Independent Component Analysis of Gravity Earth Tide Signal Based on Differential Evolution Algorithm

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Abstract. To solve the difficulty of extracting orthogonal component of tidal harmonic from gravity earth tide signals, an orthogonal decomposition model for gravity earth tide signal is proposed in this paper. The harmonic component of the gravity earth tide signal is implemented orthogonal decomposition by the model in space, in order to realize the extraction of the harmonic component frequency reflects the independent geophysical information on the space. Thus, a new feature extraction method of gravity earth tide signal is presented by combining independent component analysis and differential evolution algorithm. It can be more effective to get the independent component which is consistent with the gravity solid tidal signal decomposition model. In the experiment, the measured signal is processed and compared with the theoretical value in Kunming area. Experimental result shows that the method can reveal the relationship of tidal harmonic component in gravity earth tide signals and this relationship corresponding to the gravity earth tide signal orthogonal decomposition model. Each spectral information component contained accords with theory value.

1. Introduction
Due to the change in the relative position of the sun and moon, the gravitational force of the sun and moon will cause the earth's gravitational field to change periodically with time[1]. According to reflect this change in the vertical direction and numerical of gravity, can be divided into the ground tilt solid tide and the gravity earth tide. However, in this paper, we only study the gravity earth tide signal. The gravity earth tide has not only become an important means of seismic observation, but also is an important part of the study of seismic origin, earthquake precursor and earthquake prediction[2]. Rich tidal harmonic components of gravity earth tide signals can be divided into long period wave, diurnal wave, semi diurnal wave three harmonic line[3]. Because these harmonic components are rich in geophysical information, it is significant to analyze the harmonic component of gravity tide signal.

Research of solid earth tide to years is mainly carried out harmonic analysis [4], at present, the Venedikov harmonic analysis method and the ETERNA harmonic analysis method are two standard tidal analysis methods recommended by the international earth tide Center (ICET). In fact, Harmonic analysis is a method based on Fourier transform, which decomposes the observed information into a simple periodic function. However, Fourier transform is a kind of integral transform which is defined by the sine basis function in the infinite period. It can only analyze the stationary signal with time and the global analysis of the signal. But it cannot be analyzed the nonlinear non-stationary signal with time variation and local analysis of the signal. This shows that harmonic analysis cannot fully reveal the modulation relationship between harmonics, and it cannot realize the orthogonal decomposition of the harmonic component of gravity earth tide signal.

In order to effectively extract the geophysical information of the gravity tide signal, the modulation relation between them is revealed. The independent component analysis (ICA) algorithm based on
differential evolution (DE) algorithm is introduced in this paper, and an orthogonal decomposition model is proposed for the analysis and treatment of the observed signals. By using this method, in this paper, the corresponding relation between the independent component of tidal harmonic in gravitational earth tide signal and the gravitational tide effect among celestial bodies is revealed. A new method has been found for the study of gravity tides.

2. Gravity Solid Tide Signal Orthogonal Decomposition Model

As shown in Figure 1, an observation point $A$ on earth, which under the influence of tidal force exerted by the earth's rotation, the sun and the moon, triggered a solid tidal $F$, can be decomposed into gravity earth tide signal $F_x$. In this work, we only analyze the gravity earth tide signal $F_x$, which can be decomposed into 2 orthogonal vectors: a signal component $F_1$ parallel to the equatorial plane, a parallel to the earth's self-axis of the signal component $F_2$[7]. And $F_1$ can be decomposed into two orthogonal signal components $F_{11}$ and $F_{12}$ (in the same plane). Thus, gravity earth tide signal is orthogonal decomposition of three-dimensional orthogonal vector (see Fig 1). According to the model of 3D orthogonal vector decomposition, the $F_1$ is parallel to the equatorial plane, which means that it is related with the earth's rotation and revolution, it is mainly reflected on the harmonic components of diurnal wave $F_{11}$ and semi diurnal $F_{12}$. However, $F_2$ is parallel to the earth's axis of rotation, has nothing to do with the rotation of the earth, its weight in the tidal gravity signals for years wave, half years wave, excluding day wave, semi diurnal wave. In this way, according to the three-dimensional orthogonal decomposition model, the different harmonic components of gravity earth tide signals can be decomposed into three dimensional orthogonal vectors corresponding to each harmonic.

![Figure 1](image-url) The orthogonal decomposition model of gravity earth tide signal

3. Analysis method

3.1 Independent component analysis (ICA) algorithm

ICA is one method for performing blind signal separation that aims to find de-mixing matrix $W$ of overlapping signal $X$, so that each component of $Z = WX$ as far as possible independent of each other, in which the independent criterion function is $G$. ICA mathematical model:
Make $\tilde{x} = (x_1, x_2, ..., x_n)^T$, $\tilde{s} = (s_1, s_2, ..., s_n)^T$,

$$\tilde{x} = A \tilde{s} = \sum_{i=1}^{n} a_is_i, i = 1, 2, ..., n$$

(1)

Where $A$ is an unknown mixing matrix, and $\tilde{x}$ represents the latent source signals.

$$A^{-1} = W_1W_2$$

(2)

Where $W_1$ is an whitening matrix which is usually computed singular or eigen-value decomposition of the covariance matrix of $x$, $W_2$ is de-mixing matrix.

$$\tilde{y} = W_1 \tilde{x}$$

(3)

Here, $\tilde{y}$ is the whitening signals.

De-mixing model:

$$\tilde{z} = W_2 \tilde{y} = G \tilde{s}$$

(4)

Where $\tilde{z}$ is independent component, $G$ for global matrix / system matrix. If $\tilde{y} = \tilde{s}$, the target of separation (or) to recover the source signal is achieved.

In fact, the ICA algorithm is including two aspects:

- preprocessing (processing of $X$ to zero mean, spheroidize);
- core algorithm (separation matrix processing);

From above, We can get approximate estimates of various components of the source signals. In fact, Its essence is to use the objective function and the corresponding core algorithm (optimization algorithm)that is used to optimize the objective function, and the process of the optimal separation matrix is obtained.

The model of ICA can be depicted by Figure 2.

In this paper, in order to obtain the optimal separation matrix, we first use the negative entropy as the objective function of ICA, and then use the differential evolution algorithm to optimize the objective function.

3.2 Differential evolution (DE) algorithm

Differential evolution algorithm is a kind of group evolution algorithm which can remember the individual optimal solution and the information sharing in the population, it can solve the optimization problem by the cooperation and competition among individuals in the population, which means that it is a greedy genetic algorithm based on real number coding with the idea of preserving the best[8]. But compared with the traditional genetic algorithm, it not only retains the global searching strategy based on population, with real encoding, simple mutation differential and a survival strategy based on competition, and reduce the complexity of genetic operations.

One of the most frequently used mutation strategies, named "DE/rand/1/bin" [9],will be employed in this paper. Differential evolution’s basic steps can be described as follows:

Step 1: Generating the initial population
\[ x_i = (x_{i,1}, x_{i,2}, ..., x_{i,D}), i = 1,2, ..., NP \]  
(5)

where \( D \) is the dimension, \( NP \) is the population size.

Step 2: Mutation operation

\[ V_i(g + 1) = x_{i,1} + F \cdot (x_{i,2}(g) - x_{i,3}(g)) \]  
(6)

where \( x_{i,1}, x_{i,2}, x_{i,3} \) are randomly selected vectors, \( x_{i,2}(g) - x_{i,3}(g) \) is differential vector and \( F \) is scaling factor.

Step 3: Crossover operation

\[ u_{i,j}(g + 1) = \begin{cases} v_{i,j}(g + 1), & \text{Rand_{i,j}(0,1)} \leq CR \text{ or } j = \text{j_rand} \\ x_{i,j}(g), & \text{Otherwise} \end{cases} \quad j = 1,2, ..., D, \quad \text{j_rand} \in [1, D] \]  
(7)

Where \( \text{Rand}_{i,j} \) is a uniform random number within \([0,1]\). \( CR \) denotes the crossover constant with outcome \( \in [0,1] \). \( \text{j_rand} \) is a uniform random integer chosen index \( \in 1,2, ..., n[10] \).

Step 4: Selection operation

\[ x_i(g + 1) = \begin{cases} u_{i,g}(g + 1), & f(u_{i,g}(g + 1)) < f(x_i(g)) \\ x_i(g), & \text{Otherwise} \end{cases} \]  
(8)

where \( x_i(g + 1) \) is target vector for the next generation. Until the algorithm reaches the maximum number of iteration by doing steps from step 2 to step 4 through.

3.3 ICA and gravity earth tide signals based on DE

In the optimization process, the DE algorithm can be based on the dynamic change of the whole population to adjust the search strategy, with a strong ability of global optimization, so the optimization objective function ICA algorithm to deal with the tidal gravity signal DE algorithm(Figure3).

![Diagram](Figure.3) The processing of ICA-DE feature extraction
Step 1: Obtaining the gravity earth tide signal at the observation point. According to the gravity earth tidal model, it is known that the gravity earth tide signal is caused by the tidal force of the earth's rotation, the sun and the moon, so the gravity earth tidal wave is selected as the three signal;

Step 2: Whitening matrix $W_1$ is obtained for the signal of gravity earth tide;

Step 3: Using DE algorithm to optimize the separation matrix $W_2$;

1. Set parameter initial value: population size $NP$ and dimension $nDim$, mutation rate $F_0$, the maximum iteration number $GM$, the crossover rate $CR$, the parameter value of the initialization algebra $G$.

2. Start iteration loop: $i = 1$ to $NP$, from (5) to derive the variance vector, from (6) to cross the variance vector;

3. The particle matrix transpose, according to the fitness value function $Fitness = E[u^4] - 3(E[u^2])^2$ that the kurtosis of the separated signals in non-Gaussian of measure to calculate the particle fitness value of $Fitness$ ;

4. According to the formula (4) optimization, If $Fitness_{new} < Fitness_{old}$ (i), the termination of circulation, the matrix transpose of the particle, obtain the separation matrix $W_2$;

Step 4: According to the formula (8), the output signal $\hat{z}$;

Step 5: The harmonic component of gravity earth tide signal is extracted, and its spectral analysis is done.

4. Results and discussion

According to the orthogonal decomposition model of gravity solid tidal signals. In order to extract the harmonic component and the geophysical information of the gravity earth tide, in the period of January 2010 to June 2010, three different observation points on the same longitude ($130^\circ E, 40^\circ N$), ($130^\circ E, 50^\circ N$), ($130^\circ E, 60^\circ N$) were selected to carry out ICA-DE analysis on the gravity earth tidal signals.

DE algorithm parameter settings, as shown in Table 1.

| Parameter name | mutation rate($F_0$) | Maximum Iteration number ($GM$) | Population size($NP$) | Crossover rate ($CR$) | Initial algebra($G$) | Dimensio-n ($nDim$) |
|----------------|---------------------|---------------------------------|-----------------------|----------------------|----------------------|---------------------|
| Parameter values | 0.5                | 10                              | 100                   | 0.9                  | 1                    | m^2                 |

**Figure 4.** The input gravity earth tide signals

**Figure 5.** The ICA-DE processing results of the gravity earth tide signal
The waveform of gravity earth tide wave at the observation point is shown in Figure 4. The signal waveform after ICA-DE algorithm is processed as shown in Figure 5.

To test the experimental results, the spectral analysis of the output signal is carried out (see Figure 6). According to the expansion formula proposed by Du Sen, the angular frequency of gravity earth tide signal is analyzed, and the theoretical value of the main harmonic component of the gravity earth tidal signal is calculated. The harmonic information extracted from the output signal is compared with the theoretical value as shown in Table 2.

![Spectrum analysis](image)

**Figure.6.** Spectrum analysis of the output signal after processing by ICA-DE algorithm

**Table.2** Frequencies of observation points and theoretic values

| output signals | Observation point | Observed frequency f(\text{Hz}) | Theoretic frequency f(\text{Hz}) | Harmonic component |
|----------------|------------------|--------------------------------|---------------------------------|-------------------|
| $r_1(t)$       | $f_1$            | 2.192e-05                     | 2.1944e-05                     |                   |
|                | $f_2$            | 2.207e-05                     | 2.2e-05                        |                   |
| $r_2(t)$       | $f_3$            | 2.238e-05                     | 2.2364e-05                     | Semidiurnal waves |
|                | $f_4$            | 2.314e-05                     | 2.3148e-05                     |                   |
|                | $f_5$            | 2.322e-05                     | 2.3212e-05                     |                   |
| $r_3(t)$       | $f_1$            | 1.073e-05                     | 1.0759e-05                     |                   |
|                | $f_2$            | 1.149e-05                     | 1.1511e-05                     |                   |
|                | $f_3$            | 1.165e-05                     | 1.1637e-05                     | diurnal waves     |
|                | $f_4$            | 1.165e-05                     | 1.1606e-05                     |                   |
|                | $f_5$            | 1.203e-05                     | 1.2026e-05                     |                   |
|                | $f_6$            | 1.249e-05                     | 1.2453e-05                     |                   |
| $r_4(t)$       | $f_1$            | 7.663e-08                     | 6.33777e-08                    | half year waves   |
|                | $f_2$            | 4.598e-07                     | 4.2004e-07                     | waves month       |
|                | $f_3$            | 8.429e-07                     | 8.4725e-07                     | half month waves  |
|                | $f_4$            | 1.252e-07                     | 1.2453e-05                     |                   |
|                | $f_5$            | 1.296e-07                     | 1.2896e-07                     |                   |
|                | $f_6$            | 1.342e-07                     | 1.3353e-07                     |                   |
From table 2:
(1): The actual frequency of the 3 channel gravity earth tide signal is in agreement with the theoretical frequency;
(2): Based on the orthogonal decomposition model of gravity earth tide, the ICA-DE algorithm can separate the gravity earth tidal signal in accordance with the harmonic component.
(3): The signal components of the 3 channel gravity earth tide signals are $F_{11}$, $F_{12}$ and $F_{2}$ respectively, and are not interfered with each other.
(4): Diurnal waves, and semidiurnal waves, harmonic component belongs to the equatorial plane signal components, long period of harmonic component is part of the earth's rotation axis signal components, between them have obvious modulation.

5. Conclusions
Based on the orthogonal decomposition model of gravity earth tide, this paper uses the algorithm of ICA and DE to optimize the gravity earth tide signals, and the results are consistent with the orthogonal decomposition model of gravity earth tidal wave. Then, By means of spectrum analysis, this method can get the spectrum information which is consistent with the generation mechanism of the gravity earth tidal signal. Finally, compared with the actual observed value (after some processing), we can get some information about the earthquake. This proves that the algorithm is effective, and it is more convenient to compare with other methods. In addition, multiplicative relations are still existed in the decomposed components extracted from the gravity earth tidal signals. This provides a good entry point for the next research.

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