We show that both the weak phase $\beta$ and the strong phase $\delta_d$ can be determined from the time-dependent measurement of $B_d \to D^{(*)}\pm D^{(*)}\mp$ decays, whose final states are non-CP eigenstates. It is also possible to extract $\beta$ from $B_d \to D^{(*)}\pm D^{(*)}\mp$ transitions without doing the angular analysis. Possible final-state rescattering effects in $B_d \to D^{(*)}\pm D^{(*)}\mp$ channels are discussed by means of the isospin analysis. We emphasize that it is worthwhile to check whether the naive factorization approximation works or not for such $B$-meson decay modes into two heavy charmed mesons.

1 Introduction

Weak decay modes $B_d \to D^+D^-$, $D^{*+}D^-$, $D^{+}D^{*-}$ and $D^{*+}D^{*-}$ are interesting for the study of CP violation and final-state interactions at $B$-meson factories. The experimental result for the branching fraction of $B_d \to D^{*+}D^{-}$ is $B(D^{*+}D^{-}) = (9.9^{+4.4}_{-3.5}) \times 10^{-4}$ [1]. Recently the Belle Collaboration has reported the first measurement of $B_d \to D^{\mp}D^{\mp\mp}$ decays. The sum of their branching fractions is $3 B(D^{+}D^{-} \oplus D^{-}D^{+}) = (1.17 \pm 0.26^{+0.22}_{-0.25}) \times 10^{-3}$ (full reconstruction method) or $(1.48 \pm 0.38^{+0.28}_{-0.31}) \times 10^{-3}$ (partial reconstruction method). We hope that a measurement of $B_d \to D^+D^-$ will soon be available.

The $B_d \to D^{(*)+}D^{(*)-}$ channels are associated with the weak CP-violating phase

$$\beta \equiv \arg \left( -\frac{V_{td}^*V_{ts}}{V_{cd}^*V_{cs}} \right),$$

where $V_{ij}$ (for $i = u, c, t$ and $j = d, s, b$) are elements of the Cabibbo-Kobayashi-Maskawa matrix. A determination of $\beta$ from CP-violating asymmetries of $B_d \to D^{(*)+}D^{(*)-}$ transitions will be useful, not only to cross-check the extraction of $\beta$ from $B_d \to J/\psi K_S$, but also to shed some light on the relevant penguin effects and final-state interactions. In addition, it is important to test whether the naive factorization approximation works or not for such $B$ decay modes into two heavy charmed mesons.
2 Determining strong and weak phases in $B_d \to D^\pm D^{*\mp}$ decays

The transitions $B_d^0 \to D^\pm D^{*\mp}$ can occur through both tree-level and loop-induced (penguin) quark diagrams. The penguin contribution to the overall amplitude of each decay mode is negligible. In this good approximation, one may define two interference quantities between decay amplitudes and $B_d^0$-$\bar{B}_d^0$ mixing:

$$\lambda_{D^+D^-} = \frac{q_d}{p_d} \cdot \frac{A(\bar{B}_d^0 \to D^{*+}D^-)}{A(B_d^0 \to D^{*+}D^-)} = \frac{V_{td}V_{td}^*}{V_{tb}V_{tb}^*} \frac{V_{cb}V_{cb}^*}{V_{cb}V_{cb}^*} \zeta_d e^{i\delta_d} = \zeta_d e^{i(\delta_d + 2\beta)},$$

$$\tilde{\lambda}_{D^-D^+} = \frac{p_d}{q_d} \cdot \frac{A(B_d^0 \to D^{*-}D^+)}{A(B_d^0 \to D^{*-}D^+)} = \frac{V_{td}V_{td}^*}{V_{tb}V_{tb}^*} \frac{V_{cb}V_{cb}^*}{V_{cb}V_{cb}^*} \zeta_d e^{i\delta_d} = \zeta_d e^{i(\delta_d - 2\beta)},$$

where $q_d/p_d = (V_{td}V_{td}/V_{tb}V_{tb})$ describes the $B_d^0$-$\bar{B}_d^0$ mixing phase in the box-diagram approximation, $\zeta_d$ and $\delta_d$ denote the ratio of the real hadronic matrix elements and the strong phase difference between $B_d^0 \to D^{*+}D^-$ and $B_d^0 \to D^{*-}D^-$. In the naive factorization approximation, we have $\zeta_d = [f_D \cdot A_0(B^0 \to D^+)(m^2_D)]/[f_D \cdot F_1(B^0 \to D^+)(m^2_D)] \approx 1.04$, where the relevant decay constants and formfactors are self-explanatory. Comparing the experimental and theoretical results of $\zeta_d$ will provide a clean test of the factorization hypothesis for neutral-$B$ decays into two heavy charmed mesons.

The imaginary parts of $\lambda_{D^+D^-}$ and $\tilde{\lambda}_{D^-D^+}$ are of particular interest for the study of CP violation. It should be noted, however, that $\text{Im} \lambda_f$ and $\text{Im} \tilde{\lambda}_f$ (for $f = D^{*+}D^-$) themselves are not CP-violating observables! Only their difference $\text{Im}(\lambda_f - \tilde{\lambda}_f)$, which will vanish for $\beta = 0$ or $\pi$, measures the CP asymmetry. The time-dependent rates of $B_d \to D^\pm D^{*\mp}$ modes read as:

$$R \left[ B_d^0(t) \to D^{*+}D^- \right] \propto \left[ \frac{1}{2} + \frac{\zeta_d^2}{2} \cos(x_d \Gamma_d t) \right] \frac{\zeta_d \sin(\delta_d + 2\beta) \sin(x_d \Gamma_d t)}{\zeta_d \sin(\delta_d - 2\beta) \sin(x_d \Gamma_d t)};$$

$$R \left[ B_d^0(t) \to D^{*-}D^+ \right] \propto \left[ \frac{1}{2} + \frac{\zeta_d^2}{2} \cos(x_d \Gamma_d t) \right] \frac{\zeta_d \sin(\delta_d - 2\beta) \sin(x_d \Gamma_d t)}{\zeta_d \sin(\delta_d + 2\beta) \sin(x_d \Gamma_d t)};$$

where $x_d \approx 0.7$ is the $B_d^0$-$\bar{B}_d^0$ mixing parameter, and $\Gamma_d$ denotes the $B_d$ decay width. Then we may extract the weak phase $\beta$ and the strong phase $\delta_d$ up to a four-fold ambiguity:

$$\sin^2(2\beta) = \frac{1}{2} \left[ (1 - S_+ S_-) \pm \sqrt{(1 - S_+^2)(1 - S_-^2)} \right],$$

$$\sin^2 \delta_d = \frac{1}{2} \left[ (1 + S_+ S_-) \pm \sqrt{(1 - S_+^2)(1 - S_-^2)} \right],$$

where $S_\pm \equiv \sin(\delta_d \pm 2\beta)$. Indeed only a two-fold ambiguity in $\sin(2\beta)$ exists, as $\sin(2\beta) > 0$ has been experimentally verified within the standard model. If final-state interactions were insignificant in the decay modes under discussion, $\delta_d$ might not deviate too much from zero. In this case, $S_+ \approx -S_-$ would be a good approximation.

3 Extracting $\beta$ from $B_d \to D^{*\pm}D^{*\mp}$ decays without angular analysis

A comparison between the value of $\sin 2\beta$ to be determined from $B_d \to D^{*+}D^{*-}$ and that already measured in $B_d \to J/\psi K_S$ is no doubt important, as it may cross-check the consistency of the standard-model predictions. Towards this goal, a special attention has to be paid to possible uncertainties associated with the CP asymmetry in $B_d \to D^{*+}D^{*-}$. One kind of uncertainty comes from the penguin contamination, as the weak phase of the penguin amplitude is quite different from that of the tree-level amplitude. Another kind of uncertainty arises from the $P$-wave dilution, because the final state $D^{*+}D^{*-}$ is composed of both the CP-even ($S$- and $D$-wave)
and the CP-odd \((P\text{-wave})\) configurations. Of course an analysis of the angular distributions of \(B_d^0 \to D^{*+} D^{*-}\) transitions allows us to distinguish between the CP-even and CP-odd contributions. Here we like to emphasize that the direct measurement of \(\beta\) can be made in \(B_d \to D^{*+} D^{*-}\) decays without doing the angular analysis.

Taking the \(P\)-wave dilution and the penguin contamination into account, one may write the characteristic measurable of indirect CP violation in \(B_d \to D^{*+} D^{*-}\) as follows

\[
\Delta_d \equiv \text{Im} \left( \frac{V_{tb} V_{td}^{*}}{V_{tb} V_{td}^{*}} \frac{\langle D^{*+} D^{*-}|\mathcal{H}_{\text{eff}}|B_d^0\rangle}{\langle D^{*+} D^{*-}|\mathcal{H}_{\text{eff}}|B_d^0\rangle} \right) = P_d (1 - Q_d) \sin 2\beta ,
\]

where \(P_d\) and \(Q_d\) represent the \(P\)-wave dilution factor and the penguin-induced correction, respectively. With the help of the effective weak Hamiltonian, the naive factorization approximation and the heavy quark symmetry, we obtain

\[
P_d = \frac{m_B^3 - 3m_B m_D^2 + 10m_D^3}{m_B^3 + m_B m_D^2 + 2m_D^3},
\]

\[
Q_d = \frac{c_x + c_z}{c_x} \cdot \frac{\cos 2\beta}{\cos \beta} \cdot \left| \frac{V_{tb} V_{td}^{*}}{V_{cb} V_{cd}^{*}} \right| ,
\]

where \(c_x \approx 1.045, c_y \approx -0.031\) and \(c_z \approx -0.0014\) are the effective Wilson coefficients. Typically taking \(\beta = 26^\circ\), which is favored by current BaBar and Belle data, we find \(P_d \approx 0.89\) and \(Q_d \approx -0.021\). This result indicates that the penguin contamination in \(\Delta_d\) is negligibly small, while the \(P\)-wave dilution to \(\Delta_d\) should be taken seriously.

It is worth remarking that the approach advocated here may be complementary to the angular analysis considered in the literature. Hopefully both will soon be confronted with the new data from \(B\)-meson factories.

### 4 Final-state rescattering effects in \(B_d \to D^{(*)\pm} D^{(*)\mp}\) decays

The effective weak Hamiltonian responsible for \(B_d^- \to D^- D^0\), \(\bar{B}_d^0 \to D^+ D^-\) and \(\bar{B}_d^0 \to D^0 \bar{D}^0\) decay modes has the isospin structure \([1/2, -1/2]\). The decay amplitudes of these transitions can be written in terms of the isospin amplitudes:

\[
A^{+-} = \langle D^+ D^-|\mathcal{H}_{\text{eff}}|B_d^0\rangle = \frac{1}{2} (A_1 + A_0) ,
\]

\[
A^{00} = \langle D^0 \bar{D}^0|\mathcal{H}_{\text{eff}}|\bar{B}_d^0\rangle = \frac{1}{2} (A_1 - A_0) ,
\]

\[
A^{+0} = \langle D^+ \bar{D}^0|\mathcal{H}_{\text{eff}}|\bar{B}_d^+\rangle = A_1 ,
\]

where \(A_1\) and \(A_0\) are the isospin amplitudes with \(I = 1\) and \(I = 0\), respectively. Clearly the isospin relation \(A^{+-} + A^{00} = A^{+0}\) holds, and it corresponds to a triangle in the complex plane. Denoting \(A_0/A_1 = z e^{i\theta}\), we obtain

\[
z = \sqrt{\frac{2(|A^{+-}|^2 + |A^{00}|^2)}{|A^{+0}|^2} - 1}, \quad \theta = \arccos \left( \frac{|A^{+0}|^2}{z |A^{+0}|^2} \right) ;
\]

If \(z = 1\) and \(\theta = 0\), for example, we find that \(|A^{00}| = 0\), i.e., the decay mode \(B_d^0 \to D^0 \bar{D}^0\) is forbidden. One may get similar isospin relations for the decay modes \(B_d^+ \to D^+ \bar{D}^0\), \(B_d^0 \to D^+ D^-\) and \(B_d^0 \to D^0 \bar{D}^0\).

It is worth mentioning that the same isospin analysis can be done for \(B \to D \bar{D}^*\) and \(B \to D^* \bar{D}\) decays. Of course, the isospin parameters \(z\) (\(\bar{z}\)) and \(\theta\) (\(\bar{\theta}\)) in \(B \to D \bar{D}, D \bar{D}^*\) and
$D^*\bar{D}$ may be different from one another due to their different final-state interactions. As for $B \to D^*\bar{D}^*$, the same isospin relations hold separately for the decay amplitudes with helicity $\lambda = -1, 0, +1$.

The time-independent measurements of those decay modes mentioned above allow us to construct the relevant isospin triangles. Consequently the isospin parameters $z$ ($\bar{z}$) and $\theta$ ($\bar{\theta}$) are extractable in the absence of any time-dependent measurement. If the branching ratios of $B^0_d \to D^0\bar{D}^0$ and $\bar{B}^0_d \to D^0\bar{D}^0$ are too small to be observable, then large cancellation between the isospin amplitudes $A_1$ ($A_1$) and $A_0$ ($A_0$) must take place. In the case that $B^0_d \to D^+D^-$ and $B^+_d \to D^+\bar{D}^0$ have been measured earlier than $B^0_d \to D^0\bar{D}^0$, a lower bound on the rate of the latter decay mode is model-independently achievable from the isospin relations obtained in Eq. (7). Since $\cos \theta \leq 1$, we get from Eqs. (7) and (8) that

$$B(B^0_d \to D^0\bar{D}^0) \geq \left[ \frac{\sqrt{B(B^0_d \to D^+D^-)}}{B(B^+_d \to D^+\bar{D}^0) - 1} \right]^2 B(B^+_d \to D^+\bar{D}^0),$$

where tiny isospin-violating effects induced by the mass difference $m_{D^0} - m_{D^-}$ and the life time difference $\tau_{B^0_d} - \tau_{B^+_d}$ have been neglected. This bound is useful to set a limit for the results of $B(B^0_d \to D^0\bar{D}^0)$ obtained from specific models of hadronic matrix elements. Similarly one can find the lower bounds for the branching ratios of $B^0_d \to D^{*0}\bar{D}^0$, $D^0\bar{D}^{*0}$ and $D^{*0}\bar{D}^{*0}$.

5 Concluding remarks

Some conclusions can be drawn from our results: (a) $B_d \to D^\pm D^{\mp\pm}$ modes are useful to determine the weak CP-violating phase $\beta$ and the strong phase shift $\delta_d$; (b) $\beta$ can also be determined from $B_d \to D^{*\pm}D^{*\mp}$ transitions without doing the angular analysis; (c) it is worthwhile to investigate final-state rescattering effects in $B \to D^{(*)}\bar{D}^{(*)}$ channels, and to check whether the naive factorization approximation works or not for such $B$ decays into two heavy charmed mesons.

Similar analyses can be done for $B_s \to D^{(*)}_s\bar{D}^{(*)}_s$ decay modes.

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