Modelling Dependence Structure of Exchange Rate and Energy Price by C-Vine Copula in China

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Abstract. This study aims to investigate the co-movement between the exchange rate and the different energy prices by using the canonical vine copula approach. China needs to import large amounts of oil, natural gas, and other energy sources every year, thus, the exchange rate plays a vital role in the energy market import in China. This study uses C-vine copula to estimate the interdependence between the exchange rate and the different energy prices including gasoline price, coal price, and liquified natural gas price. The result shows that the exchange rate has a positive dependency effect on all energy prices, especially the highest degree of interdependence between the exchange rate and gasoline prices.

1. Introduction
Energy issues have always been a topic of common concern to countries around the world. With the development of modern industrialization, the consumption of energy in China is increasing, and since 2010, China's total oil consumption has ranked first in the world, in 2019, China's dependence on oil reached 72%. China has become the largest natural gas importing country in the world in 2018. However, the imported energy is associated with risk and cost, especially the risks of exchange rate fluctuations; the fluctuation of the exchange rate would result in volatile energy prices. There are a lot of studies that concentrate on the correlation of exchange rate and distinct energy prices, for instance, oil price. Beckmann [1] indicated that exchange rate and oil price are closely linked. Sadorsky [2] showed the exchange rate has a long-term cointegration relation with the futures prices of crude oil, heating oil, and unleaded gasoline. The rise in oil prices has different effects on the exchange rates of different countries, Ghosh [3] and Lizardo [4] deemed that the increase in oil price returns caused the currency to depreciate against the US dollar in India and Japan. For Fiji Island, Narayan [5] found that rising oil prices resulted in an appreciation of the Fijian dollar. However, these studies focus on the relationship between exchange rates and oil prices or other energy prices individually, scarce research inspect the fluctuation of exchange rate impact on energy price at the same time, while the volatile crude oil price might encourage people to use the alternative energy, which might affect the price of other energy products.

In the previous studies, researchers used different models and approaches to analyze exchange rates and energy prices. Hussain [6] showed there is a negative weak correlation between exchange rates and oil prices in Asian countries by detrended cross-correlation approach. Kumar [7] found that there is a nonlinear correlation between the exchange rate and oil using nonlinear ARDL tests. The linear model might result in unideal empirical result, and the research ignores the dependence relation between the exchange rate and energy price. Lu [8] showed the distribution was asymmetric, and the skewed
Student's t-distribution is more suitable than normal distribution on crude oil futures and natural gas futures, through the time-varying copula-GARCH model. Copula can flexibly capture the dependency between two variables because copula has many families, and copula can also consider whether the dependence can be asymmetric and the relationship can be nonlinear. Therefore, we aim to examine a dependency structure between exchange rates and energy prices in China, moreover we estimate the vine-copula model using monthly data from January 2014 to December 2019.

The structure of this study is as follows: The method we adopted will be described in Section 2. Section 3 summarizes the data and empirical results, and section 4 draws a conclusion.

2. Methodology

In this study, we show the basic theories of C-vine copulas and GJR-GARCH model, we used the GJR-GARCH model, and three skewness distributions will be used including skew-normal distribution (snom), skewed Student's t-distribution (sstd), and skew generalized error distribution (sged). Then imported the results of GJR-GARCH into c-vine copulas, and we applied C-vine copulas to investigate the interdependence between the exchange rate and the different energy prices.

2.1. Canonical vines Copula functions

The concept of the copula is a broken-down multivariate distribution into components by estimating marginals and dependency structure separately. The Sklar's theorem allows random variables $X_i$ with continuous marginal distribution functions $F_i$, variables $U_i = F_i(X_i)$, which are uniformly distributed random variables on $[0, 1]$, The function $C$ is exactly a copula

$$F(X_1, X_2) = P(X_1 \leq x_1, X_2 \leq x_2) = P(U_1 \leq F_1(x_1), U_2 \leq F_2(x_2)) = C(F_1(x_1), F_2(x_2)) \quad (1)$$

Joe [9] proposed vines copula and developed Canonical vines (C-vines) in Bedford and Cooke [10]; vines are a flexible model that decomposes multivariate copula into bivariate copulas in Kurowicka and Cooke [11]. Let $X = (X_1, X_2, X_3)^T$ with univariate marginal distribution functions $F_1, F_2$ and $F_3$, such that:

$$f(x_1, x_2, x_3) = f_1(x_1) f_2(x_2 | x_1) f_3(x_3 | x_1, x_2) \quad (2)$$

by Sklar's theorem we know that

$$f(x_2 | x_1) = \frac{f(x_2, x_1)}{f(x_1)} = c_{1,2}(F_1(x_1), F_2(x_2)) f_2(x_2) \quad (3)$$

$$f(x_3 | x_1, x_2) = \frac{f(x_3, x_1, x_2)}{f(x_2 | x_1)} = c_{2,3}(F_2(x_2 | x_1), F_3(x_3 | x_1, x_2)) f_3(x_3) \quad (4)$$

Vines copula arrange the $(e-1)/2$ pair-copulas of an $e$-dimensional Pair-copula construction in $e-1$ linked trees, which $e$ is the number of random variables in the model, such as $e=3$ shown in Figure 1.

![Fig.1 C-vine structure for three-dimensional.](image-url)
2.2. Glosten-Jagannathan-Runkle GARCH model
The GARCH model was put forward by Engle [12] and extended by Rockinge [13]. The marginal
distribution of each variable is characterized by GJR-GARCH model before apply copula function. The
Glosten-Jagannathan-Runkle GARCH model has an additional leverage effect, which means that the
data can be asymmetric. The GJR-GARCH model can be represented as
\[
\sigma_t^2 = \sigma_0^2 + \sum_{z=1}^{q} \beta_z \sigma_{t-z}^2 + \sum_{m=1}^{p} \alpha_m \epsilon_{t-m}^2 + \sum_{m=1}^{r} \gamma_m \epsilon_{t-m}^2 I_{t-m}
\]
(5)
where \( I_{t-m} = 0 \) if \( \epsilon_{t-m} \geq 0 \) and \( I_{t-m} = 1 \) if \( \epsilon_{t-m} < 0 \). \( \alpha_0, \alpha, \beta, \gamma \) express the parameters and the
leverage effect impact of \( \gamma \).

3. Empirical Results and Discussions

3.1. Description of data
In this study, we used the monthly closed price of USD to the RMB exchange rate (USD/CNY) and the
monthly market average price of energy prices, namely, Liquefied Natural Gas price (LNG), Anthracite
price (COAL), Gasoline price (GAS) as sourced from CEIC data. The observation period of the data is
from January 1, 2014, to December 31, 2019. The log-difference of the monthly energy prices return
and the exchange rate return \( (r_t) \) was calculated separately, which can be expressed as
\[
r_t = \log \left( \frac{p_t}{p_{t-1}} \right)
\]

Table 1 data description

|                | Exchange rate | GAS          | COAL         | LNG           |
|----------------|---------------|--------------|--------------|---------------|
| **Mean**       | 6.570137      | 7685.332747  | 1008.342466  | 3902.794521   |
| **Standard Deviation** | 0.30253862   | 1520.574265  | 149.8645888  | 837.6907405   |
| **Skewness**   | 0.020209461   | -0.655305621 | 0.063646393  | 1.223531277   |
| **Kurtosis**   | -1.273074941  | -0.281975097 | -0.921320271 | 2.799424949   |
| **Min**        | 6.06          | 3824         | 771          | 2850          |
| **Max**        | 7.16          | 10022.14     | 1322         | 7320          |
| **Median**     | 6.61          | 7955.12      | 1000         | 3780          |

Table 1 shows all of the means of variables are positive. The skewness of the exchange rate, COAL,
and LNG is positive which express the skewed distribution is right-skewed distribution; with a tail on
the right. The skewness of GAS is negative, which means the marginal distribution has a tail on the left.
All of the kurtosis of variables is negative except LNG, the negative kurtosis of LNG shows that the
marginal distribution of LNG is flatter compared with other variable distributions. The kurtosis of LNG
is the largest positive value and shows the distribution is steeper compares with other variables.

Table 2. Augmented Dickey-Fuller Test (ADF) for variables

|                | Exchange rate | GAS          | COAL         | LNG           |
|----------------|---------------|--------------|--------------|---------------|
| **ADF**        | -20.304       | -4.5133      | -3.4819      | -4.1364       |
| **Bayes factors** | 0.054         | 0.054        | 0.054        | 0.054         |

Table 2 indicates the stationary test for the return of the exchange rate and the different energy prices.
Before empirical analysis, it is necessary to test the stationary of the time series data, if the variable data
is not stable, there will be a unit root. The Bayes factors are $0.054 < 0.1$ that expresses strong evidence the variable is stationary [14]. Thus, all the variables are stationary.

3.2. Empirical results

Table 3. The estimated result for variables using GJR-GARCH model.

| Exchange rate | GAS          | COAL         | LNG          |
|---------------|--------------|--------------|--------------|
| $\omega$      | 0.001874     | 0.000387     | 0.000018     | 0.000271     |
| $\alpha$      | 0.264074     | 0.499831     | 0.474562     | 0.109544     |
| $\beta$       | 0.403145     | 1.000000     | 0.828360     | 0.286231     |
| $\gamma$      | -0.560116    | -1.000000    | -0.647137    | 0.731573     |
| Skew          | 0.100028     | 1.408684     | 1.094494     | 0.813138     |
| Distribution  | nom          | std          | std          | nom          |
| AIC           | -3.5235      | -2.2425      | -4.2467      | -4.1829      |

Table 3 shows the GJR-GARCH model to estimate the optimal parameter. According to the AIC, the return of exchange rate and LNG is the best fit for a skewed normal distribution, the return of GAS and COAL price is more suitable for skewed Student t distribution.

Table 4. The results of the Canonical vines Copula

| Copula Family | Tree1 | Tree2 | Tree3 |
|---------------|-------|-------|-------|
| C (FX, GAS)   | Survival Joe | -0.50 | Rotated Clayton 90° |
| C (FX, COAL)  | Clayton | 0.07  | -0.03 |
| C (FX, LNG)   | Joe    | 0.05  | -0.06 |
| C (GAS, COAL| Gaussian| -0.03 | -0.06 |
| C (GAS, LNG| Frank   | 0.05  | -0.04 |
| C (COAL, LNG| Rotated Clayton 90° | -0.03 | -0.06 |
| S.E.          | 0.1378 | 0.2229 | 0.2074 |
| par2          | 0.00   | 0.00   | 0.00   |
| utd           | -      | -      | -      |
| ltd           | 0.34   | -      | -      |
| AIC           | 1.4437 | 1.5957 | 0.2035 |

Table 4 shows the estimated results of the exchange rate and the GAS price, COAL price, and LNG price from C-vine copulas. The results introduce the most appropriate copula based on the estimates of Kendall’s tau, tail dependence, the estimates of copula parameters, and AIC criteria. Kendall’s tau correlation Kendall’s tau is related to the copula function and can be transformed from copula; furthermore, we can use Kendall’s tau to compare the extent of interdependence between two variables due to the different of the bivariate copula families.

The first trees report the extent and structure of interdependence between the returns of the exchange rate and the returns of prices of different energy—GAS, COAL, and LNG. The estimated C-vine copula parameters of all variables and the values of Kendall’s tau of all variables are positive, which means positive interdependence between the exchange rate and the different energy prices (GAS, COAL, and LNG). This represents co-movements between the exchange rates and the prices of different energy—GAS, COAL, and LNG. GAS has the highest interdependence with the exchange rate followed by COAL and LNG which are 0.07 and 0.05 separately. For the second trees and third trees, the Canonical vines copula parameters and the values of Kendall’s tau s are estimated to be negative for all variables, which shows the returns of the GAS price are negatively correlated with the returns of the COAL and LNG prices as given the exchange rate.

As for the dependence structure, there are 6 pairs of copulas including survival Joe copula, Clayton copula, Joe copula, Gaussian copula, Frank copula, and Clayton Rotated 90 degrees copula. For the first trees, the copula families of survival Joe and Clayton have a lower tail dependence, which suggests that the relation of the exchange rate and GAS price or COAL price tends to be lower when the exchange
rate and GAS price, COAL price rise. Only Joe copula has upper tail dependence, which points to the possibility for the exchange rate and LNG price to jointly go up when they jointly rise. For the second trees, the feature of Gaussian copulas and Frank copulas are symmetrical, and do not have tail dependence, which suggests the conditional interdependence for GAS price and COAL price, GAS price, and LNG price both are symmetric as given in the exchange rate.

4. Conclusion
In this study, we investigated the interdependence between the exchange rate and different energy prices using the C-vine copulas based on the Glosten-Jagannathan-Runkle GARCH Model. There are exist a positive interdependence between the exchange rate and the different energy prices (GAS, COAL, and LNG). All of the energy prices have been affected by the exchange rate, all Kendall's tau between -0.06 and 0.17, of these energy prices, GAS prices are the most likely to fluctuate with exchange rate fluctuations. Moreover, GAS has a negative dependency effect on other energy prices as given in the exchange rate. For the policymaker, they should avoid excessive price fluctuations, ensure the stability of the energy price market environment. For the investors, they should take the exchange rate as a consideration when they invest in the energy market.

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