Stiffness of Composite Beams with Full Shear Connection

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Abstract. This paper presents an analysis of the stiffness of various composite beams with full shear connection. The authors analysed steel-concrete composite (SCC), aluminium-concrete composite (ACC), steel-timber composite (STC) and aluminium-timber composite (ATC) beams using numerical simulations. The purpose of the analysis was to compare and contrast the stiffness of the above-mentioned composite beams. Non-linear 3D finite element (FE) models of the analysed beams were developed and used to evaluate deflections. The SCC beam demonstrated the highest stiffness and the ATC beam demonstrated the lowest stiffness.

1. Introduction
Composite steel and concrete structures are often used in multi-storey buildings [1] and bridges [2, 3]. General rules for designing these structures are presented in designers’ guide [4] and in standard [5]. Damage diagnosis of steel-concrete composite beams is shown in [6, 7]. There are also other types of composite structures. Composite concrete structures in which there are two concrete parts (a new part and an old part) may be used to strengthen an old concrete slab [8, 9]. Steel and timber composite structures [10] and timber and glass composite structures [11] are a relatively new concept and may be used in floors. Aluminium and concrete composite structures are not new, they are used in bridges [12-16] and may also be used as beams and columns [17]. Timber and concrete composite structures have been extensively investigated and used in floors [18, 19]. Aluminium and timber structures seem to be a new solution when it comes to joining materials [20].

The authors of this article tried to analyse the stiffness of steel-concrete composite (SCC), aluminium-concrete composite (ACC), steel-timber composite (STC) and aluminium-timber composite (ATC) beams using numerical simulations. The stiffness of the composite beams was also compared with the stiffness of the aluminium (A) and steel (S) beam.

2. Problem formulation
In this paper the authors compared the stiffness of the few types of composite beams. They made the following assumptions:
- Composite beams with full shear connection were analysed.
- The beams had the same geometric dimensions.
- Elastic-plastic material models were used.
- The behaviour of the composite beams was analysed until the deflection reached L/250 (L – beam length).
- Non-linear 3D finite element (FE) models of the analysed beams were developed in the Abaqus program.
The geometric configurations and details of a single composite beam are presented in figure 1.

![Figure 1. Geometrical outline and details of the composite beam.](image)

The dead loads of the analysed beams are presented in table 1.

| Beam | Dead load [kN/m] |
|------|------------------|
| A    | 0.075            |
| S    | 0.217            |
| ATC  | 0.405            |
| STC  | 0.547            |
| ACC  | 1.575            |
| SCC  | 1.717            |

The aluminium beam is 2.9 times lighter than the steel beam. The aluminium and timber composite beam is the lightest of all the composite beams.

3. Numerical models of the analysed composite beams

3.1. Material models
The behaviour of aluminium, steel and laminated veneer lumber (LVL) was modelled using an elastic-perfectly plastic model. The non-linear behaviour of concrete was captured using the concrete damaged plasticity (CDP) model available in the Abaqus program [21] and successfully used for concrete [22-26]. The behaviour of the materials is presented in figure 2. The stress-strain diagram for the analysis of the concrete subjected to compression was adopted from [27], the stress-strain diagram for the analysis of the concrete subjected to tension was taken from [28]. The compressive strength and the tensile strength of the concrete were based on own laboratory tests. The value of the fracture energy and the critical crack opening were calculated using the formulas presented in [29-31]. The parameters for LVL were based on the manufacturer's data [30]. The material parameters for metals are from the standards [33, 34]. All the materials parameters used in the numerical model are presented in tables 2-4.
Figure 2. Material models.

Table 2. Parameters of materials used in numerical calculations.

| Material     | Parameter                                      | Value       |
|--------------|------------------------------------------------|-------------|
| Concrete C50/60 | Young’s modulus $E_{cm}$ [MPa]                | 37 000      |
|              | Poisson’s ratio $\nu$ [-]                     | 0.20        |
|              | Mean value of concrete cylinder compressive strength $f_{cm}$ [MPa] | 61.8        |
|              | Mean value of axial tensile strength of concrete $f_{tm}$ [MPa] | 4.6         |
|              | Largest nominal maximum aggregate size $d_{a}$ [mm] | 16.0        |
|              | Fracture energy $G_F$ [N/m]                    | 92.0        |
|              | Parameter $n$ [-]                              | 1.2         |
|              | Dilatation angle [°]                           | 40.0        |
|              | Eccentricity [-]                               | 0.1         |
|              | $f_{00}/f_{0}$ [-]                             | 1.16        |
|              | Parameter $\kappa$ [-]                         | 0.667       |
|              | Viscosity parameter [-]                        | 0.001       |
| LVL          | Young’s modulus $E_{0,mean}$ [MPa]             | 14 000      |
|              | Poisson’s ratio $\nu$ [-]                     | 0.4         |
|              | Tension strength, parallel to grain $f_{0,k}$ [MPa] | 36.0        |
|              | Compression strength, parallel to grain $f_{0,k}$ [MPa] | 40.0        |
| S235         | Young’s modulus $E$ [MPa]                      | 210 000     |
|              | Poisson’s ratio $\nu$ [-]                     | 0.3         |
|              | Yield strength [MPa]                           | 235.0       |
| AW-6060 T6   | Young’s modulus $E$ [MPa]                      | 70 000      |
|              | Poisson’s ratio $\nu$ [-]                     | 0.3         |
|              | Yield strength [MPa]                           | 140.0       |
Table 3. Material parameters used in the CDP model for the C50/60 concrete subjected to compression.

| Concrete compression hardening | Concrete compression damage |
|-------------------------------|----------------------------|
| Stress [MPa] | Crushing strain [-] | Dc [-] | Crushing strain [-] |
| 28.82 | 0.00000000 | 0.000 | 0.00000000 |
| 43.79 | 0.00011650 | 0.000 | 0.00011650 |
| 48.87 | 0.00017927 | 0.000 | 0.00017927 |
| 51.17 | 0.00021706 | 0.000 | 0.00021706 |
| 56.96 | 0.00036056 | 0.000 | 0.00036056 |
| 59.72 | 0.00048599 | 0.000 | 0.00048599 |
| 60.70 | 0.00055935 | 0.000 | 0.00055935 |
| 61.75 | 0.00083104 | 0.001 | 0.00083104 |
| 61.35 | 0.00094196 | 0.007 | 0.00094196 |
| 60.50 | 0.0106499 | 0.021 | 0.0106499 |
| 59.14 | 0.0120157 | 0.043 | 0.0120157 |
| 57.22 | 0.0135338 | 0.074 | 0.0135338 |
| 54.67 | 0.0152237 | 0.115 | 0.0152237 |
| 51.40 | 0.0171087 | 0.168 | 0.0171087 |
| 47.30 | 0.0192162 | 0.235 | 0.0192162 |
| 42.26 | 0.0215791 | 0.316 | 0.0215791 |
| 36.12 | 0.0242372 | 0.415 | 0.0242372 |
| 28.72 | 0.0272388 | 0.535 | 0.0272388 |
| 19.82 | 0.0306438 | 0.679 | 0.0306438 |
| 9.150 | 0.0345263 | 0.852 | 0.0345263 |
| 3.050 | 0.0366750 | 0.951 | 0.0366750 |

Table 4. Material parameters used in the CDP model for the C50/60 concrete subjected to tension (n=1.2).

| Concrete tension stiffening | Concrete tension damage |
|----------------------------|-------------------------|
| Stress [MPa] | Cracking strain [-] | Dt [-] | Cracking strain [-] |
| 4.600000 | 0.00000000 | 0.000000 | 0.00000000 |
| 1.989286 | 0.0001962 | 0.10594 | 0.0001962 |
| 1.328445 | 0.0003141 | 0.13276 | 0.0003141 |
| 0.982585 | 0.0004234 | 0.26520 | 0.0004234 |
| 0.772307 | 0.0005291 | 0.42245 | 0.0005291 |
| 0.632294 | 0.0006329 | 0.52736 | 0.0006329 |
| 0.532294 | 0.0007356 | 0.60194 | 0.0007356 |
| 0.458060 | 0.0008376 | 0.65745 | 0.0008376 |
| 0.400827 | 0.0009392 | 0.70025 | 0.0009392 |
| 0.355466 | 0.0010404 | 0.73417 | 0.0010404 |
| 0.318704 | 0.0011414 | 0.76166 | 0.0011414 |
| 0.288359 | 0.0012422 | 0.78436 | 0.0012422 |
| 0.262921 | 0.0013429 | 0.80338 | 0.0013429 |
| 0.241315 | 0.0014435 | 0.81954 | 0.0014435 |
| 0.222755 | 0.0015440 | 0.83342 | 0.0015440 |
| 0.206654 | 0.0016444 | 0.84546 | 0.0016444 |
| 0.192566 | 0.0017448 | 0.85599 | 0.0017448 |
| 0.180144 | 0.0018451 | 0.86528 | 0.0018451 |
| 0.169116 | 0.0019454 | 0.87353 | 0.0019454 |
| 0.159265 | 0.0020457 | 0.88090 | 0.0020457 |
| 0.150418 | 0.0021459 | 0.88751 | 0.0021459 |
| 0.142432 | 0.0022462 | 0.89349 | 0.0022462 |
3.2. Non-linear 3D FE models

The numerical model was prepared in the Abaqus program and it consisted of a beam and a slab. The authors of this article prepared only 1/2 of the model (see figure 3) using one axis of symmetry.

The connection between the beam and the slab was modelled using the tie function (full shear connection). The slab was divided into eight-node cuboidal finite solid elements (C3D8R) and the beam was divided into four-node shell elements (S4R) (see figure 4). The size of the mesh was 20.0 mm. The total number of all elements was 31 137.

![Figure 3](image1.png)

**Figure 3.** The model of the composite beam in the Abaqus environment:
a) complete model; b) half of the model; 1 – slab, 2 – beam, 3 – plane of symmetry.

The calculations were performed using the Newton-Raphson method. Figure 5 presents the boundary conditions used in the computer model.

![Figure 4](image2.png)

**Figure 4.** Mesh used in the numerical calculations.

The calculations were performed using the Newton-Raphson method. Figure 5 presents the boundary conditions used in the computer model.
4. Results and discussions

Figure 6 shows the mid-span moments versus the mid-span deflections of SCC, STC, ACC and ATC beams from the FEA analyses.

The comparison of the moments and deflections of the composite beams and metal beams is presented in tables 5 and 6.

Table 5. Comparison of the moments and deflections of composite beams.

| Beam | SCC | STC | ACC | ATC |
|------|-----|-----|-----|-----|
| $u^a$ | $M^b$ | $M^c$ | $M^d$ | $M^b$ | $M^c$ | $M^d$ | $M^b$ | $M^c$ | $M^d$ | $M^b$ | $M^c$ | $M^d$ |
| [mm] | [kNm] | [kN] | [kNm] | [kN] | [kNm] | [kN] | [kNm] | [kN] | [kNm] | [kN] | [kNm] |
| 5.0  | 42.2 | 8440 | 39.4 | 31.4 | 6280 | 29.7 | 19.1 | 3820 | 16.4 | 14.5 | 2898 |
| 12.0 | 78.3 | 6525 | 75.5 | 65.3 | 5442 | 63.6 | 42.1 | 3508 | 39.4 | 34.6 | 2885 |

$^a$ $u$ - mid-span deflection; $^b$ $M$ - mid-span moment; $^c$ $\alpha = M / u$; $^d$ $M^d$ - mid-span moment – moment from dead load.
### Table 6. Comparison of the moments and deflections of metal beams.

| Beam | u[^a] [mm] | M[^b] [kNm] | S[^c] [kN] | M[^d] [kNm] | α[^c] | M[^d] [kNm] |
|------|------------|-------------|------------|-------------|-------|-------------|
| 5.0  | 3.4        | 681         | 2.2        | 10.2        | 2048  | 8.9         |
| 12.0 | 8.2        | 681         | 7.0        | 24.6        | 2048  | 23.2        |

[^a] u - mid-span deflection;[^b] M - mid-span moment;[^c] α = M / u;[^d] M[^d] - mid-span moment – moment from dead load.

The SCC beam showed the highest stiffness and the ATC beam showed the lowest stiffness among all the analysed composite beams. Both A and S beams showed a lower stiffness than the composite beams presented in this paper. The stiffness of the metal beams was constant up to a point where deflection reached L/250 (they showed linear elastic behaviour). The composite beams started to yield before deflection reached L/250 (12.0 mm).

### Table 7. Comparison of stiffness.

| Beam | A | S | ATC | STC | ACC | SCC |
|------|---|---|-----|-----|-----|-----|
| u[^e] [mm] | β[^e] | β[^e] | β[^e] | β[^e] | β[^e] | β[^e] |
| 5.0  | 1.0 | 3.0 | 4.3 | 9.2 | 5.6 | 12.4 |
| 12.0 | 1.0 | 3.0 | 4.2 | 8.0 | 5.2 | 9.6 |

[^e] β - mid-span moment for the proper beam / mid-span moment for the aluminium beam.

The ATC beam showed a 4.3 greater stiffness than the A beam, and demonstrated a 1.3 lower stiffness than the ACC beam. The STC beam showed a 3.1 greater stiffness than the S beam, and demonstrated 1.3 lower stiffness than the SCC beam.

### 5. Conclusions

The main conclusions of this paper are as follows:

- The analysed composite beams had a greater stiffness than the analysed metal beams. The combination of materials always provides for a greater stiffness.
- The SCC beam demonstrated the highest stiffness, followed by the ACC beam, the STC beam, the ATC beam, and the S beam. The A beam demonstrated the lowest stiffness.
- The presented composite beams have both advantages and disadvantages. Designers should take into account not only the load bearing capacity and stiffness, but also the cost and construction time of composite structures, as well as their useful properties and durability.

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