Chiral Symmetry and Axial Anomaly in Hadron and Nuclear Physics — a review —

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Abstract. The important role played by chiral symmetry and axial anomaly in QCD in nuclear physics is reviewed. Some recent topics on possible chiral restoration in hot and/or dense matter are picked up. We also discuss so called effective restoration of chiral anomaly hot and/or dense matter, as may be seen in a character change of $\eta'$ meson.

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INTRODUCTION

Nuclear physics was primarily quantum many-body physics with the nuclear force given, which is responsible for binding the nucleons against the repulsive Coulomb force between protons. Yukawa’s meson theory[1] was the first intended application of quantum-field theory to the problem of the nuclear force. The salient ingredients of the nuclear force[2] are the tensor force[3] in the long range and the short-range repulsive core[4]. The tensor force is generated by one-pion exchange between the nucleons. The one-pion-exchange potential (OPEP)[3] reads

$$V_{\text{OPEP}}(1,2) = f^2 m_\pi \frac{\tau_1 \cdot \tau_2}{3} \left[ (\sigma_1 \cdot \sigma_2) Y(m_{\pi r}) + S_{12} Z(m_{\pi r}) \right],$$

where $Y(x) = \exp(-x)/x$, $Z(x) = (1 + 3/x + 3/x^2) Y(x)$ and $S_{12} = 3(\sigma_1 \cdot \hat{r}) (\sigma_2 \cdot \hat{r}) - \sigma_1 \cdot \sigma_2 = 3\sqrt{5} [\sigma_1 \otimes \sigma_2]^{(2)} \otimes [\hat{r}_1 \otimes \hat{r}_2]^{(2)}]^{(0)}$ being the tensor operator which is constructed from the two second-rank tensors. The appearance of such an operator with a bad symmetry is due to the fact that the pion is a pseudo-scalar particle.

Owing to the transformation properties of the tensor force, it only acts to the spin-triplet state but not to the singlet state, which is the reason why deuteron exists as a proton-neutron bound system although there are no di-neutron bound system: The second-order contribution of the tensor force gives rise to an additional attraction between the triplet state. This second-order effect of the tensor force is also an essential ingredient for realizing the saturation property of the nuclear matter[5].

Then why does pion is isovector and pseudo-scalar particle with the lightest mass in the hadron world? These are all because the pion is the Nambu-Goldstone boson associated with dynamical breaking of chiral symmetry of QCD[6]. How important roles does the chiral symmetry play in nuclear physics? Some answers may be found in [7, 8]. Before answering this problem, we clarify the chiral symmetry and its spontaneous breaking in QCD.
CHIRAL INVARIANCE OF CLASSICAL QCD LAGRANGIAN

The classical QCD Lagrangian reads

\[ \mathcal{L} = \bar{q}(i\gamma^\mu D_\mu - m)q - \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu}. \]  

(2)

The classical QCD Lagrangian with vanishing quark mass \((m \to 0)\) is invariant under the chiral transformation. The chiral transformation for \(N_F\)-flavor quark field \(q_f \ (f = 1, 2, \ldots, N_F)\) is defined as a direct product of two unitary transformations \(U_L\) and \(U_R\);

\[ q_{Lf} \equiv \frac{1 - \gamma_5}{2} q_f \to (U_L)_{ff'} q_{Lf'}, \quad q_{Rf} \equiv \frac{1 + \gamma_5}{2} q_f \to (U_R)_{ff'} q_{Rf'}. \]  

(3)

Notice that the vector current \(\bar{q}\gamma^\mu q = \bar{q}_L\gamma^\mu q_L + \bar{q}_R\gamma^\mu q_R\) is invariant under the chiral transformation, although the Dirac mass term \(\bar{q}q = \bar{q}_Rq_L + \bar{q}_Lq_R\) is not. If we neglect the current quark mass term, the quark field enters QCD only as a combination \(\bar{q}\gamma^\mu D_\mu q\), and hence it becomes invariant under the chiral transformation. A warning is in order here; the axial \(U(1)\) symmetry is explicitly broken by a quantum effect, which is known as \(U(1)_A\) anomaly[9].

A quark bilinear operator \(\Phi_{ij}\) defined by \(\Phi_{ij} = \bar{q}_j (1 - \gamma_5) q_i = 2 \bar{q} r j q_i L\) is transformed as follows,

\[ \Phi_{ij} \to (U_L)_{ik} \Phi_{lk} (U_R^*)_{lj}. \]  

(4)

In the two-flavor case, the generators of the chiral transformation are given by the isospin charges \(Q^a\) and the axial charges \(Q^5_5\);

\[ Q^a = \int dx \bar{q} \gamma^0 \tau^a q/2, \quad Q^a_5 = \int dx \bar{q} \gamma^0 \gamma_5 \tau^a q/2. \]  

(5)

We note the commutation relation,

\[ [i Q^a_5, \bar{q} i \gamma_5 \tau^b q] = -\delta^{ab} \bar{q} q. \]  

(6)

Then taking the vacuum expectation value of (6), we have

\[ \langle 0 | \bar{q} q | 0 \rangle = \langle 0 | [Q^a_5, \bar{q} i \gamma_5 \tau^a q] | 0 \rangle, \]  

(7)

which implies that if \(\langle 0 | \bar{q} q | 0 \rangle \neq 0\), then \(Q^a_5 | 0 \rangle\) can not be zero for some \(a\). That is, chiral symmetry is spontaneously broken! Indeed there is a following celebrated relation due to Gell-Mann, Oakes and Renner[10],

\[ f_\pi^2 m_\pi^2 = -\frac{m_u + m_d}{2} \langle 0 | \bar{u} u + \bar{d} d | 0 \rangle, \]  

(8)

which does indicate that the chiral symmetry is spontaneously broken in the QCD vacuum, because the pion decay constant \(f_\pi \simeq 93\) MeV is finite.
POSSIBLE CHIRAL RESTORATION IN FINITE NUCLEI

One of the interesting nature of QCD is that the QCD vacuum can change along with an inclusion of external hard scale, which may be induced by baryon chemical potential, i.e., the baryon density, temperature, strong magnetic field and so on. An interesting observation is that a nucleus can provide a hard scale by its baryon density, which might cause a change of the QCD vacuum, and hence the chiral symmetry may be partially restored in a finite nucleus. Thus exploring possible evidence of partial restoration of chiral symmetry in the nuclear medium has become one of the most important and challenging problems in nuclear physics [8, 11]. Relevant experimental studies include the spectroscopy of deeply bound pionic atoms [12], low energy pion-nucleus scatterings [13], and the production of di-pions in hadron-nucleus and photon-nucleus reactions [14, 15, 16]. These experiments revealed the following anomalous properties of the pion dynamics in the nuclear medium; (i) an enhancement of the repulsion $\pi^-\text{neutron}$ interaction [12, 13], (ii) an enhanced attraction of the $\pi\pi$ interaction in the scalar-isoscalar channel [14, 15, 16].

In the theoretical side, possible relevance of the $\pi\pi$ interaction in a nuclear medium was first suggested in [17]. Weise and his collaborators showed that the reduction of the temporal part of the pion decay constant in the nuclear medium $F_t^\pi$ is intimately related to the anomalous repulsion (i) [18, 19]. It was also argued that the reduction of $F_t^\pi$ is responsible for the phenomenon (ii) [20].

Recently, Jido, Hatsuda and the present author [21] derived a novel sum rule for the quark condensate valid for all density, which sum rule is reduced to

$$\langle \bar{q}q \rangle^*/\langle \bar{q}q \rangle = (F_t^\pi/F_\pi)Z_{\pi}^{1/2}$$

in the low-density limit. Here $\langle \bar{q}q \rangle^*$ is the quark condensate, $F_t^\pi$ the (temporal) pion decay constant and the pion wave-function renormalization constant $Z_{\pi}^*$ all in the nuclear medium. It is noteworthy that the $Z_{\pi}^*$ can be estimated with the use of the the iso-singlet pion-nucleon scattering amplitude at low energy, and they found that

$$Z_{\pi}^{1/2} = \left( \frac{G_{\pi}^*}{G_\pi} \right)^{1/2} = 1 - \frac{\gamma}{\rho_0},$$

where $G_{\pi}^{(*)}$ is the (in-medium) pion coupling constant. Here the coefficient $\gamma = \beta \rho_0/2 = 0.184$ with $\beta = 2.17 \pm 0.04$fm$^3$. (Parametrically, $\beta$ is expressed as

$$\beta = \frac{\sigma_{\pi N}}{F_\pi^2 m_\pi} + \left( 1 + \frac{m_\pi}{m_N} \right) \frac{4\pi a_{\pi N}}{m_\pi^2}.$$  

Here $\sigma_{\pi N}$ and $a_{\pi N}$ are the $\pi$-$N$ sigma term and the iso-singlet scattering length, respectively.) Then the in-medium quark condensate is nicely expressed by the temporal pion decay constant and the pion coupling;

$$\langle \bar{q}q \rangle^* = -F_t^\pi G_{\pi}^*.$$  

Now the $s$-wave $\pi^-\text{nucleus}$ optical potential $U_s$ is parametrized as

$$2m_\pi U_s = -4\pi[1 + \frac{m_\pi}{m_N}](b_0^* \rho_+ - b_1^* \rho_-),$$
\[ T(\omega = m_\pi : m_\pi)\rho_+ - T(-\omega = m_\pi : m_\pi)\rho_-, \] 

(13)

where \( \rho_{\pm} = \rho_p \pm \rho_n \) with \( \rho_{p(n)} \) being the proton (neutron) density. Then the parameter \( b_1^* \) which can be extracted from the experimental data is expressed as

\[ \frac{b_1^*}{b_1^*} = \left( \frac{\rho_p}{\rho_n} \right)^2. \] 

(14)

Combining these relations, Jido et al[21] derived the following relation

\[ \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle} \simeq \left( \frac{b_1^*}{b_1^*} \right)^{1/2} \left( 1 - \gamma \frac{\rho}{\rho_0} \right). \] 

(15)

Now one sees that the experimental evidence of the repulsive enhancement as given by \( b_1^* \) implies that the absolute value of the quark condensate in the nuclei is smaller than that in the vacuum, and hence the chiral symmetry is partially restored in the nuclei.

**EFFECTIVE RESTORATION OF AXIAL SYMMETRY AT FINITE TEMPERATURE AND DENSITY**

How about other signals of the chiral restoration at finite density and/or temperature? The bottom line is that some hadrons are intimately related to the chiral symmetry and its dynamical breaking, and hence their properties may change along with the chiral transition at finite density/temperature[7]. Such hadrons include the sigma meson[22]. The chiral symmetry implies that the degeneracy of the vector and axial vector correlators as well as that in the scalar and pseudoscalar channels, which are parity partners. Thus exploring the possible tendency of the degeneracy in these opposite parity channels should be interesting in an extreme environment. The parity doubling in the baryon sector may be affected by the underlying chiral symmetry[23]. Examining the properties of the negative-parity baryons such as \( N^*(1535) \) at finite density and/or temperature should be also interesting[24]. An interesting ingredient in this subject lies in the fact that \( N^*(1535) \) is strongly coupled with \( \eta \) meson. Thus the study of \( N^*(1535) \) is automatically to explore the properties of \( \eta \) meson in nuclei. One should also note that \( \eta \) meson is a mixing partner of \( \eta'(958) \), the nature of which is intimately related with the axial anomaly of QCD[25].

One of the fundamental properties of QCD is \( U(1)_A \) anomaly or axial anomaly[9]. The ninth pseudoscalar meson \( \eta' \) which is almost flavor singlet with a mass as large as 958 MeV is a reflection of the \( U(1)_A \) anomaly and the \( \theta \) vacuum owing to the instanton configuration. The mass of \( \eta \) and \( \eta' \) and their mixing property are realized with combined effects of the anomaly, explicit and dynamical breaking of chiral symmetry. The instanton density as well as the quark condensates is expected to decrease at finite temperature and density[26, 27]. Thus one should also explore the properties of the \( \eta-\eta' \) meson sector at finite temperature and/or density[28], which may show an effective restoration of \( U(1)_A \) symmetry[29, 28, 27] as seen in the mixing properties of \( \eta \) and \( \eta' \) mesons and their masses.
The $U(1)_A$ anomaly implies that the $U(1)_A$ symmetry is explicitly broken by a quantum effect. In the context of the effective Lagrangian, there should exist a vertex which violates this symmetry. One of such an interaction is the six-quark interaction with a determinantal form as introduced by Kobayashi and Maskawa[30] in 1970,

$$\mathcal{L}_{KMT} = g_D \det \bar{q}_i(1 - \gamma_5)q_j + \text{h.c.},$$

where h.c. stands for Hermite conjugate. This vertex is contained in instanton-induced quark interaction derived by ’t Hooft in 1976[31]; see [7] for a review. It was first shown by the present author [28] using a generalised Nambu-Jona-Lasinio model incorporating the Kobayashi-Maskawa-’t Hooft term (16) that the $\eta$ and $\eta'$ mesons change their nature owing to both the temperature dependence of the quark condensates and the possible decrease in the KMT coupling constant $g_D$ with $T$. The coupling constant $g_D$ of the KMT term may be dependent on temperature and baryon chemical potential because the instanton density is dependent on them[26, 27]. Such a possible temperature dependence causes a temperature dependence of the mixing angle $\theta_\eta$ so that $\theta_\eta$ increases in the absolute value and the mixing between the $\eta$ and $\eta'$ approaches the ideal one. Although the $\eta_0$ component in the physical $\eta'$ decreases as $T$ is increased, the $\eta'$ mass decreases gradually with increasing $T$, because the $\eta_0$ tends to acquire the nature of the ninth Nambu-Goldstone boson of the $SU(3)_L \otimes SU(3)_R \otimes U(1)_A$ symmetry and decreases its mass rapidly. This is an effective restoration $U(1)_A$ symmetry first discussed by Pisarski and Wilczek[29]. Such an anomalous decrease in the $\eta'$ mass might have been observed in the relativistic heavy ion collisions at RHIC[32]. Recent studies on this problem are reviewed in [33].

**BRIEF SUMMARY**

The following is a summary what I wanted to say in this report: (1) The saturation property of the nuclear matter can be attributed eventually to chiral symmetry and its dynamical breaking in QCD. (2) Hadrons are sort of elementary excitations on top of the nonperturbative QCD vacuum, and hence may change their properties along with that of the QCD vacuum. (3) The QCD vacuum can change and even show a phase transition(s) with an increase of temperature and/or baryon density, which in turn gives rise to a change of particle pictures of the hadrons in the system.

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REFERENCES

1. H. Yukawa, Proc. Phys. Math. Soc. Jap. 17, 48 (1935).
2. R. Tamagaki, Prog. Theor. Phys. 39, 91 (1968); 
   R. V. Reid, Ann. of Phys. 50, 411 (1968); 
   M. Taketani, R. Tamagaki, W. Watari, S. Machida, S. Ogawa, T. Ueda, W. Watari, M. Yonezawa, S. 
   Furuichi and K. Nisimura, Prog. Theor. Phys. Suppl. 39 (1967).
   N. Hoshizaki and S. Otsuki, Prog. Theor. Phys. Suppl. 42 (1968).
3. M. Taketani, J. Iwadare, S. Otsuki, R. Tamagaki, S. Machida, T. Toyoda, W. Watari and K. Nishijima, 
   Prog. Theor. Phys. Suppl. 3 (1956).
4. R. Jastrow, Phys. Rev. 81, 165 (1950).
5. As review articles, see, for example, H. A. Bethe, R. Annu. Rev. Nucl. Sci. 21, 93 (1971); 
   B. Day, Rev. Mod. Phys. 50, 495 (1978).
6. Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).
7. T. Hatsuda and T. Kunihiro, Phys. Rept. 247, 221 (1994)
8. T. Hatsuda and T. Kunihiro, arXiv:nucl-th/0112027.
9. S. Weinberg, The quantum theory of fields. Vol. 2: Modern applications (Cambridge University Press, 
   UK, 1996);
   K. Fujikawa and H. Suzuki, Path integrals and quantum anomalies Oxford, UK: Clarendon, (2004).
10. M. Gell-Mann, R. J. Oakes and B. Renner, Phys. Rev. 175, 2195 (1968).
11. W. Weise, Nucl. Phys. A 805, 115 (2008) [arXiv:0801.1619 [nucl-th]].
12. K. Suzuki et al., Phys. Rev. Lett. 92, 072302 (2004); P. Kienle and T. Yamazaki, Prog. Part. Nucl. 
   Phys. 52, 85 (2004).
13. E. Friedman et al., Phys. Rev. Lett. 93, 122302 (2004) Phys. Rev. C 72, 034609 (2005).
14. F. Bonutti et al. [CHAOS collaboration], Phys. Rev. Lett. 77, 603 (1996); Nucl. Phys. A 677, 213 
   (2000); P. Camerini et al. [CHAOS collaboration], Nucl. Phys. A 735, 89 (2004).
15. A. Starostin et al. [Crystal Ball Coll.], Phys. Rev. Lett. 85, 5539 (2000); Phys. Rev. C 66, 055205 
   (2002).
16. J. G. Messchendorp et al., Phys. Rev. Lett. 89, 222302 (2002).
17. T. Hatsuda, T. Kunihiro and H. Shimizu, Phys. Rev. Lett. 82, 2840 (1999).
18. E. E. Kolomeitsev, N. Kaiser, and W. Weise, Phys. Rev. Lett. 90, 092501 (2003).
19. W. Weise, arXiv:nucl-th/0507058.
20. D. Jido, T. Hatsuda, and T. Kunihiro, Phys. Rev. D63, 011901 (2001).
21. D. Jido, T. Hatsuda and T. Kunihiro, Phys. Lett. B 670, 109 (2008). See also D. Jido, T. Hatsuda and 
   T. Kunihiro, Prog. Theor. Phys. Suppl. 168, 478 (2007) [arXiv:0706.0258 [nucl-th]].
22. T. Hatsuda and T. Kunihiro, Prog. Theor. Phys. 74, 765 (1985); 
   Phys. Lett. B 185, 304 (1987).
23. C. E. Detar and T. Kunihiro, Phys. Rev. D 39, 2805 (1989).
24. H. Nagahiro, D. Jido and S. Hirenzaki, Phys. Rev. C 68, 035205 (2003).
25. H. Nagahiro and S. Hirenzaki, Phys. Rev. Lett. 94, 232503 (2005).
26. D. J. Gross, R. D. Pisarski and L. G. Yaffe, Rev. Mod. Phys. 53, 43 (1981).
27. T. Schafer and E. V. Shuryak, Rev. Mod. Phys. 70, 323 (1998).
28. T. Kunihiro, Phys. Lett. B 219, 363 (1989); 
   Nucl. Phys. B 351, 593 (1991).
29. R. D. Pisarski and F. Wilczek, Phys. Rev. D 29, 338 (1984).
30. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 44, 1422 (1970); 
   M. Kobayashi, H. Kondo and T. Maskawa, Prog. Theor. Phys. 45, 1955 (1971).
31. G. ’t Hooft, Phys. Rev. D 14, 3432 (1976) [Errata; 18, 2199 (1978)]; Phys. Rep. 142, 357 (1986).
32. R. Vértesi, T. Csörgő and J. Sziklai, Nucl. Phys. A 830, 631C (2009): arXiv:0905.2803.
33. Roles of the $U(1)_A$ anomaly at finite temperature and/or density is reviewed in 
   T. Kunihiro, Prog. Theor. Phys. 122, 255 (2009). [arXiv:0907.3808 [hep-ph]].