Precision Determination of $|V_{ub}|$

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The last two years have seen an impressive improvement in the determination of $|V_{ub}|$, especially from inclusive decays. The error on $|V_{ub}|$ measured with inclusive decays was reduced from 18% (PDG 2004) to 8% (PDG 2006). This progress is a result of combined experimental and theoretical efforts. In this talk, the theoretical framework (BLNP) that enabled such progress is reviewed, as well as other approaches to an inclusive determination of $|V_{ub}|$ (DGE, $M_X - q^2$ etc.). The prospects of improving $|V_{ub}|$ are discussed, addressing issues of weak annihilation, implications of leptonic B decays, and determination of $|V_{ub}|$ with exclusive decays.

1. INTRODUCTION

$V_{ub}$, one of the smallest matrix element of the CKM matrix, is one of the fundamental parameters of the Standard Model. In the geometrical picture of the unitarity triangle, the side opposite to the angle $\beta (\equiv \phi_1)$ is proportional to $|V_{ub}/V_{cb}|$. Since this angle is measured to high accuracy by the $B$ factories, and the error on $|V_{cb}|$ is at a level of 2% [1], an accurate measurement of $|V_{ub}|$ is important for constraining the unitarity triangle.

$|V_{ub}|$ can be measured either through exclusive decays, e.g., $\bar{B} \rightarrow \pi l^- \bar{\nu}$, or inclusive decays: $\bar{B} \rightarrow X_u l^- \bar{\nu}$. In the former method we encounter a large theoretical uncertainty as a result of our limited knowledge of the form factors that control the decay. The latter method offers, in principle, the most accurate way to extract $|V_{ub}|$ from the total width $\Gamma (\bar{B} \rightarrow X_u l^- \bar{\nu})$. In practice, since $|V_{cb}| \gg |V_{ub}|$, the total width cannot be measured due to the large charm background from $\bar{B} \rightarrow X_u l^- \bar{\nu}$ decays. In order to eliminate the charm background one is forced to look at regions of phase space where charm cannot be produced. In these regions the theoretical description is more complicated, but thanks to recent progress in our understanding of charmless inclusive $B$ decays, the inclusive measurement of $|V_{ub}|$ gives a smaller error compared to the exclusive one.

The recent global analysis of the unitarity triangle shows a "tension" at the level of 2σ between $\beta (\equiv \phi_1)$ and $|V_{ub}|$ [2] (see the contribution by S. T’Jampens at this conference). This is a result of two effects. One, the central value of $|V_{ub}|$ has increased, especially as measured from inclusive decays, and the error bar decreased from 18% in 2004 [3] to 8% in 2006 [1]. At the same time the value of $\sin 2\beta (\equiv \sin 2\phi_1)$ has decreased. It is therefore important to understand how the value of $|V_{ub}|$ is obtained.

In this talk we will therefore focus on the theory of inclusive measurement of $|V_{ub}|$. For various experimental issues we refer the reader to the contribution by L. Gibbons at this conference, and for exclusive measurement using lattice data we refer to the contribution of C. Davies.

2. KINEMATICS

We begin our discussion by shortly reviewing the kinematics of semileptonic $B$ decays. For a “pedestrian” introduction to semileptonic $B$ decays see chapter 1 of [4].

Any $\bar{B} \rightarrow X_u l^- \bar{\nu}$ event can be described by three kinematical variables. The triple differential decay rate depends, via the optical theorem, on the hadronic tensor $W_{\mu\nu}$ (for derivation see [4]). Given a basis of two four vectors, the hadronic tensor is usually decomposed into the various possible Lorentz structures, where the coefficients of these structures are called structure functions and denoted as $W_i$.

In [5] the use of the $(v, n)$ basis was advocated.
In this basis $v$ is the four velocity of the decaying $B$ meson and $n$ is a light-like vector in the direction of the hadronic jet. In the rest frame of the decaying meson we have $v = (1, 0, 0, 0)$ and we can choose $n$ to be $n = (1, 0, 0, 1)$. One can also define a conjugate light like vector $\bar{n} = (1, 0, 0, -1)$, such that $2v = n + \bar{n}$ (in the following we assume these values for $v, n$ and $\bar{n}$). This choice of vectors motivates the following choice of kinematical variables:

\[ P_1 = M_B - 2E_t, \]
\[ P_- = \bar{n} \cdot P_X = E_X + |\bar{P}_X|, \]
\[ P_+ = n \cdot P_X = E_X - |\bar{P}_X|, \]

where $P_X$ is the four momentum of the hadronic jet and $E_t$ is the energy of the charged lepton. Other kinematical variables can be expressed in terms of this choice of variables. For example the hadronic and leptonic invariant masses are: $M_X^2 = P_+ P_-$, and $q^2 = (M_B - P_-)(M_B - P_+)$, respectively.

In terms of these variables the exact expression for the triple differential decay rate is:

\[
\frac{d^3\Gamma_u}{dP_+ dP_- dP_1} = \frac{G_F^2 |V_{ub}|^2}{16\pi^3} (M_B - P_+) \left[ (P_- - P_1)(M_B - P_+ + P_1 - P_+) W_1 \right. \\
+ (M_B - P_-)(P_- - P_+) \frac{W_2}{2} + \\
\left. (P_- - P_1)(P_1 - P_+) \left( \frac{y}{4} W_3 + \bar{W}_4 + \frac{1}{y} W_5 \right) \right],
\]

(2)

where $y = (P_- - P_1)/(M_B - P_+)$ and $W_i$ are defined in [5] and do not depend on $P_1$. This choice of variables and basis has two main advantages. The first is that the phase space has probably the simplest form possible:

\[
\frac{M_X^2}{P_-} \leq P_+ \leq P_1 \leq P_- \leq M_B.
\]

(3)

The second advantage is that there is no explicit dependence on the mass of the $b$ quark in the expression for the triple rate [2]. This allows for theoretical predictions of partial rates instead of event fractions and eliminates the large source of uncertainty from the value $m_b$. The triple rate depends on $m_b$ only through the $W_i$ functions, which we now discuss.

3. DYNAMICS

The structure functions $\bar{W}_i$ cannot be calculated exactly. Fortunately for heavy meson decays there are two small parameters we can expand $\bar{W}_i$ in: the mass of the $b$ quark (or more exactly $\Lambda_{QCD}/m_b$) and $\alpha_s$.

If we had no charm background we could integrate over $P_\pm$ up to $M_B$ and use a Heavy Quark Effective Theory (HQET) based Operator Product Expansion (OPE) to write $\bar{W}_i$ as:

\[
\bar{W}_i \sim c_0 \langle O_0 \rangle + c_2 \langle O_2 \rangle \frac{m_b^2}{m_c^2} + c_3 \langle O_3 \rangle \frac{m_b^3}{m_c^3} + \cdots.
\]

(4)

(We use a short hand notation in here. In practice there are several operators at each order and the coefficients are generalized functions and not constants). The coefficients $c_i(\mu)$ contain the short distance physics ($\mu \sim m_b$) and are calculable in perturbation theory ($\langle O_1 \rangle = 0$ as a result of HQET equations of motion [6]). Currently $c_0$ [7] is known at $O(\alpha_s)$ while $c_2$ [8] and $c_3$ [9] are known at $O(\alpha_s^2)$. Recently even $c_4$ was calculated at $O(\alpha_s^3)$ [10].

The matrix elements of the local operators $O_i$ between $B$ meson states, $\langle O_i \rangle$, are called the Heavy Quark (HQ) parameters. They contain the long distance physics and must be taken from experiment. We expect them to scale as $\langle O_i \rangle \sim \Lambda_{QCD}^i$. We have $\langle O_0 \rangle = 1; \langle O_2 \rangle$ defines two HQ parameters $\mu_2^2$ and $\nu_2^2 = 3[(M_B)^2 - (M_B)^2]/4; \langle O_3 \rangle$ defines $\rho_{3,B}^3$ and $\rho_{3,D}^3$ etc.

A similar OPE can be constructed for $B \rightarrow X_s l^- \bar{\nu}$ and $\bar{B} \rightarrow X_s \gamma$, which contains the same HQ parameters. For $B \rightarrow X_s l^- \bar{\nu}$ the OPE works very well. As a result the error on $\langle V_{ub} \rangle$ is at the level of 2% [11] and the HQ parameters can be extracted from experiment. For $\bar{B} \rightarrow X_s \gamma$ a local OPE is more problematic for two reasons. The first is that the OPE is valid for photon energies that cannot be attained at the current experiments. The second is that even for such energies when one goes beyond leading order in $1/m_b$ non-local operators arise (see [12] and references
The second moment is related to the shape function, which is related to the HQ parameters. For example, the first moment of the shape function is different. Beyond leading order this is no longer the case. Neglecting $\alpha_s$ corrections, we can define a renormalization scheme, called the “shape function scheme” [13], such that these relations hold at each order in perturbation theory. Currently, these relations are known at order $\alpha_s^2$ for $m_b$ and $\mu^2$ [20]. The high precision of these relations implies that a good knowledge of the HQ parameters helps to constrain the shape functions. There are similar relations between the subleading shape functions and the HQ parameters, although they are known only at order $\alpha_s^0$. Still, such relations help us to model the subleading shape functions.

This concludes our brief review of the dynamics of inclusive $B$ decays. We should emphasize that the above description is not a theoretical model, but a rigorous theory based on QCD and a systematic expansion in $1/m_b$. Any inclusive extraction of $|V_{ub}|$ has to be based on these ingredients.

4. INCLUSIVE EXTRACTION OF $V_{ub} -$ PRESENT

There are currently several theoretical calculations, already implemented by experiments, which combine some or all of the ingredients described in the previous section. We now review each of them briefly (the LLR approach will be discussed in the next section). For a more detailed account see the original papers.

BLNP Approach: The BLNP (Bosch-Lange-Neubert-Paz) approach [5] is a culmination of a research efforts that extends over 12 years. Its “philosophy” is to use all that is currently known about the triple differential decay rate of $B \rightarrow X_u l^- \bar{\nu}$ and $B \rightarrow X_s \gamma$, namely

- At leading order in $1/m_b$: $H_u$, $H_s$, $J$ at $O(\alpha_s)$;
- $1/m_b$ subleading shape functions at $O(\alpha_s^0)$;
- Known $\alpha_s/m_b$ terms from the OPE calculation (the part of $c_0$ in equation (4) that becomes subleading in the SF region);
- Known $1/m_b^2$ terms from the OPE calculation (part of $c_2$ in equation (4)).

The various ingredients are implemented in such a way that there is a smooth transition between the

\[
\frac{d\Gamma}{dE_{\gamma}} \sim H_s \cdot J \otimes S + \frac{1}{m_b} \sum_k h^k_s \cdot j^k_s \otimes s^k_s + \cdots . (6)
\]

Notice that the leading order jet and shape function are the same as $B \rightarrow X_u l^- \bar{\nu}$, while the hard function is different. Beyond leading order this is no longer the case.

What is the relation between the two regions? Neglecting $\alpha_s$ corrections, we can define a renormalization scheme, called the “shape function scheme” [13], such that these relations hold at each order in perturbation theory. Currently, these relations are known at order $\alpha_s^2$ for $m_b$ and $\mu^2$ [20]. The high precision of these relations implies that a good knowledge of the HQ parameters helps to constrain the shape functions. There are similar relations between the subleading shape functions and the HQ parameters, although they are known only at order $\alpha_s^0$. Still, such relations help us to model the subleading shape functions.

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SF and OPE regions, in the sense that once the kinematical variables are integrated far enough the OPE result is recovered.

Non-perturbative physics effects are contained in the leading and subleading order shape functions. BLNP uses the fact that the leading order shape function is universal, so the formalism allows the extraction of the leading order shape function from the photon spectrum in $\bar{B} \to X_s \gamma$ and its use as an input for $\bar{B} \to X_u l^- \bar{\nu}$ (beyond leading order a slight redefinition of of $S$ is needed, see [5]). The subleading shape functions are modeled in accordance with their moment constraints.

The complete error analysis consists of the following:

- The leading order shape function is taken from experiment as explained above, substantially reducing the associated error.
- The perturbative error is estimated by varying the scale of $\alpha_s$ in the different terms.
- The subleading shape function error is estimated as follows. At order $\alpha_s^3$, there are 3 subleading functions and 9 models are constructed for each of them. The $9^3 = 729$ possible combinations are then scanned to estimate the resulting error. The models for the subleading shape functions are constructed such that they respect the moment constraints (the zeroth moment vanishes, the first moment is related to $\mu_x^2$ and $\mu_y^2$, and the second moment is of order $\Lambda_Q^2$). And that the dimensionless shape functions are $O(1)$ in a region where their arguments are of order $\Lambda_Q$.

- The weak annihilation error (see below) is taken as a fixed percentage of the total rate.

The HFAG average for ICHEP 2006 [21] gives the value $|V_{ub}| = (5.02 \pm 0.26 \pm 0.37) \times 10^{-3}$ using the BLNP approach, where the first error is from experiment and the second from theory. The non experimental errors associated with this result are: 4.2% HQ error (for the leading order shape function), 3.8% for the combined perturbative and subleading shape function error and 1.9% for weak annihilation.

**BLL Approach:** The BLL (Bauer-Ligeti-Luke) approach [22] is based on the fact that if a low $q^2$ cut is imposed to eliminate charm background, an OPE expansion for the partial rate can be constructed, which is suppressed by inverse powers of $m_c$ instead of $m_b$. (This is possible since for such a cut $P_+ \sim P_- \sim m_c$). In order to optimize both the efficiency and the theoretical uncertainty, the authors of [22] suggested to use a combined $M_X - q^2$ cut. The main ingredients of the BLL approach are:

- The OPE is assumed to be valid for the combined cut.
- The LO shape function sensitivity is estimated by convoluting the tree-level decay rate with the difference between the “tree-level” shape function model and a delta function model for the shape function.
- The subleading shape function contribution is assumed to be small and is not assessed.

The HFAG average for ICHEP 2006 [21] gives the value $|V_{ub}| = (5.02 \pm 0.26 \pm 0.37) \times 10^{-3}$ using the BLL approach, where the first error is from experiment and the second from theory. The theory error includes a 3% contribution from shape function sensitivity.

**DGE Approach:** The Dressed Gluon Expansion (DGE) approach is advocated by Andersen and Gardi [23]. Similar to the BLNP approach it can be applied to various experimental cuts, but conceptually it is different from the other approaches discussed in this talk. For a less technical review of the DGE approach we refer the reader to [24]. Here we only mention the following features of this approach:

- The decay spectra is approximated by the resummed on-shell $b$-quark decay spectrum, for which the only input parameters are $m_b$ and $\alpha_s$.
- Non-perturbative effects associated with the meson structure are estimated using renormalon analysis.
The HFAG average for ICHEP 2006 [21] gives the value $|V_{ub}| = (4.46 \pm 0.20 \pm 0.20) \cdot 10^{-3}$ using the DGE approach, where the first error is from experiment and the second from theory.

**Discussion:** Let us compare the various approaches. As might be expected, the main difference between them is the estimate of non perturbative effects and power corrections, beginning with the leading order shape function.

In the BLNP approach the leading order shape function is to be extracted from experiment. In the current experimental implementation the full shape of the photon spectrum is not used and only the first two moments of the shape function are used to constrain its form. One might argue that the possible variations of the photon spectrum are already included in the range of the HQ parameters, but a verification of this assumption would be desirable. In the DGE approach there is no error associated with this issue in the BLL approach is in order. In the DGE approach there is no error from subleading shape functions. Again, it is unclear how the parameters $C$ and $f_{PV}$ in the DGE approach are related to the subleading shape function error in the other approaches.

Despite all of these differences, the inclusive measurement are all consistent with each other, even if we assume that the error bars are underestimated. We have no good explanation for this fact.

5. **IMPROVED $V_{ub}$**

The extraction of $|V_{ub}|$, impressive as it is, can be further improved. We separate the discussion of improvements that can be implemented today, using the currently available theoretical tools, and future feasible improvements.

**Improved $V_{ub}$ - Today:** In the former class we have the treatment of weak annihilation (WA) and the weight function approach.

Weak annihilation appears at order $1/m_b^2$ in the OPE. It arises from flavor specific four quark operators of the form $b \Gamma u \bar{u} \Gamma b$, and effects neutral and charged $B’s$ differently [26]. Currently there are only estimates of its magnitude [27]. For example a CLEO analysis finds the limit $\Gamma_{WA}/\Gamma_{b \rightarrow u} < 7.4\%$ at the 90% confidence level [28]. Apart from estimating the WA as a fixed percentage of the rate, a different strategy was suggested by Lange, Neubert and Paz [5]. Since this conference took place in Oxford UK, we can follow the Queen of Hearts and summarize this strategy as: “Off with its head!” [29]. More concretely, since weak annihilation is concentrated in the region of $q^2 = m_b^2$, if we cut on high $q^2 < q_{max}^2$ (e.g. $q_{max}^2 = (M_B - M_D)^2$) combined with a $M_X$ or $P_+$ cut to eliminate charm background, we would eliminate the WA error. With such a cut one loses efficiency but this might be compensated by the elimination of the WA error. Preliminary studies in [5] showed that this is indeed the case. This method is still waiting for experimental implementation.

The weight function idea is to directly relate the photon spectrum of $B \rightarrow X_s \gamma$ to $B \rightarrow X_u l^- \bar{\nu}$ spectra without a need to extract the leading order shape function, and was first suggested in [30]. The theoretical input in this approach is the weight function, which at leading order can...
be calculated in perturbation theory:

\[ W \sim \frac{\Gamma_n}{\Gamma_s} \sim \frac{H_n \cdot J[m_0(P_+ - \bar{\omega})]}{H_s \cdot J[m_0(P_+ - \bar{\omega})]} \otimes S(\bar{\omega}) + ... \] (7)

In this relation the leading order shape function cancels in the ratio. Beyond leading order this is no longer the case. BaBar has used a calculation of a weight function for the \( M_X \) spectrum by Leibovich, Low, and Rothstein (LLR) \cite{Leibovich} to measure \(|V_{ub}|\). The result is \(|V_{ub}| = (4.43 \pm 0.45 \pm 0.29) \times 10^{-3}\) \cite{Ball}, where the first error is from experiment and the second from theory.

A more recent theoretical calculation was performed by Lange, Neubert, and Paz \cite{Lange}. In this approach a weight function that relates the normalized photon spectrum in \( B \to X_u \gamma \) to the \( P_+ \) in \( B \to X_u l^- \bar{\nu} \) is constructed. The main reason to use the normalized spectrum is the better perturbative convergence of the weight function. This weight function contains two loop corrections from the ratios of jet functions, subleading shape function corrections and the known \( \alpha_s/m_b \) corrections. The combined theoretical error is at a level of 5%. The approach was generalized by Lange \cite{Lange2} and weight functions were calculated for an arbitrary \( B \to X_u l^- \bar{\nu} \) spectra. These weight functions have the potential to give the best extraction of \(|V_{ub}|\). This method is still waiting for experimental implementation.

**Improved \( V_{ub} \) - Future:** Apart from these already available calculations we can construct a wish list for feasible theoretical calculations. First, a complete subleading shape function analysis for \( B \to X_s \gamma \) is underway \cite{Hurth} and preliminary results were already reported in \cite{Hurth2}. With the approaching completion of the full order \( \alpha_s^2 \) corrections to \( C_0 \) for \( B \to X_s \gamma \) \cite{Hurth} (see also the contribution by T. Hurth at this conference) the calculation of \( H_s \) at \( \mathcal{O}(\alpha_s^2) \) is nearly done \cite{Hurth2}. In order to construct a full two loop weight function a two loop expression for \( H_s \) is needed which is not easy but feasible. The next source of uncertainty is the \( \alpha_s \) corrections for the terms containing the subleading shape functions. Such a calculation is more complicated and connected to the calculation of \( c_2 \) in equation (4) to order \( \alpha_s \), where the latter would serve as a check to the former. Finally it is not completely clear that the subleading shape functions cannot be extracted from data. With the completion of this wish list we would probably reach the boundary of the theoretical accuracy in extracting \(|V_{ub}|\).

### 6. EXCLUSIVE \(|V_{ub}|\) AND LESSONS FROM LEPTONIC \( B \) DECAYS

\(|V_{ub}|\) can also be extracted from exclusive decays such as \( B \to \pi \bar{\nu} \) (see the contribution by L. Gibbons at this conference). In order to do so there is a need for a theoretical input about the form factor \( f_+(q^2) \). Currently there are two different theoretical approaches. The first being lattice QCD calculations, valid for \( q^2 > 16 \) GeV, and the second light cone sum rules valid for \( q^2 < 16 \) GeV. For the first approach HFAG \cite{HFAG} cites three sources: an unquenched collaboration by HPQCD collaboration with \(|V_{ub}| = (3.93 \pm 0.26^{+0.59}_{-0.41}) \times 10^{-3}\) \cite{HPQCD}, an unquenched collaboration by the FNAL collaboration with \(|V_{ub}| = (3.51 \pm 0.23^{+0.61}_{-0.4}) \times 10^{-3}\) \cite{FNAL}, and a quenched calculation which can be found in \cite{HFAG}. For the second approach there is only a single source: a calculation by Ball and Zwicky \cite{Ball2} that gives \(|V_{ub}| = (3.38 \pm 0.12^{+0.36}_{-0.37}) \times 10^{-3}\). As can be seen from the above list, the exclusive predictions are typically smaller than the inclusive ones, which was also “historically” the case \cite{Inclusive}.

Recently the Belle collaboration reported on evidence for the (pure) leptonic \( B \to \tau \bar{\nu} \) decay \cite{Belle}. The branching fraction of such a decay depends on the product of \(|V_{ub}|\) and \( f_B \), the \( B \) meson decay constant. The product was measured to be \( f_B |V_{ub}| = (10.1^{+1.6}_{-1.4}(\text{stat})^{+1.3}_{-1.4}(\text{syst}) ) \times 10^{-4} \) GeV. Using the inclusive value \(|V_{ub}| = (4.39 \pm 0.33) \times 10^{-3}\) the decay constant can be extracted: \( f_B = 0.229^{+0.036}_{-0.03}(\text{stat})^{+0.034}_{-0.037}(\text{syst}) \). This value is in good agreement with the unquenched lattice value: \( f_B = 0.216 \pm 0.022 \) GeV \cite{Lattice}, or the QCD sum rule calculations \( f_B = 0.210 \pm 0.019 \) GeV \cite{Sumrule} and \( f_B = 0.206 \pm 0.020 \) GeV \cite{Sumrule2}, supporting the value of the inclusive \(|V_{ub}|\).
7. CONCLUSIONS

The last two years have seen an impressive improvement in the determination of $|V_{ub}|$, especially from inclusive decays. This is a result of combined experimental and theoretical hard work. In this talk we have reviewed some of the theoretical work. Further improvement is also possible. As we have pointed out, there are theoretical tools that still await experimental implementation, namely a cut on high $q^2$ to eliminate weak annihilation, and advanced two loop relations between the photon spectrum in $\bar{B} \to X_s \gamma$ and $\bar{B} \to X_u l^- \bar{\nu}$ spectra. Beyond these, more theoretical improvement is also feasible. The time has now come for a critical comparison of the theoretical approaches to inclusive $|V_{ub}|$, namely, comparing the underlying assumptions, the perturbative and non perturbative corrections. This is especially important if we want to take seriously the $2\sigma$ “tension” between $\sin 2\beta (\equiv \sin 2\phi_1)$ and inclusive $|V_{ub}|$

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