Modified Chirp Scaling Algorithm for Ultra-High Resolution Spaceborne-SAR

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Abstract. The hyperbolic range equation model (HREM) and equivalent squint range model (ESRM) are applied in traditional chirp scaling algorithm (CSA). However, these range models cannot describe the satellite range history in the high-resolution case accurately because of the long azimuth integration time. The non-negligible phase error caused by this will lead the targets distort. In this paper, a modified chirp scaling algorithm (MCSA) is proposed by introducing a novel high-precision range model. A more accurate signal spectrum is calculated through it. Then, the modified chirp scaling factor, range compression filter, range cell migration correction (RCMC) filter and azimuth compression filter can be derived based on this signal spectrum, and the focused target obtained at last. Finally, the experimental results, to validate the proposed algorithm, adopted by the sliding spotlight synthetic aperture radar (SAR) simulation are provided.

1. Introduction
The capability of azimuth antenna beam rotating offers the long azimuth integration time for spaceborne-SAR. Based on this, the spotlight mode and the sliding spotlight mode, which can provide the ultra-high resolution, have been developed in many advanced spaceborne-SAR system such as TerraSAR-X, Cosmo-SkyMed and GaoFen-3. For the future, the resolution will reach 0.25 m by the TerraSAR NEXT GENERATION (TerraSAR NG).

However, the significant phase error which introduced by the conventional HREM or ESRM for much longer integration time will result in distortion of the target focusing. To avoid this, Wang [1] et al. proposed a high precision range model named modified equivalent squint range model (MESRM), the azimuth velocity variation is concerned through adding an equivalent radar acceleration in ESRM to improve the precision of this range model. In the end of [1], an extended hybrid correlation algorithm is presented that can achieve 0.25 m of azimuth resolution.

The chirp scaling algorithm (CSA) is proposed in 1994 by Raney [2] et al. It is an efficient algorithm in processing RCMC by the principle of linear frequency modulation signal (LFM) scaling. But the insufficient accuracy of the signal spectrum calculated by the HREM or ESRM limits CSA’s performance in the high-resolution situation. In 2012, an extended nonlinear chirp scaling algorithm, proposed by An [3] et al, which can obtain the focused target of an azimuth resolution around 1 m with a huge squint angle. Wang [4] et al, a year later, optimized the conventional CSA by the four-order range
model and reach the azimuth resolution of 0.3 m. In 2020, Wei [5] et al. enhanced the resolution to 0.1 m by means of combining the CSA and range migration algorithm (RMA).

In this paper, a modified chirp scaling algorithm (MCSA) is proposed by introducing the MESRM into the conventional CSA. Due to the difficulty of obtaining the spectrum’s expression because of the MESRM’s complex expression while adopting the principle of stationary phase (POSP), the Taylor series approximation and Lagrange inversion is employed under the condition of ensuring accuracy.

This paper is organized as follows. The step of signal spectrum calculation and MCSA procedure is presented in Section 2. Simulation results, in Section 3, are provided and conclusions are drawn in Section 4.

2. Modified Chirp Scaling Algorithm

2.1. Signal Model

The spaceborne-SAR received signal for a point target, after demodulation to baseband, can be described as

$$s(\tau, \eta) = A_0 \omega_R \left( \tau - \frac{2R(\eta)}{c} \right) \omega_A (\eta - \eta_c) \exp \left\{ -j \frac{4\pi f_0 R(\eta)}{c} \right\} \exp \left\{ j \pi K_r \left( \tau - \frac{2R(\eta)}{c} \right)^2 \right\}$$  \hspace{1cm} (1)

In these expressions, the $A_0$ represents the signal amplitude; $\omega_R(\cdot)$ and $\omega_A(\cdot)$ donate the antenna pattern functions in the range and azimuth directions respectively; $c$ is the speed of light; $f_0$ is the signal carrier frequency; $K_r$ is the range chirp rate; $R(\eta)$ is the MESRM; $\tau$ is the range time and $\eta$ is the azimuth.

In order to avoid the difficulty of the signal spectrum computation caused by MESRM’s complex expression, the six-order Taylor approximation is applied by

$$R(\eta) \approx R_0 + R_1 \eta + R_2 \eta^2 + R_3 \eta^3 + R_4 \eta^4 + R_5 \eta^5 + R_6 \eta^6$$  \hspace{1cm} (2)

On the basis of this approximation, the signal 2-D spectrum can be obtained by the Lagrange inversion as

$$S(f_\tau, f_\eta) = A_0 \omega_R (f_\tau) \omega_A (f_\eta - f_{\eta_c}) \exp \{ j \Phi(f_\tau, f_\eta) \}$$  \hspace{1cm} (3)

$$\Phi(f_\tau, f_\eta) = -\frac{4\pi (f_0 + f_\tau)}{c} \sum_{i=2}^{7} \frac{i-1}{i} C_i \left[ P \left( f_\eta + \frac{2\pi (f_0 + f_\tau)}{c} R_1 \right) \right]^{i-1} - \frac{\pi f^2 \sqrt{K_r}}{c} - \frac{4\pi (f_0 + f_\tau) R_0}{c}$$  \hspace{1cm} (4)

while $C_i$ is the coefficients of the inversion and $P = -c f_\eta / 2(f_0 + f_\tau)$. The $f_\tau$ is the range frequency and the $f_\eta$ is the azimuth. Expand $\Phi(f_\tau, f_\eta)$ by $f_\tau$ through Taylor series, it can be expressed as

$$\Phi(f_\tau, f_\eta) = D_0(f_\eta) + D_1(f_\eta) f_\tau + D_2(f_\eta) f_\tau^2 + \cdots$$  \hspace{1cm} (5)

After compensated the high-order phase term and transformed the signal to the range time domain by a inverse fast Fourier transform (IFFT), the signal in range Doppler (RD) domain is

$$S_1(\tau, f_\eta) = A_0 \omega_R \left( \tau - F_2(f_\eta) \right) \omega_A (f_\eta - f_{\eta_c}) \exp \{ j E_0(f_\eta) \} \exp \left\{ -j E_1(f_\eta) \left( \tau - E_2(f_\eta) \right)^2 \right\}$$  \hspace{1cm} (6)

while $E_0(f_\eta) = D_0(f_\eta)$, $E_1(f_\eta) = \frac{\pi^2}{2 \Delta_2(f_\eta)}$, $E_2(f_\eta) = -\frac{D_1(f_\eta)}{2 \pi}$.  

2.2. Target Focus

According to the (6), the modified chirp scaling factor, range compression filter, RCMC filter and azimuth compression filter can be derived as

$$\Phi_{cs}(\tau, f_\eta) = \exp \left\{ -j E_1(f_\eta, R_{0, ref}) C_s(f_\eta) \left( \tau - \tau_{ref}(f_\eta) \right)^2 \right\}$$  \hspace{1cm} (7)

$$H_{rc}(f_\tau, f_\eta) = \exp \left\{ -j \frac{f_\tau^2}{E_1(f_\eta, R_{0, ref}) C_s(f_\eta)} \right\} \exp \left\{ \frac{4\pi f_\tau R_{0, ref} C_s(f_\eta)}{c} \right\}$$  \hspace{1cm} (8)
\[ H_{ac}(\tau, f_{\eta}) = \exp\{-jE_0(f_{\eta})\} \times \exp\left\{-jE_1(f_{\eta}, R_{0, ref})\left(\frac{E_2(f_{\eta}) + CS(f_{\eta})\tau_{ref}(f_{\eta})}{1 + CS(f_{\eta})}\right)^2 - E_2^2(f_{\eta}) - CS(f_{\eta})\tau_{ref}^2(f_{\eta})\right\} \]  

(9)

Where

\[ CS(f_{\eta}) = \frac{cE_2(f_{\eta} R_{0, ref})}{2R_{0, ref}} - 1 \]  

(10)

\[ \tau_{ref}(f_{\eta}) = E_2(f_{\eta}, R_{0, ref}) \]  

(11)

and \( R_{0, ref} \) represented the reference slant range of the scene center. The different RCM of the different targets located in each range door can be transformed as the same by multiply (7) and (6) in RD domain. After finished the chirp scaling operation, the range compression and RCMC is followed by transform the result to the two-dimensional frequency domain and multiply with the filter (8). Then back to RD domain through range IFFT and multiply with the azimuth compression filter (9). Finally, to finish the target focus, an azimuth IFFT is taken. The detail implementation steps are given in Figure 1.

![Figure 1. Detailed implementation of MCSA](image)

3. Simulation

To verify the proposed algorithm, the experiment result of simulation is presented in this section and compared with the imaging result gotten by the conventional CSA. As shown in Figure 2, nine point targets arranged in a scene of 2 km × 2 km evenly in the sliding spotlight mode, and the main simulation parameters are given in Table 1.

![Figure 2. The ground scene of simulation.](image)

**Table 1. Simulation Parameters**

| Description     | Value | Units | Description     | Value | Units |
|-----------------|-------|-------|-----------------|-------|-------|
| Semi-major      | 514   | km    | Bandwidth       | 1.0   | GHz   |
| \( \rho_a \)    | 0.25  | m     | \( f_s \)       | 1.2   | GHz   |
| Wavelength      | 0.03125 | m | Look angle      | 30    | deg   |
The part of simulation results of CSA and MCSA, to evaluate the image result, are as followed

(a) PT3  
(b) PT5  
(c) PT7

**Figure 3.** Interpolated results of (a) PT3, (b) PT5, (c) PT7 by the conventional CSA

(a) PT3  
(b) PT5  
(c) PT7

**Figure 4.** Interpolated results of (a) PT3, (b) PT5, (c) PT7 by the MCSA.

As seen from the results in Figure 3, the phase error which introduced by the ESRM for long integration time will lead the imaged targets distort and defocus. Whereas the MCSA imaging results shown in Figure 4, this problem has been solved by replacing the imprecise range model to the MESRM.

In order to quantify the algorithm performance, the analysis results of focused point target are listed in Table 2. The ideal peak sidelobe ratio (PSLR) is -13.26 dB and the integrated sidelobe ratio(ISLR) is -9.68 dB, with the rectangular window. The theoretical resolution is

\[
\begin{align*}
\rho_r &= \frac{c}{2B_r} \\
\rho_a &= \frac{L_a}{2} H_f
\end{align*}
\]

where \( L_a \) is represent the antenna length and \( H_f \) is the hybrid factor.

**Table 2.** Performance analysis of Point Targets Obtained by MCSA

| Target No. | \( \rho_r \) (m) | Range PSLR(dB) | ISLR(dB) | \( \rho_a \) | Azimuth PSLR(dB) | ISLR(dB) |
|-----------|-----------------|----------------|----------|--------------|-----------------|----------|
| 1         | 0.1328          | -13.224        | -10.036  | 0.2393       | -13.237         | -10.370  |
| 2         | 0.1328          | -13.257        | -10.048  | 0.2392       | -13.254         | -10.373  |
| 3         | 0.1328          | -13.225        | -10.032  | 0.2393       | -13.234         | -10.397  |
| 4         | 0.1330          | -13.233        | -10.017  | 0.2394       | -13.234         | -10.398  |
| 5         | 0.1329          | -13.247        | -10.019  | 0.2393       | -13.244         | -10.381  |
| 6         | 0.1330          | -13.198        | -10.028  | 0.2394       | -13.250         | -10.384  |
| 7         | 0.1329          | -13.212        | -10.020  | 0.2394       | -13.245         | -10.402  |
| 8         | 0.1328          | -13.266        | -10.042  | 0.2392       | -13.237         | -10.397  |
| 9         | 0.1329          | -13.218        | -10.017  | 0.2393       | -13.229         | -10.392  |
According to the results, it can be seen that the resolution error is less than 1% in range direction and less than 4% in azimuth direction. The PSLRs degradation shows in table are less than 1% both range and azimuth direction.

4. Conclusion
In order to focus the targets in the spaceborne-SAR system with ultra-high resolution, a modified chirp scaling algorithm is proposed in this paper by introduce a high-precision range model. To overcome the difficulty of signal spectrum calculate, a Taylor approximate and Lagrange inversion are employed. Based on this, the high-precision 2-D spectrum and processing steps are presented as followed. The experiment results of simulation validated this algorithm has good adaptability in practical applications.

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