Flight stability for the anti-aircraft guided projectile firing on the move

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Abstract. Aiming at the rapid manoeuvring launching requirements of anti-aircraft guided projectiles, the flight stability analysis of a guided projectile launch is carried out, and the initial disturbance parameters model based on equivalent analysis is established. The initial disturbance parameters model and the guided projectile rigid body trajectory model are combined. The relationship between the dynamics and kinematics of the guided projectile is described. The flight performance of the guided projectile is analysed by the stability theory and the criterion. The influence of the launch disturbance on the flight stability is obtained, and the allowed boundary of the guided projectile launch is determined.

1. Introduction

Anti-aircraft guided projectile is an important technical development direction due to its rapid reaction, high accuracy and low cost. As the guidance control technology develops, guided projectile plays an important role in field anti-aircraft defence. Meanwhile, aircrafts develop as well, there are new type of aircrafts known as LSS-Target (Low altitude, Slow speed Small Target), which need anti-aircraft guided projectiles with improved performance, firing on the move is one of the important indicators.

Lack of systematic research in this area, reference [1] conducted a study of dynamic stability performance of guided projectile’s controllable trajectory, which analysed how control disturbance impacts the trajectory stability. However, the article no involved the law of the influence of the moving disturbance for the stability. Reference [2] conducted a study of how trajectory correction impacts the projectile flight stability, the main emphasis is how vector corrective force influences the projectile flight stability.

Based on the above-mentioned research, this article modeled the shooting platform’s moving disturbance and analyzed the dynamic shooting disturbance, by using trajectory dynamic stability analyzing method, acquired the flight stability pattern of projectiles. The flight stability pattern provides the reference for firing on the move bounded constraints of anti-aircraft guided projectile.

2. Firing on the move disturbance model

The shooting platform motion can be categorized as centroid motion and circular swing motion around centroid. The relationship of platform centroid motion and guided projectile movement is shown in figure 1, the velocity of a projectile based on ground coordinate system is:

$$\vec{u}_0 = \vec{u}_{0r} + \vec{u}_H + \vec{u}_ω$$

(1)
in which, $\bar{u}_0$, is the muzzle velocity of guided projectile;
$\bar{u}_H$ is the shooting platform moving velocity;
$\bar{u}_w$ is the additional velocity caused by platform swing due to external disturbance.

**Figure 1.** The relationship of platform centroid motion and guided projectile movement.

Assume $\bar{u}_w = 0$, keeping the origin of ground coordinate system and the centroid of shooting platform coincident, the direction of $\alpha x_1$ and velocity vector $\bar{u}_H$ coincide. The velocity vector $\bar{u}_{0r}$ of guided projectile’s centroid relative to shooting platform gives the firing angle $\theta_{0r}$ and direction angle $q_{rc}$ in relative movement, of which $\bar{u}_H$ is the transport velocity, the projection of guided projectile velocity in ground coordinate system is:

$$
\begin{align*}
    \bar{u}_{0x1} &= u_{0r} \cos \theta_{0r} \cos q_{rc} + u_H \\
    \bar{u}_{0y1} &= u_{0r} \sin \theta_{0r} \\
    \bar{u}_{0z1} &= -u_{0r} \cos \theta_{0r} \sin q_{rc}
\end{align*}
$$

(2)

the velocity modulus of the guided projectile in absolute movement:

$$
\bar{u}_0 = \left(\bar{u}_{0x1}^2 + \bar{u}_{0y1}^2 + \bar{u}_{0z1}^2\right)^{1/2}
$$

(3)

the firing azimuth angle in absolute motion:

$$
q_e = \arctg\left(\frac{u_{0r} \cos \theta_{0r} \sin q_{rc}}{u_{0r} \cos \theta_{0r} \cos q_{rc} + u_H}\right)
$$

(4)

the firing angle in absolute motion:

$$
\theta_0 = \arccos\left(\frac{u_{0r} \cos \theta_{0r} \cos q_{rc} + u_H}{u_{0r} \cos q_e}\right)
$$

(5)
The relationship of platform swing motion around the centroid and guided projectile movement is shown in figure 2, since the platform swing motion around the centroid can affect muzzle disturbance parameters, therefore building the model of swing motion and guided projectile velocity deviation:

\[
\begin{align*}
\Delta v_{x10} &= (v_H + \dot{x}_1) \cos \theta_0 \cos \psi_0 + \dot{y}_1 \sin \theta_0 + \dot{z}_1 \cos \theta_0 \sin \psi_0 \\
&\quad + \psi_H (z_H \cos \theta_0 \cos \psi_0 - x_H \cos \theta_0 \sin \psi_0) \\
&\quad + \phi_H (x_H \sin \theta_0 - y_H \cos \theta_0 \cos \psi_0) \\
&\quad + \dot{y}_H (y_H \cos \theta_0 \sin \psi_0 - z_H \sin \theta_0) \\
\Delta v_{y10} &= (v_H + \dot{x}_1) \sin \theta_0 \cos \psi_0 + \dot{y}_1 \cos \theta_0 - \dot{z}_1 \sin \theta_0 \sin \psi_0 \\
&\quad + \psi_H (x_H \sin \theta_0 \sin \psi_0 - z_H \sin \theta_0 \cos \psi_0) \\
&\quad + \phi_H (x_H \cos \theta_0 + y_H \sin \theta_0 \cos \psi_0) \\
&\quad - \dot{y}_H (y_H \sin \theta_0 \sin \psi_0 + z_H \cos \theta_0) \\
\Delta v_{z10} &= (v_H + \dot{x}_1) \sin \psi_0 + \dot{z}_1 \cos \psi_0 \\
&\quad - \psi_H (x_H \cos \psi_0 + z_H \sin \psi_0) \\
&\quad + \phi_H y_H \sin \psi_0 + \dot{y}_H y_H \cos \psi_0
\end{align*}
\]

in which, \(v_H, \psi_0, \theta_0\) are the platform motion velocity, ideal firing direction angle and ideal firing pitch angle respectively, they are constants;

\(\dot{x}_1, \dot{y}_1, \dot{z}_1\) are ground coordinate system projections of additional velocity caused by platform swing motion;

\(\psi_H, \phi_H, \dot{y}_H\) are platform rotation angles;

\(x_H, y_H, z_H\) are shooting platform coordinate system projections of vectors of platform centroid to launching device swing motion centre.

\[\text{Figure 2. The relationship of platform swing motion around the centroid and guided projectile movement.}\]
The disturbance parameters of firing the projectile can be acquired by using the velocity deviation caused by platform swing motion:

\[
\Delta \alpha_0 = \arctan \left( \frac{\Delta v_{y10}}{v_0 + \Delta v_{x10}} \right) \\
\Delta \beta_0 = \arcsin \left( \frac{\Delta v_{z10}}{(v_0 + \Delta v_{x10})^2 + \Delta v_{y10}^2 + \Delta v_{z10}^2} \right)^{\frac{1}{2}} \\
\Delta v_0 = \left( (v_0 + \Delta v_{x10})^2 + \Delta v_{y10}^2 + \Delta v_{z10}^2 \right)^{\frac{1}{2}} - v_0
\]

This model can be used to acquire muzzle initial parameters of guided projectile, \(v_{0x1, v_{0y1}, v_{0z1}}\) are initial velocities, \(\Delta \alpha_0, \Delta \beta_0, \Delta v_0\) are initial disturbance parameters.

3. Guided projectile angular motion equations and its simplification

The projectile rotation equations:

\[
\Psi = \frac{g \sin \theta}{v} \Psi + b_y v \xi \\
\Phi = \left( k_{zz} v - \frac{i c \gamma}{A} \right) \Phi - \frac{v^2 k_x \xi}{A} \gamma \phi - k_{zz} v \phi - \dot{\theta} \\
\dot{\xi} + \left( b_y + k_{xx} - \frac{i c \gamma}{A} \right) v \xi + \left( -i \frac{c \gamma b_y}{A} - k_x \right) v^2 \xi = \frac{i c \gamma}{A} \gamma \phi - k_{xx} v \phi - \dot{\theta}
\]

The first equation is the complex drift angle equation, the second one is the rotation angle equation, the third one is the complex swing angle equation, the fourth one is the complex attack angle equation. The complex attack angle equation changes the independent variable from time \(t\) to non-dimensional trajectory arc length \(\xi, s = s/d\), then the attack angle equation is:

\[
\xi'' + (H - iP)\xi' - (M + iP\theta) = (i\frac{g}{v}) \dot{\theta} - k_{zz} \frac{d^2}{v^2} \dot{\theta} - \frac{d^2}{v^2} \theta
\]

in which, the definition of symbols is in reference [3].

\[
P = C\gamma d/(Av), H = b_y d + k_{xx} d - b_x d - g d \sin \theta/v^2, M = k_x d, T = b_y d
\]

Considering the impact of which transient disturbance of guided projectile has on attack angle character, the forced term in the right hand side of the angular motion equations can be ignored, the angular motion equations can be simplified to the homogeneous equation:

\[
\xi'' + (H - iP)\xi' - (M + iP\theta) = 0
\]

According to solution theory of differential equations, the general solution of a homogeneous equation is:

\[
\xi = K_F e^{i\phi_F} e^{i\lambda_F} s + K_S e^{i\phi_S} e^{i\lambda_S} s
\]

in which, \(K_F, K_S\) are fast and slow circular motion modal amplitudes respectively, \(\phi_F = \phi_{F0} + \phi_{F}\), \(\phi_S = \phi_{S0} + \phi_{S}\) are fast and slow circular motion initial phases respectively, \(\phi_{F}, \phi_{S}\) are fast and slow circular motion rotation frequencies respectively.
\[
\varphi_P = \frac{1}{2} \left[ P + \left( P^2 - 4M \right)^{\frac{1}{2}} \right] - \frac{1}{2} \left[ P - \left( P^2 - 4M \right)^{\frac{1}{2}} \right] \\
\lambda_F, \lambda_S \text{ are fast and slow circular motion damping coefficients respectively:}
\[
\lambda_F = -\frac{1}{2} \left[ H - \frac{P(2T-H)}{(P^2-4M)^2} \right], \quad \lambda_S = -\frac{1}{2} \left[ H + \frac{P(2T-H)}{(P^2-4M)^2} \right]
\]

4. Analysis of guided projectile firing on the move stability

The initial disturbance parameters caused by shooting platform disturbance can be calculated by using firing on the move disturbance model, the initial disturbance parameters include initial velocity from equation (2) and muzzle initial velocity and muzzle disturbance parameters from equation (7). The disturbance velocity is not always on the same direction with guided projectile velocity, which can change velocity vector and create additional attack angle, because the shooting platform motion velocity is much less than guided projectile velocity. So the additional attack angle and the impact on guided projectile stability can be ignored. The swing motion of shooting platform is a key factor of guided projectile firing on the move performance because it directly cause disturbance angular velocity which can affect guided projectile’s flight attitude.

Using equation (10) to analyse guided projectile stability, putting the additional angular velocity from platform swing motion into the equation, we can get the corresponding attack angle changing pattern, as shown in figure 3.

![Figure 3. The impact of shooting platform single direction swing motion on guided projectile.](image)

We can see from figure 3 that when the platform does not swing, the maximum attack angle is 0.03rad, when the platform side disturbance angular velocity is 0.02rad/s, the maximum attack angle is around 0.1rad, the pitch disturbance and roll disturbance are around 0.06rad.
The above analysis indicates the shooting platform’s side disturbance has a larger impact on guided projectile stability. The impact of shooting platform complex swing motion on guided projectile is shown in figure 4.

We can see from figure 4 that when the shooting platform has a complex swing motion in multiple directions, the complex swing motion coinciding with guided projectile angular motion can likely partially offset the impact on flying attitude, on the other hand, they can also accumulate to make the projectile out of balance.

In conclusion, the guided projectile’s firing on the move stability is mainly affected by shooting platform’s swing motion in the pitch and yaw directions instead of the absolute velocity.

5. Conclusion
For anti-aircraft guided projectile firing on the move ability research, this article built a disturbance model, along with guided projectile angular motion and its simplification. The disturbance model can use the guided projectile motion disturbance parameters to conclude the impact a pattern of platform disturbance has on projectile stability, the result shows that the moving velocity of platform affects the projectile stability much less than the swing motion of the platform. Combined with the above analysis, it can be concluded that the platform moving-smoothly on flat ground can keep the projectile’s stability and accuracy, the bumpy ground or moving-rapidly can affect the stability and accuracy of projectile. With the development of technology, the passive control principle can be used on shot on the anti-aircraft guided projectile to raise the allowed boundary and firing flexibility.

References
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