Magnetic Moments of Baryons with a Heavy Quark

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Abstract. We compute magnetic moments of baryons with a heavy quark in the bound state approach for heavy baryons. In this approach the heavy baryon is considered as a heavy meson bound to a light baryon. The latter is represented as a soliton excitation of light meson fields. We obtain the magnetic moments by sandwiching pertinent components of the electromagnetic current operator between the bound state wave–functions. We extract this current operator from the coupling to the photon field after extending the action to be gauge invariant.

INTRODUCTION

This talk is dedicated to Joe Schechter on the occasion of his 65th birthday. Joe has made important contributions to the descriptions of hadrons with a heavy quark [1, 2]. In particular he has successfully developed a hadron model that is valid for finite masses of the heavy quarks but naturally leads into the heavy quark effective model as these masses tend towards infinity [3, 4, 5]. In this talk we will present an application of this approach.

The basic idea is to consider an effective meson model that includes both, light degrees of freedom and mesons that contain a heavy quark. In this model the interaction of the light degrees of freedom is governed by chiral symmetry. On the other hand the heavy quark components of the heavy meson field are coupled such that the Lagrangian reproduces the heavy quark symmetry, i.e. spin and flavor independence of the interactions, as the masses of the heavy mesons tend to infinity. It is thus essential that this part of the model contains both the heavy pseudoscalar and heavy vector meson fields.

It is well established that the light sector of such a model may contain soliton solutions to the classical equations of motion [6, 7]. After appropriate quantization these solutions may be identified with baryon states as in the Skyrme model framework [8]. Essentially this is a practical realization of the large $N_C$ picture for baryons in QCD [9]. The heavy mesons interact with the light mesons according to the roles of chiral symmetry. Substituting the soliton for the light meson fields into that interaction Lagrangian generates a background potential for the heavy meson field. This potential has bound state solutions that, when combined with the soliton, describe baryons with a heavy quark [10, 11, 12]. It is then, of course, interesting to study properties of such a heavy baryon by com-

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puting matrix elements of pertinent operators. The most prominent operator that comes to mind is the one of the electromagnetic current. In particular the spatial components of this operator determine the magnetic moments of the considered baryon. We obtain this operator by incorporating the photon field such that the total action becomes gauge invariant. The current operator is then the coefficient that is linear in the photon field.

This presentation is based on on–going studies regarding electromagnetic properties of baryons with a heavy quark in the above described bound state picture. These studies will be presented elsewhere [13]. Therefore the numerical results presented here should be considered preliminary. We will commence this talk with a brief a review of the soliton picture for the light mesons when vector mesons like $\rho$ and $\omega$ are included. Such an extension has proven necessary in many aspects, as e.g. the proton–neutron mass difference [14], the predicted pion–nucleon phase shifts [15] or the axial singlet charge of the nucleon [16]. We will then discuss the occurrence of the bound state and describe the quantization of the combined soliton bound state system that leads to the heavy baryon state. Finally we will compute the magnetic moments of heavy baryons as matrix elements of such bound states.

**THE MODEL LAGRANGIAN**

In this section we review the classical, i.e. leading order part in the $1/N_C$ expansion of the bound state description for the heavy baryons in the soliton picture.

**Light Mesons**

For the sector of the model describing the light pseudoscalar and vector mesons we adopt the chirally invariant Lagrangian discussed in detail in the literature [17, 6]. This Lagrangian can be decomposed into a regular parity part

$$\mathcal{L}_S = f_{\pi}^2 \text{tr} \left[ p_\mu p^\mu \right] + \frac{m_\pi^2 f_{\pi}^2}{2} \text{tr} \left[ U + U^\dagger - 2 \right] - \frac{1}{2} \text{tr} \left[ F_{\mu\nu} (\rho) F^{\mu\nu} (\rho) \right] + m_V^2 \text{tr} \left[ R_\mu R^\mu \right]$$

(1)

and a part which contains the Levi-Civita tensor, $\epsilon_{\mu\nu\alpha\beta}$. The action for the latter is most conveniently displayed with the help of differential forms $p = p_\mu dx^\mu$, etc.:

$$\Gamma_\epsilon = \frac{2N_C}{15\pi^2} \int Tr(p^5) + \int Tr \left[ \frac{4i}{3}(\gamma_1 + \frac{3}{2} \gamma_2)Rp^3 - \frac{g}{2} \gamma_2 F(\rho)(pR - Rp) - 2ig(\gamma_2 + 2\gamma_3)R^3p \right].$$

(2)

In eqs (1) and (2) we have introduced the abbreviations

$$p_\mu = \frac{i}{2} \left( \xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right), \quad \nu_\mu = \frac{i}{2} \left( \xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi \right) \quad \text{and} \quad R_\mu = \rho_\mu - \frac{1}{g} \nu_\mu.$$
Here $\xi$ refers to a square root of the chiral field, i.e. $U = \xi^2$. Furthermore $F_{\mu\nu}(\rho) = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - ig [\rho_\mu, \rho_\nu]$ denotes the field tensor associated with the vector mesons $\rho$ and $\omega$, which are combined in $\rho_\mu = \left( \omega_\mu \mathbb{I} + \rho_\mu^a \tau^a \right) / 2$ when the reduction to two light flavors is made. The parameters $g, \gamma_1$, etc. can be determined (or at least constrained) from the study of decays of the light vector mesons such as $\omega \to 2\pi$ or $\omega \to 3\pi$ [6]. The complete determination of all parameters, however, also requires some information for the light baryon sector, see below.

The action for the light degrees of freedom ($\int \mathcal{L}_S + \Gamma_\varepsilon$) contains static soliton solutions. The appropriate ansatz are [6]

$$\xi(\vec{r}) = \exp \left( \frac{i}{2} \hat{\gamma} \cdot \hat{\gamma} \mathcal{F}(r) \right), \quad \omega_\nu(\vec{r}) = \frac{\omega(r)}{g} \quad \text{and} \quad \rho_{i,a}(\vec{r}) = \frac{G(r)}{gr} \varepsilon_{ija} \hat{r}_j$$

while all other field components vanish. The resulting non–linear Euler–Lagrange equations for the radial functions $F(r), \omega(r)$ and $G(r)$ are solved numerically subject to the boundary conditions $F(0) = -\pi, \omega'(0) = 0$ and $G(0) = -2$ while all fields vanish at radial infinity [6]. These boundary conditions are needed to obtain a consistent baryon number one configuration.

### The Relativistic Model for the Heavy Mesons

In this subsection we present the relativistic Lagrangian, which describes the coupling between the light and heavy mesons [3]

$$\mathcal{L}_H = D_\mu P(D^\mu P)^\dagger - \frac{1}{2} Q_{\mu\nu}(Q^{\mu\nu})^\dagger - M^2 PP^\dagger + M^* Q_{\mu} Q^\mu \dagger + 2iM d \left( P \rho_{\mu} Q^{\mu} - Q_{\mu} P^\mu P^\dagger \right) - \frac{d}{2} \epsilon^{\alpha\beta\mu\nu} \left[ Q_{\nu\alpha} P_\mu Q^\beta + Q_{\beta\mu} (Q_{\nu\alpha})^\dagger \right]$$

$$- \frac{2\sqrt{2}icM}{m_v} \left\{ 2Q_{\mu} F^{\mu\nu}(\rho) Q^\nu \right.$$  

$$\left. - \frac{i}{M} \epsilon^{\alpha\beta\mu\nu} \left[ D_\beta P F^{\mu\nu}(\rho) Q^\alpha + Q_\alpha F^{\mu\nu}(\rho) (D_\beta P)^\dagger \right] \right\}. \quad (5)$$

Here we have allowed the mass $M$ of the heavy pseudoscalar meson $P$ to differ from the mass $M^*$ of the heavy vector meson $Q_\mu$. Note that the heavy meson fields are conventionally defined as row vectors in isospin space with $P = (P^\dagger)^\dagger, D_\mu P = (D_\mu P^\dagger)^\dagger$ etc. The covariant derivative introduces the additional parameter $\alpha$:

$$D_\mu P^\dagger = (\partial_\mu - i\alpha g \rho_\mu - i(1 - \alpha) v_\mu) P^\dagger = (\partial_\mu - iv_\mu - ig \alpha R_\mu) P^\dagger, \quad (6)$$

$$D_\mu Q^\nu = (\partial_\mu - iv_\mu - ig \alpha R_\mu) Q^\nu. \quad (7)$$

The covariant field tensor of the heavy vector meson is then defined as

$$(Q_{\mu\nu})^\dagger = D_\mu Q^\nu - D^\nu Q^\mu. \quad (8)$$
The coupling constants $d, c$ and $\alpha$, which appear in the Lagrangian (5), have still not been very accurately determined. In particular there is no direct experimental evidence for the value of $\alpha$, which would be unity if a possible definition of light vector meson dominance for the electromagnetic form factors of the heavy mesons was adopted [2]. The other parameters in (5) will be taken [2] to be:

$$d = 0.53, \quad c = 1.60;$$
$$M = 1865 \text{ MeV}, \quad M^* = 2007 \text{ MeV}; \quad D - \text{meson};$$
$$M = 5279 \text{ MeV}, \quad M^* = 5325 \text{ MeV}; \quad B - \text{meson}. \quad (9)$$

It should be stressed that the assumption of infinitely large masses for the heavy mesons has not been made in (5). However, a model Lagrangian which was only required to exhibit the Lorentz and chiral invariances would be more general than the relativistic Lagrangian (5). Rather the coefficients of the various Lorentz and chirally invariant pieces in the relativistic Lagrangian (5) have precisely been arranged to become spin–flavor symmetric in the heavy quark limit $M, M^* \to \infty$ [3].

**Bound States**

Here we briefly review the origin of bound states in the S– and P–wave heavy meson channels. These orbital angular momentum quantum numbers refer to those of the pseudoscalar component ($P^\dagger$) of the heavy meson multiplet ($P^\dagger, Q_{\mu}^\dagger$). More details are provided in refs. [4].

For the P–wave channel the appropriate *ansatz* reads

$$P^\dagger = \frac{1}{\sqrt{4\pi}} \Phi(r) \hat{r} \cdot \tilde{\rho} e^{i\epsilon t}, \quad Q_0^\dagger = \frac{1}{\sqrt{4\pi}} \Psi_0(r) \rho e^{i\epsilon t},$$
$$Q_i^\dagger = \frac{1}{\sqrt{4\pi}} \left[ \hat{r}_i \Psi_1(r) + \frac{1}{2} \epsilon_{ijk} \hat{r}_j \hat{r}_k \right] \rho e^{i\epsilon t}. \quad (10)$$

Note that here $\rho$ refers to a properly normalized spinor which describes the isospin of the heavy meson multiplet. There exist also S–wave bound states ($P^\dagger = \frac{1}{\sqrt{4\pi}} \Phi(r) \rho e^{i\epsilon t}, \ldots$) which are, however, not relevant for the discussion of the magnetic moments. For more details see ref. [4].

We substitute the soliton background (4) for the light fields into the Lagrangian (5) and find the equations of motion for the radial functions in the ansatz (10). Due to the soliton background there are solutions to these equations with $|\epsilon| < M$. These are the bound states we are looking for.

As the relativistic Lagrangian (5) is bilinear in the heavy meson fields the resulting equations of motion are linear. Hence the overall magnitude of the solution is not fixed by the equation of motion. Nevertheless, the equations of motion for the heavy meson fields allow us to extract a metric for a scalar product between different bound states. In particular its diagonal elements serve to properly normalize the bound state wave–functions. The Lagrange function which results from substituting the *ansätze* (4) and
may generally be written as

\[
L = -M_{cl} \left[ F, G, \omega \right] + I_\varepsilon \left[ F, G, \omega; \Phi, \Psi_0, \Psi_1, \Psi_2 \right] \rho^\dagger(\varepsilon) \rho(\varepsilon) .
\]

Here \(M_{cl}\) denotes the soliton mass \([6]\) whose minimum determines the light meson profiles \(F, G\) and \(\omega\). The explicit expressions for the functionals \(I_\varepsilon\) are given in ref. \([4]\). The subscript refers to the explicit dependence on the energy eigenvalues. Upon canonical quantization the Fourier amplitudes \(\rho(\varepsilon)\) and \(\rho^\dagger(\varepsilon)\) are respectively elevated to annihilation and creation operators for a heavy meson bound state with the energy eigenvalue \(\varepsilon\). Demanding that each occupation of the bound state adds the amount \(|\varepsilon|\) to the total energy yields the normalization condition

\[
\left| \frac{\partial}{\partial \varepsilon} I_\varepsilon [\Phi, \Psi_0, \Psi_1, \Psi_2] \right| = 1
\]

in addition to the canonical commutation relation \([\rho_i(\varepsilon), \rho_j^\dagger(\varepsilon')]=\delta_{ij}\delta_{\varepsilon,\varepsilon'}\). Note that for bound states the energy eigenvalues are discretized. For the P–wave channel we obtain the normalization condition

\[
\int drr^2 \left\{ \frac{2}{r} \sin F \Psi_1 - \frac{1}{2} F'' \Psi_2 \right\} \Psi_2 + \frac{4\sqrt{2}c}{g_m} \frac{1}{r^2} \left[ G(G+2) \Psi_1 - G'r\Psi_2 \right] \Phi \right\} = 1
\]

from eq (12). For convenience we have employed the abbreviation \(R_\alpha = \cos F - 1 + \alpha(1 + G - \cos F)\). The analogous condition for the bound state wave function in the S–wave channel is given in ref. \([4]\). The radial profiles associated with these normalizations are displayed in figure \(1\) for the choice \(\alpha = -0.3\). The parameters entering the light meson Lagrangian \([12]\) are given in eq \([25]\).

CRANKING THE BOUND HEAVY MESON STATE

It can easily be verified that the field configurations for both the light mesons \([4]\) and the heavy mesons configurations \([10]\) obtained above are neither eigenfunctions of the spin–nor the isospin generators. Here we will construct such eigenstates from the soliton – bound state system.

Collective Coordinates and Their Quantization

In order to generate states which correspond to physical baryons a cranking procedure is employed. In the first step collective coordinates, which parameterize the (iso–) spin orientation of the meson configuration, are introduced via

\[
\xi \rightarrow A(t)\xi A^\dagger(t) \quad \text{and} \quad \rho_\mu \rightarrow A(t)\rho_\mu A^\dagger(t) .
\]
The time–dependence of the collective rotations is measured by angular velocities $\tilde{\Omega}$

$$A^+(t) \frac{d}{dt} A(t) = \frac{i}{2} \bar{\tau} \cdot \tilde{\Omega}. \quad (15)$$

In addition to the collective rotation of the soliton configuration, field components are induced that vanish classically. For the light vector mesons these are $[7]$

$$\omega_i = \frac{2}{r} \phi(r) \varepsilon_{ijk} \Omega_j \hat{r}_k \quad \text{and} \quad \rho^k_0 = \xi_1(r) \Omega_k + \xi_2(r) \hat{r} \cdot \vec{\Omega} \hat{r}_k. \quad (16)$$

The light meson Lagrangian then contains a term which is quadratic in the angular velocities. The constant of proportionality defines the moment of inertia $\alpha^2 [F, G, \omega; \xi_1, \xi_2, \phi]$. The radial functions $\phi(r), \xi_1(r)$ and $\xi_2(r)$ are obtained from a variational approach to $\alpha^2 [7]$. Here it is worth mentioning that $\alpha^2$ is of order $N_C$.

In analogy to eq (14) the heavy meson fields also need to be rotated in isospin space,

$$P^+ \longrightarrow A(t) P^+ \quad \text{and} \quad Q^+ \longrightarrow A(t) Q^+. \quad (17)$$

Substituting the collectively rotating configurations into the total Lagrangian finally yields (the subscript $\ell = 0, 1$ refers to S and P wave bound states)

$$L^\ell = -M^\ell + I^\ell \rho^+ \rho + \frac{1}{2} \alpha^2 \vec{\Omega}^2 + \frac{1}{2} \chi^\ell \rho^+ \vec{\Omega} \cdot \bar{\tau} \rho. \quad (18)$$

The quantity $I^\ell$ may be extracted from ref. [4]. It has already been employed to obtain the bound state profiles functions, e.g. eq. (10). The new quantity is the hyperfine parameter $\chi^\ell$ whose explicit expression is also displayed in ref. [4].

Once the Lagrangian (18) for the coupling of the collective coordinates, $A$ to the creation and annihilation operators, $\rho$ of the bound is found, its quantization proceeds...
along the bound state approach to the Skyrmion [10, 11, 12, 18]. In doing so, we have to construct Noether charges for spin and flavor. For this purpose we first have to consider the variation of the fields under infinitesimal symmetry transformations. For the isospin transformation we observe
\[
\left[ \phi, i \tau_i \frac{\partial}{\partial \Omega_j} \right] = -D_{ij}(A) \frac{\partial \phi}{\partial \Omega_j} + \ldots. \tag{19}
\]
Here \( \phi \) refers to any of the iso–rotating meson fields and the ellipses represent terms, which are subleading in \( 1/N_C \), as e.g. time derivatives of the angular velocities which might arise from eq. (16). Furthermore \( D_{ij}(A) = (1/2) \text{tr}(\tau_i A \tau_j A^\dagger) \) denotes the adjoint representation of the collective rotations \( A \). From eq (19) we conclude that the total isospin is related to the derivative of the Lagrange function with respect to the angular velocities
\[
I_i = -D_{ij}(A) \frac{\partial L_\ell}{\partial \Omega_j}. \tag{20}
\]
Next we note that the total spin operator \( \vec{J} \) contains the grand spin operator \( \vec{G} \)
\[
G_i = J_i + D_{-1}^{-1}(A) J_i = J_i - J_i^{\text{sol}}. \tag{21}
\]
with \( J_i^{\text{sol}} = \partial L_\ell / \partial \bar{\Omega} \). As a consequence of the relation (20) we have \( (\vec{J}^{\text{sol}})^2 = \vec{I}^2 = I(I + 1) \). By construction the light meson fields do not contribute to \( \vec{G} \). Even more importantly and as has been noted before, the pieces of the heavy meson wave–functions (17), which are located between the collective coordinates \( A \) and the spinor \( \rho \), have zero grand spin too. With the normalization condition (13) one then finds
\[
\vec{G} = -\rho \frac{i}{2} \bar{\tau} \rho. \tag{22}
\]
This relates the operator multiplying the hyperfine parameter in the collective Lagrangian [18] to the spin and isospin operators. The collective piece of the Hamiltonian is obtained from the Legendre transformation
\[
H_\ell^{\text{coll}} = \bar{\Omega} \cdot \vec{J}^{\text{sol}} - L_\ell^{\text{coll}} = \frac{1}{2\alpha^2} \left[ \vec{J}^{\text{sol}} + \chi_\ell \vec{G} \right]^2, \tag{23}
\]
Here \( L_\ell^{\text{coll}} \) refers to the \( \bar{\Omega} \) dependent terms in eq (18). Finally the mass formula for a baryon with a single heavy quark becomes
\[
M_\ell = M_\text{cl} + |\epsilon_\ell| + \frac{1}{2\alpha^2} \left[ \chi_\ell J(J + 1) + (1 - \chi_\ell)I(I + 1) \right], \tag{24}
\]
where contributions of \( \mathcal{O}(\chi_\ell^2) \), which apparently are quartic in the heavy meson wave–function, have been omitted for consistency because terms of that order have been excluded from the very beginning.
TABLE 1. The parameters of the mass formula (24) for $\alpha = 0.3$. Here $\omega = M - |\epsilon|$ denotes the binding energy. The subscripts $S$ and $P$ refer to the bound states in the $S$ and $P$ wave channels, respectively.

|          | $\omega_P$ | $\chi_P$ | $\omega_S$ | $\chi_S$ |
|----------|------------|----------|------------|----------|
| D-meson  | 478 MeV    | 0.114    | 247 MeV    | 0.205    |
| B-meson  | 713 MeV    | 0.0045   | 573 MeV    | 0.0052   |

TABLE 2. Heavy baryons mass differences with respect to $\Lambda_c$ or $\Lambda_b$ for $\alpha = 0.3$. Primes indicate negative parity baryons, i.e. $S$–wave bound states. All energies are in MeV.

|          | $\Sigma_c$ | $\Sigma_c'$ | $\Lambda'_c$ | $\Sigma_c''$ | $N$   | $\Lambda_b$ |
|----------|------------|-------------|--------------|--------------|-------|-------------|
| $M(B) - M(\Lambda_c)$ | 177 | 211 | 238 | 397 | 458 | -1321 | 3174 |
| empir.   | 168 | 233 | 308 | ? | ? | -1345 | 3356 ±50 |
|          | $\Sigma_b$ | $\Sigma_b'$ | $\Lambda'_b$ | $\Sigma_b''$ | $N$   | $\Lambda_b$ |
| $M(B) - M(\Lambda_b)$ | 191 | 205 | 161 | 351 | 367 | -4494 | 0 |
| empir.   | ? | ? | ? | ? | ? | -4701 ±50 |

From eq. (24) we recognize that the spin degeneracy between baryons containing a heavy quark vanishes in the heavy quark limit because $\chi_P$ approaches zero. Of course, this result is a direct consequence of the spin–flavor symmetry and would not have come out in case the various Lorentz and chirally invariant terms in eq (5) had been chosen arbitrarily.

The parameters in the light sector, eqs. (12) cannot completely be determined from properties of the corresponding mesons. The remaining (limited) parameter space is, however, more than fully constrained by a best fit to the mass differences of the low-lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons in the light sector. This yields:

$$g = 5.57, \quad m_V = 773 \text{ MeV}$$
$$\gamma_1 = 0.3, \quad \gamma_2 = 1.8, \quad \gamma_3 = 1.2.$$  \hspace{1cm} (25)

The resulting mass differences for the light baryons all agree within about 10% [19] with experiment. The corresponding moment of inertia is $\alpha_2 = 5.00 \text{ GeV}^{-1}$. The parameters in the heavy baryon mass formula (24) are listed in table 1. The resulting masses are displayed in table 2. We observe that the model represents a sensible picture of the spectrum of baryons with a heavy quark. This even more motivates the investigation of their properties, as e.g. the magnetic moments.

ELECTRO–MAGNETIC CURRENT AND MAGNETIC MOMENTS

In this part of the talk we would first like to describe the extraction of an electromagnetic current operator for bound state configuration. We then compute the magnetic moments
The functionals electromagnetic current associated with the Lagrangian (5) reads

\[ B(I, I_3; J_3) = \mathcal{N} \sum_{J^\text{sol}_{I_3}, G_3} C_{J^\text{sol}_{I_3}, G_3}^3 \frac{1}{2} G_3 \mathcal{D}_{J^\text{sol}_{I_3}, G_3}(A) |\frac{1}{2}, G_3\rangle \]  

(26)

that are obtained by coupling the collective coordinate wave–function (represented by the Wigner D–function, \(D(A)\)) together with bound state wave–function to total spin \(J\) according to eq. (21). The prefactor \(\mathcal{N}\) is a suitably chosen normalization constant.

First we introduce the photon field \(\mathcal{A}_\mu\) in close analogy to refs. \([6, 7]\) for the light sector and ref. \([2]\) for the sector of the heavy mesons. The aim is to construct a gauge invariant Lagrangian \(\mathcal{L}(\xi, \rho, P, Q, \mathcal{A})\) and to extract the current operator as the term that is linear in the photon field: \(J^{\text{em}}_{\mu} = \partial_t \mathcal{L} / \partial \mathcal{A}_\mu \bigg|_{\mathcal{A}_\mu = 0}\). For the light sector, this exercise has been done in refs. \([6, 7]\), while ref. \([2]\) provides the electromagnetic generalization of the covariant derivatives \((6, 7)\). Here we also need to gauge the derivatives in \(p_\mu\) which can be done by writing \([6, 7]\) \(p_\mu \rightarrow p_\mu + ie(\xi Q\xi^\dagger - \xi^\dagger Q\xi)\mathcal{A}_\mu\). Obviously this gauge term vanishes when light pseudoscalar fields are omitted (\(\xi = 1\)). Therefore such a contribution did not arise in the work of ref. \([2]\) as those authors were only interested in electromagnetic processes of heavy mesons that did not contain pions. To this end the electromagnetic current associated with the Lagrangian (5) reads

\[
J^{\text{em}, H}_{\mu} = \left< \text{e} \left( P\check{C}(D_\mu P)^\dagger - D_\mu P\check{C}P^\dagger \right) + \text{e} \left( Q^\nu\check{C}Q^\dagger_{\nu\mu} - Q_{\mu\nu}\check{C}Q^{\nu\dagger} \right) + \text{e} \left( Q^\nu\check{C}Q^\dagger_{\mu\nu} - \text{e} \left( \frac{1}{2} Q^\nu\check{P}Q^{\nu\dagger} - Q^\nu\check{P}Q^{\nu\dagger} \right) \right) \right. \\
+ \left. \text{e} \left( \frac{1}{2} Q^\nu\check{C}Q^\dagger_{\nu\mu} - Q^\nu\check{C}Q^{\nu\dagger} \right) \right) \right) \\
+ \text{e} \left( \frac{1}{2} Q^\nu\check{C}Q^\dagger_{\nu\mu} - Q^\nu\check{C}Q^{\nu\dagger} \right) \right) \\
+ \text{e} \left( \frac{1}{2} Q^\nu\check{C}Q^\dagger_{\nu\mu} - Q^\nu\check{C}Q^{\nu\dagger} \right) \right) \right)
\]  

(27)

where \(\check{C} = C + \frac{1}{2}(\xi Q\xi^\dagger + \xi^\dagger Q\xi)\) and \(C\) is the charge of the heavy quark that is contained in the heavy meson \(\langle C = 2/3\rangle\) and \(\langle C = -1/3\rangle\) in the charm and bottom sectors, respectively). Substituting both the (rotating) soliton and the bound state wave functions we compute

\[
\frac{1}{2} \int d^3r \left( \vec{\tau} \times \vec{J}_{\text{e.m.}} \right)_3 = \mu_0 \alpha^2 \Omega_3 + \mu_{V,0} D_{33}(A) + \mu_{S,1} \rho^\dagger \frac{\tau_3}{2} \rho + \mu_{V,1} D_{33}(A) \rho^\dagger. \]  

(28)

Here \(\mu_{S,0}, \ldots, \mu_{V,1}\) are functionals of all radial profiles that have been computed earlier. The functionals \(\mu_{S,0}\) and \(\mu_{V,0}\) do not contain the heavy meson wave functions and are listed in refs. \([7, 12]\). Note that in the first term on the right hand side we made explicit the appearance of the moment of inertia, \(\alpha^2\) to make contact with earlier work \([11, 12]\). The detailed form of the functionals \(\mu_{S,1}\) and \(\mu_{V,1}\) that contain the bound state wave functions is presented in the appendix, see also ref. \([13]\). Using the quantization rules for the collective coordinates discussed above we can sandwich this operator between baryon states,

\[
\mu_B = \frac{1}{2} \langle B | \int d^3r \left( \vec{\tau} \times \vec{J}_{\text{e.m.}} \right)_3 | B \rangle
\]  

(29)
yielding \[11, 12\]

\[
\begin{array}{cccccccccc}
\Lambda_c & \Sigma_+^+ & \Sigma_0^0 & \Sigma^- & \Sigma_+ & \Lambda_c & \Sigma_0^0 & \Sigma_0^0 & \Sigma_0^- & \Sigma_1^+ \\
0.12 & 2.56 & 0.17 & -2.22 & -2.39 & -0.02 & 2.63 & 0.21 & -2.19 & 2.41 \\
\end{array}
\]

In this table we present numerical results for the magnetic moment in eq.(30). Those results originate from a spectral sum rule analysis \[20\]. On the other hand, our predictions for the transition magnetic moments \(\Sigma_Q \rightarrow \Lambda_Q\) are slightly larger (in magnitude) than those obtained from QCD sum rules \[21\].

We furthermore observe that with increasing masses of the heavy mesons the parameters \(\chi, \mu_S, 1\) and \(\mu_1\) tend to zero. Then all magnetic moments are dominated by their light meson components \[22\]. Of course, this is merely a reflection of the heavy quark symmetry which implies that the magnetic moment of the heavy quark vanishes as its mass becomes infinite.

### CONCLUSIONS

In this talk we have presented a study on the magnetic moments of baryons with a heavy quark. The picture for such heavy baryons is that of a heavy meson being bound to a baryon that is built out of light quarks. We adopt the soliton picture for baryons in which light baryons are represented by soliton excitations of light meson fields. This is in the spirit of the Skyrme model but we have also included light vector meson degrees of freedom in the construction of the soliton. Such extensions by short range fields are needed for various phenomenological reasons.

We have obtained the heavy meson bound state from a relativistic Lagrangian that does not manifestly reflect the heavy quark symmetry. Rather this model is developed for physical values of the heavy meson masses. However, in the (academic) limit of large masses it is demanded to reflect the heavy quark symmetry. This condition strongly
constrains the number of free parameters. Also, it requires the model to contain heavy vector meson fields rather than just the pseudoscalar degrees of freedom as e.g. in the bound state approach to the Skyrme model and its application to the charm sector [12]. We have already observed in ref. [4] that such extensions are also required to properly reproduce the spectrum of baryons with a heavy quark.

Once the model has been set up, it is straightforward to extract the operator of the electromagnetic current by gauging the action with respect to the photon field. In turn we have then used the methods to of refs. [11, 12] to compute the magnetic moments of baryons with a heavy quark. At this time the results are still somewhat preliminary and in future we wish to further explore the connection with the heavy quark limit for the magnetic moments [23].

We hope that Joe considers this study not only an interesting application of ideas that he decisively helped to develop, but also worth to be reported on the occasion of his birthday.

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**APPENDIX**

In this appendix we list the explicit expressions for the parameters in eq. (28) that involve the P–wave bound state wave–function (10).

For the isoscalar case we have

\[
\mu_{S,1} = \frac{2e}{3} \left[ C + \frac{\alpha - 1}{6} \right] \int_{0}^{\infty} dr r^2 m_S(r) \\
\phi(r) = -2 \left( 1 + \frac{1}{2} R_\alpha \right) \Phi^2 \\
+ \left( \Psi_2^r + \Psi_2 + R_\alpha \Psi_1 \right) \Psi_1 - \left( 1 + \frac{1}{2} R_\alpha \right) \Psi_2 - \left[ R_\alpha \Psi_0 + (\epsilon - \frac{2}{2} \omega) \Psi_2 \right] \Psi_0 \\
- d \left( \frac{r}{2} F' \Psi_2 - \sin F \Psi_1 \right) \Psi_0 + \frac{2\sqrt{2}c}{gm_V} \left( r \rho' \Psi_2 - 2G' \Psi_0 \right) \Phi, \\
\]

where again \( R_\alpha = \cos F - 1 + \alpha \left( 1 + G - \cos F \right) \) and primes denote derivatives with respect to \( r \). For the isovector contribution we find

\[
\mu_{V,1} = \frac{e}{6} \int_{0}^{\infty} dr r^2 \left[ (\alpha - 1)m_V^{(1)}(r) + m_V^{(2)}(r) \right] \\
m_V^{(1)}(r) = 2 \left( 1 + \frac{1}{2} R_\alpha \right) \Phi^2 \cos F \\
+ \left\{ \left( \Psi_2^r + \Psi_2 + R_\alpha \Psi_1 \right) \Psi_1 + \left( 1 + \frac{1}{2} R_\alpha \right) \Psi_2 - \left[ R_\alpha \Psi_0 + (\epsilon - \frac{2}{2} \omega) \Psi_2 \right] \Psi_0 \right\} \cos F \\
+ d \sin 2F \Psi_0 \Psi_1 - \frac{2\sqrt{2}c}{gm_V} \left( r \rho' \Psi_2 - 2G' \Psi_0 \right) \Phi \cos F,
\]
\[ m_V^{(2)}(r) = -2Md \sin F \Psi_2 \Phi \\
+ d \left[ r (\Psi_0' \Psi_2' - \Psi_0' \Psi_2) + \Psi_0 \Psi_2 + 2R_\alpha \Psi_0 \Psi_1 + 2r(e - \frac{\alpha}{2}) \Psi_1 \Psi_2 \right] \]  

(32)

The analytic expressions for \( \mu_{S,0} \) and \( \mu_{V,0} \) can be found in refs. [7, 19].

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