Detection limits in space-based transit observations

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Abstract. From simulations of transit observations, it is found that the detectability of extrasolar planets depends only on two parameters: The signal-to-noise ratio during a transit, and the number of data points observed during transits. All other physical parameters describing transit configurations (planet and star size, orbital period, orbital half axis, latitude of the transit across star) can be reduced to these two parameters. In turn, once the dependency between transit detectability and these two parameters has been determined, predictions for an instrument’s ability to detect transits by any combination of physical parameters can be derived with ease. These predictions are applied to the Eddington proposal of a combined Astroseismology/Transit-detection space mission currently under study by ESA, which is described briefly.

1. Introduction

To evaluate the performance for the detection of extrasolar planets of observations with the transit method, Jenkins, Doyle & Culler (1996) introduced the method of the calculation of two distributions, one of the detection statistics for transit-less noise-only data (which may be observed or simulated), and one detection statistics for data with known transits in them. This method allows the determination of detection probabilities while at the same time setting limits for the false alarm rate - important in the case of transit detections, as one neither knows if a transit signal is in the data, nor the signal’s duration, amplitude or period. Doyle et al. (2000) used this method to set detection limits in ground based observational data, taken with the goal to detect transits across the eclipsing binary CM Draconis. The evaluations shown here were performed during the course of specifying the Eddington proposal currently pending at ESA for a combined astroseismology/planet-finding mission with a 1m-class space telescope, which is described at the end. In the following, the dependence of the
parameters describing a transit configuration (Planet’s size, star’s size, planet’s period for example) on detectability is shown.

2. Extrasolar planet detectability by transits

Planet detection performance may be expressed in terms of detectability or the detection probability \( p_d \), which is the probability that a physically present transiting planet is going to be detected by the experiment. The detectability of a planet in an observed transit sequence depends on a number of physical parameters of the planet-star system and of the observing instrument. The most obvious one is the size of the planet, with the amount of brightness-loss during a transit being proportional to the square of the planet radius, \( R_{pl} \). Similar, detectability will remain roughly constant, if the ratio \( R_{pl}/R_\ast \) is kept constant.

To assess the performance in the detection of planetary transits, simulated lightcurves with the instrument’s noise characteristics were submitted to a detection test. This detection test gives some ‘detection value’, which may be calculated by any kind of procedure that performs some evaluation between a given model lightcurve and the data. The most suitable one for data which are dominated by ‘white’ noises and without (or with correctable) zero-point drifts is a scalar multiplication between the data vector and the model vector:

\[
C = \sum d_i * m_i
\]

where \( C \) is the detection value, and \( d_i, m_i \) are data and model values. The detection test was performed in large numbers on lightcurves with – and without – simulated transits added to the noise. The highest ‘detection values’ from a large number (between \( 10^5 - 10^7 \)) of lightcurves without transits (that is, with noise only) give a threshold for detection values that constitute false alarms. The fraction of lightcurves with transits (again using a large number of randomly generated curves, but with known transit parameters) that results in detection values higher than the false alarm threshold gives the detection probability. This procedure is similar to one used for an assessment of detection limits in the ground based transit detection project ‘TEP’ (Doyle, et al. 2000).

During the course of evaluating a variety of transit configurations, it became obvious that the physical parameters governing transit lightcurves may be reduced to two parameters which solely govern the detectability in any observed sequence of transits around single stars (the case for binaries would be more complicated): the data’s S/N during a transit, and the total number of data points within transits, \( N_{tp} \). Thus, if the function \( p_d(S/N, N_{tp}) \) can be established, the detectability can be calculated for any combination of physical parameters. The physical parameters influence S/N and \( N_{tp} \) as follows:

- The Signal S depends on planet’s and star’s sizes, the latitude of transit, and on the star’s limb-darkening (mostly for high-latitude transits). For a precise calculation of model transit lightcurves (and hence S), a code dealing correctly with the planet’s ingress and egress and with differential limb-darkening under the planet’s disk should be used, such as ‘utm’ (Deeg, 1999).
- The noise $N$ depends on the noises affecting the observing instrument’s performance (which has always a dependency on a star’s apparent brightness) and on intrinsic brightness fluctuations (stellar activity, oscillations, variability).

- The number of on-transit data points, $N_{tp}$, depends on exposure-time, the number of transits observed (=observational coverage) and on the duration of a transit. The duration in turn is governed by the planet’s orbital period, by the star’s mass and size, by the latitude of the transit, and to a small amount by the size of the planet. It can be calculated by straightforward application of basic geometry and Kepler’s third law.

For detectability evaluations, a ‘standard case’ was chosen, which has a detection probability ($p_d$) of 0.99. In Fig. 1, the sequence of 6 transits of the standard case is shown. That case comprised the following setup:

- Physical parameters: K0 star ($0.85 \, R_{\odot}, \, 0.78 \, M_{\odot}$), orbited by a planet with $1.4 \, R_{\text{Earth}}$, 200d orbital period, transiting at a latitude of 45° across the star.

- Observational parameters: 3 years (1095 days) observing duration; lightcurve with a noise of 450 ppm, or approximately 0.5mmag from 600 seconds exposure time. In the case of the Eddington proposal, this is expected for a star with 16 mag.

As an example of parameter variations around the standard case, the dependency of $p_d$ on planet-size is shown:

| $R_{pl}/R_{\text{Earth}}$ | S/N  | $p_d$ |
|---------------------------|------|-------|
| 1.0                       | 0.25 | 0.16  |
| 1.3                       | 0.42 | 0.91  |
| 1.4                       | 0.48 | 0.99  |
| $\geq 1.5$                | $\geq 0.55$ | 1.00  |

More useful then detection probability is however the determination of the minimum planet size that can be detected reliably. A ‘reliable detection’ is considered to have $p_d \geq 0.99$. From the table above, the detection limit is therefore a planet of $1.4 \, R_{\text{Earth}}$. Similar tables where also generated for the ‘standard case’ but with orbital periods of 50, 100 and 365 days, thus performing an empirical determination of the planet-size $R_{pl,\text{min}}$ where $p_d \approx 0.99$. These results are shown in the next table, where besides $R_{pl,\text{min}}$ the following quantities are given: $n_{tr}$, the number of transits in the 3-year long lightcurve; $t_{tr}$, the duration of one transit; $N_{tp}$, the total number of on-transit data points at 600 seconds exposure time; and S/N, the signal to noise ratio, where the signal is taken as the maximum relative brightness variation during a transit.

| $P_{\text{orb}/\text{day}}$ | 50 | 100 | 200 | 365 |
|----------------------------|----|-----|-----|-----|
| $R_{pl,\text{min}}$       | 1.16 | 1.30 | 1.40 | 1.62 |
| $n_{tr}$                  | 22  | 11  | 6   | 3   |
| $t_{tr}/\text{day}$       | 0.186 | 0.234 | 0.295 | 0.361 |
| $N_{tp}$                  | 589 | 371 | 255 | 156 |
| S/N                       | 0.33 | 0.42 | 0.48 | 0.64 |
From above table, a linear relationship between log $S/N$ and log $N_{tp}$ was established, which can be expressed as:

$$\log S/N = 0.90 - 0.50 \log N_{tp},$$

or more generally:

$$S/N \sqrt{(N_{tp})} = \text{const for } p_d = \text{const}. $$

It should be noted, that this expression is valid only when using $C = \sum d_i * m_i$ for the determination of the ‘detection value’. Other possibilities to derive detection values, such as the calculation of $\chi^2$ values between model and data, are expected to lead to other dependencies, and may not allow the reduction of the physical transit parameters to the two parameters $S/N$ and $N_{tp}$. Above expression between the fundamental parameters governing transit detectability (using the level $p_d = 0.99$) allows the determination of $R_{\text{pl,min}}$ for any combination of physical parameters constituting a transit configuration.

In Fig. 2, $R_{\text{pl,min}}$ is given for a variety of stellar types and orbital periods and apparent magnitudes as expected by Eddington. As the calculations were performed for transits at latitudes of $45^\circ$, the given planet sizes are valid for $\approx 70\%$ of theoretically possible transit configurations. For the remaining $30\%$ of polar transits, detection probabilities may be significantly lower, and planets with $R_{\text{pl,min}}$ cannot be detected reliably. Indicated as hatched regions in Fig. 2.
Detection limits for transit observations

Figure 2. Minimum detectable planet size (in $R_{\text{Earth}}$) for a range of stars in dependence of orbital period (in days) and apparent magnitude for the Eddington proposal. A transit latitude of $45^\circ$ was used. Hatched regions indicate the circumstellar habitable zone.
is the extent of the circumstellar habitable zone based on values from Whitmire & Reynolds (1996). It can be seen, that Earth-sized planets around a solar-type star of \( m_v = 14 \) may be detected, and even stars with \( m_v = 18 \) may still allow for the detection of 'large terrestrial' planets.

3. The \textit{Eddington} proposal

This work was performed in the context of the \textit{Eddington} study. \textit{Eddington} is a combined asteroseismology and planet-detection mission, based on a 1 m class space telescope, initiated by a European consortium of scientists (Favata, Roxburgh, & Christensen-Dalsgaard 2000) in response to ESA’s F2/F3 mission call. \textit{Eddington} was successfully selected for the initial currently ongoing study phase, and will undergo a final selection in September 2000. With foreseen photometric precision of \( \simeq 1 \) part in 100 000, the major science goal for the planet-detection part will be the search for terrestrial size planets in the habitable zone. With its large mirror size (1.2 m diameter) and field of view (3° diameter), it will monitor several hundred thousand stars during the 3 years of its mission dedicated to the transit search. To enable long, uninterrupted observations within a stable thermal environment, an L2 halo orbit is being baselined. These parameters will allow \textit{Eddington} to achieve qualitatively different science goals than the small missions currently in preparation (COROT, MONS, MOST). The discovery of extrasolar terrestrial planets, in addition to being a major achievement on its own, will serve as a fundamental stepping stone for future missions specifically designed for the detection of extraterrestrial habitats, such as ESA and NASA missions IRSI-Darwin and Terrestrial Planet Finder (TPF). \textit{Eddington} will also contribute in a major way to the field of asteroseismology, enabling an accurate characterization of the stellar structure for stars spanning a wide range of masses, ages and chemical compositions.

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