SEARCHING FOR SPACE-TIME ASYMMETRIES IN PARTICLE PRODUCTION

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The possibilities of unlike particle correlations for a study of the space-time asymmetries in particle production, including the sequence of particle emission, are demonstrated.

1 Introduction

The correlations of particles at small relative velocities are widely used to study space-time characteristics of the production processes. Particularly, for non-interacting identical particles, like photons, this technique is called intensity or particle interferometry. In this case the correlations appear solely due to the effect of quantum statistics (QS).\(^1\)^\(^2\) Similar effect was first used in astronomy to measure the angular radii of stars by studying the dependence of the two-photon coincidence rate on the distance between the detectors (HBT effect \(^3\)). In particle physics the QS interference was first observed as an enhanced production of the pairs of identical pions with small opening angles (GGLP effect \(^1\)). Later on, similar to astronomy, Kopylov and Podgoretsky\(^2\) suggested to study the interference effect in terms of the correlation function.\(^a\)

The effect of QS is usually considered in the limit of a low phase-space density such that the possible multi-particle effects can be neglected. This approximation seems to be justified by present experimental data which does not point to any spectacular multi-boson effects neither in single-boson spectra nor in two-boson correlations. These effects can be however of some importance for realistic simulations of heavy ion collisions.\(^4\) They can also clearly manifest themselves in some rare events (\(e.g.,\) those with large pion multiplicities) or in the eventually overpopulated regions of momentum space (see, \(e.g.,\)\(^5\)^\(^6\) and references therein).\(^b\)

\(^a\)Note that though both the KP and HBT methods are based on the QS interference, they represent just orthogonal measurements.\(^2\) The former, being the momentum-energy measurement, yields the space-time picture of the source, while the latter does the opposite. In particular, the HBT method provides the information about the angular size of a star but, of course, - no information about the star radius or its lifetime.

\(^b\)It was shown\(^5\)^\(^6\) using the analytically solvable model with the Gaussian distribution of the emission points characterized by the dispersions \(\Delta^2\) and \(r_0^2\) in the momentum and ordinary
The particle correlations are also influenced by the effect of particle interaction in the final state (FSI). Thus the effect of the Coulomb interaction dominates the correlations of charged particles at very small relative momenta (of the order of the inverse Bohr radius of the two-particle system), respectively suppressing or enhancing the production of particles with like or unlike charges. Regarding the effect of the strong FSI, it is quite small for pions, while for nucleons it is often a dominant one due to the very large magnitude of the s-wave singlet scattering length of about 20 fm.

Though the FSI effect complicates the correlation analysis, it is an important source of information allowing one to measure the space-time characteristics of the production process even with the help of non-identical particles. Moreover, the unlike particle correlations, contrary to those of identical particles, are sensitive to the relative space-time asymmetries in their production, e.g. - to the relative time delays, thus giving an important complementary information not accessible to the standard interferometry measurements. In the following we will briefly formulate the theory of these correlations and demonstrate the possibilities of the corresponding correlation technique.

2 Formalism

As usual, we will assume sufficiently small phase-space density of the produced multi-particle system, such that the correlation of two particles emitted with a small relative velocity in nearby space-time points is influenced by the effects of their mutual QS and FSI only. We define the ideal two-particle correlation function \( R(p_1, p_2) \) as a ratio of the differential two-particle production cross section to the reference one which would be observed in the absence of the effects of QS and FSI. In heavy ion or high energy hadronic collisions we can neglect kinematic constraints and most of the dynamical correlations and construct the reference distribution by mixing the particles from different events.

Assuming the momentum dependence of the one-particle emission probabilities inessential when varying the particle 4-momenta \( p_1 \) and \( p_2 \) by the amount characteristic for the correlation due to QS and FSI (smoothness assumption), i.e. assuming that the components of the mean space-time distance between particle emitters are much larger than those of the space-time extent of the emitters, we get the well-known result of Kopylov and Podgoretsky for identical particles, modified by the substitution of the plane wave \( e^{i(p_1 \cdot x_1 + ip_2 \cdot x_2)} \) by space respectively, that a pion condensate of a small momentum dispersion of \( \Delta /2r_0 \ll \Delta^2 \) gradually develops with the increasing phase-space density.

\(^c\)This assumption may be not valid in the case of low energy heavy ion reactions when the particles are produced in a strong Coulomb field of residual nuclei. To deal with this field a quantum adiabatic approach can be used.
the nonsymmetrized Bethe-Salpeter amplitudes in the continuous spectrum of
the two-particle states $\psi_{p_1 p_2}(x_1, x_2)$, where $x_i = \{t_i, r_i\}$ are the 4-coordinates
of the emission points of the two particles and $S$ is their total spin. At equal
emission times in the two-particle c.m.s. ($t^* = t_1^* - t_2^* = 0$) this amplitude
coincides (up to an unimportant phase factor due to the c.m.s. motion) with a
stationary solution of the scattering problem $\psi_{-k^*}(r^*)$, where $k^* = p_1^* = -p_2^*$
and $r^* = r_1^* - r_2^*$ (the minus sign of the vector $k^*$ corresponds to the reverse
in time direction of the emission process). The Bethe-Salpeter amplitude can
be usually substituted by this solution (equal time approximation). Then, for
nonidentical particles,
\[
R(p_1, p_2) = \sum_S \rho_S \langle |\psi_{S}(+)(r^*)|^2 \rangle_S. \tag{1}
\]
Here the averaging is done over the emission points of the two particles in a
state with total spin $S$ populated with the probability $\rho_S$. $\sum_S \rho_S = 1$. For identical particles, the amplitude in Eq. (1) should be properly symmetrized:
\[
\psi_{-k^*}(r^*) \rightarrow [\psi_{-k^*}(r^*) + (-1)^S \psi_{k^*}(r^*)]/\sqrt{2}. \tag{2}
\]
3 Measuring the relative space-time asymmetries

The correlation function of two nonidentical particles, compared with the iden-
tical ones, contains a principally new piece of information on the relative space-
time asymmetries in particle emission. This is clearly seen in the case of
neutral particles when the two-particle amplitude $\psi_{-k^*}(r^*)$ takes the form
\[
\psi_{-k^*}(r^*) = e^{-ik^*r^*} + \phi_{k^*}^S(r^*), \tag{3}
\]
where the scattered wave $\phi_{k^*}^S(r^*)$, in the considered region of small relative
momenta, is independent of the directions of the vectors $k^*$ and $r^*$. Inserting
Eq. (3) into the formula (1) for the correlation function, we can see that the
latter is sensitive to the relative space-time asymmetry due to the odd term
\[d\]The equal time approximation is valid on condition $|t^*| \ll m_2, r^2$ for sign($t^*$) = ±1
respectively. This condition is usually satisfied for heavy particles like kaons or nucleons. But
even for pions, the $t^* = 0$ approximation merely leads to a slight overestimation (typically
< 5%) of the strong FSI effect and, it doesn’t influence the leading zero–distance ($r^* \ll |a|$)
\[e\]For unpolarized particles with spins $s_1$ and $s_2$ the probability $\rho_S = (2S+1)/[(2s_1+1)(2s_2 + 1)]$. Generally, the correlation function is sensitive to particle polarization. For example, if
two spin-1/2 particles are emitted with polarizations $P_1$ and $P_2$ then $\rho_0 = (1 - P_1 \cdot P_2)/4$
and $\rho_1 = (3 + P_1 \cdot P_2)/4$.
for the classically forbidden region becomes important and compensates the deviation of the Coulomb factor from unity except for the correlation functions \( R \) and approach 1 at large values of \( v \). As the sign of the time difference \( t = t_1 - t_2 \) is transferred to the correlation function through the odd in the Coulomb wave function is of increasing importance with a narrowing with the increasing residual charge.\(^7\)

It is clear that in the case of a dominant time asymmetry, \( v|\langle t \rangle| \gg |\langle r_L \rangle| \), a straightforward way to determine the mean time delay \( \langle t \rangle \) is to measure the correlation functions \( R_+ (k^*v \geq 0) \) and \( R_- (k^*v < 0) \). Depending on the sign of \( \langle t \rangle \), their ratio \( R_+ / R_- \) should show a peak or a dip in the region of small \( k^* \) and approach 1 at large values of \( k^* \). As the sign of the scalar product \( k^*v \) is practically equal to that of the difference of particle velocities \( v_1 - v_2 \) (this equality is always valid for particles of equal masses), the sensitivity of the correlation functions \( R_+ \) and \( R_- \) to the sign of the difference of particle

\(^7\)This factor substantially deviates from unity only at \( k^* < 2\pi/|a| \) (e.g., at \( k^* < 22 \text{ MeV}/c \) for two protons). Note that for the distances \( r^* > |a| \) the confluent hypergeometrical function becomes important and compensates the deviation of the Coulomb factor from unity except for the classically forbidden region of \( k^* < (|a|r^*/2)^{-1/2} \), narrowing with the increasing \( r^* \). Thus the FSI practically vanishes if at least one of the two particles comes from a long lived source (\( \eta, \eta', \Lambda, K^0_s, \ldots \)).
emission times has a simple classical explanation. Clearly, the interaction between the particles in the case of an earlier emission of the faster particle will be weaker compared with the case of its later emission (the interaction time being longer in the latter case leading to a stronger correlation). This expectation is in accordance with Eqs. (1) and (4) at \( k^* \to 0, \langle r^* \rangle \ll |a| \) and \( \langle |φ_{Sck}^*(r^*)| \rangle \ll 1 \), when (the arrow indicates the limit \( \nu|\langle t \rangle| \gg |\langle r_L \rangle| \)):

\[
R_+/R_- \approx 1 + 2 \frac{\langle r_L^2 \rangle}{a} \to 1 - 2 \frac{\langle γv(t_1 - t_2) \rangle}{a}.
\] (5)

The sensitivity of the \( R_+/R_- \) correlation method to the mean relative time shifts (introduced \textit{ad hoc}) was studied\(^6\) for various two-particle systems simulated in \( Pb+Pb \) collisions at SPS energy using the event generator VENUS 5.14\(^1\)\(^1\)\(^1\). The scaling of the effect with the space-time asymmetry and with the inverse Bohr radius \( a \), indicated by Eq. (5), was clearly illustrated for the \( K^+K^- \) system \((a = -110 \text{ fm})\) and for the like- and unlike-sign \( πK, πp \) and \( Kp \) systems \((a = \pm 249, \pm 226 \text{ and } \pm 84 \text{ fm respectively})\). It was concluded that for sufficiently relativistic pairs \((γv > 0.5)\) the \( R_+/R_- \) ratio can be sensitive to the shifts in the particle emission times of the order of a few \( \text{fm}/c \). Motivated by this result the \( R_+/R_- \) method was recently applied to the \( K^+K^- \) system simulated in a two-phase thermodynamical evolution model and the sensitivity was demonstrated to the production of the transient strange quark matter state even if it decays on strong interaction time scales\(^1\)\(^2\).

The method sensitivity was also studied for AGS and SPS energies using the transport code RQMD v2.3\(^1\)\(^3\). Thus at SPS energy the central \( Pb+Pb \) collisions have been simulated and the \( π^+K^+, π^+p \) and \( K^+p \) correlations have been studied\(^1\)\(^4\). To get rid of the effect of a fast longitudinal motion, the study was done in the longitudinally co-moving system (LCMS) in which the pair is emitted transverse to the reaction axis so that \( v = v_⊥, r_L = \Delta x \) and

\[
\Delta x^* = γ_⊥(\Delta x - v_⊥ Δt), \quad Δy^* = Δy, \quad Δz^* = Δz.
\] (6)

The simulated correlation functions \( R_+, R_- \) and their ratios are plotted in Fig. 1. We can see that for \( π^+p \) and \( π^+K^+ \) systems these ratios are less than unity at small values of \( q \equiv k^* \), while for \( K^+p \) system the ratio \( R_+/R_- \) practically coincides with unity. These results well agree with the mean values of \( Δt, Δx \) and \( Δx^* \) presented in Table 1 \((\langle Δy \rangle \approx \langle Δz \rangle \approx 0)\). It can be seen from Eqs. (5) and (6) that the absence of the effect in the \( R_+/R_- \) ratio for the \( K^+p \) system is due to practically the complete compensation of the space and time asymmetries leading to \( Δx^* \approx 0 \). For \( π^+p \) system the effect is determined mainly by the x-asymmetry. For \( π^+K^+ \) system both the x- and time-asymmetries contribute in the same direction, the latter contribution
being somewhat larger. The separation of the relative time delays from the spatial asymmetry is, in principle, possible (see Eq. (6)) by studying the ratio $R_+/R_-$ in different intervals of the pair velocity.

At AGS energy the Au + Au collisions have been simulated and the $\pi^+p$ correlations have been studied in the projectile fragmentation region where proton directed flow is most pronounced and where the proton and pion sources are expected to be shifted relative to each other both in the longitudinal and in the transverse (flow) directions in the reaction plane. It was demonstrated that a modification of the $R_+/R_-$ method (± corresponding now to the signs of the respective components $k_i^*$) is sufficiently sensitive to reveal these shifts.\textsuperscript{15}

At low energies, the particles in heavy ion collisions are emitted with the characteristic emission times of tens to hundreds fm/c so that the observable time shifts should be of the same order.\textsuperscript{10} In fact the $R_+/R_-$ method has already been successfully applied to study proton-deuteron correlations in several heavy ion experiments at GANIL.\textsuperscript{16,17,18} It was observed, in agreement with the coalescence model, that deuterons are on average emitted earlier than protons.

4 Conclusion

We have shown that unlike particle correlations, compared with those of identical particles, contain a principally new piece of information on the relative space-time asymmetries in particle emission, thus allowing, in particular, a measurement of the mean relative delays in particle emission at time scales as small as $10^{-23}$ s. To determine these asymmetries, the unlike particle correlation functions $R_+$ and $R_-$ have to be studied separately for positive and negative values of the projection of the relative momentum vector in pair c.m.s. on the pair velocity vector or, generally, - on any direction of interest. We have presented here the results of recent studies of these correlation functions for a number of two-particle systems simulated with various event generators. It was shown that the $R_+/R_-$ ratio is sufficiently sensitive to the relative space-time asymmetries arising due to the formation of the quark-gluon plasma and strangeness distillation and even to those expected in the usual dynamical scenarios at AGS and SPS energies. As to the detection of the unlike particles with close velocities ($p_1/m_1 \approx p_2/m_2$), there is no problem with the two-track resolution since these particles, having either different momenta or different charge-to mass ratios, have well separated trajectories in the detector magnetic field. For the same reason, however, a large momentum acceptance of the detector is required.
Acknowledgments

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Table 1: Mean values of the relative space-time coordinates in LCMS (in fm) calculated\textsuperscript{14} from RQMD (v2.3) for 158 A-GeV Pb + Pb central collisions.

| system | $\langle \Delta t \rangle$ | $\langle \Delta x \rangle$ | $\langle \Delta x - v_\perp \Delta t \rangle$ | $\langle \Delta x^* \rangle$ |
|--------|-----------------|-----------------|-----------------|-----------------|
| $\pi^+ p$ | -0.5 | -6.2 | -6.4 | -7.9 |
| $\pi^+ K^+$ | 4.8 | -2.7 | -5.8 | -7.9 |
| $K^+ p$ | -5.5 | -3.2 | -0.6 | -0.4 |

Figure 1: Unlike particle correlation functions $R_+$ and $R_-$ and their ratios simulated with RQMD for mid-rapidity particle pairs $\pi^+ p$, $\pi^+ K^+$, and $K^+ p$ in Pb + Pb collisions at SPS energy.