Soliton transverse instabilities in anisotropic nonlocal self-focusing media

Kristian Motzek and Friedemann Kaiser
Institute of Applied Physics, Darmstadt University of Technology, D-64289 Darmstadt, Germany

Wen-Hen Chu and Ming-Feng Shih
Physics Department, National Taiwan University, Taipei, 106, Taiwan

Yuri Kivshar
Nonlinear Physics Group, Research School of Physical Sciences and Engineering, Australian National University, Canberra ACT 0200, Australia

We study, both theoretically and experimentally, the transverse modulational instability of spatial stripe solitons in anisotropic nonlocal photorefractive media. We demonstrate that the instability scenarios depend strongly on the stripe orientation, but the anisotropy-induced features are largely suppressed for spatial solitons created by self-trapping of partially incoherent light.

PACS numbers:

Nonlinearity-driven instabilities have been studied in many different branches of physics, since they provide simple means to observe strongly nonlinear effects in Nature. Transverse (or symmetry-breaking) instabilities of solitary waves have been predicted theoretically almost 30 years ago [1], but only recently both transverse and spatiotemporal instabilities were observed for different types of bright and dark spatial optical solitons [2, 3].

Many of the experimental studies of spatial optical solitons in a nonlinear bulk medium are being carried out for photorefractive crystals known to exhibit an anisotropic nonlocal response characterized by an asymmetric change of the refractive index. Given the strong anisotropy of the photorefractive nonlinearities, it is crucially important whether the theoretical results obtained mostly for isotropic nonlinear media [2] can be applied, at least qualitatively, to the case of anisotropic nonlocal media. In particular, the transverse instabilities of solitary waves, that develop under the action of higher-order perturbations or temporal effects, are known to initiate a breakup of a stripe soliton, and several different scenarios of the breakup dynamics are known [2]. The most interesting scenario is the generation of new stable localized structures. In the scalar case, this corresponds to a breakup of a soliton stripe into (2+1)-dimensional bright solitons in a self-focusing medium.

The purpose of this Letter is twofold. First, we employ a simple model of an anisotropic nonlocal medium which takes into account important properties of photorefractive nonlinearity [4] and demonstrate numerically that the classical scenario of the soliton transverse instability, namely the stripe breakup and the formation of two-dimensional spatial solitons [2], depends dramatically on the stripe orientation; these results are fully confirmed by our experimental studies of the breakup of the vertical, horizontal, and tilted soliton stripes. Second, we generalize the familiar coherence-function approach, which is employed for describing the propagation of partially incoherent light in nonlinear media, and study the transverse instability of partially incoherent soliton stripes in anisotropic nonlocal media. We demonstrate theoretically and confirm experimentally that strong anisotropy-driven features of photorefractive nonlinearity are largely suppressed by spatial partial incoherence of light.

We consider the propagation of a single optical beam with the slowly varying amplitude $E$ in a biased photorefractive crystal, described by the paraxial equation

$$i \frac{\partial E}{\partial z} + \frac{1}{2} \nabla_\perp^2 E = \frac{\partial \varphi}{\partial x} E, \quad (1)$$

where $\nabla_\perp$ stands for the transverse gradient in the plane perpendicular to the propagation direction $z$ and $\partial \varphi/\partial x$ yields the electric field inside the crystal. We assume that an external electric field is applied to the crystal, and it is parallel to the direction of $x$, so that the electric potential $\varphi$ is defined by the potential equation [3]

$$\nabla_\perp^2 \varphi + \nabla_\perp \varphi \nabla_\perp \ln(1 + I) = \frac{\partial}{\partial x} \ln(1 + I), \quad (2)$$

where $I = |E|^2$ is the light intensity inside the crystal.

First, we study numerically the nonlinear evolution of a narrow stripe oriented perpendicular to the external electric field. Figures 1(a-c) show the visualized images of the nonlinear evolution at the input [(a)] and at two different propagation distances [(b),(c)]. In numerical simulations, the initial diameter of the input Gaussian beam was chosen to be close to that of a solitary solution, so the width of the vertical beam remains roughly the same. As a result, narrow beams evolve in a fashion that is very similar to the formation of quasi-one-dimensional spatial solitons. Increasing nonlinearity (i.e. the applied field) leads to a breakup of the vertical stripe due to the transverse modulation instability, first discussed for photorefractive crystals by Mamaev et al. [2, 4, 5]. At the initial stage of the breakup all spatial harmonics of the noise are small and each is amplified exponentially with its own growth rate. The fastest-growing modes become noticeable first.
FIG. 1: Numerical results for the coherent stripe propagation: (a-c) the soliton stripe is perpendicular to the external electric field, (d-f) the soliton stripe is parallel to the field, and (g-i) an intermediate orientation of a tilted stripe.

FIG. 2: Numerical results for the partially incoherent stripes. (a,b) Stripe is perpendicular to the external field, and (c,d) stripe is parallel to the field. Shown are the input [(a,c)] and output [(b,d)] beams. Insert shows the growth rate of the transverse instability vs. the coherence parameter $\theta_0$.

FIG. 3: Experimental observation of the transverse instability of coherent soliton stripe. (a,d,g) inputs for three orientations of the stripes. All other images are taken at the crystal output (length is 7 mm) for the applied voltage of (b,h) 1 kV, (c,i) 2 kV, (e) 0.5 kV, and (f) 1 kV, respectively.

We have also studied numerically the propagation of partially incoherent light stripes in an anisotropic photorefractive medium described by the model \((1, 2)\). We have extended the coherent density approach \([7]\), earlier used for the study of isotropic media (see, e.g., Ref. \([3]\)), and described the anisotropic nonlocal response according to Eq. \((2)\). The coherent density approach is based on the fact that partially incoherent light can be described by a superposition of mutually incoherent light beams that are tilted with respect to the $z$-axis at different angles. One thus makes the ansatz that the partially incoherent light stripe consists of many coherent, but mutually incoherent light stripes $E_j$: $I = \sum_j |E_j|^2$. By setting $|E_j|^2 = G(j\vartheta)I$, where $G(\vartheta) = (\pi^{1/2}\theta_0)^{-1} \exp(-\vartheta^2/\theta_0^2)$ is the angular power spectrum, one obtains a partially incoherent light stripe whose coherence is determined by the parameter $\theta_0$, i.e. less coherence means bigger $\theta_0$. Here, $j\vartheta$ is the angle at which the $j$-th beam is tilted with respect to the $z$-axis. If the light is incoherent along the $y$-axis (vertical stripes), they are tilted in $y$-direction, whereas they are tilted in the $x$-direction if the light is to be incoherent along that axis (horizontal stripes).

Figures 2(a,b) show our numerical results for the propagation of a vertical stripe close to the stability threshold with the degree of incoherence determined by $\theta_0 = 0.43^\circ$. The external electric field is perpendicular to the stripe. The most obvious difference to the scenario of the coherent-stripe decay is that the filaments are much more elongated. Furthermore, they change their profile only very slowly as they propagate and thus can be con-
FIG. 4: Experimental results for the stripes oriented perpendicular (two upper rows) and parallel (two lower rows) to the electric field. Images (a,g) and (d,j) correspond to the input and output of the coherent light propagation (7 mm), presented for comparison; whereas all other images correspond to partially incoherent light.

sidered as incoherent solitons. Larger values of \( \theta_0 \) (i.e. \( \theta_0 > 0.43^\circ \)) correspond to a complete suppression of the soliton transverse instability.

Figures (c,d) show the propagation of an incoherent light stripe parallel to the external electric field with \( \theta_0 = 0.56^\circ \). Increasing the incoherence further leads to the case where the beam diffracts before the transverse instability can set in. Obviously the filaments have the same size as in the coherent case, but there is more intensity lost to radiation. This confirms previous results that it is very difficult to obtain solitary structures that are elongated along the axis of the external field.

We performed a number of experiments to study the development of the anisotropy-driven soliton transverse instability in photorefractive crystals, for both coherent [see Fig. 3] and partially incoherent [see Fig. 4] soliton stripes. Experiments are conducted in a photorefractive SBN-61 crystal in a setup similar to that employed for the observation of the self-trapping of partially incoherent light. We also illuminate the entire crystal with a background light with intensity much stronger than that of the soliton stripe to make the nonlinearity close to a Kerr-type self-focusing nonlinearity.

The beam is made spatially incoherent by passing it through a rotating diffuser. The rotating diffuser provides a new phase and amplitude distribution every 1 ms, which is much shorter than the response time (~1 s) of the medium. We follow Anastassiou et al. and generate a beam which is very narrow and fully coherent in one direction (say x), yet uniform and partially incoherent in the other direction (say y).

Figures (a-c) and (d,a,d) show the development of the transverse instability for the coherent stripe perpendicular to the electric field direction. As the degree of incoherence grows, the instability weakens [see Fig. 4(b,e)], and then it disappears completely [see Fig. 4(c,f)] because, in order to observe the instability, the value of the nonlinearity has to exceed a threshold imposed by the degree of spatial coherence \( \xi \). The nonlinearity is turned on by applying a voltage of 3 kV to the photorefractive crystal with \( V_{33} = 250 \text{ pm/V} \). Figures (d-f) and (g,l) show the development of instability for the stripe parallel to the field. An important observation is that the strong anisotropy-driven effects observed for coherent light are largely suppressed when the degree of spatial incoherence exceeds a threshold [cf. Fig. 3(f) and 4(l)].

In conclusion, we have studied theoretically and experimentally the transverse modulational instability of coherent and partially incoherent soliton stripes in photorefractive crystals. We have demonstrated a number of novel, anisotropy-driven features of the stripe instability, as well as analyzed the effect of partial incoherence.

Y.K. thanks Glen McCarthy for valuable discussions, and the Physics Department of the Taiwan University for hospitality. This work was partially supported by the Australian Academy of Science and the National Science Council, Taiwan.

[1] V.E. Zakharov and A.M. Rubenchik, Zh. Eksp. Teor. Fiz. 65, 997 (1973) [Sov. Phys. JETP 38, 494 (1974)].
[2] See the review paper by Yu.S. Kivshar and D.E. Pelinovsky, Phys. Rep. 331, 117 (2000), and references therein.
[3] See, e.g., Yu.S. Kivshar and G.P. Agrawal, Optical Solitons: From Fibers to Photonic Crystals (Academic, San Diego, 2003), 560 pp.
[4] See, e.g., A.A. Zozulya and D.Z. Anderson, Phys. Rev. A 51, 1520 (1995).
[5] A.V. Mamaev, M. Saffman, D.Z. Anderson, and A.A. Zozulya, Phys. Rev. A 54, 870 (1996).
[6] A.V. Mamaev, M. Saffman, and A.A. Zozulya, Europhys. Lett. 35, 25 (1996).
[7] D.N. Christodoulides, T.H. Coskun, M. Mitchell, and M. Segev, Phys. Rev. Lett. 78, 646 (1997).
[8] C. Anastassiou, M. Soljacic, M. Segev, E.D. Eugenev, D.N. Christodoulides, D. Kip, Z.H. Musslimani, and J.P. Torres, Phys. Rev. Lett. 85, 4888 (2000).
[9] M. Mitchell, Z. Chen, M. Shih, and M. Segev, Phys. Rev.
Lett. 77, 490 (1996).