A damage-based approach for the fatigue design of composite structures

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Abstract. The paper illustrates a fatigue design strategy, based on the physics of the damage evolution, under development by the Composite Group at DTG-University of Padova. After a brief introduction, where the motivations of the work are analysed, examples of damage mechanisms at the microscopic scale are discussed. Then the procedures for the quantitative description of these mechanisms are illustrated.

1. Introduction

Composite materials are excellent candidates for the development of reliable lightweight components and structures complying with the needs, for instance, of decreasing fuel consumptions in the transportations field or increasing the specific energy production in the wind energy field.

Most of structural components manufactured with composite materials are subjected during their in-service life to cyclic loadings, which might lead to a progressive damage, a consequent loss of stiffness and residual strength and, eventually, to the final failure. The "design against fatigue" is therefore fundamental to improve the reliability of composite structural parts.

To meet the demand of tools for designing against fatigue of composites in many industrial fields, the author’s group is working since several years to the development of a design framework suitable to predict the initiation of damage, its evolution and the final failure of composite laminates under cyclic loadings.

As a starting point of the discussion, it is important to clarify the concept of "design against fatigue", which can be meant in different ways depending on the requirements of the part under design:

i) design against crack initiation (no damage);
ii) design against stiffness degradation (damage tolerant design);
iii) design against final failure.

In some applications (like for instance fuel and pressure vessels or fuel rails) avoiding the onset of damage is a safety requirement. In this case the capability to predict the life spent for the initiation of the first crack is essential. In other cases like automotive composite frames, turbine blades, bicycle cranks or composite rims, the global stiffness can be the design driver. Due to the several fatigue damage mechanisms, structural composite components can lose up to 30-40% of their initial stiffness (depending on lay-up and load conditions) much before the final failure. Therefore, for a more reliable and cost-effective design, it is important in these cases to estimate the stiffness degradation under the specific loading conditions.
When only the load bearing capability of the part is of interest, the final failure (separation into two or more pieces) is the design target. However, the prediction of the fatigue response of a composite part is not an easy task, particularly when variable amplitude cyclic loadings applied in multiple directions are considered.

As discussed later on in more details, the development of damage in composite laminates is characterised by multiscale and hierarchical processes, from the damage initiation at the microscopic scale to the onset and propagation of macro-cracks and delaminations leading to the final failure. These mechanisms are also dependent on the materials adopted, the lay-up and the loading type and multiaxial conditions [1].

The only way to deal with such a complicated problem and provide reliable design methods is to develop models based on the damage mechanisms actually occurring at the different length scales [1] and link them in a multiscale and multi-mechanism design procedure.

The next sections illustrate, with representative examples, the main mechanisms and damage modes leading a laminate to its degradation and failure under cyclic loadings. Afterwards the modelling strategy under development is presented.

2. Observations of damage mechanisms under fatigue loadings

In this section the damage mechanisms characterising the fatigue life of generic composite laminates are discussed, taking advantage of experimental results and observations collected within recent research programs by the Group and by other researchers in the literature. As already mentioned, the damage evolution in composite materials is a multiscale and hierarchical process, involving several length scales. Therefore the damage mechanisms will be presented at two different scales, i.e. macro- and micro-scales. In this case, the macro-scale is that related to the ply and laminate level, whereas the micro-scale is relevant to the fibres and the inter-fibre spacing.

2.1 Macro-scale mechanisms

If a unidirectional (UD) laminate is tested under a uniaxial off-axis fatigue load, it will fail suddenly, without any indication of damage development, as the number of cycles for the initiation of the first off-axis crack (macro-crack) is reached [2-4]. Here, with the term off-axis crack (or macro-crack) we mean a crack involving the entire thickness of a ply and propagating in a plane containing the fibres, along the fibres direction. Therefore, no off-axis cracks are detectable before the final failure and no stiffness degradation is measured. This does not imply necessarily the absence of damage occurring and evolving before the final failure. In fact, this occurs at a lower scale and does not influence significantly the global stiffness. Conversely, in multidirectional laminates the 90° UD plies are constrained between tougher 0° layers, which prevent the laminate separation when the first off-axis crack initiates in the weakest position. This leads to the initiation of further cracks and their stable propagation in the fibres direction.

![Figure 1](image_url)
This scenario is documented, as an example, by Figure 1 for a [0/60\,2/0/-60\,2]s glass/epoxy laminate [5]. In this case, several off-axis cracks had already initiated after 5000 cycles in both the 60° and -60° layers. Then, at 10000 cycles, the existing cracks had propagated and new ones initiated. The crack density increases until a saturation condition is reached. This is usually followed, or preceded, by the initiation of delaminations, which then propagate for the remaining part of the fatigue life. Even if the initiation of multiple off-axis cracks and delaminations is not critical for the final failure, these mechanisms lead to the degradation of the global elastic properties and can trigger the fibre failure.

Figure 2 shows the trend of the elastic modulus of a [0/50\,2/0/-50\,2]s laminate and the crack density evolution in the off-axis plies. Most of stiffness loss occurs in the earlier stages of fatigue life, where the crack density increases steeply. Then when the crack density reaches a saturated value, the rate of stiffness degradation becomes very low. This is a proof that off-axis cracks are the main responsible for the loss of stiffness, the initiation and propagation of delaminations providing, typically, a limited contribution in this sense.

A schematic of the typical stiffness trend for multidirectional laminates is shown in Figure 3.

Figure 2. Crack density evolution and stiffness degradation in a [0/50\,2/0/-50\,2]s laminate [5].

Figure 3. Schematic of the typical stiffness trend in multidirectional laminates under a fatigue load.
2.2 Micro-scale mechanisms

As discussed in the previous paragraph, the first damage event visible at the macro-scale in composite laminates is the initiation of off-axis cracks. In the case of off-axis UD laminates this event leads to the unstable propagation and sudden failure, whereas in multidirectional laminates the damage evolution process is more complicated, involving several damage mechanisms.

In both cases the initiation of an off-axis crack is preceded by the initiation and evolution of the damage at a lower scale, i.e. the micro-scale. In fact, recognising the presence of irreversible processes at the micro-scale is necessary for justifying the sensitivity of composite materials to a fatigue [6].

With the aim of identifying the microscopic damage mechanisms leading to the initiation of a macro-crack, glass/epoxy [45/-45/0], laminates were tested under a uniaxial cyclic load [7]. The top surface of the 45° layer was accurately polished in order to make the fibres and the matrix clearly visible. At regular intervals the specimens were removed from the machine and their top surface observed under a microscope. It was observed that the first event of damage at the micro-scale, before the initiation of a visible off-axis crack, was the initiation of micro-cracks in the matrix, inclined of a certain angle with respect to the fibres direction (Figure 4a). The accumulation and coalescence of these micro-cracks led then to the initiation of an off-axis crack propagating in the fibres direction. It was shown that also the propagation phenomenon occurred with a similar mechanism, i.e. by the initiation and coalescence of micro-cracks in a region in front of the macro-crack tip [7].

This scenario gives reason of the fracture surfaces observed on glass/epoxy tubes tested under combined tension-torsion loadings, leading to the presence of the transverse ($\sigma_2$) and in-plane shear ($\sigma_6$) stresses [8]. In fact, it was shown that, if the shear stress was high enough with respect to the transverse stress, the fracture surfaces were characterised by a wide presence of shear cusps (figure 4b), reasonably due to the initiation of inclined micro-cracks as those observed on the 45° ply of the [45/-45/0], laminates.

This scenario was observed only if the $\sigma_6/\sigma_2$ ratio ($\lambda_{12}$ from now on) is higher than a certain threshold value, which, for the material tested, was about 0.5.

In fact, the tubes tested with $\lambda_{12}=0$ (pure tension) and 0.5 did not show the presence of shear cusps and the fracture surfaces were quite regular and smooth, as those characterising a mode I driven failure [8].

![Figure 4](image_url)

**Figure 4.** Damage mechanisms at the microscopic scale: a) micrograph of a 45° ply in a [45/-45/0], laminate under tension-tension loadings (frontal view) [7] and b) SEM image of a fracture surface of a tube under combined tension/torsion [8].

After identifying the micro-scale damage modes leading to the initiation and propagation of an off-axis crack, the delamination and fibre failure phenomena were analysed from the microscopic point of
Fatigue tests were carried out on glass/epoxy cross-ply laminates. Again, multiple transverse cracks were seen to initiate and propagate. The crack density saturation was slightly preceded by the initiation of delaminations. Micrographs were taken on the specimens edges, revealing as a typical scenario that shown in Figure 5. As expected, the delaminations were triggered by the presence of the transverse cracks and then propagated between the resin layer at the interface and the 90° fibres.

The role that transverse cracks and delaminations play on the failure of the fibres is clear from Figure 5. In fact, 0° fibres breaks are concentrated in the vicinity of the transverse crack tip and above the delamination. It is therefore reasonable to assume that the fibre failure phenomenon occurs according to the scenario sketched in Figure 6 for a cross-ply laminate. After the first load application, some randomly distributed fibre breaks occur in the weakest positions. As a transverse crack initiates, it causes a strong stress concentration in the 0° ply, promoting the failure of the fibres in the crack tip neighbourhood. Then, as a delamination initiates and propagates, a progressively larger region of the 0° ply is subjected to a higher value of the longitudinal stress, thus leading to further fibre breaks in that region, until a critical condition is reached, which leads to the laminate separation.

![Figure 5](image1.png)

**Figure 5.** Example of delamination from a transverse crack tip with broken fibres.

![Figure 6](image2.png)

**Figure 6.** Schematic of micro-scale delamination- and fibre-related damage.
3. A damage based modelling strategy

According to the experimental evidences deduced from the damage observation at the macro- and micro-scales, the fatigue life of a composite laminate is divided into different phases, each characterized by a dominating damage mode (Figure 7):

- In the first stages, the damage initiates and accumulates at the microscopic scale in the form of micro-cracks, until an off-axis crack forms in the weakest position ($N_{i,c}$);
- Then multiple cracks initiate and propagate causing a steep stiffness reduction, until, thanks to the stress re-distribution between cracks, a saturation condition is reached ($N_{p,c}$);
- Typically, slightly before or after the crack density saturation state, delaminations initiate at the tips of the off-axis cracks ($N_{i,d}$);
- The remaining part of the fatigue life is spent for the propagation of the delaminations ($N_{p,d}$);
- During the entire life, fibre breaks occur, first randomly distributed in the weakest positions, then driven by the critical stress state due to the presence of off-axis cracks and delaminations, until a critical condition occurs and leads to the final failure ($N_f$).

According to the design requirement, for a safe and efficient design against fatigue it is necessary to predict:

i) The fatigue life spent for crack initiation (no damage);

ii) The fatigue life required to produce a certain crack density/delamination length associated to a specific stiffness reduction (damage tolerant approach);

iii) The fatigue life to the laminate failure.

As the final failure is driven by the presence of cracks and delaminations, the last point includes the prediction of the initiation and propagations of off-axis cracks and delaminations and their effect on the development of the fibre breaks.

![Figure 7: Typical sequence of damage mechanisms in multidirectional laminates under fatigue.](image)

In the following sections the models developed by the authors for the prediction of crack initiation, crack density evolution, stiffness degradation and delamination propagation are briefly introduced.

4. A damage based criterion for crack initiation

To predict the initiation of the first crack, as well as the crack density evolution, it is necessary to define a crack initiation criterion, suitable to deal with multiaxial stress states. Such a tool was developed by Carraro and Quaresimin [9] and is briefly recalled in the following. As already mentioned, at a microscopic level, a progressive and irreversible damage process takes place from the very early stages of the fatigue life. Thus, the number of cycles spent for the initiation of an off-axis crack is controlled by the damage evolution occurring at the microscopic level. As a consequence, the definition of a crack initiation criterion consists in the identification of the driving force for the damage evolution at the microscopic scale. To this aim, the concept of the local nucleation plane is introduced in the following. This is defined as the plane in which the micro-cracks initiate in the
matrix, as shown in Figure 4a. The authors have proved that the orientation of this plane is normal to the direction of the maximum principal stress in the matrix, between the fibres [7].

Accordingly, the stress component to be considered as the driving force for this kind of damage is the Local Maximum Principal Stress (LMPS) in the matrix.

When a ply is subjected to a pure transverse stress, the local nucleation plane is normal to the transverse direction. In fact, in this case no shear cusps can be observed in the matrix between the fibres [8]. In addition, Asp et al. [10] proved that in the case of pure transverse tension the local stress state at or close to the fibre-matrix interface is nearly hydrostatic. Therefore, the damage initiation occurs in the form of cavitation-induced matrix cracking and debonding. Asp et al. showed that a suitable criterion for predicting the static failure in the case of pure transverse stress is based on reaching a critical value for the dilatational energy density expressed in Eq. (1):

$$U_v = \frac{1-2\nu}{6E} I_1^2$$  \hspace{1cm} (1)

where $I_1$ is the first invariant of the local stress tensor.

If we assume to extend this finding also to the case of cyclic loadings, a change has to be expected in the leading damage mode, moving from a loading condition near the pure transverse stress to another one, characterized by the presence of a high enough shear stress component. In the former case, the micro-scale damage evolution is assumed to be driven by the Local Hydrostatic Stress in the matrix (LHS = $I_1/3$), in the latter by the LMPS.

On these basis, the estimation of the life to crack initiation under multiaxial fatigue can be made by using the S-N curves expressed in terms of two different, alternative parameters:

- the peak of the Local Hydrostatic Stress (LHS = $I_1/3$), for nearly pure transverse tension case;
- the peak of Local Maximum Principal Stress (LMPS) for high enough shear stress component.

As the LHS and LMPS are local parameters, representative of the driving forces for the microscopic damage evolution, they have to be expressed in terms of local stresses (or micro-stresses) acting in the matrix and at the fibre-matrix interface. A multiscale approach, to link the applied stresses $\sigma_1$, $\sigma_2$ and $\sigma_6$ to the local stress fields, is therefore adopted (Figure 8).

Regular fibres distributions are indeed not representative of the real microstructure in composite plies. In spite of this, it was proved that the unit cell provides reliable results for the application of the proposed initiation criterion [7, 9] even in the case of real random fibre distributions.

The micro-stresses can be calculated by means of FE analyses of a fibre-matrix unit cell subjected to the average (or macroscopic) stresses $\sigma_1$, $\sigma_2$ and $\sigma_6$. In addition, residual stresses due to the different thermal expansion of the two phases during the cooling process must be accounted for. In the FE code thermal loads have been applied as a uniform temperature gap $\Delta T = T_c - T_r$, where $T_c$ is the curing temperature and $T_r$ is the room temperature. Since tensile loading conditions are analysed here and the epoxy behaviour is usually linear elastic in that range, linear elastic FE analyses have been carried out with the software ANSYS 11® using 20 node solid elements.

At each point P of the unit cell the mechanical micro-stresses in polar coordinates ($r$, $\varphi$, $z$) can be defined in terms of stress concentration factors $k_{i,j}$ relating the macro-stress $\sigma_i$ to the local stress $\sigma_{jl}$ as in equation (2). Finally the thermal stresses are related to the temperature gap $\Delta T$ by means of thermal concentration factors $h_{i,j}$, as in Eq. (2).
The peak values of the LMPS and LHS have been found to be always at the points A or B (mostly at point A) of Figure 8 where, due to the symmetry, some stress components vanish, allowing one to adopt the following simple expressions for the LMPS and LHS (valid in the segment AB):

\[
\begin{align*}
\text{LMPS} & = \frac{1}{2} \left[ \sigma_{rr} + \sigma_{zz} + \sqrt{\sigma_{rr}^2 + 4\sigma_{rz}^2 - 2\sigma_{rr}\sigma_{zz} + \sigma_{zz}^2} \right] \\
\text{LHS} & = \frac{\sigma_{rr} + \sigma_{pp} + \sigma_{zz}}{3}
\end{align*}
\]

In Eqs. (3) and (4) the local stresses obtained from Eq. (2) must be substituted.

\[\begin{bmatrix}
\sigma_{rr} \\
\sigma_{pp} \\
\sigma_{zz}
\end{bmatrix} = 
\begin{bmatrix}
k_{1,rr} & k_{2,rr} & 0 \\
k_{1,pp} & k_{2,pp} & 0 \\
0 & 0 & k_{6,zz}
\end{bmatrix} \begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_6
\end{bmatrix} + \Delta T
\begin{bmatrix}
h_{rr} \\
h_{pp} \\
h_{zz}
\end{bmatrix}
\]

(2)

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Figure 8. Definition of the fibre-matrix unit cell for stress analysis at the microscopic scale.

As a validation of the proposed criterion, the S-N curves for the first crack initiation in glass/epoxy tubes subjected to tension/torsion loadings are considered [8]. These are shown in Figure 9a in terms of the maximum cyclic value of the transverse stress, for four values of the biaxiality ratio \(\lambda_{12}\). Four distinct curves are obtained in terms of \(\sigma_2\), but they are collected in only two scatter bands when plotted in terms of the LHS and LMPS parameters. Therefore, when designing against crack initiation with this material, under a general multiaxial stress state, one of these two curves can be adopted, depending on the fact that the ratio \(\lambda_{12}\) is higher or lower than the threshold value 0.5. The criterion validation through a bulk of experimental results from the literature can be found in Ref. [9].
5. Crack density prediction model

The crack initiation criterion described in the previous section, combined with the classical laminate theory (CLT), is enough for predicting the first crack initiation within a ply in a laminate, according to a given probability of survival. The prediction of the crack density evolution is instead a much more complicated task, which was treated according to the following basic ideas [11-13]:

- The initiation of multiple cracks can be predicted, again, using S-N curves for crack initiation in terms of the local parameters LHS and LMPS. The statistical distribution of the fatigue strength has now to be introduced, as it is the only explanation for having crack initiations at different number of cycles;
- The CLT is not suitable for calculating the stress distributions in the plies of a laminate in the presence of a high number of cracks. To overcome this limitation, Carraro and Quaresimin developed a suitable stress re-distribution model [14];
- The propagation of off-axis cracks along the fibres direction can be treated with a Paris-like curve, linking the crack growth rate to the energy release rate.

A schematic of the multiscale procedure for the prediction of the crack density evolution is shown in Figure 10, and it consists in the simulation of the fatigue life by progressively increasing the number of cycles by steps $\Delta N$.

For a given step of fatigue cycles $\Delta N$, and for every off-axis layer, the analysis is carried out as follow:

1) Calculation of ply stresses in each layer, with the possible presence of off-axis cracks, and therefore considering the stress re-distribution.

2) Calculation of micro-stresses (or local stresses) by means of a fibre matrix unit cell subjected to periodic boundary conditions (see the stress concentration factors defined previously).

3) Calculation of the local parameters LHS and LMPS.

Figure 9. First crack initiation results for glass/epoxy tubes [8]: maximum a) cyclic transverse stress, b) LMPS and c) LHS against the number of cycles for first crack initiation.
4) According to the multiaxial condition, calculation of the total density of nucleated cracks, by using the LHS of LMPS master curves for the material considered. The statistical distribution of fatigue strength and the sequence effect, due to the stress re-distribution, must be accounted for.
5) Analysis of the crack propagation phase to compute the length of all the nucleated cracks.
6) Increasing the number of cycles of a quantity $\Delta N$ and repeat steps 1)-6) until the required number of cycles or crack density saturation is reached.

It is worth mentioning that, for the calculation of the local stresses at the step 2) of the procedure, one does not need to carry out a FE analysis of the unit cell every time. In fact the micro-stresses can be simply calculated with Eq.(2), once the stress and thermal concentration factors are calculated at the beginning of the analysis.

Once the crack density is predicted in all the plies of a laminate, it can be used as an input for the stiffness degradation model developed by Carraro and Quaresimin [14]. This model is capable of dealing with generic laminates with cracks in any number of plies, also accounting for the interaction between cracks in different layers. This is important in view of avoiding underestimations of the stiffness loss due to the presence of off-axis cracks in multiple layers [14].

![Figure 10](image_url) *Figure 10. Procedure for the prediction of the crack density evolution.*

6. Prediction of the delamination propagation
It has been observed that the propagation of delaminations plays an important role in the fibres failure process. Therefore, treating this phenomenon is of great importance for the prediction of the final failure. With the aim of characterising the delamination initiation and propagation, the authors have
recently conducted a dedicated experimental investigation. Fatigue tests were conducted on glass/epoxy cross-ply laminates, focusing the attention on the initiation of multiple transverse cracks and also on the initiation and propagation of delaminations from the nucleated transverse cracks. It was observed that the crack density saturation and the initiation of delaminations occurred at a very small percentage of the total fatigue life (1-2%), so that the cycles spent for these phenomena can be even neglected if the final failure is the target of the analysis.

The delamination length was measured during the fatigue tests (figure 11a), the relevant values of the mode I and II Energy Release Rate (ERR) were computed with Finite Element analyses and the VCCT technique, and eventually related to the delamination growth rate. It has been confirmed that the propagation of the delaminations can be treated by means of Paris-like curves relating the growth rate to the mode II energy release rate (the mode I contribution is very limited or absent, at least under tensile loadings) (figure 11b).

This tool allows the delamination length to be predicted as a function of the number of cycles. In addition it is possible to estimate the stiffness loss due to the delaminations, by means of easy analytical tools as for instance a shear lag model.

![Figure 11. a) Example of delamination length evolution and b) relevant Paris-like curve.](image)

7. Conclusions

The fatigue behavior of composite laminates under fatigue loading is characterized by a progressive and complex damage evolution which involves different length scales. It was shown that, in the presence of a high enough shear stress, the first event of damage is the initiation of micro-cracks in the matrix between the fibres. The accumulation and coalescence of these cracks lead to the initiation of an off-axis crack, involving the entire ply thickness and propagating along the fibres direction. It was proved that this kind of micro-scale damage evolution is driven by the Local Maximum Principal Stress in the matrix. Conversely, when the shear stress is very low, the microscopic damage evolution is driven by the Local Hydrostatic Stress (LHS). According to these findings, a fatigue crack initiation criterion was proposed by the authors, suitable to deal with multiaxial stress states, based on the use of these two local parameters. This criterion can be used to predict the initiation of the first cracks in laminates, but also the entire evolution of the crack density, through a model which accounts also for the statistical distribution of the fatigue strength and the stress re-distribution in the presence of cracks. The predicted crack density can be used as in input for a stiffness degradation model, to estimate the laminate stiffness loss as a function of the number of cycles. The developed tool is fundamental in a damage tolerant design against fatigue.

To predict the cycles spent for the final separation of a laminate, the subsequent mechanisms of the initiation and propagation of delaminations must be addressed. An experimental campaign on cross-ply laminates was carried out for this purpose, showing that the cycles spent for initiating the delaminations and saturating the crack density is about 1-2% of the total fatigue life. It was proved that a Paris-like curve is a suitable tool for predicting the propagation of delamination, which is another fundamental step for the estimation of the total life. To reach this final goal, the influence of the presence of off-axis cracks and delamination on the development of fibre-related damage must be
modelled. The authors are currently working in this direction, to eventually provide a damage based approach of industrial applicability for the prediction of damage initiation, evolution and the final failure of composite laminates.

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