Isospin violation and meson-exchange models of the nucleon-nucleon interaction

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Abstract

Traditionally isospin violation (charge symmetry breaking and charge dependence) has been modeled by the exchange of mesons between two nucleons. Sources of isospin violation in these models include meson mass differences, meson mixing (of pseudoscalar, vector, and axial-vector mesons) and isospin violating couplings of mesons with nucleons. I will review the calculated results of this theoretical approach and the $NN$ data on isospin violation.

*invited talk given at XIIIth International Seminar on High Energy Physics Problems (ISHEPP 13), Relativistic Nuclear Physics and Quantum Chromodynamics, held at the Joint Institute for Nuclear Research, Dubna, Russia, September 2-7, 1996.
1 Experimental evidence for charge dependence and charge asymmetry

A classification scheme for isospin violation

Isospin violation in the nucleon-nucleon system is usually described in terms of classes of possible isospin operators for nucleons 1 and 2, although the classification scheme of Henley and Miller [1] applies to any baryon isodoublet system (such as $^3\text{H}$ - $^3\text{He}$). Here I review this scheme of four classes, beginning with the dominant class I isospin or charge independent interactions and ending with the class IV interactions which are the weakest. Class I forces have no isospin dependence (1) or their dependence is proportional to $\tau(1) \cdot \tau(2)$. Class II forces maintain charge symmetry but break charge independence and are proportional to $\tau_3(1)\tau_3(2)$. They are characterized as proportional to an isotensor $\tau_3(1)\tau_3(2) - \frac{1}{3} \tau(1) \cdot \tau(2)$. Class III forces break charge symmetry (and therefore necessarily charge independence) and are symmetric under the interchange of particles 1 and 2. Class III forces are proportional to $(\tau_3(1) + \tau_3(2))$. They do not cause isospin mixing in the two-body system because the third component of total isospin commutes with $I^2$. Finally the class IV forces also break charge symmetry and charge independence and are anti symmetric under the interchange of particles 1 and 2. These forces are proportional to $(\tau_3(1) - \tau_3(2))$ or to $[\tau(1) \times \tau(2)]_3$. Class IV forces connect states of $I = 0$ and $I = 1$. For the two-body system these are the spin triplet $^3L_j$ and spin singlet $^1L_j$ states. Classes II and IV forces vanish for $pp$ and $nn$ systems while class III forces vanish for the $np$ system. Class IV forces have no effect on an $nn$ or $pp$ system but cause spin-dependent isospin mixing effects in the $np$ system.

Class II and III: low energy $NN$ scattering parameters

The best evidence for Class II and III forces lies in the low energy parameters of the effective range expansion of $np$ ($I = 1$), $pp$, and $nn$ spin-singlet scattering and the $^3\text{H}$ - $^3\text{He}$ binding energy difference, after the effects of one-photon exchange have been removed. I will utilize here the results of the Nijmegen phase shift analyses which include such small effects as vacuum polarization and the Breit (relativistic correction) term in one photon exchange. The subtraction of one-photon exchange to arrive at a pure nuclear effective range expansion from the recent Nijmegen phase-shift analyses is still underway, so I will quote the low energy parameters of the recent strong $NN$ potentials [3] which match the data nearly as well as the phase-shift analysis. They are [2]

2
\[ a_{np} \approx -24.0 \text{ fm} \quad r_{np} \approx 2.68 \text{ fm} \]
\[ a_{pp} \approx -17.4 \text{ fm} \quad r_{pp} \approx 2.83 \text{ fm} \]

The experimental values of \( a_{nn} \) found from \( \pi^{-}d \rightarrow \gamma nn \) in which only the photon was detected are
\[ a_{nn} = -18.5 \pm 0.4 \text{ fm} \quad r_{nn} = 2.80 \pm 0.11 \text{ fm} \]
in excellent agreement with the kinematically complete determination of \( a_{nn} = -18.7 \pm 0.6 \) fm from the same reaction. Other compilations of the effective range parameters can be found in reviews. There is a large difference between \( np \) (\( I = 1 \)), and the average \( pp \) and \( nn \) \( 1S_0 \) scattering parameters (\( \Delta a^I \equiv |a_{np}| - \frac{1}{2}(|a_{nn}| + |a_{pp}|) \), \( \Delta r_0^I \equiv r_{np} - \frac{1}{2}(r_{nn} + r_{pp}) \))
\[ \Delta a^I \approx 6.0 \text{ fm} \quad \Delta r_0^I \approx -0.13 \text{ fm} \]
indicating a fairly strong class II force (but still at the level of a few % of the charge independent strong force).

The class III force effects on the effective range parameters (\( \Delta a^{III} \equiv |a_{nn}| - |a_{pp}| \), \( \Delta r_0^{III} \equiv r_{nn} - r_{pp} \))
\[ \Delta a^{III} \approx 1.10 \text{ fm} \quad \Delta r_0^{III} \approx -0.03 \text{ fm} \]
are not so dramatic as are those of class II. They are, however, quite consistent in sign and magnitude with the positive value for the \(^3H\) \(-\) \(^3\)He binding energy difference of 764 keV, also due in part to class III nuclear forces. The direct electromagnetic contribution to this number (static Coulomb force between the \( pp \) pair in \(^3\)He and other smaller electromagnetic effects such as vacuum polarization and the Breit interaction) must be subtracted to arrive at the effect from class III forces. The totality of these effects was estimated first in a nearly model independent manner twenty years ago and that estimate has been verified by direct Faddeev calculations with many \( NN \) potential models and by a combination of the two methods. Subtracting out these direct em contributions and a small difference in the \( nn \) and \( pp \) systems due to the neutron-proton mass difference, we find a remaining
\[ \Delta E_{\text{exp}} \equiv (^3H - ^3\text{He}) \approx 76 \pm 24 \text{ keV} \]
which is attributed to class III forces.

The positive \( \Delta a \) reflects an interaction between two neutrons which is more attractive than between two protons and more binding energy is provided for \(^3\)H as compared to \(^3\)He.
The consistency in magnitude is more interesting. It has recently been touted as a triumph of sophisticated Faddeev and quantum Monte Carlo calculations with modern $NN$ potentials that $\Delta E$ in the $A = 3$ system can be explained by a class III charge asymmetric $NN$ force which has been adjusted to match $\Delta a^{III}$ and $\Delta r_0^{III}$ in the $A = 2$ system [9, 10, 12]. However it has long been known from separable potential models that $\Delta E$ is much more sensitive to $\Delta r_0^{III}$ than to $\Delta a^{III}$ [13]. Gibson and Stephenson made a dedicated study of this dependence for both charge dependence and charge asymmetry in the bound trinucleon [14]. For central separable potentials producing the correct $^3He$ binding their charge asymmetry results can be very well fitted by

$$\Delta E_{\text{GS}} = (40\Delta a - 1600\Delta r_0) \text{ keV/fm.}$$

(A discussion of the validity of this relationship in light of current three-body technology can be found in [11]). The sensitivity of the trinucleon binding energy to $r_0$ rather than to $a$ was reviewed in the 1930’s by Bethe and Bacher [16] with the model that Thomas used to demonstrate that nuclear forces are not contact forces but have a finite (i.e., non-zero) range [15]. In that model the binding energy of the deuteron is kept fixed by increasing the depth of the potential as the range is decreased, and the $nn$ interaction is neglected. Thomas demonstrated that, as the range of the two-body force goes to zero, the three-body binding energy becomes infinite.

**Class IV: $np$ elastic scattering**

The class II and class III effects in the $NN$ system require careful subtraction of much larger isospin violating one-photon exchange (including static Coulomb) to be revealed. For example, $a_{pp}^{\exp} = -7.8014 \pm 0.0011$ fm has as a strong analogue $a_{pp} \sim 17.4$ fm after the subtraction [2]. The large static Coulomb interaction is absent in $np$ elastic scattering so the small class IV charge asymmetry is not masked as is class III. In a charge symmetric world the complete separation of isosinglet ($I = 0$) and isotriplet ($I = 1$) states leads to a decoupling of spin singlet and spin triplet states and the equality of the analyzing powers when polarized neutrons are scattered from unpolarized protons and vice versa. Any non-zero difference of the analyzing powers $\Delta A \equiv A_n - A_p$, where the subscripts denote the polarized nucleon, is evidence for the class IV forces which mix $I = 0$ and $I = 1$ states in the $np$ system. Three (challenging) measurements have been made of a non-zero $\Delta A$ (at the zero-crossing angle of the average analyzing power) with increasing precision for the later experiments:

$$\Delta A = (47 \pm 22 \pm 8) \times 10^{-4} \text{ at 477 MeV [17]}$$

$$\Delta A = (34.8 \pm 6.2 \pm 4.1) \times 10^{-4} \text{ at 183 MeV [18]}$$
When one-photon exchange is removed from the third and latest measurement, leaving only the strong \( NN \) interaction, \( \Delta A \) is five standard deviations away from zero.

2 Predictions of meson-exchange models

A single or two-meson exchange graph is reduced to a non-relativistic potential which breaks charge independence or charge symmetry. To calculate effects in \( NN \) systems the resulting potential is added to a charge independent phenomenological potential and the resulting change in the scattering parameters is compared to the experimental measures of section 1.

Class II charge dependence

The sum of class II forces from meson mass differences in single and two-meson exchange and \( \gamma - \pi \) exchange does not explain the observed charge dependence. For example, \( \Delta a_{\text{II}} \sim 3.6 \) fm from the sources discussed below falls short of the experimental value of \( \sim 6 \) fm. This statement is contrary to previous conclusions because a recent calculation of the charge dependent \( \gamma - \pi \) potential finds its effect to be an order of magnitude smaller than previous (flawed) estimates.

Single meson exchange

Much of the observed class II effect in the low energy scattering parameters is accounted for by the mass difference of the charged and neutral pion. A typical result is that found by Coon and Scadron [20] for the pion mass difference (\( \Delta m_\pi = m_\pi^+ - m_\pi^0 \)) and rho mass difference (defined in the same way):

\[
\Delta a_{\text{II}} \approx (+2.9 \text{ fm})_{\Delta m_\pi} + (+0.1 \text{ fm})_{\Delta m_\rho}
\]
\[
\Delta r_0'' \approx (-0.10 \text{ fm})_{\Delta m_\pi} + (0.0 \text{ fm})_{\Delta m_\rho}
\]

These charge dependent potentials are due solely to the mass difference and the coupling constants are assumed to be charge independent. The latter assumption is justified \textit{a posteriori} by the Nijmegen phase shift analysis which finds almost no deviation from charge independence of the pion-nucleon coupling constant (Table 1 of [22]). The poorly known vector meson mass difference is that given by the Coleman-Glashow tadpole model (to be
discussed later). In the above I quote results obtained with a charge independent (CI) potential with a super-soft core \([21]\) which I think would be close to results with more modern CI potentials.

**Simultaneous \(\pi\) and \(\gamma\) exchange**

Quite recently Friar and Coon reported the first reliable calculation of the isospin violating effect of simultaneous exchange of a pion and photon \([23]\). The calculation corresponds to a subset of leading order diagrams in chiral perturbation theory and one hopes that it corresponds to the dominant subset. They worked in the static limit \((m_N \rightarrow \infty)\) and in Coulomb gauge for the photon exchange. The two Feynman diagrams which survive under those stipulations are then reduced to charge dependent potentials each of which gives a small \(\Delta a^{II}\) of opposite sign. The total effect is \(\Delta a^{II}_{\gamma-\pi} \approx -0.15 \pm 0.03\) for a variety of short-range cutoffs and model CI potentials. To the order worked out, the only effect of this mechanism is class II charge dependence and \(\Delta a^{II}_{\gamma-\pi}\) is small (2-3%) and of the opposite sign to the empirical \(\Delta a^{II}\). Because only the first terms in a \(1/m_N\) expansion were kept, the class III or IV charge asymmetry is expected to be \(O(m_\pi/m_N)\) of the charge dependent result.

This class II \(\gamma - \pi\) force result is about an order of magnitude smaller than previous estimates of \(\Delta a^{II}_{\gamma-\pi} \approx +1\) fm which are quoted in the literature \([6, 7, 24]\).

**Meson mass difference in \(2\pi\) exchange**

Ericson and Miller \([24]\) employed the CI two-pion exchange potentials derived by Partovi and Lomon \([25]\) from covariant Feynman graphs to estimate \(\Delta a^{II}_{2\pi} \approx +0.9\) fm from \(\Delta m_\pi\) in box and cross-box diagrams with (equal mass) nucleon intermediate states. No effect due to the nucleon mass differences was considered. Coon and Scadron had earlier estimated the latter effect to be \(\approx -0.15\) fm, also with the Partovi-Lomon potentials. Chung and Machleidt \([26]\) employed a Bonn potential to establish \(\Delta a^{II}_{2\pi} \approx +0.9\) fm from nucleon and delta baryon intermediate states, only 0.2 of which came from nucleon intermediate states. The initial polemics of this agreement \([3, 27]\) can be enjoyed in Refs. \([24, 26]\).

**Class III charge asymmetry in \(\Delta a\), \(\Delta r_o\), and \(\Delta E\)**

\(\Delta I = 1\) meson mixing (single meson exchange)

The mixing of \(I = 0\) and \(I = 1\) mesons give rise to a class III \(NN\) force \([28]\). This mixing occurs in nature and is successfully described by the Coleman-Glashow picture that the \(\Delta I = 1\) hadronic processes are dominated by symmetry breaking tree level tadpole diagrams \([29, 30]\). In this picture, at the quark level, the meson-mixing matrix element \(\langle a_i^1 | H_{em} | f_1 \rangle\)
is determined by the dominant single-quark operator $H^{(3)} = \frac{1}{2}(m_u - m_d)\bar{q}\gamma_3 q$, established in Ref. [31] from the electromagnetic mass differences in the pseudoscalar mesons, vector mesons, baryon octet, and baryon decuplet. When extended to the off-diagonal $\Delta I = 1$ transitions $\langle \pi^0 | H_{em} | \eta_{NS} \rangle$, $\langle \rho^0 | H_{em} | \omega \rangle$, and $\langle a_1^1 | H_{em} | f_{1NS} \rangle$ this gives the value $-0.005 \text{ GeV}^2$ for the mixing matrix elements, independent of the particular mesons concerned. Indeed, for this one-body operator, it is to be expected that one obtains the same numerical value connecting any $I = 1$ state with an $I = 0$ nonstrange state of the same spin and parity. It also follows from this one-body operator structure (or the concomitant tadpole diagram at the hadronic level) that the $\Delta I = 1$ transition has no dependence upon the four-momentum squared of the mesons, thus allowing it to be tested against experiment on-mass-shell for time-like $q^2$ and to be employed in the spacelike momentum transfer of an $NN$ force diagram. These tests are passed for the vector $\rho\omega$ mixing [32] and pseudoscalar $\pi\eta\eta'$ mixing [33], and, of course, there is no experimental data on the $\Delta I = 1$ transitions of the axial vector mesons $a_1$ and $f_1$.

Explicit calculation shows [34, 32, 35]

\[
\begin{align*}
\Delta a_{\pi\eta\eta'}^{III} &\approx +0.26 \text{ fm} & r_o^{III} &\approx -0.02 \text{ fm} & \Delta E &\approx 32 \text{ keV} \\
\Delta a_{\rho\omega}^{III} &\approx +1.5 \text{ fm} & r_o^{III} &\approx -0.03 \text{ fm} & \Delta E &\approx 90 \text{ keV} \\
\Delta a_{a_1f_1}^{III} &\approx +0.13 \text{ fm} & r_o^{III} &\approx 0.00 \text{ fm} & \Delta E &\approx 15 \text{ keV}
\end{align*}
\]

indicating a good description of class III charge asymmetry by meson mixing alone. The two-body results quoted are with the (charge independent) de Touriel-Sprung-Rouben potential and the three-body estimate was made with the “model-independent” method [8] also used to establish the empirical $\Delta E$.

**2π exchange**

The major contribution to the difference of the $nn$ and $pp$ interactions comes from the baryon mass difference in intermediate states. Unfortunately, a recent calculation based upon non-relativistic $\pi NN$ and $\pi N\Delta$ vertices [11] finds a much stronger effect ($\Delta a^{III} \approx +1.0 \text{ fm}$, $\Delta r_0^{III} \approx -0.02 \text{ fm}$, and $\Delta E \approx 43 \text{ keV}$) than the first estimate [20] which was based upon the Partovi-Lomon potential. Neither the covariant calculation nor the non-relativistic calculation of the charge independent $2\pi$-exchange $NN$ potential have a clear chiral symmetric character. Weinberg and van Kolck have emphasized the utility of an explicit consideration of chiral symmetry in the analysis of isospin violating interactions [36, 37]. Leading order chiral $2\pi$-exchange potentials are now available [38] and could provide an alternative and more reliable foundation for future studies of this mechanism.
A second source of class III (and class IV) forces, which is significant in describing the class IV experiments [39], is baryon mass differences in non-relativistic $\pi NN$ and $\pi N\Delta$ couplings themselves. We found them to play a small role in class III effects: ($\Delta a_{III} \approx -0.1 \text{ fm}$, $\Delta r_{0}^{III} \approx 0.0 \text{ fm}$, and $\Delta E \approx -1 \text{ keV}$) [11].

Class IV charge asymmetry

In meson-exchange based $NN$ potential models, the major strong interaction contribution to $\Delta A$ stem from single-pion exchange (due to the $np$ mass difference in the charged pion-$NN$ vertex just mentioned) and $\rho \omega$ mixing. A class IV contribution from one-photon exchange (interaction of the proton current with the neutron magnetic moment) must be included in the analysis but it is comparatively small, especially at the higher energies of the TRIUMF measurements. The $\rho \omega$ mixing contribution is similar to single photon exchange in that it is due to the product of the Pauli (magnetic) coupling of the isovector $\rho$ and the Dirac coupling of the isoscalar omega to the nucleons. (Mixing of pseudoscalar mesons cannot therefore give a class IV force. Such a force could arise, in principle, from the mixing of axial vector mesons but the Pauli couplings of axial vector mesons to nucleons were neglected in Ref. [35] as being too speculative at the present time.)

The low energy IUCF measurement cannot be described without $\rho \omega$ mixing, although with slightly larger vector-nucleon couplings than used for the class III aspect of $\rho \omega$ mixing. In any event, the theoretical predictions of Holzenkamp, Holinde, and Thomas [40] and Iqbal and Niskanen [11] agree well with all three measurements (see Fig. 3 of Ref. [19]).

3 Effective field theories of isospin violation

Recently, following a suggestion by Weinberg, isospin violation has been studied in the context of a general effective Lagrangian which also displays chiral symmetry breaking [37]. The first steps have been taken in parameterizing this approach with the aid of the nuclear data [42]. The effective field approach does not depend upon the models discussed in section 2 but the conclusions of this first paper is consistent with the results of the single-meson exchange models. For lack of space, I cannot discuss further this promising approach which links in a direct way the observed isospin violation in the $NN$ interaction to the symmetries of QCD.

The meson-exchange models, unlike the effective field theories, take their strengths and structure from hadronic physics outside the $NN$ interaction and therefore make predictions. Although not directly related to QCD, they do, however, provide a reasonable picture of
class III charge symmetry breaking and are not as successful in quantitatively describing
class II charge dependence.

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