Pseudo Euclidean-Signature Harmonic Oscillator, Quantum Field Theory and Vanishing Cosmological Constant

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ABSTRACT

The harmonic oscillator in pseudo euclidean space is studied. A straightforward procedure reveals that although such a system may have negative energy, it is stable. In the quantized theory the vacuum state has to be suitably defined and then the zero-point energy corresponding to a positive-signature component is canceled by the one corresponding to a negative-signature component. This principle is then applied to a system of scalar fields. The metric in the space of fields is assumed to have signature (++++−−−−−−) and it is shown that the vacuum energy, and consequently the cosmological constant, are then exactly zero. The theory also predicts the existence of stable, negative energy field excitations (the so called ”exotic matter”) which are sources of repulsive gravitational fields, necessary for construction of the time machines and Alcubierre’s hyperfast warp drive.

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1. Introduction

Quantum field theory is a very successful theory and yet it is not free of some unresolved problems. One of the major obstacles to future progress in our understanding of the relation between quantum field theory and gravity is the problem of the cosmological constant \[1\). Since a quantum field is an infinite set of harmonic oscillators, each having a non vanishing zero-point energy, a field as a whole has infinite vacuum energy density. The latter can be considered, when neglecting the gravitational field, just as an additive constant with no influence on dynamics. However, the situation changes dramatically if one takes into account the gravitational field which feels the absolute energy density. As a consequence the infinite (or more realistically, given by Planck scale cutoff) cosmological constant is predicted which is in drastic contradiction with its observed small value. No generally accepted solution to this problem has been found so far.

In the present paper I study the system of uncoupled harmonic oscillators in a space with arbitrary metric signature. One interesting consequence of such a model is vanishing zero-point energy of the system when the number of positive and negative signature coordinates is the same and so there is no cosmological constant problem. As an example I first solve a system of two oscillators in the space with signature \((+−)\) by using a straightforward, though surprisingly unexploited approach, based on the fact that energy is here just a quadratic form consisting of positive and negative terms. Both positive and negative energy component are on the same footing, and the difference in sign does not show up, until we consider gravitation. In the quantized theory the vacuum state can be defined straightforwardly and the zero-point energies cancel out. If the action is written in a covariant notation, one has to be careful of how to define the vacuum state. Usually it is required that energy be positive, and then, in the absence of a cutoff, the formalism contains an infinite vacuum energy and negative norm states. In the proposed formalism energy is not necessarily positive, negative energy states are also stable and there is no negative norm states.

The formalism is then applied to the case of scalar fields. The extension of the spinor-Maxwell field is also discussed. Vacuum energy density is zero and the cosmological constant vanishes. However, the theory contains the negative energy fields which couple to the gravitational field in a different way than the usual, positive energy, fields: the sign of coupling is reversed. This is the prize to be paid, if one wants to get small cosmological
constant in a straightforward way. One can consider this as a prediction of the theory to be tested by suitably designed experiments. Usually, however, the argument is just the opposite to the one proposed in this paper and classical matter is required to satisfy certain (essentially positive) energy conditions [2] which can only be violated by quantum field theory.

2. The 2-dimensional pseudo euclidean harmonic oscillator

Instead of the usual harmonic oscillator in 2-dimensional space let us consider the one given by the Lagrangian

\[ L = \frac{1}{2}(\dot{x}^2 - \dot{y}^2) - \frac{\omega^2}{2}(x^2 - y^2) \] (1)

The corresponding equations of motion are

\[ \ddot{x} + \omega^2 x = 0, \quad \ddot{y} + \omega^2 y = 0 \] (2)

Note that in spite of the minus sign in front of the \( y \)-terms in the Lagrangian (1), \( x \) and \( y \) components satisfy the same type equations of motion.

The canonical momenta conjugate to \( x, y \) are

\[ p_x = \frac{\partial L}{\partial \dot{x}} = \dot{x}, \quad p_y = \frac{\partial L}{\partial \dot{y}} = -\dot{y} \] (3)

The Hamiltonian is

\[ H = p_x \dot{x} + p_y \dot{y} - L = \frac{1}{2}(p_x^2 - p_y^2) + \frac{\omega^2}{2}(x^2 - y^2) \] (4)

We see immediately that the energy so defined may have positive or negative values, depending on initial conditions. Even if the system happens to have negative energy, it is stable, since the particle moves in a closed curve around the point (0,0). Motion of the harmonic oscillator based on the Lagrangian (1) does not differ from the one of the usual harmonic oscillator. The difference occurs when one considers the gravitational fields around the two systems.

The Hamiltonian equations of motion are

\[ \dot{x} = \frac{\partial H}{\partial p_x} = \{x, H\} = p_x, \quad \dot{y} = \frac{\partial H}{\partial p_y} = \{y, H\} = -p_y \] (5)
\[ \dot{p}_x = -\frac{\partial H}{\partial x} = \{p_x, H\} = -\omega^2 x, \quad \dot{p}_y = -\frac{\partial H}{\partial y} = \{p_y, H\} = \omega^2 y \]  

(6)

where the basic Poisson brackets are \( \{x, p_x\} = 1 \) and \( \{y, p_y\} = 1 \).

Quantizing our system we have

\[
[x, p_x] = i, \quad [y, p_y] = i
\]

(7)

Introducing the non hermitian operators according to

\[
c_x = \frac{1}{\sqrt{2}} \left( \sqrt{\omega} + \frac{i}{\sqrt{\omega}} p_x \right), \quad c_\dagger_x = \frac{1}{\sqrt{2}} \left( \sqrt{\omega} - \frac{i}{\sqrt{\omega}} p_x \right)
\]

(8)

\[
c_y = \frac{1}{\sqrt{2}} \left( \sqrt{\omega} + \frac{i}{\sqrt{\omega}} p_y \right), \quad c_\dagger_y = \frac{1}{\sqrt{2}} \left( \sqrt{\omega} - \frac{i}{\sqrt{\omega}} p_y \right)
\]

(9)

we have

\[
H = \frac{\omega}{2} (c_\dagger_x c_x + c_x c_\dagger_x - c_\dagger_y c_y - c_y c_\dagger_y)
\]

(10)

From the commutation relations (7) we obtain

\[
[c_x, c_\dagger_x] = 1, \quad [c_y, c_\dagger_y] = 1
\]

(11)

and the normal ordered Hamiltonian then becomes

\[
H = \omega (c_\dagger_x c_x - c_\dagger_y c_y)
\]

(12)

The vacuum state is defined as

\[
c_x |0\rangle = 0, \quad c_y |0\rangle = 0
\]

(13)

The eigenvalues of \( H \) are

\[
E = \omega (n_x - n_y)
\]

(14)

where \( n_x \) and \( n_y \) are eigenvalues of the operators \( c_\dagger_x c_x \) and \( c_\dagger_y c_y \), respectively.

The zero-point energies belonging to the \( x \) and \( y \) components cancel out! Our 2-dimensional pseudo harmonic oscillator has vanishing zero-point energy!. This is a result we obtain when applying the standard Hamilton procedure to the Lagrangian (1).

In the \((x, y)\) representation the vacuum state \( \langle x, y|0\rangle \equiv \psi_0(x, y) \) satisfies

\[
\frac{1}{\sqrt{2}} \left( \sqrt{\omega} x + \frac{1}{\sqrt{\omega}} \frac{\partial}{\partial x} \psi_0(x, y) \right) = 0, \quad \frac{1}{\sqrt{2}} \left( \sqrt{\omega} y + \frac{1}{\sqrt{\omega}} \frac{\partial}{\partial y} \psi_0(x, y) \right) = 0
\]

(15)
which comes straightforwardly from (13). A solution which is in agreement with the probability interpretation,
\[
\psi_0 = \frac{2\pi}{\omega} \exp\left[-\frac{\omega}{2}(x^2 + y^2)\right]
\]
is normalized according to \( \int \psi_0^2 \, dx \, dy = 1. \)

We see that our particle is localized around the origin. The excited states obtained by applying \( c_x^\dagger \), \( c_y^\dagger \) on the vacuum state are also localized. This is in agreement with the fact that also according to the classical equations of motion (2), the particle is localized in the vicinity of the origin. All states \( |\psi\rangle \) have positive norm. For instance, \( \langle 0|cc^\dagger|0\rangle = \langle 0|c,c^\dagger|0\rangle = \langle 0|0\rangle = \int \psi_0^2 \, dx \, dy = 1. \)

3. Harmonic oscillator in \( d \)-dimensional pseudo euclidean space

Extending (1) to arbitrary dimension it is convenient to use the compact (covariant) index notation
\[
L = \frac{1}{2} \dot{x}_\mu \dot{x}_\mu - \frac{\omega^2}{2} x_\mu x_\mu \tag{17}
\]
where for arbitrary vector \( A_\mu \) the quadratic form is \( A_\mu A_\mu \equiv \eta_{\mu\nu} A_\mu A_\nu \). The metric tensor \( \eta_{\mu\nu} \) has signature \(+ + + \cdots - - - \). The Hamiltonian is
\[
H = \frac{1}{2} p_\mu p_\mu + \frac{\omega^2}{2} x_\mu x_\mu \tag{18}
\]
Conventionally one introduces
\[
a_\mu = \frac{1}{\sqrt{2}} \left( \sqrt{\omega} x_\mu + \frac{i}{\sqrt{\omega}} p_\mu \right), \quad a_\mu^\dagger = \frac{1}{\sqrt{2}} \left( \sqrt{\omega} x_\mu - \frac{i}{\sqrt{\omega}} p_\mu \right) \tag{19}
\]
In terms of \( a_\mu \), \( a_\mu^\dagger \) the Hamiltonian reads
\[
H = \frac{i\omega}{2} (a_\mu^\dagger a_\mu + a_\mu a_\mu^\dagger) \tag{20}
\]
Upon quantization we have
\[
[x_\mu, p_\nu] = i\delta^\mu_\nu \quad \text{or} \quad [x_\mu, p_\nu^\nu] = i\eta^{\mu\nu} \tag{21}
\]
and
\[
[a_\mu, a_\nu^\dagger] = \delta^\mu_\nu \quad \text{or} \quad [a_\mu, a_\nu^\dagger] = \eta^{\mu\nu} \tag{22}
\]
We shall now discuss two possible definitions of vacuum state. The first possibility is the one that is usually assumed, while the second possibility is the one I am proposing in this paper.

**Possibility I.** Vacuum state can be defined according to

\[ a^\mu |0\rangle = 0 \]  

and the Hamiltonian, normal ordered with respect to the vacuum definition (23), after using (22) becomes

\[ H = \omega \left( a^{\mu\dagger} a_\mu + \frac{d}{2} \right), \quad d = \eta^{\mu\nu} \eta_{\mu\nu} \]  

Its eigenvalues are all positive and there is the non vanishing zero-point energy \( \omega d/2 \). In the \( x \) representation the vacuum state is

\[ \psi_0 = \left( \frac{2\pi}{\omega} \right)^{d/2} \exp\left[ -\frac{\omega}{2} x^\mu x_\mu \right] \]  

It is a solution of the Schrödinger equation

\[ -\left( \frac{1}{2} \right) \partial_\mu \partial^\mu \psi_0 + \left( \frac{\omega^2}{2} \right) x^\mu x_\mu \psi_0 = E_0 \psi_0 \]  

with positive \( E_0 = \omega \left( \frac{1}{2} + \frac{1}{2} + \ldots \right) \). The state \( \psi_0 \) as well as excited states cannot be normalized to 1. Actually, there exist negative norm states. For instance, if \( \eta_{33} = -1 \), then

\[ \langle 0 | a^3 a^{3\dagger} | 0 \rangle = \langle 0 | a^3 | 0 \rangle = -\langle 0 | 0 \rangle. \]

**Possibility II.** Let us split \( a^\mu = (a^\alpha, a^{\bar{\alpha}}) \), where indices \( \alpha, \bar{\alpha} \) refer to the components with positive and negative signature, respectively, and define vacuum according to

\[ a^\alpha |0\rangle = 0 \quad a^{\bar{\alpha}\dagger} |0\rangle = 0 \]  

Using (22) we obtain the normal ordered Hamiltonian with respect to the vacuum definition (26)

\[ H = \omega \left( a^{\alpha\dagger} a_\alpha + \frac{r}{2} + a_\alpha a^{\bar{\alpha}\dagger} - \frac{s}{2} \right) \]  

where \( \delta^\alpha_\alpha = r \) and \( \delta^{\bar{\alpha}}_{\bar{\alpha}} = s \). If the number of positive and negative signature components is the same, i.e., \( r = s \), then the Hamiltonian (27) has vanishing zero-point energy:

\[ H = \omega (a^{\alpha\dagger} a_\alpha + a_{\bar{\alpha}} a^{\bar{\alpha}\dagger}) \]  

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2 Equivalently, one can define annihilation and creation operators in terms of \( x^\mu \) and the canonically conjugate momentum \( p_\mu = \eta_{\mu\nu} p^\nu \) according to \( c^\mu = (1/\sqrt{2}) (\sqrt{\omega} x^\mu + (i/\sqrt{\omega}) p_\mu) \) and \( c^{\mu\dagger} = (1/\sqrt{2}) (\sqrt{\omega} x^\mu - (i/\sqrt{\omega}) p_\mu) \), satisfying \( [c^\mu, c^{\nu\dagger}] = \delta^\mu_\nu \). Vacuum is then defined as \( c^\mu |0\rangle = 0 \). This is just the higher dimensional generalization of \( c_x, c_y \) (eq.(5),(6) and the vacuum definition (13).
Its eigenvalues are positive or negative, depending on which component (positive or negative signature) are excited. In $x$-representation the vacuum state is

$$\psi_0 = \left(\frac{2\pi}{\omega}\right)^{d/2} \exp\left[-\frac{\omega}{2} \delta_{\mu\nu} x^\mu x^\nu\right] \quad (29)$$

where the Kronecker symbol $\delta_{\mu\nu}$ has values $+1$ or $0$. It is a solution of the Schrödinger equation

$$-(1/2) \partial^\mu \partial_\mu \psi_0 + (\omega^2/2) x^\mu x_\mu \psi_0 = E_0 \psi_0$$

with $E_0 = \omega(\frac{1}{2} + \frac{1}{2} + \ldots - \frac{1}{2} - \frac{1}{2} - \ldots)$. One can also easily verify that there is no negative norm states.

Comparing Possibility I with Possibility II we observe that the former has positive energy vacuum invariant under pseudo euclidean rotations, while the latter has the vacuum invariant under euclidean rotations and having vanishing energy (when $r = s$). In other words, we have either (i) non vanishing energy and pseudo euclidean invariance or (ii) vanishing energy and euclidean invariance of the vacuum state. In the case (ii) the vacuum state $\psi_0$ changes under the pseudo euclidean rotations, but its energy remains zero.

The invariance group of our Hamiltonian and the corresponding Schrödinger equation consists of pseudo-rotations. Though a solution to the Schrödinger equation changes under a pseudo-rotation, the theory is covariant under the pseudo-rotations in the sense that the set of all possible solutions does not change under the pseudo-rotations. Namely, the solution $\psi_0(x')$ of the Schrödinger equation

$$-(1/2) \partial^\mu \partial_\mu \psi_0(x') + (\omega^2/2) x'^\mu x'_\mu \psi_0(x') = 0$$

in a pseudo-rotated frame $S'$ is $\psi_0(x') = (2\pi/\omega)^{d/2} \exp[-(\omega/2) \delta_{\mu\nu} x'^\mu x'^\nu]$. If observed from the frame $S$ the latter solution reads $\psi'_0(x) = (2\pi/\omega)^{d/2} \exp[-(\omega/2) \delta_{\mu\nu} L^\mu_{\rho} L^\nu_{\sigma}]$, where $x'^\mu = L^\mu_{\rho} x^\rho$. One finds that $\psi'_0(x)$ as well as $\psi_0(x)$ (eq.(29) are solutions of the Schrödinger equation in $S$ and they both have the same vanishing energy. In general, in a given reference frame we have thus a degeneracy of solutions with the same energy. This is so also in the case of excited states.

In principle it seem more naturally to adopt Possibility II, because classically energy of our harmonic oscillator is nothing but a quadratic form $E = (1/2)(p^\mu p_\mu + \omega^2 x^\mu x_\mu)$ which in the case of pseudo euclidean-signature metric can be positive, negative or zero.

4. A system of scalar fields
Suppose we have a system of two scalar fields described by the action

\[ I = \frac{1}{2} \int d^4x \left( \partial_\mu \phi_1 \partial^\mu \phi_1 - m_1^2 - \partial_\mu \phi_2 \partial^\mu \phi_2 + m^2 \phi_2^2 \right) \]  

(30)

This action differs from the usual action for a charged field in the sign of the \( \phi_2 \) term. It is a field generalization of our action for the point-particle harmonic oscillator in 2-dimensional pseudo euclidean space.

The canonical momenta are

\[ \pi_1 = \dot{\phi}_1 , \quad \pi_2 = -\dot{\phi}_2 \]  

(31)

satisfying

\[ [\phi_1(x), \pi_1(x')] = i\delta^3(x - x') , \quad [\phi_2(x), \pi_2(x')] = i\delta^3(x - x') \]  

(32)

The Hamiltonian is

\[ H = \frac{1}{2} \int d^3x \left( \pi_1^2 + m^2 \phi_2^2 - \partial_i \phi_1 \partial^i \phi_1 - \pi_2^2 - m^2 \phi_2^2 + \partial_i \phi_2 \partial^i \phi_2 \right) \]  

(33)

We use the spacetime metric with signature (+ - - -) so that

\[ -\partial_i \phi_1 \partial^i \phi_1 = (\nabla \phi)^2, \quad i = 1, 2, 3. \]

Using the expansion \( (\omega_k = (m^2 + k^2)^{1/2}) \)

\[ \phi_1 = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left( c_1(k)e^{-ikx} + c_1^\dagger(k)e^{ikx} \right) \]  

(34)

\[ \phi_2 = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} \left( c_2(k)e^{-ikx} + c_2^\dagger(k)e^{ikx} \right) \]  

(35)

we obtain

\[ H = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{\omega_k}{2\omega_k} \left( c_1^\dagger(k)c_1(k) + c_1(k)c_1^\dagger(k) - c_2^\dagger(k)c_2(k) - c_2(k)c_2^\dagger(k) \right) \]  

(36)

The commutation relations are

\[ [c_1(k), c_1^\dagger(k')] = (2\pi)^3 2\omega_k \delta^3(k - k') \]  

(37)

\[ [c_2(k), c_2^\dagger(k')] = (2\pi)^3 2\omega_k \delta^3(k - k') \]  

(38)

The Hamiltonian can be written in the form

\[ H = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{\omega_k}{2\omega_k} \left( c_1^\dagger(k)c_1(k) - c_2^\dagger(k)c_2(k) \right) \]  

(39)

\[ ^3\text{Here, for the sake of demonstration, I am using the formalism of the conventional field theory, though in my opinion a better formalism involves an invariant evolution parameter.} \]
If we define vacuum according to
\[ c_1(k)|0\rangle = 0, \quad c_2(k)|0\rangle = 0 \] (40)
then the Hamiltonian (39) contains the creation operators on the left and has no zero-point energy. However, it is not positive definite: it may have positive or negative eigenvalues. But, as it is obvious from our analysis of the harmonic oscillator (1), negative energy states in our formalism are not automatically unstable; they can be as stable as positive energy states.

Extension of the action (30) to arbitrary number of fields \( \phi^a \) is straightforward. Let us now include also the gravitational field \( g_{\mu\nu} \). The action is then
\[
I = \frac{1}{2} \int d^4x \sqrt{-g} (g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b \gamma_{ab} - m^2 \phi^a \phi^b \gamma_{ab} + \frac{1}{16\pi G} R) \] (41)
where \( \gamma_{ab} \) is the metric tensor in the space of \( \phi^a \). Variation of (41) with respect to \( g_{\mu\nu} \) gives the Einstein equations
\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu} \] (42)
where the stress-energy tensor is
\[
T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} = \left[ \partial_\mu \phi^a \partial_\nu \phi^b - \frac{1}{2} g_{\mu\nu} \left( g^{\rho\sigma} \partial_\rho \phi^a \partial_\sigma \phi^b - m^2 \phi^a \phi^b \right) \right] \gamma_{ab} \] (43)
If \( \gamma_{ab} \) has signature \((+++\ldots−−−)\) with the same number of plus and minus signs, then the vacuum contribution to \( T_{\mu\nu} \) cancel out, so that the expectation value \( \langle T_{\mu\nu} \rangle \) remains finite. In particular we have
\[
T_{00} = \frac{1}{2} (\ddot{\phi}^a \phi^b - \partial_i \phi^a \partial^i \phi^b + m^2 \phi^a \phi^b) \gamma_{ab} \] (44)
which is just the Hamiltonian \( H \) of eq.(33) generalized to an arbitrary number of fields \( \phi^a \).

An analogous procedure as before could be done for other types of fields such as charged scalar, spinor and gauge fields. The notorious cosmological constant problem does not arise in our model, since vacuum expectation value \( \langle 0 | T_{\mu\nu} | 0 \rangle = 0 \). We could reason the other way around: since experiments clearly show that the cosmological constant is small, this indicates (especially in the absence of any other acceptable explanation) that to every field there corresponds a companion field with opposite signature of the metric eigenvalue.
in the space of fields. The companion field need not be excited - and thus observed - at all. Its mere existence is sufficient to be manifested in the vacuum energy.

However, there is the prize to be paid. If negative signature fields are excited, then \( \langle T_{00} \rangle \) can be negative which implies repulsive gravitational field around such a source. Such a prediction of the theory could be considered as an annoyance on the one hand, or a virtue on the other hand. If the latter point of view is taken, then we have here the so called exotic matter with negative energy density which is necessary for construction of the stable wormholes with the time machine properties [4]. Also the Alcubierre warp drive [5] which enables faster than light motion with respect to a distant observer requires negative energy density matter.

As an example let me show how the above procedure works for spinor and gauge fields. Neglecting gravitation the action is

\[
I = \int d^4x \left[ i \bar{\psi}^a \gamma^\mu (\partial_\mu \psi^b + ie A_\mu^b \psi^c) - m \bar{\psi}^a \psi^b + \frac{1}{16\pi} F_{\mu\nu}^{ac} F^{\mu\nu}_{bc} \right] \gamma_{ab} \tag{45}
\]

where \( F_{\mu\nu}^{ab} = \partial_\mu A_\nu^{ab} - \partial_\nu A_\mu^{ab} - (A_\mu^{ac} A_\nu^{db} - (A_\nu^{ac} A_\mu^{db}) \gamma_{cd} \right). This action is invariant under local rotations \( \psi'^a = U^{ab} \phi^b \), \( \bar{\psi}'^a = U^{ab} \bar{\psi}^b \), \( A'^\mu_a b = U^{ac} A_\mu^d U^a_c + i U^{ac} \gamma_{bc} \). This action is invariant under local rotations \( \psi'^a = U^{ab} \phi^b \), \( \bar{\psi}'^a = U^{ab} \bar{\psi}^b \), \( A'^\mu_a b = U^{ac} A_\mu^d U^a_c + i U^{ac} \gamma_{bc} \). These are generalization of the usual \( SU(N) \) transformations to the case of the metric \( \gamma_{ab} = \text{diag}(1, 1, ..., -1, -1) \).

In the case when there are two spinor fields \( \psi_1, \psi_2 \) and \( \gamma_{ab} = \text{diag}(1, -1) \) the equations of motion derived from (45) admit a solution \( \psi_2 = 0, A^{12} = A^{21} = A^{22} = 0 \). In the quantum field theory such a solution can be interpreted that when the fermions of the type \( \psi_2 \) (the companion or negative signature fields) are not excited (not present) also the gauge fields \( A^{12}, A^{21}, A^{22} \) are not excited (not present). What remains are just the ordinary \( \psi_1 \equiv \psi \) fermion quanta and \( U(1) \) gauge field \( A^{a11} \equiv A^a \) quanta. The usual spinor-Maxwell electrodynamics is just a special solution to the more general system given by (45).

Although having vanishing vacuum energy such a model is consistent with the well known experimentally observed effects which are manifestations of vacuum energy. Namely, the companion particles \( \psi_2 \) are expected to be present in the earth material in small amounts at most, because otherwise the gravitational field around the Earth would be repulsive. So, when considering vacuum effects, there remain only (or predominantly) the interactions between the fermions \( \psi_1 \) and the virtual photons \( A^{a11} \). For instance, in
the case of the Casimir effect the fermions $\psi_1$ in the two conducting plates interact with the virtual photons $A_{\mu_11}$ in the vacuum and hence impose the boundary conditions on the vacuum modes of $A_{\mu_11}$ in the presence of the plates. As a result have a net force between the plates, just like in the usual theory.

When gravitation is not taken into account, the fields within the doublet $(\psi_1, \psi_2)$ as described by the action are not easily distinguishable, since they have the same mass, charge and spin. They mutually interact only through the mixed coupling terms in the action and unless the effects of this mixed coupling are specifically measured, the two fields can be misidentified as a single field. Its double character could manifest itself straightforwardly in the presence of gravitational field to which the members of a doublet couple with the opposite sign. In order to detect such doublets (or perhaps multiplets) of fields, one has to perform suitable experiments. Description of such experiments is beyond the scope of this paper which only aims to bring attention to such a possibility. Here I only mention that difficulties and discrepancies in measuring precise value of the gravitational constant might have roots in negative energy matter. The latter would affect the measured value of the effective gravitational constant, but would leave the equivalence principle untouched.

5. Conclusion

The problem of the cosmological constant is one of the toughest problems in theoretical physics. Its resolution would open the door to further understanding of the relation between quantum theory and general relativity. Since all more conventional approaches seems to have been more or less exploited without unambiguous success, the time is right for a more drastic novel approach. Such is the one which relies on the properties of the harmonic oscillator in a pseudo euclidean space. This can be applied to the field theory where the fields behave as components of a harmonic oscillator. If the space of fields has the metric with signature then the vacuum energy can be zero in the case when the number of plus and minus signs is the same. As a consequence, expectation value of the stress-energy tensor, the source of gravitational field, is finite, and there is no cosmological constant problem. However, the stress-energy tensor can be negative in certain circumstances and the matter then acquires exotic properties which are desirable
for certain very important theoretical constructions, like the time-machines [4] or faster-than-light warp drive [5]. Negative energy matter, with repulsive gravitational field, is considered here as a prediction of the theory. On the contrary, in a more conventional approach just the opposite point of view is taken. It is argued that, since for all known forms of matter gravitation is attractive, certain energy conditions (weak, strong and dominant) must be satisfied [2]. But my point of view, advocated in this paper, is that existence of negative energy matter is necessary in order to keep cosmological constant small (or zero). Besides that, such exotic matter, if indeed present in the Universe, should manifest itself in various gravitation related phenomena. Actually, we cannot claim to posses a complete knowledge and understanding of all those phenomena, especially when some of them are still waiting for a generally accepted explanation.

The theory of the pseudo euclidean-signature harmonic oscillator is possibly important also for strings. Since it eliminates the zero-point energy, it presumably eliminates also the need for the critical dimension. We may thus expect to obtain a consistent string theory in an arbitrary even dimensional spacetime with suitable signature. Several exciting new possibilities of research are thus opened.

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