Analysis of mathematical abstraction on concept of a three dimensional figure with curved surfaces of junior high school students

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Abstract. The purpose of this study is to determine the process of abstraction of junior high school students on the concept of a Three Dimensional Figure with Curved Surfaces (TDFCS). Benefits of this research are to provide input to teachers in order to know the process of student abstraction, so it can be used as consideration in preparing lesson plans. The problem that is considered is how the process of student abstraction related to the concept of TDFCS. The research method used is qualitative descriptive with the subject of research is 25 students of Junior High School Students grade 9th. The main instrument in this study is the researchers themselves, tests abstraction and interview guidelines. Furthermore, three students were selected to be interviewed (from high, average and low ability based on their prior knowledge test). In this study, the abstraction used is a reflective abstraction with the following stages (1) recognition, (2) representation, (3) structural abstraction, and (4) structural awareness. The conclusion in this study shows that junior high school students generally still do not have abstraction abilities in the concept of TDFCS, so it becomes a necessity to do further research using learning models and appropriate teaching materials.

1. Introduction
Mathematical concepts should be constructed in the minds of students through meaningful learning processes, not transferred directly, nor emphasizing students to memorize it only. The concept construction process that occurs in students' minds by using their initial experience or knowledge is called mathematical abstraction process [1]. So, mathematical abstraction is one of the abilities that students must possess, because it is the path of the emergence of mathematical concepts, and it is very crucial for student [2].

The process of abstraction occurs when one is aware of the same characteristics of an object based on the previous experiences. The similarity is the basis for classification. We can recognize a new experience by comparing it to the old one [3]. In simple terms this process is called the process of abstraction, the result of the process of abstraction is a concept. It is in line with the ideology of empiricism Aristotle.

Those experts who have reviewed the issue are Skemp [3], Ferrari [4], Mitchelmore and White [5,6], Hong and Kim [7], etc. In general, previous studies are still on topics outside of geometry, whereas geometry has a significant position on some things, for example, by studying geometry will...
improve logical thinking and accelerate students' mental development. Theoretically and based on Euclid philosophy, the relationships between objects in geometry are studied deductively, avoiding the empirical induction, and separating math from the real world. It avoids the process of abstraction and makes many students find it difficult. Based on the cognitive point of view, the main reason for failure in learning mathematics, in particular, geometry, is the abstraction [4].

In Indonesia, geometry becomes one part of the school's mathematical subject that is difficult to learn. Trends in International Mathematics and Science Study (TIMSS) in 2011 said that the dimension of Indonesian students' lowest content is geometry. Geometry is taught only as a rote and count. Students must memorize the definitions and theorems without knowing the process of finding the concept. Students also can not apply it in life and cannot transfer it into a new context and avoid the process of abstraction. Therefore, it is necessary to design an instructional experience to facilitate the process of abstraction in learning geometry.

Based on the statement from Saitta and Zucker, abstraction process in geometry can start from the observation and measurement of physical spaces and forms, moving then to the abstract axioms of the Euclidean geometry and later on to non-Euclidean geometries, farther and farther removed from the perceived physical world [8]. A three-dimensional figure with curved surfaces is a fundamental concept in geometry, especially for junior high school students of grade 9th. It is a continuation of flat surfaces. Students learn the basic concepts of this comprehensively. We can see the process of abstraction during learning.

This paper contains an analysis of the student's abstraction process when learning the concept of a three-dimensional figure with curved surfaces. This research provides information on how to generate abstraction process. Such information can be useful for further mathematics teachers and researchers in designing learning contexts that lead to mathematical abstractions.

1.1 Abstraction Ability

Piaget distinguishes three types of abstractions namely: Empirical Abstraction, Pseudo-Empirical Abstraction, and Reflective Abstraction [9]. In Empirical Abstraction, individuals acquire knowledge through the properties of objects. It means, we can gain such knowledge from the previous experiences. The acquired knowledge is internal and the results constructed internally by the subject. Pseudo-Empirical Abstraction is midway between Empirical and Reflective. This abstraction occurs when a person meets an object, then discover the properties of the object through the process of imagining an action. He tries to configure the object in the space as well as to examine possible relationships as an effort to release the material properties of an object. Reflective abstraction is the general coordination of action so that the source is a subject equipped with complete internal properties. Focusing on the idea of action and operation being a thematic object to thinking or arguably with assimilation. The result of a reflective abstraction is a knowledge scheme.

The flow of empirical abstraction processes and theoretical abstractions are different. In empirical abstractions, individuals form new concepts based on observation and experience. In theoretical abstractions, matching old concepts with experiences in individual thinking will form new concepts. Piaget's theory of reflective abstraction is a form of theoretical [10].

Based on the above notions, the process of mathematical abstraction in this study is the construction of concepts that occur in the minds of students. The construct takes advantage of the student's initial experience or knowledge associated with the theory, thus including into a reflective abstraction. Levels in the reflective abstraction are recognition, representation, structural abstraction and structural awareness [11]. The first level is recognition, the visible aspect of abstraction. Students identify the existing of mathematical structure, both in the same and previous activity. Recognising a mathematical structure occurs when a student realizes the structure that has existed and may have been used previously "attached" to the current (basic concept) mathematical problem. To achieve these objectives, they must recall the structures they have acquired in their previous activities and use them for the next. The second level is a representation, the visible aspect of the abstraction is that student using the diagram in solving a situation. The third level is a structural abstraction, the visible aspect that students reflect the previous activity. Students also able to reorganize the structures created from
the activity and interpretation to a new situation. The existing mathematical structure is projected and reorganized, thereby increasing the depth of their own knowledge. The reorganization of concepts is the activity of collecting, composing, and developing mathematical elements into new. The fourth level is structural awareness, the visible aspect of the abstraction is the student showing one ability to anticipate the results of the activity without having to finish all the thinking activities.

2. Method
This research uses a descriptive qualitative method and the subject of this study is junior high school students of grade 9th in one school in Ngamprah as many as 25 students. The main instruments in this study are the researchers themselves and also abstraction ability tests (with one question) and interview guides. Researchers would like to explore in depth how the process of abstraction (based on abstraction levels) on the concept of a three-dimensional figure with curved surfaces. For data collection, the researcher gives the abstraction test, then conducts an interview. Three students were selected to be interviewed from high (Score $\geq \bar{x} + 0.5\text{Stdv}$), average ($\bar{x} - 0.5\text{Stdv} \leq \text{Score} < \bar{x} + 0.5\text{Stdv}$) and low (Score $< \bar{x} - 0.5\text{Stdv}$) ability based on their score of prior knowledge test. The next step is to analyze the subject's answer to test the relevance of their representations by comparing data and theory. The final step is to conclude the abstraction of the studied students.

3. Result and Discussion
The Question of Mathematical abstractions test:
"A water container, the base and the top is a circle of radius 6 cm and 18 cm respectively. Determine the maximum water volume (in liters) in the container!. Write down what is known and asked! (In answering the question write down the steps and also what formulas/rules you use)".

Table 1 shows the recapitulation of abstraction test results based on their level.

| Abstraction Level in Reflective Abstraction | Characteristic and Activity | %     |
|------------------------------------------|-----------------------------|-------|
| Recognition                              | Recall previous activities and experiences related to the problem at hand. | 30    |
| Representation                            | Be able to solve problems by anticipating any source of difficulty (by first stating the thought result in the form of mathematical symbols, words or diagrams). | 13    |
| Structural Abstraction                    | Reorganise (collect, assemble, develop) mathematical elements into new elements. | 7.3   |
| Structural Awareness                      | Give reasons (formulas/rules) to the resulting decision. Be able to show a summary of their activities. | 6 5.5 |

From the table 1, students' mathematical abstraction ability, must be continuously developed. At the Recognition level, students should be able to recall previous activities and experiences related to the problem at hand. The test result is not maximum, it's 30% and looks very far from the maximum achievement. The higher the level of abstraction more complex the activity and its characteristics. Students appear to have problems for higher levels because the results achieved decreased, especially at the end of Structural Awareness level of 5.5%. It is shown that students do not have adequate mathematical abstraction ability. Furthermore, three students were selected based on their ability level.
(high, average and low based on their prior knowledge). Researchers will analyze more deeply about their mathematical abstractions. Here is the result of the recapitulation process of abstraction test work.

| Characteristic and Activity | Subject 1 | Subject 2 | Subject 3 |
|-----------------------------|------------|------------|------------|
| Recall previous activities and experiences related to the problem at hand. | Student tries to remember the concept and makes predictions to solve the question by applying the concept of comparative worth. | Students assume the problem relates to the concept of the tube and has a radius of 6 cm. | Students assume the problem is related to the concept of trapezoidal because there are parallel sides. |
| Be able to solve problems by anticipating any source of difficulty (by first stating the thought result in the form of mathematical symbols, words or diagrams). | The student sketched the image and made it into a cone, where there is a comparative concept for determining unknowns. | The student sketched the image in question. | The student sketched the image on the problem of being a trapezoid whose legs were the same length, with the intention of making an analogy. |
| Reorganise (collect, assemble, develop) mathematical elements into new elements. | Students determine the length of the truncated cone blanket with the concept of comparison, then determine the height of the truncated cone. Next, the student determines the large cone volume and the truncated cone, then determines the difference from the large cone with the small cone. The result is the volume of the water container in question. | The student assumes the image is a tube, so to determine the volume, the student directly performs the calculation with the concept of tube volume, which is the base area times height, without considering the base and the roof has different sizes. | Students consider the question to be an analogy of the trapezoid concept, so to determine the student's volume using the formula \( a + b \times \frac{1}{2} \times t \). |
| Give reasons (formulas/rules) to the resulting decision. | Students write the rules/formula to solve the problem correctly and systematically. | Students write the rules, but they are not appropriate. | Students do not write the formula and answer it directly. But the answer is wrong. |
| Be able to show a summary of their activities. | Student shows a summary of their activities, and their contents correspond to the solution of the problem. | Students write a summary of their activities, but the solution is wrong. | The student does not show a summary of activity. |
Based on the description in table 2, we will submit the results of interviews to students whose level of mathematical ability is high, medium and low (based on their prior knowledge). The transcript of the interview can we see below (R = Researcher, S1 = Subject 1/high, S2 = Subject 2/average, S3 = Subject 3/low).

R : As you read that question, what's on your mind?
S1 : After looking at the image on the question, I immediately imagine that the image is a truncated cone. From that problem, many unknown lengths (subject 1 thinks long enough), seems to be solved by comparison.
S2 : I do not think so. That's a picture of a tube, just look at the picture (while looking at the picture on the question).
R : Is it that true?
S2 : Of course, the base and top is a circle, so it's easy to set the volume, hahaha...(laugh).
S3 : Unlike you guys, I think the picture is a trapezoid with the same leg length.
R : Why do you think so?
S3 : Because, there are parallel sides, though it is not the same length.

The above conversation shows students' ability in the level of Recognition [11], where students are encouraged to recall previous activities and experiences related to the problems at hand. All the students interviewed had different views on the problem. Consequently, we have a variational solution. A student with high mathematics ability has a useful experience in solving problems. The student with average ability doesn't have a useful experience in solving problems. He incorrectly defines the tube, making it wrong in determining the settlement step. A student with low ability, experience used to solve a problem does not make sense.

Next transcript,

R : Can you solve it? If you can, how?
S1 : I can. First, I will draw the full cone. Then, look for the little cone line painters with the concept of comparison
R : Are you sure of the way you use it? Try to convince me!
S1 : On a large cone, the length of the painter's line must be proportional to the radius of the circle. Likewise on the small cone. The comparison value of both will be the same.
R : After that, what should you do?
S1 : Since the sought is the volume of the truncated cone, then I have to determine the height of the cone by using the concept of comparison again.
R : and then?
S1 : I will look for the volume of the whole cone and the small cone, then find the difference volume between whole and small cones, finished.

Here is a snippet of work S1,

![Figure 1. S1 answer for question.](image)

S2 : If I immediately solve it by using the tube volume formula, i.e. $V = \pi r^2 t$ where $r = 6$ and $t = 20$, the volume is $720\pi$ cm$^3$
R : Why use tube concept? Are you sure?
S2 : Yes I'm sure because it's a picture of a tube, where the base and top are circular.
R : Should the base and tube roof not be congruent?
S2 : (Silent), I do not know.

Here are the results of S2 work,

![Image of Figure 2. S2 answer for question.]

R : How about you, why answer like that?
S3 : Actually, I do not know. I'm just guessing if it's a trapezoid with the same leg length, but it seems to be for flat trapezium, whereas in this question, the picture has a volume (looks thinking).
R : so how do you think?
S3 : to determine its volume clearly by summing the two parallel sides then divided by 2, then multiplied by its height.

Here are the results of S3 work,

![Image of Figure 3. S3 answer for question.]

From the interviews and the figure 2 and 3, it appears that all students have sufficient Recognition ability. However, students with a medium and low ability are still mistaken for their experience. They also can't anticipate the difficulties encountered and mistaken in reorganizing its elements so that the Representation and Structural Abstraction level was passed [11]. For the level of Structural Awareness experienced the only student with high ability.

4. Conclusion
The abstraction ability of junior high school students must be continuously developed because the process of abstraction is to discover a concept. Students appear to have problems for higher levels, especially at the end of Structural Awareness. The higher level of abstraction more complex the requirement. Based on this, needs learning models and appropriate teaching materials in learning geometry that can develop the process of abstraction. It can start from the observation and measurement of physical spaces and forms, moving then to the abstract axioms of the Euclidean geometry, and, later on, to non-Euclidean geometries, farther and farther removed from the perceived physical world [8].

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