Chiral cosmic strings are naturally produced at the end of D-term inflation and they present very interesting cosmological consequences. In this work, we investigate the formation and evolution of wakes by a chiral string. We show that, for cold dark matter, the mechanism of forming wakes by a chiral string is similar to the mechanism by an ordinary string.

Keywords: Topological defects; formation of large scale structure.

PACS Nos.: 98.80.Cq, 98.70.Vc

1. Introduction

Topological defects are predicted in many gauge models as solitonic solutions resulting from spontaneous breaking of gauge or global symmetries. Among all these solutions, cosmic strings, and in particular those with conducting properties, have a number of cosmological applications. The possible scenarios for structure formation by cosmic strings depend very much in the detail of the model. One relevant mechanism to understand the structure formation by these configurations involves long strings moving with relativistic speed in the normal plane, giving rise to velocity perturbations in their wake. The mechanism of forming wakes has been considered by many authors in both Einstein and Scalar-Tensor theories of gravity in the neutral string case and in the conducting string case.

Recently, it has been shown that for strings like those proposed by Witten (i.e., current-carrying strings) there appear logarithmic terms and the mechanism of
forming wakes can breakdown. Inclusion of a current-like effect modifies tremendously the internal structure of a cosmic string in such a way that new states can be reached. This is due to the breaking of the Lorentz boost invariance along the worldsheet allowing rotating equilibrium configurations called vortons. If these vortons are stable they can overclose the Universe. The new feature pointed out in the Refs. is that a long current-carrying string can also lead to a catastrophe for the underlying theories that predict them.

Chiral strings arise when Witten’s fermionic zero mode mechanism gives rise to a purely left (or a purely right) moving modes with a null current. As a step forward in our previous investigation, and as a main goal of this paper, we study the impact of the chirality property of the string in the mechanism of wakes. For this purpose, we will use what can be called the “canonical cold dark matter model”. That is, we will mimic the mechanism of wakes based on linear adiabatic perturbations (the Zel’dovich approximation) and cold dark matter flowing past the chiral string. This letter is outlined as follows. In the section 2, we briefly recall some general properties of a chiral cosmic string, giving particular attention to its metric. In the section 3, we treat the mechanism of forming wakes by a chiral string by means of the Zel’dovich approximation. We finally end up our paper with some conclusions and future perspectives in the section 4.

2. Some General Properties of a Chiral Cosmic String and its Metric

In this section, we will briefly review some properties and the metric around a chiral cosmic string. We will mainly follow the results presented in the paper. The stress energy tensor for the infinite string is given by:

\[
T^{\mu\nu} = m^2 \text{diag}(1, 0, 0, -k) \delta(x)\delta(y),
\]

\[
T^{03} = m^2 \frac{1 - k}{2} \delta(x)\delta(y)
\]

(1)

\( k \) characterizes the state of the string: \( k = 0 \) corresponds to the maximal charged strings and \( k = 1 \) corresponds to the ordinary, neutral string. It can happen that \( k \) is not constant, but we will not deal with this case here. The off-diagonal terms are presented because of the null current of the string and they represent their momentum. If \( k = 1 \) we reduce our case to the case of an ordinary cosmic string, in which \( T^0_0 = T^3_3 \) and the off-diagonal terms vanish.

In the weak-field approximation, \( g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \) where \( |h| \ll 1 \), and in the de Donder gauge, we have:

\[
\Box h_{\mu\nu} = 16\pi G(T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T)
\]

(2)

Writing these equations in the cartesian coordinates system in which \( r^2 = x^2 + y^2 \), the solution for Eqs. (2) are:

\[
h_{tt} = -h_{tz} = h_{zz} \equiv X(r, k) = 4Gm^2(1 - k) \ln(r/r_0)
\]
where $r_0$ is an integration constant which can be interpreted as the width of the string. The metric of a chiral cosmic string can be simplified by means of the coordinate transformation:

\[(1 - Q(r, k))r^2 = (1 - 2Gm^2(1 + k))^2R^2\]  

which gives the final form for the metric as:

\[ds^2 = dt^2 (1 + X(R, k)) - dz^2 (1 - X(R, k)) - dR^2 - (1 - 2Gm^2(1 + k))^2R^2d\theta^2 - 2X(R, k)dtdz.\]  

Just as in the case of ordinary string, this metric also has a deficit angle given by

\[\delta(k) = 4\pi Gm^2(1 + k),\]  

which is now $k$-dependent.

With the metric we can easily calculate the gravitational force on moving test particles

\[F(R, k) = -\frac{4G}{R} \frac{1 - k}{1 + k},\]  

and the velocity perturbation

\[u = 8\pi Gm^2v_s\gamma + \frac{4\pi G}{v_s\gamma} \left(\frac{1 - k}{1 + k}\right),\]  

where $v_s$ is the string’s normal velocity through matter and $\gamma = (1 - v^2)^{-1/2}$. Both the force and the velocity are $k$-dependent and have their maximum for $k = 0$ (the vortons case). Thus, one might expect that chiral strings with large charge will be more effective in the formation of wakes. In the next section we will analyse the accretion problem due to this string.

3. Formation and Evolution of the Wakes and the Zel’dovich Approximation

In this section we will study the mechanism of formation and evolution of wakes by a chiral string, with metric. As usual and to simplify our problem, we will consider a situation in which cold dark matter composed by non-relativistic collisionless particles move past a long string. In this simple case, we can apply the Zel’dovich approximation which consists in considering the Newtonian accretion problem in an expanding Universe using the method of linear perturbations.

We start by considering the velocity perturbation given by Eq. In this equation, the first term is equivalent to the relative velocity of particles flowing past the string. The second term, appears because of the chirality of the string and it vanishes when $k = 1$. 

\[u = 8\pi Gm^2v_s\gamma + \frac{4\pi G}{v_s\gamma} \left(\frac{1 - k}{1 + k}\right),\]
Let us suppose now that the wake was formed at $t_i > t_{eq}$, where $t_{eq}$ is the time of equal matter and radiation. The physical trajectory of a dark particle can be written as

$$h(\vec{x}, t) = a(t)[\vec{x} + \psi(\vec{x}, t)]$$  \hspace{1cm} (9)

where $\vec{x}$ is the unperturbed comoving position of the particle and $\psi(\vec{x}, t)$ is the comoving displacement developed as a consequence of the gravitational attraction induced by the wake on the particle. Suppose, for simplification, that the wake is perpendicular to the $x$-axis (assuming that $dz = 0$ in the metric (5) and $r = \sqrt{x^2 + y^2}$) in such a way that the only non-vanishing component of $\psi$ is $\psi_x$. Therefore, the equation of motion for a dark particle in the Newtonian limit is

$$\ddot{h} = -\nabla_h \Phi$$  \hspace{1cm} (10)

where the Newtonian potential $\Phi$ satisfies the Poisson equation

$$\nabla_h^2 \Phi = 4\pi G_0 \rho$$  \hspace{1cm} (11)

where $\rho(t)$ is the dark matter density in a cold dark matter universe. For a flat universe in the matter-dominated era, $a(t) \sim t^{2/3}$. Therefore, the linearised equation for $\psi_x$ is

$$\ddot{\psi} + \frac{4}{3t} \dot{\psi} - \frac{2}{3t^2} \psi = 0$$  \hspace{1cm} (12)

with appropriated initial conditions: $\psi(t_i) = 0$ and $\dot{\psi}(t_i) = -u_i$. Eq. (12) is the Euler equation whose solution is easily found

$$\psi(x, t) = \frac{3}{5} \left[ \frac{u_i t_i^2}{t} - u_i t_i \left( \frac{t}{t_i} \right)^{2/3} \right]$$

Calculating the comoving coordinate $x(t)$ using the fact that $\dot{h} = 0$ in the “turn around”\(^a\), we get

$$x(t) = -\frac{6}{5} \left[ \frac{u_i t_i^2}{t} - u_i t_i \left( \frac{t}{t_i} \right)^{2/3} \right]$$  \hspace{1cm} (13)

With the help of (13) we can compute both the thickness $d(t)$ and the surface density $\sigma(t)$ of the wake.\(^b\) We have, then, respectively (to first order in $G$)

$$d(t) \approx \frac{48}{5} \left( \frac{t}{t_i} \right)^{3/2} \left( 2\pi G m^2 v_s \gamma + \frac{\pi G}{v_s \gamma} \left( \frac{1-k}{1+k} \right) \right)$$

$$\sigma(t) \approx \frac{8}{5} \left( \frac{t}{t_i} \right)^{3/2} \left( 2m^2 v_s \gamma + \frac{1}{v_s \gamma} \left( \frac{1-k}{1+k} \right) \right)$$  \hspace{1cm} (14)

\(^a\)The moment when the dark particle stops expanding with the Hubble flow and starts to collapse onto the wake.
Surprisingly enough, we can easily see that, for \( k = 0 \) (the vorton case), the results obtained above reduce to those already known for a wiggly string in General Relativity \(^3\) or an ordinary string in Scalar-Tensor gravity \(^6\), after some identifications of the constants in the second term in Eq. \((14)\). This means that, at least for cold-dark matter, the details of the model (e.g., whether the string is current-carrying or not or whether the underlying theory of gravity is purely tensorial or scalar-tensorial) is not relevant.

### 4. Conclusions

Even tough the recent results of the CMB satellite WMAP\(^{10}\) indicate inflationary models as the source of the large-scale structure in the Universe, cosmic string models still raise some interest in cosmology \(^{17}\). In particular, wakes produced by moving strings can provide an explanation for filamentary and sheetlike structures observed in the universe. A wake produced by the string in one Hubble time has the shape of a strip of width \( \sim v_s t_i \). In this work we have treated the accreation problem around a chiral string using the canonical cold-dark matter model. Although both the force and the velocity reach their maxima values for \( k = 0 \) (vortons), we came to the conclusion that the formation of wakes is not sensible to the chirality property of the string, which is expressed in metric \((5)\) by the mixed term \( 2X(R,k)dt dz \). We can infer that, most likely, this is due to the fact that we restricted ourselves to the cold dark matter case.

Nonbaryonic dark matter is either hot or cold depending on whether the thermal velocity of the dark particles at the time \( t_{eq} \) is large or negligible. In a model with hot dark matter (say, neutrinos) and adiabatic density perturbations, the Zel’dovich approximation is no longer valid, but requires some adaptation \(^4\). We plan to deal with this problem in a forthcoming paper.

### Acknowledgments

The authors are grateful to A. L. N. Oliveira, V. C. Andrade and D. Steer for fruitful discussions. A.A.B. and M.E.X.G. would like to thank Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP/Brazil) for partial support and the Instituto de Física Teórica (IFT/UNESP) for kind hospitality. The authors would like to thank the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq/Brazil) for partial support.

### References

1. A. Vilenkin and E. P. S. Shellard *Cosmic Strings and Other Topological Defects* (Cambridge: Cambridge University Press, 1994).
2. J. Silk and A. Vilenkin, *Phys. Rev. Lett.* **53**, 1700 (1984)
6 M. C. B. Abdalla, A. A. Bytsenko and M. E. X. Guimarães

3. T. Vachaspati, Phys. Rev. Lett. 57, 1655 (1986); A. Stebbins, S. Veeraraghavan, R. H. Brandenberger, J. Silk and N. Turok, Ap. Journ. 1, 322 (1987); N. Deruelle and B. Linet, Class. Quantum Grav. 5, 55 (1988); W. Hiscock and B. Lail, Phys. Rev. D 37, 869 (1988).

4. L. Perivolaropoulos, R. H. Brandenberger and A. Stebbins, Phys. Rev. D 41, 1764 (1990).

5. T. Vachaspati, Phys. Rev. D 45, 3487 (1992).

6. S. R. M. Masalskiene and M. E. X. Guimarães, Class. Quantum Grav. 17, 3055 (2000).

7. P. Peter and D. Puy, Phys. Rev. D 48, 5546 (1993).

8. A. L. N. Oliveira and M. E. X. Guimarães, Phys. Rev. D 67, 123514 (2003).

9. A. L. N. Oliveira and M. E. X. Guimarães, Phys. Lett. A 311, 474 (2003).

10. E. Witten, Nucl. Phys. B 249, 557 (1985).

11. B. Carter, Phys. Lett. B 224, 61 (1989); ibid 238, 166 (1990); Ann. N. Y. Acad. Sci. 647, 758 (1991).

12. P. Peter, Phys. Rev. D 45, 1091 (1992); ibid 46, 3336 (1992).

13. A. C. Davis, T. Kibble, M. Pickles and D. A. Steer, Phys. Rev. D 62, 083516 (2000); D. A. Steer, Verbier 2000, Cosmology and Particle Physics, 425 (2000); D. A. Steer, Phys. Rev. D 63, 083517 (2001).

14. S. Weinberg, Gravitation and Cosmology (John Wisley and Sons, 1972).

15. A. Vilenkin, Phys. Rev. D23, 852 (1981).

16. C. L. Bennett et al., astro-ph/0304237, Ap. J. (to appear).

17. M. Landriau and E. P. S. Shellard, Phys. Rev. D 69, 023003 (2004).