Some Issues in the Loop Variable Approach to Open Strings and an Extension to Closed Strings

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Abstract

Some issues in the loop variable renormalization group approach to gauge invariant equations for the free fields of the open string are discussed. It had been shown in an earlier paper that this leads to a simple form of the gauge transformation law. We discuss in some detail some of the curious features encountered there. The theory looks a little like a massless theory in one higher dimension that can be dimensionally reduced to give a massive theory. We discuss the origin of some constraints that are needed for gauge invariance and also for reducing the set of fields to that of standard string theory. The mechanism of gauge invariance and the connection with the Virasoro algebra is a little different from the usual story and is discussed. It is also shown that these results can be extended in a straightforward manner to closed strings.

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1 Introduction

One of the most important issues in string theory that needs elucidation is that of space time symmetries of the theory. Closed string theory, in particular, contains general coordinate invariance in its low energy limit and we do not know what the appropriate string generalization of this is. The BRST formalism\[1, 2, 3\] that has been used successfully to construct a non polynomial closed string field action probably contains the answer to this question. But at the moment it remains hidden deep inside. A formalism where the answer to this question is manifest is a crying need at this point - if only because we are unlikely to be able to do any physically significant computation otherwise. In particular the problem of background dependence of most formalisms becomes an impediment to non perturbative calculations.

In \[4\] we had speculated that some generalization of the renormalization group (in space-time) could be the space time interpretation of the symmetries of string theory. Motivated by this idea, we were able to describe the open string gauge transformation as a local (in \(\sigma\)) scale transformation. Furthermore we were able to derive the gauge invariant equations of the fields of the open string using a generalization of the sigma model \(\beta\) function (2-dimensional renormalization group) methods developed earlier \[3, 6, 7, 8\]. This generalization involves using a nonlocal loop variable defined on the boundary of a circular hole in the world sheet rather than the usual vertex operator located at a puncture. As argued in \[8, 9\] while going off shell in the sigma model approach one should maintain a finite cutoff. If one keeps a finite cutoff one must include all the irrelevant operators which, in string theory, correspond to the vertex operators of the massive modes. Furthermore one would like to do all this in a gauge invariant way. We believe that the loop variable approach \[4\] shows some promise in this regard.

However there are many curious features in that formalism that require further analysis. One is the contrast between the form of the gauge transformation used in \[4\] and that generated by the Virasoro generators (or \(Q_{BRST}\)) in usual formulations of string theory. Some aspects are discussed in \[11\]. A related puzzle was the fact that the gauge transformations in \[4\] were of the standard form that one gets in Kaluza Klein theories: Start with a massless, D+1 dimensional, gauge invariant theory and dimensionally reduce to get a gauge invariant massive D-dimensional theory. In the usual
formulations\cite{1,2}, in order to get the action and transformations to the
standard form one has to perform field redefinitions at each mass level. Fur-
thermore they work only in 26 dimensions (for the bosonic string). The
method of \cite{4} does not require any field redefinitions. Since we are requiring
conformal invariance we are restricted to D=26. However this does not show
up in a direct way as in \cite{1} Somehow it is as if the field redefinitions have been
implicitly performed at the beginning itself. Of course as an algorithm for
deriving gauge invariant equations for massive fields it works in any dimen-
sion. But only in D=26 can one make contact with standard string theory.
In other dimensions it is not the standard string theory. The connection with
the usual formulations of string theory and the question of field redefinitions,
we believe, is the most pressing one although we will not have much to say
about this in the present work.

A related feature is the partially symmetric treatment of the extra ghost
coordinate. Counting arguments \cite{1} have shown that all the extra degrees of
freedom necessary for the string field can be obtained by adding one extra
bosonic oscillator minus its zero mode and first mode. In \cite{4} we added an
extra coordinate , but integrated it out and replaced it with the Liouville
mode. This was motivated by the work of \cite{6}. It turns out that this is
unnecessary and one can treat it just as an extra dimension. Thus we seem to
have a D+1 dimensional theory. However in making contact with the vertex
operators of string theory it becomes necessary to impose some constraints on
the generalized momenta. It turns out that these constraints are essentially
the ones we were forced to impose by hand in \cite{4} in order to have gauge
invariance. These constraints were, in fact, another of the curious features
of that method.

The gauge transformations in \cite{4} had the form of scale transformation .
There was no obvious restriction that it had to be infinitesimal. Yet it was
the lowest order terms that we kept and which were equivalent to the usual
gauge transformations of string theory. The obvious question is whether
the next order terms give anything new. It turns out that the answer is
in the negative. However in the process of understanding this we will also
understand how computationally the gauge invariance actually works and
also the origin of a tracelessness condition on the gauge parameters.

Finally, it is not immediately obvious whether these techniques work
equally well for the closed string. We will show that at least for the low-
est mass level they do work.
In this paper we address some of these issues. In Sec. II we briefly review the results of [4] and describe how one can simplify many features by treating the extra coordinates in a symmetric fashion and by not integrating it out as was done there. In Sec. III we discuss the connection between the gauge transformation there and the action of the Virasoro generators as discussed in [11, 12]. In Sec. IV we discuss the mechanism of gauge invariance and also discuss the issue of higher order terms. In Sec. V we extend the discussion to closed strings.
2 Loop Variables

In this section we briefly review the results of [1] and then give a modified simpler treatment. The basic idea there is to consider a generalization of the vertex operator in the form of a loop variable.

\[ e^{ia \int_c k(t) \partial_z X(z+at) dt + i k_0 X(z)} \] (2.1)

The integral is along a contour 'C' which is taken to be a circle of radius '\( a \)' centered at the point \( z \). As explained in [12] this loop variable is actually more general than just the collection of vertex operators that is obtained when one Taylor expands \( X(z+at) \) in positive powers of 'at'. However we will ignore this for the moment and Taylor expand \( \partial X(z+at) \) and \( k(t) \) in powers of \( t \) as follows:

\[ \partial X(z+at) = \partial X(z) + at \partial^2 X + (at)^2 / 2 \partial^3 X + ... \] (2.2)

\[ k(t) = k_0 + k_1 / t + k_2 / t^2 + ... + k_n / t^n + ... \]

Just as the vertex operator is a puncture on the world sheet, the loop variable is the boundary of a circular hole of radius '\( a \)' in the world sheet, centered at the point '\( z \)'. A point on the loop is \( z' = z + at = z + ae^{i\theta} \). One has to allow for reparametrizations of \( z' \) that leave the shape of the boundary unchanged, i.e., maps that map \( z' \) at one point on the boundary to another point on the boundary [13, 14, 15, 16]. In order to implement this we introduce an einbein \( \alpha(t) \) so that equation (2.1) is modified to : (We have set \( a = 1 \))

\[ e^{i \int_c \alpha(t) k(t) \partial_z X(z+t) dt + i k_0 X} \] (2.3)

and we assume that \( \alpha(t) \) has an expansion

\[ \alpha(t) = \alpha_0 (= 1) + \alpha_1 / t + \alpha_2 / t^2 + ... + \alpha_n / t^n + .. \] (2.4)

Substituting the power series expansions (2.2) and (2.4) into (2.3) we get

\[ e^{i(k_0 Y + k_1 Y_1 + k_2 Y_2 + k_3 Y_3 + ...)} \] (2.5)

where \( Y, Y_1, Y_2...Y_n \) are defined as follows:

\[ Y = X + \alpha_1 \partial X + \alpha_2 \partial^2 X + \alpha_3 \partial^3 X / 2! + ... \] (2.6)
Define the variables $x_m$ by the relation: $\frac{\partial \alpha}{\partial x_m} = \alpha_{n-m}; \alpha_0 = 1$. Then

$$Y_n = \frac{\partial Y}{\partial x_n} = \partial^n X/(n-1)! + \alpha_1 \partial^{n+1} X/n! + ...$$ (2.7)

The expression (2.5) is the basic object we work with. Vertex operators are obtained by expanding the exponential in (2.5). Thus $e^{ik_0 Y}$ is the tachyon operator, $ik_1 Y_1 e^{ik_0 Y}$ is the spin one vertex operator, $\frac{i k_\mu Y_\mu}{2} e^{ik_0 Y}$ and $-\frac{1}{2} k_1^\mu k_1^\nu Y_1^\mu Y_1^\nu e^{ik_0 Y}$ are the vertex operators at the next mass level. The vertex operators in (2.5) contain an anomalous dependence on the Liouville mode. We can obtain a compact expression for this by defining a new Liouville field:

$$2\sigma_{new} =< YY > =< XX > +2\alpha_1 < \partial XX > + ... \equiv 2\sigma + \alpha_1 \partial \sigma + ...$$ (2.8)

Then using

$$< Y_n Y > = 1/2 \frac{\partial}{\partial x_n} < YY > = \frac{\partial \sigma}{\partial x_n}$$

$$< Y_n Y_m > = 1/2(\frac{\partial^2 \sigma}{\partial x_n \partial x_m} - \frac{\partial \sigma}{\partial x_{n+m}})$$ (2.9)

we get on including the Liouville mode dependence, in (2.5):

$$e^{ik_n Y_n - \frac{1}{2}k_n . k_m(\frac{\partial^2 \sigma}{\partial x_n \partial x_m} - \frac{\partial \sigma}{\partial x_{n+m}})}$$ (2.10)

Finally the fields are obtained as follows:

$$\phi(x) = \int [dk_n] \Phi[k_n] e^{ik_0 Y}$$

$$\phi_1^\mu(x) = \int [dk_n] k_n^\mu \Phi[k_n] e^{ik_0 Y}$$

$$\phi_{11}^{\mu\nu}(x) = \int [dk_n] k_n^\mu k_n^\nu \Phi[k_n] e^{ik_0 Y}$$ (2.11)

and so on. Here $\Phi[k_n]$ is the string field. The gauge invariant equations are obtained by requiring

$$\frac{\delta}{\delta \sigma} \int [dk_n] \Phi[k_n] e^{ik_n Y_n - \frac{1}{2}k_n . k_m(\frac{\partial^2 \sigma}{\partial x_n \partial x_m} - \frac{\partial \sigma}{\partial x_{n+m}})} = 0$$ (2.12)

In order to get the right (mass)$^2$ one must remember to replace the radius 'a' of the loop by $ae^{2\sigma}$. So far we have not discussed the ghost coordinate.
In [4] we had included an extra space time coordinate $\theta$ and a conjugate momentum $q$, so the vertex operator (2.5) becomes

$$e^{ik_n^\mu Y_n^\mu + iq_n \theta_n} \quad (2.13)$$

We then integrated out $\theta_n$ and were left with the induced Liouville terms as in (2.10). We used $\frac{\partial^2 \sigma}{\partial x_n \partial x_m}e^{ik_0 Y}$ as the vertex operator corresponding to the auxiliary degrees of freedom. Thus we treated the extra coordinate somewhat asymmetrically, but not as asymmetrically as the bosonized ghost coordinate should be treated - since the action for the ghost coordinate contains a coupling to the world sheet curvature scalar - which we do not have. The precise connection between $\theta$ and the bosonized ghost is something that needs to be worked out. In any case we will see below that the treatment of $\theta$ can, in fact, be completely symmetric. Thus we can just use the vertex operator (2.10) and require (2.12) to hold without any consideration of extra auxiliary coordinates. The equations have the invariance

$$k(t) \rightarrow k(t) \lambda(t)$$
$$\lambda(t) = \lambda_0 + \lambda_1/t + \lambda_2/t^2 + ... \quad (2.14)$$

just as in [4]. These equations are precisely those for massless D+1 dimensional gauge fields of different spin! At this point the connection with string theory is completely obscure since this is hardly the right spectrum. However one can 'dimensionally reduce' a la Kaluza-Klein and set the D+1 st component of momentum equal to the mass to get the right spectrum. However we must remember that in string theory it is the $(mass)^2$ that is an integer multiple and not the mass, as would be obtained if we really set the internal momentum equal to a mass. We will just set the variable $q_0^2 = mass$ by hand to get the right spectrum. We are effectively setting $q_0^2$ equal to the naive dimension of the vertex operator. This is equivalent to replacing the radius 'a' by $ae^{2\sigma}$ in eqn.(2.2), which is just what is required on a curved world sheet [13, 4]. However we lose the conventional geometric interpretation of $\theta$ as a D+1 st compactified space coordinate. We will illustrate this now for the level two fields. The vertex operators at the second mass level are:

$$e^{k_0^2 \sigma + ik_0 Y} [-1/2!k_1^\mu k_1^\nu Y_1^\mu Y_1^\nu + ik_2^\mu Y_2^\mu + ik_1 k_0 \frac{\partial \sigma}{\partial x_1} k_1^\mu Y_1^\mu +$$

$$\frac{1}{2} k_1.k_1(\frac{\partial^2 \sigma}{\partial x_1^2} - \frac{\partial \sigma}{\partial x_1}) + k_2.k_0 \frac{\partial \sigma}{\partial x_2} ] \quad (2.15)$$
Setting $\frac{\delta}{\delta \sigma}$[expression (2.15)] = 0 we get the following two equations:

$$[-1/2k_0^2k_1^\mu k_1^\nu + k_1.k_0k_1^\mu k_0^\nu - 1/2k_1.k_1k_0^\mu k_0^\nu]Y_1^\mu Y_1^\nu e^{i\phi_0} = 0$$

(2.16)

$$[ik_0^2k_1^\mu k_1^\nu + k_1.k_0k_1^\mu k_0^\nu - 1/2k_1.k_1k_0^\mu - ik_2.k_0k_0^\mu]Y_2^\mu e^{i\phi_0} = 0$$

(2.17)

These equations are invariant under the gauge transformations (2.14):

$$k_1 \rightarrow k_1 + \lambda_1 k_0 ; k_2 \rightarrow k_2 + \lambda_1 k_1 + \lambda_2 k_0$$

(2.18)

Let us set $k^{D+1}(t) = q(t)$, $Y^{D+1} = \theta$ and let the internal momentum $q_0$ be the mass. Then we get gauge invariant equations for a set of massive 'spin'2 and 'spin' 1 fields. Equation (2.16) reduces to a set of three equations:

$$[-1/2(k_0^2 + q_0^2)k_1^\mu k_1^\nu + (k_1.k_0 + q_1 q_0)1/2(k_1^\mu k_0^\nu + k_0^\mu k_1^\nu)]$$

$$-1/2(k_1.k_1 + q_1 q_1)k_0^\mu k_0^\nu] = 0...............(2.16a)$$

$$[-1/2(k_0^2 + q_0^2)k_1^\mu q_1 + 1/2(k_1.k_0 + q_1 q_0)(k_1^\mu q_0 + q_1 k_0^\mu)$$

$$-1/2(k_1.k_1 + q_1 q_1)k_0^\mu q_0] = 0...............(2.16b)$$

$$[-1/2(k_0^2 + q_0^2)q_1 q_1 + (k_1.k_0 + q_1 q_0)q_1 q_0 - 1/2(k_1.k_1 + q_1 q_1)q_0^2]$$

$$= [-1/2k_0^2 q_1 q_1 + k_1_k_0q_1 q_0 - 1/2k_1.k_1 q_0^2] = 0...............(2.16c)$$

which are the coefficients of $Y_1^\mu Y_1^\nu$, $Y_1^\mu \theta_1$, and $\theta_1 \theta_1$ respectively. Similarly (2.17) reduces to:

$$[(k_0^2 + q_0^2)k_2^\mu - (k_1.k_0 + q_1 q_0)k_1^\mu + (k_1.k_1 + q_1 q_1)k_0^\mu - (k_2.k_0 + q_2 q_0)k_0^\mu] = 0...........(2.17a)$$

$$[(k_0^2 + q_0^2)q_2 - (k_1.k_0 + q_1 q_0)q_1 + (k_1.k_1 + q_1 q_1)q_0 - (k_2.k_0 + q_2 q_0)q_0] =$$

$$[k_0^2 q_2 - k_1.k_0 q_1 + k_1.k_1 q_0 - k_2.k_0 q_0] = 0...............(2.17b)$$

which are the coefficients of $Y_2^\mu$ and $\theta_2$ respectively.

Equations (2.16a-c) describe a massive spin 2 field with the requisite auxiliary fields (a vector and a scalar). Thus $\phi_{11}^{\mu\nu} \sim k_1^\mu k_1^\nu$; $\phi_{11}^{\mu5} \sim k_1^\mu q_1$; $\phi_{11}^{55} \sim q_1 q_1$ describe these fields. The gauge transformations (using 2.18) are:

$$\delta \phi_{11}^{\mu\nu} = \epsilon_2^\mu k_0^\nu ; \delta \phi_{11}^{\mu5} = k_0^\mu \epsilon_2 + \epsilon^\mu ; \delta \phi_{11}^{55} = 2\epsilon_2$$

(2.19)
where we have set $\lambda_1 k'^\mu_1 \sim \epsilon_2^\mu$, $\lambda_1 q_1 \sim \epsilon$. Thus $\phi^{\mu 5}_{1 1}$ and $\phi^{5 5}_{1 1}$ can be gauged away and one recovers the Fierz Pauli equation for a massive spin 2 field. It is important to note that no field redefinitions are required.

Equation (2.17a) describes a massive spin one field $k'^\mu_2$ in addition to $k'^\mu_1 q_1$. In fact this was the equation we had in [4] where we made the identification $k'^\mu_1 q_1 \sim k'^\mu_2$. With this identification (2.17a) and (2.16b) are in fact the same equation. This condition amounts to saying that $'q'_1$ is not an entirely independent degree of freedom. In [4] it was shown by a counting argument that one should set the zeroth and first modes of a bosonic field to zero in order to reproduce the auxiliary field counting. What we are seeing here is just another version of that. In fact (2.16c) becomes identical with (2.17b) if we further set $q_1 q_1 \sim q_2$. Furthermore on adding the trace of (2.16a) with (2.16c) one reproduces the equation associated with the vertex operator $\partial^2 \sigma / \partial \overline{\sigma}_1$ in [4].

The vertex operator $k'^\mu_2 \partial^2 X^\mu$ is not covariant when one has a curved background - in fact one should have $(\partial^2 X + \partial \sigma \partial X)$. If we think of $\theta$ as somehow representing the degree of freedom $\sigma$ then the vertex operator $(Y'^\mu_2 + \theta_1 Y'^\mu_1)$ is the vertex operator at the second mass level. Thus it makes sense to identify $k'^\mu_1 q_1$ with $k'^\mu_2$. We must conclude that the identification $k'^\mu_1 q_1 \sim k'^\mu_2$ is required if one is to make contact with the usual vertex operators of string theory. If we do not make these identifications we seem to have a simpler theory, but with a bigger set of vertex operators and fields. In that case the counting of states would be quite different from that of the usual BRST formalism.

To summarize this section, if we treat the extra 'ghost' coordinate in string theory as an extra dimension and write the theory as a D+1 dimensional theory we get a set of equations for a massless spin 2 field with an auxiliary field. If we now dimensionally reduce to D dimensions, the three vertex operators $Y'^\mu_1 Y'^\nu_1$, $Y'^\mu_1 \theta_1$ and $(\theta_1)^2$ give us all the fields necessary for a covariant massive spin 2. This is similar to the usual Kaluza Klein phenomenon. The vertex operators $Y'^\mu_2$ and $\theta_2$ then give an extra set of vector and scalar fields. The theory is still gauge invariant. If we now impose $k'^\mu_1 q_1 \sim k'^\mu_2$ and $q_1 q_1 \sim q_2$ we truncate back to the usual set (spin 2, spin 1, spin 0) for a gauge invariant, manifestly Lorentz invariant massive spin 2 and get rid of the extra fields. We have the same counting of states as in BRST formalism of string theory and

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1See Sec.III for more details
the same equations of motion. However it would be nice to have a formal proof that this construction is equivalent to all orders, in 26 dimensions, (upto field redefinitions) with string theory.
3 Gauge Transformations

In this section we discuss the gauge transformations in [4] and the connection with Virasoro generators. In [4] we had the string field (we suppress the einbein $\alpha(t)$ for the moment).

$$\int Dk(t)e^{i\int c k(t)\partial_z X(z+t)dt}e^{ik_0X(z)}\Phi[k(t)] = \Phi[X(C)]$$

(3.1)

The usual point particle fields are obtained from (3.1) as in (2.11). $\Phi[X(C)]$ thus is not the usual string field which, in fact, is obtained by acting with $\Phi$ on the (SL(2,R) invariant) vacuum. Thus

$$\Psi[X(C)] = \Phi[X(C)]|0>$$

(3.2)

is the object commonly referred to as the string field. The transformation properties of $\Phi$ under the action of Virasoro generators are clearly different from those of $\Psi$ since the vacuum is only invariant under $L_{-n}$ with $n = 0, \pm 1$. These were worked out in [11, 12] in some detail. It was shown there that the action of $L_{-n}$, $n > 0$ on the loop variable (2.5) has two parts to it: The first piece is obtained by making the transformation:

$$k_{m+n} \xrightarrow{L_{-n}} k_{m+n} + m\lambda_n k_m$$

$$k_n \xrightarrow{L_{-n}} k_n + \lambda_n k_0$$

$$k_m \xrightarrow{L_{-n}} k_m, n > m$$

(3.3)

This amounts to saying that $k(t)$ transforms as a scalar and $\partial_z X(z+t)$ as a vector (or a tensor of weight one) under the diffeomorphism

$$t \rightarrow t(1 + \lambda(t))$$

(3.4)

which is clearly different from (2.14). On the other hand let us start with the loop variable

$$e^{i\int c k(t)X(z+t)}$$

(3.5)

This can be obtained from (2.1),(3.1) by rescaling the $k_n$’s:

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2 In the BRST formalism $Q_{BRST}|0> = 0$. It would seem that this distinction is irrelevant. Nevertheless the gauge transformation in the BRST formalism has terms corresponding, in the covariant formalism, to terms of the form $L_{-n}|0>$. See below.
Thus the transformations (3.3) becomes

\[ k_{m+n} \xrightarrow{L_n} k_{m+n} + (m+n)\lambda_n k_m \]  

which is equivalent to \( k(t)/t \rightarrow k(t)/t + \frac{d}{dt} [t\lambda(t)k(t)/t] \). (3.7) can be obtained by transforming ‘t’ in \( X(z+t) \) according to

\[ t \rightarrow t(1 + \lambda(t)) = t + \lambda_0t + \lambda_1 + \lambda_2/t + ... \]  

and requiring invariance of (3.5). Since \( t\lambda(t) \) is the change in \( t \), this is equivalent to requiring that \( k(t)/t \) transform as a tensor of weight one. Thus

\[
e^i \int_c dt/tk(t)X(z+t) \frac{\delta(t)}{\lambda(t)} e^i \int_c dt/tk(t)X(z+t+\delta(t))
\]

for infinitesimal \( \delta \). Thus when \( \delta(t)/t = 1 \), the second factor is the loop variable (2.1). If we now change \( \delta(t)/t \rightarrow \delta(t)\lambda(t)/t \), then the second factor becomes (for \( \delta(t)/t = 1 \)), the gauge transformed loop variable. Thus the \( k(t) \rightarrow \lambda(t)k(t) \) can be viewed as the transformation induced on (2.1) by the action of a Virasoro generator not on (2.1) but on the loop variable (3.5). Heuristically, if we think of the loop variable (3.5) as a group element \( g[k] \), then the Virasoro generators induce a transformation

\[ g[k] \rightarrow g[k + \delta k] \equiv g[k]h[\delta k] \]  

The loop variable (3.1) is obtained by translating back to the origin by \( g^{-1} \):

\[ g^{-1}[k]g[k + \delta k] = h[\delta k] \]  

[To lowest order in \( \delta \), \( h[\delta k] = 1 + g^{-1}\delta g \)]. The gauge transformation (2.14) corresponds to changing \( \delta k \rightarrow \lambda \delta k \) so that

\[ h[\delta k] \rightarrow h[\lambda \delta k] \]  

Thus the transformation (2.14) while not a diffeomorphism of \( S^1 \), is induced by one in the manner outlined. It has a simpler space time interpretation,
however - it is a rescaling of $k(t)$ and hence of $X(t)$. Since it is a pointwise multiplication, the operation $k(t) \rightarrow k(t)\lambda(t)$ has a simple geometrical interpretation (of local rescaling) and the parameter along the string, $t$, has no role to play- which is just as well since it has no physical significance. This is not the case in diffeomorphisms - where the transformation involves the derivatives with respect to $t$.

So far we have discussed only one part of the action of the Virasoro group on vertex operators \[11, 12\]. When one calculates the operator product (OP) of the group element $e^{\sum_n \lambda_n L_{-n}}$ on a vertex operator $e^{i\sum_{n>0} k_n Y_n}$, we are, in fact, computing the action of $e^{\sum_n \lambda_n L_{-n}}$ on the state created by $e^{i\sum_{n>0} k_n Y_n}$, i.e.

$$e^{\sum_n \lambda_n L_{-n}} e^{i\sum_{n>0} k_n Y_n} |0\rangle$$ (3.13)

and from \[12\] this has the form (to lowest order in $\lambda$)

$$e^{1/2Y_n \lambda_{n+m} Y_m - 1/2nk_n \lambda_{-n-m} mk_m - ink_n \lambda_{-n+m} Y_m}$$ (3.14)

The first term in the exponent comes from the action of the $L_{-n}$ on the vacuum state $|0\rangle$ since it is not invariant. The second term comes from the $L_{+n}$ and these are constraints rather than gauge transformations and will not concern us here. The third term is the action of $L_{-n}$ on the vertex operator. From our discussion thus far it is clearly the last term that is involved in the gauge transformation of the "string field" in \[4\]. That is we have been defining the transformation of the space time fields by transforming $\Phi[X(C)]$ rather than $\Psi[X(C)] = \Phi[X(C)] |0\rangle$. In the usual formalism it is the transformation of the latter field that defines the gauge transformation on the space time fields. In fact it was noted in \[1\] that the gauge transformation in the BRST formalism \[1\] had an extra $\delta^{\mu\nu}$ piece which was then gotten rid of by means of field redefinitions. This extra term is precisely of the form of the first term in the exponent of (3.14). This is one important difference between the formalism of \[1\] and the usual string field theory formalism. The precise mechanism by which, in 26 dimensions, one can perform field redefinitions that take one from $\Psi$ to $\Phi$ needs further elucidation. Some suggestions were made in \[11\].

What we have discussed thus far in this section is the connection between gauge transformations as defined by equation (2.14) and diffeomorphisms of $S^1$. Now starting from (2.14), with the vector field defined as in equation
(2.11) to get to the gauge transformation law:

$$\phi_1^{\mu}(k_0) \rightarrow \phi_1^{\mu}(k_0) + k_0^\mu \Lambda$$  

(3.15)

we have to assume that the string field $\Phi[k_n]$ depends on $\lambda$ as well as $k$ so that (3.15) comes from

$$\int [dk_n][d\lambda] \Phi[k, \lambda] k_1^\mu \rightarrow \int [dk_n][d\lambda] \Phi[k, \lambda] (k_1^\mu + \lambda_1 k_0^\mu)$$  

(3.16)

with

$$\int [dk][d\lambda] \Phi[k, \lambda] \lambda_1 \equiv \Lambda$$  

(3.17)

the gauge parameter. The inclusion of a $\lambda$-dependence inside $\Phi$ is unusual and has no counterpart in usual field theory, where the gauge parameter is separate from the fields. In our case $\lambda(t)$ multiplies $\alpha(t)$ the einbein. Thus one could just as well think of the einbein as parametrising the space of gauge transformations and then it is not unnatural to let the string field depend on $\alpha(t)$, which could play the role of the ghosts. However this issue of the $\lambda$ dependence of the string field needs further investigation.
4 Mechanism of Gauge Invariance

In this section we describe the mechanism of gauge invariance and also discuss what happens at higher orders in the gauge parameter \( \lambda \). As described in [4] we have to start with the loop variable (2.3), with an einbein \( \alpha(t) \), which is integrated over since one has to sum over reparametrizations of the boundary. Thus we have

\[
\int \mathcal{D}\alpha(t)e^{i \int_0^1 (t)k(t)\partial_z X(z + t)dt + i k_0 X}
\]  

(4.1)

Clearly at the formal level this is invariant under \( \alpha(t) \rightarrow \alpha(t)\lambda(t) \) since this is just a change of variables, provided the measure \( \mathcal{D}\alpha(t) \) is invariant. In [4] we set

\[
\mathcal{D}\alpha(t) = [dx_1 dx_2 ...]; \alpha(t) = e^{\sum_n x_n t^{-n}}
\]  

(4.2)

Thus if we parametrise \( \lambda(t) \) by:

\[
\lambda(t) = e^{\sum_n y_n t^{-n}}
\]  

(4.3)

clearly a gauge transformation corresponds to a translation of the \( x_n \)'s:

\[
x_n \rightarrow x_n + y_n
\]  

(4.4)

under which the measure (4.2) is invariant. However while this is formally true (4.1) contains an implicit dependence on \( \sigma \), reflecting the ultraviolet divergences that come about when doing the X-functional integral. Let us write all the terms upto \( Y_3 \):

\[
exp(i(k_0 Y + k_1 Y_1 + k_2 Y_2 + k_3 Y_3))exp(k_0^2 \sigma + k_1 k_0 \frac{\partial \sigma}{\partial x_1} + k_2 k_0 \frac{\partial \sigma}{\partial x_2} + \ldots)
\]  

(4.5)

\[
k_1 k_1/2(\frac{\partial^2 \sigma}{\partial x_1^2} - \frac{\partial \sigma}{\partial x_2}) + k_3 k_0 \frac{\partial \sigma}{\partial x_3} + k_2 k_1(\frac{\partial^2 \sigma}{\partial x_1 \partial x_2} - \frac{\partial \sigma}{\partial x_3})
\]

We can rearrange the terms as follows:

\[
exp(i[k_0 \sigma + iY] + k_1 \frac{d}{dx_1}(k_0 \sigma + iY) + k_2 \frac{d}{dx_2}(k_0 \sigma + iY) + \ldots)
\]  

(4.6)

\[
k_3 \frac{d}{dx_3}(k_0 \sigma + iY) + k_1 k_1/2(\frac{\partial^2 \sigma}{\partial x_1^2} - \frac{\partial \sigma}{\partial x_2}) + k_2 k_1(\frac{\partial^2 \sigma}{\partial x_1 \partial x_2} - \frac{\partial \sigma}{\partial x_3})
\]

\[\text{It is conceivable that this is true only in 26 dimensions}\]
Furthermore using

\[
\frac{d^3}{dx_1^3} \sigma = \frac{d^3}{dx_1^3} < YY >= 2[Y_2 Y > + 3 < Y_2 Y_1 >] = -2 \frac{\partial \sigma}{\partial x_3} + 3 \frac{\partial^2 \sigma}{\partial x_1 \partial x_2} \tag{4.7}
\]

we see that

\[
\frac{d}{dx_1} \left( \frac{\partial^2 \sigma}{\partial x_1^2} - \frac{\sigma}{\partial x_2} \right) = 2 \left( \frac{\partial^2 \sigma}{\partial x_1 \partial x_2} - \frac{\partial \sigma}{\partial x_1} \right) \tag{4.8}
\]

Thus we can further rewrite (4.6):

\[
\exp(i[(k_0 + k_1 \frac{d}{dx_1} + k_2 \frac{d}{dx_2} + k_3 \frac{d}{dx_3}))(k_0 \sigma + iY) + k_1, k_2 \frac{d}{dx_1} \left( \frac{\partial^2 \sigma}{\partial x_1^2} - \frac{\partial \sigma}{\partial x_2} \right)] \right) \equiv e^A \tag{4.9}
\]

If we make gauge transformations with parameter \( \lambda_1 \) we get:

\[
\exp(A + i \lambda_1 \frac{d}{dx_1} (k_0 Y + k_1 Y_1 + k_2 Y_2)) + \lambda_1 \frac{d}{dx_1} \left( k_0^2 \sigma + k_1, k_0 \frac{\partial \sigma}{\partial x_1} + k_2, k_0 \frac{\partial \sigma}{\partial x_2} + k_1, k_1 \left( \frac{\partial^2 \sigma}{\partial x_1^2} - \frac{\partial \sigma}{\partial x_2} \right) \right) = e^{A + \lambda_1 \frac{dA}{dx_1}} \tag{4.10}
\]

to this order. From (4.3) we see that to lowest order \( \lambda_1 = y_1 \) and thus we see that all we have to this order is a transformation of \( x_1 \) by an amount \( \lambda_1 = y_1 \) - which is just what we expect from (4.4). When we derive the equations of motion by varying respect to \( \sigma \) we routinely integrate by parts on the \( x_n \). This of course corresponds to adding total derivatives - which we are allowed to do inside an integral (assuming no boundary terms). Thus we rewrite \( \lambda_1 \frac{d}{dx_1} A \) as \( \sigma B \), for some \( B \). Now \( \lambda_1 \frac{d}{dx_1} A \) is a total derivative in \( x_1 \) and so must \( \sigma B \) be one [since integrating by parts corresponds to adding more total derivatives]. Clearly \( \sigma B \) cannot be a total derivative since there is no derivative on \( \sigma \). Thus the only possibility is \( B = 0 \). Thus we see that a \( \sigma \) variation of a gauge variation of \( A \) is zero. Since a \( \sigma \) variation commutes with a gauge variation we would seem to have proved that the gauge variation of an equation of motion [i.e. a \( \sigma \) variation of \( A \)] is zero. While this argument is true as it stands one has to be a little careful about applying it here since
it involves using (4.8). Identities like (4.8) and (4.7) can be used to reexpress \( \delta A \) in terms of different sets of vertex operators. For instance \( \frac{\delta}{\delta \sigma} \partial^3 \sigma \partial^3 x \) will have a term involving \( Y_1^\mu Y_1^\nu Y_1^\rho \) whereas \( \frac{\delta}{\delta \sigma} (-2 \partial^3 \sigma + 3 \partial^2 \sigma) e^{ik_0 Y} \) will have only terms quadratic in derivatives of \( Y \). Thus the argument made earlier can be applied to the sum of all terms in the variation of \( A \) (i.e. including different vertex operators). However if we want to set the coefficient of each vertex operator separately to zero our gauge invariance is necessarily more restricted. Thus the gauge variation of the terms of the type in (4.8) must necessarily be zero. Now \( \delta(k_1 k_2) = \lambda_1 k_1, k_1 + \lambda_2 k_2, k_0 \). It is the former term that involves the vertex operator \( \partial^2 \sigma \partial^2 x \), as can be seen in (4.10). Thus we must require

\[
\int [d\lambda dk] \lambda_1 k_1, k_1 \Phi[k, \lambda] = 0 \quad (4.11)
\]

This amounts to saying that the gauge parameter is traceless. This condition is familiar in higher spin gauge theories. If we look at the gauge invariance of the spin 3 equation of motion, i.e. the coefficient of \( Y_1^\mu Y_1^\nu Y_1^\rho \), we will see that this condition is indeed required. This equation is:

\[
[-ik_0^2/3!k_1^\mu k_1^\nu k_1^\rho + i/2!k_1.k_0 k_1^\mu k_1^\nu k_0^\rho - ik_1.k_1/2!k_0^\mu k_0^\nu k_0^\rho]Y_1^\mu Y_1^\nu Y_1^\rho = 0 \quad (4.12)
\]

and can be seen to be invariant when the tracelessness condition is imposed.

Having described the mechanism of gauge invariance, we can easily see that it can be extended to higher orders in \( \lambda \).

In eqn. (4.4) if we let \( y_1 \) be finite then one has higher order terms in eqn.(4.3). Thus we get

\[
1 + \lambda_1/t + \lambda_2/t^2 + \lambda_3/t^3 + \ldots = 1 + y_1/t + y_2/2!1/t^2 + y_3/3!1/t^3 + \ldots \quad (4.13)
\]

from which we conclude that

\[
y_1 = \lambda_1; \; \lambda_2 = \lambda_1^2/2!; \; \lambda_3 = \lambda_1^3/3!\ldots \quad (4.14)
\]

and the gauge transformation on the \( k_n \) become:

\[
k_1 \rightarrow k_1 + \lambda_1 k_0; \; k_2 \rightarrow k_2 + \lambda_1 k_1 + \lambda_1^2/2!k_0 \quad (4.15)
\]

If we start with (4.6) (or 4.5) and make the above substitution keeping all terms to \( O(\lambda_1^2) \) we will find, as expected, instead of (4.10)

\[
exp(A + \lambda_1 dA/dx_1 + \lambda_1^2/2!d^2A/dx_1^2 + .) \quad (4.16)
\]
In the spin 2 equations (17) one can make the substitution (4.15) and check that they are invariant order by order in $\lambda$. Note that the field $\phi_{11}^{\mu\nu}$ transforms as:

$$\int [dkd\lambda]k_1^\mu k_1^\nu \Phi[k, \lambda] \rightarrow \int [dkd\lambda](k_1^\mu k_1^\nu + \lambda_1 (k_1^\mu k_0^\nu + k_1^\nu k_0^\mu) + \lambda_1^2 k_0^\mu k_0^\nu)\Phi[k, \lambda]$$

$$\Phi_{11}^{\mu\nu} \rightarrow \Phi_{11}^{\mu\nu} + \partial^{(\mu} \Lambda_{11}^{\nu)} + \partial^\mu \partial^\nu \Lambda_{11}$$

Thus the higher order pieces in $\lambda_1$ do not result in a higher order term in $\Lambda_{11}^\mu$, but in a modification of $\Lambda_{11}^\mu$ to $\Lambda_{11}^\mu + \partial^\mu \Lambda_{11}$. Clearly (as expected) this does not result in any new symmetry. One can also check the spin 3 equations and as before we need the tracelessness condition $\lambda_1 k_1 . k_1 = 0 (4.11)$, but now also a condition $\lambda_2^2 k_1 . k_0 = 0$. This second condition follows by a gauge transformation of (4.11) so it is not unexpected.

In this section we have described the basic mechanism of gauge invariance: the freedom to add total derivatives in the $'x_n'$. This generalizes the well known freedom in $\sigma$-models to add derivatives in $z$. We have also described the origin of the tracelessness condition and also what happens at higher orders in $\lambda(t)$. One issue that needs further exploration is that of the extra degrees of freedom $\alpha(t), \lambda(t)$ or $x_n, y_n$. From (4.4) it is obvious that the only justification for treating $y_n$ differently from $x_n$ is that $y_n$ is a gauge parameter describing the change in $x_n$. On the other hand we are forced to include $y_n$ as a degree of freedom (as explained in Sec. III). Clearly there must be some way to get around this duplication.
5 Closed String

In this section we extend the results of [4] to closed strings. The vertex operators for closed strings are obtained by combining holomorphic and anti holomorphic open string vertex operators of the same mass level. All we have to do then is to apply the construction described in Sec.II and III separately to the holomorphic and anti holomorphic generalized vertex operators. Thus we can generalize the loop variable to

\[ \exp \left( i \int_c \alpha(t) k(t) \partial_z X(z + t, \bar{z} + \bar{t}) dt + \int_c \bar{\alpha}(\bar{t}) \bar{k}(\bar{t}) \partial_{\bar{z}} X(z + t, \bar{z} + \bar{t}) d\bar{t} + ik_0 X \right) \]  

(5.1)

We will Taylor expand the exponent in powers of \( t, \bar{t} \), keeping in mind that \( \partial_z \partial_{\bar{z}} X(z, \bar{z}) = 0 \). Thus the exponent becomes:

\[ k_0 [X(z, \bar{z}) + \alpha_1 \partial X + \partial^2 X + \alpha_3 \partial^3 X/2! + ... + \alpha_1 \partial X + \alpha_2 \partial^2 X + \alpha_3 \partial^3 X/2! + ...] \]

(5.2)

\[ + k_1 [\partial X + \alpha_1 \partial^2 X + \alpha_2 \partial^3 X/2! + ...] + \bar{k}_1 [\partial \bar{X} + \alpha_1 \partial^2 \bar{X} + \alpha_2 \partial^3 \bar{X}/2! + ...] \]

\[ + k_2 [\partial^2 X + \alpha_1 \partial^3 X/2! + ...] + \bar{k}_2 [\partial^2 \bar{X} + \alpha_1 \partial^3 \bar{X}/2! + ...] + ... \]

We have assumed that \( k_0 = \bar{k}_0 \). Let

\[ Y = X + \alpha_1 \partial X + \alpha_2 \partial^2 X + \alpha_3 \partial^3 X/2! + ... + \alpha_1 \partial X + \alpha_2 \partial^2 X + \alpha_3 \partial^3 X/2! + ... \]

(5.3)

Then defining \( x_n, \bar{x}_n \) in the same way as before (2.6), (2.7)

\[ \frac{\partial Y}{\partial x_1} = \partial X + \alpha_1 \partial^2 X + \alpha_2 \partial^3 X/2! + ... \]

(5.4)

\[ \frac{\partial Y}{\partial \bar{x}_1} = \partial \bar{X} + \alpha_1 \partial^2 \bar{X} + \alpha_2 \partial^3 \bar{X}/2! + ... \]

(5.5)

\[ \frac{\partial^2 Y}{\partial x_1 \partial \bar{x}_1} = 0 \]

(5.6)

The exponent becomes

\[ k_0 Y + \sum_n (k_n \frac{\partial Y}{\partial x_n} + \bar{k}_n \frac{\partial Y}{\partial \bar{x}_n}) \]

(5.7)
As before we can define $< YY > = \sigma$ to get
\[
< \frac{\partial Y}{\partial x_n} > = 1/2 \frac{\partial^2 \sigma}{\partial x_n \partial x_m} \quad (5.8)
\]
The loop variable (5.1) along with its $'\sigma'$ dependence is
\[
\begin{align*}
\exp(i(k_0 Y + \sum_n (k_n \frac{\partial Y}{\partial x_n} + \tilde{k}_n \frac{\partial Y}{\partial x_n}) + (k_0^2 + q_0^2)\sigma + (k_1. k_1 + q_1 q_1)1/2(\frac{\partial^2 \sigma}{\partial x_1^2} - \frac{\partial \sigma}{\partial x_2} ) \\
+ (k_1. k_0 + q_1 q_0) \frac{\partial \sigma}{\partial x_1} + (k_2. k_0 + q_2 q_0) \frac{\partial \sigma}{\partial x_2} + (k_1. k_1 + q_1 q_1) \frac{\partial^2 \sigma}{\partial x_1 \partial x_1} ))
\end{align*}
\] (5.9)

We have denoted the D+1 st component of 'k' by q and integrated out the D+1st component of X. (We are following the procedure of [4] rather than the Kaluza Klein approach of Sec. II in this paper.) The vertex operators at the lowest mass level are $Y_1^\mu Y_1^\nu$ and $\partial^2 \sigma / \partial x_1 \partial x_1$. We collect terms of this dimension and set the variation $\delta \sigma / \delta x_1$ to zero (evaluating the derivative at the point $\partial \sigma / \partial x_1 = 0$, just as in Ref.1).

There are three types of terms that need to be varied:

\[
I. \quad e^{(k_0^2 + q_0^2)\sigma} e^{ik_0 Y} \{(i)^2 k_1^\mu \tilde{k}_1^\nu \frac{\partial Y^\mu}{\partial x_1} \frac{\partial Y^\nu}{\partial \tilde{x}_1} \}
\] (5.10)

\[
II. \quad e^{(k_0^2 + q_0^2)\sigma} e^{ik_0 Y} \{(k_1. k_0 + q_1 q_0) \frac{\partial \sigma}{\partial x_1} i k_1^\nu \frac{\partial Y^\nu}{\partial \tilde{x}_1} \}
\] (5.11)

\[
III. \quad e^{(k_0^2 + q_0^2)\sigma} e^{ik_0 Y} \{(k_1. k_0 + q_1 q_0) (\tilde{k}_1 k_0 + \tilde{q}_1 q_0) \frac{\partial \sigma}{\partial x_1} \frac{\partial \sigma}{\partial \tilde{x}_1} + 2(k_1. \tilde{k}_1 + q_1 \tilde{q}_1)1/2(\frac{\partial^2 \sigma}{\partial x_1 \partial \tilde{x}_1}) \}
\] (5.12)

\[
\frac{\delta}{\delta \sigma} I = -(k_0^2 + q_0^2) k_1^\mu \tilde{k}_1^\nu \frac{\partial Y^\mu}{\partial x_1} \frac{\partial Y^\nu}{\partial \tilde{x}_1} e^{ik_0 Y} \] (5.13)

20
\[
\frac{\delta}{\delta \sigma} II = (k_1.k_0 + q_1q_0)k_1^\mu k_0^\nu \frac{\partial Y^\mu}{\partial \bar{x}_1} \frac{\partial Y^\nu}{\partial x_1} 
+ (\bar{k}_1.k_0 + \bar{q}_1q_0)k_1^\mu k_0^\nu \frac{\partial Y^\nu}{\partial \bar{x}_1} \frac{\partial Y^\mu}{\partial x_1}
\]

(5.14)

\[
\frac{\delta}{\delta \sigma} III = -2(k_1.k_0 + q_1q_0)(\bar{k}_1.k_0 + \bar{q}_1q_0)\frac{\partial^2 \sigma}{\partial \bar{x}_1 \partial x_1}
+ 2(k_0^2 + q_0^2)(k_1.\bar{k}_1 + q_1\bar{q}_1)\frac{\partial^2 \sigma}{\partial \bar{x}_1 \partial x_1}

+ (i)^2 k_0^\mu k_0^\nu \frac{\partial Y^\nu}{\partial \bar{x}_1} \frac{\partial Y^\mu}{\partial x_1}(k_1.\bar{k}_1 + q_1\bar{q}_1)
\]

(5.15)

Coefficient of \[
\frac{\partial Y^\nu}{\partial \bar{x}_1} \frac{\partial Y^\mu}{\partial x_1}
\]

(5.16)

\[
[-k_0^2 k_1^\mu \bar{k}_1^\nu + k_1.k_0 \bar{k}_1^\mu k_0^\nu + \bar{k}_1.k_0 k_1^\mu k_0^\nu - k_0^\mu k_0^\nu (k_1.\bar{k}_1 + q_1\bar{q}_1)] = 0
\]

Coefficient of \[
\frac{\partial^2 \sigma}{\partial \bar{x}_1 \partial x_1}
\]

(5.17)

\[
[-(k_1.k_0)(\bar{k}_1.k_0) + k_0^2(k_1.\bar{k}_1 + q_1\bar{q}_1)] = 0
\]

We have set the \((mass)^2 = q_0^2 = 0\). The fields are the graviton, the anti-symmetric tensor and the dilaton:

\[
\int [dkd\bar{k}dqd\bar{q}]k_1^\mu k_0^\nu \Phi[k, \bar{k}, q, \bar{q}] = h^{\mu\nu}
\]

(5.18)

\[
\int [dkd\bar{k}dqd\bar{q}]k_1^\mu \bar{k}_0^\nu \Phi[k, \bar{k}, q, \bar{q}] = A^{\mu\nu}
\]

and

\[
\int [dkd\bar{k}dqd\bar{q}]q_1\bar{q}_1 \Phi[k, \bar{k}, q, \bar{q}] = \eta
\]

(5.19)

The gauge transformations are

\[
k_1 \rightarrow k_1 + \lambda_1 k_0; \ \bar{k}_1 \rightarrow \bar{k}_1 + \bar{\lambda}_1 \bar{k}_0; \ q_1 \rightarrow q_1; \ \bar{q}_1 \rightarrow \bar{q}_1
\]

(5.20)

Since \(q_0\) is zero, \(q_1, \bar{q}_1\) remain invariant. This gives the usual infinitesimal transformation for the graviton

\[
h^{\mu\nu} \rightarrow h^{\mu\nu} + \partial^{(\mu} \epsilon^{\nu)} + \partial^{(\nu} \epsilon^{\mu)} = h^{\mu\nu} + \partial^{(\mu} \epsilon^{\nu)}
\]

(5.21)
where we define
\[ \epsilon^\mu_S = \epsilon^\mu + \bar{\epsilon}^\mu \] (5.22)
and
\[ A^{\mu\nu} \rightarrow A^{\mu\nu} + \partial[\mu \epsilon^\nu] + \partial[\nu \epsilon^\mu] = A^{\mu\nu} + \partial[\mu \epsilon_A^\nu] \] (5.23)
where
\[ \epsilon_A^\mu = \epsilon^\mu - \bar{\epsilon}^\mu \] (5.24)
and
\[ \delta \eta = 0 \] (5.25)
One can easily check that under the variations (5.20) equations (5.16) and (5.17) are invariant. Presumably one can reproduce these results in the Kaluza Klein approach of Sec. II. We have not attempted to do this here.
6 Conclusion

In this paper we have attempted to extend the results of [4] in several directions and also to understand a bit better the connection between these results and the usual BRST formalism - in particular the issue of the difference in the form of the gauge transformation. We have been able to understand the mechanism of the gauge invariance, the connection with the Virasoro group and also the origin of the ad-hoc constraints in [4]. We have also extended the treatment to closed strings. In the process of trying to understand some of the peculiar features there arose some open questions. Some of the outstanding questions are: How does one start from the usual BRST formalism and arrive at the equations and gauge transformations here, at the level of the string field rather than component by component? In particular what is the connection between $\alpha(t)$ and the ghost? Finally we would like to generalize these results to the interacting theory where the issue of regularization becomes important when you try to go off shell. These issues have been discussed in a related context in [7]. We hope to report on these questions in the future.

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