Simulation of forest fires based on a two-dimensional three-phase model

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Abstract. A two-dimensional three-phase mathematical model of forest fires constructed by the method of averaging over the height of the forest fuel layer is considered. The gas phase in the model is described by gas dynamics equations with the $k - \varepsilon$ turbulence model and the eddy breakup model to describe the rate of turbulent combustion and the rate of heat generation; the solid phase is described by a system of evolution equations for the change in the volume fractions of forest fuel and solid products of their pyrolysis with kinetic formulas for the rates of chemical reactions; and the disperse phase is described by the transfer equations for the mass and energy of solid burning particles in the wind field above the forest fuel layer. For the numerical implementation of the proposed model, a parallel software package is created. The results of calculations of the forest fire propagation with consideration of the basic physical-chemical processes and mechanisms typical of the real forest wildfire propagation are presented.

1. Introduction

Forest fires are a dangerous natural phenomenon that causes annually a significant economic and ecological damage. An important element for a successful forest-fire fighting is the prediction of the process of their propagation; the most efficient tool for such predictions is a mathematical modeling method that does not require considerable computational resources.

Mathematical modeling of forest fires has intensively developed over the past twenty years; many various models of forest fires are created up to the present time; a brief review of these models is presented in [1]. One of the directions of modeling of forest fires which was developed by the authors of [1-3] is the prediction of the fire-front propagation based on two-dimensional mathematical models. In the early studies of the authors of [2, 3], on the basis of the general approach to the description of forest fires as a multiphase reactive medium [4], a two-dimensional two-phase model of forest fires in which the medium consists of an air-gas (gas) phase and a solid phase was developed. This model was
based on the fundamental mass, momentum, and energy conservation laws; the equations for these laws were obtained by averaging the initial three-dimensional equations over the height of the forest fuel (FF) layer. The model considers all basic physical processes that pass in the fire zone and may be applied on modern multiprocessor computer complexes to perform practically real-time modeling of the forest wildfire propagation [2, 3]. However, this model used a simplified description of turbulent mixing; the heat generation in the gas phase (the rate of fire-front propagation depends on this parameter) was described by kinetic formulas for the rates of chemical reactions with constants assigned with a significant uncertainty. Recently, the authors have proposed a modified two-dimensional model [1] which describes turbulence by means of the known \( k − \varepsilon \) model with additional terms for the turbulence generation and dissipation in the woodland [5, 6]. In this model, the rate of turbulent combustion in the gas phase is described by the eddy breakup model, according to which the rate of combustion at high temperatures does not depend on the kinetics of chemical reactions and is determined only by the rate of turbulent mixing of the components [7-10].

In models [1-3], one of the components of the gas phase is highly dispersed soot which is transported with the gas phase. In our study, we consider a three-phase model as a development of the modified two-dimensional model [1]; in this three-phase model, the third phase consists of large burning disperse particles transported in the wind field above the FF layer. Considering the disperse particles as a separate phase moving with a velocity different from that of the gas phase, it is possible to describe the fire propagation mechanism observed in real forest fires with the help of burning particles, which are transported by the wind above the forest canopy and, precipitating, generate new sources of fire ahead of the leading edge of the fire front. The distances to which the large burning particles are transported in the horizontal direction depend on the wind velocity, burning time, mass of particles and their geometric shape that influences on their aerodynamics.

2. Mathematical model

Let us consider a one-stratum model of a forest wildfire in one FF layer of a constant thickness, assuming that the physical-chemical parameters (real density, moisture content, calorific power, etc.) are constant in this FF layer. This model considers a three-phase medium which consists of the gas, solid, and disperse phases. The gas phase consists of the following six components: combustible gas CO (with the mass concentration \( C_1 \)), oxidizer \( O_2 \) (\( C_2 \)), carbon dioxide \( CO_2 \) (\( C_3 \)), water vapor \( H_2O \) (\( C_4 \)), nitrogen \( N_2 \) (\( C_5 \)), and highly dispersed soot (\( C_6 \)). It is assumed that the particles of highly-dispersed soot move together with the gas phase; in the process of soot burning, the heat exchange passes rapidly and, thus, we can consider that the gas phase temperature is unified. The solid phase is also multicomponent; it consists of the FF (with the volume fraction \( \phi_1 \)) and the FF pyrolysis products – coke breeze (\( \phi_2 \)) and ash (\( \phi_3 \)). Coke breeze and fine-dispersed soot consist almost entirely of carbon and, thus, no solid remainder is left after the combustion. The disperse phase (with the volume fraction \( \phi_d \)) consists of burning carbon particles of the same shape and dimension (monodisperse mixture) which are transported by the wind above the FF layer and settle down due to the action of the gravity force.

The system of equations that describe the two-dimensional three-phase model was obtained by integrating the original three-dimensional equations over the FF layer (the forest stratum); it consists of the subsystem of the Favre-average gas dynamics equations for the multicomponent gas phase, the subsystem of equations for the mass of the components of the solid phase and its energy, and the subsystem of the mass and energy transfer equations for the large disperse particles.

The model equations for the gas phase have the following form:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = Q - J_\rho, \\
\frac{\partial \rho \mathbf{V}}{\partial t} + \nabla \cdot (\rho \mathbf{V} \mathbf{V}) + \nabla (\phi p) = \nabla \cdot (\mu_{eff} \nabla \mathbf{V}) + \mathbf{F} + \mathbf{F}_w - J_\mathbf{V},
\]
\[
\frac{\partial \rho E}{\partial t} + \nabla \cdot (\rho V E + \phi p V) = \nabla \cdot \left( k_{\text{eff}} \nabla T \right) + \alpha (T^4 - T) + \sigma \left( \epsilon T^3 - \epsilon T^4 + \epsilon T^4 \right) + \alpha_s (T_d - T) + \sigma \left( \epsilon T^4 - \epsilon T^4 \right) - \nabla \cdot W_i + f_i + f_{iR} + q_i W_i + q_i W_i - J_E,
\]
\[
\frac{\partial \rho C_i}{\partial t} + \nabla \cdot (\rho V C_i) = \nabla \cdot \left( \rho D_{\text{eff}} \nabla C_i \right) + Q_i + W_i + W_i' - J_{C_i}, \quad i = 1,5,
\]
\[
\frac{\partial \rho k}{\partial t} + \nabla \cdot (\rho V k) = \nabla \cdot \left( \frac{\mu_{\text{eff}}}{\sigma_k} \nabla k \right) + G + P_k - \rho e - L_k,
\]
\[
\frac{\partial \rho e}{\partial t} + \nabla \cdot (\rho V e) = \nabla \cdot \left( \frac{\mu_{\text{eff}}}{\sigma_k} \nabla e \right) + c_e G e + P_e - c_e \rho e^2 k - L_k,
\]
\[
p = \rho^\gamma RT \sum_{i=1}^{5} \frac{C_i}{M_i};
\]
for the solid phase:
\[
\rho_j \frac{\partial \phi_j}{\partial t} = R_{\phi_j}, \quad j = 1,3;
\]
\[
\frac{\partial}{\partial t} \sum_{j=1}^{3} \rho_j \phi_j c_{p_j} T_i = -\alpha (T_i - T) - \sigma \left( \epsilon T^3 \epsilon - \epsilon T^4 \right) - \alpha_s (T_i - T_d) - \sigma \left( \epsilon T^3 - \epsilon T^4 \right) + Q_i;
\]
for the disperse phase:
\[
\frac{\partial V_d}{\partial t} = -c_{\Omega_d} (V_d - V_u) |V_d - V_u|,
\]
\[
\frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d V_d) = R_d - J_d,
\]
\[
\frac{\partial \rho_d c_{p_d} T_d}{\partial t} + \nabla \cdot (\rho_d V_d c_{p_d} T_d) = -\alpha (T_d - T) - \alpha_s (T_d - T) - \sigma (\epsilon T^3 \epsilon - \epsilon T^4) - \sigma (\epsilon T^3 - \epsilon T^4) - J_{\Omega_d} + Q_{\Omega_d};
\]
the normalizing and balance relations are as follows:
\[
\phi + \sum_{j=1}^{3} \phi_j + \phi_d = 1, \quad \sum_{i=1}^{5} C_i + C_{\Omega} = 1, \quad \sum_{i=1}^{5} (Q_i + W_i + W_i') + Q_i - W_i = Q, \quad \sum_{j=1}^{3} R_{\phi_j} = -Q.
\]

The system of equations (1)–(4) is considered in the Cartesian coordinates in the rectangular domain \( \Omega = \{0 \leq x \leq l_x, 0 \leq y \leq l_y\} \) on the horizontal plane \( XY \); \( \rho \) is the partial density of the gas phase; \( p \) is the total pressure of the multiphase medium; \( V = (u,v) \) is the velocity vector of the gas phase; \( T \) is the temperature of the gas phase; \( E \) is the total energy of the gas phase, \( E = 0.5 (u^2 + v^2) + c_v T \); \( c_v \) is the specific heat capacity of the gas phase; \( \phi \) is its volume fraction; \( \rho^\gamma = \rho/\phi \) is its real density; \( R \) is the
universal gas constant; $M_i$ is the molecular mass of component $i$ of the gas phase; $F=-\rho c_d s \mathbf{V} | \mathbf{V} |$ is the volume force associated with the interphase momentum exchange (the interphase friction force); $c_d$ is the empirical vegetation resistance coefficient; $s$ is the FF specific surface; $\mathbf{F}_w=-\rho \xi (\mathbf{V}-\mathbf{V}_u)|\mathbf{V}-\mathbf{V}_u|$ is the friction force conditioned by the wind with the velocity $\mathbf{V}_u$ at the upper boundary of the FF layer; $\xi$ is the dimensionless empirical friction coefficient; $Q$ is the rate of the gas inflow into the gas medium due to chemical processes in the solid phase; $\rho_j$, $j=1,3$ is the real density of component $j$ of the solid phase, $\rho_j^0=\text{const}$; $R_h$ is the rate of the mass variation for component $j$ of the solid phase due to chemical reactions; $T_i$ is the temperature of the solid phase; $Q_h$ is the heat generation in the solid phase in the combustion process; $c_{p,j}$ is the specific heat capacity of the solid phase; $W$ is the mass rate of the CO combustion reaction; $W_i$ is the mass rate of the soot combustion; $i=1,5$ are the production and consumption rates of the gas phase components in the corresponding reactions, $\rho_i, T_i, \mathbf{V}_d=(u_d, v_d), \phi_d$ are the partial density, temperature, velocity, and volume fraction, respectively; the particle resistance coefficient $c_i$ depends on the shape and cross section of the particles which are assumed to be identical (we consider a monodisperse mixture), and the Reynolds number, $\mathbf{V}_w=(u_w, v_w)$ is the wind velocity above the woodland; $\phi_d = \rho_d / \rho_j^0$, where $\rho_j^0$ and $c_{p,j}$ are the real density and specific heat capacity of the material of disperse particles, respectively, and, since the disperse particles and coke breeze consist of carbon, $\rho_d^0 = \rho_2^0$, $c_{p,j} = c_{p2}$ and they are equal to the corresponding parameters of carbon.

The turbulent closure of the system of equations for the gas phase, is the modified model of turbulence in which the right-hand sides of equations for the turbulent kinetic energy $k$ and its dissipation rate $\varepsilon$ contain, in addition to the usual expression for the generation of the turbulence energy $G$, extra terms $P_i, P_\varepsilon$ and $L_\varepsilon, L_\varepsilon$, which describe, respectively, the turbulence generation and dissipation due to the interaction between the flow and FF [5, 6]. The effective viscosity coefficient is equal to the sum $\mu_{eff}=\mu_l+\mu_t$ of the laminar and turbulent viscosities, where the laminar viscosity is calculated by using the Sutherland formula $\mu_l = \frac{1.458 T^{3/2}}{T+110.4} \times 10^{-6}$, while the turbulent viscosity is calculated according to the formula $\mu_t = c_\mu \rho k^2 / \varepsilon$;

$$G = \mu_t \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \right] ;$$

$$P_i = \frac{1}{2} c_d s \rho |\mathbf{V}|^4, \quad P_\varepsilon = c_{1\varepsilon} c_d s \rho |\mathbf{V}|^4 \frac{\varepsilon^0}{k^0}, \quad L_\varepsilon = 2 c_d s \rho |\mathbf{V}| k, \quad L_\varepsilon = 4 c_{2\varepsilon} c_d s \rho |\mathbf{V}| \varepsilon ,$$

where the following standard set of constants of the $k-\varepsilon$ model of turbulence is used: $c_\mu = 0.09$, $c_1 = 1.44$, $c_2 = 1.92$, $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$, while for the terms $P_i, L_\varepsilon$ the following constants obtained with the help of the in-situ experiments [5, 6] are used: $c_{1\varepsilon} = 1.8$, $c_{2\varepsilon} = 0.6$; $k^0, \varepsilon^0$ are the typical values of turbulence parameters in a woodland which are also used as the initial values for $k$ and $\varepsilon$.

The effective heat conductivity coefficient $k_{eff} = k_l + k_t + k_r$ is the sum of the molecular, turbulent, and radiation heat conductivity coefficients, respectively, where $k_l = c_p \mu_l / \text{Pr}_l$, $k_t = c_p \mu_t / \text{Pr}_t$, $\text{Pr} = \text{Pr}_l = 0.7$ are the laminar and turbulent Prandtl numbers, respectively. The radiation heat conductivity model was considered as a model of radiation transport. In this model, the radiation
energy flux density is defined by the formula $W = -k \nabla T$, $k = 16\sigma l^3 / T^3$, $l$ is the radiation path length, $\sigma$ is the Stefan–Boltzmann constant.

The effective diffusion coefficient is defined by the formula $D_{\text{eff}} = \mu / \rho Sc + \mu / \rho Sc_r$, where $Sc = Sc_r = 0.7$ are the laminar and turbulent Schmidt numbers, respectively.

The combustion model for the gas phase consists of the two following single-stage irreversible CO-combustion brutto-reactions for the dispersed soot:

$$1 \text{ kg CO} + \nu_{\text{CO}_2} \text{ kg O}_2 \rightarrow \nu_{\text{CO}_2} \text{ kg CO}_2, \quad 1 \text{ kg C} + \nu'_{\text{O}_2} \text{ kg O}_2 \rightarrow \nu'_{\text{CO}_2} \text{ kg CO}_2,$$

where $\nu_i, \nu'_i$, $i = 1, 5$ are the mass stoichiometric coefficients of the gas-phase and dispersed- soot combustion reactions, respectively. The turbulent combustion rate is described by the eddy breakup model [7-10]. The mass combustion rates for CO and the soot are expressed by the formulas (the chemical formulas of the components are used instead of index $i$)

$$W = \rho A \varepsilon_{\text{min}} \left( \frac{C_{\text{CO}_2}}{\nu_{\text{CO}_2}}, \frac{BC_{\text{CO}_2}}{\nu_{\text{CO}_2}} \right), \quad W_i = \rho A \varepsilon_i \left( \frac{C_{\text{CO}_2}}{\nu_{\text{CO}_2}}, \frac{C_{\text{CO}_2}}{\nu_{\text{CO}_2} + C_{\text{CO}_2} \nu_{\text{CO}_2}} \right),$$

respectively, were $A = 4, B = 0.5$. The heat generation in the gas phase occurs due to the combustion of CO and dispersed soot; it is equal to $q_i W + q'_i W'_i$, where $q_i, q'_i$ are the combustion heat for the gas and soot, respectively; $W$ is the rate of the chemical reaction of gas combustion; $W'_i$ is the soot burn-out rate; $W_i, W'_i$, $i = 1, 5$ are the formation or consumption rates for the gas-phase components in the corresponding reactions, which are expressed via the rates $W$, $W'_i$ and the mass stoichiometric coefficients by the formulas $W_i = \pm \nu_i W$, $W'_i = \pm \nu'_i W'_i$, where the “plus” sign is valid for the reaction products and the “minus” sign is valid for the fuel and oxidizer; $\nu_{\text{CO}_2} = 1$, according to the definition of $W$.

Let us consider the coke breeze pyrolysis and combustion reactions as the basic chemical reactions in the solid phase. Let us describe the rates of these reactions according to the Arrhenius law. The rate of the FF pyrolysis reaction is described by the formula $R_0 = \rho \phi k_0 \exp(-E_0 / (RT))$ [4]. Let us introduce the following designations: $\alpha_1, \alpha_2, \alpha_3, \alpha_m$ are the portions of gas i, dispersed soot, coke breeze, and ash, respectively, which are produced due to the pyrolysis; for oxygen and nitrogen, $\alpha_2 = \alpha_m = 0$. Then, the generation of gases, soot, coke breeze, and ash due to the pyrolysis is described by the terms $Q_i = \alpha_1 R_0$, $Q_2 = \alpha_2 R_0$, $Q_3 = \alpha_3 R_0$, $Q_4 = \alpha_4 R_0$, $Q_5 = \alpha_5 R_0$, respectively; the FF burn-out rate is $R_0 = -R_0$.

The coke breeze combustion rate is described by the formula $R_i = \rho C_{\text{C}_2} k_i \exp(-E_i / (RT))$ [4]. The coke breeze combustion with the combustion rate $R_i$ is accompanied with the oxygen burn-out in the gas phase and generation of CO$_2$ which inflows into the gas phase. Then, $Q_2 = -R_i$, $Q_3 = \alpha_3 R_0 + R_i$, $R_0 = \alpha_3 R_0 - R_i$.

The change in the mass of the disperse phase is described by the formula $R_j = \alpha_j R_0 - R_j$, where the first term describes the generation of the disperse particles due to the pyrolysis; the second term, their burn-out.

The heat generation due to the breeze coke combustion is described by the formula $Q_{\text{r}_j} = q_i \phi R_j$, where $q_i$ is the heat combustion for carbon. The heat generation due to the disperse particle combustion is described by the formula $Q'_{\text{r}_j} = q'_{\text{r}_j} R_j$.

The matter, momentum, and energy flows through the upper and lower boundaries of the FF layer.
where \( h_1 \), \( h_2 \) are the heights of the upper and lower boundaries of the FF layer, respectively; \( h \) is the FF layer thickness, \( h = h_2 - h_1 \); the vertical velocity \( w \) at the upper and lower boundaries of the FF layer is defined by the semi empirical expression \[ w_{t=h_2} (\Phi) = (g (T-T_2) h / T_2^{1/2}) , \]

\( T_2 \) is the temperature above the upper boundary of the FF layer (for the top forest stratum, it is the temperature of the unperturbed atmosphere). In the considered one-stratum forest model, \( w_{t=h_1} = 0 \). For the multistratum model, the value of \( w_{t=h_1} \) has to be assigned equal to the value of \( w_{t=h_2} \) for the lower stratum.

The term \( \alpha(T_i - T) \) describes the interphase heat exchange, where \( \alpha \) is the heat-exchange coefficient; \( \sigma_\alpha(\varpi_iT_i^4 - \varpi T^4) \) is the interphase radiant energy exchange, where \( \varpi \) is the gas-phase radiation absorption coefficient, which takes into account the dispersed soot; \( \varpi_i \) is the solid-phase radiation absorption coefficient.

The fallout of burning disperse particles from the surface atmosphere layer is modeled with the help of a random number generator; the terms \( \alpha_e(T_d - T) \), \( \alpha_s(T_s - T_i) \), \( \sigma_\alpha(\varpi_sT_s^4 - \varpi T^4) \), and \( \sigma_\alpha(\varpi_dT_d^4 - \varpi T^4) \) are the point energy sources in the gas and solid phases that act at the sites of fallout deposition of burning particles up to their complete burn-out; \( \varpi_d \) is the absorption coefficient for the disperse particle radiation.

The laws for the heat exchange \( f_r \) and radiant energy exchange \( f_{rr} \) at the upper and lower boundaries of the FF layer are the same as those for the interphase heat exchange and radiant energy exchange, respectively.

**Boundary and initial conditions**

The boundary conditions for the system of equations (1)–(4) are to be set at the open lateral boundary \( \Gamma \) of the computational domain. For the parts of the boundary where the flow enters the computational domain, the unperturbed environment parameters (with index \( a \)) are set as follows:

\[
\Phi_{\Gamma} = \Phi_a , \quad \Phi = (\rho, u, v, T) , \quad (V, n) \leq 0 ;
\]

for the parts of the boundary where the gas flows out of the computational domain, the following conditions are set:

\[
\frac{\partial \Phi}{\partial n}_{\Gamma} = 0 , \quad \Phi = (\rho, u, v, E, C, \rho_s, T_d) , \quad (V, n) > 0 .
\]

The initial values of the parameters outside the fire source are the environment parameters; inside the fire source, we set the values of the parameters characteristic for the considered kind of fire and FF type \( (T_i^0, T_s^0, C_i^0, C_s^0, \rho_d^0) \); the initial velocities of the gas and disperse phases are \( V_i^0 = 0, V_d^0 = 0 \); the pressure at the initial instant of time is set to be identical to the atmospheric pressure, \( p_i^0 = p_a \); the unknown value of the initial gas-phase density can be derived from the state equation.

**3. Results of numerical simulation**

The algorithm and software for the numerical implementation of model (1)–(4) were based on the algorithm and software developed for the two-dimensional two-phase model described in [1], with inclusion of additional stages to the scheme of splitting with respect to physical processes to calculate the velocity, mass, and energy of the disperse phase. In the calculations of the interphase heat exchange and radiant energy exchange, we have considered additionally the exchange between the gas and disperse phases and between the solid and disperse phases.
Figure 1. Propagation of the forest fire front.
Numerical simulation of forest fires was carried out on the HPC MBC-15000BM in Russian Academy of Sciences, Moscow.

Figure 1 shows the results of computations of the forest fire propagation by using the two-dimensional three-phase model (1)–(4) under the conditions of inhomogeneous FF distribution and the presence of the wind.

Figure 1a corresponds to the initial time; it shows schematically three rectangular glades and two roads with no vegetation. The fire source has a circular shape. The wind velocity is \( V_w = (4,4) \) m/s. Figures 1b–1f show the propagation of the forest fire temperature front in time. The fire front propagates in the wind direction. The wind transports the burning particles which are brought by the wind ahead of the main fire front, precipitate and form new fire sources which join the main fire front. Such a mechanism of fire propagation is observed in the real forest fires. It is seen in Fig. 1 how the fire front bypasses the glades and propagates over the roads. As the FF burns out, the combustion process stops, the temperature in this region gradually decreases and the back edge of the fire front is formed.

4. Conclusions
A two-dimensional three-phase model of forest fire propagation is developed and implemented; this model describes the fire-front propagation with consideration of the main physicochemical processes typical of the real forest fire propagation, including a local inhomogeneous FF distribution, the presence of obstacles for the fire propagation, the presence of a wind, as well as with consideration of the mechanism of fire propagation due to the burning particles transported by the wind over the FF layer.

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