Stability of exomoons around the Kepler transiting circumbinary planets

Adrian S. Hamers1⋆, Maxwell X. Cai2†, Javier Roa3‡, and Nathan Leigh4,5§

1Institute for Advanced Study, School of Natural Sciences, Einstein Drive, Princeton, NJ 08540, USA
2Leiden Observatory, Leiden University, PO Box 9513, NL-2300 RA Leiden, The Netherlands
3Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Dr, Pasadena, CA 91109, USA
4Department of Astrophysics, American Museum of Natural History, Central Park West and 79th Street, New York, NY 10024, USA
5Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794-3800, USA

ABSTRACT

The Kepler mission has detected a number of transiting circumbinary planets (CBPs). Although currently not detected, exomoons could be orbiting some of these CBPs, and they might be suitable for harboring life. A necessary condition for the existence of such exomoons is their long-term dynamical stability. Here, we investigate the stability of exomoons around the Kepler CBPs using numerical N-body integrations. We determine regions of stability and obtain stability maps in the ($a_m$, $i_{pm}$) plane, where $a_m$ is the initial exolunar semimajor axis with respect to the CBP, and $i_{pm}$ is the initial inclination of the orbit of the exomoon around the planet with respect to the orbit of the planet around the stellar binary. Ignoring any dependence on $i_{pm}$, for most Kepler CBPs the stability regions are well described by the location of the 1:1 mean motion resonance (MMR) of the binary orbit with the orbit of the moon around the CBP. This can be ascribed to a destabilizing effect of the binary compared to the case if the binary were replaced by a single body, and which is borne out by corresponding 3-body integrations. For high inclinations, the evolution is dominated by Lidov-Kozai oscillations, which can bring moons in dynamically stable orbits to close proximity within the CBP, triggering strong interactions such as tidal evolution, tidal disruption, or direct collisions. This suggests that there is a dearth of highly-inclined exomoons around the Kepler CBPs, whereas coplanar exomoons are dynamically allowed.

Key words: gravitation – planets and satellites: dynamical evolution and stability – planet-star interactions

1 INTRODUCTION

One of the exotic type of planetary systems detected by the Kepler mission are transiting circumbinary planets (CBPs), i.e., planets in nearly-coplanar orbits around a stellar binary (and nearly coplanar with the plane of the sky), temporarily blocking the binary’s light and giving a generally complex light curve (e.g., Martin & Triaud 2015; Martin 2017). Currently, 10 confirmed Kepler CBPs are known in 9 binary systems (see Table 1 for an overview). These systems have revealed important clues for the formation and evolution of planets and high-order multiple systems. For example, none of the Kepler CBPs are orbiting binaries with periods shorter than 7 d, which suggests the presence of a third star (Muñoz & Lai 2015; Martin et al. 2015; Hamers et al. 2016), or could be indicative of coupled stellar-tidal evolution (Fleming et al. 2018).

A key question for the CBP systems is whether they would be able to harbour life. As shown by various authors (Orosz et al. 2012a; Quarles et al. 2012; Kane & Hinkel 2013; Haghighipour & Kaltenegger 2013; Cuntz 2014, 2015; Zuluaga et al. 2016; Wang & Cuntz 2017; Moorman et al. 2018), some of the Kepler CBPs are within the habitable zone (HZ). Some of the Kepler CBPs are giant planets and are unlikely to be able to sustain life; on the other hand, exomoons are prime candidates for celestial objects harbouring life (e.g., Lammer et al. 2014; Sato et al. 2017; Dobos et al. 2017). Although techniques to observe exomoons have been developed, they have not yet been found (e.g., Kipping 2009a,b; Kipping et al. 2012, 2013a,b, 2014; see Kipping 2014 for a review). Given the abundance of moons in the Solar system, it is reasonable to assume that they exist.

A necessary condition for the existence of exomoons in the Kepler CBPs is that they be long-term dynamically stable. In this paper, we address this issue by carrying out N-body integrations of exomoons orbiting around the Kepler CBPs (i.e., in S-type orbits around the CBPs, Dvorak 1984). We will show that stable configurations are possible for a wide range of parameters, and that the sta-
Our systems consist of a stellar binary (masses $M_1$ and $M_2$; semimajor axis $a_{bin}$) orbited by a CBP (mass $M_p$; semimajor axis $a_m$). A schematic representation (not to scale) in a mobile diagram of the system (Evans 1968) is given in Fig. 1.

We take the binary and planetary parameters for the currently-known Kepler systems from various sources. The adopted orbital parameters are given for convenience and completeness in Table 1. In most cases, the orbital angles are defined with respect to the plane of the sky. Note that the binary’s longitude of ascending node, $\Omega_m$, is defined to be zero. For several systems, the argument of periastron of the binary or the planet is not known. In this case, we set the corresponding values to zero. We do not vary the initial mean anomalies of the binary and the planet (both are set to zero), but we do vary the moon’s initial mean anomaly (see below).

Kepler 47 harbors at least two CBPs, Kepler 47b and Kepler 47c (Orosz et al. 2012a), and there may be a third CBP, Kepler 47d (Hinse et al. 2015). Here, we do not consider Kepler 47d, which has not yet been confirmed. Also, we treat the two confirmed planets in Kepler 47 as separate cases, i.e., in each case we neglect the gravitational potential of the other CBP. We have checked that this assumption is justified by running simulations with both confirmed planets included, which yielded no significantly different results. Kepler 64, also known as Planet Hunters 1 (PH1), is a quadruple-star system; the binary+CBP system is orbited by a distant stellar binary (a visual binary) at a separation of $\sim 1000 \text{ au}$ (Schwamb et al. 2013). In our integrations, we do not include the visual binary given the large separation between the two binary stars, and the relative scale of the inner CBP+planet triplet.

For each binary+planet system, we set up a fourth particle, the exomoon (henceforth simply ‘moon’), with mass $M_m = q_m M_p$, where $q_m$ is either 0.001 or 0.1, initially orbiting around the CBP (for reference, $q_m \approx 0.012$ for the Earth-Moon system, $q_m \approx 4.7 \times 10^{-5}$ for Io-Jupiter, and $q_m \approx 0.12$ for Pluto-Charon). We consider 40 values of the lunar semimajor axis, $a_m$, on an evenly-spaced grid, with the lower and upper values depending on the system; typically, $a_m$ ranges between 0.005 and 0.02 au (see the figures in Section 3.1 for the precise ranges of $a_m$ assumed for each system). The initial eccentricity is set to $e_m = 0.01$; we also carried out simulations with higher initial eccentricities, but found no significant dependence of the stability regions on $e_m$.

The lunar argument of periapsis is set to $\omega_m = 0^\circ$; the longitude of the ascending node is set to $\Omega_m = \Omega + \pi$, such that the mutual inclination between the planet and the moon is

$$\cos(i_{pm}) = \cos(i_p) \cos(i_m) + \sin(i_p) \sin(i_m) \cos(\Omega - \Omega_m)$$

$$= \cos(i_p + i_m),$$

(1)

i.e., by construction, the mutual inclination between the planet and the moon is $i_{pm} = i_p + i_m$. The inclination of the moon is set to $i_m = i_{pm} - i_p$, where $i_{pm}$ is evenly spaced from 0 to 180° with 40 values. We consider 10 different values of the lunar initial mean anomaly, $M_m$, evenly spaced between 0 and $2\pi$. In summary, for each Kepler system in Table 1, we carry out $40 \times 40 \times 10 = 16,000$ integrations with different $\omega_m$, $i_{pm}$, and $M_m$. These sets are repeated for $q_m = 0.001$ and $q_m = 0.1$ (binary case), and for $q_m = 0.001$ and with the stellar binary effectively replaced by a point mass (single case, i.e., the primary stellar mass is replaced by $M_1 + M_2$, and the secondary stellar mass is reduced to $M_2 = 1 \text{ kg}$, essentially a test particle). This implies a total of $10 \times 3 \times 16,000 = 480,000$ simulations for all 10 Kepler CBP systems.
in Section 3.3. We check for collisions of the moon with the planet and stars, taking the collision radii to be the observed physical radii. In reality, the moon is likely tidally disrupted before colliding with the CBP or with the stars. The tidal disruption radius, however, depends on the density of the moon. Here, we choose a more general approach of setting the collision radii equal to the physical radii.

We carry out 4-body integrations of the binary+CBP+moon system for a duration of \( t_{\text{end}} = 1000 \, P_p \), where \( P_p \) is the orbital period of the planet around the binary. To investigate whether this integration time is sufficiently long, we show the stability regions below in Section 3 for different integration lengths. The (absolute values of the) relative energy errors in the simulations after 1000 \( P_p \) are typically on the order of \( 10^{-12} \), with maxima of \( \sim 10^{-10} \).

We determine stability maps using the following approach. For a given integration time \( t_{\text{end}} \) and a point in the \((a_m, i_m)\) parameter space, we define the orbit as ‘stable’ if the moon remains bound to the CBP (i.e., \( a_m > 0 \) and \( 0 \leq e_m < 1 \)) at \( t_{\text{end}} \) for all 10 realizations with different \( M_m \) (green). If none of the 10 realizations of \( M_m \) yield bound orbits to the CBP, then the orbit is flagged as ‘unstable’ (red). If the orbit remains bound for some, but not all realizations with different \( M_m \), then we consider the orbit to be ‘marginally stable’ (yellow).

3 RESULTS

3.1 Stability regions

The main results of this paper are the stability maps shown in Figs 2 and 3, for Kepler 16, and the other 9 Kepler CBPs, respectively. In these maps, each point in the \((a_m, i_m)\) plane represents 10 simulations with different initial \( M_m \). Green filled circles correspond to stable systems, red crosses to unstable systems, and yellow open circles to marginally stable systems (see Section 2.2 for the definition of stable, unstable and marginally stable systems). If a collision of the moon with the stars or the CBP occurred for one or more \( M_m \)-realizations, then this is indicated with either the ‘-’ or ‘\( \ast \)’ symbols for collisions with the planet and the stars, respectively. We show the frequency for the most common collision type among the mean anomalies, and the color of these symbols encodes this frequency with respect to the 10 realizations of \( M_m \) (yellow to red for 1 to 10). For collisions of the moon with the stars, the large and small ‘\( \ast \)’ symbols correspond to collisions with the primary and secondary star, respectively.

In Fig. 2, the three panels in each row correspond to different integration times: 400, 700 and 1000 \( P_p \). In the first and third columns, the mass ratio of the moon to the planet is \( q_m = M_m/M_p = 0.001 \), whereas in the second column, \( q_m = 0.1 \). The third column corresponds to the single-star case, in which the stellar binary is replaced by a point mass. In Fig. 3, the integration time is 1000 \( P_p \), and the mass ratio is \( q_m = 0.001 \).

Generally, the maps can be characterized with a stable region at small \( a_m \), an unstable region at larger \( a_m \), and a marginally stable region in between, as can be intuitively expected. There is also a dependence on \( i_m \), the mutual inclination between the orbit of the moon around the planet, and the orbit of the planet around the inner binary — typically, high inclinations near 90° are less stable than low inclinations (near 0° and 180°), and lead to collisions. The larger instability near high inclinations can be ascribed to LK evolution, and this explored in more detail in Section 3.3. Also, retrograde orbits (\( i_m \) near 180°) tend to be more stable than prograde orbits (\( i_m \) near 0°), in the sense that the marginal stability region extends to a larger range in \( a_m \) for retrograde orbits compared to prograde orbits. This is consistent with the general notion that retrograde orbits are typically more stable than prograde orbits owing to the higher relative velocities for retrograde orbits. However, we find that the ‘binarity’ of the stellar binary is also important, especially for retrograde orbits. This is investigated in more detail in Section 3.2.

As expected, the stability regions decrease for longer integration times. However, there is little dependence on the integration time for the values shown, indicating that 1000 \( P_p \) is sufficiently long to determine stability in the majority of the parameter space.

In the figures, we show for reference with the vertical blue dashed dotted lines the Hill radius \((e.g., \text{Hamilton} & \text{Burns} 1992\); replacing the binary by a point mass), i.e.,

\[ r_H = a_p(1 - e_p) \left( \frac{M_p}{3(M_1 + M_2)} \right)^{1/3}. \]

Note that \( r_H \) does not lie within the range of \( a_m \) shown in all figures; if the vertical blue dashed dotted line is not visible, then \( r_H \) is larger than the largest value of \( a_m \) shown. Although correct within an order of magnitude, the Hill radius only captures the true stability boundary within a factor of a few. This is not very surprising given that the Hill radius strictly only applies to the three-body case (with the binary replaced by a single body), and does not capture the inclination dependence.

3.2 Mean motion resonances

As mentioned above, the Hill radius does not accurately describe the boundary between stable and unstable orbits in the \((a_m, i_m)\) plane. Here, we consider an alternative simple analytic description of the stability boundary using an interpretation based on MMRs. First, we show in the third row of Fig. 2 and in Fig. 4 stability maps for integrations in which the stellar binary was effectively replaced by a point mass. For Kepler 16, the stable regions are significantly larger in the single-star case, especially for retrograde orbits. This shows that the binary tends to destabilize the moon, as is intuitively clear.

MMRs are due to commensurate orbital periods, and they have been studied in detail for single-planetary systems (e.g., Peale 1976; Sessin & Ferraz-Mello 1984; Batygin & Morbidelli 2013; Deck et al. 2013). For small bodies, MMRs are known to be destabilizing (e.g., the Kirkwood gaps in the Asteroid belt, e.g., Wisdom 1983; Henrand & Caranicolos 1990; Murray & Holman 1997, and rings of Saturn, e.g., Goldreich & Tremaine 1978; Borderies et al. 1982). The dynamics of MMRs in quadruple systems in the 2+2 configuration, which applies to our systems, are not well-understood (see Breiter & Vokrouhlický 2018 for a pioneering study which applies to the case of four bodies with comparable masses). In our configuration, we expect MMRs of the stellar binary with the orbit of the moon around the planet to be important. Since the small-body limit applies, we also expect that these MMRs have a destabilizing effect. As shown by the ‘binary case’ stability maps, the binary indeed has a destabilizing effect, which is active for \( a_m > r_H \), until at a certain value of \( a_m \) orbits are again stable. For \( a_m < r_H \), we expect the latter value to be mainly determined by the 1:1 MMR with the stellar binary.

Using Kepler’s law, it is straightforward to show that the condition \( P_{\text{bin}} = \alpha P_n \), where \( P_{\text{bin}} \) and \( P_n \) are the binary and lunar orbital periods, respectively, and \( \alpha \) indicates the order of the MMR,
Figure 2. Stability maps for the Kepler 16 system. In each panel, each point represents 10 realizations with different initial lunar mean anomaly, which are used to define stable (green filled circles), completely unstable (red crosses), and marginally stable (yellow open circles) systems. Collisions of the moon with the planet and stars are indicated with ‘−’ and ‘⋆’, respectively, where the color encodes the frequency with respect to the 10 realizations of the initial lunar mean anomaly (yellow to red for 1 to 10). Here, we show the frequency for the most common collision type among the mean anomalies. For collisions of the moon with the stars, the large and small ‘⋆’ symbols correspond to collisions with the primary and secondary star, respectively. The three panels in each row correspond to different integration times: 400, 700 and 1000 $P_p$. In the first and third columns, the mass ratio of the moon to the planet is $q_m = M_m/M_p = 0.001$, whereas in the second column, $q_m = 0.1$. The third column corresponds to the single-star case, in which the stellar binary is replaced by a point mass. The vertical black solid lines show the location of the 1:1 MMR of the moon with the binary (equation 3), and the vertical black dashed lines show the locations of the 2:1 MMR. The red dashed lines show the boundary for collisions of the moon with the planet due to LK evolution, computed according to equation (4).
This can be understood from standard Lidov-Kozai (LK) dynamics, where the mean anomaly of the binary MMR is significantly larger than 0.5. The maximum eccentricity reached is given by

\[ e_{\text{max}} = \sqrt{\frac{5}{3} \cos^2(i_{\text{pm}})} \]  

(4)

The implied periapsis distance is \( r_{\text{peri},M} = a_{M}(1 - e_{\text{max}}) \) for \( \cos^2(i_{\text{pm}}) < 3/5 \), and \( r_{\text{peri},M} = a_{M}(1 - e_{M}) \) (where the initial value is \( e_{M} = 0.01 \)) for \( \cos^2(i_{\text{pm}}) \geq 3/5 \) (the critical angles for LK oscillations are set by \( i_{\text{pm}} = \arccos(\sqrt{3/5}) \sim 39.215^\circ \) and \( i_{\text{pm}} = \pi - \arccos(\sqrt{3/5}) \sim 140.785^\circ \)).

Table 1. Initial conditions adopted for the Kepler CBP systems. See Fig. 1 for a schematic illustration of the orbit definition for the parameters. Here, \( i \), \( \Omega \), and \( \omega \) denote inclination, longitude of the ascending node, and argument of periapsis, respectively. The initial mean anomalies for the binary and planetary orbit were set to \( M_{\text{bin}} = M_{\text{p}} = 0 \). References: 1 — Doyle et al. (2011); 2 — Welsh et al. (2012); 3 — Orosz et al. (2012b); 4 — Orosz et al. (2012a); 5 — Schwamb et al. (2013); Kostov et al. (2013); 6 — Kostov et al. (2014); 7 — Welsh et al. (2015); 8 — Kostov et al. (2016).

| Name       | \( M_1 \)  | \( M_2 \)  | \( M_3 \)  | \( \omega_1 \) | \( \omega_2 \) | \( \omega_3 \) | \( i_{\text{peri}} \) | \( \omega_{\text{peri}} \) | \( \Omega_{\text{peri}} \) | \( \Omega_{\text{bin}} \) | \( \Omega_{\text{p}} \) | \( \omega_{\text{bin}} \) | \( \omega_{\text{p}} \) | \( \text{Ref.} \) |
|------------|----------|----------|----------|---------------|---------------|---------------|----------------|----------------|---------------|---------------|---------------|----------------|---------------|---------|
| Kepler 16  | 0.6897   | 0.20555  | 0.333    | 0.22431       | 0.7048        | 0.19444       | 0.0069         | 39.3041       | 0.032          | 0.003         | 263.464       | 318.0         | 1       |
| Kepler 34  | 1.0479   | 1.0208   | 0.22     | 0.22882       | 1.0986        | 0.52087       | 0.182          | 89.8584       | 90.355         | 0.004         | 1.74          | 0             | 0       |
| Kepler 35  | 0.8877   | 0.8994   | 0.127    | 0.17617       | 0.60347       | 0.1421        | 0.042          | 90.4238       | 90.76          | 0.004         | 2             | 0             | 0       |
| Kepler 38  | 0.949    | 0.249007 | 0.00921  | 0.1469        | 0.46444       | 0.1032        | 0.002          | 0.182         | 0.004         | 208.690       | 0             | 3       |
| Kepler 47B | 1.043    | 0.362    | 0.02651  | 0.2956        | 0.2324        | 0.0355        | 0.0355         | 89.34         | 89.59          | 0.1           | 212.3         | 0             | 4       |
| Kepler 47C | 1.043    | 0.362    | 0.072877 | 0.0836        | 0.989         | 0.0234        | 0.411          | 89.34         | 89.826         | 0.106         | 212.3         | 0             | 4       |
| Kepler 64  | 1.47     | 0.37     | 0.531558 | 0.1769        | 0.642         | 0.204         | 0.1            | 87.59         | 90.0           | 0             | 214.3         | 305.0         | 5       |
| Kepler 413 | 0.820    | 0.5423   | 0.21074  | 0.10148       | 0.3535        | 0.065         | 0.1181         | 4.073         | 0             | 279.74        | 94.6          | 6       |
| Kepler 453 | 0.944    | 0.1951   | 0.006828 | 0.15839       | 0.7903        | 0.0524        | 0.0359         | 90.266        | 89.4429        | 2.103         | 263.05        | 185.1         | 7       |
| Kepler 1647| 1.2207   | 0.9678   | 1.5198   | 0.1276        | 2.7209        | 0.1602        | 0.0581         | 87.9164       | 90.0972        | 0             | 263.05        | 185.1         | 8       |

The stability maps shown above are based on Newtonian point-mass dynamics, neglecting strong interactions such as tidal effects and tidal disruption. Here, we briefly consider such interactions when the moon passes close to its parent planet (we do not consider strong interactions with the two stars, although, in principle, unstable moons can also experience strong interactions with the binary, see, e.g., Gong & Ji 2018 for the case of scattering-induced tidal capture CBPs).

We evaluate the importance of strong interactions by comparing the periapsis distances of the moon to its parent planet to (multiples of) the radius of the planet. In Fig. 5, we show for Kepler 16 in the \((a_{\text{bin}}, i_{\text{pm}})\) plane the minimum periapsis distances, \( r_{\text{peri},M} \), recorded in the simulations. Here, we determine \( r_{\text{peri},M} = a_{M}(1 - e_{M}) \) from the minimum value among the systems in the \((a_{\text{bin}}, i_{\text{pm}})\) plane that are stable (i.e., if the moon remains bound to the planet for all realizations of \( M_{\text{bin}} \)); if at least one of the realizations yielded an unbound orbit or a collision, then we set \( r_{\text{peri},M} \) to \(-0.001\, \text{au}\), which is indicated in Fig. 5 with the dark blue regions. Note that, due to a finite number of output snapshots, the true closest approach can be missed in some cases. For low inclinations, there is no excitation of \( e_{M} \), and \( r_{\text{peri},M} = a_{M}(1 - e_{M}) \), where \( e_{M,0} \) is the initial eccentricity. This is manifested in Fig. 5 as a linear relation for small inclinations. For larger inclinations, LK evolution drives high \( e_{M} \) and small \( r_{\text{peri},M} \), and collisions with the planet occur as \( i_{\text{pm}} \) approaches 90°.

Also, we show in Fig. 6 a plot of \( r_{\text{peri},M} \) versus \( i_{\text{pm}} \) for a slice in \( a_{\text{bin}} \), with \( a_{\text{bin}} = 0.005\, \text{au}\). The latter figure shows that the dependence of \( r_{\text{peri},M} \) on \( i_{\text{pm}} \) for \( r_{\text{peri},M} > R_{\text{p}} \) is well described by the canonical LK relation (equation 4; green dashed line). The red dotted and dashed lines show \( r_{\text{peri},M} = R_{\text{p}} \) and \( r_{\text{peri},M} = 3 \, R_{\text{p}} \), respectively, where \( R_{\text{p}} = 0.7538 \, R_{\text{J}} \) is the planetary radius (Doyle et al. 2011). Here, we take a factor of 3 to be indicative of the regime where tidal effects are important. Much closer in, tidal disruption is possible, or even direct collision. In the case shown for Kepler 16 and for \( a_{\text{bin}} = 0.005\, \text{au} \), tidal effects are potentially important for inclinations near 60° and 120°. Direct collision occur for inclinations between 75° and 105°.

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Figure 3. Stability maps for nine Kepler CBP systems, similar to Fig. 2. The name of the system is indicated above each panel. In all cases, $q_m = 0.001$, and the integration time is $1000 \, P_p$. The vertical black solid lines show the location of the 1:1 MMR of the moon with the binary (equation 3); the vertical black dashed lines show the locations of the 2:1 and 1:2 MMRs. The red dashed lines show the boundary for collisions of the moon with the planet due to LK evolution, computed according to equation (4). The vertical blue dashed lines (not visible in all panels) show the Hill radius, equation (2).

### 3.4 Short-range forces

Our $N$-body integrations include the Newtonian point-mass terms only. Here, we investigate, a posteriori, the importance of short-range forces (SRFs) acting on the planet-moon orbit, in conjunction with LK cycles. Such SRFs can include relativistic precession, and precession due to the quadrupole moment of the planet. In the case of Kepler 16, relativistic precession is completely unimportant since the precession time-scale (on the order of Myr) is much longer than the LK time-scale (on the order of tens of years; see also Table 3).

However, the quadrupole moment of the planet can be important depending on the assumed structure of the planet, and, more importantly, its rotation rate. In Fig. 6, we show with the blue dotted line the periapsis distance $r_{\text{pm},m}$ according to LK theory with the addition of the quadrupole moment and tidal bulges of the planet (assuming $q_m = 0.001$). We compute $e_{\text{max}}$ by using energy and angular-momentum conservation, with the expressions for the Hamiltonian adopted from Fabrycky & Tremaine (2007). For the
planet, we here assume a spin rotation period of 5 hr (a somewhat extreme case, given that for Kepler 16 the critical rotation period for centrifugal breakup is $2\pi\sqrt{GM_p/R_p^3} \approx 3.4$ hr), and an apsidal motion constant of 0.25. For these values, there is some deviation from the canonical relation equation (4), although, in this case, $r_{peri,m}$ is not much affected unless $i_{pm}$ is close to 90°.

To investigate the importance of SRF in all Kepler CBP systems, we compute the associated time-scales in Table 3. Approximating the binary as a point mass, the LK time-scale can be estimated as (e.g., Innanen et al. 1997; Kinoshita & Nakai 1999; Antognini 2015)

$$t_{LK} = \frac{P_p^2}{P_m^2} \frac{M_p}{M_{bin}} \frac{M_{bin} + M_m}{M_{bin}} \left(1 - e_p^2\right)^{3/2},$$

where $M_{bin} = M_1 + M_2$. The relativistic time-scale is (e.g., Weinberg 1972)

$$t_{1PN} = \frac{1}{6} \frac{a_{pm} c^2}{GM_p} P_m,$$

and the tidal bulges (TB) and rotation time-scales are given by (e.g.,
Due to LK evolution, computed according to equation (4). The vertical black dashed line shows the locations of the 2:1 MMR. The red dotted curve: analytic LK periapsis distance based on equation (4). Blue dotted curve: the analytic LK periapsis distance based on equation (4). The red dashed lines show the location of the 1:1 MMR with the binary (equation 3), and the vertical black solid lines show the right-hand regions of each panel. The vertical black solid lines show the location of the 1:1 MMR of the moon with the binary (equation 3), and relativistic precession (which is not important). The red dotted curve: analytic LK periapsis distance based on a semianalytic calculation taking into account the quadrupole moment and tidal bulges of the planet (assuming a spin rotation period of 5 hr and an apsidal motion constant of 0.25), and relativistic precession (which is not important). The red dotted and dashed lines show the locations of the 2:1 MMR. The red dashed lines show the boundary for collisions of the moon with the planet due to LK evolution, computed according to equation (4).

The time-scales are shown for two semimajor axes: \( a_m = a_0 \), the smallest semimajor axis considered in the simulations, and \( a_m = a_1 = a_{m,MM} \), the location of the 1:1 MMR with the binary.

As shown in Table 3, relativistic precession and tidal bulges are completely negligible for the Kepler CBP systems. Precession due to rotation, i.e., due to the quadrupole moment of the planet, are potentially important at small semimajor axes (in particular for Kepler 47b, 47c, and 453), although it should be taken into account that we assumed a somewhat extreme case of a near-critical rotation speed of the planet.

3.5 Summary plots

We summarize the results of the stability maps by determining the largest stable values of \( a_{mm} \), \( a_{m,crit} \), for each system and for a given value of \( t_{tie} \), and plotting these values as a function of the masses of the

\[
\begin{array}{cccccc}
\text{Kepler 16} & 3235 & 106 & 129 & 3366 & 79 & 110 & 3193 & 0 \\
\text{Kepler 34} & 3120 & 3 & 3 & 3161 & 1 & 2 & 3395 & 0 \\
\text{Kepler 35} & 2889 & 26 & 22 & 2685 & 22 & 23 & 4376 & 0 \\
\text{Kepler 38} & 5743 & 0 & 0 & 5690 & 0 & 0 & 6059 & 0 \\
\text{Kepler 47b} & 4056 & 0 & 0 & 4246 & 0 & 0 & 4205 & 0 \\
\text{Kepler 47c} & 3115 & 0 & 0 & 3112 & 0 & 0 & 3281 & 0 \\
\text{Kepler 64} & 2442 & 55 & 73 & 2648 & 55 & 87 & 3319 & 1 \\
\text{Kepler 413} & 3155 & 15 & 10 & 3246 & 11 & 10 & 3958 & 0 \\
\text{Kepler 453} & 3047 & 0 & 0 & 3098 & 0 & 0 & 4727 & 0 \\
\text{Kepler 1647} & 1660 & 0 & 0 & 1702 & 0 & 0 & 1977 & 0 \\
\end{array}
\]

Table 2. Number of collisions of the moon with the planet (\( N_{col,p} \)), the primary star (\( N_{col,\star 1} \)), and the secondary star (\( N_{col,\star 2} \), if applicable), in the simulations with \( q_m = 0.001 \) (binary star case), \( q_m = 0.1 \) (binary star case), and \( q_m = 0.001 \) (single star case), and for an integration time of 1000 \( P_p \). The total number of integrations for each Kepler system is 16,000.
Alternatively, one could compute $a_{\text{m, crit}}$ normalized to $a_{\text{min}}$, as a function of $M_p/M_{\text{bin}}$. Different symbols correspond to different inclinations (refer to the legend). Note that high inclinations, near 90°, are excluded since in this case there are no stable orbits in our simulations. Also plotted in the same panel with the solid black line is equation (3), the location of the 1:1 MMR. Most of the data points from the simulations match well with equation (3), again showing that the critical semimajor axis is close to the 1:1 MMR. Two notable exceptions are Kepler 47c and Kepler 1647 (indicated in the figure with arrows), which, as mentioned in Section 3.2, have relatively large planetary semimajor axes, such that the 1:1 MMR is weak and therefore does not set the stability boundary.

Alternatively, one can normalize $a_{\text{m, crit}}$ by $a_p$. The resulting points from the simulations are shown in the bottom panel of Fig. 7. Again, most of the data are well fit by a power-law function of $M_p/M_{\text{bin}}$ with a slope of 1/3, although the absolute normalization is different. The black dashed line shows the Hill radius (equation 2), which, as noted above, overestimates the critical $a_p$ for stability. However, we can obtain a relation that better describes the data by using the fact that most of the Kepler CBPs are close to the limit for stability (i.e., stability of the planet in absence of the moon). In particular,

$$a_{\text{m, crit}} = a_{\text{min}} \frac{a_{\text{bin}}}{a_{\text{bin}}/a_p} = \left( \frac{M_p}{M_{\text{bin}}} \right)^{1/3} a_{\text{min}}/a_p,$$  \hspace{1cm} (9)

where we used equation (3) with $\alpha = 1$ (the 1:1 MMR). Subsequently, we replace $a_{\text{bin}}/a_p$ by $\langle a_p/a_{\text{bin}} \rangle_{\text{HW99}}^{-1}$, where $\langle a_p/a_{\text{bin}} \rangle_{\text{HW99}}$ is the critical CBP semimajor axis in units of $a_{\text{bin}}$ determined from the analytic fits of Holman & Wiegert (1999), and where the average is taken assuming a thermal eccentricity distribution of $a_{\text{bin}}$ and a flat mass ratio distribution (i.e., flat in $q = M_2/M_1$). With these assumptions, we find $\langle a_p/a_{\text{bin}} \rangle_{\text{HW99}}^{-1} \simeq 0.252267$, such that

$$a_{\text{m, crit}} \simeq 0.252267 \left( \frac{M_p}{M_{\text{bin}}} \right)^{1/3}.$$  \hspace{1cm} (10)

Equation (10) is shown in the bottom panel of Fig. 7 with the black solid line, and agrees with most of the data from the simulations.

Table 3. Short-range precession time-scales for lunar orbits around the Kepler CBPs. The time-scales given are the LK, relativistic, TB, and rotation time-scales (see text for details), and are shown for two semimajor axes: $a_{\text{m}} = a_{\text{min}}$, the smallest semimajor axis considered in the simulations, and $a_{\text{m}} = a = a_{\text{m, MM}}$, the location of the 1:1 MMR with the binary. Note that the units for $t_{\text{LK}}$ and $t_{\text{rot}}$ are yr, and Myr for $t_{\text{1PN}}$ and $t_{\text{TB}}$.

| $q_{\text{M, bin}}$ | $t_{\text{LK}}$/Myr | $t_{\text{TB}}$/Myr | $t_{\text{1PN}}$/Myr | $t_{\text{rot}}$/yr |
|-------------------|-----------------|-----------------|-----------------|--------------------|
| K16               | 19.8            | 3.5             | 3.5             | 5.4                |
| K34               | 24.4            | 7.8             | 9.8             | 535.9              |
| K35               | 4.0             | 2.3             | 22.4            | 57.8               |
| K38               | 3.3             | 1.6             | 1661.3          | 0.001              |
| K47b              | 8.3             | 0.9             | 29.8            | 0.001              |
| K47c              | 138             | 25.6            | 0.9             | 15.2               |
| K48               | 9.0             | 2.6             | 21.0            | 33.2               |
| K443              | 14.5            | 1.2             | 12.5            | 0.003              |
| K453              | 10.6            | 5.8             | 1147            | 0.011              |
| K1647             | 349             | 297             | 3.1             | 4.0                |

Figure 7. Top panel: largest stable value of $a_{\text{m, crit}}$, normalized to $a_{\text{min}}$ and plotted as a function of $M_p/M_{\text{bin}}$, where $M_{\text{bin}} = M_1 + M_2$, for each system in the binary case with $q_{\text{m}} = 0.001$, and for a given value of $t_{\text{bin}}$ (refer to the legend). The solid black line shows equation (3), the location of the 1:1 MMR. Bottom panel: $a_{\text{m, crit}}$ normalized to $a_p$. The black dotted line shows the Hill radius (equation 2), and the black solid line shows the estimate equation (10) assuming the CBP is close to the critical boundary for stability.

4 DISCUSSION

4.1 Comparison to similar work

In the limit that the binary can be interpreted as a point mass, our results can be compared to Grishin et al. (2017), who considered the stability of inclined hierarchical three-body systems. Grishin et al. (2017) find the following expression for the limiting radii of stability as a function of the mutual inclination, i.e., in our case, $t_{\text{bin}}$.

$$r_{\text{L}}^2 \left( \theta_{\text{bin}}, \epsilon_{\text{m, max}} \right) = f_{\text{fudge}} 3^{1/3} t_{\text{bin}} g(\theta_{\text{L}})^{-2/3} f(\epsilon_{\text{m, max}}),$$  \hspace{1cm} (11)

where $g(\theta_{\text{L}}) = \cos(\theta_{\text{L}}) + \sqrt{3 + \cos^2(\theta_{\text{L}})}$, and we take the ‘corrected’ function to be $f(\epsilon_{\text{m, max}}) = 1/(1 + \epsilon_{\text{m, max}}/2)$ with $\epsilon_{\text{m, max}}$ given by equation (4). The ‘fudge factor’ $f_{\text{fudge}}$ is set to $f_{\text{fudge}} = 0.5$, consistent with Grishin et al. (2017).

For the single-star cases, we show equation (11) with the black dotted lines in the third row of Fig. 2 and in Fig. 4 for Kepler 16.
and the other Kepler CBP systems, respectively. The absolute locations of the stability boundaries in the simulations do not agree precisely with equation (11). However, the general shapes of the boundaries agree, with the \( a_m \)-boundaries increasing with increasing \( \lambda_m \) (i.e., retrograde orbits are more stable in the single-star case), and a ‘bulge’ near high inclination. Note that equation (11) does not take into account collisions of the moon with the planet, which determine the stability boundaries in the simulations for inclinations near 90°.

4.2 Implications of collisions

As shown above, most of the collisions in our simulations are collisions of the moon with the CBP, and occur at high inclination. However, some of the collisions are between the moon and the stars, and show no strong inclination dependence. Such collisions could enhance the metallicity of the stars, and/or trigger a speed-up of the stellar rotation in the case of a relative massive moon and a low-mass star. Speed-up of stellar rotation has been proposed to occur as a result of planets plunging onto their host star due to high-eccentricity migration (Qureshi et al. 2018).

5 CONCLUSIONS

We investigated the stability of moons around planets in stellar binary systems. In particular, we considered exomoons around the transiting Kepler circumbinary planets (CBPs). Such exomoons may be suitable for harboring life, and are potentially detectable in future observations. We carried out numerical \( N \)-body simulations and determined regions of stability around the Kepler CBPs. Our main conclusions are listed below.

1. We obtained stability maps in the \((a_m, \lambda_m)\) plane, where \( a_m \) is the lunar semimajor axis with respect to the CBP, and \( \lambda_m \) is the inclination of the orbit of the moon around the planet with respect to the orbit of the planet around the stellar binary. For most of the Kepler CBPs and ignoring the dependence on \( \lambda_m \), the stability regions are well described by the location of the 1:1 mean motion resonance (MMR) of the binary orbit with the orbit of the moon around the CBP (equation 3). This can be ascribed to a destabilizing effect of the binary compared to the case if the binary were replaced by a single body, and which is borne out by corresponding 3-body integrations. For the two exceptions, Kepler 47b and Kepler 1647, the CBP semimajor axis is relatively large such that the 1:1 MMR is weak, and therefore it does not set the stability boundary.

2. For stable lunar orbits and high inclinations, \( \lambda_m \) near 90°, the evolution is dominated by Lidov-Kozai oscillations (Lidov 1962; Kozai 1962). These imply that moons in orbits that are dynamically stable could be brought to close proximity within the CBP, and experience strong interactions such as tidal evolution, tidal disruption, or direct collisions. This suggests that there is a dearth of highly-inclined exomoons around the Kepler CBPs, whereas coplanar exomoons are dynamically allowed.

3. Most of the collisions in our simulations are CBP-moon collisions, occurring at high inclination. Collisions with the stars are rare. However, if such collisions do occur, the stars in the binary might be enhanced in metallicity, and/or show an anomalously-high rotation speed.

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