1. Introduction

The thin strip casting of metals and alloys requires a very high rate of heat extraction, which results in very high thermal stresses in the solidifying material. These stresses arise due to thermal gradients in the material, metallostatic pressure of the unsolidified melt pool, and friction between roller and strip. These stresses can have a deleterious effect on the quality of the cast product. Stresses, tensile in nature can lead to the formation of a variety of surface and internal cracks. Hence, it is of great importance to understand the development and nature of thermal stresses and formation of defects in the cast product due to these stresses.

The modeling of stresses in thin strip casting processes has received very little attention of the researchers in past. However, some computational work on thermal stresses in the conventional continuous casting processes is available in the literature. Li and Ruan1) presented a transient coupled thermal, fluid flow and stress model to predict thermal stresses evolving in a solidifying ingot during the initial phase of continuous casting of aluminium. They used a hypoelastic-viscoplastic constitutive relation to describe the deformations in the solidifying material. Aboutalebi et al.2) developed a three-dimensional model to analyze heat transfer and solidification in continuous casting of arbitrary sections. From the temperature field predicted by heat flow model, they carried out a quasi-non-linear stress analysis within the solidifying shell of the beam blank section. Song et al.3) developed a coupled finite element model to simulate heat transfer, phase change, and stress build up in the material. They used a viscoplastic constitutive relation to predict the evolution of plastification. Zabaras et al.4) used a front tracking finite element method to calculate temperature and stress field in a solidifying pure metal. They employed a rate dependent viscoplastic-hypoelastic constitutive model to solve the equilibrium equations. Tszeng and Kobayashi5) calculated thermal stresses, using thermal-elasto-plastic finite element method, in a solidifying body. Kristiansson6) presented a coupled numerical model to analyze thermal and mechanical behavior of the solidifying shell within the mold during continuous casting of steel. He calculated temperatures, stresses, and strains in the shell to estimate the risk for formation of longitudinal cracks. Grill et al.7) calculated thermal profile in the material during the continuous casting process using finite difference method and calculated, using the temperature profile, stresses using finite element method. Sorimachi and Brimacombe8) used more reliable mechanical property data for steel to improve upon the work done by Grill et al.

In the present work, a two-dimensional finite element model is developed to calculate the stress field in the solidified metal during two-roll melt drag thin strip casting process. The model uses a visco-plastic constitutive relation to describe the behavior of solidifying steel. A temperature dependent ultimate strength is used to define the cracking index, which indicates the susceptibility of the material to crack. Stress calculations are performed using a commercial software ANSYS.

KEY WORDS: two-roll melt drag thin strip casting; thermo-mechanical model; thermal stresses; visco-plastic constitutive relationship; cracking index.

2. Geometry of the Model and Calculation Domain

Figure 1 shows the computational domain for the mathematical model. The computational domain included both rollers, melt pool and part of the solidified strip. The geom-


3. Mathematical Model

3.1. Assumptions

The mathematical model involves the following assumptions:

1) The material is assumed to be isotropic in nature.
2) The elastic strains are small relative to the plastic strains.
3) The plastic flow of material is assumed to be isochoric (i.e. volume preserving).
4) Rolling of solidified steel is not considered.
5) There are no stresses due to friction between roll and the cast strip.

3.2. Governing Equations

The mechanical equilibrium equation for the solidifying body can be written as:

\[ \sigma_{ij, j} + f_i = 0, \quad i, j = 1, 2, 3 \quad \text{summation on } i \]  \hspace{1cm} (1)

where, \( \sigma_{ij} \) are the Cartesian components of the Cauchy stress tensor in the body and \( f_i \) represents the body force acting on the system.

The total strain tensor, \( \varepsilon_{ij} \), can be decomposed in the following way:

\[ \varepsilon_{ij} = \varepsilon_{ij}^s + \varepsilon_{ij}^p + \varepsilon_{ij}^{th} \]  \hspace{1cm} (2)

where, \( \varepsilon_{ij}^s \) is the elastic, \( \varepsilon_{ij}^p \) is the plastic, and \( \varepsilon_{ij}^{th} \) is the thermal part respectively.

According to the constitutive law for isotropic linear elasticity, the stress is related to strains by:

\[ \sigma_{ij} = E_{ijkl}(T) \varepsilon_{kl}, \quad i, j, k, l = 1, 2, 3 \]  \hspace{1cm} (3)

where, \( E_{ijkl}(T) \) is the temperature dependent Young’s modulus of elasticity.

Table 1. Material parameters for viscoplasticity model for Fe–2%Si as given in Ref. 9.

| Material Parameter | Value     |
|--------------------|-----------|
| A                  | 6.346 x 10^9 sec^-1 |
| Q                  | 312.35 kJ/mole   |
| \( \xi \)           | 3.25             |
| \( s^* \)          | 125.1 Mpa        |
| n                  | 0.06869          |
| \( h_0 \)          | 3091.5 Mpa       |
| a                  | 1.5              |

Thermal strain vector is given by:

\[ \{ \varepsilon_{ij}^{th} \} = \Delta T \{ \alpha_x, \alpha_y, 0, 0, 0 \} \]  \hspace{1cm} (4)

where: \( \alpha_x \), thermal coefficient of expansion in the x direction

\( \Delta T : T - T_{ref} \)

\( T \) is the current temperature at the point and \( T_{ref} \) is the reference (strain-free) temperature at that point.

Inelastic deformation, \( \varepsilon_{ij}^p \), is calculated according to a rate dependent viscoplastic constitutive model. The following general form\(^9\) is assumed here,

\[ \varepsilon_{ij}^p = f_q(\sigma_{ij}, q^k_{ij}, T), \quad i, j = 1, 2, 3 \]  \hspace{1cm} (5)

where \( q^k_{ij} \) denote properly defined state variables for which evolution equations of the following form are given:

\[ q_{ij}^k = g^k(\sigma_{ij}, q^l_{ij}, T), \quad i, j = 1, 2, 3 \]  \hspace{1cm} (6)

Anand’s model\(^9\) for viscoplasticity is employed in the present work. There are two basic features in Anand’s model applicable to isotropic rate-dependent constitutive model for metals. First, there is no explicit yield surface, rather the instantaneous response of the material is dependent on its current state. Secondly, a single scalar internal variable ‘\( s \)’, called the deformation resistance, is used to represent the isotropic resistance to inelastic flow of the material. The specifics of this constitutive equation are the flow equation:

\[ d^p = A e^{-Q/R_B}[\sinh(\xi s)^{1/m}] \]  \hspace{1cm} (7)

and the evolution equation:

\[ s' = \{ h_j[B^l/B]/B \} d^p \]  \hspace{1cm} (8)

where,

\[ B = 1 - s/s^* \]  \hspace{1cm} (9)

with

\[ s^* = s^*[d^p/A] e^{(Q/R_B)\mu} \]  \hspace{1cm} (10)

where:

\( d^p = \) effective inelastic deformation rate

\( \sigma = \) effective cauchy stress

\( s = \) deformation resistance

\( s^* = \) saturation value of deformation resistance

\( s' = \) time derivative of deformation resistance

\( \theta = \) absolute temperature.

The numerical values of the parameters used in this model are given in Table 1.

3.3. Special Procedure for Liquid Regions

Since both liquid and solid regions are part of calculation domain, a special procedure is employed to handle the liquid region.\(^5\) A value of Poisson’s ratio very close to 0.5 is
assigned at the nodes where temperature is above the coherence (or zero-strain) temperature. This makes the liquid phase close to being incompressible for mechanical loading. The Young’s modulus is set to a very small number at the nodes above the coherence temperature.

3.4. Finite Element Formulation

The finite element discretization process yields a set of simultaneous equations:

\[ [K] [u] = [F]^a \] .............................(11)

where: 
- \([K]\) is the coefficient matrix
- \([u]\) is the displacement vector
- \([F]^a\) is the vector of applied loads.

In the present work, the Newton–Raphson iterative method is employed to solve this highly nonlinear problem. This method can be written as:

\[ [K]^T_i \{\Delta u_i\} = [F]^a - [F]^{u_m} \] .............................(12)

\[ \{u_i\}_{+1} = \{u_i\} + \{\Delta u_i\} \] .............................(13)

Where: 
- \([K]^T_i\) = Jacobian matrix (tangent matrix)
- \(i\) = subscript representing the current equilibrium iteration
- \([F]^{u_m}\) = vector of restoring loads corresponding to the element internal loads.

Both \([K]^T_i\) and \([F]^{u_m}\) are evaluated based on the values given by \([u_i]\).

The Newton–Raphson restoring force is given by

\[ \{F^{u_m}\} = \int_{vol} [B]^T \{\sigma\} d(vol) \] .............................(14)

where, 
- \([B]\) is the strain-displacement matrix
- \([\sigma]\) is the Cauchy stress.

Cauchy stress can be decomposed into deviatoric part and the pressure part.

\[ \{\sigma\} = \{\sigma'\} - (q)P \]

where, 
- \(\{\sigma'\}\) = the Cauchy stress deviator
- \(q\) = [1 1 1 0 0 0]
- \(P\) is the hydrostatic pressure = \(- (\sigma_x + \sigma_y + \sigma_z)/3.\)

The restoring force can now be written as:

\[ \{F^{u_m}\} = \int_{vol} [B]^T \{\sigma'\} d(vol) - \int_{vol} [B]^T \{q\} P d(vol) \] .............................(16)

The incompressibility constraint during plastic flow is enforced through the augmentation of the momentum equations with the additional equation:

\[ \int_{vol} [N^p]^T \{\Delta J - \Delta \dot{J}(\Delta P)\} d(vol) = 0 \] .............................(17)

where, \([N^p]\) is the shape function associated with the independently interpolated pressure degree of freedom.

\(\Delta J\) is the determinant of the relative deformation gradient (the relative volume change).

\(\Delta \dot{J}\) is the constitutively prescribed function expressing the pressure–volume change relationship

\[ \Delta \dot{J} = \exp \frac{-\Delta P}{G} \] .............................(18)

where, 
- \(G\) = elastic bulk modulus for the material
- \(E\) = Young’s modulus
- \(\nu\) = Poisson’s ratio.

The total Cauchy stress is calculated by finding the deviatoric part from the constitutive equations using the strains calculated from nodal displacements and subtracting the separately interpolated pressure, i.e.

\[ \{\sigma\} = \{\sigma'\} - (q)P_0 \] .............................(19)

Where, \(P_0\) is interpolated from the pressure field.

The stiffness matrix is constructed by evaluating the exact Jacobian of the discretized system. This yields an equation of the form:

\[ \begin{bmatrix} K^{uu} & K^{up} \\ K^{pu} & K^{pp} \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta P \end{bmatrix} = \begin{bmatrix} F^u \\ F^p \end{bmatrix} \] .............................(20)

where, \(\{F\}\) is external node forces.

\(\{\Delta u\}, \{\Delta P\}\) are the increments of displacement and pressure, respectively.

\[ \{F^u\} = \int_{vol} [B]^T \{\sigma\} d(vol) \] .............................(21)

\[ \{F^p\} = \int_{vol} [N^p]^T \{\Delta J - \Delta \dot{J}(\Delta P)\} d(vol) \] .............................(22)

\[ K^{uu} = \frac{\partial}{\partial u} \left[ \int_{vol} [B]^T \{\sigma\} d(vol) \right] \] .............................(23)

\[ K^{up} = \frac{\partial}{\partial P} \left[ \int_{vol} [B]^T \{\sigma\} d(vol) \right] \] .............................(24)

\[ K^{pp} = \frac{\partial}{\partial P} \left[ \int_{vol} [N^p]^T \{\Delta J - \Delta \dot{J}(\Delta P)\} d(vol) \right] \] .............................(25)

4. Numerical Procedure

The finite element mesh for the present simulation is shown in Fig. 2. The temperature profile obtained from the fluid flow and heat transfer calculations, which has been presented elsewhere, is applied as a thermal load to the mechanical system. The temperature contours obtained from the fluid flow and heat transfer calculations are shown in Fig. 3. The mean of the solidus and the liquidus temperature is specified as the reference (or stress free) temperature. A value of 0.499 for Poisson’s ratio and a value very close to zero for Young’s modulus at nodes where the temperature is higher than the reference temperature. The governing equations for stress and strain are solved at each node of the mesh using the discretization procedure described in the previous section.
Results and Discussion

The stress model can be used to evaluate conditions under which cracks may appear in the solidifying body. In this study, the cracking criterion proposed by Ramacciotti is employed, in which a temperature dependent ultimate strength is used as a reference for cracking criterion:

\[ \sigma_R = C(T_{sol} - T)^{0.5} \, \text{(MPa)} \] ..........................(26)

where, \( C = 1.2^{-0.5} \, \text{(MPa)} \, (K)^{-0.5} \).

A cracking index was defined based on the principal stress as \( C.I. = (\sigma / \sigma_R)_{max} \), where, \( \sigma_p \) is the principal stress.

The temperature dependence of Young’s modulus of steel is given by the following equation:

\[ Y = \frac{(T_{sol} - T)^2}{14.752} \, \text{(MPa)} \]

A positive value of cracking index at a point means that the material is subjected to tensile stresses at that point whereas a negative cracking index would mean that the material is subjected to compressive stresses. Cracks can appear at points with the value of C.I. greater than one, i.e. the points, which are subjected to tensile stresses more than the ultimate strength, while cracks will not appear at the points under the compressive stresses.

Thermal stresses arise primarily due to thermal gradients.
in the material. The thin strip casting process results in very high stresses in the material before the ‘kissing point’ (which is the point of least separation of two rolls) and relatively lower stresses in the exiting cast strip. It is understandable that before the kissing point liquid steel comes directly into contact with chilled roller surface, which provides rapid cooling resulting in a very high thermal gradient in the material. Due to this thermal gradient, material experiences different amount of shrinkage at different locations in the melt pool and hence large thermal stresses develop in the material. For the exiting cast strip, even though the cooling of material is more, the fact that the strip is very thin (2 mm in this case) and exposed to open atmosphere on both the surfaces results in less thermal gradient and hence comparatively low thermal stresses are generated.

The magnitude of these stresses depends on the process parameters such as casting speed and melt-roll heat transfer coefficient. To investigate the effect of process parameters on thermal stresses, cracking index is plotted at different locations of cast strip in two-roll melt drag method for various casting conditions.

For a given roll gap and melt-roll heat transfer coefficient, cracking index is plotted for three values of casting speed. Similarly, for a constant value of roll gap and casting speed, melt-roll heat transfer coefficient is varied to see its effect on the stress profile in the solidified metal. For a given roll gap and melt-roll heat transfer coefficient, cracking index is plotted for three values of casting speed. Similarly, for a constant value of roll gap and casting speed, melt-roll heat transfer coefficient is varied to see its effect on the stress profile in the solidified metal.

Figure 4 shows the variation in the cracking index across the width of the cast strip at the kissing point for three casting speeds. In the finite element mesh, there are 45 nodes along the width of the strip. Point 1 represents the node at the point of contact between the upper roll and the strip, similarly point 45 represents the node at the point of contact between the lower roll and the strip. It is evident from this figure that in the vicinity of the rolls, the material is most susceptible to crack for high casting speeds. At high casting speeds, high thermal gradients exist in the vicinity of the rolls because less time is provided for homogenization of temperature. These gradients result in high thermal stresses in the solidified shell. At low casting speeds, the stresses are low at the points near the rollers but at points away from the rolls the stresses are high. At points located away from the rolls, the material is more susceptible to crack for low casting speeds. This is because at low casting speeds, the thickness of solidified shell on the rolls is much higher than that for high casting speeds. This solidified shell reduces the rate of heat transfer from the melt pool to the rolls, which results in high thermal gradients and hence high thermal stresses in the material.

Figure 5 shows the variation in the cracking index across the width of the cast strip at the kissing point for different values of melt-roll heat transfer coefficients in two-roll melt drag thin strip casting of steel ($V_{roll}=0.30 \text{ m/s}, T_{sup}=60 \text{ K}, U_{in}=0.05 \text{ m/s}$).

Figure 6 shows the variation in the cracking index along the middle layer of the strip for different casting speeds in two-roll melt drag thin strip casting of steel ($h_{melt-roll}=11 \text{,000 W/m}^2\text{K}, T_{sup}=60 \text{ K}, U_{in}=0.05 \text{ m/s}$).
the middle layer of the cast strip for different casting speeds. Cracking index is plotted at 15 nodes along the middle layer of the strip. Points 1 to 3 are upstream and 5 to 15 are downstream of the kissing point. For points 1 to 3 no stresses are present in the material because the material is still in the liquid state at these points. For a casting speed of 0.45 m/sec, there are no stresses in the middle layer of the cast strip because the core of the strip is still liquid. As the casting speed decreases the cracking index increases because of increasing thermal load in the system.

Figure 7 shows the cracking index along the middle layer of the cast strip for different melt-roll heat transfer coefficients in two-roll melt drag thin strip casting of steel ($V_{\text{roll}}=0.30$ m/s, $T_{\text{exp}}=60$ K, $U_{\text{in}}=0.05$ m/s).

Figure 8 shows the variation in the cracking index along the surface of the cast strip in contact with the upper roll for three different values of melt-roll heat transfer coefficient. The cracking index is plotted at 16 points along the surface of the solidified shell. Points 1 to 5 are located on the nodes at metal-roll contact and 6 to 16 are along the upper surface of the cast strip. As the liquid metal comes into contact with chilled rolls very high stresses develop in the material. These stresses gradually decrease as the solidification progresses.

It is evident from Fig. 8 that the susceptibility of material to fail due to thermal stresses is highest at locations before the kissing point of two rolls and is lowest at locations after the kissing point. At locations before the kissing point, these stresses are higher for high melt-roll heat transfer coefficient and are lower for low melt-roll heat transfer coefficients.

6. Conclusions

The susceptibility of material to fail due to thermal stresses is maximum at locations before the kissing point of two rolls and is minimum at locations after the kissing point. Since cracking index is significantly less along the middle layer of the strip as compared to that at the upper and lower surfaces, the upper and lower surfaces of the cast strip are more prone to cracking than the inside of the strip. Very rapid cooling should be avoided as it results in very high thermal stresses in the solidified material. Less cooling rates should also be avoided as they may result in breakouts if the solidification is not over before the strip leaves the rolls.

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Nomenclature

\begin{itemize}
  \item $a$: Strain rate sensitivity of hardening or softening (dimensionless)
  \item $A$: Pre-exponential factor ($s^{-1}$)
  \item $B$: Strain-displacement matrix
  \item $d^p$: Effective inelastic deformation rate ($s^{-1}$)
  \item $E_{\text{mod}}(T)$: Temperature dependent modulus of elasticity (Pa)
  \item $\{F^a\}$: Vector of applied loads
  \item $\{F^m\}$: Vector of restoring loads corresponding to the element internal loads.
  \item $G$: Elastic bulk modulus for the material (Pa)
  \item $h_s$: Hardening/softening constant (Pa)
  \item $K$: Coefficient matrix
  \item $m$: Strain rate sensitivity of stress (dimensionless)
  \item $n$: Strain rate sensitivity of saturation value (dimensionless)
  \item $Q$: Activation energy (J/mol)
  \item $R$: Universal gas constant (J/mol-K)
  \item $s$: Deformation resistance (Pa)
\end{itemize}
\( s^* \): Saturation value of deformation resistance (Pa)
\( s' \): Time derivative of deformation resistance (Pa/s)
\( s_0 \): Initial value of deformation resistance (Pa)
\( s^\wedge \): Coefficient for deformation resistance saturation value (Pa)
\( T_{\text{sol}} \): Solidus temperature of the alloy (K)
\( T_{\text{sup}} \): Superheat in the molten metal (K)
\( U_{\text{in}} \): Inlet velocity of the metal (m/s)
\( \mathbf{u} \): Displacement vector
\( \alpha_x, \alpha_y \): Thermal coefficient of expansion in x and y direction (°C\(^{-1}\))
\( \mathbf{e}_{ij} \): Total strain tensor
\( \mathbf{e}_{ij}^E \): Elastic component of total strain
\( \mathbf{e}_{ij}^P \): Plastic component of total strain
\( \mathbf{e}_{ij}^T \): Thermal component of total strain
\( \mathbf{\sigma}_{ij} \): Stress tensor
\( \zeta \): Multiplier of stress (dimensionless)

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