Calculation of Load-bearing Capacity of Tubula Anchor of Friction Type

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Abstract. The load-bearing capacity of a rod of a tubular anchor of a friction type (friction-type anchor) is determined by the nature of its interaction with the walls of the hole. The preliminary (estimated) calculation of this indicator is difficult because of the lack of information about the deformed state of the rod. The proposed interaction scheme assumes coupling only over a part of the cross-sectional profile. Force factors are represented by concentrated and distributed loads. On the basis of accepted assumptions, a mathematical model which indicates rod-hole force interaction is formed. The determination of the parameters characterizing the adopted interaction scheme is based on the solution of the obtained nonlinear equations. The physical simulation of the process of installing the anchor in the borehole model was performed. The accepted scheme of interaction of a rod – borehole is experimentally confirmed. The results of numerical simulation have large convergence with experimental data.

1. Introduction

Anchors of friction type of fixation in borehole become more popular [1 -7]. Load-bearing capacity is provided by means of frictional forces producing at the contact surface which is caused by the pressure produced by the anchor rod. Two methods of pressure production and correspondingly two types of anchors are used. According to the first one the interaction is provided by means of elastic deformation of the anchor rod which occurs when it is set into a borehole. In the second option the pressure is produced through the change of shape of the rod cross-section that has been already set into the borehole when its internal space is full of service fluid under excessive pressure [8, 9]. The calculation of the main parameters of a given type anchor is represented in detail [10-12].

In the first method the anchor construction includes a rod and a support plate. The rod is produced as a quill cylinder with an axial slot throughout the length [13-15]. The head of the rod is made as flattened cone and tailpiece is equipped by limit stop for interaction with the support plate.

Simple design, automation capability while mounting [16-25] caused common usage of the first type friction anchors.
The construction is based on principle when external diameter of the rod $D_R$ is bigger than the borehole diameter $D_{Bh}$ (figure 1, a). The rod deforms elastically in the process of anchor setting, and external dimensions of its cross section reduce to the borehole diameter. Fixation of the rod in the borehole is provided by means of friction forces determined by force interaction between the rod and the borehole.

![Cross sections of the anchor rod: before setting into the borehole (a); after setting (b).](image)

**Figure 1.** Cross sections of the anchor rod: before setting into the borehole (a); after setting (b).

The volume of usage of anchors that realize presented option keeps steadily growing which is determined by significant technological advantages. At the same time the calculation methodology of an anchor main features and first of all load-bearing capacity has been absent up to the present moment. Isolated publications are not applicable for analysis of the most usable anchor type since usable schemes of the rod loading are not relevant [26-27]. In this regard, acquisition of dependence of load-bearing capacity in the function of the rod parameters and strengthened kind qualities is of significant interest.

2. **Analytical model of calculation of load-bearing capacity of a tubular friction type anchor**

Different modern methods are used in the calculations of stress conditions in particular of friction anchors [26]. It is based on the usage of initial loading scheme designed with clear justifications. The task is to select the loading scheme of the anchor rod and to validate it.

The following assumptions are accepted in the formation of mathematical model that determines the force interaction of the rod set into a borehole:
- the nature of force interaction throughout the length of the rod is constant;
- the destruction of the borehole is absent in the area of contact of the rod walls and the borehole;
- the rod is deformed elastically;
- the initial cross-sectional profile of the rod and the borehole is made as a circle;
- the rod of nominally unit length is considered.

The loading scheme based on the performance of the main moments which characterize the deformation of the rod in the borehole is proposed for the following calculations. When the rod is set the borehole its initial cross-sectional profile shape (figure 1, a) is deformed and takes the shape presented in figure 1, b.
In this case the following conditions are realized:
1. The total coupling of the walls of the rod and the borehole is provided in segment $BB_I$. The outer radius of the rod in segment $BB_I$ equals the radius of the borehole $0.5D_{bh}$ (figure 1 b).
2. In segments $BC$ ($B_CI$) the contact between the rod and the borehole is absent. The outer radius of the rod is increased from $0.5D_{bh}$ to $0.5D_R$.
3. In point $C$ ($C_I$) the contact between the rod brim and the borehole is realized.

The accepted relative position of the walls of the rod and the borehole determines the loading scheme.
1. The distributed loading $q$ operates along the whole circular arc $BB_I$. The border of the interval that is point $B$ ($B_I$) is determined by angle $\beta$.
2. There are no external loadings at the interval $BC$ ($B_CI$).
3. Point forces $R_C$ ($R_{CI}$) and $R_B$ ($R_{BI}$) are applied to points $C$ ($C_I$) and $B$ ($B_I$) correspondingly.

The loading scheme (figure 2) is determined by four parameters such as $q$, $\beta$, $R_C$ and $R_B$. Identification of their dependence on geometric parameters of the borehole and the anchor rod is based on accomplishment of accepted deformation scheme of the rod cross-section and equilibrium equation.

![Figure 2. The loading scheme of the anchor rod cross-section.](image)

The change of flection of rod wall $\rho'$ from the basic value $2 \cdot D_R^{-1}$ to $2 \cdot D_{bh}^{-1}$ takes place at the interval $BB_I$

$$\rho' = \frac{2 \cdot (D_R - D_{bh})}{D_R \cdot D_{bh}}$$

Or taking into account the fact that the deformation of the rod is $S = D_R - D_{bh}$ in cross-section:

$$\rho' = \frac{2 \cdot (D_R - D_{bh})}{D_R \cdot D_{bh}} .$$

The constant value of the flection radius at the interval $BB_I$ is possible if the value of moment of flection in all sections is constant within the given interval [28]:

$$M_{BBI} = E \cdot J \cdot \rho' , \quad (1)$$
Where \( E \) is elastic modulus of the rod material, Pa;
\( J \) is the inertia moment of the rod wall \( t \), \( m^4 \) in thickness
\[
J = \frac{l \cdot t^3}{12},
\]
\( l \) is the unit length of the rod, \( l = 1 \) m,
\( t \) is the thickness of the rod wall, m.

On the basis of equilibrium condition in random section which position is determined by angle \( \gamma \) we get the following:
\[
M_{BBI} = \frac{R_C \cdot D_{bh}}{2} \sin(\gamma - \omega) + \frac{R_B \cdot D_{bh}}{2} \sin(\beta - \omega) + \frac{q \cdot D_{bh}^2}{4} \left( (\gamma - \beta) \frac{\sin(\gamma - \beta)}{2} \right),
\]
where \( \omega \) is the angle that determines the position of the rod edge after setting into the borehole:
\[
\omega = \arcsin \left( \frac{b}{2D_{bh}} \right),
\]
\( b \) is the distance between brims of the rod slot after setting, m.

The third summand of the formula (2) includes angles \( \gamma \) and \( \beta \) in an explicit form what complicates further solution in analytical form. We use a substitute used by V.I. Feodosiev in formation of formula of a moment generated by distributed loading which operates on a circular arc:
\[
(\gamma - \omega) \sin((\gamma - \beta) / 2) = \frac{1}{2} \left( \sin^2(\gamma - \beta) + (1 - \cos(\gamma - \beta))^2 \right).
\]

After making a substitution and transformation aimed to determine the constant component of a moment formula the origin equation (2) transforms to the following form:
\[
M_{BBI} = \frac{D_{bh}}{2} \left[ R_C + R_B \cos(\beta - \omega) - \frac{q \cdot D_{bh} \cdot \sin(\beta - \omega)}{2} \right] \sin(\gamma - \omega) - \\
\frac{D_{bh}}{2} \left[ R_B \sin(\beta - \omega) + \frac{q \cdot D_{bh} \cdot \cos(\beta - \omega)}{2} \right] \cos(\gamma - \omega) + \frac{q \cdot D_{bh}^2}{4}.
\]

The moment \( M_{BBI} \) will be constant if the multiplicands in square brackets in the first and the second summands equal null.
\[
\begin{align*}
R_C + R_B \cdot \cos(\beta - \omega) - \frac{q \cdot D_{bh} \cdot \sin(\beta - \omega)}{2} &= 0, \\
R_B \cdot \sin(\beta - \omega) + \frac{q \cdot D_{bh} \cdot \cos(\beta - \omega)}{2} &= 0.
\end{align*}
\]

In this case the value MBBI will be constant and equal the third summand \( \frac{q \cdot D_{bh}^2}{4} \). Taking into account formula (1) we have the following for the distributed loading:
\[
q = \frac{4 \cdot E \cdot J \cdot \rho^l}{D_{BBI}^2}.
\]

Set of equations (3) includes three unknown variables: \( \beta, RC, RB \). To determine them we use the deformation condition that is the distance between points C and A changed for value \( S \). This condition is written through the loading parameters by a known integral. Taking into account constancy of
loading on two segments and expressing elementary linear motion $ds$ by the angular $ds = \frac{D_{bh}}{2} \cdot d\gamma$

we get the following:

$$S = \int_{\omega}^{\beta} M_{CB} \cdot \frac{M_1}{E \cdot J} \cdot \frac{D_{bh}}{2} \cdot d\gamma + \int_{\beta}^{\omega} M_{BBI} \cdot \frac{M_1}{E \cdot J} \cdot \frac{D_{bh}}{2} \cdot d\gamma,$$

(5)

where $M_{CB}$ is the moment of flection on the segment $CB$, N.m,

$$M_{CB} = \frac{R_c \cdot D_{bh} \cdot \sin(\gamma - \omega)}{2},$$

$M_{BBI}$ is the moment of flection on the segment $BBI$, N.m,

$M_1$ is the moment of flection from the single load applied in point $C$ and performing towards point $A$, N*m.

$$M_1 = \frac{D_{bh} \cdot \sin(\gamma - \omega)}{2}.$$

After making corresponding substitutions in the equation (5) and integration we get:

$$S = \frac{R_c \cdot D_{bh}^3}{16EJ} \left[ (\beta - 2\omega) - \frac{1}{2} \sin(2(\beta - 2\omega)) \right] + \frac{SD_{bh}}{2D_c} \left[ \cos(\beta - 2\omega) + \cos\omega \right].$$

(6)

Solving the set of equations (3) we find $R_C, R_B$:

$$R_C = \frac{4 \cdot E \cdot J \cdot S}{D_{bh}^2 \cdot D_R} \left( \frac{1}{\sin(\beta - \omega)} \right), \quad R_B = -\frac{4 \cdot E \cdot J \cdot S}{D_{bh}^2 \cdot D_R} \cdot \ctg(\beta - \omega)$$

(7)

Acquired formulae contain angle $\beta$. Its value is determined after substitution of the value $R_C$ in the equation (6).

After transformations we get:

$$\frac{D_{bh}}{D_R} \cdot \frac{1}{\sin(\beta - \omega)} \left[ (\beta - 2\omega) - \frac{1}{2} \sin(2(\beta - 2\omega)) \right] + \frac{2D_{bh}}{D_R} \left[ \cos(\beta - 2\omega) + \cos\omega \right] = 4$$

(8)

Numerical solution of transcendental equation (8) makes it possible to determine angle $\beta$.

After substitution of the angle $\beta$ value into (7) $R_C, R_B$ are calculated by analytical formulae.

The load-bearing capacity is determined by the following formula:

$$F = 2[R_c + R_B + qD_{bh}(\pi - \beta)] \cdot f \cdot k_1,$$

where $k_1$ is the length coefficient, the ratio of the real working unit of the rod to the unit length. $f$ is the coefficient of sliding friction in ground state.

3. Experimental research

Physical simulation of the process of installing the anchor 1 m in length in the demountable borehole model that consists of five identical steel modules (figure 3) was performed to confirm the accepted assumptions and deformation scheme.
Figure 3. The borehole model with installed friction anchor: general view (a); while recording of the state of the anchor in the borehole (b).

Each module was produced separating along the centreplane. It allowed to fix the nature of the anchor- borehole model interaction (figure 4).

Figure 4. Recording of the state of the anchor rod in the borehole model.
The recording of the nature of anchor interaction was performed at the second and the third modules. The borehole model was 46 mm in diameter. Anchor of 48 mm in initial diameter was set. As a result of performed series of setting anchor into a borehole was settled that the width of axial slot $b_D$ had increased from 23 mm to 15,1-15,7 mm, angle $\beta$ had totaled $117^0 - 123^0$. Final installation effort is 48 kN (the width of the anchor wall is 3 mm, Standard 20). The force required for relocation of the anchor (load-bearing capacity) from the borehole model is 32,6 kN. ‘TIKA 3’ puller of a rock bolt was used during the project. Forces were registered using pressure gauge indication. Diagram of the variation of installation effort is presented in figure 5.

![Figure 5. The diagram of variation during installation of the anchor to the borehole model.](image)

4. The sequence of calculation of load-bearing capacity of a friction-type anchor

Not only parameters of friction anchor rod are needed to be taken into account in the process of calculation but also results of its force interaction with the borehole walls. The first stage is the preliminary calculation. Initial values of force factors: $q$, $R_C$, $R_B$ and angle $\beta$ are determined according to given data such as borehole diameter $D_{bh}$, rod diameter $D_R$, slot width $b$, wall width $t$, elastic modulus $E$.

The second stage is evaluation of the borehole perceptibility of initial values of force factors. The destruction of the borehole surface attended with insertion of edge zones to the kind and appearance of contact area is provided by point force $R_C$. The process of insertion takes place until distributed load caused by force $R_C$ reaches the limit strength of kind.

Necessary area (line) of a contact in point C is calculated according to the structural behavior of the kind material. The depth of insertion of the rod edge zones to the borehole (decrease of value $S$ to $S'$) and actual increase of the borehole diameter to $D_{bh}'$ is being determined.

The third stage is calculation of parameters $q$, $R_C$, $R_B$, $\beta$ according to corrected numerical values $S'$, $D_{bh}'$. Calculation of load-bearing capacity $F$.

5. Conclusions

The scheme of interaction of a tubular friction anchor and a borehole walls is experimentally-confirmed.
Analytical correspondences enable to calculate load-bearing capacity taking into account geometric and structural characteristics of the anchor rod are acquired. The deviation of calculated values from experimental values did not exceed the load-bearing capacity 9.6 % and angle $\beta$ 5.2 – 8.5%.

The developed technique of calculation of the load-bearing capacity of the friction type tubular anchor makes it possible to perform analysis of degree of impact of its dimensions on the main functional parameter.

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