Spin Resonance and dc Current Generation in a Quantum Wire

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We show that in a quantum wire the spin-orbit interaction leads to a narrow spin resonance at low temperatures, even in the absence of an external magnetic field. A relatively weak dc magnetic field of a definite direction strongly increases the resonance absorption. Linearly polarized resonance radiation produces dynamic magnetization as well as electric and spin currents. The effect strongly depends on the external magnetic field.

PACS numbers: 73.21.Hb, 76.20.+q, 71.70.Ej

Introduction. In recent years it has become possible to engineer nanodevices using materials with predefined properties\textsuperscript{1,2}. This development revitalized an interest in electron interactions in nanowires and led to discoveries of many fascinating phenomena. One such interaction is the spin-orbit (SO) interaction of the conduction electrons with the lattice. Even comparatively weak SO interaction changes the symmetry of electronic system and leads to numerous novel effects.

This Letter considers Electron Spin Resonance (ESR) in 1D nanowires with a SO interaction.\textsuperscript{3,4} The standard picture of ESR in metals is as follows. In 3D an external magnetic field \( B \) gives distinct up-spin and down-spin Fermi-surfaces, with Zeeman energy the same for all electrons. An applied ac-field (considered to be almost uniform) induces resonant transitions between states with the same momentum (in the spherical shell between two Fermi-surfaces) and opposite directions of spin. However, a sufficiently strong SO interaction smears out this ESR resonance. The direction of the SO induced Zeeman “internal” magnetic field \( B_{SO} \) which acts on an electron depends on the electron’s momentum.

This Letter exploits the fact that this anisotropic broadening is strongly reduced in a quantum wire where the direction of \( B_{SO} \) is the same for all \( p \). Thus ESR is narrow at low temperatures. This picture is the basis of our main results: The relative strength of the Dresselhaus and Rashba interactions sets a specific direction of \( B_{SO} \) for a wire. Even a weak dc magnetic field perpendicular to this direction increases the resonance absorption by orders of magnitude, while the resonance frequency \( \omega_r \) changes only slightly. The component of external magnetic field parallel to \( B_{SO} \) separates the resonances for left and right movers. Linearly polarized resonance radiation then produces a net magnetization and dc electric and spin currents. The magnitude of the effect is controlled by the external magnetic field \( B \).

For 1D nanowires the geometrical constraints, together with quantization of the transverse motion, strongly suppresses the most effective Dyakonov-Perel mechanism of spin relaxation,\textsuperscript{5} thus stabilizing the resulting resonance induced non-equilibrium state.

The wire can be formed by the growth process,\textsuperscript{2} or from a semiconducting film or heterojunction by a proper configuration of the gate electrodes.\textsuperscript{6} In the latter case the substrate must violate reflection symmetry.

Electronic spectrum and eigenstates. Consider the type III-V semiconductors GaAs and InGaAs.\textsuperscript{7} The electron density is assumed to be sufficiently large and the temperature sufficiently low to ensure a degenerate Fermi gas. Electron-electron Coulomb interactions, i.e. Luttinger liquid effects in a 1D electron,\textsuperscript{8} will be neglected. We also assume that the wire is narrow enough to exclude multiple channels.

In 1D the most general form of the SO interaction, including both Rashba and Dresselhaus terms, is \( H_{SO} = (\alpha \sigma_x + \beta \sigma_y) p \), where \( p \) is the 1D momentum along the wire direction \( x \) and \( \sigma \) are the Pauli spin matrices. The total Hamiltonian, without impurity scattering, also includes the kinetic energy \( p^2/2m \) and Zeeman term \( -b \sigma \), where \( b = g \mu_B B/2 \). Let us introduce a unit vector \( n \) in the direction \( \alpha x + \beta y \) of \( B_{SO} \). We also introduce the longitudinal and transverse components of magnetic field: \( b = b_{\parallel} n + b_{\perp} \). The total Hamiltonian reads:

\[
H = p^2/2m + (\gamma p - b_{\parallel}) n\sigma + b_{\perp}\sigma, \quad \gamma = \sqrt{\alpha^2 + \beta^2}. \tag{1}
\]

Its eigenvalues are

\[
E(p, \sigma) = p^2/2m + \sigma q, \quad q = \sqrt{(\gamma p - b_{\parallel})^2 + b_{\perp}^2}, \tag{2}
\]

where \( \sigma = \pm 1 \) shows the projection of the electron’s spin on the total effective Zeeman magnetic field \( B + B_{SO} \) and is the eigenvalue of the operator \( \Sigma = \gamma p - b_{\parallel}/q \left( n + b_{\perp}/\gamma p - b_{\parallel} \right) \sigma \). For a nonzero transverse magnetic field \( b_{\perp} \), the direction of spin quantization depends on momentum. The graph of energy vs. magnetic field for small magnetic fields \( b \ll p^2/2m \) is represented by two slightly distorted Rashba parabolas shifted vertically in opposite directions and with avoided crossing (Fig. 1). The parallel magnetic field \( b_{\parallel} \) is responsible for the reflection asymmetry, whereas \( b_{\perp} \) causes the avoided crossing. There are generally four Fermi moments, two left and right movers for each value of \( \sigma \).
For a typical experimental setup the SO velocity $\gamma \ll v_F = p_F/m$. If $|b| \ll \gamma p_F$ then the four Fermi momenta differ only slightly from the Fermi momentum of the wire without SO interaction and magnetic field, $p_F = \pi \hbar n/2$ ($n$ is the 1D electron density), and are given by

$$p_{\sigma \tau} = \tau p_F - \sigma m \left[ \gamma - \frac{b_{\parallel}}{p_F} + \frac{b_{\perp}^2}{2 p_F (\gamma p_F - \tau b_{\parallel})} \right], \quad (3)$$

where $\tau = \pm 1$ indicates right (R) and left (L) movers. In the ground state electrons with spin projection $\sigma$ fill a momentum interval from $p_{\sigma -}$ to $p_{\sigma +}$.

All states in the interval $(p_{-}, p_{+})$ are doubly occupied. A net spin-flip is possible only in the singly occupied intervals $(p_{+}, p_{-})$ and $(p_{-}, p_{+})$, and requires energy $E_{sf} = 2g \approx 2 (|\gamma| p - \tau b_{\parallel})$. Thus, for $b_{\parallel} \neq 0$, there are two different resonance frequencies corresponding to the left and right movers. The “lengths” of singly-occupied intervals are $2m (|\gamma| p - \tau b_{\parallel}/p_F)$. For $m\gamma \ll \hbar n$, we find that the spin-flip energies are centered at $E_{sf}^0 = 2 (\gamma p_F - \tau b_{\parallel})$ and confined to narrow bands of width $4m\gamma (|\gamma| - \tau b_{\parallel}/p_F) = 2m\gamma E_{sf}^0/p_F \ll E_{sf}^0$. Spin-flip processes can be excited by a resonant external field with frequency $\omega_c = E_{sf}^0/\hbar$. The temperature must be small $T < \hbar \omega_c/\hbar k_B$ to avoid thermal smearing.

**Transition rate by linearly polarized ac field.** Let an ac electric field linearly polarized along $x$ direction $\mathbf{E}(t) = \hat{x} E_0(t) e^{-i\omega_0 t} + \hat{x} E_0(t) e^{i\omega_0 t}$ have spectral intensity $I(\omega)$ centered about $\omega_0$ with width $\Delta \omega \ll \omega_0$. Here $E_0(t)$ describes a stationary random process with correlation time $\tau_B = 2\pi/\Delta \omega$. Averaged over a time interval $t'$ satisfying $2\pi/\omega_c \ll t' \ll \tau_B$, we have $E_0^2(t) E_0^2(t') = (2\pi)^{-1} \int_{-\infty}^{\infty} I_\omega e^{i\omega(t-t')} d\omega$. It interacts with the spin since $p$ in the Hamiltonian (4) must be replaced by $p + F A$ (other directions of polarizations do not couple to spin.) In the Weyl gauge (electric potential $\Phi = 0$) the relation between the vector-potential $A$ and electric field reads $A = -\frac{e}{c} \left( E_0^0(t) e^{-i\omega_0 t} - E_0^0(t) e^{i\omega_0 t} \right)$. Thus, the interaction between the electric field and spin gives

$$H_{nl} = -\frac{e \gamma}{\omega_0} \left( E_0^0(t) e^{-i\omega_0 t} - E_0^0(t) e^{i\omega_0 t} \right) \mathbf{n} \sigma.$$

For $b_{\perp} = 0$, the interaction Hamiltonian is proportional to the same spin projection $\mathbf{n} \sigma$ which enters the static Hamiltonian (4) and therefore does not produce spin reversal. Then only magnetic dipolar transitions can reverse electron’s spin. However, $b_{\perp} \neq 0$ makes electric field induced spin reversal not only possible, but more probable than magnetic dipolar ones. With the matrix element $\langle \sigma \mid \mathbf{n} \sigma \rangle = 2 |b_{\perp}|/E_{sf}^0$ of the operator $\sigma$ producing spin reversal between the two eigenstates of the operator $\Sigma$, time-dependent perturbation theory gives the spin-flip transition rate for an electron with momentum $p$ as

$$w = \frac{4e^2 \gamma^2}{\pi \omega_0^2} (b_{\perp}/E_{sf}^0)^2 I(2|\gamma|/\hbar - \omega_0).$$

In order of magnitude $I_\omega \approx \pi E_0^2/(\Delta \omega)$, where $\Delta \omega$ is the spectral width of the ac field, so

$$w \approx \pi e^2 E_0^2 (b_{\perp}/E_{sf}^0)^2 / p_F^2 \Delta \omega. \quad (4)$$

The ratio of the electric and magnetic transition rates is $(c b_{\perp}/v_F E_{sf}^0)^2$. The ratio $c/v_F$ is about $10^3$ for InGaAs. Thus, for a comparatively small transverse magnetic field $b_{\perp} \sim 10^{-1} E_{sf}^0$ the transition rate (4) exceeds magnetic dipolar induced rate by 4 orders, whereas the resonance frequency changes only by 1%.

Perturbation theory is valid if the average occupation number for excited electrons is small, i.e. $w \tau_{eff} \ll 1$, where $\tau_{eff}$ is a characteristic lifetime. In the ballistic regime the time of flight $\tau_f = L/v_F$ is much shorter than any collision time and plays the role of lifetime for an excited electron or hole. In 1D elastic scattering can reverse the excitations’ velocity. If the corresponding backscattering time $\tau_0$ is much less than $\tau_f$, then diffusion occurs, with lifetime $\tau_{eff} = \tau_f^2/\tau_0 \gg \tau_f$.

If $w \tau_{eff} \gtrsim 1$, recombination almost completely compensates the excitation and thus further increase of power of the external ac field becomes ineffective. If the spectral width of the external ac field $\Delta \omega$ is smaller than $\tau_{eff}^{-1}$, then Rabi oscillations occur.

**Dynamic generation of dc currents and magnetization.** If the longitudinal field satisfies the condition $b_{\parallel} \gg m\gamma^2$, the resonance lines for right and left movers are distinct and can be excited separately. Thus a resonant linearly polarized ac field can produce a magnetization as well as dc electric and spin currents. Consider a linearly polarized ac field that causes spin flips of right movers. First we obtain upper limits for the stationary density of excited electrons (and holes) and currents.

The density of right-moving states participating in the resonance spin-reversal process is $n_{+n} = n |\Delta \omega/4\omega_c|$. As these states have a lifetime $\sim \tau_{eff}$, the density of excitations can be estimated as $n_{+n} = \min(w \tau_{eff}, 1) n_{+n}$. The hole density is the same. The maximum number of excitations $n_{+n} \leq n/\tau_{eff}$ occurs for $w \tau_{eff} \geq 1$, $|\Delta \omega| = \omega_c$. The spin-flip energy is $E_{sf}^0 = 2m\gamma p_F - \tau b_{\parallel}$. The energy $E_{sf}^0$ is measured from the Fermi level and therefore the spin-flip process is possible if $E_{sf}^0 > \epsilon_F$. If $E_{sf}^0 < \epsilon_F$, the energy difference $\Delta E_{sf} = E_{sf}^0 - \epsilon_F$ must be provided by a constant magnetic field $B = \Delta E_{sf}/|\gamma| q$. The magnetic field is needed to make the spin-flip process possible. The transition rate (4) gives the number of excitations $n_{+n} \approx \left[ \frac{2\pi E_0^2 (b_{\perp}/E_{sf}^0)^2}{p_F^2 \Delta \omega} \right]^{1/2}$.
$4m\gamma^2/\hbar$. The pumped spin per electron is $s = n_{ex}/n$.

An upper limit for the electric current can be obtained on assuming that all excited electrons move with their initial velocity. The equilibrium electric current is zero. After excitation an electron velocity increases by $2\gamma$. Thus, the maximum electric current is $j_{e}^{\text{max}} = 2\gamma en_{ex}^{\text{max}}$. The maximum spin current is $j_{\uparrow}^{\text{max}} = v_F n_{ex}^{\text{max}}$. However, the upper limit usually is not reached, especially in the diffusive regime where elastic back-scattering reverses the excitations’ velocities. For $w\tau_{s} < 1$, in the ballistic regime the electric current is:

$$j_e = 2\gamma en_{ex} = ein\tau_{s}^{\gamma^2/2}/v_F$$ (5)

To show how diffusion affects the currents, for simplicity we neglect both spin-flip back-scattering and energy relaxation. A simplified set of kinetic equations reads:

$$dn_{\uparrow}/dt = \Delta n_{\uparrow} - (\tau_{\uparrow}^{-1} + \tau_{b}^{-1})n_{\uparrow} - \tau_{\uparrow}^{-1}n_{\uparrow}^{\prime}, \quad \text{(6)}$$

$$dn_{\downarrow}/dt = - (\tau_{\downarrow}^{-1} + \tau_{b}^{-1})n_{\downarrow} + \tau_{\downarrow}^{-1}n_{\downarrow}^{\prime}, \quad \text{(7)}$$

where $n_{\uparrow}$ and $n_{\downarrow}$ are the densities of right and left-moving spin up states, respectively. The ac field creates equal numbers of electrons and holes with parallel spins, and this property is maintained by the back-scattering if spin flip process are negligible. The pumped spin is polarized approximately along $n + b_1/\gamma p_F$. Its dc absolute value per unit length is $s_{eff} = 2w\tau_{s}^{\gamma/2}$. The spin current is $j_{s} = g\mu_B v_F n_{sr} w\tau_{s}^{\gamma/2}(2\gamma\tau_{s} + \gamma)$. The electric current is:

$$j_{e} = -2ein\tau_{s}^{\gamma\tau_{b}} - 2\tau_{s}^{\gamma} + \tau_{s}^{\gamma} + ein\tau_{s}^{\gamma\tau_{b}} + \gamma /\gamma p_F^{2}(2\tau_{s}^{\gamma} + \gamma)$$ (8)

Eq (3) shows that the electric current changes sign in diffusive regime at $b_1 > \gamma p_F^{2}y/2\tau_{s}^{\gamma}$. This happens because the back scattering equalizes the number of left and right moving excitations, whose velocities differ. For resonance of left movers, at frequency $\omega_{c} = 2(\gamma p_F + b_1)/\hbar$, the magnitization and currents are reversed.

Relaxation processes. Relaxation processes play an extremely important role in spin resonance phenomena. In a typical wire of 10μm length and 10×10mm² cross area the number of electrons is $\sim 1000$. Because this is small, we neglect the electron-electron interaction. At low temperature the main mechanism of energy relaxation is Cherenkov emission of phonons. If the corresponding relaxation time $\tau_{p}$ becomes comparable to or shorter than $\tau_{s}$, energy relaxation occurs before electrons and holes leave the wire. It does not change the total spin, but may decrease the excitation’s velocity. On the other hand, energy relaxation removes particles from the excited states and fills the depleted states. This makes the increase of power of the external ac field more effective.

The electron-phonon interaction is modeled by a standard Hamiltonian $H_{cp} = U \int \nabla u(x)\psi^{\dagger}(x)\psi(x)$, where $u(x)$ is the displacement vector, $\psi(x)$ is the electron field operator and $U$ is the deformation potential. Electrons in the wire are always one-dimensional, but phonons can be 1, 2, or 3-dimensional depending on the experimental setup. For example, for an electron with momentum deviating by $\xi$ from the Fermi-point, and emitting 3D phonons, a slightly simplified result for the relaxation time is $\tau_{p}^{-1} = \frac{U^2\alpha^2}{6\pi \hbar M \omega_{p}}(v_F \xi/\hbar)^3$, where $M$ and $\alpha$ are the mass and lattice constant of elementary cell; $u$ is the sound velocity.

In 2D and 3D systems elastic scattering (diffusion) leads to spin relaxation by the Dyakonov-Perel mechanism because the direction of the internal Zeeman field $B_{SO}$ depends on the direction of the electron’s momentum and is randomized by diffusion. In 1D for $b_1 = 0$ the direction of $B_{SO}$ is the same for all electrons and Dyakonov-Perel mechanism does not work. For $b_1 \neq 0$, spin flip does occur in back scattering. However, in the resonance regime $\omega_{s} \tau_{s} \gg 1$ the direction of $B_{SO}$ is not random during an oscillation. Spin-flip then has a probability of the order of $\langle b_1 / E_F \rangle^{2}$ and can be neglected. Other spin relaxation mechanisms, such as phonon emission combined with SO interaction, are much weaker.

Numerical estimates. All numerical estimates are made for In0.53Ga0.47As. We take $m = 4.3 \times 10^{-29}$g, $\alpha = 1.08 \times 10^{6} cm/s$, and $g = -0.5$. A typical 2D electron density is $2 \times 10^{12} cm^{-2}$. Let a wire width be $a = 5nm$ and length $l = 1-10 \mu m$. Then we find 1D density $n = 10^{6} cm^{-2}$, $p_F = 1.65 \times 10^{21} g cm/s$ and $v_F = 0.38 \times 10^{8} cm/s$. Assuming $\alpha = \beta$, we have $\omega_{s} = 4.8 \times 10^{12} s^{-1}$ (≈ 0.8THz) and the resonance width $\Delta \omega$ is $3 \times 10^{11} s^{-1}$. The value $E_F^{-2}$ in Eq. (6) is determined by the source power in the terahertz range. Although standard cascade lasers have power in the range 1mW - 1W, the power can be strongly enhanced by non-linear devices, and in very short pulses (1 ps) it can reach 1MW. The free-electron laser at UCSB provides continuous power 1-6 kW for the frequency range 0.9-4.75THz. On focusing, the energy flux becomes up to 40kW/cm². For the moderate flux $S = 1kW/cm²$, we find $E_F^{-2}$ = $S/c$ = 0.33erg-cm⁻³. For $B_{1} = 10T$ we have $b_1 / E_F^{1/2}$ = 0.048, and Eq. (4) yields $w = 0.8 \times 10^{9} s^{-1}$. As noted above, $w$ can be increased by changing the power or size of the focus area. For the length 1-10 μm the time of flight is $t_{f} = 2.6 \times (10^{12}-10^{11})$. The back-scattering time $\tau_b$ can be estimated from typical mobilities $\mu = e\tau / m = (2 \times 10^{4} - 4 \times 10^{5}) cm²/Vs$ in the bulk or film. Since the scattering cross-section area is much less than the wire cross-section area, $\tau$ can be identified with $\tau_b$. Typical values are $\tau_b = 2.7 \times 10^{13}-10^{12}$ s. In this case the regime is either diffusive or marginal between diffusive and ballistic. In the ballistic regime with $t_f = 2.6 \times 10^{12}$s, according to Eq. (5) the electric current equals 18mA. The spin current is $j_{s} = 8.3 \times 10^{9} s^{-1}$. In the diffusive regime ($\tau_b = 2.7 \times 10^{-12} s, \tau_f = 2.6 \times 10^{-11} s$), ac-
according to Eq. 8 the electric current \( I_e \) in the wire equals 3.6 pA and the magnetization per particle, in Bohr magnetons, is \( \frac{2e}{m^*} u \tau_{\text{eff}} \sim 0.002 \). The temperature must be maintained below \( 2\gamma p_F/k_B \approx 35 \). \( u \tau_{\text{eff}} \) is about 0.12, indicating that perturbation theory works well and there is a possibility for current and spin increase.

For energy relaxation we assume 3D phonons and take numerical values for InGaAs: deformation potential \( U = 16 eV \), sound velocity \( u = 3.3 \times 10^5 \text{cm/s} \), \( a = 5 \AA, M = 1.8 \times 10^{-22} \text{g} \) and \( \xi = m \gamma \). Then the energy relaxation time \( \tau_{\text{ep}} = 1.4 \times 10^{-12} \text{s} \). With 2D and 1D phonons the formulae differ, but numerical estimates give the same order of magnitude. This result shows that, even in the ballistic regime, \( \tau_{\text{ep}} \) is usually much shorter than \( \tau_f \) and energy relaxation is substantial, decreasing the currents.

Prior work. The generation of currents by an ac field is similar to the photogalvanic effect predicted by Ivchenko and Pikus [18] and by Belinicher [19]. More recently many clever modifications of this effect have been proposed and experimentally observed (see review [20], and article [21]). They are mostly realized in 2D systems, but more importantly, unlike our case non-resonance optical or infrared radiation is used. The resonant nature of our effect also ensures the strong dependence of the resonance line and transition probability on the transverse magnetic field.

Shechter et al. considered a similar problem in a 2D film with SO interaction. [22] They noted that Rashba SO [3] leads to what they called a chiral resonance. Transitions between states with different chirality are possible only for electrons with momenta located between two Fermi-circles corresponding to different chiralities. However, as the authors themselves noted, the Dresselhaus SO interaction [3] strongly broadens the resonance. This broadening is of the same order of magnitude as the spin-flip energy. In Ref. [22] the authors proposed to avoid this broadening by choosing a specific growth direction, or by decreasing the electron density. Both proposals require sophisticated experimental techniques.

Conclusions. We predict that in 1D the SO interaction gives rise to spin resonance, even without an external magnetic field. This happens because for a 1D degenerate Fermi gas the direction of the effective SO magnetic field is the same for all \( p \). The resonance frequency is typically in the terahertz region with relative width depending linearly on the Dresselhaus and Rashba SO constants. The external longitudinal magnetic field (parallel to the internal SO field) separates the resonance frequencies of the left and right movers, producing charge and spin currents. A perpendicular magnetic field violates reflection symmetry and couples an ac electric field to the spin which otherwise is flipped only by the weak magnetic dipolar interaction. On resonance an ac electric field linearly polarized along the wire produces dc charge current, dc spin current, and dc magnetization. The amplitude of these effects can be easily controlled by the static external magnetic field and the gate voltage.

Acknowledgements. We thank A.M. Finkelstein and M. Khodas for discussions and for pointing out their work. [22] and J. Kono for an illuminating discussion of the experimental situation. This work was supported by the Department of Energy under the grant DE-FG02-06ER46278. Ar.A. was supported by NSF0757992 and Welch Foundation (A-1678).

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