Small-$p_T$ and large-$x$ regions for Higgs transverse momentum distributions

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Abstract

It was shown recently that standard resummation of logarithms of $Q/p_T$ can be supplemented with the resummation of logarithmic contributions at large $x = Q^2/s$ in the case of a colourless final state such as Higgs produced via gluon fusion or the production of a lepton pair via Drell–Yan mechanism. Such an improved transverse momentum resummation takes into account soft emissions that are emitted at very small angles. We report on recent phenomenological studies of a combined threshold-improved $p_T$ and threshold resummation formalism to the Higgs boson produced at the LHC where small-$p_T$ and threshold logarithms are resummed up to NNLL and NNLL* respectively. We show that the effect of the modified $p_T$ resummation yields a faster perturbative convergence in the small-$p_T$ region while the effect of the threshold one improves the agreement with fixed-order calculations in the medium and large-$p_T$ regions.

Keywords: QCD, Resummation, Higgs

1. Introduction

Precise computations of differential distributions of a colour singlet object such as the Higgs boson will play a major role in the future physics programme of the LHC. In particular, the kinematic distributions of the Higgs boson produced in association with QCD radiations can be used to constrain new-physics models due to its sensitivity to modification of the Yukawa couplings [1,2].

The transverse momentum distribution of the Higgs in the Effective Field Theory (EFT) approach is currently known to very high accuracy, namely NNLO [3,4,5].

The perturbative description of the Higgs transverse momentum in certain regions of the phase space, however, requires the resummation of large logarithmic corrections to all-order in the coupling constant $\alpha_s$.

In this article, we review a formalism [6,7] that combines two regions, namely small-$p_T$ and large-$x$, for the resummation of Higgs transverse momentum spectra. This was achieved by, first modifying the standard $p_T$ resummation to take into account soft radiations that emitted at very small angles, and then combining the resulting expression with the pure contribution from the threshold.

Our starting point is the Mellin space cross section [6,7] where short and large range interactions factorizes:

$$d\sigma^{\text{res}}_{\xi_p}(N,\xi_p,\alpha_s) = \sum_{a,b} L_{ab}(N) \frac{d\sigma^{\text{res}}_{ab}}{d\xi_p}(N,\xi_p,\alpha_s),$$

where we have expressed the cross sections in terms of the dimensionless variable $\xi_p = p_T^2/Q^2$. In the sequel, the superscript "res" may describe the threshold, modified small-$p_T$, and combined resummation. This article starts with a brief description of the three resummation formalisms, followed by the phenomenological results for the Higgs boson produced at LHC. Conclusions are drawn in Section 6.

2. Threshold resummation

This section is not meant to be a review of the threshold resummation for Higgs boson production, in which
extensive literature already exists, but rather to collect the relevant equations that will be used as a comparison to the threshold-improved \( p_T \) resummation in Section 3.

Typically, when the invariant mass \( Q \) of the system approaches the partonic center of mass energy \( \sqrt{s} \), the phase space for gluon bremsstrahlung vanishes and results in large logarithmic corrections. The tower of logarithms are of the form \( \alpha_s^k \left( \ln^i(1-x)/(1-x) \right) \) for \( 0 \leq k \leq 2n-1 \) at the \( k \)-th order in perturbation theory. Here, the plus distribution regularizes the singularity at \( x = 1 \). The appearance of these large corrections spoils the perturbative convergence of the fixed-order calculations even if we are well in the perturbative regime where \( \alpha_s \ll 1 \). These logarithmic structures, however, exponentiate after taking the Mellin transform w.r.t. the relevant kinematic variable. Generally, threshold resummed expression for the production of a Higgs boson takes the form

\[
\frac{d\sigma}{d\xi_p}(N,\xi_p,\alpha_s) = C_{ab}^{(LO)}(N,\xi_p,\alpha_s) g \exp\left(\Delta(N) + \Delta(N) + J(N) + S(N,\xi_p)\right),
\]

where \( \sigma_0 \) denotes the leading order Born cross section. Notice that we have omitted the \( \alpha_s \) dependence in the exponent for simplicity. The matching function \( g \) assures that at every perturbative order, the resummed cross section agrees with the exact fixed-order calculations up to corrections of the order of \( 1/N \). The exponent collects all the enhanced terms when \( N \to \infty \) (equivalent to say \( x \to 1 \)) which in itself can be written as perturbative series \( \sum_{n=0}^{\infty} \sum_{n=0}^{\infty} \alpha_s^n \ln^n(N) \). Notice that now, the enhanced contributions are single-logarithmic in the moment \( N \). Finally, \( C_{ab}^{(LO)} \) represents the leading-order term in the \( p_T \) distribution whose gg-channel part is given by

\[
C_{gg}^{(LO)}(N,\xi_p,\alpha_s) = 2\alpha_s \frac{\sigma_0 C_A}{\pi} \sum_{m=0}^{4} (-1)^m \frac{\Gamma(1/2)\Gamma(N+m)}{\Gamma(N+m+1/2)} f_m(\xi_p) \frac{1}{2} F_1\left(1,1.5;N+m,N+m+1/2;\xi_p^2\right). \]

The functions \( f_m \) fully depend on the modified variable \( \xi_p \) and are defined as follows:

\[
\begin{align*}
f_0(\xi_p) &= 1, & f_2(\xi_p) &= \xi_p^2(1+\xi_p)(3+\xi_p), \\
f_1(\xi_p) &= \xi_p(1+\xi_p), & f_3(\xi_p) &= \xi_p^3(1+\xi_p), \\
f_4(\xi_p) &= \xi_p^4, & z &= z(\xi_p) = \sqrt{1+\xi_p^2}.
\end{align*}
\]

3. Threshold-improved \( p_T \) resummation

3.1. Description

In the context of threshold-improved \( p_T \) resummation, soft and small-\( p_T \) behaviours are jointly resummed in the new argument of the logarithm \( \chi = \left( \hat{N}^2 + \hat{b}^2 / \mu_0^2 \right) \) where \( \hat{N} = N \exp(\gamma_E) \) and \( \hat{b} = bQ \). It is now apparent that \( \chi \) interpolates between threshold and small-\( p_T \) limit in the respective limit. As shown in Eq. (2.7), the partonic \((N,\xi_p)\)-space of the hadronic resummed cross section is given by

\[
\frac{d\sigma}{d\xi_p}(\xi_p) = \frac{\sigma_0}{z(\xi_p)^N} \int_0^{\infty} db \, J_0(b \sqrt{\xi_p}) \frac{d\sigma}{d\xi_p}(b), \tag{5}
\]

where the explicit expression of the \((N,b)\)-space partonic part is given by the following:

\[
\frac{d\sigma}{d\xi_p}(N,b,\alpha_s) = \hat{H}_e\left(\frac{N^2}{\chi},\alpha_s,Q^2\right) W_{gb}(N,\alpha_s) \tag{6}
\]

The hard function \( \hat{H} \) collects all the \( \beta \)-dependence that are not enhanced in the limit \( b \to \infty \). Its expression can be written in terms of the standard hard function whose perturbative expansions are given in \[2,\]

\[
\hat{H}_e\left(\frac{N^2}{\chi},\alpha_s\right) = H_e(\alpha_s) + A_1(\alpha_s) Li_2\left(\frac{N^2}{\chi}\right) + O(\alpha_s^2). \tag{7}
\]

The function \( W \) contains both the coefficient function and the evolution of the Parton Distribution Functions (PDFs). Its expression is given by

\[
W_{gb}(N,\alpha_s) = C_{gb}(N,\alpha_s) U_{ab}(N,\alpha_s) \tag{8}
\]

where the expressions of \( C \) and \( U \) are defined as in Eq. (2.26), Eq. (2.28). As opposed to the standard \( p_T \) resummation, the coefficient and evolution functions are computed from a joint scale \( Q^2/\chi \). The logarithmic enhanced contributions are contained in \( S \) and is defined such that \( S = 1 \) when \( \alpha_s \ln \chi = 0 \). The LL term collects logarithmic contributions of the form \( \alpha_s \ln^{n+1} \chi \); the NLL part resums logarithms of the form \( \alpha_s \ln^{n} \chi \); and finally the NNLL terms resums \( \alpha_s^2 \ln^{n-1} \chi \) contributions. Therefore, the function \( S \) can be organized as follows:

\[
S_{ab}(N,\chi,\mu_0^2) = \alpha_s^{-1} \bar{g}_1(\chi) + g_2(\chi) + \alpha_s g_3(\chi). \tag{9}
\]
where
\[ g_1(\chi) = \frac{A_1}{\rho_0^2}(\lambda_x + \ln(1 - \lambda_x)) \quad \text{with} \quad A_1 = \frac{C_A}{\pi}. \] (10)

Here, we only give the expression of the leading logarithmic contributions that will be used later. The explicit expressions of \( g_1 \) and \( g_2 \) can be extracted from [6]. The \( b \) and \( N \) dependence are now embodied in \( \lambda_x = \alpha_s \rho_0 \ln \chi \) while the pure \( N \) dependence is embodied in \( \lambda_N = \alpha_s \rho_0 \ln N^2 \). It is straightforward to see that the in the small-\( p_T \) limit (or equivalently large-\( b \)), threshold-improved \( p_T \) reproduces standard transverse momentum as given in Ref. [10]. Indeed, taking \( b \to \infty \) in Eq. (6) implies that \( Q^2/\chi \to b^2_0/b^2 \), \( \chi \to b^2/b^2_0 \), and \( L_\perp (N^2/\chi) \to 0 \).

3.2. Large-\( N \) behaviour at small-\( p_T \)

In the following section, we check that the threshold-improved \( p_T \) resummation captures the correct threshold behaviour at small-\( p_T \); these are initial state soft gluons emitted at very small angles. Such a check amounts to the expansion of the resummed expression in Eq. (5) and compare it to \( C_{\text{LO}}^{\text{gg}} \) in Eq. (3). For simplicity, we only consider the LL resummation of the gg-channel with the \( \alpha_s \)-term in the perturbative expansion.

We start from Eq. [6] where at LL we approximate \( \bar{H}_g \) and the coefficient function \( C_{\text{gg}} \) to 1 and we only include in the Sudakov exponent the function \( g_1 \). The PDF evolutions are included up to LO with the leading-order anomalous dimension replaced by its large-\( N \) behaviour while the evolution factor that evolves the coefficient functions are approximated to 1. Thus, using [7] Eq. (2.28a)], we arrive to the following

\[ \frac{d\sigma^{\gamma*}_{\gamma*}}{d\xi_p}(N, b, \alpha_s) = \left( U^{\text{LO}}_{\gamma*}(\chi \leftarrow Q^2) \right)^2 \exp(g_1) \] (11)

The LO solution [7] Eq. (2.29)] to the DGLAP evolution equation in the large-\( N \) limit writes as

\[ U^{\text{LO}}_{\gamma*}(\chi \leftarrow Q^2) = -\frac{A_1}{2\alpha_s \rho_0^2 \lambda_N} \ln(1 - \lambda_N). \] (12)

Thus, Eq. (11) yields

\[ \frac{d\sigma^{\gamma*}_{\gamma*}}{d\xi_p}(N, b, \alpha_s) = \exp\left( \frac{A_1}{\alpha_s \rho_0^2} (\lambda_x + (1 - \lambda_N) \ln(1 - \lambda_x)) \right) \] (13)

Expanding the above equation and retaining only the \( \alpha_s \)-term we arrive at the following expression

\[ \frac{d\sigma^{\gamma*}_{\gamma*}}{d\xi_p}(N, b, \alpha_s) = -\alpha_s A_1 \left( \frac{\ln^2 \chi}{2} - 2 \ln N \ln \chi \right). \] (14)

Plugging back Eq. (14) into Eq. (5) and performing the Fourier back transform using [7] Eq. (2.43a), Eq. (2.43b)] we end up with a \( (N, \xi_p) \) version of the expanded expression. The final result writes as,

\[ \frac{d\sigma^{\gamma*}_{\gamma*}}{d\xi_p}(N, \xi_p, \alpha_s) = 2\alpha_s A_1 \frac{\sigma_0}{\xi_p} \chi^{1/2} \exp\left( \frac{N}{\sqrt{\xi_p}} \right) \left\{ \frac{N}{\sqrt{\xi_p}} \left. K_1(\tau) \right| (2N \sqrt{\xi_p}) \right. \] (15)

Recall that we are here interested to check the large-\( N \) behaviour of the threshold-improved transverse momentum resummation while keeping \( N p_T \) fixed in which case the expression above cannot be simplified further. Thus, in order to relate Eq. (15) with Eq. (3), we have to simplify \( C_{\text{gg}}^{\text{LO}} \) in the aforementioned limit. For this, it is sufficient to only consider the most dominant term, i.e. \( m = 4 \). Using the properties of the Gauss hypergeometric functions, we have

\[ 2F1 \left( \frac{1}{2} N + 4, N + 9, \frac{z^2}{1 - z^2} \right) = \frac{\tilde{F}_1 \left( N + 9/2, z^2/(1 - z^2) \right)}{\sqrt{1 - z^2}}. \] (16)

where we have defined \( \tilde{F}_1 \left( N + 9/2, z^2/(1 - z^2) \right) = \frac{2}{\sqrt{1/2} \Gamma(1/2) \Gamma(N + 4)} \sum_{s=0}^{\infty} g_s \left( \frac{z^2}{1 - z^2} \right) \]

\[ \Gamma(s + 1/2) \ln^2 \left( \frac{1}{2} \right) s + 1, -2N \ln z. \] (17)

We stress that this expression only holds if \( N \to \infty \) uniformly w.r.t. large values of \( z^2/(z^2 - 1) \) which is indeed the case as \( z^2/(z^2 - 1) \to \infty \) as \( \xi_p \to 0 \). In such a limit, it is sufficient to only consider first term in the sum of Eq. (17) (i.e. \( s = 0 \)) where \( g_0 \) and the 2nd Kummer function \( U \) are defined as

\[ g_0(z) = \sqrt{\frac{\ln \left( \frac{1}{z} \right)}{z}}. \] (18)

\[ U \left( \frac{1}{2}, 1, 2z \right) = \frac{1}{\Gamma(1/2)} z \exp(z) K_1(\tau). \] (19)

Using Eqs. (19) and (19) to redefine \( \tilde{F}_1 \) in Eq. (16) and plugging its expression back into Eq. (3) we arrive to the following expression

\[ C_{\text{gg}}^{\text{LO}} \left|_{\lambda_N = 4} \right. = 2\alpha_s A_1 \frac{\sigma_0}{\xi_p} \frac{\sqrt{2N} \ln \xi}{\pi N^2 \sqrt{1 - z^2}} K_1(\tau). \] (20)
with $\xi = -N \ln z$. The $z^N$-term in the denominator comes from re-rewriting the exponential in Eq. (19) in terms of the explicit expression of $z$. We can now safely take the limit $\xi_p \to 0$ in the terms that are not $N$-dependent, which yields the following simplifications $\sqrt{2 \ln z} \sim 2 \sqrt{\xi_p}$ and $\sqrt{1 - z^2} \sim 2 \sqrt{\xi_p}$. Finally, the large-$N$ limit just amounts to replacing $N + 2$ with $N$. Putting everything together and with a little bit of algebraic simplification we get

$$C_{gs}(^{\text{LO}})_{m=4} = 4\alpha_s C_A \frac{\sigma_0}{\pi} \frac{N}{\sqrt{\xi_p}} \frac{1}{2N} \frac{1}{\sqrt{\xi_p}}. \quad (21)$$

In the limit $N \sqrt{\xi_p} \to 0$, the above expression is asymptotic to expansion of the modified resummation in Eq. (15) as shown in Fig. 1. In order to generate $K_1$-terms from the leading-order $p_T$ spectra $C_{gs}(^{\text{LO}})$, the sum in Eq. (17) has to be performed beyond $s = 0$. This is computationally challenging as the form of the Kummer U does not lead directly to compact Bessel functions. However, it can be seen from the numerical checks in Fig. 1 that the threshold-improved $p_T$ resummation captures best the soft behaviours from fixed-order calculations while the standard $p_T$ resummation deviates largely from the LO as $N$ increases.

![Figure 1: Ratio between the asymptotics and the LO in Eq. (3). The Standard $p_T$ (SSpT) resummation is computed in the limit $\xi_p \to 0$. In the plots, $\alpha_s = 0.118$ and $\xi_p = 10^{-3}$.](image)

4. Combined resummed expression

The threshold-improved $p_T$ resummation described in Section 3 does not contain all the soft logarithms such as the soft logarithms that arise when soft gluons are emitted at large angle. Therefore, the pure threshold contribution has to be manually added through a profile matching function. The profile matching function is chosen such (i) that there is no double counting, and (ii) the combined result reproduces threshold-improved $p_T$ and threshold resummation at small $p_T$ and large $x$ respectively. A possible expression for such a combined resummation is given by

$$\frac{d\hat{\sigma}^{\text{ab}}(N, \xi_p, \alpha_s)}{d\xi_p} = T(N, \xi_p) \frac{d\hat{\sigma}^{\text{th}}}{d\xi_p}(N, \xi_p, \alpha_s) + \left(1 - T(N, \xi_p)\right) \frac{d\hat{\sigma}^{\text{ab}}}{d\xi_p}(N, \xi_p, \alpha_s), \quad (22)$$

where we define the profile matching function as $T(N, \xi_p) = N^k \xi_p^m/(1 + N^k \xi_p^m)$. Hence, combined resummation produces results that differ from the threshold-improved $p_T$ resummation by $O(\xi_p^m)$ corrections when $\xi_p \to 0$, and from the threshold resummation by $O(1/N^k)$ when $N \to \infty$. The values of $k$ and $m$ can be chosen arbitrarily provided that $m < k$ and can be used to assess the ambiguity of the matching.

5. Phenomenological results

In this section, we present phenomenological results for the standard model Higgs produced via gluon-gluon fusion at the LHC with a center of mass energy $\sqrt{s} = 13$ TeV. The issue related to the computation of the Fourier-Mellin back-transform from our modified $p_T$ resummation is dealt using the Borel prescription [17]. The following plots are produced using the NNPDF31_nnlo_as_0118 set [13] of parton distribution functions. The uncertainty bands are obtained from varying the renormalization and factorization scales using the seven-point prescription. For the combined results, the parameters that enter into the matching function are set to $m = 2$ and $k = 3$.

In Fig. 2 we compare the standard $p_T$ (SSpT) resummation with the threshold-improved $p_T$ (TIpT) and combined resummation. The resummed calculations are matched with the fixed-order (FO) predictions in order to reduce the effect of unjustified logarithms in the medium and large-$p_T$ regions. In contrast to the NLO predictions, resummation leads to a well-behaved transverse momentum distribution in the small-$p_T$ regions that has a peak at $p_T \sim 10$ GeV. As we go from SSpT to TIpT, we see a faster perturbative converge of the NLL+LO result while the NNLL+NLO results are similar. This is well understood as the $N/b$ corrections become negligible as the logarithmic accuracy increases. Due to the structure of the matching function, TIpT and combined are exactly similar at small-$p_T$ and the pure threshold contribution start to become relevant.
momenta. In particular, we investigated further the soft momentum distributions that is valid for all values of $p_T$. Such a formalism results in resummation of transverse momentum resummation that combines small-$p_T$ and large-$p_T$ contributions.

\[ \frac{dN}{dp_T} (\sqrt{s} = 13\text{TeV}) \] \[ \frac{d\sigma}{dp_T} (\sqrt{s} = 13\text{TeV}) \]

(a) NLL+LO

(b) NNLL+NLO

Figure 2: $p_T$ spectrum for the Higgs boson production comparing the three types of resummations matched with standard fixed-order calculations. The top panels compare the matched results with the F.O calculations, while the lower panels show the ratio w.r.t. the central value of the combined results.

at $p_T \sim 60\text{GeV}$ which yields a better agreement with the F.O. results. This is because medium and large-$p_T$ regions are dominated by soft behaviours.

6. Conclusions

In this article, we have presented a review of a formalism that combines small-$p_T$ and large-$p_T$ resummation for Higgs boson transverse momentum distribution. Such a formalism results in resummation of transverse momentum distributions that is valid for all values of momenta. In particular, we investigated further the soft logarithms that are present in the modified $p_T$ resummation by comparing it to fixed-order. We finally showed that while the modified resummation improves the convergence at small-$p_T$, combining it with the pure threshold contribution results in a more accurate prediction in the medium and large-$p_T$ regions.

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