The Rarita–Schwinger field: renormalization and phenomenology

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We discuss renormalization of propagator of interacting Rarita–Schwinger field. Spin-3/2 contribution after renormalization takes usual resonance form. For non-leading spin-1/2 terms we found procedure, which guarantees absence of poles in energy plane. The obtained renormalized propagator has one free parameter and is a straight generalization of the famous free propagator of Moldauer and Case. Application of this propagator for production of \( \Delta^{++}(1232) \) in \( \pi^+ p \to \pi^+ p \) leads to good description of total cross-section and to reasonable agreement with results of partial wave analysis.

I. INTRODUCTION

The vector-spinor Rarita–Schwinger (R.–S.) field is used to describe spin 3/2 particles in quantum field theory [1]. It is known that all theories with higher spins \( s \geq 1 \) are faced with a common problem: the corresponding field contains extra degrees of freedom related with non-leading spins \( s-1, \ldots \). Therefore some constrains should be imposed on the free field to exclude these degrees of freedom. But inclusion of interaction usually breaks these constraints and it generates the main problems. These issues for spin-3/2 were discussed in [2], [3] and in numerous following papers. Problem of consistence of constrains and interactions is discussed actively up to now [4, 5, 6, 7].

The main applications of Rarita–Schwinger formalism are related with baryon spectroscopy. But because of old theoretical problems any application for spin-3/2 baryons with necessity contains some approximations.

We prefer to investigate the dressed propagator of R.–S. field instead of equations of motion and constrains. Besides technical advantages, it allows to apply the obtained dressed propagator to description of experimental data. The full non-renormalized propagator of R.–S. field with account of all spin components was found in [8, 9]. In this paper we study the procedure of renormalization of this propagator. After it we apply the constructed renormalized propagator to production of \( \Delta^{++}(1232) \) in \( \pi^+ p \to \pi^+ p \) reaction.

II. DRESSED PROPAGATOR OF RARIA–SCHWINGER FIELD

A. Most general lagrangian of R.–S. field

Free lagrangian of the R.–S. field is defined by differential operator \( S^{\mu \nu} \) which is, in fact, the inverse propagator

\[
\mathcal{L} = \overline{\Psi} S^{\mu \nu} \Psi^\nu.
\]

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The standard form of $S_{\mu\nu}$ is (see e.g. [10])

$$S_{\mu\nu} = (\hat{p} - M)g_{\mu\nu} + A(\gamma^\mu p^\nu + \gamma^\nu p^\mu) + \frac{1}{2}(3A^2 + 2A + 1)\gamma^\mu \hat{p} \gamma^\nu + M(3A^2 + 3A + 1)\gamma^\mu \gamma^\nu.$$ (2)

Here $M$ is the mass of spin-$3/2$ particle, $p_\mu = i\partial_\mu$ and $A$ is an arbitrary real parameter. Equations of motion, following from (2), lead to constrains $p\Psi = \gamma \Psi = 0$ and therefore to exclusion of the spin-$1/2$ degrees of freedom. In other words, the corresponding terms in propagator should not have poles in energy plane.

We want to derive the most general form of the R.–S. lagrangian without additional assumption about the nature of spin-$1/2$ contributions. Depending on the choice of parameters the $s = 1/2$ components may have or have not the poles in the complex energy plane. Such construction will be useful at renormalization of the dressed R.–S. field propagator even if we require the spin-$1/2$ components to be unphysical.

Let us formulate the main requirements for lagrangian:

1. The fermion lagrangian is linear with respect to derivatives.
2. It should be hermitian $\mathcal{L}^\dagger = \mathcal{L}$ or $\gamma^0(S_{\mu\nu})^\dagger \gamma^0 = S_{\nu\mu}$.
3. The spin-$3/2$ contribution has standard pole form (to be specified below).
4. Lagrangian should not be singular at $p^2 \to 0$. This point is rather evident but it happens that some rough methods generate singularities in a propagator (see e.g. discussion in [11]).

The suitable starting point to construct the generalized lagrangian is decomposition of $S_{\mu\nu}$ in $\gamma$-matrix basis \(^{(1)}\) with arbitrary coefficients. The first requirement remains 6 complex coefficients in \(^{(1)}\)

$$S_{\mu\nu} = g_{\mu\nu} \cdot s_1 + \hat{p}g_{\mu\nu} \cdot s_4 + p^\mu \gamma^\nu \cdot s_5 + \gamma^\mu p^\nu \cdot s_6 + \sigma_{\mu\nu} \cdot s_7 + i\varepsilon_{\mu\nu\lambda\rho} \gamma_\lambda p_\rho \cdot s_{10} =$$

$$= g_{\mu\nu}(s_1 - s_7) + \hat{p}g_{\mu\nu}(s_4 - s_{10}) + p^\mu \gamma^\nu(s_5 + s_{10}) + \gamma^\mu p^\nu s_6 + s_{10}) + \gamma^\mu \gamma^\nu s_7 - \gamma^\mu \hat{p} \gamma^\nu s_{10}.$$ (3)

If we start from the $\gamma$-matrix decomposition with non-singular coefficients the fourth requirement is fulfilled automatically.

This expression satisfies the condition $\gamma^0(S_{\mu\nu})^\dagger \gamma^0 = S_{\nu\mu}$, if $s_1, s_4, s_7, s_{10}$ are real parameters while $s_5$ and $s_6 = s_5^*$ may be complex. It is convenient to introduce the new notations

$$s_1 = r_1, \quad s_4 = r_4, \quad s_7 = r_7, \quad s_{10} = r_{10}, \quad s_5 = r_5 + ia_5, \quad s_6 = r_5 - ia_5,$$

where all parameters are real.

To take into account the third requirement, we need to recognize the spin-$3/2$ part of inverse propagator. It is easy to do in the $\hat{p}$-basis (see Appendix \(^{(2)}\) for details)

$$S_{\mu\nu} = (\hat{p} - M)(P^{3/2})_{\mu\nu} + \text{(spin-1/2 contributions)}.$$ (4)

\(^{(1)}\) We use conventions of Bjorken and Drell textbook [12]: \(\varepsilon_{0123} = 1, \gamma^5 = \gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3\) except that $\sigma_{\mu\nu} = \frac{1}{2} [\gamma^\mu, \gamma^\nu]$. 

\(^{(2)}\)
Reversing the Eq. (4), we obtain propagator with the standard pole behavior of spin-$3/2$ contribution

$$G^{\mu \nu} = \frac{1}{\hat{p} - M} (P^{3/2})^{\mu \nu} + (\text{spin-1/2 contributions}).$$

Eq. (4) gives

$$S_1 = s_1 - s_7 = -M, \quad S_2 = s_4 - s_{10} = 1.$$  

So the $r_7, r_{10}$ are dependent values

$$r_7 = M + r_1, \quad r_{10} = r_4 - 1$$

and we come to four-parameter $(r_1, r_4, r_5, a_5)$ lagrangian which satisfies all the necessary requirements.

$$S^{\mu \nu} = g^{\mu \nu} (\hat{p} - M) + p^\mu \gamma^\nu (r_5 + r_4 - 1 + u_5) + p^\nu \gamma^\mu (r_5 + r_4 - 1 - u_5) + \gamma^\mu \gamma^\nu (M + r_1) - \gamma^\mu \hat{p} \gamma^\nu (r_4 - 1).$$

To build the propagator of the R.–S. field we need to reverse (7)

$$G^{\mu \nu}(p) = (S^{-1})^{\mu \nu},$$

and the Λ-basis is convenient here, see eq. (22) below.

Let us write down the denominators following from our lagrangian (7):

$$\Delta_1(W) = -M(3M + 4r_1) + 2W(Mr_5 - Mr_4 - r_1) + W^2(-3a_5^2 - 3(r_4 + r_5)^2 + 4r_5 + 2r_4),$$

$$\Delta_2(W) = \Delta_1(W \to -W).$$

The requirement the spin-1/2 contribution to be unphysical is equivalent to condition $\Delta_1 = \text{const}$ and we come to equations

$$M(r_5 - r_4) - r_1 = 0,$$

$$3a_5^2 + 3(r_4 + r_5)^2 - 4r_5 - 2r_4 = 0.$$  

One can rewrite it in terms of sum and difference $\sigma = r_5 + r_4, \delta = r_5 - r_4$

$$r_1 = M \delta,$$

$$\delta = 3(\sigma^2 - \sigma + a_5^2).$$

There exists the well-known transformation of R.–S. field

$$\Psi_\mu \to \Psi'_\mu : \quad \Psi_\mu = \theta_{\mu \nu} (B) \Psi'^{\nu},$$

where $\theta_{\mu \nu}(B) = g_{\mu \nu} + B \gamma_{\mu} \gamma_{\nu}$ and $B = b + i \beta$ is a complex parameter.

This transformation doesn’t touch the spin-$3/2$ because $(P^{3/2})^{\mu \nu}$ operator is orthogonal to $\gamma^\mu$. If one apply it to the inverse propagator (7)

$$S_{\mu \nu} \to S'_{\mu \nu} = \theta_{\mu \alpha} (B*) S^{\alpha \beta} \theta_{\beta \nu} (B),$$

We need to use different bases, so to distinguish them we use different notations: $s_i, S_i, \bar{S}_i$ for coefficients in $\gamma$-, $\hat{p}$- and Λ-basis respectively. Technical details are collected in Appendix A.
one can see that $S'_{\mu\nu}$ keeps all the properties of $S_{\mu\nu}$ (7). It means that, in fact, we have reparametrization – after transformation of the (7) we obtain the same propagator with changed parameters

$$\theta_{\mu\alpha}(B^*)S^{\alpha\beta}(r_1, r_4, r_5, a_5)\theta_{\beta\mu}(B) = S_{\mu\nu}(r_1', r_4', r_5', a_5').$$  \hspace{1cm} (13)

Using the field renormalization $\Psi'_{\mu} = \theta_{\mu\nu}(B)\Psi_{\nu}$, we can eliminate two of four parameters in free lagrangian (7). After renormalization the $\theta$-factor will appear in vertex.

Recall that the standard interaction lagrangian of $\pi N\Delta$ for R.–S. field contains this $\theta$-factor

$$\mathcal{L}_{\text{int}} = g_{\pi N\Delta}(x)\theta^{\mu\nu}(a)\Psi(x) \cdot \partial_\nu\varphi(x) + \text{H.c.}.$$  \hspace{1cm} (14)

Details of transformation can be found in [13]. Let us mention also the previous works [14, 15] where generalized R.–S. lagrangian was discussed.

### B. Dyson–Schwinger equation

The Dyson–Schwinger equation for the propagator of the R.–S. field has the following form

$$G^{\mu\nu} = G_0^{\mu\nu} + G^{\mu\alpha}\Sigma_{\alpha\beta}G_0^{\beta\nu}. \hspace{1cm} (15)$$

Here $G_0^{\mu\nu}$ and $G^{\mu\nu}$ are the free and full propagators respectively, $\Sigma^{\mu\nu}$ is a self-energy contribution. The equation may be rewritten for inverse propagators as

$$(G^{-1})^{\mu\nu} = (G_0^{-1})^{\mu\nu} - \Sigma^{\mu\nu}. \hspace{1cm} (16)$$

If we consider the self-energy $\Sigma^{\mu\nu}$ as a known value (so called ”rainbow” approximation) then the problem is reduced to reversing of relation (16).

The most convenient basis (Λ-basis) for the spin-tensor $S^{\mu\nu}(p)$ is build with use of five known tensor operators [11, 16, 17]

$$(P^{3/2})^{\mu\nu} = g^{\mu\nu} - n_1^\mu n_1^\nu - n_2^\mu n_2^\nu$$

$$(P^{1/2})^{\mu\nu} = n_1^\mu n_1^\nu, \hspace{1cm} (P^{1/2})^{\mu\nu} = n_2^\mu n_2^\nu, \hspace{1cm} (P^{1/2})^{\mu\nu} = n_2^\mu n_1^\nu. \hspace{1cm} (17)$$

Here we introduced the unit ”vectors”

$$n_1^\mu = \frac{1}{\sqrt{3}p^2}(-p^\rho + \gamma^\rho \hat{p})\hat{p}, \hspace{0.5cm} n_2^\mu = p^\mu/\sqrt{p^2}I_4, \hspace{0.5cm} (n_i \cdot n_j) = \delta_{ij}I_4, \hspace{1cm} (18)$$

$I_4$ is unit $4 \times 4$ matrix. To build a basis with good multiplicative properties we need also the off-shell projection operators $\Lambda^\pm = (1 \pm \hat{p}/\sqrt{p^2})/2$. Ten elements of Λ-basis look as

$$\mathcal{P}_1 = \Lambda^+P^{3/2}, \hspace{0.5cm} \mathcal{P}_5 = \Lambda^+P^{1/2}_{11}, \hspace{0.5cm} \mathcal{P}_5 = \Lambda^+P^{1/2}_{22}, \hspace{0.5cm} \mathcal{P}_7 = \Lambda^+P^{1/2}_{21}, \hspace{0.5cm} \mathcal{P}_9 = \Lambda^+P^{1/2}_{12},$$

$$\mathcal{P}_2 = \Lambda^-P^{3/2}, \hspace{0.5cm} \mathcal{P}_4 = \Lambda^-P^{1/2}_{11}, \hspace{0.5cm} \mathcal{P}_6 = \Lambda^-P^{1/2}_{22}, \hspace{0.5cm} \mathcal{P}_8 = \Lambda^-P^{1/2}_{21}, \hspace{0.5cm} \mathcal{P}_{10} = \Lambda^-P^{1/2}_{12}. \hspace{1cm} (19)$$

\(^3\) Below we suppose as usual parameter in vertex to be real, so $a_5 = 0$ in (7)
where tensor indices are omitted.

Decomposition of a spin-tensor in this basis has the following form:

$$S_{\mu\nu}(p) = \sum_{i=1}^{10} \mathcal{P}_{i}^{\mu\nu} \tilde{S}_{i}(p^2). \quad (20)$$

The $\Lambda$-basis has very simple multiplicative properties which are represented in the Table I.

Let us denote the inverse dressed and free propagators by $S_{\mu\nu}$ and $S_{0\mu\nu}$ respectively. Decomposing the $S_{\mu\nu}$, $S_{0\mu\nu}$ and $\Sigma_{\mu\nu}$ in $\Lambda$-basis according to (20) we reduce the equation (16) to set of equations for the scalar coefficients

$$\tilde{S}_{i}(p^2) = \tilde{S}_{0i}(p^2) - \tilde{\Sigma}_{i}(p^2), \quad i = 1 \ldots 10.$$ 

After that the reversing of the $S_{\mu\nu}$ leads to equations for the coefficients $\tilde{G}_{i}$:

$$\left( \sum_{i=1}^{10} \mathcal{P}_{i\alpha}^{\mu} \tilde{G}_{i}(p^2) \right) \cdot \left( \sum_{k=1}^{10} \mathcal{P}_{k\nu}^{\alpha} \tilde{S}_{k}(p^2) \right) = \sum_{i=1}^{6} \mathcal{P}_{i\mu\nu}, \quad (21)$$

which are easy to solve due to simple multiplicative properties of $\mathcal{P}_{i}^{\mu\nu}$:

$$\tilde{G}_{1} = 1/\tilde{S}_{1}, \quad \tilde{G}_{3} = \tilde{S}_{6}/\Delta_{1}, \quad \tilde{G}_{5} = \tilde{S}_{4}/\Delta_{2}, \quad \tilde{G}_{7} = -\tilde{S}_{7}/\Delta_{1}, \quad \tilde{G}_{9} = -\tilde{S}_{9}/\Delta_{2},$$

$$\tilde{G}_{2} = 1/\tilde{S}_{2}, \quad \tilde{G}_{4} = \tilde{S}_{5}/\Delta_{2}, \quad \tilde{G}_{6} = \tilde{S}_{3}/\Delta_{1}, \quad \tilde{G}_{8} = -\tilde{S}_{8}/\Delta_{2}, \quad \tilde{G}_{10} = -\tilde{S}_{10}/\Delta_{1}, \quad (22)$$

where $\Delta_{1} = \tilde{S}_{3}\tilde{S}_{6} - \tilde{S}_{7}\tilde{S}_{10}$, $\Delta_{2} = \tilde{S}_{4}\tilde{S}_{5} - \tilde{S}_{8}\tilde{S}_{9}$.

The $\tilde{G}_{1}$, $\tilde{G}_{2}$ terms which describe the spin-3/2 have the usual resonance form, the $\tilde{G}_{3} - \tilde{G}_{10}$ terms correspond to the spin-1/2 contributions.

**C. Matrix form of spin-1/2 sector**

The spin-1/2 related terms of R.--S. field in $\Lambda$-basis can be written in following form

$$G_{s=1/2}^{\mu\nu}(p) = \sum_{i=3}^{10} \mathcal{P}_{i}^{\mu\nu} \tilde{G}_{i} =$$

$$= (\tilde{G}_{3}\Lambda^{+} + \tilde{G}_{4}\Lambda^{-}) (\mathcal{P}_{11}^{1/2})^{\mu\nu} + (\tilde{G}_{5}\Lambda^{+} + \tilde{G}_{6}\Lambda^{-}) (\mathcal{P}_{22}^{1/2})^{\mu\nu} +$$

$$+ (\tilde{G}_{7}\Lambda^{+} + \tilde{G}_{8}\Lambda^{-}) (\mathcal{P}_{21}^{1/2})^{\mu\nu} + (\tilde{G}_{9}\Lambda^{+} + \tilde{G}_{10}\Lambda^{-}) (\mathcal{P}_{12}^{1/2})^{\mu\nu} =$$

$$= (\tilde{G}_{3}\Lambda^{+} + \tilde{G}_{4}\Lambda^{-}) n_{1}^{\mu} n_{1}^{\nu} + (\tilde{G}_{5}\Lambda^{+} + \tilde{G}_{6}\Lambda^{-}) n_{2}^{\mu} n_{2}^{\nu} +$$

$$+ (\tilde{G}_{7}\Lambda^{+} + \tilde{G}_{8}\Lambda^{-}) n_{3}^{\mu} n_{3}^{\nu} + (\tilde{G}_{9}\Lambda^{+} + \tilde{G}_{10}\Lambda^{-}) n_{4}^{\mu} n_{4}^{\nu} \quad (23)$$

Now it is convenient to shift $\Lambda^{\pm}$ in (23) to be between the unit vectors$^{4}$ $n_{i}^{\mu}\Lambda^{\pm} n_{j}^{\nu}$

After that we come to typical matrix form for mixing of two propagators where appeared two matrices $2 \times 2$ accompanied by $\Lambda^{\pm}$ projectors

$$G_{s=1/2}^{\mu\nu}(p) = \left( n_{1}^{\mu} \Lambda^{-} n_{2}^{\mu} \Lambda^{-} \right) \left( \frac{\tilde{G}_{3}}{\tilde{G}_{10}} \frac{\tilde{G}_{7}}{\tilde{G}_{6}} \right) \left( \Lambda^{-} n_{1}^{\nu} \Lambda^{-} n_{2}^{\nu} \right) +$$

$$+ \left( n_{1}^{\mu} \Lambda^{+} n_{2}^{\mu} \Lambda^{+} \right) \left( \frac{\tilde{G}_{4}}{\tilde{G}_{9}} \frac{\tilde{G}_{8}}{\tilde{G}_{5}} \right) \left( \Lambda^{+} n_{1}^{\nu} \Lambda^{+} n_{2}^{\nu} \right) \quad (24)$$

$$\equiv T^{\mu\nu}[G^{+}, G^{-}]$$

$^{4}$ There are useful properties: $\Lambda^{\pm} n_{2}^{\mu} = n_{2}^{\mu} \Lambda^{\pm}$, $\Lambda^{\pm} n_{1}^{\mu} = n_{1}^{\mu} \Lambda^{\pm}$
Here we introduce special notation for this object. Matrix form \((2 \times 4)\) has very convenient property at multiplication: if we have spin-tensors \(A^\mu_\nu(p)\) (characterized by \(A^+, A^-\) matrices) and \(B^\mu_\nu(p)\) (characterized by \(B^+, B^-\)) then multiplication of these spin-tensors is reduced to multiplication of corresponding \(2 \times 2\) matrices.

\[
T^\mu_\rho[A^+, A^-] \cdot T^\rho_\nu[B^+, B^-] = T^\mu_\nu[A^+ B^+, A^- B^-].
\] (25)

In particular, the \(\theta\)-transformation also can be written in matrix form:

\[
\theta^\mu_\nu(a) \equiv g^\mu_\nu + a_{\gamma\rho}^\mu \gamma_\nu = \theta^\mu_\nu_{s=3/2} + T^\mu_\nu[\theta^+, \theta^-],
\] (26)

where

\[
\theta^+ = \begin{pmatrix} 1 + 3a & \sqrt{3a} \\ \sqrt{3a} & 1 + a \end{pmatrix}, \quad \theta^- = \begin{pmatrix} 1 + 3a & -\sqrt{3a} \\ -\sqrt{3a} & 1 + a \end{pmatrix}.
\] (27)

## III. RENORMALIZATION OF R.–S. PROPAGATOR

### A. Renormalization of spin-3/2 terms

It is convenient to renormalize the inverse propagator \(S^\mu_\nu\). Spin-3/2 contribution corresponds to first two term in (20)

\[
S^\mu_\nu_{s=3/2} = \sum_{i=1}^{2} P^\mu_\nu_i \check{S}_i,
\] (28)

where

\[
\check{S}_1(W) = W - M - \check{\Sigma}_1(W) = W - M - \left(\Sigma_1(W^2) + W \Sigma_2(W^2)\right)
\] (29)

and \(\check{\Sigma}_2(W) = \check{\Sigma}_1(-W)\).

If to use the on-mass-shell scheme of renormalization then \(M\) is the renormalized mass. For resonance state located higher the threshold, the renormalization condition is

\[
\check{S}_1 = W - M + o(W - M) + \frac{i \Gamma}{2} \quad \text{at} \quad W \sim M.
\] (30)

Note that real part of (30) is some requirement on the subtraction constants of self-energy functions \(\Sigma_1, \Sigma_2\), whereas the imaginary part simply relates the coupling constant and width.

So for normalization we should subtract the self-energy contribution twice at this point

\[
\check{S}'_1(W) = W - M - [\Sigma_1(W) - \text{Re} \check{\Sigma}_1(M) - \text{Re} \check{\Sigma}'_1(M)(W - M)]
\] (31)

and after it \(\check{\Sigma}_2(W) = \check{\Sigma}_1(-W)\).

Renormalization of spin-3/2 sector determines the values \(S_1(0), S_2(0)\) which take part in cancellation of \(1/p^2\) singularities (A13).

### B. Renormalization of spin-1/2 sector

Let us consider the contraction of spin-1/2 sector of propagator with \(\theta(a)\), as they appear in amplitude (\(\theta(a)\) is matrix in vertex, see (14))

\[
Z_{\mu\nu} = \theta_{\mu\alpha}(a) \left(G^{\alpha\beta}\right)^{s=1/2} \theta_{\beta\nu}(a).
\] (32)
For our purpose it is convenient to write it using the introduced $2 \times 2$ matrix representation

\[ Z^{\mu\nu} = T^{\mu\nu} \left[ Z^+, Z^- \right] = T^{\mu\nu} \left[ \theta^+(a) G^+ \theta^+(a), \theta^-(a) G^- \theta^-(a) \right]. \] (33)

Let us consider in detail the matrix which is accompanied by $\Lambda^-$ projector:

\[ Z^- = \theta^-(a) G^- \theta^-(a) = \theta^-(a) \left( \tilde{S}_3 \tilde{S}_7 \right)^{-1} \theta^-(a). \] (34)

The inverse matrix is more transparent:

\[ (Z^-)^{-1} = \left( \theta^-(a) \right)^{-1} \left( \tilde{S}_3 \tilde{S}_7 \right) \left( \theta^-(a) \right)^{-1} = \left( \theta^-(a) \right)^{-1} \left[ S_0^- - \Sigma^- \right] \left( \theta^-(a) \right)^{-1}. \] (35)

Recall that by wave function renormalization we made free propagator to be 2-parametric, as a result the $\theta(a)$-factor appeared in a vertex. So the self-energy contains this parameter, as it seen from (14)

\[ \Sigma^-(p; a) = \theta^-(a) \Sigma^-(p; a = 0) \theta^-(a). \] (36)

Matrix $Z$ takes the form

\[ (Z^-)^{-1} = \left( \theta^-(a) \right)^{-1} \left[ S_0^- - \theta^-(a) \Sigma^-(p; a = 0) \theta^-(a) \right] \left( \theta^-(a) \right)^{-1} = \left( \theta^-(a) \right)^{-1} S_0^- \left( \theta^-(a) \right)^{-1} - \Sigma^-_{a=0} \] (37)

and one can see that first term is again the general three-parameter spin-1/2 propagator. So $(Z^-)^{-1}$ is, in fact, the dressed inverse propagator $S^-$. We see that after all the parameter $a$ has been disappeared from the self-energy.

1. Cancellation of $1/p^2$ singularities

First of all require the absence of $1/p^2$ singularities in the dressed propagator (37). It is convenient to subtract the self-energy at zero (even it is not necessary for convergence of the integrals)

\[ \Sigma^-(W) = \left( \frac{T_3 + WT_4}{\sqrt{3(T_8 + WT_7)}} \right) + \Sigma^-(0)_i(W). \] (38)

Here $T_i$ is subtraction constants of self-energy components in $\hat{p}$-basis and $\Sigma^-(0)_i(W)$ denotes the integral subtracted at origin,

\[ \Sigma_i(s) = \frac{1}{\pi} \int_{s_1}^{\infty} \frac{dz}{z - s} \rho_i(z) = T_i + \frac{s}{\pi} \int_{s_1}^{\infty} \frac{dz}{z(z - s)} \rho_i(z) = T_i + \Sigma^-(0)_i(s). \] (39)

Matrix (37) after it takes the form

\[ (Z^-)^{-1} \equiv S^- = S_0^- - T - \Sigma^-(0) = -T - \Sigma^-. \] (40)

\[ ^5 \text{Let us suppose here the loop integrals to be convergent, their behaviour is defined by form-factor in a vertex, see below.} \]
We can see that both matrices $S_0^-$ and $T$ have the same structure (see (7) and (A14)) and it leads only to changing of parameters. So we can omit $S_0^-$ in previous formula. After this it is easy to write down reparametrized expression which is free from $1/p^2$ singularities for fixed spin-$3/2$ sector

$$ S^- = -\left( \begin{array}{cc} 2S_1(0) + 3T_5 & \sqrt{3}(-S_1(0) - T_5) \\ \sqrt{3}(-S_1(0) - T_5) & T_5 \end{array} \right) - W \left( \begin{array}{cc} 2S_2(0) + 3T_6 - 6\tilde{T}_7 & \sqrt{3}\tilde{T}_7 \\ \sqrt{3}\tilde{T}_7 & -T_6 \end{array} \right) - \Sigma^{(0)}_\text{sub}.$$

(41)

We used here the relations (A13), (A14) between coefficients. This equation contains three parameters $T_5$, $T_6$, and $\tilde{T}_7$, while $S_1(0)$, $S_2(0)$ are fixed after renormalization of the spin-$3/2$ sector.

2. Behaviour at infinity

Let us consider determinant of (41) at $W \to \infty$.

$$ \Delta^- = \det(S^-) = W^2d_2 + Wd_1 + d_0 \quad \text{at} \quad W \to \infty. \quad (42) $$

Note that subtracted loops $\Sigma_{(0)i}(W^2)$ tends to some constants $\Sigma_{(0)i}(\infty)$ in this limit. It is convenient to introduce new notations

$$ \tau_i = T_i + \Sigma_{(0)i}(\infty). \quad (43) $$

There are appeared some natural combinations of parameters, containing $S_1(0)$, $S_2(0)$:

$$ C_1 = S_1(0) - \Sigma_{(0)5}(\infty), $$

$$ C_2 = 2S_2(0) + \Sigma_{(0)4}(\infty) - 3\Sigma_{(0)6}(\infty) + 6\Sigma_{(0)7}(\infty), $$

$$ C_3 = 2S_1(0) + \Sigma_{(0)3}(\infty) - 3\Sigma_{(0)5}(\infty). \quad (44) $$

Let us require $\Delta^- \to \text{const}$ at $W \to \infty$. It gives

$$ d_2 = -C_2\tau_6 - 3(\tau_6 - \tau_7)^2 = 0, $$

$$ d_1 = C_2\tau_7 - C_3\tau_6 + 6C_1\tau_7 = 0, $$

$$ d_0 = -3C_1^2 + \tau_5(C_3 - 6C_1). \quad (45) $$

These two equations are solved easily if to introduce the sum and difference instead of $\tau_6$, $\tau_7$: $\delta = \tau_6 - \tau_7$, $\sigma = \tau_6 + \tau_7$. Solving (45), we express all parameters through the one free parameter $\delta$.

$$ \sigma = -\frac{6\delta^2 - \delta}{C_2}, \quad \tau_5 = \frac{3\delta}{C_2^2} \left[ -\delta C_3 + 2C_2(3\delta + C_2) \right]. \quad (46) $$
Final expressions for $\tau_i$ are (we introduced here notation $\delta = -A - 1$ for historical reasons)

$$\tau_3 = \frac{(C_2 - 3A - 3)(3C_3A - 18C_1A + 3C_3 - 18C_1 + C_2C_3)}{C_2^2},$$

$$\tau_4 = \frac{(3A + 3 - C_2)^2}{C_2},$$

$$\tau_5 = -3(A + 1)(2C_1C_2 - 6C_1A - 6C_1 + C_3A + C_3),$$

$$\tau_6 = -3(A + 1)^2, \quad (47)$$

$$\tau_7 = \frac{(A + 1)(C_2 - 3A - 3)}{C_2},$$

$$\tau_8 = \frac{(3C_3 - 18C_1)(A + 1)^2 + 6C_1C_2(A + 1) + C_1C_2^2}{C_2^2},$$

$$\tau_9 = \tau_7,$$

$$\tau_{10} = -\tau_8.$$

Recall that $\tau_i$ are subtraction constants at infinity, so renormalized propagator may be written as (41)

$$S^- = -\left( \begin{array}{cc} \tau_3 + \Sigma_3(s) & \sqrt{3}(\tau_8 + \Sigma_8(s)) \\ \sqrt{3}(\tau_8 + \Sigma_8(s)) & \tau_5 + \Sigma_5(s) \end{array} \right) - W \left( \begin{array}{cc} \tau_4 + \Sigma_4(s) & \sqrt{3}(\tau_7 + \Sigma_7(s)) \\ \sqrt{3}(\tau_7 + \Sigma_7(s)) & -\tau_6 - \Sigma_6(s) \end{array} \right),$$

where $\Sigma_i(s)$ are non-subtracted integrals (39) and $\tau_i$ are defined by (47).

If in renormalized propagator (48) to turn off interaction ($\Sigma_i(s) = 0$) and to put $C_i$ to its bare values

$$C_1^0 = -M, \ C_2^0 = 2, \ C_3^0 = -2M$$

then we come back to famous one-parameter free inverse propagator [10] with unphysical spin-1/2 sector

$$S_0^- = \left( \begin{array}{cc} M(3A + 1)(3A + 2) & -\sqrt{3}M(3A^2 + 3A + 1) \\ -\sqrt{3}M(3A^2 + 3A + 1) & 3MA(A + 1) \end{array} \right) + \frac{W}{2} \left( \begin{array}{cc} -(3A + 1)^2 & \sqrt{3}(A + 1)(3A + 1) \\ \sqrt{3}(A + 1)(3A + 1) & -3(A + 1)^2 \end{array} \right)$$

(50)

Determinant of this matrix is equal to constant $\det(S^-) = -2M^2(2A + 1)^2$, so there is no poles in spin-1/2 sector.

**IV. APPLICATION TO $\pi N$ SCATTERING**

As a first step we will use the obtained renormalized R.-S. propagator for description of total cross-section $\pi^+ p \rightarrow \pi^+ p$ in vicinity of $\Delta^{++}(1232)$ resonance. The discussed problem of non-leading spin terms in R.-S. field, on the other hand, is a problem of exact form of resonance curve. As we will see below, the high accuracy $\pi N$ data allow to "feel" these non-leading spin terms due to interference with main spin-3/2 contribution even in total cross-section.
A. Amplitude $\pi N \rightarrow \Delta \rightarrow \pi N$

Standard $\pi N\Delta$ interaction lagrangian is of the form\(^6\)

$$L_{\text{int}} = g_{\pi N\Delta} \bar{\Psi}_\mu(x) \theta^{\mu\nu}(a) \Psi(x) \cdot \partial_\nu \varphi(x) + \text{H.c.}$$

Resonance contribution:

$$\mathcal{M} = g^2_{\pi N\Delta} \cdot \bar{u}(p_2) k_{2\mu} \theta_{\mu\alpha}(a) G^{\alpha\beta}(p) \theta_{\beta\nu}(a) k_{1\nu}(p_1).$$

Here $G^{\alpha\beta}(p)$ is dressed propagator of R.–S. field, $\theta^{\mu\alpha}(a) = g^{\mu\alpha} + a \gamma^\mu \gamma^\alpha$. As it was seen above (32) and (37) the $a$ parameter has been disappeared from the amplitude. So we can put $a = 0$ below.

Let us write down the matrix between spinors in (52).

1. Spin-3/2 contribution

$$R^{s=3/2} = k_{2\mu} \left( G^{\mu\nu} \right)^{s=3/2} k_{1\nu} = k_{2\mu} \left( P_1^{\mu\nu} \bar{G}_1 + P_2^{\mu\nu} \bar{G}_2 \right) k_{1\nu} =$$

$$= \Lambda^+ \left[ -\bar{G}_1 \mathbf{p}^2 \cos \theta - \bar{G}_2 \frac{[(W - m)^2 - \mu^2]^2}{12W^2} \right] + \Lambda^- \left[ -\bar{G}_2 \mathbf{p}^2 \cos \theta - \bar{G}_1 \frac{[(W + m)^2 - \mu^2]^2}{12W^2} \right].$$

Here $\mathbf{p}$ is a spatial momentum of $\pi N$ pair in c.m.s. $\mathbf{p}^2 = \lambda(s, m_N^2, m_\pi^2)/4s$.

2. Spin-1/2 contribution

$$R^{s=1/2} = k_{2\mu} \theta_{\mu\alpha}(0) \left( G^{\alpha\beta} \right)^{s=1/2} \theta_{\beta\nu}(0) k_{1\nu} = k_{2\mu} \theta_{\mu\alpha}(0) \left( \sum_{i=3}^{10} P_i^{\alpha\beta} \bar{G}_i \right) \theta_{\beta\nu}(0) k_{1\nu} =$$

$$= (k_2 \theta n_1 \Lambda^+, \; k_2 \theta n_2 \Lambda^+) \left( \bar{G}_4 \bar{G}_8 \right) \left( \Lambda^+ n_1 \theta k_1 \right) + (k_2 \theta n_1 \Lambda^-, \; k_2 \theta n_2 \Lambda^-) \left( \bar{G}_3 \bar{G}_7 \right) \left( \Lambda^- n_1 \theta k_1 \right).$$

\(^6\) We are interested only in isospin $I = 3/2$ so we omit here isotopical indices.
Projection operators and spinors turn the components of "vectors" into unit matrix \(4 \times 4\).

\[
R^{s=1/2} = \Lambda^+ \cdot \left( a_1(W), a_2(W) \right) \left( \begin{array}{cc} G_4 & G_8 \\ G_9 & G_5 \end{array} \right) \left( \begin{array}{c} a_1(W) \\ a_2(W) \end{array} \right) + \Lambda^- \cdot \left( a_1(-W), a_2(-W) \right) \left( \begin{array}{cc} G_3 & G_7 \\ G_{10} & G_6 \end{array} \right) \left( \begin{array}{c} a_1(-W) \\ a_2(-W) \end{array} \right).
\]

(55)

Here

\[
a_1(W) = \frac{1}{2\sqrt{3W}} [(W - m_N)^2 - m_\pi^2],
\]

\[
a_2(W) = \frac{1}{2W} [W^2 - m_N^2 + m_\pi^2].
\]

(56)

In more detail:

\[
R^{s=1/2} = \Lambda^+ \left[ a_2^2(W)\tilde{G}_4 + a_1(W)a_2(W)\tilde{G}_8 + a_1(W)a_2(W)\tilde{G}_9 + a_2^2(W)\tilde{G}_5 \right] + \Lambda^- \left[ a_2^2(-W)\tilde{G}_3 - a_1(-W)a_2(-W)\tilde{G}_7 - a_1(-W)a_2(-W)\tilde{G}_{10} + a_2^2(-W)\tilde{G}_6 \right].
\]

(57)

**B. Self-energy**

The one-loop self-energy contribution is

\[
\Sigma^{\mu\nu}(p) = -ig_{\pi N}\Delta \int \frac{d^4k}{(2\pi)^4} \theta^{\rho\lambda}(0) k_\rho \frac{1}{\bar{p} + k - m_N} k_\lambda \theta^{\nu\lambda}(0) \frac{1}{k^2 - m_\pi^2}.
\]

(58)

Let us calculate the discontinuity of loop contribution in \(\hat{p}\)-basis.

\[
\Delta \Sigma_1 = -\frac{g^2 I_0}{(2\pi)^2} \frac{m_N}{12s} \lambda(s, m_N^2, m_\pi^2),
\]

\[
\Delta \Sigma_2 = -\frac{g^2 I_0}{(2\pi)^2} \frac{1}{24s^2} (s + m_N^2 - m_\pi^2) \lambda,
\]

\[
\Delta \Sigma_5 = \frac{g^2 I_0}{(2\pi)^2} \frac{m_N}{4s} (s - m_N^2 + m_\pi^2),
\]

\[
\Delta \Sigma_6 = \frac{g^2 I_0}{(2\pi)^2} \frac{1}{8s^2} (s + m_N^2 - m_\pi^2)(s - m_N^2 + m_\pi^2),
\]

\[
\Delta \Sigma_3 = -\frac{g^2 I_0}{(2\pi)^2} \frac{m_N}{12s} \lambda,
\]

\[
\Delta \Sigma_4 = -\frac{g^2 I_0}{(2\pi)^2} \frac{1}{24s^2} (s + m_N^2 - m_\pi^2) \lambda,
\]

\[
\Delta \Sigma_7 = \frac{g^2 I_0}{(2\pi)^2} \frac{1}{\sqrt{3}} \frac{1}{24s} (s - m_N^2 + m_\pi^2) \lambda,
\]

\[
\Delta \Sigma_8 = 0,
\]

\[
\Delta \Sigma_9 = \Delta \Sigma_7,
\]

\[
\Delta \Sigma_{10} = 0.
\]

(59)

Here \(\lambda(a, b, c) = (a - b - c)^2 - 4bc\), arguments of \(\lambda\) are shown in the first expression. \(I_0\) is the base integral

\[
I_0 = \int d^4k \delta(k^2 - m_\pi^2) \delta((p + k)^2 - m_N^2) = \theta(p^2 - (m_N + m_\pi)^2) \pi \sqrt{\lambda(p^2, m_N^2, m_\pi^2)} (p^2).
\]

(60)
Let us write down also the imaginary parts of first components in Λ-basis:

$$\text{Im } \bar{\Sigma}_1 = \text{Im } \Sigma_1(W^2) + W \text{Im } \Sigma_2(W^2) = -\frac{g^2}{16\pi} \frac{\lambda^{3/2}}{24W^5} [(W + m_N)^2 - m^2] = -\frac{\Gamma(W)}{2}$$

$$\text{Im } \bar{\Sigma}_2 = \text{Im } \Sigma_1(W^2) - W \text{Im } \Sigma_2(W^2) = \frac{g^2}{16\pi} \frac{\lambda^{3/2}}{24W^5} [(W - m_N)^2 - m^2].$$

(61)

And last: let us introduce $W$-dependent form-factor into coupling constant (also known as centrifugal form-factor or Blatt–Weisskopf form-factor, see e.g. [18, 19]). We will make it in a simplest way to ensure the convergence of loop integrals

$$g \rightarrow g \cdot F(W) = g \cdot \frac{M^2 + s_L}{W^2 + s_L}.$$  (62)

Such modification allows to perform the renormalization procedure in the spin-1/2 sector which was described above.

C. Fit of total cross-section $\pi^+ p \rightarrow \Delta^{++}(1232) \rightarrow \pi^+ p$

Let us consider the data $\pi^+ p \rightarrow \pi^+ p$ [20] which have the best statistics in the region of $\Delta(1232)$ resonance. We can use the obtained renormalized R.–S. propagator for description of total cross-section. Results of fit in energy region from the threshold up to 1.32 GeV are presented at Fig. 1. Note that use of dressed propagator of R.–S. field with all spin components leads to good quality of description. The best-fit parameters are:

$$M_\Delta = 1232.0 \pm 0.2 \text{ MeV} \quad A = -0.576 \pm 0.014$$

$$\Gamma_\Delta = 113 \pm 1 \text{ MeV} \quad s_L = -0.607 \pm 0.012 \text{ GeV}^2$$

$$\chi^2/DOF = 0.99.$$  (63)

In Table II one can see comparison of different variants of fit, shown at Fig. 1. Evidently that account of spin-1/2 terms in dressed propagator improves essentially quality of description. For comparison at Fig. 1 one can see also the curve, corresponding to naive Breit-Wigner contribution with energy-independent mass and width.

Looking at the best-fit parameters (63) we observe at first sight unexpected fact: the parameter $s_L$ is negative, so we have pole in the vertex formfactor (62) not far from $\pi N$ threshold. We suppose that appearance of this pole imitates the one-nucleon contribution. Of course, simplest parameterization (62) does not reproduce correctly the one-nucleon term but data ”feel” the presence of rapidly changing under-threshold contribution. In principle it’s possible to improve the model of formfactor to reproduce the one-nucleon term. Recall that there exist an approach based on low-energy dynamical dispersion equations (see, e.g. [21]), which account correctly the cross-channels contributions. So we should make our amplitude to be consistent with these dynamical equations. But as a first step we restrict ourselves by rough recipe (62).

After fixing free parameters by (63), we can calculate partial waves of isospin-3/2 with use of our dressed propagator. Our amplitude contains four partial waves $P_{33}, D_{33}, S_{31}, P_{31}$, satisfying the elastic unitary condition $\text{Im } f = |f|^2$. It turns out that the so obtained partial waves are in reasonable agreement with results of the partial wave analysis [22], as it seen from Figs. 2,3.

Note that at $A \sim -0.5$ in our amplitudes takes place some changing of behaviour. This is rather natural if to remember that $A = -0.5$ is the singular value for free propagator.
V. CONCLUSIONS

Thus we found the renormalization procedure for dressed propagator of Rarita-Schwinger field, which remains the spin-1/2 degrees of freedom to be unphysical. This fact is rather non-trivial, as it seen from discussion in [23]. We think that the studying of dressed propagator is more adequate method then investigation of equations of motion and additional constrains.

Spin-3/2 contribution after renormalization takes usual resonance form. The obtained renormalized propagator in $s = 1/2$ sector has one free parameter $A$ and is a straight generalization of the standard free propagator, suggested by Moldauer and Case [10].

We found that the obtained dressed propagator describes well the total cross-section $\pi^+ p \rightarrow \pi^+ p$ in vicinity of $\Delta^{++}(1232)$ resonance. Moreover, it allows to describe the partial waves also. If to say more exactly, our amplitude describes well $J = 3/2$ waves $P_{33}$ and $D_{33}$. As for $J = 1/2$ partial waves, it describes reasonably the smooth non-resonant contributions in these waves.

So we can conclude that the concept of effective multicomponent quantum field is really working thing and it may be useful in phenomenology.

VI. ACKNOWLEDGMENTS

We thank A.M. Moiseeva for participation at the beginning of this work. This work was supported in part by RFBR grant No 05-02-17722.

APPENDIX A: DECOMPOSITION OF SPIN-TENSOR

Propagator or self-energy of the R.–S. field has two spinor and two vector indices and depends on momentum $p$. We will denote such object as $S^{\mu\nu}(p)$, omitting spinor indices, and will call it shortly as a spin-tensor. In our consideration we need to use different bases for this object.

1. $\gamma$-basis

Most evident is a $\gamma$-matrix decomposition. It’s easy to write down all possible $\gamma$-matrix structures with two vector indices. Altogether there are 10 terms in decomposition of spin-tensor, if parity is conserved.

$$S^{\mu\nu}(p) = g^{\mu\nu} \cdot s_1 + p^\mu p^\nu \cdot s_2 + \hat{p} p^{\mu\nu} \cdot s_3 + \hat{p} g^{\mu\nu} \cdot s_4 + p^\mu \gamma^\nu \cdot s_5 + \gamma^\mu p^\nu \cdot s_6 +$$

$$+ \sigma^{\mu\nu} \cdot s_7 + \sigma^{\mu\lambda} p_\lambda p^\nu \cdot s_8 + \sigma^{\nu\lambda} p_\lambda p^\mu \cdot s_9 + \gamma_\lambda \gamma^5 \epsilon^{\lambda\mu\nu\rho} p_\rho \cdot s_{10}. \quad (A1)$$

Here $s_i(p^2)$ are the Lorentz-invariant coefficients and $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$.

This is a good starting point of any consideration, since this basis is complete, nonsingular and free of kinematical constrains. But this basis is not convenient at multiplication (e.g. in Dyson summation) because elements of basis are not orthogonal to each other.
2. $\hat{p}$-basis

There is another basis (see e.g. [16]) for $S^{\mu\nu}$, which we call as $\hat{p}$-basis. Decomposition of any spin-tensor in this basis has the form

$$S^{\mu\nu}(p) = (S_1 + \hat{p}S_2)(P^{3/2})^{\mu\nu} + (S_3 + \hat{p}S_4)(P_{11}^{1/2})^{\mu\nu} + (S_5 + \hat{p}S_6)(P_{22}^{1/2})^{\mu\nu} + (S_7 + \hat{p}S_8)(P_{21}^{1/2})^{\mu\nu} + (S_9 + \hat{p}S_{10})(P_{12}^{1/2})^{\mu\nu}, \quad (A2)$$

where appeared the well-known tensor operators

$$(P_{3/2}^{\mu\nu}) = g^{\mu\nu} - n_1^\mu n_1^\nu - n_2^\mu n_2^\nu$$

$$(P_{11}^{1/2})^{\mu\nu} = n_1^\mu n_1^\nu,$$

$$(P_{22}^{1/2})^{\mu\nu} = n_2^\mu n_2^\nu,$$

$$(P_{21}^{1/2})^{\mu\nu} = n_1^\mu n_2^\nu,$$

$$(P_{12}^{1/2})^{\mu\nu} = n_2^\mu n_1^\nu. \quad (A3)$$

which are written here in a non-standard form. Here we introduced the unit "vectors"

$$n_1^\mu = \pi^\mu / \pi^2, \quad n_2^\mu = \hat{p}^\mu / \sqrt{p^2 I_4}, \quad (n_i \cdot n_j) = \delta_{ij} I_4, \quad (A4)$$

where "vector" $\pi^\mu$ is

$$\pi^\mu = \frac{1}{3p^2} (-p^\mu + \gamma^\mu \hat{p}) \hat{p}$$

with the following properties:

$$(\pi p) = 0, \quad (\gamma \pi) = (\pi \gamma) = 1, \quad (\pi \pi) = \frac{1}{3}, \quad \hat{p} \pi^\mu = -\pi^\mu \hat{p}. \quad (A5)$$

3. $\Lambda$-basis

The most convenient at multiplication basis is build by combining the $P_{\mu\nu}^i$ operators (A3) and the off-shell projection operators $\Lambda^\pm$:

$$\Lambda^\pm = \frac{1}{2} \left( 1 \pm \frac{\hat{p}^\mu}{\sqrt{p^2}} \right), \quad (A6)$$

where we assume $\sqrt{p^2}$ to be the first branch of analytical function. Ten elements of this basis look as

$$P_1 = \Lambda^+ P^{3/2}, \quad P_3 = \Lambda^+ P_{11}^{1/2}, \quad P_5 = \Lambda^+ P_{22}^{1/2}, \quad P_7 = \Lambda^+ P_{21}^{1/2}, \quad P_9 = \Lambda^+ P_{12}^{1/2},$$

$$P_2 = \Lambda^- P^{3/2}, \quad P_4 = \Lambda^- P_{11}^{1/2}, \quad P_6 = \Lambda^- P_{22}^{1/2}, \quad P_8 = \Lambda^- P_{21}^{1/2}, \quad P_{10} = \Lambda^- P_{12}^{1/2}, \quad (A7)$$

where tensor indices are omitted. Decomposition in this basis:

$$S^{\mu\nu}(p) = \sum_{A=1}^{10} \tilde{S}^A P_{\mu\nu}^A. \quad (A8)$$
The multiplication table of elements of the basis is in Table I

Coefficients of $S^\mu\nu$ in $\hat{p}$- and $\gamma$-bases are related by

\[ s_1 = \frac{1}{3}(2S_1 + S_3), \quad s_2 = \frac{1}{3W^2}(-2S_1 - S_3 + 3S_5), \]
\[ s_3 = \frac{1}{3W^2}(-2S_2 - S_4 + 3S_6 - \sqrt{\frac{3}{W}}(S_7 + S_9)), \quad s_4 = \frac{1}{3}(2S_2 + S_4), \]
\[ s_5 = \frac{1}{\sqrt{3W}}S_9, \quad s_6 = \frac{1}{\sqrt{3W}}S_7, \]
\[ s_7 = \frac{1}{3}(-S_1 + S_3), \quad s_8 = \frac{1}{3W^2}(S_1 - S_3 - \sqrt{3WS_8}), \]
\[ s_9 = \frac{1}{3W^2}(-S_1 + S_4 - \sqrt{3WS_10}), \quad s_{10} = \frac{1}{3}(-S_2 + S_4). \]

Reversed relations:

\[ S_1 = s_1 - s_7, \quad S_2 = s_4 - s_{10}, \]
\[ S_3 = s_1 + 2s_7, \quad S_4 = s_4 + 2s_{10}, \]
\[ S_5 = s_1 + W^2s_2, \quad S_6 = W^2s_3 + s_4 + s_5 + s_6, \]
\[ S_7 = \sqrt{3WS_6}, \quad S_8 = -\sqrt{\frac{3}{W}}(s_7 + W^2s_8), \]
\[ S_9 = \sqrt{3WS_6}, \quad S_{10} = \sqrt{\frac{3}{W}}(s_7 + W^2s_8). \]

Transition from $\hat{p}$- to $\Lambda$-basis:

\[ \tilde{S}_1 = S_1 + WS_2, \quad \tilde{S}_3 = S_3 + WS_4, \quad \tilde{S}_5 = S_5 + WS_6, \quad \tilde{S}_7 = S_7 + WS_8, \quad \tilde{S}_9 = S_9 + WS_{10}, \]
\[ \tilde{S}_2 = S_1 - WS_2, \quad \tilde{S}_4 = S_3 - WS_4, \quad \tilde{S}_6 = S_5 - WS_6, \quad \tilde{S}_8 = S_7 - WS_8, \quad \tilde{S}_{10} = S_9 - WS_{10}. \]  

Let us note that $\hat{p}$- and $\Lambda$-bases are singular at point $p^2 = 0$. As for branch point $\sqrt{p^2}$ appearing in different terms, it is canceled in total expression. But poles $1/p^2$ don’t cancel automatically, if we work in $\hat{p}$- or $\Lambda$-basis. First of all, one can see from (A10) that the $S_7 - S_{10}$ should have kinematical $\sqrt{p^2}$ factors:

\[ S_7 = \sqrt{3WS_9}, \quad S_8 = \sqrt{\frac{3}{W}}S_8, \quad S_9 = \sqrt{3WS_9}, \quad S_{10} = \sqrt{\frac{3}{W}}S_{10}, \]

and $\tilde{S}_1$ don’t have branch point at origin. After it we see from (A9) conditions of absence of $1/p^2$ poles:

\[ 2S_1(0) + S_3(0) - 3S_5(0) = 0, \]
\[ S_1(0) - S_3(0) - 3\tilde{S}_8(0) = 0, \]
\[ 2S_2(0) + S_4(0) - 3S_6(0) + 3(S_7(0) + \tilde{S}_9(0)) = 0, \]
\[ \tilde{S}_8(0) + \tilde{S}_{10}(0) = 0. \]

It is convenient to solve these relations in such manner:

\[ S_3 = -2S_1(0) + 3S_5(0), \]
\[ \tilde{S}_8(0) = S_4(0) - S_5(0), \]
\[ S_4(0) = -2S_2(0) + 3S_6(0) - 3(\tilde{S}_7(0) + \tilde{S}_9(0)), \]
\[ \tilde{S}_{10}(0) = -\tilde{S}_8(0). \]
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TABLE I: Multiplicative properties of the \Lambda\text{-basis}

| $P_1$  | $P_2$  | $P_3$  | $P_4$  | $P_5$  | $P_6$  | $P_7$  | $P_8$  | $P_9$  | $P_{10}$ |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|----------|
| $P_1$  | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0      | 0        |
| $P_2$  | 0      | $P_2$  | 0      | 0      | 0      | 0      | 0      | 0      | 0        |
| $P_3$  | 0      | 0      | $P_3$  | 0      | 0      | $P_7$  | 0      | 0      | 0        |
| $P_4$  | 0      | 0      | 0      | $P_4$  | 0      | 0      | 0      | $P_8$  | 0        |
| $P_5$  | 0      | 0      | 0      | 0      | $P_5$  | 0      | 0      | 0      | $P_9$    |
| $P_6$  | 0      | 0      | 0      | 0      | $P_6$  | 0      | 0      | 0      | $P_{10}$ |
| $P_7$  | 0      | 0      | 0      | 0      | 0      | $P_7$  | 0      | 0      | 0        |
| $P_8$  | 0      | 0      | 0      | 0      | $P_8$  | 0      | 0      | 0      | 0        |
| $P_9$  | 0      | 0      | 0      | $P_9$  | 0      | 0      | 0      | $P_5$  | 0        |
| $P_{10}$| 0      | 0      | $P_{10}$| 0      | 0      | 0      | $P_6$  | 0      | 0        |

TABLE II: Comparison of different models at fitting of cross section shown at Fig. I

| Model                                           | $\chi^2$/DOF |
|------------------------------------------------|--------------|
| Simplest Breit-Wigner with energy independent mass and width | 215          |
| Dressed spin-3/2 components                      | 5.6          |
| Pressed propagator of R.–S. field with all components | 0.99         |
FIG. 1: Results of fit of data [20] on $\pi^+ p$ cross-section in energy region from threshold up to 1.32 GeV.
FIG. 2: Comparison of our partial waves with results of partial wave analysis [22] (they are very close to results of previous analysis [18, 24] in this region). Parameters in our amplitudes are taken from total cross section fit (63). Our partial waves satisfy the elastic unitary condition $\text{Im } f = |f|^2$.

FIG. 3: The same as Fig. 2 for partial waves $S_{31}$ and $P_{31}$ in extended energy region.