New Method of Fuzzy Conditional Inference and Reasoning

Poli Venkata Subba Reddy

Department of Computer Science and Engineering, College of Engineering, Sri Venkateswara University, Tirupati, India

ABSTRACT

We consider fuzzy inference of the form ‘if … then …’. ‘if … then … else …’ and ‘if … and/or … and/or … then …’. Muzomoto proposed logical constructs for ‘if. then.’ by applying Godeliene and Standard Sequence methods. These method do not satisfy all the intuitions In this paper, We propose method of fuzzy inference and applied on logical constructs. We have shown that our fuzzy inference method satisfy all the intuitions under several criterions.

1. Introduction

Classical logics unable to provide a methods to reasoning uncertainty, vague, incomplete or imprecise propositions. Fuzzy logic is capable of reasoning such inexact propositions [1,2] Zadeh [3] and Mamdani [4] proposed methods for fuzzy conditional propositions of the form ‘if … then …’. Fukami et al. [5] developed logical constructs for fuzzy implications using Standard Sequence and Godeliene Sequence Methods. The Mizumoto shown that the Zadeh [1] and Mamdani [2] methods do not fit Muzimoto logical constructs. Muzimoto logical constructs are satisfy some intuitions using Standard Sequence and Godeliene Sequence Methods. We apply our method on these logical constructs and satisfy all the intuition for ‘if … then …’, ‘if … then … else …’ and ‘if.. and/or.. then …’ propositions. We show four method which fit all types of fuzzy conditional inferences.

Fukami et al. [3]. Consider the following types of inference for these propositions.

Type-1

If x is P then y is Q  
  x is P;  
  — — — — — —  
  y is ?  

If Apple is red then Apple is ripe  
  apple is very ripe  
  — — — — — —  
  y is ?

CONTACT  Poli Venkata Subba Reddy  pvsreddy@hotmail.co.in

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Type-2
If \( x \) is \( P \) then \( y \) is \( Q \) else \( y \) is \( R \)
\[ x \in P_1 \]
---
\( y \) is ?

If \( \text{Apple is red} \) then \( \text{Apple is ripe} \) else \( \text{Apple is fade} \)
\( \text{apple is very ripe} \)
---
\( y \) is ?

Type-3
If \( x \) is \( P \) and \( x \) is \( Q \) or \( x \) is \( R \) then \( y \) is \( S \)
\[ x \in P_1 \text{ and } x \in Q_1 \text{ or } x \in R_1 \]
---
\( y \) is ?

If \( x \) is red or \( x \) is ripe and \( x \) is big then \( x \) is taste
\( x \) is red or \( x \) is ripe and \( x \) is very big
---
\( y \) is ?

2. Fuzzy Conditional Inference

A fuzzy set \( P \) is defined by its characteristic function \( \int \mu_P(x)/x, x \in X \), where \( x \) is individual and \( X \) is universe of discourse.

\[ P = \int \mu_P(x)/x \]
\[ P' = 1 - \int \mu_P(x)/x \]
\[ P \lor Q = \max \{(\int \mu_P(x), \int \mu_Q(y))\} \]
\[ P \land Q = \min \{(\int \mu_P(x), \int \mu_Q(y))\} \]
\[ P \oplus Q = \min \{1, (\int \mu_P(x) + \int \mu_Q(y))\} \]

The fuzzy conditional propositions are of the form ‘if (precedent part) then (consequent part)’.

Consider the proposition of type ‘if \( x \) is \( P \) then \( y \) is \( Q \)’

Zadeh [1] fuzzy conditional inference of type ‘if \( x \) is \( P \) then \( y \) is \( Q \)’ is given by
\[ P \rightarrow Q = P' \oplus Q = \{1 \text{ and } 1 - (\int \mu_P(x) + \int \mu_Q(y))\} \]

Consider fuzzy conditional inference for ‘if \( x \) is \( P \) then \( y \) is \( Q \) else \( x \) is \( R \)’.

It may be defined as ‘if \( x \) is \( P \) then \( y \) is \( Q \lor \) if \( x \) is \( P' \) then \( x \) is \( R \)’
It is given by

if \( x \) is \( P \) then \( y \) is \( Q \) if \( x \) is \( P' \) then \( x \) is \( R \)
\[ P' \rightarrow R = P' \oplus Q = \{1 \text{ and } 1 - (\int \mu_P(x) + \int \mu_Q(y))\} \]
\[ P \rightarrow Q = P' \oplus Q = \min \{1, (\int \mu_P(x) + \int \mu_Q(y))\} \]
\[ P' \rightarrow R = P' \oplus R = \min \{1, (\int \mu_P(x) + \int \mu_R(y))\} \]

Mamdani [2] fuzzy conditional inference of type ‘if \( x \) is \( P \) then \( y \) is \( Q \)’ is given by
\[ P \rightarrow Q = P' \oplus Q = \{(\int \mu_P(x) \times \int \mu_Q(y))\} \]

Mamdani [2] fuzzy conditional inference of type ‘if \( x \) is \( P' \) then \( y \) is \( R \)’ is given by
\[ P' \rightarrow R = P' \oplus Q = \{(\int \mu_P(x) \times \int \mu_R(y))\} \]
Logical system of Standard sequence $S$ method is given by

$$v(P \rightarrow Q) = \begin{cases} 
1 & v(P) \leq v(Q) \\
0 & v(P) > v(Q) 
\end{cases}$$

Logical system of Godelian sequence $G$ method is given by

$$v(P \rightarrow Q) = \begin{cases} 
1 & v(P) \leq v(Q) \\
v(Q) & v(P) > v(Q) 
\end{cases}$$

2.1. Fuzzy Placability

Consider the causal logical inference [6,7]

**Modus Ponens**

$p \rightarrow q$

$p$

$q$

**Modus Tollens**

$p \rightarrow q$

$q'$

$p'$

**Generalization**

$p \lor q = p$

$p \lor q = q$

**Specialization**

$p \land q = P$

$p \land q = q$

**Fuzzy plausibility**

Plausibility theory will perform inconsistent information into consistent.

**Generalization**

$p \lor q, \mu = P, \mu$

$p \lor q, \mu = q, \mu$

**Specialization**

$p \land q, \mu = P, \mu$

$p \land q = q, \mu$

The inference is given using generalization and specialization

$p \land q \lor r, \mu = P, \mu$

$p \land q \lor r, \mu = q, \mu$

Consider fuzzy conditional inference

Fuzzy conditional inference for Type-1 is given by

If $x$ is $P$ then $y$ is $Q$
Fuzzy conditional inference for Type-2 is given by
If \( x \) is \( P \) then \( y \) is \( Q \) else \( y \) is \( R \)
\[
\begin{array}{c}
\text{If } x \text{ is } P \text{ then } y \text{ is } Q \\
\text{else } y \text{ is } R
\end{array}
\]
Fuzzy conditional inference for Type-3 is given by
If \( x \) is \( P \) and \( x \) is \( Q \) or \( x \) is \( R \) then \( y \) is \( S \)
\[
\begin{array}{c}
\text{If } x \text{ is } P \text{ and } x \text{ is } Q \text{ or } x \text{ is } R \text{ then } y \text{ is } S
\end{array}
\]
3. New Methods of Fuzzy Conditional Inference

The fuzzy conditional propositions is of the form ‘if (precedent part) then (consequent part)’. Mamdani [2] fuzzy conditional inference given by \( P \rightarrow Q \equiv \{ P \times Q \} \).
The consequent part is derived from precedent part for fuzzy conditional inference [8,9].
if \( x \) is \( P \) then \( y \) is \( Q = P \)

\[
\int \mu_Q(y) = \int \mu_P(x), \text{ i.e. } Q \subseteq P \text{ and } P \subseteq Q \text{ Consider fuzzy quantifiers } P^\alpha \text{ and } Q^\alpha
\]

\[
P^\alpha \subseteq Q
\]

\[
Q^\alpha \subseteq P
\]

The fuzzy conditional inference is given by using Mamdani fuzzy conditional inference

if \( x \) is \( P \) then \( y \) is \( Q = \{ P \times Q \} \)

if \( x \) is \( P \) then \( y \) is \( Q = \{ P \times P \} \)

if \( x \) is \( P \) then \( y \) is \( Q = \{ P \} \)

if \( x \) is \( P \) then \( y \) is \( Q = \{ \int \mu_P(x) \} \)

Similarly,

\[
\int \mu_R(y) = \int \mu_P(x), \text{ i.e. } R \subseteq P' \text{ and } P' \subseteq R
\]

Consider fuzzy quantifiers \( P'^\alpha \) and \( R^\alpha \)

\[
P'^\alpha \subseteq R
\]

\[
R^\alpha \subseteq P'
\]

From the Type-1, Type-2 and Type-3, we have Criterion-1 and Criterion-2

**Criterion-1**

If \( x \) is \( P \) then \( y \) is \( Q \)

\[
\begin{array}{c|c|c|c}
I-1 & x \text{ is } P & y \text{ is } Q & \text{Inference} \\
I-2 & y \text{ is } Q & x \text{ is } P & \text{Inference} \\
II-1 & x \text{ is very } P & y \text{ is very } Q & \text{Inference} \\
II-2 & y \text{ is very } Q & x \text{ is very } P & \text{Inference} \\
III-1 & x \text{ is more or less } P & y \text{ is more or less } Q & \text{Inference} \\
III-2 & y \text{ is more or less } Q & x \text{ is more or less } P & \text{Inference} \\
IV-1 & x \text{ is not } P & y \text{ is not } Q & \text{Inference} \\
IV-2 & y \text{ is not } Q & x \text{ is not } P & \text{Inference}
\end{array}
\]

**4. Verification of Fuzzy Criterions**

**4.1. Verification of Fuzzy Criterion-1**

The fuzzy intuitions are given by Table 1.

Verification of fuzzy intuitions

| Intuition | Proposition | Inference |
|-----------|-------------|-----------|
| I-1       | \( x \) is \( P \) | \( y \) is \( Q \) |
| I-2       | \( y \) is \( Q \) | \( x \) is \( P \) |
| II-1      | \( x \) is very \( P \) | \( y \) is very \( Q \) |
| II-2      | \( y \) is very \( Q \) | \( x \) is very \( P \) |
| III-1     | \( x \) is more or less \( P \) | \( y \) is more or less \( Q \) |
| III-2     | \( y \) is more or less \( Q \) | \( x \) is more or less \( P \) |
| IV-1      | \( x \) is not \( P \) | \( y \) is not \( Q \) |
| IV-2      | \( y \) is not \( Q \) | \( x \) is not \( P \) |
4.1.1. In the Case of Intuition I-1
\[ P \circ (P \rightarrow Q) \]
\[ = \int \mu_P(x) \circ (\int \mu_P(x) \rightarrow \int \mu_Q(y)) \]
Considering \( P \rightarrow Q = P \)
Considering \( Q = P \)
\[ = \int \mu_Q(y) \circ (\int \mu_Q(y)) \]
\[ = \int \mu_Q(y) \wedge (\int \mu_Q(y)) \]
Using specialization
\[ = \int \mu_Q \]
\( = y \) is \( Q \)
Intuition I-1 satisfied.

4.1.2. In the Case of Intuition I-2
\[ (P \rightarrow Q) \circ Q \]
\[ = (\int \mu_P(x) \rightarrow \int \mu_Q(y)) \circ \int \mu_Q(y) \]
Considering \( P \rightarrow Q = P \)
Considering \( Q = P \)
\[ = \int \mu_P(x) \circ \int \mu_P(x) \]
\[ = \int \mu_P(x) \wedge \int \mu_P(x) \]
Using specialization
\[ = \int \mu_P(x) \]
\( = x \) is \( P \)
Intuition I-2 satisfied.

4.1.3. In the Case of Intuition II-1
\[ \text{very} P \circ (P \rightarrow Q) \]
\[ = \int \mu_{\text{very}P}(x) \circ (\int \mu_P(x) \rightarrow \int \mu_Q(y)) \]
Considering \( P \rightarrow Q = P \)
Considering \( Q = P \)
\[ = \int \mu_{\text{very}Q}(y) \circ (\int \mu_Q(y)) \]
\[ = \int \mu_{\text{very}Q}(y) \wedge (\int \mu_Q(y)) \]
Using specialization
\[ = \int \mu_{\text{very}Q}(y) \]
\( = y \) is \( Q \)
Intuition II-1 satisfied.

4.1.4. In the Case of Intuition II-2
\[ (P \rightarrow Q) \circ \text{very} Q \]
\[ = (\int \mu_P(x) \rightarrow \int \mu_{\text{very}Q}(y)) \circ \int \mu_Q(y) \]
Considering \( P \rightarrow Q = P \)
Considering \( Q = P \)
\[ = \int \mu_P(x) \circ \int \mu_{\text{very}P}(x) \]
\[ = \int \mu_P(x) \wedge \int \mu_{\text{very}P}(x) \]
Using specialization
\[ = \int \mu_{\text{very}P}(x) \]
\( = x \) is \( P \)
Intuition II-2 satisfied.
4.1.5. In the Case of Intuition III-1
more or less $P \circ (P \rightarrow Q)$
\[ = \int \mu_{\text{more or less}}(x) \circ (\int \mu_P(x) \rightarrow \int \mu_Q(y)) \]
Considering $P \rightarrow Q = P$
Considering $Q = P$
\[ = \int \mu_{\text{more or less}}(y) \circ (\int \mu_P(x)) \]
\[ = \int \mu_{\text{more or less}}(y) \wedge (\int \mu_Q(y)) \]
Using specialization
\[ = \int \mu_{\text{more or less}}(y) \]
\[ = y \text{ is more or less } Q \]
Intuition III-1 satisfied.

4.1.6. In the Case of Intuition III-2
$(P \rightarrow Q) \circ \text{more or less } Q$
\[ = (\int \mu_P(x) \rightarrow \int \mu_{\text{more or less}}(y)) \circ \int \mu_Q(y) \]
Considering $P \rightarrow Q = P$
Considering $Q = P$
\[ = \int \mu_P(x) \circ \int \mu_{\text{more or less}}(x) \]
\[ = \int \mu_P(x) \wedge \int \mu_{\text{more or less}}(x) \]
Using specialization
\[ = \int \mu_{\text{more or less}}(x) \]
\[ = x \text{ is more or less } P \]
Intuition III-2 satisfied.

4.1.7. In the Case of Intuition IV-1
not $P \circ (P \rightarrow \overline{Q})$
\[ = \int \mu_{\overline{P}}(x) \circ (\int \mu_P(x) \rightarrow \int \mu_Q(y)) \]
Considering $P \rightarrow Q = P$
Considering $Q = P$
\[ = \int \mu_{\overline{P}}(y) \circ (\int \mu_Q(x)) \]
\[ = \int \mu_{\overline{P}}(y) \wedge (\int \mu_Q(y)) \]
Using specialization
\[ = \int \mu_{\overline{P}}(y) \]
\[ = y \text{ is not } Q \]
Intuition IV-1 satisfied.

4.1.8. In the Case of Intuition IV-2
$(P \rightarrow Q) \circ \text{not } Q$
\[ = (\int \mu_P(x) \rightarrow \int \mu_{\overline{Q}}(y)) \circ \int \mu_{\overline{Q}}(y) \]
Considering $P \rightarrow Q = P$
Considering $Q = P$
\[ = \int \mu_P(x) \circ \int \mu_{\overline{Q}}(x) \]
\[ = \int \mu_P(x) \wedge \int \mu_{\overline{Q}}(x) \]
Using specialization
\[ = \int \mu_{\overline{Q}}(x) \]
\[ = x \text{ is not } P \]
Table 2. Fuzzy intuitions for criterion-2.

| Intuition | Proposition | Inference |
|-----------|-------------|-----------|
| I'-1      | $x$ is $P'$ | $y$ is $R$ |
| I'-2      | $y$ is $R$  | $x$ is $P'$ |
| II'-1     | $x$ is very $P'$ | $y$ is very $R$ |
| II'-2     | $y$ is very $R$ | $x$ is very $P'$ |
| III'-1    | $x$ is more or less $P'$ | $y$ is more or less $R$ |
| III'-2    | $y$ is more or less $R$ | $x$ is more or less $P'$ |
| IV'-1     | $x$ is not $P'$ | $y$ is not $R$ |
| IV'-2     | $y$ is not $R$ | $x$ is not $P'$ |

Intuition IV-2 satisfied.
Criteria-1 is satisfies I-1, I-2, II-1, II-2, III-1 and III-2, IV-1, IV-2.

4.2. Verification of Fuzzy Criterion-2

The fuzzy intuitions are give by Table 2.

4.2.1. In the Case of Intuition I'-1

$$P' \circ (P' \rightarrow R)$$

$$= \int \mu_{P'}(x) \circ (\int \mu_{P'}(x) \rightarrow \int \mu_{R}(y))$$

Considering $P' \rightarrow R = P'$

Considering $R = P'$

$$= \int \mu_{R}(y) \circ (\int \mu_{R}(y))$$

$$= \int \mu_{R}(y) \land (\int \mu_{R}(y))$$

Using specialization

$$= \int \mu_{R}(y)$$

$$= y \text{ is } R$$

Intuition I'-1 satisfied.

4.2.2. In the Case of Intuition I'-2

$$(P' \rightarrow R) \circ R$$

$$= (\int \mu_{P'}(x) \rightarrow \int \mu_{R}(y)) \circ \int \mu_{R}(y)$$

Considering $P' \rightarrow R = P'$

Considering $R = P'$

$$= \int \mu_{P'}(x) \circ \int \mu_{P'}(x)$$

$$= \int \mu_{P'}(x) \land \int \mu_{P'}(x)$$

Using specialization

$$= \int \mu_{P'}(x)$$

$$= x \text{ is } P'$$

Intuition I'-2 satisfied.

4.2.3. In the Case of Intuition II'-1

very $P' \circ (P' \rightarrow R)$

$$= \int \mu_{\text{very}P'}(x) \circ (\int \mu_{P'}(x) \rightarrow \int \mu_{R}(y))$$

Considering $P' \rightarrow R = P'$

Considering $R = P'$
\[
\int \mu_{\text{very}R}(y) \circ (\int \mu_R(y))
\]
\[
= \int \mu_{\text{very}R}(y) \wedge (\int \mu_R(y))
\]
Using specialization
\[
= \int \mu_{\text{very}R}(y)
\]
= y is very \(R\)
Intuition II'-1 satisfied.

4.2.4. In the Case of Intuition II'-2

\[
(P' \rightarrow R) \circ \text{very } R
\]
\[
= (\int \mu_{P'}(x) \rightarrow \int \mu_R(y)) \circ \int \mu_{\text{very}R}(y)
\]
Considering \(P' \rightarrow R = P'\)
Considering \(R = P'\)
\[
= \int \mu_{P'}(x) \circ \int \mu_{\text{very}P'}(x)
\]
\[
= \int \mu_{P'}(x) \wedge \int \mu_{\text{very}P'}(x)
\]
Using specialization
\[
= \int \mu_{\text{very}P'}(x)
\]
= x is very \(P'\)
Intuition II'-2 satisfied.

4.2.5. In the Case of Intuition III'-1

more or less \(P' \circ (P' \rightarrow R)\)
\[
= \int \mu_{\text{more or less}P}(x) \circ (\int \mu_{P}(x) \rightarrow \int \mu_R(y))
\]
Considering \(P' \rightarrow R = P'\)
Considering \(R = P'\)
\[
= \int \mu_{\text{more or less}R}(y) \circ (\int \mu_R(y))
\]
\[
= \int \mu_{\text{more or less}R}(y) \wedge (\int \mu_R(y))
\]
Using specialization
\[
= \int \mu_{\text{more or less}R}(y)
\]
= y is more or less \(R\)
Intuition III'-1 satisfied.

4.2.6. In the Case of Intuition III'-2

\[
(P' \rightarrow R) \circ \text{more or less } R
\]
\[
= (\int \mu_{P'}(x) \rightarrow \int \mu_R(y)) \circ \int \mu_{\text{more or less}R}(y)
\]
Considering \(P' \rightarrow R = P'\)
Considering \(R = P'\)
\[
= \int \mu_{P'}(x) \circ \int \mu_{\text{more or less}P}(x)
\]
\[
= \int \mu_{P'}(x) \wedge \int \mu_{\text{more or less}P}(x)
\]
Using specialization
\[
= \int \mu_{\text{more or less}P}(x)
\]
= x is more or less \(P'\)
Intuition III'-2 satisfied.

4.2.7. In the Case of Intuition IV'-1

\[
\text{not}P' \circ (P' \rightarrow R)
\]
\[
= \int \mu_{\text{not}P}(x) \circ (\int \mu_{P}(x) \rightarrow \int \mu_R(y))
\]
Considering not$P' \to R = P'$
Considering not$R = notP' = \int \mu_{notR}(y) \wedge (\int \mu_R(y))$

Using specialization
= $\int \mu_{notR}(y)$
= $y$ is not $R$
   Intuition IV'-1 satisfied.

4.2.8. In the Case of Intuition IV'-2
($P' \to R \wedge \text{not } R$
= $(\int \mu_{P'}(x) \to \int \mu_R(y)) \circ \int \mu_{notR}(y)$

Considering $P' \to R = P'$
Considering not$R = notP' = \int \mu_{P'}(x) \wedge \int \mu_{notP'}(x)$

Using specialization
= $\int \mu_{notP'}(x)$
= $x$ is $P$
   Intuition l'-2 satisfied.
   Criteria-2 is satisfies l'-1, l'-2, II'-1, II'-2, III'-1, III'-2, IV'-1, IV'-2.
   The fuzzy conditional inferences of types may proved similar lines

5. Conclusion
We introduced fuzzy natural deduction as an easily proof procedure. The fuzzy intuitions are studied for fuzzy conditional inference with our method for the proposition containing ‘if … then …’, ‘if … and/or … and/or then …’, and ‘if … then … else …’. All the Fuzzy intuitions are satisfied with our method.

Disclosure statement
No potential conflict of interest was reported by the author(s).

Notes on contributor
Poli Venkata Subba Reddy is Professor of Computer Science and Engineering, College of Engineering, Sri Venkateswara University, Tirupati, India, He did MSc (Applied Mathematics, 1986), MPhill (DBMS, 1988), PhD (Artificial Intelligence, 1992) in same University. His areas of interest are Fuzzy Systems, Database Systems and Artificial Intelligence.

ORCID
Poli Venkata Subba Reddy  http://orcid.org/0000-0002-3531-7660
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