Abstract—It is known that the estimated energy consumption of digital-to-analog converters (DACs) is around 30% of the energy consumed by analog-to-digital converters (ADCs) keeping fixed the sampling rate and bit resolution. Assuming that similarly to ADC, DAC dissipation doubles with every extra bit of resolution, a decrease in two resolution bits, for instance from 4 to 2 bits, represents a 75% lower dissipation. The current limitations in sum-rates of 1-bit quantization have motivated researchers to consider extra bits in resolution to obtain higher levels of sum-rates. Following this, we devise coarse quantization-aware precoding using few bits for the broadcast channel of multiple-antenna systems based on the Bussgang theorem. In particular, we consider block diagonalization algorithms, which have not been considered in the literature so far. The sum-rates achieved by the proposed Coarse Quantization-Aware Block Diagonalization (CQA-BD) and its regularized version (CQA-RBD) are superior to those previously reported in the literature. Simulations illustrate the performance of the proposed CQA-BD and CQA-RBD algorithms against existing approaches.

Index Terms—Coarse quantization-aware, digital-to-analog converter, consumption, block diagonalization, Bussgang’s theorem.

I. INTRODUCTION

Recent work in wireless communications has shown a great deal of progress in massive multiple-input multiple-output (MIMO) systems, which in case of transmissions using broadcast channels employ base-stations (BS) composed of a huge number of antennas. Nonetheless, the increasing numbers of antennas at the BS result in higher costs in terms of equipment and energy consumption. Thus, the design of effective and economical MIMO-based systems to provide coverage of geographical areas and cost-effective systems will require more energy-efficient and low-cost components [1], [2], [3], [4], [5], [6].

Despite the progress in 1-bit quantization [12], [13] with the aim of reducing energy consumption in the large number of DACs used in massive MIMO, the achievable sum rates still remain low, which makes higher resolution quantizers with $b = 2, 3, 4$ bits attractive for the design of precoders and receivers. Bussgang’s theorem [7] lets us express a Gaussian precoded signal that was quantized as a linear function of the quantized input and a distortion term which has no correlation with the input [8], [9], [10]. This approach makes possible the computation of sum-rates of Gaussian data [11]. In this context, block diagonalization (BD) precoding methods and their variants [4], [6], [7], [8], [9], [10] are known to be linear transmit approaches for multiuser MIMO (MU-MIMO) systems based on singular value decompositions (SVD), which provide excellent achievable sum-rates in the case of significant levels of multi-user interference. However, BD has not been considered with coarsely quantized signals so far.

Motivated by the relatively poor performance of 1-bit quantization of precoded signals applied to massive MU-MIMO systems, we propose coarse quantization-aware BD (CQA-BD) type precoders for signals quantized with an arbitrary number of bits in broadcast channels. Then, using Bussgang’s theorem we derive expressions to compute the achievable sum-rates of the proposed CQA-BD type precoders. Simulations illustrate the excellent sum-rate performance of the proposed CQA-BD and CQA-RBD precoders against previously reported techniques.

This paper is structured as follows. Section II briefly describes the system model and background for understanding the proposed CQA-BD class algorithms. Section III presents the proposed CQA-BD type algorithms. In Section IV we present and discuss numerical results whereas the conclusions are drawn in Section V.

Notation: the superscript $H$ denotes the Hermitian transposition, $E[\cdot]$ expresses the expectation operator, $I_M$ stands for the $M \times M$ identity matrix, and $0_M$ represents a $M \times 1$ vector whose elements are all zero.

II. SYSTEM MODEL AND BACKGROUND

Let us take into account a BS containing $N_b$ antennas, which sends radio frequency (RF) signals to $N_u = \sum_{j=1}^{K} N_j$ receive antennas, where $N_j$ denotes the number of receive antennas per $j$th user $U_j$, $j = 1, \ldots, K$, as outlined in Fig. 1. We can model the input-output relation of the broadcast channel (BC) as

$$y = \mathbf{H} \mathbf{s} + \mathbf{n},$$

where $y \in \mathbb{C}^{N_u}$ contains the signals received by all users and $\mathbf{H} \in \mathbb{C}^{N_u \times N_b}$ stands for the matrix which models the assumed broadcast channel that is assumed known to the BS. The entries of $\mathbf{H}$ are considered independent circularly-symmetrical
Fig. 1. Outline of a quantized massive MU-MIMO downlink system. Upper diagram: some simplified parts of BS. Lower diagram: Bussgang’s theorem applied to the detached part of interest.

correlated Gaussian random variables $[H]_{u,b} \in \mathbb{C}N(0,1)$, $u = 1, \ldots, N_u$ and $b = 1, \ldots, N_b$. The noise vector $n \in \mathbb{C}^{N_u}$, is characterized by its i.i.d. circularly-symmetric complex Gaussian entries $n_u \in \mathbb{C}N(0, \frac{1}{2})$. We consider that the noise level is known at BS and so is the sampling rate of DACs at BS and ADCs at user equipments. Bussgang’s theorem [7], [8] allows us to express quantized signals as linear functions of the quantized information and distortion expressions, which have no correlation with the signals undergoing quantization. Therefore, the operations performed by the two blocks encompassed by the braces in the upper part of Fig. 1 can be transformed in the expression composed of the operations outlined in the lower part. Thus, the quantization $Q(\cdot)$ of a precoded symbol vector $Ms$, where $M \in \mathbb{C}^{N_b \times N_u}$ is a precoding matrix and $s \approx N^C(0, N_u)$ is the symbol vector, can be expressed by the quantized vector

$$s_q = Q(Ms) = TMs + f,$$

where the distortion term $f$ and the symbol $s$ vectors are uncorrelated. For the general case, $T \in \mathbb{R}^{N_b \times N_b}$ is the diagonal matrix expressed by

$$T_{n,n} = \frac{\alpha \gamma}{\sqrt{\pi}} \text{diag}\left(MM^H\right)^{-1/2} \cdot \sum_{l=1}^{J-1} \exp\left(-\gamma^2 \left(l - \frac{J}{2}\right)^2 \text{diag}\left(MM^H\right)^{-1}\right)$$

where $n = 1 \ldots N_b$, and $J$ and $\gamma$ stand for the number of levels and the step size of the quantizer, respectively. The regularization factor $\alpha \in \mathbb{R}$, which will be defined in Subsection II-A, has the purpose of satisfying the average power limitation

$$\mathbb{E}[\|Ms\|_2^2] \leq P$$

where $P = SNR \frac{N_u}{2}$.

A. Achievable sum-rates

In order to compute approximations of achievable sum-rates, in which $N_b$ and $N_u$ are sufficiently large and that the error resulting from the combination of multiuser interference (MUI) and the distortion from limited resolution of DACs is considered a Gaussian process, we can assume that (3) is the following scalar matrix:

$$T_{n,n} = \delta I_{N_b \times N_b}$$

where the entries of $T_{n,n}$ are given by the Bussgang scalar factor:

$$\delta = \alpha \gamma \sqrt{\frac{N_b}{\pi P}} \sum_{l=1}^{J-1} \exp\left(-\frac{N_b \gamma^2}{P} \left(1 - \frac{J}{2}\right)^2\right)$$

in which the regularization factor $\alpha$ for enforcing the power constraint (4) is obtained by

$$\alpha = \left(2N_b \gamma^2 \left(1 - \frac{J}{2}\right)^2 \right)^{-1/2} - 2 \sum_{l=1}^{J-1} \left(1 - \frac{J}{2}\right) \Xi \left(\sqrt{2N_b \gamma^2} \left(1 - \frac{J}{2}\right)\right)$$

where $\Xi(w) = \int_{-\infty}^{w} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{v^2}{2}\right) dv$ is the distributed function of a Gaussian random variable.

It can be proven via Bussgang’s theorem that assuming the system model in Section II and the identity in (5), the sum rate provided by CQA-BD and CQA-RBD precoders can be approximated by

$$C = \log_2 \left\{ \text{det}\left[I_{N_u} + \delta^2 \frac{SNR}{N_u} (HM)(HM)^H \left(1 - \delta^2 \frac{SNR}{N_u} (HM)(HM)^H + I_{N_u}\right)^{-1}\right] \right\}$$

where the $SNR$ was defined in (4) and $M$ is the precoding matrix (11), which is defined in Subsection II-B.

It must be highlighted that all process of quantization is concentrated in Bussgang’s factor $\delta$ in (5) and (6). This is one of the contributions of this work, i.e., the derivation of a closed form expression for estimating achievable sum rates based on a scalar factor which characterizes a Bussgang’s gain scalar matrix (5) that approximates the effects of multi-bit quantization. This derivation is provided in the Appendix. The second contribution of this study, which have not been considered in the literature so far, is the application of the obtained closed form expression to evaluate the performance of the achievable sum rates of our proposed CQA-BD and CQA-RBD precoding algorithms under 2, 3 and 4-bit quantization.

B. Review of BD precoding algorithms

It is known [18], [16], [14] that BD is a low-rank technique [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34], [35], [36], [37], [38], [39], [40], [41], [42], [43], [44], [45], [46], [47], [48], [49], [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [60], [61], [62], [63], [64], [65], [66], [67], [68], [69], [70], [71], [72], [73] that employs SVD to design the precoder, which can be performed in two stages. The precoder computed in the first stage suppresses (BD) or attempts to obtain a trade-off between MUI and noise (RBD).
Afterwards parallel or near-parallel single user (SU)-MIMO are calculated. The precoder computed in the second stage parallelizes the streams intended for the users. In this way, a precoding matrix \( M_j \) related to the \( j \)th user can be expressed as a product
\[
M_j = M_j^c M_j^d
\]
in which \( M_j^c \in C^{N_b \times L_j} \) and \( M_j^d \in C^{L_j \times N_j} \). The constant \( L_j \) depends on which precoding algorithm is chosen (BD or RBD). We can express the combined channel matrix \( H \) and the resulting precoding matrix \( M \) as follows:
\[
H = [H_j^T H_{j+1}^T \cdots H_{K}^T]^T \in C^{N_j \times N_b} \tag{10}
\]
\[
M = [M_j^c M_j^d \cdots M_K^c]^T \in C^{N_j \times N_u} \tag{11}
\]
where \( H_j \in C^{N_j \times N_b} \) is the channel matrix of the \( j \)th user. The expression \( M_j \in C^{N_u \times N_j} \) represents the precoding matrix of the \( j \)th user. For BD precoding algorithm [16], the first factor in (9) is given by
\[
M_j^{(BD)} = \mathbf{W}_j^{(0)} \tag{12}
\]
where \( \mathbf{W}_j^{(0)} \) is obtained by the SVD [18] of \( \mathbf{H}_j \), in which the matrix channel of \( j \)th user was removed, i.e.:
\[
\mathbf{H}_j = [H_j^T H_{j+1}^T \cdots H_{K}^T]^T \in C^{N_j \times N_b} = \mathbf{U}_j \Phi_j \mathbf{W}_j^H = \mathbf{U}_j \Phi_j \left[ \begin{array}{c}
\mathbf{W}_j^{(1)} \\
\mathbf{W}_j^{(0)}
\end{array} \right]^H \tag{13}
\]
where \( \mathbf{U}_j = N_u - N_j \). The matrix \( \mathbf{W}_j^{(0)} \in C^{N_b \times (N_b - N_j)} \), where \( \mathbf{T}_j \) is the assumed rank of \( \mathbf{H}_j \), embraces the ultimate \( N_b - \mathbf{T}_j \) zero singular vectors. In the case of RBD precoding algorithm, the first factor in (9) is given [16] by
\[
M_j^{(RBD)} = \mathbf{W}_j \left( \Phi_j^H \Phi_j + \alpha \mathbf{I}_{N_b} \right)^{-1/2} \tag{14}
\]
where \( \alpha = \frac{N_u \sigma^2}{\tilde{\mathbf{r}}_j} \) is the regularization factor and \( \mu \) is the whole average transmit power.

The second factor in (9) is obtained by SVD of the effective channel matrix for the \( j \)th user \( \mathbf{H}_{e_j} \) and power loading, respectively as follows:
\[
M_j^{(BD)} = \mathbf{W}_j^{(1)} \Gamma^{(BD)} \tag{15}
\]
\[
M_j^{(RBD)} = \mathbf{W}_j \Gamma^{(RBD)} \tag{16}
\]
where the matrix \( \mathbf{W}_j^{(1)} \) embraces the early \( \mathbf{L}_e = \text{rank} (\mathbf{H}_{e_j}) \) singular vectors obtained by the decomposition of \( \mathbf{H}_{e_j} \), as follows
\[
\mathbf{H}_{e_j} = \mathbf{H}_j \mathbf{M}_j^c = \mathbf{U}_j \Phi_j \mathbf{W}_j^H
\]
\[
= \mathbf{U}_j \left[ \begin{array}{c}
\Phi_j \\
0
\end{array} \right] \left[ \begin{array}{c}
\mathbf{W}_j^{(1)} \\
\mathbf{W}_j^{(0)}
\end{array} \right]^H \tag{17}
\]
The power loading matrix \( \Gamma \) can be obtained by a procedure like water filling (WF) [20].

C. Increasing requirement of more efficient DACs

Until recently, the use of a modest number of antennas at the BS and their required DACs were not an issue in terms of energy consumption. This is due the fact that DACs consume less energy than ADCs. Despite the diversity of research about DACs, very few allow the calculation of the increment of chip power dissipation as bit resolution increases bit-by-bit, for a fixed technology. In order to roughly compare the consumption of both equipments, we make use of Table I which contains the fabrication parameters for GaAs 4-bit Analog-to-Digital Converter (AD) and 5-bit Digital-to-Analog (DA) converters, using a 0.7-μm MEFET self-aligned gate process [21] and the expression proposed in [22].

| ADC | DAC |
|-----|-----|
| Resolution | Sampling Rate | Power dissipation |
| (bits) | (GHz) | (mW) |
| ADC 4 | 1 | 140 |
| DAC 5 | 1 | 85 |

We start with the expression [22] which relates the power consumed by an ADC to the resolution bits, as follows:
\[
P_{\text{ADC}}(b) = c \tau 2^b \tag{18}
\]
where \( b \) stands for the resolution bits, \( c \) is a constant and \( \tau \) is the sampling rate. From [18], we obtain \( P_{\text{ADC}(4)} = \frac{P_{\text{ADC}(5)}}{2} \), which allows us to estimate \( \frac{P_{\text{ADC}(5)}}{P_{\text{ADC}(5)}} \approx 30\%P_{\text{ADC}(5)} \). With the help of Table I, we obtain \( P_{\text{DAC}(5)} \approx 30\%P_{\text{ADC}(5)} \). So, the DAC consumes around 30 % of the energy of the ADC with fixed parameter. From the results obtained before, we can roughly estimate the economy in energy by assuming that similarly to ADC, DAC consumption doubles with every extra bit of resolution, i.e., of \( O (2^b) \). Therefore, a decrease in two resolution bits, for instance from 4 to 2 bits, represents a consumption 75% lower. This reduction of DAC consumption motivates our study.

III. PROPOSED CQA-BD AND CQA-RBD PRECODER ALGORITHM

In this section, we summarize the proposed precoding techniques in Algorithm I which encompasses both CQA-BD and its refined variation CQA-RBD. The algorithm starts with the use of the knowledge of the combined channel matrix (10). Then, for a fixed SNR and a fixed realization of the channel, we perform the calculations from 1 to 12 step-by-step, to obtain the precoding matrix \( M \) (11). All operations involved in these steps are detailed in Subsection IIB which reviews the BD-type precoding algorithms. Next, we calculate the conformation parameter \( \alpha \) (7), which ensures the power constraint (4), and after this, the Bussgang’s scalar factor (5), which concentrates all process of quantization in the scalar matrix (5). Finally, we can obtain the achievable sum-rates for a fixed SNR and a fixed realization of the channel.
Algorithm 1 Algorithm for estimating CQA-BD and CQA-RBD reachable sums

Require: $\mathbf{H} = [\mathbf{H}_1^T \mathbf{H}_2^T \cdots \mathbf{H}_K^T]^T \in \mathbb{C}^{N_u \times N_b}$

1: for $j = 1 : K$ do
2: $\mathbf{H}_j = [\mathbf{H}_1^T : \cdots : \mathbf{H}_{j-1}^T : \mathbf{H}_j^T : \cdots : \mathbf{H}_K^T]^T \in \mathbb{C}^{N_y \times N_b}$
3: $\mathbf{M}_j^{(BD)} = \mathbf{W}_j^{H} = \mathbf{U}_j \mathbf{S}_j \mathbf{W}_j^{H}$
4: $\mathbf{M}_j^{(RBD)} = \mathbf{W}_j \left( \mathbf{F}_j + \alpha \mathbf{I}_{N_b} \right)^{-1/2}$
5: $\mathbf{H}_j = \mathbf{H}_j \mathbf{M}_j^{H}$
6: $\mathbf{M}_j = \mathbf{M}_j \mathbf{M}_j^{T}$
7: $\mathbf{M}_j^{(RBD)} = \mathbf{W}_j \mathbf{r}^{(RBD)}$
8: $\mathbf{M}_j^{(BD)} = \mathbf{W}_j \mathbf{r}^{(BD)}$
9: $\mathbf{M}_j = \mathbf{M}_j \mathbf{M}_j^{T}$
10: end for
11: $\mathbf{M} = \mathbf{M}_1 \mathbf{M}_2^T \cdots \mathbf{M}_K^T \in \mathbb{C}^{N_u \times N_b}$
12: $\alpha = \left( \frac{2}{2N_b} \right)^{2 \left( \frac{1}{2} \right) - 1/2}$
13: $-2 \sum_{j=1}^{J} \left( 1 - \frac{j}{2} \right)^{2} \left( \frac{2N_b}{2} \right)^{\alpha} \left( \frac{1}{2} \right)$
14: $\delta = \alpha \sqrt{\frac{N_b}{2}} \sum_{j=1}^{J} \exp \left( - \frac{N_b}{2} \left( 1 - \frac{j}{2} \right)^{2} \right)$
15: $C = \log_2 \left\{ \det \left[ \mathbf{I}_{N_u} + \frac{\delta}{\sqrt{\text{SNR}/N_b}} \mathbf{H} \mathbf{M} \mathbf{H}^{H} \right] \right\}$

IV. NUMERICAL RESULTS

We focus our simulations on two scenarios, which are composed of $(N_{b1}, N_{a1}) = (32, 16)$ and $(N_{b2}, N_{a2}) = (128, 16)$, respectively. We model the channel matrix $\mathbf{H}_j$ of the $j$th user with entries given by complex Gaussian random variables with zero mean and unit variance. In addition, it is assumed that the channel is static while each packet is transmitted and that the antennas are uncorrelated. The channel estimation is considered ideal at the receive side and there is no error in the feedback of the channel information to the receiver. We set the trials to $2 \times 10^5$ and the packet length to $10^2$ symbols.

Fig. 2 depicts the sum-rates of the proposed CQA-BD and CQA-RBD algorithms, based on Bussgang’s theorem, under 2 and 3-bit quantization, corresponding to $J = \{8, 4\}$ levels of quantization, and employing the first scenario $(N_{b1}, N_{a1}) = (32, 16)$. For the purpose of comparison, we have also included the sum-rate of CQA-ZF, which represents the same technique used in CQA-BD type, applied to Zero-Forcing (ZF). They are compared to the RBD (refined variant of BD), the standard BD, and ZF, at upper side, all of them in full resolution (RBD FR, BD FR, ZF FR). They are also compared to RBD under 2 and 3-bit roughly standard quantization, i.e., not using Bussgang’s theorem. It is possible to notice four well-defined ranked groups of curves in the range $[7, 20]$ and that, in the same range, the rank of each group is also clear. Thus, the inequalities $\text{Sum\ RBD} > \text{Sum\ BD} > \text{Sum\ ZF}$ is preserved, regardless of quantization. It is also clear the influence of the levels of quantization on the the sum-rates. We highlight the clear gaps between each group of sum-rates achieved corresponding to each level of quantization. It can be noticed the significant increasing gaps from 2-3bit roughly quantized (RBDqr), at the bottom, to 2-bits and CQA-RBD, and how close 3-bit CQA-RBD is to full resolution (FR) RBD. Based on Subsection IV.C the performance of CQA-RBD and CQA-BD under 2-3 bits quantization, which increasingly approximates RBD FR and BD FR, indicating a corresponding saving in energy.

Fig. 3 illustrates the performance of the sum-rates in the second configuration mentioned before, i.e., $(N_{b2}, N_{a2}) = (128, 16)$. In this arrangement, we compare only 2,3,4-bit CQA-BD class to 2,3-bit RBDqr (roughly quantized) and RBD FR,BD FR. It can be noticed that the curves depicting CQA-RBD and CQA-BD at each level of quantization are almost similar, which can be justified by the increase of transmit antennas. However the aim of the figure is to show more clearly how 2,3,4-bit CQA-RBD and CQA-BD algorithms increase sum-rates, comparing them to a bad condition, i.e., 2,3-bit RBDqr and to a ideal condition RBD-FR. Taking RBD FR as a reference at 3.57 and 7.14 dB, the sum-rate achieved by 2-bit CQA-RBD, represents 80% and 74%, respectively, of the rates achieved with a full-resolution system. For 3-bit CQA-RBD, at the same range, the sum-rate achieves 93% and 90%, respectively, of that achieved by full resolution. This means substantially less energy dissipated at the cost of slightly lower sum-rates, which justify to the investigation of low-resolution precoding techniques using 2,3 bits as an alternative of 1-bit quantization-based precoders. Future work will include the development of detection and decoding techniques [74], [75], [76], [77], [78], [79], [80], [81], [82], [83], [84], [85], [86], [87], [88], [89], [90], [91], [92], [93], [94], [95], [96], [97], [98], [99], [100].

V. CONCLUSIONS

Founded on Bussgang’s theorem, which allows us to express quantized signal as linear functions of the quantized information and distortion, we have proposed the CQA-BD and CQA-RBD precoding algorithms for multi-bit DACs, in particular,
We can estimate the correlation matrix vector (2), as follows:

\[
B. \text{ Development}
\]

where we made use of (5) and the autocorrelation matrix of the symbol vector \( R_{ss} = E[ss^H] = \sigma^2_s I_{Nu} \), in which \( \sigma_s^2 \) is its variance. The term \( R_{ff} = E[ff^H] \) stands for the autocorrelation of the distortion vector \( f \).

Next, we can notice that in full resolution, since there is no quantization and its associated distortion, (2) turns into

\[
\text{where we make } T_{n,n} = I_{Nb \times Nb}, \text{ i.e., } \delta = 1 \text{ in (5)}, \text{ and assume that } f = 0_{Nb}.
\]

Now, we calculate the autocorrelation of the full resolution precoded symbol vector (22) as follows:

\[
R_{sqsq} = E[(Ms)(Ms)^H] = E[MM^H], \quad (23)
\]

By equating (21) and (23), we can obtain the expression of the autocorrelation of the distortion vector \( R_{ff} \):

\[
\delta^2 \sigma_s^2 MM^H + R_{ff} = \sigma_s^2 MM^H
\]

\[
\therefore R_{ff} = (1 - \delta^2) \sigma_s^2 MM^H \quad (24)
\]

By equating (21) and (23), we can obtain the expression of the autocorrelation of the distortion vector \( R_{ff} \):

\[
\delta^2 \sigma_s^2 MM^H + R_{ff} = \sigma_s^2 MM^H
\]

\[
\therefore R_{ff} = (1 - \delta^2) \sigma_s^2 MM^H \quad (25)
\]

We can then compute the autocorrelation matrix of (19), obtaining:

\[
R_{yy} = E[yy^H]
\]

\[
= (HTM) R_{ss} (HTM)^H + H R_{ff} H^H + R_{nn}, \quad (26)
\]

where \( R_{ss} = E[ss^H], R_{ff} = E[ff^H] \) and \( R_{nn} = E[nn^H] \) are the autocorrelation matrices of the signal, the distortion and the noise vectors, respectively. Similar procedure applied to the distortion-plus-noise vector (20), considering the conditions above, yields:

\[
R_{\tilde{n}\tilde{n}} = E[\tilde{n}\tilde{n}^H] = HR_{ff} H^H + R_{nn}, \quad (27)
\]

From the principles of information theory (23) and the capacity of a frequency flat deterministic MIMO channel (20), we can bound the achievable rate in bits per channel use at which information can be sent with arbitrarily low probability of error by the mutual information of a Gaussian channel, i.e.

\[
C \leq I(s, y) = \Upsilon(y) - \Upsilon(y|s)
\]

\[
= \Upsilon(y) - \Upsilon(\tilde{n})
\]

\[
= \log_2 \det(\pi e R_{yy}) - \log_2 \det(\pi e R_{\tilde{n}\tilde{n}})
\]

\[
= \log_2 \det(R_{yy}) - \log_2 \det(R_{\tilde{n}\tilde{n}})
\]

\[
= \log_2 \det(R_{yy} R_{\tilde{n}\tilde{n}}^{-1}) \quad (28)
\]

where \( \Upsilon(y) \) and \( \Upsilon(y|s) \) are the differential and the conditional differential entropies of \( y \), respectively.
By combining (26) and (27) with (28), we have:

\[ C \leq \log_2 \left\{ \det \left[ \left( \mathbf{HTM} \mathbf{R}_{ss} \mathbf{HTM}^H \right) \right. \right. \\
\left. \left. \left( \mathbf{HR}_{JJ} \mathbf{H}^H + \mathbf{R}_{nn} \right)^{-1} + \mathbf{I}_{Nu} \right] \right\} \]  

(29)

In Section II (8), we have defined the total power as \( P = SNR \frac{N_u}{N} \). From the definition of the noise vector, also in that Section, we can express its covariance matrix as \( \mathbf{R}_{nn} = \frac{N_u}{N} \mathbf{I}_{Nu} \). Combining the two previously mentioned expressions, we obtain

\[ \mathbf{R}_{nn} = \frac{P}{SNR} \mathbf{I}_{Nu} \]  

(30)

Recalling, from Section II, that \( \mathbf{R}_{ss} \approx \mathbf{I}_{Nu} \), and assuming that the total power is given by \( P = \text{trace}(\mathbf{R}_{ss}) = N_u \), (30) turns into:

\[ \mathbf{R}_{nn} = \frac{N_u}{SNR} \mathbf{I}_{Nu} \]  

(31)

By combining (29), (25) (5) and the expression of \( \mathbf{R}_{ss} \) previously mentioned with (31), followed by algebraic manipulation, we can obtain:

\[ C = \log_2 \left\{ \det \left[ \mathbf{I}_{Nu} + \delta^2 \frac{SNR}{N_u} (\mathbf{HM})^H (\mathbf{HM})^H \right. \right. \\
\left. \left. \left( 1 - \delta^2 \right) \frac{SNR}{N_u} (\mathbf{HM})^H (\mathbf{HM}) + \mathbf{I}_{Nu} \right]^{-1} \right\} \]  

(32)

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