Nucleation and Growth of Normal Phase in Thin Superconducting Strips

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(April 1, 2022)

We investigate the kinetics of normal phase nucleation and flux line condensation in the type-II superconductors by numerical study of the time-dependent Ginzburg-Landau equation. We have shown that under the sufficient transport current the normal phase nucleates in the superconducting strips in a form of the macroscopic droplets having the multiple topological charge. We discuss the stability and the dynamics of the droplets. We found that pinning suppresses the droplet formation.

PACS: 74.60.Ge,68.10.-m,05.60.+w

The study of magnetic flux penetration in type II superconductors has attracted a wide interest both in view of important technological questions and as an prototype of a general class of problems of nonlinear dynamics. Observations showed that the flux dynamics exhibits features that are similar to the viscous-fingering growth phenomenon in liquid-solid systems [1–3]. In particular recent experiments revealed dendrite flux penetration and the fingering of the remagnetization front [4,5]. The formation of vortex structure is traditionally viewed as the sequential penetration of vortices through the Bean-Livingstone surface barrier [6]. It was found recently that flux penetration may also occur via dynamic instabilities of order parameter caused by the applied current and/or magnetic field. Numerical simulations revealed the invasion of the extended macroscopic normal areas (droplets) carrying flux into the superconducting sample [6,7].

While the formation of normal areas looks natural for type I superconductors with the positive surface energy of the normal-superconductor (NS) interface, it seems surprising at the first sight that such an interface having in the static case the negative surface energy persists in type II superconductors. We see the explanation of this phenomenon in the fact that the transport current or alternating magnetic field, drives the superconductor into a strongly non-equilibrium state where the moving interface becomes stable. The idea that the free energy considerations do not apply to nonstationary processes in superconductors was put forward by Anderson et al [8] in the context of phase-slips phenomenon.

In this letter we report on our investigation of the kinetics of normal phase nucleation and flux lines condensation in the type-II superconductors. We present the results of a numerical study of the dynamics of the flux penetration into strips with the transverse dimensions less than the effective penetration length $\lambda_{ef} = \lambda^2/h$, where $h$ is the thickness of the strip and $\lambda$ is the London penetration depth. We propose that the existence of the macroscopic normal regions is the direct consequence of their motion under the transport current. A current cannot penetrate the immobile compact normal zone immersed into a superconductor, therefore the NS interface moves towards normal phase with the velocity going to infinity [11,12], and the normal droplet disappears. At the same time, the penetration of the current into a normal phase makes it stable with respect to small fluctuations [12], i.e. the transport current drives the system into a bistable state. Since the expulsion of the current from the normal regions requires a finite time, the current penetrates the moving normal droplet. The normal state develops and invades into the superconducting region provided the current $j$ flowing through the interface exceeds the stall current $j^*$ [13,14]. Thus the sufficient transport current stabilizes moving nuclei of the normal state in type II superconductors.

The process of flux penetration occurs via the depression of the order parameter on the macroscopic scale and can be viewed as the nucleation of the extended droplets of the normal phase in the superconducting sample. An adequate description of such process involving the fast variations of the order parameter on the relevant spatial scale is given by the time-dependent Ginzburg-Landau equation (TDGLE) completed by the appropriate Maxwell equations:

$$u(\partial_t + i\mu)\Psi = (\nabla - iA)^2\Psi + \left(1 - |\Psi|^2\right)\Psi,$$

$$j = |\Psi|^2(\nabla\varphi - A) - (\nabla\mu + \partial_t A),$$

$$\nabla \cdot j = 0, \quad \nabla \cdot A = 0,$$

$$\Delta A = -\frac{1}{\lambda_{ef}}\delta(z).$$

where $\Psi$ is the (complex) order parameter, $\varphi = \arg \Psi$, $A$ and $\mu$ are vector and scalar potentials, and $j$ is the current density. The value of the dimensionless material parameter $u$ is obtained from the microscopic theory [15]. The unit of length is the coherence length $\xi$, unit of time is $t_0 = \xi^2/Du$, $D = v_F l^2/3$ is the diffusion constant, $l$ is a mean free path, $v_F$ is a Fermi velocity, the field is measured in units of the upper critical field $H_{c2} = \Phi_0/2\pi\xi^2$, $\Phi_0$ is the flux quantum. The
unit of current is \( j_0 = \sigma \hbar / 2eL_0 \), where \( \sigma \) is the normal conductivity. In these units the depairing current \( j_p = 2/3\sqrt{3} \approx 0.3875 \). The condition \( \lambda_{eff} > 1 \) enables us to neglect the magnetic field created by currents \([\text{14}]\) and therefore drop the equation \([\text{6}]\). We choose the origin of the coordinate frame at the mid-point of the strip with the \( x \) axis lengthwise and the \( y \) axis in the lateral direction, so that the edges are located at \((x, -d/2)\) and \((x, d/2)\). The normal to the strip magnetic field \( B \) is associated with the vector potential \( A = (By, 0, 0) \).

We performed numerical simulations of TDGLE. We took a homogeneous superconducting state initial conditions \((\Psi = 1, \text{i.e. the state without magnetic field})\) perturbed by a small amplitude noise. We used the no-flux boundary conditions, \( \partial_y \Psi = 0 \) \( (\text{i.e. the boundary with vacuum}) \) in the transverse direction and the NS boundary conditions in the longitudinal direction \((\Psi(x, y) \to 0 \text{ for } x \to 0, L, \text{where } L \text{ is the strip length})\). We apply the split-step method described in \([\text{2,14}]\); the number of the grid points was \( 256 \times 256 \) and the time-step was \( 0.05-0.1 \).

Results of the simulations are shown in Fig. 1. The simulations were performed for \( j = 0.25, B = 0.0175 \) where as it had been shown in \([\text{14}]\), the pure superconducting state is unstable with respect to vortex nucleation (note that our equations do not contain fluctuations). The integration domain was \( 120 \times 60 \).

On the Fig. 1 the large dark droplets (for \( t = 50, 80 \)) represent the normal phase emerging at the one side of the strip and traversing toward the opposite edge. The droplets are the long-living objects and as well as vortices play a crucial role in the dissipative processes. In our simulations the topological charge of these droplets would become as big as \( 5-7 \) and even more. The droplets possess long tails (due to a finite relaxation time of the order parameter at the superconducting areas swept by the droplet). Our simulations show that new vortices appear at the edge just at the tail and then get sucked in the droplet. This can be easily understood since the formation of the new vortices is favored in the regions with suppressed order parameter. The normal phase areas can evolve in two different ways. First, the normal droplet emerges at the edge, passes through the sample and vanishes at the opposite edge of the strip. In the second scenario that occurs under elevated currents the droplet traverses a strip leaving a channel (wake) of the normal phase behind.

This scenario is shown on Fig. 1, \( t = 260 \). Than this channel traversing the sample breaks into the sequence of vortices (vortex street), which then propagate across the strip and annihilate at the edge. The nucleation and the propagation of the droplets and the vortices gives rise to non-periodic voltage oscillations along the strip.

The droplets posses the topological charge \( n \) proportional to the gain in the superconducting phase along the loop enclosing the normal area. A relationship between the characteristic size \( R \) of the nucleus and \( n \) is determined then from the condition that the supercurrent encircling the nucleus \((\sim n/R)\) becomes equal to the \( j_p \), giving \( R \approx n/j_p \). The size of the droplets in the strip can be estimated from the condition that total transport current at the distance \( R \) from the edge \( j(R) \approx j - B(R - d/2) \) is equal to the \( j_p \). It gives \( R = d/2 + (j - j_p)/B \). For the chosen parameters we obtain \( R \approx 20 \) and \( n \approx 6 - 7 \), which is in qualitative agreement with the results of simulations. We expect that the above consideration holds also for the large, \( d \gg \lambda \) samples where \( \lambda \) takes the role of the characteristic length. We observed that droplets move much faster than single vortices. Simple analysis shows that the Magnus force exerted on the droplet grows linearly with \( n \) whereas the mobility saturates for large \( n \), resulting in the velocity growth.

Shown on Fig. 1 is a sequence of snapshots demonstrating a remagnetization process (we reversed the direction of the magnetic field at \( t = 200 \)). At the first stage of remagnetization large normal phase areas develop at the edge of the strip. These areas swallow vortices corresponding to the previous direction of the magnetic field. Then the normal areas assume more complicated form and then break up into smaller droplets. For the zero applied current the Abrikosov vortex lattice is formed in the external field. In contrast to the case with nonzero applied current, vortices penetrate from both edges of the strip. When the direction of the field is reversed, large normal areas develop at both edges and swallow vortices corresponding to the initial direction of the field. After a while the new Abrikosov lattice forms with vortices along the reversed direction of the field.

To include the Hall effect in our simulations we introduce the complex material parameter \( u = 5.79 + i \). The imaginary correction to \( u \) describes the effect of the transverse Hall force on the vortices drift \([\text{17}]\). This gives rise to the Hall voltage, moreover, we observe the turn of the droplet tail. We suggest that the rotation of the droplet’s tail in the experimental work \([\text{3}]\) is caused by a significant Hall contribution.

To summarize, we have found the long living droplets of normal phase inside a superconducting phase, and observed that they may posses the topological charge than can significantly exceed unity. Note that the droplets must be distinguished from the Abrikosov vortices with multiple charge. The linear stability analysis shows that such vortices are unstable with respect to the splitting into single charged vortices. The characteristic time of the splitting is of about \( 10 - 15 \) dimensionless units and, therefore, cannot explain the existence of long-living droplets. Note that these droplets may be viewed as the result of the ”fusion" of the separate vortices.

The qualitative arguments describing the droplet dynamics can be put on the more rigorous basis for the droplets with the size well exceeding the coherence length \( \xi \). In this case the boundary of the droplet can be considered locally as slightly curved NS interface. Inside the
droplet the order parameter $\Psi$ vanishes and the field is described entirely by Laplace equation

$$\Delta \mu = 0$$

(5)

The Eq. [3] has to be completed by the boundary conditions on the interface, deduced from the continuity equation $\nabla j = 0$. It gives the relation between the components of currents normal to the interface $j_n^{(n)} = j_n^{(s)}$, where superscripts $s, n$ denotes currents in normal and superconducting regions respectively. Using Eq. [2] we arrive at the first boundary condition $-\nabla_n \mu^{(n)} = |\Psi|^2(\nabla_n \varphi - A_n) - \nabla_n \mu^{(s)}$ (here $\nabla_n$ means normal projection of the gradient). The order parameter in superconducting region near the slightly curved interface is given in "adiabatic approximation" by $|\Psi|^2 = 1 - (\nabla \varphi_0 - \Lambda)^2$.

The phase $\varphi$ of the superconducting order parameter in the leading order is described by Laplace equation

$$\Delta \varphi = 0$$

(6)

together with the equation for normal velocity of the interface. They latter can be derived from Eq. [1] for the slightly curved interface. The small curvature $\chi$ renormalizes the normal velocity $\mu_n$ of the interface according Gibbs-Thomson condition $\mu_n = \mu_0 - \chi$, where $\mu_0$ is the velocity of flat interface.

For the flat NS interface the velocity $\mu_0(j) = 0$ is a function of the transport current. The one-dimensional situation had been considered in [14], where the existence of the "stall" current $j^*$ at which the interface velocity becomes equal to zero had been established. For $u = 5.79$ the stall current was found to be $j^* = 0.335$, and $\mu_0(j \to j^*) \approx \alpha(j^* - j)$, where $\alpha = 0.6$ is the numerical factor. In two dimensional situation the topological charge of the droplet induces the circular current $j_\tau$, tangential to the interface which modifies its velocity. To account for the effect of the tangential current we take the order parameter close to the nearly flat interface in a form (the interface is parallel to $y$-axis, and we use a frame moving together with the interface along $x$-axis with the velocity $c$) $\Psi = F(x - ct) \exp[i k_y y + \phi(x - ct)]$, where $k_x = \lim_{x \to -\infty} \phi_x$, and $j_\tau = (1 - k_x^2 - k_y^2)k_y$, $j_n = (1 - k_x^2 - k_y^2)k_x$. A simple scaling analysis shows that the current renormalizes the interface velocity as

$$c(j_n, j_\tau) = c_0(\tilde{j}) \sqrt{1 - k_y^2},$$

(7)

where $\tilde{j} = j_n/\sqrt{1 - k_y^2}$. If the curvature of the interface is small (i.e $\chi \approx 1/R \ll 1$), the interface itself is defined by the additional condition that at the (flat) interface $\mu = \mu_0 = k_x c(j_n, j_\tau)$. After that the problem is completely defined.

In the superconducting phase we have the Eq. [1] completed by the boundary conditions for $\varphi$ on the strip edges. Thus the problem under study is a generalization of well-know problem of the Laplacian growth (see, e.g. [18]). A new feature is that the function $\varphi$ is a multivalued one and has branch cuts. This multivalueness means that the obtained equations contain implicitly vortex solutions: vortices can appear and/or vanish via the formation of singularity at the interface. The detailed consideration of these equations we leave for the future, for now we would like to mention that the linear stability analysis shows that the flat interface with the current flowing through is stable with respect to small perturbations. The above discussion and the results of our simulations make us to conclude that the passage of the current suppresses the NS interfaces instabilities in thin superconducting films.

To study the effects of pinning we carried out simulations of TDGLE with randomly distributed pinning centers. In the presence of the weak pinning the newly formed droplets assume the "fractal" configuration since the normal phase tries to settle at the pinning sites where the order parameter is already suppressed (see Fig. 2). The moving droplets percolates along the easy paths connecting the pinning sites, but the pinning centers impede the interface motion. As a result the current that penetrates the normal area gets smaller and cannot support the existence of the droplet any more and droplets break up. For stronger pinning the droplets do not form at all, and single vortices penetrate the strip via the jumping resembling the vortex motion through the array of linear defects [19].

Finally we discuss briefly the time scale of the observed effects. The characteristic time in dirty superconductors is $t_0 \approx 10^{-14} \div 10^{-11}$ sec depending on the temperature interval. This means that the considered phenomena develop on the nanosecond scale. However the process of the flux penetration can be considerably slowed down by pinning. In this case the characteristic time (for "dendrite" formations for example) should include macroscopic characteristics such as the size of the sample and the average pinning strength [21] and can grow up considerably.

In conclusion, we have shown that under the sufficient transport current the normal phase nucleates in the superconducting strips in a form of the macroscopic droplets which tear off at the edges and further propagate across the sample. These droplets possess the multiple topological charge related to the magnetic flux they carry. Pinning suppresses the droplet formation converting normal area into the multi-connected fractal formations which then split into the separate vortices. We believe that the observed phenomena are not specific to the thin strips, and that the same mechanism governs the normal phase formation in large samples as well.

We are grateful to A. Koshelev and U. Welp for illuminating discussions. This work was supported through U.S. Department of Energy, BES-Materials Sciences, under contract # W-31-109-ENG-38. The work of IA and
BS was partly supported by the Raschi Foundation and ISF. IA acknowledges the support by NSF office of the Science and Technology Center under contract # DMR 91-20000 at Argonne National Laboratory. The visit of VV to Israel was supported by the Rich Foundation via the Israel Ministry of Science and Arts.

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FIG. 1. Dynamics of the normal phase. The current is applied along x-axis and the magnetic field is perpendicular to the strip. Gray-coded images show $|\Psi(x,y)|$ ($|\Psi|=0$ is shown in black and $|\Psi|=1$ is shown in white). The field is reversed at $t=200$.

FIG. 2. Normal phase penetration at $t=40$, $j=0.25$, $B=0.018$ in the presence of 180 randomly distributed pinning centers, other parameters are the same as in Fig. 1.