Propagation of acoustic edge waves in graphene under quantum Hall effect

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(Dated: August 25, 2014)

We consider a graphene sheet with a zigzag edge subject to a perpendicular magnetic field and investigate the propagation of in-plane acoustic edge waves under the influence of magnetically induced electronic edge states. In particular is it shown that propagation is significantly blocked for certain frequencies defined by the resonant absorption due to electronic-acoustic interaction. We suggest that strong interaction between the acoustic and electronic edge states in graphene may generate significant non-linear effects leading to the existence of acoustic solitons in such systems.

FIG. 1. (Color online) A schematic picture of a continuous (graphene) sheet with an edge along the z-axis and an applied perpendicular magnetic field (purple). The electronic states (red) may be either localized Landau orbits in the bulk or dispersive states near the edge. Along the edge there are propagating acoustic (Rayleigh) edge waves given by a 2D displacement field (blue, amplitude exaggerated).

Since the graphene edge, which is normal to \(-\hat{e}_y\) and located at \(y = 0\), is stress-free, the elastic boundary conditions are

\[
\sigma_{fy}(x, 0) = 0, \quad j = x, y, z, \quad (1)
\]

where \(\sigma_{ij}(x, y)\) is the usual 2D stress tensor. Since the Rayleigh waves are pseudo-1D, they can be specified by the wave vector \(x\)-component \(q\) alone, which will be referred to as the wave number. Standard techniques\(^3\) give the two-component displacement field \(\mathbf{u}^{(q)}(x, y)\) for an in-plane Rayleigh wave as

\[
\mathbf{u}^{(q)}(x, y; t) = 2u_0 \left( \frac{f_x^{(q)}(y) \cos(qx - \omega t)}{\text{sgn}(q)f_y^{(q)}(y) \sin(qx - \omega t)} \right), \quad (2)
\]

where

\[
f_x^{(q)}(y) = e^{-\lambda_t|q|y} - C_x e^{-\lambda_1|q|y} \quad (3)
\]

and

\[
f_y^{(q)}(y) = -\lambda_t e^{-\lambda_1|q|y} + C_y e^{-\lambda_1|q|y}. \quad (4)
\]

The dimensionless constants are

\[
\lambda_1 = 0.81, \quad \lambda_t = 0.46, \quad C_x = 0.61, \quad C_y = 1.3, \quad (5)
\]

The discovery of graphene,\(^1\) an ultra-pure 2D crystal membrane of remarkable promise,\(^2\) has in just the past few years led to the rapid growth of a new field of research, uniting and challenging scientists from research backgrounds as diverse as the capabilities of the material itself. In addition to its astounding material properties, the existence of acoustic edge waves in graphene suggests a flexible 2D membrane, implying a topology completely normal to the 2D electron plane so that the electrons experienced no localization of acoustic energy. However, the isolation of single-layer graphene,\(^1\) a flexible 2D membrane, suggests that strong interaction between the acoustic and electronic edge states in graphene may allow for acoustic wave propagation in the \(x\)-direction.

To be concrete, we consider a 2D graphene sheet with a zigzag edge subject to a perpendicular magnetic field and investigate the propagation of in-plane acoustic edge waves under the influence of magnetically induced electronic edge states. In particular is it shown that propagation is significantly blocked for certain frequencies defined by the resonant absorption due to electronic-acoustic interaction. We suggest that strong interaction between the acoustic and electronic edge states in graphene may generate significant non-linear effects leading to the existence of acoustic solitons in such systems.

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The dimensionless constants are

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\lambda_1 = 0.81, \quad \lambda_t = 0.46, \quad C_x = 0.61, \quad C_y = 1.3, \quad (5)
\]
and depend only on the ratio of the transverse and longitudinal sound velocities in graphene, \( s_t/s_l \), or, equivalently, on the Poisson ratio. The sound velocities are taken to be \( s_t = 1.4 \times 10^4 \text{ m/s} \) and \( s_l = 2.1 \times 10^4 \text{ m/s} \). The dispersion relation is linear,

\[
\omega(q) = s_R |q|, \tag{6}
\]

with Rayleigh-wave sound velocity \( s_R = 1.2 \times 10^4 \text{ m/s} \).

The electronic subsystem is described by the standard effective-model graphene Hamiltonian

\[
\hat{H}_e = v_F (\sigma_x \hat{p}_x + \tau \sigma_y \hat{p}_y), \tag{7}
\]

where \( v_F = 1.0 \times 10^6 \text{ m/s} \) is the Fermi velocity of graphene, \( \tau = \pm 1 \) for the \( K \)-point and \( (K\')\)-point, the \( \sigma \)'s are the sublattice-space Pauli matrices, and the sublattice pseudospinor upon which the Hamiltonian acts is defined by \( \psi(x, y) = (\psi_A(x, y), \tau \psi_B(x, y))^T \).

The transverse magnetic field is represented by a vector potential \( \mathbf{A}_B = (By, 0)^T \), and then included in the Hamiltonian of Eq. (7) through the minimal coupling \( \mathbf{p} \rightarrow \mathbf{p} + e \mathbf{A} \) (the electron charge is \( -e < 0 \)).

In an infinite bulk system the electronic energies form Landau levels,\(^{21-23}\)

\[
E_n = \text{sgn}(n) E_1 \sqrt{|n|}, \quad n = 0, \pm 1, \pm 2, \ldots \\
E_1 = \sqrt{2 \hbar v_F l_B^{-1}} \propto \sqrt{B}, \tag{8}
\]

and the electronic wave functions are harmonic oscillator states centered around \( y_c = -k l_B^2 \), corresponding to closed Landau orbits, see Fig. 1. This simple picture is modified by the introduction of an edge.

In the considered system the edge at \( y = 0 \) is a zigzag edge of \( B \)-atoms, leading to the electronic boundary condition\(^{24}\)

\[
\psi_A(x, 0) = 0. \tag{9}
\]

Since the zigzag boundary condition does not mix valleys the \( K \)- and \( (K\') \)-points can be considered separately.

The edge induces a positive (negative) dispersion in the electron-like (hole-like) Landau levels as \( k \) increases and the wave function center \( y_c \propto -k \) moves toward and over the edge,\(^{11}\) pressing the oscillator wave functions against the edge and turning them into edge-localized current-carrying states. For a classical, intuitive picture of this effect, see Fig. 1.

The dispersion can be calculated by generalizing the Landau-level index \( n \) to a \textit{continuous} analogue, \( \nu = (E/E_1)^{2/3} \), and the harmonic oscillator functions to (Whittaker’s) parabolic cylinder functions \( D_n(z) \), which reduce to harmonic oscillator functions for integer \( \nu \) but allow for non-integer \( \nu \) solutions between the bulk Landau levels. The spectrum is then calculated from the boundary condition of Eq. (9).\(^{12-15}\) The dimensionless energy \( E/E_1 \equiv \tilde{E} \) is plotted against the dimensionless wave number \( kl_B \equiv \tilde{k} \) in Fig. 2 for both the \( K \)- and \( (K\') \)-points. The energy band stemming from Landau level \( n \) will be referred to as “edge band \( n \)”. When \( \tilde{k} = kl_B = 0 \) and \( y_c = 0 \) the wave function is centered on the edge.

As seen in Fig. 2, the zeroth Landau level remains dispersionless for all \( \tilde{k} \) in the \( (K\') \)-point spectrum, whereas it is seemingly split in two edge bands, one electron-like and one hole-like, in the \( K \)-point spectrum. This can be explained by extra degeneracies introduced by topological edge states; the peculiar nature of the \( n = 0 \) Landau level have been studied in other papers;\(^{14,15,25,26}\) for the purpose of this paper the schematic spectra in Fig. 2 will suffice.

The electronic pseudospinor wave functions are given in the appendix for reference. There, scaled physical coordinates \( \tilde{x}(\tilde{y}) \equiv x(y)/l_B \) are introduced, which will be employed below when considering the absorption.

The standard first-order-in-strain Hamiltonian for the electron-strain interaction in graphene is given by\(^{27}\)

\[
\hat{H}_\text{int}^e (u(x, y; t)) = g_1 (u_{xx} + u_{yy}) I + \\
+ g_2 (-\tau (u_{xx} - u_{yy}) \sigma_x + 2u_{xy} \sigma_y), \tag{10}
\]
where \( u_{ij} \) is the standard tensor strain. The diagonal elements are the scalar deformation potential, with coupling constant \( q_1 \sim 10 \text{ eV} \), and the off-diagonal elements are usually imagined as a strain-induced pseudo vector-potential, and their coupling constant is \( q_2 \sim 1 \text{ eV} \). Since the valley separation is \([K - K'] \sim a^{-1}\), interaction with the acoustic Rayleigh waves will not mix \( K \) and \( K' \) if the acoustic wave number \( q \ll a^{-1} \), which must hold for the continuous-media model to be valid. Therefore all electronic transitions induced by the acoustic waves are intra-valley and the \( K \) and \( K' \)-point spectra can still be considered separately using \( \tau \).

Inserting Eqs. (2), (3), and (4) into Eq. (10) yields the Hamiltonian for an electronic transition due to interaction with the acoustic Rayleigh waves as

\[
H_{\text{int}} \left( u^{(q)}(x, y; t) \right) = u_0 e^{i qx - i \omega t} \left\{ \frac{1}{2} \left( - \tau T^{x,l}_2 \sigma_x + i sgn(q) T^{y}_2 \sigma_y \right) e^{-\lambda_i |q| y} \right. \\
+ \left. \left[ \tau T^{x,l}_2 \sigma_x - i sgn(q) T^{y}_2 \sigma_y \right] e^{-\lambda_i |q| y} \right\} + \text{H.c.} \quad (11)
\]

where the constants are

\[
T_1 = 0.34, \quad T^x_2 = 1.6, \\
T^{x,l}_2 = 1.7, \quad T^{x,t}_2 = 1.2. \quad (12)
\]

Considering the spectra for the \( K \)- and \( K' \)-points in Fig. 2, it is evident that using a gate voltage \( V_G \) to adjust the scaled Fermi energy, \( E^F / E_1 = \tilde{E}^F \propto V_G / \sqrt{B} \), alters the number of dispersive energy bands crossing the Fermi level. If

\[
|\tilde{E}_{n-1}| < |\tilde{E}^F| < |\tilde{E}_n|, \quad (13)
\]

where \( \tilde{E}_n \) refers to the scaled energy of bulk Landau level \( n \) (see Eq. (8)), there will be \( n \) \((n-1)\) energy bands crossing the Fermi level in the \( K \)-spectrum (\( K' \)-spectrum). These crossings are the quantized conduction channels of the quantum Hall effect theory and the absolute values in Eq. (13) correspond to the electron-hole symmetry of the spectrum. The dispersionless level in the \( K' \)-spectrum never crosses the Fermi level and is therefore assumed never to be involved in transitions.

To analyze the possible transitions, consider the transition rate between levels, thereby introducing conservation laws. The transition rate \( W_{m,n} \) for an electronic jump from energy band \( n \) to energy band \( m \) due to interaction with an acoustic wave with scaled wave number \( qB \equiv \tilde{q} \) is given by the Fermi golden rule,

\[
W_{m,n} = \frac{2\pi}{\hbar} \sum_{\tilde{k}_n} \int d\tilde{E}_m \delta \left( \tilde{E}_m + \tilde{E}_R - \tilde{E}_m \right) \delta_{\tilde{k}_n, \tilde{q}, \tilde{k}_m} \times \rho(E_m) \left| \Lambda_{\tilde{k}_m, \tilde{q}, \tilde{k}_n} \right|^2 f_{\text{FD}}(E_n) \left( 1 - f_{\text{FD}}(E_m) \right). \quad (14)
\]

Here, \( \delta_{\tilde{k}_n, \tilde{q}, \tilde{k}_m} \Lambda_{\tilde{k}_m, \tilde{q}, \tilde{k}_n} \) is the matrix element of an induced transition from \( \tilde{k}_n \) to \( \tilde{k}_m \) defined by

\[
\delta_{\tilde{k}_n, \tilde{q}, \tilde{k}_m} \Lambda_{\tilde{k}_m, \tilde{q}, \tilde{k}_n} = \frac{i}{\hbar} \int_{\tilde{S}} \psi^{\dagger}_n \tilde{k}_m \cdot \mathbf{H}_{\text{int}} \left( u^{(q)} \right) \psi_{\tilde{k}_n} \tilde{k}_m d\tilde{x} d\tilde{y} \quad (15)
\]

where the interaction is given by Eq. (11) (the harmonic time dependence is accounted for by the energy conservation), the electronic wave functions are given in the appendix and the integration surface is the whole sheet \( S \) in terms of the scaled coordinates. The continuous level index is \( \nu = (E / E_1)^2 \) as before, \( f_{\text{FD}}(E(k)) \) is the Fermi-Dirac distribution function, \( \rho(E_m) \) is the density of final states, \( \tilde{k}_n \) is the scaled wave number for an electronic state in energy band \( n \) corresponding to energy \( \tilde{E}_n \), and the scaled acoustic dispersion is given by, using Eq. (6),

\[
\tilde{E}_R (\tilde{q}) = \tilde{s}_R |\tilde{q}|, \quad (16)
\]

with dimensionless speed of sound

\[
\tilde{s}_R \equiv \frac{s_R}{\sqrt{2} v_F}. \quad (17)
\]

The energy integration and the Fermi-Dirac factors confine the energy region of absorption to the vicinity of the Fermi energy, \( E_n \lesssim E^F \lesssim E_m \), and thus imply that the energies and wave numbers may be taken at the Fermi level, e.g. \( \tilde{k}_n \rightarrow \tilde{k}_n^F \). Armed with this knowledge, the picture can be simplified by linearizing the spectrum, swapping each curved energy band \( n \) for a linear band \( n \) with velocity equal to the Fermi velocity \( v_n \) of the band, see Fig. 3. Then the linearized dimensionless dispersion of band \( n \) is

\[
\tilde{E}_n \left( \tilde{k}_n^F \right) = \tilde{v}_n \left( \tilde{k}_n^F - \tilde{k}_n^F \right) + \tilde{E}^F \quad (18)
\]

where the dimensionless velocity of the band is defined analogously to Eq. (17),

\[
\tilde{v}_n \equiv \frac{v_n}{\sqrt{2} v_F}, \quad (19)
\]

and \( \tilde{s}_R \ll \tilde{v}_n \forall n \).

The above arguments together with energy and momentum conservation restrict the number of allowed transitions by imposing the requirement that

\[
\tilde{q} \approx \tilde{k}_m^F - \tilde{k}_n^F \equiv \Delta k_{m,n}, \quad (20)
\]

i.e. the acoustic wave number \( \tilde{q} \) must roughly match the \( k \)-separation of the two Fermi crossing points. Transitions occur in the vicinity of the Fermi level, not at the Fermi level, but for the purpose of this paper it is sufficient to take \( \tilde{q} = \Delta k_{m,n} \). The same above arguments also imply that there are no allowed intra-level transitions, \( n \neq m \).
The number of band-to-band transitions \( N_t(n) \) for \( n \) Fermi level crossings is then

\[
N_t(n) = \begin{cases} 
\frac{n!}{2(n-2)!} & \text{if } n \geq 2, \\
0 & \text{if } n < 2,
\end{cases}
\]  

and it must be remembered that transitions can occur in both the \( K \)- and \( K' \)-spectra.

Since the spacing \( \Delta k_{n+1,n}^F \) between neighboring Fermi crossings is approximately equal for the same energy, i.e. \( \Delta k_{n,n+j}^F \approx j \cdot \Delta k_{m,m+1}^F \), it is potentially useful to group the transitions in terms of how many bands they jump, i.e. a jump from band \( n \) to band \( n-j \) is a \( j \)-jump (the minus sign is due to Fermi crossings of higher-\( n \) bands having larger \( k \)). For the situation with \( n \) Fermi crossings in one of the valley spectra, the number of \( j \)-jumps is

\[
N_{t,j}(n) = \begin{cases} 
n-j & \text{if } n \geq 2, \\
0 & \text{if } n \leq 2,
\end{cases}
\]

Summing \( N_{t,j} \) for all \( j < n \) yields the total number of transitions in the spectrum, \( N_t \). Since all \( j \)-jumps have approximately equal \( \Delta k_{m,n}^F \), i.e. absorbed acoustic frequency, they might appear as a multi-peak in the absorption spectrum: \( N_{t,j} \) peaks close together.

The absorbed acoustic frequencies \( s \tilde{t}_B^{-1} \Delta k_{m,n}^F \) can be found by using the electronic boundary conditions to find the Fermi level crossings \( \tilde{k}_{m,n}^F \), see the appendix. These frequencies are on the order of \( s \tilde{t}_B^{-1} \sim \sqrt{B[T]} \cdot 10^{11} \text{s}^{-1} \) and depend only on the scaled Fermi energy \( \tilde{E}_F \), i.e. the relative position of the Fermi level. The periods of these acoustic frequencies must be much shorter than the acoustic decay time due to interaction with the electronic subsystem for the Fermi golden rule to remain valid.

For the linearized spectrum, Eq. (18), standard periodic boundary conditions in the \( x \)-direction yields the density of final states per unit length \( \rho(E_m) \) as

\[
\rho(E_m) = \frac{1}{2\sqrt{2\pi \hbar v F}} \frac{1}{\tilde{v}_m}.
\]  

(23)

As seen in Fig. 2, the density of states increases with edge localization, i.e. increasing \( k \).

Since transitions occur near the Fermi level, the matrix element of transition in Eq. (15) is evaluated for \( \tilde{q} = \Delta \tilde{k}_{m,n} \) and \( \tilde{E}_m = \tilde{E}_m = \tilde{E}_F \), and is then

\[
A_{\tilde{k}_m,\tilde{k}_n}^F = i\Delta \tilde{k}_{m,n} \left( \frac{u_0}{l_B} \right) (g_1 F_1 + g_2 F_2) \]  

(24)

where the dimensionless transition-dependent integrals have been separated into a scalar potential contribution \( F_1 \) and pseudo-magnetic-field contribution \( F_2 \); both given in the appendix. Normalization of the electronic wave functions causes these integrals to be at the most unity.

Inserting the above into Eq. (14), the final expression for the absorption rate per unit length is

\[
W_{m,n} = \frac{1}{\sqrt{2\pi \hbar v F}} \frac{1}{\tilde{v}_m} \left( \frac{\Delta \tilde{k}_{m,n}}{\tilde{v}_m} \right)^2 \left( \frac{u_0}{l_B} \right)^2 |g_1 F_1 + g_2 F_2|^2.
\]  

(25)

where all relevant dependencies have been included explicitly for clarity. The first factor \( \approx 1.6 \cdot 10^{14} \text{eV}^{-2} \text{s}^{-1} \text{m}^{-1} \) and consists of general constants, and the second factor consists of parameters specific to the transition in question and is \( \sim 2 \). The third is the amplitude dependence, with the amplitude scaled by the magnetic length. By assumption, the amplitude is low, \( A \ll l_B \), causing this factor to be very small. The final factor is the coupling coefficients and the transition integrals, which are less than one by normalization, meaning that the order of magnitude is set by the coupling. Inserting the definition of the magnetic length yields \( W_{m,n} \propto B \). This direct proportionality to the field comes from the \( ij \)-factor in the strain tensor and the fact that absorption occurs only for the phonon wave
numbers \( q \) which match the electro-magnetic spectrum and are thus are on the order of inverse magnetic length.

The total energy of the acoustic wave is\(^{28}\)

\[
E_{ac} = \rho_{gr} |q| \omega(q)^2 \int \int_S |u^{(q)}(x, y; 0)|^2 \text{d}x\text{d}y
\]  

(26)

where \( \rho_{gr} = 7.6 \cdot 10^{-7} \text{kg/m}^2 \) is the surface mass density of graphene.\(^{19}\) In this case

\[
\int \int_S |u^{(q)}(x, y; 0)|^2 \text{d}x\text{d}y = \frac{2L a_0^2}{|q| N_{ac}^2},
\]

(27)

and it can be shown that

\[
N_{ac} = 1.2,
\]

(28)

whereas the energy lost to each electronic transition is simply \( h\omega(q) \). The acoustic inverse decay time \( \tau_D \) due to interaction with the electronic subsystem is then given by

\[
\frac{1}{\tau_D} = \left( \frac{N_{ac}^2 (\Delta k_{F,m,n}^2)}{2 \sqrt{2}\hbar v_m \rho_{gr} t_s^2 B S R} \right) |g_1 F_1 + g_2 F_2|^2
\]

\[
= \frac{2.0 \cdot 10^7 B[T]}{s \text{ eV}^2} \left( \frac{\Delta k_{F,m,n}^2}{v_m} \right) |g_1 F_1 + g_2 F_2|^2.
\]

(29)

As an example, consider the simplest case. The gate voltage is adjusted in relation the magnetic field so that

\[
\tilde{E}^F = \frac{\tilde{E}_1 + \tilde{E}_2}{2},
\]

(30)

i.e. the Fermi level is now in the middle of the gap between Landau level 1 and 2. According to Eq. (13) there will be 2 bands crossing the Fermi level in the \( K' \)-point spectrum (1 in the \( K' \)-spectrum) and by Eq. (21) there will, trivially, be 1 possible transition (0 possible transitions). Eq. (22) specifies that this one transition will be between neighboring levels. Solving Eq. (B1) numerically returns \( \tilde{k}_1^F = -1.29 \) and \( \tilde{k}_0^F = 0.36 \), the points where the bands intersect the Fermi level. This leads to \( \Delta k_{0,1}^F = 1.65 \), which will be the acoustic wave number absorbed in the transition from edge band 1 to edge band 0. The set \( \tilde{E}^F \) means that the generalized level index is, according to Eq. (B3), \( \nu_F = \left( 1 + \sqrt{2} \right)^2 / 2 \approx 1.4571 \) and the velocity of the destination band is estimated to \( \tilde{v}_0 \approx 0.6 \) (in general, \( \tilde{v}_0 \approx 0.5 \)). Using the wave functions of Eq. (A2) with parameters \( \nu_F \) and \( \tilde{k}_n^F \) for edge band 1 (0) as well as the acoustic wave number \( \Delta k_{0,1}^F \) allows for numerical evaluation of the integrals in the appendix. The interaction integrals in Eqs. (C1) and (C2) yield \( F_1 = -0.0546 \) and \( F_2 = -0.0918 \). Inserting all known values into Eq (29) the resulting inverse decay time is

\[
\frac{1}{\tau_D} = \frac{9.1 \cdot 10^7 B[T]}{s \text{ eV}^2} |0.0546 g_1 + 0.0918 g_2|^2.
\]

(31)

With the standard values\(^{27}\) of \( g_1 \approx 20 \text{ eV} \) and \( g_2 \approx 2 \text{ eV} \), the decay time becomes \( \tau_D \approx 6.8 \text{ ns/B[T]} \), which corresponds to a characteristic decay length of \( 82 \mu \text{m}/B[T] \). The decay time is much longer than the acoustic period \( \sqrt{B[T]} \approx 10^{-11} \text{ s} \), thus validating our use of the Fermi golden rule.

In conclusion, we have shown that a stress-free graphene edge supports propagating vibrational in-plane edge modes in the form of 2D Rayleigh waves, and that interaction with such waves can cause electronic transitions between the electronic edge states induced by a perpendicular magnetic field.

Since momentum conservation requires the wavelength of the acoustic waves to be on scale of the magnetic length for transitions to occur, the magnetic field strength enters into the low-amplitude absorption rate as a simple proportionality through the strain tensor.

With the expressions given in this paper, both the absorbed acoustic frequencies and the resulting decay time can be calculated for all electronic transitions of the considered type, yielding an acoustic absorption spectrum which could be used for result confirmation in a wave-propagation experiment.

We suggest, based on comparison with similar systems,\(^{29}\) that this edge-localized interaction could result in nonlinear phenomena such as acoustic solitons propagating along the edge.

I. ACKNOWLEDGEMENTS

We thank L. Gorelik for valuable discussion, E. Cojocaru for his Matlab scripts\(^{30}\) and the Swedish Research Council (VR) for funding.

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Appendix A: Electronic wave functions

The electronic pseudospinor wave functions of the energy band $n$ and $m$, are\textsuperscript{12–15}

$$
\psi^{\tau,k}_{\nu}(x,y) = \frac{N_{\nu,k}}{\sqrt{L_B}} e^{ikx} \phi^{\tau,k}_{\nu}(y),
$$
(A1)

where $\tau$ labels the valley ($K$ or $K'$) as before. The $y$-dependent factor is

$$
\phi^{+1,k}_{\nu}(y) = \left( \frac{D_{\nu}(\sqrt{2}(\tilde{k} + \tilde{y}))}{\sqrt{D_{\nu-1}(\sqrt{2}(\tilde{k} + \tilde{y}))}} \right),
$$
(A2)

for the $K$-point and

$$
\phi^{-1,k}_{\nu}(y) = \left( \frac{\sqrt{7}D_{\nu-1}(\sqrt{2}(\tilde{k} + \tilde{y}))}{-D_{\nu}(\sqrt{2}(\tilde{k} + \tilde{y}))} \right),
$$
(A3)

for the $K'$-point. The factors $N_{\tau,n,k}/(\sqrt{L_B})$ are normalization constants.

Appendix B: Fermi level crossings

The edge boundary condition of Eq. (9) ultimately gives an equation for the electronic spectrum. At the Fermi energy $E_F$ this equation reads, for the $K$-point,

$$
D_{\nu,F}(\sqrt{2}\tilde{k}^F) = 0,
$$
(B1)

and for the $K'$-point

$$
D_{\nu,F-1}(\sqrt{2}\tilde{k}^F) = 0,
$$
(B2)

where

$$
\nu^F = \left( \tilde{E}^F \right)^2.
$$
(B3)

Solving Eqs. (B1) and (B2) for $\tilde{k}^F$ gives the Fermi crossing points $\tilde{k}_{m,n}^F$ for the given $\tilde{E}^F$. Identifying them with the different bands allows for calculation of $\Delta \tilde{k}_{m,n}^F$ and thus the absorbed acoustic frequencies. In general $\Delta \tilde{k}_{m,n}^F \sim 1$.

Appendix C: Absorption integrals

Here the dimensionless transition integrals that enter into the transition matrix element are given. Since the integrands decay into the bulk, they are easily evaluated using a cutoff. The integral giving the scalar-potential contribution to the absorption is (normalization constants have been moved to the left hand side for brevity)

$$
\frac{F_1}{N_{\nu,F}^* N_{\nu,F}} = T_1 \int_{0}^{\infty} \phi^{\tau,k_{m,n}^F\nu}(y) \phi^{\tau,k_{m,n}^F\nu}(y) e^{-\lambda \Delta \tilde{k}_{m,n}^F} d\tilde{y},
$$
(C1)

and the pseudo-magnetic-field contribution integral is

$$
\frac{F_2}{N_{\nu,F}^* N_{\nu,F}} = \tau \int_{0}^{\infty} \phi^{\tau,k_{m,n}^F\nu}(y) \sigma_x \phi^{\tau,k_{m,n}^F\nu}(y) \times 
$$
$$
\left( -T_2 e^{\lambda \Delta \tilde{k}_{m,n}^F} + T_2 e^{-\lambda \Delta \tilde{k}_{m,n}^F} d\tilde{y} + 
$$
$$
\text{sgn}(\Delta \tilde{k}_{m,n}^F) T_2 y \int_{0}^{\infty} \phi^{\tau,k_{m,n}^F\nu}(y) \sigma_y \phi^{\tau,k_{m,n}^F\nu}(y) \times 
$$
$$
\left( e^{-\lambda \Delta \tilde{k}_{m,n}^F} - e^{\lambda \Delta \tilde{k}_{m,n}^F} \right) d\tilde{y}. \quad (C2)
$$

The numerical normalization constants are given by

$$
\left| N_{\nu,F}^* \right|^2 = \frac{1}{\int_{0}^{\infty} \left| \phi^{\tau,k_{m,n}^F\nu}(y) \right|^2 d\tilde{y}}. \quad (C3)
$$