Heat transfer enhancement in H$_2$O suspended by aluminium alloy nanoparticles over a convective stretching surface

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Abstract
The purpose of this work is to investigate the heat transport on water suspended by aluminium alloy nanomaterials. The analysis is conducted by incorporating the influence of imposed magnetic field and viscous dissipation over convective surface. The self-similar version of the model is treated numerically and the results for the flow field are presented. It is perceived that the velocity of AA7072-H$_2$O and AA7075-H$_2$O declines for stronger magnetic field effects. Due to convective condition, the temperature rises abruptly. Moreover, increasing trends in the local heat transfer rate are examined for higher Biot effects.

Keywords
Heat transfer, convective surface, stretching surface, aluminium alloy, numerical scheme

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Introduction
Heat transfer analysis in viscous incompressible fluid over a stretching sheet either rotating or nonrotating is one of the potential research areas because of its variety of applications. These comprised in wire drawing, manufacturing of glass fibre, production of rubber sheets and cooling of large metallic plates like electrolyte and so on.

First of all, Crane$^1$ investigated the fluid flow past a stretching plate. Afterwards, researchers focused on the analysis of fluid over a stretchable surface by considering various physical flow conditions like convective condition and slip condition. For precedence, we can study Mukhopadhyay$^2$, Sahoo$^3$ and Rashidi et al.$^4$ The stimulations of suction/injection of flow over a stretchable surface were described by Gupta and Gupta.$^5$ The behaviour of heat and mass transfer due to various nondimensional physical quantities is comprised in their study. The analysis of rotating fluid over a stretchable surface is discussed by Wang.$^6$ They observed fascinating behaviour of parameter $\lambda$ in the flow regimes. The parameter $\lambda$ is a quotient of revolution to the...
stretching rate of the rotating sheet. They treated the model for smaller values of \( \lambda \). For the above said purpose, they adopted regular perturbation technique.

The influences of Lorentz forces on the flow regimes are very significant. In many industrial processes, flow of fluid in different channels and tubes may contain impurities. In order to purify the flowing liquid, Lorentz forces were applied on it. Due to applied Lorentz forces, the motion of fluid become slowdown and impurities remain at the bottom. Thus, researchers targeted on the analysis of heat and mass transfer by considering the effects of Lorentz forces. In 2016, Reddy and Chamkha \(^7\) reported magnetohydrodynamic (MHD) flow by encountering the influences of various parameters. Time-dependent flow of non-Newtonian fluid over a surface was reported in Ullah et al. \(^8\). They also highlighted the hidden phenomena of thermal and concentration gradients in the velocity and concentration fields. For numerical computation, they employed Keller-box technique. Very recently, Ahmed et al. \(^9\) presented novel analysis on flow of Newtonian fluid over convective nature of unsteady stretching surface. They examined the impact of thermal radiations on chemically reacting fluid. The study of fluids (including micropolar fluids) by considering the remarkable Soret and Dufour stimulations and convective boundary condition is comprised in Sharidan et al. \(^10\), Aurangaizb et al. \(^11\) and Sharidan et al. \(^12\) and references therein.

Studies made for the improvement of less thermally conductive fluids suspended by solid particles began more than a century ago. First, a theoretical model which developed to improve thermal conductivity of heterogeneous nature of solid particles is known as Maxwell model. In order to investigate thermal conductivity of the fluids diluted with solid micrometre- or millimetre-sized particles, the Maxwell model was adopted. The major issue with the use of micro-sized particles in the conventional fluids is that the particles diffuse in the liquids very rapidly. Furthermore, drop in pressure, abrasion and clogging is caused by these phenomena. In order to obtain remarkable improvement in thermal conductivities of these compositions, particles with high concentration are required.

For many industrial and technological processes, fluids are required to have rapid thermal conductivity properties. For such processes, regular liquids, such as water, kerosene oil, engine oil, ethylene glycol, tri-ethylene glycol, lubricants and bio-fluids, are failed because of poor thermal conductivity properties. To fill aforementioned gap in the industrial and technological sides, a new class of fluids developed known as nano-fluids. The term nanoparticle fluid suspensions or nano-fluids is coined by Choi \(^13\) in 1995. This newly developed class of fluids solved many technological and industrial issues. Nanofluids are the compositions of nanosized particles in the conventional liquids. The nanoparticles are oxide ceramic, metals (Al-Cu), metal carbides (SiC), nitrides and other functionalized nanoparticles. For useful study regarding nanofluids in varying geometries by considering the various flow parameters and conditions like convective and slip conditions, we can study Sheikholeslami and Seyednezhad, \(^14\) Khan et al. \(^15\) and Ahmed et al. \(^16\) Significant analysis related to the heat transport characteristics in the colloidal suspensions and regular liquids under various flow conditions in different flow scenarios was examined in Khan and colleagues. \(^17\)–\(^27\) The analysis of magnetized Maxwell liquid, micropolar fluid characteristics, stagnation MHD flow with mixed convection and the novel analysis of activation energy in Carreau nanofluid was perceived in Hsiao. \(^28\)–\(^31\)

The unique composition of characteristics provided by aluminium and its alloys makes it one of the multifarious and economical materials from engineering point of view. Aluminium has less density (2.7 g/cm\(^3\)) compared to that of steel (7.83 g/cm\(^3\)). Furthermore, weight of steel in one cubic foot is about 490 lb which is greater compared to that of aluminium alloys having weight only 170 lb. Thus, aluminium alloys are used in wheel manufacturing that are advantageous for air crafts and vehicles and all types of water-borne and land vehicles.

Aluminium alloys are alloys in which aluminium play a dominant role. Furthermore, in aluminium alloys, copper, silicon, zinc and magnesium are used as a typical alloying element. Aluminium alloys are characterized into two further sub-disciplines called heat treatable and heat non-treatable aluminium alloys. The aluminium alloys are widely used in household wiring and in manufacturing of wheels and air crafts. Among aluminium alloys, two important alloys are known as AA7072-H2O and AA7075-H2O (for precedence, we can study Khan et al. \(^32\)). These aluminium alloys contain 90% Al, 2%–3% Mg, 5%–6% Zn and 1%–2% copper with the addition of Cu, Fe and Mn. Similarly, AA7072-H2O is the colloidal suspension of 98% Al and 1% Zn with Cu, Fe and Si.

In this work, incompressible flow of water suspended by aluminium alloys particles, namely, AA7072-H2O and AA7075-H2O, is taken over a stretchable surface. The impact of resistive heating and convective nature of boundary condition is also under consideration. Section ‘Model formulation’ describes the modelling of the particular flow model. Solution procedure (Runge–Kutta scheme) is given in section ‘Solution of the model’. The stimulations of various nondimensional quantities particularly resistive heating and convective condition are embedded in section ‘Graphical results’. Finally, remarkable outcomes are described in section ‘Conclusion’.
Model formulation

The steady two-dimensional (2D) laminar flow of water suspended by AA7072 and AA7075 aluminium alloys is taken under consideration over a stretchable surface. The influence of convective flow condition and ohmic heating is taken into account. The sheet is placed in Cartesian coordinate system and x-axis aligned upwards. The sheet is stretched in opposite way with liquid that are equal in magnitude. The velocity and temperature of continuously stretching surface are $U(x) = bx$ and $T_w(x) = Ax/l$, respectively. Here, $A$ and $b$ are the constants. Furthermore, it is assumed that flow is incompressible, alloy nanoparticles and base liquid are in thermal equilibrium and no chemical reaction occurs. Physical theme of the flow model is shown in Figure 1.

The equations that govern the flow of AA7072-H$_2$O and AA7075-H$_2$O nanofluids in the existence of applied Lorentz force and ohmic heating over a stretchable surface are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u}{\partial y^2} \right) - \frac{\sigma_{nf}}{\rho_{nf}} B_0^2 u$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{nf}}{(\rho C_p)_{nf}} \frac{\partial^2 T}{\partial y^2} + \frac{\sigma_{nf}}{(\rho C_p)_{nf}} B_0^2 u^2$$

Equation (1) shows the law of conservation of mass, while momentum and energy equations in the existence of Lorentz force and resistive heating are described in equations (2) and (3), respectively. Moreover, $\mu_{nf}$ is the effective dynamic viscosity, $\rho_{nf}$ is the effective density, $\sigma_{nf}$ is the electrical conductivity, $k_{nf}$ is the effective thermal conductivity, $(\rho C_p)_{nf}$ is the specific heat capacitance and $B_0$ is the perpendicularly applied magnetic field.

The conditions at the sheet surface and away from it are as follows

$$u = U(x)$$
$$v = 0$$
$$-K \frac{\partial T}{\partial y} = h_f (T_w - T)$$

at \( y = 0 \) \hspace{1cm} (4)

$$u \to 0$$
$$T \to T_\infty$$

at \( y \to \infty \) \hspace{1cm} (5)

where $u$, $v$ and $T$ are functions of $x$ and $y$. For effective dynamic viscosity, effective thermal conductivity, effective density and effective electrical conductivity, the following nanofluid models are adopted

$$\rho_{nf} = \left( 1 - \phi \right) + \frac{\phi \rho_s}{\rho_f} \rho_f$$

$$\left( \rho C_p \right)_{nf} = \left( 1 - \phi \right) + \frac{\phi \left( \rho C_p \right)_f}{\left( \rho C_p \right)_f} \left( \rho C_p \right)_f$$

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^2}$$

$$k_{nf} = \frac{k_f \left[ \tilde{\Gamma} - 2 \phi \tilde{\Gamma}_1 \right]}{\left[ \tilde{\Gamma} + \phi \tilde{\Gamma}_1 \right]}$$

where $\tilde{\Gamma} = k_f + 2k_f \phi$, $\tilde{\Gamma}_1 = k_f - k_s$

$$\sigma_{nf} = \sigma_f \left[ 1 + \frac{3(\Pi - 1) \phi}{(\Pi + 2) - (\Pi - 1) \phi} \right]$$

where $\Pi = \frac{\sigma_s}{\sigma_f}$

In equations (6)–(8), volume fraction of aluminium alloys is denoted by $\phi$. Furthermore, $\rho_f, k_f$ and $\mu_f$ represent density, thermal conductivity and effective viscosity of the conventional liquid, respectively. Thermophysical characteristics are given in Table 1.

The similarity variables for particular flow model are defined in the following manner

$$\eta = \left( \frac{u}{b} \right)^{-1} y$$

$$u = bx F'(\eta)$$

$$v = -\sqrt{\eta} b F'(\eta)$$

$$\beta(\eta) = \frac{T - T_w}{T_w - T_\infty}$$

Using suitable differentiation and nanofluids models given in Table 1, we attained the following model
The feasible boundary conditions for both AA7072-H₂O and AA7075-H₂O models are given in equations (13) and (14).

The dimensional form for shear stress and local Nusselt number at the convective surface is in the following form

\[
C_F = -\frac{\mu_{nf}\partial u}{\rho_{nf}\partial y} \quad \text{at} \quad y = 0 \quad \text{(19)}
\]

\[
Nu = -\frac{\lambda_{nf}}{k_f(T_w - T_0)} \frac{\partial T}{\partial y} \quad \text{(20)}
\]

By entreating feasible differentiation in equations (19) and (20) and after some calculation, mathematical expressions for shear stresses and local Nusselt number transformed into the following dimensionless version

\[
C_F = \frac{(1 - \phi)^{2.5} E^{\prime\prime}(\eta)}{(Re_s)^2} \quad \text{at} \quad \eta = 0 \quad \text{(21)}
\]
\[ Nu = - (Re) \left( \frac{174.226 + 344.774\phi}{174.226 - 172.387\phi} \right) \beta' (\eta) \text{ at } \eta = 0 \]  

(22)

Here, \( Re \) is the local Reynolds number and \( Re = U_{\infty} x / \nu_{f} \).

**Solution of the model**

Both the flow models (AA7072-H\(_{2}\)O and AA7075-H\(_{2}\)O) are nonlinear. It is not easy to calculate exact solutions for these models. Thus, we imposed our attention to solve the models numerically. For this, we adopted Runge–Kutta numerical scheme (for precedence, we can study Ahmed and colleagues\(^9,15,16\)). To initiate the Runge scheme, first, one need to introduce the following transformations

\[ z_{1} = F, \quad z_{2} = F', \quad z_{3} = F'' \]  

(23)

\[ z_{4} = \beta, \quad z_{5} = \beta' \]  

(24)

Before employing the above substitutions, first, we write both the models in the following pattern.

**AA7072-H\(_{2}\)O model**

\[ F'' = \frac{(1 + 1.72791094173102\phi)(F'^{2} - FF'')}{{(1 - \phi)\frac{2.08979999 \times 10^{10}\phi}{6.966000002 \times 10^{9} - 6.965999999 \times 10^{9}\phi}}}F'' \]

\[ + \frac{Pr\left(1 - 0.4170795714376755\phi\right)F\beta'}{223.226 + 442.774\phi} \]

\[ - \frac{EcM^{2}\left(1 + \frac{2.089799999 \times 10^{10}\phi}{6.966000002 \times 10^{9} - 6.965999999 \times 10^{9}\phi}\right)F'^{2}}{223.226 + 442.774\phi} \]  

(25)

\[ \beta'' = - \frac{Pr\left(1 - 0.4170795714376755\phi\right)F\beta'}{223.226 + 442.774\phi} \]

\[ - \frac{EcM^{2}\left(1 + \frac{2.089799999 \times 10^{10}\phi}{6.966000002 \times 10^{9} - 6.965999999 \times 10^{9}\phi}\right)F'^{2}}{223.226 + 442.774\phi} \]  

(26)

Now, entreating the transformations made in equations (23) and (24), we have the following system of first-order ordinary differential equations

\[ \begin{bmatrix} z'_{1} \\ z'_{2} \\ z'_{3} \\ z'_{4} \\ z'_{5} \end{bmatrix} = \begin{bmatrix} z_{2} \\ z_{3} \\ \frac{(1 + 1.72791094173102\phi)}{(1 - \phi)^{2.5}}(z_{2} - z_{1}) + \\ M^{2}\left(1 + \frac{2.08979999 \times 10^{10}\phi}{6.966000002 \times 10^{9} - 6.965999999 \times 10^{9}\phi}\right)z_{2} \\ \frac{Pr\left(1 - 0.4170795714376755\phi\right)z_{1}z_{3}}{223.226 + 442.774\phi} \\ \frac{EcM^{2}\left(1 + \frac{2.089799999 \times 10^{10}\phi}{6.966000002 \times 10^{9} - 6.965999999 \times 10^{9}\phi}\right)z_{2}^{2}}{223.226 + 442.774\phi} \end{bmatrix} \]  

(27)

**AA7075-H\(_{2}\)O model**

The AA7075-H\(_{2}\)O model described in equations (17) and (18) can be written as

\[ F'' = \frac{(1 + 1.818172700832414\phi)(F'^{2} - FF'')}{{(1 - \phi)\frac{1.606199997 \times 10^{10}}{5.354000002 \times 10^{9} - 5.353999999 \times 10^{9}\phi}}}F'' \]

\[ + \frac{M^{2}\left(1 + \frac{1.606199997 \times 10^{10}}{5.354000002 \times 10^{9} - 5.353999999 \times 10^{9}\phi}\right)F'^{2}}{223.226 + 442.774\phi} \]  

(28)

\[ \beta'' = - \frac{Pr\left(1 - 0.3526092862409387\phi\right)F\beta'}{223.226 + 442.774\phi} \]

\[ - \frac{EcM^{2}\left(1 + \frac{1.606199997 \times 10^{10}}{5.354000002 \times 10^{9} - 5.353999999 \times 10^{9}\phi}\right)F'^{2}}{223.226 + 442.774\phi} \]  

(29)

Using substitutions, we have the following system
In Figure 2, it is obvious that for stronger magnetic parameters, the velocities of nanofluids vanish asymptotically far from the stretchable surface. Consequently, impact of magnetic parameter \( M \) on the velocity component \( F(\eta) \) is illustrated in Figure 3. It is clear that near the convective surface (i.e., \( \eta = 0 \)) variations in the velocity component are minimal. Beyond \( \eta \geq 1.0 \), the velocity component alters rapidly and maximum decrement in the velocity fields is observed for both AA7072-H\(_2\)O and AA7075-H\(_2\)O nanofluids. Drop in the velocity profile for AA7072-H\(_2\)O nanofluid is slowdown as compared to that of AA7075-H\(_2\)O nanofluid.

It is imperative to mention that volumetric fraction \( \phi \) plays significant role in the study of nanofluids. In order to analyse the influence of volumetric fraction \( \phi \) on the velocity fields, Figures 3 and 4 are depicted. The volumetric fraction \( \phi \) favours the velocities of both AA7072-H\(_2\)O and AA7075-H\(_2\)O nanofluids. The velocity field \( F(\eta) \) increases very slowly for both types of nanofluids. Figure 4 shows the effects of volumetric fraction \( \phi \) on \( F(\eta) \). It is examined that the velocity filed increases rapidly. For AA7075-H\(_2\)O nanofluid, dominating behaviour of \( F(\eta) \) is observed. It is noted that away from the convective surface, the velocity field \( F(\eta) \) becomes stable for AA7072-H\(_2\)O and AA7075-H\(_2\)O nanofluids.

**Graphical results**

The stimulations in the flow characteristics due to fluctuating quantities ingrained in the models are described in this section. Furthermore, to provoke the validation of the analysis, a fruitful comparison is carried out.

**Velocity field**

The influences of \( M \) on the velocity field \( (F(\eta) \) and \( F(\eta) \)) for AA7072-H\(_2\)O and AA7075-H\(_2\)O nanofluids are plotted in Figures 2 and 3, respectively. From Figure 2, it is obvious that for stronger magnetic parameter \( M \), the velocities of both AA7072-H\(_2\)O and AA7075-H\(_2\)O nanofluids decline. Near the convective surface, these fluctuations are almost inconsequential for both types of nanofluids. In the region above the convective surface, rapid drop in the velocities is observed. A quite rapid decrement in the velocity of AA7075-H\(_2\)O nanofluid is investigated as compared to that of AA7072-H\(_2\)O nanofluid. Moreover, the velocities of nanofluids vanish asymptotically far from the thermal field

The properties of nanofluids like effective dynamic viscosity, effective density, specific heat capacitance, thermal conductivity and electrical conductivity play a key role in the study of nanofluids. These imperative physical quantities are embedded in magnetic number \( M \), Prandtl number \( Pr \) and Eckert number \( Ec \). Furthermore, the influence of convective nature of boundary condition is also taken into account which comprised Biot’s number \( B_i \). The effects of aforementioned nondimensional physical quantities on the temperature of AA7072-H\(_2\)O and AA7075-H\(_2\)O nanofluids
are illustrated in Figures 5–8. Moreover, $Pr$ is fixed at 6.96, because carrier liquid is H$_2$O.

The influence of Hartmann number, which is a quotient of electromagnetic force to the viscous forces, is shown in Figure 5. As electrical conductivity of AA7072 alloys is greater than that of AA7075 alloys due to which temperature of AA7072-H$_2$O nanofluids increases. Furthermore, for stronger magnetic field, temperature for both AA7072-H$_2$O and AA7075-H$_2$O nanofluids rises. Near the stretchable surface, that is, at $\eta = 0$, the variations in thermal field are very prominent. From $\eta = 0$ to $\eta = 1.5$, dominating behaviour of AA7072-H$_2$O is noted. Besides, thermal field shows asymptotic behaviour. The volumetric fraction is a key ingredient to study the behaviour of nanofluids temperature. For the above said purpose, Figure 6 is depicted. It is inspected that for high-volume fraction $\phi$ of alloy nanoparticles (AA7072-H$_2$O and AA7075-H$_2$O) temperature field $\beta(\eta)$ starts increasing. For AA7072-H$_2$O nanofluids, temperature rises quite rapidly as compared to that of AA7075-H$_2$O nanofluids. These effects are due to the high thermal conductivity of AA7072-H$_2$O nanofluids. Because for smaller size of metals particles and high-volume fraction, thermal conductivity of the nanofluids could increase. Beyond $\eta \geq 1.5$ nanofluids, temperature profile become stable and vanishes asymptotically.

Eckert number, which is a relation of advective transport to the heat dissipation potential ($C_p\Delta T$), is very important to characterize heat dissipation. In our study, $\Delta T$ shows the temperature difference at the convective surface $T_w$ and the ambient temperature $T_\infty$, 
since heat capacitance of AA7072 alloys is less than AA7075 alloy nanoparticles. Due to less heat capacitance, heat dissipation potential becomes dominant and the temperature increases for AA7072-H₂O nanofluids as compared to that of AA7075-H₂O nanofluids. Because for AA7075 alloy nanoparticles, heat capacitance is greater which leads to less heat dissipation potential and consequently temperature of AA7075-H₂O nanofluids rises slowly. These effects are demonstrated in Figure 7 for varying $Ec$.

An imperative dimensionless physical quantity called Biot’s number has a wide range of applications which comprised in transient heat transfer and so on. Fluid mechanics problem having smaller Biot’s number are (less than one) thermally easy to study because of uniform thermal field inside the body. However, high Biot’s number (much greater than one) signals the non-uniformity of thermal field inside the body. The stimuli of Biot’s number $Bi$ on thermal characteristics of AA7072-H₂O and AA7075-H₂O are plotted in Figure 8. For AA7072-H₂O nanofluids, dominant effects of temperature are observed as compared to that of AA7075-H₂O nanofluids. These effects are due to smaller thermal conductivity of AA7072 alloy nanoparticles. In the vicinity of the convected surface temperature of under consideration, nanofluids increase very rapidly. High thermal conductivity of AA7075 alloy nanoparticles decreases the Biot’s number due to which temperature of nanofluids increases slowly as compared to that of AA7072-H₂O nanofluids.

**Skin friction and Nusselt number**

Shear stresses and local Nusselt number attained much interest due to vital role in industries. These quantities alter due to various flow parameters like magnetic number, $Pr$, $Ec$ and Biot’s number significantly. For the above said purpose, Figures 9–15 are plotted in which effects of aforementioned self-similar physical quantities are embedded.

Figure 9 depicts the alterations in the shear stress for increasing $M$ versus $\phi$. For stronger magnetic number $M$, shear stress starts to decrease. For AA7075-H₂O nanofluids, shear stress decreases rapidly as compared to that of AA7072-H₂O nanofluids. This is due
to low electrical conductivity of AA7075 alloy nanoparticles. The volume fraction varies horizontally and magnetic parameter alters its values curve wise. However, the impact of volume fraction $\phi$ on the shear stress of AA7072-H$_2$O and AA7075-H$_2$O nanofluids is demonstrated in Figure 10. The volume fraction $\phi$ favours the shear stress. For AA7072-H$_2$O nanofluids, skin friction coefficient increases very rapidly comparative to AA7075-H$_2$O nanofluids. Furthermore, it is investigated that for increasing magnetic parameter $M$ horizontally, changes in the shear stresses become slowdown. The stimulus of $Ec$ versus $\phi$ on the shear stress is shown in Figures 11 and 12, respectively. In both the cases (for varying Eckert number and volume fraction curve wise), shear stress decreases. For varying volume fraction $\phi$, decrement in the shear stresses is rapid comparative to increasing Eckert number $Ec$.

Figures 13–15 illustrate the variations in the local Nusselt number for Eckert number, Biot’s number and $\phi$, respectively. The Eckert number (Figure 13) and volumetric fraction $\phi$ (Figure 15) oppose the local heat transfer rate. In the case of varying $\phi$, a rapid drop in the local rate of heat transfer is examined. On the other side, for convective, surface local rate of heat transfer increases. The increase in local rate of heat transfer is almost similar for both AA7072-H$_2$O and AA7075-H$_2$O nanofluids.
Validity of the analysis

For $M = 0$ and $\phi = 0$, the model reduced into the conventional flow model. Therefore, the comparison is made in the light of aforementioned conditions and compared the results presented in Vajravelu et al.\(^{33}\) and it is examined that the presented results are reliable (Table 2). Table 3 presents the values of $F''(0)$ against multiple values of the magnetic parameters and $\phi$.

| $M$ | $\phi$ | $F''(0)$   |
|-----|--------|------------|
| 0.5 | 0.2    | -1.40207   |
| 1.0 | 0.2    | -1.60641   |
| 1.5 | 0.2    | -1.93051   |
| 2.0 | 0.2    | -2.32679   |
| 2.5 | 0.2    | -2.76203   |
| 3.0 | 0.2    | -3.21871   |

Conclusion

A novel analysis on flow of water by considering aluminium alloys (AA7072 and AA7075) over a convectively heated surface is investigated. The major output of the analysis is

1. The parameter $M$ opposes the nanofluids velocity and $\phi$ favours the velocity of AA7072-H$_2$O and AA7075-H$_2$O nanofluids.
2. For AA7075-H$_2$O nanofluids, the velocity field decreases quite rapidly because of their low electrical conductivity.
3. The nanofluids temperature increases very rapidly for more convective surface and in the case of AA7075-H$_2$O nanofluids temperature increases slowly because of their low thermal and electrical conductivities as compared to that of AA7075-H$_2$O nanofluids.
4. The presence of resistive heating in the energy equations favours the temperature of under consideration nanofluids.
5. The volumetric fraction $\phi$ and Eckert number $Ec$ lead to increase in the temperature of both types of nanofluids.
6. The shear stresses decrease for stronger magnetic parameter and increasing $\phi$.
7. The local rate of heat transfer rises for convective stretching surface.

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Appendix I

Notation

- \( A, b \) constants
- \( B_i \) Biot’s number
- \( B_0 \) magnetic field (T)
- \( C_f \) skin friction coefficient
- \( Ec \) Eckert number
- \( F(\eta) \) nondimensional velocity
- \( k_f \) thermal conductivity of base fluid (W/m K)
- \( k_s \) thermal conductivity of alloy particles (W/m K)
- \( M \) Hartmann number
- \( Nu \) local Nusselt number
- \( Pr \) Prandtl number
- \( Re_s \) local Reynolds number
- \( T \) temperature (K)
- \( T_w \) temperature at the surface (K)
- \( T_a \) ambient temperature (K)
- \( u \) velocity component in \( x \)-direction (m/s)
- \( U \) linear stretching velocity (m/s)
- \( v \) velocity component in \( y \)-direction (m/s)
- \( x, y \) coordinate axes
- \( \beta(\eta) \) dimensionless temperature
- \( \eta \) similarity variable
- \( \mu_{nf} \) dynamic viscosity of nanofluids (Pa s)
- \( \rho_f \) density of the base fluid (kg/m\(^3\))
- \( \rho_s \) density of alloy particles (kg/m\(^3\))
- \( (\rho C_p)_nf \) heat capacitance (J/kg K)
- \( \sigma_f \) electrical conductivity of base fluid (S/m)
- \( \sigma_s \) electrical conductivity of alloys particles (S/m)
- \( \phi \) volume fraction of nanoparticles