Spin-orbit coupling (SOC) effects in Bose-Einstein condensates (BECs) have been considered of major theoretical and experimental interest in the last decade [1, 2]. In particular, the significant experimental developments [3, 4] on the realization of exotic quantum phases in rotating spin-orbit-coupled BECs [5–7] in various trapping potentials. In fact, various trapping potentials may significantly affect the stationary states and dynamic behaviors of the BECs. Recently, a novel quasicrystalline optical lattice (QOL) with long-range order and high-rotational symmetry has become experimentally achievable [8]. Matter-wave interference and Bloch oscillation are predicted for a BEC in the QOL [8]. What interests us very much is the dynamic properties of rotating BECs with SOC in the QOL.

We consider a quasi-2D system of rotating pseudo-spin-1/2 BEC with Rashba SOC trapped in a QOL. In the mean-field framework, the dynamics can be well described by the dissipative coupled Gross-Pitaevskii (GP) equations [10–12]

\[(i - \gamma)\frac{\partial \psi_1}{\partial t} = \left[ \frac{-\hbar^2 \nabla^2}{2m} + V Q(r) + g_{11} |\psi_1|^2 + g_{21} |\psi_2|^2 \right] \psi_1 - \Omega L z \psi_1 + \hbar \lambda (\tilde{\kappa}_1 - i\tilde{\kappa}_2) \psi_2, \tag{1} \]

\[(i - \gamma)\frac{\partial \psi_2}{\partial t} = \left[ \frac{-\hbar^2 \nabla^2}{2m} + V Q(r) + g_{21} |\psi_1|^2 + g_{22} |\psi_2|^2 \right] \psi_2 - \Omega L z \psi_2 - \hbar \lambda (\tilde{\kappa}_2 + i\tilde{\kappa}_1) \psi_1, \tag{2} \]

where \(\psi_1\) and \(\psi_2\) are the component wave functions, \(m\) is the atomic mass, and \(\gamma\) is the dissipation parameter. The phenomenological model is a variation of that in [8] and a generalization of that of a rotating scalar BEC [10–11], and has good predictive power. In particular, this model enables one not only to find the steady state of a rotating system but also to study the whole dynamical process toward the final steady state. When the dissipation term is completely switched off, our simulation based on the time-dependent GP equations shows that there is no steady state. In addition, the motion of generated vortices remains turbulent and rather irregular, and there is no vortex lattice (or vortex necklace) formation. We take \(\gamma = 0.03\) throughout this work, corresponding to a temperature of about 0.1TE [10, 11]. In fact, the variation of nonzero \(\gamma\) only influences the relaxation time scale but does not change the dynamics of the vortex formation and the ultimate steady structure of the rotating system. The initial wave functions are normalized as \(\int |\psi_1|^2 + |\psi_2|^2 |dx dy| = N\) with \(N\) being the initial number of atoms. The atomic spin is an eightfold symmetric optical lattice plus a harmonic trap, which can be defined by

\[V Q(r) = V_0 \sum_{i=1}^{4} \cos^2 \left( \frac{G_1}{2} \cdot r \right) + \frac{1}{2} m \omega_z^2 r^2, \tag{3} \]

where \(V_0\) is the lattice strength. The reciprocal lattice vectors are \(G_1 \propto (1, 0), G_2 \propto (1, 1)/\sqrt{2}, G_3 \propto (0, 1),\) and \(G_4 \propto (-1, 1)/\sqrt{2}\), \(\omega_z\) is radial oscillation frequency, and \(r = \sqrt{x^2 + y^2}\). The interaction parameters are given by \(g_{ij} = 2\sqrt{2m\alpha_{ij}} \hbar^2/ma_z\) (\(j = 1, 2\)) and \(g_{21} = g_{22} = 2\sqrt{2m\alpha_{12}} \hbar^2/ma_z\), where \(\alpha_{ij}\) and \(\alpha_{12}\) are the s-wave scattering lengths between intra- and inter-component atoms, respectively. \(\alpha_{12} = \sqrt{\hbar/ma_z}\) is the oscillation length in the z direction. \(\lambda\) is the strength of isotropic SOC. \(\Omega\) is the rotation frequency along the z direction, and \(L_z = ih(\tilde{\kappa}_1 - i\tilde{\kappa}_2)\) is the z component of the angular momentum operator. The angular momentum is defined as \((L_z) = \int \psi^* \nabla \psi |dx dy|\) with \(\psi(r) = (\psi_1, \psi_2)^T\). The energy functional of the system is given by

\[E = \sum_{j=1,2} \int d^2 r \left[ \frac{\hbar^2}{2m} |\psi_j|^2 + V Q(r) |\psi_j|^2 + \frac{g_{j1} |\psi_j|^4 - \Omega^2 |L_z \psi_j|^2}{\hbar} \right] + \frac{1}{2} g_{2j} |\psi_1 \psi_2|^2 d^2 r, \tag{4} \]

where the tilde is omitted for simplicity. The spin texture is defined by \(S = \sum_{\sigma_{1/2}} \psi_1^\sigma \psi_2^{-\sigma_{1/2}}\) with \(\sigma_{1/2} = (\sigma_x, \sigma_y, \sigma_z)\) being the Pauli matrix, and \(X = |x_1, x_2|\) with \(x_1 = \psi_1 / \sqrt{|\psi_1|^2 + |\psi_2|^2}\) and \(\rho = |\psi_1|^2 / |\psi_2|^2\). The topological charge can be written as \(q(r) = \frac{1}{2} \text{S} \cdot \frac{\partial \phi}{\partial \sigma_y}\), and the topological charge is given by \(Q = \int q(r) d^2 r\).

We numerically solve the GP equations with the split-step Fourier method [10, 11]. The initial quantum state can be obtained by using the imaginary-time propagation method [10, 12] for \(\Omega = 0\). We consider 87Rb atoms which are confined in a \((\omega_x, \omega_z) = 2\pi \times (10, 100)\) Hz trap, and the length scale is \(a_0 = 3.41\mu m\). The numerical grids are chosen as \(256 \times 256\), and the actual system is set as \(20 \times 20a_0^2\). We introduce a ratio between inter- and intra-species interaction, i.e., \(\beta = 12/11\), and we set \(\beta_{11} = \beta_{22} = 200\) with \(a_{11} = 81.35a_0\) being the Bohr radius.

We first prepare a spin-orbit-coupled BEC trapped in a stationary QOL. Figure 1 shows the typical dynamics of the component densities and phases after the QOL begin to rotate suddenly with \(\Omega = 0.6\). For a BEC with SOC in a stationary harmonic trap, the typical quantum phases are plane-wave phase and stripe phase [2]. Here the system initially exhibits spatially separated heliciform-stripe phase [Fig. 1(a)] which is resulted from the interplay of the SOC, QOL, and the strong interspecies repulsion. Then the component densities are elongated along the x axis or the y axis, and the heliciform stripes are broken and separated into fragments [Fig. 1(b)]. When \(t = 5\), the system becomes irregular and reaches the...
minimum in the density distribution, and complex turbulent oscillations appear in the phase distribution [Fig. 1(c)], which makes the boundary of the system unstable and excites the surface waves propagating along the surfaces. With further time evolution, the central surface waves develop into visible vortices and hidden vortices [14], and the boundary ones become ghost vortices [Figs. 1(d)-1(e)].

In Fig. 2, we display the typical transitions of the topological charge density and spin texture. Our numerical calculation shows that the local topological charges in Figs. 2(a3) and 2(a4) approach −0.5, and 0, respectively. The unit length is \( a_0 \) and 1/\( ω⊥ \), respectively.

The dynamic process can also be characterized by the time evolution of the average angular momentum per atom \( ⟨L_z⟩ \), where the dependence of \( ⟨L_z⟩ \) on \( Ω, λ, δ, \) and \( V_0 \) are displayed in Fig. 3, respectively. Take the black solid curve as an example, when the system suddenly begins to rotate with \( Ω = 0.6 \), \( ⟨L_z⟩ \) increases rapidly with the time evolution (0 < \( t \) ≤ 15), and then gradually (15 < \( t \) < 100) approaches a maximum value (equilibrium value).

Physically, the combined effects of the continuous input of angular momentum, the quantum fluid nature, SOC, QOL and the dissipation lead to the formation of almost the same steady vortex structure in the two components (e.g., see Fig.1(f)).

The transition of topological charge densities and spin textures, the right two columns represent the local enlargements of spin textures, respectively. The red solid circle and blue dotted circle denote a half-skyrmion and a half-antiskyrmion, respectively. The unit length is \( a_0 \).

| External potential | Quantum phase (δ > 1) |
|--------------------|----------------------|
| Harmonic trap      | Segregated symmetry preserving condensates with a giant skyrmion, stripes, vortex and peak lattices |
| Toroidal trap      | Triangular vortex lattice, Anderson-Toulouse coreless vortex |
| Standard optical lattice | Square vortex lattice, vortex chain |
| Quasicrystalline optical | Visible vortex necklace with a giant vortex lattice |

In summary, the rotating pseudo-spin-1/2 BECs with Rashba SOC in a QOL can show interesting and unusual dynamic behaviors. For fixed parameters, the system gradually evolves from the initial heliciform-stripe phase to the steady visible vortex necklace with a giant vortex and a hidden vortex necklace. At the same time, the spin texture experiences a structural phase transition from a meron-antimeron pair into a half-antiskyrmion necklace. Furthermore, the temporal evolution of the angular momentum is revealed, where the angular momentum gradually increases to an equilibrium value. Finally, the typical quantum phases of rotating two-component BECs with SOC in different external potentials (including the different rotating results between a QOL and a standard OL) are briefly summarized in Table 1, where the ones without references are our simulation results. Our findings have provided new understanding for the physical properties of ultracold quantum gases.

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