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Higher order couplings in magnetized brane models

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ABSTRACT: We compute three-point and higher order couplings in magnetized brane models. We show that higher order couplings are written as products of three-point couplings. This behavior is the same as higher order amplitudes by conformal field theory calculations e.g. in intersecting D-brane models.

KEYWORDS: Strings and branes phenomenology, Phenomenology of Field Theories in Higher Dimensions

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1 Introduction

Extra dimensional field theories, in particular string-derived ones, play important roles in particle physics and cosmology. It is one of keypoints how to realize four-dimensional chiral theories as low-energy effective theories from such higher dimensional theories. Introducing constant magnetic fluxes in extra dimensions is one of interesting scenarios to realize four-dimensional chiral theories [1–10]. Indeed, several models have been studied in field theories and string theories. Furthermore, magnetized D-brane models are T-duals of intersecting D-brane models, and various interesting models have been constructed within the framework of intersecting D-brane models [4–6, 11–13].

Orbifolds with magnetic fluxes and other non-trivial backgrounds with magnetic fluxes have also been studied [15–18].

In magnetic background, zero-modes are quasi-localized and the number of zero modes are determined by a size of background magnetic flux. Such a behavior of zero-modes would be important in application for particle phenomenology. Couplings among those zero-modes in four-dimensional effective field theories are obtained as overlap integrals of zero-mode profiles in the extra dimensional space. Thus, if they are localized far away from each other in the extra dimensional space, their four-dimensional couplings would be suppressed and such couplings would be useful to explain suppressed couplings in particle physics such as Yukawa couplings of light quarks and leptons. Hence, computation of those couplings is quite important. Indeed, three-point couplings have been calculated and their results were found to coincide with three-point couplings in intersecting D-brane models [7, 19]. (See also [20].) Furthermore, three-point couplings could lead to realistic Yukawa matrices. (See e.g. [18].)

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\(^1\) See for a review [14] and references therein.
For further phenomenological applications, it is also important to compute higher order couplings. Indeed, higher order couplings as well as three-point couplings have been computed within the framework of intersecting D-brane models [21, 22] and heterotic orbifold models [23–27] by using conformal field theory (CFT) technique. Our purpose in this paper is to compute higher order couplings in magnetized brane models. We carry out overlap integrals of three or more wavefunctions in the extra dimensional space in order to obtain higher order couplings in four-dimensional effective field theories. It will be shown that such higher order couplings are written as products of three-point couplings. This behavior is the same as CFT calculations in intersecting D-brane models as well as heterotic orbifold models.

This paper is organized as follows. In section 2, we show our set-up by reviewing ref. [7]. In section 3, we reconsider the computation of the three-point couplings. Its result have been obtained in [7], but here we pay attention to the selection rules and rewrite the result, which is convenient to our purpose. In section 4, we compute the four-point couplings and we study its extensions to higher order couplings. In section 5, we give comments on comparison with those couplings in intersecting D-brane models. Section 6 is devoted to conclusion and discussion.

2 Set-up

We consider dimensional reduction of ten-dimensional $\mathcal{N} = 1$ super Yang-Mills theory with $U(N)$ gauge group [28], on a six torus in Abelian magnetic flux background. We factorize the six-torus into two-tori $(T^2)^3$, each of which is specified by the complex structure $\tau_d$ and the area $A_d = (2\pi R_d)^2 \text{Im} \tau_d$ where $d = 1, 2, 3$. From the periodicity of torus, the background magnetic flux is quantized as [29]

$$F_{z^d z^d} = \frac{2\pi i}{\text{Im} \tau_d} \left( \begin{array}{c} m_1^{(d)} 1_{N_1} \\ \vdots \\ m_n^{(d)} 1_{N_n} \end{array} \right), \quad d = 1, 2, 3,$$  \hspace{1cm} (2.1)

where $1_{N_a}$ are the unit matrices of rank $N_a$, $m_i^{(d)}$ are integers and $z^d$ are the complex coordinates. This background breaks the gauge symmetry $U(N) \to \prod_{a=1}^N U(N_a)$ where $N = \sum_{a=1}^N N_a$.

A magnetic flux in $(4 + 2n)$ extra dimensions can give rise to chiral fermions in four dimensions. Focusing on a submatrix consisting of two blocks,

$$F_{z^d z^d,ab} = \frac{2\pi i}{\text{Im} \tau_d} \left( \begin{array}{cc} m_a^{(d)} 1_{N_a} & 0 \\ 0 & m_b^{(d)} 1_{N_b} \end{array} \right),$$  \hspace{1cm} (2.2)

the corresponding internal components $\psi_n(z)$ of gaugino fields $\lambda(x, z)$ have the form

$$\lambda(x, z) = \sum_n \chi_n(x) \otimes \psi_n(z), \quad \psi_n(z) = \left( \begin{array}{c} \psi^{aa}_n(z) \\ \psi^{ab}_n(z) \\ \psi^{ba}_n(z) \\ \psi^{bb}_n(z) \end{array} \right),$$  \hspace{1cm} (2.3)
where \(x\) denotes the coordinates of four-dimensional uncompactified space-times, \(P^{3,1}\). The off-diagonal components of zero-modes of the Dirac equation transform as bifundamental representations \(\psi^{ab} \sim (N_a, N_b)\), \(\psi^{ba} \sim (\overline{N}_a, N_b)\) under \(\text{SU}(N_a) \times \text{SU}(N_b)\), where we omit the subscript 0 corresponding to the zero-modes, \(n = 0\). Since only either of the off-diagonal components has exclusive zero-modes, depending on the sign of the relative magnetic flux \(M^{(d)} \equiv n_a^{(d)} - n_b^{(d)}\), the spectrum is chiral; The positive helicity zero-mode provides CPT conjugate to the one with negative helicity. With an appropriate gauge fixing, the zero-modes on each \(d\)-th \(T^2\) are written as [7]

\[
\psi_{d,M^{(d)}}^j(z^d) = N_{M^{(d)}} e^{i\pi M^{(d)} z^d/|\text{Im} \tau_d|} \vartheta \begin{bmatrix} j/M^{(d)} \\ 0 \end{bmatrix} (M^{(d)} z^d, \tau_d M^{(d)}),
\]

for \(j = 1, \ldots, |M^{(d)}|\), where the normalization factor \(N_M\) is obtained as

\[
N_{M^{(d)}} = \left( \frac{2|\text{Im} \tau_d|M^{(d)}}{A_d^2} \right)^{1/4}.
\]

We have the \(|M^{(d)}|\) zero-modes labelled by the index \(j\). Note that the wavefunction for \(j = k + M^{(d)}\) is identical to one for \(j = k\). They satisfy the orthonormal condition

\[
\int d^2 z^d |\psi_{d,M^{(d)}}^j(z^d)|^2 = \delta_{jj}.
\]

The important part of zero-mode wavefunctions is written in terms of the Jacobi theta function

\[
\vartheta \begin{bmatrix} a \\ b \end{bmatrix} (\nu, \tau) = \sum_{n=-\infty}^{\infty} \exp \left[ \pi i (n + a)^2 \tau + 2 \pi i (n + a) (\nu + b) \right].
\]

It transforms under the symmetry of torus lattice and has several important properties [30]. One of them is the following product rule

\[
\vartheta \begin{bmatrix} i/M_1 \\ 0 \end{bmatrix} (z_1, \tau M_1) \cdot \vartheta \begin{bmatrix} j/M_2 \\ 0 \end{bmatrix} (z_2, \tau M_2) = \sum_{m \in \mathbb{Z}_{M_1}+M_2} \vartheta \begin{bmatrix} i+j+M_1 m \\ M_1+M_2 \\ 0 \end{bmatrix} (z_1 + z_2, \tau (M_1 + M_2)) \times \vartheta \begin{bmatrix} M_2 z_1 - M_1 z_2 + M_1 M_2 m \\ M_1 M_2 (M_1 + M_2) \\ 0 \end{bmatrix} (z_1 M_2 - z_2 M_1, \tau M_1 M_2 (M_1 + M_2)).
\]

Here \(\mathbb{Z}_M\) is the cyclic group of order \(|M|\), \(\mathbb{Z}_M = \{1, \ldots, |M|\}\) where every number is defined modulo \(M\). Although this expression looks asymmetric under the exchange between \(i\) and \(j\), it is symmetric if we take into account the summation. By using the product property (2.8), we can decompose a product of two zero-mode wavefunctions as follows,

\[
\psi_{d,M^{(d)}}^i(z^d) \psi_{d,M^{(d)}}^j(z^d) = \frac{N_{M_1} N_{M_2}}{N_{M_1+M_2}} \sum_{m \in \mathbb{Z}_{M_1}+M_2} \psi_{d,M^{(d)}}^{i+j+M_1 m,M_1+M_2}(z^d) \times \vartheta \begin{bmatrix} M_2 z_1 - M_1 z_2 + M_1 M_2 m \\ M_1 M_2 (M_1 + M_2) \\ 0 \end{bmatrix} (0, \tau_d M_1 M_2 (M_1 + M_2)).
\]
In this paper, we calculate the generalization of Yukawa couplings to arbitrary order $L$ couplings
\begin{equation}
Y_{i_1...i_Lx_{i_{L+1}}...i_L} \chi^{i_1}(x) \cdots \chi^{i_{L+1}}(x) \phi^{i_{L+1}}(x) \cdots \phi^{i_L}(x),
\end{equation}
with $L = L_\chi + L_\phi$, where $\chi$ and $\phi$ collectively represent four-dimensional components of fermions and bosons, respectively. The system under consideration can be understood as low-energy effective field theory of open string theory. The magnetic flux is provided by stacks of D-branes filling in the internal dimension. The leading order terms in $\alpha'$ are identical to ten-dimensional super-Yang-Mills theory, whose covariantized gaugino kinetic term gives the three-point coupling upon dimensional reduction [7, 19]. The higher order couplings can be read off from the effective Lagrangian of the Dirac-Born-Infeld action with supersymmetrization. The internal component of bosonic and fermionic wavefunctions is the same [7]. Therefore it suffices to calculate the wavefunction overlap in the extra dimensions
\begin{equation}
Y_{i_1i_2...i_L} = g_L^{10} \int_{T^2} d^6z \prod_{d=1}^{3} \psi_d^{i_1,M_1}(z) \psi_d^{i_2,M_2}(z) \cdots \psi_d^{i_L,M_L}(z),
\end{equation}
where $g_L^{10}$ denotes the coupling in ten dimensions.

## 3 Three-point coupling

In this section, we calculate the three-point coupling considering the coupling selection rule. As we see later, the three-point coupling provides a building block of higher order couplings.

The gauge group dependent part is contracted by the gauge invariance, so that the choice of three blocks $m_a, m_b, m_c$ in (2.1) automatically fixes the relative magnetic fluxes
\begin{equation}
(m_a - m_b) + (m_b - m_c) = (m_a - m_c), \quad \text{and} \quad M_1 + M_2 = M_3,
\end{equation}
where $M_1 = m_a - m_b$, $M_2 = m_b - m_c$ and $M_3 = m_a - m_c$. Here every $M_i$ is assumed to be a positive integer. This relation is interpreted as the selection rule, in analogy of intersecting brane case [31, 32], to which we come back later. If it is not satisfied, there is no corresponding gauge invariant operator in ten dimensions. In terms of quantum numbers the coupling has the form $(N_a, \overline{N}_b, 1) \cdot (1, N_b, \overline{N}_c) \cdot (N_a, 1, N_c)$ under $U(N_a) \times U(N_b) \times U(N_c)$.

The internal part including the wavefunction integrals on the $d$-th $T^2$ gives
\begin{equation}
y_{ijk} = \int d^2z \psi^{i,M_1}(z) \psi^{j,M_2}(z) \left( \psi^{k,M_3}(z) \right)^*,
\end{equation}
The complete three-point coupling is the direct product of those in $d = 1, 2, 3$ and $g_L^{10}$. For the moment we neglect the normalization factors $N_M$, and consider two-dimensional wavefunctions, omitting the extra dimensional index $d$. By using the relation (2.9), we can decompose the product of the first two wavefunctions $\psi^{i,M_1}(z) \psi^{j,M_2}(z)$ in terms of $\psi^{k,M_3}(z)$ and we apply the orthogonality relation (2.6). Then, we obtain
\begin{equation}
y_{ijk} = \sum_{m \in \mathbb{Z}_{M_3}} \delta_{i+j+M_1m,k} \delta \left[ \begin{array}{ccc} M_{22} & M_{23} & M_{2m} \\ M_{12} & M_{13} & M_{1m} \\ 0 & 0 & 0 \end{array} \right] (0, \tau M_1 M_2 M_3),
\end{equation}
where the numbers in the Kronecker delta is defined modulo $M_3$. This expression is symmetric under the exchange $(i, M_1) \leftrightarrow (j, M_2)$.

For $\gcd(M_1, M_2) = 1$, we solve the constraint from the Kronecker delta

$$i + j - k = M_3 l - M_1 m, \quad m \in \mathbb{Z}_{M_3}, l \in \mathbb{Z}_{M_1}. \quad (3.4)$$

Using Euclidean algorithm, it is easy to see that, in the relatively prime case $\gcd(M_1, M_2) = 1$, there is always a unique solution for given $i, j, k$. This situation is the same as one in intersecting D-brane models \[31, 32\]. The argument of the theta function in eq. (3.3) becomes

$$\frac{M_2 i - M_1 j + M_1 M_2 m}{M_1 M_2 (M_1 + M_2)} = \frac{M_2 k - M_3 j + M_2 M_3 l}{(M_3 - M_2) M_2 M_3}. \quad (3.5)$$

Therefore, the three-point coupling is written as

$$y_{ijk}(l) = \vartheta \left[ \frac{M_2 k - M_3 j + M_2 M_3 l}{M_2 M_3 (M_3 - M_2)} \right] (0, \tau (M_3 - M_2) M_2 M_3), \quad (3.6)$$

where $l$ is an integer related to $i, j, k$ through (3.4). This is called the 2-3 picture, or the $j$-$k$ picture, where the dependence on $i$ and $M_1$ is only implicit.

In the case with a generic value of $\gcd(M_1, M_2) = g$, we can show

$$y_{ijk} = \sum_{n=1}^{g} \vartheta \left[ \frac{M_2 k - M_3 j + M_2 M_3 l}{M_1 M_2 M_3} \right] \left(0, \tau \frac{M_1 M_2 M_3}{g} \right). \quad (3.7)$$

The point is that, for a given particular solution $(i, j, k)$, the number of general solutions satisfying eq. (3.4) is equal to $g$. We can use a similar argument as above, now considering $\mathbb{Z}_{M_1/g}$ and $\mathbb{Z}_{M_3/g}$ instead of the original region. There is a unique pair $(l, m)$ in $(\mathbb{Z}_{M_1/g}, \mathbb{Z}_{M_3/g})$ satisfying the constraint (3.4), i.e.,

$$\frac{i + j - k}{g} = \frac{M_2 l}{g} - \frac{M_1 m}{g}. \quad (3.8)$$

Obviously, when $(l, m)$ is a particular solution, the following pairs,

$$\left( l + \frac{M_1}{g}, m + \frac{M_3}{g} \right) \in (\mathbb{Z}_{M_1}, \mathbb{Z}_{M_3}), \quad (3.9)$$

also satisfy the equation with the same right-hand side (r.h.s.). Since $\mathbb{Z}_{M_1}$ and $\mathbb{Z}_{M_3}$ are respectively unions of $g$ identical copies of $\mathbb{Z}_{M_1/g}, \mathbb{Z}_{M_3/g}$, there are $g$ different solutions. This situation is the same as one in intersecting D-brane models \[31, 32\]. If we reflect the shift (3.9) in (3.5), we obtain the desired result (3.7).

There can be Wilson lines $\zeta \equiv \zeta_r + \tau \zeta_i$, whose effect is just a translation of each wavefunction [7]

$$\psi^{j,M}(z) \rightarrow \psi^{j,M}(z + \zeta), \quad \text{for all } j. \quad (3.10)$$
Thus the corresponding product for (2.8) is obtained as

\[
\vartheta \left[ \frac{i}{M_1} 0 \right] ((z + \zeta_1)M_1, \tau M_1) \cdot \vartheta \left[ \frac{j}{M_2} 0 \right] ((z + \zeta_2)M_2, \tau M_2)
\]

\[
= \sum_{m \in \mathbb{Z}_{M_1+M_2}} \vartheta \left[ \frac{i+j+M_1 m}{M_1+M_2} 0 \right] ((M_1 + M_2)(z + \zeta_3), (\tau (M_1 + M_2))
\]

\[
\times \vartheta \left[ \frac{M_2 (i-j+M_1 m)}{M_1 M_2 (M_1+M_2)} 0 \right] (M_1 M_2 (\zeta_1 - \zeta_2), \tau M_1 M_2 (M_1 + M_2)),
\]

(3.11)

where \( M_3 = M_1 + M_2 \) and \( \zeta_3 M_3 = \zeta_1 M_1 + \zeta_2 M_2 \).

Finally, we take into account the six internal dimensions \( T^2 \times T^2 \times T^2 \). Referring to (2.11), essentially the full coupling is the direct product of the coupling on each two-torus. The overall factor in (2.11) is the physical ten dimensional gauge coupling \( g_{YM}^{10} = g_{YM} \), since this is obtained by dimensional reduction of super Yang-Mills theory. Collecting the normalization factors (2.5) from (2.9), the full three-point coupling becomes

\[
Y_{ijk} = g_{YM} \prod_{d=1}^{3} \left( \frac{2\text{Im} \tau_d}{A_d^2} \frac{M_1^{(d)} M_2^{(d)}}{M_3^{(d)}} \right)^{1/4}
\]

\[
\times \exp \left( i\pi (M_1^{(d)} \zeta_1^{(d)} \text{Im} \zeta_1^{(d)} + M_2^{(d)} \zeta_2^{(d)} \text{Im} \zeta_2^{(d)} + M_3^{(d)} \zeta_3^{(d)} \text{Im} \zeta_3^{(d)}) / \text{Im} \tau_d \right)
\]

\[
\times \sum_{n_d = 1}^{g_d} \vartheta \left[ \frac{M_2^{(d)} k-M_1^{(d)} j+M_2^{(d)} l}{M_1^{(d)} M_2^{(d)} M_3^{(d)}} 0 \right] + \frac{n_d}{g_d} \right) (M_2^{(d)} M_3^{(d)} (\zeta_2^{(d)} - \zeta_3^{(d)}), \tau_d M_1^{(d)} M_2^{(d)} M_3^{(d)})
\]

(3.12)

Here the index \( d \) indicates that the corresponding quantity is the component in \( d \)-th direction. For later use, it is useful to visualize the three-point coupling like Feynman diagram in figure 1.
4 Higher order coupling

4.1 Four-point coupling

We calculate the four-point coupling

\[ y_{ijkl} = \int d^2 z \psi^j(z) \psi^k(z) \left( \psi^M(z) \right)^*, \]  

(4.1)

and represent it in various ways. The main result is that the four-point coupling can be expanded by three-point couplings. Thus by iteration, we can generalize it to higher order couplings.

We consider the case without Wilson lines, since the generalization is straightforward. The product of the first two wavefunctions \( \psi^j(z) \psi^k(z) \) in (4.1) is the same as in (2.9). Again, we suppose \( M_1 + M_2 + M_3 = M_4 \). Then the product of the first three wavefunctions \( \psi^j(z) \psi^k(z) \psi^M(z) \) in (4.1) gives

\[
\sum_{m \in \mathbb{Z}_{M_1+M_2}} \sum_{n \in \mathbb{Z}_{M_4}} \psi^{i+j+k+M_1 m + (M_1 + M_2) n, M_1}(z) \delta \left[ \frac{M_4-i-j-M_1-M_2 m}{M_1 M_2 (M_1+M_2)} \right] (0, \tau M_1 M_2 (M_1+M_2))
\times \delta \left[ \frac{M_4(i+j+m)+M_3 M_4}{M_1 M_2 M_3 M_4} \right] (0, \tau (M_1+M_2) M_3 M_4).
\]  

(4.2)

Now, we product the last wave function \( \psi^M(z) \) in (4.1), acting on the first factor in (4.2), yielding the Kronecker delta \( \delta_{i+j+k+(M_1+M_2) n, M} \). The relation is given modulo \( M_4 \), reflecting that \( i, j, k, l \) are defined modulo \( M_1, M_2, M_3, M_4 \), respectively. It is non-vanishing if there is \( r \) such that

\[ i + j + k + M_1 m + (M_1 + M_2) n = l + M_4 r. \]  

(4.3)

We solve the constraint equation in terms of \( n \).

For \( \gcd(M_1, M_2, M_3) = 1 \), any coupling specified by \( (i, j, k, l) \) satisfies the constraint. For a coupling \( y_{ijkl} \), fixing \( (m, r) \) there is always a unique \( n \) satisfying the constraint. This means that by solving the constraint equation in terms of \( n \), we can remove the summation over \( n \) in (4.2). The result is

\[
y_{ijkl} = \sum_{m \in \mathbb{Z}_{M_1+M_2}} \psi \left[ \frac{M_4-i-j-M_1-M_2 m}{M_1 M_2 M_3 M_4} \right] (0, \tau M_1 M_2 M) \cdot \psi \left[ \frac{M_4-i-j-k-M_3 M_4 r}{M_1 M_2 M_3 M_4} \right] (0, \tau M_3 M_4 M),
\]  

(4.4)

where \( M = M_1 + M_2 = -M_3 + M_4 \). This form (4.4) is expressed in terms of only ‘external lines’, \( i, j, k, l \), and in the ‘internal line’ \( r \) is uniquely fixed by \( m \) from the relation (4.3). This is to be interpreted as expansion in terms of three-point couplings (3.6). From the property of the theta function, we have relations like \( y_{ijkl} = y_{ijk}^* \), etc. Thus we can write

\[
y_{ijkl} = \sum_{m \in \mathbb{Z}_{M_1+M_2}} y_{ijm}(m) \cdot y_{kmn}(r),
\]  

(4.5)
Figure 2. A four-point coupling is decomposed into products of three-point couplings. It also has ‘worldsheet’ duality. We have another ‘u-channel’ diagram.

where $m$ and $r$ are uniquely related by the relation (4.3). Recall that three-point coupling can be expressed in terms of ‘two external lines’ depending on the 2-3 ‘picture.’

The result (4.4) can be written by arranging the summation of quantum numbers as follows,

$$y_{ijkl} = \sum_{s \in \mathbb{Z}_{M_1+M_2}} \vartheta \left[ \frac{M_{2s-M_j+M_2Mr}}{M_1M_2M} + \frac{\varphi}{g} \right] (0, \tau M_1M_2M) \cdot \vartheta \left[ \frac{-M_1+M_4+MM_4}{M_3M_4M} + \frac{\varphi}{g} \right] (0, \tau M_3M_4M). \quad (4.6)$$

Here, we rewrite (4.3)

$$i + j + M_1m = s + (M_1 + M_2)r,$$

$$-k + l + M_3r = s + (M_1 + M_2)n,$$

by introducing an auxiliary label $s$, defined modulo $M = M_1 + M_2 = -M_3 + M_4$. This is uniquely fixed by other numbers from (4.3) and it can be traded with $m$. Thus we arrive at the second form (4.6), which becomes

$$y_{ijkl} = \sum_{s \in \mathbb{Z}_{M_1+M_2}} y_{ijst} \cdot y_{ksli}. \quad (4.8)$$

The second expression (4.6), explicitly depends on the ‘internal line’ $s$. It is useful to track the intermediate quantum number $s$.

We saw that in the case gcd$(M_1, M_2) = 1$, there is a unique solution. Since we expand higher order coupling in terms of three-point couplings, if any of them have degeneracies as in (3.7), i.e., gcd$(M_i, M_j) = g_{ij} > 1$, we should take into account their effects. It is interpreted that each three-point coupling contains a flavor symmetry $\mathbb{Z}_{g_{ij}} [33]$. For the four-point coupling with gcd$(M_1, M_2) = g_{12}$ and gcd$(M_3, M_4) = g_{34}$ we have also gcd$(g_{12}, g_{34}) = g = \text{gcd}(M_1, M_2, M_3, M_4)$, without loss of generality (see below). Employing the ‘intermediate state picture’, or the $(j-s) \times (s-l)$ picture, in the last expression in (4.6), we have

$$\sum_{p \in \mathbb{Z}_4} \sum_{s \in \mathbb{Z}_{M_1+M_2}} \vartheta \left[ \frac{M_{2s-M_j+M_2Mr}}{(M-M_2)M_2M} + \frac{\varphi}{g} \right] (0, \tau(M - M_2)M_2M) \times \vartheta \left[ \frac{-M_1+M_4+MM_4}{M_4(M_4-M)} + \frac{\varphi}{g} \right] (0, \tau MM_4(M_4-M)). \quad (4.9)$$
It shows that the two symmetries \( Z_{g_{12}} \) and \( Z_{g_{34}} \) are broken down to the largest common symmetry \( Z_g \), due to the constraint. Otherwise we cannot put together the vertices with the common intermediate state \( s \).

Reminding that we are examining the overlap of four wavefunctions, and it does not depend on the order of product. If we change the order of the product in (4.1), namely consider the product of the second and the third wavefunctions \( \psi^{j,M_2}(z)\psi^{k,M_3}(z) \) first, we have differently-looking constraint relation which is equivalent to (4.3) undergoing the decomposition,

\[
\begin{align*}
  j + k + M_2 m' &= t + (M_2 + M_3) r', \\
  -i + l + M_1 r' &= t + (M_2 + M_3) n'.
\end{align*}
\]

This looks like the ‘\( t \)-channel’ and we have

\[
y_{ijk} \bar{\ell} = \sum_{t \in Z_{M'}} \vartheta \left( \frac{M'k + M_1M_r'}{(M' - M_3)M_3M} \right) (0, \tau(M' - M_3)M_3M') \times \vartheta \left( \frac{-M'l + M_1l + M'M_4n}{M'M_1(M_1 - M')} \right) (0, \tau M'M_1(M_1 - M')) \tag{4.11}
\]

\[
y_{ijk} \bar{t} \cdot y_{jkt},
\]

with \( M' = -M_1 + M_4 = M_2 + M_3 \). The result has a behavior like ‘worldsheet’ duality in those of Veneziano and Virasoro-Shapiro [34]. This means that, in decomposing the diagram, the position of an insertion does not matter.

If we have Wilson lines, we just replace the three-point couplings by those with Wilson lines (3.12).

### 4.2 Generic \( L \)-point coupling

We have seen that the four point coupling is expanded in terms of three-point couplings. We can generalize the result to obtain arbitrary higher order couplings. The constraint relations and the higher order couplings are always decomposed into products of three-point couplings. It is easily calculated by Feynman-like diagram.

The decompositions (4.4), (4.6), (4.11) are understood as inserting the identity expanded by the complete set of orthonormal eigenfunctions \( \{ \psi_n^{i,M} \} \) as follows. For example, we split the integral (4.1) as

\[
y_{ijk} = \int d^2 z d^2 z' \psi^{j,M_1}(z)\psi^{k,M_2}(z)\delta^2(z - z')\psi^{k,M_3}(z') \left( \psi^{l,M_4}(z') \right)^*. \tag{4.12}
\]

Then, we use the complete set of orthonormal eigenfunctions \( \{ \psi_n^{i,M} \} \) of the Hamiltonian with a magnetic flux \( M \). That is, they satisfy

\[
\sum_{s,n} \left( \psi_n^{s,M}(z) \right)^* \psi_n^{s,M}(z') = \delta^2(z - z'). \tag{4.13}
\]
Figure 3. Likewise, any amplitude with arbitrary external lines is decomposed into product of three-point amplitudes.

We insert l.h.s. instead of the delta function $\delta^2(z - z')$ in (4.12). Since $\psi^{j_1, M_1}(z)\psi^{j_2, M_2}(z)$ is decomposed in terms of $\psi_n^{s, M_1 + M_2}(z)$, it is convenient to take $M = M_1 + M_2$ for inserted wavefunctions $\left(\psi_n^{s, M}(z)\right)^* \psi_n^{s, M}(z')$. In such a case, only zero-modes of $\psi_n^{s, M}(z)$ appear in this decomposition. If we take $M \neq M_1 + M_2$, higher modes of $\psi_n^{s, M}(z)$ would appear. At any rate, when we take $M = M_1 + M_2$, we can lead to the result (4.6) and (4.5). On the other hand, we can split $y_{ijkl} = \int d^2 z d^2 z' \psi^{j_1, M_1}(z)\psi^{j_2, M_2}(z)\delta^2(z - z')\psi^{j_3, M_3}(z')\left(\psi^{j_4, M_4}(z')\right)^*$, (4.14)

and insert (4.13) with $M = M_2 + M_3$. Then, we can lead to (4.11). Furthermore, we can calculate the four-point coupling after splitting $y_{ijkl} = \int d^2 z d^2 z' \psi^{j_1, M_1}(z)\psi^{j_2, M_2}(z)\delta^2(z - z')\psi^{j_3, M_3}(z')\left(\psi^{j_4, M_4}(z')\right)^*$. (4.15)

How to split corresponds to ‘s-channel’, ‘t-channel’ and ‘u-channel’. Note that only zero-modes appear in ‘intermediate states’, when we take proper values of $M$ because of the product property.

We have considered the four-point couplings with $M_1 + M_2 + M_3 = M_4$ for $M_i > 0$. We may consider the case with $M_1 + M_2 = M_3 + M_4$ for $M_i > 0$, which corresponds to $y_{ijkl} = \int d^2 z \psi^{j_1, M_1}(z)\psi^{j_2, M_2}(z)\left(\psi^{j_3, M_3}(z)\right)^* \left(\psi^{j_4, M_4}(z)\right)^*$. (4.16)

In order to consider both of this case and the previous case at the same time, we would have more symmetric expression for the four-point coupling $y_{ijkl} = \int d^2 z \psi^{j_1, M_1}(z)\psi^{j_2, M_2}(z)\psi^{j_3, M_3}(z)\psi^{j_4, M_4}(z)$, (4.17)

by defining $\psi^{j, -M}(z) \equiv \left(\psi^{j, M}(z)\right)^*$, (4.18)

with $M_1 + M_2 + M_3 + M_4 = 0$, where some of $M_i$ are negative, and $\tilde{z} = z$ for $M > 0$ and $\tilde{z} = \bar{z}$ for $M < 0$.

We can extend the above calculation to the $L$-point coupling, $y_{i_1 i_2 \ldots i_L} = \int d^2 z \prod_{j=1}^L \psi^{j, M_j}(\tilde{z})$, (4.19)
Figure 4. Five-point coupling. No more independent Feynman-like diagram for different insertion.

with the extension as in (4.18). We have then the selection rule

$$\sum_{j=1}^{L} M_j = 0,$$  \hspace{1cm} (4.20)

where some of $M_j$ are negative. The constraint is given as

$$\sum_{j=1}^{L} \left( i_j + \left( \sum_{l=1}^{j} M_l \right) r_j \right) = 0.$$  \hspace{1cm} (4.21)

Again, it shows the conservation of the total flavor number $i_j$, reflecting the fact that each $i_j$ is defined modulo $M_j$. We can decompose $L$-point coupling into $(L-1)$ and three-point couplings

$$\sum_{j=1}^{L-3} \left( i_j + \left( \sum_{l=1}^{j} M_l \right) r_j \right) + i_{L-2} = s - Kr_{L-1},$$

$$i_{L-1} + i_L + M_{L-1} r_{L-1} = -s - Kr_{L-2},$$  \hspace{1cm} (4.22)

where

$$K = \sum_{k=1}^{L-2} M_k = -M_{L-1} - M_{L},$$  \hspace{1cm} (4.23)

is the intermediate quantum number. Therefore if $\gcd(M_1, M_2, \ldots, M_L) = 1$, by induction we see that there is a unique solution by Euclidean algorithm. By iteration

$$y_{i_1 i_2 \ldots i_L} = \sum_s y_{i_1 i_2 \ldots i_{L-2}s} \cdot y_{s i_{L-1} i_L},$$  \hspace{1cm} (4.24)

we can obtain the coupling including the normalization. Thus, we can obtain $L$-point coupling out of $(L-1)$-point coupling. Due to the independence of ordering, we can insert (or cut and glue) any node.

As an illustrating example we show the result for the five-point coupling. We employ $s$-channel-like insertions, by naming intermediate quantum numbers $s_i$ as in figure 4. We
have
\[
y_{i_1i_2i_3i_4i_5} = \prod_{j=1}^{5} \vartheta_{i_0}^{M_j} (z M_j, \tau M_j)
\]
\[
= \sum_{s_1, s_2} \vartheta \left[ \frac{M_2s_1 - (M_1 + M_2)i_0 + M_2(M_1 + M_2)l_1}{M_2(M_1 + M_2)(M_1 + 2M_2)} \right] (0, M_1 M_2 (M_1 + M_2) \tau)
\]
\[
\times \vartheta \left[ \frac{(M_1 + M_2)i_2 - M_5s_2 + M_5(M_1 + M_2)l_2}{M_3(M_1 + M_2)(M_1 + M_2 + M_3)} \right] (0, (M_1 + M_2) M_3 (M_1 + M_2 + M_3))
\]
\[
\times \vartheta \left[ \frac{(M_1 + M_2)l_3 - M_5s_3 + M_5(M_1 + M_2 + M_3)l_3}{M_4(M_1 + M_2 + M_3)(M_1 + M_2 + M_3 + M_4)} \right] (0, -(M_1 + M_5) M_4 M_5 \tau),
\]
\[(4.25)\]

where
\[s_1 \in \mathbb{Z}_{M_1 + M_2}, \quad s_2 \in \mathbb{Z}_{M_1 + M_2 + M_3}.\]

From the regular patterns of increasing orders, we can straightforwardly generalize the couplings to arbitrary order.

Now, taking into account full six internal dimensions, as in three-coupling case (3.12), we have various normalization factors besides the product of theta functions. Again, from the product relation of theta function (2.8) we have
\[
s_L g_{YM}^{L-2} \alpha^{(L-4+L_x/2)/2} \times \prod_{d=1}^{3} \left( \frac{2 \text{Im} \gamma_d}{A_d^4} \sum_{M_1^{(d)}>0} (M_1^{(d)})^{1/4} \left( \frac{2 \text{Im} \gamma_d}{A_d^4} \sum_{M_1^{(d)}>0} (M_1^{(d)})^{1/4} \right)^{-1} \prod_{i=1}^{L} \left( \frac{2 \text{Im} \gamma_d |M_i^{(d)}|}{A_d^2} \right)^{1/4}.\]
\[(4.26)\]

Recall that \(L_x\) is the number of fermions in the couplings (2.10). We have \(g_L^{0} = s_L g_{YM}^{L-2} \alpha^{(L-4+L_x/2)/2}\) in (2.11), where symmetric factor \(s_L\) comes from higher order expansions of lower-level completion of Yang-Mills theory, having also an expansion parameter \(\alpha'\). In open string theory, it is the Dirac-Born-Infeld action, and it is unknown beyond the quartic order in \(\alpha' F\) [36]. The dependence of ten-dimensional gauge coupling \(g_{YM}\) and Regge slope \(\alpha'\) can be easily accounted by order counting [35]. Note that \(g_{YM}\) is dimensionful. This factor (4.26) is non-holomorphic in the complex structure \(\tau\) and complexified Kähler modulus \(\alpha' J = B + i A/4 \pi^2\), where \(B_{\alpha \beta} = e^{A/2\pi}\) is the antisymmetric tensor field component in \(d\)-th two-torus. They are interpreted as originating from the Kähler potential [7, 19]. The product \(\prod M_1^{(d)}\) is the leading order approximation of Euler beta function and its multivariable generalization, which is the property of dual amplitude.

As an example of full expressions, we show the four-point coupling among scalar fields, \(Y_{ijlm} \phi^i \phi^j (\phi^m)^*\), where \(\phi^i\) and \((\phi^j)^*\) \((\phi^j)^*\) and \((\phi^m)^*\) correspond to the magnetic flux \(M_1^{(d)}\) \((M_2^{(d)})\). For simplicity, we consider the case with vanishing Wilson lines.
and \( \gcd(M_1, M_2) = 1 \). The full coupling \( Y_{ij\bar{m}} \) is obtained as

\[
Y_{ij\bar{m}} = g_{YM}^2 \prod_{d=1}^{3} \left( \frac{2\text{Im} \tau_d M_1^{(d)} M_2^{(d)}}{A_d^2} \right)^{1/2} \sum_{k \in \mathbb{Z}} \left( y_{ij}^{(d)} \right)^* \gamma_{k\bar{m}},
\]

(4.27)

up to \( s_L \), where

\[
y_{ij}^{(d)} = \vartheta \left[ \frac{M_2^{(d)} k - M_1^{(d)} j + M_3^{(d)} M^{(d)} r}{M_1^{(d)} M_2^{(d)} M^{(d)}} \right] (0, \tau_d M_1^{(d)} M_2^{(d)} M^{(d)}).
\]

(4.28)

This scalar coupling with \( s_L = 1 \) appears from ten-dimensional super Yang-Mills theory and satisfies the relation \( Y_{ij\bar{m}} = Y_{ij\bar{k}}(Y)^* \) for the three-point coupling \( Y_{ijk} \) in eq. (3.12).

5 Intersecting D-brane models

Here we give comments on the relation between the results in the previous sections and higher order couplings in intersecting D-brane models, i.e. CFT-calculations.

There is well-known T-duality relation between magnetized and intersecting brane models. In intersecting brane case, the wavefunctions are highly localized around intersection points, whereas magnetized brane wavefunctions are fuzzily delocalized over the entire space.

Under the ‘horizontal’ duality with respect to real axis, \( X_z \leftrightarrow 2\pi\alpha' A_z \). The parameter is changed as

\[
\tau \leftrightarrow J, \quad \zeta \leftrightarrow \nu.
\]

(5.1)

Still the translational offset \( \nu \) is the Wilson line. Thus, the magnetic flux gives the slope \( A_1^\nu = -\frac{i}{2} F_{zz}^i = \frac{\pi}{\text{Im} J} M_i \) and the corresponding quantum number is the ‘relative angle,’ for small angles,

\[
\pi \theta_i = \frac{M_i}{\text{Im} J}.
\]

(5.2)

The selection rule due to the gauge invariance becomes

\[
M_1 + M_2 = M_3 \leftrightarrow \theta_1 + \theta_2 = \theta_3.
\]

(5.3)

In the intersecting brane case, as well as heterotic string case, there have been CFT calculation of higher order amplitude [22, 24, 27] using vertex operator insertion [21, 23, 25, 31]. There are vertex operators \( V_i \) corresponding to massless modes. We compute their \( L \)-point amplitude,

\[
\langle V_1 V_2 \ldots V_L \rangle.
\]

(5.4)

We have operator product expansion (OPE),

\[
V_i(z)V_j(0) \sim \sum_k \frac{c_{ijk}}{z^{h_{ijk}}} V_k(0),
\]

(5.5)

with \( h_{ijk} = h(V_k) - h(V_i) - h(V_j) \), where \( h(V_i) \) is the conformal dimension of \( V_i \). This OPE corresponds to (2.9). Furthermore, the coefficients \( c_{ijk} \) correspond to the three-point
couplings in four-dimensional effective field theory. In ref. [7], it is shown that the above three-point coupling \( c_{ijk} \) in intersecting D-brane models corresponds to the T-dual of the three-point couplings \( Y_{ijk} \) in magnetized D-brane models.

Now, let us consider the \( L \)-point amplitude \( \langle \prod_i V_i(z_i) \rangle \). We use the OPE (5.5) to write the \( L \)-point amplitude in terms of \((L - 1)\) point amplitudes. Such a procedure is similar to one in the previous sections, where we write \( L \)-point couplings in terms of three-point couplings.

For example, the CFT calculations for the four-point couplings \( c_{ijkl} \) in the intersecting D-brane models would lead

\[
c_{ijkl} \sim \sum_s c_{ij \bar{s}} c_{skl},
\]

and

\[
c_{ijkl} \sim \sum_t c_{ik \bar{t}} c_{tjl},
\]

depending on the order of OPE’s, i.e. s-channel or t-channel. Thus, the form of the four-point couplings as well as \( L \)-point couplings \((L > 4)\) is almost the same as the results in the previous sections. Note that in eq. (2.9), a product of two wavefunctions is decomposed in terms of only the lowest modes. On the other hand, in r.h.s. of eq. (5.5), higher modes as well as lowest modes may appear. However, dominant contribution due to the lowest modes are the same, because \( c_{ijk} \) for the lowest modes \((i,j,k)\) corresponds exactly to \( Y_{ijk} \) for the lowest modes.

Let us examine the correspondence of couplings between magnetized models and intersecting D-brane models by using concrete formulae. In the intersecting D-brane models, the amplitude (5.4) is decomposed into the classical and the quantum parts,

\[
\langle V_1 V_2 \ldots V_L \rangle = Z_{qu} \cdot Z_{cl} = Z_{qu} \cdot \sum_{\{X_{cl}\}} \exp(-S_{cl}),
\]

where \( X_{cl} \) is the solution to the classical equation of motion. The classical part is formally characterized as decomposable part and physically gives instanton of worldsheet nature, via the exchange of intermediate string. That gives intuitive understanding via the ‘area rule’, where the area corresponds to one, which intermediate string sweeps.

In the three-point amplitude, the summation of the classical action \( \sum_{\{X_{cl}\}} \exp(-S_{cl}) \) becomes the theta function [31], where \( S_{cl} \) corresponds to the triangle area. When we exchange \( \tau \) and \( J \) as (5.1) in the magnetized models, the Yukawa coupling (3.6) corresponds to the following expansion

\[
y_{ijk} = \vartheta \left[ \frac{M_{2k} - M_{3j} + M_{2M3l}}{M_1 M_2 M_3} \right] \left( 0, i M_1 M_2 M_3 A/(4\pi^2 \alpha') \right)
\]

\[
= \sum_{n \in \mathbb{Z}} \exp \left[ \frac{M_1 M_2 M_3 A}{4\pi\alpha'} \left( \frac{M_{2k} - M_{3j} + M_2 M_3 l}{M_1 M_2 M_3} + n \right)^2 \right],
\]

by using the definition (2.7). We have neglected the antisymmetric tensor component \( B \). The exponent corresponds the area (divided by \( 4\pi\alpha' \)) of possible formation of triangles and
the one with $n = 0$ corresponds to the minimal triangle. Recall that the theta function part depends only $\tau$ and $J$ in magnetized and intersecting D-brane models, respectively.

We have omitted the normalization factor, corresponding to the quantum part $Z_{\text{qu}}$. It is obtained by comparing the coupling (5.9) with (3.12). We find the factor

$$2^{-9/4} \pi^{-3} e^{\phi_4/2} \prod_{d=1}^{3} \left( \text{Im} \tau_d \frac{M_1^{(d)} M_2^{(d)}}{M_3^{(d)}} \right)^{1/4},$$

in the magnetized brane side corresponds to

$$Z_{\text{qu}} = (2\pi)^{-9/4} e^{\phi_4/2} \prod_{d=1}^{3} \left( \text{Im} J_d \right)^2 \theta_1^{(d)} \theta_2^{(d)} \theta_3^{(d)}),$$

in the intersecting brane side. We obtain the four dimensional dilaton $\phi_4 = \phi_{10} - \ln |\text{Im} \tau_1 \text{Im} \tau_2 \text{Im} \tau_3|$ from the ten dimensional one $\phi_{10}$, which is related with $g_{\text{YM}}$ as $g_{\text{YM}} = e^{\phi_{10}/2} \alpha'^{3/2}$. The vacuum expectation value of the dilaton gives gauge coupling $e^{\langle \phi_4 \rangle/2} = g$.

In this case, the factor containing the angles is a leading order approximation of the ratio of Gamma function

$$\frac{\Gamma(1 - \theta_1) \Gamma(1 - \theta_2) \Gamma(\theta_3)}{\Gamma(\theta_1) \Gamma(\theta_2) \Gamma(1 - \theta_3)} \approx \frac{\theta_1 \theta_2}{\theta_3},$$

valid for small angles. Therefore, the three-point couplings coincide each other between magnetized and intersecting D-brane models. That is the observation of [7].

Now, let us consider the four-point coupling of intersecting D-brane model corresponding to the left figure of figure 5. The four-point amplitude is written as (5.8), where the classical action corresponds to the area of the left figure. However, that can be decomposed into two triangles like the right figure, that is, the classical part can be decomposed into two parts, each of which corresponds to the classical part of three-point amplitude, i.e.

$$\exp(-S_{\text{cl}}^{(4)}) = \exp(-S_{\text{cl}}^{(3)}) \exp(-S_{\text{cl}}'^{(3)}),$$

where $S_{\text{cl}}^{(4)}$ corresponds to the area of the left figure of figure 5 and $S_{\text{cl}}^{(3)}$ and $S_{\text{cl}}'^{(3)}$ correspond to the triangle areas of the right figure.

On the other hand, our results in the previous sections show that the four-point coupling in the magnetized model is also expanded as (4.6). Each of theta functions in (4.6) corresponds to the classical parts of the three-point couplings in the intersecting D-brane model.
models. This relation corresponds to the above decomposition (5.13). Thus, the theta function parts of the four-point couplings, i.e. the classical part, coincide each other between magnetized and intersecting D-brane models. That means that the holomorphic complex structure, $\tau$, dependence of the four-point couplings in the magnetized brane models is the same as the holomorphic Kähler moduli $J$ dependence in the intersecting D-brane models, since the theta function part in the magnetized (intersecting) D-brane models depends only on $\tau$ ($J$). The other part in the magnetized brane models corresponds to normalization factors $N_M$. When we take a proper normalization, these factors also coincide.

6 Conclusions

We have calculated three-point and higher order couplings of four-dimensional effective field theory arising from dimensional reduction of magnetized brane models. We have found that higher order couplings are written as products of three-point couplings. This behavior is the same as higher order amplitudes of CFT, that is, higher order amplitudes are decomposed as products of three-point amplitudes in intersecting D-brane models. Our results on higher order couplings would be useful in phenomenological applications. Numerical analysis on higher order couplings is also possible.

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