New and Strong Constraints on the Parameter Space of the MSSM from Charge and Color Breaking Directions

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Abstract

Although the possible existence of dangerous charge and color breaking (CCB) directions in the MSSM has been known since the early 80’s, only particular directions in the fieldspace have been considered, thus obtaining necessary but not sufficient conditions to avoid dangerous CCB minima. Furthermore, the radiative corrections to the potential were not normally included in a proper way, often leading to an overestimation of the restrictive power of the bounds. It turns out that when correctly evaluated, the “traditional” CCB bounds are very weak. I give here a brief survey of recent results on this subject, which represent a complete analysis, showing that the new CCB bounds are very strong and, in fact, there are extensive regions in the parameter space that become forbidden. This produces important bounds, not only on the value of $A$, but also on the values of $B$ and $M_{1/2}$. The form of strongest one of the new bounds, the so called UFB-3 bound, is explicitely given.

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It is well known\[8\], \[3\], \[4\], \[5\] that the presence of scalar fields with color and electric charge in supersymmetric (SUSY) theories induces the possible existence of dangerous charge and color breaking (CCB) minima. However, a complete study of this crucial issue is still lacking. This is mainly due to two reasons. First, the enormous complexity of the scalar potential, $V$, in a SUSY theory, which has motivated that only analyses examining particular directions in the field–space have been performed. Second, the radiative corrections to $V$ have not been normally included in a proper way. Concerning the first point, the tree-level scalar potential, $V_o$, in the minimal supersymmetric standard model (MSSM) is given by

$$V_o = V_F + V_D + V_{\text{soft}},$$

where $V_F$ and $V_D$ are the F– and D–terms respectively and $V_{\text{soft}}$ are the soft breaking terms, i.e.

$$V_{\text{soft}} = \sum_{\alpha} m_{\phi_o}^2 |\phi_o|^2 + \sum_{i=\text{generations}} \{ A_u, \lambda_u, Q_i H_2 u_i + A_d, \lambda_d, Q_i H_1 d_i + A_e, \lambda_e, L_i H_1 e_i + \text{h.c.} \} + (B \mu H_1 H_2 + \text{h.c.}) ,$$

in a standard notation. $V_o$ is extremely involved since it has a large number of independent fields and parameters. Even assuming universality of the soft breaking terms at the unification scale, there are five undetermined parameters: $m$, $M$, $A$, $B$, $\mu$, i.e. the universal scalar and gaugino masses, the universal coefficients of the trilinear and bilinear scalar terms, and the Higgs mixing mass, respectively. (Notice that $M$ does not appear explicitly in $V_o$, but it does through the renormalization group equations (RGEs) of all the remaining parameters.)

As mentioned above, the complexity of $V$ has made that only particular directions in the field-space have been explored, thus obtaining necessary but not sufficient CCB conditions to avoid dangerous CCB minima. By far the most extensively used CCB condition is the “traditional” bound, first studied in ref.\[1\], \[2\]. Namely, given a particular trilinear scalar coupling, e.g. $\lambda_u A_u Q_u H_2 u$, assuming equal vacuum expectation values (VEVs) for the three fields involved in it, i.e. $|Q_u| = |H_2| = |u|$, it turns out that a very deep CCB minimum appears unless the famous constraint

$$|A_u|^2 \leq 3 \left( m_{Q_u}^2 + m_u^2 + m_2^2 \right)$$

is satisfied. In the previous equation $m_{Q_u}^2, m_2^2, m_2^2$ are the mass parameters of $Q_u, u, H_2$, where $m_2^2$ is the sum of the $H_2$ squared soft mass, $m_{H_2}^2$, plus $\mu^2$. Further analogous constraints have been derived in the existing literature \[3\], \[4\], \[5\], \[6\].

Concerning the radiative corrections it should be noted that the usual CCB bounds (e.g. eq.(1)) come from the tree-level potential, $V_o$. However, $V_o$ is strongly dependent on the renormalization scale, $Q$ and the one-loop radiative corrections to it, namely $\Delta V_1 = \sum_{\alpha} \frac{n_{\alpha}}{64 \pi^2} M_{\alpha}^4 \log \left( \frac{M_{\alpha}^2}{Q^2} \right) - \frac{3}{2}$ are crucial to make it stable against variations of $Q$ \[6\]. \[7\]. In the previous expression $M_{\alpha}$ are the tree-level mass eigenstates, which in general are field-dependent quantities, so $\Delta V_1$ is a complicated function of all the scalar fields. However, a good approximation is to still work just with $V_o$, but at an appropriate choice for the value of $Q$, so that $\Delta V_1$ is small and the predictions of $V_o$ and $V_o + \Delta V_1$ essentially coincide. This occurs for a value of $Q$ of the order of the most significant $M_{\alpha}$ mass appearing in $\Delta V_1$, which in turn depends on what is the direction in the field-space that is being analyzed. In the usual calculations, however, the CCB bounds are imposed at any scale between $M_X$ and $M_Z$ and, therefore, their restrictive power has been overestimated.

In this talk I give a brief survey of the results of our article, ref. \[8\], where we have tried to completely classify all the possible dangerous directions in the MSSM, extracting the corre-
sponding improved (and hopefully complete) bounds and analyzing numerically their restrictive power.

It is important to keep in mind that the Higgs part of the potential must be in such a way that it develops a realistic minimum at $|H_1| = v_1$, $|H_2| = v_2$, with $v_1^2 + v_2^2 = 2M_W^2/g_2^2$, which corresponds to the standard vacuum. This requirement fixes the value of $\mu$ in terms of the other independent parameters, i.e. $m, M, A, B, \mu$. Furthermore one has to demand that all the physical particles have masses compatible with their observed values (or upper bounds).

There are two types of charge and color breaking constraints: the ones arising from directions in the field-space along which the (tree-level) potential can become unbounded from below (UFB), and those arising from the existence of charge and color breaking (CCB) minima in the potential deeper than the realistic minimum. A complete classification of the UFB and CCB constraints can be obtained from ref. [8]. Since there is no room here to list the precise form of these bounds, let us mention here their most important characteristics and what is the most important bound.

Concerning the UFB directions (and corresponding constraints), there are three of them, labelled as UFB-1, UFB-2, UFB-3 in [8]. The relevant scalar fields involved are $\{H_1, H_2\}$, $\{H_1, H_2, L\}$, $\{H_2, L, e_{Lj}, e_{Rj}\}$ respectively, where $L$ is a slepton taking the VEV along the $\nu_L$ direction and $e_{Lj}, e_{Rj}$ are selectrons of the $j$—generation. The UFB-3 bound turns out to be the strongest one of all the UFB and CCB constraints in the parameter space of the MSSM, so it deserves to be exposed in greater detail.

UFB-3

It is possible, by simple analytical minimization, to write the value of all the relevant fields along the UFB-3 direction in terms of the $H_2$ one. Then, for any value of $|H_2| < M_X$ satisfying

$$|H_2| > \sqrt{\frac{\mu^2}{4\lambda_{e_j}^2} + \frac{4m^2_{L_j}}{g^2 + g_2^2}} - \frac{|\mu|}{2\lambda_{e_j}},$$

the value of the potential along the UFB-3 direction is simply given by

$$V_{\text{UFB-3}} = (m_2^2 - \mu^2 + m_{L_j}^2)|H_2|^2 + \frac{|\mu|}{\lambda_{e_j}}(m_{L_j}^2 + m_2^2 + m_{e_j}^2)|H_2| - \frac{2m^4_L}{g^2 + g_2^2},$$

otherwise

$$V_{\text{UFB-3}} = (m_2^2 - \mu^2)|H_2|^2 + \frac{|\mu|}{\lambda_{e_j}}(m_{L_j}^2 + m_2^2 + m_{e_j}^2)|H_2| + \frac{1}{8}(g^2 + g_2^2)\left[|H_2|^2 + \frac{|\mu|}{\lambda_{e_j}}|H_2|^2\right]^2.$$

In eqs.(4,5) $\lambda_{e_j}$ is the lepton Yukawa coupling of the $j$—generation (see eq.(1)). Then, the UFB-3 condition reads

$$V_{\text{UFB-3}}(Q = 0) > V_{\text{real min}},$$

where $V_{\text{real min}} = -\frac{1}{8}(g^2 + g_2^2)(v_2^2 - v_1^2)^2$ is the realistic minimum and the $\hat{Q}$ scale is given by $\hat{Q} \sim \text{Max}(g_2|e|, \lambda_{\text{top}}|H_2|, g_2|H_2|, g_2|L|, M_S)$ with $|e| = \frac{\mu}{\lambda_{e_j}}|H_2|$ and $|L|^2 = -\frac{4m^4_L}{g^2 + g_2^2} + (|H_2|^2 + |e|^2)$. Finally $M_S$ is the typical scale of SUSY masses (normally a good choice for $M_S$ is an average of the stop masses, for more details see refs.[6,7,8]). From (4,5), it is clear that the larger $\lambda_{e_j}$, the more restrictive the constraint becomes. Consequently, the optimum choice of the $e$–type slepton should be the third generation one, i.e. $e_j = \text{stau}$.
Let us briefly turn to the CCB constraints in the strict sense, i.e. those coming from the possible existence of charge and color breaking (CCB) minima in the potential deeper than the realistic minimum. We have already mentioned the “traditional” CCB constraint \[ \text{eq}(2) \] of eq.(2). Other particular CCB constraints have been explored in the literature \[ \text{ref}[8] \]. In ref.[8] it has been performed a complete analysis of the CCB minima, obtaining a set of “improved” analytic constraints that represent the necessary and sufficient conditions to avoid the dangerous ones. For certain regions of values of the initial parameters, the CCB constraints “degenerate” into the above-mentioned UFB constraints since the minima become unbounded from below directions. In this sense, the CCB constraints comprise the UFB bounds, so the latter can be considered as special (but extremely important) limits of the former.

It is not possible to give here an account of the general CCB constraints obtained in ref.[8], so let us mention their most outstanding characteristics. First, the most dangerous, i.e. the deepest, CCB directions in the MSSM potential involve only one particular trilinear soft term of one generation. Then, for each trilinear soft term there are three possible (optimized) types of constraints, which in \[ \text{ref}[8] \] were named CCB-1,2,3. Of course these constraints, which have an analytical form not very different from the “traditional” ones (see eq.(2)), include the latter and are much more stronger than them. It is important to recall here that the CCB bounds must be evaluated at a correct renormalization scale, \( \hat{Q} \), in order to avoid an overestimation of their restrictive power. That scale is always of order \( A/\lambda \), where \( A \) and \( \lambda \) are, respectively, the coefficient of the trilinear scalar term and the Yukawa coupling constant associated to the Yukawa coupling under consideration (a more precise recipe for the value of \( \hat{Q} \) can be found in \[ \text{ref}[8] \]). The numerical analysis shows that the the “traditional” CCB bounds when correctly evaluated turn out to be very weak (see Fig.1a). On the contrary, the new improved CCB-1,2,3 constraints obtained in ref.[8] (and not explicitly written here) are much more restrictive (see Fig.1b).

Anyway, as mentioned above, the most important restrictions come from the UFB constraints, in particular from the UFB-3 one, explicitly shown in eqs.(3 -6). This is clearly exhibited in Fig.2, where we summarize all the constraints plotting also the excluded region due to (conservative) experimental bounds on SUSY particle masses. The allowed region left at the end of the day (white) is quite small.

When the whole MSSM parameter space is scanned it is observed that, as a general trend, the smaller the value of \( m \), the more restrictive the constraints become. In the limiting case \( m = 0 \) essentially the whole parameter space turns out to be excluded. This has obvious implications, e.g. for no-scale models. Let us also mention that, contrary to a common believe, the UFB and CCB constraints are very strong and put important bounds not only on the value of \( A \) (soft trilinear parameter), but also on the values of \( B \) (soft bilinear parameter) and \( M \) (gaugino masses). This is a new and interesting feature.
Figure 1: Fig.1: Excluded regions in the parameter space of the Minimal Supersymmetric Standard Model, with $B = A - m$, $m = 100$ GeV and $M_{\text{top}}^{\text{phys}} = 174$ GeV. The dark region is excluded because there is no solution for $\mu$ capable of producing the correct electroweak breaking. a) The circles and diamonds indicate regions excluded by the “traditional” CCB constraints associated with the $e$ and $d$-type trilinear terms respectively. b) The same as (a) but using our “improved” CCB constraints. The triangles correspond to the $u$-type trilinear terms.
Figure 2: Excluded regions in the parameter space of the Minimal Supersymmetric Standard Model, with $B = A - m$ and $M_{\text{phys}}^{\text{top}} = 174$ GeV. The small filled squares indicate regions excluded by our Unbounded From Below constraints, mainly the UFB-3 one. The circles indicate regions excluded by our “improved” CCB constraints. The filled diamonds indicate regions excluded by the experimental lower bounds on supersymmetric particle masses.

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