The interplay of soft and hard contributions in the electromagnetic pion form factor

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We consider various relativistic models for the valence Fock-state wave function of the pion. These models are obtained from simple instant-form wave functions by applying a Melosh rotation to the spin part and by imposing physical constraints on the parameters. We discuss how the soft and the hard (perturbative) parts of the electromagnetic form factor are affected by the choice of the model and by the Melosh rotation.

The knowledge of wave functions of the hadronic constituents allows to link hadronic phenomena in different kinematical regions. For several reasons it is advantageous to consider the composition of hadrons out of its constituents at fixed light-front time $\tau = t + z$ rather than at ordinary time $t$. The “time” evolution in $\tau$ is then determined by front-form dynamics. One of the attractive features of such an approach is that the corresponding wave functions are direct generalizations of non-relativistic wave functions in the sense that they can be interpreted as probability amplitudes for finding a particular Fock state in the hadron under consideration with the constituents carrying certain momenta, spins, etc. Like in the non-relativistic case light-front wave functions can be expressed in terms of purely internal variables (momentum fractions $x$ and transverse momenta $k_\perp$).

For a hadron form factor the, in principle, exact expression is just a sum (over all Fock states) of overlap integrals of incoming and outgoing light-front wave functions $[1]$. A widely used approximation is then to assume that the dominant contribution comes from the valence Fock state. For large momentum transfers $Q \to \infty$ the analysis of the corresponding overlap integral reveals that the one-gluon-exchange tail of the wave function can be factored out, so that one ends up with a perturbative representation of the form factor in terms of a convolution integral $[2]$. The distribution amplitude $\phi(x, Q)$ entering this convolution integral is again related to the valence-quark light-front wave function of the hadron. Its dependence on the factorization scale $\tilde{Q}$ (which in turn depends on $Q$) is given by an evolution equation which is driven by one-gluon-exchange. Practically, this means that the knowledge of the soft part of the wave function suffices, since the high-momentum tail of the wave function is determined by perturbative evolution. What we therefore want to model is only the soft part of the pion wave function.

A commonly used ansatz for the quark-antiquark light-front wave function of the pion is of harmonic-oscillator type

$$\psi(x, k_\perp) = A_\pi \chi(x, k_\perp) J(x, k_\perp) \exp \left( -\frac{M_0^2}{8\beta^2} \right) \tag{1}$$

with $M_0^2$ denoting the front-form expression for the free two-particle mass

$$M_0^2 = \frac{k_\perp^2 + m_0^2}{x(1-x)} \tag{2}$$

$A_\pi$ is a normalization constant, $\chi(x, k_\perp)$ the (light-front) spin wave function of the q-$\bar{q}$ pair, and $J(x, k_\perp)$ the square root of a Jacobian. Such a model for the pion wave function has, e.g., been proposed by Brodsky, Huang and Lepage (BHL) $[3]$. They took $J = 1$ and the usual (instant-form) expression for the spin-wave function $\chi$. Terent’ev and Karmanov (TK) $[4]$, on the other hand, chose $J(x) = \sqrt{1/2x(1-x)}$, i.e. the square root of the Jacobian relating...
the relativistic integration measures (\(d^3k/k^0\)) and 
\(d^2k_\perp dx\). The \(J(x, k_\perp)\) adopted by Chung, Co-
ester and Polyzou (CCP) \(\Box\) was of the form 
\(J(x, k_\perp) = \sqrt{M_0/4m_qx(1-x)}\), i.e. the square 
root of the Jacobian relating the non-relativistic 
integration measure \(d^3k/m_0\) to \(d^2k_\perp dx\). In 
addition to these different choices of the \(J_s\) we 
will use another simple ansatz for the s-state or-
bital function of the pion, which has a power-law 
form

\[\psi_{PL}(x, k_\perp) = A_\pi \chi(x, k_\perp)J(x, k_\perp) \left( \frac{\beta^2}{M_0^2 + \beta^2} \right)^\alpha.\]

A few words about the spin wave function 
\(\chi(x, k_\perp)\) are also in order. It is a well known dis-
advantage of the light-front formalism that the 
usual addition of angular momenta (e.g. for 2 
particles \(J = \vec{L} + \vec{S}^{(1)} + \vec{S}^{(2)}\)) holds only for 
the third component, whereas the addition law for the 
other two components is, in general, much more 
complicated. It is, however, possible to go over to 
a unitarily transformed spin operator which satisfies 
the usual spin-addition laws \(\Box\). This unit-
ary transformation is known as the “Melosh ro-
tation” (MR). The light-front spin wave function 
\(\chi(x, k_\perp)\) results thus from an inverse Melosh ro-
tation of an ordinary spin wave function. A con-
venient expression for \(\chi(x, k_\perp)\) can, e.g., be found 
in Ref. \(\Box\). Remarkably, the light-front spin wave 
function of an s-wave pion contains also helicity 
\(\pm 1\) components.

The idea behind the construction of such wave-
function models is always that the valence-Fock-
state wave function within QCD has something 
to do with the wave function of constituent 
quarks (which can be obtained from much sim-
pler dynamics). Therefore the mass \(m_q\) oc-
curring in the wave functions has to be inter-
preted as constituent-quark mass. We took \(m_q = 
330\) MeV. According to the finding in Ref. \(\Box\) 
we have also fixed the parameter \(\alpha\) in \(\psi_{PL}\) to 
be \(\alpha = 3.5\). The normalization \(A_\pi\) has then 
been adjusted such that the weak decay constant 
\(f_\pi = 2\sqrt{3} \int dx d^2k_\perp \psi = 93\) MeV is reproduced.

The dependence of the soft (overlap) contribu-
tion to \(F_\pi\) on the choice of the quark-antiquark 
wave function is displayed in Fig. \(\Box\). At very small 
\(Q^2\) (small figure) the curves nearly coincide. Fig-
ure \(\Box\) on the other hand, shows the interplay of soft 
and hard (perturbative) contributions to \(F_\pi\) 
and the influence of the MR factor. Results are 
only considered for \(\psi_{BHL}\), since the hard contribu-
tion to the form factor is nearly independent of 
the choice of the wave function. It does not even 
depend on whether the MR factor is fully taken 
into account or neglected at all. Marked differ-
ences are only observed if the MR factor is ap-
proximated like in Ref. \(\Box\). Shortly summarized, 
the following conclusions can be drawn from the 
figures (and from Table \(\Box\)):

The use of different wave functions does not have 
a significant effect on the electromagnetic pion

| wave function | \(\beta\) (MeV) | \(A_\pi\) (GeV\(^{-1}\)) | \(P_q\) | \(\sqrt{\langle k_\perp^2 \rangle}\) (MeV) |
|---------------|---------------|-----------------------|-------|-----------------|
| \(\psi_{BHL}\) (full MR) | 300 | 110. | 1.00 | 282 |
| \(\psi_{BHL}\) (no MR) | 300 | 78.2 | 0.51 | 282 |
| \(\psi_{BHL}\) (approx. MR \(\Box\)) | 300 | 65.8 | 1.85 | 323 |
| \(\psi_{TK}\) (full MR) | 291 | 77.5 | 1.00 | 270 |
| \(\psi_{CCP}\) (full MR) | 272 | 80.0 | 1.00 | 272 |
| \(\psi_{PL}\) (full MR) | 1045 | 238. | 1.00 | 266 |

\(m_q = 330\) MeV and \(f_\pi = 93\) MeV throughout.
form factor as long as these wave functions provide comparable results for $f_\pi$ and $\langle k_\perp^2 \rangle$. This holds in both, the soft and the hard regimes.

Normalizing the wave functions in such a way that $f_\pi$ remains unaltered, the soft part of $F_\pi$ (and also $P_{q\bar{q}}$) becomes considerably smaller if the MR factor is neglected. Minor changes due to the omission of the MR factor can be observed in the perturbative part of $F_\pi$. The approximations for the MR factor adopted in Ref. [11], on the other hand, entail also sizable changes in the hard part of $F_\pi$.

Comparing the soft with the perturbative contribution to $F_\pi$ one observes that the latter starts to dominate at $Q^2 \approx 5\text{GeV}^2$.

The effect of the MR factor on the pion distribution amplitude is just a slight broadening as compared to the asymptotic distribution amplitude (cf. Fig. 2). The broadening becomes much stronger if the approximations of Ref. [11] are applied to the MR factor. A recent QCD sumrule analysis of the pion distribution amplitude by Bakulev et al. [12] provides a double-humped distribution amplitude (cf. Fig. 2), but with less pronounced structure than originally proposed by Chernyak and Zhitnitsky.

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