Study of Opportunistic Relaying and Jamming Based on Secrecy-Rate Maximization for Buffer-Aided Relay Systems

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Abstract—In this paper, we investigate opportunistic relaying and jamming techniques and develop relay selection algorithms that maximize the secrecy rate for multiuser buffer-aided relay networks. We develop an approach to maximize the secrecy rate of relay systems that does not require the channel state information (CSI) of the eavesdroppers. We also devise relaying and jamming function selection (RJFS) algorithms to select multiple relay nodes as well as multiple jamming nodes to assist the transmission. In the proposed RJFS algorithms inter-relay interference cancellation (IC) is taken into account. IC is first performed to improve the transmission rate to legitimate users and then inter-relay IC is applied to amplify the jamming signal to the eavesdroppers and enhance the secrecy rate. With the buffer-aided relays the jamming signal can be stored at the relay nodes and a buffer-aided RJFS (BF-RJFS) algorithm is proposed. Greedy RJFS and BF-RJFS algorithms are then developed for relay selection with reduced complexity. Simulation results show that the proposed RJFS and BF-RJFS algorithms can achieve a higher secrecy rate performance than previously reported techniques even in the absence of CSI of the eavesdroppers.

Index Terms—Physical-layer security, relay systems, resource allocation, jamming.

I. INTRODUCTION

The broadcast nature of wireless communications makes secure transmissions a very challenging problem. Security techniques implemented at the network layer rely on encryption keys which are nearly unbreakable. However, the computational cost of such encryption algorithms is extremely high. In order to reduce such cost novel security techniques at the physical layer have been developed. Physical-layer security was first conceived by Shannon in his landmark 1949 paper [1] using an information theoretic viewpoint, where the feasibility of physical-layer security has been theoretically discussed. Later on, Wyner proposed a wire-tap channel model that can achieve positive secrecy rates under the assumption that users have statistically better channels than those of the eavesdroppers [2]. Since then further research has been devoted to the wire-tap model in broadcast and multiple-antenna channels [3], [4], [5]. Techniques to enhance the secrecy of wireless systems such as artificial noise [6], beamforming [7] and relay techniques [8], [9] have also been extensively studied.

A. Previous Work and Problems

Recently, the concept of physical-layer security with multiuser wireless networks has been thoroughly investigated and approaches based on transmit processing and relay techniques have drawn a great deal of attention [9], [10], [11], [12]. Transmit processing relies on intelligent design of precoding and signalling strategies to improve the secrecy rate performance. The use of relays [13] and the exploitation of spatial diversity can also enhance secrecy rates. Moreover, recent advances like buffer-aided relays have gained significant attention [14], [15], [16] as they can provide significant performance advantages over standard relays.

Buffer-aided relay systems with secure constraints have been investigated in half-duplex [14], [15], [17], [16] and full-duplex systems [18]. Opportunistic relay schemes have been examined with buffer-aided systems in [19], [20], [21]. In this context, inter-relay interference cancellation (IC) at relay nodes is a fundamental aspect in opportunistic relay schemes. In [19], IC has been combined with buffer-aided relays and power adjustment to mitigate inter-relay interference (IRI) and minimize the energy expenditure. Furthermore, in [20] a distributed joint relay-pair selection has been proposed with the aim of rate maximization in each time slot using a threshold to avoid increased relay-pair switching and CSI acquisition. In [21] and [22], a jammer selection algorithm and a joint relay and jammer selection technique have been investigated. The studies in [21] and [22] have shown that relaying contributes to a better transmission rate for legitimate users, whereas jamming can deteriorate the transmission to the eavesdropper. Therefore, relaying and jamming lead to an improvement in secrecy rate performance. However, opportunistic buffer-aided relay schemes with jamming techniques for improving physical layer security have not been examined so far.

B. Contributions

In this work, we propose an opportunistic relaying and jamming scheme and develop relay selection algorithms for the downlink of multiuser single-antenna and multiple-input multiple-output (MIMO) buffer-aided relay networks that maximize the secrecy rate, which is a challenging task due to the difficulty to obtain CSI of the eavesdroppers. Preliminary results of the proposed techniques have been reported in [23], where relaying and jamming selection have been examined, and in [24], where relay selection based on the secrecy rate has been studied. Here, we devise a relay selection approach for effective secrecy rate (E-SR) maximization that does not require CSI of the eavesdroppers. The proposed relaying and jamming function selection (RJFS) algorithms select multiple relay nodes as well as multiple jamming nodes to help the transmission. We also present an opportunistic relaying and jamming scheme in which relaying or jamming is performed within the same set of relays at different time slots. In the proposed RJFS algorithms, IC is employed to improve the transmission rate to legitimate users and the residual interference is used to amplify the jamming signal to the eavesdroppers. We exploit buffer-aided relays to store the jamming signals at the relay nodes and devise a buffer-aided relaying and jamming function selection (BF-RJFS) algorithm. Greedy RJFS and BF-RJFS algorithms are also developed for relay selection with...
reduced complexity. Simulations show that the proposed RJFS and BF-RJFS algorithms can outperform previously reported techniques in the absence of CSI of the eavesdroppers. In addition, the greedy RJFS and BF-RJFS algorithms achieve a performance close to that of the exhaustive search-based RJFS and BF-RJFS algorithms, while requiring a much lower computational cost. The main contributions of this work are:

- The E-SR maximization approach that does not require CSI of the eavesdroppers is proposed.
- An opportunistic relaying and jamming scheme for single-antenna and MIMO buffer-aided relay systems.
- Novel RJFS algorithms that maximize the secrecy rate are developed for buffer-aided relay systems.
- Greedy RJFS and BF-RJFS algorithms are developed to reduce the computational complexity of exhaustive search-based RJFS and BF-RJFS algorithms.
- A secrecy rate analysis of the proposed RJFS algorithms.

This paper is organized as follows. In Section II, the system model and problem formulation are introduced. A review of relay selection techniques and a novel relay selection criterion without CSI to the eavesdroppers are included in Section III. The proposed RJFS and BF-RJFS algorithms are introduced in Section IV. In Section V a secrecy analysis is carried out. In Section VI, we present and discuss the simulation results. The conclusions are given in Section VII.

C. Notation

| Notation | Description |
|----------|-------------|
| $A \in \mathbb{C}^{M \times N}$ | matrices of size $M \times N$ |
| $a \in \mathbb{C}^{M \times 1}$ | column vectors of length $M$ |
| $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ | conjugate, transpose, and conjugate transpose, respectively |
| $I_M$ | identity matrix with size $M$ |
| $\text{diag}(a)$ | diagonal matrix with the elements of $a$ along its diagonal |
| $CN(0, \sigma_n^2)$ | complex Gaussian |
| $\log(\cdot)$ | logarithm |
| $\|A\|_F$ | Frobenius norm of $A$ |
| $H_i \in \mathbb{C}^{N_i \times N_t}$ | channel matrix from the transmitter to the $i$th relay |
| $L_{\text{state}} \in \mathbb{C}^{S_{\text{total}} \times N_t \times L}$ | state matrix of the relays |
| $s^{(t)}(i) \in \mathbb{C}^{M \times N_t \times 1}$ | transmit signal at the source |
| $y_i^{(t)} \in \mathbb{C}^{N_t \times 1}$ | received signals at the $i$th relay |
| $\Gamma_i^{(t)}$ | SINR at the $i$th relay node |
| $C_r$ and $C_e$ | secrecy capacity and rate |
| $\Omega_r$ | transmit power |

II. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we introduce the multiuser MIMO buffer-aided relay system model along with details of the proposed opportunistic relaying and jamming scheme. The physical-layer security problem associated with the proposed opportunistic relaying and jamming scheme is then formulated.
to a relay scheme without buffers.

- **Buffer size** \( L > 1 \): The thresholds \( \eta_{\text{link1}} \) and \( \eta_{\text{link11}} \) that indicate the power allocation to the transmitter \( \eta_{\text{link1}} P \) or \( \eta_{\text{link11}} (2 - P) \), where \( P \) is the power, are calculated separately for Link I and Link II, which determines if the relays perform relaying or jamming.
  - If \( \eta_{\text{link1}} > \eta_{\text{link11}} \), Link I is active. It indicates that the channels from the source to the relays can provide a better transmission environment. In this scenario, the jamming signals are generated independently at the relays, which are selected to perform the jamming function. The selection of the relays which perform the relaying function can be done according to different relay selection criteria. The jamming signal will also be stored at the buffers.
  - If \( \eta_{\text{link1}} \leq \eta_{\text{link11}} \), Link II is active. It indicates that the channels from the relays to the users have better links. In this scenario, relays will forward the signals to the destination. The jamming signals in Link II are the stored jamming signals in Link I, which means that the jamming signals in Link II do not need to be generated in Link II.

If CSI remains unchanged or one link is always better than the other than the system will employ a counter, \( \eta_L \), compare it with a maximum value \( \eta_{\text{max}} \), and activate the link that has been inactive for \( \eta_{\text{max}} \) transmissions. The value \( \eta_{\text{max}} \) is set by the designer. In addition, the change of links makes it more difficult for the eavesdropper to obtain the pattern.

In this system, each relay node is equipped with \( N_i \) antennas. To indicate when the relays are performing the jamming function, the relay antenna number is represented by \( N_k \). For one relay we can have \( N_k = N_i \). At the receiver side each user and each eavesdropper is equipped with \( N_r \) and \( N_e \) receive antennas. We also assume that the eavesdroppers do not jam the transmission and the data transmitted to each user, relay, jammer and eavesdropper experience a flat-fading MIMO channel. The quantities \( H_i \in \mathbb{C}^{N_r \times N_i} \) and \( H_e \in \mathbb{C}^{N_e \times N_i} \) denote the channel matrices of the \( i \)-th relay and the \( e \)-th eavesdropper, respectively. The quantities \( H_{kr} \in \mathbb{C}^{N_e \times N_k} \) and \( H_{kr} \in \mathbb{C}^{N_r \times N_k} \) denote the channel matrices of the \( k \)-th relay to the \( e \)-th eavesdropper and the \( k \)-th relay to the \( r \)-th user, respectively. The channel between the \( k \)-th relay to the \( i \)-th relay is represented by \( H_{ki} \in \mathbb{C}^{N_r \times N_k} \).

To support the transmission of data to \( M \) users, the source is equipped with \( N_t \geq N_r M \) antennas. The total number of antennas with \( S \) relaying function nodes as well as \( K \) jamming function nodes should satisfy \( N_t S \geq N_r M \) and \( N_k K \geq N_r M \), respectively. At the same time we assume that the total number of antennas of the eavesdroppers is \( N_e N \geq N_r M \). In order to satisfy the precoding constraints [25], the number of \( N_e M \) transmit antennas is used to transmit signals to \( M \) users. The relays can estimate the channel from the jammers by assuming that there are pilots in the packet structure, that they know the jamming signals and that the eavesdroppers cannot decode the jammers. This is reasonable because the relays also perform jamming and therefore should know the jamming signals. Moreover, we also assume that CSI of the users can be obtained at the transmitter by feedback channels from the relays. Alternatively, advanced parameter estimation and relay techniques can be employed [26, 27].

In previous works [39, 40, 41, 42, 43], precoding techniques have been applied to mitigate the interference among users. In this work, we adopt for simplicity linear zero-forcing precoding whose precoding matrix can be described by

\[
U^{(t)} = H^{(t)} H^{(t)H} \cdot \frac{1}{\sigma^2} \in \mathbb{C}^{N_r \times N_i}\]  

with \( U_i \in \mathbb{C}^{N_r \times N_i} \), the total precoding matrix can be expressed as

\[
U^{(t)} = \begin{bmatrix} U_1^{(t)} & U_2^{(t)} & \cdots & U_M^{(t)} \end{bmatrix},
\]

and the channel matrix to \( S \) selected relays is given by

\[
H^{(t)} = \begin{bmatrix} H_1^{(t)} & H_2^{(t)} & \cdots & H_S^{(t)} \end{bmatrix} \in \mathbb{C}^{N_e \times N_t}.
\]

If the number of antennas equipped at each relay and each user are the same, the minimum required number of relays is \( S = M \). The channels of the selected relays forwarding the signals to the \( r \)-th user are described by

\[
H_{Kr}^{(t)} = \begin{bmatrix} H_{1r}^{(t)} & H_{2r}^{(t)} & \cdots & H_{Kr}^{(t)} \end{bmatrix} \in \mathbb{C}^{N_r \times KN_k}
\]

and the channels from the relays to the users are described by

\[
H_{M}^{(t)} = \begin{bmatrix} H_{K1}^{(t)} & H_{K2}^{(t)} & \cdots & H_{KM}^{(t)} \end{bmatrix} \in \mathbb{C}^{MN_r \times KN_k}.
\]

The selected relays also perform jamming for Link I’s transmission to the eavesdroppers, whereas the channels of the jammers to the \( i \)-th relay are given by

\[
H_{Ki}^{(t)} = \begin{bmatrix} H_{1i}^{(t)} & H_{2i}^{(t)} & \cdots & H_{Ki}^{(t)} \end{bmatrix} \in \mathbb{C}^{N_r \times KN_k}
\]

In each link, if we assume that the total jamming signals are \( J = [j_1^{T}\ j_2^{T}\ \cdots\ j_K^{T}]^{T} \), the received signal \( y_i^{(t)} \in \mathbb{C}^{N_i \times 1} \) at each relay node can be expressed by

\[
y_i^{(t)} = H_i U_i s_i^{(t)} + \sum_{j \neq i} H_i U_j s_j^{(t)} + H_{Ki}^{(t)} J + n_i
\]

In [8], \( n_i \in \mathcal{C}N(0, \sigma^2) \) and the superscript \( pt \) designates the previous time slot when the signal is stored in the buffer at the relay nodes. The quantity \( \sigma^2 \) is the noise variance for the channel and \( H_{Ki}^{(t)} J \) is regarded as the IRI among the \( i \)-th relay and the \( K \) jammers. The intended relays are selected according to different criteria, which will be explained later on. The received signals are expressed by \( y^{(pt)} = [y_1^{(pt)}\ y_2^{(pt)}\ \cdots\ y_S^{(pt)}]^{T} \). The superscript \( pt \) represents the time slot and due to the characteristics of buffer relay nodes, the values can be different for each relay node. According to the theorem in [19], IRI can be cancelled. The jammers are targeted towards the \( e \)-th eavesdropper channel described by

\[
H_{Ke}^{(t)} = \begin{bmatrix} H_{1e}^{(t)} & H_{2e}^{(t)} & \cdots & H_{Ke}^{(t)} \end{bmatrix} \in \mathbb{C}^{N_e \times KN_k}
\]
The received signal at the $e$th eavesdropper is then given by
\[ y_e^{(t)} = H_e U_j s_j^{(t)} + \sum_{j \neq i} H_e U_j s_j^{(t)} + H_{ke}^{(t)} J + n_e. \] (10)
where $n_e \sim \mathcal{CN}(0, \sigma_n^2)$ is the noise vector at the eavesdropper. For the eavesdropper, the term $H_{ke}^{(t)} J$ acts as the jamming signal, which cannot be removed without CSI knowledge from the $k$th jammer to the $e$th eavesdropper.

If we assume that the transmitted signals from the relays to the users are expressed as $s^{(t)}$, the received signal at the destination is given by
\[ y_r^{(t)} = H_M r^{(t)} + n_r. \] (11)
where $n_r \sim \mathcal{CN}(0, \sigma_n^2)$ is the noise vector at the relay nodes.

In the existing IRI scenario based on (8) when the transmitted signals $s$ are statistically independent with unit average energy $\mathbb{E}[ss^H] = I$, the SINR at relay node $i$ $\Gamma_{11i-1}^{(t)}$ is given by
\[ \Gamma_{11i-1}^{(t)} = \frac{\gamma_{s_i, R_i}}{\varphi(k,i) \gamma_{R_k, R_i} + \gamma_{s_j, R_i} + N_t}, \] (12)
where $\varphi(K, i)$ is the factor that describes the IC feasibility and $\gamma_{m,n}$ represents the instantaneous received signal power for the links $m \rightarrow n$ as described by
\[ \gamma_{s_i, R_i} = \|H_i U_i\|_F, \quad \gamma_{s_j, R_i} = \sum_{j \neq i} \|H_i U_j\|_F, \] (13)
\[ \gamma_{R_k, R_i} = \|H_k J\|_F, \] (14)
The SINR at the $e$th eavesdropper node $\Gamma_e^{(t)}$ as well as the $r$th legitimate user $I_{r}^{(t)}$ is described by
\[ I_{11i-e}^{(t)} = \frac{\gamma_{s_i, E_e}}{\gamma_{R_k, E_e} + \gamma_{s_j, E_e} + N_e}, \] (15)
and
\[ I_{11i-r}^{(t)} = \frac{\gamma_{R_k, R_r}}{\gamma_{R_k, R_r} + N_r}, \] (16)
where the terms in (13) are given by
\[ \gamma_{s_i, E_e} = \|H_i U_i\|_F, \quad \gamma_{s_j, E_e} = \sum_{j \neq i} \|H_i U_j\|_F, \] (17)
\[ \gamma_{R_k, E_e} = \|H_k J\|_F \] (18)
and
\[ \gamma_{R_k, R_r} = \|H_k J\|_F. \] (19)
Depending on the IRI cancelation (IC) at the relay nodes, two type of schemes can be applied. According to [19], if we assume $\mathbb{E}[ss^H] = I$ and $\mathbb{E}[y^{(pt)} y^{(pt)^H}] = I$, the feasibility of IC can be described by a factor $\varphi(K, i)$ which is described by
\[ \varphi(K, i) = \begin{cases} 0 & \text{if } \det \left( (H_e H_e^H + I)^{-1} H_k J H_k^H \right) \geq \gamma_0 \\ 1 & \text{otherwise}, \end{cases} \] (20)
where $\varphi(K, i) = 0$ means the interference can be cancelled from the received signal at the relays, whereas $\varphi(K, i) = 1$ means IC should not be performed. The quantity $\gamma_0$ is the threshold that indicates the feasibility of IC, which is obtained by simulation. We assume that the channels from the relays, which perform jamming, are available at the transmitter.

In the IC scenario, interference mitigation can be performed at the relay nodes by setting $\varphi(K, i) = 0$. The SINR expressions at the $i$th relay node, the $e$th eavesdropper and the $r$th receiver are respectively given by
\[ \Gamma_{11i-1}^{(t)} = \frac{\gamma_{s_i, R_i}}{\gamma_{s_j, R_i} + N_t}, \quad \Gamma_{11i-e}^{(t)} = \frac{\gamma_{s_i, E_e}}{\gamma_{R_k, E_e} + \gamma_{s_j, E_e} + N_e} \] (22)
and
\[ \Gamma_{11i-r}^{(t)} = \frac{\gamma_{R_k, R_r}}{\gamma_{R_k, R_r} + N_r}. \] (23)
Alternative interference mitigation techniques can also be considered [44], [45], [46], [47], [48], [49], [50], [51], [52], [53], [54], [55], [56], [57], [58], [59], [60], [61], [62], [63], [64], [65], [66], [67], [68], [69], [70], [71], [72], [73], [74], [75], [76], [77], [78], [79], [80], [81], [82], [83], [84], [85], [86], [87], [88], [89], [90], [91].

### B. Problem formulation

In this subsection, we describe the secrecy rate used in the literature to assess the performance of the proposed algorithms in physical-layer security systems and formulate the problem. The MIMO system secrecy capacity $C_s$ is given by
\[ C_s = \max_{Q_s \succeq 0, \text{Tr}(Q_s) = 1} \log(\det(I + H_b a Q_s H_b^H)) - \log(\det(I + H_e a Q_s H_e^H)), \] (24)
where $Q_s$ is the covariance matrix associated with the signal and $H_b a$ and $H_e a$ represent the links between the source to the users and the eavesdroppers, respectively. For relay systems [21], according to (8) and (11), with equal power $P$ allocated to the transmitter and the relays, the achievable rate of the users is given by
\[ R_e = \log(\det(I + \Gamma_e^{(t)})) \] (25)
where $\Gamma_e^{(t)}$ according to (16) is given by
\[ \Gamma_e^{(t)} = \frac{P}{\sum_{k=1}^{K} N_k} H_k J H_k^H (I + \frac{P}{N_t} H_{pt} U_{pt} U_{pt}^H H_{pt}) \] (26)
Similarly, the achievable rate of eavesdroppers is described by
\[ R_e = \log(\det(I + \Gamma_e^{(t)})) \] (27)
and the $\Gamma_e^{(t)}$ according to (15) is described by
\[ \Gamma_e^{(t)} = (I + \Delta)^{-1} \frac{P}{N_t} H_k H_k^H (I + \frac{P}{N_t} H_{pt} U_{pt} U_{pt}^H H_{pt}^H) \] (28)
where
\[ \Delta = \sum_{e=1}^{N} \frac{P}{\sum_{k=1}^{K} N_k} H_e H_e^H (I + \frac{P}{N_t} H_{pt} U_{pt} U_{pt}^H H_{pt}) \] (29)
In (28), $\Delta$ is the jamming signal to the eavesdropper. Using (28) and (27) the secrecy rate is given by
\[ R = \sum_{r=1}^{N} \sum_{e=1}^{E} [R_e - R_e^{+}] \] (30)
where \([x]^+ = \max(0, x)\). In (30), we assume that each eavesdropper will listen to the information transmitted to a particular user. However, the assumption of the availability of global CSI knowledge is impractical, especially for the eavesdroppers. For this reason, we consider partial CSI knowledge to the relays as well as to the users. The problem we are interested in solving is to select the set of relay nodes to perform relaying or jamming based on the maximization of the secrecy rate. Therefore, the proposed optimization problem can be formulated as:

\[
\begin{align*}
\text{maximize} & \quad R \\
\text{subject to} & \quad \Omega', \Omega^m \in \Psi
\end{align*}
\]

(31)

where \(\Psi\) represents the collection of relay subsets and \(\Omega'\) and \(\Omega^m\) denote the set of selected jamming function nodes and the set of relaying function nodes, respectively.

### III. Relay Selection Algorithms

In conventional relaying or jamming systems, relays always perform as the transmitter and the receiver to enhance the signal transmission from the source to the destination [92]. We first review several algorithmic solutions under this conventional relay scenario and then present the proposed relay selection based on the secrecy rate, which does not require knowledge of CSI to the eavesdroppers.

#### A. Conventional Relay Selection

Conventional relay selection does not take the jamming function of relay nodes into account and the relay nodes are selected with different selection criteria to assist the transmission between the source and the destination with only one eavesdropper [15] or without consideration of eavesdroppers [22], [93]. In [93], a max-min relay selection has been considered as the optimal selection scheme for conventional decode-and-forward (DF) relay setups. In a single-antenna scenario the relay selection is given by

\[
R^\text{max-min}_i = \arg\max_{R_i \in \Psi} \min(h_{S,R_i}, ||h_{R_i,D}||^2)
\]

(32)

where \(h_{S,R_i}\) is the channel gain between the source and the relay and \(h_{R_i,D}\) is the channel gain between the relay \(k\) and the destination. Similarly, a max-link approach has also been introduced to relax the limitation that the source and the relay transmission must be fixed. The max-link relay selection strategy can be described by

\[
R^\text{max-link}_i = \arg\max_{R_i \in \Psi} \left( \sum_{R_j \in \subset' \Psi \neq L} ||h_{S,R_i}||^2 \right)
\]

(33)

With the consideration of the eavesdropper, a max-ratio selection policy is proposed in [15] and is expressed by

\[
R^\text{max-ratio}_i = \arg\max_{R_i \in \Psi} \left( \eta_1, \eta_2 \right)
\]

(34)

with

\[
\eta_1 = \max_{R_i \in \subset' \Psi \neq L} \frac{||h_{S,R_i}||^2}{||h_{ac}||^2}
\]

(35)

The aforementioned relay selection procedure is based on knowledge of CSI.

#### B. Optimal Selection (OS)

Since conventional relay selection [22] may not support systems with secrecy constraints, we consider optimal selection (OS) which takes the eavesdropper into consideration. The SINR of OS in the downlink of multiuser MIMO relay systems under consideration can be expressed similarly to (15) and (16), as described by

\[
\Gamma^{(i)}_{s} = \frac{\gamma_{S_i,R_e}}{\gamma_{S_i,R_e} + N_e}
\]

(37)

and

\[
\Gamma^{(i)}_{r} = \frac{\gamma_{R_i,R_e}}{\gamma_{R_i,R_e} + N_r}
\]

(38)

The OS algorithm is given by

\[
R^\text{OS} = \arg\max \left( \Gamma^{(i)}_{s} - \Gamma^{(i)}_{r} \right)
\]

(39)

#### C. Proposed Effective Secrecy-Rate Relay Selection

In the previously described relay selection algorithms, the availability of CSI to the eavesdroppers is an adopted assumption in the design of relay selection algorithms with secrecy constraints. However, in the optimization problem in (31), the CSI of the eavesdroppers is not available to the transmitter and the users. In order to circumvent this limitation, we propose a novel relay selection criterion that is termed effective secrecy rate (E-SR), which does not require CSI to the eavesdroppers and is incorporated in the multiuser MIMO buffer-aided relay system under study. The proposed E-SR approach is based on the maximization of the secrecy rate and introduces a simplification in the computation of the expression that does not require the knowledge of CSI to the eavesdroppers. The proposed E-SR approach for selecting multiple relays is expressed by

\[
R^\text{E-SR} = \arg\max_{\Psi} \sum_{i=1}^{L} \left\{ \log \left( \det \left( I + \left( H_i R_i H_i^H \right)^{-1} (H_i R_d H_i^H) \right) \right) \right\}
\]

(40)

\[
\text{where the covariance matrix of the interference and the signal can be described as } R_i = (H_i)^{-1}\left(H_i^H + \sum_{j \neq i} U_j s_j^H U_j R_d \right) \text{ and } R_d = U_i s_i^H U_i R_d \text{, respectively. The details of E-SR relay selection criterion are given in the Appendix. In [40], no CSI to the eavesdroppers is required and the E-SR approach only depends on the CSI to the intended receiver and the covariance matrix of the interference and the signal. In the following proposed relaying and jamming schemes, the E-SR technique is applied to circumvent the need for global instantaneous CSI of the eavesdroppers.}

### IV. Relaying and Jamming Function Selection Algorithms

In this section, we detail the proposed RJFS and BF-RJFS algorithms along with their cost-effective greedy versions for single-antenna and multiple-antenna scenarios.
A. Relaying and Jamming Function Selection (RJFS)

We assume that the total number of relay nodes is \( S_{\text{total}} \) and \( \Omega \) is the total relay set. To apply the opportunistic scheme in the system, an initial state is set according to the channel:

\[
\Omega^m, s = \arg \max_{\Omega^m} \det \left( H_{\Omega^m} H_{\Omega^m}^H \right),
\]

(41)

where \( H_{\Omega^m} \) refers to the set of channels examined prior to selection and we assume that in the initial state the relays will not perform the jamming function. \( S \) relay nodes are selected according to the criterion as explained in Section II. With the total number of relaying and jamming nodes \( S_{\text{total}} \) and the number of selected nodes in each group \( S \), the selection operation can be expressed as:

\[
\Psi = \left( S_{\text{total}} S \right).
\]

(42)

where \( \Psi \) represents the total number of sets of \( S \) combinations and in each set there are \( S \) selected relaying or jamming nodes. For a particular set \( \Omega^m \), the channel matrix of selected sets can be described by

\[
H_{\Omega^m} = \begin{bmatrix}
H_{\Omega^m_1}^T & H_{\Omega^m_2}^T & \cdots & H_{\Omega^m_S}^T
\end{bmatrix}^T.
\]

(43)

If the total collection of selected sets is represented by \( \Psi_{\text{Relaying}} \), then for each set the relay selection is given by

\[
\Omega^m, s = \arg \max_{\Omega^m \in \Psi_{\text{Relaying}}} \sum \left\{ \log \left( \det \left( \Gamma_{\Omega^m}^{(t)} \right) \right) - \log \left( \det \left( \Gamma_{\Omega^m}^{(t)} \right) \right) \right\}
\]

(44)

where

\[
\Gamma_{\Omega^m}^{(t)} = I + (H_i R_{i}^{\Omega^m} H_i^H)^{-1}(H_i R_{i}^{\Omega^m} H_i^H)
\]

(45)

and

\[
\Gamma_{\Omega^m}^{(t)} = I + U_i^H R_{i}^{\Omega^m} U_i^H
\]

(46)

In (45) and (46), the covariance matrices \( R_{i}^{\Omega^m} \) and \( R_{i}^{\Omega^m} \) can be obtained in the same way as illustrated in (40). The only difference of the RJFS algorithm resides in the calculation of \( R_{i}^{\Omega^m} \), apart from the interference from different users, there is also existing interference from the jamming function relay nodes. With the same distributions of the channels from the jamming function relay nodes to the eavesdropper, \( R_{i}^{\Omega^m} \) can be calculated in a similar way to that in (40). In Algorithm 1 the main steps of RJFS are given. Step 1 of Algorithm 1 gives the collection of relay subsets, which contain the combinations of \( S \) relay nodes out of \( S_{\text{total}} \) relay nodes. In our definition, \( \Psi_{\text{Relaying}} \) is the same as \( \Psi \). However, to indicate the differences in the buffer relay system, we use \( \Psi_{\text{Relaying}} \) instead of \( \Psi \). Note that in both RJFS and BF-RJFS algorithms, we use \( \Psi_{\text{Relaying}} \) as a collection of relay subsets which perform the relaying function. With no buffers implemented at the relay nodes, the RJFS algorithm only selects the relays in Link I. The relays used in Link II are the same as those selected in Link I.

Algorithm 1 RJFS Algorithm

Require: \( H_i, R_i, R_d, S_{\text{total}} \) and \( S \)

1: \( \Psi_{\text{Relaying}} = \{ S_{\text{total}} S \} \)

2: \( \{ \Omega_c, \Omega_r \} = \text{size}(\Psi_{\text{Relaying}}) \)

3: for \( m = 1 : \Omega_c \) do

4: \( \Gamma_{\Omega^m}^{(t)} = I + (H_i R_{i}^{\Omega^m} H_i^H)^{-1}(H_i R_{i}^{\Omega^m} H_i^H) \)

5: \( \Gamma_{\Omega^m}^t = I + U_i^H R_{i}^{\Omega^m} U_i^H \)

6: \( \Gamma(\Omega^m) = \sum \left\{ \log \left( \det \left( \Gamma_{\Omega^m}^{(t)} \right) \right) - \log \left( \det \left( \Gamma_{\Omega^m}^t \right) \right) \right\} \)

7: \end for

8: \( \Omega^{m,*} = \arg \max_{\Omega^m \in \Psi_{\text{Relaying}}} \{ \Gamma(\Omega^m) \} \)

9: return The set of the selected relays \( \Omega^{m,*} \)

B. Buffer-Aided Relaying and Jamming Function Selection (BF-RJFS)

Here we describe the proposed BF-RJFS algorithm, which exploits relays equipped with buffers. Based on the RJFS algorithm, the selection of the \( S \) relays used for signal reception is the same as that in the buffer relay scenario. The main difference between the proposed BF-RJFS and RJFS algorithms relies on the selection of the jammer. The selection of the set of jamming and communication relays is performed simultaneously. According to (42), we assume the corresponding threshold is stored in \( \Gamma \). Given the total collection of jamming selections \( \Psi_{\text{Jamming}} \), the remaining relays are selected according to the proposed E-SR criterion as described by

\[
\Omega^{r,*} = \arg \max_{\Omega^r \in \Psi_{\text{Jamming}}} \sum \left\{ \log \left( \det \left( \Gamma_{\Omega^r}^{(t)} \right) \right) - \log \left( \det \left( \Gamma_{\Omega^r}^{(t)} \right) \right) \right\},
\]

(47)

where \( \Gamma_{\Omega^r}^{(t)} \) is given by

\[
\Gamma_{\Omega^r}^{(t)} = I + (H_{\Omega^r} R_{b}^{\Omega^r} H_{\Omega^r}^H)^{-1}(H_{\Omega^r} R_{b}^{\Omega^r} H_{\Omega^r}^H),
\]

(48)

where \( R_{b}^{\Omega^r} \) is the covariance matrix of the transmit signal from the jamming function relay nodes to the users. The jamming signal is the same as the received signal from the relays in previous time slots. The calculation of \( R_{b}^{\Omega^r} \) depends on (29) and \( R_{b}^{\Omega^r} \) relies on (17) and (18). In this procedure, the calculation of \( \Gamma_{\Omega^r}^{(t)} \) is obtained by

\[
\Gamma_{\Omega^r}^{(t)} = I + U_{r}^H R_{b}^{\Omega^r} U_{r}^H,
\]

(49)

where the relays used for jamming in the next time slot are selected. With the selection of communication relays and jamming relays the system can provide a better secrecy performance as compared to conventional relay systems. In Algorithm 2 the main steps of BF-RJFS are outlined. Steps 1 to 8 of Algorithm 2 eliminate the relay nodes with empty buffers because they cannot perform relaying function. Steps 9 to 20 eliminate relay nodes with a full buffer as the signals...
C. Proposed Greedy RJFS and BF-RJFS Algorithms

In both RJFS and BF-RJFS algorithms, exhaustive searches are implemented to select the relaying and jamming nodes. The incorporation of a greedy strategy [33] in both RJFS and BF-RJFS algorithms can significantly reduce the computational cost of the proposed exhaustive search-based RJFS and BF-RJFS algorithms. In an exhaustive search, all possible combinations are investigated to achieve optimal relay selection. Unlike an exhaustive search, a greedy search selects the relay with the best output at every iteration and then repeats the process with the remaining relays. The selection is completed when the desired number of relays are chosen. The total number of relays considered in the search is given by

$$\Omega_c = S_{\text{total}} + S_{\text{total}} - 1 + \cdots + S_{\text{total}} - S.$$  \hspace{1cm} (50)

From (50), we can see that the number of relays considered increases linearly with the total number of relays $S_{\text{total}}$ which contributes to the reduction of the computational complexity.

In the following we describe the proposed greedy RJFS algorithm. When the $K$ relays that forward the signals to the users are determined, the relays used for signal reception are chosen based on the E-SR criterion, as given by

$$m^* = \arg \max_{m \in \Omega} \left[ \log \left( \det \left( \Gamma_m^t \right) \right) - \log \left( \det \left( \Gamma^t \right) \right) \right],$$  \hspace{1cm} (51)

where $m$ represents the selected relay and $\Gamma_m^t$ corresponds to the $m$th relay which is calculated based on (12) and given by

$$\Gamma_m^t = I + (H_m R_m^t H_m^H)^{-1} (H_m R_m^t H_m^H),$$  \hspace{1cm} (52)

whereas $\Gamma^t$ is described by

$$\Gamma^t = I + U_m^H R_m^t U_m R_m^t.$$  \hspace{1cm} (53)

Instead of the exhaustive search of the selected set $\Omega^m$, the $m$th relay is computed with the aim of finding the relay that provides the highest secrecy rate based on $m$. In (52), $R_m^t$ and $R_m^t$ are obtained in the same way as in (45) and (46). The main steps are described in Algorithm 3.

### Algorithm 3 Greedy-RJFS Algorithm

Require: $H_m$, precoding matrix $U_m$, $R_m^t$, $R_m^t$, $S$, $\Omega$

1: for $t = 1 : S$ do
2: \hspace{1cm} $\Omega = \text{length}(\Omega)$
3: \hspace{1cm} for $m = 1 : \Omega$ do
4: \hspace{2cm} $\Gamma_m^t = I + (H_m R_m^t H_m^H)^{-1} (H_m R_m^t H_m^H)$
5: \hspace{2cm} $\Gamma^t = I + U_m^H R_m^t U_m R_m^t$
6: \hspace{2cm} $\Gamma(m) = \log \left( \det \left( \Gamma_m^t \right) \right) - \log \left( \det \left( \Gamma^t \right) \right)$
7: \hspace{2cm} $\{\text{Calculate the threshold for all relays}\}$
8: \hspace{1cm} end for
9: \hspace{1cm} $m^* = \arg \max_{m \in \Omega} \Gamma(m)$
10: \hspace{1cm} $\{\text{Find the relay which gives the highest value and choose this relay as one of the selected relay node}\}$
11: end for
12: $\Omega_{\text{Greedy},*} = m^*$
13: $\Omega = \Omega/m^*$
14: $\Gamma = \Gamma/m^*$
15: $\{\text{Remove the selected relay node from all relay set}\}$
16: end for
17: $\{\text{Repeat the steps again until $S$ relays are found}\}$
18: return $\Omega_{\text{Greedy},*}$

Similarly to the BF-RJFS algorithm, the Greedy-BF-RJFS algorithm substitutes the exhaustive search of all combinations with a greedy search of individual relays. The main difference lies in the jamming relay selection. For a particular user $r$, each relay performs a threshold calculation and the relay $k$ with the highest threshold is selected until $S$ relays are selected to forward the signal to all users. The details of the Greedy-BF-RJFS algorithm are given in Algorithm 4.
Algorithm 4 Greedy-BF-RJFS Algorithm

Require: $H_m$, $R_d$, $R_d$, $H_H$ and precoding matrix $U_r$, $R_m$, $H_m$, $L_{state}$, $L$, $\Omega$, $S$ and $\Omega$

1: if $L_{state}(::L) = 0$ then
2: \begin{align*}
\gamma_{link1} &= 0 \\
\{\text{The buffer is empty}\}
\end{align*}
3: else if $L_{state}(::L) \neq 0$ then
4: \begin{align*}
\Gamma^{(i)}(::m) &= I + (H_m R_d H_H)^{-1}(H_m R_d H_H) \\
\Gamma^{(i)}(::m) &= I + U_r H_r U^{-1} R_m R_d
\end{align*}
5: \begin{align*}
\Gamma^{(i)}(::m) &= I + [\log(\det(\Gamma^{(i)})) - \log(\det(\Gamma^{(i)}(::m)))]
\end{align*}
6: \begin{align*}
\gamma_{link1} &= \Gamma^{(i)}(::m)
\end{align*}
7: \begin{align*}
\gamma_{link1} &= \Gamma^{(i)}(::m)
\end{align*}
\begin{align*}
\{\text{The buffer is not empty, the threshold for Link I}\}
\end{align*}
8: end if
9: if $L_{state}(::1) = 0$ then
10: for $t = 1 : S$ do
11: for $m = 1 : \Omega$ do
12: \begin{align*}
\Gamma^{(i)}(::m) &= I + (H_m R_d H_H)^{-1}(H_m R_d H_H) \\
\Gamma^{(i)}(::m) &= I + U_r H_r U^{-1} R_m R_d
\end{align*}
13: \begin{align*}
\Gamma^{(i)}(::m) &= I + [\log(\det(\Gamma^{(i)})) - \log(\det(\Gamma^{(i)}(::m)))]
\end{align*}
14: \begin{align*}
\gamma_{link1} &= \max_m(\Gamma^{(i)}(::m))
\end{align*}
15: end for
16: $\Omega^* = \max_m(\Gamma^{(i)}(::m))$
17: end for
18: end if
19: \begin{align*}
\gamma_{link1} &= 0 \\
\{\text{The buffer is full}\}
\end{align*}
20: end if
21: else if $L_{state}(::1) \neq 0$ then
22: \begin{align*}
\gamma_{link1} &= 0 \\
\{\text{The buffer is not empty, the threshold for Link I}\}
\end{align*}
23: end if
24: if $\gamma_{link1} > \gamma_{link1}$ then
25: return The set of the selected relays $\Omega^*$ and perform Link I.
26: else if $\gamma_{link1} < \gamma_{link1}$ then
27: return The set of the selected relays $\Omega^*$ and perform Link I.
28: end if

V. Secrecy Analysis

In this section, we analyze the secrecy performance of standard single-antenna and MIMO relay systems as well as the proposed buffer-aided MIMO relay system with relaying and jamming function selection. We derive secrecy rate expressions for scenarios where CSI is available to the eavesdroppers. The expressions derived serve as benchmarks for the proposed RJFS and BF-RJFS algorithms. The overall secrecy capacity of a single-antenna relay system [13] is given by

**Definition 1.** For a selected relay $k$ and channels from source to relay $k$, relay $k$ to destination, source to eavesdropper, relay $k$ to eavesdropper expressed as $h_{srk}, h_{rkd}, h_{se}, h_{rk}$, respectively, the capacity is given by

$$C_k = \max \left\{ \frac{1}{2} \log_2 \left( \frac{1}{1 + P(h_{srk})^2} + \frac{1}{1 + P(h_{se})^2 + P(h_{rk})^2} \right) \right\}$$

Equation (54) can be rewritten as (55) in which the first part corresponds to the secrecy capacity to the user and the second part to the secrecy capacity of the eavesdropper:

$$C_k = \max \left\{ \frac{1}{2} \log_2 \left( \min\left\{ 1 + P(h_{srk})^2, 1 + P(h_{rkd})^2 \right\} \right) \right\}$$

In half-duplex MIMO relay systems, based on (26) and (28), the secrecy capacity from the source to the relay and to the eavesdropper can be respectively expressed by

$$C_i = \max \left\{ \frac{1}{2} \log_2 \left( \det(I + H_i(1)UU^H H_i(1)) \right) \right\}$$

$$C_e = \max \left\{ \frac{1}{2} \log_2 \left( \det(I + H_i(1)) \right) \right\}$$

The secrecy capacity from relay to destination is given by

$$C_r = \max \left\{ \frac{1}{2} \log_2 \left( \det(I + H_i(1)) \right) \right\}$$

With equations (56), (57) and (58) based on the overall secrecy capacity of single-antenna relay systems, we can express the overall secrecy capacity of MIMO relay systems:

$$C_k^{MIMO} = \max \left\{ \frac{1}{2} \log_2 \left( \min\left\{ M_i, M_r \right\} \right) \right\}$$

where $M_i = \det(I + H_i(1)UU^H H_i(1))$ and $M_r = \det(I + H_r(1))$. Note that the factor $\frac{1}{2}$ is due to half-duplex systems.

**Proposition 1.** With buffers of size $L$ implemented in the relay nodes, the secrecy-rate performance can be improved. The secrecy rate difference varies between $0$ to $\Delta_{BF}$.

**Proof.** In half-duplex MIMO relay systems with multiple relays, relay selection can be performed prior to transmission. If we use $\Psi$ to represent a set of relay nodes based on (59) then with relay selection the secrecy rate is expressed by

$$\max_{i \in \Psi} \left\{ \det(I + H_i(1)UU^H H_i(1)) \right\}$$

Under the condition that $\det(I + H_i(1)UU^H H_i(1)) < \det(I + H_r(1))$, relay selection can be simplified and given by

$$\max_{i \in \Psi} \left\{ \det(I + H_i(1)UU^H H_i(1)) \right\}$$

where $i_R$ represents the selected relay. In this scenario, the secrecy rate is described by

$$C_{\text{Relay}}^{(i)} = \frac{1}{2} \log_2 \left( \frac{1}{1 + P(h_{srk})^2} + \frac{1}{1 + P(h_{se})^2 + P(h_{rk})^2} \right)$$

$$C_{\text{Relay}} = \frac{1}{2} \log_2 \left( \frac{1}{1 + P(h_{srk})^2} + \frac{1}{1 + P(h_{se})^2 + P(h_{rk})^2} \right)$$

(62)
Under the condition that $\det(I + H_i^{(t)}U^H H_i^{(t)}^H) > \det(I + \Gamma_r^{(t)})$, the secrecy rate can be computed in the same way as above and the result is given by

$$C_{\text{Relay}}^{(2)} = \frac{1}{2} \log_2 \left( \left\{ \det(I + \Gamma_r^{(t)}) \right\} \right) - \frac{1}{2} \log_2 \left( \det(I + \Gamma_e^{(t)}) \right).$$

(63)

When each relay node is equipped with an infinite buffer, the signals can be stored in the buffers which means the signals can wait at the relay nodes until the condition $\det(I + H_i^{(t)}U^H H_i^{(t)}^H) < \det(I + \Gamma_r^{(t)})$ is satisfied. If we use $\det(I + \Gamma_e^{(pt)})$ to represent the condition that is experienced in the previous time slot which follows $\det(I + H_i^{(pt)}U^H H_i^{(pt)}^H) > \det(I + \Gamma_e^{(pt)})$, then the expression of the secrecy rate with infinite buffers is described by

$$\Delta_{R-BF} = \frac{1}{2} \log_2 \left( \left\{ \det(I + H_i^{(t)}U^H H_i^{(t)}^H) \right\} \right) - \frac{1}{2} \log_2 \left( \det(I + \Gamma_e^{(pt)}) \right).$$

(64)

Specifically, with a buffer with size $L$ the condition $\det(I + H_i^{(t)}U^H H_i^{(t)}^H) > \det(I + \Gamma_r^{(t)})$ will not hold and the difference of the secrecy rates will be between 0 and $\Delta_R-BF$.

In the scenarios considered, to avoid the interference in the transmission to or from the relays, a half-duplex scheme is employed. To limit the number of time slots, an opportunistic scheme can be applied to MIMO relay systems.

**Proposition 2.** An opportunistic scheme can improve the secrecy rate as compared with standard half-duplex MIMO relay systems.

**Proof.** According to [20], in the opportunistic scheme we have concurrent transmissions with all relays. This will result in IRI and as a result its effect on the relay that receives the source signal must be considered during the opportunistic scheme. In [20], it has been pointed out that IC can be performed at the relay node. To simplify the proof, we first assume IC is performed and the secrecy rate is expressed by

$$C_{\text{Opportunistic-Relay}}^{(1)} = 2 \times C_{\text{Relay}}^{(1)},$$

(65)

and

$$C_{\text{Opportunistic-Relay}}^{(2)} = 2 \times C_{\text{Relay}}^{(2)},$$

(66)

and

$$\Delta_{\text{Opportunistic-Relay-buffer}} = 2 \times \Delta_{\text{Relay-buffer}},$$

(67)

which shows that the secrecy rate of the opportunistic scheme doubles. If IC cannot be performed, based on [62], the secrecy rate is expressed by

$$C_{\text{Opf-Relay}} = \log_2 \left( \left\{ \det(I + (I + \Delta'_{iR})^{-1}H_i^{(t)}U^H H_i^{(t)}^H) \right\} \right) - \log_2 \left( \det(I + \Gamma_e^{(t)}) \right),$$

(68)

where

$$\Delta'_{iR} = \sum_{k=1}^{K} H_{kiR} H_{kiR}^{(pt)} U^{(pt)} U^{(pt)H} H_{kiR}^{(pt)H} H_{kiR}^H.$$  

(69)

which represents IRI. Then, the secrecy rate difference between a standard relay system and an opportunistic buffer-aided relay system is obtained by

$$\Delta_{\text{Opp-R-BF}} = \log_2 \left( \left\{ \det(I + (I + \Delta'_{iR})^{-1}H_i^{(t)}U^H H_i^{(t)}^H) \right\} \right) - \log_2 \left( \det(I + \Gamma_e^{(t)}) \right).$$

(70)

**A. Relaying and Jamming Function Selection**

**Theorem 1.** When $\text{SNR} \to \infty$, the secrecy rate $C_{\text{Opportunistic-Relay-buffer}} \to \infty$ and the secrecy rate with IRI cancellation outperforms that without IRI cancellation.

**Proof.** In all aforementioned systems, we have not taken any jamming signal into consideration. In the presence of systems with multiple relay nodes, some relay nodes can perform the jamming function by transmitting jamming signals to the eavesdroppers. More specifically, IRI cancellation is considered. In the RJFS algorithm, the selected relay at the current time interval is the jammer as well as the relay responsible for forwarding the data in the next time interval. The aim of the RJFS algorithm is to choose the relay that provides the highest secrecy rate performance.

According to Algorithm 1, the relay selection criterion is given by

$$R_{iR} = \arg \max_{iR \in \Psi} \left( I + I^{-1} \right) \left( I + \Gamma_e^{(t)} \right)$$

(71)

where $i_R$ represents the selected relay. Based on (71), the secrecy rate with the selected relay can be expressed as:

$$C_{\text{RJFS-IRI}} = \log_2 \left( \left\{ \det(I + \Gamma_e^{(t)}) \right\} \right) - \log_2 \left( \det(I + \Gamma_e^{(t)}) \right).$$

(72)

When IC is performed at the relay nodes, (72) can be simplified to

$$C_{\text{RJFS-IC}} = \log_2 \left( \left\{ \det(I + H_i^{(t)}U^H H_i^{(t)}^H) \right\} \right) - \log_2 \left( \det(I + \Gamma_e^{(t)}) \right).$$

(73)

Equation (73) was obtained in our previous study [39] when $\text{SNR} \to \infty$. Comparing (72) with (73), we can have $C_{\text{RJFS-IC}} > C_{\text{RJFS-IRI}}$, as indicated in Fig. 5.

**B. Buffer-aided Relay and Jammer Function Selection**

**Theorem 2.** According to Proposition 1, the secrecy-rate performance can be improved with buffers. This can also be applied to the RJFS algorithm. In the IC scenario, when more power is allocated to the transmitter the secrecy rate will suffer from a dramatic decrease.

**Proof.** In the buffer-aided RJFS algorithm, the relay selection and jamming selection can be implemented simultaneously with the following selection criterion:

$$R_{iR} = \arg \max_{iR \in \Psi} \left( I + I^{-1} \right) \left( I + \Gamma_e^{(t)} \right)$$

(74)

and

$$R_n = \arg \max_{n \in \Psi} \left( I + \Gamma_e^{(t)} \right).$$

(75)
where in both transmissions we can achieve high secrecy rate performance with separate selection from the source to the relays and from the relays to the destination. Considering power allocation, with the parameter $\eta$ indicating the power allocated to the transmitter, we assume the power allocated to the transmitter is $\eta P$ and the power allocated to the relays is $(2 - \eta)P$. When $\eta \to 0$, less power will be allocated to the transmitter according to:

$$C_{\text{BF-RJFS-IRI}}^{(1)} = \log_2 \left( \left\{ \det \left( I + \Gamma_r^{(t)} \right) \right\} \right) - \log_2 \left( \left\{ \det \left( I + \Gamma_e^{(t)} \right) \right\} \right)$$

and

$$C_{\text{BF-RJFS-IRI}}^{(2)} = \log_2 \left( \left\{ \det \left( I + \Gamma_r^{(n)} \right) \right\} \right) - \log_2 \left( \left\{ \det \left( I + \Gamma_e^{(n)} \right) \right\} \right)$$

(76)

where the secrecy rate $C_{\text{BF-RJFS-IRI}}^{(1)}$ will have an increase, while $C_{\text{BF-RJFS-IRI}}^{(2)}$ will have a decrease. As a result, the overall secrecy rate will decrease. More specifically, with less power allocated in the relay or the jammer, IC that acts as a jamming signal to the eavesdropper will have less effect on the contribution to the secrecy rate. Then, the overall secrecy rate will have a dramatic decrease.

If IC is performed, according to (76) and (77), we have

$$C_{\text{BF-RJFS-IC}}^{(1)} = \log_2 \left( \left\{ \det \left( I + H_r^{(t)} U U^H H_r^{(t)} H_r^{(t)} \right) \right\} \right) - \log_2 \left( \left\{ \det \left( I + \Gamma_e^{(t)} \right) \right\} \right)$$

and

$$C_{\text{BF-RJFS-IC}}^{(2)} = \log_2 \left( \left\{ \det \left( I + \Gamma_r^{(n)} \right) \right\} \right) - \log_2 \left( \left\{ \det \left( I + \Gamma_e^{(n)} \right) \right\} \right)$$

(78)

(79)

where more power is allocated to the transmitter and the secrecy rates $C_{\text{BF-RJFS-IC}}^{(1)}$ and $C_{\text{BF-RJFS-IC}}^{(2)}$ are less affected than those in the scenario with IC. The results in Figs. 6 and 7 indicate the change with different power allocation.

C. Greedy Algorithm

**Theorem 3.** With high SNRs, the proposed greedy BF-RJFS algorithm can achieve comparable secrecy rate performance with a dramatic reduction in the computational cost.

**Proof.** According to (42), the total number of visited sets for an exhaustive search can be expressed as:

$$\Omega^{\text{exhaustive}} = \frac{S^{\text{total}}!}{(S^{\text{total}} - S)!S!}$$

(80)

In the proposed greedy algorithms, the search is implemented in the remaining relay nodes so that the total number of visited sets in the greedy search is given by

$$\Omega^{\text{greedy}} = S^{\text{total}} - S + 1$$

$$= S^{\text{total}}S - \frac{(S - 1)!S}{2}$$

(81)

Based on (80) and (81), for a number of selected relay nodes $S$, when the total number of relay nodes $S^{\text{out}}$ increases, the total number of visited sets for the exhaustive search is much higher than those for the greedy search, that is $\Omega^{\text{exhaustive}} \gg \Omega^{\text{greedy}}$.

VI. Simulation Results

In this section, we assess the secrecy-rate performance of the proposed E-SR relay selection criterion and the RJFS and BF-RJFS algorithms against existing techniques via simulations for the downlink of a multiuser buffer-aided relay systems. In particular, the proposed E-SR relay selection criterion is compared against the impractical SR method that uses the CSI of the eavesdroppers, the SINR-based techniques and the max-ratio approach. Moreover, the proposed RJFS and BF-RJFS algorithms that employ IC are evaluated against an approach without IC. We consider both single-antenna and MIMO settings. In a single-antenna scenario, the transmitter is equipped with 3 antennas to broadcast the signal to 3 legitimate users through multiple single-antenna relays in the presence of 3 eavesdroppers equipped with a single antenna. In the MIMO scenario, the transmitter is equipped with 6 antennas and each user, eavesdropper and relay has 2 antennas. The buffer can store up to $J = 4$ packets. In both scenarios, a zero-forcing precoding technique is employed at the transmitter. 

$$H_e = R_e^{\frac{1}{2}} H R_t^{\frac{1}{2}}$$

(82)

where $R_e$ and $R_t$ are receive and transmit covariance matrices with $\text{Tr}(R_e) = N_r$ and $\text{Tr}(R_t) = N_t$. Both $R_e$ and $R_t$ are positive semi-definite Hermitian matrices. For the case of an urban wireless environment, the user is always surrounded by rich scattering objects and the channel is most likely independent Rayleigh fading at the receive side. Hence, we assume $R_e = I_{N_r}$, and we have

$$H_e = H R_t^{\frac{1}{2}}$$

(83)

To study the effect of antenna correlations, random realizations of correlated channels are generated based on the exponential correlation model such that the elements of $R_t$ are given by

$$R_t(i, j) = \begin{cases} r^{j-i} & \text{if } i \leq j, |r| \leq 1 \\ r^{i-j} & \text{if } i > j, |r| \leq 1 \end{cases}$$

(84)

where $r$ is the correlation coefficient between any two neighboring antennas.

In Figs. 2 and 3 we compare the secrecy rate performance in uncorrelated and correlated channels. The results indicate that the proposed E-SR relay selection criterion can improve the secrecy rate in both scenarios. Among the investigated relay selection criteria, E-SR is close to the SR-based scheme that employs CSI to the eavesdroppers and outperforms SINR-based techniques, which are often adopted in the literature [13, 17] and require the CSI of the eavesdroppers.

In Fig. 4 the secrecy rate performance with infinite buffer size is compared with buffer size $L = 1$ and $L = 10$. The theoretical curves are obtained with the expression obtained in Section V for the secrecy rate difference $\Delta_{\text{IRI-BF}}$. According to the results, when the buffer size is increased, the secrecy rate will improve and get close to the theoretical curves.

In Fig. 5 in a single-antenna scenario, the secrecy-rate performance with the proposed IC scheme and RJFS algorithm is better than that with the conventional algorithm without
IC. With IC, the secrecy-rate performance is better than the one without IC, as expected. Compared with the single-antenna scenario, the multiuser MIMO system contributes to the improvement in the secrecy rate as verified in Fig. 5.

In Fig. 6 and Fig. 7, a power allocation technique is considered and the parameter $\eta$ indicates the power allocated to the transmitter. If we assume in the equal power scenario that the power allocated to the transmitter as well as the relays are both $P$, then the power allocated to the transmitter is $\eta P$ and the power allocated to the relays is $(2 - \eta)P$. In Fig. 6 and Fig. 7 we can notice that with more power allocated to the transmitter the secrecy rate performance will become worse. Comparing Fig. 6 and Fig. 7 when $\eta < 1.5$ the secrecy rate performance in the scenario with IC is better than that without IC. When $\eta > 1.5$ the secrecy rate of the system without IC is better than that of the system with IC.

In Fig. 8 with a fixed number of relays, the computational complexity of the exhaustive and the greedy searches with the RJFS and BF-RJFS algorithms is examined. The results show that the greedy algorithms are substantially simpler than those with the exhaustive search and are suitable for scenarios with a higher number of relays. In Fig. 9 a comparison between the exhaustive search and the greedy algorithms is carried out. The results show that the greedy algorithms approach the same secrecy rate with a much lower complexity than that of the exhaustive search-based techniques.

VII. CONCLUSION

In this work, we have proposed the E-SR approach that allows the maximization of the secrecy rate in buffer-aided relay systems without the need for the CSI of the eavesdroppers. We have also presented algorithms to select a set of relay nodes to enhance the legitimate users’ transmission and another set of relay nodes to perform jamming of the eavesdroppers. The proposed RJFS and BF-RJFS selection algorithms can exploit the use of the buffers in the relay nodes and result in substantial gains in secrecy rate over existing techniques.

APPENDIX

PROOF OF THE PROPOSED E-SR CRITERION

In this appendix, we include the detailed steps of the derivation of the E-SR criterion.

Proof. From the original expressions for the achievable rate for users and eavesdroppers, which are shown in (25) and (27), we consider relay selection based on the secrecy rate criterion according to:

$$R_{SR} = \arg \max_{\varphi \in \Psi} \sum_{\varphi_r \in \varphi_r} \left\{ \frac{\det(I + \Gamma_r)}{\det(I + \Gamma_e)} \right\},$$

Fig. 2: Secrecy-rate performance of relay selection criteria in uncorrelated channels.

Fig. 3: Secrecy-rate performance of relay selection criteria in correlated channels.

Fig. 4: Secrecy-rate performance versus buffer size in uncorrelated channels.

Fig. 5: Secrecy rate performance in correlated channels.
Using the property of the determinant 
\[ \det(A^{-1}) = \frac{1}{\det(A)} \]
and the proposed greedy algorithms.

\[ \det(\Lambda_1)^{-1} \det[\Lambda_1 + (H_e R_l H_e^H)] \]

where \( \Lambda_1 = H_e R_l H_e^H \).

Since \( \Lambda_1 \) is assumed to be a square matrix, \( \Lambda_1 \) can be decomposed as

\[ \det[\Lambda_1] = \det[\Lambda_1 + (H_e R_l H_e^H)] \]

Using the property of the determinant \( \det(A^{-1}) = \frac{1}{\det(A)} \) [94], we have

\[ (\det[\Lambda_1])^{-1} \det[\Lambda_1 + (H_e R_l H_e^H)] \]

where

\[ \Gamma(t) = (H_e R_l H_e^H)^{-1}(H_e R_t H_e^H + H_e R_l H_e^H) \]

\( \Gamma(t) = (H_e R_l H_e^H)^{-1}(H_e R_t H_e^H) \). Note that [85] requires \( H_e \), i.e., CSI to the eavesdroppers and that channels from all relays are taken into account for selection. In what follows, we show that a designer can employ an equivalent expression to [85] without resorting to the knowledge of CSI to the eavesdroppers. This requires the assumption that several channel matrices are square. However, it can also be used even for scenarios of non-square channel matrices if the matrices are completed with zeros to ensure a square structure.

In [85], our aim is to circumvent the need for CSI to the eavesdroppers from the denominator. To this end, we assume square matrices which allows the linear algebra property

\[ \det(AB) = \det(A) \det(B) \]

Following this approach, the denominator of [85] can be expressed as

\[ \det[\Lambda_1^{-1} \Lambda_1 + \Lambda_1^{-1} (H_e R_l H_e^H)] \]

(86)

where \( \Lambda_1 = H_e R_l H_e^H \).

Fig. 6: Secrecy rate performance with power allocation and IC.

Fig. 7: Secrecy rate performance with power allocation and without IC.

Fig. 8: Number of visited sets for the exhaustive and greedy searches.

Fig. 9: Secrecy rate performance with an exhaustive search and the proposed greedy algorithms.
as
\[
\begin{align*}
(\det[A_1])^{-1}&\det[A_1 + (H_e R_d H_e^H)] = \\
(\det[H_e U_1])^{-1}&\det[\left(\sum_{j \neq i} U_j s_j^{(t)} s_j^{(t)H} U_j^H U_i^H \right)] \det[U_i^H H_e^H] = \\
(\det[H_e U_1])^{-1}&\det[\left(\sum_{j \neq i} U_j s_j^{(t)} s_j^{(t)H} U_j^H U_i^H \right) + s_i^{(t)} s_i^{(t)H}] \\
&\det[U_i^H H_e^H],
\end{align*}
\]
(91)

Using the matrix inverse property
\[
(\det[A])^{-1} \det[B] \det[C]^{-1} = (\det[C])^{-1} (\det[B])^{-1} (\det[A])^{-1}
\]
we can write
\[
\begin{align*}
(\det[A_1])^{-1}&\det[A_1 + (H_e R_d H_e^H)] = (\det[H_e U_1])^{-1} \\
(\det[U_i^H H_e^H])^{-1}&\det[H_e U_1] \det[\left(\sum_{j \neq i} U_j s_j^{(t)} s_j^{(t)H} U_j^H U_i^H \right)]^{-1} \\
&+ s_i^{(t)} s_i^{(t)H} \det[U_i^H H_e^H],
\end{align*}
\]
(92)

By observing the terms above, we notice that the 1st term can be canceled by the 4th term, and that the 3rd term can be canceled by the 6th term, resulting in
\[
(\det[A_1])^{-1}\det[A_1 + (H_e R_d H_e^H)] = (\det[U_i^H H_e^H])^{-1} \\
\det[\left(\sum_{j \neq i} U_j s_j^{(t)} s_j^{(t)H} U_j^H U_i^H \right)]^{-1} \\
+ s_i^{(t)} s_i^{(t)H} \det[U_i^H H_e^H],
\]
(93)

By substituting the result in (93) in (83), we obtain the proposed E-SR criterion given by
\[
R_{S-\text{SR}} = \arg \max_{\Phi \in \mathcal{P}} \left\{ \log \frac{\det[I + G_{\Phi}]}{\det[I + F_{\Phi}]} \right\} = \arg \max_{\Phi \in \mathcal{P}} \left\{ \log \frac{\det[I + (H_e R_d H_e^H)^{-1}(H_e R_d H_e^H)]}{\det[I + (H_e R_d H_e^H)^{-1}]} \right\} = \arg \max_{\Phi \in \mathcal{P}} \left\{ \log \frac{\det[I + (H_e R_d H_e^H)^{-1}(H_e R_d H_e^H)]}{\det[I + U_i^H R_i^{-1} U_i R_d]} \right\} = \arg \max_{\Phi \in \mathcal{P}} \left\{ \log \left( \frac{\det[I + (H_e R_d H_e^H)^{-1}(H_e R_d H_e^H)]}{\det[I + U_i^H R_i^{-1} U_i R_d]} \right) \right\},
\]
(94)

where the last expression in (94) no longer requires knowledge of CSI to the eavesdroppers $H_e$ and is equivalent to (40).

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