Effect of Investment and Disinvestment Delay in Biofuels

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Abstract: This research studies the strategic level decision-making in the biofuel industry by considering the effect of investment and disinvestment delay. Under uncertain environments, the delay causes additional risk exposure to decision makers, and the optimal decision reflects the effect of delay. This research provides the concise and comprehensive profile of the optimal investment and disinvestment policy by taking investment and disinvestment delay into consideration. The results show that the trend of fuel price plays a crucial role to determine the effect of delay. If the trend is favorable to the alternative biofuel plant, longer exercise delays provide the decision makers an incentive to act more quickly, and shorter delays provide an opposite incentive. By applying the suggested model to the bioenergy investments cases in the state of Georgia, the United States, we provide managerial insights for the strategic decision makers in the biofuel industry.

Keywords: Real options; Investment and disinvestment; Delay; Time-to-build
1 Introduction

The complicated circumstance of the biofuel industry requests the decision makers to understand investment and disinvestment opportunities comprehensively. This research focused on investment and disinvestment delay among the important but overlooked factors. An investment in biofuel industry often requires considerable time to be effective. The change in preference and technology may impose disinvestment on a certain type of biofuel. Investment and disinvestment delay cause additional risk to decision makers when the decision is irreversible. This research investigated the effect of the time lag between the time a decision is made and the time that decision takes effect, on the optimal investment decisions.

There is little debate that investments in biofuels are important. Recently, several studies show that disinvestment is also important in this industry. During the 1990s and the early 2000s, ethanol had the limelight as a possible renewable energy source. Moreover, state and federal governments provided various subsidies to stimulate investments in ethanol production [10]. The boomed demand and governments’ subsidies prompted investors to build ethanol production facilities those are remote from the major raw material sources such as the Midwestern Corn Belt. However, the change of environments around first generation biofuels (including grain-based ethanol) urges investors to consider disinvestment. In the late 2000s, environmentalists doubt the value of grain-based ethanol as a source of sustainable energy [10], and economists raise questions about its economic values [6]. Moreover, the second generation biofuels take center stage, and governments’ support moves to them as well [3]. This change asks biofuel industry to disinvest from the first generation biofuels and to invest in the alternative types of biofuels.

The investment and disinvestment take considerable time to be effective. Although investment lag and disinvestment delay are important factors to be considered, they have not gathered enough attention from the biofuel industry practitioners and researchers. There are many causes of exercise delay in the biofuel industry. For instance, the remote ethanol facilities necessitate constructing a railroad system connecting the raw material sources and ethanol markets, and it obviously takes considerable time. Acquiring governments’ support frequently involves time-consuming procedures. For disinvestment, governments’ policy may impose disinvestment delay. With the renewable fuel standard, the United States governments obligate transportation fuel sold in the United States to contain a minimum volume of renewable fuel [11]. Contracts between renewable fuel suppliers and consumers often include “grace period” that plays a role in a disinvestment delay [12]. Moreover, the energy industry is an industry requiring the longest time-to-build [2].

Real options approach is an appropriate framework to analyze the investment and disinvestment opportunities in biofuels. McCarty and Sesmero [7] analyzes the value of investment options in second-generation biofuels. The research shows the effect of price risk and irreversibility on the value of investment in a corn stover-based cellulosic biofuel plant. Gonzalez et al. [5] provides an interesting case study about bioenergy investment and disinvestment in the state of Georgia.

A number of studies extend real options approach incorporates exercise delays. Bar-Ilan and Strange [1] is a seminal research article in this area. They consider the case when the recourse decision can be abandoned during the delay, and find that the option of abandonment can hasten the option exercise time. Øksendal [8] delivers the general result for the real option problem with exercise
delay.

The primary contribution of this paper is analyzing the effect of investment and disinvestment delay and extracting managerial insights for investment and disinvestment decisions on the biofuel industry. Our model represents various types of investment or disinvestment according to the parameters’ values. Therefore, it enhances the comparability of the analysis and will be provided in Section 2. Our analysis is based on the comprehensive closed form solutions of the optimal decisions that are derived by leveraging the previous research results. Therefore, our research differs from the research based on numerical analysis and is helpful to find the insights about the effect of exercise delay on optimal decisions. For the completeness of this research, Section 3 includes the comprehensive profile of the optimal strategies, and its derivation is provided in Appendix A. We analyze the effect of exercise delay in Section 4. Our analysis starts with an assumption that the investment cost or disinvestment revenue is fixed and extended to the case that the cost or revenue is a function of exercise delay. To show the practical value of our research, Section 5 provides the application of our results to find the optimal investment or disinvestment decisions of biofuel production. We conclude in Section 5.

2 Model

The United States Energy Information Administration defines biofuels as “Transportation fuels like ethanol and diesel that are made from biomass materials.” This suggests that there is a close relationship between the price of fossil transportation fuels and the profit of biofuel plant. The market price of fossil fuel is often modeled as a geometric Brownian motion, and the profit rate of a biofuel plant is described as a linear function of the fossil fuel price [7]. Moreover, previous research supports that the market price of biofuel follows a geometric Brownian motion [5, 13]. Based on the previous studies, we assume that the fuel price at time $t$ follows a geometric Brownian motion, and the profit rate of biofuel plants is a linear function with respect to the fuel price. Let $X(t)$ denote the fuel price at time $t$. Then the dynamics of the fuel price is described by the following stochastic differential equation:

$$dX(t) = \mu X(t)dt + \sigma X(t)dB(t), \quad X(0) = x > 0, \quad t \geq 0,$$

where $\mu \in \mathbb{R}$ and $\sigma \in [0, \infty)$ are the drift and volatility parameters, and $B(t)$ is the one-dimensional standard Brownian motion. The current and alternative biofuel plants yield discounted profit rate of $e^{-\rho t}(a_1 X(t) - b_1)$ and $e^{-\rho t}(a_2 X(t) - b_2)$, respectively. The parameters, $a_1, a_2, b_1, b_2$, and $\rho$, are given constants. The discount factor $\rho$ is assumed to be positive and $\rho > \mu$ to avoid obvious solutions. The parameter $a_i$ stands for the production levels of plant $i$, and $b_i$ represents production cost of plant $i$, for $i = 1, 2$.

This setting can describe various decisions. For example, this model includes entry into the biofuel industry with $a_1 = b_1 = 0$, exit from it with $a_2 = b_2 = 0$, capacity expansion with $a_1 < a_2$, and downsizing with $a_1 > a_2$. For the sake of convenience, we will refer to the case of $a_1 > a_2$ as a disinvestment, and that of $a_1 < a_2$ as an investment. To avoid trivial solution, we assume that $a_1 \neq a_2$. Moreover, we employ the notation $\theta = \{\mu, \sigma, \rho, a_1, a_2, b_1, b_2\}$ to represent the full set of parameters.

When the decision maker decides to change the plant at time $\tau$, there is a delay $\delta$ until the alternative plant takes effect at time $\tau + \delta$. During the delay, the biofuel plant is assumed to continue
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the initial yield mode. Moreover, we assume that there exists only one opportunity to change the system. According to [8], the problem with multiple opportunities can be solved by the iterative application of the method that this research employs. Therefore, this assumption does not undermine the value of this research but assists to elucidate the effect of exercise delay.

Making the decision to change to the alternative system incurs the cost \( c(\delta) < 0 \) or revenue \( c(\delta) > 0 \); this study models the one-time cash flow as occurring at the time when the transformation is completed, \( \tau + \delta \). Investment typically incurs a cost, whereas disinvestment may produce revenue through the sale of equipment. Moreover, we assume the investment cost or salvage value depends on the length of exercise delay. For example, the time-to-build a new biofuel plant can be shortened by paying expedite-costs. Suppose that a supplier continues to produce its product due to a contract obligation after they decide to stop producing. The supplier may charge additional fees to lengthen the additional production period. We reflect these possibilities by imposing the exercise cost or revenue as a function of exercise delay.

At this point, we can formulate the performance function, \( j(x; \tau; \delta) \), and the value function, \( v(x; \delta) \):

\[
j(x; \tau; \delta) = \mathbb{E} \left[ \int_0^{\tau+\delta} e^{-\rho t} \{a_1 X(t) + b_1\} \, dt + e^{-\rho(\tau+\delta)} \tilde{c}(\delta) + \int_{\tau+\delta}^{\infty} e^{-\rho t} \{a_2 X(t) + b_2\} \, dt \right], \tag{2}
\]

\[
v(x; \delta) = \sup_{\tau \in [0, \infty)} j(x; \tau; \delta). \tag{3}
\]

We will exclude a couple of obvious cases from the further analysis. Suppose that \( a_1 < a_2 \) and \( \tilde{c}(\delta) \geq \frac{b_2-b_1}{\rho} \). Then, the company earns more revenue with lower operation cost, and the operation cost saving exceeds the investment cost. Thus, regardless of the level of \( X(t) \), it is always optimal to invest immediately. Likewise, in the case that \( a_1 > a_2 \) and \( \tilde{c}(\delta) \geq \frac{b_1-b_2}{\rho} \), disinvestment is never optimal.

### 3 Optimal Decisions and Effect of Exercise Delay

Notice that the problem described with (2) and (3) has the optimal threshold policy where the continuation region is either below the threshold when the alternative system is an upgrade or above the threshold when the alternative system is a downgrade [4]. However, according to the parameter values, the decision to exercise the investment or disinvestment option could be made immediately, when the state variable crosses an optimal threshold. We present the combinations of parameters that determine the boundaries of the optimal decisions in the following proposition.

**Proposition 1 (Optimal Investment or Disinvestment Decisions)** Define the threshold values

\[
x_i^*(\delta) = \frac{r_i(\mu - \rho)}{(a_1 - a_2)(r_1 - 1)} \left[ \frac{b_1 - b_2}{\rho} - \tilde{c}(\delta) \right] e^{-\mu \rho}, \quad \text{for } i = 1, 2, \tag{4}
\]

where

\[
r_1 = \frac{(\sigma^2 - 2\mu + \sqrt{(2\mu-\sigma^2)^2 + 8\rho \sigma^2})}{2\sigma^2} \quad \text{and} \quad r_2 = \frac{(\sigma^2 - 2\mu - \sqrt{(2\mu-\sigma^2)^2 + 8\rho \sigma^2})}{2\sigma^2}.
\]

The investment or disinvestment problem in (2) and (3) has the following closed form solutions.
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1. If \( a_1 < a_2, \ c(\delta) < \frac{b_1 - b_2}{\rho} \) and \( x^*_1(\delta) \leq x \), it is optimal to invest immediately. In this case the value of the investment option is

\[
v(x, \delta) = \frac{(a_1 - a_2)e^{(\mu - \rho)\delta} - a_1}{\mu - \rho} x - \frac{b_1 - b_2}{\rho} e^{-\rho\delta} + \frac{b_1}{\rho} + e^{-\rho\delta} c(\delta). \tag{5}\]

2. If \( a_1 < a_2, \ c(\delta) < \frac{b_1 - b_2}{\rho} \) and \( x^*_1(\delta) \geq x \), it is optimal to postpone the investment until \( \tau^*_1 = \inf \{ t \geq 0 : X(t) \geq x^*_1 \} \). In this case the value of the investment option is

\[
v(x, \delta) = \frac{e^{-\rho\delta}}{r_1 - 1} \left[ \frac{b_1 - b_2}{\rho} - c(\delta) \right] \left( \frac{x}{x^*_1(\delta)} \right)^{r_1} - \frac{a_1 - b_1}{\mu - \rho} x + \frac{b_1}{\rho}. \tag{6}\]

3. If \( a_1 > a_2, \ c(\delta) > \frac{b_1 - b_2}{\rho} \) and \( x^*_2(\delta) \geq x \), it is optimal to disinvest immediately. In this case the value of the investment option is identical to (5).

4. If \( a_1 > a_2, \ c(\delta) > \frac{b_1 - b_2}{\rho} \) and \( x \geq x^*_2(\delta) \), it is optimal to postpone the disinvestment until \( \tau^*_2 = \inf \{ t \geq 0 : X(t) \leq x^*_2 \} \). In this case the value of the investment option is

\[
v(x, \delta) = \frac{e^{-\rho\delta}}{r_2 - 1} \left[ \frac{b_1 - b_2}{\rho} - c(\delta) \right] \left( \frac{x}{x^*_2(\delta)} \right)^{r_2} - \frac{a_1 - b_1}{\mu - \rho} x + \frac{b_1}{\rho}. \tag{7}\]

**Proof:** See Appendix A

In analyzing the effect of delay on the optimal decisions, the key insight is that lengthening the delay increases the risk exposure a decision maker faces. It turns out this increased exposure has varying effects on the expected investment or disinvestment time depending both on the trend of the stochastic process and the effect of the delay itself.

To concentrate on the effect of additional risk exposure, let us begin with a constant exercise cost, \( c(\delta) = c \). By taking derivative of the threshold value \( x^*_1 \) with respect to the delay, \( \frac{d}{d\delta} x^*_1(\delta) = -\mu x^*_1(\delta) \). When the stochastic process has positive drift, \( \mu > 0 \), the future state is likely to be favorable to an investment. This expectation hastens investment given a realization of underlying uncertainty, since \( \frac{d}{d\delta} x^*_1(\delta) < 0 \), and may eventually make an immediate investment desirable. On the other hand, the positive trend in the state variable means longer delays make disinvestment increasingly dour. A longer exercise delay lowers the threshold value for disinvestment, because \( \frac{d}{d\delta} x^*_2(\delta) < 0 \), and may eventually bring the disinvestment option into the never desirable case. It implies that the longer delay defers disinvestment. In the case that \( \mu < 0 \), the exercise delay has opposite effects on the optimal policies, since \( \frac{d}{d\delta} x^*_1(\delta) > 0 \) and \( \frac{d}{d\delta} x^*_2(\delta) > 0 \).

In the interesting solution cases where the option is not currently desirable but will be exercised if the fuel price crosses some threshold, we provide the following interpretation. If the trend of the fuel price is favorable to the alternative system, the decision maker is essentially trading off the optimality of the current plant configuration in the current state against the future optimality of the alternative plant configuration in the uncertain future states. With short delays, the operator can delay execution when the trend of the fuel price is favorable to the alternate system to squeeze ever last drop out of the current plant’s optimality. With longer delays, the decision maker’s hand is forced to more quickly act upon the expected optimality of the alternative biofuel plant in future states.
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Table 1: The Effect of Exercise Delay upon Optimal Decisions.

|                | Investment \((a_1 < a_2)\) | Disinvestment \((a_1 > a_2)\) |
|----------------|-----------------------------|------------------------------|
|                | Longer delay postpones      | Longer delay hastens         |
|                | the expected exercise time.  | the expected exercise time.   |

Table 2: The Estimated Parameters for Gross Margin.

|                      | Thriving market | Recession |
|----------------------|-----------------|-----------|
| Annual growth rate \((\mu)\) | 7.9%           | -5.6%     |
| Annual rate of volatility \((\sigma)\) | 29.3%           | 30.9%     |

Even when the trend of the fuel price is unfavorable to the alternative biofuel plant, the investment or disinvestment option may be exercised. It is the case that the observed fuel price is so favorable to the alternative biofuel plant that it can harvest enough benefits from the temporary advantageous status. In this case, a longer delay requires more advantageous status to exercise the options, and it results in postponements of execution time. In the viewpoint of risk exposure, a longer delay increases the risk that the realized advantageous status becomes unfavorable against the alternative biofuel plant. Therefore, the decision maker exercises the options with a more favorable realization of underlying uncertainty to compensate the increased risk exposure due to the delay.

The above analysis can be extended to the case that the exercise cost is a function of delay to consider both risk exposure effect and cost effect of delay. Suppose that the exercise cost is once differentiable with respect to delay \((\delta)\). The first derivative of the threshold value with respect to exercise delay is expressed as:

\[
\frac{d}{d\delta} x_i^*(\delta) = - \left\{ \frac{r_i(\mu - \rho)}{(a_1 - a_2)(r_i - 1)} \right\} e^{-\mu\delta} \left\{ \mu \frac{b_2 - b_1}{\rho} - \mu \tilde{c}(\delta) + \frac{d\tilde{c}(\delta)}{d\delta} \right\}.
\]

Therefore, if \(\frac{d\tilde{c}(\delta)}{d\delta} < \mu \left\{ \tilde{c}(\delta) - \frac{b_2 - b_1}{\rho} \right\}\) then \(x_i^*(\delta)\) is increasing, otherwise \(x_i^*(\delta)\) is decreasing with respect to \(\delta\). The integrated effect of exercise delay is summarized in Table 1.

4 Application

Gonzalez et al. [5] studies bioenergy investments cases in the state of Georgia, the United States. By analyzing the data on the price of ethanol and corn, they find that the gross margin of producing ethanol follows a geometric Brownian motion. They split the sample into two periods to estimate the parameters for thriving market and recession. The annual growth rate of the gross margin was positive when the ethanol market was prosperous. However, the growth rate turns to be negative in the recession period. Table 2 provides the estimated parameters.

The annual discount rate is 15\%, i.e., \(\rho = 0.15\). We postulate entry, expansion and exit options based on the capacity and cost structure provided by the case study. The entry option is an investment opportunity to build a new ethanol plant that produces 100 million gallons per year. The operation cost of the plant is 75 cents/gallon and the building cost is estimated at 171 million dollars. Another investment opportunity, the expansion option, assumes that an ethanol plant is already built with the
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Table 3: Capacities and Costs of the Investment or Disinvestment Options.

|                      | Entry | Expansion | Exit |
|----------------------|-------|-----------|------|
| Initial Capacity     | 0     | 50        | 100  |
| (million gallon)     |       |           |      |
| Initial Operation    | 0     | 77        | 75   |
| Cost (cents/gallon)  |       |           |      |
| Alternative Capacity | 100   | 100       | 0    |
| (million gallon)     |       |           |      |
| Alternative Operation Cost | 75 | 75        | 0    |
| (cents/gallon)       |       |           |      |
| Exercise Cost        | 171   | 78.5      | 17.1 |
| (million dollars)    |       |           |      |

Figure 1: Optimal Threshold Values in the Thriving Market.

annual capacity of 50 million gallons, and it is able to be expanded for the capacity of 100 million gallons. Suppose that a plant, which yields 100 million gallons of ethanol per year, is operating. The exit option describes the disinvestment decision to terminate the operation of the plant. Table 3 summarizes the parameters of the investment or disinvestment options. For the simplicity, we assume that the exercise cost is fixed and not affected by the delay.

Figure 1 illustrates the optimal threshold values that trigger the exercise of the options under the thriving market conditions whose annual growth rate is positive. The horizontal axis stands for the length of exercise delay, and the vertical axis represents the unit gross margin of ethanol. The black solid line, the red dotted line, and the blue dashed line is the threshold values of exit, expansion and exit options, respectively.

The investment options (entry and expansion options) are exercised when the unit gross margin is above the threshold value. Therefore, we refer the region above the threshold values of investment options to investment region. Because the disinvestment option is exercised when the gross margin
drops down below its threshold value, the bottom area stands for the disinvestment region. The continuation region is the area that no option is exercised.

As shown in Section 3, the threshold values decline as the exercise delay increases, when the market is prosperous. This means that the longer delay hastens the execution of investment option and postpones that of disinvestment option. Figure 1 shows this fact evidently. Suppose that the current unit gross margin of ethanol is $1. If it takes one year to build a new plant, it is optimal to start to build the new plant with 100 million gallons capacity at the time the unit gross margin becomes $1.33. When the time-to-build is three years, $1.13 is the threshold value. It takes the shorter time for the stochastic gross margin to reach the threshold value of 3 years’ time-to-build than that of 1 years’ time-to-build. Even more, if the time to build is five years, immediate start to build the plant is optimal, since the trigger point, $0.97, is below the current gross margin. A similar phenomenon is observed in the expansion option as well. Therefore, exercise delay hastens the execution of investment options in the thriving market.

Even if the market is expected to be flourishing, disinvestment could be an appropriate decision when the current state is impoverished. In Figure 1, disinvestment region illustrates this cases. When the realized gross margin is lower than the threshold value of exit option, stopping to operate the ethanol factory is optimal. However, the effect of exercise delay to disinvestment options is opposite to investment options. For the one year, three years and five years exercise delay; it is optimal to exercise the exit option when the gross margin becomes $0.22, $0.19 and $0.16, respectively. Given a high enough current gross margin, which ensures that immediate disinvestment is not optimal, reaching lower boundary take longer time. Therefore, a longer delay postpones the execution of a disinvestment option.

In a recession, the threshold value increases as the exercise delay lengthen, due to the negative annual growth rate. Figure 2 illustrates the threshold values for the suggested options under recession and confirms the relationship between threshold values and exercise delay. Under recession, decision makers have an incentive to disinvest. We can see this incentive through the larger area of disinvestment region in Figure 2, comparing to Figure 1’s disinvestment region. Turning to the effect of exercise delay, if a firm can exit the market as soon as the decision is made, they can stay in the market longer than the case that disinvestment has exercise delay. The exit option’s threshold value elucidates this relationship. When there is no exercise delay, i.e. $\delta = 0$, the threshold value is $0.50$. On the other hand, when the exercise delay is 5 years, the threshold value is $0.66$. Suppose that the current gross margin is $1$. Then it takes longer time for the margin to drop down to $0.50$ than to $0.66$. Therefore, exercise delay hastens disinvestment under recession.

Despite the negative prospect of the future market condition, investment can occur with high gross margin. The negative expectation about future market state shrinks the investment region in Figure 2 from that in Figure 1. Moreover, the higher threshold value of an investment with longer delay implies that delay postpones investment, according to the similar line of logic we mentioned above.

### 5 Conclusion

We contribute to the strategic level decision making in the biofuel industry by considering the effect of investment and disinvestment delay. Since investment and disinvestment delays are in the biofuel
industry, appropriate consideration of exercise delay improves the performance of the strategic decisions. Our concise and comprehensive profile of the optimal investment and disinvestment policy is helpful to enhance the decision makers’ understanding.

Under uncertain environments, investment and disinvestment delay causes additional risk exposure to decision makers, and the optimal decision reflects the effect of delay. The provided model describes various investment and disinvestment problems with delays, where the fuel price process follows a geometric Brownian motion. We provided a comprehensive profile of optimal investment and disinvestment decisions according to the values of parameters as the basis of analysis.

Assuming the investment cost or the disinvestment revenue is fixed, we found that the trend of fuel price plays a crucial role to determine the effect of delay. If the trend is favorable to the alternative biofuel plant, longer exercise delays provide the decision makers an incentive to act more quickly. On the other hand, with shorter delays, the decision makers are able to wait for stronger evidence that alternative plant feature is optimal so that the option is exercised later than the case with longer delays. We extended our analysis to the case that the investment cost or disinvestment revenue is a function of exercise delays.
A Proof of Proposition 1

Proof: For notational simplicity, let $a = a_1 - a_2$ and $b = b_2 - b_1$. Applying Theorem 2.1 of [8], we can rewrite (2) and (3) as following:

$$\tilde{v}(x, \delta) = \sup_{\tau \geq 0} \mathbb{E}^x \left[ \int_0^\tau e^{-\rho t} \{ aX(t) + b \} \, dt + e^{-\rho \tau} \{ F_1 X(\tau) + F_2 \} \right],$$

where $F_1 = \frac{a(\mu - \rho)\delta - 1}{\mu - \rho}$ and $F_2 = \frac{b(1 - e^{-\rho \delta})}{\rho} + e^{-\rho \delta} \tilde{c}(\delta)$.

Utilizing Propositions 3.3 and 3.4 of [9], the set of continuation region is $U := \{ x < \frac{e^{-\mu t}(\rho \delta)(-b)}{a} \}$. If $a_1 < a_2$ and $\tilde{c}(\delta) \geq b/\rho$, then $U = \emptyset$. Therefore, it is optimal to exercise the option immediately regardless of $x$. In the case that $a_1 > a_2$ and $\tilde{c}(\delta) \leq b/\rho$, $x < \frac{e^{-\mu t}(\rho \delta)(-b)}{a}$ satisfies always. Therefore, the option would not be exercised, since the continuation region contains the entire range of $x$.

Based on [8], let $\varphi(s, x) = e^{-\rho s} \psi(x)$ be the solution of (9) using the time shift factor, $s$. In the case that $a_1 < a_2$ and $\tilde{c}(\delta) < b/\rho$, the general solution in the continuation region is known as $\psi(x) = \Lambda_1 x^{r_1} - ax/\mu + b/\rho$, where $\Lambda_1$ is a constant to be determined with boundary conditions, and $r_1$ is the positive solution of auxiliary equation, $u(r) = -\rho + rt + 1/2\sigma^2 r(r - 1) = 0$. Note that $u(r)$ has a positive solution $r_1 = (a^2 - 2b + \sqrt{(2\mu - \sigma^2)^2 + 4\rho \sigma^2})/(2\sigma^2)$ and a negative solution $r_2 = (a^2 - 2b - \sqrt{(2\mu - \sigma^2)^2 + 4\rho \sigma^2})/(2\sigma^2)$.

Moreover $r_1 > 1$, because $u(1) = \mu - \rho < 0$. Including the exercise region,

$$\psi(x) = \begin{cases} \\
\Lambda_1 x^{r_1} - \frac{a}{\mu - \rho} x + \frac{b}{\rho}, & \text{if } 0 \leq x < x_1^*, \\
F_1 x + F_2, & \text{if } x_1^* \leq x.
\end{cases}$$

Using the value matching condition and the smooth pasting condition, we can find the threshold value $x_1^*$ and the constant coefficient $\Lambda_1$ as following

$$x_1^* = \left\{ \frac{r_1 (\mu - \rho)}{(a_1 - a_2)(r_1 - 1)} \right\} \left\{ \frac{b}{\rho} - \tilde{c}(\delta) \right\} e^{-\mu \delta},$$

$$\Lambda_1 = \frac{(a_1 - a_2)e^{(\mu - \rho)\delta}}{r_1 (\mu - \rho)} (x_1^*)^{1-r_1}.$$  

Because we are interested in the current operational value of system, $s = 0$, and we can summarize the operational value function as following including $a_2x/(\rho - \mu) + b_2/\rho$ term.

$$v(x, \delta) = \begin{cases} \\
\left\{ \frac{(a_1 - a_2)e^{(\mu - \rho)\delta} - 1}{a_1 - a_2}\right\} \left\{ \frac{b_1 - b_2}{\rho} x + \frac{b_2}{\rho} \right\} e^{-\rho \delta} \tilde{c}(\delta), & \text{if } 0 \leq x < x_1^*, \\
\left\{ \frac{(a_1 - a_2)e^{(\mu - \rho)\delta} - 1}{a_1 - a_2}\right\} \left\{ \frac{b_1 - b_2}{\rho} x + \frac{b_2}{\rho} \right\} e^{-\rho \delta} \tilde{c}(\delta), & \text{if } x_1^* \leq x.
\end{cases}$$

The optimal time to exercise the option is

$$\tau_1 = \inf\{ t \geq 0 | X(t) \geq x_1^* \}.$$  

With similar procedure, the solution for the case that $a_1 < a_2$ and $\tilde{c}(\delta) \geq b/\rho$ is obtained as

$$v(x, \delta) = \begin{cases} \\
\left\{ \frac{(a_1 - a_2)e^{(\mu - \rho)\delta} - 1}{a_1 - a_2}\right\} \left\{ \frac{b_1 - b_2}{\rho} x + \frac{b_2}{\rho} \right\} e^{-\rho \delta} \tilde{c}(\delta), & \text{if } 0 \leq x < x_2^*, \\
\left\{ \frac{(a_1 - a_2)e^{(\mu - \rho)\delta} - 1}{a_1 - a_2}\right\} \left\{ \frac{b_1 - b_2}{\rho} x + \frac{b_2}{\rho} \right\} e^{-\rho \delta} \tilde{c}(\delta), & \text{if } x_2^* \leq x.
\end{cases}$$

\[\square\]


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