A Brownian Motion Model of Parametric Correlations in Ballistic Cavities

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(March 23, 2022)

Abstract

A Brownian motion model is proposed to study parametric correlations in the transmission eigenvalues of open ballistic cavities. We find interesting universal properties when the eigenvalues are rescaled at the hard edge of the spectrum. We derive a formula for the power spectrum of the fluctuations of transport observables as a response to an external adiabatic perturbation. Our formula correctly recovers the Lorentzian-squared behaviour obtained by semiclassical approaches for the correlation function of conductance fluctuations.

Pacs numbers: 72.10.Bg, 05.60.+w, 05.40.+j, 05.45.+b
I. INTRODUCTION

The theory of random matrices has been applied to many branches of physics, such as complex nuclei, disordered metallic grains, the general theory of quantum chaotic systems, disordered conductors, random surfaces, QCD and more recently strongly correlated many-particle systems. One of the most striking and robust predictions of random matrix theory (RMT), which has far reaching implications in all these different fields is the celebrated Wigner-Dyson statistics, which states that the probability distribution of level spacings (usually regarded as energy eigenvalues of some random Hamiltonian) is a universal function if the eigenvalues are measured in units of the mean level spacing. In the case of quantum chaotic systems, it is believed that the success of RMT depends on the chaotic dynamics of the equivalent classical system. This dependence seems to be so strong that many results derived from random matrix ensembles, such as the Wigner-Dyson statistics, have been used as signatures of quantum chaos.

An interesting extension of RMT is to consider variations in the energy levels of the physical system resulting from external adiabatic perturbations. These perturbations could be, for instance, a change in the Aharonov-Bohm flux through a ring, or a slight alteration in the position of some impurity in a disordered metallic grain, or a variation in the strength of an external applied magnetic field or even simply a tiny modification in the geometry of the sample. The theory, subsequently developed by Simons, Altshuler, Lee and others to account for this effect, has proven to be a significant extension of RMT, which we shall call parametric random matrix theory (PRMT). Remarkably, it has been demonstrated that PRMT, while incorporating all features of RMT, predicts new universal statistics after appropriate rescaling of the physical parameters, and thus provides an even more powerful characterization of quantum chaotic dynamics. PRMT contains only two system dependent parameters: the mean level spacing, $\Delta$, and the mean square gradient of levels, $C_0$, which is defined as the ensemble average

$$C_0 = \left\langle \left( \frac{\partial \varepsilon_i(U)}{\partial U} \right)^2 \right\rangle,$$

where $\varepsilon_i \equiv E_i/\Delta$ denote renormalized energy levels and $U$ is the parameter that controls the strength of the external perturbation. A striking prediction of PRMT is that the $n$-point correlation function for density of states fluctuations becomes a universal expression if the energy levels are measured in units of $\Delta$ and the perturbation parameter, $U$, is renormalized by $C_0$. This universality has been derived analytically for disordered metallic grains with orthogonal, unitary and symplectic symmetries and has been verified numerically in a number of chaotic systems.

We remark that the universality obtained in the framework of PRMT, and similarly that of Wigner-Dyson statistics, applies only for levels away from the edge of the support of the spectrum. This statement can, in principle, be formally justified by means of renormalization group arguments. A simple renormalization group procedure has recently been devised by Brézin and Zinn-Justin. These authors have shown that there are two kinds of fixed points...
in the renormalization group equations: a stable gaussian one governing the behavior of the system at the bulk of the spectrum and an unstable one governing a small crossover region around the endpoint of the support where the average density goes to zero as a power law. The attractive gaussian fixed point supports the general validity of the Wigner-Dyson statistics everywhere at the bulk of the spectrum. It is therefore natural to expect that the results of PRMT are of similar general validity, although an explicit proof is not yet available.

At the edge of the spectrum, however, Wigner-Dyson statistics breaks down and an entirely new regime of universal statistics emerges. There are two kinds of edges in RMT: a hard edge and a soft edge, where the eigenvalue support is bounded and unbounded respectively. The Gaussian ensembles have two soft edges, while the Laguerre ensembles exhibit a hard and a soft edge. There are also ensembles with two hard edges, like for instance the Jacobi ensembles.

The hard edge of the Laguerre ensemble is important in the description of many physical systems such as disordered metallic conductors, ballistic cavities and QCD. In a recent paper Slevin and Nagao have introduced a group of matrices, which they called Ω-matrices, that gives rise to a very powerful way of studying the Laguerre ensemble. Although they have used the mathematical structure of this group to propose a model to describe disordered metals, we believe that the hard edge of the ensemble generated from the group of Ω-matrices is actually more appropriate to describe ballistic cavities.

Ballistic cavities are of considerable current interest mainly because of important recent breakthroughs in nanolithography, which has enabled its construction in novel high-mobility semiconductor heterostructures. These systems have elastic and inelastic mean free paths exceeding the device dimensions at sufficiently low temperatures. As a consequence, transport in these structures is dominated by scattering at the boundaries of the sample rather than the more usual impurity scattering of mesoscopic metals. Two striking features of the physics of these systems are weak-localization and universal conductance fluctuations. Such typically mesoscopic phenomena appear in ballistic cavities due to the chaotic nature of the boundary scattering potential.

In this work we shall mostly be concerned with a set-up that consists of an interaction region of finite volume (the resonant cavity in a microwave experiment or the ballistic microstructure in mesoscopic physics) connected to two reservoirs by free propagation regions (wave guides for microwaves or perfectly conducting leads for electron waves) in which asymptotic scattering channels can be defined. The interaction region is assumed to "trap" the incoming waves by irregular boundary scattering thereby driving the system to a regime where the ray optic limit (or classical dynamics) is dominated by classical chaos. We shall call such a set-up an open ballistic cavity. In particular, we study the problem of parametric correlations in open ballistic cavities, that is, the response of the random spectra to an external adiabatic perturbation. Our main objective is twofold: first, we want to extend the PRMT to describe open quantum chaotic systems, such as ballistic cavities coupled to external reservoirs; and second we want to understand how the theory changes when the region of physical interest is at the hard edge of the support of the spectrum. The physics of the soft edge has been discussed elsewhere.

Parametric correlations in open mesoscopic systems has recently been the subject of many works. A concise account of some of our results has been presented previously.
In this work we provide much more details of the calculations, a different interpretation of the hydrodynamic limit and new results.

In section II we discuss the S-matrix approach to ballistic systems. In section III, we review some of the properties of the group of Ω-matrices of Slevin and Nagao. In addition, we derive a diffusion equation generated by a random walk on the Ω-matrix manifold. We propose a modified version of this equation as a Brownian motion model for parametric correlations in open ballistic cavities. In section IV, the Brownian motion model is used to derive exact non-perturbative expressions for the parameter-dependent two-point correlation function at the hard edge of the spectrum. We demonstrate that after appropriate rescaling this function becomes universal. In section V we show that in the hydrodynamic limit our theory predicts a Lorentzian-squared behaviour for the correlator of conductance fluctuations of ballistic cavities in agreement with semiclassical calculations. Finally, we derive a formula for the power spectrum of an arbitrary linear statistic on the transmission eigenvalues. A summary and conclusions are presented in section VI.

II. S-MATRIX AND BALLISTIC SYSTEMS

It is now well established that the most convenient description of quantum transport in open ballistic cavities is given by the S-matrix. By definition the S-matrix relates the incoming flux amplitudes \( I_l \) and \( I_r \) to the outgoing ones \( O_l \) and \( O_r \) through the formula

\[
S \left( \begin{array}{c} I_l \\ I_r \end{array} \right) = \left( \begin{array}{c} O_l \\ O_r \end{array} \right),
\]

where the subscripts \( l \) and \( r \) denote the left and the right sides of the sample respectively. Current conservation implies that \( S \) is unitary: \( S^{-1} = S^\dagger \). A general and rather simple explicit parametrization for \( S \) is

\[
S = \left( \begin{array}{cc} r & t' \\ t & r' \end{array} \right) = \left( \begin{array}{cc} u^{(1)} & 0 \\ 0 & u^{(2)} \end{array} \right) \left( \begin{array}{cc} -\sqrt{1-\tau} & \sqrt{\tau} \\ \sqrt{\tau} & \sqrt{1-\tau} \end{array} \right) \left( \begin{array}{cc} v^{(1)} & 0 \\ 0 & v^{(2)} \end{array} \right),
\]

where \( r \) and \( r' \) are \( N \times N \) reflection matrices; \( t \) and \( t' \) are \( N \times N \) transmission matrices; \( u^{(i)}(i = 1, \ldots, 4) \) are \( N \times N \) unitary matrices and \( \tau \) denotes an \( N \times N \) diagonal matrix with real eigenvalues \( 0 \leq \tau_\alpha \leq 1(\alpha = 1, 2, \ldots, N) \), which are called transmission eigenvalues. The total transmission coefficient can be written as

\[
T \equiv \text{tr} tt^\dagger = \sum_{\alpha=1}^{N} \tau_\alpha.
\]

Therefore, the Landauer-Büttiker conductance is simply

\[
G = G_0 \sum_{\alpha=1}^{N} \tau_\alpha.
\]

The remarkable simplicity of this formula, in comparison with the Kubo-Greenwood expression for the conductance obtained directly from linear response theory, highlights the advantage of the Landauer-Büttiker formalism. This, however, cannot be the end of the
story because in principle the transmission eigenvalues, $\tau_\alpha$, depend in a very complicated way on the underlying quantum dynamics induced by the scattering mechanism. Fortunately, in some cases of interest such as ballistic cavities and quasi-one-dimensional disordered conductors the complexity of the dynamics turns out to be such that most details of microscopic origin are irrelevant and the joint probability distribution of the $\tau_\alpha$’s can be determined from symmetry arguments which are well known in the context of random matrix theory.

In the case of ballistic cavities, it has been shown that the $S$-matrix fluctuates as a function of the incident momentum due to multiple overlapping resonances in the cavity. There is considerable numerical evidence to support the assumption that through this fluctuations the $S$-matrix covers its manifold with uniform probability. With this ergodic hypothesis in mind, we can study the fluctuations in the $S$-matrix by means of an ensemble of random matrices with a uniform distribution. In terms of the parametrization (3) one finds, after integration over the eigenvectors distributions,

$$dP_\beta(\tau) \equiv d\mu_\beta(\tau) = C_{N,\beta}J_\beta(\tau) \prod_c \tau_c^{(\beta-2)/2} \prod_a d\tau_a,$$

where $C_{N,\beta}$ is a normalization constant and $\beta$ is a symmetry index, whose value is $\beta = 1$ for systems with time-reversal symmetry (T-symmetry) in the absence of spin-orbit scattering, $\beta = 2$ for systems without T-symmetry and $\beta = 4$ for systems with T-symmetry in the presence of spin-orbit scattering. The factor $J_\beta(\tau) \equiv \prod_{a<b} |\tau_a - \tau_b|^\beta$ is ultimately responsible for transmission eigenvalue repulsion in the ensemble.

At this stage it is convenient to make the following change of variables, $\tau_i = 1/(1 + \nu^2_i)$, with $0 \leq \nu_i < \infty$. From Eq. (3) we find the following joint probability density for the variables $\{\nu_i\}$

$$P(\nu) = Z^{-1} \exp(-\beta H),$$

where

$$H = \frac{1}{2} \sum_{i \neq j} Q(\nu_i, \nu_j) + \sum_i V(\nu_i),$$

$$Q(\nu, \nu') = \ln |\nu^2 - \nu'^2|,$$

and

$$V(\nu) = \left( N + \frac{2 - \beta}{2\beta} \right) \ln(1 + \nu^2) - \frac{1}{\beta} \ln \nu.\]$$

Note that $P(\nu)$ has the form of a Gibbs distribution and $H$ plays the role of a Hamiltonian of classical particles with logarithmic pairwise repulsion, $Q(\nu, \nu')$, and a confining potential, $V(\nu)$. The dimensionless Landauer-Büttiker conductance in this new set of variables reads

$$g = \sum_{i=1}^N g(\nu_i),$$

5
where \( g(\nu) = (1+\nu^2)^{-1} \). In RMT observables of this form are called linear statistics because products of different eigenvalues do not appear in their defining expressions. Consequently, their statistical properties are the simplest. In particular, the average and variance of Eq. (11) are simply

\[
\langle g \rangle = \int_0^\infty g(\nu)\rho(\nu)d\nu,
\]

\[
\text{var}(g) = \int_0^\infty g(\nu)g(\nu')S(\nu,\nu')d\nu d\nu',
\]

in which \( \rho(\nu) \) and \( S(\nu,\nu') \) are the average level density and two-point correlation function respectively. There is a very powerful way of calculating \( S(\nu,\nu') \) which has been developed by Beenakker. It is based on the following identity

\[
S(\nu,\nu') = -\frac{1}{\beta} \frac{\partial \rho(\nu)}{\partial V(\nu')},
\]

which can be easily verified. For \( N \gg 1 \), one can show that

\[
V(\nu) = \int_0^\infty \rho(\nu')Q(\nu,\nu')d\nu'.
\]

Using (15) the functional derivatives in (14) can be performed and we find

\[
S(\nu,\nu') = -\frac{1}{\beta} Q^{-1}(\nu,\nu') = \frac{1}{\pi^2 \beta} \frac{\partial^2}{\partial \nu \partial \nu'} \ln \left[ \frac{\nu + \nu'}{\nu - \nu'} \right] + O(1/N).
\]

A remarkable consequence of this technique is the conclusion that the leading expression for \( S(\nu,\nu') \), in an expansion in inverse powers of \( N \), is independent of the potential \( V(\nu) \). Therefore, the variance of a linear statistic, such as the conductance (see Eq. (13)), is the same for all random matrix ensembles with an edge at the origin of the spectrum (since the \( \nu \)'s are all non-negative) and with an eigenvalue repulsion potential equal to \( Q(\nu,\nu') \). Consequently, if we are only interested in studying the dominant contribution to fluctuations in mesoscopic observables, we are free to choose the functional dependence of \( V(\nu) \) that we find convenient. It turns out that the simplest choice for \( V(\nu) \) can be obtained from (10) by means of the rescaling, \( \nu \rightarrow \nu/N \), and by taking the large \( N \) limit keeping \( \nu \) fixed. We then get

\[
V(\nu) = \frac{1}{N} \nu^2 - \frac{1}{\beta} \ln \nu.
\]

We remark that Eqs. (7), (8) and (9) with \( V(\nu) \) given by (17) constitute the Laguerre ensemble of random matrices. This ensemble has a number of interesting universal properties which will be discussed in section III, where it is used to build a model of parametric correlations in ballistic cavities.

We conclude this section by giving the leading terms in the expansions of \( \langle g \rangle \) and \( \text{var}(g) \) in inverse powers of \( N \)
\[ \langle g \rangle = \frac{1}{2} N + \frac{\beta - 2}{4\beta} + O(1/N), \quad (18) \]

\[ \text{var}(g) = \frac{1}{8\beta} + O(1/N). \quad (19) \]

The leading term in Eq. (18) is a direct consequence of the ergodic hypothesis, since it implies that the average transmission and reflection probabilities are equal. The second term in Eq. (18) corresponds to the weak-localization correction, which as is due to constructive interference of time-reversed backscattering trajectories. Eq. (19) is an illustration of the remarkable phenomenon of universal conductance fluctuations, which is a clear signature of the non-self-averaging nature of observables in quantum transport theory.

III. THE GROUP OF Ω-MATRICES

In this section we introduce the group of Ω-matrices as an abstract mathematical structure, which, as we shall see, is very convenient for studying the Laguerre ensemble. We remark that our approach differs considerably in philosophy from that used by Slevin and Nagao. They have motivated the group of Ω-matrices by relating it to the group of transfer matrices of a disordered conductor. We believe that to make such a connection from the outset is unnecessary and might lead to misinterpretations. The group of Ω-matrices is interesting in its own right as a powerful way of describing statistical properties of the Laguerre ensemble, which we regard as a tractable mathematical model for studying local eigenvalue correlations in systems with a hard edge in the spectrum. Furthermore, as we discussed in section II, the formula for the variance of linear statistics derived from the Laguerre ensemble applies quite generally to any maximum-entropy ensemble with a hard edge, most notably, the ensemble describing ballistic cavities.

By definition, a general complex matrix Ω satisfies

\[ \Sigma_z \Omega \Sigma_z = -\Omega. \quad (20) \]

If the system has T-symmetry we have the additional relation

\[ \Omega^* = \Sigma_x \Omega \Sigma_x. \quad (21) \]

The matrices \( \Sigma_z \) and \( \Sigma_x \) are given by

\[ \Sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (22) \]

and

\[ \Sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (23) \]

where 1 is the \( N \times N \) unit matrix for orthogonal and unitary systems, and it is the \( N \times N \) unit quaternion matrix for symplectic ensembles.
One can show that a general matrix $\Omega$ satisfying (20) and (21) can be written in the form

$$
\Omega = \begin{pmatrix} u & 0 \\ 0 & v \end{pmatrix} \begin{pmatrix} 0 & \nu' \\ \nu & 0 \end{pmatrix} \begin{pmatrix} u^\dagger & 0 \\ 0 & v^\dagger \end{pmatrix},
$$

(24)

where $u$ and $v$ are $N \times N$ unitary matrices for unitary and orthogonal systems and they are $N \times N$ quaternion unitary matrices for symplectic systems. The matrix $\nu$ is a diagonal matrix with real eigenvalues. One can easily check that the matrices $\Omega$ form a group under addition, which we call $G^\beta(\Omega, +)$.

Let $\Omega_t$ denote a point on the $\Omega$-matrix manifold that is generated from a random walk defined by

$$
\Omega_t = \sum_{s=1}^{t} \Omega_s(\delta t),
$$

(25)

where $\delta t$ is the step of the walk and $\Omega_s(\delta t)$ is a random matrix belonging to $G^\beta(\Omega, +)$. The simplest choice for $\Omega_s(\delta t)$ is

$$
\Omega_s(\delta t) = 2i\sqrt{\delta t} \begin{pmatrix} 0 & y \\ -y^\dagger & 0 \end{pmatrix},
$$

(26)

where $y$ is a complex random matrix with vanishing average and with second moment given by

$$
\langle y_{ab}y_{cd}^* \rangle = \frac{1}{\gamma(N+1)}(\delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}), \quad \beta = 1,
$$

(27)

$$
\langle y_{ab}y_{cd}^* \rangle = \frac{1}{\gamma N}\delta_{ac}\delta_{bd}, \quad \beta = 2,
$$

(28)

$$
\langle y_{aba}y_{cd\alpha'}^* \rangle = \frac{1}{2\gamma(2N-1)}(\delta_{ac}\delta_{bd} + (2\delta_{\alpha,0} - 1)\delta_{ad}\delta_{bc})\delta_{\alpha,\alpha'}, \quad \beta = 4,
$$

(29)

in which $\gamma$ is a constant. Note that for systems with symplectic symmetry ($\beta = 4$) we have used the fact that $y$ is a quaternion matrix and thus any of its matrix elements can be written as $y_{ab} = \sum_{\alpha=0}^{3} y_{aba}e_\alpha$, where $e_\alpha$ are the quaternion units.

We can rewrite (25) as $\Omega_{t+\delta t} = \Omega_t + \Omega_s(\delta t)$, which in view of (24) implies, to order $O(\delta t)$, the stochastic process

$$
\delta \nu_i = i\sqrt{\delta t}(A_{ii} - A^\dagger_{ii})/\nu_i + \frac{2\delta t}{\gamma(\beta N + 2 - \beta)} \left[ \frac{1}{\nu_i} + 2\beta \sum_{j(\neq i)} \frac{\nu_j}{\nu_i^2 - \nu_j^2} \right],
$$

(30)

where $A_{ij} \equiv (u^\dagger y v v^\dagger)_{ij}$ and $\nu_i \equiv (\nu)_{ii}$. If we take the continuum limit of (30) we find the following Fokker-Planck equation for the probability distribution $W(\nu, \tau)$
\[
\frac{\gamma}{\partial \tau} \frac{\partial}{\partial \tau} W = \sum \left( -\frac{\partial}{\partial \nu_i} D_i + \frac{\partial^2}{\partial \nu_i^2} \right) W,
\]  
(31)

where \( \tau = 2t/(\beta N + 2 - \beta) \) and

\[
D_i = \frac{1}{\nu_i} + 2\beta \sum_{j \neq i} \frac{\nu_i}{\nu_i^2 - \nu_j^2}.
\]  
(32)

It is interesting to note that the transformation \( W = JP \) implies that \( P \) satisfies

\[
\frac{\gamma}{\partial \tau} \frac{\partial}{\partial \tau} P = \frac{1}{J} \sum \frac{\partial}{\partial \nu_i} J \frac{\partial}{\partial \nu_i} P = \nabla^2 P,
\]  
(33)

where \( J = \prod_i \nu_i \prod_{i<j} |\nu_i^2 - \nu_j^2|^{\beta} \) and \( \nabla^2 \) is the Laplace-Beltrami operator of the \( \Omega \)-matrix manifold.

In the analysis of Slevin and Nagao, the distribution of \( \Omega \)-matrices was determined by a maximum-entropy criterion to be the Laguerre ensemble

\[
W^L(\nu) = C_{N,\beta} \prod_{i<j} |\nu_i^2 - \nu_j^2|^{\beta} \prod_{i=1}^N \nu_i^{\beta+1} \exp\left(-\frac{\beta}{2} c \nu_i^2\right),
\]  
(34)

where \( C_{N,\beta} \) and \( c \) are constants, \( \alpha = 0 \) and \( \beta \) is the usual symmetry index. This result suggests, in analogy with transport theory, that our Brownian motion in the \( \Omega \) manifold must satisfy the requirement that when \( \tau \to \infty \), \( W(\nu, \tau) \to W^L(\nu) \), that is the distribution function evolves to a configuration that maximizes the entropy. This can be achieved simply by adding to \( D_i \) a term corresponding to a confining force so that (31) becomes

\[
\frac{\gamma}{\partial \tau} \frac{\partial}{\partial \tau} W = \sum_{i=1}^N \frac{\partial}{\partial \nu_i} \left( W \frac{\partial \Phi}{\partial \nu_i} + \frac{1}{\beta} \frac{\partial W}{\partial \nu_i} \right),
\]  
(35)

in which

\[
\Phi(\nu) = -\left(\alpha + \frac{1}{\beta}\right) \sum_i \ln \nu_i + \frac{c}{2} \sum_i \nu_i^2 - \sum_{i<j} \ln |\nu_i^2 - \nu_j^2|.
\]  
(36)

where \( \beta = 1 \) and \( \alpha = 0 \). Eq. (35) is the main result of this section. It can be regarded as representing a Brownian motion of \( N \) classical particles at positions \( \{\nu_i(\tau)\} \) moving in a viscous fluid with friction coefficient \( \gamma^{-1} \) at temperature \( \beta^{-1} \). The parameter \( \gamma \) is a fictitious time which we shall relate to the perturbation parameter, so that (35) becomes a model to describe parametric correlations in the Laguerre ensemble. In the framework of PRMT it has been explicitly demonstrated that all the universal functions can be obtained from a Brownian motion model if one assumes that \( \tau = (\delta u^2)^{\frac{1}{2}} \), where \( \delta u \) is a parameter characterizing the strength of the external perturbation. Motivated by this result, we assume the same relation to apply to our model. We stress that the consistency of this assumption has recently been confirmed by explicit calculations of a similar model using supersymmetry. The apparently artificial parameter \( \alpha \) has been introduced to allow the classification and understanding of the universal expressions, which we derive in section IV, in the more general context of Laguerre ensembles. Note that Eq. (34) with \( \alpha > -1 \) characterizes the most general Laguerre ensemble of random matrices.
IV. UNIVERSAL CORRELATIONS AT THE HARD EDGE

A crucial difference between open ballistic systems and metallic conductors is that, in the former, the joint probability distribution of the transmission eigenvalues can be directly obtained from a maximum-entropy principle, whilst as shown in Refs. 63, 64, and 65 this is not the case for disordered conductors. A direct consequence of this fact is that the fluctuations in transport observables of ballistic systems are completely characterized by the universal logarithmic repulsion of conventional random matrix ensembles, like the Laguerre one. An outstanding common feature of the ensembles of random matrices appropriate to describe open ballistic systems and disordered conductors, though, is that the spectrum of transmission eigenvalues has a hard edge, since the eigenvalues are all non-negative. This is, as we discussed in the Introduction, also a feature of the Laguerre ensemble. In this section we shall demonstrate that the presence of this hard edge implies the existence of universal parametric correlations of a new kind.

A. Exact Mapping onto a Schrödinger Equation

Our starting point is Eq. (35)-(36), which we rewrite here in the form

\[ \gamma \frac{\partial W}{\partial \tau} = \mathcal{L}_{FP} W, \]  

(37)

where \( \mathcal{L}_{FP} \) is the Fokker-Planck operator, given by

\[ \mathcal{L}_{FP} = \sum_i \frac{\partial}{\partial \lambda_i} \left( -D_i^{(1)} + \frac{\partial}{\partial \lambda_i} D_i^{(2)} \right), \]  

(38)

where \( \lambda_i \equiv \nu_i^2 \) and

\[ D_i^{(1)} = 4 \sum_{j \neq i} \frac{\lambda_i}{\lambda_i - \lambda_j} - 2c\lambda_i + 2\alpha + 4/\beta \]  

(39)

while

\[ D_i^{(2)} = \frac{4\lambda_i}{\beta} \delta_{ij}. \]  

(40)

At this stage, it is useful to perform the following transformation

\[ P(\lambda, \tau) = \exp \left( -\frac{1}{2} \Phi(\lambda) \right) \Psi(\lambda, \tau), \]  

(41)

which implies that \( \Psi(\lambda, \tau) \) satisfies the evolution equation

\[ \frac{\partial \Psi}{\partial \tau} = \mathcal{L} \Psi, \]  

(42)

where \( \mathcal{L} = \mathcal{L}^\dagger \) is given by
\[ L = e^{\Phi/2} L_{FP} e^{-\Phi/2}. \]  

From (36), (38)-(40) and (43) we obtain 

\[ L = B - \mathcal{H}, \quad \mathcal{H} = \omega \sum_i \left[ -\frac{\partial}{\partial r_i} r_i \frac{\partial}{\partial r_i} - \frac{\beta}{4} \left( r_i + \frac{\alpha^2}{2} \right) + \frac{\beta(\beta - 2)}{4} \sum_{j(\neq i)} \frac{1}{(r_i - r_j)^2} \right], \]  

\[ B = Nc(2 + \beta(N + \alpha - 1))/(2\gamma), \quad \omega = 4c/(\beta\gamma) \text{ and } r_i = c\lambda_i. \]  

The operator \( \mathcal{H} \) can be regarded as a one dimensional Hamiltonian of fictitious hard-core particles, for which it is convenient to assign fermionic statistics. We remark that it is a general property of the mapping (41) that equilibrium fluctuations of the classical Brownian particles correspond to quantum mechanical ground-state fluctuations of these fictitious fermions. Note that for \( \beta = 2 \) the two-body interaction term vanishes and \( \mathcal{H} \) becomes the Hamiltonian of a system of free fermions. In this case we can rewrite \( \mathcal{H} \) as

\[ \mathcal{H} = \sum_{p=0}^{\infty} \varepsilon_p c_p^\dagger c_p, \]  

where \( \varepsilon_p = (2c/\gamma)(p + (\alpha + 1)/2) \) and \( c_p^\dagger (c_p) \) creates (annihilates) a fermion with quantum number \( p \). It is convenient at this stage to introduce the field operators \( \psi(\lambda) \) and \( \psi^\dagger(\lambda) \), where \( \psi(\lambda) \equiv \sum_n \phi_n(\lambda) c_n \) and \( \psi(\lambda) \equiv \sum_n \phi_n(\lambda) c_n^\dagger \), in which

\[ \tilde{\phi}_n(\lambda) = \left( \frac{\Gamma(1+\alpha_p)}{(p+1+\alpha)} \right)^{1/2} \lambda^{\alpha/2} e^{-c\lambda/2} L^n_\alpha(c\lambda), \]  

with \( L^n_\alpha(x) \) denoting the associate Laguerre polynomial.

With this notion, we turn to the calculation of some quantities of physical interest, namely the average level density and the two-point correlation function for level-density fluctuations. In second quantized language the local density operator is defined as

\[ \hat{n}(\lambda, \tau) = \psi^\dagger(\lambda, \tau) \psi(\lambda, \tau) \]  

where

\[ \psi^\dagger(\lambda, \tau) = e^{\mathcal{H} \tau} \psi^\dagger(\lambda) e^{-\mathcal{H} \tau} \]  

and

\[ \psi(\lambda, \tau) = e^{\mathcal{H} \tau} \psi(\lambda) e^{-\mathcal{H} \tau}. \]

Let \( |0\rangle \) denote the \( N \)-fermion ground-state of (45). Then the average level density is simply

\[ \bar{\rho}(\lambda, \tau) = \langle 0| \hat{n}(\lambda, \tau) |0\rangle = \sum_{p=0}^{N-1} \tilde{\phi}_p(\lambda)^2 = \tilde{\rho}(\lambda), \]  

which is independent of \( \tau \), as expected, since the Brownian particles are in a stationary state.
The two-point function for level-density fluctuations is defined as
\[ \tilde{S}(\lambda, \lambda', \tau) \equiv \langle 0|\hat{n}(\lambda, \tau)\hat{n}(\lambda', 0)|0 \rangle - \tilde{\rho}(\lambda)\bar{\rho}(\lambda). \tag{51} \]

Using (47) and Wick’s theorem we find
\[ \tilde{S}(\lambda, \lambda', \tau) = G_0(\lambda, \lambda', \tau) \sum_{q=0}^{N-1} \tilde{\phi}_q(\lambda)\tilde{\phi}_q(\lambda')e^{\varepsilon_q \tau} - \sum_{p=0}^{N-1} \sum_{q=0}^{N-1} \tilde{\phi}_p(\lambda)\tilde{\phi}_p(\lambda')\tilde{\phi}_q(\lambda)\tilde{\phi}_q(\lambda')e^{(\varepsilon_q - \varepsilon_p)\tau}, \tag{52} \]

where
\[ G_0(\lambda, \lambda', \tau) = \frac{c\exp\left(-\frac{c}{2}(\lambda + \lambda')\coth(\omega\tau/2)\right)}{2\sinh(\omega\tau/2)}I_\alpha\left(\frac{c\sqrt{\lambda\lambda'}}{\sinh(\omega\tau/2)}\right). \tag{53} \]

and \( I_\alpha(x) \) is a Bessel function. Equations (52) and (53) can be considered as a generalization to finite \( \tau \) of the standard result of RMT, which we recover by taking the limit \( \tau \to 0 \), giving
\[ \tilde{S}(\lambda, \lambda', 0) = \delta(\lambda - \lambda')\tilde{\rho}(\lambda) - \left(\tilde{K}(\lambda, \lambda')\right)^2, \tag{54} \]

where
\[ \tilde{K}(\lambda, \lambda') = \sum_{p=0}^{N-1} \tilde{\phi}_p(\lambda)\tilde{\phi}_p(\lambda'). \tag{55} \]

In the next section we show how to rescale the average level density and the two-point correlation function at the hard edge of the spectrum.

**B. Rescaling Physical Quantities at the Hard Edge**

In order to obtain the new universal behaviour anticipated in the Introduction we need to rescale all the quantities of physical interest at the hard edge of the spectrum, that is at \( \lambda = 0 \). This is done more easily if we reintroduce the original variables \( \nu_i = \sqrt{\lambda_i} \). So, the average level density becomes
\[ \rho(\nu) = \sum_{p=0}^{N-1} \phi_p(\nu)^2, \tag{56} \]

where \( \phi_p(\nu) \equiv \sqrt{2\nu}\tilde{\phi}(\nu^2) \). For large \( N \) we can use the asymptotic
\[ \phi_N(\nu) \simeq (2c\nu)^{1/2}J_\alpha\left(2(Nc)^{1/2}\nu\right), \tag{57} \]

where \( J_\alpha(x) \) is a Bessel function. Inserting (57) into (56) and taking \( N \to \infty \) and \( c \to 0 \) such that \( 2\sqrt{Nc} \to \rho_0\pi \) we find
\[ \rho(\nu) = \pi^2\rho_0^2 \int_0^1 d\nu \nu J_\alpha^2(\pi \rho_0 \nu s), \tag{58} \]
which is exactly the result obtained in Refs. 37–39 for the average level density rescaled at the hard edge of the spectrum. Note that the constant $\rho_0$ is, as can be seen from the relation $\rho(\nu) \simeq \rho_0$, valid for large $\nu$, just the bulk average level density.

A similar calculation yields for the two-point function

$$S(\nu, \nu', \delta u) = \pi^4 \rho_0^4 \int_0^1 ds \int_1^\infty ds' \nu \nu' J_\alpha(\pi \rho_0 \nu s) J_\alpha(\pi \rho_0 \nu' s') J_\alpha(\pi \rho_0 \nu' s') \exp \left( \frac{\pi^2 \rho_0^2}{2\gamma} \delta u^2 (s^2 - s'^2) \right).$$

(59)

This is the central result of this section. Note that, for $\delta u = 0$, Eq. (59) reproduces known results 37–39 for the two-point correlator of ensembles with a hard edge. Therefore, we believe that Eq. (59) represents an extension of these RMT results to account for the dispersion of the eigenvalues as a function of the perturbation parameter. Bearing in mind physical applications, consider now the particular case of the ensemble describing open ballistic systems, which can be mapped (see section 2.2) onto a Laguerre ensemble with $\alpha = 0$. Following Ref. 25 we make the rescalings $\hat{\nu} = \rho_0 \nu$, $\hat{\nu}' = \rho_0 \nu'$, $\hat{\delta} u^2 = \delta u^2 \rho_0^2 / \gamma$ and $\hat{S}(\hat{\nu}, \hat{\nu}', \hat{\delta} u) = S(\nu, \nu', \delta u) / \rho_0^2$. We can see that $\hat{S}(\hat{\nu}, \hat{\nu}', \hat{\delta} u)$ is a function that is independent of any physical parameter, such as the Fermi velocity or size of the sample, and therefore can be regarded as a universal characterization of quantum chaotic scattering in such systems. For the more general case of Laguerre ensembles, with $\alpha > -1$, we can see that the parameter $\alpha$ labels new universality classes characteristic of systems with a hard edge in the spectrum. Finally, we remark that Eq. (59) has recently been derived, for $\alpha = 0$, by Andreev, Simons and Taniguchi 62 using the supersymmetry technique. In addition, they have demonstrated that the universal behaviour at the hard edge extends to another kind of two-point function.

Another quantity of some interest is the distribution of level velocities defined as

$$K(v) \equiv \langle \delta(v - d\nu/du) \rangle.$$

(60)

One can show that

$$K(v) = \lim_{\delta u \to 0} \frac{\delta u}{\rho(\nu)} S(\nu, \nu + \nu \delta u, \delta u).$$

(61)

Now using (52) with $\tau = \delta u^2$ we find

$$S(\nu, \nu + \nu \delta u, \delta u) \simeq \frac{\rho(\nu)}{\delta u} \left( \frac{\gamma}{2\pi} \right)^{1/2} \exp \left( -\frac{\gamma v^2}{2} \right)$$

(62)

and therefore the distribution of level velocities is gaussian

$$K(v) = \left( \frac{\gamma}{2\pi} \right)^{1/2} \exp \left( -\frac{\gamma v^2}{2} \right).$$

(63)

Remarkably, this is exactly the same result that one finds at the bulk of the spectrum. We stress that this result, although simple, is by no means trivial, since the hard edge rescaling procedure introduces considerable changes in most physical quantities, such as the average level density and two-point correlation function. One interesting application of (63)
is associated with the interpretation of the rescalings discussed right after Eq. (59). The quantity $C_0 = \rho_0^2/\gamma$ can now, in view of (63), be written as

$$C_0 = \rho_0^2\langle v^2 \rangle,$$

so that the rescalings $\hat{\nu} = \rho_0\nu$ and $\delta \hat{u}^2 = \delta u^2 C_0$ mean that we get universal functions if we measure the levels in units of the average bulk level spacing $\rho_0^{-1}$ and if the perturbation parameter is rescaled by the mean square gradients of the levels $\sqrt{C_0}$.

V. THE POWER SPECTRUM FORMULA

A complete characterization of the fluctuations in transport observables in mesoscopic systems consists of a description of both their magnitude and power spectrum. One can get this information from the correlator

$$F(\delta u) = \langle \delta A(\delta u)\delta A(0) \rangle,$$  

where $A$ is an arbitrary transport observable and $\delta u$ parametrizes the external perturbation on the system which drives the fluctuations. The power spectrum $C(\omega)$ is defined as

$$C(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} F(t).$$

Microscopic diagrammatic calculations\textsuperscript{44, 45} for disordered metals suggest that $F(\delta u)$ decays as a power law as a function of $\delta u/E_c$, where $E_c$ is a correlation parameter which sets the typical scale of the spacings between the peaks and valleys in the values of $A$, for a given realization, as a function of $\delta u$. This slow power law decay of $F(\delta u)$ indicates that the system sustains some sort of long-range memory in the fluctuations of the observable. This kind of behavior must be contrasted with the exponential decay of Poisson processes and can be regarded as a manifestation of spectral rigidity, a well known phenomenon in the theory of random matrices.

It is well known\textsuperscript{64, 65} that universal fluctuations in transport observables of mesoscopic disordered conductors can be obtained by simply taking the hydrodynamic limit of the DMPK equation. We expect a similar procedure to apply to the power spectrum formula of open ballistic structures. In section II, we have demonstrated that the Laguerre ensemble can be used to describe fluctuations in transport observables of ballistic cavities. With this in mind we shall now discuss the hydrodynamic limit of Eq. (35).

Multiplying (35) by $\sum_i \delta(\nu - \nu_i)$, integrating over all $\nu_i$’s and using the definition

$$\rho(\nu, \tau) \equiv \left\langle \sum_i \delta(\nu - \nu_i) \right\rangle,$$

where $\langle \ldots \rangle_\tau$ denotes an average over the distribution $W(\nu, \tau)$, we get in the large $N$ limit the following evolution equation for the average level density

$$\gamma \frac{\partial \rho(\nu, \tau)}{\partial \tau} \simeq - \frac{\partial}{\partial \nu} \left[ \rho(\nu, \tau) \frac{\partial}{\partial \nu} \left( \int_0^\infty d\mu \rho(\mu, \tau) \ln |\nu^2 - \mu^2| - \frac{C}{2} \mu^2 \right) \right].$$
One can see from (68) that the equilibrium density satisfies
\[ \int_{0}^{\infty} \rho_{eq}(\mu) \ln |\nu^2 - \mu^2| \, d\mu = \frac{c}{2} \nu^2 + \text{const.} \]  
(69)
The constant on the right hand side of (69) is determined by the normalization condition
\[ \int_{0}^{\infty} \rho_{eq}(\nu) \, d\nu = N \]  
(70)
One can easily verify that
\[ \rho_{eq}(\nu) = \begin{cases} \left( \frac{c}{\pi} \right) \sqrt{\frac{4N}{c} - \nu^2} & \text{for } 0 \leq \nu \leq 2(N/c)^{1/2}, \\ 0 & \text{for } \nu > 2(N/c)^{1/2} \end{cases}, \]  
(71)
is the solution of (69) satisfying (70). Note that for large \( N \) at fixed \( \nu \) one finds
\[ \rho_{eq}(\nu) \simeq \frac{2}{\pi} \sqrt{Nc} = \rho_0, \]  
(72)
as expected. In the regime of universal mesoscopic fluctuations that we are concerned with, one can safely consider the average level density, \( \rho_0 \), to be much larger than the fluctuating part \( \delta \rho(\nu, \tau) \). Therefore, it makes sense to try and linearize Eq. (68) by writing \( \rho(\nu, \tau) \) as
\[ \rho(\nu, \tau) = \rho_0 + \delta \rho(\nu, \tau). \]  
(73)
Inserting (73) into (68) yields
\[ \gamma \frac{\partial \delta \rho(\nu, \tau)}{\partial \tau} \simeq -\rho_0 \frac{\partial^2}{\partial \nu^2} \int_{0}^{\infty} d\mu \delta \rho(\mu, \tau) \ln \left| \nu^2 - \mu^2 \right|. \]  
(74)
Since
\[ \ln \left| \nu^2 - \mu^2 \right| = -2 \int_{0}^{\infty} \frac{dk}{k} \cos k\nu \cos k\mu, \]  
(75)
Eq. (74) can be solved by means of Fourier cosine transform. We find
\[ \delta \rho(\nu, \tau) = \frac{2}{\pi} \int_{0}^{\infty} dk \delta \tilde{\rho}(k, 0) \exp(-\pi k\rho_0 \tau / \gamma) \cos k\nu \]  
(76)
The two-point function \( S(\nu, \nu', \delta u) \) can be obtained from (76) through the identity
\[ S(\nu, \nu', \delta u) = \langle \delta \rho(\nu, \tau) \delta \rho(\nu', 0) \rangle_{eq} \]  
(77)
where \( \langle \ldots \rangle_{eq} \) stands for an average over the equilibrium distribution (24).
It is useful to define the following double Fourier cosine transform
\[ \tilde{S}(k, \delta u) = \int_{0}^{\infty} d\nu \int_{0}^{\infty} d\nu' S(\nu, \nu', \delta u) \cos k\nu \cos k\nu' \]  
(78)
so that we get from (77)
\[ \tilde{S}(k, \delta u) = \left\langle \delta \tilde{\rho}(k, \delta u^2)\delta \tilde{\rho}(k, 0) \right\rangle_{eq} = \exp(-\pi k \rho_0 \delta u^2 / \gamma) \tilde{S}(k, 0) \]  

The Fourier component \( \tilde{S}(k, 0) \) can be obtained directly from Eq. (16), which together with (78) yields \( \tilde{S}(k, 0) = k/2\beta \), so that

\[ \tilde{S}(k, \delta u) = \frac{k}{2\beta} \exp(-\pi k \rho_0 \delta u^2 / \gamma). \]  

Let \( A = \sum_i a(\nu_i) \) denote an arbitrary observable, which can be expressed as a linear statistic. Then from (65) and (77) we get for the correlation function

\[ F(\delta u) = \int_0^\infty d\nu \int_0^\infty d\nu' a(\nu)a(\nu') S(\nu, \nu', \delta u), \]  

which by virtue of (80) yields the formula

\[ F(\delta u) = \frac{2}{\beta \pi^2} \int_0^\infty dk k \exp(-\pi k \rho_0 \delta u^2 / \gamma) \tilde{a}^2(k), \]  

where

\[ \tilde{a}(k) = \int_0^\infty \cos k \nu a(\nu) d\nu. \]  

Finally, from (82) we can obtain the power spectrum formula

\[ C(\omega) = \frac{2}{\beta \pi^2} \left( \frac{\gamma}{\rho_0} \right)^{1/2} \int_0^\infty \frac{\tilde{a}^2(k)}{\sqrt{k}} \exp\left(-\frac{\gamma \omega^2}{4\pi k \rho_0} \right) dk. \]  

Equations (82) and (84) are the principal result of this section.

As an application, which also serves as a test of our results, we consider the two-probe Landauer-Büttiker dimensionless conductance (Eq. (11))

\[ g = \sum_i \frac{1}{1 + \nu_i^2}. \]  

We find

\[ \tilde{a}_g(k) = \frac{\pi}{2} e^{-k}, \]  

thus the correlator of Eq. (82) gives

\[ F(\delta u) = \frac{1}{8\beta} \left( 1 + \delta u^2 / \mathcal{E}_c^2 \right)^{-2}, \]  

where \( \mathcal{E}_c = \sqrt{2\gamma / \pi \rho_0} \) is the correlation parameter. For the power spectrum we get

\[ C(\omega) = \frac{\pi \mathcal{E}_c}{16\beta} e^{-\omega \mathcal{E}_c} (1 + \omega \mathcal{E}_c). \]  

We would like to stress that Eqs. (87) and (88), for \( \beta = 2 \), are in complete agreement with independent calculations based on semiclassical quantization.
VI. SUMMARY AND CONCLUSIONS

In this work we have studied the effects of an external adiabatic perturbation on the transmission eigenvalue correlations of open ballistic cavities.

In particular, we have derived a Brownian motion model to describe dynamic fluctuations in the Laguerre ensemble of random matrices, which we have proposed as a model for parametric correlations of transmission eigenvalues in ballistic cavities. This model has enabled us to obtain explicit non-perturbative expressions for the two-point function of level density fluctuations at the hard edge of the spectrum. We have shown that after appropriate rescaling this function becomes system independent and can therefore be used as a signature of quantum chaos in open ballistic cavities. In the hydrodynamic limit, we have demonstrated that the two-point parametric correlator of mesoscopic conductance fluctuations in ballistic cavities is a Lorentzian-squared, in agreement with semiclassical calculations. We have also obtained a formula for the power spectrum of the fluctuations of an arbitrary linear statistic in such systems.

Parametric correlations in the S-matrix ensemble has recently been discussed in Refs. 47 and 48. They obtained exponential decay for the conductance correlator, which disagrees with equation (87) and consequently with microscopic semiclassical calculations. It is not understood at the moment why the S-matrix approach, which was so successful in describing non-parametric fluctuations, should have a hydrodynamical limit for parametric correlations that disagrees with semiclassical results. We remark that our Ω-matrix approach, on the other hand, although phenomenological in nature, has the advantage of containing the correct semiclassical limit.

The author would like to thank M. D. Coutinho-Filho for reading the manuscript and helpful comments. This research was partially supported by the Brazilian Agency CNPq.
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