Effect of Laser Shock Strengthening on Modal Frequency

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Abstract. The phenomenon of laser shock strengthening (LSP) changing the resonance frequency of a structure can be used to adjust the frequency of aero-engine blades, and has a broad application prospect. In this paper, through the combination of mechanical analysis, numerical simulation and vibration test, the effect of LSP technology on structural vibration characteristics is studied. The influence mechanism of local stiffness changes on the modal frequency of the structure is analyzed from the perspective of vibration and energy. It is found that when the laser shock is strengthened in a region with large bending deformation, the greater the contribution to the overall modal stiffness and elastic potential energy, the greater the modal frequency. The impact is also greater. Established a mapping relationship among laser shock strengthening, local stiffness and modal frequency, and finally proved that LSP technology can significantly change the local elastic modulus of TC4 titanium alloy, which provides a reliable application of the technology in the production department Theoretical support.

1. Introduction
The aeroengine blades have a harsh working environment. As one of the key components of aeroengines, the vibration problem is significant, and failures occur frequently [1-4]. Adjusting the resonance frequency to avoid the source of excitation is an important way to reduce blade vibration. Common methods for adjusting the resonance frequency mainly include: thickening or thinning the blade cross section, removing the local material of the blade body or blade tip, designing reinforcement ribs, etc [5].

Surface strengthening technology, such as ultrasonic surface strengthening[6-9], laser impact strengthening[10-15], surface shot peening [16-17], surface rolling[18-20], etc., as a post-processing technology, features It is not changing the geometry and quality characteristics of the material, and significantly improving the overall mechanical properties of the structure. It is a research hotspot in the current industrial field.

The phenomenon of structural vibration characteristics changes caused by surface strengthening is commonly seen in the test results of related literature on surface strengthening technology to improve structural vibration fatigue performance [2][3][4], but special research on the effect of surface strengthening technology on vibration characteristics is rare. In this paper, we study the effect of surface strengthening technology on vibration characteristics. Through a combination of mechanical analysis, finite element simulation and vibration test, a mapping model of laser shock strengthening-local stiffness change modal frequency change is established to use laser shock strengthening technology. Improving the vibration characteristics of the structure lays the foundation and contributes to improving the viability of the structure in the vibration environment.
2. Mechanical Analysis of the Influence of Local Stiffness on Modal Frequency

The modal frequency of the structure is closely related to the stiffness characteristics of the structure. The higher the stiffness corresponding to the modal characteristics, the higher the modal frequency. In this section, based on the beam model, the effect of local stiffness change on the overall modal frequency is obtained by means of vibration analysis and energy analysis, respectively. It is pointed out that the laser impact strengthening at the maximum bending stress has the most significant effect on the modal frequency.

2.1. Mechanical Analysis of Beam Model

As shown in Fig.3(a), the cantilever beam model is established, the x-axis direction is recorded as the axial direction and the z-axis direction is recorded as the transverse direction, and the transverse deformation and vibration of the beam model are studied. The Bernoulli-Euler beam model is adopted here, and the thickness of the beam is h. Without considering the shear deformation and the moment of inertia of the section around the neutral axis, the microelement of dx on the beam is taken, and the force analysis is shown in Fig. 3(b). The displacements of the upper point of the beam in the x and z directions are recorded as u and w respectively.

\[ D = \frac{1}{2} \int_{-h/2}^{h/2} E z^2 dz \]  

Equation (2.1)

When the beam is deformed laterally, the point strain and normal stress at a distance z from the central axis on the transverse section of the beam are:

\[ \begin{align*}
\varepsilon_z &= \frac{\partial u}{\partial x} = -z \frac{\partial^2 w}{\partial x^2} \\
\sigma_z &= E \varepsilon_z = -Ez \frac{\partial^2 w}{\partial x^2} 
\end{align*} \]

Equation (2.2)

From equation (2.2), the bending moment generated by normal stress is:

\[ M = D \frac{\partial^2 w}{\partial x^2} \]  

Equation (2.3)
Where \( \frac{\partial^2 w}{\partial x^2} \) represents the bending degree of the beam. The equilibrium equation is established by the micro-element force analysis shown in Fig.1(b):

\[
\begin{align*}
\sum F_i &= 0: -\frac{\partial Q}{\partial x} dx + q dx = 0 \\
\sum F_i &= 0: -\frac{\partial M}{\partial x} dx + Q dx = 0 
\end{align*}
\]  

(2.4)

Substituting equation (2.3) into the second equation in equation (2.4) —— the bending moment balance equation, and substituting the second equation into the first equation in equation (2.4) —— the force balance equation, eliminating \( dx \), gives:

\[
\frac{\partial^2}{\partial x^2}(D \frac{\partial^2 w}{\partial x^2}) = q 
\]  

(2.5)

When the beam model vibrates freely, embody \( q \) as the inertial force \(-\rho A \frac{\partial^2 w}{\partial x^2}\), where \( \rho \) is the density and \( A \) is the cross-sectional area of the beam:

\[
\frac{\partial^2}{\partial x^2}(D \frac{\partial^2 w}{\partial x^2}) + \rho A \frac{\partial^2 w}{\partial t^2} = 0 
\]  

(2.6)

Equation (2.6) is the fourth-order partial differential equation about the deflection function \( w \). According to the specific boundary conditions and the stiffness distribution function \( D \), the vibration mode of a certain mode, namely the deflection function \( w \), and its corresponding modal frequency can be obtained. However, it is difficult to obtain general guidance directly from the equation.

2.2. Energy Analysis of Thin Plate Model
Using the energy method to analyze the free vibration of the beam model, the elastic potential energy and kinetic energy of the beam model are expressed as:

\[
\begin{align*}
U &= \frac{1}{2} \int D \left( \frac{\partial^2 w}{\partial x^2} \right)^2 dx \\
E &= \frac{1}{2} m v^2 
\end{align*}
\]  

(2.7)

From the perspective of energy, the process of free vibration is also the process of mutual conversion between kinetic energy and elastic potential energy. When the bending stiffness of the structure changes \( \Delta D \), taking the increase as an example, the error is not found! The reference source is not found. It can be seen more clearly that the larger the position \( \Delta D \) is applied to \( \frac{\partial^2 w}{\partial x^2} \), the greater the contribution to the elastic potential energy, and \( \frac{\partial^2 w}{\partial x^2} \) is the bending degree of the beam. If the mass characteristic \( m \) does not change and the elastic potential energy increases, then only when the motion speed \( x \) increases can energy conservation be maintained, which means that the free vibration frequency increases. Therefore, it can also be concluded that: the surface impact treatment is applied to the part where the degree of bending deformation is greater, the frequency change degree is more significant.

3. Simulation Research on the Effect of Laser Shock
Based on a cantilever thin plate specimen, the influence of laser shock treatment on the modal frequency is verified by means of finite element simulation and modal vibration test, the local stiffness
changes caused by laser shock treatment are characterized, and the mapping relationship among laser shock strengthening, local stiffness of material and modal frequency of structure is established.

Firstly, finite element method was used to analyze the influence law of the stiffness change location, the stiffness change area and the stiffness change degree on the overall modal frequency of the structure, so as to verify the mechanical analysis conclusion of section 2. Then, under the guidance of the simulation results, the modal vibration test is carried out to verify the effect of laser shock intensification on the modal frequency. In combination with the simulation results, the local stiffness changes of the material are characterized.

3.1. Modal Characteristics of Cantilever Beam Specimens

The cantilever beam test piece adopts the equal section design, the material is the TC4 titanium alloy commonly used in aero engine compressor blades, its main physical and mechanical properties are shown in Table 1, and the test piece structure is shown in Figure 2. The grey area is the clamping part, and the first-order bending mode is the mode concerned by the test, which is pure bending vibration.

Table 1. Main physical and mechanical properties of TC-4.

| Property          | Value       |
|-------------------|-------------|
| Density (g/cm³)   | 4.44        |
| Elastic Modulus   | 109 GPa     |
| Poisson's ratio   | 0.34        |
| Yield Strength    | 827 MPa     |
| Melting temperature | 1630-1650°C |

The model is established based on the geometric characteristics of the cantilever thin plate specimen (hereinafter referred to as the specimen), and the simulation calculation is carried out using Abaqus finite element analysis software. Taking the first-order modal of the specimen—a bending mode as the research object, the modal analysis is performed based on the Lanczos method, and the frequency of a bending mode is 318.65 Hz. The modal characteristics are shown in Fig 3. Fig.3(a) is the distribution characteristics of the displacement field, that is the deflection function in the equation of Section 2.1.

![Sample size and sample picture](image)

Figure 2. Cantilever beam test-piece

It can be seen that the clamping position and root displacement of the specimen are the smallest, while the end position is the largest; Fig3.(b) shows the distribution characteristics of the Mises stress field at various parts of the specimen surface, and it can be seen that the maximum Mises stress at the root and the minimum Mises stress at the end. This shows that the specimen has the largest degree of bending at the root and the smallest degree of bending at the end in a bending mode; Fig.3(c)、(d) and (e) are the stress field distribution characteristics of S11, S22 and S12, respectively. S11 refers to the normal stress in the x direction, S22 refers to the normal stress in the y direction, and S12 refers to the shear stress in the xy direction. It can be seen that compared with S11 stress and S12 stress, S22 stress accounted for the main component of stress, and S22 stress was close to Mises stress distribution,
reflecting the strain characteristics of the specimen, with a larger degree of bending at the root and a smaller degree of bending at the end.

Figure 3. Bending modal characteristics of cantilever beam specimen

3.2. Simulation Analysis of Modal Frequency Changes

3.2.1. Influence of the location of the stiffness change zone on the natural frequency

The model is shown in Fig.4(a), where the blue area is the clamping area and the red area is the stiffness change area. Knowing the original stiffness $E$, the elastic modulus is increased by 10% and decreased by 10%, and $x$ is set to 15 points in the range of $[0,70]$ to obtain the inherent rigidity of the specimen when the stiffness change area is located at different positions. The effect of frequency is shown in Fig. 4 (b). It can be seen that the modal frequency increases as the stiffness increases; the modal frequency decreases as the stiffness decreases. The closer the stiffness change zone is to the root, the effect of frequency is obvious, and the modal frequency changes by 2%; the closer to the end, the weaker the frequency effect, or even close to no effect.

Figure 4. Change in natural frequency
3.2.2. The effect of the stiffness change area on the natural frequency
The model is shown in Fig. 5 (a), where the blue area is the clamping area and the red area is the stiffness change area. Knowing the original stiffness E, the elastic modulus is increased by 10% and decreased by 10%, and set x to take 15 points in the [0,80] interval, and obtain the effect of the stiffness change area on the natural frequency of the specimen as shown in Fig. 5 (b). Similarly, the stiffness increases and the modal frequency increases; the stiffness decreases and the modal frequency decreases. When the change areas are all located at the root, the larger the area, the more obvious the effect on the frequency.

![Image](image_url)

(a) Area of stiffness change area  
(b) Influence on the natural frequency of the test-piece

**Figure 5.** Change in natural frequency

3.2.3. The effect of the degree of stiffness change on the natural frequency
This section will further analysis the effect of stiffness changes on the specimen’s natural frequency, and set the original elastic modulus to E = 109GPa. The selection of the stiffness change area is shown in Fig. 5 (a), taking x = 10 and x = 20 respectively.

Set the local stiffness changes in the normal range and the limit range respectively, and perform modal analysis. The results are shown in Fig. 6 (a) and (b). The abscissa in the figure is the ratio of the local stiffness to the original stiffness, which is recorded as the stiffness ratio. The horizontal axis uses the logarithmic coordinate axis, the left vertical axis is the modal frequency after the local stiffness changes, and the right vertical axis is the response modal frequency rate of change. In Fig. 6(b), the stiffness ratio 100 (ie 1) is the original elastic modulus, and the corresponding modal frequency is the original frequency of 318.65 Hz.

![Table](table_url)

**Table 2.** Common metal materials elastic modulus (E) and Poisson's ratio (μ)

| Metallic material | W | Ni | Carbon steel | Alloy steel | Austenite stainless steel | Copper | Ti | Brass | Al | Mg |
|-------------------|---|----|--------------|------------|--------------------------|--------|----|-------|----|----|
| E(GPa)            | 407 | 207 | 207          | 205        | 200                      | 110    | 109| 97    | 72 | 45 |
| μ                 | 0.28 | 0.31 | 0.3          | 0.29       | 0.3                      | 0.34   | 0.34| 0.34  | 0.33| 0.29|

The conventional range is determined with reference to the elastic modulus of common metals, as shown in Table 2. Set the change range of the elastic modulus to [43.6, 436] ([0.4E, 4E]). In the interval, take 15 parameter points that are evenly distributed according to the logarithmic coordinates, perform modal analysis, and obtain the first-order natural frequency of the specimen is shown in Fig. 6 (a). It can be seen that the local stiffness increases, the modal frequency increases; the local stiffness decreases, and the modal frequency decreases. The modal frequency has an approximate linear relationship with the logarithm of the local stiffness change. The modal frequency changes in different
areas with the same trend and the difference is that the larger the area, the greater the slope of the curve.

![Graph](image)

- **Figure 6.** Modal characteristics at different elastic moduli

The setting of the limit change range is mainly to analyze the influence limit of the local stiffness change on the natural frequency of the structure. For the setting of the local elastic modulus, in the interval of \([E/1000, 1000E]\), 15 uniformly distributed parameter points are taken according to the logarithmic coordinates, then set the elastic modulus to 0 and \(1 \times 10^9E\) respectively, where \(1 \times 10^9E\) is used to indicate that the elastic modulus tends to infinity, and the modal analysis results are shown in Fig. 6 (b).

It can be seen that the change trend of the modal frequency with local stiffness is the same as that in Fig. 6 (a), but when the stiffness ratio increases to 20, the modal frequency tends to be stable and rises slightly. When it is increased to \(\infty\), the modal frequency is increased by 30.8% and 78.5% respectively in the region of \(x=10\) and \(x=20\), so the increase in rigidity has a limit to the increase of the specimen’s natural frequency. This is because when the stiffness is large enough, the local part can be regarded as a rigid body. If the rest of the structure can also be elastically deformed, the natural frequency of the structure is the limit of the frequency increase that can be caused by the increase in local stiffness. When the stiffness ratio drops to 0.02, the natural frequency changes close to steady and approaches 0; when the stiffness ratio drops to 0, the natural frequency is 0. This is because when the local stiffness outside the stiffness change area is reduced to 0, it is equivalent to inelastic constraints, so the natural frequency is reduced to 0. If the local stiffness is reduced to 0, the whole structure can still maintain elastic connection, the natural frequency of the structure is the frequency reduction limit caused by the local stiffness reduction.

4. **Experimental Analysis of the Influence of Local Stiffness on Modal Frequency**

4.1. **Test Plan and Error Analysis**

A modal vibration test is carried out on the cantilever beam specimen to obtain the resonance frequency of the specimen, namely the natural frequency. From the analysis in Section 3, we can see that the change in the stiffness of the root of the specimen has the most significant effect on the natural frequency of the whole structure. Therefore, as shown in Fig. 5 (a), set the area as \(x = 10\), and perform surface impact treatment on the area. The specific implementation scheme is shown in Table 3:

| Processing method | USP | LSP1 Time | LSP3 Times |
|-------------------|-----|-----------|------------|
| Number of cases   | 2   | 2         | 2          |
The test is mainly affected by processing errors and operation errors. In order to analyze the test errors and guide the design of the test plan, 8 original test pieces are selected to carry out the modal vibration test, in which each test piece is repeated three times.

Table 4. Error analysis of cantilever beam test piece.

| Frequency/Hz | ST/Hz | STA/Hz | STE/Hz | V/% | VA/% | VE/% |
|--------------|-------|--------|--------|-----|------|------|
| 285.51       | 4.50  | 7.85   | 1.45   | 1.57| 2.75 | 0.51 |

A total of 24 tests were conducted. Based on the test results, the analysis test errors are shown in Table 4. In the table, Frequency is the average value of the natural frequency of the test piece, ST, STA and STE are the standard deviations, indicating the amount of error, the unit is Hz, where ST is the overall standard deviation of the test results, STA is the standard deviation between the test results, and STE is within-group standard deviation of test results. V, VA, and VE are coefficients of variation, representing the error rate in units of %, where V is the overall coefficient of variation of the test results, VA is the coefficient of variation between groups of the test results, and VE is the coefficient of variation of the groups of the test results. Among them, ST and V represent the overall error of the test, STA and VA represent the error between different test pieces, STE and VE represent the same test piece, the error under different tests.

It can be seen from Table 4 that the intra-group error is smaller than the inter-group error, STA reaches 7.85 Hz, VA reaches 2.75%. It shows that the error between different test pieces is relatively large. If the control group is used for the test, the influence of the local stiffness on the natural frequency will be covered by the test error. Therefore, in order to ensure the accuracy of the test, the method of comparison within the group is used for the test, and the same test piece is compared before and after the treatment, which is before the surface impact treatment, three resonance search tests are performed to obtain the average value. After the surface impact treatment, the resonance test of the test piece was repeated three times to obtain the average value, and the effect of the surface impact treatment on the natural frequency was obtained by comparing the two values.

4.2. Modal Test Results and Material Performance Characterization

The amplitude-frequency response characteristic of one of the specimens is shown in Fig. 7, (a) and (b) show the sweep results of the three resonance search tests before and after the LSP1 and LSP3 treatments, respectively. It can be seen from Fig. 7 (a) that the LSP 1 treatment did not significantly affect the local stiffness of the titanium alloy. In Fig. 7 (b), the resonance peak shifts significantly to the left (the natural frequency decreases), indicating that the LSP3 treatment reduces the local stiffness of the titanium alloy, which causes the frequency of the first-order bending mode of the specimen to be reduced.

Figure 7. Effect of LSP on the amplitude-frequency response of titanium beams
The overall test results are shown in Table 5, where the natural frequencies before and after the treatment are averaged from the results of six tests of the two test pieces in total. It can be seen that after the USP treatment, the frequency of the test piece increased by 0.76 Hz, and the rate of change was 0.27%. After the LSP treatment, the frequency of the test piece decreased by 0.32 Hz, and the rate of change was -0.11%, and their absolute values were less than the STE of the test piece in Table 4. And VE value, it can be considered that USP and LSP have no effect on the rigidity of titanium alloy. After the LSP treatment for 3 times, the frequency decreased by 7.08 Hz, and the change rate was -2.47%. Compared with the STE and VE values of the test pieces in Table 4, it can be seen that the change is more significant, which can rule out the influence of the test error.

Table 5. Surface treatment result analysis

| Method      | Before/Hz | After/Hz | Change/Hz | Change rate/% |
|-------------|-----------|----------|-----------|---------------|
| USP 1 time  | 283.76    | 284.51   | 0.76      | 0.27          |
| LSP 1 time  | 290.80    | 290.48   | -0.32     | -0.11         |
| LSP 3 times | 286.32    | 279.24   | -7.08     | -2.47         |

We analysed the change in local elastic modulus, which based on the modal frequency of the test piece in Table 5 after 3 LSP drops by 2.47%. Refer to Table 2 for material parameter settings, and the original elastic modulus E=109GPa. Using standard LM (Levenberg-Marquardt) method combined with Abaqus modal analysis to identify the local elastic modulus, the local elastic modulus was 98.07GPa after LSP was obtained 3 times, which was 10.03% lower than the original elastic modulus.

5. Summary
This paper focuses on the study of the impact of LSP on the structural modal frequency. Conduct research through a combination of mechanical analysis, numerical simulation calculations and modal vibration tests, It is verified that LSP will cause the change of the material stiffness, and it is concluded that the laser shock strengthening treatment area is located at the part where the deformation is greater, and the modal frequency changes more significantly.

The influence mechanism of local elastic modulus (material stiffness) change on structural modal frequency is analyzed. From the perspective of vibration, changes in local stiffness will cause changes in modal stiffness; From an energy perspective, changes in local stiffness will cause changes in the structural elastic potential energy. When laser shock is strengthened in a region with large bending deformation, the greater the contribution to the overall modal stiffness and elastic potential energy, the greater the impact on the modal frequency.

The mapping relationship among LSP, local stiffness and modal frequency is established. Based on the cantilever beam specimen, the mapping relationship between the position, area and degree of stiffness change to the modal frequency was established by finite element analysis. Through the LSP modal vibration test, the mapping relationship between surface impact treatment and modal frequency is obtained. Combining simulation and experiment, the mapping of surface laser shock treatment to local stiffness is established, and the result of LSP tertiary shock reducing the local elastic modulus of TC-4 titanium alloy by 10.03% is obtained.

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