The scheme construction to solve the adding fractions problems using images conducted by elementary students

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Abstract. Mathematics consists of various learning segments and one of which is a fraction. Completing the adding fractions can be done through several methods, one of which is by using images. This is a less-known method of learning, especially for elementary school students which may be due to incomplete student scheme construction. This study aims to describe how elementary school students construct the scheme by using images in the adding fraction. The subject was the sixth grader who has high mathematical skills. Data were collected through interviews as well as mathematical abilities and adding fraction tests as supporting instruments. Mathematical ability tests were used to determine the students' mathematical abilities category; while adding fraction tests were used to explore the student schemes construction. Data were analyzed qualitatively based on the APOS theory framework (Action, Process, Object, Scheme). The results showed that the subject made an incomplete sketch of adding fractions through images proved by how to complete the design scheme in adding fractions through images, subject tended to change procedural solutions by using sketches. Keywords: scheme, constructions, APOS, triad.

1. Introduction

Fractions are challenging problems for elementary students. Most of them face it difficulty. They find it hard to solve fractional operation problem, differ the fractional concept and integer, understand the denominators and counters, as well as apply the correct procedure to solve certain problem [1,2]. Several factors cause the students difficulties to solve fractional problems, namely: 1) there are various complicated arithmetics in fractions compared to original numbers, 2) teachers tend to introduce alogarithm for fractional operations before students are able to understand the concepts, and 3) students less understand in facing complicated/unfamiliar fractional problems [1]. In elementary school, fraction is introduced symbolically as (1) common fraction (e.g. $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, etc.), (2) decimal (e.g. 0.2; 0.3; 0.5 etc.), (3) percentage (e.g. 5%, 15%, 3%, etc.) [3,4]. To learn the fractional concept for elementary students, teachers should employ three approaches [3,4] in order to modelize the fraction, namely: (1) width (e.g. one-third of field), (2) length (two of three meters), and (3) set (three of five teams). Learning additional operation in fraction can be modeled through image/picture which is represented by the area. For example, $\frac{2}{3} + \frac{1}{4}$ can be represented by the image as seen in Fig. 1.
Some studies indicate that introducing fractional concepts represented by part of a circle effectively construct students' understanding on fractional concepts [3,5]. Reconstructing fractional concept on elementary students is commonly started by observing object. Fractional object represented in a part of a circle effectively construct students understanding on fraction [3]. By observing fractional object as part of whole is possibly allowed students to construct the relationship among fractions by drawing the sketches.

Understanding mathematical concept is a result of construction or reconstruction from mathematical objects [6]. Mathematical object construction/reconstruction is created from the action, process, object, and scheme activities of prior knowledge [6]. These activities (Action, Process, Object, and Scheme – APOS) are reconstructed into new scheme which popularly known as APOS theory [7].

APOS theory assumes that individual Mathematics knowledge is a tendency of related individual to respond and understand the situation of mathematical problems by reflecting in social context, reconstruct them, into mathematical action, process, and object as well as apply them into scheme in order to understand the situation and solve the problems [8]. Action is a form of understanding in mathematical concept which involves the mental and physical transformation of mathematical object from external activity. Here, students only know how to operate when order is given. Process is defined as a form of understanding from mathematical concept which involves imagination in transforming mental or physical object as internal and controlled activity. In the process stage, students should be able to perform similar action or transform without external stimulation. Object is a form of understanding of mathematical concept as implementation of action and process. Scheme is a group of action, process, and object as well as interconnecting to other intended schemes which are used by students to understand mathematical concept [6, 9].

APOS theory is an adaptation from Piaget’s opinions about individual development. Knowledge on individual growth and development based on certain mechanism which is divided into three levels or known as triad, including intra level (lowest level), inter level (intermediate level), and trans level (the highest level). In the trans level, individual is able to construct all structures found interrelated/interconnected and create scheme coherently [3, 10]. Scheme construction in this research was related to action, process, object, and other scheme involved students to construct a concept in order to solve certain problem.

2. Methods

2.1. Research Type and Subject
This research was qualitative research. Data were collected through interview. Interviews were conducted to students related to the two tasks given to student (task I and task II). The second task was given in the period of one week after the first task. The data from both interviews were used in triangulation. Population was the sixth graders in State Elementary School Sampung, Kawedanan, Magetan in the academic year 2018-2019. Subject was a student with high mathematical ability. In selecting subject, researcher examined the students’ ability in solving fractional problems through drawing sketches which considered as unfamiliar method for the sixth graders.

2.2. Research Instrument
Instrument used to reveal the scheme construction in solving adding fractions was by drawing sketches of two equal fractions problems as: (1) the result of \( \frac{1}{2} + \frac{1}{4} \) in sketch, (2) the result of \( \frac{1}{4} + \frac{1}{3} \) in
sketch, (3) transform $\frac{4}{5}$ into other fractions by drawing sketches, and (4) transform $\frac{5}{6}$ into other fractions by drawing sketches.

2.3. Data Analysis
Data were analyzed based on the characteristics of APOS theory and framework with indicators defined as in Table 1.

| Component  | Indicator | Activity Observed                                                                 |
|------------|-----------|-----------------------------------------------------------------------------------|
| 1. Action  | 1.1       | The depth of subject understanding in fractional transformation is limited to action. |
|            | 1.1.1     | Adding the fractions by drawing as exemplified by teacher (by shading the intended part of sketch which generally in form of a rectangle or circle). |
| 2. Process | 2.1       | Subject is able to control the action.                                             |
|            | 2.1.1     | Being able to reflect the steps in drawing the adding fractions.                   |
| 3. Object  | 3.1       | Subject is able to decapsulate an object from its original process.                |
|            | 3.1.1     | Being able to reform fraction into adding fractions by drawing.                    |
| 4. Scheme  | 4.1       | Subject is able to connect/relate the action, process, and object of certain concept to another one. |
|            | 4.1.1     | Being able to explain the necessary action to solve the adding fraction problem through drawing. |

3. Results and Discussion
Seen in data analysis, the stages on how to solve adding fraction problem through drawing sketch are divided into that four, namely action stage, process stage, object stage, and scheme stage.

3.1. Action Stage (A)
Subject activities to solve adding fraction in interview I and II are seen in Table 2.

| Table 2. Sketches of Adding Fraction Drawn by Subject |
|------------------------------------------------------|
| Interview I for quiz $\frac{1}{2} + \frac{1}{4}$    |
| Interview II for quiz $\frac{1}{4} + \frac{1}{3}$    |
| Sketches drawn by subject                            |
### Table 3. Sketches for Fractional Transformation Drawn by Subject

| Interview I for quiz \( \frac{4}{5} \) | Interview II for quiz \( \frac{5}{6} \) |
|------------------------------------------|------------------------------------------|
| ![Sketches](image1.png)                 | ![Sketches](image2.png)                 |

According to interview I and II, it was concluded that in the first activity subject was able to solve the adding fraction procedurally by equalizing denominators, followed by drawing sketches from the fractions added and its result. In this stage, subject understanding to do fractional transformation was limited to action, subject did not understand the relationship between the fractions added and its results.

#### 3.2. Process Stage (P)

The subject activities in this stage were seen through the interview.

**Footage 1:**

Interviewer: How to make sketch of fractional transformation from \( \frac{4}{5} \) into other fractions?

Subject: Subject drew sketch as followed.

![Sketches](image3.png)

Interviewer: Explain these sketches!

Subject: In the two-fifth, the counters are shaded, the other five are not. While in four-fifth, the counters are shaded and the fifth is not.

Subject asked to draw the sketch of adding fraction with same denominator. The transformation then would be made into sketch. In this stage subject was able to reflect the steps of his actions as seen from how he drew an identical sketch from the first action by drawing a circle.

#### 3.3. Object Stage (O)

In this stage, subject did not explain the drawing steps in details from drawing adding fractions with different denominators; however, in the same denominator, subject was able to explain it clearly. Subject tried to correct the process of adding fraction by drawing sketch, but the sketch resulted was incorrect and did not reflect the adding process [11]. This stage showed how subject carried out the action on object as seen from how he deciphered the fractions into adding fraction meaning that subject was able to decipher object into process.

#### 3.4. Scheme Stage (S)

In this stage, subject activities asked to solve adding fraction by drawing sketches, when: (1) same denominator, subject directly drew the sketch, (2) different denominators, subject had to equalize the denominators by dinding the LCM (the least common multiple) before drawing the sketch. Subject should be able to connect/relate all the prior stages (action, process, and object stages as well as interconnecting to other intended scheme or finding the LCM). Based on the analysis referred to the APOS theory and the Triad Scheme Development theory, the scheme construction can be defined as followed:
4. Conclusion
In the first stage (Action stage), fractional transformation done by student by drawing sketches was carried out by reacting to the external signal. Student was able to solve the problems by making two sketches. The depth of student understanding in fractional transformation was limited to action, as he was only able to imitate the action exemplified by teachers.

In the second stage (Process stage), student was able to control the action. The control to solve the fractional problems through drawing sketches was done by equalizing the denominator (conducted procedurally).

The third stage (Object stage) shows that student was able to decipher the object into process (from third stage into second stage). Student could make use of his decapitating ability to do the action (first stage) in solving fractional problem.

In the last stage (Scheme stage), student is able to relate/connect all prior activities (action, process, and object stages as well as interconnecting to the other intended schemes or finding LCM). In this stage, student could not explain the steps in solving fractional problems through drawing sketch clearly and detail.

Generally concluded, elementary students with high mathematical ability tend to be able to do the action and process stages to construct the concept, connect/relate object with the original scheme to solve problems, and construct all structures found as interconnected/interrelated schemes coherently meaning that students ability to construct scheme is in the trans level.

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