Controlling stable tunneling in a non-Hermitian spin-orbit coupled bosonic junction

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In this paper, we study how to apply a periodic driving field to control stable spin tunneling in a non-Hermitian spin-orbit coupled bosonic double-well system. By means of a high-frequency approximation, we obtain the analytical Floquet solutions and their associated quasieenergies and thus construct the general non-Floquet solutions of the dissipative spin-orbit coupled bosonic system. Based on detailed analysis of the Floquet quasienergy spectrum, the profound effect of system parameters and the periodic driving field on the stability of spin-dependent tunneling is investigated analytically and numerically for both balanced and unbalanced gain-loss between two wells. Under balanced gain and loss, we find that the stable spin-flipping tunneling is preferentially suppressed with the increase of gain-loss strength. When the ratio of Zeeman field strength to periodic driving frequency \(\Omega/\omega\) is even, there is a possibility that continuous stable parameter regions will exist. When \(\Omega/\omega\) is odd, nevertheless, only discrete stable parameter regions are found. Under unbalanced gain and loss, whether \(\Omega/\omega\) is even or odd, we can get parametric equilibrium conditions for the existence of stable spin tunneling. The results could be useful for the experiments of controlling stable spin transportation in a non-Hermitian spin-orbit coupled system.

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I. INTRODUCTION

Over the last two decades, non-Hermitian systems have attracted increasing interest from both fundamental and application viewpoints¹ ⁴, which has spawned a great deal of research work in many branches of physics, ranging from atomic and molecular physics,¹ ⁷ to spin and magnetic systems,² ⁹, quantum computing,¹⁰ ¹¹, and mesoscopic solid-state structures.¹² ¹³. Some important applications of non-Hermitian systems have been uncovered in different fields including optics, optomechanics, and acoustics.¹⁴ ¹⁷. As is well known, the quantum dynamics of a time-dependent non-Hermitian system is usually unstable with the probability amplitudes either exponentially increasing or decaying. For example, a non-Hermitian Hamiltonian system can possess complex energy eigenvalues whose negative imaginary parts describe an overall probability decrease that can be used to stimulate decay phenomena.¹⁸ ²¹. Recently, a scheme on stabilizing non-Hermitian systems by periodic driving has been proposed,²² which is feasible for a continuous range of system parameters and originate from the fact that the eigenvalues of the so-called Floquet Hamiltonian may become all real.

Quantum transport in non-Hermitian systems have been known to be profoundly affected by non-Hermitian terms that arise from an effective description of system-environment interaction.²³ ²⁶. A simple and paradigmatic example of non-Hermitian transport was originally introduced in 1996 by Hatano and Nelson.² In this respect, a large variety of exciting phenomena of non-Hermitian transport have been revealed, including non-Hermitian unidirectional and bidirectional robust transport,²⁷ ²⁸, non-Hermitian localization,² and delocalization,²⁹, hyperballistic transport,³⁰, and topological phase transitions,³¹, to name only a few.

In condensed matter physics, spin-orbit (SO) coupling arises from the interaction between the motion and spin of a particle. It has led to a number of interesting phenomena like the spin-Hall effect,³², the persistent spin helix,³³, and topological insulators.³⁴. Recently, artificial SO-coupling of both bosonic and fermionic ultracold atoms has been realized in experiments,³⁵ ⁴⁰, which provides an ideal platform to study novel quantum phenomena of SO-coupled ultracold atomic systems with an unprecedented level of controllability over all available experimental parameters. There have been some works focusing on the intriguing dynamics of the SO-coupled cold atomic gases, including Josephson dynamics of a SO-coupled Bose-Einstein condensate in a double-well potential,⁴¹ ⁴³, collective dynamics,⁴⁸ ⁴⁴, ⁴⁵, selective coherent spin transportation in a SO-coupled bosonic junction,⁴⁶, Klein tunneling,⁴⁷, nonequilibrium dynamics of SO-coupled lattice bosons,²⁸, tunable Landau-Zener transitions in SO-coupled atomic gases,⁴⁹, controlling spin-dependent localization and directed transport in a bipartite lattice,⁵⁰, Bloch oscillations of SO-coupled cold atoms in an optical lattice,⁵¹, dynamics of SO-coupled cold atomic gases in a Floquet lattice with an

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impurity, and so on. Nevertheless, all these achievements are hitherto limited in studying the properties of quantum dynamics of SO-coupled cold atoms with Hermitian potential. Thus, many of these works cannot be applied to active systems where gain or loss arises naturally. Overcoming these limitations will not only enrich the conventional investigation in quantum dynamics of SO-coupled cold atoms but also offer new methods for controlling and engineering quantum spin transport. It is therefore highly desirable to investigate quantum spin dynamics of SO-coupled cold atomic systems that incorporate gain and loss mechanisms.

The aim of the present work is to study how to apply periodic driving to stabilize spin transportation in a non-Hermitian SO-coupled bosonic double-well system. Within a high frequency approximation, the Floquet states and quasienergies, as well as non-Floquet states of this non-Hermitian driven system are analytically obtained. Based on the stability analysis, we find the stability of spin-dependent dynamics depends on the competition and balance between the effective coupling parameters (arising from periodic driving) and the gain-loss coefficients. We have systematically explored how the interplay between periodic driving, dissipation, and other system parameters influences the stability of the spin-dependent tunneling in the SO-coupled bosonic junction, by taking into account balanced/unbalanced loss-gain between two wells. Under balanced loss and gain, the stable generalized Rabi oscillations with (without) spin-flipping are found, and stable spin-flipping tunneling is shown to be preferentially suppressed with the increase of gain-loss strength. When the ratio of Zeeman field strength to periodic driving frequency $\Omega/\omega$ is even, continuous stable parameter regions is possible to emerge with subtle domain boundaries. When $\Omega/\omega$ is odd, however, only discrete stable parameter regions are found. Under unbalanced loss and gain, whether $\Omega/\omega$ is even or odd, we also find the equilibrium conditions of parameters for the existence of stable spin tunneling. The results could be useful for the experiments of manipulating stable spin transportation via a periodic driving field in a non-Hermitian SO-coupled system.

II. ANALYTICAL SOLUTIONS IN THE HIGH-FREQUENCY APPROXIMATION

We consider a single SO-coupled ultracold boson held in a non-Hermitian driven double-well potential, in which the dynamics is governed by a non-Hermitian Hamiltonian

$$
\hat{H}(t) = -\nu (\hat{a}_t^\dagger e^{-i\pi \lambda \sigma \nu} \hat{a}_r + H.c.) + \frac{\Omega}{2} \sum_j (n_{j\uparrow} - n_{j\downarrow}) + \sum_{\sigma} [\bar{\varepsilon}(t) n_{\sigma\uparrow} - \varepsilon_r(t) n_{\sigma\downarrow}].
$$

Here $\hat{a}_j = (\hat{a}_{j\uparrow}, \hat{a}_{j\downarrow})^T$ (the superscript $T$ stands for the matrix transpose) is the two-component vector with elements being the annihilation operators of spin-up and spin-down atoms in the $j$th ($j = l, r$) well, and $\hat{a}_j^\dagger$ denotes its Hermite conjugation with elements being the creation operators. $\hat{n}_{j\sigma} = \hat{a}_j^\dagger \hat{a}_{j\sigma}$ denotes the number operator for spin $\sigma$ ($\sigma = \uparrow, \downarrow$) in well $j$, $\nu$ denotes the tunneling amplitude without SO coupling, $\lambda$ characterizes the SO coupling strength, $\sigma_y$ is the $y$ component of Pauli operators, and $\Omega$ is the effective Zeeman field intensity. The form $\varepsilon_j(t) = \varepsilon \cos(\omega t) + i\beta_j$ is taken, where the first term denotes the periodic driving with driving amplitude $\varepsilon$ and frequency $\omega$, and the latter term the gain-loss strength. Without loss of generality, we assume $\beta_j > 0$ such that the plus ($+$ is omitted by convention) and minus ($-$) signs in the last summation term of Hamiltonian are used to represent the left well experiencing gain while the right well loss. Throughout this paper, $\hbar = 1$ is adopted and the dimensionless parameters $\nu, \Omega, \varepsilon, \omega, \beta_j$ are in units of the reference frequency $\omega_0 = 0.1 E_r$, with $E_r = k_2^2/(2m)$ being the single-photon recoil energy, and time $t$ is normalized in units of $\omega_0^{-1}$. Note that the single-photon recoil energy is $E_r = k_2^2/(2m) = 22.5$ kHz and the Zeeman field $\Omega$ is set as $-40 \omega_0 \sim 40 \omega_0$ in the experiment, and the experimentally achievable system parameters can be tuned in a wide range as follows: $24, 26, 50, 53–56$.

Using the Fock basis $|0, \sigma\rangle (|\sigma, 0\rangle)$ to represent the state of a spin $\sigma$ atom occupying the right (left) well and no atom in the left (right) well, we can expand the quantum state of the SO-coupled system as

$$
|\psi(t)\rangle = a_1(t)|0, \uparrow\rangle + a_2(t)|\downarrow, 0\rangle + a_3(t)|\uparrow, 0\rangle + a_4(t)|0, \downarrow\rangle,
$$

where $a_k(t)$ ($k = 1, 2, 3, 4$) denote the time-dependent probability amplitudes of the atom being in the corresponding Fock state $|0, \sigma\rangle$ or $|\sigma, 0\rangle$ (e.g., $a_1(t)$ denotes the time-dependent probability amplitude of the atom being in state $|0, \uparrow\rangle$). The corresponding probabilities read

$$
P_k(t) = |a_k(t)|^2.
$$

Inserting equations (1) and (2) into Schrödinger equation

$$
i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = \hat{H}(t)|\psi(t)\rangle,$$

results in the coupled equations

$$
in\hat{a}_1(t) = -\nu \cos(\pi \lambda) a_3(t) - \nu \sin(\pi \lambda) a_2(t)
+ \frac{\Omega}{2} - \varepsilon \cos(\omega t) - i\beta_1 a_1(t),
$$

$$
in\hat{a}_2(t) = -\nu \cos(\pi \lambda) a_4(t) - \nu \sin(\pi \lambda) a_1(t)
+ \frac{\Omega}{2} + \varepsilon \cos(\omega t) + i\beta_1 a_2(t),
$$

$$
in\hat{a}_3(t) = -\nu \cos(\pi \lambda) a_1(t) + \nu \sin(\pi \lambda) a_4(t)
+ \frac{\Omega}{2} + \varepsilon \cos(\omega t) + i\beta_3 a_3(t),
$$

$$
in\hat{a}_4(t) = -\nu \cos(\pi \lambda) a_2(t) + \nu \sin(\pi \lambda) a_3(t)
+ \frac{\Omega}{2} - \varepsilon \cos(\omega t) - i\beta_3 a_4(t).
$$

It is hard to obtain the exact analytical solutions of equation (3) because of the periodically varying
coefficients, but the quantum motion of the system can be investigated analytically in high-frequency approximation. In the high-frequency regime where \( \omega \gg \nu, \beta_j \) and within multiphoton resonance case (\( \Omega = n\omega \) with \( n \) being integers), we introduce the slowly varying functions \( b_k(t) \) through the transformation

\[
\begin{align*}
\psi(t) &= b_1(t) e^{-i [\frac{1}{2} \epsilon \cos(\omega t)] dt} + b_2(t) e^{-i [\frac{1}{2} \epsilon \cos(\omega t)] dt} + b_3(t) e^{-i [\frac{1}{2} \epsilon \cos(\omega t)] dt} + b_4(t) e^{-i [\frac{1}{2} \epsilon \cos(\omega t)] dt},
\end{align*}
\]

where the effective coupling constants are simply written as \( J_0 = \nu \cos(\pi \lambda) J_0(\frac{\pi \lambda}{\omega}) \) and \( J_{\pm} = \nu \sin(\pi \lambda) J_{\pm}(\frac{\pi \lambda}{\omega}) \) with \( J_n(x) \) being the \( n \)-order Bessel function of \( x \). It can apparently be seen that the effective coupling constants can be controlled by adjusting the system parameters, e.g., the SO coupling strength \( \lambda \), driving amplitude \( \epsilon \), driving frequency \( \omega \), and the Zeeman field \( \Omega \).

A. Floquet states and quasienergies

Applying the well-established Floquet theorem to the periodic time-dependent equation (3), we can seek the analytical Floquet states by setting

\[
(\begin{bmatrix} b_1(t) \\ b_2(t) \\ b_3(t) \\ b_4(t) \end{bmatrix})^T = (A, B, C, D)^T \exp(-iEt),
\]

where \( (A, B, C, D)^T \) are the eigenvector and eigenvalue of the time-independent version of equation (4) respectively. In view of the relations between \( a_k(t) \) and \( b_k(t) \), the analytical solutions of equation (3) can be constructed as

\[
|\psi(t)\rangle = |\varphi(t)\rangle \exp(-iEt), \quad |\varphi(t)\rangle = Ae^{-i \int [\frac{1}{2} \epsilon \cos(\omega t)] dt} |\uparrow, 0\rangle + Be^{-i \int [-\frac{1}{2} \epsilon \cos(\omega t)] dt} |\downarrow, 0\rangle + Ce^{-i \int [\frac{2}{3} \epsilon \cos(\omega t)] dt} |\uparrow, 0\rangle + De^{-i \int [-\frac{2}{3} \epsilon \cos(\omega t)] dt} |\downarrow, 0\rangle.
\]

According to the Floquet theorem, it is well known that a Floquet state should inherit the period of the driving field. From the expression of \( |\varphi(t)\rangle \), we immediately note that \( |\varphi(t)\rangle \) is periodic with the driving period \( T = \frac{2\pi}{\omega} \), satisfying \( |\varphi(t + \tau)\rangle = |\varphi(t)\rangle \). The result implies that the solution \( |\varphi(t)\rangle \) can be viewed as the so-called Floquet states and the corresponding eigenvalues \( E \) in equation (4) as the approximate analytical quasienergies. Upon inserting the stationary solutions into equation (4), solvability of this equation gives four analytical Floquet quasienergies \( E_p \) and the corresponding eigenvector components \( A_p, B_p, C_p, D_p \) for \( p = 1, 2, 3, 4 \) as follows,

\[
\begin{align*}
B_{1,2} &= \pm iA_{1,2}\alpha_\pm(\pm\beta_r \pm \beta_t + \zeta_\pm), \\
C_{1,2} &= \mp iA_{1,2}\kappa_\pm(\pm\beta_r \pm \beta_t + \zeta_\pm), \\
D_{1,2} &= -A_{1,2}\eta, E_{1,2} = \frac{1}{2}(\beta_t - \beta_r \pm \zeta_\pm), \\
B_{3,4} &= \mp iA_{3,4}\alpha_\pm(\mp\beta_r \mp \beta_t + \zeta_\pm), \\
C_{3,4} &= \mp iA_{3,4}\kappa_\pm(\mp\beta_r \mp \beta_t + \zeta_\pm), \\
D_{3,4} &= A_{3,4}\eta, E_{3,4} = \frac{1}{2}(\beta_t - \beta_r \mp \zeta_\pm),
\end{align*}
\]

where we have set the following constants

\[
\begin{align*}
\alpha_\pm &= \frac{J_0^2 - J_{\pm}^2}{2J_0(J_0 - J_{\pm})}, \\
\kappa_\pm &= \frac{J_0^2 + J_{\pm}^2}{2J_0(J_0 - J_{\pm})}, \\
\zeta_\pm &= \sqrt{(\beta_r + \beta_t)^2 - 4J_0^2 - 4J_{\pm}^2 \pm 4|J_0(J_0 - J_{\pm})|}, \\
\eta &= \frac{|J_0(J_0 - J_{\pm})|}{2J_0(J_0 - J_{\pm})}, \\
\end{align*}
\]

which can be manipulated by adjusting the related system parameters. It is noteworthy that the quasienergies given by equation (5) may be complex because the Hamiltonian system is non-Hermitian. Given equation (5), we can write down the four Floquet states in the form

\[
|\varphi_p(t)\rangle = A_p e^{-i \int [\frac{1}{2} \epsilon \cos(\omega t)] dt} |\uparrow, 0\rangle + B_p e^{-i \int [-\frac{1}{2} \epsilon \cos(\omega t)] dt} |\downarrow, 0\rangle + C_p e^{-i \int [\frac{2}{3} \epsilon \cos(\omega t)] dt} |\uparrow, 0\rangle + D_p e^{-i \int [-\frac{2}{3} \epsilon \cos(\omega t)] dt} |\downarrow, 0\rangle
\]

for \( p = 1, 2, 3, 4 \). Here, we do not bother to normalize the Floquet states because the normalization factor will have no effect on the final result. As is well known, the Floquet state and quasienergy provide two basic concepts and tools for understanding the tunneling dynamics of periodically driven system, and all available information about the system at any time can be deduced from the two basic concepts.

B. General coherent non-Floquet solution

The Floquet solution given above, \( |\psi(t)\rangle = |\varphi(t)\rangle \exp(-iEt) \), only denotes the simplest fundamental solution of the periodically driven system, which just
acquires a phase $-iE\tau$ during the interval of one period $\tau$. In order to study the general spin tunneling dynamics starting from an arbitrary initial state, we have to consider the coherent superposition of the Floquet states. When the superposition states do not obey the well-known definition of the Floquet state, we call them the non-Floquet solutions [62]. The superposition principle of quantum mechanics indicates that the non-Floquet states can be constructed by the linear superposition of the Floquet states [63]. Directly employing equations (5) and (6) to the linear superposition yields the general non-Floquet solution

$$|\psi(t)\rangle = \sum_{p=1}^{4} \Lambda_p |\varphi_p(t)\rangle e^{-iE_p t}$$

$$= d_1 e^{-i\int |\frac{\Omega}{\omega} - \varepsilon \cos(\omega t)| dt} |0,\uparrow\rangle$$

$$+ d_2 e^{-i\int |\frac{\Omega}{\omega} + \varepsilon \cos(\omega t)| dt} |0,\downarrow\rangle$$

$$+ d_3 e^{-i\int |\frac{\Omega}{\omega} - \varepsilon \cos(\omega t)| dt} |\uparrow,0\rangle$$

$$+ d_4 e^{-i\int |\frac{\Omega}{\omega} + \varepsilon \cos(\omega t)| dt} |\downarrow,0\rangle,$$  \hspace{1cm} (7)

where $\Lambda_p$ are the superposition coefficients. In equation (7), the probability amplitudes are rearranged as $d_1(t) = \sum_{p=1}^{4} \Lambda_p A_p e^{-iE_p t}$, $d_2(t) = \sum_{p=1}^{4} \Lambda_p B_p e^{-iE_p t}$, $d_3(t) = \sum_{p=1}^{4} \Lambda_p C_p e^{-iE_p t}$, $d_4(t) = \sum_{p=1}^{4} \Lambda_p D_p e^{-iE_p t}$ with the undetermined constants $A_p, B_p$ depending on the initial conditions. Thus, the occupancy probabilities in each local state are given by $P_k = |d_k(t)|^2$ for $k = 1, 2, 3, 4$. The superposition state, equation (7), implies quantum interference among the four Floquet states and may cause dynamical enhancement or suppression of quantum spin tunneling, whose degree is governed by the values of the effective coupling parameters [64, 65] and gain-loss coefficients.

**III. STABILITY ANALYSIS AND CONTROLLING STABLE SPIN TUNNELING UNDER BALANCED AND UNBALANCED DISSIPATION**

We write Floquet quasienergies $E_p$ in the form $E_p = \text{Re}(E_p) + i \text{Im}(E_p)$ ($p = 1, 2, 3, 4$, hereafter, Re and Im stand for the real part and the imaginary part of a complex value respectively). According to the stability analysis on the linear equations (4) [66, 67], we know that the system stability is determined by the imaginary parts of quasienergies, $\text{Im}(E_p)$, whose values are associated with the following cases.

**Case A**, when all of $\text{Im}(E_p)$ are equal to zero, quasienergies of this system thus become all real. In this case, the evolutions of probabilities are time-periodic and the system is stable [68].

**Case B**, when some of $\text{Im}(E_p)$ are equal to zero and the others of $\text{Im}(E_p)$ are less than zero, the system is also stable because its total probability tends to a constant value at $t \rightarrow \infty$ [26, 69].

**Case C**, when all of $\text{Im}(E_p)$ are less than zero, all probabilities will exponentially decay to zero and the atom will be lost asymptotically.

**Case D**, when any one of $\text{Im}(E_p)$ is greater than zero, the total probability will show exponential growth and the system is unstable.

In this paper, we will mainly present the stability analysis on the non-Hermitian system by employing the stability criterions of Case A and Case B, under balanced and unbalanced gain-loss respectively.

1. **Stability analysis under balanced gain and loss**

In this part, we only focus our attention on the balanced gain-loss situation, where the loss (gain) coefficients of two wells take the same values, $\beta_r = \beta_i = \beta$. In such situation, we will analyze the dependence of the system stability on different parameters in detail for both even and odd $\Omega/\omega$, by using the stability criterion described by Case A.

(1) even $\Omega/\omega$

For this case, from equation (5) we easily infer $E_1 = -E_2 = -E_3 = E_4 = \frac{1}{2} i \rho$ with $\rho = 2\sqrt{\beta^2 - J_0^2 - J_2^2}$. According to Case A of stability analysis, by writing $\rho = \text{Re}(\rho) + i \text{Im}(\rho)$, we directly find that the system is stable when $\text{Re}(\rho) = 0$, and is unstable otherwise. In the case of even $\Omega/\omega$, evidently, the boundary between stable ($\text{Re}(\rho) = 0$) and unstable ($\text{Re}(\rho) \neq 0$) regimes can be parameterized by using the relation $\beta^2 - J_0^2 - J_2^2 = 0$.

In all stability diagrams as illustrated by Figs. 1-3, the boundaries between stable and unstable regimes are plotted as white curves. In Figs. 1 (a)-(c), we take $\nu = 1$, $\Omega = 100$, $\omega = 50$ (hence $\Omega/\omega = 2$) to plot $\text{Re}(\rho)$ as a function of $2\varepsilon/\omega$ and $\lambda$ for different gain-loss strengths with (a) $\beta = 0.2$, (b) $\beta = 0.45$, and (c) $\beta = 0.6$. As illustrated in these plots, two types of stable parameter regions are possible: when the system is stable for a continuous range of system parameters, we call the parameter regions continuous stable regions; when the system is only stable in some disconnected parameter spaces, we call the parameter regions discrete stable regions. When $\beta = 0.2$, from Fig. 1 (a), it is clearly seen that the system can be stabilized for a continuous range of parameters in the $(\lambda, 2\varepsilon/\omega)$ space, and the width of these continuous stable parameter regions becomes increasingly narrower with increasing $2\varepsilon/\omega$. The continuous stable parameter regions are found to possibly occur for the balanced gain-loss situation with $\Omega/\omega$ being even, which provides some convenience for the experimental manipulating stable spin transport in a non-Hermitian spin-orbit coupled cold atomic system. As $\beta$ gets stronger, we find that the continuous stable regions disappear and the discrete stable regions instead emerge, as shown in Fig. 1 (b). Combining Fig. 1 (a) with $\beta = 0.2$ and Fig. 1 (b) with $\beta = 0.45$, we have another observation: when $2\varepsilon/\omega$ is less than a certain value, stable spin-flipping tunneling with $\lambda = m + 0.5(m = 0, 1, 2, \ldots)$ can not occur, but stable non-spin-flipping tunneling with $\lambda = m$ can happen nevertheless. This observation implies that the available
values of $2\varepsilon/\omega$ for realizing stable spin-flipping tunneling with $\lambda = m + 0.5$ are greater than those for stable non-spin-flipping tunneling with $\lambda = m$. In Fig. 1 (c) with $\beta = 0.6$, we find that the number of discrete stable parameter regions decreases as compared to that of Fig. 1 (b) with smaller $\beta = 0.45$. Specially, it is worth noting that when $\beta = 0.6$, the stable spin-flipping tunneling with $\lambda = m + 0.5$ vanishes, but the stable tunneling without spin flipping with $\lambda = m$ still exists. This finding shows that with the increase of dissipative strength $\beta$, stable spin-flipping tunneling, when/if compared with stable non-spin-flipping tunneling, will be preferentially suppressed.

Now let us numerically illustrate the above results by directly integrating the original model equation (3). In Figs. 1 (d)-(e), we arbitrarily pick two parameter sets localized in stable parameter regions of Figs. 1 (a) and 1 (b), $(\beta, 2\varepsilon/\omega, \lambda) = (0.2, 3, 0.5)$ and $(\beta, 2\varepsilon/\omega, \lambda) = (0.45, 1, 1)$, as examples to plot the time evolutions of probabilities $P_k = |\psi_k(t)|^2$ for the particle initially occupying the state $|0, \uparrow\rangle$. As we will see in Fig. 1 (d), a stable but non-normal-preserving population oscillation with spin flipping takes place between states $|0, \uparrow\rangle$ and $|\downarrow, 0\rangle$, which represents a type of generalized Rabi oscillation. Similarly, a stable generalized Rabi oscillation without spin flipping between states $|0, \uparrow\rangle$ and $|\downarrow, 0\rangle$ is also shown in Fig. 1 (c). In Fig. 1 (f), we choose one parameter set $(\beta, 2\varepsilon/\omega, \lambda) = (0.6, 1, 0.5)$ in the unstable region of Fig. 1 (c) to illustrate the time evolutions of probabilities with the same initial condition as above. It is shown that the probabilities exponentially grow without bound which marks an unstable spin tunneling.

In Figs. 2 (a)-(c), we show the real part $\text{Re}(\rho)$ as a function of $\beta$ and $\lambda$ with (a) $2\varepsilon/\omega = 5.1356$, (b) $2\varepsilon/\omega = 2.405$, and (c) $2\varepsilon/\omega = 1.5$, for the case of $\Omega/\omega = 2$. Throughout this paper, the other two parameters are fixed as $\nu = 1, \omega = 50$ unless otherwise specified. Figs. 2 (a) and (b) describe the stability diagrams for the non-spin-flipping and spin-flipping tunneling respectively, which are associated with the adopted driving parameter $2\varepsilon/\omega = 5.1356$ and $2\varepsilon/\omega = 2.405$. The former ratio $2\varepsilon/\omega = 5.1356$ infers $J_2(2\varepsilon/\omega) = J_2(5.1356) = 0$ and $J_0(5.1356) \neq 0$, which leads to the quantum tunneling without spin flipping, while the latter ratio $2\varepsilon/\omega = 2.405$ gives $J_0(2\varepsilon/\omega) = J_0(2.405) = 0$ and $J_2(2.405) \neq 0$, which instead leads to the quantum tunneling with only spin flipping. As such, when the system parameters are selected in the discrete stable region of Figs. 2 (a) and (b), stable generalized Rabi oscillations without and with spin flipping (see Figs. 2 (d) and (c)) will arise respectively. In Fig. 2 (c), the ratio $2\varepsilon/\omega = 1.5$ implies $J_0(1.5) \neq 0$ and $J_2(1.5) \neq 0$ such that the spin-conserving and spin-flipping tunnelings will coexist. For the former two special case, where $J_0(2\varepsilon/\omega) = 0$ or $J_0(2\varepsilon/\omega) = 0$, there exist a sequence of crossings between the white boundary curve and the transverse axis. As can be seen from Figs. 2 (a)-(b), the crossings for the spin-conserving (spin-flipping) case are precisely located at $\lambda = m + 1/2$ ($\lambda = m$) respectively. At these crossing points, where $J_0 = J_2 = 0$, the quantum spin tunneling is not allowed and the system is stable if and only if $\beta_1 = \beta_2 = \beta = 0$. Indeed, the stable dynamics at these crossing points characterizes the well-known CDT phenomenon in the Hermitian system, which is not shown here. For the general case of $J_0(2\varepsilon/\omega) \neq 0$ and $J_2(2\varepsilon/\omega) \neq 0$, from Fig. 2 (c), it can be seen that the white boundary curve does not cross with the transverse axis and the stable parameter region has been split into two parts. The lower part corresponds to a continuous stable region, whose width $d$ equals to the maximal safe value (the upper limit) of $\beta$ for existence of continuous stable region and is determined essentially by the minimum value of $\sqrt{J_0^2 + J_2^2}$. In our case, the width $d$ is given by $d = \beta_{\text{max}} = |J_2| = \nu J_2(1.5) \approx 0.232088$; it corresponds to an upper limit in the sense that in the region under this limit, the system is stable for the whole range of parameters in the $(\lambda, \beta)$ space. The other (upper) part is associated with some discrete stable regions with the gain-loss strength $d < \beta < \nu J_0(1.5)$. Obviously, when the gain-loss strength is greater than the maximum value of $\sqrt{J_0^2 + J_2^2}$, namely, $\beta > |J_0| = \nu J_0(1.5) \approx 0.5118$, the system is always unstable. As an example, we take $\beta = 0.3 > d \approx 0.232088$ and $\lambda = 0.8$ in the discrete stable parameter region to plot the time evolutions of probabilities as shown in Fig. 2 (f), and a stable generalized Rabi oscillation is presented accordingly. Besides, Figs. 2 (a)-(c) have confirmed, from a different angle, the fact that these stable parameter regions are shrunk with increasing the gain-loss strength $\beta$.

In Figs. 3 (a)-(c), we show the real part $\text{Re}(\rho)$ as a function of $\beta$ and $2\varepsilon/\omega$ with (a) $\lambda = 0.5$, (b) $\lambda = 1$, and (c) $\lambda = 1.7$, for the case of $\Omega/\omega = 2$. In Fig. 3 (a), the selected SO coupling strength $\lambda = 0.5$ is associated with $J_0 = 0$, which means the quantum tunneling with spin flipping. In Fig. 3 (b), we consider the other special case with SO coupling strength $\lambda = 1$, which corresponds with $J_2 = 0$ and thus describes the quantum tunneling without spin flipping. As expected, by taking system parameters in the discrete stable region of Figs. 3 (a) and (b) respectively, we can observe the stable generalized Rabi oscillation with and without spin flipping as shown in Figs. 3 (d) and (e) correspondingly. From Figs. 3 (a)-(b), it is seen that the white boundary curve can cross with the transverse axis repeatedly. The positions of these crossing points correspond to the roots of $J_2(\frac{2\varepsilon}{\omega}) = 0$ for $\lambda = m + 1/2$ (see Fig. 3 (a)) and of $J_0(\frac{2\varepsilon}{\omega}) = 0$ for $\lambda = m$ (see Fig. 3 (b)). At these crossing points, we have $J_0 = J_2 = 0$ and $\beta = 0$, which means the occurrence of CDT (not shown in Fig. 3). In Fig. 3 (c), we discuss the general case with SO coupling strength $\lambda = 1.7$, and find a continuous stable parameter region, due to the fact there is no intersection between the white boundary curve with the transverse axis. Moreover, it is shown that the width of the continuous stable region decreases with the increase of $2\varepsilon/\omega$, which is consistent
FIG. 1: (Color online) Top panels depict the real part $\text{Re}(\rho)$ as a function of $2\epsilon/\omega$ and $\lambda$ for (a) $\beta = 0.2$, (b) $\beta = 0.45$, and (c) $\beta = 0.6$. Bottom panels show the time-evolution curves of probabilities $P_k = |a_k(t)|^2$ given by the original model (3), for (d) $\beta = 0.2$, $2\epsilon/\omega = 1$, $\lambda = 0.5$; (e) $\beta = 0.45$, $2\epsilon/\omega = 1$, $\lambda = 1$; (f) $\beta = 0.6$, $2\epsilon/\omega = 1$, $\lambda = 0.5$, starting the system with a spin-up particle in the right well. The other parameters are chosen as $\nu = 1$, $\Omega = 100$, and $\omega = 50$. Hereafter, the white curves are the boundary between stable and unstable parameter regions. Shown in Figs. 1 (d)-(f) are the numerical results of population $P_1 = |a_1(t)|^2$ in state $|\uparrow, \uparrow\rangle$ (orange thin solid line), population $P_2 = |a_2(t)|^2$ in state $|\uparrow, 0\rangle$ (green thin long-dashed line), and population $P_3 = |a_3(t)|^2$ in state $|\downarrow, 0\rangle$ (red thick long-dashed line). The squares denote the analytical correspondences obtained from the effective model (4), unless it is specially indicated. All parameter adopted in these figures are dimensionless.

FIG. 2: (Color online) Top panels depict the real parts $\text{Re}(\rho)$ as a function of $\beta$ and $\lambda$ for (a) $2\epsilon/\omega = 5.1356$, (b) $2\epsilon/\omega = 2.405$, and (c) $2\epsilon/\omega = 1.5$. The stable parameter regions are under the white boundary curves. Bottom panels show the time-evolution curves of probabilities $P_k = |a_k(t)|^2$ for (d) $2\epsilon/\omega = 5.1356$, $\beta = 0.1$, $\lambda = 0.1$; (e) $2\epsilon/\omega = 2.405$, $\beta = 0.1$, $\lambda = 0.4$; (f) $2\epsilon/\omega = 1.5$, $\beta = 0.3$, $\lambda = 0.8$, starting the system with a spin-up particle in the right well. The other parameters are the same as those of Fig. 1.

with the behaviors reflected perviously by Fig. 1(a). For the general case of $\lambda = 1.7$ and with the parameter points taken in the stable region, when the ratio $2\epsilon/\omega$ is changed to satisfy $J_0(2\epsilon/\omega) = 0$ or $J_2(2\epsilon/\omega) = 0$, the stable generalized spin-flipping or non-spin-flipping Rabi oscillation will occur correspondingly; otherwise, the stable generalized spin-flipping and non-spin-flipping Rabi oscillation will simultaneously happen, as we can see from Fig. 3 (f).

(2) odd $\Omega/\omega$

When $\Omega/\omega$ is odd, we have $E_{1,2} = \pm \frac{i}{2}\rho_-$ and $E_{3,4} = \mp \frac{i}{2}\rho_+$ with $\rho_\pm = 2\sqrt{\beta^2 - (|J_0| + |J_2|)}$. Applying the condition $J_0 = 0$ or $J_2 = 0$ will lead to the simple case: $\rho_+ = \rho_-$, $E_3 = E_4$, and $E_2 = E_3$. We write $\rho_+ + \rho_-$= $\text{Re}(\rho_+ + \rho_-)$+ $i\text{Im}(\rho_+ + \rho_-)$ and know that $\text{Re}(\rho_+ + \rho_-)$=0 will make $\text{Re}(\rho_-)$=0 and $\text{Re}(\rho_+)$=0 hold simultaneously. According to Case A of stability analysis, we can deduce that when the system parameters satisfy $\text{Re}(\rho_+ + \rho_-)$=0, quasenergies $E_p$ become all real and the spin-dependent tunneling is stable. In fact, if $\text{Re}(\rho_-)$=0 holds, both $\text{Re}(\rho_+)$=0 and $\text{Re}(\rho_+ + \rho_-)$=0 will naturally hold. As illustrated in Figs. 4-6, in all the stability diagrams with odd $\Omega/\omega$, the boundary between the stable ($\text{Re}(\rho_+ + \rho_-)$=0) and unstable ($\text{Re}(\rho_+ + \rho_-)$≠0)
0) regimes is displayed by the white curves, which can be obtained from the relation $\beta^2 - (|J_0| - |J_m|)^2 = 0$. In Figs. 4 (a)-(c), we take $\nu = 1$, $\Omega = \omega = 50$ (hence $\Omega/\omega = 1$) to show $\text{Re}(\rho_+ + \rho_-)$ as a function of $2\epsilon/\omega$ and $\lambda$, with different gain-loss strengths (a) $\beta = 0.2$, (b) $\beta = 0.45$, and (c) $\beta = 0.6$. It is shown that only discrete stable parameter regions exist for the case of odd $\Omega/\omega$. The basic explanation of this phenomenon is as follows: the value of $(|J_0| - |J_m|)^2$ can become zero as either $2\epsilon/\omega$ or $\lambda$ varies continuously, such that there does not exist a non-zero gain-loss coefficient $\beta$ below which $\rho_-$ is always a purely imaginary number (the system is stable) over the continuous range of system parameters.

From Fig. 4 (a), we see that the sizes of discrete stable parameter regions decrease with the increase of $2\epsilon/\omega$. It can also be observed that for small driving parameters $2\epsilon/\omega$, the stable spin-flipping tunneling with $\lambda = m + 0.5$ ($m=0, 1, \ldots$) cannot happen, whereas the stable generalized Rabi oscillation without spin flipping $(\lambda = m)$ can happen, which is demonstrated by time-evolution of the probabilities in Fig. 4 (d). By comparing Fig. 4 (b) with Fig. 4 (a), we can see that the number of discrete stable parameter regions is getting smaller as the gain-loss strength increases. From Fig. 4 (b), we also find that the adjustable values of $2\epsilon/\omega \in [1.03109, 2.67221]$ corresponding to the stable spin-flipping tunneling with $\lambda = m + 0.5$ are not greater than all the adjustable values $2\epsilon/\omega \in [0, 1.60947]$ corresponding to the stable non-spin-flipping tunneling with $\lambda = m$. This result is distinct from the even $\Omega/\omega$ case. In order to support this finding, for instance, we set $\lambda = 1$ and $2\epsilon/\omega = 1.4 > 1.03109$ to illustrate the time evolution of stable non-spin-flipping tunneling, as shown in Fig. 4 (e). In Fig. 4 (c) with $\beta = 0.6$, we observe that the stable spin-flipping tunneling with $\lambda = m + 0.5$ vanishes, but the stable generalized non-spin-flipping Rabi oscillation with $\lambda = m$ still exists (e.g., see the time-evolution of probabilities in Fig. 4 (f)). Based on these numerical results, a conclusion can be drawn: for odd $\Omega/\omega$, the stable spin-flipping tunneling is suppressed preferentially as the gain-loss strength is increased. This conclusion is basically the same as that reached for the case of even $\Omega/\omega$.

In Figs. 5 (a)-(c), we show the real part $\text{Re}(\rho_+ + \rho_-)$ as a function of $\beta$ and $2\epsilon/\omega$ for (a) $\lambda = 0.5$, (b) $\lambda = 1$ and (c) $\lambda = 1.7$. Bottom panels show the time-evolution curves of probabilities $P_\pm = |\alpha_\pm(t)|^2$ with $\beta = 0.2$, and (d) $\lambda = 0.5$, $2\epsilon/\omega = 3$; (e) $\lambda = 1$, $2\epsilon/\omega = 4$; (f) $\lambda = 1.7$, $2\epsilon/\omega = 7$, starting the system with a spin-up particle in the right well. The other parameters are the same as those of Fig. 1.
tion without spin flipping or with spin flipping will occur respectively. As excepted, when other values of λ are selected in the stable parameter regions, stable generalized spin-flipping and non-spin-flipping Rabi oscillation will simultaneously occur, as illustrated in Fig. 5 (f).

In Figs. 6 (a)-(b), we show the real part of \( \text{Re}(\rho_+ + \rho_-) \) as a function of \( \beta \) and \( 2\varepsilon/\omega \) with (a) \( \lambda = 0.5 \) and (b) \( \lambda = 1.7 \), for the case of \( \Omega/\omega = 1 \). As before, only the discrete stable parameter regions are found. In Fig. 6 (a), we take the SO coupling strength \( \lambda = 0.5 \) which is associated with \( J_0 = 0 \). When the system parameters are chosen in the discrete stable regions of Fig. 6 (a), stable generalized Rabi oscillation with spin flipping can be observed (e.g., see Fig. 6 (c)). In Fig. 6 (a), the crossing points of the white boundary curve and transverse axis are precisely aligned with the roots of \( J_1(\frac{2\varepsilon}{\omega}) = 0 \), at which CDT will occur (not shown). In Fig. 6 (b), for the general case of \( \lambda = 1.7 \), with the parameters in the discrete stable regions and \( 2\varepsilon/\omega \) obeying \( J_0(\frac{2\varepsilon}{\omega}) = 0 \) or \( J_1(\frac{2\varepsilon}{\omega}) = 0 \), the stable generalized Rabi oscillation with spin flipping or without spin flipping will occur respectively. In Fig. 6 (d), we have shown the stable generalized Rabi oscillation without spin flipping for the suitable chosen parameters. Meanwhile, the adjustable values of \( \beta \) for realizing stable spin tunneling are shown to exhibit a sequence of peaks. These peaks are situated at the zeros of \( J_1(\frac{2\varepsilon}{\omega}) \) and \( J_0(\frac{2\varepsilon}{\omega}) \) alternately, which stand for the non-spin-flipping tunneling and spin-flipping tunneling. The situation for \( \lambda = 1 \) is similar to the even \( \Omega/\omega \) case as shown in Fig. 3 (b) and it is not discussed here.

2. Stability analysis under unbalanced gain and loss

Now we turn to the unbalanced gain-loss situation, where the loss (gain) coefficients of two wells do not take the same values, namely, \( \beta_r \neq \beta_l \). In such situation, we will perform stability analysis based on Case B with \( \Omega/\omega \) being even and odd respectively. To guarantee the system’s stability, we find that the dissipation coefficients must obey \( \beta_r > \beta_l \).

(1) even \( \Omega/\omega \)

When \( \Omega/\omega \) is even, the quasienergies become \( \varepsilon_1 = E_1 = \frac{1}{2}i(\beta_l - \beta_r + \rho') \) and \( \varepsilon_2 = E_3 = \frac{1}{2}i(\beta_l - \beta_r - \rho') \) with \( \rho' = \sqrt{(\beta_l + \beta_r)^2 - 4J_0^2 - 4J_2^2} \). By adjusting the system parameters to satisfy the balance between the effective coupling strengths and the gain-loss coefficients, \( \beta_r \beta_l = J_0^2 + J_2^2 \), we can simplify the quasienergies as: \( E_1 = E_4 = 0 \) and \( E_2 = E_3 = i(\beta_l - \beta_r) \). According to Case B of stability analysis, for the gain-loss coefficients obeying \( \beta_r > \beta_l \), we find that the considered system is stable under the balance (equilibrium) condition of \( \beta_r \beta_l = J_0^2 + J_2^2 \). To verify the analytical results, by fixing the parameters \( \Omega = 100, \omega = 50, \nu = 1, \lambda = 1/3, \varepsilon = 75 \), and adjusting the dissipation coefficient \( \beta_r, \beta_l \) to meet the above balance condition, from the accurate model (3), we plot the time-evolution curves of all probabilities \( P_k = |a_k(t)|^2 \) and \( P = \sum_{k=1}^4 P_k \), as shown in Fig. 7. It is clearly seen that all the probabilities tend to stable values after a period of time and the system is stable [27, 69].

In Figs. 7 (a)-(b), we adopt the same parameters and yet different initial states. As shown in Fig. 7 (a), for the particle initialized in the right (loss) well, the total probability of the particle decays quickly in the initial time interval, then increases to approach a constant value of 0.4. If we instead start the particle in the left (gain) well, the total probability monotonically increases and finally reaches a higher stable value of 1.9. In Fig. 7 (c), when the special ratio \( \beta_r/\beta_l = 3 \) is set, we can see that the total probability of the particle initially prepared in the right well monotonically tends to the steady value of 1 after a temporary decay, and the system is then stabilized. In the limit of \( t \to \infty \), the special ratio between the two dis-
show the time-evolution curves of probabilities $P_\beta$ as a function of $\beta$ and $\lambda$ for (a) $2\epsilon/\omega = 3.8317$, (b) $2\epsilon/\omega = 2.4048$ and (c) $2\epsilon/\omega = 1$. Bottom panels depict the real part $\Re(P_\beta)$ as a function of $\lambda$. Top panels show the time-evolution curves of probabilities $P_\beta = |a_\pm(t)|^2$ for (d) $2\epsilon/\omega = 3.8317$, $\beta = 0.2$, $\lambda = 0.5$; (e) $2\epsilon/\omega = 2.4048$, $\beta = 0.3$, $\lambda = 0.4$; (f) $2\epsilon/\omega = 1$, $\beta = 0.1$, $\lambda = 0.1$, starting the system with a spin-up particle in the right well. The other parameters are the same as those of Fig. 4. In (a)-(c), the white curves are the boundary between $\Re(P_\beta + P_-) = 0$ and $\Re(P_\beta + P_-) \neq 0$. The stable parameter region is under the white curve.

FIG. 5: (Color online) Top panels depict the real part $\Re(P_\beta + P_-)$ as a function of $\beta$ and $\lambda$ for (a) $2\epsilon/\omega = 3.8317$, (b) $2\epsilon/\omega = 2.4048$ and (c) $2\epsilon/\omega = 1$. Bottom panels show the time-evolution curves of probabilities $P_\beta = |a_\pm(t)|^2$ for (d) $2\epsilon/\omega = 3.8317$, $\beta = 0.2$, $\lambda = 0.5$; (e) $2\epsilon/\omega = 2.4048$, $\beta = 0.3$, $\lambda = 0.4$; (f) $2\epsilon/\omega = 1$, $\beta = 0.1$, $\lambda = 0.1$, starting the system with a spin-up particle in the right well. The other parameters are the same as those of Fig. 4. In (a)-(c), the white curves are the boundary between $\Re(P_\beta + P_-) = 0$ and $\Re(P_\beta + P_-) \neq 0$. The stable parameter region is under the white curve.

FIG. 6: (Color online) Top panels show the time-evolution curves of probabilities $P_\beta = |a_\pm(t)|^2$ for (c) $\lambda = 0.5$, $\beta = 0.3$, $2\epsilon/\omega = 2$; (d) $\lambda = 1.7$, $\beta = 0.1$, $2\epsilon/\omega = 3.8317$, starting the system with a spin-up particle in the right well. In (a)-(c), the white curves are the boundary between stable ($\Re(P_\beta + P_-) = 0$) and unstable ($\Re(P_\beta + P_-) \neq 0$) regimes. The other parameters are the same as those of Fig. 4.

sipation parameters, $\beta_r/\beta_l = 3$, can make the total probability evolve in time toward unity, namely, $P(\infty) = 1$, a result which is in analogy to the non-Hermitian cold atomic system without SO coupling [26]. This nontrivial result can be demonstrated analytically from the general non-Floquet solution (7) with a given initial condition (not shown here).

(2) odd $\Omega/\omega$

When $\Omega/\omega$ is odd, the quasienergies become $E_{1,2} = \frac{1}{2}i(\beta_l - \beta_r \pm \rho_-')$ and $E_{3,4} = \frac{1}{2}i(\beta_l - \beta_r \mp \rho'_+)$ with $\rho'_\pm = \sqrt{(\beta_l + \beta_r)^2 - 4(|J_0| \pm |J_2|)^2}$. According to Case B of the stability analysis, we obtain some stable conditions which are distinguished as the following two categories.

Category 1. two of the quasienergies are 0, and the imaginary parts of the other two are less than 0.

(i) $J_2 = 0$ and $\beta_r\beta_l = J_0^2$

When the stable condition $J_2 = 0$ and $\beta_r\beta_l = J_0^2$ is fulfilled, the quasienergies can be simplified as $E_1 = E_4 = 0$ and $E_2 = E_3 = i(\beta_l - \beta_r)$, which thus produces stable non-spin-flipping tunneling for $\beta_r > \beta_l$. To verify this analytical result, we set the parameters $\Omega = \omega = 50$ ($\Omega/\omega = 1$), $\nu = 1$, $\lambda = 1/3$, $\epsilon = 3.8317 \times 50/2$, $\beta_l = 0.405538$ and $\beta_r = 0.1$ to satisfy the above stable condition (i), and from equation (3) we plot the time-evolution curves of probabilities for the initial spin-up atom occupying the right well, as shown in Fig. 8 (a). It can be seen that the total probability of the final-state will approach 0.55 after a sufficiently large time. Here, we observe that the stable non-spin-flipping tunneling occurs because the driving parameters $2\epsilon/\omega = 3.8317$ satisfy $J_1(2\epsilon/\omega) = 0$.

(ii) $J_0 = 0$ and $\beta_r\beta_l = J_2^2$

When the stable condition $J_0 = 0$ and $\beta_r\beta_l = J_2^2$ is fulfilled, the quasienergies can also be written as $E_1 = E_4 = 0$ and $E_2 = E_3 = i(\beta_l - \beta_r)$, thereby producing stable spin-flipping tunneling for $\beta_r > \beta_l$. Taking the parameters to satisfy the above stable condition (ii) as: $\Omega = \omega = 50$, $\nu = 1$, $\lambda = 1/2$, $\epsilon = 2.001 \times 50/2$, $\beta_l = 3.0332971$ and $\beta_r = 0.332971$, from equation (3) we have plotted the time-evolution curves of probabilities with the spin-up atom initially localized in the right well, as shown in Fig. 8 (b). Obviously, as time increases to infinity, namely, $t \to \infty$, the final total probability tends to 1, which also seems reminiscent of the previous result.
with $\beta_r/\beta_l = 3$ studied in the non-Hermitian bosonic junction without considering SO coupling \cite{20}. Here, we observe that the stable spin-flipping tunneling occurs because the SO coupling strength $\lambda = 1/2$ is associated with $J_0 = 0$.

(iii) $J_0 = J_{\frac{\omega}{2}} = 0$ and $\beta_l = 0$

When $J_0 = J_{\frac{\omega}{2}} = 0$ and $\beta_l = 0$ are set, we have the quasienergies: $E_1' = E_4 = 0$ and $E_2 = E_3 = -i\beta_r$. In such a case, the particle will remain frozen in the initially occupied well. Thus, under the stable condition (iii) $J_0 = J_{\frac{\omega}{2}} = 0$ and $\beta_l = 0$, we find that the system’s stability depends on its initial condition: when the initial state is state $|0, \uparrow\rangle$ or $|0, \downarrow\rangle$ or a superposition state of $|0, \uparrow\rangle$ and $|0, \downarrow\rangle$, that is, the system is initialized with the particle in the right (loss) well, the total probability of system will exponentially decay to 0; when the initial state is state $|\downarrow, 0\rangle$ or $|\uparrow, 0\rangle$ or a superposition state of $|\downarrow, 0\rangle$ and $|\uparrow, 0\rangle$, that is, the system is initialized with the particle in the left (zero gain) well, conventional CDT will occur.

For example, we take the parameters to satisfy the above stable condition (iii) as: $\Omega = \omega = 50$, $\nu = \lambda = 1$, $\varepsilon = 2.4048 \times 50/2$, $\beta_r = 0.4$, and $\beta_l = 0$, and from equation (3) we have plotted the time-evolution curves of probabilities with the particle initially localized in the right ($P_{\lambda}(0) = 1$) and left well ($P_{\beta}(0) = 1$), as shown in Fig. 8 (c) and in Fig. 8 (d) respectively. It is clearly seen that for the former initial condition ($P_{\lambda}(0) = 1$), the total probability of system (here $P = \lambda$) exponentially decays to 0, for the latter ($P_{\beta}(0) = 1$), however, the initial state is kept and conventional CDT occurs.

Category 2. one of the quasienergies is 0, and the imaginary parts of the others are less than 0.

(i) $(|J_0| - |J_{\frac{\omega}{2}}|)^2 = \beta_r \beta_l$ and $(\beta_l + \beta_r)^2 < 4(|J_0| + |J_{\frac{\omega}{2}}|)^2$

When the system parameters satisfy the stable conditions $(|J_0| - |J_{\frac{\omega}{2}}|)^2 = \beta_r \beta_l$ and $(\beta_l + \beta_r)^2 < 4(|J_0| + |J_{\frac{\omega}{2}}|)^2$, the quasienergies becomes $E_1 = 0$, $E_2 = i(\beta_l - \beta_r)$, $E_3 = \frac{1}{2}i(\beta_l - \beta_r - \rho_+)$, and $E_4 = \frac{1}{2}i(\beta_l - \beta_r + \rho_+)$ with $\rho_+$ being a purely imaginary number, and thus the system is stable for $\beta_r > \beta_l$. In Fig. 9 (a), we take the parameters to match the stable conditions $(|J_0| - |J_{\frac{\omega}{2}}|)^2 = \beta_r \beta_l$ and $(\beta_l + \beta_r)^2 < 4(|J_0| + |J_{\frac{\omega}{2}}|)^2$ as follows: $\Omega = \omega = 50$, $\nu = 1$, $\lambda = 1/4$, $\varepsilon = 100$, $\beta_r = 0.54816$, and $\beta_l = 0.1$. Starting the system with a spin-down particle in the left well. The other parameters are chosen as $\nu = 1$ and $\Omega = \omega = 50$.

Adopting such parameters, we illustrate the time evolution of all the probabilities with a spin-down particle initially localized in the left well, as illustrated in Fig. 9 (a). Obviously, after a sufficiently long time, the final
total probability tends to a steady value of 0.88.

(ii) \(|J_0| - |J_\omega|)^2 = \beta_r \beta_l \) and \(0 < \rho_+ < \beta_r - \beta_l\).

When the system parameters satisfy the stable conditions \((|J_0| - |J_\omega|)^2 = \beta_r \beta_l \) and \(0 < \rho_+ < \beta_r - \beta_l\), the quasienergies become \(E_1 = 0\), \(E_2 = i(\beta_l - \beta_r)\), \(E_3 = \frac{i}{2}(\beta_l - \beta_r - \rho_+^*)\), and \(E_4 = \frac{i}{2}(\beta_l - \beta_r + \rho_+)\), and thus stable spin-dependent tunneling can be achieved for \(\beta_r > \beta_l\). In Fig. 9 (b), we take the parameters to match the stable conditions of \((|J_0| - |J_\omega|)^2 = \beta_r \beta_l \) and \(0 < \rho_+ < \beta_r - \beta_l\) as follows: \(\Omega = \omega = 50\), \(\nu = 1\), \(\lambda = 1/4\), \(\varepsilon = 100\), \(\beta_r = 0.685209\), and \(\beta_l = 0.08\). With such parameters, we have shown in Fig. 9 (b) the time evolution of all the probabilities with a spin-down particle initially localized in the left well. It can be seen that the final total probability approaches a steady value of 0.72 after a sufficiently large time.

IV. CONCLUSION AND DISCUSSION

In summary, we have studied the stabilization of spin tunneling of a single SO-coupled atom placed in a periodically driven non-Hermitian double-well potential. By use of the high-frequency approximation, we derive the analytical expressions of quasienergies and Floquet states of the non-Hermitian system, which can give us the direct information about the system’s stability. The main results are summarized as follows, which are numerically confirmed by monitoring the time-evolution of the accurate model. When the loss (gain) coefficients of two wells take the same values, \(\beta_r = \beta_l = \beta\), we find that as the gain-loss strength is increased, the stable spin-flipping tunneling is preferentially suppressed and the stable parameter regions will become narrower. In such balanced gain-loss situation, when \(\Omega/\omega\) is even, continuous stable parameter regions can emerge; when \(\Omega/\omega\) is odd, nevertheless, only discrete stable parameter regions are found. On the other hand, when the loss (gain) coefficients of two wells does not take the same values, the parametric equilibrium conditions for realizing stable spin tunneling are presented whether \(\Omega/\omega\) is even or odd. These results may be relevant to quantum control of the spin-dependent tunneling dynamics in the realistic dissipative systems and promises some potential applications in the design of novel spintronics devices.

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