Lower Bound on $|U_{e3}|^2$ from Single and Double Beta Decay Experiments

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Abstract

We point out under the assumption of Majorana neutrinos that a lower bound on the MNS matrix element $|U_{e3}|^2$ can be derived by using constraint imposed by neutrinoless double beta decay experiments and by positive detection of neutrino mass by single beta decay experiments. We show that the lower bound exists in a narrow region of the ratio of the observables in these two experiments, $\langle m \rangle_{\beta\beta}/\langle m \rangle_{\beta}$. It means that once the neutrino mass is detected in the bound-sensitive region one must soon observe signal in neutrinoless double beta decay experiments.

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I. INTRODUCTION

There exist, by now, accumulated evidences in the atmospheric [1] and the solar neutrino [2] observations that neutrinos do oscillate. The existence of neutrino oscillation is further strengthened by the result of the first long-baseline man-made beam experiment K2K, in particular by their latest result [3]. They constitute the first compelling evidence for physics beyond the standard model of particle physics. In particular, we have leaned that an almost maximal mixing angle $\theta_{23}$ is required to account for the atmospheric neutrino anomaly, which is quite unexpected from our experience in the quark sector.

What is even more surprizing to us is that, according to the latest global analyses of the current solar neutrino data [4–8], the angle $\theta_{12}$ which is responsible for the solar neutrino oscillation is likely to be large though may not necessarily be maximal. It is in sharp contrast with the fact that the remaining mixing angle $\theta_{13}$ is constrained to be small, $s_{13}^2 < 0.03$, by the reactor experiments [4]. Given the current status of our understanding of the structure of lepton flavor mixing matrix, the MNS matrix [10], it would be nice if there are any hints on how small is the angle $\theta_{13}$.

In this paper, we try to pursue such a possibility and point out that one can derive a lower bound on $s_{13}^2 = |U_{e3}|^2$ through joint efforts by double beta decay experiments and by direct mass determination either by single beta decay or by cosmological observations. We assume in this paper that neutrinos are Majorana particle to rely on the bound imposed by double beta decay experiments.

Before getting into the bussiness, let us briefly summarize existing knowledge on how $\theta_{13}$ can be measured, or further constrained. Most optimistically, the next generation long baseline experiments, MINOS [12], JHF [13], and OPERA [14] will observe $\nu_e$ appearance events and measure the angle $\theta_{13}$. Most notably, the JHF can probe $\sin^2 2\theta_{13} \gtrsim$ several $\times 10^{-3}$ in its phase I [13]. If realized, a large-volume reactor experiment [13] can also probe the similar (to a slightly shallower) region. If the angle is smaller than the sensitivity region of these experiments, we have to wait for future supermassive detector experiments, utilizing either low energy conventional superbeams or neutrino factories. (See e.g., [16] for references cited therein.) If nature is so unkind as to tune the angle extremely small, $\sin^2 2\theta_{13} \ll 10^{-5}$, then the only way to detect its effect would be via supernova neutrinos [17].

* See e.g., [11] for a summary of remaining issues in three flavor mixing scheme of neutrinos.
II. CONSTRAINT FROM NEUTRINOLESS DOUBLE BETA DECAY

Let us start by examining constraint from double beta decay. We use throughout this paper the standard notation of the MNS matrix:

\[ U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}. \] (1)

Using the notation, the observable in neutrinoless double beta decay experiments can be expressed as

\[ \langle m \rangle_{\beta\beta} = \left| \sum_{i=1}^{3} m_i U_{ei}^2 \right| = \left| m_1 s_{12}^2 c_{13}^2 e^{-i\beta} + m_2 s_{12}^2 c_{13}^2 e^{+i\beta} + m_3 s_{13}^2 e^{i(3\gamma-2\delta)} \right|, \] (2)

where \( m_i \) (i=1, 2, 3) denote neutrino mass eigenvalues, \( U_{ei} \) are the elements in the first row of the MNS matrix, and \( \beta \) and \( \gamma \) are the extra CP-violating phases characteristic to Majorana neutrinos \[18,19\]. We have used in the second line of (2) the Majorana phases in the convention of \[20\]. There have been large number of papers quite recently which devoted to extract constraints from neutrinoless double beta decay experiments \[20–22\].

We define the neutrino mass-squared difference as \( \Delta m_{ij}^2 \equiv m_j^2 - m_i^2 \) in this paper. In the following analysis, we must distinguish the two different neutrino mass patterns, the normal (\( \Delta m_{23}^2 > 0 \)) vs. inverted (\( \Delta m_{23}^2 < 0 \)) mass hierarchies. We use the convention that \( m_3 \) is the largest (smallest) mass in the normal (inverted) mass hierarchy so that the angles \( \theta_{12} \) and \( \theta_{23} \) are always responsible for the solar and the atmospheric neutrino oscillations, respectively. We therefore sometimes use the notations \( \Delta m_{23}^2 \equiv \Delta m_{\text{atm}}^2 \) and \( \Delta m_{12}^2 \equiv \Delta m_{\odot}^2 \) to emphasize that they are experimentally (the latter to be) measured quantities. Because of the hierarchy of mass scales, \( \Delta m_{\odot}^2/\Delta m_{\text{atm}}^2 \ll 1 \), \( \Delta m_{12}^2 \) can be made always positive as far as \( \theta_{12} \) is taken in its full range \([0, \pi/2]\) \[23\].

In order to derive constraint on mixing parameters we need the classification.

Case A:

\[ \left| m_1 c_{12}^2 c_{13}^2 e^{-i\beta} + m_2 s_{12}^2 c_{13}^2 e^{+i\beta} \right| \geq m_3 s_{13}^2 \] (3)

Case B:

\[ \left| m_1 c_{12}^2 c_{13}^2 e^{-i\beta} + m_2 s_{12}^2 c_{13}^2 e^{+i\beta} \right| \leq m_3 s_{13}^2 \] (4)
Both types of mass hierarchies are allowed in the cases A and B. For a given experimental upper bound on $\langle m \rangle_{\beta\beta}$, one can derive a lower (upper) bound on $s_{13}^2$ in the case A (B). Therefore, we start with the case A.

A. Case A

In this case, the lower bound on $\langle m \rangle_{\beta\beta}$ can be obtained as in the following way;

$$\langle m \rangle_{\beta\beta} \geq c_{13}^2 \left| (m_1 c_{12}^2 + m_2 s_{12}^2) \cos \beta - i(m_1 c_{12}^2 - m_2 s_{12}^2) \sin \beta \right| - m_3 s_{13}^2$$

$$= c_{13}^2 \sqrt{m_1^2 c_{12}^4 + m_2^2 s_{12}^4 + 2m_1 m_2 c_{12}^2 s_{12}^2 \cos 2\beta - m_3 s_{13}^2}. \quad (5)$$

Noticing that the right-hand-side of (5) has a minimum at $\cos 2\beta = -1$, we obtain the inequality

$$\langle m \rangle_{\beta\beta} \geq c_{13}^2 \left| m_1 c_{12}^2 - m_2 s_{12}^2 \right| - m_3 s_{13}^2. \quad (6)$$

If a neutrinoless double beta decay experiment imposes the bound $\langle m \rangle_{\beta\beta} \leq \langle m \rangle_{\beta\beta}^{\exp}$ the lower bound on $s_{13}^2$ results;

$$s_{13}^2 \geq \frac{\left| m_1 c_{12}^2 - m_2 s_{12}^2 \right| - \langle m \rangle_{\beta\beta}^{\exp}}{\left| m_1 c_{12}^2 - m_2 s_{12}^2 \right| + m_3}. \quad (7)$$

We rewrite the lower bound into the one expressed by the heaviest neutrino mass $m_H$ and $\Delta m^2$ measured by the atmospheric and the solar neutrino experiments. We note that the neutrino masses $m_i$ (i=1,2,3) can be parametrized by $\Delta m^2_{\text{atm}} = \Delta m^2_{23}$, $\Delta m^2_{\odot} = \Delta m^2_{12}$, and a absolute mass scale $m_H$. We take $m_H = m_3$ and $m_H = m_2$ for the normal and the inverted mass hierarchies, respectively. Then,

$$m_1 = \sqrt{m_H^2 - \Delta m^2_{\text{atm}} - \Delta m^2_{\odot}}, \quad m_2 = \sqrt{m_H^2 - \Delta m^2_{\text{atm}}}, \quad m_3 = m_H, \quad (8)$$

for the normal mass hierarchy, and

$$m_1 = \sqrt{m_H^2 - \Delta m^2_{\odot}}, \quad m_2 = m_H, \quad m_3 = \sqrt{m_H^2 - |\Delta m^2_{\text{atm}}|}, \quad (9)$$

for the inverted mass hierarchy.

We express the lower bound on $s_{13}^2$ by using the dimensionless ratios

$$R_{\beta\beta} \equiv \frac{\langle m \rangle_{\beta\beta}^{\exp}}{m_H}, \quad r_{\text{atm}} \equiv \frac{\sqrt{\left| \Delta m^2_{\text{atm}} \right|}}{m_H}, \quad r_{\odot} \equiv \frac{\sqrt{\Delta m^2_{\odot}}}{m_H}. \quad (10)$$

The lower bound on $s_{13}^2$ reads for each type of mass hierarchy as follows:
(i) Normal mass hierarchy; $m_H = m_3$

\[ s_{13}^2 \geq \frac{\sqrt{1 - r_{\text{atm}}^2 - r_\odot^2 c_{12}^2} - \sqrt{1 - r_{\text{atm}}^2 s_{12}^2} - R_{\beta\beta}}{\sqrt{1 - r_{\text{atm}}^2 - r_\odot^2 c_{12}^2} - \sqrt{1 - r_{\text{atm}}^2 s_{12}^2} + 1} . \]  

(11)

(ii) Inverted mass hierarchy; $m_H = m_2$

\[ s_{13}^2 \geq \frac{|\sqrt{1 - r_\odot^2 c_{12}^2} - s_{12}^2| - R_{\beta\beta}}{\sqrt{1 - r_\odot^2 c_{12}^2} - s_{12}^2} . \]  

(12)

The parameter $r_\odot^2$ is extremely small, $r_\odot^2 = \Delta m_\odot^2/m_H^2 \lesssim 10^{-3}$ for the possible best sensitivity of $\sim 0.3$ eV. (See later.) Hence, it is an excellent approximation to ignore $r_\odot^2$ unless the sensitivity goes down to very close to $\Delta m_{atm}^2$ so that it is comparable with $1 - r_{\text{atm}}^2$ in the normal mass hierarchy case. Ignoring $r_\odot^2$, the lower bound on $s_{13}^2$ greatly simplifies:

\[ s_{13}^2 \geq \frac{|\cos 2\theta_{12}| \sqrt{1 - r_{\text{atm}}^2} - R_{\beta\beta}}{|\cos 2\theta_{12}| \sqrt{1 - r_{\text{atm}}^2} + 1} \quad \text{(normal mass hierarchy),} \]  

(13)

\[ s_{13}^2 \geq \frac{|\cos 2\theta_{12}| - R_{\beta\beta}}{|\cos 2\theta_{12}| + \sqrt{1 - r_{\text{atm}}^2}} \quad \text{(inverted mass hierarchy).} \]  

(14)

We finally note that in the degenerate mass limit $r_{\text{atm}} \to 0$ the lower bound ceases to distinguish between the mass patterns, and has a universal form

\[ s_{13}^2 \geq \frac{|\cos 2\theta_{12}| - R_{\beta\beta}}{|\cos 2\theta_{12}| + 1} \quad \text{(degenerate mass limit).} \]  

(15)

**B. Case B**

For completeness, we treat the case B, which yields the upper bound on $s_{13}^2$. Proceeding via the similar way toward (5) we obtain the inequality

\[ \langle m \rangle_{\beta\beta} \geq m_3 s_{13}^2 - c_{13}^2 |m_{1c_{12}}^2 + m_{2s_{12}}^2| \]  

(16)

which is saturated at $\cos 2\beta = +1$. Then, the upper bound on $s_{13}^2$ for a given experimental bound on $\langle m \rangle_{\beta\beta}$ entails after ignoring $r_\odot^2$ as

\[ s_{13}^2 \leq \frac{\sqrt{1 - r_{\text{atm}}^2} + R_{\beta\beta}}{\sqrt{1 - r_{\text{atm}}^2} + 1} \quad \text{(normal mass hierarchy),} \]  

(17)

\[ s_{13}^2 \leq \frac{1 + R_{\beta\beta}}{1 + \sqrt{1 - r_{\text{atm}}^2}} \quad \text{(inverted mass hierarchy).} \]  

(18)

It can give a stronger bound than the CHOOZ limit only for the normal hierarchy and if $R_{\beta\beta} < 0.03$. We do not discuss it further because it is outside of our analysis in section IV.
III. CONNECTION BETWEEN $M_H$ AND THE OBSERVABLE IN DIRECT MASS MEASUREMENTS

Before entering into the actual analysis of the bound, we clarify the relationship between the largest mass $m_H$ ($= m_3$ for normal, and $= m_2$ for inverted mass hierarchies) and the observable in direct mass measurement in single beta decay experiments. We argue that $m_H$ can be identified as the observable in such experiments in a good approximation. Suppose that the neutrino masses are hierarchical and obey either $m_3 \gg m_2 \simeq m_1$ (normal mass hierarchy), or $m_2 \simeq m_1 \gg m_3$ (inverted mass hierarchy). In the former case, it is obvious that $m_3 = m_H$ is the observable. In the latter case, it was shown in [25] that the observable in direct mass measurements $\langle m \rangle_\beta$ is given by

$$\langle m \rangle_\beta = \frac{\sum_{j=1}^{n} m_j |U_{ej}|^2}{\sum_{j=1}^{n} |U_{ej}|^2}$$

(19)

where $n$ is the dimension of the subspace of (approximately) degenerate mass neutrinos, and $n = 2$ in the case under discussion. Then, $\langle m \rangle_\beta = m_2 = m_H$ in a good approximation. In the opposite extreme, $m_i^2 \gg \Delta m_{atm}^2$ which is usually referred to as "almost degenerate neutrinos", (19) with $n = 3$ tells us that $\langle m \rangle_\beta = m_H$ in a very good approximation.

Thus, $\langle m \rangle_\beta = m_H$ holds in both extreme, hierarchical and degenerate mass neutrinos. This discussion strongly suggests that $\langle m \rangle_\beta$ is reasonably well approximated by $m_H$ even in the intermediate region.

IV. ANALYSIS OF THE LOWER BOUND ON $S_{13}^2$

We present in Fig.1 the lower bound on $s_{13}^2$ as a function of $m_H$ for various values of $\cos 2\theta_{12}$; the region lower-right to each curve is excluded. The shaded area indicates the region excluded by the CHOOZ experiment [9], which we approximate as $s_{13}^2 < 0.03$. We take two typical values of $\langle m \rangle_{\beta \beta}^{\text{exp}}$, 0.34 eV and 0.1 eV. We present in Fig. 1 only the results

† For the present status and the future prospects of this type of experiments, see e.g., the website of a recent conference devoted to the topics [24].

‡ While the precise value of the CHOOZ constraint actually depends upon the value of $|\Delta m_{atm}^2|$ [4], we do not elaborate this point in this paper.
for the normal mass hierarchy. It is because the results barely changes in the case of inverted mass hierarchy; there is practically no difference in (a) $\langle m \rangle_{\beta\beta}^{exp} = 0.34$ eV case, and the curve shifts toward the right by about 4% in (b) $\langle m \rangle_{\beta\beta}^{exp} = 0.1$ eV case at the best fit value of LMA solution (see below).

We note that the former value of $\langle m \rangle_{\beta\beta}^{exp}$ corresponds to the present 90% CL bound by Heidelberg-Moscow group \[26\], while the latter indicates a modest sensitivity to be achieved in near future by CUORE \[27\], GENIUS \[28\], and by MOON \[29\] double beta decay experiments. We would like to remind the readers that the present strongest upper limit on $\langle m \rangle_{\beta}$ is from the Mainz collaboration \[30\], $\langle m \rangle_{\beta} \leq 2.2$ eV (95% CL). A similar bound $\langle m \rangle_{\beta} \leq 2.5$ eV (95% CL) is derived by the Troitsk group \[31\]. The sensitivity of the proposed KATRIN experiment is expected to extend to $\langle m \rangle_{\beta} \leq 0.3$ eV \[32\].

In the numerical analysis in this section we assume $|\Delta m_{atm}^2| = 3 \times 10^{-3}$ eV$^2$, the best fit value \[33\] of the combined data of Super-Kamiokande \[1\] and the K2K \[3\] experiments. We ignore $\Delta m_{\odot}^2$ in most part of our analysis and its effect is detectable only in a limited region outside of the sensitivity of the KATRIN experiment, as shown in Fig. 2.

We note that the allowed region of the mixing angle $\theta_{12}$ has a large uncertainty. In particular, its value at the largest end is far more uncertain compared with the accuracy we need to determine where is the bound-sensitive region. We just quote for illustration the allowed region for the LMA solution given in \[4\]; $0.67 \geq \cos 2\theta_{12} \geq 0.19$ (95% CL), $0.099 \geq -0.024$ (99.73% CL). Then, the allowed region of the LMA solution extends even at 95% CL to the right-most contour in Fig. 1. Therefore, the Mainz and the Troitsk experiments begin to touch the parameter region already at their present sensitivities. Similarly, the 95% allowed regions of the LOW and the VAC solutions extend, very roughly to, $0.30 \geq \cos 2\theta_{12} \geq 0.026$ and $0.57 \geq |\cos 2\theta_{12}| \geq 0.30$, respectively \[4\].

We observe in Fig. 1b that for $\langle m \rangle_{\beta\beta}^{exp} = 0.1$ eV the KATRIN experiment start to cover a large portion of the parameter region of the LMA solution toward its best fit value of $\cos 2\theta_{12} = 0.48$ (0.46 in \[8\]). Therefore, if the LMA solution is confirmed by the KamLAND reactor experiment and if the KATRIN experiment detects direct neutrino mass, we would hope that we will have a lower bound on $s_{13}^2$ at the same time.

In Fig. 2 we present the similar plot as in Fig. 1 but for $\langle m \rangle_{\beta\beta}^{exp} = 0.01$ eV. In this case the cases of the normal vs. the inverted mass hierarchies differ clearly from each other, as one can see by comparing Figs. 2a and 2b. In region of $m_H \sim \sqrt{|\Delta m_{atm}^2|} \sim 0.06$ eV the lines of positive and negative $\cos 2\theta_{12}$ start to split by nonvanishing $r^2_{\odot}$ correction, as indicated by the dashed lines in Fig. 2.
The significant feature of the lower bound we have obtained is that it exists in a narrow window of $m_H$, the highest neutrino mass, whose values depend very sensitively on $\langle m \rangle_{\beta\beta}$, the observable in neutrinoless double beta decay experiment. We now try to characterize the region, the bound-sensitive region, as a function of the experimental upper limit $\langle m \rangle_{\beta\beta}^{\text{exp}}$ and $\langle m \rangle_\beta$. In the degenerate mass limit $r_{\text{atm}} \to 0$, it can readily be done by using (15). By demanding the consistency with the CHOOZ bound, we obtain

$$0.97|\cos 2\theta_{12}| - 0.03 \leq R_{\beta\beta} \simeq \frac{\langle m \rangle_{\beta\beta}^{\text{exp}}}{\langle m \rangle_\beta} \leq |\cos 2\theta_{12}|.$$

(20)

Therefore, the bound-sensitive region is characterized by the ratio of these two experimental observables in the degenerate mass approximation. This feature prevails to certain extent beyond the approximation as we will see.

In Fig. 3 we present plots of the bound-sensitive region expressed by the ratio $R_{\beta\beta} \equiv \langle m \rangle_{\beta\beta}^{\text{exp}} / \langle m \rangle_\beta$ as a function of $|\cos 2\theta_{12}|$ for the normal and the inverted mass hierarchies in regions for (a) LMA and (b) LOW solar neutrino solutions. The region covered is determined as the best fit value $\pm 10\%$ in $\tan^2 \theta_{12}$. For the LMA solutions it roughly corresponds to the accuracy to be achieved in KamLAND [34]. We do not present the case of vacuum solution because the relevant region roughly overlaps with that of LMA solution. In the degenerate mass limit we must have a universal curve (20) and the splitting between the normal and the inverted hierarchy cases in Fig. 3 represents corrections by the effect of nonvanishing $r_{\text{atm}}$. We do not plot the scaling curve (20) because it is virtually identical with the one for the inverted mass hierarchy.

Notice that the ratio $R_{\beta\beta}$ can be small for each mass pattern because of the suppression of contribution from highest mass neutrinos by small $s_{13}^2$ in the case of normal mass hierarchy, and by possible cancellation in the inverted mass hierarchy. Once $R_{\beta\beta}$ is given, it is easy for the readers to read off the lower bound on $s_{13}^2$ for a given value of $\theta_{12}$ because the function of lower bound (13) and (14) are approximately linear within the narrow strip $0 \leq s_{13}^2 \leq 0.03$.

We note, however, that several highly nontrivial requirements must be met in order to extract the lowest value of $s_{13}^2$. We see from fig. 3 that the accuracy of direct mass measurement of $\langle m \rangle_\beta$ at given $\langle m \rangle_{\beta\beta}^{\text{exp}}$ must be better than roughly $\pm 5\%$ for the LMA and $\pm 20\%$ for the LOW solutions, respectively. Furthermore, the precise values of mixing parameters, in particular, $\cos 2\theta_{12}$ must be known to better than $\pm 10\%$ level.

Finally, some remarks are in order:

(1) Suppose that in a future time the solar mixing angle $\theta_{12}$ is determined by some ingenious experiments. Assume then that we have obtained the lower bound on $s_{13}^2$ by KATRIN
observation of neutrino mass at the time of sensitivity $\langle m \rangle_{\beta\beta}^{\text{exp}} = 0.1$ eV of double beta decay experiment. Now, further suppose that the latter experiment would have been improved so that $\langle m \rangle_{\beta\beta}^{\text{exp}} \ll 0.1$ eV. Figure 3 tells us that then we are in the excluded region. What does it mean?

The answer is; it means that the double beta decay experiment must see positive events before going down to $\langle m \rangle_{\beta\beta}^{\text{exp}} \ll 0.1$ eV. If not, it is the indication that nature chooses a different lepton flavor mixing scheme from the standard one, or our assumption of Majorana neutrinos is wrong. The last possibility is a rare case of excluding Majorana hypothesis in spite of finite resolution of $\langle m \rangle_{\beta\beta}$ measurement, which takes place owing to the CHOOZ constraint. It is very exciting that a discovery in an experiment either signals discovery of another experiment, or indicate radically different views of lepton sector from the one usually accepted.

(2) We have explored in this paper features of the lower bound on $s_{13}^2$ which can be derived by using constraints imposed by the single and the double beta decay experiments. The similar consideration can be done to constrain different parameters such as solar mixing angle $\theta_{12}$, for example. A more generic analysis of the bound is in progress [35].

(3) While we have focused on beta decay experiments in this paper, the possibility of obtaining better knowledge of neutrino mass in terms of cosmological observation must be pursuit. See e.g., [36].

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FIG. 1. The lower bound on $s_{13}^2$ is displayed as a function of $m_H$ for various values of $|\cos 2\theta_{12}|$ as indicated in the figure for (a) $\langle m_{\beta\beta}^{\text{exp}} \rangle = 0.34$ eV and (b) $\langle m_{\beta\beta}^{\text{exp}} \rangle = 0.1$ eV. The region lower-right to each curve is excluded. The shaded area indicates the region excluded by the CHOOZ experiment. Only the case of normal mass hierarchy is presented. (See the text.)
FIG. 2. The same as in Fig. 1 but with $\langle m \rangle_{\beta\beta}^{\text{exp}} = 0.01 \text{ eV}$. Figures 2a and 2b are for cases of the normal and the inverted mass hierarchies, respectively. The upper and lower dashed lines around curves of $|\cos 2\theta_{12}| = 0.2$ and 0.15 in both figures are for $\cos 2\theta_{12} < 0$ and $\cos 2\theta_{12} > 0$ cases, respectively, with $\Delta m^2_\odot = 4.8 \times 10^{-5} \text{ eV}$. 
FIG. 3. The region of $R_{\beta\beta} \equiv \langle m \rangle_{\beta\beta}^{exp}/\langle m \rangle_{\beta}$ in which the lower bound exists is exhibited as a region between two lines as a function of $|\cos 2\theta_{12}|$. The solid and dotted lines correspond to the normal and the inverted mass hierarchies, respectively. Figures 3a and 3b cover roughly the parameter regions of the LMA and the LOW solar neutrino solutions, respectively.