Baryon Number Violation and String Hadronization

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Abstract. In supersymmetric scenarios with broken $R$-parity, baryon number violating sparticle decays are possible. We report on the development of a framework allowing detailed studies with special attention given to the hadronization phase. In our model, implemented in the Pythia event generator, the baryon number violating vertex is associated with the appearance of a junction in the colour confinement field. This allows us to tell where to look for the extra (anti)baryon directly associated with the baryon number violating decay.

1 Introduction

In the Minimal Supersymmetric extension of the Standard Model (MSSM), the standard particle content, extended to two Higgs doublets, is doubled up by the presence of superpartners to all normal particles. The conservation of a multiplicative quantum number called $R$-parity, defined by $R = (-1)^{2S+3B+L}$, where $S$ is the particle spin, $B$ its baryon number and $L$ its lepton number, is usually assumed, since this prevents fast proton decay and has the nice additional consequence of making the Lightest Supersymmetric Particle (LSP) stable, thus making it a WIMP type dark matter candidate.

However, the choice of $R$-parity conservation to prevent fast proton decay is not unique, and due to the distinct differences in collider phenomenology between models with and without $R$-parity conservation, it is of importance to be well prepared for all possibilities at present as well as future high-energy experiments.

With $R$-parity conserved, experimental SUSY signals would consist of jets, leptons and missing $E_T$ from escaping neutrinos and LSP’s. In scenarios with baryon number violation (BNV in the following) the main decay product is jets, with only few leptons or neutrinos, and so observability above QCD backgrounds becomes far from trivial at hadron colliders such as the Tevatron or the LHC. In order to carry out realistic studies it is therefore necessary to have a detailed understanding of the properties of both signal and background events. The prime tool for achieving such an understanding is to implement the relevant processes in event generators, where simulated events can be studied with all the analysis methods that could be used on the real events.

In this presentation, we concentrate on the possibility that baryon number may be broken, resulting in BNV sparticle decays. Sparticle production by BNV, important when the BNV couplings are large and/or the sparticles are heavy, is not considered here. In the past, BNV has been modelled \cite{1,2} and studied \cite{3} in detail in the HERWIG framework, with emphasis on the
perturbative aspects of the production process. In [4], we present a corresponding implementation in PYTHIA, summarized here, where a special effort is dedicated to the non-perturbative aspects, allowing us to address the possibility of obtaining a “smoking-gun” evidence that a BNV decay has occurred, with questions such as Could the presence of a violated baryon number be directly observed? and If so, what strategy should be used?. In addition, many other differences exist between the PYTHIA and HERWIG physics scenarios, for parton showers and underlying events, thereby allowing useful cross-checks to be carried out and uncertainties to be estimated.

2 The BNV Scenario

The most general superpotential which can be written down for the MSSM includes 4 $R$-parity odd terms:

$$W^\text{MSSM}_{\text{RPV}} = \frac{1}{2} \lambda_{ijk} \epsilon^{ab} L^i_a L^j_b \bar{E}^k + \lambda_{ijk}' \epsilon^{ab} q^i_b Q^j_b \bar{D}^k + \frac{1}{2} \lambda''_{ijk} \epsilon^{\alpha_1 \alpha_2 \alpha_3} \bar{u}^i_{\alpha_1} D^j_{\alpha_2} D^k_{\alpha_3} + \kappa_i L^i H^a_2$$  \hspace{1cm} (1)

where $i, j, k$ run over generations, $a, b$ are $SU(2)_L$ isospin indices, and $\alpha(i)$ runs over colours.

In a $B$-conserving theory like the SM or the $R$-conserving MSSM, there is no colour antisymmetric perturbative interaction term, i.e. no term with a colour structure like that of the UDD term (the third term in the above equation). Apart from extreme occurrences, like knocking two valence quarks out of the same proton in different directions, by two simultaneous but separate interactions, normal high-energy events would therefore not fully display the antisymmetric colour structure of the proton. So what is different about the UDD term is that it allows the production of three colour carriers at large momentum separation, without the creation of corresponding anticolour carriers. It is the necessary $SU(3)$ gauge connection between these three partons that will lead us in the development of the nonperturbative framework.

A further point about the UDD term is that the contraction of the $\epsilon$ tensor with $D^i D^k$ implies that $\lambda''_{ijk}$ should be chosen antisymmetric in its last two indices, since a $(j, k)$-symmetric part would cancel out.

The part of the Lagrangian coming from the UDD superpotential term in which we are interested is:

$$\mathcal{L}_{\text{BNV}} = \frac{1}{2} \lambda''_{ijk} \epsilon^{\alpha_1 \alpha_2 \alpha_3} \left( \bar{u}^i_{\alpha_1} (\bar{d}^*)^j_{\alpha_2} (d^c)^k_{\alpha_3} + \bar{d}^i_{\alpha_1} (\bar{u}^*)^j_{\alpha_2} (d^c)^k_{\alpha_3} - (j \leftrightarrow k) \right) + \text{h.c.}$$  \hspace{1cm} (2)

where we have made the choice of not yet using any of the antisymmetry requirements, so that the ordinary Einstein summation convention applies.

Combining the vertices in eq. (2) with the full MSSM Lagrangian, also decays involving one or more gauge couplings are clearly possible, e.g. neutralino decay via $\tilde{\chi}^0 \rightarrow \tilde{q}_i (\rightarrow \tilde{q}_j \tilde{q}_k) \tilde{q}_i$. The BNV SUSY decay processes currently implemented in PYTHIA, with Born level matrix elements as calculated by [4], are:

1) $\tilde{d}_{jn} \rightarrow \tilde{u}_i d_k$  \hspace{1cm} (36)
2) $\tilde{u}_{in} \rightarrow \tilde{d}_j d_k$  \hspace{1cm} (18)
3) $\tilde{\chi}^0_n \rightarrow \tilde{u}_i d_j d_k$  \hspace{1cm} (144)
4) $\tilde{\chi}^+_n \rightarrow u_i u_j d_k$  \hspace{1cm} (30)
5) $\tilde{\chi}^0_n \rightarrow \tilde{d}_i d_j d_k$  \hspace{1cm} (14)
6) $\tilde{g} \rightarrow u_i d_j d_k$  \hspace{1cm} (36)

where $n$ runs over the relevant mass eigenstates: $n \in \{L, R\}$ for the first two generations of squarks, $n \in \{1, 2\}$ for the third generation squarks and the charginos, and $n \in \{1, \ldots, 4\}$ for the
neutralinos. The numbers in brackets are the number of modes when summed over \( n, i, j, \) and \( k, \) and over charge conjugate modes for the Majorana particles.

When calculating the partial widths (and hence also the rates) into these channels, we integrate these matrix elements over the full phase space with massive \( b \) and \( t \) quarks, massive \( \tau \) leptons, and massive sparticles. All other particles are only treated as massive when checking whether the decay is kinematically allowed or not.

A feature common to the \textsc{Herwig} and \textsc{Pythia} implementations is how double-counting in the BNV three-body modes is avoided. The diagrams for these modes contain intermediate squarks which may be either on or off the mass shell, depending on the other masses involved in the process. If a resonance can be on shell, we risk doing double counting since \textsc{Pythia} is then already allowing the process, in the guise of two sequential \( 1 \to 2 \) splittings. In particular, this means that the list of \( 1 \to 3 \) BNV widths obtained by a call to \textsc{PyStat}(2) only represent the non-resonant contributions, the resonant ones being accounted for by sequences of \( 1 \to 2 \) splittings in other parts of the code.

## 3 BNV Colour Topologies

Up till now we have considered short-distance processes, where perturbation theory provides a valid description in terms of quarks, gluons and other fundamental particles. At longer distances, the running of the strong coupling \( \alpha_s \) leads to confinement and a breakdown of the perturbative description of QCD processes. The perhaps most successful and frequently model for the transition from the description in terms of quarks and gluons to a description based on hadrons is the Lund string fragmentation model \cite{5}.

This approach has not before been applied to the colour topologies encountered in BNV. Therefore we here extend the model by the introduction of a junction, where three string pieces come together, c.f. figure \ref{fig:bnv_junction}. Effectively, it is this junction that carries the (anti)baryon number that is generated by a BNV process. The hadronization in the region around the junction will therefore be of special interest.

In figure \ref{fig:bnv_junction}, the central black dot represents such a junction, and the dashed lines show the string pieces stretched between the junction and each endpoint quark, across emitted gluons, resulting in a Y-shaped topology. In the simplest picture of fragmentation, each string piece is broken by the formation of a number of \( q\bar{q} \) pairs along the string. The end-point quark of each
piece then pairs up with the closest $\bar{q}$ (in colour space) to form a meson, leaving a new unpaired $q$ which pairs up with another $\bar{q}$, and so on until almost all the energy stored in each string piece is used up. From this picture, it is evident that the fragmentation eventually produces 3 unpaired quarks, one on each side of the junction. By colour conservation, with the split off mesons being colour singlets, these 3 quarks are in a colour-antisymmetric state, i.e. a baryon. In the following, we refer to this baryon as the “junction baryon”.

It could have been interesting to contrast the junction concept with some alternatives, but we have been unable to conceive of any realistic such, at least within a stringlike scenario of confinement. The closest we come is a V-shape topology, with two string pieces, similar to the configuration in a qgq topology. This would be obtained if one e.g. imagined splitting the colour (anti-colour) of one of the final state quarks (antiquarks) into two anticolours (colours). In such a scenario the baryon would be produced around this quark, and could be quite high-momentum. Of course, such a procedure is arbitrary, since one could equally well pick either of the three quarks to be in the privileged position of producing the key baryon. Further, with two string pieces now being pulled out from one of the quarks, the net energy stored in the string at a given (early) time is larger than in the junction case, meaning the Y junction is energetically favoured over the V topology. For these reasons, the V scenario has not been pursued.

### 3.1 Fragmentation of Junction Strings

As mentioned, the kind of string configuration depicted in fig. 1 has not previously been a part of Pythia, thus we here outline the technical aspects of the fragmentation process step by step. A more comprehensive description will be contained in [4].

In the rest frame of the junction the opening angle between any pair of quarks is $120^\circ$, i.e. we have a perfect Mercedes topology. This can be derived from the action of the classical string [6], but follows more directly from symmetry arguments.

Using this requirement, the rest frame of the junction can easily be found for the case of three massless quarks (and no further gluons), but the general massive case admits no analytical solution. Rather, we use an iterative, numerical procedure.

When gluon emission is included, the junction motion need not be uniform. Consider e.g. an event like the one in fig. 1. Here the quarks each radiated a gluon, and so the strings to the junction are drawn via the respective gluons. It is the direction of these gluons that determines the junction motion at early times, and the directions of the quarks themselves are irrelevant. As a gluon moves out from the junction origin, it loses energy to the string. From the point when it has lost all its energy and onwards, it would then be the direction of the respective quark, and not of the gluon, that defines the pull on the junction, resulting in a “jittering around” of the junction. Naturally, this also applies in the general case where an arbitrary number of gluons is emitted.

Rather than trying to trace this jitter in detail — which anyway will be at or below the limit of what it is quantum mechanically meaningful to speak about — we define an effective pull of each string on the junction as if from a single particle with a four-momentum

$$p_{\text{pull}} = \sum_{i=1}^{n} p_i \exp \left( -\sum_{j=1}^{i-1} E_j / E_{\text{norm}} \right).$$

(3)

Here $i = 1$ is the innermost gluon, $i = 2$ is the next-innermost one, and so on up to $i = n$, the endpoint quark. The energy sum in the exponent runs over all gluons inside the one considered (meaning it vanishes for $i = 1$), and is normalized to a free parameter $E_{\text{norm}}$, which by default we associate with the characteristic energy stored in the string at the time of breaking. Note
that the energies $E_j$ depend on the choice of frame. A priori, it is the energies in the rest frame of the junction which should be used in this sum, yet since these are not known to begin with, we employ an iterative procedure.

Since the string junction is a very localized part of the full string system, it is not desirable that the hard part of the fragmentation spectrum of each string, i.e. the hadrons produced close to the endpoint quark, should be significantly affected by the presence of the junction. In particular, if we consider events where each of the three outgoing quark jets have large energies in the junction rest frame, the production of high-momentum particles inside a jet should agree with the one of a corresponding jet in an ordinary two-jet event. This can be ensured by performing the fragmentation from the outer end of the strings inwards, just like for the ordinary $q\bar{q}$ string. Thus an iterative procedure can be used, whereby the leading $q$ is combined with a newly produced $\bar{q}_1$, to form a meson and leave behind a remainder-jet $q_1$, which is fragmented in its turn. Flavour rules, fragmentation functions and handling of gluon-emission-induced kinks on the string are identical with the ones of the ordinary string.

While these hadronization principles as such are clear, and give the bulk of the physics, there is a catch: if all three strings are fragmented until only little energy and momentum remain in each, and then these remainders are combined to a central baryon, what guarantees that this baryon obtains the correct invariant mass it should have?

In this brief summary, we are forced to refer the reader to [4] for the technical details pertaining to the answer to this question. The end result is that a physical mass for the junction baryon is obtained by first fragmenting two of the three strings from the respective end inwards, towards a fictitious other end. In order to have a large-mass system left for the system in which energy-momentum conservation will eventually be imposed as a constraint, we prefer to pick these two to be the ones with lowest energy, as defined in the junction rest frame. As hadrons are successively produced in the fragmentation, their summed energy (in the same frame) is updated. Once the hadronic energy exceeds the string energy, the fragmentation has gone too far, i.e. it has passed the junction point of the string system, so it is stopped and the latest hadron is rejected.

When two acceptable hadronic chains have been found, the remaining four-momenta from the respective two strings are combined into a single parton (diquark), which then replaces the junction as endpoint for the third string. If the new parton does not turn out to be spacelike, the fragmentation procedure for this string is then identical with that of an ordinary string from here on. Otherwise, the fragmentation is restarted from the beginning. Note that popcorn baryon production may result in the splitting off of a meson from the initial diquark to produce a new diquark. That is, the baryon number may then migrate to higher energies than otherwise, but will still be rather centrally produced.

At this point, it is interesting to see how dependent our model is on the implicit assumptions that go into it, for example the definition of the junction pull vector, eq. (3), and the choice of the two least energetic string pieces as the ones to be fragmented first in the fragmentation scheme described above.

The variation of the CM momentum spectrum of primary hadrons and junction baryons under changes to these assumptions are shown in figure 2 from which it is apparent that the model does not suffer from stability problems. Observe also that our earlier remarks that the junction baryons would be rather centrally produced are quantified here in the much sharper peaking (notice the log scale) of the junction baryon momentum distribution as compared to that of the primary hadrons. With respect to the normalization difference between the two sets of curves, it is chiefly due to the many mesons produced in the fragmentation. The junction baryons in fact roughly double the total number of baryons in the momentum region below $\sim 2$
Figure 2: Momentum spectra of primary hadrons and junction baryons in the decay of a 96 GeV neutralino to three quarks. Results with the default implementation are compared with five alternative ones. $E_{\text{norm}}$ refers to the normalization energy in eq. (3). Average multiplicities of primary hadrons are shown in the lower right corner of the plot.

GeV. This gives us our first hint of how to search for this “smoking-gun” evidence of BNV.

As a final comment, it should be mentioned that more complicated topologies than the ones so far mentioned are possible. Specifically when two colour-connected BNV processes occur, there will either be two junctions with a string spanned between them or the two baryon numbers will cancel against each other and give rise to two unconnected $q\bar{q}$ string pieces. In the current PYTHIA implementation, we assume that the junction-junction string topology dominates over the non-junction one, essentially since we expect the string length, and hence the total string energy, to be smaller more often for the former topology than for the latter.

4 Conclusion

It has not previously been possible to study baryon number violating decays of SUSY particles within the PYTHIA framework, essentially because it lacked a hadronization mechanism for colour configurations containing non-zero baryon number. From PYTHIA 6.207 on this is now possible, and the various aspects of the implementation have been described in broad terms here. Details will be available in [4] and in the PYTHIA manual. The hadronization is based on a physical picture and shows negligible model dependence. Furthermore, it allows us to “predict” that the smoking-gun evidence of baryon number violation, an excess of baryons, should be looked for in baryons having small momenta relative to their parent sparticle.
References

[1] H.K. Dreiner, P. Richardson and M.H. Seymour, JHEP 0004 (2000) 008;

[2] S. Moretti, K. Odagiri, P. Richardson, M.H. Seymour and B.R. Webber, JHEP 0204 (2002) 028.

[3] B.C. Allanach, A.J. Barr, L. Drage, C.G. Lester, D. Morgan, M.A. Parker, B.R. Webber and P. Richardson, JHEP 0103 (2001) 0248;
    B.C. Allanach, A.J. Barr, M.A. Parker, P. Richardson and B.R. Webber, JHEP 0109 (2001) 021.

[4] T. Sjöstrand and P. Skands, in preparation.

[5] B. Andersson, G. Gustafson, G. Ingelman and T. Sjöstrand, Phys. Rep. 97 (1983) 31;
    B. Andersson, ‘The Lund Model’ (Cambridge University Press, 1998).

[6] X. Artru, Nucl. Phys. B85 (1975) 442.