Stellar-mass black holes in young massive and open stellar clusters V: comparisons with LIGO-Virgo merger rate densities

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ABSTRACT
I study the contribution of young massive star clusters (YMCs) and open star clusters (OCs) to the present day, intrinsic merger rate density of dynamically-assembled binary black holes (BBHs). The BBH merger event rate is estimated based on a set of 65 state-of-the-art evolutionary models of star clusters, as presented in Banerjee (2020b). The model clusters, initially, have masses $10^4 M_\odot - 10^5 M_\odot$, sizes 1 pc-3 pc, metallicity 0.0001-0.02, and massive stars in primordial binaries, consistently with the observed properties of YMCs and OCs. These relativistic direct many-body computed models incorporate up-to-date stellar mass loss and remnant formation ingredients. The merger-event rates are obtained by constructing a cluster population of the Universe, out of the models, taking into account mass distribution of clusters and cosmic star formation and enrichment histories, as per observations. The model BBH merger rate density ranges from a pessimistic to a reference value of $0.5 \text{yr}^{-1} \text{Gpc}^{-3}$ to $37.9 \text{yr}^{-1} \text{Gpc}^{-3}$, for a LIGO-Virgo-like detector horizon. The reference rate well accommodates the BBH merger rate densities estimated from GWTC-1 and GWTC-2 merger-event catalogues. The computed models also yield differential BBH merger rate densities that agree reasonably with those from GWTC-1 and, as well, with the much more constrained ones from GWTC-2. These results suggest that dynamical interactions in YMCs and OCs can, in principle, alone explain the BBH merger rate density and its dependence on the merging-binary properties, as inferred from to-date gravitational-wave (GW) events. The cosmic evolution of merger rate density from the computed models is also studied. The models predict a rate of $\approx 5 \text{yr}^{-1} \text{Gpc}^{-3}$ for eccentric LIGO-Virgo mergers from YMCs and OCs. The improving constraints on BBH merger rate density with mounting GW events will help constraining scenarios of star cluster formation across cosmic time and as well the relative contributions of the various compact binary merger channels.

Key words: open clusters and associations: general – globular clusters: general – stars: kinematics and dynamics – stars: black holes – methods: numerical – gravitational waves

1 INTRODUCTION

Until 2019, the LIGO-Virgo collaboration (hereafter LVC), in their first gravitational wave transient catalogue (Abbott et al. 2019a, hereafter GWTC-1), has published 11 compact binary merger events from their first and second observing runs (hereafter O1 and O2, respectively) with the ground-based interferometric gravitational wave (hereafter GW) detectors LIGO (Aasi et al. 2015) and Virgo (Akerlund et al. 2014). In 2020, the LIGO-Virgo-KAGRA (hereafter LK) collaboration has announced, in their second gravitational wave transient catalogue (Abbott et al. 2020a, hereafter GWTC-2), 39 additional candidates of compact binary coalescence events from the first half, ‘O3a’, of their recently concluded third observing run (hereafter O3). Based on the parameter estimations of these events, the vast majority of them has been designated as binary black hole (hereafter BBH) mergers with component masses ranging through $\approx 5M_\odot - 90M_\odot$ (Abbott et al. 2020a). The rest comprise likely candidates of binary neutron star (hereafter BNS) mergers and neutron star-black hole binary mergers.

Even prior to the publication of GWTC-1, the handful of then known GW events had already triggered wide debates regarding possible origins of such merging compact binaries and of the masses of the black holes (hereafter BH) and neutron stars (hereafter NS) they are made of. The issues remain as open until now. In these regards, the large jump in the number of events from GWTC-1 to GWTC-2 (consistently with the improved detector sensitivity and the observing time during O3a, see Abbott et al. 2020a) is particularly enlightening: apart from providing us with a wider variety and, as well, highly atypical GW events (e.g., Abbott et al. 2020b; Abbott et al. 2020d; Abbott et al. 2020c), the constraints on the rate of such merger events in the Universe is significantly improved in GWTC-2. Event rate is among the key aspects that would help to understand the relative contri-
butions of the various astrophysical channels (or scenarios) for general-relativistic (hereafter GR) inspiral and mergers of compact binaries. Such channels can be divided into two main categories (Benacquista 2006; Benacquista & Downing 2013; Mandel & Farmer 2017; Mapelli 2018), namely, (a), evolution of isolated massive stellar binaries (e.g., Dominik et al. 2012; Belczynski et al. 2016a,c; Marchant et al. 2016; Stevenson et al. 2017; Giacobbo et al. 2018; Spera et al. 2019; Rastello et al. 2020; Santoliquido et al. 2020; Belczynski et al. 2020; Bavera et al. 2020) and, (b), dynamical interactions among stellar remnants in various dense-stellar and dynamically-active environments or systems such as globular clusters (hereafter GC) (e.g., Breen & Heggie 2013; Morscher et al. 2013; Rodriguez et al. 2015; Askar et al. 2017; Chat-terjee et al. 2017; Askar et al. 2018; Fragnone & Kocsis 2018; Antonini & Gieles 2020b; Kremer et al. 2020), nuclear clusters (e.g., Antonini & Rasio 2016; Hoang et al. 2018; Arca Sedda 2020; Arca Sedda et al. 2020; Mapelli et al. 2020), young massive clusters (hereafter YMC), open clusters (hereafter OC) (e.g., Banerjee et al. 2010; Ziesi et al. 2014; Mapelli 2016; Banerjee 2017; Rastello et al. 2019; Di Carlo et al. 2019; Kumamoto et al. 2019; Banerjee 2020b), field hierarchical systems (e.g., Katz et al. 2011; Lithwick & Naoz 2011; Antonini et al. 2017; Silsbee & Tremaine 2017; Fragnone & Kocsis 2019; Fragnone et al. 2020), and stellar-remnant BHs trapped in gas disks in active galactic nuclei (e.g., McKernan et al. 2018; Se-cunda et al. 2019).

Dynamical interactions among stellar-remnant (or stellar-mass) BHs in YMCs and OCs has recently drawn high interest due to the channel’s natural ability to produce the unusual, highly mass-asymmetric (e.g., GW190425 and GW190814; Abbott et al. 2020b; Abbott et al. 2020d) or very massive (e.g., GW190521; Abbott et al. 2020b) BBH mergers (Di Carlo et al. 2020; Banerjee 2020b). Low and moderate mass young clusters, owing to their relatively short two-body relaxation times (Spitzer 1987; Heggie & Hut 2003) and spatial ambience, can produce BBH mergers at rates comparable to or exceeding those from GCs and isolated binary evolution (Banerjee 2018; Kumamoto et al. 2020; Di Carlo et al. 2020; Santoliquido et al. 2020). BBH merger events and rates apart, interest in younger evolutionary phases of all categories of stars clusters would grow naturally with increasing visibility horizons (Chen et al. 2017) of the forthcoming upgrades of the current GW detectors (e.g., the LIGO A+ upgrade) and future GW detectors (e.g., Voyager, Einstein Telescope, Cosmic Explorer; Reitz et al. 2019). With increasing look back time of the GW sources, one essentially rewinds to younger versions of the clusters, i.e., accesses mergers of shorter delay times.

In this work, the set of state-of-the-art N-body evolution-ary models of star clusters, as described in Banerjee (2020b), is utilized to estimate the contribution of dynamical inter-actions, in intermediate mass and massive YMCs and OCs, to the present-day BBH merger rate density. Sec. 2.1 summarizes the computed star cluster models. Sec. 2.2 describes the method used to evaluate the present-day, intrinsic BBH merger rate density and the corresponding differential merger rate densities (w.r.t. the merging binary’s primary mass, mass ratio, and eccentricity), based on the computed model set and observationally-derived cluster population properties. Sec. 3.1 presents the differential BBH merger rate densities as estimated from the computed models. Sec. 3.2 explores how the model BBH merger rate density depends on the GW detector’s horizon and event redshifts. Sec. 3.1 and Sec. 3.2 also make detailed comparisons with the BBH merger rate densities and BBH differential merger rate densities obtained from GWTC-1 and GWTC-2. The results, their various uncertainties, and caveats in the present approach are further discussed in Sec. 4. Sec. 5 summarizes the results and discusses potential next steps.

2 METHOD

Below, the evolutionary star cluster models and the method for calculating the present-day merger rate density are described.

2.1 Direct N-body star cluster-evolutionary models with up-to-date remnant formation and post-Newtonian dynamics

In this work, the 65 N-body evolutionary models of star clusters, as described in Banerjee (2020b, hereafter Paper II), are utilized. These computations and the model ingredients are described in detail in Paper II and further discussions are provided in Banerjee (2020a). Therefore, only a summary of these computations is provided in this paper as follows.

The model clusters, initially, have a Plummer density profile (Plummer 1911), are virialized (Spitzer 1987; Heggie & Hut 2003), and are unsegregated (i.e., have no radial dependence of stellar mass distribution). They, initially, have masses of $10^4 M_\odot \leq M_\bullet \leq 10^5 M_\odot$ and half-mass radii of $1 \leq r_\bullet \leq 3$ pc. They range over $0.0001 \leq Z \leq 0.02$ in metallicity and orbit in a solar-neighborhood-like external galactic field. The initial models are made of zero-age-main-sequence (hereafter ZAMS) stars with masses of $0.08 M_\odot \leq m_\star \leq 150 M_\odot$ which are distributed according to the standard initial mass function (hereafter IMF; Kroupa 2001). About half of these models have a primordial-binary population (overall initial binary fraction $\approx 5\%$ or $10\%$) where all the O-type stars (i.e., stars with ZAMS mass down to $16 M_\odot$) are initially paired among themselves according to an observationally-motivated distribution of massive-star binaries (Sana & Evans 2011; Sana et al. 2013; Moe & Di Stefano 2017). Such cluster parameters and stellar compositions are consistent with those observed in ‘fully-formed’, (near-)spherical, (near-)gas-free YMCs and medium-mass OCs (Portegies Zwart et al. 2010; Banerjee & Kroupa 2017, 2018) that continue to form and dissolve in the Milky Way and other Local-Group galaxies.

These model clusters are evolved using NBODY7, a state-of-the-art post-Newtonian (hereafter PN) direct N-body in-tegrator (Aarseth 2003, 2012; Nitadori & Aarseth 2012), that couples with the semi-analytical stellar and binary-evolutionary model BSE (Hurley et al. 2000, 2002). The integrated BSE is made up to date, in regards to prescriptions of stellar wind mass loss and formation of NSs and BHs, as described in Banerjee et al. (2020, hereafter Paper I). In particular, the NSs and BHs form according to the ‘rapid’ and ‘delayed’ SN models of Fryer et al. (2012) and pulsation pair-instability SN (PPSN) and pair-instability SN (PSN) models of Belczynski et al. (2016b). The NSs and BHs receive natal kicks based on SN fallback onto them, as in Belczynski et al. (2008). Such fallback slows down the remnants, causing
BHs of $\gtrsim 10M_\odot$ (Paper I) to retain in the clusters right after their birth. Furthermore, NSs formed via electron-capture SN (Podsiadlowski et al. 2004) also receive small natal kicks and retain in the clusters.

In NBODY7, the PN treatment is handled by the ARCHAIN algorithm (Mikkola & Tanikawa 1999; Mikkola & Merritt 2008). Such a PN treatment allows for GR evolution of the innermost NS- and/or BH-containing binary of an in-cluster (i.e., gravitationally bound to the cluster) triple or higher order compact subsystem, in tandem with the Newtonian-dynamical evolution of the subsystem (Kozai-Lidov oscillation or chaotic three-body interaction), potentially leading to the binary’s (in-cluster) GR in-spiral and merger. The treatment also undertakes the GR evolution of in-cluster NS/BH-containing binaries that are not a part of a higher-order subsystem. As discussed in Paper II (see also the references therein), the moderate density and velocity dispersion in the model clusters make them efficient factories of dynamically assembling PN subsystems, particularly, those comprising BHs. As also discussed in Paper II (see also Anagnostou et al. 2020), the vast majority of the GR mergers from these computed model clusters are in-cluster BBH mergers. As also demonstrated therein (see also Banerjee 2020a), the final inspiralling phases of such merging BBHs sweep through the LISA and deci-Hertz GW frequency bands before merging in the LVK band.

### 2.2 Calculation of intrinsic merger rate density from computed model clusters

To obtain the intrinsic merger rate density at the present cosmic epoch, ‘mock detection experiments’ are performed in a ‘Model Universe’ that is constructed out of the computed model clusters of Paper II. An ideal GW detector (a detector with zero noise floor) is considered that can detect GW arriving from all GR mergers within a comoving volume whose boundary is at a redshift $z_{\text{max}}$. $z_{\text{max}}$ represents an (artificial) horizon, for average source inclination, for a realistic GW detector like LVK. For LVC O1/O2 observing runs, $z_{\text{max}} \approx 1$ (Chen et al. 2017; Abbott et al. 2019b). However, since an ideal GW detector is considered here, $z_{\text{max}}$ will be varied to also address future, ground-based GW detectors of $>\text{Hz}$ frequency band (e.g., LIGO A+ upgrade, Einstein Telescope, Cosmic Explorer). If the GR compact-binary merger rate density (per unit comoving volume), $\Phi'$, in the Universe has an inherent dependence on the merger event redshift, $z_{\text{event}}$, then the present day, intrinsic merger rate density, $\Phi$, would depend on the detector horizon, $z_{\text{max}}$ (Abadie et al. 2010; Abbott et al. 2019b). Conversely, if $\Phi'$ is independent of $z_{\text{event}}$, then $\Phi$ will also be independent of $z_{\text{max}}$. An inherent $\Phi'(z_{\text{event}})$ dependence would arise from a non-uniform distribution of delay time (time of merger since the birth of the parent stellar population or host system), depending on the channel(s) responsible for GR mergers in the Universe (Benacquista 2006; Benacquista & Downing 2013), and as well due to the cosmic variation of star formation rate. Hereafter, for brevity, the present-day, (differential), intrinsic merger rate density, $\Phi$, will simply be referred to as (differential) merger rate density. The inherent redshift dependence of the merger rate density, $\Phi'(z_{\text{event}})$, will, hereafter, be referred to as the cosmic merger rate density.

From the computed grid (see Table C1 of Paper II), model clusters are randomly chosen with initial masses, within $2 \times 10^4M_\odot \leq M_\odot \leq 10^7M_\odot$, according to a power law distribution of index $-2$ (i.e., $\phi_{\text{CLMF}}(M_\odot) \propto M_\odot^{-2}$) as observations of young clusters in the Milky Way and nearby galaxies suggest (Gieles et al. 2006; Larsen 2009; Portegies Zwart et al. 2010; Bastian et al. 2012). Their initial sizes are chosen uniformly between 1 pc $\leq r_\odot \leq 3$ pc. Each selected cluster is then assigned a formation redshift, $z_I$, that corresponds to an age, $t_I$, of the Universe, according to the probability distribution given by the cosmic star formation history (Madau & Dickinson 2014; hereafter SFH)

$$\Phi_{\text{SFH}}(z_I) = 0.015 \frac{(1+z_I)^{2.7}}{1+[(1+z_I)/2.9]^{8.5}}M_\odot \text{yr}^{-1} \text{Mpc}^{-3}. \tag{1}$$

Since the masses of the stellar-remnant BHs (and hence of the merging BBHs) depend on their parent cluster’s metallicity, $Z$ (see Paper I and references therein), the mass dependence of the differential merger rate density would depend on the $Z$-distribution of star clusters in the Universe and the distribution’s redshift ($z$) dependence. In this work, the observationally-derived $Z$-spread and its $z$-dependence over $0 \leq z \leq 10$, as obtained by Chruslińska & Nelemans (2019) (based on their ‘moderate-$Z$’ sample), is adopted (see also Chruslińska et al. 2020). The metallicity of a model cluster of $M_\odot$, $r_\odot$, $z_I$ ($t_I$) is selected from the model grid based on this observationally-derived $Z-z$ distribution\(^2\).

In this way, the comoving volume within the detector horizon, $z_{\text{max}}$, is uniformly populated with a sample cluster population of size $N_{\text{samp}} = 5 \times 10^5$. A GR merger occurs from a cluster after a delay time, $t_{\text{mrg}}$, from the cluster’s formation when the age of the Universe is $t_{\text{event}}$, i.e.,

$$t_{\text{event}} = t_I + t_{\text{mrg}}. \tag{2}$$

If the light travel time from the cluster’s comoving (or Hubble) distance, $D$, is $t_{\text{D}}$, then the age of the Universe is

$$t_{\text{obs}} = t_{\text{event}} + t_{\text{D}} \tag{3}$$

when the (redshifted) GW signal from the merger event arrives the detector. The GW signal is considered ‘present-day’ (or ‘recent’ or ‘in the present epoch’) if

$$t_{\text{Hubble}} - \Delta t_{\text{obs}} \leq t_{\text{obs}} \leq t_{\text{Hubble}} + \Delta t_{\text{obs}} \tag{4}$$

where $t_{\text{Hubble}}$ is the current age of the Universe (the Hubble time) and $\Delta t_{\text{obs}}$ is taken to be $\Delta t_{\text{obs}} = 0.15 \text{Gyr}$. $\Delta t_{\text{obs}}$ serves as an uncertainty in the cluster formation epoch; with the above choice it is well within the typical epoch uncertainties in the observed SFH data (Madau & Dickinson 2014). In this work, SFH within $z \leq 10$ is considered to be consistent with the adopted cosmic metallicity evolution (see above).

If $N_{\text{mrg}}$ present-day mergers are obtained from $N_{\text{samp}}$ clusters, then the number of mergers per cluster, $N_{\text{mrg}}/N_{\text{samp}}$, can be scaled to infer the merger rate density from the Model

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\(^1\) Since $M_\odot \approx 10^5M_\odot$ clusters are sparse in the model set, clusters of $M_\odot \geq 2 \times 10^5M_\odot$ are considered in this work. 

\(^2\) In practice, a $100 \times 1000$ $Z-z$ matrix is generated using their publicly-available moderate_F6H_z_dR.dat and the corresponding Python script. The Anders & Grevesse (1989) solar metallicity scaling is adopted to covert their O/H-metallicity to Fe-metallicity, $Z$. A $Z$ is randomly picked from the $Z-z$ lookup table, for the tabulated $z$ that is closest to $z_I$. The model with metallicity closest to this $Z$ is then selected. See also Kumamoto et al. (2020).
Table 1. A summary of the ‘mock detection experiments’ performed in this work (Sec. 2.2). The columns from left to right are as follows: Col. 1: the total number of clusters in the sample comprising the Model Universe, $N_{\text{samp}}$. Col. 2: the total number of merger events, $N_{\text{merg}}$, from this sample at the present cosmic epoch, within a time $2\Delta t_{\text{obs}}$ (Col. 3) around the current age of the Universe, Col. 4: the instrument visibility boundary, $z_{\text{max}}$, for average source inclination, Col. 5(6): the inferred reference (pessimistic) present-day, intrinsic merger rate density from the Model Universe (see Sec. 2.2, Eqn. 5).

| $N_{\text{samp}}$ | $N_{\text{merg}}$ | $\Delta t_{\text{obs}}$/[Gyr] | $z_{\text{max}}$ | $\mathcal{R}_+$/[yr$^{-1}$Gpc$^{-3}$] | $\mathcal{R}_-$ /[yr$^{-1}$Gpc$^{-3}$] |
|------------------|------------------|------------------------|----------------|----------------|----------------|
| $5 \times 10^5$  | 16407            | 0.15                   | 1.0            | 37.9          | 0.51           |
| $5 \times 10^5$  | 26795            | 0.15                   | 2.0            | 61.9          | 0.84           |
| $5 \times 10^5$  | 27775            | 0.15                   | 3.5            | 64.1          | 0.87           |
| $5 \times 10^5$  | 24493            | 0.15                   | 5.0            | 56.6          | 0.76           |
| $5 \times 10^5$  | 19121            | 0.15                   | 7.5            | 44.1          | 0.60           |
| $5 \times 10^5$  | 16335            | 0.15                   | 10.0           | 37.7          | 0.51           |
| $5 \times 10^5$  | 13006$^a$        | 0.15                   | 1.0            | 132.2         | 0.99           |
| $5 \times 10^5$  | 33741$^b$        | 0.15                   | 1.0            | 19.5          | 0.26           |

$^a$ $\Phi_{\text{CLMF}}(M_i) = M_i^{-2.5}$  
$^b$ $M_{\text{cl},\text{low}} = 5 \times 10^5$ M$_\odot$

The second ratio of integrals in the r.h.s. of Eqn. 5 is the boost factor due to the difference in SFH between the progenitors of YMCs/OCs and what we call GCs (Harris 1996). Star clusters like YMCs (some authors refer to young clusters of $\gtrsim 10^4$ M$_\odot$ as ‘super star clusters’, conceived to be the young progenitors of present-day GCs) continue to form throughout the cosmic star formation history and evolve to become old OCs or GCs or dissolve by the present cosmic epoch. Therefore, in the numerator, $z_f$ is considered over [0.0, 10.0], the limit at $z_f = 10.0$ being due to that in the metallicity evolution history (Chruslinska & Nelmans 2019) used to construct the sample cluster population (see above). This is also why $z_{\text{max}} \lesssim 10.0$ in this work (see Fig. 3). The GCs, on the other hand (by definition), are all old objects which have formed over $3.0 \lesssim z_t \lesssim 6.0$ (El-Badry et al. 2019), setting the $z_t$ limits in the denominator.

The spatial number density of GCs (per unit comoving volume), $\rho_{\text{GC}}$, in Eqn. 5 is taken to be the observationally-determined value, $\rho_{\text{GC}} = 8.4 \, \text{h}^3 \, \text{Mpc}^{-3}$, as in Portegies Zwart & McMillan (2000), for the reference $\mathcal{R}_+$. With the dimensionless Hubble constant $h \equiv H_0/[100 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}] = 0.674$ (Planck Collaboration et al. 2020), $\rho_{\text{GC}} = 2.57 \, \text{Mpc}^{-3}$. For obtaining $\mathcal{R}_-$, $\rho_{\text{GC}} = 0.33 \, \text{Mpc}^{-3}$ is taken which is a lower limit of GC spatial density as estimated in Rodriguez et al. (2015).

The factor $R_{\text{mort}}$ in Eqn. 5 is the ‘mortality ratio’ that absorbs any inherent inefficiency, relative to GC progenitors, to become a typical gas-free young cluster from a gas-embedded, proto-cluster phase. Effects relating to star (cluster) formation mechanisms and environment (e.g., Banerjee & Kroupa 2018; Kuijken et al. 2019) would determine the success of assembling a gas-free, parsec-scale, young cluster of the kind we typically observe (and as taken as initial conditions of the model clusters). The cluster formation efficiency can depend on the gas-free initial mass, $M_i$, and formation epoch, $z_t$, which are, still, largely open questions (e.g., Renaud 2018; Krumholz et al. 2019). In this work, for simplicity, $R_{\text{mort}} = 1$ is assumed implying that, beyond $M_i \gtrsim 10^4$ M$_\odot$ (as in the models here), the cluster formation efficiency is assumed to be independent of $M_i$. For example, direct N-body models suggest that embedded clusters of $\gtrsim 10^4$ M$_\odot$ are resilient to the violent relaxation phase induced by rapid residual gas expulsion from proto-clusters (e.g., Brinkmann et al. 2017; Shukirgaliev et al. 2017).
If the normalized present-day distribution of a quantity $X$, that is measurable from the detected merger-event GW signals (e.g., the merging compact binary’s primary mass, mass ratio, eccentricity), is $\psi(X)$ over the range $[X_1, X_2]$ (i.e., $\frac{\int_{X_1}^{X_2} \psi(X) dX}{\int_{X_1}^{X_2} dX} = 1$) then the differential merger rate density w.r.t. $X$ is obtained by

$$\frac{d\mathcal{R}}{dX}(X) = \mathcal{R} \psi(X). \quad (6)$$

(Hence, $\frac{X_2^2}{X_1^2} [d\mathcal{R}/dX](X) dX = \mathcal{R}$. Here, $\mathcal{R}$ is determined from Eqn. 5. In the present mock detection experiment, $[X_1, X_2]$ is divided into $N_b$ equal-sized bins of width $\Delta X$. If the number of events detected within the $i$-th bin around $X_i$ is $\Delta N_{X,i}$, then

$$\psi(X_i) \approx \frac{\Delta N_{X,i}}{\Delta X N_{\text{msg}}}. \quad (7)$$

(Here, $\sum_{i=1}^{N_b} \mathcal{R} \psi(X_i) \Delta X = \mathcal{R}$.)

In this study, redshift, comoving distance, and light travel time are interrelated (based on a lookup table; Wright 2006) according to the $\Lambda$CDM cosmological framework (Peebles 1993; Narlikar 2002). The cosmological constants from the latest Planck results ($H_0 = 67.4$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.315$, and flat Universe for which $t_{\text{Hubble}} = 13.79$ Gyr; Planck Collaboration et al. 2020) are applied.

In summary, the reference (differential) merger rate density, $\mathcal{R}$ ($d\mathcal{R}/dX$) $^3$, is obtained from Eqn. 5 (and Eqns. 6, 7) with $[M_{\text{GC,low}}, M_{\text{GC,high}}] = [5 \times 10^5 M_\odot, 1 \times 10^6 M_\odot]$ and $\rho_{\text{GC}} = 2.57$ Mpc$^{-3}$. A (double-)pessimistic (differential) rate, $\mathcal{R}_{\text{m}}$ ($d\mathcal{R}_{\text{m}}/dX$), is obtained with $[M_{\text{CL,low}}, M_{\text{CL,high}}] = [1 \times 10^7 M_\odot, 2 \times 10^7 M_\odot]$ and $\rho_{\text{CL}} = 0.33$ Mpc$^{-3}$ in Eqn. 5 (and Eqns. 6, 7). For both evaluations, $[M_{\text{CL,low}}, M_{\text{CL,high}}] = [2 \times 10^6 M_\odot, 1 \times 10^7 M_\odot]$ (but see Sec. 4). $\Delta t_{\text{obs}} = 0.15$ Gyr is used for all the mock detection experiments $^4$, $N_b = 10$ or 20 is used.

3 RESULTS: MERGER RATE DENSITY FROM YOUNG MASSIVE AND OPEN STELLAR CLUSTERS

Table 1 provides the present-day merger counts, $N_{\text{msg}}$, out of $N_{\text{amp}} = 5 \times 10^5$ Model-University clusters and the corresponding reference (pessimistic) merger rate density, $\mathcal{R}$ ($\mathcal{R}_{\text{m}}$), in mock detection experiments (Sec. 2.2) with the detector horizon redshift, $z_{\text{max}}$, varied from 1.0 to 10.0.

3.1 Differential merger rate density

The grey-filled histograms in the panels of Fig. 1 show the differential merger rate densities, $d\mathcal{R}/dM_1$ ($d\mathcal{R}/dq$), w.r.t. the merger primary mass (mass ratio), $M_1$ ($q \equiv M_2/M_1; M_1 \geq M_2$), as obtained from the Model Universe. The upper panel is the outcome with $z_{\text{max}} = 1.0$, which visibility boundary is relevant for LVK O1, O2, and O3 observing runs (Chen et al. 2017). The lower panel is for $z_{\text{max}} = 2.0$, relevant for future

$^3$ In a few instances in this text, the symbol $\mathcal{R}$ is also used to denote the merger rate density from Eqn. 5 in general, as evident from the context. Due to only a handful of such occurrences in this text, no new symbol is invoked.

$^4$ The unit of $\mathcal{R}$ ($d\mathcal{R}/dX$) is then Gyr$^{-1}$ Mpc$^{-3}$ [M$^{-1}$] = yr$^{-1}$ Gpc$^{-3}$ [M$^{-1}$] [1].

$^5$ The data for the differential merger rate densities are obtained from the public repository of GWTC-1 at https://dcc.ligo.org/LIGO-P1800324/public.

$^6$ Due to the assumed uniform spatial density of the clusters, the probability density function of the clusters’ distance redshift, $z_D$, increases monotonically ($\propto z_D^2$, $D$ being the comoving distance corresponding to $z_D$) as $z_D$ approaches the detector horizon, $z_{\text{max}}$.

$^7$ Such BH mass distribution occurs for metallicities $Z \lesssim Z_\odot/4$. For $Z = Z_\odot$, the (retained) BH mass distribution truncates at $\approx 15 M_\odot$ and no PPSN/PSN takes place (see Paper I). However, for the cosmic metallicity evolution considered here (Chru{\l}inska & Nelemans 2019), low $Z$ clusters form at all ages of the Universe.

$^8$ The Model-University $d\mathcal{R}/dM_1$ extends continuously below $M_1 < 10 M_\odot$. For the same reason, when $M_1$ approaches $10 M_\odot$.
The filled histogram gives the present-day, differential intrinsic merger rate density (Y-axis), as obtained from Figure 1. Upper panels: the Model Universe cluster population (Sec. 2.2), as a function of merger primary mass (left panel) and mass ratio (right panel) along the X-axis. The upper and lower limits (histogram error bars) represent the reference and the pessimistic rates (Sec. 2.2, Table 1), respectively. The heights of the histogram boxes lie halfway between these two values (at approximately half of the reference value). A visibility boundary (for average source inclination) at a redshift of $z_{\text{max}} = 1.0$ is assumed. The solid lines are the differential intrinsic BBH merger rate densities as published in the LVC GWTC-1 public repository, corresponding to their Model A, B, and C (legend) for BH masses in merging BBHs (Abbott et al. 2019b). For each BH-mass model, the upper and lower lines enclose the 90% symmetric credible intervals; for Model C, the thick, blue line gives the median. Lower panels: The same as in the top panels but for $z_{\text{max}} = 2.0$.

The Model-Universe $dR/dM$ extends beyond $M_1 > 40M_\odot$ and consistently with the Model-C boundaries (see Fig. 1). Such BH mass is the outcome of star-star mergers in binaries (see Fig. 12 of Paper I), star-BH mergers in binaries via the formation of BH-Thorne Zytkow object (see Paper II), or (first-generation) BBH mergers (see Paper II). The Model-Universe $dR_{-}/dM$ and $dR_{-}/dq$ (the histogram lower limits in Fig. 1) are highly conservative estimates (see Sec. 2.2) and they fall below the lower boundaries of the Model-C differential merger rate densities. The differential rate densities are generally higher and more biased towards larger $M_1$ for $z_{\text{max}} = 2.0$ than for $z_{\text{max}} = 1.0$ (see Fig. 1; left panels). With larger $z_{\text{max}}$, the detector is able to receive signals from shorter $t_{\text{merg}}$ mergers (see above) which mergers are generally more massive and numerous (see Figs. 4 & 9 of Paper II), causing such trends. The dependence of $R$ on $z_{\text{max}}$ is further discussed in Sec. 3.2.

Fig. 2 plots the $dR/de_m$ and $dR_{-}/de_m$ (the histogram upper and lower limits, respectively) as obtained from the Model Universe cluster population. Here, $e_m$ is the maximum LVK-band eccentricity, defined as the eccentricity of the in-spiralling binary when its detector-frame (peak-power) GW frequency is $10$ Hz or the eccentricity at the minimum (peak-power) GW frequency, $f_{\text{min}}$, if the binary’s final inspiral towards the merger begins at $> 10$ Hz ($f_{\text{min}} \sim 10 - 100$ Hz in the detector frame, for such LVK eccentric inspirals obtained in the present models; see Fig. 10 and associated discussions in Paper II). In the panels of Fig. 2, $R$ ($R_{-}$) is nearly wholly concentrated over the bin at the smallest $5M_\odot$, i.e., close to, within, and below the (NS-BH) ‘mass gap’, due to the inclusion of the ‘delayed’ remnant-mass scheme in some of the models. A few of such models give rise to BBH mergers involving primaries close to and within the mass gap. See Paper II for the details. $dR/dM$ below the mass gap occurs due to the BNS mergers occurring in some of these models (Fragione & Banerjee 2020).
Since the population shows an overall increasing trend with decreasing $z$ (Fig. 3; black, filled squares). However, this competes with the fact that beyond $z \approx 3$ the redshift at which the cosmic SFH peaks (Eqn. 1; Fig. 4, left panel, grey line), the majority of the clusters form too late for the long light travel times from comoving distances approaching $z_{\text{max}}$. Indeed, in Fig. 3 (see also Table 1), $R$ increases only slightly\(^9\) as $z_{\text{max}}$ increases from 2.0 to 3.5, beyond which $R$ begins to decline. In this way, the maxima of $R$ at $z_{\text{max}} \approx 3.5$ is an outcome of two competing effects and is specific for the $t_{\text{mrg}}$ distribution obtained from the Model Universe cluster population (given the ΛCDM Universe with its currently-determined parameters and the cosmic SFH; see Sec. 2.2).

\(^9\) The Poisson error in $N_{\text{mrg}}$ leads to merger-rate uncertainties of $\Delta R/\Delta z \approx 10^{-1} \text{ yr}^{-1} \text{ Gpc}^{-3}$ ($\Delta R/\Delta z \approx 10^{-3} \text{ yr}^{-1} \text{ Gpc}^{-3}$).

Fig. 3 shows that for $z_{\text{max}} = 1$, which detector horizon is relevant for LVK O1, O2, and O3, the Model Universe reference $R(= 37.9 \text{ yr}^{-1} \text{ Gpc}^{-3}$; the black, filled square) falls moderately below the median BBH merger rate density estimated from GWTC-1 (Abbott et al. 2019b), for their Model C ($= 58.3 \text{ yr}^{-1} \text{ Gpc}^{-3}$; blue line), but lies well within the corresponding 90% credible interval (blue-shaded area). The rather broad GWTC-1 limits accommodate the Model Universe reference $R$s (black, filled squares) for all $z_{\text{max}}$. On the other hand, the Model Universe $R$ at $z_{\text{max}} = 1$ nearly coincides with the 90% credible upper limit of the significantly more constrained GWTC-2 BBH merger rate density (The LIGO Scientific Collaboration et al. 2020; orange lines). This means that with suitable choices of astrophysical quantities in Eqn. 5, e.g., $M_{\text{GC,low}}, M_{\text{cl,low}}, \rho_{\text{GC}}$, and $R_{\text{mott}}$, the Model Universe cluster population can reproduce the GWTC-2 median BBH merger rate density. This is discussed further in Sec. 4.

It would be worth looking into the inherent dependence of merger rate density on merger-event redshift, $z_{\text{event}}$ (Sec. 2.2), as obtained from the Model Universe cluster population. This cosmic merger rate density function, $R(z_{\text{event}})$, is shown in Fig. 4 (left panel, blue line). Note that $R(z_{\text{event}})$ is simply the collective redshift distribution of the merger events from the Model Universe cluster population, without taking light travel times into account, as opposed to $R(z_{\text{max}})$ in Fig. 3 (see above). The $R(z_{\text{event}})$ function in Fig. 4 is constructed based on a sample cluster population (Sec. 2.2) of $N_{\text{samp}} = 10^6$, the mergers from which are distributed among 100, equal-sized log$_{10}(z_{\text{event}})$ bins, over $0 \leq z_{\text{event}} \leq 10$. The merger rate density at each bin is then obtained from Eqn. 5 with $\Delta N_{\text{obs}}$ replaced by $\Delta \rho_{\text{age}}$, where $\Delta \rho_{\text{age}}$ is the Universe-age difference corresponding to the redshift difference across the bin. The reference values of $[M_{\text{GC,low}}, M_{\text{GC,high}}]$ and $\rho_{\text{GC}}$ are applied (Sec. 2.2), i.e., $R(z_{\text{event}})$ in Fig. 4 is the reference cosmic merger rate density. The distribution of the merger delay times, $t_{\text{mrg}}$, for the Model Universe, as obtained from this sample cluster population, is also shown in Fig. 4 (right panel). Due to the predominance of shorter $t_{\text{mrg}}$, $R(z_{\text{event}})$ has a global

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**Figure 2.** The filled histograms give the differential merger rate density (Y-axis), as obtained from the Model Universe cluster population (Sec. 2.2), as a function of maximum LIGO-Virgo-KAGRA-band eccentricity (Sec. 3.1), $e_m$, along the X-axis. The upper and lower limits (histogram error bars) represent the reference and the pessimistic rates (Sec. 2.2, Table 1), respectively. The heights of the histogram boxes lie halfway between these two values (at approximately half of the reference value). The left (right) panel corresponds to the visibility boundary (for average source inclination) at the redshift $z_{\text{max}} = 1.0$ ($z_{\text{max}} = 2.0$).
Figure 3. The black, filled squares give the reference present-day intrinsic merger rate density (Y-axis), from the Model Universe cluster population (Sec. 2.2), as a function of the detector visibility boundary (for average source inclination) redshift, \( z_{\text{max}} \) (X-axis). The grey, filled triangle gives the merger rate density, for \( z_{\text{max}} = 1 \), for a Model Universe cluster population with the cluster initial mass function being a power law of index \( \alpha = -2.5 \) (Sec. 4). The grey, filled circle gives the merger rate density, for \( z_{\text{max}} = 1 \), for a Model Universe cluster population with the cluster initial mass function truncated at the lower limit of \( M_{\text{cl,low}} = 5 \times 10^4 M_{\odot} \) (Sec. 4). The blue line indicates the median value of the BBH merger rate density estimated from GWTC-1 (Abbott et al. 2019b), for their Model C, and the blue-shaded background represents the corresponding 90% credible interval. The three orange lines are the median and the 90% credible limits for the BBH merger rate density estimated from GWTC-2 (The LIGO Scientific Collaboration et al. 2020).

Figure 4. Left panel: The blue line gives the evolution of the (reference) cosmic merger rate density \( \dot{R}(z_{\text{event}}) \) with merger-event redshift, \( z_{\text{event}} \) (X-axis), as obtained from the Model Universe cluster population (Sec. 3.2). For visual comparison, the gray line shows the variation of cosmic star formation rate with redshift (Madau & Dickinson 2014, not to scale along the Y-axis). Right panel: the delay time \( t_{\text{mrg}} \) distribution of the Model Universe cluster population. The \( \dot{R}(z_{\text{event}}) \) function and the \( t_{\text{mrg}} \) distribution are constructed based on a sample of \( N_{\text{samp}} = 10^6 \) clusters. To construct \( \dot{R}(z_{\text{event}}) \), \( 0 \leq z_{\text{event}} \leq 10 \) is divided into 100, equal-sized \( \log_{10} \) bins.

peak at a redshift \( z_{\text{event}} = z_{\text{peak}} \) very close to the cosmic SFH peak redshift, \( i.e., \) at \( z_{\text{peak}} \approx 1.85 \) (Fig. 4, left panel) \(^\text{10}\); see also Santoliquido et al. (2020). In contrast, \( \dot{R}(z_{\text{max}}) \) peaks at \( z_{\text{max}} \approx 3.5 \) (see above; Fig. 3). Since within a volume enclosed by a spherical boundary at \( z_{\text{max}} \) the sources are located closer

distribution. The distinct peak of \( \dot{R}(z_{\text{event}}) \) at \( z_{\text{event}} \approx 10 \) (Fig. 4, left panel) is caused by the distinct peak in the smallest \( t_{\text{mrg}} \) bin (Fig. 4, right panel).
to $z_{\text{max}}$ with a higher probability (for a uniform spatial density; see Sec. 3.1), $\mathcal{R}(z_{\text{max}})$ will lie in between $\mathcal{R}'(0)$ (Eqn. 5(peak)) and $\mathcal{R}'(z_{\text{max}})$ for $z_{\text{max}} \leq z_{\text{peak}}$ (for $z_{\text{max}} > z_{\text{peak}}$). Hence are the $\mathcal{R}(z_{\text{max}})$ values (Fig. 3).

4 DISCUSSIONS: UNCERTAINTIES IN MERGER RATE DENSITY

The uncertainties in the (differential) merger rate density, as obtained in this study, is mainly driven by the various astrophysical limits and factors in Eqn. 5. While the ‘laws’ that enter Eqn. 5, i.e., $\Phi_{\text{CLMF}}(M_d)$ and $\Phi_{\text{SFH}}(z_d)$ (plus, implicitly, the CDM Universe with the Planck Collaboration et al. 2020 parameters that partly determines $N_{\text{arg}}/N_{\text{samp}}$), are based on observations (see Sec. 2.2 and references therein), $\mathcal{R}$ strongly depends on $M_{d,\text{low}}$ and $M_{\text{GC,low}}$. $\Phi_{\text{CLMF}}$ being a power law of index $\alpha = -2$. Also, $\mathcal{R}$ simply proportionates with $\rho_{\text{GC}}$. The strong dependence on these quantities is clear from the large difference in value between the reference $\mathcal{R}$ and its pessimistic counterpart, $\mathcal{R}_-$. (see Sec. 2.2; Table 1).

Although $\alpha = -2$ is the ‘widely accepted’ value of the cluster birth mass function index (based mainly on photometric mass estimates of young, gas-free clusters; see, e.g., Gieles et al. 2006; Larsen 2009; Portegies Zwart et al. 2010; Bastian et al. 2012), observations also suggest potential moderate variations of $\alpha$ (e.g., Ryon et al. 2015). To probe the dependence of $\mathcal{R}$ on moderate alterations of $\alpha$, a sample cluster population is constructed as described in Sec. 2.2 but out of a $\Phi_{\text{CLMF}}$ with $\alpha = -2.5$ ($z_{\text{max}} = 1$ is assumed). Despite the resulting $N_{\text{arg}}/N_{\text{samp}}$ is somewhat smaller compared to that with $\alpha = -2$ (as expected, since less massive clusters, which are more predominant for $\alpha = -2.5$, tend to produce less number of mergers per cluster; see Table C1 of Paper II), the corresponding $\mathcal{R}$ is $\approx 3.5$ times higher (compare between the $z_{\text{max}} = 1$ entries in Table 1). This rate is also indicated in Fig. 3 (the grey, filled triangle), which exceeds the 90% credible upper limit from GWTC-1.

It would also be of interest to examine the impact of the lower mass cutoff, $M_{d,\text{low}}$, of $\Phi_{\text{CLMF}}$ on $\mathcal{R}$. With $R_{\text{mon}} = 1$ (Sec. 2.2, Eqn. 5), $M_{d,\text{low}}$ serves as an effective cutoff: clusters either fail to assemble efficiently as gas-free, gravitationally-bound young clusters or preferentially get destroyed after successful assembly due to environmental effects (e.g., interactions with molecular clouds) with initial masses below $M_{d,\text{low}}$. A sample cluster population is constructed with $M_{d,\text{low}} = 5 \times 10^4 M_\odot$, but the other ingredients being as default (see Sec. 2.2) and $z_{\text{max}} = 1$. Despite the resulting $N_{\text{arg}}/N_{\text{samp}}$ is nearly doubled (as expected, since more massive clusters tend to produce a larger number of mergers per cluster; see Table C1 of Paper II), the corresponding $\mathcal{R}$ is nearly halved (compare between the $z_{\text{max}} = 1$ entries in Table 1). This rate is indicated in Fig. 3 (the grey, filled circle), which lies close to the GWTC-2 median value.

With the default sample cluster population (Sec. 2.2), $\mathcal{R}$, at $z_{\text{max}} = 1$, is close to the 90% credible upper limit of the GWTC-2 BBH merger rate density (Sec. 3.2, Fig. 3). Nevertheless, the median GWTC-2 value is obtained by calculating the reference $\mathcal{R}$ from Eqn. 5 with a slightly lower $M_{\text{GC,low}} = 3.9 \times 10^5 M_\odot$ instead of the default reference lower mass limit of GC progenitors ($M_{\text{GC,low}} = 5 \times 10^5 M_\odot$; Sec. 2.2). This altered $M_{\text{GC,low}}$ is still consistent with being progenitors of present-day GCs (Kremer et al. 2020). Fig. 5 shows the Model Universe $d\mathcal{R}/dM_1$ and $d\mathcal{R}/dq$, for $z_{\text{max}} = 1$, corresponding to the integrated $\mathcal{R}$ matching the GWTC-2 median rate (23.9 yr$^{-1}$Gpc$^{-3}$). As seen in Fig. 5, the Model Universe (renormalized) reference differential merger rate densities agree reasonably with the GWTC-2 median and 90% credible limit merger rate densities, for their ‘power law + peak’ BH mass model (The LIGO Scientific Collaboration et al. 2020) $^{12}$. This prior for BH masses in merging BBHs is similar to the Model C prior (Sec. 2.2) used to obtain the GWTC-1 differential merger rate densities (Abbott et al. 2019b).

The Model Universe $d\mathcal{R}/dM_1$, for $z_{\text{max}} = 1$, is somewhat focussed towards $M_1 \gtrsim 20 M_\odot$ whereas the corresponding GWTC-2 differential merger rate density has a dominant peak below $M_1 \lesssim 10 M_\odot$ (Fig. 5, left panel). As discussed in Sec. 3.1, the mass distribution of BHs, which retain in the present model clusters after birth and which pair-up via dynamical interactions, is far from being a well defined, simple function such as a power-law plus peak. With the inclusion of BBH merger events involving primaries closer to the mass gap and a larger number of BNS mergers, the GWTC-2 differential merger rate density extends down to $M_1$ that is very similar to that from the Model Universe (see Fig. 5, left panel; Sec. 3.1). It is clear that with increasingly improved constraints on (differential) compact-binary merger rate density from LVK observations, it would be possible to provide constraints on widely debated issues regarding star cluster formation and large scale structure formation, e.g., mass dependence of cluster formation efficiency, $R_{\text{mon}}(M_1)$, and lower and upper mass limits, $[M_{\text{GC,low}}, M_{\text{GC,high}}]$, of GC progenitors (see, e.g., Rodriguez et al. 2015; Banerjee & Kroupa 2018; Kruijssen et al. 2019; El-Badry et al. 2019; Krumholz et al. 2019). The differential merger rate density profiles would also help constraining the relative contributions of the various other channels for producing compact binary mergers, e.g., dynamical evolution of field hierarchical systems, mergers via Kozai-Lidov mechanism in galactic nuclei, and evolution of field massive binaries.

In this context, it is important to note that the present (differential) merger rate density estimates, based on the Model Universe sample cluster population (Sec. 2.2), do not assume any absolute (cosmic) star cluster formation efficiency as a result of the cosmic star formation processes. What goes into the rate estimates is the number of gas-free, bound clusters forming per unit comoving volume, beyond a certain threshold (birth) mass $M_{d,\text{low}}$, relative to the observed number density of present-day GCs, $\rho_{\text{GC}}$. In other words, the formation efficiency of lower mass clusters relative to the massive progenitor clusters of what we now call GCs. The present $\text{11}$ Interestingly, this GC-progenitor lower limit is very close to that in Antonini & Gieles (2020a).

$\text{12}$ The data for the differential merger rate densities are obtained from the public repository of GWTC-2 at https://dcc.ligo.org/LIGO-P2000434/public. A modified version of the Python script provided in the directory Fig-3-m1-ppd is utilized to extract the relevant data from the dataset located in the directory Multiple-Fig-Data.
The histograms are the same as in the top panels of Fig. 1 (i.e., corresponds to the default sample cluster population with $z_{\text{max}} = 1$; see Sec. 2.2, Table 1) but renormalized so that the reference values (histogram upper limits) sum up to the median BBH merger rate density as estimated from GWTC-2 (The LIGO Scientific Collaboration et al. 2020). The moderate renormalization corresponds to applying a somewhat smaller GC-progenitor lower mass limit, $M_{\text{GC,low}} = 3.9 \times 10^3 M_\odot$ (Sec. 4), in obtaining the reference $\mathcal{R}$ (using Eqn. 5) instead of the default reference lower limit ($M_{\text{GC,low}} = 5 \times 10^3 M_\odot$; Sec. 2.2). The orange lines are the median (central line) and the 90% credible limits (upper and lower lines) of the differential BBH merger rate density as obtained for GWTC-2 by The LIGO Scientific Collaboration et al. (2020, their ‘power law + peak’ model).

The present estimate incorporates an additional boost factor of $\approx 3.4$ due to YMCs and OCs undergoing a much longer SFH, up to $z = 0$, in contrast to the progenitors of present-day GCs (Eqn. 5; Sec. 2.2). The time evolution of the cluster mass function, beginning from $\Phi_{\text{CLMF}}$, is naturally incorporated in the present work since the sample cluster population is built based on long term evolutionary model clusters (Sec. 2.2; Paper II).

The Model Universe merger rate densities and differential merger rate density profiles, as obtained from the present computed cluster model sets, can be moderately affected by the incomplete model grid (Paper II). Nevertheless, it is ensured that the intended $\Phi_{\text{CLMF}}(M_\odot)$ and $\Phi_{\text{SFH}}(z)$ are achieved for each sample cluster population. The present model clusters do not incorporate any ‘relic’ of the pre-assembly violent-relaxation phase, e.g., initial substructures and initial mass segregation. However, such details are unlikely to have an impact on the merger yield of the model clusters as discussed in Banerjee (2020a, Sec. II. A. and references therein).

The present mock detection experiments do not incorporate detector sensitivity curves and GW strain for the mergers’ luminosity distances (Paper II), since only intrinsic merger rate densities are evaluated. Such a mock detection experiment can be straightforwardly extended to include a signal-to-noise-ratio threshold, as described in Banerjee (2020a), which will be taken up in a future study to estimate the merger detection counts with ground-based GW detectors.

Although the $\Lambda$CDM cosmological framework is adopted in the present work, any alternative Universe framework can also be incorporated in the current approach with alternative simulation lies below the ‘turn over’ mass of the present-day GC mass distribution, which mass range primarily gets depleted in the ‘turning over’ process and, hence, dominantly contributes to the boost factor.

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13 This is expected since $[M_{\text{cl,low}}, M_{\text{cl,high}}]$ for the sample cluster pop-
interrelations between redshift, Universe age, and light travel time.

5 SUMMARY AND OUTLOOK

This work estimates the present-day intrinsic merger rate density and its derivatives (i.e., the differential merger rate densities) w.r.t. the merging binary’s primary mass, mass ratio, and eccentricity from GR compact binary mergers occurring due to dynamical interactions in YMCs and OCs. To that end, a set of computed model clusters, with up to date stellar-evolutionary and stellar remnant formation schemes and PN treatment of compact binary merger events (Paper I; Paper II; Sec. 2.1), is utilized to construct sample cluster populations in a CDM Model Universe (Sec. 2.2). From such a sample cluster population, merger GW signals are accumulated at the present cosmic epoch ($z = 0$) in an idealized LVK-type detector (Sec. 2.2). The model clusters, initially, have masses spanning over $2 \times 10^3 M_\odot \leq M_\odot \leq 10^5 M_\odot$, half-mass radii over $1 \text{ pc} \leq r_h \leq 3 \text{ pc}$, and metallicity over $0.0001 \leq Z \leq 0.02$ and are composed of stars following a standard IMF with the O-type stars being in an observationally-motivated distribution of primordial binaries (Sec. 2.1; Paper II: Sana & Evans 2011; Moe & Di Stefano 2017). Such initial model cluster properties are consistent with those observed in fully-grown, (near-)gas-free, (near-)spherical YMCs. Each sample cluster population is constructed (Sec. 2.2) following initial cluster mass distribution $\rho_{\text{CLMF}}(M_A) \sim M_A^{-2}$, cosmic SFH (Madau & Dickinson 2014), and cosmic metallicity evolution (Chruslinska & Nelemans 2019) that are all derived from observations. The member clusters of a sample (of size $N_{\text{samp}} = 5 \times 10^5$) are distributed in the Model Universe with a uniform spatial distribution and within a detector visibility horizon located at a redshift $z_{\text{max}}$. That way, sample cluster populations are obtained with $z_{\text{max}}$ varying from 1.0 to 10.0, which cluster populations produce merger event counts, $N_{\text{merg}}$, at the present epoch (Sec. 3, Table 1). The resulting $N_{\text{merg}}/N_{\text{samp}}$ values are then scaled to estimate the merger rate densities and differential merger rate densities of the Model Universe (Sec. 2.2).

For $z_{\text{max}} = 1$, which represents the detector horizon for LVK O1-O3 observing runs, the Model Universe reference (pessimistic) BBH merger rate density is evaluated to be $\mathcal{R} = 37.9 \text{ yr}^{-1}\text{Gpc}^{-3}$ ($\mathcal{R} = 0.51 \text{ yr}^{-1}\text{Gpc}^{-3}$) (Sec. 3, Table 1). This merger rate density range well accommodates the GWTC-1 BBH merger rate density (Abbott et al. 2019b) and as well the much more constrained GWTC-2 BBH merger rate density (The LIGO Scientific Collaboration et al. 2020), given the median values and the 90% credible intervals of the GWTC estimates (Sec. 3; Fig. 3). Also, the $z_{\text{max}} = 1$ Model Universe differential merger rate densities w.r.t. primary mass and mass ratio $[d\mathcal{R}_- / dM_1, d\mathcal{R}_- / dM_2]$ and $[d\mathcal{R}_- / dq, d\mathcal{R}_- / dq]$, respectively) well accommodate the corresponding differential merger rate densities estimated from GWTC-1 and GWTC-2 (Fig. 1; Sec. 3.1; Fig. 5; Sec. 4). The large difference between the reference $\mathcal{R}$ and the pessimistic $\mathcal{R}_-$ (and, hence, between $d\mathcal{R}_- / dX$ and $d\mathcal{R}_- / dX$) arises due to large uncertainties in the spatial number density of GCs, $\rho_{\text{GC}}$, that is used as a ‘tracer’ to estimate the population of clusters formed, until the present epoch, within the (comoving) volume enclosed by the detector horizon at $z_{\text{max}}$ (Sec. 2.2; Sec. 4). The difference arises as well due to the uncertainties in the lower mass limit, $M_{\text{GC,low}}$, of the progenitors of present-day GCs (Sec. 2.2; Sec. 4). The Model Universe $\mathcal{R}$ also depends on the power law index, $\alpha$, and on the lower mass cutoff, $M_{\text{cl,low}}$, of the cluster birth mass function (Fig. 3; Sec. 4). With improving constraints on (differential) merger rate density from further observations of compact binary merger events, such widely debated quantities related to large scale structure formation and cosmic star formation can be better constrained. The Model Universe yields eccentric LVK mergers from YMCs and OCs at the current epoch, with a (reference) merger rate density of $\mathcal{R}_{\text{ec}} \approx 5.0 \text{ yr}^{-1}\text{Gpc}^{-3}$ for $z_{\text{max}} = 1$ (Fig. 2; Sec. 3.1).

The Model Universe $\mathcal{R}$ depends on $z_{\text{max}}$, maximizing to $\mathcal{R} = 64.1 \text{ yr}^{-1}\text{Gpc}^{-3}$ at $z_{\text{max}} \approx 3.5$, most of the growth in $\mathcal{R}$ occurring within $z_{\text{max}} \lesssim 2.0$ (Fig. 3; Table 1; Sec. 3.2). This $z_{\text{max}}$ dependence is due to the cosmic variation of star formation rate (the SFH) and the distribution of merger delay time, $t_{\text{merg}}$, of the Model Universe (Fig. 4, right panel), resulting in an inherent dependence, $\mathcal{R}(z_{\text{even}})$, of merger rate density on merger-event redshift (the cosmic merger rate density function; Fig. 4, left panel; Sec. 3.2).

The Model Universe (differential) merger rate density will further improve as the computed model grid (Paper II) gets more complete and refined. Additional N-body computations to that end are ongoing and the updated results will be presented in a future paper. It would also be worth comparing the outcomes of the present Model Universe with those from Universes that incorporate alternative cosmic metallicity evolution, e.g., those of Rafelski et al. (2012); Madau & Fragos (2017). For a more complete and consistent treatment of the formation and evolution of star cluster population in the Universe over cosmic time (and, hence, of dynamical merger rate estimates from them), in relation to the present-day GC population, it is necessary to combine evolutionary models of clusters as in here (of $\approx 10^4 M_\odot - 10^5 M_\odot$) with those of lower mass clusters as in, e.g., Rastello et al. (2019); Di Carlo et al. (2019); Kumamoto et al. (2019) and of much higher mass GC-progenitor clusters as in, e.g., Askar et al. (2017); Kremer et al. (2020). GWTC-2 has been released whilst preparing this manuscript which is why the GWTC-2 data is addressed somewhat briefly here and a more elaborate comparison (as in, e.g., Paper II, which addresses the GWTC-1 merger-event data) would be worth undertaking. The present work motivates such future lines of research.

This study suggests that with reasonable astrophysical inputs based on Local Universe and cosmological observations (Sec. 2.2; Sec. 4), dynamical interactions in YMCs and OCs can, in principle, explain the BBH merger rate density and the corresponding differential rate densities as estimated, so far, from LVK GW events, without invoking additional channels for producing compact binary mergers. As the (differential) merger rate density gets increasingly constrained with forthcoming merger-event detections, the relative role of the various channels of compact binary mergers will be better understood.

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DATA AVAILABILITY

The GWTC-1 data utilized in this article is publicly available at the URL https://dcc.ligo.org/LIGO-P1800324/public and is described in the paper Abbott et al. (2019b). The GWTC-2 data utilized in this article is publicly available at the URL https://dcc.ligo.org/LIGO-P2000434/public and is described in the paper The LIGO Scientific Collaboration et al. (2020). The redshift-metallicity relation data is obtained from the public repository provided in Chruslińska & Nelemans (2019). Further details on how these data are accessed are provided in the text. The simulation data underlying this article will be shared upon reasonable request to the corresponding author.

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