BREAKING DEGENERACY OF DARK MATTER MODELS BY THE SCALE-SCALE CORRELATIONS OF GALAXIES

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ABSTRACT

Recently, scale-scale correlations have been detected in the distributions of quasars' Lyα absorption lines and the maps of cosmic temperature fluctuations. This paper investigates the scale-scale correlations in galaxy distributions. Using samples of mass field given by N-body simulation, we first show that the scale-scale correlation of two-dimensional and three-dimensional mass distributions at present day are capable of breaking the degeneracy between the SCDM (standard cold dark matter model) and OCDM (open CDM model) or LCDM (flat CDM model) and even show the difference between the OCDM and LCDM. Using biased galaxy samples produced in an appropriate bias model, we show that the scale-scale correlation of galaxy distribution at zero redshift is still a powerful tool to break the degeneracy in the parameter space, including both cosmological and biasing parameters. We analyze the scale-scale correlations of the APM bright galaxy catalog and compare it with the mock catalog in the SCDM, OCDM, and LCDM models. We find that all the spectra of local and nonlocal scale-scale correlations predicted by the LCDM model are in excellent agreement with those of the APM-BGC sample, while the SCDM and LCDM mock catalog appear to have somewhat weaker scale-scale correlations than the observation. 

Subject headings: cosmology: theory — galaxies: statistics — large-scale structure of universe

1. INTRODUCTION

A problem in current study of the large-scale structures of the universe is the so-called degeneracy among viable models. That is, structure formation models with different cosmological parameters are able to reproduce the same features as the observed large-scale structure. For instance, in the cold dark matter (CDM) cosmogony, the standard CDM model (SCDM) predicts about the same abundance and two-point correlation functions of the present \(z = 0\) galaxy clusters as the open CDM model (OCDM) and the low-density flat model (LCDM). This is sometimes called the degeneracy in parameter space of \(\Omega\) and \(\sigma_8\), where \(\Omega\) is the present value of the cosmological density parameter and \(\sigma_8\) the normalization of the power spectrum on an 8 \(h^{-1}\) Mpc scale. Considering the redshift evolution of clusters, the degeneracy between the SCDM and OCDM (or LCDM) is broken (e.g., Jing & Fang 1994; Bahcall, Fan, & Cen 1997). However, the models of OCDM and LCDM remain in degeneracy for the second and lower order statistical features of clusters throughout redshifts as high as \(z \sim 0.5\).

In terms of the clustering of galaxies, model degeneracy is more common. For instance, the mock catalog of the Sloan Digital Sky Survey (SDSS) and 2dF galaxy redshift surveys were generated by a bias sampling of the simulated mass fields. The parameters used for the bias sampling were selected by fitting the two-point correlation function of the bias-sampled galaxies with observations (Cole et al. 1998). Thus, a majority of the dark matter models are found to be able to produce a mock catalog that is basically in agreement with the observed power spectrum and two-point correlation function of galaxies if the biasing parameters are appropriately selected. Namely, to a first approximation, all the models considered are degenerate. This is a degeneracy in the space of both dark matter and biasing parameters.

It is largely believed that the nonlinear behaviors of cosmic clustering would be useful for the degeneracy breaking. In this paper, we investigate the model degeneracy breaking by a nonlinear measure, the local scale-scale correlations. The hierarchical scenario of structure formation is characterized by a rule that determines how the small-scale massive halos are related to their parent halos on the large scale. The relation between parent and daughter halos leads to a local scale-scale correlation. Different models are characterized by different relations between massive halos on different scales. Therefore, the local scale-scale correlation is sensitive to models. This motivated us to study the possibility of breaking the model degeneracy by use of the local scale-scale correlations.

Moreover, the local scale-scale correlations have been detected in one-dimensional samples, such as Lyα forests (Pando et al. 1998), and two-dimensional samples from the COBE Differential Microwave Radiometer (DMR) cosmic temperature map on angular scales of 10°–20° (Pando, Valls-Gabaud, & Fang 1998). Namely, the current data of large-scale structures are ready for detecting scale-scale correlations. In this paper, we show that the scale-scale correlations of the APM-BGC galaxies can provide meaningful information for degeneracy breaking of cosmological models.

The outline of this paper is as follows. Based on the discrete wavelet transform (DWT), §2 gives a brief description of the basic idea and algorithm for the scale-scale correlations. In §3 the scale-scale correlations are computed in simulated samples for three typical CDM models. The scale-scale correlation of underlying dark matter and biased galaxies in real space as well as redshift space are investi-
gated in detail. The comparison of the scale-scale correlations between the APM-BGC galaxies and the model predictions are presented in § 4. Finally, in § 5, we briefly summarize the conclusions drawn from our calculations.

2. BASIC FORMULAE OF SCALE-SCALE CORRELATIONS

In the hierarchical clustering scenario, the evolution of massive halos is prescribed by an evolutionary tree of halos, such as smaller halos merging into a bigger one. This process leads to correlations between density perturbations on different scales, but which are spatially localized. The correlation can be described easily in phase space, i.e., the position-wavevector space. For phase-space analysis, a density field \( \rho(x) \) decomposed into perturbations at phase-space points \((k, x)\), where \( k \) and \( x \) denote, respectively, the wavevector and position of a volume element \( d^3xd^3k \approx 1 \) in phase space. The localized scale-scale correlation characterizes the correlations between the density perturbations at two phase-space points with the same spatial coordinate \( x \) but different scale coordinates \( k_1 \) and \( k_2 \).

One way to realize the phase-space decomposition is by the discrete wavelet transform (DWT) (Fang & Thews 1998; Xu, Fang, & Wu 1998). With the DWT decomposition, a three-dimensional density field \( \rho(x) \), \( x = (x_1, x_2, x_3) \), in the range \( 0 < x_i < L_i \), \( i = 1, 2, 3 \) can be expanded as

\[
\rho(x) = \tilde{\rho} + \sum_{j} \sum_{l=0}^{\infty} \tilde{\psi}_{j,l} \psi_{j,l}(x) \tag{1}
\]

where the normalized summation notation \( \sum_j \equiv \prod_{i=1}^3(1/2^j) \sum_{l=0}^{2^j-1} \) was used. The last step of equation (3) is caused by the fact that the bases of wavelet are admissible, or \( \int \psi(x)dx = 0 \). Thus, the variance or the power spectrum with respect to the DWT decomposition is determined by (Pando & Fang 1998)

\[
P_{DWT}(j) = \frac{1}{2^{l_1+j_2+j_3}} \langle \tilde{\epsilon}_j \tilde{\epsilon}_j \rangle = \sum_{j} \tilde{\epsilon}_{j,l}^2. \tag{4}
\]

where the quantity in square brackets denotes the integer part of the quantity. Because \( L_1/2^j = L_2/2^{j+1} \), the position \( l_j \) at the scale \( j \) is the same as the positions \( 2l_1 \) and \( 2l_1 + 1 \) at the scale \( j + 1 \). Therefore, equation (5) measures the correlation between scales \( j \) and \( j + 1 \) at the same physical point \( x \) and \( x' \), i.e., it is a localized scale-scale correlation. If perturbations on scales \( j \) and \( j + 1 \) are independent, we have \( \langle \tilde{\epsilon}_j \tilde{\epsilon}_{j+1} \rangle \propto \langle \tilde{\epsilon}_j \tilde{\epsilon}_{j+1} \rangle = 0 \). It is interesting to point out that the statistic of equation (5) is of the second order.

The statistical properties of \( \tilde{\epsilon}_{j,l} \) with respect to \( l \) generally are independent from the statistics on \( j \). For instance, one can have distributions for which \( \tilde{\epsilon}_{j,l} \) is Gaussian in its one-point distribution with respect to \( l \) while scale-scale correlated in terms of \( j \) (Greiner, Lipa, & Carruthers 1995). A simple example is as follows. Consider a distribution of \( \tilde{\epsilon}_{j,l} \), which is Gaussian in \( j \) at a given scale \( j \). Namely, higher order correlations of \( \tilde{\epsilon}_{j,l} \) are zero at this scale. Let us consider the perturbation \( \tilde{\epsilon}_{j,l} \) at scale \( j \). If at a given physical point \( x \), \( \tilde{\epsilon}_{j,l} \) is always proportional to \( \tilde{\epsilon}_{j,1} \) at the same point, the distribution of \( \tilde{\epsilon}_{j,l} \) is also Gaussian in \( l \) distribution. However, the perturbations on scales \( j_1 \) and \( j_2 \) are strongly correlated. Obviously such localized scale-scale correlations can be detected only by a scale-space decomposition.

Instead of equation (5), we will employ a normalized scale-scale correlation defined as

\[
C^{p,p}_{j,l} = \frac{\sum_{l=0}^{\infty} \tilde{\psi}_{j,l} \tilde{\psi}_{j,l+1,1} \sum_{l=0}^{\infty} \tilde{\psi}_{j,l+1,1} \sum_{l=0}^{\infty} \tilde{\psi}_{j,l} \tilde{\psi}_{j,l+1,1}}{\sum_{l=0}^{\infty} \tilde{\psi}_{j,l} \tilde{\psi}_{j,l} \sum_{l=0}^{\infty} \tilde{\psi}_{j,l} \tilde{\psi}_{j,l}}. \tag{6}
\]

where \( p \) is an integer. In the case of odd \( p \), the denominator would be zero (eq. [3]); therefore, we will use even \( p \) only. The statistics \( C^{p,p}_{j,l} \) measures the \( p \)-order correlations between local density perturbations with wavevectors \( k/2^j = (2^{l_1}/L_1, 2^{l_2}/L_2, 2^{l_3}/L_3) \) and \( k'/2^j = (2^{l_1+1}/L_1, 2^{l_2+1}/L_2, 2^{l_3+1}/L_3) \). In the case of \( p = 2 \), \( C^{2,2}_{j,l} \) is actually the correlation between the DWT spectra (eq. [4]) on scales \( j \) and \( j + 1 \), i.e., it is a local spectrum-spectrum correlation.

Generally, we can measure the \( p \)-order correlation between perturbations at two phase-space points \((j, l)\) and \((j + 1, l + \Delta l)\) by

\[
C^{p,p}_{j,l+\Delta l} = \frac{\sum_{l=0}^{\infty} \tilde{\psi}_{j,l} \tilde{\psi}_{j,l+1,1+\Delta l} \sum_{l=0}^{\infty} \tilde{\psi}_{j,l} \tilde{\psi}_{j,l+1,1+\Delta l}}{\sum_{l=0}^{\infty} \tilde{\psi}_{j,l} \tilde{\psi}_{j,l} \sum_{l=0}^{\infty} \tilde{\psi}_{j,l} \tilde{\psi}_{j,l}}. \tag{7}
\]

For \( p = 2 \) and \( \Delta l \neq 0 \), \( C^{2,2}_{j,l+\Delta l} \) describes nonlocal spectrum-spectrum correlations.

Obviously, \( C^{p,p}_{j,l+1,1} = 1 \) for Gaussian fields. The inequality \( C^{p,p}_{j,l+1,1} > 1 \) correspond to a positive scale-scale correlation, and \( C^{p,p}_{j,l+1,1} < 1 \) to the negative case. Despite the statistics of equations (6) and (7) being higher order for \( p \geq 2 \), they are essentially the two-point statistics in phase space.

In this paper, we will consider only diagonal components of scale-scale correlation, i.e., \( L_1 = L_2 = L_3, j_1 = j_2 = j_3 = j \) and \( \Delta l_1 = \Delta l_2 = \Delta l_3 = \Delta l \).

3. SCALE-SCALE CORRELATION OF SIMULATION SAMPLES

3.1. Samples of N-body Simulation

To demonstrate the scale-scale correlation as a tool of degeneracy breaking, we first calculate the scale-scale corre-
loration of mass fields given by $N$-body simulation. We used the modified version of AP3M code (Couchman 1991). The cosmological models considered here are the standard cold dark matter model (SCDM), the low-density open model (OCDM), and the low-density flat model (LCDM). The model parameters ($\Omega_0$, $\Omega_\Lambda$, $\Gamma$, $\sigma_8$) are taken to be (1.0, 0.0, 0.5, 0.55), (0.3, 0.0, 0.25, 0.85), and (0.3, 0.7, 0.25, 0.85) for the SCDM, OCDM, and LCDM, respectively.

The simulations were performed by evolving $128^3$ particles in a periodic cubical box of side $256 \, h^{-1}$ Mpc. The soft parameter is taken to be 15% of grid spacing, i.e., $300 \, h^{-1}$ kpc, which remains unchanged in the comoving coordinates, and the force resolution approximates $100 \, h^{-1}$ kpc for a corresponding Plummer force law. The initial perturbations were generated by the Zeldovich approximation applying to a particle distribution with a "glass" configuration (White 1994). The time steps are 400 for the SCDM evolving from $z = 15$ to the present and 600 for the LCDM and OCDM from $z = 20$.

These models are degenerate at low redshift $z \sim 0$, in the sense that they produce almost the same abundance and power spectrum of clusters at $z = 0$. The degeneracies of SCDM-OCDM and SCDM-LCDM may be broken if the abundance of clusters at high redshifts is considered. However, both LCDM and OCDM predict almost the same redshift evolution of cluster abundance until redshift $z \sim 0.5$.

3.2. Scale-Scale Correlation of Three-dimensional Mass Distribution

To probe the scale-scale correlation, we randomly placed 1000 cubic boxes of $64^3 \, h^{-3}$ Mpc$^3$ in the $256^3 \, h^{-3}$ Mpc$^3$ simulation space. These boxes form an ensemble of our statistics. Each $64^3$ box is divided into $128^3$ grids, and the $128^3$ three-dimensional matrix of density distribution is then obtained by the triangular-shaped cloud (TSC) mass assignment scheme. We subject the three-dimensional matrix of the density field to a Daubechies 4 transform and find the WFCs where $j = 1, 2 \ldots 6$, which corresponds to scales $(64/2^j) \, h^{-1}$ Mpc.

The scale-scale correlation defined by equation (6) is calculated for the ensemble of $64^3 \, h^{-3}$ Mpc$^3$ boxes in all three cosmological models. Figure 1 plots the histogram of the distribution of $\log C_{j,2}^{2,2}$ for $j = 2$ to 5 in the OCDM model. It shows that the scale-scale correlation increases with the $j$, i.e., the scale-scale correlation on smaller scales is greater than that on larger scales. The distribution of $C_{j,2}^{2,2}$ at $j = 2$, or $\sim 16 \, h^{-1}$ Mpc, appears to close a lognormal distribution. However, the mean value of $C_{j,2}^{2,2}$ already departs from one. At $j = 3$, i.e., at the spatial scale $\sim 8 \, h^{-1}$ Mpc, the confidence level of $C_{j,2}^{2,2} > 1$ is greater than 95%. Down to the minimum structure resolved here, the distribution of $C_{j,2}^{2,2}$ is peaked as high as about 30, which indicates that the non-linear evolution of the density field on small scales leads to a significant positive scale-scale correlation.

We now examine the model dependence of the scale-scale correlation. Since the distributions of $\log C_{j,2}^{2,2}$ for $j \geq 3$ are highly non-Gaussian, the mean value and variance of $\log C_{j,2}^{2,2}$ is not enough to describe their statistical behavior. We introduce the cumulative distribution of $\log C_{j,2}^{2,2}$, defined by

$$ F(\log C_{j,2}^{2,2}) = \int_{\log C_{j,2}^{2,2}}^{\infty} N(\log C_{j,2}^{2,2}) d \log C_{j,2}^{2,2} \int_{\log C_{j,2}^{2,2}}^{\infty} N(\log C_{j,2}^{2,2}) d \log C_{j,2}^{2,2} . $$

Figure 2 compares the cumulative distribution $F(\log C_{j,2}^{2,2})$ of simulation samples in the SCDM, OCDM, and LCDM models at $z = 0$. This figure shows clearly that the curve $F(\log C_{j,2}^{2,2})$ of the SCDM is significantly different from those of the LCDM and OCDM. That is, low-redshift ($z \sim 0$) data are already capable of breaking the degeneracy.
between SCDM and LCDM, as well as OCDM, if the scale-scale correlation of mass distribution can be detected. Moreover, Figure 2 also shows that the curves $P(< \log C_{5}^{2,2})$ for the LCDM and OCDM are distinguishable from each other at low redshift, despite the difference between LCDM and OCDM being relatively small. These results, at least, demonstrate the possibility of LCDM-OCDM degeneracy breaking by the DWT scale-scale correlations.

The power of scale-scale correlation increases from the SCDM to the LCDM and then to the OCDM. This is consistent with the sequence of nonlinear evolution. The SCDM clustering is less nonlinear than that of the LCDM and OCDM, and the OCDM model shows somewhat more compact structures than the LCDM (Jing et al. 1995).

3.3. Scale-Scale Correlation of Two-dimensional Mass Distributions

To measure the scale-scale correlation in two-dimensional distribution, we make an ensemble of samples by randomly placing 1000 $128 \times 128 \times 16$ $h^{-3}$ Mpc$^3$ slabs in the simulation space. A mass distribution in a two-dimensional space of $128 \times 128$ ($h^{-1}$ Mpc)$^3$ can be obtained by summing up all masses along the dimension of 16 $h^{-1}$ Mpc. The density distribution is tabulated by the TSC mass assignment scheme on 1024$^2$ grids in two dimensions. Accordingly, the $j$ runs 1, 2, ..., 9, which corresponds to scales (128/2$^j$) $h^{-1}$ Mpc.

Figure 3 presents the histogram of the log $C_{5}^{2,2}$ distribution for the two-dimensional samples in the OCDM model. Obviously, the figure shows behavior similar to a three-dimensional scale-scale correlation. The power of the two-dimensional scale-scale correlation $C_{5}^{2,2}$ is almost the same as that for the three-dimensional samples but with slightly smaller values than in the three-dimensional case. Generally, the ordinary two-point correlation strength in two dimensions is significantly smaller than in three dimensions because of the projection effect. However, the $C_{j}^{2,2}$ measures a ratio between localized correlations, thus the effect of projection is weaker. Therefore, detecting scale-scale correlations in two-dimensional samples would be as effective as in three-dimensional cases.

The model dependence of $C_{5}^{2,2}$ in the two-dimensional samples is shown in Figure 4, which compares the cumulative distribution $F(< \log C_{5}^{2,2})$ between the SCDM, OCDM, and LCDM models at $z = 0$. Similar to the three-dimensional case, the curve of $F(< \log C_{5}^{2,2})$ for the SCDM is substantially different from that of the LCDM and OCDM models. The curves for the LCDM and the OCDM are also distinguishable. This result does not depend very sensitively on the thickness of the slab. Using the slab of $128 \times 128 \times 32$ $h^{-3}$ Mpc$^3$, we found very similar results. Thus, to break the degeneracy between the SCDM, LCDM, and OCDM models, the two-dimensional data at low redshift ($z \sim 0$) are as powerful as that of the three-dimensional samples.

3.4. Scale-Scale Correlation of Biased Galaxy Distributions

Now we turn to the scale-scale correlation of simulation samples of galaxies. To produce galaxy samples comparable with the mock samples of the SDSS and 2dF galaxy redshift surveys in Cole et al. (1998), we employ the so-called two-parameter Lagrangian bias model to identify the galaxies from N-body simulation mass distributions. Namely, the density field is first smoothed by a Gaussian window function of width $3$ $h^{-1}$ Mpc. The biased galaxies distribution is then drawn from the smoothed density field $\rho(r)$ by a selection probability defined as

$$P(v) \propto \begin{cases} \exp \left(\alpha v + \beta v^{3/2}\right) & \text{if } v \geq 0, \\ \exp(\alpha v) & \text{if } v \leq 0, \end{cases}$$

where the dimensionless variable $v$ is defined by $v(r) = \delta(r)/\sigma; \delta(r) = (\rho(r) - \bar{\rho})/\bar{\rho}$ is the density contrast of the density field $\rho(r)$, and $\sigma^2 = \langle \delta_j^2 \rangle$ is its variance. The two parameters $\alpha$ and $\beta$ are determined by the least-squares fitting of the observed variance of galaxies count in a cubic cell of $5$ $h^{-1}$ Mpc and $20$ $h^{-1}$ Mpc. The adopted values $\sigma_5 = 2.0$ and $\sigma_{20} = 0.67$ were obtained for APM galaxy samples by Baugh & Efstathiou (1994). The two-parameter bias models are more flexible in matching both the amplitude and the slope of the galaxy two-point correlation function on scales of $1$–$10$ $h^{-1}$ Mpc, though their underlying matter distribution may be quite different. That is, all models used to produce the biased galaxy samples are degenerate in terms of the low-order statistics, such as the two-point correlation or power spectrum of galaxies.

Proceeding in the same way as in § 3.3, we randomly placed 1000 cubic boxes of $64^3$ $h^{-3}$ Mpc$^3$ in the simulation space. The biased galaxy samples from these $64^3$ boxes form an ensemble for our statistics of the scale-scale correlations. The distributions of $C_{5}^{2,2}$ for the OCDM model are presented in Figure 5. The $C_{5}^{2,2}$ shows the behavior of the scale dependence as similar to the mass distribution, i.e., the scale-scale correlation increases with $j$. However, the values of $C_{5}^{2,2}$ are less than their parent mass distributions.

We also produced biased galaxy samples in redshift space. For simplicity, the redshift distortion is applied along one axis of the three-dimensional Cartesian coordinates, which is in fact equivalent to the assumption of an infinitely distant observer. The 1000 randomly placed $64^3$ $h^{-3}$ Mpc$^3$...
boxes form an ensemble for calculating scale-scale correlations. The results are shown in Figure 5. The scale-scale correlations in redshift space are generally less than those in real space, i.e., redshift distortion suppresses the scale-scale correlation. The difference of the scale-scale correlation between the redshift and real spaces disappears gradually with increasing scales.

This result is as we expected. The local scale-scale correlations are sensitive to the accuracy of distance or position measurement. A radial distance uncertainty in redshift space is caused by the peculiar velocity. Therefore, the scale-scale correlation will be significantly contaminated on scales comparable with the uncertainty of radial distance, while the correlation on a large scale will not be affected by the redshift distortion.

To reduce the effect of redshift distortion, one can measure the scale-scale correlation in two-dimensional samples, as a two-dimensional sample is not affected by redshift distortion. As shown in § 3.3, the power of the scale-scale correlation of the two-dimensional mass distribution is about the same as the three-dimensional sample in real space. Therefore, two-dimensional samples probably are more suitable than three-dimensional samples to detect scale-scale correlations on scales where the redshift distortion is significant.

3.5. Nonlinearity of Bias

An advantage of the statistics $C_{j}^{2,2}$ is that they are free of linear-bias parameters. The linear-bias parameter $b$ is defined by

$$\delta(x)_g = b \delta(x)_m ,$$

where $\delta(x)_g$ and $\delta(x)_m$ are the density contrasts of galaxies and density dark matter, respectively. Since the DWT is linear, one has

$$\tilde{\delta}_{j,1g} = b \tilde{\delta}_{j,1m} .$$

Therefore, we have

$$C_{j}^{2,2} = C_{j}^{2,2} .$$
If the linear bias is introduced by the variance of perturbations, i.e.,

$$\delta_{j,1}^2 = b\delta_{j,1m}^2,$$  \hspace{1cm} (13)

we still have equation (12).

 Accordingly, one can detect the nonlinearity of a bias model by comparing the $C_{j,2}^{2,2}$ of the biased galaxy samples with the underlying unbiased matter distribution. As shown in Figures 1 and 5, the values of $C_{j,2}^{2,2}$ of the biased galaxies are significantly less than their parent mass distributions. Therefore, the bias model equation (9) is highly nonlinear.

Figure 6 gives the spectra of $C_{j,2}^{2,2}$ for both biased and unbiased galaxy samples in the OCDM model. Obviously, the unbiased sample has a much stronger scale-scale correlation than biased galaxies created by equation (9).

4. SCALE-SCALE CORRELATIONS OF APM-BGC GALAXIES

4.1. Samples of APM-BGC Galaxies

Now we attempt to measure the scale-scale correlation in the observational galaxy samples and then compare the results with the model predictions. We analyzed the two-dimensional samples of galaxies listed in the APM bright galaxies catalog (Loveday et al. 1992), which gives positions, magnitudes, and morphological types of 14,681 galaxies brighter than $16^{m}44$ over a $4180^\circ$ area in 180 Schmidt survey fields of south sky. Completeness is about 96.3% with a standard deviation of 1.9% inferred from 12 carefully checked fields. Therefore, it is large enough and sufficiently uniform for a two-dimensional DWT analysis.

To carry out a two-dimensional DWT analysis, we chose two fields, S1 and S2, from the entire survey area as shown in Figure 1 of Fang, Deng, & Xia (1998). S1 and S2 do not overlap each other. These two fields are selected to be squares on an equal-area projection of the sky. The equal-area projection keeps the surface number density of galaxies on the plane the same as that on the sky. The sizes of S1 and S2 have been taken to be as large as possible so as to cover the whole area of the survey. Each field has an angular size of about $37^\circ \times 37^\circ$. S1 and S2 contain 1095 and 1055 ellip-
tical and lenticular (EL) galaxies and 2186 and 2092 spiral (SP) galaxies, respectively. The galaxies in the regions S1 and S2 are completely independent.

We divide each square region into $2^{10} \times 2^{10} (1024^2)$ cells, the angular scale labeled by $j$ is $2^{10-j}$ in units of the size of a cell, and $j$ runs from 1 to 10. Using the estimation of the mean depth of the sample given by the luminosity function from the Stromlo-APM redshift survey (Loveday et al. 1992), the cell size is found to be about $65 h^{-1}$ kpc, which is fine enough to detect statistical features on scales larger than $1 h^{-1}$ Mpc.

In doing statistical analysis of the APM-BGC, the effect of the 1456 holes drilled around big bright objects should be taken into account. For instance, random samples are generated in the area with the same drilled holes as found in the original survey. This may lead to a underestimation of the scale-scale correlations. We also removed a few galaxies placed in the holes drilled.

4.2. Sales-Scale Correlations of EL and SP Galaxies

Figure 7 plots the $C_{j}^{2,2}$ of the EL and SP galaxies in fields I and II. The confidence level is estimated from 1000 randomized samples with the same number of galaxies as real data. Both randomized and bootstrap resampling give about the same confidence level (Fang, Deng, & Xia 1998). Figure 6 shows that the randomized distributions are also substantially scale-scale correlated for $j = 5, 6$, i.e., $C_{5}^{2,2}, C_{6}^{2,2} > 1$. This is because the random sample on small scales ($j = 5, 6$) essentially is a Poisson process that is non-Gaussian and thus leads to scale-scale correlation. On larger scales ($j \leq 4$), the mean number of galaxies in a $j$ cell is high enough, and their one-point distribution of random samples is Gaussian.

Despite the shot noise, Figure 7 shows that the distributions of the EL and SP galaxies of the APM bright galaxies catalog are highly scale-scale correlated at $j = 5, 6$. The scale-scale correlations are also significant (greater than 95% confidence level) on scales of $j = 3$ and 4, which corresponds to the scale $\sim 8 h^{-1}$ Mpc. Basically, the EL and SP galaxies have about the same behavior in terms of the $j$-dependence of $C_{j}^{2,2}$. It increases with the increase of $j$. This is qualitatively consistent with our simulation results (§ 3). In addition, ELs generally have a higher power of than SPs. This is consistent with the segregation between ELs and SPs. ELs concentrate in clusters, having higher clustering, while the field galaxy SPs are lower.

4.3. Mock Samples of Galaxies

For a quantitative confrontation between observed and model-predicted scale-scale correlations, we use the mock samples for the SDSS redshift survey (Cole et al. 1998). The radial selection effect is prescribed by the Schechter form of the luminosity function with the parameters drawn from the Stromlo-APM bright galaxy survey (Loveday et al. 1992). Therefore, it is appropriate to use these mock samples for demonstrating the model degeneracy breaking by the APM-BGC galaxy scale-scale correlations.

We consider the three popular cosmological models of the SCDM, LCDM, and OCDM here (referred to as E4, L3S, and O3S, respectively, in Cole et al. 1998). To mimic the APM-BGC sample, we extract 10 subsamples out of the SDSS mock catalog by imposing a magnitude limit of $B_{J} \leq 16.44$ for each model of the SCDM, LCDM, and OCDM. The size of the 10 fields is the same as the size of fields I and II. The 10 fields were chosen to cover the projected survey area as much as possible but with less overlap from each other. This selection ensures the statistical independence to a maximum extent.

The surface number density of galaxies for all the mock samples is found to be approximately the same as that obtained for APM-BGC data: the difference between the number densities of the real data and mock samples is less than 3%. The two-point angular correlation functions of the mock samples in the three cosmological models are also found to be in good agreement with that of APM-BGC. That is, in terms of the first- (number density) and the second-order (two-point correlation function) statistics, the mock samples of the SCDM, LCDM, and OCDM are degenerate.
4.4. The APM-BGC Scale-Scale Correlation and Cosmological Models

Since the mock samples do not contain the information on morphology, we calculated the scale-scale correlations of APM-BGC galaxies consisting of both ELs and SPs in fields I and II. The results are shown in Figure 8. The 95% confidence level is estimated from 1000 Poisson-distributed samples with the same number of galaxies as the real data. The scale-scale correlations of the 10 mock samples in the SCDM, LCDM, and OCDM models are displayed by the scatter symbols in Figure 8, respectively.

Figure 8 shows first that fields I and II give almost the same spectrum of $C^2_2$ versus $j$. The data are qualified for model discrimination. At first glance, all mock samples of the three CDM cosmogony models show the same shape of the $C^2_2$ spectrum and are generally consistent with the observed $C^2_2$. However, SCDM, LCDM, and OCDM predicted different $C^2_2$ at $j = 5$. The OCDM is in good agreement with the entire spectrum of APM-BGC's $C^2_2$ from $j = 2$ to 5, i.e., scales $\sim 2-16 \ h^{-1} \ Mpc$. However, the values of $C^2_2$ of SCDM and LCDM are somewhat lower than the observation at 90% confidence level. Considering the possible underestimation of the APM-BGC’s scale-scale correlation because of the holes drilled around big bright objects, the difference between observed results and the SCDM and LCDM prediction might be more significant than 90%. Thus, the test of scale-scale correlation seems to be in favor of the OCDM model.

It might be too early to say this conclusion is concrete, as the data is still not perfect enough. However, this result showed that the scale-scale correlation is effective and fea-
ible as a tool of degeneracy breaking. The correlations reveal not only the difference between the SCDM and OCDM, but also between the LCDM and OCDM.

All $C^{j^2}_{l^2}$ shown in Figures 1–8 are calculated by taking $\Delta l = 0$ (see eq. [7]). It describes the correlations between $j$ and $j + 1$ perturbations at the same physical coordinates, i.e., it is a local scale-scale correlation. We also calculated nonlocal scale-scale correlations by taking $\Delta l = 1$, 2, and 3. It describes the correlation between perturbations on scale $j$ at position $l$ and on scale $j + 1$ at $l \pm \Delta l$. Figure 9 compares the nonlocal correlations $C^{j^2}_{l^2, \Delta l = 1, 2, 3}$ of APM-BGC data and the OCDM mock sample. It shows that the OCDM result is in excellent agreement with observed data.

5. DISCUSSIONS AND CONCLUSIONS

The ordinary two-point correlation function is a two-point statistic in physical space. The scale-scale correlation essentially is two-point statistics both in physical space and wavenumber space, i.e., in a position-scale phase space. Moreover, the scale-scale correlation describes the feature of the nonlinear evolutions of the hierarchical clustering. Therefore, the scale-scale correlation is probably the most basic measure of the cosmic clustering in the phase space.

The abundance and two-point correlation functions of galaxies and clusters are unlikely to discriminate among the CDM family of models, which are degenerate in cosmological parameter space. In this paper, using the mass distribution generated by $N$-body simulation, we showed that the scale-scale correlation of mass distributions is sensitive to the cosmological parameters. The scale-scale correlation of biased galaxy distribution is sensitive to cosmological and biasing parameters. Therefore, scale-scale correlation of galaxy distributions provides us with a tool for discriminating among various cosmological models, which are degenerate in the parameter space, including both cosmological and bias parameters.

Using APM-BGC data, we found that the $j$ spectrum of local and nonlocal scale-scale correlations can generally be matched by the mock galaxy samples of the CDM cosmogonies. Among the models, the best fitting is given by the OCDM model. Both the SCDM model and LCDM model show somewhat weaker scale-scale correlations than the OCDM on small scales. We can conclude that the OCDM plus the biasing algorithms (eq. [9]) are favored by the current detection of APM-BGC’s scale-scale correlations. This showed that the scale-scale correlation is a capable tool for degeneracy breaking.

We found that the scale-scale correlation in two-dimensional samples is comparable with that in three-dimensional samples. Because the redshift distortion contaminates the scale-scale correlation in three-dimensional samples, but the two-dimensional samples are free of redshift distortion, the scale-scale correlation measured in a three-dimensional redshift galaxy survey will be lower than the two-dimensional scale-scale correlation. Therefore, the difference between the two-dimensional scale and the three-dimensional scale may provide us with a new way to study the large-scale velocity field.

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