Structural model updating study in consideration of complex pre-stress distribution

Chen Luyun¹, Huang Xichun² and Yi Hong¹

Abstract
In the present work, to improve the accuracy of the numerical model, the structural model updating problem is investigated in consideration of the influence of pre-stress. In order to eliminate the influence of complex pre-stress on the calculation scale and calculation precision for the continuous structures, the structural dynamic model updating is implemented by using the structural optimization strategy. The optimization function is established, in which the residual value of natural frequency parameter between the finite element model with/without complex pre-stress distribution is defined as objective function under certain constraints. Finally, the thin cylindrical shell structure with welding residual stress distribution, for example, the structural model updating is carried out, and the numerical results verify the feasibility and effectiveness of the proposed method.

Keywords
Cylindrical shell structure, model updating, natural frequency, pre-stress matrix, welding residual stress, weight coefficient

Introduction
Thin cylindrical shell structures are widely used in the engineering structures, such as aerospace, marine, mechanical, and civil constructs. The vibro-acoustic problem of cylindrical shells structure has been a hot topic over the recent years. Theoretical analysis, numerical calculation, and experimental investigations are the three main approaches to analyze the structural dynamic problem. Among them, the finite element method (FEM) based on the use of locally supported simple polynomials as shape functions within elements, is an accuracy and efficiency method for numerical calculation. An accurate FEM modeling is necessary to analyze and predict the dynamic performance of the complex structural systems. However, despite the remarkable improvements of the FE analysis techniques, the behavior of constructed structures and that of the FEM models built according to the same design drawings still differ greatly.¹,² The mismatch is mainly caused by inaccurate model simplification, the uncertainty of the material and geometrical parameters, and so on. In order to obtain an accurate FEM modeling, the structural model updating method is an effective tool. Structural model updating methods include matrix updating method and parameter updating method.³,⁴ The matrix updating method involves the direct updating of element matrix components, such as stiffness, mass, and materials damping.⁵,⁶ The parameter updating method involves modified the structural design parameters, such as geometry parameter, materials parameter, boundary conditions, and shapes of topology structure.⁷,⁸ In the structural model updating, such as natural frequencies, mode shapes, and dynamic response parameters, are initially more interesting than the

¹State Key Laboratory of Ocean Engineering, MOE Key Lab Marine Intelligent Equipment & System, Shanghai Jiao Tong University, Shanghai, PR China
²China Ship Development and Design Center, Wuhan, PR China

Corresponding author:
Chen Luyun, State Key Laboratory of Ocean Engineering, Shanghai Jiao Tong University, Shanghai 200240, PR China.
Email: cluyun@sjtu.edu.cn

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utilization of other parameters used in the structural model updating because such an approach allows a convenient model interpretation.\textsuperscript{9,10} The structural model updating problem is essentially a structural optimization problem and it involves minimizing the given objective function under certain constraints condition, and it is necessary to define the corresponding optimization strategies in the structural model updating.\textsuperscript{11–13}

The pre-stress (initial stress) often exists in the structures before undertaking the external loading, which includes welding residual stress, structural manufacturing defects, material thermal effects, static loading, and so on, these types of pre-stress are defined as complex pre-stress.\textsuperscript{14} The pre-stress includes linear and nonlinear parts, and the superposition principle is carried for linear parts.\textsuperscript{15,16} The existence of pre-stress gives a great influence on the local stiffness matrix and global stiffness matrix of the structures, and the natural frequencies of the structure will increase or decrease due to the pre-stress distribution.\textsuperscript{17,18} The pre-stress can resist or aid in structural deformation and alter the structural static and dynamic properties. Among them, welding residual stress is a typical nonuniform distributed pre-stress, which cannot be completely eliminated by conventional treatment process.\textsuperscript{19} Therefore, to consider the influence of welding residual stress on the dynamic response problem is worth studying.\textsuperscript{20}

In most current FE models, except hydrostatic pressure, the other forms of pre-stress are often omitted, which reduces the accuracy of the FE model in the structural dynamic analysis. On the other hand, if the influence of pre-stress is taken into account in the FE model, the complexity of structural analysis will be obviously increased, and the application scope of the numerical model may be limited. In order to improve the universality of the FE model with pre-stress distribution, it is necessary to transform the FE model with pre-stress distribution into a general FE model, while the mode shapes and natural frequency properties of the transformed FE model remain unchanged. This transformation process is design parameter reconstructing process, and the reconstructed FE model no longer contains pre-stress distribution while the dynamic properties are consistent, especially for the structural stiffness matrix and structural mass matrix. By reconstructing the FE model, the calculation scale and complexity of the FE model are reduced, and it effectively expands the application scope of the reconstructed FE model, this FE model transformation process is essentially a procedure of structural model updating. By consideration of the influence of natural frequency parameter and dynamic response parameter, the optimization formulation for dynamic model updating is established.

Based on the reasons mentioned above, the main objectives of the present research are to provide an efficient method to deal with the structural model updating problem of the FE model with pre-stress distribution, with which the application range of FE model can be effectively increased by structural model updating. The remainder of the present article is organized as follows: In Motion equation of cylindrical shell structure with complex pre-stress section, the motion equation of cylindrical shell structure with pre-stress distribution is presented. In Structural model updating formulation section, the structural model updating function is established, in which the influence of pre-stress on the natural frequency parameter is considered. In Numerical analysis section, the welding residual stress on the thin cylindrical shell structure, for example, the feasibility and effectiveness of the proposed method are illustrated by numerical analysis. Finally, some major conclusions are summarized in Conclusion section.

**Motion equation of cylindrical shell structure with complex pre-stress**

In this section, the motion equation of cylindrical shell structure with pre-stress distribution is established. At first, several hypotheses are defined as follows: fluid-structure coupling problem is omitted; the pre-stress and structural stresses caused by vibration satisfy the linear superposition principle; the vibration satisfies the small elastic deformation condition; the pre-stress is distributed uniformly in the thickness direction; the structural stress is perpendicular to the cross section in vibration; the pre-stress remains constant during the structural vibration; the material and mechanical properties are uniform along the overall of cylindrical shell structure.

**Pre-stress model**

The thin cylindrical shell structure, for example, the pre-stress distribution problem is discussed in this section. The structural stress $\sigma$ in the cylindrical shell structure can be expressed as

$$\sigma = \sigma_0 + \sigma_f$$  \hfill (1)
where \( \sigma = [\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}, \tau_{r\theta}, \tau_{rz}, \tau_{\theta z}]^T, \sigma_f = [\sigma_{frr}, \sigma_{f\theta\theta}, \sigma_{fzz}, \tau_{fr\theta}, \tau_{frr}, \tau_{f\theta z}]^T, \) and \( \sigma_0 = [\sigma_{0rr}, \sigma_{0\theta\theta}, \sigma_{0zz}, \tau_{0r\theta}, \tau_{0rr}, \tau_{0\theta z}]^T \) are the structural stress vector, dynamic stress vector caused by external dynamic loading and pre-stress vector according to the cylindrical coordinate system, respectively. \( \sigma_0 = 0 \) indicates without pre-stress distribution. According to the cylindrical shell Flügge theory and structure elasticity theory, if only two principal directions stress are considered, the structural stress vector is represented as \( \sigma = [\sigma_{rr}, \sigma_{\theta\theta}, \tau_{r0}]^T \), and the pre-stress vector can be stated as \( \sigma_0 = \tau_{0rr} = \tau_{0\theta\theta} = \tau_{0rz} = \tau_{0\theta z} = 0 \). The relationship between the strain and stress for the homogeneous and isotropic materials can be expressed as

\[
\epsilon_{xx} = \frac{E}{1-\mu^2} (\epsilon_{xx} + \mu \epsilon_{\theta\theta}), \quad \sigma_{00} = \frac{E}{1-\mu^2} (\epsilon_{00} + \mu \epsilon_{xx}), \quad \tau_{c0} = \frac{E}{2(1+\mu)} \gamma_{c0},
\]

respectively. \( \epsilon_{xx} \) and \( \epsilon_{\theta\theta} \) are the structure–strain displacement in axial and circumferential directions, \( \gamma_{c0} \) is the structure shear strain, respectively. \( E \) is the material Young’s modulus, and \( \mu \) is the Poisson ratio. In addition, if only the radial direction pre-stress \( \sigma_{0,rr} \) and circumferential pre-stress direction \( \sigma_{0,\theta\theta} \) are considered, then the pre-stress vector can be stated as \( \sigma_0 = [\sigma_{0,rr}, \sigma_{0,\theta\theta}]^T \).

The structural deformation problem which subject to external loading can be defined as a procedure with large structural deformation and small strain, the stress–strain relationship between the stress and strain vectors can be obtained as follows

\[
\sigma = D\epsilon = \sigma_0 + D\epsilon
\]  

where \( \sigma_0 \) is the pre-stress vector, viz. \( \sigma_0 = [\sigma_{0,rr}, \sigma_{0,\theta\theta}]^T \), and \( D \) represents the shell stress–strain matrix. For the thin cylindrical shell structure with isotropic material, the matrix \( D \) can be expressed as

\[
D = \frac{E}{(1+\mu)(1-2\mu)} \begin{bmatrix}
1 - \mu & \mu & 0 & 0 & 0 \\
\mu & 1 - \mu & 0 & 0 & 0 \\
\mu & \mu & 1 - \mu & 0 & 0 \\
0 & 0 & 0 & 0.5 - \mu & 0 \\
0 & 0 & 0 & 0 & 0.5 - \mu
\end{bmatrix}
\]  

Motion formulation of thin cylindrical shell structure

In an element body of the cylindrical shell structure, the structural geometric equation can be expressed as

\[
\epsilon = B\delta
\]  

where \( B \) is the strain matrix of structural element, \( \delta \) is the displacement vector.

Defining the vector sum of the internal force caused by pre-stress and external force \( F_{\text{out}} \) are \( F \). In the structural domain \( \Omega_S \), according to the principle of virtual work, the work done by internal force on virtual displacement \( \delta^* \) is equal to the work done by structural stress \( \sigma \) on virtual strain \( \epsilon^* \), then the virtual work can be expressed as

\[
\delta^*^T(F - F_{\text{out}}) = \int_{\Omega_S} \epsilon^*^T \sigma d\Omega_S
\]

Substituting equation (4) into equation (5), according to the arbitrariness of the virtual displacement \( \delta^* \), the general equilibrium equation of the geometrically nonlinear problems can be obtained as follows

\[
F - F_{\text{out}} = \int_{\Omega_S} B^T \sigma d\Omega_S
\]
If pre-stress is taken into account in equation (6), substituting equation (2) into equation (6), then equation (6) can be written as

$$\mathbf{F} - \mathbf{F}_{\text{out}} = \int_{\Omega} \mathbf{B}^T \mathbf{D} \mathbf{e} d\Omega_s + \int_{\Omega} \mathbf{B}^T \mathbf{\sigma}_0 d\Omega_s \tag{7}$$

Assuming that the internal force and external force acting on the structural system are conservative force, and then equation (7) can yield the following equation as

$$d\mathbf{F} = \int_{\Omega} d(\mathbf{B}^T \mathbf{D} \mathbf{e}) d\Omega_s + \int_{\Omega} d(\mathbf{B}^T \mathbf{\sigma}_0) d\Omega_s \tag{8}$$

Substituting equation (4) into equation (8), the first item of the right side of equation (8) can be expressed as

$$\int_{\Omega} d(\mathbf{B}^T \mathbf{D} \mathbf{e}) d\Omega_s = \int_{\Omega} d(\mathbf{B}^T \mathbf{D} \mathbf{B}) d\Omega_s d\delta = \mathbf{K}^e_{\delta} d\delta \tag{9}$$

where \( \mathbf{K}^e_{\delta} \) is stiffness matrix without pre-stress distribution.

Similarly, substituting equation (2) into equation (8), the second item of the right side of equation (8) can be expressed as

$$\int_{\Omega} d(\mathbf{B}^T \mathbf{\sigma}_0) d\Omega_s = \int_{\Omega} d(\mathbf{GSG}) d\Omega_s = \mathbf{K}^{\sigma_0}_{\delta} d\delta \tag{10}$$

where \( \mathbf{K}^{\sigma_0}_{\delta} \) is the additional stiffness matrix caused by the pre-stress distribution, \( \mathbf{G} \) is the strain transformation matrix, and \( \mathbf{S} \) is the pre-stress transformation matrix. The transformation matrix \( \mathbf{S} \) can be written as

$$\mathbf{S} = \begin{bmatrix} \sigma \mathbf{I}_2 & \tau_{0,0} \mathbf{I}_2 & \tau_{0,0} \mathbf{I}_2 \\ \tau_{0,0} \mathbf{I}_2 & \sigma_{0,0} \mathbf{I}_2 & \tau_{0,0} \mathbf{I}_2 \\ \tau_{0,0} \mathbf{I}_2 & \tau_{0,0} \mathbf{I}_2 & \sigma_{0,0} \mathbf{I}_2 \end{bmatrix}$$

where \( \mathbf{I}_2 \) is a unit matrix of order 2. Substituting the pre-stress \( \mathbf{\sigma}_0 = \begin{bmatrix} \sigma_{0,0} \mathbf{I}_2 & 0 & 0 \\ 0 & \sigma_{0,0} \mathbf{I}_2 & 0 \\ 0 & 0 & \sigma_{0,0} \mathbf{I}_2 \end{bmatrix} \) into the transformation matrix, then the transformation matrix \( \mathbf{S} \) can be written as

$$\mathbf{S} = \begin{bmatrix} \sigma_{0,0} \mathbf{I}_2 & 0 & 0 \\ 0 & \sigma_{0,0} \mathbf{I}_2 & 0 \\ 0 & 0 & \sigma_{0,0} \mathbf{I}_2 \end{bmatrix}$$

Substituting equations (9) and (10) into equation (8), then the differential function can be expressed as

$$d\mathbf{F} = (\mathbf{K}^e + \mathbf{K}^{\sigma_0}) d\delta \tag{11}$$

As shown in equation (11), the stiffness matrix of structural element body in considering of the pre-stress is \( \mathbf{K}^e = \mathbf{K}^e + \mathbf{K}^{\sigma_0} \). Since the vibration structure is meshed into many FEs, then the mass matrix and the stiffness matrix of the vibration structure can be written as: \( \mathbf{K} = \sum_{e=1}^{Ne} \mathbf{K}_e + \mathbf{K}^{\sigma_0} \), where \( \mathbf{K}_e = \sum_{e=1}^{Ne} \mathbf{K}^e \) and \( \mathbf{K}^{\sigma_0} = \sum_{e=1}^{Ne} \mathbf{K}^{\sigma_0}_e \). \( \mathbf{Ne} \) is the total number of FEs. \( \mathbf{K}^e \) is the elemental stiffness matrix of cylindrical shell without consideration of pre-stress distribution; \( \mathbf{K}^{\sigma_0} \) is the additional part of the elemental stiffness matrix of the cylindrical shell with pre-stress distribution, and it is defined as the pre-stress stiffness matrix. Equation (11) has shown that the distribution of pre-stress does not affect the mass matrix, but directly changes the stiffness matrix of the structure.

In the structural dynamic response analysis, if the structure damping is considered, the differential equation of the continuous structure under external harmonic loading can be expressed as

$$[\mathbf{M}]\{\mathbf{U}\} + [\mathbf{C}]\{\mathbf{U}\} + [\mathbf{K}]\{\mathbf{U}\} = \mathbf{F}(x, t), \quad x \in \Omega^S, \quad t > 0 \tag{12}$$

where \( \{\mathbf{U}\} \) indicates the nodal displacement vector matrix, \( [\mathbf{M}] \) is the structural mass matrix, \( [\mathbf{K}] \) is the structural stiffness matrix include the pre-stress stiffness matrix, and \( [\mathbf{C}] \) is the viscous damping matrix. External harmonic loading can be written as \( \mathbf{F}(x, t) = f(\omega) e^{i\omega t} \), where \( f(\omega) \) is the magnitude of the harmonic loading and \( \omega \) is the
circular frequency. Substituting the displacement equation and the external loading function into equation (12), and the expression of the nodal displacement matrix for the structure can be written as $\mathbf{u}(\omega) = [\mathbf{A}(\omega)]^{-1}f(\omega)$, where $[\mathbf{A}(\omega)] = -\omega^2[\mathbf{M}] + i\omega[\mathbf{C}] + [\mathbf{K}]$.

**The influence of pre-stress on structural mode**

In order to analyze the influence of pre-stress on natural frequencies of the cylindrical shell structure, the structural mode function can be rewritten as

$$
\omega^2 \mathbf{I} \varphi = \mathbf{M}^{-1} \mathbf{K} \varphi
$$

where $\varphi$ is the mode vector.

By introducing the structural pre-stress model, defining the pre-stress stiffness matrix $\mathbf{K}^0$ as a perturbation of the structural stiffness matrix for the vibration structure, there exists a natural frequency square perturbation $\Delta \omega^2$ for the natural frequency of the free vibration structure, and the structural mode function can be expressed as follows

$$(\omega^2 + \Delta \omega^2) \mathbf{I} \varphi = \mathbf{M}^{-1}(\mathbf{K}^0 + \mathbf{K}^0) \varphi$$

By substituting equation (13) into equation (14), then the frequency square perturbation $\Delta \omega^2$ is obtained as follows

$$
\Delta \omega^2 \mathbf{I} \varphi = \mathbf{M}^{-1} \mathbf{K}^0 \varphi
$$

Equation (15) has shown the influence of the pre-stress on the natural frequency for the free vibration structure. Two sides of equation (14) dividing the item $\mathbf{M}^{-1}(\mathbf{K}^0 + \mathbf{K}^0) \varphi$, then the sensitivity function of the natural frequency square by the pre-stress distribution can be obtained as follows

$$
\frac{\Delta \omega^2}{\omega^2} = \frac{||\mathbf{M}^{-1} \mathbf{K}^0 \varphi||}{||\mathbf{M}^{-1} \mathbf{K}^0 \varphi||}
$$

As shown in equation (16), the deviation of the natural frequency square of the same mode caused by pre-stress is related to the structure stiffness, the perturbation in the stiffness matrix caused by pre-stress and the mass matrix. It shows that the natural frequency square deviation value is magnified to the structural stiffness deviation value.

**Structural model updating formulation**

The FE models are updated by selecting the geometric and/or material properties of the FE model as design variables. The objective functions are modified through iterations to minimize the residual values of the dynamic response properties (e.g., natural frequency, vibration mode, and dynamic response) between the numerical results with/without pre-stress distribution under certain constraints.

**Model updating formulation based on natural frequency**

Structural natural frequency is the inherent dynamic parameter of an elastic structure; and natural frequency parameter is sensitive to alterations in structural stiffness and mass matrices. Natural frequency can be measured with high precision conveniently. Therefore, natural frequency should be used in the structural model updating. The structural model updating problem is essentially an inverse problem of structural optimization. The residual values of natural frequency between the numerical results with/without pre-stress distribution are defined as objective function in the dynamic model updating. In the low–middle frequency band, the residual value function for model updating can be written as follows

$$
R_i(x) = \sum_{i=1}^{N} c_i \frac{f_{\text{with},i}(x) - f_{\text{without},i}(x)}{f_{\text{with},i}(x)}, \quad (i = 1, 2, \ldots, N)
$$
where \( N \) is the number of modes of the concerned structure; \( f_{\text{with},i} \) and \( f_{\text{without},i} \) denote the \( i \)th natural frequency of the numerical analysis result with/without pre-stress distribution, respectively; \( c_{fi} \) is the weight coefficient to be imposed on \( i \) orders of natural frequency; and \( x \) is design variable. In the practical engineering, the natural frequency varies in orders, corresponds to different weighting coefficient.

Model updating formulation consideration of complex pre-stress distribution

The structural model updating with pre-stress distribution is to eliminate the pre-stress parameters in the numerical model, in which the influence of pre-stress on structural vibration properties is considered. In the structural model updating, the residual value function for the natural frequency between the FE model with welding residual stress distribution and FE model without welding residual stress distribution is defined as objective function, those two FE models have the same boundary conditions. If the FE models are updated, the influence of pre-stress on structural vibration properties can be reflected by the modification of other design parameters. These objective functions are modified through iterations to minimize the residual values of the dynamic response characteristics, and the FE model without welding residual stress distribution is reconstructed. The structural model updating formulation is a structural optimization formulation, and it can be written as follows

\[
\text{Find: } \mathbf{X} = (x_1, x_2, \ldots, x_i)^T, \quad (i = 1, 2, \ldots, N) \\
\text{Min: } R(x) = \sum_{i=1}^{N} c_{fi} \frac{f_{\text{with},i}(x) - f_{\text{without},i}(x)}{f_{\text{with},i}(x)} \\
\text{S.t.: } h_p(x_i) \leq 0, \quad (i = 1, 2, \ldots, N) \\
x_i^l \leq x_i \leq x_i^u, \quad (i = 1, 2, \ldots, N)
\]

where \( \mathbf{X} = (x_1, x_2, \ldots, x_i)^T \) is the \( T \) dimensional design variable vectors; \( R(x) \) is the residual value function for the natural frequency parameters; \( h_p(x_i) \) is the constraint condition; \( x_i^l \) and \( x_i^u \) are the upper and lower bounds of the design variables, respectively. \( c_{fi} \) is the weight coefficient to be imposed on different orders of natural frequency. According to equation (11), the pre-stress only affects the stiffness matrix of the element structure, it is more reasonable to define the elastic modulus of the structural material in the pre-stress distribution area as the design variable, and the elastic modulus in the pre-stress distribution area be defined as “equivalent elastic modulus.”

For a continuous structure, the results of model updating are directly affected by weight coefficient parameters. Therefore, weight coefficient parameters are key factor to the establishment of model updating or optimization equations. How to define the weight coefficients values are very important for the comprehensive evaluation of the results of dynamic model updating. The weight coefficients factors for the different orders depend on the inherent properties of the FE model, the goal of the designer, and so on. The normalized model of weight coefficients is proposed, and the weight coefficient equation can be obtained as follows

\[
\sum_{i=1}^{N} c_{fi} = 1
\]

where the weight coefficient satisfies \( c_{fi} \geq 0 \). The equations show a remarkable balance between the residual values of the natural frequency parameter.

Numerical analysis

Model description

A stiffened cylindrical shell structure, for example, the welding residual stress is defined as complex pre-stress, and the structural dynamic model updating with welding residual stress distribution is carried out. The FE model with/without welding residual stress distribution has same design sizes and boundary conditions. In the present work, all FEM predictions are calculated by using MSC.Patran/Nastran 2012.2, this commercial software is a widely used numerical tool for structural dynamic response analyses. The FE model is shown in Figure 1.
The design parameters of cylindrical shell structure are as follows: radius $R = 1$ m, length $L = 6$ m, shell plate thickness $t = 0.01$ m. There are 10 T-shaped circumferential ribs with spacing $l = 0.6$ m. The type of T-shaped ribs is $\frac{8}{10}$ to $\frac{12}{10}$.

The cylindrical shell structure and rib are made of structural steel with the following mechanical performance parameters: density, $\rho = 7800$ kg/m$^3$; modulus of elasticity, $E = 210$ GPa; Poisson ratio, $\mu = 0.3$.

**Model of welding residual stress distribution**

The ribs structure and cylindrical shell structure are joined by welding technology. The welding residual stress exists in the welding area. The welding residual stress in the cylindrical shell structure has self-balanced characteristic, it includes compressive stress part and tensile stress part in different location. The maximum amplitude value is close to the yield limit of the material. The welding residual stresses on the cylindrical shells structure include circumferential direction welding residual stresses $\sigma_{0,0}$ which along the weld direction and axial direction welding residual stresses $\sigma_{0,xx}$ which vertical the weld direction.

There are 10 welding seam on the cylindrical shells structure, as shown in the red area in Figure 2. The width of the welding residual stress area is about 60 mm. The elastic modulus of the shell structural material in the welding stress distribution area is defined as design variable.

There are two methods to gain the distribution of welding residual stress, namely numerical calculation and experimental measurement.\textsuperscript{21,22} In this study, the distribution characteristic of welding residual on the cylindrical shells structure is gained by numerical analysis with MSC. Marc code. The trigonometric function is used to fit the circumferential and radial welding residual stresses.

The variation of the welding residual stress in the thickness direction is neglected for simplicity. The maximum welding residual stress amplitude value in circumferential direction is 150 MPa, and the maximum welding residual stress amplitude value in axial direction is 250 MPa. The positive and negative values are the tensile and compressive stresses, respectively, as shown in Figure 3.
Natural frequency analysis

The welding residual stress is applied in the FE model to obtain the corresponding natural frequencies and modes of the cylindrical shells structure through the numerical analysis. The 1–10 orders of the natural frequencies and mode shapes of cylindrical shells structure with/without considering of welding residual stress are compared, as shown in Table 1.

Table 1 has shown that the welding residual stress has a considerable effect on natural frequency, especially in the first order. Meanwhile, the relative influence of the welding residual stress on the natural frequency is decreased with the increase in modal order. Therefore, at different modes, the natural frequencies of the structure will increase or decrease, which reflects the difference of the frequencies of different orders. This result can be attributed to the decrease in the overall structural strength of the circular plate caused by the existing pre-stress, particularly near the seam welding.

Structural model updating analysis with welding residual stress distribution

From Table 1, it can be seen that the difference of natural frequencies of cylindrical shells is relatively large, especially in the low-frequency band, due to the existence of welding residual stress. In order to obtain a FE model with welding residual stress, structural dynamic parameters need to be updated.

In the optimization equation, in the present research, the materials parameters (modulus of elasticity $E$) are defined as design variables. This process aims to minimize the residual values of differences between the FEM
The analysis result and the experimental data, and the residual values function is defined as objective function. The optimization equations for the dynamic model updating with FEM can be written as follows

\[ \text{Find: } X = (E_1, E_2, \ldots, E_{10})^T \]

\[ \text{Min: } R(x) = \sum_{i=1}^{10} c_{f,i} \frac{f_{\text{with},i}(x) - f_{\text{without},i}(x)}{f_{\text{with},i}(x)} \]

\[ E_l \leq E_i \leq E_u, \quad (i = 1, 2, \ldots, 10) \]

\[ \sum_{i=1}^{10} c_{f,i} = 1 \]

(20)

### Table 2. Weight coefficient parameter.

| Mode order | Weight coefficient |
|------------|--------------------|
| 1          | 0.25               |
| 2          | 0.15               |
| 3          | 0.12               |
| 4          | 0.1                |
| 5          | 0.1                |
| 6          | 0.08               |
| 7          | 0.06               |
| 8          | 0.05               |
| 9          | 0.05               |
| 10         | 0.04               |

### Table 3. The design variable.

| Design variables | Initial elastic modulus of structure (MPa) | Updated elastic modulus of structure (MPa) |
|------------------|--------------------------------------------|--------------------------------------------|
| $E_1$            | 2,10,000                                   | 3,75,000                                   |
| $E_2$            | 2,10,000                                   | 3,08,000                                   |
| $E_3$            | 2,10,000                                   | 3,42,000                                   |
| $E_4$            | 2,10,000                                   | 3,68,000                                   |
| $E_5$            | 2,10,000                                   | 3,55,000                                   |
| $E_6$            | 2,10,000                                   | 3,31,000                                   |
| $E_7$            | 2,10,000                                   | 3,28,000                                   |
| $E_8$            | 2,10,000                                   | 3,15,000                                   |
| $E_9$            | 2,10,000                                   | 3,37,000                                   |
| $E_{10}$         | 2,10,000                                   | 3,66,000                                   |

### Table 4. Comparison of the natural frequency after updated.

| Mode order | Natural frequency (with welding residual stress)/Hz | Natural frequency of updated FE model/Hz | Difference (%) |
|------------|---------------------------------------------------|----------------------------------------|----------------|
| 1          | 64.494                                            | 64.860                                 | 0.568          |
| 2          | 63.540                                            | 63.868                                 | 0.516          |
| 3          | 60.730                                            | 61.001                                 | 0.446          |
| 4          | 63.981                                            | 63.705                                 | -0.432         |
| 5          | 105.224                                           | 105.664                                | 0.418          |
| 6          | 101.461                                           | 101.105                                | -0.351         |
| 7          | 161.749                                           | 162.517                                | 0.475          |
| 8          | 160.379                                           | 159.954                                | -0.265         |
| 9          | 161.692                                           | 161.157                                | -0.331         |
| 10         | 155.116                                           | 154.675                                | -0.284         |
Figure 4. Mode shapes of the cylinder shell structure.
where $X = (E_1, E_2, \ldots, E_{10})^T$ is the 10 dimensional design variable vectors; $R(x)$ is the residual value function for the natural frequency parameters; parameters $E_{ui}^u$ and $E_{i}^l$ are the upper and lower bounds of the design variables, respectively. $c_{fj}$ is the weight coefficient to be imposed on different orders of natural frequency.

In the structural model updating, in order to search for the global optimum, genetic algorithm (GA) is introduced. The GA simultaneously considers multiple candidate solutions to the problem of minimizing the objective function and iterating it by moving this population of solutions toward a global optimum. According to equation (20), structural dynamic optimization is performed by using the iSIGHT optimization platform in which the GA code is embedded.

In the structural model updating, the objective functions involved are natural frequencies, and the first 10 orders are used for calculation. Considering that the first-order frequency reflects the inherent properties of the structure most directly, the first-order frequency weighting coefficient is the largest. In practical engineering, the weight coefficients value of the natural frequency is defined according to the inherent characteristics of the complex structure and the goal of the designer, and the effect of model updating varies if the weight coefficients value are changed. The value of the weighting coefficient is shown in Table 2.

**Structural model updating results**

The structural model updating for stiffened cylindrical shells are carried out. The parameters are modified and optimized, and the design variables are shown in Table 3.

Table 3 shows that the elastic modulus of cylindrical shell structure materials increases in the welding residual stress distribution region, which enhances the overall stiffness of the cylindrical shell structure.

An optimal structure is obtained after the dynamic optimization analysis, and the natural frequencies and corresponding modes of cylindrical shells structure are obtained by structural model updating, as shown in Table 4.

As shown in Table 4, the natural frequency of the FE model of cylindrical shell structure is basically in agreement with the results of welding residual stress by structural model updating. The influence of welding residual stress distribution is considered in the updated FE model. Compared with the original FE model, the structural stiffness of the current FEM is improved.

The mode shapes of the structure are compared, as shown in Figure 4. In Figure 4, “A” is the mode shapes of the structure without welding residual stress distribution; “B” is the mode shapes of the structure with welding residual stress distribution; and “C” is the mode shapes of the structure for updated FE model.

As shown in Figure 4, the mode shapes of the cylindrical shell structure are basically in agreement with the structural response parameters of the FE model with welding residual stress through structural model updating. Compared with the updating FE model, it shows that the reconstructed FE model has been greatly improved which achieves the goal of structural model updating and provides a more accurate FE model for dynamic numerical analysis for the future.

**Conclusion**

In this study, by using the structural model updating theory, the influence of welding residual stress on continuous structure is transformed into the influence of design parameters. A novel method for dynamic model updating is proposed in which the influence of welding residual stress distribution is considered, and the numerical results of the updated models are in agreement between with/without welding residual stress distribution. The numerical results of the updated models have validity of the proposed method. Through numerical calculation, the following conclusions can be drawn: (1) the complexity of the FE model can be effectively reduced and the calculation efficiency can be improved by implementing model updating; (2) in the structural model updating, the selection of weight coefficient is of great significance for structural model updating; (3) the structural model updating can provide a more accurate numerical model for dynamic response analysis for the future; (4) the optimization techniques have been extensively applied in the structural model updating.

**Declaration of conflicting interests**

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ORCID iD

Chen Luyun https://orcid.org/0000-0002-7749-9052

References

1. Altunel F, Çelik M and Çalışkan M. A correlation improvement technique for model updating of structures. *Int J Strab Dyn* 2016; 16: 1–19.
2. Zhai X, Fei CW, Wang JJ, et al. Parametric modeling and updating for bolted joints of aeroengine casings. *J Mech Eng Sci* 2016; 230: 2940–2951.
3. Mottershead JE, Link M and Friswell MI. The sensitivity method in finite element model updating: a tutorial. *Mech Syst Signal Process* 2011; 25: 2275–2296.
4. Zhou LR, Wang L, Chen L, et al. Structural finite element model updating by using response surfaces and radial basis functions. *Adv Struct Eng* 2016; 19: 1446–1462.
5. Lepoittevin G and Kress G. Finite element model updating of vibrating structures under free-free boundary conditions for modal damping prediction. *Mech Syst Signal Process* 2011; 25: 2203–2218.
6. Esfandiari A, Rahai A, Sanayei M, et al. Model updating of a concrete beam with extensive distributed damage using experimental frequency response function. *J Bridge Eng* 2016; 21: 1597–1614.
7. Xu WR and Chen GL. Inverse problems for (R, S)-symmetric matrices in structural dynamic model updating. *Comput Math Appl* 2016; 71: 1074–1088.
8. Qiao DS, Li Q, Khan I, et al. A novel finite element model updating method based on substructure and response surface model. *Eng Struct* 2015; 103: 147–156.
9. Mishra AK and Chakraborty S. Inverse detection of constituent level elastic parameters of FRP composite panels with elastic boundaries using finite element model updating. *Ocean Eng* 2016; 111: 358–368.
10. Yuan YX, Zhao WH and Liu H. Analytical dynamic model modification using acceleration and displacement feedback. *Appl Math Model* 2016; 40: 9584–9593.
11. Nehete DV, Modak SV and Gupta K. Coupled vibro-acoustic model updating using frequency response functions. *Mech Syst Signal Process* 2016; 70/71: 308–319.
12. Wang JT, Wang CJ and Zhao JP. Frequency response function-based model updating using Kriging model. *Mech Syst Signal Process* 2017; 87: 218–228.
13. Jensen HA, Millas E, Kusanovic D, et al. Model-reduction techniques for Bayesian finite element model updating using dynamic response data. *Comput Methods Appl Mech Eng* 2014; 279: 301–324.
14. Wu HY and Zhou SJ. Free vibrations of sensor diaphragm with residual stress coupled with liquids. *J Appl Phys* 2014; 115: 1–5.
15. Khan I and Zhang S. Effects of welding-induced residual stress on ultimate strength of plates and stiffened panels. *Ships Offshore Struct* 2011; 6: 297–309.
16. Paik JK and Sohn JM. Effects of welding residual stresses on high tensile steel plate ultimate strength: nonlinear finite element method investigations. *ASME Int Conf Ocean* 2012; 134: 46–51.
17. Chen LY and Liu Y. Acoustic characteristics analysis of cylindrical shell with prestress in local areas. *Int J Acoust Vibrat* 2016; 21: 301–307.
18. Liu Y and Chen LY. The effect of weld residual stress on the free vibrational characteristics of cylindrical shell through the analytical method. *J Vibroeng* 2016; 18: 334–349.
19. Gannon L, Liu Y, Pegg N, et al. Effect of welding-induced residual stress and distortion on ship hull girder ultimate strength. *Marine Struct* 2012; 28: 25–49.
20. Yang N, Chen LY, Yi H, et al. The effect of welding residual stress on the free vibration of underwater cylindrical shell. *J Vibroeng* 2016; 18: 2016–2030.
21. Guo YJ, Chen LY, Wang HD, et al. Experimental study of welding residual stress of high-strength shipbuilding steel. *Brodgrodajna/Shipbuilding* 2019; 70: 17–32.
22. Salerno G, Bennett C, Sun W, et al. An experimental and numerical investigation on the process efficiency of the focused-tungsten inert gas welding of Inconel 718 thick plates. *J Eng Manufact* 2019; 233: 823–833.