ | $4.6$  | $2.2$  | $-0.1$ | $-0.2$ | $6.4$    |
| $K_4^R(M_\rho)$ | $-9.9$ | $4.4$  | $-0.2$ | $-0.5$ | $-6.2$   |
| $K_5^R(M_\rho)$ | $13.7$ | $6.6$  | $-0.4$ | $0$   | $19.9$   |
| $K_6^R(M_\rho)$ | $14.8$ | $-6.6$ | $0.4$ | $0$   | $8.6$    |
| $K_7^R \ldots K_{10}^R$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $K_{11}^R(M_\rho)$ | $2.8$ | $-2.2$ | $0$ | $0$ | $0.6$ |
| $K_{12}^R(M_\rho)$ | $-4.7$ | $-4.4$ | $0$ | $0$ | $-9.2$ |
| $K_{13}^R$ | $19.0(\ast)$ | $-4.7(\ast)$ | $0$ | $0$ | $14.2(\ast)$ |
| $K_{14}^R$ | $0$ | $2.4(\ast)$ | $0$ | $0$ | $2.4(\ast)$ |

Units: $\times 10^{-3}$