The Impact of Line Misidentification on Cosmological Constraints from Euclid and Other Spectroscopic Galaxy Surveys

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Abstract

We perform forecasts for how baryon acoustic oscillation (BAO) scale and redshift-space distortion (RSD) measurements from future spectroscopic emission line galaxy surveys such as Euclid are degraded in the presence of spectral line misidentification. Using analytic calculations verified with mock galaxy catalogs from lognormal simulations, we find that constraints are degraded in two ways, even when the interloper power spectrum is modeled correctly in the likelihood. First, there is a loss of signal-to-noise ratio for the power spectrum of the target galaxies, which propagates to all cosmological constraints and increases with contamination fraction, $f_c$. Second, degeneracies can open up between $f_c$ and cosmological parameters. In our calculations, this typically increases BAO scale uncertainties at the 10%–20% level when marginalizing over parameters determining the broadband power spectrum shape. External constraints on $f_c$ or parameters determining the shape of the power spectrum, for example, from cosmic microwave background measurements, can remove this effect. There is a near-perfect degeneracy between $f_c$ and the power spectrum amplitude for low $f_c$ values, where $f_c$ is not well determined from the contaminated sample alone. This has the potential to strongly degrade RSD constraints. The degeneracy can be broken with an external constraint on $f_c$, for example, from cross-correlation with a separate galaxy sample containing the misidentified line or deeper subsurveys.

Key words: cosmology: observations – distance scale – large-scale structure of universe

1. Introduction

Measurements of cosmic microwave background (CMB) anisotropy, particularly from the Wilkinson Microwave Anisotropy Probe (WMAP) and Planck satellite missions, have precisely constrained the parameters of the standard $\Lambda$ cold dark matter ($\Lambda$CDM) model and limited or ruled out many possible modifications or extensions (Hinshaw et al. 2013; Planck Collaboration VI 2018). The next decade will see many experimental collaborations aiming to take advantage of the vast amount of cosmological information encoded in large-scale structure (LSS), building on recent galaxy clustering and weak gravitational lensing measurements (e.g., Alam et al. 2017; Dark Energy Survey Collaboration 2018b; van Uitert et al. 2018; Hikage et al. 2019). Examples include the Dark Energy Spectroscopic Instrument5 (Levi et al. 2013), Euclid6 (Laureijs et al. 2011), the Large Synoptic Survey Telescope (LSST)7; LSST Dark Energy Science Collaboration 2012), and the Wide Field Infrared Survey Telescope (WFIRST8; Spergel et al. 2015). A key motivation for these experiments is to detect or tightly constrain deviations from cosmological constant dark energy behavior (see Weinberg et al. 2013, for review of observational methods). Additional goals include testing general relativity (GR) on cosmological scales (see Clifton et al. 2012 for a review of modified gravity theories and, e.g., Jeong & Schmidt 2015 for testing GR with LSS), measuring neutrino mass through the suppression of small-scale clustering (e.g., Font-Ribera et al. 2014b; Boyle & Komatsu 2018), and improving on Planck’s constraints on primordial non-Gaussianity (Planck Collaboration XVII 2016).

The baryon acoustic oscillation (BAO) scale is understood to be the most robust observable in LSS clustering, and BAO measurements over a range of redshifts provide valuable dark energy constraints (see Section 4 of Weinberg et al. 2013 for a review and Alam et al. 2017; Bautista et al. 2017; Dark Energy Survey Collaboration 2017; du Mas des Bourboux et al. 2017, and Ata et al. 2018 for the latest results). The BAO measurements also tightly constrain portions of the $\Lambda$CDM parameter space, particularly in conjunction with CMB data (e.g., Hinshaw et al. 2013; Aubourg et al. 2015; Addison et al. 2018; Planck Collaboration VI 2018). They play an important role in the current Hubble constant ($H_0$) tension, providing evidence for $H_0 < 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ in joint fits with the CMB or primordial deuterium abundance measurements within $\Lambda$CDM (e.g., Addison et al. 2013, 2018; Aubourg et al. 2015; Bernal et al. 2016; Planck Collaboration XIII 2016; Dark Energy Survey Collaboration 2018a), while the latest local distance ladder measurement is $H_0 = (73.52 \pm 1.62) \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Riess et al. 2018).

The Baryon Oscillation Spectroscopic Survey (BOSS9; Dawson et al. 2013) has provided 1%–2% BAO scale measurements using luminous red galaxies over redshift $0.2 < z < 0.75$, as well as lower-precision measurements from the Ly$\alpha$ forest along sight lines to quasars at $z \geq 2$ (see Alam et al. 2017; Bautista et al. 2017; du Mas des Bourboux et al. 2017, for final Data Release 12 constraints). Euclid and

5  https://www.desi.lbl.gov/
6  https://www.euclid-ec.org/
7  https://www.lsst.org/
8  https://wfirst.gsfc.nasa.gov/
9  http://www.sdss3.org/surveys/boss.php
2. Power Spectra from Catalogs Containing Misidentified Lines

2.1. Simplifying Assumptions

Since we are interested in how line misidentification degrades cosmological constraints, rather than the overall constraining power or optimal analysis choices for future surveys, we make a number of simplifying assumptions.

(i) We assume the ELGs are linear tracers of the linear dark matter density fluctuations. We approximately account for the impact of nonlinearity by varying a maximum cutoff scale in \( \ell \), beyond which we assume no cosmological information can be recovered. We note that the impact of nonlinearity is relatively small at the BAO scale \( \ell \approx 150 \text{ Mpc} \) and smaller at the redshifts we are considering than for BAO surveys like BOSS at \( z \approx 0.7 \).

(ii) We perform simulations and calculations for sky patches of up to \( 10^3 \text{ deg}^2 \), where the sky can be well approximated as flat, and assume that constraints from larger sky areas can be obtained by simply combining the information from multiple patches. This is a reasonable approximation for scales much smaller than the patch size, including the BAO feature for ELGs at \( z > 0.7 \), which is the focus of this work, but throws away information from scales comparable to or larger than the patch size.

(iii) We ignore complications in the survey geometry and weighting or masking and approximate each bin in redshift as a comoving cuboid. If survey depth varies significantly with position, then spatial variation in the line misidentification rate could also be introduced. An investigation of this effect for specific surveys is left to future work.

(iv) We neglect any time evolution within a redshift bin, for instance, in matter clustering or ELG properties.

(v) We assume the likelihood function for the galaxy power spectra can be approximated as Gaussian and, further, that non-Gaussian contributions to the power spectrum covariance can be neglected.

2.2. ELG Power Spectrum

We use the redshift-space multipole power spectrum of fluctuations in the ELG overdensity as the observable that directly enters the cosmological likelihood. To forecast constraints, we therefore need to compute the mean and covariance of the multipole power spectrum as a function of cosmological and ELG parameters.

In Appendix A we connect the ELG power spectrum to the underlying density fluctuations and compile some analytic results from the literature for the multipoles of the linear theory redshift-space power spectrum and the covariance between different multipoles. Given the Fourier coefficients of the galaxy density field at wavevector \( k \), \( \delta_g(k) \), we write down an estimator for the ELG multipole power spectrum at multipole \( \ell \) as

\[
\hat{P}_{g,\ell}(k) = \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu_k \delta_g(k)\delta_g^*(\mu_k)P_\ell(\mu_k) - \frac{\delta_{10}}{n_g},
\]

where \( \mu_k = k\ell/|k| \) is the cosine of the angle between \( k \) and the line of sight, \( P_\ell \) is a Legendre polynomial, and \( n_g \) is the number density of galaxies, so that \( 1/n_g \) is the shot-noise contribution to the power spectrum, which is subtracted for the monopole, \( \ell = 0 \) (\( \delta_{10} \) here is the Kronecker delta). The mean and covariance of this estimator can be computed analytically for linear theory RSDs (Kaiser 1987) and are nonzero only for \( \ell = 0, 2, 4 \). Full expressions are provided in Appendix A. The mean of the monopole, for example, is given by

\[
\langle \hat{P}_{g,\ell=0}(k) \rangle = \left( 1 + \frac{2}{3}\beta + \frac{1}{5}\beta^2 \right) b_g^2 P_m(k),
\]
where $\beta = f/b_g$, $f$ is the derivative of the cosmological growth rate, $b_g$ is the galaxy bias, and $P_m(k)$ is the linear matter power spectrum.

### 2.3. Adding Interlopers with Misidentified Lines

We follow the process described by P16 and Leung et al. (2017) for adding interloper galaxies with misidentified spectral lines. The observed emission line wavelength $\lambda$ is related to the rest-frame wavelength $\lambda_0$ by $\lambda = \lambda_0(1+z)$. A misidentified line with rest-frame wavelength $\lambda_{\text{int}}$ is observed at redshift $z_{\text{int}}$ such that $\lambda = \lambda_{\text{int}}(1+z_{\text{int}})$. When angular coordinates and redshift are transformed to three-dimensional coordinates in order to estimate the ELG power spectrum, the interloper coordinates are calculated incorrectly. The coordinates are also remapped anisotropically, because transverse separations scale like the proper-motion distance,\(^{13}\) $D_M(z)$, while line-of-sight separations scale like $D_L(z) = (1+z)c/H(z)$. We follow P16 by introducing transverse and line-of-sight remapping parameters, $\gamma_\perp$ and $\gamma_\parallel$, given by

\[
\gamma_\parallel = \frac{D_M(z)}{D_M(z_{\text{int}})}
\]

\[
\gamma_\perp = \frac{D_M(z)}{D_M(z_{\text{int}})} = \frac{(1+z)/H(z)}{(1+z_{\text{int}})/H(z_{\text{int}})} = \frac{\lambda_{\text{int}}H(z_{\text{int}})}{\lambda_0H(z)},
\]

and a contamination fraction, $f_c$, such that $f_c$ is the fraction of the total number of galaxies in the catalog where line misidentification has occurred. Writing the total number density as $n = n_{\text{t}} + n_{\text{int}}$, with the subscripts “t,” “g,” and “int” denoting total, target galaxy, and interloper, respectively, the number density of interlopers is

\[
n_{\text{int}} = f_c n_t = \frac{f_c}{1-f_c} n_g.
\]

Note that $n_{\text{int}}$ here is calculated using the target ELG survey volume. The ELG overdensity in the contaminated catalog is

\[
\delta_t(x) = (1-f_c)\delta_t(x) + f_c\delta_{\text{int}}(x//\gamma_\perp, x//\gamma_\parallel),
\]

where $x$ is the three-dimensional position vector, and the volume integral in the Fourier transform picks up a factor $\gamma_\perp^2\gamma_\parallel$, so we have

\[
\delta_t(k) = (1-f_c)\delta_t(k) + f_c\gamma_\perp^2\gamma_\parallel\delta_{\text{int}}(\gamma_\parallel k_{\perp}, \gamma_\parallel k_{\parallel}).
\]

One factor of $\gamma_\perp\gamma_\parallel$ is used in remapping the coordinates of the Dirac delta in the covariance of $\delta_{\text{int}}(k)$ (Equation 8 of P16):

\[
\langle \delta_{\text{int}}(k)\delta_{\text{int}}(k') \rangle = \gamma_\perp^2\gamma_\parallel \delta_3(k-k')P_{\text{int}}(\gamma_\parallel k_{\perp}, \gamma_\parallel k_{\parallel}).
\]

Assuming there is no correlation between the target and interloper populations (i.e., no redshift overlap), we can calculate the mean of the estimator in Equation (1) for the contaminated case,

\[
\langle \hat{P}_{t,\ell}(k) \rangle = (1-f_c)^2P_{t,\ell}(k) + f_c^2\gamma_\perp^2\gamma_\parallel P_{\text{int},\ell}(\gamma_\parallel k_{\perp}, \gamma_\parallel k_{\parallel}).
\]

where

\[
P_{\text{int},\ell}(\gamma_\parallel k_{\perp}, \gamma_\parallel k_{\parallel}) = \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu_k P_{\text{int}}(\gamma_\parallel k_{\perp}, \gamma_\parallel k_{\parallel}) P_\ell(\mu_k)
\]

and

\[
P_{\text{int}}(\gamma_\parallel k_{\perp}, \gamma_\parallel k_{\parallel}) = (1 + \beta \mu_k^2) \frac{\gamma_\parallel^2 k_{\parallel}^2}{\sqrt{\gamma_\perp^2 k_{\perp}^2 + \gamma_\parallel^2 k_{\parallel}^2}}.
\]

Expessions for the covariance $\langle (\hat{P}_{t,\ell}(k) - \langle \hat{P}_{t,\ell}(k) \rangle)(\hat{P}_{t,\ell}(k') - \langle \hat{P}_{t,\ell}(k') \rangle) \rangle$ can be derived in an analogous way to Equation (31), although there are no closed-form expressions analogous to Equation (32). There are separate contributions from the target galaxy sample variance, interloper sample variance, and shot noise, as well as cross terms. Note that the strong scaling of the covariance contributions with $f_c$, e.g., the target ELG galaxy sample variance scales as $(1-f_c)^2$, means that approximating the contaminated covariance with the pure target ELG covariance may be a poor approximation unless $f_c \ll 1$.

For models close to $\Lambda$CDM, $D_M$ increases monotonically with redshift. The transverse remapping parameter $\gamma_\perp$ is thus greater than 1 for lower-redshift interlopers and less than 1 for higher-redshift interlopers. The quantity $(1+z)/H(z)$ increases at low redshift but peaks at $z \simeq 0.7$ before decreasing, eventually falling off like $(1+z)^{-1/2}$ at redshifts where the universe is essentially completely matter-dominated and the dark energy density is negligible. For the target and contaminant lines relevant to Euclid, HETDEX, or WFIRST, $\gamma_\parallel$ differs from unity only at the $10\%$–$15\%$ level, while $\gamma_\perp$ can vary by an order of magnitude when the difference between target and contaminant redshifts is large.

In the simple but unrealistic case where $\gamma_\parallel = \gamma_\parallel = \gamma$, the interloper coordinate remapping is isotropic, and when we measure power at wavenumber $k$, we are in fact measuring interloper power from wavenumber $\gamma k$ instead. In other words, the measured power spectrum at each multipole contains a “squashed” or “stretched” contribution from the interloper power at that same multipole.

The realistic anisotropic case where $\gamma_\parallel \neq \gamma_\parallel$ is more complicated. The power at wavenumber $k$ in the contaminated sample contains contributions from a range of scales in the interloper spectrum (scales between $\gamma k$ and $\gamma k$), and the integral in Equation (9) no longer has a closed-form solution. The anisotropy causes power to be transferred between multipoles. For example, interlopers at higher redshift than the target ELGs have their quadrupole and hexadecapole power enhanced relative to the monopole. The effects of coordinate remapping are discussed in more detail for specific combinations of target and interloper lines in Sections 4 and 5.

### 2.4. Approximations for Finite Volume

In practice, when we are considering a finite survey volume, only discrete $k$-modes are available, and the expressions in

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\(^{13}\) Also referred to as the comoving angular diameter distance. We follow recent BAO literature in using the $D_M$ notation (e.g., Alam et al. 2017).
Sections 2.2 and 2.3 need to be modified to account for this. We work with bins in $|k|$ and write down an estimator for the binned power spectrum as a weighted sum over $k$-modes,

$$
\hat{P}_{i,j}(b) = \frac{2\ell + 1}{N_b} \sum_{k \in b} \delta_i(k) \delta_j^*(k) \mathcal{P}(\mu_k) - \frac{\delta^{(0)}}{n_t},
$$

where $N_b$ is the number of independent modes in bin $b$ and the sum runs over these modes. Provided the power spectrum does not vary significantly over the modes within a given bin $b$, we can make the approximation that

$$
(\hat{P}_{i,j}(b)) \simeq P_{i,j}(k_b),
$$

where $k_b$ is the central wavenumber in the bin. The expressions for the covariance of the estimator (Equations (31) and (32)) and their analogs for the interlopers are similarly evaluated at $k_b$ and multiplied by a factor $2/N_b$, with the factor of 2 arising because of only counting independent modes (the reality of the density field means that only half the Fourier modes are independent).

A second complication is that the integrals over $\mu_k$ in Equations (1) and (9) should be replaced by a sum over the discrete set of $\mu_k$ values corresponding to the $k$-modes falling in each bin. To simplify the calculations for different survey volumes, we continue to use the integrals and ignore the exact configuration of modes. This is a reasonable approximation, provided there are enough modes in each bin to provide roughly uniform coverage in $\mu_k$.

We compared our calculations to the results obtained from mock galaxy catalogs from redshift-space lognormal simulations of the cosmological density field. This is an important check of both the finite volume approximations and results when interlopers are included and integrals no longer have closed-form solutions. The method and code to generate the simulations are described by Agrawal et al. (2017). More details are provided in Appendix B. We also use this code to compute $N_b$ for each bin and survey.

### 2.5. Forecasting Methodology

Our main goal is to forecast how cosmological parameter constraints are degraded in different interloper scenarios and examine the extent to which the impact of interlopers can be mitigated. We emphasize that this is different from the approach described in Section 2 of P16, where the focus is on estimating the bias in cosmological parameters when the interlopers are present but not accounted for in the fitting.

The steps in our calculations are as follows: (i) choose the fiducial values of the cosmological parameters and parameters relating to the ELG and interloper populations (e.g., $f_c$, $b_q$); (ii) specify a redshift range for the target ELGs and calculate the dimensions of the comoving volumes (cuboids) containing the target and interloper ELGs, as well as the number of Fourier modes in each $k$ bin; (iii) calculate the mean and covariance of the estimator defined in Equation (12), including target and interloper ELGs, as described above; and (iv) calculate the Fisher matrix, $\mathcal{F}$, for the parameters of interest (including nuisance parameters like galaxy bias) by computing the numerical derivatives of the multipole power spectra with respect to the parameters:

$$
\mathcal{F}_{ij} = \sum_{\ell \ell'} \sum_b \frac{\partial P_{\ell \ell'}(k_b)}{\partial \theta_i} \{ C_{\ell \ell'}(k_b, k_b) \}^{-1} \frac{\partial P_{\ell \ell'}(k_b)}{\partial \theta_j}.
$$

Note that we include correlations between multipoles but not $k$ bins, as described in Appendix B. The sample variance contribution to the covariance $C_{\ell \ell'}(k_b, k_b)$ also depends on the cosmological and ELG parameters. We found that this dependence impacts the results for the Euclid [O III] sample in Section 4 at the percent level and so neglect it, evaluating the covariance only for the fiducial parameters. See, for example, Heavens (2009) or Verde (2010) for more discussion of Fisher matrices in the context of cosmological analysis.

To facilitate calculating numerical derivatives and removing or rescaling the BAO “wiggles” in the power spectrum, we calculated linear matter power spectra using code from the same package used to generate lognormal simulations in Appendix B. The code implements the approximations and fitting functions described by Eisenstein & Hu (1998). This is less accurate than the matter power spectrum produced by the Code for Anisotropies in the Microwave Background (CAMB; Lewis et al. 2000) but adequate to assess the loss of information in the presence of interlopers. Our results were calculated assuming a cosmology with $\{\Omega_m, \Omega_b, h, n_s, \sigma_8\} = \{0.0456, 0.274, 0.704, 0.963, 0.809\}$, based on WMAP analysis (Komatsu et al. 2011). Despite the precision of future spectroscopic surveys, changing the input parameters—for example, using more recent constraints from Planck—does not significantly impact our findings, which are largely based on the comparison between contaminated and pure ELG samples rather than overall constraining power.

We performed calculations with bin width $\Delta k = 0.005 \, h \, \text{Mpc}^{-1}$ and used bin centers covering $k_{\text{min}} = 0.005 \, h \, \text{Mpc}^{-1}$ to $k_{\text{max}} = 0.3 \, h \, \text{Mpc}^{-1}$. The choice of these bounds is fairly arbitrary. Arguably, the upper limit should depend on the redshift of the target ELGs, since the impact of nonlinearity is redshift-dependent, for example. In Section 4.5 we show that even large changes to the range of scales do not change our main conclusions regarding the effect of interlopers on BAO and RSD constraints.

### 2.6. Recovery of Isotropic BAO Scale

The BAO scale imprinted at recombination is a key observable in LSS clustering and an important driver for the ELG number density and volume surveyed in current and future surveys. A range of methods have been developed to robustly extract the BAO scale (e.g., Eisenstein et al. 2005; Sánchez et al. 2008, 2013; Beutler et al. 2011; Padmanabhan et al. 2012; Kazin et al. 2014). Our approach is motivated by the method used in the multipole power spectrum analysis of the final BOSS release (Sections 7.2 and 7.3 of Beutler et al. 2017; see references in that paper for earlier work). We consider a shift in the location of the BAO “wiggles” rather than a shift in the full power spectrum, however, since we are also interested in the effect of marginalizing over parameters determining the broadband shape.

We consider a shift in the apparent position of the BAO peak in the galaxy correlation function relative to a fiducial cosmological model of $r_{\text{BAO}} \rightarrow r_{\text{BAO}}^\alpha$. If the fiducial cosmological model is correct, then $\alpha = 1$, with other values indicating a difference between the data and the fiducial model in either the conversion of ELG angular position and redshift to
three-dimensional coordinates or the absolute sound horizon at decoupling, \( r_p \). Following Eisenstein et al. (2005), we have

\[
\alpha = \frac{[D_V(z)/r_d]}{[D_V(z)/r_d]_{\text{fid}}},
\]

where \( D_V(z) \) is an angle-averaged combination of \( D_M(z) \) and \( 1/H(z) \):

\[
D_V(z) = \left[\frac{D_M^2(z) \frac{c^2}{H(z)}}{H(z)}\right]^{1/3}.
\]

To apply this shift in the BAO peaks in Fourier space, we write the linear matter power spectrum as

\[
P_m(k) = P_{nw}(k) + P_w(k),
\]

where \( P_{nw} \) is the “no wiggles” power spectrum computed using a transfer function without the baryonic oscillatory features (Equation (30) of Eisenstein & Hu 1998), and \( P_w \) is the power spectrum of the “wiggles” (i.e., BAO). We then have

\[
P_w(k, \alpha) = P_{nw}(k) + \frac{1}{\alpha^3} [P_m(k/\alpha) - P_{nw}(k/\alpha)].
\]

We forecast constraints on the isotropic BAO scale by treating \( \alpha \) as a model parameter and numerically computing derivatives using Equation (14). Note that we only consider a shift in the BAO scale for the target ELGs and do not attempt to use the interloper power to constrain the BAO or other cosmological parameters.

2.7. Recovery of Anisotropic BAO Scale

Current state-of-the-art BAO surveys like BOSS have the constraining power to measure the BAO scale along the line of sight and in the transverse direction simultaneously instead of simply an angle-averaged isotropic scale (e.g., Anderson et al. 2014; Font-Ribera et al. 2014a; Alam et al. 2017). As already discussed in the context of interloper coordinate remapping, line-of-sight separations scale with \((1+z)/H(z)\), while transverse separations scale with \( D_M \). Separate constraints on these quantities from anisotropic BAO contain additional cosmological information over a single angle-averaged measurement (e.g., Section 3.2 of Addison et al. 2018). To investigate how anisotropic BAO constraints from future surveys are impacted by interlopers, we introduce separate transverse and line-of-sight BAO dilation parameters, \( \alpha_\perp \) and \( \alpha_\parallel \), so that

\[
P_m(k_\perp, k_\parallel, \alpha_\perp, \alpha_\parallel) = P_{nw}(k_\perp, k_\parallel)
+ \frac{1}{\alpha_\perp^3} [P_m(k_\perp/\alpha_\perp, k_\parallel/\alpha_\parallel) - P_{nw}(k_\perp/\alpha_\perp, k_\parallel/\alpha_\parallel)].
\]

As in the isotropic case above, departure from unity in these parameters indicates that the data prefer either a different conversion from angles and redshifts to three-dimensional coordinates or a different absolute sound horizon scale compared to the fiducial model. Specifically, we have

\[
\alpha_\perp = \frac{D_M(z)/r_d}{[D_M(z)/r_d]_{\text{fid}}}, \quad \alpha_\parallel = \frac{D_H(z)/r_d}{[D_H(z)/r_d]_{\text{fid}}} = \frac{[H(z)/r_d]_{\text{fid}}}{H(z)}.
\]

where \( D_M(z) \) and \( D_H(z) \) are defined in Section 2.3. We forecast constraints on the anisotropic BAO scale measurements by treating \( \alpha_\perp \) and \( \alpha_\parallel \) as additional model parameters and numerically computing derivatives using Equation (14). Note that \( \alpha_\perp \) and \( \alpha_\parallel \) are always varied together as a pair.

2.8. Measuring Growth of Structure with RSDs

There is a complete degeneracy between the linear galaxy bias, \( b_g \), and the amplitude of the matter power spectrum, \( P_m \), from the monopole power spectrum alone (Equation (2)). Adding the quadrupole provides a constraint on \( \beta = f/b_g \) (Equation (29)). Since the matter power spectrum amplitude is proportional to \( \sigma_8^2(z) \), the constraints on \( \beta \) and \( b_g \) can be combined to produce a constraint on the quantity \( f\sigma_8(z) \), removing the dependence on the bias. This is the approach used to obtain cosmological constraints from RSD in recent surveys (e.g., Beutler et al. 2012; Howlett et al. 2015; Alam et al. 2017; Pezzotta et al. 2017). It is well suited to measurements over a modest range of scales, where a fiducial model for the shape of the power spectrum can be assumed without significantly impacting the results.

For the RSD Fisher forecasts discussed in Sections 4 and 5 we did not assume a fiducial power spectrum shape and instead varied \( b_g \), \( \sigma_8 \), and \( \Omega_m \) (in \( \Lambda \)CDM, \( \Omega_m \) determines \( f \)) separately, along with additional parameters like \( h \) and \( n_s \) that determine \( P_m(k) \). We also investigated holding the power spectrum shape fixed and varying \( \beta \) and \( b_g \sigma_8 \), as described above, and did not find any qualitatively different behavior.

2.9. Power Spectrum Signal-to-noise Ratio

In addition to considering the effect of interlopers on cosmological parameter determination, we also found it helpful to examine their impact at the power spectrum level. We define an overall power spectrum signal-to-noise ratio, \( S \), where “signal” is the power spectrum of the target ELGs, by summing over all of the multipoles and power spectrum bins,

\[
S^2 = \sum_{\ell \ell'} \sum_b (1 - f_{\ell \ell'})^2 P_{\ell \ell'}(k_b) \cdot [C_{\ell \ell'}(k_b, k_b)]^{-1} \cdot P_{\ell \ell'}(k_b),
\]

where \( k_b \) denotes the \( k \)-modes in bin \( b \), and we take the bins as independent (Appendix B). The covariance \( C \) includes sample variance and shot noise for both the target and interloper ELGs.

The value of \( S \) corresponds to the significance at which the target ELG power spectrum is measured to be nonzero, assuming perfect knowledge of the target and interloper power spectra, and \( f_{\ell \ell'} \). It is equivalent to the Fisher uncertainty on the overall power spectrum amplitude (proportional to \( \sigma_8^2 \)) while keeping all other parameters fixed. Comparing the Fisher forecasts to \( S \) can be useful for assessing whether cosmological constraints are degraded in the presence of interlopers due to the loss of information in the power spectrum or some additional parameter degeneracy opening up, for example, with \( f_{\ell \ell'} \). We note, however, that the loss of power spectrum signal-to-
noise defined in this way does not represent a strict lower bound on the degradation of BAO or other parameter uncertainties.

### 3. Experiment Properties

Properties for the surveys and emission lines we consider are listed in Table 1 and discussed in more detail below. We show the results in Section 4 for the *Euclid* [O III] survey contaminated by Hα, motivated by the fact that severe contamination is possible in this case. Figure 15 of P16 shows that the WFIRST [O III] survey may have Hα contamination at the level of tens of percent, even after using secondary line identification. The problem is likely to be more severe for *Euclid*, given the lower signal-to-noise line detection threshold. We discuss our conclusions regarding the recovery of BAO and RSD information as a function of contamination fraction in the context of other surveys and target lines in Section 5.

One of the strengths of spectroscopic redshift surveys for dark energy constraints is being able to make BAO and RSD measurements in narrow, possibly overlapping redshift bins (e.g., Wang et al. 2017). In this work, we focus on comparing cosmological constraints from contaminated ELG samples, including misidentified lines with corresponding constraints from pure samples, in order to directly assess the impact of interlopers. As a result, the exact choices of redshift binning, redshift range, or effective ELG bias do not significantly impact our conclusions, and for simplicity, we show results without subdividing the ELG samples by redshift.

#### 3.1. Euclid

The *Euclid* mission design includes a spectroscopic galaxy survey over around 15,000 deg$^2$ using its Near Infrared Spectrometer and Photometer (NISP) instrument (Laureijs et al. 2011). Its red grisms cover 1250–1850 nm, corresponding to a redshift range of roughly $0.9 < z < 1.8$ for the primary line targeted in the survey, Hα (6563 Å), with a blue grism covering shorter wavelengths. A secondary cosmological target is the [O III] doublet at 5007 and 4959 Å, which is observed in the red grism for $1.5 < z < 2.7$. Other lines, including Lyα, [O II], Hβ, and [S II], from ELGs at other redshifts will also fall into the red grism wavelength range. These lines may also be misidentified as Hα or [O III], depending on how much additional information (for instance, from equivalent widths or photometry) is brought to bear when constructing ELG catalogs (P16). Note that the NISP has the resolution to resolve the [O III] doublet, but the 4959 line flux is only a third of the 5007 flux, meaning that for low signal-to-noise spectra, a noise fluctuation may either render the 4959 line undetectable or create a false doublet when the detected line is actually Hα.

The Hα source density of 3900 deg$^{-2}$ in Table 1 is taken from the lower range of recent forecasts by Merson et al. (2018) for the Hα+[N II] blended flux limit of $2 \times 10^{-16}$ erg s$^{-1}$ cm$^{-2}$. The [O III] number density of 282 deg$^{-2}$ is from predictions from the Hubble Space Telescope Wide Field Camera 3 Infrared Spectroscopic Parallels (WISP) program (Colbert et al. 2013) and a flux limit of around $3 \times 10^{-16}$ erg s$^{-1}$ cm$^{-2}$. To match the predictions from Colbert et al. (2013), we restrict the redshift ranges to those shown in Table 1 and do not include high-redshift Hα ELGs at $1.5 < z < 1.8$ or [O III] ELGs at $2.3 < z < 2.7$. There are substantial uncertainties in the source density predictions, and we examine the implications of large changes in these values in Section 5.1.

#### 3.2. HETDEX

The HETDEX survey is designed to observe 840,000 Lyα-emitting galaxies (LAEs) over $1.9 < z < 3.5$ (3500–5500 Å) in a 300 deg$^2$ field (Hill et al. 2008). An additional smaller field will also be observed; however, we perform calculations for the main field only, following Leung et al. (2017). The main interloper line is the [O II] doublet around 3727 Å, which will not be resolved with the HETDEX spectrograph. The interloper ELGs in this case are at $z < 0.5$, a much lower redshift than the target lines, which leads to $\gamma_{\perp} > 1$ and a more pronounced anisotropic coordinate remapping than for the other surveys and lines we consider. Leung et al. (2017) forecast a fractional contamination of the HETDEX LAE sample by [O II] of up to few percent based on a Bayesian classification scheme using equivalent width distributions. Note that there will be no contamination for LAEs at $z < 2.065$ because this would require observing [O II] at wavelengths shorter than 3727 Å (i.e., a blueshift). We follow Leung et al. (2017) and assign a linear bias of $b_x = 2.0$ for the LAEs and $b_{\text{int}} = 1.0$ for the low-redshift [O II] interlopers.

### Table 1

| Experiment | Euclid | Euclid | HETDEX |
|------------|--------|--------|--------|
| Target line | [O III] 5007+4959 Å | Hα 6563 Å | Lyα 1216 Å |
| Redshift range | $1.5 < z < 2.3$ | $0.9 < z < 1.5$ | $1.9 < z < 3.5$ |
| Effective redshift | 1.9 | 1.2 | 2.7 |
| ELG bias, $b_x$ | 1.7 | 1.5 | 2.0 |
| Surface density, $n_x$ [deg$^{-2}$] | 282 | 3900 | 2800 |
| Interloper line | Hα 6563 Å | [O III] 5007 + 4959 Å | [O III] 3726 + 3729 Å |
| Interloper redshift range | $0.9 < z < 1.5$ | $1.5 < z < 2.3$ | $0 < z < 0.5$ |
| Interloper effective redshift | 1.2 | 1.9 | 0.2 |
| Interloper ELG bias, $b_{\text{int}}$ | 1.5 | 1.7 | 1.0 |
| Interloper surface density, $n_{\text{int}}$ [deg$^{-2}$] | 3900 | 282 | 3333 |
| Transverse remapping factor, $\gamma_{\perp}$ | 1.3 | 0.7 | 7.3 |
| Line-of-sight remapping factor, $\gamma_{||}$ | 0.9 | 1.1 | 0.9 |
| Volume remapping factor, $\gamma_{\perp}^2\gamma_{||}$ | 1.7 | 0.6 | 47.6 |

**Note.** The $\gamma$ remapping factors are defined in Equation (3) of Section 2.3. The two *Euclid* columns correspond to the two target lines, [O III] and Hα.
The planned WFIRST high-latitude spectroscopic survey covers 227 deg^2 and will target the same emission lines as Euclid, H\(\alpha\) at 1.06 < \(z\) < 1.88 and [O III] at 1.88 < \(z\) < 2.77 (Section 2.2.4 of the Science Definition Team (SDT) report; Spergel et al. 2015). The forecast source densities in the SDT report are around 7400 deg\(^{-2}\) for H\(\alpha\) and 600 deg\(^{-2}\) for [O III], although Merson et al. (2018) forecast a higher H\(\alpha\) density of 10,400–15,200 deg\(^{-2}\) for the same flux cut of 1 x 10^{-16} erg s\(^{-1}\) cm\(^{-2}\), including blended H\(\alpha\)+[N II] flux. The impact of H\(\alpha\)–[O III] line misidentification on the WFIRST cosmological constraints is discussed in Section 5.1.

3.3. WFIRST

The planned WFIRST high-latitude spectroscopic survey covers 2227 deg^2 and will target the same emission lines as Euclid, H\(\alpha\) at 1.06 < \(z\) < 1.88 and [O III] at 1.88 < \(z\) < 2.77 (Section 2.2.4 of the Science Definition Team (SDT) report; Spergel et al. 2015). The forecast source densities in the SDT report are around 7400 deg\(^{-2}\) for H\(\alpha\) and 600 deg\(^{-2}\) for [O III], although Merson et al. (2018) forecast a higher H\(\alpha\) density of 10,400–15,200 deg\(^{-2}\) for the same flux cut of 1 x 10^{-16} erg s\(^{-1}\) cm\(^{-2}\), including blended H\(\alpha\)+[N II] flux. The impact of H\(\alpha\)–[O III] line misidentification on the WFIRST cosmological constraints is discussed in Section 5.1.

4. Results

4.1. Contaminated Power Spectra

The top left panel of Figure 1 shows a forecast of the monopole, quadrupole, and hexadecapole power from [O III] ELGs at \(z = 1.9\) for a 15,000 deg^2 Euclid galaxy survey (where, as stated earlier, we approximate the constraining power of the full survey by imagining combining separate constraints from 600 deg^2 patches). The top right panel shows the power spectrum from the same [O III] ELGs contaminated with interloper H\(\alpha\) ELGs for a contamination fraction \(f_c = 0.2\). The interlopers contribute anisotropic power and suppress the monopole. Bottom left: shape of the H\(\alpha\) ELG power spectra without coordinate remapping. Differences in overall and relative amplitudes of the different multipoles compared to [O III] are due to differences in galaxy bias (1.7 for [O III], 1.5 for H\(\alpha\)) and growth of structure between \(z = 1.9\) and 1.2. Bottom right: shape of the H\(\alpha\) ELG power spectra for galaxies that are misidentified as [O III] and have coordinates remapped. The quadrupole and hexadecapole are enhanced relative to the monopole for lower-redshift interlopers. Each \(k\) bin in the [O III] coordinates receives contributions from a range of \(k\) in the true H\(\alpha\) coordinates, causing a smearing out of BAO wiggles in the monopole power spectrum.
power in the second term in Equation (8) has roughly compensated for the \((1-f_c)^2\) loss.

4.2. Recovery of BAO Scale

Figure 2 shows the forecast increase in uncertainty in the BAO scale parameters \(\alpha\), \(\alpha_{\perp}\), and \(\alpha_{||}\), defined in Section 2.7, as a function of the input \(f_c\) for the Euclid \([\text{O III}]\) survey. Note that \(\alpha_{\perp}\) and \(\alpha_{||}\) are always varied together. The different panels show results for different assumptions about the broadband power spectrum, discussed below. In all cases, we plot the ratio of the uncertainty obtained from inverting the Fisher matrix defined in Equation (14) to the corresponding uncertainty in a forecast where the sample is pure \([\text{O III}]\) and \(f_c\) is known to be zero. Here we assume perfect knowledge of the remapped interloper power spectrum. This assumption is discussed in more detail in Section 4.4.

We first considered an optimistic scenario in which the matter power spectrum is taken to be known perfectly (top left panel of Figure 2). In this case, the parameters that are varied in the Fisher forecast are either \(\alpha\) (isotropic) or \(\alpha_{\perp}\) plus \(\alpha_{||}\) (anisotropic), as well as \(b_2\) and \(f_c\). The uncertainties in the BAO scale closely follow the loss of overall constraining power in the target \([\text{O III}]\) power spectrum (dashed black line, defined in Equation (21)).

Second, we considered a more pessimistic scenario in which all of the \(\Lambda\)CDM parameters \((\Omega_b, \Omega_m, h, n_s, \text{ and } \sigma_8)\) are marginalized over, which acts to modify the shape and amplitude of \(P_m(k)\), as well as the contrast (sharpness) of the BAO wiggles. In existing surveys, the power spectrum is often modeled using a fixed fiducial model spectrum multiplied by a low-order polynomial function and exponential (e.g., Anderson et al. 2014). Marginalizing over the polynomial coefficients and
The exponential cutoff scale allows the BAO scale to be extracted while allowing for imperfect modeling of the broadband power spectrum, particularly nonlinearities. We expect marginalizing over the parameters determining $P_m(k)$ (without any external constraints, for example, from the CMB) to achieve approximately the same effect in our forecasts.

Clearly, the BAO scale constraints will be substantially degraded when the $\Lambda$CDM parameters are marginalized over, since a phenomenological shift in the BAO scale can be largely compensated for by shifts in the $\Lambda$CDM parameters, especially for a limited range of $k$ values. In other words, the BAO scale is a large part of how the galaxy power spectrum constrains the $\Lambda$CDM parameters. Again, here we are interested in how the BAO constraints are further degraded in the presence of interlopers, not the absolute precision of the BAO recovery.

Varying $\Omega_m$ or $h$ also changes how angular position and redshift transform into three-dimensional position and thus leads to anisotropic rescaling of the whole target ELG power spectrum through changes to $D_{ML}(z)$ and $D_H(z)$ relative to the fiducial model. Mathematically, this effect is equivalent to the interloper remapping in Section 2.3. Since anisotropic information is important for identifying the presence of interlopers, one might imagine that including the changes in $D_{ML}(z)$ and $D_H(z)$ leads to a stronger degeneracy between $f_c$ and $\Omega_m$ or $h$. This is indeed the case, and a partial degeneracy between $\alpha$ and $f_c$ also opens up (top right panel of Figure 2).

The uncertainties in the BAO scale, particularly the transverse scale, are increased beyond the loss of information in the target power spectrum, even if the true contamination is small or zero.

To verify the importance of this coordinate remapping effect, we forecast constraints without including it, so that varying $\Omega_m$ and $h$ only impacts the shape of the matter power spectrum. The results are shown in the bottom left panel of Figure 2. While there is a large degeneracy among the $\Lambda$CDM parameters determining the power spectrum shape, the addition of interlopers does not degrade the BAO constraints beyond the power spectrum signal-to-noise loss.

The bottom right panel of Figure 2 shows the results where all of the $\Lambda$CDM parameters are varied but $f_c$ is held fixed to the true value (approximating the case where we have a precise external constraint on $f_c$). In this case, despite the freedom allowed in the shape of $P_m(k)$ and the anisotropic rescaling of the matter power spectrum with changes in $\Omega_m$ and $h$, discussed above, the BAO uncertainties increase with $f_c$ following the power spectrum uncertainties. This illustrates that degeneracy with $f_c$ is what causes the increased uncertainties in the top right panel.

We finally performed a forecast where $\Omega_b$ and $n_s$ are held fixed, so only $\Omega_m$, $h$, and $\sigma_8$ are varied. The results are not shown in Figure 2 but are virtually indistinguishable from the bottom right panel. Fixing $\Omega_b$ and $n_s$ substantially reduces degeneracies between the parameters determining the shape of the ELG power spectrum. A change in the BAO scale cannot be compensated for by a change in the other parameters in the way that is possible when all of the $\Lambda$CDM parameters are free. Fixing $\Omega_b$ and $n_s$ is motivated by the fact that the ELG power spectrum over a modest range of scales does not precisely constrain either parameter, while they are both determined extremely precisely by modern CMB power spectrum constraints. Additionally, the CMB constraints on these parameters are fairly robust to modifications in the cosmological model, particularly low-redshift modifications such as the evolution in dark energy density that the BAO surveys are aiming to probe (see, e.g., Table 5 of Planck Collaboration VI 2018). Strictly speaking, the CMB spectra are sensitive to $\Omega_8 h^2$ rather than $\Omega_b$; however, the Fisher forecasts for the BAO scale are the same in either case.

In summary, if a perfect template for the interloper power is available, BAO constraints are not degraded beyond the loss of information in the power spectrum, provided an external constraint on either $f_c$ or the broadband power spectrum shape (here we considered fixing $\Omega_b$ and $n_s$) are available.

4.3. Recovery of Cosmological Constraints from RSDs

Figure 3 shows forecast constraints on the RSD parameter $f_{\sigma 8}$ in a similar way to Figure 2, comparing uncertainties against a pure [O III] ELG survey with $f_c$ fixed to zero. Note that we are not jointly varying the BAO scale parameters $\alpha$, $\alpha_{\perp}$, or $\alpha_{\parallel}$ in these fits. Also, fractional errors on $f_{\sigma 8}(z)$ and $\sigma_8(z = 0)$ are equal in our forecasts, since these quantities are related by a factor that is a function of $\Omega_m$ only (the combination of $f$ and the linear growth factor that determines the redshift dependence of $\sigma_8$). Results are shown for $f_{\sigma 8} = 0.15$ in addition to the values used in Figure 2.

When the $\Lambda$CDM parameters are all varied, the $f_{\sigma 8}$ constraints are completely degraded for low values of $f_c$ and somewhat exceed the power spectrum constraints for larger values (red points and line in Figure 3). When the true value of $f_{\sigma 8}$ is small, the presence of interlopers cannot be detected at high significance from the contaminated power spectra alone. Consequently, there is close to a complete degeneracy between the amplitude of the matter power spectrum (here parameterized by $f_{\sigma 8}$) and $(1 - f_c)^2$, which multiplies the target ELG power in the first term of Equation (8). When the true value of
$f_c$ is larger, the interlopers are detected; however, a partial degeneracy with the power spectrum amplitude remains.  

The blue circles in Figure 3 show that, when $f_c$ is fixed, the uncertainty in $f_{\sigma_8}$ follows the power spectrum uncertainty. Since the $f_{\sigma_8}$ constraints depend on anisotropic information—in our case, from the relative amplitudes of the different multipoles of the power spectrum—one would expect introducing additional anisotropic contributions from interlopers to impact constraints more strongly than in the BAO case. We have shown that this is indeed the case. The fact that the degeneracy gets worse for lower values of $f_c$, strongly motivates exploring external constraints on $f_c$.

We also considered a range of alternatives not shown in Figure 3, including (i) fixing $\Omega_b$ and $n_s$, (ii) fixing $b_g$, and (iii) removing the anisotropic rescaling effect of $\Omega_m$ and $h$, that is, assuming that the conversion between angular and three-dimensional coordinates is known perfectly. None of these changes remove the strong degeneracy between $f_c$ and $f_{\sigma_8}$ when $f_c$ is small. Fixing $b_g$, for example, does improve the precision of $f_{\sigma_8}$ recovery; however, it does so in roughly the same way in both the pure [O III] and contaminated cases and so does not bring the red points and line in Figure 3 into agreement with the dashed black line.

The Fisher forecasts do not account for the fact that $f_c$ must be positive and may also be unreliable when there is a severe degeneracy, since the response of the data to a small change in parameter values is assumed to be linear. We therefore wanted an independent verification of the behavior shown for low $f_c$ in Figure 3. We examined the relationship between $f_{\sigma_8}$ and $f_c$ by drawing 1000 Gaussian realizations of $P_{\ell}(k_0)$ from the covariance matrix, $C_{\ell\ell}(k_0, k_0)$, and numerically finding the maximum-likelihood values of $f_{\sigma_8}$ and $f_c$ in each case, taking all other parameters as fixed, and requiring $f_c \geq 0$. We tested the case where $f_c = 0.2$ and found good agreement between the uncertainty in $f_{\sigma_8}$ obtained from the Fisher forecast and the spread in the maximum-likelihood values from the 1000 realizations. We then tested the case where there is no contamination, $f_c = 0$, but $f_c$ is still varied, generating 1000 pure [O III] realizations. In around half of the realizations, the best-fit $f_c$ was essentially zero, and the spread in values of $f_{\sigma_8}$ was consistent with the case where only $f_{\sigma_8}$ was varied. In the other half of the realizations, however, the best-fit value of $f_c$ was nonzero, and $f_{\sigma_8}$ was biased high to compensate. For these realizations, the spread in $f_{\sigma_8}$ values was over 10 times the spread with $f_c$ held fixed to zero, confirming the degeneracy indicated by the Fisher calculations.

One might ask whether the bias on $\sigma_8$ or $f_{\sigma_8}$ from ignoring the contamination in the fitting might be acceptable when $f_c$ is small. Taking partial derivatives of the power spectrum, the bias in $\sigma_8$ from ignoring the contamination when $f_c$ is small is given by $\partial \sigma_8 / \partial \sigma_8 = -f_c + O(f_c^2)$. Note that there is no dependence on the interloper power to first order in $f_c$ (Equation (9)). Our Fisher forecast predicts a 1σ statistical uncertainty of $\partial \sigma_8 / \sigma_8 = 0.021$ for the Euclid [O III] survey in the case where other CDM parameters are marginalized over. While this forecast is optimistic (Section 2.1), it implies that a significant bias could result from ignoring interlopers if $f_c$ is not known to be smaller than a fraction of a percent. This is consistent with the calculations of P16, who found that even a 0.15%-0.30% interloper fraction could bias the growth rate measurements by more than 10% of the statistical error.

4.4. Constraining the Interloper Power Spectrum and $f_c$ in Cross-correlation

In the calculations above, we fixed the interloper power spectrum, motivated in part by the fact that, for the Euclid or WFIRST [O III] surveys—which are among the most prone to redshift misidentification (e.g., P16)—the primary contaminant is Hα ELGs, which are the main cosmology target of these experiments. As a result, measurements of the Hα power spectrum will be obtained at a higher precision than the contaminated [O III].

If a separate catalog of the ELGs that were misidentified is available, a cross-correlation with the contaminated catalog can also provide a precise constraint on $f_c$. Consider taking the Euclid Hα catalog from the same patch of sky as the [O III] catalog and remapping the three-dimensional coordinates “by hand,” as if the ELGs in the main Hα sample had all been misidentified as [O III]. Assuming the target and interloper populations do not overlap in redshift, the expected cross-power spectrum between this remapped Hα sample and the contaminated [O III] sample will be given by

$$\langle \hat{P}_{\alpha\ell}(k) \rangle = f_c \gamma_2 \gamma_3 \langle \hat{P}_{\text{int},\ell}(\gamma_\perp k_\perp, \gamma_{||} k_{||}) \rangle, \quad (22)$$

that is, like the second term of Equation (8), except scaling like $f_c$, rather than $f_c^2$. The auto-power spectrum of the remapped Hα catalog, on the other hand, does not depend on $f_c$:

$$\langle \hat{P}_{\alpha,\ell}(k) \rangle = \gamma_2 \gamma_3 \langle \hat{P}_{\text{int},\ell}(\gamma_\perp k_\perp, \gamma_{||} k_{||}) \rangle. \quad (23)$$

The combination of the contaminated power spectra, $\hat{P}_{\alpha\ell}$; remapped contaminated cross-spectrum, $\hat{P}_{\alpha,\ell}$; and remapped auto-spectrum, $\hat{P}_{\alpha,\ell}$, can simultaneously constrain the shape of the interloper power, $f_c$, and the parameters we are trying to measure from the contaminated sample. We demonstrate this for constraints on CDM parameters from a Euclid-like [O III] survey with $f_c = 0.2$ in Figure 4. The results are similar for the BAO parameters. Here we do not assume a particular model for the Hα power spectrum and instead consider the power spectrum in each bin of the monopole, quadrupole, and hexadecapole power spectrum as a free parameter. Even in this more conservative case, the addition of the $\hat{P}_{\alpha\ell}$ and $\hat{P}_{\alpha,\ell}$ spectra allow cosmological constraints to be recovered from the contaminated [O III] sample as if $f_c$ was known perfectly. In fact, there is a small (percent level) improvement in constraints over the forecast for the contaminated [O III] sample with $f_c$ fixed. In other words, we can do a little better than the black lines in Figures 2 and 3 would suggest. This is because adding precise measurements of the Hα power from galaxies tracing the same modes of the density field as the misidentified Hα ELGs also effectively removes the Hα sample variance as an error source in the contaminated power spectra. Unfortunately, the improvement is small because the Hα interloper sample variance is only a small contribution to the total power spectrum error budget.

One possible concern with this approach is uncertainty in the coordinate remapping that we should apply by hand to the separate catalog of Hα ELGs (the $\gamma_\perp$ and $\gamma_{||}$ factors). We cannot completely ignore this uncertainty, since, if the remapping was known perfectly in advance, there would be little motivation to make the BAO measurement from secondary samples like the Euclid [O III] in the first place. Fortunately, the remapping can be determined empirically simply by varying the transverse and line-of-sight rescaling and...
recomputing the remapped contaminated cross-spectrum. If either of the rescaling factors is substantially incorrect, the modes of the remapped ELG overdensity field will not fall in the same power spectrum bin as the corresponding modes of the interlopers in the contaminated sample, and the cross-spectrum will be consistent with zero. Since coarse binning of the power spectrum is already undesirable for BAO constraints, the power spectrum will be consistent with zero. Since coarse binning of the interlopers in the contaminated sample, and the cross-spectrum are also marginalized over in this case. These Fisher forecasts were performed for a Euclid-like [O III] survey with contamination fraction $f_c = 0.2$ and an external pure H$\alpha$ catalog.

4.5. Effect of Changing Range of $k$ Values

We investigated the effect of changing the range of scales used in the forecasting. We considered lowering $k_{\text{max}}$, representing excluding information from small scales that are challenging to model due to nonlinear clustering. We also considered increasing $k_{\text{min}}$, which has less physical motivation and was done primarily to test the assumptions discussed in Section 2.4. The main conclusions regarding the BAO and RSD constraints discussed earlier in this section still held, and no new behavior was observed. Note that we are always comparing against pure [O III] constraints with the same $k_{\text{min}}$ and $k_{\text{max}}$.

Quantitatively, the main effect we found from changing the range of scales was a worsening of the degeneracy between $\sigma_8$ and $f_c$ in the anisotropic BAO case, where the $\Lambda$CDM parameters are varied. This is not surprising, given that we are removing information by restricting the available k-modes. We give some example results here for $f_c = 0.2$. For the $k_{\text{max}} = 0.3 h$ Mpc$^{-1}$ case shown in the top right panel of Figure 2, the $\alpha_\perp$ error is increased beyond the loss of information in the power spectrum by 10%. For $k_{\text{max}} = 0.225 h$ Mpc$^{-1}$, the corresponding increase is 19%. For $k_{\text{max}} = 0.15 h$ Mpc$^{-1}$, the $\alpha_\parallel$ error is increased by 46%. Increasing $k_{\text{min}}$ from 0.005 up to 0.1 h Mpc$^{-1}$ (holding $k_{\text{max}}$ fixed to 0.3 h Mpc$^{-1}$) similarly resulted in an increase of 46%. For $\alpha_\parallel$, or $\alpha$ in the isotropic BAO case, the results are consistent with Figure 2 within a few percent. Furthermore, the increase in BAO scale (isotropic or anisotropic) for the cases with $\Omega_0$ and $n_s$ fixed or with $f_c$ fixed are consistent with the increase in power spectrum uncertainties within a few percent, also similar to Figure 2, even for $k_{\text{max}} = 0.15 h$ Mpc$^{-1}$. External constraints on portions of the $\Lambda$CDM parameter space or $f_c$ are effective at breaking the $f_c - \alpha_\parallel$ degeneracy, even when the range of scales is limited.

We found that reducing $k_{\text{max}}$ did not significantly impact the degeneracy between $f_c$ and $\sigma_8$ shown in Figure 3. A more detailed analysis of the effect of including nonlinearity in the ELG power spectrum and marginalization over parameters that describe how the nonlinearity is modeled, particularly for constraints on neutrino mass, are left to future analysis including survey-specific systematic effects.

5. Discussion

In Section 4 we performed calculations for a Euclid-like [O III] ELG survey ($1.5 < z < 2.3$) contaminated by H$\alpha$ (0.9 $< z < 1.5$) and identified two ways interlopers degrade cosmological constraints, even when included in the likelihood modeling: loss of signal-to-noise in the target power spectrum and additional degeneracy with the contamination fraction, $f_c$.

In this section, we expand our analysis to consider implications for other lines and surveys.

5.1. Varying Galaxy Density and Implications for WFIRST

We repeated the calculations from Section 4 with higher [O III] source densities, up to 10 times the original 282 deg$^{-2}$. This would roughly correspond to a flux limit of $1.0 \times 10^{-16}$ erg s$^{-1}$ cm$^{-2}$ based on Table 2 of Colbert et al. (2013), which predicted 3056 deg$^{-2}$. While the absolute constraining power of the survey would dramatically increase, the effect of interlopers for a given $f_c$ is very similar to the results shown in Figures 2 and 3. Compared to the increase in power spectrum errors, the BAO errors for the case where $\Lambda$CDM parameters are all varied are slightly increased with the higher galaxy density. For example, the increase in $\alpha_\parallel$ error is around 20% larger than the increase in power spectrum error, whereas in Figure 2, the difference is more like 10%–15% (compare red cross to black line). The degeneracy between $f_c$ and power spectrum amplitude for the RSD constraint is present in the same way.

We have not repeated the calculations for the WFIRST High Latitude Survey volume and expected galaxy density (Section 3.3), but since the number density of [O III] ELGs is around twice the density we used in Section 4 we do not expect any substantially different behavior for a given $f_c$. That said, the higher signal-to-noise cut for ELG detection in WFIRST would both reduce the probability of misidentification occurring and make dealing with additional complications, such as spatial variation in $f_c$, less challenging.
5.2. Impact for Different Target Lines

The top row of panels of Figure 5 shows the effect of interlopers on the multipole power spectra for three different ELG samples: a Euclid-like [O III] sample contaminated by Hα (z = 1.2), a Euclid-like Hα sample contaminated by [O III], and a HETDEX-like Lyα sample (z = 2.7) contaminated by [O II] (z = 0.2). A contamination fraction $f_c = 0.12$ is shown in each case. The contaminated power spectra have been divided by the overall suppression factor $(1 - f_c)^2$ (see Figure 8) to make the impact of interlopers on different multipoles clearer. Bottom row: increase in BAO scale errors for $f_c = 0.01, 0.03, 0.05, 0.07,$ and $0.09$ when $f_c$ are marginalized over (compare to top right panel of Figure 2). The loss of the signal-to-noise ratio in the target ELG power spectra is shown for comparison (dashed lines; see Equation (21)).

![Figure 5: Top row: comparison of pure (dashed lines) and contaminated (solid lines) multipole power spectra for (left to right) a Euclid-like [O III] sample (z = 1.9) contaminated by Hα (z = 1.2), a Euclid-like Hα sample contaminated by [O III], and a HETDEX-like Lyα sample (z = 2.7) contaminated by [O II] (z = 0.2). A contamination fraction $f_c = 0.12$ is shown in each case. The contaminated power spectra have been divided by the overall suppression factor $(1 - f_c)^2$ (see Equation (8)) to make the impact of interlopers on different multipoles clearer. Bottom row: increase in BAO scale errors for $f_c = 0.01, 0.03, 0.05, 0.07,$ and $0.09$ when $f_c$ are marginalized over (compare to top right panel of Figure 2). The loss of the signal-to-noise ratio in the target ELG power spectra is shown for comparison (dashed lines; see Equation (21)).](image)

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ELGs are at higher redshift than the targets (e.g., when [O III] contaminates Hα), the quadrupole and hexadecapole power contributed by the interlopers is suppressed relative to the monopole and may become negative. This arises because γ_⊥ < 1, while (as in all cases considered) γ_∥ ≈ 1. Transverse modes receive an enhancement because the matter power spectrum $P_m(k)$ is falling with $k$ for $k > 0.02 \, h \, \text{Mpc}^{-1}$ (past the peak of the power spectrum, note that we show $k^3P(k)$ in plots, not $P(k)$ itself). Apart from the first few $k$ bins, then, the power at $γ_∥k$ is larger than the power at $k$. This effect counteracts the usual RSD enhancement of line-of-sight power, which is what produces positive $ℓ = 2$ and $ℓ = 4$ power for analysis in the correct coordinate system and why the $ℓ = 2$ and $ℓ = 4$ spectra are largely unchanged by interlopers in the middle panel.

The converse happens for interlopers at a lower redshift than the targets, and the power at $ℓ = 2$ and $4$ is enhanced relative to $ℓ = 0$ as a result. This is clearly apparent in the HETDEX case because of the large difference in redshift between the targets ($1.9 < z < 3.5$) and interlopers ($z < 0.5$). The remapping factors are $γ_∥ = 7.3$ and $γ_1 = 0.9$, so that modes at $γ_∥k$ have much lower power than $γ_1k$.

The panels in the bottom row of Figure 5 show the forecast increase in BAO scale constraints as a function of contamination fraction $f_c$, for the case where the ΛCDM parameters are marginalized over in addition to $f_c$ itself. The results for Euclid Hα contaminating [O III] are the same as in the top right panel of Figure 2. To highlight the impact of interlopers, we show fractional increases in BAO error forecasts over pure samples for each target line, as in Section 4. We include the fractional loss of overall power spectrum signal-to-noise ratio for comparison (Equation (21); dashed black lines).

A given $f_c$ leads to a far smaller increase in BAO scale uncertainty for the case where [O III] is contaminating Hα, compared to when the Hα is contaminating [O III]. The degeneracy between $f_c$ and the ΛCDM parameters, particularly $Ω_m$ and $h$, is weaker. In other words, the increase in the monopole power caused by the higher-redshift [O III] interlopers in the middle panel of the top row of Figure 5 is harder to mimic through rearranging other parameters. We note that the number density of Hα ELGs in our Euclid-like forecasts is almost 14 times higher than [O III] (3900 deg$^{-2}$ compared to 282 deg$^{-2}$; Table 1), which means the shot-noise errors on the power spectrum bins are far lower. We performed calculations with the Hα density decreased by hand to match [O III] to understand how this number density difference impacts the BAO error forecasting. Even with this change, the increase in BAO uncertainty for the [O III] targets was worse than for the Hα targets by a factor of 2–4 for a given $f_c$, verifying the importance of the relationship between target and interloper redshift and the $γ_∥$ and $γ_1$ factors in determining the degeneracy between BAO α factors and $f_c$. As in Section 5.1, we emphasize here that looking at the ratio of forecast uncertainties for the contaminated sample to the pure sample substantially reduces the importance of effects like number density that impact both.

The forecasts for the HETDEX BAO uncertainties are fairly similar to those for the Euclid [O III] targets despite the large difference in volume factor $γ_1^2/γ_∥^2$ mentioned above. While a given $f_c$ produces a larger difference in the multipole power spectra for HETDEX, the effect on the power spectrum errors is fairly small in either case for $f_c < 0.1$. We experimented with varying the HETDEX and [O III] ELG number densities and found that this did not have a large impact on the results shown in Figure 5. We note that the impact of HETDEX interlopers for a given $f_c$ may be inaccurate here for two reasons. First, our use of the linear matter power spectrum in calculations is a poor approximation for the remapped [O III] ELGs because the large transverse dilation factor $γ_∥ ≈ 7$ means that smaller, more highly nonlinear scales are being probed for a given $k$ in the coordinates of the Lyα targets. Additionally, the value of $γ_1$ itself is highly sensitive to the effective redshift adopted for the [O II] sample, since the sample extends to redshift zero; for example, Leung et al. (2017) found $γ_1$ between 5.3 and 29.8 for [O II] galaxies at redshift 0.305 and 0.044, respectively.

Fixing $f_c$ brings the increase in BAO scale uncertainty into agreement with the black dashed lines for the points shown in the left and right panels of the bottom row of Figure 5, as in Figure 2. There is little effect for the points shown in the middle panel because the degeneracy with $f_c$ is weaker. In fact, one can see in this panel that some of the BAO points lie below the black dashed line even when $f_c$ is varied. The results for RSD are not shown in Figure 5, but the degeneracy between $f_c$ and $f_{g8}$ is present and degrades the constraints by a factor of at least a few for $f_c < 0.1$, as in Figure 3. While the details of the interloper remapping and absolute constraining power of the surveys vary significantly, an external constraint on $f_c$ is essential for recovering the RSD information in all cases we considered.

5.3. Future Directions

We have focused on the BAO scale and RSD as the primary cosmological observables from ELG surveys and performed forecasts for the impact of line misidentification with some simplifying assumptions. In the future, it would be useful to extend the investigation to other aspects of galaxy clustering, including BAO reconstruction (e.g., Padmanabhan et al. 2012), constraints from the bispectrum and other higher-order statistics, choice of redshift binning, and clustering on scales larger than the BAO (including associated systematics; e.g., Kalus et al. 2019).

We have highlighted the importance of an external constraint on $f_c$ for RSD constraints. In Section 4.4 we described how a cross-correlation with a separate catalog containing the misidentified line can achieve this; however, such a catalog may not be available for low-level contaminants. Another approach is to use deeper subsurveys, where multiple lines are detected and unambiguously identified in each ELG spectrum. An estimate of the expected contamination rate arising from the Hα contamination rate arising from the misidentification rate can be obtained for each potential interloper line, for example, by adding artificial noise to the deeper spectra. The Euclid survey strategy includes observing several small regions (tens of deg$^2$) at a depth greater than the main survey (Laureijs et al. 2011). Demonstrating that low-level contaminants can be constrained sufficiently well to break the degeneracy between $f_c$ and $f_{g8}$ using realistic simulated spectra would be valuable.

Simulated data are also required to understand the impact of spatial variation in misidentification rate arising from the complex weighting or masking applied to real spectroscopic surveys, for example, to deal with contamination from bright stars, zodiacal dust emission, or variability in optical
performance across the field of view of the instrument. One approach would be to extend galaxy weighting schemes, already adopted in current BAO surveys like BOSS (e.g., Ross et al. 2012), in order to down-weight ELGs with spectra more prone to misidentification (e.g., with lower signal-to-noise ratio) before computing clustering statistics. In principle, this method could help recover some of the information lost in the presence of interlopers, although again, this should be demonstrated quantitatively.

6. Conclusions

We used Fisher forecasts to investigate the impact of contamination of ELG catalogs due to spectroscopic line misidentification. We used redshift-space multipole power spectra to describe the anisotropic distortion of the interloper power spectrum due to incorrect mapping of the galaxy angular position and redshift to a three-dimensional position. Using calculations performed for a Euclid-like [O III] survey contaminated by Hα interlopers as an example, we found that cosmological constraints on the BAO scale and RSD parameter $f_{\sigma_{8}}(z)$ are degraded in the presence of interlopers in two ways.

(i) The presence of interlopers decreases the signal-to-noise ratio of the target ELG power spectra, even if $f_{c}$ and the shape of the interloper power are known perfectly. This is because the contaminated sample contains two populations tracing LSS at different redshifts, which do not correlate (neglecting small corrections, e.g., from gravitational lensing). Recovering this information requires additional information or weighting on a spectrum-by-spectrum basis. This will be investigated further in future work.

(ii) Degeneracy with $f_{c}$ can further degrade constraints. This increases BAO errors at the 10%−20% level for the case where the ΛCDM parameters determining the broadband power spectrum shape are marginalized over, although this assumes that there is an external template for the interloper power. For the RSD, there is a near-complete degeneracy between $f_{c}$ and the power spectrum amplitude ($\sigma_{8}$ or $f_{\sigma_{8}}$) when $f_{c}$ is too small for the presence of interlopers to be detected at high significance from the contaminated power spectrum alone.

For the BAO, we found that external constraints on a portion of the ΛCDM parameter space (e.g., $\Omega_{b}$ and $n_{s}$ from CMB measurements), or on $f_{c}$, remove the degeneracy with $f_{c}$. In the RSD case, only a constraint on $f_{c}$ achieved this, indicating that an estimate for the contamination fraction is important even for low-level contaminants. We considered constraining $f_{c}$ using cross-correlation with a separate catalog containing the misidentified ELGs. This appears to be an effective approach for constraining both $f_{c}$ and the shape of the interloper power for the Euclid and WFIRST Hα and [O III] ELGs, where both emission lines are cosmology targets, but may be challenging for minor contaminants.

More realistic calculations, including systematic effects that may impact line identification, such as spatial variability in spectral quality and signal-to-noise ratio, are necessary to quantify the impact of line misidentification more accurately. The formalism and results we have presented will help guide these future efforts.

The analysis in this paper complements that recently presented by Grasshorn Gebhardt et al. (2019), which focuses on calculations in two-dimensional Fourier space, starting from Equation (6). Our results focus on BAO and RSD forecasts. Grasshorn Gebhardt et al. (2019) examined the cross-correlation method discussed in Section 4.4 and addressed the case of two-way contamination for HETDEX and WFIRST. Note that the remapping factors we call $\gamma_{i}$ and $\gamma_{j}$ are denoted $\alpha$ and $\beta$ by Grasshorn Gebhardt et al. (2019). The two projects arose from some earlier common discussion, but the calculations were performed and manuscripts prepared independently.

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Appendix A

Power Spectrum Formalism

The redshift dependence of the quantities below is omitted for brevity. The fractional matter overdensity, $\delta_{m}(x)$, is defined as

$$\delta_{m}(x) = \frac{\rho_{m}(x) - \bar{\rho}_{m}}{\bar{\rho}_{m}},$$

(24)

where $\rho_{m}(x)$ is the matter density at position $x$ and $\bar{\rho}_{m}$ is the mean density (averaged over position). The Fourier coefficients of the matter overdensity field are denoted $\delta_{m}(k)$. For statistically isotropic density fluctuations, the matter power spectrum $P_{m}(k)$ is defined as

$$\langle \delta_{m}(k) \delta_{m}^{*}(k') \rangle = \delta_{D}^{3}(k - k')P_{m}(k),$$

(25)

where the angle brackets represent averaging over a large number of realizations of the density field, $\delta_{D}^{3}$ is the three-dimensional Dirac delta, and $k = |k|$.

Galaxies are biased tracers of the underlying matter density. In this work, we assume that galaxies are linear tracers, meaning that the galaxy and matter overdensities are related by a single scale-independent bias factor, $b_{g}$. For our purposes, galaxies are discrete objects, and the two-point product of galaxy overdensities produces a shot-noise component $1/n_{g}$, where $n_{g}$ is the galaxy number density:

$$\langle \delta_{g}(k) \delta_{g}^{*}(k') \rangle = \delta_{D}^{3}(k - k')\left[P_{g}(k) + \frac{1}{n_{g}}\right].$$

(26)

This shot-noise contribution is subtracted off whenever we are estimating the galaxy power spectrum; however, it still contributes to the power spectrum uncertainties. We neglect any difference between the actual number density of galaxies in some surveyed volume and the mean number density in calculations.

Galaxy clustering is anisotropic due to the peculiar motion of galaxies relative to the Hubble flow. Consequently, $P_{g}(k)$ depends on the direction of $k$ rather than just the magnitude. In linear theory, the three-dimensional power spectrum is given by
(e.g., Kaiser 1987; Hamilton 1998)

$$P_g(k) = (1 + \beta \mu_k^2)^2 P_g(k),$$  \hspace{1cm} (27)

where $\beta = f/b_v$, with $f$ the derivative of the cosmological growth rate, and $\mu_k$ is the cosine of the angle between the line of sight and $k$ (i.e., $\mu_k = k_z/k$). To account for this anisotropy, we work with the multipole galaxy power spectra, $P_{g,\ell}(k)$, which are related to the full three-dimensional power spectrum, $P(k)$, by

$$P_{g,\ell}(k) = \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu_k \delta_g(k) \delta_g^*(k) P_\ell(\mu_k) P(\mu_k),$$  \hspace{1cm} (28)

where $P_\ell$ are Legendre polynomials. In linear theory, only the monopole, quadrupole, and hexadecapole moments ($\ell = 0, 2, 4$) of the multipole power spectra are nonzero. The integrals in Equation (28) can be evaluated analytically and produce

$$P_{g,0}(k) = \left(1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2 \right) b_v^2 P_m(k),$$

$$P_{g,2}(k) = \frac{4}{3} \beta b_v^2 P_m(k),$$

$$P_{g,4}(k) = \frac{8}{35} \beta^2 b_v^2 P_m(k).$$  \hspace{1cm} (29)

To estimate the multipole power spectra given a particular realization of the galaxy overdensity field $\delta_g$, we define an estimator,

$$\hat{P}_{g,\ell}(k) = \frac{2\ell + 1}{2} \int_{-1}^{1} d\mu_k \delta_g(k) \delta_g^*(k) P_\ell(\mu_k) - \frac{\delta_{10}}{n_g}. $$  \hspace{1cm} (30)

Thanks to the shot-noise subtraction, this results in an unbiased estimate, so that $\langle \hat{P}_{g,\ell}(k) \rangle = P_{g,\ell}(k)$. We can then compute the covariance of the estimator as

$$C_{\ell,\ell'}(k, k') = \langle \hat{P}_{g,\ell}(k) \rangle \langle \hat{P}_{g,\ell'}(k') \rangle \langle \hat{P}_{g,\ell}(k') \rangle \langle \hat{P}_{g,\ell'}(k) \rangle$$

$$= \frac{(2\ell + 1)(2\ell' + 1)}{4} \int_{-1}^{1} d\mu_k d\mu_k'$$

$$\times \left[ \langle \delta_g(k) \delta_g^*(k) \rangle \langle \delta_g(k') \delta_g^*(k') \rangle \right] P_\ell(\mu_k) P_{\ell'}(\mu_k')$$

$$= \frac{(2\ell + 1)(2\ell' + 1)}{4} \int_{-1}^{1} d\mu_k d\mu_k'$$

$$\times \left[ \delta_{\ell\ell'}(k - k') + \delta_{\ell\ell'}(k + k') \right]$$

$$\times \left[ \langle 1 + \beta \mu_k^2 \rangle^2 b_v^2 P_m(k) + \frac{1}{n_g} \right] P_\ell(\mu_k) P_{\ell'}(\mu_k).$$  \hspace{1cm} (31)

These integrals also have analytic solutions that can be found, for example, in the Appendix of Yoo & Seljak (2015).

For convenience, we provide them here:

$$C_{0,0}(k, k) = \frac{1}{n_g^2} + \frac{2}{n_g} \left(1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2 \right) b_v^2 P_m(k)$$

$$+ \left(1 + \frac{4}{3} \beta + \frac{6}{5} \beta^2 + \frac{4}{7} \beta^3 + \frac{1}{9} \beta^4 \right) b_v^2 P_m^2(k),$$

$$C_{0,2}(k, k) = \frac{8}{n_g} \beta \left(\frac{1}{3} + \frac{1}{7} \beta \right) b_v^2 P_m(k)$$

$$+ 8 \beta \left(\frac{1}{3} + \frac{3}{7} \beta + \frac{5}{21} \beta^2 + \frac{5}{9} \beta^3 \right) b_v^2 P_m^2(k),$$

$$C_{0,4}(k, k) = \frac{16}{35n_g} \beta^2 b_v^2 P_m(k) + 48 \beta^2$$

$$\times \left[ \frac{1}{35} + \frac{2}{7} \beta + \frac{1}{143} \beta^2 \right] b_v^2 P_m^2(k),$$

$$C_{2,2}(k, k) = \frac{5}{n_g^2} + \frac{10}{n_g} \left(1 + \frac{22}{21} \beta + \frac{3}{7} \beta^2 \right) b_v^2 P_m(k)$$

$$+ 5 \left(1 + \frac{44}{21} \beta + \frac{18}{7} \beta^2 \right)$$

$$+ \frac{340}{231} \beta^3 + \frac{415}{1287} \beta^4 \right) b_v^2 P_m^2(k),$$

$$C_{2,4}(k, k) = \frac{16}{17n_g} \beta (3 + \frac{17}{11} \beta^2) b_v^2 P_m(k) + 16 \beta$$

$$\left(3 + \frac{51}{77} \beta + \frac{435}{1001} \beta^2 + \frac{15}{143} \right) b_v^2 P_m^2(k),$$

$$C_{4,2}(k, k) = \frac{9}{n_g^2} + \frac{18}{n_g} \left(1 + \frac{78}{77} \beta + \frac{1929}{5005} \beta^2 \right) b_v^2 P_m(k)$$

$$+ 9 \left(1 + \frac{156}{77} \beta + \frac{11,574}{5005} \beta^2 \right)$$

$$+ \frac{1308}{2431} \beta^3 + \frac{711}{2431} \beta^4 \right) b_v^2 P_m^2(k).$$  \hspace{1cm} (32)

For a pure galaxy sample (before interlopers), then, the multipole power spectra and covariance depend on the galaxy bias, $b_v$; galaxy number density, $n_g$; and cosmological parameters that determine $\beta$ and $P_m(k)$. Note that the sample variance and shot-noise contributions to the covariance do not separate because the power spectrum covariance is a four-point function of the density field.

**Appendix B**

**Comparison with Lognormal Simulations**

Here we provide some more details of the comparison with mock galaxy catalogs from lognormal simulations using a publicly available code\textsuperscript{14} described by Agrawal et al. (2017). The code generates catalogs of three-dimensional galaxy position and velocity vectors by Poisson sampling the density field given an input matter power spectrum, galaxy bias, mean ELG density, and choice of resolution scale, using fast Fourier transform (FFT). We first produce two separate ELG catalogs for the target and interloper lines within cuboid volumes with side lengths specified by the sky area and redshift range for the survey under

\textsuperscript{14} https://bitbucket.org/komatsu5147/lognormal_galaxies
consideration. We then randomly select galaxies from the interloper line catalog to be misidentified according to the specified contamination fraction, \( f_c \), remap their three-dimensional positions as \( (x_{\text{int}}, y_{\text{int}}, z_{\text{int}}) \rightarrow (x_{\text{int}}, y_{\text{int}}, \gamma_x z_{\text{int}}, \gamma_y z_{\text{int}}) \), and add them to the catalog of target ELGs. Finally, the code computes the redshift-space multipole power spectra of the contaminated catalog.

Note that some coupling exists between the multipoles of the power spectrum computed from the simulations, as described in Appendix D.2.3 of Agrawal et al. (2017). Recovering an unbiased estimate for each multipole to compare with our analytic calculations then requires a deconvolution with a multipole-mixing matrix. On large scales \( k \), this deconvolution is not always numerically stable, which can result in fluctuations in the multipole power spectra at low \( k \). An alternative option in the code is to embed the cuboid survey volume in a larger cube, then perform FFT on the cube, with the density field set to zero outside the survey box. While this means that we avoid the numerical issue associated with the multipole deconvolution, it has the downside of introducing coupling between \( k \) bins. We opted to leave the \( k \) bins uncorrelated and accept some numerical instability in the recovery of the large-scale power in the lognormal simulations.

The main goal of computing the simulated power spectra was to verify the analytic calculations used in our Fisher forecasts, particularly to check that each of the terms in the power spectrum covariance was being computed correctly. The green crosses in Figure 6 show the sample variance of the monopole power spectrum estimated from 500 simulations for a Euclid-like [O III] survey with H\( \alpha \) interlopers (Table 1) and \( f_c = 0.2 \). The scatter in the points is due to the finite number of simulations, except for the second and fourth bins, where the deconvolution effect mentioned above causes larger scatter. The solid lines show the variance computed using the approximations described in Section 2.4. The contribution from the [O III] variance is given by the first line of Equation (32) multiplied by \( (1-f_c) \) and scaled by the number of modes in each \( k \) bin. The contribution from the interloper H\( \alpha \) ELGs is computed from Equation (9) and includes the effect of coordinate remapping. Note that because the power spectrum variance is a four-point product of the density field, there is also a contribution from two factors of [O III] ELG overdensity and two H\( \alpha \) labeled “[O III]–H\( \alpha \),” even though the populations are independent. All of the lines shown in Figure 6 include both sample variance and shot noise. The agreement between the simulated and calculation covariance is similar for other multipoles.

The Fisher forecasts assume that the power spectrum estimated for each bin is Gaussian-distributed. We verify that this is a reasonable approximation by examining the histogram of \( P_{m}(k_b) \) values from the lognormal simulations. Figure 7 shows the distribution of the first two and last two bins of the monopole power spectrum from the same 500 realizations as in Figure 6. The agreement indicates that the likelihood is sufficiently Gaussian even for the large scales where the number of modes in each bin is smallest.

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