CP Violation in Supersymmetric Theories:

\( \tilde{t}_2 \to \tilde{t}_1 HH, \tilde{t}_2 \to \tilde{t}_1 ZZ, \tilde{t}_2 \to \tilde{t}_1 W^+W^-, \tilde{t}_2 \to \tilde{t}_1 ZH \)

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Abstract

We study the decays \( \tilde{t}_2 \to \tilde{t}_1 HH, \tilde{t}_2 \to \tilde{t}_1 ZZ, \tilde{t}_2 \to \tilde{t}_1 W^+W^- \) and \( \tilde{t}_2 \to \tilde{t}_1 ZH \), with an eye towards measuring the CP-violating supersymmetric parameters contained in these processes. We find that \( \tilde{t}_2 \to \tilde{t}_1 HH \) tends to have the largest CP asymmetry and width, and is perhaps the most favourable experimentally. These decays are sensitive primarily to \( \phi_{A_t} \), the phase of the trilinear coupling \( A_t \).
Supersymmetry (SUSY) is widely thought to be the physics that lies beyond the standard model (SM). SUSY theories typically contain many new parameters, some of which are complex, and hence violate CP. Should SUSY be found experimentally, we will want to find the values of all of these parameters. In particular, it will be important to measure the CP-violating SUSY phases.

CP-violating SUSY effects have been studied extensively in meson mixing [1], in CP violation related to the $B$-meson system [2] and in electric dipole moments (EDMs) [3]. In particular, EDMs provide quite stringent constraints on the low-energy CP-violating SUSY phases of the superparticle couplings. For example, if sfermion masses are taken to be of order the weak scale and complex SUSY parameters have phases of order unity, theoretical predictions for EDMs are not generically in agreement with experimental limits. Nevertheless, the so-called SUSY CP problem can be avoided in several SUSY scenarios [4] as calculations of EDMs are highly model-dependent. In this paper, all SUSY parameters used as inputs in the numerical simulations are assumed not to violate EDM constraints. The approach adopted here is that our observables may offer an independent measurement of relevant CP-odd SUSY parameters.

In a recent paper [5], we showed that the decay $\tilde{t}_2 \rightarrow \tilde{t}_1 \tau^- \tau^+$ is particularly sensitive to $\phi_{A_t}$, the phase of the trilinear coupling $A_t$. (The “stops” $\tilde{t}_1$ and $\tilde{t}_2$ are the two mass eigenstates of the scalar superpartners of the top quark, with $m_{\tilde{t}_2} > m_{\tilde{t}_1}$.)

In the present paper, we examine how some other decay processes depend on the SUSY phases. Throughout we assume that SUSY has been discovered, and that the CP-conserving parameters (e.g. masses of SUSY particles) are known independently. All effects which violate CP require the interference of (at least) two amplitudes. For a given decay, there are two types of CP-violating signals. Direct CP asymmetries (spin-independent or spin-dependent) are proportional to $\sin \delta$, where $\delta$ is the relative strong (CP-even) phase between the interfering amplitudes. Triple-product (TP) asymmetries take the form $\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)$ (each $v_i$ is a spin or momentum), and are proportional to $\cos \delta$. One can therefore have a nonzero TP even if $\delta = 0$. It is also possible to have a nonzero TP with only a single decay amplitude if, for example, the intermediate particle has both scalar and pseudoscalar couplings [5, 6].

Strong phases can be generated in one of two ways. First, one can have the exchange of gluons between the particles involved in the decay, leading to QCD-based strong phases. Unfortunately, we do not know how to calculate the strong phases in this case. Alternatively, the strong phases can be generated by the (known) widths of the intermediate particles in the decay. Given that we want to measure the SUSY CP phases, and not simply detect the presence of CP violation, we must consider decays in which the strong phases are generated in the second way. Thus, the decay processes cannot contain too many particles which couple to gluons.

It is therefore quite natural to consider the decays $\tilde{t}_2 \rightarrow \tilde{t}_1 HH$, $\tilde{t}_2 \rightarrow \tilde{t}_1 ZZ$, $\tilde{t}_2 \rightarrow \tilde{t}_1 W^+ W^-$ and $\tilde{t}_2 \rightarrow \tilde{t}_1 Z H$, where $H$ is a neutral Higgs boson. There are several points here which we should note in relation to these processes. First, SUSY theories...
involve two Higgs doublets which contain (in the gauge basis) two neutral scalars
and one pseudoscalar. In the mass basis, these particles mix and one obtains three
mass eigenstates \( H_1, H_2 \) and \( H_3 \). All four decays receive contributions from several
diagrams, including those with an intermediate \( H_i \) (\( i = 1, 2, 3 \)). The amplitudes
are typically dominated by those diagrams in which an intermediate particle can
go on shell. In SUSY theories, the lightest mass is \( m_{H_1} = O(100) \) GeV and we
therefore take the final-state \( H \) to be \( H_1 \). When it appears as an internal line in
a diagram, the \( H_1 \) is too light to decay (on shell) to \( H_1H_1, ZZ, W^+W^- \) or \( ZH_1 \).
Nevertheless, we shall retain such diagrams since they can in principle give non-
negligible contributions.

Second, triple products are due to terms of the form \( \text{Tr}[\gamma_\alpha\gamma_\beta\gamma_\rho\gamma_\sigma\gamma_5] \) in the square
of the amplitude. Since no fermions are involved in the decay processes, no CP-
violating TP’s can arise here, and we have only direct CP asymmetries. The final-
state particles do not couple to gluons and any exchange of gluons between \( \tilde{t}_2 \) and \( \tilde{t}_1 \)
only serves to renormalize the couplings of the stops. Thus, the strong phase arises
only due to the widths of the intermediate particles \( H_2 \) and \( H_3 \). For a given set
of SUSY parameters, the widths \( \Gamma_2 \) and \( \Gamma_3 \) are calculable (as are the ‘off-diagonal
widths’ associated with transitions \( H_i \leftrightarrow H_j \)). Thus, the measurement of CP
violation in these decays (due to direct CP asymmetries) will allow us to extract
and/or constrain the SUSY parameters, including the CP-violating SUSY phases.

Third, there can be significant interference between the two decay amplitudes
only if the masses of \( H_2 \) and \( H_3 \) are similar. Fortunately, it is relatively common in
SUSY theories that \( m_{H_2} \simeq m_{H_3} \). Since it is assumed that the masses are known,
we will know beforehand whether or not CP violation is likely in these decays.

Finally, we make a comment regarding our previous work, which examined CP
asymmetries in \( \tilde{t}_2 \rightarrow \tilde{t}_1\tau^-\tau^+ \). The process \( \tilde{t}_2 \rightarrow \tilde{t}_1\tau^-\tau^+ \) is interesting since it is
theoretically clean (no strong phases from gluons) and it can have large CP asymme-
tries while simultaneously having a moderately large branching fraction. This pro-
cess is particularly attractive if the intermediate Higgs bosons are too light to decay
to heavier final states, such as those considered in the present work (\( W^+W^-, H_1H_1, \) etc.). Nevertheless, as noted in Ref. [5], the simple rate asymmetry for \( \tilde{t}_2 \rightarrow \tilde{t}_1\tau^-\tau^+ \) is extremely small. Thus, sizeable CP asymmetries are only expected in cases where
the spin of one or both of the final-state leptons is measured. The processes con-
sidered in the present work have an advantage over \( \tilde{t}_2 \rightarrow \tilde{t}_1\tau^-\tau^+ \) in that they do not require the measurement of any spins – the regular rate asymmetries can have
relatively large values.

There are five classes of diagrams that contribute to the four processes under
consideration, although not every class of diagram contributes to each process. The
first class is shown in Fig. 1. In this case the heavier stop decays to the lighter
stop and emits a Higgs boson. The Higgs then decays to the final state \( f_1f_2 \) (where
\( f_1f_2 = H_1H_1, \) etc.) This process can proceed resonantly if the intermediate Higgs
boson(s) can go on shell. Figure 2 shows the other four possibilities. In diagram
(a) the stop decays to particle $f_2$ and a squark $\tilde{q}_j$ (either a stop or a sbottom), which subsequently decays into $f_1$ and the lighter stop. Diagram (b) is a crossed version of diagram (a). Diagram (c) is similar to that shown in Fig. 1 but with the intermediate Higgs replaced by a $Z$. Finally, diagram (d) shows the contribution due to a possible quadrilinear vertex.

Of all the diagrams shown in Fig. 2, there is only one case in which the process can proceed resonantly. This occurs for the decay $\tilde{t}_2 \rightarrow \tilde{t}_1 f_1 f_2$ with $f_1 f_2 = H_1 H_1, ZZ, W^+ W^-$ and $Z H_1$. We define two invariant masses as follows:

\begin{align}
M^2 &= (p_1 + p_2)^2, \\
\rho^2 &= (p_1 + p_{\tilde{t}_1})^2,
\end{align}

where $p_{1,2}$ are the four-momenta of $f_1$ and $f_2$, respectively, and $p_{\tilde{t}_1}$ is the four-momentum of the $\tilde{t}_1$. All dot products that arise in the calculation may be written in terms of $M^2$, $\rho^2$ and the various particle masses.

As noted above, there are no TP’s in this process. In principle there are polarization-dependent CP-violating observables similar to the single-spin CP asymmetry defined in Ref. [5]. It turns out that, in the limit in which one can neglect the non-Higgs-mediated diagrams, such polarization-dependent observables are sensitive to the same combinations of underlying SUSY parameters as are the rate
asymmetries. Thus, in this limit, one gains no new information by measuring polarizations. In principle the cross terms of resonant and non-resonant amplitudes can give new contributions to polarization-dependent CP-odd observables, but such contributions are expected to be suppressed. Given the difficulty of the related measurements, and the possible suppression, we will ignore such observables and sum over the polarizations of the final-state particles.

Let us consider first the “processes” \( \tilde{t}_2^- \rightarrow \tilde{t}_1^- f_1 f_2 \) (as opposed to the “anti-processes,” involving the decay of a \( \tilde{t}_2^+ \), which will be considered in a moment). (Note that the indices \( \pm \) associated with the \( \tilde{t} \)'s indicate that they have charge \( \pm 2/3 \).) The Higgs-exchange diagrams shown in Fig. 1 contribute for all four final states. The following are some technical details for each process regarding the diagrams shown in Fig. 2.

1. \( \tilde{t}_2^- \rightarrow \tilde{t}_1^- H_1 H_1 \): Diagrams (a), (b) and (d) in Fig. 2 contribute. (The \( H_1^- H_1^- Z \)}
coupling is zero.) The intermediate $\tilde{q}_j$ are stops. These stops cannot go on shell.

2. $\tilde{t}_2 \rightarrow \tilde{t}_1 ZZ$: Diagrams (a), (b) and (d) contribute. The intermediate $\tilde{q}_j$ in diagrams (a) and (b) are stops; they cannot go on shell.

3. $\tilde{t}_2 \rightarrow \tilde{t}_1 W^+W^-$: Diagrams (a), (c) and (d) contribute ($f_1 = W^+$ and $f_2 = W^-$). The intermediate squarks in diagram (a) are sbottoms. In principle the sbottoms could go on shell, although in practice we choose parameters such that this does not occur\(^1\).

4. $\tilde{t}_2 \rightarrow \tilde{t}_1 ZH_1$: Diagrams (a), (b) and (c) contribute. The intermediate top squarks in diagrams (a) and (b) cannot go on shell.

The amplitudes for the four processes may be written in the following manner:

\[
\begin{align*}
A_{HH} &= B_{HH} , \\
A_{ZZ} &= \left( B_{ZZ} g^{\mu\nu} + C_{ZZ} p_2^\mu p_1^\nu + D_{ZZ} p_2^\mu p_1^\nu + E_{ZZ} p_2^\mu p_1^\nu \right) \epsilon^\lambda_1^* \epsilon^\lambda_2^* , \\
A_{WW} &= \left( B_{WW} g^{\mu\nu} + C_{WW} p_2^\mu p_1^\nu + D_{WW} p_2^\mu p_1^\nu + E_{WW} p_2^\mu p_1^\nu \right) \epsilon^\lambda_1^* \epsilon^\lambda_2^* , \\
A_{ZH} &= \left( B_{ZH} p_2^\mu + C_{ZH} p_1^\mu \right) \epsilon^\lambda_1^* ,
\end{align*}
\]

where the $\epsilon^{\lambda_1,\lambda_2}_{\mu,\nu}$ are the polarization tensors for the final-state vector mesons. The parameters $B_{f_1f_2}, \ldots, E_{f_1f_2}$ are functions of the various coupling constants and mixing matrices. In each case, $B_{f_1f_2}$ contains the Higgs-mediated contributions, as well as one or more other contributions coming from diagram(s) in Fig. 2 that is,

\[
B_{f_1f_2} = B_{f_1f_2}^{\text{Higgs}} + \delta B_{f_1f_2} ,
\]

where $\delta B_{f_1f_2}$ denotes the non-Higgs-mediated contributions. Expressions for $\delta B_{f_1f_2}$, $C_{f_1f_2}$, $D_{f_1f_2}$ and $E_{f_1f_2}$ may be found in the Appendix.

We include here the expressions for the Higgs-mediated contributions, $B_{f_1f_2}^{\text{Higgs}}$, since they are of the most interest to us (they tend to dominate, and they are also

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\(^1\) If one or both sbottoms go on shell the calculation becomes more complicated. In particular, the asymmetry would depend on the total widths of the sbottoms. Furthermore, gluon exchange between $\tilde{t}_2$ and $\tilde{t}_1$ in diagram (a) could give rise to additional strong phases. Even if one were to include the on-shell sbottoms diagrams, the resulting asymmetry would tend to decrease. The sbottom diagrams are unlikely to interfere with each other, since sbottom masses are generically not close to each other. Also, since sbottoms do not proceed via the $s$-channel, the on-shell interference between Higgs bosons and sbottoms is restricted to a very small region in the Dalitz plot.
the source of the required strong phases). These contributions are given by,

\[ B_{HH}^{\text{Higgs}} = -v^2 \sum_{i,j} g_{H_i Z_i} D_{ij}(M^2) g_{H_i H_i \eta_j}, \]  
\[ B_{ZZ}^{\text{Higgs}} = -vgm_W \cos^2 \theta_W \sum_{i,j} g_{H_i Z_i} D_{ij}(M^2) g_{H_i \nu \nu}, \]  
\[ B_{WW}^{\text{Higgs}} = -vgm_W \sum_{i,j} g_{H_i Z_i} D_{ij}(M^2) g_{H_i \nu \nu}, \]  
\[ B_{ZH}^{\text{Higgs}} = \frac{ivg}{\cos \theta_W} \sum_{i,j} g_{H_i Z_i} D_{ij}(M^2) g_{H_i H_i Z}, \]

where we have included factors \( \eta_1 = 6 \) and \( \eta_2,3 = 2 \) in order to correctly account for the manner in which the \( g_{H_i H_i H_k} \) are defined in Ref. [9]. The Higgs propagator matrix is given to a good approximation by [7, 10]

\[ iD(M^2) = \]

\[
\begin{pmatrix}
M^2 - m_{H_1}^2 + i \text{Im} \hat{\Pi}_{11} & i \text{Im} \hat{\Pi}_{12} & i \text{Im} \hat{\Pi}_{13} \\

i \text{Im} \hat{\Pi}_{12} & M^2 - m_{H_2}^2 + i \text{Im} \hat{\Pi}_{22} & i \text{Im} \hat{\Pi}_{23} \\
i \text{Im} \hat{\Pi}_{13} & i \text{Im} \hat{\Pi}_{23} & M^2 - m_{H_3}^2 + i \text{Im} \hat{\Pi}_{33}
\end{pmatrix}^{-1}
\]

Expressions for the absorptive parts of the Higgs-boson self-energies, \( \text{Im} \hat{\Pi}_{ij}(M^2) \), may be found in Ref. [7]. The Appendix of the present work contains a brief discussion of the various couplings \( g_{H_i Z_i}, g_{H_i H_i H_k}, g_{H_i \nu V} \) and \( g_{H_i H_i Z} \). Of these couplings, only \( g_{H_i Z_i} \) is complex. We adopt the notation of Ref. [9] for these couplings, except in the case of \( g_{H_i H_i Z} \).

To obtain the width for \( t_2 \to t_1 f_1 f_2 \), we multiply the respective amplitude by its complex conjugate, sum over the polarization states of the vector boson(s) in the final state (if appropriate) and integrate over the squares of the two invariant masses, \( M^2 \) and \( \rho^2 \), to obtain

\[
\Gamma_{f_1 f_2} = \frac{S_F}{256 \pi^3 m_t^3} \int \left( \sum_{\text{pol}} |A_{f_1 f_2}|^2 \right) dM^2 d\rho^2,
\]

where \( S_F = 1 \) for \( f_1 f_2 = WW, ZH \) and \( S_F = \frac{1}{2} \) for \( f_1 f_2 = H_1 H_1, ZZ \). For the \( H_1 H_1 \) case there is no sum over polarizations and the calculation is relatively straightforward. For the \( ZZ \) and \( WW \) cases there are two sums over polarization states (one for each vector particle in the final state), leading to ten separate terms, each with its own kinematical factor. The terms are proportional to \( |B|^2, \text{Re}(BC^*) \), etc. The \( ZH_1 \) final state only requires one sum over polarization states, resulting in three separate terms. The appropriate range of integration for \( M^2 \) and \( \rho^2 \) may be found, for example, in Ref. [11].
The width $\Gamma_{f_1f_2}$ can be similarly defined for the CP-conjugate process $\tilde{t}_2^+ \rightarrow \tilde{t}_1^+ f_1 f_2$. The only difference compared to the expression in Eq. (13) is that one must complex conjugate the weak phases in $B_{f_1f_2}, \ldots, E_{f_1f_2}$, which amounts to making the replacements $g_{H_i t_k^* t_k} \rightarrow g_{H_i t_k^* t_k}^*$ and $U^T \leftrightarrow U^{T*}$ in the various expressions, as well as $i \rightarrow -i$ in Eq. (11). Some further discussion may be found in the Appendix. ($U^\dagger$ is the stop mixing matrix.) For the rate asymmetries to be non-zero, we require (at least) two interfering amplitudes, and these must have a non-zero relative strong phase as well as a non-zero relative weak phase. The weak phases appear in the various couplings and the strong phases are provided by absorptive pieces in the Higgs propagator matrix. (In principle, there are also strong phases associated with the widths of the $Z$ and the squarks in the diagrams in Fig. 2. The effects of these widths are suppressed, however, since the associated (s)particles are always off-shell in our calculation.)

The rate asymmetries for the four cases are then defined to be

$$A_{CP}(\tilde{t}_2 \rightarrow \tilde{t}_1 f_1 f_2) = \frac{\Gamma_{f_1f_2} - \Gamma_{f_1f_2}}{\Gamma_{f_1f_2} + \Gamma_{f_1f_2}}.$$  \hspace{1cm} (14)

Our previous paper \cite{5} contains an extended discussion of the behaviour of CP asymmetries as functions of the invariant mass $M$. The analysis in the present case is complicated by the non-negligible contributions of diagrams that are not in the $s$-channel. Nevertheless, one could still define differential widths as functions of $M$, and these would still be expected to exhibit resonant peaks near $M \approx m_{H^{\pm}}$.

We now turn to a numerical investigation of rate asymmetries for $\tilde{t}_2 \rightarrow \tilde{t}_1 H_1 H_1, \tilde{t}_1 ZZ, \tilde{t}_1 W^+W^-$ and $\tilde{t}_1 ZH_1$. In our numerical work we have made extensive use of the computer program CPsuperH \cite{9}. Note that since we are using the widths of the Higgs bosons to provide the required strong phase, the rate asymmetries will only be non-negligible if there is a significant overlap of the interfering resonances (see Ref. \cite{5} for further discussion of this point). Fortunately, as noted above, it is not uncommon to have $m_{H_2} \approx m_{H_3}$ in SUSY.

Figure 3 shows several scatter plots of the rate asymmetries for $\tilde{t}_2 \rightarrow \tilde{t}_1 H_1 H_1, \tilde{t}_1 ZZ, \tilde{t}_1 W^+W^-$ and $\tilde{t}_1 ZH_1$. In these plots we have allowed several SUSY parameters to vary in specified ranges, taking $m_{H^{\pm}} \in (160-500)$ GeV, $\tan \beta \in (1-15)$, $\mu \in (200-1400)$ GeV (without loss of generality, the $\mu$ parameter is taken to be real and positive – see Ref. \cite{5} for further discussion), $m_{\tilde{Q}_3}, m_{\tilde{U}_3}, m_{\tilde{D}_3} \in (300-700)$ GeV, $|A_t| \in (400-2000)$ GeV and $\phi_{A_t} \in (0^\circ-360^\circ)$. All other input parameters have

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2 Since the Higgs bosons are typically mixtures of scalar and pseudoscalar states when CP is broken, some of the final states that we consider are not eigenstates of CP. Nevertheless, the widths we calculate for what we are calling the “CP-conjugate” processes are in fact the widths that would be measured experimentally. Furthermore, the asymmetries that we calculate are zero in the CP-even limit.
been assigned fixed values and have been taken to be real.\footnote{For completeness, we list here some of the other parameter choices, in CPsuperH notation: $M_1 = 100$ GeV, $M_2 = 200$ GeV, $m_{\tilde{g}} = M_3 = 1000$ GeV, $m_{\tilde{L}_3} = 150$ GeV, $m_{\tilde{E}_3} = 600$ GeV, $A_b = 1000$ GeV and $A_{\tau} = 750$ GeV. With these parameter choices, and after applying cuts noted subsequently in the text, some of the supersymmetric particles have the following mass ranges: $m_{\tilde{\nu}_\tau} \in (136 - 138)$ GeV, $m_{\tilde{t}_1} \in (199 - 558)$ GeV, $m_{\tilde{\chi}^0_1} \in (468 - 883)$ GeV, $m_{\tilde{g}_1} \in (255 - 685)$ GeV, $m_{\tilde{t}_1} \in (146 - 156)$ GeV, $m_{\tilde{\chi}_1^\pm} \in (196 - 200)$ GeV and $m_{\tilde{\chi}_1^0} \in (100 - 101)$ GeV. Also, the mass ranges for the three Higgs bosons are $m_{H_1} \in (100 - 126)$ GeV, $m_{H_2} \in (161 - 490)$ GeV and $m_{H_3} \in (161 - 495)$ GeV.} We have insisted that the mass of the lightest Higgs boson be greater than or equal to 100 GeV and have chosen values for the charged Higgs mass and $\tan \beta$ that are consistent with the recent bound from Belle, $\tan \beta / m_{H^\pm} \lesssim 0.146$ GeV$^{-1}$ \cite{12}. We have also insisted that all supersymmetric particles have masses greater than or equal to 100 GeV and that the lighter stop be kinematically allowed to decay by at least one of the following two modes: $\tilde{t}_1 \to t \tilde{\chi}_1^0$ or $\tilde{t}_1^\pm \to b \tilde{\chi}_1^\pm$. For the purpose of these plots we have set the widths of intermediate stops and sbottoms to be 10 GeV \cite{13}. (Varying these widths does not seem to have a significant effect on the numerical values obtained.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{scatter_plots}
\caption{Scatter plots showing rate asymmetries for $\tilde{t}_2 \to \tilde{t}_1 H_1 H_1, \tilde{t}_1 ZZ, \tilde{t}_1 W^+ W^-$ and $\tilde{t}_1 Z H_1$. The horizontal axis in each plot gives the sum of the widths for the process and the anti-process for the decay in question.}
\end{figure}
for the asymmetries, as long as the sbottoms are not allowed to go on shell.) The horizontal axes in these plots show the sum of the widths for the process and the anti-process. If we assume a typical stop width to be approximately 10 GeV, we see that it is possible to have large rate asymmetries while simultaneously having relatively large branching ratios. For example, in $\tilde{t}_2 \rightarrow \tilde{t}_1 H_1 H_1$ it is possible to have an asymmetry with magnitude of order 20–30% when $\Gamma_{H_1 H_1} + \Gamma_{H_1 H_1} \simeq 0.2$ GeV. Among the decays considered, $\tilde{t}_2 \rightarrow \tilde{t}_1 H_1 H_1$ is clearly favoured in that it tends to have a larger width for a given CP asymmetry. Experimentally, however, some of the other decay channels may be favoured due to increased detection efficiencies. Note that the $WW$ final state has fewer data points compared to the other plots because parameter sets for which a sbottom would have gone on shell have been discarded for that plot.

In Figure 3 we have allowed the phase of the trilinear coupling $A_t$ to vary, but have set all other phases to zero. In addition to the results shown in Fig. 3, we have also performed a limited analysis of the effects on the CP asymmetries due to the phases of other SUSY parameters. The strongest effects come from $\phi_{A_b}$ and $\phi_{M_3}$, the phases of the trilinear coupling $A_b$ and gluino mass $M_3$, respectively. The phase $\phi_{A_b}$ can affect the asymmetries due to the involvement of $A_b$ in the mixing of the Higgs bosons. The phase of $M_3$ comes into play through its effect on the Higgs-bottom-bottom effective vertex\footnote{Even though the decays under consideration do not involve this effective vertex in a direct way, the strength of this vertex can affect the widths of the two heavier Higgs bosons, whose values can undergo large variations for certain values of $\phi_{M_3}$. This could lead, for instance, to a larger overlap between the two heavier Higgs bosons in the available phase space, giving rise to an enhancement of the CP asymmetries.}. The phase $\phi_{A_b}$ only seems to produce a non-negligible effect when $|A_b|$ is greater than 5 TeV (due to the difference in the top and bottom Yukawa couplings). Allowing $\phi_{M_3}$ to assume a nonzero value can lead to changes in the CP asymmetries of order $\pm 0.1$ if $|M_3|$ is taken to be approximately 1 TeV.

To summarize, many physicists believe that supersymmetry (SUSY) is the physics that lies beyond the standard model, and that it will be found at a future high-energy collider. Assuming that this is the case, we will want to measure the CP-violating SUSY parameters. With this in mind, in a previous paper \cite{5}, we studied the decay $\tilde{t}_2 \rightarrow \tilde{t}_1 \tau^- \tau^+$. We found that the CP asymmetries could be large, but that they require the measurement of the spin of one or both of the final-state $\tau$'s.

In the present paper we have studied the decays $\tilde{t}_2 \rightarrow \tilde{t}_1 H H$, $\tilde{t}_2 \rightarrow \tilde{t}_1 ZZ$, $\tilde{t}_2 \rightarrow \tilde{t}_1 W^+W^-$ and $\tilde{t}_2 \rightarrow \tilde{t}_1 ZH$, constructing rate asymmetries that are sensitive to CP violation in the underlying SUSY theory. There are several ways in which the current work complements that performed in our previous study. First, as we have shown, the CP asymmetries considered here do not require the measurement of any spins – the rate asymmetries alone are measurable and they can be relatively large. Second, we had found in our previous work that $\tilde{t}_2 \rightarrow \tilde{t}_1 \tau^- \tau^+$ is sensitive primarily to $\phi_{A_t}$.\footnote{Even though the decays under consideration do not involve this effective vertex in a direct way, the strength of this vertex can affect the widths of the two heavier Higgs bosons, whose values can undergo large variations for certain values of $\phi_{M_3}$. This could lead, for instance, to a larger overlap between the two heavier Higgs bosons in the available phase space, giving rise to an enhancement of the CP asymmetries.}
the phase of the trilinear coupling \( A_t \). As noted above, the decays considered in this work are also sensitive primarily to \( \phi_{A_t} \), although the phases associated with the trilinear coupling \( A_b \) and the gluino mass \( M_3 \) can also affect the asymmetries. Third, it is worth noting that the decays considered here tend to involve a different range of Higgs masses than those considered in our previous work. For example, in the present case we need to have \( m_{H_{1,2,3}} > 1.2 m_W \) in order for \( \tilde{t}_2 \to \tilde{t}_1 W^+ W^- \) to yield an appreciable asymmetry along with a moderately large branching fraction. In our former work, non-negligible asymmetries and branching fractions could be obtained for lower Higgs masses.

Of the processes studied in this work, we find that \( \tilde{t}_2 \to \tilde{t}_1 H H \) tends to have the largest CP asymmetry and width, and so it is the process which is perhaps most amenable to study. (Note that experimental detection efficiencies may make other decay modes more favourable.) As we have emphasized, the decays of stops depend primarily on \( \phi_{A_t} \), and so do not provide a sensitivity to a large number of phases. It would be of considerable interest to investigate CP asymmetries in analogous decays of sbottoms or staus to assess their sensitivity to various SUSY phases (\( \phi_{A_b}, \phi_{A_t}, \phi_{A_s} \), and probably also \( \phi_{A_1} \)). We are currently beginning such a study to determine if such decays might provide complementary tools for the study of CP violation in SUSY.

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**APPENDIX: EXPRESSIONS FOR COUPLING CONSTANTS AND AMP\-PLITUTES**

In this Appendix we describe some of the Higgs couplings in Eqs. (8)-(11) in more detail. We also provide the full expressions for the amplitudes referred to in Eqs. (12)-(16).

The couplings between the stops and Higgs bosons are defined as follows [9],

\[
v g_{H_{i,j,k}} = \left( \Gamma^{\alpha i i} \right)_{\beta \gamma} O_{\alpha i} U_{\beta j} U_{\gamma k},
\]

where \( O \) and \( U_i \) denote the Higgs and stop mixing matrices [9], respectively, and where \( i = 1, 2, 3 \) and \( j, k = 1, 2 \). As usual, \( v = \sqrt{v_1^2 + v_2^2} \) is defined in terms of the
vacuum expectation values of the two Higgs doublets. Expressions for the $2 \times 2$ matrices $\Gamma^{\alpha\tilde{t}}$ may be found in Appendix B of Ref. [9]. These matrices depend on the SUSY parameters $A_t, \mu, \cos \beta$ and $\sin \beta$ (where $\tan \beta \equiv v_2/v_1$). It is interesting to note that the couplings involving scalar Higgs bosons are real and those involving the pseudoscalar Higgs are purely imaginary. These couplings are generally complex if CP is broken.

For the Higgs-Higgs-Z vertex we adopt the same notation as in Ref. [14] (this differs somewhat from that employed in Ref. [9]):

$$L_{HHZ} = \frac{g}{2 \cos \theta_W} \sum_{j>i} g_{H_iH_jZ} Z^\mu H_i \partial_\mu H_j , \quad (A.2)$$

where

$$g_{H_iH_jZ} = \begin{cases} O_{3i}(c_\beta O_{2j} - s_\beta O_{1j}) - O_{3j}(c_\beta O_{2i} - s_\beta O_{1i}), & j > i \\ 0, & \text{otherwise}, \end{cases}$$

with $c_\beta = \cos \beta$ and $s_\beta = \sin \beta$. The Higgs-$W$-$W$ and Higgs-$Z$-$Z$ vertices are both proportional to the coupling $g_{H,VV}$, defined as follows [9]:

$$g_{H,VV} = c_\beta O_{1i} + s_\beta O_{2i} . \quad (A.3)$$

Finally, expressions for the trilinear Higgs couplings $g_{H_iH_jH_k}$ are given in Ref. [9] and references therein.

We now turn to a consideration of the full expressions for the amplitudes in Eqs. (3)-(6). The parameters $B_{f_1f_2}$ always include the Higgs-mediated pieces, but also include other contributions with similar kinematical structures. In the text these have been parameterized as

$$B_{f_1f_2} = B_{f_1f_2}^{\text{Higgs}} + \delta B_{f_1f_2} .$$

The expressions for the Higgs-mediated pieces, $B_{f_1f_2}^{\text{Higgs}}$, are given in Eqs. (8)-(11). In the expressions below we employ the Breit-Wigner form of the propagator for sbottom- and stop-mediated graphs, defining

$$i\tilde{D}(p^2, m^2, \Gamma) \equiv \frac{i}{p^2 - m^2 + i\Gamma m} , \quad (A.4)$$

where we have used a tilde to distinguish the Breit-Wigner propagator from the $3 \times 3$ Higgs propagator matrix defined in Eq. (12). We also use a Breit-Wigner-type propagator for $Z$-mediated graphs. Working in Unitary gauge, we take the propagator to be $i\tilde{D}(p_2^2, m_Z^2, \Gamma_Z) \left(-g^{\alpha\beta} + \frac{p_2^\alpha p_2^\beta}{m_Z^2} \right)$. We employ the following definitions for the various combinations of momenta that appear in the propagators,

$$M^2 = (p_1 + p_2)^2 ,$$
$$\rho^2 = (p_1 + p_i)^2 ,$$
$$\xi^2 = (p_2 + p_i)^2 ,$$

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where we note that $\xi^2$ can be written as a function of $M^2$, $\rho^2$ and various masses.

The Feynman rules used to build many of the following amplitudes were extracted from Ref. [15] (with appropriate modifications to allow for scalar-pseudoscalar mixing among the Higgs bosons). Amplitudes corresponding to the charge conjugated decays can be obtained from those listed below [and in Eqs. (8)-(11)] by taking the complex conjugate everywhere except in the propagator functions $D_{ij}$ and $\tilde{D}$. This is equivalent to making the replacements $g_{H_1i\tilde{t}j\tilde{t}_k} \to g_{H_1i\tilde{t}j\tilde{t}_k}^*$ and $U_{\tilde{t}} \leftrightarrow U_{\tilde{t}}^*$ in the various expressions, as well as $i \to -i$ in Eq. (11).

1. $\tilde{t}_2^\rightarrow \tilde{t}_1^- H_1 H_1$

The amplitude for $\tilde{t}_2 \to \tilde{t}_1^- H_1 H_1$ is given by

$$B_{HH} = B_{HH}^{\text{Higgs}} - v^2 \sum_j g_{H_1i\tilde{t}_j} g_{H_1i\tilde{t}_k} \left[ \tilde{D} \left( \rho^2, m_{\tilde{t}_j}^2, \Gamma_{\tilde{t}_j} \right) + \tilde{D} \left( \xi^2, m_{\tilde{t}_j}^2, \Gamma_{\tilde{t}_j} \right) \right]$$

$$\left( 1 - \frac{8}{3} \sin^2 \theta_W \right) \left( (O_{11})^2 - (O_{21})^2 + (O_{31})^2 \left( s_\beta^2 - c_\beta^2 \right) \right) \eta_{HH}, \quad (A.5)$$

where

$$\eta_{HH} = \frac{g^2}{4 \cos^2 \theta_W} U_{12}^{\tilde{t}i} U_{11}^{\tilde{t}i} \quad (A.6)$$

2. $\tilde{t}_2^\rightarrow \tilde{t}_1^- ZZ$

The amplitudes for $\tilde{t}_2 \to \tilde{t}_1^- ZZ$ are as follows:

$$B_{ZZ} = B_{ZZ}^{\text{Higgs}} + \left( 2 - \frac{16}{3} \sin^2 \theta_W \right) \eta_{ZZ}, \quad (A.7)$$

$$C_{ZZ} = -4 \sum_j \left( |U_{1j}|^2 - \frac{4}{3} \sin^2 \theta_W \right) \tilde{D} \left( \xi^2, m_{\tilde{t}_j}^2, \Gamma_{\tilde{t}_j} \right) \eta_{ZZ}, \quad (A.8)$$

$$D_{ZZ} = -4 \sum_j \left( |U_{1j}|^2 - \frac{4}{3} \sin^2 \theta_W \right) \tilde{D} \left( \rho^2, m_{\tilde{t}_j}^2, \Gamma_{\tilde{t}_j} \right) \eta_{ZZ}, \quad (A.9)$$

$$E_{ZZ} = C_{ZZ} + D_{ZZ}. \quad (A.10)$$

Each of the non-Higgs-mediated pieces is proportional to the same complex quantity $\eta_{ZZ}$:

$$\eta_{ZZ} = \eta_{HH} = \frac{g^2}{4 \cos^2 \theta_W} U_{12}^{\tilde{t}i} U_{11}^{\tilde{t}i} \quad (A.11)$$
3. $\tilde{t}_2 \rightarrow \tilde{t}_1 W^+ W^-$

The amplitudes for $\tilde{t}_2 \rightarrow \tilde{t}_1 W^+ W^-$ are given by,

$$B_{WW} = B_{WW}^{Higgs} + \left[1 + \left(\rho^2 - \xi^2\right)^2 \widetilde{D} \left(M^2, m_Z^2, \Gamma_Z\right)\right] \eta_{WW},$$

(A.12)

$$C_{WW} = 4 \widetilde{D} \left(M^2, m_Z^2, \Gamma_Z\right) \eta_{WW},$$

(A.13)

$$D_{WW} = -4 \left[\widetilde{D} \left(M^2, m_Z^2, \Gamma_Z\right) + \sum_j \left|U_{ij}^\dagger\right|^2 |V_{tb}|^2 \widetilde{D} \left(\rho^2, m_{t_b}^2, \Gamma_{t_b}\right)\right] \eta_{WW} ,$$

(A.14)

$$E_{WW} = C_{WW} + D_{WW} = -4 \sum_j \left|U_{ij}^\dagger\right|^2 |V_{tb}|^2 \widetilde{D} \left(\rho^2, m_{t_b}^2, \Gamma_{t_b}\right) \eta_{WW} .$$

(A.15)

In the numerical work we set $V_{tb} = 1$. Note that each of the non-Higgs terms is proportional to the same complex quantity $\eta_{WW}$, which is defined as follows:

$$\eta_{WW} = \frac{g^2}{2} U_{12}^t U_{11}^t .$$

(A.16)

4. $\tilde{t}_2 \rightarrow \tilde{t}_1 ZH_1$

The amplitudes for $\tilde{t}_2 \rightarrow \tilde{t}_1 ZH_1$ are as follows:

$$B_{ZH} = B_{ZH}^{Higgs} - \frac{2}{m_Z} \left(m_{t_2}^2 - m_{t_1}^2 - m_Z^2\right) \widetilde{D} \left(M^2, m_Z^2, \Gamma_Z\right) g_{H_1 V V} \eta_{ZH}$$

$$- \frac{v g}{\cos \theta_W} \sum_j \left(U_{12}^t U_{1j}^t - \frac{4}{3} \sin^2 \theta_W \delta_{2j}\right) \widetilde{D} \left(\xi^2, m_{t_j}^2, \Gamma_{t_j}\right) g_{H_1 t_j t_j} ,$$

(A.17)

$$C_{ZH} = 4 m_Z \widetilde{D} \left(M^2, m_Z^2, \Gamma_Z\right) g_{H_1 V V} \eta_{ZH}$$

$$- \frac{v g}{\cos \theta_W} \sum_j \left(U_{12}^t U_{1j}^t - \frac{4}{3} \sin^2 \theta_W \delta_{2j}\right) \widetilde{D} \left(\xi^2, m_{t_j}^2, \Gamma_{t_j}\right) g_{H_1 t_j t_j}$$

$$- \frac{v g}{\cos \theta_W} \sum_j \left(U_{1j}^t U_{11} - \frac{4}{3} \sin^2 \theta_W \delta_{1j}\right) \widetilde{D} \left(\rho^2, m_{t_j}^2, \Gamma_{t_j}\right) g_{H_1 t_j t_j} ,$$

(A.18)

where

$$\eta_{ZH} = \eta_{HH} = \frac{g^2}{4 \cos^2 \theta_W} U_{12}^t U_{11}^t .$$

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