Nonfactorizable Contribution to B-Meson Decays to s-Wave Mesons

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ABSTRACT

Two-body weak decays of bottom mesons into two pseudoscalar and pseudoscalar and vector mesons, are examined under isospin analysis to study nonfactorizable contribution.

Keywords: Weak Hadronic decays, Nonfactorization, Isospin formalism

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1. Introduction

There has been a growing interest in studying the nonfactorizable terms [1-4] of weak hadronic decays of charm and bottom mesons. We study the nonfactorizable contributions to various Cabibbo–Kobayashi–Maskawa (CKM) favored decays of B-mesons. Unfortunately, it has not been possible to calculate such contributions from the first principle, as these are non-perturbative in nature. Earlier attempts involved to find how much nonfactorizable contributions are required from the empirical details for weak charm hadronic decays [5-7]. We determine these contributions in the respective isospin $I=1/2$ and $3/2$ amplitudes for $B \to \pi D / B \to \rho D$ and $B \to \pi D^*$ decay modes by taking $N_c = 3$ to calculate the factorizable terms. The ratio of the nonfactorizable amplitude in these channels also seems to follow a universal value for all the above decay modes.

2. Methodology

The effective weak Hamiltonian for Cabibbo enhanced B-mesons decays is given by

$$H_w = \frac{G_F}{\sqrt{2}} V_{ub} V^*_{cd} \left[ c_1 \left( \bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2 \right) + c_2 \left( \bar{q}_2 q_1 \right) \right],$$

where $\bar{q}_1 q_2 = \bar{u}_1 \gamma_\mu (1 - \gamma_5) d_2$ denotes color singlet $V-A$ Dirac current and the QCD coefficients at bottom mass scale [4] are

$$c_1(\mu) = 1.12, \quad c_2(\mu) = -0.26. \quad (2)$$

where $\mu = m_B^2$, the values of $c_1$ and $c_2$ are taken from [5], and Fierz transforming the product of two Dirac currents of (1) in $N_c$ color- space, we get

$$\left( \bar{c} u \right) \left( \bar{d} b \right) = \frac{1}{N_c} \left( \bar{c} u \right) \left( \bar{d} b \right) + \frac{1}{2} \left( \lambda^a u \right) \left( d \lambda^a b \right) \quad (3)$$

And similar term for $\left( \bar{c} u \right) \left( \bar{d} b \right)$, where $\lambda^a$ are the Gell-Mann matrices. By using (3) and its analogue we reduced the effective Hamiltonian to describe color-favored (CF) and color-suppressed (CS) decays, respectively.

3. Results and Discussion

We applied the isospin formalism, and express decay amplitudes in terms of isospin reduced amplitudes $(A_{12}^{12}, A_{12}^{10})$ and as final-state interaction phase difference

$$\delta = \left( \delta_{12} - \delta_{14} \right).$$
There are many calculations for form factors and decay constants, such as light-cone sum rules [8], perturbative QCD approach, and lattice QCD [9-13] etc. We write nonfactorizable part in terms of isospin C. G. coefficients as scattering amplitudes for spurion + $B \rightarrow \pi D$ process:

$$A^{\pi'} (\bar{B}^0 \rightarrow \pi^- D^+ ) = \frac{1}{\sqrt{3}} e^{i\theta_0} \left[ A_{3/2}^{\pi D} + \sqrt{2} A_{1/2}^{\pi D} e^{i\theta} \right] ,$$

$$A^{\pi} (\bar{B}^0 \rightarrow \pi^0 D^0 ) = \frac{1}{\sqrt{3}} e^{i\theta_0} \left[ \sqrt{2} A_{1/2}^{\pi D} - A_{3/2}^{\pi D} e^{i\theta} \right] ,$$

$$A^D (\bar{B}^- \rightarrow \pi^- D^0 ) = \sqrt{3} A_{1/2}^{\pi D} e^{i\theta_0} .$$

Branching ratio for two body $B$-meson decays to pseudoscaler mesons is related to decay amplitude

$$B (\bar{B} \rightarrow P_1 P_2 ) = \frac{\tau_B}{\sqrt{2}} G_{V, P_1, P_2} \left[ \frac{p}{8 \pi m_B^2} \right]^{1/2} \left| A (\bar{B} \rightarrow P_1 P_2 ) \right|^2 \tag{5}$$

where $\tau_B$ is the life time of $B$-meson, $G_{V, P_1, P_2}$ is the product of the CKM matrix elements [1], $p$ is the magnitude of the 3-momentum of the final state particles in the rest frame of $B$-meson and $A (\bar{B} \rightarrow P_1 P_2 )$ is the decay amplitude. We have calculated isospin reduced amplitudes, $A_{3/2}^{\pi D}$ and $A_{1/2}^{\pi D}$

$$\left| A_{3/2}^{\pi D} \right|^{\exp.} = (1.272 \pm 0.065) \text{GeV}^3 ,$$

$$\left| A_{1/2}^{\pi D} \right|^{\exp.} = (1.323 \pm 0.018) \text{GeV}^3 , \tag{6}$$

using the experimental value [1], where the factorizable parts are calculated by using BSW model [3], expressed as

$$A^I (\bar{B}^0 \rightarrow \pi^- D^+ ) = a_{1/2} f_{3/2} (m_B^2 - m_B^2)^{F_{3/2}^{BD} (m_B^2)} = 2.178 \pm 0.009 \text{GeV}^3 ,$$

$$A^I (\bar{B}^0 \rightarrow \pi^0 D^0 ) = - \frac{1}{\sqrt{2}} a_{1/2} f_{3/2} (m_B^2 - m_B^2)^{F_{3/2}^{BD} (m_B^2)}$$

$$= - (0.139 \pm 0.025) \text{GeV}^3 \tag{7}$$

$$A^I (\bar{B}^- \rightarrow \pi^- D^0 ) = a_{1/2} f_{3/2} (m_B^2 - m_B^2)^{F_{3/2}^{BD} (m_B^2)} + a_{3/2} f_{1/2} (m_B^2 - m_B^2)^{F_{1/2}^{BD} (m_B^2)}$$

$$= 2.377 \pm 0.009 \text{GeV}^3$$

Table 1: Comparison of final results for all the decay modes.

| Decay modes | $\bar{B} \rightarrow \pi D$ | $\bar{B} \rightarrow \rho D$ | $\bar{B} \rightarrow \pi D^*$ |
|-------------|-----------------------------|-----------------------------|-----------------------------|
| $A^{\pi D}_{1/2}$ | $-0.730 \pm 0.065$ | $-0.081 \pm 0.024$ | $-0.064 \pm 0.011$ |
| $A^{\pi D}_{3/2}$ | $-2.492 \pm 0.018$ | $-0.317 \pm 0.009$ | $-0.272 \pm 0.004$ |
| $\alpha = A^{\pi D}_{1/2} / A^{\pi D}_{3/2}$ | $0.293 \pm 0.026$ | $0.256 \pm 0.078$ | $0.237 \pm 0.043$ |

Summary and Conclusions

The motivation for the exploration of nonfactorizable term has been the failure of the large-$N_c$ limit, which was supposed to be supported by the $D$-meson phenomenology, especially when extended to the $B$-meson sector. For instance, $D$-decays demand a negative value for $\alpha$, indicating $N_c \rightarrow \infty$ limit, whereas $B$-meson decays clearly favor positive value for $\alpha$. Therefore, it has been suggested to investigate the effect of...
nonfactorizable terms in the heavy quark decays keeping the real value of color $N_c = 3$.

We determine $A_{nf}^{12}$ and $A_{nf}^{32}$ (as shown in table), for all the decay modes, $\bar{B} \to \pi D / \bar{B} \to \rho D$ and $\bar{B} \to \pi D^*$. We notices that the non-factorizable amplitudes shows as increasing pattern with decreasing momenta available to the final state particles, i.e.,

$$|A^e(\bar{B} \to \pi D^*)| > |A^e(\bar{B} \to \rho D)| > |A^e(\bar{B} \to \pi D)| \quad (11)$$

This behavior is understandable, since low momentum states are likely to be affected more through the exchange of soft gluons and can acquire larger non-factorizable contributions [8]. We observe that in all the decay modes, the non-factorizable isospin amplitude $A_{nf}^{12}$ bears the same ratio, with in the experimental errors, as well as same sign, $A_{nf}^{32}$ amplitude.

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