Structure and evolution of low-mass W UMa type systems – II. with angular momentum loss

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ABSTRACT
In a preceding paper, using Eggleton’s evolution code we have discussed the structure and evolution of low-mass W UMa type contact binaries without angular momentum loss (AML). The models exhibit cyclic behavior about a state of marginal contact on a thermal time-scale. Part of the time of each cycle is spent in contact and part in a semi-detached state. According to observations, W UMa systems suffer AML. We present the models of low-mass contact binaries with AML due to gravitational wave radiation (GR) or magnetic stellar wind (MSW) are presented. We find that gravitational radiation cannot prevent the cyclic evolution of W UMa systems, and the effect of gravitational radiation on the cyclic behavior of contact binary evolution is almost negligible. We also find that the most likely AML mechanism for W UMa systems is magnetic braking, and that magnetic braking effects can increase the period of the cyclic evolution, and shorten the fraction of the time spent in the poor thermal contact state exhibiting EB light curve. If W UMa stars do not undergo cyclic evolution, and their angular momentum loss is caused simultaneously by MSW of both components, we find that the value of the parameter, $\lambda$, should be taken a larger value in comparison with those derived from observations of single stars. This indicates that the AML efficiency in W UMa systems may be lowered in comparison with non-contact stars because of the less mass contained in the convective envelopes of the components in W UMa systems. If W UMa systems lose their angular momentum at a constant rate, an angular momentum rate of $\frac{d\ln J}{dt} \approx 1.6 \times 10^{-9} \text{yr}^{-1}$ can prevent the cyclic behaviour of the model, and the model can keep in good contact with an essentially constant depth of contact.

Key words: stars: binaries: close–stars: rotation-stars: activity–stars: evolution

1 INTRODUCTION
In a preceding paper (Li et al. 2004, hereafter Paper I), the structure and evolution of low-mass W UMa systems have been discussed by us using the stellar evolution code developed by Eggleton (1971, 1972, 1973), with updated physics (Pols et al. 1995). Our models for low-mass W UMa systems were based on the conservative assumption, namely that the total mass of the system and its orbital angular momentum remain preserved. As a result, our models exhibit cyclic behavior about a state of marginal contact with a period of about $10^7$ yr. Part of the cycle is spent in contact and part in the semi-detached state. In any cycle, a larger temperature difference ($\Delta T_{\text{diff}} > 300$K) between the two components occurs in a part of a cycle (about 30-35 per cent time of a cycle). So most previous investigators had thought that this requires there to be as many short-period binaries with EB light curves as with EW light curves, and that the TRO models for W UMa type systems cannot explain the light curve paradox according to the investigation of Lucy & Wilson (1979). Ruciński (2002) finds 13 EWs and 5 EBs [and 14 ellipsoidal (ELL) variables, which have too small an amplitude to be classified as EWs or EBs]. It is reasonable to identify the EWs as contact binaries, and the EBs as semi-detached. The ratio of 5/13 is not much out of line with TRO theory (Lucy 1976; Flannery 1976; Robertson & Eggleton 1977) indicating that zero-age contact systems are not thermally stable, but suffer cyclic, thermally unstable mass transfer.

The TRO models for W UMa systems (Rahunen 1981, see also Paper I) has shown that the predicted ratio of the timescales characterizing the phases in which the oscillating systems will exhibit W UMa-type (EW) and $\beta$ Lyrae-type (EB) light curves is $\tau_{\text{EW}}/\tau_{\text{EB}} \leq 2$ (in the mass
ratio range \(0.5 \leq q \leq 0.7\). But the observations give \(\tau_{\text{EW/GR}} \approx 5\) to 6, suggesting that W UMa systems probably suffer angular momentum loss. It is well known that loss of angular momentum from the binaries consisting of two point masses via the emission of gravitational wave radiation (GR) causes the separation between the components to decrease. The possible importance of gravitational radiation for short period binaries was first pointed out by Kraft, Mathews, & Greenstein (1963). In the evolution of W UMa systems, it has been discussed by Faulkner (1971) and Paczyński (1967). Huang (1967) suggested that magnetic torques could bring together the separate components of a detached binary. This is Schatzman’s (1962) and Mestel’s (1968) mechanism of magnetic braking. If the rapid rotation leads to strong dynamos in the convective zones of the solar stars in W UMa systems, effective magnetic braking may result and the angular momentum loss by magnetic stellar wind (MSW) may control the contact binary evolution (Van’t Veer 1979). Meanwhile, there is increasing evidence of strong magnetic activity in short-period binaries, including W UMa stars (spots, flares, strong chromospheres and coronae, etc.). This suggests indirectly that W UMa systems suffer angular momentum loss via MSW. Okamoto & Sato (1970) have considered the formation of W UMa stars by magnetic braking in more detail. Van’t Veer (1979, 1981) has further developed this idea. The angular momentum loss in contact binaries has also been treated by Moss (1972), Webbink (1977), Robertson & Eggleton (1977), Rahnenführer (1981) and Villu (1982).

The observational data and theory of contact binaries have been reviewed in three extensive papers by Mochanand, Villu (1981) and Smith (1984). Currently the prevailing opinion is that the AML phenomenon is either the main property of or an ingredient in the proper explanation of the existence and evolution of contact binaries of late spectral types (Van’t Veer 1978, Rucin ski 1982). There is ample evidence that the W UMa-type systems possess all the chromospheric and coronal phenomena normally (i.e. in the solar analogue) related to the existence of magnetic fields. Robertson & Eggleton (1977) have found that a steady loss of angular momentum drives the system towards smaller mass ratios. Rahnenführer (1981) has found that it is possible to keep zero-age models in good thermal contact if the orbital angular momentum of the binary is allowed to decrease and the timescale of the required angular momentum loss is about \(5 \times 10^8\) yr.

In this paper, we restrict our attention to the general nature of the effects of angular momentum loss (i.e. \(J < 0\)) and to the influence of the matter accreted by the secondary on its evolution in a semi-detached state and present models of low-mass W UMa systems with angular momentum loss via gravitational radiation or magnetic braking. We find that the influence of gravitational radiation on the cyclic evolution of contact binaries is almost negligible because the gravitational radiation life-time is long compared with the thermal timescale of the primary of our models. Systemic angular momentum loss via gravitational radiation only accelerates the evolution towards more extreme mass ratios, rather than leading to orbital collapse. The most likely angular momentum loss mechanism for W UMa systems is magnetic braking. It can increase the period of the cyclic evolution and shorten the fraction of the time spent in the state exhibiting EB light curves, and a suitable angular momentum loss rate can prevent the cyclic behaviour of the model. Meanwhile, if we consider an energy source at secondary’s atmosphere provided by the accreting matter from the primary in semi-detached evolution, and find that the expansion in radius of the secondary is hastened by the accreting matter which should have a higher entropy than the original star, it can shorten the time spent in the state exhibiting EB light curve. We compare the convective envelopes of a semi-detached model with those of a contact one which has the same total mass and mass ratio as the semi-detached one. We find that the convective envelope of the secondary in the contact system is much thinner than that in the semi-detached one and the convective envelope of the primary in the contact system is slightly thicker than that in semi-detached one. But the mass contained in the convective envelopes of two components of contact system, which is related to the dynamo action, is less than that contained in the envelopes of the semi-detached one. This is caused by the energy transfer between the two components in contact systems. As a result, W UMa systems show the lower activity in comparison with the non-contact systems.

2 CONTACT CONDITION AND LUMINOSITY TRANSFER

2.1 Contact condition

In Paper I, based on Roche geometry, we have given a radius grid of contact binaries. The radius grid gives the relative radii of both components of contact binaries with different mass ratios \((0.02, 0.04,...,1.0)\) and different depth of contact \((0.0, 0.025, 0.05,...,1.0)\) and it has been used as the surface condition of contact binaries to ensure the surfaces of both components lie on the same equipotential by interpolation. Here we use the same contact condition as Paper I. If the mass is transferred from star 2 to star 1, the rate of mass transfer is expressed as,

\[
\frac{dm}{dt} = -C \text{Max}[0, (\ln R_{20})^3]
\]

in which

\[
R_{20} = \frac{3}{4} \sqrt{\frac{g_1 R_2^3 + g_2 R_2^3}{g_1 R_1^3 + g_2 R_2^3}}.
\]

The explanations of Eqs. (1) and (2) are as in Paper I. If mass is transferred from star 1 to star 2, the rate of mass transfer can be derived by the same way.

2.2 Luminosity transfer

2.2.1 Luminosity transfer due to mass transfer

In the semi-detached phase, the mass above the critical lobe, together with the energy (including gravitational energy, heat energy, and radiative energy) in the transferred mass, is transferred to the secondary. The effect of heating the photosphere of a low-mass ZAMS stars (with a deep convective envelope or fully convective), by the kinetic energy of the infalling matter, is very important for their response (Prialnik & Livio 1985). Since the radiative energy is so small that it can be neglected relative to gravitational
energy and heat energy, the accretion luminosity due to accreting mass is
\[ L_{\text{acc}} = \beta (\psi_1 - \psi_2 + s_1 - s_2) M_2 \] (3)
where \( \psi_1 \) and \( s_1 \) are, respectively, the gravitational potential and specific entropy at the surface of the component \( i \), \( M_2 \) is the rate of mass accretion of the secondary and \( \beta \) is a coefficient which describes the efficiency of heating of the photosphere by the shock wave region.\(^\text{[Kähler 2002a]}\) took the parameter \( \beta \) to be 1.0, i.e. he thought that the bulk kinetic energy gained by falling through the potential is completely transformed into thermal energy and it, together with the thermal energy carried by infalling matter, is regarded as an energy source in the outermost layer of the gainer without energy loss. However, some of this energy may be dissipated dynamically in a shock, but the dissipated ratio of this energy is unknown, so we used a value of \( \beta = 0.5 \text{[Sarna & Fedorova 1989]} \). This suggests that 50 per cent of the transferred energy is lost from the system.

Since the accretion luminosity is caused by energy sources in the gainer’s outermost layers, we modify the surface boundary condition of the secondary to model the influence of the accretion luminosity on the evolution of the secondary, i.e. the surface temperature of the secondary may be approximated by the formula, \( L_{\text{in}} + L_{\text{acc}} = 4\pi R^2 \sigma T^4 \), where \( L_{\text{in}} \) is the luminosity coming to the photosphere from the stellar interior, \( R \) is the radius of the secondary, and \( \sigma \) is the Stefan-Boltzmann constant.

### 2.2.2 Luminosity transfer due to circulation currents in the common envelope

\textit{Struve (1948)} first recognized the unusual mass-luminosity relationship of the secondary components of W Ursae Majoris systems, i.e. the two components have identical temperatures and hence luminosities scaling as \( L \propto R^3 \propto M \) for Roche geometry in spite of differing masses. This Suggests that it might be causally related to a possible common envelope.\(^\text{[Osaki 1964]}\) noted that von Zeipel’s theorem would require the observed approximate constancy of radiative flux over the surface of a system with a radiative common envelope in order to maintain hydrostatic equilibrium. The fact that most W UMa systems appear to have convective envelopes, however, led \textit{Lucy (1968)} to propose that the anomalous mass-luminosity relationship possessed by W UMa binaries can be explained if energy transfer is assumed to take place between the two components of the systems well below the atmosphere.

A luminosity transfer by circulation in the common envelope is applied in the outermost layers which are close to the surface of each component. The amount of the luminosity transfer by circulation must satisfy the requirement of W UMa systems’ observations.\(^\text{[Robertson & Eggleton 1977]}\), it can be written as
\[ \frac{L_1 - \Delta L_0}{m_1} = \frac{L_2 + \Delta L_0}{m_2}, \] (4)
where \( L_{1,2} \) are the core luminosities (including nuclear and thermal luminosities) of both components, and \( m_{1,2} \) the masses of the components. Since transfer is not fully efficient at all phases, an arbitrary factor \( f \) is introduced, which varies through the cycle and goes to zero with the depth of contact. Thus we take:
\[ \Delta L = f \cdot \Delta L_0, \quad 0 \leq f \leq 1, \] (5)
where \( f \) is the efficient factor of energy transfer. We take
\[ f = \text{Min}(1, \alpha (d^2 - 1)) \] (6)
in which
\[ d = \text{Max}[1, \text{Min}(\frac{r_1}{R_{\text{crit}1}}, \frac{r_2}{R_{\text{crit}2}})] \] (7)
where \( r_{1,2} \) are the radii of both stars, \( R_{\text{crit}1,2} \) the Roche critical radii of both stars. The parameter \( \alpha \) is expected to be moderately large, so that heat transfer becomes fully efficient for stellar radii exceeding the Roche radii by some standard small amount. In this work, we take \( \alpha = 15 \).

The luminosity transferred by circulation currents from the primary to the secondary adopted by\(^\text{[Kähler 2002a]}\) is
\[ \Delta L = \int_0^{M_2} \sigma_{\text{ex},2} dm_2 = - \int_0^{M_1} \sigma_{\text{ex},1} dm_1, \] (8)
where \( \sigma_{\text{ex},1} \) is the source (when positive) or sink (when negative) of energy per unit of mass caused by interaction of the components. Although the luminosity increment is applied in the adiabatic portion of the envelope of each star by most of the previous investigators, according to the energy transfer model\(^\text{[Robertson 1980]}\) there is no essential distinction between convective and radiative envelopes. Meanwhile, Li et al. (Paper I) argued that the energy transfer may take place in the outermost layers for low-mass W-subtype systems and the transfer takes place in the base of the common envelope for A-subtype systems. It follows that convection is by no means essential to heat transport in contact envelopes, although it may well have an important influence. Therefore, we assume that the luminosity transfer takes place in the outermost layers of the common envelope for low-mass W UMa systems, and the luminosity increment is applied in the 10 meshpoints which are close to the surfaces of both components as Paper I.

### 3 GRAVITATIONAL RADIATION AND MAGNETIC BRAKING VIA MSW

It is well known that loss of angular momentum from the binary systems consisting of two point masses due to the emission of gravitational radiation causes the separation between the components to decrease.\(^\text{[Landau & Lifschitz 1962, Faulkner 1971, and Taam, Flannery, & Faulkner 1980]}\) showed that gravitational radiation, by removing orbital angular momentum, could stimulate mass transfer from the main sequence red component at a rate which causes the binary to evolve more rapidly than nuclear evolution would dictate.\(^\text{[Faulkner 1971, and Chau & Lauterborn 1977]}\) have both found very significant effects of \( J \) via gravitational radiation (GR) in their computations on the evolution of low-mass close binary systems.\(^\text{[Chau 1978]}\) has considered the effects of loss of orbital angular momentum via gravitational radiation on the period changes and the implications for mass-transfer rate determinations.

We adopt the standard Einstein theory which predicts that the rates of change of semi-major axis and orbital angular momentum are, respectively,\(^\text{[Landau & Lifschitz 1962]}\).
\[ \frac{dA}{dt} = -\frac{64G^3}{5c^5A^4} M_1 M_2 (M_1 + M_2) \]  

and

\[ \frac{dln J}{dt} = -\frac{32G^3}{5c^5} \frac{M_1 M_2 (M_1 + M_2)}{A^4}, \]

where \( J \) is the orbital angular momentum, \( A \) is the semimajor axis of the system, \( M_{1,2} \) are the masses of both components, \( G \) is the gravitational constant, \( c \) is the light velocity, and \( t \) is the time. The rate of angular momentum loss associated with gravitational radiation is also given by (Webbink 1973),

\[ \frac{dln J}{dt} = -8.1 \times 10^{-10} \frac{M_1 M_2 (M_1 + M_2)}{A^4} \text{ yr}^{-1} \]

where \( J \) is the orbital angular momentum in erg \( \cdot \) s, \( M_{1,2} \) are the masses of both components in units of solar masses \( M_\odot \), \( A \) is the semimajor axis of the system in units of solar radii \( R_\odot \), and \( t \) is the time in years.

Magnetic braking of the rotation of the cool stars in the binaries is an attractive mechanism. The idea that in stars with convective envelopes a dynamo is operating, which has efficiency that decreases with decreasing rotation rate, is consistent with observational data which indicate that the chromospheric (i.e. Ca II H and K) and coronal emissions are strongest in rapidly rotating stars (Zwaan 1981). The magnetic braking is presumably due to the stellar wind which streams out along the coronal magnetic field lines, which are rooted in the convection zone (Schatzman 1962, 1963). The loss of angular momentum per unit mass in the stellar wind is very large since the outflowing matter is forced by magnetic field to corotate with the star out to large distances. The magnetic stellar wind (MSW) as a possible driving force for the evolution of low-mass binaries was proposed first by Huang (1966). Observational support for the action of such a wind in low-mass spectroscopic binaries has been provided by Kraicheva et al. (1978). Theoretical studies of the effect of MSW on the evolution of close binaries consisting of a low-mass component with a convective envelope filling its Roche lobe and a compact accreting component that is a degenerate dwarf or a neutron star have been conducted by Verbunt & Zwaan (1983), Rappaport, Verbunt & Joss (1983), Taam (1983), Patterson (1984), Spruit & Ritter (1983), Tutukov (1984), and Sarna & Fedorova (1985), among others.

The most likely angular momentum loss mechanism for W UMa systems is magnetic braking. The observational evidence and theoretical reasons have been presented for believing that low-mass W UMa binaries can lose their orbital angular momentum efficiently as a consequence of a MSW from one or both components. The Einstein soft X-ray observations and IUE ultraviolet observations (Vairana 1980; Vilhu 1982; Eaton 1983; Rucinski & Vilhu 1984) showed that W UMa systems are strong sources. This suggests surface activity of the kind observed on our Sun, and so the presence of magnetic field. A magnetic stellar wind would cause braking. In the absence of an explicit theory for the angular momentum loss by MSW, we employ a Skumanich (1972) law, \( V_{\text{rot}} \approx 10^{14} A t^{-0.5} \), where \( V_{\text{rot}} \) is the rotating velocity in unit of cm s\(^{-1} \), \( t \) the age in seconds, and \( A \) a free parameter. Observations give \( \lambda = 0.73 \), according to Skumanich (1972), and \( \lambda = 1.78 \), according to Smith (1970). This estimates the rate of angular momentum loss by a low-mass rotating star in a binary system. Assuming that tidal coupling is perfect, so that the wind-emitting star rotates synchronously with the orbital period, and neglecting the spin angular momentum of each component relative to the orbital angular momentum, we can take into account the Tutukov (1983) and Rappaport, Verbunt & Joss (1983) formalism for the loss of angular momentum by a MSW in the following form

\[ \frac{dln J}{dt} = -9.6 \times 10^{-15} \frac{R_2^2 (M_1 + M_2)^2}{A^2 A^4 M_1} \text{ s}^{-1} \]

\[ = -3.03 \times 10^{-7} \frac{R_2^2 (M_1 + M_2)^2}{A^2 A^4 M_1} \text{ yr}^{-1} \]

where \( J \) is the orbital angular momentum of the system in erg \( \cdot \) s, \( M_{1,2} \) are the masses of both components in solar masses \( M_\odot \), \( R_2 \) is the radius of the donor in solar radii \( R_\odot \), and \( A \) is the semimajor axis of the system in solar radii \( R_\odot \).

So far we have only indirect evidence of the magnetic activity in W UMa-stars. Rapid period changes and disturbances in light curves can be interpreted as being caused by big flares and spots (Binnendijk 1971; Mullar 1973; Van’t Veer 1978).

4 EVOLUTION INTO CONTACT

4.1 The initial model

Our initial model consists of two zero-age main-sequence (ZAMS) stars of Population I (\( X = 0.70, Z = 0.02 \)) with a total mass of \( 1.8 M_\odot (1.23 + 0.57 M_\odot) \) and an initial mass ratio of 0.4634. The convective mixing length is twice the pressure scale height. We take an initial period of 0.3766 d and orbital angular momentum, \( J \), of 5.161 \( \times \) 10\(^{51} \) erg \( \cdot \) s \[ \log(J/\text{erg} \cdot \text{s}) = 51.713 \]. The initial separation between two components is about 1.8574 \( \times \) 10\(^{11} \) cm (i.e. 2.67\( R_\odot \)). The initial model is a detached binary, and the surface of the primary lies only a short way inside its Roche lobe which it fills after about 7.3 \( \times \) 10\(^{5} \) yr of nuclear evolution.

4.2 Contact evolution without angular momentum loss

4.2.1 Evolution without luminosity transfer due to mass transfer in semi-detached phase

Our models in Paper I exhibit cyclic behaviour on a thermal timescale, with a period of about 10\(^{5} \) yr. Part of the time of each cycle is spent in a contact phase and part in a semi-detached phase. At first we only consider the luminosity transfer by circulation in the common envelope under the assumption that no energy transfer due to mass transfer takes place in the semi-detached phase, the system evolves in thermal cycles without loss of contact. The changes of characteristic quantities (including the contact depth, surface temperature, central temperature, and central density of the primary and the secondary, together with the mass ratio and orbital period of the system) during the cycles are shown in Figure 1.

As seen from Figure 1, beginning at the initial model, the primary fills its Roche lobe after about 7.3 \( \times \) 10\(^{5} \) yr of nuclear evolution. It evolves into a semi-detached state and it loses mass to its companion. Since the addition mass to the secondary causes it to expand, and its effective temperature and luminosity to increase, after a total of about 1.5 \( \times \) 10\(^{7} \) yr,
the secondary has swollen to fill its Roche lobe, so that the system evolves into a contact system. The contact binary formed at this point has masses of 1.0 and 0.8$M_\odot$, and a mass ratio of about 0.8. Thereafter, the model exhibits cyclic evolution on a thermal timescale (about $6 \times 10^7$ yr) without loss of contact. Part of time of each cycle is spent in good thermal contact, with EW light curves and part in poor thermal contact, with EB light curves. As seen from Figure 1b, the ratio of the timescales characterizing the phases in which the oscillating systems exhibit W UMa-type (EW) and $\beta$ Lyrae-type (EB) light curves is $\tau_{\text{EW}}/\tau_{\text{EB}} < 1$.

It is seen in Figures 1c and d that the central temperatures and central densities of both components of the binary exhibit cyclic evolution. This causes the cyclic evolution of the core luminosities of both components of the binary.

4.2.2 Evolution with luminosity transfer due to mass transfer in semi-detached phase

We now consider the evolution of the binary system from the initial model with luminosity transfer due to mass transfer included in the manner described in Sect. 2.2.1. The system also evolves in thermal cycles without loss of contact. The evolution of some characteristic quantities is shown in Figure 2. It is seen in Figure 2 that the system evolves into a contact binary from a detached one after a total of about $1.35 \times 10^7$ yr of evolution from the main sequence and the contact binary formed at this time has masses of about 1.045 and 0.755$M_\odot$ with a mass ratio of about 0.72. So this model spent shorter time in the semi-detached phase than the model in which the luminosity transfer caused by mass transfer had been neglected in the semi-detached state. This is because the energy source at the secondary’s atmosphere, provided by accreting mass from the primary in the semi-detached phase, can hasten the expansion of the sec-
secondary and so shorten the time spent in the semi-detached evolution.

It is seen in Figure 2b that the ratio the timescales (τ_{EW}/τ_{EB}) characterizing the phases in which the oscillating systems exhibits W UMa-type (EW) and β Lyrae-type (EB) is still less than 1.0. It is much smaller than its observational value (5 to 6) derived by Rahunen (1981) and even smaller than a smallest value (13/5) derived by Ruciński (2002). This suggests that the real W UMa systems may suffer angular momentum loss. Therefore, we restrict the following discussions to the possible evolutionary consequences of the models for W UMa systems with angular momentum loss.

4.3 Contact evolution with angular momentum loss

4.3.1 Evolution with angular momentum loss by GR

Assuming that the angular momentum of W UMa-type systems is lost only by gravitational wave radiation, we obtain a model for W UMa-type systems that exhibits cyclic evolution without loss of contact as the models mentioned above. The contact degree and surface temperature of both components, together with the mass ratio and the orbital period of the system, are shown in Figures 3a–6a. It is seen in Figures 3a–6a that the model exhibits cyclic evolution on a thermal timescale (with a period of about $6 \times 10^6$ yr) without loss of contact.

As seen from Figure 4a, the ratio of the timescales (τ_{EW}/τ_{EB}) is still smaller than 1.0, this is still much smaller than the value given by Vilhu (1981) according to observations, and even smaller than a smallest value (13/5) found by Ruciński (2002). This suggests that, although the gravitational radiation has a significant influence on the evolution of W UMa-type systems and other close binaries, it still can not prevent the cyclic behaviour of the model for W UMa systems. Vilhu (1981) had concluded that if the timescale of angular momentum loss for W UMa stars is shorter than the secondary’s thermal timescale (except at extreme mass ratios where the primary nuclear evolution dominates) the system will soon fill the outer critical surface and then prob-
ably coalesces into a single star, if the timescale is longer than the primary nuclear timescale, the effect will be negligible. If the angular momentum loss rate falls between these limits, it makes an important contribution to the contact evolution. The rate of angular momentum loss associated with gravitational radiation is $10^{-11}$ to $10^{-10}$ yr$^{-1}$, corresponding to a timescale of about $10^{10}$ to $10^{11}$ yr. It is clear that the gravitational radiation life-time is long compared with the nuclear timescale of the primary. Systemic angular momentum loss due gravitational radiation must accelerate the evolution toward more extreme mass ratios (see Figure 5a) rather than lead to orbital collapse, so that the effect of gravitational radiation on the cyclic evolution is almost negligible. The most likely angular momentum loss mechanism for W UMa systems is a magnetic stellar wind. There is considerable observational and theoretical evidence that stars which possess convective envelopes and, at the same time, rotation can amplify primordial magnetic fields, emit a stellar wind and lose angular momentum as a consequence of the interaction between the fields and the wind.

### 4.3.2 Evolution with angular momentum loss by MSW

Equation (12) gives a rate of angular momentum loss (via MSW) of a close binary system in which the mass is transferred from the star 2 (with a mass of $M_2$ and a radius of $R_2$) to star 1. This suggests that the rate of angular momentum loss by MSW is directly related to the direction of the mass transfer. Assuming that the mass is always transferred from the secondary to the primary in W UMa systems, and taking $\lambda=1.8$, 1.2, or 0.8, which are in its observational range, we investigate the influence of the parameter $\lambda$ on the evolution of the models for W UMa systems. In these cases the evolution of contact degree and the surface temperature of both components, together with the mass ratio and the orbital period of the system, are also plotted in Figure 3b–6b, c, and d, respectively.
It is seen in Figures 3–6 that our models still exhibit cyclic behaviour without loss of contact on the thermal timescale of the secondary. This is similar to the case of constant angular momentum with the exception that the cycles are slowly shifted towards more extreme mass ratios. And that the smaller the parameter $\lambda$ is taken to be (i.e. the higher the rate of angular momentum loss is assumed to be), the longer is the period of cyclic evolution, and the larger is the ratio of the timescales characterizing EW and EB phases ($\tau_{\text{EW}}$ and $\tau_{\text{EB}}$). If we take $\lambda = 0.8$, the ratio of the timescales ($\tau_{\text{EW}}$ and $\tau_{\text{EB}}$) has risen up to 2 to 3 which is consistent with the result obtained by Ruciński (2002).

Therefore, the most likely angular momentum loss mechanism for W UMa systems is MSW, and the value of $\lambda$ is of about 0.8 for W UMa systems if the mass is transferred from the secondary to the primary in W UMa systems. Meanwhile, as seen from Figures 3–6, the mean mass ratio in a cycle becomes smaller and smaller as the evolution of the system proceeds, in any case with angular momentum loss, the mean orbital period of the system becomes longer and longer. I.e. the system drifts to smaller and smaller mass ratios and to longer and longer periods along contact branches of the cyclic evolution. So W-type W UMa systems evolve into A-type W UMa systems with low total masses. Since the systems with $P < 0.35$ d are apparently not observed to have EB-type light curves, the fraction of time spent in the broken-contact phase or in poor thermal contact phase is uncertain and possibly small. This suggests that the poor thermal contact or semi-detached state probably appears very rarely (or never appears) during the evolution of the real W UMa systems. In effect, the loss of angular momentum due to MSW is used here to gently squeeze the systems and thus prolong the contact phase of the TRO model to such a degree that the broken-contact or poor thermal contact phase appears very rarely (or never appears).

If the mass is transferred from the primary to the secondary, the rate of angular momentum loss should be expressed as
Figure 5. Evolution of mass ratio \( q = \frac{M_2}{M_1} \) of the binary with angular momentum loss by GR or by MSW of the secondary (with the different values of \( \lambda \)).

\[
\frac{\text{d} \ln J}{\text{d} t} = -3.03 \times 10^{-7} \frac{R_1^2 (M_1 + M_2)^2}{\lambda^2 A^5 M_2} \text{ yr}^{-1}. \tag{13}
\]

We take \( \lambda = 1.8 \) which is the largest value obtained by the observations of single stars, and corresponds to the smallest rate of angular momentum loss due to MSW for single stars, and find that the model soon fills its outer critical Roche lobe after a total of about \( 2.5 \times 10^7 \) yr of nuclear evolution. The system loses its mass and angular momentum through the outer Lagrangian point. Then it probably coalesces into a single star. Since the direction of mass transfer may change with time, we take \( \lambda = 1.8, 1.2, \) and \( 0.8 \), respectively, and calculate the rate of angular momentum loss via Eq. (12) when mass is transferred from the secondary to the primary, otherwise via the Eq. (13). In these cases, the models rapidly drift to extreme mass ratios, and they cannot be in good thermal contact when the mass ratio of the models becomes sufficiently small. Because, once the mass ratio of the system is very small, the mass is transferred from the primary to the secondary when the model is in poor thermal contact, then the rate of angular momentum due to MSW of the primary is very large, and the model evolves towards good thermal contact rapidly, but the direction of mass transfer has been reversed before the models are in good thermal contact, and the rate of angular momentum loss due to MSW of the secondary is too small to further force the model to evolve towards deep contact. Therefore, once the mass ratio of the models is sufficiently small the model is always in poor thermal contact even if we take the parameter, \( \lambda = 0.7 \) (see Figure 7) although the models exhibit thermal cyclic behavior with a short period and a small amplitude.

In effect, W UMa systems are surrounded by a common envelope and the loss of angular momentum is probably not related to the direction of the mass transfer and may be simultaneously caused by MSW of both components. We take \( \lambda = 3.8 \), and the loss rate of angular momentum of the model is taken as the sum of loss rates predicted by Eqs (12) and (13). The evolution of some quantities is shown in Figure 8. The thermal structure of the two stars of the
Evolution of the orbital period \( (P) \) of the binary with angular momentum loss by GR or by MSW of the secondary (with the different values of \( \lambda \)).

This suggests that the magnetic braking in W UMa systems is weaker than that in the similar single stars or components of non-contact binaries. The observations have shown that both in X-rays (Crudace & Dupree 1984; Vilhu 1984) and in the radio (Hughes & McLean 1984, 1985) the W UMa systems are definitely less active than are similar single stars or components of non-contact binaries. The lower coronal activity in W UMa systems is probably caused by the internal rotation of their components or by their convective zone which is thinner than indicated by their surface temperatures (Rucinski 1984). The relative weakness of coronae in W UMa systems directly tells us that the large-scale magnetic structures are smaller in W UMa systems and contain less plasma in them than in other similar but non-contact stars. Therefore, the magnetic-braking effects in W UMa systems are weaker than in other similar but non-contact stars. Meanwhile, we can see from Figure 8b that the surface temperature of the secondary is unstable, so that its luminosity is unstable. This is consistent with the
properties of W-type W UMa systems with variable light curves. Moreover, we can see from Figure 8c that the radius of the primary is expanding rapidly while the radius of the secondary appears to be essentially constant although the model is drifting to smaller and smaller mass ratios. This is very similar to the result predicted by observations (Wang 1995). This suggests that W phenomenon is likely caused by the expansion of the primary (Paper I) because the expansion of the radius of the primary depletes a part of the core luminosity so that the temperature of the secondary exceeds that of the primary.

As seen from Figure 9, the outermost layers of the common envelopes of both components are so similar that the difference is slight. Meanwhile, we can see in Figure 9b that the profile of the specific entropy in the surface layers of the secondary is very different from the result given in Paper I. It clearly rises up to a higher value in the surface layers of the secondary because of the energy transfer due to mass transfer. Figure 12 shows the distribution of the specific entropy in both components as a function of the mass (Figure 12a) and the radius (Figure 12b) during a phase of semi-detached evolution and of contact evolution with efficient energy transfer. The semi-detached model has the same total mass and mass ratio as the contact one. The flat parts in outer envelopes of the components indicate the convective regions. It is seen in Figure 12 that the convective envelope of the primary in the contact system is slightly thicker than that of the primary in the semi-detached one and the convective envelope of secondary in contact system is much thinner than that of the secondary in the semi-detached one (see Figure 12b), but the total mass contained in the convective envelopes of the contact system, which is directly related to the dynamo action (Hurley, Tout & Pols 2002), is much less than that contained in the envelopes of the semi-detached one (see Figure 12a). We conclude that this is caused by luminosity transfer between the two components. Because part of the luminosity in the envelope of the primary is transported to the envelope of the secondary in the contact state, the temperature of the envelope of the primary decreases and the temperature of the envelope of
Figure 8. With angular momentum loss due to MSW ($\lambda = 3.8$) of both components, the same quantities against Age.

the secondary increases. The decrease in the temperature of the envelope of the primary would then drive the radiative temperature gradient, $\nabla_r$, to increase and thence lead part of the radiative zone to be convective so that the convective envelope of the primary becomes thicker in the contact state. The increase in the temperature of the envelope of the secondary would drive the radiative temperature gradient, $\nabla_r$, to decrease and thence lead part of the convection zone to be in radiative equilibrium. As a result, in contact systems, the mass in the convective envelope of the primary increases by about $0.01\, M_\odot$, and the mass in the convective envelope of the secondary decreases by about $0.08\, M_\odot$ in comparison with in the semi-detached one (see Figure 12a). Since the decrease of the mass in the convective envelope of the secondary is much more than the increase of the mass in the convective envelope of the primary, the total convective mass in the envelope of the contact system is less than that in the envelopes of the semi-detached one. As a result of the less total mass contained in the convective envelopes of the W UMa systems, because of energy transfer, W UMa systems show lower activity than non-contact stars.

We also investigate the evolution of the models which lose angular momentum steadily at the different rates, and find that the system can keep in shallow contact (with a degree of about 20%, see Figure 10) by a hypothetical angular momentum loss $\frac{d\ln J}{dt} \approx 1.6 \times 10^{-9}\, \text{yr}^{-1}$, on the average, corresponding to a timescale of about $6 \times 10^8\, \text{yr}$ which is in agreement with the result obtained by Rahunen (1981). If we assume a somewhat larger rate, the system soon reaches the outer critical surface. This was confirmed with a steady loss of angular momentum with a rate that was slightly larger than the critical loss rate (see also Robertson & Eggleton, 1977). After reaching the outer critical surface, the system loses its mass and its angular momentum through the outer Lagrangian points, so that the system probably coalesces into a single star [Webbink 1976, Rahunen 1981]. On the other hand, if the assumed loss rate is smaller than the critical rate, the system exhibits cyclic evolution with alternating good contact and poor contact phases on the secondary’s
Figure 9. The thermal structure of the primary (solid lines) and the secondary (dashed lines) during a phase of semi-detached evolution (SD) and of contact evolution (C) with efficient energy transfer.

Figure 11. The evolution of the massive component of the binary in the period-colour diagram.
With a steady angular momentum loss, $\frac{d\ln J}{dt} \approx 1.6 \times 10^{-9} \text{yr}^{-1}$, the same quantities as figure 1 change with Age.

Figure 10. With a steady angular momentum loss, $\frac{d\ln J}{dt} \approx 1.6 \times 10^{-9} \text{yr}^{-1}$, the same quantities as figure 1 change with Age.

5 DISCUSSION AND SUMMARY

In semi-detached phase, the mass is transferred from the primary to the secondary, and the energy (including the gravitational energy, thermal energy, and radiative energy) in the transferred mass is also transferred to the surface of the secondary. The effect of the accreting matter on the radii of the accretors is related to their envelopes being radiative or convective, and calculations involving mass accretion on to a star with thick convective or fully convective envelope (Whyte & Eggleton 1985), have shown that the accreting star shrinks (under the assumption that the entropy of the accreting matter is the same as the surface entropy of the original star). However, Sarna & Fedorova (1989) thought that the newly accreting matter has substantially higher entropy than the original star and so the photosphere of the secondary in close semi-detached binaries is heated by the infalling matter. Due to this process, the accreting secondary should significantly increase in radius (Prialnik & Livio 1985). That is to say, the energy source at secondary’s atmosphere provided by the accreting matter from the primary hastens the expansion of the secondary and so shortens the time spent in the semi-detached evolution. Therefore, the luminosity transfer due to mass transfer plays an important role in the origin and evolution of contact binaries. Meanwhile, if the energy transfer due to mass transfer is applied in the surface layer of the secondary as a surface condition of the secondary, we can see that this energy source has a significant influence on the thermal structure in the surface layers of the secondary (see figure 9b). As seen from figure 9b, the entropy profile in the surface layers of the secondary in the semi-detached phase rises. It is very different from the result in Paper I (see Paper I’s Figure 6), and the difference arises from the luminosity transfer due to mass transfer.

W UMa systems have been divided by Binnendijk.
into A-type and W-type systems according to whether the primary minimum in the light curve is a transit or an occultation. This suggests that the temperature of the more massive component is higher in A-type systems while the temperature of the less massive star is higher in W-type ones, i.e., the so-called W phenomenon. It is seen in Figure 8c that the radius of the primary is expanding in good contact while the secondary almost maintains an essentially constant radius in low-mass W UMa systems. Since the expansion of the primary depletes a part of the core luminosity, the temperature of the secondary exceeds that of the primary. This suggests that W-phenomenon is caused by expansion of the primary (see Figure 8b and c). It is similar to the statistical result derived by Wang (1995) from the observational data.

Our model exhibits cyclic behavior without loss of contact, with a period of about $6 \times 10^6$ yr if we do not consider angular momentum loss from the system. And the ratio of the timescales characterizing the phases in which the oscillating systems exhibit W UMa-type (EW) and $\beta$ Lyrae-type (EB) light curves is $\tau_{\text{EW}}/\tau_{\text{EB}} < 1.0$. This is much smaller than its observational value (5 to 6, Rahnen 1981 and 2 to 3, Rucinski 2002). Meanwhile, all constant angular momentum loss models need a formation mechanism which can produce unequal components because, with constant angular momentum, it is not possible to cover a large range of mass ratios (Vilhu 1981; Rahnen 1981). This suggests that the real W UMa systems should suffer angular momentum loss during their evolution. The rate of angular momentum loss associated with gravitational radiation is about $10^{-11}$ to $10^{-10}$ yr$^{-1}$ for W UMa systems (Robertson & Eggleton 1977; Webbink 1977; Rahnen 1981). This is two orders of magnitude smaller than the rate of the hypothetical angular momentum loss needed to keep the systems in good thermal contact. The effect of gravitational wave radiation on the cyclic evolution is so small that the model with angular momentum loss due to GR is not consistent with the observations of W UMa systems. The most likely angular momentum loss mechanism is MSW. In fact, there are some
indications of strong magnetic activity in W UMa systems. Van’t Veer (1978) thinks that magnetic braking may control the evolution of solar-type contact binaries. Eqs. (12) and (13) give the rate of angular momentum loss by MSW of the secondary and the primary, respectively, and the observations give the value of parameter, $\lambda = 0.73$ to 1.78 for single stars. According to Eqs. (12) and (13) the rate of angular momentum loss caused by MSW is directly related to the direction of the mass transfer in a close binary system. If we assume that the mass is always transferred from the secondary to the primary, and take $\lambda = 0.8, 1.2$, and 1.8, respectively. We find that our models exhibit cyclic evolution which is similar to the case of constant angular momentum with the exception that the cycles are slowly shifted to the more extreme mass ratio, and the larger the rate of angular momentum loss is assumed to be, the more rapidly the model evolves towards the extreme mass ratios, and the larger is the ratio of the timescales ($T_{\text{EW}}/T_{\text{EM}}$). If the parameter, $\lambda$, is taken as 0.8, the ratio of the timescales ($T_{\text{EW}}/T_{\text{EM}}$) is about 2 to 3 which is in agreement with observations given by Rucinski (2002). Meanwhile, we can see from Figures 3-6 that the mean mass ratio of a cycle becomes smaller and smaller and the mean period becomes longer and longer as the evolution of the system proceeds. This indicates that it slowly evolves towards a system with an extreme mass ratio and a longer period. If the mass is assumed to be transferred from the primary to the secondary, even if the parameter, $\lambda$, is taken to be 1.8, the model soon fills its outer Roche lobe. The star would then lose its mass and angular momentum through the outer Lagrangian point, and the system would soon evolve into a single star. If the direction of the mass transfer is changed with time, and the angular momentum loss of the system is caused by MSW of the secondary when the mass is transferred from the secondary to the primary or by MSW of the primary when the mass is transferred from the primary to the secondary, no matter how the parameter $\lambda$ is chosen between 0.73 and 1.78, the model cannot be in good contact when it evolves to a system with a sufficiently small mass ratio.

Our models for low-mass W UMa systems show that the magnetic braking effects in W UMa systems are likely to be weaker than that in other similar but non-contact stars. This is in agreement with observations. Although there were both observational and theoretical indications (Roberts 1974; Knobloch et al. 1982) of a power law ($B = B_0\Omega^\delta$, where $\Omega$ is the Keplerian angular velocity, $B$ the average poloidal field, with the most probable value of $\delta \approx 1$). However, observation has shown that the strengths of the chromospheric and transition-region emissions increase very slowly with the increase of the inverse to Rosseynumber $\tau/P$ (Rucinski 1986; Stepien 1993). This can be explained as a result of the total saturation of stellar surfaces by small active regions. Meanwhile, the observations have shown that, both in X-rays and in the radio, the W UMa systems are definitely less active than are similar single stars or components of non-contact binaries (Rucinski 1986). The lower coronal activity in W UMa systems is likely caused by the internal rotation of their components or because their convective zone is thinner than indicated by their surface temperatures. The observations directly tell us that in W UMa systems the largest-scale magnetic structures are smaller and contain less plasma in comparison with the similar but non-contact stars so that the efficiency of the angular momentum loss in W UMa systems may be lowered in comparison with non-contact stars of the same $\tau/P$. Figure 12 shows the distribution of the specific entropy of both components during a phase of a semi-detached state and of a contact state. It is seen in Figure 12 that the convective envelope of the primary in the contact system is slightly thicker than that in the similar semi-detached one and the convective envelope of the secondary in the contact state is much thinner than that in the semi-detached state, but the total mass contained in the convective envelopes of contact systems, which is related to dynamo action (Hurley, Tout & Pols 2002), is less than that contained in the convective envelopes of the semi-detached stars because of energy transport from the primary to the secondary (see Figure 12). This is in a good agreement with the prediction of observations mentioned above. As result of the less total mass in convective envelopes of W UMa systems, W UMa systems show the lower activity in comparison with the single or non-contact stars.

Our models indicate that W UMa systems evolve into contact binaries with the extreme mass ratios or even into single stars if they suffer angular momentum loss due to MSW. Vilhu (1981) thought that W UMa systems most probably evolve into single stars, but the way (and the timescale) in which this evolution proceeds is not clear. However, it is argued that due to a longer timescale to reach a contact, and also to the longer timescale of contact evolution, the ultimate appearance of low-mass binaries after merging of both components will be distinctly different from that of massive binaries. After an initial mass exchange of a massive contact system, it is very likely that the main-sequence (MS) lifetime of a more massive component becomes shorter than the duration of the contact phase of evolution, and that the coalescence results in the formation of a fast-rotating giant, similar to FK Com. In the case of low-mass binary, the MS lifetime of a more massive component may still be longer than the duration of the contact phase. That would result in the formation of a single fast-rotating MS star, similar to single blue straggler (Stepien 1993). The contact systems with most extreme mass ratios (few known due to the low probability of eclipses) are important for testing models of contact binary evolution. But most W UMa stars with low mass ratios (such as RR Cen and ε CrA, see Vilhu 1981) seem to have an evolved primary and both components should deviate from thermal equilibrium.

Although we can keep our models in good contact under some assumptions, there is still one observational point which seems to be in contradiction with the scenarios presented above, because it produces a peculiar mass ratio distribution showing a large excess of the systems with small mass ratios. This is due to the fact that the mass transfer occurs on the thermal timescale of the secondary, which grows strongly as the secondary mass and therefore as the mass ratio decreases. Thus systems with large mass ratios initially evolve in less than $10^8$ yr towards mass ratios less than about 0.4 (see Figures 8 and 10). The scenarios in which contact-binary evolution continues into a single star on the secondary thermal timescale produce a rather steep period-distribution. It can be shown quite easily that the resulting mass ratio distribution does not agree with the mass ratio distribution deduced from observation (Van’t Veer 1978; Vilhu 1981). Meanwhile, W UMa-
type contact binaries are found in stellar aggregates of a widely differing age, in very old clusters like NGC188 and in moderately old ones like M67 and Praesepe. This cannot be interpreted by these models. The decrease in the coronal (X-rays and radio) activity of contact binaries, which is indeed observed (Crudace & Dupree 1984; Vilhu 1984; Hughes & McLean 1984), is used as an argument that the AML efficiency in contact is relatively low and the contact stage is considerably prolonged relative to adjacent stages. This small modification to the AML models is capable of explaining why so many different contact binaries are observed in old systems like NGC188. Therefore, a feedback mechanism is crucial to the success of these models. Vilhu (1981) envisaged a situation in which increasing depth of contact causes increased mixing in the common envelope which tends to bury the strong surface magnetic field. The magnetic braking is then weakened, leading to a decreased loss rate of angular momentum and decreased depth of contact again. It is clear that this picture is very crude and speculative. On the observational side one should try to measure the magnetic fields and to study the chromospheres and coronae more closely in the UV- and X-ray region. On the theoretical side one should clarify the differential rotation and dynamo action in rapidly rotating stars in connection with the circulation pattern in the common envelope, a task which is not easy (Vilhu 1981). Rucinski (1982) explained the period gap at periods just longer than those for contact systems by a phase of very rapid magnetic braking in detached binaries which ends when the stars come into contact. However, it is unclear that this mechanism is sensitive enough to maintain an essentially constant depth of contact.

The spin angular momentum of both components and the loss of the system’s mass have not been considered in our present work. Indeed the spin angular momentum of both components is not small compared with the orbital angular momentum when the mass ratio becomes extreme (Van’t Veer 1979). If the spin angular momentum of both components is considered, angular momentum loss required to spin up the stars to keep them in corotation as the orbit shrinks, so the spin angular momentum of both components increases and the orbital angular momentum of the system decreases, the system might then suffer Darwin’s instability (i.e., the tendency for an orbit to desynchronize if the spin angular momentum of both stars is more than a third of the orbital angular momentum, Eggleton & Kiseleva-Eggleton 2001) when so much angular momentum has been removed that there is no longer sufficient in the orbit to bring the stars into corotation and evolution proceeds on a tidal timescale (i.e., \( \tau_{\text{tid}} = \frac{\Omega_{\text{spin}}}{\Omega_{\text{tid}}^{\text{min}}} \)) where the dot indicates a time derivative, \( \Omega_{\text{spin}} \) is the spin angular velocity, Bagot 1996). As a result, angular momentum loss would ultimately lead to a lack of corotation and thence to a failure of the Roche model. Meanwhile, magnetic braking is always accompanied by the mass loss from the system, and the mass loss has a significant influence on the evolution of W UMa systems. We will consider the spin angular momentum and mass loss of the system to construct a true non-conservative model for low-mass W UMa systems in our future work.

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