Flat-space picture of gravity vs. General Relativity: a precision test for present ether-drift experiments

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Abstract

Modern ether-drift experiments in vacuum could in principle detect the tiny refractive index that, in a flat-space picture of gravity, is appropriate for an apparatus placed on the Earth’s surface. In this picture, in fact, if there were a preferred reference frame, light on the Earth would exhibit a slight anisotropy with definite quantitative differences from General Relativity. By re-analyzing the data published by two modern experiments with rotating optical resonators, and concentrating on the part of the signal that should be free of spurious systematic effects, we have found evidences that would support the flat-space scenario.
1. Introduction

The present generation of ether-drift experiments, using rotating optical resonators, is currently pushing the relative accuracy of the measured frequency shifts to the level $O(10^{-16})$. As we shall try to illustrate, this level of accuracy is crucial to obtain fundamental informations on the space-time structure of the vacuum and the possible existence of a preferred reference frame.

In this paper, we’ll present a re-analysis of the observations reported in Ref.[1] for the anisotropy of the speed of light in the vacuum. This re-analysis, that improves with a higher statistics on our previous work [2] based on the data of Refs. [3, 4], leads to two conclusions: i) the experiments exhibit a non-zero daily average for the amplitude of the signal ii) the magnitude of this average amplitude is entirely consistent with the theoretical anisotropy parameter [5] [6] [7]

$$|B_{th}| \sim 3(N_{\text{vacuum}} - 1) \sim 42 \cdot 10^{-10}$$  (1)

that, in the presence of a preferred frame, enters the two-way speed of light in the vacuum to $O(v^2/c^2)$

$$\frac{\bar{c}(\theta) - \bar{c}(0)}{c} \sim B \frac{v^2}{c^2} \sin^2 \theta$$  (2)

In Eq.(1) $N_{\text{vacuum}}$ indicates the effective vacuum refractive index that is appropriate, in a flat-space picture of gravity, for an apparatus placed on the Earth’s surface.

We emphasize that the prediction Eq.(1) refers to experiments performed in the vacuum, i.e. in the highest vacua attainable with present technology. Therefore, in principle, it should not be compared with other type of experiments as those of Ref.[8] where about 100% of the electromagnetic energy propagates within a dielectric medium with $N \sim 3$ [9].

After this general Introduction, the plan of the paper is as follows. In Sect.2, we shall first illustrate the basic formalism and report the experimental data of Ref. [1]. Then, in Sect.3, we shall use these published data to deduce a basic quantity of any ether-drift experiment: the daily average amplitude of the signal $A_0$ that, so far, has never been reported by the experimental groups. Further, in Sect.4, we shall try to estimate the possible systematic uncertainty on our value of $A_0$, by also comparing with Ref.[4]. Then, in Sect.5, we shall point out that the experimental values of $A_0$ for the two experiments, besides being well consistent with each other, are also in good agreement with the theoretical prediction expected in a flat-space picture of gravity, in the presence of a preferred reference frame. Finally, in Sect.6, we shall present our summary and conclusions.
2. Basic formalism and experimental data

The experimental data reported in Ref. [1] refer to 27 short-period observations, performed by the Berlin group [3] during a total period of 392 days. The starting point for our analysis is the expression for the relative frequency shift of two optical resonators at a given time \( t \). For the Berlin experiment [3], this can be expressed as

\[
\frac{\Delta \nu(t)}{\nu_0} = S(t) \sin 2\omega_{\text{rot}} t + C(t) \cos 2\omega_{\text{rot}} t
\]

where \( \omega_{\text{rot}} \) is the rotation frequency of one resonator with respect to the other which is kept fixed in the laboratory and oriented north-south. We observe that the sine component of the signal was denoted as \( S(t) \) in Ref. [3] and as \( B(t) \) in Ref. [1]. Here, we shall maintain the notation of Ref. [3] where the Fourier expansions of \( S(t) \) and \( C(t) \) are expressed as

\[
S(t) = S_0 + S_{s1} \sin \tau + S_{c1} \cos \tau + S_{s2} \sin(2\tau) + S_{c2} \cos(2\tau)
\]

\[
C(t) = C_0 + C_{s1} \sin \tau + C_{c1} \cos \tau + C_{s2} \sin(2\tau) + C_{c2} \cos(2\tau)
\]

\( \tau = \omega_{\text{sid}} t \) being the sidereal time of the observation in degrees and \( \omega_{\text{sid}} \sim \frac{2\pi}{23h56'} \). Introducing the colatitude of the laboratory \( \chi \), and the unknown average velocity, right ascension and declination of the cosmic motion with respect to a hypothetical preferred frame (respectively \( V, \alpha \) and \( \gamma \)), one finds the expressions reported in Table I of Ref. [3] in the RMS formalism [10]

\[
C_0 = - \frac{K \sin^2 \chi}{8}(3 \cos 2\gamma - 1)
\]

\[
C_{s1} = \frac{1}{4}K \sin 2\gamma \sin \alpha \sin 2\chi \quad C_{c1} = \frac{1}{4}K \sin 2\gamma \cos \alpha \sin 2\chi
\]

\[
C_{s2} = \frac{1}{4}K \cos^2 \gamma \cos 2\alpha(1 + \cos^2 \chi) \quad C_{c2} = \frac{1}{4}K \cos^2 \gamma \cos 2\alpha(1 + \cos^2 \chi)
\]

where

\[
K = |B| \frac{V^2}{c^2}
\]

is proportional to the light anisotropy parameter. The corresponding \( S \)–quantities are also given by \( (S_0 = 0) \)

\[
S_{s1} = - \frac{C_{c1}}{\cos \chi} \quad S_{c1} = \frac{C_{s1}}{\cos \chi}
\]

\[
S_{s2} = - \frac{2 \cos \chi}{1 + \cos^2 \chi} C_{c2} \quad S_{c2} = \frac{2 \cos \chi}{1 + \cos^2 \chi} C_{s2}
\]
To compare with the similar Düsseldorf experiment of Ref. [4], one should just re-nominate the two sets

\[
(C_0, C_{s1}, C_{c1}, C_{s2}, C_{c2}) \rightarrow (C_0, C_1, C_2, C_3, C_4) \quad (12)
\]

\[
(S_0, S_{s1}, S_{c1}, S_{s2}, S_{c2}) \rightarrow (B_0, B_1, B_2, B_3, B_4) \quad (13)
\]

and introduce an overall factor of two for the frequency shift since, in this case, two orthogonal cavities are maintained in a state of active rotation.

As suggested by the same authors of Refs. [3,4], it is safer to concentrate on the observed time modulation of the signal, i.e. on the quantities \(C_{s1}, C_{c1}, C_{s2}, C_{c2}\) and on their \(S\)-counterparts. In fact, the constant components \(C_0\) and \(S_0 \equiv B_0\) are likely affected by spurious systematic effects. The experimental \(C_k\) and \(S_k\) coefficients, as extracted from Fig. 2 of Ref. [1], are reported in our Tables 1 and 2. The quoted errors are both statistical and systematical. For the quantities we are considering, according to Ref. [3], this latter component should be very small.

### 3. The daily average amplitude of the signal

For our analysis, we shall re-write Eq. (3) as follows

\[
\frac{\Delta \nu(t)}{\nu_0} = A(t) \cos(2\omega_{\text{rot}} t - 2\theta_0(t)) \quad (14)
\]

with

\[
C(t) = A(t) \cos 2\theta_0(t) \quad S(t) = A(t) \sin 2\theta_0(t) \quad (15)
\]

\(\theta_0(t)\) representing the instantaneous direction of a hypothetical ether-drift effect in the plane of the interferometer.

Eq. (14), while fully equivalent to Eq. (3), introduces in the analysis a positive-definite quantity, the amplitude of the signal \(A(t)\). The interest in a computation of \(A(t)\) can be easily understood by comparing the two elementary amplitudes \(C(t)\) and \(S(t)\) with the cartesian coordinates \(x\) and \(y\) of a 2-dimensional particle motion and \(A(t)\) with the radial coordinate \(r = \sqrt{x^2 + y^2}\), so that \(x = r \cos \theta\) and \(y = r \sin \theta\). For definiteness let us denote by \(x_i \pm \Delta x_i\) and \(y_i \pm \Delta y_i\) the individual measurements of the particle position and assume that, to a good accuracy, the average values are \(\bar{x} = \bar{y} = 0\). It goes without saying that this situation does not determine all properties of the motion since there might be a plenty of non-trivial rotationally invariant motions that differ for the average radius. To determine this other parameter, one has to compute the radial distances \(r_i = \sqrt{x_i^2 + y_i^2}\) for the individual measurements and
finally take their average. Of course, since now one deals with the positive-definite quantities
\( r_i \), their mean value \( \bar{r} \) will be definitely different from \( \sqrt{x^2 + y^2} = 0 \) and will depend on the experimental accuracy of the individual values \( r_i \pm \Delta r_i \). Assuming that the errors for the elementary entries \( x_i \) and \( y_i \) belong to a normal distribution, the errors \( \Delta r_i \) can be computed according to standard error propagation for composite observables, namely
\[
\Delta r_i = \sqrt{\cos^2 \theta_i (\Delta x_i)^2 + \sin^2 \theta_i (\Delta y_i)^2} \tag{16}
\]
Therefore, the final answer about a possible non-zero \( \bar{r} \) will depend on the confidence level of the point \( r = 0 \), i.e. on the distributions of the ratios \( R_i = r_i / \Delta r_i \). At the same time, when using the vectorial observables \( S(t) \) and \( C(t) \), noise tends to average out to zero for large sample of data. With a positive-definite quantity as \( A(t) \) this does not occur. Therefore one has to estimate the effect of noise that might mimic a true signal. This point will be discussed in Sect.4.

After these preliminary considerations, let us return to our basic amplitude \( A(t) \) by observing that, in the framework where the amplitudes \( S(t) \) and \( C(t) \) are represented as in Sect.2, it can be expressed in terms of \( v(t) \), the magnitude of the projection of the cosmic Earth’s velocity in the plane of the interferometer, and of the light anisotropy parameter as
\[
A(t) = \frac{1}{2} |B| \frac{v^2(t)}{c^2} \tag{17}
\]
To compute \( v(t) \), we shall use the theoretical relations given by Nassau and Morse \[11\] from which \( A(t) \), \( S(t) \) and \( C(t) \) can be obtained up to an overall proportionality constant. These expressions are valid for series of short-period observations, as those performed in Refs.[1, 3, 4], where, within each experimental session, the kinematical parameters of the cosmic velocity \( \mathbf{V} \) are not appreciably modified by changes in the Earth’s orbital motion around the Sun.

In this approximation, by introducing the magnitude \( V \) of the full Earth’s velocity with respect to a hypothetic preferred frame \( \Sigma \), its right ascension \( \alpha \) and angular declination \( \gamma \), we get
\[
\cos z(t) = \sin \gamma \sin \phi + \cos \gamma \cos \phi \cos(\tau - \alpha) \tag{18}
\]
\[
\sin z(t) \cos \theta_0(t) = \sin \gamma \cos \phi - \cos \gamma \sin \phi \cos(\tau - \alpha) \tag{19}
\]
\[
\sin z(t) \sin \theta_0(t) = \cos \gamma \sin(\tau - \alpha) \tag{20}
\]
\[
v(t) = V \sin z(t), \tag{21}
\]
where \( z = z(t) \) is the zenithal distance of \( V \), \( \phi \) is the latitude of the laboratory and again \( \tau = \omega_{\text{sid}} t \) denotes the sidereal time of the observation in degrees. As one can check, by using the above relations, together with Eqs. (15) and (17), and replacing \( \chi = 90^\circ - \phi \), one re-obtains the expansions for \( C(t) \) and \( S(t) \) reported in Eqs. (4)-(8).

After having obtained this first consistency check, we can now replace Eq. (21) into Eq. (17) and, by adopting a notation of the type in Eqs. (4)-(5), express \( A(t) \) as

\[
A(t) = A_0 + A_1 \sin \tau + A_2 \cos \tau + A_3 \sin(2\tau) + A_4 \cos(2\tau) \quad (22)
\]

where

\[
A_0 = \frac{1}{2} |K| \left( 1 - \sin^2 \gamma \cos^2 \chi - \frac{1}{2} \cos^2 \gamma \sin^2 \chi \right) \quad (23)
\]

\[
A_1 = -\frac{1}{4} |K| \sin 2\gamma \sin \alpha \sin 2\chi \quad A_2 = -\frac{1}{4} |K| \sin 2\gamma \cos \alpha \sin 2\chi \quad (24)
\]

\[
A_3 = -\frac{1}{4} |K| \cos^2 \gamma \sin 2\alpha \sin^2 \chi \quad A_4 = -\frac{1}{4} |K| \cos^2 \gamma \cos 2\alpha \sin^2 \chi \quad (25)
\]

On the basis of the above relations, it is now possible to extract the average amplitude \( A_0 \) from the published data of Ref. [1]. On the other hand, no information on the phase of the signal \( \theta_0(t) \) can be obtained. To this end, in fact, one would need the full amplitudes \( S(t) \) and \( C(t) \) at the sidereal times of the observations and these are not available.

To obtain \( A_0 \), we observe that the daily averaging of the signal (here denoted by \( \langle .. \rangle \)), when used in Eq. (22) produces the relation

\[
\langle A^2(t) \rangle = A_0^2 + \frac{1}{2} (A_1^2 + A_2^2 + A_3^2 + A_4^2) \quad (26)
\]

On the other hand, using Eqs. (4), (5) and (15), one also obtains

\[
\langle A^2(t) \rangle = \langle C^2(t) + S^2(t) \rangle = C_0^2 + S_0^2 + \frac{1}{2} (C_{11}^2 + S_{11}^2 + C_{22}^2 + S_{22}^2) \quad (27)
\]

where we have introduced the combinations

\[
C_{11} \equiv \sqrt{C_{s1}^2 + C_{c1}^2} \quad C_{22} \equiv \sqrt{C_{s2}^2 + C_{c2}^2} \quad (28)
\]

\[
S_{11} \equiv \sqrt{S_{s1}^2 + S_{c1}^2} \quad S_{22} \equiv \sqrt{S_{s2}^2 + S_{c2}^2} \quad (29)
\]

As one can check, replacing the expressions (23)-(25), Eq. (26) gives exactly the same result that one would obtain replacing the values for the C- and S-coefficients in Eq. (27). Therefore, one can combine the two relations and get

\[
A_0^2(1 + r) = C_0^2 + S_0^2 + \frac{1}{2} (C_{11}^2 + S_{11}^2 + C_{22}^2 + S_{22}^2) \equiv C_0^2 + S_0^2 + Q^2 \quad (30)
\]
where

\[ Q = \sqrt{ \frac{1}{2} \left( C_{11}^2 + S_{11}^2 + C_{22}^2 + S_{22}^2 \right) } \]  

(31)

and

\[ r \equiv \frac{1}{2A_0^2} (A_1^2 + A_2^2 + A_3^2 + A_4^2) \]  

(32)

To evaluate \( A_0 \) we shall proceed as follows. On the one hand, we shall compute the ratio \( r = r(\gamma, \chi) \) using the theoretical expressions Eqs. (23)-(25). This gives

\[ 0 \leq r \leq 0.4 \]  

(33)

for the latitude of the laboratories in Berlin [3] and Düsseldorf [4] in the full range \( 0 \leq |\gamma| \leq \pi/2 \). On the other hand, we shall adopt the point of view of the authors of Refs. [1, 3, 4] that, even when large non-zero values of \( C_0 \) and \( S_0 \) are obtained (compare with the value \( C_0 = (-59.0 \pm 3.4 \pm 3.0) \cdot 10^{-16} \) of Ref. [4] and with the large scatter of the data reported in in Fig. 2 of Ref. [1] or in Fig. 3 of Ref. [3]), tend to consider these individual determinations as spurious effects.

This means to set in Eq. (30)

\[ S_0 = \langle A(t) \sin 2\theta_0(t) \rangle \sim 0 \]  

(34)

\[ C_0 = \langle A(t) \cos 2\theta_0(t) \rangle \sim 0 \]  

(35)

We can thus define a daily average amplitude, say \( \hat{A}_0 \), which is determined in terms of \( C_{11}, S_{11}, C_{22} \) and \( S_{22} \) alone as

\[ \hat{A}_0 = \frac{Q}{\sqrt{1 + r}} \sim (0.92 \pm 0.08)Q \]  

(36)

Here the uncertainty takes into account the numerical range of \( r \) in Eq. (33). This value provides, in any case, a lower bound to its true experimental value since

\[ A_0 = \frac{C_0^2 + S_0^2 + Q^2}{1 + r} \geq \hat{A}_0 \]  

(37)

The data for the various coefficients are reported in our Table 3 together with the quantity \( Q \). The errors on the various coefficients have been computed, according to standard error propagation for composite observables, starting from the errors for the basic entries \( C_k \) and \( S_k \) reported in our Tables 1 and 2.
4. The value of $A_0$ and its experimental uncertainty

In this section we shall now try to deduce the average value of $A_0$ (or more precisely its lower bound) for the Berlin experiment [1, 3]. To this end, a first simple choice could be to take the weighted average $\bar{Q}$ of the 27 values of $Q$ reported in Table 3 and then use Eq.(36). This straightforward strategy gives

$$\bar{Q} = (13.0 \pm 0.7) \cdot 10^{-16} \quad (38)$$

In the above relation the quoted error should be understood as purely statistical. In fact, by taking the weighted average of a large number $N$ of measurements, one obtains an error on the mean that vanishes as $1/\sqrt{N}$ in the limit $N \to \infty$. To estimate a possible systematic error to be added in Eq.(38) we shall make the reasonable assumption that systematic uncertainties affect the value $Q_i$ of the i-th experimental session by an amount $Q_i^{sys}$ which is comparable to $\Delta Q_i$. To get a numerical value, we shall follow three different methods that turn out to give similar results.

In a first approach, one could estimate the systematic error on the mean by simply averaging the 27 determinations $Q_i^{sys} \sim \Delta Q_i$, reported in Table 4. This gives the value

$$\Delta Q^{(1)} \sim \frac{1}{27} \sum_i Q_i^{sys} \sim 4.4 \cdot 10^{-16} \quad (39)$$

As a second approach, one can estimate a mean systematic component by using the weighted average expression

$$\Delta Q^{(2)} \sim \sum_i \frac{Q_i^{sys}}{(\Delta Q_i)^2} \left( \sum_i \frac{1}{(\Delta Q_i)^2} \right)^{-1} \quad (40)$$

In this case, by using again the data reported in Table 4, we obtain

$$\Delta Q^{(2)} \sim 3.5 \cdot 10^{-16} \quad (41)$$

Finally, one can also consider the values of the signal and estimate the noise for the i-th experimental session from the ratio $R_i = Q_i/\Delta Q_i$. Taking the arithmetic mean of the 27 determinations $R_i$ reported in Table 4 gives a value $\bar{R} \sim 3.6$ that, when using the weighted average $\bar{Q} = 13.0 \cdot 10^{-16}$, gives the other estimate

$$\Delta Q^{(3)} \sim \frac{\bar{Q}}{\bar{R}} \sim 3.6 \cdot 10^{-16} \quad (42)$$

Therefore, by averaging the three different determinations in Eqs.(39), (41) and (42), one obtains the value

$$\Delta Q \sim 3.8 \cdot 10^{-16} \quad (43)$$
that we shall take as our final estimate of the systematic error on the mean thus replacing
Eq. (38) with
\[
\bar{Q} = (13.0 \pm 0.7 \pm 3.8) \cdot 10^{-16} \tag{44}
\]
From this, by using Eq. (36), we get
\[
\hat{A}_0 = (12.0 \pm 1.0 \pm 3.5) \cdot 10^{-16} \text{ Ref. [1]} \tag{45}
\]
where the former error is due to the variation $0 \leq r \leq 0.4$ and the latter reflects our estimate of the noise.

As a further control of the validity of our Eq. (44), we report in Tables 5-7 the $C$ and $S$ coefficients of Ref. [4]. As one can check the value $Q = (13.1 \pm 2.1) \cdot 10^{-16}$ is in excellent agreement with our result (44) and corresponds to
\[
\hat{A}_0 = (12.1 \pm 1.0 \pm 2.1) \cdot 10^{-16} \text{ Ref. [4]} \tag{46}
\]
in perfect agreement with (45).

5. An effective refractive index for the vacuum

In this section, we shall point out that the two experimental values in Eqs. (45) and (46) are well consistent with the theoretical prediction
\[
A_{0}^{\text{th}} = \frac{1}{2} |B_{\text{th}}| \frac{v_0^2}{c^2} \sim (9.7 \pm 3.5) \cdot 10^{-16} \tag{47}
\]
of Refs. [6, 7]. This was obtained, in connection with the anisotropy parameter \[5\] $|B_{\text{th}}| \sim 42 \cdot 10^{-10}$, after inserting the average cosmic velocity (projected in the plane of the interferometer) $v_0 = (204 \pm 36)$ km/s that derives from a re-analysis \[6, 7\] of the classical ether-drift experiments. Due to this rather large theoretical uncertainty, the different locations of the various laboratories and any other kinematical property of the cosmic motion can be neglected in a first approximation.

For a proper comparison, we also remind that in Refs. [6, 7] the frequency shift was parameterized as
\[
\frac{\Delta \nu(\theta)}{\nu_0} = |B_{\text{th}}| \frac{v^2}{c^2} \cos 2\theta \tag{48}
\]
This relation is appropriate for a symmetrical apparatus with two rotating orthogonal lasers, where the difference $\bar{c}(\theta) - \bar{c}(0)$ gets replaced by $\bar{c}(\pi/2 + \theta) - \bar{c}(\theta)$ as in the Düsseldorf experiment \[4\], and gives an average amplitude
\[
A_0^{\text{symm}} = 2A_0^{\text{th}} \sim (19 \pm 7) \cdot 10^{-16} \tag{49}
\]
The theoretical prediction for the anisotropy parameter was obtained starting from the many analogies that one can establish, in the weak-field limit, between General Relativity and a flat-space description where gravity leads to re-defined masses, space-time units and an effective vacuum refractive index. This alternative approach, see for instance Wilson [12], Gordon [13], Rosen [14], Dicke [15], Puthoff [16] and even Einstein himself [17], before his formulation of a metric theory of gravity, in spite of the deep conceptual differences, produces an equivalent description of the phenomena in a weak gravitational field.

The substantial phenomenological equivalence of the two approaches was well summarized by Atkinson as follows [18]: "It is possible, on the one hand, to postulate that the velocity of light is a universal constant, to define natural clocks and measuring rods as the standards by which space and time are to be judged and then to discover from measurement that space-time is really non-Euclidean. Alternatively, one can define space as Euclidean and time as the same everywhere, and discover (from exactly the same measurements) how the velocity of light and natural clocks, rods and particle inertias really behave in the neighborhood of large masses."

This type of analogy, which is preserved by the weak-field classical tests, is interesting in itself and deserves to be explored. In fact, "...it is not unreasonable to wonder whether it may not be better to give up the geometric approach to gravitation for the sake of obtaining a more uniform treatment for all the various fields of force that are found in nature" [14].

To distinguish between the two interpretations, one can start from the zeroth order approximation to the problem embodied in the Equivalence Principle. As it is well known, this basic principle, introduced before General Relativity, does not necessarily rely on the notion of a curved space-time (see e.g. Ref.[19] or even the critical point of view of Ref.[20]). According to it, for an observer placed in a freely falling frame, local Lorentz invariance is valid to first order. Therefore, given two space-time events that differ by \( (dx, dy, dz, dt) \), and the space-time metric

\[
ds^2 = c^2 dt^2 - (dx^2 + dy^2 + dz^2)\tag{50}
\]

one gets from \( ds^2 = 0 \) the same speed of light that one would get in the absence of any gravitational effect.

However, to a closer look, an observer placed on the Earth’s surface is equivalent to a freely-falling frame up to the presence of the Earth’s gravitational field. In this case, both General Relativity and the flat-space approach predict the weak-field, isotropic form of the
metric
\[ ds^2 = c^2 dt^2 (1 - \frac{2GM}{c^2 R}) - (1 + \frac{2GM}{c^2 R})(dx^2 + dy^2 + dz^2) = c^2 d\tau^2 - dl^2 \] (51)

\( G \) being Newton’s constant and \( M \) and \( R \) the Earth’s mass and radius. Here \( d\tau \) and \( dl \) denote respectively the elements of ”proper” time and ”proper” length in terms of which, in General Relativity, one would again deduce from \( ds^2 = 0 \) the same universal value \( \frac{dl}{d\tau} = c \).

On the other hand, in the flat-space approach the condition \( ds^2 = 0 \) is interpreted in terms of an effective refractive index for the vacuum
\[ N_{\text{vacuum}} - 1 \sim \frac{2GM}{c^2 R} \sim 14 \cdot 10^{-10} \] (52)
as if Euclidean space would be filled by a very rarefied ”gravitational medium” (let us recall that a moving dielectric medium acts on light as an effective gravitational field, see for instance Refs.\[13, 21\]).

Now, since in the flat-space approach light can be seen isotropic by only one inertial frame \[22\], say \( \Sigma \), the ether-drift experiments can clarify whether \( \Sigma \) coincides with the Earth’s frame or with a hypothetical preferred frame. In the former case, corresponding to no anisotropy of the two-way speed of light in the vacuum, the equivalence between General Relativity and the gravitational-medium picture would persist. In the latter case, for the observer sitting on the Earth, there would be off-diagonal elements in the metric
\[ g_{0i} \sim (N_{\text{vacuum}}^2 - 1) \frac{V_i}{c} \] (53)
and an anisotropy of the speed of light. Its value can be estimated directly in flat space, by Lorentz transforming from the isotropic value \( c/N_{\text{vacuum}} \) in \( \Sigma \), and one predicts an anisotropy of the two-way speed of light
\[ \frac{\bar{c}(\theta) - \bar{c}(0)}{c} \sim B_{\text{th}} \frac{V^2}{c^2} \sin^2 \theta \] (54)
governed by the parameter \[5, 6, 7\]
\[ |B_{\text{th}}| \sim 3(N_{\text{vacuum}} - 1) \sim 42 \cdot 10^{-10} \] (55)

For this reason, assuming a typical value \( V^2/c^2 \sim 10^{-6} \), the present ether-drift experiments, with their \( O(10^{-16}) \) accuracy, represent precision probes of the vacuum and of its space-time structure. In this respect, it is interesting that, independently of our re-analysis of the data, a \( 2 - 3 \sigma \) non-zero signal in the photon sector \( \tilde{\kappa}_{e}^{XZ} = (-10.3 \pm 3.9) \cdot 10^{-16} \) (see Table II of Ref.\[1\]) was reported by the authors of Ref.\[1\] by using the SME parametrization \[23\].
6. Summary and conclusions

In this paper, we have presented a re-analysis of the Berlin [1, 3] and Düsseldorf [4] ether-drift experiments that, by employing rotating optical resonators, attempt to establish the isotropy of the speed of light in the vacuum to a level of accuracy \( \mathcal{O}(10^{-16}) \). In our re-analysis we have extracted from the published data a basic observable of an ether-drift experiment, namely the daily average for the amplitude of the signal. By denoting with \( \langle \cdot \rangle \) the daily average of any given quantity, this is defined as \( A_0 = \langle A(t) \rangle \) after re-writing the frequency shift

\[
\frac{\Delta \nu(t)}{\nu_0} = S(t) \sin 2\omega_{rot} t + C(t) \cos 2\omega_{rot} t \quad (56)
\]

in the form

\[
\frac{\Delta \nu(t)}{\nu_0} = A(t) \cos(2\omega_{rot} t - 2\theta_0(t)) \quad (57)
\]

By further assuming, as the authors of Refs. [3, 4] do, that experimental results providing large non-zero values for either \( \langle C(t) \rangle = C_0 \) or \( \langle S(t) \rangle = S_0 \) in Eqs. (4) and (5) should be interpreted as spurious effects (e.g. due to thermal drift, non-uniformity of the rotating cavity speed, misalignment of the cavity rotation axis,...), \( A_0 \) can be approximated by its lower bound

\[
\hat{A}_0 \sim (0.92 \pm 0.08) Q \quad (58)
\]

where

\[
Q = \sqrt{\frac{1}{2} (C_{11}^2 + S_{11}^2 + C_{22}^2 + S_{22}^2)} \quad (59)
\]

is given in terms of the coefficients \( C_{11}, C_{22}, S_{11}, S_{22} \) defined in Eqs. (28)-(29). These coefficients, that reflect the time modulation of the signal, are much less affected by spurious effects (as compared to \( C_0 \) and \( S_0 \)) and so should be the value of \( \hat{A}_0 \).

Now, the two resulting experimental determinations in Eqs. (45) and (46), namely \( \hat{A}_0 = (12.0 \pm 1.0 \pm 3.5) \cdot 10^{-16} \) from Ref. [1] and \( \hat{A}_0 = (12.1 \pm 1.0 \pm 2.1) \cdot 10^{-16} \) from Ref. [4], besides being in excellent agreement with each other (and with the previous lower-statistics re-analysis of the Berlin experiment reported in [2]) also agree with the theoretical prediction \( (9.7 \pm 3.5) \cdot 10^{-16} \) of Refs. [6, 7]. This prediction was obtained in a flat-space description of gravity, by assuming an average speed with respect to a hypothetical preferred frame of the type suggested by a re-analysis of the classical ether-drift experiments. Therefore this non-trivial level of consistency, between different experiments and with a theoretical prediction formulated before the experiments were performed, supports the conclusion that a non-zero anisotropy of the speed of light in the vacuum has actually been measured with values of
the anisotropy parameter that are *one order of magnitude larger* than those quoted by the authors of Refs. [3, 4].

A first obvious possibility for this discrepancy is that in our average we might have grossly underestimated the effect of the residual noise, even in the *time-dependent* part of the signal. In this case, our error in Eq. (44) would be too small. For instance, by insisting on a zero signal and deciding that a vanishing amplitude requires consistency with the value \( Q = 0 \) to some definite confidence level, say 10% one side, one might argue that the correct final estimate for the Berlin experiment should be \( \bar{Q} \sim (13 \pm 10) \cdot 10^{-16} \). To this end, however, the individual determinations in Table 4 should exhibit a typical value \( R_i = Q_i / \Delta Q_i \sim 1.3 \) which is quite different from their average result \( R_i \sim 3.6 \). Analogous considerations apply if the confidence level were pushed to 5%, again one side. In this case, the final estimate for the Berlin experiment should be \( \bar{Q} \sim (13 \pm 8) \cdot 10^{-16} \) with individual determinations \( R_i = Q_i / \Delta Q_i \sim 1.6 \) in Table 4.

Now, the individual central values \( Q_i \) and their errors \( \Delta Q_i \) reported in Table 4 were computed, starting from the values of the elementary entries \( C_k \) and \( S_k \) reported in Tables 1 and 2, according to the standard statistical rules for composite observables. Errors such as those needed to obtain \( \bar{Q} \sim (13 \pm 10) \cdot 10^{-16} \) or \( \bar{Q} \sim (13 \pm 8) \cdot 10^{-16} \) would be respectively five or four times larger than that extracted from the data of Ref. [4] after a single experimental session (see Table 7). This would be equivalent to deny the assumed high precision of the Berlin experiment.

On the other hand, by accepting our estimate of the errors, the reasons for the discrepancy with the anisotropy parameter reported in Refs. [3, 4] have to be searched elsewhere. To this end, we observe preliminarily that introducing \( C_{11}, C_{22}, S_{11} \) and \( S_{22} \) represents the simplest way to obtain *rotationally invariant* combinations, out of the elementary coefficients \( C_{s1}, C_{c1} \) and of their \( S \)-counterparts. Their use can eliminate possible spurious effects, depending on the relative phases, that enter the delicate splitting of the signal in its various Fourier components (see the discussion given by the authors of Ref. [3] and their note [13]).

Now, suppose that for some reason, during the \( i \)-th experimental session the coefficients \( C_{s1} \) and \( C_{c1} \) were erroneously rotated by an angle \( \varphi_i \) with respect to their true value. This means erroneous values

\[
C_{s1}(i) = C_{s1} \cos \varphi_i - C_{c1} \sin \varphi_i
\]

(60)

and

\[
C_{c1}(i) = C_{c1} \cos \varphi_i + C_{s1} \sin \varphi_i
\]

(61)
with analogous effects for the other parameter pairs \((C_s2, C_c2)\), \((S_s1, S_c1)\) and \((S_c2, S_s2)\) and an overall shift in the value of the right ascension \(\alpha \rightarrow \alpha + \varphi_i\).

Nevertheless, even if this happens, our rotationally invariant combinations would remain unchanged, i.e.

\[
C_{11}(i) = \sqrt{C_{s1}^2(i) + C_{c1}^2(i)} = \sqrt{C_{s1}^2 + C_{c1}^2} = C_{11}
\]

with similar results for \(C_{22}, S_{11}\) and \(S_{22}\). Therefore, even substantial levels of phase error, that can produce zero intersession averages for the various \(C_k\) and \(S_k\) vectorial coefficients (and thus a vanishing anisotropy parameter from these averages), would not change our determination of \(\hat{A}_0\). In this sense, \(\hat{A}_0\) represents a robust indicator. Analogous considerations apply to the separation of the signal in its \(S(t)\) and \(C(t)\) parts which is obtained by extracting the components proportional to \(\sin 2\omega_{\text{rot}} t\) and \(\cos 2\omega_{\text{rot}} t\). Again, the combination \(A^2 = S^2 + C^2\) is not modified by replacing \(\omega_{\text{rot}} t \rightarrow \omega_{\text{rot}} t + \Omega_i\). This type of argument would also explain the remarkable agreement between our final estimate Eq. (44) from Ref. [1] and the value \(Q = (13.1 \pm 2.1) \times 10^{-16}\) from Ref. [4], in spite of the completely different \(C_k\) and \(S_k\) coefficients obtained by the two experiments.

Clearly, the simplest way to check our result would be that the authors of Refs. [1, 3, 4] could repeat their analysis of the data, replacing Eq. (3) with Eq. (14), and compute from scratch \(A_0\) (and its lower bound \(\hat{A}_0\)) for the various experimental sessions. This computation, that would only require the elementary algebraic relations used in this paper, could also provide a powerful consistency check of the whole experiment. At the present, since this has not been done, by accepting the errors of Refs. [1, 3, 4] for the elementary entries \(C_{s1}, C_{c1}\)… and their S-counterparts, our values of \(A_0\) are the only existing estimate of this basic physical quantity.

For this reason, since with our analysis one would be driven to deduce non-trivial consequences, such as the existence of a preferred frame and a flat-space description of gravity, we emphasize the importance of comparing different points of view and approaches to the data to finally achieve a full understanding of the underlying fundamental physical problem.

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Table 1: The experimental $C_k$ coefficients as extracted from Fig.2 of Ref.[1].

| $C_{s1}[\times 10^{-16}]$ | $C_{c1}[\times 10^{-16}]$ | $C_{s2}[\times 10^{-16}]$ | $C_{c2}[\times 10^{-16}]$ |
|--------------------------|--------------------------|--------------------------|--------------------------|
| 9.2 ± 4.6                | 10.0 ± 5.4               | −1.5 ± 4.6               | 0.0 ± 5.4                |
| −2.3 ± 6.9               | −4.6 ± 6.2               | −5.8 ± 6.5               | −0.8 ± 6.2               |
| −2.3 ± 3.8               | 0.0 ± 3.1                | −2.3 ± 3.8               | −4.6 ± 3.8               |
| 4.6 ± 4.6                | 11.5 ± 3.8               | −14.6 ± 3.8              | −0.8 ± 4.6               |
| 5.4 ± 8.5                | −11.5 ± 8.5              | 11.5 ± 8.5               | −4.6 ± 6.9               |
| −5.4 ± 5.4               | −6.2 ± 6.2               | 6.9 ± 5.4                | 1.5 ± 5.4                |
| −10.8 ± 6.2              | −5.4 ± 5.4               | −6.2 ± 6.2               | −4.6 ± 5.4               |
| −3.1 ± 9.2               | 7.7 ± 9.2                | −10.8 ± 9.2              | 7.7 ± 8.5                |
| −6.2 ± 6.2               | 6.2 ± 7.7                | −2.7 ± 6.5               | 6.9 ± 6.2                |
| −1.5 ± 4.6               | −9.2 ± 4.6               | −10.0 ± 5.4              | −3.1 ± 5.4               |
| 2.3 ± 3.8                | 4.6 ± 3.1                | −3.1 ± 3.1               | 0.0 ± 3.8                |
| 13.8 ± 7.7               | −10.0 ± 6.9              | 10.0 ± 6.9               | 3.8 ± 7.7                |
| −20.0 ± 4.6              | −5.4 ± 5.4               | 4.6 ± 4.6                | 9.2 ± 5.4                |
| −2.3 ± 3.8               | 6.2 ± 3.1                | −3.8 ± 3.8               | 1.5 ± 3.8                |
| −16.2 ± 6.9              | 22.3 ± 6.9               | −3.1 ± 6.2               | 10.0 ± 6.2               |
| −6.2 ± 7.7               | 18.5 ± 9.2               | 6.2 ± 9.2                | 13.1 ± 7.7               |
| 3.1 ± 3.1                | 3.8 ± 3.8                | −10.8 ± 3.1              | 5.4 ± 3.1                |
| −10.0 ± 8.5              | 1.5 ± 7.7                | −1.5 ± 7.7               | 3.1 ± 8.5                |
| 0.8 ± 8.5                | 6.9 ± 8.5                | −13.1 ± 8.5              | 2.3 ± 9.2                |
| −16.2 ± 6.9              | 4.6 ± 7.7                | 0.8 ± 6.9                | 26.2 ± 8.5               |
| 1.5 ± 6.2                | −21.5 ± 6.2              | 12.3 ± 6.2               | −6.2 ± 5.4               |
| 0.8 ± 6.9                | −1.5 ± 7.7               | −7.7 ± 7.7               | −13.1 ± 7.7              |
| 7.7 ± 9.2                | −14.6 ± 8.5              | 14.6 ± 8.5               | 3.8 ± 9.2                |
| 0.0 ± 4.6                | 6.9 ± 3.8                | 2.3 ± 3.8                | 7.7 ± 3.8                |
| −16.2 ± 6.9              | −10.0 ± 8.5              | 1.5 ± 7.7                | 11.5 ± 7.7               |
| −1.5 ± 4.6               | 0.8 ± 3.8                | −3.1 ± 4.6               | 1.5 ± 3.8                |
| −5.4 ± 10.0              | 10.8 ± 12.3              | −20.8 ± 10.0             | −14.6 ± 10.8             |
Table 2: The experimental $S_k \equiv B_k$ coefficients as extracted from Fig. 2 of Ref. [1].

| $S_{c1}[x10^{-16}]$ | $S_{c1}[x10^{-16}]$ | $S_{c2}[x10^{-16}]$ | $S_{c2}[x10^{-16}]$ |
|----------------------|----------------------|----------------------|----------------------|
| 2.3 ± 3.8            | −10.8 ± 4.6          | −4.6 ± 4.6           | −4.6 ± 4.6           |
| 14.6 ± 6.9           | −10.0 ± 6.9          | −6.9 ± 6.9           | 1.5 ± 6.2            |
| −1.5 ± 4.6           | −5.8 ± 3.5           | 3.8 ± 3.8            | −2.3 ± 3.8           |
| −8.5 ± 3.8           | 8.5 ± 3.8            | −10.0 ± 3.8          | 4.6 ± 4.6            |
| 3.1 ± 7.7            | −3.8 ± 8.5           | −3.8 ± 6.9           | −6.9 ± 6.9           |
| 1.5 ± 6.2            | 3.1 ± 6.2            | −10.8 ± 6.2          | 1.5 ± 6.2            |
| 9.2 ± 6.2            | 3.8 ± 5.4            | 5.4 ± 5.4            | −1.5 ± 6.2           |
| −3.8 ± 8.5           | −2.3 ± 8.5           | −16.2 ± 8.5          | −14.6 ± 8.5          |
| −4.6 ± 6.2           | 10.0 ± 6.9           | 9.2 ± 6.2            | 1.5 ± 6.2            |
| 3.8 ± 5.4            | −4.6 ± 4.6           | 5.8 ± 5.0            | 4.6 ± 4.6            |
| 3.1 ± 3.1            | 1.5 ± 3.1            | −3.8 ± 3.8           | −1.5 ± 3.1           |
| 3.1 ± 7.7            | 2.3 ± 6.9            | 2.3 ± 6.9            | 0.8 ± 6.9            |
| −3.8 ± 3.8           | 0.4 ± 5.0            | 5.8 ± 3.8            | −2.3 ± 3.8           |
| 5.4 ± 3.8            | 1.5 ± 3.1            | 6.2 ± 3.1            | 3.1 ± 3.1            |
| −14.6 ± 5.4          | 20.0 ± 6.2           | 3.8 ± 5.4            | 6.5 ± 5.8            |
| 10.0 ± 6.9           | −19.2 ± 8.5          | 16.9 ± 7.7           | −16.2 ± 6.9          |
| −2.3 ± 3.8           | 2.3 ± 3.8            | 8.5 ± 3.8            | −8.5 ± 3.8           |
| 2.3 ± 6.9            | −10.0 ± 6.9          | −8.5 ± 6.9           | 1.5 ± 7.7            |
| −10.0 ± 8.5          | 3.8 ± 8.5            | −5.4 ± 8.5           | −3.1 ± 9.2           |
| 0.0 ± 7.7            | 6.2 ± 7.7            | 10.8 ± 7.7           | −4.6 ± 9.2           |
| 2.3 ± 5.4            | −29.2 ± 6.2          | 9.2 ± 6.2            | −8.5 ± 5.4           |
| −6.9 ± 6.9           | −0.8 ± 6.9           | −11.5 ± 6.9          | −16.2 ± 6.9          |
| −2.3 ± 9.2           | −7.7 ± 9.2           | 10.8 ± 9.2           | −4.6 ± 9.2           |
| 5.4 ± 3.8            | 8.5 ± 3.8            | −8.5 ± 3.8           | 0.8 ± 3.8            |
| 7.7 ± 7.7            | −1.5 ± 9.2           | −8.5 ± 8.5           | 5.4 ± 8.5            |
| −0.8 ± 3.8           | −0.8 ± 3.8           | −2.3 ± 3.8           | 3.8 ± 3.8            |
| −16.2 ± 11.5         | 6.2 ± 13.8           | −25.4 ± 11.5         | −16.2 ± 11.5         |
Table 3: The experimental values of Ref. [1] for the combinations of $C-$ and $S-$ coefficients defined in Eqs. (28)-(29) and the resulting $Q$ from Eq. (31). For simplicity, we report symmetrical errors.

| $C_{11}$[x10⁻¹⁶] | $C_{22}$[x10⁻¹⁶] | $S_{11}$[x10⁻¹⁶] | $S_{22}$[x10⁻¹⁶] | $Q$[x10⁻¹⁶] |
|-------------------|-------------------|-------------------|-------------------|-------------|
| 13.6 ± 5.0        | 1.5 ± 4.6         | 11.0 ± 4.6        | 6.5 ± 4.6         | 13.3 ± 3.4  |
| 5.2 ± 6.3         | 5.8 ± 6.5         | 17.7 ± 6.9        | 7.1 ± 6.9         | 14.6 ± 4.8  |
| 2.3 ± 3.8         | 5.2 ± 3.8         | 6.0 ± 3.5         | 4.5 ± 3.8         | 6.6 ± 2.6   |
| 12.4 ± 4.0        | 14.6 ± 3.8        | 12.0 ± 3.8        | 11.0 ± 4.0        | 17.8 ± 2.8  |
| 12.7 ± 8.5        | 12.4 ± 8.3        | 4.9 ± 8.2         | 7.0 ± 6.9         | 14.0 ± 5.8  |
| 8.1 ± 5.9         | 7.1 ± 5.4         | 3.4 ± 6.2         | 10.9 ± 6.2        | 11.1 ± 4.2  |
| 12.0 ± 6.0        | 7.7 ± 5.9         | 10.0 ± 6.0        | 5.6 ± 5.4         | 13.0 ± 4.2  |
| 8.3 ± 9.2         | 13.2 ± 9.0        | 4.5 ± 8.5         | 21.8 ± 8.5        | 19.2 ± 6.1  |
| 8.7 ± 7.0         | 7.4 ± 6.2         | 11.0 ± 6.8        | 9.4 ± 6.2         | 13.0 ± 4.7  |
| 9.4 ± 4.6         | 10.5 ± 5.4        | 6.0 ± 4.9         | 7.4 ± 4.9         | 12.0 ± 3.5  |
| 5.2 ± 3.2         | 3.1 ± 3.1         | 3.4 ± 3.1         | 4.1 ± 3.7         | 5.7 ± 2.4   |
| 17.1 ± 7.4        | 10.7 ± 7.0        | 3.8 ± 7.4         | 2.4 ± 6.9         | 14.6 ± 5.2  |
| 20.7 ± 4.7        | 10.3 ± 5.2        | 3.9 ± 3.9         | 4.5 ± 3.8         | 16.9 ± 3.3  |
| 6.6 ± 3.2         | 4.1 ± 3.8         | 5.6 ± 3.8         | 6.9 ± 3.1         | 8.3 ± 2.4   |
| 27.5 ± 6.9        | 10.5 ± 6.2        | 24.8 ± 5.9        | 7.6 ± 5.7         | 27.7 ± 4.5  |
| 19.5 ± 9.1        | 14.5 ± 8.0        | 21.7 ± 8.2        | 23.4 ± 7.3        | 28.3 ± 5.7  |
| 4.9 ± 3.6         | 12.0 ± 3.1        | 3.3 ± 3.8         | 12.0 ± 3.8        | 12.7 ± 2.5  |
| 10.1 ± 8.4        | 3.4 ± 8.3         | 10.3 ± 6.9        | 8.6 ± 6.9         | 12.1 ± 5.3  |
| 7.0 ± 8.5         | 13.3 ± 8.5        | 10.7 ± 8.5        | 6.2 ± 8.7         | 13.7 ± 6.0  |
| 16.8 ± 7.0        | 26.2 ± 8.5        | 6.2 ± 7.7         | 11.7 ± 8.0        | 23.9 ± 5.7  |
| 21.6 ± 6.2        | 13.8 ± 6.0        | 29.3 ± 6.1        | 12.5 ± 5.8        | 28.9 ± 4.3  |
| 1.7 ± 7.5         | 15.2 ± 7.7        | 7.0 ± 6.9         | 19.9 ± 6.9        | 18.4 ± 5.1  |
| 16.5 ± 8.6        | 15.1 ± 8.5        | 9.9 ± 9.2         | 11.7 ± 9.2        | 19.2 ± 6.2  |
| 6.9 ± 3.8         | 8.0 ± 3.8         | 10.0 ± 3.8        | 8.5 ± 3.8         | 11.9 ± 2.7  |
| 19.0 ± 7.4        | 11.6 ± 7.7        | 7.8 ± 7.8         | 10.0 ± 8.5        | 18.1 ± 5.4  |
| 1.7 ± 4.5         | 3.4 ± 4.5         | 1.1 ± 3.8         | 4.5 ± 3.8         | 4.2 ± 2.9   |
| 12.0 ± 11.9       | 25.4 ± 10.3       | 17.3 ± 11.9       | 30.1 ± 11.5       | 31.6 ± 7.9  |
For each experimental session, we report the values $Q_i$, $\Delta Q_i$ and the ratio $R_i = Q_i/\Delta Q_i$ from Table 3.

| $Q_i$[x10^{-16}] | $\Delta Q_i$[x10^{-16}] | $R_i = Q_i/\Delta Q_i$ |
|-------------------|--------------------------|------------------------|
| 13.3              | 3.4                      | 3.9                    |
| 14.6              | 4.8                      | 3.0                    |
| 6.6               | 2.6                      | 2.5                    |
| 17.8              | 2.8                      | 6.3                    |
| 14.0              | 5.8                      | 2.5                    |
| 11.1              | 4.2                      | 2.6                    |
| 13.0              | 4.2                      | 3.1                    |
| 19.2              | 6.1                      | 3.1                    |
| 13.0              | 4.7                      | 2.8                    |
| 12.0              | 3.5                      | 3.4                    |
| 5.7               | 2.4                      | 2.4                    |
| 14.6              | 5.2                      | 2.8                    |
| 16.9              | 3.3                      | 5.1                    |
| 8.3               | 2.4                      | 3.4                    |
| 27.7              | 4.5                      | 6.2                    |
| 28.3              | 5.7                      | 5.0                    |
| 12.7              | 2.5                      | 5.1                    |
| 12.1              | 5.3                      | 2.3                    |
| 13.7              | 6.0                      | 2.3                    |
| 23.9              | 5.7                      | 4.2                    |
| 28.9              | 4.3                      | 6.7                    |
| 18.4              | 5.1                      | 3.6                    |
| 19.2              | 6.2                      | 3.1                    |
| 11.9              | 2.7                      | 4.4                    |
| 18.1              | 5.4                      | 3.3                    |
| 4.2               | 2.9                      | 1.4                    |
| 31.6              | 7.9                      | 4.0                    |
Table 5: The experimental $C$–coefficients as reported in Ref.[4].

| $C_s1[\times10^{-16}]$ | $C_c1[\times10^{-16}]$ | $C_s2[\times10^{-16}]$ | $C_c2[\times10^{-16}]$ |
|------------------------|------------------------|------------------------|------------------------|
| $-3.0 \pm 2.0$         | $11.0 \pm 2.5$         | $1.0 \pm 2.5$          | $0.1 \pm 2.5$          |
Table 6: The $S-$coefficients of Ref.[4]. The values for the $S$-coefficients, constrained by the authors of Ref.[4] in their fits to the data to the theoretical predictions Eqs.(10) and (11), have been deduced from Table 5 using these relations.

\[
\begin{array}{ccccc}
S_{s1}[x_{10^{-16}}] & S_{c1}[x_{10^{-16}}] & S_{s2}[x_{10^{-16}}] & S_{c2}[x_{10^{-16}}] \\
-14.1 \pm 3.2 & -3.8 \pm 2.6 & -0.1 \pm 2.5 & 1.0 \pm 2.5 \\
\end{array}
\]
Table 7: The values of Ref. [4] for the combinations of $C$– and $S$– coefficients defined in Eqs. (28)–(29) and the resulting $Q$ defined in Eq. (31), as computed from Tables 5 and 6. For simplicity, we report symmetrical errors.

| $C_{11}$ [x10^{-16}] | $C_{22}$ [x10^{-16}] | $S_{11}$ [x10^{-16}] | $S_{22}$ [x10^{-16}] | $Q$ [x10^{-16}] |
|------------------------|------------------------|------------------------|------------------------|----------------|
| 11.4 ± 2.5             | 1.0 ± 2.5              | 14.6 ± 3.3             | 1.0 ± 2.5              | 13.1 ± 2.1     |