An improved constrained multi-objective optimization evolutionary algorithm for carbon fibre drawing process

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\textbf{ABSTRACT}

In this paper, an improved $\varepsilon$CMOEA/D-DE is proposed to improve the performance of CMOEA algorithm and achieve parameter optimization in carbon fibre drawing process. In order to avoid overusing the infeasible solutions, two repair operators are introduced in the population evolution model. More specially, during the evolutionary process, when the constraint violation of infeasible solution exceeds a tolerance threshold, the proposed two repair operators are used to find a better solution to repair the infeasible solution. On the other hand, in order to enhance the convergence rate of the Differential Evolution (DE), a modified DE is proposed. Then, an $\varepsilon$CMOEA/D-mDE-RO is proposed by incorporating these two improved strategies into the $\varepsilon$CMOEA/D-DE. Subsequently, the performance of the proposed $\varepsilon$CMOEA/D-mDE-RO is evaluated on the constrained test problem series and the experiment shows that the proposed algorithm outperforms the existing $\varepsilon$CMOEA/D-DE. Finally, in order to further illustrate the application potential, the proposed algorithm is successfully applied in optimizing carbon fibre drawing process and the optimal draw ratio vector is obtained.

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\section{1. Introduction}

The multi-objective optimization problem is common in the practical engineering area such as knapsack problems (Ponsich, Jaimes, & Coello, 2013), power system frequency control (Wang et al., 2017), carbon fibre drawing process (Chen, Ding, Jin, & Hao, 2013), and so on. It is well known that the evolutionary algorithm is an effective tool to solve the multi-objective optimization problem and, in the past decades, a variety of evolutionary algorithms have been proposed/developed such as genetic algorithm (Deb, Pratap, Agarwal, & Meyarivan, 2002), differential evolution (DE) algorithm (Bandyopadhyay & Mukherjee, 2015; Wang, Liao, Zhou, & Cai, 2014), extremal optimization algorithm (Li, Lu, Zeng, Wu, & Chen, 2016; Lu, Zhou, Zeng, & Du, 2018), PSO algorithm (Zeng \textit{et al.}, 2018; Zeng, Wang, & Zhang, 2016; Zeng, Wang, Zhang, \& Alsaadi, 2016; Zeng, Zhang, Liu, Liang, \& Alsaadi, 2017), and immune algorithm (Khaleghi, Farsangi, Nezamabadi-Pour, \& Lee, 2011). Note that the evolutionary algorithms mentioned above are only able to deal with the multi-objective optimization problems without constraints or with box-constraints. However, in most practical engineering applications, the objective functions to be optimized are accompanied with the complex equality or inequality constraints. Therefore, it is of practical significance to find a suitable constraint handling technology (CHT) so as to obtain an optimal solution to the constrained multi-objective problem (CMOP).

In recent years, a great deal of effort has been made on the CHTs. The developed CHTs mainly focus on the constrained single-objective optimization problem and the CHTs for the CMOP are relatively few. Among the existing CHTs for the CMOP, the penalty function is the most commonly used and the CMOP is usually solved by transforming it into an unconstrained multi-objective optimization problem. For example, in Riche, Knopf-Lenoir, and Haftka (1995), a genetic algorithm based segregated penalty function has been proposed and it has been shown that the performance is less sensitive to the penalty parameters. In Carlson and Shonkwiler (1998), an annealing penalty has been proposed by using the simulated annealing algorithm and a variable penalty function related to the temperature. Furthermore, in Hamida and Schoenauer (2002), an adaptive penalty function has been proposed which is helpful for making the algorithm search in the regions around the boundary parts. Nevertheless, it should be pointed out that in the algorithms based on the above penalty function,
the infeasible solutions are still overused (Jan, Khanum, & Tairan, 2016).

On the other hand, the \(\varepsilon\)-constraint theory based approach is capable of tuning parameters appropriately. For example, in Becerra and Coello (2016), an \(\varepsilon\)-constraint theory based hybrid multi-objective optimization algorithm has been proposed by using the evolutionary single-objective optimizer and the diversity of Pareto non-dominated solutions has been enhanced. Recently, in Yang, Cai, and Fan (2014), the \(\varepsilon\)-constraint theory and decomposition technology have been combined and the \(\varepsilon\)-CMOEA/D-DE has been proposed to solve the CMOP. However, in the \(\varepsilon\)-CMOEA/D-DE, the infeasible solutions have still been overused in the evolutionary process which may reduce the searching efficiency and the convergence rate of algorithm.

As analysis above, for a CMOP, two important performance indices should be concerned, i.e. searching efficiency and convergence rate. In order to enhance the searching efficiency, an alternative method is to use the infeasible solutions reasonably. In this paper, two repair operators are proposed to avoid overusing the infeasible. Specifically, during the evolutionary process, when the constraint violation of infeasible solution exceeds a tolerance threshold, the proposed two repair operators are employed to find a better solution to repair the infeasible one. With respect to convergence rate, we propose a modified DE, which extends the traditional DE in \(\varepsilon\)-CMOEA/D-DE in order to improve the convergence rate. By combining the proposed repair operators and modified DE, an improved \(\varepsilon\)-CMOEA/D-DE, i.e. the \(\varepsilon\)-CMOEA/D-mDE-RO is proposed to enhance the algorithm performance for the COMPs.

On the other hand, the carbon fibre drawing process can be viewed as a typical CMOP. Generally, the drawing process consists of six drawing steps, i.e. spinneret drawing, air drawing, DMF coagulation drawing, hot water drawing, boiling water drawing and third-class drawing (Chen, Ding, & Hao, 2013). In order to realize the optimization in the characters of the carbon fibre such as density, strength and breaking elongation ratio, the multiple draw ratios must be designed simultaneously. Actually, for multi-objective optimization problem for the carbon fibre drawing process, there have been a number of advanced approaches available in the existing literatures, see e.g. (Chen et al., 2013; Kalantari, Dong, & Davies, 1999; Salmalian, Nariman-Zadeh, Gharababei, Haftchenari, & Varvani-Farahani, 2010). In order to test the performance of the \(\varepsilon\)-CMOEA/D-mDE-RO proposed in this paper, we try to apply the \(\varepsilon\)-CMOEA/D-mDE-RO into the multi-objective optimization problem for the carbon fibre drawing process.

Based on the above discussion, the main contributions of this paper can be summarized in the following two aspects.

1. A new \(\varepsilon\)-CMOEA/D-mDE-RO is proposed where the improved two repair operators and the modified DE is capable of (a) tackling the problem of overusing infeasible solutions during the evolutionary process; (b) speeding up significantly the searching convergence; and (c) achieving a proper balance between the use of infeasible solutions and the optimal performance in the searching process.

2. The proposed \(\varepsilon\)-CMOEA/D-mDE-RO is successfully employed in the optimization of the carbon fibre drawing process and a set of optimal drawing ratio vector has been obtained which provides a reference for the practical production of the carbon fibre.

The remainder of the paper is organized as follows. Section 2 provides the basic knowledge of COMP. Section 3 gives the description of \(\varepsilon\)-CMOEA/D-DE. Section 4 details the proposed \(\varepsilon\)-CMOEA/D-mDE-RO. In Section 5, the comparative experimental results are provided and discussed. Section 6 show the comparative experimental results for the carbon fibre drawing process by applying the proposed \(\varepsilon\)-CMOEA/D-mDE-RO. Finally, conclusions are presented in Section 7.

2. Preliminaries

The CMOP under consideration is described as follows

\[
\begin{align*}
\min \quad & f(x) = [f_1(x)\ f_2(x)\ \cdots\ f_m(x)]^T \\
\text{s.t.} \quad & g_i(x) \leq 0, \quad i = 1, 2, \ldots, p \\
& h_j(x) = 0, \quad j = 1, 2, \ldots, q \\
& x_{\text{lower}} \leq x_i \leq x_{\text{upper}}, \quad l = 1, 2, \ldots, n,
\end{align*}
\]

(1)

where \(f_i(x) (i = 1, 2, \ldots, m)\), \(g_i(x) (i = 1, 2, \ldots, p)\), and \(h_j(x) (j = 1, 2, \ldots, q)\) are the real-scalar valued functions, \(x = [x_1\ x_2\ \cdots\ x_n]^T \in \mathbb{R}^n\) is an \(n\) dimensional decision vector and \(x_{\text{lower}}\) and \(x_{\text{upper}}\) are the lower and upper bounds of all decision variables \(x_i (i = 1, 2, \ldots, n)\).

In solving the above MOPs, the Pareto theory is often used. Some notions associated with the Pareto theory are given as follows (Martinez & Coello, 2014; Miettinen, 1999).

**Definition 2.1**: \(\forall x, y \in \Omega\), \(x\) is said to dominate \(y\) or \(y\) is said to be dominated by \(x\) (denoted as \(x \prec y\)) if and only if \(f_i(x) \leq f_i(y)\) for all \(i = 1, 2, \ldots, m\) and there is at least one \(i (i = 1, 2, \ldots, m)\) such that \(f_i(x) < f_i(y)\).
Definition 2.2: \( \forall x^* \in \Omega, x^* \) is said to be a Pareto non-dominated solution or Pareto optimal solution if and only if there is no other solution \( y \in \Omega \) such that \( y \prec x^* \).

Definition 2.3: The Pareto optimal set is expressed as \( \text{PS} = \{x \in \Omega | x \text{ is Pareto non-dominated solution} \} \), and the Pareto optimal front is defined by \( \text{PF} = \{F(x) | x \in \text{PS} \} \).

3. The description of \( \varepsilon \)-CMOEA/D-DE

As mentioned above, the proposed algorithm is improved based on \( \varepsilon \)-CMOEA/D-DE (Martinez & Coello, 2014). Thus, in this subsection the \( \varepsilon \)-CMOEA/D-DE is described firstly. The \( \varepsilon \)-CMOEA/D-DE integrates classical MOEA/D-DE (Zhang & Li, 2007) and an \( \varepsilon \)-constraint theory based CHT. The main idea of \( \varepsilon \)-CMOEA/D is to transform a CMOP into several scalar problems by using the Tchebycheff approach and then to obtain the Pareto optimal solutions by using the \( \varepsilon \)-constraint theory. Meanwhile, the DE operator and the polynomial mutation (PLM) operator (Tang & Tseng, 2013) are used to obtain the corresponding PS and make the optimization objectives converge to the obtained PF. Some relevant theories and methods involved in the algorithm are outlined as follows.

The Tchebycheff method is adopted to transform a CMOP into several scalar problems. Specifically, by using the Tchebycheff approach, a CMOP is decomposed into \( N \) scalar optimization subproblems by the \( \varepsilon \)-constraint theory. Meanwhile, the DE operator and the polynomial mutation (PLM) operator (Tang & Tseng, 2013) are used to obtain the corresponding PS and make the optimization objectives converge to the obtained PF. Some relevant theories and methods involved in the algorithm are outlined as follows.

The \( \varepsilon \)-constraint theory is used to evaluate the individual quality. The evaluation function sequence \((f, \Phi)\) is composed of objective function \( f \) and constraint violation \( \Phi \). Hence, for any \( \varepsilon \geq 0 \), the \( \varepsilon \)-comparison \( '\leq_\varepsilon' \) between the sequence \((f_1, \Phi_1)\) and the sequence \((f_2, \Phi_2)\) can be defined by

\[
(f_1, \Phi_1) \leq_\varepsilon (f_2, \Phi_2) \leftrightarrow \begin{cases} f_1 \leq f_2 & \text{if } \Phi_1, \Phi_2 \leq \varepsilon \\ f_1 \leq f_2 & \text{if } \Phi_1 = \Phi_2 \\ \Phi_1 < \Phi_2 & \text{otherwise} \end{cases} \tag{8}
\]

Note that, when \( \varepsilon = \infty \), \( \varepsilon \)-comparison \( '\leq_\varepsilon' \) is simplified as the unconstrained multi-objective optimization problem where only the objective functions are compared.
The DE operator and the PLM operator are used to make the optimization objectives converge to PF. The DE operator (Storn & Price, 1997) is defined as

\[
y_k = \begin{cases} 
  x_k^1 + F \cdot (x_k^2 - x_k^3) & \text{with probability } CR \\
  x_k^1 & \text{with probability } 1 - CR 
\end{cases},
\]

where \( CR \) and \( F \) are two empirical parameters, \( x_k^1 \) is a current individual in the \( k \)th generation, and \( x_k^2 \) and \( x_k^3 \) are taken from the neighbourhood whose indexes are \( r_2 \) and \( r_3 \), respectively.

The PLM operator (Tang & Tseng, 2013) is defined as:

\[
y_k' = y_k + \delta \cdot \delta_{\text{max}},
\]

where

\[
\delta = \begin{cases} 
  (2r)^{(1/q+1)} - 1 & \text{if rand}(0, 1) < 0.5 \\
  1 - [2(1 - r)]^{(1/q+1)} & \text{otherwise},
\end{cases}
\]

\( q \) is a positive number, \( r \in \text{rand}(0, 1) \), \( \delta_{\text{max}} = \max \{x_k(t) - x_k^L, x_k^U - x_k(t)\} \), and \( x_k^L \) and \( x_k^U \) represent the lower and upper bounds of \( x_k \), respectively.

### 4. An novel \( \varepsilon \)CMOEA/D-DE algorithm: \( \varepsilon \)CMOEA/D-mDE-RO

In this section, a new \( \varepsilon \)CMOEA/D-mDE-RO algorithm is proposed to further improve the search ability of the original \( \varepsilon \)CMOEA/D-DE algorithm. The main novelty of the proposed \( \varepsilon \)CMOEA/D-mDE-RO lies in the introduction of two repair operators and a modified DE operator into the population evolution model. More specifically, when the constraint violation of infeasible solution exceeds the tolerance threshold during the evolutionary process, the proposed two repair operators are used to find a better solution to repair the infeasible one. Furthermore, in order to overcome the disadvantage of the low convergence rate in traditional DE, a modified DE is proposed to improve the convergence rate. Compared to the original \( \varepsilon \)CMOEA/D-DE algorithm, the newly introduced two repair operators and the modified DE operator in the population evolution model make it possible for us to (1) make better use of infeasible solutions during the evolutionary process; (2) pursue strong convergence capability; (3) keep an adequate balance between the convergence and the diversity. As such, the proposed \( \varepsilon \)CMOEA/D-mDE-RO could not only exploit the infeasible solutions more reasonably than the \( \varepsilon \)CMOEA/D-DE algorithm, but also has higher convergence rate.

### 4.1. The proposed two repair operators

#### 4.1.1. The first repair operator

If the constraint violation \( \Phi \) of the \( i \)th individual \( \chi^i \) (the individual \( a \)) satisfying \( \Phi^i > \gamma \times \varepsilon \), we perform the repair operator on this individual. In the neighbourhood of \( \chi^i \), if the ratio of feasible solutions (RFS) satisfies RFS > 0.9, we consider this neighbourhood to be high-quality, and then we adopt the strategy that a solution \( y \) (the individual \( b \)) is selected via the minimum objective value \( g^b \) from this neighbourhood, as shown in Figure 1(a). The RFS indicator refers to the number of feasible solutions found in the neighbouring \( R \). This practice can be described as follows

\[
R = \{x^i | l \in E\},
\]

\[
y = \{x^i | \min(g^b | x^j \in T, z^0 \in R\},
\]

where \( E \) consists of the the subscript of each neighbour individual, \( R \) consists of each neighbour individual and \( y \) is the solution with minimum objective value \( g^b \) in the neighbouring \( R \).

If RFS \( \leq 0.9 \), we select a set of solutions satisfying the constraint violation \( \Phi^i \) or \( \Phi^i \) is denoted by \( T \). Then we use the objective function sorting to find a solution \( y \) (the individual \( b \)) via the minimum objective value \( g^b \), that is, the solution \( y \) is the one nearest to \( x^i \) in \( T \). Finally, let the solution \( y \) repair \( a \). This practice can be described as follows

\[
T = \{x^i | 0 < \Phi^i < \gamma \times \varepsilon, l \in E\},
\]

\[
y = \{x^i | \min(g^b | x^j \in T, z^0 \in R\},
\]

The principle of above approach is to make the algorithm concentrate to search in the infeasible region beside the boundary parts and then realize the purpose of using infeasible solutions efficiently and reasonably, which is depicted in Figure 1(b). However, if \( T \) couldn’t be found, it means that the constraint violation \( \Phi \) of all solutions in neighbour \( E \) satisfies \( \Phi^i \) or \( \Phi^i = 0 \) and the number of infeasible solutions is not less than two (if \( M = 20 \)). Therefore, we consider the neighbour \( E \) to be low-quality relatively, and then select a solution which is the farthest away from \( x^i \).

In Figure 1, the solid line in upper picture is the feasible region (FR), the solid line in the lower picture is the infeasible region (IR). Figure 1(a) shows the repair process of the case of RFS > 0.9 where the individual \( b \) replaces the individual \( a \). Figure 1(b) shows the replacement process satisfying preconditions that RFS < 0.9 where the neighbouring \( T \) can be found and then the individual \( b \) repairs the individual \( a \). The region between solid line and dotted line is the infeasible region beside the boundary parts, and the infeasible solutions in this region are deemed the
high-quality ones. By this approach, the infeasible solutions are used reasonably and the algorithm performance is hence improved.

### 4.1.2. The second repair operator

The main idea of second repair operator is to inhibit the excessive convergence in the process of population evolution. If constraint violation $\Phi^i$ of the infeasible solution $x^i$ exceeds $\gamma \times \varepsilon$, an better solution $y$ (the individual $b$) is taken from its neighbouring $U$ to repair it. More concretely, the solution $y$ that is the closest to the Pareto optimal solution in the previous generation is selected, and then repairs the inferior $x^i$. The main steps are given as follows.

1. If $\Phi^i > \gamma \times \varepsilon$, we select a set of solutions which satisfies the constraint violation $\Phi < \Phi^i$, denoted by neighbouring $U$.
2. Then, we use objective function sorting to find a solution $y$ via the minimum objective value $g^{te}$.
3. Finally, the solution $y$ repairs the original solution $x^i$.

The notations $U$ and $y$ are formulated as follows:

$$U = \{x^i | 0 < \Phi^i < \Phi^*_i, i \in E\}, \quad (16)$$

$$y = \{x^i | \min(g^{te}(x^i|w^i, z^*)), x^i \in U\}. \quad (17)$$

In the above equations, the weight of $x^i$, $w^i$ remains to be constant which is different from the one in Equation (13) and Equation (15) where the weight are varying.

The advantages of this repair operator are that the algorithm is inhibited to generate individuals within excessive constraint violations at the beginning of the algorithm evolutionary process. Additionally, the algorithm will accumulate the beneficial effect in the subsequent generations, and eventually improve the algorithm performance. The replacement process is depicted in Figure 2.

### 4.2. The modified DE operator

We propose a modified DE operator which has better search efficiency and convergence rate according to the design requirement of practical engineering problems. The modified DE operator and the two repair operators proposed above are combined in the proposed algorithm. The results on the CTP-series test instances show that the proposed DE operator can effectively speed up the convergence of the algorithm while improve the expense of diversity.

The main characteristic of the DE operator is mutation which is also seen as a variation or perturbation modelled by a random variable. The DE operator widely used in MOEA/D is described as follows.

$$y_k = \begin{cases} 
  x_k^{i_1} + F \cdot (x_k^{i_2} - x_k^{i_3}) \quad &\text{with probability } CR \\
  x_k^{i_1} \quad &\text{with probability } 1 - CR
\end{cases} \quad (18)$$

where $F$ is the scaling factor, $x_k^{i_1}$ is the decision variable vector, $x_k^{i_2}$ and $x_k^{i_3}$ are the donor vectors, $r_2$ and $r_3$ are
the indexes that are randomly selected from the
neighbourhood $E$ and $y_k$ is a trial vector generated by the DE
operator. It can be seen that, if $\text{rand}(0, 1) < CR$, the DE
operator is performed on $x^i_k$, see Figure 3(a).

Considering the design requirement of the practical
engineering problems, we propose a modified DE op-
erator which has better search efficiency and convergence
rate. The strategy can be outlined as follows. A base vector
$x_{\text{best}}$ is selected as the best solution in current population.
Then, by using DE operator generates the intermediate
vector $y_k^i$, let $y_k^i$ inherit any of decision variables from $x^i_k$
(the dimension $n$ of decision variables satisfying $n \leq 10$).
Finally, the operator generates the trial vector $y_k(x_1)$ or
$y_k(x_2)$, as shown in Figure 3(b).

Figure 3(a) represents the search space of a simple
DE operator in 2-D and Figure 3(b) represents the search
space of a modified DE operator. In Figure 3, $y_k(x_1)$ and
$y_k(x_2)$ are the trial vectors which inherits $x^i_k$ and $y^i_k$ is an
intermediate vector.

4.3. The framework of $\varepsilon$CMOEA/D-mDE-RO

The framework of $\varepsilon$CMOEA/D-mDE-RO1 is given in
Algorithm 1. Line 15 and Line 29 $\sim$ Line 33 are the modi-
ﬁed parts where Line 15 describes the modiﬁed DE oper-
ator (mDE) and Line 29 $\sim$ Line 38 are the pseudo-code of
the ﬁrst repair operator (RO1) that we proposed. Line 1 $\sim$
Line 28 and Line 34 $\sim$ Line 41 are the $\varepsilon$MOEA/D-mDE
where if Line 15 performs the DE operator on ($x^{i1}$, $x^{i2}$
and $x^{i3}$), the algorithmic framework can be simpliﬁed as
the $\varepsilon$MOEA/D-DE. Speciﬁcally, When the constraint
violation $\Phi$ of an individual satisﬁes $\Phi > \gamma \times \varepsilon$ (Line 29)
and RFS satisﬁes $\text{RFS} > 0.9$ (Line 30), the neighbouring $R$
(Line 31) is obtained in terms of the speciﬁed condition,
and then a superior individual is selected from the neigh-
bouring $R$ (Line 32). If $\text{RFS} \leq 0.9$ (Line 33), the neighbour-
ing $T$ (Line34) is obtained by the speciﬁed condition and
then a superior individual with the minimum objective
value $g^e$ is obtained from the neighbouring $T$ (Line 35).
At last, the superior individual repairs the original indi-
vidual (Line 9).

The framework of $\varepsilon$MOEA/D-mDE-RO2 can be
described by replacing the pseudo-code part of Line 29 $\sim$
Line 38 in Algorithm 1 with Algorithm 2. When the con-
straint violation $\Phi$ of an individual exceeds $\gamma \times \varepsilon$ (Line 1),
the neighbouring $U$ (Line 2) is obtained by the speciﬁed
condition. Then, a superior individual with the minimum
objective value $g^e$ is obtained from this neighbouring $U$
(Line 3). Finally, the superior individual repairs the original
one (Line 4).

Algorithm 1: The Framework of $\varepsilon$MOEA/D-mDE-RO1

| Input: |
|--------|
| $G$: the maximum iterations |
| $N$: the number of subproblems in $\varepsilon$MOEA/D-bDE-CHT2 |
| $W$: a group of weight vectors $w^1, \ldots, w^N$ |
| $M$: the number of weight vectors contained in the neighbourhood |
| $\delta$: a probability decides whether parent solutions are chosen |
| from the neighbourhood $B(i)$ or not |

| Output: |
|--------|
| $P$: the ﬁnal population found by $\varepsilon$MOEA/D-bDE-CHT2 |

1: Initialize a random population $P$ $\leftarrow x^1, \ldots, x^N$
2: $P' \leftarrow F(x^i)$
3: for $i$ $\leftarrow 1$ to $N$ do
4: $B(i) \leftarrow l_1, \ldots, l_M$, where $w^{i1}, \ldots, w^iM$ are the $M$ closest weight vectors to $w^i$
5: end for
6: $z^e \leftarrow (z^e_1, \ldots, z^e_\text{max})^T$
7: while the maximum iterations $G$ is not met do
8: for $i$ $\leftarrow 1$ to $N$ do
9: if $\text{rand}(0, 1) < \delta$ then
10: $E \leftarrow B(i)$
11: else
12: $E \leftarrow \{1, 2, \ldots, N\}$
13: end if
14: $r_1 \leftarrow i$ and select two indexes $r_2, r_3 \in E$ randomly
15: $y_{\text{c}} \leftarrow \text{Differential Evolution operator on } (x^{r_1}, x^{r_2}, x^{r_3})$ and $x_{\text{best}}$. $y_{\text{c}}$ is the best solution in current population
16: $y \leftarrow \text{Polynomial mutation operator on } y_{\text{c}}$
17: Calculate $f(y)$
18: for $j$ $\leftarrow 1$ to $m$ do
19: if $f(y) \leq z_j$ then
20: $z_j \leftarrow f(y)$
21: end if
22: end for
23: $c \leftarrow 0$
24: for $l$ $\leftarrow 1$ to $M$ do
25: $\Phi^l(i) \leftarrow \Phi_l(x^i)$
26: $\Phi_{\text{max}}(l) \leftarrow \max_i\{\Phi^l(i)\}$
27: $\Phi_{\text{min}}(l) \leftarrow \min_i\{\Phi^l(i)\}$
28: $\varepsilon \leftarrow \Phi_{\text{min}}(l) + \alpha \times (\Phi_{\text{max}}(l) - \Phi_{\text{min}}(l))$
29: $\text{if } \Phi_l(y) > \gamma \times \varepsilon \text{ then}
30: \text{if } \text{RFS} > 0.9 \text{ then}
31: $R = \{x^i \mid x^i \in E\}$
32: $y^d_{\text{min}} = \{x^i \mid \min_j(g^e(x^i|w^j, z^e_j)) \leq \varepsilon \}$
33: else
34: $T = \{x^i \mid \Phi_l(x^i) < \Phi_l(y), x^i \in E\}$
35: $y^d_{\text{min}} = \{x^i \mid \min_j(g^e(x^i|w^j, z^e_j)) \leq \varepsilon \}$
36: end if
37: $y \leftarrow y^d_{\text{min}}$
38: end if
39: if $g^e(y|w^i, z^e) \leq \varepsilon$, $\Phi_l(y) \leq \varepsilon$ and $c \leq n_i$ then
40: $x^i \leftarrow y$
41: $P' \leftarrow F(y)$
42: $c \leftarrow c + 1$
43: end if
44: end for
45: end for
46: end while

Algorithm 2: The second repair operator (RO2)

1: if $\Phi_l(y) > \gamma \times \varepsilon$ then
2: $U = \{x^i | \Phi_l(x^i) < \Phi_l(y), x^i \in E\}$
3: $y^d_{\text{min}} = \{x^i \mid \min_j(g^e(x^i|w^j, z^e_j)) \leq \varepsilon \}$
4: $y \leftarrow y^d_{\text{min}}$
5: end if
5. Experiment and result analysis

5.1. Experimental setting

In this subsection, we first introduce the experiment circumstance including the test instances with related parameters, the definition of performance index, the parameters used in the proposed algorithm and the operating circumstance.

5.1.1. Test instances and performance index

We choose eight test instances CTP1 ∼ CTP8 from the bi-objective CTP-series (Deb, Pratap, & Meyarivan, 2001; Liu, Peng, Gu, & Wen, 2016), which are shown as follows:

\[
\begin{align*}
\text{CTP1} & : \\
& \begin{cases}
    f_1(x) = x_1 \\
    f_2(x) = g(x) \exp(-f_1(x)/g(x)) \\
    g_1(x) = 0.858 \exp(-0.541f_1(x)) - f_2(x) \leq 0 \\
    g_2(x) = 0.728 \exp(-0.295f_1(x)) - f_2(x) \leq 0
  \end{cases}
\end{align*}
\]

\begin{equation}
\text{CTP2} \sim \text{CTP8} : \\
\begin{cases}
    f_1(x) = x_1 \\
    f_2(x) = g(x)(1 - f_1(x)/g(x)) \\
    g_1(x) = -a \sin b \pi [\sin(\theta)(f_2(x) - e) + \cos(\theta)f_1(x)]^2 + \cos(\theta)[f_2(x) - e] - \sin(\theta)f_1(x) \geq 0
  \end{cases}
\end{equation}

where \( g(x) = 1 + \sum_{j=3}^{n} (x_j - 2x_2 \sin(6\pi x_1 + j\pi/n))^2, x \in [0,1]^2 \times [-2,2]^{n-2}, n = 10 \). The parameters of CTP2 ∼ CTP8 are given in Table 1.

Additionally, we use the popular performance metric Hypervolume (HV) (Liu et al., 2016) as the performance index. HV metric can demonstrate both the convergence and diversity of Pareto non-dominated solutions in a sense (see in Figure 4). In Figure 4, we assume that \( Q = A, B, C, D \) is the Pareto non-dominated solutions set and thus the HV is the \( ABCDW \) enclosed by the discontinuous boundary, where \( W \) is the reference point. If the reference point is \((0,0)\) and test instances are minimized, then the smaller value is the HV metric, the better convergence and diversity are the algorithm. This can be described by the following formula:

\[
HV = \text{volume} \left( \bigcup_{i=1}^{Q} v_i \right), 
\]

where \(|Q|\) is the number of the Pareto non-dominated solutions, \(v_i\) is the diagonal corners of hypercube constructed with the reference point \(W\) and solutions \(Q\).

5.1.2. Parameters setting in \(\varepsilon\)CMOEA/D-mDE-RO and operating circumstance

In this experiment, the parameters are given as follows: population size \(N = 200\), the dimension of decision variable \(n = 10\), the adjustable threshold \(\tau\) of constraint violation degree \(\varepsilon = 0.2\), the initial reference point \(z^v = (+\infty, \ldots, +\infty)^T\), the neighbourhood size \(M = 0.1N = 20\), the maximum number of replacements in the neighbourhood \(n_r = 0.01N = 2\), the empirical parameter in Equation (5) \(\alpha = 0.9\), the empirical parameters in DE operator \(CR = 1.0\) and \(F = 0.5\), the empirical parameter and implemented probability of PLM \(q = 2\) and \(p_m = 1/n\), and a user-defined maximum number of generations \(G = 50\).

We independently run the algorithm for 30 times for each
Figure 4. Illustration the HV metric in 2-D space

test instance on an identical computer (Inter(R) Core(TM) i7−6700 CPU @ 3.40GHz, Matlab 9.0).

5.2. Experimental results and analysis

In this section, we implement three comparative experiments to evaluate the performance of the introduced $\varepsilon$CMOEA/D-mDE-RO algorithm. The first experiment is to test the performance of the proposed two repair operators by comparison $\varepsilon$CMOEA/D-DE incorporated with the first repair operator (termed as $\varepsilon$CMOEA/D-DE-RO1) and $\varepsilon$CMOEA/D-DE incorporated with the second repair operator (termed as $\varepsilon$CMOEA/D-DE-RO2, that is, $\varepsilon$CMOEA/D-DE). The second experiment is to demonstrate the performance of the modified DE operator by comparison $\varepsilon$CMOEA/D-DE equipped with modified DE (termed as $\varepsilon$CMOEA/D-mDE) and $\varepsilon$CMOEA/D-DE. The third experiment is to verify performance of the proposed $\varepsilon$CMOEA/D-mDE-RO by comparing with $\varepsilon$CMOEA/D-DE.

5.2.1. Comparison $\varepsilon$CMOEA/D-DE-RO and $\varepsilon$CMOEA/D-DE

In order to test the sensibility of the tolerance value $\gamma$, we set the parameter $\gamma = 2$ and $\gamma = 10$. The simulation results are shown in Figure 5. It can be seen from Figure 5 that, equipped with the two repair operators as the CHT approach, the algorithm performance is significantly improved in CTP6 $\sim$ CTP8; and the second repair operator approach is better than the first repair operator approach.

The corresponding HV values are listed in Table 2. From Table 2, it is seen that the value with an unserscore is a better result obtained by $\varepsilon$CMOEA/D-DE-RO1, the italic value is a better result obtained by $\varepsilon$CMOEA/D-DE-RO2 and the value with overstriking is the best result in each CTP test instance. Table 2 indicates that $\varepsilon$CMOEA/D-DE-RO2 ($\gamma = 10$) has the best algorithm performance in terms of HV values.

5.2.2. Comparison $\varepsilon$CMOEA/D-mDE and $\varepsilon$CMOEA/D-DE

By using the modified DE operator, the convergence is enhanced comparing with the traditional DE. The corresponding results are shown in Figure 6. It can be seen from Figure 6 that the $\varepsilon$CMOEA/D-mDE has higher search efficiency and convergence rate than $\varepsilon$CMOEA/D-DE.

5.2.3. Comparison $\varepsilon$CMOEA/D-mDE-RO and $\varepsilon$CMOEA/D-DE

After the above experimental results and analysis, we incorporate the proposed two strategies into $\varepsilon$MOEA/D (i.e. the $\varepsilon$CMOEA/D-mDE-RO and the simulation results of the $\varepsilon$CMOEA/D-mDE-RO and the traditional $\varepsilon$CMOEA/D-DE are given in Figures 7 and 8. From Figure 7, we can see that the proposed algorithm has better performance in all CTP-series test instances. Figure 8 shows a group of Pareto optimal fronts (PFs) obtained by $\varepsilon$CMOEA/D-mDE-RO, when the number of generations satisfying $C = 200$ and from Figure 8, it can be seen that the negative effects of the traditional DE operator disappears when the number of generations is large. That is, the algorithm deeply converges to the global optimal solutions.

6. The application of $\varepsilon$CMOEA/D-mDE-RO into optimization of carbon fibre drawing process

The carbon fibre drawing process is a typical CMOP. In the carbon fibre production process, precursor fabrication is a critical factor to ensure carbon fibre products to meet high-quality standards, and the drawing process is a crucial working procedure during precursor fabrication. Generally, the drawing process consists of six drawing steps, i.e. spinneret drawing ratio, air drawing ratio, DMF coagulation drawing ratio, hot water drawing ratio, boiling water drawing ratio and third-class drawing ratio (Chen et al., 2013; Wang, Ni, & Liu, 2000). In order to realize the purpose of optimizing the fundamental characters, e.g. density, strength and breaking elongation ratio, the multiple draw ratios should be selected reasonably. In this section, the proposed algorithms are employed to optimize the drawing ratios in carbon fibre drawing process, as an application example of the proposed algorithms.

6.1. Optimization problem

As analysis in Chen et al. (2013) and Wang et al. (2000), the optimization problem for the density and strength of
Figure 5. Average HV-metric values of $\varepsilon$ CMoEA/D-DE-RO on CTP-series test instances: (a) CTP1, (b) CTP2, (c) CTP3, (d) CTP4, (e) CTP5, (f) CTP6, (g) CTP7, (h) CTP8

Table 2. HV-metric results of CTP-series test instances.

| Algorithm          | CTP1   | CTP2   | CTP3   | CTP4   | CTP5   | CTP6   | CTP7   | CTP8   |
|--------------------|--------|--------|--------|--------|--------|--------|--------|--------|
| $\varepsilon$ CMoEA/D-DE      | 0.2167 | 0.4383 | 0.4522 | 0.6414 | 0.4266 | 1.7002 | 0.9423 | 2.0768 |
| $\varepsilon$ CMoEA/D-DE-RO1, $\gamma = 2$ | **0.2095** | 0.4382 | 0.4147 | 0.5790 | 0.4164 | 1.5087 | 0.6496 | 1.4416 |
| $\varepsilon$ CMoEA/D-DE-RO1, $\gamma = 10$ | 0.2209 | 0.4382 | 0.4359 | 0.6159 | 0.4178 | 1.5782 | 1.3403 | 1.5052 |
| $\varepsilon$ CMoEA/D-DE-RO2, $\gamma = 2$ | 0.2279 | 0.4112 | 0.4055 | 0.4702 | 0.4023 | 1.4013 | 0.5501 | 1.3707 |
| $\varepsilon$ CMoEA/D-DE-RO2, $\gamma = 10$ | 0.2136 | **0.4051** | **0.3979** | **0.4688** | **0.3921** | 1.4221 | 0.5901 | 1.3707 |

carbon fibre can be described as follows

$$\max F(x) = (f_1(x), f_2(x))^T$$

s.t. $x_{mul} = \prod_{k=1}^{6} x_l \in [10, 18]$, $l = 1, 2, \ldots, 6$

$$f_2(x) \geq f_{2,LTB}$$

$$1 \leq x_1 \leq 4.5, 1 \leq x_2 \leq 2$$

$$1 \leq x_3 \leq 2, 1 \leq x_4 \leq 6$$

$$1 \leq x_5 \leq 1.3, 1 \leq x_6 \leq 2.5,$$  \hspace{1cm} (22)

where $x_l$ is the drawing ratio of six drawing stages, $x_{mul}$ is the total drawing ratio, $f_{2,LTB}$ is the lower boundary and $f_1$, $f_2$ are, respectively, the objective functions representing the density, strength of carbon fibre which are given by

$$f_1 = \sum_{l=1}^{6} w_l \times 0.364 x_l^{0.99} \exp(-5.49/x_l),$$

$$f_2 = \sum_{l=1}^{6} w_l \times 0.37 x_l^{0.99} \exp(14.28/x_l).$$  \hspace{1cm} (23)
Figure 6. Average HV-metric values of $\varepsilon$CMOEA/D-DE and $\varepsilon$CMOEA/D-mDE on CTP-series test instances: (a) CTP1, (b) CTP2, (c) CTP3, (d) CTP4, (e) CTP5, (f) CTP6, (g) CTP7, (h) CTP8

Figure 7. Average HV-metric values of $\varepsilon$CMOEA/D-mDE-RO on CTP-series test instances: (a) CTP1, (b) CTP2, (c) CTP3, (d) CTP4, (e) CTP5, (f) CTP6, (g) CTP7, (h) CTP8
In this example, the weight of drawing process is taken as 
\[ w = [0.3102, 0.1703, 0.0895, 0.1703, 0.1703, 0.0895] \] and the lower boundary is chosen as \( f_2^{LTH} = 10 \times 10^3 \).

6.2. Experimental results

In this subsection, we solve the optimization problem and obtain the optimal parameters of drawing ratio and the Pareto fronts (PFs). Then, the obtained parameters are compared with the values given in Chen et al. (2013). Because in Chen et al. (2013) the results are obtained after iteration times \( G = 50 \), we set the same iteration times in all our experiments, and the algorithms are independently run for 30 times. Note that the optimization problem (22) is maximized. Therefore, the bigger average HV-metric values, the better algorithm performance.

Firstly, we test the convergence on the carbon fibre drawing optimization problem, the results of all the proposed modified algorithms and \( \varepsilon \)CMOEA/D-DE are shown in Figure 9.

It is seen from Figure 9 that all the modified algorithms have better convergence than the traditional \( \varepsilon \)CMOEA/D-DE and the \( \varepsilon \)MOEA/D-mDE-RO2 has the best performance in terms of efficiency and convergence. Secondly, in Figure 10, all PFs obtained by modified and \( \varepsilon \)CMOEA/D-mDE-RO are presented. It can be seen from Figure 10 that, compared with \( \varepsilon \)CMOEA/D-DE, the modified algorithms obtain better optimal solutions.

In Figure 10, points A, B, C are three optimal ones found by \( \varepsilon \)CMOEA/D-mDE-RO1 where points A and C represent the best \( f_2 \) and \( f_1 \), respectively and B is an intermediate point. Points D, E, F are the corresponding ones obtained by the \( \varepsilon \)CMOEA/D-mDE-RO2. Furthermore, we compare them with the unconstrained optimal points obtained by SICSA, i.e. the two unconstrained optimal points G and H taken from (Chen et al., 2013) and (Chen et al., 2013), respectively. Comparison results are shown in Table 3. It can be seen from Table 3 that all points obtained by our proposed modified algorithms are better than the ones obtained by SICSA. Therefore, it is believed that the obtained drawing ratios can serve as a reference and a guidance for the practical production of carbon fibre, which also verifies the effectiveness of the proposed algorithms in this paper.
Table 3. Objective functions and drawing ratios obtained by different algorithms.

| Optimal Points | Results |
|----------------|---------|
|                | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ |
| A              | 1     | 1     | 1.2   | 5.9   | 1     | 1.9   | 0.1468 | 428271 |
| B              | 1.7   | 1     | 1.1   | 5.8   | 1     | 1.9   | 0.1654 | 345171 |
| C              | 2.9   | 1     | 1.1   | 5.6   | 1     | 1     | 0.1948 | 277403 |
| D              | 1     | 1     | 1.1   | 5.7   | 1     | 1.8   | 0.1386 | 416081 |
| E              | 1.6   | 1.1   | 1     | 5.8   | 1.1   | 2     | 0.163  | 333747 |
| F              | 3.1   | 1     | 1     | 5.4   | 1     | 1     | 0.1868 | 265708 |
| G              | 1.4   | 1.1   | 1     | 2.1   | 1.3   | 1.1   | 0.0147 | 70947  |
| H              | 4.5   | 2     | 2     | 4     | 1.3   | 2.5   | 0.232  | 5104   |

Figure 10. Obtained PFs by modified algorithms and $\varepsilon$CMOEA/D-DE

7. Conclusion

In this paper, the improved $\varepsilon$CMOEA/D-DE has been proposed to solve the CMOP. In order to avoid overusing the infeasible solutions in the evolutionary process, three repair operators have been proposed, and the DE operator has been modified to improve the search ability of convergence. Then, the newly proposed repair operators and the modified DE operator have been incorporated into the $\varepsilon$CMOEA/D-DE and an improved CMOP has been proposed. Then, three experiments have been implemented to demonstrate the performance of the improved CMOP. Moreover, the proposed algorithm has been successfully applied in optimizing carbon fibre drawing process and the optimal draw ratio vector has been obtained. Future works would include (1) how to further improve the diversity of the proposed $\varepsilon$CMOEA/D-DE; and (2) how to apply the proposed algorithms to solve other maximized problems in carbon fibre and the more common industrial systems.

Disclosure statement

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References

Bandyopadhyay, S., & Mukherjee, A. (2015). An algorithm for many-objective optimization with reduced objective computations: A study in differential evolution. *IEEE Transactions on Evolutionary Computation, 19*(3), 400–413.

Becerra, R. L., & Coello, C. A. C. (2016). Solving hard multiobjective optimization problems using $\varepsilon$-constraint with cultured differential evolution. *Parallel Problem Solving from Nature – PPSN IX*, 4093, 543–552.

Carlson, S. E., & Shonkwiler, R. (1998). Annealing a genetic algorithm over constraints. *IEEE International Conference on Systems, Man, And Cybernetics*, pp. 3931–3936.

Chen, J. J., Ding, Y. S., & Hao, K. R. (2013). The optimization of carbon fiber drawing process based on cooperative immune clonal selection algorithm. *Advanced Materials Research, 681*, 304–308.

Deb, K., Pratap, A., Agarwal, S., & Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE Transactions on Evolutionary Computation, 14*(10), 1722–1730.

Deb, K., Pratap, A., & Meyarivan, T. (2001). Constrained test problems for multi-objective evolutionary optimization. *1st International Conference on Evolutionary Multi-Criterion Optimization (EMO 2001)*, 1993, 284–298.

Hamida, S. B., & Schoenauer, M. (2002). ASCHEA: New results using adaptive segregational constraint handling. *IEEE World Congress on Computational Intelligence (WCCI)*, 1, 884–889.

Jan, M. A., Khanum, R. A., & Tairan, N. M. (2016). Performance of a constrained version of MOEA/D on CTP-series test instances. *International Journal of Advanced Computer Science and Applications (IJACSA)*, 7(6), 496–505.

Khalil, H., & Gibbens, I. J. (2011). Pareto-optimal design of damping controllers using modified artificial immune algorithm. *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, 41(2), 240–250.

Khaleghi, M., Farsangi, M. M., Nezamabadi-Pour, H., & Lee, K. Y. (2011). Pareto-optimal design of damping controllers using modified artificial immune algorithm. *IEEE Transactions on Systems, Man, and Cybernetics, Part C (Applications and Reviews)*, 41(2), 240–250.

Li, L. M., Lu, K. D., Zeng, G. Q., Wu, L., & Chen, M. R. (2016). A novel real-coded population-based extremal optimization algorithm with polynomial mutation: A non-parametric statistical study on continuous optimization problems. *Neurocomputing*, 174, 577–587.

Liu, H. L., Peng, C. D., Gu, F. Q., & Wen, J. (2016). A constrained multi-objective evolutionary algorithm based on boundary search and archive. *International Journal of Pattern Recognition and Artificial Intelligence*, 30(1), 3058–3063.
Lu, K. D., Zhou, W. N., Zeng, G. Q., & Du, W. (2018). Design of PID controller based on a self-adaptive state-space predictive functional control using extremal optimization method. *Journal of the Franklin Institute*, 355(5), 2197–2220.

Martinez, S. Z., & Coello, C. A. C. (2014). A multi-objective evolutionary algorithm based on decomposition for constrained multi-objective optimization. *IEEE Congress on Evolutionary Computation*, pp. 429–436.

Miettinen, K. (1999). *Nonlinear multiobjective optimization*. Boston: Kluwer Academic.

Ponsich, A., Jaimes, A. L., & Coello, C. A. C. (2013). A survey on multiobjective evolutionary algorithms for the solution of the portfolio optimization problem and other finance and economics applications. *IEEE Transactions on Evolutionary Computation*, 17(3), 321–344.

Riche, G. Le., Knopf-Lenoir, C., & Haftka, R. T. (1995). A segregated genetic algorithm for constrained structural optimization. *Proceedings of the 6th International Conference on Genetic Algorithms*, pp. 558–565.

Salmalian, K., Nariman-Zadeh, N., Gharababei, H., Haftchenari, H., & Varvani-Farahani, A. (2010). Multi-objective evolutionary optimization of polynomial neural networks for fatigue life modelling and prediction of unidirectional carbon-fibre-reinforced plastics composites. *Proceedings of the Institution of Mechanical Engineers Part L-Journal of Materials-Design and Applications*, 224(2), 79–91.

Storn, R., & Price, K. (1997). Differential evolution – a simple and efficient heuristic for global optimization over continuous spaces. *Journal of Global Optimization*, 11(4), 341–359.

Takahama, T., & Sakai, S. (2010). Constrained optimization by the ε constrained differential evolution with an archive and gradient-based mutation. *IEEE Congress on Evolutionary Computation*, 3809(3), 1–9.

Tang, P. H., & Tseng, M. H. (2013). Adaptive directed mutation for real-coded genetic algorithms. *Applied Soft Computing*, 13(1), 600–614.

Wang, J. H., Liao, J. J., Zhou, Y., & Cai, Y. Q. (2014). Differential evolution enhanced with multiobjective sorting-based mutation operators. *IEEE Transactions on Cybernetics*, 44(12), 2792–2805.

Wang, X. F., Ni, R. Q., & Liu, Q. (2000). Preparing for high performance PAN precursor. *Synth Fiber China*, 29(4), 23–27.

Wang, H., Zeng, G. Q., Dai, Y. X., Bi, D. Q., Sun, J. L., & Xie, X. Q. (2017). Design of a fractional order frequency PID controller for an islanded microgrid: A multi-objective extremal optimization method. *Energies*, 10(10), 1502.

Yang, Z. X., Cai, X. Y., & Fan, Z. (2014). Epsilon constrained method for constrained multiobjective optimization problems: Some preliminary results. *Genetic and Evolutionary Computation Conference*, 14(10), 1181–1186.

Zeng, N., Qiu, H., Wang, Z., Liu, W., Zhang, H., & Li, Y. (2018). A new switching-delayed-PSO-based optimized SVM algorithm for diagnosis of Alzheimer’s disease. *Neurocomputing*, 320, 195–202.

Zeng, N., Wang, Z., & Zhang, H. (2016). Inferring nonlinear lateral flow immunoassay state-space models via an unscented Kalman filter. *Science China Information Sciences*, 59(11), 112204.

Zeng, N., Wang, Z., Zhang, H., & Alsaidi, F. E. (2016). A novel switching delayed PSO algorithm for estimating unknown parameters of lateral flow immunoassay. *Cognitive Computation*, 8(2), 143–152.

Zhang, Q. F., & Li, H. (2007). MOEA/D: A multiobjective evolutionary algorithm based on decomposition. *IEEE Transactions on Evolutionary Computation*, 11(6), 712–731.