Study on permeability coefficient of saturated cohesive soil based on fractal theory

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Abstract: Because of its extremely low permeability, cohesive soil is often regarded as a very important impermeable material in geotechnical projects. Therefore, further study of the mechanism of cohesive soil infiltration has a very far-reaching significance in the field of civil engineering. In this paper, with the concept of effective void ratio, the traditional formula of soil permeability coefficient is modified. Based on the fractal dimension of fractal theory, the function relationship between void ratio and fractal dimension was established. Based on this function, the fractal dimension was used to describe the effective void ratio of saturated clay, and the fractal theory is used to correct the empirical formula of permeability coefficient of saturated cohesive soil. The formula can be used to study the permeability of saturated cohesive soil and improve the theory of percolation and the solution of permeability coefficient.

1. Introduction
Soil is a porous, loose three-phase medium consisting of soil particles and water and air between pores. According to the different particle sizes of the soil particles, the soil can be divided into coarse-grained soil and cohesive soil. The coarse-grained soil particles have a large particle size, and the particles are in direct contact with each other and the adhesion is weak. The cohesive soil is composed of soil particles with extremely small particle size, and there is an electrostatic field force between the soil particles, so that a layer of combined water film exists on the surface of the soil particles of the cohesive soil. In general, the permeability coefficient of cohesive soil is smaller than that of coarse-grained soil. Many people mistakenly believe that the reason is that the cohesive soil pore ratio is much smaller than that of coarse-grained soil. In fact, the relative density of soil particles of cohesive soil and coarse-grained soil is approximately equal. For the traditional empirical formula of permeability coefficient, the permeability coefficient increases with the increase of void ratio, but the pore ratio of clay soil is approximately equal to the pore ratio of coarse-grained soil, and its permeability coefficient is much smaller than that of coarse-grained soil. Wang Xiuyan and others[1-5] have done a lot of experiments to study the permeability characteristics of clay soil, but the phenomenon that the results measured by different methods differ several times or even tens of times is very common. L.N. Reddi and S. Thangavadi et al.[6] used stochastic networks and percolation theory to calculate the permeability coefficient of clayey soils; H. Komine et al.[7,8] got the formula for calculating the permeability coefficient of expansive soils according to Poiseuille's law; Liang Jianwei and Fang Yingguang[9] discussed the seepage characteristics of very fine granular Clays Based on the constant head test and the micro-electric field effect. The expression of equivalent permeability coefficient is derived. The above methods of calculating permeability coefficients are closely related to
the theory and experiment, but there is also a problem that the required parameters can not be easily obtained, so the practicability needs to be improved. It is often different from actual results to estimate permeability coefficient of cohesive soil by empirical formula of permeability coefficient of coarse grained soil.

Therefore, it is meaningful to study the influence of invalid pores on soil permeability from the microscopic point of view and to modify the traditional empirical formula of soil permeability coefficient. Based on the concept of effective void ratio\(^{[10]}\), the five traditional empirical formulas of permeability coefficient are modified, and the empirical formula of permeability coefficient suitable for saturated clay is obtained. The empirical formula of permeability coefficient suitable for saturated clay is obtained. According to the pore number-size fractal model\(^{[11,12]}\), the functional relationship between porosity and fractal dimension is derived. Based on the initial slope method, the numerical relationship between the effective void ratio and the fractal dimension of saturated clay is derived. An empirical formula for the permeability coefficient of saturated clay is based on fractal dimension.

2. Correct the traditional penetration coefficient empirical formula

Natural cohesive soil is a three-phase system, including soil particles and water and air present in the pores, and the pore water includes free water and combined water. For cohesive soils, the internal binding water is highly viscous, has little fluidity and is not affected by its own gravity. Although the bound water in the pores of the clay soil occupies a considerable part of the pore volume, it cannot produce or transmit the pore water pressure, nor can it produce seepage.

As shown in Figure 1, the ratio of the volume of the bound water in the soil to the volume of the soil particles can be defined as the void ratio \(e_0\), and the ratio of the residual volume to the volume of the soil in the soil except the bound water is defined as effective. The void ratio is \(e_u\). It can be seen that the relationship between the two and the total pore ratio of the soil is \(e = e_0 + e_u\). For the saturated clay soil studied in this paper, there is no gas phase in the internal pores. Therefore, the effective pore ratio can be used to replace the total pore ratio in the empirical formula of the traditional permeability coefficient, and the empirical formula of the traditional permeability coefficient can be corrected to satisfy both the coarse-grained soil and the viscous soil. Empirical formula for the permeability coefficient of soil.

\[
k = 2e^2d_{10}^2
\]  

In the formula, \(d_{10}\) indicates the particle size reached by the cumulative percentage in the soil, that
is, the effective particle size; \( e \) indicates the total porosity of the soil; \( k \) indicates the soil permeability coefficient.

The formula of the permeability coefficient of Terzaghi is corrected by the concept of effective void ratio, and the empirical formula of the permeability coefficient of saturated clay is obtained, namely

\[
k^t = 2e_u^2d_{10}^2
\]  

(2)

For the empirical formula of the permeability coefficient based on the Darcy theorem\[14\], the permeability coefficient can be expressed as

\[
k = \frac{\beta \gamma_{wz}}{\lambda \eta} \cdot \frac{e^2}{1 + e} \cdot d^2
\]  

(3)

In the formula, \( \beta \) indicates the spherical coefficient of the soil particles in the soil, assuming that the soil particles are regular spheres, \( \beta = \pi / 6 \); \( \gamma_{wz} \) indicates the free water gravity in the pores; \( \eta \) indicates the free hydrodynamic viscosity coefficient in the pores; \( \lambda \) indicates the influence coefficient of the adjacent soil particles, for saturation Soil, assuming that the soil particles are regular spheres, \( \lambda = 3\pi \); \( d \) indicates the particle size of the soil particles in the soil.

The permeability coefficient formula is modified by the effective void ratio concept to obtain the empirical formula of the saturated cohesive soil permeability coefficient, namely

\[
k^t = \frac{\beta \gamma_{wz}}{\lambda \eta} \cdot \frac{e_u^2}{1 + e_u} \cdot d^2
\]  

(4)

For Stokes's empirical equation for pore flow in coarse-grained soils\[15\], the permeability coefficient can be expressed as

\[
k = \frac{\gamma_{wz}R^2}{8\eta} \cdot \varphi = \frac{\gamma_{wz}R^2}{8\eta} \cdot \frac{e}{1 + e}
\]  

(5)

In the formula, \( R \) represents the capillary radius in the pores of the soil, \( \varphi \) indicates the porosity of the soil.

The Stokes pore permeability coefficient formula is modified by the effective void ratio concept to obtain the empirical formula of the saturated cohesive soil permeability coefficient, namely

\[
k^t = \frac{\gamma_{wz}R^2}{8\eta} \cdot \frac{e_u}{1 + e_u}
\]  

(6)

The empirical formula for the permeability coefficient derived from the consolidation formula\[16\], namely

\[
k = C_v m_v \gamma_{wz} = C_v \gamma_{wz} \cdot \frac{a_v}{1 + e}
\]  

(7)

In the formula, \( m_v \) represents the volume compression coefficient of the soil; \( C_v \) represents the consolidation coefficient of the soil; \( a_v \) represents the compression coefficient of the soil.

The empirical formula of the permeability coefficient based on the consolidation degree formula is modified by the concept of effective void ratio to obtain the empirical formula of the permeability coefficient of saturated clay soil, namely

\[
k^t = C_v \gamma_{wz} \cdot \frac{a_v}{1 + e_u}
\]  

(8)

For the Carson-Carmen permeability coefficient empirical formula\[17\], namely

\[
k = \frac{c_2 \rho \omega e^3}{s^3\eta(1 + e)}
\]  

(9)

In the formula, the coefficient \( c_2 \) related to the shape of the soil particles in the soil and the direction.
of pore water flow is usually 0.125; $s$ indicates the specific surface area of the soil particles.

The empirical formula of Carson-Carmen permeability coefficient is modified by the effective void ratio concept to obtain the empirical formula of the permeability coefficient of saturated clay soil, namely

$$k' = \frac{c_2 \rho_w e_i^3}{s^2 \eta (1 + e_o)}$$  \hspace{1cm} (10)

The above various empirical formulas for the permeability coefficient of saturated clay based on effective void ratio have important significance for understanding the permeability mechanism of cohesive soil. However, in order to make the above formula have practical application value, it is very important to derive the calculation method of effective void ratio. Based on the fractal theory, the numerical relationship between soil void ratio and fractal dimension is established. Then, the effective slope ratio solution method of the above various clay soil permeability coefficient formulas is deduced and elaborated by the initial slope method.

3. Calculation formula of void ratio based on fractal theory

The pore-solid fractal model was formed in the 1990s. As early as 1989, Neimark developed a self-similar multi-scale percolation system for characterizing disordered discrete porous media with a pore-solid phase interface as a fractal. In 1994, based on the number-size distribution of fractal pores and solids, Perrier\textsuperscript{[18]} independently proposed a multi-scale model of soil structure. Based on the definition given by Neimark, Perrier et al\textsuperscript{[19]} further systematically established a pore-solid fractal model. The model can be used to characterize porous media with surface fractals, fractal pore size distribution, fractal solid mass, and fractal pore quality characteristics\textsuperscript{[20,21]}. Because the geotechnical material is microscopically a porous medium composed of soil particles of different sizes, the construction process of such materials is very similar to the fractal. Therefore, the porosity of geotechnical materials can be quantitatively analyzed based on the fractal model.

3.1 Calculation of fractal dimension

Based on the definition given by the pore-solid fractal model, in a self-similar system, there is a power-law relationship between the cumulative number of measurements of the fractal object and the cell size\textsuperscript{[22]}:

$$N(r_i) = k (r_i)^{-D} \quad (i = 0, 1, 2, \ldots, \infty)$$  \hspace{1cm} (11)

Then the fractal dimension is

$$D = \frac{\lg \left( \frac{N(r_i)}{k} \right)}{\lg \left( \frac{1}{r_i} \right)}$$  \hspace{1cm} (12)

In the formula, $N(r_i)$ is the number of units with a length of $r_i$, $k$ is the initial number of units per unit length, $i$ is the number of iterations and $D$ is the fractal dimension. The above formula is a generalized number-size distribution of pores and solids based on fractal theory, which is independent of the initial element dimension and construction method in the self-similar system.

3.2 Calculation of void ratio

In 1992, Tyler and Wheatcraft pointed out that there may be both pore number-size and solid number-size fractal power law relationships in soil materials\textsuperscript{[23]}, but there is no theory explaining this phenomenon at the time. The pore-solid fractal model proposed by Perrier et al\textsuperscript{[4]}, provides a clear theoretical model. Based on this model, Zheng et al\textsuperscript{[24]} fractalized the pore structure of porous
materials. Based on the construction process of the pore-solid fractal model, the relationship between the porosity of the soil material and the fractal dimension is derived as follows:

Assuming that the initial element is a cube whose length is \( R \), in the first iteration, the initial element is equally divided into \( n \) equal parts, and \( n^3 \) small cubes with side lengths of \( R/n \) can be obtained. Randomly remove the \( m \) small cubes, then the number of remaining cubes is \((n^3-m)\). According to this method, iteratively iterates, after iteration \( i \), you can get \((n^3-m)^i\) small cubes with side length \( R_i = R/n^i \). Then the remaining solid volume in the sample can be expressed as

\[
V_s = \left( n^3 - m \right)^i \left( R / n^i \right)^3
\]  

(13)

Then the pore volume can be expressed as

\[
V_p = R^3 - \left( n^3 - m \right)^i \left( R / n^i \right)^3 = R^3 \left[ 1 - \left( \frac{n^3 - m}{n^3} \right)^i \right]
\]  

(14)

Then the porosity can be expressed as

\[
\varphi = \frac{V_p}{V} = 1 - \left( \frac{n^3 - m}{n^3} \right)^i
\]  

(15)

Assuming that the initial element length is \( R=1 \), the cube fractal dimension can be derived from equation (12)

\[
D = \frac{\lg(n^3 - m)}{\lg(n)}
\]  

(16)

In the formula, the intuitive meaning of \( n \) is the ratio of the sample side length to the highest order pore average diameter in the length direction of one side of the initial meta-sample, that is, the measurement scale of the sample; and \( m \) indicates the number of the highest order pores [18]. It can be seen that the fractal dimension \( D \) can reflect the relationship between the size of the pores in the sample and its distribution space.

It can be obtained from equation (16), \( m=n^3-n^D \), substituting equation (15) for the mathematical relationship between porosity and fractal dimension

\[
\varphi = 1 - \left( n^{D-3} \right)^i
\]  

(17)

Take the cube initial element length \( R=1 \), \( i = \frac{\lg(1/r)}{\lg(n)} \), \( r \) represents the length of the small cube after the \( i \)th iteration, which can characterize the measurement scale of the fractal structure. Porosity can be expressed as

\[
\varphi = 1 - \left( n^{D-3} \right)^{\frac{\lg(1/r)}{\lg(n)}}
\]  

(18)

The numerical relationship between void ratio and porosity is

\[
e = \frac{\varphi}{1 - \varphi}
\]  

(19)

Substituting equation (18) into equation (19)

\[
e = \frac{1}{\left( n^{D-3} \right)^{\frac{\lg(1/r)}{\lg(n)}} - 1}
\]  

(20)

It can be seen from equation (20) that the pore ratio of the soil can be obtained by the fractal dimension when the measurement scale is constant. Based on this formula, the fractal dimension can
be used to quantitatively describe the effective void ratio of cohesive soil.

4. Calculation of Effective Pore Ratio of Saturated Cohesive Soil

In this paper, based on the initial slope method, the calculation formula of effective void ratio of saturated clay based on fractal dimension is established by the relationship between fractal dimension and void ratio. For saturated sand, the seepage velocity of pore water in the soil is proportional to the hydraulic gradient, namely \( v = k \cdot i \). For saturated clay, due to the existence of a layer of water film around the soil particles, it is considered that there is an initial hydraulic gradient \( i_0 \) for the saturated clay, and the pore water needs to overcome the initial hydraulic gradient to seepage. As shown in Figure 2, for saturated clay, when the hydraulic gradient \( i < i_0 \), the combined water film in the pores occupies the seepage pores between the cohesive soil particles, which greatly restricts the free water between the pores. Flow, so the seepage in the soil does not meet Darcy's law. When the hydraulic gradient gradually increases to \( i = i_0' \), under the action of pore water pressure, the weakly bound water in the combined water is converted into free water in the pores and flows, and the seepage begins. When the hydraulic gradient is \( i \geq i_0 \), the seepage velocity \( v \) and the hydraulic gradient \( i \) in the saturated clay are in accordance with Darcy's law, which is expressed as\(^{[10]} \)

\[
v = k \left( i - i_0 \right)
\]  

The seepage in the soil tends to be stable.

For the convenience of calculation, the initial hydraulic gradient is assumed to be \( i_0 \), and the part occupied by the water film in the pores of the saturated clay is invalid. At this time, the modified Darcy law formula of the saturated clay can be expressed as

\[
v = k^* i
\]  

In the formula, \( k^* \) represents the effective permeability coefficient of saturated clay, and the simultaneous equation (21) and equation (22) formulas are available.

\[
k^* = k \left( 1 - i_0 / i \right)
\]  

It can be seen from equation (23) that the effective permeability coefficient \( k^* \) of saturated clay varies with the hydraulic gradient \( i \), and the numerical relationship between the effective void ratio \( e_u \) and the hydraulic gradient can be established by the effective permeability coefficient of saturated clay.

For the theory of soil infiltration in the base of Terzaghi, the formulas equation (1) and equation (2) are substituted into the equation (23).

\[
2e_u^2 d_{10}^2 = 2e^2 d_{10}^2 \left( 1 - i_0 / i \right)
\]  

Substituting equation (20) into equation (24) and simplifying to obtain effective void ratio based on fractal dimension.
For the empirical formula of the permeability coefficient based on Darcy's law, substituting equation (3) and equation (4) into equation (23),

\[
e_u = \left[ \frac{1}{n^{D-3}} \right] \cdot (1 - i_b / i)^{1/2}
\]  

(25)

Simplify the effective void ratio

\[
e_u = \frac{e^2 (1 - i_b / i)}{2(1 + e)} + \frac{(1 - i_b / i)^2}{4(1 + e)^2}
\]  

(26)

In the formula, \( e = \frac{1}{n^{D-3}} - 1 \), equation (27) is the effective void ratio based on the fractal dimension.

For the empirical formula of the permeability coefficient based on Stokes, substituting equation (5) and equation (6) into equation (23),

\[
\frac{\gamma_u R^2 e_u}{8 \eta (1 + e_u)} = \frac{\gamma_u R^2 e}{8 \eta (1 + e)} (1 - i_b / i)
\]  

(28)

Substituting equation (20) into equation (28) and simplifying to obtain effective void ratio based on fractal dimension

\[
e_u = \frac{1}{1 - (1 - i_b / i) \left[ \frac{1}{n^{D-3}} \right] - 1}
\]  

(29)

For the empirical formula of the permeability coefficient derived by the degree of consolidation, substituting equation (7) and equation (8) into equation (23) can be obtained

\[
C_v \frac{d_v}{1 + e_u} = C_v \frac{d}{1 + e} (1 - i_b / i)
\]  

(30)

Substituting equation (20) into equation (30) and simplifying to obtain effective void ratio based on fractal dimension

\[
e_u = \frac{1}{\left[ \frac{1}{n^{D-3}} \right] - 1} - 1
\]  

(31)

For the Carson-Carmen permeability coefficient empirical formula, substituting equation (9) and equation (10) into equation (23)

\[
\frac{c_2 \rho u e_u^3}{s^2 \eta (1 + e_u)} = \frac{c_2 \rho e^3}{s^2 \eta (1 + e)} (1 - i_b / i)
\]  

(32)

Simplified effective void ratio

\[
e_u = \frac{e^2 (1 - i_b / i)}{2(1 + e)} + \frac{e^2 (1 - i_b / i)^2}{4(1 + e)^2} - \frac{e^2 (1 - i_b / i)^3}{27(1 + e)^2} + \frac{e^2 (1 - i_b / i)^2}{2(1 + e)} - \frac{e^2 (1 - i_b / i)^3}{4(1 + e)^2} - \frac{e^2 (1 - i_b / i)^3}{27(1 + e)^2}
\]  

(33)
In the formula, $e = \frac{1}{\left( n^{D-3} \right) \log(\frac{r}{r_0})} - 1$, equation (33) is the effective void ratio based on the fractal dimension.

The above five calculation methods establish the numerical relationship between the effective void ratio of saturated clay and the fractal dimension and the initial hydraulic gradient, and realize the correction of the empirical formula of the permeability coefficient of saturated clay by fractal dimension. The initial hydraulic gradient of cohesive soil can be determined by percolation test. Therefore, when calculating the effective void ratio of saturated clay, the initial hydraulic gradient is constant, and the fractal dimension is established by the concept of effective void ratio. Saturated clay soil permeability coefficient function.

5. Numerical analysis of seepage flow
In this paper, the correction formula of the permeability coefficient of the Terzaghi is taken as an example. A silty clay dam is selected as a numerical example to verify the practicability and accuracy of the modified effective permeability coefficient function of saturated clay. In this example, saturated soil seepage parameters are used to iteratively calculate the saturation line. According to the previous calculation results, the location of the saturation overflow point is constantly modified until the accuracy requirements are met.

Figure 3 is a two-dimensional earth dam geometric model in numerical examples. In this example, the top width of earth dam is 4M, the ratio of upstream to downstream slope is 1:2, the total height is 12M, and the bottom width is 52M. The upstream water depth is 8M and there is no water downstream. The density of saturated silty clay is 2.3KN/m³, elastic modulus is 1500Mpa, Poisson's ratio is 0.35, internal friction angle is 34°, and cohesion is 33Kpa. In the example, the permeability coefficient of earth dam is set as a function related to the fractal dimension of silty clay. Assuming the dam is homogeneous silty clay, the permeability coefficient function of saturated clay in the example can be expressed as follows

$$k = 2 \left[ \frac{1}{\left( n^{D-3} \right) \log(\frac{r}{r_0})} - 1 \right] \cdot \left(1 - \frac{i_o}{i} \right) d_1^2$$

(34)

In the formula, the relevant parameters $n$ and $D$ of silty clay can be obtained from the fitting results of soil water characteristic curve data[26].

Figure 4 is a pressure head cloud map of earth dam under saturated soil condition in this paper. For different geological conditions, only the fractal dimension of soil and other related parameters are substituted into ANSYS command flow to analyze the seepage in the soil. Based on the fractal theory and the concept of effective porosity, the effective permeability coefficient function of saturated clay is
deduced in this paper. Compared with the traditional formula of permeability coefficient, it can more accurately reflect the seepage of clay dam in saturated state. The permeability coefficient function derived from fractal dimension can also be used to study the seepage of pore water in different geological conditions and conditions through the corresponding fractal dimension of different types of soil, which has important reference value for practical engineering.

6. Conclusion
Based on the fractal theory, the micro-permeability and permeability coefficient of saturated clay are studied in this paper. The following conclusions are obtained:

1. Based on the related parameters of fractal theory, the functional relationship between fractal dimension and pore ratio of clayey soil is derived in this paper.
2. Based on the deduced pore ratio function of clayey soil, the formula for calculating the effective pore ratio of clayey soil is deduced through the hydraulic gradient and other related parameters in the initial slope method.
3. Based on the formula of effective void ratio of clayey soil, five traditional empirical formulas of permeability coefficient of coarse-grained soil are modified, and the empirical formulas of permeability coefficient of saturated clayey soil considering the influence of combined water film are obtained. When the invalid void ratio equals 0, the formula becomes the empirical formula of the permeability coefficient of coarse-grained soil, which realizes the unification of the empirical formula of the permeability coefficient of coarse-grained soil and clayey soil.
4. A silty clay dam is selected, and the permeability coefficient function derived in this paper is substituted into ANSYS command flow, and the seepage numerical simulation is carried out under its saturated state to verify the practicability of the empirical formula of clay permeability coefficient.

The formula obtained in this paper can be used to study the permeability problem of cohesive soil from the microscopic point of view, and to link the macroscopic mechanical parameters of saturated clay soil with the microscopic parameters such as water content in the pore. The microscopic mechanism and control factors affecting the permeability of saturated clay can be explored, and the permeability theory of cohesive soil and the solution method of permeability coefficient are improved and enriched.

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