Neutron Rich Hypernuclei in Chiral Soliton Model

Vladimir B. Kopeliovich
Institute for Nuclear Research, Russian Academy of Sciences, Moscow 117312, Russia

Abstract

The binding energies of neutron rich strangeness $S = -1$ hypernuclei are estimated in the chiral soliton approach using the bound state rigid oscillator version of the $SU(3)$ quantization model. Additional binding of strange hypernuclei in comparison with nonstrange neutron rich nuclei takes place at not large values of atomic (baryon) numbers, $A = B \leq \sim 10$. This effect becomes stronger with increasing isospin of nuclides, and for "nuclear variant" of the model with rescaled Skyrme constant $e$. Total binding energies of $^{8}_ΛHe$ and recently discovered $^{6}_ΛH$ satisfactorily agree with experimental data. Hypernuclei $^{7}_ΛH$, $^{9}_ΛHe$ are predicted to be bound stronger in comparison with their nonstrange analogues $^7H$, $^9He$; hypernuclei $^{10}_ΛLi$, $^{11}_ΛLi$, $^{12}_ΛBe$, $^{13}_ΛBe$ etc. are bound stronger in the nuclear variant of the model.

1 Introduction

Studies of nuclear states with unusual properties — nontrivial values of flavor quantum numbers (strangeness, charm or beauty), or large isospin (so called neutron rich nuclides) are of permanent interest. They are closely related to the problem of existence of strange quark matter and its fragments, strange stars (analogues of neutron stars), and may be important for astrophysics and cosmology. Recently new direction of such studies, the studies of neutron rich hypernuclei with strangeness $S = -1$, got new impact due to discovery of the hypernucleus $^6_ΛH$ (heavy hyperhydrogen) by FINUDA Collaboration [1] which followed its search during several years [2].

Theoretical discussion of such nuclei took place during many years, beginning with the work by R.H. Dalitz and R. Levi-Setti, [3] - [7], in parallel with experimental searches [8, 9, 10, 2]. It has been noted first in [3] that the Lambda particle may act as additional glue for the nuclear matter, increasing the binding energy in comparison with nucleus having zero strangeness. Here we confirm this observation within the chiral soliton approach (CSA). Moreover, this effect becomes stronger for the neutron rich nuclei, with increasing excess of neutrons inside the nucleus.

The important advantage of the CSA proposed by Skyrme [11], in comparison with traditional approaches to this problem, is its generality, i.e. the possibility to consider different nuclei on equal footing, and considerable predictive power. (Some early descriptions of this model can be found in [12]). The drawback of the CSA is its relatively law accuracy in describing the properties of each particular nucleus. In this respect the CSA cannot compete with traditional approaches and models like shell model, Hartree-Fock method, etc. [3] — [6].
The quantization of the model performed first in the $SU(2)$ configuration space for the baryon number one states [13], somewhat later for configurations with axial symmetry [14] and for multiskyrmions [15], allowed, in particular, to describe the properties of nucleons and $\Delta$-isobar [13] and, more recently, some properties of light nuclei, including so called ”symmetry energy” [16], and some other properties of nuclei [18].

The $SU(3)$ quantization of the model has been performed first within the rigid rotator approach [19] and also within the bound state model [20]. The binding energies of the ground states of light hypernuclei have been described in [21] within a version of the bound state chiral soliton model [22], in qualitative, even semiquantitative agreement with empirical data [23]. The collective motion contributions, only, have been taken into account in [21] (single particles excitations should be added), and special subtraction scheme has been used to remove uncertainties in absolute values of masses intrinsic to the CSA [24, 25]. This investigation has been extended to the higher in energy (excited) states, with baryon number $B = 2$ and 3, some of them may be interpreted as antikaon-nuclei bound states [26]. Some of the states obtained in [26] are bound stronger than predicted originally by Akaishi and Yamazaki [27, 28]. These states could overlap and appear in experiment as a broad enhancement, in qualitative agreement with data obtained by FINUDA collaboration [29] and more recently by DISTO collaboration [30].

To estimate the total binding energies of neutron rich hypernuclei we are using here one of possible $SU(3)$ quantization models, the rigid oscillator version of the bound state model [22] which seems to be the simplest one. In section 2 our approach, the CSA, is shortly described, section 3 contains the formulas summarizing the CSA results for strange hypernuclei and numerical results for the binding energies of neutron rich hypernuclei with neutron excess $N - Z = 3$ and 4, atomic number $A \leq 17$. Final section contains conclusions and discussion of perspectives.

2 Features of the CSA applied to hypernuclei

The CSA is based on few principles and ingredients incorporated in the truncated effective chiral lagrangian [11, 12, 13]:

$$L^{\text{eff}} = -\frac{F^2}{16} Tr l_\mu l_\mu + \frac{1}{32e^2} Tr [l_\mu l_\mu]^2 + \frac{F^2 m^2_\pi}{8} Tr (U + U^\dagger - 2),$$  \hspace{1cm} (1)

the chiral derivative $l_\mu = \partial_\mu U U^\dagger$, $U \in SU(2)$ or $U \in SU(3)$- unitary matrix depending on chiral fields, $m_\pi$ is the pion mass, $F_\pi$- the pion decay constant known experimentally, $e$ - the only parameter of the model in its minimal variant proposed first by Skyrme [11].

The chiral and flavor symmetry breaking term in the lagrangian density depends on kaon mass and decay constant $m_K$ and $F_K$ ($F_K/F_\pi \simeq 1.22$ from experimental data):

$$L^{\text{FSB}} = \frac{F^2_K m^2_K - F^2_\pi m^2_\pi}{24} Tr (U + U^\dagger - 2) (1 - \sqrt{3} \lambda_8) -$$

$$-\frac{F^2_K - F^2_\pi}{48} Tr (Ul_\mu l_\mu + l_\mu l_\mu U^\dagger) (1 - \sqrt{3} \lambda_8)$$  \hspace{1cm} (2).

\(^2\text{Recently the neutron rich isotope }^{18}B \text{ has been found to be unstable relative to the decay }^{18}B \rightarrow^{17}B + n [17], \text{ in agreement with prediction of the CSA [16].}\)
This term defines the mass splittings between strange and nonstrange baryons (multibaryons), modifies some properties of skyrmions and is crucially important in our consideration. The whole lagrangian given by (1), (2) is proportional to the number of colors of underkying QCD, \( L^{eff} \sim N_c \), which is one of justifications of the model.

The mass term \( \sim F_\pi^2 m_\pi^2 \), changes asymptotics of the profile \( f \) and the structure of multiskyrms at large \( B \), in comparison with the massless case. For the \( SU(2) \) case

\[
U = \cos f + i (\vec{n} \vec{r}) \sin f,
\]

the unit vector \( \vec{n} \) depends on 2 functions, \( \alpha, \beta \). Three profiles \( \{ f, \alpha, \beta \} (x, y, z) \) parametrize the 4-component unit vector on the 3-sphere \( S^3 \).

The topological soliton (skyrmion) is configuration of chiral fields, possessing topological charge identified with the baryon number \( B \) [11] (for the nucleus it is atomic number \( A \): \( B = A \)):

\[
B = \frac{1}{2\pi^2} \int s_f s_\alpha I ((f, \alpha, \beta)/(x, y, z)) d^3r,
\]

where \( I \) is the Jacobian of the coordinates transformation, \( s_f = \sin f \), \( s_\alpha = \sin \alpha \). So, the quantity \( B \) shows how many times the unit sphere \( S^3 \) is covered when integration over 3-dimentional space \( R^3 \) is made.

The important feature of the CSA is that multibaryon states including nuclei and hypernuclei can be considered on equal footing with the \( B = 1 \) case.

| \( B \) | \( \Theta_I \) | \( \Theta_J \) | \( \Theta_0 \) | \( \Theta_S \) | \( \Gamma \) | \( \bar{\Gamma} \) | \( \mu_S \) | \( \omega_S \) |
|---|---|---|---|---|---|---|---|---|
| 1 | 5.55 | 5.55 | 2.05 | 2.636 | 4.80 | 14.9 | 3.155 | 307 |
| 3 | 14.4 | 49.7 | 6.34 | 8.049 | 14.0 | 27.0 | 3.069 | 289 |
| 4 | 16.8 | 78.3 | 8.27 | 10.47 | 18.0 | 31.0 | 2.975 | 283 |
| 5 | 23.5 | 127 | 10.8 | 13.71 | 23.8 | 35.0 | 3.098 | 288 |
| 6 | 25.4 | 178 | 13.1 | 16.64 | 29.0 | 38.0 | 3.125 | 287 |
| 7 | 28.9 | 221 | 14.7 | 18.64 | 32.3 | 44.0 | 3.009 | 283 |
| 8 | 33.4 | 298 | 17.4 | 22.15 | 38.9 | 47.0 | 3.125 | 288 |
| 9 | 37.8 | 376 | 20.6 | 26.25 | 46.3 | 47.5 | 3.269 | 292 |
| 10 | 41.4 | 455 | 23.0 | 29.35 | 52.0 | 50.0 | 3.289 | 293 |
| 11 | 45.2 | 547 | 25.6 | 32.74 | 58.5 | 52.4 | 3.340 | 295 |
| 12 | 48.5 | 637 | 28.0 | 35.83 | 64.1 | 54.6 | 3.348 | 295 |
| 13 | 52.1 | 737 | 30.5 | 39.07 | 70.2 | 56.8 | 3.372 | 296 |
| 14 | 56.1 | 865 | 33.7 | 43.15 | 78.2 | 58.9 | 3.460 | 299 |
| 15 | 59.8 | 987 | 36.3 | 46.69 | 85.1 | 60.9 | 3.498 | 301 |
| 16 | 63.2 | 1110 | 38.9 | 50.07 | 91.5 | 62.8 | 3.517 | 302 |

Table 1. Characteristics of classical skyrmion configurations which enter the nuclei — hypernuclei binding energies differences. The numbers are taken from [31, 32]: moments of inertia \( \Theta, \Sigma \)-term \( \Gamma \) and \( \bar{\Gamma} \) - in units \( GeV^{-1} \), \( \omega_S \) - in \( MeV \), \( \mu_S \) is dimensionless (see next sections for explanation). All these quantities have the lower index \( B \) which is omitted for the sake of brevity. Parameters of the model \( F_\pi = 186 MeV; e = 4.12 \) [31, 32, 21].

Minimization of the mass functional \( M_\epsilon \) provides 3 profiles \( \{ f, \alpha, \beta \} (x, y, z) \) and allows to calculate moments of inertia \( \Theta_I, \Theta_F \), the \( \Sigma \)-term (we call it \( \Gamma \)) and some other characteristics of chiral solitons which contain implicitly information about interaction
between baryons. In Table 1 we present numerical values of the moments of inertia and other quantities taken from [31, 32, 33] where analytical expressions for them can be found as well. The moment of inertia $\Theta_S$ given in Tables 1 and 2 is certain combination of $\Theta_F^0$ and sigma term $\tilde{\Gamma}$:

$$\Theta_S = \Theta_F^0 + \frac{1}{4} \left( \frac{F_K^2}{F_F^2} - 1 \right) \Gamma. \quad (5)$$

The strangeness excitation energies $\omega_S$ given in Tables 1, 2 are somewhat overestimated, especially for nuclear variant of the model — this is an artefact of the CSA. However, this overestimation is cancelled in the nuclear binding energies differences considered below.

| $B$ | $\Theta_I$ | $\Theta_F^0$ | $\Theta_S$ | $\Gamma$ | $\tilde{\Gamma}$ | $\mu_S$ | $\omega_S$ |
|-----|------------|--------------|------------|---------|----------------|--------|-----------|
| 1   | 12.8       | 4.66         | 5.893      | 10.1    | 19.6          | 6.407  | 344       |
| 6   | 62.6       | 30.7         | 38.60      | 64.7    | 50.6          | 6.728  | 334       |
| 7   | 69.6       | 34.9         | 43.75      | 72.5    | 54.4          | 6.500  | 330       |
| 8   | 79.9       | 41.3         | 51.97      | 87.4    | 58.2          | 6.785  | 334       |
| 9   | 88.9       | 47.1         | 59.43      | 101     | 61.7          | 6.927  | 337       |
| 10  | 97.4       | 52.6         | 66.40      | 113     | 64.9          | 6.957  | 336       |
| 11  | 106        | 58.5         | 73.88      | 126     | 67.9          | 7.038  | 337       |
| 12  | 114        | 63.8         | 80.65      | 138     | 70.8          | 7.049  | 337       |
| 13  | 122        | 69.5         | 87.94      | 151     | 73.6          | 7.102  | 338       |
| 14  | 132        | 76.3         | 96.81      | 168     | 76.3          | 7.289  | 341       |
| 15  | 140        | 82.3         | 104.5      | 182     | 78.8          | 7.353  | 342       |
| 16  | 148        | 88.1         | 112.0      | 196     | 81.2          | 7.402  | 343       |

Table 2. Same as in Table 1 for rescaled (nuclear) variant of the model with constant $e = 3.0$ [16, 33].

The characteristics given in Tables 1, 2 have the following scaling properties: $\Theta_I$, $\Theta_J$, $\Theta_F$, $\Theta_S$, $\Gamma$, $\tilde{\Gamma} \sim N_c$; $\mu_S$, $\omega_S \sim N_c^0 \sim 1$. The properties of the $B = 2$ toroidal skyrmion, not included in Tables 1,2, have been considered in details previously, [26] and references therein. The rational map approximation [34] simplifies considerably calculations of various characteristics of multiskyrmions presented in Tables 1, 2.

## 3 Spectrum of strange hypernuclei in the rigid oscillator model

The observed spectrum of strange multibaryon states (hypernuclei) is obtained by means of the $SU(3)$ quantization procedure and depends on the quantum numbers of multibaryons and characteristics of skyrmions presented in Tables 1, 2. Within the bound state model [20, 22, 21] the antikaon field is bound by the $SU(2)$ skyrmion. The mass formula takes place

$$M = M_{cl} + \omega_S + \bar{\omega}_S + |S|\omega_S + \Delta M_{HFS} \quad (6)$$

where strangeness and antistrangeness excitation energies

$$\omega_S = N_c(\mu_S - 1)/8\Theta_S, \quad \bar{\omega}_S = N_c(\mu_S + 1)/8\Theta_S, \quad (7)$$
\[ \Theta_S = \Theta_F^0 + \frac{1}{4} \left( \frac{F_R^2}{F_\pi^2} - 1 \right) \Gamma, \quad \mu_S = \sqrt{1 + \frac{m_R^2}{M_0^2}} \cong 1 + \frac{m_R^2}{2M_0^2}, \]

\[ M_0^2 = N_c^2/(16\Theta_S) \sim N_c^0, \quad \bar{m}_R^2 = m_R^2 F_R^2/F_\pi^2. \]

(8)

The hyperfine splitting correction to the energy of the baryon state, depending on hyperfine splitting constants \( c_S, \bar{c}_S \), observed isospin \( I \), ”strange isospin” \( I_S \), the isospin of skyrmion without added antikaons \( \bar{I}_r \) and the angular momentum \( J \), equals in the case when interference between usual space and isospace rotations is negligible or not important [22], see also [32, 33]:

\[ \Delta M_{HFS} = \frac{J(J+1)}{2\Theta_J} + \frac{c_SI_r(I_r+1) - (c_S-1)I(I+1) + (\bar{c}_S-c_S)I_SI_S(I_S+1)}{2\Theta_I} \]

(9)

The hyperfine splitting constants are equal

\[ c_S = 1 - \frac{\Theta_I}{2\Theta_S \mu_S} (\mu_S - 1), \quad \bar{c}_S = 1 - \frac{\Theta_I}{\Theta_S \mu_S} (\mu_S - 1), \]

(10)

Strange isospin equals \( I_S = 1/2 \) for \( S = \mp 1 \), for negative strangeness in most cases of interest \( I_S = |S|/2 \) which minimizes this correction (but generally it can be not so).

We recall that body-fixed isospin \( \bar{I}^{bf} = \bar{I}_r + \bar{I}_S \) [22], [32, 33]. \( \bar{I}_r \) is quite analogous to the so called ”right” isospin within the rotator quantization scheme [19]. When \( I_S = 0 \), i.e. for nonstrange states, \( I = I_r \) and this formula goes over into \( SU(2) \) formula for multiskyrmions. Correction \( \Delta M_{HFS} \sim 1/N_c \) is small at large \( N_c \), and also for heavy flavors [20, 32].

The mass splitting within \( SU(3) \) multiplets is important for us here. The unknown for the \( B > 1 \) solitons Casimir energy [24, 25] cancels in the mass splittings. For the difference of energies of states with strangeness \( S \) and with \( S = 0 \) which belong to multiplets with equal values of \( (p, q) \)-numbers \( (p = 2I_r) \), we obtain, using the above expressions for the constants \( c_S \) and \( \bar{c}_S \) (it is first subtraction):

\[ \Delta E(p, q; I, S; I_r, 0) = |S| \omega_S + \frac{\mu_S - 1}{4\mu_S \Theta_S} [I(I+1) - I_r(I_r+1)] + \frac{(\mu_S - 1)(\mu_S - 2)}{4\mu_S \Theta_S} I_SI_S(I_S+1). \]

(11)

For the difference of binding energies of the hypernucleus with strangeness \( S = -1 \), isospin \( I = I_r - 1/2 \) and nonstrange nucleus with isospin \( I = I_r \) (the neutron excess \( N - Z = 2I_r \)) we obtain from here (second subtraction):

\[ \Delta \epsilon = \omega_{S,1} - \omega_{S,B} - \frac{3 \mu_{S,1} - 1}{8 \mu_{S,1} \Theta_{S,1}} + \left( I_r + \frac{1}{4} \right) \frac{\mu_{S,B} - 1}{4 \mu_{S,B} \Theta_{S,B}} - \frac{3 (\mu_{S,B} - 1)(\mu_{S,B} - 2)}{16 \mu_{S,B} \Theta_{S,B}}. \]

(12)

At \( I_r = 1/2 \) we obtain from (12) Eq. (9) of previous paper [21]. The term \( \sim (I_r + 1/4) \) in Eq. (12) is responsible for the additional binding of neutron rich hypernuclei in comparison with \( S = 0 \) neutron rich nuclei (same values of \( A \) and \( Z \)). The values of the quantities which enter the above formulas are shown in Tables 1, 2, the results of calculations are presented in Tables 3 and 4.

Experimental data on total binding energies of nonstrange neutron rich nuclides presented in first numerical columns of Tables 3 and 4 are taken from [35]. The experimental value of binding energy of hyperhydrogen shown in Table 3, \( \epsilon(\Lambda H) = 10.8 \text{ MeV} \) is the sum of the binding energy of \( ^6\Lambda H \) relative to \( ^5H + \Lambda \), measured in [1], \( \epsilon(\Lambda H) = (4.0 \pm 1.1)\text{MeV} \) and the binding energy of \( ^5H \) measured in [36], \( \epsilon(\Lambda H) \approx 6.78 \text{MeV} \).
These data have been indicated to the author by D.E. Lansky.

The value of the binding energy of $^8\Lambda$He shown in Table 3 is the sum of the $\Lambda$ separation energy $7.16 \pm 0.70 \text{ MeV}$ measured in $^8\Lambda$He [8] and the total binding energy of the $^7\Lambda$He nucleus, $\epsilon(7\Lambda e) \simeq 28.82 \text{ MeV}$. The values marked with $^*$, $\Delta \epsilon^{th*}$ and $\epsilon^{th*}$, here and in Table 4 denote the theoretical values obtained in rescaled (nuclear) variant of the model with Skyrme constant $c = 3.0$. This variant allowed to satisfactorily describe mass splittings of nuclear isotopes, including neutron rich nuclides, with the mass numbers between $\sim 10$ and 30 [16]. The binding energies of the ground states of hypernuclei with moderate atomic numbers can be described within this variant of the model better than in the original variant ($e = 4.12$) [21] (these results will be presented in next publications).

| $A - \Lambda A$ | $\epsilon_2^{exp}$ | $\epsilon_2^{th}$ | $\Delta \epsilon_{2,3/2}^{th}$ | $\Delta \epsilon_{3/2}^{th}$ | $\Delta \epsilon_{th*}$ | $\epsilon_2^{th*}$ |
|-----------------|---------------------|------------------|--------------------------|--------------------------|------------------|------------------|
| $^6H - ^6\Lambda H$ | 5.8 | 10.8 | 9.0 | 14.8 | 11.2 | 16.0 |
| $^8\Lambda He - ^8\Lambda He$ | 31.4 | 36.0 | 3.4 | 34.8 | 8.9 | 40.0 |
| $^{10}\Lambda Li - ^{10}\Lambda Li$ | 45.3 | -4.7 | 40.6 | 4.8 | 50.0 |
| $^{12}\Lambda Be - ^{12}\Lambda Be$ | 68.6 | -9.3 | 59.3 | 2.7 | 71.0 |
| $^{14}\Lambda B - ^{14}\Lambda B$ | 85.4 | -15.0 | 70.4 | -1.7 | 84.0 |
| $^{16}\Lambda C - ^{16}\Lambda C$ | 111 | -18.5 | 92.3 | -3.9 | 107.0 |

Table 3. The total binding energies and binding energies differences $\Delta \epsilon_{2,3/2}^{th} = \epsilon_{3/2}^{th} - \epsilon_2$ between hypernuclei with isospin $I = 3/2$ and nonstrange isotopes with $I = 2$, $N - Z = 4$ (in $\text{MeV}$) for the original variant, $e = 4.12$, and for the variant with rescaled constant, $e = 3$ (numbers with the $^*$). Experimental values of binding energy are available only for $^8\Lambda$He [8] and $^8\Lambda H$ [1].

| $A - \Lambda A$ | $\epsilon_5^{exp}$ | $\Delta \epsilon_{5/2}^{th}$ | $\epsilon_2^{th}$ | $\Delta \epsilon_{5/2}^{th*}$ | $\epsilon_2^{th*}$ |
|-----------------|---------------------|--------------------------|------------------|------------------|------------------|
| $^7\Lambda H$ | 8 | 15 | 23 | 16.4 | 24.0 |
| $^9\Lambda He - ^9\Lambda He$ | 30.3 | 0.1 | 30 | 7.0 | 37.0 |
| $^{11}\Lambda Li - ^{11}\Lambda Li$ | 45.6 | -5.0 | 41 | 5.0 | 51.0 |
| $^{13}\Lambda Be - ^{13}\Lambda Be$ | 68.1 | -9.0 | 59 | 3.0 | 71.0 |
| $^{15}\Lambda B - ^{15}\Lambda B$ | 88.2 | -16 | 72 | -2.0 | 86.0 |
| $^{17}\Lambda C - ^{17}\Lambda C$ | 111 | -17 | 94 | -2.7 | 108.0 |

Table 4. Same as in Table 3 for odd atomic numbers $A$, hypernuclei with $I = 2$ and nonstrange isotopes with $I = 5/2$, $N - Z = 5$. Experimental data are not available, still.

The value $8\text{MeV}$ for the binding energy of $^7\Lambda H$ is preliminary result published in [37]. It follows from Tables 4 and 3 that hypernuclei $^7\Lambda H$, $^9\Lambda He$, $^{11}\Lambda Li$, $^{13}\Lambda Be$ and $^{15}\Lambda B$ are stable relative to the decay into $\Lambda$-hyperon and nuclei $^6\Lambda H$, $^8\Lambda He$, $^{10}\Lambda Li$, $^{12}\Lambda Be$ and $^{14}\Lambda B$, in nuclear variant of the model. In view of our former results [16] just the nuclear variant of the CSA should be considered as most reliable.

We did not include the correction to the binding energies difference depending on the spin of the nucleus $J$ by following reasons. First, this correction is small in any case because the moment of inertia $\Theta_J$ shown in Table 1 is large, generally $\Theta_J \sim B^2$ and $\Theta_J > B \Theta_I$. Besides, in some cases of interest spins of nucleus and hypernucleus coincides, and in any case the spins of neutron rich hypernuclei are not known presently. The

---

3These data have been indicated to the author by D.E. Lansky.
decrease of values of $\Delta e^{th}_{5/2,2}$ with increasing atomic number may be connected with limited applicability of the rational map approximation [34] for describing multiskyrmions at larger baryon (atomic) numbers.

4 Conclusions and prospects

We have calculated the difference of total binding energies of neutron rich hypernucleus with atomic, or baryon number $A$, strangeness $S = -1$, charge $Z$ (i.e. containing $Z$ protons), isospin $I = (N - Z - 1)/2$, and the zero strangeness nucleus with same atomic number $A$, $Z$ protons and $N = A - Z$ neutrons, which has isospin $I = (N - Z)/2$. Within the chiral soliton approach this quantity contains the smallest uncertainty.

This calculation does not contain any free parameters to be fitted. We performed calculations for two values of the Skyrme constant, $e = 4.12$, and for $e = 3.0$ (rescaled, or nuclear variant) which allowed to describe the mass splittings of nuclear isotopes with atomic numbers up to $\sim 30$ [16]. The total binding energies of the ground states of hypernuclei with $A \geq 7$ are described better with rescaled constant $e$ than it was made previously in [21] with the standard value $e = 4.12$. Both variants of the model provide close results for $^6\Lambda H$ and $^7\Lambda H$, but for greater atomic numbers the difference becomes considerable. Results of the rescaled nuclear variant seem to be more reliable for greater atomic numbers, $A \geq 10$. Calculations performed in present paper may be extended easily to hypernuclei with arbitrary excess of neutrons in nuclei.

The uncertainty of our estimates is considerably greater than that of traditional methods [3] - [6], several $MeV$, at least. For the nucleus $^6\Lambda H$ our result for the total binding energy is between $14.8 MeV$ and $16 MeV$ in comparison with the value $10.8 \pm 1.1 MeV$ which can be extracted from data [1], see Table 3 and its discussion. The results of [3] and [5] are in much better agreement with data. In case of the $^8\Lambda He$ our results are in satisfactory agreement with previously obtained data [8]. However, the advantage of the CSA is that it provides a general look at nuclei with different excess of neutrons and great variety of atomic numbers. We hope that results presented here may be useful for planning of future experiments aimed to find new neutron rich hypernuclei.

Hypernuclei with quantum number beauty are expected to be bound considerably stronger than strange hypernuclei, at least by several MeV, in some cases by few tens of MeV [21]. For charmed hypernuclei the binding is not so strong, due to increase of electric charge of the nucleus by unity. More detailed calculation of binding energies in the case of beauty and charm will be presented elsewhere.

The author is indebted to D.E.Lanskoy for reading the manuscript and useful remarks and suggestions.

References

1. M. Agnello et al (FINUDA Collab.) Phys.Rev.Lett.108:042501,2012; arXiv:1203.1954 [nucl-ex]
2. M. Agnello et al (FINUDA Collab.) Phys.Lett. B640, 145 (2006); nucl-ex/0607019
3. R.H. Dalitz and R. Levi-Setti, Nuovo Cimento 30, 489 (1963)
4. T.Yu. Tretyakova and D.E. Lanskoy, Eur.Phys.J. A5, 391 (1999); Nucl.Phys. A691, 51c (2001); Phys.At.Nucl. 66, 1651 (2003)
5. S. Shimura, K.S. Myint, T. Harada and Y. Akaishi, J.Phys. G28, L1 (2002); Y. Akaishi, Prog.Theor.Phys. Suppl. 186, 378 (2010)
6. L. Majling, AIP Conf. Proc. 893, 493 (2006); ibid. 1012, 392 (2008); L. Majling and S. Gmuca, Phys.Atom.Nucl. 70, 1611 (2007)
7. M.T. Win, K. Hagino and T. Koike, Phys. Rev. C83, 014301 (2011)
8. M. Juric, G. Bohm, J. Klabuhn et al, Nucl. Phys. B35, 160 (1971)
9. K. Kubota et al, Nucl.Phys. A602, 327 (1996)
10. P.K. Saha et al, Phys.Rev.Lett. 94, 052502 (2005)
11. T.H.R. Skyrme, Proc. Roy. Soc. of London, A260, 127 (1961); Nucl. Phys. 31, 556 (1962)
12. G. Holzwarth and B. Schwesinger, Rept.Prog.Phys. 49, 825 (1986); I. Zahed and G.E. Brown, Phys.Rept. 142, 1 (1986)
13. G. Adkins, C. Nappi, and E. Witten, Nucl. Phys. B228, 552 (1983)
14. V.B. Kopeliovich, Sov.J.Nucl.Phys. 47, 949 (1988)
15. P. Irwin, Phys. Rev. D61, 114024 (2000); S. Krusch, Annals Phys. 304, 103 (2003); Proc. Roy. Soc. Lond. A462, 2001 (2006).
16. V. Kopeliovich, A. Shunderuk and G. Matushko, Phys.Atom.Nucl. 69, 120 (2006)
17. A. Spyrou et al, Phys.Lett. B683, 129 (2010)
18. R. Battye, N. Manton, P. Sutcliffe and S.W. Wood, Phys.Rev. C80, 034323 (2009)
19. E. Guadagnini, Nucl.Phys. B236, 35 (1984); V.B. Kopeliovich, J.Exp.Theor.Phys. 85, 1060 (1997); A.M. Shunderuk, Phys.Atom.Nucl. 67, 748 (2004)
20. C.G. Callan and I.R. Klebanov, Nucl. Phys. B262, 365 (1985); N. Scoccola, H. Nadeau, M. Nowak and M. Rho, Phys. Lett. B 201, 425 (1988); C. Callan, K. Hornbostel, and I. Klebanov, Phys. Lett. B 202, 269 (1988).
21. V. Kopeliovich, J.Exp.Theor.Phys. 96, 782 (2003); Nucl. Phys. A721, 1007 (2003); V. Kopeliovich and A. Shunderuk, Eur.Phys.J. A33, 277 (2007)
22. K.M. Westerberg, I.R. Klebanov, Phys. Rev. D50, 5834 (1994); Phys.Rev. D53, 2804 (1996)
23. H. Bando, T. Motoba and J. Zofka, Int.J.Mod.Phys. 21, 4021 (1990); O. Hashimoto and H. Tamura, Prog.Part.Nucl.Phys. 57, 564 (2006)
24. B. Moussalam, Ann. of Phys. (N.Y.) 225, 264 (1993)
25. F. Meier, H. Walliser, Phys. Rept. 289, 383 (1997); H. Walliser, Phys.Lett. B432, 15 (1998)
26. V. Kopeliovich and I. Potashnikova, Phys.Rev. C83, 064302 (2011)
27. Y. Akaishi and T. Yamazaki, Nucl. Phys. A684, 409 (2001); Phys. Rev. C65, 044005 (2002)
28. T. Yamazaki and Y. Akaishi, Phys. Rev. C76:045201 (2007); Y. Akaishi, A. Dote and T. Yamazaki, Phys. Lett. B613, 140 (2005)
29. M. Agnello et al (FINUDA Collab.) Phys.Rev.Lett. 94, 212303 (2005)
30. T. Yamazaki et al (DISTO Collab.), Phys.Rev.Lett. 104, 132502 (2010); M. Maggiora et al, Nucl.Phys. A835, 43 (2010)
31. V.B. Kopeliovich and W.J. Zakrzewski, JETP Lett. 69, 721 (1999); Eur.Phys.J. C18, 369 (2000)
32. V.B. Kopeliovich, J.Exp.Theor.Phys. 93, 435 (2001); JETP Lett. 67, 896 (1998)
33. V.B. Kopeliovich and A.M. Shunderuk, J.Exp.Theor.Phys.100, 929 (2005)
34. C. Houghton, N. Manton, P. Sutcliffe, Nucl. Phys. B510, 507 (1998)
35. Atom. Data Nucl. Data Tables 59, 185 (1995)
36. A.A. Korsheninnikov et al, Phys.Rev.Lett. 87, 092501 (2001)
37. A.A. Korsheninnikov et al, Phys.Rev.Lett. 90, 082501 (2003)