Wavelength shifting of intra-cavity photons: Adiabatic wavelength tuning in rapidly wavelength-swept lasers

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Abstract: We analyze the physics behind the newest generation of rapidly wavelength tunable sources for optical coherence tomography (OCT), retaining a single longitudinal cavity mode during operation without repeated build up of lasing. In this context, we theoretically investigate the currently existing concepts of rapidly wavelength-swept lasers based on tuning of the cavity length or refractive index, leading to an altered optical path length inside the resonator. Specifically, we consider vertical-cavity surface-emitting lasers (VCSELs) with microelectromechanical system (MEMS) mirrors as well as Fourier domain mode-locked (FDML) and Vernier-tuned distributed Bragg reflector (VT-DBR) lasers. Based on heuristic arguments and exact analytical solutions of Maxwell’s equations for a fundamental laser resonator model, we show that adiabatic wavelength tuning is achieved, i.e., hopping between cavity modes associated with a repeated build up of lasing is avoided, and the photon number is conserved. As a consequence, no fundamental limit exists for the wavelength tuning speed, in principle enabling wide-range wavelength sweeps at arbitrary tuning speeds with narrow instantaneous linewidth.

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1. Introduction

Over the last 10 years, rapidly and widely wavelength tunable lasers with sweep rates of several 10kHz and relative wavelength tuning ranges of more than 10% have revolutionized optical imaging and sensing in many applications. The most important impact of these sources is in biomedical imaging. In optical coherence tomography (OCT), especially in highly scattering tissue, such sources have triggered a dramatic increase in imaging speed [1–6]. Already today, numerous companies use swept source OCT (SS-OCT) in their imaging engines and it is common sense that in the future many more OCT applications will see swept laser based systems. Depending on the laser, SS-OCT can provide superior performance with respect to imaging speed [7], imaging range [8,9], and sensitivity [10–13]. Other biomedical imaging applications include fast beam steering in endoscopes [14] and label free molecular imaging with Raman contrast [15].

Besides biomedical applications, these swept laser types are also applied for spectroscopy [16–18], non-destructive testing [19–21], and dynamic sensing [22,23]. For many applications, a further increase of sweep rate well above 1MHz is highly desirable [7, 10, 12, 24, 25]. However, classical tuning concepts based on wavelength-scanning filters inside the laser cavity are inherently limited in their tuning speed by the build up time of the lasing field from amplified spontaneous emission (ASE) [1, 2, 16]. Such lasers basically tune by hopping between the cavity modes. In the semiconductor optical amplifier, the intracavity field experiences an additional frequency shift due to four-wave mixing, which can have a beneficial effect on the sweeping action [1, 2, 26]. The formation of a discrete mode structure is sometimes intentionally suppressed to achieve continuous tuning [16], but the fundamental limit to tuning speed still exists. A reduction in laser cavity length to increase the optical roundtrip frequency is only beneficial to a certain degree, because shorter laser cavities have a wider mode spacing which prevents dense spectral sampling. Furthermore, the resulting small number of active modes can lead to increased noise with intensity fluctuations of up to 100% [26]. Hence, highly integrated wavelength-swept optical coherence tomography (OCT) lasers sometimes have an external fiber end mirror [27] to provide a sufficiently narrow mode spacing of about 1GHz [28]. Thus, these lasers have an optical roundtrip length of approximately 30cm even though the package is only 10 – 20mm. In addition, these classical tuning concepts suffer from time-bandwidth limitations, i.e., the more rapidly the laser build up dynamics happens the wider the instantaneous spectrum is.

As a consequence, the preferred solution to implement a rapidly swept laser with a sufficiently narrow instantaneous linewidth is to avoid mode hopping. This can be achieved by employing a laser configuration that remains in the same instantaneous cavity eigenmode during sweeping operation, which is here referred to as adiabatic wavelength tuning. Importantly, this does not imply that the frequency tuning is limited to low sweep rates; rather, an active mechanism is employed to continuously wavelength-shift the intracavity light field. At moderate sweep speeds sufficient wavelength shifting can be accomplished by a separate intracavity element [1]. For higher tuning speeds inherent shifting by changing the intracavity optical path length, given by the product of geometric length and cavity refractive index, can be used. In the case of laser tuning by an axial movement of one end mirror the wavelength shift is caused by the Doppler effect [29]. Notably, also so-called Fourier Domain Mode Locked (FDML) lasers take advantage of this concept, as discussed further below. However, not all implementations
of changing the laser cavity length lead to a wavelength conversion and enable adiabatic tuning [16]. On the other hand, wavelength shifting can also be achieved by tuning the refractive index inside the laser cavity [30–32], a case which is even less obvious.

Currently all rapidly wavelength-swept OCT lasers with narrow linewidth, relative wavelength tuning ranges of $\sim 10\%$ and sweep rates substantially above 100kHz use inherent intracavity wavelength shifting in some form. Although this mechanism is essential for the wavelength-swept laser sources of the next generation OCT systems, it has up to now not been considered and analyzed in detail. Thus, we will give a comprehensive theoretical treatment and interpretation of the physical mechanisms involved, considering the various concepts used to achieve high sweep speeds. The first class of very rapidly wavelength tunable OCT lasers are vertical-cavity surface-emitting lasers (VCSELs) with a microelectromechanical system (MEMS) mirror. They are based on a Fabry-Pérot resonator with tunable cavity length, and the semiconductor quantum well laser gain medium is located inside the resonator. Widely tunable MEMS-based VCSELs have achieved up to $\sim 1\text{MHz}$ sweep rate and tuning ranges of $\sim 100\text{nm}$ [8, 33–36]. A second, related concept are Fourier domain mode-locked (FDML) lasers where the Fabry-Pérot cavity serves as an intracavity tunable bandpass filter element, but is separated from the laser gain medium. The resulting optical light field which contains the full wavelength sweep imposed by the Fabry-Pérot filter is stored in an additional delay fiber [37]. With FDML lasers sweep rates of 5.2MHz [7] and tuning ranges of 160nm [38] have been demonstrated. Finally as third approach, Vernier-tuned distributed Bragg reflector (VT-DBR) laser structures achieve wavelength tuning with 200kHz sweep rate and 40nm sweep width based on dynamic refractive index tuning [31, 32], which has recently attracted considerable interest also for other applications such as wavelength conversion of external optical fields [39–44]. Although both the tuning of mirror position and refractive index aim at varying the optical path length inside the resonator, there are also fundamental differences between those two cases [45]. Passive concepts such as swept sources based on temporally stretching ultrashort laser pulses [46, 47] do not contain any active tuning element and are not considered here.

Based on heuristic arguments and exact analytical solutions of Maxwell’s equations for a fundamental resonator model without gain and loss, we investigate the underlying mechanisms of rapid wavelength tuning for the above mentioned active concepts. Specifically, we show that the resonance wavelengths are adiabatically adjusted, i.e., a repeated build up of the optical resonator field is avoided, and the photon number is an adiabatic invariant of the tuning process [41, 48]. Thus, the optical gain only compensates for mirror outcoupling and other loss mechanisms as in fixed-wavelength lasers, while the change in photon energy is provided by the tuning process itself. As a practical consequence, we demonstrate that no fundamental limit exists for the tuning speed which determines the achievable sweep rate and range.

2. Heuristic description of adiabatic wavelength tuning

Heuristic descriptions of the tuning effects complement rigorous physical models and exact calculations by providing intuitive insight. In particular, they are helpful for understanding which tuning methods involving a change of the intracavity optical path length feature implicit wavelength conversion and enable adiabatic tuning. Figure 1 contains schematic illustrations of three methods for wavelength tuning by changing the intracavity optical path length. In Figs. 1(a) and 1(b), cavity length and refractive index tuning is shown, respectively. As we will see in the following, both methods feature intrinsic wavelength conversion and are thus not limited in speed by repeated build up of the lasing field. In Fig. 1(c), an example of changing the intracavity optical path length without wavelength conversion is shown.
Fig. 1. Schematic representation of various methods for wavelength tuning by changing the intracavity optical path length: (a) Resonator with moving end mirror; (b) resonator with time dependent refractive index; (c) grating-based wavelength tuning mechanism.

2.1. Resonator with moving mirror

Here we investigate a linear Fabry-Pérot resonator with moving end mirror, schematically illustrated in Fig. 1(a). This serves for example as a model for wavelength tuning in MEMS-VCSELs [33, 34]. For a cavity with length $L$, refractive index $n = 1$ and mirror reflection coefficients $r_{1,2} = |r_{1,2}| \exp(i \phi_{1,2})$, the resonance frequency of the $m$th longitudinal mode is given by $\omega = [m\pi - (\phi_1 + \phi_2)/2]c/L$ [29], where $c$ denotes the vacuum speed of light. Moving the right mirror with a velocity $v$ leads to a cavity length change $\delta L = vT_R$ during one roundtrip with roundtrip time $T_R = 2L/c$, which results in an associated resonance frequency change $\delta \omega = -[m\pi - (\phi_1 + \phi_2)/2]c\delta L/L^2 = -2\omega v/c$. Upon reflection at the moving mirror, the electromagnetic wave experiences a Doppler shift which is in non-relativistic approximation given by $\delta \omega = -2\omega v/c$ [49] and thus compensates for the change in resonance frequency. In this way, the mode wavelength and the corresponding photon energy are adiabatically tuned, and a repeated build up of the lasing field is not necessary [29]. From energetic considerations, the force exerted on the moving mirror due to radiation pressure is $F = 2P/c$ [50], where $P = W/T_R = Wc/(2L)$ is the optical power incident on the mirror, and $W$ is the intracavity electromagnetic field energy. The change in energy of the system is thus

$$\delta W = -F \delta L = -W \frac{\delta L}{L}. \quad (1)$$

Moving the mirror in $-z$ direction leads to a positive $\delta W$ in Eq. (1), i.e., mechanical work against the radiation pressure has to be applied which is transferred to the electromagnetic field. This results in an increased photon energy and frequency which corresponds to the Doppler shift discussed above.

2.2. Resonator with time dependent refractive index

Besides changing the geometric cavity length, the optical path length can also be varied by tuning the intracavity refractive index [30–32]. This is for example achieved by exploiting the linear or quadratic electro-optic effect [50], or through current injection [31, 51]. An intuitive description of refractive index tuning can be obtained based on an extended Lorentz oscillator model of light-matter interaction, as illustrated in Fig. 1(b). The atomic response to an electric field is here described by a classical equation of motion $m_e \ddot{x} = -eE(t) - eE_0 + F(x)$, where $x$ is the displacement between the nucleus and the center of the electron cloud [52]. Furthermore, $e$ and $m_e$ denote the elementary charge and electron mass, respectively. The damping term has been neglected, i.e., gain and loss are here not considered. $E(z,t) = \hat{E}(k) \sin(kz) \cos(\omega t)$ is the time-varying cavity field with the amplitude $\hat{E}$ in $x$ direction, frequency $\omega_0$, and wavenumber $k$, which is a multiple of $\pi/L$ in a cavity of length $L$ [29]. The corresponding field energy in the...
cavity with volume \( V \) and refractive index \( n \) is given by \( W = V \varepsilon_0 n^2 E^2 \),

\[ W = V \varepsilon_0 n^2 E^2, \quad (2) \]

where \( \varepsilon_0 \) denotes the vacuum permittivity. For ease of interpretation, we assume that \( n \) is tuned using the linear electro-optic effect, i.e., by applying an additional electric field \( E_0 \). The restoring force is modeled by a spring-like ansatz,

\[ F(x) = -k_0 x + k_1 x^2, \quad (3) \]

where in addition to the linear term with spring constant \( k_0 \) a nonlinear term with constant \( k_1 \) is included to describe the linear electro-optic effect. The field \( E_0 \) shifts the point of equilibrium to \( x_0 \approx -k_0^{-1} e E_0 \), and the equation of motion for small-amplitude oscillations around \( x_0 \) is given by \( m_e \ddot{x} = -e E(t) - k'(x - x_0) \), with the modified spring constant

\[ k' = -\frac{dF(x_0)}{dx} = k_0 - 2k_1 x_0. \quad (4) \]

From the solution \( x(t) = x_0 + \hat{x} \sin(kz) \cos(\omega_0 t) \) with \( \hat{x} = 2e \hat{E} / (\alpha_0^2 m_e - k') \), the refractive index is obtained as \[ n = \left( 1 + \chi - \frac{Ne}{V\varepsilon_0 2E} \right)^{1/2} = \left( 1 + \chi + \frac{Ne^2}{V\varepsilon_0 k' - \alpha_0^2 m_e} \right), \quad (5) \]

where \( N \) is the number of atoms in the cavity volume \( V \), and \( \chi \) is the background susceptibility. Now we consider a small change of \( E_0 \) leading to a shift of \( x_0 \) by \( \delta x \). This results with Eqs. (5) and (4) in a refractive index change

\[ \delta n = k_1 \frac{V\varepsilon_0}{Ne^2 n} \left( n^2 - 1 - \chi \right)^2 \delta x. \quad (6) \]

Assuming that this process extends over many optical periods, the associated energy change in the system is given by \(-W \delta x\). Since the force \( F \) in Eq. (3) is space and time dependent, the expectation value has to be taken, yielding

\[ -N \frac{\alpha_0}{2\pi L} \delta x \int_0^{2\pi/\alpha_0} \int_0^L F(x_0 + \hat{x} \sin(kz) \cos(\omega_0 t)) dz dt = -N \left( -k_0 x_0 + k_1 x_0^2 + \frac{1}{4} k_1 \hat{x}^2 \right) \delta x. \quad (7) \]

The last term in Eq. (7) becomes with Eqs. (5), (6) and (2)

\[ \delta W = -\frac{1}{4} N k_1 \hat{x}^2 \delta x = -W \frac{\delta n}{n}, \quad (8) \]

describing the energy exchanged with the optical field. Reducing the refractive index, \( \delta n < 0 \), leads to a positive \( \delta W \), i.e., energy is transferred to the electromagnetic field. Since the photon number is conserved, this results in an increased photon energy and frequency. Both Eq. (8) and Eq. (1) can be expressed as \( \delta W / W = -\delta L_{\text{op}} / L_{\text{op}} \), where \( L_{\text{op}} = L n \) is the optical path length. Thus \( W \) and the frequency are inversely proportional to \( L_{\text{op}} \), while the vacuum wavelength is directly proportional to \( L_{\text{op}} \).
2.3. Changing the optical path length without wavelength conversion

Laser wavelength tuning by changing the intracavity optical path length does not always involve intrinsic wavelength conversion [16], which is however necessary to avoid a repeated build up of the optical resonator field. For cavity length tuning by a moving mirror discussed in Section 2.1, the wavelength shift is provided by the Doppler effect. In Fig. 1(c), a method of changing the intracavity optical path length without intrinsic wavelength conversion is illustrated [16]. Here the cavity contains a tilt mirror which directs the laser beam onto a blazed grating, designed to suppress unwanted zeroth-order diffraction [16]. For a certain wavelength which depends on the mirror tilt angle [50], the light is back-reflected onto the mirror, thus enabling wavelength selection. Additionally, the optical path length is changed by tilting the mirror [see Fig. 1(c)]. However, since the laser beam hits the mirror on its rotation axis, no mechanical work is performed against the radiation pressure. Hence, the photon energy remains unchanged, and the wavelength-detuned laser field has to build up again. Such a setup can thus not be used for adiabatic wavelength tuning; rather, the rapid change in cavity length has been exploited to suppress the formation of resonant modes [16]. This clearly demonstrates that not all approaches to change the cavity length and with it the resonance frequencies of the longitudinal modes also generate synchronous wavelength shifting of the intracavity light field.

3. Rigorous theoretical model

The primary goal of the calculations presented in this section is to rigorously show that no fundamental limits in tuning speed exist for the adiabatic tuning concepts considered here. Thus we restrict ourselves to fundamental resonator models which allow for exact analytical solutions of Maxwell’s equations. We consider one-dimensional optical propagation in an ideal Fabry-Pérot resonator, where the optical path length is changed by a moving end mirror [Fig. 2(a)] or by a homogeneous cavity medium with a time dependent refractive index [Fig. 2(b)]. The former case applies to MEMS-VCSELs or FDML lasers, while the latter case serves as a model for VT-DBR lasers. Gain and loss are not considered; thus, perfectly reflecting end mirrors with amplitude reflection coefficients $r_{1,2} = \exp(i\phi_{1,2})$ are assumed except for the FDML laser, where the Fabry-Pérot resonator is part of a fiber ring cavity. Furthermore, undesirable effects such as dispersion and jitter are not taken into account, assuming that they can in principle be avoided, and also ASE is ignored in our analysis. Magnetization effects are assumed to be negligible at the optical frequencies considered. The evolution of the electromagnetic field inside the cavity is then described by Maxwell’s equations of the form

\[
\begin{align*}
\partial_t H_y & = -\mu_0^{-1} \partial_z E_x, \\
\partial_t D_x & = -\partial_z H_y,
\end{align*}
\]

where $z$ and $t$ are the position and time coordinate, respectively. $H_y(z,t)$ denotes the transverse $y$ component of the magnetic field. $E_x(z,t)$ and $D_x(z,t)$ are the transverse $x$ component of the electric field and the electric displacement, respectively. Furthermore, $\mu_0$ denotes the vacuum permeability. The dielectric medium inside the resonator is assumed to be homogeneous, but may be time dependent, so that the constitutive relation between $D_x$ and $E_x$ is given by

\[
D_x(z,t) = \varepsilon_0 n^2(t) E_x(z,t).
\]

Here, $\varepsilon_0$ is the vacuum permittivity, and $n(t)$ is the refractive index. Differentiation of Eq. (9b) with respect to $t$ and insertion of Eq. (9a) yields with Eq. (10) the wave equation [53]

\[
n^2(t) \partial_t^2 D_x = c^2 \partial_z^2 D_x,
\]

where $c = (\mu_0 \varepsilon_0)^{-1/2}$ denotes the vacuum speed of light.
For ease of interpretation and to obtain compact analytical solutions, we assume a linear change of the optical path length with time, i.e., a uniform tuning of mirror position or refractive index, resulting in a linear wavelength sweep corresponding to a ramp. This waveform is commonly used for VT-DBR lasers and constitutes the preferred operating mode for FDML lasers [31, 54]. We note that this model can also be approximately applied to more general sweep functions by local linearization of the corresponding drive waveform.

3.1. Resonator with moving mirror

In the following, a linear Fabry-Pérot resonator with moving end mirror is considered, as employed for wavelength tuning in both MEMS-VCSELs [33, 34] and FDML lasers [37]. As shown in Section 2.1, the Doppler shift at the moving end mirror exactly compensates for the change in resonance frequency due to the change in cavity length. In this way, the mode wavelength and the corresponding photon energy are adiabatically changed, and a repeated build up of the lasing field is not necessary [29]. This concept is contrary to the strategy of rapidly varying the cavity length without changing the photon momentum, sometimes used for conventional wavelength-tuned lasers specifically to prevent mode formation [16].

3.1.1. MEMS-VCSEL

The exact analytical treatment of the MEMS-VCSEL cavity is based on the Fabry-Pérot resonator model shown in Fig. 2(a), comprising a fixed mirror positioned at $z_1 = -L_0$ and a mirror moving with constant velocity $v$, thus located at $z_2(t) = vt$. The reflection coefficients are $r_1$ and $r_2$, respectively. Furthermore, we here assume a cavity refractive index $n = 1$. The field inside the resonator is then obtained by solving Eq. (11), where the general (d’Alembert) solution is with $D_x = \epsilon_0 E_x$ given by [49]

$$E_x = E_x^+ (t - z/c) + E_x^- (t + z/c).$$

(12)

$E_x^+$ and $E_x^-$ are arbitrary functions representing right- and left-travelling waves, respectively. Furthermore, the boundary conditions at the mirrors have to be considered. For the fixed mirror at $z_1 = -L_0$, the left- and right-travelling field components are related to each other by

$$E_x^+ (t - z/c) = r_1 E_x^- [t - (z + 2L_0)/c].$$

(13)

For a right-travelling plane wave $E_x^+ = A \exp[-i\omega (t - z/c)]$ incident on the moving mirror at $z_2(t) = vt$, the reflected field is given by $A\gamma r_2 \exp[-i\gamma \omega (t + z/c)]$ with $\gamma = (1 - v/c) / (1 + v/c)$, where the frequency change is due to the Doppler shift [49]. More generally, by representing an arbitrary waveform $E_x = E_x^+ (t - z/c)$ as a superposition of plane waves it can be shown that the reflected field is

$$E_x^- (t + z/c) = \gamma r_2 E_x^+ [\gamma (t + z/c)].$$

(14)
From Eqs. (13) and (14), we thus obtain with $\tau = t - z/c$ the condition

$$E_x^+(\tau) = \gamma r_1 r_2 E_x^+ [\gamma(\tau - 2L_0/c)]. \quad (15)$$

In the cavity, wavelength tuning is obtained by linearly changing the optical path length. Thus, we expect that also the resonance wavelength changes linearly, $\lambda = \lambda_0 (1 + bt)$, resulting in a time dependent instantaneous frequency $\omega(t) = \omega_0 / (1 + bt)$. This corresponds to a complex electric field ansatz

$$E_x^+(\tau) = \hat{E}_x \exp \left( -i \int_0^\tau \omega(t^\prime) \, dt^\prime \right) = \hat{E}_x \exp \left[ -i \omega_0 b^{-1} \ln (1 + b\tau) \right], \quad (16)$$

where $\hat{E}_x$ denotes the amplitude. Inserting Eq. (16) in Eq. (15), we see that above ansatz actually solves the problem if we allow for complex values of $\omega$. For perfectly reflecting mirrors $r_{1,2} = \exp (i\phi_{1,2})$, Eq. (15) yields

$$b = \frac{1 - \gamma}{2L_0\gamma} = \frac{c v}{(c - v)L_0}, \quad \omega_0 = -b \left( \frac{2\pi m - \phi_1 - \phi_2}{\ln \gamma} + i \right), \quad (17)$$

where $m$ is the mode number. The physical electric field inside the resonator is then given by the real part of the corresponding complex quantity. From Eqs. (12) and (14), we obtain with Eqs. (16) and (17)

$$E_x = \Re \left\{ E_x^+ (t - z/c) + E_x^- (t + z/c) \right\} = |\hat{E}_x| \left\{ \frac{\cos [-\delta \ln (1 + bt - bz/c) + \phi]}{1 + bt - bz/c} + \frac{\cos [\gamma^{-1} + bt + bz/c] - \phi_1 + \phi]}{\gamma^{-1} + bt + bz/c} \right\}, \quad (18)$$

where $\delta = \Re \{ \omega_0 \} / b$ and $\phi = \angle \hat{E}_x$. The magnetic field is found from Eq. (9) as

$$H_y = \frac{|\hat{E}_x|}{c \mu_0} \left\{ \frac{\cos [-\delta \ln (1 + bt - bz/c) + \phi]}{1 + bt - bz/c} - \frac{\cos [\gamma^{-1} + bt + bz/c] - \phi_1 + \phi]}{\gamma^{-1} + bt + bz/c} \right\}. \quad (19)$$

Based on above exact solution of Maxwell’s equations, quantities such as the instantaneous wavelength, intracavity field energy and photon number can be calculated, see Sections 4.1 and 4.2.

3.1.2. FDML laser

As for the MEMS-VCSEL discussed above, the FDML laser is again based on a linear Fabry-Pérot configuration with moving end mirror, which here serves as a rapidly tuned bandpass filter for the wavelength-swept light stored in an external fiber ring cavity of up to a few km in length [37]. In contrast to conventional rapidly wavelength-tuned lasers [1, 2, 16], the filter does here not impose a hopping between the cavity modes associated with a repeated build up of lasing, but rather enables the formation of a stationary wavelength-swept cavity field which can in principle circulate undisturbed.

We consider a basic model of the FDML laser shown in Fig. 3(a), consisting only of a lossless optical fiber storing the light field and an ideal tunable bandpass filter which imposes the wavelength sweep. The optical fiber length is chosen so that the propagation time corresponds
to the duration of a full (forward and backward) sweep of the bandpass filter, ensuring that the wavelength of the optical field incident on the bandpass filter matches the resonance condition at all times. For ideal filter transmission, the wavelength-swept optical field circulating in the fiber system is then completely transmitted by the bandpass filter, thus resulting in a lossless system with adiabatic wavelength tuning.

The Fabry-Pérot bandpass filter considered in Fig. 3(b) is a generalization of the Fabry-Pérot resonator in Fig. 2(a), in that the end mirrors are now partially transmitting with reflection and transmission coefficients \( r_{1,2} = |r_{1,2}| \exp (i \phi_{1,2}) \) and \( t_{1,2} \), respectively. Thus, electromagnetic radiation is allowed to enter and exit the Fabry-Pérot filter cavity. The electromagnetic field solution for the ideal FDML model described above is given by the wavelength-swept field which is completely transmitted by the bandpass filter. We assume a left-incident wave \( E_{x,0}^+(t = t - z/c) \) of the form Eqs. (16), (17). The field entering through the left mirror at \( z = -L_0 \) is then \( E_{x,0}^+(\tau) = t_1 E_{x,1}^+ (\tau) \). The portion outcoupled through the right mirror located at \( z = vt \) is \( t_2 E_{x,0}^+(\tau) \), and the reflected component circulates in the filter cavity. The corresponding field after one roundtrip is with Eqs. (15), (16) and (17) given by

\[
E_{x,1}^+(\tau) = r_1 r_2 E_{x,0}^+ (\gamma (\tau - 2L_0/c)) r_1 r_2 E_{x,0}^+ (\tau),
\]

and for the field component after the \( m \)th roundtrip, we consequently obtain

\[
E_{x,m}^+(\tau) = |r_1 r_2|^m E_{x,0}^+ (\tau). \tag{20}
\]

The total transmitted field is with Eq. (20) given by the sum of all transmitted components,

\[
E_{x,t}^+(\tau) = t_2 \sum_{m=0}^{\infty} E_{x,m}^+(\tau) = E_{x,0}^+ (\tau) t_1 t_2 \sum_{m=0}^{\infty} |r_1 r_2|^m = E_{x,1}^+ (\tau) \frac{t_1 t_2}{1 - |r_1 r_2|}. \tag{21}
\]

Full transmission of the wavelength-swept light is obtained for non-absorbing mirrors with identical reflectivity, i.e., \( |r_1| = |r_2| = r \) and \( |t_1| = |t_2| = t \) with \( r^2 + r^2 = 1 \). The transmitted field is then given by \( E_{x,t}^+(\tau) = E_{x,1}^+ (\tau) \exp (i \theta) \) with \( \exp (i \theta) = t_1 t_2 /r^2 \), i.e., apart from a phase shift the wavelength-swept field passes the filter completely unaffected. For the limiting case of non-moving mirrors, \( v \rightarrow 0 \), the incoming field Eqs. (16), (17) becomes a monochromatic wave with angular frequency \( \omega_0 \). In this case, we recover from Eq. (17) the standard resonance condition

\[
\omega_0 = c (2 \pi m - \phi_1 - \phi_2) / (2L_0).
\]

and Eq. (21) gives the transmission for the monochromatic field at resonance [29].
The physical electric field inside the filter cavity becomes with Eqs. (14) and (21)

\[
E_x = \sum_{m=0}^{\infty} |r_1 r_2|^m \Re \left\{ E_{x,0}^+(t-z/c) + \gamma r_2 E_{x,0}^+ [\gamma(t+z/c)] \right\}
\]

\[
= \frac{1}{1 - |r_1 r_2|^2} \Re \left\{ E_{x,0}^+(t-z/c) + \gamma r_2 E_{x,0}^+ [\gamma(t+z/c)] \right\}
\]

\[
= |\hat{E}_x| \left\{ \cos \left[ -\delta \ln \left( 1 + bt - bz/c \right) + \phi \right] \right\} + |r_2| \cos \left[ \frac{-\delta \ln (1 + \gamma t + b + bz/c + \phi)}{\gamma - 1 + bt + bz/c} \right],
\]

\[
(22)
\]

where \( \delta = \Re \{ \omega_0 \}/b, \hat{E}_x = E_{x,\mu}/(1 - |r_1 r_2|) \) with the incoming field amplitude \( \hat{E}_{x,i} \), and \( \phi = \angle \hat{E}_x \). For the magnetic field, we obtain from Eq. (9)

\[
H_x = \frac{|\hat{E}_x|}{c \mu_0} \left\{ \cos \left[ -\delta \ln \left( 1 + bt - bz/c \right) + \phi \right] \right\} - |r_2| \cos \left[ \frac{-\delta \ln (1 + \gamma t + b + bz/c + \phi)}{\gamma - 1 + bt + bz/c} \right].
\]

\[
(23)
\]

As expected, for \( |r_2| = 1 \) Eqs. (22) and (23) are identical to the solution for the lossless Fabry-Pérot resonator, Eqs. (18) and (19).

3.2. Resonator with time dependent refractive index

In the VT-DBR laser, the wavelength is continuously tuned by changing the optical cavity path length, which is partly achieved by altering the effective refractive index inside the cavity through current injection \([31, 51]\). Although the achievable refractive index change is limited to \( \sim 1\% \) \([51]\), sweep widths of 40nm are obtained by additionally switching between different longitudinal cavity modes and stitching the data after acquisition \([31, 32]\).

In the following, we are only interested in the continuous tuning process due to the refractive index change of the medium inside the cavity, which is here modeled by a homogeneous dielectric with a time dependent refractive index \( n(t) \), see Fig. 2(b). As in Section 3.1, we assume again a linear optical path length change with time in the resonator, achieved by a refractive index \( n(t) = n_0(1 + bt) \). For solving Eq. (11), we make the substitutions \( F = (n_0/n)^{1/2} D_x \), \( \tau = n_0 \int_0^t n^{-1}(t') \, dt' \) \([53]\), and arrive at

\[
-\frac{n_0 b^2}{4c^2} F = \partial_t^2 F - \frac{n_0^2}{c^2} \partial_t^2 F.
\]

Eq. (24) is solved by the plane wave ansatz \( F = A \exp (\pm ikz - i\omega_0 \tau) \). Insertion yields

\[
k = \frac{n_0}{c} \omega_0 \left( 1 + \frac{1}{4} \frac{b^2}{\omega_0^2} \right)^{1/2},
\]

\[
(25)
\]

and resubstituting \( \tau = b^{-1} \ln (1 + bt) \) yields with \( E_x = D_x e_0^{-1} n^{-2} = F e_0^{-1} n_0^{-1/2} n^{-3/2} \)

\[
E_x^\pm(z,t) = \hat{E}_x^\pm (1 + bt)^{-3/2} \exp \left[ \pm ikz - i\omega_0 b^{-1} \ln (1 + bt) \right].
\]

(26)

As for the moving mirror field solution Eq. (16), the linear tuning of the optical path length leads again to a linear wavelength change over time, and thus to an instantaneous frequency \( \omega(t) = \omega_0 / (1 + bt) \). However, unlike for the resonator with moving mirror, the wavenumber \( k \) in Eq. (25) is here constant \([55]\). From the standard dispersion relation \( k = n\omega/c \) this is to be expected since the time dependence of \( n(t) \) and \( \omega(t) \) cancel out. However we note that \( k \) in
Eq. (25) deviates from this conventional dispersion relation by a factor \((1 + \omega_0^2 b^2 / 4)^{1/2}\). The boundary conditions are given by the mirrors at positions \(z = -L_0\) and \(z = 0\) with reflection coefficients \(r_1\) and \(r_2\), respectively. We obtain

\[
E_+^+(z,t) = r_1 E_-^-(z - 2L_0, t),
\]

\[
E_-^-(z,t) = r_2 E_+^+(z,t),
\]

yielding

\[
E_+^+(z,t) = r_1 r_2 E_+^+(z + 2L_0, t).
\]

For perfectly reflecting mirrors \(r_{1,2} = \exp(i\phi_{1,2})\), Eq. (28) gives with Eq. (26) the condition

\[
k = (2\pi m - \phi_1 - \phi_2) / (2L_0)
\]

and thus with Eq. (25) the resonance frequencies

\[
\omega_0 = \left[ c^2 n_0^2 \left( \frac{2\pi m - \phi_1 - \phi_2}{2L_0} \right)^2 - \frac{1}{4} b^2 \right]^{1/2},
\]

where \(m\) is the mode number. This differs from the resonance condition in a standard Fabry-Pérot resonator by the additional term \(\sim b^2/4\) in Eq. (30).

The physical electric field inside the resonator is given by the real part of the corresponding complex quantity. With Eqs. (26) and (27), we obtain

\[
E_x = \Re \{ E_+^+(z,t) + E_-^-(z,t) \}
\]

\[
= \frac{2|\hat{E}_x|}{(1 + bt)^{3/2}} \cos \left( kz - \frac{\phi_2}{2} \right) \cos \left[ -\frac{\omega_0}{b} \ln (1 + bt) + \phi + \frac{\phi_2}{2} \right],
\]

where \(k\) and \(\omega_0\) are given by Eqs. (29) and (30), respectively, and \(\hat{E}_x^+ = |\hat{E}_x| \exp(i\phi)\). The magnetic field is found from Eq. (9) as

\[
H_y = -\frac{2\omega_0 n_0^2 |\hat{E}_x|}{k (1 + bt)^{3/2}} \sin \left( kz - \frac{\phi_2}{2} \right)
\]

\[
\times \left\{ \omega_0 \sin \left[ -\frac{\omega_0}{b} \ln (1 + bt) + \phi + \frac{\phi_2}{2} \right] + \frac{b}{2} \cos \left[ -\frac{\omega_0}{b} \ln (1 + bt) + \phi + \frac{\phi_2}{2} \right] \right\}.
\]

In Sections 4.1 and 4.2, the field solutions Eqs. (31) and (32) are used to calculate quantities such as the instantaneous wavelength, intracavity field energy, and photon number.

4. Results

4.1. Instantaneous wavelength

Both the tuning of mirror position and refractive index aim at varying the optical path length inside the resonator. As shown in Sections 3.1 and 3.2, for a uniform change in cavity length \(L(t) = L_0 (1 + vt)\) or refractive index \(n(t) = n_0 (1 + bt)\) the exact solution of Maxwell’s equations yields linear tuning with the instantaneous vacuum wavelength

\[
\lambda(t) = 2\pi c / \omega(t) = 2\pi c (1 + bt) / |\Re \{ \omega_0 \}|.
\]

The angular frequency \(\omega(t)\) in Eq. (33) is obtained from the temporal derivative of the argument of the cosine function in Eq. (18) and Eq. (31), with \(\omega_0\) given by Eqs. (17) and (30), respectively.
For cavity length tuning, Eq. (33) applies to the right-propagating field component at position \( z = 0 \). Importantly, the wavelength tuning speed \( \partial t \lambda = v_i = 2\pi cb / |\Re\{\omega_0\}| \) is only restricted by the tuning mechanism itself, rather than by the build up time of the lasing field from ASE.

This linear wavelength sweep can intuitively be explained based on the conventional resonance condition [29]

\[
\lambda_m = 2L_{op} \left( m - \frac{\phi_1 + \phi_2}{2\pi} \right)^{-1},
\]

where \( \lambda_m \) is the vacuum wavelength of mode \( m \) for a linear cavity with optical length \( L_{op} = Ln \). Although Eq. (34) is only valid for the stationary cavity, we show that it can in practice also be used for rapidly wavelength-tuned resonators. In the case of cavity length tuning, Eq. (33) yields with Eq. (17) the instantaneous vacuum wavelength

\[
\lambda(t) = \ln \left( \frac{c + v}{c - v} \right) \left( \frac{c - v}{v} L_0 + ct \right) \left( m - \frac{\phi_1 + \phi_2}{2\pi} \right)^{-1}.
\]

In the limit \( v \ll c \) which is fulfilled for all practical purposes, the conventional resonance condition Eq. (34) is recovered from Eq. (35), with the time dependent optical path length \( L_{op} = L_0 + vt \) as intuitively expected. In the case of refractive index tuning, the instantaneous vacuum wavelength is with Eqs. (33) and (30) obtained as

\[
\lambda(t) = (1 + bt) \left[ \lambda_m - \frac{b}{4\pi c} \right]^{-1/2},
\]

where \( \lambda_m \) is the resonance frequency of the unswept laser cavity, given by Eq. (34) with \( L_{op} = L_0n_0 \). For \( b\lambda_m / (4\pi c) \ll 1 \), Eq. (36) assumes the form of the conventional resonance condition Eq. (34) with a time dependent optical path length \( L_{op} = L_0 n(t) = L_0 n_0 (1 + bt) \). This assumption is always fulfilled in practice; e.g., for a sweep rate of 1 MHz over a range of 100 nm, we obtain \( b\lambda_m / (4\pi c) \approx 2.65 \times 10^{-11} \).

Above results show that for a uniform change in cavity length or refractive index, a linear wavelength sweep is obtained as intuitively expected, and the wavelength tuning speed is not fundamentally restricted. While the exact expression for the instantaneous wavelength deviates from the conventional resonance condition Eq. (34), this relation can be used for all practical purposes by inserting the time dependent optical path length into Eq. (34). Thus, the laser configuration remains in the same instantaneous cavity eigenmode during sweep operation, enabling rapid wavelength tuning without repeated build up of the optical resonator field.

4.2. Electromagnetic field energy and photon number

The electromagnetic field energy in the resonator is obtained by spatial integration over the energy density of the electromagnetic field \( (\varepsilon E_x^2 + \mu_0 H_y^2) / 2 \) [41, 49], yielding

\[
W = \frac{A}{2} \int_{-L_0}^{z_2} \left( \varepsilon E_x^2 + \mu_0 H_y^2 \right) dz.
\]

where \( A \) is the field cross-sectional area. Furthermore, \( \varepsilon = \varepsilon_0 \) and \( z_2 = vt \) for cavity length tuning [Fig. 2(a)], while \( \varepsilon = \varepsilon_0 n^2(t) \) and \( z_2 = 0 \) for refractive index tuning [Fig. 2(b)].

For the three cavity configurations discussed in Sections 3.1 and 3.2, Eq. (37) is evaluated by inserting the corresponding field solutions given by Eqs. (18), (19), Eqs. (22), (23), and Eqs. (31), (32), respectively. The resulting field energy, averaged over an optical cycle, can be brought into the form

\[
W = \varepsilon_0 n_0^2 |\hat{E}_x|^2 \frac{1 + |r_2|^2}{2} \frac{AL_0}{1 + bt},
\]
where \( v \ll c \) and \( h\lambda_{\text{th}}/(4\pi c) \ll 1 \) have been assumed for cavity length and refractive index tuning, respectively (see also Section 4.1). Here, \(|r_2| < 1 \) for the Fabry-Pérot filter cavity used in the FDML laser, and \( |r_2| = 1 \) otherwise. Furthermore, \( n_0 \) is set to 1 for cavity length tuning. The obtained expression for the electromagnetic field energy \( W \) is consistent with the heuristically derived results in Eqs. (8) and (1). \( W \) varies with time in the same way as the photon energy \( hc/\lambda(t) \), see Eq. (33), where \( h \) is Planck’s constant. Thus, as expected in the absence of nonlinear effects \([48]\), the intracavity photon number is conserved, i.e., the photon number is an adiabatic invariant of the tuning process \([41, 48]\). After one full bidirectional sweep cycle, the initial electromagnetic energy is recovered. For cavity length tuning, the wavelength shift is provided by the Doppler effect \([29]\), and the corresponding energy is extracted (for \( v > 0 \)) or supplied (for \( v < 0 \)) by the mechanical displacement work of the mirror \([48]\) (see also Section 2.1). In the case of refractive index tuning, the energy is provided by the external agent causing the refractive index change in the medium \([44]\), as exemplified in Section 2.2.

For the FDML laser discussed in Section 3.1.2, Eq. (38) refers to the Fabry-Pérot filter, while basically the complete wavelength-swept field is stored outside the filter in the fiber ring cavity. Ideally, the filter consists of two lossless mirrors with identical reflectivity. As shown in Section 3.1.2, it then serves as a tunable bandpass filter fully transmitting the field with the proper wavelength sweep, which might be referred to as adiabatic wavelength scanning. For FDML operation, the sweep duration has to be synchronized to the cavity roundtrip time, determined by the fiber length \([37]\). This enables the formation of a stationary wavelength-swept cavity field which can circulate undisturbed, and the photon number conservation applies to the filter cavity as well as to the total FDML system. In practice, this ideal dynamics is perturbed not only by cavity losses, but also by optical nonlinearities and especially fiber dispersion \([56, 57]\).

5. Discussion

5.1. Differences between cavity length and refractive index tuning

As discussed in Sections 4.1 and 4.2, there are many common features of cavity length and refractive index tuning. Most importantly, inherent wavelength shifting allows for wavelength sweeps without hopping between the cavity modes, thus avoiding a repeated build up of the optical resonator field and the associated limitation in tuning speed. However, there are also fundamental differences between those two processes \([45]\). Specifically, cavity length tuning relies on the Doppler shift at the moving end mirror which provides an abrupt wavelength change. By contrast, for refractive index tuning the shift is induced over an extended region. Thus, if the refractive index change occurs throughout the whole cavity, as assumed for the model calculation in Section 3.2, the instantaneous wavelength inside the cavity is position independent, as is the wavenumber \( k \) [see Eq. (25)]. From a practical point of view, cavity length tuning can provide an arbitrary sweep range. On the other hand, refractive index tuning has the advantage that no moving parts are needed \([31, 32]\), but the achievable refractive index change is limited to \( \sim 1\% \) \([51]\). Thus, additional switching between different longitudinal cavity modes is required to increase the tuning range in VT-DBR lasers \([31, 32]\).

5.2. Comparison to conventional rapidly wavelength-swept lasers

For conventional rapidly wavelength-swept lasers, the maximum wavelength tuning speed \( \partial_t \Delta \lambda = v_t \) where the lasing activity can still fully build up from the ASE background is estimated as \( v_{t,\text{max}} = \Delta \lambda / T_b \) \([2, 16]\). Here, \( \Delta \lambda \) is the linewidth or spectral resolution of the laser, which is for conventional rapidly wavelength-swept lasers determined by the filter width of the tuning element \([2, 16]\). The oscillation build up time from ASE is given by \( T_b \approx T_R \ln(I_s/I_0) / \ln G_{\text{net}} \) \([2, 29]\), where \( T_R = L_R/c \) is the cavity roundtrip time with the optical cavity roundtrip length \( L_R \). In a linear cavity with length \( L \) and refractive index \( n \), we have \( L_R = 2Ln \). Furthermore,
Furthermore assuming For conventional rapidly wavelength-tuned lasers, a typical value is increase the coherence length, it becomes clear that wavelength tuning does not apply to FDML lasers. Combined with ongoing efforts to further $L_c$ of MEMS-VCSELs, FDML and VT-DBR lasers discussed above. With a typical coherence length $G_{\text{net}}$ of 20nm and $G_{\text{net}} \approx 100nm$ with at least tens of mW output power [9, 34], a tuning speed exceeding $10^7$ nm/ms which is smaller by a factor of $10^4$, clearly showing that this speed limitation for conventional wavelength tuning does not apply to FDML lasers. Combined with ongoing efforts to further increase the coherence length, it becomes clear that $v_{t, \text{max}}$ in Eq. (39) poses a serious limitation to conventional wavelength tuning even for short cavity lasers with $L_R$ in the cm range. As demonstrated in Sections 3.1 and 3.2, adiabatic wavelength tuning is not subject to this limit, and $v_t$ is in principle only restricted by the tuning mechanism itself, i.e., the modulation speed of mirror position or refractive index. Furthermore, the linewidth is here not set by the filter or cavity bandwidth, but rather single mode operation is ideally obtained [31, 34]. For MEMS-VCSELs, Eq. (39) yields $v_{t, \text{max}} \approx 1.5 \times 10^5$ nm/ms for typical parameter values of $L_R \approx 1 \mu m$ and $G_{\text{net}} \approx 1.01$ [33, 34], i.e., $v_{t, \text{max}}$ is much larger than for FDML lasers due to the short cavity length. Nevertheless, Doppler-induced intracavity wavelength conversion also plays an important role for the performance of MEMS-VCSELs, as discussed in the following section.

5.3. Fundamental time-bandwidth limits: Experimental evidence for Doppler shift

The previous considerations mainly referred to typical elements and implementations used to build today’s swept lasers. One could infer that improved designs with, e.g., reduced cavity length or increased gain of the laser medium might enable high tuning speed in classical lasers. Accordingly, it has been argued that the reason why the MEMS-VCSELs can generate very rapid wavelength sweeping is due to the micron scale cavity length and the resulting fast roundtrip time, enabling a very rapid build up of lasing [62]. For example, if in Eq. (39) a cavity roundtrip length of $L_R \approx 10 \mu m$ is used, wavelength tuning speeds exceeding $10^7$ nm/ms corresponding to sweep rates of many MHz over a 100 nm tuning range should be possible. Thus, the fast build up is often used as the only explanation for the rapid tuning of VCSELs. However, this would be impossible and violate the fundamental time-bandwidth relation as the following consideration will show.

The wavelength tuning speed in Eq. (39) is the maximum value where the lasing activity can still fully build up from the ASE background, i.e., higher sweep rates can only be obtained...
at the expense of output power [1]. Additionally, an upper tuning speed limit exists which cannot be exceeded for fundamental reasons. Grulkowski et al. [63] demonstrated OCT with a MEMS-VCSEL up to 1.52 m imaging range corresponding to $L_c = 3.04$ m roundtrip delay, yielding with Eq. (40) an instantaneous optical linewidth of $\Delta_{\lambda} = 0.168$ pm or equivalently $\Delta_f = 43.5$ MHz at $\lambda = 1075$ nm. Using the time-bandwidth product (for a Fourier-limited Gaussian pulse) $\Delta_f/\Delta_t = 0.44$, we obtain a minimum time uncertainty of $\Delta_t \approx 10$ ns. The wavelength sweep rate was 20 kHz, corresponding to a sweep duration of 50 $\mu$s. The wavelength sweep range was 9 nm, corresponding to 2.3 THz at 1075 nm. This means, 2.3 THz/43.5 MHz = 52874 spectral points could be resolved. The time duration for the laser to sweep over a 43.5 MHz window is then 50 $\mu$s/52874 $\approx$ 1 ns. This value is clearly below the time uncertainty of $\approx 10$ ns, i.e., it is impossible that the laser radiation in the VCSEL builds up from noise, because this would at minimum take 10 times longer. However if the light field is already present in the laser and is just shifted in wavelength a much longer photon lifetime is possible, enabling the observed extremely long coherence lengths. So the results in [63] are a direct proof for the adiabatic intracavity wavelength shift discussed in this paper and indicate that the Doppler shift becomes essential for rapidly tunable lasers above a certain tuning rate.

The same considerations of course also apply inversely to the stitching procedure presented by Bonesi et al. [32]. The Vernier tunable laser only sweeps piecewise continuously which necessitates stitching the data after acquisition, see Section 3.2. Over 40 nm sweep range 450 resets of the laser are required. If the same coherence performance should be achieved as presented in [63] the maximum sweep rate would be limited to $1/(450 \times 10$ ns) = 220 kHz.

6. Conclusion

We have theoretically analyzed the physical key mechanism to enable very rapid wavelength sweeping in the most recent actively tuned high speed OCT lasers. The MEMS-VCSELs, FDML and VT-DBR lasers represent the next generation OCT light sources, featuring several 100 kHz sweep repetition rate over wavelength ranges of $\approx 100$ nm. In all concepts an inherently synchronous wavelength shift of the intracavity light is achieved by changing the optical path length of the laser cavity. This shift ensures that the laser light in the cavity always matches the continuously changing resonance frequency of the corresponding longitudinal mode during wavelength sweep operation. The MEMS-VCSEL and FDML laser employ a linear Fabry-Pérot resonator with a movable end mirror, where the necessary wavelength shift is provided by the Doppler effect. The VT-DBR laser uses a change of the intracavity refractive index to tune the optical path length.

Based on heuristic arguments and exact solutions of Maxwell’s equations, we have shown that a linear change of the optical path length with time by uniform tuning of mirror position or refractive index results in a linear wavelength sweep. Adiabatic wavelength conversion of the intracavity field with conservation of photon number is obtained, and the change in photon energy is compensated by the tuning mechanism itself. Thus, the optical gain only compensates for mirror outcoupling and other losses as in fixed-wavelength lasers. A closely related concept employed in FDML lasers is adiabatic wavelength scanning, where a tunable Fabry-Pérot bandpass filter imposes a wavelength sweep onto an optical field circulating in a fiber ring cavity, which can then pass the filter undisturbed.

In conclusion, it is found that for very rapidly swept lasers inherent wavelength shifting is necessary to avoid limitations in tuning speed. Current implementations of such lasers use the Doppler effect or intracavity refractive index tuning for adiabatic wavelength conversion. In this way, a repeated build up of lasing as in conventional swept sources is avoided, and in principle arbitrary wavelength tuning speeds at narrow instantaneous linewidths can be obtained.
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