Transverse spin effects and higher
twist in the singlet channel

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Abstract

The effects, originating from the longitudinal gluon polarization, 
are considered. We derive the analogue of the Wandzura-Wilczek 
relation for the light-cone distributions of polarized gluons in a trans-
versely polarized nucleon. The short distance cross-section is entirely 
due to the intrinsic transverse momenta of gluon in the nucleon, in 
complete analogy to the quark case. Numerical estimate for the double 
spin transverse asymmetries are presented. The distribution of longi-
tudinal gluons in a nucleon, introduced earlier by Gorsky and Ioffe, 
is estimated from below by making use of the positivity of density 
matrix. The similar upper bound for the nonperturbative polarized charm is discussed.

1 Introduction

It is now well known that the spin properties of gluons and quarks are 
fairly different. In particular, there is no analogue of the twist-two transver-
sity distributions for massless gluons and their contribution to the transverse 
asymmetry starts at the twist-three level. Also, longitudinal massless glu-
ones do not exist. However, due to the confinement property, gluons should
acquire an average mass and/or a transverse momentum of the order of the inverse of the hadron size. As a result, one can have a nonzero longitudinal gluon distribution. Generally speaking, it is suppressed by the gluon mass squared and contributes at the twist-four level. However, in the case of Deep Inelastic Scattering (DIS) this is cancelled by the pole in the box diagram. This effect was studied in details for longitudinal gluons by Gorski and Ioffe [1]. It was shown to be related to the conformal anomaly, and one may wonder, if it could be observed in other processes.

The gluon mass and/or its intrinsic transverse momentum should result also in a nonzero transverse gluon distribution. It was recently studied in [2] where the twist-two approximation was derived. The resulting double transverse spin asymmetries $A_{TT}$, for low mass dijet or low $p_T$ direct photon production, at RHIC are rather small ($\leq 1\%$) due to a kinematic suppression factor. It seems also promising to study the double asymmetries in open charm or heavy quarkonia leptoproduction by a longitudinally polarized lepton beam off a transversely polarized target. The suppression factor that enters in this case is to the first power of $M/\sqrt{s}$, contrary to the second power for some double transverse spin asymmetries. The situation seems especially favorable for the case of diffractive charmonium production, investigated recently [3], when the partonic c.m. energy $\sqrt{s}$ is of the order of the charm quark mass. This would make possible the measurement of the transverse gluon distribution.

The present paper is devoted to the discussion of the manifestation of longitudinal gluons in the polarized and unpolarized nucleons and the relations between them.

2 Gluonic contribution to the transverse polarization of nucleon

The absence of the twist-two transversity distribution for gluons, mentioned above, is leading generally speaking, to the relative suppression of gluon transverse asymmetries with respect to the quark ones, which was used recently to formulate the selection rule [4] in QCD, which is of direct relevance for the physics programme of the future polarized $pp$ collider at BNL-RHIC.
However, the detailed analysis of the quark contribution to the double transverse spin asymmetries \( A_{TT} \) using the Monte-Carlo simulation [5], resulted in rather small numbers of the order of 1\%, and therefore it seems natural to question the role of gluon corrections.

The transverse polarization effects are arising from two basic sources: the leading twist transversity distribution, resulting in the correlation of transverse polarizations, and the twist-three parton correlations, suppressed by the hadron mass. While the first are absent for gluons, the gluon correlations are, generally speaking, rather complicated [6]. At the same time, the experimental data on the \( g_2 \) structure function [7] do not deviate strongly from the twist-two approximation, suggested by Wandzura and Wilczek (WW) [8], whose physical meaning is just the dominance of the effect of transverse motion of the quark over that of the gluon field [3]. This is the reason, why we are suggesting here a generalization of the WW approximation to the case of gluons.

To do this, let us start from the light-cone density matrix of gluon, namely:

\[
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p, s | A^\rho(0) A^\sigma(\lambda n) | p, s \rangle \sim \frac{1}{2}(e_1^\rho e_1^\sigma + e_2^\rho e_2^\sigma)G(x) + \frac{i}{2}(e_1^\rho e_2^\sigma - e_2^\rho e_1^\sigma)\Delta G(x) + \frac{i}{2}(e_1^\rho e_L^\sigma - e_L^\rho e_1^\sigma)\Delta G_T(x) + G_L(x)e_L^\rho e_L^\sigma, \tag{1}
\]

where \( n \) is the gauge-fixing light-cone vector such that \( np = 1 \), and we define two transverse polarization vectors, \( e_{1T} \) and \( e_{2T} \). One of them, namely \( e_{2T} \) for definiteness, is chosen to be parallel to the direction of the transverse component of the polarization, so that’s why only the vector \( e_{1T} \) enters in the contribution of \( \Delta G_T(x) \). Also, we introduce the longitudinal polarization vector \( e_L \). We denote by \( s_\mu \) the covariant polarization vector of the proton of momentum \( p \) and mass \( M \) and we have \( s^2 = -1, sp = 0 \). Here \( G(x) \) and \( \Delta G(x) \) are the familiar unpolarized gluon distribution and gluon helicity distribution, respectively. The transverse gluon distribution \( \Delta G_T(x) \) is the most natural way to measure its transverse polarization, analogous to the quark structure function \( g_T = g_1 + g_2 \), since in the quark case we have:

\[
\frac{1}{4M} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p, s | \bar{\psi}(0) \gamma^\mu \gamma^5 \psi(\lambda n) | p, s \rangle = g_1(x)(sn)p^\mu + g_T(x)s_T^\mu. \tag{2}
\]
The quantity $g_T$ was shown to be the good variable to study the generalized Gerasimov-Drell-Hearn sum rule, and the $x$ dependence of the anomalous gluon contribution \[10\]. The latter result was recently confirmed \[11\].

The light-cone distributions $\Delta G$ and $\Delta G_T$ can be easily obtained by the projection of gluon density matrix, so we have,

$$\Delta G(x) = \frac{ix}{4M(sn)} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p, s | A_{\rho}(0) A_{\sigma}(\lambda n) | p, s \rangle \epsilon^{\rho\sigma mn},$$

$$\Delta G_T(x) = \frac{ix}{4M(s^2)} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle p, s | A_{\rho}(0) A_{\sigma}(\lambda n) | p, s \rangle \epsilon^{\rho\sigma ps}. \quad (3)$$

Now by making use of the axial gauge $A_n = 0$, one may express their moments in terms of gluon field strength $G_{\mu\nu}$, according to

$$\int_0^1 dx x^k \Delta G(x) = \frac{1}{4M(sn)} \langle p, s | G_{\rho\mu n}(0)(i\partial n)^{k-1} G_{\sigma n}(0) | p, s \rangle \epsilon^{\rho\sigma mn},$$

$$\int_0^1 dx x^k \Delta G_T(x) = \frac{1}{4M(s^2)} \langle p, s | G_{\rho\mu n}(0)(i\partial n)^{k-1} G_{\sigma n}(0) | p, s \rangle \epsilon^{\rho\sigma ps}. \quad (4)$$

We denote here $G^{\mu n} = G^{\mu\nu} n_\nu, \partial n = n_\mu \partial^\mu$, and we recall that in configuration space $x^k = (i\partial n)^k$. The kinematical identities, implied by the vanishing of the totally antisymmetric tensor of rank 5 in four-dimensional space,

$$n^\mu \epsilon_{\rho\sigma mn} + n^\nu \epsilon_{\sigma\mu pn} = \epsilon_{\rho\sigma mn} \quad \text{and} \quad n^\mu \epsilon_{\rho\sigma ps} + n^\nu \epsilon_{\sigma\mu ps} = \epsilon_{\rho\sigma \mu T} \quad (5)$$

allow one to come to the standard gluonic operators, used in the operator product expansion for spin-dependent case \[15\]

$$\int_0^1 dx x^k \Delta G(x) = \frac{i^{k-1}}{2M(sn)} \langle p, s | \tilde{G}_{\sigma\alpha}(0) \partial^{\mu_1} \cdots \partial^{\mu_{k-1}} G_{\sigma\beta}(0) | p, s \rangle n^\alpha n_\beta n_{\mu_1} \cdots n_{\mu_{k-1}},$$

$$\int_0^1 dx x^k \Delta G_T(x) = \frac{i^{k-1}}{2M(s^2)} \langle p, s | \tilde{G}_{\sigma\alpha}(0) \partial^{\mu_1} \cdots \partial^{\mu_{k-1}} G_{\sigma\beta}(0) | p, s \rangle s^{\alpha}_T n_\beta n_{\mu_1} \cdots n_{\mu_{k-1}}, \quad (6)$$

\(^{1}\)The first moment requires to take into account the non-local operator \[12, 13\]. At the same time, the non-local operators found in the renormalization of the gluon contribution to $g_2$ \[3\] should be equal to zero, when one uses the gauge invariance and equations of motion.
where \( \tilde{G}_{\sigma \alpha} = \frac{1}{2} \epsilon_{\sigma \alpha \mu \nu} G_{\mu \nu} \). Taking the totally symmetric part of the matrix element

\[
i^{k-1} \langle p, s | \tilde{G}_{\sigma \alpha}(0) \partial^\mu_1 \cdots \partial^\mu_{k-1} G_{\sigma \beta}(0) | p, s \rangle = a_k S_{\alpha \beta \mu_1 \cdots \mu_{k-1}} S_\alpha^\mu p_\beta^\mu p_{\mu_1} \cdots p_{\mu_{k-1}},
\]

(7)

where \( S \) denotes the total symmetrization and \( a_k \) is the scalar constant, one immediately obtains the relation

\[
\int_0^1 dx x^k \Delta G(x) = (k + 1) \int_0^1 dx x^k \Delta G_T(x),
\]

(8)

which is equivalent to the WW formula:

\[
\Delta G_T(x) = \int_x^1 \frac{\Delta G(z)}{z} dz.
\]

(9)

The existence of this relation is very natural because of the similarity between quark and gluon density matrices (see eqs. (2) and (1)). Actually, such a relation is emerging due to the symmetry properties of the operators and does not depend neither on the fields (quark or gluons) nor on the coefficient function (c.f. [16]).

Our present knowledge on \( \Delta G(x) \), which is not very precise, allows a great freedom, so several different parametrizations have been proposed in the literature [17, 18, 19, 20]. In ref.[2], we have shown in Figs.1a and 2a, some possible gluon helicity distributions \( x \Delta G(x) \) and in Figs.1b and 2b, the corresponding \( x \Delta G_T(x) \) obtained by using (9). It is worth recalling, from these pictures that, in all cases \( x \Delta G(x) \) and \( x \Delta G_T(x) \) are rather similar in shape and magnitude.

Let us now move to the calculation of short-distance subprocess. For this, it is instructive to compare the two terms in the gluon density matrix (11). While the longitudinal term is in fact a two-dimensional transverse antisymmetric tensor and corresponds to the density matrix of a circularly polarized gluon

\[
\Delta G(x) \epsilon^{\rho \sigma \mu \nu} = \Delta G(x) \epsilon^\rho_\sigma T T,
\]

(10)
the transverse polarization term generates the circular polarization in the plane, defined by one transverse and one longitudinal direction

$$M \Delta G_T(x) \epsilon^{\rho\sigma} s^m = \Delta G_T(x) \epsilon^{\rho\sigma}_{TL} ,$$

and therefore corresponds to the circular transverse polarization of gluon. Such a polarization state is clearly impossible for on-shell collinear gluons. They should have either nonzero virtuality, or nonzero transverse momentum. Note that one of these effects is required to have nonzero anomalous contribution to the first moment of the structure function $g_1$ [21, 22]. One may consider this similarity as supporting the mentioned relations between $\Delta G_T$ and anomalous gluon contribution [10, 11].

We should adopt the second possibility, namely a nonzero transverse momentum, because the gluon remains on-shell and the explicit gauge invariance is preserved. In this case, the transverse polarization of nucleon may be converted to the longitudinal circular polarization of gluon. The similar effect was discussed earlier for quarks [23, 2] and for photons in QED [24].

To calculate now the asymmetry in short-distance subprocess, it is enough to find the effective longitudinal polarization by projecting the transverse polarization onto the gluon momentum:

$$s_L = \frac{-s_T \hat{k} \cdot k}{|k|} = s_T \frac{k_T}{k_L} .$$

The partonic longitudinal-transverse and double transverse asymmetries can be easily obtained from the longitudinal one according to,

$$\hat{A}_{LT} = \frac{k_{T1}}{k_{L1}} \hat{A}_{LL}, \quad \hat{A}_{TT} = \frac{k_{T1} k_{T2}}{k_{L1} k_{L2}} \hat{A}_{LL} .$$

By neglecting the transverse momentum dependence of $\hat{A}_{LL}$ one has

$$\hat{A}_{TL} \sim \frac{\langle k_T \rangle}{\sqrt{s}} \hat{A}_{LL}, \quad \hat{A}_{TT} \sim \frac{\langle k_T^2 \rangle}{\hat{s}} \hat{A}_{LL} ,$$

where $\hat{s}$ is the partonic c.m. energy.
The most promising effects, as it was mentioned above, seem to be the diffractive $J/\psi$ production, where the suppression factor should be of the order of 0.3. However, even the case of the longitudinal polarization, the large value which seems to be provided by the analogue of the Landau theorem, requires further investigation.

3 Positivity bound for the condensate of longitudinal gluons

Note that $\Delta G_T(x)$ may be also considered as an analogue of transversity for quark, since for gluons there is no such a difference, caused for quarks by the two Dirac projections (chiral-even and chiral-odd). It is proportional to the following gluon-nucleon matrix element

$$\Delta G_T(x) = \langle +, 1|-, 0 \rangle \equiv M_{+-} ,$$

where $(+, -)$ and $(1, 0)$ are the nucleon and gluon helicities, respectively.

At the same time we have

$$G_L(x) = \langle +, 0|+, 0 \rangle = \langle -, 0|-, 0 \rangle \equiv M_0 .$$

To establish the connection with the existing positivity relations, let us consider first the forward $\gamma^*p$ elastic scattering ($\gamma^*$ is a massive photon), which allows to calculate the DIS structure functions. It is described in terms of four helicity amplitudes: $M_0, M_{+-}$ defined above and

$$M_+ = \langle +, 1|+, 1 \rangle, \quad M_- = \langle -, 1|-, 1 \rangle .$$

There is a well-known condition established long time ago by Doncel and de Rafael [25], written in the form

$$|A_2| \leq \sqrt{R} ,$$

where $A_2$ is the usual transverse asymmetry and $R = \sigma_L/\sigma_T$ is the standard ratio in DIS. It reflects a non-trivial positivity condition one has on the photon-nucleon helicity amplitudes, which read using the above notations
\[(M_{+\text{\smash{\_}}})^2 \leq 1/2(M_{+} + M_{-})M_{0}. \quad (19)\]

One can apply this result to the similar case of gluon-nucleon scattering, adding to the definitions (4,5,6), the following ones

\[G(x) = M_{+} + M_{-}, \; \Delta G(x) = M_{+} - M_{-}. \quad (20)\]

As a result, the positivity relation (8) leads to

\[|\Delta G_T(x)| \leq \sqrt{1/2G(x)G_L(x)}. \quad (21)\]

It is most instructive to use this relation to estimate \(G_L\) from below.

\[G_L(x) \geq 2[\Delta G_T(x)]^2/G(x) = 2\lambda(x)G(x), \quad (22)\]

where \(\lambda(x) = [\Delta G_T(x)/G(x)]^2\).

Note that given the data on \(R\) in DIS, one obtains from (7) an upper bound on \(|A_2|\), which is satisfied by polarized DIS data \[26\] and is far from saturation. However \[22\] provides a lower bound on \(G_L(x)\) since \(G(x)\) is known from unpolarized DIS or direct photon production and \(\Delta G_T(x)\) can be evaluated \[2\], in the twist-two approximation if one uses eq.(3). One obtains \(\lambda(x) \simeq 0.01\) for \(x \simeq 0.1\), or so and our lower bound gives \(G_L(x) \geq 0.3\) or so. For lower \(x\) values, due to the rapid rise of \(G(x)\), \(\lambda(x)\) is much smaller and, for example, for \(x \simeq 10^{-3}\), we find \(G_L(x) \geq 10\) or so. At the same time, for very large \(x \to 1\), as \(\Delta G_T(x)\) is similar to \(\Delta G(x)(1-x)\), \(\lambda\) is close to zero, and \(G_L(x)/G(x) \sim 0\).

Such a relation is of special interest, since it relates, at least formally, different twist structures. This is by no means surprising, because, in the case of polarized DIS, if we would have known \(A_2\) before \(R = \sigma_L/\sigma_T\), we could have also estimated the latter from below. Note that physically the existence of such a relation is due to the fact, that transverse and longitudinal gluon distributions are generated by the same source, the gluon mass.
4 Conclusions

Up to now, we did not discuss the entire twist three effects, emerging from the QCD long-range interactions. In the singlet channel, they are described by the complicated set of the three-gluon correlations [1].

They may lead to large single spin asymmetries in the fragmentation region of the unpolarized particle [2]. This effect was studied quantitatively [27], by making use of a nonperturbative estimate of the correlations. The latter was performed by considering a semi-classical long-wave gluonic field [28]. The contributions of these correlations in DIS were also studied and the Burkhardt-Cottingham sum rule was found to be valid [14].

Another interesting manifestation of the three-gluon correlations in spin physics [29] was suggested recently. Namely, it was shown that the nonperturbative contribution of charmed quarks to the proton spin is related to the matrix element of the operator $G_{\mu\nu}\tilde{G}^{\nu\alpha}G^{\alpha\mu}$, which was estimated to be very large. The authors argue, that such strong effects are absent in the vector channel, which is studied in the unpolarized scattering.

However, the positivity conditions may be applied here also, and leads to the relation $|<p,s|\bar{c}(o)\gamma_\mu\gamma_5c(z)|p,s>| \leq |<p,s|\bar{c}(o)\gamma_\mu c(z)|p,s>|$. It might not be a useful bound for the local matrix element, since the r.h.s. which corresponds to the first moment of the unpolarized sea distribution, may be infinite. However if we apply it to the $x$ distributions, one gets $|\Delta c(x)| \leq c(x)$. The quantity $c(x)$ (unpolarized intrinsic charm) is known to be small, and the most precise data comes from DIS at HERA. Therefore, the polarized charm should be small, in the very small $x$ region, which, by no means, has been investigated by the existing polarized DIS experiments and therefore has nothing to do with the present available data. If the analysis [29] turns out to be valid, this would lead to a dramatic increase of charmed quark contribution at small $x$. However, from the more recent evaluation of the nonperturbative polarized charm [30], it results a much smaller contribution.

In the case where the correlations are not strong, the kinematic approximation we used is rather natural. It is also supported by the fact, that all the current high-twist calculations are compatible with such a ”kinematical dominance” [31]. In the singlet channel, it results in the self-consistent picture of the massive gluons, contributong to the polarized and unpolarized structure functions.
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