Study regarding the dynamic loads upon the track at failure of the dampers in the primary suspension of the railway vehicle

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Abstract. This paper investigates the dynamic loads on the track from a two-axle bogie, while considering the particular case of a failure in a damper in the suspension of an axle. To this purpose, a model of the bogie-track system is taken into account, which looks at the track elasticity, the contact of the wheel-rail, and the bounce and pitch vertical vibrations of the bogie. The study relies on the results from numerical simulations concerning the dynamic wheel-rail contact forces, corresponding to one of the wheels, due to the failure in the damper. The dynamic wheel-rail contact forces generated during the running of a bogie on a track with vertical irregularities described by both a harmonic function and a pseudo-random function are being examined.

1. Introduction
The track geometry is not perfect, as it is changed by a number of irregularities mainly coming from the building imperfections, track exploitation, modification in the track infrastructure due to the environment factors or soil movements [1]. The track irregularities represent deviations from the design geometry, a result of the lateral and vertical dislocation of each rail from its nominal position. These will be added to the irregularities in the rolling surfaces or the discontinuities of the rails (joints, switches, crossings) [2]. Dynamic contact forces occur in the wheel-rail interface while a railway vehicle is running on a track with deviations from the design geometry or over the irregularities or discontinuities of the rails [3 - 7]. Also, the dynamic wheel-rail contact forces develop when one of the vehicle wheels travels over a local defect of its own rolling surface (eg. the flat point) [8] or when this wheel shows deviations from the circular form [9].

In dependence on their dimension, the dynamic wheel-rail contact forces can trigger important loads of the running gear and the track, where these loads have an impact upon the wheel resistance and of other components of the running gear and the rail [10, 11]. The wear of the rolling surface of the wheels and rails [12] and the occurrence of the rolling noise [13] can be noticed. Under such conditions, the dynamic forces limitation is imperative, since it is one of the approval criteria for the railway vehicle as for the dynamic behaviour in terms of the track fatigue [14, 15].

The magnitude of the contact dynamic forces is mainly dependent on the velocity and the characteristics of the track irregularities, as well as a number of parameters of the railway vehicle aiming at the suspended and non-suspended masses or the suspension parameters [3, 4].
This paper investigates the dynamic forces on the track from a two-axle bogie, while considering the particular case of a failure in a damper in the suspension of an axle. To calculate the dynamic forces, a model of the bogie-track system is taken into account, which looks at the track elasticity, the wheel-rail contact, and the bounce and pitch vertical vibrations of the bogie. As for the vertical irregularities of the track, they are considered to have a harmonic or random shape. The study relies on the results from applications of numerical simulations, where the failure of the damper is simulated by various degrees of reduction in the damping constant of the suspension corresponding to one of the wheels, compared with the reference value.

2. The mechanical model of the bogie-track system
To study the dynamic loads upon the track at the failure of the dampers in the primary suspension of the railway vehicle, a two-axle bogie is considered, running at a constant speed \(V\) on a track with vertical irregularities.

Figure 1 presents the model of the bogie-track system. The model of the bogie includes 3 rigid bodies that help with modelling the bogie chassis and the two axles connected between them by Kelvin-Voigt type systems that model the primary suspension corresponding to each axle. The elastic element of the wheel suspension has the constant \(k_b\) and the damping elements has the constant \(c_{b1}\) and \(c_{b2}\), respectively. When neither of the dampers is failure, the damping constants of the suspension in the two axles are equal (\(2c_{b1} = 2c_{b2} = 2c_b\), where \(2c_b\) represents the reference value of the damping constant in the axle suspension).

The rigid vibration modes of the bogie in the vertical plan, namely bounce (\(z_b\)) and pitch (\(\theta_b\)) are considered. The bogie parameters are: \(m_b\) – bogie mass, \(2a_b\) – bogie wheelset, \(J_b = m_b i_b^2\) - inertia moment, with \(i_b\) – the gyration radius of the bogie.

The axles of mass \(m_w\) operate a translation motion on the vertical direction (\(z_{w1,2}\)).

While overlooking the coupling effects between the wheels due to the propagation of the bending wavelength, in the frequency domain specific to the vertical vibrations of the vehicle, an equivalent model with lumped parameters will be selected. Against each axle, the track is represented as an oscillating system with one degree of freedom that can move on the vertical direction, with the displacement \(z_{r1,2}\).

The equivalent model of the track has mass \(m_r\), stiffness \(2k_r\) and the damping coefficient \(2c_r\). The vertical irregularities of the track are described against each axle by the functions \(\eta_{1,2}\).
The elasticity of the wheel-rail contact will be taken into account by introducing certain elastic elements with a linear characteristic. The calculation of the stiffness in the contact elastic elements - \(2k_H\) for a wheel-rail system – is done based on the theory of contact between two Hertz’s elastic bodies by applying the linearization of the relation of contact deformation against the deformation corresponding to the static load on the wheel.

3. The equations of motion for the bogie-track system

The vertical motions of the bogie-track system are described by six motion equations, corresponding to the vibration modes of the bogie – bounce and pitch, the vertical displacements of the wheels and of the rails.

The equations (1) and (2) define the bounce and pitch motions of the bogies

\[
m_b \ddot{z}_b = \sum_{i=1}^{2} F_{bi} - F_s
\]

\[
J_b \ddot{\theta}_b = a_b \sum_{i=1}^{2} (-1)^{i+1} F_{bi}
\]

where \(F_s\) represents the force in the secondary suspension and \(F_{bi}\) (equation (3)) are the forces in the primary suspension of the axles \(i\) (for \(i = 1, 2\)),

\[
F_{b1,2} = -2c_{b1,2}(\dot{z}_h \pm a_b \dot{\theta}_b - \dot{z}_{w1,2}) - 2k_b(z_h \pm a_b \theta_b - z_{w1,2})
\]

The equations (4) of the motions on the vertical direction of each axle are:

\[
m_w \ddot{z}_{w1,2} = 2Q_{d1,2} - F_{b1,2}
\]

where \(Q_{d1,2}\) are the dynamic forces of contact; the dynamic forces on the wheels of an axle is considered to be equal. To calculate the dynamic forces, the hypothesis of the Hertzian linear contact between the wheel and the rail has been opted for, equation (5):

\[
Q_{d1,2} = -k_H[z_{w1,2} - z_{r1,2} - \eta_{1,2}]
\]

in which \(k_H\) is the stiffness of the wheel-rail contact.

The vertical displacements of the rails are to be found in equation (6):

\[
m_w \ddot{z}_{r1,2} = F_{r1,2} - 2Q_{d1,2}
\]

where \(F_{r1,2} = -2c_r \dot{z}_{r1,2} - 2k_r z_{r1,2}\)

After transformations, the equations of motion for the bogie-track system become, (equations (8-11)):

\[
m_b \ddot{z}_b + 2c_{b1}(\dot{z}_b + a_b \dot{\theta} - \dot{z}_{w1}) + 2c_{b2}(\dot{z}_b - a_b \dot{\theta} - \dot{z}_{w2}) + 2k_b[2z_b - (z_{w1} + z_{w2})] - F_s = 0
\]

\[
J_b \ddot{\theta}_b + 2c_{b1}(a_b \dot{\theta}_b + \dot{z}_b - \dot{z}_{w1}) + 2c_{b2}(a_b \dot{\theta}_b - \dot{z}_b - \dot{z}_{w2}) + 2k_b[a_b \theta_b - (z_{w1} - z_{w2})] = 0
\]

\[
m_w \ddot{z}_{w1,2} + 2c_{b1,2}(\dot{z}_{w1,2} - \dot{z}_b \mp a_b \dot{\theta}_b) + 2k_b(z_{w1,2} - z_{b} \mp a_b \theta_b) + 2k_H(z_{w1,2} - z_{r1,2} - \eta_{1,2}) = 0
\]

\[
m_w \ddot{z}_{r1,2} + 2c_r \dot{z}_{r1,2} + 2k_r z_{r1,2} + 2k_H(z_{r1,2} - z_{w1,2} + \eta_{1,2}) = 0
\]

A system of six differential equations of second order was thus obtained, in which the variables of state, displacements and velocities are introduced, as such, (equation (12)): 
\[
q_{2k+1} = p_k, \quad q_{2k} = \dot{p}_k, \quad \text{for } k = 1 \ldots 6
\] (12)

where \(p_1 = z_b, p_2 = \theta_b, p_{3,4} = z_{x1,2}, p_{5,6} = z_{r1,2}\).

The result will be a system of 12 differential equations of first order that can be written in a matrix-like form, (equation (13)):

\[
\dot{\mathbf{q}} = \mathbf{Aq} + \mathbf{B}
\] (13)

where \(\mathbf{q}\) is the vector of the state variables, \(\mathbf{A}\) the matrix of the system and \(\mathbf{B}\) – the vector of the non-homogeneous terms. The system of equations (13) can be solved by a numeric integration, applying the Runge-Kutta algorithm.

4. The results of the numerical simulations

This section deals with the results derived from numerical simulations regarding the dynamic loads generated by the bogie upon the track in the case of failure in a damper in the primary suspension of one of the axles. The failure of a damper is simulated by the reduction of the damping constant \((c_b)\) of the suspension corresponding to one of the wheels in the front axle, compared to the reference value \((c_0)\). The dynamic forces of wheel-rail contact generated during the running of the bogie at velocity of 200 km/h on a track with vertical irregularities of a harmonic and random form. The parameters of the numerical model are shown in table 1.

| Bogie mass       | \(m_b = 3200\) kg |
|------------------|-------------------|
| Axle mass        | \(m_a = 1650\) kg |
| Rail mass (under the axle) | \(M_r = 175\) kg |
| Bogie wheelset   | \(2a_b = 2.56\) m |
| Inertia moment   | \(J_b = 2.05 \times 10^4\) kg\(\cdot\)m² |
| Elastic constant of the suspension corresponding to a wheel | \(k_b = 1.10\) MN/m |
| Damping constant of the suspension corresponding to a wheel | \(c_b = 13.05\) kNs/m |
| Rail vertical stiffness | \(k_r = 70\) MN/m |
| Vertical damping of the track | \(c_r = 20\) kNs/m |
| Stiffness of the wheel-rail contact | \(k_H = 1500\) MN/m |

During a first stage, the vertical irregularities of the track are considered to have a harmonic form with the wavelength \(\Lambda\) and amplitude \(\eta_0\) (equation (14)):

\[
\eta_{1,2}(x_{1,2}) = \eta_0 \cos(2\pi/\Lambda)x_{1,2}, \quad \text{for } x_{1,2} > 0
\] (14)

with \(x_1 = Vt; \quad x_2 = Vt - 2a_b\).

The vertical irregularities of the track against the two axles can also be written as, (equation (15)):

\[
\eta_1(t) = \eta_0 \cos \omega t, \quad \eta_2(t) = \eta_0 \cos \omega (t - 2a_b/V)
\] (15)

where \(\omega = 2\pi f/\Lambda\) stands for the pulsation induced by the track excitation. The value for \(\eta_0\) is 1 mm, and the wavelength of the track irregularities is in such a way selected that the excitation frequency matches the eigenfrequencies of the bogie vibrations: \(\Lambda = 9.41\) m for 5.9 Hz (the frequency of the bogie bounce), \(\Lambda = 5.91\) m for 9.4 Hz (the frequency of the bogie pitch).

Figure 2 features the dynamic wheel-rail contact forces generated during the travelling of the bogie on a track with irregularities of a harmonic shape, with the characteristics above. Neither of the dampers was considered failed and, therefore, the damping constants in the suspension of the two axles are equal. The dynamic wheel-rail contact forces are noticed to be unequal, due to the phase shifting in the vertical displacements of the two axles. In both cases, the contact force is higher against the rear axle, but this is more visible if the excitation frequency is equal with the bounce frequency of...
the bogie. On the other hand, the higher values of the dynamic contact forces can be seen if the excitation frequency has the same value with the pitch frequency of the bogie.

Figure 2. The dynamic forces during the travelling on a track with irregularities in a harmonic shape: (a) $\Lambda = 9.41$ m; (b) $\Lambda = 5.91$ m.

Figure 3. The dynamic forces during the travelling on a track with irregularities in a harmonic shape: (a) and (a’) $\Lambda = 9.41$ m; (b) and (b’) $\Lambda = 5.91$ m.
Generally speaking, the failure of a damper in the front axle of the bogie triggers a significant increase in the dynamic wheel-rail contact forces. This is shown in diagrams in figure 3 and results in table 2, with the maximum values of the dynamic contact forces. There are exceptions, however, when the dynamic wheel-rail contact forces decrease – for track irregularities with the wavelength of 5.91 m, the dynamic forces in the front axle go down along with the failure in the damper, but this decrease does not exceed 10%.

**Table 2.** The maximum values of the dynamic contact forces upon travelling on a track with vertical irregularities of a harmonic shape.

|             | \(\lambda = 9.41\) m | \(\lambda = 5.91\) m |
|-------------|------------------------|------------------------|
| \(c_b = c_{b2} = c_b\) | 1605                   | 4671                   |
| \(c_b = c_{b2}/2; c_{b2} = c_b\) | 2031                   | 5130                   |
| \(c_b = c_{b4}; c_{b2} = c_b\) | 2572                   | 5477                   |

Further on, there will be an analysis of the dynamic forces generated by the bogie travelling at velocity of 200 km/h on a track with vertical irregularities of a random shape.

The vertical irregularities of the track are described against each axle by a pseudo-random function in the form of equation (16), [16]:

\[
\eta_{1,2}(x_{1,2}) = K_n f(x_{1,2}) \sum_{k=0}^{N} U_k \cos(\Omega_k x_{1,2} + \varphi_k), \text{ for } x_{1,2} > 0
\]  
(16)

with

\[
K_n = \frac{\eta_{adm}}{\max\left| f(x_{1,2}) \sum_{k=0}^{N} U_k \cos(\Omega_k x_{1,2} + \varphi_k) \right|}
\]  
(17)

where \(K_n\) is a scale coefficient of the amplitudes in the vertical irregularities of the track, \(\eta_{adm}\) is the maximum value of the track vertical irregularities according to which UIC 518 Leaflet [15]; \(f(x_{1,2})\) is a smoothing function applied on the distance \(L_0\), in the form of , (equation (18)):

\[
f(x_{1,2}) = \left[ 6 \left( \frac{x_{1,2}}{L_0} \right)^5 - 15 \left( \frac{x_{1,2}}{L_0} \right)^4 + 10 \left( \frac{x_{1,2}}{L_0} \right)^3 \right] H(L_0 - x_{1,2}) + H(x_{1,2} - L_0)
\]  
(18)

where \(H(.)\) is Heaviside step function; \(U_k\) is the amplitude of the spectral component corresponding to the wave number \(\Omega_k\), and \(\varphi_k\) is the phase shifting of the spectral component, \(k\) for which a uniform random distribution will be selected.

The amplitude of each spectral component is established based on the power spectral density of the track vertical irregularities according to ORE B176 [17] and the specifications included in UIC 518 Leaflet regarding the geometric quality of the track by the levels of QN1 and QN2 quality.

Figure 4 shows the track vertical irregularities on a 2000-meter distance for a track of QN1 quality, as well for a track defined by the QN2 quality. To design the track geometry, the contribution of 300 spectral components \((k = 300)\) with wavelengths ranging from 3 and 120 m has been considered.

Figures 5 and 6 feature the maximum values of the dynamic wheel-rail contact forces generated during the travelling of a bogie on a track of QN1 and QN2 quality levels, for various degrees of failure in a damper of the front axle. To this purpose, different degrees of reduction in the damping constant \(c_{b(1)}\) will be considered, versus the reference value \(c_b\).
The first observation is the much higher values of the dynamic contact forces derived during the travelling on a QN2 quality track, compared to a QN1 track. Another one would be that the damper failure leads, in all cases, to the increase in the dynamic wheel-rail contact forces (see table 3).
5. Conclusions
The causes prompting the dynamic forces at the wheel-rail interface are multiple – the track geometric deviations, the irregularities or discontinuities in the rolling surfaces of the rails, the local defects of the wheel rolling surface, the deviations of the wheel from the circular shape. Should these forces exceed certain limit values, they can lead to the deterioration in the track and the rolling device, the wear of the wheel and of the rail and the amplification in the rolling noise.

This paper has dealt with the dynamic wheel-rail contact forces in a particular case of the damper failure in the suspension of a two-axle bogie during its travelling on a track with vertical irregularities. The vertical irregularities of the track are analytically described, both via harmonic and pseudo-random functions.

To conduct the study, there will be used the results from the numerical simulations developed based on a model of the bogie-track system, which considers the elasticity of the rolling track and of the wheel-rail contact, besides the vertical vibrations of bounce and pitch in the bogie. The dynamic wheel-rail contact forces for various scenarios of damper failure have been calculated, namely different degrees of reduction in the damping constant versus the reference value.

Upon looking at the results concerning the dynamic wheel-rail forces generated during running on a track with vertical irregularities in a harmonic shape, the dynamic wheel-rail contact forces in the two axles have been pointed out at being unequal, a fact explained by the phase shifting in the vertical displacements of these axles. Similarly, the dynamic forces have been shown to have higher values when the excitation frequency is equal with the pitch frequency in the bogie, compared to when the former frequency is equal with the bounce frequency of the bogie. Another point was made in the significant increase of the dynamic contact forces due to the damper failure.

In terms of the results from the bogie running on a track with vertical irregularities in a random shape, they have confirmed the dependence of the dynamic forces on the track geometric quality and their increase during the failure of the damper.

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6. References
[1] Pombo J and Ambrósio J 2012 An alternative method to include track irregularities in railway vehicle dynamic analyses Nonlinear Dynamics 68(1-2) 161-176
[2] Steenbergen M 2006 Modelling of wheels and rail discontinuities in dynamic wheel-rail contact analysis Vehicle System Dynamics 44(10) 763-787
[3] Dumitriu M and Sebeşan I 2017 Influence of the vertical track irregularities upon the wheel-rail dynamic forces Journal of Engineering Science and Technology Review 10(1) 160-167
[4] Dumitriu M 2013 Influence of the primary suspension damping on the vertical dynamic forces at the passenger railway vehicles UPB Scientific Bulletin, Series D: Mechanical Engineering 75(1) 25-40
[5] Mazilu T 2009 Analysis of infinite structure response due to moving wheel in the presence of irregularities via Green’s functions method Proceedings of the Romanian Academy, Series
A: Mathematics, Physics, Technical Sciences, Information Science

[6] Mazilu T 2009 On the dynamic effects of wheel running on discretely supported rail Proceedings of the Romanian Academy, Series A: Mathematics, Physics, Technical Sciences, Information Science 10 139-150

[7] Spiroiu M A 2016 Wheel-rail dynamic forces induced by random vertical track irregularities IOP Conf. Series: Materials Science and Engineering 147 012117

[8] Wu T X and Thompson D J 2002 A hybrid model for the noise generation due to railway wheel flats Journal of Sound and Vibration 251 115-139

[9] Mazilu T, Dumitriu M, Tudorache C and Sebeşan M 2011 Wheel/rail interaction due to the polygonal wheel UPB Scientific Bulletin, Series D, Mechanical Engineering D(3) 95-108

[10] Gullers P, Andersson L and Lundén R 2008, High-frequency vertical wheel–rail contact forces Field measurements and influence of track irregularities Wear 265 (9-10) 1472–1478

[11] Karttunen K, Kabo E and Ekberg A 2014 The influence of track geometry irregularities on rolling contact fatigue Wear 314 78–86

[12] Oostermeijer K H 2008 Review on short pitch rail corrugation studies Wear 265 1231-1237

[13] Thompson D J 1988 Wheel/rail contact noise: development and detailed evaluation of a theoretical model for the generation of wheel and rail vibration due to surface roughness, ORE DT 204 - C 163 (Utrecht)

[14] EN 14363 2013 Railway applications - Testing and Simulation for the acceptance of running characteristics of railway vehicles - Running behaviour and stationary tests

[15] UIC 518 Leaflet 2009 Testing and approval of railway vehicles from the point of view of their dynamic behaviour – Safety – Track fatigue – Running behaviour

[16] Dumitriu M 2016 Numerical synthesis of the track alignment and applications. Part I: The synthesis method Transport Problems 11(1) 19-28

[17] C 116 1971 Interaction between vehicles and track, RP 1, Power spectral density of track irregularities, Part I: Definitions, conventions and available data, Utrecht