DENSITIES AND ECCENTRICITIES OF 139 KEPLER PLANETS FROM TRANSIT TIME VARIATIONS

SAM HADDEN1,2 AND YORAM LITHWICK1,2

1 Department of Physics & Astronomy, Northwestern University, Evanston, IL 60208, USA
2 Center for Interdisciplinary Exploration and Research in Astrophysics (CIERA), Northwestern University, Evanston, IL 60208, USA

Received 2013 October 28; accepted 2014 April 4; published 2014 May 6

ABSTRACT

We extract densities and eccentricities of 139 sub-Jovian planets by analyzing transit time variations (TTVs) obtained by the Kepler mission through Quarter 12. We partially circumvent the degeneracies that plague TTV inversion with the help of an analytical formula for the TTV. From the observed TTV phases, we find that most of these planets have eccentricities of the order of a few percent. More precisely, the rms eccentricity is 0.018^{+0.005}_{-0.004} and planets smaller than 2.5 R_⊕ are around twice as eccentric as those bigger than 2.5 R_⊕. We also find a best-fit density–radius relationship ρ ≈ 3 g cm^{-3} \times (R/3 R_⊕)^{-2.3} for the 56 planets that likely have small eccentricity and hence small statistical correction to their masses. Many planets larger than 2.5 R_⊕ are less dense than water, implying that their radii are largely set by a massive hydrogen atmosphere.

Key words: planets and satellites: composition – planets and satellites: dynamical evolution and stability

Online-only material: color figures, machine-readable tables

1. INTRODUCTION

The Kepler telescope has provided an unprecedented look into the world of extrasolar planetary systems. It has detected the transits of thousands of planetary candidates, most of which are smaller than Neptune and orbit with periods ≤100 days (Batalha et al. 2013; Borucki et al. 2011, 2010). A transiting planet’s orbital period is trivially deduced from the time between transits, and its radius from the depth of transit. However, its other physical properties are much more difficult to obtain, including mass and eccentricity. Knowledge of a planet’s mass—and hence density—can inform us about its composition; and knowledge of its eccentricity can constrain its dynamical history. Both are key clues toward understanding how the surprising planetary systems discovered by Kepler formed and evolved.

A handful of Kepler planets have had their masses measured with radial velocity (RV) follow-up (Koch et al. 2010; Batalha et al. 2011; Cochran et al. 2011; Gautier et al. 2012; Gilliland et al. 2013). However, RV measurements are very difficult to obtain for the bulk of low-mass Kepler candidates. In multi-planet systems, one can take advantage of the fact that mutual gravitational perturbations alter the times of transit. If these transit time variations (TTVs) can be detected, they can be used to characterize planets (Agol et al. 2005; Holman & Murray 2005; Steffen et al. 2013, 2012; Fabrycky et al. 2012; Nesvorný et al. 2012; Lithwick et al. 2012; Xie 2013). TTVs are exquisitely sensitive to the planets’ masses and eccentricities, and can probe planets that are too far from their star or too small to yield a detectable RV signal. However, they depend on planet properties in a non-trivial way and can suffer from important degeneracies, making it a challenge to invert the TTV signal to infer the planets’ properties.

3 Inclinations can be deduced statistically from the relative numbers of transiting planets in different systems. The inclination dispersion is found to be very small—a few degrees (e.g., Lissauer et al. 2011b; Figueira et al. 2012; Tremaine & Dong 2012; Fang & Margot 2012; Johansen et al. 2012; Weissbein et al. 2012). There may be a second population of highly inclined systems, although that is strongly degenerate with the spacing distribution (Lissauer et al. 2011b).
4 After this paper was submitted, Marcy et al. (2014) reported an additional 16 Kepler planets with RV-measured masses.

5 We follow the convention of using primes to denote quantities corresponding to the outer planet.
6 The typical free eccentricities that we deduce in this paper are much larger than the planets’ forced eccentricities, and hence nearly equal to the total eccentricities. We shall therefore call the free eccentricity simply the eccentricity, with the understanding that it is really the free eccentricity that is deduced from TTV measurements.

To overcome this challenge, we perform the inversion with the help of simple analytical formulae for the TTV, derived in Lithwick et al. (2012). We focus on near-resonant pairs, which have particularly large TTV signals. To a good approximation, the TTV of each planet in such a pair is sinusoidal, with period P′/|jΔ| (which we call the “superperiod”), where Δ = P′/j − 1). If ω is the normalized distance to the nearest j:1 resonance, and P and P′ are the average orbital periods of the inner and outer planet.

\[ V \times e^{-i2\pi j/P'} = \frac{P}{\pi} \frac{\mu}{j^{2/3} (j - 1)^{1/3}} \left( -f - \frac{3}{2\Delta} Z_{\text{tree}} \right) \]

\[ V' = \frac{P'}{\pi} \frac{\mu}{j\Delta} \left( -g + \frac{3}{2\Delta} Z_{\text{tree}}^* \right) \]
with 68% confidence, based on the covariance matrix generated. The amplitudes are inconsistent with 0 at the predicted superperiods of "well-detected" TTVs, which we take to mean that their TTV magnitudes are inconsistent with zero within the error bar. In other columns, $P$ and $P'$ are orbital periods, $j$ is the nearest $j:j - 1$ resonance, and $\Delta$ is the normalized distance to resonance. Dashes appear for Keplerian periods shorter than 3 days. We exclude the middle planet's TTV (indicated by a blank entry in Table 1), because the effect of its two partners cannot be disentangled. In addition, we exclude the following systems from further consideration: KOIs 500 and 730 because of multiple degeneracies; KOIs 262 and 1858 because their expected superperiods are too long; and KOI 738 because it is strongly perturbed by the 9:7 resonance.

### 3. Eccentricities and Masses from TTV

We seek to extract the planets' eccentricities and masses from the values of $V$ and $V'$ by inverting Equation (2). However, there is a strong degeneracy: for any pair, one may scale down the masses without affecting the TTV, as long as one correspondingly scales up the eccentricities. Nonetheless, this degeneracy can be partially lifted with a large sample of TTVs. We proceed in two steps.

#### 3.1. Eccentricity Distribution

In the first step, we work with the phases of $V$ and $V'$ to determine the distribution of eccentricities. The phases depend only on the eccentricities, not the masses (Equation (2)). For a single planet pair, the phases cannot be used to infer the eccentricities because they depend on the unknown orientation of the planets' Keplerian ellipses ($\sigma$). However, the orientation should be random with respect to the line of sight, i.e., the combined phase of $Z_{\text{free}}$ should be random. Therefore, Equation (2) implies that if all the planets have sufficiently large eccentricities, $Z_{\text{free}}$ should be random. Alternatively, Equation (2) implies that if all the planets have sufficiently large $e$ (equivalent to $|\Delta|$), the TTV phases will be uniformly distributed between $-180^\circ$ and $180^\circ$. Conversely, if the planets have small eccentricities, $Z_{\text{free}}$ will be highly correlated with $\Delta$. Thus, we use the distribution of the phases of $V$ and $V'$ to estimate the distribution of eccentricities. The eccentricities depend only on the observable, uncorrelated eccentricities of the planets, not on the masses (Equation (2)).

### Table 1

#### TTV Amplitudes

| KOI | P (days) | P' (days) | j | $\Delta$ | $|V|$ (minutes) | $|V'|$ (minutes) | $\phi$ (deg) | $\phi'$ (deg) |
|-----|---------|-----------|---|---------|---------------|---------------|-------------|-------------|
| 82.02/01 | 10.31 | 16.15 | 3 | 0.044 | ... | 1.4$^{+0.5}_{-0.5}$ | ... | 159$^{+25}_{-25}$ |
| 82.04/02 | 7.07 | 10.31 | 3 | $-0.028$ | 16.0$^{+5.8}_{-5.8}$ | ... | 10$^{+9}_{-17}$ | ... |
| 85.01/03 | 5.86 | 8.13 | 4 | 0.041 | ... | 0.2$^{+3.6}_{-0.2}$ | ... | 4.1$^{+2.8}_{-2.8}$ | 33$^{+180}_{-180}$ | 102$^{+38}_{-45}$ |
| 111.01/02 | 11.43 | 23.67 | 2 | 0.036 | 1.1$^{+2.3}_{-1.1}$ | ... | 3.3$^{+2.1}_{-2.1}$ | ... | 5$^{+180}_{-180}$ | 2$^{+42}_{-43}$ |
| 115.01/02 | 5.41 | 7.13 | 4 | $-0.013$ | 1.3$^{+3.0}_{-0.8}$ | ... | 3.1$^{+2.5}_{-2.5}$ | ... | 148$^{+180}_{-180}$ | 116$^{+54}_{-52}$ |
| 137.01/02 | 7.64 | 14.86 | 2 | $-0.028$ | 5.3$^{+0.4}_{-0.4}$ | ... | 4.1$^{+0.3}_{-0.3}$ | ... | 8$^{+5}_{-5}$ | 170$^{+5}_{-5}$ |
| 148.01/02 | 4.78 | 9.67 | 2 | 0.012 | 4.0$^{+1.2}_{-1.2}$ | ... | 3.4$^{+0.8}_{-0.8}$ | ... | 6$^{+16}_{-16}$ | 159$^{+16}_{-16}$ |
| 152.02/01 | 27.40 | 52.09 | 2 | $-0.050$ | 1.0$^{+4.0}_{-1.0}$ | ... | 5.7$^{+13}_{-13}$ | ... | 17$^{+180}_{-180}$ | 99$^{+34}_{-33}$ |
| 152.03/02 | 13.48 | 27.40 | 2 | 0.016 | 8.2$^{+2.8}_{-2.8}$ | ... | 21.7$^{+3.9}_{-3.9}$ | ... | 36$^{+18}_{-18}$ | 135$^{+8}_{-8}$ |
| 156.01/03 | 8.04 | 11.78 | 3 | $-0.024$ | 1.3$^{+1.3}_{-1.3}$ | ... | 3.5$^{+0.7}_{-0.7}$ | ... | 5$^{+180}_{-180}$ | 150$^{+12}_{-12}$ |

Notes. Each line gives the TTV amplitudes for the inner and outer member of a pair of interacting planets, obtained by fitting the transit times cataloged in Mazeh et al. (2013). The TTV amplitudes (complex numbers $V$ and $V'$) are denoted here by their magnitudes ($|V|$ and $|V'|$) and phases ($\phi$ and $\phi'$), with error bars at 68% confidence. Our primary sample consists of planets whose TTV magnitudes are inconsistent with zero within the error bar. In other columns, $P$ and $P'$ are orbital periods, $j$ is the nearest $j:j - 1$ resonance, and $\Delta$ is the normalized distance to resonance. Dashes appear for middle planets in degenerate triples (see footnote 7). (This table is available in its entirety in a machine-readable form in the online journal. A portion is shown here for guidance regarding its form and content.)
...Furthermore, the points cluster near, but not always at, the point (0, 0) in the observed values of \( z \) eccentricity observed phases under the assumed eccentricity distribution. Inner and outer planets have anticorrelated phases, as implied by Equation (2). (unshaded). Nearly all the red points lie along the dashed diagonal line, i.e., the panels show histograms of the red points (shaded histograms) and gray points have a single measured TTV, such that each point (with error bar) corresponds...9 We define the TTV phase as \( \phi = \Delta(V \times \text{sgn} \Delta) \) for the inner planet, and the same for the outer (Lithwick et al. 2012). The extra factor of \( \text{sgn} \Delta \) = ±1 allows pairs narrow and wide of resonance to be treated symmetrically.

To be more precise, we assume that the eccentricities of the planets are drawn from the same underlying distribution, independent of any assumption.

To be more precise, we assume that the eccentricities of the planets are drawn from the same underlying distribution, independent of any assumption. The likelihood that a given planet in our sample has observed TTV phase \( \phi_{\text{obs}} \) is the convolution

\[
I(\phi_{\text{obs}} | \sigma_e) = \int P(\phi | \sigma_e) e^{-(\phi - \phi_{\text{obs}})^2 / (2 \sigma_e^2)} d\phi,
\]

where \( \phi \) is its true TTV phase; the first factor in the integrand is the probability that the phase as determined by Equation (2) is \( \phi \), assuming that the planets’ \( e \) values are randomly drawn from the Rayleigh distribution; and the second factor is the probability that the noise generates the observed phase from \( \phi \), where \( \sigma_e \) is the 68% confidence error on \( \phi_{\text{obs}} \) (averaging the asymmetric error bars). Note that \( P(\phi | \sigma_e) \) also depends on the (known) value of \( \Delta \) but not on the masses. We compute the total likelihood by multiplying together the likelihoods for all pairs, where the phase for a pair is taken to be the phase of its inner planet, which in turn is taken to be either \( \phi \) or \( \phi - 180^\circ \) depending on which planet has the smaller phase error bar. Maximizing the total likelihood, we find

\[
\sigma_e = 0.018^{+0.005}_{-0.004}.
\]

where the error bars delimit a decrease in the total likelihood by \( \Delta \ln L = -0.5 \) which would constitute the 68% confidence interval for a normally distributed error. The solid curves in the side panels of Figure 1 show the phase distribution that results from the above eccentricity dispersion, using the observed values of \( \Delta \). The dependence of the total likelihood on \( \sigma_e \) is shown in Figure 2.

Figure 3 shows the distributions of TTV phases for large and small planets separately, taking 2.5 \( R_\oplus \) as the dividing line between large and small. Larger planets tend to have smaller phases, which are indicative of lower eccentricities. To be quantitative, the phase distribution of pairs of large planets differs from that of small planets at 97% confidence, based on a K-S test of the absolute phases. Fitting for the eccentricity distributions of the two subsamples separately, we find that the large planets are around half as eccentric, with eccentricity dispersions given by

\[
\sigma_e = \begin{cases} 
0.017^{+0.009}_{-0.005} & \text{for } R \text{ and } R' < 2.5 \ R_\oplus \text{ (green). Dashed horizontal lines indicate decreases in } \ln L \text{ of } -0.5, -2, \text{ and } -4.5 \text{ which correspond to nominal } 1, 2, \text{ and } 3 \sigma \text{ confidence limits. (A color version of this figure is available in the online journal.)}
\end{cases}
\]

We detail below how we infer planet radii.

...Hadden & Lithwick 2014 May 20, 787:80 (7pp), The Astrophysical Journal
be due to measurement error. Nonetheless, we feel it likely that smaller planets are truly more eccentric, both because the error bars in Equation (5) seem sufficiently small to distinguish the two groups, and because another indicator of eccentricity—the nominal densities (see below)—also supports this conclusion. An additional concern is that higher eccentricities boost the TTV signal, making it easier to detect. So while the $\sigma_e$ we report for small planets with detected TTVs might be correct, we are biased against planets with smaller $e$. The same concern does not apply to the large planets because they have $e \lesssim |\Lambda|$, and hence most cannot have much of a boost. A more careful investigation of the underlying distributions must await future investigation.

### 3.2. Planet Masses and Densities

The masses are encoded in the absolute values of $V$ and $V'$. We define the nominal mass as the mass that would be inferred if one assumed (incorrectly, in general) that the (free) eccentricity vanishes,

$$m_{\text{nom}} = M_\oplus \frac{V' |\Lambda|}{P' \pi}$$

$$m'_{\text{nom}} = M_\oplus \frac{V |\Delta|}{P' \pi} (j - 1)^{2/3}$$

(Equation (2)). Since eccentricities do not vanish in general, the true mass $m$ differs from the nominal one by an eccentricity-dependent factor:

$$m = m_{\text{nom}} \left| 1 - 3Z_{\text{free}}/(2g\Delta) \right|$$

with an analogous expression for $m'$. If planets in a pair have very low eccentricities ($e \ll |\Delta|$), the nominal masses are the true masses, whereas if the eccentricities are much higher than

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Histograms of inner-planet TTV phases of the large ($R$ and $R' > 2.5 R_\oplus$), small ($R$ and $R' < 2.5 R_\oplus$), and mixed (one large, one small) planet pairs. In the lower panels, the individual phases are shown. The phases of the large planets are more concentrated around $0^\circ$, indicating they have smaller eccentricities. The phase distributions resulting from the best-fit $\sigma_e$ (Equation (5)) are plotted as solid curves after folding in the observed values of $\Delta$.

(A color version of this figure is available in the online journal.)

Table 2

| Planet | $m_{\text{nom}}$ | $R_\oplus$ | $e$-flag | $M_\oplus$ | $R_\oplus$ |
|--------|------------------|-----------|----------|------------|-----------|
| 82.02  | $2.6^{+1.3}_{-1.1}$ | $1.2^{+0.1}_{-0.1}$ | h | 0.73 | 0.755 |
| 85.01  | $2.6^{+2.0}_{-1.8}$ | $2.6^{+0.04}_{-0.04}$ | h | 1.27 | 1.424 |
| 111.01 | $19.6^{+14.5}_{-12.4}$ | $3.1^{+0.5}_{-0.7}$ | h | 0.81 | 1.361 |
| 115.01 | $5.1^{+6.3}_{-4.1}$ | $5.3^{+3.0}_{-1.1}$ | l | 1.09 | 1.332 |
| 115.02 | $3.0^{+9.2}_{-2.0}$ | $1.9^{+1.0}_{-0.4}$ | l | 1.09 | 1.332 |
| 137.01 | $18.4^{+2.7}_{-2.3}$ | $5.2^{+0.4}_{-0.3}$ | l | 0.88 | 1.055 |
| 137.02 | $15.7^{+2.0}_{-1.4}$ | $6.6^{+0.5}_{-0.4}$ | l | 0.88 | 1.055 |
| 148.01 | $14.3^{+4.3}_{-4.4}$ | $2.6^{+0.2}_{-0.1}$ | h | 0.96 | 0.852 |
| 148.02 | $9.8^{+3.3}_{-3.3}$ | $3.6^{+0.4}_{-0.1}$ | h | 0.96 | 0.852 |
| 152.02 | $13.7^{+4.5}_{-4.5}$ | $2.6^{+0.2}_{-0.2}$ | h | 1.08 | 1.037 |

**Notes.**

* Planet nominal mass, given by Equation (6).

* Flag indicating whether the planet is classified as high-$e$ ("h") or low-$e$ ("l") according to criteria listed in Section 3.2.

(This table is available in its entirety in a machine-readable form in the online journal. A portion is shown here for guidance regarding its form and content.)

that ($e \gg |\Delta|$), the nominal masses are overestimates of the true masses. For intermediate eccentricities ($e \sim |\Delta|$), the nominal and true masses differ by an order unity factor—they can be either smaller or larger depending on the phase of $Z_{\text{free}}$.

Table 2 lists the nominal masses of the planets in our primary sample, and the top panel of Figure 4 plots these against planet radii. For planets with multiple TTV partners, we use the TTV amplitude with the smallest fractional uncertainty to compute its nominal mass. The quoted error on nominal mass combines the 68%-confidence error in TTV amplitude with that in stellar mass. To determine planet radii and stellar host parameters, we
take star masses and radii from Huber et al. (2014), as listed at the NASA Exoplanet Archive.\textsuperscript{12} We obtain planet radius by multiplying the star’s radius by the planet-to-star size ratio reported on the Exoplanet Archive; the latter come from the literature if available, and otherwise from the Kepler pipeline. There are 11 host stars among our primary sample for which Mann et al. (2013) have published revised parameters based on spectroscopic models calibrated to nearby late K and M-dwarfs. We use their values of stellar radius and mass along with planet radii where applicable.

In order to focus on pairs that likely have small eccentricities, and thus true masses close to their nominal masses, we split our sample into “low-e” and “high-e” planets. The nominal masses of the former are shown in the lower panel of Figure 4, and the nominal densities in Figure 5. A pair is categorized as high-e if it satisfies any of the following criteria.

1. The TTV phase differs, at 68% confidence, by more than 30° from its zero-e value, i.e., from 0° for the inner planet or 180° for the outer, using whichever planet has the smaller phase error.
2. The nominal density exceeds that of pure iron composition at 68% confidence. Since true densities are likely not so high, these planets presumably have large e and hence a large correction to their nominal masses. This conjecture is supported by the fact that many planets nominally denser than iron possess large TTV phase (Figure 4).
3. One or both planets have transit durations that imply $e > |\Delta|$ at 68% confidence (Figure 6). This criterion could be triggered by an underestimated radius for the planet’s host star, rather than the planet’s high e. But for such planets, the nominal density would also exceed the true density.

We find 83 planets fall into the high-e sample, and 56 in the low-e sample.

\begin{equation}
    m_{\text{nom}} \approx 14.9^{+3.4}_{-2.8} M_{\oplus} \times \left(\frac{R}{3 R_{\oplus}}\right)^{0.65 \pm 0.14}.
\end{equation}

We determine a mass–radius relationship for the low-e planets\textsuperscript{13} by performing a linear least-squares fit in log–log space, yielding

\begin{equation}
    \rho_{\text{nom}} \approx 3 \text{ g cm}^{-3} \times \left(R/3 R_{\oplus}\right)^{-2.3}.
\end{equation}

This agrees reasonably well with what was found in Wu & Lithwick (2013) based on a smaller sample of TTVs. It is also in adequate agreement with results from planets with RV-determined masses (Figure 5), which were not included in the fit. Weiss & Marcy (2014) report $m \approx 7 M_{\oplus} \times (R/3 R_{\oplus})^{0.93}$, for planets in the size range $1.5 R_{\oplus} < R < 4 R_{\oplus}$, based primarily on RV measurements.

Large planets ($>2.5 R_{\oplus}$) appear to be distinct from small ones ($<2.5 R_{\oplus}$) in a number of ways. First, larger planets tend to be less dense. Many large ones are less dense than water, indicating that their radii must be set to a large extent by a gaseous hydrogen atmosphere. By contrast, nearly all small ones are denser than rock. Second, small ones tend to be closer to their star: the median orbital period for small planets in our primary sample is 10.1 days, whereas that of the large ones is 15.3 days. Such a correlation for Kepler candidates has been noted in Wu & Lithwick (2013) and Owen & Wu (2013). As argued there, all planets might have started out with gaseous envelopes, and then the ones closer to their star

\textsuperscript{12} http://exoplanetarchive.ipac.caltech.edu/index.html

\textsuperscript{13} We exclude Kepler-51c (KOI 620.03) from our mass–radius fits, as it is significantly larger than the other planets ($9.3 R_{\oplus}$).
Figure 5. Nominal planet density vs. radius for the 56 planets in our low-$e$ sample. These planets likely have true densities $\rho \approx \rho_{\text{nom}}$. The color scheme for the circles is as described in Figure 4. The best fit relation is plotted as a black dashed line (Equation (9)). Pink stars are planets with RV-determined masses, including those listed in Table 3 of Wu & Lithwick (2013), Kepler planets listed in Table 2 of Marcy et al. (2014), as well as Kepler-68c (Gilliland et al. 2013), and HD97658b (Dragomir et al. 2013).

(A color version of this figure is available in the online journal.)

Figure 6. Selecting high-$e$ planets based on transit duration. The $x$-axis shows the planets' nominal density, and the $y$-axis shows the ratio of their observed transit durations to what the duration would be if the planet were on a circular orbit with zero impact parameter. When that ratio exceeds unity, the eccentricity is bounded from below by $e > T_{\text{dur}}/T_{\text{dur},0} - 1$ (Equation (1) of Ford et al. 2008, to linear order in $e$). The dashed line corresponds to a minimum eccentricity of 0.06, which is greater than 99.7% of eccentricities belonging to our best-fit eccentricity distribution. The color/symbol scheme is as described in Figure 4. Error bars are based on uncertainty in stellar radius, and are suppressed in the $x$-direction. Five planets are selected as high-$e$ based on transit duration alone and are emphasized with bold red error bars. In addition, many of the planets selected as high-$e$ based on TTV phase or nominal density also have large transit durations, corroborating their membership in the high-$e$ sample.

(A color version of this figure is available in the online journal.)

Figure 7. Nominal planet density of the low-$e$ subsample vs. orbital period. The color and symbol scheme is the same as described in Figure 4. While planet densities are quite scattered, there is a hint of closer-in planets being denser. The RV planets probe significantly shorter periods than the majority of TTV planets.

(A color version of this figure is available in the online journal.)

Third, as shown above, small ones are around twice as eccentric (Equation (5); Figure 3). This is corroborated by the fact that planets in Figure 4 with nominal densities higher than iron have $R < 2.5 R_\oplus$.

It would be of interest to know whether Kepler planets are mostly water. If so, it would suggest that the planets formed beyond the ice line and then migrated inward; if not, it would argue for in situ formation. Unfortunately, the water content is difficult to deduce, since the density is very sensitive to the presence of hydrogen; e.g., a 5 $M_\oplus$ rocky planet with a hydrogen atmosphere that is only $\approx 1\%$ of its mass would have...
have the same density as a pure water planet (Adams et al. 2008; Wu & Lithwick 2013). However, we conjecture that the majority of planets are nearly water-free, based on the fact that planets of a given mass span such a wide range of densities—from less dense than water to denser than rock, without much evidence for a pileup of planets near the water-density curve. Furthermore, the fact that small planets tend to be closer to their star suggests that those might have lost their gaseous envelope by photoevaporation, exposing the rocky core underneath (Lopez et al. 2012; Wu & Lithwick 2013; Owen & Wu 2013).

4. SUMMARY

We have extracted the eccentricities and masses—and hence densities—of a large sample of Kepler planets, starting from the Mazeh et al. (2013) catalog of transit times through Quarter 12. We found 139 planets suitable for our analysis. In order to invert the observed TTVs, we used an analytical formula for the TTVs of near-resonant pairs. Inverting the analytical formula is not only much faster than the more typically used N-body inversion (Holman et al. 2010; Lissauer et al. 2011a, 2013; Cochran et al. 2011), but it also allows one to break the strong degeneracy between mass and eccentricity in a statistical way. We largely followed the approach of Lithwick et al. (2012) and Wu & Lithwick (2013), but applied it to a significantly larger sample and for nearly twice the duration. Our main results are as follows.

1. From the TTV phases, the planets’ eccentricities are of the order of a few percent, with rms \( \sigma_e = 0.18^{+0.05}_{-0.04} \) when fit with a Rayleigh distribution. Small planets (<2.5 \( R_\oplus \)) are around twice as eccentric as the larger ones.

2. The nominal masses—and hence densities—were extracted from the TTV amplitudes. The nominal densities of the low-e planets, which are expected to be comparable to the true planet densities, are displayed in Figure 5 (see also Table 2). The mass–radius relation we infer from TTVs largely agrees with that found from RV. Large planets (>2.5 \( R_\oplus \)) appear distinct from small ones, in that many of them are so underdense that they must be covered in gas, whereas most small ones are as dense as water or rock. In addition, the large ones are typically less eccentric and further from their host star.

The untimely malfunction of the Kepler satellite makes it unlikely that the number of planets with TTV-measured eccentricities and densities will increase substantially in the near future. However, there are a number of prospects for improving upon the results of this paper. First, Kepler has released light curves up to Quarter 16, and the present study may be extended to incorporate them. Second, the radii of stars in the Kepler catalog are uncertain. This directly translates into uncertainty in the planet radii—and hence densities. For a crude estimate of the error, we compare the radii of the 38 stars in our primary sample that have been followed up (e.g., with spectroscopy or asteroseismology) with their pre-follow-up values, and find that the median error is 12%; 31 of the stars have errors <25%, but three have errors larger than 50%. In the future, more accurate measurements of stellar radii will yield more accurate planet densities.

Third, the TTV signals sometimes have components in addition to the dominant one of Equation (1), especially when a planet pair lies near a higher-order resonance. If these are detected and decoded, they can be used to help break the mass-eccentricity degeneracy. Fourth, RV measurements will continue to expand our knowledge of sub-Jovian planets.

We thank Will Farr, Eric Gaidos, Jason Steffen, and Yanqin Wu for helpful discussions. We are also grateful to the Kepler team for acquiring and publicly releasing such spectacular results. Y.L. acknowledges support by NSF grant AST-1109776 and NASA grant NNX14AD21G.

REFERENCES

Adams, E., Seager, S., & Elkins-Tanton, L. 2008, ApJ, 673, 1160
Agol, E., Steffen, J., Sari, R., & Clarkson, W. 2005, MNRAS, 359, 567
Batalha, N. M., Borucki, W. J., Bryson, S. T., et al. 2011, ApJ, 729, 27
Batalha, N. M., Rowe, J. F., Bryson, S. T., et al. 2013, ApJS, 204, 24
Borucki, W. J., Koch, D., Basri, G., et al. 2010, Sci, 327, 977
Borucki, W. J., Koch, D. G., Basri, G., et al. 2011, ApJL, 736, 19
Cochran, W. D., Fabrycky, D. C., Torres, G., et al. 2011, ApJS, 197, 7
Dragomir, D., Matthews, J. M., Eastman, J. D., et al. 2013, ApJL, 772, L2
Fabrycky, D. C., Ford, E. B., Steffen, J. H., et al. 2012, ApJ, 750, 114
Fang, J., & Margot, J.-L. 2012, ApJ, 761, 92
Figueira, P., Marmier, M., Boué, G., et al. 2012, A&A, 541, A139
Ford, E. B., Quinn, S. N., & Veras, D. 2008, ApJ, 678, 1407
Fortney, J., Marley, M., & Barnes, J. 2007, ApJ, 659, 1661
Gaidos, E., & Mann, A. W. 2013, ApJ, 762, 41
Gautier, T. N., III, Charbonneau, D., Rowe, J. F., et al. 2012, ApJ, 749, 15
Gilliland, R. L., Marcy, G. W., Rowe, J. F., et al. 2013, ApJ, 766, 40
Holman, M. J., Fabrycky, D. C., Agol, E., & Wu, Y. 2009, ApJ, 701, 1288
Huber, D., Aguirre, V. S., Matthews, J. M., et al. 2013, ApJL, 711, 2
Johansen, A., Davies, M. B., Church, R. P., & Holmén, V. 2012, ApJ, 758, 39
Koch, D. G., Borucki, W. J., Rowe, J. F., et al. 2010, ApJL, 713, L131
Lissauer, J. J., Fabrycky, D. C., Ford, E. B., et al. 2011a, Natur, 470, 53
Lissauer, J. J., Jontof-Hutter, D., Rowe, J. F., et al. 2013, ApJ, 770, 131
Lissauer, J. J., Ragozzine, D., Fabrycky, D. C., et al. 2011b, ApJS, 197, 8
Lithwick, Y., Xie, J., & Wu, Y. 2012, ApJ, 761, 122
Lopez, E. D., Fortney, J. J., & Miller, N. 2012, ApJ, 761, 59
Mann, A. W., Gaidos, E., & Ansdel, M. 2013, ApJ, 779, 188
Marcy, G. W., Isaacson, H., Howard, A. W., et al. 2014, ApJS, 210, 20
Mazeh, T., Nachmani, G., Holczer, T., et al. 2013, ApJS, 208, 16
Nesvorný, D., Kipping, D. M., Buchhave, L. A., et al. 2012, Sci, 336, 1133
Owen, J. E., & Wu, Y. 2013, ApJ, 775, 105
Steffen, J. H., Fabrycky, D. C., Agol, E., et al. 2013, MNRAS, 428, 1077
Steffen, J. H., Fabrycky, D. C., Ford, E. B., et al. 2012, MNRAS, 421, 2342
Tremaine, S., & Dong, S. 2012, AJ, 143, 94
Weiss, L. M., & Marcy, G. W. 2014, ApJL, 783, L6
Weissbein, A., Steinberg, E., & Sari, R. 2012, arXiv:1203.6072
Wu, Y., & Lithwick, Y. 2013, ApJ, 772, 74
Xie, J.-W. 2013, ApJS, 208, 22
Xie, J.-W. 2014, ApJS, 210, 25