An Image Denoising Method Based on Improved Wavelet Thresholding

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Abstract. This paper proposes an improved-threshold function aimed to enhance the denoising performance of wavelet thresholding method. Firstly, the theory of wavelet transforms and characteristic of soft-threshold function and hard-threshold function was introduced. Then, an improved-threshold function was proposed. The new method overcomes the discontinuous in hard threshold de-noising method and reduces the permanent bias in soft threshold de-noising method. Last, the improved-threshold algorithm, soft-threshold algorithm, hard-threshold algorithm, disperse wavelet transform (DWT) and wiener filtering are used to reduce the noise in the same image. The experiment result show that the improved-threshold algorithm can get a better denoising effect than the traditional soft and hard thresholding de-noising algorithms.

1. Introduction

Noise exist extensively in the process of image acquisition and transmission. It reduces the image quality and the image will not be conducive to subsequent analysis and processing. The goal of denoising is to remove the noise while retaining as much as possible the important signal features [3]. Many researchers focus on this problem in the past 50 years or so and addressed this problem from many points of view. Statistical estimators of all sorts, spatial adaptive filters, stochastic analysis, partial differential equations, transform-domain methods, splines and other approximation theory methods, morphological analysis, order statistics, and more, are some of the many directions explored in studying this problem [13].

Wavelet transform is a signal processing technique which can display the signals on in both time and frequency domain [4]. Wavelet transform is a superior approach to other time-frequency analysis tools because its time scale width of the window can be stretched to match the original signal, especially in image processing studies. This makes it particularly useful for no stationary signal analysis, such as noises and transients. The method of wavelet thresholding for removing noise in an image has been researched extensively due to its excellence, simplicity and effectiveness. Donoho and Johnstone proposed a denoising method based on wavelet thresholding in [1] which can get a good visual effect, then much of the literatures have focused on how to develop the effectiveness. Many of them tried to develop the best uniform threshold or best basis selection.

In recent years there has been a plethora of work on using wavelet thresholding for image denoising, due to its effectiveness and simplicity [3-6]. This technique reduces the noise in the orthogonal wavelet domain, where each coefficient is threshold by comparing with a threshold.
In this paper, we proposed a new approach based on wavelet theory to provide an enhanced denoising algorithm.

The paper is organized as follows: firstly, the theory of wavelet transform and characteristic of soft-threshold function and hard-threshold function was introduced in section II. Then, an improved-threshold function was proposed in section III. And next, the improved-threshold algorithm, hard-threshold algorithm, disperse wavelet transform (DWT) and wiener filtering are used to reduce the noise in the same image and the de-noising effect of the image is evaluated with Mean-square error, Peak signal noise ratio and Structural similarity.

2. Wavelet Thresholding

Wavelets, like Fourier transform, can be used to approximate an underlying signal or image [7]. Assumes $\psi(t) \in L^2(R)$ and its Fourier transform $\hat{\psi}(\omega)$ meet the admissibility conditions

$$\int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < +\infty$$

(1)

Then, we call $\psi(t)$ the basic wavelet.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}} \psi_{a,b} \left( \frac{t-b}{a} \right)$$

(2)

Where $a,b \in R, a \neq 0$ and the continuous wavelet transform of $f(t)$

$$W_f(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{+\infty} f(t) \psi\left( \frac{t-b}{a} \right) dt$$

(3)

Thresholding function is the wavelet shrinkage function which determines how the threshold is applied to wavelet coefficient. There are two thresholding methods frequently used. The soft-threshold function and hard-threshold function. The hard-threshold function can be expressed as:

$$\hat{w}_{j,k} = \begin{cases} w_{j,k}, & \text{if } |w_{j,k}| \geq \lambda \\ 0, & \text{if } |w_{j,k}| < \lambda \end{cases}$$

(4)

Where $w_{j,k}$ denotes the wavelet coefficient, $\lambda$ is the threshold. $\hat{w}_{j,k}$ Represents the estimated value of the threshold. The hard-threshold set the elements lower than the threshold to zero and keep the elements whose value is greater.

The soft-threshold function can be expressed as:

$$\hat{w}_{j,k} = \left\{ \begin{array}{ll}
\text{sgn} (w_{j,k}) \cdot \left( |w_{j,k}| - \lambda \right), & |w_{j,k}| \geq \lambda \\
0, & |w_{j,k}| < \lambda
\end{array} \right.$$

(5)
Where \( \text{sgn}(\bullet) \) the symbolic function and the soft-threshold is is an extension of hard-threshold which set the elements lower than the threshold to zero while shrinking the other coefficients.

The wavelet thresholding procedure removes noise by thresholding only the wavelet coefficients of the detail sub bands, while keeping the low-resolution coefficients unaltered.

Although the methods are widely used in practice and get good effect, there are potential drawbacks in these methods. For hard-threshold function, there is a discontinuous point under the case of \( \hat{w}_{j,k} = \pm \lambda \), so, it may yield abrupt artifacts in the recovered images especially when the noise energy is significant.

For soft-threshold function, there will be deviations between \( \hat{w}_{j,k} \) and \( w_{j,k} \) when, \( w_{j,k} > \lambda \) which will lead to expand the deviation between the reconstructed image and real image.

3. **Image Denoising Based on Improved Wavelet Thresholding**

The improved-threshold function can be expressed as:

\[
\hat{w}_{j,k} = \begin{cases} 
  w_{j,k} - \text{sgn}(w_{j,k}) \cdot \left( \cos \left( \frac{\pi}{2} \left( 1 - \frac{e}{|w_{j,k}|} \right) \right) \right)^a, & \text{if } |w_{j,k}| \geq \lambda \\
  0, & \text{if } |w_{j,k}| < \lambda
\end{cases}
\]

(6)

Where \( a \) is adjustment factor. The improved algorithm consists of a cosine function and an exponential function. It improves the continuity of the threshold function and it’s flexible with the adjustment factor \( a \), the function is approximately similar to soft-threshold function when \( a \to 0 \) and hard-threshold function when \( a \to \infty \). (The improved-threshold function is approximately the same as hard-threshold function when \( a > 30 \) from the experimental data.)

Fig.1 displays the thresholds. We can see that the improved-threshold function is continuous. It overcome the pseudo-gibbs effect caused by discontinuity and make the signal smoothly. This threshold function not only combines the advantages of soft-threshold and hard-threshold function, but also improves the effect of denoising.

![Figure 1. Threshold function](image-url)
We can denoise the image by taking the following steps:

1. Perform the wavelet transform to the image and yield a set of wavelet coefficients $W_{j,k}$
2. Calculate the estimated value of the threshold $\hat{w}_{j,k}$ via function (6) in high-frequency and retain the threshold value in the low-frequency.
3. Reconstruct the image with the new coefficients via wavelet reconstruction.

4. Experimental Results and Discussion

4.1. Objective Quality Assessment Parameters

The de-noising effect of the image is evaluated with the following objective quality assessment parameters:

1. Mean-square error:

$$MSE = \frac{1}{M \times N} \left[ \sum_{i=0}^{M} \sum_{j=0}^{N} (f_{ij} - g_{ij})^2 \right]$$

(7)

Where $M$ and $N$ is the size of the image, $f_{ij}$ is the original image, $g_{ij}$ is the quantified image. The lower the value is, the better denoising effect is.

2. Peak signal noise ratio:

$$PSNR = 10 \log \left( \frac{A^2}{\sum_{i=0}^{M} \sum_{j=0}^{N} (f_{ij} - g_{ij})^2} \right)$$

(8)

The higher the value is, the better denoising effect is.

3. Structural similarity [2]:

$$SSIM (x, y) = \frac{\left(2\mu_x \mu_y + C_1\right)\left(2\sigma_{xy} + C_2\right)}{\left(\mu_x^2 + \mu_y^2 + C_1\right)\left(\sigma_x^2 + \sigma_y^2 + C_2\right)}$$

(9)

The detail of SSIM was shown in [2]. The higher the value is, the better denoising effect is.

4.2. Experiment

Fruits.jpg (480*512) was considered as the research object, and was processed and quantified via improved-threshold function, hard-threshold function, soft-threshold function, disperse wavelet transform (DWT) [8] and wiener filtering [12] with different standard deviation of Gaussian noise.

Fig.2 displays the de-noising effect of different algorithm with Gaussian noise $\sigma = 0.06$. There are noise points in the de-noising image via soft and hard-threshold algorithm and DWT algorithm. The Wiener filtering de-noising image appears blurred. Instead, the improved-threshold de-noising image possesses low distortion, the less number of noise points, smooth and natural transition of image details.
From Table 1 and Fig.3-5, compared with objective assessment parameter values of different algorithm, the improved-threshold algorithm is more optimal with lower MSE and higher PSNR.

From Fig.3-5, we can also find that when the standard deviation $\sigma$ of the Gaussian noise is low, the five algorithm have good effect, as the standard deviation $\sigma$ of the Gaussian noise increases, the effect get poorer. As the standard deviation $\sigma$ of the Gaussian noise increases, the MSE of soft-threshold and hard-threshold increase rapidly and PSNR and SSIM decrease rapidly, while these value of improved-threshold and Weiner filtering change slowly. It indicates that the improved-threshold algorithm and Weiner filtering are robust.

**Table 1. Contrast of Objective Assessment Parameters**

| De-noising algorithm          | MSE   | PSNR | SSIM |
|-------------------------------|-------|------|------|
| Improved -threshold           | 153.08| 26.28| 0.47 |
| hard -threshold               | 933.63| 18.43| 0.10 |
| soft-threshold                | 854.30| 18.81| 0.11 |
| DWT                           | 221.59| 24.68| 0.54 |
| Weiner filtering              | 430.75| 21.79| 0.15 |

Figure 2. Image de-noising effect diagram
5. Conclusion
The soft-threshold function and hard-threshold function have the disadvantages while reducing noise of an image. We have developed an improved-threshold algorithm for wavelet thresholding. In our experiment, the de-noising quality objective assessment parameter values indicates that the improved-threshold algorithm can get lower MSE, higher PSNR and SSIM with a good visual effect. We therefore conclude that the improved-threshold algorithm is an effective tool with image denoising.

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