A D2-brane in the Penrose limits of $\text{AdS}_4 \times \text{CP}^3$

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Abstract

We consider a D2-brane in the pp-wave backgrounds obtained from $\text{AdS}_4 \times \text{CP}^3$ when electric and magnetic fields have been turned on. Upon fixing the light-cone gauge, light-cone Hamiltonian and BPS configurations are obtained. In particular we study BPS configurations with electric dipole on the two sphere giant and a giant graviton rotating in transverse directions. Moreover we show that the gauge field living on the D2-brane is replaced by a scalar field in the light-cone Hamiltonian. We also present a matrix model by regularizing (quantizing) 2-brane theory.
1 Introduction

AdS$_5$/CFT$_4$ correspondence identifies $\mathcal{N} = 4$ SU(N) superconformal gauge theory to type IIB superstring theory on the maximally supersymmetric AdS$_5 \times S^5$ background. This correspondence is a weak/strong coupling corresponding and this makes it a powerful tool to compute the strong coupling region of either theory using the weak coupling of the other. Albeit helpful, this property makes difficult to test AdS/CFT duality explicitly since neither type IIB superstring on AdS$_5 \times S^5$ background nor strong coupling gauge theory are well-understood.

Another maximal supersymmetric solution of type IIb supergravity is pp-wave and it can be obtained by taking Penrose limit of AdS$_5 \times S^5$. The superstring theory on this background was explicitly solved [1]. Therefore in the pp-wave background we know the string spectrum and can check that whether the same spectrum exists on the gauge theory side. Then we first need to understand how this specific limit translates to the gauge theory side. It was argued that the Penrose limit corresponds to considering a certain section of operators namely BMN operators [2]. Study of AdS/CFT correspondence in this specific limit opens a new way to test this conjecture more precisely.

Another example of AdS/CFT duality is AdS$_4$/CFT$_3$. $\mathcal{N} = 8$ CFT$_3$ was an open question for years and it was finally written in [3], the so-called BLG theory which is a $\mathcal{N} = 8$ three dimensional superconformal Chern-Simon theory. AdS$_4$/CFT$_3$ tells us that this theory is a suitable candidate to describe multiple M2-branes. But after a while it was shown that the BLG theory describes two coincident M2-branes [4]. Based on the BLG model, ABJM theory has been nominated to describe low energy of multiple M2-branes and to be dual to M-theory on AdS$_4 \times S^7/Z_k$ [5]. The ABJM model is a $\mathcal{N} = 6$ three dimensional superconformal $U(N) \times U(N)$ Chern-Simon theory of level $k$ and $-k$. The duality between ABJM model and type IIA string theory on AdS$_4 \times \text{CP}^3$ has been found when $N^{1/5} \ll k \ll N$ [5].

Pp-wave background has been also studied in the AdS$_4$/CFT$_3$ context. A Penrose limit of AdS$_4 \times \text{CP}^3$ with zero space-like isometry was obtained in [6] and string spectrum and BMN-like operators were obtained. Also, pp-wave metrics with one flat direction and two space-like isometries were found in [7, 8]. In this paper we consider a D2-brane in a general pp-wave background [7] and will then find light-cone (LC) Hamiltonian and BPS configurations with electric field. This paper is organized as follows. In the next section we
will review pp-wave backgrounds and in section 3 the LC Hamiltonian for a
D2-brane in pp-wave backgrounds will be obtained by using LC gauge. Then
we replace the gauge field on the D2-brane by a scalar field and find a matrix
model by applying a suitable prescription. In section 4, BPS configurations
are given. The last section is devoted to discussion.

2 Pp-wave backgrounds

In this section we will review three pp-wave backgrounds which are coming
from AdS$_4 \times$CP$^3$ by taking Penrose limit. One of the differences between
them is concerned with the number of space-like isometries. A general form
of these metrics has been written in [7] which leads to three pp-wave back-
grounds by choosing appropriate parameters. It is important to notice that
the only meaningful pp-wave backgrounds in AdS/CFT context are those
which are derived from Penrose limit of AdS$_4 \times$CP$^3$. The general form of
pp-wave geometry is given by

$$ds^2 = -4dx^+dx^- + \sum_{i=1}^{4} \left( du_i^2 - u_i^2(dx^+)^2 \right) + \sum_{a=1}^{2} \left[ dx_a^2 + dy_a^2 \right]$$
$$+ \left( \xi_a^2 - \frac{1}{4} \right) (x_a^2 + y_a^2)(dx^+)^2 + 2 \left( (\xi_a - 2C_a)x_a dy_a - (\xi_a + 2C_a)y_a dx_a \right) dx^+),$$

(1)

and by the following parameters we have

no flat direction $\leftrightarrow \xi_a = C_a = 0$, \hspace{1cm} (2a)

one flat direction $\leftrightarrow \xi_1 = \frac{1}{2}, \xi_2 = b + \frac{1}{2}, C_1 = \frac{1}{4}, C_2 = 0$, \hspace{1cm} (2b)

two flat directions $\leftrightarrow \xi_a = -\frac{1}{2}, C_a = \frac{1}{4}$, \hspace{1cm} (2c)

where $b$ is an arbitrary parameter. In addition in AdS$_4 \times$CP$^3$ background
there are two- and four-form RR fields which after taking Penrose limit be-
come

$$C_{+ij} = -\frac{1}{g_s} \epsilon_{ijk} u_k, \hspace{1cm} C_+ = -\frac{1}{g_s} u_4,$$

(3)

where $i, j = 1, 2, 3$ and $g_s$ is IIA string coupling constant.
AdS$_4 \times S^7$ is a maximally supersymmetric background. After taking the $Z_k$ orbifolding of $S^7$ and reducing the M-theory background AdS$_4 \times S^7/Z_k$ to type IIA string background AdS$_4 \times \text{CP}^3$, 24 out of 32 killing spinors remain [9]. It was shown that the case (2a) also preserves 24 supercharges [10].

More supersymmetric pp-waves in M-theory and their dimensional reduction to D0-brane or pp-waves in type IIA and T-dualisation to solutions in type IIB theory are studied in [11]. Moreover, in each case of the above pp-wave backgrounds coming from AdS$_4 \times \text{CP}^3$, the minimum bosonic symmetry is a $SO(3)$ rotation acting on $u^i$ as well as the translation symmetry in $x^+$ and $x^−$ directions.

3 Light-cone Hamiltonian

The low energy effective action for a D2-brane in general form of pp-wave background is

$$S = \int d\tau d^2\sigma \sqrt{-\text{det} N} + \int C^{(3)} + \int C^{(1)} \wedge F$$

$$= \int d\tau d^2\sigma L,$$  

(4)

where

$$g_{\hat{\mu}\hat{\nu}} = -4\partial_\hat{\mu}x^- \partial_\hat{\nu}x^+ + \left[ (\xi_a^2 - \frac{1}{4})(x_a^2 + y_a^2) - u_i^2 \right] \partial_\hat{\mu}x^+ \partial_\hat{\nu}x^+ + \partial_\hat{\mu}x^i \partial_\hat{\nu}x^i \quad (5a)$$

$$+ 2(\xi_a - 2C_a)x_a \partial_\hat{\mu}y_a \partial_\hat{\nu}x^+ - 2(\xi_a + 2C_a)y_a \partial_\hat{\mu}x_a \partial_\hat{\nu}x^+, \quad (5b)$$

$$F_{\hat{\mu}\hat{\nu}} = \partial_\hat{\mu}A_{\hat{\nu}} - \partial_\hat{\nu}A_{\hat{\mu}},$$

$$N_{\hat{\mu}\hat{\nu}} = g_{\hat{\mu}\hat{\nu}} + F_{\hat{\mu}\hat{\nu}}, \quad (5c)$$

and $x^i = \{ u^i, x^a, y^a \}$. In the LC gauge we fix a part of the area preserving diffeomorphism invariance which mix world-volume time and spatial coordinates. In order to fix LC gauge we separate the space and time indices on the brane world-volume as $\sigma^\mu = (\tau = \sigma^0, \sigma^r)$, $r = 1, 2$. The LC gauge is fixed by choosing

$$x^+ = \tau.$$  

(6)

In order to ensure that the above condition is respected by dynamics we use the time-space mixing part of area preserving diffeomorphism and set [12]

$$N^{0r} + N^{r0} \equiv G^{0r} = G_{0r} = (g - F g F)_{0r} = 0.$$  

(7)
$G_{\hat{\mu}\hat{\nu}}$ is the symmetric part of $N_{\hat{\mu}\hat{\nu}}$ which has the interpretation of open string metric \[13\] with $G_{\hat{\mu}\hat{\nu}}$ its inverse.

We noted that in the pp-wave background, $x^+$ and $x^-$ are cyclic variables and their conjugate momenta are constants of motion. Then

$$p^+ = \frac{\partial L}{\partial (\partial_\tau x^-)} = -\frac{2}{g_s} \sqrt{-\det NN_{00}},$$  \hspace{1cm} (8)

and LC Hamiltonian is

$$H_{lc} = p^- = \frac{\partial L}{\partial (\partial_\tau x^+)}.$$  \hspace{1cm} (9)

From the above equation we have

$$H_{lc} = p^+ \left( \partial_\tau x^- \frac{1}{2} \left[ (\xi_a^2 - \frac{1}{4})(x_a^2 + y_a^2) - u_i^2 \right] \right)$$

$$- \frac{1}{2} (\xi - 2C_a) x_a \dot{y}_a + \frac{1}{2} (\xi + 2C_a) y_a \dot{x}_a$$

$$- \frac{1}{2p^+ g_s} \epsilon_{ijk} u^i \{ u^j, u^k \} \right) - \frac{2B u^4}{3p^+ g_s}.$$  \hspace{1cm} (10)

Next we should eliminate $\partial_\tau x^-$. Using (8a), $N_{00}$ is

$$N_{00} = -4 \partial_\tau x^- (\xi_a^2 - \frac{1}{4})(x_a^2 + y_a^2) - (\dot{u}^i)^2 + (\dot{x}^i)^2 + 2(\xi - 2C_a) x_a \dot{y}_a$$

$$- 2(\xi + 2C_a) y_a \dot{x}_a.$$  \hspace{1cm} (11)

Let us recall the definition of $\det N$ which is

$$\det N = \det(N_{rs})(N_{00} - N_{0r} N_{rs} N_{s0}),$$  \hspace{1cm} (12)

where $N^{rp} N_{ps} = \delta^r_s$. It is important to note that $N^{00} \neq \frac{1}{N_{00}}$ because of off-diagonal electric-magnetic fields and hence

$$N^{00} = \frac{\det(N_{rs})}{\det N}.$$  \hspace{1cm} (13)

The above two equations together with (8) lead to

$$N_{00} = -\left( \frac{2}{p^+ g_s} \right)^2 \det(N_{rs}) + N_{0r} N^{rs} N_{s0}.$$  \hspace{1cm} (14)
By means of (14) the LC Hamiltonian (10) becomes

\[
\mathcal{H}_{lc} = p^+ \left( \left( \frac{1}{p^+ g_s} \right)^2 \det N_{rs} - \frac{1}{4} N_{0r} N^{rs} N_{s0} + \frac{1}{4} (\dot{x}^f)^2 \right)
- \frac{1}{4} \left[ (\xi_a^2 - \frac{1}{4}) (x_a^2 + y_a^2) - (\dot{u}^i)^2 \right] - \frac{1}{2p^+ g_s} \epsilon^{ijk} u^i \{ u^j , u^k \} - \frac{2Bu_4}{3p^+ g_s},
\]

where Chern-Simon terms has been added. In the case of D2-brane the first term in the Hamiltonian is

\[
\det N_{rs} = \det g_{rs} + \det F_{rs} = \frac{1}{2} \{ x^I , x^J \}^2 + B^2,
\]

where \( B = F_{12} \) and \( \{ F, G \} = \epsilon^{rs} \partial_r F \partial_s G \). The second term of (15) can be simplified by using the momentum conjugate to the gauge field which is

\[
p^E_r = \frac{\partial \mathcal{L}}{\partial \partial_0 F^r} = \frac{1}{2g_s} \sqrt{- \det NN^{0r}},
\]

and one can then show

\[
-p^+ N_{0r} N^{rs} N_{s0} = \frac{16}{p^+} p^E_r g_{rs} p^E_s
= \frac{16}{p^+} p^E_r \partial_0 X^I p^E_s \partial_s X^I = \frac{(4p^E_I)^2}{p^+}.
\]

Putting all these together we find the LC Hamiltonian to be

\[
\mathcal{H}_{lc} = \frac{(2p^E_I)^2}{p^+} + \frac{p^+ \dot{u}^i}{p^+} + p^+ \left( \frac{2p_a^0}{p^+} - (\xi_a + 2C_a) y_a \right)^2
+ \frac{p^+}{4} \left( \frac{2p_a}{p^+} + (\xi_a - 2C_a) x_a \right)^2 + \frac{1}{2p^+ g_s^2} \{ x^I , x^J \}^2 + \frac{B^2}{p^+ g_s^4}
\]

\[
- \frac{p^+}{4} \left[ (\xi_a^2 - \frac{1}{4}) (x_a^2 + y_a^2) - (\dot{u}^i)^2 \right] - \frac{1}{2g_s} \epsilon^{ijk} u^i \{ u^j , u^k \} - \frac{2Bu_4}{3g_s},
\]

where the third term in (15) was replaced by the following conjugate mo-
menta
\[
p^i = \frac{\partial L}{\partial (\partial_\tau u^i)} = -\frac{p^+}{2} \dot{u}_i,
\]
\[
p^a_x = \frac{\partial L}{\partial (\partial_\tau x^a)} = -\frac{p^+}{2} [\dot{x}_a - (\xi_a + 2C_a)y_a],
\]
\[
p^a_y = \frac{\partial L}{\partial (\partial_\tau y^a)} = -\frac{p^+}{2} [\dot{y}_a + (\xi_a - 2C_a)x_a].
\]

Matrix model

BMN (BFSS) matrix model is an interesting candidate for DLCQ of M-theory in terms of D0-branes in maximally supersymmetric eleven dimensional pp-wave background (flat space) \[2, 15\]. The Hamiltonian of this model is obtained as a regularized version of M2-brane LC hamiltonian in eleven dimensional pp-wave background \[16, 17\]. Moreover another matrix model describing DLCQ of type IIB string theory on the maximally supersymmetric ten dimensional pp-wave background has been introduced in \[18\], namely TGMT (Tiny Graviton Matrix Model). By regularizing spherical D3-brane in the ten dimensional pp-wave background, the Hamiltonian of the TGMT matrix model is obtained. In the following, the gauge field on the D2-brane is replaced by a scalar field and, by using the logic of \[16, 17\], a matrix model is introduced.

The gauge field living on a D2-brane has only one physical degree of freedom and it can be replaced by a scalar field in three dimensions. We are going to replace electric and magnetic fields in the LC hamiltonian by derivative of scalar field. In the case of D2-brane, it is easy to show that
\[
\mathcal{L}_{DBI} = \sqrt{\det g} \left( 1 + \frac{1}{2} F_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}} \right)
\]
\[
= -\frac{1}{2p} \det g + \frac{p}{2} \left( 1 + \frac{1}{2} F_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}} \right),
\]
where \(p\) is a Lagrangian multiplier. Let us define
\[
F_{\hat{\mu}\hat{\nu}} = \beta \epsilon_{\hat{\mu}\hat{\nu}\hat{\alpha}\hat{\gamma}} t_{\hat{\alpha}},
\]
where \(\beta\) and \(t_{\hat{\alpha}}\) are arbitrary constant and vector respectively. By using the equation of motion for the gauge field coming from (21) together with (22)
we find
\[ t_\dot{\alpha} = \partial_\dot{\alpha} \varphi, \]
\[ P_\dot{E}^\alpha = \beta \epsilon^{\alpha} \partial_\nu \varphi, \]
\[ B = \beta \dot{\varphi}. \]  \hspace{1cm} (23)

Terms including electric and magnetic fields in LC Hamiltonian are thus simplified as follows
\[ \frac{(2p_\dot{E}^I)^2}{p^+} = \frac{1}{2p^+ g_s^2} \{ x^I, \varphi \}^2, \]
\[ \frac{B^2}{p^+ g_s^2} - \frac{2Bu_4}{3g_s} + \frac{p^+_4}{4} u_4^2 = p^+ \left( \frac{p_\varphi}{p^+} - \frac{1}{3} u_4 \right)^2 + \frac{5p^+_4}{36} u_4^2, \]  \hspace{1cm} (24)

where \( \beta = \frac{1}{2\sqrt{2g_s}} \) and \( p_\varphi = \frac{\dot{\varphi}}{\beta g_s} \). Since we are looking for DLCQ description we need to compactify \( x^- \) on a circle of radius \( R_- \)
\[ x^- \equiv x^- + 2\pi R_- . \]  \hspace{1cm} (25)

This leads to the quantization of the LC momentum \( p^+ \)
\[ p^+ = \frac{J}{R_-} . \]  \hspace{1cm} (26)

By following [17, 18], we replace \( x^I, p^I \) with \( J \times J \) matrices, i.e.
\[ x^i \leftrightarrow X^i, \quad p^i \leftrightarrow J P^i, \]  \hspace{1cm} (27)

together with
\[ p^+ \int d^2 \sigma \leftrightarrow \frac{1}{R_-} \text{Tr}, \]
\[ \{ F, G \} \leftrightarrow J[F, G], \]  \hspace{1cm} (28)

where \( x^i = (x^i, \varphi) \). Equation (19) then becomes
\[ H = R_- \text{Tr} \left[ (p^i)^2 + \frac{1}{4} \left( 2P_x^a - \frac{1}{R_-} (\xi_a + 2C_a) Y_a \right)^2 + \left( P_\varphi - \frac{1}{3R_-} U_4 \right)^2 \right. \]
\[ \left. + \frac{1}{4} \left( 2P_y^a + \frac{1}{R_-} (\xi_a - 2C_a) X_a \right)^2 + \frac{1}{2g_s^2} [X^i, X^j]^2 + \frac{5}{36R_-^2} U_4^2 \right. \]
\[ \left. - \frac{1}{4R_-} \left[ (\xi_a - \frac{1}{4})(X_a^2 + Y_a^2) - (U^i)^2 \right] - \frac{1}{2R_- g_s} \epsilon^{ijk} U^i [U^j, U^k] \right]. \]  \hspace{1cm} (29)
Inspired by [17, 18], this matrix model describes DLCQ of M-theory on the uplifted pp-wave backgrounds obtained from AdS$_4 \times$CP$^3$.

## 4 BPS configuration

In this section we study BPS configurations involving electromagnetic fields. The case of our interest is the static electromagnetic fields. Our solutions include giant graviton and deformed giant graviton. Moreover a giant graviton rotating in transverse directions will be found as a BPS state.

### Giant-like solution

We start with the case where $u^i \neq 0$ while other fields set to be zero. In this case the LC Hamiltonian is

$$
\mathcal{H}_{lc} = \frac{p^+}{4} \left( u_i^2 + \frac{2}{(p^+ g_s)^2} u^i \{ u^i, u^j \}^2 - \frac{2}{p^+ g_s} \epsilon_{ijk} u^i \{ u^j, u^k \} \right) \\
= \frac{p^+}{4} \left( u^i - \frac{1}{p^+ g_s} \epsilon^{ijk} \{ u^j, u^k \} \right)^2.
$$

(30)

We consider the following ansatz

$$
u^i = \alpha \frac{p^+ g_s}{2} J^i,
$$

(31)

where $\alpha$ is a constant and $J^i$'s satisfy

$$
\{ J^i, J^j \} = \epsilon^{ijk} J^k,
$$

(32)

which specifies a two-sphere whose radius is one. By substituting (31) in the LC Hamiltonian we then have

$$
\mathcal{H}_{lc} = \frac{1}{16} (p^+)^3 g_s^2 \alpha^2 (1 - \alpha)^2
$$

(33)

The usual BPS argument tells us that $\mathcal{H}_{lc}$ is minimized when

$$
\alpha = 0 \quad \text{or} \quad \alpha = 1
$$

(34)

The above solutions (34) are graviton ($\alpha = 0$) and giant graviton ($\alpha = 1$) where their radii are zero and $\frac{1}{2} p^+ g_s$ respectively. These are $\frac{1}{2}$BPS (12 out

\[ \text{\footnotesize \textsuperscript{1}Giant graviton in the AdS$_4 \times$CP$^3$ background is discussed in [14].} \]
of 24 in the case \((2a)\) configurations whose LC energy is zero and preserve \(SO(3)\) symmetry. One can turn on a constant magnetic field on the spherical D2-brane \(i.e. B = F_{12} = \text{constant}\). This magnetic field doesn’t change the spherical shape and the radius of giant graviton but moves its center of mass from \(u_4 = 0\) to \(u_4 = \frac{3B}{p^+g_s}\).

**BIGGons solution**

For the pure electric field, \((19)\) simplifies to

\[
\mathcal{H}_{lc} = \frac{4}{(p^+)^2} \left( (P_i^E)^2 + (\tilde{u}^i)^2 \right)
= \frac{4}{(p^+)^2} \left( (\tilde{u}^i \pm R^{ij} P_j^E)^2 \mp 2\tilde{u}^i R^{ij} P_j^E \right),
\]

where \(\tilde{u}^i = \frac{(p^+)^{3/2}}{4} (u^i - \frac{1}{p^+g_s} \epsilon^{ijk} \{u^j, u^k\})\) and \(R^{ij}\) is a \(SO(3)\) rotation. Hence, the BPS equation is

\[
\tilde{u}^i = R^{ij} P_j^E. \tag{36}
\]

This BPS equation was discussed in section 3.1 of \([12]\) for the case of a three sphere giant graviton where the electric field is turned on. There, the shape deformation induced by the electric field sourced by two equally and opposite point charges placed on the North and South poles of the three spherical brane was obtained. The findings of \([12]\) generalize Blons \([19]\) to spherical D3-brane BIGGons. Remarkably, \((36)\) and its solutions are the same to those found in \([12]\). In other words we have found BIGGons solutions for spherical D2-branes. Physically these family of solutions describe open strings ending on two sphere giant graviton.

**Rotating giant graviton solution**

Another family of solutions that we consider are rotating giant gravitons. We turn on \(u^i(\tau, \sigma^r), x^a(\tau)\) and \(y^a(\tau)\) fields and the LC Hamiltonian thus becomes

\[
\mathcal{H}_{lc} = \frac{p^+}{4} \left( u^i - \frac{1}{p^+g_s} \epsilon^{ijk} \{u^j, u^k\} \right)^2 + \left( p_a^\alpha \pm \frac{1}{2} p^+ \alpha_+ y_a \right)^2 + \left( p_a^\alpha \pm \frac{1}{2} p^+ \alpha_- x_a \right)^2
+ (\mp \alpha_+ - \xi_a - 2C_a) p_a^\alpha y_a + (\xi_a - 2C_a \pm \alpha_-) p_a^\alpha x_a, \tag{37}
\]
where
\[ \alpha^2_{\pm} = (\xi_a \pm 2C_a)^2 - (\xi_a^2 - \frac{1}{4}). \] (38)

If the coefficients of the last two terms in (37) are equal they will show an angular momentum. Let us consider case (2a). In this case \( \alpha_{\pm} = \frac{1}{2} \) and hence
\[ H_{lc} = \frac{p^+}{4} \left( u^i - \frac{1}{p^+ g_s} \epsilon^{ijk} \{ w^j, u^k \} \right)^2 + \left( p^a_x \pm \frac{1}{4} p^+ y_a \right)^2 + \left( p^a_y \mp \frac{1}{4} p^+ x_a \right)^2 \] (39)

The BPS equations are given by
\[ u_i = \frac{1}{p^+ y_a} \epsilon^{ijk} \{ u^j, u^k \}, \]
\[ p^a_x = \pm \frac{1}{4} p^+ y_a, \] (40)
\[ p^a_y = \mp \frac{1}{4} p^+ x_a, \]

and the LC Hamiltonian is
\[ H_{lc} = \frac{1}{16} p^+ (x_a^2 + y_a^2) = L_{x_a y_a}. \] (41)

The above solution (40) describes a giant graviton rotating in \( x^a - y^a \) plane whose angular momentum is \( \frac{1}{4} p^+ (x_a^2 + y_a^2) \). This configuration is \( \frac{1}{4} \)BPS and preserves \( SO(3) \times U(1) \times U(1) \). Obviously one can also consider a giant graviton rotating in \( x_1 - x_2 \) or \( y_1 - y_2 \) plane.

## 5 Conclusion

There are three different pp-wave backgrounds coming from \( \text{AdS}_4 \times \text{CP}^3 \) where they have a different number of space-like isometry. We consider a D2-brane in these pp-wave backgrounds and the LC hamiltonian of this system is found by applying LC gauge fixing. There is a contribution coming from the gauge field living on the D2-brane in the LC Hamiltonian considered as a electric

\[ ^2 \text{Similar solution exists for the case } \alpha_{\pm} = -\frac{1}{2}. \]
and magnetic fields. We show that in three dimensions these fields are replaced by derivative of a scalar field. Using the idea of matrix model [16][17], we propose a matrix theory describing M-theory on the uplifted pp-wave backgrounds.

We then find BPS configurations. Half-BPS solutions are graviton and giant graviton with $SO(3)$ symmetry. For pure electric field, we reproduce BIGGons configurations describing open strings ending on giant graviton. These are $\frac{1}{4}$BPS configurations.

A giant graviton rotating in transverse directions is another $\frac{1}{4}$BPS configuration. Our solution has $SO(3) \times U(1) \times U(1)$ symmetry and rotates in $x^a - y^a$ plane. Rotation can be easily extended to other planes in transverse directions.

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