See-saw Enhancement of Neutrino Mixing due to the Right-handed Phases

Morimitsu TANIMOTO

Institut für Theoretische Physik, Universität Wien
Boltzmanngasse 5, A-1090 Wien, AUSTRIA

ABSTRACT

We study the see-saw enhancement mechanism in presence of the right-handed phases of the Dirac neutrino mass matrix and the Majorana mass matrix. The enhancement condition given by Smirnov is modified. We point out that the see-saw enhancement could be obtained due to the right-handed phases even if the Majorana matrix is proportional to the unit matrix. We show a realistic Dirac mass matrix which can use the see-saw enhancement.

\[^1\]Permanent address:Science Education Laboratory, Ehime University, 790 Matsuyama, JAPAN
The see-saw mechanism of the neutrino mass generation gives a very natural and elegant understanding for the smallness of neutrino masses[1]. The recent observed solar neutrino deficit[2] and muon neutrino deficit in the atmospheric neutrino flux[3] stimulate the systematic study of the neutrino mixings[4]. In the standpoint of the quark-lepton unification in most GUT groups, the Dirac mass matrix of neutrinos is similar to the one of quarks, therefore, the neutrino mixings turn out to be typically of the same order of magnitude as the quark mixings. However, some authors pointed out that the large neutrino mixing between the different generations could be obtained in the see-saw mechanism as a consequence of certain structure of the right-handed Majorana mass matrix[5,6,7]. That is the so called see-saw enhancement[7] of the neutrino mixing due to the cooperation between the Dirac and Majorana mass matrices, which needs a higher representation of the Higgs fields such as $126$ in SO(10).

In this paper, we modify the enhancement conditions[7], which were given by Smirnov in presence of the right-handed phases of the Dirac mass matrix and the Majorana mass matrix. So, we point out that the see-saw enhancement could be obtained even if the Majorana matrix is proportional to the unit matrix, i.e., there is no hierarchy in the Majorana mass matrix. This enhancement is caused by the right-handed phases, which never appear in the case of the quark mixing. The large enhancement is obtained for some textures of the Dirac mass matrix.

In order to see the role of the right-handed phases of the Dirac mass matrix clearly, we work in the nearest neighbour interaction(NNI) basis[8], which is one specific weak basis and does not imply any loss of generality for the Dirac mass matrix. In this basis, the Dirac mass matrix with three generations is written as follows:
\[ D_i = \begin{pmatrix} 0 & A_i e^{i\alpha} & 0 \\ A_i' e^{i\alpha'} & 0 & B_i e^{i\beta} \\ 0 & B_i' e^{i\beta'} & C_i e^{i\gamma} \end{pmatrix}, \]  
\[ (1) \]

where \( i \) denotes quarks or leptons, and \( A, B, \) and \( C \) are positive real numbers. Using the diagonal phase matrices \( P_L \) and \( P_R \), the Dirac mass matrix turns to be real one such as

\[ D_i = P_L^* \begin{pmatrix} 0 & A_i & 0 \\ A_i' & 0 & B_i \\ 0 & B_i' & C_i \end{pmatrix} P_R = P_L^* R_i P_R, \]

where

\[ P_L = \begin{pmatrix} e^{-i(\alpha-\beta')} & 0 & 0 \\ 0 & e^{-i(\beta-\gamma)} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad P_R = \begin{pmatrix} e^{i(\alpha'-\beta+\gamma)} & 0 & 0 \\ 0 & e^{i\beta'} & 0 \\ 0 & 0 & e^{i\gamma} \end{pmatrix}. \]

These phase matrices are removed by re-defining the left-handed and right-handed fermion fields. Then, the phase matrix \( P_L \) appears in the expression of the quark mixing matrix. The right-handed phase matrix \( P_R \) never appear in the quark mixings since the charged current is the left-handed one.

However, neutrino mixings with the see-saw mechanism are affected by the right-handed phase matrix \( P_R \) because the Majorana mass matrix is the right-handed one. This point is an important difference from the case of the quark mixings. In the basis of the real Dirac mass matrix, the see-saw mass matrix is written as follows:

\[ \begin{pmatrix} 0 & R_N \\ R_N^T & P_R^* M_R P_R^* \end{pmatrix}, \]

where \( M_R \) is a right-handed Majorana matrix. It should be noticed that \( M_R \) is multiplied by \( P_R^* \) in both sides because \( M_R \) is Majorana mass. Since the matrix \( M_R \) may be a complex off-diagonal matrix, it is generally written as \( P_R^* \tilde{U}_R M_R^{\text{diag}} \tilde{U}_R^T P_R^* \), where \( P_R^*, \tilde{U}_R \) and \( M_R^{\text{diag}} \) are the phase matrix, the orthogonal one and the diagonal one, respectively. If we move to the diagonal basis of the Dirac mass matrix by a bi-orthogonal
transformation such as \( U^T_L R_N U_R \to R^\text{diag}_N \), the see-saw mass matrix turns to be

\[
\begin{pmatrix}
0 \\
R^\text{diag}_N \\
U^T_R P_R^* \tilde{P}_R^* U_R M^\text{diag}_R \tilde{U}^T_R P_R^* \tilde{P}_R^* U_R \\
\end{pmatrix}.
\] (5)

Then, the light neutrino mass matrix \( m_\nu \) is given as follows:

\[
m_\nu = R^\text{diag}_N (U^T_R P_R^* \tilde{P}_R^* U_R M^\text{diag}_R \tilde{U}^T_R P_R^* \tilde{P}_R^* U_R)^{-1} R^\text{diag}_N
\]

\[
= R^\text{diag}_N U^T_R Q_R \tilde{U}_R (M^\text{diag}_R)^{-1} \tilde{U}^T_R Q_R U_R R^\text{diag}_N,
\] (6)

where \( P_R \tilde{P}_R \) is replaced with a single phase matrix \( Q_R \). This mass matrix is a diagonal one only if \( U^T_R Q_R \tilde{U}_R (M^\text{diag}_R)^{-1} \tilde{U}^T_R Q_R U_R \) is proportional to the unit matrix. In this case, the lepton mixing matrix is obtained by diagonalizing the charged lepton mass matrix. Therefore, the mixing is expected to be of the same order of magnitude as the quark mixings. However, even if \( M^\text{diag}_R \) is proportional to the unit matrix, i.e., the case of the three degenerated right-handed Majorana masses, the mass matrix \( m_\nu \) is not diagonal one unless the phase matrix \( Q_R \) is a real matrix except for an over all phase. Since the phase elements in \( Q_R \) are independent of ones in \( P_L \) in eq.(3), these parameters are new freedoms of the lepton mixings in contrast to the case of the quark mixings.

Generally, the lepton mixing matrix is given as \( N^T_L U^T_L P_L C_L \), where \( N_L \) is the unitary matrix to diagonalize \( m_\nu \) in eq.(6) and \( C_L \) is the orthogonal matrix to diagonalize the real charged lepton mass matrix in the NNI basis. The CP violating phase could be in both \( P_L \) and \( N_L \).

In order to show the effect of the phase matrix \( Q_R \), we consider two-generation case. We parametrize following matrices in the two generations,

\[
U_R = \begin{pmatrix}
\cos \theta^D_R & \sin \theta^D_R \\
-\sin \theta^D_R & \cos \theta^D_R
\end{pmatrix}, \quad \tilde{U}_R = \begin{pmatrix}
\cos \theta^M_R & \sin \theta^M_R \\
-\sin \theta^M_R & \cos \theta^M_R
\end{pmatrix},
\]

\[
Q_R = \begin{pmatrix}
1 & 0 \\
0 & e^{i\phi}
\end{pmatrix}, \quad M^\text{diag}_R = \begin{pmatrix}
M_1 & 0 \\
0 & M_2
\end{pmatrix}, \quad R^\text{diag}_N = \begin{pmatrix}
m_1 & 0 \\
0 & m_2
\end{pmatrix}.
\] (7)
The light neutrino mass matrix $m_\nu$ of eq.(6) is given by six parameters, $m_1$, $M_1$, $m_2^2/M_2$, $	heta^D_R$, $\theta^M$ and $\pi h$. Since this matrix is very long matrix and somewhat complicated one, we show it in the basis of $\theta^M = 0$, in other words, the diagonal basis of the Majorana mass matrix $M_R$. Defining mass hierarchy parameters such as

$$
\epsilon^D \equiv \frac{m_1}{m_2}, \quad \epsilon^M \equiv \frac{M_1}{M_2},
$$

we get

$$
m_\nu = \left( \begin{array}{ccc}
|m_{11}| e^{i\alpha} & |m_{12}| e^{i\beta} & \\
|m_{12}| e^{i\beta} & |m_{22}| e^{i\gamma} & \end{array} \right),
$$

where

$$
|m_{11}| = \frac{(\epsilon^D m_2)^2}{\epsilon^M M_2} \left[ \cos^4 \theta^D_R + (\epsilon^M)^2 \sin^4 \theta^D_R + 2 \epsilon^M \sin^2 \theta^D_R \cos^2 \theta^D_R \cos 2\phi \right]^{\frac{1}{4}},
$$

$$
|m_{21}| = \frac{\epsilon^D m_2^2}{2 \epsilon^M M_2} \left[ 1 + (\epsilon^M)^2 - 2 \epsilon^M \cos 2\phi \right]^{\frac{1}{2}} \sin 2\theta^D_R,
$$

$$
|m_{22}| = \frac{m_2^2}{\epsilon^M M_2} \left[ \sin^4 \theta^D_R + (\epsilon^M)^2 \cos^4 \theta^D_R + 2 \epsilon^M \sin^2 \theta^D_R \cos^2 \theta^D_R \cos 2\phi \right]^{\frac{1}{4}},
$$

$$
\tan \alpha = \frac{\epsilon^M \sin^2 \theta^D_R \sin 2\phi}{\cos^2 \theta^D_R + \epsilon^M \sin^2 \theta^D_R \cos 2\phi},
$$

$$
\tan \beta = -\frac{\epsilon^M \sin 2\phi}{1 - \epsilon^M \cos 2\phi},
$$

$$
\tan \gamma = \frac{\epsilon^M \cos^2 \theta^D_R \sin 2\phi}{\sin^2 \theta^D_R + \epsilon^M \cos^2 \theta^D_R \cos 2\phi}.
$$

Some of the phases in the matrix of eq.(9) are removed by the phase rotation of the light neutrino fields as follows:

$$
m_\nu \Rightarrow \left( \begin{array}{cc}
e^{-\frac{i}{2}(\beta-\omega)} & 0 \\
0 & e^{-\frac{i}{2}(\beta+\omega)} \end{array} \right) m_\nu \left( \begin{array}{cc}
e^{-\frac{i}{2}(\beta-\omega)} & 0 \\
0 & e^{-\frac{i}{2}(\beta+\omega)} \end{array} \right) = \left( \begin{array}{c}
|m_{11}| e^{i\varphi_1} \\
|m_{12}| \\
|m_{22}| e^{i\varphi_2} \end{array} \right),
$$

where $\varphi_1 = \alpha - \beta + \omega$, $\varphi_2 = \gamma - \beta - \omega$, and $\omega$ is fixed in order to give the following relation:

$$
\text{Im}\{|m_{11}| e^{i(\alpha-\beta+\omega)}\} = \text{Im}\{|m_{22}| e^{i(\gamma-\beta-\omega)}\},
$$

(12)
which gives
\[ \tan \omega = \frac{|m_{22}| \sin(\gamma - \beta) - |m_{11}| \sin(\alpha - \beta)}{|m_{22}| \cos(\gamma - \beta) + |m_{11}| \cos(\alpha - \beta)}. \] (13)

Since the phase matrix in eq.(11) is absorbed into the charged lepton fields finally[9], it does not affect the observable quantities such as \( CP \) violation. Now, we can diagonalize the neutrino mass matrix in eq.(11) by using the orthogonal matrix, while those mass eigenvalues are complex ones. One phase in the eigenvalues cannot be removed away, and then, causes \( CP \) violation even in the two generation case[10].

We get the angle \( \theta_{ss} \) in the orthogonal matrix, which was called as the see-saw angle by Smirnov[7], as follows:

\[ \tan 2\theta_{ss} = -2 \tan(-\theta^D_R) \frac{\epsilon^D[(1 - \epsilon^M)^2 + 2\epsilon^M(1 - \cos 2\phi)]^{\frac{1}{2}}}{[(\tan^2(-\theta^D_R) + \epsilon^M)^2 - 2\epsilon^M \tan^2(-\theta^D_R) + (1 - \cos 2\phi)]^{\frac{1}{2}} \cos \varphi_2 - \delta}, \] (14)

where \( \delta \) is a small term of second order in the Dirac mass hierarchy:

\[ \delta = (\epsilon^D)^2[(\epsilon^M \tan^2(-\theta^D_R) + 1)^2 - 2\epsilon^M \tan^2(-\theta^D_R)(1 - \cos 2\phi)]^{\frac{1}{2}} \cos \varphi_1. \] (15)

In eqs.(14) and (15), the minus sign in front of \( \theta^D_R \) is taken in order to compare with eq.(5) in ref.[7]. The mixing matrix of the leptonic charged currents can be written in the form[9]

\[ \begin{bmatrix} \cos(\theta^D_L - \theta^L + \theta_{ss}) & \sin(\theta^D_L - \theta^L + \theta_{ss}) \\ -\sin(\theta^D_L - \theta^L + \theta_{ss}) & \cos(\theta^D_L - \theta^L + \theta_{ss}) \end{bmatrix} e^{i\rho \tau_3}, \] (16)

where \( \theta^D_L (\theta^L) \) is the angle of the rotation of the left-handed neutrino(charged lepton) components, which diagonalizes the Dirac mass matrix of neutrinos(charged leptons), and \( \rho \) is the \( CP \) violating phase, which could be written in terms of six parameters in eq.(7). If we choose \( \phi = 0 \), the \( CP \) violating phase \( \rho \) disappears, and our eqs. (14) and (15) are reduced to eqs.(5) and (6) in ref.[7] in the basis of \( \theta^M = 0 \). However, the
see-saw enhancement conditions given by Smirnov[7] should be modified because the phase $\phi$ is not generally non-zero.

Let us consider modified conditions of the see-saw enhancement. According to Smirnov’s discussion, there are two possibilities of the see-saw enhancement: (i) all terms in the denominator of the RHS of the eq.(14) are very small, (ii) there is a strong cancellation in the denominator. For the case (i), the effect of the $\phi$ is very small because both $\epsilon^M$ and $\tan(-\theta^D_R)$ must be small. On the other hand, the phase $\phi$ plays an important role for the case (ii). The strong cancellation in the denominator is occurred at the minimum value of the following term

$$
(tan^2(-\theta^D_R) + \epsilon^M)^2 - 2\epsilon^M \tan^2(-\theta^D_R)(1 - \cos 2\phi)
$$

$$
= (tan^2(-\theta^D_R) + \epsilon^M \cos 2\phi)^2 + (\epsilon^M)^2(1 - \cos^2 2\phi) .
$$

(17)

The minimum value is obtained such as $(\epsilon^M)^2(1 - \cos^2 2\phi)$ at

$$
\tan^2(-\theta^D_R) = -\epsilon^M \cos 2\phi .
$$

(18)

Then, the see-saw mixing is given as follows:

$$
\tan 2\theta_{ss} = \frac{-2|\epsilon^M \cos 2\phi|^{\frac{1}{2}}[(1 - \epsilon^M)^2 + 2\epsilon^M(1 - \cos 2\phi)]^{\frac{1}{2}}}{\epsilon^M(\epsilon^D)^{-1}(1 - \cos^2 2\phi)^{\frac{1}{2}} \cos \varphi_2 - \epsilon^D[1 + (\epsilon^M)^2((\epsilon^M)^2 - 2) \cos^2 2\phi]^{\frac{1}{2}} \cos \varphi_1} .
$$

(19)

Eqs.(18) and (19) reduce to eqs.(10) and (11) of ref.[7] in the case of $\phi = 0$ and $\epsilon^M \ll 1$.

It is remarked that the see-saw angle $\theta_{ss}$ is not zero even in the case of $\epsilon^M = 1$ as seen in eqs.(14) and (19). Especially, if the condition $\tan^2(-\theta^D_R) = -\cos 2\phi$ with $\epsilon^M = 1$ is satisfied, the see-saw enhancement is obtained as follows:

$$
\tan 2\theta_{ss} = \frac{-2|\cos 2\phi|^{\frac{1}{2}}[2(1 - \cos 2\phi)]^{\frac{1}{2}}}{(\epsilon^D)^{-1}(1 - \cos^2 2\phi)^{\frac{1}{2}} \cos \varphi_2 - \epsilon^D[1 - \cos^2 2\phi]^{\frac{1}{2}} \cos \varphi_1} .
$$

(20)
Then, the maximal mixing $\theta_{ss} = \pi/4$ is obtained at $\phi = \pi/2$, which gives $\cos \varphi_1 = \cos \varphi_2 = 1$ and $\tan(-\theta_R^D) = 1$. This situation is contrast to the one in ref.[7], where $\theta_{ss}$ is precisely zero in the case of $e^M = 1$.

Our formula in eq.(14) is given in the basis of $\theta^M = 0$ for simplicity. Although the general formula is complicated one in the basis of $\theta^M \neq 0$, it is noticed that the case of $\theta^M = \theta_R^D$ does not lead to $\theta_{ss} = 0$ due to the phase $\phi$. If we take $\phi = \pi$ tentatively, $\tan 2\theta_{ss}$ is given as the same one as eq.(5) in ref.[7] except replacing $\theta^M$ with $-\theta^M$. Thus, the right-handed phase affects the see-saw angle $\theta_{ss}$ drastically.

Modification of the see-saw enhancement condition in ref.[7] is minor in the case of $\epsilon^M \ll 1$. Therefore, we focus on only the case of $\epsilon^M \simeq 1$ in the following discussions. In this case, the see-saw enhancement gives the maximal angle $\theta_{ss} = \pi/4$ at $\phi = \pi/2$ with $\tan(-\theta_R^D) = 1$ as discussed above. Therefore, we need the Dirac mass matrix to satisfy the condition $\tan(-\theta_R^D) = 1$. If we take, for example, the Fritzsch texture[11] for the Dirac mass matrix, $\theta_R^D$ is small due to the hierarchical fermion masses such as $\sin \theta_R^D \simeq \sqrt{m_1/m_2}$. So, the see-saw enhancement is not obtained. However, we know at least two textures of the Dirac mass matrix which lead to the large $\theta_R^D$: One is the democratic mass matrix[12] and the other is the non-Hermitian mass matrix, which is the modified Fritzsch texture, proposed by Branco and Silva-Marcos[13]. We can easily present a realistic mass matrix model with above condition.

Let us show an example by using the non-Hermitian Dirac mass matrix, which was proposed by Branco and Silva-Marcos[13]. In NNI basis of eq.(1), the assumptions of $A' = A$ and $B' = B$ give the Fritzsch mass matrix[11], which implies correlations between quark masses and mixings. However, the large top-quark mass is unfavourable to the Fritzsch ansatz. Branco and Silva-Marcos have taken another ansatz[13], $A' = A$
and \( B' = C \), which give the non-Hermitian mass matrix. This new mass matrix seems to be consistent with the observed quark mixing matrix and quark masses[14]. After absorbing phases with \( P_L \) and \( P_R \) in eq.(3), the 3 \( \times \) 3 real Dirac mass matrix \( R_i \) is written as

\[
R_i = \begin{pmatrix}
0 & A_i & 0 \\
A_i & 0 & B_i \\
0 & C_i & C_i
\end{pmatrix},
\]

which is diagonalized by a bi-orthogonal transformation such as \( U_L^T R_i U_R \). The orthogonal matrices can be expressed approximately in terms of quark masses as follows:

\[
U_L \simeq \begin{pmatrix}
1 & -2^{-\frac{1}{2}} \left( \frac{m_1}{m_2} \right)^{\frac{1}{2}} & 2^{-\frac{1}{2}} \left( \frac{m_1 m_2}{m_3} \right)^{\frac{1}{2}} \\
2^{-\frac{1}{2}} \left( \frac{m_1}{m_2} \right)^{\frac{1}{2}} & 1 & -\frac{m_2}{m_3} \\
-2^{-\frac{1}{2}} \left( \frac{m_1 m_2}{m_3^2} \right)^{\frac{1}{2}} & -\frac{m_2}{m_3} & 1
\end{pmatrix},
\]

\[\text{nonumber} \tag{22}\]

\[
U_R \simeq \begin{pmatrix}
1 & -2^{\frac{1}{2}} \left( \frac{m_1}{m_2} \right)^{\frac{1}{2}} & 2^{\frac{1}{2}} \left( \frac{m_1 m_2}{m_3^2} \right)^{\frac{1}{2}} \\
-2^{\frac{1}{2}} \left( \frac{m_1}{m_2} \right)^{\frac{1}{2}} & 1 & \frac{1}{\sqrt{2}} \\
-2^{-\frac{1}{2}} \left( \frac{m_1}{m_2} \right)^{\frac{1}{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix},
\]

\[\tag{23}\]

where \( m_1, m_2 \) and \( m_3 \) denote the fermion masses of the first, second and third generations, respectively. As seen in eq.(22), the left-handed orthogonal matrix \( U_L \) has a hierarchical structure, which is consistent with the observed quark mixings. On the other hand, the right-handed orthogonal matrix \( U_R \) has a hierarchical structure between the first generation and the second one, but a democratic structure between the second generation and the third one. If an unknown right-handed relative phase between the second generation and the third one is around \( \pi/2 \), the neutrino mixing angle between the second generation and the third one becomes large due to the see-saw enhancement in the case that there is no hierarchy in the Majorana mass matrix. Thus, the see-saw enhancement due to the right-handed phase is possible in the realistic Dirac mass matrix.

Summary is given as follows: We studied the see-saw enhancement mechanism
in presence of the right-handed phases of the Dirac neutrino mass matrix and the Majorana mass matrix. The enhancement conditions given by Smirnov is modified. We pointed out that the see-saw enhancement could be obtained due to the right-handed phases. In that case, the hierarchical structure of the right-handed Majorana mass matrix is not required to get the see-saw enhancement. We showed a realistic example by using the non-Hermitian Dirac mass matrix, which was proposed by Branco and Silva-Marcos. More phenomenological analyses are helpful to confirm this enhancement mechanism in other Dirac matrix textures.

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