Stability of Thick-Walled Elastic Anisotropic 3-Dimensional Cylindrical Shells Under Axial Pressure Load

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Abstract. On the basis of the refined Timoshenko-Mindlin beam theory, an approach to calculate the stability of three-dimensional anisotropic cylindrical shells of geometrically nonlinear subcritical stress-strain state, in which the extreme layers are of higher rigidity than the average, is presented. The shell material has symmetry properties in a plane that could match the median surface. Numerical solutions are performed with the use of discrete orthogonalization methods. In the paper the plots illustrating the influence of stacking sequence and lay-up angle of layered fibrous composites on magnitude of critical values of axial compression are presented.

1. Introduction
The possibilities of creating, for instance, optimal multilayer composite structures and their use in aviation and space industries, mechanical engineering, etc., when using fibrous reinforced materials in layered composite cylindrical shells, are considerably expanded. The effectiveness of the introduction of thin-walled layered structures made of advanced composite materials greatly depends on the experimental and theoretical study of issues related to determining the bearing capacity of such structures. It is known that its exhaustion is often associated with the stability loss.

Analysing the latest achievements in mechanics of thin-walled structures, we can assert, that there is a large number of directions to study the stability of composite shell structures [1, 2, 3, 4, 5, 6]. Anisotropic shells are usually produced by winding or calculating on individual composite layers mandrel with a slight thickness [4, 7, 8]. This suggests that the lowest level of material property symmetry of such layers is when there is one plane in which mechanical properties are symmetric. In the monograph the results of study of thin-walled anisotropic shells made of the materials of elastic symmetry plane are presented [9]. In the paper the methods of calculation within the geometrically nonlinear and Kirchhoff-Love plate theory, and results of study of thin-walled anisotropic shells stability of zero, positive and negative Gaussian curvatures made of composite materials with one plane of elastic symmetry are offered.

However, there are a lot of unsolved problems regarding the stability of thick-walled anisotropic shells. It is generally well known that refined higher-order theories to calculate the stability of thick-walled anisotropic shells should be applied [2, 10, 11, 12] and the refined Timoshenko beam theory is among the most commonly-used approaches.
The paper presents an approach to calculate the stability of thick-walled anisotropic cylindrical shells using the refined Timoshenko beam theory.

2. Geometrically nonlinear subcritical stress-strain state of thick-walled anisotropic cylindrical shells under axially symmetric loads.

To construct equations determining the shells critical state with regard to bifurcation phenomenon, it is used the canonical system of non-linear stress-strain equations with symmetrically loaded thick-walled anisotropic shells see figure 1.

![Figure 1. Cylindrical shells under axial compression](image)

Let’s represent the displacement $u$, $v$, $w$ in accordance with Timoshenko beam hypotheses as linear scheduling in the coordinate $z$:

$$
\begin{align*}
    u &= u_0 + z\theta_1, \\
    v &= v_0 + z\theta_2, \\
    w &= w \\
    u_\alpha &= u_{0\alpha} + z\theta_\alpha, \\
    u_1 &= u + z\theta_1, \\
    v_2 &= v + z\theta_2, \\
    w_1 &= w
\end{align*}
$$

where $u$, $v$, $w$ - the displacement of the shell median surface, $z$ - the coordinate to specify the shell thickness changes, $\theta_\alpha$, ($\alpha = 1, 2$) - rotate angles relative to the surface normal of the coordinate axes.

The interrelations defined by Hooke’s Law for anisotropic material with one plane of elastic symmetry in accordance with the source [1]:

$$
\begin{align*}
    \sigma_{11} &= a_{11}\varepsilon_{11} + a_{12}\varepsilon_{22} + a_{13}\varepsilon_{33} + a_{16}\varepsilon_{12}, \\
    \sigma_{33} &= a_{13}\varepsilon_{11} + a_{23}\varepsilon_{22} + a_{33}\varepsilon_{33} + a_{36}\varepsilon_{12}, \\
    \sigma_{13} &= a_{45}\varepsilon_{23} + a_{55}\varepsilon_{13}, \\
    \sigma_{12} &= a_{16}\varepsilon_{11} + a_{26}\varepsilon_{22} + a_{36}\varepsilon_{33} + a_{46}\varepsilon_{12}
\end{align*}
$$

where $\sigma_{ij}$ - normal and shear stresses, $\varepsilon_{ij}$ - deformation of tension and displacement ($i, j = 1, 2, 3$), $a_{ij}$ - elastic steels ($i = 1, 2, 3; j = 1, 6$).

To obtain nonlinear equations within quadratic terms [2]:

$$
\begin{align*}
    \varepsilon_{11} &= \varepsilon_{11} + zk_{11} + z^2\nu_{11}, \\
    \varepsilon_{22} &= \varepsilon_{22} + zk_{22} + z^2\nu_{22}, \\
    \varepsilon_{12} &= \varepsilon_{12} + zk_{12} + z^2\nu_{12}
\end{align*}
$$

where

$$
\begin{align*}
    \varepsilon_{11} &= \varepsilon_{1} + \frac{1}{2}\theta_1^2, \\
    \varepsilon_{22} &= \varepsilon_{2} + \frac{1}{2}\theta_2^2, \\
    \varepsilon_{12} &= \theta_1\theta_2, \\
    k_{11} &= \chi_1 + \varepsilon_1\chi_1, \\
    k_{22} &= \chi_2 + \varepsilon_2\chi_2, \\
    k_{12} &= \tau_1 + \tau_2
\end{align*}
$$
\[ \nu_{11} = \frac{1}{R_1} k_{11} + \frac{1}{2} (\chi_1^2 + \tau_1^2), \quad \nu_{22} = \frac{1}{R_2} k_{22} + \frac{1}{2} (\chi_2^2 + \tau_2^2), \quad \nu_{12} = \frac{1}{R_1} \tau_1^* + \frac{1}{R_2} \tau_2^* + \chi_1 \tau_2, \]

in which \( \chi_1 = k_1 + \frac{\varepsilon_1}{R_1}, \quad \chi_2 = k_2 + \frac{\varepsilon_2}{R_2}, \quad \tau_1 = t_1, \quad \tau_2 = t_2, \quad \tau_1^* = \tau_1 + \varepsilon_2 \tau_1, \quad \tau_2^* = \tau_2 + \varepsilon_2 \tau_2. \)

The deformations and increments of the curvatures and torsion are as follows:

\[ \varepsilon_{11} = \varepsilon_1 = \frac{1}{A_1} \frac{\partial u}{\partial \alpha_1} + a_1 v - \frac{w}{R_1}, \quad \varepsilon_{22} = \varepsilon_2 = \frac{1}{A_2} \frac{\partial v}{\partial \alpha_2} + a_2 v - \frac{w}{R_2}, \quad \varepsilon_{12} = \theta_1 + \theta_2, \]

\[ k_{11} = k_1 + \varepsilon_1, \quad k_1 = \frac{1}{A_1} \frac{\partial \theta_1}{\partial \alpha_1} + a_1 \varepsilon_1 - \frac{\theta_2}{R_1}, \quad k_{22} = k_2 + \varepsilon_2, \]

\[ k_2 = \frac{1}{A_2} \frac{\partial \theta_2}{\partial \alpha_2} + a_2 \varepsilon_2 - \frac{\theta_1}{R_2}, \quad k_{12} = t_1 + t_2, \quad t_1 = \frac{1}{A_1} \frac{\partial \theta_1}{\partial \alpha_1} - a_1 \varepsilon_1, \quad t_2 = \frac{1}{A_2} \frac{\partial \theta_2}{\partial \alpha_2} - a_2 \varepsilon_2, \]

\[ \nu_{11} = \frac{1}{R_1} k_{11}, \quad \nu_{22} = \frac{1}{R_2} k_{22}, \quad \nu_{12} = \frac{1}{R_1} \tau_1 + \frac{1}{R_2} \tau_2. \]

\( A_1, A_2 \) – Lame parameters (coefficients of the first quadratic form of the coordinate surface),

\[ a_1 = \frac{1}{A_1 A_2} \frac{\partial A_1}{\partial \alpha_2}, \quad a_2 = \frac{1}{A_1 A_2} \frac{\partial A_2}{\partial \alpha_1}. \]

Angles of rotation and curvature \( \kappa_{ij} \) are as follows

\[ \theta_1' = \frac{1}{A_1} \frac{\partial w}{\partial \alpha_1} + \frac{u}{R_1}, \quad \theta_2' = \frac{1}{A_2} \frac{\partial w}{\partial \alpha_2} + \frac{u}{R_2}, \]

\[ \kappa_{11} = k_1 + \frac{1}{R_1} \varepsilon_{11}, \quad \kappa_{22} = k_2 + \frac{1}{R_2} \varepsilon_{22}, \quad \kappa_{12} = 2 \tau + \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \varepsilon_{12} \]

In terms of torsion \( \kappa_{12} \) we have the functions \( \tau \) i \( \nu_{ij} \):

\[ t_1 = t_2 = \tau, \quad \nu_{11} = \frac{1}{R_1} \kappa_{11}, \quad \nu_{22} = \frac{1}{R_2} \kappa_{22}, \quad \nu_{12} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \tau. \]

In the source [9] an approach to construct a system of equations that describes the axisymmetric stress-strain state of anisotropic shells is presented. With this approach, we obtain the equations of the nonlinear deformation of symmetrically loaded thick-walled anisotropic shells.
3. Mathematical models and algorithms for studying anisotropic shells stability

The system of nonlinear equations (10) can be used to construct equations determining the shells critical state with regard to bifurcation phenomenon. After entering the notations

\[ y_1 = u, \quad y_2 = v, \quad y_3 = w, \quad y_4 = \theta_1, \quad y_5 = \theta_2, \]
\[ y_6 = T_{11}, \quad y_7 = T_{12}, \quad y_8 = T_{13}, \quad y_9 = M_{11}, \quad y_{10} = M_{12}^*. \]

We obtain the system of equations (10):

\[ \frac{1}{A_i} \frac{\partial y_i}{\partial \alpha_i} = L_i(y) + q_i, \]

where \( y \) - vector, components of which are functions \( y_i \), \( q_i \) - load components, \( L_i \) - nonlinear differential operators, \( i = 1, \ldots, 10 \).
On the main deformation trajectory equations (12) are as follows

$$\frac{1}{A_i} \frac{\partial y_{i,o}}{\partial \alpha_i} = L_i(y_o) + q_i. \quad (13)$$

On a related path they should be written as follows

$$\frac{1}{A_i} \frac{\partial (y_{i,o} + y_j)}{\partial \alpha_i} = L_i(y_o + y) + q_i. \quad (14)$$

According to the Euler’s criterion, $y_j$ - is infinitesimal disturbance of the ground state. Therefore, using the Fréchet derivative concept, in Taylor's series it is enough to use two variables

$$L_i(y_o + y) = L_i(y_o) + L_{i,j}(y_o)y \quad (15)$$

$L_{i,j}$ - Fréchet derivatives of operators $L_i$ of the argument $y_j (j = 1, ..., 10)$.

The equation (14) will be:

$$\frac{1}{A_i} \frac{\partial y_{i,o}}{\partial \alpha_i} + \frac{1}{A_i} \frac{\partial y_j}{\partial \alpha_i} = L_i(y_o) + L_{i,j}(y_o)y + q_i. \quad (16)$$

Taking into consideration that the loading $q_i$ does not change, and functions with the index «0» satisfy the equation (13), and from the equation (16) we obtain the linearized equation with respect to the increments of the function at the point of bifurcation.

$$\frac{1}{A_i} \frac{\partial y_j}{\partial \alpha_i} = L_{i,j}(y_o)y \quad (17)$$

A system of differential equations will be as follows:

$$\frac{1}{A_i} \frac{\partial y_1}{\partial \alpha_i} = -\frac{1}{A_2} \frac{\partial T_{21}^*}{\partial \alpha_2} - a_1(y_2 + T_{21}^*) - a_2(y_1 - T_{21}^*) + \frac{1}{R_1} y_3;$$

$$\frac{1}{A_i} \frac{\partial y_2}{\partial \alpha_i} = -\frac{1}{A_2} \frac{\partial T_{22}^*}{\partial \alpha_2} - a_2(y_2 + T_{21}^*) + a_1(y_1 - T_{22}^*) + \frac{1}{R_2} T_{23}^*;$$

$$\frac{1}{A_i} \frac{\partial y_3}{\partial \alpha_i} = -\frac{1}{A_2} \frac{\partial T_{23}^*}{\partial \alpha_2} - a_3y_3 + a_4 T_{23}^* - \frac{1}{R_1} y_1 - \frac{1}{R_2} T_{22}^*;$$

$$\frac{1}{A_i} \frac{\partial y_4}{\partial \alpha_i} = -\frac{1}{A_2} \frac{\partial M_{12}}{\partial \alpha_2} - 2a_4 M_{12} - a_5(y_4 - M_{12}) T_{13};$$

$$\frac{1}{A_i} \frac{\partial y_5}{\partial \alpha_i} = -a_5 y_2 + \frac{1}{R_1} y_7 - \epsilon_1 \epsilon_{1,0} - \theta_1 ^{1}, T_{11} + A_1 T_{11} + A_{12} T_{12} + A_{13} y_4 + d_1 \epsilon_{1,22} + d_1 \kappa_{22} + d_1 \kappa_{12};$$
\[
\frac{1}{A_1} \frac{\partial y_6}{\partial \alpha_1} = a_1 y_5 - \theta_{1,0} \theta_2 - \theta_{2,0} + A_{21} T_{11} + A_{22} T_{12} + A_{23} y_4 + d_1 \varepsilon_{22} + d_2 \kappa_{22} + d_3 \kappa_{12};
\]
\[
\frac{1}{A_3} \frac{\partial y_5}{\partial \alpha_3} = -\frac{1}{R_1} y_5 - y_6;
\]
\[
\frac{1}{A_4} \frac{\partial y_4}{\partial \alpha_4} = a_{0,2} + A_{21} T_{11} + A_{32} T_{12} + A_{33} y_4 + d_3 \varepsilon_{22} + d_4 \kappa_{22} + d_5 \kappa_{12};
\]
\[
\frac{1}{A_5} \frac{\partial y_3}{\partial \alpha_5} = T_{23}^* + d_1 \varepsilon_{22} + d_2 \kappa_{22} + d_3 \kappa_{12},
\]
(18)

where
\[
y_6 = T_{11} (1 + \varepsilon_{1,0}) + T_{11,0} \varepsilon_1, \quad y_7 = T_{12} (1 + \varepsilon_{2,0}) + T_{12,0} \varepsilon_2 - \frac{2}{R_2} M_{12},
\]
\[
y_8 = T_{13} + T_{11,0} \theta_1 + T_{11,0} \theta_2 + T_{12,0} \theta_2, \quad y_9 = M_{11}, \quad y_{10} = M_{12}^*,
\]
\[
T_{21}^* = T_{12} (1 + \varepsilon_{2,0}) + T_{22,0} \varepsilon_2,
\]
\[
T_{22}^* = \frac{1}{A_2} \frac{\partial \varepsilon_{22}}{\partial \alpha_2} - a_1 (y_4 - M_{22}) + 2 a_2 M_{12} + T_{11,0} \theta_1 + T_{11} \theta_1 + T_{12} \theta_2 + T_{12} \theta_2.
\]
(19)

Loads \(T_{22}, T_{23}\), and moments \(M_{22}, M_{12}\) are passive variables and are used to denote the function in some neighborhood of the point

\[
T_{22} = d_{11} T_{11} + d_{21} T_{12} + d_{31} M_{11} - (C_{22} - C_{22}^0) \varepsilon_{22} - (B_{22}^* - B_{22}^0) \kappa_{22} - (B_{26}^* - B_{26}^0) \kappa_{12},
\]
\[
M_{22} = d_{12} T_{11} + d_{22} T_{12} + d_{32} M_{11} - (B_{22}^* - B_{22}^0) \varepsilon_{22} - (D_{22} - D_{22}^0) \kappa_{22} - (D_{26} - D_{26}^0) \kappa_{12},
\]

(20)

\[
M_{12} = d_{13} T_{11} + d_{23} T_{12} + d_{33} M_{11} - (B_{26}^* - B_{26}^0) \varepsilon_{22} - (D_{26} - D_{26}^0) \kappa_{22} - (D_{66} - D_{66}^0) \kappa_{12}.
\]

Equations (18) are nonhomogeneous second order linear equations. Taking into account the frequency of desired function, we can approximate them by trigonometric Fourier series and thereby transform system (18) to the system of homogeneous first order linear equations.

The circular cylindrical shells have been considered, therefore, the solutions for the functions are periodic in a circular coordinate \(\alpha_2\) or \(\varphi\). We represent the quantum functions in the form of Fourier series in a complex form

\[
y_j = \sum_{n=\pm\infty} y_{j,n} e^{in\varphi}, \quad \varphi = \alpha_2, \quad 0 \leq \alpha_2 \leq 2\pi,
\]
(21)

where \(y_{j,n}\) - complex functions, \(j = 1,...,10\), \(n\) - number of waves of the circular motion.

After the substitution (21) into the system of stability equations (18) we obtain a system of ordinary differential equations that for every positive-valued function have the form
\[
\frac{1}{A_i} \frac{d y_{j,n}}{d \alpha_i} = -i n_a (T_{12,n}) + \psi_2 (T_{22,n} - T_{11,n}) + \frac{1}{R_1} (y_{3,n} - i n_a M_{12,n});
\]
\[ \frac{1}{A_i} \frac{dy_{2,n}}{d\alpha_i} = - \left( T_{22,n} - \psi_2(2T_{12,n}) + \frac{3}{R_2} - \frac{1}{R_1} \right) \psi_2 M_{12,n} + \frac{1}{R_2} \left( - T_{12,n} y_{8,n} + T_{22,n} \theta_2,n - T_{12,n} y_{8,n} + T_{22,n} \theta_2,n + \frac{1}{R_1} \left( T_{22,n} \right) \right); \]
\[ \frac{1}{A_i} \frac{dy_{3,n}}{d\alpha_i} = - \left( \frac{T_{12,n} y_{8,n} + T_{22,n} \theta_2,n - T_{12,n} y_{8,n} + T_{22,n} \theta_2,n + 2 \psi_2 M_{12,n} \right) \psi_2 y_{3,n} - \frac{1}{R_1} y_{1,n} - \frac{1}{R_2} \left( T_{22,n} \right); \]
\[ \frac{1}{A_i} \frac{dy_{4,n}}{d\alpha_i} = - \left( \frac{1}{R_1} y_{1,n} + \psi_2 y_{3,n} + y_{8,n} \theta_2,n + y_{8,n} \psi_2 - T_{11,n} y_{8,n} + T_{11,n} y_{8,n} - T_{12,n} \theta_2,n - S_1 \theta_2,n; \right) \]
\[ \frac{1}{A_i} \frac{dy_{5,n}}{d\alpha_i} = \frac{1}{R_1} \left( y_{7,n} - y_{8,n} \psi_2 + A_1 T_{11,n} + A_2 T_{12,n} + A_3 y_{4,n} - d_1 \varepsilon_{22,n} - d_2 \varepsilon_{22,n} - d_3 \varepsilon_{22,n} - d_4 \varepsilon_{22,n}; \right) \]
\[ \frac{1}{A_i} \frac{dy_{6,n}}{d\alpha_i} = \frac{1}{R_1} \left( y_{5,n} - y_{8,n} \psi_2 + y_{8,n} \theta_2,n + A_1 T_{11,n} + A_2 T_{12,n} + A_3 y_{4,n} - d_1 \varepsilon_{22,n} - d_2 \varepsilon_{22,n} - d_3 \varepsilon_{22,n} - d_4 \varepsilon_{22,n}; \right) \]
\[ \frac{1}{A_i} \frac{dy_{7,n}}{d\alpha_i} = \frac{1}{R_1} \left( \left( y_{5,n} - y_{8,n} \psi_2 + A_1 T_{11,n} + A_2 T_{12,n} + A_3 y_{4,n} - d_1 \varepsilon_{22,n} - d_2 \varepsilon_{22,n} - d_3 \varepsilon_{22,n} - d_4 \varepsilon_{22,n}; \right) \right); \]
\[ \frac{1}{A_i} \frac{dy_{8,n}}{d\alpha_i} = \frac{1}{R_1} \left( \left( y_{5,n} - y_{8,n} \psi_2 + A_1 T_{11,n} + A_2 T_{12,n} + A_3 y_{4,n} - d_1 \varepsilon_{22,n} - d_2 \varepsilon_{22,n} - d_3 \varepsilon_{22,n} - d_4 \varepsilon_{22,n}; \right) \right); \]
\[ \frac{1}{A_i} \frac{dy_{9,n}}{d\alpha_i} = \frac{T_{23,n} + d_1 \varepsilon_{22,n} + d_2 \varepsilon_{22,n} + d_3 \varepsilon_{22,n}; \right) \]
\[ \frac{1}{A_i} \frac{dy_{10,n}}{d\alpha_i} = M_{12,n} + A_2 T_{12,n} + d_3 \varepsilon_{22,n} - d_4 \varepsilon_{22,n} - d_5 \varepsilon_{22,n}; \right) \]

in which \( n_a = n / A_2 \).

Thus, the problem of static stability of a symmetrically loaded elastic anisotropic shell within the rotation circular shell theory is considered as a system of ten homogeneous differential equations in the normal Cauchy form (18) with variable coefficients and homogeneous boundary conditions on the path \( \alpha_1 = \alpha_0; \ B_\alpha y_n = 0; \) on the path \( \alpha_1 = \alpha_1; \ B_\alpha y_n = 0; \) (23)

The minimal initial value of the homogeneous boundary value problem (22), (23) characterizes the transition from the symmetric principal equilibrium state to the asymmetric, which is characterized by the corresponding number of waves that undergo circular motion. This equilibrium state is completely characterized by the following values: \( y_{1,n}, \ldots, y_{10,n}, T_{11,n}, T_{22,n}, T_{12,n}, M_{12,n}, M_{22,n}, \varepsilon_{1,n}, \varepsilon_{2,n}, \varepsilon_{22,n}, \theta_{1,n}, \theta_{2,n}, k_{22,n}, k_{12,n} \), and by subcritical parameters \( T_{11}^0, T_{22}^0, T_{12}^0, \theta_1^0, \theta_2^0 \).

The minimum initial number is calculated with a consistent loading increase, when the determinant of the matrix boundary conditions is zero. When determining the system of differential equations with complex coefficients, the determinant is also complex. The solution of a system of homogeneous algebraic equations with complex coefficients can be found, if the real and imaginary parts of the
determinant are simultaneously equal to zero. The methods of the considered boundary value problems solutions are based on a numerical discrete orthogonalization method.

4. Studying the stability of three-dimensional cylindrical shells
To represent the proposed method, we consider the problem of the thick-walled cylindrical shell stability under axial compression in which the ratio of length $L$ to the shell radius is equal to one ($L/R = 1$). One end of the shell is fixed hingedly $v = w = M_{11} = M_{12} = 0$, $T_{11} = -T_{11}^{0}$, and the other end is hinged-motionless ($u = v = w = M_{11} = M_{12} = 0$).

The material of the layered shell is boron fiber reinforced plastic. A material is orthotropic relatively to own axes. With respect to the coordinate axes on the median surface of the shell, the orthotropic axis can be rotated by an arbitrary angle $\psi$. When $\psi \neq 0^\circ, 90^\circ$, the material loses orthotropic
properties and deformed material behaves as a plane elastic material symmetry. The shell is composed of three layers. The layout of composite packets is illustrated in the Figure 2. For this variant of reinforcement, it is assumed that the elasticity modulus of the middle layer is smaller than the extreme

\[ E'_c = 0.1E_c \].

5. The results of the stability of three-dimensional cylindrical shells study

The estimated data was evaluated using the basic theory and the solving system of equations is presented in the form (22). The comparison of the received critical loads will be carried out with numerical calculations on the stability of anisotropic shells, based on the method of Kirchhoff-Love plate theory [9,13]. Figure 3 shows the results of calculations of anisotropic cylindrical shells stability with different shell thickness ratios \( h \) to their radius \( R \): a) \( h / R = 0.01 \); b) \( h / R = 0.1 \); c) \( h / R = 0.2 \). Curve 1 is the critical values obtained by the use of basic theory. Chart 2 is based on the method of Kirchhoff-Love plate theory [9,13].

The main proposed approach to calculate the cylindrical anisotropic shells stability leads to obtaining smaller values of critical loads compared to the Kirchhoff-Love plate theory. The difference between the values of the critical loads, calculated within two approaches for thin-walled shells, is insignificant and does not exceed 7.3% under the axial compression. With an increase of shell thickness, the difference between the magnitudes of critical loads obtained within different approaches increases, too. For axial compression, the discrepancy in the results is respectively: a) \( h/R=0.01 \) – 18.7%, b) \( h/R=0.1 \) – 59.1%, and c) \( h/R=0.2 \) – 103.0%.

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