Observer-Based Optimal Zero-Sliding Midcourse Guidance for Missiles with TVC and DCS

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This paper addresses observer-based optimal zero-sliding midcourse guidance for missiles with thrust vector control (TVC) and a divert control system (DCS) for the interception of a theater ballistic missile. First, a three degree-of-freedom observer-based optimal zero-sliding midcourse guidance law is designed to minimize the control effort and to narrow the distance between the missile and the target. Then, the exponential stability of the overall system is analyzed thoroughly via the Lyapunov stability theory. Finally, successful simulations are developed to verify the effectiveness of the proposed guidance law.

Key Words: Optimal Control, Zero Sliding, Observer Based, Target Tracking

Nomenclature

- $a$: acceleration vector
- $dp$: pitch angle of propellant
- $dy$: yaw angle of propellant
- $F$: thrust vector
- $g$: gravitational acceleration vector
- $l$: distance between nozzle and gravity center
- $L_b = \begin{bmatrix} -1 & 0 & 0 \end{bmatrix}^T$ displacement vector
- $m$: mass of missile
- $N$: magnitude of thrust
- $q$: quaternion
- $r$: position vector
- $\hat{r}$: estimate vector of $r$
- $\hat{\hat{r}}$: unit vector of $\hat{r}$
- $|\hat{r}|$: magnitude of $\hat{r}$
- $t$: present time
- $t_g = t - t_e$ time-to-go until intercept
- $T$: intercepting time
- $v$: velocity vector
- $\hat{v}$: estimate vector of $v$

Subscripts

- $b$: body coordinate frame
- $d$: desired
- $i$: inertial coordinate frame
- $M$: missile
- $p$: perpendicular to LOS
- $S$: divert control
- $T$: target/thrust

1. Introduction

During the missile midcourse guidance phase$^1$, which lasts from the end of the launch phase$^2$ to the beginning of the terminal phase$^3$, the target can be locked-on to by the missile’s on-board sensor. The midcourse task is to deliver the missile to some place near the target with some predetermined conditions, such as suitable final velocity or appropriate attitude. For an upper-tier defender such as the theater high altitude area defense (THAAD) system$^5$, the midcourse phase lasts for a long period of time, and therefore the variation of the missile mass during the travel period cannot be neglected to guarantee that the intercepting of the missile is achievable during its flight.

Based on the concept of the PN guidance law$^6$, constant bearing guidance is often used on for bank-to-turn missiles$^7$, whereas a different kind of guidance law, namely, the zero-sliding guidance law aims at eliminating the sliding velocity between the missile and the target in the direction normal to the line of sight (LOS)$^3$. Roddy et al.$^8$ apply the linear quadratic optimal control theory to the problem of bank-to-turn control in a CLOS guidance system so that the optimality of conventional control ideas is assessed. A CLOS guidance law is developed for head-on high-speed maneuvering targets using lateral acceleration demand and normalized missile acceleration.$^9$

To generate a linear parametric state-space model, an optimal linear-fractional-transformation-based robust stability analysis and control design is presented by Pfifer and Hecker.$^{10}$ Basin et al.$^{11}$ derive the optimal regulator for a linear system with time delay in control input using the duality principle and then the optimal regulator is proved using the maximum principle.

For filtering motion disturbances, Sadhu and Ghoshal$^{12}$ propose sight line rate estimation using a noninvasive seeker filter based on the disturbance observer concept to extract the target sight line rate signal from the raw signal. A disturbance observer is developed to estimate disturbances based on the robust control scheme by solving linear matrix inequalities based on Chen and Chen.$^{13}$ To grasp unknown

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states or disturbances, an observer has been applied to nonlinear networked control systems by Mao et al.\textsuperscript{14} to synchronous motors by Lee et al.\textsuperscript{15} and to nonlinear mechanical systems by Bonnabel.\textsuperscript{16}

It is quite often that quaternion representation has been adopted to describe the attitude of missiles,\textsuperscript{17,18} because it is recognized as a kind of global attitude representation. To deal with the non-ideal factors under attitude control and to strengthen the robust property of the controller, the sliding mode control has been employed for spacecrafts by Pukdeboon et al.\textsuperscript{19} for missiles by Zhi\textsuperscript{20} and for mechanical systems by Sankaranarayanan and Mahindrakar.\textsuperscript{21} All the above research works address the issue on attitude control, which is mainly used to achieve the goal of attitude tracking.

A missile equipped with thrust vector control (TVC) and a divert control system (DCS) can effectively control its acceleration direction\textsuperscript{3,22,23} even when the missile’s fins cannot be used. This in turn implies that the maneuverability/controllability of the missile can be greatly enhanced by TVC during the stage when the speed of the missile is slow and/or the air density around the missile is low. In addition to missile applications, there are also a number of other applications of TVC. For instance, Rahman and Whidborne\textsuperscript{24} improved the flying qualities for blended-wing-body aircraft while operating at low airspeeds with nominal static margins. Wie\textsuperscript{25} deals with a mathematical formulation and a practical solution of the TVC control design problem of solar-sail spacecraft, which maintains the proper orientation of a spacecraft to provide its desired thrust vector pointing/steering. There are also some other application instances in the areas of launch vehicles and the transportation industry.

In this paper, we investigate observer-based optimal zero-sliding midcourse guidance for missiles equipped with TVC and DCS so that the missile is able to reach some place near the target for the purpose of successful interception of an inbound target in the follow-up homing phase. A general analysis is then conducted to investigate the stability property of the overall midcourse system. Several numerical simulations have been provided to validate the excellent target tracking property.

2. Equations of Motion for Missiles

Consider the translational motions; the equations of motion for a missile can be described as

$$\mathbf{v}_M = \mathbf{a}_M + \mathbf{g}_M, \quad \mathbf{r}_M = \mathbf{v}_M, \quad (1)$$

where all the variables are defined in Nomenclature.

Assume that the vector $L_b$, defined as the relative displacement from the missile’s center of gravity to the center of the nozzle, satisfies $|L_b| = l$, as referred to in Fig. 1. Note that the force of the DCS, also depicted in Fig. 1, can be produced by the thrusts located around the center of gravity such that the extra translation force, over and above that produced by TVC, can be provided.

![Fig. 1. Missile with TVC and DCS actuators.](image)

For practical application, this paper will assume that the missile can only gain constant thrust force during the flight. Referring to Fig. 1, the force exerted on the missile can be expressed in the body coordinate frame as

$$\mathbf{F}_{Mb} = \mathbf{F}_{Tb} + \mathbf{F}_{Sh} = N \begin{bmatrix} \cos d_p \cos d_t & \cos d_p \sin d_t & 0 \\ \cos d_p \sin d_t & \sin d_p & 0 \\ 0 & 0 & F_{S_B} \end{bmatrix}. \quad (2)$$

Here, the rotation matrix $B_t$ denotes the transformation from the body coordinate frame to the inertial coordinate frame. Thus, the force exerted on the missile observed in the inertial coordinate system is

$$\mathbf{F}_{Mi} = B_t \mathbf{F}_{Mb}, \quad (3)$$

where

$$B_t = I_{3 \times 3} + 2 \langle \mathbf{q} \times \rangle \langle \mathbf{q} \times \rangle + q_4 \langle \mathbf{q} \times \rangle$$

$$= \begin{bmatrix} 1 - 2q_2^2 - 2q_3^2 & 2(q_1q_2 - q_2q_1) & 2(q_1q_3 + q_3q_1) \\ 2(q_1q_2 + q_2q_1) & 1 - 2q_1^2 - 2q_3^2 & 2(q_2q_3 - q_3q_2) \\ 2(q_1q_3 - q_3q_1) & 2(q_2q_3 + q_3q_2) & 1 - 2q_2^2 - 2q_1^2 \end{bmatrix},$$

(4)

whereas

$$\langle \mathbf{q} \times \rangle = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix}$$

is a skew-symmetric matrix. According to Euler’s rotation theory,\textsuperscript{17-19} for two arbitrary coordinate frames $B_1$ and $B_2$ with respective origins being coincident, there exist a unit vector $n$ and an angle $\phi$ such that when $B_1$ undergoes rotation of an angle $\phi$ with respect to $n$, $B_1$ will coincide with $B_2$. Quaternion, otherwise named as Euler parameters, can be defined as four parameters $\mathbf{q} = [q_1 \ q_2 \ q_3 \ q_4]^T = [\mathbf{q}^T \ q_4]^T$ involving $n$ and $\phi$, i.e.,

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = n \sin(\phi/2)$$

$$q_4 = \cos(\phi/2). \quad (5)$$

From Eqs. (1) to (3), the motion model of missiles can then be derived as

\[\text{Trans. Japan Soc. Aero. Space Sci. Vol. 56, No. 1}\]
\[ \dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u}, \quad \mathbf{x}(0) = \mathbf{x}_0, \quad \mathbf{r} = \mathbf{C}^T \mathbf{x}, \]

where

\[
\begin{align*}
\mathbf{A} &= \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}, \\
\mathbf{B} &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\
\end{bmatrix}, \\
\mathbf{C}^T &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\
\end{bmatrix}, \\
\mathbf{u} &= \mathbf{a}_M(t).
\]

Here we assume that \( \mathbf{u} \) is a piecewise continuous and bounded function of time, that the matrix \( \mathbf{A} \) is not a stable matrix, and that the initial state vector \( \mathbf{x}_0 \) is unknown, but that the state observation errors are required to be converged to zero. The following observer is used as

\[ \dot{\mathbf{\hat{x}}} = \mathbf{A} \mathbf{\hat{x}} + \mathbf{B} \mathbf{\hat{u}} + \mathbf{K} (\mathbf{r} - \dot{\mathbf{r}}), \quad \dot{\mathbf{\hat{x}}}(0) = \mathbf{\hat{x}}_0, \]

\[ \dot{\mathbf{r}} = \mathbf{C}^T \mathbf{\hat{x}}, \]

where the relative state estimate \( \mathbf{\hat{x}} = [\dot{\mathbf{r}} \quad \mathbf{\hat{u}}]^T \) and \( \mathbf{K} \) is a matrix to be chosen.

The state observation error \( \mathbf{\hat{x}} \) for Eqs. (8) and (9) satisfies

\[ \dot{\mathbf{\hat{x}}} = (\mathbf{A} - \mathbf{K} \mathbf{C}^T) \mathbf{\hat{x}} = \mathbf{\hat{A}} \mathbf{\hat{x}}, \quad \mathbf{\hat{x}}(0) = \mathbf{x}_0 - \mathbf{\hat{x}}_0, \]

\[ \dot{\mathbf{r}} = \mathbf{C}^T \mathbf{\hat{x}}. \]

We can choose

\[ \mathbf{K} \mathbf{C}^T = \mathbf{A} - \mathbf{\hat{A}}, \]

where \( \mathbf{K} \) can be chosen arbitrarily so that the rate of convergence of \( \mathbf{\hat{x}} \) may be designed to be satisfied to meet the observer requirements.

The optimal control theory\(^8,10,11,26\) is then adopted for design of the guidance law in the aforementioned interception problem, where our objective is to compute the necessary missile acceleration \( \mathbf{a}_M \) at the present time \( t \) in terms of estimates \( \dot{\mathbf{r}}(t) \) and \( \ddot{\mathbf{r}}(t) \) so that a minimum effort interception occurs at some terminal time \( T \geq t \). Therefore, the cost function is given by

\[ J = \frac{\gamma}{2} \mathbf{r}(T)^T \mathbf{r}(T) + \frac{1}{2} \int_{t_0}^{T} \mathbf{a}_M(t) \mathbf{a}_M(t) dt, \]

where \( T > 0 \) is an appropriate weighting and \( t_0 \) is some starting time.

Using the optimal control theory, the optimal acceleration command \( \mathbf{a}_M \) can be found using the state feedback method\(^{26} \) as

\[ \mathbf{a}_M(t) = \frac{3}{t_f^2} \left[ \dot{\mathbf{r}}(t) + t_f \ddot{\mathbf{r}}(t) \right], \]

where \( t_f \) denotes the time-to-go from the current time \( t \) to the intercepting time \( T \).

However, an observer-based optimal zero-sliding guidance law without exact estimation of time-to-go can be designed; that is, for the proposed midcourse guidance, the terminal time \( T \) can be arbitrarily chosen as long as \( T > t \) is guaranteed, where \( t \) is the present time. In this paper, the terminal time \( T \) is determined as the initial condition for the subsequent terminal phase guidance and control to guarantee that the target missile can be locked-on to by its own sensor of the interception missile, and its task is to deliver the missile to some place near the target. The observer-based optimal zero-sliding midcourse guidance law can still force the interception system to be exponentially stable. This is based on the component of the relative velocity normal to the LOS, i.e., \( \dot{\mathbf{\hat{v}}} = \dot{\mathbf{v}} - (\mathbf{v}^T \mathbf{\hat{r}}) \mathbf{\hat{r}} \) (see Fig. 2). To proceed, we first derive the estimate equation of the relative motion perpendicular to the LOS as

\[ \dot{\mathbf{\hat{v}}} = -\mathbf{a}_M - \frac{d}{dt} (\mathbf{v}^T \mathbf{\hat{r}}) \mathbf{\hat{r}} - (\mathbf{v}^T \mathbf{\hat{r}}) \mathbf{\hat{r}}, \]

\[ = - [\mathbf{a}_M - (\mathbf{a}_M^T \mathbf{\hat{r}}) \mathbf{\hat{r}}] - \frac{1}{|\mathbf{\hat{r}}|} |\mathbf{\hat{v}}| |\mathbf{\hat{r}}| \mathbf{\hat{r}} - \mathbf{v}^T \mathbf{\hat{r}} \mathbf{\hat{v}}. \]

where \( \mathbf{a}_p = \mathbf{a}_M - (\mathbf{a}_M^T \mathbf{\hat{r}}) \mathbf{\hat{r}} \) denotes the missile’s acceleration perpendicular to the LOS. Then, for employing the optimal guidance principle, the perpendicular acceleration component \( \mathbf{a}_p \) is set as

![Fig. 2. Relative motion along the LOS.](image-url)
\[ a_p = -u - \frac{\hat{\mathbf{a}}^T \mathbf{r}}{|\mathbf{r}|} \hat{\mathbf{q}}, \]  
(15)

which leads to the equation of the normal component of the relative motion as

\[ \frac{\dot{\mathbf{q}}}{|\mathbf{r}|} = u - \frac{1}{|\mathbf{r}|} \hat{\mathbf{q}}^T \mathbf{r}, \]

where \( u \) is calculated by minimizing the quadratic cost function defined as

\[ J = \frac{1}{2} \gamma(T) \hat{\mathbf{q}}^T(T) \hat{\mathbf{q}}(T) \]

\[ + \frac{1}{2} \int_{t_0}^{T} \left( \alpha \hat{\mathbf{q}}^T(t) \hat{\mathbf{q}}(t) + \rho u^T(t) u(t) \right) dt, \]

where \( \gamma(T) \geq 0, \alpha \geq 0, \rho > 0, \) and \([t_0, T] \) is the time interval. Using the optimal control theory, the Riccati equation \( 26 \) can be derived as

\[ \gamma(t) = \frac{\gamma(t)}{\rho} - \sigma, \]

(17)

where \( \gamma(t) \) is the solution subject to the final condition \( \gamma(T) \). Using separation of variables and setting \( \gamma(T) \) as a very large value, \( \gamma(t) \) via backward integration of the Riccati equation can be computed as

\[ \gamma(t) = \sqrt{\gamma(0)} \left( 1 + \frac{2}{e^{2\sqrt{\gamma(0)}(T-t)}} - 1 \right). \]

(18)

(see Appendix) which leads to the optimal control \( u \) as

\[ u(t) = \left[ \frac{\gamma(0)}{\rho} \left( 1 + \frac{2}{e^{2\sqrt{\gamma(0)}(T-t)}} - 1 \right) \right] \hat{\mathbf{q}}. \]

(19)

Thus, the acceleration component perpendicular to the LOS is derived as

\[ a_p(t) = \left[ \frac{\gamma(0)}{\rho} \left( 1 + \frac{2}{e^{2\sqrt{\gamma(0)}(T-t)}} - 1 \right) \right] \hat{\mathbf{q}}^T \mathbf{r}^T. \]

(20)

so that the equation of relative motion in Eq. (14) becomes

\[ \dot{\hat{\mathbf{q}}} = \left[ \frac{\gamma(0)}{\rho} \left( 1 + \frac{2}{e^{2\sqrt{\gamma(0)}(T-t)}} - 1 \right) \right] \hat{\mathbf{q}} - \frac{1}{|\mathbf{r}|} \hat{\mathbf{q}}^T \mathbf{r}. \]

(21)

From Eq. (20), which is well defined unless \( \mathbf{r} = 0 \), generally, \( |\mathbf{r}| \neq 0 \) is always true throughout the whole midcourse phase, but not in the terminal phase. To verify that the observer-based optimal zero-sliding midcourse guidance law will cause the system to be exponentially stable, Theorem 1 is proposed to reach the verification goal.

**Theorem 1:** Let the estimate equation of relative motion perpendicular to the LOS be given by Eq. (14) and the observer-based optimal zero-sliding guidance law be given by Eq. (20). If \( \hat{\mathbf{q}} \) has no component on the normal direction of the LOS and if \( \hat{\mathbf{q}} \mathbf{r} \leq \alpha < 0 \) with \( \hat{\mathbf{q}} \) being bounded away from zero, then the observer-based optimal zero-sliding midcourse guidance system will ensure target tracking.

**Proof:** Taking

\[ V_G = \frac{m}{2} \hat{\mathbf{q}}^T \hat{\mathbf{q}}, \]

as a Lyapunov function, we obtain

\[ \frac{m}{4} \hat{\mathbf{q}}^T \mathbf{r} \leq V_G \leq m \hat{\mathbf{q}}^T \hat{\mathbf{q}}. \]

(22)

Hence, \( V_G \) is positive definite, decrement and radially unbounded. The time derivative of \( V_G \) is given by

\[ V_G = -m \hat{\mathbf{q}}^T \left[ \frac{\gamma(0)}{\rho} \left( 1 + \frac{2}{e^{2\sqrt{\gamma(0)}(T-t)}} - 1 \right) \right] \hat{\mathbf{q}} + \frac{m}{2} \hat{\mathbf{q}}^T \hat{\mathbf{q}} \hat{\mathbf{q}} + \frac{m}{2} \hat{\mathbf{q}}^T \hat{\mathbf{q}} \hat{\mathbf{q}} \]

\[ \leq -m \frac{\gamma(0)}{\rho} \hat{\mathbf{q}}^T \hat{\mathbf{q}} \hat{\mathbf{q}}, \]

(23)

where \( \hat{\mathbf{q}} \mathbf{r} = 0 \) and \( \sqrt{\gamma(0)}/\rho \) is a positive constant and \( \mathbf{m} \) is negative. Thus, \( V_G \) is apparently negative definite, and using the Lyapunov stability theory, we can conclude that the origin of \( \hat{\mathbf{q}} \) is globally exponentially stable.

Accordingly, to verify that the intercepting missile will gradually approach the target, we take \( V_G = (1/2)\hat{\mathbf{q}}^T \mathbf{r} \) as another Lyapunov function. Thus, we have \( V_G \) as positive definite, decrement and radially bounded, and the time derivative as

\[ \dot{V}_G = \hat{\mathbf{q}}^T \left( \hat{\mathbf{q}} - \hat{\mathbf{q}} \right) \hat{\mathbf{q}} = \hat{\mathbf{q}}^T \mathbf{r} \leq \alpha < 0, \]

(24)

where \( \hat{\mathbf{q}} = \hat{\mathbf{q}} - \hat{\mathbf{q}} \), so that \( \dot{V}_G \) is negative definite. Therefore, via the Lyapunov stability theory and constant bearing condition, \( 3 \) the observer-based optimal zero-sliding guidance will cause the origin of the missile interceptions system to be globally exponentially stable.

**Remark 1:** To define the zero-sliding guidance law, if the relative velocity components perpendicular to the LOS of the relative velocity from the missile to the target are zero; that is, the direction of \( \mathbf{r} \) remains fixed thus fulfilling the constant bearing condition, \( 3 \) and then the approaching velocity between the missile and the target is always on the LOS. Therefore the zero-sliding guidance condition is satisfied such that the missile will hit the target eventually.

Given that \( \hat{\mathbf{q}} \mathbf{r} \leq \alpha < 0 \), we are now ready to prove the above claim as follows.

**Remark 2:** The relative velocity between the target and the missile is depicted in Fig. 3, where \( \hat{\mathbf{u}}, \hat{\mathbf{i}} \), and \( \hat{\mathbf{n}} \) are the present relative velocity between the missile and the target, the component of \( \hat{\mathbf{q}} \) perpendicular to the LOS and the component of \( \hat{\mathbf{q}} \) in the LOS direction, respectively. It is assume that the thrust of the missile is large enough during the midcourse phase to overcome the results of the aerodynamic effect, the gravity and the wind forces such that the magnitude of the relative velocity, \( |\hat{\mathbf{q}}| \), will be a nondecreasing function with respect to time. By defining the angle, \( \theta \), between \( \hat{\mathbf{u}} \) and \( \hat{\mathbf{n}} \), as
we can conclude that $|\hat{\theta}_p|$ will decay exponentially referring to Eqs. (22) and (23), and $|\hat{\theta}_r| = |\theta - \hat{\theta}_p| \geq |\theta| - |\hat{\theta}_p|$ due to $\theta = \hat{\theta}_p + \theta_r$ referring to Fig. 3. Therefore, $|\hat{\theta}_r|$ will be an increasing function with respect to time, which in turn implies that the angle $\theta$ will be monotonically decreasing as time goes on. Hence, we can conclude that $\hat{\theta}^T \hat{r} < 0$ for all $t \geq 0$ since the starting condition $\hat{\theta}^T(t_0) \hat{r}(t_0) < 0$ in the midcourse phase can be guaranteed, meaning that the condition justifies our assumption in Theorem 1. Therefore, the target tracking objective can be achieved as claimed by the aforementioned theorem.

4. Simulations

To validate the proposed observer-based optimal zero-sliding guidance law for missiles, we provide thorough and convincing computer simulations in this section. We assume the target is launched from a distance of 600 km. The missile has a sampling period of 10 ms. The bandwidth of the TVC is 20 Hz and the two angular displacements are both limited to 5°. For simulation, the variation of missile’s mass is $\dot{m} = -1$ kg/s for the initial mass $m = 600$ kg. Some positive definite values of $\rho = \sigma = 1$ for minimizing the quadratic cost function (see Eq. (16)), and the magnitude of the total thrust $N = 30,000$ N for the constant magnitude of the thrust (see Eq. (2)) are given.

In the simulation scenario (Table 1), the feasibility of the presented approach is satisfactorily demonstrated by the 3D engagement, and by the locations of the missile and of the target, in Figs. 4 and 5, respectively. The total simulation flight time is 93 s with 0.01 s increment time. The relative velocity perpendicular to the LOS is shown in Fig. 6. Figure 7 reveals that the pitch angle and the DCS control inputs of the missile, where the larger forces are found during the flight time of the missile with larger curvature. For observer design, the rate convergence of the observer is predetermined to ensure that the state observer errors converge to zero as time approaches to infinity. The relative distance and relative velocity perpendicular to the LOS between the missile and the target for the entire interception system are narrowed and shown in Fig. 8. The current and desired quaternions of the missile can be computed using Eq. (5). The displacements are computed respectively from the inertial coordinate frame to the current attitude $X_b$ and the LOS direction $X_d$ and are shown in Fig. 9.

![Fig. 3. Relative velocity between target and missile.](image3.png)

![Fig. 4. 3D engagement of missile and target.](image4.png)

![Fig. 5. Locations of missile and target.](image5.png)

![Fig. 6. Relative velocity perpendicular to LOS.](image6.png)

![Fig. 7. Pitch angle and DCS control inputs of the missile.](image7.png)

![Fig. 8. Relative distance and relative velocity perpendicular to the LOS.](image8.png)

![Fig. 9. Current and desired quaternions of the missile.](image9.png)
5. Conclusions

This paper focused on the midcourse-phase intercept system, which is a period of time lasting until the missile is close enough to the target such that the sensor equipped on the missile can lock-on to the target. Considering the properties of the TVC, DCS and unknown states during the midcourse phase, we employed the optimal zero-sliding guidance law with observer, where the time-to-go of the missile does not have to be estimated. We proved the stability of the observer-based optimal zero-sliding midcourse guidance via the Lyapunov stability theory. Realistic simulation has been developed to verify the feasibility of the observer-based optimal zero-sliding midcourse guidance with TVC and DCS. The results were quite satisfactory and encouraging.

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**Appendix**

From Eq. (17) via using separation of variables, we obtain

\[
\int_{y(t)}^{y(T)} \frac{dy}{y^2 - \sigma} = \int_t^T \frac{1}{\rho} \, dt.
\]

\[
\Rightarrow \frac{1}{2} \sigma \left( \sqrt{y - \sqrt{\rho \sigma}} - \sqrt{y + \sqrt{\rho \sigma}} \right) + \frac{1}{\rho} \left( y^2 - \sigma \right) \left( y^2 - \sigma \right) = e^{2\sqrt{\rho \sigma}} - 1.
\]

If we want to ensure that the optimal control drives the component of the terminal estimate velocity perpendicular to LOS, \(v_p(T)\), exactly to zero, we can let \(y(T) \to \infty\) to weight \(v_p(T)\) more heavily in Eq. (14). Under this limit, we obtain

\[
y(t) = \sqrt{\rho \sigma} \left( 1 + \frac{2}{e^{2\sqrt{\rho \sigma}(T-t)} - 1} \right).
\]