The return branch of viscous fingers

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We report a simple experiment of two-dimensional pattern formation in a circular Hele-Shaw cell, showing the appearance of a return branch that is equivalent to the upward-connecting leader of lightning. Injecting water from the center into a foam filled cell, we obtained patterns similar to dendrites of two-dimensional dielectric breakdown experiments. When we repeat this experiment allowing the presence of water in the outer (low pressure) region, dendrites grow initially as in a normal experiment, but when a branch is near the outer boundary, the low pressure water begins to penetrate the foam against the pressure field, forming several return branches.

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Typical experiments in a two-dimensional region between two parallel plates or Hele-Shaw cell[1], consist of injecting a less viscous fluid into a more viscous fluid. These systems develop patterns that were discovered for the first time when water was injected to extract the remaining oil deposited in the bottom of natural wells. Contrary to what was expected, water invaded the oil forming patterns and frustrated the extraction. It is very easy to produce these patterns in the laboratory, and many experiments have been reported for many different geometries and fluids[2]. We report a new class of patterns obtained in a radial geometry Hele-Shaw cell, injecting water from the center into a foam that fills the two-dimensional region between the plates. The novel ingredient is to allow the presence of water at atmospheric pressure at the outer boundary.

In thunderstorms[3, 4], air currents separate negative and positive charges producing regions of high electric fields, when this field exceeds the value for breaking the air, a lightning stroke is produced. A typical negative cloud to ground stepped leader has an overall duration of 35 ms and a current between 100-200 A. As the tip of this negative leader get close to the earth surface, one or more upward-moving discharges are initiated from the ground, and an attachment with the downward-moving leader occurs some tens of meters above ground. In this work, we obtained the equivalent of this effect in a simple experiment using fluids under pressure, this experiment shows that the return branch is universal, and provides an essential step towards the quest for an unified mechanism that explains the basic features of different pattern forming systems. To our knowledge, no experiment has ever been reported showing the appearance of a return branch in a system different from lightning.

There are previous experimental works showing the appearance of universal patterns in completely different systems: The Couder bubble[2, 5] provided the mechanism for patterns typically formed in solidification[6, 7, 8], to be found in experiments injecting a less viscous fluid towards a more viscous one in a Hele-Shaw cell; Carving a lattice in one of the plates of a Hele-Shaw cell, produce dendritic patterns that look like snow crystals[9]; Bacterial colonies in a Petri dish[10, 11] under some conditions of nutrients and hardness of the gel, produce beautiful patterns resembling fractals and spirals. Nature also provides many examples of patterns showing some kind of universality[12, 13, 14]: The blood vessels in our retina, the form of plants and simple animals, the fracture patterns in solids, dielectric breakdown, the river networks, etc.

Motivated by the possibility for the existence of an unified mechanism relating the appearance of dendrites in dielectric breakdown[15] and in viscous fingers, we began to explore the idea of trying to obtain the return branch that is typically found in natural lightning, in simple viscous fingers experiments. In a normal viscous finger experiment using a radial Hele-Shaw cell, a low viscosity fluid (water for example) is injected from the center towards a high viscosity fluid that fills the gap between the upper and lower plates. This setup is not favorable for our purposes, because if we put water at the outer boundary, the continuous outwards flux of the high viscosity fluid will prevent water to advance against this flux. Some years ago people began to explore the effects of using more complex fluids in viscous fingers experiments[2]. Experiments where the high viscosity fluid was replaced by foams revealed dendritic growth
in experiments using foams, because in real lightning the main branch measures several hundred meters and the return branch is in the order of thirty meters.

We constructed our radial Hele-Shaw cell using a 44 x 44 cm acrylic tray with raised edges of 5 cm to contain two circular plates and the low viscosity fluid, that will be present at the outer boundary of the plates (see fig. 1). A circular glass plate 0.8 cm thick and 38 cm in diameter was glued to the bottom of the tray, a similar glass plate with a 0.8 mm hole at its center was put on top of the lower plate and was fixed to the bottom of the tray by four handles, a small cylindrical container for the low viscosity fluid was attached to the central hole and connected to a constant pressure source. The main characteristics of this setup are: to allow the presence of the low viscosity fluid at the boundary, a clean boundary because the handles are attached to the upper side of the top glass plate, the surfaces in contact with the foam are flat and rigid. We used four calibrated spacers to set the separation between the plates, then we adjusted the four handles, removed the spacers and fixed the handles.

Figure 2 shows a pattern obtained injecting colored water from the central hole, the circular boundary is under water at atmospheric pressure, the gap between the plates was set to a separation of 0.2 mm and filled with a commercial shaving foam, a constant pressure of 47 mm of Hg was used in this experiment. This pattern is similar to that shown in figure 1 of ref \[16\] corresponding to a leader discharge in a gas confined between two plates, the fractal dimension \[17\] measured for that experiment was \(D = 1.7\). During this experiment we obtained a series of high resolution images using a digital camera, in the image shown, we have cut the region outside the circular boundaries of the plates and filled that region with a gray color, the inner region was converted to gray levels and only the brightness and contrast were modified.

Using the previous parameters, a typical experiment lasts for about thirty seconds, this slow evolution makes very easy to observe how patterns grow and to capture images with a camera. When the left branch shown in fig. 2 is near the boundary, water from the low pressure region begin to invade the foam, but as this branch advances towards the boundary it swept the foam outwards, preventing the return branch to be formed. Nevertheless, in the upper region (see the arrow) near the tip of this branch, there is clear evidence for the appearance of the return branch, in this region and in the lower region near the tip, one can see the water that invaded the foam. The effect is more evident when one analyze the sequence of high resolution images, advancing forwards and backwards in time makes clear the two competing effects: water from the low pressure region invades the foam and the foam is swept outwards by the advancing main branch. To obtain the return branch, it is crucial to eliminate this second effect. By this reason, we used water and foam and not water and oil as in a normal viscous finger experiment. We expect that by using other kind of foams we could make the effect more impressive and more similar to the equivalent upward-connecting leader of lightning, we have tried many separations between the plates and several combinations of water, oil, gels and white of egg, with no success.

The return branch is very easy to produce, in fact at early stages of our experiment we used two acrylic plates instead of the glass plates, and the return branch was formed under similar conditions. This return branch induce to think that there is an universal mechanism that governs lightning and viscous fingers. The appearance of the return branch in this experiment is important because it shows universality, but it also goes against intuition, water from the outer boundary advances against the pressure field to meet the main branch. Our explanation for the appearance of this return branch, is that the system is lowering its energy by releasing the stress, and the return branch is a local effect that allow the system to lower its energy globally. Although this experiment was motivated by numerical results from a model of one of the authors \[16\], we have not included that model in this paper to explain the appearance of the return branch, because the theory behind the model is controversial.
We used thick glass plates to eliminate the possibility of bending. To be sure that the return branch was not caused by an induced separation of the plates, at late stages of some experiments we applied an upwards force to a point near the boundary of the upper plate, this force was similar in strength to the force driving the constant pressure source used in our experiments, no effect on the already formed pattern or in the water at the boundary was observed.

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