Rose windows are of special significance in gothic cathedrals. These are large circular windows. They are also exquisite works of masonry. In terms of their construction, the sectioning of the circle into smaller, equal segments presented a particular challenge. The division of a circle into equal segments or the ratio between the diameter and the circumference was an old mathematical problem, which had practical consequences for architects and masons. Their answers ranged from simple to more complex solutions. Analysis has shown that the segmentation of rosettes was simpler in the early period. It later evolved into more complex segmentation, which was probably also based on anthropometric measuring systems. Perhaps the most complex segmentation of the rosette can be found in the rose windows of the Basilica of Saint Clare and the Basilica of Saint Francis in Assisi, which are presented separately.
1. INTRODUCTION

The Gothic period left its most significant mark with the great cathedrals of its time. It was the successor of the Romanesque period, which was the first pan-European style of art (Lah, 2019). Despite the recognisable uniformity of its architectural style, the Gothic period began at different times in various parts of Europe and lasted different periods of time. Interestingly, Gothic art developed as a common style in an otherwise very divided society. At that time Europe was marked by the end of the unity of church and state. It was separated into numerous small states, riven by the ambitions of individual rulers and dynasties. This was also the time of the unification of France and its rise as a dominant political and cultural force. The period coincided with an increase in population and growing urbanisation, a rising significance of cities and trade, the formation of educational centres, and scientific development. The most important theological masters of the time were the two Scholastic philosophers, the Dominicans Thomas Aquinas and Albertus Magnus. The previous image of security centred around the figure of the emperor was beginning to wane. Francis of Assisi, for instance, foregrounds the significance of the human individual, your fellow brother or sister. This was also the time of the Crusades which, despite their intention and end result, still represented a form of inter-European cultural exchange.

Arguably the most important figure in the development of Gothic art is Suger, the Abbot of Saint-Denis (1122-51). His reforms of the abbey of Saint-Denis and his influence over the French King led to a new understanding of Architecture and the role of the cathedral (Koch, 1999, pp. 146).

We cannot fully understand the significance of the Gothic cathedral without understanding the religious spirit which inspired it. According to Pope Benedict XVI (2009), Gothic cathedrals represent a synthesis of faith and art, harmonically expressed in a universal, enchanting language of beauty, which still fascinates today. A new technique of arch construction enabled a significant increase in height, while at the same time liberating the building of its massive walls. Its vertical lines express the soul's longing for God and invite prayer. The "unencumbered" surface of the wall was transformed into a window and decorated with stained glass. Windows became marvellous illuminated images intended to educate people in faith. The messages incorporated into the stained glass enabled the dissemination of saints' lives, Biblical events and similar content. The mystical light flooding through the stained glass windows onto believers made them a part of the story of redemption. The cathedral churchgoers thus understood their history in the context of a common history of salvation. The deeper, comprehensive meaning of the Gothic cathedral is perhaps best encapsulated by the words engraved on the central portal of Saint-Denis in Paris, which exhorts the viewer extolling the beauty of the entrance not to be deceived by its outer glory – rather, they should keep in mind the painstaking labour needed to complete it. The resplendent image of the church is therefore a representation of illuminated truth guiding us towards true light, which is Christ (Samper and Herrera, 2015).

In terms of beauty and harmony, Gothic churches are particularly interesting for their circular windows – rose windows. These conveyed multifaceted meanings. As already mentioned, stained glass windows carry a strong theological and pastoral significance and contribute to the overall theological sense of the building. The masonry of a rose window is generally the result of excellent artisanship, while its segmentation depended on the knowledge of mathematical (Havermann and Fellner, 2004; Samper and Hererra, 2015) and compositional principles. Sanctuaries have always been spaces of transcendence. On the one hand, they reflect human longing for the transcendent, and on the other, they embody the power of religion. The function of sacral buildings is thus distinctly subservient to their beauty and composition. According to Crow (2000), sacral buildings occupy the third place in the list of structures in which form supersedes function, right after monuments and cemeteries. The evolution of rose windows is closely tied with the development of the architectural design of churches, which was based on contemporary theological guidelines, the zeitgeist, and technical capabilities of the age. Circular or so-called "wheel windows" were already present in the Romanesque period, but these were usually smaller and simpler architectural elements, such as for instance in the Saint Cyriacus cathedral in Ancona, which dates to the period between the 11th and 13th centuries. Rose windows flourished in Gothic art. Rose windows which favour exquisite masonry over stained glass elements, which is to say that the stained glass merely complements the stone elements, are generally more interesting in terms of composition.

Rose windows were usually quite large and therefore complex to execute. This is why the window opening was usually divided into smaller parts. The segments were sometimes separated with the aid of iron frames, but most often by stone tracery. Because of the symbolic, compositional, and visual significance of the windows, special attention was devoted to their composition and the execution of the tracery. This primarily involved segmentation into smaller, equal or compositionally complementing parts. With rose windows this often entailed the division of a window into one or more circular rings. These circular rings were then further divided into equal segments. The division of a circular ring into halves, quarters, thirds, sixths, etc., is generally not that complex. It can be done using a compass and a ruler. Because any circular arc may be halved with the use of a compass and ruler, this provides us with different combinations of circle segmentation. The most basic of these is the division of a circle into two equal parts. We can then continue halving the segments and thus divide the circle into 4, 8, 16, 32, 64, 128 ... parts. The other geometrical division involves segmenting the circle into 6 parts. With further addition and halving the circle can then be divided into 3, 6, 12, 24, 48, 96 ... equal segments. A circle can also be geometrically divided into 5 equal parts, which results in further segmentation into 10, 20, 40, 80... equal parts. Dürer's copperplate engravings (1525) also feature some other geometrical divisions of the circle, which shows that they were already known during that time.

A different and more complex segmentation of a circle or ring is based on the ratio between the circle’s circumference and diameter. This involves working with approximations, which is only acceptable in practice, when the margin of error is acceptably small. It was well known since ancient history that the ratio between circumference and diameter was constant. Perhaps the oldest known description of this is in the Biblical Book of First Kings (1 Kings 7:23), which describes the Temple of Jerusalem. There the ratio between the diameter and circumference of the basin of water was 30:10. The error for this ratio is 4.5070%.

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A better approximation, 25:8, was known by the Babylonians, where the error is 0.5282%. A similarly small error was made by the ancient Egyptians, who used the ratio 256:81 to describe the ratio between the circumference and diameter (the error here is 0.6016%) (Kušar, 2008). Greater progress in rational approximations was achieved by the ancient Greeks. With the aid of inscribed and circumscribed polygons, Archimedes discovered that the true values of the ratio between the circumference and the diameter were between 223:71 and 22:7. Both values represent an error smaller than 0.0403%. The ratio was more accurately described by Ptolemy (cca. 150 BC) with the ratio 377:120, with an error of merely 0.0023% (Davide, 1984). Experience backed up by mathematical knowledge showed that the best approximations of the ratio between circumference and diameter were 16:5, 19:6, 22:7, 25:8, 28:9... We know today that the ratio between circumference and diameter corresponds to the irrational number π.

In practice it is possible to connect this ratio and the applied measurement system with the use of an appropriate module or modular system. The modular system has to be compatible with the applied measurement system; until the introduction of the metric system, the latter was based on the so-called anthropometric system.

2.1 Anthropometric measurement system

The ratio between the length, width, and height of a building and between its architectural elements is one of the fundamental issues to challenge architecture since early times. Geometric analyses show that builders always kept to specific ratios or proportions throughout their constructions. They were based on smaller elements and the human body. The human body was used as a template for various dimensions (size of palm, feet, step,...), as a building “machine” (arm span, hauling strength,...), or as scale for the size of spaces, ceiling height, seats, etc. This was the basis for the development of a so-called anthropometric measurement system, where measurements were determined by the human body and were thus often also named after body parts. A measurement system is present in cases, where the relations between larger and smaller measures may be expressed with the ratios of smaller integers. Usually these ratios are 1:2, 1:3, 1:4, 1:5, 1:6, 1:10 and 1:20. Some systems also include more unusual ratios, such as 1:7 or 1:51. One cubit was based on smaller elements and the human body. The human body was used as a template for various dimensions (size of palm, feet, step,...), as a building “machine” (arm span, hauling strength,...), or as scale for the size of spaces, ceiling height, seats, etc. This was the basis for the development of a so-called anthropometric measurement system, where measurements were determined by the human body and were thus often also named after body parts. A measurement system is present in cases, where the relations between larger and smaller measures may be expressed with the ratios of smaller integers. Usually these ratios are 1:2, 1:3, 1:4, 1:5, 1:6, 1:10 and 1:20. Some systems also include more unusual ratios, such as 1:7 or 1:51. One cubit was 3/2 feet or 6 palms or 18 thumbs.

Apart from the use of a measurement system, more complex problems (in larger constructions) required the specification of new main measures, the so-called modules. Modules were based on a measurement system. Just like the measurement system, the modules also had to be composite, i.e. capable of being joined into a whole. In principle, the number of modules in a composition has to be small and proportional to the composition. Consequently, the module of the whole is larger than the module of the element, which makes up the whole.

Most anthropometric systems in the Mediterranean and Europe were based on the Roman anthropometric system (Figure 1). The eventual demise of the Roman state and the founding of smaller states led to the establishment of different measurement systems. Because of their common basis, however, they are similar to a certain degree. They mainly differ in the length of their basic unit. Anthropometric systems were in use in Europe until the introduction of the metre in the second half of the 19th century. On the one hand, the introduction unified metrology; on the other hand, it resulted in the loss of certain advantages of the old systems. The most famous architect preoccupied with the question of how to bring together the advantages of the new and old systems was Le Corbusier. His efforts resulted in the Modulor (2004).

The goal of this research is the analysis of the rosettes found in cathedrals across Europe. We wished to determine both simpler and mathematically more complex forms of segmentation and identify a potential rational similarity between individual cases. This was complemented by the modular analysis of rosettes in the Basilica of Saint Clare and the Basilica of Saint Francis in Assisi. We tried to determine the connection between the ratio of the equal segments of the rim of the circle and its diameter. By placing relevant modules in the applied measurement system, we tested their usefulness for the segmentation of a circle into several equal parts.

2. METHOD

306 churches across Europe were examined. The buildings were chosen based on the list of important Gothic architecture (List of Gothic architecture, 2020). We determined the number of rings and the number of elements in each ring based on images and church presentations published online (Rose Window). Even though some windows seemed to be (or were) composed of multiple rings, we focused on the segmentation of rings into smaller, equal parts. If the multiple rings of a single window contained an equal number of smaller segments, we considered all of these rings as a single ring of the individual window.

The criterion for the inclusion of a building or a window into our analysis was the presence of a large circular window. These were found in 67 churches. We analysed the central rose window. This was usually the rose window above the entrance, on the main façade. Some of the rose windows were not placed on the front façade, but rather on the front façades of the main transept. In these cases we analysed the transept windows.
The detailed compositional analysis of the rosettes was based on the measurement and modular analysis of the two most interesting rosettes. These were the rosettes of the basilicas of Saint Francis and Saint Clare in Assisi (Italy), which contain multiple rings divided into different numbers of equal segments. The modular analysis was made with the use of the anthropometric measurement system used in the region during the time of construction. The measurement system allowed us to determine the compositional key of the window following the system devised by Prof. Kurent at the Faculty of Architecture, University of Ljubljana. The system enables the identification of the module, which served as the basis for work. The module must correspond to the applied measurement system (Kurent, 1970). We used archival material stored at the Faculty of Architecture, University of Ljubljana. Data on circular compositions have been collected by prof. Kurent. He used them also in his works Architektov zvezek I, II and III. (2002, 2005, 2006). Our research was conducted with the aid of Autocad software. Modules were determined on the basis of compositional and measurement analyses and the application of an anthropometric system. They were identified on the basis of the segmentation of a circle into equal parts. We looked at the dimensions of equal circular segments and their ratio to diameter. We then tried to place it into a relevant measurement system and discover whether it was based on corresponding principles or not.

3. RESULTS

Our research encompassed 306 Gothic churches in 31 countries, mostly in France and Great Britain. 67 of these churches had rose windows. Instead of rose windows, the other churches featured tall or lancet windows. The majority of rose windows were found in France, Italy, and Portugal. Analysis has shown that there were fewer rose windows in particular in Germany, Spain, Belgium and other countries in Central and Eastern Europe.

The majority of the analysed rosettes had one (29 churches) or two (33 churches) rings divided into equal segments, even though these were sometimes seemingly different. Despite the fact that some rosettes give the impression of having multiple rings, detailed analysis shows that the segmentation of these “different” rings is in fact the same. A good example of this is the rosette in the Chartres Cathedral (France), where a cursory glance reveals three rings. However, all three rings are segmented into 12 equal parts. Three of the churches had differently segmented rings: the Basilica of Saint Clare in Assisi (Italy) has 5, and the Basilica of Saint Francis in Assisi (Italy) has 6 rings.

The analysis of the segmentation of the rings into equal parts reveals as many as 20 different approaches. The predominant method of segmentation included division into 1, 4, 6, 8, 12, 16, or 24 equal segments. This is unsurprising, since it involves the relatively simple segmentation of a circle or further division of segments (4-8-16 or 6-12-24). The rarer segmentations, into 3, 5, 7, 10, 14, 15, 20, 22, 30, 44, 46, 50 and 108 segments, are of greater interest in terms of composition. As already mentioned above, geometric segmentation can also result in the division of a circle into 5 equal segments, with further halving resulting in 10 and 20 equal segments.

Segmentation into 14, 22 and 44 parts results in a ratio between circumference and diameter, which approximates π. This can be achieved through modular composition, which will be presented below with the example of the basilicas of Francis and Clare in Assisi.

3.1 Rosette of the Basilica of Saint Francis in Assisi

The Basilica of Saint Francis in Assisi was under construction from 1228 to 1253 (Basilica of St. Francis of Assisi, 2019; Bonsanti, 1998). It actually includes two basilicas, upper and lower. The front façade is divided into three parts: the triangular tympanum with a circular opening in the centre, a central part with a richly decorated rose window, and a bottom part with a monumental portal and twin entrance doors. The rest of the front façade is relatively modest in comparison with other Gothic churches (of a later period). The rosette itself is very interesting, since it is composed of 5 circular rings, divided into 12, 14, 46, 44 and 108 equal segments. Two different measurements were available regarding the dimension of the rosette. Muhić (1987) mentions that the rosette is 31 Roman feet wide, which equals 9.17 m. The second dimension, based on the archival analysis conducted by Kurent (2020) mentions a diameter spanning 7.19 m. Photograph analysis seems to confirm the latter dimension.
The division of inner rings is of particular interest, since they are segmented into 12, 14, 44, 46 and 108 equal parts (Figure 3). The compositional analysis demonstrated the use of various modules, which were 12, 22, 26, 28 and 44 unciae (sing. uncia) long. During the construction the region was under the dominion of the Papal States, which had a unique measurement system that was included in the analysis. The use of these modules expressed a ratio between the circumference and diameter, an approximation of 22:7 (44:14) and 28:9. The rosette is specific because of the segmentation of a ring into 46 equal parts. We do not know the reasons for this segmentation, we can only guess. It is interesting that generally in windows of this type the number of segments in a ring increases with the distance from the centre. However, in the rosette of the Basilica of Saint Francis a ring is first segmented into 46 parts and only then into 44. The symbolic meaning of number 46 in the Christian tradition is not as commonly known as are the meanings of some other numbers. Even so, the Old Testament is composed of 46 books. It also took the Jews 46 years to build Temple of Jerusalem.

Modular units 12, 12, 20, 22, 26, 28 and 44 unciae, which appear in the analysis, are relatively easy to translate into the anthropometric system (Figure 4).

3.2 Rosette in the Basilica of Saint Clare in Assisi

The Basilica of Saint Clare in Assisi (Italy) was built under the direction of Filippo Campello in the 13th century. It was supposedly completed in 1260, as that is when they moved the remains of Saint Clare from the Church of Saint George into the newly built basilica. It is a simple church with a Gothic interior and large flying buttresses on the outside (Basilica di Santa Chiara, 2019). The front façade is equally modest and decorated with a rose window.

The rose window of the Basilica of Saint Clare is similar in size and age to the one in the nearby Basilica of Saint Francis; unlike the rest of the façade, it is characterised by intricate masonry. The rosette measures 7.04 m in diameter, and the inner, segmented part 5.64 m. It consists of four rings segmented into different numbers of equal parts. The rings are thus divided into 6, 15, 30 and 50 parts (Figure 5).

The segmentation into 15 and 30 equal parts is of particular interest, since this division of the circle is more complex than segmentation into 6 or 50 elements. Namely, the division into 50 segments is the result of doubling the ratio 25:8 (50:16). The main compositional principle of the rosette is based on the measurement system in use in the Papal States. The diameter of the whole ornament is one catena, while the inner radius corresponds to one canna, as well as the diameter of the innermost ring.
The segmentation of the rosette into 15 or 30 parts is more demanding than the more common segmentations. With the module \( M = 2 \text{ palmi} = 24 \text{ uncia} \) and the ratio between diameter and circumference of \( 10 : 30 \), the error is relatively large (4.33% or 2.02 cm per segment). With module \( M(c) = 25/2 \text{ uncia} \) and \( M(d) = 12 \text{ uncia} \) the error is minimised (0.16 cm or 0.34%) to a degree where it no longer affects the inclusion of the rosette in a wall (Figure 5). The combination of different measures seen above is supported by the very measurement system of the Papal States, since this was not a decimal system, but rather a mixture of different systems, such as decimal, duodecimal, pentadecimal, and other systems. Catena is a particularly interesting measuring unit, comprising 25.5 palms. In this case, modular units are simple to convert into an anthropometric system (Figures 4 and 5).

4. DISCUSSION

Apart from the exceptional artisanship of the masonry, rose windows, which were a unique feature of Gothic churches, also display mathematical and compositional knowledge reflected in the segmentation of circular rings into equal parts. Even though most examples of rosettes are based on compositionally simple segmentations, some of them follow more complex principles.

These cases show a rational approximation of the ratio between the circumference and diameter, expressed as small integers. The ratio of 22:7 explains the segmentation of circular rings in the Basilica of Saint Francis in Assisi into 12, 14, and 44 parts.

With the aid of a module it was possible to divide a circle into 7 or 14 equal segments. In angular degrees this is an equivalent of 25.714° or 16.364°, which was probably unusable in practice. This shows that more complex segmentation of the circle was carried out based on the principles of modular composition. A simple transposition of modular measurements into anthropometric measurement systems proved their efficiency in the design and construction of rosettes. It is possible to segment a ring into 46 parts with the aid of a 20 uncia module and the ratio between circumference and diameter of 46:15. The segmentation of the rosette in the Basilica of Saint Clare is more interesting. It probably entailed the use of two modules, which differed by 1 uncia. This resulted in a minimal error for the segmentation of the ring into 15 or 30 parts. The segmentation into 50 parts displays the ratio of 50:16, which is an error of only 0.5%.

Complex segmentations are evidence of great geometric and mathematical knowledge, which remained a well-kept secret. On the one hand, this indicated the knowledge of individual masters; on the other hand, it was a well-guarded guild secret. In view of the harsh punishment for the betrayal of guild (artisan) secrets and the organisation and educational structure of guilds, it is not surprising that the secrets were eventually lost and are not a part of the written record. With today’s knowledge of mathematics and geometry we can thus only speculate about the techniques that were in use.

Because rose windows were such demanding artisan elements, they were probably built on the ground. They...
probably first transposed the design of the window in 1:1 scale on an adequately prepared surface, and then divided it into rings and segmented them based on knowledge of geometry and ensuing guidelines. A thus executed working “draft” also enabled them to control the precision of the stone carved elements and their conjoining. The completed “whole” was then taken apart and re-assembled at its location in the construction.

In today’s awareness, we think of Gothic art in terms of height, arches, silence, darkness, and mystical light. These spaces imbue us with the presence of the transcendent. However, these are also buildings whose manner of construction, stability, and resistance to the ravages of time and elements continue to astonish us today. Even more fascinating, however, is the extraordinary mathematical and compositional knowledge hidden in cathedral details, such as rosettes, which is oftentimes obscured from the cursory inspection of modern-day admirers.

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