Competition between $\alpha$-decay and $\beta$-decay for Heavy and Superheavy Nuclei

SHENG Zong-Qiang$^{1,1)}$  SHU Liang-Ping$^1$  MENG Ying$^3$
HU Ji-Gang$^2$  QIAN Jian-Fa$^3$

$^1$ School of Science, Anhui University of Science and Technology, Huainan 232001, China
$^2$ School of Electronic Science and Applied Physics, Hefei University of Technology, Hefei 230009, China

Abstract: In this work, the $\beta$-stable region for $Z \geq 90$ is proposed. The calculated $\beta$-stable nuclei in the $\beta$-stable region are in good agreement with the ones obtained by Möller et al. The half-lives of the nuclei close to the $\beta$-stable region are calculated and the competition between $\alpha$-decay and $\beta$-decay is systematically investigated. The calculated half-lives and the suggested decay modes are well in line with the experimental results. The predictions for half-lives and decay modes of the nuclei with $Z = 107-110$ are presented.

Key words: decay, $\beta$-stable region, half-life, superheavy nuclei

PACS: 21.10.Tg, 23.60.+e, 27.90.+b

1 Introduction

After Becquerel discovered nuclear radioactivity in 1896, scientists began to investigate nuclear decay and found several decay modes. $\alpha$-decay and $\beta$-decay are two of the most important decay modes. $\alpha$-decay was first explained by Rutherford in 1908 [1]. In the 1930s, Fermi proposed the basic theory of $\beta$-decay [2]. From then on, $\alpha$-decay and $\beta$-decay have been widely studied theoretically and experimentally [3–10], and a large number of research results and publications have come out. Generally speaking, $\alpha$-decay mostly occurs in heavy and superheavy nuclei, while $\beta$-decay can occur throughout the whole periodic Table.

In the early stage of the development of nuclear physics, scientists could only study the properties of the nuclei very close to the $\beta$-stable line. As a result, many nuclear phenomena, laws, formulae, methods, and models were based on the long-lived nuclei or stable nuclei close to the $\beta$-stable line. It is much easier to find and synthesize new nuclei close to the $\beta$-stable line. Nowadays, with the development of radioactive nuclear beams, many nuclei far from the $\beta$-stable line have been studied. Many new experimental phenomena have been discovered. At present, the stable nuclei with $Z < 83$ are very clear and definite. On the other hand, most nuclei beyond $Z = 83$ are unstable, and their half-lives are usually short. In this article, the $\beta$-stable region for $Z \geq 90$ will be proposed. The half-lives of the nuclei close to the $\beta$-stable region will be calculated and the competition between $\alpha$-decay and $\beta$-decay will be investigated. Then the decay modes can be suggested by the results of competition.

This article is organized in the following way. In Sec. 2, the $\beta$-stable region for $Z \geq 90$ is proposed. In Sec. 3, the half-lives of the nuclei close to the $\beta$-stable region are calculated and the competition between $\alpha$-decay and $\beta$-decay is studied. A summary is given in Sec. 4.

2 The $\beta$-stable region for $Z \geq 90$

The $\beta$-stable line for $Z < 83$ has been well studied by physicists. For heavy and superheavy nuclei with $Z \geq 90$, most of them can occur $\alpha$-decay and $\beta$-decay simultaneously, and their half-lives are usually short. For this reason, it is more important to study the $\beta$-stable region than to study the $\beta$-stable line for these heavy and superheavy nuclei. In this section, we investigate the boundary of the $\beta$-stable region based on a successful binding energy formula.

To accurately measure and calculate the ground-state nuclear binding energies (or masses) is an important goal of nuclear physicists. The binding energy plays a crucial role for the nuclear stability on $\beta$-decay, $\alpha$-decay and spontaneous fission of heavy-mass region with $Z \geq 90$. 

---

Received 14 March 2009

* Supported by National Natural Science Foundation of China under Grant No. 11247001, by the Research Foundation of Education Bureau of Anhui Province, China under Grant Nos. KJ2012A083 and KJ2013Z066, and by Fundamental Research Funds for the Central Universities of China under Grant No. 2012HGZ004.

1) E-mail: zqsheng@aust.edu.cn

©2013 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd
In Ref. [11], Dong and Ren proposed a binding energy formula for heavy and superheavy nuclei. One can accurately reproduce the binding energies for the known heavy and superheavy nuclei with this formula. This formula is useful for accurately estimating the binding energies of unknown superheavy nuclei. Its form is the following:

\[
B(Z, A) = a_v A - a_s A^{2/3} - a_e Z^2 A^{-1/3} - a_a \frac{(A/2 - Z)^2}{A} + a_p A^{-1/2} + \frac{a_6|A-252| - a_7|N-152|}{A} + \frac{a_8|N-Z-50|}{A}.
\]  

The best fit parameters are

\[
\begin{align*}
    a_v &= 15.8032 \text{ MeV,} \\
    a_s &= 17.8147 \text{ MeV,} \\
    a_e &= 0.71478 \text{ MeV,} \\
    a_a &= 97.6619 \text{ MeV,} \\
    a_6 &= 5.33 \text{ MeV,} \\
    a_7 &= 21.0 \text{ MeV,} \\
    a_8 &= -15.25 \text{ MeV.}
\end{align*}
\]  

The coefficients of the pairing energy are

\[
 a_p = \begin{cases} 
    12.26 \text{ MeV, } & \text{even-even nuclei,} \\
    3.0 \text{ MeV, } & \text{even-odd nuclei,} \\
    0 \text{ MeV, } & \text{odd-odd nuclei,} \\
    -8.8 \text{ MeV, } & \text{odd-odd nuclei.}
\end{cases}
\]  

The mass formula has the form:

\[
M(Z, A) = ZM_H + Nm_n - B(Z, A) = AM_n + Z(M_H - m_n) - B(Z, A),
\]  

where \((M_H - m_n) = -0.782 \text{ MeV.}\)

The decay energies of \(\beta^\text{-decay}\) and \(\beta^+\text{-decay}\) can be written as:

\[
E_d(\beta^-) = M(Z, A) - M(Z + 1, A),
\]  

\[
E_d(\beta^+) = -1.804 - a_6(2Z+1)A^{-1/3} - a_a(A - 2Z - 1)
\]

\[
+ a_7 \left( \frac{|A-Z-152|}{A-Z} - \frac{|A-Z-151|}{A-Z+1} \right)
\]

\[
+ a_8 \left( \frac{|A-2Z-52| - |A-2Z-50|}{A} \right).
\]  

If the values of \(E_d(\beta^-)\) and \(E_d(\beta^+)\) are set to zero, one can get the limits of \(\beta^-\text{-decay}\) and \(\beta^+\text{-decay}\) for each isotopic chain. For each fixed proton number \(Z\), one can get two different mass numbers for the limits of \(\beta^-\text{-decay}\) and \(\beta^+\text{-decay}\), respectively. For all the proton numbers from \(Z = 90\) to \(Z = 126\), two sets of mass numbers for the limits of \(\beta^-\text{-decay}\) and \(\beta^+\text{-decay}\) can be obtained. Connecting two sets of mass numbers for the limits of \(\beta^-\text{-decay}\) and \(\beta^+\text{-decay}\) in the coordinate space \((Z, A)\), the boundary of the limits of \(\beta^-\text{-decay}\) and \(\beta^+\text{-decay}\) can be obtained. The calculated results are plotted in figure 1.

In figure 1, the two curves denote the limits of \(\beta^-\text{-decay}\) and \(\beta^+\text{-decay}\), respectively. They are almost parallel. The calculated \(\beta\)-stable region is a long and narrow region between the two curves. According to the calculations, the nuclei above the calculated \(\beta\)-stable region can occur \(\beta^-\text{-decay}\) and the nuclei below the region can occur \(\beta^+\text{-decay}\). The nuclei in the \(\beta\)-stable region are the possible \(\beta\)-stable nuclei. Because there are few experimental data, we compare our calculated results with the results given by Möller et al. [12]. The comparison between our calculated results and Möller’s results are shown in table 1.

Table 1.  The possible \(\beta\)-stable nuclei in the calculated \(\beta\)-stable region for \(Z \geq 90\). The corresponding \(\beta\)-stable nuclei calculated by Möller et al. [12] are listed for comparison.
stable nuclei in their isotopic chains except for $Z = 115$, the results calculated by Möller et al. Möller’s results show that there are only one or two nuclei from Möller’s results, and vice versa. For odd $Z$, nuclei from our calculations, but they are not $\beta$-stable calculated almost the same. On the whole, the range by Möller their isotopic chain. For even $Z$, the two results are al-

| $Z$ | Mass number $A$ of $\beta$-stable nuclei |
|-----|----------------------------------------|
| 90  | 224–230, 224, 226–230, 232             |
| 91  | 227–232, 231                           |
| 92  | 230–235, 230, 232–236, 238             |
| 93  | 233–238, 237                           |
| 94  | 236–241, 236, 238–242, 244             |
| 95  | 238–244, 241#, 243#                    |
| 96  | 241–247, 240, 242–246, 248             |
| 97  | 244–250, 247                           |
| 98  | 247–252, 246, 248–252, 254             |
| 99  | 250–255, 253                           |
| 100 | 252–258, 252, 254–258, 260, 262        |
| 101 | 255–261, 259                           |
| 102 | 258–264, 258, 260–264, 266             |
| 103 | 261–267, 265                           |
| 104 | 264–270, 264, 266–268, 270, 272, 274  |
| 105 | 267–273, 269, 271                      |
| 106 | 270–276, 268, 270, 272–276, 278, 280  |
| 107 | 273–279, 277                           |
| 108 | 276–282, 274, 276, 278–284, 286        |
| 109 | 279–286, 283#, 285#                    |
| 110 | 282–289, 282, 284–288, 290             |
| 111 | 285–292, 289                           |
| 112 | 288–295, 288, 290–294, 296             |
| 113 | 291–298, 293#, 295#                    |
| 114 | 294–301, 292, 294–298, 300, 302, 304   |
| 115 | 298–304, 299, 301, 303                 |
| 116 | 301–308, 300, 302, 304–306, 308, 310–312, 314 |
| 117 | 304–311, 307, 309                      |
| 118 | 307–314, 304, 306, 308, 310–314, 318  |
| 119 | 310–317, 315                           |
| 120 | 313–320, 312, 316–318, 320, 322, 324   |
| 121 | 316–324, 319                           |
| 122 | 320–327, 318, 320, 321, 323–326, 328, 330 |
| 123 | 324–330, 322, 327                      |
| 124 | 327–334, 317, 323, 324, 326, 328–332, 334, 336, 338 |
| 125 | 329–337, 325, 327, 333                 |
| 126 | 332–340, 326, 330, 332, 334–338        |

In table 1, the calculated possible $\beta$-stable nuclei and the results calculated by Möller et al. [12] are listed in the second and the third columns, respectively. The mass numbers with # denote that the nuclei with these mass numbers are $\beta$-stable nuclei by estimated from systematic trends in neighboring nuclei. Our calculated results show that there are several (from six to nine) $\beta$-stable nuclei in each isotopic chain and they are continuous in their isotopic chain. For even $Z$, the two results are almost the same. On the whole, the range by Möller et al. is slightly larger than our calculated results. The calculated $\beta$-stable nuclei by Möller et al. are not continuous in their isotopic chains. Some nuclei are $\beta$-stable nuclei from our calculations, but they are not $\beta$-stable nuclei from Möller’s results, and vice versa. For odd $Z$, Möller’s results show that there are only one or two $\beta$-stable nuclei in their isotopic chains except for $Z = 115, 125$. It is different from our results. But it can be seen that the $\beta$-stable nuclei from Möller’s results are all in the middle of our calculated $\beta$-stable region except for $Z = 123, 125$. From the above discussions, it can be said that the calculated $\beta$-stable region are in good agreement with the Möller’s results. For further comprehending the calculated $\beta$-stable region, the $\beta$-stable nuclei from both the calculated results and Möller’s results are drawn in figure 2.

In figure 2, the hollow squares denote the possible $\beta$-stable nuclei obtained by our calculations, and the dark circles are the ones from Möller’s results. For even $Z$, the $\beta$-stable nuclei obtained from Möller’s results almost cover the calculated $\beta$-stable nuclei. For odd $Z$, except $Z = 123, 125$, the $\beta$-stable nuclei obtained from Möller’s results are included in the calculated $\beta$-stable nuclei. It is in line with the above discussions.

3 Competition between $\alpha$-decay and $\beta$-decay of the nuclei close to the $\beta$-stable region

In the previous section, the $\beta$-stable region for $Z \geq 90$ has been proposed. Most nuclei with $Z \geq 90$ can occur $\alpha$-decay, $\beta$-decay and spontaneous fission simultaneously. In this section, we will calculate the half-lives of the nuclei close to the calculated $\beta$-stable region, and study the competition between $\alpha$-decay and $\beta$-decay of them. It is a very interesting topic. There are plenty of experimental half-lives and the decay modes of many nuclei are very explicit in this region. The calculated results can be compared with these experimental data and the reliability of the calculated results can be tested. On the other hand, the predictions are useful for quickly estimating
the decay modes and half-lives of future superheavy experiments. Before calculating the half-lives, we firstly introduce several successful formulae for calculating.

\( \alpha \)-decay is a very general decay mode for the ground states of heavy and superheavy nuclei. In Ref. [13], Ni et al. proposed a unified formula of half-lives for \( \alpha \)-decay and cluster radioactivity. For \( \beta \)-decay, in Ref. [14], Zhang et al. proposed a reliable formula to calculate the \( \beta \)-decay half-lives. It is written as:

\[
\log_{10} T_{1/2} = 2a\sqrt{\mu}(Z-2)Q_o^{-1/2} + b\sqrt{\mu}[2(Z-2)]^{1/2} + c,
\]

where \( \mu = 4(A-4)/A \), \( T_{1/2} \) is the half-life of \( \alpha \)-decay (in seconds), and \( Q_o \) is \( \alpha \)-decay energy (in MeV). \( A \) and \( Z \) are the mass number and the proton number of the parent nuclei respectively. The values of the parameters are \( a = 0.39961 \), \( b = -1.31008 \). Parameter \( c \) is determined to be \( c_{e-e} = -17.00698 \) (for even-even nuclei), \( c_{e-o} = -16.26029 \) (for even-odd nuclei), \( c_{o-e} = -16.40484 \) (for odd-even nuclei), and \( c_{o-o} = -15.85337 \) (for odd-odd nuclei).

\( \beta \)-decay is also a very important decay mode for rich-neutron or rich-proton nuclei. For \( \beta^- \)-decay, in Ref. [15], Zhang et al. proposed a reliable formula to calculate the \( \beta^- \)-decay half-lives. It is written as:

\[
\log_{10} T_{1/2} = (c_1Z + c_2)N + c_3Z + c_4 + \text{shell}(Z,N),
\]

where

\[
\text{shell}(Z,N) = c_5e^{-((N-29)^2/15 + e^{-(N-50)^2/37}} + e^{-(N-85)^2/9 + e^{-(N-131)^2/3}} + c_6e^{-(Z-51.5)^2 + (N-80.5)^2}/1.9
\]

is the shell correction term. \( Z \) and \( N \) are the proton number and neutron number of the parent nuclei. \( T_{1/2} \) is the half-life of \( \beta^- \)-decay (in seconds). The parameters are \( c_1 = 3.37 \times 10^{-4} \), \( c_2 = -0.2558 \), \( c_3 = 0.4028 \), \( c_4 = -1.0100 \), \( c_5 = 0.9039 \), and \( c_6 = 7.7139 \).

For \( \beta^+ \)-decay, in Ref. [16], Zhang et al. proposed a similar formula to the Eq. (10). It is written as:

\[
\log_{10} T_{1/2} = (c_1Z + c_2)N + c_3Z + c_4.
\]

For different order (the allowed \( \beta^+ \)-transition, the first and the second forbidden \( \beta^- \)-transition), the parameters are different. The even-odd effect has been taken into account in the above equation. The best fit parameters are displayed in Table 2.

### Table 2: The parameters of the Eq. (12). The word “order” in the first column denotes the order of the \( \beta^+ \)-decay from ground state to ground state. The even-odd effect has been included.

| order | \( c_1 \) | \( c_2 \) | \( c_3 \) | \( c_4 \) | \( e-o \) | \( o-e \) | \( o-o \) |
|-------|--------|--------|--------|--------|------|------|------|
| allowed | -0.00179 | 0.4233 | -0.3405 | -0.6443 | -1.7389 | -2.1323 |
| first | -0.00127 | 0.3992 | -0.4183 | 3.8215 | 3.7969 | 4.0364 |
| second | -0.00162 | 0.3980 | -0.3286 | -0.1618 | -0.4854 | 0.0267 |

For a given proton number \( Z \), we select ten continuous isotopes nearest to the top and bottom of the \( \beta^- \)-stable region, respectively. Thus there are 20 nuclei for each isotopic chain. Because only the half-lives of the allowed \( \beta^- \)-transition, the first and the second forbidden \( \beta^+ \)-transition can be calculated by the Eq. (12), the nuclei with higher forbidden \( \beta^- \)-transition are not included. Because the formula (9) can only calculate the half-lives of the nuclei with \( Z \geq 84 \) and \( N \geq 128 \), the nuclei with \( N < 128 \) are not included also. So the number of the calculated nuclei of each isotopic chain may be less than 20. We calculate the half-lives of the nuclei from \( Z = 90 \) to \( Z = 126 \), and predict the decay modes of them. Because the calculated data are too many, we firstly compare the calculated results with the available experimental data [16]. The selected region for comparison is from \( Z = 90 \) to \( Z = 103 \), because there are many experimental data in this region. The results are listed in Table 3.

### Table 3: The comparison of the half-lives and decay modes between the calculated results and the experimental data by Audi et al. [13] from \( Z = 90 \) to \( Z = 103 \). Here \( C = \log_{10}(T_{1/2}^{cal}/T_{1/2}^{expt}) \).

| \( Z \) | \( A \) | \( T_{1/2}^{cal} \) | \( T_{1/2}^{cal} \) | Calculated decay modes and intensities(%) | Experimental decay modes and intensities(%) | \( T_{1/2}^{expt} \) | \( C \) |
|-------|-------|---------------|---------------|---------------------------------|---------------------------------|--------------|------|
| 90    | 218   | 9.14 \times 10^{-5} | 718.8 | \( \alpha = 100 \) | \( \alpha = 100 \) | 1.09 \times 10^{-7} | -0.08 |
| 90    | 219   | 3.11 \times 10^{-5} | 844.5 | \( \alpha = 100 \) | \( \alpha = 100 \) | 1.05 \times 10^{-6} | 0.47 |
| 90    | 221   | 5.67 \times 10^{-4} | 3136.2 | \( \alpha = 100 \) | \( \alpha = 100 \) | 1.68 \times 10^{-3} | -0.47 |
| 90    | 222   | 0.0028 | 9912.9 | \( \alpha = 100 \) | \( \alpha = 100 \) | 2.05 \times 10^{-3} | 0.14 |

(Continued on next page)
| $Z$ | $A$ | $T_{\text{cal}}^{40K}$ | $T_{\beta}^{40K}$ | Calculated decay modes and intensities(%) | Experimental decay modes and intensities(%) | $T_{1/2}^{exp}$ | $C$ |
|-----|-----|----------------|----------------|---------------------------------|---------------------------------|----------------|-----|
| 90  | 223 | 0.93           | 1.16×10⁴       | $\alpha = 100$, $\beta^- = 100$ | $\alpha = 100$ | 0.6            | 0.19 |
| 231 | 1.63×10¹⁷ | 2823.1         | $\beta^- = 100$ | 9.19×10⁴                         | -1.51                          |
| 233 | 2.13×10²¹ | 999.5          | $\beta^- = 100$ | 2.08×10⁶                         | -3.54                          |
| 234 | 2.02×10²¹ | 594.7          | $\beta^- = 100$ | 432                             | -0.09                          |
| 235 | 3.78×10²⁴ | 353.9          | $\beta^- = 100$ | 2250                            | -1.03                          |
| 236 | 2.44×10²⁴ | 210.6          | $\beta^- = 100$ | 288                             | -0.36                          |
| 237 | 4.61×10²⁹ | 125.3          | $\beta^- = 100$ | 564                             | -0.88                          |
| 238 | 2.87×10²⁹ | 74.6           | $\beta^- = 100$ | 5.90×10⁶                         | 0.53                           |
| 239 | 2.02×10⁻⁵ | 424.7          | $\beta^- = 100$ | 5.10×10⁻³                         | 0.09                           |
| 240 | 0.0063 | 1568.1         | $\alpha = 100$, $\beta^+ < 0.001$ | 1.7                              | 0.58                           |
| 241 | 6.5   | 5789.2         | $\alpha = 100$ | 108                             | 0.78                           |
| 242 | 694.0 | 1.05×10⁴       | $\alpha = 94$, $\beta^+ = 6$ | 2.33×10⁶                         | -2.69                          |
| 243 | 7.47×10⁶ | 4741.5         | $\beta^+ = 100$ | 4.14×10⁴                         | -0.93                          |
| 244 | 1.15×10¹⁰ | 2823.4         | $\beta^- = 100$ | 1470                            | 0.06                           |
| 245 | 3.53×10¹⁰ | 1681.3         | $\beta^- = 100$ | 546                             | 0.26                           |
| 246 | 1.35×10²² | 1001.2         | $\beta^- = 100$ | 522                             | 0.06                           |
| 247 | 7.10×10²⁷ | 355.0          | $\beta^- = 100$ | 136                             | 0.42                           |
| 248 | 8.19×10²³ | 211.4          | $\beta^- = 100$ | 6.48×10³                         | -1.49                          |
| 249 | 3.83×10⁻⁴ | 212.4          | $\alpha = 100$, $\beta^+ = 0.2$ | 2.10×10⁻⁵                       | 1.26                           |
| 250 | 0.18  | 779.4          | $\alpha = 100$ | 0.061                           | 0.47                           |
| 251 | 0.34  | 2449.3         | $\alpha = 100$ | 0.269                           | 0.10                           |
| 252 | 99.1  | 2860.9         | $\alpha = 97$, $\beta^+ = 3$ | 66                               | 0.16                           |
| 253 | 659.4 | 4508.8         | $\alpha = 87$, $\beta^+ = 13$ | 546                             | 0.02                           |
| 254 | 8.59×10⁴ | 2888.3         | $\alpha = 85$, $\beta^+ = 15$ | 3.48×10³                         | -0.10                          |
| 255 | 3.37×10⁵ | 4753.1         | $\beta^+ = 100$ | 7.39×10¹⁴                        | 0.16                           |
| 256 | 1.02×10¹⁸ | 2832.6         | $\beta^+ = 100$ | 5.83×10⁵                         | -2.30                          |
| 257 | 1.62×10¹⁰ | 1006.0         | $\beta^+ = 100$ | 1.41×10³                         | -0.15                          |
| 258 | 8.57×10²⁰ | 599.5          | $\beta^+ = 100$ | 5.08×10⁴                         | -1.92                          |
| 259 | 4.35×10²³ | 212.9          | $\beta^+ = 100$ | 1.01×10³                         | -0.68                          |
| 260 | 1.34  | 385.2          | $\alpha = 100$, $\beta^+ = 0.05$ | 0.51                             | 0.42                           |
| 261 | 118.8 | 695.3          | $\alpha = 85$, $\beta^+ = 15$ | 61.4                             | 0.22                           |
| 262 | 1033.1 | 1405.5         | $\alpha = 85$, $\beta^+ = 15$ | 61.4                             | 0.22                           |
| 263 | 3.00×10⁴ | 64.6           | $\beta^+ = 100$ | 276                             | -0.63                          |
| 264 | 4.85×10⁵ | 1354.4         | $\beta^+ = 100$ | 2.93×10³                         | -0.34                          |
| 265 | 1.23×10¹⁶ | 4779.5         | $\beta^+ = 100$ | 2.04×10⁵                         | -1.63                          |
| 266 | 6.17×10¹⁶ | 2850.5         | $\beta^+ = 100$ | 3.71×10³                         | -0.11                          |
| 267 | 8.45×10¹⁸ | 1700.1         | $\beta^+ = 100$ | 834                             | 0.31                           |
| 268 | 2.05×10¹⁹ | 1013.9         | $\beta^+ = 100$ | 132                             | 0.86                           |
| 269 | 4.96×10¹⁹ | 604.7          | $\beta^+ = 100$ | 111                             | 0.74                           |
| 270 | 3.11×10²⁴ | 360.7          | $\beta^+ = 100$ | 137                             | 0.42                           |
| 271 | 24.3  | 189.2          | $\alpha = 69$, $\beta^+ = 11$ | 120                              |                               |
| 272 | 1.78×10⁴ | 194.6          | $\alpha = 1$, $\beta^+ = 99$ | 516                             | -0.43                          |
| 273 | 1.10×10⁶ | 629.9          | $\beta^+ = 100$, $\beta^+ = 0.12$ | 1.25×10³                         | -0.30                          |
| 274 | 1.86×10⁶ | 2038.6         | $\beta^+ = 100$, $\beta^+ = 0.003$ | 1.52×10³                         | 0.13                           |
| 275 | 1.72×10¹³ | 4821.1         | $\alpha = 100$ | 4.93×10¹¹                        |                               |
| 276 | 2.30×10¹⁵ | 2877.5         | $\beta^+ = 100$ | 1.78×10⁴                         | -0.79                          |
| 277 | 1.85×10¹⁵ | 1717.5         | $\alpha = 100$, SF = 0.12 | 1.05×10⁴                         |                               |
| 278 | 2.50×10¹⁷ | 1025.1         | $\beta^+ = 100$ | 3.78×10⁴                         | -1.57                          |
| 279 | 4.72×10¹⁷ | 611.9          | $\beta^+ = 100$ | 9.36×10⁵                         | -3.18                          |

(Continued on next page)
The calculated half-lives are obtained from the formulae (9), (10) and (12). When using the formula (9) to calculate half-lives of α-decay and β+-decay, we need α-decay energies $Q_\alpha$, spins and parities. Here, all the values of $Q_\alpha$ are taken from Ref. [16][17]. If there are no experimental data, we use the calculated data obtained by Möller et al. [12]. The calculated α-decay half-lives and the calculated β-decay (including β- decay and β+- decay) half-lives (in seconds) are listed in the third and the fourth columns in table 3. The fifth column is the calculated decay modes and intensities (in %). The decay mode can be regarded as a competition between α-decay and β-decay. Here we define a symbol $R$ to denote the ratio of α-decay half-life and β-decay half-life. It is defined as: $R = T_{\alpha 1/2}^{cal}/T_{\beta 1/2}^{cal}$. If the α-decay half-life is shorter than the β-decay half-life by 100 times (i.e., $R < 0.01$) in a nucleus, we can say the decay mode of this nucleus is α-decay. If the α-decay half-life is longer than the β-decay half-life by 100 times (i.e., $R > 100$) in a nucleus, we can say the decay mode of this nucleus is β-decay. If $0.01 < R < 100$, the decay mode can be regarded as a coexistence state of both α-decay and β-decay, and we use the symbol $\alpha + \beta^+$ to denote the coexistence state of both α-decay and β+-decay. The sixth and seventh columns are the experimental decay modes and intensities (in %) and half-lives. The data marked with # denote the values from systematic trends in neighboring nuclei. The symbol $C$ in the last column is the ratio of calculated half-life and experimental one, and it is in the form of $C = \log_{10}(T_{\alpha 1/2}^{cal}/T_{\beta 1/2}^{expt})$. The nuclei which do not have explicit experimental decay modes and intensities (in %) are not included in table 3. There are alto-
gether 91 nuclei. It can be seen that the predicted decay modes are in excellent agreement with the experimental ones. There are only six nuclei whose decay modes are not in line with the predicted decay modes. The proton numbers and the mass numbers of these six nuclei are marked in bold italic type in table 3. It indicates that numbers and the mass numbers of these six nuclei are not in line with the predicted decay modes. The proton modes are in excellent agreement with the experimental together 91 nuclei. It can be seen that the predicted decay modes are in excellent agreement with the experimental data.

After having compared the calculated results with the experimental data from Z = 90 to Z = 103, we predict some half-lives and decay modes for some heavier proton number Z. Synthesizing superheavy nuclei is a hot topic of nuclear physics. At present, Chinese physicists are endeavoring to synthesize some superheavy nuclei around Z = 110, and they have obtained some successful results [18]. Next, we select the region from Z = 107 to Z = 110 to make some predictions. There are many researches in this region [18–27]. We hope that our predictions will be useful for future experiments on heavy and superheavy nuclei. Because there are almost no explicit spins and parities for the nuclei with Z ≥ 107 except even-even nuclei, it is difficult for us to judge the orders of β+ -decay. When using the formula (12) to calculate half-lives of β+ -decay, we suppose all the orders of β+ -decay are the first β+ -transition for simplifying the calculations. The calculated results are listed in table 4.

Table 4: The calculated half-lives and the predicted decay modes of the nuclei from Z = 107 to Z = 110. Some available experimental half-lives of α-decay [18, 19, 22, 27] are listed for comparison. Here $D = \frac{T_{\alpha 1/2}^{cal}}{T_{\alpha 1/2}^{expt}}$.

| Z  | A     | Cal. $T_{\alpha 1/2}^{cal}$ (s) | Cal. $\beta 1/2$ (s) | Calculated decay modes and intensities(%) | $T_{\alpha 1/2}^{expt}$ (s) | D     |
|----|-------|-------------------------------|---------------------|----------------------------------------|---------------------|-------|
| 107| 264   | 0.16                          | 2.40                | $\alpha = 94, \beta^+ = 6$             | 0.9 [27]            | 0.18  |
| 107| 265   | 1.99                          | 4.64                | $\alpha = 70, \beta^+ = 30$           | 0.94 [18]           | 2.12  |
| 107| 266   | 20.9                          | 8.04                | $\alpha = 28, \beta^+ = 72$           | 5 [16]             | 4.18  |
| 107| 267   | 36.3                          | 15.6                | $\alpha = 30, \beta^+ = 70$           | 17 [22]            | 2.14  |
| 107| 268   | 55.3                          | 27                  | $\alpha = 33, \beta^+ = 67$           |                     |       |
| 107| 269   | 86.5                          | 52.4                | $\alpha = 38, \beta^+ = 62$           |                     |       |
| 107| 270   | 63.7                          | 90.9                | $\alpha = 59, \beta^+ = 41$           | 61 [19]            | 1.04  |
| 107| 271   | 0.91                          | 176.3               | $\alpha = 100$                        |                     |       |
| 107| 272   | 34.1                          | 305.5               | $\alpha = 90, \beta^+ = 10$           | 9.8 [19]           | 3.48  |
| 107| 273   | 56.2                          | $\beta$-stable      | $\alpha = 100$                        |                     |       |
| 107| 274   | 3.99×10^3                     | $\beta$-stable      | $\alpha = 100$                        |                     |       |
| 107| 275   | 108.5                         | $\beta$-stable      | $\alpha = 100$                        |                     |       |
| 107| 276   | 4.00×10^4                     | $\beta$-stable      | $\alpha = 100$                        |                     |       |
| 107| 277   | 4.58×10^6                     | $\beta$-stable      | $\alpha = 100$                        |                     |       |
| 107| 278   | 9.12×10^5                     | $\beta$-stable      | $\alpha = 100$                        |                     |       |
| 107| 279   | 1.07×10^7                     | $\beta$-stable      | $\alpha = 100$                        |                     |       |
| 107| 280   | 2.49×10^10                    | 1.19×10^4           | $\beta^− = 100$                       |                     |       |
| 107| 281   | 3.24×10^11                    | 7.16×10^5           | $\beta^− = 100$                       |                     |       |
| 107| 282   | 3.42×10^13                    | 4.31×10^3           | $\beta^− = 100$                       |                     |       |
| 107| 283   | 1.52×10^12                    | 2.60×10^3           | $\beta^− = 100$                       |                     |       |
| 107| 284   | 1.16×10^12                    | 1.57×10^3           | $\beta^− = 100$                       |                     |       |
| 107| 285   | 9.45×10^11                    | 945.6               | $\beta^− = 100$                       |                     |       |
| 107| 286   | 4.41×10^15                    | 570.1               | $\beta^− = 100$                       |                     |       |
| 107| 287   | 5.48×10^14                    | 343.7               | $\beta^− = 100$                       |                     |       |
| 107| 288   | 7.69×10^15                    | 207.2               | $\beta^− = 100$                       |                     |       |
| 107| 289   | 1.43×10^15                    | 124.9               | $\beta^− = 100$                       |                     |       |
| 108| 266   | 2.72×10^−3                    | 1.83                | $\alpha = 100, 2.3×10^−3$ [22]         |                     | 1.18  |
| 108| 267   | 0.129                         | 2.04                | $\alpha = 94, \beta^+ = 6$            | 0.058 [22]         | 2.22  |
| 108| 268   | 0.0376                        | 6.12                | $\alpha = 100$                        |                     |       |
| 108| 269   | 9.44                          | 6.82                | $\alpha = 42, \beta^+ = 58$           | 9.7 [22]           | 0.97  |
| 108| 270   | 1.88                          | 20.4                | $\alpha = 92, \beta^+ = 8$            | 3.6 [22]           | 0.52  |

(Continued on next page)
| \(Z\) | \(A\) | \(T_{\alpha 1/2}^{cal}\) (s) | \(T_{\beta 1/2}^{cal}\) (s) | Calculated decay modes and intensities(\%) | \(T_{\alpha 1/2}^{exp}\) (s) | \(D\) |
|-----|-----|-----------------|-----------------|---------------------------------|-----------------|-----|
| 108 | 271 | 0.21            | 22.8            | \(\alpha = 100\)               |                  |     |
| 108 | 272 | 0.011           | 68.3            | \(\alpha = 100\)               |                  |     |
| 108 | 273 | 0.21            | 76.2            | \(\alpha = 100\)               |                  |     |
| 108 | 274 | 0.49            | 228.4           | \(\alpha = 100\)               |                  |     |
| 108 | 275 | 0.31            | 254.6           | \(\alpha = 98, \beta^+ = 2\)  | 0.19 [19]        | 1.63 |
| 108 | 276 | 66.3            | \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 108 | 277 | 8.00 \times 10^3| \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 108 | 278 | 516.2           | \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 108 | 279 | 9.31 \times 10^4| \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 108 | 280 | 1.98 \times 10^6| \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 108 | 281 | 4.57 \times 10^{10}| \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 108 | 282 | 1.60 \times 10^{10}| \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 108 | 283 | 3.50 \times 10^{11}| 1.25 \times 10^4| \(\beta^- = 100\)              |                  |     |
| 108 | 284 | 1.60 \times 10^{10}| 7.54 \times 10^3| \(\beta^- = 100\)              |                  |     |
| 108 | 285 | 1.71 \times 10^{10}| 4.55 \times 10^3| \(\beta^- = 100\)              |                  |     |
| 108 | 286 | 8.25 \times 10^9  | 2.48 \times 10^3| \(\beta^- = 100\)              |                  |     |
| 108 | 287 | 9.52 \times 10^{13}| 1.66 \times 10^3| \(\beta^- = 100\)              |                  |     |
| 108 | 288 | 4.23 \times 10^{12}| 999.3          | \(\beta^- = 100\)              |                  |     |
| 108 | 289 | 1.09 \times 10^{14}| 602.9          | \(\beta^- = 100\)              |                  |     |
| 108 | 290 | 2.22 \times 10^{13}| 363.8          | \(\beta^- = 100\)              |                  |     |
| 108 | 291 | 1.09 \times 10^{14}| 219.5          | \(\beta^- = 100\)              |                  |     |
| 108 | 292 | 4.83 \times 10^{12}| 132.5          | \(\beta^- = 100\)              |                  |     |
| 109 | 269 | 7.33 \times 10^{-3} | 0.891        | \(\alpha = 100\)               |                  |     |
| 109 | 270 | 3.62 \times 10^{-4} | 1.54          | \(\alpha = 88, \beta^+ = 12\) | 5 \times 10^{-3} [22] | 0.07 |
| 109 | 271 | 0.073           | 2.96           | \(\alpha = 98, \beta^+ = 2\)  |                  |     |
| 109 | 272 | 0.018           | 5.1            | \(\alpha = 100\)               |                  |     |
| 109 | 273 | 1.45 \times 10^{-3} | 9.84        | \(\alpha = 100\)               |                  |     |
| 109 | 274 | 1.14            | 17             | \(\alpha = 94, \beta^+ = 6\)  | 0.445 [19]       | 2.56 |
| 109 | 275 | 9.82 \times 10^{-3} | 32.7        | \(\alpha = 100\)               | 9.7 \times 10^{-3} [19] | 1.01 |
| 109 | 276 | 1.57            | 56.3           | \(\alpha = 97, \beta^+ = 3\)  | 0.72 [19]        | 2.18 |
| 109 | 277 | 4.28            | 108.9          | \(\alpha = 96, \beta^+ = 4\)  |                  |     |
| 109 | 278 | 240.6           | 187.2          | \(\alpha = 44, \beta^+ = 56\) |                  |     |
| 109 | 279 | 1.29 \times 10^3 | \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 109 | 280 | 3.21 \times 10^4 | \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 109 | 281 | 1.90 \times 10^5 | \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 109 | 282 | 1.94 \times 10^9 | \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 109 | 283 | 2.06 \times 10^9 | \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 109 | 284 | 1.70 \times 10^{10}| \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 109 | 285 | 8.22 \times 10^8 | \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 109 | 286 | 4.41 \times 10^8 | \(\beta\)-stable| \(\alpha = 100\)               |                  |     |
| 109 | 287 | 1.69 \times 10^9 | 7.97 \times 10^3| \(\beta^- = 100\)              |                  |     |
| 109 | 288 | 1.40 \times 10^{12}| 4.81 \times 10^3| \(\beta^- = 100\)              |                  |     |
| 109 | 289 | 2.02 \times 10^{12}| 2.90 \times 10^3| \(\beta^- = 100\)              |                  |     |
| 109 | 290 | 1.47 \times 10^{13}| 1.75 \times 10^5| \(\beta^- = 100\)              |                  |     |
| 109 | 291 | 7.61 \times 10^{12}| 1.06 \times 10^5| \(\beta^- = 100\)              |                  |     |
| 109 | 292 | 1.88 \times 10^{13}| 639.6          | \(\beta^- = 100\)              |                  |     |
| 109 | 293 | 1.00 \times 10^{12}| 386.3          | \(\beta^- = 100\)              |                  |     |
| 109 | 294 | 1.62 \times 10^9  | 233.2          | \(\beta^- = 100\)              |                  |     |
| 109 | 295 | 9.23 \times 10^8  | 140.8          | \(\beta^- = 100\)              |                  |     |
| 109 | 296 | 1.40 \times 10^{10}| 85.1           | \(\beta^- = 100\)              |                  |     |
| 110 | 272 | 1.02 \times 10^{-4} | 1.15        | \(\alpha = 100\)               |                  |     |
| 110 | 273 | 2.16 \times 10^{-4} | 1.28        | \(\alpha = 100\)               | 1.7 \times 10^{-4} [22] | 1.27 |

(Continued on next page)
In table 4, the calculated half-lives and the suggested decay modes of the nuclei from Z = 107 to Z = 110 can be clearly seen. The available experimental half-lives of α-decay are listed in the sixth column, and the corresponding references are also listed there. The symbol \(D\) in the last column is the ratio of calculated half-life of α-decay and experimental one, and it is in the form of \(D = T_{\alpha, cal}/T_{\alpha, expt}\).

For \(\beta^+\)-decay, the values of half-life vary from 10\(^4\) s to 10\(^8\) s for all Z. The nearer the nuclei are close to the \(\beta^-\)-stable region, the longer their half-lives are. For \(\beta^-\)-decay, on the whole, it is similar to the case of \(\beta^-\)-decay. For α-decay, the half-lives of α-decay approximately vary from 10\(^{-4}\) s to 10\(^{16}\) s in this region. The half-life of α-decay of a nucleus above the \(\beta^-\)-stable region is much longer than its half-life of \(\beta^-\)-decay on the whole. Thus the nuclei above the \(\beta^-\)-stable region occur \(\beta^-\)-decay mainly. However, for most nuclei below the \(\beta^-\)-stable region, their half-lives of α-decay are slightly less than the ones of \(\beta^-\)-decay. So the decay modes of these nuclei are mainly \(\alpha\) or \(\alpha + \beta^+\)-decay. There are 18 experimental half-lives of α-decay in this region. It can be seen that the calculated half-lives of α-decay are in agreement with the experimental ones. Except for \(^{270}\)Mt (\(D = 0.07\), the values of \(D\) vary from 0.18 to 4.18. It is a good approximation. It must be pointed out that the half-life is very sensitive to the α-decay energy. A small change in α-decay energy will lead to a very large difference in half-life. There are few experimental α-decay energies in this region, and most α-decay energies used for calculation are the estimated data \(^\text{[17]}\) or the calculated results \(^\text{[12]}\).

To clearly understand the competition between α-decay and \(\beta^-\)-decay of the nuclei close to the calculated \(\beta^-\)-stable region, we draw the predicted decay modes from \(Z = 90\) to \(Z = 126\) in figure 3.
and decay modes of the nuclei with \( Z = 107 \) results from Audi’s Table. The predictions for half-lives systematically study the competition between \( \alpha \)-decay and \( \beta^- \)-decay (\( \alpha + \beta^- \)) simultaneously when \( Z \geq 112 \). It is a very interesting phenomenon. The competition between \( \alpha \)-decay and \( \beta^- \)-decay is very complex and drastic below the \( \beta^- \)-stable region. The calculated results on the half-lives and the decay modes of the nuclei close to the calculated \( \beta^- \)-stable region are useful for the future experiments on heavy and superheavy nuclei.

5 References

References

In figure 3, one can clearly see the decay modes of the nuclei close to the calculated \( \beta^- \)-stable region. The dark asterisks denote \( \alpha \)-decay. The dark circles denote \( \alpha + \beta^- \)-decay. The hollow circles denote \( \beta^- \)-decay. The decay modes are mostly \( \beta^- \)-decay above the \( \beta^- \)-stable region. Especially for \( Z \leq 111 \), all the decay modes are \( \beta^- \)-decay. The decay modes are very complex below the \( \beta^- \)-stable region. All the three cases of decay mode can occur from \( Z = 90 \) to \( Z = 126 \). It indicates that the competition between \( \alpha \)-decay and \( \beta^- \)-decay is very complex and drastic below the \( \beta^- \)-stable region. It can be seen that the nuclei above the \( \beta^- \)-stable region can occur \( \alpha \)-decay and \( \beta^- \)-decay (\( \alpha + \beta^- \)) simultaneously when \( Z \geq 112 \). It is a very interesting phenomenon, because there is not the decay mode of \( \alpha + \beta^- \) according to experimental results by Audi et al. for all \( Z \).

4 Conclusions

In summary, we propose the \( \beta^- \)-stable region for \( Z \geq 90 \). The predicted \( \beta^- \)-stable nuclei in the calculated \( \beta^- \)-stable region are in good agreement with the ones obtained by Möller et al. We calculate the half-lives of the nuclei close to the calculated \( \beta^- \)-stable region and systematically study the competition between \( \alpha \)-decay and \( \beta^- \)-decay. The calculated half-lives and the suggested decay modes are in good agreement with the experimental results from Audi’s Table. The predictions for half-lives and decay modes of the nuclei with \( Z = 107 \)–110 are presented. We draw the predicted decay modes from \( Z = 90 \) to \( Z = 126 \) in a figure. We find the nuclei above the \( \beta^- \)-stable region can occur \( \alpha \)-decay and \( \beta^- \)-decay (\( \alpha + \beta^- \)) simultaneously when \( Z \geq 112 \). The predicted half-lives and the suggested decay modes are in good agreement with the experimental results by Audi et al. for all \( Z \).

Fig. 3. The predicted decay modes of the nuclei close to the calculated \( \beta^- \)-stable region. The dark asterisks denote \( \alpha \)-decay. The dark circles denote \( \alpha + \beta^- \)-decay. The hollow circles denote \( \beta^- \)-decay.

**References**

1. Rutherford E, Geiger H. Proc. Roy. Soc. A, 1908, **81**: 162
2. Fermi E. Z Phys., 1934, **88**: 161
3. Hofmann S, Münzenberg G. Rev. Mod. Phys., 2000, **72**: 733
4. Wilk P A, Gregorich K E, Türl A et al. Phys. Rev. Lett., 2000, **85**: 2697
5. Haxton W C, Johnson C. Phys. Rev. Lett., 1990, **65**: 1325
6. Koonin S E. Nature, 1991, **354**: 468
7. ZHANG X P, REN Z Z. Phys. Rev. C, 2006, **73**: 014305
8. REN Z Z, XU G O. Phys. Rev. C, 1987, **36**: 456
9. ZHANG X P, REN Z Z. Phys. Rev. C, 2006, **73**: 041301(R)
10. DONG T K, REN Z Z. Phys. Rev. C, 2008, **77**: 064310
11. Möller P, Nix J R, Kratz K -L. At. Nucl. Data Tables, 1997, **66**: 131
12. Ni D D, REN Z Z, DONG T K et al. Phys. Rev. C, 2008, **78**: 044310
13. Audi G, Bersillon O, Blachot J et al. Nucl. Phys. A, 2003, **729**: 106
14. Möller P, Nix J R, Kratz K -L. At. Nucl. Data Tables, 1997, **66**: 131
15. Audi G, Bersillon O, Blachot J et al. Nucl. Phys. A, 2003, **729**: 106
16. XU C, REN Z Z. Phys. Rev. C, 2006, **73**: 162
17. Audi G, Wapstra A H, Thibault C. Nucl. Phys. A, 2003, **729**: 106
18. GAN Z G, GUO J S, WU X L et al. Eur. Phys. J. A, 2004, **19**: 348
19. ZHANG X P, REN Z Z. Phys. Rev. C, 2006, **73**: 014305
20. ZHANG X P, REN Z Z. Phys. Rev. C, 2006, **73**: 014305
21. Möller P, Nix J R, Kratz K -L. At. Nucl. Data Tables, 1997, **66**: 131
22. Audi G, Bersillon O, Blachot J et al. Nucl. Phys. A, 2003, **729**: 106
23. Audi G, Bersillon O, Blachot J et al. Nucl. Phys. A, 2003, **729**: 106
24. GAN Z G, GUO J S, WU X L et al. Eur. Phys. J. A, 2004, **19**: 348
25. ZHANG X P, REN Z Z. Phys. Rev. C, 2006, **73**: 014305
26. ZHANG X P, REN Z Z. Phys. Rev. C, 2006, **73**: 014305
27. ZHANG X P, REN Z Z. Phys. Rev. C, 2006, **73**: 014305