Machine Condition Classification by Using Wavelet Packet Decomposition and Multi-scale Entropy

Hongkun Li\textsuperscript{1, a}, Shuai Zhou\textsuperscript{2, b}, Yuzhen Chen\textsuperscript{3, c}

\textsuperscript{1, 2, 3}School of Mechanical Engineering, Dalian University of Technology, Dalian 116024, China
\textsuperscript{1}State Key Lab. of Structural Analysis for Industrial Equipment, China, 116023
\textsuperscript{a}lihk@dlut.edu.cn, \textsuperscript{b}zhoushuaidlut@yahoo.cn, \textsuperscript{c}cyz90@126.com

Keywords: Wavelet Packet Decomposition, Multi-scale Entropy, Rolling Bearing, Condition Classification.

Abstract. A new condition classification method is put forward based on the analysis of vibration signals. Machine working condition can be recognized by the combination of wavelet packet decomposition (WPD) and multi-scale entropy (MSE). Firstly, vibration signal of machine is decomposed by wavelet packet with the appropriate decomposition layer. Then, each sub-signal in different frequency band is analyzed with the multi-scale entropy. Through analyzing the multi-scale entropy distribution curves of sub-signals for different operating conditions in each frequency band, entropy of certain frequency bands and scales will be chosen as the feature vector, which is used to distinguish different machine conditions. This method presents a novel perspective for rolling bearing default diagnosis and is tested to be very effective to classify different bearing operating conditions through series of experiments.

Introduction

Many methods have been investigated on machine pattern recognition and fault diagnosis. How to effectively extract the characteristics of the fault signals is the pivotal problem. Commonly employed methods are fault diagnosis by combination Empirical Mode Decomposition with support vector machine [1], machine condition identification based on support vector machine and information spectrum entropy [2], and etc. With the development of the wavelet analysis since 1980, it is broadly used to process vibration signals and the effect is remarkable. As the extension and development of wavelet transform, Wavelet Packet Decomposition (WPD) is multi-dimensioned. Entropy is very suitable for characterizing time series’ complexity. Researchers, including Richman [3], put forward sampling entropy based on the improvement of approximate entropy, presented by Pincus [4]; Costa [5] put forward the algorithm of Multi-scale Entropy (MSE).

In this paper, a new method for machine operating condition classification is presented using WPD and MSE. WPD is made toward machine operating signals, and the decomposed sub-signals in different frequency bands are analyzed by MSE. In order to identify machine’s operating conditions, multi-scale entropy distribution curves of different machine operating signals in a certain frequency band are compared. The feature vector for operating condition is established by entropy with proper frequency bands and scales. A rolling bearing’s working condition pattern recognition was carried out to testify the effectiveness of this method.

Theory and Method

Multi-scale Entropy. MSE has been widely used in the analysis of physiological time series. Based on Sample Entropy, it can be employed to describe the irregularity of MSE.

For \( \{x_1, x_2, \ldots, x_n\} \), which is a discrete time series with a length of \( N \). Time series \( \{y^{(s)}\} \) can be acquired through coarse-graining transformation of Eq. 1.

\[
y^{(s)} = \frac{1}{\tau} \sum_{i=(j-1)\tau+1}^{i} x_i.
\]  

(1)

where \( \tau \) is time scale.
The length of coarse-grained time series is \( M = \text{int}(N / \tau) \). When \( \tau = 1 \), the coarse-grained time series is identical to the original time series. The sampling entropy for different sampling interval \( \tau \) can be calculated as follows. Construct a \( m \)-dimension vector, according to the time series of length \( M \), which is transformed under time scale \( \tau \):

\[
X_m(i) = \{ y_{i+k} : 0 \leq k \leq m-1 \}.
\]

(2)

For each single \( i \), calculate the distances between \( X_m(i) \) and each of the remaining vectors \( X_m(j) \):

\[
d\left[ X_m(i), X_m(j) \right] = \max \left[ y'_{i+k} - y'_{j+k} \right], \quad (0 \leq k \leq m-1; \ i, j = 1 \sim M - m + 1; i \neq j).
\]

(3)

Set \( r \ (r>0) \) as a clear threshold for the matching process. For each \( i \), calculate \( B^m(i) \) which meets the condition of \( d\left[ X_m(i), X_m(j) \right] < r, (i, j = 1 \sim M - m + 1; i \neq j) \), and define the ratio of \( B^m(i) \) to the total distance as:

\[
C^m,\tau (r) = B^m(i) / (M - m).
\]

(4)

Calculate the average of \( C^m,\tau (r) \):

\[
C^{m,\tau} (r) = (M - m + 1)^{-1} \sum_{i=1}^{M-m+1} C^m,\tau (r).
\]

(5)

Increase the dimension to \( m+1 \) and repeat the steps above, calculate the average of \( C^{m+1,\tau} (r) \):

\[
C^{m+1,\tau} (r) = (M - m)^{-1} \sum_{i=1}^{M-m} C^{m+1,\tau} (r).
\]

(6)

When \( M \) is finite, estimated sampling entropy of series with a length of \( M \) can be got following the steps above:

\[
\text{SampEn}(m, r, M) = -\ln \left[ \frac{C^{m+1,\tau} (r)}{C^{m,\tau} (r)} \right].
\]

(7)

Sampling entropy for different \( \tau \) can be calculated by repeating the steps above.

The self-similarity of time series increases with the decrease of sampling entropy, while series’ complexity increases with the increase of sampling entropy. Entropy is suitable for characterizing system complexity. Based on sample entropy, MSE has the following merits: it can be used for multi-scale analysis; sample entropy of different signals can be analyzed on different scales [6].

**Wavelet Packet Decomposition.** The core idea of WPD put forward by R.Coifman and V.Wickerhauser is to re-decompose the wavelet subspace series after multi-resolution analysis, and re-devise the frequency window, which becomes wider with the decrease of dimension. Thus, the resolution of signal’s high-frequency part will increase.

According to signal filter, orthogonal wavelet packet decomposition is to filter the signal through a low pass filter and a high pass filter respectively. A group of low frequency signals and a group of high frequency signals can be obtained. Actually, wavelet packet decomposition is a frequency separating method. Some sub-signals in certain frequency bands are chosen and analyzed according to actual requirement. As different types have different optimum decomposition layers, it needs further investigation. The following is the diagram of 3-layer wavelet packet decomposition.
Analytical Method. In order to classify different machine conditions, WPD is used to decompose original vibration signal. Sub-signals with the main feature information of the machine condition will be acquired. MSE analysis toward signals can provide frequency bands and scales, where entropy can effectively distinguish different machine conditions. Machine condition classification can be carried out as following: First, WPD is used to determine sub-signals in different frequency band for monitored vibration signal. Then, MSE of resulted sub-signals are calculated according to above equations. Scales and frequency bands are selected, where Sampling entropy of different working conditions can be distinguished between each other. In the end, entropies of the selected frequency bands and scales are used to construct original signal’s feature vector for condition classification. An experiment carried on in the lab is shown in detail for this method.

Experimental Test

In this paper, a bearing is an example to examine the effectiveness of this method in different working conditions with different speed. The type of experiment rolling bearing is N205. There are four kinds of condition, normal, outer ring wearing, inner ring wearing, and ball wearing. Two acceleration vibration signals radical located at the bearing block were monitored by hardware produced by BK signal monitoring system and its type is 3560. Fig.2 shows the pictures during the experiment in the lab. The time domain signal of rolling bearing is shown in Fig.3.

Data Analysis

MSE Analysis of Original Signal. When motor speed is 1200RPM, five groups of data are collected for each of the four types of abrasion. The results of MSE analysis are presented in Fig. 4. From the results, only the multi-scale entropy of normal condition is obviously distinguished from other
conditions, while the entropies of other three conditions are too closed to be distinguished from each other on the scale from 2 to 15. The four operating conditions can’t be identified just based on MSE of the original data.

![MSE Analysis of Four Rolling Bearing Working Conditions](image)

**Fig. 4.** MSE analysis of four rolling bearing working conditions

**Wavelet Packet Decomposition.** The signals of rolling bearing are always mixed with noise signals from other parts. In the most circumstance, analysis of original signal will not get ideal results. In order to reduce noise disturbance and effectively distinguish the four different operating conditions, WPD is carried on toward original data. The decomposition layer is set three and wavelet base is db10. Fig.5 shows that the vibration signal of ball wearing condition decomposed into eight sub-signals frequency bands. Sub-graphs of S30, S31, … S37 are corresponding to the frequency bands from low level to high level.

![Time Domain of the Decomposed Sub-Signals](image)

**Fig. 5.** Time domain of the decomposed sub-signals

**MSE Analysis of Sub-signals.** Each original data is decomposed into sub-signals in eight frequency bands, which is analyzed by MSE method, and the results are presented in Fig.6. Sub-graphs of (a), (b), … (h) are corresponding to the frequency bands from low frequency to high frequency.
Subgraphs (a)(b)(c)(d)(e)(f)(g)(h) correspond to the decomposition of the eight sub-signals from low frequency to high frequency.

Fig. 6. MSE analysis of four rolling bearing working conditions in eight frequency bands.

In the frequency bands 1, 5, 6 and 8, MSE curves of four working conditions are similar and can’t be effectively distinguished. In the frequency band 2 with scales 2, 3 and 4, frequency band 3 with scales 2, 3, 4, 5 and 8, and also frequency band 4 with scale 6 and 7, MSE curves of ball wearing and normal condition can be distinguished from other, but outer ring wearing can’t be distinguished from inner ring wearing. In the frequency band 7 with scale of 2, 3 and 4, MSE curves of inner ring wearing can be distinguished from outer ring wearing, but ball wearing can’t be distinguished from normal condition. From the above analysis, it is hard to totally distinguish the four conditions just using sampling entropies of sub-signals in one frequency band and one scale. So two sub-signals sampling entropies in different frequency bands and scales needs to be selected to construct original signal’s character vector $M(x, y)$, and the detailed combination is listed in Table 1.

| Table 1 Combination of character vector $M(x,y)$ |
|-----------------------------------------------|
| Sampling entropy | $x$ | $y$ |
| Frequency band | 2 | 3 | 4 | 7 |
| Scale | 2, 3, 4 | 2, 3, 4, 5, 8 | 6, 7 | 2, 3, 4 |
**Condition Classification.** Based on overall consideration, the sampling entropies of the frequency band 3 with scale 4 and the frequency band 7 with scale 4 are selected to construct the character vector for condition classification. For each condition, 30 sets of data are collected. Character vector $M(x,y)$ is constructed from sampling entropy $x$ of band 3 with scale 4 and the sampling entropy $y$ of band 7 with scale 4, where 3-layer wavelet packet decomposition is done toward each signal. The distribution of character vectors of each vibration signal in $x$-$y$ plane is showed in Fig. 7. It is clear that the four conditions can be effectively distinguished. And the method presented in this paper is verified very effective.

![Fig. 7. Feature vectors distribution of four rolling bearing working condition](image)

**Summary**

A new method for machine operating condition identification is presented based on the research of wavelet packets decomposition and multi-scale entropy analysis. Sub-band signals from WPD of the original signal set a good foundation for the extraction of characteristics and condition identification. MSE is done toward the resulted sub-signals; characteristic vector composed from entropies which can effectively distinguish different rolling bearing conditions. From the experiments, four different operating conditions of rolling bearing can be effectively indentified. How to determine the best characteristic parameters on frequency band and scale is the focus for further study.

**Acknowledge**

The support from Chinese National Science Foundation (Grant No: 50805014) ,the Fundamental Research Funds for the Central Universities (Grant No: DUT11NY04) and State Key Laboratory of Structural Analysis for Industrial Equipment, Dalian University of Technology (Grant No. GZ0817) for this research are gratefully acknowledged.

**References**

[1] YANG Yu, Research on Fault Diagnosis Methods for Rotating Machinery Based on Empirical Mode Decomposition and Support Vector Machine, Hunan University,2005.

[2] PAN Mingqing, ZHOU Xiaojun, YANG Chenlong, PANG Mao, Application of Support Vector Machine Based on Information Spectrum Entropy in Machine State Identification, Chinese journal of sensors and actuators, 18 (2) (2005) 277-280.

[3] J.S. Richman, J.R. Moorman, Physiological time-series analysis using approximate entropy and sample entropy, Am Physiological Soc, 278(6) (2000) 2039-2049.

[4] S.M. Pincus, Approximate entropy as a complexity measure, Chaos, 5(1995) 110-117.

[5] M. Costa, A.L. Goldberger, C.K. Peng, Multi-scale entropy analysis of complex physiologic time series, Physical Review Letters, 89(6) (2002) 068102.

[6] Zheng Guibo, Jin Ningde, Multi-scale entropy and dynamic characteristics of two-phase flow patterns, Acta Physica Sinica, 58(7) (2009), 4485-4491.