Clusters of galaxies are the largest virialised structures in the universe [1]. They form via a hierarchical sequence of mergers and accretion of smaller systems driven by dark matter, that dominates the gravitational field. The majority of baryonic matter in clusters is in the form of a hot (∼$10^8$ K) and tenuous (∼$10^{-4}$ cm$^{-3}$) gas, the intra-cluster medium (ICM).

The ICM is a unique environment for plasma physics due to the combination of its weakly collisional nature and high plasma beta. It also provides situations for cosmic ray (CR) acceleration and dynamics that differ significantly from those in other astrophysical environments. First of all, the bulk of CRs are confined and accumulated in the very large volumes of galaxy clusters for Hubble time [2–5]. Second, the lifetime of CRs in the ICM is very long, as CRs diffuse in a dilute high-beta plasma [5, 6]. Under these conditions particle acceleration mechanisms that are notoriously inefficient, such as stochastic Fermi-II mechanisms, become important because they can act for very long time-scales and on very large volumes. Another point is that the combination of CR confinement and long lifetime with the complex dynamics of the ICM implies that CRs can experience phases of cooling and (re)acceleration by different mechanisms. This results in a complex energy and spatial distribution of the CRs in clusters [5, 7].

Clearly, galaxy clusters contain several discrete sources of CRs, including galaxies and AGNs, however there is much more. Indeed, radio observations show cluster-scale diffuse synchrotron sources in a fraction of galaxy clusters; the most prominent ones are giant radio halos that cover Mpc$^3$ volumes [8]. These emissions are not directly associated with discrete sources and require in situ mechanisms of particle acceleration...
in the ICM [5]. Radio halos are observed in cluster mergers [8–11], suggesting that a fraction of the kinetic energy of large-scale motions can be channelled into electromagnetic fluctuations and particle acceleration at smaller scales in the ICM. This point is telling us that a complex hierarchy of processes is active in the ICM at different scales, and offers a possibility to constrain fundamental aspects of the microphysics of the ICM.

The synchrotron spectra of radio halos are steep, with spectral slopes measured at gigahertz frequencies $\alpha \sim 1–2$ (flux $\propto \nu^{-\alpha}$), suggesting that the underlying acceleration mechanisms are not very efficient [5]. A popular scenario for the origin of radio halos is based on the possibility that turbulence generated during cluster–cluster mergers (re)accelerates seed electrons distributed on Mpc$^3$ volumes to energies of a few giga-electron volts or more, that are required to produce the synchrotron radiation observed in the radio band [12–20]. This scenario allows us to explain the very extended and diffuse nature of the observed emission of radio halos and the tight connection between radio halos and the dynamics of the hosting clusters. On the other hand crucial ingredients in this scenario are poorly known. Primarily, the challenge is to understand the chain of mechanisms that transport energy from large scales to collisionless small scales in the ICM.

In this paper we focus on CRs acceleration by compressive turbulence driven at large scales in the ICM. Specifically we explore the changes in the efficiency of acceleration that are induced by different assumptions on the turbulent spectrum and ICM microphysics.

2. A brief overview of turbulence in the ICM

In this section we briefly review the current view of the properties of turbulence in galaxy clusters. Galaxy clusters contain many potential sources of turbulence [5, 21]; the most important for large-scale turbulence are mergers between clusters. Mergers deeply stir and rearrange the cluster structure, generating turbulence through the sloshing of cluster cores, through shearing instabilities in the ICM and via the complex patterns of interacting shocks that form during mergers, and structure formation more generally. Such a complex ensemble of mechanisms should drive both compressive and incompressive turbulence, as also supported by the analysis of numerical (fluid) simulations [18, 22, 23].

Large-scale motions that are driven during cluster–cluster mergers and dark matter subhalo motions are expected on scales comparable to cluster core scales, $L_0 \sim 100–400$ kpc, and might have typical velocities $v_B \sim 300–700$ km s$^{-1}$ [15, 21, 23–26]. These motions are subsonic, typically with $M_v = \delta V l_c \approx 0.2–0.5$, but super-Alfvénic, with $M_A = \delta V / V_A \approx 2–8$. An open question is whether these motions generate an efficient turbulent cascade at smaller scales or if they are dissipated at larger scales. The Coulomb mfp of particles in the ICM is very large, $l_C \approx 10$ kpc, which in turns implies a moderate value of the Reynolds number in the ICM, $\mathcal{R} e \sim 100$, for typical parameters. Such a moderate Reynolds number would not guarantee per se an efficient turbulent cascade. However the ICM is a magnetised and weakly collisional high-beta medium, that is notoriously unstable to several instabilities [27–32], implying that collisionless effects govern microphysics and make the medium more turbulent. The combination of these facts suggests that the ICM is turbulent. We note that this conclusion is also supported by similarities with the IPM, that is also a weakly collisional, magnetised and (moderately) high-beta plasma. In fact, the Reynolds number (considering Coulomb collisions) of the IPM would not be large, but the IPM is observed to be turbulent [33].

At large scales, where the bulk of turbulence is generated by cluster dynamics in the ICM, turbulence is hydrodynamic with kinetic energy of motions being in excess of magnetic energy. At scales smaller than the MHD scale, $l_A = \Lambda_0 M \delta A^3$ (using Kolmogorov scaling for hydro-turbulence), turbulence becomes MHD provided that the ICM behaves as a fluid and that collisionless effects play a role only at smaller scales. MHD turbulence can be described by the Goldreich–Sridhar model with Alfvén and slow modes developing an anisotropic spectrum [34, 35]. On one hand we can think that solenoidal and compressive turbulence are generated at large scales and cascade at smaller scales; at the same time however we should expect that turbulence can also be generated directly at smaller scales in response to plasma/kinetic instabilities and coherent wave phenomena driven by the cascade of strong MHD turbulence [28, 31, 36, 37]. The energy associated with small-scale turbulence is however subdominant. Both large-scale motions (and their cascading at smaller scales) and the variety of waves excited at small scales should play crucial roles in governing the micro-physics of the ICM through the scattering of particles and the perturbation and amplification of the magnetic field. However, the efficiency of the transport of turbulent energy from large to small scales and the efficiency of generation of waves at smaller scales are still open issues.

The nonlinear interplay between particles and turbulent waves/modes induces a stochastic process that drains energy from plasma turbulence to particles [38–40]. Turbulent acceleration is invoked for the origin of giant radio halos. Acceleration of CRs directly from the thermal pool to relativistic energies by MHD turbulence in the ICM is very inefficient and faces serious problems due to associated energy arguments [41]. Consequently, turbulent acceleration in the ICM is rather a matter of reacceleration of pre-existing (seed) CRs rather than ab initio acceleration of CRs. This poses the problem of the origin of the seed particles, a problem that is currently a subject of active discussion [7, 17], but will not be addressed in this paper.

Presumably the many types of wave generated/excited in the ICM, at both large and very small scales, jointly contribute to the scattering process and (re)acceleration of CRs. In recent years, much attention has been devoted to CR reacceleration due to compressible turbulence that is driven at large scales in the ICM from cluster mergers and that cascades to smaller scales [16, 20]. This is the simplest scenario that can be thought of; nevertheless it naturally predicts a direct connection between cluster mergers and radio halos. In fact several
studies suggest that turbulent reacceleration by compressive modes provides a plausible explanation for radio halos [5, 7, 16–19], although several ingredients of the physics of this mechanism are still poorly known.

In the following we will discuss transit-time-damping (TTD) resonance (section 3) and non-resonant acceleration by turbulent compressions (section 4), exploring the effects induced on the efficiency of these mechanisms by different assumptions.

3. Transit time acceleration mechanism: turbulent spectrum and mfp

The compressible component of the magnetic field of compressible modes (i.e. the component along \( B_0 \) in the case of oblique propagation) can interact with particles through TTD resonance [42, 43]. The condition for resonance is

\[
\omega - k_0 v_\parallel = 0
\]

(1)

where \( k_0 \) and \( v_\parallel \) are the components of the wavenumber and particle velocity parallel to the magnetic field; \( \omega = c k \) for fast modes (magnetosonic waves) in the ICM. This interaction is essentially a coupling between the magnetic moment of particles and the (parallel) magnetic field gradients. This interaction, in combination with additional/external sources of pitch-angle scattering/isotropisation during acceleration [16, 44, 45], is considered as a fundamental way to accelerate particles in different astrophysical environments, including the ICM.

Stochastic acceleration can be described as a diffusion in the momentum space of particles. For TTD the expression of diffusion coefficient, assuming quasi-isotropic turbulent cascade and high-beta plasma (conditions suitable for the ICM), is given in [16]:

\[
D_{pp}(p) = \frac{\pi^2}{2} \frac{p^2 c_s^2}{2eB_0^2} \int_{k_0}^{k_{\text{cut}}} dk W_B(k) k^2
\]

(2)

where \( \mu \) is the cosine of the pitch angle, \( H(x) \) is the Heaviside step function (1 for \( x > 0 \) and 0 otherwise), \( W_B \) is the energy spectrum of magnetic field fluctuations and \( k_0 \) and \( k_{\text{cut}} \) are the injection and cut-off scales (wavenumber) of the fluctuations.

The most important ingredients in equation (2) are the energy density of electromagnetic fluctuations and their spectrum, and the cut-off scale of the turbulent (magnetic fluctuations) spectrum \( k_{\text{cut}} \). In the simple scenario adopted in this paper, where turbulence is generated at a large scale and cascades at smaller scales, the minimum scale \( k_{\text{cut}} \) is the scale where the mechanisms of damping of turbulence become faster than turbulent cascade. During acceleration, energy goes into particles increasing the damping of turbulence due to the particles themselves (the cut-off scale \( 1/k_{\text{cut}} \) increases) and reducing the acceleration efficiency. Constraining \( k_{\text{cut}} \) is thus critical to obtain meaningful estimates of the acceleration efficiency.

To do this we need to calculate the damping of turbulence in the ICM and the efficiency of turbulent cascade. Following the motivations given in [16] we assume that collisionless dampings with particles are the dominant ones in the ICM. The damping is obtained assuming quasi-linear theory [39, 40]:

\[
\Gamma = -i \left( \frac{E_0^2 k_0^2 E_j}{16\pi W} \right) \omega_\parallel \delta \omega
\]

(3)

where \( K_j^2 \) is the anti-Hermitian part of the plasma dielectric tensor [39], \( W \) is the total energy in the modes, \( E_j \) is the electric field (fluctuations) and \( \omega_\parallel \) is the real part of the mode frequency.

TTD determines the strongest collisionless interaction between particles and compressive fast modes in the ICM. The collisionless damping with thermal electrons and protons is [16]

\[
\Gamma_{\text{el}}(k, \theta) = \frac{\pi}{8} \left( \frac{B_0}{W(k, \theta)} \right) \left(1 - \frac{c_e}{c} \frac{k}{|k||} \right)^2 \left( \frac{e}{c} \left( \frac{k}{|k||} \right) \right)^2 \times \left( m_e c_s^2 \right)^{1/2} N_{\text{el}} \exp \left( \frac{m_e c^2}{2 k_0 T} \left[ 1 - \left( \frac{c_k}{c_{k||}} \right)^2 \right] \right)
\]

(4)

where the ratio of magnetic field fluctuations and total energy density is calculated in the collisionless regime in [16], \( \beta_{el} = 2c_e^2/V_A^2 \), and \( \langle B_0^2 / W \rangle \approx 16\pi/\beta_{el} \langle \ldots \rangle \) is the average over pitch-angles.

The other source of damping of fast modes in the ICM is due to TTD interaction with CRs [16]:

\[
\Gamma_{\text{el}}(k, \theta) = - \frac{\pi}{8} \left( \frac{B_0}{W(k, \theta)} \right) \left( \frac{k}{|k||} \right)^2 \left( \frac{e}{c} \left( \frac{k}{|k||} \right) \right)^2 \times \left( m_e c_s^2 \right)^{1/2} N_{\text{el}} \exp \left( \frac{m_e c^2}{2 k_0 T} \left[ 1 - \left( \frac{c_k}{c_{k||}} \right)^2 \right] \right)
\]

(5)

where \( N_{\text{el}} \left( p \right) \) is the distribution function of CRE\( p \) in the momentum space. The second element that is necessary to constrain \( k_{\text{cut}} \) is the time-scale of turbulent cascade. We shall use MHD as a guide and derive the cascading of isotropic fast modes using the Kraichnan treatment. In this case the wave–wave diffusion coefficient in \( k \)-space is [46]

\[
D_{k} \sim k^4 W(k)/(\rho c_s)
\]

and the resulting cascading time is

\[
\tau_{k} \sim \frac{k^3}{(\partial / \partial k ) (D_{k} / \Delta k)^2} \sim \frac{c_s}{9} \frac{\delta V^2}{\delta V^2} (k_0) \sim 1/2.
\]

(6)

In equation (6) and in the following we assume \( W(k_0) k_0 \sim \delta V^2 ; \delta V \) is the velocity of large-scale eddies.

---

1 indeed the \( n = 0 \) resonance changes only the component of particle momentum parallel to the seed magnetic field that would increase the degree of anisotropy of particle distribution and decrease the acceleration efficiency with time.
Having derived damping and cascading coefficients, the cut-off scale is obtained requiring $\tau_{\alpha} \sim t_{\alpha}^{-1}$:

$$k_{\text{cut},K} \approx \frac{81}{4} \left( \frac{\delta V^2}{\epsilon} \right)^2 \frac{k_0}{\left( \sum_s (\Gamma_s \epsilon_s)^{-1} \right)^2}$$

(7)

where $\langle ... \rangle$ marks pitch-angle averaging.

Given the uncertainties in the ICM microphysics, we consider two scenarios.

(i) Assume that the interaction between turbulent modes and both thermal and CRs is fully collisionless. This happens when particles' collision frequency in the ICM is $\omega_i < \omega = k_0^2$; for example this is the case where ion–ion collisions in the thermal ICM are due to Coulomb collisions. Under this condition the damping of compressive modes is dominated by TTD with thermal particles (equation (4)) [16]. For large $\beta_{pl}$ the dominant damping rate is due to thermal electrons and can be approximated by $\Gamma_{e} \approx c k \sqrt{3 \pi (m_i/m_p) 20 \mu^2} \exp(-5(m_i/m_p)/(3/\mu^2)) (1 - \mu^2)$. Combining pitch-angle averaging of this expression with equation (7) gives $k_{\text{cut},K} \sim 10^{4} k_0 M_0^4$.

Assuming a Kraichnan spectrum of magnetic field fluctuations, $W_0 \propto k^{-3/2}$, the resulting acceleration time-scale (from equation (2)) is

$$\tau_{\text{acc}} = \frac{p^2}{4 D_{pp}} \simeq 2.5 \left( \frac{\beta_{pl}|B|}{16 \pi W} \right)^{-2} \left( \frac{M_0}{1/2} \right)^{-4} \left( \frac{L_{\text{obs}}/300 \text{ kpc}}{c/1500 \text{ km s}^{-1}} \right) \text{(Myr)}$$

(8)

where $\langle ... \rangle \sim 1$ independent of scale, $x = c_s/c$ and

$$f_x = \frac{\pi}{4} \left( \frac{x^4}{4} + x^2 (1 + 2x^2) \ln (x) - \frac{5}{4} \right) = 0.02 .$$

(9)

For typical conditions this scenario predicts acceleration time-scales of about 100 Myr that are sufficient to explain radio halos, and is commonly adopted to calculate turbulent acceleration in the ICM [7, 16, 20, 47].

(ii) The other possibility is that thermal particles do not contribute very much to the collisionless dampings. The ICM is a weakly collisional high-beta plasma that is unstable to several instabilities (see section 2). Scattering induced by magnetic field perturbations driven by instabilities may increase the collision frequencies in the thermal plasma, because charged particles can be randomised if they interact with the perturbed magnetic field. This process can be viewed as the collective interaction of an individual ion with the rest of the plasma. Under this condition one may think that the interaction between modes and thermal particles behaves as collisional. In this case the dominant collisionless damping in the ICM is due to the CRs [30], and combining equations (5) and (7) we have

$$k_{\text{cut},K} \approx \left( \frac{18 \pi}{\sqrt{\pi}} \right)^2 M_0^2 f_x^{-2} \left( \frac{\rho_{\text{cr}}^2}{\rho_e} \right)^{1/2} k_0^{-1}$$

(10)

Under typical conditions in the ICM this is $k_{\text{cut},K} \sim 1000 k_0 M_0^2 (\epsilon_{\text{ICM}}/\epsilon_{\text{CR}})^2$, where the ratio of thermal and CR energy densities is $\epsilon_{\text{ICM}}/\epsilon_{\text{CR}} \sim 100$. It implies that the turbulent cascade reaches scales that are much smaller than those in case (i) and consequently the acceleration rate is higher (equations (2) and (10)):

$$\tau_{\text{acc}} = \frac{p^2}{4 D_{pp}} \approx 6 \left( \frac{\beta_{pl}|B|}{16 \pi W} \right)^{-2} \left( \frac{M_0}{1/2} \right)^{-4} \left( \frac{L_{\text{obs}}/300 \text{ kpc}}{c/1500 \text{ km s}^{-1}} \right)\times \left( \frac{c}{\rho_e^{1/2} \epsilon_{\text{cr}}^{1/2}} \right)^{0.04} \text{(Myr)} .$$

(11)

The acceleration efficiency is inversely proportional to the energy density of CRs. In this scenario CRs drain energy efficiently from the turbulent cascade; however as the CR energy density increases, the acceleration efficiency is reduced. From the practical point of view this is simply because CRs can obtain a constant energy flux from turbulence, implying that the effect on their spectrum is smaller for increasing values of the CR energy density. As a consequence of this back reaction, fast, acceleration cannot be maintained for long time. Assuming typical conditions in the ICM, the value of the acceleration time averaged on a time-period of few hundred megayears is generally found to be $\tau_{\text{acc}} \sim 10$ Myr [30]. This is up to ten times more efficient than in case (i), resulting in harder spectra of both CRs and synchrotron radiation [20, 30].

All the results discussed above are based on the Kraichnan spectrum of fast modes, $W_{k}(k) \propto k^{-3/2}$. On the other hand, part of the energy of the MHD cascade of fast modes may be dissipated into weak shocks, producing a steeper spectrum [48].

In general, if the spectrum of compressive turbulence becomes steeper, the turbulent acceleration rate decreases. Assuming a spectrum $W_{k}(k) \propto k^{-5/2}$ in equation (2), the acceleration rate will be changed with respect to that evaluated using a Kraichnan spectrum by a factor $3$ of about $1/2 \left( k_{\text{cut}/s}^{-1} 2^{-s/2} k_{\text{cut},s}^{-1} 2^{-s/2} k_{\text{cut},K}^{-1} \right)$, where $k_{\text{cut},s}$ is the cut-off in the turbulent spectrum assuming a slope $s$. In order to evaluate this $k_{\text{cut},s}$ we follow a simple procedure. We still assume the Kraichnan diffusion coefficient $D_k \sim k^2 W(k)/\rho_e$ to model wave–wave coupling at scale $k$, but we use a spectrum $W(k) \propto k^{-s}$. The resulting cascading time-scale of fast modes is

$$\tau_{k} \approx \frac{c_s}{5} \frac{k^{-2}}{5 - s} \frac{k_{\text{cut},K}}{k_0} = \frac{7/2}{5 - s} \tau_{\text{ic}} \left( \frac{s}{3} \right)^{1/2} \left( \frac{k}{k_0} \right)^{-1}$$

(12)

3 It should also be mentioned that scatterings induced by instabilities decrease the effective mfp of thermal particles, decreasing the effective viscosity in the ICM and increasing the effective Reynolds number of the ICM.
and the cut-off, obtained from the condition $\tau_{kk} \sim 1/\sum(\Gamma)$, is

$$k_{cut} = \left[\frac{2}{7} (5 - s) k_{cut,K}^{1/2} k_{0}^{-3/2}\right]^{1/7}$$

(13)

where $k_{cut,K}$ refers to the cut-off in both equations (7) and (10).

Figure 1 shows the effect on the acceleration rate due to different assumptions for the turbulent spectrum. The acceleration rate decreases with increasing slope. For Burgers spectra ($s = 2$) the acceleration is ten times less efficient that in the Kraichnan case. This has important consequences; for example in the collisional case (i) steep spectra of magnetic field fluctuations produce acceleration rates via TTD that are generally too small to explain radio halos [20].

4. Stochastic acceleration by large-scale compression: turbulent spectrum and mfp of relativistic particles

Relativistic particles diffusing through large-scale compressible turbulence experience a statistical acceleration effect [16, 49, 50]. In the case of subsonic turbulence, $\delta V^2 \ll c_s^2$, and provided that turbulence has correlation scales much longer than the particles mfp, the diffusion coefficient in the particle momentum space is

$$D_{pp} = \frac{2}{9} \frac{p^2 D}{k_{0}^{2}} \int_{k} \frac{d^{2}K(y)}{c_{s}^2 + y^2 D^2}$$

(14)

where $D$ is the particle spatial diffusion coefficient and $K$ is the kinetic spectrum of turbulence, $k_0 K(k_0) \sim \delta V^2/2$.

The spatial diffusion coefficient of CRs in the ICM is unknown and we shall consider it as a free parameter. We assume $D = \frac{1}{2} c_{s} \lambda_{mfp}$ with a CR mfp $\lambda_{mfp} = \xi l_{b}$, where $l_{b}$ is the minimum scale of magnetic field reversal in the ICM. Under these assumptions equation (14) reads

$$D_{pp} = \frac{p^2}{3} \frac{\delta V^2}{c_{B}} \frac{\tau_{cut}}{2 \pi} \int_{k} \frac{d^{2}x}{x^2 + \epsilon^2 x^2}$$

(15)

where $x_{k} = k_{cut} l_{b}$.

The acceleration rate depends on the turbulent energy and spectrum, but also on the CRs mfp. This mechanism is characterised by two regimes: fast diffusion, for $\xi^2 \gg (3 c_{s} l_{b})^2/(2 \pi c_{B})^2$, in which case particles leave the eddies before they turn over and the acceleration is dominated by the largest eddies, and slow diffusion, for $x_{k}^2 \xi^2 \ll (3 c_{s} l_{b})^2/(2 \pi c_{B})^2$, in which case the acceleration is mainly dominated by the smallest eddies. In the two regimes the dependences of the acceleration rate on the physical parameters are different: $D_{pp} \propto p^2 M_{0}^{2} D_{ff}^{1} k_{cut}^{-1}$ in the slow regime and $D_{pp} \propto p^2 M_{0}^{2} D_{ff}^{2} D_{fast}$ in the fast regime (see also [50]).

The total (turbulent advection and diffusion) spatial diffusion coefficient of particles diffusing and interacting with compressible/acoustic turbulence is [16, 49, 50]

$$D_{a} = \frac{1}{3} l_{b} c \xi \left[1 + \frac{3}{2 \pi} \left(\frac{L_{0} \delta V}{l_{b} c}\right)^{2} \int_{1}^{\infty} \frac{d^{2}x}{x^2 + \epsilon^2 x^2}\right]$$

(16)

the time-scale necessary to diffuse on scale $L^*$ is $\tau_{diff} \approx L^*/4D_{a}$.

In order to quantify the acceleration rate we should estimate $l_{b}$. This reversal depends on turbulent properties. We estimate $l_{b}$ following two hypotheses: (a) We assume that both solenoidal and compressible turbulence are generated in the ICM at scale $L_{0}$ with similar energies, $\delta V_{L}^2 \approx \delta V^2$, and use solenoidal turbulence to estimate $l_{b} = l_{x0}$, where $l_{x0} = L_{0} M_{x}^{2}$ is the MHD scale assuming Kolmogorov spectrum of the (super-Alfvénic) solenoidal turbulence. (b) We assume that turbulence in the ICM is only compressible; in this case $l_{b} = \max\{l_{x0}, 2 \pi k_{cut}\}$ where $l_{x0} = L_{0} M_{x}^{4}$ is the MHD scale (for Kraichnan turbulence) and $k_{cut}$ is given in section 3.

Figure 2 shows the acceleration rate versus $\xi$ assuming different slopes of the kinetic turbulent spectrum. The cut-off scale is derived according to section 3, case (i). Although $\xi$ is a free parameter, we note that mfp $\gg r_{L}$ implies $\xi > 10^{-6}$ for multi-giga–tera-electron volt particles in the ICM. At the same time the upper bound of $\xi$ is set by the condition that CRe must be confined in Mpc volumes for gigayears or more in order to generate giant radio halos; this typically requires $\xi < 0.1$. For $\delta V \sim 700–800$ km s$^{-1}$ we conclude that the acceleration time ranges between $10^7$ and a few times $10^9$ years depending on the slope of the turbulent spectrum and on the mfp of CRs.

In figure 3 we show the effect of increasing plasma collisionality in the ICM (i.e. case (ii) in section 3) by assuming a cut-off $k_{cut} = \beta_{cut,L}$. This increases the acceleration rate in the branch of the slow diffusion regime. The effect is stronger for...
flatter spectra of the turbulence; for instance $f = 30$ implies a boost of the acceleration efficiency by more than two orders of magnitude in the Kraichnan case.

5. Discussion and conclusions

A popular scenario that is adopted to explain giant radio halos is based on turbulent reacceleration, with turbulence generated during cluster–cluster mergers. If true, this scenario implies that a hierarchy of complex mechanisms drains a fraction of the energy of the large-scale motions that are generated by the process of cluster formation into electromagnetic fluctuations and collisionless mechanisms of particle acceleration at much smaller scales. It has been realised that the existence of these complex collisionless mechanisms opens new prospects to understand the micro-physics of the ICM [5, 18, 20].

In this paper we have focused on the problem of stochastic acceleration of CRs by compressible turbulence in the ICM. The efficiency of acceleration depends on the particles’ mfp and on the spectrum of compressible turbulent motions, in particular on that of the electromagnetic fluctuations. The extent and the shape of this spectrum in turn depend on the processes of plasma damping and on the way turbulence is
generated and transported at smaller scales. We have explored this subject by considering the two mechanisms that are commonly adopted to explain giant radio halos, TTD due to fast modes (section 3) and non-resonant acceleration due to turbulent compressions (section 4).

In the case of TTD we analysed two extreme situations that differ in the efficiency of collision frequencies between thermal particles in the ICM, in particular whether collisions occur via Coulomb scattering or mainly via collective processes induced by plasma instabilities. In the latter case collisionless damping of compressive turbulence is dominated by CRs and the acceleration is more efficient than in the other case. We also discussed the changes in the acceleration rate that are induced by different slopes of the turbulent spectrum.

In the case of non-resonant acceleration we explored the combined effect induced on the acceleration rate by different assumptions for the turbulent spectra and mfp of CRs. The latter parameter is very uncertain, but plays a crucial role as it determines the regime of diffusion of CRs in the turbulent field and consequently has the potential to strongly change the acceleration efficiency.

In conclusion we have shown that the uncertainties about the ICM microphysics induce substantial variations in the acceleration efficiency of both mechanisms. On the other hand however this also implies that radio halos and the non-thermal properties of galaxy clusters are effective probes of the complex microphysics of the ICM.

In this paper we do not investigate reacceleration by solenoidal/incompressible turbulence. Several works have attempted to model this situation to explain radio halos [14, 51, 52]. We note however that these calculations did not take into account the scale-dependent anisotropies in Alfvénic turbulence, and consequently addressing the role of Alfvénic turbulence in the acceleration process in the ICM requires further investigation.

At this point we believe that future advances in the field will derive from studies that aim to address the generation of plasma instabilities in the ICM and from attempts to model self-consistently the way these instabilities generate small-scale fluctuations, and affect particles’ mfp and acceleration rates. The importance of this step has been clearly highlighted in section 3 and 4 where we have compared acceleration rates obtained in the collisional and collisionless cases. The situation may be even more complicated because, similar to the IPM, it can be thought that the collisional properties of the ICM may evolve with time and space. Finally we believe that magnetic reconnection in the ICM and its interplay with turbulence is another piece of this complex puzzle and it may play a role in the acceleration and reacceleration of CRs.

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