Compositional Freeze-Out of Neutron Star Crusts

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ABSTRACT
We have investigated the crustal properties of neutron stars without fallback accretion. We have calculated the chemical evolution of the neutron star crust in three different cases (a modified Urca process without the thermal influence of a crust, a thick crust, and a direct Urca process with a thin crust) in order to determine the detailed composition of the envelope and atmosphere as the nuclear reactions freeze out. Using a nuclear reaction network up to technetium, we calculate the distribution of nuclei at various depths of the neutron star. The nuclear reactions quench when the cooling timescale is shorter than the inverse of the reaction rate. Trace light elements among the calculated isotopes may have enough time to float to the surface before the layer crystallizes and form the atmosphere or envelope of the neutron star. The composition of the neutron-star envelope determines the total photon flux from the surface, and the composition of the atmosphere determines the emergent spectrum. Our calculations using each of the three cooling models indicate that without accretion of fallback the neutron star atmospheres are dependent on the assumed cooling process of the neutron star. Each of the cooling methods have different elements composing the atmosphere: for the modified Urca process the atmosphere is $^{28}\text{Si}$, the thick crust has an atmosphere of $^{50}\text{Cr}$, and the thin crust has an atmosphere of $^{40}\text{Ca}$. In all three cases the atmospheres are composed of elements which are lighter than iron.

Key words: pulsar: general — stars: neutron — stars

1 INTRODUCTION
Neutron stars are the end product of the core collapse of a star with a mass greater than $8\text{M}_\odot$ (Lattimer & Prakash, 2004). A canonical neutron star has a radius of 10 km and a mass of $1.4\text{M}_\odot$, resulting in an average density greater than that found in an atomic nucleus. A neutron star is comprised of five major regions: core, inner crust, outer crust, envelope, and atmosphere (Lattimer & Prakash, 2004). The crustal layer extends 1-2 km below the surface inward to a density of around $10^{14}\text{g/cm}^3$. Here we define the neutron-star envelope to be the upper layer of the crust that throttles the heat flow running from a density of $10^7\text{g/cm}^3$ outward (Hernquist & Applegate, 1984; Heyl & Hernquist, 2001). The atmosphere lies at relatively low density and comprises a column density of about $1\text{g/cm}^3$.

The envelope and atmosphere contain a negligible amount of the neutron-star mass (or the mass of the crust for that matter) but play a crucial role in shaping observations of neutron stars. In order to fully interpret the observed emission, we need to understand the crustal composition especially the lightest trace elements that can float to the surface to form the envelope and atmosphere. The atmosphere shapes the emergent spectrum, understanding the composition could yield predictions of spectral features in the neutron star thermal emission. The envelope, in turn, influences the transport and release of thermal energy, a change in its composition changes the thermal conductivity and inferred surface temperature of the neutron star (Lattimer & Prakash, 2004).

Without fallback from the supernova or other accretion, a neutron star is expected to have an atmosphere of iron-group elements (Chiu & Salpeter, 1964). An example of a family of neutron stars which are closest to this idealized situation are the isolated neutron stars. The isolated neutron star RX J185635-3754 has been studied extensively. The surface composition of RX J185635-3754 has been examined by fitting the x-ray spectra to various model atmospheres, including models attributed to fallback accretion. These models have included: black body, hydrogen, helium, iron, silicon-ash atmospheres (Pons et al., 2002), and later extended to two black bodies, pure silicon, low iron silicon ash and magnetic hydrogen atmospheres (Walter et al., 2004). The model atmospheres that fit the spectrum of RX J185635-3754 the best are those with heavy el-
ments, though the predicted absorption lines are not seen (Pons et al., 2002; Walter et al., 2004).

Here we calculate the mass fractions that exist in the crust after a neutron star cools long enough for the nuclear reactions to be quenched. These mass fractions are calculated using a 489 isotope reaction network, torch (the code is available at: http://cococubed.asu.edu/code_pages/nse.shtml and the reaction rates in torch).

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2 NEUTRON STAR COOLING

In order to calculate the crustal abundances three different cooling curves were used: a modified Urca process, a thick crust, and a thin crust. Each of the cooling curves start near a temperature of $10^{10}$K and cool until the nuclear reactions are quenched.

In the case of the modified Urca process the nuclear reactions are quenched before a year has past. The core cooling does not affect the reactions in the crust. The thin crust cooling curve is appropriate for a normal neutron star crust and the total thermal energy of the crust is $\Delta$U = 1yr $kT \frac{M_{cr}}{m_a}$ (Shapiro & Teukolsky, 1986), where $k$ is Boltzmann’s constant, $T$ is the temperature of crustal bremsstrahlung for the first month. After a month has past the direct Urca process dominates. We take the temperature of the crust to be $T = T_{brems}(e^{-(t/\text{month})^2} + T_{DU}(1 - e^{-(t/\text{month})^2})$ (2)

where $T_{brems}$ is the temperature of crustal bremsstrahlung and $T_{DU}$ is the temperature at a specific time step using the equation for direct Urca cooling. The $T_{brems}$ starts at a temperature of $10^{10}$K and decreases by 0.1% each step. The time, or age, of the star is given by the crustal bremsstrahlung equation for a thin crust:

$$t = \frac{3}{10} \frac{M_{\odot}}{k} \frac{2 \times 10^{-31}}{K} \frac{2}{erg} [1 - T_{10}^5]$$

for a neutron startnning at a temperature of $10^{10}$K, where $T_{10}$ is the temperature in units of $10^{10}$K.

3 CRUSTAL MASS FRACTIONS

3.1 Running torch

The code torch is a reaction network written by Frank X. Timmes and is available on his website (http://cococubed.asu.edu). The software follows the abundances of 489 isotopes to technetium. The construction of the reaction network is covered in detail in Timmes et al. (2000) and summarized below. The reaction network starts by determining the mass fraction of an isotope, $i$: $X_i = \rho_i/\rho$ and the corresponding molar abundances of the isotope: $Y_i = X_i/\rho_i$. A set of partial differential equations are constructed from the continuity equation of the isotope:

$$\frac{dY_i}{dt} + \nabla \cdot (Y_i V_i) = \dot{R}_i$$

(5)

where $\dot{R}_i$ is the total reaction rate and $V_i$ is the mass diffusion velocity, which is set to zero. This results in a set of ordinary differential equations that comprise the reaction network:

$$\frac{dY_i}{dt} = \dot{R}_i.$$  

(6)
ties at 10
dance calculations. The methods for cooling include the modi-
Figure 1.

These curves are appropriate for d ensi-
fied Urca, thick crust (labelled Lattimer 1994) and a thin crust

Lattimer et al. (1994), and the thin crust is appropriate for a

strange star.

\[ P = \frac{1.2435 \times 10^{15}}{\mu_e^{4/3}} \left( \frac{\rho}{1 \text{ g/cm}^3} \right)^{4/3} \text{dyne/cm}^2, \]  

where we have assumed that relativistic electrons dominate the pressure and that \( \mu_e = 2 \). For the case where we look at a density of 10^6 g/cm^3 the pressure is given by:

\[ P = 1.42180 \times 10^{25} \phi(x) \text{dyne/cm}^2 \]

3.2 Mass Fraction Calculations

For each of the densities investigated the system is started in nuclear statistical equilibrium. The temperature and the length of time at which the system burns is determined by the cooling curve. After the reactions have quenched the abundances at a specific density are calculated and the iso-
topes which are sufficiently abundant to compose the atmo-
sphere or envelope are determined.

In order for the isotope to form the atmosphere it needs to have a surface density greater than 1 g/cm^2 \( \sim \sigma_T/m_p \), the ratio of the Thompson cross-section to the mass of the proton. The first step in determining which isotopes rise to the surface is to calculate the total pressure at the density of the calculation. For densities greater than 10^6 g/cm^3 the pressure is given by:

\[ \phi(x) = [x\sqrt{1+x^2}(2x^2/3 - 1) + \ln(x + \sqrt{1+x^2})] \]  

and \( x = p_f/m_e c \), the ratio of Fermi momentum \( (p_f) \) to the product of the mass of the electron and the speed of light [Shapiro & Teukolsky, 1986].

With the pressure calculated the next step is to calcu-
late the column density between the particular density and the neuron star surface: \( P/\rho_{NS} \). Where \( \rho_{NS} \) is the surface gravity given by:

\[ \rho_{NS} = \frac{GM}{R^2} \left( 1 - \frac{2GM}{c^2R} \right)^{-1/2}. \]

For a 10km and 1.4M_\odot neutron star the surface gravity is 2.43 \times 10^{14} cm/s^2. Finally, the minimum mass fraction re-
quired for an isotope to rise to form the atmosphere is given by the ratio of 1 g/cm^2 to the calculated column density.

For a mass fraction sufficient to form the envelope we are looking for isotopes which have abundances on the order of parts per million over the entire crust, versus in the atmosphere where the isotopes need to have abundances on the order of parts per billion or less. In order to calculate the minimum mass fractions required for an isotope to float to form the envelope we scale according to the column densi-
ties, we use the typical column density of the envelope about 4 \times 10^9 g/cm^2 and 1 g/cm^2 for the atmosphere.

4 FREEZE-OUT

To calculate the neutron star freeze-out we consider three different timescales: the cooling time (\( \tau_c \), for example Eq. (1) for the modified Urca case), the settling time (\( \tau_s \)), and the nuclear reaction timescale (\( \tau_{rxn} \)). The composition of the neutron star depends on the timescales; for the case \( \tau_{rxn} < \tau_c < \tau_s \) we expect the particular species to be in nu-
uclear statistical equilibrium (NSE); for \( \tau_c < \tau_{rxn} < \tau_s \) the particular nuclear reactions are quenched – the star is cooling faster than the reactions can occur; for the case \( \tau_s < \tau_{rxn} < \tau_c \) the isotopes can float up faster than the reactions bring them to NSE. For the case \( \tau_s < \tau_{freeze} \) the light isotopes can float all the way up to the top before the layer freezes.

The time when the particular layer of the neutron star freezes is determined by comparing the potential energy be-
between the ions that compose the crust to their kinetic energy [Shapiro & Teukolsky, 1986].

\[ \Gamma = \frac{\text{Potential Energy}}{a T k_b} \]

where \( e \) is the electron charge and \( a \) is the ion radius such that the product of \( (4/3)\pi a^3 \) and the ion number density is unity. When \( \Gamma > 180 \) the layer freezes out, or crystallizes, and the light species can no longer float upward.

4.1 Settling timescale

A second question is whether the light isotopes can reach the surface before the layer freezes or before we look at the star. On the other hand the gravitational settling could be so efficient that the light isotopes could float up before the layer
cools and nuclear reactions quench. For these two reasons an estimate of the settling time is crucial. Brown et al. (2002) calculate the sedimentation or settling timescale for a neutron-star atmosphere,

$$\tau_s \approx 10^5 \frac{Z_1^{4.9} Z_2^{3.5} \rho_1^{1.3}}{A_1^{1.8} g_1^{4.7} T_1^{9.3} (A_2 Z_1 - A_1 Z_2)}$$

This is an estimate of the time for a nuclide to settle down over a pressure scale height — a negative value means that the nuclide ascends. This timescale is typically around a few years for the envelope and about $10^8$ yr for the outer crust ($\rho < 10^{12}$ g cm$^{-3}$) for Silicon-28 in a background of Iron-56. In particular until the bulk of the Nickel-56 has decayed the settling time for Silicon-28 is much larger because $A_2 Z_1 - A_1 Z_2 \ll 1$; consequently, the nucleons differentiate gravitationally after the nuclear reactions effectively cease, i.e. $\tau_{nse} < \tau_s$.

4.2 Reaction timescale

For the modified Urca case we also calculated the reaction rate timescales by making use of the subroutines in torch. Each of the rate calculations depends only on the input temperature and the densities of the various species. As there are many different ways to make a specific isotope, e.g. $^{28}$Si, the rates which lead to the creation of the isotope are added together to get the timescale of the reaction rate, $\tau_{nse}$.

In order to calculate the abundance of the alpha particles and the other species we make use of the nuclear statistical Saha equations as implemented in nse. We assume that a particular species freezes out of equilibrium when the reaction timescale exceeds the cooling timescale. The abundance in nuclear statistical equilibrium at the freeze out temperature gives an alternative estimate of the final abundance of the nuclides.

5 RESULTS

In order to determine the expected composition of the neutron star atmosphere, in the cases of the modified Urca and the thick crust we examined a density of $10^7$ g/cm$^3$. For the thin crust a density of $10^8$ g/cm$^3$ would crystallize before any of the isotopes had time to reach the surface, so we examined a density of $10^9$ g/cm$^3$. The results from the nuclear reaction network are compared with those of a semi-analytic freeze-out calculation in the modified Urca case. Each of the three cases are discussed below.

5.1 Case 1: Modified Urca

5.1.1 Mass Fractions: Atmosphere

At a density of $10^7$ g/cm$^3$ the corresponding pressure is: $1.1 \times 10^{24}$ dyne/cm$^2$. At this pressure the column density to the surface is: $4.4 \times 10^9$ g/cm$^2$. The resulting required minimum mass fraction required for an isotope to be optically thick on the surface is: $2.3 \times 10^{-10}$.

Isotopes with a mass fraction greater than $2.3 \times 10^{-10}$ will have a surface density of 1 g/cm$^2$. The lightest elements to be optically thick on the surface and have time to reach the surface before crystallization of the layer occurs are shown in Figure 2 where the horizontal line indicates the minimum mass fraction required to be optically thick on the surface. The lightest elements to rise to the surface which are optically thick are: $^{28}$Si, $^{32}$S, $^{34}$Ar in nuclear statistical equilibrium using nse are depicted for comparison by the nearly vertical, heavy lines.

5.1.2 Freeze-Out

Using the steps outlined in §4 we have calculated the cooling ($\tau_c$), settling ($\tau_s$), and the nuclear reaction ($\tau_{nse}$) timescales for the case of $^{28}$Si. The results of these calculations are displayed in Figure 3. These calculations compare the age of the neutron star to the settling, crystallization temperature, creation and destruction timescales of $^{28}$Si for two densities: $10^7$ g/cm$^3$ and $10^{12}$ g/cm$^3$. The temperatures at which the layers crystallize are $4.8 \times 10^8$ K and $2.2 \times 10^9$ K, for the densities of $10^7$ g/cm$^3$ and $10^{12}$ g/cm$^3$, respectively. The creation and destruction rates are $\frac{d\ln X_i}{dt}$, where $X_i$ is the abundance of the $^{28}$Si isotope. These rates are calculated by using the routines in torch to determine the energy release per unit mass, these are then multiplied by the abundances calculated from the nse code. The abundances were also calculated using the nuclear Saha equation and are the output of the nse routine. The two different methods for calculating the relative abundances, the output from torch and using nse are displayed in Figure 2. It is clear that the results from Figure 2. Lightest isotopes for which the mass fraction abundance would be great enough that the isotope will be optically thick on the neutron star surface. All of these isotopes have time to reach the surface before the layer crystallizes. These are the mass fractions for the density of $10^9$ g/cm$^3$. The corresponding pressure and column density for this neutron star density are $1.1 \times 10^{24}$ dyne/cm$^2$ and $4.4 \times 10^9$ g/cm$^2$, respectively. The horizontal line indicates the minimum abundance required for an isotope to have a surface density of 1 g/cm$^2$. The abundances for $^{28}$Si, $^{32}$S, and $^{36}$Ar in nuclear statistical equilibrium using nse are depicted for comparison by the nearly vertical, heavy lines.
torch do not follow nuclear statistical equilibrium precisely.

Using Figure 3 we can find the quenching temperature for the silicon reactions. The reactions become quenched when the reaction rate time becomes longer than the age of the neutron star. From the plot the quenching temperature for the reactions which destroy silicon are $2.12 \times 10^9$K and $2.15 \times 10^8$K for the densities of $10^{12}$g/cm$^3$ and $10^{10}$g/cm$^3$, respectively. The reactions which create silicon are quenched at $3.05 \times 10^9$K for both of the densities. These approximately give the temperature range over which the abundance of silicon levels out in Figure 2. The relative abundances of $^{28}$Si at the quenching temperatures for the destructive reactions are $4.86 \times 10^{-13}$ and $2.36 \times 10^{-10}$, for densities of $10^{12}$g/cm$^3$ and $10^{10}$g/cm$^3$, respectively. The relative abundances of silicon for the creation reactions are $5.82 \times 10^{-9}$ at a density of $10^{12}$g/cm$^3$ and $1.84 \times 10^{-6}$ at a density of $10^{10}$g/cm$^3$.

5.2 Case 2: Thick Crust

In the case of the thick crust, the density of $10^7$g/cm$^3$ was examined for the isotopes which can rise to the surface. The pressure, column density to the surface and the minimum mass fraction required to be optically thick are the same for the modified Urca case. For the thick crust the lightest isotopes which are optically thick and can rise to the surface are $^{50}$Cr, $^{53}$Mn, $^{54}$Fe, $^{55}$Fe, and $^{57}$Co, as shown in Figure 4. These isotopes include $^{40}$Ca, $^{50}$Cr, $^{53}$Mn, $^{54}$Fe, and $^{55}$Fe.

5.3 Case 3: Thin Crust

For the thin crust the isotopes which had the possibility to be optically thick could not reach the surface before the layer crystallized for densities at $10^6$g/cm$^3$ and higher; consequently, we examined the layer at a density of $10^6$g/cm$^3$. At this density the corresponding pressure is $2.3 \times 10^{22}$dyne/cm$^2$. The column density to the surface at this pressure is $9.6 \times 10^7$g/cm$^2$, requiring a minimum mass fraction of $1.0 \times 10^{-8}$ for an isotope to optically thick on the surface. The isotopes which are not only optically thick, but can also rise to the surface are shown in Figure 5. These isotopes include $^{40}$Ca, $^{50}$Cr, $^{53}$Mn, $^{54}$Fe, and $^{55}$Fe.

6 CONCLUSIONS

In the case of cooling by the modified Urca process, without the thermal influence of a crust, our results (e.g. Fig. 3) show that silicon has sufficient time to float to the top from a density of $10^7$g/cm$^3$ before the layer freezes or we observe it. However, the settling time from $10^{12}$g/cm$^3$ is too long for light species to float up before the crust freezes; therefore, we can conclude that the atmosphere in this case is likely to be composed of silicon but the envelope is likely to be composed of iron-group elements that have been chemically separated by gravitational settling. Deeper layers are unlikely to be chemically separated at least by gravity.

We have used the torch code in order to calculate the
The lightest isotopes which would rise to the neutron star surface and have compared these results to semi-analytic calculations. Using the torch code we calculated the mass fractions at a density of $10^6 \text{g/cm}^3$ for a neutron star with a thin crust. The corresponding pressure and column density at this density are $2.3 \times 10^{22} \text{dyne/cm}^2$ and $9.6 \times 10^7 \text{g/cm}^2$, respectively. The horizontal line indicates the minimum mass fraction required in order for an isotope to be optically thick.

We also did semi-analytic calculations of the freeze-out of $^{28}\text{Si}$, for the modified Urca case, assuming local thermodynamic equilibrium until the cooling rate exceeds the reaction rate. We used the rates from torch and the abundances from the code nse in order to calculate the rates of nuclear reactions involving $^{28}\text{Si}$; the reactions become quenched when the reaction time is longer than the age of the neutron star. We found the creation reactions are quenched at a temperature of $3.05 \times 10^9 \text{K}$ for both the densities of $10^7 \text{g/cm}^3$ and $10^{12} \text{g/cm}^3$. The reactions which destroy silicon are quenched at $2.12 \times 10^9 \text{K}$ and $2.15 \times 10^9 \text{K}$ at a density of $10^{12} \text{g/cm}^3$ and $10^{7} \text{g/cm}^3$, respectively. The calculated quenching temperatures of the silicon reactions agree with the results calculated using the torch code directly.

In the case of the thick crust, the atmosphere could be formed by $^{50}\text{Cr}$ rising to the surface from a density of $10^7 \text{g/cm}^3$. For a neutron star with a thin crust and direct Urca cooling the layers at $10^7 \text{g/cm}^3$ and higher crystallize before the isotopes could have time to reach the surface. The atmosphere in the thin crust case could be formed from $^{40}\text{Ca}$ rising to the surface from a layer at a density of $10^6 \text{g/cm}^3$.

Unless there has been significant accretion either from the supernova debris, the interstellar medium or a companion, neutron-star atmospheres are unlikely to be composed of iron, helium or hydrogen. The type of isotope composing the atmosphere depends on the cooling mechanism at work in the star: $^{26}\text{Si}$ for modified Urca, $^{50}\text{Cr}$ for a thick crust, and $^{40}\text{Ca}$ for a thin crust and direct Urca cooling. The formation of these atmospheres can provide additional justification, with fallback accretion, for isolated neutron stars fit with intermediate atmosphere models. Understanding how this novel composition of the atmospheres affects the neutron-star emission may provide new insights on the observed spectra of neutron stars.

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