A Proposal on Some Microscopic Aspects of the AdS/CFT Duality

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Abstract
We suggest a model of the large N limit $\mathcal{N} = 4 \ D = 4 \ SU(N)$ SYM as a gas of 3-branes in a 10 dimensional space. Field theory analysis suggests that this 10 dimensional space does not carry the usual gravity dynamics but rather a contraction of it. Using a non-local transformation some aspects of the dynamics of this system are mapped to the dynamics of standard gravitons on $AdS_5 \times S^5$. In particular some of the correspondence between operator in the CFT and states on $AdS$ is more transparent.
1. Introduction

A recurring problem in black hole physics is the following. Suppose we have a particle falling towards a cluster of D-branes which form a black hole, then this process has 2 dual descriptions. The first is in terms of the particle as an object in the field theory of the branes, and the other is in terms of a particle moving in the well defined near horizon geometry of the black hole. The puzzle is to directly relate these two descriptions.

The understanding of this and related issues for the case of D3-branes has been the subject of intensive research, [1], and was crystallized in [2] where it was suggested that the large N limit of $\mathcal{N} = 4$ U(N) SYM in 3+1 dimensions is equivalent to type IIB gravity on $AdS_5 \times S^5$. A quantitative procedure of extracting CFT dimensions from the analysis of this sugra vacuum is given in [3] and [4].

One way of trying to understand the microscopic aspects of the $AdS/CFT$ duality is along the lines of [5]. We will take another approach in which when $N \to \infty$ we will try and pass to a collective coordinates description of the gauge theory system (almost, we will be more precise about this below) as a gas of 3-branes fluctuating in a 10 dimensional target space. This 10 dimensional space is not the same as $AdS_5 \times S^5$, rather it is related to it by a non-local transformation. In this respect the construction is reminiscent of 2d gravity constructions [6][7].

Suppose we start with the theory on a sphere and $N$ is small. We can try and understand the structure of the vacuum in the following way. We would like to separate the variables in the path integral into slow and fast variables and integrate over the fast ones. When the number of branes is small one can suggest the following separation. The slow variables are $N$ copies of a $U(1)$ theories (along what would like to be the flat directions\(^1\)) and the fast variables are the $W$ bosons. The (Euclidean) action contains [3]

$$\int d^4x (g^{\mu\nu} Tr(\partial_\mu X^\alpha \partial_\nu X^\alpha) + R(g) Tr(X^2))$$ (1.1)

where the indices $\mu$ and $\nu$ refer to coordinates on the sphere, $X$ are the 6 adjoint scalar fields, the index $\alpha$ runs from 1 to 6, and $R(g)$ is the curvature of the metric. Since the flat directions have been lifted the branes are confined to a finite region, $B$, in the space of eigenvalues of $X$. Since the number of branes is small then most of the measure in the path

\(^1\) It is important to emphasize that we are not working along real flat directions. These are lifted by the $RX^2$ term
integral is in configurations in which the branes are separated (but still within $B$) and we can therefore use a gas of brane approximation. When we integrate out the $W$’s, the fast variables, we generate a long range force between the separated branes. The singularity (when branes meet) in the this long range force is not a conceptual problem in our case since for most of the time the branes are separated and do not feel this singularity.

In the following we will try and extend this picture to a large number of branes. In this picture the dominant configurations in the field theoretic path integral will be approximated by a density of 3-branes in a 10 dimensional space made out of 4 $x$ coordinates and 6 eigenvalues of $X^\alpha$, $\alpha = 1..6$. We will call this space the $\lambda - x$ space. Treating the singularity in the effective action when branes intersect will then be crucial, and the burden of the proof is now shifted to controlling the corrections to the effective Lagrangian and to the dynamics of the system. We do not claim to have complete control over these, but we can obtain some level of qualitative understanding of some aspects of the AdS/CFT duality.

Similar ideas have been proven useful in the discussion of Matrix black holes [8][9]. In particular, references [9] discuss the description of Schwarzschild black holes using an effective description in terms of a gas of 0-branes[2]. One important distinction, however, should be made. Whereas most of these papers discuss a form of mean field approximation of the gas, we will try and analyze a large enough class of configurations in the path integral, since we are in the vacuum of the theory and not in a high temperature semiclassical regime. Nevertheless, for the lack of a better name, we shall still refer to the picture as that of a gas of branes.

The $\lambda - x$ space, in which the 3-branes fluctuate, can then be related to $AdS_5 \times S^5$. This is done in a way that reproduces some of the rules of associating operators in the field theory with states in the bulk [3][4]. One should emphasize, however, that two space are not identical. Rather, the transformation between them is non-local (as in [6][7]). This makes precise the idea that the branes “are everywhere in $AdS$”.

The framework suggested above seems to have the following implication. Within the gas of branes picture one expects to find a well behaved static supergravity description of supersymmetric field theories only if the gas of branes has a well defined stationary state. In order for the gas to have a ground state we need to lift the flat directions, or at least not allow, by any other means, for the cloud of branes to disperse. In our

\[2\] or that a gas of 0-branes is the dominant component in the black hole.
case this is done by the $R\lambda^2$ term but it can also be done using high temperature. Both such configurations have well behaved supergravity descriptions \[3\] and this is to be contrasted with situations in which the gas is allowed to dissolve such as the system of 0-branes where the supergravity solution is singular.

The organization of this paper is the following. Section 2 contains some scaling arguments in $AdS$ that hint towards the interpretation as a gas of branes. Section 3 describes some aspects of the non-local transformation from $R^4 \times R^6$ to $AdS_5 \times S^5$. Section 4 sets up part of the machinery in the context of the free theory, before we include the effects of the $W$ bosons. Section 5 describes corrections to the free theory, and the introduction of what will become the supergravity fields. Section 6 describes the transformation of these fields to the $AdS$ and the matching between some operators and excitations on $AdS$. Section 7 contains some conclusion and open problems.

One important point of notation that we will use extensively is the following. When we will think of the 6 scalars as $N \times N$ matrices we will denote them by $X$. When we will think about $6N$ scalar fields which are the $N$ eigenvalues in the gas of branes picture, we will denote these fields by $\lambda(x)$. When we will consider these $6N$ scalar fields as embedding, for every point $x$, into an additional 6D space then we will also use $\lambda$ to denote coordinates on this space. In this case we will denote use the notation $\lambda$ without any $x$ dependence.

As this paper was prepared for submission, related work appeared in \[11\].

2. The UV-IR relation

Let us briefly review the relation \[12\] between the IR regulator of $AdS$ and the UV cutoff of the field theory on the boundary. In the coordinates in which $AdS_5$ is

$$\frac{R^2}{(1-z^2)^2} \sum_{i=1}^{5} dz_i^2$$

one imposes a cutoff $\sum z_i^2 = 1 - \delta$. This is an IR cutoff on the $AdS$ and a UV cutoff on the field theory which lives at $z^2 = 1 - \delta$.

Another coordinate system, which will be more convenient for our purposes, is the one in which the metric takes the form (neglecting numerical coefficients)\[2\]

$$ds^2 = \alpha' \left( \frac{1}{\sqrt{g^2 N}} U^2 dy^2 + \sqrt{g^2 N} \frac{dU^2}{U^2} \right)$$

(2.1)
where $g$ denotes $g_{YM}$. The IR-UV relation in [12] can be transferred to the description in terms of these coordinates. In this case we fix $U_0$ as our sugra IR regulator, and this defines a UV regulator, $L_{uv}$, in the field theory. The relation that one obtains is

$$U_0 L_{uv} = \sqrt{g^2 N}.$$  

(2.2)

This relation is given by the following analysis. If we define $U = \sqrt{g^2 N} \hat{U}$ then the metric is such that all the $N$ dependence is in a factor in front of the metric, and the rest is $\hat{U}^2 dy^2 + \frac{d\hat{U}^2}{U^2}$. Since the computation in [12] is that of geodesics in the metric then the UV-IR relation is $\hat{U}_0 L_{uv} = 1$, which is (2.2).

Loosely, we can think of the coordinate $U$ as some characteristic size or scale [2] associated with the scalar fields on the brane, which usually parameterize its position. One therefore would like to know what is the significance, in the regulated field theory, of $U_o$ as a value of the scalar fields. The normalization that we will use is such that the action (for a single brane) is

$$\mathcal{L} \sim \frac{1}{g^2} (\partial X)^2$$

where $X$ denotes the scalar fields.

The interpretation of this scale is the following. Let us regulate the theory in the IR by some scale which we will take as 1 and in the UV by a length cutoff which we will denote by $\delta \rightarrow 0$. This is the same as we had on the sphere (we will also neglect the conformal coupling $RX^2$ since the zero-mode of $X$ will not play an important role in what follows). Expanding in momentum modes we obtain that the action for a $U(1)$'s worth of $X$ is

$$\mathcal{L} \sim \frac{1}{g^2} \sum_{n,l} n^2 X_{n,l}^2$$  

(2.3)

where the factor of $n^2$ comes from the kinetic term. $n$ labels the total momentum of the mode and $l$ parameterizes the states in that momentum shell ($l = 1..d_n$).

If we now wish to evaluate the dispersion of the values of $X$, we can compute the quantity $<X(x)^2>$. For a single field this yields

$$<X(x)^2> = g^2 \sum_{n=1}^{d_n} \frac{1}{n^2}$$

$$= g^2 \sum_{n=1}^{d_n} \frac{n^3}{n^2} \propto g^2 \frac{1}{\delta^2}$$  

(2.4)
which is the same (up to constants, and for \( N = 1 \)) as the relation (2.2) between \( U_\alpha \) and \( \delta \).

For a large number of branes we have an \( N \times N \) matrix. If we take one of the \( X \) matrices then we can diagonalize it. When we do so the distribution of eigenvalues scales like \( \sqrt{N} \). In our normalization, where for a large number of branes \( \mathcal{L} \sim \int Tr(\partial X^2) \), the distribution of eigenvalues is governed by the scale \( \frac{1}{N} Tr X^2 \sim \frac{N}{\delta^2} \) which sets a scale \( \frac{1}{\delta} \sqrt{N} \).

This scaling relation supports the framework that was suggested before. One may be able to think of the region in AdS inside the cutoff \( \delta \) as corresponding to the region of eigenvalues in which the field theory scalars fluctuate. The sugra excitations should then be explained as objects living on this range in the space of eigenvalues. The rest of the paper is a speculative step in this direction.

3. The transformation from \( \lambda - x \) space to \( AdS_5 \times S^5 \)

3.1. The \( \lambda - x \) space and the transformation from \( R^4 \)

Even though it is important to have the term \( R\lambda^2 \) in order to have a well defined supergravity dual, we can choose to work for most purposes with the metric of \( R^4 \), which is what we will do from now on.

In the gas of brane picture we have \( N \) embeddings of D3-branes into a 10 dimensional space. This space is given by 4 coordinates \( x^\mu \) along the brane and by 6 coordinates \( \lambda^\alpha \) which are associated with 6 scalar fields. This seems to be the usual IIB D3-brane in flat space picture but this is not quite so. The important difference is that the \( \lambda \) coordinates parameterize the flat directions of the field theory and are distinct from the \( x \) coordinates, and therefore the dynamics in the different direction can be significantly

\[ \text{In fairness we should note that there might be another scaling. We may argue that each of the } N \text{ branes is positioned in } \lambda \text{ according to the expectation value of the scalar field. In this case the dispersion of the branes in the eigenvalue space is much smaller, and should only be calculated by the zero mode. We believe that this is not the correct scaling because there is no energetic reason to impose first the restriction to the zero mode whereas the expression (2.3) above is more accurate from that respect. We will also see below that we do not use the expectation values, rather the analysis will be in each value of } x \text{ independently. A more serious problem is that our estimate for the actual dispersion is not really correct on configurations that dominate the path integral due to correlations, the commutator term in the action or because of quantum correction. However, it is not clear how to estimate this effect.} \]
different. In particular we will not obtain standard gravitational dynamics on the $\lambda - x$ space. Rather we would like to argue that some aspects of the dynamics on the $\lambda - x$ space will be equivalent to dynamics on $AdS_5 \times S^5$ after a non-local transformation. We will discuss the dynamics on the $\lambda - x$ space at greater length later, and for now we will and construct a map from $\lambda - x$ space to $AdS_5 \times S^5$.

In this subsection we will discuss the transformation law of scalar functions on $R^4 \times R$ (the distinct $R$ component is a single $\lambda$ coordinate) to functions on $AdS_5$. We will later restore the full $S^5$. $R \times R^4$ will be parameterized by a single $\lambda$ and four $x$, and $AdS_5$ will be parameterized by $y$ and $U$ as in equation (2.1).

The transformation that we will look for is the most general linear transformation of the form

$$\tilde{f}(u, y) = \int d\lambda d^4 x f(\lambda, x) K(\lambda, u, x, y)$$

such that it intertwines the action of the conformal algebra on both sides.

Since we know the transformation laws of $x$ and $\lambda(x)$, we know the geometric action of the conformal group on the space parameterized by $x$ and $\lambda$. We also know the action of the conformal group on $U$ and $y$. Taking care of rotational and translational symmetries in $R^4$ is an easy matter, and the non-trivial restrictions come from $D$ and $K^\rho$ (the special conformal transformations). The action of these generators on functions $f(\lambda, x)$ and $\tilde{f}(U, y)$ is given by

$$D f = (\lambda \partial \lambda - x^\mu \partial x^\mu) f \quad \text{(3.1)}$$

$$\tilde{D} \tilde{f} = (U \partial U - y^\mu \partial y^\mu) \tilde{f}$$

$$K^\rho f = (-2x^\rho \lambda \partial \lambda + (2x^\rho x^\mu - \delta^\rho_\mu x^2) \partial x^\mu) f \quad \text{(3.2)}$$

$$\tilde{K}^\rho \tilde{f} = (-2y^\rho U \partial U + (2y^\rho y^\mu - \delta^\rho_\mu (y^2 + \frac{1}{U^2}) \partial y^\mu) \tilde{f}$$

Once we have identified the action of these generators, we can require that the map intertwines them. Using (3.1) and (3.2) This fixes the form of the transform to be

$$\tilde{f}(u, y) = \int d\lambda d^4 x \lambda^3 f(\lambda, x) K\left(\lambda u((x - y)^2 + \frac{1}{u^2})\right) \quad \text{(3.3)}$$

We are able to determine the transform up to a function of a single variable. This is the best we expect to do if we use only the invariance under the conformal group because of

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4 One immediately sees that the $\lambda - w$ plane can not be simply identified with the $\rho, v$ space since they transform differently under conformal transformation.
the following reason. Functions on AdS can be decomposed according to eigenvalues under the AdS Laplacian, which we will denote by \( \Delta_{AdS} \), and this value does not change under the action of conformal transformations. On the \( \lambda - x \) there will be another differential operator, which we will also refer to as a “Laplacian” (although there is no metric on that space), and denote by \( \Delta_\lambda \) which plays a similar role. The transform maps these two operators to each other, i.e., it maps a function with a given eigenvalue under \( \Delta_{AdS} \) to a function with the same eigenvalue under \( \Delta_\lambda \), but we can allow the transform to map it to such a function times an arbitrary constant. An independent multiplication is allowed for every value of the eigenvalue under the \( \Delta \) operators and hence the transform is determined up to a function of a single variable.

3.2. The “Laplacian” on the \( \lambda - x \) plane

The “Laplacian”, \( \Delta_\lambda \), on the \( \lambda - x \) side will play an important role in what follows. Returning to the \( 6 \)’s, it turns out that this operator is

\[
\Delta_\lambda = \left( \sum_\beta \lambda^{\beta^2} \right) \sum_\alpha \frac{\partial^2}{\partial \lambda^{\alpha^2}}
\]

(3.4)

The computation is the following. The Laplacian on the \( U, y \) coordinates is

\[
\frac{1}{U^3} \partial_u U^5 \partial_u + \frac{1}{U^2} \partial_y^2.
\]

We can transform it to the \( \lambda - x \) plane using the kernel above, and obtain that

\[
\int d\lambda d^4 x \lambda^3 \phi(\lambda, x) \left( \frac{1}{U^3} \partial_U U^5 \partial_U + \frac{1}{U^2} \partial_y^2 \right) K \left( \lambda u \left( t^2 + \frac{R^4}{u^2} \right) \right) =
\]

\[
= \int d\lambda d^4 x \lambda^3 \phi(\lambda, x) \left( \lambda^2 \partial_\lambda^2 + 5 \lambda \partial_\lambda \right) K =
\]

\[
= \int d\lambda d^4 x \lambda^5 \left( \partial_\lambda^2 + \frac{5}{\lambda} \partial_\lambda \right) \phi(\lambda, x) K.
\]

We are now interested in restoring the \( 6 \) instead of having only one. To do so we need to add an \( S^5 \) angular part to the kernel, which we will not discuss in great detail. More interesting is the effect on \( \Delta_\lambda \). Choosing an eigenvalue \( \omega^2 \) for the \( S^5 \) Laplacian, the operator \( \frac{1}{U^3} \partial_U U^5 \partial_U + \frac{1}{U^2} \partial_y^2 + \omega^2 \) is now mapped to

\[
= \int \frac{\lambda^6 d^4 x}{\lambda^2} \lambda^2 \left( \partial_\lambda^2 + \frac{5}{\lambda} \partial_\lambda + \frac{\omega^2}{\lambda^2} \right) \phi(\lambda, x) K,
\]

where we have separated the scale invariant measure from the operator. One now recognizes the operator as \( \lambda^{\beta^2} \partial_\alpha^2 \). A more group theoretic approach that yields the same result is to realize that the \( \Delta \) operators are the Casimir operators for the conformal group. This allows their immediate computation in both spaces.
3.3. Locality on the boundary

Even though we do not have an explicit form for the kernel of the transformation one can see that it is local on the boundary for physical states in the sense that if we take a function that satisfies the Laplacians on both sides and is supported at a given \( x \) then it will be supported at \( y = x \) as \( u \to \infty \). This can be derived by using only group theory properties of the transformation.

On the \( \lambda - x \) side we will choose a function of the form \( P(\lambda)\delta(x) \) where \( P(\lambda) \) is a symmetric traceless polynomial, \( \Delta_\lambda P(\lambda) = 0 \). This function is also annihilated by \( K_\mu \).

Since the transformation commutes with \( SO(5,1) \), the same will be true for image of the function under the transformation. These gives us two equations (taking into account the \( SO(6) \) quantum numbers, which determines the mass \( m \), and \( SO(4) \) quantum numbers)

\[
\left( \Delta_{AdS} + m^2 \right) \tilde{f}(U, y) = 0
\]

\[
K_{AdS}^\mu \tilde{f} = -2y^\mu U \partial_U + \left( 2y^\mu y^\nu - \delta^{\mu\nu} \left( y^2 + \frac{1}{U^2} \right) \right) \partial_\nu \tilde{f}(U, y) = 0.
\]

The solutions (for every \( m \)) to these equation are the propagators from the boundary to the bulk described in [3] and [4]. These functions are such that as we go to the boundary the support of the function is at \( y \to x \).

3.4. A brief look at what’s ahead

The axion-dilaton pair are scalar fields on \( \mathcal{M} = AdS_5 \times S^5 \), and their linearized equation of motion is \( \Delta_{\mathcal{M}} \Phi = 0 \). If we would like to describe similar fields on the \( \lambda - x \) space their equation of motion will be

\[
\frac{\partial^2}{\partial \lambda^2} \Phi(\lambda^\alpha, x^\mu).
\]

Note that the field is 10 dimensional but its equations of motion are 6 dimensional - only derivatives in the \( \lambda \) directions appears. Since the \( \lambda \) space is the space of flat directions we expect such a result. As we will see later the field \( \Phi \) participates in the mediation of long range forces in the \( \lambda - x \) space. Since these long range forces are supposed to generate an overall field theoretic effect, they should have a correct expansion in terms of series of local (in \( x \)) terms. This implies that the long range forces should be such that they connect branes at different point in \( \lambda \) but at the same value in \( x \). This is exactly what the 6D equation of motion achieves.
This transformation also improves the “locality” of the description. We see that using the transform we can make a function that is spread in \( y \) (except in the boundary) into a function that is localized in \( x \). This will be the key to obtaining, after we have laboriously crunched through all the gas description and several additional approximations, the correspondence between spread states on \( AdS \) and local operators in the field theory.

4. The free “Gas” of branes

The picture that we will describe is that of \( N \) branes fluctuating in the \( \lambda - x \) space. For each brane we have the fields \( \lambda_i^\alpha(x) \), \( A_i^\mu(x) \) and fermions, where \( i = 1..N \), \( \alpha = 1..6 \) and \( x \) are four dimensional coordinates. When writing down the action we will focus on the bosonic part.

The lowest order action “along the flat direction” is

\[
\sum_{i=1}^{N} \int d^4x \left( \partial^\mu \lambda_i^\alpha(x) \partial_\mu \lambda_i^\alpha(x) + \frac{1}{g_0^2} F_i^2 + R \lambda(x)^2 \right)
\]  

(4.1)

(The term in the brackets depends on whether we are on the sphere or not. For the most part we will work on \( R^4 \)).

Had these been a large number of particles moving in a confined region, we would have known what to do. The correct prescription would be to go to collective coordinates of this gas, i.e. to variables such as local density and local average velocity. A generalization of these quantities appears in this problem as well. For example \( \partial_\mu \lambda^\alpha \) is a generalization of velocity. Furthermore, since we have additional degrees of freedom in the field theory we will have additional collective degrees of freedom in the \( \lambda - x \) space, such as \( A^\mu(\lambda, x) \).

The main assumption that we need to make is that when the branes are close together they fluctuate roughly the same such that it is useful to describe them using collective excitations.

The quantities that we will find useful to work with are \( L_i^\alpha \), which is roughly \( \partial_\mu X^\alpha \), \( A_\mu \) and \( \psi(\lambda, x) \). The function \( L_i^\alpha(\lambda, x) \) measures the following: Suppose we are in a point \( \lambda_0 \) and \( x_0 \). There is a 3-brane that passes through this point. Let us denote the trajectory of this brane by \( U^\alpha(x; \lambda_0, x_0) \) and we will then define \( L_i^\alpha(x_0, \lambda_0) = \partial_{x^\nu} U^\alpha(x_0; \lambda_0, x_0) \), so the L-matrix measures the gradients of the branes at a given point. From it we can obtain \( \lambda^\alpha(x) \) of the brane everywhere but this is a non-local transformation and we will avoid using it. If the only \( \mu \) coordinate was the time coordinate then this quantity would have
the velocity profile of the gas. Since we require that $L^\alpha_\mu$ has integral surfaces, then not all the components of $L$ are independent. This is take care of below by the Lagrange multiplier $M^\mu_{\alpha\nu}$.

The definitions for $A_\mu(\lambda, x)$ and $\psi(\lambda, x)$ are simpler. If brane $k$ passes through point $(x_0, \lambda_0)$ then we will define $A_\mu(x_0, \lambda_0) = A^k_\mu(x_0)$, and a similar definition holds for the fermions.

Another object that will be useful is a derivative along the brane. This is given by

$$D_\mu = \frac{\partial}{\partial x^\mu} + L^\alpha_\mu \frac{\partial}{\partial \lambda^\mu}. \quad (4.2)$$

This expression is obtained when lifting $\frac{\partial}{\partial x^\mu}$ to the $\lambda - x$ space.

In order to write the action we need one more ingredient which is the density of branes in each point in the $\lambda - x$ space. We will denote this function by $\rho(\lambda, x)$. It is a density only on the $\lambda$ coordinate and is normalized to 1 at each $x$ separately

$$\int d^6 \lambda \rho(\lambda, x) = 1.$$ 

This function is not a new independent variable since it depends on $L^\alpha_\mu$ (although not completely determined by it). If we start from a point $(\lambda_0, x_0)$ and follow the integral surface determined by $L$ then we can deduce how the density of branes changes along such surfaces. Since this information is encoded both in $\rho$ and in $L$, there is some redundancy between these quantities. The redundancy will be of the form that the variation of $\rho$ as we change $x$ and go along integral lines of $L$ should match a change in a volume element which is determined by $L$. This constraint is taken care of by the Lagrange multiplier $M^\mu$.

The action that one obtains is (up to numerical coefficients)

$$N \int d^6 \lambda d^4 x \rho(\lambda, x) \left( L^\alpha_\mu L^\alpha_\mu + \frac{1}{g_0^2} F_{\mu\nu} + i \bar{\psi} \sum \partial \psi \right) +$$

$$\int d^6 \lambda d^4 x \left( M^\mu_{\alpha\nu} \partial_\mu L^\alpha_\nu + M^\mu (\partial_\mu (\rho) + \partial_\alpha (\rho L^\alpha_\mu)) \right). \quad (4.3)$$

4.1. Supersymmetry

Since this system is derived from the system of N 3-branes it possesses the full superconformal symmetry. We will, however, make this slightly more explicit. The path integral that we now do is over configurations of N branes fluctuating in the $\lambda - x$ space and we
would like to show how this configuration transforms under some of the superconformal
transformation.

In this section we will briefly discuss the some aspects 16 supersymmetries of the
model. Some aspects of the conformal symmetry will be discussed in the next subsection.
Overall they will generate the full superconformal algebra.

The supersymmetries of the model are easy to guess. The only change one needs to
do is to replace $\partial_\mu$ by $D_\mu$ from (4.2). An N=1 worth of supersymmetries can be described
using a superspace formalism. As usual one adds the usual fermionic coordinates $\theta$ and $\bar{\theta}$.
The only difference is in the definition of the operators $Q, \bar{Q}, D, \bar{D}$. These are defined such
that $D_\mu$ replaces $\partial_\mu$.

There are two features that one needs to preserve in order to use the conventional
superspace construction of N=1 theories. The first is that

$$ [D_\mu, D_\nu] = 0, $$

and this is true after we solve the constraint on $L_\mu^\alpha$ (implemented using a Lagrange mul-
tiplier) which reconstructs the embedding of the brane into $\lambda$ space. The 2nd property is
the ability to integrate by part. This is also true in our case, i.e.,

$$ \int d^4x d^6\lambda \rho(\lambda, x)f_1(\lambda, x)D_\mu f_2(\lambda, x) = -\int d^4x d^6\lambda \rho(\lambda, x)(D_\mu f_1(\lambda, x))f_2(\lambda, x) $$

because of the dependence between $\rho$ and $L_\mu^\alpha$ (which is again implemented with a Lagrange
multiplier).

Using this superspace formulation, the appropriate definition of a chiral superfield is
by the condition

$$ D_\alpha \Phi(\lambda, x) = 0. $$

The chiral fields that we will use are the chiral fields whos lowest component is the coordi-
nate $\lambda^\alpha$ (made into 3 complex pairs) which we will by $\Lambda^\alpha$. We will also have a $U(1)$ vector
multiplet $W$ (whos chirality properties are also defined by $D$).

The last ingredient that we need is the transformation law of $\rho$. As we will see
momentarily, these supersymmetry transformation laws also include a flow term because
$\lambda$ changes under the supersymmetry. We require that $\rho$ transforms as a density under this
flow and this defines its susy transformation. The Lagrangian that one obtains is

$$ \sim \int d^6\lambda d^4x \rho(\lambda, x) \left( \int d^4\theta \Lambda^\alpha \Lambda^\alpha \right) + \int d^2\theta W^2 + c.c. \right) $
The expansion of the chiral field $\Lambda^\alpha$ is the following

$$\Lambda^\alpha(\lambda, x) = \lambda^\alpha + \theta^a \psi_a^\alpha(\lambda, x) + \theta \sigma^{\mu} \bar{\theta} L^\alpha_{\mu} + \ldots \tag{4.5}$$

The supersymmetry variation of this multiplet (up to numerical coefficients) is

$$(\zeta^a Q_a + \bar{\zeta}_a \bar{Q}^a)\Lambda^\alpha \sim \Lambda^\alpha + \zeta^\alpha \psi_a^\alpha(\lambda, x) + \theta^a (\psi_a + \sigma^{\mu} a \bar{a} L^\alpha_{\mu}) + \ldots \tag{4.6}$$

The lowest component in the expansion signifies the position of the field, therefore (4.6) encodes the fact that we changed the embedding of the D3 brane into $\lambda$ space when we do a susy transformation. As functions on the $\lambda - x$ space the physical degrees of freedom in the $\Lambda^\alpha$ multiplet transform as

$$\delta L^\alpha_{\mu} \sim -((\zeta^{\beta} \psi^{\beta}) \frac{\partial}{\partial \lambda^\beta} + (\bar{\zeta}^{\bar{\beta}} \bar{\psi}^{\bar{\beta}}) \frac{\partial}{\partial \bar{\lambda}^{\bar{\beta}}}) L^\alpha_{\mu} + \zeta D_{\mu} \psi^\alpha \tag{4.7}$$

$$\delta \psi^\alpha = -((\zeta^{\beta} \psi^{\beta}) \frac{\partial}{\partial \lambda^\beta} + (\bar{\zeta}^{\bar{\beta}} \bar{\psi}^{\bar{\beta}}) \frac{\partial}{\partial \bar{\lambda}^{\bar{\beta}}}) \psi^\alpha + + \sigma^{\mu} \bar{\zeta} L^\alpha_{\mu}$$

and a similar transformation, which we will not make explicit, for the vector multiplet.

4.2. Conformal Symmetry

We would like to show how the conformal symmetries act on fields in the $\lambda - x$ space. The action of the conformal group has two pieces to it. The first is a geometric action in the $\lambda - x$ space and the 2nd is the standard dimension dependent rescaling. The geometric action is the following:

$$P_{\mu} = \partial_{x^{\mu}} \tag{4.8}$$

$$M_{\mu \nu}^{(g)} = x^{\mu} \partial_{x^{\nu}} - x^{\nu} \partial_{x^{\mu}}$$

$$D^{(g)} = \lambda^\alpha \partial_{\lambda^\alpha} - x^{\mu} \partial_{x^{\mu}}$$

$$K^{(g)}_{\rho} = -2 x^{\rho} \lambda^\alpha \partial_{\lambda^\alpha} + (x^{\rho} x^{\nu} - \delta^{\rho \nu} x^2) \partial_{x^{\nu}}$$

And after the inclusion of the dimension dependent rescaling, the final action on a field $\Phi$ who’s dimension is $d$ is

$$P_{\mu} \Phi(\lambda, x) = \partial_{x^{\mu}} \Phi(\lambda, x) \tag{4.9}$$

$$M_{\mu \nu} \Phi(\lambda, x) = (x^{\mu} \partial_{x^{\nu}} - x^{\nu} \partial_{x^{\mu}} + \Sigma_{\mu \nu}) \Phi(\lambda, x)$$
\[ \mathcal{D}\Phi(\lambda, x) = (-d + \lambda^\alpha \partial_{\lambda^\alpha} - x^\mu \partial_{x^\mu}) \Phi(\lambda, x) \]

\[ K_\rho \Phi(\lambda, x) = (-2x^\rho (\lambda^\alpha \partial_{\lambda^\alpha} - d) + (x^\rho x^\nu - \delta^\rho_\nu x^2) \partial_{x^\nu} - x^\mu \sigma_{\mu \rho}) \Phi(\lambda, x) \]

These transformation rules may be derived as follows. Given a field \( \Phi_i(x) \) on \( x \) with dimension \( d \), its variation on the \( \lambda - x \) space can also be split into the following two parts. The first, which we will denote by \( \delta_0 \), is just its 4D variation pulled back to the \( \lambda - x \) space, and the second is a transport term that arises because we change \( \lambda(x) \) which is the position of the brane. The total variation is therefore

\[ \delta \Phi(\lambda, x) = \delta_0 \Phi - (\delta_0 \lambda^\beta) \frac{\partial}{\partial \lambda^\beta} \Phi \] (4.10)

where for scale transformation

\[ \delta_0 \Phi = (-x^\mu \mathcal{D}_\mu + d) \Phi \]

\[ \delta_0 \lambda^\beta = -x^\mu L^\beta_\mu + \lambda^\beta \]

and for special conformal transformations

\[ \delta_0 \Phi = (2x^\rho x^\mu \mathcal{D}_\mu - x^2 \mathcal{D}_\rho + dx^\rho) \Phi \]

\[ \delta_0 \lambda^\beta = 2x^\rho x^\mu L^\beta_\mu - x^2 L^\beta_\rho + x^\rho \lambda^\beta. \]

Inserting these into (4.10) yields the expression above. Note that in \( \delta_0 \) one needs to use \( \mathcal{D}_\mu \) and not \( \partial_\mu \). The former is the correct extension of a derivative along the brane when going to the \( \lambda - x \) picture.

In this notation \( A_\mu \) has dimension 1, and the worldvolume fermion has dimension \( \frac{3}{2} \). \( \rho \) is a density and so transforms in way that \( d^6 \lambda \rho(\lambda, x) \) has dimension 0 (so \( \rho \) has dimension -6). Since \( L^\alpha_\mu \) corresponds to \( \partial_\mu \lambda^\alpha(x) \) its transformation law is slightly different but is still an immediate consequence of the field theory transformation law.

Once we have demonstrated conformal invariance and supersymmetry we obtain the entire superconformal symmetry.
5. Factorizing the low-energy action

We have seen that we can write down the action for the $N$ free D3 branes in the \( \lambda - x \) space. However, in order to get a realistic description of the dynamics one needs and take into account the interactions of the branes along the flat direction due to the W-bosons. Our approach is to start with the system at low density, write the corrections in this regime and try and understand what happens when we increase the density. After we have identified the degrees of freedom at large density, we would like to write down an effective action that controls these degrees of freedom.

Also, we will focus only on corrections along the flat directions which are of the form $F^4$ or its susy partners. This is again very similar to what is done in Matrix black holes. Since in $\mathcal{N} = 4$ D=4 SYM this interaction is protected by supersymmetry, it will be the most reliable to use in extrapolating between the different regimes. These will actually suffice in matching some operators on the boundary and excitations in the bulk.

5.1. The $F^4$ term along the flat directions

In this section we will briefly discuss some field theoretic aspects of the $F^4$ term. First let us discuss the interaction between a pair of branes. In this case the relevant degrees of freedom are a single $U(1)$ multiplet on each of the two branes. The sum decouples, but the difference is corrected at one loop, which is the infamous $F^4$ term (and it superpartners). We will mainly interested in the $F^4$. The term that one obtains is\[^{[13]}\]

\[
\frac{(F_{+,1} - F_{+,2})^2(F_{-,1} - F_{-,2})^2}{(<\lambda_1> - <\lambda_2>)^4}
\]

where $F_{+(-)}$ is the (anti)self-dual part of the field strength, and the index 1, 2 is the index of the brane.

We are interested in the analogue of this term when the branes are embedded in a more arbitrary way in the $\lambda - x$ space, i.e., when the vev of each of the branes is varying (but st non-zero separation). The full term under these circumstances is not known, but we can guess some parts of it when the branes do not bend too much. In that case we certainly expect that there will be a good expansion in local terms. More precisely, since the original action was invariant under Weyl rescaling (and up to C-number anomaly the same is true quantum mechanically), we require the same of this term in the effective action. Our conventions for the Weyl rescaling are

\[
g_{\mu\nu} \to e^{2w(x)}g_{\mu\nu}, \quad X^\alpha(x) \to e^{-w}X^\alpha(x), \quad F(x) \to F(x).
\]
Since we know how the fields transform under Weyl rescaling and what the term is when the vev’s are flat, we obtain that at least part of the term for a varying vev is of the form

$$\int d^4x \sqrt{g} \frac{(F_1(x) - F_2(x))^4}{\langle \lambda_1 \rangle - \langle \lambda_2 \rangle} , \quad (5.1)$$
i.e., the difference of the expectation value is evaluated at each point.

Another point that we need to understand is what happens to this term when there are many branes. In this case there is no proven non-ren. theorem for the $F^4$ term, although one expects that the term would still be restricted. As for the one loop contribution, in this case it is the sum over the interaction between all pairs. Suppose we have a loop with some external massless fields attached to it (fields from the flat directions). All of these carry charge 0 under the $U(1)$’s that are unbroken along the flat directions. The particles that we are integrating out can not change their charge in the loop. Since they are charged under a pair of the $U(1)$’s, say $k_1$ and $k_2$, the contribution of this diagram will be the same as if we had only branes $k_1$ and $k_2$ which means that it is included in the sum over all pairs.

In fact, the t’hooft double line notation lends itself to easy manipulation along the flat directions. Since the external, particles are associated with one of $N U(1)$ multiplets, the diagram becomes a surface with several holes, each associated with an excitation of a given brane (along the flat directions). This is quite clear from open-closed string duality in the description of D-branes. A diagramatic analysis along these lines is in fact expected to be much simpler than usual (as for example in [14][15]) because the vertex operator does not change the Chan-Paton index.

5.2. Resolution of the $F^4$ singularity

The singularity tells us that we have integrated out degrees of freedom. These are the $W$ bosons. However, for our purposes we are not very interested in the precise nature of these degrees of freedom. Since we are working in a gas of branes approximation, a generic pair of branes is separated from each other, the $F^4$ term is not singular and the mass of the $W$’s is not zero. If most of the effect of the correction term comes primarily from this regime, which we will assume is the case, then any way that we choose to mimic this term will give the same result\footnote{For some processes at least} - whether we choose to reintroduce the $W$’s, put it by hand (as
is done in many cases for Matrix black hole applications) or generate it by other means. The rest of the construction relies on this freedom.

It is important to emphasize that we have made a very strong assumption. As explained, this assumption is that the path integral is dominated by configuration which are approximately a gas of separated branes, and that the effect of the W’s that become light can be encoded in an effective Lagrangian of these branes. It is difficult to prove these assumption without a detailed dynamical analysis which we do not know how to do. Rather we will assume it and examine whether we can obtain any insight into the AdS/CFT correspondence under this assumption. As we have mentioned before, this assumption has yielded in the context of Matrix theory useful insight into Schwarzschild black holes which are just as good as a classical sugra vacuum.

Other than the W-bosons, an exchange of a a sugra multiplet between separated branes can also generate the $F^4$ term. By an $\mathcal{N} = 4$ Non-ren. theorem their contribution is the same. Our purpose is therefore to introduce fields that are similar to the gravity fields. Using these, one expects that we will be able to reproduce the essential ingredients of the interaction between separate branes in a similar fashion to the DBI action. These field, to which we will refer as pseudo-gravity, live on the $\lambda - x$ space and will become the 10D sugra fields only after we perform the transformations described in section 3.

More precisely what we will do is the following. For every configuration in the path integral, i.e., a configuration of of the D3-branes embedded in the $\lambda - x$ space, we will define a set of fields, the pseudo-gravity fields, that satisfy a set of differential equations with the fields on the branes ($L, F$ etc.) as sources. As example of such an equation will be that of the “dilaton-axion” fields

$$\partial^2_\lambda \phi(\lambda, x) = \rho(\lambda, x) F^2(\lambda, x)$$

where $\phi$ is a pseudo-sugra field that will become a combination of dilaton and axion after the transformation in section 3. These fields are such that when we insert their value into the action we obtain the $F^4$ correction to the action. The pseudo-gravity fields should be thought of as auxiliary fields that satisfy their equations of motion with the off-shell configuration of the 3-branes as a source. We would then like to show, for a large enough class of configurations in the path integral, how the matching between some operators and states in $AdS/CFT$ correspondence comes about.

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6 There might be an off-shell extension of the pseudo-gravity fields but we will not need one.
There might also be a description in which the pseudo-gravity fields arise from dressing up collective excitations of the gauge theory and in particular of the $W$’s. The procedure that we will discuss does so but only indirectly for most of the pseudo-sugra multiplet. Returning to equation (5.2), if we are using only $\phi$ that satisfy this equation of motion we could think of it as a definition of $\phi$ in terms of variables $F^2$ and hence it is an object in the field theory. A similar approach for the case of dynamics of D0 branes was taken in [16] [17].

We would like to emphasize again that the pseudo-sugra fields are NOT 10 dimensional sugra on the $\lambda - x$. There are several ways of seeing this. One of the ways is to notice that we want the interaction to be propagated only in $\lambda$, at constant $x$. This is different from 10D sugra fields that propagate in all 10 dimensions. A related reason is that in the vacuum of the theory we expect to retain the conformal symmetry, but there is no conformally invariant metric in the $\lambda - x$ space (in fact, we will never really use a metric structure on the entire $x - \lambda$ space).

As we have explained above, and will make more explicit below, these fields will become regular supergravity fields only after we use the non-local transformation to map from the $\lambda - x$ space to the $u - y$ space. Under such a transformation the non-sugra equations of motion for the pseudo-gravity multiplet will be transformed into supergravity equations of motion on $AdS_5 \times S^5$ (at least, for the cases that we have checked).

In that sense, in the analysis that we will present here there is really no fundamental reason to go to the AdS, since the discussion is consistent in the $\lambda - x$ space. It is therefore not clear which of the two spaces is more fundamental. This question may be resolved by examining higher order interactions, or if we want to formulate the theory as a string theory.

5.3. Contraction of Gravity

Since the fields that we will introduce will not satisfy exactly the IIB supergravity equations of motion, and the gas of D3 branes is not a gas of D3 branes in sugra, one needs to explain what equations they do satisfy and how to couple them to the $U(1)^N$ field theory.

One would further like to couple them in a way that will generate the $F^4$ term. Since the sugra couplings to D3 branes do reproduce this term by tree level sugra exchange, we will take the usual sugra and its D3 brane as a starting point and deform it until we obtain the desired action for the pseudo-sugra fields.
In this section we will discuss the behavior of the pseudo-sugra fields. In the next one we will discuss its coupling to the brane. The deformation that we will discuss is a sort of “contraction” of gravity, and is closely related to going to the near horizon geometry.

Our initial starting point is that of D3-branes fluctuating in $\lambda^\alpha - x^\mu$ space. In order to make this space into a 10D space where all the coordinates are on equal footing we need to choose a dimensionful parameter\footnote{This is not the same as $l_p$ on $AdS$. Since we will be dealing with the sugra equations of motion at the linearized level (free, apart from sources) we will not be able to determine what is $l_p$ on $AdS$.}, which we will denote by $l'_p$ and define new coordinates

$$x^\alpha = \lambda^\alpha l'_p^2$$

where now all the coordinates are of mass dimension $-1$.

To construct the pseudo-gravity multiplet on this space, we begin with the 10D sugra multiplet and its equations of motion. Our coordinates naturally split into a set of 4 coordinates ($x^\mu$) and another set of 6 ($x^\alpha$). The equation of motion for the scalars field in gravity is (assuming $g^{\mu\alpha} = 0$)

$$\frac{1}{\sqrt{g}}(\partial_\alpha g^{\alpha\beta} \sqrt{g}\partial_\beta + \partial_\mu g^{\mu\nu} \sqrt{g}\partial_\nu)\phi(x^\alpha, x^\mu) = 0$$

(5.3)

In order to obtain the equations of motion on the $x^\alpha - x^\mu$ space we will take

$$g^{\mu\nu} = \epsilon \to 0, \ g^{\alpha\beta} = O(1).$$

(5.4)

This is similar to taking the limit of the near horizon geometry since in this limit the invariant separation between two point in fixed $\lambda - x$ shrinks in the $x^\alpha$ direction relative to the $x^m u$ directions.

When we do this scaling we would like to keep the N=4 SYM finite, i.e., without any powers of $\epsilon$. In order to do so we can identify the above rescaling with a constant Weyl rescaling in the SYM. From this one deduces that the scaling of the SYM fields is the following:

$$F \sim O(1), \ L^\alpha_\mu \sim O(\epsilon^{\frac{1}{4}}), \ \Psi \sim O(\epsilon^{\frac{3}{4}}), \ \eta \sim \epsilon^{-\frac{1}{4}}$$

(5.5)

where $\eta$ is a parameter of one of the 16 susy (which are the only ones that we will check in this section). One more requirement that we will impose is that certain couplings of
D3-brane fields to the sugra fields remains finite as we take $\epsilon \to 0$. We will list those when we will use them.

This information is enough to determine the scaling of all the pseudo-sugra fields. This is done by requiring that there are no terms in the deformed IIB susy transformations that are singular in $\epsilon$. An example of such a computation is the following. Let us denote by $\Gamma = \Gamma^{0123}$, which defines projection operators $P_L = \frac{1}{2}(1 - i\Gamma)$, $P_R = \frac{1}{2}(1 + i\Gamma)$. The will take the unbroken susy to be $\eta = \eta_R$.

The susy transformation laws in type IIB include the transformations (for the complete transformation as well as other relevant conventions see [18][19]):

$$\delta B^i_{\mu\nu} = V^i_+ \bar{\eta}^* \Gamma_{\mu\nu} \epsilon^\rho_{\mu} \epsilon^\bar{\mu}_{\nu} (\lambda_R)^* + \ldots \quad (5.6)$$

$$\delta \lambda_R \sim \Gamma^{\bar{\mu}_1 \bar{\mu}_2 \bar{\mu}_3} \eta \epsilon^\mu_{\mu_1} \epsilon^\mu_{\mu_2} \epsilon^\mu_{\mu_3} G_{\mu_1 \mu_2 \mu_3} + \ldots \quad (5.7)$$

where $G_{\mu_1 \mu_2 \mu_3} \sim V_i \partial_{\mu_1} B^i_{\mu_2 \mu_3}$, and $i$ is an $SU(1,1)$ index ($B^i$ denotes the entire $SL(2,\mathbb{Z})$ multiplet of $B^{NSNS}$ and $B^{RR}$).

The $V$'s are determined by the axion-dilaton scalars. Because we would like to keep their leading coupling to the brane ($\int \phi F^2$) and the operator to which they couple does not scale with $\epsilon$, then the $\epsilon$-dimension of $V$ is zero. We have also determined the scaling of the vielbein so the only unknown variables in (5.6) and (5.7) are the scaling dimensions of $B_{\mu\nu}$ and of $\lambda_R$. We will denote these by $[B_{\mu\nu}]_\epsilon$ and $[\lambda_R]_\epsilon$.

The basic requirement is that there would be negative powers of $\epsilon$ in the susy transformation. Otherwise the procedure will not give a well defined end result. This requirement imposes the two following inequalities.

$$(5.6) \Rightarrow [B_{\mu\nu}]_\epsilon \leq -\frac{5}{4} + [\lambda_L]_\epsilon$$

$$(5.7) \Rightarrow [\lambda_L]_\epsilon \leq \frac{5}{4} + [B_{\mu\nu}]_\epsilon,$$

which gives an equality

$$[B_{\mu\nu}]_\epsilon = -\frac{5}{4} + [\lambda_L]_\epsilon. \quad (5.8)$$

In this way one can obtain equations between the dimensions of different fields in the sugra multiplet. Another relation that involves $B_{\mu\nu}$ which we will use comes from the requirement that the term $k^{\frac{1}{2}} \int F \wedge B$ has no $\epsilon$ dependence. This gives a relation

$$\frac{1}{2} [k]_\epsilon + [B_{\mu\nu}]_\epsilon = 0$$

$k$ is Planck’s constant to some power. In our notation $F, B$ and $k$ has mass dimension 2, 4 and -4 respectively.
The $\epsilon$-dimensions that one obtains using this procedure are:

$$[\Psi_\alpha]\epsilon = \frac{9}{4}, \quad [\Psi_\mu]\epsilon = \frac{7}{4}, \quad [\lambda]\epsilon = \frac{9}{4}, \quad [k]\epsilon = -2$$ (5.9)

$$[e_\mu^\alpha]\epsilon = [e_\mu^\alpha]\epsilon = -\frac{1}{2}, \quad [e_\alpha^\lambda]\epsilon = [e_\alpha^\lambda]\epsilon = 0$$ (5.10)

$$[B_{\mu\nu}]\epsilon = 1, \quad [B_{\mu\alpha}]\epsilon = 1\frac{1}{2}, \quad [B_{\alpha\beta}]\epsilon = 2$$ (5.11)

$$[A_{\alpha\beta\gamma\delta}]\epsilon = 2, \quad [A_{\alpha\beta\gamma\mu}]\epsilon = 1\frac{1}{2}, \quad [A_{\alpha\beta\mu\nu}]\epsilon = 1, \quad [A_{\alpha\mu\nu\rho}]\epsilon = \frac{1}{2}, \quad [A_{\mu\nu\rho\eta}]\epsilon = 0$$ (5.12)

We also need to specify how we choose the parameter $l'_p$. Since we have not analyzed the interaction of the pseudo-sugra fields it is not clear how to choose it precisely but we can get a bound on it value, which will be useful in what follows. One obvious choice is that $l'_p$ be smaller than our cut-off $L_{uv}$, but one can do better.

We are interested in mapping the system of branes parameterized by $\lambda_\epsilon(x)$ to the $x^{\alpha} - x^\mu$ space. The width in the $\lambda$ direction is $\sqrt{N} l_{uv}$. After the map the width in the $x^\alpha$ direction is $l'_p \frac{\sqrt{N}}{L_{uv}}$. This is the region in $x^\alpha$ space that is relevant for our discussion. We would like that the effect of the 10D equations of motion as we transverse from one side of the $x^\alpha$ plane to another is such that (in Minkowski space) will not be bigger than the cut-off $L_{uv}$. Otherwise our theory is not $\mathcal{N} = 4$ SYM even below the cut-off. The condition for that can be derived from the spread of a massless particle with a source at one side of this space. We would like that the spread of the propagator when it reaches the other side of the $\lambda$ space will be small enough. The result is that the following condition has to be satisfied

$$\frac{l'_p \sqrt{N}}{L_{uv}} < L_{uv}.$$ (5.13)

5.4. Coupling the D3-branes to the Pseudo-gravity

The purpose of the contraction was to generate the 10D pseudo-gravity which has a chance of exactly reproducing the 1-loop corrected effective action, and which will become usual sugra on on AdS$_5$ after an appropriate transformation (at least for some of the fields). There are two elements to this which are how the pseudo-gravity fields couple to the D3-brane, and what are their new equations of motion in the $\lambda - x$ space. In this section we will discuss the former. We will also restrict attention to a set of simple couplings, some of which will play a role later on, and only at the linearized level. Our starting point before the contraction are the couplings of the standard D3 brane to the
standard IIB supergravity. We would then like to trace what couplings remain when we take $\epsilon \to 0$.

Before we proceed to the computations, it is worth mentioning that although we are deriving the correspondence between operators and fields in the bulk from a variant of the D3-brane action, the procedure outlined here is different than in [20]. In our case we have a configuration of branes filling an entire 10 dimensional space (which is not $AdS$) rather than $N$ branes on top of each other at a point in $AdS$.

1. Coupling of the Axion-Dilaton pair

The (linear) couplings of the axion-dilaton pair to the D3-brane is given by

$$
\int d^4x (\phi F^2 + \bar{\phi} F^2)
$$

where in Minkowski space $\bar{\phi} = \phi^*$. When going to the gas picture the coupling becomes

$$
\int d^4x d^6\lambda\rho(\lambda, x)(\phi F^2 + \bar{\phi} F^2).
$$

This coupling does not change as $\epsilon \to 0$.

2. The 2-form fields

The terms in the couplings of the D3-brane to gravity that are linear in the $B$ fields (with polarization parallel to the brane) are

$$
\int d^4x k^{\frac{1}{4}} \left( F_{\mu\nu} B_{\mu_1\nu_1}^{NSNS} g^{\mu_1\nu_1} \sqrt{g} + k^{\frac{1}{4}} \epsilon^{\mu_1\ldots\mu_4} F_{\mu_1\mu_2} B_{\mu_3\mu_4}^{RR} + k^{\frac{3}{4}} (F^3 g^{\rho\sigma3})_{\mu_1\nu_1} B_{\mu_1\nu_1}^{NSNS} g^{\mu_1\nu_1} \sqrt{g} + \ldots \right)
$$

(Although we were not careful with the contraction of the indices, we were careful to include all the appearances of the metric since it has a non-trivial $\epsilon$ dimension. $F^3 g^{\rho\sigma3}$ stands for a specific quantity made out of 3 $F_{\mu\nu}$ and 3 $g$ with upper indices).

The total $\epsilon$-dimension of these terms is 0, which means that they survive the contraction. We will rewrite the term as

$$
\int d^4x k^{\frac{1}{4}} \left( F_{\mu\nu} B_{\mu_1\nu_1}^{+} g^{\mu_1\nu_1} \sqrt{g} + k^{\frac{3}{4}} (F^3 g^{\rho\sigma3})_{\mu_1\nu_1} (B^{-} + B^{+})_{\mu_1\nu_1} g^{\mu_1\nu_1} + \ldots \right)
$$

where $B^\pm = B_{NSNS} \pm *_4 B_{RR}$, $*_4$ denotes the 4D Hodge $\ast$. 

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Next we would like to keep the couplings of $B^+$ to $F$ and of $B^-$ to $F^3$. The quantities that we would like to keep fixed are the $\hat{B}$

$$B^+ = k^{-\frac{1}{2}}\hat{B}^+$$

$$B^- = k^{-\frac{3}{2}}\hat{B}^-$$

Since $k$ is a smaller than the cutoff we would like to take it to 0. This will leave us only the coupling of $\hat{B}^-$ to $F^3$.

Although not necessary in the linearized approximation, it is interesting to discuss singularities in $\epsilon$ that might arise when we perform the contraction. We do not have a complete analysis but we can rule out some simple occurrences of such singularities. The argument is as follows. If we are interested in the coupling of the brane to several pseudo-gravity fields then because the pseudo gravity fields carry spin and mass dimension we are restricted as to how they can correct existing terms in the action. For example we have a term of the form $\int d^4 x B F$ and we may as whether we can try and add another $B$ field and write a correction of the form $\int d^4 x B^2 F$. The question is then whether such a term will have a singularity in $\epsilon$. The answer is that such terms will not have any singularities in $\epsilon$. To show this we build out of each pseudo-gravity field a quantity that is dimensionless (by multiplying power of $k$) and carries no Einstein indices (but may carry Lorentz indices. This is done by contraction with the vielbein). It turns out the the $\epsilon$ dimension of these composite fields is 0. Hence there will be no $\epsilon$ singularity when correcting non-singular terms by higher powers of the pseudo-gravity fields.

Another issue are the couplings between between fields in the bulk and higher dimension operators on the brane. An example for such terms, which will require special treatment in the next section, would be the expansion

$$\sum_k (l'_p)^4(k-1) \int d^4 x \sqrt{g} (\partial_\mu x^\alpha)^k(g^{\mu\mu})^k(g_{\alpha\alpha})^k$$

where we have indicated only schematically the location and type of important indices. Although it is difficult to rule out systematically all such terms, we can rule some on a case by case analysis. For example, this specific term is such that it is actually an expansion in $\frac{l'_p}{L_{uv}}$. The reasons is that the kinetic term $\partial x^\alpha$ is actually derived from the field theory quantity $\partial \lambda$ and is therefore of order $1/L_{uv}^2$ at most, and the expansion is therefore in $l'_p^2 \partial \lambda \sim \frac{l'_p^2}{L_{uv}^2}$.
5.5. Contraction of the Equations of Motion

The main purpose of the contraction was to obtain fields whose wave equation is only 6D. In this section we will briefly discuss how that comes about. We will not discuss the most general equation of motion but rather assume that $g_{\alpha\mu} = 0$. This will be the case that we will need in the following.

1. The Axion-Dilaton pair.

The equation of motion of the Axion-Dilaton pair was in fact used before to motivate the construction. Let us briefly repeat the argument. The equation of motion, in the linearized approximation, was

$$\frac{1}{\sqrt{g}} \left( \frac{\partial}{\partial x^\alpha} g^{\alpha\beta} \sqrt{g} \frac{\partial}{\partial x^\beta} + \frac{\partial}{\partial x^\mu} g^{\mu\nu} \sqrt{g} \frac{\partial}{\partial x^\nu} \right) \Phi(\lambda, x) = 0. \quad (5.17)$$

Under the contraction the 2nd term is $O(\epsilon)$ compared to the first and therefore we obtain, at the linearized level for $\Phi$, that the equation of motion for is

$$\frac{1}{\sqrt{g}} \partial_\alpha g^{\alpha\beta} \sqrt{g} \partial_\beta \Phi(\lambda, x) = 0 \quad (5.18)$$

which is a 6D equation of motion.

2. The 2-form fields

The linearized equation of motion for the 2-form fields in IIB sugra is \[18\] \[19\]

$$D_P G_{MNP} = -\frac{2}{3} i k F_{MNPQR} G^{PQR} \quad (5.19)$$

where $M, N, P, Q, R$ run from 0 to 9\[14]. When we divide the coordinate we obtain that the LHS (neglecting the various $\sqrt{g}$ and $g^{\alpha\mu}$ which will not change the argument\[15]) are

$$g^{\mu\nu} D_\mu G_{[\nu\rho\gamma]} + g^{\alpha\beta} D_\alpha G_{[\beta\mu\nu]}$$

$$g^{\mu\nu} D_\mu G_{[\nu\rho\gamma]} + g^{\alpha\beta} D_\alpha G_{[\beta\rho\gamma]}$$

$$g^{\mu\nu} D_\mu G_{[\nu\gamma\delta]} + g^{\alpha\beta} D_\alpha G_{[\beta\gamma\delta]}$$

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\[9\] G was described above. It is a combination of the $dB$ that depends on the scalar fields

\[10\] $\sqrt{g}$ terms appear both in the numerator and the denominators, as in equation (5.17) and therefore do not contribute to the $\epsilon$-scaling. These terms, however, will be important later where we will treat them more carefully.
These, however, simplify under the contraction.

Suppose we focus on a certain $G_{MNP}$. The scaling of the B fields is such that the more $\alpha$ indices they have, the higher is their $\epsilon$ dimensions. This implies that the only term that remains under the contraction is the one in which the derivative (in $G \sim dB$) is such that it is in the $x^\alpha$ directions. Furthermore, the contraction with $g^{\mu\nu}$ adds another power of $\epsilon$ and makes the term disappear even faster as $\epsilon \to 0$. The final result is that the 2nd order term in the equations of motion are 6D and are qualitatively (up to factors of $\sqrt{g}$ which we will restore later)

$$g^{\alpha\beta} D_\alpha \partial_\beta B_{[\mu\nu]}$$

$$g^{\alpha\beta} D_\alpha \partial_\beta B_{[\gamma\rho]}$$

$$g^{\alpha\beta} D_\alpha \partial_\beta B_{[\gamma\delta]}$$ (5.20)

Similar arguments also show that on the RHS, one obtains only derivatives with respect to $x^\alpha$, so the overall result is that these equations are also 6D.

3. The RR self-dual 4-form

Since the equations of motion of the 4-form field are of slightly different nature, it is worthwhile to check them as well. The same mechanism that helped us in contracting the equations of motion for the 2-form field strength is again at work for the self-dual 4-form. The self duality equation are a set of equations for the components

$$F_{\alpha_1 \mu_1 \mu_2 \mu_3 \mu_4}, F_{\alpha_1 \alpha_2 \mu_1 \mu_2 \mu_3}, F_{\alpha_1 \alpha_2 \alpha_3 \mu_1 \mu_2}$$

which relates them to their dual. However, each component of F is dominated (as $\epsilon \to 0$) by the allowed component of $A_4$ with the least number of $\alpha$ indices. If we denote by $d_6$ the exterior derivative using on the 6 $\lambda$ coordinates then this implies that

$$F_{\alpha_1 \mu_1 \mu_2 \mu_3 \mu_4} \to d_6 A_{\mu_1 \mu_2 \mu_3 \mu_4}$$

and its dual satisfies

$$F_{\alpha_1 \alpha_2 \alpha_3 \alpha_5 \alpha_5} \to d_6 A_{\alpha_1 \alpha_2 \alpha_3 \alpha_4}.$$ (5.21)

If we denote the components of $A_4$ by $A_{p,q}$ where $p$ denotes the number of $\mu$ indices and $q$ the number of $\alpha$ indices then the contracted equations of motion are

$$d_6 A_{4,0} = * d_6 A_{0,4}$$

$$d_6 A_{3,1} = * d_6 A_{1,3}$$

$$d_6 A_{2,2} = * d_6 A_{2,2}$$

which are again 6 dimensional equations of motion.
6. Transforming back to the AdS

We have so far obtained a set of fields which, at least for some of them, after transforming to the AdS would become the supergravity multiplet on that space. We would now like to see how perturbing the field theory by a local operator corresponds to turning on a space dependent field in the sugra multiplet on AdS. The two classes of operators that we will discuss are those that couple to the axion-dilaton pair and part of those that couple to the NSNS and RR 2-form fields. We will discuss only rudiments of this map, and there are clearly many more details to check.

Ideally one would like to show that for any configuration of the branes in the path integral, turning on a perturbation in the field theory corresponds to turning on a field on the sugra. In this case the statement will be true when we coherently integrate over all the configuration, which is the classical sugra vacuum that we observe at the end of the day. It is not clear how to show this for an arbitrary configuration but we will show it, for some of the sugra fields, under some dynamical assumptions on the configurations that dominate the path integral.

6.1. The dynamical assumptions

Since we have not analyzed the entire non-linear couplings of the branes to the pseudo-gravity, we are restricted for the most part to the regime where we can treat the pseudo-sugra fields as small perturbation. This is possible for all the sugra fields except $A_{\mu_1\mu_2\mu_3\mu_4}$, $g_{\mu_1\mu_2}$ and $g_{\alpha_1\alpha_2}$, and we will assume that the rest of the fields are indeed small. For example, in order not to excite a large $g_{\alpha\mu}$, we are restricted to look at brane configurations which are almost flat. For $A_{\mu_1\mu_2\mu_3\mu_4}$, $g_{\mu\mu}$ and $g_{\alpha\alpha}$, we can not assume that they are small because the effect of the branes on them is large.

If we neglect the effect of the bending of the branes on $g_{\alpha\mu}$, and set it to 0 at leading order, then the branes are roughly parallel and we can solve for the back reaction on the metric and on the self-dual 4-form. Note that we are in better shape than if we had tried to use the same argument with the standard sugra multiplet. The reason is that the equations here are 6D, i.e. only in the $\lambda$ space, so $g_{\mu\alpha}$ is determined only by $L_\mu^\alpha$ at the same value of $x$ whereas is in the usual sugra 10D equations of motion it would have been determined by the behavior of the gas of branes at far values of $x$. 
Under this assumption one can write down the solution for the metric and 4-form, which is a continuum version of the 2-cluster solution described in [2]. One defines a function $f$ which satisfies
\[ \partial_{x^\alpha} f = N \rho(x^\alpha, x^\mu) \] (6.1)
(note that it is a 6D equation of motion) and the metric is then given by [21]
\[ g_{\mu\nu} = f^{-\frac{1}{2}} \delta_{\mu\nu}, \quad g_{\alpha\beta} = f^{\frac{1}{2}} \delta_{\alpha\beta}, \quad F_{0123\alpha} = -\frac{1}{4} \partial_{\alpha} f^{-1} \] (6.2)

We can try and justify the assumption regarding the fluctuation of the brane in the following way. Since the source for $g_{\mu\alpha}$ is $L_\alpha^\mu$ we expect that these metric elements will be proportional to $l_p^2 \partial_{\mu} \lambda^\alpha \sim \frac{l_p^2}{l_{uv}^\alpha} << 1$.

6.2. The axion-dilaton pair

We have seen that the coupling of the axion-dilaton pair to the D3-branes persists after the contraction. Before we do the contraction, the action for the axion-dilaton pair and the coupling of sugra to a density of D3-branes is of the form
\[ \int d^4x^\mu d^6x^\alpha \sqrt{g} g^{ij} \partial_i \phi \partial_j \bar{\phi} + \] (6.3)
\[ N \int d^4x^\mu d^6x^\alpha \rho(\phi F^2_+ + \bar{\phi} F^2_-). \]

When we do the contraction, insert the ansatz from the previous section, and go back to the $\lambda$ coordinates the equation of motion for $\Phi$ that we obtain is
\[ \sum_{\alpha=1}^6 \frac{\partial^2}{\partial \lambda^\alpha^2} \phi = -N \rho F^+_{\mu\nu} F^+_{\mu_1 \nu_1} \eta^{\mu \mu_1} \eta^{\nu \nu_1} \] (6.4)

Let us now perturb the field theory by a chiral operator of the form
\[ \int d^4x \alpha(x) Tr(F^2_+ P(\lambda)), \] (6.5)
where $P(\lambda)$ is a symmetric traceless polynomial on of $SO(6)$. In the “gas of brane” approximation we are adding to the action in the $\lambda - x$ space a term
\[ \int d^4x d^6\lambda \rho(\lambda, x) \alpha(x) F^2_+(\lambda, x) P(\lambda). \] (6.6)
The action now (in addition to the tree level action) is
\[ \int d^4 x d^6 \lambda \rho(\lambda, x) (\phi + \alpha(x) P(\lambda)) F^2_+(\lambda, x). \] (6.7)
The equation of motion (6.4) is not modified.

We can now see how \( \Phi(\lambda, x) \) is turned on in AdS. If we define \( \tilde{\phi}(\lambda, x) = \phi(\lambda, x) + \alpha(x) P(\lambda) \), then this field satisfies (6.4) with the same source terms. In fact, to this order \( \tilde{\phi} \) appears in the same way as that \( \phi \) appeared in the system before the perturbation. The reason for this is that the tracelessness condition on \( P(\lambda) \) is equivalent to the statement that
\[ \frac{\partial^2}{\partial \lambda^2} \alpha(x) P(\lambda) = 0. \]
This implies that whatever field we have on the AdS (which should give \( <\phi> = 0 \) in the vacuum) now changes by the image of \( \alpha(x) P(\lambda) \) under the transform. Since this function satisfies “Laplace equation” in the \( \lambda - x \) plane, it will clearly satisfy Laplace equation, which is the equation of motion for this field on AdS.

Using arguments as in section 3.4 one see that the solution is exactly as describes in \( \text{[3][4]} \). Shifting the value of a scalar field on the \( \lambda \) space by \( P(\lambda) \delta(x) \) exactly corresponds to turning on the correct boundary-bulk propagator in the AdS bulk.

6.3. The RR and NSNS B fields

One can repeat the analysis for the NSNS and RR 2 form fields. Combinations of these fields couple to \( F \) and to \( F^3 \) on the D3 brane. We have also analyzed part of their equations of motion on the \( \lambda - x \) plane. Let us focus on the couplings which are
\[ \int d^4 x d^6 \lambda \rho B^{\mu\nu} F_{\mu\nu}, \] (6.8)
appended by the equation of motion
\[ f^{-\frac{3}{2}} \frac{\partial^2}{\lambda \alpha^2} B^+_\mu = \text{source terms} \] (6.9)
which is what we obtain when we use ansatz (6.2) in the contracted equations of motion.

We again add a perturbation of the form
\[ \int d^4 x d^6 \lambda \rho C^{\mu\nu}(x) F_{\mu\nu} P(\lambda) \] (6.10)
and, as before, can reabsorb this term by a shift of \( B \) that is compatible with the equation of motion if \( P(\lambda) \) is a symmetric traceless polynomial. The spectrum of eigenvalues of the AdS Laplacian is \( k(k + 4) \), \( k > 1 \) and it corresponds the operators \( F_{\mu\nu} P(\lambda) \) which agrees with the AdS analysis \( \text{[4]} \).

\[ ^{11} \text{The case K=1 corresponds to adding a perturbation in the decoupled U(1)}. \]
7. Discussion

In this paper we started from an effective description of the field theory and constructed fields that seem to have properties of sugra fields on $AdS_5 \times S^5$. We then showed how one can use this construction to understand some aspects of the matching between operators on the boundary and fields in the bulk. In particular one sees that perturbing the field theory by a local operator corresponds immediately to turning on a field on $AdS$.

There are several caveats to this construction. The first is that in order for it to work a large number of details have to work, for example, the matching of all operators. Most of these will have to await future investigation, although we expect that many will work due to supersymmetry and the fact that we have established them for at least one field in the multiplet. Another serious problem is that we were able to calculate the transform only for linear fluctuations around the $AdS$, but already in our analysis we required non-linear analysis of the pseudo-sugra multiplet when we analyzed the back reaction of the metric and 4-form field strength. Extending the transformation to strong field strengths is a prerequisite before discussing the full non-linear dynamics of sugra. For example we would like to calculate 2-pt functions using this prescription. This already requires analyzing the Lagrangian (which is not a linear functional of the fields). Another issue might be to try and study non-linear field theoretic corrections to the interactions of the pseudo-sugra fields with the branes or with themselves, and map these to $AdS$.

There are also several improvements that one may try and examine. The most interesting is the following. Even for the case of D3-branes on a sphere one expects that there is probably a more complete story in which off-diagonal terms are taken into account. One possible extension will be the following. In our description we kept the quantity $F_{\mu\nu}(\lambda, x)$. There could be a picture in which one keeps also local non-abelian terms, for configurations which are almost abelian. Such an object would be for example an effective $[X_\mu, X_\nu](\lambda, x)$ calculated on branes that passes through the point $\lambda, x$ up to a certain uncertainty, i.e., the matrices are almost diagonal and this term will measure the deviation from being diagonal. From the sugra point of view it is natural to have such terms for the following reason. The brane field $F_{\mu\nu}$ can probably be thought of as equivalent under a gauge transformation to $B_{\mu\nu}$ - we are familiar with this when we have a single brane in 10D spacetime, so when the gas of branes fills spacetime these might be thought of as being gauge equivalent to the other throughout spacetime. A non-abelian extension of the local degrees of freedom, of the form above, might be similarly related to $B_{\mu\nu}$.
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