Full-field broadband invisibility through reversible wave frequency-spectrum control: supplementary material

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This document provides supplementary information to “Full-field broadband invisibility through reversible wave frequency-spectrum control,” https://doi.org/10.1364/OPTICA.5.00779. The contents of this document include a brief description of the time-frequency duality of the Talbot effect (which is the basis of the reported work), a description of the signal metrics and characterization methods used in the reported work, as well as five supplementary figures.

1. THE TIME-FREQUENCY DUALITY OF THE TALBOT EFFECT

The temporal and spectral phases involved in the Talbot effect are the key pieces of the proposed energy-preserving wave-spectrum control method and spectral invisibility cloak. The relationship between such temporal and spectral phases was recently formalized [1], and it is known as the time-frequency duality of the Talbot effect.

Temporal Talbot self-imaging of a pulse train with period $t_r$ (corresponding to a frequency comb with free spectral range, FSR, $\nu_r = t_r^{-1}$) is achieved by propagation through a group-velocity dispersive medium with a predominantly second-order dispersion (linear group delay as a function of frequency) that satisfies the following condition [1, 2],

$$2\pi |\beta_2| z = \frac{P}{m} \nu_r^2, \quad (S1)$$

where $\beta_2$ is the second-order dispersion coefficient of the medium, i.e., the amount of second-order dispersion per unit length of the medium [3], $z$ is the propagation length through the medium, and $P$ and $m$ are two mutually prime natural numbers ($P = 1$ in the reported experiments). After propagation, the comb FSR remains unaltered, but the $k$-th comb line acquires a phase given by,

$$\phi_{k,P,m} = \sigma \pi \frac{P}{m} k^2, \quad (S2)$$

where $\sigma$ is the sign of the parameter $\beta_2$ ($\sigma = 1$ if $\beta_2 > 0$, and $\sigma = -1$ if $\beta_2 < 0$).

Under these conditions, the pulses of the corresponding temporal waveform are recovered without distortion at the output of the dispersive medium, but at a repetition rate $m$ times faster than the input train (output pulse period of $m^{-1} t_r$). Furthermore, $n$-th output pulse of the obtained sequence acquires a phase given by [1],

$$\varphi_{n,P,m} = -\sigma \pi \frac{s}{m} n^2, \quad (S3)$$

where $s$ is a natural number, mutually prime with $m$, so that [4],

$$sp = 1 + m \epsilon_m \pmod{2m}, \quad (S4)$$

where $\epsilon_m$ is the parity of the parameter $m$ ($\epsilon_m = 0$ if $m$ is even, and $\epsilon_m = 1$ if $m$ is odd).

The proposed spectral invisibility cloaking method relies on calculating the temporal Talbot phase associated to a specific Talbot condition for a given value of $m$, and cancelling it through the use of a phase modulation mechanism (details in Fig. S1a). This results in an increase of the comb FSR by the pulse rate multiplication factor $m$, or, more generally, in the reported formation of frequency gaps.

As shown above, the sign of the required temporal phase modulation sequence is determined by the sign of the second-order dispersion coefficient, $\beta_2$. We note that in the case of illu-
Although the reported demonstration uses a single RF tone driving signal to implement the phase modulation operations, it is important to note that this corresponds to a first-order approximation to the ideal temporal Talbot phase pattern with \( m = 2 \) (see Fig. S2b). The ideal phase modulation profiles resulting from Talbot conditions with arbitrary \( m \) factors are periodic sequences of phase steps. From a practical viewpoint, these can be implemented by an electronic arbitrary waveform generator. Fig. S3 shows a set of numerical simulations illustrating the formation of frequency gaps for different values of \( m \), and the associated temporal Talbot phase modulation sequences, satisfying Eq. S3.

Additionally, Fig. S2c shows a comparison between the expected power spectra of the frequency gaps obtained with the prescribed ideal Talbot phase for \( m = 2 \), and the single-tone approximation used in the reported experiments. The results suggest that the cloaking bandwidth could be improved through the use of phase modulation sequences closer in shape to the ideal Talbot pattern.

### 3. CROSS-CORRELATION COEFFICIENT

The cross-correlation coefficient is a widely-employed metric for quantitative comparison of real-valued signals (used here to quantify the similarity between the illumination wave and the waves observed at the output of the cloaking device). It corresponds to the zero-lag sample of the cross-correlation between two signals, normalized to the zero-lag sample of the auto-correlations of each signal. For two real-valued signals \( x(t) \) and \( y(t) \), the definition of the cross-correlation coefficient, \( r_{x,y} \), is:

\[
r_{x,y} = \frac{\int_{-\infty}^{\infty} x(\tau) y(\tau) d\tau}{\sqrt{\int_{-\infty}^{\infty} |x(\tau)|^2 d\tau \int_{-\infty}^{\infty} |y(\tau)|^2 d\tau}}.
\]

This coefficient takes values between \(-1\) and \(1\). Two real-valued signals satisfying \( x(t) = y(t) \) yield a cross-correlation coefficient \( r_{x,y} = 1 \), while the value of the coefficient becomes \(-1\) when \( x(t) = -y(t) \). If the two signals are real-valued and positive, the cross-correlation coefficient is defined between \(0\) and \(1\). The closer this coefficient is to \(0\), the more dissimilar the signals \( x(t) \) and \( y(t) \) are. The similarity between the signals \( x(t) \) and \( y(t) \) is then higher the closer the value of \( r_{x,y} \) is to \(1\).

### 4. CHARACTERIZATION OF THE COMPLEX TEMPORAL ENVELOPE

The indirect reconstruction of the complex envelope of the involved pulses is achieved through spectral measurements, using a frequency-domain version of the PROUD technique (phase reconstruction through optical ultrafast differentiation), referred to as SPROUD (spectral phase reconstruction through optical ultrafast differentiation) [5].

The method reconstructs the complex temporal envelope (including both amplitude and phase profiles) of a signal under test (SUT), \( x(t) = |x(t)|e^{i\phi(t)} \), where \( i \) is the imaginary unit and \( |x(t)| \) and \( \phi(t) \) are the temporal amplitude and phase profiles of the SUT, respectively. This reconstruction requires the measurement of two power spectra: \(|X(\omega)|^2\), the power spectrum of the SUT, (where \( \omega = 2\pi f \) is the radial frequency variable, measured in rad), and \(|Y(\omega)|^2\), the power spectrum resulting from modulating the amplitude of \( x(t) \) with a linear monotonic function of time. Following the properties of the Fourier transform, the result of such a modulation translates into a differentiation of the Fourier spectrum of the SUT, \( X(\omega) \).

In the reported experiments, this spectral differentiation was achieved through amplitude temporal modulation of the incoming optical waveform (SUT) in a LiNbO\(_3\) Mach-Zehnder intensity modulator (MZM). The electronic modulation driving signal was a sinusoidal function, slowly-varying as compared to the temporal envelope of the SUT, where the required linear monotonic function was approximated by the rising slope of the sinusoidal cycle. For this purpose, the MZM was biased at the quadrature point (i.e., in the linear operation region, half way between the maximum and minimum transmission points). The group-delay frequency distribution of \( X(\omega) \) (the derivative of the phase of \( X(\omega) \) with respect to radial frequency) can be numerically reconstructed by means of the following equation,

\[
\frac{d}{d\omega} <X(\omega) = \frac{1}{4\Omega} \left(1 - \frac{1}{|X(\omega)|} \sqrt{\frac{2}{T_0} |Y(\omega)|^2 - \left(\frac{A\Omega}{d\omega} X(\omega)\right)^2}\right),
\]

where \(|X(\omega)|^2\) and \(|Y(\omega)|^2\) are the measured power spectra of the SUT and the optical signal at the output of the MZM modulator, and the derivative of \(|X(\omega)|\) on the right-hand side of Eq. S6 is performed numerically on the measured input amplitude spectrum. The parameters \( A, \Omega \), and \( T_0 \) are associated to the temporal modulation processing: \( A \) is the half-amplitude of the RF tone referred to the half-wave voltage of the MZM, \( \Omega \) is the frequency of the RF tone, and \( T_0 \) is the maximum throughput of the MZM. The values of these parameters in the reported experiment are: \( A\Omega \approx 7.8 \) ns\(^{-1}\) and \( T_0 \approx 1 \).

The spectral phase profile of the SUT, \(<X(\omega)\rangle\), is then obtained (except for an additive constant term) by numerical integration of the calculated group delay profile. The time-domain waveform of the SUT is finally reconstructed by simply calculating the inverse Fourier transform of the measured spectral amplitude, \(|X(\omega)|\), with the spectral phase profile, \(<X(\omega)\rangle\), calculated numerically, such that \( X(\omega) = |X(\omega)|e^{i<X(\omega)\rangle} \).
**Fig. S1.** Energy-preserving transformations for spectral invisibility cloaking based on Talbot effects, including phase-only manipulations along the time and frequency domains (temporal phase modulation, PM, and group-velocity dispersion, GVD, respectively) for: (a) a frequency comb illumination wave, shown here to explain the proposed concept, and (b) the most general case of an illumination wave with a continuous spectrum, where no interaction occurs between neighbouring pulses in the cloaking device at any given time. Parameters defined in the main text; $t$ holds for time and $\nu$ holds for frequency.
Fig. S2. Comparison between the ideal discrete Talbot phase modulation and the employed single tone phase modulation. (a) Photodetected temporal amplitude profile of the dispersed pulse (after the first dispersive fiber section, see experimental setup in Fig. 2), shown with respect to the estimated pulse time-width of the original illumination wave, measured by a 28 Gsa/s real-time oscilloscope equipped with a 43 GHz photodiode. (b) Periodic temporal Talbot phase profile associated to a Talbot condition with $m = 2$. $V_{\pi}$ represents the half-wave voltage of the electro-optic phase modulator. (dashed line) Prescribed phase obtained from the theory of Talbot effect. (solid line) Measured phase modulator drive voltage used in the experiments, approximating the Talbot phase by a single frequency component. The number of periods of the phase sequence occurring within the temporal duration of the dispersed pulse approximately equals the number of generated frequency gaps in the illumination spectrum. (c) Numerical simulation showing a comparison between the expected power spectra of the frequency gaps obtained with the prescribed Talbot phase (dashed line), and the single-tone approximation (solid line). The measured spectrum of the frequency gaps is shown for reference (shaded area). The results suggest that the cloaking bandwidth could be improved through the use of phase modulation sequences closer in shape to the ideal Talbot pattern.
Fig. S3. Formation of frequency gaps in the spectrum of a broadband wave for different values of the parameter $m$ (numerical simulation). (left) Temporal Talbot phase sequences, satisfying Eq. S3; (right) resulting power spectra. See text for definitions of parameters.
Fig. S4. Complete characterization of the complex spectral transfer function of the object. Phase (top) and log-scale magnitude of the transmission (bottom) spectral profiles of the linear optical filter, used as the object to be concealed in the reported experiments. The measurements are performed with an optical vector analyzer (Luna Innovations OVA 5000) with a frequency resolution of 200 MHz. In this representation, the transmission spectral response corresponds to the reciprocal of the object absorption as a function of frequency; this way, a transmission value of 0 dB indicates transparency, while a transmission value of $-35$ dB corresponds to $35$ dB of optical power absorption. The object consists of a set of 11 resonances spaced $38$ GHz apart, which sets the value of $\nu_c$ that is used for design in the reported transformations for spectral invisibility cloaking. Each resonance has a 3-dB bandwidth of $17.5$ GHz. The peak absorption is measured to be approximately $35$ dB. Each resonance introduces an additional phase shift of $\pi$ rad.
**Fig. S5.** Frequency gaps measured with different spectral resolutions. The solid line corresponds to a power spectral density measurement with a spectral resolution of 140 MHz, the dashed line shows the same measurement with a spectral resolution of 2 GHz. (a) Detail of the illumination spectrum. (b) Detail of the spectrum of the frequency gaps. The illumination wave has a repetition rate of 250 MHz (observable in the high resolution measurements in the form of spectral lines with a 250-MHz frequency spacing). The designed spectral transformations introduce frequency gaps in the complex envelope of the spectrum, regardless of the pulse repetition rate. The design parameters correspond to the same implementation described in the Main text, with $v_r = 19$ GHz and $m = 2$. 
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