LTV-FIR command prefiltering for vibration reduction of a flexure-jointed X-Y motion stage with payload variation

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Abstract
This paper investigates the application of a linear-time-varying (LTV) command prefilter for vibration reduction in motion control of LTV systems. A model-based time-varying prefilter function is constructed that can be convolved with arbitrary command input signals to achieve residual vibration cancellation in a finite time. The prefilter is implemented on an experimental X-Y micro-positioning stage with a low-compliance flexure-jointed mechanism. Model coefficients within the LTV prefilter realization are varied in real-time according to measured platform position. As the dynamic model is also dependent on platform payload, a method is introduced to compensate for changes of payload mass based on the interpolation of model coefficients for extremum values. The results reveal that residual vibration can be cancelled effectively with settling time reduced from over 2 seconds to less than 0.4 seconds. Rapid point-to-point maneuvers over large travel distances, and with different known values of payload mass, can be achieved with high accuracy.

Keywords: Motion control, Vibration control, Nonlinear system, Input shaping

1. Introduction
Compliant mechanisms have certain advantages over conventional mechanism, especially for high precision motion tasks where the elimination of friction and backlash is required [1-3]. However, the performance of a compliant mechanism is often limited by undesired vibration during fast operation. One of the greatest challenges for controller design is dealing with nonlinear and/or time-varying characteristics of the mechanism dynamics [4-6].

For feedback controller synthesis, nonlinear dynamics can be treated by linear parameter varying (henceforth LPV) formulations. The main disadvantage of using LPV models for feedback control synthesis is the computational complexity and numerical fragility for high order system [7]. An alternative approach is to use feed-forward control (command prefiltering) to compensate for nonlinear effects and suppress vibration. For the FIR command prefiltering approach, a key feature is the ability to cancel residual vibration exactly within a finite time [8]. Additionally, command prefiltering can be easily implemented in practice without the risk of feedback-induced instability. To date, several studies have investigated impulse-based (time-delay) command prefilters, applied to LTI systems, where knowledge only of the natural frequency and damping ratio is required [8]. Model-based feedback may also be applied with command prefilters to improve the response of the system when subject to
parameter variation or uncertainty [9, 10]. A drawback of impulse-based command prefiltering is that excitation of unmodeled high-frequency modes can be amplified [11]. Therefore, in [12], an FIR command prefilter was introduced having smooth impulse response function and minimum $H_2$-norm that reduces excitation of high frequency modes by removing discontinuities from the command input.

This paper investigates a recently proposed time-varying FIR prefilter for settling time reduction in nonlinear motion control systems [13]. The technique is based on a second-order LTV system model and involves constructing the filter impulse response function in order to achieve exact residual vibration cancellation. An experimental study is undertaken involving an X-Y motion stage, where LTV prefiltering is applied via position-dependent scheduling of the prefilter function. A modified version of the LTV prefilter is also devised to compensate for changes in payload mass.

2. Theoretical foundation

2.1 Prototype LTI prefilter

A command prefilter for a second-order LTV system may be constructed by first considering the following undamped LTI system having infinite duration impulse response $g(t) = \sin(t)$:

$$\ddot{y}(t) + y(t) = y_d(t)$$  \hspace{1cm} (1)

To achieve exact residual vibration cancellation, a kinematic command input $y_d(t)$, can be prefiltered by convolving with a function $h(t)$ having finite duration $T_s$. Zero residual vibration will be achieved with arbitrary command input if $h(t)$ satisfies a condition of orthogonality with respect to the functions $\sin t$ and $\cos t$ (see [12],[13]). Here, we consider a prefilter function with the general form $h(t) = A + Bt + C \cos t + D \sin t$. For the case with $T_s = \pi$, the solution is obtained as [14]

$$h(t) = \frac{2\pi}{(\pi^2 - 8t^2)} \left[ \pi(\pi - t) - 4(1 + \cos t) - \frac{2}{\pi}(\pi^2 - 8) \sin t \right], \ t \in [0, \pi]$$  \hspace{1cm} (2)

The impulse response of the overall system, including prefilter, is then finite duration and is given by

$$f(t) = \int h(\tau) g(t - \tau) \, d\tau$$  

$$= \frac{2}{(\pi^2 - 8t^2)} \left[ \pi(\pi^2 - \pi t - 4) - (\pi^3 - \pi^2 t - 4\pi + 8t) \cos t - 2(\pi t - 4) \sin t \right], \ t \in [0, \pi]$$  \hspace{1cm} (3)

1.1. LTV prefilter formulation

To apply the prefilter function given by equation (2) in the case of an LTV system described by

$$\ddot{y}(t) + C(t)\dot{y}(t) + K(t)y(t) = K(t)y_d(t)$$  \hspace{1cm} (4)

the following construction is used. First, defining $\omega(t) = \sqrt{K(t)}$ and $\nu(t) = \omega(t)y_d(t)$, we obtain

$$\ddot{y}(t) + C(t)\dot{y}(t) + \omega^2(t) = \omega(t)v(t)$$  \hspace{1cm} (5)

The prefiltering is then defined by convolution of $v(t)$ with time-varying function $\tilde{h}$ given by

$$\tilde{h}(t, q) = \omega(t)h(n(t, q)) + (\dot{\omega}(t)/\omega(t) + C(t))f'(n(t, q))$$  \hspace{1cm} (6)

Here, $q$ is the local time variable and $n(t, q)$ is a time-warping function that depends on the history of $\omega(t)$ over $(t - q, t)$:

$$n(t, q) = \int_{t-q}^{t} \omega(\tau) d\tau$$  \hspace{1cm} (7)

The filtered input $\tilde{y}_d$ is obtained by convolving $\tilde{h}(t, q)$ with $v(t) = \omega(t)y_d(t)$, over $q$:

$$\tilde{y}_d(t) = \frac{1}{\omega(t)} \int (t-q) \tilde{h}(t, q) \, dq$$  \hspace{1cm} (8)
It can be seen that, when substituting $v = \tilde{h}$ in equation (5), the solution must be $y = f$ because equation (5) then reduces to

$$f'' + f = h$$

(9)

Consequently, a finite duration input-output pair satisfying equation (1) can be used in the LTV prefilter function given by equation (6). Further details of the proof are given in [13]. The implementation of the LTV prefilter is provided in Figure 1. To see an example of the time evolution of the prefilter function, $\tilde{h}$, see Figure 9 in the experimental results section (Section 4).

### 3. System description and dynamic behavior

#### 3.1 System setup

The LTV-FIR prefiltering method was applied to an experimental micro-positioning X-Y stage (see Figure 2). The system was designed based on 4 parallel RRR chains attached to the end-platform, but with the revolute joints replaced by thin flexures. The system is driven by two voice coil actuators with rotation measured by absolute magnetic encoders. The position of the end platform is measured by laser distance sensors. The overall size is approximately 250 x 250 mm and the operating space is 20 x 20 mm square. More details of the optimized design and kinematic behavior can be found in [15]. For the purpose of experiments, the motion of the system was restricted to translation along the Y-axis only. To test the system with different payloads, an additional mass is clamped at the center of the motion stage. The system dynamics are strongly dependent on platform position and this must be accounted for in the prefilter formulation, as described in Section 2. All control operations and prefiltering were implemented in discrete-time on PC-based hardware with sampling frequency 3000 Hz.
Figure 3. Controlled system with inverse kinematic model and actuator PD feedback control

Figure 4. Normalized stiffness and damping coefficients ($K = \omega^2$ and $C$): Variation with platform position is shown for different platform payloads.

3.2 Inverse kinematics for control implementation
Initially, proportional-derivative (PD) feedback control was applied locally at each actuator to stabilize the system, as shown in Figure 3. It should be noted that high D gain cannot be used because flexible mechanism is prone to noise excitation and instability. Because the kinematic behavior of the mechanism is affected by joint compliance, the rigid body inverse kinematic (RBIK) model is modified by a correction function $\phi(y_d)$ so that the demand values for the actuated joints under feedback control are $\theta_d = RBIK(y_d) + \phi(y_d)$.

This correction function is chosen as a 6th order summation of Chebyshev polynomials:

$$\phi(y_d) = \sum_{i=0}^{6} a_i T_i(y_d/L)$$

where $T_i = 0, ..., 6$ are Chebyshev polynomials of the first kind. The constant coefficients $a_i$ were obtained by experimental identification such that zero positioning error was achieved at the set of Chebyshev nodes given by

$$y_i = L \cos \left(i + \frac{1}{2}\right) \frac{\pi}{7}, \ i = 0, ..., 6$$

where $L = 10.2$ mm.

3.3 System dynamics
The dynamic properties of the system were established by applying linear system identification routines at the set of Chebyshev nodes given by Equation (11). The system identification was performed by response-matching for small step input commands. The model structure was set to match the 2nd order system model in Equation (4). Hence, the transfer function identified for each operating point (Chebyshev node) has the following form

$$\frac{Y(s)}{Y_d(s)} = \frac{\hat{K}^i}{s^2 + C^i s + \hat{K}^i}$$

where $i \in \{0, ..., 6\}$. The parameter $\hat{K}^i$ was set independently to $K^i$ even though, with perfect kinematic behavior, the values should be equal (as $y = y_d$ in the steady state). Four cases were considered, with different platform payloads of 0, 5, 10 and 15 grams. The main results of system identification are shown
in Figure 4, together with the Chebyshev interpolation curves (6th order polynomials). These are used within the prefilter implementation to set the values of \( \omega = \sqrt{K} \) and \( C \) based on the current platform position, as shown by the block diagram in Figure 1. The stiffness coefficient \( K \) is additionally scheduled based on the payload mass by using linear interpolation between the two sets of extremum values (for no payload and 15-gram payload). The damping coefficient \( C \) is not affected greatly by the payload mass. Therefore, it is scheduled according to the current platform position only.

4. Experimental results and discussion
Experiments were conducted to evaluate the performance of the LTV-FIR prefilter with both position- and mass-dependent scheduling of the dynamic coefficients. Initial results for testing with no payload can be seen from Figure 5. The platform motion is produced by a change in command signal involving 20 mm/sec ramp and overall move distance of 2 mm. Clearly, the LTV-FIR prefilter was able to suppress vibration effectively and with short settling time compared with the case without prefiltering. However, with an increase in payload mass to 15 grams, significant transient oscillations occurred. These initial results show the importance of having accurate knowledge of the dynamic coefficient values. They also indicate that it is desirable to adjust the values in relation to the payload mass.

The results in Figure 5 also show that some static positioning error occurred due to residual errors in the inverse kinematic model (which is also affected by platform payload). Additional compensation for positioning error was introduced by using model reference feedback of platform position with low gain PID control according to the block diagram in Figure 6 below. The model reference feedback acts to reduce the error between the actual system output \( y(t) \) and predicted output \( y_p(t) \), which is calculated by convolving \( y_d(t) \) with the target system impulse response \( (y_p = y_d * f) \). Due to the fact that \( f \) has unity steady-state gain, the use of integral feedback ensures that \( y = y_d \) at steady state. The results with model reference feedback are also provided in Figure 7. These show that the model reference feedback helps to eliminate static positioning error by shifting the command input. However, an improved insensitivity to model error is also observed.

![Figure 5. The result of LTV-FIR prefilter with and without payload scheduling](image)

![Figure 6. The control implementation with model reference feedback](image)
To investigate the performance of the LTV-FIR prefilter with payload scheduling, tests were performed where the platform undergoes a series of motion intervals from $y = -8$ mm to $y = 8$ mm. Each change in command value involves a constant speed (ramp) of 20 mm/sec with a travel distance of 2 mm, followed by a 5 second dwell period. Figure 8 compares the results obtained with and without prefiltering for four cases with different payload mass values. Although there are significant differences in natural frequency for each payload value, the vibration cancellation effect from the prefiltering is maintained. Moreover, the position-dependent scheduling of the prefilter ensures effective control over the full range of motion.

Further results shown in Figure 9 are based on constant-speed ramp commands that produced motion of the platform between extremum points with $y = \pm 8$. It can be seen that the LTV-FIR command filtering was still effective, even though the large range maneuvers involved rapid changes in model coefficient values. The time-varying filter impulse response function $\tilde{h}$ during these maneuvers is presented in Figure 9b.
5. Conclusions

This paper has investigated an LTV-FIR prefiltering approach that can prevent undesired oscillations in the motion control of LTV systems. The prefiltering method was applied to a flexure-based micro-positioning stage, where the system dynamics were determined by linear identification at a set of calibration nodes. Polynomial interpolation of model coefficients based on measured platform position was used to schedule the LTV prefilter function in real-time. To deal with changes in platform payload, a linear interpolation was adopted based on extremum payload values. This gave significant improvements in vibration cancellation performance compared with cases without payload scheduling.

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