Feasible Generalized Least Squares for Panel Data with Cross-sectional and Serial Correlations

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Abstract

This paper considers generalized least squares (GLS) estimation for linear panel data models. By estimating the large error covariance matrix consistently, the proposed feasible GLS (FGLS) estimator is robust to heteroskedasticity, serial correlation, and cross-sectional correlation. It is more efficient than the OLS estimator. To control serial correlation, we employ the banding method. To control cross-sectional correlation, without knowing the clusters, we suggest using the thresholding method. We establish the consistency of the proposed estimator. A Monte Carlo study is considered. The proposed method is applied to an empirical application.

Keywords: Panel data, efficiency, thresholding, banding, cross-sectional correlation, serial correlation, heteroskedasticity

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1 Introduction

We consider a linear panel regression model with fixed-effects defined by

\[ y_{it} = x_{it}'\beta + \alpha_i + \mu_t + u_{it}, \]

where \( \alpha_i \) and \( \mu_t \) are unknown individual and time fixed-effects, \( x_{it} \) is a \( k \times 1 \) vector of explanatory variables, \( u_{it} \) is an unobservable error component, \( y_{it} \) and the fixed effects are scalars, and \( \beta \) is \( k \times 1 \) vector.

In the econometric literature on panel data, clustering and autocorrelation problems have long been considered. A common unobserved shock at the group level causes the clustering issue, and the autocorrelation problem arises when the individual or the group level shocks are serially correlated. In the presence of heteroskedasticity, serial and/or cross-sectional correlations, there are two approaches to proceed. One is the ordinary least squares (OLS) estimation, but with a robust standard error (e.g. Arellano (1987), Liang and Zeger (1986) and Petersen (2009)). Hansen (2007a) studied an asymptotic theory that are robust to the heteroskedasticity and serial correlation in the case of large-\( N \) large-\( T \). Driscoll and Kraay (1998) suggested a standard error that controls for arbitrary cross-sectional dependence. In addition, Vogelsang (2012) generalized the results of Hansen (2007a), which is robust to heteroskedasticity, autocorrelation and spatial correlation. Recently, Bai, Choi, and Liao (2019) proposed a robust standard error for the OLS estimator. Their robust standard error allows heteroskedasticity, serial correlation, and cross-sectional correlation of unknown form.

The present paper focuses on the second approach, which is feasible generalized least squares (FGLS). FGLS is known to be more efficient than OLS in the presence of heteroskedasticity, serial and/or cross-sectional correlations (see Cameron and Miller (2015) and Wooldridge (2010)). Hansen (2007b) studied FGLS estimation that takes into account serial correlation and clustering problems in fixed effects panel and multilevel models. By obtaining bias-corrected estimates of the AR(\( P \)) coefficients, Hansen (2007b) considered FGLS estimation and showed efficiency gains relative to OLS. However, it requires the cluster structure to be known. This gives motivation to our paper. We assume the unknown cluster structure, and control heteroskedasticity, both serial and cross-sectional correlations by estimating the large error covariance matrix consistently.

In this paper, we consider the setting of large-\( N \) large-\( T \) case, and serial and cross-sectional correlations, but unknown structure of clusters. We introduce a modified FGLS estimator that eliminates the cross-sectional and serial correlation bias by proposing a high-dimensional error covariance matrix estimator. In addition, our proposed method is applicable when the knowledge of clusters is not available. Following an idea suggested in Bai and Liao (2017), in this paper, the FGLS involves estimating an \( NT \times NT \) dimensional inverse covariance matrix.
\Omega^{-1}, \text{ where }

\Omega = (E_{t}u_{t}')

where each block \( E_{t}u_{t}' \) is an \( N \times N \) autocovariance matrix. Here parametric structures on the serial or cross-sectional correlations are not imposed. By assuming weak dependences, we apply nonparametric methods to estimate the covariance matrix. To control the autocorrelation in time series, we employ the idea of Newey-West truncation. This method, in the FGLS setting, is equivalent to “banding”, previously proposed by Bickel and Levina (2008b) for estimating large covariance matrices. We apply it to banding out off-diagonal \( N \times N \) blocks that are far from the diagonal block. In addition, to control for the cross-sectional correlation, we assume that each of the \( N \times N \) block matrices are sparse, potentially resulting from the presence of cross-sectional correlations within clusters. We then estimate them by applying the thresholding approach of Bickel and Levina (2008a). We apply thresholding separately to the \( N \times N \) blocks, which are formed by time lags \( E_{t}u_{t+h} : h = 1, 2, \ldots \) This allows the cluster-membership to be potentially changing over-time. An alternative approach would be to apply thresholding universally over all lags, assuming that the cluster-membership does not change over time. This potentially would increase the accuracy of identifying the cluster-membership, so called “sparsistency”, but it will not change the first-order asymptotics (rate of convergence of the FGLS estimator, or its asymptotic distribution). A contribution of this paper is the theoretical justificaiton for estimating the large error covariance matrix.

For the FGLS, it is crucial for the asymptotic analysis to prove that the effect of estimating \( \Omega \) is first-order negligible. In the usual low-dimensional settings that involve estimating optimal weight matrix, such as the optimal GMM estimations, it has been well known that consistency for the inverse covariance matrix estimator is sufficient for the first-order asymptotic theory, e.g., Hansen (1982), Newey (1990), Newey and McFadden (1994). However, it turns out that when the covariance matrix is of high-dimensions, not even the optimal convergence rate for estimating \( \Omega^{-1} \) is sufficient. In fact, proving the first-order equivalence between the FGLS and the infeasible GLS (that uses the true \( \Omega^{-1} \)) is a very challenging problem under the large \( N \), large \( T \) setting. We provide a new technical argument to achieve this goal.

The rest of the paper is organized as follows. In Section 2 we describe the model and the large error covariance matrix estimator. Also we introduce a new FGLS estimator and
its limiting distribution. Section 3 presents Monte Carlo studies evaluating the finite sample performance of the estimators. In Section 4 we apply our methods to study the US divorce rate problem. Conclusions are provided in Section 5. All proofs are given in Appendix A.

Throughout this paper, let $\nu_{\text{min}}(A)$ and $\nu_{\text{max}}(A)$ denote the minimum and maximum eigenvalues of matrix $A$ respectively. Also we use $\|A\| = \sqrt{\nu_{\text{max}}(A'A)}$, $\|A\|_1 = \max_i \sum_j |A_{ij}|$ and $\|A\|_F = \text{tr}(A'A)$ as the operator norm, $\ell_1$-norm and the Frobenius norm of a matrix $A$, respectively.

2 Generalized Least Squares

Consider the following linear regression model:

$$y_{it} = x_{it}'\beta + u_{it}$$

(2.1)

where $\beta$ is a $k \times 1$ vector of unknown coefficients, $x_{it}$ is a $k \times 1$ vector of regressors, and $u_{it}$ is the unobservable error term, often known as the idiosyncratic component. This model incorporates the standard fixed effects models as in Hansen (2007a). After removing the nuisance parameters from the equation, $x_{it}, y_{it}$ and $u_{it}$ can be interpreted as variables. It is straightforward to allow additive fixed effects by demeaning procedure, such as first-differencing to remove the fixed effects.

Then model (2.1) can be stacked and represented in full matrix notation as

$$Y = X\beta + U,$$

(2.2)

where $Y = ((y_1', \cdots, y_T')'$ is the $NT \times 1$ vector of $y_{it}$ with each $y_t$ being an $N \times 1$ vector; $X = ((x_1', \cdots, x_T')'$ is the $NT \times d$ matrix of $x_{it}$ with each $x_t$ being an $N \times d$; $U = ((u_1', \cdots, u_T')'$ is the $NT \times 1$ vector of $u_{it}$ with each $u_t$ being an $N \times 1$ vector.

Let $\Omega = \text{Var}(U)$ be the $NT \times NT$ covariance matrix. We consider the following (infeasible) GLS estimator of $\beta$:

$$\tilde{\beta}_{\text{GLS}}^{\text{inf}} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y.$$ 

(2.3)

Note that $\Omega$ is a high-dimensional conditional covariance matrix, which is very difficult to estimate. Note that $\Omega = ((E_{ut}u_s')_{N \times N})$ is a $NT \times NT$ matrix, with the $(t,s)$ th block as an $N \times N$ covariance matrix $E_{ut}u_s'$. We aim to achieve the following: (i) obtain a "good" estimator of $\Omega^{-1}$, allowing an arbitrary form of weak dependence in $u_{it}$, and (ii) show that the effect of replacing $\Omega^{-1}$ by $\hat{\Omega}^{-1}$ is asymptotically negligible.

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1 Consider the underlying model defined by $y_{it} = \alpha_i + \mu_t + x_{it}'\beta + u_{it}$. To control the fixed-effects, let $y_{it} = y_{it} - \tilde{y}_i - \tilde{y}_t + \tilde{y}$, $x_{it} = x_{it} - \tilde{x}_i - \tilde{x}_t + \tilde{x}$, with $\tilde{y}_i = T^{-1} \sum_{t=1}^{T} y_{it}$, $\tilde{y}_t = N^{-1} \sum_{i=1}^{N} y_{it}$, $\tilde{y} = (NT)^{-1} \sum_{i=1}^{N} \sum_{t=1}^{T} y_{it}$, and $\tilde{x}_i, \tilde{x}_t$ and $\tilde{x}$ are defined similarly.
2.1 Estimating $\Omega$ for a general GLS problem

To gain the intuitions, we start with a population approximation for $\Omega$. Then, we suggest the estimator for $\Omega$ that takes into account both correlations problem.

2.1.1 Population approximation

We start with a “banding” approximation to control serial correlations. Recall that $\Omega = (Eu_t u'_s)$, where the $(t, s)$ block is $Eu_t u'_s$. Due to the stationarity, $\Omega_{t,s}$ depends on $(t, s)$ only through $h = |t - s|$. So with slight abuse of notation, we can write $\Omega_{t,s} = \Omega_h = Eu_t u'_{t+h}$.

Under the stationarity and weak serial correlation assumption, $\Omega = (Eu_t u'_s)$, where the diagonal blocks are all $Eu_t u'_s$, and off-diagonal blocks $Eu_t u'_{t+h}$ decay to the “zero matrix” as $h \to \infty$. In the Newey-West spirit, $\Omega$ can be approximated by: $\Omega_{NW} = (\Omega_{ts}^{NW})$, where $\Omega_{ts}^{NW}$ depends on $(t, s)$ only through $h = |t - s|$, so we write $\Omega_{ts}^{NW} = \Omega_h^{NW}$. Here $\Omega_h^{NW}$ is an $N \times N$ block matrix, defined as

$$\Omega_h^{NW} = \begin{cases} Eu_t u'_{t+h}, & \text{if } |h| \leq L \\ 0, & \text{if } |h| > L, \end{cases}$$

for some pre-determined $L \to 0$. We can set $L$ equal to $4(T/100)^{(2/9)}$ as Newey and West (1987) suggested. We regard $\Omega_h^{NW}$ as the “population banding approximation”.

Next, we focus on the $N \times N$ block matrix $\Omega_h = Eu_t u'_{t+h}$ to control cross-sectional correlations. Under the intuition that $u_{it}$ is cross-sectional weakly dependent, we assume $\Omega_h$ is a sparse matrix, that is, $\Omega_h_{ij} = Eu_{it} u_{j,t+h}$ is “small” for “many” pairs $(i, j)$. Then $\Omega_h$ can be approximated by a sparse matrix (Bickel and Levina (2008a)):

$$(\Omega_h^{BL})_{ij} = \begin{cases} Eu_{it} u_{j,t+h}, & \text{if } |Eu_{it} u_{j,t+h}| > \tau_{ij} \\ 0, & \text{if } |Eu_{it} u_{j,t+h}| \leq \tau_{ij}, \end{cases}$$

for some pre-determined threshold $\tau_{ij} \to 0$. We regard $\Omega_h^{BL} = (\Omega_h^{BL})_{ij}$ as the “population sparse approximation”.

In summary, we approximate $\Omega$ by an $NT \times NT$ matrix $(\tilde{\Omega}_{ts}^{NT})$, where each block $\tilde{\Omega}_{ts}^{NT}$ is an $N \times N$ matrix, defined as:

$$\tilde{\Omega}_{ts}^{NT} := \begin{cases} \Omega_h^{BL}, & \text{if } |h| \leq L \\ 0, & \text{if } |h| > L, \end{cases} \quad h = t - s.$$ 

Therefore, we use “banding” to control the serial correlation, and “sparsity” to control the cross-sectional correlation.
2.1.2 The estimator of $\Omega$ and FGLS

Given the intuition of the population approximation, we construct the large covariance estimator as follows. We first estimate the $N \times N$ block matrix $E_{UtU_{t+h}}$. To do so, let

$$\bar{R}_{h,ij} = \frac{1}{N} \sum_{t=1}^{T-h} (\tilde{u}_{it}\tilde{u}_{jt+h} + \tilde{u}_{i,t+h}\tilde{u}_{jt}),$$

where $\tilde{u}_{it} = y_{it} - x_{it}'\hat{\beta}_{OLS}$. Define

$$\tilde{\sigma}_{h,ij} = \begin{cases} \bar{R}_{h,ii}, & \text{if } i = j \\ s_{ij}(\bar{R}_{h,ij}), & \text{if } i \neq j \end{cases},$$

where $s_{ij}(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ is a “soft-thresholding function” with an entry dependent threshold $\tau_{ij}$ such that

$$s_{ij}(z) = 0 \text{ if } |z| < \tau_{ij}; s_{ij}(z) = \text{sgn}(z)(|z| - \tau_{ij}) + \text{ if } |z| > \tau_{ij}.$$  

For the threshold value, we specify

$$\tau_{ij} = M\gamma_{NT}\sqrt{|\bar{R}_{0,ii}| |\bar{R}_{0,jj}|},$$

for some pre-determined value $M > 0$, where $\gamma_{NT} = \sqrt{\frac{\log(LN)}{T}}$ is such that $\max_{h \leq L} \max_{i,j \leq N} |E_{UtU_{i,t+h}} - \bar{R}_{h,ij}| = O_P(\gamma_{NT})$. In practice, $M$ can be chosen by multifold cross-validation, which is explained in next subsection. Then define

$$\hat{\Omega}_h = (\tilde{\sigma}_{h,ij})_{N \times N}.$$  

Next, we define the $(t,s)$th block $\hat{\Omega}_{t,s}$ as an $N \times N$ matrix:

$$\hat{\Omega}_{t,s} = \begin{cases} \omega(h, L)\hat{\Omega}_h, & \text{if } |t - s| = h, h \leq L \\ 0, & \text{if } |t - s| = h, h > L \end{cases}.$$  

Here $\omega(h, L)$ is the kernel function. As Newey and West (1994) suggested, let $\omega(h, L) = 1 - h/(L + 1)$ be the Bartlett kernel function, where $L$ is the bandwidth. Our final estimator is a $NT \times NT$ matrix:

$$\hat{\Omega} = (\hat{\Omega}_{t,s}).$$  

Here $\hat{\Omega}$ is a nonparametric estimator, which does not require an assumed parametric structure on $\Omega$. Given $\hat{\Omega}$, we propose the feasible GLS (FGLS) estimator of $\beta$ as

$$\hat{\beta}_{FGLS} = [X'\hat{\Omega}^{-1}X]^{-1}X'\hat{\Omega}^{-1}Y.$$  

Remark 2.1 (Universal thresholding). We apply thresholding separately to the $N \times N$ blocks, $(\tilde{\sigma}_{h,ij})_{N \times N}$, which are estimated lagged blocks: $E_{UtU_{t+h}} : h = 1, 2, \ldots$. This allows the cluster-membership to be potentially changing over-time, that is, the “sparsistency”
(identification of nonzeros) of $Eu_{t}u_{t+h}$ can change over $h$. But still we can consistently recover the membership. If it is known that the cluster-membership is time-invariant, then one would set $\tilde{\sigma}_{h,ij} = 0$ if $\max_{h \leq L} |\tilde{R}_{h,ij}| < \tau_{ij}$. This potentially would increase the finite sample accuracy of identifying the cluster-membership, but it will not change the first-order asymptotics (rate of convergence of the FGLS estimator, or its asymptotic distribution).

### 2.2 Choice of tuning parameters

Our suggested covariance matrix estimator, $\hat{\Omega}$, requires the choice of tuning parameters $L$ and $M$, which are the bandwidth and the threshold constant respectively. To choose the bandwidth $L$, we suggest using $L = 4(T/100)^{2/9}$, which is proposed by Newey and West (1994). For the small size of $T$, we also recommend $L \leq 3$.

In practice, the thresholding constant, $M$, can be chosen through multifold cross-validation, which is also used in Bickel and Levina (2008) and Fan, Liao, and Mincheva (2013). After obtaining the estimated $N \times 1$ vector residuals $\hat{u}_t$ by OLS, we split them randomly into two subsets, denoted by $\{\hat{u}_t\}_{t \in J_1}$ and $\{\hat{u}_t\}_{t \in J_2}$; $J_1$ represents the training data set, and $J_2$ represents the validation data set. Let $T(J_1)$ and $T(J_2)$ be the sizes of $J_1$ and $J_2$, which satisfy $T(J_1) + T(J_2) = T$ and $T(J_1) \approx T$. For example, as suggested by Bickel and Levina (2008), we can choose $T(J_1) = T(1 - \log(T)^{-1})$ and $T(J_2) = T/\log(T)$.

We repeat this splitting procedure multiple times, say $P$ times. At the $p$th split, we denote by $\tilde{\Omega}_0^p$ the sample covariance matrix based on the validation set, defined by $\tilde{\Omega}_0^p = T(J_2)^{-1}\sum_{t \in J_2} \hat{u}_t\hat{u}_t'$. Let $\tilde{\Omega}_0^{H,p}(M)$ be the hard-thresholding estimator with threshold constant $M$ using the training data set. Finally, we choose the constant $M^*$ by minimizing a cross-validation objective function

$$M^* = \arg \min_{c < M < C} \frac{1}{P} \sum_{j=1}^{P} \|\tilde{\Omega}_0^{H,j}(M) - \tilde{\Omega}_0^p\|_F^2.$$  

Then the resulting estimator is $\hat{\Omega}(M)$. Here $c$ is the minimum constant that $\tilde{\Omega}_0(M)$ is positive definite for $M > c$, and $C$ is a large constant such that $\tilde{\Omega}_0(C)$ is a diagonal matrix. We find that setting $c = 1$ and $C = 2$ works well. Hence the minimization procedure is obtained over $M \in (1, 2)$ through a grid search.

The proposed procedure modifies that of Bickel and Levina (2008) and Fan, Liao, and Mincheva (2013). Due to the nature of time series data, we use a consecutive block for the validation set, so that the serial correlation is not perturbed. Hence we first divide the data into $P = \log(T)$ blocks with block length $T/\log(T)$. Similar to the K-fold cross-validation, each $T(J_2)$ is taken as one of the $P$ blocks when computing $\tilde{\Omega}_0^p$.  

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2.3 The effect of $\hat{\Omega}^{-1} - \Omega^{-1}$

A key step of proving the asymptotic property for $\hat{\beta}_{FGLS}$ is to show that it is asymptotically equivalent to $\hat{\beta}_{GLS}^{inf}$, that is:

$$\frac{1}{\sqrt{NT}}X'(\hat{\Omega}^{-1} - \Omega^{-1})U = o_P(1). \quad (2.7)$$

In the usual low-dimensional settings that involve estimating optimal weight matrix, such as the optimal GMM estimations, it has been well known that consistency for the inverse covariance matrix estimator is sufficient for the first-order asymptotic theory, e.g., [Hansen (1982), Newey (1990), Newey and McFadden (1994)]. It turns out, when the covariance matrix is of high-dimensions, not even the optimal convergence rate of $\|\hat{\Omega} - \Omega\|$ is sufficient. In fact, proving equation (2.7) is a very challenging problem.

**A special example.** To illustrate the key technical issue, we start with a simple and ideal case where $u_{it}$ is known, and independent across both $i$ and $t$, but with cross-sectional heteroskedasticity. In this case, the covariance matrix of the $NT \times 1$ vector $U$ is a diagonal matrix, with diagonal elements $\sigma_i^2 = Eu_{it}^2$:

$$\Omega = \begin{pmatrix} D & & \\ & D & \\ & & D \end{pmatrix}, \text{ where } D = \begin{pmatrix} \sigma_1^2 \\ \sigma_2^2 \\ & \ddots \\ & & \sigma_N^2 \end{pmatrix}.$$  

Then a natural estimator for $\Omega$ is

$$\hat{\Omega} = \begin{pmatrix} \hat{D} & & \\ & \hat{D} & \\ & & \hat{D} \end{pmatrix}, \text{ where } \hat{D} = \begin{pmatrix} \hat{\sigma}_1^2 \\ \hat{\sigma}_2^2 \\ & \ddots \\ & & \hat{\sigma}_N^2 \end{pmatrix},$$

and $\hat{\sigma}_i^2 = \frac{1}{T} \sum_{t=1}^{T} u_{it}^2$, because $u_{it}$ is known. Then the GLS becomes:

$$\left( \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} x_{it}x_{it}' \hat{\sigma}_i^{-2} \right)^{-1} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} x_{it}y_{it} \hat{\sigma}_i^{-2}.$$  

A key technical step is to prove that the effect of estimating $D$ is asymptotically negligible:

$$\frac{1}{\sqrt{NT}} \sum_{i=1}^{N} \sum_{t=1}^{T} x_{it}u_{it}(\hat{\sigma}_i^{-2} - \sigma_i^{-2}) = o_P(1).$$
It can be shown that the problem reduces to proving:

\[
A \equiv \frac{1}{\sqrt{NT}} \sum_{i=1}^{N} \sum_{t=1}^{T} x_{it} u_{it} \sigma_{i}^{-2} \left( \frac{1}{T} \sum_{s=1}^{T} (u_{is}^{2} - E(u_{is}^{2})) \right) \sigma_{i}^{-2} = o_{P}(1). 
\]

In fact, straightforward calculations yield

\[
EA = \frac{\sqrt{NT}}{T} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} E(x_{it} E(u_{it}^{3} | x_{it} )) \sigma_{i}^{-4}. 
\]

Generally, if \( u_{it} | x_{it} \) is non-Gaussian and asymmetric, \( E(u_{it}^{3} | x_{it} ) \neq 0 \). Hence we require \( N/T \to 0 \) to have \( EA \to 0 \). Hence, to allow for non-Gaussian and asymmetric conditional distributions, in the GLS setting we adopt the assumption that \( N = o(T) \).

Next, even if \( N = o(T) \), the problem is still non-trivial. The optimal rate of convergence under the operator norm is

\[
\max_{i} |\tilde{\sigma}_{i}^{-2} - \sigma_{i}^{-2}| = \|\tilde{\Omega}^{-1} - \Omega^{-1}\| = O_{P}(\sqrt{\log N} / T).
\]

Then to apply the optimal rate above, we would use the following bound:

\[
\left\| \frac{1}{\sqrt{NT}} X'(\tilde{\Omega}^{-1} - \Omega^{-1}) U \right\| \leq \frac{1}{\sqrt{NT}} \|X\| \|\tilde{\Omega}^{-1} - \Omega^{-1}\| \|U\|
\leq \frac{1}{\sqrt{N}} O_{P}(NT) O_{P}(\sqrt{\log N}) = O_{P}(\sqrt{N \log N}),
\]

which is not sufficient to prove that it is \( o_{P}(1) \) even if \( N/T \to 0 \). Hence, a more careful analysis is required to prove (2.7).

In the general case when both cross-sectional and serial correlations are present, our strategy is to use a careful expansion for \( \frac{1}{\sqrt{NT}} X'(\tilde{\Omega}^{-1} - \Omega^{-1}) U \). We shall proceed in two steps:

Step 1: Show that \( \frac{1}{\sqrt{NT}} X'(\tilde{\Omega}^{-1} - \Omega^{-1}) U = \frac{1}{\sqrt{NT}} W'(\tilde{\Omega} - \Omega) \epsilon + o_{P}(1) \), where \( W = \Omega^{-1} X \), and \( \epsilon = \Omega^{-1} U \).

Step 2: Show that \( \frac{1}{\sqrt{NT}} W'(\tilde{\Omega} - \Omega) \epsilon = o_{P}(1) \).

Now we suppose \( \omega(h, L) = 1, \Omega \approx \Omega^{NW} \) and let \( A_{bh} = \{ (i, j) : |E u_{it} u_{j,t+h} | \neq 0 \}, A_{sh} = \{ (i, j) : |E u_{it} u_{j,t+h} | = 0 \} \). As for Step 2, we shall show,

\[
\frac{1}{\sqrt{NT}} W'(\tilde{\Omega} - \Omega) \epsilon \approx \frac{1}{\sqrt{NT}} \sum_{h=1}^{L} \sum_{i,j \in A_{bh}} \sum_{t=1}^{T} w_{it} \epsilon_{j,t+h} \frac{1}{T} \sum_{s=1}^{T-h} (u_{is} u_{j,s+h} - E u_{it} u_{j,t+h}). \quad (2.8)
\]

Here \( w_{it} \) is defined such that, we can write \( W = (w_{1}', \ldots, w_{T}')' \) with \( w_{t} \) being an \( N \times d \) matrix.
of \(w_{it}; \varepsilon_{it}\) is defined similarly. We then further argue that the right hand side of (2.8) is \(o_p(1)\) when \(N = o(T)\).

### 2.4 Asymptotic results of FGLS

We impose the following technical conditions, regulating the sparsity and serial weak dependence.

**Assumption 2.1.** (i) \(E x_{it}^4 < \infty, \) and \(E u_{it}^4 < \infty.\)

(ii) \(\gamma_T(L) \equiv \max_t \sum_{|h| > L} \|Eu_{it}u_{i+h}\|_{sp} \rightarrow 0.\)

(iii) For each fixed \(h, f_T(L) \equiv L \max_{h < L} |1 - \omega(h, L)| \rightarrow 0\) and \(\max_{h \leq L} |\omega(h, L)| \leq C\) for some \(C > 0.\)

**Assumption 2.2.** Let \(Q_{hv}^{imp} \equiv \sum_{t=1}^{T} w_{it} \sum_{j=1}^{N} (\Omega^{-1})_{t+h,m+v,jp}.\)

Then, \(\max_{i,p,h,v} |\frac{1}{NT} \sum_{q=1}^{N} \sum_{m=1}^{T} \varepsilon_{mq} Q_{hv}^{imp}| = O_p(\sqrt{\log(LN)NT}).\)

**Assumption 2.3.** Let \(m_q = \max_{j \leq N} \frac{1}{N} \sum_{i=1}^{N} (Eu_{it}u_{j,t+h})^q\) and \(\gamma_{NT} = \sqrt{\frac{\log(LN)}{NT}}.\)

(i) \(\sum_{h > L} \| (\Omega)_{h} \|_1 \leq L^{-\alpha}\)

(ii) \(\max_{i,t} \sum_{m=1}^{T} \sum_{j=1}^{N} |(\Omega^{-1})_{im,ij}| = O(1).\)

(iii) \(L^{2}m_q^2(\log(LN))^{1.5-qT^{-1}} \rightarrow 0.\)

(iv) \(L^{1-\alpha} NT^{\gamma_{NT}^{-q}} \rightarrow 0, \) and \(L^{-\alpha} T^{\gamma_{NT}^{-q}} m_q^{1-q} \rightarrow 0.\)

(v) Let the \(\gamma^* = L m_q^{1-q} + f_T(L) + \gamma_T(L).\) Then \(\sqrt{NT}\gamma^3 \rightarrow 0.\)

Assumption 2.1 is a weak serial dependence condition to the high-dimensional case in panel data literature. In addition, this condition allows us to prove the convergence rate of the covariance matrix estimator. Assumption 2.2 is required to prove Step 1 in the previous section. Because \(\| \Omega^{-1} \|_1 < \infty,\) we know \(E(Q_{hv}^{imp})^2 < \infty.\) Hence this assumption holds under mixing conditions with exponential tails. Assumption 2.3 (i)-(ii) require the weak cross-sectional correlations. Assumption 2.3 (iii)-(v) is the sparsity assumption as discussed in Section 2. In addition, the sparsity assumptions assume that \(m_q\) should not be too large. Assumption 2.4 allows us to prove Step 2 in Section 2.3.

Note in the special case of serial independence and finite clusters where each individual \(u_{it}\) is correlated with only finite cross-sectional units, \(m_q = O(1), q = 0\) and \(L = O(1),\) then the condition \(\sqrt{NT}\gamma^3 \rightarrow 0\) reduces to \(N = o(T^2).\) We need an upper bound for \(N\) to ensure the accuracy of estimating each of the \(N \times N\) block matrix in \(\Omega_{st}.\)

The following theorem shows the convergence rate of the estimated large covariance matrix.
Theorem 2.1. Under the Assumption 2.1, when \(\|\Omega^{-1}\| = O(1)\), for \(\gamma_{NT} = \sqrt{\frac{\log(LN)}{T}}\) and some constant \(q \in [0, 1)\),

\[
\|\hat{\Omega} - \Omega\| = O_P(L\gamma_{NT}^q m_q) + f_T(L) + \gamma_T(L) = \|\hat{\Omega}^{-1} - \Omega^{-1}\|.
\]

Proposition 2.1. Under the Assumption 2.1-2.3,

\[
\sqrt{NT}(\hat{\beta}_{FGLS} - \beta) = \Gamma^{-1}\left(\frac{1}{\sqrt{NT}} X'\Omega^{-1}U\right) + \Gamma^{-1}\left(\frac{1}{\sqrt{NT}} X'\Omega^{-1}(\hat{\Omega} - \Omega)\Omega^{-1}U\right) + o_P(1),
\]

where \(\Gamma = E(X'\Omega^{-1}X/NT)\).

Now we shall show that middle part of right hand side equation above is \(o_P(1)\), which is the same as equation (2.8). Let \(G_{T,ij}^1(h) = \frac{1}{\sqrt{T}} \sum_{s=1}^{T-h} (u_{is}u_{j,s+h} - Eu_{is}u_{j,t+h})\) and \(G_{T,ij}^2(h) = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} w_{it}\varepsilon_{j,t+h}\). Here both \(G_{T,ij}^1(h)\) and \(G_{T,ij}^2(h)\) are empirical processes. Then the right hand side of equation (2.8) can be written as

\[
\frac{1}{\sqrt{N}} \sum_{i,j} A_{bh} G_{T,ij}^1(h) G_{T,ij}^2(h),
\]

where \(A_{bh} = \{(i,j) : \|Eu_{it}u_{j,t+h}\| \neq 0\}\).

Assumption 2.4. Let \(A_{bh} = \{(i,j) : \|Eu_{it}u_{j,t+h}\| \neq 0\}\). Then

\[
\frac{1}{\sqrt{N}} \sum_{h=1}^{L} \sum_{i,j \in A_{bh}} G_{T,ij}^1(h) G_{T,ij}^2(h) = o_P(1),
\]

where \(G_{T,ij}^1(h) = \frac{1}{\sqrt{T}} \sum_{s=1}^{T-h} (u_{is}u_{j,s+h} - Eu_{is}u_{j,t+h})\) and \(G_{T,ij}^2(h) = \frac{1}{\sqrt{T}} \sum_{t=1}^{T} w_{it}\varepsilon_{j,t+h}\).

A simple sufficient condition for Assumption 2.4 is that \(\frac{1}{\sqrt{N}} \sum_{h=1}^{L} \sum_{i,j \in A_{bh}} G_{T,ij}^p(h)^2 = O_P(1)\), for \(p = 1, 2\) and \(NL^2 = o(T)\). Then we have the following limiting distribution by using the result of Theorem 2.1

Theorem 2.2. Suppose \(\text{Var}(U|X) = \Omega\). Under the Assumptions 2.1-2.4 as \(N, T \to \infty\),

\[
\sqrt{NT}(\hat{\beta}_{FGLS} - \beta) \overset{d}{\to} N(0, \Gamma^{-1}),
\]

where \(\Gamma = E(X'\Omega^{-1}X/NT)\). The consistent estimator of \(\Gamma\) is \(\hat{\Gamma} = X'\hat{\Omega}^{-1}X/NT\).

The asymptotic variance of the FGLS estimator is \(\text{Avar}(\hat{\beta}_{FGLS}) = \Gamma^{-1}/NT\), and an estimator of it is \((X'\hat{\Omega}^{-1}X)^{-1}\).
2.5 FGLS Standard Error Estimation

In this subsection, we discuss in detail the estimation of the limiting covariance matrix. Our proposed FGLS method gives more precise estimation compared to OLS estimation. According to theory, the standard errors in Section 2.4 are of non-sandwich form. On the other hand, in the literature, the form of sandwich estimators is used for robust inference as discussed in Romano and Wolf (2017) and Miller and Startz (2018). By using diagonal matrix, heteroskedasticity-robust standard errors are most commonly used. Based on the simulation study, we find that the FGLS standard errors are sometimes underestimated using the non-sandwich form. To deal with this underestimated standard error, we also propose an estimator of the asymptotic variance of the FGLS estimator with a sandwich form as follows:

\[
(X'\hat{\Omega}^{-1}X)^{-1}(X'\hat{\Omega}^{-1}\hat{\Sigma}\hat{\Omega}^{-1}X)(X'\hat{\Omega}^{-1}X)^{-1},
\]

where \(\hat{\Sigma}\) is the \(NT \times NT\) covariance matrix estimator using FGLS residuals. When estimating \(\hat{\Sigma}\), we use a procedure similar to estimating \(\hat{\Omega}\), but use a different thresholding constant, \(M'\). Therefore, one should employ another cross-validation procedure as discussed in Section 2.2 to find \(M'\).

Using two tuning parameters is analogous to the lasso literature, one for the original lasso estimation, the other for debiasing in order to do inference, for example Javanmard and Montanari (2014) and Van de Geer, Bühlmann, Ritov, Dezeure, et al. (2014). Here we use one tuning parameter for estimation, and another tuning parameter for standard error estimation. In practice, however, there is a small probability that the standard error might be overestimated using the sandwich form when the cross-validation choose a small threshold constant, \(M\).

Based on our Monte Carlo simulation study, we find that the standard errors with the sandwich form and the non-sandwich form are quite similar when OLS and FGLS estimators are close to each other. In addition, both standard errors are sometimes underestimated, especially when \(T\) is relatively smaller than \(N\). Hence, the t-statistics of FGLS tend to over-reject. To control this issue, we recommend using \(\hat{\Sigma}^* = I_{NT} \times \text{diag}(\hat{\Sigma})\), which is the diagonal matrix with the diagonal elements of \(\hat{\Sigma}\), instead of using \(\hat{\Sigma}\) in the variance matrix estimator (2.9) of FGLS. This indeed gives larger standard errors, so that one would have a more conservative confidence interval. Also it might be regarded as the counterpart of the White-HAC estimator in OLS.
3 Monte Carlo evidence

3.1 DGP and methods

In this section we compare the proposed FGLS estimator with OLS estimator. We consider the simple linear regression model

$$y_{it} = \beta_0 x_{it} + u_{it},$$

where the true $\beta_0 = 1$. The data generating process (DGP) allows for serial and cross-sectional correlation in both $x_{it}$ and $u_{it}$, which are generated by $(NT) \times (NT)$ covariance matrices, $\Omega_X$ and $\Omega_U$, as follows: let $R_\eta = (R_{\eta,ij})$ denote an $N \times N$ block diagonal correlation matrix. We fix the number of clusters as $G = 25$. Hence, each diagonal block is a $N/G \times N/G$ matrix with the off-diagonal entries $(i,j)$ in the same cluster, $R_{\eta,ij}$ for $i \neq j$, which are generated from i.i.d. Uniform$(0, \gamma)$. In this study, we set the level of cross-sectional correlation in each cluster as $\gamma = 0.3$ or $0.7$. For the cross-sectional heteroskedasticity, let $D = \text{diag}\{d_i\}$, where $\{d_i\}_{i \leq N}$ are i.i.d. Uniform$(1, m)$. Finally, we define the $N \times N$ covariance matrix of $u_t$ as $\Sigma_u = DR_\eta D$. In this case, we report results when $m = \sqrt{5}$. For the covariance matrix of the regressor, we simply set $\Sigma_x = R_\eta$, which does not have heteroskedasticity.

Now we introduce $i$-dependent serial correlation for the regressor and the error as follows: first let $\sigma_{ii} = \rho_i$ if $i = j$ and $\sigma_{ij} = \rho_i \rho_j$ if $i \neq j$. Then we define the $(NT) \times (NT)$ covariance matrix, $\Omega_U = (\Omega_{t,s})$. The $(t,s)$th block is an $N \times N$ covariance matrix, given by $\Omega_{t,s} = (\Omega_{t,s}(i,j))$, where $\Omega_{t,s}(i,j) = \Sigma_{t,s}^{[t-s]}$. The large covariance matrix of the regressor, $\Omega_X$, is generated similarly. The level of $i$-dependent $\rho_i$ of the regressor and the error is generated from i.i.d. Uniform$(0, 0.6)$, separately.

Note that the $(t,s)$th block covariance decays exponentially as $|t-s|$ increases. Finally we generate the $NT \times 1$ vectors $(u'_1, \ldots, u'_T)' = \Omega_U^{1/2} \zeta$, where $\zeta$ is an $NT \times 1$ vector, whose entries are generated from i.i.d. $\mathcal{N}(0, 5)$. Similarly, the regressor is generated by $(x'_1, \ldots, x'_T)' = \Omega_X^{1/2} \xi$, where $\xi$ is an $NT \times 1$ vector, whose entries are generated from i.i.d. $\mathcal{N}(0, 1)$. Note that $x_{it}$ is uncorrelated with $u_{it}$.

In this simulation, we use sample sizes $N = 50, 100$ and $T = 30, 60, 100$. For each $\{N,T\}$ combination, we set the bandwidth $L = 3$ in all cases. The threshold constant, $M$, is obtained by the cross-validation method as explained in Section 2.2. In general, the cross-validation chooses $M$ between 1.4 and 1.8. Interestingly, as the level of cross-section correlation increases, the cross-validation tends to choose smaller $M$, so that the number of non-thresholded elements increases. Hence it takes into account the strength of cross-sectional correlation. We use the Bartlett kernel for our FGLS estimator. Results are summarized in Tables 1-2.
3.2 Results

Tables 1-2 present the simulation results, where each table corresponds to a different level of cross-sectional correlation, $\gamma = \{0.3, 0.7\}$. In each table, the mean and standard deviation of the estimators are reported. FGLS(Diag) refers to the FGLS estimator using the diagonal covariance matrix, which only takes into account heteroskedasticity. RMSE is the ratio of the mean squared error of FGLS to that of OLS. The mean and standard deviation of the estimated standard errors for OLS and FGLS are also reported. The robust unknown clustered standard error, suggested by Bai, Choi, and Liao (2019), is used for OLS. For FGLS, we use the standard error of the sandwich form as discussed in Section 2.5. The difference between the standard deviation of the estimators and the mean of standard error can be explained as the bias of the estimated standard error. In addition, we present null rejection probabilities for the 5% level tests using the traditional $N(0, 1)$ critical value based on each standard errors.

According to Tables 1-2, we see that both methods are almost unbiased, while our proposed FGLS has indeed smaller standard deviation of $\hat{\beta}$ than that of OLS and FGLS(Diag). In all cases, the RMSE of our proposed method is significantly smaller than one. Hence the results confirm that the FGLS estimator is more efficient than the OLS and the FGLS(Diag) estimators in presence of heteroskedasticity, serial and cross-sectional correlations. Regarding the $t$-test, in Table 1, the rejection probabilities of FGLS and OLS are close to 0.05 when $T$ is large, while those of FGLS(Diag) tend to over-reject. Since the FGLS(Diag) estimator does not take into account the serial and the cross-sectional correlations, its standard errors are underestimated. On the other hand, in Table 2, we find that the standard errors of all estimators are underestimated and the $t$-test rejection probabilities are much larger than 0.05, especially when $N = 100$. This is due to the strong cross-sectional correlation within clusters. Also $T$ is relatively smaller than $N$. However, the rejection probabilities of FGLS and OLS are much smaller than those of FGLS(Diag). In addition, they become close to 0.05 as $T$ increases, which corresponds to our sufficient condition for Assumption 2.4, which is $NL^2 = o(T)$. In summary, FGLS does improve efficiency in terms of mean squared error; also we obtain unbiased standard error estimator and appropriate rejection rate when $T$ is large.

4 Empirical study: Effects of divorce law reforms on divorce rates

In the literature, the cause of the sharp increase in the U.S. divorce rate in the 1960-1970s is an important research question. During 1970s, more than half of states in the U.S. liberalized the divorce system, and the effects of reforms on divorce rates have been investigated by
many such as Allen (1992) and Peters (1986). With controls for state and year fixed effects, Friedberg (1998) suggested that state law reforms significantly increased divorce rates. Also, she assumed that unilateral divorce laws affected divorce rates permanently. However, divorce rates from 1975 have been subsequently decreasing according to empirical evidence. Therefore the question of whether law reforms also affect the divorce rate decrease has arisen. Wolfers (2006) revisited this question by using a treatment effect panel data model, and identified only temporal effects of reforms on divorce rates. In particular, he used dummy variables for the first two years after the reforms, 3-4 years, 5-6 years, and so on. More specifically, the following fixed effect panel data model was considered:

$$y_{it} = \alpha_i + \mu_t + \sum_{k=1}^{8} \beta_k X_{it,k} + \delta_i t + u_{it},$$  \hspace{1cm} (4.1)

where $y_{it}$ is the divorce rate for state $i$ and year $t$, $\alpha_i$ a state fixed effect, $\mu_t$ a time fixed effects, and $\delta_i t$ a linear time trend with unknown coefficient $\delta_i$. $X_{it,k}$ is a binary regressor which denotes the treatment effect $2k$ years after the reform. Wolfers (2006) suggested that “the divorce rate rose sharply following the adoption of unilateral divorce laws, but this rise was reversed within about a decade”. He also concluded that “15 years after reform the divorce rate is lower as a result of the adoption of unilateral divorce, although it is hard to draw any strong conclusions about long-run effects.

Both Friedberg (1998) and Wolfers (2006) used a weighted model by multiplying all variables by the square root of state population. In addition, they used ordinary OLS standard error, which does not take into account heteroskedasticity, serial and cross-sectional correlations. However, standard errors might be biased when one disregards these correlations. Therefore, we re-estimated the model of Wolfers (2006) using the OLS with robust standard error, proposed by Bai, Choi, and Liao (2019), and our suggested FGLS estimators.

The same dataset as in Wolfers (2006) is used, which includes the divorce rate, state-level reform years, binary regressors, and state population. Due to missing observations around divorce law reforms, we exclude Indiana, New Mexico and Louisiana. As a result, we obtain balanced panel data from 1956 to 1988 for 48 states. We fit the models both with and without linear time trend, and use OLS and FGLS in each model to estimate $\beta$. The FGLS standard errors with non-sandwich form are reported. In the FGLS estimation, we set bandwidth $L = 3$ as proposed by Newey and West (1994) ($L = 4(T/100)^{2/9}$). The thresholding values are chosen by the cross-validation method as discussed in Section 2.2 more specifically, $M = 1$ for the model with linear time trends, and $M = 1.2$ for the model without linear time trends. The Bartlett kernel is used in the OLS robust standard error and FGLS estimation. The estimated $\beta_1, \cdots, \beta_8$ with and without linear time trend and standard errors are summarized in Table 3 below.
The OLS and FGLS estimates in both models are similar to each other. The results show that divorce rates rose soon after the law reform. However, within a decade, divorce rates had fallen over time. Interestingly, FGLS confirms the negative effects of the law reforms on the divorce rates, specifically, 13-15+ years after the reform in the model with state-specific linear time trends, and 11-15+ years after the reform in the model without state-specific linear time trends. In addition, the FGLS estimates for 1-4 and 1-6 years are positive and statistically significant in the models with and without linear time trends, respectively. For OLS, the coefficient estimates for 3-4 and 9-15+ are significant in the model without linear time trends. In contrast, the OLS estimates are statistically significant only for 1-4 years when a linear time trend is added, which is a more conservative conclusion.

Based on OLS and FGLS estimation results with and without a linear time trend, we make the following conclusion: in the first 8 years, the overall trend of divorce rate is increasing, but the law reform reduces the divorce rate after 3-4 years. However, 8 years after the reform, we observe that the law reform has a negative effect on divorce rate. These estimates are closely comparable to the conclusion of Wolfers (2006).

5 Conclusions

In this paper, we propose a large covariance matrix estimator and a modified version of FGLS that takes into account both serial and cross-sectional correlations in linear panel models that are robust to heteroskedasticity, serial and cross-sectional correlations. The covariance matrix estimator is asymptotically unbiased with an improved convergence rate. It is shown to be more efficient than other existing methods in panel data literature. From simulated experiments, we confirmed that our FGLS estimates are more efficient than OLS estimates.
Table 1: Performance of estimated \( \beta_0 \); true \( \beta_0 = 1 \); both serial and weak cross-sectional correlations (\( \gamma = 0.3 \)). The threshold value, \( M \), is chosen through the K-fold cross-validation method. For the bandwidth, we set \( L = 3 \).

| N   | T   | OLS   | FGLS   | OLS   | FGLS   | OLS   | FGLS   |
|-----|-----|-------|--------|-------|--------|-------|--------|
|     |     | Diag  | Our    | Diag  | Our    | Diag  | Our    |
|     |     | mean(\( \hat{\beta} \)) | std(\( \hat{\beta} \)) | RMSE |
| 50  | 30  | 0.995 | 0.994  | 0.994 | 0.114  | 0.101 | 0.092  | 1.000 | 0.784 | 0.649 |
| 60  |     | 0.995 | 0.996  | 0.996 | 0.076  | 0.070 | 0.062  | 1.000 | 0.860 | 0.677 |
| 100 |     | 1.000 | 1.000  | 1.000 | 0.058  | 0.053 | 0.048  | 1.000 | 0.815 | 0.677 |
| 100 | 30  | 1.001 | 1.000  | 0.999 | 0.074  | 0.067 | 0.064  | 1.000 | 0.826 | 0.754 |
|     | 60  | 0.999 | 0.999  | 0.999 | 0.054  | 0.049 | 0.045  | 1.000 | 0.847 | 0.692 |
|     | 100 | 0.998 | 0.998  | 0.998 | 0.042  | 0.037 | 0.034  | 1.000 | 0.780 | 0.653 |
|     |     | mean(s.e.) | std(s.e.) | t-test rejection prob. |
| 50  | 30  | 0.106 | 0.086  | 0.084 | 0.007  | 0.003 | 0.003  | 0.070 | 0.098 | 0.081 |
| 60  |     | 0.073 | 0.061  | 0.060 | 0.003  | 0.002 | 0.001  | 0.061 | 0.090 | 0.060 |
| 100 |     | 0.058 | 0.050  | 0.049 | 0.002  | 0.001 | 0.001  | 0.052 | 0.063 | 0.044 |
| 100 | 30  | 0.071 | 0.058  | 0.057 | 0.003  | 0.001 | 0.001  | 0.066 | 0.096 | 0.079 |
|     | 60  | 0.051 | 0.041  | 0.041 | 0.002  | 0.001 | 0.001  | 0.060 | 0.110 | 0.075 |
|     | 100 | 0.040 | 0.033  | 0.032 | 0.001  | 0.000 | 0.000  | 0.055 | 0.084 | 0.054 |

**Note:** OLS and FGLS comparison. RMSE is the ratio of the mean squared error of FGLS to that of OLS. The t-test rejection prob. is t-test rejection rates for 5% level tests. Robust standard error suggested by Bai, Choi, and Liao (2019) is used for OLS. Reported results are based on 1000 replications.
Table 2: Performance of estimated $\beta_0$; true $\beta_0 = 1$; both serial and strong cross-sectional correlations ($\gamma = 0.7$). The threshold value, $M$, is chosen through the K-fold cross-validation method. For the bandwidth, we set $L = 3$.

|     | OLS Diag | OLS Our | FGLS Diag | FGLS Our | OLS Diag | OLS Our | FGLS Diag | FGLS Our |
|-----|---------|---------|-----------|----------|---------|---------|-----------|----------|
| N   |         |         |           |          |         |         |           |          |
|     | mean(\hat{\beta}) | std(\hat{\beta}) | RMSE     |          |         |         | mean(s.e.) | std(s.e.) |          |
| 50  | 1.000   | 1.001   | 0.108     | 0.098    | 0.092   | 1.000   | 0.832     | 0.723    |          |
| 60  | 1.000   | 1.000   | 0.079     | 0.072    | 0.067   | 1.000   | 0.850     | 0.726    |          |
| 100 | 1.000   | 1.004   | 0.060     | 0.053    | 0.047   | 1.000   | 0.800     | 0.633    |          |
| 100 | 1.003   | 1.003   | 0.091     | 0.082    | 0.078   | 1.000   | 0.799     | 0.732    |          |
| 100 | 1.000   | 0.999   | 0.062     | 0.058    | 0.052   | 1.000   | 0.870     | 0.716    |          |
| 100 | 1.002   | 1.002   | 0.047     | 0.043    | 0.039   | 1.000   | 0.800     | 0.658    |          |

|     |         |         |           |          |         |         | t-test rejection prob. |          |
| 50  | 0.101   | 0.082   | 0.007     | 0.003    | 0.003   | 0.073   | 0.099     | 0.083    |          |
| 60  | 0.074   | 0.062   | 0.003     | 0.002    | 0.002   | 0.055   | 0.087     | 0.066    |          |
| 100 | 0.056   | 0.046   | 0.002     | 0.001    | 0.001   | 0.062   | 0.100     | 0.055    |          |
| 100 | 0.074   | 0.060   | 0.003     | 0.002    | 0.002   | 0.114   | 0.141     | 0.125    |          |
| 100 | 0.053   | 0.045   | 0.002     | 0.001    | 0.001   | 0.095   | 0.127     | 0.082    |          |
| 100 | 0.043   | 0.034   | 0.001     | 0.000    | 0.001   | 0.060   | 0.117     | 0.064    |          |

**Note:** OLS and FGLS comparison. RMSE is the ratio of the mean squared error of FGLS to that of OLS. The $t$-test rejection prob. is $t$-test rejection rates for 5% level tests. Robust standard error suggested by *Bai, Choi, and Liao (2019)* is used for OLS. Reported results are based on 1000 replications.
Table 3: Empirical application: effects of divorce law reform with state and year fixed effects: US state level data annual from 1956 to 1988, dependent variable is divorce rate per 1000 persons per year. OLS and FGLS estimates and standard errors (using state population weights).

| Effects: | $\hat{\beta}_{OLS}$ | $se_{OLS}$ | $\hat{\beta}_{FGLS}$ | $se_{FGLS}$ |
|----------|----------------------|-------------|----------------------|-------------|
| Panel A: Without state-specific linear time trends | | | | |
| 1-2 years | 0.256 | 0.148 | 0.155 | 0.065* |
| 3-4 years | 0.209 | 0.089* | 0.248 | 0.052* |
| 5-6 years | 0.126 | 0.069 | 0.279 | 0.055* |
| 7-8 years | 0.105 | 0.040* | 0.112 | 0.059 |
| 9-10 years | -0.122 | 0.054* | -0.037 | 0.060 |
| 11-12 years | -0.344 | 0.075* | -0.302 | 0.065* |
| 13-14 years | -0.496 | 0.062* | -0.401 | 0.059* |
| 15+ years | -0.508 | 0.077* | -0.370 | 0.064* |
| Panel B: With state-specific linear time trends | | | | |
| 1-2 years | 0.286 | 0.140* | 0.403 | 0.042* |
| 3-4 years | 0.254 | 0.126* | 0.373 | 0.055* |
| 5-6 years | 0.186 | 0.143 | 0.010 | 0.058 |
| 7-8 years | 0.177 | 0.146 | 0.063 | 0.067 |
| 9-10 years | -0.037 | 0.154 | -0.047 | 0.072 |
| 11-12 years | -0.247 | 0.183 | -0.088 | 0.079 |
| 13-14 years | -0.386 | 0.209 | -0.394 | 0.083* |
| 15+ years | -0.414 | 0.243 | -0.486 | 0.095* |

Note: Standard errors with asterisks indicate significance at 5% level using $N(0,1)$ critical values.
A Appendix

Throughout the proof, $\max_i$, $\max_t$, $\max_h$, $\max_{ij}$, and $\max_{it}$ denote $\max_{i \leq N}$, $\max_{t \leq T}$, $\max_{h \leq L}$, $\max_{i \leq N, j \leq N}$, and $\max_{i \leq N, t \leq T}$ respectively.

A.1 Proofs of Theorem 2.1-2.2.

Lemma A.1. If Assumption 2.1 (i) holds, then

$$\max_{h \leq L} \max_{i,j \leq N} \| \frac{1}{T} \sum_{t=1}^{T-h} x'_{i,t+h} u_{jt} \| = O_P(\sqrt{\frac{\log(LN)}{T}}).$$

Proof. Let $\gamma_{ij,t,h} = x_{i,t+h} u_{jt} 1_{t \leq T-h}$. Note that

$$\max_{ij} \max_h \frac{1}{T} \sum_{t=1}^{T} \operatorname{Var}(\gamma_{ij,t,h}) = \max_{ij} \max_h \frac{1}{T} \sum_{t=1}^{T-h} \operatorname{Var}(x'_{it} u_{jt+h})$$

$$\leq \max_h \frac{1}{T} \sum_{t=1}^{T-h} C = O(1).$$

Hence, by using Bernstein inequality, we assume the following rate to allow the time dependence. Then $\max_{ij} \max_h \| \frac{1}{T} \sum_{t=1}^{T-h} x'_{it} u_{jt+h} \| = O_P(\sqrt{\frac{\log(LN^2) \max_{ij,h} \sum_{t=1}^{T} \operatorname{Var}(\gamma_{ij,t,h})}{T}}) = O_P(\sqrt{\frac{\log(LN^2)}{T}}) = O_P(\sqrt{\frac{\log(LN)}{T}}).$ \hfill \Box

Lemma A.2. If assumption of Lemma A.1 holds, then

$$\max_{h \leq L} \max_{i,j \leq N} | \tilde{\sigma}_{h,ij} - E u_{it} u_{jt+h} | = O_P(\sqrt{\frac{\log(LN)}{T}}),$$

which is a thresholding rate.
Proof.

\[
\max_h \max_{ij} |\tilde{\sigma}_{h,ij} - E u_{it} u_{j,t+h}| \leq \max_h \max_{ij} \frac{1}{T} \sum_{t=1}^{T-h} \hat{u}_{it} \hat{u}_{j,t+h} - u_{it} u_{j,t+h} \\
+ \max_h \max_{ij} \frac{1}{T} \sum_{t=1}^{T-h} u_{it} u_{j,t+h} - E u_{it} u_{j,t+h} \\
+ \max_h \max_{ij} \frac{L}{T} |E u_{it} u_{j,t+h}| \\
:= a_1 + a_2 + a_3.
\]

\(a_1\) is bounded by \(a_{11} + a_{12} + a_{13}\), and it is \(O_P(\frac{1}{T\sqrt{\log(\frac{L}{N})}})\).

\[
a_{11} := \max_h \max_{ij} \frac{1}{T} \sum_{t=1}^{T-h} (\hat{u}_{it} - u_{it})(\hat{u}_{j,t+h} - u_{j,t+h}) \\
a_{12} := \max_h \max_{ij} \frac{1}{T} \sum_{t=1}^{T-h} (\hat{u}_{it} - u_{it}) u_{j,t+h} \\
a_{13} := \max_h \max_{ij} \frac{1}{T} \sum_{t=1}^{T-h} u_{it}(\hat{u}_{j,t+h} - u_{j,t+h})
\]

Here,

\[
a_{11} \leq \max_h \max_{ij} \frac{1}{T} \sum_{t=1}^{T-h} \|x_{it}\|\|x_{j,t+h}\|\|\hat{\beta} - \beta\|^2 \\
\leq O_P(\frac{1}{NT}) \max_i \frac{1}{T} \sum_{t=1}^T \|x_{it}\|^2 = O_P(\frac{1}{NT}),
\]

and

\[
a_{12} \leq \max_h \max_{ij} \frac{1}{T} \sum_{t=1}^{T-h} x_{it}' u_{j,t+h}\|\|\beta - \hat{\beta}\| \\
\leq O_P(\frac{1}{\sqrt{NT}}) \max_h \max_{ij} \frac{1}{T} \sum_{t=1}^{T-h} x_{it}' u_{j,t+h} \|\|\beta - \hat{\beta}\| \\
= O_P(\frac{1}{T\sqrt{\log(\frac{L}{N})}}).
\]

Similarly, \(a_{13}\) is bounded using same argument.

By using Bernstein Inequality, strong-mixing and exponential tail conditions, \(a_2 = O_P(\sqrt{\frac{\log(LN)}{T}})\).

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In addition, \( a_3 = O_P(\frac{1}{T}) \), which can be proved easily. Together,
\[
\max_h \max_{ij} \left| \frac{1}{T} \sum_{t=1}^{T-h} \hat{u}_{it} \hat{u}_{j,t+h} - Eu_{it}u_{j,t+h} \right| = O_P(\sqrt{\frac{\log(LN)}{T}}).
\]

Then we have following lemma.

**Lemma A.3.** If Lemma A.2 holds, then
\[
\max_{t,s \leq T} \| \hat{\Omega}_{t,s} - \Omega_{t,s} \|_1 = \max_{h < L} \| \hat{\Omega}_h - \Omega_h \|_1 = O_P\left( m_q \left( \frac{1}{T} \right)^{(1-q)/2} \right),
\]
where \( m_q = \max_j (Eu_{it}u_{j,t+h})^q \).

**Proof.** It can be shown by sparsity property. \( \square \)

**Proof of Theorem 2.1.** For any \( NT \times NT \) blocked matrix \( M = (m_{t,s}) \) where the \( m_{t,s} \) is the \( (t, s) \)th block \( N \times N \) matrix, and any \( 0 \leq L < T \), define, \( B_L(M) = [(m_{ij})_{N \times N}1(|t - s| \leq L)] \).

Then we can write
\[
\| \hat{\Omega} - \Omega \| \leq \| B_L(\Omega) - \Omega \| + \| \hat{\Omega} - B_L(\Omega) \|.
\]

For the first part,
\[
\| B_L(\Omega) - \Omega \|_{sp} \leq \max_t \sum_{s:|s-t| > L} \| Eu_{t}u'_{s} \|_{sp} = \max_t \sum_{|h| > L} \| Eu_{t}u'_{t+h} \|_{sp} := \gamma_T(L).
\]

Next
\[
\| \hat{\Omega} - B_L(\Omega) \|_{sp} \leq \max_{|h| < L} \| (\hat{\sigma}_{h,ij})_{N \times N} \omega(h, L) - Eu_{t}u'_{s} \|
\leq \max_{|h| < L} \| (\hat{\sigma}_{h,ij})_{N \times N} - Eu_{t}u'_{s} \| \omega(h, L)\| + L \max_{|h| < L} \| Eu_{t}u'_{s}(1 - \omega(h, L))\|
:= (i) + (ii),
\]
where \( \hat{\sigma}_{h,ij} = \frac{1}{T} \sum_{t=1}^{T-h} \hat{u}_{it} \hat{u}_{j,t+h} \) and \( h = |t - s| \), which is an estimator for \( Eu_{it}u_{j,s} \).
From the result of Lemma A.3, we have
\[(i) \leq C L \| \hat{\Omega}_{t,s} - \Omega_{t,s} \| = O_P(L m_q \gamma_{NT}^{1-q}),\]
where \(m_q = \max_j \sum_i (E u_i u_{j,t+h})^q\).
Therefore, under the Assumption 3.1(ii), \((ii) = O_P(f_T(L)),\)
\[\| \hat{\Omega} - \Omega \| = O_P(L m_q \gamma_{NT}^{1-q} + f_T(L) + \gamma_T(L)).\]

We now show the second statement of Theorem 2.1. We have
\[
\| \hat{\Omega} - \Omega^{-1} \| \leq \| (\hat{\Omega}^{-1} - \Omega^{-1})(\hat{\Omega} - \Omega)\Omega^{-1} \| + \| \Omega^{-1}(\hat{\Omega} - \Omega)\Omega^{-1} \|
\leq \| \hat{\Omega}^{-1} - \Omega^{-1} \| \| \Omega^{-1}(\hat{\Omega} - \Omega)\Omega^{-1} \| + \| \Omega^{-1} \| \| \hat{\Omega} - \Omega \|
= O_P(L m_q \gamma_{NT}^{1-q} + f_T(L) + \gamma_T(L))\| \hat{\Omega}^{-1} - \Omega^{-1} \| + O_P(L m_q \gamma_{NT}^{1-q} + f_T(L) + \gamma_T(L)).
\]
Hence we have \((1 + o_P(1))\| \hat{\Omega}^{-1} - \Omega^{-1} \| = O_P(L m_q \gamma_{NT}^{1-q} + f_T(L) + \gamma_T(L)),\) that implies the result. \(\boxdot\)

**Proof of Proposition 2.1.** First the left hand side of equation (2.7) can be extended as
\[
\frac{1}{\sqrt{NT}} X'(\hat{\Omega}^{-1} - \Omega^{-1})U = \frac{1}{\sqrt{NT}} X'\Omega^{-1}(\hat{\Omega} - \Omega)\Omega^{-1}U
+ \frac{1}{\sqrt{NT}} X'\Omega^{-1}(\hat{\Omega} - \Omega)\Omega^{-1}(\hat{\Omega} - \Omega)\Omega^{-1}U
+ \frac{1}{\sqrt{NT}} X'(\hat{\Omega}^{-1} - \Omega^{-1})(\hat{\Omega} - \Omega)\Omega^{-1}(\hat{\Omega} - \Omega)\Omega^{-1}U
:= a + b + c.
\]
Now we shall show that \(\frac{1}{\sqrt{NT}} X'(\hat{\Omega}^{-1} - \Omega^{-1})U = \frac{1}{\sqrt{NT}} X'\Omega^{-1}(\hat{\Omega} - \Omega)\Omega^{-1}U + o_P(1).\) Under
Assumptions 2.2–2.3 we can show that $b = o_P(1)$ as follows:

$$b = \frac{1}{\sqrt{NT}} \sum_{t=1}^{T} \sum_{h<L} \sum_{v<L} \sum_{m=1}^{T} w'_t(\hat{\Omega} - \Omega)_{h,t} (\Omega^{-1})_{t+h,m+v} (\Omega - \Omega)_{v} \varepsilon_m$$

$$- \frac{1}{\sqrt{NT}} \sum_{t=1}^{T} \sum_{h<L} \sum_{v>L} \sum_{m=1}^{T} w'_t(\hat{\Omega} - \Omega)_{h,t} (\Omega^{-1})_{t+h,m+v} \Omega_v \varepsilon_m$$

$$- \frac{1}{\sqrt{NT}} \sum_{t=1}^{T} \sum_{h>L} \sum_{v>L} \sum_{m=1}^{T} w'_t(\Omega^{-1})_{t+h,m+v} (\hat{\Omega} - \Omega)_{v} \varepsilon_m$$

$$:= b_1 + b_2 + b_3.$$

We have

$$b_1 = \frac{1}{\sqrt{NT}} \sum_{h<L} \sum_{v<L} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{p=1}^{N} \sum_{q=1}^{N} (\hat{\Omega} - \Omega)_{h,i} (\hat{\Omega} - \Omega)_{v,p} \sum_{t=1}^{T} \sum_{m=1}^{T} w_{ti} \varepsilon_{mq} (\Omega^{-1})_{t+h,m+v,jp}$$

$$\leq \frac{1}{\sqrt{NT}} \max_h \max_j \sum_{i=1}^{N} (\hat{\Omega} - \Omega)_{h,i} \max_v \sum_{p=1}^{N} (\hat{\Omega} - \Omega)_{v,p}$$

$$\times \max_i \max_p \left| \sum_{h=1}^{L} \sum_{v=1}^{L} \sum_{j=1}^{N} \sum_{q=1}^{N} \sum_{t=1}^{T} \sum_{m=1}^{T} w_{hi} \varepsilon_{mq} (\Omega^{-1})_{t+h,m+v,jp} \right|$$

$$\leq \frac{1}{\sqrt{NT}} \max_h \|\hat{\Omega} - \Omega\|_2^2 \max_{i,p} \left| \sum_{h=1}^{L} \sum_{v=1}^{L} \sum_{j=1}^{N} \sum_{q=1}^{N} \sum_{t=1}^{T} \sum_{m=1}^{T} \varepsilon_{mq} Q_{hv}^{h_v} \right|$$

$$\leq \frac{1}{\sqrt{NT}} \max_h \|\hat{\Omega} - \Omega\|_2^2 L^2 NT \max_{i,p,h,v} \left| \frac{1}{NT} \sum_{q=1}^{N} \sum_{m=1}^{T} \varepsilon_{mq} Q_{hv}^{h_v} \right|$$

$$= O_P(L^2 m_\eta^2 (\log(LN))^{1.5-q} T^{-1}) = o_P(1).$$
and

\[
b_2 = \frac{1}{\sqrt{NT}} \sum_{t=1}^{T} \sum_{h=1}^{L} \sum_{v>h}^{T} \sum_{m=1}^{L} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{p=1}^{N} \sum_{q=1}^{N} w_{ti}(\hat{\Omega} - \Omega)_{h,ij}(\Omega^{-1})_{t+h,m+v,jp}(\Omega)_{v,pq} \varepsilon_{mq}
\]

\[
\leq \frac{L}{\sqrt{NT}} \max_{h} \max_{j} \sum_{i=1}^{N} |\hat{\Omega} - \Omega|_{h,ij} \left[ \sum_{p=1}^{N} \max_{q=1}^{\sum \varepsilon_{pq}} |(\Omega)_{v,pq}| \right]
\]

\[
\times \max_{i} \sum_{t=1}^{T} |w_{ti}| \max_{m} \sum_{v>L}^{N} \sum_{p=1}^{N} \max_{j} \sum_{h>L}^{N} \sum_{i=1}^{T} \sum_{p=1}^{N} \sum_{q=1}^{N} |(\Omega^{-1})_{t+h,m+v,jp}|
\]

\[
\leq \frac{L}{\sqrt{NT}} \max_{h} \|\hat{\Omega}_h - \Omega_h\|_v \sum_{v>L}^{T} \|\Omega\|_v \cdot T \cdot N \cdot O(1)
\]

\[
= O_P(L^{1-a} \sqrt{NT} m_{q}(\log(LN)\sqrt{T})^{(1-q)/2}) = o_P(1).
\]

In addition,

\[
b_3 = \frac{1}{\sqrt{NT}} \sum_{t=1}^{T} \sum_{m=1}^{T} \sum_{h=L}^{T} \sum_{k=1}^{T} \sum_{j=1}^{N} \sum_{p=1}^{N} \sum_{q=1}^{N} w_{ti}(\hat{\Omega})_{h,ij}(\Omega^{-1})_{t+h,m+v,jp}(\hat{\Omega} - \Omega)_{km,pq} \varepsilon_{mq}
\]

\[
\leq \frac{T}{\sqrt{NT}} \max_{h} \max_{j} \sum_{i=1}^{N} |\hat{\Omega} - \Omega|_{h,ij} \left[ \sum_{p=1}^{N} \max_{q=1}^{\sum \varepsilon_{pq}} |(\Omega)_{h,ij}| \right]
\]

\[
\times \max_{i} \sum_{t=1}^{T} |w_{ti}| \max_{m} \sum_{v>L}^{N} \sum_{p=1}^{N} \max_{j} \sum_{h>L}^{N} \sum_{i=1}^{T} \sum_{p=1}^{N} \sum_{q=1}^{N} |(\Omega^{-1})_{t+h,m+v,jp}|
\]

\[
\leq \frac{T}{\sqrt{NT}} \max_{h} \|\hat{\Omega}_h - \Omega_h\|_v \sum_{h>L}^{T} \|\Omega\|_h \cdot T \cdot N \cdot O(1)
\]

\[
= O_P(L^{-\alpha} \sqrt{NT} m_{q}(\log(LN)\sqrt{T})^{(1-q)/2}) = o_P(1).
\]

Under the Assumption 2.3 (v), we have

\[
c = \frac{1}{\sqrt{NT}} X'(\hat{\Omega}^{-1} - \Omega^{-1})(\hat{\Omega} - \Omega)\Omega^{-1}(\hat{\Omega} - \Omega)\Omega^{-1} U
\]

\[
\leq \frac{1}{\sqrt{NT}} \sqrt{NT} \gamma^3 \sqrt{NT} = O_P(\sqrt{NT} \gamma^3).
\]

Therefore, we have

\[
\frac{1}{\sqrt{NT}} X'(\hat{\Omega}^{-1} - \Omega^{-1}) U = \frac{1}{\sqrt{NT}} X' \Omega^{-1}(\hat{\Omega} - \Omega)\Omega^{-1} U + o_P(1).
\]

From Theorem 2.1, it is easy to that \( \frac{1}{\sqrt{NT}} X' \hat{\Omega}^{-1} X = \frac{1}{\sqrt{NT}} X' \Omega^{-1} X + o_P(1) \). Also, \( \frac{1}{\sqrt{NT}} X' \Omega^{-1} X \) =
\[ \Gamma^{-1} + o_P(1), \text{ where } \Gamma = E(\frac{1}{\sqrt{NT}} X' \Omega^{-1} X). \text{ Then} \]

\[
\sqrt{NT}(\hat{\beta}_{FGLS} - \beta) = \left(\frac{1}{\sqrt{NT}} X' \Omega^{-1} X\right)^{-1} \left(\frac{1}{\sqrt{NT}} X' \hat{\Omega}^{-1} U\right) + o_P(1)
\]

\[
= \left(\frac{1}{\sqrt{NT}} X' \Omega^{-1} X\right)^{-1} \left(\frac{1}{\sqrt{NT}} X' \Omega^{-1} U + \frac{1}{\sqrt{NT}} X' (\hat{\Omega}^{-1} - \Omega^{-1}) U\right) + o_P(1)
\]

\[
= \Gamma^{-1} \left(\frac{1}{\sqrt{NT}} X' \Omega^{-1} U + \frac{1}{\sqrt{NT}} X' (\hat{\Omega}^{-1} - \Omega^{-1}) U\right) + o_P(1)
\]

\[
= \Gamma^{-1} \left(\frac{1}{\sqrt{NT}} X' \Omega^{-1} U\right) + \Gamma^{-1} \left(\frac{1}{\sqrt{NT}} X' (\hat{\Omega}^{-1} - \Omega^{-1}) U\right) + o_P(1). \Box
\]

**Proof of Theorem 2.2.** It suffices to prove \( \frac{1}{\sqrt{NT}} X' \Omega^{-1} (\hat{\Omega} - \Omega)^{-1} U = o_P(1) \).

Let \( W = \Omega^{-1} X, \text{ and } \varepsilon = \Omega^{-1} U. \text{ Then under the Assumption 2.4,} \)

\[
\frac{1}{\sqrt{NT}} X' \Omega^{-1} (\hat{\Omega} - \Omega)^{-1} U = \frac{1}{\sqrt{NT}} W' (\hat{\Omega} - \Omega) \varepsilon
\]

\[
\approx \frac{1}{\sqrt{NT}} \sum_{h=1}^{L} \sum_{i,j \in A_h} \sum_{t=1}^{T} w_{it} \varepsilon_{j,t+h} \frac{1}{T} \sum_{s=1}^{T-h} (u_{is} u_{j,s+h} - E u_{is} u_{j,t+h})
\]

\[
= \frac{1}{\sqrt{T}} \sum_{h=1}^{L} \frac{1}{\sqrt{N}} \sum_{i,j \in A_h} G_{T,ij}^1(h) G_{T,ij}^2(h) = o_P(1).
\]

In addition, if the sufficient conditions as mentioned in Section 2.4 holds, it is indeed

\[
\frac{1}{\sqrt{NT}} X' \Omega^{-1} (\hat{\Omega} - \Omega)^{-1} U \approx \frac{1}{\sqrt{T}} \sum_{h=1}^{L} \frac{1}{\sqrt{N}} \sum_{i,j \in A_h} G_{T,ij}^1(h) G_{T,ij}^2(h)
\]

\[
\leq \frac{L \sqrt{N}}{\sqrt{T}} \sum_{h=1}^{L} \frac{1}{N} \sum_{i,j \in A_h} G_{T,ij}^1(h)^2 \sqrt{\frac{1}{N} \sum_{i,j \in A_h} G_{T,ij}^2(h)^2}
\]

\[
\leq \frac{L \sqrt{N}}{\sqrt{T}} \left( \frac{1}{LN} \sum_{h=1}^{L} \sum_{i,j \in A_h} G_{T,ij}^1(h)^2 \right) \left( \frac{1}{LN} \sum_{h=1}^{L} \sum_{i,j \in A_h} G_{T,ij}^2(h)^2 \right)
\]

\[
\leq O_P\left(\frac{L \sqrt{N}}{\sqrt{T}}\right) = o_P(1).
\]
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