Flight Reliability Assessment of Assisted Torpedo Based on Probabilistic Risk Assessment

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Abstract. In this paper, a reliability assessment method based on probabilistic risk assessment is proposed to solve the difficulty in the flight reliability evaluation of assisted torpedo. An event chain model is established through the analysis of missions and functions. The relationship between functional events and mission process reliability is described on the basis of event tree. The relationship between component reliability and functional event reliability is analyzed on the basis of fault tree. For the uncertainty of component failure probability, triangle fuzzy number is used to describe the interval representation of component failure probability, and fuzzy fault tree is used to represent the reliability of each functional event. The reliability of the entire flight process is comprehensively evaluated to obtain a realistic flight torpedo flight reliability assessment result.

1. Introduction

Torpedo mission reliability is an important index of torpedo combat effectiveness and an important indicator of torpedo combat effectiveness [1,2]. The actual aviation reliability for ship (submarine) torpedoes is mission reliability, and the mission reliability for fly aid torpedoes includes flight reliability and actual aviation reliability [3,4].

At present, most of them rely on the land test of torpedo components and torpedo lake sea test and other relevant information to evaluate the actual navigation reliability of torpedo. However, the fly aid torpedo cannot conduct a large number of field tests, and the flight reliability is difficult to estimate because of the continuous improvement of its reliability and its components and the constraints of time and funds and other factors. Thus, a model Type B should be established. The flight reliability of the assisted torpedo is scientifically and reasonably estimated to determine whether it can meet the operational requirements.

Many uncertainties are found in the system, especially the lack of sufficient data to determine the specific probability of the bottom event, because of the influence of mission time, human factors, and other factors. The analysis based on traditional fault tree will lead to large errors in the calculation results. Therefore, fuzzy mathematics theory should be introduced to deal with the fuzzy uncertainty in flight reliability analysis.

In this paper, a flight reliability prediction model based on probability risk evaluation method is proposed to solve the problems existing in the flight reliability evaluation of assisted torpedo. An event chain model is established through the analysis of the mission and function of the torpedo. The relationship between the functional events and flight reliability is described on the basis of
event tree. At the same time, a fuzzy operator is introduced into the fault tree model. On the one hand, the relationship between the component reliability and functional events is described on the basis of fault tree analysis. On the other hand, the component failure is represented in terms of fuzzy set and its cut set, which is the fuzzy probability value. On the basis of the reliability analysis of each functional event, the flight reliability of the assisted torpedo is obtained through the comprehensive evaluation of reliability to provide an effective method support for the flight reliability evaluation of the assisted torpedo.

2. Overview of probabilistic risk assessment (PRA)

2.1. Definition

PRA is a structured and integrated logical analysis method used to identify and evaluate the reliability and security risks of complex systems [5,6]. PRA does not stick to a single method but uses event tree, fault tree, and other methods to build an event chain model, integrates information of different levels (components or systems) and different sources (test data, expert opinions, etc.), and uses Bayes, Monte Carlo, and other methods to conduct quantitative analysis on the model [7].

2.2. PRA-based task reliability assessment

PRA adopts the event chain modeling method for multistage task system that reflects the different systems or components needed to complete different task functions in different stages of the task system. In the data analysis method, the probability evaluation method can use stress intensity model method, Bayesian network method, Monte Carlo simulation, and other data analysis methods to comprehensively analyze the task process and uncertainty. PRA can integrate data information from different levels, such as component level and from different sources, such as test data, simulation data, and expert prediction, making the prediction practical.

The task reliability prediction based on PRA mainly includes four steps, namely, event tree analysis, fault tree analysis, event reliability analysis, and task process reliability prediction.

(1) Event tree analysis
An event tree is used to describe the event sequence of the entire task process, the relationship between the normal completion of each event or function, and the success of the task;

(2) Fault tree analysis
Event sequence is determined by the event tree, and each function event is analyzed by the fault tree to clarify the logical relationship between the normal function of each event and the reliability of components;

(3) Event reliability analysis
The probability of the bottom event of each function event is determined on the basis of the data information of different levels and different sources, and the reliability evaluation of each function event is completed in accordance with the fault tree structure;

(4) Mission process reliability prediction
The mission success probability is obtained using reliability evaluation method or Monte Carlo simulation technology to synthesize the reliability of each event in the event chain.

3. Reliability analysis of functional events based on fuzzy fault tree

The fuzzy fault tree method is used to analyze the functional event reliability of the task process for reasonably describing the uncertainty of the failure probability of the bottom event and improve the accuracy of the system reliability analysis. The main idea is to take the function event as the top event. On the basis of the established fault tree, the occurrence probability of the bottom event is expressed by fuzzy number, and the fuzzy reliability of the top event is calculated using the relevant operation rules of fuzzy number.
3.1. Selection of fuzzy operators

Two types of fuzzy numbers, namely, linear and nonlinear, are used. Triangle fuzzy number is usually used for linear fault tree, and normal fuzzy number is usually used for nonlinear fault tree when analyzing fault tree based on fuzzy number [8]. When analyzing the reliability of various functional events in torpedo mission process, the triangle and normal membership functions have small difference in the calculation results [9] and have minimal influence on the analysis of flight reliability prediction results. Triangle fuzzy number is used to express the occurrence probability of bottom event because its calculation is relatively simple.

Let L, R be the fuzzy reference function [10], then

\[ \mu_{\tilde{A}}(x) = \begin{cases} 
L\left(\frac{m-x}{\alpha}\right), & x \leq m, \alpha > 0 \\
R\left(\frac{x-m}{\beta}\right), & x \geq m, \beta > 0 
\end{cases} \]  

(1)

where \( \mu_{\tilde{A}}(x) \) is a membership function, and \( \tilde{A} \) is called the L-R fuzzy number, which is recorded as \( \tilde{A} = (m, \alpha, \beta)_{LR} \). \( m \) is the mean value of \( \tilde{A} \), and \( \alpha \) and \( \beta \) are the upper and lower confidence limits of \( \tilde{A} \), respectively. \( \tilde{A} \) is constant when \( \alpha = 0 \), \( \beta = 0 \), \( \tilde{A} \) is fuzzy when \( \alpha \) and \( \beta \) are large.

Assume that

\[ \begin{align*} 
L\left(\frac{m-x}{\alpha}\right) &= \max\left(0, 1 - \frac{m-x}{\alpha}\right), \quad x \leq m, \alpha > 0 \\
R\left(\frac{x-m}{\beta}\right) &= \max\left(0, 1 - \frac{x-m}{\beta}\right), \quad x > m, \beta > 0 
\end{align*} \]

Then, the membership function of \( \tilde{A} \) is

\[ \mu_{\tilde{A}}(x) = \begin{cases} 
1 - \frac{m-x}{\alpha}, & m - \alpha \leq x \leq m \\
1 - \frac{x-m}{\beta}, & m < x \leq m + \beta 
\end{cases} \]  

(2)

When \( \tilde{A} = [(m-\alpha), m, (m+\beta)] \), its \( \hat{\lambda} \) cut set is \( \tilde{A}_\lambda = [(m-\alpha) + \alpha \cdot \hat{\lambda}, m, (m+\beta) - \beta \cdot \hat{\lambda}] \), where \( \hat{\lambda} \) is the confidence level. The membership function of triangle fuzzy number is shown in Figure 1.

![Figure 1. Membership function of triangular fuzzy numbers.](image)

For any fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \), the operation rules in accordance with the expansion theorem [11] are:
\[ \tilde{A}_\lambda + \tilde{B}_\lambda = \left[ L^i_\lambda + L^j_\lambda, R^i_\lambda + R^j_\lambda \right] \]
\[ \tilde{A}_\lambda - \tilde{B}_\lambda = \left[ L^i_\lambda - L^j_\lambda, R^i_\lambda - R^j_\lambda \right] \]
\[ \tilde{A}_\lambda \times \tilde{B}_\lambda = \left[ L^i_\lambda \times L^j_\lambda, R^i_\lambda \times R^j_\lambda \right]. \]
\[ \tilde{A}_\lambda / \tilde{B}_\lambda = \left[ L^i_\lambda / L^j_\lambda, R^i_\lambda / R^j_\lambda \right] \]

3.2. Fuzzy fault tree analysis

Using fuzzy fault tree to analyze the system reliability, the main process is to calculate the fuzzy number of the top event occurrence probability and the importance of the bottom event on the basis of the fault tree structure and the fuzzy number of the bottom event occurrence probability.

(1) Fuzzy probability of top event

When occurrence probability \( i \) of the bottom event is a triangular fuzzy number, its \( \lambda \) cut set is:

\[ \tilde{F}_i^\lambda = [(m_i - \alpha_i) + \alpha_i \lambda, (m_i + \beta_i) - \beta_i \lambda] \]
\[ \tilde{F}_2^\lambda = [(m_2 - \alpha_2) + \alpha_2 \lambda, (m_2 + \beta_2) - \beta_2 \lambda] \]
\[ \ldots \]
\[ \tilde{F}_n^\lambda = [(m_n - \alpha_n) + \alpha_n \lambda, (m_n + \beta_n) - \beta_n \lambda] \]

Therefore, the AND gate and OR gate operators of \( \lambda \) cut set can be calculated, which can be expressed as follows:

AND gate:

\[ \tilde{F}_{i_{\text{and}}}^\lambda = \prod_{i=1}^{n} \tilde{F}_i = \prod_{i=1}^{n} [ (m_i - \alpha_i) + \alpha_i \lambda, (m_i + \beta_i) - \beta_i \lambda ] \]
\[ \times \prod_{i=1}^{n} [ (m_i - \alpha_i) + \alpha_i \lambda, (m_i + \beta_i) - \beta_i \lambda ] \]
\[ \times \ldots \times \prod_{i=1}^{n} [ (m_n - \alpha_n) + \alpha_n \lambda, (m_n + \beta_n) - \beta_n \lambda ] \]
\[ = \prod_{i=1}^{n} [ (m_i - \alpha_i) + \alpha_i \lambda, (m_i + \beta_i) - \beta_i \lambda ] \]
\[ = \prod_{i=1}^{n} \prod_{j=1}^{n} [ (m_i - \alpha_i) + \alpha_i \lambda, (m_i + \beta_i) - \beta_i \lambda ] \]

OR gate:

\[ \tilde{F}_{i_{\text{or}}}^\lambda = 1 - \prod_{i=1}^{n} (1 - \tilde{F}_i) \]
\[ = [1, 1] - \prod_{i=1}^{n} \left[ [1, 1] - [(m_i - \alpha_i) + \alpha_i \lambda, (m_i + \beta_i) - \beta_i \lambda] \right] \]
\[ = \left[ 1 - \prod_{i=1}^{n} [1 - (m_i - \alpha_i) - \alpha \lambda, 1 - (m_i + \beta_i) + \beta \lambda] \right] \]

(2) Fuzzy importance of the bottom event

In fault tree analysis, the bottom events are assumed to be independent from each other. Therefore, the occurrence of the bottom event and its effect on the top event are related to the fault tree structure and to the reliability of the bottom event itself.

In accordance with the relevant definition of Hideo Tanaka [12], the fuzzy importance of the bottom event can be written as:

\[ \tilde{I}_b(j) = \frac{\partial \tilde{h}(p)}{\partial p_j}, \quad j = 1, 2, \ldots, n. \]
where \( p_j \) represents the fuzzy occurrence probability of bottom event \( x_j \), and \( h(p) \) is the fuzzy function of the top event.

4. Flight reliability evaluation of flying torpedo based on PRA

4.1. Air mission and function analysis of assisted torpedo

4.1.1. Task analysis. The air ballistics of the assisted torpedo includes the flying aid and parachute ballistics, which starts from the ignition of the engine and ends when the combat load enters the water. In accordance with the working process of the torpedo, the mission to be completed mainly includes the three following aspects:

(1) Air flight out of the box: the rocket engine ignites to complete the flight and control of the torpedo from launch to separation point and ensure that the attitude, speed, position, and height of the torpedo and the arrow during separation meet the specified requirements when the launch control console presses the launch button.

(2) Separation of lightning and arrow: the separation command of lightning and arrow is received to realize the separation of lightning and arrow. During the separation, the setting cable plug, the waiting rope of water activated battery and the pulling pin of the delay umbrella opening mechanism of the air stabilization device are pulled out.

(3) Parachute in the air: after the separation of the lightning and the arrow, the parachute is opened in accordance with the predetermined procedure to reaize the stability and deceleration of the combat load to ensure the stability of the air trajectory of the lightning parachute, to meet the requirements of the parameters, such as entering speed and attitude, and to separate the lightning parachute when entering the water.

4.1.2. Function analysis. The functional analysis of the components and systems that make up the assisted torpedo in ensuring that the assisted torpedo can complete air missions, such as flight out of the box and separation of the torpedo and parachute, is as follows:

(1) Power propulsion function: this function is mainly completed by the power system of the flying aid rocket to provide the power of the entire mine flying aid section, realize the thrust termination under different range conditions, and bears various static and dynamic loads on the ground, on the ship, and in active flight.

(2) Navigation, guidance, and attitude control functions: these functions are mainly completed by the control system of the flying aid rocket to complete the power supply and distribution of the equipment on the mine, flight sequence control, moving base alignment information transmission, signal acquisition, transmission, transformation, and other functions.

(3) Separation function of mine and arrow: this function is mainly completed by the separation module of the separation system on the mine, the nonelectric explosion system, the control subsystem of the strap and the flying aid rocket, and the initiating explosive device. The control subsystem of the flying aid rocket sends out the command of separating the thunder from the arrow and detonates the initiating explosive device and the separating system functions to realize the separation of the thunder from the arrow.

(4) Protection function: the head cap and propeller protective cover in the air drop accessory system can reduce the aerodynamic resistance of air flight, prevent the propeller from rotating under the aerodynamic force, and perform other functions in accordance with.

(5) After the separation of the lightning and the arrow: this function is mainly completed by the air stabilization device by realizing the parachute opening in accordance with the predetermined procedure, stabilizing and decelerating the combat load, and ensuring the air ballistic stability of the lightning parachute to meet the requirements of parameters, such as water speed and attitude.
4.2. Event tree analysis of assisted torpedo

Event tree analysis can describe the relationship between the reliability of functional events that complete each function during the flight mission and the reliability of the entire mission process. Using event tree analysis to analyze the flight mission process of the assisted torpedo, the main event in the air flight trajectory is the main line, and an event tree is established from the initial event through a series of intermediate mission events until the mission flight mission is successful, as shown in Figure 2.

| Initial Event | Middle Event |
|---------------|--------------|
| Mission Start | Air flight out of the box |
|               | Separation of lightning and arrow |
|               | Parachute in the air |
|               | Numbering |
| MS            | AF         |
| SL            | PA         |

| State of consequences |
|-----------------------|
| 1 Mission Success     |
| 2 Mission Failure     |
| 3 Mission Failure     |
| 4 Mission Failure     |

**Figure 2.** Flight mission event tree.

4.3. Fuzzy fault tree analysis of functional events

4.3.1. Establishment of fault tree. Taking the “separation of mine and arrow” as an example, the nonelectric detonation system can reliably cut off the front, middle, and rear hoops at the same time when receiving the separation instruction from the control subsystem of the rocket system to realize the separation of mine and arrow. In the separation process, the battle load setting cable plug, the water activated battery waiting line, and the air stability device delay umbrella opening mechanism pull pin are pulled out. The main action processes of the separation of mine and arrow are sending command, band unlocking, separation of mine and arrow, unlocking insurance, and end of separation.

Taking the “failure of separation of thunder and arrow” as the top event, qualitative analysis is conducted, and the fault tree is obtained, as shown in Figure 3.

**Figure 3.** Thunderbolt separation fault tree.

The meaning of each symbol in the fault tree of the above event is shown in Table 1.

| Symbol | Event name                        | Symbol | Event name                        |
|--------|-----------------------------------|--------|-----------------------------------|
| $T_2$  | Thunderbolt failed to separate    | $X_2$  | Non-telegraphic explosion         |
| $M_1$  | Separation system failure         | $X_3$  | System failure                    |
| $M_2$  | Assisted rocket system failure    | $X_4$  | Cuff failure                      |
| $X_1$  | Split compartment failure         |        | Power subsystem failure           |
4.3.2. Fuzzy fault tree analysis. Through the qualitative analysis of fault tree, we can obtain the expression:

\[ T_2 = M_1 + M_2 = X_1 + X_2 + X_3 + X_4. \]  

Therefore, the minimum cut set of the fault tree is \( \{X_1\}, \{X_2\}, \{X_3\}, \{X_4\} \).

In accordance with the relevant data, the order of magnitude of the occurrence probability of the bottom event of the fault tree of each functional event in the process of torpedo mission is 10^{-3}, and the order of magnitude of deviation is 10^{-4}. The occurrence probability of each bottom event in \( T_2 \) event is shown in Table 2.

| Symbol | Bottom event                                      | m (10^{-3}) | \( \alpha \beta \) value (10^{-4}) |
|--------|--------------------------------------------------|-------------|------------------------------------|
| \( X_1 \) | Split compartment failure                        | 2.2         | 2.4                                 |
| \( X_2 \) | Non-telegraphic explosion system failure         | 1.8         | 2.0                                 |
| \( X_3 \) | Cuff failure                                     | 2.0         | 2.2                                 |
| \( X_4 \) | Power subsystem failure                          | 4.5         | 3.6                                 |

The triangle fuzzy number is used to express the occurrence probability of the bottom event as follows:

\[ \tilde{F}_1 = [(0.0022 - 0.00024) + 0.00024\lambda, \]
\[ (0.0022 + 0.00024) - 0.00024\lambda] \]  

\[ \tilde{F}_2 = [(0.0018 - 0.00020) + 0.00020\lambda, \]
\[ (0.0018 + 0.00020) - 0.00020\lambda] \]  

\[ \tilde{F}_3 = [(0.0020 - 0.00022) + 0.00022\lambda, \]
\[ (0.0020 + 0.00022) - 0.00022\lambda] \]  

\[ \tilde{F}_4 = [(0.0045 - 0.00036) + 0.00036\lambda, \]
\[ (0.0045 + 0.00036) - 0.00036\lambda] \]  

The fuzzy probability of the top event and the fuzzy importance of the bottom event can be analyzed.

1. Fuzzy probability of top event

Given that \( T_2 = X_1 + X_2 + X_3 + X_4 \), the fuzzy number of \( T_2 \) occurrence probability of the top event is calculated, with 5 decimal places reserved, which can be expressed as:

\[ \tilde{F}_{T_2} = [0.00945 + 0.00105\lambda, 0.01147 - 0.00101\lambda]. \]  

The conclusion can be expressed as follows:

The occurrence probability of bottom event is a certain value, and that of top event \( T_2 \) is 0.01046 when \( \lambda = 1 \). The occurrence probability of bottom event is a fuzzy number, and that of top event \( T_2 \) is \( [0.00945, 0.01147] \) when \( \lambda = 0 \).

2. Fuzzy importance of bottom events

The triangle fuzzy number is used to express the occurrence probability of bottom event, and the logical relationship of fault tree is or gate. The following formula can be obtained,
Therefore, the calculation formula of the fuzzy importance of the bottom event corresponding to top event $T_2$ is:

\[
\hat{I}_b(j) = \frac{\partial \hat{h}(p)}{\partial \hat{p}_j} = \frac{\partial \left\{1 - \prod_{i=1}^{\hat{p}_j} (1 - \tilde{F}_i)\right\}}{\partial \hat{p}_j} = \prod_{i=1}^{\hat{p}_j} (1 - \tilde{F}_i)
\]

(12)

The fuzzy importance of each bottom event when $\lambda = 0$ is calculated, as shown in Table 3. Therefore, the order of fuzzy importance of top event $T_2$ is

\[
\hat{I}_b(4) > \hat{I}_b(1) > \hat{I}_b(3) > \hat{I}_b(2).
\]

(13)

Table 3. Vague importance of bottom events.

| Symbol | Fuzzy importance       | Symbol | Fuzzy importance       |
|--------|------------------------|--------|------------------------|
| $X_1$  | [0.99094, 0.99250]     | $X_2$  | [0.99051, 0.99214]     |
| $X_3$  | [0.99073, 0.99232]     | $X_4$  | [0.99335, 0.99467]     |

Similarly, in accordance with the fuzzy failure rate of other components, the probability of the top event of the two functional events of “flying out of the box” and “parachuting in the air” can be calculated as follows:

\[
\tilde{F}_{\bar{R}}^a = \left[0.02478 + 0.00239, 0.02927 - 0.00210\right],
\]

(14)

and

\[
\tilde{F}_{\bar{R}}^a = \left[0.00547 + 0.00062, 0.00674 - 0.00068\right].
\]

(15)

4.4. Comprehensive evaluation of flight reliability of assisted torpedo

On the basis of the event chain model (fault and event trees) and in accordance with the evaluation results of each functional event and component, the flight mission process of assisted torpedo is comprehensively estimated through Monte Carlo simulation. In accordance with the fault tree calculation results of three functional events in Section 3.3, namely, “flying out of the box in the air,” “separation of thunder and arrow,” and “parachuting in the air,” the reliability of functional events is calculated accordingly. The entire mission process is integrated by Monte Carlo simulation, and the calculation results of flight reliability are as follows:

\[
\tilde{R}^i = \left[0.95313 + 0.00366, 0.96072 - 0.00393\right].
\]

(16)

The fuzzy probability of the entire mission success of the torpedo is shown in Figure 4.
The calculation results show that the reliability of the entire flight mission is an interval value. In particular, the fuzziness of failure probability of components is ignored, and the flight reliability of assisted torpedo is the only determined value $RF = 0.95679$ when $\lambda = 1$. The value of flight reliability is between $[0.95313, 0.96072]$ when $\lambda = 0$. As shown in Fig. 4, the larger the threshold value $\lambda$ corresponding to different membership functions, the smaller the reliability interval of the flight-assisted torpedo, and the closer the boundary probability value in the interval to the result of conventional reliability calculation. Corresponding to different membership functions, the smaller the threshold value $\lambda$ is, the greater the reliability interval of the flight-assisted torpedo, and the value of the probability of being close to the boundary in the interval deviates from the results of conventional reliability calculation but consistently contains them. From the analysis results, the flight reliability of assisted torpedoes is between $[0.95313, 0.96072]$ and is likely around 0.95679. The specified value of the torpedo flight reliability index is 0.95, and the interval for calculating the flight reliability is greater than 0.95. The reliability index still meets the launch requirements after the torpedoes undergo multistate conditions.

5. Conclusion
Considering the current situation of flight reliability evaluation and the demand of flight reliability prediction of assisted torpedo, this paper proposes a method of flight reliability evaluation of flying torpedo based on probability risk evaluation. Triangle fuzzy number is used to solve the uncertainty of component failure probability, and the reliability of functional events and fuzzy importance of components in the process of flight mission are quantitatively analyzed. This method realizes the “qualitative analysis, quantitative calculation, and calibration evaluation,” provides technical support for the flight reliability evaluation of assisted torpedo, and has better engineering application value.

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