Determination of the satellite attitude motion according to the onboard measurements data

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Abstract. The methods for reconstruction of the rotational motion of the Earth scientific satellites from measurements of onboard sensors are presented in this article. The goal of this reconstruction is to gain the data for the analysis of the residual microaccelerations that were affecting during space experiments. It is sufficient to know the quasi-static (low frequency) component of microacceleration, to analyze the results of many experiments with gravitationally sensitive systems and processes. The quasistatic component of the microacceleration has frequencies less than 0.01 Hz and is determined from information of the satellite motion by calculation.

1. Introduction
The methods based on the complete (dynamic and kinematic) equations of the satellite's rotational motion were applied in these articles [1–7]. These methods were used only one type of measurements: the Earth’s Magnetic Field (EMF) [1, 2, 6, 7], or the satellite’s angular velocity [3], or the satellite’s microacceleration [5]. The motion was reconstructed by the least squares method using the solutions to the equations of satellite rotational motion. The time intervals in which these equations make reconstruction possible were from one to five orbital revolutions. To get an idea on microaccelerations and satellite motion during an entire flight, the motion was reconstructed in several tens of such intervals [1, 2, 6, 7]. And in the paper [7] were proposed a method for motion reconstruction suitable for an interval of arbitrary length. The method is based on the Kalman filter.
The complete equations contain explicit expressions for the moments of external forces applied to the satellite, and are intended to describe uncontrolled motion. Such expressions may not be accurate enough. And results of methods, which are based on complete equations, may be checked by the presented methods. Presented methods are based only on the kinematic equations and are used the measurements data of both types – the EMF and the satellite’s angular velocity. The possibility of constructing a technique for determining rotational motion on the basis of only kinematic equations has been known for a long time. In the article [8] was submitted, that the onboard measurements of the Earth magnetic field and the angular velocity, provide the opportunity for the real-time satellite's motion determining. In this article are presented two views to the determining of the satellite’s rotational motion: the integral-statistic method, which based on the least squares method, and the method, based on the Kalman Filter. The rotational motion of the satellites Foton-12 [9], Foton M-2 [10] and Foton M-3 [11] were constructed, using the integral-statistic method. An updated version of this method is represented in this paper.
The monitoring method, based on kinematic equations, will become the main one for processing experimental data, received from perspective satellites, which will be flying in the oriented mode. In this article is presented method to solve the problem, which was mentioned above, of constructing
rotational motion during an entire flight. This method is based on the Kalman Filter and the kinematic equations. The methods of reconstructing uncontrolled rotational motion of a low-orbit satellite are described using the example of Foton M-3.

2. The angular rate and the Earth’s magnetic field strength measurement data
On board the satellite were four three-component magnetometers included in the DIMAC equipment [6, 7, 11]. The equipment was designed to measure microaccelerations on board the satellite. Its main sensors were accelerometers.

Let $Ox_1x_2x_3$ is the instrumental coordinate system rigidly fixed to the satellite. The EMF and the satellite’s angular velocity onboard measurement data are interpreted in this system. Point $O$ is the satellite’s center of mass, axis $Ox_1$ is parallel to the satellite’s length wise axis and is directed from the descending vehicle toward the instrument module.

The DIMAC system carried out 9 sessions of angular velocity measurements that covered 84-minute lengths of time. The measurement data were transmitted to the Earth by the telemetric channel, after the session.

The measurement data obtained during one session is a collection of numbers:

$$t_0^{(\Omega)}, \Omega_1^{(n)}, \Omega_2^{(n)}, \Omega_3^{(n)} (n = 0, 1, \ldots, N_\Omega),$$

where $\Omega_i^{(n)}$ $(i = 1, 2, 3)$ – the values of the satellite angular velocity components at the time $t_0^{(\Omega)}$, $t_{n+1}^{(\Omega)} - t_n^{(\Omega)} = 12$ s in the instrumental coordinate system. These data were approximated by discrete Fourier series, independently for each vector component, for a posteriori approximation of satellite motion [9–11].

Magnetic measurements were carried out during the flight. Measurements of different magnetometers were digitized for combined time instants, the intervals between which were varied from 1 to 12 s; on average, they were about 5 s. The data were represented as a set of numbers:

$$t_0^{(H)}, h_1^{(n)}, h_2^{(n)}, h_3^{(n)} (n = 0, 1, \ldots, N_H),$$

where $h_i^{(n)}$ $(i = 1, 2, 3)$ are the measured values of the vector components of the magnetic field strength at instant $t_0^{(H)}$, $t_1^{(H)} < t_2^{(H)} < \ldots < t_{N_H}^{(H)}$. We believe that these components with an accuracy up to constant shifts, as well as small errors of measurements and coordinate transformations, coincide with the components of the EMF strength in the $Ox_1x_2x_3$ coordinate system.

3. Kinematic equations of the satellite motion
We assume satellite is a rigid body. The motion of the center of mass of the satellite is the Kepler ellipse. The elements of this motion are determined from the data of trajectory measurements [9–11]. We will use the instrumental coordinate system $Ox_1x_2x_3$ and the Greenwich coordinate system $CY_1Y_2Y_3$ for writing the equations of motion of the satellite relative to the center of mass and the formulas for the measurement data processing. Point $C$ is the center of mass of the Earth, plane $CY_1Y_2$ contains the equator, the positive semiaxis $CY_1$ intersects the Greenwich meridian, and the $CY_3$ axis is directed toward the North Pole. The position of the coordinate system $Ox_1x_2x_3$ relative to the Greenwich coordinate system will be given by the unit quaternion $Q$, when the points $C$ and $O$ are connected.

The kinematic equations for the components of the quaternion $Q$ takes form
\begin{equation}
2\dot{Q}_0 = -\sum_{i=1}^{3} Q_{0i} \omega_i, \quad 2\dot{Q}_j = Q_{0j} \omega_j + \sum_{k=1}^{3} e_{jk} Q_{0k} \omega_k, \quad \omega_i = \Lambda_{\alpha\alpha} + \chi_i(t - \tau) - \omega_e g_{3i} \quad (i = 1, 2, 3). \tag{1}
\end{equation}

Here \( e_{jk} \) is the Levi-Civita symbol, \( \omega_i \) is the components of the absolute angular velocity vector of the satellite in the system \( Ox_1x_2x_3 \), \( \chi_i(t) \) is the trignometric polynomials, approximating the angular velocity measurements, \( \Lambda_{\alpha\alpha} \) are the constants of the bias in these measurements, \( \tau \) is the offset of the time scale of the DIMAC instrument with respect to the time scale of the satellite motion control system, \( \omega_e \) is the angular velocity of the rotation of the Earth, \( \| g_{ij} \|_{i,j=1}^3 \) is the transition matrix from the system \( Ox_1x_2x_3 \) to the system \( CY_1Y_2Y_3 \).

4. Reconstruction of the satellite motion by the least squares method, using kinematic equations

Let \( I_{\Omega}(\tau) = [t_0^\Omega + \tau, t_{N_{\alpha}}^\Omega + \tau] \). The equations (1) are considered for values \( \tau \in t \), satisfying conditions \( I_{\Omega}(\tau) \subset [t_0^\Omega, t_{N_{\alpha}}^\Omega] \), \( t \in I_{\Omega}(\tau) \). As an approximation to the actual motion of the satellite on the \( I_{\Omega}(\tau) \) was taken the solution to the equations (1), giving a minimum to the functional

\[
\Phi = \sum_{i=1}^{3} \left\{ \sum_{n \in U(\tau)} \left[ h_i(t_n^\alpha) - \tilde{h}_i(t_n^\alpha) \right]^2 - N_{\tau} \tilde{\Lambda}_{Hj} \right\},
\]

\[
\tilde{\Lambda}_{Hj} = \frac{1}{N_{\tau}} \sum_{n \in U(\tau)} \left[ h_i(t_n^\alpha) - \tilde{h}_i(t_n^\alpha) \right], \quad U(\tau) = \{ n : t_n^H \in I_{\Omega}(\tau) \}.
\]

Here, \( \tilde{\Lambda}_{Hj} \) are estimates of constant shifts in the EMF measurements, \( \tilde{h}_i(t) = \sum_{j=1}^{3} H_j(t) g_{ji} \), \( H_j(t) \) is the calculated values of the components of the EMF strength, and \( N_{\tau} \) is the number of elements of the set \( U(\tau) \). \( \Phi \) is minimized according to the initial conditions of the solution \( Q(t_0^\Omega + \tau) \) and shifts \( \tilde{\Lambda}_{Hj} \ (i = 1, 2, 3) \), \( \tau \) taking into account the normalization condition \( \| Q(t_0^\Omega + \tau) \| = 1 \).

The motion of the Foton M-3 satellite was restored in 9 time intervals, corresponding to the angular velocity measurements, using a method, based on the least squares method. An example of approximating the actual motion of the satellite Foton M-3 is shown in the figure 1. The time in this figure is measured in minutes from the moment \( t_0^\Omega + \tau \) to \( t_{N_{\alpha}}^\Omega + \tau \). The graphs of expressions \( \chi_i(t) \), approximating the angular velocity measurements, and broken lines, passing through the points \((t_n^\alpha, \Omega_n^\alpha)\), were placed in the left part of the figure. The graphs of functions \( \tilde{h}_i(t) + \tilde{\Lambda}_{Hj} \) and broken lines, passing through the points \((t_n^H, h_n^\alpha) \ n \in U(\tau) \), were placed in the right part of the figure. Each broken line and the plot approximating its function are shown in the universal coordinate system. The broken lines and their approximating graphs are almost identical, with the chosen scale. The results of the constructed reconstructions are consistent with the results obtained in the articles [9–11]. The difference between the above method and the technique used [9–11] is the refusal of EMF measurement data compression. All EMF measurements are included in the processing, integration of the motion equations is carried out with an optimal step in the presented method. The calculated analogs of the measurements are calculated in the measurement points, using polynomials that interpolate the computed solution within the integration step [7]. Such polynomials for Runge-Kutta methods have entered the wide practice of computations in the last quarter of the 20th century [12]. Interpolation polynomials, enlargement of personal computers speed and memory capacity have made it possible to modify the software of integral-statistic methods.
Figure 1. Foton M-3 motion reconstructed by least squares method. Time instants $t = 0$ in plots correspond to 12:27:24 DMT on September 22, 2007, $N_e = 1109$, $\sigma = 567\gamma$.

5. Reconstruction of the satellite motion, using the Kalman filter

The satellite motion mathematical model and the measurement process in the nonlinear Kalman filter, is represented in the form [13]

$$x_n = F_n(x_{n-1}) + \eta_n, \quad y_n = G_n(x_n) + \xi_n \quad (n = 1, 2, \ldots).$$

(2)

The state vector components $x_n$ are independent parameters, by which the variables of the satellite motion equations (1) values are expressed in the nodes of the time grid $\{t_n\}$, $t_n < t_{n+1}$ ($n = 0, 1, 2, \ldots$). Shifts in measurements of the angular velocity and the magnetic field, which are constants on the $t_{n-1} < t \leq t_n$, are included in the set of the vector $x_n$ components, thus $\dim x_n = 10$.

All measurements of the EMF, falling within the $t_{n-1} < t \leq t_n$, are included into the measurement vector $y_n$. The dimension of the vectors $y_n$ depends on the $n$. These vectors may be missing, for some values of $n$, but typically $\dim y_n = 6$. The functions $F_n(x)$ and $G_n(x)$ are calculated by integrating the equations of the satellite rotational motion in the interval $t_{n-1} < t \leq t_n$. These functions are smooth, their Jacobi matrices have full rank, under the assumptions were made. And, in the (2), $\xi_n$ is the vector of random measurement errors, $\eta_n$ is the vector of random disturbances of the system. Let the vectors $\xi_m$ and $\eta_n$ are uncorrelated for all values of $m$ and $n$. The vectors $\xi_m$ and $\xi_n$, as well as $\eta_m$ and $\eta_n$, are uncorrelated for all $m \neq n$. The expected values and covariance matrices of these vectors are determined by the formulas

$$M\xi_n = 0, \quad M\eta_n = 0, \quad M\xi_n\eta_n^T = K_n, \quad M\eta_n\eta_n^T = L_n.$$
Here, the $K_n$ and $L_n$ are positive definite matrices; the order of the matrix $K_n$ is equal the dimension of the vector $y_n$, the order of the matrix $L_n$ is 9.

An estimate of the vector $x_n$ by the measurements $y_k$ ($k=1,2,\ldots,n$) is given by the expected values $\hat{x}_n$ and the covariance matrix $P_n$. An estimate of the vector $x_n$ by the measurements $y_k$ ($k=1,2,\ldots,n$) is given by the expected values $\hat{x}_n$ and the covariance matrix $P_n$. Recurrent formulas, for calculating these values, can be obtained by the least squares method. It is more comfortable to work with inverse values of the covariance matrices, if variable dimensionality of measurement vectors is a large. Let $\hat{x}_{n-1}$ и $P^{-1}_{n-1}$ are known. The vector's $x_n$ prediction has a expected value and a covariance matrix $P^*_n$, where

$$P^{-1}_n = Z_n - Z_n(L_n^{-1} + Z_n)^{-1}Z_n, \quad Z_n = (A_n^{-1})^T P^{-1}_{n-1} A_n^{-1}, \quad A_n = \frac{\partial F_n(\hat{x}_{n-1})}{\partial x}.$$

The refined value of the vector, $x_n$ by the new set of measurements $y_n$, is found from the condition of the minimum of the function

$$f(x_n) = (x_n - x^*_n)^T P^{-1}_n (x_n - x^*_n) + [y_n - G_n(x_n)]^T K^{-1}_n [y_n - G_n(x_n)],$$

following the least squares method, i.e. $\hat{x}_n = \text{argmin} f(x_n)$. The refined covariance matrix satisfies equation

$$P^{-1}_n = P^{-1}_{n-1} + B_n^T K^{-1}_n B_n, \quad B_n = \frac{\partial G_n(\hat{x}_n)}{\partial x}.$$

The nodes of the time grid $\{t_n\}$ are the midpoints of the $[t_{n-1}, t_n]$. The function is minimized by the Gauss-Newton [14] method, the initial point is the $\hat{x}_{n-1}$. The normal equations of the least squares method are

$$(P^{-1}_n + B_n^T K^{-1}_n B_n) \Delta x_n = B_n^T K^{-1}_n [y_n - G_n(x^*_n)] - P^{-1}_n (x^*_n - x^*_n),$$

$$\Delta x_n = x_n - x^*_n, \quad B_n = \frac{\partial G_n(x^*_n)}{\partial x}.$$

Here $x^*_n$ is the current approximate value of $\hat{x}_n$, which after the solution of the normal system is assigned the value $x^*_n + \Delta x_n$. The refinement of $x^*_n$ stops, when the norm $||\Delta x_n||$ becomes an acceptable small. Finally, $\hat{x}_n = x^*_n$ is accepted.

The prediction is carried out by integrating the equations of rotational motion and the corresponding equations in variations from the point $t_{n-1}$ to the point $t_n$. The iteration refinement of the function $f(x_n)$ minimum point was carried out by repeatedly integrating these equations from the point $t_{n-1}$ to the point $t_n$.

The equations of rotational motion are integrated by the difference scheme

$$Q(t_n) = Q(t_{n-1}) + \mathbf{k}_n,$$

$$\mathbf{k}_n = \left( \frac{16 - \tau_n^2}{16 + \tau_n^2} \left| \omega(t_n^\Omega) \right|^2, \quad \frac{8\tau_n \omega(t_n^\Omega)}{16 + \tau_n^2} \left| \omega(t_n^\Omega) \right|^3 \right), \quad \omega(t_n^\Omega) = \omega(t_n^\Omega) + \Delta \gamma(t_n), \quad \tau_n = t_n - t_{n-1}.$$

This scheme saves the quaternion norm, its local truncation error is on the order of $O(\tau_n^3)$. The equations in variations are integrated by a similar scheme with order local truncation error is on the order of $O(\tau_n^3)$. 


An example of approximating the motion of the satellite by the Kalman Filter is demonstrated in the figure 2. This approximation is constructed in the same time segment as the reconstruction in the figure 1. The graphs of broken lines, passing through the points \((t_n^H, \Omega_n^{(H)} + \Delta_n(t_n))\), were placed in the left part of the figure. These broken lines demonstrate the angular velocity used to construct the rotational motion by the Kalman filter. The graphs of broken lines, passing through the points \((t_n^H, \hat{h}_n(t_n^H) + \Delta_H(t_n))\), were placed in the right part of the figure 2. The broken lines are almost identical, with the chosen scale. The results of the reconstruction by the Kalman filter are also completely consistent with the results obtained by integral-statistic method.

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\[
\omega_1, \omega_2, \omega_3 \text{ (deg/s)} \quad h_1, h_2, h_3 \text{ (\gamma)}
\]

\[t \text{ (min)}\]

**Figure 2.** Foton M-3 motion reconstructed with Kalman filter. Time instants \(t = 0\) in plots correspond to 12:27:18 DMT on September 22, 2007.

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