KAPITSA RESISTANCE IN DEGENERATE QUANTUM GASES WITH BOGOLYUBOV ENERGY EXCITATIONS IN THE PRESENCE OF BOSE – EINSTEIN CONDENSATE

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The linearized kinetic equation modelling behaviour of the degenerate quantum bose - gas with the frequency of collisions depending on momentum of elementary excitations is constructed. The general case of dependence of the elementary excitations energy on momentum according to Bogolyubov formula is considered. The analytical solution of the half–space boundary problem on temperature jump on border of degenerate bose - gas in the presence of a Bose — Einstein condensate is received. Expression for Kapitsa resistance is received.

Keywords: degenerate quantum Bose gas, collision integral, Bose — Einstein condensate, Bogolyubov excitations, temperature jump, Kapitsa resistance.

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1. INTRODUCTION

Nonequilibrium properties of the quantum gases in limited space give rise to interest last years as well as the equilibrium properties. In particular, important significance has such phenomenon, as temperature jump [1] on the border gas – condensed (in particular - solid) body in the presence of a thermal flux normal to a surface.

* N. N. Bogolyubov founded in 1945 department of theoretical physics in Moscow State Regional University
Such temperature jump is frequently called the Kapitsa temperature jump [2].

The problem of temperature jump is one of the major in the kinetic theory [3]. The analytical solution of this problem for a case of the rarefied one-atomic gas is received in [4].

The problem of temperature jump of electronic gas in metal was considered in our works [5, 6]. In these works the analytical solution of this problem of the temperature jump, caused by heat flux to a surface is received.

The behavior of quantum gases has aroused heightened interest in recent years. In particular, this is related to the development of experimental procedures for producing and studying quantum gases at extremely low temperatures [7]. The bulk properties of quantum gases have been studied in the majority of papers [8] and [9].

At the same time, it is obviously important to take boundary effects on the properties of such systems into account. We mention a paper where the thermodynamic equilibrium properties of quantum gases in a half-space were considered [10].

We note that up to now, the Kapitsa jump has been calculated in the regime where only phonon scattering at the boundary between two media was taken into account and phonon scattering in the bulk was neglected [11].

In the work [12] the problem of temperature jump in the quantum Fermi–gas was considered. The analytical solution under arbitrary degeneration degree of the gas has been received. In the work [13] the similar problem was considered for Bose–gas. But gas was assumed to be nondegenerate,
i.e. in the absence of Bose — Einstein condensate [14].

The present work is devoted to the problem of temperature jump in degenerate Bose–gas analysis. Presence of Bose — Einstein condensate leads to essential improvement of the problem statement, and its method of solution as well.

Thus for the description of kinetic processes near to a surface the kinetic equation with model collision integral will be used. We will assume, that boundary conditions on the surface have specular–diffusive character.

In the present work we consider the kinetic equation, in which Bogolyubov general dependence on momentum for elementary excitations energy of bose - gas in volume is taken into account.

Character of scattering of the elementary excitations on the surface is considered thus by introduction of phenomenological factor of reflectivity of scattering on the surface. Thus, the given approach is additional to [11].

In our work [15] the presence fonon component in Bogolyubov formula was neglected, and in work [16], on the contrary, was considered, that phonon component prevails in the elementary excitations of bose - gas. In the given work the general case of dependence of energy of elementary excitations on an momentum is considered.

2. DERIVATION OF THE KINETIC EQUATION

To describe the gas behavior, we use a kinetic equation with a model collision integral analogous to that used to describe a classical gas. We take the quantum character of the Bose gas and the presence of the Bose — Einstein condensate into account.

For a rarefied Bose gas, the evolution of the gas particle distribution
function $f$ can be described by the kinetic equation

$$\frac{\partial f}{\partial t} + \frac{\partial \mathcal{E}}{\partial \mathbf{p}} \nabla f = I[f],$$

(1)

where $\mathcal{E}$ is the kinetic energy of gas particles, $\mathbf{p}$ is the gas particle momentum, and $I[f]$ is the collision integral.

In the case of the kinetic description of a degenerate Bose gas, we must take into account that the properties of the Bose—Einstein condensate can change as functions of the space and time coordinates, i.e., we must consider a two–liquid model (more precisely, a ”two-fluid” model because we consider a gas rather than a liquid). We let $\rho_c = \rho_c(\mathbf{r}, t)$ and $\mathbf{u}_c = \mathbf{u}_c(\mathbf{r}, t)$, thus we denote the density and velocity of the Bose—Einstein condensate.

We then can write the expressions

$$\mathbf{j} = \rho_c \mathbf{u}_c, \quad Q = \frac{\rho_c u_c^2}{2} \mathbf{u}_c,$$

$$\Pi_{ik} = \rho_c u_{ci} u_{ck},$$

for the densities $\mathbf{j}$, $Q$, and $\Pi_{ik}$ of the mass, energy, and momentum fluxes of the Bose—Einstein condensate (under the assumption that the chemical potential is zero) respectively.

The conservation laws for the number of particles, energy, and momentum require that the relations

$$\nabla \mathbf{j} = - \int I[f] d\Omega_B,$$

$$\nabla Q = - \int \mathcal{E}(\mathbf{p}) I[f] d\Omega_B,$$

$$\nabla \Pi = - \int \mathbf{p} I[f] d\Omega_B,$$
are satisfied in the stationary case. Here,
\[ d\Omega_B = \frac{(2s + 1)d^3p}{(2\pi\hbar)^3}, \]
where \( s \) is the particle spin, \( \mathcal{E}(\mathbf{p}) \) is the energy, \( \hbar \) is the Planck constant and \( I[f] \) is the collision integral in Eq. (1).

In what follows, we are interested in the case of motion with small velocities (compared with thermal velocities). We note that for the Bose—Einstein condensate, the quantities \( Q \) and \( \Pi_{ik} \) depend on the velocity nonlinearly (they are proportional to the third and second powers of the velocity). Therefore, in the linear approximation by the velocity \( u_c \), the energy and momentum conservation laws can be written as
\[
\int \mathcal{E}(\mathbf{p}) I[f] d\Omega_B = 0, \\
\int \mathbf{p} I[f] d\Omega_B = 0.
\]

According to the Bogolyubov theory, the following relation for the excitation energy \( \mathcal{E}(p) \) is true for a weakly interacting Bose gas \([14]\)
\[
\mathcal{E}(p) = \left[ u_0^2 p^2 + \left( \frac{p^2}{2m} \right)^2 \right]^{1/2},
\]
where
\[
u_0 = \left( \frac{4\pi\hbar^2 an}{m^2} \right)^{1/2},
\]
a is the scattering length for gas particles, \( n \) is the concentration, \( m \) is the mass, and \( \mathbf{p} \) is the momentum of elementary excitations, \( u_0 \) is the sound velocity. The parameter \( a \) characterizes the interaction force of gas molecules and can be assumed to be small for a weakly interacting gas.

In our previous paper \([15]\), we considered the case where the relation
\[
u_0^2 \ll \frac{kT}{m}
\]
is satisfied for sufficiently small \( a \), where \( k \) is the Boltzmann constant and \( T \) is the temperature. In that case, the first term in the brackets in (2) can be neglected. The expression for the energy \( \mathcal{E}(p) \) takes the same form as in the case of noninteracting molecules:

\[
\mathcal{E}(p) = \frac{p^2}{2m}.
\]

In this case

\[
\frac{d\mathcal{E}(p)}{dp} = v.
\]

In our work [16], we considered the case, where in expression (2) phonons component is prevails, i.e., when \( T \ll \frac{mu_0^2}{k} \). In this case according to (2) is received, that

\[
\mathcal{E}(p) = u_0|p|.
\]

Hence,

\[
\frac{d\mathcal{E}(p)}{dp} = u_0 \frac{p}{|p|}.
\]

Now we will consider the general case, when neither the first, nor the second component in elementary excitations can be neglected. In this case

\[
\frac{d\mathcal{E}(p)}{dp} = \alpha(p)p,
\]

where

\[
\alpha(p) = \frac{u_0^2 + p^2/(2m^2)}{\mathcal{E}(p)}.
\]

When considering the kinetic equation (1) under gas particles it is necessary to understand elementary excitations of the bose - gas with the spectrum energy (2). Character of the elementary excitations is shown
in properties of integral of collisions. We take in the equation (1) \(\tau\)–approximation as integral of its collisions. Then the character of elementary excitations will be shown in dependence of frequency of collisions on the momentum of excitations [14], [1]

\[
\frac{\partial f}{\partial t} + \alpha(p)\frac{\partial f}{\partial r} = \nu(p - p_0)(f_B^* - f). \tag{3}
\]

Here, \(f\) is the distribution function,

\[
\nu(p - p_0) = \nu_0 |p - p_0|^\gamma
\]
is the dependence of the collision frequency on the excitation momentum, and \(\gamma\) is a constant. In the case where the phonon component dominates in the elementary excitations, \(\gamma \geq 3\) [14]. We have \(p_0 = m\nu_0\), where \(\nu_0\) is the velocity of the normal component of the Bose gas, \(f_B^*\) is the equilibrium function of the Bose — Einstein distribution

\[
f_B^* = \left[ \exp \left( \frac{E(p - p_*)}{kT_s} \right) - 1 \right]^{-1},
\]

and \(\nu_0\) is a model parameter \(\nu_0 = \nu_1/(mkT_s)^{1/2}\), having the meaning of the inverse mean free path \(l\), \(\nu_0 \sim 1/l\), \(T_s\) is the temperature of gas in certain point at the surface, \(k\) is the Boltzmann constant,

\[
E(p - p_*) = \left[ u_0^2(p - p_*)^2 + \left( \frac{(p - p_*)^2}{2m} \right)^2 \right]^{1/2}.
\]

The parameters in \(f_B^*\), namely, \(T_s = T_s(r, t)\) and \(p_* = p_*(r, t)\), can be determined from the requirement that the energy and momentum conservation laws

\[
\int \nu(p - p_0)p \left[ f - f_B^* \right] d^3p = 0, \tag{4a}
\]

\[
\int \nu(p - p_0)E(p) \left[ f - f_B^* \right] d^3p = 0 \tag{4b}
\]
are applicable.

These parameters we will call an effective temperature and an momentum respectively.

The conservation law for the number of particles is inapplicable here because of the transition of a fraction of particles to the Bose — Einstein condensate.

We now assume that the gas velocity is much less than the mean thermal velocity and the typical temperature variations along the mean free path \( l \) are small compared with the gas temperature. Under these assumptions, the problem can be linearized.

Let’s start the linearization of equation (3).

Let’s begin with linearization of the effective temperature:

\[
T_* = T_s + \delta T_* = T_s \left( 1 + \frac{\delta T_*}{T_s} \right).
\]

The function of Bose — Einstein distribution is function of the momentum \( p \) and parameters \( p_* \) and \( \delta T_*/T_s \). Its linearization we will implement by two last parameters:

\[
f^*_B(p, p_*, \frac{\delta T_*}{T_s}) = f^*_B(p, 0, 0) + \frac{\partial f^*_B}{\partial p_*}(p, 0, 0)p_* + \frac{\partial f^*_B}{\partial \left( \frac{\delta T_*}{T_s} \right)}(p, 0, 0)\delta T_*/T_s.
\]

As a result of such linearization we obtain:

\[
f^*_B(p, p_*, \frac{\delta T_*}{T_s}) = f_B(p) + g(p)\alpha(p)\frac{p p_*}{kT_s} + g(p)\frac{\mathcal{E}(p)}{kT_s} \frac{\delta T_*}{T_s}.
\]

Here

\[
f_B(p) \equiv f^*_B(p, 0, 0) = \frac{1}{\exp \left( \frac{\mathcal{E}(p)}{kT_s} \right) - 1},
\]
\[ g(p) = \frac{\exp \left( \frac{\mathcal{E}(p)}{kT_s} \right)}{ \left[ \exp \left( \frac{\mathcal{E}(p)}{kT_s} \right) - 1 \right]^2}. \]

The linearization of the distribution function according to (5) we will carry out as follows:

\[ f(r, p, t) = f_B(p) + g(p)h(r, p, t). \quad (6) \]

From (5) and (6) we find, that

\[ f_B - f = g(p) \left[ \alpha(p) \frac{p \mathcal{P}^*}{kT_s} + \frac{\mathcal{E}(p) \delta T_s}{kT_s} \right]. \]

Let’s return to the equation (3). We will realize linearization of this equation according to (5).

We will notice, that in linear approximation the quantity \( \nu(p - p_0) \) in the equation (3) it is possible to replace by \( \nu(p) = \nu_0 p^\gamma \).

Then the equation (3) as a result of the linearization has the following form

\[ \frac{\partial h}{\partial t} + \alpha(p)p \frac{\partial h}{\partial r} = \nu_0 p^\gamma \left[ \alpha(p) \frac{p \mathcal{P}^*}{kT_s} + \frac{\mathcal{E}(p) \delta T_s}{kT_s} \right]. \quad (7) \]

In the equation (7) we will introduce a dimensionless momentum (velocity)

\[ C = \frac{v}{v_T} = \frac{p_T}{p_T}, \]

where \( v_T = \sqrt{kT_s/m} \) is the thermal velocity of gas particles, \( p_T \) is their momentum.

Then

\[ \mathcal{E}(p) = kT_s \mathcal{E}(C), \]
where $\mathcal{E}(C) = \sqrt{w_0^2 C^2 + C^4/4}$, and $w_0 = u_0/v_T$ is the dimensionless sound velocity.

Besides that, we will notice, that $\alpha(p) = \alpha(C)/m$, where

$$\alpha(C) = \frac{w_0^2 + C^2/2}{\sqrt{w_0^2 C^2 + (C^2/2)^2}}.$$  

We will introduce dimensionless time and coordinate $\tau = \nu_0 t$ and $r_1 = \nu_0 \sqrt{\frac{m}{kT_s}} r$.

Now it is clear, that the equation (3) (in dimensionless variables) has the following form:

$$\frac{\partial h}{\partial \tau} + \alpha(C) C \frac{\partial h}{\partial r_1} =$$

$$= C^\gamma \left[ \alpha(C) C C_*(r_1, \tau) + \mathcal{E}(C) \frac{\delta T_s^*}{T_s}(r_1, \tau) - h(r_1, C, \tau) \right]. \quad (8)$$

In laws of preservation (4) we will also implement such linearization. As a result we get the laws of preservation of the momentum and energy in the following form:

$$\int p^\gamma p \left[ \alpha(p) \frac{pp^*}{kT_s} + \frac{\mathcal{E}(p) \delta T_s^*}{kT_s T_s^*} - h(r_1, p, \tau) \right] g(p) d^3 p = 0,$$

$$\int p^\gamma \mathcal{E}(p) \left[ \alpha(p) \frac{pp^*}{kT_s} + \frac{\mathcal{E}(p) \delta T_s^*}{kT_s T_s^*} - h(r_1, p, \tau) \right] g(p) d^3 p = 0,$$

In these equalities we will pass to integration on a dimensionless momentum. We obtain:

$$\int C^\gamma C \left[ \alpha(C) C C_*(r_1, \tau) + \mathcal{E}(C) \frac{\delta T_s^*}{T_s} - h(r_1, C, \tau) \right] g(C) d^3 C = 0,$$

$$\int C^\gamma \mathcal{E}(C) \left[ \alpha(C) C C_*(r_1, \tau) + \mathcal{E}(C) \frac{\delta T_s^*}{T_s} - h(r_1, C, \tau) \right] g(C) d^3 C = 0,$$
where
\[ g(C) = \frac{e^{\mathcal{E}(C)}}{(e^{\mathcal{E}(C)} - 1)^2}. \]

From these laws of conservation of energy and momentum it is found:
\[ C^*(r_1, \tau) = \frac{\int C^\gamma C h(r_1, C, \tau) g(C) \, d^3C}{\int C^\gamma C_x^2 \alpha(C) g(C) \, d^3C}, \]

and
\[ \frac{\delta T^*_s}{T_s}(r_1, \tau) = \frac{\int C^\gamma \mathcal{E}(C) h(r_1, C, \tau) g(C) \, d^3C}{\int C^\gamma \mathcal{E}^2(C) g(C) \, d^3C}. \]

Let’s calculate the integrals standing in denominators of the two last equalities. We have:
\[ \int C^\gamma C_x^2 \alpha(C) g(C) \, d^3C = \frac{4\pi}{3} g_1. \]

and
\[ \int C^\gamma \mathcal{E}^2(C) g(C) \, d^3C = 4\pi g_2, \]

Here
\[ g_1 = \int_0^\infty C^{\gamma+2} \alpha(C) g(C) \, dC, \quad g_2 = \int_0^\infty C^{\gamma+2} \mathcal{E}^2(C) g(C) \, dC. \]

Thus, equation parameters are equal definitively
\[ C^*(r_1, \tau) = \frac{3}{4\pi g_1} \int C^\gamma C h(r_1, C, \tau) g(C) \, d^3C, \quad \text{(9)} \]
\[ \frac{\delta T^*_s}{T_s}(r_1, \tau) = \frac{1}{4\pi g_2} \int C^\gamma \mathcal{E}(C) h(r_1, C, \tau) g(C) \, d^3C. \quad \text{(10)} \]
By means of equalities (9) and (10) we will present the equation (8) in the standard form for the transport theory:

\[
\frac{\partial h}{\partial \tau} + \alpha(C) \frac{\partial h}{\partial r_1} + C' \gamma h(r_1, C, \tau) = \]

\[
= \frac{C'}{4\pi} \int K(C, C') h(r_1, C', \tau) C' \gamma g(C') d^3C'.
\]

(11)

Here \( K(C, C') \) is the kernel of equation,

\[
K(C, C') = \frac{3\alpha(C)CC'}{g_1} + \frac{E(C)E(C')}{g_2}.
\]

3. PROBLEM STATEMENT

In the problem under consideration, a degenerate Bose gas occupies the half-space \( x > 0 \) above a planar surface where the heat exchange between the condensed phase and the gas occurs. Therefore, the function \( h(\tau, r_1, C) \) can be regarded as

\[
h(\tau, r_1, C) = h(x, \mu, C).
\]

in what follows.

Here \( \mu \) is cosine of a corner between a direction of a vector of velocity \( C \) and an axis \( x \) in spherical system of velocities of Bose – particles, \( C_x = \mu C \). The equation (11) for function \( h \) will be written in a form:

\[
\frac{\mu \alpha(C) }{C \gamma - 1} \frac{\partial h}{\partial x} + h(x, \mu, C) = \frac{3\alpha(C)C\mu}{2g_1} W_1(x) + \frac{E(C)}{2g_2} W_2(x).
\]

(12)

In this equation

\[
W_1(x) = \int_{-1}^{1} \int_{0}^{\infty} C'^{\gamma+3} \mu' h(x, \mu', C') g(C') d\mu' dC'.
\]
and

\[ W_2(x) = \int_{-1}^{1} \int_{0}^{\infty} \mathcal{E}(C') h(x, \mu', C') C'^{r+2} g(C') d\mu' dC'. \]

The problem consists in finding of quantity of the relative temperature jump \( \varepsilon_T = \Delta T/T_s \), \( \Delta T = T_s - T \), as function \( Q_x \) which is the quantity of a heat flow projection to an axis \( x \). Considering the linear character of the problem, it is possible to write down:

\[ \varepsilon_T = R Q_x. \]

The dimensionless factor \( R \) of the temperature jump is called Kapitsa resistance.

It is obvious that equation (12) has the particular solutions:

\[ h_1(x, \mu, C) = \alpha(C) C \mu \]

and

\[ h_2(x, \mu, C) = \mathcal{E}(C), \]

and the Chapman — Enskog distribution function is

\[ h_{as}(x, \mu, C) = B^+ \alpha(C) C \mu - \varepsilon_T \mathcal{E}(C), \]

where the quantity \( B^+ \) is proportional to the heat flux \( Q_x \).

Assuming that the reflection of the elementary excitations from the wall is specular–diffuse, we now formulate the boundary conditions

\[ h(0, \mu, C) = q h(0, -\mu, C), \quad 0 < \mu < 1, \quad (13) \]

and

\[ h(x, \mu, C) = B^+ \alpha(C) C \mu - \varepsilon_T \mathcal{E}(C) + o(1), \quad x \to +\infty, -1 < \mu < 0, \quad (14) \]
where $q$ is the specular reflection coefficient.

The problem is to solve Eq. (12) with boundary conditions (13) and (14). Finding the value of the temperature jump $\varepsilon_T$ is of special interest.

4. REDUCTION TO THE INTEGRAL EQUATION

We continue the function $h(x, \mu, C)$ to the half–space $x < 0$ symmetrically:

$$h(x, \mu, C) = h(-x, -\mu, C), \quad x < 0.$$  

For $x < 0$, we then have the following Chapman — Enskog distribution

$$h_{as}(x, \mu, C) = B^{-} \mu - \varepsilon_T C,$$

with $B^{+} = -B^{-}$.

Now we will extract the Chapman — Enskog distribution of the function $h(x, \mu, C)$, considering at $\pm x > 0$:

$$h(x, \mu, C) = B^{\pm} \alpha(C) C \mu - \varepsilon_T \mathcal{E}(C) + h_{c}(x, \mu, C).$$

For the function $h_{c}(x, \mu, C)$, we formulate the boundary conditions for the lower and upper half–spaces:

$$h_{c}(+0, \mu, C) =$$

$$= -(1 + q)B^{+} \alpha(C) C \mu + (1 - q)\varepsilon_T \mathcal{E}(C) + q h_{c}(+0, -\mu, C),$$

where $0 < \mu < 1$, and

$$h_{c}(-0, \mu, C) =$$

$$= -(1 + q)B^{-} \alpha(C) C \mu + (1 - q)\varepsilon_T \mathcal{E}(C) + q h_{c}(-0, -\mu, C),$$
where $-1 < \mu < 0$, and

$$h_c(+\infty, \mu, C) = 0, \quad h_c(-\infty, \mu, C) = 0.$$ 

We include these boundary conditions in the kinetic equation. We obtain the equation

$$\mu \frac{\partial h_c}{\partial x} + \frac{C^{\gamma-1}}{\alpha(C)} h_c(x, \mu, C) =$$

$$= \frac{C^{\gamma-1}}{\alpha(C)} \left\{ \frac{3\alpha(C)C\mu}{2g_1} W_1(x) + \frac{\mathcal{E}(C)}{2g_2} W_2(x) +$$

$$+ |\mu| \left[ - (1 + q) B^+ \alpha(C) C |\mu| + (1 - q) \varepsilon_T \mathcal{E}(C) -$$

$$-(1 - q) h_c(\mp 0, \mu, C) \right] \delta(x) \right\}.$$ (15)

Here, $\delta(x)$ is the Dirac delta function.

Equation (15) actually combines two equations. The point is that the term $h_c(-0, \mu, C)$ corresponds to positive $\mu : 0 < \mu < 1$ and the term $h_c(+0, \mu, C)$ corresponds to negative $\mu : -1 < \mu < 0$.

We seek the solution of equations (15) in the form of Fourier integrals:

$$h_c(x, \mu, C) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} \Phi(k, \mu, C) dk, \quad \delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} dk.$$

$$W_1(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} E_1(k) dk,$$

$$W_2(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} E_2(k) dk.$$
Let’s begin with search of unknown boundary values \( h_c(\pm 0, \mu, C) \). We will express these values in terms of spectral densities \( E_1(k) \) and \( E_2(k) \). We will consider for this purpose the equation (15) for \( x < 0 \) and \( x > 0 \). In both cases component with \( \delta \)-function drops out of the equation (15).

Solving Eq. (15) with \( x > 0 \) and \( \mu < 0 \), assuming that the right–hand side of this equation is known, and assuming that the boundary conditions are satisfied far from the wall, we obtain

\[
h_c^+(x, \mu, C) = -\frac{1}{\mu} \exp \left( -\frac{x C^{\gamma-1}}{\mu \alpha(C)} \right) \int_x^{+\infty} \exp \left( \frac{t C^{\gamma-1}}{\mu \alpha(C)} \right) W(t, \mu, C) dt, \tag{16}
\]

where

\[
W(t, \mu, C) = \frac{C^{\gamma-1}}{\alpha(C)} \left[ \frac{3\alpha(C)C\mu}{2g_1} W_1(t) + \frac{E(C)}{2g_2} W_2(t) \right].
\]

For \( x < 0 \), \( \mu > 0 \) we similarly receive:

\[
h_c^-(x, \mu, C) = \frac{1}{\mu} \exp \left( -\frac{x C^{\gamma-1}}{\mu \alpha(C)} \right) \int_{-\infty}^{x} \exp \left( \frac{t C^{\gamma-1}}{\mu \alpha(C)} \right) W(t, \mu, C) dt.
\]

Let’s underline, that in the equation (15) boundary values of the required functions \( h_c(\pm 0, \mu, C) \) are boundary values of the represented above functions \( h_c^\pm(x, \mu, C) \) at \( x \to \pm 0 \) from corresponding semi–planes.

From two last equalities for integrals Fourier follows, that

\[
W(t, \mu, C) = \frac{C^{\gamma-1}}{\alpha(C)} \left[ \frac{3\alpha(C)C\mu}{2g_1} \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikt} E_1(k) dk + \frac{E(C)}{2g_2} \cdot \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikt} E_2(k) dk \right].
\]
After simple calculations in (16), we obtain

\[
h_c^+(x, \mu, C) = \frac{C^{2(\gamma-1)}}{2\pi} \int_{-\infty}^{+\infty} e^{ikx} \left[ \frac{3\alpha(C)C\mu}{2g_1} E_1(k) + \right.
\]

\[+ \frac{\mathcal{E}(C)}{2g_2} E_2(k) \left. \right] \frac{dk}{C^{2(\gamma-1)} + k^2\mu^2\alpha^2(C)}.
\]

It is similarly possible to show, that for function \(h_c^-(x, \mu, C)\) precisely the same expression is received. Hence, considering evenness by \(k\) of the functions \(E_1(k)\) and \(E_2(k)\), we receive, that

\[
h_c^\pm(0, \mu, C) = \frac{C^{\gamma}}{\pi} \int_{0}^{+\infty} \left[ \frac{3\alpha(C)C\mu}{2g_1} E_1(k) + \right.
\]

\[+ \frac{\mathcal{E}(C)}{2g_2} E_2(k) \left. \right] \frac{dk}{C^{2(\gamma-1)} + k^2\mu^2\alpha^2(C)}.
\]  

(17)

So, boundary values of required function \(h_c(\pm 0, \mu, C)\) from the equation (15) are boundary values of \(h_c^\pm(0, \mu, C)\), defined by equality (17).

By means of the relation (17) it is visible, that two equations (15) are possible to be united in one:

\[
\frac{\mu \alpha(C)}{C^{\gamma-1}} \frac{\partial h_c}{\partial x} + h_c(x, \mu, C) = \frac{3\alpha(C)C\mu}{2g_1} W_1(x) + \frac{\mathcal{E}(C)}{2g_2} W_2(x) +
\]

\[+ |\mu| \left\{ - (1 + q)B^\pm |\mu| \alpha(C) + (1 - q)\varepsilon_T \mathcal{E}(C) \right\} \delta(x) -
\]

\[- (1 - q)|\mu| \delta(x) \frac{C^{\gamma}}{\pi} \int_{0}^{\infty} \left[ \frac{3\alpha(C)C\mu}{2g_1} E_1(k) + \right.
\]

\[+ \frac{\mathcal{E}(C)}{2g_2} E_2(k) \left. \right] \frac{dk}{C^{2(\gamma-1)} + k^2\mu^2\alpha^2(C)}.
\]  

(18)
5. CHARACTERISTIC SYSTEM OF THE EQUATIONS

We pass to the Fourier integrals in equation (18) and obtain the equation

$$\left[C^\gamma + ik\mu\alpha(C)C\right]\Phi(k, \mu, C) =$$

$$= \frac{3\alpha(C)C^{\gamma+1}\mu}{2g_1}E_1(k) + \frac{\mathcal{E}(C)C^{\gamma}}{2g_2}E_2(k) -$$

$$(1 + q)B^+\mu^2C^\gamma + (1 - q)\varepsilon_TC^{\gamma+1}\mu| - (1 - q)|\mu|\frac{C^{3\gamma}}{\pi} \times$$

$$\times \int_0^\infty \left[\frac{3\alpha(C)C\mu}{2g_1}E_1(k) + \frac{\mathcal{E}(C)}{2g_2}E_2(k)\right] \frac{dk}{C^{2\gamma} + k^2\mu^2\alpha^2(C)C^2}. \quad (19)$$

It is simple to find expressions for $E_1(k)$ and $E_2(k)$:

$$E_1(k) = \int_0^1 \int_0^\infty C^{\gamma+3}\mu\Phi(k, \mu, C)g(C)d\mu dC$$

and

$$E_2(k) = \int_0^1 \int_0^\infty \mathcal{E}(C)C^{\gamma+2}\Phi(k, \mu, C)g(C) d\mu dC.$$

We solve equation (19) for $\Phi(k, \mu, C)$ and substitute it in the last two expressions.

We will introduce following designations:

$$T_{r,s}^{m,n}(k) = \int_0^1 \int_0^\infty \frac{\alpha^r(C)\mathcal{E}^s(C)C^m\mu^n}{C^{2\gamma} + k^2\mu^2\alpha^2(C)C^2} d\mu dC, \quad r, s = 0, 1, 2, \cdots,$$

and

$$J_{r,s}^{m,n}(k, k_1) = \int_0^1 \int_0^\infty \frac{\alpha^r(C)\mathcal{E}^s(C)C^m\mu^n g(C) d\mu dC}{(C^{2\gamma} + k^2\mu^2\alpha^2(C)C^2)(C^{2\gamma} + k_1^2\mu^2\alpha^2(C)C^2)}.$$
where \( r, s = 0, 1, 2 \cdots \).

Between these integrals the following relation is obvious:

\[
J_{m,n}^{r,s}(k,0) = T_{m-2\gamma,n}^{r,s}(k), \quad J_{m,n}^{r,s}(0,k_1) = T_{m-2\gamma,n}^{r,s}(k_1).
\]

By means of the entered designations we receive characteristic system, consisting of two equations:

\[
\left[ 1 - \frac{3}{g_1} T_{3\gamma+4,2}^{1,0}(k) \right] E_1(k) + \frac{ik}{g_2} T_{2\gamma+4}^{1,1}(k) E_2(k) =
\]

\[
= 2(1 + q) B^+ T_{2\gamma+4,4}^{1,0}(k) - 2(1 - q) \varepsilon T_{2\gamma+5,3}^{1,0}(k) - 
\]

\[
-3 \frac{1 - q}{g_1 \pi} \int_0^\infty J_{5\gamma+4,3}^{1,0}(k, k_1) E_1(k_1) dk_1 +
\]

\[
+ ik \frac{1 - q}{g_2 \pi} \int_0^\infty J_{4\gamma+4,3}^{1,1}(k, k_1) E_2(k_1) dk_1,
\]

and

\[
\frac{3ik}{g_2} T_{2\gamma+4,2}^{2,1}(k) E_1(k) + \left[ 1 - \frac{1}{g_2} T_{3\gamma+2,0}^{0,2}(k) \right] E_2(k) =
\]

\[
= -2(1 + q) B^+ T_{3\gamma+2,2}^{0,1}(k) + 2(1 - q) \varepsilon T_{3\gamma+3,1}^{0,1}(k) +
\]

\[
+ 3ik \frac{1 - q}{g_1 \pi} \int_0^\infty J_{4\gamma+4,3}^{2,1}(k, k_1) E_1(k_1) dk_1 -
\]

\[
- \frac{1 - q}{g_2 \pi} \int_0^\infty J_{5\gamma+2,1}^{2,0}(k, k_1) E_2(k_1) dk_1.
\]

We introduce the dispersion matrix function

\[
\Lambda(k) = \begin{bmatrix}
1 - \frac{3}{g_1} T_{3\gamma+4,2}^{1,0}(k) & \frac{ik}{g_2} T_{2\gamma+4,2}^{1,1}(k) \\
\frac{3ik}{g_1} T_{2\gamma+4,2}^{2,1}(k) & 1 - \frac{1}{g_2} T_{3\gamma+2,0}^{0,2}(k)
\end{bmatrix}.
\]
From spectral densities $E_1(k)$ and $E_2(k)$ we form the vector column

$$E(k) = \begin{bmatrix} E_1(k) \\ E_2(k) \end{bmatrix},$$

and we introduce two vector columns of arbitrary terms

$$T_1(k) = \begin{bmatrix} ik T_{2\gamma+4,4}^{1,0}(k) \\ -T_{3\gamma+2,2}^{0,1}(k) \end{bmatrix}, \quad T_2(k) = \begin{bmatrix} ik T_{2\gamma+5,3}^{1,0}(k) \\ -T_{3\gamma+3,1}^{0,1}(k) \end{bmatrix}.$$  

We will combine the received system of the scalar equations in one vector equation

$$\Lambda(k) E(k) = 2(1 + q) B^+ T_1(k) - \frac{2(1 - q) \varepsilon T_2(k)}{\pi} + \int_0^\infty J(k, k_1) E(k_1) dk_1. \quad (20)$$

Here $J(k, k_1)$ is the matrix kernel of integral equation (20),

$$J(k, k_1) = \begin{bmatrix} -\frac{3}{g_1} J_{5\gamma+4,3}^{1,0}(k, k_1) & \frac{ik}{g_2} J_{4\gamma+4,3}^{1,1}(k, k_1) \\ \frac{3ik}{g_1} J_{4\gamma+4,3}^{2,1}(k, k_1) & -\frac{1}{g_2} J_{5\gamma+2,1}^{0,2}(k, k_1) \end{bmatrix}.$$  

We will consider obvious equalities

$$1 - \frac{3}{g_1} T_{3\gamma+4,2}^{1,0}(k) = \frac{3k^2}{g_1} T_{\gamma+6,4}^{3,0}(k),$$

$$1 - \frac{1}{g_2} T_{3\gamma+2,0}^{0,2}(k) = \frac{k^2}{g_2} T_{\gamma+4,2}^{2,2}(k),$$

and we will present the dispersion matrix function in the following form:

$$\Lambda(k) = \begin{bmatrix} \frac{k^2}{g_2} T_{\gamma+4,2}^{2,2}(k) & \frac{3ik}{g_1} T_{2\gamma+4,2}^{2,1}(k) \\ \frac{ik}{g_2} T_{2\gamma+4,2}^{1,1}(k) & \frac{3k^2}{g_1} T_{\gamma+6,4}^{3,0}(k) \end{bmatrix}.$$
6. METHOD OF SUCCESSIVE APPROXIMATIONS

We seek the solution of Eq. (20) in the form

$$\varepsilon_T = \frac{1 + q}{1 - q} \left[ \varepsilon_0 + \varepsilon_1(1 - q) + \varepsilon_2(1 - q)^2 + \cdots \right], \quad (21)$$

$$E(k) = 2(1 + q) \left[ E^{(0)}(k) + E^{(1)}(k)(1 - q) + E^{(2)}(k)(1 - q)^2 + \cdots \right]. \quad (22)$$

We substitute these equalities in the characteristic equation. We obtain the countable system of equations

$$\Lambda(k) E^{(0)}(k) = -B^+ T_1(k) + \varepsilon_0 T_2(k), \quad (23)$$

$$\Lambda(k) E^{(1)}(k) = \varepsilon_1 T_2(k) - \frac{1}{\pi} \int_0^\infty J(k, k_1) E_0(k_1) dk_1, \quad (24)$$

$$\Lambda(k) E^{(2)}(k) = \varepsilon_2 T_2(k) - \frac{1}{\pi} \int_0^\infty J(k, k_1) E_1(k_1) dk_1, \cdots \quad (25)$$

We call the determinant of the dispersion matrix the dispersion function:

$$\lambda(z) \equiv \det \Lambda(z) = k^2 \omega(k),$$

where

$$\omega(k) = \frac{3}{g_1 g_2} \left[ k^2 T_{\gamma+4,6}^{3,0}(k) T_{\gamma+4,2}^{2,2}(k) + T_{2\gamma+4,2}^{1,1}(k) T_{2\gamma+4,2}^{2,1}(k) \right].$$

The inverse matrix to the dispersion matrix is

$$\Lambda^{-1}(k) = \frac{D(k)}{\lambda(k)},$$

where

$$D(k) = \begin{bmatrix} \frac{k^2}{g_2} T_{\gamma+4,2}^{2,2}(k) & -\frac{i k}{g_2} T_{2\gamma+4,2}^{1,1}(k) \\ -\frac{3 i k}{g_1} T_{2\gamma+4,2}^{2,1}(k) & \frac{3 k^2}{g_1} T_{\gamma+4,6}^{3,0}(k) \end{bmatrix}.$$
We consider the construction of series (20). We designate

\[ E^{(m)}(k) = \begin{bmatrix} E_1^{(m)}(k) \\ E_2^{(m)}(k) \end{bmatrix}, \quad m = 0, 1, 2, \cdots. \]

From the equation (23) we obtain

\[ E_1^{(0)}(k) = \frac{ik}{\omega(k)g_2}T_{2\gamma+4,2}^{2,2}(k) \left[ B^+T_{2\gamma+4,4}^{1,0}(k) + \varepsilon_0 T_{2\gamma+5,3}^{1,0}(k) \right] - \]

\[ + \frac{i}{k\omega(k)g_2}T_{2\gamma+4,2}^{1,1}(k) \left[ B^+T_{3\gamma+2,2}^{0,1}(k) - \varepsilon_0 T_{3\gamma+3,1}^{0,1}(k) \right], \quad (26) \]

and

\[ E_2^{(0)}(k) = \frac{3}{\omega(k)g_1}T_{2\gamma+4,2}^{2,1}(k) \left[ B^+T_{2\gamma+4,4}^{1,0} - \varepsilon_0 T_{2\gamma+5,3}^{1,0}(k) \right] + \]

\[ + \frac{3}{\omega(k)g_1}T_{\gamma+4,6}^{3,0} \left[ - B^+T_{3\gamma+2,2}^{0,1}(k) + \varepsilon_0 T_{3\gamma+3,1}^{0,1}(k) \right]. \quad (27) \]

The quantity \( E_2^{(0)}(k) \) exists for all values of \( k \) (has no singularities). But quantity \( E_1^{(0)}(k) \) has a simple pole at point \( k = 0 \). Eliminating the singularity at zero, we take

\[ \varepsilon_0 = \frac{B^+T_{3\gamma+2,2}^{0,1}(0)}{T_{3\gamma+3,1}^{0,1}(0)}. \]

Here

\[ T_{3\gamma+2,2}^{0,1}(0) = \int_0^1 \int_0^\infty \mathcal{E}(C)C^{\gamma+2} \mu^2 d\mu dC = \frac{1}{3} \int_0^\infty \mathcal{E}(C)C^{\gamma+2} dC \]

and

\[ T_{3\gamma+3,1}^{0,1}(0) = \int_0^1 \int_0^\infty \mathcal{E}(C)C^{\gamma+3} \mu^2 d\mu dC = \frac{1}{2} \int_0^\infty \mathcal{E}(C)C^{\gamma+3} dC. \]
We designate now
\[ g_{\varepsilon,2}(\gamma) = \int_{0}^{\infty} \mathcal{E}(C) C^{\gamma+2} g(C) dC, \]
\[ g_{\varepsilon,3}(\gamma) = \int_{0}^{\infty} \mathcal{E}(C) C^{\gamma+3} g(C) dC, \]
\[ \mathcal{E}(C) = \sqrt{w_{0}^{2} c^{2} + (C^{2}/2)^{2}}, \quad g(C) = \frac{e^{\mathcal{E}(C)}}{(e^{\mathcal{E}(C)} - 1)^{2}}. \]

Now we have
\[ \varepsilon_{0} = B^{+} \frac{2g_{\varepsilon,2}(\gamma)}{3g_{\varepsilon,3}(\gamma)}. \]

The received expression for \( \varepsilon_{0} \) we will substitute in (25) and (26). Let’s substitute expressions (25) and (26) in (23). From the received equation we find \( E^{(1)}(k) \). Then we will substitute \( E^{(1)}(k) \) in the equation (24). From the received equation we find \( E^{(2)}(k) \). Continuing this process beyond all bounds, let’s construct all members of the series (20) and (21).

7. TEMPERATURE JUMP AND KAPITSA RESISTANCE

We will find the quantity \( \varepsilon_{0} \) in an explicit form. The quantity \( B^{+} \) is proportional to the heat flux:
\[ Q = \int f(x, p) \frac{\partial \mathcal{E}(p)}{\partial p} \mathcal{E}(p) d\Omega_B. \]

We transform this expression as
\[ Q = \frac{(2s + 1)}{(2\pi\hbar)^3} \int h(x, p) g(p) \alpha(p) p \mathcal{E}(p) d^3p. \]

Let’s pass in this expression to integration by the dimensionless to momentum. For this purpose we will notice, that
\[ \alpha(p) p \mathcal{E}(p) d^3p = \frac{kT_s}{m} (kT_s m)^2 \alpha(C) C \mathcal{E}(C). \]
We receive that

\[ Q = \frac{(2s + 1)(kT_s)^3m}{(2\pi \hbar)^3} \int h(x, C)\alpha(C)C\mathcal{E}(C)g(C) \, d^3C. \]

We replace here the function \( h(x, C) \) with its Chapman — Enskog expansion \( h_{as}(x, C) \). As a result, we have for the \( x \)-component of the thermal flux

\[ Q_x = \frac{(2s + 1)(kT_s)^3m}{(2\pi \hbar)^3} \int_{-1}^{1} \int_{0}^{2\pi} \int_{0}^{\infty} \left[ B^+ \alpha(C)C\mu - \varepsilon_T\mathcal{E}(C) \right] \times \]

\[ \times \alpha(C)C\mu\mathcal{E}(C)C^2g(C)d\mu dC d\chi = \frac{(2s + 1)(kT_s)^3m}{(2\pi \hbar)^3} \cdot \frac{4\pi}{3} g_{\alpha\varepsilon}(\gamma) B^+. \]

Here

\[ g_{\alpha\varepsilon}(\gamma) = \int_{0}^{\infty} \alpha^2(C)\mathcal{E}(C)C^4g(C) \, dC. \]

We hence have

\[ B^+ = Q_x \frac{6\pi^2\hbar^3}{(2s + 1)m(kT_s)^3g_{\alpha\varepsilon}(\gamma)}. \]

Thus, the quantity \( \varepsilon_0 \) is equal:

\[ \varepsilon_0 = Q_x \frac{6\pi^2\hbar^3}{(2s + 1)m(kT_s)^3} \cdot \frac{g_{\varepsilon,2}(\gamma)}{3g_{\varepsilon,3}(\gamma)g_{\alpha\varepsilon}(\gamma)}. \quad (28) \]

Returning to the formula for the temperature jump

\[ \Delta T = R Q_x, \]

we find from expression (28) that the Kapitsa resistance in the zeroth approximation is

\[ R = C(\gamma, q) \frac{\hbar^3}{(2s + 1)k^3T_s^2m}. \]
Here

\[ C(\gamma, q) = \frac{2\pi^2 g_{\varepsilon,2}(\gamma)}{g_{\varepsilon,3}(\gamma) g_{\alpha\varepsilon}(\gamma)} \cdot \frac{1 + q}{1 - q} \]  

is the (dimensionless) coefficient of the temperature jump.

The plus sign in formula (28) indicates that the wall temperature is higher than the phonon component temperature.

The graphs of the behavior of the temperature jump coefficient as a function of the parameter \( \gamma \) and the specular reflection coefficient \( q \) are shown in Figs. 1 – 4.

So, on fig. 1 the dependence of temperature jump coefficient on the parameter \( \gamma \) in case of zero factor of reflectivity \((q = 0)\) is presented. Curves of 1, 2, 3 correspond to values of dimensionless velocity \( w_0 = 1, 2, 3 \).

On fig. 2 the dependence of this factor on reflectivity factor \( q \) in a case, when dimensionless velocity of the sound \( w_0 = 10 \) is presented. Curves of 1, 2, 3 correspond to values of the parameter of collisions \( \gamma = 1, 3, 10 \).

On fig. 3 for the case when \( \gamma = 1 \) the dependence of the temperature jump coefficient on reflectivity factor \( q \) is given. Curves of 1, 2, 3 correspond to values dimensionless velocity \( w_0 = 5, 10, 20 \).
On fig. 4 for the case when $q = 0.5$ the dependence of the coefficient on the dimensionless velocity $w_0$ is shown. Curves of 1, 2, 3 correspond to values of parameter $\gamma = 1, 5, 10$.

From the graphs shown it is seen, that at the fixed values of the parameters the quantity of the factor $C(\gamma, q)$:

- Monotonously decreases with the growth of the parameter $\gamma$,
- Monotonously grows with growth of parameter of reflectivity $q$,
- Monotonously grows with growth of dimensionless quantity of velocity of sound $w_0$.

Besides, under convergence of reflectivity coefficient $q$ to unit the quantity $C(\gamma, q)$ increases unlimitedly, as in this limit the heat exchange between the wall and gas adjacent to it becomes impossible.

8. CONCLUSION

For degenerate quantum bose-gas with frequency of collisions, depending on momentum of elementary excitations of bose-gas, the kinetic equation is constructed. The general case of dependence of energy of elementary ex-
citations of bose - gas on momentum is considered. Boundary conditions are assumed to be specular – diffusive. The solution of a semi–spatial boundary problem of jump of temperature on border degenerate bose - gas in the presence of Bose — Einshtein condensate is received. The formula for finding of jump of temperature and calculation of Kapitsa resistance is deduced. Sufficiently general method of the solution of the kinetic equations with specular – diffusive boundary conditions, for the first time offered in [17] in the problem of skin – effect is developed.

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