Coulomb gauge approach for charmonium meson and hybrid radiative transitions.

Peng Gou\textsuperscript{1,2}, Tochtl Yépez-Martínez\textsuperscript{2}, Adam P Szczepaniak\textsuperscript{1,2,3}

\textsuperscript{1} Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA
\textsuperscript{2} Center For Exploration of Energy and Matter, Indiana University, Bloomington, IN 47408, USA.
\textsuperscript{3} Physics Department, Indiana University, Bloomington, IN 47405, USA
E-mail: pguo@jlab.org, tyepezma@indiana.edu, aszczepa@indiana.edu

Abstract.
We consider the lowest order interaction of the Foldy-Wouthuysen QED and QCD Hamiltonian in the Coulomb gauge approach, to describe radiative transitions between conventional and hybrids charmonium mesons. The results are compared to potential quark models and lattices calculations.

1. Introduction
It has long been expected that excitation of the gluon field should contribute to the spectrum of hadrons. Hybrid resonances, \textit{i.e.} states that contain both quark and gluon excitations, were considered in various models, \cite{1, 2, 3, 4, 5, 6}, and recent lattice simulations \cite{7, 8, 9} confirm model expectations. Furthermore, in recent years several new resonances, in particular in the charmonium spectrum have been identified and may include hybrids, \textit{e.g.} the $\Upsilon(4260)$. Conventional heavy quarkonia are well described by non-relativistic QCD \cite{10}. Thus it is reasonable to expect that hybrids containing heavy quarks could be treated in a similar way, \textit{i.e.} by considering gluon excitations in presence of slowly moving quarks.

To a good approximation heavy quarks interact with photons as bare Dirac particles. Thus radiative transitions can be used to explore quarkonium dynamics. We assume that this phenomenology may be extended to quarkonium hybrids \cite{11, 12}. Over the years several radiative transitions have been measured \cite{13, 14, 15} and extensive theoretical studies were performed \cite{16, 17, 18, 19}. More recently lattice gauge simulations have become available \cite{20, 21} that also include predictions for transitions involving hybrid mesons \cite{22, 23}.

In this contribution we present the radiative transitions involving the lowest mass conventional charmonia and the lowest mass multiplet of charmonium hybrids. We also discuss current matrix elements involving states of identical charge conjugation. These vanish when photon couples to both the quark and the antiquark but are in general finite when the current operator acts on a single quark. They are well defined within the model and can also be computed on the lattice.

2. Coulomb gauge approach and the basic ingredients
The QCD Hamiltonian $H_{QCD}$, which describes non-relativistic quarks interacting with (relativistic) gluons can be constructed from the full QCD Hamiltonian in the Coulomb gauge
by applying Foldy-Wouthuysen transformation [24]. This Hamiltonian was used to study the glueball spectrum [11] and the low mass charmonia and bottomonia including hybrids [12]. In addition to the strong interaction part, here we also consider the minimal coupling of the photon to the quarks, which in the non-relativistic limit is given by

\[ H_{QED} = \frac{e_q}{2m} \int dx \bar{\Psi}^i(x) \beta [2i A_\gamma(x) \cdot \nabla - \Sigma \cdot B_\gamma(x)] \Psi(x) \]  

(1)

where \( A_\gamma \) and \( B_\gamma \) are the photon vector potential and magnetic field, respectively. The quark fields are given by

\[ \Psi^i(x) = \sum_{\lambda=\pm 1/2} \int \frac{dk}{(2\pi)^3} e^{ik\cdot x} [u_\lambda b(k, \lambda, i) + v_\lambda d^i(-k, \lambda, i)] \]  

(2)

with \( u, v \) being the Dirac spinors in the non-relativistic limit. Given an approximate solution of the Schrödinger equation \( H_{QCD} |N[cc]⟩ = E_N |N[cc]⟩ \), within the Fock sector containing only the heavy quark-antiquark pair, the QED interaction of Eq. (1) determines the radiative transition matrix element,

\[ \mathcal{M}_{N→N'} \propto \langle N'[cc] | H_{QED} | N[cc] \rangle \]  

(3)

between ordinary charmonia. In the case of transitions involving hybrids, which are given by solutions of \( H_{QCD} |N'[cc\bar{c}]⟩ = E_N |N'[cc\bar{c}]⟩ \) in the sector containing in addition to the \( cc \bar{c} \) pair a transverse quasi-gluon, the radiative transition to an ordinary meson state can be accompanied by gluon absorption. To lowest order in the heavy quark mass expansion the latter is determined by the instantaneous Coulomb interaction that changes the gluon number, \( ⟨cc\bar{c} | H_C | cc\bar{c} \rangle \). Here \( H_C \) is given by

\[ H_C = -\frac{q^2}{2} \int dx dy \rho^a(x) K_{a,b}(x,y,A_\gamma) \rho^b(y) \]  

(4)

and \( \rho^a(x) = \bar{\Psi}^i(x) T^a \Psi(x) \) is the quark color charge density and the gluon field \( A_\gamma \) is related to the quasi-gluon particle operators by

\[ A_\gamma^a(x) = \frac{1}{\sqrt{2\omega(k)}} \int \frac{dk}{(2\pi)^3} e^{ik\cdot x} [\bar{\epsilon}(k, \lambda) a(k, \lambda, a) + \bar{\epsilon}^T(-k, \lambda) a^T(-k, \lambda, a)], \]

(5)

with \( \omega(k) \) the quasi-gluons dispersion function, \( (\lambda, a) \) being the helicity and color indices, respectively and \( \bar{\epsilon}(k, \lambda) \) the helicity vectors. In the variational model the Coulomb kernel is replaced by its vacuum expectation value and the operator which changes the gluon number by one becomes,

\[ K_{a,b} = f_{abc} \int \frac{dk}{(2\pi)^3} \frac{dq}{(2\pi)^3} e^{ik\cdot x - iq\cdot y} k \cdot A_\gamma^c(k - q) K^1(k, q) \]  

(6)

with the scalar function \( K^1(k, q) \) obtained from a solution of a series of Dyson-Schwinger equations [25, 26, 27, 28, 29, 30, 31]. The model has been used successfully [32, 33] in the study of excited adiabatic potentials between static quarks [34], which can be used to determine the single gluon orbitals in Eq. (5). Combining Eqs. (1,4) leads to an effective operators for radiative transitions between hybrid and ordinary quarkonia

\[ \mathcal{M}_{N→N',\gamma} \propto \langle N'[cc\bar{c}] | H_{QED}^{\text{eff}} | N[cc\bar{c}] \rangle \]  

(7)

where \( H_{QED}^{\text{eff}} = H_C H_{QED} \Delta E \) with \( 1/\Delta E \) representing the Green’s function of the \( cc \bar{c} \) pair. In the following we calculate the decay widths and transition amplitudes for several mesons and hybrid states.
2.1. Meson basis.

We represent the N-th quarkonium state of spin and its projection J, M, with parity P and charge conjugation C and total momentum P by

$$| P; J M P C N \rangle = \sum_{\alpha, m_1, m_2} \int \frac{d q}{(2 \pi)^3} \psi_{c\bar{c}}^{K, \alpha} (q) \chi_{m_1, m_2}^{J M P C} (P, \bar{q}, \alpha) b^\dagger (p_c, m_1, i_1) \delta_{i_1, i_2} d^\dagger (p_{\bar{c}}, m_2, i_2) |0\rangle.$$

(8)

Here \( \alpha = (L, S) \), and q is the magnitude of relative momentum between quark and antiquark. \( p_c = \frac{P}{2} + q \) and \( p_{\bar{c}} = \frac{P}{2} - q \) are the quark and antiquark momenta, respectively. The meson spin-orbital wave function is written using the relative momentum of the quark-antiquark, respectively,

$$\chi_{m_1, m_2}^{J M P C} (P, \bar{q}, \alpha) = \sum_{M_S, M_L} Y_{M_L} (q) \langle \frac{1}{2} m_1 | \frac{1}{2} m_2 | S M_S \rangle \langle S M_S ; L M L | J M \rangle \times \frac{1 + C (-1)^{L+S} 1 + P (-1)^{L+1}}{2}.$$

(9)

We consider the spin-orbital wave function \( \chi_{m_1, m_2}^{J M P C} (q, \alpha) \) for the charmonium mesons \( \eta_c (0^{-+}), \quad J / \psi (1^{--}), \quad h_c (1^{++}), \quad \chi_c (0^{-+}), \quad \chi_c (1^{++}) \) and \( \chi_c (2^{++}) \). The states are normalized according to \( \langle P'; J'M'P'C'N' | P; J M P C N \rangle = 2 E_{c\bar{c}} (2 \pi)^3 \delta^3 (P - P') \delta_{L, L'} \delta_{M, M'} \delta_{P, P'} \delta_{C, C'} \delta_{N, N'} \).

2.2. Hybrid basis

In construction of hybrid wave functions we thus follow the coupling scheme optimized for glue lump studies [11]. The hybrid state with total spin and projection J, M, parity P, charge conjugation C is then given by

$$| J M P C N \rangle = \sum_{\alpha = (S, L, i)} \int \frac{d k}{(2 \pi)^3} \frac{d q}{(2 \pi)^3} \psi_{c\bar{c}}^{k, \alpha} (k, q) \sum_{m_1, m_2, \sigma} \frac{1}{\sqrt{C_F N_C}} \chi_{m_1, m_2, \sigma}^{J M P C} (k, \bar{q}, \alpha) \times b^\dagger (\frac{k}{2} + q, m_1, i_1) T^a_{i_1, i_2} d^\dagger (\frac{k}{2} - q, m_2, i_2) a^\dagger (-k, \sigma, a) |0\rangle.$$

(10)

The spin-orbital wave function \( \chi_{m_1, m_2, \sigma}^{J M P C} (k, \bar{q}, \alpha) \) describes the \((L + J_g) + S\) coupling, with \( J_g \) the total spin of the gluon and \( \sigma = \pm 1 \) the gluon helicity

$$\chi_{m_1, m_2, \sigma}^{J M P C} (k, \bar{q}, \alpha) = \sqrt{\frac{2 J_g + 1 + C (-1)^{J_g+S} + (-1)^{J_g} \sqrt{2}}{4 \pi}} \sum_{M_g, M_S, M_L, m} Y_{M_L} (q) \langle \frac{1}{2} m_1 | \frac{1}{2} m_2 | S M_S \rangle \times \langle J_g M_g, L M_L | j m \rangle \langle j m, S M_S | J M \rangle D^*_{M_g, -\sigma} (\bar{k}) \delta_{\sigma, 1} + P (-1)^{J_g + L + 1} \delta_{\sigma, -1} \rangle.$$

(11)

Here \( q \) is the relative momentum between the quark-antiquark and \( k \) is the momentum of the gluon in the overall center of mass frame. The parity and charge conjugation are given by \( P = \xi (-1)^{J_g+S+1}, \quad C = (-1)^{L+S+1} \), respectively. Here \( \xi = +1 \) corresponds to the TM (natural parity) and \( \xi = -1 \) for TE (unnatural parity) gluon state that are given by \( | \sigma = +1 \rangle + \xi | \sigma = -1 \rangle \) combinations of gluon helicity states. The state is normalized in the same way as the normalization of conventional meson state.

For the lowest four hybrids we are considering [12], \((L, J_g P_g C_g) = (0, 1^{-})\), which correspond to the gluon in the TE mode. Coupling the TE gluon with the color octet \( \bar{Q}Q \) state in \( L = 0 \) produces a hybrid state with the intermediate angular momentum \( j = L + J_g = 1 \). Adding the quark spin \( S = 0, 1 \), and ignoring hyperfine splitting we obtain four low lying hybrids with quantum numbers, \( J^{PC} = Y (1^{-}) \) for \( S = 0 \) and \( J^{PC} = 0^{+}, \eta_c (1^{++}), 2^{-} \) for \( S = 1 \). It is worth noting that the hybrid with exotic quantum numbers \( 1^{-} \) appears in this lowest multiplet and is predicted to have the \( \bar{Q}Q \) pair in spin-1. These states has been described by lattice calculations [20, 22] and is expected to have a mass around 4.3 GeV.
3. Decay widths, transition amplitudes and predictions.

3.1. Meson-to-meson radiative transitions

We have considered fifteen transitions between conventional charmonia. Even though some of the transitions considered here vanish due to charge conjugation, we investigate the underlying matrix elements with photon attached to only one of the quarks. Some of these C-violating results can be compared with lattice results reported in [22], and others constitute our predictions. The matrix elements are obtained by using the model described in Sec. 2 and decay widths computed from

\[
\Gamma(N \rightarrow N'\gamma) = \int d\Omega_\gamma \frac{1}{32\pi^2} \frac{k_\gamma}{m_N^2} \frac{1}{2J_N + 1} \sum_{\sigma, M_N, M_{N'}} |M_{N \rightarrow N'\gamma}|^2. \tag{12}
\]

The photon momentum for each transition is given by \(k_\gamma = (M_N^2 - M_{N'}^2)/2M_N\). For the radial wave functions we use a harmonic oscillator approximation with a width parameter \(\beta = 0.5 \text{ GeV}\). This leads to some differences with respect to the potential-quark results of [19], where a Coulomb plus linear plus and hyperfine interactions were used to compute the wave functions. In Figure 1, our Coulomb gauge approach is compared with PDG reports [35] and non-relativistic (NR) potential-quark results [19].

![Figure 1](image-url)

**Figure 1.** Decay widths for \(c\bar{c}\)-mesons, a) electric and b) magnetic dipole transitions.

We calculate the transition amplitudes \(TA = |\hat{V}|, |\hat{F}_1|\) introduced in [20, 22] in the context of analysis of lattice data. Here \(\hat{F}_1\) represents either electric, \(\hat{E}_1\) or magnetic, \(\hat{M}_1\) dipole transition and \(\hat{V}\) is the magnetic dipole transition involving a vector and a pseudoscalar meson,

\[
|\hat{F}_1|^2 = \frac{1}{8e^2} \sum_{\sigma, M_N, M_{N'}} |M_{N \rightarrow N'\gamma}|^2, \quad |\hat{V}|^2 = \frac{(m_N + m_{N'})^2}{32e^2 m_N^2 k_\gamma} \sum_{\sigma, M_N, M_{N'}} |M_{N \rightarrow N'\gamma}|^2. \tag{13}
\]

In Figure 2, we compare the transition amplitudes in the Coulomb gauge approach with the available lattice computations [20, 22].

Furthermore, we found other three non-zero transitions, not reported so far, which decay widths and TA correspond to, \(\Gamma(\chi_{c2} \rightarrow h_c\gamma) = 0.1 \text{ keV}, |\hat{F}| = 0.12 \text{ GeV}, \Gamma(h_c \rightarrow \chi_{c1}\gamma) = 0.001 \text{ keV}, |\hat{F}| = 0.01 \text{ GeV} \) and \(\Gamma(h_c \rightarrow \chi_{c0}\gamma) = 0.6 \text{ keV}, |\hat{F}| = 0.13 \text{ GeV}\), these constitute our predictions. Finally, we calculate the transition amplitudes for charge conjugation violating transitions, \(|\hat{F}|/2\). The factor of two is introduced to account for the fact that photon couples to
We studied radiative decays of conventional charmonia and charmonium hybrids. We used the ordinary $c\bar{c}$-mesons $\eta_c(0^{++}), J/\psi(1^{--}), \chi_{c0}(0^{++}), \chi_{c1}(1^{++}), h_c(1^{--}), \chi_{c2}(2^{++})$, where we considered the minimal coupling of the photon to the non-relativistic quarks. We have compared the ordinary $c\bar{c}$-mesons $\eta_c(0^{++}), J/\psi(1^{--}), \chi_{c0}(0^{++}), \chi_{c1}(1^{++}), h_c(1^{--}), \chi_{c2}(2^{++})$, where we considered the minimal coupling of the photon to the non-relativistic quarks. We have compared

$\chi_{c2} \rightarrow J/\psi \gamma$ and $\chi_{c1} \rightarrow \chi_{c0} \gamma$ and their TA are given by $\tilde{F} = 0.10$ GeV and $\tilde{F} = 0.17$ GeV, respectively, which also constitute predictions of the model.

3.2. Hybrid-to-meson radiative transitions

We have studied 24 possible hybrid to meson radiative transitions, including matrix elements for C-violating modes, using $m_{hyb} = 4.35$ GeV for the spin-averaged mass of the lowest hybrid multiplet $Y(1^{--}), 0^{+-}, \eta_c(1^{++}), 2^{-+}$.

Lattice simulations [22] reports two decay widths $\Gamma(Y \rightarrow \eta_c \gamma) = 42$ keV, $\Gamma(\eta_c \rightarrow J/\psi \gamma) = 115$ keV and both correspond to a magnetic dipole transition. These decay widths are obtained from

$$\Gamma(Y \rightarrow \eta_c \gamma) = \frac{\alpha k^3_2 \tilde{F}^2}{27 (m_Y + m_{\eta_c})^2} \quad \text{and} \quad \Gamma(\eta_c \rightarrow J/\psi \gamma) = \frac{16 \alpha \tilde{F}_2}{27 m_{\eta_c}}$$

with $\tilde{V} = 0.28$ and $\tilde{F} = 0.69$ GeV, respectively.

We found four non-zero hybrid to meson transition amplitudes in our model, i.e., $Y \rightarrow \eta_c \gamma$, $0^{+-} \rightarrow J/\psi \gamma$, $\eta_c \rightarrow J/\psi \gamma$ and $2^{-+} \rightarrow J/\psi \gamma$. All of them depend on a single factor that is determined by the hybrid meson wave function. Therefore, the four decay widths can be written as $\Gamma = \alpha k_2^3 \tilde{F}_2^2$ or $\Gamma = \alpha \kappa_1 \kappa_2$, showing explicitly a symmetry relation implied by the light hybrid multiplet. The common factors $\kappa_1$ and $\kappa_2$ can be normalized by the two magnetic dipole matrix elements $\tilde{V}$ and $\tilde{F}$. To minimize sensitivity to the hybrid wave functions we calculate the ratio of hybrid decay widths. In general the model predicts $R = \frac{\Gamma(Y \rightarrow \eta_c \gamma)}{\Gamma(\eta_c \rightarrow J/\psi \gamma)} \approx 1$ for the whole hybrid multiplet while lattice reports $R = \frac{\Gamma(Y \rightarrow \eta_c \gamma)}{\Gamma(\eta_c \rightarrow J/\psi \gamma)} = 0.37$.

For the C-violating hybrid to meson matrix elements $\mathcal{M}_{CV}$ we found that all of them are proportional to a common factor i.e., $\mathcal{M}_{CV} \propto \kappa_3$. We use the transition amplitude $|\tilde{E}|_{1^{--}0^{++} \gamma} = 0.34$ GeV from [22] to normalize the relevant wave function overlap factor $\kappa_3$ in our model. Therefore, we predict seven non-zero C-violating transitions amplitudes $|\tilde{E}_1|$ within the range of 0.30 – 0.66 GeV and correspond to the transitions $Y \rightarrow h_c \gamma$, $0^{+-} \rightarrow \chi_{c1} \gamma$, $\eta_c \rightarrow \chi_{c0} \gamma$, $\eta_c \rightarrow \chi_{c2} \gamma$, $2^{-+} \rightarrow \chi_{c1} \gamma$ and $2^{-+} \rightarrow \chi_{c2} \gamma$.

4. Summary

We studied radiative decays of conventional charmonia and charmonium hybrids. We used the ordinary $c\bar{c}$-mesons $\eta_c(0^{++}), J/\psi(1^{--}), \chi_{c0}(0^{++}), \chi_{c1}(1^{++}), h_c(1^{--}), \chi_{c2}(2^{++})$, where we considered the minimal coupling of the photon to the non-relativistic quarks. We have compared

![Figure 2. Transition amplitudes for $c\bar{c}$-mesons.](image-url)
our results with non-relativistic quark model and lattice calculations and found a reasonable agreement. A few new predictions for transition amplitudes were presented including charge violating transitions amplitudes.

To describe hybrid decays we considered a model based on an effective QCD Hamiltonian that describes non-relativistic quarks interacting with (relativistic) gluons and is constructed from the QCD in the Coulomb gauge by applying Fuld-Wouthuysen transformation. We considered decays of states from the hybrid multiplet $1^−−, (0; 1; 2)_{+}^+ \, \text{with} \, 1^+ \text{being the exotic state. There are 24 possible radiative transitions between this multiplet and ground state charmonia. We found four non-zero decay widths for which the model predicts } R = \frac{\Gamma_N(N'\gamma)}{\Gamma_{N(1^+\gamma)}} \approx 1 \text{ for the whole hybrid multiplet while lattice reports } R = \frac{\Gamma(N(1^-\gamma)\rightarrow J/\psi\gamma)}{\Gamma_{N(1^+\gamma)\rightarrow J/\psi\gamma}} = 0.37. \text{ We also investigated C-violating matrix elements involving hybrids. The model predicts several of such matrix elements to be non-zero and of the same order of magnitude as the reported transition } 1^- \rightarrow 0^{++}\gamma \text{ by lattice calculations.}

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