Research Article

Increasing Utilization of Redundant Virtual Array for DOA Estimation Based on Coprime Array

Chenghong Zhan,1,2 Guoping Hu,1 Zixin Zhang,1,2 and Ziang Feng1

1Air and Missile Defense College, Air Force Engineering University, Xi’an 710051, China
2Graduate College, Air Force Engineering University, Xi’an 710051, China

Correspondence should be addressed to Guoping Hu; mma18207171897@163.com

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In this paper, we initiated a method to estimate the direction of arrival (DOA) of far-field, narrowband, and incoherent targets using coprime array. First, we proposed a coprime array structure and analyzed the distribution of difference coarray (DCA). The degrees of freedom (DOF) of the proposed coprime array became clearer by referring to the DCA conception. However, previous algorithm only uses the continuous virtual array, which causes the virtual array elements in the repeated position being abandoned. Therefore, the paper analyses the distribution of virtual array based on DCA conception and averages the receiving signal on these redundant virtual array elements to increase the utilization of receiving data. As a result, the algorithm has high precision in parameter estimation. Simulation results have shown the superiority of the proposed algorithm.

1. Introduction

Because sparse arrays can overcome the limitation of interelement spacing, direction of arrival (DOA) estimation based on it has attracted great attention. Compared to the uniform linear arrays (ULAs), sparse arrays, such as minimum redundancy array (MRA) [1], nested array (NA) [2, 3], and coprime array (CPA) [4], have great advantages in improving estimation accuracy, enhancing angle resolution, and reducing physical cost [5–8]. Larger array element spacing expands the virtual aperture, which leads to the rise in the number of detectable sources and angle resolution. In addition, it can also effectively suppress the mutual coupling effect of array elements and the coherence of received noise. Thus, improving the estimation, low redundancy can effectively reduce wastage of resources and accelerate operation [9–11].

Coprime arrays have more DOFs than ULAs and have no mutual coupling problems compared with nested arrays [12]. Furthermore, it has closed form expression of freedom extension degree and systematic array structure design. Therefore, DOA estimation based on coprime arrays has become a research hotspot. Most algorithms with coprime arrays derive extended virtual array and operate the corresponding signals to estimate DOA [13–16]. In [17], the paper proposes a modified Toeplitz matrix reconstruction method using a random nonzero row of the covariance matrix in the virtual uniform linear array. Besides, a full rank covariance matrix is obtained by restructuring a Toeplitz matrix. In [18], the author recovered the Toeplitz matrix by solving a nuclear norm minimization problem and decorrelated the signal. In addition, the discrete virtual array elements are utilized by interpolating virtual elements. However, the virtual arrays in repeated position were abandoned. In [16], a generalized coprime array configuration for DOA estimation is proposed. Based on generalized sum and difference coarray (GSDC) concept, the proposed configuration enlarges the interelement space, and hence, the DOFs of the arrays are increased. Besides, there are a large number of virtual arrays in the same position which are not utilized. On the other hand, in [19], an algorithm with low computational complexity and high estimation accuracy was initiated. The method first used estimates signal parameters via rotational invariance.
technique (ESPRIT) algorithm. Then, the search scope was narrowed down based on the initial results. Finally, more accurate estimation results were obtained by multiple signal classification (MUSIC). Similarly, in order to reduce the computational cost, the authors of [20] propose a successful signal subspace fitting (SSF) method from a computationally efficient perspective. All of the algorithms above focused either on improving the array structure to obtain higher DOFs and larger virtual array aperture or on reducing the complexity by avoiding spatial spectrum search and so on. Obviously, the redundant virtual elements in the same position were not utilized, and their data are wasted.

In this paper, we take the coprime array with the difference coarray (DCA) as an example to illustrate the redundant virtual array elements distribution and then average the virtual elements in the repeated position to increase the utilization efficiency of received signal data. Consequently, the snapshots of received signal have increased, and the DOA estimation performance of targets is improved noticeably.

2. Coprime Array Signal Model

As shown in Figure 1, a coprime array structure consists of two uniform linear subarrays. Subarray 1 has \( N \) sensors with an interelement spacing \( N \). The other subarray has \( 2M - 1 \) sensors with interelement spacing \( M \). \( M \) and \( N \) are a pair of coprime integers \( (M < N) \). We assume the unit of interelement spacing is \( d \), where \( d = \lambda / 2 \) is half-wavelength.

According to Figure 1, sensors’ location is expressed as

\[
S = \{Mn, 0 \leq n \leq N - 1\} \cup \{Nm, 0 \leq m \leq 2M - 1\}. \tag{1}
\]

We assume narrowband and uncorrelated sources in the far-field from distinct angles impinging on the array with sensors. The output of the received signal after the matched filter can be denoted as

\[
X(t) = AS(t) + N(t). \tag{2}
\]

In the above equation, \( S(t) = [s_1(t), s_2(t), \ldots, s_K(t)]^T \), where \( s_k(t) \) is the reflection coefficient of the \( k \)th target. \( N(t) \) is the complex-valued additive white Gaussian noise matrix with mean value 0 and variance \( \sigma^2 \). \( A \) is the steering vector which can be denoted as

\[
A = [a(\theta_1), a(\theta_2), \ldots, a(\theta_K)], \tag{3}
\]

where \( a(\theta_k) \) can be expressed as

\[
a(\theta_k) = \left[ 1, e^{-j\pi \sin(\theta_k)}, e^{-j\pi M \sin(\theta_k)}, \ldots, e^{-j\pi (N-1)M \sin(\theta_k)}, e^{-j\pi N \sin(\theta_k)}, \ldots, e^{-j\pi (2M-1)N \sin(\theta_k)} \right]^T. \tag{4}
\]

Then, the covariance matrix can be given as

\[
\mathbf{R} = \mathbb{E}[X(t)X^H(t)] = \mathbf{AR} \mathbf{A}^H + \sigma^2 \mathbf{I}_{N \times N}, \tag{5}
\]

where \( \mathbf{R} = \mathbb{E}[S(t)S^H(t)] \) represents the target covariance matrix. \( \mathbb{E}[\cdot] \) denotes mathematical expectation; \( [\cdot]^H \) denotes conjugate transpose; \( \sigma^2 \) denotes the energy of the noise; and \( \mathbf{I}_{N \times N} \) is \( N \times N \) dimensional identity matrix. Because the signal is incoherent, \( \mathbf{R} = \mathbb{E}[S(t)S^H(t)] \) can be given as

\[
\mathbf{R} = \mathbb{E}[S(t)S^H(t)] = \text{diag} [\sigma_1^2, \sigma_2^2, \ldots, \sigma_K^2], \tag{6}
\]

where the \( \sigma_k^2 \) denotes the power of the signal.

In practice, the signal covariance matrix is often estimated with \( L \) snapshots.

\[
\hat{\mathbf{R}} = \frac{1}{L} \sum_{l=1}^{L} X(t)X^H(t). \tag{7}
\]

\( \hat{\mathbf{R}} \) needs to be vectorized to obtain the virtual array elements. Consequently, the vectorizing covariance matrix of signal can be expressed as

\[
S_{\text{CPA}} = \{d_n - d_{n'}, d_n, d_{n'} \in P_{\text{CPA}}\} = S_p \cup S_e,
\]

where

\[
S_p = \{\pm Mn, \quad n = 0, \ldots, N - 1\} \cup \{\pm Nm, \quad m = 0, \ldots, 2M - 1\},
\]

\[
S_e = \{\pm (Mn - Nm), \quad m = 0, 1, \ldots, 2M - 1, n = 0, 1, \ldots, N - 1\}. \tag{10}
\]

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where \( S_c \) denotes the self-difference set and \( S_e \) denotes the cross-difference set.

From formula (11), DOFs of the virtual array is \( O(MN) \) that only needs \( 2M + N - 1 \) physical sensors. It is obvious that the virtual array elements are discontinuous. For example, when \( M = 2, N = 3 \), the continuous range of virtual elements is \([-7, 7]\). Next, the distribution and application of virtual redundant elements are discussed.

### 3. Improved Algorithm for Coprime Array

In this section, we will compare traditional algorithms based on difference coarray and reuse the redundant virtual elements.

#### 3.1. Review of Traditional Algorithm

In equations (8) and (9), the virtual elements formed by the vectorization of the signal covariance matrix are reordered, and redundant virtual elements are deleted. Then, a new steering vector of the single snapshot can be acquired and is marked as signal \( \tilde{B} \), which is expressed as

\[
\tilde{B} = \begin{bmatrix}
e^{-jnN_{\text{Max}} \sin \theta_1} & e^{-jnN_{\text{Max}} \sin \theta_2} & \cdots & e^{-jnN_{\text{Max}} \sin \theta_k} \\
e^{j\pi \sin \theta_1} & e^{j\pi \sin \theta_2} & \cdots & e^{j\pi \sin \theta_k} \\
e^{jnN_{\text{Max}} \sin \theta_1} & e^{jnN_{\text{Max}} \sin \theta_2} & \cdots & e^{jnN_{\text{Max}} \sin \theta_k} \\
\end{bmatrix}
\]

where \( N_{\text{Max}} = MN + M - 1 \) denotes the maximum position of extended continuous virtual elements. Correspondingly, the range of the virtual array elements is \([-N_{\text{Max}}, N_{\text{Max}}]\).

Figure 2 shows the distribution of virtual array elements when \( M = 2 \) and \( N = 3 \).

Figure 1: Coprime array MIMO radar structure.

In order to making full use of redundant virtual array elements and increase the utilization of received signal data,
data of redundant elements are compiled into a new received signal. Section 3.2 will show how the algorithm works.

3.2. Improved Algorithm Based on Reusing the Redundant Virtual Array Elements. In order to effectively utilize the received signal information in the redundant virtual array element, we take the average redundant elements in the repeated positions. Moreover, we replace the virtual elements in the original positions with the averaging redundant elements. The process is as follows:

\[
\begin{aligned}
&\{ -7, \ldots, -3, -3, -3, -2, -2, -2, -1, -1, 0, 0, 0, 0, 0, 1, 1, 2, 2, 2, 3, 3, 3, \ldots, 7 \}.
\end{aligned}
\]  

(16)

Therefore, the DOAs of the targets can be expressed by the following formula:

\[ \theta_i = \arcsin \left( \frac{\lambda}{2\pi d_0} \arg(z_i) \right), \quad i = 1, \ldots, K. \]  

(21)

3.3. Algorithm Steps. In order to better illuminate the algorithm ideas, we add detailed algorithm steps in Algorithm 1.

4. Simulation Results

In this section, in the coprime array with \( M = 2 \) and \( N = 3 \), we assume that both subarrays start from zero. Obviously, the total physical sensors are \( 2M + N - 1 = 6 \). We compared the performance of improved algorithm with traditional algorithm based on coprime array and ULA with six \((M = 6)\) physical sensors.

Simulation will show advantages of reusing the redundant virtual array elements in terms of root means square error and spatial spectral resolution.
4.1. Simulation of Spatial Spectral Resolution. In the simulation, a pair of coprime integers are assumed as $M = 2$ and $N = 3$. Similarly, the uniform linear array has $M = 6$ physical sensors. Considering three equal power targets impinging on the coprime array from the directions in $[-20^\circ, 0^\circ, 20^\circ]$ with SNR = 0 dB, the snapshots is $L = 500$. The following formula helps achieve the spatial spectrum:

$$f_{\text{MUSIC}}(\theta) = \frac{1}{\| \mathbf{a}^H (\theta) \mathbf{U} \|^2},$$  \hspace{2cm} (22)

where $\mathbf{a}(\theta)$ denotes the steering vector of the signal and $\mathbf{U}$ consists of eigenvectors corresponding to $N_{\text{Max}} - K$ eigenvalues.

Figure 3 shows the resolution performance of the three algorithms. It can be seen that the spectrum peak of the algorithm with redundant virtual array element (red solid line) is sharper, and the side lobe is lower. Thus, the red line represents the best performance. Similarly, the algorithm resolution with conventional CPA structure is better than the ULAs referring to the higher array aperture.

4.2. Relationship between SNR and RMSE. We assume three incoherent signals with equal power impinging on the arrays from the directions $\theta_1 = 10^\circ$, $\theta_2 = 20^\circ$, and $\theta_3 = 30^\circ$. Then, the SNR varies from $-10 \text{ dB}$ to $10 \text{ dB}$ with an interval of $2 \text{ dB}$. Meanwhile, the number of snapshots is $L = 500$, which is the same with that of Monte-Carlo trials. The noise is assumed to be Gaussian white noise with a mean value of 0 and variance of $\sigma^2$. In the next all simulations, we will use the Root-MUSIC algorithm to calculate the DOAs of the targets. The formula for calculating root mean square error is defined as

$$\text{RMSE} = \sqrt{\frac{1}{LK} \sum_{i=1}^{500} \sum_{k=1}^{K} (\hat{\theta}_k - \theta_k)^2},$$  \hspace{2cm} (23)

where $K$ denotes the number of the targets, while $L$ denotes the snapshots of the received signal $(i = 1, 2, \ldots, K)$.

As shown in Figure 4, under the same SNR condition, the root mean square error of the proposed algorithm is the smallest. With the increase of SNR, the algorithm obtains...
higher estimation accuracy and smaller root mean square error value.

4.3. Relationship between Snapshots and RMSE. Based on the same assumption in part B, the SNR is 0 dB. The number of snapshots $L$ varies from 100 to 1000 with an interval of 100. Moreover, the number of Monte-Carlo trials is 500. The noise is assumed to be Gaussian white noise with a mean value of 0 and variance of $\sigma^2$.

From Figure 5, with the increase of snapshots, the root means square error plummets. Under the same number of snapshots, the proposed algorithm has better performance.
5. Conclusions

In this paper, a new algorithm via reusing the redundant virtual array elements is proposed for DOA estimation of uncoherent targets. This method averages the virtual array elements in the repeated position. In this way, a new co-variance matrix of signal is constructed, which increases the utilization of the received signal data. In addition, Root-MUSIC algorithm is employed to estimate the DOAs of these targets. Unlike the existing algorithm, the improved algorithm makes full use of the information in the redundant virtual array elements. Consequently, the proposed algorithm has greatly improved the resolution performance and reduced estimation error. Furthermore, the algorithm can also be introduced to the MARs and NAs based on the conception of difference coarray and sum-difference coarray. Simulation results show that our algorithm provides better performance for uncoherent signals than conventional methods.

Data Availability

The data supporting the conclusion of the article are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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