Confinement/Deconfinement and Gravity-Assisted Emergent Higgs Mechanism in Quintessential Cosmological Model

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Motivated by the ideas of Jacob Bekenstein concerning gravity-assisted symmetry breaking, we consider a non-canonical model of $f(R) = R + R^2$ extended gravity coupled to neutral scalar “inflaton”, as well as to $SU(2) \times U(1)$ multiplet of fields matching the content of the bosonic sector of the electroweak particle model, however with the following significant difference – the $SU(2) \times U(1)$ iso-doublet Higgs-like scalar enters here with a standard positive mass squared and without quartic selfinteraction. Strong interaction dynamics and, in particular, QCD-like confinement effects are also considered by introducing an additional coupling to a strongly nonlinear gauge field whose Lagrangian contains a square-root of the standard Maxwell/Yang-Mills kinetic term. The latter is known to produce charge confinement in flat spacetime.

The principal new ingredient in the present approach is employing the formalism of non-Riemannian spacetime volume-forms – alternative generally covariant volume elements independent of the spacetime metric, constructed in terms of auxiliary antisymmetric tensor gauge fields of maximal rank. Although being almost pure-gauge, i.e. not introducing any additional propagating degrees of freedom, their dynamics triggers a series of physically important features when passing to the Einstein frame: (i) Appearance of two infinitely large flat regions of the effective “inflaton” scalar potential with vastly different energy scales corresponding to the “early” and “late” epochs of the Universe; (ii) Dynamical generation of Higgs-like spontaneous symmetry breaking effective potential for the $SU(2) \times U(1)$ iso-doublet scalar in the “late” Universe, and vanishing of the symmetry breaking in the “early” Universe; (iii) Dynamical appearance of charge confinement via the “square-root” nonlinear gauge field in the “late” Universe and deconfinement in the “early” Universe.

Keywords: non-Riemannian volume-forms; quintessential evolution; confining gauge theories, dynamical generation of electroweak symmetry breaking.

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1. Introduction

Jacob Bekenstein was a remarkable scientist and person. He had both the creativity and the courage to look at physics from a different perspective. That was evident from his very early idea that black holes must have entropy. Bekenstein presented very strong arguments to support this idea but he was confronted with the opposition of S. Hawking, who strongly argued against his idea. The issue was later resolved in favor of Jacob when Hawking showed that the black holes emit radiation in a way consistent with the entropy proposal.

Jacob had also his original approach to other fundamental physics problems, like the idea that gravity must be modified in order to reproduce effects of dark matter, without the dark matter being really present.

Another topic Jacob was fascinated with, and on which we will focus here, was the search for new avenues for spontaneous symmetry breaking. One of us (E.G.) was for example involved in a joint research with Jacob on how symmetry breaking can be generated by density effects, even for a theory without spontaneous symmetry breaking when no background densities were present. During this collaboration E.G. learned lots of physics, of course, but also he understood how it is possible for a great mind to be simultaneously a remarkable human being.

Bekenstein’s search for new mechanisms for symmetry breaking led him also to consider gravity-induced symmetry breaking effects instead of density effects. In an intriguing paper from 1986 he proposed the remarkable idea of a gravity-assisted spontaneous symmetry breaking of electroweak (Higgs) type without invoking unnatural (according to his opinion) ingredients like negative mass squared and a quartic self-interaction for the Higgs field. By considering a model of gravity interacting with a standard Klein-Gordon scalar field (with small positive mass squared and without selfinteraction) coupled conformally to the scalar curvature he managed to obtain a prototype of dynamically induced Higgs-like spontaneous symmetry breaking scalar potential. A similar approach was further worked out in Ref. 10.

Motivated by Bekenstein’s idea, we wrote an essay to the gravity research foundation, where we considered a non-canonical model of gravity coupled to a neutral scalar “inflaton” as well as to a set of $SU(2) \times U(1)$ iso-doublet scalar and gauge fields corresponding to the bosonic sector of the electroweak particle model. Here the iso-doublet scalar field was introduced with a standard positive mass squared and without selfinteraction.

The essential non-standard feature of the model in Ref. 11 is its construction in terms of non-Riemannian spacetime volume-forms (alternative metric-independent generally covariant generally volume elements) defined in terms of auxiliary antisymmetric tensor gauge fields of maximal rank. The latter were shown to be almost pure-gauge – apart from few arbitrary integration constants they do not produce propagating field-theoretic degrees of freedom (see Appendices A of Refs. 11,13). Yet the non-Riemannian spacetime volume-forms trigger a series of important physical features unavailable in ordinary gravity-matter models with the standard Riemann-
nian volume-form (given by the square-root of the determinant of the Riemannian metric):

(i) The “inflaton” $\varphi$ develops a remarkable effective scalar potential in the Einstein frame possessing an infinitely large flat region for large negative $\varphi$ describing the “early” universe evolution;

(ii) In the absence of the $SU(2) \times U(1)$ iso-doublet scalar field, the “inflaton” effective potential has another infinitely large flat region for large positive $\varphi$ at much lower energy scale describing the “late” post-inflationary (dark energy dominated) universe;

(iii) Inclusion of the $SU(2) \times U(1)$ iso-doublet scalar field $\sigma$ introduces a drastic change in the total effective scalar potential in the post-inflationary universe – the effective potential as a function of $\sigma$ dynamically acquires exactly the electroweak Higgs-type spontaneous symmetry breaking form.

In the present paper we will extend the above model by introducing in the initial action a $R^2$-gravity term as well as coupling to an additional strongly nonlinear gauge field whose Lagrangian contains a square-root of the standard Maxwell/Yang-Mills kinetic term. The latter is known to describe charge confinement in flat spacetime as well as in curved spacetime for static spherically symmetric field configurations (Appendix B in Ref. 13; see also Eq. (10) below). Thus, the addition of the “square-root” nonlinear gauge field will simulate the strong interactions QCD-like dynamics and, therefore, our extended model represents qualitatively a quintessential cosmological model incorporating the full bosonic content of the standard particle model. Now, in the physical Einstein frame alongside with the Bekenstein-inspired gravity-assisted dynamical generation of Higgs-type electroweak spontaneous symmetry breaking in the “late” universe, while there is no electroweak breaking in the “early” universe, we obtain gravity-assisted dynamical generation of charge confinement in the “late” universe as well as gravity-suppression of confinement, i.e., deconfinement in the “early” universe.

2. Non-Canonical Gravity Coupled to a Confining Nonlinear Gauge Field and the Bosonic Sector of the Electroweak Standard Model

2.1. Non-Standard $f(R)$-Gravity Model with Non-Riemannian Spacetime Volume-Forms

We start with the following non-canonical $f(R) = R + R^2$ gravity-matter action constructed in terms of two different non-Riemannian volume-forms (generally covariant metric-independent volume elements) generalizing the actions in Refs. 11, 13 (for simplicity we use units with the Newton constant $G_N = 1/16\pi$):

\[
S = \int d^4x \Phi(A) \left[ R + L_1(\varphi, X) + L_2(\sigma, Y) - \frac{1}{2} f_0 \sqrt{-F^2} \right] + \\
\int d^4x \Phi(B) \left[ \epsilon R^2 - \frac{1}{4\epsilon^2} F^2 - \frac{1}{4g^2} F^2(A) - \frac{1}{4g'^2} F^2(B) + \frac{\Phi(H)}{\sqrt{-g}} \right].
\] (1)
Here the following notations are used:

- $\Phi(A)$ and $\Phi(B)$ are two independent non-Riemannian volume-forms given in terms of the dual field-strengths of rank 3 antisymmetric tensor gauge fields $A_{\nu\kappa\lambda}$ and $B_{\nu\kappa\lambda}$.

$$\Phi(A) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu A_{\nu\kappa\lambda}, \quad \Phi(B) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu B_{\nu\kappa\lambda}. \quad (2)$$

- $\Phi(H)$ is the dual field-strength of an additional auxiliary tensor gauge field $H_{\nu\kappa\lambda}$, whose presence is crucial for the consistency of the model (1):

$$\Phi(H) = \frac{1}{3!} \varepsilon^{\mu\nu\kappa\lambda} \partial_\mu H_{\nu\kappa\lambda}. \quad (3)$$

- We particularly emphasize that we start within the first-order Palatini formalism for the scalar curvature $R$ and the Ricci tensor $R_{\mu\nu}$:

$$R = g^{\mu\nu} R_{\mu\nu}(\Gamma),$$

where $g_{\mu\nu}$, $\Gamma_\lambda^{\mu\nu}$ – the metric and affine connection are apriori independent.

- $L_1(\varphi, X)$ is the “inflaton” Lagrangian:

$$L_1(\varphi, X) = X - f_1 e^{-\alpha \varphi}, \quad X \equiv -\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi, \quad (4)$$

where $\alpha, f_1$ are dimensionful positive parameters.

- $\sigma \equiv (\sigma_a)$ is a complex $SU(2) \times U(1)$ iso-doublet Higgs-like scalar field with Lagrangian:

$$L_2(\sigma, Y) = Y - m_0^2 \sigma_a^* \sigma_a, \quad Y \equiv -g^{\mu\nu} (\nabla_\mu \sigma)_a^* \nabla_\nu \sigma_a, \quad (5)$$

where gauge-covariant derivative acting on $\sigma$ reads:

$$\nabla_\mu \sigma = \left( \partial_\mu - \frac{i}{2} \tau_A A_\mu^A - \frac{i}{2} B_\mu \right) \sigma, \quad (6)$$

with $\frac{1}{2} \tau_A$ ($\tau_A$ – Pauli matrices, $A = 1, 2, 3$) indicating the $SU(2)$ generators and $A_\mu^A (A = 1, 2, 3)$ and $B_\mu$ denoting the corresponding electroweak $SU(2)$ and $U(1)$ gauge fields.

- The electroweak gauge field kinetic terms are of the standard Yang-Mills form (all $SU(2)$ indices $A, B, C = (1, 2, 3)$):

$$F^2(A) \equiv F^A_{\mu\nu} F^A_{\nu\lambda} (A) g^{\mu\nu} g^{\rho\lambda}, \quad F^2(B) \equiv F_{\mu\nu}(B) F_{\nu\lambda}(B) g^{\mu\nu} g^{\rho\lambda}, \quad (7)$$

$$F^A_{\mu\nu}(A) = \partial_\mu A_\nu^A - \partial_\nu A_\mu^A + \epsilon^{ABC} A_\mu^B A_\nu^C, \quad F_{\mu\nu}(B) = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (8)$$

Finally, there is an additional coupling in the action (1) to another strongly nonlinear (Abelian) gauge field $A_\mu$ with the square-root Maxwell term $-\frac{1}{2} f_0 \sqrt{-F^2}$ alongside the standard kinetic term $-\frac{1}{4} F^2$:

$$F^2 \equiv F_{\mu\nu} F_{\kappa\lambda} g^{\mu\nu} g^{\rho\lambda}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \quad (9)$$

As shown in Appendix B of Ref. 13 for static spherically symmetric fields in a static spherically symmetric spacetime metric the square-root term $-\frac{1}{2} f_0 \sqrt{-F^2}$ produces
an effective “Cornell”-type confining potential\(^{15,16}\) \(V_{\text{eff}}(L)\) between charged quantized fermions, \(L\) being the distance between the latter:

\[
V_{\text{eff}}(L) = \sqrt{2ef_0} L - \frac{e^2}{2\pi L} + (L-\text{independent const}) ,
\]

i.e., \(f_0\) and \(e\) play the role of a confinement-strength coupling constant and of a “color” charge, respectively.

In fact, we could equally well take the “square-root” nonlinear gauge field \(A_{\mu}\) to be non-Abelian – for static spherically symmetric solutions the non-Abelian model effectively reduces to the abelian one.\(^{12}\) Thus, the “square-root” gauge field will simulate the QCD-like confining dynamics.

Let us note that the structure of action (1) is uniquely fixed by the requirement for invariance (with the exception of the regular mass term of the isospin-doublet scalar \(\sigma_a\)) under the following global Weyl-scale transformations:

\[
g_{\mu\nu} \rightarrow \lambda g_{\mu\nu}, \quad \varphi \rightarrow \varphi + \frac{1}{\alpha} \ln \lambda, \quad A_{\mu\nu\kappa} \rightarrow \lambda A_{\mu\nu\kappa}, \quad B_{\mu\nu\kappa} \rightarrow \lambda^2 B_{\mu\nu\kappa}, \quad (11)
\]

where \(\lambda\) is arbitrary dimensionful and \(\alpha\) arbitrary dimensionless integration constants. The algebraic constraint Eqs.(12)-(14) are the Lagrangian-formalism counterparts of the Dirac first-class Hamiltonian constraints on the auxiliary tensor gauge fields \(A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, H_{\mu\nu\lambda}\).\(^{11,13}\)

The equations of motion of the initial action \(^{11}\) w.r.t. auxiliary tensor gauge fields \(A_{\mu\nu\lambda}, B_{\mu\nu\lambda}\) and \(H_{\mu\nu\lambda}\) yield the following algebraic constraints:

\[
R + L_1(\varphi, X) + L_2(\sigma, Y) - \frac{1}{2} f_0 \sqrt{-F^2} = -M_1 = \text{const} ,
\]

\[
\epsilon R^2 - \frac{1}{4\epsilon^2} F^2 - \frac{1}{4g^2} F^2(A) - \frac{1}{4g^2} F^2(B) + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const} ,
\]

\[
\frac{\Phi(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const} ,
\]

where \(M_1\) and \(M_2\) are arbitrary dimensional and \(\chi_2\) arbitrary dimensionless integration constants. The algebraic constraint Eqs.(12)-(14) are the Lagrangian-formalism counterparts of the Dirac first-class Hamiltonian constraints on the auxiliary tensor gauge fields \(A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, H_{\mu\nu\lambda}\).\(^{11,13}\)

The equations of motion of the initial action \(^{11}\) w.r.t. affine connection \(\Gamma^\mu_{\nu\lambda}\) (recall – we are using Palatini formalism):

\[
\int d^4x \sqrt{-g} g^{\mu\nu} \left( \frac{\Phi_1}{\sqrt{-g}} + 2 \epsilon \frac{\Phi_2}{\sqrt{-g}} R \right) \left( \nabla_{\kappa} \delta \Gamma^\mu_{\nu\kappa} - \nabla_{\mu} \delta \Gamma^\kappa_{\nu\kappa} \right) = 0
\]

yield a solution for \(\Gamma^\mu_{\nu\lambda}\) as a Levi-Civita connection:

\[
\Gamma^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda}(\bar{g}) = \frac{1}{2} g^{\mu\kappa} \left( \partial_{\nu} \bar{g}_{\lambda\kappa} + \partial_{\lambda} \bar{g}_{\nu\kappa} - \partial_{\kappa} \bar{g}_{\nu\lambda} \right) ,
\]

w.r.t. to the following Weyl-rescaled metric \(\bar{g}_{\mu\nu}\):

\[
\bar{g}_{\mu\nu} = \left( \chi_1 + 2\epsilon\chi_2 R \right) g_{\mu\nu} , \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} ,
\]

\(2.2. \text{ Derivation of the Einstein-Frame Action}\)

Solutions of the equations of motion of the initial action \(^{11}\) w.r.t. auxiliary tensor gauge fields \(A_{\mu\nu\lambda}, B_{\mu\nu\lambda}\) and \(H_{\mu\nu\lambda}\) yield the following algebraic constraints:

\[
R + L_1(\varphi, X) + L_2(\sigma, Y) - \frac{1}{2} f_0 \sqrt{-F^2} = -M_1 = \text{const} ,
\]

\[
\epsilon R^2 - \frac{1}{4\epsilon^2} F^2 - \frac{1}{4g^2} F^2(A) - \frac{1}{4g^2} F^2(B) + \frac{\Phi(H)}{\sqrt{-g}} = -M_2 = \text{const} ,
\]

\[
\frac{\Phi(B)}{\sqrt{-g}} \equiv \chi_2 = \text{const} ,
\]

where \(M_1\) and \(M_2\) are arbitrary dimensional and \(\chi_2\) arbitrary dimensionless integration constants. The algebraic constraint Eqs.(12)-(14) are the Lagrangian-formalism counterparts of the Dirac first-class Hamiltonian constraints on the auxiliary tensor gauge fields \(A_{\mu\nu\lambda}, B_{\mu\nu\lambda}, H_{\mu\nu\lambda}\).\(^{11,13}\)

The equations of motion of \(^{11}\) w.r.t. affine connection \(\Gamma^\mu_{\nu\lambda}\) (recall – we are using Palatini formalism):

\[
\int d^4x \sqrt{-g} g^{\mu\nu} \left( \frac{\Phi_1}{\sqrt{-g}} + 2 \epsilon \frac{\Phi_2}{\sqrt{-g}} R \right) \left( \nabla_{\kappa} \delta \Gamma^\mu_{\nu\kappa} - \nabla_{\mu} \delta \Gamma^\kappa_{\nu\kappa} \right) = 0
\]

yield a solution for \(\Gamma^\mu_{\nu\lambda}\) as a Levi-Civita connection:

\[
\Gamma^\mu_{\nu\lambda} = \Gamma^\mu_{\nu\lambda}(\bar{g}) = \frac{1}{2} g^{\mu\kappa} \left( \partial_{\nu} \bar{g}_{\lambda\kappa} + \partial_{\lambda} \bar{g}_{\nu\kappa} - \partial_{\kappa} \bar{g}_{\nu\lambda} \right) ,
\]

w.r.t. to the following Weyl-rescaled metric \(\bar{g}_{\mu\nu}\):

\[
\bar{g}_{\mu\nu} = \left( \chi_1 + 2\epsilon\chi_2 R \right) g_{\mu\nu} , \quad \chi_1 \equiv \frac{\Phi_1(A)}{\sqrt{-g}} ,
\]
\( \chi_2 \) as in \(^{(14)}\). Upon using relation \(^{(12)}\) and notation \(^{(13)}\) Eq. \(^{(17)}\) can be written as:

\[
g_{\mu\nu} = \left[ \chi_1 - 2\epsilon \chi_2 \left( L_1(\varphi, X) + L_2(\sigma, Y) - \frac{1}{2} f_0 \sqrt{-F^2} + M_1 \right) \right] g_{\mu\nu} . \tag{18} \]

Varying \(^{(1)}\) w.r.t. the original metric \( g_{\mu\nu} \) and using relations \(^{(12)}-\(^{(14)}\) we have:

\[
\chi_1 \left[ R_{\mu\nu} + \frac{1}{2} \left( g_{\mu\nu} L^{(1)} - T^{(1)}_{\mu\nu} \right) \right] - \frac{1}{2} \chi_2 \left[ T^{(2)}_{\mu\nu} + g_{\mu\nu} \left( \epsilon R^2 + M_2 \right) - 4\epsilon R R_{\mu\nu} \right] = 0 , \tag{19} \]

with \( \chi_1 \) and \( \chi_2 \) as in \(^{(14)}\) and \(^{(13)}\), and \( T^{(1,2)}_{\mu\nu} \) being the canonical energy-momentum tensors:

\[
T^{(1,2)}_{\mu\nu} = g_{\mu\nu} L^{(1,2)} - 2\frac{\partial}{\partial g^{\mu\nu}} L^{(1,2)} . \tag{20} \]

of the scalar+gauge field Lagrangians in the original action \(^{(1)}\):

\[
L^{(1)} \equiv L_1(\varphi, X) + L_2(\sigma, Y) - \frac{1}{2} f_0 \sqrt{-F^2} , \quad L^{(2)} \equiv - \frac{1}{4\epsilon^2} F^2 - \frac{1}{4g^2} F^2(A) - \frac{1}{4g^2} F^2(B) . \tag{21} \]

Taking the trace of Eqs. \(^{(19)}\) and using again relation \(^{(12)}\) we solve for the ratio \( \chi_1 \) \(^{(17)}\):

\[
\chi_1 \equiv 2\chi_2 T^{(2)}/4 + M_2 \frac{T^{(1)}/4 + M_1}{L^{(1)} - T^{(1)} - M_1} , \tag{22} \]

where \( T^{(1,2)} = g^{\mu\nu} T^{(1,2)}_{\mu\nu} \). Explicitly we obtain from \(^{(22)}\):

\[
\chi_1 = \frac{1}{2\chi_2 M_2} \left( \frac{f_1 \epsilon^2 - \alpha \varphi + m_0 \sigma^* \sigma - M_1} {1 + 2\epsilon \chi_2 \left( X + Y - \frac{1}{2} f_0 \sqrt{-F^2} \right)} \right) . \tag{23} \]

The Weyl-rescaled metric \( \bar{g}_{\mu\nu} \) \(^{(18)}\) can be written explicitly as:

\[
\bar{g}_{\mu\nu} = \chi_1 \Omega g_{\mu\nu} , \quad \Omega \equiv \frac{1 + \frac{f_0}{\chi_2} \left( \frac{f_1 \epsilon^2 - \alpha \varphi + m_0 \sigma^* \sigma - M_1} {1 + 2\epsilon \chi_2 \left( X + Y - \frac{1}{2} f_0 \sqrt{-F^2} \right)} \right) + 2\epsilon \chi_2 \left( Y - \frac{1}{2} f_0 \sqrt{-F^2} \right)} , \tag{24} \]

\[
X \equiv - \frac{1}{2} g^{\mu\nu} \partial_{\varphi} \partial_{\varphi}, \quad Y \equiv - \bar{g}^{\mu\nu} \nabla_{\nu} \sigma^* \nabla_{\nu} \sigma , \quad F^2 \equiv F_{\mu
u} F_{\nu\lambda} \bar{g}^{\mu\nu} \bar{g}^{\nu\lambda} . \tag{25} \]

Now, we can bring Eqs. \(^{(19)}\) into the standard form of Einstein equations in the second-order formalism for the Weyl-rescaled metric \( \bar{g}_{\mu\nu} \) \(^{(21)}\), i.e., the Einstein-frame equations:

\[
R_{\mu\nu}(\bar{g}) - \frac{1}{2} \bar{g}_{\mu\nu} R(\bar{g}) = \frac{1}{2} L^{\text{eff}}_{\mu\nu} \tag{26} \]

with effective energy-momentum tensor corresponding according to the definition \(^{(20)}\):

\[
L^{\text{eff}}_{\mu\nu} = g_{\mu\nu} L_{\text{eff}} - 2\frac{\partial}{\partial g^{\mu\nu}} L_{\text{eff}} \tag{27} \]

to the following effective Einstein-frame matter Lagrangian (using short-hand notations \(^{(21)}\) and with \( \chi_1 \) as in \(^{(23)}\) and \( \Omega \) as in \(^{(24)}\)):

\[
L_{\text{eff}} = \frac{1}{\chi_1 \Omega} \left( L^{(1)} + M_1 + \frac{\chi_2}{\chi_1 \Omega} \left[ L^{(2)} + M_2 + \epsilon (L^{(1)} + M_1)^2 \right] \right) . \tag{28} \]
The full Einstein-frame action, where all quantities defined w.r.t. Einstein-frame metric \(\bar{g}\) are indicated by an upper bar, explicitly reads:

\[
S = \int d^4x \sqrt{-\bar{g}} \left[ R(\bar{g}) + L_{\text{eff}}(\varphi, \bar{X}; \bar{Y}; \bar{F}, \bar{F}(A)^2, \bar{F}(B)^2) \right],
\]

where \(\bar{X}, \bar{Y}, \bar{F}^2\) are as in (25) (and similarly for \(\bar{F}(A)^2, \bar{F}(B)^2\)), and where:

\[
L_{\text{eff}} = \left(\bar{X} + \bar{Y}\right)(1 - 4e\chi^2 U(\varphi, \sigma)) + e\chi^2 \left(\bar{X} + \bar{Y}\right)^2 \left(1 - 4e\chi^2 U(\varphi, \sigma)\right)
\]

\[
- (\bar{X} + \bar{Y}) \sqrt{-F^2} e\chi^2 f_{\text{eff}}(\varphi, \sigma) - \frac{1}{2} f_{\text{eff}}(\varphi, \sigma) \sqrt{-F^2}
\]

\[
= -U(\varphi, \sigma) - \frac{1}{4e^2 f_{\text{eff}}(\varphi, \sigma)} F^2 - \frac{\chi^2}{4g'^2} F^2(A) - \frac{\chi^2}{4g^2} F^2(B)
\]

In (30) the following notations are used:

- \(U(\varphi, \sigma)\) is the effective scalar field (“inflaton” + Higgs-like) potential:
  \[
  U(\varphi, \sigma) = \left(\frac{f e^{-\alpha \varphi} + m_0 \sigma^* \sigma - M_1}{2} \right)^2.
  \]

- \(f_{\text{eff}}(\varphi, \sigma)\) is the effective confinement-strength coupling constant:
  \[
  f_{\text{eff}}(\varphi, \sigma) = f_0 \left(1 - 4e\chi^2 U(\varphi, \sigma)\right).
  \]

- \(e_{\text{eff}}^2(\varphi, \sigma)\) is the effective “color” charge squared:
  \[
  e_{\text{eff}}^2(\varphi, \sigma) = \left[ 1 + e^2 f_0^2 \left(1 - 4e\chi^2 U(\varphi, \sigma)\right) \right]^{-1}
  \]

Note that (30) is of quadratic “\(k\)-essence” type\(^{17–20}\) w.r.t. “inflaton” \(\varphi\) and the Higgs-like \(\sigma\) fields.

3. Quintessence, Confinement/Deconfinement and Gravity Assisted Emergent Higgs Mechanism

The nonlinear “confining” gauge field \(A_\mu\) develops a nontrivial vacuum field-strength:

\[
\left. \frac{\partial L_{\text{eff}}}{\partial F^2} \right|_{X,Y=0} = 0
\]

explicitly given by:

\[
\sqrt{-F^2}_{\text{vac}} = f_{\text{eff}}(\varphi, \sigma) e_{\text{eff}}^2(\varphi, \sigma)
\]

Substituting (35) into (36) we obtain the following total effective scalar potential (with \(U(\varphi, \sigma)\) as in (31)):

\[
U_{\text{total}}(\varphi, \sigma) = \frac{U(\varphi, \sigma)(1 - e^2 f_0^2) + e^2 f_0^2/4\chi^2}{1 + e^2 f_0^2 \left(1 - 4e\chi^2 U(\varphi, \sigma)\right)}.
\]

\(U_{\text{total}}(\varphi, \sigma)\) \(^{30}\) has few remarkable properties.
First, $U_{\text{total}}(\varphi, \sigma)$ possesses two infinitely large flat regions as function of $\varphi$ when $\sigma$ is fixed:

(a) (-) flat “inflaton” region for large negative values of $\varphi$,
(b) (+) flat “inflaton” region for large positive values of $\varphi$,

respectively, as depicted on Fig.1 (for $m_0\sigma^*\sigma \leq M_1$) or Fig.2 (for $m_0\sigma^*\sigma \geq M_1$).

(i) In the (-) flat “inflaton” region:
• The effective scalar field potential reduces to:

\[ U(\phi, \sigma = \text{fixed}) \simeq \frac{1}{4\epsilon\chi_2} \rightarrow U_{\text{total}} \simeq \frac{1}{4\epsilon\chi_2}, \quad (37) \]

implying that all terms containing \( \phi \) and \( \sigma \) disappear from the Einstein-frame Lagrangian \(^{[29]}\), i.e., there is no electroweak spontaneous breakdown in the (-) flat “inflaton” region.

• From \(^{[32]}\) the first relation \(^{[37]}\) implies \( f_{\text{eff}} = 0 \), i.e., there is no confinement in the (-) flat “inflaton” region.

(ii) In the (+) flat “inflaton” region:

• The effective scalar field potential becomes:

\[ U(\phi, \sigma) \simeq U^{(+)}(\sigma) = \left( m_0^2\sigma^*\sigma - M_1 \right)^2 \]

\[ \rightarrow U_{\text{total}}(\phi \sigma) \simeq U^{(+)}_{\text{total}}(\sigma) = \frac{U^{(+)}(\sigma)(1 - \epsilon e^2 f_0^2) + e^2 f_0^2 / 4\chi_2}{1 + \epsilon e^2 f_0^2 (1 - 4\epsilon\chi_2 U^{(+)}(\sigma))} \]

\[ \quad \text{(39)} \]

producing a dynamically generated nontrivial vacuum for the Higgs-like field:

\[ |\sigma_{\text{vac}}| = \sqrt{M_1 / m_0}, \quad (40) \]

i.e., we obtain “gravity-assisted” electroweak spontaneous breakdown in the (+) flat “inflaton” region.

• At the Higgs vacuum we have dynamically generated vacuum energy density (cosmological constant):

\[ U^{(+)}_{\text{total}}(\sigma_{\text{vac}}) \equiv 2\Lambda^{(+)} \simeq \epsilon e^2 f_0^2 \left[ 4\epsilon\chi_2 \left( 1 + \epsilon e^2 f_0^2 \right) \right]^{-1}. \quad (41) \]

• The effective confinement-strength coupling constant:

\[ f_{\text{eff}} \simeq f^{(+)} = f_0 \left( 1 - 4\epsilon\chi_2 U^{(+)}(\sigma) \right) > 0, \quad (42) \]

therefore we obtain “gravity-assisted” charge confinement in the (+) flat “inflaton” region.

As seen from Fig.1 or Fig.2, the two flat “inflaton” regions of the total scalar potential given by \( U^{(-)}_{\text{total}} = \frac{1}{4\epsilon\chi_2} \) \(^{[34]}\) and \( U^{(+)}_{\text{total}}(\sigma_{\text{vac}}) \equiv 2\Lambda^{(+)} = \epsilon e^2 f_0^2 \left[ 4\epsilon\chi_2 (1 + \epsilon e^2 f_0^2) \right]^{-1} \) \(^{[11]}\), respectively, can be identified as describing the “early” (“inflationary”) and “late” (today’s dark energy dominated) epoch of the universe provided we take the following numerical values for the parameters in order to conform to the PLANCK data\(^{[21,22]}\):

\[ U^{(-)}_{\text{total}} \sim 10^{-8} M_{\text{Pl}}^4 \rightarrow \epsilon\chi_2 \sim 10^{8} M_{\text{Pl}}^{-4}, \quad \Lambda^{(+)} \sim 10^{-122} M_{\text{Pl}}^4 \rightarrow \frac{\epsilon^2 f_0^2}{\chi_2} \sim 10^{-122} M_{\text{Pl}}^4, \quad (43) \]
where $M_{Pl}$ is the Planck mass scale.

From the Higgs v.e.v. $|\sigma_{\text{vac}}| = \sqrt{M_1/m_0}$ and the Higgs mass $M_1, m_0^2$ resulting from the dynamically generated Higgs-like potential $U_{\text{total}}^{(+)}(\sigma)$ we find:

$$m_0 \sim M_{\text{EW}}, \quad M_{1.2} \sim M_{\text{EW}}^4,$$

where $M_{\text{EW}} \sim 10^{-16} M_{Pl}$ is the electroweak mass scale.

4. Conclusions and Outlook

Here we have proposed a non-canonical model of $f(R) = R + R^2$ gravity coupled to non-standard matter incorporating two main building blocks – employing the formalism of non-Riemannian spacetime volume forms (generally covariant metric-independent volume elements) as well as introducing a special strongly non-linear gauge field with a square-root of the usual Maxwell/Yang-Mills kinetic term simulating QCD-like confinement dynamics. Due to the special interplay of the dynamics of the above principal ingredients our model is capable of producing in the Einstein frame:

- (i) Unified “quintessential” description of the evolution of the “early” and “late” universe due to a natural dynamical generation of vastly different vacuum energy densities thanks to the auxiliary non-Riemannian volume-form antisymmetric tensor gauge fields;
- (ii) Gravity-assisted dynamical generation of Higgs-like electroweak spontaneous symmetry breaking effective scalar potential in the “late” universe, as well as gravity-assisted charge confinement mechanism through the “square-root” nonlinear gauge field;
- (iii) Gravity-induced suppression of electroweak spontaneous symmetry breaking, as well as gravity-induced deconfinement in the “early” universe.

The non-Riemannian volume-form formalism has further physically relevant applications such as producing a novel mechanism for supersymmetric Brout-Englert-Higgs effect in supergravity through dynamical generation of a cosmological constant triggering spontaneous supersymmetry breaking and dynamical gravitino mass generation.\cite{23-24}

Similarly, the QCD-simulating “square-root” nonlinear gauge field when interacting with gravity produces several other interesting effects:

- (a) black holes with an additional constant background electric field exercising confining force on charged test particles even when the black hole itself is electrically neutral\cite{24}
- (b) Coupling to a charged lightlike brane produces a charge-“hiding” light-like thin-shell wormhole, where a genuinely charged matter source is detected as electrically neutral by an external observer\cite{25-26}
(c) Coupling to two oppositely charged lightlike brane sources produces a two-“throat” lightlike thin-shell wormhole displaying a genuine QCD-like charge confinement, i.e., the whole electric flux is trapped within a tube-like spacetime region connected the two charged lightlike branes.

(d) Charge confining gravitational electrovacuum shock wave.

The model here presented needs further amendments in order to avoid getting an unnaturally small value for the effective confinement strength coupling constant $f_0$ in the “late” universe resulting from the second relation (43) (condition for compatibility with the PLANCK data for the value of today’s cosmological constant).

Further obvious extension of the present model must be inclusion of the fermions in order to incorporate more faithfully the full standard particle model. To this end we can follow the steps outlined in several previous papers by some of us devoted to the study of modified gravity within the non-Riemannian volume element formalism coupled to fermionic matter fields, such as Ref. 28 (on the geometric origins of fermionic families), Ref. 29 (fermionic families and dark energy and dark matter), Ref. 30 (exotic low density fermionic states and neutrino dark energy).

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