Variation of Apparent Cohesion and Friction Angle Under Polyaxial Stress Conditions in Concrete

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Abstract. Amongst many mechanical properties, cohesion (c) and angle of internal friction (φ) are probably the most widely used rock and rock-like material strength design parameters. However, unlike Mohr-Coulomb (MC) failure criterion that assumes cohesion and friction angle are intrinsic material properties and are not affected by the applied stress level, the Hoek-Brown (HB) criterion predicts a continuous change of apparent cohesion and friction angle if the induced normal stress on the fracture plane changes. That is, at low values of normal stresses, the instantaneous angle of friction will be relatively large, whereas cohesion will be a small value. As the applied normal stress value on the fracture plane increases (moving ‘up’ the non-linear Hoek-Brown strength envelope), the angle of friction reduces, and the cohesion increases. This is an important result from the HB failure model and allows a more realistic estimate of shear strength to be made at low values of normal stress, preventing potential over-design problems. Nevertheless, the HB model neglects the effect of the intermediate principal stress on material properties. Limited studies on the variation of apparent cohesion and friction angle under polyaxial stresses in concrete are available in the literature. Therefore, this paper aims to investigate the effect of true triaxial stresses on concrete cohesion and friction angle using a polyaxial strength criterion developed by Mogi. The results of concrete show that the intermediate principal stress has a pronounced effect on cohesion degradation and the mobilization of internal friction angle as the ratio of the intermediate to the minor principal stress changes. The results are expected to provide a framework for a more realistic design of underground concrete structures at depth.

1. Introduction

Proper and reliable estimation of rock, concrete and rock-like material strength and deformation characteristics are necessary to develop a safe and sustainable design for open-pit mines, rock slopes, foundations, tunnels, and underground excavations. The most widely used mechanical parameters to characterize the material strength parameter are cohesion (c) and angle of internal friction (φ). One of the earliest and widely adopted failure criteria in rock mechanics solely based on c and φ is Mohr-Coulomb [1]. In 1776, the basis of Coulomb failure criterion was introduced on the assumption that rock failure in compression occurs when the shear stress (τ) on an arbitrarily oriented failure plane inside the rock mass becomes sufficiently large to overcome the rock cohesion and the frictional force that opposes the sliding motion and shear displacement on that plane [2]. The MC model is simply a linear shear-based strength relationship describing the failure of intact rock (or a smooth joint) under the major (σ1) and minor (σ3) principal stresses, but also has representation in the form of normal (σ2) and shear stresses (τ) acting on the fracture plane or the joint surface [3, 4]. Despite the success of the
MC criterion in rock engineering, it has several practical shortcomings. For instance, the MC model (i) ignores the non-linear response of rock especially under high confinements, (ii) neglects the effect of the intermediate principal stress ($\sigma_2$), (iii) assumes that cohesion and friction angle are intrinsic material properties and are not affected by the applied stress level, (iv) postulates that both cohesion and friction angle acts simultaneously during the rock progressive yield process, and (v) overestimates tensile strength of rock and discontinuities [5, 6, 7, 8]. Over the past 250 years or so since the introduction of Coulomb failure theory, many researchers have attempted to develop new rock failure criteria and address the limitations of previously available models. To overcome the above difficulties with the MC model, the well-known Hoek-Brown (HB) failure criterion was introduced in the early 1990s and derived from the results of intact rock failure conducted by Hoek on a model of jointed rock mass studies carried by Brown [9]. [10] suggested that the cohesive strength estimated from the linear MC tends to overestimate the actual rock strength. Unlike the MC failure criterion, the HB criterion predicts a continuous change of apparent cohesion and friction angle as the induced normal stress on the fracture plane changes. That is, at lower normal stresses, the instantaneous angle of internal friction is relatively large. By moving up the HB strength envelope, the applied normal stress value on the fracture plane increases, and therefore the angle of friction reduces while the cohesion increases. This is an important result of the HB model which allows a more realistic estimate of rock shear strength to be made at low values of normal stresses, preventing potential over-design problems. Nevertheless, the HB model: (i) neglects the effect of the intermediate principal stress on rock strength properties, (ii) overestimates rock strength at low confinements (usually $\sigma_3 < 10\%$ UCS) where spalling and rockburst can occur in high-stress deep mines, and (iii) is incapable of capturing strength-hardening at high confinements thus underestimates rock mass strength at confinements above a critical threshold. The fundamental HB assumption of $\sigma_2 = \sigma_3$ is therefore only valid for a limited range of (rock-dependant) confining stresses between the two critical values above in (ii) and (iii). To resolve the second problem with the HB model, in particular, a non-linear Cohesion Weakening Friction Strengthening (CWFS) model is becoming popular in recent years [11, 12] to capture more realistically the process of brittle failure at laboratory and field scales where cohesion degrades and friction angle mobilizes while damage accumulates in brittle rocks. The CWFS model emphasizes that using straight lines for cohesion degradation and friction mobilization (as a function of plastic strain) can provide an implausible stress-strain curve, hence a smooth curve for the degradation of cohesion and mobilization of friction is suggested where both parameters change simultaneously depending on the loading rate. However, the CWFS model is yet a two-dimensional framework, and one needs to adopt a more appropriate 3D model, e.g. Lade-Duncan, Wiebols and Cook, 3D Hoek-Brown, Mogi–Coulomb, Matsuoka-Nakai, Tridimensional Griffith, von Mises, and Stassi-D’Alia, among others; for the study of cohesion and friction angle variation under true triaxial stresses.

The failure mode of concrete under polyaxial stresses is addressed in the literature [13]. However, the cohesion degradation and friction mobilization of concrete is yet a mystery. Therefore, this study aims to quantify the variation of apparent cohesion and instantaneous friction angle of concrete under true triaxial stress conditions using the Mogi-Coulomb failure model concept. The Mogi-Coulomb model [14] is favoured in this study as it showed a better agreement with the true triaxial experimental results of concrete [13], as described in the next section.

2. Mogi-Coulomb Failure Criterion

Early attempts on polyaxial testing of rock specimens through a true triaxial testing apparatus was carried out by Mogi where he investigated the impact of intermediate principal stress on rock strength properties. The work of [14] concluded that the fracture plane is resisted by the mean normal stress ($\sigma_{m,2}$) rather than the octahedral mean stress ($\sigma_{oct}$). It means:

$$\sigma_{oct} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}$$

$$\tau_{oct} = a + b\sigma_{m,2}$$
\[ \sigma_{m,2} = \frac{\sigma_1 + \sigma_3}{2} \]  

The octahedral shear stress \( \tau_{oct} \) is the shear stress component acting on the plane where its normal vector forms equal angles with the principal coordinate axes. The parameters \( a \) and \( b \) are curve fitting constants which can be related directly to material cohesion (\( c \)) and angle of internal friction (\( \varphi \)) under triaxial stress states given by the following expressions [2, 15]:

\[ a = \frac{2\sqrt{2}}{3} c \cos \varphi, \quad b = \frac{2\sqrt{2}}{3} \sin \varphi \]

3. Curve Fitting of Mogi-Coulomb Failure Criterion

Given that the available 3D-failure criteria are functions of principal stresses (\( \sigma_1, \sigma_2, \sigma_3 \)), unconfined compressive strength (\( \sigma_c \)), and material strength parameters (\( c, \varphi \)), fitting the experimental data into a 3D model could be challenging sometimes. To do so, experimental data could be fitted either explicitly or implicitly. The first step in fitting the true triaxial data through a Mogi-Coulomb failure model in an explicit fashion is by plotting the linear relationship between the octahedral shear stress (\( \tau_{oct} \)) versus the effective mean normal stress (\( \sigma_{m,2} \)) based on the applied principal stresses. Then, using linear curve-fitting procedures to find \( a \) and \( b \), the corresponding cohesion and angle of internal friction are calculated based on equation 4. Consequently, a polynomial equation as a function of the actual principal stresses (\( \sigma_1, \sigma_2, \sigma_3 \)), \( c \) and \( \varphi \) can be obtained and solved through the EXCEL solver function. Alternatively, to fit a non-linear Mogi-Coulomb curve into a series of true triaxial experimental points implicitly, values for \( c \) and \( \varphi \) shall be first selected based on the material type and then the root means square error (RMSE) of the difference between the actual and applied stresses need to be calculated in each trial. The best-fit curve is then selected where the least RMSE is obtained through a grid-search-based technique [16]. The root means square error (RMSE) is the square root of the difference between the actual applied stress (\( \sigma_i \)) and the calculated stress (\( \sigma_i' \)) for \( n \) values as follows:

\[ RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\sigma_i - \sigma_i')^2} \]

4. Experimental testing

To investigate the evolution of the cohesion and friction angle as a function of \( \sigma_2 \), a number of true triaxial loading and unloading tests were carried out on carefully casted concrete samples. In this study, 20 concrete cubes were cast, cured, cut to 50 mm, and ground to ensure smoothness and perpendicularity in all six surfaces. The samples were then tested under uniaxial, biaxial, generalized triaxial compression (\( \sigma_2=\sigma_3 \)), generalized triaxial tension (\( \sigma_1=\sigma_2 \)) and true-triaxial (\( \sigma_1 > \sigma_2 > \sigma_3 \)) by a newly developed true triaxial testing system at the Geotechnical Engineering Center within the University of Queensland [13, 17, 18, 19]. The newly developed true triaxial is a multifunction testing apparatus with the ability to perform: true-triaxial loading through independently controlled orthogonal loading platen, hydraulic fracking at up to 52 MPa on cubic specimens, permeability testing at up to 10 MPa pressure, thermo-mechanical coupling models at an elevated temperature of up to 100 degrees, and the possibility to accommodate acoustic emission or ultrasonic for the assessment of progressive damage as shown in figure 1. The load applied through the true triaxial testing system through servo-controlling actuators where each actuator is equipped with three Linear Variable Differential Transformer (LVDTs) in order to measure the volume change precisely as suggested by [20]. The loading gap can cause non-uniform distribution of intermediate principal stress on specimens and hence affects the rock strength [21]. Therefore, the apparatus has been designed to avoid loading gaps and using specimens with chamfered edges to ensure uniform distribution of the load. To avoid sample eccentricity while loading, a seating load of 5 kN was maintained in all three directions before the testing commencement. During the loading, a loading rate of 15 kN/min was applied to satisfy the ISRM recommendations. After that, the principal, intermediate and minor stresses were increased simultaneously. The stress-path of the loading was carried out as suggested by [20] by simultaneously increasing all the stresses from the seating load up to the desired minor and intermediate principal stresses and then increasing the major principal stresses to the sample failure.
The test results are presented in figure 2 and summarised in table 1. It can be seen that the peak strength of concrete at different intermediate principal stress ($\sigma_2$) values increase from the generalized triaxial compression stress state to the generalized triaxial tensile stress state for constant values of minor principal stress ($\sigma_3$) of 3 and 5 MPa. The angle of the fracture plane, measured from the horizontal reference axis, was further obtained in the range of 55°-70° under biaxial, triaxial and true triaxial loadings as listed in table 2.

**Figure 1.** True triaxial testing facility at the Geotechnical Engineering Center of the University of Queensland [17]

**Figure 2.** Uniaxial, biaxial, triaxial and true triaxial loading conditions of concrete specimens [13]
Table 1. Experimental results of testing concrete cubes under true triaxial stresses

| Loading Condition | σ₁ (MPa) | σ₂ (MPa) | σ₃ (MPa) | σ₁ (MPa) | σ₂ (MPa) | σ₃ (MPa) |
|-------------------|----------|----------|----------|----------|----------|----------|
| Uniaxial          | 50       | 0        | 0        | 164      | 17       | 17       |
| Triaxial          | 70       | 0        | 0        | 264      | 41       | 41       |
| Biaxial           | 80       | 11       | 0        | 167      | 51       | 3        |
|                   | 105      | 22       | 0        | 212      | 100      | 3        |
|                   | 113      | 41       | 0        | 214      | 151      | 3        |
|                   | 120      | 66       | 0        | 205      | 212      | 3        |
|                   | 132      | 115      | 0        | 158      | 30       | 5        |
| Triaxial          | 83       | 3        | 3        | 199      | 62       | 5        |
|                   | 119      | 5        | 5        | 244      | 121      | 5        |
|                   | 139      | 10       | 10       | 241      | 201      | 5        |

Table 2. Fracture angle under different loading conditions

| Sample Photo | Fracture Angle (°) | Loading Condition |
|--------------|--------------------|-------------------|
|              | 60                 | Biaxial           |
|              | σ₁ = 105 MPa       |
|              | σ₂ = 22 MPa        |
|              | σ₃ = 0 MPa         |
|              | 70                 | Triaxial          |
|              | σ₁ = 83 MPa        |
|              | σ₂ = 3 MPa         |
|              | σ₃ = 3 MPa         |
|              | 70                 | True Triaxial     |
|              | σ₁ = 214 MPa       |
|              | σ₂ = 151 MPa       |
|              | σ₃ = 3 MPa         |

5. Test Results and Discussion

As discussed above, the MC failure criterion assumes that the failure of an isotropic material is linear where the cohesion and friction angle are intrinsic material properties and can be calculated as follows:

\[ σ₁ = σ_c + σ₃ \tan δ \]  \hspace{1cm} (6)

\[ \tan δ = \tan^2 \left( 45 + \frac{φ}{2} \right) \]  \hspace{1cm} (7)
\[ \sigma_c = \frac{2c \cos \varphi}{1 - \sin \varphi} \]  

Based on the above equations and referring to table 1, it can be found that \( c = 16.42 \text{ MPa} \) and \( \varphi = 42^\circ \) if the MC model is fitted to the experimental data. To verify whether Mogi-Coulomb under triaxial conditions (where \( \sigma_2 = \sigma_3 \)) provides similar estimations of \( c \) and \( \varphi \) to that of the MC model, cohesion and angle of internal friction were calculated by plotting \( \tau_{oct} \) versus \( \sigma_{m,2} \) for the triaxial data only of the test results shown in table 1. The curve-fitting parameters are \( a = 13.135 \) and \( b = 0.6139 \), and the corresponding cohesion and angle of internal friction can be measured as 19.85 MPa and 41.8\(^\circ\); respectively (see also table 3). It means, MC and Mogi-Coulomb under triaxial stresses both provide similar strength parameters. In the next step and to investigate the variation of apparent cohesion and friction angle by moving up the curve from triaxial to true triaxial stress states, the curve-fitting parameters \( a \) and \( b \) and their relevant apparent cohesion and instantaneous friction were calculated either explicitly (through equations 1 to 4) or implicitly by minimising the RMSE as discussed in Section 3 above. Results are presented in table 3 and figure 3. It is obvious that a pronounced variation of the curve-fitting parameters between triaxial and true triaxial loading conditions can be observed. The apparent cohesion decreases from 19.85 MPa to 13.85 MPa while the friction angle increased from 41.8\(^\circ\) to 49.3\(^\circ\) by moving-up the curve where the intermediate principal stress increased. It means a cohesion degradation of up to 31 % and friction mobilization of up to 18 % increase is confirmed as a consequence of increasing the intermediate principal stress under polyaxial stress conditions. The dashed curves in figure 4 is the fitting curve based on the triaxial strength parameters, while the straight-line curves are the best-fitting curves for the true triaxial strength parameters - the circle dots are also the actual experimental data. It can be seen that the Mogi-Coulomb criterion is in relatively good agreement in fitting the concrete true triaxial test results when the intermediate principal stress is not very high [13] with a relative error of less than 16%. From the results, it can be deduced that ignoring the degradation of cohesion and mobilization of friction as a result of increasing the intermediate principal stress (which is not considered in MC, HB and CWFS models) can lead to a miss-fitting of a true triaxial testing data hence providing unrealistic concrete strength properties.

### Table 3. Apparent cohesion and friction angle at triaxial and true triaxial loading condition

| Loading Condition | c (MPa) | \( \varphi \) | a         | b         | \( R^2 \) |
|-------------------|--------|--------------|-----------|-----------|----------|
| Triaxial Data     | 19.85  | 41.85        | 13.135    | 0.6139    | 0.99     |
| All Data          | 13.85  | 49.32        | 8.513     | 0.715     | 0.95     |

**Figure 3.** Mogi-Coulomb fitting parameters \((a, b)\) under triaxial and true triaxial loading conditions
6. Conclusion

In this study, a series of uniaxial, biaxial, triaxial, and true triaxial tests were performed on concrete cubes to investigate the variation of cohesion and friction angle as a function of changing the intermediate principal stress. The Mogi-Coulomb was preferred as a suitable 3D failure criterion to fit the concrete experimental data. Both explicit (i.e. analytically or through Excel Solver) and implicit (through grid-search method to minimise the Root Means Square Error) techniques were used. Within the range of $\sigma_2$ variation in this study from 0-3×UCS, a degradation of 31% in cohesion and an increase of up to 18% in internal friction angle was reported. The results suggest that the intermediate principal stress can make a profound impact on the apparent cohesion and friction angle in concrete materials, which is ignored in MC, HB and CWFS models.

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