Generalized Analysis of Weakly-Interacting Massive Particle Searches

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We perform a generalized analysis of data from WIMP search experiments for point-like WIMPs of arbitrary spin and general Lorenz-invariant WIMP-nucleus interaction. We show that in the non-relativistic limit only spin-independent (SI) and spin-dependent (SD) WIMP-nucleon interactions survive, which can be parameterized by only five independent parameters. We explore this five-dimensional parameter space to determine whether the annual modulation observed in the DAMA experiment can be consistent with all other experiments. The pure SI interaction is ruled out except for very small region of parameter space with the WIMP mass close to 50 GeV and the ratio of the WIMP-neutron to WIMP-proton SI couplings $-0.77 \leq f_n/f_p \leq -0.75$. For the predominantly SD interaction, we find an upper limit to the WIMP mass of about 18 GeV, which can only be weakened if the constraint stemming from null searches for energetic neutrinos from WIMP annihilation the Sun is evaded. None of the regions of the parameter space that can reconcile all WIMP search results can be easily accommodated in the minimal supersymmetric extension of the standard model.

PACS numbers: 95.35.+d,

I. INTRODUCTION

A weakly-interacting massive particle (WIMP) is perhaps the most natural candidate for the dark matter that permeates our Galactic halo \cite{1, 2}. Simply stated, if new physics at the electroweak scale introduces a neutral stable particle, then that particle has a cosmological density comparable to that contributed by halo dark matter. The most widely studied possibility for the WIMP is the neutralino, a linear combination of the supersymmetric partners of the photon and $Z^0$ and Higgs boson, the lightest superpartner (LSP) in
many minimal supersymmetric extensions of the standard model (MSSMs). However, other possibilities include the sneutrino (the neutrino’s superpartner), heavy neutrinos, particles in non-minimal supersymmetric models, and Kaluza-Klein modes in models with universal extra dimensions; there may be others as well.

If such WIMPs exist in the Galactic halo, they may be detected directly via detection of the $O(30 \text{ keV})$ recoil energy imparted to a nucleus in a low-background detector when a halo WIMP elastically collides with the nucleus. They may also be detected indirectly via observation of energetic neutrinos (i.e., $E_\nu \gtrsim 10 \text{ GeV}$) produced by annihilation of WIMPs that have accumulated in the Sun and/or Earth. A third possibility is observation of cosmic-ray antiprotons, positrons, or gamma rays produced by annihilation of dark-matter particles in the Galactic halo, although the certainty with which we can predict the fluxes of such cosmic rays is limited by considerable uncertainties in the distribution of dark matter in the halo as well as in cosmic-ray propagation.

Of the several direct dark-matter-detection experiments underway, one—the DAMA collaboration—has reported an annual modulation in their NaI detector that they attribute to the difference in the WIMP flux incident on the Earth between the summer and winter. Since their announcement, several other experiments have reported null results that they argue conflict with the DAMA detection. In order to come to this conclusion, however, it is assumed that the WIMP has only scalar (spin-independent; SI) interactions with nucleons, and with equal interaction strength with neutrons and protons. Although such interactions are favored in most of the supersymmetric models that appear in the literature, they are by no means universal to the MSSM, nor to WIMP models more generally. Ref. studied the possibility that the DAMA results might be due to a WIMP with an axial-vector (spin-dependent; SD) coupling to either protons or to neutrons, and showed that the former possibility conflicts with null searches for energetic neutrinos from the Sun, while the latter is inconsistent with other direct-detection experiments. However, although the realm of possibilities that have been explored has been expanded, it is still not universal.

In this paper we expand further on prior analyses of WIMP-detection experiments and explore whether the DAMA modulation may be consistent with null searches in a more generalized parameter space. We suppose that the WIMP is a pointlike particle, but with arbitrary spin. We consider particles that are either self-charge-conjugate, or that have
an antiparticle, either with or without a particle-antiparticle asymmetry. We begin in the
next Section by showing that from the point of view of dark-matter detection, the Dirac
structure of the interaction Lagrangian of all such particles is such that the interactions
relevant for dark-matter detection can be parameterized by only five quantities: the particle
mass, SI couplings to neutrons and protons, and SD couplings to protons and neutrons. In
Section III we review the cross sections. Section IV discusses the experimental results we
use from the DAMA NaI experiment, the DAMA Xe experiment, EDELWEISS, ZEPLIN-I,
and energetic-neutrino searches. In Section V we search the five-dimensional parameter
space for regions that are consistent with all of the experimental results. The results are
summarized in Section VI, the Conclusions. The bottom line is that there are regions in
this five-dimensional parameter space for both SD and SI interactions that are consistent
with all current experimental results, but these regions are unlikely to occur in any MSSM
and do not apply to any of the other WIMP candidates that have appeared in the recent
literature.

II. GENERALIZED WIMP-NUCLEON INTERACTION

WIMP searches have usually been interpreted within the framework of the minimal su-
persymmetric standard model (MSSM) because it contains an excellent WIMP candidate,
the lightest neutralino. Interactions of the neutralino with quarks (and ultimately with
nucleons and nuclei) have been extensively studied in the literature (see e.g. [1, 13]). For
slow-moving Majorana neutralinos, the neutralino-nucleon interaction has only two terms,
\[
\mathcal{L}_\chi = \bar{\chi} \gamma^\mu \gamma_5 \chi \mathcal{J}_\mu^5 (x) + \bar{\chi} \chi S (x) ,
\]
\[
\mathcal{J}_\mu^5 (x) = \sqrt{2} G_F \left( a_p \bar{p} \gamma_\mu \gamma_5 p + a_n \bar{n} \gamma_\mu \gamma_5 n \right) ,
\]
\[
S (x) = f_p \bar{p} p + f_n \bar{n} n ,
\]
where \( G_F \) is the Fermi constant, and \( a_{p,n} \) and \( f_{p,n} \) are, respectively, the proton and neutron
coupling constants given in Ref. [1]. The term containing \( S (x) \) gives rise to the spin-
independent (SI) interaction, whereas the $J_{\mu}^S(x)$ term is responsible for the spin-dependent (SD) interaction.

Although Eq. (1) was derived assuming that WIMPs are slow-moving Majorana spin-1/2 fermions within the MSSM, it can be generalized to particles of arbitrary spin, independent of whether they are self-charge-conjugate or not. Consider a general Lorentz-invariant interaction for the WIMP-nucleon coupling,

$$\mathcal{L}_{\chi} = (S_{\chi} + P_{\chi}) \left( G_s \bar{N} N + G_p \bar{N} \gamma_5 N \right) + \left( V_\mu^\chi + A_\mu^\chi \right) \left( G_v \bar{N} \gamma_\mu N + G_a \left( \frac{q_\mu}{m_\pi^2 - q^2} \bar{N} \gamma_5 N \right) \right) + \left( T_\mu^\chi \gamma^\rho + D_\mu^\chi \right) \left( G_t \bar{N} \sigma_{\rho\mu} N + G_d \bar{N} \sigma_{\rho\mu} \gamma_5 N \right).$$  \hspace{1cm} (2)

Here, $S_{\chi}$ and $P_{\chi}$ are linear combinations of scalar and pseudoscalar operators built from the $\chi$ field, $V_\mu^\chi$ and $A_\mu^\chi$ are the corresponding vector and axial-vector operators, and $T_\mu^\chi$ and $D_\mu^\chi$ are the tensor and pseudotensor operators. In the Weyl representation, the Dirac structures appearing in Eq. (2) are (Latin indices indicate spatial components),

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$\sigma^{\mu\nu} = \frac{i}{2} \{ \gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu \},$$  \hspace{1cm} (3)

where $\sigma^i$ are Pauli matrices. In order to use this interaction to calculate matrix elements for WIMP-nucleus scattering, one must sum over all nucleons in the target nucleus.

The WIMP-nucleon interaction is determined by the underlying WIMP-quark interaction, and to obtain Eq. (2) from the underlying interaction one must take its matrix elements between one-nucleon states. In the limit $|q| \ll M_N$ relevant for dark-matter detection, one can do this by simply replacing the quark fields with the nucleon fields and rescaling the corresponding coupling constants. For example, $g_{V}^q \langle N | \bar{q} \gamma_\mu q | N \rangle = G_V^q \bar{u} N \gamma_\mu u_N$, where $u_N$ is the nucleon spinor. This procedure holds for all operators except for the quark axial-vector current whose matrix element has a pole:

$$\langle N | J_\mu^{A5} | N \rangle = \bar{N} \left( g_A \gamma^\mu \gamma^5 + g_P(q^2) q^\mu \gamma_5 \right) N,$$

$$g_P(q^2) = g_A \frac{2 M_N}{m_\pi^2 - q^2},$$  \hspace{1cm} (4)

where the second line implements the PCAC hypothesis, and $m_\pi$ is the pion mass. This argument justifies the appearance of the particular structure multiplying $G_a$ in Eq. (2). The
second term in the first line of Eq. (4) becomes important if \( |\vec{q}| \gtrsim m_\pi \). For simplicity, we do not explicitly keep the pion pole term in the following. However, it should be understood that this term always appears in the form given by the last term in the second line of Eq. (2).

Except for the pion pole term, Eq. (2) does not contain operators with explicit factors of momentum transfer \( q \). Such terms are considered subleading. As will be shown below, such operators are suppressed by powers of \( q/M_p \) or \( q/M_\chi \). It is easy to show that in the frame where the nucleus is initially at rest the components of \( \vec{q} \) are related by

\[
2q_0 M_{\text{nuc}} + q^2 = 0,
\]

which also shows that \( q_0 \ll |\vec{q}| \). Therefore, we find,

\[
\frac{q}{M_p} \sim \sqrt{\frac{q_0 M_{\text{nuc}}}{M_p^2}} = \sqrt{\frac{q_0 A}{M_p}} \lesssim 0.01 \sqrt{A} \lesssim 0.1 ,
\]

(5)

where \( A \lesssim 100 \) is the atomic number of the nucleus and \( q_0 \) is the recoil energy, which we assume not to significantly exceed 100 keV.

Let us look at each of the one-nucleon operators appearing in Eq. (2) and determine which of them have non-vanishing nuclear matrix elements in the non-relativistic limit.

**Scalar operators.** The scalar operator \( \bar{N}N \) simply counts nucleons; it obviously survives in the non-relativistic limit. On the other hand, nuclear matrix elements of the pseudoscalar operator \( \mathcal{P}_N = \sum_k \bar{N}_k(x + r_k)\gamma_5 N_k(x + r_k) \) are suppressed: \( \langle f | \mathcal{P}_N | i \rangle \sim |\vec{q}|/M_p \). Here, \( x \) and \( r_k \) correspond, respectively, to the center of mass of the nucleus and to the position of the \( k \)th nucleon relative to the center of mass. Since our discussion is not affected by the fact that the nucleus has a finite size we can consider it to be a point particle and set all \( r_k \) to zero. In a more accurate treatment one can account for finite size \( R \) by introducing a form-factor \( F(|\vec{q}|R) \). Defining an “axial-vector current” operator \( \hat{A}_5^a(x) = \int x^\mu \mathcal{P}_N(x) dx^\mu \) as an intermediate step we obtain:

\[
\langle f | \mathcal{P}_N(x) | i \rangle = \langle f | \partial_\mu \hat{A}_5^a(x) | i \rangle = i q_\mu e^{i q \cdot x} \langle f | \hat{A}_5^a(0) | i \rangle
\]

\[
\approx i q_\mu e^{i q \cdot x} P \langle f | \hat{J}_N^i | i \rangle + \mathcal{O}(q^2) ,
\]

(6)

where \( \hat{J}_N^i \) is the nuclear spin operator. In the last step we used the fact that spin is the only axial vector describing a state of the nucleus that does not vanish as \( q \to 0 \). The scalar constant \( P \) has the dimension of inverse mass. The largest quantity that has this property and is not singular in the limit \( q \to 0 \) is the inverse mass of the nucleon. Therefore, \( P \sim 1/M_p \), and matrix elements of \( \mathcal{P}_N \) are suppressed according to Eq. (5). Another way to see that \( P \sim 1/M_p \) (and, e.g., not \( 1/M_{\text{nuc}} \)) is to first consider the one-nucleon operator \( \bar{N}_i \gamma_5 N \)
using free Dirac spinors for the nucleon fields. Using free spinors is a good approximation
because the binding energy of a nucleon in a nucleus is much smaller than the nucleon mass,
which means that nucleons are only slightly off-shell. A direct calculation yields,

\[ \bar{N}(p + q)\gamma_5 N(p) = \frac{q_i}{M_p} \eta^i \sigma_i \eta, \]

where \( \eta, \eta' \) are Weyl spinors characterizing the states of the nucleon with the initial momentum \( p \), and \( \sigma_i \) is the vector of Pauli matrices. In a nucleus one obtains

\[ \frac{q_i}{M_p} \langle f | \sum_k \eta^i_k \sigma_i \eta_k | i \rangle \sim \frac{q_i}{M_p} \langle f | \hat{J}^i_N | i \rangle, \]

in agreement with Eq. (6).

**Vector operators.** For \( \mathcal{V}_N^\mu = \sum_k \bar{N}_k \gamma^\mu N_k \) we must have:

\[ \langle f | \mathcal{V}_N^\mu | i \rangle = V_N^+ (p_f + p_i)^\mu + V_N^- (p_f - p_i)^\mu, \]

where \( V_N^+ \) and \( V_N^- \) are constants. Here, \( p_i = (M_{\text{nuc}}, 0) \) and \( p_f = q + p_i \). Just like for \( \mathcal{P}_N \), we may conclude on dimensional grounds that \( V_N^+, V_N^- \sim 1/M_p \). Then the time component of \( \mathcal{V}_N \) is of order unity, whereas the spatial components are suppressed as \( \mathcal{V}_N^i \sim q^i/M_p \). The time component \( \mathcal{V}_N^0 \) multiplies \( \mathcal{V}_x,0 \) and \( A_x,0 \) in Eq. (2), which transform, respectively, as a scalar and a pseudoscalar under the extended rotational group. Therefore, they can be effectively absorbed into \( S_x \) and \( \mathcal{P}_x \) in the non-relativistic case.

For the axial-vector operator \( \mathcal{A}_N^\mu = \sum_k \bar{N}_k \gamma^\mu \gamma_5 N_k \) the situation is reversed. The time component transforms as a pseudoscalar under the extended rotational group and its matrix elements are suppressed for the same reason as those of \( \mathcal{P}_N \). The spatial components transform like a pseudovector leading to

\[ \langle f | \mathcal{A}_N^i | i \rangle = A_N \langle f | \hat{J}_N^i | i \rangle, \]

where \( A_N \) is a dimensionless constant, which can in general be of order unity.

**Tensor operators.** Let us first consider the operator \( \mathcal{T}_N^{ij\mu} = \sum_k \bar{N}_k \sigma^{ij\mu} N_k \). Nonzero components of \( \sigma^{ij\mu} \) are \( \sigma^{0i} \) and \( \sigma^{ij}, i, j = 1, 2, 3 \). Under the extended rotational group, \( \sigma^{0i} \) transforms as a polar vector. Therefore, in analogy to Eq. (9),

\[ \mathcal{T}_N^{0i} = T_N^+ (p_f + p_i)^i + T_N^- (p_f - p_i)^i, \]

which is suppressed similarly to \( \mathcal{V}_N^i \) in Eq. (9). To analyze \( \mathcal{T}_N^{ij} \) consider its dual \( \mathcal{T}_N^i = \epsilon^{ijk} \mathcal{T}_N^{jk} = \sum_k \bar{N}_k \epsilon^{ijk} \sigma_{jk} N_k \equiv 2 \sum_k \bar{N}_k \sigma^i N_k \), where \( \epsilon^{ijk} \) is the three-dimensional Levi-Civita
We obtain
\[
\langle f | T^i_{N,j} | i \rangle = \frac{1}{2} \epsilon^{ijk} T^j_{N,k} \sigma_i N_k | i \rangle = T_N \epsilon^{ijk} \langle f | \tilde{J}_{N,k} | i \rangle ,
\]  
(12)

where \( T_N \) is a dimensionless constant. Therefore, \( T^i_{N,j} \) survives in the non-relativistic limit.

The operator
\[
D^{\mu \nu}_{N} = \sum \bar{N}_k \sigma^{\mu \nu \gamma_5} N_k
\]
can be expressed as \((i/2) \epsilon^{\mu \nu \rho \lambda} \sigma_{\rho \lambda} \), which follows from the identity
\[
\sigma^{\mu \nu \gamma_5} = (i/2) \epsilon^{\mu \nu \rho \lambda} \sigma_{\rho \lambda} .
\]

Since only \( T^i_{N,j} \) survives in the non-relativistic limit we conclude that \( D^0_{N,i} \) are the only non-vanishing components of \( D^{\mu \nu}_{N} \) in this limit:
\[
D^0_{N,i} = \frac{i}{2} \epsilon^{0ijk} T^j_{N,k} = \frac{i}{4} \epsilon^{0ijk} \epsilon_{jkl} T^l_{N} = \frac{i T_N}{2} \langle f | \tilde{J}_{N} | i \rangle .
\]  
(13)

The antisymmetric tensor \( \epsilon_{ijk} \) in Eq. (12) is contracted with spatial components of \( T^{\mu \nu}_{\chi} \) and \( D^{\mu \nu}_{\chi} \) in Eq. (2). The corresponding quantities \( T^i_{\chi,j} \epsilon_{ijk} \) and \( D^i_{\chi,j} \epsilon_{ijk} \) transform, respectively, as an axial and a polar vector under the extended rotational group. Therefore, in the non-relativistic limit \( T^i_{\chi,j} \epsilon_{ijk} \) can be absorbed into \( A_{\chi,k} \) and \( D^i_{\chi,j} \epsilon_{ijk} \) into \( V_{\chi,k} \). Similarly, taking the Levi-Civita structure \( \epsilon^{0ijk} \epsilon_{jkl} \) from Eq. (13) we conclude that \( T_{\chi,0l} \epsilon^{0ijk} \epsilon_{jkl} \equiv -2 T_{\chi,0l} \) transforms as a polar vector and \( D_{\chi,0l} \epsilon^{0ijk} \epsilon_{jkl} \equiv -2 D_{\chi,0l} \) as an axial vector. They can be absorbed into, respectively, \( V_{\chi,l} \) and \( A_{\chi,l} \). With the above considerations in mind Eq. (2) can be effectively rewritten in the non-relativistic limit as
\[
L_{\chi} = (S_{\chi} + P_{\chi}) G_s \bar{N} N + \left( V^{i}_{\chi} + A^{i}_{\chi} \right) G_s \bar{N} \gamma_i \gamma_5 N ,
\]  
(14)

where \( S_{\chi}, P_{\chi}, V^{i}_{\chi}, \) and \( A^{i}_{\chi} \) have been redefined to absorb the contributions from the vector and tensor interactions. Moreover, since the general arguments used for \( P_N \) and \( V^{i}_{N} \) also apply to \( P_{\chi} \) and \( V^{i}_{\chi} \) (with the suppression \( |q|/M_{\chi} \sim 10^{-3} \) in this case) we can safely neglect the latter.

The scalar density \( S_{\chi}(x) \) in Eq. (14) can be written as \( \chi^{\dagger} \chi \). For a WIMP of spin \( J_{\chi}, \chi \) is a \((2J_{\chi} + 1)\)-component non-relativistic spinor (e.g. in the MSSM, \( \chi \) is a two-component Weyl spinor). The spatial components of the axial-vector density \( A^{i}_{\chi}(x) \) in the non-relativistic limit are proportional to the spin density \( \chi^{\dagger} \tilde{S}^{i}_{\chi} \chi \). Finally, we arrive at the following form of the WIMP-nucleon interaction Lagrangian in the non-relativistic limit,
\[
L_{\chi} = 4 f_N \chi^{\dagger} \chi \eta^{\dagger}_{N} \eta_{N} + 16 \sqrt{2} G_F \alpha_{N} \chi^{\dagger} \bar{S}_{\chi} \lambda_{N} \tilde{S}_{N} \eta^{\dagger}_{N} \eta_{N} + O \left( \frac{q}{M_{\rho,\chi}} \right) ,
\]  
(15)
where $\eta_N$ is the two-component Weyl spinor for the nucleon (initial and final state spinors may be different), $N = n, p$, $\vec{S}_N = \vec{\sigma}/2$, and the notation for the couplings has been adjusted to match that of Eq. (1). In this form, Eq. (15) is valid for non-relativistic point-like WIMPs of arbitrary spin.

The operators neglected in Eq. (15) are suppressed by a factor of $0.01\sqrt{A}$ relative to the leading order terms [see Eq. (3)]. Strictly speaking, in neglecting such terms we made an implicit assumption that the couplings $f_N$ and $G_F a_N$ in Eq. (15) are not suppressed relative to the other couplings in Eq. (2), such as $G_p$. If it turned out that $G_p \gtrsim 100\sqrt{A} f_N$ then one would have to consider the $\bar{N}\gamma_5N$ operator even though it is formally suppressed in the non-relativistic limit. In this paper we ignore this possibility, and modulo the aforementioned assumption our analysis is model-independent.

A. Some Examples

Below we consider some examples of theories that have WIMPS with spin-0, spin-1/2 (Dirac particle), and spin-1 and explicitly show that in each case the WIMP-nucleon interaction reduces to Eq. (15).

1. Spin-0

It is possible that in a supersymmetric theory the lightest SUSY particle is the sneutrino $\tilde{\nu}$, the scalar partner of the neutrino. In the non-relativistic limit, such a particle interacts with a nucleon as [3],

$$\mathcal{L}_{\tilde{\nu}} = -\sqrt{2} G_F J_{\tilde{\nu}}^\mu J_{N,\mu},$$

$$J_{\tilde{\nu}}^\mu = \left( p_{\tilde{\nu}}^\mu + p_{\tilde{\nu}}'^\mu \right) \bar{\tilde{\nu}} \tilde{\nu},$$

$$J_N^\mu = \left( T_N^3 - 2Q_N \sin^2 \theta_W \right) \bar{N} \gamma^\mu N,$$

where $p_{\tilde{\nu}}, p_{\tilde{\nu}}'$ are the initial and final momentum of $\tilde{\nu}$, $T_N^3$ and $Q_N$ are the isospin and the electric charge of the nucleon, and $\theta_W$ is the Weinberg angle. For a slow-moving Dirac fermion of mass $M$ the leading terms in the $v/c$ expansion of the $u$- and $v$-spinors are (in
the Weyl representation),

$$u(p) = \begin{pmatrix} 1 + \frac{\vec{p} \cdot \vec{\sigma}}{2M_p} \eta \\ 1 - \frac{\vec{p} \cdot \vec{\sigma}}{2M_p} \eta \end{pmatrix}, \quad v(p) = \begin{pmatrix} (1 + \frac{\vec{p} \cdot \vec{\sigma}}{2M_p}) \eta \\ -(1 - \frac{\vec{p} \cdot \vec{\sigma}}{2M_p}) \eta \end{pmatrix},$$

where \( p \) is the fermion momentum and \( \eta \) is a Weyl spinor. Using the above expressions one can explicitly verify that only the time component of the nucleon vector current survives in the limit \( \vec{q} \to 0 \). Therefore, one is left with a purely SI interaction, which is a special case of Eq. (15) with \( f_N = -\sqrt{2}G_F \left( T^3_N - 2Q_N \sin^2 \theta_W \right) \) and \( a_N = 0 \).

2. Spin-1/2

In theories with extra dimensions Kaluza-Klein (KK) excitations of the SM particles can produce viable dark matter candidates. As an example, consider a Dirac KK neutrino, which interacts with nucleons via \( Z^0 \) exchange as

$$L_\nu = -\sqrt{2}G_F \left( T^3_N - 2Q_N \sin^2 \theta_W \right) \bar{\nu}_{KK} \gamma_\mu \nu_{KK} \bar{N} \gamma^\mu N.$$  

(18)

Just like in the previous example, only time components of currents remain in the non-relativistic limit, and we again obtain Eq. (15) with \( f_N = -\sqrt{2}G_F \left( T^3_N - 2Q_N \sin^2 \theta_W \right) \) and \( a_N = 0 \).

3. Spin-1

Kaluza-Klein excitations of the SM gauge bosons could constitute cold dark matter. If, for example, dark matter is composed of excitations of the hypercharge gauge boson \( B_1 \) the WIMP-quark interaction has the form:

$$L_B = -(\beta_q + \gamma_q) B^\dagger_1 \mu B_{1\mu} \bar{q} q - i\alpha_q B^\dagger_{1\mu} B_{1\nu} \epsilon^{\mu\nu\rho\sigma} \bar{q} \gamma_\rho \gamma_5 q,$$

(19)

$$\alpha_q = \frac{e^2}{2 \cos^2 \theta} \left[ \frac{Y_{qL}^2 m_{B_1}}{m_{q_L}^2 - m_{B_1}^2} + (L \to R) \right],$$

$$\beta_q = E_q \frac{e^2}{2 \cos^2 \theta} \left[ \frac{Y_{qL}^2 m_{B_1}^2 + m_{q_L}^2}{m_{q_L}^2 - m_{B_1}^2} + (L \to R) \right],$$

$$\gamma_q = m_q \frac{e^2}{4 \cos^2 \theta} \frac{1}{m_h^2},$$

(20)
where $Y_q$ is the hypercharge of the quark with mass $m_q$ and energy $E_q$, $m_\chi$ is the mass of the quark’s first KK excitation, and $m_h$ is the Higgs boson mass. The first term in Eq. (13) corresponds to the SI interaction in Eq. (15):

$$\langle N|\bar{q}q|N\rangle = \frac{M_N}{m_q} f_{T_q}^N \bar{N}N = 2 \frac{M_N}{m_q} f_{T_q}^N \eta_t \eta_N,$$

$$f_N = \frac{1}{2} \sum_{u,d,s,...} \left( \beta_q + \gamma_q \right) \frac{M_N}{m_q} f_{T_q}^N,$$

where $f_{T_q}^N$ for various quark flavors can be found e.g. in Refs. [1, 13], and $f_N$ is normalized to reproduce Eq. (15). The second term produces a SD WIMP-nucleon interaction:

$$i\epsilon^{0ijk} B_{ij}^1 (\bar{q}\gamma_k \gamma_5 q) \langle N|\bar{N}\gamma_t \gamma_5 N\rangle = -4 \Lambda_q^N \langle B_1|\tilde{S}_B|B_1\rangle \bar{N}S_N N,$$

$$a_N = \frac{1}{4\sqrt{2}} \frac{G_F}{\sum_{u,d,s,...}} \alpha_q \Lambda_q^N,$$

where $(\tilde{S}_B)_{ij} = i\epsilon^{0kij}$ is the matrix of the spin operator for $B_1$, $\Lambda_q^N$ is a dimensionless constant of order unity, and $a_N$ is normalized to reproduce Eq. (15). A recent analysis gives $\Delta_u = \Delta_d = 0.78 \pm 0.02$, $\Delta_d = \Delta_u = -0.48 \pm 0.02$, and $\Delta_s = \Delta_s = -0.15 \pm 0.02$ [16].

### III. WIMP-NUCLEUS CROSS SECTION

The SI and SD WIMP-proton cross sections at low momentum transfers are easily calculated from Eq. (15)³,

$$\sigma_{\chi p}^{SI} = \frac{4 f_p^2}{3\pi} M_{\chi p}^2 (M_p) ,$$

$$\sigma_{\chi p}^{SD} = \frac{128J_\chi (J_\chi + 1) J_p (J_p + 1) G_F^2 a_p^2}{3\pi} M_{\chi p}^2 (M_p) ,$$

$$M_{\chi p} = \frac{M_\chi M}{M_\chi + M} ,$$

where $M_\chi$ and $J_\chi$ are the WIMP mass and spin. In our convention, the non-relativistic spinors $\psi$ for both the WIMP and and the nucleon are normalized as $\psi_{\chi p}^\dagger \psi_{\chi p} = M_{\chi p}$. The SI and SD WIMP-nucleus cross sections at asymptotically small energies, $\sigma_{0\chi N}^{SI}$ and $\sigma_{0\chi N}^{SD}$, can be expressed in terms of the corresponding WIMP-proton cross section,

$$\sigma_{0\chi N}^{SI} = \frac{M_{\chi p}^2 M_{\text{nuc}}^2}{M_{\chi p}^2 (M_p)} \left[ Z + (A - Z) \frac{f_n}{f_p} \right]^2 \sigma_{\chi p}^{SI} ,$$

³ For a non-self-charge-conjugate WIMP, such as heavy Dirac neutrino, the WIMP-proton cross section must be multiplied by an extra factor of $1/4$. This modification does not affect Eq. (24).
\[ \sigma^{SD}_{0\chi N} = \frac{M^2_{\text{red}}(M_{\text{nuc}})}{M^2_{\text{red}}(M_p)} \frac{4(J + 1)}{3J} \left[ \langle S_p \rangle + \langle S_n \rangle \frac{a_n}{a_p} \right]^2 \sigma^{SD}_{\chi p}. \] (24)

Here, \( A \) and \( Z \) are the mass number and the charge of the nucleus with spin \( J \), and \( f_p(f_n) \) and \( a_p(a_n) \) are the SI and SD WIMP-proton(neutron) couplings, respectively. The quantities \( \langle S_p \rangle \) and \( \langle S_n \rangle \) are the average spins of the proton and the neutron in the nucleus. At finite momentum transfer one must average the one-nucleon operators from Eq. (24) over the given nucleus using some nuclear structure model. Including the WIMP velocity distribution (see Ref. [1]), one then obtains for the elastic scattering rate of WIMPs on the nucleus (per unit detector mass),

\[ \frac{dR}{dE} = \frac{\rho_\chi}{4v_0EM_{\text{nuc}}^2} \left[ \text{erf} \left( \frac{v_{\text{min}} + v_E}{v_0} \right) - \text{erf} \left( \frac{v_{\text{min}} + v_E}{v_0} \right) \right] \times \left[ \sigma^{SI}_{0\chi N}F^2_{SI}(E) + \sigma^{SD}_{0\chi N}S_A(E) \right], \]

\[ v_{\text{min}} = \sqrt{\frac{EM_{\text{nuc}}}{2M^2_{\text{red}}(M_{\text{nuc}})}}, \] (25)

where \( \rho_\chi \) is the local Galactic halo density \(^4\), \( E \) is the recoil energy of the nucleus with mass \( M_{\text{nuc}} \), and \( v_E \) and \( v_0 \) are, respectively, velocities of the Earth and the Sun in the Galactic frame.

The form-factors \( F_{SI}(E) \) and \( S_A(E) \) in Eq. (25) depend on the nuclear structure. The SI form-factor can be well approximated by \([13]\)

\[ F_{SI}(E) = \frac{3j_1(qR_1)}{qR_1} e^{-(qs)^2/2}, \]

\[ q = \sqrt{2M_{\text{nuc}}E}, \]

\[ R_1 = \sqrt{1.44A^{2/3} - 5s^2} \text{ fm}, \]

\[ s \approx 1 \text{ fm}, \] (26)

where \( j_1(x) \) is the spherical Bessel function. Unfortunately, there is no such universal expression for \( S_A(E) \). It can be parameterized in terms of three nucleus-dependent functions:

\[ S_A(E) = \frac{1}{4} \left[ (a_p + a_n)^2 S_{00}(E) + (a_p - a_n)^2 S_{11}(E) + (a_p^2 - a_n^2) S_{01}(E) \right] \] (27)

where \( S_{ij}(E) \) for most nuclei used in WIMP searches can be found in Ref. [17].

\(^4\) We take \( \rho_\chi = 0.3 \text{ GeV/cm}^3 [1]. \)
It is apparent from Eqs. (24) and (25) that the detection rate for non-relativistic WIMPs depends on only five parameters (in addition to nucleus-dependent constants). We choose these parameters to be $\sigma_{SI}^{\chi_p}$, $\sigma_{SD}^{\chi_p}$, $f_n/f_p$, $a_n/a_p$, and $M_\chi$. Various extensions of the SM generally reduce the number of parameters (e.g., in the MSSM one normally has $f_n/f_p \approx 1$). In our analysis we do not impose such restrictions.

IV. EXPERIMENTAL DATA

A. DAMA NaI

In utilizing the data published by the DAMA collaboration in Ref. \[6\] we adopt the same approach as in Refs. \[11, 18\]. In particular, we define the quantity,

$$\kappa = \sum_i \left[ \left( \frac{S_{th,0,i} - S_{th,0,i}^{exp}}{\Delta S_{th,0,i}^{exp}} \right)^2 + \left( \frac{S_{th,m,i} - S_{th,m,i}^{exp}}{\Delta S_{th,m,i}^{exp}} \right)^2 \right],$$

(28)

where $S_{th,(exp)}^{th,0,i}$ and $S_{th,(exp)}^{th,m,i}$ are, respectively, the theoretical (experimental) average value and the annual-modulation amplitude of the detection rate in the $i$th energy bin. Theoretical predictions for these quantities as functions of $M_\chi$, $\sigma_{SI}^{\chi_p}$, $\sigma_{SD}^{\chi_p}$, $f_n/f_p$, and $a_n/a_p$ can be obtained using Eq. (25). In this work, we used the experimental values for $S_{th,(0,m),i}^{exp}$ given in the first two columns of Table 1 in Ref. \[6\]. The DAMA preferred region, shown in Fig. 4a) of Ref. \[6\], is well reproduced for $\kappa \approx 100$.

B. DAMA $^{129}$Xe

The latest results on WIMP searches with a liquid xenon target by the DAMA collaboration have appeared in Ref. \[10\]. We take the limits on counts per detector per unit mass per day (detector rate unit, $dru$) for various energy bins from Fig. 4 and Table I of Ref. \[10\]. In calculating the limits on the WIMP cross sections we follow the approach of Ref. \[19\]. We take the central values for the $dru$’s in all energy bins to be zero and the 90\% CL upper limits on the $dru$’s to be equal to 1.3 times the total error bar on the $dru$’s. We verified that the 90\% CL upper limits appearing in the last column of Table I in Ref. \[10\] are reproduced in this way with the error bars from the second column of the same table.
In this approach, the 90% CL upper limits on the dru in each energy bin result in an upper limit on the WIMP-nucleon cross section $\sigma_{\chi N}^{\max}(k)$ with the help of Eq. (25). The combined upper limit from all energy bins is obtained using Eq. (15) of Ref. [19],

$$\frac{1}{(\sigma_{\chi N}^{\max})^2} = \sum_k \frac{1}{(\sigma_{\chi N}^{\max}(k))^2}.$$  

(29)

We verified that our calculation reproduces the exclusion curves shown in Figs. 6(a,b) of Ref. [10] for both the SI and the SD cases.

C. EDELWEISS

We used Ref. [8] to incorporate the latest results from the EDELWEISS experiment that used a heat-and-ionization cryogenic Ge detector. In order to extract the 90% CL limit on the average expected number of events $N$ from the null experimental result we used the Bayesian approach described in Ref. [20]. For our analysis, we adopt the following prior distribution for $N$,

$$\pi(N) = \begin{cases} 0 & \text{for } N < 0, \\ 1 & \text{for } N \geq 0, \end{cases}$$  

(30)

which leads (after normalization) to a PDF for $N$ of the form $p_0(N) = e^{-N}$. Therefore, the probability for $N$ to be below some value $N_0$ is:

$$P_0(N < N_0) \equiv \int_0^{N_0} p_0(N)dN = 1 - e^{-N_0}. $$  

(31)

Equating the above probability to the confidence level of the EDELWEISS constraint (90%), we obtain $N_0 = \ln 10 \approx 2.3$.

The predicted number of events in the EDELWEISS detector for fixed values of WIMP parameters can be related to the detection rate by integrating Eq. (25) over the range of energies accepted by EDELWEISS (20 keV to 64 keV) and multiplying by the total effective exposure of 11.7 kg-days [8]. In order to properly normalize the detection rate we multiply it by a factor $C_E$. This factor is adjusted such that the following predictions given in Ref. [8] for numbers of events are reproduced for the pure SI case: for $M_\chi = 44$ GeV and $\sigma_\chi^S = 5.4 \times 10^{-6}$ pb one has $N = 6.2$, and for $M_\chi = 52$ GeV and $\sigma_\chi^S = 7.2 \times 10^{-6}$ pb one has $N = 9.8$. Note that since there is only one coefficient to determine, the second data point is redundant, and can thus be used for a consistency check. We find that for $C_E \approx 0.76$ both test points and
the EDELWEISS exclusion curve given in Fig. 5 of Ref. [8] are well reproduced. In all the Figures shown below, the constraints from EDELWEISS data are obtained by setting the predicted number of events in the detector equal to $N_0 = \ln 10$.

D. ZEPLIN-1

The ZEPLIN-1 experiment (see Ref. [9]) used liquid Xenon as a medium and relied on pulse-shape discrimination analysis. Unfortunately, at present there is no publication available containing detailed experimental results. The absence of such data prevented us from treating ZEPLIN-1 results in the same manner as those from the DAMA experiment with liquid $^{129}$Xe (see above). Instead, we chose an approach similar to the one we used for the EDELWEISS experiment. We deduced the parameters necessary for the calculation from Ref. [9]. In particular, we chose the visible energy to be between 4 keV and 30 keV, with quenching factor $q_{Xe}(Zeplin) = 0.2$. These parameters correspond to recoil energies between 20 keV and 150 keV. In order to calibrate our calculation we introduced an effective statistics factor, $C_Z$. We found that the SI 90% CL ZEPLIN-1 limit shown in Ref. [9] is exceptionally well reproduced for $C_Z \approx 8$ kg-days.

Due to the absence of published results, the ZEPLIN-1 limit is the most uncertain input in our calculation. However, the results of our analysis are robust and are unlikely to change when the full data is properly included. Indeed, no change in position of the ZEPLIN-1 curve in Fig. 2 will move the upper limit on the WIMP mass above about 25 GeV, which is still in conflict with the present lower limit on the MSSM lightest-neutralino mass of 37 GeV. Similarly, even if the ZEPLIN-1 constraint is entirely removed from the analysis one would still require $f_n/f_p < 0$ to achieve even marginal agreement among all data for the predominantly SI case (see explanation to Fig. 3). As shown below $f_n/f_p < 0$ would be very unusual in the MSSM.

E. Energetic neutrino searches

A promising method for indirect detection of WIMPs is the search for neutrinos from WIMP annihilation in the Sun and/or the Earth. Such neutrinos produce upward muons in the Earth via charged-current interaction. Measurement of (or constraint on) the flux of
such muons indirectly constrains the annihilation rate of WIMPs, which is related to the WIMP capture rate by the Sun and/or the Earth. A detailed review of such indirect WIMP detection with extensive list of references on the subject can be found in Ref. [1]. In this work, we use the limits on the fluxes of neutrino-induced upward muons near the surface of the Earth obtained by Super-Kamiokande [12]

\[
\Gamma_\mu(\text{sun}) \sim \Gamma_\mu(\text{Earth}) \lesssim 10^{-2} \text{ m}^{-2} \text{ year}^{-2}.
\]  
(32)

Limits from other experiments, such as IMB, Baksan, MACRO, AMANDA, etc., are very similar (see Refs. [1, 11] and references therein). The upward muon flux is related to the WIMP-proton cross section as

\[
\Gamma_{SI}^\mu \approx 1.96 \times 10^{-13} d \tanh^2 \left( \frac{t}{\tau} \right) \xi(M_\chi) f'(M_\chi) \left( \frac{M_\chi}{\text{GeV}} \right)^2 \left( \frac{\sigma_{SI}^{\chi p}}{10^{-40} \text{ cm}^2} \right),
\]

\[
\Gamma_{SD}^\mu(\text{sun}) \approx 1.6 \times 10^{-2} \tanh^2 \left( \frac{t}{\tau} \right) \xi(M_\chi) S(M_\chi/M_p) \left( \frac{M_\chi}{\text{GeV}} \right) \left( \frac{\sigma_{SD}^{\chi p}}{10^{-40} \text{ cm}^2} \right),
\]  
(33)

where \(d = 3.3 \times 10^8 \text{ m}^{-2} \text{ year}^{-2}\) for the Sun and \(d = 1.7 \times 10^8 \text{ m}^{-2} \text{ year}^{-2}\) for the Earth, and \(\tau\) is the time scale for equilibration between WIMP capture and WIMP annihilation. For the Sun, we can take \(\tanh(t/\tau) \approx 1\), whereas for the Earth it is likely substantially smaller than unity [1]. The function \(\xi(M)\) is given by Eq. (9.54), and \(S(x)\) by Eq. (9.21) in Ref. [1]. The function \(f'(M)\) is the generalization of the function \(f(M)\) given by Eq. (9.28) of the same reference,

\[
f'(M) = \sum_i f_i \phi_i S(M_\chi/M_{\text{nuc}}^i) F_i(M_\chi) \frac{M_{\text{nuc}}^i M_\chi^3}{M_p^2 (M_\chi + M_{\text{nuc}}^i)^2} \left[ \frac{Z_i}{A_i} + \left( 1 - \frac{Z_i}{A_i} \right) \frac{f_n}{f_p} \right]^2,
\]  
(34)

where the sum runs over the nuclei with mass \(M_{\text{nuc}}^i\), charge \(Z_i\), and atomic number \(A_i\). The quantities \(f_i\) and \(\phi_i\) are given in Tables 8 and 9 of Ref. [1]. The form-factor suppression \(F_i(M_\chi)\) is only important for iron in the Earth, where it is given by Eq. (9.23) of Ref. [1].

For \(M_\chi \gg M_p\) the muon flux \(\Gamma_{SD}^\mu\) is roughly independent of \(M_\chi\), which leads to the constraint \(\sigma_{\chi p}^{SD} < 1.2 \times 10^{-4}/\xi \text{ pb}\). In the MSSM one generally has \(\xi \geq 0.03 [1]\), and we obtain \(\sigma_{\chi p}^{SD} < 4 \times 10^{-3} \text{ pb}\). The constraints on \(\sigma_{SI}^{\chi p}\) from energetic solar neutrinos and on \(\sigma_{\chi(p,n)}^{SD}\) from terrestrial neutrinos are too weak to be interesting. On the other hand, the constraint on \(\sigma_{SI}^{\chi p}\) from energetic neutrinos originating in the Earth is potentially non-trivial.
For the case $f_n/f_p \approx -0.76$ and $M_\chi \approx 50$ GeV (see Section V B) one obtains

$$\sigma^{SI}_{\chi p} < \frac{3.6 \times 10^{-6}}{\xi \tanh^2 \left[ t/\tau_\oplus \right]} \text{ pb} .$$

(35)

As shown below, one needs $\sigma^{SI}_{\chi p} \approx 0.0035$ pb for this case. In addition, one can deduce from Ref. [21]

$$\left( \frac{t}{\tau_\oplus} \right)^2 = 2.4 \times 10^3 f'(50 \text{ GeV}) \frac{\sigma_A v}{10^{-26} \text{ cm}^3 \text{s}^{-1}} \frac{\sigma^{SI}_{\chi p}}{1 \text{ pb}} ,$$

(36)

where $\sigma_A$ is the WIMP annihilation cross section in the limit of zero relative WIMP velocity $v$. In order to have a WIMP relic density of order unity we must have $\sigma_A v \approx 10^{-26} \text{ cm}^3 \text{s}^{-1}$ [21], and for $f_n/f_p = -0.76$ one has $f'(50 \text{ GeV}) = 0.85$. Therefore, $(t/\tau_\oplus)^2 \approx 2 \times 10^3 \left( \sigma^{SI}_{\chi p}/1 \text{ pb} \right) \approx 8$, or $\tanh^2 \left[ t/\tau_\oplus \right] \approx 1$. Now, Eq. (35) can be simply rewritten as a constraint on $\xi$: $\xi < 0.001$ for $f_n/f_p = -0.76$. For the largest value $\xi = 0.001$ allowed in this scenario the energetic neutrino constraint on the WIMP-proton SD cross section is $\sigma^{SD}_{\chi p} < 0.12$ pb.

Can the constraints coming from non-observation of upward muons from energetic solar neutrinos be evaded? Below we consider two possibilities. In the first case, the MSSM WIMPs predominantly decay into light fermions. Because the annihilation rate $\Gamma(\chi\chi \to f^+f^-)$ is proportional to the fermion mass squared $m_f^2$, direct annihilation into neutrinos is virtually impossible, and energetic neutrinos appear in the decay chain of the initial annihilation products. The total flux of the neutrinos originating from the branch $\Gamma(\chi\chi \to f^+f^-)$ inherits the suppression by $m_f^2$, and in the case where only light fermions appear during annihilation the flux may be orders of magnitude below the conventional estimates [11]. Effectively, in this case $\xi(M_\chi)$ is substantially smaller than estimated in Ref. [1], which weakens constraints on $\sigma^{SI,SD}_{\chi p}$. In the second case, the WIMP is not identical to the anti-WIMP, and only WIMP–anti-WIMP annihilation is allowed. Although direct annihilation into neutrinos may be possible (leading to stronger energetic neutrino signals), the annihilation rate may still be significantly suppressed in the presence of significant WIMP-anti-WIMP asymmetry.

1. **WIMPs only decay into light fermions**

As pointed out in Ref. [11] the energetic-neutrino constraint could be evaded if the WIMPs annihilated to $u\bar{u}$, $d\bar{d}$, $s\bar{s}$, $e^+e^-$, and/or $\mu^+\mu^-$ pairs but not $c\bar{c}$, $b\bar{b}$, nor $\tau\bar{\tau}$ pairs.
Such a situation can in principle be achieved in the MSSM by fine tuning the sfermion masses and the WIMP composition. Let us write the lightest neutralino field as

$$\chi = Z_1 \tilde{B} + Z_2 \tilde{W}_3 + Z_3 \tilde{H}_1 + Z_4 \tilde{H}_2 ,$$

(37)

where $Z_i$ are constants subject to $\sum |Z_i|^2 = 1$, $\tilde{B}$ and $\tilde{W}_3$ are superpartners of the $B$ and $W_3$ gauge bosons, and $\tilde{H}_{1,2}$ are superpartners of the neutral Higgs bosons. Examination of the general expressions for the axial-vector and scalar neutralino-fermion couplings (Eq. (3.6) in Ref. [13]) shows that if $|Z_3| = |Z_4|$ and $Z_2 = \tan \theta_W Z_1$ the scalar coupling vanishes and the axial coupling $a_f$ is inversely proportional to the fermion superpartner mass squared,

$$a_f \sim \frac{1}{M_{\tilde{f}}^2}$$

(38)

where $M_{\tilde{f}}$ is the mass of the superpartner of the fermion $f$. Choosing the superpartner mass to be very large for the charm and bottom quarks, as well as for the $\tau$ lepton, one can force the neutralinos to annihilate into the light quarks and leptons only. However, choosing such flavor non-universal masses for the scalars may be problematic in the MSSM. Indeed, one must obey the existing stringent constraints on the size of flavor-changing neutral currents (FCNC) (see e.g. Ref. [22] for a review of FCNC constraints on the MSSM spectrum). Since evading energetic-neutrino constraints in a way we just described requires significant flavor non-universality among certain entries of the squark mass matrices, whereas smallness of the FCNC prefers the opposite, one may expect that fine tuning would be required to make any such scenario phenomenologically viable. We note that even for non-MSSM WIMPs flavor non-universality of the WIMP couplings is needed to evade the energetic-neutrino constraint in the described manner, and experimental limits on FCNC are likely to present complications for any WIMP candidate.

2. The WIMP is not identical to anti-WIMP

Another way to evade the energetic-neutrino constraint is to consider WIMPs that are not identical to their antiparticles. Indeed, for the total number density of the dark matter particles $N_0$ one has

$$n_0 + \bar{n}_0 = N_0 ,$$

$$n_0 - \bar{n}_0 = W ,$$

(39)
where \( n_0 \) and \( \bar{n}_0 \) are the WIMP and anti-WIMP densities in the vicinity of the Solar system, and \( W \) is the “WIMP number” density, which may be generated in the presence of some T-, C- and CP-violating physics. The (anti-)WIMP densities \( n \) and \( \bar{n} \) inside the solar core obey the equations [23],

\[
\dot{n} = A n_0 - B n \bar{n} - \Gamma_{\text{esc}} ,
\]

\[
\dot{\bar{n}} = \bar{A} \bar{n}_0 - B n \bar{n} - \bar{\Gamma}_{\text{esc}} ,
\]

(40)

where the first and the second terms on the RHS of both equations represent, respectively, the capture rate of WIMPs incident on the Sun and the annihilation of WIMPs and anti-WIMPs in the solar core. The constants \( A \) and \( \bar{A} \) are proportional, respectively, to the WIMP-proton and anti-WIMP–proton scattering cross sections. The last terms represent the losses of WIMPs due to evaporation (see e.g. Ref. [24]).

Consider the case where \( n_0 \gg \bar{n}_0 \). In the stationary state \( \dot{n} = 0 \), and we have for the annihilation rate \( \Gamma_{\text{ann}} = \bar{A} \bar{n}_0 \). We make a reasonable assumption that annihilation is the dominant mechanism for anti-WIMP loss if \( \bar{n} \ll n \) because the evaporation rate is suppressed by a small Boltzmann-like factor \( e^{-E_e/kT_W} \). Here, \( E_e \) is the escape energy and \( T_W \) is the effective WIMP temperature. On the other hand, for neutralinos in the MSSM we have \( (\Gamma_{\text{esc}} \) can be neglected here as well),

\[
\dot{N} = A^{\text{MSSM}} N_0 - B^{\text{MSSM}} N^2 .
\]

(41)

In the stationary state this would give for the annihilation rate \( \Gamma^{\text{MSSM}}_{\text{ann}} = A^{\text{MSSM}} N_0 \). We find that

\[
\Gamma_{\text{ann}} = \frac{\bar{A}}{A^{\text{MSSM}}} \frac{\bar{n}_0}{N_0} \Gamma^{\text{MSSM}}_{\text{ann}} = \frac{\bar{A}}{2 A^{\text{MSSM}}} \Gamma^{\text{MSSM}}_{\text{ann}} \left( 1 - \frac{W}{N_0} \right)
\]

(42)

We generally expect \( A \sim \bar{A} \), and since the WIMP-proton cross section is constrained by DAMA we expect \( A \sim A^{\text{MSSM}} \). Therefore, we conclude that \( \Gamma_{\text{ann}} \sim \Gamma^{\text{MSSM}}_{\text{ann}} (1 - W/N_0) \).

The flux of upward muons in the Super-Kamiokande detector is proportional to the WIMP annihilation rate: \( \Gamma_\mu \sim \xi(M_\chi) \Gamma_{\text{ann}} \). Therefore, we obtain,

\[
\Gamma_\mu \sim \frac{\xi(M_\chi)}{\xi^{\text{MSSM}}(M_\chi)} \left( 1 - \frac{W}{N_0} \right) \Gamma^{\text{MSSM}}_\mu .
\]

(43)

Since the WIMP is not identical to its anti-WIMP, direct WIMP annihilation into neutrinos may be possible, which may yield \( \xi(M_\chi)/\xi^{\text{MSSM}}(M_\chi) > 1 \). On the other hand, for \( 1 -
W/N_0 \ll 1$, the muon detection rate in the Sun can still be significantly below the one predicted by Eq. (5) in Ref. [11], and the energetic-neutrino constraint may not apply. In this work, we first perform the analysis without this constraint, and then add it later on.

V. ANALYSIS

A. The energetic-neutrino constraints are not applicable

Let us first consider the constraints on the WIMP parameter space without the energetic-neutrino bounds. In this case an agreement between all direct search experiments can be achieved for a predominantly SD WIMP-nucleon interaction for a wide range of WIMP masses and couplings. A similar conclusion was originally obtained in Ref. [11]. In Fig. 1 we plot the allowed region in $\sigma_{\chi p}^{SI}$ vs. $\sigma_{\chi p}^{SD}$ plane for $f_p/f_n = 1$, $a_n/a_p = 0$ and $M_\chi = 50$ GeV. The shaded region is allowed by all direct search experiments described in Section IV. Since a substantial SD interaction is required, the WIMP cannot be a scalar particle in this case.

B. The energetic-neutrino constraints are applicable

We now include the energetic-neutrino constraint. As is well known, for the usual case $f_n/f_p \approx 1$ the results from all direct search experiments cannot be reconciled for the pure SI case at about 3$\sigma$ level, and some SD interaction appears necessary. With the WIMP-proton SD cross section limited from above by the energetic-neutrino searches, a relatively large WIMP-neutron SD cross section is needed to accommodate the DAMA/NaI result. The situation is similar to the one described in Ref. [11] for the pure WIMP-neutron interaction, and the allowed region for this case is shown in Fig. 2. For this figure, we set the WIMP-proton SD cross section to zero thereby automatically satisfying the energetic-neutrino constraint. In this scenario, there is an upper limit on the WIMP mass. The strongest limit, about 18 GeV, will likely be provided by the ZEPLIN-1 experiment. We found that this limit does not significantly change even if some SI interaction is allowed. If the ZEPLIN-1 result is not included the upper limit on the WIMP mass increases to about 25 GeV.

It is interesting to note that MSSM neutralinos with mass below 37 GeV are excluded by direct collider searches [20]. Although the analysis of such searches is not completely general
FIG. 1: Constraints in the $\sigma_{SI}^{\chi p}$ vs. $\sigma_{SD}^{\chi p}$ plane due to various direct WIMP searches provided that the energetic-neutrino constraint is not included. In this plot, $f_n/f_p = 1$, $a_n/a_p = 0$, and $M_\chi = 50$ GeV. The shaded area is the $3\sigma$ allowed region consistent with the annual modulation observed by the DAMA collaboration in their NaI detector \cite{6}. The region below the dashed curve is the region allowed by the DAMA $^{129}$Xe experiment \cite{10}. The region below the dotted line is that allowed by the EDELWEISS experiment \cite{8}, and region below the dash-dotted line is allowed by ZEPLIN-I experiment \cite{9}, since it assumes gaugino and sfermion mass unification at the GUT scale \cite{20}, evading this limit, if at all possible, would require fine tuning of the MSSM parameters.

Up to this point, we always maintained the condition $f_n/f_p = 1$. It turns out that relaxing this constraint allows one to achieve marginal agreement among all data for $M_\chi$ in the vicinity of 50 GeV. However, with the ZEPLIN-I result included this occurs for only a very narrow range of $f_n/f_p$. The possibility arises because the neutron-to-proton ratio differs slightly from one nucleus to the other. Specifically, if $-0.77 \lesssim f_n/f_p \lesssim -0.75$, then the WIMP coupling to the neutron in Xe cancels that from the proton, allowing a null result in ZEPLIN-1 to be consistent with a modulation in DAMA. If ZEPLIN-1 is not considered...
FIG. 2: Constraints in the $\sigma_{SD}^\chi_n$(pb) vs. $M_\chi$ (GeV) plane due to various direct WIMP searches. To illustrate, we took $a_p/a_n=0$ (pure WIMP-neutron interaction). This choice automatically ensures that the energetic-neutrino constraints are satisfied. Notation is the same as in Fig. 1.

The allowed range is increased but the consistency range for $f_n/f_p$ is still very narrow. The situation for $f_n/f_p = -0.76$ is shown in Fig. 3. The small region allowed by all the data is centered around $\sigma_{SI}^\chi_p \approx 0.0035$ pb. Note that, due to significant cancellation between the WIMP-neutron and the WIMP-proton scattering amplitudes, the individual WIMP-nucleon cross sections are about $10^2$ times larger than in the case where $f_n/f_p = 1$. The inverse of this number, 0.01, reflects the amount of fine tuning required for this solution to work.

From the standpoint of the MSSM, this solution is in fact “doubly fine-tuned”. In addition to making sure that $f_n/f_p$ is carefully selected to fit all the data one has to tune the model of SUSY breaking to even obtain $f_n/f_p < 0$ (normally, one has $f_n \approx f_p$ in the MSSM [1]). It is interesting to ask, therefore, whether a significant deviation from the approximate equality $f_n/f_p \approx 1$ is at all possible within the MSSM. In general, we have,

$$f_N = f_0 + f_1 \tilde{t}_3 ,$$

(44)
FIG. 3: Constraints in the $\sigma_{\chi p}^{SI}$ (pb) vs. $\sigma_{\chi n}^{SD}$ (pb) plane due to various direct WIMP searches. We took $f_n/f_p=0.76$. Notation is the same as in Fig. 1. The energetic-neutrino constraint for this case, $\sigma_{\chi p}^{SD} < 0.12$ pb (see Section IV E), is satisfied.

where $N = p, n$ and $\tilde{\tau}_3$ is the usual Pauli matrix in the strong isospin space. The Feynman diagrams and detailed expressions for $f_{p,n}$ can be found in Ref. 1. Since only the up and down quarks have non-zero strong isospin, the isovector part of $f_N$ is entirely due to coupling of the neutralino bilinear to isovector operators built from the up and down quarks. The isoscalar part arises due to coupling to various isoscalar operators built from the up and down quarks, the strange and all heavy quarks, and the gluons. The scalar coupling of the neutralino to a quark has the form 5,

$$\mathcal{L}_{\chi,q} = G_F \frac{m_q}{M_W} \bar{\chi} \chi \bar{q} q \left( A_q \frac{M_{\tilde{q}_L}}{M_{\tilde{h}_L}} + B_q \frac{M_{\tilde{h}_L}}{M_{\tilde{q}_L} - (M_{\chi} + m_q)^2} \right) + O(\frac{1}{M_{\tilde{q}}}) \approx m_q f_q \bar{\chi} \chi \bar{q} q, \quad (45)$$

where $G_F$ is the Fermi constant, $A_q$ and $B_q$ are dimensionless constants, $m_q$ is the quark mass, $M_{\tilde{q}}$ is the mass of the superpartner of $q$, $M_{\tilde{h}}$ is the lightest Higgs-boson mass, and $M_{\chi}$

---

5 We assume that the squark mass matrices are diagonal in the flavor space.
is the neutralino mass. We can now write the couplings \( f_{p,n} \) in the form,

\[
f_{p,n} = m_{p,n} \left[ f_{T_u}^{p,n} f_u + f_{T_d}^{p,n} f_d + f_{T_s}^{p,n} f_s + \cdots \right],
\]

where \( \cdots \) stand for both the isovector terms of order \( 1/M_\tilde{q}^2 \) and all remaining isoscalar contributions. Here, \( m_N f_N^F = \langle N | m_q \tilde{q} q | N \rangle \). Ignoring the remaining terms we obtain (see Table 6 in Ref. [1]),

\[
\left| \frac{f_1}{f_0} \right| \leq \left| \frac{f_u(f_{T_u}^p - f_{T_u}^n) + f_d(f_{T_d}^p - f_{T_d}^n)}{2f_{T_s} f_s + f_u(f_{T_u}^p + f_{T_u}^n) + f_d(f_{T_d}^p + f_{T_d}^n)} \right| \approx \frac{-0.014 f_u + 0.025 f_d}{f_s + 0.15 f_u + 0.27 f_d}. \]

(46)

In most models of SUSY breaking \( f_{u,d} \sim f_s \), and although the above range is nothing more than an estimate it at least shows that \( |f_1/f_0| \geq 1 \) (which would lead to a negative ratio \( f_n/f_p \)) is generically disfavored in the MSSM. However, it is possible in principle to fine-tune the parameters to obtain \( |f_1/f_0| \geq 1 \). One scenario arises when \( |f_s| \ll |f_{u,d}| \). This can be achieved if \( M_\tilde{q} \) is very large for all squarks except for \( \tilde{u} \) and \( \tilde{d} \), all physical Higgs bosons, except the lightest one, are very heavy, and coupling between the WIMP and the lightest Higgs boson is tuned to zero. In this case, one finds (we use the MSSM Feynman rules given in Ref. [25] and bring the notation in correspondence with Ref. [13]),

\[
\begin{align*}
  f_u &= -\frac{g^2}{4M_W M_\tilde{q}^2} \sin \beta Z_1 (Z_2 - \tan \theta_W Z_1), \\
  f_d &= \frac{g^2}{4M_W M_\tilde{q}^2} \cos \beta Z_3 (Z_2 - \tan \theta_W Z_1), \\
  f_s &\approx 0.
\end{align*}
\]

(47)

Here, \( g \) is the \( SU(2)_L \) gauge coupling, and \( \tan \beta \) is the ratio of the expectation values for the two Higgs boson doublets in the MSSM. We find

\[
\left| \frac{f_1}{f_0} \right| \sim \frac{0.014 + 0.25 \tan^2 \beta \frac{M_\tilde{q}^2}{M_\tilde{q}^2}}{-0.15 + 0.27 \tan^2 \beta \frac{M_\tilde{q}^2}{M_\tilde{q}^2}}.
\]

(48)

If \( M_\tilde{q}/M_\tilde{q} \approx 1.3 \tan \beta \) then one can have \( |f_1/f_0| > 1 \), which may lead to \( f_n/f_p < 0 \). At present, \( \tan \beta \gtrsim 3 \) is favored by precision data \([20]\). Therefore, one needs mild hierarchy between the up- and down-squark masses to obtain \( f_n/f_p < 0 \): \( M_\tilde{q} \approx 3.9 M_\tilde{q} \).

There might be other scenarios yielding \( f_n \) substantially different from \( f_p \). However, they will all have to share the same property: the first generation of squarks must be singled out from the rest to enhance the up- and down-quark contributions to \( f_{0,1} \). Reconciling such
significant flavor non-universality of the squark flavors with stringent constraints from FCNC will generally require fine-tuning. In this sense, having significantly different values for $f_n$ and $f_p$ is unnatural, although potentially possible, in the MSSM. There is an additional complication, however. For this scenario to work, the WIMP-proton scattering cross section must be roughly 100 times larger than for the case $f_n \approx f_p$. On the other hand, one must have $\xi < 0.001$ in this case (requiring $Z_2 - \tan \theta_W Z_1 \ll 1$, see Section IV E 1), which suppresses $f_{u,d}$ according to Eq. (18). Partly due to this suppression, we have found no scenario where sufficient enhancement of $f_{p,n}$ would occur in the MSSM for $M_\chi \approx 50$ GeV together with $f_n/f_p \approx -0.76$.

We point out that $f_n/f_p \approx -0.76$ does not hold for any of the dark-matter candidates we considered. For scalar neutrinos and KK neutrinos one finds (see Subsection II A) $f_n/f_p = T^3_3/(T^3_p - 2Q_p \sin^2 \theta_W) \approx -10$. A detailed study of this ratio for KK excitation of the hypercharge gauge boson shows that it is at most few percent away from unity. In summary, although there exists a small phenomenologically allowed region of parameter space where predominantly SI WIMP-nucleon interactions can marginally account for all data available on WIMP searches this region appears to be out of reach for all of the WIMP candidates we considered.

VI. CONCLUSIONS

In this work we performed a generalized analysis of the dark-matter detection experiments listed in Section IV. Our analysis is formulated in terms of the WIMP mass $M_\chi$, the SI and SD WIMP-proton cross sections $\sigma_{SI,SD}^{\chi p}$, and coupling ratios $f_n/f_p$ and $a_n/a_p$. We found several regions in this parameter space that allow for agreement among all data.

- If the energetic-neutrino constraint does not apply it is possible to reconcile all data with a predominantly SD WIMP-proton interaction. This scenario can in principle be tested in a direct dark matter detection experiment with an odd-proton target, such as e.g. $^{19}$F [26]. Evading the energetic-neutrino constraint is problematic in the MSSM because it generally requires highly flavor non-universal squark masses. Another possibility is a non-self-charge-conjugate WIMP, which would indicate physics other than the MSSM. In order for this possibility to be realistic, such physics must accommodate WIMP-number–violating operators and possess a sufficient amount of
C and CP violation to generate a fractional WIMP asymmetry near unity.

- If the energetic-neutrino constraint applies then the SD WIMP-proton cross section is constrained to be $\sigma_{\chi p}^{SD} < 1.2 \times 10^{-4}/\xi$ pb, and for a wide range of $f_n/f_p$, a SD WIMP-neutron cross section $\sigma_{\chi n}^{SD} \gtrsim 30$ pb is required. In addition, $M_\chi$ is constrained to be below 18 GeV, which is inconsistent with the present lower limit on the lightest neutralino mass within most models of SUSY breaking [20].

- A heavier WIMP (about 50 GeV) is allowed for a narrow range $-0.77 \lesssim f_n/f_p \lesssim -0.75$ with essentially no SD interaction present. In addition, one must have $\sigma_{\chi p}^{SI} \approx 0.0035$ pb. None of the WIMP candidates we considered could satisfy both requirements at the same time.

We see that although it is possible to reconcile the direct and indirect dark-matter search experiments listed in Section [IV] the resulting parameters are in general unnatural for the MSSM with most models of SUSY breaking that appear in the literature. Either significant flavor non-universality among the SUSY breaking parameters (e.g. to evade the energetic-neutrino constraint) or some delicate relationships among them (to avoid the lower limit on the lightest neutralino mass) appear necessary. Generating the WIMP parameters in any of the allowed regions may be problematic in a theory with non-MSSM WIMPs as well. For instance, evading the energetic-neutrino constraint in a theory with non-self-charge-conjugate WIMPs may require a significant amount of CP-violation to generate a fractional WIMP number close to one. Such CP-violating interactions may be strongly constrained by existing limits on e.g. neutron and atomic electric dipole moments [20]. If the energetic-neutrino constraint is adopted, one would have to explain how very light WIMPs ($M_\chi \lesssim 18$ GeV) with significant WIMP-quark couplings have so far escaped detection.

On the other hand, if the DAMA result is removed from the analysis the MSSM neutralino (as well as other WIMP candidates) will be compatible with the remaining experiments and still be capable of being produced with sufficient abundance to account for all dark matter [20]. In view of this situation, one can hope that forthcoming direct dark-matter search experiments, potentially capable of improving the present sensitivity by several orders of magnitude, will be able to conclusively confirm or exclude the results published by DAMA based on their observation of annual modulation of the detection rate.
Acknowledgments

We thank Petr Vogel and Gary Prézeau for useful comments. This work was supported in part by NASA NAG5-9821 and DoE DE-FG03-92-ER40701 and DE-FG03-02ER41215.

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