Propagation of the trajectories for reentry spherical debris including rotation, melting fragmentation and voxel method

J O Murcia P, U T V Guedes and A F B D Prado

Engineering and Space Technology Division. National Institute for Space Research INPE, Av. dos Astronautas 1758 Jd. Da Granja, São José dos Campos, SP, Brazil.

jhoanthan.pineros@inpe.br

Abstract. It is estimated that more than 22,000 objects are in orbit around the Earth, with a total mass of 8,400,000 kg. These numbers consider only objects with dimensions above 10 cm and some non-operational, but still orbiting satellites without control (debris). The debris represent a hazard to operational satellites and aerospace operations due to the high probability of collisions. Due to the interaction of the debris with the atmosphere of the Earth and the solar activity, they began to lose energy and finally decay. During the de-orbit process, the debris fall into the Earth’s atmosphere at hypersonic speeds and these objects can break-up and/or be fragmented due to the aerodynamics, thermal and structural loads. It is important to obtain the trajectory and attitude of any fragment to determine the possible survival mass, impact area, hazard conditions and risks to the population, the air traffic control, and infrastructure. In this case, it is implemented a computational code to integrate the equations of motion to propagate the dynamics and kinematics of spherical debris or propellant tanks. It is also analyzed the results of trajectories with six degrees of freedom, atmospheric winds, and Magnus effect. A voxel method is implemented to analyze the tanks heat transfer, surface temperature and structures stress. To determine and observe the influence of the rotation and the Magnus force in six reentry spherical bodies, three materials are selected; aluminum alloy, due to its application in many aerospace structures; titanium and graphite epoxy I, due to their highest melting point and specific heat. Generally, these materials are used in tanks and rocket motors. More than 62 trajectories were simulated. The mathematical model and computational code were validated in three degrees of freedom. Results are compared with data from other computational tools available in the scientific literature. The results show a good approximation with reported cases of study. New results are generated in the simulations of rotational bodies, due to the influence of aerodynamic forces in the trajectory and the changes in the stagnation regions. Due to the implementation of wind and rotation of the debris, the fragments increased the survivability and the dispersion area.

1. Introduction

The final phase of a space mission around the Earth is known as the disposal or de-orbit phase. In this phase the spacecraft and/or debris (if it is nonoperational and without control) begins to decay due to the combination of the atmospheric drag and gravitational forces acting in the center of mass of the body. At altitudes below 120 km from the Earth’s surface, the air density is strong enough to generate significant drag and breaking the spacecraft. Then the spacecraft velocity becomes lower than the orbital velocity at its altitude and the new trajectory is of re-entry.

During the re-entry, the spacecraft decay rapidly into a higher density atmosphere, increasing the drag, the dynamic pressure and the temperature in its surface. Hypersonic, supersonic, transonic and/or
subsonic flight can be obtained, according to the spacecraft geometry, materials, initial conditions, flight path angle and atmospheric fluid dynamics. The reentry trajectories are classified in controlled and uncontrolled. The controlled trajectories are developed for manned spacecraft (Ex. Apollo, Space Shuttle) or larger bodies (Ex. Mir Space Station, Skylab, Tiangong-1). Uncontrolled reentries are generated due to the orbit decay at the end of the lifetime and is applied to bodies like satellites, rocket stages or debris.

The increase in the space activities in the last two decades has increased exponentially the number of objects in Low Earth Orbits (LEO), increasing the number of orbital debris. The orbital debris are a risk to de operational satellites, because they increase the probability of collisions and, in the same way, the incremented of uncontrolled reentries [1], which affect the air traffic, and, in some cases represent a risk for land facilities and human population. It is estimated that around 100 uncontrolled debris reentry in aleatory conditions yearly [2], but only the largest bodies are monitored and observed. The Aerospace corporation website presents a summary of more than 74 events of recovered reentry debris, with eleven objects recovered in Brazil since 1962.

Space agencies and organizations has been proposing activities to reduce the risk of the reentry debris, like the regularizations of space activities with laws, the development of a space traffic control, and the creation of operational satellites to capture orbital debris. There are two activities to determine the re-entry of orbital debris. The first-one is the monitoring of space objects with optical telescopes, radar and operational satellites. The second-one are the simulations of the reentry trajectories. The last-one is the focus of the paper, because the goal of this investigation is the modeling and propagation of trajectories and attitude of reentry spherical debris including fragmentation, Magnus effect, voxel methods and distributive simulations to determine their survivability and possible collision areas.

Then, the mathematical model of the problem is described in section 2, the methodology implemented is discussed in section 3 and the results of the re-entry simulations are presented in section 4.

2. Mathematical model

2.1. Dynamic

The debris in the final orbit around the Earth has an inertial velocity vector ($\vec{V}$), a relative velocity vector ($\vec{v}$) at the position vector ($\vec{r}$), measured from the Earth’s center of mass.

The two forces that determine the dynamics of the system during the re-entry are the weight ($W$), that is proportional to the spacecraft mass and Earth’s geopotential ($U$), and the aerodynamic force ($D$), which is proportional to the area and shape of the spacecraft and the atmospheric density. To complement the aerodynamic force due to the angular velocity of the debris, there is also the Magnus Force ($M_F$). The force diagram with origin in the center of mass of the spacecraft is showed in figure 1.

Then, the general equation of motion for the spacecraft is:

$$\ddot{\vec{r}} = \vec{V}U + \frac{\vec{A}_F}{m}$$ (1)

In this case $m$ represents the mass of the spacecraft [3-5]. The aerodynamics force has three components in the Wind Reference System (WRS):

$$\vec{A}_F = \vec{L} + \vec{D} + \vec{M}_F,$$ (2)

Where the Lift $\vec{L}$ is perpendicular to the spacecraft-atmospheric relative velocity $\vec{V}_{W'}$; $\vec{D}$ is the Drag, in direction opposite to spacecraft-atmospheric relative velocity and $\vec{M}_F$ the Magnus force. The Drag is proportional to the projected surface of the body (S), the drag coefficient ($C_D$), the atmospheric density ($\rho$) and the relative wind velocity. The Lift force is perpendicular to the wind velocity and it depends
on the lift coefficient ($C_L$), area and dynamic pressure (air density and squared wind velocity). The wind velocity is equal to the spacecraft relative velocity to the rotational atmosphere plus the local horizontal winds.

$$D = \frac{1}{2} \rho S C_D V_w^2,$$

(3)

$$L = \frac{1}{2} \rho S C_L V_w^2,$$

(4)

The physical relation between air density and the wind velocity is known as dynamic pressure ($q_\infty$), and it is the pressure acting under the spacecraft structure [6].

$$q_\infty = \frac{1}{2} \rho V_w^2.$$  

(5)

The aerodynamic coefficients are functions of the Angle of Attack ($\alpha$) (AOA), the banking angle ($\beta$), wind velocity, flow properties, body shape and others. Generally, the coefficients are calculated and validated using wind tunnel tests. Acceptable approximations can be obtained implementing Computational Fluid Dynamics (CFD). To complete the aerodynamic force, it is necessary to include the Magnus force, which is only present in the body if it has a rotation. In spherical bodies at high Reynolds the Magnus force ($M_F$) is derived from [7], see Equation (6). The area of the spherical debris with radius $r_B$ is $\pi r_B^2$, and the spin parameter determined from the angular velocity is: $2\pi \omega_B r_B / V_W$. Effects of turbulence and boundary layers are negligible. Then, the Magnus force is:

$$\vec{M}_F \approx (\pi^2 r_B^3 \rho) \vec{\omega}_B \times \vec{V}_W,$$

(6)

The distribution of the forces acting in the instantaneous center of mass of the debris, in the body reference system, are presented in the figure 1.

Complementing the translation motion, for propagations of the trajectories in 6 Degrees of Freedom (DOF), it is take into account the rotational motion as a function of the angular velocities, torques and inertia. The differential equation in the body reference system, for the changes of the angular velocity ($\vec{\omega}_B$) due to the inertia tensor ($\vec{I}$), variable mass system with inertia changes ($\vec{I}_I$) (Euler’s equation) and with the total toques applied to the body ($\vec{T}_{ord}$) is [3-5]:

$$\vec{\omega}_B \vec{I} = \vec{I}_I - \vec{\omega}_B \times (\vec{I} \vec{\omega}_B) - \vec{\omega}_B \vec{I},$$

(7)
Equations 1 and 7 are numerically integrated to obtain the debris position and attitude for each instant of time.

2.2. Aerodynamics and aerothermodynamics

The principal characteristic of the re-entry, that differs from the other aerospace problems, is the possibility to reach hypersonic, supersonic, transonic and subsonic velocities during the altitude decay. These rapid changes in the flight conditions affects the aerodynamic coefficients and the heat transfer to the debris walls. During the hypersonic trajectories in the upper atmosphere at rarefied flow, it is present the first break-up and parts like antennas and solar arrays are melted. Secondary break-ups from survival fragments are presented in the high-density atmospheric region, in the stratospheric zone at supersonic velocities. The increase of the density, heat transfer, dynamic pressure and deceleration in the debris causes shockwaves and vibrations. In this case the fluid is continuum. Finally, if the fragments survive the supersonic stratospheric flight, the deceleration due to the increment in the density reduce the velocity to continuum subsonic flow, where the heat transfer is reduced, and the body present the lower dynamic pressure at low speeds.

To calculate the aerodynamics coefficients, the heat transference, the mechanic and thermic fragmentation, the variation of mass, the changes in the inertia tensor and the positions of the pressure and center of mass, it was implemented a transformation of the geometry to small boxes with same dimensions like units of volume. In computational sciences this is known as Voxels. Using the fixed element size model (Voxel), with volume $\Delta X_B \Delta Y_B \Delta Z_B$ and the debris material, it is possible to determine the quantity of mass in each volume element $\Delta m$, as a function of the material density. The voxel-based meshing is used to construct finite elements models of textiles to find the stress and strain in the composite fibers. The advantages of the voxel meshed is the generation on unit-cell volumes that can be automated, and it requires less human interaction than the traditional finite elements mesh [8]. Another advantage of the voxel technic is the excellent results obtained from the implemented voxel mapped in surfaces, topography, composites and medicine. It allows the voxel to be modelled by complex geometries [9]. Voxel discretization was implemented in the analysis of fragmentation of ceramics to find the fracture conditions. The results show a good agreement between the numerical data with the experiments [10].

The Voxels distribution is presented in figure 2, where $n$ is the vector normal to the voxel surface, and $\Theta$ is the incident fluid angle necessary to compute the pressure coefficients and to determine the total aerodynamic coefficients [11].

![Voxel and panel methods](image_url)

To calculate the aerodynamic coefficients and heat transfer in rarefied flow, it was used the mathematical model presented in [11-13] with convective heat transfer. In the case of continuum supersonic flow, it was used the modified Newton method with Detra’s formula to correlate the heat transfer in the stagnation point [3, 13 - 16] and, in the transition flow, the empirical bridged function [17, 18]. The total heat storage in each voxel results from the sum of the convective heat and the
radiation heat (for voxels in the debris surface), and the conductive heat in 1D (from the stagnation point to the opposite point).

3. Methodology
To determine the influence of rotation in the re-entry trajectories, was development a computational code to integrate numerically the equations of motion. The numerical propagator for simulate the re-entry trajectories is written in FORTRAN. The code solves the dynamical equations of motion in 6DOF in the Inertial Earth Reference System. The atmospheric model selected was the NRLMSISE-00 [20, 21], with the Earth Geopotential model EGM-08 to 100th order in the WGS-84 system [22]. The rotational atmosphere model is complemented with the horizontal wind model HWM93 [23] and the Runge-Kutta-Fehlberg 7/8 (RKF-7/8) numerical integrator [24]. Compared to other re-entry simulation tools showed in [17], the present propagator implements a more accuracy numerical integrator compared with the traditional RK 4/5, the debris is modelled like a 3D solid allowing simulations in 6DOF (only a few tools present the option), including the recent atmospheric and local winds models, and contains a higher order gravitational model. Another point is that the total aerodynamic force is modelled with the Magnus force and the solid body is mathematically modelled by voxels to generate the automatic mesh, control of volumes and to determine, with a better approximation the fragmentation, heat transfer and aerodynamic coefficients. The voxels allow the studies of different body shapes. Debris resulted from fragmentation and break-ups are propagated in parallel and/or using computational distribution. The model was validated in 3DOF with results from simulations in computational tools used by NASA and ESA. After the validation, the trajectories were propagated for the rotational debris in 6DOF.

4. Results
After the code development, the first step is the verification and validation of the trajectory propagator. It was selected a study case using data from the ORSAT and SCARAB software to compare the results, like showed in [25]. It is compared the re-entry trajectory of a spherical tank of 0.125 m outside radius and 0.075 m of inside radius, with a mass of 10.070 kg. The material of the sphere is Graphite Epoxy I, because of the resistance to the ablation and/or fragmentation during the re-entry. The initial propagation conditions are presented in [17, 25].

The altitude as a function of time and the altitude as a function of the velocity are related and they agree with the results reported using the other computational tools [17, 25]. Only these variables are compared, because these are the only data reported in the scientific publications. Two special cases where selected to observe the influence of the angular velocity during the debris re-entry. Initially, the inertial axis and the body axis are aligned, and it is applied an initial angular velocity in the Z-axis, orthogonal to the plane of the trajectory. The angular velocity selected is 1200 RPM, in same order of projectiles angular velocity to observe easily the effect of the Magnus force during the trajectory. It was applied in the prograde and retrograde direction.

In figure 3 it is observed the altitude of the trajectories as a function of time. The data obtained from the ORSAT and SCARAB trajectories is adapted from [17, 25] and more information about of the results of the simulations is available in [19]. Initially, all the trajectories have the same behaviour. They decay linearly, around 90 km of altitude, where the atmospheric density increases and then they begin an exponential decay. Differences between the trajectories reported used the ORSAT, SCARAB with the trajectories simulated from the propagator, are associated with the differences between the mathematical models, but all the results presented a good agreement in velocity and altitude (figure 4). The four trajectories propagated are similar until the 60 km of altitude (see figure 3), where the trajectories that influence the initial rotational motion move away from the trajectories with 3DOF and 6DOF. The difference is more significant at altitudes below 20 km, in the highest density zone. Due to the influence of the Magnus force and attitude changes, the final trajectories have a higher flight time than the trajectories in 3DOF and 6DOF, also changing the impact zones (figures 3 and 5). It is
possible to observe that there are not significant differences between the trajectories with 3DOF and 6DOF (figures 3 to 5), due to the absence of initial angular velocity, and the symmetry of the body that reduce the aerodynamics torques. The trajectory of the 6DOF with angular velocity equal to zero, present a null influence of the Magnus effect and is equal to the 3DOF trajectory. Trajectories with induced rotation generate Magnus force in a radial direction, reducing the vertical velocity and increasing the time of flight (figure 3). In the case of positive rotation (blue line), the direction of the rotation reduces the relative air velocity, reducing the effect of the Magnus force and having a time of flight lower than the negative rotation trajectory (red line), where the direction of rotation increases the effects of the Magnus force (figure 3 and 5). This type of phenomena is not observed in the other trajectory’s tools, because in the data obtained from the scientific literature, they do not consider the rotation with Magnus force [17]. The differences between the propagated trajectories in 3DOF and 6DOF with the references results (from computational tools [25]) are generated due to the differences in the dynamical, atmospheric, winds, aerodynamics, solid and numerical integration models.

The variation of the relative velocity with the altitude shows the same behaviour and correlation for all the re-entry cases. The velocity presents a small variation in altitudes between 90 km to 120 km, due to the low density of the air. The increment of density with the reduction of the altitude generates a breaking of around 7 km/s in altitudes between 80 km to 20 km. The debris velocity is subsonic in the troposphere. The impact velocity is inferior to 100 m/s due to the influence of the highest density zone. In subsonic flow the drag coefficient is the minimum (see figure 4) and it reduces the velocity losses.
The four cases to compare are the sphere re-entry in 3DOF, the sphere re-entry in 6DOF without initial angular velocity and the rotating spherical tank initially at 1200 RPM in the plane of the trajectory in clockwise and counter clockwise directions. Initially, the four trajectories have the same behaviour but, in figure 5, it can be observed that the trajectory with the sphere at 1200 RPM hits back of the trajectories with 3DOF and 6DOF. The trajectory with -1200 RPM shows a larger displacement, like the presented by rotational balls in sports influenced by the Magnus force. The Magnus force is orthogonal to the angular and translational velocities. This direction makes trajectories with positive rotation to generate a component in the opposite direction compared to the trajectories with negative rotation and without rotation. This behaviour is only visible at low altitudes because of the air density increases. With the results shown in those figures, it is possible to observe that the unknown of the attitude and rotation of the re-entry debris can increase the impact zone in a ratio of 80 km around of the 3DOF and 6DOF trajectories. With the increase of the landing zone, the hazard probability is increased, making the debris more dangerous.

![Graph showing spherical debris impacts coordinates.](image)

**Figure 5.** Spherical debris impacts coordinates.

### 5. Conclusions

It is a fact that the increment in the population of space objects due to the space industry increases. A consequence of this increment is the accelerated increase of re-entry debris, generally uncontrolled objects. Possible collisions and fragmentation of orbital debris will generate more objects to re-entry, which is a complex problem and a risk to the satellite population and Earth’s activities, like air traffic control.

The implementation of the Voxels simplifies the meshing process, reduce the computational cost and allows to model any type of geometric shape with good agreement, because the size of the object is fixed and don’t require additional computational process to recalculate the geometry in case of fragmentation. Each voxel stores information of the temperature, heat, structural stress and fluid interaction, simplifying the structural analyses and the reconfiguration of the new meshes. In this case, the voxels were used to model spherical debris. The results of voxel implementation present good agreement with the object-oriented model from ORSAT and can generate a spacecraft-oriented model like the SCARAB. Those computational tools were developed for more than 20 years and are currently receiving updates. The present computational propagator generates equivalent results like was showed in the results. The results of propagations in 6DOF with initial rotation allows observing significant changes from the trajectories propagated in 3DOF and in 6DOF without rotation, like an increase in...
the collision area when the Magnus effect was significant. The increase of the angular velocity increases the Magnus force, but its influence is only perceived in the low atmosphere, below 20 km of altitude, due to the atmospheric density. In higher altitudes the rotational motion doesn’t show significant variations with the trajectories without the Magnus influence.

Acknowledgments

The authors wish to express their appreciation for the support provided by the grants # 406841/2016-0 and 301338/2016-7 from the National Council for Scientific and Technological Development (CNPq); grants # 2016/24561-0 and 301338/2016-7 from São Paulo Research Foundation (FAPESP), to the financial support from the National Council for the Improvement of Higher Education (CAPES) and to the National Institute for Space Research (INPE).

References

[1] Patera R and Ailor W 1998 Proc. AAS/AIAA space flight mechanics meeting (Monterey) vol 8
[2] Ailor W and Wilde P 2008 Proc. IAASS conference: building a safer space together (Rome) vol 3
[3] Gallais P 2007 Atmospheric re-entry vehicle dynamics (Berlin, Springer)
[4] Weiland C 2010 Computational spaceflight mechanics (Berlin, Springer)
[5] Zipfel P 2007 Modeling and Simulation of Aerospace Vehicle Dynamics (Reston, AIAA)
[6] Tewari A 2007 Atmospheric and space flight dynamics (Boston, Birkhauser)
[7] Thorsten K, Jorg F and Wolfram F 2012 J. of Wind Energy and Industrial Aerodynamics 110 19
[8] Kim H and Swan C 2003 Int. J. for Numerical Methods in Engineering 56 977
[9] Green S et al 2014 Composite Structures 118 284
[10] Sapozhnikov S, Kudryavtsev O and Dolganina N 2015 Materials & Design 88 1042
[11] Schaaf S and Chambre P 1961 Flow of rarefied gases no. 8 (New Jersey, Princeton University)
[12] Tewari A 2009 J. of Spac. and R. AIAA 46 299
[13] Padilla J and Boyd I 2006 Proc. AIAA/ASME joint thermophysics and heat transfer conference (San Francisco) vol 9
[14] Viviani A and Pezzella G 2015 Aerodynamic and aerothermodynamics analysis of space mission vehicles (New York, Springer)
[15] Hankey W 1998 Re-entry aerodynamics (Virginia, AIAA)
[16] Detra R and Hidalgo H 1961 ARS Journal 31 318321
[17] Lips T and Fritsche B 2005 Acta Astronautica 57 312323
[18] Koppenwallner G and Legge H 1985 Proc. Thermophysical Aspects of Re-Entry Flows vol 103
[19] Murcia J 2018 Trajectory and attitude modeling and propagation for reentry debris with fragmentation implementing voxels meshes (Thesis, INPE)
[20] ANSI 2004 Guide to reference and standard atmospheric models (Reston, AIAA)
[21] Picone J et al 2002 Journal of Geophysical Research: Space Physics 107 116
[22] Kuga H and Carrara V 2013 Proc. simpósio Brasileiro de sensoriamento remoto (Foz de Iguazu) vol 16
[23] Hedin A et al 1996 Journal of Atmospheric and Terrestrial Physics 58 14211447
[24] Fehlberg E. 1968 Classical fifth-, sixth-, seventh-, and eighth-order Runge-Kutta formulas with step size NASA Report TRS-287 (Washington, NASA)
[25] Park S and Park G 2017 Advances in Space Research 60 893906