Probing the homogeneous spectral function of a strongly interacting superfluid atomic Fermi gas in a trap using phase separation and momentum resolved rf spectroscopy

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It is of central importance to probe the local spectral function \( A(k, \omega) \) of a strongly interacting Fermi gas in a trap. Momentum resolved rf spectroscopy has been demonstrated to be able to probe the trap averaged \( A(k, \omega) \). However, the usefulness of this technique was limited by the trap inhomogeneity. Independent of a specific theory, here we propose that by studying the momentum resolved rf spectra of the minority fermions of a phase separated, population imbalanced Fermi gas at low temperature, one can effectively extract \( A(k, \omega) \) of a homogeneous superfluid Fermi gas (at the trap center). In support, we present calculated spectral functions and spectral intensity maps for various cases from BCS through BEC regimes using different theories.

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I. INTRODUCTION

Ultracold atomic Fermi gases has emerged as a rapidly developing research field, bridging condensed matter and atomic physics. Owing to their easy tunabilities such as the effective two-body interaction strength, population imbalance, and dimensionality, atomic Fermi gases can be viewed as a quantum simulator of many important condensed matter systems, such as the Hubbard model and high Tc superconductors.

Of central importance in a many-body system is the single particle spectral function \( A(k, \omega) \), especially in the strongly interacting regime (where different theories often do not agree); it can be used to calculate essentially all other physical quantities, and to test various theories. For example, one important question, which has been long standing for high Tc superconductors, is whether a pseudogap exists and how it evolves in the superfluid state. Because measurements often involve integration of the entire momentum space and the entire trap, the intrinsic spatial inhomogeneity introduced by the trapping potential constitutes a severe problem. It has been a long-sought goal to extract the local or homogeneous properties, especially \( A(k, \omega) \), from measurements in the trap.

In this paper, we propose that one can use momentum resolved rf spectroscopy in conjunction with a high population imbalance to measure the homogeneous spectral function \( A(k, \omega) \), for the density and chemical potential at the trap center.

Since the first experimental realization of BCS-BEC crossover, the most direct experimental probe of the single particle properties is arguably the rf spectroscopy. The first generation of rf measurements involves integration over the entire trap and the whole momentum space, which led to a two-peak structure of the rf spectra at an intermediate temperature \( T \) below \( T_c \). Due to the integrations, controversies arose regarding the origin of this two-peak structure. Tomographic rf spectroscopy involves integration over the entire momentum space, and therefore cannot be used to probe the spectral function. The recent Ho-Zhou scheme allows one to calculate the density and fermionic chemical potential \( \mu \) as a function of the radius \( r \). However, it relies on the assumption of a strict harmonic trapping potential, often not satisfied. More importantly, it does not provide any microscopic information such as the spectral function.

The controversies regarding the origin of the aforementioned double peak structure have largely been cleared by the recent experiments on \(^{40}\)K using momentum resolved rf spectroscopy. With momentum resolution, the rf spectroscopy essentially measures the centrally important \( A(k, \omega) \), which is of central importance in a many-body system. However, this is true only for a homogeneous Fermi gas. The inherent spatial inhomogeneity severely limits the quantitative resolution of the extracted spectral function in the experiment of Ref. \[7\], which involves integration over the entire trap.

Here we address this inhomogeneity issue and show that it can be largely avoided utilizing phase separation at high population imbalances. In what follows we will use two different theories to do computations. Despite some differences in quantitative details, we emphasize that the validity of our proposal is model independent and can be applied in other theories as well. Most importantly, it is a proposal for experiment.

As mentioned elsewhere, the use of \(^{40}\)K rather than \(^{6}\)Li is crucial in extracting \( A(k, \omega) \), since there are no complications from final state interactions near the usual Feshbach resonance around 202 G. It is for this reason that the rf spectral intensity for \(^{40}\)K is simply proportional to the spectral function \( 40A(k,\omega) \). For \(^{6}\)Li, extra efforts are needed in order to extract \( A(k, \omega) \) from the rf spectral measurements.

The spectral function \( A(k, \omega) \) can be used to uniquely construct the Green’s function and the single particle self energy, for which different theories often give different results. Therefore, the more accurate the measurement of \( A(k, \omega) \), the easier it is to test different theories. For example, different BCS-BEC crossover approaches show different dispersive behavior of \( A(k, \omega) \). The theoretical scheme used in Ref. \[8\] exhibits a clear downward dispersion in the spectral intensity map near \( T_c \), manifesting an existing pseudogap before the onset of superfluidity at lower \( T = T_c \), different from those \([12, 13]\) that follow the approach of Nozières and Schmitt-Rink (NSR) \([14]\). A quantitatively accurate measurement of \( A(k, \omega) \) should serve to unambiguously test these different theories.
II. THEORETICAL FORMALISM

In a typical rf spectroscopy measurement, pairing takes place between two low-lying hyperfine states, which we refer to as levels 1 and 2. An rf field of frequency $\nu$ is used to excite the atoms in hyperfine level 2 to another hyperfine state, which is unoccupied initially and referred to as level 3. It has been shown previously [8] that for $^{40}$K we have for the momentum-resolved RF current

$$I(k, \nu) = \frac{1}{2\pi} A(k, \omega) f(\omega) \bigg|_{\omega = \xi_k - \nu},$$

where $A(k, \omega) = -2 \text{Im} G(k, \omega + i0)$, $f(x)$ the Fermi distribution function, with the transition matrix element set to unity. Here $G$ is the Green’s function of level 2 atoms, which we take to be the spin down or minority species. Similar to angle-resolved photoemission spectroscopy in a usual condensed matter system, the rf current measures $A(k, \omega)$ directly. Note that here $\nu$ is the rf detuning. The frequency $\omega = \xi_k - \nu$ corresponds to the energy of level 2 atoms measured with respect to their Fermi level, where $\xi_k = k^2/2m - \mu$, $\mu$ is the chemical potential of atoms in level 2. We take $\hbar = 1$. It is obvious that when level 2 atoms are free, we have $\nu = 0$ and $\omega = \xi_k$; the former gives the sharp peak at zero detuning in previous rf spectra in Ref. [3] whereas the latter gives the free atom dispersion in the spectral intensity map in the $\omega - k$ plane in Ref. [2]. As has been used in the experiment [2], the angle-integrated “occupied spectral intensity” is given by

$$I_{\text{photo}}(k, \omega) \equiv \frac{k^2}{2\pi^2} A(k, \omega) f(\omega).$$

The central issue here is to calculate the spectral function or equivalently the Green’s function $G(K)$ for the minority (level 2) atoms. Here we present calculations using two different formalisms. In the first, pairing fluctuation, approach, in the presence of population imbalance, detailed calculations of the superfluid phase diagram, the Green’s function and density profiles in each phase can be found in Refs. [15] and [16]. In the case of phase separation in a trap, as shown in Fig. 2 of Ref. [15], there is a BCS superfluid core (without population imbalance), surrounded by the majority atoms, as confirmed experimentally [17, 18]. To a first order approximation, the density profile of the minority atoms can be obtained by truncating the density profile of an unimbalanced Fermi gas at the (sharp) phase separation boundary. With and without population imbalance, the normal state self energy $\Sigma_{\text{pg}}$ follows a rather simple BCS-like form [8, 19]. In the superfluid state, the self energy $\Sigma(K)$ contains two terms, associated with the condensed ($\Sigma_{sc}$) and noncondensed fermion pairs ($\Sigma_{pg}$), respectively:

$$\Sigma(k, \omega) = \Sigma_{pg}(k, \omega) + \Sigma_{sc}(k, \omega),$$

where

$$\Sigma_{pg}(k, \omega) = \frac{\Delta^2_{pg}}{\omega + \xi_k + i\gamma} - i\Sigma_0,$$

$$\Sigma_{sc}(k, \omega) = \frac{\Delta^2_{sc}}{\omega + \xi_k}.$$  

Here $\Delta_{pg}$, $\Delta_{sc}$ and $\Delta = \sqrt{\Delta^2_{sc} + \Delta^2_{pg}}$ are the pseudogap, superfluid gap and total excitation gap, respectively. The broadening $\gamma \neq 0$ and “incoherent” background contribution $\Sigma_0$ can be determined by fitting the experimentally measured rf spectra. The precise values of $\gamma$ and $\Sigma_0$, and their $T$-dependencies are not particularly important for the present purposes. Although the pseudogap contribution becomes less pronounced when the temperature is low enough to exhibit phase separation, it still makes a significant difference in comparison with a strict BCS theory.

In the second, mean-field, approach, as a test case, we use the same density profiles as obtained from the first approach, but take a simple broadened BCS self energy,

$$\Sigma_{BCS}(k, \omega) = \frac{\Delta^2}{\omega + \xi_k + i\gamma} - i\Sigma_0.$$  

Both approaches give the same form of quasiparticle dispersion $E_k = \sqrt{\xi_k^2 + \Delta^2}$ albeit with different meanings of $\Delta$. Obviously, they become equivalent above $T_c$, when we take $\gamma = \Sigma_0$. In contrast, they differ dramatically below $T_c$ [19] because one contains a pseudogap while the other does not. In both approaches, $\gamma$ and $\Sigma_0$ scale with local $E_F(r)$. As in Ref. [8] here we will take $\Sigma_0$ as $T$ independent, $\gamma$ linear in $T/T_c$, where $T_c$ is calculated using the first approach. The rf spectra are finally convoluted with a Gaussian broadening function with a standard deviation $\sigma$, reflecting the instrumental resolution in experiment caused by e.g. finite energy and momentum resolution of both the rf pulse and the time of flight imaging technique.

III. RESULTS AND DISCUSSIONS

We first study the radial profile of the excitation gap in the superfluid phase. While throughout this paper we study each case in a harmonic trap with a local density approximation using both approaches, here we show in Fig. [1] the result from the pairing fluctuation approach Eq. (2). It suffices to
calculate for cases without population imbalance, since upon phase separation the superfluid core becomes unimbalanced so that the gap profile of minority atoms can be obtained by cutting off the curves in Fig. 1 at different radii for different population imbalances \( p \equiv (N_\downarrow - N_\uparrow)/(N_\downarrow + N_\uparrow) \). For simplicity, here the data were all calculated at \( T = 0.5T_c \), below which phase separation takes place at unitarity [15]. The curves, as labeled, correspond to \( 1/k_F^0a = -0.5 \) (BCS case), 0 (unitary), 0.5 (pseudogap) and 1.0 (BEC) cases, respectively, where \( a \) the inter-fermion s-wave scattering length, \( E_F^0 \equiv k_BT_F^0 \equiv k^2(k_F^0)^2/2m \) and \( k_F^0 \) are the global Fermi energy and Fermi momentum in the noninteracting limit for the majority atoms. Figure 1 reveals that for \( r < 0.3R_{TF} \), the gap is nearly flat from BCS through BEC regimes, where \( R_{TF} \) is the Thomas-Fermi radius. Since the pseudogap increases with pairing strength and it is already comparable with the zero \( T \) gap at unitarity [2, 7], the total gap profile \( \Delta(r) \) remains nearly flat for small \( r \) even at higher \( T \approx T_c \) at unitarity and in the BEC regime [20].

With this knowledge, now we study the occupied spectral intensity maps of the minority atoms of a population imbalanced \( \omega + \mu(r) \) plane for different degrees of population imbalance \( p \), as characterized by the radius \( R_{\downarrow} \) of the minority atomic gas. For illustration purpose, we present the result from the pairing fluctuation approach Eq. (2) for the unitary case with phase separation in Fig. 2 for different \( R_{\downarrow} \). Note here that, as in Ref. [3], we use \( \omega + \mu(r) \) instead of \( \omega \) in the vertical axis, since the former combination is independent of \( r \) representing the single particle energy measured from the bottom of the band. While Fig. 2(a) corresponds to an extremely high population imbalance, which may not be readily realizable in experiment, Fig. 2(c)-(d) are certainly accessible experimentally [17]. For example, for \( ^{6}\text{Li} \), Ref. [18] reports a phase separation boundary at about \( 0.3R_{TF} \) at \( p = 0.54 \). The (magenta) dashed curves represent the local (or homogeneous) dispersion at the trap center, and the (white) solid curves are the quasiparticle dispersion given by the loci of the peak location of the EDC. The broadening parameters \( \Sigma_0 = \gamma(T_c) \) at the trap center are \( (a) 0.1E_F^0 \), (b)-(c) \( 0.2E_F^0 \), and (d) \( 0.35E_F^0 \), respectively. \( T_c \) is taken from the pairing fluctuation approach. The instrumental broadening \( \sigma = 0.2E_F^0 \). For all cases, the two sets of curves are essentially indistinguishable.

Shown in Fig. 3 are the occupied spectral intensity maps of the minority atoms of population imbalanced, phase separated atomic Fermi gases in a harmonic trap with minority radius \( R_{\downarrow} = 0.3R_{TF} \) for different pairing strength from BCS through BEC. Here we show the results calculated using the mean field self energy Eq. (2). Since the atomic cloud shrinks with increasing \( 1/k_F^0a \), these same \( R_{\downarrow} \) corresponds to an imbalance of about \( p = 0.9 \sim 0.7 \). Here the parameters \( \gamma \) and \( \Sigma_0 \) increase from BCS to BEC, reflecting an increasing excitation gap. Clearly, for all cases shown, the dispersions ex-
The spectral peaks at \( r = 0 \) are intrinsic spectral lines, whereas the broad ones are convoluted with an instrumental broadening function. The complete spectral function \( A(k, \omega) \) can be obtained by dividing the unconvoluted lines by \( f(\omega) \).

Indeed, a comparison between Fig. 2(c) and Fig. 3(b) reveals that the spectral lines from self-energy Eq. (3) have a broader background than that from Eq. (2). Precise spectral function measurement as shown in Fig. 4 is expected to tell them apart.

**IV. CONCLUSIONS**

In conclusion, in the presence of phase separation for a highly population imbalanced, strongly interacting Fermi gas, the trap averaged quasiparticle dispersion of the minority atoms is very close to that at the trap center. Therefore, one can use the former dispersion to extract effectively not only the excitation gap and the fermionic chemical potential but also the centrally important spectral function at the trap center. This scheme does not depend on our particular BCS-BEC crossover theory, nor does it depend on the strict harmonicity of the trapping potential. For higher \( T \) or deeper BEC regime where phase separation is prohibited, one may use an rf pulse with a narrow cross section (as has been reported experimentally \([23]\)) without population imbalance to achieve similar results. In this way, the narrow rf beam mimics the effect of artificial phase separation by picking up signals only from the central part of the trap \([24]\).

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[1] A. J. Leggett, Nat. Phys., 2, 134 (2006).
[2] Q. J. Chen, J. Stajic, S. N. Tan, and K. Levin, Phys. Rep., 412, 1 (2005).
[3] C. Chin, M. Bartenstein, A. Altmeyer, S. Riedl, S. Jochim, J. Hecker-Denschlag, and R. Grimm, Science, 305, 1128 (2004).
[4] Y. Shin, C. H. Schunck, A. Schirotzek, and W. Ketterle, Phys. Rev. Lett., 99, 090403 (2007).
[5] T.-L. Ho and Q. Zhou, Nat. Phys., 6, 131 (2009).
[6] Confirmed by T.-L. Ho. Private communications.
[7] J. T. Stewart, J. P. Gaebler, and D. S. Jin, Nature (London), 454, 744 (2008).
[8] Q. J. Chen and K. Levin, Phys. Rev. Lett., 102, 190402 (2009).
[9] S. Basu and E. Mueller, Phys. Rev. Lett. 101, 060405 (2008); Y. He, C.-C. Chien, Q. J. Chen and K. Levin, ibid. 102, 020402 (2009); A. Perali, P. Pieri, and G. C. Strinati, ibid. 100, 010402 (2008); M. Punk and W. Zwerger, ibid. 99, 170404 (2007); Z. Yu and G. Baym, Phys. Rev. A 73, 063601 (2006).

[10] K. Levin, Q. J. Chen, Y. He, and C.-C. Chien, Ann. Phys. 325, 233 (2010); Q. J. Chen, Y. He, C.-C. Chien, and K. Levin, Rep. Prog. Phys. 72, 122501 (2009).

[11] S.-Q. Su, D. E. Sheehy, J. Moreno, and M. Jarrell, Phys. Rev. A. 81, 051604(R) (2010).

[12] A. Perali, P. Pieri, G. C. Strinati, and C. Castellani, Phys. Rev. B. 66, 024510 (2002).

[13] P. Massignan, G. M. Bruun, and H. T. C. Stoof, Phys. Rev. A, 77, 031601(R) (2008).

[14] P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys., 59, 195 (1985).

[15] C.-C. Chien, Q. J. Chen, Y. He, and K. Levin, Phys. Rev. Lett., 98, 110404 (2007).

[16] Y. He, C.-C. Chien, Q. J. Chen and K. Levin, Phys. Rev. A75, 021602(R) (2007); Phys. Rev. B76, 224516 (2007).

[17] G. B. Partridge, W. H. Li, R. I. Kamar, Y. A. Liao, and R. G. Hulet, Science 311, 503 (2006); M. W. Zwierlein, A. Schirotzek, C. H. Schunck, and W. Ketterle, ibid. 311, 492 (2006).

[18] Y. Shin, C. H. Schunck, A. Schirotzek, and W. Ketterle, Nature (London), 451, 689 (2008).

[19] Q. J. Chen, K. Levin, and I. Kosztin, Phys. Rev. B, 63, 184519 (2001).

[20] In the BEC regime, the gas cloud shrinks so that phase separation boundary also moves toward the trap center.

[21] Q. J. Chen, APS March Meeting, 2009, [http://meetings.aps.org/link/BAPS.2009.MAR.A16.5](http://meetings.aps.org/link/BAPS.2009.MAR.A16.5).

[22] J. P. Gaebler, J. T. Stewart, T. E. Drake, D. S. Jin, A. Perali, P. Pieri, and G. C. Strinati, Nat. Phys., 6, 569 (2010).

[23] T. Drake et al., APS March Meeting, 2011, [http://meetings.aps.org/link/BAPS.2011.MAR.P45.3](http://meetings.aps.org/link/BAPS.2011.MAR.P45.3).

[24] It should be noted that the boundary of a focused rf pulse may not be as sharp as the interface of phase separation, making our phase-separation method a better choice at low T.