QCD analysis of $\bar{p}N \rightarrow \gamma^* \pi$ in the scaling limit.

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Abstract

We study the scaling regime of nucleon - anti-nucleon annihilation into a deeply virtual photon and a meson, $\bar{p}N \rightarrow \gamma^* \pi$, in the forward kinematics, where $|t| \ll Q^2 \sim s$. We obtain the leading twist amplitude in the kinematic region where it factorizes into an antiproton distribution amplitude, a short-distance matrix element related to nucleon form factor and the long-distance dominated transition distribution amplitudes which describe the nucleon to meson transition. We give the $Q^2$ evolution equation for these transition distribution amplitudes. The scaling of the cross section of this process may be tested at the proposed GSI intense anti-proton beam facility FAIR with the PANDA or PAX detectors. We comment on related processes such as $\pi N \rightarrow N'\gamma^*$ and $\gamma^* N \rightarrow N'\pi$ which may be experimentally studied at intense meson beams facilities and at JLab or HERMES, respectively.

1 Introduction

In a recent paper [1], we have shown that factorization theorems [2] for exclusive processes apply to the case of the reaction $\pi^-\pi^+ \rightarrow \gamma^*\gamma$ in the kinematical regime where the virtual photon is highly virtual (of the order of the energy squared of the reaction) but the momentum transfer $t$ is small. We also advocated the extension of this approach to the reaction $\bar{p}p \rightarrow \gamma^*\gamma$ and to virtual Compton scattering in backwards kinematics. This enlarges the successful description of deep exclusive reactions in terms of distribution
amplitudes (DA) and/or generalized parton distributions (GPD) on the one side and perturbatively calculable coefficient functions describing hard scattering at the partonic level on the other side. We want here to describe along the same lines the reaction

\[ \bar{p}(k)N(p) \to \gamma^*(q)\pi(p') \tag{1} \]

in the near forward region and for large virtual photon invariant mass \(Q\), which may be studied in detail at GSI. Such an extension of the GPD framework has already been advocated in the pioneering work of [7].

In Ref. [1], we defined the \(\pi \to \gamma\) leading twist transition distribution amplitudes (TDAs) from the matrix elements

\[ \langle \gamma | \bar{q}^\alpha(\bar{z}_1) [z_1; z_0] q^\beta(z_0) | \pi \rangle \bigg|_{z_i^+ = 0, z_i^- = 0} \tag{2} \]

where the Wilson line \([y; z] \equiv P \exp \left[ i g (y - z) \int_0^1 dt \ A_\mu(ty + (1-t)z) \right]\) provides the QCD-gauge invariance for non local operators and equals unity in a light-like (axial) gauge. In a similar way, we shall define in section 2 the nucleon to meson TDAs from the matrix elements

\[ \langle \pi | q^\alpha(z_1) [z_1; z_0] q^\beta(z_2) [z_2; z_0] q^\gamma(z_3) [z_3; z_0] | p \rangle \bigg|_{z_i^+ = 0, z_i^- = 0} \tag{3} \]
The $\bar{p}N \rightarrow \gamma^*\pi$ amplitude at small momentum transfer is then proportional to the TDAs $T(x_i, \xi, t)$, where $x_i$ (i=1,2,3) denote the light cone momentum fractions carried by participant quarks, and $\xi$ is the skewedness parameter connected with $x_B$ by

$$\xi \approx \frac{x_B}{2 - x_B}$$

in the Bjorken limit. It reads schematically

$$M(Q^2, \xi, t) = \int dxdy\phi(y_i, Q^2)T_H(x_i, y_i, Q^2)T(x_i, \xi, t, Q^2),$$

where $\phi(y_i, Q^2)$ is the antiproton distribution amplitude and $T_H$ the hard scattering amplitude, calculated in the colinear approximation. We shall show in section 3 that these TDAs obey QCD evolution equations, which, as always, follow from the renormalization group equation of an appropriate operator, in our case of the three quark operator. Their $Q^2$ dependence is thus completely under control. We calculate in section 4 the hard amplitude and derive some phenomenological model-independent predictions of our picture.

In section 5, we comment on processes related by crossing, such as $\pi N \rightarrow N'\gamma^*$ and $\gamma^*N \rightarrow N'\pi$ which may be experimentally studied at intense meson beams facilities and at JLab or Hermes, respectively.

2 The $N \rightarrow \pi$ transition distribution amplitude

Let us take a closer look at the transition distribution amplitudes from a nucleon to a pseudoscalar meson. A similar description of the antiproton to meson TDA may be straightforwardly deduced from our study. For their definition we introduce light-cone coordinates $v^\pm = (v^0 \pm v^3)/\sqrt{2}$ and transverse components $v_T = (v^1, v^2)$ for any four-vector $v$. The skewedness variable $\xi = -\Delta^+/2P^+$ with $\Delta = p' - p$ and $P = (p + p')/2$ describes the loss of plus-momentum of the incident hadron in the proton $\rightarrow$ meson transition. We parametrize the quark momenta as shown on Fig. 1. The fractions of $+$ momenta are labelled $x_1, x_2$ and $x_3$, and their supports are within $[-1+\xi, 1+\xi]$. Momentum conservation implies (we restrict to the case $\xi > 0$):

$$\sum_i x_i = 2\xi.$$
The fields with positive momentum fractions, \( x_i \geq 0 \), describe creation of quarks, whereas those with negative momentum fractions, \( x_i \leq 0 \), the absorption of antiquarks. The spinorial and Lorentz decomposition of the matrix element follows the same line as in the case of the baryon distribution amplitude \([9, 10]\). Because of that let us first recall the definition of the proton DA at leading twist

\[
4 \langle 0 | \epsilon^{ijk} u^i_\alpha(z_1 n) u^j_\beta(z_2 n) d^k_\gamma(z_3 n) | B(p, s) \rangle \tag{7}
= f_N \left[ V(\hat{p} C)_{\alpha\beta}(\gamma^5 B)_{\gamma} + A(\hat{p} \gamma^5 C)_{\alpha\beta} B_{\gamma} + T(p' i \sigma_{\mu\nu} C)_{\alpha\beta}(\gamma^\mu \gamma^5 B)_{\gamma} \right] ,
\]

where \( i, j, k \) are color indices and \( n \) is the light cone + direction. The vector \( n^\mu \) is a light-like vectors \( (n^2 = 0) \) which together with \( p^\mu \) defines the light-cone kinematics.

We then define the leading twist TDAs for the \( p \to \pi^0 \) transition as:

\[
4 \langle \pi^0(p') | \epsilon^{ijk} u^i_\alpha(z_1 n) u^j_\beta(z_2 n) d^k_\gamma(z_3 n) | p(p, s) \rangle \bigg|_{z^+_0 = 0, z_T = 0} \tag{8}
= -\frac{f_N}{2f_\pi} \left[ V_1^0(\hat{P} C)_{\alpha\beta}(B)_{\gamma} + A_1^0(\hat{P} \gamma^5 C)_{\alpha\beta}(\gamma^5 B)_{\gamma} + 3 T_1^0(P^{\mu' i} \sigma_{\mu\nu} C)_{\alpha\beta}(\gamma^\mu B)_{\gamma} \right] \\
+ V_2^0(\hat{P} C)_{\alpha\beta}(\hat{\Delta}_T B)_{\gamma} + A_2^0(\hat{P} \gamma^5 C)_{\alpha\beta}(\hat{\Delta}_T \gamma^5 B)_{\gamma} + T_2^0(\Delta^\mu_\nu P^{\mu' \sigma_{\mu\nu} C})_{\alpha\beta}(B)_{\gamma} \\
+ T_3^0(P^{\mu' \sigma_{\mu\nu} C})_{\alpha\beta}(\sigma^{\mu\nu} \Delta^\rho_\nu B)_{\gamma} + \frac{T_4^0}{M}(\Delta^\mu_\nu P^{\mu' \sigma_{\mu\nu} C})_{\alpha\beta}(\hat{\Delta}_T B)_{\gamma} ,
\]

where \( \sigma^{\mu\nu} = i/2[\gamma^\mu, \gamma^\nu] \), \( C \) is the charge conjugation matrix and \( B \) the nucleon spinor. \( \hat{P} = P^{\mu' \gamma_\mu} \), the vector \( \Delta = p' - p \) has - in the massless limit - the transverse components

\[
\Delta^\mu_\nu = \left( g^{\mu\nu} - \frac{1}{P_n}(P^{\mu} n^\nu + P^{\nu} n^\mu) \right) \Delta ,
\]

\( f_\pi \) is the pion decay constant \( (f_\pi = 93 \text{ MeV}) \) and \( f_N \) is the constant which determines the value of the nucleon wave function at the origin, and which has been estimated through QCD sum rules to be of order \( 5.3 \cdot 10^{-3} \text{ GeV}^2 \) \([11]\). The first three terms in (8) are the only ones surviving the forward limit \( \Delta_T \to 0 \). The constants in front of these three terms have been chosen in reference to the soft pion limit results (see below). With these conventions each function \( V(z_i P \cdot n) \), \( A(z_i P \cdot n) \), \( T(z_i P \cdot n) \) is dimensionless. Finally let us note that the number of leading twist TDAs in (8) corresponds to
eight independent helicity amplitudes related to the matrix element in \((2^4/2 = 8)\).

For the \(n \to \pi^−\) TDA the analogous expression has the form

\[
4(\pi^−(p'))|\epsilon^{ijk}u_\alpha(z_1 n)u_\beta(z_2 n)d_\gamma(z_3 n)|n(p, s)\rangle\bigg|_{z^+_0, z_\gamma=0} = \frac{f_N}{\sqrt{2} f_\pi} \left[ V^{-}_1(\hat{P}C)_{\alpha\beta}(B)\gamma + A^{-}_1(\hat{P}\gamma^5 C)_{\alpha\beta}(\gamma^5 B)\gamma + T^{-}_1(P^\mu\sigma_{\mu\nu} C)_{\alpha\beta}(\gamma^\mu B)\gamma \right] + V^{-}_2(\hat{P}C)_{\alpha\beta}(\hat{\Delta}_{T}B)\gamma + A^{-}_2(\hat{P}\gamma^5 C)_{\alpha\beta}(\hat{\Delta}_{T}^\gamma B)\gamma + T^{-}_2(P^\mu\sigma_{\mu\nu} C)_{\alpha\beta}(\sigma^\mu\rho\Delta_{T}^\rho B)\gamma + T^{-}_3(T^\mu P^\nu\sigma_{\mu\nu} C)_{\alpha\beta}(\Delta_{T}^\mu B)\gamma + \frac{T^{-}_4}{M}(\Delta_{T}^\mu P^\nu\sigma_{\mu\nu} C)_{\alpha\beta}(\Delta_{T}^\mu B)\gamma .
\]

(9)

One might reexpress \(V^0_i\) and \(V^-_i\) (respectively \(A^0_i\) and \(A^-_i\), respectively \(T^0_i\) and \(T^-_i\)) in terms of the isospin 1/2 and 3/2 quantities \(V^i_{1/2}\) and \(V^i_{3/2}\) (respectively \(A^i_{1/2}\) and \(A^i_{3/2}\), respectively \(T^i_{1/2}\) and \(T^i_{3/2}\)). Simple isospin rotation enables to deduce the TDAs for \(p \to \pi^0\) and \(n \to \pi^0\) transitions from Eqs. (8) and (9). We do not write down the \(p \to \pi^-\) transition (which is pure isospin 3/2 exchange) since it does not contribute to the process under study.

Each TDA can then be Fourier transformed to get the usual representation in terms of the momentum fractions, through the relation

\[
F(z_iP \cdot n) = \int_{-1+\xi}^{1+\xi} d^3x \delta(x_1 + x_2 + x_3 - 2\xi)e^{-iPn\Sigma x_i z_i} F(x_i, \xi)
\]

(10)

where \(F\) stands for \(V_i, A_i, T_i\) and \(\int d^3x \equiv \int dx_1 dx_2 dx_3 \delta(2\xi - x_i - x_2 - x_3)\).

3 Evolution equations

QCD radiative corrections lead as usual to logarithmic scaling violations. The scale dependence of the proton to meson TDAs is governed by evolution equations which are an extension of the evolution equations for usual DAs and GPDs \[3, 4, 5\]. The derivation of evolution equation for TDAs proceeds in an analogous way as for DAs and for GPDs therefore we sketch only essential steps.
The non-local three-quark operators relevant for TDAs and their evolution, see [10] for details and notation which we follow, involve quark fields having definite chirality or helicity
\[ q^{\uparrow (\downarrow)} = \frac{1}{2} \left( 1 \pm \gamma^5 \right) q. \]  
(11)

The separation of “minus” components of quark fields [9] leading to the dominant twist-2 contribution is achieved by the substitution \( q \to \hat{n}q \), with \( \hat{n} = n^\mu \gamma_\mu \). There are two relevant operators in our problem: the first one corresponds to the case where the three quarks have total helicity \( 1/2 \)
\[ B^{1/2}_{\alpha \beta \gamma}(z_1, z_2, z_3) = \epsilon^{ijk}(\hat{n}q_i^\dagger)_\alpha(z_1n)(\hat{n}q_j^\dagger)_\beta(z_2n)(\hat{n}q_k^\dagger)_\gamma(z_3n) \]  
(12)
and the second one with the total helicity \( 3/2 \)
\[ B^{3/2}_{\alpha \beta \gamma}(z_1, z_2, z_3) = \epsilon^{ijk}(\hat{n}q_i^\dagger)_\alpha(z_1n)(\hat{n}q_j^\dagger)_\beta(z_2n)(\hat{n}q_k^\dagger)_\gamma(z_3n). \]  
(13)

For simplicity, we have assumed that all quarks in (12), (13) have different flavours. The conditions imposed by flavour symmetry do not influence the evolution equations but lead to certain symmetry properties of TDAs. Since operators (12), (13) belong to different representations of the Lorentz group they do not mix with each other.

The operators \( B \), (12) and (13), satisfy the renormalisation group equation
\[ \mu \frac{d}{d \mu} B = H \cdot B \]  
(14)
with \( H \) being an integral [10] operator
\[ H = -\frac{\alpha_s}{2\pi} \left[ (1 + 1/N_c) \mathcal{H} + 3C_F/2 \right] \]  
(15)
in which the second term \( \sim C_F \) corresponds to the self-energy corrections of each quark field. The operator \( \mathcal{H} \) acts in different way on the operator (13) and on the operator (12). In the first case it is determined by contributions from one loop Feynman diagrams describing in the Feynman gauge the vertex corrections corresponding to gluon exchanges between quark fields and gluons forming Wilson lines as
\[ \mathcal{H}_{3/2} = \mathcal{H}_1^v + \mathcal{H}_2^v + \mathcal{H}_3^v. \]  
(16)

\[ ^1 \text{We restored in Eq. (15) the factor } -\alpha_s/(2\pi) \text{ absent in Eq. (2.23) of [10].} \]
where
\[
\mathcal{H}_{12}^\nu B(z_i) = -\frac{1}{\alpha} \int_0^1 d\alpha \{ \bar{\alpha} [B(z_{12}^\alpha, z_2, z_3) - B(z_1, z_2, z_3)] \\
+ \bar{\alpha} [B(z_1, z_{21}^\alpha, z_3) - B(z_1, z_2, z_3)] \},
\]
(17)
with \(\bar{\alpha} = 1 - \alpha\), \(z_{ik}^\alpha = z_i \bar{\alpha} + z_k \alpha\). In the case of (12) the operator \(\mathcal{H}\) is determined not only by above contributions but also by those ones which correspond to Feynman diagrams with gluon exchange between quark lines having opposite chiralities (i.e. in our case between lines (1,2) and (2,3)) and it can be written as
\[
\mathcal{H}_{1/2} = \mathcal{H}_{3/2} - \mathcal{H}_{12}^e - \mathcal{H}_{23}^e,
\]
(18)
where
\[
\mathcal{H}_{12}^e B(z_i) = \int D\alpha B(z_{12}^{\alpha_1}, z_{12}^{\alpha_2}, z_3),
\]
(19)
with
\[
\int D\alpha = \int_0^1 d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3).
\]
(20)

From the renormalisation group equation (14) and (15) we derive the corresponding equation for the matrix element of operators \(B\) between states relevant for the process under study. The subsequent use of parametrisations of these matrix elements in terms of different TDAs, as those given by Eqs. (8) or (9), results in the evolution equations for TDAs \(V_i, A_i, T_i\).

As a definite example we present the evolution equation for the set of TDAs denoted as \(F^{\uparrow\downarrow\uparrow}(x_i)\) which are related to the matrix element of three quark operator with total helicity 1/2
\[
\langle \pi^0(p')|e^{ijk}(\hat{n}q^i_\alpha)(z_1 n)(\hat{n}q^j_\beta)(z_2 n)(\hat{n}q^k_\gamma)(z_3 n)|N(p, s)\rangle
\]
being parametrised according to Eq. (8). The evolution equation has the form
\[
Q \frac{d}{dQ} F^{\uparrow\downarrow\uparrow}(x_i) = -\frac{\alpha_s}{2\pi} \left( \frac{3}{2} C_F F^{\uparrow\downarrow\uparrow}(x_i) - \left( 1 + \frac{1}{N_c} \right) \right)
\]
(21)
The support of integrands is defined by functions $\rho(x, y) = \theta(x \geq y \geq 0) - \theta(x \leq y \leq 0)$, with $\theta(x \geq y \geq 0) = \theta(x \geq y)\theta(y \geq 0)$. This function $\rho(x, y)$ is a generalization of analogous one which appears in equations describing the pure evolution ERBL [4]. The second condition is the requirement that although not denoted as a variable of integration the variables $x_i'$ must satisfy the condition that $x_i' \in [-1 + \xi, 1 + \xi]$, e.g. in the first integral over $x_1'$ on the rhs of (21) the variable $x_2' = 2\xi - 3 - x_i'$ must belong to the interval $x_2' \in [-1 + \xi, 1 + \xi]$.

The evolution equation for the set of TDAs $F^{\uparrow\uparrow\uparrow}(x_i)$, which correspond to the case where three quarks have total helicity $3/2$, is obtained from (21) by neglecting two last lines.

The results of this evolution are different in the various $x_i$ sectors. In particular, when all $x_i > 0$ one is in the same kinematics as the usual ERBL equation for the baryons, with the simple $x_i \rightarrow x_i/\xi$ rescaling. The solutions of the Eq. (21) in this ERBL region are thus well known and are expressed in terms of Appell polynomials $P_n(x_i/2\xi)$:

$$F(x_i, \xi, \mu^2) = x_1 x_2 x_3 \delta(x_1 + x_2 + x_3 - 2\xi) \sum_n \phi_n P_n(x_i/2\xi) \left( \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_n/\mu_0}, \quad (22)$$
where \( b_0 = \frac{11}{3}N_c - \frac{2}{3}n_f \), \( \gamma_n \) are the corresponding anomalous dimensions and \( \phi_n(\mu_0) \) are dimensionless nonperturbative parameters.

When one or two \( x_i < 0 \), the solutions of evolution equation (21) are unknown: they will deserve further study. Although asymptotic solutions are simple, one should not use them without caution in phenomenological studies, since it is known that the asymptotic solution of the proton distribution amplitude (namely, the DA proportional to \( x_1x_2x_3 \) like the first term in (22)) does not allow for a good description of form factors at accessible values of \( Q^2 \). Thus one should not insist on the asymptotic form but instead use nonperturbative techniques such as QCD sum rules or lattice calculations to get boundary values to insert in the evolution equations, and maybe solve them by applying some methods based on conformal symmetry used in [10] or by methods proposed in [12].

4 Hard amplitude and cross section estimates

Since the leading twist antiproton distribution amplitude selects the helicity \( \pm \frac{1}{2} \) state for the hard scattered quarks, and since the photon coupling does not modify the helicity of these quarks, the three quarks extracted from the proton by the TDA have also a total helicity of \( \pm \frac{1}{2} \). Moreover, in this first study, we concentrate on terms which are not vanishing at zero \( \Delta_T \), \( i.e. \) to the contributions of the three first TDAs in Eq. (8), namely \( V_1, A_1, T_1 \).

The hard amplitude is then at leading twist straightforwardly deduced from the studies of proton form factors at high \( Q^2 \). At leading order in \( \alpha_s \), the amplitude \( M^\mu \) for the reaction

\[
\bar{p}(k, \lambda)p(p, s) \rightarrow \gamma^*(q)\pi^0(p')
\]

may be read off from baryonic form factor calculations \[3, 15\] as

\[
M^\mu = -ie_p F(Q^2, \xi, t) \bar{v}(k, \lambda)\gamma^\mu\gamma^5 u(p, s)
\]

\[
F(Q^2, \xi, t) = \frac{f_\pi^2}{f_\pi} \frac{(4\pi\xi\alpha_s(Q^2))^2}{27Q^4} \int_{1+\xi}^{1} d^3x \int_{0}^{1} d^3y \sum_{\alpha=1}^{10} T_\alpha(x_i, y_j)
\]

(23)

with

\[
T_1 = \frac{4}{3} \frac{\mathcal{V} + \mathcal{T}}{(2\xi - x_1 + i\epsilon)^2(x_3 + i\epsilon)(1 - y_1)^2y_3}
\]
\[ T_2 = \frac{4\mathcal{T}}{3(x_1 + i\epsilon)(2\xi - x_2 + i\epsilon)(x_3 + i\epsilon)y_1(1 - y_2)y_3} \]
\[ T_3 = \frac{4\mathcal{V}}{3(x_1 + i\epsilon)(2\xi - x_3 + i\epsilon)(x_3 + i\epsilon)y_1(1 - y_1)y_3} \]
\[ T_4 = \frac{4\mathcal{V}}{3(x_2 + i\epsilon)(2\xi - x_3 + i\epsilon)(x_3 + i\epsilon)y_2(1 - y_1)y_3} \]
\[ T_5 = \frac{2\mathcal{V}}{3(2\xi - x_3 + i\epsilon)^2(1 - y_3)^2} \left( \frac{1}{(x_1 + i\epsilon)y_1} + \frac{1}{(x_2 + i\epsilon)y_2} \right) \]
\[ T_6 = \frac{2\mathcal{V} + \mathcal{T}}{3(2\xi - x_1 + i\epsilon)^2(x_2 + i\epsilon)(1 - y_1)^2y_2} \]
\[ T_7 = \frac{2\mathcal{V} + \mathcal{T}}{3(2\xi - x_1 + i\epsilon)^2(x_2 + i\epsilon)(1 - y_1)^2y_2} \]
\[ T_8 = \frac{2\mathcal{V}}{3(x_1 + i\epsilon)(x_2 + i\epsilon)(2\xi - x_3 + i\epsilon)y_1(1 - y_1)y_2} \]
\[ T_9 = \frac{2\mathcal{V}}{3(x_2 + i\epsilon)(2\xi - x_1 + i\epsilon)(x_2 + i\epsilon)y_1(1 - y_1)y_2} \]
\[ T_{10} = \frac{2\mathcal{V}}{3(x_1 + i\epsilon)(x_2 + i\epsilon)(2\xi - x_1 + i\epsilon)x_3y_1y_2(1 - y_3)} \]

with

\[ \mathcal{V}(x_j, y_i, \xi, t) = [V(y_i) - A(y_i)] \cdot [V_1(x_j, \xi, t) - A_1(x_j, \xi, t)] \]
\[ \mathcal{T}(x_j, y_i, \xi, t) = -12[T(y_i)][T_1(x_j, \xi, t)] \]

and \( \int d^3y \equiv \int dy_1dy_2dy_3\delta(1 - y_1 - y_2 - y_3) \). Note that the electromagnetic vector current has been replaced in the amplitude (23) by an axial vector current, due to the presence of the outgoing pseudo scalar meson \( \pi \)-meson in the TDA (8).

A model independent result of our analysis is the scaling law for the amplitude:

\[ \mathcal{M}(Q^2, \xi) \sim \frac{\alpha_s(Q^2)^2}{Q^4} \]

valid up to logarithmic corrections due to DA and TDA anomalous dimensions. On the other hand the ratio:

\[ \frac{d\sigma(\bar{p}p \to l^+l^-\pi^0)/dQ^2}{d\sigma(\bar{p}p \to l^+l^-)/dQ^2} \]
should be almost $Q^2$ independent.

Another interesting consequence of our framework is the dominance of the transverse polarization of the virtual photon, which results in a specific angular distribution of the lepton pair in its rest frame, namely

$$\frac{d\sigma(p\bar{p} \to l^+l^-\pi)}{\sigma d\theta} \sim 1 + \cos^2\theta \quad (28)$$

where $\theta$ is the usual emission angle of the lepton in the virtual photon center of mass frame.

Since the electromagnetic form factor $F_1^p$ is obtained with the same expressions as Eqs. (23), (24) with the replacement of the axial vector current by vector one and with $2f_\pi \to 1$, and $\mathcal{T} \to 4[T(y_i)] \cdot [T(x_j)]$ one may obtain an estimate of the threshold cross section in terms of the electromagnetic form factor, once a reasonable form of $V$, $A$ and $T$ are chosen. We shall not enter more the phenomenology in this letter but will study it in a forthcoming work.

## 5 Related processes and conclusions

We have defined the new transition distribution amplitudes $N \to \pi$, i.e. which parametrize the matrix elements of light-cone operators between a baryon and a meson states; this generalizes the concept of GPDs for non-diagonal transitions. Similar matrix element was introduced also in Ref. [7] in discussion of the exclusive production of forward baryons off nucleons. Obviously related processes are

- Firstly, the exclusive lepton pair production

$$\pi N \to N'\gamma^* \quad (29)$$

in the kinematical regime where the outgoing nucleon is almost colinear to the incoming meson, which is the backward region of the reaction $\pi N \to \gamma^*N'$ studied in Ref. [13], and which may be studied in an intense pion beam facility such as the project JPark.

- Secondly, the same framework may be applied to

$$\gamma^* N \to N'\pi \quad (30)$$
in the kinematical regime where the outgoing meson is almost colinear to the incoming proton, which is the spacelike analog of the previous case.

- Thirdly, the crossed \((t \rightarrow s)\) version of the \(N \rightarrow \pi\) TDA describes the exclusive fragmentation of three colinear quarks in a Baryon-meson pair. Analogously to the two meson generalized distribution amplitude [4], it is an interesting tool to access reactions where an emerging isolated baryon is replaced by a baryon-meson system. An example of its usefulness is the calculation of the impact factor of a hard diffractive reaction (at large values of \(t\)) where a baryon projectile is transformed into a baryon-meson ejectile. In this way, the crossed TDA describes the partonic content of a continuum or resonating baryon-meson state.

Many other processes may be studied in the same way, where the \(\pi\) meson is replaced with other mesons, or/and the outgoing nucleon is replaced with other baryons.

The introduction of transition distribution amplitudes thus allows to study the reactions

\[ \bar{p}p \rightarrow l^+l^-\pi^0 \quad \bar{p}n \rightarrow l^+l^-\pi^- \]

along the same lines as the timelike electromagnetic baryon form factors. The applicability of a perturbative QCD approach to the proton form factor at accessible energies has been controversial for years [14]. We pretend here neither that experimental data have shown that the perturbative approach is successful, nor that next to leading order corrections will succeed in describing them. We just want to point out that a new phenomenology of related but different reactions is now possible to try to understand the present puzzle. Moreover, the study of form factors has been the subject of many developments after the pioneering papers [3] [15]. In particular, the calculation of next to leading logarithm corrections [16], the proposal of an optimal renormalization scale fixing [17], the importance of soft gluon resummation [18] and the discussion of the timelike vs spacelike aspects [19], have put to a firmer basis the QCD description of these objects (for a short review, see [20]). These interesting developments ought to be applied now to the meson production processes. This necessary effort should help a phenomenological analysis to be more constrained so that one may clearly see if a (quite sophisticated) perturbative QCD approach is indeed relevant to the
kinematical range which may be accessible at GSI-FAIR, with $\sqrt{s} \approx 32\text{GeV}^2$ in the target rest frame and $\sqrt{s} \approx 207\text{GeV}^2$ in the accelerator mode. On the other hand, an alternative more phenomenological point of view has been also developped to describe form factors in terms of generalized parton distributions and the so-called ”Feynman mechanism” \cite{21}. This framework has recently been enlarged \cite{22} to describe reactions \cite{1} at fixed angle through generalized distribution amplitudes \cite{23}.

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