All the fifty years of conscious brooding
have brought me no closer to the answer to the question,
“What are light quanta?”
Of course today every rascal thinks he knows the answer,
but he is deluding himself.
A. Einstein, 1951

Structure Functions for the Virtual and
Real Photons\textsuperscript{1}

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Abstract. Development of concepts related to the photon and its high energy hadronic
interaction is briefly reviewed. A photon considered as an ideal probe of hadron struc-
ture, paradoxically is also considered as an ideal target to test the perturbative QCD.
The present status of the theoretical and experimental analysis of the structure func-
tions for a virtual and real photon is presented.

100 YEARS OF LIGHT QUANTA

The photon is the best known boson and one of the oldest elementary particles.
This year is special - we celebrate 100 years of light quanta; below I list the crucial
dates in the development of notion of the photon \cite{1}:
\begin{itemize}
  \item 1900 - Planck’s hypothesis of quantum of electromagnetic energy, \( E = h\nu \)
  \item 1905 - Einstein’s hypothesis of quantum of light (\( \gamma \)), \( E = h\nu = pc \)
  \item 1915 - Millikan’s experiment: photo-emission from metal
  \item 1922 - Compton experiment: \( \gamma e \rightarrow \gamma e \)
\end{itemize}
In the next years (1925-7) the Quantum Electrodynamics (QED) was invented by
Born, Heisenberg, Jordan, Dirac and others \cite{2}, with photon playing the role of a
gauge particle of electromagnetic interaction. Few years later (1931) Wigner gave
the complete group theoretical description of angular-momentum states, according
to which photon means helicity states of spin 1 massless particle \cite{3}. The current
name: the photon was given in 1926 by the chemist G.N. Lewis \cite{4}.

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Photon-hadron interaction

The photon properties, as follows from Quantum Electrodynamics, are well known: the photon is a massless, chargeless object with a pointlike coupling to the charged fundamental particles. As such, it is an ideal tool for probing structure of more complicated objects, for example hadrons, acting as a microscope with the resolution given by its wavelength.

However, in the high energy photon-hadron interaction there are phenomena which can (should?) be interpreted in terms of “hadronic (partonic) structure” of the photon. This way one can effectively describe leading contributions to the rates of certain processes. Some early ideas and facts are recalled below:

• 1960-72 - observation of hadronic properties of the photon in soft processes, the $\rho(\omega, \phi)$-photon analogy, Vector Dominance Model (VDM) (also GVDM) [5]
• 1969-71 - an importance of $\gamma\gamma \rightarrow$ hadrons processes in $e^+e^-$ collisions [6]
• 1970-74 - a deep inelastic scattering on the real photon [7]a) and Parton Model predictions for structure functions $F_1^{\gamma(*)}, F_2^{\gamma(*)}, g_1^{\gamma}$ [7]b)
• 1977-80 - asymptotic (point-like) solution in QCD (LO [8] and NLO [9] results); structure functions of a real photon as a unique test of QCD
• 1979-84 - singularities in the asymptotic solutions for a real photon at small $x$; negative $F_2^{\gamma}$ in the NLO QCD analysis [9–12]
• 1981-84 - hadronic contribution to $F_2^{\gamma}$ as a cure of the problem at small $x$ [11,12]
• 1981-84 - structure functions of virtual photons (singularity free LO and NLO predictions) - a unique test of QCD [10,12]
• 1989-93 - relation of spin-dependent structure functions of photons to QED and QCD anomaly [13].

Starting from 1981 structure functions of unpolarized real and virtual photons, $F_2^\gamma$ and $F_{\gamma eff}^\gamma$, respectively, are being measured. Ten years later the first data on the production of large $p_T$ particles and jets in the photon-induced processes, so called resolved photon processes, have appeared for both real and virtual photons. Review of data can be found in [14], [15].

The photon A.D.2000

The above short outline of historical development of basic concepts related to the hadronic ”structure” of the photon shows how in the past high expectations were followed by deep defeats. Even nowadays the situation is far from being clear. Although for many hadronic processes involving photons there exist already NLO QCD calculations, a proper way of describing the photon interaction is still a subject of ongoing discussions, e.g. how to count the order of the perturbation [16]. Still, after so many years of photon physics there is a lot of confusion, even terminology seems to be inadequate and generates additional problems, e.g. [17]. The fact that a photon has a double face - being a probe and a target, sometimes in the same process, obviously does not help.
The main source of data on the strong (hadronic, partonic) properties of the real photon comes from DIS $e\gamma$ experiments in the $e^+e^-$ collisions. Recent results on the structure function $F_2^\gamma$, based on few years runs at LEP1 and TRISTAN at the CM energy $\sim 90$ and $60$ GeV, respectively, are now available. There are also new data taken at LEP at higher energies. Altogether the existing data cover a wide range of the (average) $Q^2$ from 0.2 to $\sim 706$ GeV$^2$ [18]. The range of the (center of bin) $x_{Bj}$ variable extends from $\sim 0.001$ [19] to 0.98. Recently the first dedicated measurement of $F_2^\gamma$ has been performed at LEP2, see [20]. In addition there exist data of the leptonic structure functions, see e.g. [14].

The complementary data on the photon structure are coming from measurements of the resolved real photon processes, i.e. production of large $p_T$ jets (also individual hadrons, photons, heavy quarks), in $\gamma\gamma$ collisions at $e^+e^-$ machines and in the photoproduction at the $e^\pm p$ collider HERA ($\sqrt{s} \sim 300 - 320$ GeV), see [21].

The determination of the hadronic structure function $F_2^\gamma$ for a real photon in $e^+e^-$ collision relies on the unfolding, therefore precision of these data depends crucially on the accurate description of the hadronic final state by the Monte Carlo models. The improvement in the unfolding in DIS-type experiments has been obtained recently, still the dependence on the chosen MC used in the analysis cannot be avoided [22], see also [23]a-b). In the modeling of the final state, one includes [24,25] various initial “states” of the target photon: the photon as a $q\bar{q}$ pair and the photon as a $\rho$ meson, with a relevant structure. The virtual photon (a probe) can interact by its constituents as well, see e.g. [26]. So, in the DIS-type analyses also processes with a resolved $\gamma$ ($\gamma^*$) are being involved.

During the last years data on the “structure of the virtual photon” have appeared, mostly from the resolved virtual photon processes at the $ep$ collider HERA (with the virtuality of the photon from 0.1 to 85 GeV$^2$), see [21]. Fifteen years after the first measurement of the DIS$_{e\gamma}$ events by PLUTO collaboration [27], a new measurement has just been performed at LEP (L3 Coll. [28]). Extraction of the effective structure function or of effective parton densities in a virtual photon from the $e^+e^-$ and $ep$ data is a difficult task, especially if interference terms are large as observed in the OPAL experiment for double-tag leptonic events [29,14].

Basic QCD predictions for processes with real and virtual photon are definitely in agreement with the data. However the discrepancies between the data and predictions of Monte Carlo models for various distributions in the photon-induced processes are observed both in $ep$ and $e^+e^-$ collision. Implementation of the modified transverse momenta distribution of the partons in the $\gamma$ and taking into account multiple parton interaction in the MC programs [23]c) help to describe the data. Nevertheless, the existing data, also these on heavy quark production [30], seem to give us a message that the partonic content of the photon is not properly described by existing parametrizations.

Taking all these facts into account I think it is sensible to start from scratch with a basic introduction to the concept of photon structure functions. I use as a model the “leptonic structure” of a real photon. Next I discuss the $e^+e^-$ environment of the DIS-type experiments for photons, where our basic knowledge comes from. At
this stage the leptonic structure of the virtual photon can be introduced. Then the same steps will lead us to the concept of hadronic structure functions of the photon.

THE "STRUCTURE" OF THE LIGHT QUANTA

In the quantum field theory, a photon, as any elementary particle, can fluctuate into various states consisting of leptons, quarks, $W^\pm$ bosons, hadrons. "...through an interaction with a Coulomb field the photon could materialize as a pair of electrons, $\gamma \rightarrow e^+e^-$. Although not usually thought of in these terms, this phenomenon was the earliest manifestation of photon structure" [5]b).

Leptonic structure of the photon

the target = $\gamma$

One can test leptonic structure of photon in the deep inelastic scattering, $e\gamma \rightarrow e + \text{leptons}$. Let us take a (unpolarized) real photon ($p^2 = 0$) as a target and assume that the probe, highly virtual photon $\gamma^*$, with large virtuality $-q^2 = Q^2$, couples directly to the electric charge of the fundamental particles. To describe this process one can introduce the corresponding structure functions, as for the proton case. The contribution of the lowest order QED process, $\gamma^*(q)\gamma(p) \rightarrow l^+l^-$, Fig.1(left), to $F_2^\gamma|_{l}$ is given by (the box contribution) [6]:

\[
F_2^\gamma|_{l} = \frac{\alpha}{\pi} Q_l^4 x_{Bj} \left[ (-1 + 8 x_{Bj} (1 - x_{Bj}) - x_{Bj} (1 - x_{Bj}) \frac{4m_l^2}{Q^2}) \beta \\
+ [x_{Bj}^2 + (1 - x_{Bj})^2 + x_{Bj} (1 - 3 x_{Bj}) \frac{4m_l^2}{Q^2} - x_{Bj}^2 \frac{8m_l^2}{Q^2}] ln \frac{1 + \beta}{1 - \beta} \right],
\]

where $x_{Bj} = Q^2/2pq$, $m_l$ - lepton mass, $Q_l$ - electric charge, $Q_l = 1$, and $\beta$ is the lepton velocity in the $\gamma^*\gamma$ CM system. For large invariant mass of the $l^+l^-$ system,
\( W \gg m_t, x_{Bj} \) not too close to 0 and 1, and neglecting terms \( \sim m_t^2/Q^2 \) one gets (keeping the leading logarithmic (LL) term)

\[
F_2^\gamma |_{l} = \frac{\alpha}{\pi} Q_l^4 x_{Bj} [x_{Bj}^2 + (1 - x_{Bj})^2] \ln \frac{Q^2}{m_l^2}.
\] (2)

The obtained \( F_2^\gamma |_{l} \) is solely due to the QED interaction. It is large at large \( x_{Bj} \) and it has the characteristic logarithmic rise with \( Q^2 \) from the collinear configuration in the \( \gamma \to l^+l^- \) splitting. In principle one can also introduce a leptonic density in the real photon: in the LLA one gets \( l^+ (x_{Bj}, Q^2) = \frac{\alpha}{\pi} Q_l^2 x_{Bj} [x_{Bj}^2 + (1 - x_{Bj})^2] \ln \frac{Q^2}{m_l^2} \) (the same for \( l^- \)). Here \( x_{Bj} = x \), where \( x \) - the part of the four-momentum of the initial photon taken by its leptonic constituent (Fig.1 (right)).

Leptonic structure of the real photon \( F_2^\gamma |_{l} \) can be measured at future colliders, where beams of energetic real photons can be obtained in the backward Compton scattering (Photon Colliders) [31]. Nowadays the leptonic structure functions of a real photon are measured in various experiments at \( e^+e^- \) colliders. Here (quasi) real photons with a Weizsäcker-Williams energy spectrum play a role of a target. The \( F_2^\gamma |_{l} \) data for \( l = \mu \) together with QED predictions are summarized in Fig.2, from [14].

The basic features of the lowest QED order result (2) will be modified only

**FIGURE 2.** Summary of existing \( F_2^\gamma |_{l}/\alpha \) data, for \( \mu^+\mu^- \) final state, shown with QED predictions for \( P^2=0 \), (left) - as a function of \( x_{Bj} \) for broad \( < Q^2 > \) range, (right) - as a function of \( Q^2 \) for fixed \( x_{Bj} \) bins, from [14].
softly by higher QED corrections. Formally leading logarithmic QED corrections, powers of $\alpha \log Q^2/m_e^2$, can be summed up using the evolution equation in $Q^2$ [32], with an inhomogenous term due to the log $Q^2$-dependence present already in the lowest order QED prediction (2). The resulting collinear QED cascade included in the LLA in the leptonic density $l^+(x, Q^2)$ is represented in Fig. 1(center). Here, starting from the first splitting of the initial photon all emission processes up to the interaction with a probe are based on point-like couplings.

the target = $\gamma^*$

Leptonic structure functions can also be introduced for a virtual photon, i.e. with $|p^2| = P^2 \neq 0$, to be measured in the lepton beams collision. Let us discuss the production of an arbitrary state $X$ in the process $e(p_1)e(p_2) \rightarrow e(p'_1)e(p'_2)X$, via the $\gamma^*(q_1)\gamma^*(q_2)$ collision ($q_1 = p_1 - p'_1, q_2 = p_2 - p'_2$), Fig.3 (left). The corresponding cross section for the unpolarized lepton beams, assuming $|q_{1,2}^2| \gg m_e^2$, and typical conditions in present experiments, is given by (see [6], also [14])

$$E_1E_2 \sigma_{ee \rightarrow eeX} \frac{d^3p'_1d^3p'_2}{d^3p_1d^3p_2} = L_{TT}(\sigma_{eff} + \frac{1}{2}\tau_{TT} \cos 2\phi - 4\tau_{TL} \cos \phi), \tag{3}$$

where helicity states of photons are denoted by $T$ - transverse (+ or -) and $L$ - longitudinal (0). An effective cross section is defined as $\sigma_{eff} = \sigma_{TT} + \sigma_{LT} + \sigma_{TL} + \sigma_{LL}$, where $\sigma_{TT,LT,LT,LL}$ (the first subscript is for the photon with $q_1$) denote the corresponding cross sections, $\tau_{TT,TL}$ - the interference terms. The $\phi$ is the angle between two scattering planes of the scattered electrons in the $\gamma^*\gamma^*$ CM system.

FIGURE 3. The $\gamma^*\gamma^* \rightarrow X$ process in $e^+e^-$ collision (left). The cross section for the $\gamma^*\gamma^* \rightarrow X$ scattering and its relation to the imaginary part of the forward $\gamma^*\gamma^* \rightarrow \gamma^*\gamma^*$ amplitude (right).

One can relate these quantities to the imaginary part of the forward helicity amplitudes $W_{\lambda'_1\lambda'_2,\lambda_1\lambda_2}$ for the process $\gamma^*(\lambda_1)\gamma^*(\lambda_2) \rightarrow \gamma^*(\lambda'_1)\gamma^*(\lambda'_2)$ [6], see Fig.3 (right). Amplitudes with $\lambda_{1(2)} = \lambda'_{1(2)}$ are related to corresponding cross sections; the interference terms correspond to helicity-flip forward amplitudes: $\tau_{TT} = AW_{++,-,-}$ and $\tau_{TL} = A(W_{++,00} - W_{0,+,-})/2$, where $2A = ((q_1q_2)^2 - q_1^2q_2^2)^{1/2}.$
To measure structure functions of virtual photon $\gamma^*$ the double-tag events with $|q_1^2| \gg |q_2^2| \neq 0$ are used. Below I will concentrate on DIS $e\gamma^*$ events with $l^+l^-$ final state, where $Q^2 \gg P^2 \gg \mu^2$ (using a standard notation for a probe $q \equiv q_1$ and $Q^2 = -q^2$, and for a target $p \equiv q_2$ and $P^2 = -p^2$), with $\mu$ - a characteristic scale for studied phenomena (here $m_l$). The structure functions for a polarization-averaged virtual photon target, $F_{\gamma^*1}, F_{\gamma^*2}$ and $F_{\gamma^*L} = F_{\gamma^*2} - 2xF_{\gamma^*1}$ can be introduced, e.g.

$$F_{\gamma^*} = \frac{Q^2}{4\pi^2\alpha} A (\sigma_{TT} + \sigma_{LT} - \frac{1}{2}\sigma_{LL} - \frac{1}{2}\sigma_{TL}).$$

If, after integration of the differential cross section (3) over $\phi$, the contributions of the terms $\tau_{TT}$ and $\tau_{TL}$ vanish, one can relate the measured cross section to an effective structure function for a virtual photon, $F_{\gamma^*eff} \sim \sigma_{eff}$ [6]. This is not the case in the recent OPAL measurement of the $\mu$-pair production [29,14], as can be seen in Fig.4. Here the Monte Carlo measurement of the $\mu$-pair production [29,14], as can be seen in Fig.4. Here the Monte Carlo predictions based on the QED calculations, also for the options with $\tau_{TT}=0$ and $\tau_{TT} = \tau_{TL}=0$, to test relevance of the interference terms, are displayed.

Large (negative) interference terms found for $x > 0.1$ in double-tag leptonic events at LEP1, as apparent from Fig. 4, make the extracting of the corresponding leptonic structure function for $\gamma^*$ unfeasible, and also shed a light on potential problems in extracting the hadronic structure function for $\gamma^*$.

Note that for $P^2 \to 0$ only one interference term, $\tau_{TT}$, remains in cross section (3). It is called also the $F_3^\gamma$ structure function [7][b]. In the lowest order QED it has the scaling property, $F_3^\gamma | = -\alpha/\pi Q^4 x_Bj$, so it should not influence significantly the extraction of $F_2^\gamma$ for a real photon at large $Q^2$.

**FIGURE 4.** Differential cross section for $\mu^+\mu^-$ production in $\gamma^*(Q^2)\gamma^*(P^2)$ collision at LEP1 (OPAL Coll.). The thick line is a QED prediction of Vermaseren Monte Carlo; predictions represented by the solid line and dot-dashed (dashed) line were obtained by using GALUGA MC program for all contributions and assuming $\tau_{TT} = 0$ ( $\tau_{TT} = \tau_{TL} = 0$), from [29].
Hadronic structure function of the photon

In principle one can introduce also “partonic structure of the photon” since the photon could materialize itself as a pair of quarks. The splitting $\gamma \rightarrow q\bar{q}$ leads to the corresponding photon structure functions already in the lowest order QED, equivalent here to the Parton Model (PM).

The Parton Model prediction for the deep inelastic scattering $e\gamma \rightarrow e + \text{hadrons}$ is based on the process $\gamma^* \gamma \rightarrow q\bar{q}$. Prediction for the (hadronic) $F_2^\gamma$ is given by the formula as for a leptonic final state (eqs. 1 and 2), with some modifications: $m_l \rightarrow m_q$, $Q_l \rightarrow Q_q$ and the color factor, $N_c = 3$, has to appear. For a final state with heavy quarks this is the modification (QPM formulae), for light quarks one usually uses the massless approximation, with the QCD parameter, $\Lambda_{QCD}$, as an argument in the leading logarithm. So, we have in the Parton Model (in LLA) the following expressions for hadronic $F_2^\gamma$ and for the (light) quarks densities:

$$F_2^\gamma = \sum_{q,\bar{q}} Q^2_q x_{Bj} q^\gamma(x_{Bj}, Q^2), \quad (5)$$

$$q^\gamma(x_{Bj}, Q^2) = \frac{\alpha}{2\pi} Q^2_q N_c [x_{Bj}^2 + (1 - x_{Bj})^2] \ln \frac{Q^2}{\Lambda_{QCD}^2}. \quad (6)$$

As previously for leptons, $F_2^\gamma$ has been calculated within the QED - it is proportional to $\alpha$! Both the $x_{Bj}$ and the $Q^2$ dependence are obtained, both are the same as for leptonic final state: large $F_2^\gamma$ value at large $x_{Bj}$, a logarithmic rise with $Q^2$ (here called a scaling violation).

The leading logarithmic QCD corrections introduce logarithmic ($Q^2$) modifications of the basic predictions of the Parton Model (5),(6), and include also a gluonic content of the photon. These corrections can be summmed up by solving the corresponding inhomogeneous evolution equations. By solving them without any input (boundary condition), assuming only that the particular solution of the equation has the $Q^2$ dependence as in PM (eqs. 5-6) one obtains the so called asymptotic solution [8]. It corresponds to the collinear configuration of successive emissions of quarks and gluons (as in Fig. 1 (center)), all of them based on the point-like couplings of QED (the first and the last one), and of QCD.

However, the asymptotic, purly perturbative solutions suffer from power singularities. For example the moments $(f^n(Q^2) = \int dxx^{-1}f(x, Q^2))$ of the non-singlet structure function are given by $f^n(Q^2)|_{\text{asym}} \sim \log Q^2/(1 - d_n)$, where $d_n$ are proportional to the $n$th -moment of the PM term: $[x^2 + (1 - x)^2]$. Simple poles occur for $d_n = 1$, leading after inverting the moments to the following small $x$ behavior of $F_2^\gamma$: $(1/x)^{n=1.596}$ (LLA) and $(1/x)^{n=2}$ (NLLA) [9–12]. The singularities become increasingly severe with higher order of QCD calculation [12]. Already in the NLLA they lead to the negative value of $F_2^\gamma$ [9b].
FIGURE 5. The deep inelastic scattering on the photon in the $\rho$ state.

To cure the problem of singularities in the (asymptotic) structure functions for a real photon, one should include in the calculation also the hadron-like (non-perturbative (NP)) contribution [11,12]. The hadronic properties of the photon are apparent in the soft photon-hadron interaction, where the similarity between photon and vector mesons $\rho, \omega, \phi$ interaction is observed (VMD model [5]). This NP component can be included e.g. in a boundary condition at $Q^2_0$ scale, [11]c,

$$ f^n(Q^2) = \frac{4\pi}{\alpha_s(Q^2)} [1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(Q^2_0)} \right)^{1-d_n}] \frac{\tilde{a}_n}{1-d_n} + \left[ \frac{\alpha_s(Q^2)}{\alpha_s(Q^2_0)} \right]^{-d_n} f_n(Q^2_0), \quad (7) $$

where the input at scale $Q^2_0$ (even $f^n(Q^2_0)=0$!) regularizes the bad behaviour present for $d_n \to 1$, since $\frac{1}{\epsilon} (1 - w^\epsilon) \to -\log w$ for $\epsilon \to 0$. By doing this we get rid of the power singularities for the real photon structure functions, at the same time we lose an “absolute” predictivity of QCD for this quantity.

Equation (7) shows why it is customary to treat the structure functions of the quark densities in the photon as being $\sim \alpha/\alpha_s$ although the primary log $Q^2$ dependence present in the Parton Model (egs. 5-6), which remains also after QCD corrections, has nothing to do with $\alpha_s$. This way of counting changes organization of the perturbation expansion in the QCD calculations, see [16].

In some approaches one treats the photon in the hadronic mode almost as an independent object. Probing the “structure” of the photon in, say, state of $\rho$ can be performed in an analogous way as testing the structure of other hadrons, e.g. in the deep inelastic scattering [25], see Fig.5. The general behaviour of the partonic content of the $\gamma$ in the $\rho$ mode is known - the scaling property in the PM and the logarithmic scaling violation due to the QCD corrections. The corresponding DGLAP evolution equation are homogeneous as for the proton, and an input at some scale is needed to solve the equation, etc...

Various parton parametrizations were constructed for a real photon in the past (there are about 20 of them, see compilations in [14], [15]b). The earlier ones were based on a simple Parton Model formula (for quarks) or the asymptotic solutions. The later parametrizations were based on approaches incorporating hadronic-like (NP) contributions at some stage. Recently parametrizations for a real photon are obtained from parametrizations for virtual photon for $P^2 \to 0$ [25], [34]b).
FIGURE 6. A compilation of the photon structure function $F^\gamma_2/\alpha$ data as a function of $x_{Bj}$ in bins of $Q^2$ (left) and as a function of $Q^2$ for $<x_{Bj}>$ bins (right) compared to the GRV NLO (solid line) and SaS1D (LO) (dashed line) parametrizations of parton distributions in the photon, from [33].

The compilations of the all existing data for the structure function for the real photon [33] are presented in Fig. 5 in comparison with two parton parametrizations, obtained in two different approaches to the hadron-like contributions, GRV [34]a) and SaS [25]. The general behaviour of the $F^\gamma_2$ measured for hadronic and leptonic final states is very similar, compare Fig.5 and Fig.2.

An additional information on the “structure” of the photon is coming from the production of heavy quarks in photon-induced processes. The recent DIS-type measurement at LEP led to the extraction, for the first time, of the charm contribution to $F^\gamma_2$, $F^\gamma_2,c$, see [20]. The QCD description of heavy quark production in processes induced by photons is not satisfactory [30], however similar problem with proper description of a heavy quark production exists also in pure hadronic processes.

the target = $\gamma^*$

The structure function of the virtual photon can be calculated in the Parton Model (QED!) from the $\gamma^*\gamma^* \rightarrow q\bar{q}$ process. For $Q^2 \gg P^2 \gg m_q^2 \left(x_{Bj} = Q^2/2p \cdot q\right)$ one obtains

$$F^\gamma_2(x_{Bj}, Q^2, P^2) = N_c \sum_{q, \bar{q}} \frac{\alpha}{\pi} Q_i^q x_{Bj} \left[ x_{Bj}^2 + (1 - x_{Bj})^2 \right] \ln \left( \frac{Q^2}{P^2 x_{Bj}^2} \right) + 6 x_{Bj} (1 - x_{Bj}) - 2, \quad (8)$$

to be compared with the eq. (1). The corresponding PM quark density in the virtual photon defined in the LL approximation has the form:
\[ q^\gamma(x_{Bj}, Q^2, P^2) = \frac{\alpha}{2\pi} Q^2 N_c \left[ x_{Bj}^2 + (1 - x_{Bj})^2 \right] \ln \frac{Q^2}{P^2}. \]  

(9)

The QCD evolution equations in \( Q^2 \) for the virtual photon are analogous to those for the real photon with the inhomogeneous term given by the corresponding PM expression. In the case of the virtual photon one can solve the evolution equation without the initial conditions. Assuming that for \( Q^2 \gg P^2 \gg \Lambda_{QCD}^2 \) the nonperturbative effects are absent (see ref. [10]) one obtains for moments of the non-singlet structure function

\[ f^\gamma(Q^2) = \frac{4\pi}{\alpha_s(Q^2)} \left[ 1 - \left( \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{1-d_n} \right] \frac{\tilde{a}_n}{1-d_n} \sim \log \frac{Q^2}{P^2}. \]  

(10)

So, without additional experimental or model assumption the definite, singularity free (asymptotic) predictions can be derived for both the \( x \) and the \( Q^2 \) dependence - a unique situation in QCD. Note however that in all recent analyses nonperturbative component in \( F_2^{\gamma^*} \) is introduced [34][b], [25].

Measurements of the structure functions of \( \gamma^* \) can be performed in \( e^+e^- \) collision, as discussed for a leptonic final state. The DIS \( e\gamma^* \) events with hadronic final state were studied experimentally by PLUTO Coll. [27], new data have appeared from LEP (L3 Coll. [28]). In Fig.6 the results from both experiments, corresponding to effective structure functions (cross sections), are presented in comparison with predictions of the QPM, soft VDM, GRS [34][b] models, see however [35].

There are already few parton parametrizations for a virtual photon (see collections in [14], [15][b]), they are valid for \( 0 \leq P^2 \) and become the corresponding parametrizations for the real photon in the limit \( P^2 \to 0 \). All these parametrizations deal with the transversely polarized virtual photon, with one exception of the Chýla parametrization [36] for a longitudinal virtual photon, see also [37].

New insight into the photon structure may come from spin-dependent structure functions [38], not measured so far. Especially the structure function \( g_1^\gamma \) is of great importance, since its first moments (a sum rule for the “spin “ of the photon) involve strong and electromagnetic anomalies, and it is deeply connected with the chiral properties of QCD [13]. It maybe studied at future Linear Colliders [39].

**CONCLUSION AND OUTLOOK**

A photon, considered as an ideal probe of hadron structure, paradoxically is also considered as an ideal target to test the perturbative QCD. Both a real and a virtual photon may reveal “inner structure” in the interaction with other particles. An apparent hadronic structure of the photon is clearly seen in the data. However, there is no full agreement between the standard QCD predictions and experiment for various distributions for hadronic processes induced both by real and virtual photons. During last years we learned that an improvement of the description of the data can be obtained by introducing extra \( p_t \) for constituents in the target
FIGURE 7. The data for the $F_{\gamma\gamma}^\gamma/\alpha$ and comparison with various predictions for the virtual photon. PLUTO Coll. data for $< Q^2 > = 5$ GeV$^2$, and $< P^2 > = 0.35$ GeV$^2$ [27], and L3 Coll. data for $< Q^2 > = 120$ GeV$^2$ and $< P^2 > = 3.7$ GeV$^2$ [28]. The $x$-dependence (upper panels) and $P^2$-dependence (lower panels) are presented.

A virtual proton even more basic questions are open. How important are interference terms, both for $\gamma^*\gamma^*$ and $\gamma^*p$ processes? Do we see more than just the PM content of the virtual photon in present data? If yes, do we need a “structure” of the longitudinal virtual photon?

We have a lot to improve in our description of the photon-hadron interaction.

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