Scattering of phonons on two-level systems in disordered crystals

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received 9 April 2008; accepted in final form 10 July 2008
published online 1 September 2008

PACS 61.43.−j – Disordered solids
PACS 62.40.+i – Anelasticity, internal friction, stress relaxation, and mechanical resonances
PACS 63.20.kp – Phonon-defect interactions

Abstract – We calculate the scattering rates of phonons on two-level systems in disordered trigonal and hexagonal crystals. We apply a model in which the two-level system, characterized by a direction in space, is coupled to the strain field of the phonon via a tensor of coupling constants. The structure of the tensor of coupling constants is similar to the structure of the tensor of elastic stiffness constants, in the sense that they are determined by the same symmetry transformations. In this way, we emphasize the anisotropy of the interaction of elastic waves with the ensemble of two-level systems in disordered crystals. We also point to the fact that the ratio \(\gamma_l/\gamma_t\) has a much broader range of allowed values in disordered crystals than in isotropic solids.

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Introduction. – The “universality” of the glass-like properties of amorphous solids has been pointed out almost four decades ago [1]. Some of these properties are: heat conductivity which is almost independent of the chemical composition of the solid and proportional to \(T^2\) (where \(T\) is the temperature), specific heat proportional to \(T\), and a long-time heat release [1–3]. All these properties are described theoretically with reasonable quantitative accuracy by assuming that the amorphous solid contains dynamical defects that can be described at low temperatures as an ensemble of two-level systems (TLSs) [4,5]. Nevertheless, glass-like properties have been found also in disordered crystals [6–14] and quasicrystals, [15–17] only that in these materials they are not as universal as in amorphous solids and, even more, they exhibit anisotropy.

The deep nature of the glass-like properties — and therefore of the ensemble of the TLSs— remains elusive, despite the long and intensive efforts invested into their study. This makes the study of disordered crystals especially interesting, since there, knowing the structure of the unit cell and its modifications due to disorder, we may know which are the tunneling entities and therefore we may have additional information about the TLSs. Moreover, the observed anisotropy of the glass-like properties, although unexplained, represents additional information for the theoretical description, which may help to improve the microscopic model.

In general, the thermal properties of a dielectric glass are determined by the ensemble of TLSs, the phonon gas, and the interaction between them. In the standard tunneling model (STM) the TLS is described in a basis that diagonalizes the interaction Hamiltonian between the TLS and the phonon. In this basis, the Hamiltonian of the free TLS and the interaction Hamiltonian are

\[
H_{\text{TLS}} = \frac{1}{2} \begin{pmatrix} \Delta & -\Lambda & -\Lambda & \Delta \\ -\Lambda & -\Delta & -\Lambda & \Delta \\ -\Lambda & -\Lambda & -\Delta & \Delta \\ \Delta & \Delta & \Delta & -\Delta \end{pmatrix}
\]

and

\[
H_I = \frac{1}{2} \begin{pmatrix} \delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix},
\]

respectively. The interaction element, \(\delta\), is linear in the strain field of the phonon, \([S]\), namely

\[
\delta = 2\gamma_{ij}S_{ij},
\]

where we assumed summation over the repeated subscripts. The symmetric second-rank tensor \([\gamma]\) characterizes the TLS and its “deformability” under elastic strain. For the convenience of the calculations we work in the abbreviated subscript notations and we write \([S]\) and \([\gamma]\) as the six-dimensional vectors \(S = (S_{11}, S_{22}, S_{33}, 2S_{23}, 2S_{13}, 2S_{12})^t\) and \(\gamma = (\gamma_{11}, \gamma_{22}, \gamma_{33}, \gamma_{23}, \gamma_{13}, \gamma_{12})^t\), respectively (where \(^t\) denotes the transpose of a matrix or a vector). To go further and obtain deeper information about the TLSs,

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in ref. [18] \( \gamma \) was written as a product of two tensors: the first is a symmetric second-rank tensor describing the “free” TLS — we call it \( T \) in abbreviated subscript notations — and the second one is a fourth-rank tensor, describing the coupling between \( T \) and \( S \). In the abbreviated subscript notations, the fourth-rank tensor is a \( 6 \times 6 \) matrix which we shall call \([R]\) and represents the matrix of TLS-phonon coupling constants. So we write eq. (2) in matrix notations as [18]

\[
\delta = 2 T^t \cdot [R] \cdot S.
\]  

(3)

The advantage of eq. (3) is that it separates the tensor \( T \), which contains only the characteristics of the TLS, from the matrix of coupling constants, \([R]\), in which the characteristics of the interaction are embedded and which has a general structure determined by the symmetries of the lattice [18,19]. In general, the tensor \( T \) was taken in the simple form, \( T = (t_1^r t_2^r t_3^r 2t_2 t_3 2t_1 t_3 2t_1 t_2)^r \), where \( t_1, t_2, \) and \( t_3 \) are the components of the unit vector \( t \), which determines the “direction” of the TLS. This simplification will be used also in this paper. In the form (3), the symmetries of the lattice are imposed on the matrix \([R]\) by coordinates transformations that leave the lattice invariant, whereas the distribution over the elements of \( T \) is determined by the distribution over the “directions”, \( t \). Through the properties of \([R]\), this model predicts anisotropic glass properties of a crystal, even for an ensemble of TLSs isotropically oriented.

In ref. [20] we applied this model to study the anisotropy of the glass properties in a disordered cubic crystal and we compared our calculations with the experimental results of Topp and coworkers [11,13,14]. Unfortunately the experimental data published to date is not enough to check the model of ref. [18] or even to determine its parameters. From the available data, in ref. [18] we merely obtained a relation between these parameters, which should be confirmed or not by future experiments.

In this paper we extend our calculations to two other classes of crystal symmetries: trigonal 32 and hexagonal. The former symmetry class corresponds to (neutron-irradiated) quartz and the latter to Na-doped \( \beta \)-Al\(_2\)O\(_3\). Both materials show glass-like properties at low temperatures and strong anisotropy in the TLS-phonon coupling [7–9].

It is known that in isotropic amorphous materials the coupling of TLSs with the phonon modes is described by the scalar coupling constants, \( \gamma_l \) and \( \gamma_t \), obtained by averaging the transition rates over the isotropic distribution of the TLS orientations. In this way, from very general considerations, one gets [18,21,22]

\[
(\gamma_l/\gamma_t)^2 \geq 4/3.
\]  

(4)

But in the model that we use here, this relation is affected by the symmetry of the lattice and therefore it does not necessary hold in a disordered crystal. This motivated us to discuss at the end of the next section the range of \( \gamma_l/\gamma_t \) for a crystal with cubic symmetry.

**Phonon scattering rates in trigonal and hexagonal lattices.**

**General considerations.** The transition amplitude from a quantum state consisting of an unexcited TLS and \( n_{k\sigma} + 1 \) phonons of wave vector \( k \) and polarization \( \sigma (\sigma = l, t) \), \( |n_{k\sigma} + 1, \downarrow \rangle \), into the state of \( n_{k\sigma} \) phonons and excited TLS, \( |n_{k\sigma}, \uparrow \rangle \), is

\[
\langle n_{k\sigma} + 1 | \hat{H} | n_{k\sigma} + 1, \downarrow \rangle = -\frac{\Lambda}{\epsilon} \sqrt{n_{k\sigma}} T^t \cdot [R] \cdot S_{k\sigma},
\]  

(5)

where \( \epsilon = \sqrt{\Lambda^2 + \Lambda^2} \) is the excitation energy of the TLS. Therefore, the phonon scattering rate by a TLS in its ground state is

\[
\Gamma_{k\sigma}(t) = \frac{2\pi A^2 n_{k\sigma}}{\epsilon^2} [T^t \cdot [R] \cdot S_{k\sigma}]^2 \delta (\epsilon - \hbar \omega).
\]  

(6)

The main characteristic of the TLS-phonon interaction is contained in the quantity \( M_{k\sigma}(t) \equiv T^t \cdot [R] \cdot S_{k\sigma} \). As explained in the introduction, the TLS-phonon interaction bears an intrinsic anisotropy through the matrix \([R]\), on which the symmetries of the lattice are imposed. To calculate the average scattering rate of a phonon by the ensemble of TLSs, we have to average over the distribution of \( t \). To reduce the number of degrees of freedom of the problem, in what follows we shall assume that \( t \) is isotropically oriented.

**Trigonal lattice.** For a trigonal lattice of symmetry class 32 (the symmetry of quartz), the matrix \([R]\) has the form [23]

\[
[R] = \begin{pmatrix}
  r_{11} & r_{12} & r_{13} & r_{14} & 0 & 0 \\
  r_{12} & r_{11} & -r_{14} & 0 & 0 & 0 \\
  r_{13} & -r_{14} & r_{13} & 0 & 0 & 0 \\
  r_{14} & 0 & 0 & r_{14} & r_{11} & -r_{13} \\
  0 & 0 & 0 & r_{14} & r_{14} & r_{11} \\
  0 & 0 & 0 & r_{14} & r_{14} & r_{11}
\end{pmatrix},
\]  

(7)

similar to that of the tensor of elastic stiffness constants, \( [c] \), with \( c_{ij} \) replaced by \( r_{ij} \) [18,19]. The system of coordinates that we use here is such that the \( z \) and \( x \) axes are the 3-fold and 2-fold rotational symmetry axes, respectively, while the \( y \)-axis is perpendicular to both \( x \) and \( z \). Solving the Christoffel equation we find that the crystal can sustain pure longitudinal waves propagating along the \( x \) and \( z \) axes, and pure transversal waves propagating along the \( y \) and \( z \) axes. The sound velocities of the longitudinal waves propagating in the \( x \) and \( z \) directions are \( v_{k\sigma} = \sqrt{|c_{11}|/\rho} \) and \( v_{k\sigma} = \sqrt{|c_{33}|/\rho} \), respectively, where \( \rho \) is the density of the material. The transversal waves propagating in the \( z \)-direction have a sound velocity, \( v_{k\sigma} = \sqrt{|c_{44}|/\rho} \), independent of the polarisation direction. The pure transversal waves propagating in the \( z \)-direction should be polarized only in the \( z \)-direction and have a sound velocity \( v_{k\sigma} = \sqrt{(c_{11} - c_{12})/2\rho} \) — transversal waves polarized in other directions are not eigenvectors of the Christoffel equation. If we define the direction \( t \) by the two
Euler angles $\theta$ and $\phi$, $\hat{t} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)^T$, then for the longitudinal waves propagating in the $\hat{x}$ and $\hat{z}$ directions we have

$$M_{k,t}(\theta, \phi) = ik[r_{11} \sin^2(\theta) \cos^2(\phi) + r_{12} \sin^2(\theta) \sin^2(\phi) + r_{13} \cos^2(\theta) + r_{14} \sin(2\theta) \sin(\phi)]$$

(8a)

and

$$M_{k,t}(\theta, \phi) = ik[r_{13} \sin^2(\theta) + r_{33} \cos^2(\theta)],$$

(8b)

respectively, whereas for the transversely polarized waves propagating in the $\hat{y}$ and $\hat{z}$ directions we have

$$M_{k,y,t}(\theta, \phi) = ik \sin(\theta)[r_{14} \sin(\theta) \cos(2\phi) + 2r_{44} \cos(\theta) \sin(\phi)]$$

(8c)

and

$$M_{k,y,t}(\theta, \phi) = 2ik \sin(\theta) \cos(\theta)[r_{14} \cos(\theta) + r_{14} \sin(\theta) \sin(\phi)],$$

(8d)

respectively. For the transversal wave propagating in the $\hat{z}$-direction we choose the polarization along the $x$-axis — this choice becomes irrelevant after averaging over the directions $\hat{t}$, which we shall do next.

The phonon absorption rates are calculated by averaging (6) over the ensemble of TLSs. If we denote by $f(\theta, \phi)$ the distribution over the angles of $\hat{t}(\theta, \phi)$, then we have

$$\tau_{k,t}^{-1} = \frac{P_0 \tanh \left( \frac{\epsilon}{2k_B T} \right) n_{k,t}}{2h} \int_0^{2\pi} d\theta \int_0^\pi d\phi \sin \theta \times |M_{k,t}(|\hat{t}(\theta, \phi)|)^2 f(\theta, \phi) \equiv \frac{2\pi P_0 \tanh \left( \frac{\epsilon}{2k_B T} \right)}{h} \times n_{k,t}(\text{operator})^2.$$  

(9)

As mentioned before, we shall use the isotropic distribution, $f(\theta, \phi) = 1$. Plugging eqs. (8) one by one into (9), we get the scattering rates

$$\tau_{k,x,t}^{-1} = \frac{1}{15} \left( 3r_{11}^2 + r_{12}^2 + 2r_{13}r_{11} + 3r_{12}^2 + 2r_{12}r_{13} + 3r_{13}^2 + 4r_{44}^2 \right) \frac{2\pi P_0 N^2 n^2 k}{h} \tanh \left( \frac{\epsilon}{2k_B T} \right),$$

(10a)

$$\tau_{k,x,t}^{-1} = \frac{8r_{13}^2 + 4r_{12}r_{33} + 3r_{33}^2}{15} \frac{2\pi P_0 N^2 n^2 k}{h} \tanh \left( \frac{\epsilon}{2k_B T} \right),$$

(10b)

$$\tau_{k,y,t}^{-1} = \frac{4(r_{14}^2 + r_{44}^2)}{15} \frac{2\pi P_0 N^2 n^2 k}{h} \tanh \left( \frac{\epsilon}{2k_B T} \right),$$

(10c)

$$\tau_{k,z,t}^{-1} = \frac{4(4r_{14}^2 + r_{44}^2)}{15} \frac{2\pi P_0 N^2 n^2 k}{h} \tanh \left( \frac{\epsilon}{2k_B T} \right),$$

(10d)

where by $N$ we denoted the normalization constant of the phonon ($N = \sqrt{h/(2V \omega)}$) and by $n$ the thermal population of the phonon mode ($n = [\exp(\beta h \omega) - 1]^{-1}$); $V$ is the volume of the solid. Comparing eqs. (10) with the standard formula for the phonon scattering rates,

$$\left( \tau^{-1}(\text{STM}) \right)_{k,t} = \gamma_{k,t}^2 \frac{2\pi P_0 N^2 n^2 k}{h} \tanh \left( \frac{\epsilon}{2k_B T} \right),$$

(11)

we obtain the anisotropic values of the $\gamma$ parameters,

$$\gamma_{k,t}^2 = \frac{2(r_{11}^2 + r_{12}^2 + r_{13}^2) + (r_{11} + r_{12} + r_{13})^2 + 4r_{44}^2}{15},$$

(12a)

$$\gamma_{k,t}^2 = \frac{8r_{13}^2 + 4r_{12}r_{33} + 3r_{33}^2}{15},$$

(12b)

$$\gamma_{k,t}^2 = \frac{4(r_{14}^2 + r_{44}^2)}{15},$$

(12c)

$$\gamma_{k,t}^2 = \frac{4(4r_{14}^2 + r_{44}^2)}{15}.$$  

(12d)

**Hexagonal lattice.** The difference between the trigonal lattice of symmetry 32 and the hexagonal lattice is that $r_{14}$ and $c_{14}$ are zero. This enhancement of symmetry allows propagation of pure longitudinal and transversal waves in all the three directions, $x$, $y$, and $z$. The sound velocities of the longitudinal waves in these three directions are $\sqrt{c_{11}/\rho}$ for $x$ and $y$ directions, and $\sqrt{c_{33}/\rho}$ for the $z$-direction. For the transversal waves propagating in the $x$-direction, the transversal waves propagating in the $y$-direction are similar to the ones propagating in the $x$-direction: the waves polarized in the $x$-direction have a sound velocity of $\sqrt{(c_{11} - c_{12})/\rho}$, whereas the ones polarized in the $z$-direction have a sound speed of $\sqrt{c_{44}/\rho}$. Finally, the transversal waves propagating in the $z$-direction have all the same sound velocity, $\sqrt{c_{44}/\rho}$.

For the quantities $M$, we get

$$M_{k,t}(\theta, \phi) = ik[r_{11} \sin^2(\theta) \cos^2(\phi) + r_{12} \sin^2(\theta) \sin^2(\phi) + r_{13} \cos^2(\theta)],$$

(13a)

$$M_{k,x,t}(\theta, \phi) = ik \sin^2(\theta) \sin(2\phi) \frac{r_{11} - r_{12}}{2},$$

(13b)

$$M_{k,y,t}(\theta, \phi) = ik \sin(2\theta) \cos(\phi) r_{44},$$

(13c)

$$M_{k,y,t}(\theta, \phi) = ik[r_{11} \sin^2(\theta) \sin^2(\phi) + r_{12} \sin^2(\theta) \cos^2(\phi) + r_{13} \cos^2(\theta)],$$

(13d)

$$M_{k,y,t}(\theta, \phi) = ik \sin^2(\theta) \sin(2\phi) \frac{r_{11} - r_{12}}{2},$$

(13e)

$$M_{k,z,t}(\theta, \phi) = ik \sin(2\theta) \sin(\phi) r_{44},$$

(13f)

$$M_{k,z,t}(\theta, \phi) = ik(r_{13} \sin^2(\theta) + r_{33} \cos^2(\theta)),$$

(13g)

$$M_{k,x,t}(\theta, \phi) = ik \sin(2\theta) \cos(\phi) r_{44},$$

(13h)

$$M_{k,y,t}(\theta, \phi) = ik \sin(2\theta) \sin(\phi) r_{44},$$

(13i)
in obvious notations: the first subscript indicates the propagation direction while the second one is used only for transversal waves and denotes the direction of polarization. We plug these formulae into eq. (9) to get

\[
\left( \frac{\gamma_{l}}{\gamma_{t}} \right)_{\text{eq.(H)}} = \frac{2 \pi P_{0} N^{2} n k^{2}}{\hbar} \tanh \left( \frac{\epsilon}{2 k_{B} T} \right),
\]

where the superscript (H) stands for hexagonal and is used to make the difference between these quantities and the ones calculated in the preceding subsection. As before, we get the \( \gamma \) constants:

\[
\begin{align*}
(\gamma_{l}^{(H)})^{2} &= \frac{2(\gamma_{11}^{2} + \gamma_{12}^{2} + \gamma_{22}^{2} + \gamma_{11}^{2} + \gamma_{12}^{2} + \gamma_{22}^{2})}{15}, \\
(\gamma_{k}^{(H)})^{2} &= \frac{8\gamma_{13}^{2} + 4\gamma_{13} \gamma_{33} + 3\gamma_{33}^{2}}{15}, \\
(\gamma_{k_{y}^{(H)}}^{(H)})^{2} &= \frac{(\gamma_{11}^{(H)} - \gamma_{12}^{(H)})^{2}}{15}, \\
(\gamma_{k_{x}^{(H)}}^{(H)})^{2} &= \frac{(\gamma_{13}^{(H)} - \gamma_{23}^{(H)})^{2}}{15}.
\end{align*}
\]

(15a)  
(15b)  
(15c)  
(15d)

We notice that the constants \( \gamma_{l}^{(H)}, \gamma_{k}^{(H)}, \gamma_{k_{y}}^{(H)} \) and \( \gamma_{k_{x}}^{(H)} \) are equal to \( \gamma_{l}, \gamma_{k}, \gamma_{k_{y}} \) and \( \gamma_{k_{x}} \), respectively (eqs. (12)) if in the latter ones we set \( r_{14} = 0 \).

**Range of \( \gamma_{l}/\gamma_{t} \).** We notice also that in general the relation (4), valid for isotropic media, is not necessarily valid for crystals, which have lower symmetry. For the lattices studied above, the ratio \( \gamma_{l}^{2}/\gamma_{t}^{2} \) in any of the three directions has complicated expressions in terms of the components of \( [R] \). For the trigonal lattice \( [R] \) has six independent components, whereas for the hexagonal lattice it has five. Therefore, a discussion about the ranges of \( \gamma_{l}^{2}/\gamma_{t}^{2} \) for such symmetries would be too general to be of much use.

The simplest lattice we can discuss is the cubic lattice; its \([c]\) and \([R]\) matrices have only three independent components: \( c_{11}, c_{12}, \) and \( c_{44} \) for \([c]\) and \( r_{11}, r_{12}, \) and \( r_{44} \) for \([R]\). The \( \gamma_{l}^{(c)} \) and \( \gamma_{t}^{(c)} \) (we use the superscript \( (c) \) to refer to the cubic lattice) constants have been calculated in ref. [20] for longitudinal and transversal waves propagating in the \( (100), (110), \) and \( (111) \) crystallographic directions and for an isotropic distribution of TLS orientations. Using the results of ref. [20], we calculate the ratios \( (\gamma_{l}^{(c)}/\gamma_{t}^{(c)})^{2} \) for the waves propagating in the three directions mentioned above. Denoting \( \xi \equiv r_{12}/r_{11} \) and \( \xi \equiv r_{44}/r_{11} \), we obtain

\[
\left( \frac{\gamma_{l}}{\gamma_{t}} \right)^{2} = \left( \frac{\gamma_{l}}{\gamma_{t}} \right)^{2} = \left( \frac{\gamma_{l}}{\gamma_{t}} \right)^{2} = \frac{2 + 6\xi + 7\xi^{2} + 4\xi^{2}}{4\xi^{2}},
\]

(16a)  
(16b)  
(16c)  
(16d)

Note that in the \( (110)-\)direction there are two transversal elastic waves, of reciprocally perpendicular polarization, propagating with different sound velocities. The isotropy condition for \( [R] \) is \( \zeta + 2\xi = 1 \), which sets the range of \( (\gamma_{l}^{(c)}/\gamma_{t}^{(c)})^{2} \) to \([4/3, \infty)\), as stated in eq. (4). If the lattice has lower symmetry, then \( \zeta + 2\xi \neq 1 \) and we introduce the parameter \( K \) to quantify the anisotropy, by imposing \( \zeta + 2K\xi = 1 \); therefore \( K = 1 \) corresponds to the isotropic case. We calculate the dependence on \( K \) of the ranges of \( (\gamma_{l}^{(c)}/\gamma_{t}^{(c)})^{2} \) in the three crystallographic directions of eqs. (16). Replacing \( \zeta \) by \( 1 - 2K\xi \) into (16), we get

\[
\begin{align*}
\left( \frac{\gamma_{l}}{\gamma_{t}} \right)^{2} &= \frac{15}{4} - \frac{10K}{\xi} + 8K^{2}, \\
\left( \frac{\gamma_{l}}{\gamma_{t}} \right)^{2} &= \frac{15}{4} - \frac{7K^{2}}{\xi} + 7K^{2} + 1, \\
\left( \frac{\gamma_{l}}{\gamma_{t}} \right)^{2} &= \frac{15}{4K} + \frac{1}{\xi} + 7 + \frac{1}{K^{2}}.
\end{align*}
\]

(17a)  
(17b)  
(17c)  
(17d)

Obviously, the condition \( K = 1 \) restores the isotropic equation for \( (\gamma_{l}/\gamma_{t})^{2} \) [18]. What is interesting to note is that all eqs. (17) are quadratic in \( 1/\xi \) and attain their minima at

\[
\frac{1}{\xi}_{\text{min}} = \frac{2K^{2} + 1}{2K^{2} + 1},
\]

and we obtain the following constraints on \( (\gamma_{l}/\gamma_{t})^{2} \) in the three propagation directions of the cubic crystal:

\[
\begin{align*}
\left( \frac{\gamma_{l}}{\gamma_{t}} \right)^{2} &\geq \frac{4K^{2}}{3}, \\
\left( \frac{\gamma_{l}}{\gamma_{t}} \right)^{2} &\geq \frac{K^{2} + 3}{3}, \\
\left( \frac{\gamma_{l}}{\gamma_{t}} \right)^{2} &\geq \frac{K^{2} + 3}{3K^{2}}.
\end{align*}
\]

(19a)  
(19b)  
(19c)
\[ \left( \frac{\gamma_i^{(c)}}{\gamma_t^{(c)}} \right)^2_{(111)} \geq \frac{4}{2K^2 + 1}. \]  (19d)

Now we can see that although for \( K = 1 \) all the conditions become identical, namely \((\gamma_i^{(c)}/\gamma_t^{(c)})^2 \geq 4/3\), for \( K \neq 1 \) the lower limits of \((\gamma_i^{(c)}/\gamma_t^{(c)})^2\) vary differently. For example for \( K \gg 1 \), the lower limits for \((\gamma_i^{(c)}/\gamma_t^{(c)})^2_{(100)}\) and \((\gamma_i^{(c)}/\gamma_t^{(c)})^2_{(110)}, 1\) become very big \((\propto K^2)\), the lower limit of \((\gamma_i^{(c)}/\gamma_t^{(c)})^2_{(110), 2}\) converges to 1/3, whereas the lower limit of \((\gamma_i^{(c)}/\gamma_t^{(c)})^2_{(111)}\) converges to zero.

If \( K \ll 1 \), the situation is the other way round. The limit (19a) converges to zero, (19b) and (19d) to 1 and 4, respectively, whereas the limit value (19c) converges to infinity, like \( 1/K^2 \). Therefore, in a cubic crystal, \( \gamma_i^{(c)} \) can become smaller than \( \gamma_t^{(c)} \) if the matrix \([\mathcal{R}]\) deviates significantly from the isotropic condition.

Conclusions. — We calculated the average phonon scattering rates on TLSs in trigonal and hexagonal crystals, to emphasize the anisotropy imposed by the lattice symmetry. The parameters of the model may be obtained by measuring \( \gamma_i \) and/or \( \gamma_t \) in some crystallographic directions and this enables one to calculate the coupling of TLSs with phonons propagating in any other direction. The number of \( \gamma_i \)'s and \( \gamma_t \)'s needed, depends on the number of independent parameters of the tensor of coupling constants, \([\mathcal{R}]\), which is determined by the symmetry of the lattice.

We showed that the allowed limits of the ratio \( \gamma_i^2/\gamma_t^2 \) in crystals with different symmetries are different from the one imposed in isotropic materials, which is \( \gamma_i^2/\gamma_t^2 \geq 4/3 \). In principle \( \gamma_i^2/\gamma_t^2 \) in crystals may take any value.

The calculations can be extended easily to disordered crystals of any symmetry. Moreover, although we used in our calculations an isotropic distribution over the TLS orientations, the comparison of our calculations with experimental data would enable one to find if our assumption is true or not. If it is not true, one can determine, at least in principle, the distribution over the orientations of the TLSs.

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We are grateful to Prof. R. Pohl, Prof. K. A. Topp for very useful and motivating correspondence. This work was partially supported by the NATO grant, EAP.RIG 982080.

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