Role of antikaon condensation on the universality relations of hot and rapidly rotating neutron stars

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Abstract. We study cold as well as hot neutron star (NS) at finite entropy using density dependent relativistic mean field model in the presence of nucleons and antikaon condensates. The parameters like gravitational mass \(M\), radius \(R\), moment of inertia \(I\) and quadrupole moment \(Q\) are calculated as a function of rotation frequency for a NS with fixed baryonic mass. Next, we investigate the relation of normalized \(I\) with compactness \((M/R)\). Finally, we extend our study to calculate the tidal deformability parameter and tidal love number and show their variation with compactness.

1. Introduction

Hot NSs are formed in core-collapse supernovae explosions of massive main sequence stars. They cool down gradually over a time scale of \(\sim 100\) ms to form a stable and cold NS. The entropy per baryon of the star remain constant throughout the adiabatic collapse [1]. Various studies show the appearance of Bose-Einstein condensates of particles like antikaons \((K^-)\) in highly dense NS cores [2]. Pulsars (PSRs) are the rapidly rotating NSs with rotational periods of approximately \(1\) ms to \(10\) s [3]. A young PSR rotates rapidly whereas an old one rotates slowly. Depending on the observed rotation period, PSRs can be classified into two different classes i) normal PSRs with rotational period of the order of \(\sim 1\) s, ii) millisecond PSRs with rotational period of \(\sim 0.001\) s. The fast rotation can lead to deformation in their shape.

Physically the moment of inertia quantifies how fast a fixed angular momentum star rotates, while the quadrupole moment reflects how much a spherically symmetric star is deformed. The tidal deformability \((\lambda)\) quantifies how easily a star can be deformed [4]. It is believed that the universal relations between normalized \(I\) and compactness exists for various equations of states (EoSs), both nucleonic [5, 6, 7, 8] as well as exotic matter for NS [9] and protoneutron stars (PNS) with hyperons [10]. The tidal love number \((k_2)\) has been measured by Advanced LIGO and Advanced Virgo [11]. Combining them with the universal relations, one can obtain the \(I\) and \(Q\) which are not so easy to obtain from gravitational wave (GW) observations [4].

In this paper we study the role of \(K^-\) condensates on the structural properties of NSs using both cold as well as hot EoS. The NS is considered to be at constant entropy per baryon, so that the core remains hotter than the crust. In the following section we discuss briefly the isentropic EoS and the model. We also mention the structural properties of the rotating stars. Our results for the static and rotating stars are discussed in the Section 3.
Table 1. Maximum gravitational mass (in $M_{\odot}$) and the corresponding radius (in km) and central density ($n_c$) of static stars with np(npK) EoS. The threshold density ($n^K$) for $K^-$ condensates are for the npK ($U_K = -140$ MeV) matter.

| Entropy per baryon | $M$ ($M_{\odot}$) | $R$ (km) | $n_c$ ($fm^{-3}$) | $n^K$ ($fm^{-3}$) |
|--------------------|------------------|----------|------------------|------------------|
| $T=0$              | 2.42[2.16]       | 11.87[12.01] | 0.851[0.881]     | 0.416            |
| $s_B=1$            | 2.42[2.16]       | 12.07[12.10] | 0.836[0.881]     | 0.449            |
| $s_B=2$            | 2.43[2.22]       | 12.74[13.12] | 0.791[0.796]     | 0.535            |
| $s_B=3$            | 2.46[2.30]       | 14.20[14.83] | 0.708[0.702]     | 0.674            |

2. EoS and Tidal deformability of rotating NS
We consider two EoSs with different compositions, one with only nucleons (np) and other with nucleons, thermal kaons and antikaon condensates (npK) [12], constructed using a density-dependent relativistic mean field (DDRMF) model. At low densities, the NS contains neutrons (n), protons (p), electrons (e) and muons ($\mu$). At higher densities, the threshold condition $\mu_K = \mu_n - \mu_p = \mu_e$ is satisfied and $K^-$'s appear and replace the leptons in the system. Here $\mu$'s are the chemical potential of the respective particles. The DDRMF model satisfies the constraints on nuclear symmetry energy and its slope parameter as well as the incompressibility from the nuclear physics experiments [13]. The density-dependent meson-baryon couplings give rise to the rearrangement term in the pressure term, that accounts for the energy-momentum conservation and thermodynamic consistency of the system [14]. However, the kaon-nucleon couplings are not density dependent. The condensates do not contribute to pressure, but implicitly change the rearrangement term via the interacting meson fields.

We study rapidly rotating NS with these np and npK EoS using LORENE [15], which is primarily formulated for cold EoS, or a barotropic EoS of the form $\epsilon = \epsilon(H)$ and $P = P(H)$ [16], where $\epsilon$, $P$ and $H$ are energy density, pressure and enthalpy respectively. For a rigidly rotating star, the star is considered as circular fluid motion around the rotation axis that leads to the constant first integral $H + \nu - ln\Gamma$, where the log-enthalpy $H = ln(\frac{h}{\Gamma M_B})$, $M_B$ being the mean baryon mass $\sim 1.66 \times 10^{-27}$ kg, and the enthalpy per baryon is related to EoS via $h = \frac{\epsilon + P}{n_B}$ [17].

At finite temperature the equilibrium equation is modified to $\partial_i(H + \nu - ln\Gamma) = T exp(-H)\partial_i s_B$, where $s_B$ is the entropy per baryon in the units of Boltzmann constant $k_B$ and $T$ is the temperature. This enables us to use our isentropic EoS and compute rotating NS configurations. The rotation causes deformation in the spherically symmetric static star, that can be quantified by the deformation parameters like quadrupole moment and tidal deformability. We calculate the tidal deformability of an isolated NS, defined as $\lambda = -\frac{Q_{ij}}{E_{ij}}$, where $Q_{ij}$ is the induced quadrupole moment of the NS under the influence of a static external tidal field $E_{ij}$. The dimensionless, $l = 2$, electric-type love number, $k_2$ is related to tidal deformability by $\lambda = \frac{1}{2}k_2R^5$, where $R$ is the radius of the NS. They can be calculated in general relativity together with Tolman Oppenheimer Volkoff equations for a static NS EoS, following the prescription of Hinderer et. al [18, 19]. We assume $G = c = k_B = 1$ throughout this article, where $G$, $c$, and $k_B$ denote the gravitational constant, the speed of light, and Boltzmann constant respectively.

3. Results
We report our results for np and npK matter (with optical potential depth of $U_K = -140$ MeV). Throughout the paper, we use different colors for the EoSs (red for np and blue for npK) and different line styles for different values of $s_B$ i.e. $T=0$ in solid, $s_B=1$ in dash dot dashed, $s_B=2$ in dashed and $s_B=3$ in dotted.

We plot energy density($\epsilon$) profile in Fig. 1a for the cold star as well as the isentropic star at
different $s_B$ (=1, 2, 3). We observe more-or-less similar profiles for both the np and npK EoS for a particular entropy per baryon. The appearance of $K^-$ condensates lowers the total energy of the system. Since $K^-$ form s-wave Bose condensates, they do not directly contribute to the pressure. On the other hand, as they replace the negatively charged electrons, the drop in lepton pressure results in a softer EoS. However, the EoS-dependence is not so significant towards the surface of the NS, where the density is less and the $K^-$ condensates do not appear at all. Also, hotter the star, larger is its radius due to increasing thermal pressure.

The mass-radius profile for static, $\beta$-equilibrated and charge neutral with different compositions and $s_B$ are displayed in Fig. 1b. The thermal pressure can support more mass. Therefore, maximum mass of the star increases with increase in $s_B$. The maximum mass and corresponding radius for these static stars are noted in Table 1. Our model clearly satisfies the $2M_{\odot}$ observation limit [20, 21, 22]. The central density($n_C$) and the threshold number density($n_K$) of $K^-$ condensates are also listed in Table 1. The central density of the NS with the condensates is slightly less than that of nucleons-only star, due to larger size of the former. However, there is an inherent uncertainty with the radius values for NS at finite temperature as the surface pressure never really goes to zero.

An NS with a fixed baryonic mass of $\sim 2.3M_{\odot}$ contains an appreciable fraction of $K^-$ condensates. To study the role of $K^-$ condensates on the rotating star, we calculate the properties, $M$, $R$, $I$, and $Q$ of the NS, with fixed baryon mass of $2M_{\odot}$ starting from a static configuration up to the mass-shedding frequency. In Fig. 2, all the parameters are observed to be monotonically increasing with frequency. This was noted for other EoSs in earlier work also [23, 10]. The gravitational mass is almost independent of the constituent matter. However, the radius value differs for np and npK EoS, especially for the highest $s_B$. This affects the EoS-dependence of $I$ and $Q$. Both $I$ and $Q$ values are nearly close for np and npK cold EoS. However, for higher $s_B$ a marked difference is visible. This can be attributed to the thermal pressure which is quite large for the hadrons and leptons at higher $s_B$, and always nil for the condensates. Hence the increase in NS size is more for the np matter compared to the npK one.

Fig. 3 shows the variation of dimensionless moment of inertia ($I/M^3$) with compactness ($M/R$) for a star rotating at two different frequencies. Left panel shows the relation for the frequency 100 Hz where as right side panel is for 600 Hz. We observe universality at lower compactness. A slight deviation due to EoS is noticed at high compactness, for stars at different $s_B$. However, the lines are distinctly separate for different temperature and entropies.
Figure 2. Properties of NS with fixed baryonic mass $2.3M_{\odot}$ as function of frequency.

Figure 3. Dimensionless moment of inertia ($I/M^3$) as function of compactness ($M/R$).

The variation of tidal love number $k_2$ with compactness is shown in Fig. 4a. The love number is least for higher entropy stars i.e. the thermal energy resist the deformation. The cold NS can however, be deformed easily as evident from Fig. 4a. It increases up to a maximum value, then drop monotonically at higher $M/R$. At lower $M/R$, for a fixed $s_B$ with both np and npK EoS, $k_2$ is same. However their differences become significant at higher $M/R$, where the fraction of $K^-$ is maximal.

In Fig. 4b, we plot the tidal deformability parameter $\lambda(=\frac{2}{5}k_2R^5)$ with compactness. As $\lambda \propto R^5$, radius calculation is very important. Fortin et. al has shown that the uncertainty in radius calculations can be as high as $\sim 4\%$ unless the core-crust EoS are obtained from the same many-body theory [24]. We take the low density part of the EoS from Hempel and
Figure 4. a) Love number ($k_2$) versus compactness b) The tidal deformability parameter $\lambda$ variation with compactness ($M/R$) for both np and npK EoS.

Schaffner-Bielich [25], where the interaction among the unbound nucleons are described by the same Lagrangian density as in the high density core and using the density dependent formalism [26, 25]. We notice that higher the entropy per baryon, higher is the corresponding $\lambda$. However, this difference almost vanishes at higher compactness. On the other hand, at a fixed $s_B$, $\lambda$ is independent of the composition of the star for lower compactness. At the higher compactness region, $\lambda$ varies considerably for np and npK EoS, finally approaches zero at the black hole limit of compactness 0.5.

4. Discussion and Conclusion
We study various structural properties of NSs, that are closely related to the dynamics and hence the observable properties of pulsars for two types of EoSs, one with nucleons only constituents and other one involves Bose-Einstein condensates of $K^-$. We study the parameters like $M$, $R$, $I$ and $Q$ for a NS with fixed baryonic mass ($M_b = 2.3M_{\odot}$), which contains a considerable fraction of antikaon condensates. We observe that all the variables increase with frequency and they have similar nature for both the EoSs. However, their values are noticed to be different for np and npK EoS at higher $s_B$.

We notice universality at lower compactness for normalized $I$, which deviates slightly for different compositions at high compactness. Finally, from the love number versus compactness plot, we conclude that, it depends on EoS at higher $M/R$ and the value is maximum for the cold EoS. This indicates that the thermal pressure counterbalances the deformation due to rotation. Finally, we investigate the correlations between compactness and tidal deformability for a single NS, and conclude that it is independent of the chosen np or npK EoS. But, it is quite high for NS with larger $s_B$ at low compactness. This can be attributed to the thermal effect. At higher $M/R$, $\lambda$ does not depend on either EoS or $s_B$ and eventually becomes zero as the NS approaches the typical compactness of a black hole.

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