Recent results on short-range gravity experiment

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Abstract. According to the ADD model [1], deviation from Newton's inverse square law is expected at below sub-millimeter scale. Present study is an experimental investigation of the Newton's gravitational law at a short range scale. We have developed an experimental setup using torsion balance bar, and succeeded to confirm the inverse square law at a centimeter scale. In addition, composition dependence of gravitational constant $G$ is also tested at the centimeter scale, motivated to test the weak equivalence principle.

1. Physics Motivation

1.1. Test of Inverse Square Law

Present study is to test the Newton’s inverse square law at a short range scale, motivated on one of the most fundamental questions in the modern physics, that why gravitational force is so weak comparing to other three gauge interactions. Based on the ADD model [1], there is a possibility of dilution of gravitational field into bulk space towards directions of the large extra dimensions as illustrated in Figure 1.1

![Figure 1.1: Image of dilution of the gravitational field towards extra-dimension world.](image-url)
If “\( n \)” large extra dimensions exist, gravitational potential should be modified because of the Gauss’s law, if the size of the extra dimension \( \Lambda \) is as large as the experimental scale. From the naive picture, most simple form of the expected gravitational potential is written as:

\[
V_{r \geq \Lambda}(r) = -G \frac{Mm}{r},
\]

\[
V_{r < \Lambda}(r) = -G' \frac{Mm}{r^{1+n}}.
\]

(1.1)

Here, \( n \) is the number of the extra dimensions, and \( G' = \Lambda^n G \). In Figure 1.2, modified gravitational force is plotted for a case of \( \Lambda = 2cm \) and \( n = 2 \), being compared with that for pure Newtonian gravity. It can be noticed that the gravitational force is enhanced at distances below \( r < \Lambda \).

The functional form of the modified gravitational force is historically expressed using a Yukawa interaction form instead of (1.1);

\[
V(r) = -G \frac{Mm}{r} \left( 1 + \alpha \cdot \exp \left[ -\frac{r}{\lambda} \right] \right).
\]

(1.2)

Here, \( \lambda \) is the range and \( \alpha \) is the coupling constant of the “new” interaction. In this expression, the new Yukawa interaction term is interpreted as a new interaction arising from an exchange of a KK graviton, which has non-zero momentum component of the extra dimensions, therefore, observed as a massive graviton in our three dimensional world. The Compton wave length \( \lambda \) is obtained from the KK graviton mass \( m_{kk} \) as \( \lambda = \hbar / m_{kk} c \).

For decades, existence of the new Yukawa term is experimentally searched in a wide range from cm to astrophysical scales. In Figure 1.3, experimentally excluded regions are shown. As shown in the Figure 1.3, precision measurement is performed only at around the Earth - Mars distance scale. Therefore, it can be said that there are huge room for the existence of the new Yukawa term at short range scale, which is theoretically predicted as described above.
In the present study, we are aiming to explore the existence of non-zero $\alpha$ around 4, based on a model of toroidal-compactification which predicts $\alpha = 2n$ [3]. In this sense, we are aiming to perform the highest precision test of the inverse square law at around 0.1mm scale, where $\alpha = 4$ is not excluded experimentally yet. Present study is a first step experiment towards such sub-millimeter test aiming to confirm the performance of our experimental devices in centimeter scale measurement.

1.2. Test of Weak Equivalence Principle

Our device can investigate not only the inverse square law, but also can test weak equivalence principle using the same experimental setup. The weak equivalence principle is expressed as followings; a ratio between inertial mass and gravitational mass is independent of its composition, source interaction of the mass energies. From the definition of the inertial mass $I = \frac{F}{a}$ and that of gravitational mass $g \propto m_s$, gravitational acceleration must satisfies $\alpha \propto \frac{m_g}{m_i}$. Therefore, if the ratio $m_g/m_i$ is constant for all the materials, universality of free fall must be expected. Thus, test of the universality of free fall is interpreted as a test of weak equivalence principle. Indeed, numbers of experiments have been performed in order to test the universality of free fall in this sense. However, it should be noticed that a violation of the universality of free fall do not directly implies the violation of weak equivalence principle. It is because existence of not only weak equivalence principle violating gravitational force, but also a new non-gravitational force can violate the universality of free fall, whereas weak equivalence principle can live with such non-gravitational force.

Direct experimental confirmations of the weak equivalence principle have been performed, for example, by measuring the ratio between gravitational force from the Earth/the Sun, and the inertial force by the rotation of the Earth. From these results, weak equivalence principle can be believed with high precision in such long range regions.
Figure 1.4: Many experiments confirmed that the ratio between gravitational force and inertial force is independent for the different compositions in the Earth system.

On the other hand, there are no experimental investigations at around the short range regions yet. In order to directly perform the test measurement, we have checked the weak equivalence principle by testing universality of the gravitational constant $G$ for different compositions. Numbers of experimental tests confirmed that ratio between inertial mass for different compositions is same for the ratio between gravitational mass.

$$\frac{m_I(A)}{m_I(B)} = \frac{m_{g-long}(A)}{m_{g-long}(B)}.$$  \hspace{1cm} (1.3)

Here, $A$ and $B$ represent material names. $m_{g-long}$ denotes gravitational mass determined using long range gravitational force. This is same as the test of weak equivalence principle at long rage regions. Indeed, (1.3) can be rewritten as

$$\frac{m_I(A)}{m_{g-long}(A)} = \frac{m_I(B)}{m_{g-long}(B)},$$ \hspace{1cm} (1.4)

which directly implies the weak equivalence principle. Then, if we measure the gravitational mass at short range scales, we can test the new relation

$$\frac{m_{g-long}(A)}{m_{g-long}(B)} = \frac{m_{g-short}(A)}{m_{g-short}(B)},$$ \hspace{1cm} (1.5)

which leads

$$\frac{m_I(A)}{m_{g-short}(A)} = \frac{m_I(B)}{m_{g-short}(B)},$$ \hspace{1cm} (1.6)

by assuming (1.3). If we suppose that the gravitational mass is independent of the force range, scale and composition dependence can be included into the gravitational “constant” $G$. Therefore, short range test of composition dependence of the gravitational constant $G$ can be interpreted as a test of weak equivalence principle for a short range gravitational force.

There is a theoretical model which predicts a violation of the weak equivalence principle. A new interaction which couples to baryon number, instead of gravitational mass is proposed by Lee and Yang [4], motivated for providing existence reason for the observed baryon number conservation law. If such new interaction exists, the weak equivalence principle seems to be violated, because the “gravity-like” force, which is sum of the gravitational force and the new baryon number coupling force, is no more proportional to the gravitational mass. The baryon number coupling force must be similar to the gravitational force because mass density is roughly proportional to the baryon number. However, gravitational mass is slightly different from baryon number time nucleon mass, because of the mass defect and proton/neutron mass difference.
As shown in Figure 1.5, ratios between baryon number and gravitational mass are not constant for different materials, therefore, ratio between gravitational force and baryon number coupling force must be slightly different for different materials. Therefore, existence of the new baryon number coupling force can be examined by measuring the “gravity-like” force between different substances.

In this model [4], the new baryon number coupling term is added using new coupling constant \( f \).

\[
V(r) = -G \frac{m_A m_B}{r} \rightarrow -G \frac{m_A m_B}{r} + \frac{f^2 B_A B_B}{2 r} \tag{1.7}
\]

Now, for we are interested in a new interaction which violates weak equivalence principle only at short range, (1.7) should be modified using Yukawa form [2]:

\[
V(r) = -G \frac{m_A m_B}{r} \rightarrow -G \frac{m_A m_B}{r} \left( 1 - \frac{\varepsilon_B}{\mu_A \mu_B} \frac{B_A B_B}{\mu_A \mu_B} \exp \left( -\frac{r}{\lambda} \right) \right) \tag{1.8}
\]

Here, \( B_A \) is baryon number for material \( A \), and \( \mu_A \) is gravitational mass in hydrogen mass unit. Therefore, violation of the weak equivalence principle at short range scale is appeared as an existence of composition depending \( \alpha \) term in the extra dimension model (1.2). This relation is represented as,

\[
\alpha = \alpha_{AB} = \frac{\varepsilon_B B_A B_B}{\mu_A \mu_B} \tag{1.9}
\]

In the above expressions, gravitational constant \( G \) is supposed to be constant. Then, in a different expressions where we suppose inverse square law, but do not suppose universality of the gravitational constant \( G \), i.e.,

\[
V = -G_{AB} \frac{m_A m_B}{r} \tag{1.10}
\]

the new composition depending gravitational constant is defined using \( \varepsilon_B \):

\[
G_{AB} = G( 1 + \varepsilon_B \frac{B_A B_B}{\mu_A \mu_B} \exp \left( -\frac{r}{\lambda} \right) ) \tag{1.11}
\]

Therefore, by measuring the composition dependence of the gravitational constant \( G \), we can test the existence of non-zero \( \varepsilon_B \).
Apart from the baryon number coupling force, it can be shown that our test on the gravitational constant has sensitivity on weak equivalence principle, not concerning only for different composition, but also for different type of the mass source interactions. It is because both of the electromagnetic and strong interactions contribute on the nuclear mass, and the contribution fractions of them are varying as function of atomic numbers. For example, Coulomb energy contribution on a nuclear binding energy is about 20% for gold, whereas that for aluminium is only 5%. Therefore, we can say that our test on the composition dependence on the gravitational constant is a model independent test of the universality of free fall, for different source of the mass energy, too.

2. Experiment

In our experiment named Newton-I, we use a torsion balance bar as described in [5]. Sample CCD camera images of the torsion balance bar are shown in Figure 2.1. In Figure 2.1, torsion balance bar is pictured at the centre. Attractor positions are also shown. In this experiment, angular displacement of the torsion balance bar is the gravity signal. For example, expected angular displacement is about 0.1 degrees for the case of Figure 2.1. The tiny angular displacement is extracted by digital image analysis [6].

![Figure 2.1: Shape of the torsion balance bar and sample CCD images. Tiny angular displacement of the torsion balance bar is extracted by digital image analysis.](image)

Using the torsion balance bar system, we have performed a series of experiment, Newton-I, at around centimeter scales. Typical results from Newton-I experiment are shown in Figure 2.2, where time evolution of torsion balance bar angle is plotted for the cases of lead, copper and aluminum attractors. The expected gravity signal is clearly observed for all the attractor materials, which are in good agreement with the Newton gravity predictions in absolute scale. From this result, density dependence can be also confirmed.

Position dependence is examined by measuring the attractor setting angle dependence on the torsion balance bar angular displacement. The results are shown in Figure 2.3. We can clearly confirm the position dependences, which is also consistent with Newton gravity predictions quantitatively. The error bars are statistical error, which is obtained by standard deviations of multiple measurements. The largest systematic error in this measurement is electric and magnetic influences. They are evaluated by a separate dedicated measurement. From the results, electric influences are negligible thanks to using the electric shield. Although using non-magnetic materials, magnetic influences are observed. Typical magnetic effect is also shown in Figure 2.3.
Figure 2.2: Typical results in Newton-I experiment. Time evolution of torsion balance bar angle is plotted for the cases of lead, copper and aluminum attractors. Newton gravity estimations are also shown as references.

Figure 2.3: Position dependence of the torsion balance bar angle is shown. Solid line represents the Newton gravity prediction. Size of the small square shows a typical size of magnetic effects.

3. Discussion

Obtained results are compared with the previous experiments. In Figure 3.1, current upper limit of the Yukawa term is plotted as function of position scale in short range regions. It should be noticed
that there are no precision measurement at sub-millimeter scales. Although our result is not the highest precision data, it is approaching to the current upper limits. In order to perform improved precision measurement, we have built a next generation experimental device, Newton-III, using completely different measurement principle.

![Figure 3.1: Current upper limit on α in short range scales.](image)

As for the substance dependence of gravitation constant $G$, we obtained them for between brass target and aluminum, copper and lead attractors. The results for $G_{\text{Brass-Lead}} / G_{\text{Brass-Copper}}$ and $G_{\text{Brass-Aluminum}} / G_{\text{Brass-Copper}}$ are shown in Figure 3.2 From the composition dependence on gravitation constant $G$, we can extract the upper limit on $\xi_B$. The results are also shown in Figure 3.2. Note that our data from Newton-I experiment gives the most precise limit on the baryon number coupling $\xi_B$ at the shortest scale. In the Newton-I experiment, the largest systematic error on the composition dependence on gravitation constant $G$ comes from attractor replacement. In order to cancel out the systematic effects and to obtain more precise results, we are building next generation experiment Newton-II, which is described in detail in [5].

![Figure 3.2: Results on composition dependence of the gravitational constant $G$ and the corresponding upper limit on the baryon number coupling force. Data from [2] is used to plot $\xi_B$.](image)

The results on the test of inverse square law can be compared with the sensitivities of high energy collider search. In the ADD model, Planck-mass in “4+n” dimensional world is modified if scale of extra dimension $\Lambda$ is large.
Here, $m_{4+n}$ is the modified Planck mass in $4+n$ dimensional world, $m_{pl}$ is normal Planck mass. High energy collider can search gravitational effects if their leptonic/partonic energy is reaching the modified Planck energy $m_{4+n}c^2$, by searching creation of small black holes or KK particles. On the other hand, table top experiments which directly examine the inverse square law can see a deviation if the experimental scale is smaller than the size of the extra dimension $\Lambda$. Considering (3.1), they can be compared with each others. In Figure 3.3, sensitivities of them are plotted for comparison. It can be noticed that our results are competitive to that of highest energy colliders, including LHC. In Newton-III experiment, we are aiming to reach highest sensitivity, by utilizing the new experimental techniques.

![Figure 3.3: Comparing plot of sensitivity on extra dimension in ADD model, for high energy collider and table top experiments.](image)

### 4. Summary

We have built a prototype apparatus Newton-I, and succeeded to demonstrate its ability to test the Newton’s inverse square law in centimeter scale. For the detector resolution is better enough comparing to the current requirement, we will be able to access well below sub-millimeter scale, where even LHC cannot be accessible. We will also be able to test the weak equivalence principle in the shortest range scale. In order to utilize the full ability of our detector system, we are trying to build a new experimental setup, Newton-III, based on a new detection principle without using torsion balance pendulum.

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