Failure Criterion and Parameter Calibration Methods of Aluminum Brass Alloy based on J-C Model

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Abstract. As the impact resistance of metal varies from different material property, therefore, the failure criterion has been adopted as theoretical or empirical formula to describe the capacity of ultimate load and deformation for materials. The paper has improved Johnson-Cook failure criterion equation and identified parameters of failure criterion through combination of experimental data, analogue simulation and numerical fitting, so as to lay the foundation for simulated analysis.

1. Introduction
In terms of the fracture or failure of ductile material, the failure criterion has always been established through uncoupling approach to describe the degradation law or damage fracture failure of material property. John and Cook modified the failure criterion\cite{1} presented by relevant scholars in 1985 and put forward Johnson-Cook failure criterion (hereinafter referred J-C failure criterion) and expression\cite{2} is:

\[ \varepsilon_f = [D_1 + D_2 \exp(D_3 \sigma^*)] \left( 1 + D_4 \ln \dot{\varepsilon}_{eq} \right) \left( 1 + D_5 T^* \right) \]  

(1)

In the equation, \( \varepsilon_f \) ——The equivalent plastic strain while fracturing(failure strain); \( D_1\sim D_5 \)——Reflect the influencing parameters for stress triaxiality to material; \( D_4 \)——Sensitive parameters of strain rate; \( D_5 \)——Influencing parameters of temperature; \( \sigma^* \)——Stress triaxiality

2. Impact of Stress Triaxiality
2.1. Cylindrical Compression Test
Cylindrical compression test was conducted with Instron 5569 electronic universal material testing machine and cylindrical compression specimens with a diameter of 10mm and with respective height-diameter ratio 1.0, 1.50, 1.75 and 2.0 have been prepared.

Table 1 refers to measurements of cylindrical specimens with different height-diameter ratio before and after compression as well as the fracture strain values through calculation formulas\cite{3} and then the value of stress triaxiality of the specimens -0.333 and the value of fracture strain 0.407 were acquired through averaging.
Table 1. Cylindrical compression test data

| Specimen No. | Height-diameter ratio | Initial diameter (mm) | Initial height (mm) | Post-test height (mm) | Fracture strain $\varepsilon_f$ |
|--------------|-----------------------|-----------------------|---------------------|-----------------------|-------------------------------|
| Y1           | 2.02                  | 9.96                  | 20.12               | 13.34                 | 0.411                         |
| Y2           | 1.74                  | 10.02                 | 17.48               | 11.67                 | 0.404                         |
| Y3           | 1.51                  | 9.98                  | 15.02               | 10.09                 | 0.398                         |
| Y4           | 1.01                  | 9.96                  | 10.04               | 6.62                  | 0.416                         |

2.2. Notched Tensile Test
With regard to the cylindrical notched specimens with unidirectional tension, according to the calculation formula provided by Bridgman[4], the stress triaxiality for cylindrical specimens with different notched radius were evaluated:

$$\sigma^* = \frac{1}{3} \ln \left[ 1 + \frac{a}{(2R)} \right]$$  \hspace{1cm} (2)

In the equation, $\alpha$ refers to the radius of the gap on the specimen, $R$ refers to the radius of the notch. According to the dimensions of notched tensile specimen, unidirectional tension cylindrical specimens with $R$ as 3mm and 9mm were prepared.

As per the calculation of equation (2), the stress triaxiality for notched cylindrical specimens respectively was 0.739 ($R=3$mm) and 0.488 ($R=9$mm) and 0.333 for smooth cylindrical specimen. The unidirectional tension notched tensile test was conducted with Instron 5569 electronic universal material testing machine at room temperature of 30°C and a length extension with 30mm gauge length was tracked by video-extensometer.

![Figure 1. Fracture of notched tensile specimen](image)

The load-displacement curve of different notched specimens was obtained through unidirectional tension, the fracture morphology is as shown in Figure 1. From the micro morphology, the fracture appears to be dark grey fibrous and it has tensile isometric dimples. Micro-crack was formed at the interface between strengthening phase and matrix or at adjacent crystal boundary and end up with fracture. The fracture mode of aluminum brass HAL66-6-3-2 alloy is classified as ductile fracture.

The calculation formula of fracture strain when conducting notched tensile test is:

$$\varepsilon_f = \ln \left( \frac{D_0}{D_f} \right)$$  \hspace{1cm} (3)

In the equation, $D_0$ refers to the initial diameter of notch, $D_f$ refers to the diameter when the specimen fractures. Values of fracture strain acquired by measuring and calculating different notch radius have been provided in Table 2.
Table 2. Equivalent plastic strain of notched tensile specimen

| Notch type | Stress triaxiality | Initial diameter (mm) | Fracture diameter (mm) | Fracture strain εf | PEEQ (fracture time) |
|------------|--------------------|-----------------------|------------------------|-------------------|----------------------|
| R=3mm      | 0.739              | 6.14                  | 5.59                   | 0.094             | 0.096                |
| R=9mm      | 0.488              | 5.99                  | 5.08                   | 0.165             | 0.172                |
| R=∞mm      | 0.333              | 6.07                  | 4.60                   | 0.278             | 0.265                |

2.3. Cylindrical Torsion Test

The stress triaxiality of torsion cylindrical specimen was 0. As the dimension in Figure 2, smooth torsion cylindrical specimens were prepared and torsion test was conducted with NDW-31000 model micro-controlled electronic torsion machine, the torque-angle curve is shown as Figure 3.

![Figure 2. Torsion specimen size](image1)

![Figure 3. Torque-angle curve comparison](image2)

As can be seen from Figure 3, the torsion specimen fractured when the load angle reached to 27.52 rad (which is 1576.78°). As it is hard to obtain the equivalent plastic strain at the moment of fracture through torsion test, therefore, a finite element model of smooth torsion cylindrical specimen was established as usual by ABAQUS. The equivalent plastic strain of the specimen was constantly monitored in the analogue simulation process. Figure 4 is the nephogram of the equivalent plastic strain of torsion specimen near the fracture time. The final fracture strain value 0.342 was obtained by extracting the maximum equivalent plastic strain of the torsion specimen at the moment of fracture.

![Figure 4. Equivalent plastic strain of torsion specimen near the fracture time](image3)

2.4. Impact of Stress Triaxiality

Under reference strain rate and reference temperature, J-C failure criterion can be simplified as:

\[
\epsilon_f = D_1 + D_2 \exp(D_3 \sigma')
\]  

(4)

As can be seen from equation (4), it is exponential relationship between fracture strain and stress triaxiality. The values of fracture strain corresponding to different stress triaxiality that obtained from above research are shown in Figure 5 in scatter. As can be seen from the figure, as the increase of the stress triaxiality, the fracture strain of aluminum brass is in a non-linear downward trend.
Fitting the stress triaxiality and fracture strain with first-order exponential decay fitting method, then, the fitting formula is:

$$y = y_0 + A_1 e^{-x/D_1}$$

(5)

According to formula (5), $D_1=0.565$, $D_2=-0.221$, $D_3=1.05$ was obtained by fitting solving and fitting residual is $8.82714E^{-4}$.

3. Impact of Temperature Change

Fracture morphology of tensile specimen at high temperature at various temperature was obtained after high temperature tension test, as shown in Figure 6. As can be seen from the figure, the fracture of specimen occurred at the necking position and with significant dimples and a morphology of cone-shape, which shows that the failure mode of high-temperature specimen is also classified as ductile fracture. Based on the results of unidirectional room temperature tensile test and high-temperature tensile test, as per the fracture strain formula (3), fracture strain of specimens at various temperature can be obtained, among which, the one at room temperature was obtained by analogue simulation. Based on the solving procedure of failure parameter $D_5$, the values of fracture strain of specimens at various dimensionless temperature can be obtained through further conversion and shown in Figure 7 in scatter. Through the observation of data in scatter in the figure, as the increase of dimensionless temperature, the fracture strain of specimens is in a non-linear downward trend.

![Fracture strain curve](image)

**Figure 5.** Variation curve of fracture strain versus stress triaxiality

In J-C failure criterion, the temperature softening term is $(1+D_5T^*)$, which demonstrates the linear relation between fracture strain and dimensionless temperature. The straight line as shown in Figure 7 that obtained through linear fitting by making use of fracture strain data is obviously non-conform to the scatter data as shown in figure.
Therefore, the temperature parameter obtained by fitting of fracture strain and dimensionless temperature with polynomial fitting method is $D_5=-2.0902$, $D_6=-4.4367$. As can be seen from Figure 7, the fitting curve better reflects the variation curve of fracture strain with the change of dimensionless temperature, which is basically consistent with test data. As a result, the paper modified the original J-C failure criterion and finally acquired the improved J-C failure criterion by making use of the change of fracture strain with temperature represented by quadratic polynomial and with the premise of same damage failure formula[5] Among which, the expression of fracture strain is modified to:

$$\varepsilon_f = \left[ D_1 + D_2 \exp(D_3 \sigma^*) \right] \left[ (1 + D_4 \ln \dot{\varepsilon}_f^*) (1 + D_5 T^* + D_6 (T^*)^2) \right]$$  \hspace{1cm} (6)

### 4. Impact of Strain Rate

All parameters of J-C failure criterion, except for sensitive parameter of strain rate $D_4$, have been obtained through test data and fitting solving. Therefore, the fracture strain of dynamic tensile specimens shall be obtained by SHTB test.

![Fracture of dynamic tensile specimens](image)

(a) fracturing moment (1000 s$^{-1}$)  \hspace{0.5cm} (b) fracturing moment (2500 s$^{-1}$)  \hspace{0.5cm} (c) specimens of fracture

**Figure 8.** Fracture of dynamic tensile specimens

The values of fracture strain of specimens at different dimensionless strain rate were obtained through calculation per equation (3) by measuring dimension of specimens before and after the dynamic test, which is shown in Table 3.

| Number | 1    | 2    | 3    | 4    | 5    |
|--------|------|------|------|------|------|
| Strain rate (s$^{-1}$) | $10^3$ | $1.0 \times 10^7$ | $2.5 \times 10^3$ | $4.0 \times 10^3$ | $5.5 \times 10^3$ |
| Dimensionless strain rate ($\dot{\varepsilon}_f^*$) | $1 \times 10^3$ | $1.0 \times 10^6$ | $2.5 \times 10^6$ | $4.0 \times 10^6$ | $5.5 \times 10^6$ |
| Fracture strain      | 0.0803 | 0.1031 | 0.1109 | 0.1162 | 0.1244 |

**Table3.** Numerical data of fitting parameter $D_4$
temperature (T=30°C) and with same stress state of dynamic tensile specimens, namely, when the stress triaxiality is 0.333, J-C failure criterion can be simplified as:

$$\varepsilon_f = [D_1 + D_2 \exp(D_3 \sigma^*)](1 + D_4 \ln \dot{\varepsilon}_{eq}^*)$$  \hspace{1cm} (7)

In the equation, $D_1$, $D_2$, $D_3$ and $\sigma^*$ are the known parameters, therefore, the equation (7) can be further converted to:

$$\frac{\varepsilon_f}{D_1 + D_2 \exp(D_3 \sigma^*)} - 1 = D_4 \ln \dot{\varepsilon}_{eq}^*$$  \hspace{1cm} (8)

Obviously, equation (8) indicates that the linear relation between dimensionless strain rate and fracture strain of specimens. Please refer to Figure 9 for the linear fitting curve and $D_4$ is 0.0183 through fitting solving.

![Figure 9. Variation curve of fracture strain versus strain rate](image)

5. Conclusion
The room temperature tensile, compression, torsion, notched tensile, high-temperature and SHTB dynamic mechanics tests have been conducted for aluminium brass HAL66-6-3-2 alloy material. Based on the model of original material, the improved J-C failure criterion has been put forward. J-C failure criterion parameters have been identified based on test data, numerical simulation and fitting method and ultimately obtained the expression of the improved J-C failure criterion:

$$\varepsilon_f = [0.565 - 0.221 \exp(1.05 \sigma^*)](1 + 0.018 \ln \dot{\varepsilon}_{eq}^*)\left[1 - 2.09 T^* - 4.437(T^*)^2\right]$$  \hspace{1cm} (9)

6. References
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