Neutron skin thickness of $^{208}$Pb determined from reaction cross section for proton scattering

Shingo Tagami, Tomotsugu Wakasa, Jun Matsui, and Masanobu Yahiro
Department of Physics, Kyushu University, Fukuoka 819-0395, Japan

Maya Takechi
Niigata University, Niigata 950-2181, Japan
(Dated: June 1, 2021)

Abstract: The reaction cross section $\sigma_R$ is useful to determine the neutron radius $R_n$ as well as the matter radius $R_m$. The chiral (Kyushu) $g$-matrix folding model for $^1$H+$^2$H scattering on $^9$Be, $^{12}$C, $^{27}$Al targets was tested in the incident energy range of $30 \leq E_{in} \leq 400$ MeV, and it is found that the model reliably reproduces the $\sigma_R$ in $30 \leq E_{in} \leq 100$ MeV and $250 \leq E_{in} \leq 400$ MeV.

I. INTRODUCTION

Horowitz et al. proposed a direct measurement for neutron skin $R_{\text{skin}} = R_n - R_p$, where $R_n \equiv \langle r_n^2 \rangle^{1/2}$ and $R_p \equiv \langle r_p^2 \rangle^{1/2}$ are the root-mean-square (rms) radii of point neutrons and protons, respectively. The measurement consists of parity-violating (PV) and elastic electron scattering. The neutron radius $R_n$ is determined from the former experiment, whereas the proton radius $R_p$ is from the latter.

Very recently, by combining the original Lead Radius Experiment (PREX) result with the updated PREX-II result, the PREX collaboration reported the following value:

$$R_{\text{skin}}^{PV} = 0.283 \pm 0.071 \text{ fm}$$

where the quoted uncertainty represents a 1σ error and has been greatly reduced from the original value of $ \pm 0.177$ fm (quadratic sum of experimental and model uncertainties). The $R_{\text{skin}}^{PV}$ value is most reliable at the present stage, and provides crucial tests for the equation of state (EoS) of nuclear matter as well as nuclear structure models. For example, Reed et al. report a value of the sloop parameter $L$ of the EoS and examine the impact of such a stiff symmetry energy on some critical neutron-star observables. It should be noted that the $R_{\text{skin}}^{PV}$ value is considerably larger than the other experimental values which are significantly model dependent.

As an exceptional case, a nonlocal dispersive-optical-model (DOM) analysis of $^{208}$Pb deduces $r_{\text{DOM}}^{\text{skin}} = 0.25 \pm 0.05$ fm, which is consistent with $R_{\text{skin}}^{PV}$. It is the aim of this paper to present the $R_{\text{skin}}$ value with a similar precision of $R_{\text{skin}}^{PV}$ by analyzing the reaction cross section $\sigma_R$ for $p + ^{208}$Pb.

The reaction cross section $\sigma_R$ is a powerful tool to determine matter radius $R_m$. One can evaluate $R_{\text{skin}}$ and $R_m$ by using the $R_m$ and the $R_p$ determined by the electron scattering. The $g$-matrix folding model is a standard way of deriving microscopic optical potential for not only proton scattering but also nucleus-nucleus scattering. Applying the folding model with the Melbourne $g$-matrix for interaction cross sections $\sigma_I$ for Ne isotopes and $\sigma_R$ for Mg isotopes, we discovered that $^{31}$Ne is a halo nucleus with large deformation, and deduced the matter radii $r_m$ for Ne isotopes and for Mg isotopes. The folding potential is nonlocal, but is localized with the method of Ref. [17]. The validity is shown in Ref. [20]. For proton scattering, the localized version of $g$-matrix folding model yields the same results as the full folding $g$-matrix folding model of Ref. [20], as shown by comparing the results of Ref. [31] with those of Ref. [20].

Recently, Kohno calculated the $g$-matrix for the symmetric nuclear matter, using the Brueckner-Hartree-Fock method with chiral 4th-order ($N^3$LO) nucleon-nucleon ($N N$) forces (2NFs) and 3rd-order (NNLO) three-nucleon forces (3NFs). He set $c_D = -2.5$ and $c_E = 0.25$ so that the energy per nucleon can become minimum at $\rho = \rho_0$, see Fig. [1] for $c_D$ and $c_E$. Toyokawa et al. localized the non-local chiral $g$-matrix into three-range Gaussian forms. Using the localization method proposed by the Melbourne group, the resulting local $g$-matrix is called "Kyushu $g$-matrix".

The Kyushu $g$-matrix folding model is successful in reproducing $\sigma_R$ and differential cross sections $d\sigma/d\Omega$ for $^4$He scattering at $E_{in} = 30$–200 MeV/nucleon. The success is true for proton scattering at $E_{in} = 65$ MeV [23]. Lately, we predicted neutron skin $r_{\text{skin}}$ and proton, neutron, matter radii, $R_n$, $R_p$, $R_m$ from interaction cross sections $\sigma_I$ for $^{42}_{25}$Ca+$^{12}$C scattering at $E_{in} = 280$ MeV/nucleon, using the Kyushu $g$-matrix folding model with the densities calculated...
with Gongny-D1S HFB (GHFB) with and without the angular momentum projection (AMP) [26].

In Ref. [26], we tested the Kyushu $g$-matrix folding model for $^{12}$C scattering on $^9$Be, $^{12}$C, $^{27}$Al targets in $30 \leq E_{\text{in}} \leq 400 \text{ MeV}$, comparing the theoretical $\sigma_R$ with the experimental data [36]. We found that the Kyushu $g$-matrix folding model is reliable for $\sigma_R$ in $30 \leq E_{\text{in}} \leq 100 \text{ MeV}$ and $250 \leq E_{\text{in}} \leq 400 \text{ MeV}$. This indicates that the Kyushu $g$-matrix folding model is applicable in $30 \leq E_{\text{lab}} \leq 100 \text{ MeV}$, although the data on $p^{+}\text{Pb}$ scattering are available in $21 \leq E_{\text{lab}} \leq 180 \text{ MeV}$.

In this paper, we present the determination of $R_{\text{skin}}^{\text{GHFB}}$ from the measured $\sigma_R$ for $p +^{208}\text{Pb}$ scattering in $30 \leq E_{\text{in}} \leq 100 \text{ MeV}$ [37], using the Kyushu $g$-matrix folding model with the GHFB+AMP densities. As mentioned above, the Kyushu $g$-matrix folding model is applicable in $30 \leq E_{\text{in}} \leq 100 \text{ MeV}$, although the data on $p^{+}\text{Pb}$ scattering are available in $21 \leq E_{\text{in}} \leq 180 \text{ MeV}$. In Sec. II, we briefly describe our model. Section III presents the results and a comparison with $R_{\text{skin}}^{\text{GHFB}}$, and discussion follows. Finally, Sec. IV is devoted to a summary.

II. MODEL

Our model is the Kyushu $g$-matrix folding model [25] with the densities calculated with GHFB+AMP [26]. In Ref. [25], the Kyushu $g$-matrix is constructed from chiral interaction with the cutoff $\Lambda = 550 \text{ MeV}$. The model was tested for $^{12}$C scattering on $^9$Be, $^{12}$C, and $^{27}$Al targets in $30 \leq E_{\text{in}} \leq 400 \text{ MeV}$. It is found that the Kyushu $g$-matrix folding model is good in $30 \leq E_{\text{in}} \leq 100 \text{ MeV}$ and $250 \leq E_{\text{in}} \leq 400 \text{ MeV}$ [26].

The brief formulation of the folding model itself is shown below. For nucleon-nucleus scattering, the potential is composed of the direct and exchange parts, $U_{\text{DR}}$ and $U_{\text{EX}}$ [25]:

\[
U_{\text{DR}}(\mathbf{R}) = \sum_{\mu,\nu} \int \rho_{T}^{\mu}(r_{T}) g_{\mu\nu}(s; \rho_{\mu\nu}) dr_{T},
\]

\[
U_{\text{EX}}(\mathbf{R}) = \sum_{\mu,\nu} \int \rho_{T}^{\mu}(r_{T}, r_{T} + s) \times g_{\mu\nu}^{\text{EX}}(s; \rho_{\mu\nu}) \exp \left[-i \mathbf{K}(\mathbf{R}) \cdot s/M \right] dr_{T},
\]

where $\mathbf{R}$ is the relative coordinate between a projectile (P) and a target (T), $s = -r_{T} + \mathbf{R}$, and $r_{T}$ is the coordinate of the interacting nucleon from T. Each of $\mu$ and $\nu$ denotes the z-component of isospin; $1/2$ means neutron and $-1/2$ does proton. The nonlocal $U_{\text{EX}}$ has been localized in Eq. (2b) with the local semi-classical approximation [17], where $\mathbf{K}(\mathbf{R})$ is the local momentum between P and T, and $M = A/(1 + A)$ for the target mass number $A$; see Ref. [20] for the validity of the localization. The direct and exchange parts, $g_{\mu\nu}^{\text{DR}}$ and $g_{\mu\nu}^{\text{EX}}$, of the $g$-matrix depend on the local density

\[
\rho_{\mu\nu} = \rho_{T}^{\mu}(r_{T} + s/2),
\]

at the midpoint of the interacting nucleon pair; see Ref. [28] for the explicit forms of $g_{\mu\nu}^{\text{DR}}$ and $g_{\mu\nu}^{\text{EX}}$.

The relative wave function $\psi$ is decomposed into partial waves $\chi_{L}$, each with different orbital angular momentum $L$. The elastic $S$-matrix elements $S_{L}$ are obtained from the asymptotic form of the $\chi_{L}$. The total reaction cross section $\sigma_{R}$ is calculable from the $S_{L}$ as

\[
\sigma_{R} = \frac{\pi}{K^{2}} \sum_{L} (2L + 1) (1 - |S_{L}|^{2}).
\]

The proton and neutron densities, $\rho_{p}(r)$ and $\rho_{n}(r)$, are calculated with GHFB+AMP. As a way of taking the center-of-mass correction to the densities, we use the method of Ref. [28], since the procedure is quite simple.

III. RESULTS

Figure 2 shows the proton $\rho_{p}^{\text{GHFB}}$, neutron $\rho_{n}^{\text{GHFB}}$, and matter $\rho_{m}^{\text{GHFB}} = \rho_{p}^{\text{GHFB}} + \rho_{n}^{\text{GHFB}}$ densities as a function of $r$. The experimental point-proton distribution extracted from the electron scattering data is also shown. The theoretical proton distribution $\rho_{p}^{\text{GHFB}}$ reproduces the experimental $\rho_{p}^{\text{exp}}$ reasonably well.

The Kyushu $g$-matrix folding model with the GHFB+AMP densities underestimates the $\sigma_{R}$ data in $30 \leq E_{\text{in}} \leq 100 \text{ MeV}$ only by a factor of 0.97, as shown in Fig. 3. The proton radius $R_{p}^{\text{GHFB}} = 5.444 \text{ fm}$ calculated with GHFB+AMP agrees with the experimental value of $R_{p}^{\text{exp}} = 5.444 \text{ fm}$ [42]. Because of $\sigma_{R} \propto R_{m}^{2}$, the observed discrepancy of $\sigma_{R}$ is attributed to the underestimation of $\rho_{m}^{\text{GHFB}}$ originating from the underestimation of $\rho_{n}^{\text{GHFB}}$. Small deviation makes it possible to scale the GHFB+AMP densities for the neutron density so as to reproduce $\sigma_{R}^{\text{exp}}$ in $E_{\text{in}} = 30–100 \text{ MeV}$. The result of the scaling is $R_{n}^{\text{exp}} = 5.722 \pm 0.035 \text{ fm}$ leading to

\[
R_{\text{skin}}^{\text{exp}} = 0.278 \pm 0.035 \text{ fm}.
\]
FIG. 2. \( r \) dependence of densities, \( \rho_p(r), \rho_n(r), \rho_m(r) \), for \(^{208}\text{Pb} \) calculated with GHFB+AMP. Three dashed lines from the bottom to the top denote \( \rho_p(r), \rho_n(r), \rho_m(r) \), respectively. The experimental point-proton (unfolded) density \( \rho_p \) is taken from Refs. [37–39].

FIG. 3. \( E_{\text{in}} \) dependence of reaction cross sections \( \sigma_R \) for \( p + ^{208}\text{Pb} \) scattering. The solid line stands for the results of the Kyushu g-matrix folding model with GHFB+AMP densities. The data are taken from Refs. [37–39].

This result is consistent with \( r_{\text{skin}}^{\text{PV}} = 0.283 \pm 0.071 \) fm.

Now we show a simple derivation of \( R_{\text{skin}}^{\text{PV}} \) in the limit of \( K^{\exp} = K^{th} \). The experimental and theoretical (GHFB+AMP) reaction cross sections, \( \sigma_R^{\exp} \) and \( \sigma_R^{th} \), can be expressed as

\[
\sigma_R^{\exp} = K^{\exp} \left( (R_p^{\exp})^2 \frac{Z}{A} + (R_n^{\exp})^2 \frac{N}{A} \right), \quad (6a)
\]

\[
\sigma_R^{th} = K^{th} \left( (R_p^{th})^2 \frac{Z}{A} + (R_n^{th})^2 \frac{N}{A} \right), \quad (6b)
\]

where \( Z, N, \) and \( A \) are proton, neutron, and atomic numbers of \(^{208}\text{Pb} \), respectively, and \( K \) is a proportional coefficient between \( \sigma_R \) and \( R_i^2 = R_i^2(Z/A) + R_i^2(N/A) \). By using \( K^{\exp} = K^{th} \) and \( R_p^{\exp} = R_p^{th} \), the experimental neutron radius \( R_p^{\exp} \) can be deduced as

\[
R_p^{\exp} = \sqrt{\frac{Z(R_p^{\exp})^2 + N(R_n^{\exp})^2}{N\sigma_R^{th}}} \sigma_R^{\exp} - (\sigma_p^{\exp})^2 \frac{Z}{N}, \quad (7)
\]

from the experimental \( \sigma_R^{\exp} \) and \( R_p^{\exp} \) data and the theoretical \( R_{\text{skin}}^{th} \) in GHFB+AMP.

Figure 4 shows the \( R_{n}^{\exp} \) results as a function of incident energy \( E_{\text{in}} \). The deduced \( R_{n}^{\exp} \) values are almost independent of \( E_{\text{in}} \), and the present folding model is reliable [28]. By combining the eight data in this energy region, the neutron radius of \(^{208}\text{Pb} \) becomes \( R_{n}^{\exp} = 5.735 \pm 0.035 \) fm as shown by the filled band in Fig. 4. This result shows that the neutron skin thickness of \(^{208}\text{Pb} \) is \( R_{n}^{\exp} = 0.291 \pm 0.035 \) fm with \( R_{n}^{\exp} = 5.444 \) fm [42]. The limit of \( K^{\exp} = K^{th} \) is thus good, since \( R_{n}^{\exp} = 0.291 \pm 0.035 \) fm is close to Eq. (5). Equation (7) is quite useful when \( \sigma_R^{\exp} \approx \sigma_R^{th} \) and \( R_p^{\exp} \approx R_p^{th} \).

IV. SUMMARY

The proton radius \( R_p \) calculated with GHFB+AMP agrees with the precise experimental data of 5.444 fm. In \( 30 \leq E_{\text{in}} \leq 100 \) MeV, we can obtain \( r_{n}^{\exp} \) from \( \sigma_R^{\exp} \) by scaling the GHFB+AMP neutron density so as to reproduce \( \sigma_R^{\exp} \) for each \( E_{\text{in}} \), and take the weighted mean and its error for the resulting \( r_{n}^{\exp} \). From the resulting \( R_{n}^{\exp} = 5.722 \pm 0.035 \) fm and \( r_{n}^{\exp} = 5.444 \) fm, we can get \( R_{n}^{\exp} = 0.278 \pm 0.035 \) fm.

In conclusion, our result \( R_{n}^{\exp} \) is consistent with a new result \( r_{\text{skin}}^{\exp} \) (PREX-II) = 0.283 ± 0.071 fm of PREX-II.
[42] A. B. Jones and B. A. Brown, Phys. Rev. C 90, 067304 (2014)