TODIM method based on bipolar intuitionistic fuzzy soft set

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Abstract. This paper deals with TODIM (an acronym in Portuguese for iterative Multi-criteria decision making) method based on bipolar intuitionistic fuzzy soft set (BIF SS). Entropy measure on BIF SS is developed and it serves as a tool in computing the weight values. Further, a score function is defined and based on it a score matrix is constructed. A BIFS normalized euclidean distance is developed and by using this a dominance degree matrix is computed. An illustration is given to show the applicability of this method in solving the decision making problem.

1. Introduction
TODIM method is a multi criteria method based on the prospect theory. This method was first initiated by Gomes and Lima [1]. In order to handle the uncertainties, Krohling et al.[2, 3] developed the fuzzy TODIM method also extended the method in terms of intuitionistic fuzzy set. Dong Zhang et al. [4] have proposed TODIM method based on novel score function on intuitionistic fuzzy set. Guiwu Wei[5, 6] proposed bipolar fuzzy hamacher aggregation operators and also developed TODIM method based on picture fuzzy set for solving MCDM problem. [8] developed an entropy measure on spherical fuzzy set. Based on these concepts, we define entropy measure on BIF SS. We propose a score function on BIF SS for constructing score matrix. An illustration is provided to explain this method.

2. BIFS entropy measure
Definition of BIFS set and BIFS matrix are given in [7]. In this section, BIFS entropy measure, weight function and a score function are defined, also some theorems based on these entropy measure and score function are established.

Definition: 2.1 For a BIFS set, the entropy measure is given by

\[ EM_j(BF_{x_j}(x_i)) = \frac{1}{m} \sum_{j=1}^{q} (1 - \frac{1}{4}(|\mu_{BF_{x_j}}^2(x) - \nu_{BF_{x_j}}^2(x)| + |1 - \pi_{BF_{x_j}}(x)|)) + (|\mu_{BF_{x_j}}^2(x) - \nu_{BF_{x_j}}^2(x)| + |1 - \pi_{BF_{x_j}}(x)|)), \]

where, \( \pi_{ij}^n = -1 - \mu_{ij}^n - \nu_{ij}^n \) and \( \pi_{ij}^p = 1 - \mu_{ij}^p - \nu_{ij}^p \), respectively.
and \( \pi^p_i \in [-1,0] \) and \( \pi^p_i \in [0,1] \).

**Theorem: 2.2** Let \((BF,E)\) be a BIFSS. The entropy measure \( EM(BF_e(x)) \) satisfies the following.

(i) \( EM(BF_e(x)) = 0 \)

(ii) \( EM(BF_e(x)) = 1 \) iff \( \mu_p^{BF_e}(x) = \nu^{BF_e}(x), \mu^{BF_e}(x) \) and \( \pi^{BF_e} = -1, \pi^{BF_e} = 1 \).

**Proof:**

(i) Let \((BF,E)\) be a BIFSS, then we have,

\[
\begin{align*}
\mu_p^{BF_e}(x) &= 1, \nu^{BF_e}(x) = 0, \pi^{BF_e}(x) = 0 \quad \text{or} \\
\mu^{BF_e}(x) &= 0, \nu^{BF_e}(x) = 1, \pi^{BF_e}(x) = 0 \quad \text{and} \\
\mu_p^{BF_e}(x) &= 0, \nu^{BF_e}(x) = 0, \pi^{BF_e}(x) = 1, \text{or}
\end{align*}
\]

\[
EM(BF_e(x)) = \frac{1}{m} \sum_{i=1}^{m} (1 - \frac{1}{4} \{((\mu^{BF_e}(x) - \nu^{BF_e}(x)) | - 1 - \pi^{BF_e}(x))\})
\]

\[
= \frac{1}{m} \sum_{i=1}^{m} (1 - \frac{1}{4} (|1 - 0| + |1 - 0|)) = 0
\]

(ii) Let \( \mu^{BF_e}(x) = \nu^{BF_e}(x), \pi^{BF_e}(x) = -1 \) and \( \mu_p^{BF_e}(x) = \nu_p^{BF_e}(x), \pi^{BF_e}(x) = 1 \) for all \( x \in U \), then,

\[
EM(BF_e(x)) = \frac{1}{m} \sum_{i=1}^{m} (1 - \frac{1}{4} \{((\mu^{BF_e}(x) - \nu^{BF_e}(x)) | - 1 - \pi^{BF_e}(x))\})
\]

\[
= 1
\]

Conversely, suppose that \( EM(BF_e(x)) = 1 \) then,

\[
\frac{1}{m} \sum_{i=1}^{m} (1 - \frac{1}{4} \{((\mu_p^{BF_e}(x) - \gamma^{BF_e}(x)) | - 1 - \pi^{BF_e}(x))\}) = 0
\]

\[
|\mu^{BF_e}(x) - \gamma^{BF_e}(x)| = | - 1 - \pi^{BF_e}(x))| = 0 \quad \text{then,}
\]

\[
\mu_p^{BF_e}(x) = \gamma^{BF_e}(x), \pi^{BF_e}(x) = -1 \quad \text{and} \quad \mu_p^{BF_e}(x) - \gamma^{BF_e}(x), \pi^{BF_e}(x) = 1 \quad \text{for all} \quad x \in U.
\]

**Definition: 2.3** The weight value of each criteria \( w_i \) is defined as

\[
w_j = \frac{1 - EM_j}{\sum_{j=1}^{m} (1 - EM_j)}
\]

The relative weight \( w_{jk} \) is computed as follows,

\[
w_{jk} = \frac{w_j}{w_k}, \text{ where } w_k = \max w_j.
\]

**Definition: 2.4** For a bipolar fuzzy set,

\[
S_B(A) = \frac{(\mu^{B_p}_i)^{(1 + \sqrt{1 - \mu_i^{B_p}})^2}}{2} + (\nu^{B_p}_i)^{(1 + \sqrt{1 - \nu_i^{B_p}})^2}
\]

**Definition: 2.5** For a bipolar intuitionistic fuzzy soft set \((BF,E) = \{(\mu^{B_i}_j, \nu^{B_i}_j, \pi^{B_i}_j)\} \), the score function is defined as,

\[
S_B(BF_e(x)) = \frac{(\mu^{B_i}_j - \nu^{B_i}_j)^{-1 + \sqrt{1 - \mu^{B_i}_j - \nu^{B_i}_j}^2}}{2} + (\nu^{B_i}_j - \nu^{B_i}_j)^{1 + \sqrt{1 - \nu^{B_i}_j - \nu^{B_i}_j}^2}
\]

**Theorem: 2.6** For a BIFSS the score function \( S_B(BF_e(x)) \) satisfies the following:

(i) \( S_B(BF_e(x)) = -1 \) if \((BF,E) = ((0,0),(-1,1))\)
\( (ii) S_B(BF_e(x)) = 1 \) if \( (BF, E) = ((-1, 1), (0, 0)) \)

**Proof:**

(i) If \( (BF, E) = ((0, 0), (-1, 1)) \) is a BIFSS, then

\[
S_B(BF_e(x)) = \frac{(\mu_{ij}^+ - \mu_{ij}^-)(-1 + \sqrt{(-1 - \mu_{ij}^+ - \mu_{ij}^-)^2}) + (\mu_{ij}^+ - \mu_{ij}^-)(1 + \sqrt{(-1 - \mu_{ij}^+ - \mu_{ij}^-)^2})}{2}
\]

\[
= \frac{(0+1)(-1 + \sqrt{(-1-0+1)^2}) + (0-1)(1 + \sqrt{(-1-0+1)^2})}{2}
\]

\[
= -1.
\]

(ii) If \( S_B(BF_e(x)) = 1 \) if BIFSS, \( (BF, E) = ((-1, 1), (0, 0)) \)

\[
S_B(BF_e(x)) = \frac{(\mu_{ij}^+ - \mu_{ij}^-)(-1 + \sqrt{(-1 - \mu_{ij}^+ - \mu_{ij}^-)^2}) + (\mu_{ij}^+ - \mu_{ij}^-)(1 + \sqrt{(-1 - \mu_{ij}^+ - \mu_{ij}^-)^2})}{2}
\]

\[
= \frac{(-1-0)(-1 + \sqrt{(-1-0+0)^2}) + (0-1)(1 + \sqrt{(-1-0+0)^2})}{2}
\]

\[
= 1.
\]

Some of the existing score functions of bipolar fuzzy set are as follows.

1. Wei et al.[6] score function:

\[
E(S_i) = \frac{1 + \mu_{ij}^+ + \mu_{ij}^-}{2}, \quad E(S_i) \in [0, 1].
\]

2. Mahmood et al. [9] score function:

\[
M(B_i) = \mu_{B_i}^+ + \mu_{B_i}^-, \quad \text{where} \ M(B_i) \in [-1, 1]
\]

Shortcomings of the existing score functions.

The following examples shows that the score function listed above are unable to rank the alternatives in the decision making problems.

**Example: 2.7** Let \( B_1 = (-0.2, 0.35) \) and \( B_2 = (-0.14, 0.3) \) be two bipolar fuzzy sets, then by (1) we get,

\[
E(B_1) = 0.575
\]

\[
E(B_2) = 0.575
\]

In this case, the score function is unable to rank the alternatives.

For the same values of \( B_1 \) and \( B_2 \), using Mahmood et al. score function we get

\[
M(B_1) = 0.15
\]

\[
M(B_2) = 0.15
\]

Here also, score function fails to rank the alternatives.

Proposed score function

By using definition 2 in the same example we get.

\[
S_B(B_1) = 0.6325
\]

\[
S_B(B_2) = 0.5175
\]

\[
S_B(B_1) > S_B(B_2)
\]

So, our proposed score function is very efficient in ranking the alternatives.

**3. BIFS normalized euclidean distance**

**Definition: 3.1** Let \( U = \{x_1, x_2, ..., x_n\} \) be an universal set, \( E = \{e_1, e_2, ..., e_q\} \) be a set of parameters and \( (BF, E), (BG, E) \) be two BIFSS over \( U \). Then the normalized euclidean distance between \( (BF, E) \) and \( (BG, E) \) is defined as,

\[
\mathcal{ND}_E((BF, E),(BG, E)) = \left\{ \frac{1}{2}((\mu_{BF(e)}(x_i) - \mu_{BG(e)}(x_i))^2 + (\mu_{BF(e)}(x_i) - \mu_{BG(e)}(x_i))^2)
\right.

\[
+ (\nu_{BF(e)}(x_i) - \nu_{BG(e)}(x_i))^2 + (\nu_{BF(e)}(x_i) - \nu_{BG(e)}(x_i))^2\right\}^{\frac{1}{2}}.
\]

**Theorem: 3.2** Let \( BIFSS(U) \) be the set of all \( BIFSS \) over \( U \). Then the distance function \( \mathcal{ND}_E \) from \( BIFSS(U) \) to the set of real numbers is a metric.

**Proof:** Let \( (BF, E), (BG, E) \) and \( (BH, E) \) be three \( BIFSS \) over \( U \).

(i) \( \mathcal{ND}_E((BF, E),(BG, E)) > 0 \) follows from Definition 2.1

(ii) \( \mathcal{ND}_E((BF, E),(BG, E)) = 0 \)

\[
\Leftrightarrow (\mu_{BF(e)}(x_j) - \mu_{BG(e)}(x_j))^2 + (\mu_{BF(e)}(x_j) - \mu_{BG(e)}(x_j))^2
\]
Therefore, and some of its concepts are defined. 

\[ \nu_{BF}(x_j) - \nu_{BG}(x_j) = \nu_{BF}(x_j) - \nu_{BG}(x_j) \]

\[ \Leftrightarrow \mu_{BF}(x_j) = \mu_{BG}(x_j), \quad \mu_{BF}(x_j) = \mu_{BG}(x_j), \]

\[ \nu_{BF}(x_j) = \nu_{BG}(x_j), \quad \nu_{BF}(x_j) = \nu_{BG}(x_j) \]

\[ \pi_{BF}(x_j) = \pi_{BG}(x_j) \quad \text{and} \quad \pi_{BF}(x_j) = \pi_{BG}(x_j) \]

\[ \Rightarrow (BF, E) = (BG, E). \]

(iii) \( \mathcal{N}_{\Delta E}((BF, E), (BG, E)) = \left\{ \frac{1}{4} \left( (\mu_{BF}(x_j) - \mu_{BG}(x_j))^2 + (\nu_{BF}(x_j) - \nu_{BG}(x_j))^2 \right) \right\}^{1\over2}. \]

It follows that, \( \mathcal{N}_{\Delta E}((BF, E), (BG, E)) = \mathcal{N}_{\Delta E}((BG, E), (BF, E)). \)

(iv) Assume that \((BF, E), (BG, E)\) and \((BH, E)\) are BIFSS over \(U\). 

Then for all \(i \in \{1, 2, \ldots, m\}, \quad j \in \{1, 2, \ldots, q\} \)

\[ (\mu_{BF}(x_j) - \mu_{BG}(x_j))^2 + (\nu_{BF}(x_j) - \nu_{BG}(x_j))^2 \]

\[ + (\mu_{BF}(x_j) - \mu_{BH}(x_j))^2 + (\nu_{BF}(x_j) - \nu_{BH}(x_j))^2 \]

\[ + (\mu_{BF}(x_j) - \mu_{BH}(x_j))^2 + (\nu_{BF}(x_j) - \nu_{BH}(x_j))^2 \]

\[ + (\mu_{BF}(x_j) - \mu_{BH}(x_j))^2 + (\nu_{BF}(x_j) - \nu_{BH}(x_j))^2 \]

\[ + (\mu_{BF}(x_j) - \mu_{BH}(x_j))^2 + (\nu_{BF}(x_j) - \nu_{BH}(x_j))^2 \]

\[ \leq (\mu_{BF}(x_j) - \mu_{BG}(x_j))^2 + (\nu_{BF}(x_j) - \nu_{BG}(x_j))^2 \]

\[ + (\mu_{BF}(x_j) - \mu_{BH}(x_j))^2 + (\nu_{BF}(x_j) - \nu_{BH}(x_j))^2 \]

\[ + (\mu_{BF}(x_j) - \mu_{BH}(x_j))^2 + (\nu_{BF}(x_j) - \nu_{BH}(x_j))^2 \]

\[ = \mathcal{N}_{\Delta E}((BF, E), (BH, E)) + \mathcal{N}_{\Delta E}((BH, E), (BG, E)). \]

Therefore, \( \mathcal{N}_{\Delta E}((BF, E), (BG, E)) \leq \mathcal{N}_{\Delta E}((BF, E), (BH, E)) + \mathcal{N}_{\Delta E}((BH, E), (G, E)). \)

This shows that \( \mathcal{N}_{\Delta E} \) satisfies the triangle inequality.

Therefore, \( \mathcal{N}_{\Delta E} \) is a metric.

4. TODIM method for BIFSS

Here TODIM technique for solving MCDM problem using bipolar intuitionistic fuzzy soft set and some of its concepts are defined.

**Problem statement:**

Let \( U = \{r_1, r_2, \ldots, r_m\} \) be a set of alternatives to be ranked based on the criteria \( E = \{e_1, e_2, \ldots, e_q\} \). Each alternative \( r_i \) is described by a BIFSS over \( U \).

\[ r_i = \{ (\mu_{i1}, \nu_{i1}), (\mu_{i2}, \nu_{i2}), \ldots, (\mu_{iq}, \nu_{iq}) \}, \]

\[ i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, q. \] Taking the data as BIFSS the best alternative is determined using TODIM method.

**Definition:** 4.1 The BIFS dominance degree of one alternative \( r_i \) over each other alternative \( r_j \) based on each criteria \( e_j \) is:
the loss factor, such that $B \theta > \frac{n}{r}$

Step 2: The procedure of TODIM method to solve MCDM problem is:

Procedure 4.4:

Step 4: The weight $w_j$ of each criteria by using Definition 2.1

Step 5: Evaluate the $BIFS$ dominance degree of alternative $r_i$ over each other alternative $r_t$ under criterion $e_j$ using Definition 4.1

Step 6: The $BIFS$ overall dominance degree of alternative $r_i$ over alternative $r_t$, by using Definition 4.2

Example: 4.5.

A standard production firm decides to evaluate the best selling product. Let $D = \{D_1, D_2, D_3, D_4, D_5\}$ represent the set of alternatives (products). The sales performance of the product are analyzed based on the following criteria $E = \{e_1, e_2, e_3, e_4\}$ where, $e_1$ is profit, $e_2$ = market value, $e_3$ = cost, $e_4$ = product quality respectively.

Step 1. $BIFS$ decision matrix is:

$B\Phi(r_i, r_s) = 0$, if $s_{ij} = s_{tj}$ and $\frac{w_{jk} \cdot ND\xi(r_i, r_j)}{\sum_{j=1}^{n} w_{jk}}$, if $s_{ij} > s_{tj}$

$B\Phi_j = \{B\Phi_j(r_1, r_2), \ldots, B\Phi_j(r_1, r_n)\}$

Definition: 4.2 The $BIFS$ overall dominance degree of alternative $r_i$ over each other alternative $r_s$ is defined as:

$B\delta(r_i, r_s) = \sum_{j=1}^{n} B\Phi_j(r_i, r_t) \quad i, t \in m$

$B\delta(r_i, r_s)$ represents the dominance degree measurement.

Definition: 4.3 The $BIFS$ global value of alternative $r_i$ is defined as:

$B\xi(r_i) = \frac{\max \{\sum_{j=1}^{m} B\delta(r_i, r_i)\} - \min \{\sum_{j=1}^{n} B\delta(r_i, r_i)\}}{\sum_{j=1}^{m} B\delta(r_i, r_i)} \quad i = 1, 2, ..., m.$

Procedure 4.4:

The procedure of TODIM method to solve MCDM problem is:

Step 1: Frame a $BIFS$ decision matrix.

Step 2: Compute the $BIFS$ entropy measure $(EM_j)$ of $BIFS$ corresponding to the criteria $e_j$, by using Definition 2.1

Step 3: Determine the $BIFS$ weight $w_{jk}$ of each criteria by using Definition 2.3

Step 4: Establish the $BIFS$ score matrix by using Definition 2.5

Step 5: Calculate the $BIFS$ overall dominance degree of alternative $r_i$ over alternative $r_t$, by using Definition 4.2

Step 6: Compute the $BIFS$ global value of alternative $B\xi(r_i)$ using Definition 4.3

Step 7: Rank the global values, and the alternative which has maximum value is the best one.

$w_{1k} = 1 \ w_{2k} = 0.59 \ w_{3k} = 0.484 \ w_{4k} = 0.52$
Step 4. The BIFS score matrix is as follows:

\[
(B_s) = \begin{pmatrix}
0.167 & 0.152 & 0.07 & 0.172 \\
0.104 & 0.214 & 0.131 & 0.044 \\
0.106 & 0.235 & 0.227 & 0.192 \\
0.201 & 0.118 & 0.09 & 0.174 \\
0.105 & 0.154 & 0.13 & 0.08
\end{pmatrix}
\]

Step 5. The BIFS dominance degree matrices based on the criteria \( e_j \) are:

Here \( B\theta = 1 \)

\[
(B\Phi_1) = \begin{pmatrix}
0 & 0.272 & 0.23 & -0.46 & 0.215 \\
-0.709 & 0 & -0.753 & -0.777 & -0.597 \\
-0.599 & 0.289 & 0 & -0.755 & 0.205 \\
0.175 & 0.298 & 0.29 & 0 & 0.275 \\
-0.559 & 0.229 & -0.456 & -0.794 & 0
\end{pmatrix}
\]
Step 6. $BIFS$ overall dominance degree matrix of alternative $D_i$ over alternative $D_k$ for $B\theta = 1$

$$(B\Phi_2) = \begin{pmatrix} 0 & -0.688 & -0.939 & 0.182 & -0.685 \\ 0.172 & 0 & -0.791 & 0.158 & 0.235 \\ 0.206 & 0.216 & 0 & 0.158 & 0.272 \\ -1.02 & -1.18 & -0.668 & 0 & -0.659 \\ 0.256 & -0.893 & -1.114 & 0.154 & 0 \end{pmatrix}$$

$$(B\Phi_3) = \begin{pmatrix} 0 & -0.676 & -1.336 & -0.889 & -1.06 \\ 0.126 & 0 & -1.17 & 0.181 & 0.248 \\ 0.217 & 0.274 & 0 & 0.176 & 0.165 \\ 0.169 & -0.91 & -0.97 & 0 & -1.474 \\ 0.197 & -1.33 & -0.95 & 0.247 & 0 \end{pmatrix}$$

$$(B\Phi_4) = \begin{pmatrix} 0 & 0.256 & -1.46 & -1.25 & 0.309 \\ -1.06 & 0 & -0.63 & -1.09 & -1.128 \\ 0.231 & 0.285 & 0 & 0.249 & 0.159 \\ 0.233 & 0.213 & -0.794 & 0 & 0.189 \\ -0.134 & 0.126 & -1.167 & -0.947 & 0 \end{pmatrix}$$

$B\theta = 2$

$$(B\delta_1) = \begin{pmatrix} 0 & -0.842 & -3.51 & -2.417 & 0.899 \\ -1.471 & 0 & -3.344 & -1.528 & -1.128 \\ 0.055 & 1.064 & 0 & -0.172 & 0.801 \\ -0.443 & -1.579 & -2.142 & 0 & -1.669 \\ -0.24 & -1.87 & -3.69 & -1.34 & 0 \end{pmatrix}$$

$$(B\delta_2) = \begin{pmatrix} 0 & -0.017 & -1.266 & -0.858 & 0.174 \\ -0.411 & 0 & -1.338 & -0.408 & -1.128 \\ 0.414 & 1.064 & 0 & 0.281 & 0.801 \\ 0.17 & -0.325 & -0.684 & 0 & -0.389 \\ -0.306 & -0.533 & -1.473 & -0.296 & 0 \end{pmatrix}$$
$B\theta = 2.5$

$$(B\delta_3) = \begin{pmatrix} 0 & -0.154 & -1.64 & -1.119 & -0.348 \\ -0.585 & 0 & -1.673 & -0.595 & -1.128 \\ 0.354 & 1.064 & 0 & 0.205 & 0.801 \\ 0.068 & -0.534 & -0.923 & 0 & -0.602 \\ -0.494 & 0.754 & -1.842 & -0.469 & 0 \end{pmatrix}$$

$B\theta = 3$

$$(B\delta_4) = \begin{pmatrix} 0 & 0.074 & -1.017 & -0.684 & -0.057 \\ -0.293 & 0 & -1.114 & -0.284 & -1.128 \\ 0.455 & 1.064 & 0 & 0.331 & 0.801 \\ 0.238 & -0.185 & -0.521 & 0 & -0.246 \\ -0.179 & -0.384 & -1.257 & -0.179 & 0 \end{pmatrix}$$

$B\theta = 4$

$$(B\delta_5) = \begin{pmatrix} 0 & 0.187 & -0.705 & -0.468 & 0.088 \\ -0.146 & 0 & -0.836 & -0.128 & -1.128 \\ 0.504 & 1.064 & 0 & 0.394 & 0.801 \\ 0.323 & -0.011 & -0.318 & 0 & -0.07 \\ -0.02 & -0.2 & -0.92 & -0.034 & 0 \end{pmatrix}$$

**Step 7.** The ranking order of the alternatives are:

From the global value $B\xi(D_i)$, the alternative $D_3$ has the maximum value so it is chosen as the desirable one.

| $D_i$ | $B\xi(D_1)$ | Ranking | $B\xi(D_2)$ | Ranking | $B\xi(D_3)$ | Ranking | $B\xi(D_4)$ | Ranking | $B\xi(D_5)$ | Ranking |
|-------|-------------|---------|-------------|---------|-------------|---------|-------------|---------|-------------|---------|
| $D_1$ | 0.25        | 2       | 0.24        | 3       | 0.112       | 3       | 0.21        | 3       | 0.27        | 3       |
| $D_2$ | 0.5         | 4       | 0.35        | 2       | 0.31        | 2       | 0.38        | 2       | 0.43        | 2       |
| $D_3$ | 1           | 1       | 1           | 1       | 1           | 1       | 1           | 1       | 1           | 1       |
| $D_4$ | 0.16        | 4       | 0.16        | 4       | 0.2        | 4       | 0.15        | 4       | 0.21        | 4       |
| $D_5$ | 0.04        | 4       | 0.16        | 4       | 0.2        | 4       | 0.15        | 4       | 0.21        | 4       |
Figure 1. *BIFS* decision graph.

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