Research on Thermal Stress Birefringence of Longitudinal Modulated OVS

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Abstract. Thermal stress birefringence is a key factor affecting the stability of optical voltage sensors. The study of thermal stress birefringence is of great significance for the development of optical voltage sensors. In this regard, this article first analyzes the thermal stress birefringence theory systematically based on the theory. Then, by adopting finite element method, ANSYS software is used to establish a model of the longitudinal modulation optical voltage sensor, and the thermal stress in the BGO crystal is simulated and analyzed. Finally, the distribution and change law of stress birefringence in crystal are obtained by combining theory and simulation. The results show that the average linear birefringence on the central clear path is $4.8004 \times 10^{-7}$, and the resulting phase delay is 0.0476 (2.7273 deg). When the clear path is off-center, the birefringence will increase, and increasing the spot size also increase the birefringence.

1. Introduction
Optical voltage transformer (OVT) is one of the important equipment for power system measurement. Compared with traditional electromagnetic transformer, it has the advantages of good insulation performance, high sensitivity, high measurement accuracy, small size, digital output and so on, and it also has broad development prospects. Optical voltage sensor (OVS) is the core of OVT. Temperature and stress birefringence are the key factors affecting its stability. Therefore, studying the thermal stress birefringence distribution of OVS has important significance for its development. Generally, temperature and stress birefringence are coupled to each other, which makes OVS output results appear random and volatile. To compensate for measurement errors caused by temperature and birefringence, researchers have proposed the following methods:

Self-healing OVT [1] uses a light source, a reference voltage source, and two identical clear paths. One is the measurement optical path, which is sensitive to the voltage to be measured; the other is the reference optical path, which is sensitive to the reference voltage. Real-time correction of the measurement optical path is performed using the known knowledge of the sensitive parameters of the reference optical path to achieve temperature compensation.

The self-calibrating OVT [2] uses the reference voltage source and the measured voltage source in series to act on the same optical voltage sensor, and corrects the measurement result of the optical voltage sensor through the self-calibration coefficient.

The dual optical path compensation method [3]-[4] uses a polarization beam splitter as an analyzer to generate two signal lights with orthogonal polarization directions. By averaging the two signals, the DC component including the stress birefringence can be eliminated, and the stress birefringence...
compensation can be realized. This method can optimize the stability in the temperature range of -2 °C ~ 65 °C from plus or minus 7% to 0.75%.

The double crystal method [5] uses two BGO crystals with the same optical properties. Crystal 1 is located between the high-voltage electrode and the ground electrode and is used to measure the electro-optic phase delay. Crystal 2 is located below the ground electrode to compensate for stress birefringence. A half-wave plate is placed in the optical path between the two crystals, and the fast axis direction is 45 degrees from the fast and slow axis direction of the BGO crystal. If the stress birefringence distributions of the two crystals are identical, the stress birefringence cancel each other out. With this compensation method, OVS can achieve a ratio error of less than 0.2% and a phase error of 15 minutes.

The above schemes have a good effect on compensating for temperature effects and stress birefringence. But so far, the related theoretical analysis and simulation research on stress birefringence is very limited. In this regard, this paper first conducts a systematic theoretical analysis of thermal stress and birefringence, then uses the finite element method as the main means, and uses ANSYS software to simulate the thermal stress of the longitudinally modulated BGO crystal at a temperature change of 40 °C, and the distribution and change law of thermal stress is analyzed. The research results have important guiding significance for the design and optimization of OVS.

2. Analysis of thermal stress

2.1. Theoretical analysis of thermal stress

The displacement from the original position can be described by the vector function \( U(r) \), and the components of the strain tensor can be described as:

\[
\varepsilon_{ij} = \frac{1}{2} \left[ \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right]
\]

According to Hooke's law, the stress tensor is related to the strain tensor and temperature:

\[
\sigma_{ij} = \frac{E}{1-\nu} \left[ \varepsilon_{ij} + \nu \frac{\partial \varepsilon_k}{\partial x_k} \right] - \frac{1}{1-2\nu} \left[ \frac{1+\nu}{1-2\nu} \alpha T \delta_{ij} \right]
\]

The above \( E \) is Young's modulus, \( \nu \) is Poisson's ratio, \( \text{Tr}[\varepsilon] \) is trace of matrix \( \varepsilon \), \( \delta_{ij} \) is Kronecker delta, \( T \) is temperature, and \( \alpha \) is a coefficient of thermal expansion.

In the equilibrium state, the internal stress should be balanced in any volume element. Assuming that the crystal has no internal force, the equilibrium conditions are:

\[
\frac{\partial \sigma_{ij}}{\partial x_j} = 0
\]

The above uses the Einstein summation convention. Substituting equation (2) into equation (3), the equation of the volume element displacement vector function \( U \) is:

\[
\nabla^2 U + \frac{1}{1-2\nu} \nabla (\nabla \cdot U) = \frac{2(1+\nu)}{1-2\nu} \alpha T
\]

Simple boundary condition can be described as:

\[
\sigma_{ij} n_j = 0
\]
2.2. Simulation model
The simulation model is a sensing unit in a Longitudinal modulation OVS. Its components are: ground electrode, BGO crystal, high-voltage electrode, and SF6 gas filled between the high voltage electrode and the ground electrode. The cross-sectional view of its y-z plane is shown in figure 1, both BGO and electrodes are cylindrical. The electrode has a diameter of 50mm, a thickness of 10mm, and the material is aluminum. The BGO crystal has a diameter of 10mm and a height of 10mm. The air gap distance between the BGO crystal and the high voltage electrode is 20mm. The BGO clear direction is the 001 direction, which is the z direction of the model coordinate system. Its 100 and 010 directions are the x and y directions of the coordinate system, respectively. The O point in figure 1 is the origin of the coordinate system, the red arrow line represents the clear path, which is the center line of the BGO cylinder.

![Figure 1. Schematic diagram of OVS simulation model.](image)

2.3. Simulation parameters
The initial simulation temperature is 20°C, and there is no stress inside the crystal and the electrode at this time. The entire OVS sensing unit is slowly heated to 80 °C. It is assumed that the temperature of the sensing unit is uniform throughout the heating process.

Given the elastic matrix element of the BGO crystal, \( c_{11} = 1.158 \times 10^{11} \) N/m\(^2\), \( c_{12} = 0.27 \times 10^{11} \) N/m\(^2\), and \( c_{44} = 0.436 \times 10^{11} \) N/m\(^2\). The relationship between the elastic matrix element, Young's modulus and Poisson's ratio is given below:

\[
\begin{align*}
    c_{11} &= \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} \\
    c_{12} &= \frac{E\nu}{(1+\nu)(1-2\nu)} \\
    c_{44} &= \frac{E}{2(1+\nu)}
\end{align*}
\]

From the above, the Young's modulus \( E \) of BGO is calculated to be \( 1.0599 \times 10^{11} \) Pa, and the Poisson's ratio \( \nu \) is 0.1891. Other parameters of BGO are: density: \( 7.13 \times 10^3 \) kg/m\(^3\), thermal expansion coefficient: \( 6.44 \times 10^{-6} \) K\(^{-1}\) (300K), specific heat capacity: \( 380 \) J/mol K\(^{-1}\) (300K), thermal conductivity: \( 25.9 \) mW/cm K\(^{-1}\).

The simulation parameters of aluminum are: Young's modulus: \( 7 \times 10^{10} \) Pa, Poisson's ratio: 0.33, density 1600 kg/m\(^3\) (2700), expansion coefficient: \( 23 \times 10^{-6} \) K\(^{-1}\).
2.4. Thermal stress simulation results

Figure 2 shows the stress distribution of the cylindrical BGO crystal on the y-z section. As can be seen from the figure: (1) Normal stress $\sigma_{11}, \sigma_{22}, \sigma_{33}$ is significantly larger than shear stress $\sigma_{23}, \sigma_{13}, \sigma_{12}$ (the average value is an order of magnitude larger), and the normal stress is dominant. (2) At the junction of the BGO crystal and the ground electrode, especially at the corners, the stress is greatest. The stress above the crystal is less than the stress below (with $z$ increasing, the stress tends to decrease). (3) The stress distributions of normal stresses $\sigma_{11}$ and $\sigma_{22}$ are closer, and the stress distribution of $\sigma_{33}$ is completely different from the two former stresses. (4) The shear stress has a larger value at the lower edge, but a smaller value inside the crystal.

![Figure 2](image)

Figure 2. Distribution of six stress components on the y-z section of BGO crystal.
Figure 3 shows the distribution of the six stress components on the clear path. As can be seen from the figure: (1) The distributions of $\sigma_{11}$ and $\sigma_{22}$ are almost equal, which is determined by the symmetry of the simulation model structure. (2) As $z$ increases, the distance from the ground electrode increases, $\sigma_{11}$ and $\sigma_{22}$ decrease continuously, decrease to 0 when $z = 4.4\text{mm}$, and then become negative. As $z$ increases, $\sigma_{33}$ first decreases to 0, then increases to a maximum at $z = 3.125$, and then decreases. The change law of $\sigma_{33}$ is obviously different from $\sigma_{11}$ and $\sigma_{22}$. (4) Shear stress is 4 orders of magnitude smaller than normal stress.

![Stress distribution on the clear path.](image)

Figure 3. Stress distribution on the clear path.

3. Analysis of stress birefringence

3.1. Theoretical analysis of stress birefringence

The key parameters of relative dielectric impermeability $B$ are defined as:

$$B = n^{-2}, i, j = 1, 2, 3$$  \hspace{1cm} (7)

The above $n_{ij}$ is a refractive index tensor.

The changes in $B_{ij}$ caused by stress are described as:

$$
\begin{bmatrix}
B_{11} \\
B_{22} \\
B_{33} \\
B_{23} \\
B_{13} \\
B_{12}
\end{bmatrix} =
\begin{bmatrix}
B_{0,11} \\
B_{0,22} \\
B_{0,33} \\
0 \\
0 \\
0
\end{bmatrix} +
\begin{bmatrix}
q_{11} & q_{12} & q_{12} & 0 & 0 & 0 \\
q_{12} & q_{11} & q_{12} & 0 & 0 & 0 \\
q_{12} & q_{11} & q_{11} & 0 & 0 & 0 \\
0 & 0 & 0 & q_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & q_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & q_{44}
\end{bmatrix}
\begin{bmatrix}
\sigma_{11} \\
\sigma_{22} \\
\sigma_{33} \\
\sigma_{23} \\
\sigma_{13} \\
\sigma_{12}
\end{bmatrix}
$$  \hspace{1cm} (8)

Among them, $B_{0,11} = B_{0,22} = B_{0,33} = n_0^{-2}$ is the relative dielectric impermeability of the BGO crystal without stress. $q_{ij}$ is a piezo-optic tensor, and $\sigma_{ij}$ is the stress component. It can be known from equation (8):

$$
\begin{align*}
B_{11} &= n_0^{-2} + q_{11}\sigma_{11} + q_{12}\sigma_{22} + q_{12}\sigma_{33} \\
B_{22} &= n_0^{-2} + q_{12}\sigma_{11} + q_{11}\sigma_{22} + q_{12}\sigma_{33} \\
B_{12} &= q_{44}\sigma_{12}
\end{align*}
$$  \hspace{1cm} (9)

When the clear direction is the 001 direction (the system coordinate system is the z direction), the clear path is shown in figure 1, and the incident light passes through the center of the cylindrical BGO. BGO generates birefringence under the action of stress and forms the fast and slow axes, the incident
light is decomposed into fast and slow light. Let its refractive index be \( n_+ \) and \( n_- \), respectively, according to reference [6]:

\[
n_+ = B_+^{\frac{1}{2}}
\]

\[
B_+ = \frac{1}{2} \left\{ (B_{11} + B_{22}) \pm \left[ (B_{11} - B_{22})^2 + 4B_{12}^2 \right]^{\frac{1}{2}} \right\}
\]

Let the angle between the slow axis and the x-axis of the crystal coordinate system be \( \theta \):

\[
\tan(2\theta) = \frac{2B_{12}}{B_{11} - B_{22}}
\]

The phase delay due to stress birefringence is:

\[
\delta = \frac{2\pi}{\lambda} L(n_+ - n_-)
\]

### 3.2. Calculation of stress birefringence

Given the thermal stress distribution in a BGO crystal, to calculate stress birefringence, the values of parameters \( q_{11}, q_{12}, \) and \( q_{44} \) are also required. However, only the parameters are given in reference [7]: \( q_{11} - q_{12} = -2.995 \times 10^{-13} \text{ m}^2/\text{N}, \) \( q_{44} = -1.365 \times 10^{-12} \text{ m}^2/\text{N}, \) which cannot be directly substituted into the equation for calculation. By deriving the formula, it is found that stress birefringence can still be calculated using the parameters provided in the reference.

Substituting equation (9) into equation (11):

\[
B_+ - B_- = \left[ ((q_{11} - q_{12})(\sigma_{11} - \sigma_{22}))^2 + 4(q_{44}\sigma_{12})^2 \right]^{\frac{1}{2}}
\]

According to equation (10):

\[
n_+ - n_- = \frac{B_+ - B_-}{B_+^{\frac{1}{2}} - B_-^{\frac{1}{2}}} \left( B_+ + B_-^{\frac{1}{2}} \right)^{\frac{1}{2}}
\]

Since \( |B_+ - B_0| \ll B_0, \ 11, \) and \( B_0, 11 = n_0^{-2} \):

\[
n_+ - n_- = \frac{n_0^2(B_+ - B_-)}{2}
\]

Substituting equation (9) into equation (12):

\[
\tan(2\theta) = \frac{2B_{12}}{B_{11} - B_{22}} = \frac{2q_{44}\sigma_{12}}{(q_{11} - q_{12})\sigma_{11} - (q_{11} - q_{12})\sigma_{22}}
\]

According to equation (17), the difference in refractive index on the clear path can be calculated, and it is known that \( \sigma_{11} \) and \( \sigma_{22} \) are equal in the clear path, so:

\[
n_+ - n_- = \left| n_0 q_{44} \sigma_{12} \right|
\]
Therefore, the curve of \( n^+ - n^- \) is consistent with the shape of \( \sigma_{12} \) absolute value. If the clear direction deviates from the center line, then \( \sigma_{11} - \sigma_{22} \neq 0 \), and the resulting birefringence and phase delay will increase significantly.

### 3.3. Stress birefringence in the clear path

The distribution of \( n^+ - n^- \) in the clear path is shown in figure 4. Theoretically, it should be consistent with the shape of the absolute value of \( \sigma_{12} \), that is, the minimum value appears near zero and is zero. But the minimum value is not zero, the reason is that the simulated \( \sigma_{12} \) curve is composed of limited points, and the points in it have not been taken to zero, so the corresponding \( n^+ - n^- \) extreme values are not zero.

![Figure 4. Distribution of birefringence on the clear path.](image)

### 3.4. Stress birefringence with the clear path off-center

When the clear path is 2.5mm away from the cylindrical center of the BGO crystal (the clear path is \( y = -2.5\text{mm}, x = 0\text{mm}, z = 0\text{mm} \sim 10\text{mm} \)), the stress distribution on the clear path is shown in figure 5, and the stress birefringence distribution on the clear path is shown in figure 6. The analysis shows that:

1. \( \sigma_{11} \) and \( \sigma_{22} \) no longer completely coincide, and the stress birefringence caused by normal stress increases significantly.
2. \( \sigma_{23} \) is significantly larger than \( \sigma_{13} \) and \( \sigma_{12} \), and the magnitude is close to normal stress.
3. The distribution of stress birefringence is no longer consistent with \( \sigma_{12} \), and the average value of \( n^+ - n^- \) is \( 1.8999 \times 10^{-6} \), which is 3.96 times without deviating from the center direction.

![Figure 5. Stress distribution at 2.5 mm off-center on the clear path.](image)
3.5. Effect of spot size on stress birefringence

Considering the effect of the spot size on the stress birefringence, thermal stress simulation and birefringence calculation are performed in the range of 4mm and 1mm from the center. The birefringence distributions are shown in figure 7 and figure 8, respectively. The analysis shows that as the range from the center increases, the stress birefringence increases. And at 1mm, the average is \(5.6724 \times 10^{-7}\), which is 18.17% larger than the center path; at 4mm, the average is \(5.2158 \times 10^{-6}\), which is 10.87 times the center path.

In addition, take the transverse path and perform stress birefringence analysis on \(z = 0, z = 2\text{mm}, z = 4\text{mm}, z = 6\text{mm}, z = 8\text{mm}, z = 10\text{mm} (x = 0, y = -5\text{mm} \sim 5\text{mm})\), such as figure 9. The analysis shows that as the distance from the center increases, the stress birefringence increases, and the increase is more pronounced near the ground electrode.
3.6. Stress Birefringence Analysis with Temperature Drop to -40 ℃
In the above analysis, the temperature is uniformly increased from 20 ℃ to 80 ℃, and then the temperature is uniformly decreased from 20 ℃ to -40 ℃ for analysis. Just change the temperature, repeat the above experiment, the results are shown in figure 10 ~ figure 14. The analysis shows that the stress distribution law is similar to that at 80 ℃, but the change trend is opposite; and the stress birefringence distribution law is the same as at 80 ℃.

![Stress distribution on the clear path.](image1)

![Distribution of birefringence in the clear path.](image2)

![Distribution of birefringence in the 4mm range.](image3)

![Birefringence distribution in the range of 1mm.](image4)

![Birefringence distribution in the transverse direction.](image5)
4. Conclusion

In this paper, the stress birefringence theory is deduced and analyzed. The finite element method is used to simulate the distribution of thermal stress in the BGO crystal when the temperature of the vertically-modulated OVS sensor head rises and falls uniformly by 40°C. Based on this, the stress birefringence in the clear path is calculated. The analysis shows that: (1) When the clear direction is 001 direction (longitudinal modulation) and the clear path passes through the center of the BGO crystal, the normal stress is significantly larger than the shear stress, and the average value of the former is 4 orders of magnitude larger than the latter, stress birefringence is mainly caused by $\sigma_{12}$. The average linear birefringence is 4.8004×10$^{-7}$, and the phase delay is 0.0476(2.7273deg) when the clear path is 10mm. (2) When the clear path is off-center, $\sigma_{11}$ and $\sigma_{22}$ are no longer equal, and normal stress generates stress birefringence dominates, stress birefringence increases significantly, and increases with increasing distance; the average linear birefringence increases by 18.17% from the center path at a deviation of 1mm, and its average linear birefringence at a deviation of 4mm 10.87 times the center path. (3) The larger the spot, the greater the birefringence it produces.

In summary, in order to reduce the stress birefringence caused by temperature changes, the following measures can be taken: (1) Ensure that the incident light passes through the center of the BGO crystal, and the stress birefringence will increase significantly after deviating from the center. (2) The selected light spot is as small as possible, and the increase of the light spot will increase the generated birefringence.

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