Mechanical behaviour of DEM crushable grains with fines removal

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ABSTRACT

In order to understand the fundamental behaviour of crushable materials with fines removal in wide stress range, a numerical investigation was conducted. The investigation was carried out using DEM crushable materials because of representation of crushing phenomena. Firstly the volume percentage and microscopic stress of smallest grains regarded as fines were discussed during \(K_0\) compression. Then the mechanical behaviour were examined on the void ratio variation and the evolution of particle size of the material during \(K_0\) and plane strain compressions after fines removal. The effects of removal was dependent on the stress level. For higher stress level, a reproduction of smaller grains and the corresponding grading change were found out during \(K_0\) compression and PSC after the removal. Then the materials indicated same compression and critical state conditions to original material.

Keywords: erosion, particle crushing, DEM, mechanical behaviour

1. INTRODUCTION

Internal erosion is a phenomenon in which particles flow out of a soil matrix at a higher permeation field. This phenomenon is observed in embankment, dike, fill-dam and so on. The particles eroded are smaller particles of the soil and generally, are considered to be fine particles. This erosion would leads to a volume reduction simultaneously with the increase in a void ratio. Furthermore, a severe surface subsidence and stability problem in these soil structure might be caused (Sterpi, 2003, Cividini, et al. 2009, Uzuoka, et al., 2012). So, it is essential to understand the mechanical property of the soil with fines removal. Wood & Maeda (2006) investigated the mechanical behaviour of the soil with several kinds of particle size distribution using DEM analysis, and compared the mechanics behaviour after removing small particles. Moreover, Wood et al. (2010) proposed the mechanics constitution model to be able to evaluate the behaviour of the soil removed fines.

In 2013, MH production test on Pacific Ocean was conducted in Japan. This test was adopted with the de-compressing method to dissolve into methane gas from MH and to collect. The actual had the reduction of the water pressure from 12MPa to 4MPa at 300m of the depth from the sea bottom. This extreme water pressure reduction implies movement of particles. It is possible that movement of this particle not only leads to a production obstacle, but causes subsidence of the circumference foundation. For this, it is required to evaluate the mechanics of the soil which carried out particle removal on the wide range pressure region beyond the past researches.

Numerical simulations would be one of the tools to clarify the mechanical properties of the material with fines removal in wide stress range. Here, the wide stress region means the pressure region accompanied with particle crushing. So the simulations should be incorporated into the phenomenon of particle crushing. The Discrete Element Method (DEM) can simulate the behaviour of crushable soils. Robertson (2000) modelled crushable ‘grains’ (agglomerates) numerically by bonding elementary spheres in probabilistically flawed ‘crystallographic’ arrays. McDowell & Harireche (2002a) also validated the use of DEM in modelling soil particle fracture. Not only could DEM simulate the crushing strength of a real sand grain, with diametral breakage of the bonded agglomerates between flat platens, it could also reproduce realistic Weibull distributions of crushing strength in a batch of flawed agglomerates. McDowell & Harireche (2002b) reported that the one-dimensional compression curves normalised by crushing strength were effectively identical. Cheng et al. (2003) applied this DEM approach to simulate the compression and shearing behaviour of an element of crushable soil (silica sand). Cheng et al. (2004) continued the fore-going research with particular focus on the fundamentals of yielding and plastic deformation.

This paper presents the simulation results on the mechanical behaviour of DEM crushable materials with fines removal. Firstly a conventional \(K_0\) and plane strain...
compressions simulations will be presented. The volume percentage and microscopic stress on smallest grains will be discussed. Then the volume contraction behavior and the variation of void ratio with fines removal will be shown in $K_0$ and the subsequent plane strain compressions simulations. Furthermore, the evolution of particle size of the material after removal of particles will be analytically investigated. Since the crushable model is reproduced as group of elements, the smallest particle in this model is particle of only one element. So, removal of one-element particles as fines will be supplied in this numerical simulation.

2 SIMULATION FOR FINES REMOVAL

2.1 Crushable grain modelling

Agglomerates were formed after the fashion of Robertson (2000), being based on a regular agglomerate of 57 spheres in hexagonal close packing without initial overlap. In order to provide a sand-like variability to the strength and shape of the agglomerates, each elementary sphere was given a probability of existence of only 80% when it was created. These flawed agglomerates contained an average of 46 spheres. Stiffness, bonding and slip models are included in the constitutive representation of contact points between the elementary spheres. The normal stiffness and the shear stiffness are computed assuming that the corresponding linear stiffnesses of the two contacting objects act in series. These stiffnesses are here given the same value $K$. The simple contact bond can be envisioned as a pair of elastic springs at a bonding point. The maximum tensile force and the maximum shear force that it can withstand are here made equal to bond strength $BS$. This approach follows Robertson and Bolton (2001), McDowell and Harireche (2002) and Cheng, et al.(2003). Slip occurs between unbonded objects in contact, according to Coulomb friction with a coefficient $\mu$.

Table 1 gives a typical set for DEM parameters for crushable grains. The values of bond strengths $BS$ and contact stiffnesses $K$ used in the simulations are 1N and 0.5MN/m. The ratio of bond strength to contact stiffness becomes $2*10^{-6}$m, the same as those in Cheng, et al. (2003 & 2004). The ratio $BS/K$ controls the contact displacement when the bond breaks. Agglomerates of spheres sharing the same bond strength to stiffness ratio possess the same triaxial compression behaviour under static loading when the stress is normalised by the crushing strength (Nakata, et al., 2005).

The procedure developed by Robertson (2000) and described in Cheng, et al. (2003) was used to create an assembly of randomly oriented agglomerates with random flaws. Firstly 378 exospheres of 1 mm diameter were created in a space of (6.66 mm)$^3$ so as to be in equilibrium. The exospheres were replaced by 378 numerical agglomerates having the same centres. Then the 6 walls were moved inwards at 0.05 m/sec speed until the particulate media is at a void ratio of 2.0.

| Table 1. DEM parameters used |
|-----------------------------|
| Diameter of agglomerate     | 1.0mm |
| Diameter of sphere          | 0.2mm |
| Density of sphere           | 2650 kg/m$^3$ |
| Maximum number of spheres   | 57 |
| Maximum number of bonds in an agglomerate | 228 |
| Normal and shear bond strength $BS$ | 1N |
| Normal and shear stiffness at contact $K$ | 0.5 MN/m |
| Frictional coefficient of sphere | 0.5 |
| Percentage of spheres removed at random | 20% |

Fig. 1. Crushable grains (agglomerates) assembly (Cheng et al, 2003).

2.2 Simulation for $K_0$ & plane strain compressions

$K_0$ compression was computed with upper and lower wall movements of 0.05m/sec until a vertical stress of 40MPa was reached. Following this unloading was also simulated. During this process, four of the side walls were fixed. Plane strain compression (PSC) tests were then simulated in the cubical cell, from different stress levels on the $K_0$ virgin compression line. In PSC tests, one pair of side walls was fixed and another pair was controlled to keep a constant mean principal stress using appropriate feedback algorithms. In other words the simulation was similar to a laboratory test in its force and displacement control. One advantage of computer simulation is that the storage of data files makes it possible to reproduce sample initialisations exactly. Other is that the microscopic considerations can be conducted very carefully.

Figure 2 indicates the relations between a void ratio and mean principal stress called as “state path” during $K_0$ and PS compressions. The maximum curvatures are at about $p \approx 3.0$MPa for the $K_0$ compression curve. The stress level corresponds to 0.25 times of the characteristics of single crushing strength (Nakata, et al., 2005). Immediately after this the curve become almost straight “normal compression lines” with a gradient $\lambda =$.
The figure also indicates the simulation result of an isotropic consolidation and subsequent triaxial compression tests. The simulations for isotropic and triaxial compression were carried out by Nakata, et al. (2005). The result for $K_0$ compression simulation was drawn below the isotropic-consolidation curve, and took the smaller value by 0.1 of void ratio at the mean principal stress larger than 3MPa. It could be said that the gradient $\lambda$ for $K_0$ compression is equivalent to that for isotropic compression.

The PSC were simulated until the deviator strain reached 0.6. The samples showed the variation of void ratio depending on the mean stress. The void ratio of $p=0.56$MPa increased from 1.8 to 2.1. The void ratio of $p=3.07$MPa decreased from 1.55 to 1.35. For the void ratio of $p=6.75$MPa, the reduction from 1.2 was calculated by 0.25. The plane strain simulation results under the constant $\sigma_3$ conditions after $K_0$ were drawn on this figure by Nakata, et al. (2005). The possible critical state of all PSCs and TCs converged to one curve irrespective with stress path and stress history. This indicates the existence of the critical state line for the DEM model. In order that calculation is performed on the conditions confined by the walls without friction, it is hard to say that the simulation is exactly same conditions with the actual experiment covered by rubber membrane.

![Critical state line](image)

Fig. 2 Critical state for DEM simulation in void ratio and mean stress relations.

After successive fragmentation generations, the size distribution of the agglomerates on the $K_0$ and PSC compressions are shown in Fig 3. The horizontal axis is the representative size, defined as the 1/3 power of the current solid-volume of each individual agglomerate, while the vertical axis is the percentage by volume of sizes smaller than a size. The curves correspond conceptually to the particle size distribution curves in real sieving analyses. Initially, the agglomerates were nearly uniform in size. As mean stress increased, the agglomerates became well-distributed in size. This is similar to the evolution of real particle size distributions. The smallest size clearly increased after exceeding 3.1MPa of mean stress. Also the more evolution of size distribution during PSC was obviously observed at the simulation for $p=6.8$MPa. So, the shear loading produced more grain crushing.

![Size distribution](image)

Fig. 3. Size distribution of the agglomerates.

### 2.3 Removal of smallest grains

Removal of one-element particles as fines is supplied in this numerical simulation since the grains were modeled as the groups of elements. So, more insights would be focused on the properties of smallest grains.

Figure 4 indicates a variation of the percentage of smallest grains with increasing mean stress under $K_0$ compression. In addition, variation of the percentage of the particles on which zero force is acting is also shown. The percentage of the smallest particles does not change at 4.2% up to yield stress of 3MPa. The percentage of no-forced particles was about 3.3%. That is, most of the smallest particles do not transmit any load. In the case of the mean stress exceeded the yield stress, the smallest particles gradually increased, but the proportion of no-forced particles decreases conversely. For example, at the mean stress of 6.9MPa, the percentage of the smallest particles...
particles reaches to 5.7%, and the percentage of no-forced particles decreases to 2.7%. 52% of fine particles transmit a certain load in this stress stage. In the case of the maximum stress level obtained by calculation, the percentage of the smallest particles reached to 14.9%, and the proportion of the smallest particles which have received load reached even to 93%.

Fig. 4. Variation of percentages for smallest grains and no-forced grains.

Figure 5 indicates the average mean stress of the micro stress on the smallest particles which load is acting on. The micro stress which is acting on particles is given by the following formula.

$$\bar{\sigma}_{ij}^p = \frac{1}{V_p} \sum \mathbf{N}_c x_i^c F_{ij}^c$$  \hspace{1cm} (1)

Here, \(V_p\) is the volume of particle, \(\mathbf{N}_c\) is the coordination number, \(x_i^c\) is the location vector at each contacts and \(F_{ij}^c\) is the force acting at each contacts. By using this stress, the mean stress \(\bar{\sigma}^p\) of the micro stress (it is henceforth described to be mean micro stress) which acts on particles is calculated.

$$\bar{\sigma}^p = \frac{1}{3} \delta_{ij} \bar{\sigma}_{ij}^p$$  \hspace{1cm} (2)

Here, \(\delta_{ij}\) is Kronecker delta. In the case of the macro mean stress lower than 3MPa, the average value of \(\bar{\sigma}^p\) indicated about 1/9 of macro mean stress. Although the average value of \(\bar{\sigma}^p\) increases gradually, the ratio against macro mean stress decreases gradually. For example, in the macro mean stress of 1.0MPa, the average value of \(\bar{\sigma}^p\) was 9MPa. That is, the micro stress was about 9 times of the macro mean stress. In the macro mean stress of 3.0MPa corresponding to yield stress, the average value of \(\bar{\sigma}^p\) was 10MPa and was 3.3 times of the macro mean stress. Furthermore, for 6.8MPa which is the stress regions which surpassed yield stress level, the average value of \(\bar{\sigma}^p\) became 3.0 times of the macro mean stress.

Removal of smallest grains were simulated at five stress levels. Table 1 shows the percentages of smallest grains removed. All smallest particles were removed numerically as shown in Figure 6. After removal of smallest grains, \(K_0\) stress condition was kept. Then plane strain compression simulation were conducted by wall speed constant.

Table 1. Variation of fines before and after removal.

| Vertical stress (MPa) | 1.0 | 2.0 | 5.0 | 7.0 | 10.0 |
|----------------------|-----|-----|-----|-----|------|
| Mean stress (MPa)    | 0.56| 1.13| 3.07| 4.47| 6.88 |
| Fines (%)            | 4.2 | 4.2 | 4.2 | 4.6 | 5.8  |

Fig. 5. Variation in void ratio during and after removal.

Fig. 6. Numerical model before (a) and after (b) removal of smallest grains.

3 MECHANICAL BEHAVIOUR AFTER FINES REMOVAL

3.1 Behaviour after fines removal

Figure 7 indicates the variation of void ratio during holding \(K_0\) state after removing the smallest particles. After removal of smallest particles, sudden increase of a void ratio was recognized. The amount of sudden increase was dependent on the percentage of the grains shown in Table 1. After the sudden increase, the decrease of the void ratio were observed. The decrease was dependent on mean stress. For 3.1MPa of mean stress, the decrease in void ratio was 0.05 which is half of the sudden increase. While in the case of 6.9MPa of mean stress...
stress, the decrease in $e$ exceeded the sudden increase.

![Fig. 7. Variation in void ratio during and after removal.](image)

State paths under $K_0$ compression after the removal are indicated in Figure 8. It is understood that the void ratio behavior after the removal is dependent on a pressure level. Especially, the void ratio at higher stress level once increase and then decrease closing to the original compression line. Although the result of the mean stress of 0.56MPa is accompanied by sudden increase of the void ratio, there is small change in void ratio after the sudden increase. In the case of 6.7MPa, after the sudden increase, the larger decrease in void ratio was calculated and then the final void ratio reached the original $K_0$ compression.

![Fig. 8. Variation of void ratio after removal.](image)

The relation between the void ratio at the end of plane strain compression test simulation and mean principal stress is indicated in Figure 9. The figure indicates all the cases accompanied with removal and $K_0$ compression curve shown in Fig. 8. The final void ratio conditions are clearly influenced by the removal in the result of 0.60MPa and 1.1MPa lower than yield stress. Wood et al. (2006) indicates the similar result which reaches the higher final void ratio using a model without crushing. In the case of 3.1MPa, 4.5MPa and 6.9MPa, the final void ratio was the same as that without removal.

![Fig. 9. Effect of removal on $K_0$ compression and critical state conditions.](image)

3.2 Change of grading after removal

Figure 10 shows the change of grading for the mean stress of 6.9MPa. The horizontal axis is the representative size, defined as the $1/3$ power of the current solid-volume of each individual agglomerate, while the vertical axis is the percentage by volume of sizes smaller than a size. The curves correspond conceptually to the particle size distribution curves in real sieving analyses. The smallest value in horizontal axis is corresponding to smallest grain which has one element. The removal provided the percentage of smallest grains to be naturally zero. Then the grading change was observed during $K_0$ conditions after the removal. The percentage of smallest grains became 1.0%. In addition, further grading change was obtained during PSC. The final grading was close to the grading without removal.

![Fig. 10. Change of grading after removal.](image)

The variation of the percentage of smallest grains is shown in Figure 11. The value of the mean stress of 6.9MPa were corresponding to the result in Fig.10. The production of smallest grains due to crushing was clearly recognised at higher stress level than 3MPa. Removal of fine particles will create redistribution of the stress which acts on particles. The redistribution can cause new crushing to particles. Thereby, the further generation of fine particles is brought. If fine particles are continually eroded, it is presumed that the chain reaction mentioned above arises repeatedly.
4 CONCLUSIONS

In order to understand the fundamental behaviour of crushable materials in wide stress range after fines removal, a numerical investigation was conducted. The investigation was carried out using DEM crushable materials because of representation of crushing phenomena. Firstly the volume percentage and microscopic stress of smallest grains regarded as fines were discussed during $K_0$ compression. Then the mechanical behaviour were examined on the void ratio variation and the evolution of particle size of the material during $K_0$ and plane strain compressions after fines removal.

(1) Micro properties for the smallest grains had clear difference between lower and higher stress. For the lower stress, most of the smallest grains did not transmit any load. While there existed a few of grains transmitting intensive load. On the other hand, in the case of higher stress, most of the smallest grains transmit the load which corresponded to three times of the macro stress.

(2) The effects of removal was dependent on the stress level. For lower stress level, any void ratio change and particle crushing did not be observed during $K_0$ compression after the removal.

(3) For higher stress level, a reproduction of smaller grains and the corresponding grading change were found out during $K_0$ compression and PSC after removal. Then the materials indicated same compression and critical state conditions to original material.

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