QCD analysis of $xF_3$ structure function data and power correction to $\alpha_s$

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Abstract

Power corrections both to the strong coupling constant and to the structure function itself are estimated on the basis of the LO, NLO and NNLO QCD analysis of $xF_3$ structure function data. The sign of correction to the coupling constant is found to be negative. The x-shape of a higher twist contribution to the structure function is stable up to NNLO.

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Power corrections to perturbation QCD predictions both for $Q^2$-evolution of the running coupling constant and the structure function itself have intensively discussed recently [1, 2]. At present, precise measurements of structure functions (SF) and detailed theoretical calculations of QCD predictions for scaling violations provide an important means of accurate comparison of QCD with experiment. On this basis, during the last years the important information on the x-shape of power corrections to the structure function of a nucleon was obtained by the QCD analysis of data on deep-inelastic scattering of leptons [3] - [6].

In the present note, detailed LO, NLO and NNLO QCD analyses of precise $xF_3$ structure function data [7] have been carried out in order to evaluate the power correction to the QCD running coupling constant

$$\alpha_s(Q^2) = \alpha_s^{QCD}(Q^2) + \frac{A_2}{Q^2}$$

(1)
where in NNLO the coupling constant $\alpha_{s}^{pQCD}(Q^2)$ can be expressed in terms of inverse powers of $L = \ln(Q^2/\Lambda_{MS}^2)$ as

\[
\alpha_{s}^{pQCD}(Q^2) = \frac{1}{\beta_0 L} - \frac{\beta_1 \ln(L)}{\beta_0^2 L^2} + \frac{1}{\beta_0^3 L^3} \left[ \beta_1^2 \ln^2(L) - \beta_1^2 \ln(L) + \beta_2 \beta_0 - \beta_1^2 \right]
\]

Notice that $\beta_0 = 11 - 0.6667 f$, $\beta_1 = 102 - 12.6667 f$, $\beta_2 = 1428.50 - 279.611 f + 6.01852 f^2$.

We perform the QCD analysis using the method of the Jacobi polynomial expansion of structure functions. This method of solution of the DGLAP equation was proposed in [8] and developed both for unpolarized [9] and polarized cases [10]. The main formula of this method allows approximate reconstruction of the structure function through a finite number of Mellin moments of the structure function

\[
x F_3^{N_{\text{max}}}(x, Q^2) = \frac{h(x)}{Q^2} + x^\alpha (1-x)^\beta \sum_{n=0}^{N_{\text{max}}} \Theta_{n}^{\alpha,\beta}(x) \sum_{j=0}^{n} c_{j}^{(n)}(\alpha, \beta) M_{j+2}(Q^2)
\]

The $Q^2$-evolution of $M_N(Q^2)$ is defined by the perturbative QCD

\[
M_{N}^{QCD}(Q^2) = \left[ \frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]^{d_N} H_{N}(Q_0^2, Q^2) M_{N}^{QCD}(Q_0^2), \quad N = 2, 3, ...
\]

\[
d_N = \frac{\gamma^{(0),N}}{2\beta_0}
\]

Here $\alpha_s(Q^2)$ is the strong interaction constant, $\gamma^{(0),N}$ are nonsinglet leading order anomalous dimensions, and the factor $H_{N}(Q_0^2, Q^2)$ contains next and next-to-next to leading order QCD corrections. Power corrections to the coupling constant are introduced formally in accordance with (1).

Unknown coefficients $M_{N}^{QCD}(Q_0^2)$ in (4) could be parametrized as the Mellin moments of some function:

\[
M_{3}^{QCD}(N, Q_0^2) = \int_{0}^{1} dx x^{-N-2} a x^{b}(1-x)^c, \quad N = 2, 3, ...
\]

The shape of the function $h(x)$ as well as parameters $A_2$, $a$, $b$, $c$, and $\Lambda_{\text{MS}}$ are found by fitting the experimental data on the $xF_3(x, Q^2)$ structure function [8]. Detailed description of the fitting procedure could be found in [8]. Both terms $h(x)/Q^2$ and
$A_2/Q^2$ are considered as pure phenomenological. For a possible analytic expression see \[11\]. The target mass corrections are taken into account to the order $o(M_{\text{nucl}}^4/Q^4)$.

The results of the fit are presented in Table 1 and Figure 1.

The values of $A_2$ in Table 1 are opposite in sign to the lattice results for NLO and NNLO \[12\] $A_2^{\text{NLO}} = 0.22(2) \text{GeV}^2$ and $A_2^{\text{NNLO}} = 0.21(2) \text{GeV}^2$ obtained in different renormalization schemes. Notices also that in \[12\] ”lattice data” for the coupling constant were analysed, whereas in the QCD analysis of structure functions the anomalous dimensions and coefficient functions were involved too. Even in the leading order, the first factor in the right-hand side of (4) reproduces the terms with powers differing from $-2$, which initially appears in \[11\]. However, the absolute value of the parameter $A_2$ and a large value of the scale parameter $\Lambda_{\text{MS}}$ are in qualitative agreement with \[12\]. On the other hand, the negative value of $A_2$ is in agreement with predictions of \[11\].

|       | $A_2 \text{ [GeV}^2\text{]}$ | $\Lambda_{\text{MS}} \text{ [MeV]}$ | $\chi^2$   |
|-------|-------------------------------|-----------------------------------|------------|
| h(x) - free |                                |                                   |            |
| LO    | -0.261 ± 0.053                 | 1140 ± 110                        | 70.5 / 96  |
| NLO   | -0.130 ± 0.027                 | 788 ± 92                          | 71.7 / 96  |
| NNLO  | -0.123 ± 0.017                 | 561 ± 46                          | 73.4 / 96  |
| h(x)=0 |                                |                                   |            |
| LO    | -0.137 ± 0.011                 | 834 ± 31                          | 115.6 / 96 |
| NLO   | -0.049 ± 0.012                 | 584 ± 69                          | 116.5 / 96 |
| NNLO  | -0.046 ± 0.013                 | 561 ± 74                          | 103.2 / 96 |
| NLO   | -0.011 ± 0.008                 | 267 ± 36                          | 135.4 / 96 |
| NNLO  | -0.023 ± 0.005                 | 290 ± 36                          | 125.5 / 96 |

Table 1. The results of the LO, NLO ($N_{\text{Max}} = 10$) and NNLO ($N_{\text{Max}} = 6$) QCD fit (with TMC) of $xF_3$ data \[7\]. ($Q_0^2 = 3 \text{GeV}^2$, $Q^2 > 3 \text{GeV}^2$, f=4).

The bottom two lines correspond to substitution of (4) into the moments of the coefficient function only.

The values of constants $A_2$ for NLO and NNLO are approximately the same in agreement with the statement that the $1/Q^2$ corrections to all orders in $\alpha_s$ are of the same order \[11][12].

$^1$The LO result should not be considered on this matter because a nontrivial contribution of the
The shape of \( h(x) \) slightly differs from the results of analysis in Ref. [6] with \( A_2 = 0 \). The effect of decreasing the power correction to the structure function \([3, 5]\) while going from LO to NNLO of the perturbative QCD does not exist, as can be seen from Fig.1.

A special fit for the case \( h(x) = 0 \) gives a negative value for the parameter \( A_2 \). The increase of the \( \chi^2 \) parameter shows that using only power corrections to coupling constant, one could not reach a good description of experimental data and it is necessary to introduce additional power corrections to the structure function itself. A large difference between the value of \( A_2 \) for \( h(x) = 0 \) and \( h(x) \neq 0 \) indicates strong correlations between two power terms: \( A_2/Q^2 \) and \( h(x)/Q^2 \).

Even higher \( \chi^2 \) is obtained when the formal substitution of (11) is applied to coefficient functions and is not applied to the anomalous-dimension-dependent factor of structure function moment. This result is presented in the bottom two lines of Table 1. The values of \( A_2 \) are small and negative, but parameter the \( \Lambda_{\overline{MS}} \) is in agreement with [6].

In conclusion it should be noted that the presented values of \( A_2 \) should be taken with a great caution. One should make use of strict analytic expressions for the \( Q^2 \) evolution of the structure function moments with power corrections to coupling constant for a reliable fit of data. The nuclear and threshold effects should be taken into account, as well.

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Fig. 1 The results of the LO, NLO and NNLO extractions of twist-4 contributions of $h(x)$. The power correction to the QCD running coupling constant $\alpha_S(Q^2) = \alpha_S^{QCD}(Q^2) + A_2/Q^2$ is included.