A Spectral Approximate Strategy for Energy Management of Hybrid AC/DC System With Uncertainty

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\textbf{ABSTRACT} High proportion of renewable energy and AC-DC hybrid structure are the challenges that the power system has faced. In order to cope with the high-dimensional uncertainties in the hybrid AC/DC system, this work developed a novel method of combining the uncertainty quantification and stochastic optimization to solve a hybrid AC-DC energy management problem. The problem is approximately reformulated by a conic quadratic optimization model, while a surrogate model of random variables and polynomial manipulation for a risk constraint are proposed to handle the tricky. Given a set of samples in the sparse random space generated by Smolyak's algorithm, thus the problem could be handled by using a high-performance solver. Simulation results verify that our proposed approach not only reduces the computational cost for energy management under uncertainties but also provides more accurate statistical information for risk assessment.

\textbf{INDEX TERMS} Renewable energy, uncertainty quantification, stochastic dispatch, spectral approximation.

I. INTRODUCTION

With the rapid growth of the Distributed Generators (DG), uncertainty is becoming a big issue in power system operations and planning. In order to improve the utilization rate of renewable energy, it can be seen great values in practical engineering that there are studies on actively energy management of the flexible hybrid AC/DC grid architecture. In modeling of the energy management, it makes the optimization intractable considering both constraints and uncertainties in the nonlinear power flow equalities of a power injections. Therefore, many researchers concentrate on solving the above tricky issues [1], [4].

To handle the nonlinear constraints, the cuckoo search algorithm has been successfully used to solve the nonconvex optimization problem of power dispatch [2], [3]. To handle the uncertainties, there are numerous statistical model and physically-based prediction algorithms [5] have been developed for forecast the renewable power. But even so it is still not sufficient for the accurate forecasting of renewable power. Therefore, the energy management seeks to minimize the voltage fluctuation obtained and the cost by considering uncertainties in the hybrid AC/DC system. [6] presents the economic dispatch considering the model of connecting equipment between the AC and DC region. Developing the uncertainty set based on the assessment of the underlying renewable power curtailment risks [7], the economic optimal solution is obtained for energy management in hybrid AC/DC system. The researches in [32] established an optimal scheduling framework of coordination between utility and supply in hybrid AC/DC system. It gives robust optimal solution performing best with respect to the worst-case situation under multiple uncertainties. In order to optimize the AC-DC network, the stochastic optimization framework is proposed in [8], [33] where both costs and reliability are evaluated. The stochastic energy management problem is described for dispatching power to minimize the expected average of operation cost, while the random variables are approximated by a group of probabilistic scenarios [9]. In addition, chance-constraint is introduced to the...
optimization, so that decision makers have a desired trade-off between cost and reliability [10], [11]. The bilinear reformulation is a fast algorithm, which is developed in [36], to handle the chance constraint. The strengthened bilinear algorithm is adopted for solving the energy planning in power system with uncertainty [34], [35]. Its disadvantage is that it is computational tractability in the case of large system. Particularly, it can not provide perfect information of samples in the random space for high-dimensional random variables.

The generalized polynomial-chaos (gPC) is an expansion of orthonormal basis functions, it is generally studied as a approximation of the solution space of the problem with random input [12]. In addition, various statistical information (such as moments and probability density) of the solution can be provided with help of the gPC expansion. Avoiding computationally expensive sampling, [13] proposed a first order polynomial description of uncertainty in the hybrid AC-DC system. Applying this polynomial approximation of the random variables, it gives a new idea for solving the stochastic Optimal Power Flow (OPF) problem [14], [15]. It is worth mention that the structurally equivalent deterministic model is reformulated to approximate the stochastic problem with high dimensional random inputs. And not only that, the second order cone constraint of gPC coefficients is proved to be a reformulation of the chance constraint by means of Galerkin projection [16], so long as the coefficients are regarded as the auxiliary decision variables in the problem. However, the high-dimensional random variables need large number of basis functions in the gPC. The needed set of samples of high-dimensional random variables is large for the coefficients construction, and this involves an unavoidable computational cost. Some researchers struggle to tackle the issues, they propose techniques, such as compressed sensing [17], reduced basis [18], low rank decomposition [19], to reduce computation cost of resolving the high-dimensional stochastic problem.

In our manuscript, the energy management problem of the hybrid AC-DC system is formulated as a multi-period AC-DC OPF with risk-based chance constraints. It seeks economic optimal solution while considering the uncertainty over a finite operation horizon. To handle the multiple uncertainties, the surrogate model of uncertainty based on polynomial approximation is proposed. And then the random variable is substituted by the linear combination of basis functions, the basis functions are chosen to be orthogonal according to the probability distribution of the random variables. It is proved that the Chebyshev generating function gives a approximate expression of the chance-constraint. According to the orthogonality of the polynomial basis, the constraint of the probability of some collision below a specified threshold is relaxed by the conic constraint of polynomial coefficients. To break the curse of dimensionality, we build sparse gPC approximation of the stochastic optimal power flow upon the reformulation proposed in [14]. Given a set of samples which is one of the collocation points chosen in terms of sparse tensor product spaces, the optimal solution is obtained by solving the deterministic equivalent optimization problem. The most noteworthy is that the number of samples is reduced dramatically, while preserving a high level of accuracy and thus being able to successfully compete with Monte Carlo. The main contribution of this research is listed:

- A novel spectral strategy for energy management with uncertainties is proposed considering the risk constraint in the hybrid AC/DC system.
- With the surrogate model of stochastic variables and polynomial manipulation of the risk constraint, the energy management problem is reformulated by the determinist conic quadratic optimization problem. To handle the tricky of high-dimensional uncertainties in the optimization, the stochastic collocation method is utilized to reconstruct the gPC from the sparse grid points.
- In terms of the stochastic spectral model, the proposed strategy in this study not only obtains the optimal strategy but also give more statistic information of variables and objectives under uncertainty.

This paper is organized as follows. Section II gives the stochastic spectral strategy for the uncertainty quantification in power flow. In section III, the energy management model is developed based on the spectral representation of stochastic variables and risk constraints. And then, the case studies are discussed in section IV for estimating the proposed spectral energy management strategy and section V gives the conclusion of this paper.

II. UNCERTAINTY QUANTIFICATION IN POWER FLOW

A. POWER FLOW IN HYBRID AC-DC NETWORK

As shown in Fig.1, the hybrid AC-DC network is composed of AC and DC regions, and the two regions are connected via converters. In this study, the AC-DC power conversion is accomplished by the voltage source converter (VSC). In the energy management, it is particularly important to guarantee the power balances between the AC-DC supply and demand. Therefore, the power flow equations in the hybrid AC/DC system not only consider the AC/DC power but also the exchanging power between the two regions. The model of the converter coupled by its controlling strategy brings more challenges in the AC-DC hybrid power flow analysis.

For the power network, we use $\psi_j := (\pi_j, j)$ to represent the $j$th branch from bus $\pi_j$ to bus $j$, where the $\pi_j$ denotes the parent node of the $j$th branch. A tree graph $(N, \Psi)$ is utilized to represent the hybrid AC-DC network, where the vertex set $N$ and the edge set $\Psi$ denotes the buses and the branches respectively.

1) AC POWER EQUATIONS

In the AC region, the power flow equations of the AC branches are built according to the Ohm’s law and power balance at each AC bus [23]. Let $\Psi_{AC}$ be the edge set of AC branches.
For $\forall j \in \Psi_{AC}$,

$$P_{j,t} = \sum_{k:(j,k) \in \Psi_{AC}} P_{k,t} - R_j I_{j,t}^2 = P_{j,t}^D - P_{j,t}^G$$

(1)

$$Q_{j,t} = \sum_{k:(j,k) \in \Psi_{AC}} Q_{k,t} - X_j I_{j,t}^2 = Q_{j,t}^D - Q_{j,t}^C$$

(2)

$$V_{\pi,j,t}^2 - V_{j,t}^2 - 2(R_j P_{j,t} + X_j Q_{j,t}) + (R_j^2 + X_j^2) I_{j,t}^2 = 0$$

(3)

$$P_{j,t}^2 + (Q_{j,t})^2 = I_{j,t}^2 V_{\pi,j,t}^2$$

(4)

where $\psi_{jk} := (j,k)$ denotes the branch from bus $j$ to $k$ and the parent node $\pi_j$ is bus $j$. $R_j(X_j)$ is the resistance(reactance) of the $j$th branch. $P_{j,t}(Q_{j,t})$ is the sending-end active(reactive) power of the $j$th branch. $I_{j,t}$ is the current passing through the $j$th branch. $V_{j,t}$ is the voltage of bus $j$. $P_{j,t}^G(Q_{j,t}^G)$ and $P_{j,t}^D(Q_{j,t}^D)$ are the active(reactive) power supplied and consumed at bus $j$ respectively.

2) DC POWER EQUATIONS

In this study, the DC branch is represented as a special AC branch with none reactance [24]. The power flow equations of DC region are given by, for $\forall j \in \Psi_{DC}$,

$$P_{j,t} = \sum_{k:(j,k) \in \Psi_{DC}} P_{k,t} - R_j I_{j,t}^2 = P_{j,t}^D - P_{j,t}^G$$

(5)

$$V_{\pi,j,t}^2 - V_{j,t}^2 - 2(R_j P_{j,t})I_{j,t} = 0$$

(6)

$$P_{j,t}^2 = I_{j,t}^2 V_{\pi,j,t}^2$$

(7)

where $\Psi_{DC}$ is the edge set of the branches in DC network region.

3) THE INTERLINKING CONVERTER

The equivalent model of VSC is described as AC branch with a impedance $Z_{vsc} = R_e + jX_e$ and a controllable branch device [25], [26]. As shown in Fig.2, it can also supply or consume reactive power from the AC region to hold a constant voltage. Thus the power flow calculation of VSC is divided into two parts. At the AC sides of VSC, the active-power injections $e_{p,e}$, reactive-power injections $e_{q,e}$, the output AC voltage $V_{\text{out}}$, the controllable active and reactive power $p_{e,t}, q_{e,t}$ satisfy the AC power flow equations (1)~(4).

Besides, the control strategy of the VSC is described by:

$$p_{e,t} = e_{p,e}$$

(8)

$$p_{e}^{\text{min}} \leq e_{p,e} \leq p_{e}^{\text{max}}$$

(9)

$$q_{e}^{\text{min}} \leq e_{q,e} \leq q_{e}^{\text{max}}$$

(10)

$$V_{\text{out}} \leq \rho_v e_{p,e}$$

(11)

where $\rho_v$ is the modulation factor of the voltage. $p_{e}^{\text{min}}(q_{e}^{\text{min}})$ and $p_{e}^{\text{max}}(q_{e}^{\text{max}})$ are the limits of the controllable active(reactive) power of VSCs. At the DC side of VSC, the DC voltage $V_{e,t}$, current $e_{t}$ and the active power injection $e_{p,e}$ satisfy the DC power equations.

4) THE CONIC APPROXIMATION

From what has been mentioned above, we may find that the power equations are nonlinear. The conic approximation of the nonlinear equality make its optimization problem tractable for calculation with most commercial optimization solver. Let $V_{j,t} := V_{j,t}^2, I_{j,t} := I_{j,t}^2$, we substitute them into the power flow equations (1)~(11) and relax the equality (4), (7) into the inequality:

$$(P_{j,t})^2 + (Q_{j,t})^2 \leq I_{j,t} V_{\pi,j,t}$$

(12)

$$(P_{j,t})^2 \leq I_{j,t} V_{\pi,j,t}$$

(13)

And, thenceforth, taking into account the conic hybrid AC-DC power flow equation (1)~(3), (5)~(6), (8)~(11), (12), (13) as described above, the energy management problem are convex.

B. SPECTRAL MODEL OF UNCERTAINTY IN POWER FLOW

The hybrid AC-DC system contains renewable-based generators, there are uncertainties in the injected power not only due to the random characteristic of wind and solar radiation but also due to the load forecast. According to

$$p_{j,t}^G = p_{j,t}^G + \omega_{j,t}(\xi)$$

(14)

Although many efficient algorithms give good forecasting results, the actual power value $p_{j,t}^G$ randomly fluctuates near the forecasted value $p_{j,t}^G$. The $\omega_{j,t}(\xi)$ in above equation is used to depict the fluctuating part of the injected power, it can be a Gaussian or other distribution.
The uncertainties of injected power further influence the voltage and branch current in the power system. The truncated generalized polynomial-chaos (gPC) approximation is commonly used as a tractable model of random variable.

\[ V_{j,t} \approx V_{j,t}^o(\xi) = \sum_{\alpha=1}^{N} \tilde{V}_{j,\alpha}(t) \phi_{\alpha}(\xi) \]  

(15)

in which \( \phi_{\alpha}(\xi) \) are the polynomial basis and orthonormal, \( \alpha \in \mathbb{N}_0^d : \sum_{i=1}^{d} \alpha_i \leq \alpha, ||\alpha|| \leq d \) is the set of multi-indices. If the dimension of \( \xi \) is \( d \), the number of expansion terms is \( N=\frac{(\alpha+d)!}{\alpha!d!} \). There are basis functions given by the distribution of the random variable \( \xi \), such as Hermite basis of normal variable, Legendre basis of uniform variable. And the above gPC model can be used for other state variable in power system.

The coefficient of gPC is calculated via Galerkin projection method or collocation method [27], and the well-studied compress sensing approach is popularly used to get sparse coefficients in the range of the errors permitted. Given a set of unstructured realization \( \{\xi_k\}_{k=1}^N \), the matrix \( \Phi \) is

\[ \Phi = \begin{pmatrix} \phi_{1}(\xi_1) & \phi_{2}(\xi_1) & \cdots & \phi_{N}(\xi_1) \\ \phi_{1}(\xi_2) & \phi_{2}(\xi_2) & \cdots & \phi_{N}(\xi_2) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{1}(\xi_K) & \phi_{2}(\xi_K) & \cdots & \phi_{N}(\xi_K) \end{pmatrix} \]  

(16)

and the column vectors

\[ V_{j,t}^o = [V_{j,t}^o(\xi_1), \ldots, V_{j,t}^o(\xi_K)]^T \]  

(17)

\[ \tilde{V}_{j,1}(t) = [\tilde{V}_{j,1}(t), \ldots, \tilde{V}_{j,N}(t)]^T \]  

(18)

For the set of samples \( \{\xi_k\}_{k=1}^K \), the solutions \( V_{j,t}^o \) are obtained via solving the power flow equations. Considering the data noise or truncated error, we can get the gPC approximation via solving the following least squares optimization,

\[ \min \| V_{j,t}^o - \Phi \tilde{V}_{j,1}(t) \|_2 \]  

(19)

Under certain assumptions, the sparse solution is get via solving the following \( l_1 \) minimization,

\[ \min \| \tilde{V}_{j}(t) \|_1 \]  

(20a)

s.t. \( \| V_{j,t}^o - \Phi \tilde{V}_{j,1}(t) \|_2 \leq \sigma_{cs} \)  

(20b)

The superiority of this method is that the set of realization is allowed incomplete, which is especially suitable for the high-dimensional random \( \xi \).

C. POLYNOMIAL MANIPULATION OF RISK EVALUATION

With a random vector \( \xi \) supported on a set \( \Xi \sub \mathbb{R}^d \), the power system take the evaluation of the probability of some collision (such as voltage limit) below a specified threshold very seriously. That is given by

\[ Pr\{V_{j,t}^o(\xi) \leq V_{limit}\} \geq 1 - \eta \]  

(21)

where \( \eta < 1 \) is used to be a very small value, indicating that the event of the voltage fluctuations in the specified threshold is with high probability.

As the voltage is represented by the gPC in Eq.(15), the mean and variance can be represented by the polynomial coefficients,

\[ \mathbb{E}[V_{j,t}^o(\xi)] = \tilde{V}_{j,1}(t) \]  

(22)

\[ \text{Var}[V_{j,t}^o(\xi)] = \sum_{a=2}^{N} \tilde{V}_{j,a}^2(t) \]

Theorem 1: The risk assessment equation (21) is equivalent to the quadratic constraint of polynomial coefficients, that is,

\[ V_{\text{limit}} - \tilde{V}_{j,1}(t) \geq \frac{1 - \eta}{\eta} \cdot \sqrt{\tilde{V}_{j,2}^2(t) + \ldots + \tilde{V}_{j,N}^2(t)} \]  

(23)

Proof: For the random variable \( V_{j,t}^o(\xi) \), it can be standardized by

\[ \delta_v(\xi) = \frac{V_{j,t}^o(\xi) - \mu_v}{\sigma_v} \]  

(24)

where \( \mu_v = \mathbb{E}[V_{j,t}^o(\xi)] \) and \( \sigma_v^2 = \text{Var}[V_{j,t}^o(\xi)] \).

Thus, the risk assessment (21) is reformulated by the following constraint

\[ \text{Pr}\{ \frac{V_{j,t}^o(\xi) - \mu_v}{\sigma_v} \leq \frac{V_{\text{limit}} - \mu_v}{\sigma_v} \} \geq 1 - \eta \]  

(25)

Substituting the (24) into the above constraint, that is

\[ \text{Pr}\{ \delta_v(\xi) \leq \frac{V_{\text{limit}} - \mu_v}{\sigma_v} \} \geq 1 - \eta \]  

(26)

Suppose the distribution of \( \delta_v(\xi) \) is \( C \), we define the function \( f_C(k) \) that

\[ f_C(k) = \text{Pr}\{ \delta_v(\xi) < k \} \]  

(27)

And the inverse function

\[ f_C^{-1}(\lambda) = \inf\{k \mid f_C(k) \geq \lambda\} \]  

(28)

Therefore, the constraint (26) is equivalent to the following inequality

\[ \frac{V_{\text{limit}} - \mu_v}{\sigma_v} \geq f_C^{-1}(1 - \eta) \]  

(29)

Since

\[ f_C(k) = \text{Pr}\{ \delta_v(\xi) < k \} = 1 - \text{Pr}\{ \delta_v(\xi) \geq k \} \]  

(30)

and according to the One-side Chebyshev’s inequality,

\[ \text{Pr}\{ \delta_v(\xi) \geq k \} \leq \frac{1}{1 + k^2} \]  

(31)

Thus,

\[ f_C(k) \geq 1 - \frac{1}{1 + k^2} \]  

(32)

and in terms of the definition of \( f_C^{-1}(1 - \eta) \) in (28), we have

\[ f_C^{-1}(1 - \eta) = \sqrt{\frac{1 - \eta}{\eta}}, \quad \text{if} \quad 0 < \eta < 1 \]  

(33)
Therefore,
\[ \frac{V_{\text{limit}} - \mu_v}{\sigma_v} \geq \sqrt{\frac{1 - \eta}{\eta}} \]  
(34)

In terms of the gPC approximation of voltage, the mean and variance are represented by the coefficients. The risk constraint is reformulated by
\[ V_{\text{limit}} - V_{j,t}(t) \geq \sqrt{\frac{1 - \eta}{\eta}} \cdot \sqrt{\frac{V_{j,2}(t)}{\eta} + \cdots + \frac{V_{j,N}(t)}{\eta}} \]
which is the quadratic inequality constraint of polynomial coefficients. So this concludes the proof.

III. ENERGY MANAGEMENT WITH SPECTRAL MODEL OF UNCERTAINTIES

As can be seen in Fig. 1, it is the general architecture of hybrid AC-DC system. As we know that some renewable energy based distributed generators (DGs) are not dispatchable, such as wind and photovoltaic (PV) power plants. It is necessary that the consumer’s demand power should be satisfied by the operation of power system. Since the inherent uncertainty of renewable energy exits, the crucial issue in this condition is to keep the balance between power generated and consumed at all time.

This paper proposes a stochastic optimization framework for scheduling the controllable power to maximize the economic benefit over a period in the hybrid AC-DC power system. In this framework, the input random parameters which come from the power prediction error are characterized by finite dimensional random space. And then, a surrogate model of the tricky chance constraint and uncertainty in the economic dispatch problem is explored.

A. STOCHASTIC OPTIMIZATION MODEL OF ENERGY MANAGEMENT

In this study, the goal of energy management in the hybrid AC-DC power system is to maintain the power balance between the supply and demand, to improve the voltage stability, and to minimize the operation cost with the uncertainty of injected power. Based on the system states, the objective functions is built that the overall objective is to get the solution.

where the index \( g = G, b, D \) denotes the dispatching power of controllable generator, energy storage and controllable load respectively. The constraints (35c) and (35f) denote the State of charge (SoC) constraints, \( \rho_b \) is the charging-discharging efficiency of battery and \( E_{j,t}^B \) is the stored energy of battery at time \( t \) [30].

In time period \( t \), the operation cost \( C_t(\xi) \) is composed of four terms: the fuel consumption cost, the ESS operation cost, the controllable load cost, and the exchanged power cost.

\[ C_t(\xi) = C_{t}^{f}(\xi) + C_{t}^{OM}(\xi) + C_{t}^{DP}(\xi) + C_{t}^{E}(\xi) \]  
(36)

where \( C_{t}^{f} = \sum_{i=1}^{n} \rho_{i}^{f} P_{i,t}^{g}(\xi) \Delta t \) is the fuel cost, \( C_{t}^{OM} = \sum_{i=1}^{n} \rho_{i}^{m} P_{i,t}^{m}(\xi) \Delta t \) is the operation and maintenance cost, \( C_{t}^{E} = \rho_{t}^{grid}(\xi) \Delta t \) is the cost of purchasing electricity from grid, \( \rho_{i}^{f}, \rho_{i}^{m} \) are the corresponding cost coefficient of generator; \( \rho_{t}^{grid} \) is the electricity price at time \( t \).

As can be seen, the optimization problem (35) is nonlinear stochastic optimization problem that is hard to obtain the optimal solution without treatment of the nonlinear equality constraints and uncertain part in the problem. It is well studied that the nonlinear power flow equations are relaxed and transformed into convex constraints [28]. MC method, robust optimization method and stochastic optimization are commonly used as a strategy to handle the uncertainty part.

B. SPECTRAL APPROXIMATION OF UNCERTAINTY IN ENERGY MANAGEMENT

As proposed above, the optimization strategy for the model of energy management in Eq.(35) is bedeviled by the curse of the dimensionality. For the Monte-Carlo method, the problem (35) needs a set of samples \( \{\xi_k\}_{k=1}^{K} \) to transform into the deterministic optimization problem approximately. When the dimension of the random variable \( \xi \) is high, \( d \gg 1 \), the sample set needed is too large that it takes a long time to get the solution.

In order to overcome the curse of the dimensionality, the polynomial manipulations as presented in Eq.(15) and Eq.(23) are a substitute for the random variables and the risk constraints into the problem. So the equivalent model is as follows:

\[ \min \sum_{t \in \mathcal{T}} \tilde{C}(t) \]  
(37a)

s.t. \( \forall \psi_j \in \Psi, t \in \mathcal{T}, \xi \in \Xi : \)

\[ (1) \sim (3), (5) \sim (6), (8) \sim (11), (12), (13). \]  
(35b)

\[ Pr[V_{j,2}(\xi) \geq V_{\text{limit}}] \leq 1 - \eta \]  
(35c)

\[ p_{min}^g \leq p_{j,t}^g \leq p_{max}^g, \quad g = G, B, D \]  
(35d)

\[ E_{j,t}^B = E_{j,0} + \sum_{t=1}^{T} \rho_b \cdot p_{j,t}^B, \]  
(35e)

\[ E_{\text{min}}^B \leq E_{j,t}^B \leq E_{\text{max}}^B \]  
(35f)
the Theorem 1 proposed above. The constraint (37d) considers the truncation error or the data noise of the samples in the practical application. Therefore, the energy management problem is reformulated into a deterministic nonlinear optimization problem. The surrogate model of uncertainty used in the optimization problem can keep the accuracy while overcoming the course of dimensionality.

C. SPARSE SPECTRAL STRATEGY

Based on the collocation points in the random space \( \Xi \), the problem (37) is solved deterministically. For the choice of the collocation points, it has become popular to use points which lie on a sparse grid in the random space generated by Smolyak’s algorithm.

The sparse points are linear combinations of product formulas with the following key properties. Then for the \( d \)-dimensional sparse grid, the points are computed by using the Clenshaw-Curtis rule. The Smolyak’s algorithm is given by \( A(d+k,d) \) [31], where \( k \) is the order of the integration rule and \( d \) is the dimension. Thus the value at the sparse points are described by:

\[
\mathcal{H}(d+k,d) = \bigcup_{i=1}^{d} (\Xi_{i1} \times \cdots \times \Xi_{id})
\]

\[
\Xi_i = \{\xi_i^1, \ldots, \xi_i^{m_i}\}
\]

where \( i = 1, \ldots, d \). The sparse grid is the set of points on \( \mathbb{N}^d \). For any choice of \( m_i > 1 \) these points are computed by:

\[
\xi_i^j = -\cos \left( \frac{\pi(j-1)}{m_i-1} \right), \quad j = 1, \ldots, m_i.
\]

where the knots \( m_i \) used in the formulas is chosen by

\[
m_i = \begin{cases} 1, & \text{for } i = 1 \\ 2^{i-1} + 1, & \text{for } i > 1 \end{cases}
\]

The collocation points \( \tilde{\xi} \in \mathcal{H}(d+k,d) \) are the hyperbolic cross points in the sparse grid \( \mathcal{H}(d+k,d) \). And the set of the hyperbolic cross points \( \{\tilde{\xi}^k\}_{k=1}^{24} \) are chosen to form the realization matrix \( \Phi \) in (16). In this way, we substitute the matrix into the Eq.(37d) to reconstruct the gPC which is the spectral model of uncertainties in the optimization problem (37).

IV. NUMERICAL RESULTS

The proposed strategy is tested on the 33-buses benchmark example which has been modified to a hybrid AC-DC system. The data of the system is given in [29]. There are four DC networks connected to the 33-buses AC network at bus 25, bus 33, buss 22, bus 14 via the VSC. The Table.1 gives the specification of the renewable-based DGs (WT and PV), Energy storage system (ES), MTs in the modified 50-buses hybrid AC-DC system. The simulations are implemented on a 64-bite PC, and the MOSEK toolbox in MATLAB is used to solve the optimization problem. The optimization horizon considered in the proposed energy management, \( T = \{1, 2, \cdots, 24\} \), is taken to be

24 hours in a day. Here, the order generalized Polynomial-chaos (gPC) expansion is taken to be three for approximating the corresponding random state variables in the distribution system. \( \eta = 5\% \), the acceptable probability of the violation of voltage limit is set to 5\%. Fig.3 gives the price curve of electricity purchase of power network, we expect the lowest cost in electricity purchase that the renewable energy is given a full play when the price is high. In Fig.4, there are the forecasted daily power curves of load, PV-based generator and WT-based generator respectively. The wind and solar energy power is complementary to play full advantages of renewable energy.

A. RESULTS OF ENERGY MANAGEMENT

Based on the sparse spectral model of energy management developed in this paper, the Monte Carlo (MC) simulation is introduced to compare the accuracy of the proposed algorithm.

The test results are summarized Table.2. The operation cost of the proposed energy management strategy is $4221.43, which is similar to the result of MC simulation method. It indicates the rationality of the pseudospectral model proposed in this work for approximation of the optimization problem.

| Type of Unit | Rate power (kW) | Bus No. |
|-------------|----------------|--------|
| WT          | 200            | 7,8,14,16,30,31 |
| PV          | 100            | 39,49  |
|             | 200            | 35,41  |
|             | 400            | 45     |
| MT          | 300            | 15,25  |
| ES          | 200            | 35,41,45 |

FIGURE 3. The price of electricity.

FIGURE 4. The power curve of load, PV and WT.

FIGURE 4. The power curve of load, PV and WT.
problem. Compared with the MC simulation which needs $10^7$ samples to get the solution, the proposed strategy only needs 220 samples that the strategy has more advantages in accomplishing the stochastic energy management and reach required accuracy. In the fifth column of the table, it is the maximum observed probability ($\eta_J$) of constraint violation for the different methods. It is can be seen that the probability of meeting voltage constraint is not strictly hold as expected. This inaccuracies is due to non-normally distributed samples, the difference is however not particularly large.

The operation cost is minimized through implementing the proposed energy management strategy. The curve of the purchased electricity throughout the day as can be seen in Fig.5, the purchase cost is reduced when the price is high.

Fig.6 gives the transmitted power of the VSCs which is passed between the AC region and DC region, the red dash dot line is the VSC between bus 14 and bus 34, the blue dash line is the VSC between bus 22 and bus 38, the green line is the VSC between bus 25 and bus 43, and the purple dot line is the VSC between bus 33 and bus 47. Since the output power of the PV-based generator increases when the solar energy is sufficient, the transmitted power from the AC side of VSCs decreases. As an interlinking unit, the converter stabilizes the DC region voltage.

### Table 2. Comparison of different approaches.

| Method       | Cost ($) | time (s) | sample size | $\eta_J$ (%) |
|--------------|----------|----------|-------------|---------------|
| Proposed algorithm | 4221.43  | 18.3     | 221         | 5.1           |
| MC           | 4218.73  | 5879     | $10^7$      | 5             |

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**FIGURE 5.** The daily purchased electricity of the hybrid AC-DC system.

**FIGURE 6.** The transmitted active power of VSCs.

**FIGURE 7.** Approximate gPC obtained by proposed algorithm vs. the reference.

**FIGURE 8.** Comparison of RMSE computed via Monte Carlo.

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**B. PERFORMANCE OF THE ASSOCIATE UNCERTAINTY QUANTIFICATION**

As can be seen in Fig.7, it displays the accuracy of the surrogate model obtained from the proposed algorithm in this study. In this figure, the blue squares represent the reference value of coefficients in the gPC approximation of the random voltage (take the bus 32 voltage as an example), the red asterisks circle the recovered coefficients of the proposed sparse algorithm. The reference is the gPC coefficient obtained using a Garlerkin projection, as compared with the recovered coefficients, resulted in an approximation error below $10^{-8}$.

To measure the performance of the approximation of the surrogate model, the Root-Mean-Square Error (RMSE) is used in this study. Fig.8 shows the RMSE in the polynomial approximations of different method when number of samples is increasing, the green line is MC method and the red dash dot line is the proposed sparse strategy. With increasing sample number, the RMSE of the sparse polynomial approximated by using the proposed method decrease sharply while the MC method converges slowly. Therefore, it is a feasible way choice to accomplish the stochastic dispatching of the hybrid AC-DC system example by using limited 221 samples of Smolyak sparse grid. Especially, in the case that the samples of uncertain variables are incomplete, the proposed algorithm result in smaller RMSE as compared with the MC.

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**V. CONCLUSION**

This work introduced a sparse polynomial approximation of uncertainty in the energy management of the hybrid AC-DC system. To approximate the stochastic solution, a large number of simulation samples are needed for solving the problem with high-dimensional uncertainties. Therefore, the gPC is
used as the surrogate model for representing the random variables, and the chance constraint for the risk management is relaxed into the conic constraint of the gPC coefficients. Constructing a set of samples in the sparse grid space of random solutions, the optimal solution is obtained via solving the equivalent deterministic optimization problem. The use of the sparse spectral approximation considered in this study allows us to treat effectively stochastic optimization problems with a moderately large number of random variables. Compared with the Monte Carlo sampling method which is a general method for approximately solving the chance-constraint stochastic programming problem, the proposed strategy not only can yield reduction in computational cost for solving energy management of the hybrid AC–DC system but also provide more accurate statistical information of risk assessment.

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