Probing Psuedoscalars with Pulsar Polarisation Data Sets

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Abstract

Recently a data set containing linear and circular polarisation information of a collection of six hundred pulsars has been released. The operative radio wavelength for the same was 21 cm. Pulsars radio emission process is modelled either with synchroton / superconducting self-Compton route or with curvature radiation route. These theories fall short of accounting for the circular polarisation observed, as they are predisposed towards producing, solely, linear polarisation. Here we invoke (pseudo)scalars and their interaction with photons mediated by colossal magnetic fields of pulsars, to account for the circular part of polarisation data. This enables us to estimate the pseudoscalar parameters such as its coupling to photons and its mass in conjunction as product. To obtain these values separately, we turn our attention to recent observation on 47 pulsars, whose absolute polarisation position angles have been made available. Except, a third of the latter set, the rest of it overlaps with the expansive former data set on polarisation type & degree. This helps us figure out, both the pseudoscalar parameters individually, that we report here.

1 Introduction

In the last two decades, scenarios in which pseudoscalar particles and photons couple and subsequently mix in the presence of magnetic fields have received a lot of attention, both phenomenologically and observationally. This is of particular interest in astrophysics, where this mixing of photons with pseudoscalars could make the universe transparent, change the polarisation properties of light and is be potentially responsible for effects such as Supernovae dimming or Large-scale coherent orientation of the universe, also known as ‘Hutsemekers’ effect. The best-known light pseudoscalar particle, the axion, was introduced long ago to explain the absence of CP violation in Quantum Chromodynamics (QCD). One postulated the existence of a new spontaneously broken continuous Peccei-Quinn symmetry, so that the axion was a pseudo Goldstone boson. It was soon realised that one needed to introduce a very large scale in the theory in order to suppress the interactions of the axion, while preserving the Peccei-Quinn mechanism. The invisible axion emerges at a unification scale, and the effective coupling is suppressed by this scale. The invisible axion, being closely related to QCD, has definite and interrelated expressions for its mass and coupling strength to other particles, given a specific model. Various cosmological and astrophysical bounds can be used to further constrain the parameters, and the allowed parameters do not lead to observable effects over cosmological scales. The mass of the pseudoscalar particle needs to be very close to the photon effective mass in order to mix in the rather weak magnetic fields of the extra galactic space. However, generic pseudoscalars or axion-like particles (ALPs) have been hypothesized by many extensions of the standard model of particle physics. Theories such as Supergravity and Superstring theory contain many broken U(1) symmetries, that can lead to very light scalar, or pseudoscalar, particles.

Pulsars, discovered fifty years back, are a fusion fuel less state of a two to three solar mass star, wherein surmounting inward gravitational pull, in absence of a commensurate radiation pressure from fusion,
makes it collapse into a tiny object. Two effects follow: the protons and neutrons coalesce together making the Pulsars synonymous with neutron stars; and, also during this compression phase the magnetic flux is conserved, thereby promoting the magnetic induction field inside it to a colossal value. Other effects such as the ‘pulsating’ nature of the ‘star’ in its last phase of stellar evolution, leading to the nomenclature & discovery of the same, won’t be pursued here.

![Figure 1: Axion Photon Mixing In The Polar Cap Region](image)

Pulsars have been harnessed to estimate the column density, by observing Pulsar dispersion measure. Also, the magnetic field of the interstellar medium (henceforth ISM) along the line of sight can be estimated by observing its rotation measure. Pulsars were traditionally, observed on earth inside the radio frequency window specifically, from 100 MHz – 100 GHz. However, over time, pulsars became known for emission in other wavebands like X-ray & γ-ray, etc. Despite, half a century of efforts, the mechanisms for such types of radiation and properties thereof, such as polarisation etc. are not very well understood. This in turn banks heavily on the fact that pulsar atmosphere or its magnetosphere models are still in its infancy. There are competing contenders as preferred models for pulsed emission and continuum radiation. Curvature radiation, synchrotron radiation, inverse Compton radiation, superconducting self Compton radiation etc. are at the forefront, but none fits all observational features of pulsars. We shall, however, restrict ourselves, polarisation properties of radiation inside the pulsar atmosphere without looking into the radiation origin. Here we shall harness the two pulsar properties the size, and magnetic field which in turn is deduced from period and associated derivative, to estimate the pseudoscalar parameters like the mass and its coupling to photons, with the help of 21 cm observations.

In section no. we describe the polarimetric data set on six hundred pulsars, along with the quantities that can be derived from these observed parameters assuming a basic pulsar model. The observation of circular polarisation is hitherto unexplained by radiation models, theoretical & statistical alike, so far. Thereafter, in the section no. , we invoke the light quanta to pseudoscalar interaction to step wise calculate the correlators, ab initio, between the three degrees of freedom. Thereby, in section no. we digress to stokes parameters; the experimental interface with theoretical quantities like ellipticity parameter and polarisation position angle, using the definition of correlators.
In the next section 5, we discuss the extraction process of pseudoscalar parameters, for mixing case only, discarding another two cases and leaving the general case open that might arise, naturally. Also in this segment we estimate the values of regression parameters derived from statistical analysis of the data set tables. In the next section we present our results. Thereafter, we conclude by projecting the feasibility of our result and scope in future directions.

2 Observation

The following data shown in table no. (1) is a small part of the data obtained from the reference no. [51]. It contains the spin down luminosity \( \dot{E} \) and pulsar spin period \( P \) all six hundred of them. Following the basic pulsar model \([46,56]\) we may derive pulsar parameters such as spin period time derivative \( \dot{P} \) and the magnetic field \( B_s \).

\[
B_s = 3.2 \times 10^{19} \times \sqrt{PP} \\
\dot{P} = \frac{\dot{E}P^3}{4\pi^2I}
\]

Also, from the ratio between the percentage of circular to linear polarisation provides us with the ellipticity parameter \( \chi \).

\[
\tan(2\chi) = \frac{V}{P_{\text{lin}}}
\]

We have extracted & separately tabulated these derived values for further use in section no. 5.

3 Pseudoscalar Photon Mixing

The following mixing matrix (eqn. no. \([3.1]\)) provides for the necessary ingredient of photon pseudoscalar mixing mediated by a magnetic field \([57]\). Also this reference assumes a free space, for calculation, hence there are no Faraday effect \((M_{21} \& M_{12} \text{ entries})\) terms coupling the two photon polarisations. Inside pulsar magnetosphere this could hardly be the case. However, we may still ignore the Faraday terms. The reason being the smallness of it inside spaces with large magnetic fields; as shown in by one of the coauthors \([58]\), by deriving the limiting propagation frequency, below which Faraday effect holds significance.

\[
\omega_L = \frac{\omega_p^2 \omega B (\omega_p^2 - m_a^2)}{|B|^2} \frac{\cos \Theta}{\sin^2 \Theta}
\]

for the values derived from the pulsar database, such as the magnetic field, and the plasma frequency from literature \([59]\), which is much smaller than the pseudoscalar mass, we see that Faraday effect can safely be neglected at the operating frequency of 1.4 GHz \((\gg \omega_L)\), with which the observations were made.

\[
M = \begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_2 & T \\ 0 & T & -B \end{pmatrix}, \quad (3.1)
\]

Where the symbols in the matrix \([3.1]\) stands for

\[
A_1 = 4\omega^2 \xi \sin^2 \Theta + \omega_p^2 \\
A_2 = 7\omega^2 \xi \sin^2 \Theta + \omega_p^2 \\
B = m_a^2 \\
T = gB \omega.
\]

(3.2)

where, \( B \) is the magnetic field, \( \Theta \) is the angle between the \( \vec{k} \) and the magnetic field \( B \), \( m_a \) the axion mass, and \( \xi = \frac{\alpha}{4\pi} \left( \frac{eB}{m_f^2} \right)^2 \), with \( m_f \) \([57]\) the lightest Fermion mass.

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Table 1: Pulsar polarisation properties at 1.4 GHz (sample)

| Jname      | Period (ms) | $\dot{P}$ (s$^{-1}$) | log($\dot{E}$) | L/I % | V/I % | $|V|/I$ err | B GeV$^2$ | $\chi$ rad |
|------------|-------------|----------------------|----------------|-------|-------|-----------|-----------|------------|
| J0034-0721 | 943         | 4.24E-16             | 31.3           | 10.7  | 7.7   | 7.5       | 3         | 4.44E-08   |
| J0051+0423 | 354.7       | 7.14E-18             | 30.8           | 13.1  | -2.3  | 11.2      | 3         | 3.53E-09   |
| J0108-1431 | 807.6       | 8.43E-17             | 30.8           | 76.7  | 15.5  | 13.1      | 3         | 1.83E-08   |
| J0134-2937 | 137         | 8.21E-17             | 33.1           | 45.3  | -17.2 | 16.9      | 3         | 7.44E-09   |
| J0151-0635 | 1464.7      | 3.99E-16             | 30.7           | 29.1  | -1.7  | 4.2       | 3         | 5.36E-08   |
| J0152-1637 | 832.7       | 1.16E-15             | 31.9           | 15.1  | 1.1   | 6         | 3         | 6.90E-08   |
| J0206-4028 | 630.6       | 1.27E-15             | 32.3           | 10.6  | 9.3   | 9.9       | 3         | 6.27E-08   |
| J0211-8159 | 1077.3      | 3.17E-16             | 31             | 17    | 11.7  | 15.4      | 5.5       | 4.10E-08   |
| J0255-5304 | 447.7       | 2.86E-17             | 31.1           | 7.3   | -4.1  | 5.5       | 3         | 7.94E-09   |
| J0304+1932 | 1387.6      | 1.35E-15             | 31.3           | 33.4  | 15.1  | 14.8      | 3         | 9.60E-08   |
| J0343-3000 | 2597        | 5.59E-17             | 29.1           | 14.3  | 3.1   | 3.9       | 3         | 2.67E-08   |
| J0401-7608 | 545.3       | 1.64E-15             | 32.6           | 28.6  | -0.1  | 4.7       | 3         | 6.63E-08   |
| J0448-2749 | 450.4       | 1.46E-16             | 31.8           | 23.9  | -13.3 | 11.8      | 3         | 1.80E-08   |
| J0450-1248 | 438         | 1.07E-16             | 31.7           | 25.3  | 2.5   | 6         | 3         | 1.52E-08   |
| J0452-1759 | 548.9       | 5.28E-15             | 33.1           | 18.9  | 3.6   | 4.2       | 3         | 1.19E-07   |
| J0459-0210 | 1133.1      | 1.47E-15             | 31.6           | 10.4  | -12.9 | 9.6       | 3         | 9.05E-08   |
| J0520-2553 | 241.6       | 2.84E-17             | 31.9           | 18.2  | -4.3  | 5         | 3         | 5.81E-09   |
| J0525+1115 | 354.4       | 7.12E-17             | 31.8           | 10.6  | 12.5  | 15.5      | 3         | 1.11E-08   |
| J0528+2200 | 3745.5      | 4.21E-14             | 31.5           | 36.9  | -4.9  | 4.6       | 3         | 8.81E-07   |
| J0533+0402 | 963         | 1.8E-16              | 30.9           | 13.3  | 4     | 5.5       | 3         | 2.92E-08   |
| J0536-5433 | 1245.9      | 6.17E-16             | 31.1           | 48.8  | -11.1 | 11        | 3         | 6.15E-08   |
| J0540-7125 | 1286        | 8.55E-16             | 31.2           | 14.2  | 3.1   | 15.8      | 4.8       | 7.35E-08   |
| J0543+2329 | 246         | 1.5E-14              | 34.6           | 45.2  | -8.2  | 7.9       | 3         | 1.35E-07   |
| J0601-0527 | 396         | 1.25E-15             | 32.9           | 30.9  | 4.4   | 11.3      | 3         | 4.94E-08   |
| J0614+2229 | 335         | 6.01E-14             | 34.8           | 72    | 20.3  | 20.1      | 3         | 3.15E-07   |

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The non diagonal 2x2 matrix, in eqn. 3.1 is given by,

\[ M_2 = \begin{pmatrix} A_2 & T \\ T & -m_a^2 \end{pmatrix}. \]  

(3.3)

One can solve for the eigen values of the eqn. 3.4 from the determinant equation,

\[ \begin{vmatrix} A_2 - \lambda & T \\ T & -m_a^2 - \lambda \end{vmatrix} = 0, \]  

(3.4)

and the roots are,

\[ M_\pm = \frac{A_2 - m_a^2}{2} \pm \frac{1}{2} \sqrt{[(A_2 + m_a^2)^2 + 4T^2]}. \]  

(3.5)

\[ M_\pm = \frac{A_2 - m_a^2}{2} \pm \frac{1}{2} \sqrt{[(A_2 + m_a^2)^2 + 4T^2]}. \]  

(3.6)

### 3.1 Equation of Motion

The equation of motion for the axion photon mixing, in the non diagonal basis gets decoupled and can be written in the matrix form as,

\[ \begin{pmatrix} \omega^2 + \partial_z^2 \end{pmatrix} \begin{pmatrix} \mathbf{I} + \mathbf{M} \end{pmatrix} \begin{pmatrix} \begin{pmatrix} A_{\perp} \\ A_{\parallel} \end{pmatrix} \end{pmatrix} = 0. \]  

(3.7)

where \( \mathbf{I} \) is a 3 \times 3 identity matrix and \( \mathbf{M} \) is the mixing matrix.

The uncoupled and the coupled equations can further be written as,

\[ \begin{pmatrix} \omega^2 + \partial_z^2 \end{pmatrix} + A_1 \begin{pmatrix} \begin{pmatrix} A_{\perp} \end{pmatrix} \end{pmatrix} = 0. \]  

(3.8)

and

\[ \begin{pmatrix} \omega^2 + \partial_z^2 \end{pmatrix} \begin{pmatrix} \mathbf{I} + M_2 \end{pmatrix} \begin{pmatrix} \begin{pmatrix} A_{\parallel} \\ a \end{pmatrix} \end{pmatrix} = 0. \]  

(3.9)

It is possible to diagonalise eqn. 3.9 by a similarity transformation (we would denote the diagonalising matrix by \( O \)), leading to the form,

\[ \begin{pmatrix} \omega^2 + \partial_z^2 \end{pmatrix} \begin{pmatrix} \mathbf{I} + M_D \end{pmatrix} \begin{pmatrix} \begin{pmatrix} \bar{A}_{\parallel} \\ \bar{a} \end{pmatrix} \end{pmatrix} = 0. \]  

(3.10)

when the diagonal matrix \( M_D \) is given by:

\[ M_D = \begin{pmatrix} M_+ & 0 \\ 0 & M_- \end{pmatrix} \]  

(3.11)
3.2 Dispersion Relations

Defining the wave vectors in terms of $k_i$’s, as:
\[
\begin{align*}
    k_{\perp} &= \sqrt{\omega^2 + A_1} \\
    k_+ &= \sqrt{\omega^2 + M_+} \\
    k_- &= -\sqrt{\omega^2 + M_+}
\end{align*}
\]

and
\[
\begin{align*}
    k'_+ &= \sqrt{\omega^2 + M_-} \\
    k'_- &= -\sqrt{\omega^2 + M_-}
\end{align*}
\]

3.3 Solutions

The solutions for the gauge field and the axion field, given by (3.10) as well as the solution for eqn. for $A_\perp$ in $k$ space can be written as,
\[
\begin{align*}
    \bar{A}|\parallel(z) &= \bar{A}|\parallel(0)e^{ik_+z} + \bar{A}|\perp(0)e^{-ik_-z} \\
    \bar{a}(z) &= \bar{a}(0)e^{ik'_+z} + \bar{a}(-0)e^{-ik'_-z} \\
    A_\perp(z) &= A_{\perp}(0)e^{ik_\perp z} + A_{\perp}(0)e^{-ik_\perp z}
\end{align*}
\]

The diagonal matrix can be written as
\[
M_D = O^TM_2O
\]

when
\[
O = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \equiv \begin{pmatrix} c & -s \\ s & c \end{pmatrix}.
\]

in short hand notation.

3.4 Similarity Transformation

The diagonal matrix
\[
M_D = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} c & -s \\ s & c \end{pmatrix},
\]

With $M_{11} = A_2$, $M_{12} = T$, $M_{21} = T$ lastly $M_{22} = -B$.

The value of the parameter $\theta$, is fixed from the equality,
\[
M_D = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} c & -s \\ s & c \end{pmatrix} = \begin{pmatrix} M_+ & 0 \\ 0 & M_- \end{pmatrix},
\]

leading to,
\[
\begin{pmatrix} c^2M_{11} + s^2M_{22} + 2csM_{12} \\ M_{12}(c^2-s^2) + cs(M_{22}-M_{11}) \end{pmatrix} \begin{pmatrix} M_{12}(c^2-s^2) + cs(M_{22}-M_{11}) \\ s^2M_{11} + c^2M_{22} - 2csM_{12} \end{pmatrix} = \begin{pmatrix} M_+ & 0 \\ 0 & M_- \end{pmatrix},
\]

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Equating the components of the matrix equation \[3.22\], one arrives at:

\[
\tan(2\theta) = \frac{2M_{12}}{M_{11} - M_{22}} = \frac{2T}{A_2 - m_a^2}.
\] (3.23)

### 3.5 Correlation Functions

The solutions for propagation along the +ve z axis, is given by,

\[
\bar{A}_\parallel(z) = \bar{A}_\parallel(0)e^{ik_+z}
\]

\[
\bar{a}(z) = \bar{a}(0) e^{ik_+z}
\] (3.24)

that can further be written in the following form,

\[
\begin{pmatrix}
\bar{A}_\parallel(z) \\
\bar{a}(z)
\end{pmatrix} =
\begin{pmatrix}
 e^{ik_+z} & 0 \\
 0 & e^{ik'_+z}
\end{pmatrix}
\begin{pmatrix}
\bar{A}_\parallel(0) \\
\bar{a}(0)
\end{pmatrix}.
\] (3.27)

Since,

\[
\begin{pmatrix}
\bar{A}_\parallel(z/0) \\
\bar{a}(z/0)
\end{pmatrix} = O^T
\begin{pmatrix}
 A_\parallel(z/0) \\
a(z/0)
\end{pmatrix}.
\] (3.28)

it follows from there that,

\[
\begin{pmatrix}
 A_\parallel(z) \\
a(z)
\end{pmatrix} = O
\begin{pmatrix}
 e^{ik_+z} & 0 \\
 0 & e^{ik'_+z}
\end{pmatrix} O^T
\begin{pmatrix}
 A_\parallel(0) \\
a(0)
\end{pmatrix}.
\] (3.29)

Using eqn. \[3.29\] we arrive at the relation,

\[
A_\parallel(z) = \left[e^{ik_+z}\cos^2\theta + e^{ik'_+z}\sin^2\theta\right] A_\parallel(0) + \left[e^{ik_+z} - e^{ik'_+z}\right] \cos\theta \sin\theta \ a(0)
\]

\[
a(z) = \left[e^{ik_+z} - e^{ik'_+z}\right] \cos\theta \sin\theta \ A_\parallel(0) + \left[e^{ik_+z}\sin^2\theta + e^{ik'_+z}\cos^2\theta\right] a(0)
\] (3.30)

If the axion field is zero to begin with, i.e

\[
a(0) = 0.
\] (3.32)

Then the solution for the gauge fields take the following form,

\[
A_\parallel(z) = \left[e^{ik_+z}\cos^2\theta + e^{ik'_+z}\sin^2\theta\right] A_\parallel(0)
\]

\[
A_\perp(z) = e^{ik_+z} A_\perp(0).
\] (3.33) (3.34)

The correlations of different components take the following form:

\[
< A^*_\parallel(z) A_\parallel(z) > = \left[\cos^2\theta + \sin^2\theta + 2\sin^2\theta \ \cos^2\theta \ \cos \left[(k_+ - k'_+)z\right]\right] < A^*_\parallel(0) A_\parallel(0) >
\] (3.35)

\[
< A^*_\parallel(z) A_\perp(z) > = \left[\cos^2\theta e^{ik_+z} + \sin^2\theta e^{ik'_+z}\right] < A^*_\parallel(0) A_\perp(0) >
\] (3.36)

\[
< A^*_\perp(z) A_\perp(z) > = < A^*_\perp(0) A_\perp(0) >
\] (3.37)
4 Stokes Parameters

Using the definitions of the Stokes parameters, in terms of the correlators:

\[
I = \langle A^*_{||}(z)A_{||}(z) \rangle + \langle A^*_{\perp}(z)A_{\perp}(z) \rangle, \quad (4.1)
\]
\[
Q = \langle A^*_{||}(z)A_{||}(z) \rangle - \langle A^*_{\perp}(z)A_{\perp}(z) \rangle, \quad (4.2)
\]
\[
U = 2\text{Re} \langle A^*_{||}(z)A_{\perp}(z) \rangle, \quad (4.3)
\]
\[
V = 2\text{Im} \langle A^*_{||}(z)A_{\perp}(z) \rangle. \quad (4.4)
\]

Using the relations for the corresponding correlators, the Stokes parameters turn out to be

\[
I = \cos^4\theta + \sin^4\theta + 2\sin^2\theta \cos^2\theta \cos\left[(k_+ - k'_+)z\right] < A^*_{||}(0)A_{||}(0) > + \langle A^*_{\perp}(0)A_{\perp}(0) >
\]
\[
Q = \cos^4\theta + \sin^4\theta + 2\sin^2\theta \cos^2\theta \cos\left[(k_+ - k'_+)z\right] < A^*_{||}(0)A_{||}(0) > - \langle A^*_{\perp}(0)A_{\perp}(0) >
\]
\[
U = 2 \left(\cos^2\theta\cos\left[(k_+ - k'_+)z\right] + \sin^2\theta\cos\left[(k_+ - k'_+)z\right]\right) < A^*_{||}(0)A_{\perp}(0) >
\]
\[
V = 2 \left(\cos^2\theta\sin\left[(k_+ - k'_+)z\right] + \sin^2\theta\sin\left[(k_+ - k'_+)z\right]\right) < A^*_{||}(0)A_{\perp}(0) >
\]

(4.5)

The Stokes parameters are also expressed as such

\[
I = I_p \quad (4.6)
\]
\[
Q = I_p\cos2\psi\cos2\chi \quad (4.7)
\]
\[
U = I_p\sin2\psi\cos2\chi \quad (4.8)
\]
\[
V = I_p\sin2\chi \quad (4.9)
\]

where \(\chi\) & \(\psi\) are are usual ellipticity parameter and the polarisation position angle. The degree of (linear) polarisation is given by,

\[
p = \sqrt{Q^2 + U^2 + V^2}
\]
\[
\rho_{lin} = \frac{\sqrt{Q^2 + U^2}}{I_p} \quad (4.10)
\]

and the linear polarisation angle is given by

\[
\tan2\psi = \frac{U}{Q}
\]
\[
\tan2\chi = \frac{V}{\rho_{lin}} \quad (4.11)
\]

It has been noted in \[60\], that in case, we make any coordinate transformation around the axis of photon propagation the two linear polarisation become mixed. Hence, we need to be careful, as our solution process entails a similarity transformation. To see this we define the density matrix

\[
\rho(z) = \begin{pmatrix}
A^*_{||}(z)A_{||}(z) \\
A^*_{\perp}(z)A_{\perp}(z)
\end{pmatrix}
\begin{pmatrix}
A_{||}(z)A^*_{||}(z) \\
A_{\perp}(z)A^*_{\perp}(z)
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
I(z) + Q(z) & U(z) - iV(z) \\
U(z) + iV(z) & I(z) - Q(z)
\end{pmatrix}
\]

(4.12)

if we rotate the density matrix by an amount \(\alpha\) about an axis perpendicular the plane containing \(A_{||}(z)\) & \(A_{\perp}(z)\), the density matrix transforms as \(\rho(z) \rightarrow \rho'(z)\) given such to be

\[
\rho'(z) = \frac{1}{2}R(\alpha) \begin{pmatrix}
I(z) + Q(z) & U(z) - iV(z) \\
U(z) + iV(z) & I(z) - Q(z)
\end{pmatrix} R^{-1}(\alpha)
\]

(4.13)
where,

\[ R(\alpha) = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \] (4.14)

Under such transformation the \( I(z) \) & \( V(z) \) remains unaltered. However, the \( Q(z) \) & \( U(z) \) starts mixing with each other by the following

\[ \begin{pmatrix} Q'(z) \\ U'(z) \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix} \begin{pmatrix} Q(z) \\ U(z) \end{pmatrix} \] (4.15)

We conclude this section by mentioning that in such a case the ellipticity parameter remains unaltered but the polarisation position angle changes by \( 2\alpha \) as given below

\[ \tan(2\chi') = \tan(2\chi) \]
\[ \tan(2\psi') = \tan(2\alpha + 2\psi) \] (4.16)

5 Ellipticity Parameter & Polarization Position Angle

As a follow-up to the analytical expressions given in the previous section/s, we consider two special case of the stokes parameter where either one of the two effects, namely, the mixing effect or, the vacuum birefringence effect would be absent. Thereafter we shall consider the general formula. In each case, we would like to obtain the value of the ellipticity angle \( \chi \) after propagation a fixed distance \( z \) of light and determine it’s frequency dependence. For all the three cases we shall assume the light to be completely plane polarised in the transverse direction, or \( U \) polarised. This is common observance in Pulsar polarisation cases.

5.1 Case - I: Mixing Only

Here we assume that the vacuum birefringence terms (i.e. \( \xi \) term inside the diagonal ones \( A_1, A_2 \)) are absent. We also assume a pseudoscalar mass which is much less than the plasma frequency here. This greatly simplifies calculation without being much deviant from the reality, if we consider the parameters of the pulsar environment. Next we consider how the circular polarisation varies in this case. Assuming \( \theta \ll 1 \) one have

\[ V = \left\{ \sin[(k_\perp - k_+) z] + \frac{g B \omega}{\sqrt{\omega_p^2 + m_\perp^2}} \right\}^2 \sin\left[ (k_\perp - k_+)' z \right] < A_1^\dagger(0)A_1(0) > \] (5.1)

Following the set of eqns. 3.12-3.13 we can simplify the arguments of the remaining sinusoids of eqn. no. 5.1 as given below:

\[ k_\perp - k_+ = \left\{ \frac{(gB\omega)^2}{2(\omega_p^2 + m_\perp^2)\omega} \right\} \]
\[ k_\perp - k_+ = \left\{ \frac{(gB\omega)^2}{2(\omega_p^2 + m_\perp^2)\omega} \right\} + \frac{m_\perp^2}{2\omega} \] (5.2)

So, if \( \xi = 0 \), then the ellipticity parameter to its lowest order (\( \propto \theta^2 \)) is found to be as follows, which matches well with \[60, 61\] though the later most probably has a typo \[62\].

\[ \chi \approx \frac{1}{90\omega}(gBm_\perp)^2\theta^2 \] (5.3)

Similarly, we may now turn our attention to two linear polarisation degrees of freedom, where the mixing angle \( \theta \ll 1 \), is small, to figure out the polarisation position angle.

\[ \tan(2\psi) = \frac{U}{Q} \] (5.4)

However, in the beginning of this section we have already mentioned that \( U \simeq 1 \). This is true for the parameters of interest used here and the observational cases to be discussed later. This makes the polarisation position angle inversely proportional to \( Q \). But before we evaluate the expression for \( Q \), we note that in the case of mixing the beam

\[ (1) \text{It claimed concurrence with the former but is actually at variance, with it} \]
| Coefficients | Mean | Std.-Error | F-statistics | t-value | Pr(>|t|) |
|--------------|------|------------|--------------|---------|---------|
| Slope        | 2.404e-25 | 4.567e-26 | 27.7196 | 5.265 | 1.21e-05 |

Table 2: Regression Result for the coupling of the pseudoscalar

is assumed to propagate at an angle $\frac{\pi}{4}$ as compared to the magnetic field of the Pulsar. Hence we need to change our expression for polarisation position angle accordingly. As discussed during derivation of eqn. no. (4.10), we have:

$$\tan \left(2\psi + \frac{\pi}{2}\right) = \frac{1}{Q}$$  \hspace{1cm} (5.5)

Next, we evaluate $Q$ keeping in mind the approximations made before. Keeping terms up to order $\theta^2$ in the expression for $Q$, we have,

$$Q = -2\theta^2 \left[ \sin^2 \left( \langle k_+ - k_+' \rangle z \right) \right]$$  \hspace{1cm} (5.6)

Again, following the set of Eqns. 3.12-3.13 we have

$$\langle k_+ - k_+' \rangle \approx \frac{m_e^2}{2\omega}$$  \hspace{1cm} (5.7)

Substituting, one gets, in conjunction with [61]

$$\psi = \frac{1}{16} (g^2 B z)^2$$  \hspace{1cm} (5.8)

However, unlike the circular polarisation, which was attributed to its entirety, to the mixing effect, one can not ascribe the entire pulsar linear polarisation [62] to this tiny mixing effect, where the mixing angle $\theta \ll 1$. So, we note that the pulsar radio emission is inherently linearly polarised to a large degree, due to curvature, synchrotron and superconducting self Compton effects thereof. We use $U \simeq 1$ and only the $Q$ part is modelled via pseudoscalar photon mixing; where

$$Q = \frac{1}{8} (g^2 B z)^2$$  \hspace{1cm} (5.9)

along with the definitive couple of eqn. (4.11) to note that the linear polarisation observed is equal to

$$p_{lin} = Q \sec \left(2\psi + \frac{\pi}{2}\right)$$  \hspace{1cm} (5.10)

We note that the determination process of absolute pulsar polarisation [63, absolute PPAs] position angles, is now experimentally feasible and the same values have already been scraped out for 30 (thirty) odd pulsars. The literature contains a little less than fifty absolute PPAs from [64, absolute polarisation position angles for 47 odd pulsars], out of which only 30 (thirty) cross matched with that of our old set of 537 data, used to calculate the ellipticity parameter. The expression for $Q$ has only one unknown, the coupling of pseudoscalar with photons. Hence, we may do a regression analysis here, too, to estimate the same. The summary table is given in table no. (2).

For the sake of brevity, we post a small segment of total 47 pulsar given in reference [64] in table no. (3). The pulsar names here are catalogued in B1950 almanac standard, which were then converted to J2000 almanac standard and cross matched with the original & usable 537 strong population data on pulsar polarisation. Thirty (30) odd samples of them were found to be common in both.

Now, we turn our attention back to the ellipticity parameter given in equation no. (5.3). Being a small angle, we have relegated the tangent as equivalent to its angular argument. The regression analysis thus to be undertaken is between circular and linear random variables, lying on LHS (ellipticity parameter) and RHS (magnetic field) respectively. Suffice it to say that the LHS is readily read off from the table no. (1). The result of correlation study is given in the table no. (4)
5.2 Case - II: Vacuum birefringence Only

Here if we assume the mixing to be absent then we get \( \theta = 0 \) and hence we get the circular polarisation as

\[
V = 2 \left( \sin [(k_\perp - k_\parallel)z] \right) < \Lambda^*_\parallel(0) \Lambda_\perp(0) >
\]  

(5.11)
Table 3: Small Sample of Absolute Pulsar Polarisation Position Angles, from [64].

| Pulsar     | $PA_V$ (deg) | $PA_0$ (deg) | $\Psi$ (deg) |
|------------|--------------|--------------|--------------|
| B0011+47  | +136(3)      | 43(7)        | –87(8)       |
| B0136+57  | –131(0)      | 43(3)        | 6(3)         |
| B0329+54  | 119(1)       | 20(4)        | 99(4)        |
| B0355+54  | 48(1)        | –41(4)       | 89(5)        |
| B0450+55  | 108(0)       | –23(16)      | –94(16)      |
| B0450–18  | 40(5)        | 47(3)        | –7(6)        |
| B0540+23  | 58(19)       | –85(3)       | –37(19)      |
| B0628–28  | 294(2)       | 26(2)        | 88(3)        |
| B0736–40  | 227(5)       | –44(5)       | 91(7)        |

Coefficients | Mean | Std.-Error | F-statistics | t-value | Pr(>|t|) |
|-------------|------|------------|--------------|---------|---------|
| Slope       | 7.471e-38 | 2.251e-38 | 11.0164      | 3.319   | 0.000964 |

Table 4: Regression Result for the mass of the pseudoscalar

Here, we need to evaluate only one argument and it is the same as given above:

\[ k_\perp - k_+ = \frac{1}{2\omega} \{ -3\xi \} \]  \hspace{1cm} (5.12)

We note, that, the mass of the pseudoscalar cancels and the mixing term is assumed zero. We see, that, the circular polarisation has now become inversely proportional to frequency assuming the argument to be considerably small as in the other case. Given that no circular polarisation would be produced in this case, it would be uninteresting to ponder over here.

5.3 Case III: Limiting Case

We note the essential non-linearity resulting from the two effects taken in conjunction. Since we can not just add the two effects separately, even if they both are small and perturbative, to obtain the final result. We also note that the two effects shall be competing with each other when the following condition is met

\[ \omega \xi \approx gB \]

Leaving aside the numerical prefactors -tentatively we see that, unless the value of magnetic field is much larger than normal pulsars (as in magnetars) and the beam frequency used in experiment is quite high (unlike the present case), even the modest values coming from astronomical bounds on pseudoscalar may not be comparable with the vacuum birefringence effect & the former is in fact larger in effect.

5.4 Case IV: General Case [65]

Here in this subsection we calculate the amount of circular polarisation, vide the stokes parameter $V$, without resorting to any of the approximations made in the preceding two subsections, for completeness. Here the expression for the $V$ becomes

\[ V = \left\{ \cos^2 \left[ \frac{gB\omega}{7\xi\omega^2 \sin^2 \alpha + \omega_p^2 + m_0^2} \right] \sin \left[ (k_\perp - k_+) z \right] + \sin^2 \left[ \frac{gB\omega}{7\xi\omega^2 \sin^2 \alpha + \omega_p^2 + m_0^2} \right] \sin \left[ (k_\perp - k_+) z \right] \right\} < A_{||}(0)A_\perp(0) > \]  \hspace{1cm} (5.13)
Hence at lowest order the first term, in $\theta \ll 1$ limit, would not change anything from what the first term, for the case of the pure mixing effect, did. But the second term, even at the lowest order shall render the expression for $V$ qualitatively different from what is was then at pure mixing effect. Needless to say that pure vacuum birefringence effect does not match, even qualitatively, with any of them, either. For completeness, we write down the values of the wave vectors again.

$$k_\perp - k_\perp = \frac{1}{2\omega} \left\{ 4\xi \left( 4 \cos^2 \alpha - 7 \sin^2 \alpha \right) + \left[ \frac{(gB\omega)^2}{7\xi \omega^2 \sin^2 \alpha + m_a^2} \right] \right\}$$

$$k_\perp - k_\perp = \frac{1}{2\omega} \left\{ 4\xi \cos^2 \alpha + m_a^2 - \left[ \frac{(gB\omega)^2}{7\xi \omega^2 \sin^2 \alpha + m_a^2} \right] \right\}$$

Thus far we have only shown the difference of results of all three separate cases in terms of the $V$ parameters depicting circular polarisation. This can be done with other two linear polarisation degrees of freedom, too. We leave this for a future endeavour.

6 Result

By the careful analysis of [51, containing 600 Pulsar polarisation data, of which 537 are used here] and that of [64, containing 47 absolute PPAs for pulsars, 30 of which are common to the above], we came to the following result given in table no. [5]. We however note that more data samples on absolute PPAs are required to obtain a more statistically significant result on the coupling, which is deduced, from this parameter. Currently a little over fifty (50) pulsars are amenable to this type of absolute PPA studies. The second quantity, namely the mass, has the numbers (> 500) on its side. Nonetheless, its extraction from the ellipticity parameter, in turn, hinges on the coupling value, indirectly affecting the confidence interval found from the population. Also, for the sake of thoroughness we mention that the degree of linear polarisation is claimed to be dependent on frequencies in which they are observed [66]. The PPAs that are quoted in [64, Absolute PPAs for 47 pulsars], are for various radio frequencies, e.g. 327 MHz, 691 MHz, 3.1 GHz etc., including that of 1.4 GHz, which corresponds to 21 cm. Since, there has been no connection to PPAs are made with frequency, to our knowledge to this date, we did not investigate this further.

| Parameters | Values | Significance Level |
|------------|--------|-------------------|
| $g_{\phi \gamma \gamma}$ | $4.903 \times 10^{-13}$ GeV$^{-1}$ | $\approx 0.001\%$ |
| $m_a$ | $2.733 \times 10^{-10}$ eV | $< 0.1\%$ |

Table 5: The result of this analysis

7 Discussion & Outlook

Taking advantage of new age of data explosion arising out from newer observational techniques and that of machine tools, we tried to estimate pseudoscalar particle mass and its coupling to photons. The results thus obtained do not match any standard axion models such as DFSZ or KSVZ etc. Hence these finding must be accommodated in the fold of axion like particles (ALPs) outside of the QCD realm. Surprisingly, our bottom up study, has automatically, led us to values, that are comparable and between the contemporary theories on cosmic axion background radiation (CAB), leading to soft X-ray excesses observed from comma cluster [67] & that of the extra-galactic background light (EBL) to ALPs conversion & oscillation, leading to an observed anomalous $\gamma$ ray transparency of the universe [68]. Fortunately, the previous constraints set on the mass and coupling of pseudoscalars, either by the changes in of quasar polarisation, hypothetically by ALPs [69], or by the $\gamma$ ray burst SN1987A [70], occurring through a so called ALPs burst, are not in conflict with our results, either.

As mentioned in section no. [5] a future incorporation of vacuum birefringence effect into this study, may be performed, so as to see how the result on these estimates may change, for better or worse. These parameters may also be harnessed for devising CDM/WDM models and to obtain their relic densities.
Acknowledgement

The authors acknowledge the software R [71] for easing out the statistical analysis. SM is thankful of the help of K. Bhattyacharya regarding the same.

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