Simple Quantum Model of Learning Explains the Yerkes-Dodson Law in Psychology

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We propose the simple model of learning based on which we derive and explain the Yerkes-Dodson law - one of the oldest laws of experimental psychology. The approach uses some ideas of quantum theory of open systems (QTOS) and develops the method of statistical description of psychological systems that was proposed by author earlier.

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I. INTRODUCTION

The Yerkes-Dodson Law (YDL) is one of the oldest laws of psychology of motivation. It was discovered experimentally more than one hundred years ago in 1908[1]. The main goal of original Yerkes-Dodson experiments was to examine to what extent motivation and emotion arousal have an influence on learning process and facilitate acquisition of habits which are needed for individual in solving everyday problems. The main result obtained by Yerkes and Dodson as well as similar more late experiments with various species such as rats, chickens, cats and also with men (with specific tasks for each species of course) turned out to be universal. In all cases there is optimal level of arousal which provides the most successful execution of required task. Besides the hallmark of the YDL is the fact that optimal level of arousal depends on the complexity of the task, namely, more complex the task the lower optimal level of arousal should be. In classical handbooks of experimental psychology, (see e.g. [2]) as the verbal explanation of the YDL usually the following argument was drawn: since excessive raising of arousal leads to essential disorganization of behavior of individual therefore the execution of task in such situation should be worse.

Although such argument seems to be quite reasonable but a physicist who is interested in psychology undoubtedly would like to have some explicit model of learning that taking into account influence of arousal on the process allows him to derive (from clearly articulated assumptions) the YDL together with its hallmark. This is just the main goal of present paper. The rest of the paper is organized as follows. In Sect.2 we briefly remind the basic points of statistical method for the description of simple psychological phenomena which was proposed by author earlier[3]. This method based on some ideas of QTOS is the starting tool for our further analysis. In Sect.3 we formulate simple phenomenological model of learning and discuss to what extent it can be considered as reasonable from psychological point of view. In Sect.4 we demonstrate that the model proposed let one rigorously derive the YDL together with its hallmark. In conclusion we are discussing the possibility of experimental verification of the model proposed. Now let us go to the detail presentation of the paper.

II. PRELIMINARY INFORMATION

In this part we want to remind the main points of the method which was proposed by author earlier[3] for the statistical description of simple psychological phenomena (both individual behavior and group processes). First of all we want to emphasize that the question is only about statistical description. The fact is that due to extraordinary complexity of various psychological phenomena according to views of many modern psychologists see e.g. [4] just the statistical approach to behavior description is the most relevant. Therefore we will assume the task of behavior description is solved if we can find the probabilities of all psychological states which are possible in situation under study. We believe that for this purpose analogy with QTOS where such approach is inherent from the very beginning may be very productive (for more detail argumentation in favour of this analogy see [5]). As one knows in QTOS fundamental concept of density matrix of the system $\rho$ let one correctly describe such state of affairs. Later in this paper we use the analogy between psychology and QTOS only at phenomenological level and we will show that this analogy significantly helps one in explaining the YDL. Note the only information from QTOS that we needed for our purpose is the Lindblad equation. This equation derived in QTOS under rather general restrictions imposed on the system of interest (see [5]) let one explicitly describe the required density matrix evolution in time, that is the "behavior" of the system.

The Lindblad equation in QTOS has the following form:

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [\hat{H}, \rho] + \sum_i \left[ \hat{R}_i \rho \hat{R}_i^\dagger \right] + h.c.$$  \hspace{1cm} (1)

In Eq. (1) $\hat{H}$ is hermitian operator ("hamiltonian" of open system) and $\hat{R}_i$ are a set of nonhermitian operators that specify all links of open system in question with its
environment. If initial state of the system \( \rho(0) \) is known Eq. (1) let one to find all desired information about the system at any time. In order to apply Eq. (1) to concrete psychological system of interest we must firstly define its possible psychological states and in addition to define the form of all operators \( \hat{R}_i \) incoming in Eq. (1). To this end we must know all stimuli i.e. all psychological forces acting on the system. In other words applied to our problem to explain the YDL within the framework of Eq. (1) we must formulate relevant mathematical model of the learning process taking into account the influence of arousal on this process. Let us now begin to adress this issue.

### III. PHENOMENOLOGICAL MODEL OF LEARNING

In present paper we offer as a basis the next phenomenological model of learning. We assume that the space of psychological states of trained individual (its life space according terminology of K. Lewin’s psychological field theory [6]) can be represented as linear superposition of three basic states: state \( |1\rangle \equiv (0\ 0\ 1)^T \) - that corresponds to untrained individual, state \( |2\rangle \equiv (0\ 1\ 0)^T \)- pre-trained state (in this state individual has acquired some elements and features of the habit, but the skill for the task completion still lacks) and the trained state \( |3\rangle \equiv (1\ 0\ 0)^T \) in which individual is able successfully execute required task. Proposing such model we have in mind as correct the hypothesis of functional equivalence between perception process and other cognitive processes. Remind that according to modern cognitive psychology (see e.g. [7]) sensory information processing in brain especially visual information processing consists of two consecutive stages: the first stage (segmentation) is the formation of coherent clusters of perception at which groups of similar features of the perceived object combine together and the second stage (binding) on which fragments of perception are integrated into complete image. In fact our model of learning assumes that state \( |2\rangle \) corresponds to segmentation stage and state \( |3\rangle \) to the binding stage. In addition we believe the learning process can be represented as the set of four transitions between basic states: a) transition from \( |1\rangle \) to \( |2\rangle \)- process of pre-learning b) transition from \( |2\rangle \) to \( |3\rangle \)- process of habit acquisition, c) transition from \( |3\rangle \) to \( |2\rangle \)- process of partial loss of the habit due to the influence of different noise and d) transition from \( |3\rangle \) to \( |1\rangle \)- natural process of "forgetting" of the habit when it is not claimed for a long time. We believe that all these transitions can be described on the basis of the Lindblad equation with the help of four relevant operators \( \hat{R}_i \). Speaking more precisely: transition a) can be described by the operator \( \hat{R}_1 = \sqrt{\frac{1}{3}} \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \), transition b)- by the operator \( \hat{R}_2 = \sqrt{\frac{1}{3}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \), transition c) by the operator \( \hat{R}_3 = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \) and transition d) by the operator \( \hat{R}_4 = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \). Coefficients \( a,b,c,d \) incoming in operators \( \hat{R}_i \) reflect the power of corresponding transitions.Probabilities \( \rho_1, \rho_2, \rho_3 \) of finding the individual in one of basic states in learning process one can consider as diagonal elements of density matrix \( \hat{\rho} = \begin{pmatrix} \rho_1 & 0 & 0 \\ 0 & \rho_2 & 0 \\ 0 & 0 & \rho_3 \end{pmatrix} \).

In the paper [3] it was shown that if at initial time density matrix \( \rho(0) \) has diagonal form then under certain restrictions on the form of operators \( \hat{R}_i \) such diagonal form will be conserved in time. It is easy to check that with above-mentioned choice of operators \( \hat{R}_i \) in the model just this case occurs. Now having in hands all relevant tools we can move to a direct derivation of the YDL.

### IV. DERIVATION OF THE YERKES-DODSON LAW

In this part we assume that the main features of learning process efficiency in the light of arousal influence can be correctly described in the framework of foregoing model with the help of the Lindblad equation Eq. (1). Substituting in it operators \( \hat{\rho} \) and \( \hat{R}_i \) \((i = 1,2,3,4)\) in the form specified above after simple algebra we obtain the next system equations of motion for required probabilities \( \rho_1, \rho_2, \rho_3 \):

\[
\begin{align*}
\frac{d\rho_1}{dt} &= -a\rho_1 + b\rho_3 \\
\frac{d\rho_2}{dt} &= a\rho_1 - b\rho_2 + c\rho_3 \\
\frac{d\rho_3}{dt} &= b\rho_2 - (c + d)\rho_3
\end{align*}
\]

In the remainder of the paper we are interested only in stationary states of individual who trains therefore in Eq. (2) we equate all \( \frac{\partial \rho_i}{\partial t} \) to zero and taking into account normalization condition \( \rho_1 + \rho_2 + \rho_3 = 1 \) obtain expressions for required probabilities \( \rho_i \):

\[
\begin{align*}
\rho_1 &= \frac{bd}{a(b + c + d) + bd} \\
\rho_2 &= \frac{a(c + d)}{a(b + c + d) + bd} \\
\rho_3 &= \frac{ab}{a(b + c + d) + bd}
\end{align*}
\]
Note that until now we did not take into account in explicit form influence of arousal on learning process. At this point we are going to do it. Because of psychological reasons it can be assumed that increase of arousal facilitates the transitions a) and c) and has negligible effect on transitions b) and d). Therefore the easiest way to take into account arousal level in our model is to put \( c = a t \) and then to consider coefficients \( a, b, d, t \) as independent parameters. Now we believe that value of \( a \) exactly determines the arousal level of individual in the learning process. Let us see that there is optimal level of arousal for successful training in the model. Indeed if we write the extremal condition for probability of trained state \( \rho_3 \) that reads as \( \frac{\partial \rho_3}{\partial a} = 0 \) we obtain that \( a_{ext}^2 = \frac{bt}{d} \). Clearly this value of \( a \) corresponds exactly to the optimal level of arousal which facilitate the task realization. Besides the hallmark of the YDL is immediately implies from above expression for \( a_{ext} \) because it is clear more difficult task than the less coefficient \( b \) should be.

In conclusion let us discuss briefly the possibility to verify experimentally the validity of learning model proposed and hence (indirectly) the reasonableness of original analogy between psychology and QTOS. To this end one must reproduce experiments similar to Yerkes and Dodson but using statistically significant ensemble of learning individuals. In this situation it is possible to get reliable values of probabilities \( \rho_1, \rho_2, \rho_3 \) and to check the model proposed. In fact there is some problem of checking expressions Eq. 3 as long as values of phenomenological coefficients \( a, b, c, d \) are unknown. To avoid this obstacle it is useful to find some relations that would not depend on values of these coefficients. To this end let us write down expressions Eq. 3 in the optimal point: \( a^2 = \frac{bt}{d} \) where they take the next form

\[
\begin{align*}
\rho_1 &= \frac{\sqrt{bdt}}{b + d + 2\sqrt{bdt}}; \\
\rho_2 &= \frac{d + \sqrt{bdt}}{b + d + 2\sqrt{bdt}}; \\
\rho_3 &= \frac{b}{b + d + 2\sqrt{bdt}}.
\end{align*}
\]

We see that in optimal point the inequality: \( \rho_2 \geq \rho_1 \) always holds for any values of \( b, d, t \). If we assume in addition that ratio \( \frac{d}{b} \ll 1 \) (what means that forgetting of acquired habit is rather slow) then we obtain from Eq. 4 approximate equality: \( 2\rho_1 \approx 1 \) or \( \rho_1 \approx \rho_2 \). Thus we come to the conclusion: if \( \rho_1, \rho_2, \rho_3 \) can be measured directly in statistical learning experiment we have actual possibility to verify the validity of model proposed.

Let us summing up the results of our consideration. Starting from attractive analogy between QTOS and based on simple statistical model of learning we obtain expressions for chances of individual to reach one or another level of task execution under the conditions of changing arousal. We have shown that this approach let one strictly derive the Yerkes-Dodson Law as well as its hallmark. We would like to express a hope that new experiments help to confirm (or to disprove) the statistical approach to learning process proposed in present paper.

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