Changing domination in fuzzy graph

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Abstract
In this paper the concept of changing vertex removal, changing edge removal and changing edge addition in fuzzy graph with examples are discussed and some standard theorems are proved.

Keywords
Changing vertex removal, changing edge removal and changing edge addition in fuzzy graph.

AMS Subject Classification
05C72.

1 Introduction .................................................. 386
2 Preliminaries ................................................... 386
3 Changing Domination in Fuzzy Graph .................... 387
4 Conclusion ...................................................... 388
References .......................................................... 388

1. Introduction

Many authors discussed the fuzzy logic and applications of the fuzzy logic in the fuzzy field. The washing machine, air conditioner etc. are worked under the fuzzy logic. The road map, location of health center and the traffic signal light are worked the concept of domination number in graph theory. The application of fuzzy logic in graph theory produce several ideas such as allocation of time table, neural network etc, in this way the changing vertex removal, changing edge removal and changing edge addition are important in fuzzy graph.

2. Preliminaries

Definition 2.1. A Fuzzy Subset of a nonempty set V is a Mapping $\sigma : V \to [0,1]$. A Fuzzy Relation on V is a Fuzzy subset of $V \times V$. A Fuzzy Graph $G(\sigma, \mu)$ on $G(V,E)$ is a pair of functions $\sigma : V \to [0,1]$ and $\mu : V \times V \to [0,1]$ where for all $u, v \in V$ we have $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 2.2. A fuzzy graph $H(\tau, v)$ is called fuzzy sub graph of $G(\sigma, \mu)$ if $\tau(u) \leq \sigma(v)$ for all $u \in V$ and $v(u,v) \leq \mu(u,v)$ for all $u, v \in V$. A fuzzy sub-graph $H(\tau, v)$ is said to be a spanning fuzzy graph of $G(\sigma, \mu)$ if $\tau(u) = \sigma(u)$ for all $u$. In this case, two graphs have same vertex set, they differ only in the arc weight.

Definition 2.3. A fuzzy graph $H(V_1, E_1)$ is called a subgraph of $G(V,E)$ if $V_1 \subseteq V$ and $E_1 \subseteq E$. If $H$ is called an induced subgraph of $G$ if $H$ is the maximal subgraph of $G$ with point set $V_1$. Thus if $H$ is an induced subgraph of $G$, two points are adjacent in $H$ if and only if they are adjacent in $G$. If $V_1 \subseteq V_2$ then the induced subgraph of $G$ with point set $V_2$ is called the subgraph of $G$ induced by $V_2$ and its denoted by $G(V_2)$.

Definition 2.4. Let $G(\sigma, \mu)$ be a fuzzy graph. $N(x) = \{y \in V / \mu(x,y) = \sigma(x) \wedge \sigma(y)\}$ is called the open neighbourhood of $x$. $N[x] = N(x)U\{x\}$ is the closed neighbourhood of $x$. A vertex $u$ of a fuzzy graph is said to be an isolated vertex if $\mu(u,v) < \sigma(u) \wedge \sigma(v)$ for all $v \in V$, $N(u) = \emptyset$.

Definition 2.5. The private neighborhood set of a vertex $v \in S$ to be $pn[v,S] = N[v] = N[S - \{v\}]$. If $pn[v,S] \neq \emptyset$ for some vertex, then every vertex in $pn[v,S]$ is called a private neighbor of $v$. And also define the private neighborhood set of a set $S$ to equal $pn(S) = \{|v : pn[v,S] \neq \emptyset\}$. Finally, define the private neighbor count of a set $S$ to equal $pnc(S) = |pn(S)|$.

Definition 2.6. A fuzzy graph $G(\sigma, \mu)$ on $G^*(\sigma^*, \mu^*)$ is called a strong fuzzy graph if $\mu(x,y) = \min(\sigma(x), \sigma(y))$ for all $(x,y) \in \mu^*$ and is complete fuzzy graph if $\mu(x,y) = \sigma(x) \wedge \sigma(y)$ for all $x, y \in \sigma^*$. The vertices $u$ and $v$ are said to be neighbors if $\mu(u,v) > 0$.

Definition 2.7. A path $P$ in a fuzzy graph $G(\sigma, \mu)$ is a sequence of distinct vertices $x_0, x_1, ..., x_n$ such that $\mu(x_i-1,x_i) > 0$, $1 \leq i \leq n$. Here $n \geq 1$ is called the length of a path. The consecutive pairs $(\mu_{i-1}\mu_i)$ are called the arcs of the path.
Definition 2.8. Let $G = (\sigma, \mu)$ be a fuzzy graph. A subset $D$ of $V$ is said to be fuzzy dominating set of $G$ if for every $v \in V - D$. There exists $u \in D$ such that $\mu(u, v) = \sigma(u) \wedge \sigma(v)$.

Definition 2.9. A fuzzy dominating set $D$ of a fuzzy graph $G$ is called minimal dominating set of $G$, if for every vertex $v \in D$, $D - \{v\}$ is not a dominating set. The domination number $\gamma(G)$ is the minimum cardinalities taken over all minimal dominating sets of vertices of $G$. The upper domination number $\Gamma(G)$ of the fuzzy graph $G$ is the maximum cardinality of a minimal dominating set of fuzzy graph $G$.

Definition 2.10. A fuzzy star graph consists of two vertex set $V$ and $U$ with $\sum_{i=1}^{n} d(v_i) = 1$ and $\sum_{i=1}^{n} d(u_i) > 1$ such that $\mu(v, u_i) > 0$ and $\mu(u_i, u_{i+1}) = 0$ for $1 \leq i \leq n$. The union of fuzzy star graph is fuzzy galaxy.

3. Changing Domination in Fuzzy Graph

Definition 3.1. The deletion of a single edge from a fuzzy graph $G$ increases the fuzzy domination number by atmost one does not decrease the fuzzy domination number, $\gamma(G - uv) = \gamma(G) + 1$ for $uv \in E$.

Definition 3.2. Adding an edge into the fuzzy graph $G$ decreases the fuzzy domination number by atmost one and does not increases the domination number $\gamma(G + uv) = \gamma(G) - 1$ for $uv \in E(G)$.

Definition 3.3. The deletion of a single vertex from a fuzzy graph $G$ increases the fuzzy domination number by atmost one and does not decreases the domination number, $\gamma(G - v) = \gamma(G) + 1$ for $v \in V$.

Definition 3.4. The fuzzy domination number of a fuzzy graph $G$ becomes changed when a vertex is removed from $G(CVR)$. The vertex set $V = V^+ \cup V^-$ for $V^0$ is non empty for a tree. Hence, no tree is in CVR.

The vertex set of fuzzy graph $G$ into three sets depending upon their removal affects of $\gamma(G)$.

Let $V = V^+ \cup V^-$ for $V^0 = \{a \in V : \gamma(G - a) = \gamma(G)\}$.

$V^+ = \{a \in V : \gamma(G - a) > \gamma(G)\}$

$V^- = \{a \in V : \gamma(G - a) < \gamma(G)\}$

Similarly the edge set splits into

$E^0 = \{xy \in E : \gamma(G - xy) = \gamma(G)\}$

$E^+ = \{xy \in E : \gamma(G - xy) > \gamma(G)\}$

Definition 3.5. A Fuzzy Graph for which the Domination number changes when an arbitrary vertex is removed (CVR) has $V = V^- \cup V^+$, $\gamma(G - v) \neq \gamma(G)$ for $v \in V$.

A Fuzzy Graph for which the Domination number is unchanged when an arbitrary vertex is removed (UVR) has $V = V^0$, $\gamma(G - v) = \gamma(G)$ for $v \in V$.

Definition 3.6. The fuzzy domination number of fuzzy graph $G$ becomes changed if a single edge is deleted from $G$ has $E = E^+ \gamma(G - xy) \neq \gamma(G)$ for $xy \in E$.

The fuzzy domination number of fuzzy graph $G$ becomes changed if a single edge is added into $G$ (CEA) has $E = E^+ \gamma(G + xy) \neq \gamma(G)$ for $xy \in E(G)$

Theorem 3.7. For a fuzzy graph $G(\sigma, \mu)$, a vertex $v$ is in $V^-$ if and only if $pn[v, S] = \{v\}$ for some $\gamma$ set $S$ containing $v$.

Proof. Given that $G(\sigma, \mu)$ be fuzzy graph and let $v \in V^-$ = $\{v \in V : \gamma(G - v) < \gamma(G)\}$, $D$ be a minimal dominating set of $G - v$. Then $S = D \cup \{v\}$ is a minimal dominating set of $G$. If $D$ contains a vertex of $N(v)$, then $D$ is a minimal dominating set of $G$. This contradicting the assumption that that $v \in V^- = \{v \in V : \gamma(G - v) < \gamma(G)\}$. Since $D$ is minimal dominating set $G$ and $G - v$. Thus $pn[v, S] = \{v\}$.

Conversely suppose that $pn[v, S] = \{v\}$ for some $\gamma$ set containing $v$. Obviously $S - \{v\}$ is a dominating set of $G - v$, so $v \in V^- = \{v \in V : \gamma(G - v) < \gamma(G)\}$.

Theorem 3.8. A fuzzy graph $G(\sigma, \mu) \in CER$ if and only if $G(\sigma, \mu)$ is a galaxy.

Proof. $\gamma$ set of $G$ is $S = \{u, w\}$ then $\gamma_f(G) = 0.6\gamma$ set of $G - b_2$ is $S^1 = \{u, w, b_2\}$ then $\gamma_f(G - b_2) = 0.8, \gamma_f(G - b_2) > \gamma_f(G)$.

Let $G(\sigma, \mu)$ is a galaxy then $G$ is the union of fuzzy star graph. A fuzzy star graph consists of two vertex set $V$ and $U$. Figure 3.1. Fuzzy graph with $V^0 = \{u_1, u_2, u_3\} \cup \{v\}$, $V^+ = \{u\}, V^- = \{w\}, E^0 = \{uv, vw\}, E^+ = \{uu_1, uu_2, uu_3\}$.

Figure 3.2. Fuzzy graph with $V^0 = \{u_1, u_2, u_3\} \cup \{v\}$, $V^+ = \{u\}, V^- = \{w\}, E^0 = \{uv, vw\}, E^+ = \{uu_1, uu_2, uu_3\}$. 

Figure 3.3. Fuzzy graph with $V^0 = \{u_1, u_2, u_3\} \cup \{v\}$, $V^+ = \{u\}, V^- = \{w\}, E^0 = \{uv, vw\}, E^+ = \{uu_1, uu_2, uu_3\}$. 

Figure 3.4. Fuzzy graph with $V^0 = \{u_1, u_2, u_3\} \cup \{v\}$, $V^+ = \{u\}, V^- = \{w\}, E^0 = \{uv, vw\}, E^+ = \{uu_1, uu_2, uu_3\}$. 

Figure 3.5. Fuzzy graph with $V^0 = \{u_1, u_2, u_3\} \cup \{v\}$, $V^+ = \{u\}, V^- = \{w\}, E^0 = \{uv, vw\}, E^+ = \{uu_1, uu_2, uu_3\}$. 

Figure 3.6. Fuzzy graph with $V^0 = \{u_1, u_2, u_3\} \cup \{v\}$, $V^+ = \{u\}, V^- = \{w\}, E^0 = \{uv, vw\}, E^+ = \{uu_1, uu_2, uu_3\}$. 

Figure 3.7. Fuzzy graph with $V^0 = \{u_1, u_2, u_3\} \cup \{v\}$, $V^+ = \{u\}, V^- = \{w\}, E^0 = \{uv, vw\}, E^+ = \{uu_1, uu_2, uu_3\}$.
with which the Domination number changes when an arbitrary vertex is removed. Therefore there can be no edge between two vertices in \( S \). Removal of any edge incident with the vertices \( S \) will change the fuzzy domination number and \( \gamma_f(G - e) \neq \gamma_f(G) \). Hence \( G(\sigma, \mu) \in \text{CEA} \).

Let \( G(\sigma, \mu) \) be a fuzzy graph and the underlying crisp graph \( G^*(V, E) \). Let \( G(\sigma, \mu) \in \text{CEA} \). Fuzzy Graph for which the Domination number changes when an arbitrary edge is removed (CEA) has \( E = E^+ \gamma_f(G - e) \neq \gamma_f(G) \) for \( e \in E \) and \( S \) be minimal dominating set of a fuzzy graph \( G \). Therefore there can be no edge between two vertices in \( V - S \) or between the vertices in \( S \). Since \( S \) still dominates \( V - S \) if such an edge is removed. Therefore there can be no path in connecting two vertices in \( S \). Hence \( G \) is a union of fuzzy star graph. Then \( G \) is a galaxy.

**Theorem 3.9.** If a fuzzy graph \( G \) belongs to CEA then the fuzzy subgraph induced by \( V_0 \) is fuzzy complete.

**Proof.** This result is obvious when \( \sum_{i=1}^{n} d(V_0^i) = 1 \). Assume that \( G \) is in CEA and \( u, v \in V^0 \) and not satisfy the condition \( \mu(u, v) > 0 \). Therefore \( u \) is not a neighbor of \( v \) in the fuzzy graph \( G \). Then \( \gamma_f(G + uv) < \gamma_f(G) \). Hence, there exists a fuzzy minimal dominating set \( S \) for \( G + uv \) can decrease by the fuzzy domination number by atmost one and does not increase the the domination number \( \gamma_f(G + e) = \gamma_f(G) - 1 \) that contains exactly one of \( u \) and \( v \) say \( v \). Then \( S \cup u \) is a minimal dominating set for \( G \) implies \( u \in V^- \) which is a contradiction for \( u, v \in V^0 \). Therefore, every pair of vertices \( u, v \in V^0 \) must satisfy the conditions \( \mu(u, v) = \sigma(u) \cup \sigma(v) \) for all \( u, v \in \sigma^*, \mu(uv) > 0 \). Hence the fuzzy graph \( G \) is is complete.

### 4. Conclusion

In this paper the concept of changing domination in fuzzy graph are analyzed. The changing vertex removal, changing edge removal, changing edge addition are discussed and some theorems based on the concept of changing domination are proved. The changing domination number in fuzzy graph are very useful for electrical networks.

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