The Measurement Process in Local Quantum Theory and the EPR Paradox

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Abstract

We describe in a qualitative way a possible picture of the Measurement Process in Quantum Mechanics, which takes into account:
1. the finite and non zero time duration $T$ of the interaction between the observed system and the microscopic part of the measurement apparatus;
2. the finite space size $R$ of that apparatus;
3. the fact that the macroscopic part of the measurement apparatus, having the role of amplifying the effect of that interaction to a macroscopic scale, is composed by a very large but finite number $N$ of particles.

The conventional picture of the measurement, as an instantaneous action turning a pure state into a mixture, arises only in the limit $N \to \infty, T \to 0, R \to \infty$.

The limit where $N$ tends to infinity has been often discussed as the origin of decoherence. We argue here that, as a consequence of the Principle of Locality (requiring that observables measured in mutually spacelike separated regions of spacetime should commute with one another), before those three limits are taken, no long range entanglement between the values of observables which are spacelike separated far away can be detected (although, as it is well known, entangled states for such observables will exist in local theories: simple examples are given in an Appendix). In order to detect correlations, one of the observers has to wait until he enters the future causal shadow of the region employed by the apparatus of the other. Accordingly, in this picture of the measurement process there would be no Einstein Podolski Rosen Paradox. (Similar views had been proposed already [6], [22]). A careful comparison with the growing experimental results of the recent decades might settle the question.
1 Introduction

The unitary, reversible nature of the Schroedinger time evolution of a quantum system seems foreign to the sudden irreversible jump from a pure state $\omega$ (here a state is a positive linear normalised expectation functional over the algebra $\mathfrak{A}$ of observables) to the statistical mixture, where the expectation value of an observable $B$ is $\omega(E_1 B E_1 + \ldots + E_n B E_n)$, caused by the measurement of an observable (taken for simplicity with finite spectrum $\lambda_1, \ldots, \lambda_n$) $A = \sum_j \lambda_j E_j$. The reduction of wave packets which occurs when one particular result $\lambda_j$ is recorded, is supposed to take place instantaneously everywhere. We propose that this credo should not be taken literally as an Axiom of Quantum Mechanics, but as an oversimplified limit of the picture of realistic interaction processes between the System $S$ and the measurement apparatus $A$.

The first important step in the direction of reconciling the Measurement Process in Quantum Mechanics with the Schroedinger time evolution had been taken since the early years of Quantum Mechanics by John von Neumann (cf the exposition in [1]). The observed system $S$ and the measurement apparatus $A$ are described quantum mechanically by the tensor product of their Hilbert spaces of pure state vectors. The state vector of the composed system before the measurement is a product vector; the interaction between the two parts during the measurement takes that vector, by the unitary time evolution of the coupled system $S + A$, to another vector in the product Hilbert space, which is no longer a product vector, but a linear combination of such vectors. Therefore, the corresponding expectation functional (which is a pure state on the composed system) becomes a statistical mixture when restricted to the observables of the system alone.

Why have we to forget the observables of the measurement apparatus $A$, thus loosing all the correlations between the possible outcomes? And how do we restrict to just one of those? The answer von Neumann gave invoked the consciousness of the observer, a solution leading to well known paradoxes.

A scenario, which in our view is quite satisfactory, emerged first in a complete shape in the work of Daneri, Loinger and Prosperi [3]. The point, expressed in our own language, is that the measuring device $A$ must consist of a microscopic part, that we will call $\mu A$, and of a macroscopic part, that we will call $MA$, which amplifies the results to our observed scale. The part $MA$ interacts appreciably only with $\mu A$, and is composed of $N$ particles.

As long as $N$ is kept finite, the whole system $S + \mu A + MA$ has to be treated quantum mechanically, and the full state remains a pure state of
the composed system under the Schroedinger evolution. But the interference terms between the different outcomes will be so small, that the relative phases will be completely out of our experimental reach if $N$ is, as in practice, very large. Actually in concrete examples (bubble chamber, Geiger counters and their modern descendants, etc) the amplification is effectively an irreversible process (and as such *in principle* reversible only if you take into account the fact that the number $N$ of the particles involved is finite; while we can well take it to be infinite for all practical purposes).

The wisdom on this point of view, built on previous arguments of Ludwig, later enriched considerably due to many works (by K. Hepp, D. H. Zeh, W. Zurek, G. C. Ghirardi, R. Gambini, and many others; we refrain from trying to give a complete reference list, which can be found e.g. in \[4\]) up to the recent work of G. Sewell \[5\]. Decoherence arises by the fast decrease with $N$ of the interference terms.

But while in the von Neumann picture one *assumes* that there is an interaction potential between $S$ and $A$ which makes the measurement possible, in Quantum Field Theory the interaction is described from the start by the time evolution, as already discussed in some of the above references.

The only point of the present note is to suggest that, if we want to measure a *local observable* $A$, say localised in a space volume $V$ between time $t$ and $t+T$, then the operation of “adding to a state the *microscopic part* $\mu A$ of the apparatus which measures $A$” should be *localised in the same* (or possibly in a larger but finite) spacetime region; while the amplifying *macroscopic part* $MA$ will be localised in a much larger, macroscopic neighbourhood, which would be infinitely extended (at least in some direction) in the limit $N \to \infty$ (but practically extending e.g. from few microns to few centimeters, and from few microseconds to few seconds, around $V$ and $t$). But the interaction between $S$ and $MA$ will be neglected as inessential (the ionising cosmic ray crossing a bubble chamber does not interact with the bubble which develops as an effect of the ionisation of a single molecule, caused by one of its collisions), thus the decoherence, caused by the interaction between $\mu A$ and $MA$, is *not* the central point here.

At this point we take into account the *locality principle*, which requires that observables which can be measured in spacelike separated regions of spacetime should commute with each other; this is an expression of Einstein Causality: no physical effect can be transmitted faster than light.

Accordingly, our picture of the measurement of an observable $A$ *localised in some bounded spacetime region* entails that, if we test an arbitrary state with observables $B$ localised far away from that region in a spacelike direction, we will find *the same result* in the given state as in that obtained.
adding to that state the microscopic part \( \mu A \) (as well as adding also the macroscopic part \( MA \), if we are doing our test really far away) of the apparatus which measures \( A \). Similar conclusions have been proposed long ago in the description of measurements in Local Quantum Field Theory by Hellwig and Kraus [6].

For example, if Bob measures now, in a laboratory in the Andromeda Galaxy, the polarisation of a photon emitted long ago half way between here and there in an EPR gedanken experiment [2], and Alice measures here and now the polarisation of the companion photon, repeated measurements over many such events will show that Alice sees a superposition of left and right handed polarisations, whatever results Bob finds; correlations would show up only if Alice waits more than two million years, the time necessary for her to enter the future causal shadow of the laboratory of Bob.

These statements do not depend upon a choice of a Lorentz frame, so that no conflict with Special Relativity arises. They contrast, however, the most spread opinion, namely that:

a) EPR is a real effect;

b) the locality principle is not violated;

c) there is no contradiction, since the device of the EPR gedanken experiment cannot be used to transmit any physical effect, nor even information.

Our discussion instead points to a contradiction between a) and b); we claim that b) holds (as long as the gravitational forces between elementary particles can be neglected, namely at scales much larger than the Planck distance, i.e. at energy ranges well below the Planck energy; cf the exposition in [18] and references therein), and a) does not.

Are these conclusions in contradiction with the current experiments, especially by A.Aspect and collaborators [13]?

It is not obvious that the answer is affirmative, as one might think at first sight. For, the whole experimental setting, consisting among other things of optical fibres which guide the photon waves in preferred directions, seems to be strongly selective of a specific Lorentz frame; it might well effectively destroy the local character of the interaction with \( \mu A \), which was remarkably achieved using very rapidly varying polarisation devices.

Thus a careful analysis is still needed; it might well call for further experiments, where the entangled photons propagate in the vacuum, for instance in a setting where the entangled photons have to reach counters placed on different satellites.
The pictures of the measurement process

Let \( A = \sum_j \lambda_j E_j \) be an observable of our system \( S \) with finite spectrum \( \lambda_j, j = 1, ..., n \), and spectral selfadjoint projections \( E_j \). The Hilbert space of the state vectors for the composed system, consisting of the observed system \( S \) and the measurement apparatus \( A \), is the tensor product \( \mathcal{H}_S \otimes \mathcal{H}_A \) of the respective Hilbert spaces. In the von Neumann picture, the measurement of \( A \) takes place from time 0 to time \( T \), due to the interaction \( V \) between the system \( S \) and the measuring apparatus \( A \); the total Hamiltonian is

\[
H = H_S \otimes I + I \otimes H_A + V;
\]

the duration \( T \) is long enough for the measurement to take place, but so short (essentially instantaneous) that we can neglect the evolution of \( S \) and \( A \) on their own during the interval from time 0 to time \( T \); there is a state vector \( \Psi_R \) describing the state “ready to measure \( A \)” of \( S \), such that the time evolution of the composed system from time 0 to time \( T \) has the effect

\[
e^{iVT} E_j \Phi \otimes \Psi_R = E_j \Phi \otimes \Psi_j,
\]

where the \( \Psi_j \) are mutually orthogonal state vectors of \( A \). “Reading” which of the \( \Psi_j \) is the outcome, we know the value of \( A \) in the state of \( S \).

Thus on a generic state vector \( \Phi \) of \( S \) the result of the interaction with \( A \) is

\[
e^{iVT} \Phi \otimes \Psi_R = \sum_j E_j \Phi \otimes \Psi_j \equiv \Psi'
\]

so that, on any observable \( B \) of \( S \), we have

\[
(\Psi', B \otimes I \Psi') = \sum_{j,k} (E_j \Phi \otimes \Psi_j, B \otimes IE_k \Phi \otimes \Psi_k) = \\
= \sum_{j,k} (E_j \Phi, BE_k \Phi)(\Psi_j, \Psi_k) = \\
= \sum_j (E_j \Phi, BE_j \Phi)
\]

which was the core of von Neumann’s explanation why pure states are turned into mixtures.

The operation of “adding the apparatus to the state of our system” is described by an isometry \( W \) of the Hilbert space \( \mathcal{H}_S \) of the system into that of the composed system, \( \mathcal{H}_S \otimes \mathcal{H}_A \), given by

\[
W \Phi = \Phi \otimes \Psi_R
\]
which obviously commutes with all the observables of $\mathcal{S}$, and which is changed by the evolution from time 0 to time $T$ to

$$\alpha_T(W) \equiv e^{iHT}W e^{-iH_S T} = e^{iV T}W = \sum_j W_j E_j,$$

where we turned to the Heisenberg picture, we neglected the independent evolutions of the system and of the apparatus over the time duration of the measurement, and we introduced the isometries $W_j: \Phi \mapsto \Phi \otimes \Psi_j$, which commute with all observables of $\mathcal{S}$ as well.

Now in the spirit of the Daneri, Loinger and Prosperi philosophy, we have to work rather with the Hilbert space $\mathcal{H}_S \otimes \mathcal{H}_{\mu A} \otimes \mathcal{H}_{MA}$ describing the composed system $\mathcal{S} + \mu A + MA$, with the evolution governed by a Hamiltonian

$$H = H_S \otimes I \otimes I + I \otimes H_{\mu A} \otimes I + V \otimes I + I \otimes V' + I \otimes I \otimes H_{MA}$$

where $V'$ describes the interaction of the macroscopic part of the measuring apparatus only with the microscopic part (the interaction of $MA$ with the system itself being neglected), interaction which is responsible for the amplification of the outcome $\Psi_j$ to a state vector $\Psi_j'(N)$ of $MA$.

The crucial point for decoherence is that, in the limit $N \to \infty$ of very large number of molecules composing the macroscopic part of the measuring apparatus, the vectors $\Psi_j'(N)$ will tend weakly to 0, while the states they induce on the algebra of the macroscopic part of the measuring apparatus, will converge (in the weak* topology) to vector states of mutually disjoint representations.

As those limiting states on the algebra of all observables of $MA$ are disjoint from one another, tensoring them with vector states in the fixed representation on $\mathcal{H}_S \otimes \mathcal{H}_{\mu A}$ of the observables of the system plus the microscopic part of the measuring apparatus will give states $\phi_j$ on the observable algebra of the three fold composed system which are pairwise disjoint as well. In any representation of that algebra, which contains among its vector states the states $\phi_j$, linear combinations of the corresponding state vectors will thus induce a mixed state, with no non vanishing term of interference.

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1. More precisely, it is an intertwining operator between their actions on $\mathcal{H}_S$ and on $\mathcal{H}_S \otimes \mathcal{H}_A$.

2. That means representations of the algebra which cannot be connected by any nonzero intertwining operator. Note that we do not have to treat the macroscopic part of the measuring apparatus as a classical system in that limit: the parameters distinguishing different states in the thermodynamical limit (liquid versus gas in a bubble, etc) are classical parameters in their own, even if the description of the infinite system is quantum.
between the components with different j’s (for convenience of the reader, we reproduce the very elementary argument in Appendix II).

We want to memorise this essential role of $M\mathcal{A}$, but concentrate on the composed system $S + \mu \mathcal{A}$, on which the evolution describing the measurement takes pure states to pure states (more precisely, according to von Neumann, product state vectors to the entanglement of different product state vectors).

We now think of a Quantum Field Theory, which potentially describes all interactions which are relevant to the measurement of its observables, as already encoded in one and the same time evolution, and whose (vector) states include those describing the measurement apparatus (the microscopic part $\mu \mathcal{A}$, and, as long as $N$ is kept finite, $M\mathcal{A}$ as well; but in the limit where $N$ goes to infinity, we would have to use vector states of disjoint representations).

Thus the Hilbert spaces $\mathcal{H}_S$, $\mathcal{H}_S \otimes \mathcal{H}_{\mu \mathcal{A}}$, and $\mathcal{H}_S \otimes \mathcal{H}_{\mu \mathcal{A}} \otimes \mathcal{H}_{M\mathcal{A}(N)}$ have to be identified with one and the same Hilbert space $\mathcal{H}$, generated from the vacuum by the observables, or possibly that generated acting also with field operators which carry nonzero superselection charges.

The operation of “adding to a state the microscopic part of the measuring apparatus” (as well as that pertaining to its macroscopic part, if $N$ is kept finite) are accordingly to be described by isometries of $\mathcal{H}$ into itself, say $W_j$ creating the state where “$\mu \mathcal{A}$ is ready to measure $A$”, and $W_j$ creating the state “$\mu \mathcal{A}$ has read the value $\lambda_j$ of $A$”.

With this proviso, their role should remain as in (6), i.e., if $T$ denotes the duration of the measurement, $\alpha_t(A) = U(t)AU(-t)$ the time evolution in the Heisenberg picture, in the spirit of the approximations above (the measurement is so fast that the evolution of the $E_j$ in that time interval can be neglected), we have

$$\alpha_T(W) = \sum_j W_j E_j, \quad W_j^* W_k = \delta_{j,k}. \quad (8)$$

In the setting of (e.g. one particle) Quantum Mechanics discussed above, the $W, W_j$ commuted with (better said, intertwined) all observables. If this were still the case in the present situation, they ought to change the vacuum state vector by a phase only. This is not possible, the result ought to be state vectors describing $\mu \mathcal{A}$ alone, in the “ready” state or in the state “$\mu \mathcal{A}$ has read the value $\lambda_j$ of $A$”. In other words, $\mu \mathcal{A}$ (as well as $M\mathcal{A}$) has to be “somewhere”.

Now we come to the crucial point. We are interested in a Local Quantum Field Theory [7, 8], where the observables form an algebra $\mathfrak{A}$ which is
generated (technically as a C* Algebra) by the subalgebras \( \mathcal{A}(\mathcal{O}) \), whose self-adjoint elements describe those observables which can be measured within the spacetime limits of the bounded region \( \mathcal{O} \).

The correspondence from bounded regions\(^3\) to algebras should preserve inclusions, span an irreducible algebra in the vacuum sector, be covariantly acted upon by the Poincare’ (or at least the translation) group, fulfill the Spectrum Condition, and, most notably, fulfill the locality postulate, i.e. the measurements of two spacelike separated observables must be compatible, so that they commute with each other:

\[
\mathcal{A}(\mathcal{O}_1) \subset \mathcal{A}(\mathcal{O}_2)' \quad \text{if} \quad \mathcal{O}_1 \subset \mathcal{O}_2'
\]

where the prime on a set of operators denotes its commutant (the set of all bounded operators commuting with all the operators in the given set) and on a set in Minkowski space denotes the spacelike complement.

At least in theories where there are no massless particles, we can without any loss of generality suppose that \( \mathcal{A} \) is included in a larger algebra \( \mathfrak{F} \) of fields, with a similar structure of local subalgebras, each generated by field operators fulfilling normal Fermi/Bose commutation relations at spacelike separations, according to whether the superselection sectors they reach acting on the vacuum are both paraFermi or not \([9]\) (the notion of *Statistics of a Superselection Sector* being intrinsically defined solely in terms of the local observables \([10],[11]\); for an expository account, see e.g. \([19]\)).

These fields can be assumed to generate, acting on the vacuum state vector, all the superselection sectors of the theory (defined as the collection of all representations of \( \mathcal{A} \) which describe, in an appropriate precise sense, elementary perturbations of the vacuum). All this is just encoded in the observable algebra \( \mathcal{A} \).

Now, if we think of the measurement process of a local observable, say \( A \) in \( \mathcal{A}(\mathcal{O}) \), (for simplicity taken with finite spectrum as above), we expect that the measurement process is local!

It is then natural to assume that the isometries \( W,W_j \) fulfilling \([5]\), are elements of the same local subalgebra \( \mathcal{A}(\mathcal{O}) \) of observables, or of fields, \( \mathfrak{F}(\mathcal{O}) \). At the only price, of course, of replacing \( \mathcal{O} \) by a slightly larger region which contains, together with \( \mathcal{O} \), its time translates by amounts not exceeding \( T \).

We might well call “locally measurable observables” those local observables for which this is possible. (But it might also happen that the \( W,W_j \)

\(^3\)This correspondence may be limited to nice regions as the intersections of a future and a past open light cone, which form a collection which is globally stable under Poincare’ transformations, each region in the collection coinciding with the spacelike complement of its own spacelike complement.
have to be localised in a larger region; or they might fail to be isometric; or they might be quasilocal and not strictly local. In the last events, most conclusions would hold only asymptotically, by the cluster property.

For example if we think of the measurement of a (suitable spectral projection of a) charge density smeared over a small spacetime neighbourhood, mimicked, in practice, say by a single ionisable molecule in the metastable liquid in a bubble chamber, then $W = W_0$ and $W_1$ would be the creation operators from the vacuum of the strictly localised states of the appropriate baryon number, describing in a reasonable approximation the non ionised respectively ionised molecule, localised in $O$. Of course, the $W$’s cannot commute with all observables. In analogy with the non local picture of non relativistic Quantum Mechanics, they should lie in a subalgebra of $\mathfrak{A}(O)$ (or of $\mathfrak{F}(O)$; more precisely, they should lie in a type $I_\infty$ subfactor) commuting with the measured observable $A$.

Similarly, if we want to take care of the amplifying part of the measurement apparatus, we ought to apply to the state vector where we want to measure $A$, not only $W$, but also some operator $W'(N)$ (localised in some larger macroscopic region including $O$). But for all practical purposes a macroscopic region of the size few microns to few centimeters will suffice: that is to say, it is practically pointwise if we think of astrophysical separations.

In quite the same way as mentioned above, the interaction between the $W_j$’s and the $W'(N)$’s will explain the decoherence, occurring exponentially for large $N$’s.  

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4The important point is that they should create, acting on the vacuum, a state essentially localised in the desired region $O$. An isometry from $\mathfrak{F}(O)$ does that exactly; an operator from $\mathfrak{F}(O)$, normalised so that its action on the vacuum gives a unit vector, does that with the better approximation the closer to one is its norm; without the last condition, it might give a result as close as we want to any vector, by the Reeh - Schlieder Theorem [7].

5The field algebras $\mathfrak{F}(O)$ are actually generated by isometric field operators which, acting on the vacuum, create strictly localized states with specific values of the superselection quantum numbers [9]. But, strictly speaking, such an exactly localised state ought to have a completely indefinite number of particles, and, in order to describe with a reasonable approximation the state of a single molecule, it ought to be localised in a region not smaller than some minimal size; moreover, for this example to be correct, we ought to leave out from the description of the time evolution precisely the electromagnetic forces; otherwise, the localisation could only be approximate; replacing the relevant identities with approximate ones which become more and more exact after finite but larger and larger space distances, however, should not alter the essence of our point.

6... and extending to infinity, at least in some direction, when the number $N$ of atomic constituents of the amplification device tend to infinity.
But the W’s are localised in O; if we add to any state our apparatus µA, the state vector, Φ, is changed to WΦ; the expectation value of any observable B which is localised in a spacelike separated region will not change:

\[(WΦ, BWΦ) = (Φ, W^*BWΦ) = (Φ, BW^*WΦ) = (Φ, BΦ),\]  

(10)

where we used the local commutativity of W and B, and the isometric nature of W, i.e. \(W^*W = I\).

Since, by our choice of a slightly larger O, also \(α_T(W)\) is localised in O, the same applies after the measurement took place.

We conclude that the state described by the vector Φ, whether or not we add to it the microscopic measurement apparatus evolved in time within some interval which includes that of the measurement of A, looks exactly the same to an observer which is spacelike away from our region O. If that observer were macroscopically far away in a spacelike direction, the same would be true even if we added also the amplifying part of the measurement apparatus - provided we kept the number of its microconstituents finite.

3 Concluding remarks

The conventional picture of the measurement process in Quantum Mechanics, as an instantaneous jump from a pure state to a mixture, which affects the state all over space at a fixed time in a preferred Lorentz frame, appears, in the scenario we outlined, as the result of several limits:

1. the time duration T of the interaction giving rise to the measurement (which, in an exact mathematical treatment, would involve the whole interval from minus infinity to plus infinity, as all scattering processes) is set equal to zero;

2. the number of microconstituents of the amplifying part of the measurement apparatus is set equal to infinity, thus allowing exact decoherence;

3. the volume involved by the measurement apparatus in its interaction with the system (thus occupied by the microscopic part of the apparatus) tends to the whole space, allowing the reduction of wave packets to take place everywhere;

in the last limit, we can conceive that the local creation operators for the microscopic part of the apparatus converge (in the weak* topology of their action on the states) to the von Neummann maps of the Hilbert space into its tensor product with that of the microscopic part of the apparatus (identifying, for all values of the volume, the V dependent type I factor in \(A(O_V)\), where the W are located, with all bounded operators on a fixed
Hilbert space $\mathcal{H}_{\mu A}$; thus, actually, a deformation of $\mathfrak{A}$ to $\mathfrak{A} \otimes B(\mathcal{H}_{\mu A})$ is involved here; this point too would deserve a more precise mathematical discussion.

One should compare the necessity of the last limit with the results of the Araki Yanase analysis [12], whereby the additivity of angular momentum is compatible with the conventional description of the measurement process only in the limit where the measurement involves a volume extended to the whole space.

Strictly speaking, taking those limits independently from one another does not seem physically possible: if we are interested in the limit where decoherence becomes exact, the time needed for the evolution of $MA$ to one of the final disjoint states has to be taken into account as well, and that time will be infinite (especially in a relativistic theory, since if $MA$ contains infinitely many microscopic constituents, it has to be infinitely extended, and the return to equilibrium would require infinite time). But an appropriately tuned joint limit might well exist.

The Einstein Podolski Rosen Paradox seems to arise as the result of all those limits; and certainly the limit $R \to \infty$ destroys the local character of our measurement. If they are not taken, any physical state would be found totally unaffected by a local measurement as far as that state would be tested with observables which are localised spacelike away from the region of that local measurement.

But, as discussed in the Introduction, a careful comparison is needed with the setting of the experiments, notably by A.Aspect [13]; especially with the tests “of the third generation”, of Bell’s inequalities, with pairs of entangled photons, with spacelike separated detection events, where violations by 9 standard deviations were found.

At first sight those experimental results seem to contradict our picture, but one ought to understand whether the actual size of the experimental setting should not play the role of the localisation of our $W$’s, and whether the preferred Lorentz system is not precisely chosen by the whole experimental setting, rather than by an individual detection event. In these cases, of course, there would be no contradiction.

On the other side, the picture of the operation ”adding the measurement apparatus to the observed system” as a localised operation opens the door to other paradoxes.

For instance, as it is implicit in the foregoing discussion, in the EPR type experiment where the two photons are in the superposition of two states where they are both with right handed resp. left handed circular polarisations, spacelike separated measurements might well find one left and
the other right handed polarised; a single charged particle coming from a far away source in a state which can well be approximated as a plane wave over a large spacetime region, which includes two spacelike separated regions where two counters are located and activated, might well be detected by both counters, with a practically negligible but non zero probability.

Similar conclusions sound so striking and absurd because we mix in our thoughts the picture of the nonrelativistic or idealised world of single (or finitely many) particle physics, with the picture of localised operations in spacetime. The first picture requires implicitly a global knowledge of our states, and leaves room only to a formal, nonrelativistic notion of localisation (Newton-Wigner and alike) which has no relation to the relativistic notion of localised operations and observables, as formulated in Quantum Field Theory; while in the second picture the notion of single particle, or of number of particles, appears as a global, asymptotic notion which can be specified locally only in approximate terms (cf e.g. [21]).

The discussion above is but a qualitative picture of a possible scenario; beyond the comparison with experiments, to make it a bit more precise, many mathematical questions should be answered: one should replace (8) by a suitable approximation, and investigate the nature of the convergence for large time values, in order to justify the assumption that we can pick a finite value of $T$ as a reasonable approximation. Furthermore, the mathematical meaning of the three limits above should be studied carefully.

But, in the above picture, the interaction described by the time evolution is the same as the interaction between our system and the apparatus; one could expect that, e.g., in a free field theory there should be, in the sense described here, no observables at all!

More generally, in a full and precise theory of the measurement process in Local Quantum Physics, the subset of those observables localised in a region $\mathcal{O}$ which are “locally measurable” in the same region $\mathcal{O}$ might be a proper subset; it would coincide with all the observables localised in that same region only in model theories which offer a description of a complete set of interactions.

4 Appendix I. Entangled states and the Locality Principle: no contrast

The simplest, well known example of entangled state can be obtained picking two orthogonal pairs of unit vectors, say $\Psi_j$ and $\Phi_j$, $j = 1, 2$ respectively in the Hilbert spaces $\mathcal{H}$ and in $\mathcal{K}$, and considering e.g. the state (on all
bounded operators of the tensor product $H \otimes K$ induced by $(1/2^{1/2})(\Psi_1 \otimes \Phi_1 + \Psi_2 \otimes \Phi_2)$.

Such states play a central role in the discussions of the EPR paradox, whereby the conclusion is often drawn that “Quantum Mechanics is non local”.

A common wisdom in Local Quantum Field Theory says that very long range correlations might well appear as a property of some particular states, although locality is exactly respected, in the form of local commutativity of observables (as a fundamental manifestation of Einstein Causality: no perturbation can propagate faster than light).

In particular, entangled states as above may arise in Local Quantum Field Theory, expressing correlations between observables localised in regions which are far away spacelike to each other. For convenience of the reader we give here some elementary examples. For a more complete discussion, see e.g. [17], [20].

An immediate example is obtained considering two non trivial (neither 0 nor $I$) selfadjoint projections (“questions”) $E$ and $F$ which are localised in regions which are well spacelike separated from each other. By a result of H.J.Borchers, $EF$ and $(I - E)(I - F)$ can’t be 0; if $\Psi$ and $\Phi$ are vectors in the range of $EF$ and $(I - E)(I - F)$ respectively, say with square norm equal to $1/2$, the unit vector $\Psi + \Phi$ induces, on the whole quasilocal algebra, a pure state where the values of $E$ and $F$ are entangled. Of course, no contradiction with local commutativity.

But the reader might feel cheated: he does not see the nice tensor product decomposition between the two subsystems in such an example.

Just for the sake of leaving this little corner of our room clean, we can slightly modify our example so that it looks much closer to the example in elementary Quantum Mechanics we started with.

In a local quantum theory defined, as we outlined above in this note, in terms of algebras of local observables $\mathfrak{A}(\mathcal{O})$, a tighter form of locality (called the “Split Property”, verified in the mathematical models given by free field theories, but expected to hold in any reasonable theory [15, 16]) implies that, whenever $\mathcal{O}_1$ and $\mathcal{O}_2$ are nice regions well separated apart in spacelike directions, there is in $\mathfrak{A}$ a subalgebra, say $\mathfrak{B}$, included in some $\mathfrak{A}(\mathcal{O})$, with an appropriate $\mathcal{O}$, just slightly bigger than the union of $\mathcal{O}_1$ and $\mathcal{O}_2$, which is isomorphic to all bounded operators on a Hilbert space $H \otimes K$, in such a way that elements of $\mathfrak{A}(\mathcal{O}_1)$, respectively $\mathfrak{A}(\mathcal{O}_2)$, are mapped to (some)

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7This means that the modulus of the (negative) Minkowski distance between their points has a non zero lower bound
bounded operators on $\mathcal{H}$, respectively $\mathcal{K}$, which act as the identity on the other factor.

The vector states of $\mathfrak{A}(\mathcal{O}_1)$, respectively of $\mathfrak{A}(\mathcal{O}_2)$, as well as of $\mathfrak{B}$, in the vacuum representation of the whole quasilocal algebra, are the same as those of there images in the above representations on $\mathcal{H}, \mathcal{K}$, respectively, for the first two, or including those associated with the representation on $\mathcal{H} \otimes \mathcal{K}$ for the third.

Thus we may consider vector states $\psi_j$ and $\phi_j$, $j = 1, 2$ respectively of $\mathfrak{A}(\mathcal{O}_1)$ and $\mathfrak{A}(\mathcal{O}_2)$, where $\psi_j$ and $\phi_j$ take the values 0 respectively 1 on $E_j$ respectively $I - E_j$, where the $E_j$ are local projection operators in $\mathfrak{A}(\mathcal{O}_j)$, $j = 1, 2$; pick orthogonal pairs of vectors $\Psi_j$ and $\Phi_j$, $j = 1, 2$ respectively in the Hilbert spaces $\mathcal{H}$ and in $\mathcal{K}$ which induce them in the corresponding representations, and extend the vector state of $\mathfrak{B}$ induced by $(1/2^{1/2})(\Psi_1 \otimes \Phi_1 + \Psi_2 \otimes \Phi_2)$ in its representation on $\mathcal{H} \otimes \mathcal{K}$ to a vector state, say $\omega$, of $\mathfrak{A}$ in its vacuum representation.

We thus constructed, from the given states a state where the values of the $E_j$ are entangled, no matter how far away separated in a spacelike direction are the regions $\mathcal{O}_1$ and $\mathcal{O}_2$; and we achieved this with the entanglement of states which, when restricted to appropriately localised observables, become exact product states; thus mimicking as much as possible the first example in Quantum Mechanics we recalled at the beginning of the appendix.

This shows that entanglement, in the form it is usually thought to appear in the EPR experiment, does not conflict with the locality principle; on the contrary, it is a possibility which is enforced by a stronger form of the locality principle, namely the split property.

## 5 Appendix II. Disjoint states cannot be superposed

We reproduce, for the convenience of the reader, the very elementary argument which prevents superposition of two states $\phi$ and $\psi$ if they induce disjoint representations of a C* algebra $\mathfrak{A}$.

Let $\pi$ be a representation of $\mathfrak{A}$ on a Hilbert space $\mathcal{H}$, and $\xi$, $\eta$ two vectors in $\mathcal{H}$ which induce $\phi$ and $\psi$ as vector states of the representation $\pi$. For instance, $\pi$ could be the direct sum of the two GNS representations. Let $E, F$ denote the orthogonal projections on the closed subspaces obtained applying

---

8Technically this is due to the fact that each of those algebras (except of course the irreducible representation of $\mathfrak{B}$ acting on $\mathcal{H} \otimes \mathcal{K}$) have an infinite commutant in the relevant representations, so that normal states and vector states coincide.
the whole representation to the vectors $\xi$, $\eta$ respectively. These subspaces are stable under the representation hence these projections commute with it. The product $EF$ is a linear bounded operator from the range of $F$ to the range of $E$, and commutes with the representation $\pi$, since $E$ and $F$ do. Their ranges are invariant subspaces for the representation $\pi$, hence $EF$ is an intertwining operator between the restrictions of $\pi$ to those ranges.

But those restrictions are unitarily equivalent to the GNS representations of $\phi$ and $\psi$, which we assumed to be disjoint. Thus those restrictions are disjoint, and they have no nonvanishing intertwining operator. We conclude that $EF = 0$.

Now let $\zeta = a\xi + b\eta$ be a (normalised) linear combination of our vectors; it induces a vector state $\rho$ on $\pi$ whose value on any element $A$ of $\mathfrak{A}$ is

$$
\rho(A) = (\zeta, \pi(A)\zeta) = (a\xi + b\eta, \pi(A)a\xi + b\eta)
= a^*a(\xi, \pi(A)\xi) + b^*b(\eta, \pi(A)\eta) + a^*b(\xi, \pi(A)\eta) + ab^*(\eta, \pi(A)\xi) =
= a^*a(\xi, \pi(A)\xi) + b^*b(\eta, \pi(A)\eta)
= (|a|^2\phi + |b|^2\psi)(A),
$$

(11)

where we used the fact that the last two terms in the expansion of the scalar product, the interference terms, vanish for all $A$ in $\mathfrak{A}$, since $E\xi = \xi, F\eta = \eta$ by construction, $E$ and $F$ commute with $\pi(A)$, and $EF = 0$. Therefore, for any choice of the representation vectors of our states, the attempt to construct a superposition fails, and gives only a statistical mixture.

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