The halo mass function in alternative dark matter models

M. R. Lovell1*

1Center for Astrophysics and Cosmology, Science Institute, University of Iceland, Dunhagi 5, 107 Reykjavik, Iceland

ABSTRACT

The claimed detection of large amounts of substructure in lensing flux anomalies, and in Milky Way stellar stream gaps statistics, has lead to a step change in constraints on simple warm dark matter models. In this study we compute predictions for the halo mass function both for these simple models and also for comprehensive particle physics models of sterile neutrinos and dark acoustic oscillations. We show that the mass function fit of Lovell et al. underestimates the number of haloes less massive than the half-mode mass, $M_{hm}$ by a factor of 2, relative to the extended Press-Schechter (EPS) method. The alternative approach of applying EPS to the Viel et al. matter power spectrum fit instead suggests good agreement at $M_{hm}$ relative to the comprehensive model matter power spectra results, although the number of haloes with mass $< M_{hm}$ is still suppressed due to the absence of small scale power in the fitting function. Overall, we find that the number of dark matter haloes with masses $< 10^8 \text{M}_\odot$ predicted by competitive particle physics models is underestimated by a factor of $\sim$ 2 when applying popular fitting functions, although careful studies that follow the stripping and destruction of subhaloes will be required in order to draw robust conclusions.

Key words: dark matter – galaxies:haloes

1 INTRODUCTION

Recent observational studies have provided a new generation of constraints on the amount of small scale structure in the Universe. Two studies of flux anomalies in multiply imaged lensed quasars have inferred the presence of a large number of dark matter subhaloes (Gilman et al. 2019; Hsueh et al. 2019), and a recent study of Milky Way stellar streams has claimed a similar detection (Banik et al. 2019). These studies report the existence of a minimum number of subhaloes in a given mass range around a target host halo – massive elliptical galaxy haloes for flux anomalies and our Milky way halo in the case of stellar streams – and can therefore place limits on models of dark matter in which the abundance of haloes is suppressed by the presence of a cut-off in the linear matter power spectrum.

This cut-off can occur in models of sterile neutrino (Ns) dark matter (Dodelson & Widrow 1994; Shi & Fuller 1999; Laine & Shaposhnikov 2008; Boyarsky et al. 2009b; Lovell et al. 2016) and models with dark radiation interactions in the early Universe (Buckley et al. 2014; Brehm et al. 2014; Cyr-Racine et al. 2016; Schewtschenko et al. 2016; Vogelsberger et al. 2016). These models show a rich phenomenology of matter power spectra, including sharp cut-offs, shallow cut-offs and dark acoustic oscillations (DAOs). This wide variety of power spectrum options is difficult to constrain systematically with any observational probe, including flux anomalies measurements or gaps in stellar streams, and so these observational studies typically instead place constraints on the simple warm dark matter (WDM) thermal relic model first proposed by Bode et al. (2001) and later expanded by Viel et al. (2005) (hereafter V05). This model contains a single parameter, the thermal relic WDM particle mass $m_{\text{WDM}}$, which is related directly to the half mode wavenumber, $k_{hm}$, defined as the wavenumber at which the square root of the ratio of the WDM linear power spectrum to the cold dark matter (CDM) linear matter power spectrum – otherwise known as the transfer function – is suppressed by a factor of 2. $k_{hm}$ can be used to define a characteristic mass scale, the half-mode mass, $M_{hm}$. It is then simple to parametrize the halo mass function through the combination of $M_{hm}$, a fitting formula and the CDM mass function. The fitting formula used is typically either that derived for field galaxies in trial WDM cosmologies by Schneider et al. (2012) or for the local halo and MW subhalo populations by Lovell et al. (2014) (hereafter L14). Both of which these fits were made to N-body simulations that assumed the V05 model, and do not reflect the different environments of interest to observational studies, such as low mass field dwarfs and satellites of lensing elliptical galaxies.

In this Letter we examine under what conditions two of the approximations outlined above – the V05 approximation to the linear matter power spectrum and the L14 halo fit to the subhalo mass function – are appropriate fits to the predictions of a set of well-motivated Ns models, and also to the ETHOS model of interacting dark matter that features DAOs (Vogelsberger et al. 2016). These

* E-mail:lovell@hi.is

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predictions are restricted to simple calculations for field haloes; therefore, we will not make comparisons to observational data as these are almost invariably influenced by the abundance and radial distributions of subhaloes. We present our methods and results in Section 2 and draw conclusions in Section 3.

2 METHOD AND RESULTS

We consider four Ns models in this study. Each uses an Ns with a mass of 7.0 keV, and a unique value of the generation mechanism lepton asymmetry, $L_0 = 8, 9, 10$ and 12; for a discussion of the relationship between $L_0$ and the matter power spectrum properties see Lovell et al. (2016). Throughout this paper we refer to these four models as LA8, LA9, LA10 and LA12 respectively. LA9 is the model with the highest wavenumber cut-off that is consistent with a dark matter decay origin for the 3.55 keV line reported in M31 (Boyarsky et al. 2014), stacks of galaxy clusters (Bulbul et al. 2014) and the Galactic centre/Milky Way halo (Cappelluti et al. 2018; Hofmann & Wegg 2019). LA12 is somewhat warmer than the lowest cut-off in agreement with the line and LA10 is an intermediate case. LA8 is the coldest model of any 7 keV Ns, and was used to perform some of the hydrodynamical simulations in Despali et al. (2019), thus we can compare our results to theirs as a check for our method. In addition to these four Ns models we also use the ETHOS4 model of SIDM (hereafter simply ‘ETHOS’), which was tuned to obtain a rough match between MW satellite simulation predictions and observations (Vogelsberger et al. 2016); it was shown in Lovell et al. (2018) that this power spectrum had the same peak wavenumber, $k_{\text{peak}}$, as the 7 keV Ns with $L_0 \sim 9$.

The momentum distribution functions of the Ns models were computed initially for Lovell et al. (2016), and used cosmological parameters consistent with the Planck Collaboration et al. (2014) results.

We discuss an approximation to these linear matter spectra using the V05 thermal relic fit, $P(k)_{\text{WDM}}$, which takes the form:

$$P_{\text{WDM}}(k) = \left(1 + \left(\frac{\nu k}{\nu_0} \right)^{\alpha} \right)^{-10/\nu} P_{\text{CDM}}(k),$$  

(1)

where $\nu = 1.12$ and $P_{\text{CDM}}$ is the CDM power spectrum. $\alpha$ is related to the WDM particle mass as shown by equation 7 of V05, and in turn sets the half-mode wavenumber, $k_{\text{hm}}$, and half-mode mass, $M_{\text{hm}}$, as discussed above. In this study we will test the hypothesis that a V05 thermal relic of a given $M_{\text{hm}}$ returns a good approximation to the more complex physics models of Ns and DAOs (hereafter referred to as ‘Boltzmann-derived’). We note that Murgia et al. (2017) have presented a more comprehensive set of fits to these theories, but for the purpose of this study we consider only the V05 fit as this is the fit most commonly applied in the literature. We therefore calculate $k_{\text{hm}}$ for each of our CAMB-derived power spectra and compute the V05 power spectra specified by the same $k_{\text{hm}}$. We present our results in Fig. 1.

Our four Ns Boltzmann-derived power spectra have shallower slopes than their V05 counterparts, and this difference correlates with $L_0$. The V05 fit overestimates the power of the LA12 model for $k < k_{\text{hm}}$ by up to 10 per cent and the LA8 model by 20 per cent. It then follows that at scales smaller than $k_{\text{hm}}$ V05 progressively underestimates the power, and by a factor of more than two at $2 \times k_{\text{hm}}$. ETHOS instead presents a power spectrum cut-off even more abrupt than its V05 counterpart, possessing 20 per cent more power at $k \sim 20 h/$Mpc before dropping rapidly. We note that the $k_{\text{hm}}$ of ETHOS is almost identical to that of the LA10 model, even though it has the same $k_{\text{peak}}$ as LA8: we will therefore be able to make a statement about the degree to which $k_{\text{peak}}$ and $k_{\text{hm}}$ influence the mass function.

The matter power spectra presented here are typically evolved forward in time into halo mass functions using $N$-body simulations of structure formation, from the linear regime to the present day. This is a computationally expensive process for probing a two dimensional parameter space, especially when the target observable is the abundance of dwarf haloes in the local Universe. We therefore adopt three alternative methods: (i) evolving the Boltzmann-derived power spectra forward using the extended Press-Schechter (EPS) method (Press & Schechter 1974; Bond et al. 1991; Benson et al. 2013), (ii) repeat the EPS process with the V05 counterparts, and (iii) apply the $M_{\text{hm}}$ for each model to the fitting function presented in L14. For CDM and the Ns models we apply the sharp-$k$ space window filter to obtain the EPS mass functions, whereas for ETHOS we instead apply the smooth $k$-space cut-off introduced by Sameie et al. (2019); we subsequently renormalise the ETHOS mass function to have the same value as
CDM at $8 \times 10^{10} M_\odot$, in order to compensate for this change in window function. For all five V05 fits we use the sharp-$k$ space cut-off.

We begin our comparison between fits and the Boltzmann code-EPS (B-EPS) results with the L14 halo fits. This fit is given by the L14 equation 7, which we reproduce here:

$$n_{WDM}/n_{CDM} = (1 + M_{hm}M_{sub}^{-1})^\beta,$$

(2)

where $n_{WDM}$ and $n_{CDM}$ are the WDM and CDM differential mass functions respectively, $M_{sub}$ is the bound mass of the halo/subhalo as defined by the halo finder and $\beta = -1.3$. In addition to the L14 fit we compute the mass function fitted by Despali et al. (2019) to the subhalo mass functions of a series of four LA8 hydrodynamical simulations, which instead uses the functional form that L14 derived using an extra parameter (their equation 8):

$$n_{WDM}/n_{CDM} = (1 + \gamma M_{hm}M_{sub}^{-1})^\beta,$$

(3)

originally with $\beta = -1.3$ and $\gamma = 2.7$. Despali et al. (2019) refit $\gamma = 0.35$ for LA8; this model has $M_{hm} = 1.28 \times 10^8 M_\odot$. We compute our results for the top-hat measure of halo mass, $M_{TH}$, as this is the measure computed by EPS. One complication to this method is that the L14 fit instead derived for $M_{sub}$, rather than $M_{TH}$ or the more common $M_{200}$, defined as the mass contained within the radius of mean density 200 times the critical density for collapse, since the latter two measures of mass are not defined for subhaloes. We therefore make the first order assumption that $M_{200} \approx M_{sub}$, and then $M_{TH} = 1.2M_{200}$ as is typically found in N-body simulations. We present our results in Fig. 2, first as ratios with respect to CDM and second as the ratio of the L14 fit to B-EPS.

There is significant disagreement between the B-EPS and the L14 predictions. The latter predicts 40 per cent (30 per cent) fewer haloes at $M_{hm}$ than do the B-EPS Ns (ETHOS) calculations. This disagreement worsens towards lower masses, particularly for ETHOS as L14 cannot account for the first DAO bump, although at masses below a tenth of $M_{hm}$ the B-EPS mass function for Ns becomes shallower than the L14 prediction. The Despali et al. (2019) fit instead suggests excellent agreement with the LA8 B-EPS results in the $10^7-10^9 M_\odot$ mass range crucial for lensing and stream gap studies, at the expense of predicting up to 15 per cent more haloes than B-EPS at higher masses.

We repeat this exercise with the V05 fits evolved using EPS (V05-EPS) and present the results in Fig. 3. The agreement between B-EPS and V05-EPS at $M > M_{hm}$ is better than 10 per cent, and what discrepancy there is corresponds to an overprediction of haloes as one would expect from the excess power at $k < k_{hm}$ relative to the Boltzmann calculations (Fig. 1). The agreement at $M_{hm}$ itself is much better than 1 per cent. At lower masses the loss of power in the V05 fit is apparent in the over-suppression of haloes, by at least 80 per cent at $10^7 M_\odot$ for all models. The ETHOS V05-EPS fit instead produces 20 per cent more haloes at $M_{hm}$ than the B-EPS counterpart, although it inevitably misses the bump due the first DAO at $3 \times 10^7 M_\odot$. Finally, we note that the mass function of ETHOS overall bears a stronger affinity to that of the LA10 Ns than that of the LA9, thus we conclude that $k_{hm}$ is a better predictor of the output mass function than $k_{peak}$, in so far as one wavenumber is able to specify the entire mass function.
3 CONCLUSIONS

Recent observational studies of lensing flux anomalies (Gilman et al. 2019; Hsueh et al. 2019) and stellar stream gaps (Banik et al. 2019) have reported strong constraints on the properties of dark matter, including the presence or otherwise of a matter power spectrum cut-off predicted by well-motivated particle physics models. These studies are constrained to test single parameter models and it has therefore not been clear whether the underlying particle physics models, in their full complexity, are in tension with the data.

In this Letter we have used some simple analyses of sterile neutrino (Ns) and dark acoustic oscillations (DAO, specifically ETHOS) to test some of the fits found in the literature. We have found that the Viel et al. (2005) (V05) counterpart to the linear matter spectrum, as defined by the half-mode wavenumber $k_{hm}$ is accurate to within 10 per cent for wavenumbers $k < 30 \ h/Mpc$ for Ns models consistent with being the origin of the reported 3.55 keV line (Boyarsky et al. 2014; Bulbul et al. 2014), although the coldest model (L8) is overpredicted by up to 20 per cent (Fig. 1). At much smaller scales the fit progressively underpredicts power spectrum. The ETHOS model is instead steeper than its V05 counterpart.

We then computed $z = 0$ halo mass functions using three methods: the fitting function of Lovell et al. (2014) (L14) assuming the half-mode mass $M_{hm}$ associated with each matter power spectrum, applying the extended Press-Schechter (EPS) formalism to the V05 fits (V05-EPS), and again applying the EPS formalism to the original Boltzmann code-derived power spectra (B-EPS). We showed that the L14 fit applied to this scenario underpredicts the number of haloes at $M_{hm}$ relative to the B-EPS measurement, whereas V05-EPS agrees with B-EPS to better than 5 per cent at the same mass. However, in the $[10^7,10^8] M_\odot$ band that is crucial for lensing and stream gap studies, both L14 and V05-EPS underestimate the amount of substructure in all five models, relative to the B-EPS calculation, by at least 50 per cent over much of that range. It is therefore crucial to determine accurate models, whether improved fits (Boyarsky et al. 2009a; Murgia et al. 2017) or to use EPS models / $N$-body models, when determining whether a given model has been ruled out.

We caution that attempts to constrain these models must account for other phenomena that we do not consider here. These include the different spatial distribution of WDM subhaloes within a host halo compared to CDM (Lovell et al. 2014; Bose et al. 2017; Despali et al. 2019) and likely altered destruction rates due to the lower concentrations of WDM haloes. One must account for the different definitions of the halo / subhalo mass, especially when subhaloes within a host halo are expected to contribute a significant part of the signal, and model the impact of baryon physics on any change to the mass. It will therefore be imperative for studies that place competitive constraints on generic matter power spectrum parameters to be followed up with dedicated simulations, that will in turn ascertain whether a given model has been ruled out.

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