Potential precision of terrain measurement using space lidars

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Abstract. Laser remote sensing technologies are being actively developed and used, for instance, for modelling the terrain of the surface of the Earth, the Moon and other planets of the Solar System. Theoretical aspect of these projects is underdeveloped and not related to the measurement precision assessment. The article studies the calculation of the potential precision of range measurement and presents a method of calculation of the a space-based lidar parameters based on the condition of providing the set range measurement precision. It produces the equations that connect the potentially achievable precision of terrain (range) measurement and the receiver output signal-to-noise ratio to the lidar parameters and the parameters that determine the external conditions of such measurement. These equations form the foundation for a method of determining the lidar parameters and the requirements acting as the input data for a lidar development. It demonstrates that the space lidar altimeters are rationally associated with the term of the "energy space resolution" as the minimum size of the area of the terrain under resolution under the established range measurement error. The calculations carried out with the said method made it possible to assess the precision parameters of the currently used ATLAS space lidar. According to our data, a lidar is capable of providing the range measurement precision of several centimetres, provided that the objects are located within the area of several tens of cm². The collected results bring the conclusion of the convergence of the results of the lidar precision calculation, carried out with the developed method and using the data published by NASA.

1. Introduction
Within the past decade, space-based lidar altimeters are becoming more and more common for creating models of terrestrial surface. As a rule, any published materials of such works [1-3] do not cover any theoretical aspects, and do not contain any information on the precision of the measurement of distance and height profile of the terrain. However, requirements to the range measurement precision are critical in the development of space-based lidar altimeters, since they are a determinant factor for the technical profile of the lidar and the choice of the main components of its structure.

The article studies the calculation of the potential precision of range measurement and a method of calculation of the a space-based lidar parameters based on the condition of providing the set range measurement precision.

The space lidars use the pulse range measurement method. In this method, the potential precision of the terrain profile measurement is determined by the time tag (delay time) of the received pulses with noises on condition that optimal signal filtration is provided at the receiver output. This noise (fluctuation) component of the measurement error is fundamentally unremovable; the level of which is the minimum level of the resulting measurement error.

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2. Method

In the optimal filtration conditions, the root-mean-square error of detecting the signal (pulse) time tag in the white Gaussian noise environment at the receiver output (input of the electric path) is determined with the proportion [4]:

\[ \sigma_t = \frac{1}{\mu \Delta f_0}, \]

(1)

where \( \Delta f_0 \) is the effective width of the spectrum, \( \mu \) is the signal-to-noise ratio at the receiver output, Value \( \Delta f_0 \) may be taken as equal to

\[ \Delta f_0 = \frac{1}{t_c}, \]

where \( t_c \) is the length of the pulse.

Then the potential range measurement precision of terrain scanning is determined with the root-mean-square value:

\[ \sigma_l = \frac{c}{\sqrt{\mu \Delta f_0}}. \]

(2)

The signal-to-noise ratio at the receiver output may be expressed through the number of signal and noise electrons as

\[ \mu = \frac{n_{c\lambda}}{n_{ux\lambda}} = \frac{n_{c\lambda}}{\sqrt{n_{c\lambda} + n_{p\lambda} + n_{f\lambda}}}, \]

(3)

\( n_{c\lambda} \) - number of signal electrons; 
\( n_{p\lambda} \) - number of photoelectrons determined by the photon noise; 
\( n_{f\lambda} \) - number of photoelectrons generated by the background radiation.

The number of signal photoelectrons is determined by the radiation flux reflected from the scanned surface. When laser radiation is pointed at nadir, the exposure of the surface element \( \Delta A \) equals to

\[ E_\lambda = \frac{\Phi_0 R_\lambda \tau_\lambda \tau_0 \lambda A_{ex}}{\Delta A}. \]

Radiance of the area equals to

\[ L_{e\lambda} = E_\lambda R_\lambda. \]

Considering the radiator to be the entire exposed area \( \Delta A \), studied as an areal radiator, we get that the radiation flux emitted by the area equals to [5].

\[ \Phi_\lambda = \frac{\Phi_{0\lambda} R_\lambda \tau_\lambda \tau_0 \lambda A_{ex}}{l^2}, \]

(4)

\( \Phi_{0\lambda} \) - flux radiated by the lidar at the wave length; \( R_\lambda \) - function of distribution of the bidirectional reflection from the scanned surface; \( \tau_\lambda \) - spectral transmission of the atmosphere at the operating wave length; \( \tau_{0\lambda} \) - spectral transmission of the optical receiving system; \( A_{ex} \) - area of the entrance pupil of the optical system; \( l \) - track length (carrier flight height). Then the number of the signal photoelectrons at the receiver output will be equal to

\[ n_{c\lambda} = \frac{\Phi_{2\lambda} \eta}{h \nu} = \frac{\Phi_{0\lambda} R_\lambda \tau_\lambda \tau_0 \lambda A_{ex} \eta}{l^2 h \nu}, \]

(5)

where \( \eta \) is the quantum efficiency of the radiation receiver, \( h \) is the quantum energy on the operating wave length.

The photon (radiation) noise, resulting from the discreet nature of radiation, is described by the Poisson statistics, according to which the number of noise photoelectrons is

\[ n_{p\lambda} = \sqrt{n_{c\lambda}} = \frac{\Phi_{0\lambda} R_\lambda \tau_\lambda \tau_0 \lambda A_{ex} \eta}{l^2 h \nu}. \]

(6)

The background radiation is generated by the laser radiation, dispersed in the atmosphere in the
direction of the receiving optical system of the lidar $Φ_{αλ}$ and the background radiation of the earth surface $Φ_{фλ}$.

Dispersed in the atmosphere and returned to the receiving system, the laser radiation flux equals to:

$$Φ_{αλ} = \frac{q_{αλ}β_λ(λ)Δlτ_α^2(λ)τ_{αι}A_{αι}}{l^2},$$

(7)

where $β_λ$ is the volume backscattering coefficient;

$Δl = cτ_{αι}/2$ is the length of the atmosphere section generating the backscattering signal.

Product $β_λ(λ)Δl$ in (7) is basically the same as $R_λ$ in formula (4). Comparing $R_λ$ and $β_λ(λ)Δl$ we can see, that at the pulse length of about $10^{-9}$ and the standard value $β_λ(λ) = 10^{-5}$ m$^{-1}$ mean$^{-1}$, $Δl = 0.15$ m, the contribution made into the total noise by the noise created by the laser radiation scattered by the atmosphere in the direction of the receiving optical system of the lidar is negligibly minor.

The background radiation of the earth surface, presented as a Lambert reflector of the solar radiation, creates the input flux

$$Φ_{фλ} = \frac{E_{фλ}ΔλΔт_фτ_{фλ}A_{фλ}}{π},$$

(8)

where $Ω$ is the field angle of the receiving optical system of the lidar, $E_{фλ}$ is the spectrum density of the exposure of the earth surface, and $Δλ$ is the bandwidth of the optical input filter.

Number of photoelectrons created by flux $Φ_{фλ}$:

$$n_{фλ} = \frac{E_{фλ}ΔλΔт_фτ_{фλ}A_{фλ}f_φη}{nhν},$$

(9)

Taking (5), (6) and (9) into account, we get a developed formula for the signal-to-noise ratio at the radiation receiver output, which, after some simple transformations, looks as follows:

$$μ = \frac{Φ_{αλ}R_λ\sqrt{πτ_α^2τ_{αι}A_{αι}f_φη}}{lνhν}\sqrt{\frac{πΦ_{фλ}R_λτ_фA_{фλ}f_фη}{2πΦ_{фλ}R_λΔт_фτ_{фλ}A_{фλ}f_фη}},$$

(10)

Then, the potential precision of range measurement during terrain scanning will be determined with the following expression:

$$σ_l = \frac{cλν\sqrt{πΦ_{фλ}R_λτ_фA_{фλ}f_фη}}{2πΦ_{фλ}R_λΔт_фτ_{фλ}A_{фλ}f_фη\sqrt{πτ_α^2τ_{αι}A_{αι}f_φη}},$$

(11)

Equation (11) connects the potentially achievable precision of the terrain (range) measurement to the lidar parameters ($Φ_{фλ}$, $A_{αι}$, $η$, $Ω$, $Δλ$, $l$, $τ_{αι}$, $Δl$, $τ_ф$, $l_φ$) and the parameters determining the external conditions of the measurement. The acquired actual signal-to-noise ratio is related to the reflecting surface, completely filling up the field angle of the optical receiving system of the lidar.

However, at the sufficient value of the laser radiation flux, it is possible to receive a response (echo-signal) from the reflecting elements of uneven height within the field angle.

To register such echo-signals, the flux reflected from the terrain element with the area $A_i$, shall create the signal-to-noise ratio equal or exceeding the threshold value $μ_i$, determined from (2), required by the value $σ_l$. Obviously, the same signal from the area $A_i$ will be less than that from the entire exposed area by as many times, as the area $A_i$ is less than the entire exposed area $A_0$. For this reason, calculation under (10) and (11) requires introduction of the coefficient $m = \frac{A_i}{A_0}$.

Moreover, it is important to consider that in the multibeam lidars, the radiation flux from the laser $Φ_{фλ}$ is divided into $N$ channels using the optical diffraction elements (splitters). Flux loss in the splitter is characterized with the transmission factor $τ_{cn}$. Then the evaluation formulae (10) and (11) get the following look:
\[
\mu = \frac{m \tau c \phi R_\lambda}{N \sqrt{h} \sqrt\frac{\frac{1}{2} \pi m \tau c \phi R_\lambda \tau \lambda + i^2 E_{\phi R_\lambda} \Delta \lambda \Omega}} \tag{12}
\]

\[
\sigma_l = \frac{N l c \sqrt{h} \sqrt\frac{\frac{1}{2} \pi m \tau c \phi R_\lambda \tau \lambda + i^2 E_{\phi R_\lambda} \Delta \lambda \Omega}}{2 \pi m \tau c \phi R_\lambda \Delta f_0 \sqrt\frac{1}{2} \pi m \tau \lambda \Delta \lambda \eta} \tag{13}
\]

3. Results and discussion

As the input data, let us use the parameters of the ATLAS lidar, commissioned by NASA in September 2018 (table 1).

| Parameter                        | Designation | Value          | Size   |
|----------------------------------|-------------|----------------|--------|
| Radiation energy                 | \( W_r \)   | \( 1 \times 10^{-3} \) J |        |
| Pulse length                     | \( t_i \)   | 1 ns           |        |
| Accumulation time                | \( t_a \)   | \( 10^{-4} \) s |        |
| Wave length                      | \( \lambda \) | 532 nm         |        |
| Pulse frequency                  | \( f_i \)   | \( 10^4 \) Hz  |        |
| Laser beam divergence            | \( \Theta \) | \( 20 \times 10^{-6} \) rad |        |
| Diameter of the optical system   | \( D_{\text{entr}} \) | 0.8 m          |        |
| entrance pupil                   |             |                |        |
| Bandwidth                        | \( \Delta \lambda \) | \( 10^{-3} \) mm |        |
| Carrier flight velocity and height | \( V, H \) | 7.500 km/s, km |        |
| Number of beams                  | \( N \)     | 6 l           |        |

Let us accept the standard values of \( \tau_\lambda = 0.5, \tau_0 \lambda = 0.8, R_\lambda = 0.5, E_{\phi R_\lambda} = 2 \times 10^3 \) W/\( \mu \)m\( ^2 \) [6]. Then, from (13) we see, that at the reflection from the area \( \Delta A = 25 \) cm\(^2\), signal-to-noise ratio at the radiation receiver output \( \mu = 4.54, \sigma_l = 3.3 \) cm.

Based on the equations (12) and (13), it is possible to build the method of the parameter calculation and determination of the requirements, acting as the input data for the lidar development, i.e. to solve the opposite problem: using the given error value \( \sigma_l \), determine the combination of the lidar parameters that would satisfy the range measurement precision requirements.

For solving the problem, an algorithm and a calculation program were developed. The program organizes the free parameters' combination cycles, under which the actual signal-to-noise ratio exceeds the threshold value of \( \mu_0 \), corresponding to the given error \( \sigma_l \). Upon the results of the calculation, it is possible to reveal some mutual dependencies between the free parameters, such as the dependency of the potential precision (root mean square error of the range measurement) on the reflecting area size at different diameters of the entrance pupil of the optical system; or the dependency of the potential precision on the laser radiation flux.

It should be noticed that the size of the area, the signal of which provides the threshold signal-to-noise ratio \( \mu_0 \), does not characterize the coordinate measurement precision, as the area may be located in any position within the field angle of the lidar optical system, but does characterize the spatial resolution from the point of view of the energy proportions required for achieving the given error \( \sigma_l \).

For this reason, for the space-based lidar altimeters it makes sense to use the "energy spatial resolution" term as a minimum size of the area under resolution on the terrain, unlike the passive scanners which are characterized with a geometrical spatial resolution determined with the size of the focal matrix pixel and the focal distance of the lens.
4. Conclusion
This way, the calculations carried out with the said method make it possible to assess the precision parameters of the currently used ATLAS space lidar. The calculations demonstrate that, used as the input data, the ATLAS parameters are rationally balanced, and the lidar is capable of ensuring the range measurement error of several centimetres at the location of the objects in the area of over tens of cm². The collected results bring the conclusion of the convergence of the results of the lidar precision calculation, carried out with the developed method and using the real data. Thus, the ATLAS lidar has been declared to have a resolution ability of the measurement (monitoring) of the ice layer height up to 10 cm and the error of measurement of the altitude of ice above sea level of 3 cm, which generally matches the estimated data, and demonstrates the credibility and the practical relevance of the developed method.

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