Galaxy-halo size relation from Sloan Digital Sky Survey Data Release 7 and the ELUCID simulation

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ABSTRACT

Based on galaxies in the Sloan Digital Sky Survey Data Release 7 (SDSS DR7) and dark matter haloes in the dark matter only, cosmological and constrained ELUCID simulation, we investigate the relation between the observed radii of central galaxies with stellar mass \( \gtrsim 10^8 h^{-2} M_{\odot} \) and the virial radii of their host dark matter haloes with virial mass \( \gtrsim 10^{10.5} h^{-1} M_{\odot} \), and the dependence of galaxy-halo size relation on the halo spin and concentration. Galaxies in observation are matched to dark matter (sub-)haloes in the ELUCID simulation using a novel neighborhood subhalo abundance matching method. For galaxy 2D half-light radii \( R_{50} \), we find that early- and late-type galaxies have the same power-law index 0.55 with \( R_{50} \propto R_{\text{vir}}^{0.55} \), although early-type galaxies have smaller 2D half-light radii than late-type galaxies at fixed halo virial radii. When converting the 2D half-light radii \( R_{50} \) to 3D half-mass radii \( r_{1/2} \), both early- and late-type galaxies display similar galaxy-halo size relations with \( \log r_{1/2} = 0.55 \log(R_{\text{vir}}/210 h^{-1} \text{kpc}) + 0.39 \). We find that the galaxy-halo size ratio \( r_{1/2}/R_{\text{vir}} \) decreases with increasing halo mass. At fixed halo mass, there is no significant dependence of galaxy-halo size ratio on the halo spin or concentration.

Key words: large-scale structure of universe – methods: statistical – cosmology: observations

1 INTRODUCTION

In the standard galaxy formation model, galaxies form in the center of the dark matter haloes (White & Rees 1978). Therefore, the properties of galaxies are expected to be influenced by the properties of their host dark matter haloes. The physical connection between galaxies and haloes is an interesting and challenging problem in the study of galaxy formation and evolution.

From the observational data, such as the Sloan Digital Sky Survey (SDSS; Abazajian et al. 2009) and the Galaxy And Mass Assembly survey (GAMA; Driver et al. 2011), it’s well established that the sizes of galaxies depend not only on the luminosity and stellar mass, but also on other intrinsic properties of galaxies, such as the galaxy concentration or morphology types (Shen et al. 2003; Lange et al. 2015; Zhang & Yang 2019). Unlike the mass-size relation of galaxies in observation (van der Wel et al. 2014), the scaling relations between galaxies and dark matter haloes are not well understood (Dutton & van den Bosch 2009; Wechsler & Tinker 2018; Zanisi et al. 2020; Rohr et al. 2022), since haloes are more than one order of magnitude larger than galaxies.

The angular momentum conservation model (Fall & Efstathiou 1980; Mo et al. 1998) has been widely used to set the galaxy sizes in various semi-analytic models (Cole et al. 2000; Croton et al. 2006; Somerville et al. 2008) and to explain the empirical constraints of the galaxy-halo size relation by the abundance matching technique (Kravtsov 2013; Somerville et al. 2018). Using the abundance matching method, Kravtsov (2013) investigated the galaxy-halo size relation for hundreds of galaxies, which include dwarf galaxies from the HST and VLT/FORS1 observation and 220 massive galaxies from the SDSS observation, spanning more than eight order of magnitude in stellar mass \( (M_\star \sim 10^5-12 M_{\odot}) \). They found that the galaxy-halo size relation follows an approximately linear relation \( r_{1/2} = 0.015 R_{200}, \) where \( r_{1/2} \) is the galaxy 3D half-mass radius and \( R_{200} \) is the radius enclosing the over-density of 200 times the critical density \( \rho_{\text{crit}} \) of the universe. Combining the sizes of galaxies \( (M_\star > 10^7 M_{\odot}) \) from the CANDELS survey with the inferred sizes of dark matter haloes by the abundance matching method with four different stellar mass-halo mass relations, Huang et al. (2017) confirmed the linear galaxy-halo size relation found by Kravtsov (2013) at \( z = 0 \), and extended to the redshift range \( 0 < z < 3 \). In addition, they found that early- and late-type galaxies follow a roughly parallel galaxy-halo size relation offset by \( \sim 0.2-0.3 \) dex. Combining galaxies \( (M_\star \gtrsim 10^9 M_{\odot}) \) in the GAMA survey and subhaloes

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from the Bolshoi–Planck simulation by subhalo abundance matching method, Somerville et al. (2018) claimed that the ratio of galaxy 3D half-mass radius to halo virial radius is consistent with being roughly independent of the stellar mass, with a linear relation $r_{1/2} = 0.018R_{\text{vir}}$ for galaxies from the GAMA survey at $z = 0.1$. It is noteworthy that in their Figure 3, the ratio of galaxy projected half-light radius (or 3D half-mass radius without including scatter) to the halo virial radius decreases significantly with the increase of the galaxy stellar mass, although this trend is mitigated due to the projection effect from projected to 3D size or the including scatter in the stellar-to-halo mass relation in their subhalo abundance matching method. Using galaxies ($M_\ast \geq 10^7\,M_\odot$) in SDSS DR10 and subhaloes in Bolshoi–Planck simulation, Hearin et al. (2019) studied the size dependence of galaxy clustering and found that small galaxies cluster more strongly than large galaxies of the same stellar mass. They claimed that the size dependence of galaxy clustering can be well reproduced based on a simple linear galaxy-halo size relation with $R_{50} = 0.0175r_{\text{vir}}$, where $R_{50}$ is the galaxy 2D half-light radius, and $r_{\text{vir}}$ is the halo virial radius when halo mass reached its maximum over the merger history. Using galaxies ($M_\ast \geq 10^6\,M_\odot$) from the SDSS DR7, Rodriguez et al. (2021) combined galaxy sizes and stellar masses with their group finder algorithm to construct the relations between galaxy sizes and halo masses, and compared with predictions from the IllustrisTNG hydrodynamical simulation. They demonstrated that the results of central galaxies are in excellent agreement with the linear galaxy-halo size relation proposed by Kravtsov (2013), while for satellite galaxies, the results are more consistent with the model of Hearin et al. (2019).

In the angular momentum conservation model, the galaxy size is assumed to be determined by the angular momentum of its host halo and the galaxy spin is strongly correlated with that of its host halo, resulting in $r_{1/2} \propto \lambda R_{\text{vir}}$. However, recent studies based on cosmological hydrodynamical simulations challenge the angular momentum conservation model and indicate that there is almost no correlation between the spins of a galaxy and its host halo (Desmond et al. 2017; Jiang et al. 2019; Yang et al. 2021; Stiskalek et al. 2022). Generally, these numerical studies suggest that the baryon physics of cooling and feedback process is more significant in setting galaxy sizes. Using data from the EAGLE hydrodynamical simulation, Desmond et al. (2017) found that the sizes of galaxies ($M_\ast \geq 10^7\,M_\odot$) only weakly correlate with halo spin or concentration at fixed stellar mass with the Spearman rank coefficients $0.17$ and $-0.19$, which are consistent with $0$ within $3\sigma$. Using 15 Milky Way-mass galaxies in the hydrodynamical, zoom-in FIRE-2 simulation, Garrison-Kimmel et al. (2018) claimed that galaxy sizes are poorly correlated with the halo spin, formation time or merger histories. They found that the spin of the gas at the time the galaxy formed half of its stars is the best predictor of galaxy size. Using 34 galaxies with halo mass $M_{\text{vir}} \sim 2 \times 10^{11} - 12 \,M_\odot$ and $\sim 100$ galaxies with $M_{\text{vir}} \sim 10^{9.5} - 12.3 \,M_\odot$ from zoom-in cosmological hydrodynamical VELA and NIHAO simulations, Jiang et al. (2019) found that the galaxy spin is barely correlated with the spin of its host halo. Yang et al. (2021) also found that there is almost no correlation between galaxy size and the halo spin parameter for dwarfs in the EAGLE and APOSTLE simulations, although they claimed that there is a correlation for Milky Way sized galaxies in IllustrisTNG simulation (see their Figure 4). Using low-mass galaxies ($M_\ast \sim 10^7 - 9 \,M_\odot$) in hydrodynamical simulation in the FIRE project, Rohr et al. (2022) found that the scatter of the galaxy-halo size relation does not correlate with the halo spin or concentration.

These numerical studies show that galaxy sizes are weakly correlated with halo spin and concentration, which disagrees with the angular momentum conservation model, although Rodriguez-Gomez et al. (2022) claimed that there is a strong correlation between galaxy size and halo spin in the IllustrisTNG hydrodynamical cosmological simulation (see also Grand et al. 2017). Based on the abundance matching method, Zanisi et al. (2020) examined their models in setting galaxy sizes from their host haloes, including the MMW model in Mo et al. (1998), the empirical model in Kravtsov (2013) and the concentration model $r_{1/2} = 0.02(c/10)^{-0.7}R_{\text{vir}}$ in Jiang et al. (2019). They found that the angular momentum conservation model can not provide a good fit to the size distributions of SDSS galaxies. They suggested that the galaxy-halo size relation can be mediated by the galaxy angular momentum, not the halo spin parameter.

In general, the galaxy-halo size relation has not been well understood based on the cosmological hydrodynamical simulation or the abundance matching method. In a series of recent studies, with the help of dark matter only, cosmological and constrained ELUCID simulation, the mass and positions of dark matter haloes of galaxy groups in observation can be well produced in the ELUCID simulation, since the initial condition of the ELUCID simulation is constrained by the density field of galaxy distribution in observation (Wang et al. 2012, 2014, 2016; Zhang et al. 2021a,b). Using the ELUCID simulation, galaxies in observation can be well linked to dark matter (sub-)haloes in simulation according to their mass and positions by a neighborhood subhalo abundance matching method (Yang et al. 2018). In this work we further investigate the galaxy-halo size relation for observed central galaxies linked to the corresponding haloes in the ELUCID simulation. In particular, we aim to provide an empirical constraint of the galaxy-halo size relation.

This paper is organized as follows. In Section 2, we describe the observational galaxy sizes from SDSS DR7 and the halo properties from the ELUCID simulation. Besides, we describe the neighborhood subhalo abundance matching method linking galaxies in observation to haloes in constrained simulation. In Section 3, we investigate the relation between the observed radii of central galaxies in SDSS DR7 and the virial radii of their host haloes in the ELUCID simulation. In addition, we study how the galaxy-halo size relation depends on the halo spin and concentration. Finally, we summarize our results in Section 4. Throughout the paper, the adopted cosmological parameters are $\Omega_m = 0.258$, $\Omega_b = 0.044$, $\Omega_\Lambda = 0.742$, $h = 0.72$, $\sigma_8 = 0.796$, and $n_s = 0.963$. 

2 DATA AND METHOD

The observational data used in this paper is from the spectroscopic galaxy sample of the Sloan Digital Sky Survey Data Release 7 (SDSS DR7; Abazajian et al. 2009), from which Blanton et al. (2005) constructed the the New York Univer-
From the catalogs of Simard et al. (2011) and Meert et al. (2015), 580, 472 galaxies can be cross identified in the NYU-VAGC catalog. Figure 1 shows the r-band half-light radius as a function of the stellar mass for the cross matched galaxies. The black solid line shows the median values of galaxy sizes from NYU-VAGC catalog. The blue dotted and red dashed lines show the size-mass relation using galaxy sizes from Simard et al. (2011) and Meert et al. (2015), respectively. The cyan solid line shows the size-mass fitting relation provided by Mowla et al. (2019), who used 918 massive galaxies with mass $M_*>2\times10^{11}M_\odot$ at $0.1<z<3.0$ from the COSMOS-DASH and HST ACS/WFC surveys, and combined them with 30,958 galaxies with mass $M_*>10^9M_\odot$ from the 3D-HST/CANDELS sample of van der Wel et al. (2014). As shown in Figure 1, at fixed stellar mass the galaxy sizes in NYU-VAGC catalog agree well with those in the catalogs of Simard et al. (2011) and Meert et al. (2015), especially for low-mass galaxies. Throughout the paper, the galaxy sizes from the NYU-VAGC catalog are used unless stated otherwise. In this paper, we mainly focus on a sample of 276, 463 central galaxies with stellar mass $M_*\gtrsim10^8h^{-2}M_\odot$, corresponding to haloes with the virial mass $M_{\text{vir}}\gtrsim10^{10.8}h^{-1}M_\odot$ in the ELUCID simulation. In this sample, the median values of galaxy stellar mass and the corresponding halo virial mass are $10^{10.3}h^{-2}M_\odot$ and $10^{11.8}h^{-1}M_\odot$, respectively.

2.1 Simulation data

The ELUCID simulation (Wang et al. 2014, 2016; Tweed et al. 2017; Wang et al. 2018; Yang et al. 2018; Chen et al. 2019) is a dark matter only cosmological constrained simulation, which contains 3072$^3$ dark matter particles within a box that is $500\,h^{-1}\text{Mpc}$ on a side, and has a particle mass of $3.1\times10^8\,h^{-1}M_\odot$. The gravitational softening length is $3.5\,h^{-1}\text{kpc}$. The cosmological parameters adopted in the ELUCID simulation are $\Omega_m=0.258$, $\Omega_b=0.044$, $\Omega_{\Lambda}=0.742$, $h=0.72$, $\sigma_8=0.796$, and $n_s=0.963$.

Dark matter haloes are identified using the standard friends-of-friends (FOF) algorithm (Davis et al. 1985) with a linking length of $b=0.2$ times the mean particle separation. Using the SUBFIND algorithm (Springel et al. 2001), a given host halo is decomposed into a set of gravitational bound subhaloes, in which the most massive one is called central subhalo, and the others are referred to satellite subhaloes.

For a given host halo, the virial mass $M_{\text{vir}}$ is computed as the interior mass within a sphere of radius $R_{\text{vir}}$ centered on the position of the minimum potential particle, where $R_{\text{vir}}$ is the radius within which the over-density is $\Delta_{\text{vir}}$ times the critical density $\rho_{\text{crit}}$ of the universe. Then the virial mass and radius are related by

$$M_{\text{vir}} = \frac{4}{3}\pi\Delta_{\text{vir}}\rho_{\text{crit}} R_{\text{vir}}^3,$$  

where $\Delta_{\text{vir}}$ is dependent on the cosmological parameters and given by the fitting function in Bryan & Norman (1998).

For each halo in the simulation, the dimensionless spin pa-

\[ \text{http://sdss.physics.nyu.edu/vagc-dr7/vagc2/} \]
rameter $\lambda$ is calculated by the formula (Peebles 1969)\(^2\)

$$\lambda = \frac{J|E|^{1/2}}{GM_{\text{vir}}^{5/2}},$$  \hspace{1cm} (2)

where $J$ is the total angular momentum, $E$ is the total energy of the halo, and $G$ is Newton’s gravitational constant.

For a given halo with virial radius $R_{\text{vir}}$, the concentration of the halo is defined as $c_{\text{vir}} = R_{\text{vir}}/r_s$, where $r_s$ is the scale radius in the density profiles fitted by the simple formula (Navarro et al. 1997)

$$\rho_{\text{NFW}}(r) = \frac{\delta_c}{(r/r_s)(1+r/r_s)^2},$$  \hspace{1cm} (3)

where $\delta_c$ is a characteristic density contrast, which can be expressed as $\delta_c = \Delta_{\text{vir}} c_{\text{vir}}^3/(3*(\ln(1+c_{\text{vir}}) - c_{\text{vir}}(1+c_{\text{vir}})))$ (Macciò et al. 2007; Neto et al. 2007; Zhao et al. 2009).

Based on the haloes from the ELUCID simulation, the density profiles $\rho(r_i)$ is measured in $N$ spherical shells of uniform logarithmic bins from the radius where the bin contains at least 20 particles to $R_{\text{vir}}$. The best fitting concentration parameter is calculated by minimizing the rms deviation $\sigma$ between $\rho(r_i)$ and the density profile $\rho_{\text{NFW}}(r_i)$ in Equation 3

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (\log \rho(r_i) - \log \rho_{\text{NFW}}(r_i))^2.$$  \hspace{1cm} (4)

### 2.2 Neighborhood subhalo abundance matching

Since the initial condition of the ELUCID simulation is constrained by the matter density extracted from the galaxy distribution in SDSS DR7, the spatial distribution of the haloes in simulation is tightly correlated with the distribution of galaxies from SDSS DR7. Obviously, a set of gravitational bound subhaloes in simulation are more suitable to link galaxies in observation than FOF haloes. Therefore, we make use of a neighborhood subhalo abundance matching method (Yang et al. 2018) to link galaxies in observation to subhaloes in the ELUCID simulation according to the likelihood

$$P = M_{\text{peak}} \exp \left( -\frac{r_p^2}{2\sigma_{\text{off}}^2} \right) \exp \left( -\frac{\pi^2}{2\sigma_{\text{off}}^2} \right),$$  \hspace{1cm} (5)

where $r_p$ and $\pi$ are the separations between the galaxy and the subhalo in the perpendicular and parallel to the line-of-sight directions, respectively. $M_{\text{peak}}$ is the subhalo peak mass over the merger histories. Compared with the current mass at $z = 0$, the peak values are better traces of the potential well which shapes the galaxy statistical properties (Reddick et al. 2013; Guo et al. 2016; Zhang et al. 2021b). The free parameters $r_{\text{off}}$ and $\sigma_{\text{off}}$ are set to be $r_{\text{off}} = 5 \text{ h}^{-1}\text{Mpc}$ and $\sigma_{\text{off}} = 1000$ km/s. Note that $M_{\text{peak}}$ is the dominant variable in Equation 5, which will degrade to the traditional subhalo abundance matching method if $r_{\text{off}} = \infty$ and $\sigma_{\text{off}} = \infty$.

In what follows, central galaxies are linked to central subhaloes in simulation and satellite galaxies to satellite subhaloes. This matching criteria results in a total of 396,069 galaxy-subhalo pairs in the continuous Northern Galactic Cap (NGC) region of the range $90^\circ < \alpha < 283^\circ$ and $-7^\circ < \delta < 75^\circ$. Based on the galaxy-subhalo pairs in the matching catalog, the neighborhood subhalo abundance matching method accurately reproduced the stellar to halo mass relation, the satellite fraction, the conditional stellar mass function, the biases of galaxies (Yang et al. 2018). In addition, the matching catalog has been successfully applied to study the galaxy-halo alignments (Zhang et al. 2021a), and the physical connections between galaxy properties and halo formation time in different large-scale environments (Zhang et al. 2021b).

Unlike satellite galaxies which lose their mass due to tidal stripping after accretion into a larger system, central galaxies are more closely correlated with their host haloes in the ELUCID simulation (Yang et al. 2018; Zhang et al. 2021a,b). Therefore, in the following study, we mainly focus on the galaxy-halo size relation for a total of 276,463 central galaxies.

### 3 GALAXY-HALO SIZE RELATION

Based on galaxies from SDSS DR7 and dark matter haloes from dark matter only cosmological ELUCID simulation, we investigate the galaxy-halo size relation for a total of 276,463 central galaxies. In addition, we investigate how the galaxy-halo size relation depends on the halo spin $\lambda$ and concentration $c_{\text{vir}}$.

#### 3.1 Galaxy 2D half-light radius versus halo virial radius

At given stellar mass, galaxy size is strongly dependent on the specific star formation rate (van der Wel et al. 2014). Star-forming galaxies are on average larger than quiescent galaxies at all redshifts (Mowla et al. 2019). Besides, the galaxy sizes are also dependent on other properties of galaxies, such as concentration, morphology, and bulge fraction (Shen et al. 2003; Lange et al. 2015; Zhang & Yang 2019; Irodout et al. 2019). The galaxy concentration index, $c = R_{\text{so}}/R_{\text{vir}}$, is tightly correlated with galaxy morphological types (Deng 2013; Deng & Yu 2015; Calette et al. 2018). Following Deng (2013), galaxies are separated into early- and late-types according to $c \geq 2.85$ and $c < 2.85$, resulting in 78,238 early-types and 198,225 late-types.

Figure 2 shows the projected 2D half-light radii $R_{\text{ho}}$ of galaxies as a function of their host halo 3D virial radii $R_{\text{vir}}$. The galaxy 2D half-light radii $R_{\text{so}}$ and halo virial radii $R_{\text{vir}}$ are extracted from the NYU-VAGC catalog in observation and the halo catalog in ELUCID simulation, respectively. The red, blue and black symbols with error bars show the results for early-type, late-type and total galaxies, respectively. Obviously, the galaxy-halo size relation scales approximately linearly in logarithmic space over one order of magnitude in galaxy radius. For total galaxies in Figure 2, the black solid line shows the best-fitting relation

$$\log R_{\text{ho}} = 0.46 \log (R_{\text{vir}}/210 \text{ h}^{-1}\text{kpc}) + 0.40$$

with an average scatter $\langle \sigma \rangle = 0.14$ dex, where the average scatter $\langle \sigma \rangle$ is calculated by the mean absolute deviation from the best-fitting relation, and $210 \text{ h}^{-1}\text{kpc}$ is the virial radius of the halo with mass $M_{\text{vir}} = 10^{11.8} \text{ h}^{-1}\text{M}_\odot$, which is the median value in our sample. For early-type galaxies, the best-fitting relation is $\log R_{\text{ho}} = 0.55 \log (R_{\text{vir}}/210 \text{ h}^{-1}\text{kpc}) + 0.33$ with $\langle \sigma \rangle = 0.10$ dex indicated by the red dotted line, while for late-type galaxies, the galaxy-halo size relation is

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\(^2\) An alternative definition is $\lambda = J/(\sqrt{3} M V R)$ by Bullock et al. (2001).
Figure 2. Galaxy 2D half-light radius $R_{50}$ as a function of the host halo virial radius $R_{\text{vir}}$. Galaxies are separated into early- and late-types according to the concentration index $c \geq 2.85$ and $c < 2.85$. The red, blue and black symbols with error bars show the results for early-type, late-type and total galaxies, respectively. The black solid line shows the best-fitting relation $\log R_{50} = 0.46 \log(R_{\text{vir}}/210 \, h^{-1}\text{kpc}) + 0.40$ for total galaxies. The red dotted lines shows the relation $\log R_{50} = 0.55 \log(R_{\text{vir}}/210 \, h^{-1}\text{kpc}) + 0.33$ for early-type galaxies, while the blue dashed line represents the best-fitting relation of the form $\log R_{50} = 0.55 \log(R_{\text{vir}}/210 \, h^{-1}\text{kpc}) + 0.45$ for late-type galaxies.

Figure 3. Galaxy 3D half-mass radius $r_{1/2}$ as a function of the host halo virial radius $R_{\text{vir}}$. The red and blue symbols with error bars show the results for early- and late-type galaxies, respectively. The dotted and dashed lines shows the same best-fitting relation of the form $\log r_{1/2} = 0.55 \log(R_{\text{vir}}/210 \, h^{-1}\text{kpc}) + 0.39$ for early- and late-type galaxies. The black solid line shows the galaxy-halo size relation obtained by Kravtsov (2013).

Figure 4. Ratio of galaxy radius $r_{1/2}$ to halo virial radius $R_{\text{vir}}$ as a function of halo virial mass $M_{\text{vir}}$. The solid dashed line shows the best-fitting relation $r_{1/2}/R_{\text{vir}} = 0.01 \times (4.7 - 0.29 \log M_{\text{vir}})$.

log $R_{50} = 0.55(\log R_{\text{vir}}/210 \, h^{-1}\text{kpc}) + 0.45$ with $\langle \sigma \rangle = 0.14$ dex shown by the blue dashed line. Generally, early- and late-type galaxies have almost the same power-law index of 0.55 with $R_{50} \propto R_{\text{vir}}^{0.55}$ and offset by 0.12 dex.

3.2 Galaxy 3D half-mass radius versus halo virial radius

In this section, we investigate the size relation between 3D galaxy half-mass radius $r_{1/2}$ and halo virial radius $R_{\text{vir}}$. Inspired by Somerville et al. (2018), the relation between the projected 2D half-light radius $R_{50}$ and the 3D half stellar mass radius $r_{1/2}$ is expressed by

$$r_{1/2} = f_i f_m R_{50}, \quad (6)$$

where $f_i$ corrects for the integration effect from the intrinsic properties in 3D space to the projected distribution of the surface brightness (Jaffe 1983; Hernquist 1990), and $f_m$ refers to the conversion from the half-light radius to the half-mass radius.

The parameter $f_i$ depends significantly on the shape and structure of galaxies (Kravtsov 2013; Somerville et al. 2018). For spheroidal galaxies with s´ersic profiles, lots of studies have investigated the 3D density distributions for deprojecting the 2D density profiles in observation (Lima Neto et al. 1999; de Nicola et al. 2020;
Vitral & Mamon 2020; van de Ven & van der Wel 2021). Actually, there is no analytical deprojection for the sésic profiles. In the numerical computations for spheroidal galaxies of de Vaucouleurs profiles with sésic index $n = 4$, $f_1 = 1.35$ by integrating the 3D luminosity density along line-of-sight (Young 1976). In the analytical model proposed by Hernquist (1990), $f_1 = 1.33$ by integrating the 3D spherical density distribution as the form $\rho(r) \propto r^{-2}(r + a)^{-2}$, which closely approximates the de Vaucouleurs profile in 2D density distribution. For spheroidal galaxies with a wide range of sésic index $n$, Lima Neto et al. (1999) used a modified form of the 3D density profile given by Mellier & Mathez (1987), and derived the parameter $f_1 = 1.356 - 0.0293\nu + 0.0023\nu^2$, where $\nu = 1/n$, resulting in $f_1 = 1.35$ for de Vaucouleurs profile.

For disk galaxies, stars are assumed to be in a thin disk, and hence $f_1 = 1$ is adopted for late-type galaxies in the studies of Kravtsov (2013) and Somerville et al. (2018). In this study, early-type galaxies classified by galaxy concentration $c \geq 2.85$ are spheroid-dominated galaxies, while most of late-type galaxies with $c < 2.85$ are fairly pure disk galaxies. In this paper, we adopt $f_1 = 1.32$ for early-type galaxies, and $f_1 = 1$ for late-type galaxies.

The parameter $f_m$ seems to be weakly dependent on the galaxy morphological types (Dutton et al. 2011; Szomoru et al. 2013; Lange et al. 2015; Chan et al. 2016; Suess et al. 2019a,b; Ibarra-Medel et al. 2022). Using hundreds of massive galaxies above $10^{10.7} M_\odot$ from HST and SDSS observation, Szomoru et al. (2013) found that the half stellar mass radii are on average 25% smaller than the $g$-band half-light radii and this difference of 25% does not correlate with galaxy morphology or star forming activity. Using galaxies from GAMA survey in the redshift range $0.01 < z < 0.1$, Lange et al. (2015) found that both early- and late-type galaxies show a decrease in sizes of 13% from $g$ to $K_s$ band at stellar mass $\sim 10^{10} M_\odot$. The mass-weighted radius is assumed to be the same as the $K_s$ band light effective radius, and then $f_m = 0.87$ for the GAMA sample in Lange et al. (2015). Using a sample of local early-type galaxies selected from the SPIDER survey, Chan et al. (2016) found that the mass-weighted sizes are on average $\sim 13\%$ smaller than the $r$-band sizes, corresponding to $f_m = 0.87$ in the mass range $10.0 \leq \log(M_*/M_\odot) \leq 11.6$ for the SPIDER sample.

In this study, $f_m = 0.87$ is adopted for early- or late-types, resulting in $f_1 f_m = 1.32 \times 0.87 = 1.15$ for early-types, and $f_1 f_m = 1 \times 0.87 = 0.87$ for late-types in Equation 6. Figure 3 show the galaxy 3D half-mass radius $r_{1/2}$ as a function of the halo virial radius $R_{\text{vir}}$. The red and blue symbols with error bars show the results for early- and late-type galaxies, respectively. The red dotted and blue dashed lines are the best-fitting galaxy-halo size relations for early- and late-types. As shown in Figure 3, there is no significant difference of galaxy-halo size relations between early- and late-type in 3D space after correction for the integration effects. Early- and late-type galaxies have the same best-fitting relation as the form $\log r_{1/2} = 0.55 \log(R_{\text{vir}}/210 h^{-1} \text{kpc}) + 0.39$ with the average scatter ($\sigma$) = 0.13 dex.

It is noteworthy that the radius difference between early- and late-types is 0.12 dex in the 2D case as shown in Figure 2. Therefore, the adopted values of $f_1 = 10^{0.12} = 1.32$ for early-types ($f_1 = 1$ for late-types) can lead to the elimination of the radius difference in 3D space as shown in Figure 3. We have repeated our entire calculation assuming $f_1 = 1.35$ for early-type galaxies, in which case the 3D half-mass radius difference is 0.01 dex between early- and late-type galaxies. Note that according to Equation 6, the power-law index of 0.55 is not changed from the 2D half-light radius $R_{200}$ to the 3D half-mass radius $r_{1/2}$ either for early-types or for late-types.

However, there is an important caveat that the parameters $f_1$ and $f_m$ may be dependent on other galaxy properties beyond the morphological types in the conversion of 2D half-light radius to 3D half-mass radius. Using $\sim 16500$ galaxies with stellar masses $10^{9.6} h^{-1} M_\odot \leq M_\star \leq 10^{11.5} h^{-1} M_\odot$ in the CANDELS fields at $0 \leq z \leq 2.5$, Suess et al. (2019a,b) investigated the galaxy color gradients and quantified the ratios of half-mass to half-light radii at different redshifts. They found that the ratios are dependent on the stellar mass or stellar mass surface density for early- or late-type galaxies (see their Figure 5 of Suess et al. (2019a)). In addition, Suess et al. (2019b) claimed that the mass-to-light size ratios at fixed mass evolve rapidly with $1 \leq z < 2.5$, and remain roughly constant at $z \leq 1$. Using 537 classical elliptical galaxies $(M_\star \geq 10^8 h^{-1} M_\odot)$ from the MaNGA/SDSS-IV DR15 survey, Ibarra-Medel et al. (2022) also found the evolution of the mass-to-light size ratio and the dependence of the ratio on the galaxy stellar mass.

In the analytical model for deprojecting sésic profiles for stellar systems of arbitrary triaxial shapes, van de Ven & van der Wel (2021) claimed that the projected semi-major radius is an unbiased proxy for the radius of a sphere that contains 50% of the total luminosity or mass. Thus, we have also repeated our entire calculation using the semi-major radius from the catalog provided by Simard et al. (2011). We find that the power-law index 0.55 is not changed and the overall scatter becomes slightly larger with $\langle \sigma \rangle = 0.17$ dex based on the semi-major radius instead of circular radius. However, in a previous study, Kravtsov (2013) found that the index of the relation between $r_{1/2}$ and $R_{\text{vir}}$ is $\sim 1.0$, which is shown as the black solid line in Fig. 3 for comparison. This disagreement may be due to the difference of the detailed methodology in the abundance matching techniques (Kravtsov 2013; Yang et al. 2018), where in estimating the halo mass for each galaxy, Kravtsov (2013) adopted an abundance matching method without taking into account the scatter in the stellar-to-halo mass relation.

Figure 4 shows the galaxy-to-halo size ratio as a function of the halo virial mass. The solid dashed line shows the best-fitting relation $r_{1/2}/R_{\text{vir}} = 0.01 \times (4.7 - 0.29 \log M_{\text{vir}})$. Remarkably, the galaxy-halo size ratios depend significantly on the halo mass. Galaxies in more massive haloes have smaller ratios. In addition, we also calculate the galaxy-to-halo size ratio as a function of the galaxy stellar mass $M_\star \geq 10^9 h^{-2} M_\odot$, and find that the relation can be approximated by the relation $r_{1/2}/R_{\text{vir}} = 0.01 \times (4.8 - 0.35 \log M_\star)$.

As shown in Figure 3, the galaxy size $r_{1/2}$ is equal to that
from the best-fitting relation of Kravtsov (2013) at fixed halo radius $R_{\text{vir}} = 250 h^{-1} \text{kpc}$, where the corresponding halo mass is $M_{\text{vir}} = 10^{12.5} h^{-1} M_\odot$ and $r_{1/2}/R_{\text{vir}} = 0.011$. Note that the normalization factor is 0.015 for $R_{200}$ in Kravtsov (2013), which corresponds to the factor 0.011 for $R_{\text{vir}}$ in this study. As shown in Figure 4, the ratios of $r_{1/2}/R_{\text{vir}}$ are smaller (larger) than 0.011 for haloes more (less) than $10^{12.5} h^{-1} M_\odot$, therefore in Figure 3, at fixed halo radii the galaxy sizes are smaller (larger) than those given by Kravtsov (2013) for haloes more (less) than $10^{12.5} h^{-1} M_\odot$. This trend leads to the power-law index of this study is significantly lower than that of Kravtsov (2013). For haloes with mass larger than $\sim 10^{12} h^{-1} M_\odot$, the stellar-to-halo mass ratios decrease with increasing halo mass (Zhang et al. 2021b), thus it’s reasonable that the galaxy-halo size ratios are dependent on the halo mass. In fact, the mass dependence of galaxy-halo size relation is also detected using the 2D galaxy size in Somerville et al. (2018), although they claimed that the mass dependence is mitigated by the conversion from 2D half-light radius to 3D half-mass radius (see their Figure 3).

### 3.3 Halo spin dependence

In this section, we investigate how the galaxy-halo size relation depends on the halo spin.

The spin values are calculated using dark matter haloes from the ELUCID simulation according to Equation 2. The left panel of Figure 5 shows the probability distribution of the spin parameter $\lambda$ of dark matter haloes with mass larger than $10^{11} h^{-1} M_\odot$, which contain at least 300 particles in order to obtain a reliable measurement of halo properties, such as halo spin and concentration. The median value of the spin parameter is $\langle \lambda \rangle = 0.037$, and the standard deviation is $\sigma_{\log \lambda} = 0.24$. Obviously, the median value and the scatter of the spin parameter are in good agreement with previous works (Bullock et al. 2001; Somerville et al. 2018; Jiang et al. 2019; Yang et al. 2021).

The right panel of Figure 5 shows the spin parameters as a function of the halo virial mass. The solid symbols are the median values, which can be fitted by the linear relation $\log \lambda = -0.049 \log M_{\text{vir}} - 0.865$. In simulation, the halo spin parameter is weakly dependent on halo mass, and larger haloes have slightly smaller spin parameters. Therefore, in the following analysis, galaxies are separated into three subsamples with their corresponding halo mass $\log(M_{\text{vir}}/(h^{-1} M_\odot))$ in the ranges of $(11, 12)$, $(12, 13)$ and $(13, 14)$, in order to probe the spin dependence of the galaxy-to-halo size ratio. As pointed out in Zhang et al. (2021b), the matching between central galaxies and subhaloes can be contaminated by nearby similar mass subhaloes which may impact the link between the galaxy size and the spin parameters. However, the matched galaxy-subhalo pairs are quite reliable for those subhaloes with mass $\geq 10^{12} h^{-1} M_\odot$.

Figure 6 shows galaxy-to-halo size ratio as a function of the spin parameter in different mass ranges. The solid symbols with different colors are the median values of $r_{1/2}/R_{\text{vir}}$. Different types of lines show the linear fitting functions in different mass ranges $(11, 12)$, $(12, 13)$ and $(13, 14)$. Compared with the significant halo mass dependence, these results do not show any prominent correlation between $r_{1/2}/R_{\text{vir}}$ and $\lambda$ at fixed mass ranges, including the two massive bins where the matches are more reliable. In general, these results are consistent with recent studies based on the hydrodynamical simulations (Desmond et al. 2017; Jiang et al. 2019; Yang et al. 2021; Rohr et al. 2022; Stiskalek et al. 2022).

Using two suites of zoom-in hydrodynamical simulation, Jiang et al. (2019) claimed that there is almost no correlation between galaxy-to-halo size ratio and the spin parameter, for 34 galaxies with halo mass $\log(M_{\text{vir}}/M_\odot)$ in the mass range of $(11.3, 12.3)$ from the VELA simulation, and for $\sim 100$
galaxies in the halo mass range (9.5, 12.3) from the NIHAO simulation, respectively. Using four suits of cosmological hydrodynamical simulation, Yang et al. (2021) investigated the relation between $r_{1/2}/R_{\text{vir}}$ and halo spin parameter $\lambda$. They found that for dwarfs with halo mass $\log(M_{\text{halo}}/M_\odot) \leq 11.2$, there is almost no correlation between $r_{1/2}/R_{\text{vir}}$ and $\lambda$. For Milky Way sized galaxies in the mass range (11.7, 12.3), the correlation is very weak in the EAGLE simulation, although the correlation can be detected in the IllustrisTNG simulation. This indicates that different hydrodynamic solvers in different simulations could affect the relevant results. Using low-mass central galaxies ($M_* \sim 10^{7-9}M_\odot$) from the hydrodynamical FIREbox simulations, Rohr et al. (2022) found that the galaxy-halo size relation has almost no correlation with the halo spin, which agrees with the result in this study.

In the canonical disk formation model of (Fall & Efstathiou 1980; Mo et al. 1998), it is assumed that the angular momenta of disk galaxies are fixed fractions of those of their surrounding haloes. Therefore, in the angular momentum conservation model the galaxy spins are strongly correlated with those of their host haloes, resulting in the galaxy sizes being determined by the spins of their host haloes. However, in this study, we find that there is almost no correlation between galaxy size and the halo spin. In addition, we have repeated our entire calculation for early- and late-type sub-samples, respectively. There is also no correlation for early or late-type galaxies. Our findings disagree with the canonical disk formation model in this regard.

3.4 halo concentration dependence

In this section, we investigate how the galaxy-halo size relation depends on the halo concentration, which has been widely used to describe the internal structure of haloes.

The halo concentration is defined as $c_{\text{vir}} = R_{\text{vir}}/r_s$, where $r_s$ is the scale radius of the NFW profile given by Equation 3. The best-fitting values are calculated by minimizing the rms deviation between the NFW density profiles and the halo density profiles from the ELUCID simulation according to Equation 4. Figure 7 shows the measured concentration $c_{\text{vir}}$ as a function of the virial mass of the haloes matched to galaxies in observation. The solid symbols are the median values, which can be fitted by the relation $\log c_{\text{vir}} = -0.09 \log M_{\text{vir}} + 2.03$, where the power-law index $-0.09$ is very close to the index $-0.1$ of the relaxed haloes in Neto et al. (2007). Using $\sim 30$ million haloes ($M_{\text{halo}} \geq 2 \times 10^{12}M_\odot$) of at least 2000 particles from two very large dark matter only cosmological simulations, Child et al. (2018) investigated the concentration–mass relation in the redshift range $0 \leq z \leq 4$. For haloes at $0 \leq z \leq 1$ fitted by the power-law relation (see their Table 2), the power-law index $-0.089$ in Child et al. (2018) agrees well with the index $-0.09$ in this study.

Obviously, the halo concentration is strongly dependent on the halo mass. Thus, in order to study the concentration dependence of galaxy-to-halo size ratio, galaxies are separated into five subsamples with their corresponding halo mass $\log(M_{\text{halo}}/(h^{-1}M_\odot))$ in the ranges of (11, 11.5), (11.5, 12), (12, 12.5), (12.5, 13) and (13, 13.5). Figure 8 shows the galaxy-to-halo size ratio as a function of the halo concentration in different mass ranges. The solid symbols with different colors are
the median values, and different types of lines show the best fitting linear relations in different mass ranges. Based on these results, especially in the three high massive ranges where the matches are more reliable, we do not see any prominent correlation between $r_{1/2}/R_{\text{vir}}$ and $c_{\text{vir}}$ at fixed mass ranges.

In this work, the independence of galaxy-halo size relation on halo concentration agrees with the results for low-mass central galaxies in the hydrodynamical simulation from the FIRE project (Rohr et al. 2022), although Jiang et al. (2019) claimed that the galaxy-to-halo ratio is dependent on the halo concentration with $r_{1/2}/R_{\text{vir}} \propto c_{\text{vir}}^{-0.7}$ based on the VELA and NIHAO simulation.

4 SUMMARY

Based on a sample of 276,463 central galaxies ($M_\star \gtrsim 10^9 h^{-2} M_\odot$) in SDSS observation and their corresponding haloes ($M_{\text{vir}} \gtrsim 10^{10.5} h^{-1} M_\odot$) in the ELUCID simulation, we have investigated the galaxy-halo size relation in this study. Galaxy sizes are extracted from the NYU-VAGC catalog (Blanton et al. 2005), and the halo sizes are the virial radii ($R_{\text{vir}} \sim 60 – 2000 h^{-1} \text{kpc}$) of dark matter haloes from the ELUCID simulation. Galaxies in observation are matched to (sub-)haloes in the constrained simulation using the neighborhood subhalo abundance matching method (Yang et al. 2018). We then investigate the galaxy-halo size relation for central galaxies, which are more reliable than satellite galaxies in the matching pairs (Zhang et al. 2021a,b). In addition, we investigate how the galaxy-halo size relation depends on the halo spin and concentration. The following are our main findings:

(i) Early-type galaxies have smaller 2D half-light radii $R_{50}$ than late-type galaxies at fixed halo virial radii $R_{\text{vir}}$. They have the same power-law index 0.55 in the galaxy-halo size relation with $R_{50} \propto R_{\text{vir}}^{0.55}$.

(ii) There is no significant difference of galaxy 3D half-mass radius $r_{1/2}$ between early- and late-types. The vertical offset between early- and late-types is eliminated after correction from $R_{50}$ to $r_{1/2}$. The galaxy-halo size relation can be expressed as $\log r_{1/2} = 0.55 \log (R_{\text{vir}}/210 h^{-1} \text{kpc}) + 0.39$. Note that the power-law index of 0.55 is significantly lower than the values of $\sim 0.9 – 1.0$ in the previous studies (Kravtsov 2013; Somerville et al. 2018; Rohr et al. 2022).

(iii) In this study, the galaxy-halo size ratios $r_{1/2}/R_{\text{vir}}$ depend on the halo mass where galaxies in more massive haloes have smaller ratios, although Kravtsov (2013) and Somerville et al. (2018) claimed that there is no mass dependence of the galaxy-halo size ratios.

(iv) At fixed halo mass, there is no significant dependence of galaxy-to-halo size ratio on the halo spin or concentration, which agrees with recent works from hydrodynamical simulations (Desmond et al. 2017; Jiang et al. 2019; Yang et al. 2021; Rohr et al. 2022), but disagrees with the classical disk formation theory (Fall & Efstathiou 1980; Mo et al. 1998).

These results can provide guidelines for linking galaxy sizes to the sizes of their host haloes in the theoretical and empirical models.

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

References

Abazajian K. N., et al., 2009, ApJS, 182, 543
Bell E. F., McIntosh D. H., Katz N., Weinberg M. D., 2003, ApJS, 149, 289
Blanton M. R., et al., 2005, AJ, 129, 2562
Bryan G. L., Norman M. L., 1998, ApJ, 495, 80
Bullock J. S., Dekel A., Kolatt T. S., Kravtsov A. V., Klypin A. A., Porciani C., Primack J. R., 2001, ApJ, 555, 240
Calette A. R., Avila-Reese V., Rodríguez-Puebla A., Hernández-Toledo H., Papastergis E., 2018, Rev. Mex. Astron. Astrofís., 54, 443
Chan J. C. C., et al., 2016, MNRAS, 458, 3181

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