STATUS OF THE HADRONIC $\tau$ DETERMINATION OF $|V_{us}|$

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We update the extraction of $|V_{us}|$ from hadronic $\tau$ decay data in light of recent BaBar and Belle results on the branching fractions of a number of important strange decay modes. A range of sum rule analyses is performed, particular attention being paid to those based on “non-spectral weights”, developed previously to bring the slow convergence of the relevant integrated $D = 2$ OPE series under improved control. Results from the various sum rules are shown to be in good agreement with one another, but $\sim 3\sigma$ below expectations based on 3-family unitarity. Important avenues for future study are briefly discussed.

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I. INTRODUCTION

Once B factory data is fully analyzed, the precision of the hadronic $\tau$ decay determination of $|V_{us}|$ is expected to match or exceed that of $K_{3\ell^-}$ and $\Gamma[K_{\mu 2}] / \Gamma[\pi_{\mu 2}]$-based methods \cite{1,2}, providing important tests of three-family unitarity and the Standard Model (SM) scenario for weak semi-leptonic $\Delta S = 1$ interactions. The analysis involves weighted integrals of the spectral functions of the vector (V) and/or axial vector (A) current-current correlators, examples of which are the normalized branching function ratios $R_{V/A;ij} = \Gamma[\tau^- \to \nu_\tau \text{hadrons}_{V/A;ij} (\gamma)] / \Gamma[\tau^- \to \nu_\tau e^- \bar{\nu}_e (\gamma)]$ associated with inclusive flavor $ij = ud, us$, V/A-current-induced hadronic $\tau$ decays.

The ratios $R_{V/A;ij}$ are related to the $J = 0,1$ spectral functions, $\rho_{V/A;ij}^{(J)} (s)$, of the

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corresponding current-current correlators via \( \Pi \),

\[
R_{V/A;ij} = 12\pi^2 |V_{ij}|^2 S_{EW} \int_0^{m^2} \frac{ds}{m^2} \left[ u^{(0)}_T(y_T) \rho^{(0+1)}_{V/A;j}(s) - w^{(0,0)}_L(y_T) \rho^{(0)}_{V/A;j}(s) \right] \tag{1}
\]

where \( y_T = s/m^2 \), \( u^{(0)}_T(y) = (1 - y)^2(1 + 2y) \), \( w^{(0,0)}_L(y) = 2y(1 - y)^2 \), \( V_{ij} \) is the flavor \( ij \) CKM matrix element, and \( S_{EW} \) is a short-distance electroweak correction. \((0 + 1)\) denotes the sum of \( J = 0 \) and \( 1 \) contributions.

Using the general finite energy sum rule (FESR) relation,

\[
\int_{th}^{s_0} ds \ w(s) \rho(s) = -\frac{1}{2\pi i} \oint_{|s|=s_0} ds \ w(s) \Pi(s) , \tag{2}
\]

valid for any correlator without kinematic singularities and any analytic weight, \( w(s) \), the RHS in Eq. (1) can be replaced by an integral around \( |s| = s_0 = m^2 \) involving the correspondingly weighted linear combination of \( \Pi^{(0,1)}_{V/A;j}(s) \), the \( J = 0, 1 \) scalar parts of the relevant current-current correlator. For large enough \( s_0 \), this integral can be evaluated using the OPE [4]. Similar FESRs can be constructed for any \( s_0 < m^2 \), either of \( \Pi^{(0+1)}_{V/A;j}(s) \) or \( s\Pi^{(0)}_{V/A;j}(s) \) and general analytic \( w(s) \). The corresponding spectral integrals will be denoted generically by \( R_{ij}(s_0) \) in what follows. We will also use the term “longitudinal” to refer to the \( J = 0 \) contribution in any \( J = 0 + 1/J = 0 \) decomposition.

Flavor-breaking differences of the \( ij = ud, us \) spectral data,

\[
\delta R^w(s_0) = \left[ \frac{R^w_{ud}(s_0)}{|V_{ud}|^2} \right] - \left[ \frac{R^w_{us}(s_0)}{|V_{us}|^2} \right] , \tag{3}
\]

have OPE representations, \( \delta R^w_{\text{OPE}}(s_0) \), beginning with a \( D = 2 \) contribution \( \propto m^2 \). Taking \( |V_{ud}| \) and any OPE parameters from other sources, Eq. (3) provides a determination of \( |V_{us}| \) for any \( w(s) \), and any \( s_0 < m^2 \) sufficiently large that (i) convergence of the integrated series of known OPE terms is good and (ii) contributions from insufficiently well-known higher \( D \) OPE contributions are small. Explicitly,

\[
|V_{us}| = \sqrt{\frac{R^w_{us}(s_0)}{|V_{us}|^2} - \delta R^w_{\text{OPE}}(s_0)} . \tag{4}
\]

For the weights used in the literature, at scales \( s_0 \sim 2 - 3 \text{ GeV}^2 \), \( \delta R^w_{\text{OPE}}(s_0) \) is much smaller than \( R^w_{ud,us}(s_0) \) (at the few-to-several-\% level). The fractional uncertainty on \( |V_{us}| \) \( \sim \Delta(\delta R^w_{\text{OPE}}(s_0))/2R^w_{ud}(s_0) \) produced by an uncertainty \( \Delta(\delta R^w_{\text{OPE}}(s_0)) \) in \( \delta R^w_{\text{OPE}}(s_0) \) is thus much smaller than that on \( \delta R^w_{\text{OPE}}(s_0) \) itself, allowing modest precision in the determination of \( \delta R^w_{\text{OPE}}(s_0) \) to be turned into a high precision determination of \( |V_{us}| \) [1].

Two important theoretical complications, however, must be dealt with.

The first complication is the very bad convergence of the integrated longitudinal \( D = 2 \) OPE series [3] and strong spectral-positivity-constraint violations produced by all truncation schemes employed in the literature for this badly converging series [6]. These
problems necessitate removing longitudinal contributions from the experimental spectral distributions so that one can work exclusively with sum rules based on the better-behaved $J = 0 + 1$ OPE combination. Fortunately, for a combination of chiral and kinematic reasons, the spectral subtraction is strongly dominated by the well-known $\pi$ and $K$ pole contributions. The remaining non-pole, or “continuum”, contributions are doubly chirally suppressed, by factors of $O\left([m_d \pm m_u]^2\right)$ (respectively, $O\left([m_s \pm m_u]^2\right)$) for the flavor $ud$ (respectively $us$) V/A channels. The $ij = ud$ non-pole longitudinal subtraction is thus numerically negligible. For $ij = us$, the continuum part of the longitudinal subtraction, though small, is not entirely negligible, but can be determined using input from sum rule and dispersive studies of the $us$ scalar and pseudoscalar channels [7, 8]. The normalization of these results is strongly constrained by independent information on $m_s$ from the lattice and hence under good control theoretically. The resulting $us$ continuum longitudinal subtraction is, as expected, numerically small.

The second complication is the less than optimal convergence of the $J = 0 + 1 D = 2$ OPE series. Defining $\Delta \Pi(s) \equiv \left[\Pi^{(0+1)}_{V+A;ud}(s) - \Pi^{(0+1)}_{V+A;us}(s)\right]$, one has explicitly, with $\alpha_s(Q^2)$ and $m_s(Q^2)$ the running coupling and strange quark mass in the $\overline{MS}$ scheme, and $\bar{a} = \alpha_s(Q^2)/\pi$ [9],

$$\left[\Delta \Pi(Q^2)\right]^{OPE}_{D=2} = \frac{3}{2\pi^2} \frac{m_s(Q^2)}{Q^2} \left[1 + \frac{7}{3} \bar{a} + 19.93\bar{a}^2 + 208.75\bar{a}^3 + (2378 \pm 200)\bar{a}^4 + \cdots\right], \quad (5)$$

where the $\bar{a}^4$ coefficient is an estimate obtained using methods which previously provided a rather accurate estimate of the $\bar{a}^3$ coefficient in advance of the actual calculation of its value [9]. Since independent high-scale determinations of $\alpha_s(M_Z)$ correspond to $\bar{a}(m^2_t) \simeq 0.10$, convergence at the spacelike point on $|s| = s_0 < m^2_t$ is marginal at best. While the decrease of $|\alpha_s(Q^2)|$ on moving away from the spacelike point along the contour does allow the convergence of the integrated series to be improved through judicious choices of the sum rule weight $w(s)$, for weights not chosen specifically with this purpose in mind very slow $D = 2$ convergence is to be expected. Such slow convergence is, for example, seen explicitly in the case of the so-called $(k, m)$ spectral weights [9]. Because of the growth of $\alpha_s$ with decreasing scale, premature truncation of such a slowly converging series will produce $|V_{us}|$ values with an unphysical $s_0$-dependence. In view of the intrinsically slow convergence of the underlying $D = 2$ correlator series, a study of the $s_0$-dependence of the output $|V_{us}|$ (and demonstration of the existence of an $s_0$-stability window) is thus crucial to establishing the reliability of any hadronic-$\tau$-decay-based FESR extraction of $|V_{us}|$. Analyses in which such a study has not been performed thus typically contain additional theoretical systematic uncertainties, which may be significantly larger than the theoretical errors explicitly studied.
TABLE I: $\tau^- \to X^- \nu_\tau$ branching fractions for strange hadronic states $X^-$. Column 2 contains the PDG2006 world averages, column 3 the 2007 Babar and/or Belle results, and column 4 the resulting new world averages, with PDG-style $S$ factors included where needed.

| $X^-_{us}$ | $B_{WA,2006}$ (%) | $B_{2007}$ (%) | $B_{WA,2007}$ (%) |
|------------|-------------------|----------------|-------------------|
| $K^- \ [\tau \text{ decay}]$ | $0.691 \pm 0.023$ | $0.691 \pm 0.023$ | $0.691 \pm 0.023$ |
| $([K_{\mu2}])$ | $(0.715 \pm 0.003)$ | $(0.715 \pm 0.003)$ | $(0.715 \pm 0.003)$ |
| $K^- \pi^0$ | $0.454 \pm 0.030$ | $0.416 \pm 0.018$ | $0.426 \pm 0.016$ |
| $K^0 \pi^-$ | $0.878 \pm 0.038$ | $0.808 \pm 0.026$ | $0.831 \pm 0.028$ ($S = 1.3$) |
| $K^- \pi^0 \pi^0$ | $0.058 \pm 0.024$ | $0.058 \pm 0.024$ | $0.058 \pm 0.024$ |
| $K^0_\tau \pi^-$ | $0.360 \pm 0.040$ | $0.360 \pm 0.040$ | $0.360 \pm 0.040$ |
| $K^- \pi^- \pi^+$ | $0.330 \pm 0.050$ | $0.273 \pm 0.009$ | $0.280 \pm 0.016$ ($S = 1.9$) |
| $K^- \eta$ | $0.027 \pm 0.006$ | $0.0162 \pm 0.0010$ | $0.016 \pm 0.002$ ($S = 1.8$) |
| $(\bar{K}3\pi^-)$ (est’d) | $0.074 \pm 0.030$ | $0.074 \pm 0.030$ | $0.074 \pm 0.030$ |
| $K_1(1270) \to K^- \omega$ | $0.067 \pm 0.021$ | $0.067 \pm 0.021$ | $0.067 \pm 0.021$ |
| $(\bar{K}4\pi^-)$ (est’d) | $0.011 \pm 0.007$ | $0.011 \pm 0.007$ | $0.011 \pm 0.007$ |
| $K^{*-\eta}$ | $0.029 \pm 0.009$ | $0.0113 \pm 0.0020$ | $0.012 \pm 0.004$ ($S = 2.0$) |
| $K^- \phi$ | $0.00370 \pm 0.00025$ | $0.00370 \pm 0.00025$ | $0.00370 \pm 0.00025$ |

**TOTAL** | $2.979 \pm 0.086$ | $2.830 \pm 0.074$ | $2.830 \pm 0.074$ |

| $X^-_{us}$ | $B_{WA,2006}$ (%) | $B_{2007}$ (%) | $B_{WA,2007}$ (%) |
|------------|-------------------|----------------|-------------------|

**II. THE UPDATED DETERMINATION OF $|V_{us}|$**

Our results for $|V_{us}|$ are based on the ALEPH $ud$ [10] and $us$ [11] spectral data, for which information on the relevant covariance matrices is publicly available. Since no V/A separation has been performed for the $us$ data, we work with the V+A combination (the $ud$ spectral integral errors are also reduced by working with the V+A combination). The older $us$ data [11] has been modified to incorporate the recent (2007) BaBar [12] and Belle [13] updates of a number of the strange branching fractions, $B_{X_{us}}$. The latter measurements, together with the previous and resulting world averages, are shown in Table I. Since the re-measurement of the strange spectral distribution is not yet complete, we follow Ref. [14] and obtain a partially updated inclusive sum by rescaling the 1999 ALEPH distribution [11], mode-by-mode, by the ratio of new to old $B$ values.

With the above $ud$ and $us$ V+A spectral distributions as input, $|V_{us}|$ is determined, as a function of the upper integration limit, $s_0$, in Eq. (2), using $J = 0 + 1$ FESRs based on (i) the three improved-convergence, non-spectral weights, $w_{10}$, $w_{10}$ and $w_{20}$, introduced in Ref. [15], and (ii) the kinematic weight $w_7^{(0, 0)}$ (expected to display problematic integrated OPE convergence behavior and hence $s_0$-instability for $|V_{us}|$ [2]). Standard OPE input, the correlator form of the $D = 2$ contributions, and $K_{\mu2}$ input for the $K$ pole contribution to the $us$ spectral integrals are employed. More details of the analysis will be reported elsewhere [17]. The resulting $s_0$ dependence of $|V_{us}|$ is shown in Fig. II.
The figure shows reasonable $s_0$-stability for $w_{10}$, $\hat{w}_{10}$ and $w_{20}$ in the region above $s_0 \sim 2.5$ GeV$^2$, but significant $s_0$-instability for $w_T^{(0,0)}$. The $s_0 = m_T^2 |V_{us}|$ values,

\[
\begin{align*}
0.2156 \pm 0.0028_{\text{exp}} \pm 0.0022_{\text{th}} & \quad \text{for } w_{20} \\
0.2154 \pm 0.0032_{\text{exp}} \pm 0.0015_{\text{th}} & \quad \text{for } \hat{w}_{10} \\
0.2149 \pm 0.0033_{\text{exp}} \pm 0.0010_{\text{th}} & \quad \text{for } w_{10} \\
0.2144 \pm 0.0030_{\text{exp}} \pm 0.0017_{\text{th}} & \quad \text{for } w_T^{(0,0)},
\end{align*}
\]

are in good agreement, but lie $\sim 3\sigma$ below the expectations of 3-family unitarity \[16\]. The theoretical errors have been estimated using the scheme discussed in Refs. \[2\], which is, by design, more conservative than that employed in previous analyses. In spite of this, the $s_0$-instability of the results from the $w_T^{(0,0)}$ analysis suggests that the theoretical error in this case may still be insufficiently conservative. The $\sim 0.0050$ downward shifts in $|V_{us}|$, compared to the 2006 versions of these analyses \[1, 2\], result dominantly from the $-0.075\% (-0.050\%)$ shifts in $B_{K\pi}$ ($B_{K^-\pi^+\pi^-}$), which represent, respectively, $2.6\% (1.8\%)$ decreases in the total strange branching fraction and hence, from Eq. \[1\], $\sim 1.3\% (0.9\%)$ downward shifts in $|V_{us}|$.

In Figure 2 we compare the results above for $|V_{us}|$ to those of a number of other determinations, from various sources. The result from $K_{\ell 3}$ decay is based on the most recent KLOE $K$ decay data \[18\], together with the RBC/UKQCD lattice determination of $f_+(0)$ \[19\], that for Marciano’s method (which employs the ratio of $K_{\mu2}$ to $\pi_{\mu2}$ decay widths in combination with lattice input for $f_K/f_\pi$ \[20\]) on the FlaviaNet Kaon Working Group assessment of the experimental ratio \[21\] and Juttner’s Lattice 2007 plenary assessment of the status of lattice determinations of $f_K/f_\pi$ \[22\], while that from hyperon decays, included for completeness, is Jamin’s assessment as of Electroweak Moriond 2007 (which is 0.0010 higher, and with larger errors, than the result quoted in Ref. \[23\]). The first of the $\tau$ decay results represents a range of assessments based on the $w_T^{(0,0)}$ FESR and pre-2007 $us$ data. The next two $w_T^{(0,0)}$ FESR assessments are from Ref. \[24\] and incorporate most, but not all, of the new 2007 $us$ data. The sizeable difference produced by switching from $K_{\mu2}$ input for the $K$ pole contribution (the “pred. $\tau \to K\nu$” result) to the central measured value (the “meas. $\tau \to K\nu$” result) is a reflection of the high degree of cancellation between the $ud$ and $us$ spectral integrals \[25\]. Note that the $\tau$ decay results just mentioned, as well as those of Refs. \[26\], employ the 4-loop-truncated Adler function evaluation of the $D = 2$ OPE integral contribution. The remaining $\tau$ decay results shown in the figure are the ones obtained by us above, which, in contrast, employ the 4-loop-truncated correlator evaluation of the $D = 2$ contribution. The non-trivial difference between the results obtained using the two different methods (which are formally equivalent, up to contributions of $O(\alpha_s^4)$ and higher) provides further evidence for the slow convergence of the integrated $D = 2$ series for the $w_T^{(0,0)}$ weight, seen already in Fig. \[1\].

While the results above are compatible with small beyond-the-SM contributions, such a conclusion is obviously premature until BaBar and Belle have cross-checked each other’s recent results and also filled in the missing entries in column 3 of Table \[1\] (especially for
FIG. 1: $|V_{us}|$ versus $s_0$ for, from top to bottom, $w_{20}$, $\hat{w}_{10}$, $w_{10}$ and $w_T^{(0,0)}$.

the sizable $\bar{K}^0\pi^0\pi^-$ and previously estimated, but unmeasured, $\bar{K}3\pi$ and $\bar{K}4\pi$ modes). Results for higher multiplicity modes, with branching fractions down to the few $\times 10^{-5}$ level, are also needed for a determination at the desired level of accuracy. A finalized version of the results will require completion of the re-measurement of the full $us$ spectral distribution, work on which is in progress. Note that improved precision on the measurement of the ratio of branching fractions of the single-particle $\pi$ and $K$ modes is also of interest for the determination of $|V_{us}|$ since it would allow an independent determination in the $\tau$ sector analogous to that based on $\Gamma[K_{\mu2}]/\Gamma[\pi_{\mu2}]^{[20]}$. Experimental input on the
chirally-suppressed continuum $us$ $J = 0$ spectral contributions, needed for performing the longitudinal subtraction, appears potentially experimentally feasible, and would also be most welcome.

Finally, we note that a re-measurement of the $ud$ distribution is also of relevance, particularly in light of the known discrepancy between the $\pi\pi$ and $4\pi$ distributions measured in $\tau$ decay and those implied by electroproduction cross-sections (via CVC and known
were the electroproduction expectations to be correct, the values of $|V_{us}|$ obtained from the above analyses would be increased by $\sim 0.0018$. It is worth noting, in this regard, that two recent analyses of non-strange hadronic $\tau$ decay data \cite{27, 28}, one designed specifically to incorporate an improved treatment of $D > 4$ OPE contributions which were problematic for earlier determinations, yield values of $\alpha_s(M_Z)$ in excellent agreement with the results, 0.1190(26) and 0.1191(27), obtained from updates to the global fit to electroweak observables at the Z scale (quoted in Refs. \cite{29, 30}, respectively). Two recent updates \cite{31, 32} of the earlier lattice determination \cite{33}, employing slightly different implementations of the original approach, also obtain values (0.1183(7) and 0.1192(11), respectively) which are higher than the original result and now in excellent agreement with the results of the $\tau$ extraction and global electroweak fit. Since electroproduction cross-sections imply a significantly lower value, $\alpha_s(M_Z) \sim 0.1140 - 0.1150$ \cite{34}, we consider it unlikely that the $ud \tau$ data is significantly in error, but nonetheless a re-measurement from the B factory experiments should be pursued, given the potential impact on both SM expectations for $(g - 2)_\mu$ and on the value of $|V_{us}|$ obtained in the hadronic $\tau$ decay analyses.

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