Detecting dark matter waves with a network of precision measurement tools

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Virialized Ultra-Light Fields (VULFs) while being viable cold dark matter candidates can also solve the standard model hierarchy problem. Direct searches for VULFs due to their non-particle nature require low-energy precision measurement tools. While the previous proposals have focused on detecting coherent oscillations of the measured signals at the VULF Compton frequencies, here we exploit the fact that VULFs are essentially dark matter waves and as such they carry both temporal and spatial phase information. Thereby the discovery reach can be improved by using distributed networks of precision measurement tools. We find the expected dark-matter signal by deriving the spatio-temporal two-point VULF correlation function. Based on the developed understanding of coherence properties of dark-matter fields, we propose several experiments for dark matter wave detection. In the most basic version, the modifications to already running experiments are minor and only require GPS-assisted time-stamping of data. We also derive the expected dark matter line profile for individual detectors.

Exacting the microscopic nature of dark matter (DM) is one of grand challenges of modern physics and cosmology [1, 2]. The challenge lies in extrapolating from the observational galactic to the laboratory or microscopic scales. It was recently realized [3] that ultralight fields, while being cold DM candidates, can also solve the hierarchy problem elevating such fields to the important class of “economic” DM models such as weakly-interacting massive particles and axions [1, 2]. We will refer to such ultralight fields as VULFs (Virialized Ultra-Light Fields). Direct searches for VULFs due to their non-particle nature require low-energy precision measurements. Such measurements, with their exquisite precision, have been historically important in powerfully constraining new physics beyond the Standard Model (SM). Individual VULF DM search proposals cover a broad range of experiments [4–10]: atomic clocks, magnetometers, accelerometers, interferometers, cavities, resonators, permanent electric-dipole and parity-violation measurements, and extend to gravitational wave detectors. The sought DM signature in these proposals is DM-induced oscillations of the measured signals at the VULF Compton frequencies at the device location.

An important point is that VULFs are waves, and while they do induce an oscillating in time signal at a given spatial location, DM signals at different locations have a fixed phase relation (see Fig. 1(a)), i.e., the signals are correlated. Based on this observation, here we argue that a wider discovery reach can be gained by sampling the DM wave at several locations via a network of precision measurement tools. Further, the VULF signal is composed out of interfering waves traveling at different velocities and in different directions; thereby the problem of relating signals at different space-time locations requires computations of dark-matter correlation functions, derived here. Based on these ideas and derivations, we propose a number of DM wave detection experiments. In the most basic version, the modifications to already running experiments are minor and only require simple GPS-assisted time-stamping of data taking [11]. Previously a network of precision measurement devices have been proposed for detecting topological defects sweeping through the networks [12, 13]; here we show that such networks can be also used as discovery tools for VULF dark matter.

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In the VULF models, dark matter is composed of ultralight spin 0 bosonic fields, oscillating at their Compton frequency \( \omega_\phi = m_\phi c^2/\hbar \), where \( m_\phi \) is the boson mass (see review [14]). The frequencies can span many orders of magnitude: \( 10^{-10} \text{ Hz} \lesssim f_\phi = \omega_\phi / (2\pi) \lesssim 10^{14} \text{ Hz} \) for \( 10^{-24} \text{ eV} \lesssim m_\phi \lesssim 1 \text{ eV} \). Here the lower bound comes

![FIG. 1. (a) Dark-matter wave observatory based on a global network of existing low-energy precision measurement laboratories (red dots) around the globe; (b) Satellite mission for probing VULF DM correlation function; both the distance between the satellites and the angle between galactic velocity \( v_g \) and separation \( d \) vectors can be varied. (c) Terrestrial experiment with fixed nodes utilizes the daily variation of the angle between galactic velocity and two-node separation vector.](http://example.com/fig1.png)
from requiring that the Compton wavelength is smaller than the galactic size and the upper limit from requiring that number of particles per de Broglie volume is macroscopic (see Supplementary Materials). The proposals [4–10] have focused on searching for an oscillating signal at the Compton frequency. Unfortunately, in a laboratory environment, an observation of an oscillating signal could be ascribed to some mundane ambient noise and it desirable to establish additional DM signatures. Here we derive VULF spatiotemporal correlation functions to probe the DM coherence. Qualitatively, the coherence times and coherence lengths of the signal are related to DM properties. Indeed, in the standard halo model (see, e.g., [15, 16]), during the galaxy formation, as DM constituents fall into the gravitational potential, their velocity distribution in the galactic reference frame becomes quasi-Maxwellian with a characteristic dispersion (viral) velocity \( v_{\text{vir}} = \xi_c, \xi \approx 10^{-3} \) and a cut-off at the galactic escape velocity. This velocity distribution leads to spectral broadening of oscillations (dephasing) characterized by the coherence time \( \tau_c \equiv \langle \xi^2 \omega^2 \rangle^{-1} \). The velocity distribution also results in a spatial dispersion of individual wave packets, leading to the coherence length \( \lambda_c \equiv h/(m_\phi c) \).

Additional commonality of all the enumerated proposals is the coupling of DM fields to particles and fields in terms of the so-called portals, when the gauge-invariant operators of the SM fields \( O_X \) are coupled to the operators involving DM fields \([13, 17]\). One of possibilities is the linear coupling (Higgs or linear portal),

\[
-L_{\text{lin}} = \sqrt{\xi_c} \phi(t, r) \sum_X \frac{O_X}{\Lambda_X},
\]

Here \( O_X \) are various pieces from the SM Lagrangian, \( -L_{\text{SM}} = \sum X O_X \), such as the fermion rest mass energies, electromagnetic field tensor contribution, gluon field contribution, etc. \( \Lambda_X \) are the energy scales, characterizing the strength of the coupling. Quite naturally this portal when combined with the SM Lagrangian, leads to variation of fundamental constants, e.g., the electron rest mass \( m_e \) is modulated by DM field as \( m_e(t, r) = m_e \times \left( 1 + \sqrt{\xi_c} \phi(t, r)/\Lambda_{m_e} \right) \) or the fine structure constant \( \alpha(t, r) = \alpha \times \left( 1 + \sqrt{\xi_c} \phi(t, r)/\Lambda_\alpha \right) \), where \( m_e \) and \( \alpha \) are unperturbed quantities. VULF fields oscillate at Compton frequencies, leading to oscillating corrections to fundamental constants \([4]\).

A typical apparatus measures a signal \( S_X(t, r) \) associated with a piece \( O_X \) of the SM Lagrangian (or their combination). For example, an optical clock measures transition frequencies that depend on \( \alpha \). Then the measured quantity has a DM-induced admixture \( \Delta S_X(t, r) \) that is proportional to the field value \( \phi(t, r) \) at the device location. Thereby in the assumption of Higgs portal, the correlation between two devices DM signals can be expressed in terms of the two-point DM field correlation function \( g(\tau, \mathbf{d}) = \langle \phi(t' = t + \tau, \mathbf{r} = \mathbf{r}' + \mathbf{d}) \phi(t, \mathbf{r}) \rangle \):

\[
\langle \Delta S_{X'}(t', \mathbf{r}') \Delta S_X(t, \mathbf{r}) \rangle \propto \frac{1}{\Lambda_X - \Lambda_{X'}} \langle \phi(t', \mathbf{r}') \phi(t, \mathbf{r}) \rangle.
\]

We could also introduce a correlation function for space-time variation of fundamental constants, e.g. \( \langle \alpha(t', \mathbf{r}') \alpha(t, \mathbf{r}) \rangle / \alpha^2 \approx 1 + \frac{k_g}{c^2} \left( \pi \right) \Lambda_{\alpha}^2 \); it is expressed in terms of DM field correlation function.

We derive the VULF correlation function \( g(\tau, \mathbf{d}) \) by generalizing the formalism of quantum optics to massive spin-0 bosons and quasi-Maxwellian velocity distribution of DM fields. The derivation is given in Supplementary Materials. The resulting correlation function reads

\[
g(\tau, \mathbf{d}) = \frac{1}{2} \phi_0^2 \frac{A(\tau, \mathbf{d}) \cos (\omega_\phi \tau - \mathbf{k}_g \cdot \mathbf{d} + \Phi(\tau, \mathbf{d}))}{(\omega_\phi^2 + \mathbf{k}_g^2/2m_e \xi_c)^{3/4}},
\]

Here \( \omega_\phi \) is the Doppler-shifted value of the Compton frequency \( \omega_\phi = \phi_0 + m_\phi v_g^2/2h \) and \( \mathbf{k}_g = m_\phi v_g/h \) is the “galactic” wave vector associated with the apparatus motion through the dark matter halo. The effective field amplitude \( \phi_0 = \frac{\hbar}{m_\phi c} \sqrt{2 \rho_{\text{DM}}} \), and correlation amplitude \( A(\tau, \mathbf{d}) \) and phase \( \Phi(\tau, \mathbf{d}) \) are defined as

\[
A(\tau, \mathbf{d}) = \exp \left( -\frac{|\mathbf{d} - \mathbf{v}_g \tau|^2}{2 \lambda_c^2} \right) \frac{1}{1 + (\tau/\tau_c)^2}^{3/4},
\]

\[
\Phi(\tau, \mathbf{d}) = -\frac{|\mathbf{d} - \mathbf{v}_g \tau|^2}{2 \lambda_c^2} \frac{\tau/\tau_c}{1 + (\tau/\tau_c)^2} + \frac{3}{2} \tan^{-1} (\tau/\tau_c),
\]

where coherence time \( \tau_c \equiv \langle \xi^2 \omega^2 \rangle^{-1} \approx 10^6/\omega_\phi \) and length \( \lambda_c \equiv h/(m_\phi c) \) are expressed in terms of the viral velocity \( \xi_c \approx 10^{-3} c \). Thereby the correlation function encodes the priors on VULFs and DM halo, such as the DM energy density in the vicinity of the Solar system \([18]\), \( \rho_{\text{DM}} \approx 0.4 \text{ GeV/cm}^3 \), motion through the DM halo at \( v_g \) and the viral velocity \( \xi_c \). As such it provides an improved statistical confidence in the event of an observation of DM signal.

Now with the correlation function at hand, we explore the vast VULF parameter landscape. Typical values of various parameters are compiled in Table I. Coherence time is roughly a million of Compton periods. The coherence length can be interpreted as the de Broglie wavelength of a particle moving at virial velocity and it is thousand times larger than the Compton wavelength. The number density of VULF particles \( \rho_{\text{DM}}/(m_\phi c^2) \) ranges from \( 10^8 \) to \( 10^{32} \text{ cm}^{-3} \) for the indicated masses, i.e., a typical device interacts with a macroscopic number of DM particles. Compton frequencies range from nHz to PHz. Notice that one oscillation per year corresponds to \( \phi_0 = 3 \times 10^{-18} \text{ Hz} \) \( (m_\phi \sim 10^{-22} \text{ eV}) \). As points of reference for the coherence length, the size of our galaxy is \( \sim 10^{18} \text{ km} \) and the Earth diameter \( \sim 10^4 \text{ km} \).
In the limit when both coherence length \( \lambda_c \) and time \( \tau_c \) are infinitely large one recovers the fully coherent wave correlation function

\[
g_{\text{coh}}(\tau, \mathbf{d}) = \frac{1}{2} \phi_0^2 \cos(\omega_0 \tau - \mathbf{k}_g \cdot \mathbf{d}).
\] (3)

For a single geographic location the associated power spectral density (PSD) in frequency space is a spike at the Doppler-shifted Compton frequency. Searching for spectral density (PSD) in frequency space is a spike at the Doppler-shifted Compton frequency. For a single geographic location the associated power

\[
& 10^{-24} \quad 2 \times 10^{-10} \quad 7 \times 10^{14} \quad 2 \times 10^{17} \\
& 10^{-20} \quad 2 \times 10^{-6} \quad 7 \times 10^{10} \quad 2 \times 10^{13} \\
& 10^{-15} \quad 2 \times 10^{-1} \quad 7 \times 10^{5} \quad 2 \times 10^{8} \\
& 10^{-10} \quad 2 \times 10^{4} \quad 7 \times 10^{3} \quad 2 \times 10^{8} \\
& 10^{-5} \quad 2 \times 10^{9} \quad 7 \times 10^{-5} \quad 2 \times 10^{-2} \\
& 1 \quad 2 \times 10^{14} \quad 7 \times 10^{-10} \quad 2 \times 10^{-7}
\]

**TABLE I.** Parameters of VULF dark matter for a range of masses \( m_\phi \): Compton frequency \( f_\phi \), coherence time \( \tau_c \) and length \( \lambda_c \) and the inverse galactic wave-vector \( k_g^{-1} \sim \lambda_c \) associated with our motion through DM halo.

Another approach is to work in the frequency space by the Fourier transform of the absolute 

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S_{k\phi}(\omega) \cos(\omega_0 \phi - \mathbf{k}_g \cdot \mathbf{d}/\hbar) d\omega d\mathbf{k}_g.
\]

For a single geographic location the associated power spectral density (PSD) in frequency space is a spike at the Doppler-shifted Compton frequency. Searching for such resonances has been proposed [4] for atomic clocks with fixed positions of net-works [20] infrastructure can serve as a natural host for the VULF DM observatory. One could also use clocks on numerous navigational satellites, such as GPS, to search for correlation patterns [13]; these have an advantage of half-a-day orbits (GPS) and a large \( d \sim 50,000 \) km aperture. eLISA gravitational wave mission [21] (a network of three satellites) can also be used for VULF detection. Another point is that the network does not need to be homogenous, and various precision measurement tools can be included in the network. Indeed, as long as the DM portal is linear, Eq.(1), such a global observatory can “cast a much wider net” on possible DM couplings.

Compared to an individual device, the statistical sensitivity of a correlated network is improved by the factor \( \sqrt{N_{\text{nodes}}} \), where \( N_{\text{nodes}} \) is the number of nodes. It is further improved through the sampling of the \( \mathbf{k}_g \cdot \mathbf{d} \) phase by \( \sqrt{N_{\text{kd}}} \) (\( N_{\text{kd}} \) measurements of correlation function). All individual detector proposals [4–10] searching for Compton frequency oscillations can substantially constrain yet unexplored parameter space. There is no need to reproduce projected sensitivities of these proposals here; one could simply rescale them by the factor \( 1/\sqrt{N_{\text{nodes}}}N_{\text{kd}} \).

For co-located devices (or the same apparatus), the “local” temporal correlation function is

\[
g(\tau, \mathbf{d} = 0) = \frac{1}{2} \phi_0^2 \exp\left(-\frac{1}{2} \left(\frac{\omega_0 \tau}{\lambda_c} \right)^2 \right)^{3/4} \cos(\omega_0 \tau + \Phi(\tau, 0)).
\]

Notice the presence of the coherence length in the combination \( v_g \tau/\lambda_c \); it arise due to our motion through DM halo over the lag time \( \tau \) thereby sampling DM fields \( v_g \tau \approx 10^{-3} c \tau \) distance apart. For \( \tau = 1 \) s this translates into the ~300 km distance. Considering that \( v_g \approx v_{\text{vir}}, v_g \tau/\lambda_c \approx \tau/\tau_c \). The signal primary oscillation frequency also depends on \( v_g \). Since the Earth velocity changes seasonally, annual velocity modulations are imprinted in the correlation function.

In practice, one would obtain a time series of measurements \( \{S_n\} \) at \( t_n = n t_0 \), compute auto-correlation function \( G_k = \langle S_n S_{n+k} \rangle \) and see if it fits Eq. (4). Another approach is to work in the frequency space and examine the power spectral density of \( S_n \) given by the Fourier transform of \( G_k \). To facilitate a comparison with \( G_k \), we can define the DM-induced line-shape as a Fourier transform of the correlation function \( f(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \mathcal{G}(\tau, d = 0) e^{i\omega \tau} \), where \( F(\omega) \) is normalized as \( \int_{-\infty}^{\infty} F(\omega) d\omega = 1/2 \). With \( \eta = v_g/v_{\text{vir}} = v_g/(\xi c) \approx 1 \) and \( \omega_0 = \omega_0 + m_\phi v_g^2/2 \) the resulting “DM line-shape” reads

\[
F(\omega) = (2\pi)^{-1/2} \tau_c \eta^{-1} e^{-\eta^2} e^{-(\omega-\omega_0) \tau_c} \sinh\left(\eta \sqrt{\eta^2 + 2 (\omega - \omega_0) \tau_c}\right).
\]

\( \xi \)
This holds for detunings $\omega - \omega' > -\eta^2/(2\tau_c)$, otherwise $F(\omega) = 0$. The profile is shown in Fig. 2. As expected the line-width is $\sim \tau_c^{-1}$. The profile is strongly asymmetric due to parabolic dispersion relation for massive non-relativistic bosons. For $\eta = 1$, the maximum $F(\omega)$ value of $\approx 0.18\tau_c$ is attained at $\omega \approx \omega'_c + 0.22/\tau_c$, and the width at half-maximum is $\Delta \omega/\omega \approx 2.5/\tau_c$. It is broad in the spectroscopic sense as $\Delta \omega/\omega \approx 3 \times 10^{-6}$.

Now we examine a practical example of a network: a network of atomic clocks. As an illustration we focus on two nodes each hosting identical devices in the configuration of Fig. 1(b), i.e., orientation with respect to galactic velocity $v_g$ is static yet we can vary the distance $d$ between the nodes. Extension to rotating reference frame for terrestrial networks is straightforward. In our preceding discussion we assumed that the measurements were instantaneous; in practice there is always a finite interrogation time $t_0$ for a single measurement. We assume that the next measurement is taken right after the previous one was completed (no “dead” times) and that the measurements at different nodes are synchronized. We form a time series of fractional frequency excursions $\{S_n^{(m)} \equiv (\omega_n - \omega)/\omega\}$ taken at $t_n = n t_0$, $n = 1, 2, \ldots$, for a fixed distance $d$, with $m$ labeling the node and $\omega_c$ being the nominal clock frequency (strictly speaking one needs to form frequency ratios of two co-located clocks [4]). The VULF contribution to $S_n^{(m)}$ can be expressed in terms of sensitivity coefficients $K_X = \partial \ln \omega_c/\partial \ln X$ and single-measurement accumulated clock phase

$$S_n^{(m)} = \left( \sqrt{\hbar c} \sum_X \frac{K_X}{\Lambda_X} \right) \int_{t_n-t_0}^{t_n} \phi(r, t') dt'/t_0.$$ 

Notice the integral of the VULF field time evolution history over the interrogation duration. The two-node correlator $G(d) = \langle S_n^{(1)} S_n^{(2)} \rangle$ reduces to a double integral over the field correlation function $g(\tau, d)$: 

$$G(d) = \left( \sqrt{\hbar c} \sum_X \frac{K_X}{\Lambda_X} \right)^2 I(d),$$

with 

$$I(d) = \int_{-1/2}^{1/2} \int_{-1/2}^{1/2} \sum_{\xi_1, \xi_2} g((\xi_2 - \xi_1)t_0, d).$$

While $I(d)$ can be numerically evaluated with Eq. (2), we demonstrate the essential features in the limit of infinite coherence lengths and times $(d \ll \lambda_c, t_0 \ll \tau_c)$, i.e. with $g_{coh}$, Eq. (3):

$$G_{coh}(d) = \frac{\hbar^3 \rho_{DM}}{m^2 c^4} \left( \sum_X \frac{K_X}{\Lambda_X} \right)^2 W(\omega'_c t_0) \cos(k_g \cdot d).$$

The dependence on the distance $d$ is transparent. The “window” function $W(\omega'_c t_0) = \sin^2(\omega'_c t_0/2)/(\omega'_c t_0/2)$ emphasizes the dependence on the ratio of interrogation time to the period of VULF oscillation. If VULF oscillations are slow compared to $t_0$, then $A(\omega'_c t_0 \ll 1) \approx 1$ and if they are fast, the effect tends to average out, $W(\omega'_c t_0 \gg 1) \approx 2(\omega'_c t_0)^{-2}$. For a typical $t_0 \sim 1$ s, the separation between the two regimes occurs for $f_0 \approx 1$ Hz ($\omega'_c \sim 10^{-14}$ eV). Another consideration is synchronization accuracy: for fast oscillations, the uncertainty comes from $\sin^2(\omega'_c t_0/2)$ in $W$; if $t_0$ is uncertain $\sigma_{t_0} \gtrsim 1/\omega'_c$, the $W$-factor in Eq. (6) has uncontrolled oscillatory behavior invalidating the entire analysis. Ref. [23] has demonstrated open-air synchronization of optical clocks at the level of fs. Thereby we expect that a network should remain sensitive to $f_0 \ll 10^{12}$ Hz; at such frequencies the smallness of coherence time on the interrogation timescale becomes more important. Similar considerations apply to fiber networks [24] and GPS time transfer [25].

So far we have discussed large-scale experiments: a satellite mission where the distance $d$ between the devices can be in the order of $10^6$ km and terrestrial networks where the separations can be as large as $\sim 10^4$ km. Finally, it is worth pointing out that small-scale experiments can probe a complementary parameter space of large masses (see Table I). For example, the coherence length of sub-meV VULFs is of a meter scale, while eV VULFs have $\lambda_c \sim 100 \mu$m. These lengths suggest tabletop experiments or measurements with optical tweezers [26] where the distance between individual atoms can be varied. Additional advantage of tabletop experiments is a better synchronization as the common laser source can be used to probe both locations and the possibility of using rotating tables [27] that would compensate for the earth rotation with respect to the galactic wave vector.

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SUPPLEMENTARY MATERIALS

We derive the VULF correlation function $g(\tau, \mathbf{d})$ by generalizing the formalism of quantum optics to massive spin-0 bosons and quasi-Maxwellian velocity distribution of DM fields. We use the natural units and start the derivation in the galactic reference frame and then transform the result into the moving device frame.

$$g(\tau, \mathbf{d}) = \text{Trace} \left( \hat{\rho} \hat{\phi} (t', \mathbf{r}') \hat{\phi} (t, \mathbf{r}) \right),$$

where $\hat{\rho}$ is the density matrix and $\hat{\phi}$ are the field operators, $\hat{\phi}(x) = \sum_k (\hat{a}_k e^{-ik.x} + \hat{a}_k^\dagger e^{ik.x}) / \sqrt{2V\omega_k}$. Here $V$ is the quantization volume, $k$ and $x$ are 4-momentum and 4-position vectors, $\hat{a}_k^\dagger$ and $\hat{a}_k$ are bosonic creation and annihilation operators. The summation is carried out over the field modes with frequencies $\omega_k = \sqrt{m^2 + \mathbf{k}^2} \approx m_\phi + k^2/2m_\phi$. Taking into account that the mode occupation numbers $\bar{n}_k$ are macroscopic, $g(\tau, \rho) \approx \frac{1}{(2\pi)^3} \int d^3k \frac{1}{2\omega_k} \text{Tr} \left[ \hat{\rho} \hat{n}_k \cos (k \cdot (x - x')) \right]$. Further, the mode occupation numbers is related to the local DM energy density $\rho_{DM}$ and DM velocity distribution $f_{DM}(\mathbf{k})$ as $\bar{n}_k = (2\pi)^3 \rho_{DM} f_{DM}(k) / m_\phi$. The requirement that the occupation numbers are macroscopic, $\bar{n}_k \gg 1$, leads to $m_\phi \ll 10eV$. This explains the upper limit on VULF masses of 1 eV used in the main text.

The resulting two-point correlation function reads (restoring fundamental constants)

$$g(\tau, \mathbf{d}) = \left( \frac{\hbar}{m_\phi c} \right)^2 \rho_{DM} \int d^3v \frac{f_{DM}(v)}{1 + \frac{1}{2} (v/c)^2} \times \cos \left( \frac{m_\phi v^2}{\hbar} \tau - \frac{m_\phi v}{\hbar} \cdot \mathbf{d} + \frac{m_\phi v^2}{2\hbar} \tau \right).$$

(7)

Given the DM velocity distribution [28] $f_{DM}(v)$ this expression can be evaluated numerically. Analytically result can be obtained by taking the Maxwellian distribution of the standard halo model, $f_{DM}(v) = (2\pi)^{-3/2} (\xi c)^{-3} \exp \left( -\frac{(v - v_g)^2}{2(\xi c)^2} \right)$, where $\xi c$ is the virial velocity and $v_g \approx 10^{-3}c$ is the Earth’s velocity in the galactic reference frame. We further take the galactic escape velocity cutoff to be infinite and neglect the non-relativistic kinetic energy correction in the denominator. The resulting correlation function is given by Eq. (2) of the main text.

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