Simulation of barchan dynamics with inter-dune sand streams

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Abstract. A group of barchans, crescent sand dunes, exhibit a characteristic flying-geese pattern in deserts on Earth and Mars. This pattern implies that an indirect interaction between barchans, mediated by an inter-dune sand stream, which is released from one barchan’s horns and caught by another barchan, plays an important role in the dynamics of barchan fields. We used numerical simulations of a recently proposed cell model to investigate the effects of inter-dune sand streams on barchan fields. We found that a sand stream from a point source moves a downstream barchan laterally until the head of the barchan is finally situated behind the stream. This final configuration was shown to be stable by a linear stability analysis. These results indicate that flying-geese patterns are formed by the lateral motion of barchans mediated by inter-dune sand streams. By using simulations we also found a barchan mono-corridor generation effect, which is another effect of sand streams from point sources.

The morphological shape of a sand dune found in a desert is determined by the direction of the wind and the amount of sand available on the ground [1, 2]. Among sand dunes, certainly the most impressive structures are crescent sand dunes, called barchans. The characteristic crescent shape of a barchan consists of a slip face at the lee surface and two horns that point in a downwind direction [1, 3, 4].

The size of barchans in deserts ranges from 1 to 30 m in height, from 10 to 300 m in width and from 10 to 300 m in length. It is empirically known that both the width and length of a barchan are proportional to its height [3–11]. They are highly mobile, and man-made
structures, such as roads and pipelines that lie in their path, are occasionally buried in sand. This unstoppable march of barchans has been a long-standing issue for inhabitants of dry environments. Field observations have found that the migration velocity of a barchan is inversely proportional to its height, and small barchans move faster than large ones. This velocity difference causes collisions between barchans. Such collisions between two barchans have recently attracted researchers’ interest [12–15]. Elbelrhit et al [16] have reported a collision between barchans observed in the field. Endo et al [17], Hersen [19] and Douady [18, 19] have performed laboratory experiments using water tanks to investigate the collision processes between two barchans. Katsuki et al [14, 20] created computer simulations of such collisions using a simplified cell model, and Hermann’s group [13, 15] created a more detailed model. Katsuki et al have shown that a variety of collision patterns observed by water tank experiments can be successfully reproduced by their cell model.

Looking at aerial photographs of a field of barchans, it can be seen that a group of them form a characteristic pattern in which the horns of each barchan point to the center of the barchan(s) on its leeward side(s). As a result, they form a convoy with a triangular pattern similar to that of flying geese or align themselves in a slanted line (figure 1). Barchans are also found in many other places such as on the sea floor and on the surface of Mars, and this pattern has also been seen there.

The formation of the triangular pattern should be attributed to some interaction between a given barchan and its leeward barchan neighbors. A candidate for such an interaction between barchans is the indirect interaction mediated by an inter-dune sand stream, which is released by a given barchan and captured by another one situated downwind of it. The effects of this indirect interaction, however, have largely been overlooked and are still poorly understood. In this paper, we focus on this inter-dune interaction and investigate whether it provides a mechanism for the dune pattern.

Since the typical time scale of dune dynamics is several decades or even longer, a very-long-term observation period is required to investigate how barchans interact with each other in deserts. Performing numerical simulation is thus an effective method for the study.
of dune dynamics, and to do so, it is desirable to have a good numerical model. Recently, Katsuki et al proposed a new numerical model based on the cell model to simulate dune dynamics [14], [21–25]. It has been shown that the model captures qualitative features of dune dynamics well and can reproduce diverse phenomena observed in both deserts and water tanks, such as collisions between two barchans and the emergence of barchans from a sand bed. We employ this model here to investigate indirect interactions between barchans.

In the cell model, the dune field was divided into square cells [14], [21–25]. Each cell represented an area of sandy ground that was sufficiently larger than a grain of sand. A field variable $h(x, y, t)$, expressing the local surface height, was assigned to each cell; $t$ denoted discrete time steps, and spatial coordinates $x$ and $y$ were the central positions of the cell in the flow direction and the lateral direction, respectively.

The model consists of the following two elementary processes: saltation and avalanche. Saltation is a process in which sand grains are transported by a flowing fluid. This process is modeled by a simple transportation rule without considering the details of fluid dynamics. In each time step, a mass $q$ is transported downstream from a cell $(x, y)$ to the target cell $(x + L, y)$. Hence, the height of the taking-off cell changes as $h(x, y) \rightarrow h(x, y) - q$ and the height of the target cell changes as $h(x + L, y) \rightarrow h(x + L, y) + q$. According to field studies of sand transport, the saltation length $L$ is dependent on the height $h$ and a grain having taken off high off the ground travels a long distance before it lands. We used a specific functional form for the height dependence of the saltation length:

$$L(h) = a + bh(x, y, t) - ch^2(x, y, t),$$

where $a = 1.0$, $b = 1.0$ and $c = 0.01$ are phenomenological parameters. The saltation mass $q$ was fixed as 0.1 for simplicity. We further assumed that saltation occurs only on upwind faces.

Avalanche is another process, in which sand slides down along the steepest slope when a local slope becomes steeper than the angle of repose. The simulation steps of avalanche are as follows:

(i) Mark the pair of local slopes that exceeded the angle of repose from all the nearest-neighbor and next-nearest-neighbor pairs of cells.

(ii) Select the lowest cell $(x'_{nn}, y'_{nn})$ around a marked cell $(x, y)$.

(iii) Half of the excess sand of the marked cell, $q_A$, is transported to the cell $(x'_{nn}, y'_{nn})$:

$$q_A = 0.5(h(x, y, t) - h(x'_{nn}, y'_{nn}, t) - \tan 34^\circ).$$

This procedure is repeated until all the cells finally satisfy the stability condition. One time step consisted of saltation at all the cells performed at once, followed by avalanche. The angle of repose is fixed at $34^\circ$ in the simulations.

Our main interest lies in how a sand stream from a given barchan affects the motion of another one on its leeward side that captures the sand. Considering that the sand is released mainly from the tips of the barchan’s horns, we investigated the effects of sand streams from a point source instead of explicitly placing a barchan on the windward side of another one. The sand source was set at the upwind end of the simulation field. A leeward barchan was initially situated so that its center line was displaced laterally from the point source (see figure 2). In order to maintain the size of the barchan, the same amount of sand as that escaping from the downwind end of the numerical field was supplied from the point source. In this simulation, the effect of the collision of barchans and the lateral diffusion of the sand flux from the upwind...
Figure 2. Snapshots of the lateral motion of a barchan behind the point source. The distance from the point source is $\delta x_l = 0.6W_l$, where $W_l$ is the length of the barchan. The amount of sand supplied is the same as that out of the numerical field. The barchan moves laterally toward the sand stream.

Figure 3. Lateral motion of the center of mass of a barchan for different initial positions. The sand source is set at $y = 100 (= Y_{\text{max}}/2)$, $\delta x_l = 0.0$ ($\triangledown$), $0.2L_l$ ($\bigcirc$), $0.4L_l$ ($\bigtriangleup$) and $0.6L_l$ ($\Box$). The barchan moves under the point source. The solid line shows the solution of equation (4) for the same initial positions. Inset: typical sand mass distribution of the barchan cumulated along the flow direction ($x$-direction). We used $\phi(y_s) = 0.358$ 854 as the supplied sand given from numerical simulation.

horns were not taken into account. The simulation field consisted of 200 (windward) by 200 (lateral) cells. The center of the field was set as the origin of the barycentric coordinate. The initial barchan was made from a Gaussian sand pile with a variance $\sigma = 50$ and a height $h_G = 5$.

Figure 2 shows snapshots of the motion of the barchan for the initial lateral displacement from the point source $\delta y = 0.3w$, where $w$ is the width of the barchan. Figure 3 shows the lateral motion of a barchan for several different initial positions. In all cases, the barchan migrates laterally with time and its shape and size are kept unchanged. When it reaches right behind the
sand source, the lateral motion terminates and the barchan stays there irrespective of its initial position. In order to discuss the mechanism of the lateral migration, we measured the amount of sand released from each of the two horns. Figure 4 shows the ratio of the sand mass from the left horn to the total mass released at each time step, and this is given by

$$Q_{\text{ratio}} = \frac{Q_{\text{left}}}{Q_{\text{left}} + Q_{\text{right}}},$$

where $Q_{\text{left}}$ is the mass of sand from the left horn and $Q_{\text{right}}$ is that from the right horn, respectively. We found that about the same amount of sand is released from both horns. At first glance, this result appears to be counterintuitive. Consider the left half and the right half of the barchan separately. We term the right half of the barchan, as can be seen at $t = 0$ in figure 2, as the supplied part, and we term the left half as the non-supplied part, because the sand stream from the sand source is captured by the right half. If most of the supplied sand is deposited on the supplied part, it may naturally be expected that more sand would be released from the horn of the supplied part than from that of the non-supplied part. However, the simulation shows that this is not the case. What the result actually suggests is that the sand captured by the supplied part is sufficiently mixed inside the barchan to finally produce a symmetric sand stream from the two horns. Therefore, some amount of sand in the supplied part is transported to the non-supplied part. In the present model, this lateral movement of sand takes place only through avalanches. As seen in the figures, the morphology of the barchan dune remains stable, and this is almost certainly a result of this reconfiguration of the sand.

Based on the above simulation results for a point sand source, we next developed a phenomenological theory for the lateral motion of a barchan driven by a sand stream. For this purpose, we extended the model introduced by Lima et al [26] so that the barchan can take an arbitrary shape. As was seen in the simulation, the barchan retains its shape unchanged as it
Figure 5. Snapshots of the sand stream from the point source. (a) No barchan appears with little sand ($\phi(y) = 1$). (b) As the sand supply increases, barchans start to appear ($\phi(y) = 10$). (c) When more sand is supplied, a number of barchans are generated ($\phi(y) = 30$). The simulation field consists of 2048 (windward) by 128 (lateral) cells with an open boundary condition.

The lateral velocity of the center of the barchan, $\dot{y}_c$, is written as follows:

$$\dot{y}_c = \frac{\int_{y_c-w/2}^{y_c+w/2} y \{\phi(y) + f(y - y_c)\} \, dy}{\int_{y_c-w/2}^{y_c+w/2} \{\phi(y) + f(y - y_c)\} \, dy} - y_c,$$

where $w$, $\phi(y)$ and $f(y)$ are the width of the barchan, the sand supply per unit time and the cumulative sand mass distribution of the barchan, respectively. Equation (4) agrees with the model of Lima et al. at $|y| < w/2$ and $f(y) = 0$ otherwise. The initial distribution of the barchan $f(y)$ is given from numerical simulation. The inset of figure 3 shows the distribution of the sand cumulated along the flow direction ($x$-direction) after the sand supply $\phi(y)$ becomes steady because of the quasi-periodic boundary condition. The solid lines shown in figure 3 express calculation results of equation (4). This theory is in good agreement with the simulation.

The linear stability analysis around the fixed point $\bar{y} = y_s$ is straightforward. Since the shape of the barchan is symmetrical, we assumed that the sand distribution of the barchan $f(y)$ with respect to the center line $y = y_c$ is also symmetrical and invariant with time. According to the linear stability analysis, the fixed point is always stable provided $q > 0$. In conclusion, the barchan in this simulation was found to be driven laterally by a sand stream toward the sand source and to stay right behind it.

The effects of sand streams from a point source are not restricted to lateral shifts of barchans. Mono-corridors of barchans generated from a point-like source have been found by aerial observation [27]. Thus we also used simulations to investigate what would occur if there were only the point sand source. Our simulation showed that when there is a small amount of sand supplied, it merely slides across the field without forming a barchan (figure 5(a)). As the amount of supplied sand increases, bumps start to appear in places and they eventually form barchans (figure 5(b)). When more sand is supplied, many barchans appear and grow as they move downwind (figure 5(c)). A mono-corridor resembling that observed in the field is thus produced from the point sand source. This result also implies that barchans are bred from the horns of a large barchan, since the tip of a horn can be regarded as a point sand source.

In summary, we investigated the interaction dynamics of barchan dunes, focusing on the effects of indirect interactions mediated by an inter-dune sand flow. We showed that a barchan can be driven laterally with a sand stream to right behind the point source. The principal mechanism of this motion is a fast mixing of sand in a barchan that keeps the symmetric shape...
unchanged. This result implies that the flying-geese-like pattern (the triangular pattern) of the barchans seen in dune fields is a consequence of self-organization induced by the sand streams from barchans ‘horns’, and a network formed by horn-head sand streams will be key to the understanding of barchan field dynamics.

The configuration of two barchans such that the center of the leeward barchan is located near the horn of the windward one can be formed by other mechanisms. In fact, such patterns have been observed both in water-tank experiments and in numerical simulations of barchan collisions when the relative size and configuration of two barchans are appropriately set [15], [17–19]. Fragmentation of megabarchans due to wind fluctuations on the surface [16] also leads to a similar pattern. In both cases, the pattern is formed through a process in which a new barchan is created from a horn of another barchan. The size of the new barchan, however, is in general much smaller than that of the upwind one. Moreover, it is unlikely that a long slanted line of many barchans is formed by successive collisions. We thus think that the interaction mediated by the sand stream plays an important role in forming the flying-geese pattern.

We also showed that a mono-corridor of barchans can emerge from a point sand source when a sufficient amount of sand is supplied. So far, in most cases there has not been a focus on non-uniformity in sand streams in field observations. As was shown in this paper, however, it can have a critical effect on the dynamics of barchan fields. The present results on inter-dune sand streams will also be useful in analyzing geographical and environmental conditions on Earth and on the surface of Mars. Much work needs to be done in this direction, especially with field observations and water-tank experiments.

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