1 A Brief Biography

John Stewart Bell (1928–90) was born in Belfast, Northern Ireland, son of John Bell and Elizabeth Mary Ann Bell (née Brownlee and always called Annie), whose families had lived in Northern Ireland for several generations. His middle name, Stewart, was a Scottish family name in his mother’s lineage, and until he went to university he was called by that name. The family’s religion was Anglican (Church of Ireland), but no religious bias prevented friendships with the Catholic community. The Bell family was of the working class, but with a high evaluation of education. Although only John remained in school beyond age fourteen, each of his younger brothers advanced socially, one (who resumed his studies after marrying and having children) to become a professor of electrical engineering and the other to become a successful businessman.

By age eleven John, who read intensively at the public library, announced his intention to become a scientist. He performed very well on his entrance examination for secondary education, but the family could only afford to send him to the Belfast Technical High School, which – usefully for his later career – provided both academic and practical courses.

Finishing the Technical School in 1944 at age sixteen, Bell found a job as a junior technical assistant in the Physics Department of the Queen’s University at Belfast, under the
supervision of Prof. Karl Emeleus and Dr. Richard Sloane. They recognized his talent, loaned him books, and allowed him to attend first year lectures even though he was not enrolled as a student. After one year as a technician he became a student and was awarded modest scholarships, graduating in 1948 with a first class Honours BSc in Experimental Physics. He then stayed another year, receiving a first class Honours BSc in Mathematical Physics in 1949. Bell considered himself especially fortunate to have as his primary teacher in mathematical physics Peter Paul Ewald, a refugee from Nazi Germany, who was one of the pioneers of X-ray crystallography. Although Bell received good training in basic physics, he was not happy with the explanations of quantum mechanics, for reasons that will given in some detail in Sect. 2. In general, he was an extraordinarily able and assiduous student, showing inquisitiveness, persistence, and independence of mind that foreshadowed the great investigations of his later years.

From the middle of 1949 until 1960 John Bell worked in England, mainly at the UK Atomic Energy Research Establishment (A.E.R.E.) at Harwell. He first worked there for about two months on a reactor problem, then went to Malvern late in 1949 to join the accelerator group, but returned to Harwell in 1951 to continue work on accelerators and to do research on nuclear physics. At Harwell in 1949 John Bell first met his future wife Mary Ross. By that time she had been working on reactors for several years. Later, in 1950, she also went to Malvern to join the accelerator group. This was her second stay in Malvern, as she had been called up during World War II to work on radar at T.R.E. Malvern. When the war ended, she returned to Glasgow to take her doctorate in physics and mathematics and then went to Harwell early in 1947. They were married in 1954 and pursued careers together, often collaborating in research, first at Harwell and later at CERN until his death in 1990. There is a very moving tribute to her in the last paragraph of the preface to John Bell’s Speakable and Unspeakable in Quantum Mechanics [5], the collection of his papers on that subject: “In the individual papers I have thanked many colleagues for their help. But here I renew very especially my warm thanks to Mary Bell. When I look through these papers again I see her everywhere.”

John Bell was given a paid leave of absence during 1953 and 1954 to do graduate studies in the Physics Department at the University of Birmingham, under the direction of Paul Matthews and Rudolf Peierls, completing his doctoral thesis in 1956 after returning to Harwell. At Birmingham Bell quickly acquired a knowledge of current theoretical physics, including quantum field theory, and his doctoral thesis included a proof of the PTC theorem, on which he lost priority because Lüders published a proof of the theorem before his own paper [1] appeared in the Proceedings of the Royal Society. As a member of Tony Skyrme’s
nuclear group at Harwell he made a number of powerful applications of field theory and theory of symmetries to nuclear problems [2, 3, 4].

In the late fifties, John and Mary Bell were troubled that Harwell was becoming less devoted to research in fundamental physics, and they began to look elsewhere for a locus of work. In 1960 they resigned their tenured positions and took nontenured positions at CERN, he in the Theory Division and she in the Accelerator Research Group. They lived in the city of Geneva. Bell’s working hours at CERN were devoted primarily to theoretical particle physics. He facetiously listed his specialty in an official CERN document as “quantum engineering.” Some of his leisure time was devoted to the foundations of quantum mechanics, which he called his “hobby”. A year of leave in 1963–4, spent at the Stanford Linear Accelerator Center, the University of Wisconsin, and Brandeis University, gave him time to write his two pioneering articles [5, 6] on the foundations of quantum mechanics. Ref. [6] demonstrated a conflict concerning observable quantities between the family of local hidden variables theories and quantum mechanics. This famous result became known as “Bell’s Theorem” (see Sect. 2 below).

On his return to CERN, Bell continued his studies of the foundations of quantum mechanics privately, but his professional work was in particle physics, mostly in the framework of the then popular scheme of current algebra. Within these current algebra investigations Bell established his famous result on the anomalous nonconservation of the axial vector current, based on the anomalous breaking of chiral symmetry [8] (see Sect. 3 below).

Recognition of John Bell as a great scientist came in several stages. In the early fifties his contributions at Malvern to accelerator design, based on his expertise in particle dynamics and electromagnetic theory, were highly regarded; at age 25 he was a consultant when CERN was being established. From the time of Bell’s graduate studies at Birmingham he acquired a high reputation among particle theorists, which grew and spread worldwide after he joined the Theory Group at CERN. There was, however, relatively little attention to his work on the foundations of quantum mechanics until experiments (e.g., [9]–[15]) inspired by Bell’s Theorem were reported. Most of the experimental results were conservative, in that they strongly supported quantum mechanics and disconfirmed the implications of the local hidden variables theories, but whatever the results might have been the experiments surprised the physics community by showing that hypotheses about hidden variables and locality, generally considered to be philosophical since the famous Einstein-Bohr debate [16, 17], were amenable to empirical investigation. “Bell’s Theorem” became a topic in the guide to authors of the American Institute of Physics, and interest in the Theorem spread to philosophers and (unfortunately with frequent exaggeration and distortion) to the general public.
Bell was honored fairly early by election to the Royal Society in 1972, proposed by Paul Matthews and Rudolf Peierls. In the eighties many more honors came to him: the Reality Foundation Prize (shared with J.F. Clauser, 1982), Honorary Fellow of the American Academy of Arts and Sciences (1987), Dirac Medal of the Institute of Physics (1988), honorary doctorates at the Queen’s University of Belfast (1988) and at Trinity College in Dublin (1988), the Dannie Heineman Prize for Mathematical Physics (1989), and the Hughes Medal of the Royal Society (1989). His contributions to the foundations of quantum mechanics were regularly cited along with his discoveries in main line physics. These honors were enhanced by a universal recognition that John Bell was a man whose character equalled his intellect.

John Bell died suddenly of a stroke on October 1, 1990. He was still at the height of his powers just before he was stricken, and the loss to science was undoubtedly very great. The world also lost a man of exemplary dedication, integrity, courage, modesty, generosity, and humanity.

2 Foundations of Quantum Mechanics

2.1 Early Thoughts

Even as an undergraduate Bell was seriously troubled by the foundations of quantum mechanics and unconvinced of the dominant opinion at his University and in the physics profession generally. Information about this period can be found in Quantum Profiles \(^\text{[18]}\) of Jeremy Bernstein, who interviewed Bell, and in biographical articles of Andrew Whitaker \(^\text{[19]}\), who obtained information from Lesley Kerr, a friend of Bell one year behind him in the Physics Department at Queen’s. Quantum mechanics was taught in the context of Dr. Sloane’s course on atomic spectra, with emphasis on practical applications rather than conceptual foundations. Bell questioned Sloane skeptically and indeed aggressively, and was dissatisfied not only with Sloane’s exposition but with what he learned generally of the Copenhagen point of view. The argument with Sloane

\[
\text{...had convinced Bell of the central importance of the concept of measurement in any meaningful discussion of quantum theory. It was necessary to identify this as a central element, in order to be able to criticise the fact by pointing out the illegitimacy of using the idea of measurement as a primary term....}
\]

At the time the aspect of this that most concerned Bell was the divide or ‘Heisenberg cut’ between quantum and classical. He fully accepted Bohr’s acknowledg-
ment that the apparatus must be treated classically, but definitely did not accept his arguments about the relation between quantum and classical. Actually it would be true to say that he did not recognise where Bohr’s actual argument was supposed to be, though he knew that Bohr claimed to have solved the problem. If possible Bell wished to eliminate the distinction between quantum and classical altogether, and the most obvious way to do attempt to do that was by hidden variables. [27], Sect. 5

2.2 Ideas on Measurement and Physical Reality: “Beables”

The discontent concerning the Copenhagen Interpretation – and of quantum mechanics itself, if it is taken to be a complete theory – expressed in these reconstructions of Bell’s somewhat confused early ideas persisted throughout his career, but was sharpened, clarified, and linked to a variety of constructive programs. He felt that Bohr and Heisenberg were profoundly wrong in giving observation a fundamental role in physics, thereby letting mind and subjectivity permeate or even replace the stuff of physics, however much they obscured that they were doing this. Observation is indispensable to the process of obtaining knowledge about the physical world, but Bell always maintained that what is there to be known has an objective status independently of being observed. Here is a sample of statements to this effect from publications over a period of a quarter of a century:

1967: It is easy to imagine a state vector for the whole universe, quietly pursuing its linear evolution throughout all of time and containing somehow all possible worlds. But the usual interpretive axioms of quantum mechanics come into play only when the system interacts with something else, is ‘observed’. For the universe there is nothing else, and quantum mechanics in its traditional form has simply nothing else to say. It gives no way of, indeed no meaning of, picking out from the wave of possibility the single unique thread of history . . . . In any case it seems that the quantum mechanical description will be superseded. In this it is like all theories made by man. But to an unusual extent its ultimate fate is apparent in its internal structure. It carries in itself the seeds of its own destruction. [27]

1973: It is interesting to speculate on the possibility that a future theory will not be ambiguous and approximate. Such a theory would not be fundamentally about ‘measurements’, for that would again imply incompleteness of the system and unanalyzed interventions from outside. Rather it should again be possible to say of a system not that such and such may be observed to be so but that such and such be so. [23]
1975: **The theory of local beables.** This is a pretentious name for a theory that hardly exists otherwise, but which ought to exist . . . . The terminology, be-able as against observ-able, is not designed to frighten with metaphysic those dedicated to realphysic. It is chosen rather to help in making explicit some notions already implicit in, and basic to, ordinary quantum theory . . . . ‘Observables’ must be made somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables. \[23\]

1981: Where is the ‘measurer’ to be found? . . . . If the theory is to apply to anything but idealized laboratory operations, are we not obliged to admit that more or less ‘measurement-like’ processes are going on more or less all the time more or less everywhere. \[24\], Sect. I

1990: But experiment is a tool. The aim remains to understand the world. To restrict quantum mechanics to be exclusively about piddling laboratory operations is to betray the great enterprise. \[23\]

### 2.3 Logical Investigations on Hidden Variables Theories

As noted above in the quotation from Whitaker’s memoir \[24\], Bell was already attracted in his undergraduate years by the idea of a hidden variables theory supplementing the description of a physical system given by quantum mechanics. (He objected later to the term “hidden variables”, which suggests inaccessibility to any experimental probing, and preferred “beables”, as indicated in one of the quotations above.) Even without knowing what these hidden variables are in detail, one might construct models of them, which would illuminate some of the conceptual difficulties of quantum mechanics. For instance, fixing hidden variables might in principle determine the values of all the observable quantities recognized by quantum mechanics, though the chaotic behavior of the physical world at the level of the hidden variables – analogous to thermal chaos in kinetic theory of gases – would preclude experimental control, thereby accounting for Heisenberg’s uncertainties. According to Whitaker, however, Bell’s interest in hidden variables was dampened by reading Max Born’s *Natural Philosophy of Cause and Chance* \[26\], which maintained in general terms the difficulty of constructing a theory of this kind that would recover the known quantum statistics, and in addition stated that von Neumann \[27\] had given a rigorous mathematical proof for the impossibility of such a construction. Not knowing German, Bell could not assess the validity of von Neumann’s proof, but was inclined for a while to believe Born.
We do not seem to have information about the incubation of Bell’s ideas on quantum mechanics between from the time he left Queen’s University until 1952, but in that year he developed them radically. In the Acknowledgments of Bell’s pioneering paper in Reviews of Modern Physics, “On the Problem of Hidden Variables in Quantum Mechanics” he says, “The first ideas of this paper were conceived in 1952. I warmly thank Dr. F. Mandl for intensive discussion at that time.” Mandl was a refugee from Germany and was able to tell him the content of von Neumann’s proof, as yet untranslated. Furthermore, David Bohm’s “A Suggested Interpretation of the Quantum Theory in Terms of ‘Hidden’ Variables” was published, providing an instance of just the sort of model that von Neumann’s theorem allegedly showed to be impossible. From his reading and his discussions with Mandl it became clear to Bell that von Neumann’s theorem, though mathematically correct, was physically weak. One of von Neumann’s premisses concerned expectation values $\mathcal{E}(A), \mathcal{E}(B)$, etc. of the observables $A, B$, etc. recognized by quantum mechanics. He required that in any ensemble of systems of the specified kind, even an ensemble determined by fixing the hypothetical hidden variables – which would be dispersion-free since each observable would be given a definite value by the fixed hidden variables – the expectation values would be additive:

$$\mathcal{E}(A + B) = \mathcal{E}(A) + \mathcal{E}(B) \quad (1)$$

where A and B are observables of the system quantum mechanically represented by self-adjoint operators, not necessarily commuting. Bell’s analysis, not published until 1966, is devastating:

The essential assumption can be criticized as follows. At first sight the required additivity of expectation values seems very reasonable, and it is rather the non-additivity of allowed values (eigenvalues) which requires explanation. Of course the explanation is well known: A measurement of a sum of noncommuting observables cannot be made by combining trivially the results of separate observations on the two terms – it requires a quite different experiment. For example the measurement of $\sigma_x$ for a magnetic particle might be made with a suitably oriented Stern-Gerlach magnet. The measurement of $\sigma_y$ would require a different orientation, and of $(\sigma_x + \sigma_y)$ a third and different orientation. But this explanation of the nonadditivity of allowed values also established the nontriviality of the additivity of expectation values. The latter is a quite peculiar property of quantum mechanical states, not to be expected a priori. There is no reason to demand it individually of the hypothetical dispersion free states, whose func-
tion it is to reproduce the *measurable* peculiarities of quantum mechanics *when averaged over*. [3], Sect. 3

In addition to criticizing von Neumann’s premisses, Bell constructed a family of hidden variables models for a spin-1/2 particle in each of which definite values are consistently assigned to every spin observable, and the additivity of expectation values holds for observables represented by commuting operators. Each such model can be taken as a hidden variable $\lambda$ in a space $\Lambda$ of hidden variables, $\lambda(A)$ being the value assigned by this hidden variable to the observable $A$. Bell showed, furthermore, that every quantum mechanical state $\phi$ of the spin-1/2 system can be constructed by averaging over $\Lambda$ with an appropriate probability measure $\rho$ (independent of the observables):

$$\langle \phi | A | \phi \rangle = \int_{\Lambda} A(\lambda) \, d\rho . \tag{2}$$

Bell’s construction is quite simple, and it will be seen momentarily that the dimension two of the Hilbert space associated with the spin-1/2 system is crucial. Bell notes that hidden variables model differs conceptually from Bohm’s by using only the algebraic structure of the observables of the spin-1/2 system, without any consideration of the apparatus used for measuring.

An important part of Ref. [7] was inspired by a theorem of Andrew Gleason [29], which was not published until 1957 and not known by Bell until conversations with Josef Jauch after moving to Geneva. Gleason did not directly address the question of hidden variables, but considered which probability measures are definable on the lattice of projection operators (which is isomorphic to the lattice of closed linear subspaces) on a Hilbert space. The conditions on a probability measure $m$ are

1. $m(Q)$ is a nonnegative real number for any projection operator $Q$ on the Hilbert space;
2. $m(I) = 1$, where $I$ is the identity operator;
3. If $\{Q_i\}$ is a finite or countably infinite set of mutually orthogonal projection operators, then $m(\sum Q_i) = \sum m(Q_i)$.

Gleason’s theorem asserts that if the Hilbert space has dimension three or greater then a probability measure must be determined either by a quantum mechanical pure state,

$$m(Q) = \langle \phi | Q | \phi \rangle \tag{3}$$
for some vector $\phi$ in the Hilbert space with norm unity, or else by a convex combination of measures thus determined; equivalently, $m$ must be determined in the standard quantum mechanical manner,

$$m(Q) = \text{Tr}(QW)$$  \hspace{1cm} (4)

where $W$ is a statistical operator, i.e., a positive self-adjoint operator of trace unity.

One easily sees that $\text{Tr}(QW)$ cannot have values restricted to 0 or 1 for all projection operators $Q$, from which in turn it follows that the state represented by $W$ cannot be dispersion-free for each observable, that is, each self-adjoint operator. Consequently, a corollary of Gleason’s theorem is that no system associated quantum mechanically with a Hilbert space of dimension three or greater can admit a dispersion-free state; or in von Neumann’s locution, there can be no dispersion-free ensemble of such systems. And of course, since the set of dispersion-free states is empty, one cannot recover a quantum mechanical state $\phi$ by averaging appropriately over them. Note that the hidden variables model constructed by Bell earlier in Ref. [7] does not conflict with Gleason’s theorem and its corollary, since the Hilbert space in that case has dimension two.

The proof given by Gleason of his theorem is intricate; Bell said (at a conference) that he knew he had either to read Gleason’s proof or to construct his own proof of the corollary that interested him, and the latter alternative was obviously easier. And this he did, elegantly and simply, in Sect. 5 of Ref. [7]. An independent proof of the corollary was published the following year by Simon Kochen and Ernest Specker [30].

After this accomplishment, the end of Bell’s Ref. [7] is a remarkable surprise. Having exhibited that the hidden variables program, construed as it had been in the past, is mathematically impossible except in the trivial case of dimension two, Bell resuscitated the program by a well-motivated relaxation of conditions. He slyly remarked, “That so much follows from such apparently innocent assumptions leads us to question their innocence.” What is not innocent, he pointed out, is the tacit assumption that when the hidden variable $\lambda$ is specified, “the measurement of an observable must yield the same value independently of what other measurements may be made simultaneously.” If observable $A$ and $B$ commute, then quantum mechanics requires that in principle both can be measured simultaneously, because there exists an observable $C$ such that $A$ can be expressed as a function $A(C)$ and $B$ as a function $B(C)$. But $A$ can commute with two observables $B$ and $B'$ that are noncommuting, and then a $C$ of which $A$ and $B$ are functions necessarily differs from a $C'$ of which both $A$ and $B'$ are functions. Thus a different procedure is required to measure $A$ along with $B$ from the one used to measure $A$ along with $B'$. This observation opens up a new family
of hidden variables theories, which have come to be called “contextual”, in which the value of an observable $A$ is a function not only of the hidden variable $\lambda$ but also the context of measurement. Bohm’s model \cite{28} was an instance of this idea, but he did not present the idea explicitly and generally. By a judo-like maneuver Bell elicits unwitting cooperation in the resuscitation of the hidden variables program from Niels Bohr, a dedicated opponent of it, by citing Bohr’s thesis of the “impossibility of any sharp distinction between the behavior of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear” \cite{31}.

### 2.4 Bell’s Theorem

The most influential of Bell’s papers on the foundations of quantum mechanics was “On the Einstein-Podolsky-Rosen Paradox” \cite{6}. Published in 1964, its content presupposes \cite{7}, which was published two years later. The explanation for this chronological peculiarity is the misplacement of \cite{7} in the offices of *Reviews of Modern Physics*. Ref. \cite{6} is the pioneering presentation of what has come to be called “Bell’s Theorem,” which (roughly) asserts that no hidden-variables theory that satisfies a certain locality condition can reproduce all the predictions of quantum mechanics. The name “Bell’s Theorem” actually applies to a family of theorems with the general character just stated, but we shall review only three variants that Bell himself proved. In all of Bell’s variants a space of complete specifications – the hidden variables – of the system is envisaged (not the case in other variants, notably the controversial ones of Henry Stapp \cite{32,33} and other expositions), and the results are independent of the detailed character of the hidden variables. What is important is (i) the way in which observable quantities depend upon the hidden variables, and (ii) the fact that the space of hidden variables admits probability distributions. In Ref. \cite{6} the hidden variable is taken to be a single continuous parameter, and the system studied for the purpose of proving the Theorem consists of two well-separated particles 1 and 2. A family of quantities parameterized by $a$ is measurable on 1, each, for simplicity, assumed to have only two possible outcomes $+1$ and $-1$; likewise, a family of quantities parameterized by $b$ is measurable on 2, with the same assumption regarding outcomes. The outcome of a measurement on 1 is a function

\[
A(a, \lambda) = \pm 1
\]  

(5a)

regardless of what quantity is measured on 2; and likewise the outcome of a measurement on 2 is a function

\[
B(b, \lambda) = \pm 1
\]  

(5b)
regardless of what quantity is measured on 1. Since the outcomes are definite when the quantity specified by \( a \) (respectively \( b \)) is given along with \( \lambda \), the hidden variables theory is deterministic. It may appear that Bell has neglected the possibility of contextuality, which he introduced in Ref. [6] to resuscitate the hidden variables program, but that impression is due to the notation. As a matter of fact, two different parameters \( a \) and \( a' \) for particle 1 could refer to the same measurable quantity measured along with different contexts concerning particle 1, though with no reference to what is measured on 2; and likewise for \( b \) and \( b' \). Contextuality is tacitly allowed in this version of Bell’s theorem, but it is restricted by a locality requirement, which is vital: the result \( A \) does not depend upon what is measured on 2, and the result \( B \) does not depend upon what is measured on 1. Bell justifies this requirement with a quotation from Einstein.

But on one supposition we should, in my opinion, hold absolutely fast: the real factual situation of the system \( S_2 \) is independent of what is done with the system \( S_1 \), which is spatially separated from the former. [6], footnote 2

In Ref. [6] the well-separated particles 1 and 2 are assumed to have spin-1/2 and to be in the quantum mechanical singlet state. If \( a \) and \( b \) are vectors in ordinary three-space, then \( \sigma_1 \cdot a \) is the quantity parameterized by \( a \), and \( \sigma_2 \cdot b \) is the quantity parameterized by \( b \). The anti-correlation of spins of particles in the singlet state guarantees that if \( a \) and \( b \) are equal then the outcomes of the measurements of the corresponding quantities of 1 and 2 are opposite, or equivalently, the product of their measurement outcomes is \(-1\). For the hidden variables theory to agree with this quantum mechanical prediction there must be a probability distribution \( \rho \) over the space of hidden variables (normalized to unity) such that

\[
\int d\lambda \rho(\lambda)A(a, \lambda)B(a, \lambda) = -1. \tag{6}
\]

But this is possible only if

\[
A(a, \lambda) = -B(a, \lambda) \tag{7}
\]

for all but a set of measure zero in the space of hidden variables. All the concepts and assumptions of the theorem in Ref. [6] have now been introduced, and the remainder of Bell’s work is mathematics, which we shall condense. Using Eqs. (6), (7), and (7) he easily proves the inequality

\[
1 + P(b, c) \geq |P(a, b) - P(a, c)| \tag{8}
\]
where

\[ P(a, b) = \int d\lambda \rho(\lambda) A(a, \lambda) B(b, \lambda) \]  

(9)

(that is, the expectation value using the probability distribution \( \rho \) of the product of the outcomes of measurement of the spin components parameterized by \( a \) and \( b \)). Inequality (8) is the prototype of a family of inequalities that are now collectively called “Bell’s Inequalities.” It is a well-known consequence of the quantum mechanical singlet state that the expectation value of the product of an arbitrary spin component of particle 1 and an arbitrary spin component of particle 2 satisfies

\[ \langle \sigma_1 \cdot a \sigma_2 \cdot b \rangle = -a \cdot b. \]  

(10)

The discrepancy between the hidden variables inequality (8) and the quantum mechanical expectation value (10) is immediate, thus proving the theorem of Ref. [6].

There are important additional points in this paper. The title, “On the Einstein-Podolsky-Rosen [EPR] Paradox”, signals the importance of EPR’s paper [16] in focusing on a pair of well-separated particles that has been properly prepared to ensure strict correlations between certain of their observable quantities. Bell recognizes in Sect. 2 that the deterministic character of the hidden variables theory that EPR recommend is not an additional assumption, but is the consequence of EPR’s assumptions of locality, exact agreement with the quantum mechanical predictions of strict correlation, and their well-known sufficient condition for the existence of an element of physical reality (but his acknowledgment of the third assumption is tacit rather than explicit). Next, in recognition of the inaccuracies of actual measurement, Bell asks in Sect. 4 of Ref. [6] whether a deterministic local hidden variables theory can agree approximately with the quantum mechanical predictions, and he provides a negative answer by deriving a generalization of inequality (8). He also briefly mentions generalizations of his theorem to pairs of systems each associated with a Hilbert space of dimension greater than two and having observables with more than two possible values. Finally, and most remarkably, in Sect. 6 he mentions the possibility that quantum mechanics is of limited validity, applying only when “the settings of the instruments are made sufficiently in advance to allow them to reach some mutual rapport by exchange of signals with velocity less than or equal to that of light.” If so, the locality requirement assumed in the theorem would not be physically justified and would not be a consequence of the locality of special relativity theory. In order to check this conjecture, “experiments of the the type proposed by Bohm and Aharonov...in which the settings are changed during the flight of the particles, are crucial” [34]. Thus, in this pioneering paper of 1964 Bell already suggests the desirability of an experiment like that carried out by Aspect, Dalibard, and Roger in 1982 [15].
In 1971 Bell [35] published a new version of his Theorem, which is an improvement over the first in two respects. (a) It dispenses with reliance upon quantum mechanics, as in Eq. (2) above, for the purpose of deriving a crucial inequality. In so doing it follows a paper by Clauser et al. [36] of 1969. If the resulting inequality conflicts with experiments, one can conclude not only the falsity of the conjunction of quantum mechanics with the assumptions of the local hidden variables theory but the falsity of those assumptions themselves without conjoining quantum mechanics. (b) It weakens the assumption (5) that the hidden variable deterministically governs the outcome of each observable quantity of particles 1 and 2, asserting instead that $\lambda$ fixes the expectation value of the outcome of measuring the quantities of 1 parameterized by $a$, or equivalently the probability of each possible measurement outcome, and likewise for the quantities of 2 parameterized by $b$. The locality requirement is that the expectation value of the former is independent of $b$ and conversely. In Sect. 4 Bell says that the failure of deterministic equations like (5) above may be due to hidden variables of the instruments, but in Footnote 10 he adds that this failure may be due to a more fundamental indeterminism, persisting even when the hidden variables both of the particles and the instruments are specified. The character of the indeterminism does not affect the mathematics which leads to the inequality

$$|P(a, b) - P(a, b')| + |P(a', b') + P(a', b)| \leq 2.$$  

(11)

It is easy to show the inconsistency of Inequality (11) with the ideal quantum prediction expressed by Eq. (10) and – obviously valuable in experimental investigations – even with approximations to the ideal prediction. In view of the assumptions upon which Inequality (11) is based, its disconfirmation by experiment would throw doubt upon the entire family of deterministic and nondeterministic local hidden variables theories.

Bell’s third variant of his Theorem appeared in a somewhat rough form in “The theory of local beables” [23], but a better presentation, meeting some criticisms, appeared in “La Nouvelle Cuisine” [37], written too late to be included in the collection *Speakable and Unspeakable in Quantum Mechanics*. What is interesting about this variant is a greater explicitness about the locality requirement than in its predecessors. He supposes that the measurements of the quantities of quantities parameterized respectively by $a$ and $b$ are performed in regions 1 and 2 with space-like separation. (For the careful use of special relativity theory this description of the ideal experimental arrangement is clearly more precise than “well-separated particles”.) Region 3 is bounded by two nonintersecting space-like surfaces, both cutting through the backward lightcones of both 1 and 2 in such a way that the intersections of the later surface with the two backward lightcones are nonoverlapping, while the intersections...
of the earlier surface with the two backward lightcones overlap. In region 3, let \( c \) stand for any set of physical quantities describing the experimental arrangement and recognized by quantum mechanics in that part of region 3 that blocks the two backward lightcones and \( \lambda \) be all other variables needed in addition to \( c \) to describe completely this part of region 3. The probability of joint outcomes \( A \) and \( B \) of the two measurements reasonably depends only upon \( a, b, c, \) and \( \lambda \) and hence can be written as \( \{ A, B \mid a, b, c, \lambda \} \), which by standard probability theory can be factorized into

\[
\{ A, B \mid a, b, c, \lambda \} = \{ A \mid a, b, c, \lambda \} \{ B \mid a, b, c, \lambda \} .
\]

(12)

Then, Bell reasons,

Invoking local causality, and the assumed completeness of \( c \) and \( \lambda \) in the relevant parts of region 3, we declare redundant certain of the conditional variables in the last expression, because they are at space-like separation from the result in question. Then we have

\[
\{ A, B \mid a, b, c, \lambda \} = \{ A \mid a, c, \lambda \} \{ B \mid b, c, \lambda \} .
\]

(13)

Bell remarks that Eq. (13) “exhibits \( A \) and \( B \) as having no dependence on one another, nor on the settings of the remote polarizers (\( b \) and \( a \) respectively), but only on the local polarizers (\( a \) and \( b \) respectively) and on the past causes, \( c \) and \( \lambda \).” Eq. (13) is essentially the locality requirement used in Ref. [35], but here he does not take it as the starting point of his analysis but rather as a consequence of two quite different locality assumptions used in passing from Eq. (12) to Eq. (13). (There is similar reasoning in Sects. 2 and 4 of Ref. [23] and in a paper by J. Jarrett [38].) Once Eq. (13) is derived, and it is further reasoned that the probability distribution for the hidden variable \( \lambda \) is independent of the freely chosen variables \( a \) and \( b \) and therefore dependent only upon \( c \), one obtains a factorizability condition for expectation values:

\[
\mathcal{E}(a, b, c) = \sum_{\lambda} \sum_{A,B} AB \{ A \mid a, c, \lambda \} \{ B \mid b, c, \lambda \} \{ \lambda \mid c \} .
\]

(14)

Inequality (11) can then be derived essentially as in Refs. [35] and [36], and Bell omits the details.

### 2.5 Is There Nonlocality in Nature?

In Sect. 6.7 of [37] Bell offers the following formulation of the principle of local causality: “The direct causes (and effects) of events are near by, and even the indirect causes (and
effects) are no further away than permitted by the velocity of light.” He then reviews Bell’s Theorem and the experiments which it inspired to investigate whether the principle of local causality holds in quantum mechanics and in nature. He pays attention to attempts to give a positive answer to this question, but concludes that these are desperate and unconvincing. Bell’s own negative answer led him to controversial but tentative proposals, which will be presented below.

The ideal predictions of quantum mechanics in certain situations violate Inequality (11) – for instance, when the linear polarization of photon 1 of a pair of photons is measured in region 1 of Ref. [37] and the linear polarization of photon 2 of the pair is measured in region 2, the pair being prepared in the polarization state

$$\Psi = \frac{1}{\sqrt{2}} \{ |h\rangle_1 |h\rangle_2 + |v\rangle_1 |v\rangle_2 \}$$

(15)

where $|h\rangle$ is a state of horizontal polarization and $|v\rangle$ a state of vertical polarization. Consequently, quantum mechanics must deny one or both of the two omissions of variables in the transition from Eq. (12) to Eq. (13). As a matter of fact, it is immediate, in view of the correlations of Eq. (10), that quantum mechanics does not permit the substitution of $\{ A \mid a, c, \lambda \}$ for $\{ A \mid B, a, b, c, \lambda \}$ (where $\lambda$ is simply the quantum state $\Psi$ in quantum mechanics). On the other hand, the substitution of $\{ B \mid b, c, \lambda \}$ for $\{ B \mid a, b, c, \lambda \}$ has often been shown to be valid in quantum mechanics; a sketch of an argument in the case of relativistic quantum mechanics – using the commutativity of Heisenberg operators at space-time points that are space-like separated – is given by Bell in Secs. 6.5 and 6.11 of Ref. [37]. If this substitution were not allowed, then a superluminal signal could be transmitted from region 1 to region 2, since dependence of the probability of outcome $B$ upon the freely chosen variable $a$ provides (probabilistically) some information about that choice. Alternatively, the dependence – entailed by quantum mechanics – of the probability of the outcome $A$ in region 1 upon the outcome $B$ in region 2 does not permit transmission of information about a choice, because the outcome $B$ is a chance event not chosen by the experimenter in region 2. In other words, although quantum mechanics violates Inequality (11), which is a consequence of any local hidden variables theory, deterministic or nondeterministic, it does so in such a way that the violation cannot be used to provide superluminal communication. Accordingly, some writers on the problem (included one of us, A.S.) have suggested that, despite tension between quantum mechanics and special relativity theory, there is “peaceful coexistence” between the two theories. Furthermore, if one accepts the auxiliary assumption that the pairs of particles detected in an actual correlation experiment is a fair sample of the pairs emerging from polarization analyzers, even though the detectors are quite inefficient –
an assumption that Bell regards as very reasonable ([37], Sect. 6.12) – then in view of the overwhelming favoring of quantum mechanics in correlation experiments there is a kind of nonlocality in nature, but such as to coexist peacefully with relativistic locality.

Bell is scornful of this optimistic and conciliatory proposal. The reason is his unwillingness to admit that something complex, special, and anthropocentric can be a fundamental principle of physics – the same reason that is pervasive in his polemic “Against ‘Measurement’”, cited in Sect. 2.2 above.

Do we then have to fall back on ‘no signaling faster than light’ as the expression of the fundamental causal structure of contemporary theoretical physics? That is hard for me to accept. For one thing we have lost the idea that correlations can be explained, or at least this idea awaits reformulation. More importantly, the ‘no signaling . . . ’ notion rests on concepts that are desperately vague, or vaguely applicable. The assertion that ‘we cannot signal faster than light’ immediately provokes the question:

Who do we think we are?

We who can make ‘measurements’, we who can manipulate ‘external fields’, we who can ‘signal’ at all, even if not faster than light? Do we include chemists, or only physicists, plants, or only animals, pocket calculators, or only mainframe computers? ([37], Sect. 6.12)

This polemic against the fundamental significance of signaling implies that the failure according to quantum mechanics of the reduction of \( \{ A \mid B, a, b, c, \lambda \} \) to \( \{ A \mid a, c, \lambda \} \), in the transition from Eq. (12) to Eq. (13) is as serious a violation of local causality as the failure of the transition \( \{ B \mid a, b, c, \lambda \} \) to \( \{ B \mid b, c, \lambda \} \) would have been.

Among the logical possibilities that Bell tentatively explores in order to escape from this disagreeable conclusion is to envisage a theory in which the apparent free will of the experimenters in choosing the variables a and b is an illusion. But he comments,

Perhaps such a theory could be both locally causal and in agreement with quantum mechanical predictions. However I do not expect to see a serious theory of this kind. I would expect a serious theory to permit ‘deterministic chaos’, or ‘pseudorandomness’, for complicated subsystems (e.g., computers) which would provide sufficiently free variables for the purpose at hand. But I do not have a theorem about that . . . . ([37], Sect. 6.10)
Another logical possibility (mentioned in Ref. [37], Sect. 6.12) after a preliminary discussion of the concept of entropy is that “causal structure emerges only in something like a ‘thermodynamic’ approximation, where the notions ‘measurement’ and ‘external field’ become legitimate approximations.” But he cannot believe that this is the whole story, because “local commutativity [which is invoked for the derivations of dispersion relations in quantum field theory] does not for me have a thermodynamic air about it. It is a challenge now to couple it with sharp internal concepts, rather than vague external ones. Perhaps there is already a hint of this in ‘quantum mechanics with spontaneous wave function collapse.’” In Sect. 2.6 we shall discuss some of Bell’s conjectures on the quantum mechanical measurement problem, for which spontaneous wave function collapse is a recent proposed solution. For the present, we merely note Bell’s entertainment of the idea that there is an intimate connection between the locality problem and the measurement problem.

Another logical possibility considered by Bell is a reversion to the pre-Einsteinian relativity theory of Fitzgerald, Larmor, Lorentz, and Poincaré, in which there is a preferred reference system (an “aether”), but forces dependent upon velocity relative to the preferred frame are responsible for Fitzgerald-Lorentz contraction, hence for the null result of the Michelson-Morley experiment, hence for the cover-up that prevents the identification of the preferred frame. In this pre-Einsteinian theory there is no kinematical obstacle to superluminal causal influences. Consequently, the transition above from Eq. (12) to Eq. (13) would be blocked because \( \{ A \mid B, a, b, c, \lambda \} \) could indeed depend upon the variable \( b \), and upon the chance outcome of the experiment parameterized by \( b \), even though that variable is freely chosen in a region with space-like separation from the region where variable \( a \) is freely chosen. Hence there would be a causal relation between outcomes \( A \) and \( B \), even though no light signal could connect the events. Whether \( A \) is the cause and \( B \) the effect or conversely depends upon the time order in the preferred reference system. Although this method of permitting a causal connection between two regions with space-like separation sacrifices the conceptual beauty of Einstein’s special theory of relativity, much and perhaps all of the experimental consequences of that theory is recovered in a sufficiently clever reversion to the pre-Einsteinian theory, and Bell’s paper “How to Teach Special Relativity” [39] argues, nonpolemically, that there are some pedagogical advantages in this reversion. But the main motivation for going back to Fitzgerald et al., once the peculiarities of quantum mechanical correlations have been established experimentally, is that we would have a space-time theory that accommodates these peculiarities. Having entertained this solution to the problem of quantum mechanical nonlocality, however, Bell cannot endorse it without serious reservations:
As with relativity before Einstein, there is then a preferred frame in the formulation of the theory... but it is experimentally indistinguishable.... It seems an eccentric way to make a world. [7], last paragraph

We suspect that the passage in Bell’s writings that most accurately expresses his attitude towards the apparent nonlocality of quantum theory, which he more than any one else brought into sharp focus, is an unspecific and open conjecture about the physics of the future: “It may be that a real synthesis of quantum and relativity theories requires not just technical developments but radical conceptual renewal” ([41], last sentence).

2.6 Unpromising and Promising Approaches to the Measurement Problem

The quantum mechanical measurement problem is generated by the unitarity of the quantum mechanical time-evolution operators together with the assumption that quantum mechanics governs the entire physical world. As a consequence, when a physical apparatus interacting with an object of interest physically indicates (e.g., by a “pointer reading” or some generalization thereof) the eigenvalue of an observable quantity whenever the object is initially in an eigenstate of that quantity with that eigenvalue, then in the more general case of an initial object state which is a superposition of eigenstates with differing eigenvalues the “pointer” of the apparatus at the conclusion of the measurement has an indefinite reading. How then can there be definite outcomes of measurements, whose probabilities are supposed to be predicted by the formalism of quantum mechanics? Bell comments in some detail on four different attempts to solve this problems, two of which he considers to be unsuccessful and two at least promising.

(i) The Copenhagen Interpretation

The most unsuccessful, in Bell’s opinion, is historically the most widely accepted – the Copenhagen interpretation of quantum mechanics in one version or another. “Against ‘Measurement’” [25] is Bell’s polemic against this interpretation, represented by the versions of L.D. Landau and E.M. Lifshitz [42], K. Gottfried [13], and N.G. van Kampen [14], which he regards as sufficiently strong and characteristic to represent adequately the entire school. Each version makes, explicitly or implicitly, certain assumptions and approximations that are good “for all practical purposes” – a phrase which he abbreviates with the biting acronym “FAPP”. Landau and Lifshitz (LL) emphasize the macroscopic constitution of the apparatus actually used in measurement, in virtue of which it behaves as a classical physical system.
But that is consistent with the quantum mechanics only if there is a spontaneous transition of the state of the apparatus into an eigenstate of its “pointer reading”. But how in detail is this transition – irreversible according to LL – compatible with the universal principles of quantum mechanics? Bell concludes,

The LL formulation, with vaguely defined wave function collapse, when used with good taste and discretion, is adequate FAPP. It remains that the theory is ambiguous in principle, about exactly when and exactly how the collapse occurs, about what is microscopic and what is macroscopic, what quantum and what classical. We are allowed to ask: is such ambiguity dictated by experimental facts? Or could theoretical physicists do better if they tried harder?

An essential element in Gottfried’s solution to the measurement problem is first to represent the final state of the object and apparatus (neglecting the interaction of the apparatus with the rest of the world) by a density matrix \( \rho \) in the pointer-reading basis, which will in general contain off-diagonal terms, and then to retain only the diagonal elements of \( \rho \), yielding the density matrix

\[
\hat{\rho} = \sum_n |c_{nn}|^2 \Psi_n \Psi_n^*.
\]

Bell is willing to allow the harmlessness FAPP of replacing of \( \rho \) by \( \hat{\rho} \), because of the “practical elusiveness, even the absence FAPP, of interference between macroscopically different states”. But he objects strenuously to Gottfried’s assertion that the right-hand side of Eq. (16) endows the coefficients \(|c_{nn}|^2\) with an interpretation as probabilities.

I am quite puzzled by this. If one were not actually on the lookout for probabilities, I think the obvious interpretation of even \( \hat{\rho} \) would be that the system is in a state in which the various \( \Psi \)s somehow coexist . . . . This is not at all a probability interpretation, in which the different terms are seen not as coexisting, but as alternatives . . . . The idea that the elimination of coherence, in one way or another, implies the replacement of ‘and’ by ‘or’ is a very common one among solvers of the ‘measurement problem’. It has always puzzled me.

Bell goes on to argue that in spite of Gottfried’s pretense at obtaining irreversibility out of the reversible exact quantum dynamics by an approximation, he is modifying the rules of quantum mechanics – surreptitiously and without precision.

Van Kampen divides the entire world \( W \) into “system” \( S \), “apparatus” \( A \), and the rest \( R' \), so that \( W \) can be written as \( S + A + R' = S' + R' \), and he cleverly focuses on the
S'/R' interface. One treatment of the interface is rigorously quantum mechanical. The other treatment takes into account measurements that can actually be done, and these FAPP show no interference between macroscopically different states of S'. Bell attributes to van Kampen a conjunction of these points of view with the consequence,

It is as if the ‘and’ in the superposition had already, before any such measurements, been replaced by ‘or’. So the ‘and’ has already been replaced by ‘or’. It is as if it were so . . . so it is so.

Bell concludes that either by a quantum mechanical theorem or by a change of theory van Kampen has reached a conclusion like that attributed to LL: that a superposition of macroscopically different states somehow decays into one of its members. But no theorem is given, and Bell finds it unlikely that one could be given, because the only procedure of calculation available for proving such a theorem would require a further shift of the slippery split between system and the rest of the world, and the indefiniteness of the split precludes precise calculation. This negative conclusion points, for Bell, in the direction of changes of theory, samples of which will be sketched in (iii) and (iv) below.

(ii) H. Everett’s Interpretation

There appear to be two reasons why Bell discusses Everett’s interpretation at some length in two papers [15, 24]. The first is positive – that Bell appreciates Everett’s rejection of a division of the world into a part treated quantum mechanically and a part treated classically (which is Bell’s bête noir in the Copenhagen interpretation), for Everett simply postulates a universal quantum mechanical wave function. Moreover, Everett eliminates the baffling duality of temporal development, one continuous and unitary governed by the Schrödinger equation, and the other discontinuous and stochastic occurring when a reduction of the wave packet occurs, since he denies the occurrence of the latter. The second is that Bell recognizes some kinship between the Everett interpretation and the hidden variables theory of de Broglie and Bohm, as no one previously seems to have done, and the exploration of this kinship allows him to highlight certain virtues of the latter. Everett’s uncompromising adherence to quantum mechanics drives him to a radical account of the measuring process: If \{\phi_n\} are a complete set of eigenstates of an observable A of an object, and the object interacts with a measuring apparatus that is accurate in the sense of “recording” the eigenstate by entering a corresponding state \chi_k whenever the object is prepared in a definite \phi_k, then the final state of object plus apparatus, when the object is prepared in the superposition \sum c_n \phi_n, is \sum c_n \phi_n \chi_n. For Everett the final superposition is unequivocally interpreted as “and”, not as
“or”. He introduces the terminology “relative state” and says that relative to the apparatus state $\chi_k$ the object state is $\phi_k$, but there is no selection from among those $\{\phi_n\}$ that occur with a nonzero coefficient in the initial superposition. Each “branch” in the final state of object plus apparatus is as real as any other. Does branching occur only when there is a measurement process, the branches being labeled by the eigenstates of the “pointer reading” of the apparatus, or does it occur relative to an arbitrary basis in the Hilbert space associated with the apparatus? Bell finds an ambiguity in Everett’s text on this matter, and he raises objections to either possible answer. If the former, then

Everett is indeed following an old convention of abstract quantum measurement theory – that the world does fall neatly into such pieces – instruments and systems. . . . I think that fundamental physical theory should be so formulated that such artificial divisions are manifestly inessential. In my opinion Everett has not given such a formulation, and de Broglie has. [43]

If the latter answer is accepted, with relative states defined for arbitrary bases, then “it becomes obscure to me that any physical interpretation has either emerged from, or been imposed on, the mathematics.” (Ref. [24], footnote 9). Returning to the first answer regarding branching, we note that Bell finds implicit in it a curious view of the past:

[I]n our interpretation of the Everett theory there is no association of the particular present with any particular past. And the essential claim is that this does not matter at all. For we have no access to the past. We have only our ‘memories’ and our ‘records’. But these memories and records are in fact present phenomena. . . . Everett’s replacement of the past by memories is a radical solipsism – extending to the temporal dimension the replacement of everything outside my head by my impressions, of ordinary solipsism or positivism. Solipsism cannot be refuted. But if such a theory is taken seriously it would hardly be possible to take anything else seriously. So much for the social implications. [24]

(iii) The Hidden Variables Theory of de Broglie and Bohm

In 1927 Louis de Broglie [46] proposed a specific hidden variables interpretation that was modified and amplified by David Bohm [28] in 1952. (We shall not analyze the similarities and differences of the theories of the two men, but rather shall focus on what we shall call “BB”, which is Bell’s own exposition of their combined work.) BB applies quantum mechanics to the entire physical world, and hence admits a universal wave function. Their hidden variables
are the positions of all particles, all positions being objectively definite. The universal wave function evolves under the Schrödinger equation, uninfluenced by the positions of particles. If this wave function is expressed in the polar form

\[ \psi(q, t) = R(q, t) \exp[iS(q, t)/\hbar] \] (17)

where \( q \) is position in configuration space, \( R \) and \( S \) being real-valued functions, then BB postulates that the generalized momentum conjugate to \( q \) is

\[ p = \text{grad}_q S \] (18)

\( \psi \) is often called “the guiding field” and Eq. (18) “the guiding equation”. Although positions are the only explicit hidden variables, BB is able to ascribe, with the following strategy, a definite result to a physically described measurements of any quantity represented in quantum mechanics by a self-adjoint operator: The quantity of interest is coupled with an apparatus whose “pointer reading” is assumed to be a position (hence a component of \( q \)), in such a way that an eigenstate of the quantity is strictly correlated with a value of this position at the completion of the measurement. Definiteness of \( q \), which is postulated by the theory, entails a definite inference regarding the value of the measured quantity. The statistical predictions of BB are the same as those of standard quantum mechanics if one supposes that the Born rule for probability density,

\[ \text{prob}(q, t_0) = |\psi(q, t_0)|^2 \] (19)

holds for some initial time \( t_0 \), its holding for all other times being a consequence of the dynamics.

Bell makes a number of complimentary remarks about BB. The theory does not take an anthropocentric concept like observables as fundamental, but postulates an ontology consisting of (i) a universal wave function, thought of as physically existing like the electromagnetic field, and (ii) the positions \( q \), which are the “beables”. Indeed, these beables ought not be thought of as hidden variables, because positions are manifest quantities, whereas the wave function is shadowy and shows its existence only via its effect upon the positions. There is only one dynamical law governing the wave function and no mysterious “reduction of the wave packet” to account for a result of measurement. There are definite measurement results, but they are the values of certain components of \( q(t) \), whose temporal development depends only on an initial value, the wave function, and the guiding equation. Although BB shares with Everett’s interpretation the virtue of avoiding two different laws for the evolution of the wave function, it differs – to its advantage – by attributing a definite configuration to the
world instead of a multiplicity of branches with different configurations. It is also a virtue
of BB that nonlocality enters in a natural way: When the wave function of the world – or
of a small number of systems in an idealizing approximation – is an entangled state, the
components of the generalized momentum $p$ in the guiding Eq. (18) can be correlated even
though the corresponding components of $q$ are far separated:

That the guiding wave, in the general case, propagates not in ordinary three-
space but in a multidimensional-configuration space is the origin of the notorious
‘nonlocality’ of quantum mechanics. It is the merit of the de Broglie-Bohm version
to bring this out so explicitly that it cannot be ignored. \[47\], last paragraph

Finally, the essential idea of BB is more general than the detailed exposition by its
authors, for there is no reason in principle to restrict the beables to particle positions,
suspiciously reminiscent of classical mechanics. There are other possible choices of beables,
suggested by recent discoveries in elementary particle physics, which would be coupled with
the universal wave function by an appropriate guiding equation. In “Beables for Quantum
Field Theory” \[40\] – dedicated to Bohm – Bell suggests a version of BB that takes fermion
number density as the beables, since that “includes the positions of instruments, instrument
pointers, ink on paper . . . and much much more.” The resulting theory is BQFT (“de Broglie-
Bohm beable quantum field theory”), which has to be compared with OQFT (“‘ordinary’
‘orthodox’ ‘observable’ quantum field theory”). There is good agreement between BQFT and
two versions of OQFT that Bell presents. In spite of this empirical parity, at least FAPP, Bell
prefers BQFT conceptually, because it is designed to provide a detailed temporal account of
the measurement process.

After thus making his case for a suitably modified BB, Bell concludes with a strong
reservation.

BQFT agrees with OFQT on the result of the Michelson-Morley experiment,
and so on. But the formulation of BQFT relies heavily on a particular division
of space-time into space and time. Could this be avoided? \[40\], Sect. 5

After a brief discussion of ways to achieve Lorentz invariance, Bell concludes skeptically:

I am unable to prove, or even formulate clearly, the proposition that a sharp
formulation of quantum field theory, such as the one set out here, must disrespect
serious Lorentz invariance. But it seems to me that this is probably so. \[ibid.\]
(iv) Theories of Spontaneous Reduction of the Wave Packet

Bell was aware of a number of proposals to solve the measurement problem by modifying the time-dependent Schrödinger equation, by the addition of small nonlinear terms or by introducing stochasticity into the dynamics (see footnote 6 of Ref. [24]). The detailed proposal of this kind that impressed him most was that of G.C. Ghirardi, A. Rimini, and T. Weber (GRW) [48], which he discusses in detail in “Are There Quantum Jumps?” [49]. GRW postulate no hidden variables but consider the quantum state to be a complete description of a physical system. The quantum state satisfies Bell’s criterion for a “beable” formulation of quantum mechanics, because they describe the measurement process explicitly and clearly in terms of the fundamental dynamical law; their paper is, in fact, entitled “Unified Dynamics of Microscopic and Macroscopic Systems.” GRW’s innovation is a stochastic reduction which may occur for any particle at random infrequent times (the mean waiting time for reduction to occur being a new constant of nature $\tau$, clearly large and postulated by GRW to be $10^{15}$ s). The stochastic reduction is described in the position representation, and it can occur even when the particle is part of a composite system described by an entangled state,

$$\psi(r_n, x', t) = \sum_i c_{ni}(t)\phi_i(r_n)\Phi_i(x')$$

(20)

where $r_n$ is the position of the $n$th particle, $x'$ is the configuration of all particles except the $n$th, and $i$ is an index in a superposition of terms. When the stochastic reduction happens for the $n$th particle, the result is to replace every $\phi_i(r_n)$ by $j(r - r_n)\phi_i(r_n)/R_n(r)$, where $j$ is a “jump factor”, Gaussian for convenience with width $a$ of the order of $10^{-5}$ cm, and $R_n$ is a normalization factor. After a jump the $n$th particle will be well localized, since all wave functions $\phi_i$ with supports not close to $r$ will be annihilated by the jump factor. One virtue of this scheme is that for a single particle or a system of few particles, the stochastic modification of the Schrödinger equation causes such infrequent jumps that they will not be experimentally noticeable. On the other hand, for a system with a large number of particles, of the order of $10^{20}$, the mean waiting time for reduction of some particle or another of the system will be small, of the order of $10^{-5}$ s, and therefore – because of the algebra of entanglement – the position of the center of mass will quickly be localized within a region of diameter $10^{-5}$ cm. Thus a macroscopic system will not have a grossly unlocalized position more than a short time, and the pointer reading is quickly definite. Classical dynamics is approximately recovered for macroscopic systems by an explicit calculation based upon the dynamics of microscopic systems. Although GRW’s experimental disagreements with standard quantum are small, they are nonzero and in principle testable (but so far no feasible test has been devised).
The most troublesome feature of GRW, according to Bell, is the apparent action-at-a-distance that occurs when one of the particles of a composite system in an entangled state undergoes a spontaneous jump. A particle remote from the one undergoing the jump may be constrained to be localized, without any locally describable causal process. To be sure, no message can be transmitted by this correlation, but Bell’s reasons for thinking this fact is not crucial were presented in Section 2.3 above. What concerns him is whether the theory can be modified so as to be Lorentz invariant in spite of spooky correlations. The question cannot be directly investigated in GRW’s theory, which is nonrelativistic, but Bell does answer a related question. He proves that if a different time coordinate is used for each particle, a relative time translation invariance holds in this theory. He therefore concludes tentatively, but somewhat optimistically, concerning the prospects of further developments of GRW:

I see the GRW model as a very nice illustration of how quantum mechanics, to become rational, requires only a change which is small (on some measures!). And I am particularly struck by the fact that the model is as Lorentz invariant as it could be in the nonrelativistic version. It takes away the ground of my fear that any exact formulation of quantum mechanics must conflict with fundamental Lorentz invariance. [42], Sect. 5

It is regrettable that Bell did not live to develop further a program that he found so promising.

3 Field Theory and Elementary Particle Physics

John Bell’s public profession (also his employment) from the mid-1950s was that of a theoretical nuclear and later particle physicist, a progression that reflects the historical development of the subject. The framework within which he worked was quantum field theory. During his time, two competing ideas were also popular among particle theorists: S-matrix/Regge theory in the early years, string theory in later years. Apparently these did not appeal to Bell, who remained within field theory for all his investigations. His first paper in this area, in 1955, followed five active years of research in accelerator physics, which produced 21 publications. Drawn from his doctoral thesis at Birmingham University, it addresses an important and classic issue. Bell entitled the work “Time reversal in field theory” [1]. In fact he established the PTC theorem, which states that any local, relativistically invariant quantum field theory is invariant against the simultaneous inversion of space-time coordinates and reversal of all charges. Unknown to him this had already been shown the year before by Lüders using an argument markedly different from Bell’s [50]. Yet Bell is hardly ever cred-
ited for his independent derivation, presumably because he was not in the circle of formal field theorists (Pauli, Wigner, Schwinger, Jost, etc.) who appropriated and dominated this topic. Today Bell’s “elementary derivation” is more accessible than the formal field theoretic arguments. The subject of time reversal remained an important theme in his subsequent research \[51\], especially when it became clear that time inversion (T) (unaccompanied by space inversion and charge conjugation) is not a symmetry of Nature, and neither is space inversion conjoined with charge conjugation (PC) (unaccompanied by time inversion). Together with his friend, the experimentalist Steinberger, he wrote an influential review on the phenomenology of PC-violating experiments \[52\], and with Perring proposed a “simple model” theory to explain that effect \[53\]. They postulated the existence in the universe of a hitherto unobserved long-range field, which would catalyze the PC symmetry breaking. Though speculative, the suggestion was truly physical, hence falsifiable, and it was soon ruled out by further experiments. Nevertheless, it remains a bold and beautiful idea that continues to intrigue theorists.

Bell’s immediate postdoctoral research concerned nuclear physics, and he worked closely with Skyrme on nuclear magnetic moments \[2\], though apparently not on Skyrme’s prescient ideas on solitonic models for the nucleons. Bell contributed variously to the many-body theory of nuclei \[3\], notably the derivation with Squires of an effective one-body “optical” potential for the scattering off a complex target \[4\]. The nuclear physics work, though not producing any breakthrough in that field, prepared him well for dealing with problems that he would eventually encounter at CERN, involving nuclear and particle processes. Examples of this later work are his nuclear optical model (again!) for pi-mesons \[54\], and investigations of neutrino reactions with nuclei \[55\], where the hypothesis of a “partially conserved axial vector current” (PCAC) connects neutrino processes with pi-meson emission.

In 1960, Bell joined CERN and remained there to the end of his life. Although primarily a center for particle physics accelerator experiments, CERN houses Europe’s preeminent particle theory group, and Bell became active in that field. Typically particle theorists are divided into phenomenologists – people who pay close attention to experimental results, and interact professionally with experimentalists – and formalists, who explore the mathematical and other properties of theoretical models, propose new ideas for model building, and usually are somewhat removed from the reality of the experiments. Although Bell’s time-reversal paper \[1\] belongs forcefully in the formalist category, at CERN he was very much also a particle physics phenomenologist, drawing on his previous experience with nuclear physics. Indeed, with characteristic conscientiousness, Bell felt an obligation to work on subjects related to the activities of the laboratory. But his readiness to discuss and study any topic
in physics ensured that he would pursue highly theoretical and speculative issues as well.

Theoretical understanding of particle physics at that time was hampered by two obstacles. No single model had been identified as giving a correct account for the fundamental interactions of elementary particles, with the exception of electromagnetic interactions, which were adequately described by quantum electrodynamics. Moreover, competing models could not be assessed because their extremely complicated dynamical equations could not be solved, so the predictions of the models were unknown and could not be compared with experiments.

To overcome this impasse, Gell-Mann advanced the idea of “current algebra”, with which one could obtain explicit and testable results. Current algebra is a particle physics/quantum field theory reprise of an old technique used in quantum physics for atoms, namely, the Thomas-Reiche-Kuhn sum rule, or the Bethe energy loss sum rule \[56\]. Here one identifies relevant operators whose matrix elements govern transitions between atomic levels, and the total transition probability is a sum over all final-state levels. In favorable circumstances, the operators are manipulated by using the operator quantum commutation relations, which are deduced from the fundamental canonical commutators, and the sum can be evaluated using the completeness of states. All this can be achieved without determining individual wave functions, which would involve the daunting task of solving completely the interacting Schrödinger equation; indeed, even explicit knowledge of the potential function governing the interactions is not needed.

In the particle physics analogue, the relevant operators governing transitions are the relativistic 4-vector and axial 4-vector currents, generalizations of the electromagnetic 4-vector current, but carrying internal symmetry group \([SU(2) \text{ or } SU(3)]\) labels. In an article in the same volume of the now-defunct journal Physics in which Bell published his famous EPR paper, Gell-Mann postulated a form for the commutators of current components using a hypothesis about canonical structure and plausible ideas from group theory \[57\]. Also it was necessary to know the covariant divergences of the 4-vector and axial 4-vector currents. One took the vector current divergence to vanish, that is, that current is conserved just as is the electromagnetic 4-vector current, while the axial 4-vector current was taken to be partially conserved (PCAC) and related to pi-meson processes.

Gell-Mann’s proposal \[57\] successfully bypassed the two obstacles: ignorance about specific dynamics, and inability to unravel proposed dynamical equations. Testable results were soon obtained by Adler, Fubini, Furlan, Weisberger, and others \[58\], and the predictions of current algebra theory appeared to agree very well with experiment. Consequently almost all particle physics theorists began researching the foundations, extensions, and applications of Gell-Mann’s current algebra proposal. So also did Bell, and it is within this area that he
made his principal contributions to particle physics.

While the canonical formalism in a quantum field theory provides one starting point for deriving the current algebra, it suffers from the shortcoming that canonical commutation relations are postulated for the unrenormalized operators. But quantum field theory is notoriously polluted by various infinities, which must be renormalized, and it is not clear whether relations obtained by manipulating unrenormalized quantities accurately describe what the model entails. Bell illuminated these issues by studying the commutators in a “trivial” completely solvable but unrealistic model, which nevertheless requires renormalization (Lee model) [59]. He demonstrated that indeed canonical relations must be used with caution, because in various circumstances they will lead to values for particle physics sum rules (analogs of the atomic physics Thomas-Reiche-Kuhn or Bethe sum rules) that do not agree with the explicit summation of the explicitly calculated amplitudes. Also, under the influence of related work by his friend and colleague Veltman [60], Bell proposed a different basis for the current algebra. Rather than relying on canonical commutation relations for unrenormalized operators, he showed that invoking the non-Abelian gauge principle, which lies at the heart of Yang-Mills theory—a subject Veltman was intensely studying—will also yield the desired current commutation relations [61].

After the successes of current algebra built on the SU(2) or SU(3) group, attempts were made to employ other groups [U(4), U(6), SU(6), SL(6, C), U(6) × U(6), U(12), etc.]. The extended theory had an initial success in predicting the ratio between neutron and proton magnetic moments to be $-\frac{2}{3}$, which agrees well with experiment. But soon it became evident that a consistent relativistic theory could not incorporate such groups as symmetry groups, and Bell contributed to the critique [62].

When this generalization fell out of favor, most people left the subject, but Bell kept an eye on the topic, owing to his longtime interest (since his time with Skyrme) in nuclear magnetic moments, and his belief that the good experimental agreement should not be accidental. Thus when the subject reappeared years later in the guise of the “Melosh transformation”, Bell researched and lectured on this new direction [63], but results remained inconclusive.

In addition to formal, theoretical investigations and critiques of current algebra, Bell derived and refined various useful relations relevant to experiment: beta-decay of pi-mesons [64], low-energy Compton scattering [65], soft pi-meson emission [66]. But in these studies he also encountered results that failed to agree with experiment, even though the derivations made use of what appeared to be the most reliable aspects of the theory. First there was the current algebra and PCAC analysis that seemed to forbid the decay of the eta-meson into three pi-mesons, even though the process is seen experimentally [67]. This complemented the
result of Sutherland and Veltman [68] that a very direct application of current algebra and PCAC prohibits the decay of the neutral pi-meson into two photons, again contradicting experimental observation. Although these were small defects of the generally successful current algebra story, John Bell did not put them aside, since he would not abide imprecision and incompleteness in physical theory.

The two streams of Bell’s current algebra research – investigations on the reliability of the postulated algebraic structures, and attention to apparent failures of the theory – came together on what turned out to be his most far-reaching contribution to particle physics, published in his most-quoted scientific paper. Responding to a request by Jackiw (who in 1967–8 was a visiting scientist at CERN) for a research problem, Bell suggested analyzing current algebra’s failure in the pi-meson/two-photon process. This was a vexing puzzle, because the theoretical argument prohibiting the decay seemed to be direct and elementary. No new ideas were forthcoming, until a casual observation (over coffee in the CERN café) by Steinberger, Bell’s experimentalist collaborator on their PC review article [52], pointed the way. Steinberger remarked that years earlier he had computed the amplitude in a once popular model field theory, and gotten a nonvanishing result, which moreover agreed well with experiment [69]. Bell and Jackiw realized that Steinberger’s calculation would coincide with what one would find in the $\sigma$-model, a field theory constructed by Gell-Mann and Lévy to exhibit explicitly current algebra and PCAC [70]. So here was the test: On the one hand, a direct calculation of the decay amplitude in the $\sigma$-model should reproduce Steinberger’s nonvanishing result. On the other hand, an indirect calculation based on current algebra/PCAC, which appear to be present in the $\sigma$-model, should give a vanishing result. The resolution of this conflict would display what is happening, and is reported in their paper, “A PCAC Puzzle: $\pi^0 \rightarrow \gamma\gamma$ in the $\sigma$-Model” [8].

The direct calculation involves the now famous fermion triangle graph

![Fermion Triangle Graph](image)

where the bottom two vertices denote electromagnetic vector currents to which the two photons couple, while in the apex there is the axial vector current, which according to PCAC governs the pi-meson coupling. The lines joining the vertices denote fermion propagators, which in the Steinberger calculation were protons [69], while in contemporary theory they are quarks.

The three-current amplitude is a three-index tensor, carrying a space-time index for each
of the three currents. Direct evaluation of the Feynman graph reproduces Steinberger’s non-vanishing decay amplitude \[69\]. On the other hand, current algebra and PCAC demand that with massless fermions propagating between the vertices, the three-index tensor should be transverse in each of its indices. These transversality conditions are examples of Ward-Takahashi identities satisfied by field-theoretic amplitudes. Generically, the task of current algebra and PCAC was to establish such identities, from which physically relevant consequences could be drawn. In the pi-meson/two photon application, the threefold transversality of the three-current amplitude implies that the pi-meson decay amplitude vanishes \[68\]. Since the explicit calculation provides a nonvanishing amplitude, the Ward-Takahashi identity must fail and indeed the explicit calculation of the triangle produces a nontransverse result \[8\].

This observation put into evidence a previously unsuspected quantum field theoretic phenomenon: With massless fermions, the axial vector current appears to be conserved. (That is why the three-current amplitude is expected to be transverse in the axial vector index, just as it is in the vector indices, owing to the conservation of the vector electromagnetic current.) Current conservation is evidence that a symmetry holds: conservation of the electromagnetic current is correlated with gauge invariance. With massless fermions, a conserved axial vector current would indicate that a further symmetry is present: the so-called chiral symmetry. Chiral symmetry is understood in the following fashion. Massless spin-\(\frac{1}{2}\) fermions can be viewed as consisting of two species, which do not mix. In one species the spin is aligned along the direction of propagation; in the other, the spin points opposite to the propagation. Chiral symmetry then is the ability of performing separate and independent gauge transformations on each species. Noninteracting fermions, described by a massless Dirac equation, do indeed exhibit this chiral symmetry. Superficially, it appears that interaction with photons should preserve the chiral symmetry. However, the triangle graph calculation demonstrates that this is not so \[8\]. The technical reason for this unexpected violation of the symmetry is that the integral that evaluates the triangle graph is singular (divergent) and a prescription must be given how to handle the singularity to get a noninfinite result. It turns out that no matter what prescription is given, chiral symmetry is lost. Therefore, if we accept that quantum field theoretic infinities are a necessary consequence of the quantization procedure, the Bell and Jackiw discovery established a novel method of symmetry breaking, called anomalous or quantal symmetry breaking, which arises from the quantization procedure itself: symmetries of an unquantized theory need not survive quantization. This new mode of symmetry breaking supplements previously known possibilities: symmetry breaking by nonsymmetric dynamics or symmetry breaking by energetic instabilities, as in spontaneous
symmetry breaking with the Higgs mechanism. Moreover, this quantal symmetry breaking mechanism also violates the correspondence principle of quantum physics.\[71\]

The situation can be summarized by the anomalous divergence equation. Rather than being conserved, the axial vector current has a nonzero divergence, given by

\[ \partial_\mu J_\mu^A \propto \ast F^\mu\nu F^\mu\nu. \] (21)

On the left there is the divergence of the axial vector current \( J_\mu^A \), on the right is the anomaly responsible for the quantum mechanical nonconservation. Here \( F^\mu\nu \) is the electromagnetic Maxwell tensor, \( \ast F^\mu\nu \) its dual equal to \( \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \), and their product coincides with the scalar product of the electric and magnetic fields with which the fermions interact. The proportionality constant in the above anomalous divergence equation is determined by the number and charges of the fermions and fixes the strength of the \( \pi \)-meson/two-photon amplitude.

The implications of this symmetry breaking phenomenon are many, and they are widely spread in physical theory. Here is a list.

1. Working independently, and slightly later, Adler found that the axial vector current in quantum electrodynamics behaves similarly. When he learned of the Bell-Jackiw considerations about the \( \pi \)-meson/two-photon amplitude, he complemented their work, and proved (with Bardeen) that no other graphs contribute to the anomaly.\[72\] Therefore, a precise value for the \( \pi \)-meson/two-photon amplitude can be predicted. Numerical agreement with the experimental value requires that if the fermions propagating in the legs of the triangle graph are quarks, carrying conventional quark charges, there must be three species – “colors”.

2. Current commutators reflect the underlying symmetry of the theory. If the symmetry is broken quantum mechanically, there must be quantal correction to current algebra. Such correction were found and they provide an alternative viewpoint why Ward-Takahashi identities fail: anomalous divergences of currents and anomalous commutators are two sides of the same coin.\[73\]

3. It soon became apparent that anomalous divergences can arise in other currents as well. However, if gauge fields couple to currents, consistency of the gauge principle requires that such gauge currents be anomaly free. This can be arranged, provided the number and charges of participating fermions are such that their contributions to the proportionality constant in the anomalous divergence of the gauge current sum to zero. In the present day “Standard Model” of electroweak interactions this cancellation of
anomalies requires equal numbers of quarks and leptons, with charges taking precisely the values of the Standard Model [74]. Moreover, anomalies also endanger consistency of string theories, and the demand of anomaly cancellation strongly limits the possible string theories. This limitation led to a revival of string theory research [75].

4. Polyakov and his collaborators realized that the quantity $\ast F_{\mu\nu} F^{\mu\nu}$ and its non-Abelian generalization are topological entities, whose four dimensional integral measures the topological twist in the underlying gauge fields. This is the Chern-Pontryagin number. They identified fields with nonvanishing twist – the instantons – even though they made no reference to the anomaly, which evidently was largely unappreciated at that time in the Soviet Union [76].

5. 't Hooft, a student of Bell’s friend Veltman, and well instructed by him about Bell’s anomaly work, showed that in the Standard Model certain combinations of fermion number currents do not couple to gauge fields and are afflicted by the anomaly. As a consequence protons can decay, but with an exponentially vanishing small probability, thus not endangering the stability of our world [77]. Related to this was the observation that owing to the anomaly, there is the possibility of quantum mechanical tunneling in the Standard Model field theory. This creates a Bloch-like band structure, where states are labeled by a hitherto unsuspected PC-violating parameter – the $\theta$ vacuum angle [78]. How to fix the magnitude of this $\theta$-angle is still an open question.

6. Discussion of anomalies brought physicists into contact with mathematical entities, like the previously mentioned Chern-Pontryagin term, and the Chern-Simons term [79] (whose exterior derivative gives the Chern-Pontryagin entity). As it happened, mathematicians were studying precisely these same topics, at roughly the same time. Thus, the anomaly seeded a remarkable physics/mathematics collaboration and cross-fertilization, which is still flourishing, especially in string theory research [80].

7. It was appreciated that another symmetry, which relies on masslessness, is beset by anomalies. This is scale and conformal invariance, which would require the energy momentum tensor to be traceless. But in fact superficially scale and conformally invariant theories (like the Standard Model in the absence of its Higgs sector) acquire upon quantization a nonvanishing trace – a trace anomaly – thereby breaking quantum mechanically the scale and conformal symmetries [81]. This of course is very fortunate, because Nature certainly is not scale invariant, and could not be described by a scale-invariant theory. Moreover, it was appreciated that the proper way to deal with
anomalously broken scale and conformal symmetry in quantum theory is through the Gell-Mann–Low renormalization group [82].

8. Beyond particle physics theory, in condensed matter theory and in gravity theory, physicists have realized that a mathematical discussion of physical effects can have a topological component, which is related to structures first seen in quantal anomalies. Examples are descriptions of fractional fermions by spectral flow of the Dirac operator, the theory of the quantum Hall effect based on the Chern-Simons term.

The direct physical relevance of the axial vector anomaly in accounting for the decay of the neutral pi-meson, and in explaining quark and lepton patterns that ensure absence of anomalies in gauge currents, is evidence that not only theoretical/mathematical physicists but also Nature knows and makes use of the anomaly mechanism. Moreover, the unexpected connections that the anomaly makes within physics and with mathematics suggests that we are dealing with an as yet not completely understood wrinkle in the mathematical description of physical phenomena.

Once it was appreciated that the anomaly phenomenon is not merely an obscure pathology of quantum field theory, many people wrote many papers providing various and alternative derivations of the result. But not John Bell. It seems that he was satisfied with what he stated in his only paper on this topic [8]. He did follow the subsequent developments and elaborations, but apparently he preferred the “simple” triangle-diagram calculation that started the subject. One reason for this is that he felt that the diagrammatic approach, leading to a singular integral, very explicitly exhibits the arbitrariness and ambiguity in the calculation, which can only be resolved when some additional, external information is added. For example, the three-current triangle graph cannot be transverse in all three vertices, but there is no information in which vertex transversality fails since different methods for handling the singularity produce different results. A unique value, fixing the lack of transversality in the axial vector vertex, emerges only after requiring the vector vertices be transverse, because gauge particles – photons – couple to these vertices. On the other hand in the standard electroweak model, with the same triangle graph, the vector vertex describes a nongauged fermion number current, and in this context the anomaly can reside there, giving rise to ’t Hooft’s proton decay scenario [77].

After the anomaly paper, Bell returned to more immediately practical investigations like the already-mentioned neutrino-nucleus work with Lewellyn-Smith [53], and the Melosh reprise of “higher” symmetries [63].

This was also the period when non-Abelian gauge theories became accepted as the correct
field theoretic description of fundamental processes in the Standard Model, with the Higgs mechanism providing a mass for the carriers of the weak interaction forces. Although he was an early proponent of non-Abelian gauge symmetry in current algebra [61], Bell did not work extensively in this newly expanding field, writing only on two topics. He showed that the Higgs mechanism ensured good high-energy behavior in theories with massive vector fields (such as those that carry the weak interaction forces) [83]. Then with Bertlmann and others he examined various ideas about bound states in quantum chromodynamics – the hadronic sector of the Standard Model [84]. These were the last particle physics studies carried out by Bell. The final scientific papers that he wrote all deal with accelerator physics and with his quantum physics “hobby”, which late in his life became widely appreciated. There were many demands on him to explain and develop these ideas.

Two other investigations are noteworthy. As if anticipating the great excitement felt these days about experimental/theoretical discrepancies in the anomalous magnetic moment of the muon \( a_\mu = \frac{1}{2} (g_\mu - 2) \), Bell and de Rafael obtained an upper bound on the hadronic contribution:

\[
|a_\mu|_{\text{hadronic}} < 10^{-6} \tag{85}
\]

This is certainly satisfied by today’s accepted value, \( a_\mu |_{\text{hadronic}} \sim 7 \times 10^{-8} \). But a change in the accepted number consistent with the Bell-de Rafael bound could very well affect the experimental/theoretical discrepancy \( \Delta a_\mu \sim 4 \times 10^{-9} \).

A peculiar effect seen in quantum field theory, which challenges conventional ideas, and has a realization in condensed matter physics attracted Bell’s interest. It was observed that the measured value for the number operator of a fermion moving in the background of a topological soliton, as for example created by a domain wall in a solid-state substance, would be a fraction [86]. Moreover, it was alleged that this phenomenon is physically realized in polyacetylene – a one-dimensional polymer [87]. One naturally wonders whether the observed fraction is an expectation value, or an eigenvalue without fluctuations. Only in the latter case would this represent a truly novel and unexpected phenomenon.

On a visit to India, Bell’s host Rajaraman described to him this effect, but Bell doubted that the fraction could be a sharp observable. Nevertheless, he wanted to find out and in two papers they established that indeed the fractions were eigenvalues of a number operator, “which is defined with some sophistication” [88].

This last-mentioned work illustrates well John Bell’s attitude to his research on fundamental physical questions. Rather than advancing new theoretical models, his publications are infused with a desire to know and explain existing structures, preferably in “simple terms”, in a “simple model” – phrases that occur frequently in his papers. If physicists come in two types, those who try to read the book of Nature and those who try to write it, Bell belonged to the first category. He was conservative when it came to speculative and uncon-
ventional suggestions; he would prefer that unexpected contradictions not arise, that ideas flow along clearly delineated channels. But this would not prevent him from establishing what exactly is the case and accepting, albeit reluctantly, even puzzling results. This tension is seen in the anomaly paper, where after describing the phenomenon in the first part, a later section is devoted to an attempt at removing the anomaly, saving the chiral symmetry [8]. (Years later it was understood that this attempt yields a construction with scalar fields of the Wess-Zumino anomalous effective Lagrangian, which can be used to compensate for the fermion-induced anomaly [89].) Even in his quantum mechanical investigations, Bell would have preferred to side with the rational and clearly spoken Einstein rather than with the murky pronouncements of Bohr. But once he convinced himself where the truth lies, he would not allow his investigations be affected by his inclinations, even if he remained disturbed by their outcome. Such a commitment to “truth” – as he saw it – marked John Bell’s activity in science and in life.

4 Accelerators

From the time that Bell came to Harwell in 1949 until his leave for graduate work at the University of Birmingham in 1953–4 his research almost entirely concerned accelerator design, in a group directed by William Walkinshaw. The first twenty-one items in his bibliography, from 1950 to 1954, concerned aspects of accelerator design. Of these, only two were published; the others are internal A.E.R.E. reports at Harwell, but these reports were considered authoritative and were much consulted and cited. He continued to make contributions to accelerator design during his entire career. Since neither of the authors of this essay is an expert on accelerators, we shall do little more than note the topics of Bell’s research. But this part of his work must be kept in mind even by readers primarily interested in his contributions to foundations of quantum mechanics and high energy physics, because it shows the practicality and concreteness of his mind, which undoubtedly deeply influenced even his highly theoretical and philosophical thinking.

In the Royal Society memorial essay on Bell by P.G. Burke and I.C. Percival [90], Walkinshaw is quoted concerning collaboration with him:

We in Malvern had finished our work on small electron machines, so had turned our attention to high-energy machines. The situation was fluid and we looked at all sorts of possibilities. One of these was a disc-loaded waveguide for electron accelerators. This was the first project that John worked on with me [91] . . . another possibility was a high-energy proton linac, and we looked at variety of waveguide
structures . . . Here was a young man of high calibre who soon showed his independence on choice of project, with a special liking for particle dynamics. His mathematical talent was superb and elegant.

Mary Bell [92] pointed out that his mathematical skill enabled him to carry design problems to the point where the calculations required could be performed on the primitive desk calculators available at that time.

When the group began to consider the design of a proton synchrotron, Bell studied the application of the strong focusing principle, which was the topic of his first published paper [93]. According to the memorial essay, [94] he “wrote a seminal report on the algebra of strong focusing [94] . . . read by all accelerator designers of the day.” Other topics dealt with in internal reports were “Linear Accelerator with Spiral Orbits”, the “Stability of Perturbed Orbits in the Synchrotron,” and “Linear Accelerator Phase Oscillations.”

The collaboration between Mary and John Bell on accelerator problems, that had commenced at Harwell, continued at CERN although he was in the Theory Division and she was in the Accelerator Research Group. They wrote several papers on electron cooling [95, 96] when CERN’s Super Proton Synchrotron was being designed, and even though a less expensive stochastic cooling method was adopted for that accelerator, the Bells’ cooling method (also proposed by some Soviet physicists) was later used for the LEAR ring. They also published several papers together [97, 98] on Bremsstrahlung, a practically important phenomenon because it contributes substantially to energy loss in electron-positron linear colliders. Richard Hughes, a visitor at CERN who collaborated with Bell, commented [99], “It was perhaps from his work in this field that John acquired his rigorous understanding of classical mechanics and special relativity” (though we suspect an earlier acquisition of this expertise), and he cited published lectures by John Bell on Hamiltonian mechanics at the CERN Accelerator School [100].

One of the most remarkable of Bell’s achievements was the result of combining considerations from accelerator physics with quantum field theory. In 1976 William Unruh [101] published the derivation of a beautiful “effect” (which became known as the “Unruh Effect”): An idealized detector uniformly accelerated through the electromagnetic vacuum will experience radiation with the spectral distribution of black body radiation at a temperature proportional to the acceleration; specifically

$$kT_U = \frac{\hbar a}{c}. \quad (22)$$

Jon Leinaas, who was a fellow at CERN in the early eighties, discussed this effect with Bell, and asked whether elementary particles could be used as detectors of this effective black
body radiation [102]. They showed that existing linear accelerators, which approximated the conditions of the Unruh effect, produce effects too small to be seen experimentally. However,

As suggested by John Bell, one might ask whether the temperature effect could be related to a polarization effect which was known to exist for electrons circulating in a magnetic field. It had already been established that electrons in a storage ring polarized spontaneously, but not completely...; the maximum polarization had been found to be \( P = 0.92 \). [102]

They investigated this question and found a qualified positive answer. Circulating electrons experience spin transitions due to quantum fluctuations of the vacuum field, just as transitions are induced in linearly accelerated detectors. But there are complications in the case of circular motion. The Thomas precession has to be taken into account. The excitation spectrum is not universal as in the linear Unruh effect but depends upon characteristics of the detector. There is also a narrow resonance involving vertical fluctuations and the spin motion, with a peculiar influence on polarization: as a function of \( \gamma = (1 - \beta^2)^{-1/2} \) the calculated polarization close to the point of resonance first decreases strongly, then increases to 0.99 before decreasing again approximately to the observed saturation polarization [103]. These fine points enhance their extraordinary achievement of converting a highly theoretical Gedanken experiment into a real experiment.
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