Relational Reality in Relativistic Quantum Mechanics

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Up to now it has been impossible to find a realistic interpretation for the reduction process in relativistic quantum mechanics. The basic problem is the dependence of the states on the frame within which collapse takes place. A suitable use of the causal structure of the devices involved in the measurement process allows us to introduce a covariant notion for the collapse of quantum states.

Relativistic quantum mechanics does not provide us with a covariant notion for the collapse of a quantum state in a measurement process. The basic problem is the dependence of the states on the frame within which the collapse is stipulated to occur. On the other hand, it is well known that measurements of local observables, which commute at space-like separations, yield the correct covariant probabilities independently of the Lorentz frame used. In that sense, it does not matter if the results of the experiments are described by different Lorentz observers as different and non-covariant quantum processes at the level of the states. The reduction postulate has been controversial in many ways especially due to its well known consequences, i.e., the loss of unitarity, the micro-macro world’s division, and the non-local character of the theory that leads, due to the inclusion of relativity, to non-covariant processes. These problems have led many physicists to adopt an instrumentalist point of view. Even if we assume that the measurement problem has been solved, we still have to understand its non-covariant character. As was pointed out by D’Espagnat: “Within the world view we are looking for, a state should collapse covariantly if it collapses at all.” However, as there was not, up to now, a covariant notion of the reduction process consistent with local as well as non-local properties he concluded “that even if the measurement problem is considered as solved, the conception according to which the world is made of quantum states is not consistent with the whole of our physical knowledge and must be therefore given up.” Here we shall show that it is possible to introduce a covariant notion of the reduction process in accordance to the previous requirement.

Realistic interpretations of the quantum theory have found major difficulties with the inclusion of relativity. The main problem is the lack of a single description of the quantum state. In the non-relativistic domain, a realistic interpretation already exists. It was first suggested by Heisenberg and developed by Margenau and Jordan, and is known as the real tendency interpretation. It is important to remark that it only provides a picture of the reduction process, but it does not solve the measurement problem. Within this approach a quantum state is a real entity that characterizes the disposition of the system, at a given value of the time, to produce certain events with certain probabilities during the interaction with a set of macroscopic measurement devices. Due to the uniqueness of the non-relativistic time, the set of alternatives among which the system chooses is determined without ambiguities. In fact, they are simply associated to observables corresponding to certain decomposition of the identity. For each value of the time where measurement takes place, the system coupled with the measurement devices “makes a decision” and produces events with probabilities given by the state of disposition of the system. The evolution of this state is also perfectly well defined. For instance, if we adopt the Heisenberg picture, evolution is given by a sequence of states of disposition. The dispositions of the system change during the measurement processes according to the reduction postulate, and remain unchanged until the next measurement. Of course, the complete description is covariant under Galilean transformations. However, up to now, it has been impossible to extend these properties to the relativistic domain, and consequently all the attempts of finding a tentative description of reality based on standard quantum mechanics have been found incomplete.

Hellwig and Krauss (H-K) proposed a covariant description of the reduction process many years ago, their basic assumption being that the collapse occurs along the backward light cone of the measurement event. However, as Aharonov and Albert have shown, their description is not consistent with the measurement of non-local observables. Even more important is the fact that it does not allow us to preserve the non-relativistic interpretation in the evolution of the system as a well-defined sequence of states of disposition. Indeed, in order to define the states on a given space-like or light-like surface the H-K prescription requires the knowledge of all the future measurements to which the system will be subject.

Hence, so far it has been impossible to have any realistic interpretation of relativistic quantum mechanics. It is meaningful to notice that quantum field theory has not been of help for solving this problem. In this paper, we are going to propose a covariant description of the reduction process that will allow us to preserve a realistic interpretation in the relativistic domain. However, we shall see that only a relational kind of realism can be entertained. In order to assign probabilities to properties...
of the coupled system, one needs to identify the set of properties among which the system makes a decision. In non-relativistic quantum mechanics, the system coupled with the macroscopic objects chooses among properties (alternatives) that may always be included among a decomposition of the identity at a certain time $t_0$. The lack of a unique time variable in the relativistic domain produces the noticed ambiguities. Thus, our first objective is the identification of an intrinsic criterion for the ordering of the alternatives. An intrinsic order may be introduced by making use of the partial order of events induced by the causal structure of the theory. Let us now be more specific. Let us consider an experimental arrangement of measurement devices, each of them associated with the measurement of certain property over a space-like region of space-time at a given proper time. No special assumption is made about the state of motion of each of them. Indeed different proper times could emerge from this description due to the different local reference systems of each device. Thus, we may label each detector in an arbitrary system of coordinates by an open three-dimensional region $R_a$, and its four-velocity $u_a$. One may introduce a partial order in the following way: The instrument $A_{R_1,u_1}$ precedes $A_{R_2,u_2}$ if the region $R_2$ is contained in the forward light cone of $R_1$. \(^1\) Let us suppose that $A^0_{R,a}$ precedes all the others. In other words, let us assume that all the detectors are inside the forward light cone coming from this initial condition. That would be the case, for instance, of the instrument that prepares the initial wave packet in a two-slit experiment.

Then, it is possible to introduce a strict order without any reference to a Lorentz time as follows. Define $S^1$ as the set of instruments that are preceded only by $A^0$. Define $S^2$ as the set of instruments that are preceded only by the set $S^1$ and $A^0$. In general, define $S^i$ as the set of instruments that are preceded by the sets $S^j$ with $j < i$ and $A^0$. Notice that any couple of elements of $S^i$ is separated by space-like intervals. This procedure defines a covariant order based on the causal structure of the devices involved in the measurement process. Now we have to introduce the operators associated with each device belonging to the set $S^i$. The crucial observation is that all the measurements on $S^i$ can be considered as "simultaneous". In fact, they are associated with local measurements performed by each device, and hence represented by a set of commuting operators. Since we here intend to stay within the realm of relativistic quantum mechanics we are going to implement these operators by noticing that a relativistic system may be considered as a generally covariant system \([2][3]\). These are constrained systems invariant under general transformations of the evolution parameter. The Hamiltonian is a linear combination of the constraints and the quantum observables are constants of the motion. Time is identified with some internal clock variable $T$, and what is actually measured is not the value of a physical variable $Q$ for certain value of the parameter $\tau$ but the value $Q(T)$ taken by the physical variable when the clock variable takes the value $T$. This procedure, using the clock variables as the proper time of each device, allows us to describe all the measurements belonging to the set $S^i$ in terms of a commuting set of operators (Rovelli’s evolving constants) in a generalized Heisenberg picture (G-H-P) defined on a physical Hilbert space $\mathcal{H}_{phys}$ of solutions of the constraint \([1][4][5]\). The commutativity, and self-adjointness, of the projectors on a “simultaneous” set $S^i$, associated to different local measurement devices, assures that all of them can be diagonalized on a single option. These conditions insure that the quantum system has a well-defined disposition with respect to the different alternatives of the set $S^i$. Let us call $\psi^0$ the state of the system in the G-H-P prepared by $S^0$. Hence, after the observation of a set of events $E = \cup_a E_{R,a}$, each one associated with a local measurement in the region $R_a$ belonging to $S^1$, the state will collapse into a projected state belonging to the Hilbert space $\mathcal{H}_{phys}$, given by the normalized projection of $\psi_0$. The projector $P_E$, connecting $\psi^0$ with $\psi_E^1$, is constructed as a product of local projectors related with each individual event. This product does not depend on the order due to the commutation of the projectors. If there is not an event on the $R_a$ region, in other words if nothing is detected in $R_a$, one needs to project on the orthogonal complement of each possible event that may occur in this particular region. It is now clear the relational character of our approach. The quantum systems keep a dispositional character with respect to the alternatives belonging to $S^i$ because they are covariantly defined by an intrinsic order. The change of these dispositions is also well defined, because once the interaction with the devices belonging to $S^i$ has concluded, the state collapses into a projected state belonging to the Hilbert space $\mathcal{H}_{phys}$.

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\(^1\)The case where one has devices such that only a portion of the region $R_2$ is contained in the forward light cone of $R_1$, leads to a subtler causal structure which has interesting consequences on the global aspects of the relational tendency interpretation. The details will be discussed in a forthcoming paper.

\(^2\)We are defining event as the macroscopical result of the interaction of the system with the measurement device.
space $\mathcal{H}_{\text{phys}}$ where the state of disposition lives. The dispositions are relationally defined with respect to the set of alternatives given by the measurement devices. Furthermore, the probabilities and states are computed by using the standard rules of quantum mechanics based on the existence of self-adjoint, commutative, local projectors. The methods developed in [15] for the treatment of generally covariant systems allow the completion of a detailed analysis of the process sketched above. Covariance follows from the existence of a natural inner product in the physical space of states $\mathcal{H}_{\text{phys}}$, such that the local projectors are self-adjoint operators, and from the unitary implementation of the Lorentz transformations that insures that the mean value of the projector is a scalar quantity. Up to now we have not specified the explicit form of the observables. In fact we have introduced a general framework applicable for a wide kind of relativistic systems. 3

Let us be more specific and consider the Klein-Gordon quantum mechanics. In this case we have shown that it is possible to define a relational position observable that coincides with the Newton-Wigner operator in the Feshbach-Villars representation [15]. Indeed, it is not difficult to show that the corresponding local projectors exist and commute, up to a Compton wavelength. Furthermore, there is a natural covariant inner product in the physical Hilbert space constructed in Reference [15]. A detailed analysis of the properties of this observable may be found in the same reference. The physical Hilbert space is constructed by scalar space-time wave functions $\psi(z,z^0)$ in the generalized Heisenberg picture, which are annihilated by the constraint (i.e. solutions of the K-G equation). These states do not evolve, instead they describe the whole space-time history of the system. As we have said before, the time variable should not be identified with $z^0$. Within this framework, the evolution is described by relational observables which in this case are $X(T):=\hat{q}+\hat{p}T/\hat{p}_0$ and $\hat{p}_0:=\frac{\hat{p}}{\sqrt{\hat{p}^2+m^2}}$ where $\hat{q},\hat{p},\hat{p}_0$ are the perennials quantum operators associated with the initial position, momentum and energy of the system, and $T$ the clock variable. The first observable is the relational position operator which represents the value of $x$ when $x^0=T$, the second is the sign of the energy. As we have shown it is possible to construct a covariant inner product without any reference to any particular Lorentz time, such that these observables are well defined self-adjoint operators. Hence it is not difficult to construct local projectors associate with these observables on each local reference system in the region $R^j_a$ of the $S^j$ alternative. The local projector will be $P^j_a = \int_{R_a} |x,T,+><x,T,+|$, where $|x,T,+>$ are the eigenstate of $\hat{X}(T), \hat{c}$ with eigenvalues $x,+$. $T$ is the proper time when the measurement occurs. These projectors are defined on each local Lorentz system. In principle, we could have different Lorentz time variables on different regions belonging to the same $S^1$ alternative. This procedure allows us to represent the local position measurements on $S^1$ in terms of local projectors on a covariant Hilbert space. The projectors associated with different devices on each $S^j$ commute up to lambda Compton corrections. The reduction postulate transforms the physical state into the normalized projection that we have already defined on each set of alternatives $S^j$.

In general, our description may be extended to any relativistic quantum theory, like a Dirac particle or even for quantum field theory. Hence, our approach should be taken as a general framework for the relativistic domain. An important consequence is the following: non-local observables are also well defined in the relativistic case. In fact, since non-local properties are measured by means of local observations on a system of measurement devices separated by space-like intervals [15], they may be included in a set of alternatives $S^j$ and therefore correspond to a single covariant reduction process. Thus we have shown that relativistic quantum mechanics admits a natural realistic interpretation of the quantum states. Quantum states are multi-local relational objects that give us the disposition of the system for producing certain events with certain probability among a particular and intrinsic set of alternatives $S^j$. The evolution of this disposition is a well-defined covariant process on the physical Hilbert space in the generalized Heisenberg picture. The main difference with the non-relativistic case is that here, in each measurement, the system provides a result in devices that may be located on arbitrary space-like surfaces. It is important to notice that the above proposed description neither refers to any particular choice of the evolution parameter nor corresponds to any foliation of space-time. Therefore, one does not have a natural Schroedinger picture on this approach. As it was shown in quantum field theory [16], the global Schroedinger picture could not exist due the foliation-dependence of the global state evolution. However, as was pointed out by Dirac: “Heisenberg mechanics is the good mechanics”, and this is also the case here. The only remaining problem is that the evolving constants defined on a global, generic curved, Cauchy surface could

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3 In a forthcoming paper we shall analyze the relational description of the measurement process in Quantum Field Theory.
not be self-adjoint operators. But since we are working with the standard proper Lorentzian time of each local measurement device belonging to \( S^i \), the corresponding local operators are self-adjoint. Hence the relational local point of view adopted here allows us to avoid this problem. Nevertheless, the Schroedinger picture is well defined in a first quantized relativistic theory. In this picture, dispositions are associated to multi-local states. Each multi-local state \( \psi_i \) is given by a class whose elements are the wavefunctions computed on each space-like surface containing the measurement devices belonging to \( S^i \). One obtains each of the elements of the class \( \psi_i \) by evolving with the wave equation the initial state. Reduction takes place on any particular space-like surface containing the covariant alternatives. Notice that contrary to what happens with the standard Lorentz dependent description, here the conditional probabilities of further measurements are unique. It is in that sense that the dispositions of state to produce further results has an objective character.

We have developed the relational interpretation adopting the real tendency theory. Nevertheless, the relational point of view could be within the context of any realistic interpretation, providing a covariant reduction process. The main result we have found is that it is possible to introduce a set of local projectors, covariantly ordered, and a covariant Hilbert space with in which these operators are well defined self-adjoint operators and they commute for a given \( S^i \) set. The reduction postulate is now covariantly defined, this is an important step toward a complete realist theory. The relational nature of reality should be taken as a general feature of a relativistic world. In fact, a paradigmatic example of relational theory is general relativity that establishes the relational character of space, time and matter. Space and time are nothing but the dependence of the phenomena on one other. At the quantum level, systems do not have properties before the measurement: events are a product of the interaction of the system with the measurement devices. An even more striking piece of evidence, in quantum field theory in curved space-time, the very notion of particle depends upon the motion of the detectors \[7\]. In that sense, a system is given by the set of its behaviors with respect to others. An isolated system does not have properties or attributes, since all the “properties” result from its interaction with other systems. It is important to remark that this is a strongly objective description in the sense of D’Espagnat and it does not make any reference to operations carried out by human observers \[5,6\]. Once a quantum system and a set of measurement devices are given, the evolution of the state is uniquely and covariantly defined. A final observation is in order. As one can immediately note, the initial condition has a deep relevance in the construction of the covariant alternatives. In many cases the preparation of the system is central for the determination of the initial condition. As one already knows, a quantum system involve entangled objects, therefore in a complete quantum theory one has to take the whole universe as the system. There, the relational point of view is the only way for describing the evolution. In this domain, a quantum object may not have a natural beginning beyond the big bang. If one is describing a particular portion of the universe within a given time interval, then one can consider a partial initial condition given by a particular set of events that contain the forthcoming alternatives in the forward light cone. Hence, one naturally falls into a sort of statistical mixture, as it is the case in non-complete measurements.

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