Extraordinary galvanomagnetic effects in polycrystalline magnetic films

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Abstract – Based on the general requirement that all nature laws can be expressed in a tensor form, three new extraordinary galvanomagnetic effects in polycrystalline magnetic films, which differ from the known anomalous Hall effect and anisotropic magnetoresistance, are predicted. In general, a current perpendicular to a film can generate an electric field along the magnetization direction. Reversely, a current parallel to the magnetization can generate an electric field in the vertical direction of a film. The third extraordinary galvanomagnetic effect is that the longitudinal resistivity should depend on the magnetization component perpendicular to a film. One of the fingerprints of these effects is that the longitudinal resistance of a film is different when the current reverses its direction, and/or the in-plane longitudinal resistance is different when the perpendicular component of magnetization reverses its direction.

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Galvanomagnetic effects are electrical and thermal phenomena due to an electric current flow in a metal or a semiconductor in the presence of magnetic field or magnetization [1–3]. The galvanomagnetic effects include many well-known phenomena such as classical and quantum Hall effect [4], anomalous Hall effect (AHE) [5–8], anisotropic magnetoresistance (AMR) [9,10], and de Haas-Shubnikov oscillations [11], as well as recently discovered spin Hall and inverse spin Hall effect [12–17]. These effects are important not only in revealing the microscopic process and electronic properties of matter, but also in device applications. For magnetic materials in the absence of a magnetic field, AMR, AHE and the spin Hall effect are the only known galvanomagnetic effects. Most of the well-known galvanomagnetic effects were first found in experiments with spin Hall effect as an exception [12]. In this letter, three new extraordinary galvanomagnetic effects in polycrystalline magnetic films are predicted. Our predictions are based on the general requirement that all physical quantities must be tensors (scalars as rank-0 tensors; vectors as rank-1 tensors) and all physics equations can be written in tensor forms. Our analysis shows that all possible galvanomagnetic effects for polycrystalline magnetic bulks are already known. However, three new galvanomagnetic effects, which are not known to the best of our knowledge, exist in polycrystalline magnetic films due to their two-dimensional nature.

Consider a piece of ferromagnetic metal with an electric current density $\mathbf{J}$ passing through it. The galvanomagnetic effects of the system in the linear response region are best described by the generalized Ohm’s law that for polycrystalline ferromagnetic metals is [18–21]

$$ E = \rho_\perp \mathbf{J} + \frac{\Delta \rho}{M^2} (\mathbf{J} \cdot \mathbf{M}) \mathbf{M} - R_0 \mathbf{J} \times \mathbf{H} - R_1 \mathbf{J} \times \mathbf{M}, \quad (1) $$

where $M$ is the magnitude of magnetization $\mathbf{M}$ and $\mathbf{H}$ is an applied external magnetic field. $E$ is the electric field induced by $\mathbf{J}$. $\rho_\perp$ is the longitudinal resistivity when $\mathbf{M}$ and $\mathbf{J}$ are perpendicular to each other. $\Delta \rho = \rho_\parallel - \rho_\perp$ is the difference between $\rho_\perp$ and the longitudinal resistivity

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\( \rho_{1\parallel} \) when \( \mathbf{M} \) is parallel to \( \mathbf{J} \). This term is usually called the AMR. This AMR follows the famous universal angular dependence of the longitudinal resistivity \( \rho_{xx} = \rho_1 + \Delta \rho \cos^2 \theta \), where \( \theta \) is the angle between \( \mathbf{M} \) and \( \mathbf{J} \). \( R_0 \) and \( R_1 \) are the ordinary and anomalous Hall coefficients. This generalized Ohm’s law is the basis of the electrical detection of ferromagnetic resonance (FMR) [17–20].

Equation (1) is in fact the most general linear response of a polycrystalline magnetic bulk material. It could be understood by the following reasoning. For simplicity, let us assume that there is no external magnetic field and \( \mathbf{M} \) is the only vector variable available in the system\(^1\). For the linear response of the system to the electric current density \( \mathbf{J} \), the most general expression of the induced electric field must be \( \mathbf{E} = \tilde{\rho}(\mathbf{M}) \cdot \mathbf{J} \), where \( \tilde{\rho}(\mathbf{M}) \) is a rank-2 Cartesian tensor of the form \( f(\mathbf{M}) \mathbf{M} \mathbf{M} \). \( f \) is a function of \( \mathbf{M} \) whose exact sample dependence is particular. It is well known that a Cartesian tensor of rank 2 is reducible and can be reduced into a direct sum of three irreducible spherical tensors of rank 0, 1, and 2 (see footnote \(^2\)). Thus, \( \tilde{\rho}(\mathbf{M}) \) can be decomposed into a sum of a scalar of function of \( \mathbf{M} \), a vector that is a function of \( \mathbf{M} \) multiplying \( \mathbf{M} \), and a traceless symmetric tensor that is a function of \( \mathbf{M} \) multiplying \( \mathbf{M} \mathbf{M} - \mathbf{M}^2/3 \). Thus, the most general expression of \( \mathbf{E} \) is

\[
\mathbf{E} = \left( \rho_1 + \frac{\Delta \rho}{3} \right) \mathbf{J} + R_1 \mathbf{M} \times \mathbf{J} + \frac{\Delta \rho}{M^2} \left( \mathbf{M} \mathbf{M} - \frac{\mathbf{M}^2}{3} \right) \cdot \mathbf{J}.
\]

This is exactly eq. (1). \( \rho_{xx} \equiv (\mathbf{E} \cdot \mathbf{J})/J^2 = \rho_1 + \Delta \rho \cos^2 \theta \) is exactly the universal angular dependence of the AMR.

Although no new physics is obtained from this reasoning, this analysis is capable of deriving all galvanomagnetic effects for the bulk of polycrystalline magnetic metals. Interestingly, a much more tedious and lengthy argument was used before to derive the AMR [22].

Encouraged by the above success, we carry out a similar analysis for polycrystalline magnetic films, lying in the \( xy \)-plane as shown in fig. 1. Although a polycrystalline film is isotropic in the film plane, \( \hat{z} \) is an available vector and \( \tilde{\rho}(\mathbf{M}, \hat{z}) \) should be a function of both \( \mathbf{M} \) and \( \hat{z} \). Of course, the sense of the positive \( z \)-direction must be determined by \( \mathbf{J} \) and \( \mathbf{M} \) (see footnote \(^3\)). Since only three vectors \( (\mathbf{M}, \hat{z}, \mathbf{M} \times \hat{z}) \) and three traceless symmetric tensors

\(^1\)The spin vector of the itinerant electrons is ignored here. It could involve in the transport through the spin-orbit interactions.

\(^2\)For a rank-2 Cartesian tensor \( A_{ij} \), there are 9 independent elements \( A_{ij} \) (\( i, j = 1, 2, 3 \)). Define \( S = \sum_{i} A_{ii}/3 \), \( V_i = \sum_{j} \epsilon_{ijk} A_{kj}/2 \), and \( T_{ij} = (A_{ij} + A_{ji})/2 - \sum_{k} A_{ik}/3 \), where \( \epsilon_{ijk} \) is the Levi-Civita symbol. Then, \( S, V_i, \) and \( T_{ij} \) respectively have 1, 3, and 5 independent elements so that \( 9 = 1 + 3 + 5 \), and behave like a scalar, a vector, and a tensor of rank 2 under rotation transformations.

\(^3\)The \( z \)-direction should be determined by the current flow direction and the in-plane component of the magnetization if the current flows in the plane. One would not be able to distinguish the \(+z\)-direction from the \(-z\)-direction if the in-plane component of \( \mathbf{M} \) is parallel to current density in this case. For current perpendicular to the film, the \(+z\)-direction can be determined by the current flow direction.

\( (\mathbf{M} \mathbf{M} - \mathbf{M}^2/3, \mathbf{M} \hat{z} + \hat{z} \mathbf{M} - 2 \mathbf{M} \hat{z}/3, \hat{z} \hat{z} - 1/3) \) can be constructed out of \( \mathbf{M} \) and \( \hat{z} \), we have, with a similar reasoning as that for the bulk of polycrystalline magnetic metals,

\[
\mathbf{E} = \left( \rho_1 + \frac{\Delta \rho}{3} \right) \mathbf{J} + \left( R_1 \mathbf{M} \times \mathbf{J} + \frac{\Delta \rho}{M^2} \left( \mathbf{M} \mathbf{M} - \frac{\mathbf{M}^2}{3} \right) \right) + \left( R_2 (\mathbf{M} \hat{z} + \hat{z} \mathbf{M} - \frac{2 \mathbf{M} \hat{z}}{3}) + \rho_2 \left( \hat{z} \hat{z} - \frac{1}{3} \right) \right) \cdot \mathbf{J} = \rho_1 \mathbf{J} + R_1 \mathbf{M} \times \mathbf{J} + \frac{\Delta \rho}{M^2} (\mathbf{M} \cdot \mathbf{J}) \mathbf{M} - \rho_1 \mathbf{J} \times \hat{z} + R_2 J_z \mathbf{M} + R_3 (\mathbf{M} \cdot \mathbf{J}) \hat{z} - R_4 J_z \mathbf{M} + \rho_2 J_z \hat{z},
\]

where \( R_2 = R_0 - R_2, R_3 = R_0 + R_2 \) and \( R_4 = (2/3)R_0 \).

The \( \rho_1 \)-term can be interpreted as a kind of Hall effects if itinerant electrons can sense a fictitious magnetic field along the \( z \)-direction. Obviously, \( \rho_2 \) contributes to the longitudinal resistivity along the \( z \)-direction. Interestingly, one obtains three new terms. The \( R_2 \)-term says that a current perpendicular to the film induces an electric field in the \( \mathbf{M} \) direction. The \( R_3 \)-term describes the reverse effect: a current along the \( \mathbf{M} \) direction generates an electric field in the \( z \)-direction. Thus, the vertical voltage drop should be proportional to \( \cos \theta \), where \( \theta \) is the angle between \( \mathbf{M} \) and \( \mathbf{J} \). The \( R_4 \)-term says that the longitudinal resistivity of the film depends linearly on \( M_z \). In fact, different longitudinal resistances are indeed often observed in magnetic films [23] for \( M_z \) and \( -M_z \). This is the fingerprint feature of this term. Since the sense of the positive \( z \)-direction is determined by \( \mathbf{M} \) and \( \mathbf{J} \), this term destroys the symmetry of \( \mathbf{E}(\mathbf{M}, -\mathbf{J}) = -\mathbf{E}(\mathbf{M}, \mathbf{J}) \) [24] that eq. (1) possesses. Like a semiconductor diode, this term leads to a rectification effect. It should be pointed out that the coefficients in eq. (3), in principle, depend on \( M \) as well as other possible system variables or parameters such as the temperature and impurities. Generally speaking, all new terms should exist in magnetic films. Of course, their magnitudes depend on microscopic interactions that lead to these terms.

In order to find out how to experimentally verify the new effects, let us examine two experimental configurations involving a polycrystalline magnetic film as schematically shown in fig. 1. In fig. 1(a), an electric current density

Fig. 1: (Color online) Schematic illustration of in-plane (a) and perpendicular (b) transport measurement. A polycrystalline magnetic film is laid in the \( xy \)-plane. The \( z \)-axis is perpendicular to the film. \( \mathbf{J} \) is the electric current density and \( \mathbf{M} \) is the magnetization of the film.
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J flows in the film, say \( J = J \hat{x} \), while in fig. 1(b), \( J \) is perpendicular to the film, \( J = J \hat{z} \). In the case of fig. 1(a), the electric fields \( \mathbf{E} \) in the \( x- \), \( y- \) and \( z- \)directions are proportional to \( \rho_{xx}, \rho_{yz} \) and \( \rho_{zz} \). According to eq. (3), \( \rho_{xx} = \rho_{xx} + (\Delta \rho/M^2) M_x^2 - R_1 M_2 \). The first two terms describe the known AMR while the third term linear in \( M_2 \) is an additional contribution to the longitudinal resistivity. This contribution vanishes when \( \mathbf{M} \) lies in the plane of the magnetic film. All terms in \( \rho_{yy}, \rho_{yz} \) and \( \rho_{zz} \) depend on \( R \) and can be small. In any case, one can use this configuration to simplify the analysis, it is better to keep \( M \) small. From this consideration, \( \rho_{xx} \) may not be easy to measure because the voltage signal in the vertical direction for a micron-thick film should ensure the relevant voltage signal. From this consideration, one can measure the voltage drop in the \( x- \) direction for a micron-thick film should be small. In any case, one can use this configuration to measure \( R_4 \) by using a single-domain thin film so that the voltage signal would not be averaged. To simplify the analysis, it is better to keep \( \mathbf{M} \) in the \( yz \)-plane as illustrated in fig. 1(a). Then \( \rho_{xx} = \rho_{xx} - R_4 M_2 \). By reversing the current direction, one has, in this configuration, \( \rho_{xx} = \rho_{xx} + R_4 M_2 \). Thus, the difference in voltage drop for the two opposite current directions is proportional to \( R_4 \). One can also study the \( M_2 \)-dependence of the voltage drop in the \( x- \)direction at a fixed \( J \). One may use an external magnetic field to tune \( M_2 \).

In the case of fig. 1(b) and when one chooses the \( x \)-axis parallel to the in-plane component of \( \mathbf{M} \), \( (M_y = 0) \), then the \( x- \), \( y- \) and \( z- \)components of \( \mathbf{E} \) are proportional to \( \rho_{xx}, \rho_{yz} \) and \( \rho_{zz} \). This may be detected by measuring the voltage drops in the \( x- \), \( y- \) and \( z- \)directions when a tunneling current is passing through a multilayer sample in which the thin single-domain magnetic layer is sandwiched between two non-magnetic layers. According to eq. (3), \( \rho_{xx} = (\Delta \rho/M^2) M_x^2 + R_2 M_x \), \( \rho_{yz} = -R_1 M_x \) and \( \rho_{zz} = \rho_{xx} + (\Delta \rho/M^2) M_z^2 + (R_2 + R_3 - R_4) M_z \). Obviously, the voltage drop in the \( x \)-direction reverses its sign when \( \mathbf{M} = M \hat{x} \) changes to \( \mathbf{M} = -M \hat{x} \). The difference of the two voltage drops is proportional to \( R_2 \). This is the signature of \( R_2 \). To obtain \( R_3 \), one can measure the tunneling resistances for \( \mathbf{M} = \pm M \hat{z} \). The difference of the two tunneling resistances is proportional to \( R_2 + R_3 - R_4 \).

The new galvanomagnetic effects reported here come from the two-dimensional nature of films. The interfacial effects are not new in physics [25–27] and exist in both magnetic and non-magnetic systems. Our general analysis points out that the new galvanomagnetic effects should exist in magnetic films although it reveals neither microscopic origins of these effects nor their possible magnitudes. To utilize these new effects as material probes or to search for materials with larger effects, one needs to understand how microscopic interactions generate these new effects. Here, we would like to discuss possible microscopic interactions that could lead to these new galvanomagnetic effects. Obviously, the interactions should involve electron motion, \( \hat{z} \), as well as magnetization in order to generate these effects. Consider itinerant electrons of canonical momentum operator \( \mathbf{p} \) moving in a magnetic film, one can naturally add terms like \( \mathbf{M} \cdot (\mathbf{p} \times \hat{z}) \) (or \( \hat{z} \cdot (\mathbf{p} \times \mathbf{M}) \) or \( \mathbf{p} \cdot (\mathbf{M} \times \hat{z}) \)) to the electron Hamiltonian. Electrons, originally along \( \mathbf{p} \), are deflected in the transverse direction due to \( \mathbf{p} \times \hat{z} \) in \( \mathbf{M} \cdot (\mathbf{p} \times \hat{z}) \). The deflection senses the existence of \( \mathbf{M} \) that could generate an electrical field along \( \mathbf{M} \) (\( R_2 \)-term). For a similar reason, such terms can generate an electrical field along \( \hat{z} \) (\( R_3 \)-term).

Another interesting problem is to generalize the analysis to crystalline materials because it can help us to understand those electron transport measurements on magnetic single crystals. Since a crystal can provide intrinsic crystal vectors or tensors, \( \hat{p}(\mathbf{M}, \hat{x}_1, \hat{x}_2, \ldots) \) shall depend not only on \( \mathbf{M} \), but also on other tensor and/or vector variables \( (\hat{x}_1, \hat{x}_2, \ldots) \) that the crystal possesses. Then, one needs to construct all possible vectors and traceless symmetric tensors out of these available variables in order to find out all possible galvanomagnetic effects.

Clearly, all three new terms involve the coupling between the magnetization and the electric current so that they can generate dc-voltages in the electrical detection of FMR just like what AMR and AHE do [17–20]. The electrical detection has been used in recent years to study FMR-related physics [17–20]. Obviously, the new galvanomagnetic effects should have important implications on the interpretation of the dc-voltage signal. Furthermore, our results can naturally explain the overlooked phenomenon that the longitudinal resistance of a magnetic film is sensitive to the current direction [24] as well as to the direction of the perpendicular magnetization component [23].

In conclusion, we present a general analysis of the electric field generated by an electric current in polycrystalline magnetic films. Three extraordinary galvanomagnetic effects are predicted. A current perpendicular to the film generates an electric field along the magnetization direction. Reversely, a current along the magnetization generates an electric field perpendicular to the film. The longitudinal resistivity of a film depends linearly on the magnetization component perpendicular to the film. The typical features of these extraordinary galvanomagnetic effects are analyzed and possible ways of experimentally verifying them are suggested. The implications of these effects are also discussed.

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