Stabilization of Structure-Preserving Power Networks with Market Dynamics

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Abstract: This paper studies the problem of maximizing the social welfare while stabilizing both the physical power network as well as the market dynamics. For the physical power grid a third-order structure-preserving model is considered involving both frequency and voltage dynamics. By applying the primal-dual gradient method to the social welfare problem, a distributed dynamic pricing algorithm in port-Hamiltonian form is obtained. After interconnection with the physical system a closed-loop port-Hamiltonian system of differential-algebraic equations is obtained, whose properties are exploited to prove local asymptotic stability of the optimal point.

Keywords: electric power systems, Lyapunov stability, distributed control, nonlinear systems, optimal power flow, gradient method, frequency regulation, passivity, dynamic pricing.

1. INTRODUCTION

The future power network needs to operate reliably in the face of fluctuations resulting from distributed energy resources and the increased variability in both supply and demand. One of the feedback mechanisms that have been identified for managing this challenge is the use of real-time dynamic pricing. This feedback mechanism encourages consumers to modify their demand when it is difficult for system operator to achieve a balance between supply and demand (Borenstein et al., 2002). In addition, real-time dynamic pricing allows to maximize the total social welfare by fairly sharing utilities and costs associated with the generation and consumption of energy among the different control areas (Kiani and Annaswamy, 2010).

Many of the existing dynamic pricing algorithms focus on the economic part of optimal supply-demand matching (Kiani and Annaswamy, 2010; Roozbehani et al., 2010). However, if market mechanisms are used to determine the optimal power dispatch (with near real-time updates of the dispatch commands) dynamic coupling occurs between the market update process and the physical response of the power network dynamics (Alvarado et al., 2001).

Consequently, under the assumption of market-based dispatch, it is essential to consider the stability of the coupled system incorporating both market operation and electromechanical power system dynamics simultaneously. While on this subject a vast literature is already available, we focus on a more accurate and higher order model for the physical power network than conventionally used in the literature. In particular, a structure-preserving model for the power network with a third-order order model for the synchronous generators including voltage dynamics is used. As a result, market dynamics, frequency dynamics and voltage dynamics are considered simultaneously.

1.1 Literature review

The coupling between a high-order dynamic structure-preserving power network and market dynamics has been studied before in Alvarado et al. (2001). Here a fourth-order model of the synchronous generator is used in conjunction with turbine and exciter dynamics, which is coupled to a simple model describing the market dynamics. The results established in Alvarado et al. (2001) are based on an eigenvalue analysis of the linearized system.

It is shown in Trip et al. (2016) that the third-order (flux-decay) model for the synchronous generator, as used in the present paper, admits a useful passivity property that allows for a rigorous stability analysis of the interconnection with optimal power dispatch controllers, even in the presence of time-varying demand. In Trip and De Persis (2015) a structure-preserving power network model is considered with turbine dynamics where a similar internal-model controller is applied, which also has applications in microgrids, see De Persis et al. (2016).

Another commonly used approach to design optimal distributed controllers in power grids is the use of the primal-dual gradient algorithm (Arrow et al., 1958), which has
been proven useful in network flow theory (Feijer and Pagani, 2010). The problem formulation varies throughout the literature on power systems, with the focus being on either the generation side (Li et al., 2014; Seungil and Lijun, 2014), the load side (Mallada and Low, 2014; Zhao et al., 2015; Mallada et al., 2014) or both (Zhang et al., 2015; Zhang and Papachristodoulou, 2015).

Many of these references focus on linear power system models coupled with gradient-method-based controllers (Li et al., 2014; Seungil and Lijun, 2014; Mallada and Low, 2014; Mallada et al., 2014; Zhang et al., 2015; Zhao et al., 2014). In these references the property that the linear power system dynamics can be formulated as a gradient method applied to a certain optimization problem is exploited. This is commonly referred to as reverse-engineering of the power system dynamics (Zhang et al., 2015; Li et al., 2014; Seungil and Lijun, 2014). However, this approach falls short in dealing with models involving nonlinear power flows.

Nevertheless, Zhang and Papachristodoulou (2015); Zhao et al. (2015) show the possibility to achieve optimal power dispatch in structure-preserving power networks with nonlinear power flows using gradient-method-based controllers. On the other hand, the controllers proposed in Zhang and Papachristodoulou (2015) have restrictions in assigning the controller parameters and in addition require that the topology of the physical network is a tree.

1.2 Main contributions

The contribution of this paper is to propose a novel energy-based approach to the problem that differs substantially from the aforementioned works. We proceed along the lines of Stegink et al. (2015, 2016), where a port-Hamiltonian approach to the design of gradient-method-based controllers in power networks is proposed. In those papers it is shown that both the power network as well as the controller designs admit a port-Hamiltonian representation which are then interconnected to obtain a closed-loop port-Hamiltonian system. In the present paper we extend some of these results to structure-preserving power networks.

First it is shown that the dynamical model describing the power network as well as the market dynamics admit a port-Hamiltonian representation. Then, following Stegink et al. (2015, 2016), it is proven that all the trajectories of the coupled system converge to the desired synchronous solution and to optimal power dispatch.

Since our approach is based on passivity and does not require to reverse-engineer the power system dynamics as a primal-dual gradient dynamics, it allows to deal with more complex nonlinear models of the power network. More specifically, the physical model for describing the power network in this paper admits nonlinear power flows and time-varying voltages, and is more accurate and reliable than the classical second-order model (Machowski et al., 2008; Kundur, 1993; Sauer and Pai, 1998; Bergen and Hill, 1981). In addition, a distinction is made between generator nodes and loads nodes, resulting in a system of differential-algebraic equations.

The results that are established in the present paper are valid for the case of nonlinear power flows and cyclic networks, in contrast to Zhang et al. (2015); Li et al. (2014); Seungil and Lijun (2014); Zhao et al. (2014), where the power flows are linearized and Zhang and Papachristodoulou (2015) where the physical network topology is a tree. Moreover, in the aforementioned references the voltages are assumed to be constant.

While the third-order model for the synchronous generators has been studied before using passivity based techniques (Trip et al., 2016; Stegink et al., 2016), the combination of gradient method based controllers with structure-preserving power network models is novel. In addition, the stability analysis does not rely on linearization and is based on energy functions which allow us to establish rigorous stability results. Moreover, we do not impose any restrictive condition on controller design parameters for guaranteeing asymptotic stability, contrary to Zhang and Papachristodoulou (2015).

The remainder of this paper is organized as follows. In Section 2 the preliminaries are stated. Thereafter, the power system dynamics is introduced in Section 3 and a port-Hamiltonian representation of the system of differential-algebraic equations is given as well in this section. Then the dynamic pricing algorithm in port-Hamiltonian form is presented in Section 4. The closed-loop system is analyzed in Section 5 and local asymptotic stability of the optimal points is proven. Finally, the conclusions and the future research directions are discussed in Section 6.

2. PRELIMINARIES

2.1 Notation

Given a symmetric matrix $A \in \mathbb{R}^{n \times n}$, we write $A > 0$ ($A \geq 0$) to indicate that $A$ is a positive (semi-)definite matrix. The set of positive real numbers is denoted by $\mathbb{R}_{>0}$ and likewise the set of nonnegative real numbers is denoted by $\mathbb{R}_{\geq0}$. The notation $I_n \in \mathbb{R}^{n \times n}$ is used for the vector whose elements are equal to 1. The $n \times n$ identity matrix is denoted by $I_n$. Given an ordered set $\mathcal{I} = \{i_1, i_2, \ldots, i_k\}$ and a vector $v \in \mathbb{R}^k, k \leq n$, then $\text{col}_{i \in \mathcal{I}}(v_i)$ denotes the $k$-column vector, respectively $k \times k$ diagonal matrix whose entries are given by $v_{i_1}, v_{i_2}, \ldots, v_{i_k}$. Likewise, given vectors $v_1, v_2$ then $\text{col}(v_1, v_2) := [v_1; v_2]$. Let $f(x, y)$ be a differentiable function of $x \in \mathbb{R}^n, y \in \mathbb{R}^m$, then $\nabla f := \text{col}(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})$ and $\nabla_x f := \frac{\partial f}{\partial x}$ denotes the gradient of $f$ with respect to $x$. Given a twice-differentiable function $f : \mathbb{R}^n \to \mathbb{R}^n$ then the Hessian of $f$ evaluated at $x$ is denoted by $\nabla^2 f(x)$.

2.2 Differential algebraic equations

Let us consider a system of differential-algebraic equations (DAE’s) of the form

$$\begin{align*}
\dot{x} &= f(x, y), \\
0 &= g(x, y),
\end{align*}$$

where $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$.

Definition 1. (De Persis et al. (2016)). Let $\mathcal{D} \subset \mathbb{R}^n \times \mathbb{R}^m$ be an open connected set. The algebraic equation $0 = g(x, y)$ is regular if the Jacobian of $g$ w.r.t. $y$ has constant full rank on $\mathcal{D}$, that is,

$$\text{rank} (\nabla_y g(x, y)) = m \quad \forall (x, y) \in \mathcal{D}. $$
If the DAE-system (1) is regular on $\mathcal{D}$ then by Hill and Mareels. (1993) the existence and uniqueness of solutions of (1) in $\mathcal{D}$ over an interval $\mathcal{I} \subseteq \mathbb{R}_{\geq 0}$ for any $(x(x_0, y_0, t), y(x_0, y_0, t))$ is guaranteed.

By extending the usual LaSalle’s invariance principle for ordinary differential equations, we obtain an invariance principle that can be used for the stability analysis of DAE’s, see De Persis et al. (2016).

**Theorem 2.** Suppose the DAE (1) is regular on $\mathcal{D}$ and $f, g$ are continuous differentiable functions. Let $(x, y) = (\bar{x}, \bar{y})$ be an equilibrium of (1). Let $V(x, y) : \mathcal{D}_V \to \mathbb{R}_{\geq 0}$ be a smooth positive definite function on a neighborhood $\mathcal{D}_V \subseteq (x, y)$ such that $V(x, y) \leq 0$. Let $S = \{(x, y) \in \mathcal{D}_V | \dot{V} = 0\}$, and suppose that no solution can stay forever in $S$, other than the trivial solution $(\bar{x}, \bar{y})$. Then $(\bar{x}, \bar{y})$ is locally asymptotically stable.

### 3. POWER NETWORK MODEL

Consider a power grid consisting of $n$ buses. The network is represented by a connected and undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Its associated node set, $\mathcal{V} = \{1, \ldots, n\} = \mathcal{V}_g \cup \mathcal{V}_l$, is partitioned in the set of generator nodes $\mathcal{V}_g$ with cardinality $n_g$, and the set of load nodes $\mathcal{V}_l$ with cardinality $n_l$. The set of edges, $\mathcal{E} = \{1, \ldots, m\} \subseteq \mathcal{V} \times \mathcal{V}$, corresponds to the set of transmission lines connecting the buses. Each bus represents either a synchronous generator or a frequency-dependent load (Bergen and Hill, 1981). It is assumed that the synchronous generators are governed by a smooth positive definite function on a neighborhood $\mathcal{D}_V \subseteq (x, y)$, such that $V(x, y) \leq 0$. Let $\mathcal{S} = \{(x, y) \in \mathcal{D}_V | \dot{V} = 0\}$, and suppose that no solution can stay forever in $\mathcal{S}$, other than the trivial solution $(\bar{x}, \bar{y})$. Then $(\bar{x}, \bar{y})$ is locally asymptotically stable.

#### Assumption 3.** By using the power network model (2) the following assumptions are made, where most of them are standard in a broad range of literature on power network dynamics (Machowski et al., 2008).

- Lines are purely inductive, i.e., the conductance is zero. This assumption is generally valid for the case of high voltage lines connecting different control areas.
- The grid is operating around the synchronous frequency, say 50 Hz or 60 Hz.

#### Definition 4.** Set of buses connected to bus $i$.

#### Assumption 3.** By using the power network model (2) the following assumptions are made, where most of them are standard in a broad range of literature on power network dynamics (Machowski et al., 2008; Trip et al., 2016). The voltages satisfy $E_i > 0, i \in \mathcal{V}$ for all time $t \geq 0$ and the reactive powers at the loads $Q_i^* \geq 0, i \in \mathcal{V}_l$ are constant.

- The excitation voltage $E_{fi}$ is constant for all $i \in \mathcal{V}$. Define the angular momenta $p_i = M_i \omega_i, i \in \mathcal{V}_g$. Based on the energy stored in the generators and the transmission network, the Hamiltonian is defined by $H_p = \frac{1}{2} \sum_{i \in \mathcal{V}_g} \left( M_i^{-1} p_i^2 + (E_i - E_{fi})^2 \right) / X_{di}^2$.

#### Table 1. Parameters and state variables of model (2).

| Parameter | Description |
|-----------|-------------|
| $\delta_i$ | voltage angle |
| $\omega_i$ | frequency deviation w.r.t. nominal frequency |
| $E_i$ | transient internal voltage |
| $E_{fi}$ | excitation voltage |
| $P_{gi}$ | active power generation and demand |
| $Q_{gi}$ | reactive power generation and demand |
| $M_i$ | moment of inertia |
| $N_i$ | set of buses connected to bus $i$ |
| $A_i$ | asynchronous damping constant |
| $B_{ij}$ | negative of the susceptance of transmission line $(i, j)$ |
| $X_{di}$ | d-axis synchronous reactance of generator $i$ |
| $X_{di}^*$ | d-axis transient reactance of generator $i$ |
| $T_i$ | open-circuit transient time constant |

Using this modified set of coordinates, and defining the state variable $x_p := \text{col}(\varphi, p_g, E_g, E_l)$, the system (2) can be written in the form $W_p(x_p, \omega_l) = H_p(x_p) + U_p(\omega_l)$, $U_p(\omega_l) = \frac{1}{2} \omega_l^T \omega_l$.
col_{i \in V} \{\omega_i\} and \(E_g, P_g, P_i, p_g\) are defined likewise. The system (5) has external ports \((P_g, \omega_g), (P_i, \omega_i)\) which will be interconnected to the dynamic pricing algorithm introduced in the following section.

Remark 5. The system (5) has a slightly different form compared to conventional port-Hamiltonian DAE-systems, see van der Schaft and Jeltsema (2014). In particular, \(H_g\) is the Hamiltonian of the system while \(U_g\) can be interpreted as an auxiliary energy function which is not used as part of the (shifted) storage function to prove passivity. However, by exploiting the special structure of the system (5), the stability analysis becomes convenient as we will show in Section 5.

4. DYNAMIC PRICING ALGORITHM

The social welfare is defined as \(S(P_g, P_i) := U(P_i) - C(P_g)\), which consists of a utility function \(U(P_i)\) of the power consumption \(P_i\) and the cost \(C(P_g)\) associated to the power production \(P_g\). We assume that \(C(P_g), U(P_i)\) are strictly convex and strictly concave functions respectively. The objective is to maximize the social welfare while achieving zero frequency deviation. By analyzing the equilibrium of (2), it follows that a necessary condition for zero frequency deviation is \(\Omega^T P_g = \Omega^T P_i\). In other words, the total supply must match the total demand. It can be noted that \(P_g, P_i\) satisfy this power balance if and only if there exists a vector \(v \in \mathbb{R}^{m_c}\) such that

\[
- \left[ \begin{array}{c} D_{cg} \vspace{1mm} \\ D_{cl} \end{array} \right] v + \left[ \begin{array}{c} P_g \\ -P_i \end{array} \right] = 0.
\]

where \(D_c \in \mathbb{R}^{n \times m_c}\) is the incidence matrix of some connected communication graph with \(m_c\) edges.

\[
\max_{P_g, P_i, v} U(P_i) - C(P_g)
\]

s.t. \[
- \left[ \begin{array}{c} D_{cg} \\ D_{cl} \end{array} \right] v + \left[ \begin{array}{c} P_g \\ -P_i \end{array} \right] = 0.
\]

The corresponding KKT optimality conditions amount to

\[
\begin{align*}
\nabla C(P_g) - \lambda_g & = 0 \\
-\nabla U(P_i) + \lambda_i & = 0 \\
[D_{cg}^T D_{cl}^T] \lambda_g & = 0 \\
[D_{cg}^T D_{cl}^T] \lambda_l & = 0
\end{align*}
\]

(7)

An important observation is that the dynamic pricing algorithm (8) can be written in port-Hamiltonian form as

\[
x_c = \left[ \begin{array}{c} 0 & 0 & 0 & I_{n_g} & 0 \\ 0 & 0 & 0 & 0 & -I_{n_i} \\ -I_{n_g} & 0 & D_{cg} & 0 & 0 \\ 0 & I_{n_i} & D_{cl} & 0 & 0 \end{array} \right] \lambda_g \\
\n+ \nabla S(\tau_c^{-1} x_c), \quad H_c = \frac{1}{2} x_c^T \tau_c^{-1} x_c
\]

with

\[
\begin{align*}
x_g & = \left[ \begin{array}{c} \tau_g \\ \tau_d \\ \tau_v \\ \tau_{\lambda_g} \end{array} \right] \\
x_v & = \left[ \begin{array}{c} 0 \\ \tau_i \\ \tau_e \\ \tau_{\lambda_l} \end{array} \right] \\
x_{\lambda_g} & = \left[ \begin{array}{c} \tau_{\lambda_g} \\ \tau_{\lambda_l} \end{array} \right]
\end{align*}
\]

Since \(S\) is a concave function it satisfies the following dissipation inequality

\[
(x_c - \bar{z}_c)^T (\nabla S(z_c) - \nabla S(\bar{z}_c)) \leq 0
\]

for all \(z_c, \bar{z}_c \in \mathbb{R}^{2n+m_c}\). This property implies that the system (9) is passive with respect to its steady states, see also Stegink et al. (2016).

5. STABILITY OF THE CLOSED-LOOP SYSTEM

It is observed that, by construction of the dynamic pricing algorithm, there is two-way coupling between the physical power network (2) and the market dynamics (8) through the ports \((P_g, \omega_g), (P_i, \omega_i)\). In fact, the interconnection between (2) and (8) is power-preserving. As a result, the closed-loop system, obtained by combining the systems (5) and (9), takes the form

\[
\begin{align*}
\tau_g \dot{P}_g & = -\nabla C(P_g) + \lambda_g - \omega_g \\
\tau_d \dot{P}_d & = \nabla U(P_i) - \lambda_i + \omega_i \\
\tau_v \dot{v} & = -D_{cg} \lambda_g - D_{cl} \lambda_l \\
\tau_{\lambda_g} \dot{\lambda}_g & = D_{cg} v - P_g \\
\tau_{\lambda_l} \dot{\lambda}_l & = D_{cl} v + P_i
\end{align*}
\]

where \(\tau_g, \tau_d, \tau_v, \tau_{\lambda_g}, \tau_{\lambda_l} > 0\) are (controller design) parameters. Observe that the dynamics (8) has a clear economic interpretation (Kiani and Annaswamy, 2010; Alvarado et al., 2001): each power producer aims at maximizing their own profit which, under the assumption of perfect competition, occurs whenever their individual marginal cost equals the local price \(\lambda_{gi} - \omega_{gi}\), which depends on the local frequency \(\omega_{gi}\) of the physical network. At the same time, each consumer maximizes its own utility but is penalized by the local price \(\lambda_{li} - \omega_{li}\).

Remark 6. The idea to use the frequency deviation in the pricing mechanism stems from our previous work (Stegink et al., 2016), and helps to compensate for the power supply-demand imbalance. Moreover, it allows for a power-preserving interconnection with the physical model (2).
Next, we examine the equilibria of the coupled system (10). Theorem 7. (12), see next page. After elimination of the algebraic variable \( \phi \) and suitable Lyapunov function. in De Persis and Monshizadeh (2015) a sufficient condition for the model-

Proposition 9. Suppose that \( \bar{x} \) satisfies

\[
\frac{1}{X_{d_i} - X_{d_i}^2} + B_{ii} - \sum_{j \in N_i} B_{ij} \bar{E}_i + \bar{E}_i \sin^2 \delta_{ij} > 0, \quad i \in V_g
\]

\[
\frac{1}{X_{d_i} - X_{d_i}^2} + B_{ii} - \sum_{j \in N_i} B_{ij} \bar{E}_i + \bar{E}_i \cos \delta_{ij} > 0, \quad i \in V_l
\]

with \( \delta_{ij} \in (-\pi/2, \pi/2), \forall (i,j) \in E \), and \( \bar{E}_i > 0, \forall i \in V \). Then \( \nabla^2 \hat{H}(\bar{x}) > 0 \).

Remark 8. While the communication graph is assumed to be a tree in Theorem 7, the convergence result can be extended to the case of general (cyclic) connected communication graphs. However, due to space constraints, this is beyond the scope of the paper.

In this paper an energy-based approach to the modeling and stability analysis of structure-preserving power networks with markets dynamics has been established. In particular, local convergence of the coupled system of differential-algebraic equations to the set of optimal points of the social welfare problem have been proven using a suitable Lyapunov function.

A possible extension to the established results is to consider the more complex case that the loads are not frequency dependent. Another direction for future research is
to design additional controllers for the physical power network that achieve optimal reactive power sharing and/or voltage regulation. In addition, an extension can be made to include generator limits and line congestion. Finally, the influence of a possible delay in the communication of the dynamic pricing algorithm on the stability of the closed-loop has to be investigated.

REFERENCES

Alvarado, F., Meng, J., DeMarco, C., and Mota, W. (2001). Stability analysis of interconnected power systems coupled with market dynamics. IEEE Transactions on Power Systems, 16(4), 695–701.

Arrow, J., Hurwicz, L., Uzawa, H., and Chenery, H. (1958). Studies in linear and non-linear programming. Stanford University Press.

Bergen, A. and Hill, D. (1981). Structure preserving model for power system stability analysis. IEEE Transaction on Power Apparatus and Systems, PAS-100(1), 25–35.

Borenstein, S., Jaske, M., and Rosenfeld, A. (2002). Dynamic pricing, advanced metering, and demand response in electricity markets. Center for the Study of Energy Markets.

De Persis, C. and Monshizadeh, N. (2015). Bregman storage functions for microgrid control. arXiv preprint arXiv:1510.05811. Submitted to IEEE Transactions on Automatic Control.

De Persis, C., Monshizadeh, N., Schiffer, J., and Dörfler, F. (2016). A Lyapunov approach to control of microgrids with a network-preserved differential-algebraic model. In IEEE 55th Conference on Decision and Control (CDC), 2595–2600.

Dörfler, F., Simpson-Porco, J., and Bullo, F. (2014). Breaking the hierarchy: Distributed control & economic optimality in microgrids. arXiv preprint arXiv:1401.1767.

Feijer, D. and Paganini, F. (2010). Stability of primal–dual gradient dynamics and applications to network optimization. Automatica, 46, 1974–1981.

Hill, D.J. and Mareels, I.M. (1993). Stability theory for differential/algebraic systems with application to power systems. Sadhana, 18(5), 731–747.

Kiani, A. and Annaswamy, A. (2010). The effect of a smart meter on congestion and stability in a power market. In 49th IEEE Conference on Decision and Control. Atlanta, USA.

Kundur, P. (1993). Power System Stability and Control. Mc-Graw-Hill Engineering.

Li, N., Chen, L., Zhao, C., and Low, S.H. (2014). Connecting automatic generation control and economic dispatch from an optimization view. In American Control Conference, 735–740. IEEE.

Machowski, J., Bialek, J., and Bumby, J. (2008). Power System Dynamics: Stability and Control. John Wiley & Sons, Ltd, second edition.

Mallada, E. and Low, S. (2014). Distributed frequency-preserving optimal load control. In IFAC World Congress.

Mallada, E., Zhao, C., and Low, S. (2014). Optimal load-side control for frequency regulation in smart grids. In Communication, Control, and Computing (Allerton), 2014 52nd Annual Allerton Conference on, 731–738. IEEE.

Roozbehani, M., Dahleh, M., and Mitter, S. (2010). On the stability of wholesale electricity markets under real-time pricing. In 49th IEEE Conference on Decision and Control (CDC), 1911–1918.

Sauer, P. and Pai, M. (1998). Power system dynamics and stability. Prentice-Hall.

Seungil, Y. and Lijun, C. (2014). Reverse and forward engineering of frequency control in power networks. In Proc. of IEEE Conference on Decision and Control, Los Angeles, CA, USA.

Stegink, T.W., De Persis, C., and van der Schaft, A.J. (2015). Port-Hamiltonian formulation of the gradient method applied to smart grids. IFAC-PapersOnLine, 48(13), 13–18.

Stegink, T., De Persis, C., and van der Schaft, A. (2016). A unifying energy-based approach to stability of power grids with market dynamics. IEEE Transactions on Automatic Control. doi:10.1109/TAC.2016.2613901.

Trip, S., Bürger, M., and De Persis, C. (2016). An internal model approach to (optimal) frequency regulation in power grids with time-varying voltages. Automatica, 64, 240–253.

Trip, S. and De Persis, C. (2015). Optimal frequency regulation in nonlinear structure preserving power networks including turbine dynamics: an incremental passivity approach. arXiv preprint arXiv:1509.07617.

van der Schaft, A. and Jeltsema, D. (2014). Port-Hamiltonian systems theory: An introductory overview. Foundations and Trends in Systems and Control, 1(2-3), 173–378.

Willems, J. (1971). Direct method for transient stability studies in power system analysis. IEEE Transactions on Automatic Control, 16(4), 332–341.

Zhang, X. and Papachristodoulou, A. (2015). A real-time control framework for smart power networks: Design methodology and stability. Automatica, 58, 43–50.

Zhang, X., Li, N., and Papachristodoulou, A. (2015). Achieving real-time economic dispatch in power networks via a saddle point design approach. In Power & Energy Society General Meeting, 2015, 1–5. IEEE.

Zhao, C., Mallada, E., and Low, S. (2015). Distributed generator and load-side secondary frequency control in power networks. In 49th Annual Conference on Information Sciences and Systems (CISS), 1–6. IEEE.

Zhao, C., Topcu, U., Li, N., and Low, S. (2014). Design and stability of load-side primary frequency control in power systems. Automatic Control, IEEE Transactions on, 59(5), 1177–1189.