Detecting service provider alliances*

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Abstract

We present an algorithm for detecting service provider alliances. To perform this, we modelize a cooperative game-theoretic model for competitor service providers. A choreography (a peer-to-peer service composition model) needs a set of services to fulfill its requirements. Users must choose, for each requirement, which service providers will be used to enact the choreography at lowest cost. Due to the lack of centralization, vendors can form alliances to control the market. We propose a novel algorithm capable of detecting alliances among service providers, based on our findings showing that this game has an empty core, but a non-empty bargaining set.

1 Introduction

The current ubiquity and pervasiveness of the Internet has been leading researchers and practitioners to imagine the Future Internet [2]. The Future Internet, as a particular case of Ultra-Large Scale (ULS) systems, constitutes a futuristic vision of a yet-to-come Internet whose scale changes everything.

On such scenario, systems are usually modeled and described by a service-oriented architecture that are completely distributed [5]. Ultra-Large Scale systems will require the transition from the current service orchestration model for service composition — where the ensemble of services are composed as an executable business process, controlled by a single party (the orchestrator) — to the service choreography composition model [23] — that describes a non-executable protocol for peer-to-peer interactions from several different parties. In other words, distributed scheduling devised by the applications itself (according to their own performance objectives) will be preferred.

This new model is not only more robust and scalable, but also more collaborative. The service choreography model enforces interoperability and loose coupling by reflecting obligations and constraints between parties. Each party involved will be required to clearly state its role in a standardized manner.

Actually, some improvements on the collaboration of service providers are already a reality for some specific services, such as Cloud Computing platforms. Frameworks like Eucalyptus [22], OpenNebula [18], and OpenStack [26] already allows developers to choose different cloud providers without any additional change on their software. A cloud computing user can choose (at any time) one or more providers from a large set of options. The choice can be driven by different non-functional criteria such as total cost, quality-of-service, etc.

Beyond the complexities of the management of composite services (design, provisioning, etc.), we consider the problem from an economic point of view. Since service vendors are not regulated, both cooperative and non-cooperative behavior may be expected. This can potentially lead to the formation of alliances — i.e., the formation of groups of similar independent parties, who join together to control prices and/or limit competition.

In this work we use tools and techniques from cooperative game theory [7] in order to analyze the stability and the alliance formation on the Future Internet scenario; a problem we call the choreography enactment pricing game.

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2 Problem statement

Providing different types of services to end-users is a very promising business model. Companies can provide access to complex systems as a service, where end-users can pay to execute an operation. It can scale from small companies (e.g., a travel company that sells airline tickets for end-users) — to large companies that offers a huge number of computational resources (like public cloud computing providers, that provide their services in a pay-as-you-go manner.)

In order to execute a complex choreography, a user must select a set of service vendors that commercialize the services that fulfills all choreography requirements. A same type (or role) of service can be offered from different vendors at different prices. Also, the same vendor can commercialize more than one type of service. It is the end-user that must choose (either manually or on an automated manner) which service provider will be in charge of the execution of services that fulfills each role. This process is known in systems literature as choreography enactment [15].

We use a graph of precedence constraints to model all possible ways to enact a choreography. The vertices of the graph indicate all known public services that fulfill the needs of the user and the edges represents the order in which those services must be executed.

More formally, all possible combinations of services that can enact a given choreography is modeled as a directed acyclic graph $G = (V \cup \{s, t\}, E)$, such that $w \in V$ is a service that users can pay to use, and the existence of an edge $\{w, x\} \in E$ indicates that the service $w$ must be executed before $x$. Let $m$ be the number of public services available to the user.

Vertices $s$ and $t$ are artificial vertices added to represent, respectively, the start and the end of the choreography. Thus, for all services $x$ with in-degree equal to zero, we have an additional edge $\{s, x\}$. Similarly, for all services $w$ with out-degree zero, we also have an edge $\{w, t\}$.

A successfully enacted choreography can be represented by a path from $s$ to $t$ on the graph $G$.

Figure 1 shows a graph depicting different forms of enacting a given choreography. In this case, the user can choose among three different cloud providers and two data serialization services. Any path $s \rightarrow t$ is valid, but a user would choose the cheaper one.

Users must pay a price to the service provider in order to use its services. Each service node $w$ charge $c_w$ for its use. This value is fixed and known in advance by all providers. We define the costs of vertices $s$ and $t$ as being zero.

The total price paid by a user is straightforward defined as the sum of the costs of the vertices in the path $s \rightarrow t$ chose by the user. Let $c$ be the vector $(c_1, \ldots, c_m)$ of all services costs and $c^k$ the vector $(c_1, \ldots, c_{k-1}, c_{k+1}, \ldots, c_m)$ of costs for all services except service $k$.

Users also have a budget, i.e., to set the maximal price $p$ that can be payed to realize an enactment. We assume that a user will always choose the path with minimal cost. If this minimal cost is greater than $p$, then the user simply refuses to execute its choreography.

From the service vendors point of view, we have a set of services that are offered to users at some given price (its operating costs plus a profit). A set of service vendors $S$ compete among each other to offer those services. We use the function $owner : V \Rightarrow S$ to denote the mapping from a given service $w$ to its vendor, $owner(w)$.

In a free market, however, nothing prevents vendors from creating alliances. Those alliances could be used to control the market by fixing the smallest price that would lead a user to choose their own services (price wars). The price can be decreased as long as the total operating cost (the cost to execute the services) of the coalition can be covered by the price payed by the user, and the profit can then be split between the members of the
alliance — a concept known in game-theory as transferable utility.

To formalize these alliances (or coalitions), we use the notion of cooperative games. We assume the following hypothesis for our model:

- a path (a combination of services) is offered to the user at a unique price;
- a user always selects a path with smaller cost;
- the revenue sharing mechanism must ensure coalition stability. In other words, providers are never tempted to leave the coalition;
- the coalition must offer at least one valid path, with at least two services, composed only by providers belonging to that coalition — a unitary path would create a monopoly controlled by its owner;
- the coalition can profit only if its path is guaranteed to be selected. Either there is no other valid path costing lesser than the maximum price \( p \), or other paths are definitely more expensive.

Before describing the game that models those properties, we will recall some important concepts from game theory that will be used in this paper.

3 Cooperative game theory [20]

3.1 Transferable utility games and Characteristic functions

A cooperative game with transferable utility (TU game) is a pair \((S, v)\) where \(S = \{1, \ldots, n\}\) is a finite set of players and a characteristic function \(v : 2^{|S|} \to \mathbb{R}\) which associates each subset \(C \subseteq N\) to a real number \(v(C)\) (such that \(v(\emptyset) = 0\)). Each subset \(v(C)\) of \(S\) is called a coalition. The function \(v\) is a characteristic function of the game \((V, v)\) and the value of coalition \(C\) denoted by \(v(C)\) is the value that \(C\) could obtain if they choose to cooperate. In TU games, the value of a coalition can be redistributed among its members in any possible way.

3.2 Revenue sharing mechanism

The challenge of a revenue sharing mechanisms is to find how to split the payoff \(v(C)\) among the players in \(C\) while ensuring the stability of the coalition. A vector \(x = (x_1, \ldots, x_{|V|})\) is said to be a payoff vector for a \(k\)-coalition \(C_1, \ldots, C_k\) if \(x_i \leq 0\) for any \(i \in V\) and \(\sum_{i \in C_j} x_i \leq v(C_j)\) for any \(j \in \{1, \ldots, k\}\). We will focus on some particular payoff vectors, namely imputations.

**Definition 1.** A payoff vector \(x\) for a \(k\)-coalition \(C_1, \ldots, C_k\) is said to be an imputation if it is efficient, — i.e., \(\sum_{i \in C_j} x_i \leq v(C_j)\) for any \(j \in \{1, \ldots, k\}\) — and if it satisfies the individuality rationality property — i.e., \(x_i \geq v(\{j\})\) for any player \(j \in S\).

The objective is to find a fair distribution of the value of the coalition (the payoff of each player corresponds to his actual contribution to the coalition) and also to ensure a stable coalition in such a way that no player or subset of players have incentive to leave the coalition.

The Core [11]

Let \((S, v)\) be a cooperative game and \(x\) a payoff vector of this game. A pair \((P, y)\) is said to be objection of \(i\) against \(j\) if:

- \(P\) is a subset of \(S\) such that \(i \in P\) and \(j \notin P\) and
- if \(y\) is a vector in \(\mathbb{R}^S\) such that \(y(P) \leq v(P)\), for each \(k \in P\), \(y_k \geq x_k\) and \(y_i > x_i\) (agent \(i\) strictly benefits from \(y\), and the other members of \(P\) do not do worse in \(y\) than in \(x\)).

In this case, we say that Imputation \(x\) is dominated by \(y\).
Definition 2. The core is the set of imputations for which there is no objection. That is imputations that are not dominated.

Another equivalent definition, states that the core is a set of payoff allocations $x \in \mathbb{R}^S$ satisfying:

- Efficiency: $\sum_{i \in S} x_i = v(S)$;
- Coalitional rationality: $\sum_{i \in C} x_i \geq v(C), \forall C \subseteq S$.

When the core is nonempty the grand coalition, the coalition created by all service providers, $S$ is stable as there is no objection threatening to leave it.

The core is a strong concept. Sometimes the core is empty and this requirement should be relaxed. The following concept considers situations where objections are not justified and may be neutralized.

The bargaining set. A pair $(Q, z)$ is said to be a counter-objection to an objection $(P, y)$ if:

- $Q$ is a subset of $S$ such that $j \in Q$ and $i \notin Q$ and
- if $z$ is a vector in $\mathbb{R}^S$ such that $z(P) \leq v(P)$, for each $k \in Q \setminus P$, $z_k \geq x_k$ and, for each $k \in Q \cap P$, $z_k \geq y_k$ (the members of $Q$ which are also members of $P$ get at least the value promised in the objection).

Let $(S, v)$ be a game with a coalition structure. A vector $x \in \mathbb{R}^S$ is stable iff for each objection at $x$ there is a counter-objection.

Definition 3. The bargaining set $B((S, v))$ of a cooperative game $(S, v)$ is the set of stable payoff vectors that are individually rational, that is, $x_i \geq v(\{i\})$.

Note that it is sufficient to use the notion of imputations since the payoffs are individually rational.

The bargaining set concept requires objections to be immune to counter-objections, otherwise they are not considered as credible threats.

Now, we will go back to our problem and try to define appropriate characteristic function that reflects the outcome expected from coalition formation as described earlier in the problem statement section (Section 2). Note, however, that even if the core is empty, the bargaining set is not.

4 The choreography enactment pricing game

The choreography enactment game models the cooperative game played by service providers. Their main objective is to form coalitions in order to create the best (cheaper) choice for a user that wants to deploy (enact) a given choreography. The utility (profit) of a service provider is given as a function of the price $d_w$ that the vendor announced for service $w$ — if this coalition is chosen by the user — and the operating cost of this service. Formally, the utility $v(w)$ received by service $w$ (vertex in graph $G$) is given by:

$$v(w) = \begin{cases} p_w - d_w & \text{if } w \text{ is in the chosen path} \\ 0 & \text{otherwise} \end{cases}$$

(1)

Now, from the utility $v_w$ received by service $w$, we derive the utility $v(a)$ received by player $a$ (set of vertices in graph $G$):

$$v(a) = \sum_{w \in V : \text{owner}(w) = a} v(w)$$

(2)

We can generalize this notation for a coalition. If $X$ is a set of service providers forming a coalition, its imputation can be given by:

$$v(X) = \sum_{a \in X} v(a)$$

(3)
We note the path leading to the lowest cost offered by \( X \) as \( SP^X \) and the cost of the shortest path as \( c^X \). Similarly, we note the shortest path/lowest-cost \( k \)-avoiding path from \( s \) to \( t \) as \( SP^{-k} \) and its total cost as \( c^{-k} \). If no such path exists, its cost is defined to be \( \infty \).

This characteristic function satisfies the properties stated in Section 2. A coalition must offer at least one valid path, otherwise its value is 0. The coalition may have some profit only if its path is guaranteed to be \( \lambda \)-avoiding. Nodes \( \delta \) and \( \gamma \) are important because other mechanisms could be used in order to reach \( p \) otherwise. We can derive that:

\[
v(w) = \begin{cases} 
\min(p, c^{-X}) - c^X & \text{if } c^{-X} \geq c^X \\
0 & \text{otherwise}
\end{cases}
\]

(4)

**Corollary 1.** The value of the grand coalition \( S \) in the choreography enactment game is given by: \( v(S) = (p - c_{SP}) \)

This result is straightforward. Since there is no path outside the grand coalition, we have that \( \min(p, c^{-S}) = p \). The lowest cost path inside \( S \) is necessarily the absolute shortest path \( SP \).

We will see later that, in the general case, it is not necessary to be in the grand coalition to reach \( p \) as the cost of the coalition.

**Example:** Consider the following example, illustrated in Figure 2. In this example, different vertices’ shapes represents different owners. Player \( \Lambda \) owns two vertices, \( \alpha \) and \( \lambda \), while others own only one vertex each. For the sake of simplicity, the set \( S \) of players is \( \{\Lambda, \beta, \delta, \gamma\} \). We assume that the user budget is \( p = 34 \). Since the cost \( c_{SP} = 6 \), \( v(S) = p - c_{SP} = 28 \).

To compute the value \( v(\{\Lambda, \gamma\}) \), we must compute the difference between the cost of the shortest path not containing \( \{\Lambda, \gamma\} \), which is equal to 23, and the shortest path containing only \( \{\Lambda, \gamma\} \), which is equal to 6.

We now show that the core of this example is empty. Its characteristic function is given by:

\[
v(\{\Lambda, \delta, \gamma, \beta\}) = 34 - 6 = 28; \quad v(\{\Lambda, \gamma, \delta\}) = 34 - 6 = 28; \\
v(\{\Lambda, \gamma\}) = 23 - 6 = 17; \quad v(\{\Lambda, \gamma, \beta\}) = 34 - 6 = 28; \\
v(\{\gamma, \delta\}) = 20 - 12 = 8; \quad v(\{\delta, \gamma, \beta\}) = 34 - 6 = 28; \\
v(\{\Lambda, \beta, \delta\}) = 34 - 20 = 14;
\]

For all other coalitions the value must be zero; either it does not contain a valid path or it can be beaten by a shortest path outside the coalition.

Suppose, by contradiction, that there exists an imputation \( x = (x_\Lambda, x_\gamma, x_\delta, x_\beta) \) that belongs to the core. By Definition 2 from the efficiency property, we have that \( x_\Lambda + x_\gamma + x_\delta + x_\beta = 28 \). From the group rationality property of the coalition \( \{\Lambda, \gamma, \delta\} \), we have that \( x_\Lambda + x_\gamma + x_\delta \geq 28 \), so definitely \( x_\beta = 0 \). Applying the same argument on coalition \( \{\Lambda, \gamma, \beta\} \) results that \( x_\delta = 0 \). Substituting \( x_\delta \) and \( x_\beta \) using the group rationality property on coalition \( \{\Lambda, \beta, \delta\} \) results that \( x_\lambda \geq 14 \). Similarly, on coalition \( \{\delta, \gamma, \beta\} \), it yields \( x_\gamma \geq 28 \). Thus \( x_\lambda + x_\delta \geq 28 + 14 = 42 \) which contradicts the efficiency property. Therefore, the core of this game is empty.

From this example we can notice that both coalitions \( \{\Lambda, \gamma, \delta\} \) and \( \{\Lambda, \gamma, \beta\} \) have the same value of the grand coalition. Nodes \( \delta \) and \( \beta \) both play the same role, but are still important to the coalition. In the core, their imputation is zero, by symmetry. Still, they are important because other mechanisms could be used in order to reach \( p \) instead of \( SP \).
to redistribute them according to their contributions. In the general case that a cut of the graph (including, or in addition to the shortest path) are sufficient to get the same value as the grand coalition. As we have stated above when the core in empty, it is still interesting to consider the Bargaining set.

5 Stable coalitions on the choreography enactment pricing game

The commerce of services is not regulated. This free market implies that service vendors are free to create alliances that could potentially allow them to control the entire market. In the previous section we saw that the core of the choreography enactment pricing game is empty. In this section we show that the bargaining set is non-empty and we derive an algorithm that is capable of detecting the formation of such alliances.

In graph theory, a vertex cut \( C \) for vertices \( s \) and \( t \) is a set of the vertices such that its removal from graph \( G \) separates \( s \) and \( t \) into distinct connected components. We focus on a vertex cut \( C \) and some useful properties about function \( v(.) \).

**Lemma 1.** Let \( C \) be a vertex cut in graph \( G \). Let \( SP \) be the shortest path between nodes \( s \) and \( t \). Let \( C_{SP} = C \cup SP \) be a set of vertices. Let consider \( w \notin C_{SP} \) a vertex. We have the following property for the vertex’s utility function:

\[
v(C_{SP}) = v(C_{SP} \cup \{w\})
\]

**Proof.** It is sufficient to notice that outside \( C_{SP} \) there is no path between nodes \( s \) and \( t \) in graph \( G \) thus \( v(C_{SP}) = v(V) \). \( \square \)

This implies in the following property about the value function:

**Lemma 2.** Let \( X \) be a set of players such that \( C_{SP} = \{w : \text{owner}(w) \in X\} \) contains a vertex cut \( C \) and \( a \) be the shortest path \( SP \) between nodes \( s \) and \( t \). Let us consider \( a \notin X \) a player. Then, the following property for the value function holds:

\[
v(X) = v(X \cup \{w\})
\]

**Lemma 3.** Let \( G(G,v,p) \) be a game. Let \( x \) be a feasible stable imputation. For each player \( j \) in \( S \) such that \( c^j \geq p \), we have \( x_j = 0 \).

**Proof.** We prove this lemma by contradiction. Assume that there exists a player \( j \) in \( S \) such that \( c^j \geq p \) and \( x_j > 0 \).

There exists at least a player \( i \) such that \( c^i \leq p \) (otherwise, the user would not choose any path since all prices of the path are greater than \( p \): \( x_j = 0 \) for all \( j \in S \)). Therefore \( c_{SP} \leq p \) and there exists at most one vertex \( w \) such that \( v(w) = c_{SP} \). So player \( \text{owner}(w) \) has \( c_{owner}(w) = c_{SP} \).

Let \( i^* \) be a player such that \( c^{-i} \geq c^{-k} \) for each \( k \in S \) and such that \( c^k = c_{SP} \). First, Equations 2 and 3 state that, for a given coalition \( X \), we have \( v(X) > 0 \) if \( x^X < c^{-X} \), otherwise \( v(X) = 0 \).

Since \( c^j \geq p \), each coalition \( X \) satisfies the following property: \( v(X) = v(X \setminus \{j\}) \).

Player \( i^* \) could make an objection of \((S \setminus \{j\}, y)\) against node \( j \) such that \( y_k = x_k + \frac{x_j}{|S| - 1} \) for \( k \in S \setminus \{j\} \). Note that for any \( k \), \( y_k > x_k \) since \( x_j > 0 \). Now consider two cases:

1. \( c^{-i^*} = c_{SP} \). This implies that \( v(S \setminus \{j\}) = 0 \) and \( v(S \setminus \{i^*\}) = 0 \).

2. \( c^{-i^*} > c_{SP} \). This implies that \( c(S \setminus \{i^*\}) > c_{SP} \). As consequence of the definition of \( v(.) \), we have \( v(S \setminus \{i^*\}) = 0 \).

Thus, \( v(S \setminus \{i^*\}) = 0 \). Player \( j \) cannot make a counter-objection \((Q, z)\) against \( i \) because \( v(Q \setminus \{i^*\}) = 0 \) for all coalitions \( Q \) not containing \( i^* \). This contradicts the assumption that \( x \) is stable. \( \square \)

Let’s focus on service providers that will receive non null retribution.

**Lemma 4.** Let \( G(G,v,p) \) be a game. Let \( x \) be a feasible imputation. Let \( i \) (similarly \( j \)) a player such that \( c^i < p \) (similarly \( c^j < p \)). Let \( O = S \setminus \{j\} \).

Let \( (O, y) \) be an objection of \( i \) against \( j \). In order to have a counter-objection to \((Q, z)\), with \( Q = S \setminus \{i\} \) of \( j \) against \( i \), a sufficient condition is:

\[
x_j - x_i \leq (c^{-i} - c^{-j})
\]

(5)
Equation (7) can be rewritten as: 

\[ c^{-j} \leq \min(p, c^{-O}). \] (6)

Now, we will look for a counter-objection of player \( j \) using \((Q, z)\), where \( Q = S \setminus \{i\} \). Its value of coalition depends on the lowest-price \( j \)-avoiding path from \( s \) to \( t \): \( c^Q = c^{-j} \). Since \( v(Q) = \min(p, c^{-O}) - c^Q \), we have:

\[ c^O \leq c^{-Q} \text{ and } v(Q) = \min(p, c^{-O}) - c^{-i} \]

By combining the two previous equations with Equation (6), we obtain:

\[
\begin{align*}
 v(Q) - v(O) & = \min(p, c^{-O}) - c^{-i} - (\min(p, c^{-O}) - c^Q) \\
 v(Q) - v(O) & \leq (c^{-j} - c^{-i} - \min(p, c^{-Q})) + \min(p, c^{-O}) 
\end{align*}
\] (7)

From the definition of the game, player \( i \) (resp. \( j \)) cannot be simultaneously adjacent to \( s \) and \( t \) (otherwise a monopoly would be possible.) This implies that \( \min(p, c^{-O}) = \min(p, c^{-Q}) \).

Let \( N \) be a set of vertices such that \( N = S \setminus \{i, j\} \). Since \( v(O) = \sum_{k=1}^{|N|} y_k + y_i \) and \( v(Q) = \sum_{k=1}^{|N|} z_k + z_y \), Equation (7) can be rewritten as:

\[ \sum_{k \in Q} z_k - \sum_{k \in O} y_k \leq (c^{-j} - c^{-i}) \] (8)

Let us now show that \( Q \) is an counter-objection where, for each \( k \in Q \setminus O \), \( z_k \geq x_k \) and for each \( k \in S \setminus \{i, j\} \), \( z_k \geq y_k \). It is sufficient to consider that:

\[ z_j - y_i \leq (c^{-j} - c^{-i}) \] (9)

From the definition of objection, it is sufficient to have: \( x_j - x_i \leq (c^{-i} - c^{-j}) \). This concludes the proof of the lemma. \qed

Lemma 3 shows that a feasible imputation \( x \) is stable if, for each player \( j \) in \( S \) such that \( c^j \geq p \), we have \( x_j = 0 \). We will now compute the value of \( x_j \) for all players \( j \) such that \( c^j \geq p \). We will define the set of players \( A \) such that \( A = \{ j \in S : c^j \leq p \} \).

**Theorem 1.** Let \( G(G,v,p) \) be a game. Let \( A \) be a subset of players \( \{ j \in S : c^j \leq p \} \). There exists a unique stable imputation \( x \) if \( x \) fulfils all the three following conditions:

1. \( \forall j \in S \setminus A, x_j = 0; \)
2. \( \forall j \in A, x_j = \frac{v(S)}{|A|} + \left( c^{-j} - \frac{\sum_{k \in A^{-j}} c^{-k}}{|A|} \right); \) and,
3. \( \forall j \in A, c^{-j} \geq \frac{c_{SP} - p + \sum_{k \in A} c^{-k}}{|A|}. \)

**Proof.** Property (1) can be straightforward deduced from Lemma 3.

The bargaining set is the set of all imputations that do not admit a justified objection. So, if we apply Theorem 4 to \( i, j \) and then to \( j, i \), then we can derive that for any couple \( (i, j) \in S^2 \), we have \( x_i - x_j = (c^{-i} - c^{-j}) \).

Let \( j \) be a player in \( A \). By summing all the previous equations, we obtain:

\[ \sum_{k \in A} x_i - |A|x_j = \left( \sum_{k \in A} c^{-k} - |A|c^{-j} \right) \] (10)
From the properties of the value of the coalition and by computation, we can rewrite Equation (10) as:

\[
\forall j \in A, \quad x_j = \frac{v(S)}{|A|} + \left( c^{-j} - \frac{\sum_{k \in A} c^{-k}}{|A|} \right) \tag{11}
\]

Property (3) can be deduced from the fact \( x_j \geq 0 \) and from Equation (11).

Those results are the technical framework that allows the detection of coalitions on our game. Given a game \( \mathcal{G}(G, v, p) \), we can detect if a set of service providers \( S \) — whose operational costs are given by \( c \), but the announced prices are \( d \) — are currently forming a coalition. Algorithm 1 presents the pseudo-code for the coalition detection for a given game.

**Algorithm 1:** Coalition detection algorithm.

- **Input:** \( \mathcal{G}(G, v, p) \), \( S \), \( c \), and \( d \)
- **Output:** Whether there is a coalition or not.

1. compute the lowest cost path \( \mathcal{SP} \);
2. forall players \( a \in S \) do
3. compute \( p^{-a} \)
4. compute \( A = \{ a \in S \mid c^{-a} \leq p \} \);
5. coalition = true;
6. forall players \( a \in A \) do
7. compute \( x_a \);
8. if \( x_a \neq c_a - d_a \) then
9. coalition = false
10. return coalition

Now we will establish the relation between the price and the different costs when there is a stable imputation. Using Theorem 1 we will compute the lower bound for non-empty bargaining sets.

**Theorem 2.** Let \( \mathcal{G}(G, v, p) \) be a game. There exists a stable imputation \( x \) if and only if:

\[
p \geq \sum_{k \in B} c^{-k} - (|B| - 1)c_{\mathcal{SP}} \tag{12}
\]

where \( B = \{ j \in B : c^j = c_{\mathcal{SP}} \land c^{-j} > c_{\mathcal{SP}} \} \).

Proof. Let \( A = \{ j \in S : c^j \leq p \} \). From Theorem 1 we have:

\[
\forall j \in A, \quad c^{-j} \geq \frac{c_{\mathcal{SP}} - p + \sum_{k \in A} c^{-k}}{|A|} \tag{13}
\]

Let \( B \) be a subset of \( A \) such that \( j \in B \) if \( c^{-j} > c_{\mathcal{SP}} \). Note that each player \( j \) belongs to a shortest path. Let \( i \) be a player not in \( B \). Thus, we have:

\[
c^{-i} \geq \frac{c_{\mathcal{SP}} - p + \sum_{k \in A} c^{-k}}{|A|} \tag{13}
\]

Since \( c^{-i} = c_{\mathcal{SP}} \), the previous equation can be rewritten as:

\[
|A|c_{\mathcal{SP}} \geq c_{\mathcal{SP}} - p + \sum_{k \in A} c^{-k} \tag{14}
\]

From the definition of set \( B \):

\[
p \geq \sum_{k \in B} c^{-k} - (|B| - 1)c_{\mathcal{SP}} \tag{15}
\]
To illustrate, let us reconsider the example of Figure 2. In this example, all players on set $A = \{\Lambda, \delta, \beta, \gamma\}$ engage in the coalition.

$$c_{SP} = 6 \quad p = 34 \quad v(S) = 28 \quad c^\Lambda = 12 \quad c^\delta = 6 \quad c^\beta = 6 \quad c^\gamma = 20 \quad \sum_{k \in A} c^k |A| = 44 \quad x_\Lambda = 8 \quad x_\delta = 2 \quad x_\beta = 2 \quad x_\gamma = 16$$

The unique stable imputation for this example is: $(x_\Lambda, x_\delta, x_\beta, x_\gamma) = (8, 2, 2, 16)$. Note that in this example, $B = \{\Lambda, \gamma\}$, in order to have a imputation on the bargaining set, the threshold to have a non-empty bargaining set must be $p \geq 20$.

### 6 Comparison with a truthful mechanism for lowest-cost routing

Several works (see [9], for example) focus on the problem of inter-domain routing from a mechanism-design point of view. The mechanism-design principles applied for the routing problem is the subject of seminal works by Nisan and Ronen [21] and Hershberger and Suri [13].

Feigenbaum et al. [9] provided a polynomial-time strategy proof mechanism for optimal route selection in a centralized computational model (inspired from [21]). In their formulation, the network is modeled as an abstract graph $G = (V, E)$. Each vertex $v$ of the graph is an agent and has a private type $t_v$, which represents the cost of a message transit through this node. The mechanism-design goal is to find a lowest-cost path between two designated nodes $s$ and $t$. The valuation of an agent $v$ is $-t_v$ if $v$ is part of $P$ and 0 otherwise.

Nisan and Ronen give the following simple mechanism for the problem: the payment to agent $v$ is equal to 0 if $v$ is not in $P$, and is equal to $d_{G|c_v=\infty} - d_{G|c_v=0}$ if $v$ is in $P$ where $d_{G|c_v=0}$ is the cost of the lowest-cost path through $G$ when the cost of $v$ is $\alpha$.

This mechanism ensures that the dominant strategy for each agent $v$ is to always report its true type $t_v$ to the mechanism. Such a mechanism is said to be truthful. When all agents honestly report their costs, the lowest-cost path is selected. This algorithmic mechanism design problem is solved using the well-known VCG mechanism [24] [12] [6].

We focus on the lowest-cost routing problem: the instance is composed of $G = (V, E)$ and a type vector $t = (t_1, \ldots, t_m)$. The goal is to find a lowest-cost path $P$ between two designated nodes $s$ and $t$. We will build a coalition game $G(G, v, p)$. Each node $v$ in $S$ has its cost $c_v$ equal to $t_v$.

**Theorem 3.** The total payment of a truthful mechanism in the lowest-cost path between a end-user and a destination is equal to the problem of finding the minimal value for $p$, such that there exists a stable coalition for the choreography enactment pricing game.

**Proof.** If each agent $v$ reports its true type $t_v$ to the mechanism, then a lowest-cost path $P$ is chosen. The total payment is equal to $\sum_{v \in P} d_{G|c_v=\infty} - d_{G|c_v=0}$.

By definition, $d_{G|c_v=\infty} = c^{-v}$. Therefore:

$$\sum_{v \in P} \left( d_{G|c_v=\infty} - d_{G|c_v=0} + t_v \right) = \sum_{v \in P} \left( c^{-v} - d_{G|c_v=0} + t_v \right) = \sum_{v \in P} c^{-v} - |P|c_{SP}$$

The total payment is given by $\sum_{v \in P} c^{-v} - (|P| - 1)c_{SP}$.

Note that if $v \in P$, then $d_{G|c_v=0} + t_v = c^v = c_{SP}$. Moreover, if $d_{G|c_v=\infty} = c^{-j} = c_{SP}$, then the payment is equal to 0.

Let $B = \{ j \in P : c^j = c_{SP} \land c^{-j} > c_{SP}\}$. Only payments given to agents in $B$ are strictly greater than 0. So the total payment is equal to the minimal value such that there exists a stable coalition for game $G(G, v, p)$, where $p = \sum_{v \in P} c^{-v} - |P|c_{SP}$. \qed

In the general case we have shown that there exists a unique imputation for which there is no justified objection. This imputation exists, provided that the end-user is willing to pay a maximum price that makes this
imputation possible. This price is exactly the total payment that agents would have received if an auction had taken place and a truthful mechanism had been used.

7 Related work

The choreography enactment pricing game is similar to fair resource allocation and networking games. Fragnelli et al. [10] studied a related cooperative game that they called the shortest path games. Their game models agents willing to transport a good through a network from a source to a destination. Using a graph model, and letting agents to control the nodes, they have studied how profits should be allocated according to the core of the cooperation.

Compared to the shortest path game, our model extends the problem by considering that agents could possibly control several nodes — a vendor could offer several different services. Also, in our game, no node can be part of all shortest paths at the same time, the opposite of the notion of s-veto players introduced by Fragnelli et al. They have focused their studies on the conditions for the existence of a non-empty core and on the Shapley value of the game.

Maintaining the assumption of s-veto players, Voorneveld and Grahn [25] extended the shortest path game and proved that the core allocations coincide with the payoff vectors in the strong Nash equilibria of the associated non-cooperative shortest path game.

Several subsequent papers [4, 19] studied computability and complexity aspects of this game. Some properties of graphs and games guaranteeing the existence of a core have been proposed and the computability complexity of computing cores have been established (NP-complete and #P-complete). Other variants (different payoffs and players controlling arcs) have been considered, but mostly focusing on the existence and complexity of cores, whereas this work mostly focus on the construction of the bargaining set in polynomial time.

The flow game can be view as maximum multicommodity flow problem in a cooperative setting. This model can be used to identify the set of demands to satisfy and to route this demand on the network. In this context, players own network resources and share a capacity to deliver commodities. Kalai et al. [14] first considered flow games for network with a single commodity, where a unique player owns an arc. Several studies (for example, [8, 16, 1]) extended this seminal work. Those extensions encompass variations on the number of arcs a player can control, if the player controls all or a part of the capacity of the arc, if players control vertices, etc. Those papers mainly focus on how to obtain the optimal flow in the network and then on how to allocate the revenue using core allocation techniques (since those games have non-empty cores).

8 Conclusion

This work presents a game-theoretic model for the problem we call the choreography enactment pricing problem. Vendors offer different services to users, but the lack of regulation of the market can lead to the formation of alliances.

We show that this game may have an empty core, but still has a non-empty bargaining set. The study of the conditions that can lead our cooperative game to a unique stable imputation resulted in a new coalition detection algorithm. We also show that finding the minimal user budget that leads to a stable coalition in the choreography enactment pricing game is equivalent to the problem of finding the total payment of a truthful mechanism in the lowest-cost problem between a end-user and a destination.

As future work, we are investigating the impact of allowing server multiitenancy. We are working on adding capacities to the services so that a user can potentially enact more than one instance of the service without paying more. This result would generalize the problem for services on any kind of cloud computing platform.

References

[1] Agarwal, R., Ergun, Ö.: Mechanism design for a multicommodity flow game in service network alliances. Oper. Res. Lett. 36(5), 520–524 (2008)
[2] Álvarez, F., Cleary, F., Daras, P., Domingue, J., Galis, A., Garcia, A., Gavras, A., Karnourskos, S., Krco, S., Li, M.S., Lotz, V., Müller, H., Salvadori, E., Sassen, A.M., Schaffers, H., Stiller, B., Tselentis, G., Turkama, P., Zahariadis, T. (eds.): The Future Internet, Lecture Notes in Computer Science, vol. 7281. Springer (2012)

[3] Aumann, R.J., Maschler, M.: The bargaining set for cooperative games. Advances in Game Theory pp. 443–447 (1964)

[4] Aziz, H., Sørensen, T.B.: Path coalitional games. CoRR abs/1103.3310 (2011), http://arxiv.org/abs/1103.3310

[5] Ben Hamida, A., Kon, F., Ansaldi Oliva, G., Dos Santos, C.E.M., Lorré, J.P., Autili, M., De Angelis, G., Zarras, A., Georgantas, N., Issarny, V., Bertolino, A.: An integrated development and runtime environment for the future internet. In: Álvarez et al. [2], pp. 81–92

[6] Clarke, E.H.: Multipart pricing of public goods. Public Choice (1971)

[7] Courcoubetis, C., Weber, R.: Pricing Communication Networks: Economics, Technology and Modelling (Wiley Interscience Series in Systems and Optimization). John Wiley & Sons (2003)

[8] Derks, J.J.M., Tijs., S.H.: Stable outcomes for multicommodity flow games. Methods of Operations Research 50(493–594) (1985)

[9] Feigenbaum, J., Papadimitriou, C., Sami, R., Shenker, S.: A BGP-based mechanism for lowest-cost routing. Distributed Computing 18(1), 61–72 (2005)

[10] Fragnelli, V., García-Jurado, I., Méndez-Naya, L.: On shortest path games. Mathematical Methods of Operations Research 52(2), 251–264 (2000)

[11] Gillies, D.B.: Solutions to general non-zero-sum games. Contributions to the Theory of Games IV, 47–85 (1959)

[12] Groves, T.: Incentives in teams. Econometrica pp. 617–631 (1973)

[13] Hershberger, J., Suri., S.: Vickrey prices and shortest paths: What is an edge worth? In: Proceedings of the 42nd Symposium on the Foundations of Computer Science (2001)

[14] Kalai, E., Zemel, E.: Generalized network problems yielding totally balanced games. Operations Research 30(5), 998–1008 (1982)

[15] Leite, L., Moreira, C.E., Cordeiro, D., Gerosa, M.A., Kon, F.: Deploying large-scale service compositions on the cloud with the choreos enactment engine. In: IEEE International Symposium on Network Computing and Applications. pp. 121–128 (Aug 2014)

[16] Markakis, E., Saberi, A.: On the core of the multicommodity flow game. In: Proceedings of the 4th ACM Conference on Electronic Commerce. pp. 93–97. ACM, New York, NY, USA (2003)

[17] Mas-Colell, A.: An equivalence theorem for a bargaining set. Journal of Mathematical Economics 18, 129–139 (1989)

[18] Milojicic, D., Llorente, I.M., Montero, R.S.: OpenNebula: A cloud management tool. IEEE Internet Computing 15(2), 11–14 (2011)

[19] Nebel, F.: Graph-based coalitional games: an analysis via characteristics. In: 5th International ICST Conference on Performance Evaluation Methodologies and Tools Communications. pp. 476–485 (May 2011)

[20] von Neumann, J., Morgenstern, O.: Theory of Games and Economic Behavior. Princeton Univ. Press (1944)

[21] Nisan, N., Ronen., A.: Algorithmic mechanism design. Games and Economic Behavior pp. 166–196 (2001)
[22] Nurmi, D., Wolski, R., Grzegorczyk, C., Obertelli, G., Soman, S., Youseff, L., Zagorodnov, D.: The eucalyptus open-source cloud-computing system. In: IEEE/ACM CGRID. pp. 124–131 (May 2009)

[23] Peltz, C.: Web services orchestration and choreography. Computer 36(10), 46–52 (2003)

[24] Vickrey, W.: Counterspeculations, auction, and competitive sealed tenders. The Journal of Finance 16(1), 8–37 (1961)

[25] Voorneveld, M., Grahn, S.: Cost allocation in shortest path games. Mathematical Methods of Operations Research 56(2), 323–340 (2002)

[26] Wen, X., Gu, G., Li, Q., Gao, Y., Zhang, X.: Comparison of open-source cloud management platforms: Openstack and opennebula. In: International Conference on Fuzzy Systems and Knowledge Discovery. pp. 2457–2461 (May 2012)