Abstract. The CKM paradigm has been proven to be successful in explaining the flavour structure of the standard model and the non-trivial imaginary phase of the CKM matrix is the only known source of CP-violation. B-meson decays allow us to precisely determine the fundamental parameters of the CKM matrix and put stringent constraints on the models of New Physics. I present some of the most recent measurements related to the CKM Unitarity Triangle performed by the BABAR experiment, located at the SLAC National Accelerator Laboratory. Most results are based on the final BABAR dataset, consisting of $467 \times 10^6$ $B\bar{B}$ pairs.

1. Introduction

In the standard model, the CKM matrix [1] describes the couplings, through charged weak currents, of up-type quarks with down-type quarks. The 3 × 3 unitary matrix is determined by four parameters: three of those can be interpreted as mixing angles between the three pairs of generations, while the fourth parameter is a non-trivial complex phase which is the only known source of CP violation in the standard model.

The following among the unitarity constraints of the CKM matrix:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \quad (1)$$

can be used to construct a triangle in the complex plane, the so-called Unitarity Triangle. One of the sides of the triangle has unitary length by construction, whereas the others have lengths:

$$R_u \equiv \left| \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right|, \quad R_t \equiv \left| \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} \right|. \quad (2)$$

The angles are defined as:

$$\alpha \equiv \arg \left[ \frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*} \right], \quad \beta \equiv \arg \left[ \frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right], \quad \gamma \equiv \arg \left[ -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} \right]. \quad (3)$$

In the following, a few of the most recent measurements performed by the BABAR Collaboration, relevant for the determination of the elements of the CKM matrix, will be presented. Most of those are based on the full $\Upsilon(4s)$ dataset available to the experiment, consisting of $467 \times 10^6$ $B\bar{B}$ pairs.

Published in arXiv:1005.4431.
2. **CKM sides**

The element $|V_{cb}|$ can be extracted from the measurement of the branching fraction of $B \rightarrow D \ell \nu$ decays [2]. These decays are searched for on a sample where one of the two $B$’s is fully reconstructed in one of many hadronic final states. The measurement of the branching fractions is performed in bins of $w$, where $w$ is the product of the four-velocities of the $B$ and $D$ mesons. The signal yield is extracted from a maximum likelihood fit to the missing mass squared of the unreconstructed $B$ candidate, which peaks at zero for signal decays (see Fig. 1). The measured branching fractions are:

$$B(B^- \rightarrow D^0 \ell^- \nu) = (2.31 \pm 0.08 \pm 0.09)\%,$$
$$B(B^0 \rightarrow D^+ \ell^- \nu) = (2.23 \pm 0.11 \pm 0.11)\%,$$

where the first error is statistical and the second systematic. From this, using the calculations from Unquenched Lattice QCD [3], the value of $|V_{cb}|$ is extracted:

$$|V_{cb}| = (39.8 \pm 1.8 \pm 1.3 \pm 0.9) \times 10^{-3},$$

where the first error is statistical, the second is the experimental systematic, and the third is the error from the theory.

![Figure 1](image1.png)

**Figure 1.** Left plot: missing mass squared for the $B \rightarrow D \ell \nu$ analysis in two different bins of $w$. Right plot: simultaneous fit of data and theoretical predictions for the extraction of $|V_{ub}|$ from the branching fractions of $B \rightarrow \pi (\rho) \ell \nu$. The results on $|V_{ub}|$ are preliminary.

In a similar way, $|V_{ub}|$ is extracted from the measurement of the branching fractions of $B \rightarrow \pi (\rho) \ell \nu$ [4]. These decays are searched for inferring from the missing energy and momentum of the event the energy and momentum of the unreconstructed neutrino. We measure the branching fraction of the four (charged and neutral $\pi$ and $\rho$) modes with a simultaneous maximum likelihood fit, imposing the conservation of the isospin for the $\pi$ and $\rho$ channels. The results are:

$$B(B^0 \rightarrow \pi^- \ell^+ \nu) = (1.41 \pm 0.05 \pm 0.07) \times 10^{-4},$$
$$B(B^0 \rightarrow \rho^- \ell^+ \nu) = (1.75 \pm 0.15 \pm 0.27) \times 10^{-4},$$

where the first quoted error is statistical and the second systematic. Several methods can be employed to extract $|V_{ub}|$: theoretical predictions from Lattice QCD or Light Cone Sum Rules on the form factor of the decays can be used, integrating part of the $q^2$ spectrum ($q$ is the four-momentum of the virtual $W$ boson exchanged in the decay), or, following an innovative approach, experimental data and theoretical predictions can be fitted in a simultaneous fit (see Fig. 1). Using the results from the FNAL and MILC Collaboration [5] we obtain:

$$|V_{ub}| = (2.95 \pm 0.31) \times 10^{-3}. $$

(8)
3. CKM angles

Information about the angles of the Unitarity Triangle can be obtained by looking for several different CP violating phenomena in B decays.

Concerning $\gamma$, the Babar Collaboration recently presented some results based on the measurement of the branching fractions and charge asymmetries of $B^- \to D^{(*)0}K^{*-}$ decays. In [6], the GLW method [7] is used to get some non-trivial constraints on the angle $\gamma$ from the $B^- \to D^0K^{*-}$ decays. Analogous constraints are obtained from the ADS method [8] applied to $B^- \to D^{(*)0}K^-$ decays, where the first evidence for an ADS signal is seen [9]. Though useful, these results exclude at the 95% C.L. only a small range and are not competitive with the extraction of $\gamma$ exploiting a Dalitz Plot analysis of the $D^0$ to self-conjugate states.

The final measurement of $\beta$ from a time dependent analysis of the golden modes $B^0 \to (c\bar{c})K^0$ [10] gives:

$$\sin 2\beta = 0.687 \pm 0.028$$

(9)

The $\sim 3\sigma$ discrepancy between the golden modes and the modes dominated by penguin amplitudes which was seen in 2004, has shrunk considerably, especially in the theoretically cleanest modes, like $\phi K^0$ and $\eta'K^0$. Another determination of $\beta$ has been obtained from a Dalitz Plot analysis of the $B^0 \to K_S^{0}\pi^+\pi^-$ decay. The two solutions found (see Fig. 2) are in good agreement with the result from the golden modes and we see evidence of CP-violation in the $f_0K^0_S$ mode.

![Figure 2.](image)

The $\alpha$ angle can be extracted, within an 8-fold ambiguity, from an isospin analysis of $B \to \rho^+\rho^-, \rho^0\rho^0, \rho^+\rho^0$ decays, as proposed in [13]. This analysis was performed after Babar obtained the result on the measurement of the branching fraction and direct CP-violation of $B^+ \to \rho^+\rho^0$, based on the full dataset [14]. The measured branching fraction:

$$B(B^+ \to \rho^+\rho^0) = (23.7 \pm 1.4 \pm 1.4) \times 10^{-6}$$

(10)

flattens the isospin triangles, thus allowing the removal of some of the ambiguities with respect to the previous analysis. The result of the isospin analysis is (discarding the solution close to zero):

$$\alpha = (92.4_{-6.5}^{+6.0})^\circ.$$  

(11)
4. Conclusions
At the end of the extensive experimental campaign carried on by the B-factories BABAR and Belle, the CKM mechanism has proven to be successful in explaining all the Flavour Physics phenomena. The global fits combining all the measurement relevant for the determination of the parameters of the CKM matrix show no significant discrepancy between sets of measurements [15]. There are some tensions at the 2σ level, for example between the measured value of β and the one predicted using the ratio $|V_{ub}|/|V_{cb}|$. Although not yet significant, the investigation of these discrepancies constitute one of the motivations for further pursuing the precision measurements on Flavour Physics at the hadronic colliders (Tevatron and LHC) and at the next generation of $e^+e^-$ colliders.

5. References
[1] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[2] BABAR Collaboration, B. Aubert et. al., Phys. Rev. Lett. 104, 011802 (2010).
[3] M. Okamoto et. al., Nucl. Phys. 140, 461 (2005).
[4] BABAR Collaboration, P. del Amo Sanchez et. al., arXiv:1005.3288 [hep-ex].
[5] FNAL and MILC Collaboration, J. Bailey et. al., Phys. Rev. D79, 054507 (2009).
[6] BABAR Collaboration, B. Aubert et. al., Phys. Rev. D80, 092001 (2009).
[7] M. Gronau and D. London, Phys. Lett. B253, 483 (1991).
[8] D. Atwood, I. Dunietz, and A. Soni, Phys. Rev. D63, 036005 (2001).
[9] Presented by N. Lopez-March, at The 2009 Europhysics Conference on High Energy Physics, 16-22 July 2009 Krakow, Poland.
[10] BABAR Collaboration, B. Aubert et. al., Phys. Rev. D79, 072009 (2009).
[11] Heavy Flavour Averaging Group, http://www.slac.stanford.edu/xorg/hfag/triangle/summer2009/index.shtml
[12] BABAR Collaboration, B. Aubert et. al., Phys. Rev. D80, 112001 (2009).
[13] M. Gronau and D. London, Phys. Rev. Lett. 65, 3381 (1990).
[14] BABAR Collaboration, B. Aubert et. al., Phys. Rev. Lett. 102, 141802 (2009).
[15] The CKM Fitter Collaboration, http://ckmfitter.in2p3.fr, The UTFit Collaboration, http://www.utfit.org.