QCD and nuclear physics. How to explain the coincidence between the radius of the strong interaction of nucleons and the characteristic scale of neutron-neutron electrostatic interaction?

A.A. Godizov

Institute for High Energy Physics, 142281 Protvino, Russia

Abstract

An attempt is made to interpret, in the framework of QCD, the coincidence of two observable scales which characterize the interaction between neutrons.

1. Introduction

The discovery of vector meson $J/\psi$ in 1974 and the explanation of its decay width narrowness in the framework of perturbative quantum chromodynamics (QCD) \[ \] induced a drastic rise of interest to this quantum-field model from the scientific community, and, for more than 30 years, QCD is the only world-wide recognized candidate for the position of the fundamental theory of strong interaction. At present, a variety of very effective theoretical approximations exists: the QCD sum rules, chiral perturbation theory, relativistic mean field theory, the Skyrme model, lattice QCD, nonrelativistic models with “realistic” potentials, contact interaction models, bag models etc. However, in spite of the impressive successes in development of these approaches, many important problems related to putting QCD into agreement with experiment have not been solved yet.

In this eprint an attempt is made to get within QCD an answer to the question from the title (just for curiosity). At this, we will indispensably touch upon the problem of accordance between the global color structure of chromodynamics and the dynamics of low-energy nuclear systems.

2. Comparison of the two characteristic scales

The available experimental data on the neutron charge form factor allow, in principle, a straight model-independent estimation of the characteristic scale of the electrostatic interaction between neutrons, but, for simplicity, we use some test parametrization.

The electric charge distribution in nucleon can be represented as the superposition of two functions which determine the distributions of the quarks $u$ and $d$. Let us set the charge density

*E-mail: anton.godizov@gmail.com
inside proton as \( \rho_p^{(p)}(r) = \frac{4}{3} e \exp(-r/b)/(8\pi b^3) - \frac{2}{3} e \exp(-r/a)/(8\pi a^3) \) and, correspondingly, the charge density inside neutron as \( \rho_n^{(n)}(r) = \frac{2}{9} e \exp(-r/a)/(8\pi a^3) - \frac{2}{9} e \exp(-r/b)/(8\pi b^3) \), where \( a = 0.2 \text{ fm} \) and \( b = 0.225 \text{ fm} \). Such a description of the charge structure of nucleons looks acceptable, since in the range \( 0 < |\vec{q}| c < 1 \text{ GeV} \) the resulting charge form factor of proton, \( F^{(p)}(q^2) = \frac{4}{3(1+0.71 q^2/\hbar^2)^2} - \frac{1}{3(1+0.71 q^2/\hbar^2)^2} \), almost coincides with the phenomenological approximation \( F^{(p)}_{\text{phen}}(q^2) = (1 + \frac{q^2}{0.71 \text{ GeV}^2})^{-2} \) usually exploited in literature, and the test charge form factor of neutron, \( F^{(n)}(q^2) = \frac{2}{3(1+a^2 q^2/\hbar^2)^2} - \frac{2}{3(1+b^2 q^2/\hbar^2)^2} \), is consistent with the experimental measurements (see, for instance, Fig. 13 in [2]). The corresponding radial distributions of electric charge in nucleons are presented in Fig. 1 on the left.

Figure 1: Approximate radial distributions of electric charge in proton and neutron (on the left) and the potential energy \( V^{(nn)}_E(R) \) of neutron-neutron electrostatic interaction as function of the distance \( R \) between their centers of mass (on the right).

Having got the information on the electric charge radial distribution in neutron, we are able to calculate, in the leading approximation, the potential energy \( V^{(nn)}_E(R) \) of neutron-neutron electrostatic interaction as function of the distance \( R \) between their centers of mass (Fig. 1 on the right). The position of this function’s minimum, \( R_{\text{min}} \approx 1.4 \text{ fm} \), is not noticeably different from the action radius of nuclear forces. Namely, one can speak about the coincidence of these two scales characteristic for neutron.

The very fact of such a coincidence is highly nontrivial, because the structure of the strong and electrostatic interaction between neutrons at the distances \( 1 \div 3 \text{ fm} \) is, admittedly, quite different. Let us try to explain this effect in terms of chromodynamics.

### 3. The state vector of nucleon

In quantum theory, nucleon is described by means of the state vector \( |\psi >_{\text{nucleon}} \) which is understood as the ground configuration of three interacting fermions (the constituent quarks) possessing some electric charge, color, and flavor. This state vector is usually represented in the following form [3]:

\[
|\psi >_{\text{nucleon}} = |\psi >_{\text{color}} \otimes |\psi >_{\text{spin, flavor}} \otimes |\psi >_{\text{space}} =
\]

![Figure 1](image-url)
\[ \frac{1}{\sqrt{6}} (|1, 2, 3>_c + |2, 3, 1>_c + |3, 1, 2>_c - |2, 1, 3>_c - |1, 3, 2>_c - |3, 2, 1>_c) \otimes (1) \]
\[ \otimes \frac{1}{\sqrt{18}} [2 (u \uparrow u \uparrow d \downarrow) - (u \uparrow u \downarrow + u \downarrow u \uparrow) d \uparrow \downarrow + permutations] \otimes |\psi>_{space} , \]

where \(|\psi>_{space}\) denotes the coordinate part of the state vector.

Some evident arguments can be put forward in favor of such a description:

- the existence of the wave function strict antisymmetry characteristic for the systems of identical fermions (Fermi-Dirac statistics);
- the presumption about the adequacy of the isotopic symmetry approximation wherein \(u\)-quark and \(d\)-quark are considered as two states of the same particle with different values of the isospin projection \(I_3\);
- the requirement of the nucleon local colorlessness (a strict consequence of the confinement hypothesis);
- the compatibility of \(|\psi>_{spin,flavor}\) in (1) with the empirical relation \(\mu^{(n)}/\mu^{(p)} \approx -2/3\) for the magnetic moments of nucleons.

First of all, we should note that, in the framework of the Standard Model, quarks \(u\) and \(d\) are not identical. The isospin symmetry is an effective approximate symmetry of strong interaction, and not a fundamental symmetry of QCD. As well, the confinement is an observable phenomenon and not a primordial attribute of the QCD Lagrangian. Therefore, the requirement of \(a\ priori\) antisymmetry of the nucleon wave function regarding the permutations of the \(u\)-quark and \(d\)-quark quantum numbers is redundant.

Let us formulate the hypothesis which underlies the further reasoning.

**Basic hypothesis 1.**

- In quantum chromodynamics, there exist a gauge (accomplished, if necessary, by the proper choice of the global color representation) and a renormalization scheme consistent with this gauge, wherein the state vector of isolated nucleon in rest has the following structure, in the leading approximation:
  \[ |\psi>_{nucleon} = |\psi>_{flavor} \otimes |\psi>_{color} \otimes |\psi>_{spin} \otimes |\psi>_{space} = (2) \]
  \[ = |\psi>_{flavor} \otimes \frac{1}{\sqrt{2}} (|1, 2, 3>_c - |2, 1, 3>_c) \otimes \frac{1}{\sqrt{6}} [2 (\uparrow\uparrow\downarrow) - (\uparrow\downarrow + \downarrow\uparrow) \uparrow] \otimes |\psi>_{space} , \]

where \(|\psi>_{flavor} = |u u d>\) for proton and \(|\psi>_{flavor} = |d d u>\) for neutron.

Note, that such a structure of the nucleon state vector is in agreement with the observed relation \(\mu^{(n)}/\mu^{(p)} \approx -2/3\).

State (2) is colorless just integrally. The local colorlessness is violated. Consequently, the gluon confinement hypothesis is violated as well, since the currents of nucleons become a sources of gluon field. The reasons for the experimental non-observation of gluons are discussed below, but at this point we notice that the following equalities are valid for the state vector (2):

\[ <\psi|\hat{t}_{a}^{(i)}|\psi> = 0 \quad (a = 1, \ldots, 7; \ i = 1, 2, 3) , \]
\[ <\psi|\hat{t}_{8}^{(1, 2)}|\psi> = \frac{1}{2\sqrt{3}} , \quad <\psi|\hat{t}_{8}^{(3)}|\psi> = -\frac{1}{\sqrt{3}} , \]

where \(\hat{t}_a \equiv \lambda_a/2\) are the generators of the \(SU(3)\) group fundamental representation (\(\lambda_a\) are the Gell-Mann matrices) and the superscript denotes the quark’s number.

Relations (3) have several important consequences:
the currents of nucleons are the sources of the gluon octet 8-th component only;

under the exchanges by separate gluons, nucleons do not change their color structure at all, owing to the matrix $\lambda_8$ diagonality;

in the tree level, the interaction between two nucleons is reduced to the superposition of the constituent quark Coulomb interactions with the only distinction that the effective charge of the $u$-quarks in proton and of the $d$-quarks in neutron is equal to $-\frac{g_s}{\sqrt{3}}$, and the effective charge of the $d$-quark in proton and of the $u$-quark in neutron is equal to $+\frac{g_s}{\sqrt{3}}$, where $g_s$ is the elementary charge of QCD ($\alpha_s \equiv \frac{g_s}{\sqrt{3}}$), i.e., the distribution of the nuclear interaction effective charge in nucleon is proportional to the electric charge distribution in neutron.

In other words, the following physical pattern takes place. The operator of the total energy of nucleon-nucleon interaction can be represented as

$$\hat{V} = \sum_{i,j} \hat{V}^{gl}_{ij} + \hat{V}^r,$$

where $\hat{V}^{gl}_{ij}$ are the weakly connected parts of the effective operators of pair interaction between those constituent quarks which are contained in different nucleons, and $\hat{V}^r$ denotes other contributions (reggeon exchanges). If the isolated nucleon state vector takes form (1), then, in view of the structure of generators $\hat{t}_a$, the contribution of the weakly connected parts of the constituent quark pair interactions into the potential energy of the nuclear system are equal to zero (the confinement case). If the isolated nucleon state vector takes form (2), then the weakly connected part of quark-quark interaction can be effectively represented as

$$\hat{V}^{gl}_{ij} = \frac{\alpha_s}{12} Q^{(1)} Q^{(2)} \left[ \hat{V}^{CS}_{ij} + \hat{V}^{CM}_{ij} \right], \quad (4)$$

where $Q = -1$ for the $u$-quarks in proton and for the $d$-quarks in neutron, and $Q = +2$ for the $d$-quark in proton and for the $u$-quark in neutron. The structure of the operators $\hat{V}^{CS}_{ij}$ and $\hat{V}^{CM}_{ij}$ does not depend on the quark colors and, in the leading approximation, on their flavors. Operator $\hat{V}^{CS}_{ij}$ is related to the effective chromostatic potential $V^{CS}(r)$ which includes the influence of the QCD vacuum polarization. Operator $\hat{V}^{CM}_{ij}$ is related to the effective potential $V^{CM}(\vec{r}, \vec{n}_1, \vec{n}_2)$ of quark-quark chromomagnetic interaction (the unit vectors $\vec{n}_1$ and $\vec{n}_2$ determine the quark spin orientations).

For the phenomenological description of nucleons as the 3-fermion systems with the effective charge structure $\{-1, -1, +2\}$, the dominance of the weakly connected parts of the quark pair interactions over other contributions is necessary. Therefore, in addition to basic hypothesis 1, we need

Basic hypothesis 2.

In the systems of low-energy nucleons, the interaction between nucleons at the distances higher than 1 fm is dominated by the weakly connected parts of the effective quark-quark interaction.

Note, that although the presumed effective structure (2) of the nucleon state vector is related to some concrete renormalization scheme associated with a certain gauge, the content of the basic hypothesis 2 itself is not related to any gauge or renormalization scheme.

Under such an approach, the short-rangeness of the chromostatic interaction of nucleons is conditioned, mainly, by their integral neutrality (like the short-rangeness of the interaction
between helium atoms), in contrast to the models with pion and other meson exchanges wherein the short-rangeness is related to the meson field massiveness. Besides, at the distances \( r \gg 1 \) fm the effective potential \( V^{CM}(\vec{r}, \vec{n}_1, \vec{n}_2) \) should decrease significantly faster than \( r^{-3} \), for the faster drop of the nonscreened chromomagnetic interaction in comparison with the magnetic interaction of the quarks.

4. Discussion

4.1. Qualitative form of the effective nuclear potential

The densities of the constituent quark distributions can be extracted from the proton and neutron experimental form factors. Then, in accordance with the aforesaid, the potential energy of nucleon-nucleon interaction could be introduced as the sum of the energies of the constituent quark pair interactions. In Fig. 2 the result is presented for the test case \( \alpha_s = 3000, V^{CS}(r) = 1/r \) and \( V^{CM}(\vec{r}, \vec{n}_1, \vec{n}_2) = 0 \) (certainly, the true effective potential of quark-quark chromostatic interaction could essentially differ from the Coulomb one).

![Figure 2: The nucleon-nucleon interaction potential energy for the test case \( \alpha_s = 3000, V^{CS}(r) = 1/r \) and \( V^{CM}(\vec{r}, \vec{n}_1, \vec{n}_2) = 0 \).](image)

4.2. The range of applicability

Basic hypothesis 1 states that structure (2) corresponds to isolated nucleon in rest. Therefore, the adequacy of the considered approximation is expected for the systems of low-energy nucleons only. The color structure of state vector (2) could, in principle, take place for the hyperons \( \Sigma \) and \( \Xi \) of the lightest baryon octet. But it is absolutely unsuitable, say, for the decuplet of the light spin-3/2 baryons.
4.3. The isospin symmetry of nuclear forces

Owing to the equivalence of the proton and neutron color structures, the effective interaction of nucleons possesses the isospin symmetry, in the leading approximation. However, we would like to point out that the color of the $u$-quark in neutron is different from the colors of the $u$-quarks in proton. The same can be said about the $d$-quarks. Therefore, the exchange contributions are absent in the interaction between proton and neutron (they are distinguishable entirely), in contrast to the interaction of identical nucleons. But, due to the rather weak overlap of the nucleon wave functions, the exchange interaction turns out to be just a small corrective effect which violates the isospin symmetry on the level with the quark-quark electromagnetic interaction and the difference between $m_u$ and $m_d$.

4.4. Quark confinement

As the expected range of the considered approximation validity is restricted by low-energy nuclear systems, so the foregoing reasoning is not in contradiction with the quark confinement hypothesis.

4.5. On the experimental non-observation of gluons

In the proposed approach, nucleons are a gluon field sources. Hence, nucleon collisions are inevitably accompanied by radiation of real gluons. These gluons can interact between themselves (including the color exchanges). As well, they are able to annihilate into photons or to be absorbed by nucleons, but, in view of relations (3), only the 8-th component of the gluon octet interacts with nucleons. Let us remind, that, under such interaction, the structure of the color part of the nucleon state vector (2) keeps unchanged, i.e. the color exchange between nucleons does not occur.

The density of effective nuclear charge in nucleon is proportional to the electric charge density in neutron. Consequently, the energy spectra of the gluons radiated in nonrelativistic nuclear collisions are alike to the energy spectra of the photons radiated in neutron-neutron scattering. The significant divergence is in the multiplicities only, due to the difference between the elementary charge values of chromodynamics and electrodynamics. It is quite possible that such a “disguise” of gluons as soft photons is the main reason of the non-observation of them in low-energy nuclear reactions (especially, in the reactions with the participation of protons). However, gluons do not interact with leptons. Therefore, a principal feasibility exists to distinguish them from photons.

Concerning the non-observation of gluons even at higher energies (up to the energies characteristic for quark-gluon plasma), one should pay attention to the fact that strong interaction is several orders stronger than electromagnetic one. As, in addition, color can be transferred by gluons of arbitrarily small energy, so the gluon confinement hypothesis does not prevent numerous discussions on the gluon jet production in high-energy collisions.

5. Conclusion

Thus, an answer to the question from the eprint title can be given in terms of the global color structure of QCD. The attractiveness of such an explanation is determined as by its simplicity, so by the fact that the first and second basic hypotheses do not introduce any notions irrelevant for QCD. Namely, these hypotheses just presume the negligibility (for the systems of low-energy nucleons) of certain contributions which have an unambiguous interpretation in the framework
of QCD. Nevertheless, only the explicit comparison of the detailed modelling outcomes with the experimental data on the nonrelativistic scattering phase shifts and on the light nuclei spectra will allow to confirm or disprove the practical adequacy of the proposed approach to description of nuclear forces and, consequently, the correctness of the explanation of the coincidence between the two observable scales.

References

[1] F.J. Ynduráin, Quantum Chromodynamics. Springer-Verlag, New York–Berlin–Heidelberg–Tokyo 1983

[2] S. Platchkov et al., Nucl. Phys. A 510 (1990) 740

[3] F. Halzen and A.D. Martin, Quarks and Leptons. John Wiley & Sons, New York–Chichester–Brisbane–Toronto–Singapore 1984