A review to explore different meshless methods in various Structural problems

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Abstract. In this work, authors are actively showcasing the finding of recent papers using meshless method in different structure problems which are either 1-D, 2-D, 3-D problems or problems related to composite material. Also presents the comparison of research going in that field of most common and widely used method, i.e. most widely used method of finite element (FEM). Over the past few decades meshless methods where implemented into many application areas which are ranging from long established problems related to astral physics to that of solid mechanics problem also in engineering and mathematical models, vibration and fluid mechanics investigation and optimization of the numerical results of the equation of partial differential problems. From critical reviews of different meshless methods, authors have suggested the applicability of different meshless method to improve computational efficiency.

Keywords: Meshless method; EFGM; MLPG; collocation method; FEM;

1. Introduction

Like most traditional finite element method together with the meshless methods are also taking place in computational mechanics in the recent years. All the Meshless articulations have a similar quality of not utilizing the pre-characterized networks and cross section availability among the hubs, however the record just a bunch of dispersed hubs in the issue space least for the field variable approximations.

"B. Nayroles et. al." was the first of his time to utilize the moving least square (MLS) procedure to develop shape capacities for their Diffuse Element Method (DEM) or simple Diffuse approximation [1]. The idea was hinge on smoothed particle hydrodynamics. In this approach diffused approximation method which is a function’s approximation from a bunch of gathered focuses. This method got a better upper hand over finite element methods. Based on the DEM, the Belytschko was the one and there associate developed a contrasting perspective, called the EFG method [2]. The EFG method uses the MLS method of working to imprecise displacements and Galerkin plan of action to exhibit weak forms of system equations. The studies of the numerical which are based on above method have a greater influence of factor comparatively number of nodes, quadrature points, support size domain, etc. Besides this, Liu along with Y.T. Gu turned out a source code for analysis of the EFG method in FORTRAN[3]. Element-Free Galerkin Method (EFGM) is highly accurate and precise than any other method, and computation was relatively lower than any other method. Authors have done regressive study on literature related to meshless methods in the field of structural analysis, which are shown in Table 1.
Table 1.1: The works and finding of different researchers are as follows:

| S.No. | Reference number | Findings                                                                                                                                                                                                                                                                                                                                 |
|-------|------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1.    | [4]              | Author has combined two ongoing used methodology i.e. ICVEFG and DSM method and introduce a new type of method that was more efficient compare and another of its type of method that was HCVEFG method that were very useful in the problem like 3 dimensional which was mainly for solid mechanics problem and there character based problem. The effects of measuring conditions were most of them were depended on node number and penalty factors as being discussed in the HCVEFG method. The method also shows high computational efficiency as compared that to any other method discovered so far. |
| 2.    | [5]              | Researcher define the recent use of the simple boundary problem method for the practical uses in the structural problem and comes to the result that the efficiency of the finite element method for a solid mechanics or for a hollow cavity or a large solid. FEM convey for slim construction where the strong models are hard to get, the SBM approaches can another option, particularly when high exactness is wanted. |
| 3.    | [6]              | The deflection inside a beam having a uniform pressure has been computed by the smooth hydrodynamics model (SPH), thus by using SPH meshless method. The results were very precise for analysis of large changes in FGB. When depth ratio and gradient index gets increased the deflection in transverse decreases. And also concluded by the approach as very work efficient to get the deflection in large profile element. |
| 4.    | [7]              | Here author showed that MLPG method is developed to obtain three-dimensional static solutions for thick functionally graded (FG) plates. Several numerical examples containing the stress and deformation analysis of thick FG plates with various boundary conditions under different loading conditions are presented. The Young's modulus is considered to be graded through the thickness of plates by an exponential function while the Poisson's ratio is assumed to be constant. The obtained results have been compared with the available analytical and numerical solutions in the literature and an excellent consensus is seen. The local symmetric weak formulation is derived using the 3D equilibrium equations of elasticity. |
| 5.    | [8]              | During this research the researcher gather result in its mathematical terms that were given in the past in this area shows the comparative accuracy of DMLPG and MLPG methods. DMLPG methods uses a high CPU time consuming and have a correlation with the first MLPG methods and draw the result that it has a less time taking process then method discussed in the method. |
| 6.    | [9]              | The ELEMENT FREE GALERKIN METHOD is used for debilitating differential conditions in a scaled limit technique. The moving least square method is used to fabricate shape. The Galerkin method can perform functions which cannot be preceded by MLPG scaled technique. In numerical comparison |
and verification, the EFG approach brings ultimate accuracy and instant convergence in comparison to SBFEM and MLPG condition.

7. [10] A 3D and completely meshless approach have been created for the static investigation of thick functionally graded (FG) plates with various boundary conditions. The 3D equilibrium equations have been considered and the Young’s modulus of the plate was assumed to vary exponentially through the thickness, while the other material properties were assumed to be constant. The two meshless methods including MLPG1 and MLPG5 have been extended to solve the bending problem of FG plates. Brick-shaped local domains were considered in the arrangement system. It is seen that the methodology functions admirably in examination of FG plates.

8. [11] In this paper author showed that we can use MDP-FEM model and solve both momentum and kinetic energy conservation based problem as well as deflecting beam shape changing plastic type problems, weak form numerical integration is done on necking problems by applying tensile load on solid with the help of finite element mesh.

9. [12] The mathematic expression of GEFG is described like FEM, but the elements are shaped by subjectively choosing neighbourhood hubs encompassing the collocation hub in GFEM. As an outcomes just a single component is required for one collocation hub, the bandwidth of the system coefficient matrix is extremely narrower compared with traditional FEM.

10. [13] By using the modified Solid Isotropic Material with Penalization (SIMP) method and Optimality Criteria (OC) method, the densities and material properties of SPH particles are renewed iteratively. Meshless Smoothed Particle Hydrodynamics (SPH) method is improved to accurately and stably solve solid mechanics problem.

11. [14] MLPG approach is applied on plate bowing issues in orthorhombic (QCs) plate which are in the presence of transient and static powerful load condition and presumed that Reissner–Mindlin hypothesis diminishes the first 3D thick plate issue to a 2D plate issue. Mathematical outcomes portrayed that the coupling of material boundaries impact the phonon diversion to that of the genuine estimations of material boundaries.

12. [15] EFG method is an more advance method after getting involved it with MATLAB program approach to analyze plain stress problem results are compared with analytical and FEM results. And study of varying cross sectional area together with cantilever beam problem which is exposed to the point load at its free end. The applied burden is ductile which brings about expansion or uprooting of the bar. And then results is interpreted as the displacement result of an analytical approach to that of FEM approach for the same cases of stress and interpret as the result that EFG method can be a suitable approach to get better results.

13. [16] Whenever a result is calculated by FPM approach it is found to be inaccurate, and it may require some adjustment in its parameters, and it can be done by adjusting the quantity of focuses in the help of supporting domain, and after that
|   |   |
|---|---|
| 14. | [17] Here author showed that Kriging interpolation meshless model is very easy to solve. Compared with FEM based reanalysis methods, the main superiority of meshless-based reanalysis method is to disconnect the limitation of mesh connection. Therefore, a local reanalysis method which only needs to calculate the local stiffness matrix in the influence domain is suggested to improve the efficiency further. Considering the computational cost of meshless methods, the reanalysis method improves the efficiency of the full meshless method significantly. However, compared with the FEM–based reanalysis method, the critical challenge is to use much more nodes in the influence domain due to high order interpolation. |
| 15. | [18] Here author aims at using radial point interpolation meshless methods to study the stress distributions and predict the strength of composite adhesive single lap joints (SLJ). The stress distribution obtained using meshless methods was compared to one obtained using the well-established Finite Element Method (FEM), resulting in a similar behavior. Compared to traditional joining techniques, such as bolts, welding or riveting, adhesive bonding has some advantages, namely, they are low weight, have high strength and have a more uniform stress distribution. |
| 16. | [19] The author uses EFG method to solve the problem of the linear hyperbolic function in its two dimensional state where they showcase the usefulness of the primitive used method to that of the method used in recent era of the EFG and draw result for the EFG method is more useful and convenient and efficient in its basic state. |
| 17. | [20] The convergence and the accuracy of result of both the method i.e. MLPG and LRPIM depend well on the size of supporting domain, number of nodes, spline functions, radial basis function RBF (MQ, EXP and TPS) for LRPIM method. The result also shows dependence on the number of nodes that has its upper end. These results agree with the analytical solution. The result shows the domain of convergence is independent of the choice of the studied materials like steel, zinc, copper and aluminium, etc. |
| 18. | [21] MLPG method is one of the efficient methods to solve problem on convection domain flow it could be 1D or 2D problem with the help of unwinding concept which is very flexible like GFEN method, MLPG is also a good method for low pellet number flow but not works well high pellet number flow. MLPG 2 method is very easy to apply on multidimensional flow problem but as compared to SUPG method MLPG 2 method is little expensive as no cost effective integration method is found yet. It is also verified by numerical test that MLPG 2 method gives better solution than SUPG method. |
| 19. | [22] The approach of element-free Galerkin (EFG) is taken in account for solving the problem based on two-dimensional linear hyperbolic problems. The meshless property of EFG is the most important advantage of this scheme over the traditional mesh-dependent techniques such as the finite element method and the
boundary element method. And hence these approach were highly useful to
differentiate the efficiency of both the method and superiority of both the
method are shown.

| 20. | [23] Under static and dynamic load The meshless Petrov–Galerkin method MLPG has been applied to 1 dimensional orthorhombic QC plate banding problem. In elasto dynamic case for phason governing equation the Bak and elasto hydro dynamic model are applied, the phason displacement depend on in – plane co ordinates of mean plate surface when it is orthorhombic QC and first order shear deformation. In considered plate, nodal point has been distributed on mean surface randomly. On small fictitious subdomain Matrix form is satisfied by governing partial differential equation. By the help of MLS method small changes of phonon and phason displacement is approximated and we get ODEs for unknown nodes, and second order ODE's is solved using Houbolt Finite Differences method. |
| 21. | [24] To introduce unwinding concept [also in multi dimensional case] is very easy in MLPG method due to its general nature. To face problem of convection dominated flow. In this paper, 1D and 2D problem based upwinding concept are given and applied to solve problem based on steady convection diffusion by combining MLPG method with upwinding concept gives very efficient result also in high peclet number flow which prove that MLPG method is a good way to solve convection dominated flow and fluid mechanics problem. |
| 22. | [25] The parametric meshless Galerkin technique (PMGM), which has as of late arose in the field of meshless strategies, upgrades the effectiveness of meshless strategies using meshless shape capacities which are planned from a parametric space to the actual space. This property assists with saving the running-time expected to compute the meshless shape capacities, which is a tedious and convoluted assignment. In any case, the meshless shape capacities should be recovered in the region of the limits just as some different areas subsequent to planning. In this paper, another augmentation of the PMGM, called expanded parametric meshless Galerkin technique (X-PMGM) is introduced. |

2. Methodology
After critical review of recent work in the field of structure problems, various meshless methods have been explored. Among all the derived method so far the Element-Free Galerkin Method (EFGM) is highly accurate and precise than any other method, and computation was relatively lower than any other method. For easy understanding of EFGM, the comparison flowchart of EFGM and FEM is shown in Fig.1.
Flowchart (1): Differentiating FEM and Meshless methods

| FEM | Meshless method |
| --- | --- |
| Element mesh generation | Nodal generation |
| Creation of shaped function based on already defined element | Creation of shape function based on local nodes |
| Discretization of system equations based on elements | Discretization of system equations based on cells |

Flowchart (1): Differentiating FEM and Meshless methods

Figure (1): FEM Model [3]

Figure (2): Meshless Model [3]

Figure (3): Support domain [26]
The discrete form of EFGM can be derived by using MLS approximation and Lagrange multipliers as;

\[
\begin{bmatrix}
K & G \\
G^T & 0
\end{bmatrix}
\begin{bmatrix}
u \\
\lambda
\end{bmatrix}
= 
\begin{bmatrix}
f \\
q
\end{bmatrix}
\]

(1)

where, 

- \(K\) = Stiffness matrix,
- \(u\) = displacement vector,
- \(f\) = force vector,
- \(G\) = shape function derivatives:

\[
G_i = \int_{\Omega} B_i^T DB_i d\omega
\]

(2)

Here, \(i\) and \(j\) are \(i^{th}\) and \(j^{th}\) nodes and \(B_i\) are the shape function derivatives:

\[
B_i = 
\begin{bmatrix}
\phi_{i,x} & 0 \\
0 & \phi_{i,y} \\
\phi_{i,y} & \phi_{i,x}
\end{bmatrix}
\]

(3)

\[
G_{ik} = \int_{\Gamma u} \phi_i N_k d\Gamma
\]

(4)

\[
q_k = -\int_{\Gamma u} N_k \bar{u} d\Gamma
\]

(5)

\[
N_k = 
\begin{bmatrix}
N_k & 0 \\
0 & N_k
\end{bmatrix}
\]

(6)

Where,

- \(N_k\) = Lagrange interpolant for node \(K\).

\[
f_i = \int_{\Omega} \phi_i b d\omega + \int_{\Gamma_e} \phi_i \bar{t} d\Gamma
\]

(7)

3. **Result and discussion**

Among various meshless methods, the Element Free Galerkin (EFG) method has retained its advantages such as rapid convergence, higher accuracy percentage and good stability rate. The Meshless Local Petrov-Galerkin (MLPG) is also developed to refine the efficiency of the local frail to that form which being established at the nodal points, for the stresses, by using the collocation method. MLPG mixed collocation method possesses an all around built union rate, and is more effective than the other MLPG usage, along with the MLPG limited volume method. In EFG strategy, moving least squares (MLS) approximations are thought of. The crucial boundary conditions are included by means of Lagrange multipliers.
4. Conclusion

Hence in the current work researchers have contemplated the different meshless strategies and the works and discoveries of writing identified with the utilization of meshless techniques in the field of primary issues. Out of all meshless methods, EFGM, MLPG, MLS method, parametric meshless Galerkin method (PMGM), Houbolt Finite Differences method and SPH are most widely used methods. It is discovered that all the strategies have significant use in specific domain. Especially, EFGM and MLPG are more suitable for structural problems and used in place of most widely used commercial method i.e. method of Finite Element (FEM). These methods are used when complexity of the problem increases. Meshless methods can overcome the limitations of FEM furthermore; carry out outcomes with extraordinary exactness with no misfortune in expense and figuring time.

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