Let $M$ be a three-manifold and $K_t(M)$ be its Kauffman bracket skein module, that is, the $\mathbb{C}[t, t^{-1}]$-module generated by the isotopy classes of framed links in $M$ modulo the relations of the Kauffman bracket.

If $K$ is a knot in the three-sphere and $M$ is its complement, then $K_t(T^2 \times I)$ acts from the left on $K_t(M)$ with $T^2$ a torus. The peripheral ideal $I_t(K)$ is defined to be the left ideal of $K_t(T^2 \times I)$ annihilating the empty link $\emptyset$ in $M$. The $A$-ideal, which is shown to be a generalization of the $A$-polynomial, can be defined by $I_t(K)$ [C. Frohman, R. Gelca, and W. LoFaro, Trans. Am. Math. Soc. 354, No. 2, 735-747 (2002; Zbl 0980.57002)]. Here the $A$-polynomial is a two-variable polynomial invariant of a knot defined by using the character variety of $SL(2; \mathbb{C})$-representations of $\pi_1(M)$ [D. Cooper, M. Culler, H. Gillet, D. D. Long, and P. B. Shalen, Invent. Math. 118, No. 1, 47-84 (1994; Zbl 0842.57013)].

A pairing $K_t(D^2 \times S^1) \times K_t(M) \rightarrow \mathbb{C}[t, t^{-1}]$ is defined by gluing the solid torus $D^2 \times S^1$ to the knot complement $M$. Let $S_n(\alpha)$ be the skein obtained as a Chebyshev polynomial of $\alpha = \{0\} \times S^1 \subset D^2 \times S^1$. Then $(S_n(\alpha), \emptyset)$ defines the $n$th colored Kauffman bracket, a version of the colored Jones polynomial.

Using these facts the author proves that for a knot $K$ and a nonzero element $\psi \in I_t(K)$ there exists a number $\nu$ such that the first $\nu$ colored Kauffman brackets of $K$ and $\psi$ determine all the other colored Kauffman brackets. He also gives a technical condition that the $A$-ideal of a knot determines all the Kauffman brackets. As an example a recursive formula for the colored Kauffman brackets of the trefoil knot is given.

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MSC:

57M27 Invariants of knots and 3-manifolds (MSC2010)
46L85 Noncommutative topology
57M25 Knots and links in the 3-sphere (MSC2010)

Keywords:

Kauffman bracket; noncommutative geometry; Kauffman skein module

Full Text: DOI arXiv

References:

[1] C. Blanchet, N. Habegger, G. Masbaum, and P. Vogel, Topological quantum field theories derived from the Kauffman bracket, Topology 34 (1995), no. 4, 883 – 927. - Zbl 0887.57009 · doi:10.1016/0040-9383(94)00051-4
[2] Doug Bullock, Rings of \(\mathbb{C}[t, t^{-1}]\)?-characters and the Kauffman bracket skein module, Comment. Math. Helv. 72 (1997), no. 4, 521 – 542. - Zbl 0907.57010 · doi:10.1007/s000140050032
[3] D. Cooper, M. Culler, H. Gillet, D. D. Long, and P. B. Shalen, Plane curves associated to character varieties of 3-manifolds, Invent. Math. 118 (1994), no. 1, 47 – 84. - Zbl 0842.57013 · doi:10.1007/BF01231526
[4] Charles Frohman and Razvan Gelca, Skein modules and the noncommutative torus, Trans. Amer. Math. Soc. 352 (2000), no. 10, 4877 – 4888. - Zbl 0951.57007
[5] C. Frohman, R. Gelca, W. LoFaro, The $A$-polynomial from the noncommutative viewpoint, preprint. - Zbl 0980.57002
[6] R. Gelca, Noncommutative trigonometry and the $A$-polynomial of the trefoil knot, preprint. - Zbl 1017.57002
[7] Jim Hoste and Józef H. Przytycki, The \(2/\text{finity}\)-skein module of lens spaces; a generalization of the Jones polynomial, J. Knot Theory Ramifications 2 (1993), no. 3, 321 – 333. - Zbl 0796.57005 · doi:10.1142/S0218216593000180
[8] Vaughan F. R. Jones, A polynomial invariant for knots via von Neumann algebras, Bull. Amer. Math. Soc. (N.S.) 12 (1985), no. 1, 103 – 111. - Zbl 0564.57006
[9] Louis H. Kauffman, State models and the Jones polynomial, Topology 26 (1987), no. 3, 395 – 407. - Zbl 0622.57004 · doi:10.1016/0040-9383(87)90009-7
[10] W. B. R. Lickorish, The skein method for three-manifold invariants, J. Knot Theory Ramifications 2 (1993), no. 2, 171 – 194. · Zbl 0793.57003 · doi:10.1142/S0218216593000118

[11] Józef H. Przytycki and Adam S. Sikora, Skein algebra of a group, Knot theory (Warsaw, 1995) Banach Center Publ., vol. 42, Polish Acad. Sci. Inst. Math., Warsaw, 1998, pp. 297 – 306. · Zbl 0902.57005

[12] N. Reshetikhin and V. G. Turaev, Invariants of 3-manifolds via link polynomials and quantum groups, Invent. Math. 103 (1991), no. 3, 547 – 597. · Zbl 0725.57007 · doi:10.1007/BF01239527

[13] V. G. Turaev, Quantum invariants of knots and 3-manifolds, De Gruyter Studies in Mathematics, vol. 18, Walter de Gruyter & Co., Berlin, 1994. · Zbl 0812.57003

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