Strong forms of generalized closed sets in ditopological texture spaces

Hariwan Z. Ibrahim

Department of Mathematics, Faculty of Science, University of Zakho, Kurdistan-Region, Iraq.

Abstract

The purpose of this paper is to introduce the new concepts namely, $\alpha\text{-}g$-closed, pre-$g$-closed, semi-$g$-closed, $b$-$g$-closed, $\beta$-$g$-closed, $\alpha$-$g$-open, pre-$g$-open, semi-$g$-open, $b$-$g$-open and $\beta$-$g$-open sets in ditopological texture spaces. The relationships between these classes of sets are also obtained. Also some properties and several characterizations are investigated.

Keywords: Texture, ditopology, $g$-closed, $\alpha$-$g$-closed, pre-$g$-closed, semi-$g$-closed, $b$-$g$-closed and $\beta$-$g$-open.

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1. Introduction

Textures and ditopological texture spaces were first introduced by L. M. Brown as a point-based setting for the study fuzzy topology. $\alpha$-open sets, pre-open sets, semi-open sets, $b$-open sets, $\beta$-open sets and $g$-closed sets in ditopological texture spaces were studied by [2], [5], [3], [7], [4] and [6], respectively. In this paper, the notions of $\alpha$-$g$-closed, pre-$g$-closed, semi-$g$-closed, $b$-$g$-closed, $\beta$-$g$-closed, $\alpha$-$g$-open, pre-$g$-open, semi-$g$-open, $b$-$g$-open and $\beta$-$g$-open sets sets are introduced. The fundamental properties of such sets are studied.

2. Preliminaries

The following are some basic definitions of textures we will need later on.

Texture space: Let $S$ be a set. Then $\varphi \subseteq P(S)$ is called a texturing of $S$, and $S$ is said to be textured by $\varphi$ if

(1) $(\varphi, \subseteq)$ is a complete lattice containing $S$ and $\emptyset$ and for any index set $I$ and $A_i \in \varphi$, $i \in I$, the meet $\bigwedge_{i \in I} A_i$ and the join $\bigvee_{i \in I} A_i$ in $\varphi$ are related with the intersection and union in $P(S)$ by the equalities

$$\bigwedge_{i \in I} A_i = \bigcap_{i \in I} A_i$$

$$\bigvee_{i \in I} A_i = \bigcup_{i \in I} A_i$$
for all $I$, while
\[ \bigvee_{i \in I} A_i = \bigcup_{i \in I} A_i \]
for all finite $I$.

(2) $\varphi$ is completely distributive.

(3) $\varphi$ separates the points of $S$. That is, given $s_1 \neq s_2$ in $S$ we have $L \in \varphi$ with $s_1 \in L$, $s_2 \notin L$, or $L \notin \varphi$ with $s_2 \in L$, $s_1 \notin L$.

If $S$ is textured by $\varphi$ then $(S, \varphi)$ is called a texture space, or simply a texture.

**Complementation:** \[ 1 \] A mapping $\sigma : \varphi \to \varphi$ satisfying $\sigma(\sigma(A)) = A$, $\forall A \in \varphi$ and $A \subseteq B \Rightarrow \sigma(B) \subseteq \sigma(A)$, $\forall A, B \in \varphi$ is called a complementation on $(S, \varphi)$ and $(S, \varphi, \sigma)$ is then said to be a complemented texture.

**Ditopology:** \[ 2 \] A dichotomous topology on a texture $(S, \varphi)$, or ditopology for short, is a pair $(\tau, k)$ of subsets of $\varphi$, where the set of open sets $\tau$ satisfies

(1) $S, \phi \in \tau$,

(2) $G_1, G_2 \in \tau \Rightarrow G_1 \cap G_2 \in \tau$, and

(3) $G_i \in \tau, i \in I \Rightarrow \bigvee_i G_i \in \tau$,

and the set of closed sets $k$ satisfies

(1) $S, \phi \in k$,

(2) $K_1, K_2 \in k \Rightarrow K_1 \cup K_2 \in k$, and

(3) $K_i \in k, i \in I \Rightarrow \bigcap_i K_i \in k$.

Hence a ditopology is essentially a "topology" for which there is no a priori relation between the open and closed sets. For $A \in \varphi$ we define the closure $[A]$ and the interior $]A[$ of $A$ under $(\tau, k)$ by the equalities

\[ [A] = \bigcap \{K \in k : A \subseteq K\} \]

and

\[ ]A[ = \bigvee \{G \in \tau : G \subseteq A\}. \]

We refer to $\tau$ as the topology and $k$ as the cotopology of $(\tau, k)$. If $(\tau, k)$ is a ditopology on a complemented texture $(S, \varphi, \sigma)$, then we say that $(\tau, k)$ is complemented if the equality $k = \sigma[\tau]$ is satisfied. In this study, a complemented ditopological texture space is denoted by $(S, \varphi, \tau, k, \sigma)$. In this case we have $\sigma([A]) = [\sigma(A)]$ and $\sigma([A]) = [\sigma(A)]$. We denote by $O(S, \varphi, \tau, k)$, or when there can be no confusion by $O(S)$, the set of open sets in $\varphi$. Likewise, $C(S, \varphi, \tau, k)$, $C(S)$ will denote the set of closed sets.

**Definition 2.1.** Let $(S, \varphi, \tau, k, \sigma)$ be a ditopological texture space. A set $A \in \varphi$ is called:

(1) $\alpha$-open ($\alpha$-closed) \[ 2 \] if $A \subseteq [A][\bigcap [A] \subseteq A]$.

(2) pre-open (pre-closed) \[ 5 \] if $A \subseteq [A][\bigcap [A] \subseteq A]$.

(3) semi-open (semi-closed) \[ 3 \] if $A \subseteq [A][\bigcap [A] \subseteq A]$.

(4) $b$-open ($b$-closed) \[ 7 \] if $A \subseteq [A][\bigcup [A] \bigcap [A] \subseteq A]$.

(5) $\beta$-open ($\beta$-closed) \[ 4 \] if $A \subseteq [A][\bigcup [A] \bigcap [A] \subseteq A]$.

We denote by $O_\alpha(S, \varphi, \tau, k)$ (resp. $PO(S, \varphi, \tau, k)$, $SO(S, \varphi, \tau, k)$, $bO(S, \varphi, \tau, k)$ and $\beta O(S, \varphi, \tau, k)$), or when there can be no confusion by $O_\alpha(S)$ (resp. $PO(S)$, $SO(S)$, $bO(S)$ and $\beta O(S)$), the set of $\alpha$-open (resp. pre-open, semi-open, $b$-open and $\beta$-open) sets in $\varphi$. Likewise, $C_\alpha(S, \varphi, \tau, k)$ (resp. $PC(S, \varphi, \tau, k)$, $SC(S, \varphi, \tau, k)$, $bC(S, \varphi, \tau, k)$ and $\beta C(S, \varphi, \tau, k)$), or $C_\alpha(S)$ (resp. $PC(S)$, $SC(S)$, $bC(S)$ and $\beta C(S)$) will denote the set of $\alpha$-closed (resp. pre-closed, semi-closed, $b$-closed and $\beta$-closed) sets.

**Definition 2.2.** \[ 6 \] Let $(S, \varphi, \tau, k)$ be a ditopological texture space. A subset $A$ of a texture $\varphi$ is said to be generalized closed ($g$-closed for short) if $A \subseteq G \in \tau$ then $[A] \subseteq G$. 
Definition 2.3. [6] Let \((S, \varphi, \tau, k, \sigma)\) be a complemented ditopological texture space. A subset \(A\) of a texture \(\varphi\) is said to be generalized open \((g\text{-open for short})\) if \(\sigma(A)\) is \(g\)-closed.

We denote by \(gc(S, \varphi, \tau, k)\), or when there can be no confusion by \(gc(S)\), the set of \(g\)-closed sets in \(\varphi\). Likewise, \(go(S, \varphi, \tau, k, \sigma)\), or \(go(S)\) will denote the set of \(g\)-open sets.

3. Strong Forms of Generalized Closed Sets

Definition 3.1. Let \((S, \varphi, \tau, k)\) be a ditopological texture space. A subset \(A\) of a texture \(\varphi\) is said to be \(\alpha-g\)-closed \((\text{resp.}\; \text{pre-}g\text{-closed, semi-}g\text{-closed, b-}g\text{-closed and } \beta-g\text{-closed})\) if \(A \subseteq G \in O_{\alpha}(S)\) \((\text{resp.}\; A \subseteq G \in \beta O(S), A \subseteq G \in SO(S), A \subseteq G \in bO(S)\) and \(A \subseteq G \in \beta O(S))\) then \([A] \subseteq G\).

We denote by \(\alpha gc(S, \varphi, \tau, k)\) \((\text{resp.}\; \text{pre} gc(S, \varphi, \tau, k), \text{-semi} gc(S, \varphi, \tau, k), \text{b} gc(S, \varphi, \tau, k)\) and \(\beta gc(S, \varphi, \tau, k))\), or when there can be no confusion by \(\alpha gc(S)\) \((\text{resp.}\; \text{pre} gc(S), \text{sem} gc(S), \text{b} gc(S)\) and \(\beta gc(S))\), the set of \(\alpha-g\)-closed \((\text{resp.}\; \text{pre}-g\text{-closed, semi-}g\text{-closed, b-}g\text{-closed and } \beta-g\text{-closed})\) sets in \(\varphi\).

Proposition 3.2. For a given ditopological texture space \((S, \varphi, \tau, k)\), the following properties hold:

1. \(\beta gc(S) \subseteq b gc(S)\).
2. \(b gc(S) \subseteq \text{pre} gc(S)\).
3. \(b gc(S) \subseteq \text{semi} gc(S)\).
4. \(\text{pre} gc(S) \subseteq \alpha gc(S)\).
5. \(\text{semi} gc(S) \subseteq \alpha gc(S)\).
6. \(\alpha gc(S) \subseteq g c(S)\).
7. \(C(S) \subseteq \beta gc(S)\).

Proof. (1) Let \(A \in \beta gc(S)\) and let \(A \subseteq G \in bO(S)\) for \(A \in \varphi\). Since \(bO(S) \subseteq \beta O(S)\), then \([A] \subseteq G\) and therefore \(A \in b gc(S)\).

(2) Let \(A \in b gc(S)\) and let \(A \subseteq G \in PO(S)\) for \(A \in \varphi\). Since \(PO(S) \subseteq bO(S)\), then \([A] \subseteq G\) and therefore \(A \in \text{pre} gc(S)\).

(3) Let \(A \in b gc(S)\) and let \(A \subseteq G \in SO(S)\) for \(A \in \varphi\). Since \(SO(S) \subseteq bO(S)\), then \([A] \subseteq G\) and therefore \(A \in \text{semi} gc(S)\).

(4) Let \(A \in \text{pre} gc(S)\) and let \(A \subseteq G \in O_{\alpha}(S)\) for \(A \in \varphi\). Since \(O_{\alpha}(S) \subseteq PO(S)\), then \([A] \subseteq G\) and therefore \(A \in \alpha gc(S)\).

(5) Let \(A \in \text{semi} gc(S)\) and let \(A \subseteq G \in O_{\alpha}(S)\) for \(A \in \varphi\). Since \(O_{\alpha}(S) \subseteq SO(S)\), then \([A] \subseteq G\) and therefore \(A \in \alpha gc(S)\).

(6) Let \(A \in \alpha gc(S)\) and let \(A \subseteq G \in O(S)\) for \(A \in \varphi\). Since \(O(S) \subseteq O_{\alpha}(S)\), then \([A] \subseteq G\) and therefore \(A \in g c(S)\).

(7) Let \(A \in C(S)\) and let \(A \subseteq G \in \beta O(S)\) for \(A \in \varphi\). Since \(A = [A]\), then \([A] \subseteq G\) and therefore \(A \in \beta gc(S)\).

Remark 3.3. The following diagram hold for a ditopological texture space \((S, \varphi, \tau, k)\):

\[
\begin{array}{ccc}
gc(S) & \xleftarrow{\alpha gc(S)} & \text{pregc}(S) \\
& & \\
& \downarrow \quad \downarrow & \\
& \beta gc(S) & \xleftarrow{\beta gc(S)} \quad \text{C}(S) \\
& & \\
& \downarrow & \\
& \text{semigc}(S) & \\
\end{array}
\]

Remark 3.4. For a given ditopological texture space \((S, \varphi, \tau, k)\):
Example 3.5. Let \( S = \{a, b, c\} \), \( \varphi = P(S) \), \( \tau = \{\phi, S\} \) and \( k = P(S) \). Then \((S, \varphi, \tau, k)\) is a ditopological texture space and we have:

\[
O_\alpha(S) = PO(S) = SO(S) = bO(S) = \beta O(S) = \{\phi, S\}
\]

and

\[
\alpha gc(S) = pregc(S) = semige(S) = bgc(S) = \beta gc(S) = P(S).
\]

Example 3.6. Let \( S = \{a, b, c\} \), \( \varphi = P(S) \), \( \tau = P(S) \) and \( k = \{\phi, S\} \). Then \((S, \varphi, \tau, k)\) is a ditopological texture space and we have:

\[
O_\alpha(S) = PO(S) = SO(S) = bO(S) = \beta O(S) = P(S)
\]

and

\[
\alpha gc(S) = pregc(S) = semige(S) = bgc(S) = \beta gc(S) = \{\phi, S\}.
\]

Remark 3.7. For a given ditopological texture space \((S, \varphi, \tau, k)\):

1. \( \alpha gc(S) \) is incomparable with \( C_\alpha(S) \).
2. \( pregc(S) \) is incomparable with \( PC(S) \).
3. \( semige(S) \) is incomparable with \( SC(S) \).
4. \( bgc(S) \) is incomparable with \( bC(S) \).
5. \( \beta gc(S) \) is incomparable with \( \beta C(S) \).

Example 3.8. Let \( S = \{a, b, c\} \), \( \varphi = P(S) \), \( \tau = \{\phi, S, \{a\}\} \) and \( k = \{\phi, S, \{a, b\}\} \). Then \((S, \varphi, \tau, k)\) is a ditopological texture space and we have:

\[
O_\alpha(S) = \{\phi, S, \{a\}\},
\]

\[
C_\alpha(S) = \{\phi, S, \{a, b\}\},
\]

\[
\alpha gc(S) = \{\phi, S, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\},
\]

\[
PO(S) = \{\phi, S, \{a\}, \{c\}, \{a, c\}, \{b, c\}\},
\]

\[
PC(S) = \{\phi, S, \{b\}, \{c\}, \{a, b\}, \{b, c\}\},
\]

\[
pregc(S) = \{\phi, S, \{a, b\}\},
\]

\[
SO(S) = \{\phi, S, \{a\}, \{a, b\}\},
\]

\[
SC(S) = \{\phi, S, \{a\}, \{a, b\}\},
\]

\[
semige(S) = \{\phi, S, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\},
\]

\[
bO(S) = \{\phi, S, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\},
\]

\[
bC(S) = \{\phi, S, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}\},
\]

\[
bgc(S) = \{\phi, S, \{a, b\}\},
\]

\[
\beta O(S) = P(S), \quad \beta C(S) = P(S), \quad \beta gc(S) = \{\phi, S, \{a, b\}\}, \quad \text{and}
\]

\[
gc(S) = \{\phi, S, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}.
\]
Example 3.9. Let $S = \{0, 1, 2, 3\}$, $\varphi = P(S)$, $\tau = \{\phi, S, \{0\}, \{0, 1\}, \{0, 1, 2\}\}$ and $k = \{\phi, S, \{0, 2\}, \{1, 3\}\}$. Then $(S, \varphi, \tau, k)$ is a ditopological texture space and we have:

$$O_\alpha(S) = \{\phi, S, \{0\}, \{0, 1\}, \{0, 1, 2\}, \{0, 1, 3\}\},$$
$$C_\alpha(S) = \{\phi, S, \{1\}, \{3\}, \{0, 2\}, \{1, 3\}\},$$
$$\alpha gc(S) = \{\phi, S, \{2\}, \{3\}, \{0, 2\}, \{1, 3\}, \{2, 3\}, \{0, 2, 3\}, \{1, 2, 3\}\},$$
$$PO(S) = \{\phi, S, \{0\}, \{0, 1\}, \{1, 2\}, \{2, 3\}, \{0, 3\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 2, 3\}, \{1, 2, 3\}\},$$
$$PC(S) = \{\phi, S, \{1\}, \{2\}, \{3\}, \{0, 2\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{0, 2, 3\}, \{1, 2, 3\}\},$$
$$pregc(S) = \{\phi, S, \{0, 2\}, \{1, 3\}\},$$
$$SO(S) = \{\phi, S, \{0\}, \{0, 1\}, \{0, 2\}, \{0, 1, 2\}, \{0, 1, 3\}\},$$
$$SC(S) = \{\phi, S, \{0\}, \{1\}, \{3\}, \{0, 2\}, \{1, 3\}\},$$
$$semi gc(S) = \{\phi, S, \{2\}, \{3\}, \{0, 2\}, \{1, 3\}, \{2, 3\}, \{0, 2, 3\}, \{1, 2, 3\}\},$$
$$bO(S) = \{\phi, S, \{0\}, \{0, 1\}, \{1, 2\}, \{2, 3\}, \{0, 3\}, \{0, 2, 1\}, \{0, 1, 3\}, \{0, 2, 3\}, \{1, 2, 3\}\},$$
$$bC(S) = \{\phi, S, \{0\}, \{1\}, \{2\}, \{3\}, \{0, 2\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{0, 2, 3\}, \{1, 2, 3\}\},$$
$$bgc(S) = \{\phi, S, \{0\}, \{0, 2\}, \{1, 3\}\},$$
$$\beta O(S) = \{\phi, S, \{0\}, \{2\}, \{0, 1\}, \{1, 2\}, \{2, 3\}, \{0, 3\}, \{0, 2\}, \{0, 1, 2\}, \{0, 1, 3\}, \{0, 2, 3\}, \{1, 2, 3\}\},$$
$$\beta C(S) = \{\phi, S, \{0\}, \{1\}, \{2\}, \{3\}, \{0, 2\}, \{0, 3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{0, 2, 3\}, \{1, 2, 3\}\},$$
$$\beta gc(S) = \{\phi, S, \{0\}, \{0, 2\}, \{1, 3\}\},$$
$$gc(S) = \{\phi, S, \{2\}, \{3\}, \{0, 2\}, \{0, 3\}, \{1, 3\}, \{2, 3\}, \{0, 1, 3\}, \{0, 2, 3\}, \{1, 2, 3\}\}.$$

Theorem 3.10. Let $(S, \varphi, \tau, k)$ be a ditopological texture space. If $A, B \in \alpha gc(S)$ (resp. $\alpha gc(S)$, $\alpha gc(S)$, $\alpha gc(S)$, $\alpha gc(S)$), then $A \cup B \in \alpha gc(S)$ (resp. $\alpha gc(S)$, $\alpha gc(S)$, $\alpha gc(S)$, $\alpha gc(S)$).

Proof. Assume that $A \cup B \subseteq G \in O_\alpha(S)$ (resp. $PO(S)$, $SO(S)$, $bO(S)$ and $\beta O(S)$), then $A \subseteq G \in O_\alpha(S)$ (resp. $PO(S)$, $SO(S)$, $bO(S)$ and $\beta O(S)$) and $B \subseteq G \in O_\alpha(S)$ (resp. $PO(S)$, $SO(S)$, $bO(S)$ and $\beta O(S)$). Since $A$ and $B$ are $\alpha$-g-closed (resp. $\alpha$-g-closed, semi-$g$-c-g-closed, b-$g$-closed and $\beta$-$g$-closed), then $[A] \subseteq G$ and $[B] \subseteq G$. Therefore $[A] \cup [B] \subseteq G$ and hence $[A \cup B] \subseteq G$. Consequently $A \cup B \in \alpha gc(S)$ (resp. $\alpha gc(S)$, $\alpha gc(S)$, $\alpha gc(S)$).

In the following example, we show that in general $A \cap B \notin \alpha gc(S)$ (resp. $\alpha gc(S)$) if $A, B \in \alpha gc(S)$ (resp. $\alpha gc(S)$).

Example 3.11. Let $S = \{a, b, c\}$, $\varphi = P(S)$, $\tau = \{\phi, S, \{c\}, \{a, b\}\}$ and $k = \{\phi, S, \{a, c\}\}$. Then $(S, \varphi, \tau, k)$ is a ditopological texture space. Let $A = \{a, c\}$ and $B = \{b, c\}$. Then $A, B \in \alpha gc(S)$ (resp. $\alpha gc(S)$) but $A \cap B = \{c\} \notin \alpha gc(S)$ (resp. $\alpha gc(S)$). In fact, $A \cap B = \{c\} \subseteq \{c\} \in O_\alpha(S)$ (resp. $SO(S)$), but $\{c\} = \{a, c\}$ and $\{a, c\}$ is not a subset of $\{c\}$.

Theorem 3.12. Let $(S, \varphi, \tau, k)$ be a ditopological texture space. If $A \in \alpha gc(S)$ (resp. $\alpha gc(S)$, $\alpha gc(S)$, $\alpha gc(S)$) and $A \subseteq B \subseteq [A]$. Then $B \in \alpha gc(S)$ (resp. $\alpha gc(S)$, $\alpha gc(S)$, $\alpha gc(S)$).

Proof. Let $A \subseteq B \subseteq [A]$ and $B \subseteq G \in O_\alpha(S)$ (resp. $PO(S)$, $SO(S)$, $bO(S)$ and $\beta O(S)$). Since $A \subseteq B$, then $[A] \subseteq G$ because $A$ is $\alpha$-g-closed (resp. $\alpha$-g-closed, semi-$g$-c-g-closed, b-$g$-closed and $\beta$-$g$-closed). Since $[A] \subseteq B \subseteq [A]$, then $B \subseteq G$. Hence, $B$ is $\alpha$-g-closed (resp. $\alpha$-g-closed, semi-$g$-c-g-closed, b-$g$-closed and $\beta$-$g$-closed).

Definition 3.13. Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. A subset $A$ of a texture $\varphi$ is called $\alpha$-g-open (resp. $\alpha$-g-open, semi-$g$-open, $b$-$g$-open and $\beta$-$g$-open) if $\sigma(A)$ is $\alpha$-g-closed (resp. $\alpha$-g-closed, semi-$g$-c-g-closed, b-$g$-closed and $\beta$-$g$-closed).
We denote by $\alpha go(S, \varphi, \tau, k, \sigma)$ (resp. $\text{prego}(S, \varphi, \tau, k, \sigma)$, $\text{semigo}(S, \varphi, \tau, k, \sigma)$, $\beta go(S, \varphi, \tau, k, \sigma)$ and $\beta go(S, \varphi, \tau, k, \sigma)$), or when there can be no confusion by $\alpha go(S)$ (resp. $\text{prego}(S)$, $\text{semigo}(S)$, $\beta go(S)$ and $\beta go(S)$), the set of $\alpha$-open (resp. pre-$\alpha$-open, semi-$\alpha$-open, b-$\alpha$-open and $\beta$-$\alpha$-open) sets in $\varphi$.

**Remark 3.14.** The following diagram hold for a complemented ditopological texture space $(S, \varphi, \tau, k, \sigma)$:

\[
\begin{array}{ccc}
\text{prego}(S) & \leftarrow & \alpha go(S) \\
& \downarrow & \\
& \beta go(S) & \leftarrow O(S) \\
\end{array}
\]

**Proposition 3.15.** Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological space. If $A \in \tau$ then $\sigma(A) \in \beta gc(S)$.

**Proof.** Since $A = \lfloor A \rfloor$ and $\sigma(A) = \sigma(\lfloor A \rfloor) = [\sigma(A)]$ then $\sigma(A) \in C(S) \subseteq \beta gc(S)$.

**Remark 3.16.** For a complemented ditopological space $(S, \varphi, \tau, k, \sigma)$:

1. If $A \in \tau$ then $\sigma(A) \in bgc(S)$.
2. If $A \in \tau$ then $\sigma(A) \in \text{pregc}(S)$.
3. If $A \in \tau$ then $\sigma(A) \in \text{semigc}(S)$.
4. If $A \in \tau$ then $\sigma(A) \in \alpha gc(S)$.
5. If $A \in \tau$ then $\sigma(A) \in gc(S)$.

**Proposition 3.17.** Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological space. If $A \in k$ then $\sigma(A) \in \beta go(S)$.

**Proof.** Since $A = \lfloor A \rfloor$ and $\sigma(A) = \sigma(\lfloor A \rfloor) = \lfloor \sigma(A) \rfloor$ then $\sigma(A) \in O(S) \subseteq \beta go(S)$.

**Remark 3.18.** For a complemented ditopological space $(S, \varphi, \tau, k, \sigma)$:

1. If $A \in k$ then $\sigma(A) \in bgo(S)$.
2. If $A \in k$ then $\sigma(A) \in \text{prego}(S)$.
3. If $A \in k$ then $\sigma(A) \in \text{semigo}(S)$.
4. If $A \in k$ then $\sigma(A) \in \alpha go(S)$.
5. If $A \in k$ then $\sigma(A) \in go(S)$.

**Theorem 3.19.** Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. A subset $A$ of $\varphi$ is $\alpha$-open (resp. pre-$\alpha$-open, semi-$\alpha$-open, b-$\alpha$-open and $\beta$-$\alpha$-open) iff $F \subseteq A$, $F \in C_\alpha(S)$ (resp. $PC(S)$, $SC(S)$, $bC(S)$ and $\beta C(S)$) implies $F \subseteq \lfloor A \rfloor$.

**Proof.** Assume that $A \in \varphi$ is $\alpha$-open (resp. pre-$\alpha$-open, semi-$\alpha$-open, b-$\alpha$-open and $\beta$-$\alpha$-open) and $F \subseteq A$, $F \in C_\alpha(S)$ (resp. $PC(S)$, $SC(S)$, $bC(S)$ and $\beta C(S)$). Then $\sigma(A)$ is $\alpha$-closed (resp. pre-$\alpha$-closed, semi-$\alpha$-closed, b-$\alpha$-closed and $\beta$-$\alpha$-closed) and $\sigma(F) \subseteq \sigma(F) \in O_\alpha(S)$ (resp. $PO(S)$, $SO(S)$, $bO(S)$ and $\beta O(S)$). Hence $\sigma([A]) \subseteq \sigma(F) \in O_\alpha(S)$ (resp. $PO(S)$, $SO(S)$, $bO(S)$ and $\beta O(S)$) and therefore $F \subseteq \sigma([\sigma(A)]) = \lfloor \sigma(\sigma(A)) \rfloor$.

Conversely, we show $A$ is $\alpha$-open (resp. pre-$\alpha$-open, semi-$\alpha$-open, b-$\alpha$-open and $\beta$-$\alpha$-open) (that is, $\sigma(A)$ is $\alpha$-closed (resp. pre-$\alpha$-closed, semi-$\alpha$-closed, b-$\alpha$-closed and $\beta$-$\alpha$-closed)). Suppose that $\sigma(A) \subseteq G \subseteq O_\alpha(S)$ (resp. $PO(S)$, $SO(S)$, $bO(S)$ and $\beta O(S)$), then $\sigma(G) \subseteq A$ and $\sigma(G) \subseteq O_\alpha(S)$ (resp. $PC(S)$, $SC(S)$, $bC(S)$ and $\beta C(S)$). By hypothesis $\sigma(G) \subseteq \lfloor A \rfloor$ and so $\lfloor \sigma(A) \rfloor \subseteq G$. Thus $\sigma([A]) \subseteq G$ and consequently $\sigma(A)$ is $\alpha$-closed (resp. pre-$\alpha$-closed, semi-$\alpha$-closed, b-$\alpha$-closed and $\beta$-$\alpha$-closed). Hence $A$ is $\alpha$-open (resp. pre-$\alpha$-open, semi-$\alpha$-open, b-$\alpha$-open and $\beta$-$\alpha$-open).
Two subsets $A$ and $B$ of texture space $(S, \varphi)$ is said to be separated in a ditopological texture space $(S, \varphi, \tau, k)$ if $[A] \cap B = A \cap [B] = \emptyset$.

**Theorem 3.20.** Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space and $A, B \in \alpha g(S)$ (resp. $\text{pre} g(S)$, $\text{semigo}(S)$, $bgo(S)$ and $\beta g(S)$). If $A$ and $B$ are separated, then $A \cup B \in \alpha g(S)$ (resp. $\text{pre} g(S)$, $\text{semigo}(S)$, $bgo(S)$ and $\beta g(S)$).

**Proof.** Suppose that $A$ and $B$ are separated and $\alpha g$-open (resp. $\text{pre} g$-open, $\text{semi} g$-open, $b g$-open and $\beta g$-open) sets. We show $A \cup B$ is $\alpha g$-open (resp. $\text{pre} g$-open, $\text{semi} g$-open, $b g$-open and $\beta g$-open). Let $F \subseteq A \cup B$, $F \in C_{\alpha}(S)$ (resp. $PC(S)$, $SC(S)$, $bc(S)$ and $\beta C(S)$). Since $A$ and $B$ are separated, then $A \cap [B] = [A] \cap B = \emptyset$. Hence $F \cap [A] \subseteq A$, $F \cap [B] \subseteq B$, $F \cap [A] \in C_{\alpha}(S)$ (resp. $PC(S)$, $SC(S)$, $bc(S)$ and $\beta C(S)$), and $F \cap [B] \in C_{\alpha}(S)$ (resp. $PC(S)$, $SC(S)$, $bc(S)$ and $\beta C(S)$). Since $A$ and $B$ are $\alpha g$-open (resp. $\text{pre} g$-open, $\text{semi} g$-open, $b g$-open and $\beta g$-open), then $F \cap [A] \subseteq [A]$ and $F \cap [B] \subseteq [B]$. Therefore $(F \cap [A]) \cup (F \cap [B]) \subseteq [A] \cup [B]$ and so $F \cap (([A] \cup [B]) \subseteq [A] \cup [B]$. Hence $F \cap ([A] \cup [B] \subseteq [A] \cup [B]$ and thus $F \subseteq [A] \cup [B]$. Consequently, $A \cup B$ is $\alpha g$-open (resp. $\text{pre} g$-open, $\text{semi} g$-open, $b g$-open and $\beta g$-open).

**Corollary 3.21.** Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space and $A, B \in \alpha g(S)$ (resp. $\text{pre} g(S)$, $\text{semigo}(S)$, $bgo(S)$ and $\beta g(S)$). If $\sigma(A)$ and $\sigma(B)$ are separated, then $A \cap B \in \alpha g(S)$ (resp. $\text{pre} g(S)$, $\text{semigo}(S)$, $bgo(S)$ and $\beta g(S)$).

**Proof.** Let $A \cap B \subseteq G \in O_{\alpha}(S)$ (resp. $PO(S)$, $SO(S)$, $bO(S)$ and $\beta O(S)$). Since $A$ and $B$ are $\alpha g$-closed (resp. $\text{pre} g$-closed, $\text{semi} g$-closed, $b g$-closed and $\beta g$-closed), then $\sigma(A)$ and $\sigma(B)$ are $\alpha g$-open (resp. $\text{pre} g$-open, $\text{semi} g$-open, $b g$-open and $\beta g$-open) sets. Since $\sigma(A)$ and $\sigma(B)$ are separated, then $\sigma(A) \cup \sigma(B)$ is $\alpha g$-open (resp. $\text{pre} g$-open, $\text{semi} g$-open, $b g$-open and $\beta g$-open) set. Therefore $\sigma(\sigma(A) \cup \sigma(B)) = \sigma(\sigma(A) \cap B)) = A \cap B$ is $\alpha g$-closed (resp. $\text{pre} g$-closed, $\text{semi} g$-closed, $b g$-closed and $\beta g$-closed).

**Corollary 3.22.** Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space and $A, B \in \alpha g(S)$ (resp. $\text{pre} g(S)$, $\text{semigo}(S)$, $bgo(S)$ and $\beta g(S)$), then $A \cap B \in \alpha g(S)$ (resp. $\text{pre} g(S)$, $\text{semigo}(S)$, $bgo(S)$ and $\beta g(S)$).

**Proof.** Let $A$ and $B$ be $\alpha g$-open (resp. $\text{pre} g$-open, $\text{semi} g$-open, $b g$-open and $\beta g$-open) sets, then $\sigma(A)$ and $\sigma(B)$ are $\alpha g$-closed (resp. $\text{pre} g$-closed, $\text{semi} g$-closed, $b g$-closed and $\beta g$-closed). Hence $\sigma(A) \cup \sigma(B)$ is $\alpha g$-closed (resp. $\text{pre} g$-closed, $\text{semi} g$-closed, $b g$-closed and $\beta g$-closed) and therefore $A \cap B$ is $\alpha g$-open (resp. $\text{pre} g$-open, $\text{semi} g$-open, $b g$-open and $\beta g$-open).

**Theorem 3.23.** Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. If $[A] \subseteq A \subseteq A \in \alpha g(S)$ (resp. $\text{pre} g(S)$, $\text{semigo}(S)$, $bgo(S)$ and $\beta g(S)$), then $B \in \alpha g(S)$ (resp. $\text{pre} g(S)$, $\text{semigo}(S)$, $bgo(S)$ and $\beta g(S)$).

**Proof.** Since $[A] \subseteq A$ and $A \in \alpha g(S)$ (resp. $\text{pre} g(S)$, $\text{semigo}(S)$, $bgo(S)$ and $\beta g(S)$), then $\sigma(A)$ is $\alpha g$-open (resp. $\text{pre} g$-open, $\text{semi} g$-open, $b g$-open and $\beta g$-open). Then $\sigma(A)$ is $\alpha g$-closed (resp. $\text{pre} g$-closed, $\text{semi} g$-closed, $b g$-closed and $\beta g$-closed) and $\sigma(A) \subseteq \sigma(B) \subseteq \sigma([A]) = [\sigma(A)]$. Then $\sigma(B)$ is $\alpha g$-closed (resp. $\text{pre} g$-closed, $\text{semi} g$-closed, $b g$-closed and $\beta g$-closed) and consequently $B$ is $\alpha g$-open (resp. $\text{pre} g$-open, $\text{semi} g$-open, $b g$-open and $\beta g$-open).

**Definition 3.24.** Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. For $A \in \varphi$, we define the $\alpha g$-closure (resp. $\text{pre} g$-closure, $\text{semi} g$-closure, $b g$-closure and $\beta g$-closure) $[A]_{\alpha g}$ (resp. $[A]_{\text{pre} g}$, $[A]_{\text{semi} g}$, $[A]_{b g}$ and $[A]_{\beta g}$) and the $\alpha g$-interior (resp. $\text{pre} g$-interior, $\text{semi} g$-interior, $b g$-interior and $\beta g$-interior) $A[\alpha g]$ (resp. $A[\text{pre} g]$, $A[\text{semi} g]$, $A[b g]$ and $A[\beta g]$) of $A$ under $(\tau, k)$ by the equalities:

1. $[A]_{\alpha g} = \bigcap \{K \in \alpha g(S) : A \subseteq K\}$ and $\{A[\alpha g] = \bigcup \{G \in \alpha g(O(S) : G \subseteq A\}.$
2. $[A]_{\text{pre} g} = \bigcap \{K \in \text{pre} g(S) : A \subseteq K\}$ and $\{A[\text{pre} g] = \bigcup \{G \in \text{pre} g(S) : G \subseteq A\}.$
3. $[A]_{\text{semi} g} = \bigcap \{K \in \text{semi} g(S) : A \subseteq K\}$ and $\{A[\text{semi} g] = \bigcup \{G \in \text{semi} g(S) : G \subseteq A\}.$
Remark 3.25. Let $(S, \varphi, \tau, k, \sigma)$ be a complemented ditopological texture space. Then:

1. $A \subseteq [A]_g \subseteq [A]_{\alpha-g} \subseteq [A]_{\text{pre}-g} \subseteq [A]_{\beta-g} \subseteq [A]$.
2. $[A]_{\alpha-g} \subseteq [A]_{\text{semi}-g} \subseteq [A]_{b-g}$.
3. $[A]_{\beta-g} \subseteq [A]_{b-g} \subseteq [A]_{\text{pre}-g} \subseteq [A]_{\alpha-g} \subseteq [A]_g \subseteq A$.
4. $[A]_{b-g} \subseteq [A]_{\text{semi}-g} \subseteq [A]_{\alpha-g}$.

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