Thermoelectric power in between two plateaus in quantum Hall effect.

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We have considered the response at two energies corresponding to two plateaus in the quantum Hall effect. Since the thermoelectric power involves the derivative of conductivity with respect to energy, we introduce the concept of a line width and hence an activation energy. We then use the Hall conductivity to define the thermoelectric power at the centre of two plateaus, which is found to vary monotonically as $T^2$ at low temperatures with fixed magnetic field. At elevated temperatures, the thermoelectric power varies as $T \exp(-\Delta/k_B T)$.

(Key words: Thermoelectric power, Hall effect, Mott’s formula.)

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1. Introduction

The phenomenon of quantum Hall effect has been described for a long time\cite{1,2}. The thermoelectric power involves the derivative of conductivity as a function of energy so that at the plateau in the Hall resistivity, where energy is fixed, it is not well defined. On the other hand, if we look at the centre of two plateaus, at a fixed magnetic field, we can define this derivative by using a relaxation process. The thermoelectric power can then give information about the relaxation mechanism in the quantum Hall effect away from the plateaus. In the transverse resistivity there are minima at the same magnetic fields where there are plateaus in the longitudinal resistivity. These fields are described by integers as well as fractions which are predicted by us\cite{3}.

In this communication, we obtain the thermoelectric power at the centre of two plateaus. This is an important point because the system becomes metallic just before the plateaus and hence there is a peak in the resistivity at a point in the centre of two plateaus. The system is some times referred to as “insulator” near the peak. Thus we are able to find the thermoelectric power in the insulating phase.

2. Theory

We consider a three level system in which the energies are labeled as $E_o$, $E_1$ and $E_2$. It absorbs energies $E_2-E_o$ and $E_1-E_o$ so that there are two lines in the absorption as a function of energy. If these lines are far apart, there is no overlap and hence there is no region where the absorption depends on line width. We select a problem where there is a region in which the absorption depends on the line width. The width of the line at $E_1$ is $\delta \Omega_1$ and that at $E_2$ is $\delta \Omega_2$. The response function in this case is given by,

$$
\chi'' = \frac{\delta \Omega_1}{\{(E - E_1)/\hbar\}^2 + (\delta \Omega_1)^2} + \frac{\delta \Omega_2}{\{(E - E_2)/\hbar\}^2 + (\delta \Omega_2)^2}.
$$

(1)
If \( E_2 - E_1 >> h\delta\Omega_1 \), the lines are well resolved and peaked at well defined energies such as \( E_1 \) and \( E_2 \). On the other hand if \( E_2 - E_1 < h\delta\Omega_1 \), the lines overlap and there is absorption in the centre of two spectral lines. Since we are interested in the thermoelectric power in the centre of two lines, we consider the case of overlap between two lines. We suppose that the two lines are separated by \( \delta_1 \) such that \( \delta_1 \) is smaller than either \( \Omega_1 \) or \( \Omega_2 \), i.e.,

\[
E_2 - E_1 = \delta_1
\]

\[
\delta_1 << \delta\Omega_1
\]

\[
\delta_1 << \delta\Omega_2.
\]

At \( E = E_1 \), the first term in the response function is peaked so that the response becomes,

\[
\chi'' = \frac{1}{\delta\Omega_1} + \frac{\delta\Omega_2}{((E_1 - E_2)/h)^2 + (\delta\Omega_2)^2}
\]

but the line at \( E_2 \) is very near the one at \( E_1 \) so that the lines overlap. Substituting (2) in (5) we find,

\[
\chi'' = \frac{1}{\delta\Omega_1} + \frac{\delta\Omega_2}{(\delta_1/h)^2 + (\delta\Omega_2)^2}.
\]

For convenience of algebra, we assume \( \delta\Omega_1 \approx \delta\Omega_2 \) so that the centre of the lines between \( E_1 \) and \( E_2 \) is located at \( E_1 + \frac{1}{2}\delta_1 \). If the relaxation occurs due to interaction with phonons, the width depends on the phonon correlation function, \( 2n + 1 \), where \( n = [\exp(\Delta/k_BT) - 1]^{-1} \). It is also possible to say that \( n \approx \exp(-\Delta/k_BT) \) at low temperatures so that \( \Delta \) becomes the activation energy,

\[
\delta\Omega_1 = c_1(2n + 1) \approx 2c_1\exp(-\Delta/k_BT)
\]

In the symmetric case,

\[
\delta\Omega_1 = \delta_1.
\]

Let us consider the old Hall effect. In the quantum Hall effect the conductivity is quantized as \( \sigma = ie^2/h \) where \( i \) may be an integer or a fraction. We believe that this fraction depends on spin. If there are two plateaus, they occur at \( i = \nu_1 \) and \( i = \nu_2 \). The conductivity is quantized at both \( \nu_1 \) as well as at \( \nu_2 \) but we are neither interested in quantization at \( \nu_1 \) nor at \( \nu_2 \). We are interested in the point which is exactly in the middle of \( \nu_1 \) and \( \nu_2 \) so that there is no quantization at this point.

The current is determined by \( j = nev \) where \( n \) is the concentration of electrons, \( e \) the charge and \( v \) the velocity. We replace the charge by energy \( E_c - E_F + \frac{3}{2}k_BT \) and divide the resulting expression by the current to define the Peltier coefficient,

\[
\Pi = -\frac{E_c - E_F + \frac{3}{2}k_BT}{e}
\]

where \( E_c \) is the electron energy in the conduction band. The negative sign in the above has come from the sign of the charge, -e. The thermoelectric power, \( S \), may be defined as,

\[
\Pi = ST
\]
where $T$ is the temperature. If $|\vec{E}|$ is the electric field, another definition of the thermoelectric power is obtained by,

$$|\vec{E}| = S \text{grad} T$$

(11)

or

$$S = \frac{|\vec{E}|}{\text{grad} T}.$$  

(12)

We can replace the $\text{grad}$ by $d/dx$ but then the temperature gradient has to be evaluated. Apparently, Mott has solved[4] this problem by defining the thermoelectric power by using the energy derivative of the logarithm of the conductivity,

$$S = \frac{\pi^2 k_B^2 T}{3e} \frac{d}{dE} \ln \sigma |_{E=E_F}.$$  

(13)

This means that we should know the conductivity as a function of energy, then only we can use the Mott formula. As we described above, we are interested in a point which is mid way between two quantized values, we use the old result according to which the Hall resistivity is $\rho = nec/B$ and hence the Hall conductivity is,

$$\sigma = \frac{neec}{B},$$  

(14)

where $n$ is the electron concentration, $e$ is the charge, $B$ is the magnetic field and $c$ is the velocity of light. The transition energies in the system described above are $E_1 = \frac{1}{2}g\mu_B B_1$ and $E_2 = \frac{1}{2}g\mu_B B_2$. Our transitions occur at $E_1$ and $E_2$ and the centre is at $E_1 + \frac{1}{2}\delta_1$. Thus there is quantization at $E_1$ and at $E_2$ but not at $E_1 + \frac{1}{2}\delta_1$. Therefore, the conductivity becomes,

$$\sigma = \frac{1}{2}g\mu_B^nec \frac{1}{E}$$  

(15)

where $E = \frac{1}{2}g\mu_B B$. Taking the logarithm of conductivity, we obtain,

$$\ln \sigma = \ln \left(\frac{1}{2}g\mu_B^nec\right) - \ln E$$  

(16)

so that

$$\frac{d}{dE} \ln \sigma = -\frac{d}{dE} \ln E = -\frac{1}{E}. $$  

(17)

If the conductivity is thermally activated,

$$\sigma = \sigma_0 \exp(E/k_BT)$$  

(18)

we obtain,

$$\frac{d}{dE} \ln \sigma = \frac{1}{k_BT}. $$  

(19)

In this case, the thermoelectric power will be independent of temperature. Therefore, we do not consider this case but we consider the thermally activated relaxation rate. We go back to the Mott formula to find the thermoelectric power as,

$$S = -\frac{\pi^2 k_B^2 T}{3e} \frac{1}{E}. $$  

(20)
Where the negative sign means that power reduces as the energy is increased. When energy is either $E_1$ or $E_2$, the conductivity is quantized but at the mid point, $E = E_1 + \frac{1}{2} \delta_1$, the conductivity is not quantized so that at this point,

$$S = -\frac{\pi^2 k_B^2 T}{3eE_1(1 + \frac{1}{2E})}. \quad (21)$$

If $\delta_1$ is very small compared with $E_1$ we can write the term in the denominator as,

$$S = \frac{\delta_1 \pi^2 k_B^2 T}{6eE_1^2} - \frac{\pi^2 k_B^2 T}{3eE_1} . \quad (22)$$

Thus there is a large negative term which is linear in $T$ and a small correction which depends on the relaxation[5]. The distance $\frac{1}{2}\delta_1$ must be equal to half width so that,

$$\delta_1 = \frac{1}{2\tau_1} = c_1(2n + 1). \quad (23)$$

If the system is thermally activated, $2n + 1 \simeq 2n \simeq 2\exp(-\Delta/k_BT)$. Therefore, the first term in the thermoelectric power varies as,

$$S_1 = \frac{\pi^2 k_B^2 T c_1(2n + 1)}{6eE_1^2} \simeq \frac{\pi^2 k_B^2 T c_1 T \exp(-\Delta/k_BT)}{3eE_1^2} \quad (24)$$

in which the temperature dependence is determined by

$$S_1 \simeq c_2 T \coth \frac{\Delta}{2k_BT} \quad (25)$$

which at high temperatures varies as,

$$S_1 \simeq \frac{2c_2 k_BT^2}{\Delta}. \quad (26)$$

There is also a large negative term,

$$S = S_1 + S_2 \quad (27)$$

$$S_2 = -\frac{\pi^2 k_B^2 T}{3eE_1}. \quad (28)$$

Thus we have two terms in the thermoelectric power. One of these terms is negative and linear in temperature. This describes the emission of power under favourable conditions such as large relaxation times in the excited states. The other term is positive and depends approximately on $T^2$. It may be described as $T \coth(\Delta/2k_BT)$. It is known that relaxation in quantum Hall effect is thermally activated so that the thermopower may appear as $T \exp(-\Delta/k_BT)$ as given by (24). In these results, the magnetic field is held constant so that there is no phase transition by varying temperature.
The variation of conductivity as a function of length or thickness of the sample is well known and it is related to a critical exponent. Polyakov and Shklovskii[6] have treated \( \delta_1 \) or an equivalent quantity as if there is a phase transition with a critical temperature. In that case, the width is described by “scaling exponents”,

\[
\delta_1 \simeq T^\kappa
\]  

(29)

where \( \kappa \simeq 0.4 \). It has been possible to correlate the exponent \( \kappa \) with \( \gamma \) which describes the coherence length. The band filling factor is called \( \nu \) and the coherence length varies as,

\[
\xi(\nu) = \xi(0)|\nu - \nu_o|^{-\gamma}
\]

(30)

with \( \gamma = 2.3 \) or \( \kappa = 1/\gamma = 0.435 \). Long time ago[5], the value of \( \kappa \) was thought to be 0.5. We write the scaling width as,

\[
\delta_1 = c_2 T^\kappa
\]

(31)

which shows that the positive term of the thermoelectric power varies as,

\[
S_1 = \frac{\pi^2 k_B^2 c_2}{6eE_1^2} T^{\kappa+1}
\]

(32)

so that the exponent \( \kappa + 1 \) can be determined from the thermoelectric power as a function of temperature. The critical exponents are a characteristic of a phase transition. When \( T_c \simeq 0 \), \( (T - T_c)^\kappa = T^\kappa \) so that Polyakov’s result is applicable to a phase transition at zero temperature. When we go from one level to another, we call it a phase transition. However, if we keep the magnetic field a constant at the middle of two plateaus, then there is no phase transition and we need not expect any critical exponent.

Possanzini et al[7] have measured the thermoelectric power for two different densities. In both the cases, the thermoelectric power monotonically increases with increasing temperature from 0.4 to 4 K. An effort is made to fit the data with \( S = \alpha T^{1/3} + \eta T^4 \) which is interpreted to arise from a variable range hopping model or from \( S_{xx} = \gamma + \eta T^4 \).

Naturally, a term can be added in which the exponent has any value between 1/3 and 4 without disturbing the agreement between the model and the data. The expression (26) shows that the thermoelectric power varies as \( T^2 \). When the magnetic field is kept constant and the temperature is varied, it is reasonable to expect that there is no phase transition so the thermoelectric power varies as \( T^2 \). At a temperature of 0.32 K, the largest positive value of the diagonal thermoelectric power is about 5\( \mu \)V/K and at very small magnetic fields, the value is negative of the order of 10\( \mu \)V/K. Thus these observed features are in accord with the theoretical expressions.

3. Conclusions

In conclusions, we find that when field is fixed in between two plateaus, the thermoelectric power varies as \( T^2 \) at low temperatures. At slightly elevated temperatures, we expect a thermally activated relaxation process so that \( T exp(-\Delta/k_B T) \) type variation is predicted.

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