Does Bulk Viscosity Create a Viable Unified Dark Matter Model?

Baojiu Li\textsuperscript{1,}\textsuperscript{*} and John D. Barrow\textsuperscript{1,}\textsuperscript{†}

\textsuperscript{1}DAMTP, Centre for Mathematical Sciences, University of Cambridge, Cambridge CB3 0WA, United Kingdom

(Dated: May 11, 2009)

We investigate in detail the possibility that a single imperfect fluid with bulk viscosity can replace the need for separate dark matter and dark energy in cosmological models. With suitable choices of model parameters, we show that the background cosmology in this model can mimic that of a ΛCDM Universe to high precision. However, as the cosmic expansion goes through the decelerating-accelerating transition, the density perturbations in this fluid are rapidly damped out. We show that, although this does not significantly affect structure formation in baryonic matter, it makes the gravitational potential decay rapidly at late times, leading to modifications in predictions of cosmological observables such as the CMB power spectrum and weak lensing. This model of unified dark matter is thus difficult to reconcile with astronomical observations. We also clarify the differences with respect to other unified dark matter models where the fluid is barotropic, i.e., \( p = \rho \gamma \) (the same assumption as \[7\]). In Ref. \[18\] the authors obtained analytical solutions for \( \rho + p = B \rho^\lambda \), which is just \( \gamma = 0 \) and \( m = \lambda \), or other functional forms \( p(\rho) \) \[5\]. The Chaplygin gas models are simple, with no new dynamical degrees of freedom, and yet produce an interesting time-evolution for the dark energy. However, it was shown subsequently that the density perturbation of this fluid will either blow up or experience rapid oscillatory damping at late times, so the models can be stringently constrained by the matter power spectrum \[16\]. This distinctive behaviour arises because there is a minimum possible total density of the universe at late times.

There have been some explicit investigations in the bulk viscous cosmology in connection with the dark energy problem. Fabris \textit{et al.} \[17, 18\] investigated the case in which the viscosity coefficient has the form \( \eta(\rho) = \alpha \rho^m \) and considered both background and perturbed evolutions of a universe dominated by viscous matter. However, these studies were limited by certain simplifications. In Ref. \[17\], for example, the authors obtained analytical solution for \( \rho_\nu \) (where the subscript \( \nu \) denotes viscous matter) as a function of scale factor \( a \), under the assumption that no other matter species exist in the universe (the same assumption as \[2\]). In Ref. \[18\] the authors added a baryonic matter species and estimated the cosmological parameters in detail using Bayesian statistics; but their calculation was largely confined to the special case \( m = 0 \) and used only supernovae data to derive the constraints. We believe that such an analysis needs to

---

\textsuperscript{*}Email address: b.li@damtp.cam.ac.uk
\textsuperscript{†}Email address: j.d.barrow@damtp.cam.ac.uk
be generalized. First, as will be seen below, $m = 0$ is not necessarily the best fit for these models, even for the background cosmology. Second, no radiation component was included in previous analysis. This is a reasonable simplification as long as we are only concerned with the late-time cosmic expansion and only using the supernova data. But when observables are also related to the high-redshift features, such as the CMB shift parameter, the radiation must be taken into account. In fact, the properties of the viscosity could significantly modify the early-time evolution of the universe if the late-time evolution is consistent with the supernova data, and so the CMB shift parameter could be effective in constraining the model. Third, a detailed study of the linear perturbations in the model is needed as a critical check of the model’s feasibility. As we will see below, the UDM model based on viscous matter has several distinctive predictions regarding the perturbations when compared to the ΛCDM paradigm or the Chaplygin gas model. Such a feature is very similar to the prediction of the ΛCDM paradigm.

parameters and show that these give a background cosmology not necessarily the best fit for these models, even for the background cosmology. Second, no radiation component was included in previous analysis. This is a reasonable simplification as long as we are only concerned with the late-time cosmic expansion and only using the supernova data. But when observables are also related to the high-redshift features, such as the CMB shift parameter, the radiation must be taken into account. In fact, the properties of the viscosity could significantly modify the early-time evolution of the universe if the late-time evolution is consistent with the supernova data, and so the CMB shift parameter could be effective in constraining the model. Third, a detailed study of the linear perturbations in the model is needed as a critical check of the model’s feasibility. As we will see below, the UDM model based on viscous matter has several distinctive predictions regarding the perturbations when compared to the ΛCDM paradigm or the Chaplygin gas model. Such a feature is very similar to the prediction of the ΛCDM paradigm.

parameters and show that these give a background cosmology not necessarily the best fit for these models, even for the background cosmology. Second, no radiation component was included in previous analysis. This is a reasonable simplification as long as we are only concerned with the late-time cosmic expansion and only using the supernova data. But when observables are also related to the high-redshift features, such as the CMB shift parameter, the radiation must be taken into account. In fact, the properties of the viscosity could significantly modify the early-time evolution of the universe if the late-time evolution is consistent with the supernova data, and so the CMB shift parameter could be effective in constraining the model. Third, a detailed study of the linear perturbations in the model is needed as a critical check of the model’s feasibility. As we will see below, the UDM model based on viscous matter has several distinctive predictions regarding the perturbations when compared to the ΛCDM paradigm or the Chaplygin gas model. Such a feature is very similar to the prediction of the ΛCDM paradigm.
Eqs. (5, 8, 9) involves both background and first-order perturbed quantities (such as $A_a$, $q_a$). To obtain equations for the background cosmology it is sufficient to neglect all first-order terms, while to obtain corresponding equations for the perturbation evolution we need to take the spatial derivatives $\nabla_a$ [cf. Eq. (11)] of these equations. For the perturbation analysis it is then more convenient to work in the $k$ space because we shall confine ourselves to the linear regime where different $k$-modes decouple. Following [19], we make the following harmonic expansions of our perturbation variables:

$$\nabla_a \rho = \sum_k \frac{k}{a} \xi^k \nabla_a \rho = \sum_k \frac{k}{a} \xi^k \nabla_a \rho$$

$$q_a = \sum_k q^k a \xi^k \pi_{ab} = \sum_k \Pi Q^k_{ab}$$

$$\hat{\nabla}_a \theta = \sum_k \frac{k^2}{a^2} \xi^k \theta A_a = \sum_k \frac{k}{a} \xi^k Q^k a$$

in which $Q^k$ is the eigenfunction of the comoving spatial Laplacian $a^2 \nabla^2$ satisfying

$$\nabla^2 Q^k = \frac{k^2}{a^2} Q^k, \quad (10)$$

and $Q^k$, $Q^k_{ab}$ are given by $Q^k = \frac{1}{k} \nabla_a K^k$, $Q^k_{ab} = \frac{1}{k} \nabla_a (\xi^k Q^k b)$. The perturbed version of Eq. (8) is

$$\dot{\chi}_a + (\rho + p)(Z_a - a \theta A_a) + (\chi_a + \chi^p a \theta + a \nabla_a \xi^b q_b) \xi^a = \sum_k \xi^k (Q^k a, Q^k, Q^k_{ab}). \quad (10)$$

In § IV we shall use the harmonic expansion coefficients of Eqs. (5, 8, 9, 11) to derive the evolution equation for the density perturbation of the viscous dark matter.

### III. THE BACKGROUND EVOLUTION

The field equations governing the background cosmic expansion are the Friedmann equation, the Raychaudhuri equation and the conservation equations for energy densities of the different matter species (baryons, radiation, and viscous dark matter). Not all these equations are independent, and we choose the Friedmann equation (here $H = \dot{a}/a = \theta/3$)

$$3H^2 = 8\pi G (\rho_D + \rho_B + \rho_R), \quad (12)$$

and the conservation equations

$$\dot{\rho}_B + 3H \rho_B = 0, \quad (13)$$
$$\dot{\rho}_R + 4H \rho_R = 0, \quad (14)$$
$$\dot{\rho}_D + 3H (\rho_D + p_D) = 0 \quad (15)$$

as our starting point. In these equations $\rho_B$, $\rho_R$ and $\rho_D$ are respectively the energy densities of baryonic matter, radiation and the (viscous) dark matter. The pressure $p_D = (\gamma - 1)\rho_D - 3aH\rho_D^0$ is the effective pressure of the dark matter, with $\gamma, \alpha, m$ being our free model parameters.

We shall not follow [18] by dividing $\rho_D$ into different components with different equation of states (EoS), although it can be useful mathematically, since that might hide the fact that there is only a single fluid with varying EoS. When it comes to the perturbative evolution of this fluid, it will be misleading if one thinks that part of the fluid behaves as a cosmological constant which has no perturbation, while another part simply clusters as CDM, there being no interactions between them. We also note that the Hubble expansion rate $H$ appears in the expression of $\rho_D$, which means that the EoS of the viscous dark matter depends also on the existence and properties of other matter species, and so we cannot neglect the effects of baryonic matter and radiation (at early times).

Next, we estimate the free model parameters $\gamma, \alpha$ and $m$. We will set $\gamma = 1$ in all the calculations below because we want the viscous dark matter to behave like CDM at early times when the correction term is not important. For $m$, earlier studies [7] showed that when $m > 1/2$ the universe will start from a de Sitter phase and finally evolve towards power-law perfect-fluid dominated expansion, while if $m < 1/2$ it is just the opposite, with late-time approach to de Sitter evolution; the special $m = 1/2$ case corresponds to power law evolution throughout. In fact, from the expression $\rho_D = -3aH\rho_D^0$ we can see that if $\rho_D$ dominates over other matter species (so that $H \propto \rho_D^{1/2}$) then $\rho_D$ will be a constant if $m = -1/2$. We can estimate that the best fit value of $m$ is around $-1/2$. In what follows, $m$ will be taken as a free parameter to be constrained by data. Finally, $\alpha$ is obviously a dimen-
sional constant. To determine its value, we note that to explain the SNe data we need the candidate for dark energy to contribute an effective energy density and pressure of the same order as $\rho_A$ in the $\Lambda$CDM model. Thus, we have

$$\rho_D0 \simeq \rho_{CDM0} + \rho_A,$$

$$3\alpha H_0\rho_{DM0}^m \simeq \rho_A$$

in which the quantities on the left-hand sides are for our model while those on the right hand sides are for the $\Lambda$CDM model. A subscript $0$ here denotes the present value of a quantity. Using the results $\Omega_{CDM} \simeq 0.20$ and $\Omega_A \simeq 0.76$, we calculate from the above two equations that $\beta \equiv \alpha H_0\rho_{DM0}^{m-1} \simeq 0.26$. This gives us a sense about the magnitude of the dimensionless quantity $\beta$, which is chosen as another model parameter instead of $\alpha$ and will be constrained below.

With the above preliminaries, it is now straightforward to rewrite the conservation equation of the viscous dark matter, Eq. (16), as

$$\dot{\varrho} + 3\gamma \varrho = 9\beta \varrho^n \left[ \frac{\theta + \rho_{bd}e^{-3N} + \rho_{rd}e^{-4N}}{1 + \rho_{bd} + \rho_{rd}} \right]^{1/2}$$

where we have defined $\varrho \equiv \rho_{DM}/\rho_{DM0}$, $\rho_{bd} \equiv \rho_{BD}/\rho_{DM0}$, $\rho_{rd} \equiv \rho_{RD}/\rho_{DM0}$, and also used the Friedmann equation Eq. (12) to substitute for the Hubble parameter, $H$. The star denotes a derivative with respect to $N = \log a$. Because there are in general no closed-form solutions to these cosmological equations, we will use Eq. (16), together with Eqs. (12), 13, 14, in our subsequent numerical calculation. Note that $\rho_{bd}$ and $\rho_{rd}$ are constants which are fixed once $\Omega_B$ and $\Omega_R$ are known (using the fact that the universe is spatially flat, so that $\Omega_{DM} = 1 - \Omega_B - \Omega_R$):

$$\rho_{bd} = \frac{\Omega_B}{1 - \Omega_B - \Omega_R}, \quad \rho_{rd} = \frac{\Omega_R}{1 - \Omega_B - \Omega_R}. \quad (17)$$

A natural choice of the initial (final) condition of Eq. (16) is $\varrho(N = 0) = 1$.

We have used the supernovae luminosity distance data and CMB shift parameter to constrain the model parameters $m$ and $\beta$, analytically marginalizing the current Hubble expansion rate $H_0$ and assuming the baryon density today as fixed by BBN and measurements of the light element abundances. The result is shown in Fig. 1 and the best-fit parameters we find are $(m, \beta) = (-0.4, 0.236)$ which lie close to our estimate above $(m = -0.5, \beta = 0.264)$. Fig. 2 shows the cosmic evolution of the fractional energy densities in the bulk viscous model with the above best-fit parameters. It can be seen there that the viscous model mimics the concordance $\Lambda$CDM paradigm extremely well all through its cosmic history. The model therefore appears to work well as a description of the background cosmology.

**IV. THE EVOLUTION OF FIRST-ORDER DENSITY PERTURBATIONS**

Despite the excellent coincidence between the viscous dark matter and $\Lambda$CDM models for the background evolution found in the last section, we should also investigate the formation of large-scale structure to test whether the viscous model is also a feasible model for dark energy. In this section we show that, generally, a bulk viscosity depending on the energy density, i.e., $p_D = -\eta(p_D)\nabla^2u_a$, will significantly influence the formation and evolution of large-scale cosmological structure. This very different prediction from that of the $\Lambda$CDM model indicates that stringent constraints can be placed on the viability of the bulk viscosity models using observational data on the CMB spectrum, matter power spectrum, and weak gravitational lensing.

Let us first concentrate on the special case considered above, with $p_D = -\alpha \rho_D^4 \nabla^2 u_a = -3\alpha H_0^2 \rho_D^4$. For an observer comoving with dark matter particles (with 4-velocity $u_a$), the energy momentum tensor could be written as

$$T_{ab} = \rho_D u_a u_b - p_D (g_{ab} - u_a u_b). \quad (18)$$

In the $\Lambda$CDM paradigm, if one chooses the observer to be comoving with the dark matter particles as above, then
obviously the peculiar velocity is zero, \( v_D = 0 \), which implies, by the conservation of energy-momentum tensor, that \( A_n = 0 \) for this observer, and there is no acceleration. In the bulk viscous model, however, \( v_D = 0 \) and \( A_n = 0 \) are different choices of frame which can be used in numerical calculations. Here, for convenience, we will choose the \( v_D = 0 \) frame, in which, again from the conservation of energy, \( A_n \neq 0 \), which is easy to understand because the dark-matter particles themselves have interactions (the pressure \( p_D \)) and cannot be acceleration-free.

Taking the spatial derivative of \( p_D \) and picking the harmonic coefficients, we obtain

\[
C_1(a) = \mathcal{H} + \frac{k p_D a^2}{2 \mathcal{H}} w + \left( \frac{\mathcal{H}' - \mathcal{H}}{\mathcal{H}} \right) \frac{w}{1 + w} \left( \frac{w'}{1 + w} - 3w \mathcal{H} \right); \tag{25}
\]

\[
C_2(a) = - \frac{w}{3(1 + w)} \mathcal{H} \tag{26};
\]

\[
C_3(a) = - \frac{k p_D a^2}{2} \left[ (1 + w)(1 + 3mw) + 3w^2 \right] - 3w \mathcal{H} \mathcal{H}' - 3w \mathcal{H}^2 \left( m + \frac{w}{1 + w} \right); \tag{27}
\]

\[
C_4(a) = w \left( m + \frac{w}{1 + w} \right); \tag{28}
\]

It is straightforward to check that when \( w = 0 \) (whether or not \( m = 0 \)), this reduces to the evolution equation for the CDM density contrast, which does not depend on \( k \), indicating that the evolution of the CDM density contrast is scale-independent. For the viscous dark matter, however, we see that the equation depends on \( k \) in the coefficients of both \( \Delta' \) and \( \Delta \). Eq. (23) is effectively an equation for a damped simple oscillator, with
time-varying and scale-dependent frequency and damping force, and no driving force. On small scales \((k \gg \mathcal{H})\) and at late times (when \(w \sim \mathcal{O}(-1)\)), the \(C_2(a)\) and \(C_4(a)\) terms dominate the coefficients of \(\Delta'\) and \(\Delta\), respectively: it is then easy to show that the oscillator is over-damped so that its amplitude decays to zero rapidly with no oscillations. This qualitative feature can be seen in the upper panel of Fig. 3 where we plot the evolution of the dark matter density contrast, \(\Delta_D\), for four different length scales, \(k = 0.005, 0.01, 0.05, 0.1 \text{ Mpc}^{-1}\), for our best-fit model parameters \((m = -0.4, \beta = 0.236)\). We see that the evolution of \(\Delta_D\) deviates from that predicted by the \(\Lambda\)CDM model only at late times, but nonetheless significantly. The scale-dependent evolution of density contrast is actually a general feature in the UDM models without new dynamical degrees of freedom [16, 21, 22].

The lower panel of Fig. 3 displays the evolution of the baryon density contrast \(\Delta_B\). Because viscous dark matter clusters more weakly than cold dark matter, the gravitational potential that the baryons lie in is also weaker, making the baryons less clustered. However, since this effect is indirect, the deviation of \(\Delta_B\) from the \(\Lambda\)CDM prediction is considerably smaller than that of \(\Delta_D\). Note that the galaxy surveys actually measure the clustering of luminous (baryonic) matter, rather than that of dark matter. Consequently, these measurements can only be applied to \(\Delta_B\), which is just weakly dependent on the model parameters [23]. Nevertheless, as in the bulk viscous model, both the amplitude and the shape of baryonic matter power spectrum are distinct from those predicted by the \(\Lambda\)CDM paradigm, and therefore we expect that stringent constraints can be obtained from it.

The analysis above is generalizable to an arbitrary choice of the viscosity function \(\eta(\rho_D)\). To do this, we just need to define

\[ \tilde{m}(a) = \frac{\rho_D}{\eta(\rho_D)} \frac{\partial \eta(\rho_D)}{\partial \rho_D} \]  

(29)

and replace the constants \(m\) in Eqs. [24, 28] with \(\tilde{m}(a)\). Then, Eq. [24] describes the evolution of the dark matter density contrast in the general case of \(\eta(\rho_D)\).

There are a couple of points to be noted about Eq. [24]. First, it is clear that \(m = 0\) does not guarantee that \(C_2\) and \(C_4\) are zero, and the EoS of the viscous dark matter \(w\) is important. Since \(w \neq 0\), if the viscous dark matter is responsible for accelerating the expansion of the universe, our conclusion that the evolution of dark matter density contrast will be sensitively scale-dependent and deviate significantly from \(\Lambda\)CDM will hold in general. Note that this is different from the analysis of Ref. [18], which considers the special case of \(m = 0\) and concludes that the matter power spectrum in this model behaves well. Second, Eq. [24] is qualitatively different from its counterparts in other models which unify dark matter and dark energy, such as the (generalized) Chaplygin gas model. In the latter we have generally \(C_2 = 0\), and so the equation describes an under-damped oscillator rather than an over-damped one. Consequently, the dark matter density contrast oscillates with (rapidly) decaying amplitude (or blows up if \(C_4\) is negative), in contrast to the monotonic decay in our model here. Tracing the derivation of Eq. [24], it is easy to find that the \(C_2\) term comes from the term with \(Z\) in Eq. [19], which is created by the fact that \(p_D \propto \nabla \cdot \mathbf{u}_a\). We also note that Eqs. [24]...
are independent of $m$ and hence independent of the functional form of $\eta(\rho_D)$. As long as $C_4 > 0$, we will obtain a qualitative picture similar to the one given in Fig. 3.

Although the very different evolution of $\Delta_D$ from that in $\Lambda$CDM is not directly reflected in the observed galaxy power spectrum, it will modify the gravitational potential and subsequently affect observables such as the CMB, weak lensing and CMB-galaxy cross-correlation. As the viscous dark matter contributes the majority of the total energy in the universe, a decay in its density contrast as in Fig. 3 will also drive the gravitational potential $\phi$ to decay significantly. This point is verified in Fig. 4 which clearly shows that the decay of $\phi$ is much faster than that in the $\Lambda$CDM model.

The fast decay of $\phi$ will enhance the integrated Sachs-Wolfe (ISW) effect, which then contributes a source term $\propto \int_0^\infty \phi' j_\ell(k[\tau_0 - \tau])d\tau$ to the CMB fluctuations, where $j_\ell(k\tau)$ is the spherical Bessel function and $\phi'$ the (conformal) time derivative of $\phi$. This corresponds to more power in the CMB spectrum on large scales, as displayed in Fig. 5. Note that the positions of the acoustic peaks for the two models are the same in Fig. 5 because their background evolutions are almost indistinguishable. We can see that comparison with CMB data could provide strong restrictions on the present model as well. Again, because the damping in $\Delta_D$ is a general feature in UDM models based on bulk viscosity, so are the enhancements of the ISW effect and the low-$\ell$ CMB power. In principle, the weak-lensing convergence power spectrum, which reflects the (projected) potential distribution along the line of sight, will also be modified (as compared to the $\Lambda$CDM result) significantly by the rapid decay of the potential. This is not considered here because the CMB spectrum itself already places stringent constraint on the model.

V. DISCUSSION AND CONCLUSIONS

To summarize, in this work we have investigated the background cosmological evolution and large-scale structure formation in the bulk viscous models designed as alternatives to dark energy. A bulk viscosity generates an effective pressure $p = -3\eta(\rho)H$ which, when the function $\eta(\rho)$ is appropriately chosen, could drive the acceleration of the cosmic expansion. Our numerical calculation focuses on a particular choice $\eta(\rho) = \alpha \rho^m$ with $\alpha, m$ being constants. Such a choice has been considered before in the literature, in the contexts of both inflationary and late-time accelerating universes. It is interesting to note that with suitable values of $m$, the viscous matter behaves as cold dark matter at early times and as a cosmological constant in the future; thus this model naturally unifies dark matter and dark energy, at least at the background level.

We have used the measurements of supernovae luminosity distance and CMB shift parameter to constrain the model parameters and found that for the best-fitting parameters, $m = -0.4$ and $\beta = 0.236$ ($\beta \propto \alpha$), the background evolution of the universe is almost the same as that predicted by the $\Lambda$CDM model. From this viewpoint it seems that the model is a feasible alternative to an explicit cosmological constant.

However, when it comes to the linear perturbations, the viscous dark matter starts to behave very differently from cold dark matter. The pressure of the viscous dark matter tends to resist the growth of the density contrast, and when it becomes significant (at late times) this effect can rapidly damp out the density perturbations, particularly on small scales. Even though this smoothing of the viscous dark matter density perturbation cannot be seen directly, it could reduce the growth rate of density perturbations in the luminous matter (which can be measured by the galaxy power spectrum) and drive the fast decay of the gravitational potential fluctuations, which subsequently modifies the large-scale CMB spectrum, weak lensing and CMB-galaxy cross-correlations.

Note that the model we consider has no explicit $\Lambda$CDM limit, i.e., one cannot adjust the model parameters $\alpha$ and $m$ to make the model reduce to $\Lambda$CDM exactly, unless a cosmological constant is added and the limit $\alpha \rightarrow 0$ is taken. The latter case, in which we have both an explicit cosmological constant and viscous matter, is not particularly appealing because it will introduce more complexity without solving any of the problems of $\Lambda$CDM. Rather, we are interested in whether the viscous dark matter alone could replace $\Lambda$CDM completely. This means that the EoS parameter $w$ will necessarily be of order $-1$ at late times. According to our analysis in § IV (cf. Eqs. 24–28), it is the value of $w$ that determines the evolution of $\Delta_D$, and so we expect that all the qualitative pictures given in § IV will remain in place in any attempts to replace dark matter and dark energy with bulk viscous matter alone. Viscous matter is therefore not a successful contender for the dark sector.

The bulk viscosity model is another candidate to explain the dark energy without introducing dynamical degrees of freedom. In order to explain the accelerating cosmic expansion, one needs a negative (effective) pressure. If no new dynamical degree of freedom is introduced, the negative pressure should be either a constant, or a function of $\rho_{DE}$ (the energy density of the newly added matter), $\rho_m$ (the energy density of the existing matter) or $H$, or combinations of these variables, which are all the possible variables in the background cosmology (see 26, 27) for a counterexample however, where a new degree of freedom is added but is made non-dynamical). Hence, we have the following conclusions for each of the allowed prescriptions for a form of newly added matter 26–29, 31:

1. The case $p = \text{const.}$ is simply a cosmological constant.

2. An example of the case $p = p(\rho_{DE})$ is the (generalized) Chaplygin gas model [14, 15]. The large-
Examples for the case $p = p(H)$ includes bulk viscosity model with $\eta(\rho) = \text{const}$. considered in $[10, 12, 42, 43, 44]$, which is a special case for the general model we considered in this work, with $p = p(\rho_{\text{DE}}, H)$. Here, the dependence of $p$ on $H$ results in the evolution equation of $\Delta$ for the newly added matter acquiring both a term proportional to $k^2\Delta$ and one proportional to $k^2\Delta'$. Consequently, on small scales at late times, $\Delta$ will decay rapidly without oscillation like an over-damped oscillator. We have shown that the UDM model based on this is generally unable to pass several cosmological tests. Note that as in the case of $p = p(\rho_{\text{DE}})$, the averaging issue again needs to be taken into account if more precise predictions are to be obtained $[31, 32]$, though its full effect and significance need careful nonlinear studies such as N-body simulations.

In all such cases (except $p = \text{const}$), we have seen that the evolution of $\Delta$ (for either the existing matter or the newly added matter) becomes very irregular. If the matter with irregular perturbation evolution makes a significant contribution to the total energy of the universe, then this will lead to large modifications to the $\Lambda$CDM predictions for the matter power spectrum, CMB, weak lensing and similar tests. This indicates that a viable dynamical UDM model is likely to involve extra dynamical degrees of freedom in contrast to that provided by a (generalized) Chaplygin gas or bulk viscosity $[45, 46, 47, 48]$.

**Acknowledgments**

We thank Niayesh Afshordi for kindly pointing out Ref. [26, 27] to us. BL acknowledges financial supports from an Overseas Research Studentship, the Cambridge Overseas Trust, the DAMTP and Queens’ College at Cambridge.

[1] A. G. Riess et al., Astron. J. 116, 1009 (1998); S. Perlmutter et al., Astrophys. J. 517, 565 (1999);
[2] A. G. Riess et al., Astrophys. J. 607, 665 (2004);
[3] D. N. Spergel et al., Astrophys. J. Suppl. 170, 377 (2007).
[4] M. Tegmark et al., Phys. Rev. D 69, 103501 (2004).
[5] E. J. Copeland, M. Sami and S. Tsujikawa, Int. J. Mod. Phys. D15, 1753 (2006).
[6] G. L. Murphy, Phys. Rev. D 8, 4231 (1973); M. Heller, Z. Klimek, and L. Suszycki, Astrophys. Sp. Sci. 205, 205 (1973); V. A. Belinskii and I. Khalatnikov, Sov. Phys. JETP Letts 21, 99 (1975) and Sov. Phys. JETP 42, 205 (1976).
[7] J. D. Barrow, Phys. Lett. B, 180, 335 (1986). J. D. Barrow, Nucl. Phys. B 310, 743 (1988); J.D. Barrow, String-driven Inflation, in The Formation and Evolution of Cosmic Strings, eds. G. Gibbons, S.W. Hawking & T. Vachaspati, pp 449-464, CUP, Cambridge (1990).
[8] M. Szydłowski and O. Hrycyna, Annals of Physics322.
2745 (2007).
[9] D. Pavon, J. Bafaluy and D. Jou, Class. Quant. Grav. 8, 347 (1991).
[10] D. Pavon and W. Zimdahl, Phys. Lett. A 179, 261 (1993).
[11] W. Zimdahl, D. J. Schwarz, A. B. Balakin and D. Pavon, Phys. Rev. D 64, 063501 (2001).
[12] N. Turok, Phys. Rev. Lett. 60, 549 (1988).
[13] J.D. Barrow, Phys. Lett. B, 235, 40 (1990).
[14] A. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B 511, 261 (2001).
[15] W. Zimdahl, D. J. Schwarz, A. B. Balakin and D. Pavon, Phys. Rev. D 64, 063501 (2001).
[16] N. Turok, Phys. Rev. Lett. 60, 549 (1988).
[17] J.D. Barrow, Phys. Lett. B, 235, 40 (1990).
[18] A. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B 511, 265 (2001).
[19] W. Zimdahl, D. J. Schwarz, A. B. Balakin and D. Pavon, Phys. Rev. D 64, 063501 (2001).
[20] N. Turok, Phys. Rev. Lett. 60, 549 (1988).
[21] J.D. Barrow, Phys. Lett. B, 235, 40 (1990).
[22] A. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B 511, 265 (2001).
[23] W. Zimdahl, D. J. Schwarz, A. B. Balakin and D. Pavon, Phys. Rev. D 64, 063501 (2001).
[24] The other contribution to the enhanced large-scale CMB power comes from the photon density perturbation, which is also affected by the evolution of $\Delta_D$.
[25] If $p = p(\rho_m, \rho_{DE})$, then using the conservation equation we can reexpress $p$ as $p = p(\rho_m, \rho_{DE}, H)$. Further possibilities $p = p(H, \cdots)$ are too complex to be considered here, but by the same logic as used in deriving Eq. (23) we could show that these will also lead to scale-dependent evolution of density perturbation. For example, if $p = p(\rho, H, H)$ then we can find that there are terms proportional to $k^2$ in the coefficients of $\Delta$, $\Delta'$ and also $\Delta''$. 
[26] N. Afshordi, D. J. H. Chung and G. Geshnizjani, Phys. Rev. D 75, 083513 (2007).
[27] N. Afshordi, D. J. H. Chung, M. Doran and G. Geshnizjani, Phys. Rev. D 75, 123509 (2007).
[28] S. Nojiri and S. D. Odintsov, Phys. Rev. D 72, 023003 (2005).
[29] S. Nojiri, S. D. Odintsov and S. Tsujikawa, Phys. Rev. D 71, 063004 (2005).
[30] S. Capozziello et. al., Phys. Rev. D 73, 043512 (2006).
[31] P. P. Avelino et. al., Phys. Rev. D 69, 043010 (2004).
[32] L. M. G. Beça and P. P. Avelino, Mon. Not. Roy. Astron. Soc 376, 1169 (2007).
[33] P. Gondolo and K. Freese, Phys. Rev. D 68, 063509 (2003).
[34] D. N. Vollick, Phys. Rev. D 68, 063510 (2003).
[35] T. Koivisto, H. Kurki-Suonio and F. Ravndal, Phys. Rev. D 71, 064027 (2005).
[36] T. Koivisto, Phys. Rev. D 73, 083517 (2006).
[37] B. Li and M. C. Chu, Phys. Rev. D 74, 104010 (2006).
[38] B. Li, K. C. Chan and M. C. Chu, Phys. Rev. D 76, 024002 (2007).
[39] B. Li, J. D. Barrow and D. F. Mota, Phys. Rev. D 76, 104047 (2007).
[40] B. Li, D. F. Mota and D. J. Shaw, Phys. Rev. D 78, 064015 (2008).
[41] B. Li, D. F. Mota and D. J. Shaw, Class. Quant. Grav. 26 (2009) 055018.
[42] A. Avelino, U. Nucamendi and F. S. Guzmán [arXiv:0801.1680 [gr-qc]].