Freeze-in self-interacting dark matter in warped extra dimension

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A classically scale-invariant scalar singlet can be a MeV-scale dark matter, with a feeble Higgs portal coupling at $O(10^{-10})$. Besides, an $O(0.1)$ self-interaction coupling could further serve to alleviate the small-scale problems in the Universe. We show that, such a dark matter candidate can naturally arise in the warped extra dimension, with the huge span of parameter space predicted well within $O(1)$ fundamental parameters.

\textbf{Introduction:} Warped extra-dimension (WED) theory in the Randall-Sundrum (RS) model has shown a glorious insight to solve the hierarchy problem \cite{1}. The original RS solution to the large span between the Planck and electroweak scales set the Standard Model (SM) Higgs field to be brane-localized on a boundary point of the compactified extra dimension (the so-called IR or TeV brane), in which all the SM particles also populate. Later it was found that, if the fermions propagate in the fifth dimension, the feebleness and hierarchies of Yukawa couplings can also be addressed in a natural way \cite{2–4}. WED subsequently becomes a common stilt to explain hierarchies of couplings and scales existed in various phenomenological models.

The successful freeze-in production of dark matter (DM) with a feeble portal coupling has been studied intensively in recent years (see, e.g., refs. [5–9]). Depending on the specification of DM mass, an $O(10^{-12}) - O(10^{-9})$ portal coupling is generically required to fit the DM relic abundance. Among the possible freeze-in DM models, a real scalar singlet $S$ is arguably the simplest candidate, which interacts with the SM via the quartic Higgs portal $H^1 H S^2$. While such a framework is quite simple, the question about the origin of feeble portal coupling remains to be solved. Thus far, this has stirred up an increased attention on sourcing the tiny portal \cite{10–14}.

In the real scalar singlet models, it was found earlier in ref. \cite{15} that, if the DM mass stems entirely from the Higgs vacuum, i.e., the DM has a classical scale-invariance, its mass and relic density would share a common origin \cite{16}. In addition, if the DM self-interaction coupling resides at $O(0.1)$, an order similar to the SM Higgs quartic coupling, the self-interacting DM could further alleviate the small-scale discrepancies between the collisionless cold DM simulations and the structure observations in the Universe \cite{15} (see ref. \cite{17} for the original proposal, as well as the recent review \cite{18} and references therein). This simple DM model, however, remains a huge coupling span ranging from $O(0.1)$ to $O(10^{-10})$, and it seems that the explanation of DM relic density and small-scale problems relies on an unnaturally numerical coincidence.

We will show in this paper that, the classically scale-invariant DM can arise in the WED as the zero-mode Kaluza-Klein (KK) scalar. The DM production can be approximately ascribed to a two-step freeze-in mechanism. The zero-mode profile induced from the classical scale-invariance plays the key role to generate the numerical coincidence mentioned above, from which the huge span of parameter space is unified via natural $O(1)$ couplings in the WED. In the following, we begin by embedding a scale-invariant scalar singlet in the WED with a five-dimensional (5D) self-interaction and a Higgs portal localized on the IR brane, then we apply the simple freeze-in mechanism with a two-step production before and after the electroweak symmetry breaking (EWSB) under some proper approximations, and calculate the DM relic density as well as the ratio of cross section over mass before we finally conclude.

\textbf{Scale-invariant singlet in the WED:} The starting point to consider the RS model [1] is featured by the 5D metric

$$ds^2 = e^{-2\sigma}(\phi) \eta_{\mu\nu} dx^\mu dx^\nu - r_c^2 d\phi^2, \quad (1)$$

where the fifth dimension is compactified on an $S^1/Z_2$ orbifold, with the compactification radius $r_c$ and angular coordinate $\phi \in [-\pi, \pi]$. The $Z_2$ symmetry on the orbifold dictates a $\phi$-periodicity where $(x, \phi)$ is identified with $(x, -\phi)$, $e^{-2\sigma(\phi)}$ is the warped factor, with $\sigma(\phi) \equiv kr_c|\phi|$. The induced 4D Planck mass is related to the 5D fundamental scale $M$ via $M_{Pl}^2 = M^5(1 - e^{-2kr_c\pi})/k$, with the curvature $k$ of the fifth dimension at $k \simeq M \simeq 10^{19}$ GeV.

If the SM Higgs sector is localized on the $\phi = \pi$ brane, i.e., the so-called IR (TeV) brane, from which the action is constructed as

$$S_H = \int d^4x \sqrt{g} \left( \tilde{g}^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda (|H|^2 - v_0^2)^2 \right), \quad (2)$$

where $\tilde{g}_{\mu\nu} = e^{-2kr_c\pi} g_{\mu\nu}$, with the flat limit of $g_{\mu\nu}$ given by the Minkowskian metric signature $\eta_{\mu\nu} = (1, -1, -1, -1)$. The canonical normalization is realized by Higgs wavefunction rescaling, $H \to e^{kr_c\pi}H$, which
makes the EWSB scale $v_0$ shifted to the physical one via $v = e^{-kr_c \pi} v_0$. It can then be found that, with $kr_c \approx 12$, the electroweak vacuum $v \approx 246 \text{ GeV}$ is induced from the non-hierarchical scale $v_0 \approx M$ in the fundamental 5D regime, and hence the hierarchy problem can be explained.

In the 5D perspective, a natural suppression of Higgs portal coupling implies a bulk scalar DM propagating in the fifth dimension with an exponential-like profile. Let us consider the DM scale solely generated by the electroweak vacuum [15]. In this case, the 4D scalar potential would be of the form

$$V(S, H) = -\mu_H^2 |H|^2 + \frac{\lambda_H}{4} |H|^4 + \lambda_P S^2 |H|^2 + \frac{\lambda_S}{4} S^4,$$

where the EWSB occurs via the Higgs mechanism. To induce the above potential in the 5D framework, we propose a scalar action with a brane-localized Higgs portal: $S_{\text{scalar}} = S_\Phi + S_H$, in which

$$S_\Phi = \int d^4x \, d\phi \, e^{\sigma r_c} \left( \frac{1}{2} G^{AB} \partial_A \Phi \partial_B \Phi - \frac{1}{2} M_\Phi^2 \Phi^2 + \frac{\lambda_5}{4} \Phi^4 + \frac{\lambda_5}{4} P \Phi^2 |H|^2 \partial_5 \right),$$

with $G^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}$, $G_{\Phi\phi} = -1/r_c^2$, $\gamma_{\mu\nu} \equiv \delta(\phi - \pi)$, and a 5D bare mass term [4]: $M_\Phi^2 = a k^2 + b r_c^2 + c r_c^4$, where $a, b, c \approx O(1)$ are dimensionless parameters, and

$$\sigma''(\phi) = 2k(\delta(\phi) - \delta(\phi - \pi)).$$

Applying the KK decomposition

$$\Phi(x, \phi) = \sum_n S_n(x) \frac{f_n(\phi)}{\sqrt{r_c}},$$

we can see that, the 5D portal coupling $\lambda_5^{(5)} \approx O(1)$ is dimensionless while $\lambda_5^{(5)}$ has dimension $-1$, which, without loss of generality, can be parametrized as $\lambda_5^{(5)} = \xi/k$, with $\xi \approx O(1)$.

The equation of motion (EoM) for the free scalar profile has been obtained earlier in ref. [19], which reads

$$\frac{1}{r_c^2} \partial_\phi \left( e^{-4\sigma(\phi)} \partial_\phi f_n \right) - M_\Phi^2 e^{-4\sigma(\phi)} f_n = -m_n^2 e^{-2\sigma(\phi)} f_n.$$  (7)

For the zero mode with a classical scale invariance, we have $m_0 = 0$. Solving eq. (7) with the following boundary conditions [4]

$$f_n'(\phi) - b a f_n(\phi) \big|_{\phi=0, \pi} = 0,$$

we obtain two possible solutions for the zero mode

$$b = \pm \sqrt{a^2 + 1},$$

Given the orthonormal condition

$$\int_{-\pi}^{\pi} d\phi e^{-2\sigma(\phi)} f_n(\phi) f_m(\phi) = \delta_{mn},$$  (10)

it is easy to check that, only the portal coupling obtained in the $b = -\sqrt{a^2 + 1}$ direction can have an exponential-like scaling, corresponding to a normalized zero-mode profile

$$f_0(\phi) = \sqrt{\frac{kr_c(\sqrt{a^2 + 1} - 1)}{1 - e^{-2kr_c(\sqrt{a^2 + 1} - 1)}}} e^{-kr_c(\sqrt{a^2 + 1} - 1)}.$$  (11)

The induced 4D portal coupling $\lambda_{P,0}$ and quartic coupling $\lambda_{S,0}$ of the zero mode are given respectively by

$$\lambda_{P,0} = \lambda_5^{(5)} e^{-2\sigma(\pi)} f_0^2(\pi)$$

$$\simeq \lambda_5^{(5)} kr_c(\sqrt{a^2 + 1} - 1) e^{-2kr_c(\sqrt{a^2 + 1} - 1)},$$

$$\lambda_{S,0} = \frac{\xi}{kr_c} \int_{-\pi}^{\pi} d\phi e^{-4\sigma(\phi)} f_0^2(\phi),$$

$$\simeq \frac{1}{2} \xi(\sqrt{a^2 + 1} - 1) \coth[kr_c(\sqrt{a^2 + 1})].$$

It can readily be seen that, $\lambda_{P,0}$ is exponentially suppressed provided that $a \geq -3$. Besides, the hyperbolic cotangent indicates that the quartic coupling $\lambda_{S,0}$ would be a stable function of parameter $a$ even though it controls the exponential scaling of portal coupling $\lambda_{P,0}$.

Some remarks are made here about the EoM in eq. (7). The identification of 4D KK scalar mass $m_n$ can be obtained from the 5D EoM, which, after including the self-interaction in the bulk, gives

$$\frac{1}{\sqrt{G}} \partial_A (\sqrt{G} G^{AB} \partial_B \Phi) + M_\Phi^2 \Phi - \lambda_5^{(5)} \Phi^3 = 0,$$

or in terms of the KK decomposition, reads

$$0 = \sum_n \left( e^{4\sigma(\phi)} \partial_\phi (e^{-4\sigma(\phi)} f_n) - M_\Phi^2 f_n \right) S_n$$

$$- \frac{\lambda_5^{(5)}}{r_c} \sum_n f_n S_n \sum_m f_m f_l S_m S_l.$$  (15)

The 4D EoM for a scalar of mode $n$ can be projected out by multiplying a factor of $e^{-4\sigma f_n}$ on both sides of eq. (15) and integrating out the fifth dimension. Using the orthonormal condition, eq. (10), we can see that, eq. (15) reduces to

$$0 = \partial^2 S_n + m_n^2 S_n - \sum_{nml} \lambda_{S,\tilde{n}mnl} S_n S_m S_l.$$  (16)
where the effective 4D mass $m_\tilde{h}$ is identified as
\[ m_\tilde{h}^2 \equiv -\int_{-\pi}^{\pi} d\phi f_{\tilde{h}} \left( \frac{1}{\tau_c} \frac{\partial}{\partial \phi} (e^{-4\sigma} \partial_\phi f_{\tilde{h}}) - e^{-4\sigma} M_\Phi^2 f_{\tilde{h}} \right) \]
\[ = \int_{-\pi}^{\pi} d\phi f_{\tilde{h}} \left( e^{-2\sigma} m_\tilde{h}^2 f_{\tilde{h}} \right), \tag{17} \]
with the orthonormal condition applied to the last step, and the quartic coupling is defined by
\[ \lambda_{S,\text{ann}} \equiv \frac{\lambda_S^{(5)}}{\tau_c} \int_{-\pi}^{\pi} d\phi e^{-4\sigma} f_0 f_n f_m f_l. \tag{18} \]

Eq. (17) is nothing but the EoM given by eq. (7). Therefore, even after including the self-interaction in the 5D EoM, the 4D mass identification is no affected, and the previous EoM for obtaining the profile is valid. Moreover, eq. (13) can be readily reproduced via eq. (18).

It should be mentioned that, the derivation of eq. (7) follows the philosophy of KK decomposition [20], which is defined in the free action, while interactions are treated perturbatively after inserting the KK decomposition [20]. To validate this perturbative treatment, we apply the higher-mode profiles [4] and find that, for the zero-mode EoM concerned here, the quartic couplings associated with $S_0$ are well within the perturbative region. In particular, the couplings of $S_0 S_i S_j S_k$, $S_0^2 S_i S_j$ and $S_0^3 S_i$ for low-lying $i, j, k > 0$ are many orders-of-magnitude smaller than $\lambda_{S,\text{ann}} \equiv \lambda_{S,0}$, which will be shown later to be $\lambda_{S,0} \lesssim O(0.1)$.

In the following, we proceed with the DM evolution by a simple two-step freeze-in production under some proper assumptions, and show that eqs. (12) and (13) can successuflly predict the large parameter span in terms of the $O(1)$ 5D parameters: $a, \xi, \lambda_P^{(5)}$.

**Uniting freeze-in self-interacting DM:** For simplicity, we consider the case where the KK scalars $S_n$ have an unbroken $Z_2$ symmetry in the 4D regime such that the vacuum expectation values vanish ($S_0 = 0$). In this regime, there is no mass mixing between the zero mode $S_0$ and the physical Higgs boson $h$. Being the lightest KK scalar, $S_0$ can be a stable DM candidate, with the mass set by the SM Higgs vacuum
\[ m_{S_0} = \sqrt{\lambda_P^{(5)} v}, \tag{19} \]
which would be fixed once the portal coupling is pinned down by fitting the observed relic abundance from freeze-in production. To wit, the mass and relic density of the classically scale-invariant DM share a common origin from a portal coupling [15, 16].

It should be mentioned that, in explicit DM models with an implicitly assumed initial reheating mechanism, the production and evolution history of the scalar DM can be more involved than the conventional freeze-in pattern [9, 21, 22]. Generically, at the gauge-symmetric phase, the DM interacts with the Higgs doublet $H$ via $2 \rightarrow 2$ scattering $HH \rightarrow S_0 S_0$. At the gauge-broken phase, the dominant production comes from the Higgs boson decay $h \rightarrow S_0 S_0$. As temperature goes down to the Higgs boson mass $T \lesssim m_h$, the $h$-particle density becomes Boltzmann suppressed and the DM finally freezes in. Note that, if the DM has relatively strong self-interactions, the particle density may get modified via particle-changing $2 \leftrightarrow 4$ processes [23], and the DM could thermalize itself with a different temperature $T'$ and further undergo the dark freeze-out [24, 25]. However, the dark freeze-out process depends on the initial temperature ratio of the dark sector and the visible SM bath, $\chi \equiv T/T'$, and it could be absent if the initial ratio is too large ($\chi \gg 1$) to establish DM self-thermalization, which may originate from an explicit reheating dynamics [24, 26, 27].

On the other hand, additional production channels of the DM $S_0$ are also possible from the portal interaction $S_0 S_i H^+ H^-$, with $i > 0$. For the low-lying KK modes, their masses are given by [4]
\[ m_{S_i} \simeq i + \frac{\sqrt{a + 4}}{2} - \frac{3}{4} k \pi e^{-kr_\pi} \gtrsim 2 \text{ TeV}, \tag{20} \]
and the portal coupling between $S_0$ and $S_i$ turns out to be
\[ \lambda_{P,0i} = \lambda_P^{(5)} e^{-2\sigma_0} f_0(\pi)f_i(\pi) \simeq O(10^{-5}). \tag{21} \]

In the early Universe, the productions via $HH \rightarrow S_0 S_i$ before EWBS and $S_i \rightarrow h S_0$ after EWBS could contribute to the final $S_0$ relic density given that the portal coupling $\lambda_{P,0i}$ is sufficiently larger than $\lambda_{P,0} \simeq O(10^{-10})$. Nevertheless, the influence may be small or even negligible, if the initial dynamics predicts such a quite low reheating temperature $T_{\text{reh}}$ that the thermal mass $M_H(T)$ cannot significantly exceed $m_{S_0}/2$. In this case, the $HH \rightarrow S_0 S_i$ channel is endothermic and thus suppressed, or the production of heavier KK scalars could even not occur. In some complicated cases, on the other hand, the heavier KK scalars may decay or annihilate away quickly through gravitational effects [28], thus produce negligible $S_0$ yield.

Given the above uncertainties, we will assume in the following for simplicity that the dark freeze-out process is absent and the DM relic abundance is fixed by the two-step production in the early Universe: $HH \rightarrow S_0 S_0$ before EWBS and $h \rightarrow S_0 S_0$ after EWBS. The Boltzmann equations for the two processes can then be written as
\[ \frac{dY_1}{dT} = -\frac{2\gamma_{HH \rightarrow S_0 S_0}}{s_{\text{SM}} H T}, \quad \frac{dY_2}{dT} = -\frac{2\gamma_{h \rightarrow S_0 S_0}}{s_{\text{SM}} H T}, \tag{22} \]
where the factor 2 accounts for the fact that DM is pair-produced. The Hubble rate is given by $H(T)$ =
and the entropy density of the SM plasma $s_{\mathrm{SM}} = \frac{2\pi^2}{3}\frac{g_\ast^2 T^3}{45}$, with the relativistic degrees of freedom $g_\ast^2 \approx g_c^2 \approx 106.75$ during the production epoch. We use $Y_{1,2}$ to denote the DM yields produced before and after the EWSB. The collision terms $\gamma_{H \to S_0 S_0}$ and $\gamma_{h \to S_0 S_0}$ are determined to be

$$
\gamma_{H \to S_0 S_0} = \frac{2\pi^2 T}{(2\pi)^6} \int_{4M_H^2}^\infty d\hat{s} \sqrt{\hat{s} - 4M_H^2} K_1(\sqrt{\hat{s}/T}) \times \sigma_{H \to S_0 S_0},
$$

$$
\gamma_{h \to S_0 S_0} = \frac{m_h^2 T}{2\pi^2} \Gamma_{h \to S_0 S_0} K_1(m_h/T),
$$

respectively. Here $K_1$ is the first modified Bessel function of the second kind. $m_h \approx 125$ GeV is the vacuum mass of the SM Higgs boson and the thermal mass $M_H$ at $T > T_c$ is given by [29, 30]

$$
M_H(T) = \left( \frac{3}{16} g_2^2 + \frac{1}{16} g_Y^2 + \frac{1}{4} m_Y^2 + \frac{1}{8} \lambda_H \right) (T^2 - T_c^2),
$$

where $g_2$ ($g_Y$) is the $SU(2)_L$ ($U(1)_Y$) gauge coupling and $m_Y$ the top-quark Yukawa coupling. $T_c \approx 164$ GeV denotes the critical temperature of electroweak phase transition. The cross section $\sigma_{H \to S_0 S_0}$ and decay rate $\Gamma_{h \to S_0 S_0}$ are given by

$$
\sigma_{H \to S_0 S_0} = \frac{g_H^2 \lambda_{P,0}^2}{8\pi \sqrt{s}},
$$

$$
\Gamma_{h \to S_0 S_0} = \frac{\lambda_{P,0}^2 v^2}{8\pi m_h},
$$

where $g_H = 2$ accounts for the two gauge components of the Higgs doublet, and we have neglected the scalar DM mass since $m_{S_0}$ is expected to be at MeV scale.

Integrating $Y_1$ from $T_c \to \infty$ gives the yield produced before EWSB, and integrating $Y_2$ from $T = 0$ to $T_c$ gives the yield produced by the Higgs decay after EWSB. Note that, besides the subdominant contribution from scattering processes after EWSB, we have also neglected a small yield produced from a short period of time around $T \approx T_c$ [9]. The total yield is then found to be

$$
Y_f = Y_1 + Y_2 \approx (1.28 \times 10^{10} + 8.58 \times 10^{12}) \lambda_{P,0}^2.
$$

It can be seen that, the primary production comes from the Higgs decay after EWSB. Using the current values of entropy ($s_{\mathrm{SM,0}}$) and critical energy ($\rho_c$) densities [31], we can estimate the DM relic density via

$$
\Omega_{S_0} h^2 = \frac{m_{S_0} Y_f s_{\mathrm{SM,0}}}{\rho_c h^2} \approx 2.74 \times 10^5 \left( \frac{m_{S_0}}{\text{MeV}} \right) Y_f.
$$

It becomes clear that, the DM mass $m_{S_0}$, relic density $\Omega_{S_0} h^2$ and the Higgs portal coupling $\lambda_{P,0}$ can all be well estimated by a single $O(1)$ $a$ since their exponential behavior is controlled by $a$. We plot this feature in figure 1 by setting $\lambda_{P,0}^2 = 1$ to show the $a$-dependence of the portal coupling $\lambda_{P,0}$ (red) and relic abundance $\Omega_{S_0} h^2$ (magenta). Besides, by varying $\lambda_{P,0}^2$, we further show in the subfigure the correlation between $\lambda_{P,0}^2$ and $a$ in generating the DM relic density, where the dark-red region denotes $\Omega_{S_0} h^2 \lesssim 0.12$. It can be seen that, the fitting of $\Omega_{DM} h^2 = 0.12$ [32] exhibits a weak dependence on the parameter $\lambda_{P,0}^2$. This property allows us to obtain a good estimate of the parameter $a$ for varying $\lambda_{P,0}^2$ in the natural regime $O(0.1 - 1)$. As a reference point, we set $\lambda_{P,0}^2 = 1$ to obtain $a \approx -2.26$, as shown by the vertical line in figure 1. Then using the estimated value of $a$, we can predict the portal coupling and mass to be

$$
\lambda_{P,0} = 1.34 \times 10^{-10} \lambda_{P,0}^2, \quad m_{S_0} = 2.85 \sqrt{\lambda_{P,0}^2} \text{ MeV}.
$$

In addition, we also show the $a$-dependence of the self-interacting coupling $\lambda_{S,0}$ (blue) by fixing $\xi = 1$. It can be readily seen that $\lambda_{S,0}$ is rather stable in terms of varying $a$, which can also be inferred from eq. (13). This property implies that, we can fix $a \approx -2.26$ obtained from fitting the relic density to exploit the DM self-interaction in light of the free parameters $\lambda_{P,0}^2, \xi$. In this context, a similar order with respect to $\lambda_H = 2m_H^2/v^2$ is induced: $\lambda_{S,0} \approx 0.16\xi$. Remarkably, it predicts a ratio of $S_0 S_0 \to S_0 S_0$ cross section over mass $m_{S_0}$ as

$$
\frac{\sigma_{S_0 S_0 \to S_0 S_0}}{m_{S_0}} \approx \frac{22}{\lambda_{P,0}^2} \left( \frac{\lambda_{P,0}^2}{(\lambda_{P,0}^2 \xi)^{3/2}} \right) \text{ cm}^2/\text{g}.
$$

We show in figure 2 the region of $\sigma_{S_0 S_0 \to S_0 S_0}/m_{S_0}$ that fits the favored range $0.1 - 10 \text{ cm}^2/\text{g}$, validating the natu-
ralness criterion on these fundamental parameters. Note here that, the range is simply taken as a moderate estimate to infer the ability of solving the small-scale issues, such as the too-big-to-fail and missing satellites problems. A detailed simulation for these explanations requires taking various uncertainties from other observational constraints. For more details, one can consult from the recent review [18]. In short, we have demonstrated that, the classically scale-invariant scalar DM from the WED can simultaneously exhibit an exponentially suppressed Higgs portal to implement the freeze-in production, and a self-interacting pattern: $\sigma_{S_0S_0\rightarrow S_0S_0}/m_{S_0} \in [0.1 - 10]$ cm$^2$/g to alleviate the small-scale problems [17] from 5D natural couplings.

**Summary:** We have presented an embedding of a classically scale-invariant DM singlet in the warped extra dimension. With an exponentially suppressed profile in the extra dimension, a feeble portal coupling at $O(10^{-10})$ and a relatively large quartic coupling at $O(0.1)$ can be naturally generated by well-controlled fundamental parameters. Under some proper initial assumptions, the DM relic abundance can be approximately generated by a two-step freeze-in process. In this regime, we have shown that, it successfully explains the numerical coincidence for freeze-in self-interacting DM, which on one hand fits the current relic density, and on the other hand, alleviate the small-scale problems via considerable self-interactions.

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