Anderson localization of light in a random configuration of nanocolumns

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Abstract. We show pseudogap maps of Anderson localization of light adopting the parameters of self-organized nanocolumn samples, which consist of random arrays of parallel nanosized columnar semiconductor crystals. The maps indicate the parametric dependence of the localization effect. To obtain the maps, we simulated light propagation in open random media using the two-dimensional finite-difference time-domain method and analyzed the simulation results by Fourier transformation. We found that the shape of the pseudogaps is close to the one of bandgaps in photonic crystals. We conclude that strong localization of light occurs because of interference by average Bragg diffraction, not strong Mie resonant peaks.

1. Introduction

The possibility of Anderson localization of light, the electromagnetic wave localization similar to electron localization in disordered solids, in random dielectric systems has been discussed [1, 2]. However, theoretical and experimental studies for light propagation in random media are much more difficult than those in photonic crystals. As a result, the light localization effect is not well understood. Here we obtain pseudogap maps for self-organized nanocolumn samples [3] composed of nanosized columnar semiconductor crystals. A scanning electron microscope (SEM) image of nanocolumns is shown in figure 1.

The column diameter varies in the range from about 50 to 400 nm, and the height is around 1 µm. Nanocolumns have been fabricated using gallium nitride (GaN), indium nitride (InN), and a mixed crystal of the two, indium gallium nitride (InGaN). The band gap energies of GaN and InN are about 3.50 eV (near-ultraviolet emission) and 0.65 eV (near-infrared emission), respectively. Therefore, we can control the emission wavelength to be within near ultraviolet to near infrared by adjusting the composition of In x Ga1−x N. Because nanocolumns show stronger photoluminescence than thin-films [4], they have attracted much attention as novel materials for visible light-emitting devices. We have conducted a numerical study on the localization of electromagnetic waves in nanocolumn ensembles.

Conventional nanocolumn samples are random media where the positioning of the columns is disordered, as shown in figure 1. To observe the localization effect in such systems, the radius of the nanocolumns (r), the filling fraction of nanocolumns (Φ), and the angular frequency of electromagnetic
waves ($\omega$) must be properly tuned. We have simulated light propagation in nanocolumn ensembles with various parameters and studied the parametric dependence of the localization behavior.

![Figure 1. SEM image of GaN nanocolumns.](image1.png)

![Figure 2. Schematic of the 2D FDTD model for nanocolumns.](image2.png)

2. Calculation

We have used the two-dimensional (2D) finite-difference time-domain (FDTD) method [5] to simulate the temporal propagation of electromagnetic waves in nanocolumn ensembles allocated within a finite area. The FDTD method is a numerical method in which Maxwell’s equations are discretized using central difference approximations of the space and time partial derivatives. A schematic view of the 2D FDTD model is illustrated in figure 2. The electromagnetic waves propagate in the $x$-$y$ plane. Maxwell’s equations in the case of transverse-magnetic (TM) field and transverse-electric (TE) field can be described by

\begin{align}
\varepsilon(r)\varepsilon_0 \frac{\partial}{\partial t} E_z(r,t) &= \frac{\partial}{\partial x} H_y(r,t) - \frac{\partial}{\partial y} H_x(r,t) \tag{1a} \\
\mu_0 \frac{\partial}{\partial t} H_x(r,t) &= -\frac{\partial}{\partial y} E_z(r,t) \tag{1b} \\
\mu_0 \frac{\partial}{\partial t} H_y(r,t) &= \frac{\partial}{\partial x} E_z(r,t) \tag{1c}
\end{align}

\begin{align}
\mu_0 \frac{\partial}{\partial t} H_z(r,t) &= -\frac{\partial}{\partial x} E_y(r,t) + \frac{\partial}{\partial y} E_x(r,t) \tag{2a} \\
\varepsilon(r)\varepsilon_0 \frac{\partial}{\partial t} E_x(r,t) &= \frac{\partial}{\partial y} H_z(r,t) \tag{2b} \\
\varepsilon(r)\varepsilon_0 \frac{\partial}{\partial t} E_y(r,t) &= -\frac{\partial}{\partial x} H_z(r,t) \tag{2c}
\end{align}

respectively, where $\varepsilon_0$ and $\mu_0$ are the electric permittivity and magnetic permeability of vacuum, respectively, and $\varepsilon(r)$ is the relative electric permittivity. The refractive index $n(r)$ is related to $\varepsilon(r)$ by $\varepsilon(r) = n^2(r)$. The space increment is $\Delta x = \Delta y = 5$ nm, and the time increment is $\Delta t = 1 \times 10^{-17}$ s $< \Delta x/\sqrt{2}$, where $c$ is the velocity of light in vacuum. To model open systems, we used Berenger’s perfectly matched layer [6] for the boundary conditions in the FDTD simulation.

The sample area is $4.5 \mu m$ square and consists of random arrays of parallel nanocolumns with a constant radius ($r = 50$ nm) and a constant refractive index ($n = 2.35$, the value for GaN in the visible range). Nanocolumns do not touch each other and they are embedded in vacuum. Figure 3 shows an example of a simulated system with $\Phi = 0.4$. 


Figure 3. Example of a simulated system with $\Phi = 0.4$. The circles are nanocolumns, and the crosses are antennas. In the actual simulation, 400 antennas are evenly spaced in the system.

We have adopted several values of $\Phi$ ranging from 0.05 to 0.90, at intervals of 0.05, and calculated for the respective systems. In each case, we have irradiated a Gaussian white pulse ($\delta$-function pulse) onto the whole sample area at $t = 0$. After irradiation, light waves gradually escape through the boundaries because the simulation system is open. In the FDTD simulation, the temporal evolution of the electromagnetic field at each location is recorded via an array of 400 antennas evenly spaced in the system. From the recorded electromagnetic signals, we can obtain the localized light spectrum by averaging several power spectra that are Fourier transformed from each signal recorded in a time window $[T_1, T_2]$. Figure 4 shows an example of a localized light spectrum with TM mode, where $\Phi = 0.4$ and the time window is $[2 \text{ ps}, 4 \text{ ps}]$. The intensity was normalized by the intensity of incident light. We found five strong localization ranges designated as “frequency windows.” The shape of this spectrum depends only on $T_1$, $T_2$, and $\Phi$. In the case of other random configurations with the same parameters, the change in the shape is very small. Furthermore, we can obtain the Q-factor spectrum, which characterizes the localization effect, by taking the localized light spectra of different time windows and deriving the intensity decay times as a function of the frequency.

Figure 4. Example of a localized light spectrum with TM mode, where $\Phi = 0.4$ and the time window is $[2 \text{ ps}, 4 \text{ ps}]$ (note the logarithmic vertical scale).

3. Results and Discussion

From the simulation and analysis above, we have succeeded in obtaining pseudogap maps using these Q-factor spectra. Figures 5 and 6 show the pseudogap maps for a random medium. The maps indicate the parametric dependence of the localization effect. This is the first detailed report on Anderson localization of light. We found that the shapes of the pseudogap maps are similar to one another.
We also calculated photonic band structures by adopting the same parameters using the plane wave expansion method [7] and compared them with the present calculation for random media. The gap map for a hexagonal photonic crystal with TM mode is shown in figure 5, where the areas enclosed by the dashed lines indicate the photonic band gaps. In the case of TE polarization, photonic band gaps are not formed. We conclude that the shape of the pseudogap map with TM mode is close to the gap map for the corresponding photonic crystal. It has been pointed out that the frequency windows do not correspond to the maxima of the scattering cross section in the Mie resonances of a single cylinder, but the detailed physical origin has not been clarified [8, 9]. In the case of photonic crystals, band gaps are formed if Bragg diffraction conditions are satisfied. Therefore, from figure 5 we conclude that light localization phenomenon is very effective under the condition of “average Bragg diffraction” and that the influence of Mie scattering is much smaller than the influence of average Bragg diffraction.

We have observed random lasing with TE polarization, $\Phi = 0.45$ and $\omega r/c = 1.13$ in a self-organized GaN nanocolumn sample at the emission wavelength [10]. This phenomenon occurs because the localization effect in the random medium plays the role of an optical resonator and nanocolumns also act as an efficient gain medium at the resonant wavelength. In fact, we find that the optimal parameter $\omega r/c$ for strong light localization with $\Phi = 0.45$ ranges from about 1.05 to 1.35 from figure 6. On the other hand, we have not observed random lasing in other samples which do not indicate high-Q at the emission wavelength on figure 6. In this way, we can estimate optimal conditions for obtaining high-Q samples from pseudogap maps.

![Figure 5. Pseudogap map for a random medium with TM polarization, and gap map for a hexagonal photonic crystal (photonic band gaps are inside dashed lines).](image)

![Figure 6. Pseudogap map for a random medium with TE polarization.](image)

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