On the Generalized Degrees of Freedom of the $K$-User Symmetric MIMO Gaussian Interference Channel

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Abstract—This work derives inner and outer bounds on the generalized degrees of freedom (GDOF) of the $K$-user symmetric MIMO Gaussian interference channel. For the inner bound, an achievable GDOF is derived by employing a combination of treating interference as noise, zero-forcing at the receivers, interference alignment (IA), and extending the Han-Kobayashi (HK) scheme to $K$ users, depending on the number of antennas and the INR/SNR level. An outer bound on the GDOF is derived, using a combination of the notion of cooperation and providing side information to the receivers. Several interesting conclusions are drawn from the bounds. For example, in terms of the achievable GDOF in the weak interference regime, when the number of transmit antennas $(M)$ is equal to the number of receive antennas $(N)$, treating interference as noise performs the same as the HK scheme and is GDOF optimal. For $K > N/M + 1$, a combination of the HK and IA schemes performs the best among the schemes considered. However, for $N/M < K \leq N/M + 1$, the HK scheme is found to be GDOF optimal.

Keywords: Interference channel, Generalized degrees of freedom, MIMO, capacity bounds.

I. INTRODUCTION

Interference management in multiuser interference channels has been a major focus of research in the past few years, and it is now widely recognized that straightforward methods such as orthogonalizing users in time or frequency is suboptimal. The concept of Generalized Degrees Of Freedom (GDOF) was introduced in [1], as a means for quantifying the extent of interference management in terms of the number of free signaling dimensions in a two-user Interference Channel (IC). In a multiuser MIMO setup, the multiple antennas at the transmitters and receivers can provide additional dimensions for signaling, which can in turn improve the GDOF performance of the IC. Characterizing the GDOF performance of a multiuser MIMO IC is therefore an important problem, and is the focus of this work.

Although there exist many methods to mitigate the effect of interference, two major approaches have emerged in the literature. The first is based on the notion of splitting the message into private and public parts (also known as the Han-Kobayashi (HK) scheme) [2]; and the second is based on the idea of Interference Alignment (IA) [3] - [5]. These schemes are two different approaches for interference management: the former allows part of the interference to be decoded and canceled at unintended receivers, while the latter makes the interfering signals cast “overlapping shadows” [5] at unintended receivers, allowing them to project the received signal in an orthogonal direction and remove the effect of interference. The HK-scheme originated in the case of a 2-user IC, whereas IA is applicable when there are more than two users. The GDOF performance of the two-user MIMO IC was characterized in [6]. It was extended to the $X$ channel and the $K$-user SISO IC in [7] and [8], respectively. Also, in [9], the idea of message splitting was used to derive the GDOF in a SIMO setting when $K = N + 1$, where $N$ is the number of receive antennas at each user. The idea of IA was extended to derive the Degrees Of Freedom (DOF) in a multiuser MIMO scenario in [10], when $\alpha$, the ratio of the logarithm of the Interference to Noise Ratio (INR) to the logarithm of the SNR, is equal to 1.

In the context of the above, the major contributions of this paper are as follows:

• A new outer bound is derived for the GDOF of the $K$ user symmetric MIMO Gaussian IC using a combination of user cooperation and providing noisy side information to the receivers.

• An inner bound is derived for the symmetric IC as a combination of the HK scheme, IA, Zero-Forcing (ZF), and treating interference as noise. To the best of the authors’ knowledge, the extension of the HK scheme to the multiuser MIMO scenario presented here is new.

• The interplay between the HK scheme and IA is explored from an achievable GDOF perspective. The conditions under which the different achievable schemes are optimal in terms of the GDOF are established.

Proofs of most of the theorems are omitted due to lack of space; these are made available at [11].

The rest of the paper is organized as follows. In Section II, the system model is described. In Section III, the three outer bounds are presented. Section IV focuses on the achievable GDOF. In Section V, some numerical examples are presented to graphically illustrate the inner and outer bounds.

II. PRELIMINARIES

Consider a MIMO IC with $K$ transmitter-receiver pairs, with $M$ antennas at each transmitter and $N$ antennas at each receiver. Let $H_{ji}$ represent the $N \times M$ channel gain matrix from transmitter $i$ to receiver $j$. The time-varying
random channel coefficients are assumed to be drawn from a continuous distribution such as the Gaussian distribution. Since the channel is random, the GDOF results derived in this paper hold in an almost-sure sense. The signal received at the \( j \)-th receiver, denoted \( y_j \), is modeled as

\[
y_j = \sqrt{\rho^{\alpha_j}} \mathbf{H}_{jj} x_j + \sum_{i=1, i \neq j}^{K} \sqrt{\rho^{\alpha_i}} \mathbf{H}_{ji} x_i + \mathbf{z}_j,
\]

where \( \mathbf{z}_j \) is the complex symmetric Gaussian noise vector, distributed as \( \mathbf{z}_j \sim \mathcal{CN}(0, 1) \) and \( x_j \) is the signal transmitted by the \( i \)-th user, satisfying \( \mathbb{E} \left\{ \mathbf{x}_i^H \mathbf{x}_i \right\} = 1 \). Also, \( \rho^{\alpha_j} \) represents the received signal power relative to the noise power at receiver \( j \) due to the signal from user \( i \). In this work, for analytical tractability, attention is restricted to the symmetric IC, where \( \alpha_{jj} = 1 \) and \( \alpha_{ji} = \alpha, i \neq j \), for \( i, j = 1, \ldots, K \). Here, \( \alpha > 0 \) represents the ratio of the logarithm of the INR to the logarithm of the SNR, and is one of the key parameters that determine the GDOF. Also, for deriving the inner bounds, it is assumed that \( M \leq N \).

The GDOF, introduced in [1], is defined as

\[
d_{\text{sym}}(\alpha) = \frac{1}{K} \lim_{\rho \to \infty} C_{\Sigma}(\rho, \alpha) \log \rho,
\]

and \( C_{\Sigma}(\rho, \alpha) \) is the sum capacity of the \( K \) user IC defined above. When \( \alpha = 1 \), the GDOF reduces to the Degrees of Freedom (DOF) defined in [10].

III. OUTER BOUNDS

In this section, three outer bounds on the GDOF for the symmetric IC are stated as Theorems 3.1, 3.3 and 3.4. The overall outer bound is obtained by taking the minimum of the three outer bounds and the interference-free GDOF bound of \( M \) per user. The outer bounds are obtained by first deriving general outer bounds on the sum rate of the \( K \)-user MIMO IC, and then specializing the general outer bounds to the case of the symmetric MIMO Gaussian IC. The general sum rate bounds are presented only for the case corresponding to Theorem 3.3, due to lack of space.

The first outer bound is in similar flavor to the DOF outer bound in [12] and is obtained by considering cooperation among subsets of users and providing partial side information to the receivers to convert the system to a MIMO Z-IC, whose capacity cannot be worse than the original MIMO IC. Then, an outer bound on the Z-IC is derived, and the minimum of the outer bounds obtained by considering all possible combinations of cooperating users forms an outer bound on the sum rate of the MIMO IC.

**Theorem 3.1:** The per-user GDOF \( d(\alpha) \) of the symmetric MIMO Gaussian IC is upper bounded as follows. For \( \alpha \leq 1 \),

\[
d(\alpha) \leq \min_{L_1, L_2} \frac{1}{L} \left[ L_1 M + \min \{ r, L_1 (N - M) \} \alpha + (L_2 M - r)^+ + \min \{ r, L_2 N - (L_2 M - r)^+ \} (1 - \alpha) \right],
\]

For \( \alpha > 1 \),

\[
d(\alpha) \leq \min_{L_1, L_2} \frac{1}{L} \left[ r \alpha + \min \{ L_1 M, L_1 N - r \} + (L_2 M - r)^+ \right],
\]

where \( r \triangleq \min \{ L_2 M, L_1 N \} \), \( L_1, L_2 \) are non-negative integers satisfying \( L_1 + L_2 = L \leq K \), and \( (x)^+ \triangleq x \) if \( x > 0 \) and \( 0 \) otherwise. The second outer bound is based on providing side information to the receivers in a carefully chosen manner, in the form of a noisy version of the intended message. The outer bound is stated in the theorem below.

**Theorem 3.2:** The sum rate of the MIMO Gaussian IC is upper bounded as follows.

\[
R_1 + 2 \sum_{i=2}^{K-1} R_i + R_K \leq \sum_{i=1}^{K-1} \log |I_{N_i}| + \sum_{i=1}^{K} \Phi_{i,j} + \Psi_{i+1, i} + \sum_{i=2}^{K} \log |I_{N_i}| + \sum_{j=1, j \neq i}^{K} \Phi_{i,j} + \Psi_{i-1, i} + \epsilon_n.
\]

In the above, \( \Phi_{i,j} \triangleq \mathbf{H}_{ij} \mathbf{P}_{j} \mathbf{H}_{ji}^H \) and \( \Psi_{j,i} \triangleq \mathbf{H}_{ii} \mathbf{P}_{j}^{1/2} \left\{ \mathbf{I}_M + \mathbf{P}_{j}^{1/2} \mathbf{H}_{ij}^H \mathbf{H}_{ji} \mathbf{P}_{j}^{1/2} \right\}^{-1} \mathbf{P}_{j}^{1/2} \mathbf{H}_{ii}^H \), where \( \mathbf{P}_{j} \) is the transmit covariance matrix of the \( j \)-th user. The sum rate bound above is general in the sense that it is valid for all possible channel parameters. Specializing the bound to the case of the symmetric MIMO Gaussian IC, the following is obtained.

**Theorem 3.3:** The per-user GDOF of the \( K \) user MIMO Gaussian IC in the symmetric case is upper bounded as \( d(\alpha) \leq \)

\[
\begin{cases}
M(1 - \alpha) + \min \{ L, N - M \} \alpha, & 0 \leq \alpha \leq \frac{1}{2} \\
L \alpha + \min \{ M, N - L \} (1 - \alpha), & \frac{1}{2} \leq \alpha \leq 1 \\
L \alpha, & \alpha \geq 1
\end{cases}
\]

where \( L \triangleq \min \{ N, (K-1)M \} \).

The third outer bound is based on providing each receiver with side information comprising a noisy version of a carefully chosen part of the interference experienced by it. For the SIMO case, this bound reduces to the outer bound presented in [9].

**Theorem 3.4:** When \( N/M < K \leq N/M + 1 \), the per-user symmetric GDOF of the \( K \)-user MIMO Gaussian IC is upper bounded as \( d(\alpha) \leq \)

\[
\begin{cases}
M(1 - \alpha) + \min \{ L, N - M \} \alpha/(K - 1), & 0 \leq \alpha \leq \frac{1}{2} \\
[L \alpha + (K - 2)M(1 - \alpha)
+ \min \{ M, N - L \} (1 - \alpha)]/(K - 1), & \frac{1}{2} \leq \alpha \leq 1 \\
L \alpha/(K - 1), & \alpha > 1,
\end{cases}
\]

where \( L \triangleq \min \{ N, (K-1)M \} \), as before.

IV. INNER BOUNDS

Before stating the inner bounds, the following known results on the achievable DOF using IA and Zero-Forcing (ZF) reception are recapitulated. In [9], it is shown that for a time-varying channel, the achievable symmetric DOF per user using IA, denoted \( d^{IA} \), is

\[
d^{IA} = MN/(M + N), \quad \text{for } N < KM.
\]

The achievable DOF per user with ZF reception is

\[
d^{ZF} = \min \{ M, N/K \}.
\]
Note that the relative strength between the signal and interference (i.e., $\alpha$) does not matter for IA or ZF reception to work. Now, the new inner bounds on the GDOF of the $K$-user symmetric Gaussian IC are presented as Theorems 4.1, 4.2, 4.4 and 4.5 in this section. The final inner bound is obtained by taking the maximum of all the inner bounds.

A. Treating Interference as Noise

Treating interference as noise is one of the simplest methods of dealing with interference, and may work well when the interference is weak. The following theorem summarizes the GDOF obtained by treating interference as noise when the input covariance matrix is assumed to be full rank.

**Theorem 4.1:** The following per user GDOF is achievable for the $K$-user symmetric MIMO Gaussian IC:

1. When $N/M < K \leq N/M + 1$, $d(\alpha) \geq M + \alpha(N - KM)$.
2. When $K > N/M + 1$, $d(\alpha) \geq M(1 - \alpha)$.

In the above, the condition $K \leq N/M$ is omitted, because in that case, (5) achieves the maximum possible GDOF of $M$ per user, i.e., ZF is optimal.

B. The Han-Kobayashi (HK) Scheme

In this section, the HK-type message splitting scheme employed in [1], [6] and [9] is extended to the $K$-user symmetric Gaussian MIMO IC. Extension of the HK-scheme to the multiuser MIMO case is neither straightforward nor unique. Such one generalization is shown here. Three interference regimes are considered: the strong interference case ($\alpha \geq 1$), the moderate interference case ($1/2 \leq \alpha \leq 1$), and the weak interference case ($0 \leq \alpha \leq 1/2$). The input covariance matrix is assumed to be full rank in all these regimes.

1) **Strong Interference Case:** When the interference is strong, each receiver can decode both the unintended messages as well as the intended message. Hence, a $K$ user MAC channel is formed at each receiver, and the achievable rate region is the intersection of the $K$ MAC regions obtained. This results in the following inner bound on the per user GDOF.

**Theorem 4.2:** In the strong interference regime ($\alpha \geq 1$), the following per user GDOF is achievable by the HK scheme:

1. When $N/M < K \leq N/M + 1$,
   
   $d(\alpha) \geq \min\{M, (K - 1)M + N - (K - 1)M\}/K$.

2. When $K > N/M + 1$,
   
   $d(\alpha) \geq \min\{M, N\alpha/K\}$.

**Proof:** Due to lack of space, only the proof for $K > N/M + 1$ is provided below. The proof below requires the following minor variation of Lemma 1 in [6]:

**Lemma 4.3:** Let $Q_1$ and $Q_2$ be $N \times N$ covariance matrices with rank $r_1$ and $r_2$, respectively. Then, for $\eta \geq \beta$,

$$J_1 \triangleq \log |I + \rho^\alpha Q_1 + \rho^\beta Q_2| = r_1 \eta \log \rho + \min(r_2, N - r_1) \beta \log \rho + O(1),$$

where $O(1)$ is the approximation error, bounded above by some constant.

Due to the symmetry of the problem considered, it is sufficient to consider the GDOF achieved by any particular user, say user 1. Also consider the user subset $S \subseteq \{2, \ldots, K\}$, and let $S' \triangleq S \cup \{1\}$, i.e., $S$ is a subset of users excluding user 1, while $S'$ always includes user 1 and $|S|$ denotes the number of users in the set $S$. Now, using the MAC channel formed at the receiver of user 1 with the signals from $S$, the achievable sum rate is bounded as

$$\sum_{j \in S} R_j \leq \log |I + \rho \sum_{j \in S} H_{1j} H_{1j}^H|. \quad (7)$$

Using Lemma 4.3 and the fact that the rates are symmetric, the above reduces to

$$R_j \leq \min\{|S| M, N \log \rho + O(1), \quad |S| M \leq N, \quad |S| M > N, \quad (8)$$

Similarly, using the MAC channel formed at the receiver of user 1 with the signals from the user set $S'$, the achievable sum rate is bounded as

$$\sum_{j \in S'} R_j \leq \log |I + \rho H_{11} H_{11}^H + \rho^\alpha \sum_{j \in S} H_{1j} H_{1j}^H|. \quad (9)$$

Again, using Lemma 4.3, the above simplifies to

$$\sum_{j \in S} R_j \leq \min\{|S|M, N\alpha / |S|\log \rho + O(1), \quad |S|M \leq N, \quad (10)$$

When $N/M \leq |S| \leq K - 1$, (10) reduces to

$$R_j \leq (\alpha N)/(|S| + 1) \log \rho + O(1). \quad (11)$$

When $N/M - 1 \leq |S| \leq N/M$, (10) becomes

$$R_j \leq \left[ (\alpha - 1)M + \frac{N + M(1 - \alpha)}{|S| + 1} \right] \log \rho + O(1). \quad (12)$$

Also, when $0 \leq |S| \leq N/M - 1$, (10) simplifies to

$$R_j \leq M(|S| + 1)/(|S| + 1) \log \rho + O(1). \quad (13)$$

Finally, taking the minimum of (8), (11), (12) and (13) over the range $|S| = 0, 1, \ldots, K - 1$ yields the desired result.

2) **Moderate Interference Case:** In this regime, one achievable scheme based on the HK-type message splitting is as follows. The transmitter $j$ splits its message $W_j$ into two sub-messages, a common message $W_{c,j}$ that is required to be decodable at every receiver, and a private message $W_{p,j}$ that is only required to be decodable at the desired receiver. The common message is encoded using a Gaussian codebook with rate $R_{c,j}$ and power $P_{c,j}$. Similarly, the private message is encoded using a Gaussian codebook with rate $R_{p,j}$ and power $P_{p,j}$. The code words are transmitted using superposition coding. Further, it is assumed that the rates are symmetric, i.e. $R_{c,j} = R_c$ and $R_{p,j} = R_p$. Also, $P_{c,j} = P_c$ and $P_{p,j} = P_p$. The power on the private and common messages satisfy the constraint $P_c + P_p = 1$. Similar to [6], the power in the private message is set such that it is received at the noise floor of the unintended receivers, resulting in $INR_p = 1$. Coupled with the transmit power constraint at each of the users, the SNRs of the
common and private parts at the desired signal (denoted SNR_c and SNR_p) and the INRs of the common and private parts at unintended receivers (denoted INR_c and INR_p) are given by

\[ SNR_c = \rho - \rho^{1-\alpha}, \quad SNR_p = \rho^{1-\alpha}, \quad INR_c = \rho^\alpha - 1, \quad INR_p = 1. \]

The decoding order is such that the common message is decoded first, followed by the private message. The GDOF is contributed by both the private and public parts of the message:

\[ d(\alpha) = d_p(\alpha) + d_c(\alpha), \tag{14} \]

where \( d_p(\alpha) \) and \( d_c(\alpha) \) are the GDOF contributed by the private and public parts of the message, respectively. The following theorem summarizes GDOF achievable by this scheme.

Theorem 4.4: In the moderate interference regime (1/2 ≤ \( \alpha \leq 1 \)), the \( K \)-user symmetric MIMO Gaussian IC achieves the following per user GDOF:

1) When \( N/M < K \leq N/M + 1 \), \( d(\alpha) \geq M(1-\alpha) + \min \left\{ \frac{N\alpha}{K}, \frac{[M(2K-1)-K] + N(1-\alpha)]}{K-1} \right\} \).

2) When \( K > N/M + 1 \), \( d(\alpha) \geq M(1-\alpha) + \min \left\{ \frac{N\alpha}{K}, \frac{[N\alpha - (M-1)]}{K-1} \right\} \).

3) Weak Interference Case: An achievable GDOF in the weak interference regime can be obtained by using the same HK-type scheme described in the previous subsection. The result is summarized in the following theorem.

Theorem 4.5: In the weak interference regime (0 ≤ \( \alpha \leq 1/2 \)), the following GDOF is achievable per user in a symmetric MIMO Gaussian IC, with \( K > N/M \):

\[ d(\alpha) \geq M(1-\alpha) + (N - M)\alpha/(K-1). \]

V. DISCUSSION

First, some observations on where the inner and outer bounds on the GDOF derived above stand in relation to existing work are as follows:

- By setting \( M = 1 \) and \( K = N + 1 \), the Theorem 3.4 and the HK-type achievable scheme in Sec. IV reduce to the corresponding SIMO GDOF results in [9].
- By setting \( K = 2 \), the inner and outer bounds derived here reduce to the corresponding two-user symmetric GDOF result in [6].
- By setting \( M = N = 1 \) and \( K = 2 \), the inner and outer bounds derived here reduce to the corresponding GDOF results derived in [1].
- By setting \( M = N = 1 \), the inner bounds derived here match with the result in [8] only in the weak interference regime. In other regimes, [8] assumes the constant IC model and uses lattice coding to achieve a higher GDOF.
- By setting \( \alpha = 1 \), the cooperative outer bound of Theorem 3.1 matches with the DOF outer bound in [12] for many cases of \( K, M \) and \( N \) (e.g., \( K = 3, M = 2, N = 5 \)). Theorem 3.1 uses genie-aided message sharing in addition to cooperation, to handle the \( \alpha \neq 1 \) cases. The bound in [12] only requires cooperation, due to which it is lower for some values of \( M, N \) and \( K \). Due to marginal difference between the outer bounds, the outer bound in [12] is not taken into account at \( \alpha = 1 \), in the plots below. Next, some specific examples are considered to get a better insight on the inner and outer bounds for different cases of \( K, M, N \), and \( \alpha \).

In Fig. 1, the per user GDOF is plotted versus \( \alpha \) for \( K = 3 \) and \( M = N = 2 \). In the weak interference regime, the inner bound coincides with the outer bound. In this case, treating interference as noise performs as well as the HK scheme. Also, IA and ZF are suboptimal in this case. At \( \alpha = 1/2 \), IA, HK and treating interference as noise all coincide. In the moderate interference case, the flat segment is due to IA, and it is optimal for \( \alpha = 1/2 \) and \( \alpha = 1 \). In terms of outer bounds, initially, the side-information based bound of Theorem 3.3 is active, and as \( \alpha \) increases, the cooperative bound of Theorem 3.1 is active. In the high interference regime, IA initially performs the best, and as \( \alpha \) increases, the HK-scheme performs the best, and finally achieves the interference free GDOF. There exists a gap between the inner and outer bounds in the moderate and high interference regimes.

Figure 2 shows the per user GDOF plot for \( K = 3 \), \( M = 2 \) and \( N = 3 \). In the weak interference regime, the HK scheme outperforms treating interference as noise. In the moderate interference regime, the HK scheme initially performs the best, while the IA scheme performs better for larger \( \alpha \). In the high interference regime, the observations are similar to Fig. 1.

In Fig. 3, the per user GDOF is plotted versus \( \alpha \) for \( K = 3 \), \( M = 2 \) and \( N = 5 \). In this case, the inner bound and the outer bound coincide, and HK scheme is GDOF optimal. At \( \alpha = 1 \), the ZF scheme coincides with HK scheme. The outer bound of Theorem 3.4 is active in the weak interference regime and in the initial part of the moderate interference regime.

The observations above can be easily inferred from the GDOF bounds derived in the previous sections. In summary, some of the new insights that can be obtained from the \( K \)-user GDOF bounds derived in this paper are as follows:

1) When \( N = M \), treating interference as noise in the weak interference regime is GDOF optimal and Theorem 3.4 establishes this fact, which was not known before.
2) When \( N/M < K \leq N/M + 1 \), the HK scheme is GDOF optimal for all regimes of interference.
3) When \( K > N/M + 1 \), a combination of the HK scheme and the IA outperforms the other schemes considered.
4) When \( N > M \), in the weak interference regime, the HK scheme outperforms treating interference as noise.
5) The ZF bound is found to be DOF optimal at \( \alpha = 1 \) when \( N/M < K \leq N/M + 1 \). Moreover, while the ZF bound is GDOF optimal for \( \alpha = 1 \) and \( \alpha = 1/2 \) when \( K = 2 \) [6], this is not the case when \( K > 2 \).

VI. CONCLUSION

This work derived inner and outer bounds on the GDOF of the \( K \)-user symmetric MIMO Gaussian interference channel as a function of \( \alpha = \log \text{INR}/\log \text{SNR} \). The outer bound was
based on a combination of three schemes, one of which was derived using the notion of cooperation, and the two other outer bounds were based on providing partial side information at the receivers. The inner bound was derived using a combination of ZF-receiving, treating interference as noise, IA, and the HK scheme. Several interesting insights were obtained from the derived bounds. For example, it was found that when $M = N$, treating interference as noise performs as well as the HK scheme and outperforms both IA and the ZF bound in the weak interference regime. However, when $N > M$, treating interference as noise is always suboptimal. For $K > N/M + 1$, a combination of HK and IA performs the best in the moderate interference regime. Finally, when $N/M < K \leq N/M + 1$, the HK scheme is GDOF optimal for all values of $\alpha$.

Acknowledgements: The authors thank B. Sundar Rajan and the anonymous reviewers for helpful suggestions.

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