Dynamical picture for the exotic XYZ states

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Abstract. We present a dynamical approach for description of the multi-quark states that is based on an effective interaction Lagrangian describing the coupling of hadrons to their constituent quarks. First, we explore the consequences of treating the $X(3872)$ meson as a tetraquark bound state. We calculate the decay widths of the observed channels and conclude that for reasonable values of the size parameter of the $X(3872)$ one finds consistency with the available experimental data. Then we have critically checked the tetraquark picture for the $Z_c(3900)$ state by analyzing its strong decays. We found that $Z_c(3900)$ has a much more stronger coupling to $DD^*$ than to $J/\psi\pi$ that is in discord with experiment. As an alternative we have employed a molecular-type four-quark current to describe the decays of the $Z_c(3900)$ state as the charged particle in the isotriplet. We found that a molecular-type current gives the values of the above decays in accordance with the experimental observation. By using molecular-type four-quark currents for the recently observed resonances $Z_b(10610)$ and $Z_b(10650)$, we have calculated their two-body decay rates into a bottomonium state plus a light meson as well as into B-meson pairs.

1 Introduction

The most reliable prediction of the quark model is the spectrum of conventional hadrons composed either from quark-antiquark (mesons) or from three quarks (baryons). These states have been observed in experiments and their properties have been carefully studied by experimentalists as well as theorists for a long time. However, this situation has been changed since 2003 with the discovery of many charmonium- and bottomonium-like XYZ states that do not fit the simple quark-antiquark interpretation. The specific feature of these states is that their masses are close to meson-meson thresholds. There are several theoretical interpretations of them, e.g., as hadronic molecules, tetraquarks, threshold cusps, etc. (for review, see Refs. [1–3]).

The $Y$ states are neutral with quantum numbers $J^{PC} = 1^{--}$ of charmonium but not the simple $c\bar{c}$ charmonium. Among the observed $Y$ states are $Y(4005)$, $Y(4260)$, $Y(4360)$, etc. The $Z$ states ($Z_c$ and $Z_b$) are basically charged, for example, $Z_c^+$ has quark content $c\bar{c}ud$ so it is certainly exotic state. The $Z_b^+$ has quark content $b\bar{b}ud$. The $X$ states are the non-$q\bar{q}$ mesons but other than the $Y's$ and $Z's$. The most famous is the $X(3872)$ with quantum numbers $J^{PC} = 1^{++}$.

We present a dynamical approach for description of the multi-quark states that is based on an effective interaction Lagrangian describing the coupling of hadrons to their constituent quarks.
quarks. We give a brief sketch of our results on the decay widths of some exotic four-quark states obtained within this framework and published in series of our papers of Refs. [4–9].

First, we explore the consequences of treating the $X(3872)$ meson as a tetraquark bound state. We calculate the decay widths of the observed channels and conclude that for reasonable values of the size parameter of the $X(3872)$ one finds consistency with the available experimental data. Then we have critically checked the tetraquark picture for the $Z_c(3900)$ state by analyzing its strong decays. We found that $Z_c(3900)$ has a much more stronger coupling to $DD^*$ than to $J/ψπ$ that is in discord with experiment. As an alternative we have employed a molecular-type four-quark current to describe the decays of the $Z_c(3900)$ state as the charged particle in the isosinglet. We found that a molecular-type current gives the values of the above decays in accordance with the experimental observation. By using molecular-type four-quark currents for the recently observed resonances $Z_{b}(10610)$ and $Z_{b}(10650)$, we have calculated their two-body decay rates into a bottomonium state plus a light meson as well as into B-meson pairs.

2 Dynamical picture for multiquark states: covariant confined quark model

The main assumption of approach is that hadrons interact via quark exchange only. The interaction Lagrangian is written as

$$\mathcal{L}_{\text{int}} = g_H \cdot H(x) \cdot J_H(x),$$

where the quark currents are given by

$$J_M(x) = \int dx_1 \int dx_2 F_M(x; x_1, x_2) \cdot \bar{q}_1^a(x_1) \Gamma_M q_2^a(x_2) \quad \text{Meson}$$

$$J_B(x) = \int dx_1 \int dx_2 \int dx_3 F_B(x; x_1, x_2, x_3) \times \Gamma_1 q_1^{a_1}(x_1) \left( q_2^{a_2}(x_2) C \Gamma_2 q_3^{a_3}(x_3) \right) \cdot \epsilon^{a_1 a_2 a_3}$$

$$J_T^a(x) = \int dx_1 \ldots \int dx_4 F_T(x; x_1, \ldots, x_4) \times \left( q_1^{a_1}(x_1) C \Gamma_1 q_2^{a_2}(x_2) \right) \cdot \left( q_3^{a_3}(x_3) C \Gamma_2 q_4^{a_4}(x_4) \right) \cdot \epsilon^{a_1 a_2 a_3}.$$  

Here, the $F(x; x_1, \ldots, x_n)$ are the vertex functions characterizing the distribution of quarks inside the hadron. The matrix elements of the physical processes are described by the Feynman diagrams which are convolution of the quark propagators and vertex functions. The details of all calculations may be found in our published papers of Refs. [4–9]. For illustration, we show in Fig. 1 the tetraquark self-energy diagram which is needed for the determination of coupling constant $g_T$ in the interaction Lagrangian.

![Figure 1. Tetraquark self-energy diagram](image-url)
A narrow charmonium–like state $X(3872)$ was observed in 2003 in the exclusive decay process $B^+ \rightarrow K^+ \pi^+ \pi^- J/\psi$ [10]. A mass was found to be very close to the $D^0 D^{*0}$ threshold and width less than 2.3 MeV. The state was confirmed in B-decays by the BaBar experiment [11] and in $p\bar{p}$ production by the Tevatron experiments CDF [12] and DØ[13]. From the angular analysis performed by several collaborations it was shown that the $X(3872)$ state has the quantum number $J^{PC}=1^{++}$. Then it was found that the branching ratios of the modes $X \rightarrow \pi^+ \pi^- J/\psi$ and $X \rightarrow \pi^+ \pi^- \pi^0 J/\psi$ are almost the same that imply strong isospin violation.

There are several different interpretations of the $X(3872)$ in the literature: a molecule bound state ($D^0 D^{*0}$), threshold cusps, hybrids and glueballs.

We employed the diquark-antidiquark interpretation suggested in Ref. [14] and performed an independent analysis of the properties of the $X(3872)$ within our covariant confined quark model in the papers Refs. [4, 5]. The quark content was chosen as $[cq]_{S=0} [\bar{c}\bar{q}]_{S=1} + [cq]_{S=1} [\bar{c}\bar{q}]_{S=0} , (q = u, d)$ [14]. The physical states are supposed to be a linear superposition of the $X_u$ and $X_d$ states according to

$$X_t = X_{\text{low}} = X_u \cos \theta + X_d \sin \theta, \quad X_h = X_{\text{high}} = -X_u \sin \theta + X_d \cos \theta.$$  

The mixing angle $\theta$ is determined from fitting the ratio of branching ratios of the $X$–decays into $J/\psi + 2\pi$ and $J/\psi + 3\pi$.

The diagrams describing the strong decays $\Gamma(X \rightarrow J/\psi + n \pi)$ ($n=2,3$) and $X \rightarrow D^0 \bar{D}^{*0} \pi^0$ are shown in Fig. 2. The diagrams describing the radiative decays $\Gamma(X \rightarrow J/\psi + \gamma)$ are shown in Fig. 3.

*Figure 2. Diagrams describing the strong X-decays*

The only free parameter is the so-called size parameter $\Lambda_X$ which appears in the vertex function $F_T(x; x_1, \ldots, x_4)$. We plot the dependence of the calculated decay widths on this parameter in Fig. 4 in quite large range of $\Lambda_X \in [2.5 - 4]$ GeV.

Finally, we compare the obtained results for the ratios of decay widths with the available experimental data. One can see that there is reasonable agreement in the case of the strong decays and very good agreement in the case of the radiative decays.

$$\frac{\Gamma(X \rightarrow D^0 \bar{D}^{*0} \pi^0)}{\Gamma(X \rightarrow J/\psi + n \pi)} = \begin{cases} 4.5 \pm 0.2 & \text{theor} \\ 10.5 \pm 4.7 & \text{expt} \end{cases}, \quad \frac{\Gamma(X \rightarrow J/\psi + \gamma)}{\Gamma(X \rightarrow J/\psi + 2\pi)} = \begin{cases} 0.15 \pm 0.03 & \text{theor} \\ 0.14 \pm 0.05 & \text{expt} \end{cases}.$$  

4 $Z_c(3900)$

The BESIII Collaboration [15] observed a structure in the $\pi^\pm J/\psi$ mass spectrum of the process $e^+ e^- \rightarrow \pi^\pm \pi^- J/\psi$. It was named as the $Z_c(3900)$ state with a mass near 3.9 GeV and a
width around of 50 MeV. This state was confirmed by the Belle Collaboration [16]. Recently, the D0 Collaboration presented evidence for the exotic charged $Z_c(3900)$ in semi-inclusive weak decays of $b$-flavored hadrons [17]. This structure can be interpreted as a new charged charmonium-like state. Mode with $D\bar{D}$ in the final state was studied by the BESIII Collaboration [18]. A distinct charged structure was observed in the $(D\bar{D})^\pm$ invariant mass distribution of the process $e^+e^-\rightarrow \pi^\pm (D\bar{D})^\mp$. The angular distribution of the $\pi Z_c(3885)$ system favors a $J^P = 1^+$ quantum number assignment for the new structure. The ratio of partial widths was determined as

$$\frac{\Gamma(Z_c(3885) \rightarrow D\bar{D}^\pm)}{\Gamma(Z_c(3900) \rightarrow \pi J/\psi)} = 6.2 \pm 1.1 \pm 2.7$$ (1)
Assuming that the $Z_c$ the charged partner of the $X(3872)$ state one can write down a tetraquark-type current:

$$J^\mu = \frac{i}{\sqrt{2}} \varepsilon_{abc} \varepsilon_{dec} \left[ (u_a^T C\gamma_5 c_b)(\bar{d}_d \gamma^\mu C\bar{c}_c) - (u_a^T C\gamma_5 c_b)(\bar{d}_d \gamma^\mu C\bar{c}_c) \right]$$  \hspace{1cm} (2)

In Ref. [6] we have used the nonlocal generalization of this tetraquark current to calculate a number of the $Z_c(399)$ two-body decay widths of the process $1^+(p,\mu) \rightarrow 1^-(q_1, \nu) + 0^-(q_2)$. It was found that, in our model, the leading Lorentz metric structure in the matrix elements describing the decays $Z_c(3900) \rightarrow DD^*$ vanishes analytically. This results in a significant suppression of these decay widths by the smallness of the relevant phase space factor $|\mathbf{q}|^5$.

Since the experimental data [18] show that the $Z_c(3900)$ has a much more stronger coupling to $DD^*$ than to $J/\psi \pi$, one has to conclude that the tetraquark-type current for the $Z_c(3900)$ is in discord with experiment.

As an alternative we have employed a molecular-type four-quark current to describe the decays of the $Z_c(3900)$ state:

$$J^\mu = \frac{1}{\sqrt{2}} \left[ (\bar{d}_d \gamma_5 c)(\bar{c} \gamma^\mu u) + (\bar{d}_d \gamma_5 c)(\bar{c} \gamma^\mu u) \right].$$  \hspace{1cm} (3)

In this case we found that for a relatively large size parameter $\Lambda_{Z_c} \sim 3.3 \text{ GeV}$ one can obtain the partial widths of the decays $Z_c^+(3900) \rightarrow DD^*$ at the order $\sim 15 \text{ MeV}$ for each mode. At the same time the partial widths for decays $Z_c^+(3900) \rightarrow J/\psi \pi^+$, $\eta_c \rho^*$ are suppressed by a factor of $6 - 7$ in accordance with experimental data [18]. If the $\Lambda_{Z_c}$ is varied in the limits $\Lambda_{Z_c} = 3.3 \pm 1.1 \text{ GeV}$ then one has

$$\Gamma(Z_c^+ \rightarrow J/\psi + \pi^+) = (1.8 \pm 0.3) \text{ MeV}, \quad \Gamma(Z_c^+ \rightarrow D^0 + D^{*+}) = (10.0^{+1.7}_{-1.4}) \text{ MeV},$$
$$\Gamma(Z_c^+ \rightarrow \eta_c + \rho^+) = (3.2^{+0.5}_{-0.4}) \text{ MeV}, \quad \Gamma(Z_c^+ \rightarrow D^{*0} + D^{*+}) = (9.6^{+1.6}_{-1.3}) \text{ MeV}.$$

Preliminary data from BESIII cited in [19] were reported for the ratio

$$R(Z) = \frac{B(Z_c(3900) \rightarrow \rho \eta_c)}{B(Z_c(3900) \rightarrow \pi J/\psi)} = 2.1 \pm 0.8.$$  \hspace{1cm} (4)

They agree very well with our finding $R(Z) = 1.8 \pm 0.4$.

## 5 $Z_b(10610)$ and $Z'_b(10610)$

A few years ago the Belle Collaboration [20] reported on the observation of two charged bottomoniumlike resonances in the mass spectra of $\pi^+\Upsilon(nS)$ ($n = 1, 2, 3$) and $\pi^+ h_b(mP)$ ($m = 1, 2$) in the decays $\Upsilon(5S) \rightarrow \Upsilon(nS)\pi^+\pi^-$, $h_b(mP)\pi^+\pi^-$. The measured masses and widths were given by

$$M_{Z_b} = (10507.2 \pm 2.0) \text{ MeV}, \quad \Gamma_{Z_b} = (18.4 \pm 2.4) \text{ MeV},$$
$$M_{Z'_b} = (10652.2 \pm 1.5) \text{ MeV}, \quad \Gamma_{Z'_b} = (11.5 \pm 2.2) \text{ MeV}.$$  

and the quantum numbers are $I^G(J^P) = 1^+(1^+)$. The existence of these two states was later confirmed by the same collaboration [21, 22] in differing decay channels. In the paper [22] the Belle Collaboration reported on the results of an analysis of the three-body processes $e^+e^- \rightarrow BB\pi^\pm$, $BB^*\pi^\pm$, and $B^*B^*\pi^\pm$. It was found that the transitions $Z_b^\pm(10610) \rightarrow [BB^* + \text{c.c.}]^\pm$ and $Z'_b(10650) \rightarrow [B^*B^*]^\pm$ dominate among the corresponding final states.
Table 1. $Z_b(10610)$ and $Z_b'(10610)$: numerical results

| Channel         | $Z_b(10610)$ | $Z_b'(10610)$ |
|-----------------|--------------|---------------|
| $\Upsilon(1S)\pi^+$ | 5.9 ± 0.4    | 9.5$^{+0.7}_{-0.6}$ |
| $h_b(1P)\pi^+$   | (0.14 ± 0.01) · 10$^{-1}$ | 0.74$^{+0.05}_{-0.04}$ · 10$^{-3}$ |
| $\eta_b\rho^+$   | 4.4 ± 0.3    | 7.5$^{+0.6}_{-0.5}$ |
| $B^+\bar{B}^0 + \bar{B}^0B^{*-}$ | 20.7$^{+1.6}_{-1.5}$ | – |
| $B^{*-}B^0$      | –            | 17.1$^{+1.5}_{-1.4}$ |

Table 2. $Z_b(10610)$ and $Z_b'(10610)$: Total widths, MeV

|                  | Theory | Belle Expt. |
|------------------|--------|-------------|
| $Z_b(10610)$     | 30.9$^{+2.3}_{-2.1}$ | 25 ± 7 |
| $Z_b'(10650)$    | 34.1$^{+2.8}_{-2.5}$ | 23 ± 8 |

Since the masses of the $Z_b^*$ (10610) and $Z_b'(10650)$ are very close to the respective $B^*\bar{B}$ (10604 MeV) and $B^*\bar{B}^*$ (10649 MeV) thresholds, it was suggested in Ref. [23] that they have molecular-type binding structures.

$$J_{Z_b}^\mu = \frac{1}{\sqrt{2}} \left[ (\bar{d}\gamma_5b)(\bar{b}\gamma^\mu u) + (\bar{d}\gamma^\mu b)(\bar{b}\gamma_5u) \right], \quad J_{Z_b'}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta}(\bar{d}\gamma_\alpha b)(\bar{b}\gamma_\beta u).$$ \hspace{1cm} (5)

Such a choice guarantees that the $Z_b$-state can only decay to the $[\bar{B}^*B + c.c.]$ pair whereas the $Z_b'$-state can decay only to a $\bar{B}^*B^*$ pair. Decays into the $BB$-channels are forbidden. Due to $G$-parity conservation $Z_b \to \Upsilon + \rho$, $Z_b \to \eta_b + \pi$, $Z_b \to \chi_{b1} + \pi$, $Z_b \to h_b + \rho$. The decay $Z_b \to \chi_{b1} + \rho$ is not allowed kinematically.

There are therefore only the three allowed decays: $Z_b^+ \to \Upsilon + \pi^+$, $Z_b^+ \to h_b + \pi^+$ and $Z_b^+ \to \eta_b + \rho^+$. The only two new parameters are the size parameters of the two exotic $Z_b(Z_b')$ states. As a guide to adjust them we take the experimental values of the largest branching fractions presented by Belle:

$$\mathcal{B}(Z_b^+ \to [B^+\bar{B}^0 + \bar{B}^0B^{*-}]) = 85.6^{+1.5}_{-2.0-2.1}$ \%, \quad \mathcal{B}(Z_b^+ \to \bar{B}^0B^{*-}) = 73.7^{+3.4}_{-4.4-3.5}$ \% \hspace{1cm} (6)

By using the central values of these branching rates and total decay widths we find the central values of our size parameters $\Lambda_{Z_b} = 3.45$ GeV and $\Lambda_{Z_b'} = 3.00$ GeV. Allowing them to vary in the interval $\Lambda_{Z_b} = 3.45 \pm 0.05$ GeV and $\Lambda_{Z_b'} = 3.00 \pm 0.05$ GeV we obtain the values of various decay widths.

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