A Local Update and Refinement method of the Global Terrain Nature Grid

Wang Hongbin¹, Zhao Junwu², Liu Yuanlin¹, Ma Jiayi¹
¹Division of Environmental Engineering, CNNC Beijing Research Institute of Uranium Geology, Beijing, 100029, China
²College of Geoscience and Surveying Engineering, China University of Mining and Technology, Beijing, 100083, China
*Corresponding author’s e-mail: whbcumtb@163.com

Abstract. As an important data model for the exploration and interpretation of large range natural phenomena such as hydrological analysis, climatic analysis, simulation of sea level rise, Global Terrain Nature Grid (GTNG) partitions the earth surface into nature cells based on Morse complex, directly revealing the required global terrain morphological information at different scales. Recently, with the development of data acquisition technology, high temporal/spatial resolution terrain data sets are being collected. However, there is no top-down local refinement operation for GTNG, which is very important to the representation of Morse features dealing with the ever-growing global massive terrain data. On the other hand, due to the updating of local terrain data, GTNG needs a corresponding local reconstruction method to suit the present situation. To this aim, an accurate and efficient method to update and refine GTNG locally is presented. First, local terrain is updated through constraint Delaunay triangulation. Then, local cells of GTNG are computed and updated according to the topological relation between the terrain triangles and Morse cells. By this method, local cells of GTNG can not only be updated but also be refined successively using terrain data with higher resolution than the underlying mesh. In the end, an experiment is done to validate the correctness and feasibility of this method.

1. Introduction
Terrain morphology consists of feature points (pits, peaks and passes), feature lines (such as ridges and ravines) or segmentation of the terrain in regions of influence of pit and peak or in regions of uniform gradient field [Floriani et al. 2010]. Based on Morse complex [Floriani et al. 2010; Danovaro et al. 2007], Global Terrain Nature Grid (GTNG) partitions the earth surface into nature cells and directly reveals the global terrain morphological information at different scales, which is usually required in the exploration and interpretation of some extensive natural phenomena such as environmental monitoring, hydrological analysis, climatic analysis, simulation of sea level rise, etc.

Over the years, lots of the algorithms to construct Morse or Morse-Smale complex have been proposed [Floriani et al.2010; Pascucci 2004; Bajaj and Shikore 1998; Takahashi et al. 1995; Magillo et al. 2009], and most of them begin with feature points extraction from the constructed mesh and are applied to local terrain, human organs or small animals. As a result, since the data size may be small, both the underlying mesh and Morse complex can be rebuilt easily using the overall datasets if update occurs. On the other hand, in the hierarchy of Morse complexes[Iuricich et al. 2015; Čomić 2014; Szymczak 2012; Čomić & Floriani 2008; Bremer & Pascucci 2005; Bremer 2004; Edelsbrunner et al.
2003; Čomić & Floriani 2011; Floriani et al. 2010; Danovaro et al. 2007; Bremer et al. 2004; Danovaro et al. 2010; Iuricich & De Floriani 2017], an operator called cancellation [Milnor 1963] is used, and each refinement (also called anti-cancellation) performs an undo of the corresponding cancellation. As a result, Morse features can not be refined any more until at the initial Morse complex.

Recently, with the development of data acquisition technology, high temporal/spatial resolution terrain datasets are being collected, resulting in big size and frequent updates. As a result, it is time consuming to reconstruct GTNG from the global massive terrain data, where update often occurs at local. Additionally, there is no top-down approach to construct the hierarchy of Morse complexes in the literature, where local refinement can be achieved continuously. Actually, the top-down approach is very important to GTNG, since the extraction, representation or visualization of Morse features are based on the ever-growing global massive terrain data, and the the complexity is mainly affected by the resolution of the mesh and not by the size of the Morse Complexes [Iuricich and De Floriani 2017]. Therefore, in order to deal with the challenge of data-acquisition technology development, a local update and refinement method of GTNG is presented in this paper.

The remainder of this paper is organized as follows. In Section 2, some basic notions of Morse Theory and Morse complex are introduced briefly. In Section 3, we review the related works on both construction and hierarchical representation of Morse complexes. After that, a local update and refinement method of GTNG is proposed. Following is an experiment designed and developed to validate this algorithm. Finally, some concluding remarks are drawn.

2. Morse theory and Morse complex

Morse theory is a powerful tool to capture the morphology of manifold on which a scalar function $f$ is defined. Let $f$ be a $C^2$-differentiable real-valued function defined over a domain $D$ in the 2D space. A point $p$ of $D$ is a critical point of $f$ if and only if the gradient of $f$ vanishes on $p$. Function $f$ is said to be a Morse function when all its critical points are non-degenerate (i.e., if and only if the Hessian matrix $\text{Hess} f$ is non-singular: the determinant of $\text{Hess} f$ is not zero ). This implies that the critical points of $f$ are isolated. The number of negative eigenvalues of $\text{Hess} f$ is called the index of a critical point $p$. There are three types of non-degenerate critical point $p$: a minimum (pit), a saddle (pass), or a maximum (peak) if and only if $p$ has index 0, 1 or 2, respectively. Points which are not critical are said to be regular.

An integral line of $f$, going through a point $p$, is a maximal path which is everywhere tangent to the gradient of $f$. Integral lines that converge to (originate from) a critical point $p$ of index 1 form an i-cell ((2-i)-cell) called the descending (ascending) cell of $p$. The collection of all descending cells form a Euclidean cell complex, called descending Morse complex (Figure 1a), and the collection of all ascending cells also form a Euclidean cell complex, called ascending Morse complex (Figure 1b), which is dual with respect to the descending Morse complex. A Morse function $f$ satisfying the Morse-Smale condition, that is, descending and ascending Morse complexes intersect only transversely, is called a Morse-Smale function. This means that the intersection of the edges of the descending and ascending cells are a saddle point. In this case, we can define a complex, called the Morse-Smale complex, whose cells are the connected components of the intersections of the descending and ascending cells (Figure 1c). Integral lines connecting saddle points to other critical points are called separatrices. All separatrices forms a network, called the critical net. It is the 1-skeleton of the Morse-Smale complex.

In terrain analysis, the separatrix starting from saddle to peak is called a ridge line, while the one starting from saddle to pit is called a ravine line. Essentially, the surface network[Pfaltz 1976; Schneider 2005] consisting of ridges and ravines is the critical net.
3. Related work

In many disciplines such as computer graphics, cartography, physics, biology, chemistry, lots of algorithms have been proposed to decompose the domain of a scalar field into an approximation of a Morse complex (or of a Morse-Smale complex). Based on the approach they use, existing algorithms can be classified as [Floriani et al. 2015; Čomić 2014]: boundary-based methods[Takahashi et al. 1995; Bajaj and Shikore 1998; Eldersvbrunner et al. 2003; Schneider 2005], region-growing methods[Magillo et al. 2009], watershed methods[Soille 2004; Mangan and Whitaker 1999; Stoev and Strasser 2000], and Forman-based methods[King et al. 2005; Gyulassy et al. 2008; Robins et al. 2011; Weiss et al. 2013]. Generally, there are mainly three steps in the construction of Morse complex: constructing the underlying mesh, extracting feature points and constructing the Morse complex. From the view of research objects and experimental data in the literature, they are mainly applied into the small such as animals, human organs, molecule or local terrain. Since the data size may be small, it can be convenient to reconstruct the underlying mesh and Morse complex when update occurs. Conversely, in the exploration and interpretation of large range natural phenomena, the whole earth surface is considered and the terrain data is huge. What’s more, data acquisition and update is often limited at local regions as some natural disaster happens, taking the landslide or earthquake for instance. In this case, it is time consuming to reconstruct GTNG from the global massive terrain data. To suit the present situation of GTNG, a local reconstruction method is required.

To reduce topological noise and data redundancy, Morse complexes can be simplified by applying an operator called cancellation [Matsumoto 2002]. A cancellation removes two critical points of consecutive index which are connected by a separatrix line. Given a sequence of cancellations, a hierarchical model is built by organizing into a hierarchy. Over the years, a large amount of discussion about hierarchical representation of Morse complexes has been done in the literature, such as [Iuricich et al. 2015; Čomić 2014; Szymczak 2012; Čomić & Floriani 2008; Bremer 2004; Edelsbrunner et al. 2003; Čomić & Floriani 2011; Floriani et al. 2010; Danovaro et al. 2007]. In those models, the underlying mesh is always kept at full resolution. As a result, it is a serious issue when working with big datasets [Iuricich & De Floriani 2017]. To relate a multi-resolution morphological representation with its geometrical representation, a parametric remeshing approach is proposed by [Bremer et al. 2004] to adapt the geometrical mesh at full resolution by performing a smoothing process in order to delete those critical points that are simplified in the morphological model. Alternatively, the Multi-resolution Morse Triangulation (MMT) and Hierarchical Forman Triangulation (HFT) are proposed by [Danovaro et al. 2010] and [Iuricich & De Floriani 2017] respectively. However, most of those hierarchical Morse complexes are built from bottom to up, each refinement (also called anti-cancellation) performing an undo of the corresponding cancellation. As a result, Morse features can not be refined any more until the full resolution of the underlying mesh is reached. If more finer details are required, both mesh and Morse complex need to be reconstructed from the datasets with higher resolution. As mentioned above, this is a very serious issue to GTNG. According to our knowledge, there is no top-down approach for GTNG in the literature to achieve continuous local refinement.
4. A Local Update and Refinement method of GTNG

In this section, in order to update and refine GTNG locally, an accurate and efficient method based on local constrained triangulation is addressed in details.

Our work is assumed that an initial GTNG model consisting of ascending or descending Morse cells and that a local terrain dataset with higher resolution than that of the underlying mesh is given. In fact, extensive or global terrain dataset can be divided into subsets, according to a certain geospatial range or point number. The initial GTNG model is is built from a global terrain triangular network with lower resolution by the method proposed in [Wang et al. 2016], where a point is whether critical or not depends on the elevation comparison with its neighbors. Generally speaking, our approach of updating GTNG locally mainly consists of two steps: the first is local update of the global terrain triangular network and the following is local reconstruction of Morse complexes.

4.1 local update of global terrain triangular network

First of all, according to the given dataset, the corresponding update range needs to be confirmed and the triangles of the initial global triangular network in this range needs to be extracted. To this aim, the method of a point positioning on spherical surface in [Tong et al. 2009] is used.

As shown in figure 2, \(A, B, C\) are three vertexes of a triangle of the initial global triangular network and \(P\) is a point of the local dataset. Here, all the elevations of them are not considered. As a result, these four points can consist of four triangles on spherical surface, and the area of triangle \(ABC, ABP, BCP\) and \(ACP\) is computed respectively according to the formula (4.1) [Zhao et al. 2016]. Here, \(X_1, X_2\) and \(X_3\) are the three vertex vectors of triangle ABC on spherical surface and \(R\) is the sphere radius to simulate the earth. If the total area of triangle \(ABP, BCP\) and \(ACP\) is equal to the area of triangle \(ABC\), then the triangle \(ABC\) of the initial global triangular network is the object to be extracted.

\[
S = R^2 \left[ \sum_{i=1}^{3} \alpha - \pi \right] \quad (4.1)
\]

\[
\alpha = \cos^{-1} \left( \frac{X_i \times X_{i+1} \cdot X_j \times X_{j+1}}{|X_i \times X_{i+1}| \cdot |X_j \times X_{j+1}|} \right) \quad (4.2)
\]

\(i=1, 2, 3; \quad X_1 = X_0; \quad X_3 = X_1\)

Using this method, triangles are finally accurately picked out from the initial global mesh according to the given dataset, and the boundary of the local region consisting of these triangles can be extracted. For simplicity, the extracted triangles and region are written as \(T\) and \(R_T\) respectively, with the boundary of \(R_T\) as \(B\), as shown in Figure 3. To achieve the local update of global terrain, Delauney triangulation is done using the given dataset, where \(B\) is regarded as the constraint. The terrain of the region \(R_T\) is update.
4.2 local reconstruction of global Morse complexes

After the underlying mesh is update, GTNG requires to be reconstructed to suit the present situation. Here, according to the relationship between triangles and a vertex on the boundary \( B \), the exterior triangles along with the constraint boundary \( B \) (those in brown color as shown in Figure 3) are extracted and added into \( T \). Following, in the initial GTNG model, only the Morse cells including any triangle of \( T \) are reconstructed by the STD algorithm [Magillo et al. 2009], after the vertexes in the local update terrain are reclassified into the critical or the regular. The principle of this operation is as follows:

In the GTNG model, a Morse cell consists of some triangles of the global terrain mesh. Meanwhile, a critical point is conformed by elevation comparison with its neighbors. So, when the local constraint triangulation is done, only the elevation and neighborhood of vertexes within the region \( R_T \) (including the boundary) changes, which may produce new critical or regular points. Taking the descending cell \( C_a \) and \( C_b \) as shown in figure 3 for instance, point \( p_2 \) is a peak on the boundary of the cell \( C_b \) before local update, and may be regular when its neighborhood with other vertexes changes after local constraint triangulation is done. Therefore, cell \( C_b \) needs to be reconstructed, according to the fact that it consists of some triangles in the set \( T \) associating to point \( p_2 \). Therefore, the exterior triangles along with the boundary \( B \) are extracted. On the contrary, the boundary vertexes of the cell \( C_a \), taking \( p_1 \) for example, do not locate on the boundary \( B \) or the internal of the region \( R_T \). Both its elevation and neighborhood with other vertexes change nothing after the local terrain update, to any triangle of cell \( C_a \). Therefore, it need not to reconstruct this cell.

The main flowchart of our local update operation for GTNG is shown in Figure 4.
Alternatively, local terrain can be refined by the restrictive quad-tree algorithm [Wang et al. 2016]. However, a large amount of redundant triangles will be generated around the local update region. As a result, the calculation cost gets higher and the affected range becomes larger. On the contrary, in this operation, although the exterior triangles along with the constraint boundary are extracted and added into the computation of new critical points and cells, the other parts of GTNG don’t need to be updated.

5. Experiments and Analysis

In this section, in order to validate the feasibility and correctness of our method, an experimental prototype system is designed and developed by C++ and OSG (Open Scene Graph). Here, the source data of the initial GTNG model is GTOPO30 elevation dataset (http://www1.gsi.go.jp/geowww/globalmap-gsi/gtopo30/gtopo30.html) and the local test terrain data is selected at Qinghai-Tibet Plateau from the SRTM dataset (http://srtm.csi.cgiar.org). Respectively, the horizontal resolution of both is about 1000m and 90m.

The figure 5a shows an initial GTNG model consisting of descending Morse cells, which is constructed by the method [Wang et al. 2016]. Here, the earth surface is uniformly partition into the diamond grid at level 3 first of all, and then each diamond node is refined recursively to the max level 7, if the maximum elevation difference of the diamond vertexes exceeds the given threshold 10.0m. As a result, the nearest tow vertexes of the initial terrain triangular network are separated by about 0.61°.

Meanwhile, the local test terrain data is picked out from the SRTM dataset, where two adjacent points are separated by about 0.1°, as shown in green color in the figure 5b. What’s more, by the method of a point positioning on spherical surface proposed by [Tong et al. 2009], triangles are extracted from the initial global terrain triangular network according to the local terrain dataset, as shown in red color. The boundary of this region is conformed, also as shown in bulle color in the right picture of the figure 5a. Additionally, the cells which need to be rebuilt of the initial GTNG are shown in purple color.

Local terrain is update as shown in the figure 5c by the way of constraint Delaunay triangulation. From the local enlarged map, we can see that there are some long and narrow triangles but no cracks as well as T-junctions at the boundary, which is beneficial to the critical points extraction. The figure
5d shows the local reconstruction of the GTNG. From the right local enlarged view, we can see that all the descending cells are correct. More importantly, from the comparison between (a) and (d), especially in the local enlarged view, we can see that Morse features of the GTNG are refined locally. It indicates that our approach is feasible and correct. Using this operation iteratively, local Morse features can be continuously refined.
6. Concluding Remarks
With the development of data acquisition technology, high temporal/spatial resolution terrain datasets are being collected, resulting in big size and frequent updates. However, there is no top-down operation for GTNG in the literature to achieve continuous local refinement, which is very important to the representation of Morse features dealing with the ever-growing global massive terrain data. Additionally, GTNG needs a corresponding model reconstruction method to suit the present situation. To this aim, a local update and refinement approach of GTNT is proposed here. The main contribution of this work is as follows:

1) It can achieve accurate local update of GTNG and avoid the cells reconstruction outside of the update region. As a result, it reduces computation cost, comparing with the global reconstruction.

2) It can directly refine the Morse features locally with terrain dataset of higher resolution than the full resolution of underlying mesh.

Terrain Morphological representation with coastline will be considered in our future work.

Acknowledgements
This work has been supported by the China National Science Foundation under grant number 41601433.

References
[1] Floriani L. D., Magillo P., Vitali M.. (2010) Modeling and Generalization of Discrete Morse Terrain Decompositions. In: The 20th International Conference on Pattern Recognition. Turkey. pp.999-1002.
[2] Danovaro E., Floriani D. L.,and Vitali M., et al.. (2007) Multi-scale dual Morse complexes for representing terrain morphology. Proceedings of the 15th ACM International Symposium on Advances in Geographic Information Systems. pp.220-227.
[3] Pascucci V.. (2004) Topology diagrams of scalar fields in scientific visualization. In: Rana S., (Eds.), Topological Data Structures for Surfaces. John Wiley & Sons Ltd. pp.121-129.
[4] Bajaj C. L., Shikore D. R.. (1998) Topology preserving data simplification with error bounds. Computers and Graphics, 22(1):3-12.
[5] Takahashi S., Ikeda T., Kunii T. L., and Ueda M.. (1995) Algorithms for extracting correct critical points and constructing topological graphs from discrete geographic elevation data. In Computer Graphics Forum, 14:181-192.
[6] Magillo P., Danovaro E., De Floriani L., Papaleo L., and Vitali M.. (2009) A discrete approach to compute terrain morphology. Computer Vision and Computer Graphics Theory and Applications, 21:13-26.

[7] Iuricich F., Fugacci U., De Floriani L.. (2015) Topologically-consistent simplification of discrete Morse complex. Computers & Graphics, 51:157-166.

[8] Čomić L.. (2014) Operators for Multi-Resolution Morse and Cell Complexes. PHD thesis, University of Novi Sad.

[9] Szymczak A.. (2012) Hierarchy of Stable Morse Decompositions. IEEE Transactions on Visualization and Computer Graphics, 19(5):799-810.

[10] Čomić L., De Floriani L.. (2008) Multi-scale 3D Morse complexes. In: International Conference on Computational Sciences and Its Applications (ICCSA 2008). pp.441-451.

[11] Bremer P.-T., Pascucci V., Hamann B.. (2005) Maximizing adaptivity in hierarchical topological models. In: International Conference on Shape Modeling and Applications. California. pp.300-309.

[12] Bremer P.-T., Edelsbrunner H., Hamann B., and Pascucci V.. (2004) A topological hierarchy for functions on triangulated surfaces. IEEE Transactions on Visualization and Computer Graphics, 10(4):385–396.

[13] Edelsbrunner H., Harer J., Zomorodian A.. (2003) Hierarchical Morse–Smale Complexes for Piecewise Linear 2-Manifolds. Discrete & Computational Geometry, 30:87-107.

[14] Čomić L., De Floriani L.. (2011) Dimension-independent simplification and refinement of Morse complexes. Graphical Models, 73(5):261-285.

[15] Danovaro E., De Floriani L., Magillo P., Vitali M.. (2010) Multiresolution Morse triangulations. In: Proceedings of the fourteenth ACM symposium on solid and physical modeling-SPM’10. New York, NY, USA. pp.183.

[16] Iuricich F., De Floriani L.. (2017) Hierarchical Forman Triangulation: A multiscale model for scalar field analysis. Computers & Graphics, 66:113-123.

[17] Milnor J. W.. (1963) Morse theory. Princeton University Press. New Jersey.

[18] Pfaltz J. L.. (1976) Surface networks, Geographical Analysis, 8: 77-93.

[19] Schneider, B.. (2005) Extraction of hierarchical surface networks from bilinear surface patches. Geographical Analysis, 37(2):244-263.

[20] Floriani D. L., Fugacci U., Iuricich F., Magillo P.. (2015) Morse Complexes for Shape Segmentation and Homological Analysis: Discrete Models and Algorithms. Computer Graphics Forum. 34(2):761-785.

[21] Soille P.. (2004) Morphological Image Analysis: Principles and Applications. Springer-Verlag, Berlin and New York.

[22] Mangan A., Whitaker R.. (1999) Partitioning 3D surface meshes using watershed segmentation. Transactions on Visualization and Computer Graphics, 5(4):308-321.

[23] Stoel S. L., Strasser W.. (2000) Extracting regions of interest applying a local watershed transformation. In: Proc. IEEE Visualization’00, ACM Press, pp.21-28.

[24] King H., Knudson K., and Mramor N.. (2005) Generating discrete Morse functions from point data. Experimental Mathematics, 14(4):435-444.

[25] Gyulassy A., Bremer P.-T., Hamann B., and Pascucci V.. (2008) A practical approach to MorseSmale complex computation: Scalability and generality. IEEE Transactions on Visualization and Computer Graphics, 14(6):1619-1626.

[26] Robins V., Wood P. J., and Sheppard A. P.. (2011) Theory and algorithms for constructing discrete Morse complexes from grayscale digital images. IEEE Transactions on Pattern Analysis and Machine Intelligence, 33(8):1646-1658.

[27] Weiss K., Iuricich F., Fellegara R., and De Floriani L.. (2013) A primal/dual representation for discrete Morse complexes on tetrahedral meshes. Computer Graphics Forum, 32(3):361-370.
[28] Matsumoto Y. (2002) An Introduction to Morse Theory, volume 208 of Translations of Mathematical Monographs. American Mathematical Society.

[29] Wang H., Zhao X., Zhu X., Li J. (2016) A Global “Natural” Grid Model Based on the Morse complex. In: IOP Conference Series: Earth and Environmental Science. IOP Publishing, 46(1):012021.

[30] Tong X., Ben J., Qin Z., Zhang Y. (2009) The Subdivision of Partial Grid Based on Discrete Global Grid Systems. Acta Geodaetica et Cartographica Sinica, 38(6):506-513.

[31] Zhao Xuesheng, Yuan Zhengyi, Zhao Longfei, et al. (2016) An Improved QTM Subdivision Model with Approximate Equal-area. Acta Geodaetica et Cartographica Sinica, 45 (1): 112-118.