Coarse-grained model for spring friction study of micron-scale iron by smoothed particle hydrodynamics

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Abstract – The paper presents the construction of a coarse-grained model to investigate dry sliding friction of the body-centered-cubic Fe micron-scale system by smoothed particle hydrodynamics simulations and examines influences of the spring force on the characters of friction. The $N_{\text{atom}} = 864 \times 10^{12}$ atoms Fe system is coarse-grained into the two different simple-cubic particle systems, one of 432000 and the other of 16000 particles. From the detection of stick-slip motion, friction coefficient, dependence of friction coefficient on isotropy or anisotropy of the spring force and externally applied normal load, we find that the coarse-grained model is a reasonable modeling process for the study of the friction of the Fe system, and the anisotropic behavior presents better friction of the system than the isotropic one.

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Introduction. – Coarse-graining (CG) has become one of the most known choices in simulations of micron-scale systems. The number of degrees of freedom decreases many times in a CG system compared to that of a corresponding atomistic system; therefore, one has to solve only a few equations of motion for a CG system. Furthermore, since the mass of a CG particle is much larger than that of an atom, a simulation time step can be chosen as a longer one leading to the fact that a CG system can be simulated in a long time. In a theoretical approach, Kinjo et al. derived the equation of motion of the coarse-grained particles by the projection operator method [1]. There were a few simulation studies of the mechanical properties of CG solid materials. The simulations of Rudd et al. indicated that CG molecular dynamics provided a better description of the elastic waves than that provided by the finite element modeling, and the elastic wave scattering was more benign in CG molecular dynamics than in finite element modeling for the small periodic CG solid systems [2,3]. Kobayashi et al. supposed a CG model named the recursive CG particle method to investigate large realistic systems [4]. Although Kobayashi et al. well applied their model to investigate the deformation of the Ar and Al nanorods, they simulated the two- or three-dimensional systems of sub-micron size. One needs to investigate a micron-scale system to be able to observe the mechanical properties of devices. Subsequently, Yang et al. [5] coarse-grained a Cu substrate by estimation of its new face-centered-cubic (fcc) lattice constant based on the formula of the nearest-neighbor distance of the fcc Lennard-Jones substrate established by Stevens [6]. The limit of this coarse-graining is only one permitted CG time leading to the fact that the particle number of the system does not decrease much. This is not reasonable for considering large systems. The simulation results of the CG Cu systems exhibited a good agreement of change of stress between a CG system and an atomistic one [7]. However, by choosing the surfaces of particle oriented along the $\{111\}$ close-packed planes, Xiong et al. calculated mass of each particle in approximation and designed a complex CG method. A CG model should also fully meet the simplicity of the modeling process.

At atomistic level, we have been interested in the origin of kinetic friction by calculating phonon dissipation of bulk atoms [8–10]. Our recent work carried out sliding
friction of the GC graphene sheets [11]. Thermal escape motion resulting in low-friction mechanism in the particle model [11] is similar to that confirmed in atomistic systems. Our present work presents a simple CG method of a three-dimensional crystal system where the mass of the particle is taken exactly. In this method, a body-centered-cubic (bcc) crystal system is first coarse-grained into a simple-cubic (sc) crystal system. The obtained sc system is then coarse-grained into a larger sc one. This CG model is completely based on the symmetry of lattice structures and can easily be modeled for a large system. The GC model is utilized to investigate the sliding friction of a Fe micron-scale system by smoothed particle hydrodynamics (SPH) simulations. Up to now, we have only found a study of friction of a Fe nano-scale system at sliding velocities of 100–350 m/s [12], while most studies were focused on that of Fe alloys [13,14]. In recent years, research groups have reported friction or tribology of various materials at high sliding velocities, 100 m/s [15], 400 m/s [16] and 120 m/s [17]. Along with these studies, the present work investigates the friction of the Fe system at a sliding velocity of 100 m/s. Moreover, effects of the spring force on friction are also carefully discussed via the spring constant and the isotropy or anisotropy of the spring force.

The CG model and SPH simulation. – Coarse-graining is performed in two steps: firstly, each unit cell of the initially bcc atomistic Fe system is coarse-grained into one particle that has mass 2m (m = 55.845 g/mol) and is positioned at the center of the unit cell. This coarse-graining exchanges the atomistic system (fig. 1(a)) to the sc particle one (fig. 1(b)) having a lattice constant \(a_{BCC} = 2.85\) Å; finally, the obtained sc particle system is continuously coarse-grained into a larger sc particle one by grouping a cubic region of \(N_3\) unit cells (\(N_3\) in each direction \(x, y, \) or \(z\)) into one particle. The final sc particle system (fig. 1(c)) has the following characters: mass of each particle \(M_{CG} = 2N_3m\), lattice constant \(a_{CG} = N_3a_{BCC}\) and number of particles \(N_{CG} = 2N_3\).

Due to its well-known calculations, the SPH method is briefly presented by the governing equations in the present work. The time-evolution of the density \(ρ_i\), velocity \(v_i^0\) and internal energy \(u_i\) of the \(i\)-th particle is described by the following equations:

\[
\frac{dρ_i}{dt} = \sum_{j=1}^{N} m_j (\vec{v}_j - \vec{v}_i) \vec{∇}_j W (\bar{r}_i - \bar{r}_j, h),
\]

(1)

\[
\frac{dv_i^0}{dt} = \sum_{j=1}^{N} m_j \left( \frac{σ_{ij}^0}{ρ_i^0} + \frac{σ_{ij}^0}{ρ_j^0} + Π_{ij} \right) \times \vec{∇}_j W (\bar{r}_i - \bar{r}_j, h),
\]

(2)

\[
\frac{du_i}{dt} = \frac{1}{2} \sum_{j=1}^{N} m_j \left( \frac{σ_{ij}^0}{ρ_i^0} + \frac{σ_{ij}^0}{ρ_j^0} + Π_{ij} \right) \times (v_j^0 - v_i^0) \vec{∇}_j W (\bar{r}_i - \bar{r}_j, h),
\]

(3)

where \(σ_{ij}^0\) and \(Π_{ij}\) are the spring force and friction force respectively. Here \(h_1 = (\frac{m}{\rho_i})^{1/3}\) is the smoothed length of the \(i\)-th particle and \(n\) is a parameter; the frictional viscosity function is

\[
Π_{ij} = \begin{cases} -A\epsilon_{ij}\mu_{ij} + B\epsilon_{ij}^2, & \vec{v}_{ij} \cdot \vec{r}_{ij} \leq 0, \\ 0, & \vec{v}_{ij} \cdot \vec{r}_{ij} > 0, \end{cases}
\]

(4)

where \(A\) and \(B\) are parameters, \(\epsilon_{ij} = \frac{1}{2}(\epsilon_i + \epsilon_j)\) and \(\sigma = \frac{\sqrt{3}h_0}{\epsilon}\) is the speed of sound of the particle, \(\rho_{ij} = \frac{1}{2}(\rho_i + \rho_j)\) and \(\mu_{ij} = \frac{h_0}{\epsilon}p_{ij}^2\) is the pressure of the particle and \(p_{ij} = (p_{ij} - 1)\mu a\) is a parameter; and the stress tensor

\[
σ_{ij}^{\alpha\beta} = -p_\delta^{\alpha\beta} + S_{ij}^{\alpha\beta},
\]

(5)

where \(δ^{\alpha\beta}\) is the Kronecker symbol, \(S_{ij}^{\alpha\beta}\) is the deviatoric stress calculated from the equation

\[
\frac{dS_{ij}^{\alpha\beta}}{dt} = 2\mu \left( ε_{ij}^{\alpha\beta} - \frac{1}{3} δ^{\alpha\beta} ε_{ij}^{γγ} \right) + S_{ij}^{\alpha\beta} R_{ij}^{\beta\gamma} + S_{ij}^{\beta} R_{ij}^{\alpha\gamma},
\]

(6)

in which \(γ ≡ x, y, z\) and \(μ\) is the shear modulus of materials, \(ε_{ij}^{\alpha\beta}\) is the tensor of the rate of deformations defined by

\[
ε_{ij}^{\alpha\beta} = \frac{1}{2} \sum_{j=1}^{N} m_j (v_j^\alpha v_i^\beta - v_i^\alpha v_j^\beta) \vec{∇}_j W (\bar{r}_{ij}, h),
\]

(7)

\[
+ (v_i^\alpha - v_j^\alpha) \vec{∇}_i W (\bar{r}_{ij}, h),
\]

(8)
and \( R^{\alpha\beta}_{i} \) is the tensor of stress rotation defined by

\[
R^{\alpha\beta}_{i} = \frac{1}{2} \sum_{j=1}^{N} \frac{m_{j}}{\rho_{j}} [(v_{j}^{\alpha} - v_{i}^{\alpha}) \nabla_{i}^{\beta} W(\vec{r}_{ij}, h) - (v_{j}^{\beta} - v_{i}^{\beta}) \nabla_{i}^{\alpha} W(\vec{r}_{ij}, h)].
\] (9)

We consider adding the dissipation force on each particle to compensate the energy dissipation caused by friction during the sliding

\[
F_{\text{dis}, i} = \begin{cases} -m_{i} \gamma_{\text{dis}} (v_{i}^{x} - V_{\text{dis}}), & \text{the } x\text{-direction,} \\ -m_{i} \gamma_{\text{dis}} v_{i}^{y}, & \text{the } y\text{-direction,} \\ -m_{i} \gamma_{\text{dis}} v_{i}^{z}, & \text{the } z\text{-direction,} \end{cases}
\] (10)

where \( \gamma_{\text{dis}} \) is a parameter of the model and \( V_{\text{dis}} = 0 \) for particles of the substrate and \( V_{\text{dis}} = V \), which is a constant sliding velocity of the slider in the \( x\)-direction, for particles of the slider. By using the Prandtl-Tomlinson model, we also add a spring force on each particle of the slider

\[
F_{\text{spr}, i} = \begin{cases} K(x_{0,i} + Vt - x_{i}), & \text{the } x\text{-direction,} \\ K(y_{0,i} - y_{i}), & \text{the } y\text{-direction,} \\ K(z_{0,i} - z_{i}), & \text{the } z\text{-direction,} \end{cases}
\] (11)

where \( K \) is the spring constant, \( t \) is the sliding time, \( x_{0,i}, y_{0,i} \) and \( z_{0,i} \) are the equilibrium/initial coordinates of the \( i\)-th particle in the \( x\), \( y\) and \( z\)-directions, respectively. Interaction between the slider and the substrate is presented by the interaction between particles of the lowest layer of the slider and particles of the highest layer of the substrate. Two particles, one of each layer, interact with each other via the following spring force:

\[
F_{\text{int}, ij} = \begin{cases} -k_{a}(r - h) \frac{\vec{r}_{ij}}{r}, & 0 < r < h, \\ 0, & r > h, \end{cases}
\] (12)

where \( k_{a} \) is the spring constant that is considered being isotropy or anisotropy in the present study. The friction force \( F_{\text{fr}} \), the normal force \( F_{\text{nor}} \) and the friction coefficient \( \mu_{\text{cof}} \) are defined as follows:

\[
F_{\text{fr}} = \sum_{i=1}^{N_{s}} (F_{\text{spr}, i} + F_{\text{int}, ij}),
\] (13)

\[
F_{\text{nor}} = \sum_{i=1}^{N_{s}} (F_{\text{spr}, i} + F_{\text{int}, ij}),
\] (14)

\[
\mu_{\text{cof}} = \frac{F_{\text{fr}}}{F_{\text{nor}}},
\] (15)

where \( N_{s} \) is the number of the particles of the lowest layer of the slider, \( F_{x} \) and \( F_{z} \) are the force components in the \( x\)- and \( z\)-directions, respectively.

The \( N_{\text{atom}} = 864 \times 10^{12} \) atoms bcc Fe system including the slider of \( 17.10 \times 17.10 \times 8.55 \) (\( \mu m^{3} \)) and a substrate of \( 51.30 \times 17.10 \times 8.55 \) (\( \mu m^{3} \)) is modeled into the two different CG systems (each of them includes the slider and the substrate), one of \( N_{s} = 1000 \) with \( N_{CG} = 432000 \) (fig. 1(d)) and the other of \( N_{s} = 3000 \) with \( N_{CG} = 16000 \) (not shown). The advantage of choosing these \( N_{s} \) values is because of the statistical meaning given by the difference of the particle number of the two GC systems. The initial distance between the two objects of each CG system is equal to \( a_{CG} \). The lowest two layers of the substrate are fixed during the simulations. The following parameters are used in the simulations: \( \mu = 52.5 \) GPa [18], \( \eta = 1.2 \) and \( h_{cr} = 1.4 \) [19], \( A = B = 0.1 \) and \( \gamma_{\text{dis}} = 10^{3} \) \( 1/s \) [20], \( \epsilon = 0.01, \rho = 7.86 g/cm^{3}, V = 100 \) m/s, a time step \( dt = 285 \) ps and \( K = 0.051 nN/nm \) that was used as a spring constant of the conventional cantilevers in the study of fresh iron particles [21]. We modify the FDPS open source developed by Iwasawa et al. [19] to create our simulation program.

Results and discussions. – Figure 2 shows the sliding-time dependence of the friction force, normal force and friction coefficient (CoF) of the systems in the behavior of a load of 0.1 \( \mu N \) and \( k_{x} = k_{y} = 0.1k_{x} \) with \( k_{x} = 0.2K \). A regular occurrence of the stick-slip motion is clearly observed from the curves. In spite of the sliding at high velocity, the stick-slip motion still appears because the chosen values of the spring constants always guarantee a steady state of the particle layers at the interface and the objects are stably maintained by the damping parameter \( \gamma_{\text{dis}} \). Sang et al. found a stick-slip type of instantaneous friction force in the sliding of a support on a hard substrate [22]. The hard model in our considerations can be seen from a well-repeated periodicity of the saw-tooth shape. We find that the distance between the nearest peaks of each curve is exactly equal to the lattice constant of each system, \( a_{CG} = 0.855 \) \( \mu m \) for the 16000-particles system and \( a_{CG} = 0.285 \) \( \mu m \) for the 432000-particles system. This result is consistent with the observations of the lattice constant of various materials in the following experimental studies of sliding friction. Morita et al. obtained...
a lattice constant of 3.16 Å for MoS$_2$ from measurements of the friction force parallel to a MoS$_2$ surface acting on a Si$_3$N$_4$ tip [23]. The lattice constant of mica was detected by the lateral force image of its (0001) surface [24]. Stick-slip motion with the periodicity of the KF unit cell was seen from the lateral force image of KF (001) cleaved and imaged in UHV with a silicon nitride tip [25]. This scenario was also found for NaCl from measurements of the lateral force of a Si AFM tip sliding forward and backward in the (100) direction over the NaCl (001) surface [26]. The two systems are close to each other in oscillation amplitude and mean value of friction coefficient. This indicates that the CG model can be a reasonable process. In addition, a (mean) friction coefficient of ~0.3 monitored from the two curves (fig. 2) is in accordance with that of Fe as a dry sliding friction coefficient of about 0.15–0.40 under a pressure of 9–45 kg/cm$^2$ [27] or 0.18–0.65 in sliding distance of 0–30 m [28]. These obtained results indicate that this CG model can be used to investigate the sliding friction of the Fe micron-scale system with the above simulation condition.

Figure 3 displays the friction coefficient dependent on the value of $k_z$ in the behaviors with an applied load of 0.1 μN, $k_z = k_y = 0.1k_x$ with $k_x = K$ (fig. 3(a)) and $k_y = 0.1K$ (fig. 3(b)). The friction coefficient increases with decreasing the spring constant $k_z$, around 0.058, 0.3 (the above discussion) and 0.565 corresponding to $k_z = K$, 0.2$K$ and 0.1$K$. The friction force is strongly dependent on $k_z$ as seen in eq. (13) while the normal force is significantly dependent on both $k_z$ and applied load; therefore, the change of $k_z$ will strongly result in friction coefficient of the system. The value of 0.058 is much smaller than that reported in the previous studies [27,28], whereas the value of 0.565 is larger than a static friction coefficient (0.51) of Fe sliding on Fe in dry or unlubricated condition [29]. The limits of these considered cases can also be seen from the strong oscillation (fig. 3(a)) and instability (fig. 3(b)) of the friction coefficient. However, regularity of the stick-slip motion is observed in all the simulations, seeming to be not dependent on the value of the spring constant $k_z$. Figure 4 depicts the effects of isotropy or anisotropy of the spring interaction force $F_{int,ij}$ on the friction coefficient in the behavior with an applied load of 0.1 μN, $k_z = k_y$ and $k_x = 0.2K$. The friction coefficient grows as the anisotropy increases or the ratio $k_z/k_x$ decreases. This result is that by fixing $k_z$ and increasing $k_x$ the friction force almost does not change while the normal force increases. Lemul et al. reported that considering the isotropy of materials in the numerical models would lead to minor differences between experimental and numerical results for the friction coefficient [30]. Our result shows that the friction coefficient presents a small difference between the isotropic and anisotropic behaviors when the anisotropy is low, $k_z/k_x ≥ 0.6$; however, it has an abrupt growth as the anisotropy is high, for example a $k_z/k_x$ drop from 0.2 to 0.1. The friction coefficient reaches to the experimental value [27,28] in the most anisotropic behavior $k_z/k_x = 0.1$. This result indicates that the isotropic or anisotropic consideration of the spring interfacial force strongly influences the friction coefficient of a system and supposes that the anisotropy should be carefully detected during collection of the friction coefficient. In the past, most studies utilized the spring force to monitor the stick-slip configuration of the friction force, whereas there were few studies of friction coefficient by using this force. Maveyraud et al. reported the friction
Fig. 5: Dependence of the friction coefficient of the systems on the externally applied normal load.

coefficient of CG solid rocks by using the spring friction force; however, they did not mention effects of isotropy or anisotropy of this force on the friction coefficient [20]. It is also important to note that there is very good coincidence of change of the friction coefficient of the two systems.

Figure 5 displays a decrease of the friction coefficient with an increase of the externally applied normal load in both the isotropic and anisotropic behaviors of the spring force with the simulation condition $k_x = 0.2K$ and $k_z = k_y$. Experimental studies have reported this state [31–36]. The cause of the decrease was explained by growing the roughening of the interface and forming a large quantity of wear debris [31]. The friction coefficient of the a-C: H coating and highly oriented pyrolytic graphite sample corresponding to the steady state reduces with increasing normal load because of elastoplastic deformation, wear, materials transfer and an increased real area of contact [32]. The slip of local precursors prior to the onset of bulk sliding led to the decrease of the macroscopic static friction coefficient [35]. It is well known that increasing the load can lead to the deformation of the contact. A curve approaching the linear one can be seen in the isotropic behavior $k_z/k_x = 1.0$ or low anisotropy $k_z/k_x = 0.5$. However, a nonlinear dependence of the friction coefficient via the load becomes more explicit in the case $k_z/k_x = 0.1$. We find that the change of the curve, the increment of the friction coefficient goes down with increasing load, is similar to that found in experimental studies of dry or lubricated sliding friction of various materials [32,33,35,36]. The friction coefficient is consistent with that of the experimental reports [27,28] in the range of the considered load in the case $k_z/k_x = 0.1$. The results also show that the two systems are in accordance with each other in the applied load dependence of the friction coefficient.

Conclusions. – This study uses smoothed particle hydrodynamics simulations to investigate the sliding friction of the Fe micron-scale system by the CG model established by us and the spring friction force. A good agreement of the obtained results of the two particle systems demonstrates that the CG model is a reasonable modeling process. The spring constant and the isotropy or anisotropy of the spring force cause strong influences on the monitored friction quantities. In the anisotropic case with $k_z/k_x = 0.1$ and $k_x = 0.2K$, the CG model well presents the sliding friction characters of the Fe micron-scale system including the collection of the friction coefficient and the changes of the friction force and coefficient via the sliding time or the externally applied normal load.

REFERENCES

[1] Kinjo T. and Hyodo S., Phys. Rev. E, 75 (2007) 051109.
[2] Rudd R. E. and Broughton J. Q., Phys. Rev. B, 58 (1998) R5893.
[3] Rudd R. E. and Broughton J. Q., Phys. Rev. B, 72 (2005) 144104.
[4] Kobayashi R., Nakamura T. and Ogata S., Mater. Trans., 49 (2008) 2541.
[5] Yang S. and Qu J., Model. Simul. Mater. Sci. Eng., 22 (2014) 065011.
[6] Stevens M. J., Macromolecules, 34 (2001) 2710.
[7] Xiong L., Tucker G., Mcdowell D. L. and Chen Y., J. Mech. Phys. Solids, 59 (2011) 160.
[8] Kajita S., Washizu H. and Ohmori T., EPL, 87 (2009) 66002.
[9] Kajita S., Washizu H. and Ohmori T., Phys. Rev. B, 82 (2010) 115424.
[10] Kajita S., Washizu H. and Ohmori T., Phys. Rev. B, 86 (2012) 07545.
[11] Washizu H., Kajita S., Toyama M., Ohmori T., Nishino N., Teranishi H. and Suzuki A., Faraday Discuss., 156 (2012) 279.
[12] Chen M. Y., Hong Z. H., Fang T. H., Kang S. H. and Kuo L. M., T. Can. Soc. Mech. Eng., 37 (2013) 927.
[13] Sharma G., Limaye P. K., Ramanujan R. V., Sundararaman M. and Prabhu N., Mater. Sci. Eng. A, 386 (2004) 408.
[14] Taban E., Gould J. E. and Lippold J. C., Mater. Des., 31 (2010) 2305.
[15] Guerra R., Tartaglino U., Vanossi A. and Tosatti E., Nat. Mater., 9 (2010) 634.
[16] Liu Y. L., Grey F. and Zheng Q. S., Sci. Rep., 4 (2014) 4875.
[17] Kajita S., Phys. Rev. B, 94 (2016) 033301.
[18] Rayne J. A., Phys. Rev., 122 (1961) 1714.
[19] Iwasawa M., Tanikawa A., Hosono N., Nitadori K., Muranushi T. and Makino J., Publ. Astron. Soc. Jpn., 68 (2016) 54.
[20] Maveyraud C., Benz W., Sornette A. and Sornette D., J. Geophys. Res., 104 (1999) 28709.
[21] Pensini E., Sleep B. E., Yip C. M. and O’Carroll D., Colloids Surf. A: Physicochem. Eng. Asp., 433 (2013) 104.
[22] Sang Y., Dubé M. and Grant M., Phys. Rev. E, 77 (2008) 036123.
[23] Morita S., Fujisawa S. and Sugawara Y., Surf. Sci. Rep., 23 (1996) 1.
[24] Carpick R. W., Flater E. E., Sridharan K., Ogletree D. F. and Salmeron M., JOM, 56 (2004) 48.
[25] Carpick R. W. and Salmeron M., Chem. Rev., 97 (1997) 1163.
[26] Szlufarska I., Chandross M. and Carpick R. W., J. Phys. D: Appl. Phys., 41 (2008) 123001.
[27] Kuznetsov V. D., Metal Transfer and Build-up in Friction and Cutting (Pergamon Press, London) 1966.
[28] Furlan K. P., Prates P. B., Santos T. A. D., Dias M. V. G., Ferreira H. T., Neto J. B. R. and Klein A. N., J. Alloys Compd., 652 (2015) 450.
[29] Blau P. J., Friction Science and Technology: From Concepts to Applications (CRC Press, New York) 2009.
[30] Lemul H. G. and Trzepieciński T., J. Mech. Eng., 59 (2013) 41.
[31] Bhushan B., Tribology and Mechanics of Magnetic Storage Devices (Springer-Verlag, New York) 1996.
[32] Liu E., Blanpain B., Celis J. P. and Roos J. R., J. Appl. Phys., 84 (1998) 4859.
[33] Chan S. M. T., Neu C. P., Komvopoulos K. and Reddi A. H., J. Biomech., 44 (2011) 1340.
[34] Chowdhury M. A., Nuruzzaman D. M., Mia A. H. and Rahaman M. L., Tribol. Ind., 34 (2012) 18.
[35] Katano Y., Nakano K., Otsuki M. and Matsukawa H., Sci. Rep., 4 (2014) 6324.
[36] Alarcón H., Salez T., Poulard C., Bloch J. F., Raphaël E., Veress K. D. and Restagno F., Phys. Rev. Lett., 116 (2016) 015502.