Modified LCM’S Approximation Algorithm for Solving Transportation Problems

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Abstract

In this paper, Modified LCM’s Approximation Algorithm for Solving Transportation Problems has been developed in order to gain foremost fundamental capable solution of transportation issues where entity cut down the transportation expensive. The proposed algorithm is correlate with popular presenting methods corralling NWCM, LCM and improved algorithm and purposed algorithm found that yield to better results. Algorithm is quickest and effective. Few examples are tested by using modified algorithm is correlate with open literature.

Key Words: Transportation problem, Initial Basic Feasible Solution, Optimal Solution

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1. Introduction

Transportation problem is extremely important of linear programming issues in the area of applied mathematics and also in operation research of linear programming and also compulsory mathematical tool that cope with the use of limited resources. The type of linear programming problems which may be resolved by applying a classified version of the easier procedure which can be called as transportation issues.

Balanced Transportation in which demand and supply are equal where as demand and supply are different in unbalance transportation problem. The main purpose of transportation problem is to minimize the transportation cost. To reach the feasible and the optimal solution of transportation problems.

The transportation was introduced in 1941 by Hitchcock in order to distribute of production of several numerous localities. In 1947, T.C. Koopmans presented a study called ‘Optimum Utilization of the Transportation System’. These two contributions are the fundamentals for the progress of transportation problem. These methods such as LCM, NWCM, IM, VAM and ILCM.

The basic initial solution Methods to solve transportation problems are;

- Northwest Corner Method (NWCM)

This is the first to be solved transportation issues, is simple way to minimize the cost. At beginning northwest corner method, we can count the northwest cell in table of transportation by crossing out the rows with nothing to supply or demand by this procedure, continues till we can minimize obtain cost.

- Least Cost Method (LCM)

In this method we can count low cost in table of transportation by crossing out columns with nothing to supply and demand, we continue this procedure till we can obtain minimize cost.
- Improved Algorithm (IM)

To sum up 1st about penalty by taking difference between lower to next lower cost. Taking difference between the largest and smallest in column and in row. This procedure is continued till get optimal result.

Let \( X_{nk} \geq 0 \) be the quantity shipped from the inception “n” to the emplacement “K”. The mathematical formulation of the problem is given below.

Minimize \\
\[ Z = \sum_{n=1}^{N} \sum_{k=1}^{K} m_{nk} x_{nk} \] (Total Transportation cost)

Subject to \\
\[ \sum_{k=1}^{K} x_{nk} = q_n \text{(Supply from inception)} \]
\[ \sum_{n=1}^{N} x_{nk} = s_k \text{(Demand from emplacement)} \]
\[ x_{nk} \geq 0 \text{ for all } n \text{ and } k. \]

where 
\( Z \): Total transportation cost to be reduced
\( C_{nk} \): Unit transportation cost of the commodity from each inception n to emplacement k
\( X_{nk} \): Number of units of commodity sent from each inception n to emplacement k.

\( Q_n \): level of supply at each inception n.
\( S_k \): level of demand at each emplacement k.

Note: Transportation model is balanced if supply \( (\sum_{n=1}^{N} q_n) = \text{Demand (} \sum_{k=1}^{K} s_k) \)
Otherwise unbalanced if supply \( (\sum_{n=1}^{N} q_n) \neq \text{Demand (} \sum_{k=1}^{K} s_k) \)

Table of the general transportation Problem

| Destinations | Origins | \( l_1 \) | \( l_2 \) | \( \ldots \) | \( l_i \) | \( \ldots \) | \( l_k \) | Supply: \( q_n \) |
|--------------|---------|---------|---------|----------|---------|----------|---------|----------------|
| \( p_1 \)    |         | \( m_{11} \) | \( m_{12} \) | \( \ldots \) | \( m_{1i} \) | \( \ldots \) | \( m_{1k} \) | \( q_n \) |
| \( p_2 \)    |         | \( m_{21} \) | \( m_{22} \) | \( \ldots \) | \( m_{2i} \) | \( \ldots \) | \( m_{2k} \) | \( q_n \) |
| \( \vdots \) |         | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) |
| \( p_N \)    |         | \( m_{N1} \) | \( m_{N2} \) | \( \ldots \) | \( m_{Ni} \) | \( \ldots \) | \( m_{Nk} \) | \( q_n \) |
| \( \vdots \) |         | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) | \( \vdots \) |
| \( p_R \)    |         | \( m_{c1} \) | \( m_{c2} \) | \( \ldots \) | \( m_{ci} \) | \( \ldots \) | \( m_{ck} \) | \( q_n \) |

Demand: \( p_k \)
\( p_1 \) \( p_2 \) \( \ldots \) \( p_k \) \( \ldots \) \( p_r \)

\[ \sum_{n=1}^{N} q_n = \sum_{k=1}^{K} S_k \]

The total number of variables is O.R. The total number of constraints is \((o + r)\) while the total number of locations \((o + r - 1)\) should be in feasible solution. Here the letter indicates the number of rows and indicate the number of columns.
2. Methodology

\[
\begin{array}{cccccc}
C_{11} & C_{12} & C_{13} & \cdots & C_{1n} \\
C_{21} & C_{22} & C_{23} & \cdots & C_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
C_{m1} & C_{m2} & C_{m3} & \cdots & C_{mn} \\
\end{array}
\]

The cost cells where i = 1, 2, 3, ..., n and j = 1, 2, 3, m.

1) Transportation problem ought to be \(\sum_{i=1}^{n} q_n - \sum_{i=1}^{n} s_i\). In case it is not balanced a dummy variable needs to be added in order to balance it.

2) Select large number from each columns of each cells and subtract that number from each entry of that cell. Suppose \(c_{31}\) is large number in \(C_{11}\).

3) Allocate largest absolute number of each column corresponding column with respect to supply and demand.

4) If there are similar in column \(m_1 = m_2 = m_n\) then the minimum cell has to be selected from the given cells finally there to be applied \(S_i\) and \(q_n\).

5) If \(S_i\) and \(q_n\) of the current row are completed we shall move towards the next row repeat step 1-4 till all quantities are exhausted.

3. NUMERICAL ILLUSTRATION

In this paper, esteem four dissimilar size cost minimizing transportation problems, chosen from literature. We also describe these examples to perform a comparative study of proposed algorithm with north and west corner and least cost methods. We solve example 1 step-by-step continuous.

Example No: 1

| Destination source | \(D_1\) | \(D_2\) | \(D_3\) | Supply |
|-------------------|---|---|---|---|
| \(S_1\)           | 6 | 4 | 1 | 50 |
| \(S_2\)           | 3 | 8 | 7 | 40 |
| \(S_3\)           | 4 | 4 | 2 | 60 |
| Demand            | 20| 95| 35| \(\Sigma\) 150 |

Table 2: Solve step by step problem 1

Step 1: Count on the different column by receiving large number from each columns of each cells and subtract from each entry of that cell of each columns Table 2.1.

| Destination source | \(D_1\) | \(D_2\) | \(D_3\) | Supply |
|-------------------|---|---|---|---|
| \(S_1\)           | 6 | 4 | 1 | \(35\) |
| \(S_2\)           | 3 | 8 | 7 | 40 |
| \(S_3\)           | 4 | 4 | 2 | 60 |
| Demand            | 20| 95| 35| \(\Sigma\) 115 |

Table 2.1

Step 2: Using table 2.1 Taking largest absolute number in columns
**Table 2.2**

| Destination source | D₁ | D₂ | Supply          |
|--------------------|----|----|-----------------|
| S₁                 | 6  | 4  | 15 – 15 = 0     |
| S₂                 | 3  | 8  | 40              |
| S₃                 | 4  | 4  | 60              |
| Demand             | 20 | 95 - 15 = 80 | ∑ 100           |

Step 3: Using table 2.2 Taking largest absolute number in columns

\[ X_{11} = \max (6, 3, 4) = 0, \quad X_{12} = \max (4, 8, 4) = 4 \]

4 is largest absolute number in column S₁ is 4 allocated supply and demand S₁ is detected because its supply is zero.

**Table 2.3**

| Destination source | D₁ | D₂ | Supply          |
|--------------------|----|----|-----------------|
| S₂                 | 3  | 8  | 40 - 20 = 20    |
| S₃                 | 4  | 4  | 60              |
| Demand             | 20 | 80 | ∑ 80            |

Step 4: Using table 2.3 Taking largest absolute number in columns

\[ X_{21} = \max (3, 4) = 1, \quad X_{22} = \max (8, 4) = 0 \]

1 is largest absolute number then detect D₁ column because its supply is zero.

**Table 2.4**

| Destination source | D₂ | Supply          |
|--------------------|----|-----------------|
| S₂                 | 8  | 20 – 20 = 0    |
| S₃                 | 4  | 60              |
| Demand             | 80 | 20 – 20 = 60   |

\[ ∑ 60 \]
Step 5: Using table 2.4 Taking largest absolute number in column $X_{22} = \max \{8,4\} = 8$ so here 8 is largest absolute number in row $S_3$ is detected because supply is zero.

| Destination source | D_2 | Supply |
|--------------------|-----|--------|
| S_3                | 4   | 60     |
| Demand             | 60 – 60 | Σ 0   |

Table 2.5

In last, we have allocated demand and supply and then detect the exclusive matrix due to nothing to supply and demand.

Steps 6: applying table 1 all allocates hints for gaining minimized cost.

| Destination source | D_1 | D_2 | D_3 | Supply |
|--------------------|-----|-----|-----|--------|
| S_1                | 6   | 4   | 1   | 50     |
| S_2                | 3   | 8   | 7   | 40     |
| S_3                | 4   | 4   | 2   | 60     |
| Demand             | 20  | 95  | 35  | Σ 150  |

$Z = 1 \times 35 + 4 \times 15 + 3 \times 20 + 4 \times 60 = 35 + 60 + 60 + 160 + 240 = 555$

4. Optimality Test of Example 1

Taking initial basic feasible solution due to proposed method, we now proceed for optimality using modified Distribution methods here calculate $u_i$ and $v_j$ for occupied basic cell using $u_i + v_j = c_{ij}$

Initial we take $u_1 = 0$

$C_{12} = 41 + v_2 = 4 => V_2 = 4$

$C_{13} = 41 + v_3 = 1 => 0 + v_3 = 1 => v_3 = 1$

$C_{21} = u_2 + v_1 = 3 => 4 + v_1 = 3 => v_1 = -1$

$C_{22} = u_2 + v_2 = 8 => u_2 + 4 = 8 => u_2 = 4$

$C_{32} = u_3 + v_2 = 4 => u_3 + 4 = 0 => u_3 = 0$
| Destination Source | D_1 | D_2 | D_3 | Supply | \( U_i \) |
|---------------------|-----|-----|-----|--------|--------|
| S_1                 | 6   | 45  | 135 | 50     | 4_1 = 0 |
| S_2                 | 3   | 20  | 8   | 20     | 4_2 = 4 |
| S_3                 | 4   | 4   | 2   | 60     | 4_3 = 0 |
| Demand              | 20  | 95  | 35  | \( \sum 150 \) |

\begin{align*}
D_{11} &= c_{11} - (U_1 + U_2) = 6 - (0 + 1) = 6 - 1 = 5 \\
D_{23} &= c_{23} - (U_2 + V_3) = 7 - (4 + 1) = 7 - 5 = 2 \\
D_{31} &= c_{31} - (U_3 + V_1) = 4 - (0 + 1) = 4 - 1 = 3 \\
D_{11} &= c_{33} - (U_3 + V_3) = 2 - (0 + 1) = 2 - 1 = 1 \\
D_{11} &= 5, D_{23} = 2, D_{31} = 3, D_{33} = 1 \\
\text{Check the all unoccupied cell single}
\end{align*}

| Destination Source | D_1 | D_2 | D_3 | Supply |
|---------------------|-----|-----|-----|--------|
| S_1                 | 6   | 45  | 135 | 50     |
| S_2                 | 3   | 20  | 8   | 20     |
| S_3                 | 4   | 4   | 2   | 60     |
| Demand              | 20  | 95  | 35  | \( \sum 150 \) |

\begin{align*}
V_j &= V_1 = 1, V_2 = 4, V_3 = 1
\end{align*}

All unoccupied Single are \( D \geq 0 \) then optimal Solution is
\[ Z = 4 \times 5 + 1 \times 35 + 3 \times 20 + 8 \times 20 + 4 \times 60 = 555 \text{ Answer.} \]
The same process is adopted on various examples given below.

| Example-2 | Destination | Source | D1 | D2 | D3 | Supply | Optimal Solution |
|-----------|-------------|--------|----|----|----|--------|------------------|
|           | S1          | 6      | 8  | 4  | 14 |        | 143              |
|           | S2          | 4      | 9  | 8  | 12 |        |                  |
|           | S3          | 1      | 2  | 6  | 5  |        |                  |
| Demand    | 5           | 10     | 15 | 15 | 31 |        |                  |

| Example-3 | Destination | Source | D1 | D2 | D3 | D4 | D5 | Supply | Optimal Solution |
|-----------|-------------|--------|----|----|----|----|----|--------|------------------|
|           | S1          | 4      | 1  | 2  | 4  | 4  | 4  | 60     | 273              |
|           | S2          | 2      | 3  | 2  | 2  | 2  | 2  | 35     |                  |
|           | S3          | 3      | 5  | 2  | 4  | 4  | 4  | 40     |                  |
| Demand    | 22          | 45     | 20 | 18 | 30 | 135|     |        |                  |

| Example-4 | Destination | Source | D1 | D2 | D3 | D4 | Supply | Optimal Solution |
|-----------|-------------|--------|----|----|----|----|--------|------------------|
|           | S1          | 7      | 5  | 9  | 11 | 30 |       | 430              |
|           | S2          | 4      | 3  | 8  | 6  | 25 |       |                  |
|           | S3          | 3      | 8  | 10 | 5  | 20 |       |                  |
|           | S4          | 2      | 6  | 7  | 3  | 15 |       |                  |
| Demand    | 30          | 30     | 20 | 10 | 30 | 90 |       |                  |

| Example-5 | Destination | Source | D1 | D2 | D3 | D4 | Supply | Optimal Solution |
|-----------|-------------|--------|----|----|----|----|--------|------------------|
|           | S1          | 3      | 1  | 7  | 4  | 300|       | 2850             |
|           | S2          | 2      | 6  | 5  | 9  | 400|       |                  |
|           | S3          | 8      | 3  | 3  | 2  | 500|       |                  |
| Demand    | 250         | 350    | 400| 200| 1200|     |       |                  |
5. RESULTS AND DISCUSSION

It has been examined that the performance of proposed Algorithm in comparison to LCM, NWCM and Improved Algorithm by examining five examples the result gained using proposed algorithm in examples 1, 2, 3 and 5 was same as the optimal outcome. While in example 4 it is close to the optimal solution. It could be clearly seen that the proposed algorithm gave reliable results which was against with the flourishing methods- NWCM, LCM and Improved Algorithm.

| No. of Example | Type of problem | Result of NWCM | Result of LCM | Result of IM | Result of proposed Methods | Optimal Solution |
|----------------|----------------|----------------|---------------|--------------|---------------------------|------------------|
| Example-1      | 3 * 3          | 730            | 555           | 555          | 555                       | 555              |
| Example-2      | 3 * 3          | 228            | 163           | 144          | 143                       | 143              |
| Example-3      | 3 * 5          | 363            | 278           | 273          | 273                       | 273              |
| Example-4      | 4 * 4          | 540            | 435           | 415          | 430                       | 410              |
| Example-5      | 3 * 4          | 4400           | 2900          | 2850         | 2850                      | 2850             |

6. CONCLUSION

In this paper, for achieving fundamental capable solution of transportation issues, Modified LCM’S approximation Algorithm has been developed. The proposed algorithm has been looked over for optimality. A comparison of proposed algorithm has been made with Least Cost Method, North West Corner Method and An Improved Algorithm by examining 5 numerical examples. It is examined that the proposed algorithm has succumbed capable of outcome which were opposite to the conventional methods.

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