Interband impact ionization and nonlinear absorption of terahertz radiations in semiconductor heterostructures*

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We have theoretically investigated nonlinear free-carrier absorption of terahertz radiation in InAs/AlSb heterojunctions. By considering multiple photon process and conduction-valence interband impact ionization (II), we have determined the field and frequency dependent absorption rate. It is shown that (i) electron-disorder scatterings are important at low to intermediate field, and (ii) most importantly, the high field absorption is dominated by II processes. Our theory can satisfactorily explain a long standing experimental result on the nonlinear absorption in THz regime.

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Many terahertz (THz) related physical phenomena, such as THz-radiation-induced dc current suppression [1], multiphoton-assisted resonant tunneling [2], and multiphoton-assisted free-carrier absorption [3], have recently been reported. A few years ago, THz transmissions, reflections, and dc photoconductivity measurements were made to study multiphoton process and impact ionization (II) caused by THz radiations in InAs/AlSb heterojunctions (HJ). These experimental advances motivated new theoretical studies [7, 8, 9, 10, 11] of THz physics. However, many interesting phenomena observed experimentally remained unexplained due to lack of well controlled theoretical models in the nonlinear regime. For example, the ac field dependent absorption coefficient [4] increases with the field slowly at low field intensity, then increases rapidly at intermediate field intensities and levels off at high intensities. To date, this behavior is still not fully understood even within the picture of multiphoton absorption mechanism.

In this letter, we propose to explain this interesting absorption process for the first time by studying the multiphoton-assisted free-carrier absorption and electron-hole (e-h) generation in THz-driven InAs/AlSb HJ’s. Most existing theoretical work emphasize the importance of the electron-impurity (e-i) [5] or the electron-phonon (e-p) interaction [3, 6] in nonlinear absorption. However, both these two scattering mechanisms lead to a decreasing absorption in the high field limit and thus unable to explain the observed phenomena. The parameter that controls the absorption is the electro-optic coupling factor, \( r = eE_{ac}/(m\omega^2) \), where \( e \) is the carrier charge, \( m \) is the effective electron mass, \( E_{ac} \) is the amplitude, and \( \omega = 2\pi f_{ac} \) is the angular frequency with \( f_{ac} \) the radiation frequency. In what follows we shall show that the dominant contribution to the nonlinear absorption in the intermediate to high field region is from the interband II processes. The calculated free-carrier absorption percentages of THz radiations in InAs/AlSb HJ are in excellent agreement with the experimental data [2].

Consider a semiconductor HJ with a two-dimensional (2D) energy-wavevector relation \( \varepsilon_q(k) \), where \( s \) is the subband index. According to the balance-equation theory, when a uniform dc (or slowly varying) electric field \( E_0 \) and a uniform sinusoidal radiation field, \( E(t) = E_0 + E_{ac}\sin(2\pi f_{ac}t) \), are applied in the direction parallel to HJ interface, the transport state can be described by the following equations for the force, the energy, and the carrier number balance [6, 7, 8, 9, 10, 11],

\[
\frac{dv}{dt} = eE_0 \cdot \mathbf{K} + A_{ei} + A_{ep} + A_{II} -gv ,
\]

\[
\frac{dh_{ec}}{dt} = eE_0 \cdot v - W_{ep} - W_{II} - gh_{ec} + S_i + S_p + S_{II} ,
\]

\[
\frac{dN}{dt} = gN ,
\]

where \( v \) is the average electron velocity, and \( \mathbf{K} \) is the inverse effective mass tensor. The electron sheet density is \( N = 2 \sum_{s,k} f \left( (\varepsilon_q(k)) - \mu \right) / T_e \) with \( T_e \) the electron temperature and \( \mu \) the electron chemical potential. \( f(x) = 1/\left[ \exp(x) + 1 \right] \) is the Fermi distribution function. \( g \) is the net electron-hole generation rate by balancing II generation and Auger recombination. \( A_{ei}, A_{ep}, \) and \( A_{II} \) are the frictional accelerations respectively due to e-i scattering, e-p scattering, and interband II process. \( W_{ep} \) and \( W_{II} \) are the energy-loss rates respectively due to e-p scattering and interband II process. \( S_i, S_p \) and \( S_{II} \) are the energy-gain rates of the electron system from the radiation field through the multiphoton process \( (n = \pm 1, \pm 2, \ldots) \) in association with e-i interaction, e-p interaction, and II process, respectively. The expression of \( S_{II} \) is as follow,

\[
S_{II} = \frac{2}{N} \sum_{s',s,k} \left| \mathbf{M}_{s'\mathbf{p},s}^{\mathbf{II}} \left( \mathbf{q}_p \right) \right|^2 \sum_{n=-\infty}^{\infty} f^2_n \left( \mathbf{q}_p \cdot \mathbf{r}_w \right) n\omega
\]

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where $T$ is the lattice temperature, $\alpha_{n, \kappa} = (\omega_0, q_0)$ is the 2D phonon wave vector, $\omega_1 = k_1 v$, $\omega_2 = \omega_1 + \frac{m^2}{2} + \varepsilon_{b}^{\frac{h}{k}} - n \omega$; $\varepsilon_{b}^{\frac{h}{k}} = \varepsilon_{b}^{\frac{h}{k}} - \mu^h$, $\varepsilon_{b}^{\frac{h}{k}} = E_g + k_2^2/(2m_h)$ is the hole dispersion with $m_h$ the effective hole mass. $n(x) = 1/(\exp(x) - 1)$ is the Bose function, and $E_g$ is the band gap. $\Pi \omega(s, s, q)$ is the imaginary part of electron-electron correlation function \[9\]. $J_n(x)$ is the $n$th-order Bessel function. $\tilde{M}^\Pi_{s, s}(q)$ is the Fourier representation of the band-band Coulomb interaction matrix element \[13\] for the II and Auger processes in the 2D semiconductor system. The total energy-gain rate is defined by

$$S = S_1 + S_2 + S_3.$$  

It is useful to write $S$ as a sum of all orders of $n$-photon contributions, $S = \sum_{n=1}^{\infty} S_n$, where $S_n$ is the total contribution from terms having index $n$ and $-n$. We introduce the quantity $\alpha$ which is the ratio of the electromagnetic (EM) energy loss through the 2D sheet to the energy of the incident EM wave, $\alpha$ is a measure of the absorption of the radiation which can be written as,

$$\alpha = \frac{2NS}{\sqrt{k\varepsilon_0E_{ac}^2}},$$  

where $c$ is the light speed in vacuum and $k$ is the dielectric constant of the semiconductor. To see the role of individual multiphoton processes, we define $\alpha_n = 2NS_n/(\sqrt{k\varepsilon_0E_{ac}^2})$ and write $\alpha = \sum_{n=1}^{\infty} \alpha_n$.

In the steady-state calculations for THz-driven InAs/AlSb HJ’s, we consider the electron-acoustic-phonon scattering (via the deformation potential and the piezoelectric couplings), electron-polar-optical-phonon scattering (via the Fröhlich coupling), and elastic scattering both from the remote charged impurities and from the background impurities. For InAs, the Kane-type non-parabolic factor is $2.73$ eV$^{-1}$ and $E_g = 0.22$ eV. The three lowest electron subbands ($\varepsilon_0 = 0$, $\varepsilon_1 = 35$ meV, and $\varepsilon_2 = 200$ meV) are included. Up to 10 hole subbands are taken into account. We set the sheet density of InAs well $N_0 = 5 \times 10^{12}$ cm$^{-2}$, depletion layer charge density $N_{dep} = 5 \times 10^{10}$ cm$^{-2}$, background impurity $n_I = 6.86 \times 10^{13}$ cm$^{-3}$, and remote impurities in AlSb barrier $N_I = 1.53 \times 10^{11}$ cm$^{-2}$ located at a distance of $l = 10$ nm from the HJ interface. In the whole paper, we set $T = 300$ K, and the dc field is assumed to be $E_0 = 1.5$ V/m, same as that used in the experiments \[3\]. The orders of the Bessel functions, $n$, in Eq. \[1\] is set to be large enough so that the integral is convergent within the accuracy of $10^{-4}$.

In Fig. \[1\] (a) we show the e-h pair generation (solid lines) vs $E_{ac}$ with the II process at $f_{ac} = 0.64$, 1, and 1.7 THz, respectively. The dash line in Fig. \[1\] (a) shows no changes of the carrier number when the II process is excluded. The generation rate is essentially zero at low fields and high frequencies. If we define that the onset of the II process occurs at the threshold field $E_c(f_{ac})$. The calculations indicate that $E_c(f_{ac})/f_{ac}$ is nearly constant for all frequencies (see the inset of Fig. \[1\] (a), $E_c(f_{ac})/f_{ac} \approx 3.59$). The physical meaning of this linear dependence is such that for II process to take place, $r_\omega$, times the photon energy $\hbar \omega$ must exceed a minimum value, $r_\omega \hbar \omega = \text{constant}$. At the high field $N$ depends on $E_{ac}$ exponentially. We shall show below that the exponentially growing generation rate plays a crucial role in the total absorption percentage. The Fig. \[1\] (b) shows the effect of II processes on the electron temperature. While the mechanisms of the temperature increase with the coupling parameter $r_\omega$ is generally understood \[14\], our results show a surprisingly large contribution to $T_e$ from the interband II processes. The interband II processes account for about 50 percent of $T_e$. It is interesting to note that II processes lead to a net increase of $T_e$. In many cases, II processes act as a cooling mechanism. For example, for a 3D system under a dc field, $T_e$ with II process is lower than that without II process. In this case the electron heating is only due to electron accelerating and scattering under the dc field. For the present system under an ac field, the II processes still act as a cooling mechanism. However, large number of electrons created in II processes immediately causes a rapid increase in photon absorption due to e-i, e-p, and more importantly, e-h scattering. This absorption is further enhanced by nonlinear multiphoton absorption under an ac field. This subsequent electron scattering and absorption of photon energy lead to an increase of $T_e$ which exceeds the initial reduction of $T_e$ due to II processes. As a result, $T_e$ is higher with II processes.

In Fig. \[2\] (a) we show the calculated total absorption
percentage $\alpha$ vs $E_{ac}$ at $f_{ac} = 0.64, 1, 1.7$ THz, respectively. The solid circles are the experimental results for $f_{ac} = 0.64$ THz from Ref. 3, where $\alpha = 1 - T - R$ with $T$ the transmission and $R$ the reflection. Excellent agreement is obtained between the calculated results and the experimental data. It’s seen from Fig. 2(a) that, lower frequency and/or higher radiation intensity lead to more absorption. For $f_{ac} = 0.64$ THz, the absorption rate increases slowly from low electric field to about $E_{ac} = 3$ kV/cm, then goes fast before reaching its saturation value. The significance of II process can be clearly seen in the measurements and in the current theory. Without the II process, the multiphoton absorption is due to e-i scattering and e-p scattering. The absorption rate due to these mechanism only increases very slowly at low $E_{ac}$. In Fig. 2(b) we show the phonon-induced ($S_p$), II-induced ($S_{II}$), and the total energy gains ($S_p + S_{II}$) of the electron system from the radiation fields. At intermediate to high $E_{ac}$, both e-i and e-p scatterings are actually suppressed by the strong electro-optic coupling. As a result one observes a rapidly decreasing absorption in the high field limit. As we have seen in Fig. 1(b) the effect of strong field on the II process is quite opposite to that of electron disorder scattering because the e-h pair generation is a fundamentally different process than the scattering process. Scattering depends on the ability of electrons to simultaneously exchange momentum and energy with the scatters and photon. Strong coupling with photons reduce the probability of momentum exchange with the impurities or phonons. This momentum exchange is not required in an e-h pair generation process. Annihilation of multiple photon in a generation process leads to a strong total absorption percentage. The very rapid increase of II process at high field (see Fig. 1(a)) is largely offset by the fast decreasing of impurity and phonon scattering. The resulting total absorption percentage increase modestly in the high field limit. The curve at $f_{ac} = 0.64$ THz can be directly compared to experimental result. At this frequency, II mechanism begins to make contribution to the total absorption after about $E_{ac} = 3$ kV/cm. The carrier number increases with increasing $E_{ac}$. For $f_{ac} = 0.64$ THz, when $E_{ac} = 6.5$ kV/cm, the total electron number $N$ increases up to about six times the initial number $N_0$ due to the II process. The agreement between the present theory and the experiment is excellent. In the inset of Fig. 2(b) we show the $n$-photon absorption percentage $\alpha_n$ vs the order $n$ of the Bessel function at $f_{ac} = 1.7$ THz. $E_{ac}$ is changed from 2 to 6 kV/cm with a step of 0.5 kV/cm. For each $E_{ac}$ there is a maximal of $\alpha_n$, and the position of the peaks shifts to higher order $n$ with increasing $E_{ac}$. It means that higher order multiphoton processes play more important role on the total absorption with increasing $E_{ac}$. Generally, it can be seen that the role of THz radiations with larger $E_{ac}$ and lower $f_{ac}$ on carrier transport increases with increasing the electric-optic coupling factor $r_w$.

In Fig. 3 we show the total absorption percentage $\alpha$ vs the wavelength $\lambda$ at $E_{ac} = 2$ kV/cm (triangles), 5.8 kV/cm (circles), and 9 kV/cm (squares), respectively. The absorption percentage $\alpha$ increases with increasing $E_{ac}$ and/or increasing $\lambda$. The relation between $\alpha$ and $\lambda$ can be fitted by the function (see the lines in Fig. 3): $\alpha(\lambda) = \alpha_0 \lambda^b$. When $E_{ac} = 2$ kV/cm the fitted function is $\alpha(\lambda) = 1.70 \times 10^{-6} \lambda^2$, which is the classical absorption-wavelength relation. When $E_{ac}$ increases, however, the present nonlinear theory predicts that the square relation will be broken. For $E_{ac} = 5.8$ kV/cm we have $(\alpha_0, b) = (7.19 \times 10^{-8}, 2.6)$, and for $E_{ac} = 9$ kV/cm we have $(\alpha_0, b) = (3.31 \times 10^{-8}, 2.8)$. The physical reason is the exponential growth of the e-h pair and the more importance of the multiphoton process with increasing $r_w$. In the inset of Fig. 3 we show the first five order multiphoton contribution $\alpha_n$ $(n = 1, 2, ..., 5)$ vs $\lambda$ at $E_{ac} = 2$ kV/cm. It can be seen that, for each order of multiphoton channels there is a critical wavelength $\lambda_p$ at which the multiphoton process makes maximal contribution to the total absorption. With increasing $n$, the position of $\lambda_p$ shifts to longer wavelength. The physical origin of the

**FIG. 2:** (a) Absorption percentage $\alpha$ vs $E_{ac}$ at $f_{ac} = 0.64, 1,$ and 1.7 THz, respectively. The circles are the experimental results from Ref. 3. (b) The corresponding phonon-induced ($S_p$), II-induced ($S_{II}$), and total energy gains ($S_p + S_{II}$) vs $E_{ac}$. The inset shows $n$-photon absorption percentage $\alpha_n$ vs the order $n$ of the Bessel function at $f_{ac} = 1.7$ THz.
we have directly seen from Eq. (4) and the inset of Fig. 2(b) that the term does not contribute to the total absorption. When the electron mass is replaced by the hole mass in $J^2_n(q_\parallel \cdot r_\omega)$ (for $|n| > 0$), this quantity will be reduced by a factor of around $25^{-2} = 0.0016$. This makes the hole contribution at least two to three orders of magnitude smaller than that of the electrons.

(3) We have used a strict 2D model for charged carriers and neglected the effect of real space transfer (RST) to bulk electrons. For InAs/AlSb HJ system, the confining potential is $\Delta E_c = 1350$ meV [14]. This is about $15666$ K. The maximum $T_e$ in our calculation is around 20 times the lattice temperature (about 6000 K). Since the case $f_{ac} = 0.64$ THz represents our central result, we estimate the RST effect. At $E_{ac} = 6$ kV/cm, $T_e$ is about $127$ or $3600$ K, which is less than a quarter of the confining potential. If the RST is due to over-the-barrier activation, the probability of this transfer is $\approx \exp(-\Delta E_c/T_e) \approx 0.0129$, which is less than 1.5 percent. On the other hand, in terms of photon energy of 1 THz (about 4 meV), the barrier height is about 330 times photon energy. This makes the RST by gaining enough energy from the THz field also negligible. Actually, it is clear from the inset of Fig. 2(b) that the probability of absorbing more than 30 photons is practically zero. Therefore we neglect the RST effect.

In conclusion, we have shown that II process plays an important role in the free-carrier absorption of THz radiations in InAs/AlSb HJs. Our results can explain a long standing experimental result on the free carrier absorption in the THz regime.

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FIG. 3: Calculated total absorption percentage $\alpha$ vs wavelength $\lambda$ at $E_{ac} = 2$ kV/cm (triangles), 5.8 kV/cm (circles), and 9 kV/cm (squares), respectively. They are fitted by the function (lines): $\alpha(\lambda) = \alpha_0 \lambda^n$. The inset shows the first five critical wavelengths at a given photon number is that, for $n$-photon assisted process decreases almost like $\omega_0^{2n}$, while the energy absorbed from the radiation at $\omega_0$ [see Eq. (4)] is only important at large $q_f$ regime. There must be a critical wavelength $\lambda_p$ at which the energy absorption reaches a maximum.

Several physical mechanisms employed in our theoretical model and numerical calculation need to be further discussed. (1) We have neglected the absorption of bulk electrons in the substrate because the total absorption is dominated by the 2D channel. The Coulomb interaction matrix for the 2D channel is much stronger than the corresponding interaction matrix of 3D system except in the small wavevector regime. On the other hand the electron-laser coupling $q_\parallel \cdot r_\omega$ [see Eq. (4)] is only important at large $q_f$ regime. As a result, the THz-induced II in the bulk plays a negligible role in the present system. (2) Although a large number of holes are being created in II processes, the hole contribution to the total absorption is ignored in our model. The effective mass ($m_h = 0.94m_0$) in our system is about 25 times heavier than that of the electron ($m = 0.038m_0$), here $m_0$ is the mass of the free electron. From Eq. (4), one can see that $S_{II}$ is mainly determined by $J_n^2(q_\parallel \cdot r_\omega)$. For a small $r_\omega$, i.e., larger mass $m$ or higher frequency $\omega$, we have $J_n^2(q_\parallel \cdot r_\omega) \propto m^{-2|n|}$ for $|n| > 0$. For $n = 0$, we can directly see from Eq. (4) and the inset of Fig.

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InAs/AlSb HJ

(a) $N/N_0$ vs $E_{ac}$ (kV/cm)

(b) $E_{c}$ (kV/cm) vs $f_{ac}$ (THz)

InAs/AlSb HJ

T=300K

with II

without II

$E_{ac}$ (kV/cm)

$E_{c}$ (kV/cm)

$f_{ac}$ (THz)

$N/N_0$
InAs/AlSb HJ

T=300K

with II

without II

Cal. @ 0.64 THz with II

Exp. @ 0.64 THz

InAs/AlSb HJ

T=300K

$\alpha$ vs $E_{ac}$ (kV/cm)

$E_{ac}$ = 2 to 6 kV/cm

step = 0.5 kV/cm

$S_p$, $S_{II}$

$T=300K$, with II

$S_p + S_{II}$

$E_{ac}$ (kV/cm)

$S_{p}$, $S_{II}$ (nW/electron)

$S_{p}$

$S_{II}$

$S_{p} + S_{II}$

$E_{ac}$ (kV/cm)

$S_{p}$

$S_{II}$

$S_{p} + S_{II}$

$E_{ac}$ (kV/cm)

$S_{p}$

$S_{II}$

$S_{p} + S_{II}$

$f_{ac} = 1.7$ THz

$E_{ac} = 2$ to $6$ kV/cm

step = 0.5 kV/cm

$\alpha = 10^{-3}$
Fitted by $\alpha(\lambda) = \alpha_0 \lambda^b$

- Theory, 2kV/cm, $b=2$
- Theory, 5.8kV/cm, $b=2.6$
- Theory, 9kV/cm, $b=2.8$

InAs/AlSb HJ

$T=300$ K