Jupiter’s Obliquity and a Long-lived Circumplanetary Disk

Ignacio Mosqueira\textsuperscript{1,2} and Paul Estrada\textsuperscript{1}

\textsuperscript{1} NASA Ames Research Center, \textsuperscript{2} SETI Institute

Abstract

It has been claimed (Canup and Ward 2002; Ward 2003) that a long-lived massive (compared to the mass of the Galilean satellites) circumplanetary gas disk is inconsistent with Jupiter’s low obliquity. Such a constraint could be downplayed on the basis that it deals with a single observation. Here we argue that this argument is flawed because it assumes a solar system much like that of the present day with the one exception of a circumjovian disk which is then allowed to dissipate on a long timescale ($10^6 - 10^7$ yrs). Given that the sequence of events in solar-system history that fit known constraints is non-unique, we choose for the sake of clarity of exposition the orbital architecture framework of Tsiganis \textit{et al.} (2005), in which Jupiter and Saturn were once in closer, less inclined orbits than they are at present, and show that Jupiter’s low obliquity is consistent with the SEMM (solids-enhanced minimum mass) satellite formation model of Mosqueira and Estrada (2003a,b).

1 Introduction

While there may be good reason to believe that planet-disk interactions may excite the eccentricities of at least isolated planets (Goldreich and Sari 2003; Ogilvie and Lubow 2003) there are currently no studies to guide our understanding of likely outcomes in the case of multiple planet (or satellite) systems. Therefore, one may consider circular, coplanar giant planets as a starting condition. Tsiganis \textit{et al.} (2005) have recently argued that evolution through the 1:2 Jupiter-Saturn mean motion resonance (MMR) of a quasi-circular, coplanar and compact solar system that is allowed to evolve by planetesimal scattering is consistent with the observed semi-major axes, eccentricities and mutual inclinations of Jupiter, Saturn, Uranus and Neptune. In a companion publication, Gomes \textit{et al.} (2005) argue that this resonance crossing may be linked to the Late Heavy Bombardment of the terrestrial planets taking place $\sim 700$ Myr after their formation (Hartmann \textit{et al.} 2000). For such a scenario to work the bulk of the divergent migration between Jupiter and Saturn and the resonance passage itself must take place following gas dissipation, and also later than simulations with an evenly spread disk of planetesimals would indicate. Whether or not the identification with the Late Heavy Bombardment is correct, in this viewpoint the solar system must have been more compact and regular prior to gas dissipation (during its first $10^6 - 10^7$ years) than it is today. The authors also briefly consider the potentially disruptive consequences of such a scenario for the regular and irregular satellites of the
giant planets, but conclude that at least the regular satellites might have “survived” unscathed.\(^1\)

Jupiter’s low obliquity \(\sim 3^\circ\) may be indicative of its formation by hydrodynamic gas accretion. Yet, it has been noted that secular spin-orbit resonances can complicate this straightforward interpretation. In particular, adiabatic passage through a resonance matching the spin axis precession rate to the \(\nu_{16}\) precession frequency of the orbit plane due to the gravitational perturbation of Saturn (Hamilton and Ward 2002; Canup and Ward 2002; Ward 2003) and \(\nu_{17}\) due to the gravitational perturbation of Uranus (Hamilton, pers. comm.) may result in obliquities significantly larger than observed. For Jupiter the amplitude of the \(\nu_{16}\) term (with a period of \(\sim 50,000\) years) is \(\sim 0.36\) and the \(\nu_{17}\) term (with a period of \(P_{16} \sim 450,000\) yrs) is \(\sim 0.055\), which could have resulted in obliquities of up to \(\sim 26^\circ\) and \(\sim 14^\circ\), respectively, had these resonances been crossed adiabatically as the circumplanetary gas disk was dissipated (Ward 2003). This leads Canup and Ward (2002) to argue that the circumplanetary disk must have viscously evolved in a timescale sufficiently short \((O(10^5)\) yr\) as to preclude adiabatic passage, resulting in a gas-starved (or gas-poor [Mosqueira et al. 2000; Estrada and Mosqueira 2005]) satellite disk.

The issue we tackle here is whether our decaying turbulence\(^2\) SEMM model (Mosqueira and Estrada 2003a,b; hereafter MEa,b) is especially susceptible to secular spin-orbit resonances, and inconsistent with Jupiter’s low obliquity. In particular, we focus on the \(\nu_{16}\) term. The reasons for this are: First, the period of the orbital precession \(\nu_{17}\) \((P_{17} \sim 450,000\) yrs\) is already very close to the precession period of Jupiter’s spin axis due to the solar torque on the Galilean satellites (which are locked to the Jupiter’s equator plane by this planet’s oblateness; Goldreich 1965). A slight adjustment of satellite or planet positions might be enough to place Jupiter spin axis precession period in one side or the other of this resonance, so that any formation model is apt to be affected. Second, even if the resonance is crossed the limiting obliquity for adiabatic passage is significantly smaller than that for the \(\nu_{16}\) term. Furthermore, the timescale for adiabatic passage in the case of the \(\nu_{17}\) secular spin-orbit resonance may be longer \((O(10^7)\) yrs\) than the timescale for gas dissipation by photoevaporation\(^3\). Third, it is likely that Uranus and Neptune formed after Jupiter and Saturn, once most of gas in the planetary and circumplanetary disks had already dissipated. Thus, from here on we focus on \(\nu_{16}\) and consider Jupiter and Saturn only.

\(^1\)Though the issue is not spelled out in detail (presumably due to space limitations), in the case of the Saturnian satellite system Iapetus’ relatively low eccentricity \(e \sim 0.03\) may be the main constraint for such a scenario. For this reason, it would appear unlikely that Titan could owe its eccentricity to a close encounter between Saturn and another giant planet.

\(^2\)Consistent with numerical simulations that show turbulence decay in the absence of a “stirring” mechanism (Hawley et al. 1999).

\(^3\)Shu et al. (1993) conclude that EUV from the central star may photoevaporate a T Tauri disk in \(\sim 10^7\) yrs outside of a gravitational radius \(r_g \sim 10\) AU. But there is considerable uncertainty in this estimate. For instance, Adams et al. (2004) argue that the disk is likely to be photoevaporated by FUV radiation from neighboring stars. Furthermore, these authors state that photoevaporation is effective to a significantly smaller radius \(0.1 - 0.2r_g\)
2 Secular Perturbation Theory

In a solar system consisting of Jupiter and Saturn it is straightforward to construct a Laplace-Lagrange secular solution (e.g., Brouwer and Clemence 1961; Murray and Dermott 1999). For $I \ll 1$, the orbital inclination $I$ and the longitude of the ascending node with respect to the invariant plane $\Omega$ are given by

\[ I \sin \Omega = I_1 \sin \gamma_1 + I_2 \sin(f_2 t + \gamma_2), \]

and

\[ I \cos \Omega = I_1 \cos \gamma_1 + I_2 \cos(f_2 t + \gamma_2), \]

where $f_2$ is the eigenfrequency, $I_1$ and $I_2$ are eigenvector components, and $\gamma_1$ and $\gamma_2$ are phases. If we use parameters for Jupiter and Saturn as observed, we obtain $f_2 = -7.06 \times 10^{-36}$ yr\(^{-1}\) and $I_2 = -6.30 \times 10^{-3}$ (in radians). This yields a secular oscillation with period of $P_{16} \sim 51,000$ yr. These values are not too dissimilar from the secular solution of the planetary system (e.g., Murray and Dermott 1999).

Let us now consider a time early on before Jupiter and Saturn had passed through the 1:2 MMR but after most of the planetary gas disk had dissipated by photoevaporation in a timescale $10^6 - 10^7$ yr. Since the planetary nebula may shield it to some
degree, it may be appropriate to assume that the much denser subnebula takes longer to dissipate. If so, at this time both the precession of Jupiter’s spin axis and orbital plane may be significantly faster than they are today. Because of the scattering on nearby planetesimals (Gomes et al. 2005) and the dissipation of the nebula, Saturn would dominate the precession of Jupiter’s orbital plane. We take as nominal values for the orbital parameters of Jupiter and Saturn the starting conditions in Tsiganis et al. (2005), namely, $a_J = 5.45$ AU, $a_S = 8.50$ AU (a few tenths of an AU inside the 1:2 MMR) and $\sin I_{JS} \approx 10^{-3}$, where $I_{JS}$ is the relative inclination of the orbital planes of Jupiter and Saturn. With these parameters we obtain $f_2 = -1.60 \times 10^{-20}$ yr$^{-1}$ and $I_2 = -2.72 \times 10^{-4}$ (in radians). As shown in Fig. 1, in this case the period of Jupiter’s orbital plane precession is $P_{16} \sim 23,000$ yrs.

3 Obliquity Variation

Jupiter’s current spin axis precession period is $\sim 4.5 \times 10^5$ yrs due mostly to the solar torque exerted on the Galilean satellites (e.g., Ward 1975). However, a massive circumplanetary gas disk would result in a much shorter precession period. The inner parts of the disk ($r < r_t = (2M_p/M_\odot J_2 R_p a_p^3)^{1/5} \approx 40R_J$, where $M_p = M_J$, $a_p = 5.45$ AU, $R_p \approx 2R_J$, and $J_2 \propto 1/R_p \approx 0.008$ are the planetary mass, semi-major axis, radius and quadrupole gravitational harmonic following envelope collapse [which may take place in a fast $10^4 - 10^5$ yrs timescale, Hubickyj O., pers. comm.], and $M_\odot$ and $R_J$ are the Sun’s mass and Jupiter’s present radius) would orbit in the plane of the planet’s equator and precess as a unit with the planet, whereas the outer parts of the disk ($r > r_t$) would not (Goldreich 1966). The spin axis $\mathbf{s}$ then precesses around the orbit normal $\mathbf{n}$ at a rate given by (e.g., Tremaine 1991)

$$\frac{d\mathbf{s}}{dt} = \alpha(\mathbf{s} \cdot \mathbf{n})(\mathbf{s} \times \mathbf{n}),$$

where $\mathbf{s}$ and $\mathbf{n}$ are unit vectors, and the precession constant$^7$ is given by

$^4$Ward (1981) treats the two-orbit/nebula problem as dispersal takes place. Here we consider the case when the nebula (but not the subnebula) has already been dissipated by some means.

$^5$An estimate for the gap opening timescale may be obtained using the tidal torque formula (Lin and Papaloizou 1993) $\tau_{gap} \approx (\Delta/a)^3 P/\mu^2$, where $\Delta$ is the gap’s half-width, $\mu$ is the mass ratio of the secondary to the primary, and $P$ is the orbital period of the secondary. Using $\Delta \sim (a_S - a_J)/2 \sim 1.5$ AU, we find that Jupiter and Saturn would have cleared the gas disk in between in a short timescale $t \sim 10^4$ yrs. In particular, this timescale is shorter than satellite formation timescale $10^4 - 10^6$ (depending on location; Mosqueira et al. 2001) in MEa,b.

$^6$We obtain a precession period of $4.8 \times 10^5$ yrs using a moment of inertia for Jupiter of $K = 0.26$, which yields a spin angular momentum $J_p = 4.4 \times 10^{45}$ g cm$^2$/s. There is some uncertainty in the moment of inertia $\sim 5\%$ (Fortney J., pers. comm.) but this doesn’t affect the argument. What is important to note, however, is that about 33% of the precession constant $\alpha$ is contributed by the torque of the Sun directly on the planet because of its oblateness. Adding a circumplanetary gas disk decreases this fraction, which we then ignore.

$^7$In actuality, there should be another term added to this precession rate due to the torque of
\[ \alpha = \frac{3\pi \Omega_p^2}{2H} \int_{R_p}^{r_D} \Sigma(a) a^3 da, \]  

where the surface density of circumplanetary disk \( \Sigma(a) \) is assumed to drop-off sharply at \( r_D \sim 2r_c \), where \( r_c = R_H/48 \sim 15R_J \) is the centrifugal radius (MEa), \( \Omega_p \) is the planet’s orbital frequency and the total angular momentum of the precessing system is given by

\[ H = J_p + 2\pi (GM_p)^{1/2} \int_{R_p}^{r_D} \Sigma(a) a^{3/2} da, \]

where \( G \) is the gravitational constant and \( J_p \) is the spin angular momentum of the planet. For \( \Sigma \propto 1/a \), we can write \( \alpha = M_D \Omega_p^2 r_D^2 / (4H) \) and \( H = J_p + 2/3M_D \sqrt{GM_p r_D} \), where \( M_D \) is the disk mass. The precession period is given by \( T = 2\pi / (\alpha \cos \theta) \), where \( \cos \theta = \hat{s} \cdot \hat{n} \) is the obliquity. Given that in our SEMM model \( M_D \sim 10M_{sats} \), where \( M_{sats} \sim 4 \times 10^{26} \) g is the mass of the Galilean satellites\(^8\), the spin of the planet provides \( \sim 90\% \) of the angular momentum of the system \( H \sim 5 \times 10^{45} \) g cm\(^2\)/s (see MEa Table 3). Using an obliquity of \( \theta \sim 3^\circ \) and \( r_D = 40R_J \) (which implies a surface density \( \Sigma \sim 2 \times 10^4 \) g/cm\(^2\) at \( 15R_J \) consistent with the value obtained by applying the inviscid gap-opening criterion to Ganymede in a disk with aspect ratio \( \sim 0.1 \) [MEb]), we obtain a precession period of \( T \sim 4 \times 10^4 \) yrs, which is slightly shorter than Jupiter’s current orbital plane secular precession period \( P_{16} \sim 5 \times 10^4 \) yrs, but it is longer than Jupiter’s orbital plane precession period when Saturn was inside the 1:2 MMR, i.e., \( P_{16} \sim 2 \times 10^4 \) yrs. Hence, in our decaying turbulence SEMM model it is possible for Jupiter either to have crossed the secular spin-orbit resonance before the Keplerian disk reached its quiescent phase (and satellites formed) at a time when the viscous evolution of the disk was likely driven by Roche-lobe gas inflow and the resonance passage was non-adiabatic, or not to have crossed this resonance at all. However, it may be possible to alter this conclusion by choosing different parameters, such as a more massive and extended circumplanetary disk. Thus, next we consider the case of resonance passage.

The limiting obliquity \( \theta_{max} \) that could be generated by the obliquity “kick” incurred during adiabatic resonance passage in the non-capture direction (\( \dot{\alpha} < 0 \)) (which implies resonance crossing as the circumplanetary gas disk is dissipated and the spin-axis precession period increases) is given by (Henrard and Murigande 1987; Ward and Hamilton 2004)

\[ \theta_{max} = \frac{\alpha_{max}}{\Omega_p}, \]

the extended part of the disk \( r > r_t \) on the inner part of disk. However, the contribution of this extended region drops rapidly with distance, and in our model the gas surface density drops-off at a radial location \( r_D \lesssim r_t \).

\(^8\)There is some ambiguity here because in the model of MEa,b a significant fraction of the mass of Callisto is derived from the extended part of the disk, whereas \( M_D \) is the mass of the inner disk out to about Callisto. On the other hand, Io and Europa should be reconstituted for unaccreted volatiles.
\[
\cos \theta_{\text{max}} = \frac{2}{(1 + \tan^{2/3}|I_2|)^{3/2}} - 1, \tag{6}
\]

which yields \(\theta_{\text{max}} \sim 26^\circ\) using the current value for \(|I_2| \sim 0^\circ.36\), but a significantly smaller value of \(\theta_{\text{max}} \sim 9.1^\circ\) using \(|I_2| \sim 0^\circ.015\) obtained from our nominal, low mutual inclination case. Furthermore, the minimum time for adiabatic crossing is (Ward and Hamilton 2004)

\[
\tau_{\text{min}} \approx P_{16} \left( \frac{\theta}{2\pi|I_2|} \right)^2. \tag{7}
\]

Taking \(\theta \sim 9^\circ\), we find \(\sim 2 \times 10^8\) yrs for our nominal case, which is much longer than the gas dissipation timescale. This means that at least for the nominal case the crossing must be non-adiabatic and the final obliquity is rate dependent. Taking \(\theta \sim 3^\circ\), we calculate \(\tau_{\text{min}} \sim 9 \times 10^4\) yrs for current solar-system parameters, but it is \(\tau_{\text{min}} \sim 2 \times 10^7\) yrs (here \(P_{16} \sim 23,000\) yrs is about a factor of two shorter than the present value, but this is more than compensated by the much smaller value of \(|I_2|\) for our nominal case, which is comparable to or longer than the gas dissipation timescale. We may estimate the resulting obliquity to be \(\theta \lesssim \tan^{1/3}(|I_2|) \lesssim 4^\circ\) for our nominal case, which is consistent with Jupiter’s observed value. At any rate, this would all be irrelevant if the resonance were not crossed.

4 Conclusions

We have briefly investigated the consequences for Jupiter’s obliquity of a satellite formation model in which a massive (compared to the Galilean satellites yet enhanced in solids by a factor of \(\sim 10\) compared to the solar composition minimum mass model) subnebula is allowed to dissipate in a long timescale \((10^6 - 10^7\) yrs) after the dispersal of the nebula itself. For the sake of specificity, we have adopted the model of Tsiganis et al. (2005) in which the solar system was more compact and regular before Jupiter and Saturn crossed the 1:2 MMR than it is today. We find that such a combined scenario (by no means unique) does not imply the likelihood of a larger obliquity for Jupiter than is observed. This is both because the secular \(\nu_{16}\) spin-orbit resonance may not be crossed, and because the resulting obliquity may be consistent with Jupiter’s value even if it is crossed. We conclude that Jupiter’s low obliquity is compatible with our SEMM satellite formation model (MEa,b) provided one allows for solar-system conditions early-on unlike those presently observed.

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