3-dimensional holographic trace anomaly from AdS/CFT correspondence

Chong Oh Lee  
Department of Physics,  
Chonbuk National University, Jeonju 561-756, Republic of Korea  
cohlee@chonbuk.ac.kr

Chaiho Rim  
Department of Physics and Research Institute of Physics and Chemistry,  
Chonbuk National University, Jeonju 561-756, Republic of Korea  
rim@chonbuk.ac.kr

Abstract

We explicitly obtain energy-momentum tensor at the asymptotic 3-dimensional region of Schwarzschild AdS$_4$ and Taub-NUT-(A)dS$_4$ using the so-called 'counter-term subtraction method' in Fefferman-Graham coordinate. The energy momentum tensor is presented in a closed form for the AdS$_4$ and for the special case of Taub-NUT-dS and in an asymptotic series for other cases. The result suggests that in light of AdS/CFT correspondence, the 3-dimensional trace anomaly can be expressed in terms of the 3-dimensional volume and Ricci scalar.
1 Introduction

The AdS/CFT correspondence conjecture \[1, 2, 3, 4, 5\] asserts there is an equivalence between a gravitational theory in the bulk and a conformal field theory in the boundary. The correspondence is shown to hold for AdS \[6\], S-AdS \[7, 8, 9\], Taub-NUT-AdS \[10, 11, 12\] and for the generalized version of Taub-NUT-AdS \[13, 14\] whose metric is suggested by \[15\].

The AdS action is given as

\[ A = A_B + A_{\partial B} + A_{ct}, \]

where \( A_B \) is the bulk action \( A \) in \( d + 1 \)-dimensional Manifold \( \mathcal{M} \) and \( A_{\partial B} \) is Gibbons-Hawking action,

\[
A_B = \frac{1}{2k^2} \int_{\mathcal{M}} d^{d+1}x \sqrt{-G} (\mathcal{R} - 2\Lambda),
\]

\[
A_{\partial B} = -\frac{1}{k^2} \int_{\partial \mathcal{M}} d^d x \sqrt{-\gamma} \Theta.
\]

(1.2)

Here \( k^2 = 8\pi G_{d+1} \) with \( d+1 \)-dimensional gravitational constant \( G_{d+1} \), \( \Lambda = -d(d-1)/2l^2 \) is a negative cosmological constant and \( \Theta \) is the trace of extrinsic curvature. The boundary action \( A_{\partial B} \) is added to make equations of motion well behaved at the boundary and boundary energy-momentum (BEM) tensor is expressed by \[16\]

\[
\frac{2}{\sqrt{-\gamma}} \frac{\delta A_{\partial B}}{\delta \gamma^{ab}} = \Theta_{ab} - \gamma_{ab} \Theta.
\]

(1.3)

The counter-term action \( A_{ct} \) is added to remove the infrared divergence which appears when the boundary goes to infinity. The first few terms were explicitly evaluated in \[7, 8\] and the universality of the divergent structure determines the counter-term action \( A_{ct} \) in arbitrary dimensions \[9\]

\[
A_{ct} = -\frac{1}{k^2} \int_{\partial \mathcal{M}} d^d x \sqrt{-\gamma} \left\{ \frac{d - 1}{l} - \frac{lR}{2(d - 2)} \mathcal{F}(d - 3) - \frac{l^3}{2(d - 2)^2(d - 4)} \left( R_{ab} R^{ab} - \frac{d}{4(d - 1)} R^2 \right) \mathcal{F}(d - 5) \right.
\]

\[
+ \frac{l^5}{(d - 2)^3(d - 4)(d - 6)} \left( \frac{3d + 2}{4(d - 1)} R R_{ab} R^{ab} - \frac{d(d + 2)}{16(d - 1)^2} R^3 \right) \mathcal{F}(d - 7) + \cdots \right\},
\]

(1.4)

where \( \mathcal{F}(d) \) is a step function, 1 when \( d \geq 0 \), 0 otherwise.
The counter-term action $A_{ct}$ is best understood in Fefferman-Graham (FG) coordinate. The AdS metric near the boundary is given as
\[ ds^2 = G_{\mu\nu} dx^\mu dx^\nu = \frac{l^2}{z^2} dz^2 + \frac{l^2}{z^2} g_{ab}(x,z) dx^a dx^b, \]
where the $r = \infty$ is put to $z = 0$. The Greek indices $\mu$ and $\nu$ refer to volume coordinates, $1, \cdots, d + 1$ and the roman $a$ and $b$ to boundary coordinates, $1, \cdots, d$ and $\gamma_{ab}$ in (1.3) is given as $\frac{l^2}{z^2} g_{ab}$. The Einstein equation for the bulk metric has two different types of asymptotic forms for most of models (see counterexample in [17]); when the boundary dimensions $d$ is odd
\[ g_{ab}(x,z) = \sum_{p=0}^{(d-1)/2} g_{ab}^{(2p)}(x) z^{2p} + g_{ab}^{(d)}(x) z^d + \mathcal{O}(z^{d+2}), \]
and when $d$ is even
\[ g_{ab}(x,z) = \sum_{p=0}^{d/2} g_{ab}^{(2p)}(x) z^{2p} + h_{ab}^{(d)}(x) z^d \ln z^2 + \mathcal{O}(z^{d+2}), \]
where $g_{ab}^{(j)}(x)$ and $h_{ab}^{(j)}(x)$ contain $j$ number of derivatives with respect to $x^a$. This shows that the odd powers of $z$ do not appear in $g_{ab}(x,z)$ up to the order of $z^d$. Explicit calculation has been done for AdS$_5$ in [18, 19]. This also fits for Schwarzschild AdS (S-AdS$_4$) and Taub-NUT-(A)dS$_4$ as shown in the next sections.

Dimensional analysis of the extrinsic curvature allows the terms of order $p < d/2$ only in (1.6) and (1.7) to the divergent part of the action [7]. Therefore, the divergent terms of the BEM tensor is given as
\[ \tilde{T}_{ab} = \sum_{p=0}^{[(d-1)/2]} \tilde{T}_{ab}^{(p)}, \]
where $[x]$ is the Gauss number (greatest integer less than or equal to $x$). $\tilde{T}_{ab}^{(p)}$ has the power of $l^{2p-1}$ and is determined through the Gauss-Codazzi equations [9, 20],
\[ \frac{1}{d-1} \tilde{T}^2 - \tilde{T}_{ab} \tilde{T}^{ab} = \frac{d(d-1)}{l^2} + R. \]
Noting that $\tilde{T}_{ab}$ is derived from the counter-term action
\[ \tilde{T}_{ab} = \frac{2}{\sqrt{-\gamma}} \delta A_{ct} \delta \gamma_{ab} = \frac{2}{\sqrt{-\gamma}} \delta \int d^d x \sqrt{-\gamma} L_{ct}, \]
and the counter-term action should be invariant under the local Weyl variations $\delta_W \gamma_{ab} = \sigma \gamma_{ab}$ according to the AdS/CFT conjecture, one has [9]

$$(d - 2p)L_{ct}^{(p)} = T_{ab}^{(p)} \gamma_{ab}. \quad (1.11)$$

Thus the full BEM tensor $T_{ab}$ is given by

$$T_{ab} = \frac{2}{\sqrt{-\gamma}} \frac{\delta(A_{\theta B} + A_{ct})}{\delta \gamma_{ab}} = \frac{1}{8 \pi G_{d+1}} \left[ \Theta_{ab} - \Theta_{\gamma_{ab}} \right] + \sum_{p=0}^{[(d-1)/2]} \tilde{T}_{ab}^{(p)}, \quad (1.12)$$

where a few terms of $\tilde{T}_{ab}^{(p)}$'s are explicitly given by

$$\tilde{T}_{ab}^{(0)} = - \frac{d-1}{l} \gamma_{ab},$$

$$\tilde{T}_{ab}^{(1)} = \frac{l}{d-2} \left( R_{ab} - \frac{1}{2} \gamma_{ab} R \right) \mathcal{F}(d-3),$$

$$\tilde{T}_{ab}^{(2)} = \frac{l^3}{(d-2)^2(d-4)} \left\{ -\frac{1}{2} \gamma_{ab} \left( R_{cd} R_{cd} - \frac{d}{4(d-1)} R^2 \right) - \frac{d}{2(d-1)} RR_{ab} ight. + \frac{d}{2(d-1)} \nabla_a \nabla_b R + \square R_{ab} - \frac{1}{2} \frac{d}{(d-1)} \gamma_{ab} \square R \right\} \mathcal{F}(d-5).$$

(1.13)

To understand the role of the BEM tensor, let us introduce a UV cut-off for the boundary theory with $z = \mu \neq 0$. The ultraviolet effect, on the other hand, corresponds to the infrared effect in AdS space due to the so-called `UV/IR connection' [21]: $r(\mu)$ plays the role of IR regulator in the dual bulk theory. Thus, in light of AdS/CFT correspondence, the renormalized AdS action will lead to the vacuum expectation value (VEV) of the BEM. If one rescales the BEM tensor $t_{ab}$ as

$$t_{ab} \equiv \frac{2}{\sqrt{-g}} \frac{\delta(A_{\theta B} + A_{ct})}{\delta g^{ab}} = \left( \frac{l}{z} \right)^{d-2} T_{ab}, \quad (1.14)$$

the vacuum expectation value at the boundary ($z = 0$) is given as [22, 23].

$$t_{ab} = \begin{cases} \frac{d^{d-1}}{16 \pi G_{d+1}} g_{ab}^{(d)} , & \text{when } d = \text{odd}, \\ \frac{d^{d-1}}{16 \pi G_{d+1}} \left( g_{ab}^{(d)} + X_{ab}[g^{(n)}] \right) , & \text{when } d = \text{even}. \end{cases} \quad (1.15)$$

It is noted that at $z = 0$, $\sum_{a,b} t_{ab} g^{(0)ab} = 0$. Hence, the odd dimensional boundary is conformal. In even dimensional boundary, $X_{ab}[g^{(n)}]$ is a function of $g^{(n)}$ with $n < d$ and reflects the conformal anomalies at the boundary.
In the presence of regulator $z = \mu \neq 0$, the BEM tensor may hint at the effective theory at given scale $\mu$, which may originate from high-energy effect. In this paper, we are going to investigate the counter-term action for 4-dimensional AdS, S-AdS and Taub-NUT-(A)dS explicitly and find BEM tensor at 3-dimensional regulated boundary. To do this, we employ the FG coordinates system and the so-called ‘counter-term subtraction method’. Even though, 3-dimensional trace anomaly vanishes at $z = 0$, the quantity does not vanish at $z \neq 0$ and is expressed in terms of geometric quantity, the boundary volume and the Ricci scalar. Explicitly, the trace can be written as

$$\sum t_{ab} g^{ab} = \sum_{i=0}^{\infty} \left( \alpha_i R^{i-1} \sqrt{\frac{g_0}{\gamma}} + \beta_i R^{2i+1} \right),$$  \hspace{1cm} (1.16)$$

where $g_0$ is the determinant of the metric at $z = 0$, and $\alpha_i$ and $\beta_i$ are certain constants depending on the manifold. The S-AdS$_4$ on the Poincare patch is the exception, whose the trace anomaly is of the form

$$\sum t_{ab} g^{ab} = \frac{\alpha_0}{\sqrt{-\gamma}}.$$  \hspace{1cm} (1.17)$$

It seems that this special case is due to the flat boundary metric at any $z$.

This paper is organized as follows: S-AdS$_4$ is considered in section 2 and Taub-NUT-(A)dS is considered in section 3. BEM tensor is explicitly presented in FG coordinates and its trace anomaly is shown to reduce to the form (1.16) and (1.17). The BEM tensor is presented in a closed-form for the S-AdS$_4$ on a Poincare patch and for Taub-NUT-dS$_4$ when cosmological constant satisfies so-called ‘massless-NUT condition’ (see section 3.3 for the definition). For other types of manifolds the BEM tensor is presented in a asymptotic series expansion of $z$. Section 4 is the conclusion.

2 S-AdS$_4$

In this section, we construct the closed form of the transformation law from $r$ to $z$ in FG coordinate using S-AdS$_4$ on the Poincare patch to find the closed-form of the BEM tensor. The trace anomaly is given exactly as (1.17). S-AdS$_4$ in the Global coordinate are also considered. Even though, closed form of the BEM is not presented, one can confirm that the trace anomaly satisfies (1.16).
2.1 S-AdS\textsubscript{4} on the Poincare patch

The metric of S-AdS\textsubscript{4} in the Poincare coordinate is given as

\[ ds^2 = -f_P(r)dt^2 + \frac{dr^2}{f_P(r)} + \frac{r^2}{l^2} \left( dx_1^2 + dx_2^2 \right), \]  \hspace{1cm} (2.1)

where \( f_P(r) \) is given as

\[ f_P(r) = \frac{r^2}{l^2} - \frac{m}{r}, \]  \hspace{1cm} (2.2)

and \( m \) is a geometric mass. The asymptotic form of BEM tensor in (1.1 2) has the form as \( r \to \infty \),

\[ 8\pi G_4 T_{tt} = \frac{m}{lr} + \frac{ml}{2r^3} + \mathcal{O}(\frac{1}{r^4}), \]

\[ 8\pi G_4 T_{ii} = \frac{m}{2lr} - \frac{3ml^3}{4r^3} + \mathcal{O}(\frac{1}{r^4}). \]  \hspace{1cm} (2.3)

A conserved mass is given by the integration over a \( d-1 \)-surface \( \Sigma \) \textsuperscript{[16]}:

\[ M(r) = \int_{\Sigma} d^{d-1}x \sqrt{\gamma} T_{ab} u^a \xi^b N_{\text{lapse}}, \]  \hspace{1cm} (2.4)

where the timelike unit normal vector \( u^a \), the timelike Killing vector \( \xi^a \), and the square root of the lapse function \( N_{\text{lapse}} \) are respectively

\[ u^a = \left( \frac{1}{\sqrt{f(r)}}, 0, \cdots \right), \quad \xi^a = (1, 0, \cdots), \quad N_{\text{lapse}} = \frac{1}{\sqrt{f(r)}}. \]  \hspace{1cm} (2.5)

where \( f(r) \) denotes the metric function in black hole solutions in (1.2). At \( r = \infty \), the finite piece of (2.4) becomes the ADM mass \( M \) \textsuperscript{[7]}

\[ M = \frac{mV_2}{8\pi G_4 l^2}. \]  \hspace{1cm} (2.6)

One may put the system in FG coordinate and investigate the asymptotic behavior systematically. The transformation is given in terms of the black hole line element

\[ \int \frac{dr}{\sqrt{f_P(r)}} = -l \int \frac{dz}{z}, \]  \hspace{1cm} (2.7)

whose solution is given as

\[ r(z) = \frac{l(1 + \bar{m}c_4^3 z^3) \frac{2}{3}}{2 \bar{m}c_4^2}, \]  \hspace{1cm} (2.8)
where $\bar{m} = m/l$. $c_4$ is a dimensionless constant due to the scaling degree of freedom in FG coordinate and may be fixed $c_4 = 1$. Thus, the metric in FG coordinates is written as

$$ds^2 = -A_P(z)dt^2 + B_P(z) \left( dx_1^2 + dx_2^2 \right) + \frac{l^2}{z^2}dz^2,$$

$$A_P(z) = \frac{(-1 + \bar{m}z^3)^2}{2\bar{m}^2(1 + \bar{m}z^3)^{\frac{3}{2}}}, \quad B_P(z) = \frac{(1 + \bar{m}z^3)^{\frac{1}{2}}}{2\bar{m}^2}.$$  \hspace{1cm} (2.9)

From this, one finds the BEM tensor using the relation (1.14),

$$8\pi G_4 t_{tt} = \frac{2\bar{m}^2l^2z^3}{(1 + \bar{m}z^3)^{\frac{3}{2}}}, \hspace{1cm} (2.11)$$

$$8\pi G_4 t_{ii} = -\frac{\bar{m}(1 + \bar{m}z^3)^{\frac{1}{2}}(1 + 2\bar{m}z^3)}{\sqrt{2}(-1 + \bar{m}z^3)}. \hspace{1cm} (2.12)$$

As a check, one may integrate (2.4) using the explicit form of (2.11) in FG coordinate to find the ADM mass in (2.6).

The BEM tensor in (2.11) results in the trace

$$\sum t_{ab}g^{ab} = \frac{12\bar{m}^2l^2z^3}{8\pi G_4(1 - \bar{m}^2z^6)} = \frac{l^2}{\sqrt{A_PB_P^2}} \left( \frac{\bar{m}^2}{8\pi G_4} \right). \hspace{1cm} (2.13)$$

This shows that the trace anomaly is inversely proportional to the boundary volume as claimed in (1.17) where $\alpha_0 = \bar{m}^2/8\pi G_4$ and other coefficients vanishes. The expression (2.13) is also checked using the relation with the extrinsic curvature (1.12, 1.13)

$$\sum t_{ab}g^{ab} = \frac{l^3}{8\pi G_4z^3} \left( -2\Theta - \frac{6}{l} \right), \hspace{1cm} (2.14)$$

and the explicit expression of $\Theta$ in FG coordinates.

### 2.2 AdS$_4$ on the Global patch

Since AdS$_4$ in the Poincare coordinate is trivial, we consider only AdS$_4$ on the Global patch

$$ds^2 = - \left( 1 + \frac{r^2}{l^2} \right)dt^2 + \frac{dr^2}{1 + \frac{r^2}{l^2}} + r^2d\Omega_2,$$

where $d\Omega_2$ denotes the 2-sphere metric $d\Omega_2 = d\theta^2 + \sin^2\theta d\phi^2$. The asymptotic form of BEM tensor is

$$8\pi G_4 T_{tt} = \frac{l}{4r^2} + \mathcal{O} \left( \frac{1}{r^4} \right), \hspace{1cm} (2.15)$$

$$8\pi G_4 T_{\theta\theta} = \frac{l^3}{4r^2} + \mathcal{O} \left( \frac{1}{r^4} \right), \quad T_{\phi\phi} = T_{\theta\theta} \times \sin^2\theta. \hspace{1cm} (2.16)$$
and the ADM mass vanishes, \( M = 0 \). Using the coordinate transformation, \( r(z) = -\frac{l(z^2 - 1)}{2z} \) one has the FG metric

\[
ds^2 = -\mathcal{A}_G(z)dt^2 + \mathcal{B}_G(z)d\Omega_2^2 + \frac{l^2}{z^2}dz^2,
\]

\[
\mathcal{A}_G(z) = \frac{(2^{3/2}z^2 + 1)^2}{4(2^{3/2}z^2)^2}, \quad \mathcal{B}_G(z) = \frac{l^2((1/3)^{1/2}z - 1)^2((1/3)^{1/2}z + 1)^2}{4(2^{3/2}z^2)^2},
\]

and the BEM tensor

\[
8\pi G_4 t_{tt} = \frac{2^{3/2}z(1 + 2^{3/2}z^2)^2}{(-1 + 2^{3/2}z^2)^2}, \quad 8\pi G_4 t_{\theta\theta} = \frac{2^{3/2}z l^2((1 - 2^{3/2}z^2)^2)}{1 + 2^{3/2}z^2}, \quad t_{\phi\phi} = t_{\theta\theta} \sin^2 \theta.
\]

Note that at \( z = 0 \), \( t_{tt} = 0 \) leads to \( M = 0 \), consistent with (2.6), S-AdS_4 result with \( m = 0 \).

The close form of the trace of BEM is given as

\[
\sum t_{ab}g^{ab} = \frac{l^3}{8\pi G_4 z^3} \left( -2\Theta - 6 \frac{l}{l} - \frac{l}{2} R \right) = \frac{2^{3/2}z(1 - 3(2^{3/2})^2)l^2}{8\pi G_4(1 + 2^{3/2}z^2)(-1 + 2^{3/2}z^2)^2}.
\]

This reduces to the form suggested in (1.16)

\[
\sum t_{ab}g^{ab} = \frac{\sqrt{g_0}}{\sqrt{\mathcal{A}_G\mathcal{B}_G}} \left( \frac{l^2}{\pi G_4 R} - \frac{l^4}{4\pi G_4} \right),
\]

where \( \alpha_0 = l^2/\pi G_4 \) and \( \alpha_1 = -l^4/4\pi G_4 \) and other coefficients vanish.

### 2.3 S-AdS_4 on the Global patch

The metric of S-AdS_4 on the Global patch is written as

\[
ds^2 = -f_G(r)dt^2 + \frac{dr^2}{f_G(r)} + r^2d\Omega_2,
\]

\[
f_G(r) = \left( 1 + \frac{r^2}{l^2} - \frac{m}{r} \right).
\]

The BEM tensor is given asymptotically as [7]

\[
8\pi G_4 T_{tt} = \frac{m}{l} + \frac{1}{4lr^2} + \frac{ml}{2r^3} + \mathcal{O}\left( \frac{1}{r^4} \right),
\]

\[
8\pi G_4 T_{\theta\theta} = \frac{ml}{2r} + \frac{l^3}{4r^2} - \frac{3ml^3}{4r^3} + \mathcal{O}\left( \frac{1}{r^4} \right),
\]

\[T_{\phi\phi} = T_{\theta\theta} \times \sin^2 \theta,
\]

and the ADM mass (2.4) is

\[
M = \frac{m}{2G_4}.
\]
On the other hand, the FG metric

\[ ds^2 = -A_G(z)dt^2 + B_G(z)d\Omega^2 + \frac{l^2}{z^2}dz^2, \]  

(2.26)
is given in term of the elliptic function. Thus, we may expand \(1/\sqrt{f_G(r)}\) in power series of \(1/r\) to have

\[ \frac{1}{\sqrt{f_G(r)}} = l \left( \frac{1}{r} - \frac{l}{2r^3} + \frac{ml^2}{r^4} \right) + O\left(\frac{1}{r^5}\right), \]

(2.27)

and the relation (2.7) shows

\[ r(z) = l \left[ \frac{5^{1/3}}{z} - \frac{z}{2(5^{1/3})} + \frac{2\tilde{m}z^2}{3(5^{2/3})} - \frac{3z^3}{20} + O(z^4) \right], \]

(2.28)
and

\[ A_G(z) = \frac{1}{z^2} \left( \frac{5^{2/3}}{4} + \frac{z^2}{2} - \frac{4\tilde{m}z^3}{3(5^{1/3})} - \frac{z^4}{2(5^{2/3})} - \frac{8\tilde{m}z^5}{15} + O(z^6) \right), \]

\[ B_G(z) = \frac{l^2}{z^2} \left( \frac{5^{2/3}}{4} - \frac{z^2}{2} + \frac{3\tilde{m}z^3}{3(5^{1/3})} - \frac{z^4}{2(5^{2/3})} - \frac{2\tilde{m}z^5}{15} + O(z^6) \right). \]

(2.29)
The BEM tensor in FG coordinate is given as

\[ 8\pi G_4 t_{tt} = \frac{2\tilde{m}}{5^{1/3}} - \frac{2z}{5^{2/3}} + \frac{14\tilde{m}z^2}{15} + O(z^3), \]

\[ 8\pi G_4 t_{\theta\theta} = l^2 \left( \frac{2\tilde{m}}{5^{1/3}} + \frac{4z}{5^{2/3}} - \frac{11\tilde{m}z^2}{15} + O(z^3) \right), \quad t_{\phi\phi} = t_{\theta\theta} \times \sin^2 \theta. \]

(2.30)

From the BEM tensor one can put the series expansion of the trace into the form

\[ \sum t_{ab}g^{ab} = \frac{l^3}{8\pi G_4 z^3} \left( -2\Theta - \frac{6}{l} - \frac{l}{2}\frac{R}{R} \right) = \frac{\sqrt{g_0}}{R\sqrt{A_GB_G}} \left( \frac{l^2}{\pi G_4} \right) + \left( -\frac{\tilde{m}l^4}{20\pi G_4} \right) R + O(z^3), \]

(2.31)
which agrees with (2.21) when \(m = 0\). This result shows that the leading term is inversely proportional to the boundary volume and the next leading term is proportional to the Ricci scalar squared, which corresponds to \(\alpha_0 = l^2/\pi G_4\) and \(\beta_0 = -\tilde{m}l^4/20\pi G_4\) in (1.16).

3 Taub-NUT-(A)dS

In this section, we consider the Taub-NUT-AdS4 on the Poincare and on the Global patch. The BEM tensor and its trace anomaly is given asymptotically in terms of FG coordinates. It is noted that if one considers Taub-NUT-dS metric with NUT mass zero condition, then BEM tensor and its trace anomaly is given in a rational function. We will investigate this possibility in subsection 3.3 Taub-NUT-dS on the Global patch.
3.1 Taub-NUT-AdS$_4$ on the Poincaré patch

The metric of Taub-NUT-AdS on the Poincaré patch is given as [8, 10]:

\[ ds^2 = -g_P(r) \left( dt + \frac{n}{l^2} (x_1 dx_2 - x_2 dx_1) \right)^2 + \frac{dr^2}{g_P(r)} + \frac{r^2 + n^2}{l^2} (dx_1^2 + dx_2^2), \]

\[ g_P(r) = \frac{r^4 + 6n^2r^2 - 3n^4}{l^2(r^2 + n^2)} - \frac{mr}{r^2 + n^2}. \]  

(3.1)

The BEM tensor is given asymptotically as

\[
8\pi G_4 T_{tt} = \frac{m}{lr} + \frac{105n^4}{4l^3r^2} - \frac{5mn^2}{2l^3r} + \cdots ,
\]

\[
8\pi G_4 T_{tx} = -\frac{mnx_2}{l^3r} + \frac{105n^5x_2}{4l^5r^2} + \frac{5mn^3x_2}{2l^3r^3} + \cdots ,
\]

\[
8\pi G_4 T_{zx} = \frac{mnx_1}{l^3r} + \frac{105n^5x_1}{4l^5r^2} - \frac{5mn^3x_1}{2l^3r^3} + \cdots ,
\]

\[
8\pi G_4 T_{xx} = \frac{m(l^4 + 2n^2x^2_2)}{2l^5r} + \frac{n^4(63l^4 + 105n^2x^2_2)}{4l^7r^2} - \frac{mn^2(21l^4 + 10n^2x^2_2)}{4l^5r^3} + \cdots ,
\]

\[
8\pi G_4 T_{zz} = \frac{m(l^4 + 2n^2x^2_1)}{2l^5r} + \frac{n^4(63l^4 + 105n^2x^2_1)}{4l^7r^2} - \frac{mn^2(21l^4 + 10n^2x^2_1)}{4l^5r^3} + \cdots ,
\]

\[
8\pi G_4 T_{xz} = \frac{mn^2x_1x_2}{l^5r} - \frac{105n^6x_1x_2}{4l^7r^2} + \frac{5mn^4x_1x_2}{2l^5r^3} + \cdots ,
\]  

(3.2)

where \( \cdots \) denotes the terms of \( O(1/r^4) \). The ADM mass is given as

\[
M = \frac{mV_2}{8\pi G_4 l^2},
\]  

(3.3)

where \( V_2 \) refers to the 2-dimensional volume. From (2.7) \( r(z) \) is written in powers of \( z \)

\[
r(z) = l \left( \frac{1 - 5\bar{n}z^2}{4} + \frac{\bar{m}z^2}{6} - \frac{75\bar{n}^4z^3}{32} + O(z^4) \right),
\]  

(3.4)

where \( \bar{n} = n/l \), and the FG metric is written as

\[
ds^2 = -E_P(z) \left( dt + \frac{n}{l^2} (x_1 dx_2 - x_2 dx_1) \right)^2 + F_P \left( dx_1^2 + dx_2^2 \right) + \frac{l^2}{z^2} dz^2,
\]  

(3.5)

\[
E_P(z) = \frac{1}{z^2} \left( 1 + 5\bar{n}z^2 \right) + \frac{2\bar{m}z^2}{3} - \frac{89\bar{n}^4z^4}{8} - \frac{2\bar{m}\bar{n}^2z^5}{3} + O(z^6) \right),
\]

\[
F_P(z) = \frac{l^2}{z^2} \left( 1 - 3\bar{n}z^2 \right) + \frac{\bar{m}z^2}{3} - \frac{25\bar{n}^4z^4}{8} - \frac{5\bar{m}\bar{n}^2z^5}{12} + O(z^6) \right).
\]  

(3.6)

The BEM tensor is

\[
8\pi G_4 t_{tt} = \bar{m} - \frac{\bar{n}^4z^2}{2} - \frac{13\bar{m}\bar{n}^2z^2}{12} + O(z^3),
\]
\[
\begin{align*}
8\pi G_4 t_{x_1} &= -\frac{\bar{m}}{2} - \frac{n^5}{2} + \frac{13\bar{m}n^3}{12} + O(z^3), \\
8\pi G_4 t_{x_2} &= \frac{\bar{m}}{2} n x_1 - \frac{13\bar{m}n^3}{12} + O(z^3), \\
8\pi G_4 t_{x_1 x_1} &= \frac{\bar{m}}{2} (1 + 2n^2 \bar{x}_3^2) + \frac{1}{2} n^4 (85 - \bar{n}^2 \bar{x}_3^2) - \frac{13\bar{m}}{2} (115 + 26\bar{n}^2 \bar{x}_3^2) - \frac{24}{2} + O(z^3), \\
8\pi G_4 t_{x_2 x_2} &= \frac{\bar{m}}{2} (1 + 2n^2 \bar{x}_1^2) + \frac{1}{2} n^4 (85 - \bar{n}^2 (115 + 26\bar{n}^2 \bar{x}_3^2) - \frac{24}{2} + O(z^3), \\
8\pi G_4 t_{x_1 x_2} &= -\frac{\bar{m}}{2} n^2 \bar{x}_1 \bar{x}_2 + \frac{n^4}{2} \bar{x}_1 \bar{x}_2 \bar{z} + O(z^3), \\
\end{align*}
\]

where \(\bar{x}_1 = x_1/l\) and \(\bar{x}_2 = x_2/l\). \(t_{ul} = \frac{\bar{m}}{8\pi G_4}\) gives the VEV in boundary CFT and provides the mass in \((3.3)\).

The trace anomaly has the form

\[
\sum t_{ab} g^{ab} = \frac{1}{R \sqrt{E_p F_p}} \left( \frac{171 n^6 l^2}{8 \pi G_4} \right) + \left( -\frac{9 \bar{m} l^4}{32 \pi G_4} \right) R + O(z^3),
\]

which reproduces \((1.16)\), where \(\alpha_0 = 171 n^6 l^2 / 8 \pi G_4\) and \(\beta_0 = -9 \bar{m} l^4 / 32 \pi G_4\).

### 3.2 Taub-NUT-AdS\(_4\) on the global patch

The metric of Taub-NUT-AdS\(_4\) on the global patch is given by \([15 21]\)

\[
\begin{align*}
\sum\! t_{ab} g^{ab} &= -g_G(r) (dt + 2n \cos \theta d\phi)^2 + \frac{dr^2}{g_G(r)} + (r^2 + n^2) d\Omega_2, \\
g_G(r) &= \frac{l^2 r^2 - n^2 l^2 + r^4 + 6n^2 r^2 - 3n^4}{l^2 (r^2 + n^2)} - \frac{m_G r}{r^2 + n^2}.
\end{align*}
\]

The EM tensor is

\[
\begin{align*}
8\pi G_4 T_{ul} &= \frac{m l}{4l^3 r^2} + \frac{l^4 + 30n^2 l^2 + 105n^4}{2l^3 r^2} + O(\frac{1}{r^4}), \\
8\pi G_4 T_{u\phi} &= \cos \theta \left( \frac{mn(6l - 4)}{l^2 r} + \frac{n(l^4 + 30n^2 l^2 + 105n^4)}{2l^3 r^2} + \frac{m n(l^2 - 5n^2)}{l^4 r^2} + O(\frac{1}{r^4}) \right), \\
8\pi G_4 T_{\phi\phi} &= \frac{m(8n^2 + (l^2 - 8n^2) \sin^2 \theta)}{2lr} + \frac{4n^2 (l^4 + 30n^2 l^2 + 105n^4) + (l^2 + 4n^2)(l^4 + 12n^2 l^2 - 105n^4) \sin^2 \theta}{4l^3 r^2} \\
&\quad - \frac{m (-8n^2 (l^2 - 5n^2) + (3l^4 + 29n^2 l^2 - 40n^4) \sin^2 \theta)}{4l^3 r^2} + O(\frac{1}{r^4}), \\
8\pi G_4 T_{\theta\theta} &= \frac{ml}{r} + \frac{l^4 + 20n^2 l^2}{4l^2 r^2} + \frac{ml(3l^2 + 21n^3)}{2r^3} + O(\frac{1}{r^4}),
\end{align*}
\]
and the ADM mass \([2.4]\) is
\[
M = \frac{m}{2G_4}.
\] (3.12)

Using the asymptotic form of \(r(z)\)
\[
r(z) = l \left(1 - \frac{(1 + 5n^2)z}{4} + \frac{\bar{m}z^2}{6} - \frac{(3 + 30\bar{n}^2 + 75\bar{n}^4)z^3}{32} + \mathcal{O}(z^4)\right),
\] (3.13)
one has the metric in the FG
\[
d\sigma^2 = -E_G(z)(dt + 2n \cos \theta d\phi)^2 + F_G(z) d\Omega^2 + \frac{l^2}{z^2} dz^2,
\] (3.14)
\[
E_G(z) = \frac{1}{x^2} \left(1 + \frac{(1 + 5n^2)z^2}{2} - \frac{2\bar{m}z^3}{3} + \frac{(1 + 26\bar{n}^2 + 89\bar{n}^4)z^4}{8} + \frac{\bar{m}(1 + 2\bar{n}^2)z^5}{3} + \mathcal{O}(z^6)\right),
\]
\[
F_G(z) = \frac{l^2}{z^2} \left(1 - \frac{(1 + 3n^2)z^2}{2} + \frac{2\bar{m}z^3}{3} - \frac{(1 + 5\bar{n}^2)z^4}{8} - \frac{\bar{m}(1 + 5\bar{n}^2)z^5}{6} + \mathcal{O}(z^6)\right).
\]
The BEM tensor in FG coordinates is asymptotically given as
\[
8\pi G_4 \: t_{tt} = \frac{2\bar{m} - (4\bar{n}^2 + \bar{n}^4 + 1)z}{2} + \frac{\bar{m}(-13\bar{n}^2 + 7)}{6} z^2 + \mathcal{O}(z^3),
\]
\[
8\pi G_4 \: t_{t\phi} = l \left(2\bar{m} \bar{n} \cos \theta - \bar{n}(1 + 4\bar{n}^2 + \bar{n}^4)z + \frac{\bar{m}\bar{n}(7 - 13\bar{n}^2)}{6} z^2 + \mathcal{O}(z^3)\right),
\]
\[
8\pi G_4 \: t_{\phi\phi} = l^2 \left(\frac{\bar{m}(8\bar{n}^2 + (1 - 8\bar{n}^2) \sin^2 \theta)}{2} + \frac{[-4\bar{n}^2(1 + 4\bar{n}^2 + \bar{n}^4) + (1 + 4\bar{n}^2)(2 + 25\bar{n}^2 + \bar{n}^4) \sin^2 \theta]z}{4} - \frac{\bar{m}[-56\bar{n}^2 + 104\bar{n}^4 + (11 + 171\bar{n}^2 - 104\bar{n}^4) \sin^2 \theta]}{24}z^2 + \mathcal{O}(z^3)\right),
\]
\[
8\pi G_4 \: t_{\theta\theta} = l^2 \left(\frac{\bar{m}}{2} + \frac{(2 + 29\bar{n}^2 + 85\bar{n}^4)z}{2} - \frac{\bar{m}(11 + 115\bar{n}^2)z^2}{24} + \mathcal{O}(z^3)\right).
\] (3.15)
This BEM tensor leads to the VEV in boundary CFT \(t_{tt} = \bar{m}/(8\pi G_4)\), which reproduces the mass \([3.12]\).

This shows that the trace of BEM has the form
\[
\sum t_{ab} g^{ab} = \frac{\sqrt{g_0}}{R \sqrt{E_G F_G}} \left(\frac{(1 + \bar{n}^2)l^2(5 + 62\bar{n}^2 + 171\bar{n}^4)}{8\pi G_4}\right)
\]
\[
+ \left(\frac{\bar{m}l^4(1 + 9\bar{n}^2)}{16\pi G_4(1 + \bar{n}^2)}\right) R + \mathcal{O}(z^3),
\] (3.16)
which results in \(\alpha_0 = (1 + \bar{n}^2)l^2(5 + 62\bar{n}^2 + 171\bar{n}^4)/8\pi G_4\) and \(\beta_0 = -\bar{m}l^4(1 + 9\bar{n}^2)/16\pi G_4(1 + \bar{n}^2)\) in \([1.11]\).
3.3 Massless Taub-NUT-dS

Taub-NUT-dS metric is obtained from the Taub-NUT-AdS metric by replacing $l^2 \to -l^2$. On the global patch one has \[25, 26, 27, 24\]

\[
ds^2 = g_{G,\text{ds}}(r)(dt + 2n \cos \theta d\phi)^2 - \frac{dr^2}{g_{G,\text{ds}}(r)} + (r^2 + n^2)d\Omega_2, \tag{3.17}
\]

\[
g_{G,\text{ds}}(r) = \frac{-l^2r^2 + n^2r^2 + r^4 + 6n^2r^2 - 3n^4}{l^2(r^2 + n^2)} + \frac{m_{G,\text{ds}}r}{r^2 + n^2}.
\]

Due to the signature change of the metric compared with that of AdS, one may view this dS metric as the one describing the region outside of the cosmological event horizon. The future infinity ($r \to \infty$) corresponds to the asymptotic boundary we are paying attention to.

Let us consider the metric with $n^2 = l^2/4$ and $m_{G,\text{ds}} = 0$. In this case, the metric is simplified

\[
g_{G,\text{ds}}(r) = \frac{r^2 + l^2/4}{l^2}. \tag{3.18}
\]

Using $r(z) = l \left(\frac{1 - l^2z^2/4}{2z}\right)$ one finds the metric in FG coordinates

\[
ds^2 = E^{(1)}_{G,\text{ds}}(z)(dt + 2n \cos \theta d\phi)^2 + F^{(1)}_{G,\text{ds}}(z)d\Omega_2 - \frac{l^2}{z^2}dz^2, \tag{3.19}
\]

\[
E^{(1)}_{G,\text{ds}}(z) = \left(\frac{4 + z^2}{8z}\right)^2, \quad F^{(1)}_{G,\text{ds}}(z) = l^2 \left(\frac{4 + z^2}{8z}\right)^2.
\]

It is noted that the conditions, $n^2 = l^2/4$ and $m_{G,\text{ds}} = 0$ are not independent in the Wick-rotated Euclidean section. To see this, one Wick-rotates the time ($t \to it_E$) and NUT charge ($n \to iN$) simultaneously from (3.17) and obtains the metric

\[
-g_{E,\text{ds}}(r)(dT + 2N \cos \theta d\phi)^2 - \frac{dr^2}{g_{E,\text{ds}}(r)} + (r^2 - N^2)d\Omega_2, \tag{3.20}
\]

\[
g_{E,\text{ds}}(r) = \frac{-l^2r^2 - N^2l^2 + r^4 - 6N^2r^2 - 3N^4}{l^2(r^2 - N^2)} + \frac{m_{G,\text{ds}}r}{r^2 - N^2}.
\]

Requiring $g_{G,\text{ds}}(r)|_{r=N} = 0$ one has the relation

\[
m_{G,\text{ds}} = \frac{2N(l^2 + 4N^2)}{l^2}. \tag{3.21}
\]

Thus, if one puts $n^2 = -N^2 = l^2/4$, then $m_{G,\text{ds}} = 0$. Because of this relation, we will call $n^2 = l^2/4$ the massless-NUT condition. In addition, if one requires the regularity of the
metric at the cosmological event horizon \((r = N)\), one finds \(^{28}\)

\[
\left. \frac{4\pi}{|\partial g_{G, dS}^E(r) / \partial r|} \right|_{r = N} = 8\pi N,
\]

which is interpreted as the inverse temperature at the cosmological event horizon.

The BEM tensor is given from the minimal counter-term subtraction \(^{11, 12} \) \(^{25, 26} \) \(^{27} \)

\[
T_{ab} = \frac{1}{8\pi G_4} \left[ \Theta_{ab} - \Theta \gamma_{ab} - \frac{2}{l} \gamma_{ab} \gamma - l(R_{ab} - \frac{1}{2} \gamma_{ab} \gamma) \right],
\]

where the last term switches the sign to negative from the one in AdS case. In \(r\)-coordinate system, the BEM tensor is given asymptotically as

\[
8\pi G_4 T_{tt} = \frac{-n}{32 r^2} + \frac{n^3}{64 r^4} + O(\frac{1}{r^6}),
8\pi G_4 T_{t\phi} = \frac{-n^2 \cos \theta}{r^2} + \frac{n^4 \cos \theta}{32 r^4} + O(\frac{1}{r^6}),
8\pi G_4 T_{\phi\phi} = 8\pi G_4 T_{\theta\theta} = \frac{-n^3}{r^2} + \frac{n^5}{16 r^4} + O(\frac{1}{r^6}),
\]

and the ADM mass vanishes \(M = 0\). In \(z\)-coordinates, the BEM is given in a closed form

\[
8\pi G_4 t_{tt} = \frac{-\bar{n} z}{8},
8\pi G_4 t_{t\phi} = \frac{-ln^2 z \cos \theta}{4},
8\pi G_4 t_{\phi\phi} = 8\pi G_4 t_{\theta\theta} = \frac{-ln^3 z}{2}.
\]

The trace of BEM tensor is obtained as

\[
\sum t_{ab} g^{ab} = \frac{6\bar{n}^6 l^2 z}{\pi G_4 (1 + \bar{n}^2 z^2)^2}
= \frac{\sqrt{y_0}}{R \sqrt{\mathcal{E}_{G, dS}^{(1)}(\mathcal{F}_{G, dS}^{(1)})^2}} \left( \frac{9\bar{n}^5 l^2}{4\pi G_4} - \frac{3\bar{n}^7 l^2}{4\pi G_4} \right).
\]

This shows that \(\alpha_0 = \frac{9\bar{n}^5 l^2}{4\pi G_4}, \alpha_1 = -\frac{3\bar{n}^7 l^2}{4\pi G_4}\) and others vanish in \(^{1, 16} \).

It is remarked that the massless-NUT condition can also be imposed on the Taub-NUT-dS metric in arbitrary even dimensions. The Taub-NUT-dS with \(k S_2\) spheres is given in Euclidian section as \(^{21} \)

\[
ds^2 = -g_{G, dS}^E(r) \left[ dT + 2N \sum_{i=1}^{k} \cos(\theta_i) d\phi_i \right]^2 - \frac{dr^2}{g_{G, dS}^E(r)} + (r^2 - N^2) \sum_{i=1}^{k} d\Omega_{2}^{(i)},
\]

\[
g_{G, dS}^E(r) = \frac{r}{(r^2 - N^2)^k} \int^r \left[ (2k + 1)(s^2 - N^2)^{k+1} - (s^2 - N^2)^k \right] ds + \frac{m_{G}^{(k)} r}{(r^2 - N^2)^{k+1}}.
\]
where \( d\Omega_{2}^{(i)} \) refers to the \( i \)-th unit sphere measure. In this manifold, the NUT mass is given as

\[
m^{(k)}_{G} = \Gamma \left( \frac{3 - 2k}{2} \right) \frac{\Gamma(k + 1)N^{2k - 1}[l^{2} + (2k + 2)N^{2}]}{2\sqrt{\pi}l^{2}(2k - 1)}, \tag{3.28}
\]

where \( \Gamma(k) \) is the gamma function. Massless-NUT condition is

\[
l^{2} = -(2k + 2)N^{2} = (2k + 2)n^{2}, \tag{3.29}
\]

and the metric in Minowski section with this massless-NUT condition is written as

\[
ds^{2} = g_{G,\text{ds}}^{(k)}(r) \left[ dt + 2n \sum_{i=1}^{k} \cos(\theta_{i})d\phi_{i} \right]^{2} - \frac{dr^{2}}{g_{G,\text{ds}}^{(k)}(r)} + (r^{2} + n^{2}) \sum_{i=1}^{k} d\Omega_{2}^{(i)}, \tag{3.30}
\]

\[
g_{G,\text{ds}}^{(k)}(r) = \frac{r^{2} + n^{2}}{(2k + 2)n^{2}}.
\]

The metric is compactly written in FG coordinates

\[
ds^{2} = E_{G,\text{ds}}^{(k)}(z) \left[ dt + 2n \sum_{i=1}^{k} \cos(\theta_{i})d\phi_{i} \right]^{2} + F_{G,\text{ds}}^{(k)}(z) \sum_{i=1}^{k} d\Omega_{2}^{(i)} - \frac{l^{2}}{z^{2}}dz^{2}, \tag{3.31}
\]

\[
E_{G,\text{ds}}^{(k)}(z) = \left( \frac{2k + 2 + z^{2}}{4(k + 1)z} \right)^{2}, \quad F_{G,\text{ds}}^{(k)}(z) = l^{2} \left( \frac{2k + 2 + z^{2}}{4(k + 1)z} \right)^{2},
\]

where \( r(z) = l \left( \frac{1 - n^{2}z^{2}}{2z} \right) \) is used.

## 4 Conclusion and discussion

Employing the counter-term subtraction method, we have investigated the boundary energy momentum tensor and its trace anomaly for the various cases of metric, including AdS\(_{4}\), S-AdS\(_{4}\) and Taub-NUT-(A)dS\(_{4}\). Using the asymptotic expansion in FG coordinates, we find that the trace of the BEM tensor has a special form \( (1.16) \) in general (and \( (1.17) \) for S-AdS\(_{4}\) on the Poincaré metric only).

The 3-dimensional holographic trace anomaly vanishes at \( r \to \infty \) (or \( z \to 0 \)) boundary. However, the trace is non-vanishing when \( r \) is finite, which corresponds to the UV cut-off at the boundary field theory point of view. Thus, if the AdS/CFT duality holds in this case, these anomalies can be interpreted as the breaking of conformal symmetry of the 3-dimensional field theory. Thus, it will be interesting to find the effective field theory whose trace anomaly matches with the power correction in \( z \) as the high-energy effect.
Finally, it is noted that a different regularized action in AdS$_4$ has been obtained in [29, 30] where Gauss-Bonnet action with topological term is chosen instead of the boundary action (1.14), assuming that the Riemann curvature tensor is given as a constant negative curvature at the boundary. This action allows anomaly free conserved quantity at the boundary. Thus, one may naively expect that (1.16) and (1.17) originates from the topological term in [30]. However, it turns out that the 3-dimensional trace anomaly does not lead to the boundary action. Thus, the difference seems to arise due to the boundary condition imposed in [30] and the Dirichlet boundary condition employed in this work. The two different results allude that the AdS/CFT correspondence is sensitive to the boundary condition imposed on the CFT boundary.

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