A Liouville String Approach to Microscopic Time and Cosmology

John Ellis\textsuperscript{a}, N.E. Mavromatos\textsuperscript{b}, D.V. Nanopoulos\textsuperscript{c,\diamond}

Abstract

In the non-critical string framework that we have proposed recently, the time $t$ is identified with a dynamical local renormalization group scale, the Liouville mode, and behaves as a statistical evolution parameter, flowing irreversibly from an infrared fixed point - which we conjecture to be a topological string phase - to an ultraviolet one - which corresponds to a static critical string vacuum. When applied to a toy two-dimensional model of space-time singularities, this formalism yields an apparent renormalization of the velocity of light, and a $t$-dependent form of the uncertainty relation for position and momentum of a test string. We speculate within this framework on a stringy alternative to conventional field-theoretical inflation, and the decay towards zero of the cosmological constant in a maximally-symmetric space.

\textsuperscript{a} Theory Division, CERN, CH-1211, Geneva, Switzerland,
\textsuperscript{b} Theoretical Physics Laboratory, ENSLAPP, Chemin de Bellevue, Annecy-le-Vieux, France, and
S.E.R.C. Advanced Fellow, University of Oxford, Dept. of Physics (Theoretical Physics), 1 Keble Road, Oxford OX1 3NP, United Kingdom,
\textsuperscript{c} Center for Theoretical Physics, Dept. of Physics, Texas A & M University, College Station, TX 77843-4242, USA, and
Astroparticle Physics Group, Houston Advanced Research Center (HARC), The Mitchell Campus, The Woodlands, TX 77381, USA

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1 Introduction

The subject of fundamental constants in field or string theory is a fascinating challenge for any physicist. Unfortunately, despite vigorous attempts, the topic remains controversial today. We would not exaggerate if we said that it is still at a very primitive stage of understanding. Following Weinberg [1], we can define as “fundamental constants” a list of constants whose “value we cannot calculate in terms of more fundamental constants . . . because we do not know of anything more fundamental . . .” . Thus, Weinberg concludes that “the membership of such a list reflects our present understanding of fundamental physics.”

We shall not attempt in this talk to offer any solutions to this fundamental problem in field theory. Instead, we shall examine the consequences of a recently-proposed approach [2] to the origin of time in non-critical String Theory [3, 4] for some “constants” in string theory, namely the speed of light $c$, and the fundamental (minimal) length $\lambda_s$ derived from the uncertainty principle [5].

According to Veneziano [6], string theory appears to have no fundamental constants per se, but these constants arise as a result of spontaneous breaking of symmetries in the selection of a ground state, thereby leading a (string) physicist to conclude that [6] “the constants of the vacuum are the constants of the world”. To prove or disprove this statement is a formidable task. Presumably, to understand, in String Theory, the origin of the fundamental constants that characterize the vacuum would require first an answer to the important and still unresolved issue of what (if anything) lifts the huge degeneracy that appears at present to characterize the string ground states. This question is tied up with the question on the origin of space-time.

As a step in this direction, we have formulated [2, 7] time in string theory as a renormalization group scale on the world-sheet, which we identify with the Liouville mode. The connection of time with the Liouville field in non-critical string theory has been considered in [8, 9] in order to incorporate an expanding Universe in string theory. In [2, 7] we took one step further, and interpreted the Liouville mode as a local renormalization group scale, which flows irreversibly according to a generalization of Zamolodchikov C-theorem [8]. This scale is introduced in theories with target space-time singularities or, more generally, event or particle horizons, as a result of the breaking of conformal invariance by the truncation of the matter theory to observable local low-energy modes. In the toy example of a two-dimensional black hole that we used to develop these ideas [2, 7], these low-energy propagating modes are coupled to quasi-topological delocalized higher-level string modes, as a result of infinite level-mixing gauge stringy symmetries [10]. These discrete modes are believed to be remnants of a topological phase in string theory [11]. The information carried by these modes, which is hidden from a low-energy observer who ‘measures’ using local scattering methods, is the root of the flow of time and the
non-equilibrium nature of the low-energy field theory derived from the string. As
we have noted [2], the fact that information is lost to these hidden modes causes
apparent decoherence of the observable modes, reflected in a collapse of their (part
of the full) wave function that is rapid for more complex systems.

In this scenario, the resulting low-energy two-dimensional string-inspired theory
appears not to have any ‘fundamental’ constants, however there are dimensionful
parameters that relax to their finite critical-string values asymptotically as the Liou-
ville time $t \to \infty$. These quantities cannot be calculated in terms of anything more
fundamental in the model. According to Weinberg’s definition, then, the asymptotic
values of these quantities, which in this two-dimensional example include the speed
of light and the minimal uncertainty in length, or $\hbar$, would characterize the vacuum
state and hence would be the dimensionful fundamental constants of that ground
state. In addition, there appears in the vacuum an arbitrary expectation value of
the dilaton field, which could be considered as an arbitrary dimensionless constant,
the analogue of the fine structure constant in four-dimensional models. In the non-
equilibrium model the dilaton field is a time-dependent scale factor that relaxes to
its asymptotic (vacuum) value. We find it interesting, but also quite natural, that the
non-equilibrium nature of the non-critical Liouville string induces the appearance of
constant quantities only in the asymptotic vacuum state via a (quantum) relaxation
process. After all, the best way of understanding an equilibrium state in statistical
mechanics is the study of the approach to it by perturbing the relevant system away
from equilibrium. This is precisely the way we studied time in string theory. Using
the concept of ‘measurement’ through localized fields, we have perturbed the confor-
mal string theory by operators that spoil conformal invariance and, thus, introduced
the concept of time as an evolution parameter of the relaxation process[2, 7, 9].

As a byproduct of our approach we speculate on cosmological applications, with
emphasis on the possibility for a string alternative to conventional field-theoretical
inflation. We also deal with the issue of the target-space cosmological constant in
generic $D$-dimensional maximally-symmetric spaces, in models with a dilaton such as
certain no-scale supergravities [12], and find [9] that it relaxes to zero asymptotically.

2 String-Inspired Density Matrix Mechanics

In this section we review briefly the formalism of first-quantized non-critical
strings, and its connection with the Density Matrix Mechanics that we proposed
recently [4]. We emphasize the restoration of criticality by turning a $(1, 1)$ deforma-
tion into an exactly marginal one by Liouville dressing, thereby leading, in our
interpretation, to time-dependent backgrounds. Consider a two-dimensional critical
conformal field theory model described by an action $S_0(r)$ on the world sheet, where
the $\{r\}$ are matter fields spanning a $D$-dimensional target manifold of Euclidean
signature, that we term “space”. Consider, now, a deformation
\[ S = S_0(r) + g \int d^2z V_g(r) \] (1)

Here \( V_g \) is a \((1,1)\) operator, i.e. its anomalous dimension vanishes, but it is not *exactly marginal* in the sense that the operator-product expansion coefficients \( C_{ggg} \) of \( V_g \) with itself are non-zero in any renormalization scheme, and hence ‘universal’ in the Wilsonian sense. The scaling dimension \( \alpha_g \) of \( V_g \) in the deformed theory (1) is, to \( O(g) \) [13],
\[ \alpha_g = -gC_{ggg} + \ldots \] (2)

Liouville theory [14] requires that scale invariance of the theory (1) be preserved. The non-zero scaling dimension (2) would jeopardize this, but scale invariance is restored if one dresses \( V_g \) gravitationally on the world-sheet as
\[ \int d^2z V_g(r) \to \int d^2z e^{\alpha_g \phi} V_g(r) = \int d^2z g V_g(r) - \int d^2z g^2 C_{ggg} V_g(r) \phi + \ldots \] (3)

where \( \phi \) is the Liouville field. The latter acquires dynamics through integration over world-sheet covariant metrics, \( \gamma_{\alpha\beta} \), after conformal gauge fixing \( \gamma_{\alpha\beta} = e^{-\phi} \hat{\gamma}_{\alpha\beta} \) in the way discussed in [14, 15]. Scale invariance is guaranteed through the definition of renormalized couplings \( g_R \), given in terms of \( g \) through the relation
\[ g_R \equiv g - C_{ggg} \phi g^2 + \ldots \] (4)

This equation leads to the correct \( \beta \)-functions for \( g_R \)
\[ \beta_g = -C_{ggg} g_R^2 + \ldots \] (5)

implying a renormalization-group scale \( \phi \)-dependence of \( g_R \). The reader might have noticed that above we viewed the Liouville field as a local scale on the world-sheet [2, 7]. Local world-sheet scales have been considered in the past [16, 17], but the crucial difference of our Liouville approach is that this scale is made dynamical by being integrated out in the path-integral. In this way, in Liouville strings the local dynamical scale acquires the interpretation of an additional target coordinate. If the central charge of the matter theory is \( c_m > 25 \), the signature of the kinetic term of the Liouville coordinate is opposite to that of the matter fields \( r \), and thus the Liouville field is interpreted as Minkowski target time [3, 4]. In this interpretation the renormalization group evolution of the matter system alone, ignoring Liouville dynamics, is interpreted as time evolution, in which the time appears as an ‘external’ parameter. As argued in ref. [18, 7], the actual time flow is opposite to the usual renormalization group flow, the latter being from ultraviolet to infrared fixed points. This has to do with metastability interpretation of the renormalization group flow in Liouville strings [18, 7], where the infrared fixed point is viewed as a ‘bounce’ point of the flow [18].

It is instructive to review the properties of such a flow.
(i) Due to the assumed unitarity of the matter system, the flow is **irreversible**, as can be seen using the Zamolodchikov $C$-theorem \[8\]. The effective central charge (Zamolodchikov $C$-function \[8\]) $C(g)$ varies with time as

$$\partial_t C(g) = \beta^i < V_i V_j > \beta^j \equiv \beta^i G_{ij} \beta^j \geq 0$$

(6)

where $G_{ij}$ plays the rôle of a metric in coupling constant space. This implies a time **arrow** in all such models.

(ii) Defining a density matrix for the system, in a coupling constant phase space $g^i$, $p_i$, one has the following modified Liouville equation \[2, 20\]

$$\partial_t \rho = \{H, \rho\} + \beta^i G_{ij} \frac{\partial \rho}{\partial p_j}$$

(7)

where $\{,\}$ denote the appropriate Poisson bracket and $H$ is a Hamiltonian function. The latter is defined in terms of the effective action, which in string theories coincides with the Zamolodchikov $C$-function \[21\]

$$C(g) = \int dt \rho^\dagger i^i - H$$

(8)

The total probability $P = \int dg^i dp_i tr \rho(g, p, t)$ is conserved, since its rate of change is a boundary term in phase space, and the latter is assumed to have no boundary.

(iii) Quantization of the coupling constants $g^i$ is achieved as follows: the $g^i$ actually represent target-space background fields in string theory, and as such they are quantum field operators in a string field theory concept, where summation over genera is assumed. Hence the situation is similar to a two-dimensional wormhole calculus. From a quantum point of view, the Poisson brackets $\{,\}$ are then replaced by appropriate commutators, and the couplings $g^i$ are viewed as Heisenberg operators \[2, 4\].

(iv) Entropy $S$ is defined in terms of the density matrix $\rho$ as usual, by

$$S = -Tr \rho ln \rho$$

(9)

and it increases with time $t$

$$\partial_t S = \beta^i G_{ij} \beta^j S \geq 0$$

(10)

thereby implying, if any $\beta^i \neq 0$, non-equilibrium for the non-critical matter theory. The latter approaches asymptotically its critical state as an ultraviolet fixed point of the flow.
Energy $H$ is conserved on the average, in this quantum version, as a result of the renormalizability of the $\sigma$-model

$$\partial_t \langle H \rangle = \partial_t \text{Tr} H \rho = \text{Tr}(\rho \partial_t H) + \text{Tr}(H \partial_t \rho) = \partial_t (p_i \beta^i) = 0 \quad (11)$$

where we used (7) and (8). The vanishing result is a consequence of the fact that neither $p_i = \frac{\delta}{\delta g^i}$ nor $\beta^i$ have an explicit scale dependence in renormalizable theories. However, fluctuations in the energy are time-dependent. For instance,

$$\partial_t \langle H^2 \rangle = \langle \frac{d\beta^i}{dt} G_{ij} \beta^j \rangle \neq 0 \quad (12)$$

in general, which affects energy uncertainty relations for the string.

(vi) The above picture leads naturally to collapse of part of the wave function describing observable modes, in a way similar to that proposed in ref. [22] within a version of the wormhole calculus, namely the asymptotic vanishing (as $t \to \infty$) of the off-diagonal elements in space of the matter density matrix $\rho$:

$$\rho_{\text{out}}(x - x') = \rho_{\text{in}}(x, x') e^{-D(x - x')^2 + ...} \quad (13)$$

where $x, x'$ are target spatial coordinates, and $D$ is proportional to the sum of the squares of the anomalous dimensions of the relevant perturbations of the $\sigma$-model.

3 The Two-Dimensional String Black Hole Model

As an illustration of our approach to non-critical string theory, we now discuss the two-dimensional black hole model of ref. [23]. We regard it as a toy laboratory that gives us insight into the nature of time in string theory and contributes to the physical effects mentioned in the previous section.

The action of the model is

$$S_0 = \frac{k}{2\pi} \int d^2 z \left[ \partial r \bar{r} - \tanh^2 r \partial t \bar{t} \right] + \frac{1}{8\pi} \int d^2 z R^{(2)} \Phi(r) \quad (14)$$

where $r$ is a space-like coordinate and $t$ is time-like, $R^{(2)}$ is the scalar curvature, and $\Phi$ is the dilaton field. The customary interpretation of (14) is as a string model with $c = 1$ matter, represented by the $t$ field, interacting with a Liouville mode, represented by the $r$ field, which has $c < 1$ and is correspondingly space-like. As an illustration of the approach outlined in the previous section, however, we re-interpret (14) as a fixed point of the renormalization group flow in the local scale variable $t$. In our interpretation, the “matter” sector is defined by the spatial coordinate $r$, and has central charge $c_m = 25$ when $k = 9/4$ [23]. Thus the model describes a critical string in a dilaton/graviton background. The fact that this is static, i.e. independent of $t$, reflects the fact that one is at a fixed point of the renormalization group flow.
We now outline how one can use the machinery of the renormalization group in curved space, with \( t \) introduced as a local renormalization scale on the world sheet, to derive the model (14). A detailed technical description is given in [9, 7]. There are two contributions to the kinetic term for \( t \) in our approach, one associated with the Jacobian of the path integration over the world-sheet metrics, and the other with fluctuations in the background metric.

To exhibit the former, we first choose the conformal gauge \( \gamma_{\alpha\beta} = e^\rho \hat{\gamma}_{\alpha\beta} \) [14, 15], where \( \rho \) represents the Liouville mode. We will later identify \( \rho \) with an appropriate function of \( \phi \), thereby making the local scale \( \phi \) a dynamical \( \sigma \)-model field. Ref. [15] contains an explicit computation of the Jacobian using heat-kernel regularization, which yields

\[
- \frac{1}{48\pi} \left[ \frac{1}{2} \partial_\alpha \rho \partial^\alpha \rho + R^{(2)}(\rho) + \frac{\mu}{\epsilon} e^\rho + S_G \right]
\]  

(15)

where the counterterms \( S'_G \) are needed to remove the non-logarithmic divergences associated with the induced world-sheet cosmological constant term \( \frac{\mu}{\epsilon} e^\rho \), and depend on the background fields. This procedure reproduces the critical string results of ref. [23] when one identifies the Liouville field \( \rho \) with \( 2\alpha' \phi \). Equation (15) contains a negative (time-like) contribution to the kinetic term for the Liouville (time) field, but this is not the only such contribution, as we now show.

We recall that the renormalization of composite operators in \( \sigma \)-models formulated on curved world sheets is achieved by allowing an arbitrary dependence of the couplings \( g^i \) on the world-sheet variables \( z, \bar{z} \) [16, 17]. This induces counterterms of "tachyonic" form, which take the following form in dimensional regularization with \( d = 2 - \epsilon \) [17]:

\[
\int d^2z \Lambda_0
\]

(16)

where

\[
\Lambda_0 = \mu^{-\epsilon}(Z(g)\Lambda + Y(g))
\]

(17)

Here \( Z(g) \) is a common wave function renormalization that maps target scalars into scalars, \( \Lambda \) is a residual renormalization factor, and the remaining counterterms \( Y(g) \) can be expanded as power series in \( 1/\epsilon \), with the the one-loop result giving a simple pole. Simple power-counting yields the following form for \( Y(g) \):

\[
Y(g) = \partial_\alpha g^i G_{ij} \partial^\alpha g^j
\]

(18)

where \( G_{ij} \) is the analogue of the Zamolodchikov metric [8] in this formalism, which is positive for unitary theories. It is related to the divergent part of the two-point function \( \langle V_i V_j \rangle \) [17] that cannot be absorbed in the conventional renormalization of the operators \( V_i \). We need to consider a \( \sigma \)-model propagating in a graviton background \( G_{MN} \), in which case a standard one-loop computation [17] yields the following result for the simple \( \epsilon \)-pole in \( Y \):

\[
Y^{(1)} = \frac{\lambda}{16\pi \epsilon} \partial_\alpha G_{MN} \partial^\alpha G^{MN}
\]

(19)
where $\lambda \equiv 4\pi\alpha'$ is a loop-counting parameter. We note that the wave-function renormalization $Z(g)$ vanishes at one-loop. In ref. [17], $G_{MN}$ was allowed to depend arbitrarily on the world-sheet variables, and all world-sheet derivatives of the couplings were set to zero at the end of the calculation. In our Liouville mode interpretation, we assume that such dependence occurs only through the local scale $\mu(z, \bar{z})$, so that

$$\partial_\alpha g^i = \hat{\beta}^i \partial_\alpha \phi(z, \bar{z})$$

(20)

where $\hat{\beta}^i = \epsilon g^i + \beta^i(g)$ and $\phi = \ln \mu(z, \bar{z})$. Taking the $\epsilon \to 0$ limit, and separating the finite and $O(1/\epsilon)$ terms, we obtain for the former

$$O(1) - \text{terms}: \quad \text{Res}Y^{(1)} = \alpha'^2 R \partial_\alpha \phi \partial^\alpha \phi$$

(21)

where $R$ is the scalar curvature in target space, and we have used the fact that the one-loop graviton $\beta$-function is

$$\beta_{MN}^G = \frac{\lambda}{2\pi} R_{MN}$$

(22)

The non-logarithmic divergent terms

$$\frac{1}{\epsilon} \beta^i G^{(1)}_{ij} \beta^j$$

(23)

do not contribute to the renormalization group, and can be removed explicitly by target-space metric counterterms

$$S_G = \frac{1}{\epsilon} G_{\phi\phi} \partial_\alpha \phi \partial^\alpha \phi + \delta S(\phi, r)$$

(24)

where the coefficients $G_{\phi\phi}$ are fixed by the requirement of cancelling the $1/\epsilon$ terms (23). The $\delta S$ denotes arbitrary finite counterterms, which are invariant under the simultaneous conformal rescalings of the fiducial world-sheet metric, $\hat{\gamma} \to e^\sigma \hat{\gamma}$, and local shifts of the scale $\phi \to \phi - \sigma$. This last requirement arises as in the conventional approach to Liouville gravity [14, 15], where the local renormalization scale $\phi$ is identified with the Liouville mode $\rho$, after appropriate normalization. In our interpretation one is forced to treat the scale $\phi$ simultaneously as the target time coordinate.

In the case of the Minkowski black hole model of ref. [23], the Lorentzian curvature is

$$R = \frac{4}{\cosh^2 r} = 4 - 4\tanh^2 r,$$

(25)

which we substitute into equation (21) to obtain the form of the second contribution to the kinetic term for the Liouville field $\phi$. Combining the world-sheet metric Jacobian term in (15) with the background fluctuation term (21) (25), we finally obtain the following terms in the effective action

$$\frac{1}{4\pi\alpha'} \int d^2z [\partial_\alpha r \partial^\alpha r - \tanh^2 r \partial_\alpha \phi \partial^\alpha \phi + \text{dilaton - terms}]$$

(26)
Thus we recover the critical string $\sigma$-model action \eqref{14} for the Minkowski black-hole. Dilaton counterterms are incorporated in a similar way, yielding the dilaton background of \cite{23}. In addition, as standard in stringy $\sigma$-models, one also obtains the necessary counterterms that guarantee target-space diffeomorphism invariance of the Weyl-anomaly coefficients \cite{16}. Details are given in ref. \cite{9}.

It should be noticed that the renormalization group yields automatically the Minkowski signature, due to the $c_m = 25$ value of the matter central charge [3, 4]. However, as we remarked in ref. [9, 7], one can also switch over to the Euclidean black hole model, and still maintain the identification of the compact time with some appropriate function of the Liouville scale $\phi$ that takes into account the compactness of $t$ in that case. This Euclidean version is better studied from the point of view of discussing exactly marginal deformations that turn on matter in the model \eqref{14}. In ref. \cite{24} it was argued that the exactly marginal deformation that turned on a static tachyon background for the black hole of ref. \cite{23} necessarily involved the higher-level topological string modes, which are non-propagating delocalized states. This is a consequence of the operator product expansion of the tachyon zero-mode operator $F^{c \ -\frac{1}{2}, 0}$ \cite{24}:

\[ F^{c \ -\frac{1}{2}, 0} \circ F^{c \ -\frac{1}{2}, 0} = F^{c \ -\frac{1}{2}, 0} + W^{h w}_{-1, 0} + W^{l w}_{-1, 0} + \ldots \]  

(27)

where we only exhibit the appropriate holomorphic part for reasons of economy of space. The $W$ operators and the $\ldots$ denote level-one and higher string states. The latter cannot be detected in local scattering experiments, due to their delocalized character. From a formal field-theoretic point of view such states cannot exist as asymptotic states to define scattering, and also cannot be integrated out in a local path-integral. They can only exist as marginal deformations in a string theory. An ‘experimentalist’ therefore sees necessarily a truncated matter theory, where the only deformation is the tachyon $F^{c \ -\frac{1}{2}, 0}$, which is a $(1,1)$ operator in the black hole $\sigma$-model \eqref{14}, but is not exactly marginal. This truncated theory is non-critical, and hence Liouville dressing in the sense of \eqref{3} is essential, thereby implying time-dependence of the matter background. Due to the fact that the appropriate exactly-marginal deformation associated with the tachyon in these models involves all higher-level string states, one can conclude that in this picture the ensuing non-equilibrium time-dependent backgrounds are a consequence of information carried off by the unobserved topological string modes. The rôle of the space-time singularity\footnote{We would like to stress that the notion of ‘singularity’ is clearly a low-energy effective-theory concept. The existence of infinite-dimensional stringy symmetries associated with higher-level string states ($W_\infty$-symmetries \cite{10}) ‘smooth out’ the singularity, and render the full string theory finite.} was crucial for this argument. Indeed, in flat target-space matrix models \cite{25} the tachyon zero-mode operator $F$ is exactly marginal. As we shall argue later on, these flat models can be regarded as an asymptotic ultraviolet limit in time of the Wess-
Zumino black hole. Hence, any time-dependence of the matter disappears in the vacuum, leading to equilibrium.

In ref. [7] an additional deformation was considered in parallel with the tachyon. This was the instanton vertex

\[ V_I = g^I \int d^2 \rho \rho e^{\rho \partial(\sinh \rho - it)} + hc \]  

(28)

where \( g^I \ll 1 \) in a dilute-gas approximation. The physics behind such a deformation lies in the world-sheet interpretation of the Euclidean (Minkowski) black hole as representing a world-sheet vortex (spike) [27]. The charge of such objects is proportional to the black-hole mass, and hence the instantons, that induce transitions between defects of different charge, can represent mass changes of black holes. Higher-genus (quantum) effects are known [28] to produce black hole decay, and it is sensible to consider instanton effects as being related to these. In addition, from a conformal field theory point of view, it is known that deformations consisting purely of higher-level string modes do induce global rescalings to the black-hole metric [24], and hence shifts in its mass. Therefore we associate instanton deformations with local renormalization counterterms representing collectively higher-level and higher-genus, i.e. quantum string, effects.

Instantons in these models appear to have logarithmically-divergent world-sheet actions and topological charge, but their contribution to the path integral is finite. From a renormalization point of view they are by themselves irrelevant operators. As we mentioned before, their effect is similar to that of higher genera [29]. They induce world-sheet ultraviolet-divergent (logarithmic) shifts to the dilaton field, and when considered in the model deformed by a tachyon, their effect is to lead to extra infinities that cannot be absorbed in conventional renormalization of the tachyon fields. The presence of matter is essential in this framework, given that the instantons alone cannot affect the renormalization group flow induced by relevant operators (tachyons). Instead, in the matter-instanton deformed theory one has a renormalization of the Wess-Zumino level parameter, which, thus becomes scale-dependent. It can be shown [4, 9] that close to the ultraviolet fixed point the renormalized \( k_R \) in the dilute-gas approximation is given by

\[ k_R(t) \simeq e^{-4\pi \beta^I T_{0t}} \]  

(29)

where \( t \) is the target time, and \( T_0 \) denotes the coupling of the (relevant) tachyon perturbation. The instanton \( \beta \)-function itself depends on \( k \), e.g. \( \beta^I = -\frac{k}{2} g^I \) in the case of large \( k \), where the instantons form a dilute gas, giving an exponential increase of \( k_R \) with \( t \) [7, 9].
4 Time-Dependent “Fundamental Constants”

The consequences of the renormalization effect \((29)\) of the effective level parameter \(k_R(t)\) are dramatic for the physics of the string. First, we recall that the mass \(M\) of the black hole is given in units of the Planck mass by \([23, 7]\)

\[
M/M_{\text{planck}} = \sqrt{\frac{1}{k_R(t) - 2} e^{\text{const}}} \tag{30}
\]

implying asymptotically a vanishing-mass black-hole, i.e. flat target-space time \([23, 10]\). This makes the connection with the flat \(c = 1\) matrix model at the ultraviolet fixed point of the flow.

Secondly, we note that the exact target-space background metric of the Wess-Zumino \(\sigma\)-model (for finite \(k\)) has the following asymptotic form for large \(r \rightarrow \infty\) \([31]\)

\[
ds^2 = 2(k - 2)(dr^2 - \frac{k}{k - 2} dt^2) \tag{31}
\]

In view of \((29)\), the above relation implies an effective time dependence of the velocity of light:

\[
c_q = c \sqrt{\frac{k(t)}{k(t) - 2}} \tag{32}
\]

where \(c \equiv c(\infty)\).

The result \((32)\), implying an increase of the velocity of light as one approaches the infrared fixed point, can be re-derived from a local-renormalization-group viewpoint. Consider the two-dimensional string model \((14)\) perturbed by an irrelevant deformation \(g^i\). As shown in ref. \([4]\) and reviewed in section 3, following the local-renormalization-scale formalism of ref. \([16, 17]\) in dimensional regularization \(d = 2 - \epsilon\), an induced metric counterterm for the time-component of the metric tensor assumes the form,

\[
g^i \mathcal{G}_{ij}^{(1)} \beta^i \partial \phi \overline{\partial \phi} \tag{33}
\]

where \(\phi\) is the (time-like) Liouville field, and \(\mathcal{G}_{ij}^{(1)}\) is the residue of the first pole in \(\epsilon\) of the Zamolodchikov metric in coupling constant space. This term contributes to the wave function renormalization of \(\phi\). For irrelevant deformations, like instantons in the black-hole, the \(\beta^i < 0\) and the counterterm \((33)\) is negative. Taking into account the discussion in section 3 showing that in the case of two-dimensional black-hole backgrounds the Liouville mode induces a kinetic term for \(\phi\) of the form \(-\tanh^2 r \partial \phi \overline{\partial \phi}\) \([4]\), we observe that \((33)\) corresponds to an additive shift in the time-component of the background metric. Hence, in the limit \(r \rightarrow \infty\) this would imply an increase in the velocity of light. The presence of a relevant operator, on the other hand, leads to a decrease of the effective velocity of light. In the Wilsonian procedure of renormalization, one integrates out the irrelevant deformations, in the sense that
one sets the respective $\beta$-functions to zero and solves the irrelevant couplings in terms of the relevant ones in order to obtain the known Gell-Mann-Low $\beta$-functions for the latter. In the $\sigma$-model approach, this would correspond to considering the tachyon perturbations in the presence of an instanton background.

We now remark that in the background defined by (31) one may consider (low-energy) particle excitations and derive the conventional Lorentz transformations for coordinates $r$ and time $t$ and particle velocities $u < c_q$. The arrow of time does not appear in these Lorentz transformations: to see the arrow of time we must consider transitions among theories characterized by different $k_R$, in which case one abandons flat space-times and equilibrium situations. Such transitions are induced by the interaction of the propagating light string modes with the ‘ether’ of the topological string states. Then Lorentz transformations are only approximate, and should be replaced by general coordinate transformations appropriate for non-critical string theory. Such transformations are viewed as local redefinitions of $\sigma$-model fields, and undergo renormalization group flow. The limit $c_q \to \infty$ appears ‘Galilean’, but it should be remembered that in this limit the space-time picture changes completely: since the back-reaction effects are enormous as one approaches the topological phase. The dominance of delocalized topological modes with discrete energies and momenta in such a case is consistent with the instantaneous influence implied by $c_q \to \infty$. However, the above perturbative framework, is certainly inapplicable near the topological phase. Thus, until we understand the theory in the infrared, the above remarks should be considered as speculative.

The result (32) is reminiscent of the well-known variation of the velocity of light in a medium with its density and temperature \[32\]. Indeed, there the effective velocity of light decreases as the temperature of the heat-bath increases. However, the mechanism here is different, since the ‘medium’ of non-local topological string modes does not yield a plasmon mass.

The time-dependence of the string as it approaches the ultraviolet fixed point is reflected in a computation of the string position-momentum uncertainty relation in the $\sigma$-model deformed by tachyons and instantons. The result for the position-momentum uncertainty, defined appropriately to incorporate curved gravitational backgrounds \[4\], can be expressed as

\[
(\Delta X \Delta P)_{\text{min}} \equiv \hbar_{\text{eff}}(t) = \bar{\hbar}(1 + O(1/k(t)))
\]

(34)

where $\Delta A \equiv \langle A^2 \rangle - \langle A \rangle^2$ signifies $\sigma$-model vacuum expectation values, and $\bar{\hbar}$ is the critical-string Planck’s constant. The string uncertainty relation introduces a minimum length $\lambda_s$ \[4\], that in our case decreases with time too:

\[
\lambda_s(t) \equiv \left(2\frac{\hbar(t)\alpha'(t)}{c_q(t)^2}\right)^{\frac{1}{2}} = \lambda_s^0(1 + O(1/k(t)))
\]

(35)
where the superscript 0 is denotes quantity evaluated at the ultraviolet fixed point in the critical string theory. We also mention the following important relation that can be proven \[9\] while deriving (35):

\[
\frac{\alpha'(t)}{\alpha'_0} = \frac{c^2_q(t)}{c^2}
\]

(36)

which stems from the relation between \(k\) and the Regge slope in the model \([4]\) \([7, 9]\). The relation (36) is consistent with the topological phase transition that we conjecture occurs at the infrared fixed-point of the flow, where \(\alpha'(t) \to \infty\) \([11]\).

5 Applications to Inflation and the Cosmological “Constant” problem

We saw in the previous section that fundamental constants vary in the non-critical string Universe, because of the interaction of the low-energy world with the topological string modes. Fundamental “constants” attain fixed values asymptotically, in the particular string vacuum which is attained as the ultraviolet fixed point of the \(\sigma\)-model renormalization group flow. So what physical meaning is attached to the concept of the “running” velocity of light \(c_q(t)\) \([12]\) or of the Planck “constant” \((34),(35)\) ?

The velocity of light can always be absorbed in a rescaling of the time coordinate, provided that it depends only on time and that this time dependence does not imply causality violations. For instance, in the case of the asymptotic black-hole metric \([31]\), one can redefine time \(t \to t'(t)\) in such a way that

\[
dt' = \sqrt{\frac{k}{k-2}} \ dt
\]

(37)

This rescaling implies that such a time dependence of the velocity of light will be unobservable locally, in the sense that in a co-moving frame the velocity of light will appear as constant that can always be set to unity. From a renormalization-group point of view, the redefinition \([37]\) represents a local scheme change. As long as there is a finite maximum speed in the low-energy world, this will not have physical consequences locally. However, we conjecture that the velocity of light becomes ill-defined at the topological phase transition in the infrared limit, where the normal concepts of space and time break down.

However, the evolution of the velocity of light may have important cosmological consequences. If we consider a Friedmann-Robertson-Walker Universe, the horizon distance \(d\) in co-moving coordinates over which an observer can look back is

\[
d = \int dt c_q(t) = \int dt \sqrt{\frac{k(t)}{k(t) - 2}}
\]

(38)
which is larger than the naive estimate $d = ct$, because the effective $k(t)$ was smaller at earlier times. Indeed, the horizon distance could even become infinite if $k(t) \to 2$ in a suitable way as $t \to 0$, but this conjecture takes us beyond the dilute-gas approximation where we can compute reliably. Moreover, the time-dependence of the minimum string length indicates that the unit of length in the low-energy world appears to shrink with time. Both these features are reminiscent of inflationary cosmology, although we do not have an additional inflaton field.

The correspondence to inflation is reinforced by the possible appearance of Jeans-like instabilities, which may affect low-energy string modes at finite $k$, as we now argue. We start from the observation that the $SL(2, R)/U(1)$ quantum $\sigma$-model metric contains an overall scale factor

$$a(t)^2 = 2(k(t) - 2)$$

which grows with time $t$. This scale factor supplements any time-dependent dilaton scale factor (cf the first, simpler, linear example in ref. [3]) that could be absorbed in the metric background after appropriate redefinition. For large times, and hence large $k \to \infty$, the background is identical after the time redefinition $t \to e^{\delta t}$, $\delta \equiv -4\pi \beta^I T_0$, to the linearly-expanding universe of ref. [33], for which there are no Jeans-like instabilities. However, this is no longer the case if $\beta^I T_0$ is not constant and $kR(t)$ is no longer a simple exponential, as could occur away from the ultraviolet fixed point when instanton effects become strong. As is shown elsewhere [34], for general $kR(t)$, the evolution equation for $a(t)$ in the cosmic time frame is

$$\frac{\ddot{a}}{a} = \frac{1}{2kR(kR - 2)} \left\{ \partial_t^2 kR - \frac{kR - 1}{kR(kR - 2)} (\partial_t kR)^2 \right\}$$

where the dot denotes differentiation with respect to the cosmic time $t_c$, defined as

$$dt_c = a(t')dt'$$

The first factor in the square brackets in (40) might dominate the second near the infrared fixed point, in which case $\frac{\ddot{a}}{a} > 0$ and a Jeans-like instability occurs.

We re-emphasize, though, that a physical expansion of the Universe occurs whether or not Jeans instabilities appear. It is natural to enquire into their physical meaning, should they appear. To understand this, it is better to go to higher dimensions, where there are transverse modes. It can be shown that the temporal evolution of these modes is similar to the longitudinal ones. The point is that due to the existence of these unstable modes, the proper size of the string does not grow in the same way as the expansion of the Universe. Hence strings stretch as one goes back in time. We expect that at early times, close to the topological string phase

\[ \text{There is no stretching in two space-time dimensions. However, even in that case the existence of unstable modes in the longitudinal amplitude, makes things similar to the higher-dimensional case.} \]
there will be string regeneration through the breaking of strings larger than the Hubble horizon size $R_0$ \cite{35}. In our picture, these effects would be attributed to purely quantum effects of the string, given the connection of the instanton perturbations with higher-genus instabilities of the black hole background \cite{28}. This quantum string regeneration scenario is similar to the string-driven inflation proposed by Turok \cite{35}. Indeed, were it not for this regeneration phenomenon, the exponential expansion of the Universe in an inflationary scenario would imply a strong suppression of the string density, due to conformal stretching of the string size which grows like the scale factor of the Universe.

In this simplified scenario, one can have a qualitative picture of the entropy production rate in the Universe. In our framework, the rate of entropy increase with time is given by \cite{2, 27}

$$\partial_t S = \beta^i G_{ij} \beta^j S \quad ; \quad G_{ij} = \langle V_i V_j \rangle$$

where the unitarity requirement of the world-sheet theory implies the positivity of the Zamolodchikov metric \cite{8} $G_{ij} > 0$. Using the C-theorem \cite{8}, especially in its string formulation \cite{36} on the fiducial-metric world-sheet, one may write

$$\beta^i G_{ij} \beta^j = -\frac{1}{12} \partial_t C(g) \quad ; \quad C(g) = \int d^D y \sqrt{G} e^{-2\Phi} < T T > + \ldots$$

In this expression, the $y$ denote target spatial coordinates, $\Phi$ is the dilaton field, and $T \equiv T_{zz}$ is a component of the world-sheet stress tensor. The ... denote the remaining two-point functions that appear in the Zamolodchikov C-function \cite{8}, which involve the trace $\Theta$ of the stress tensor, i.e. $< T \Theta >$ and $< \Theta \Theta >$. Taking into account the off-shell corollary of the C-theorem, $\frac{\delta C(g)}{\delta g^i} = G_{ij} \beta^j$, it can readily be shown \cite{37} that such terms can always be removed by an appropriate renormalization-scheme choice, that is by appropriate redefinitions of the renormalized couplings $g^i$, and hence play no rôle in the physics. Thus, one can solve \cite{12} for the entropy $S$ in terms of the Zamolodchikov C-function

$$S(t) = S_0 e^{-\frac{1}{12} \int_0^t \int d^D y \sqrt{G} e^{-2\Phi} < T T > + \ldots}$$

where the minus sign in the exponent indicates the opposite flow of the time $t$ with respect to the renormalization-group flow. Expression \cite{14} reduces a complicated target-space computation of entropy production in an inflationary scenario to a conformal-field-theory computation of two-point functions involving components of the stress tensor of a first-quantized string. We observe from \cite{12} that the rate of entropy increase is maximized on the maximum-$\beta^i$ surface in coupling constant space. At late stages of the inflationary era, i.e. close to the ultraviolet fixed point, the rate of change of $S$ is strongly suppressed, due to the smallness of the $\beta^i$. 
The non-critical string scenario for the expanding Universe described in the preceding paragraphs offers the prospect of solving the three basic problems of the standard-model cosmology in a manner reminiscent of conventional inflation \[38\]. The *horizon problem* could be solved by the enhanced look-back distance \(38\), and/or the breakdown of the normal concepts of space and time in a transition to a topological phase close to the infrared fixed point. The *flatness problem* could be solved by an epoch of exponential expansion, induced by a Jeans-like instability in \(40\). The *entropy problem* could be solved by the enhanced rate of entropy production \(44\) at early times. However, the crucial difference in our approach is that the fundamental scalar field, usually termed the *inflaton*, is replaced by a world-sheet field, the Liouville mode, in our approach. Fluctuations of this field create the renormalization group flow of the system that leads to the generation of propagating matter, in the way described above and in previous works \[27, 7\]. Of course, this mode is associated with the appearance of a target space scalar, the dilaton, but the latter is part of the metric background. This can be seen clearly in the two-dimensional Wess-Zumino string theory, which may be considered as a prototype for the description of a spherically-symmetric \(s\)-wave four-dimensional Universe \[39\]. In this model the dilaton belongs to the graviton level-one string multiplet, which is a non-propagating \(\text{discrete}\) string mode, and as such can only exist as a background, in contrast to a massless ‘tachyon’ mode, which propagates and scatters.

As a final comment, we discuss the cosmological “constant” problem in our framework. We adopt a toy-model approach, and concentrate on the one-loop \(\beta\)-functions for the graviton and dilaton fields only. Tachyonic effects are summarized by the running of the level parameter, and we shall not deal with the explicit form of the tachyon potential. As argued in \[40\], its particular form is renormalization-scheme dependent and, thus, irrelevant for our purposes. We also remark that, as will become clear from the analysis below, our results can be extended to any number of *dimensions for the target space*, provided one maintains the requirement of maximal space symmetry.

Consider the following one-loop results for the dilaton and graviton \(\beta\)-functions in bosonic \(\sigma\)-models \[11, 17\]:

\[
\beta^\Phi \equiv \frac{d\Phi}{d\phi} = -\frac{2}{3} \frac{\delta c}{\alpha'} + \nabla^2 \Phi - (\nabla \Phi)^2 \\
\beta^G_{MN} \equiv \frac{dG_{MN}}{d\phi} = -R_{MN} - 2\nabla_M \nabla_N \Phi
\]

where \(\phi\) denotes our covariant Liouville cutoff \(\text{c.f. the relative minus sign compared with the notation of ref. \[41, 17\] where the cutoff is defined with the dimensions of mass})\), and \(\delta c = c - 26\), the 26 coming from the space-time reparametrization ghosts. If the central charge of the theory is not 26, as is the case of non-critical bosonic
strings, then a cosmological constant term appears in the target space effective action. The form of this target-space action, whose variations yield the $\beta$-functions \((45)\), reads:

$$I = \frac{2}{\alpha'} \int d^D y \sqrt{G} e^{-\Phi} \left\{ \frac{1}{3} \delta c - \alpha' (R + 4(\nabla \Phi)^2 + \ldots) \right\} \quad (46)$$

where the \ldots denote other fields in the theory that we shall not use explicitly. We now notice that the effects of the tachyons in our two-dimensional target-space string model amount to a shift of the level parameter \(k(\phi)\) with the renormalization group scale. This is the result of the combined effects of tachyon and instanton deformations, the latter representing higher-genus instabilities. The instantons alone, as irrelevant deformations, produce an initial instability by inducing an increase of the central charge, which then flows downhill towards 26 in the presence of relevant matter (tachyon) couplings. Hence there is a running central charge \(c(\phi) > 26\), according to the C-theorem \(\text{[8]}\), that will, in general, imply a non-vanishing, time-varying (running), positive cosmological constant, \(\Lambda(\phi)\), for the background of \((31)\). Its precise form is determined by consistency with the equations \((45)\).

For simplicity, we assume that the only effect of the dilaton is a constant contribution to the scale anomaly, which is certainly the case of interest. This allows one to decouple \(\Phi\) in the field equations obtained from \((46)\). Then the latter read

$$\frac{\delta I}{\delta \Phi} = \Lambda(\phi) - R$$

$$\frac{\delta I}{\delta G_{MN}} = -R_{MN} + \frac{1}{2} G_{MN} R \quad (47)$$

In two dimensions the second equation is satisfied identically. Decoupling of the dilaton field also implies that the first equation yields

$$R = \Lambda(\phi) \quad (48)$$

We note at this stage that the fact that the background equation for graviton is satisfied automatically implies that the metric in coupling constant space of the full two-dimensional target-space string is singular. This seems to be in agreement with the time-like signature of the Liouville field, which implies non-unitary partial contributions to the $\sigma$-model, and should be compared with the situation of ref. \(\text{[3]}\), between compact spaces and four dimensional uncompactified Minkowski space. In the present case, the conformal field theory associated with the space coordinate \(r\) and the running coupling \(G_{rr}(\phi)\) is unitary and satisfies the C-theorem \(\text{[8]}\). The non-unitarity of the Liouville part is essential in assuring the Weyl invariance of the full theory.

The metric background \((31)\) has a maximal symmetry in its space part. To make the analysis more general, we extend the background to \(d = 2 + \epsilon\) dimensions,
keeping the maximal symmetry in the spatial part of the metric \( G_{ij} \):

\[
G_{ij} = a^2(\phi) \hat{G}_{ij}(x) \quad ; \quad R_{ij} = \frac{R}{d-1} G_{ij}
\]  

(49)

where the scale factor \( a^2(\phi) \) is related to \( k(\phi) \) as in (31). The curvature of \( G_{ij} \) is assumed to be that of a scaled sphere

\[
R = \frac{b}{d-2 a^2(\phi)} = \frac{b}{\epsilon a^2(\phi)}
\]  

(50)

where \( b \) is a constant. From (48) one finds the following relation between the cosmological constant and scale factor

\[
a^2(\phi) = \frac{b}{(d-2) \Lambda(\phi)}
\]  

(51)

Substituting into the \( \beta \)-function equation (45) for the spatial component of \( G_{ij} \), it is immediate to see that the \( d \)-dependent factors cancel out, leaving an effective equation for the running cosmological constant

\[
\frac{1}{\Lambda^2} \frac{d \Lambda}{d \phi} = \frac{1}{d-1}
\]  

(52)

which should be applicable to higher dimensions. As we have explained above, the actual target-time flow is opposite to that of the renormalization group, and hence it is the negative real \( \phi \)-axis that is relevant, given also that \( \phi = \ln(a^2/L^2) \), where \( a \) and \( L \) are ultraviolet and infrared length cut-offs respectively. This yields as a solution for \( \Lambda(\phi) \)

\[
\Lambda(t) = \frac{\Lambda(0)}{1 + t \frac{\Lambda(0)}{a-1}} \quad ; \quad t \equiv -\phi > 0
\]  

(53)

which for positive \( \Lambda \) implies an asymptotically-free cosmological constant \( \beta \)-function, thereby leading to a vanishing cosmological constant at the ultraviolet fixed point on the world-sheet.

The rate of the decrease of \( \Lambda(\phi) \) is determined by its initial value at the infrared fixed point, where we conjectured that the theory makes a transition to a topological (twisted \( N = 2 \) supersymmetric) \( \sigma \)-model. It is of great interest to estimate this value in our two-dimensional model. This can be done by noticing that

\[
\Lambda(0) = \frac{2}{\alpha'(0)} \left( \frac{c(0)}{3} - 26 \right)
\]  

(54)

\footnote{The \( G_{00} \)-component depends at most on the time \( \phi \) and can be absorbed in a redefinition of the time variable. It will not be of interest to us here.}

\footnote{We remark that a similar equation has also been considered in ref. [42], but the flow of time in that reference coincides with the renormalization group flow. In such a case, one gets sensible results only for negative initial values of the cosmological constant, contrary to our case where we have a vanishing cosmological constant asymptotically, starting from positive initial values.}
where \( c(0) - 26 = \frac{3k(0)}{k(0)^2} - 27 \simeq \frac{3k(0)}{k(0)^2} \), given that \( k(\phi) \rightarrow 2 \) as \( \phi \rightarrow 0 \). Thus, taking into account (58) one observes that \( \Lambda(0) \) is determined by the critical string tension, as it should be, given that \( \alpha'_0 \) is the only scale in the problem (or equivalently the minimal string length): the result is

\[
\Lambda(0) = \frac{2}{\alpha'_0}
\]  

(55)

The latter result implies a really fast decay of the cosmological constant in this model. Notice that the finite initial value of \( \Lambda(0) \) implies from (48) no curvature singularity in the Euclidean model at the origin of target space \( r = 0 \), as is indeed the case of the two-dimensional black-hole model of ref. [23], given that this point is a pure coordinate singularity. The above analysis for the cosmological constant, therefore, applies most likely to singularity-free inflationary universes. It is understood that until the precise behaviour of the running couplings near the infrared fixed point is found, there will always be uncertainties in the above estimates. String perturbation theory is not applicable near the topological phase transition, and the infinities we get in the various running couplings constitute an indication of this. In the complete theory, these infinities should be absent.

At this stage, we notice that such a scenario for an asymptotic (in time) vanishing of the cosmological constant was conjectured in ref. [3], but no explicit example was provided. In our case, it is the association of the Liouville mode with a (local) renormalization group scale, flowing irreversibly, that makes possible a realization of such a scenario via (53).

In order to assess the physical consequences of the equation (53) for the asymptotic vanishing of the cosmological “constant”, we also need to explore further the relationship between the renormalization group time \( t = -\phi \) and the physical time measured, for instance, by atomic clocks. As already mentioned, a time-dependent scale factor appears (39) already in the \( \sigma \)-model metric, supplementing any possible time-dependent dilaton scale factors which may appear (3) in the “physical” metric. The latter is defined as a co-moving frame, where such a time-dependent scale factor is absorbed in an appropriate redefinition (37,41) so as to yield a conventional Friedmann-Robertson-Walker form for the target-space metric:

\[
ds^2 = 2(k(t'') - 2)dr^2 - (dt'')^2
\]

(56)

where \( t'' \) denotes the redefined time variable. In the previous two-dimensional example, where the Wess-Zumino model’s dilaton field is space like outside the horizon [23], the redefined time \( t'' \) (54) is given simply by the scale factor (39)

\[
dt'' = \sqrt{2k(t)}dt
\]

(57)

To integrate (57), we need to know the full \( t \)-dependence of \( k(t) \), which is lacking away from the ultraviolet limit.
It is premature to discuss quantitatively the implications of the vanishing mechanism [53], for this and other reasons. However, we finish this talk by noting a few relevant points. One is that the present age of the Universe in “physical” time is about $10^{60}$ in natural (Planck) units. Another is that one-loop calculations at the point-like field theory level in no-scale supergravity models [12, 43] yield a negative contribution to the cosmological constant that is $O((m_w/M_{\text{Planck}})^4) \approx 10^{-60}$ in Planck units. Finally, we note that astrophysical and cosmological observations are compatible with a present-day value of the cosmological constant that is about $10^{-120}$ in Planck units. Moreover, a cosmological constant of this order of magnitude could even be a welcome adjunct to Cold Dark Matter models. Thus it may even be desirable that the cosmological “constant” has not yet completely relaxed.

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