Solution to the Strong CP Problem:  
Supersymmetry with Parity

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Abstract

We find that supersymmetry with parity can solve the strong CP problem in many cases including the interesting cases of having the minimal supersymmetric standard model or some of its extensions below the Planck/GUT/intermediate scales, as well as for the case where we have a low-energy SUSY left-right model.
The smallest dimensionless parameter in the standard model is the strong CP phase, \( \bar{\theta} = \theta + ArgDet(M) \) where \( \theta / 32\pi^2 \) is the coefficient of the \( F\bar{F} \) term in the QCD lagrangian and \( M \) is the quark mass matrix. Experimental bounds on the electric dipole moment of the neutron imply that \( \bar{\theta} \leq 10^{-9} \). There is no symmetry by which \( \bar{\theta} \) can be made naturally small (or zero) at the level of the standard model, and this has been called the strong CP problem.

Two elegant solutions have been proposed. If the up quark were massless or if there were a \( U(1) \) Peccci-Quinn (PQ) symmetry \([1]\), then \( \bar{\theta} \) can be rotated away. However the up quark seems to be massive, and the PQ symmetry leads to an axion which is severely constrained by experiments. Other solutions like spontaneous CP violation or the Nelson-Barr mechanism \([2]\) require heavy quarks. Solutions based on spontaneous P violation have so far required mirror families \([3]\) or CP symmetry as well \([4]\). While none of the existing solutions are completely ruled out, nevertheless a solution to the strong CP problem continues to occupy our minds.

In the supersymmetric extension of the standard model, not only has no new solution to the strong CP problem been found, but also a new problem gets generated - namely the small SUSY phase problem \([5]\). Even if the strong CP problem were solved, for example by the PQ symmetry, direct contributions to the dipole-moment of the neutron constrain many other CP violating phases in the theory which could be apriori of the order 1. Also the Nelson-Barr mechanism does not seem to generalize to the supersymmetric extension of the standard model even with universal soft SUSY breaking terms at the Planck/GUT scales \([4]\). Thus solutions to the strong CP problem based on spontaneous CP violation or Nelson-Barr mechanism have not been extended to MSSM and this is grounds for serious concern. From an experimental point of view, while the MSSM is very predictive on things like the Higgs mass, it does very poorly on the important question of additional CP phases. At least two new independent phases in the \( A, B, \mu, m_{1/2} \) terms are expected and so far there is no theoretical prediction on their values \([7]\).

In this letter we show that if the minimal supersymmetric standard model (MSSM) \([8]\) is extended to include parity (which can then be broken to MSSM at any scale between \( M_{SUSY} \) and \( M_{Pl} \)) a new solution to the strong CP problem is obtained. Further the small SUSY phase problem is also automatically solved. An important prediction emerges that there are no phases in the ratio of the Higgs vevs, \( \mu, B, A \), or \( m_{1/2} \) of MSSM. We also study the solution for low-energy SUSY left-right symmetric model.

**Strong CP problem with Parity:** To be specific we include parity by extending the standard model to the left-right symmetric model \( SU(2)_L \times SU(2)_R \times SU(3) \times U(1)_{B-L} \) \([9]\). The matter spectrum consists of the usual quarks and leptons, \( Q_i(2, 1, 3, 1/3) \), \( Q^c_i(1, 2, -\bar{3}, -1/3) \), \( L_i(2, 1, 1, -1) \) and \( L^c_i(1, 2, 1, 1) \) where \( i \) is the generation index and runs from 1 to 3. One or more (indexed by \( a \)) bidoublet Higgs fields \( \phi_a(2, 2, 1, 0) \) are introduced to break the theory down to electromagnetism. The \( \phi_a \) are each represented by \( 2 \times 2 \) complex matrices while the doublet quark and lepton fields by \( 2 \times 1 \) column vectors. Under parity \( x \rightarrow -x \), \( Q_i \leftrightarrow Q_i^{\ast} \), \( L_i \leftrightarrow L_i^{\ast} \) and \( \phi_a \leftrightarrow \phi^a \). Invariance under parity of the Yukawa term \( (h_{ij}^a Q_i^a \phi_a Q_j^j + h.c.) \) implies that \( h_i^{\ast} = h_j^{\ast} \), (ie) the Yukawa matrix is hermitian. The mass
matrix is the product of the Yukawa matrix and the vacuum expectation values (VEVs) \( \langle \phi_a \rangle \). Therefore the mass matrix will have a real determinant if we can prove that the matrices \( \langle \phi_a \rangle \) are real. This would then lead to a solution of the strong CP problem since the coupling \( \theta \) of the parity odd \( \theta/32\pi^2 \) \( F \bar{F} \) term is zero due to parity.

\( \langle \phi_a \rangle \) are determined by minimizing the Higgs potential and can be naturally real only if all the coupling constants involving \( \phi_a \) in the Higgs potential are real. We begin by making an important observation that terms involving only \( \phi_a \) are of the form \( m_{ab} Tr \phi_a^\dagger \phi_b \), \( \mu_{ab} Tr(\tau_2 \phi_a^{\dagger T} \tau_2 \phi_b) \), etc. (in general traces of products of \( \phi_a, \phi_a^\dagger, \phi_a^T \) and \( \tau_2 \)). By comparing every term and its hermitian conjugate, it is easy to see that invariance under P (\( \phi_a \leftrightarrow \phi_a^\dagger \)) implies that all the constants \( m_{ab}, \mu_{ab}, \) etc. are real! If we have additional gauge singlet Higgs fields \( \sigma \), such that under P: \( \sigma \leftrightarrow \sigma^\dagger \), then all coupling constants of terms involving \( \phi_a \) and \( \sigma \) will also be real.

In order to break SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y \) at a scale \( M_R \) (which can be anywhere between \( M_P \) and the Planck scale \( M_P \)), we need to introduce Higgs triplet or doublet fields, namely, either \( \Delta(3, 1, 1, 2), \Delta^c(1, 3, 1, -2) \) OR \( \chi(2, 1, 1, 1), \chi^c(1, 2, 1, -1) \) such that under P: \( \Delta \leftrightarrow \Delta^c, \chi \leftrightarrow \chi^c \) and give a VEV to the right-handed fields. There will be coupling terms between \( \phi_a \) and \( \Delta^c \) or \( \chi^c \). Terms of the form \( \lambda (\Delta^{c \dagger} \Delta^c Tr(\tau_2 \phi_a^{\dagger T} \tau_2 \phi_a) + \Delta^{c \dagger} \Delta^c Tr(\tau_2 \phi_a^{\dagger T} \tau_2 \phi_a)) \) are invariant under P, and \( \lambda \) can be complex. We note that this complex term is the source of the strong CP problem in left-right symmetric theory.

If there is supersymmetry, as we shall see, these terms coupling \( \Delta^c \) to \( \phi \) with complex coupling constants are naturally absent and we are led to a solution to the strong CP problem. The rest of the paper analyzes the tree level and loop effects of the solution in SUSY left right models \[10, 11, 12\] spontaneously breaking to MSSM.

**Tree level Solutions: SUSY with Parity**

**Case 1: Minimal left-right model** - The superpotential for the minimal model is given by \[10, 11, 12\]

\[
W = M \; Tr \; \Delta^c \bar{\Delta} + M^* \; Tr \; \Delta \bar{\Delta} + \mu_{ab} \; Tr \; \tau_2 \phi_a^{\dagger T} \tau_2 \phi_b
\]  

(1)

There is no coupling between the \( \Delta^c \) and the \( \phi_a \). This is the case even for the most general soft SUSY breaking terms since they are given by the most general analytic cubic polynomials in the scalar fields of the theory. Since these have the same form as \( W \) (but with arbitrary coefficients), there is no coupling between \( \Delta^c \) and \( \phi_a \) in these terms either. The D-terms only involve real gauge coupling constants. As explained in the previous section, \( \mu_{ab} \) and coupling constants of the quadratic soft SUSY breaking terms involving \( \phi_a \), are all real due to parity. Hence all coupling constants in the Higgs potential wherever \( \phi_a \) occurs are real. Thus \( \langle \phi_a \rangle \) is naturally real and at the tree level there is no Strong CP problem.

We would like to preserve this nice feature of the minimal model while extending to non-minimal models. The main reasons to extend are that we need to break the left-right symmetric theory to MSSM at a high scale- so we have to introduce other fields. Also as it stands this model will break \( Q_{em} \) spontaneously unless R-parity is broken by giving the sneutrino a VEV \[10\]. We would like to keep R-parity unbroken, so as not to introduce
the complex phases in the lepton sector and make the problem more complicated. This is another reason to consider non-minimal models. From now on we will interchangeably use $\phi$ for $\phi_a$ since the generalization to more than one doublet is now obvious. Also, in the following we will not explicitly write the squark or slepton fields as their VEVs are zero.

**Case 2: Breaking to MSSM + Singlet** - In order to solve the $\mu$ problem, MSSM with a singlet $\sigma$ has been considered previously in the literature. A discrete $Z_3$ symmetry $\phi, \sigma, Q, Q^c, L, L^c \rightarrow e^{i2\pi/3}(\phi, \sigma, Q, Q^c, L, L^c)$ prevents the direct $\mu$-term. We will show that SUSY left-right symmetric theory can naturally break to this low-energy theory with zero tree-level strong CP phase. The most general left-right symmetric superpotential is

$$W = M Tr \Delta^c \bar{\Delta} + M^* Tr \Delta \bar{\Delta} + \beta \left( h_{\beta} Tr \Delta^c \bar{\Delta} + h_{\beta}^* Tr \Delta \bar{\Delta} \right) + f(\beta)$$

$$+ h_{a}\sigma Tr \tau_2 \phi^T \tau_2 \phi + \lambda \sigma^3$$

where $f(\beta)$ is any cubic polynomial, and under $Z_3$, $\Delta, \Delta^c \rightarrow e^{i2\pi/3}(\Delta, \Delta^c)$, $\bar{\Delta}, \bar{\Delta}^c \rightarrow e^{-i2\pi/3}(\bar{\Delta}, \bar{\Delta}^c)$ and $\beta \rightarrow \beta$. Under parity $\sigma \rightarrow \sigma^\dagger$ and $\beta \rightarrow \beta^\dagger$. The soft SUSY breaking terms can have their most general form consistent with Parity and $Z_3$. F-terms are obtained by taking the partial derivative of $W$ with respect to each of the fields in the superpotential (denoted here by $A_i$), so that

$$V_F = \Sigma_i \left| \frac{\partial W}{\partial A_i} \right|^2.$$ 

It is easy to see that there are solutions for $V_F \approx M_{SUSY}^4$ such that $\Delta^c, \bar{\Delta}^c \approx M_R$ and $\phi, \sigma$ are less than $M_{SUSY}$. This implies that we can break the theory down to MSSM + singlet at a high scale $M_R$. Once again since there is no coupling terms between the $\Delta^c$ and $\phi$ fields, the coupling constants in the Higgs potential for all the terms which contain $\phi$ are real due to parity. Likewise, all coupling constants for terms involving $\sigma$ are also real. Thus $\langle \phi \rangle$ and $\langle \sigma \rangle$ are naturally real and there is no strong CP phase at the tree level. A point to note is that this model has not been considered in Reference [10]. Therefore the result of that paper does not apply and there can be $Q_{em}$ conserving and parity breaking vacuua without needing R-parity breaking. This is because a complex $h_{\beta}$ leads to a complex VEV for $\beta$, thereby breaking parity. The quartic F-term in $\phi$ stabilizes the $Q_{em}$ conserving vacuum.

**Case 3: Breaking to MSSM** - We introduce singlet (they could be in general triplet or other fields too) fields $\alpha, \beta$ and $\gamma$. Under Parity they go to their Hermitian conjugate fields. The most general super potential is given by:

$$W = h_{\alpha} \alpha Tr \Delta^c \bar{\Delta} + h_{\beta} \beta \Delta^c \bar{\Delta}^c + m_{\alpha} \alpha \gamma + m_{\beta} \beta \gamma +$$

$$\lambda_1 \gamma^3 + \lambda_2 \alpha^3 + \lambda_3 \beta^3 + \mu_{ab} Tr \tau_2 \phi_a^T \tau_2 \phi_b + (\Delta \text{ terms})$$

where in order to prevent couplings between $\Delta^c$ and $\phi$ in the F-term, we have imposed a discrete symmetry (D) such that $\gamma, \bar{\Delta}, \Delta^c \rightarrow e^{i2\pi/3}(\gamma, \bar{\Delta}, \Delta^c)$, $\alpha, \beta \rightarrow e^{-i2\pi/3}(\alpha, \beta)$ and
the rest of the fields are invariant. The singlets allow us to break the $SU(2)_R \times U(1)_{B-L}$ symmetry at a high scale ($M_R$) to MSSM. This is important and can be checked by writing out all the F-terms obtained by differentiating $W$ with each and every field. The crucial point is that there are solutions for $V_F = 0$, with the singlets and the right-handed fields alone picking up VEVs. $m_\alpha$ and $m_\beta$ set the scale for $M_R \gg M_{SUSY}, M_W$. Once again all coupling constants in terms wherever $\phi$ occurs are real, and hence $\langle \phi \rangle$ is real and there is no strong CP problem. In order to avoid the bound $M_R \leq M_{SUSY}/f$ of reference [13] we can introduce $B-L = 0$ triplet fields $\omega, \omega^c$ such that under $D$: $\omega, \omega^c \to e^{-i2\pi/3} (\omega, \omega^c)$. This does not change the result that $\langle \phi \rangle$ is real.

This case has the advantage over case 2 since the discrete symmetries (Parity as well as the discrete symmetry $D$) can be broken at a high scale so that domain walls associated with the breakdown of discrete symmetries can be inflated away. In case 2 since there is a residual low-energy $Z_3$ symmetry this problem exists (since this symmetry breaks only when $\phi$ picks up a VEV).

We have shown three illustrative cases where there is a natural solution to the strong CP problem at the tree level. Other non-minimal models can be easily accommodated, in a similar manner. Now we will study the loop effects.

The Complete Solution

If $M_R \gg M_{SUSY}, M_W$ then the effective theory below $M_R$ will be SUSY $SU(2)_L \times SU(3) \times U(1)$ (and in particular in case 3 it will be the MSSM). The MSSM superpotential and the soft SUSY breaking terms are given by [8, 14, 15]:

$$W = \mu H^T \tilde{H} + h_u^i Q_i^T H u_i^c + h_d^i Q_i^T \tilde{H} d_i^c$$

$$V_S = m H^T \tilde{H} + A_u^i \tilde{Q}_i^T H u_i^c + A_d^i \tilde{Q}_i^T \tilde{H} d_i^c + M_3 \tilde{G}\tilde{G} + M_2 \tilde{W}_L \tilde{W}_L + M_Y \tilde{Y}\tilde{Y} + \text{quad. scalar masses.}$$

(5) (6)

where, $\tilde{G}$ and $\tilde{W}_L$ are the Gluino and Left-Handed Gaugino (wino) respectively and where the standard model Higgs doublets are denoted by $H(2,1,-1)$ and $\tilde{H}(2,1,1)$. These doublets are the light elements of the bidoublet fields $\phi_a$. Due to parity and since we have shown that all couplings in terms containing $\phi_a$ are real (in cases 1,2 and 3 above even after $\Delta^c$ picks up VEV), the following boundary conditions emerge at $M_R$:

$$\mu = \mu^*, \ h_u^i = h_u^{i*}, \ h_d^i = h_d^{i*}, \ A_u^i = A_u^{i*}, \ A_d^i = A_d^{i*}, \ m = m^*, \ M_3 = M_3^*$$

(7)

We now use the two loop renormalization group equations [14] to run the above coupling constants to the $\text{TeV}$ scale. Our results follow.

Result 1: With universal soft SUSY breaking terms - If the soft SUSY breaking terms come from a supergravity sector, there are further constraints that soft SUSY breaking terms
can satisfy \( [15] \). These constraints can both be derived from some supergravity theories, and can be motivated by low energy flavour phenomenology. We will first consider the most constrained MSSM that has received the maximum attention with the following universality conditions at the SUGRA breaking (or Planck) scale.

\[
A_{ij}^u = A h_{ij}^u, \quad m = B \mu, \quad M_3 = M_{2L} = M_{2R} = M_{B-L} = m_{1/2}.
\] (8)

Now using equation 7 it is easy to see that \( A, B \) and \( m_{1/2} \) real. The quadratic scalar masses are also universal. Therefore the only complex phase in the theory is the standard model CKM matrix phase. It is easy to see using the 2-loop renormalization group equations (RGE) of reference \([14]\) that every coupling constant (coupling matrix) and its hermitian conjugate evolve according to the same RGE if the above conditions are met. Therefore Hermitian matrices remain Hermitian and real couplings remain real. Thus at the weak scale the Yukawa and squark matrices are Hermitian. The Higgs doublet coupling constants are all real, and the Gluino, Bino and Wino mass terms are real. Hence the expectation value of \( \bar{H} \) and \( H \) is real, and the quark mass matrix is Hermitian and the Strong CP phase is zero. In this case, the loop effects at the weak scale will induce a negligibly small strong CP phase consistent with \( \bar{\theta} << 10^{-9} \), and we have the solution to the strong CP problem. Note that we have implicitly assumed that \( M_R \) is approximately equal to the SUGRA breaking scale (or \( M_{Pl} \)) and we will relax this condition later (see Result 3).

Result 2: With universality only for Gauginos - There are supergravity models where only some but not necessarily all universality conditions are predicted. Also in string theory we may have non-universal terms \([16]\). The only universality condition that is really needed for us is that of gaugino phases, namely,

\[
\text{Arg} M_3 = \text{Arg} M_{2L} = \text{Arg} M_{2R} = \text{Arg} U(1)_{B-L} = \text{Arg} M
\] (9)

We will not assume any other universality condition and so the rest of the soft SUSY breaking terms can be general. Even in this case, and using equation 4, it is easy to see that the RGE preserve the Hermitian and real nature of the respective coupling constants and therefore, just as in Result 1 it follows that there is no strong CP problem. In addition to supergravity models already included in the first result, such a universality condition can be obtained from models where the gaugino mass term ratios depend only on real numbers like the structure constants of the gauge groups \([10]\). It can also happen due to an underlying grand-unified group.

Result 3: Bound on Wino phase, accessible strong CP - Parity only relates the left and the right Wino phases but does not set them to zero. If instead of at \( M_{Pl} \), \( SU(2)_R \) is broken at an intermediate scale \( M_R \) then even if wino phases are real at \( M_{Pl} \) they will pick up a complex value from the complex terms in the \( \Delta \) sector, due to renormalization group running from \( M_{Pl} \) to \( M_R \). This phase will in turn give rise to a gluino phase because while
the left handed wino contributes to the renormalization group running from $M_R$ to $M_W$, the right handed wino does not. Both effects are at the two loop level \[14\]. Hence the gluino mass term picks up a phase of the order $(1/16\pi^2)^2 \times (1/16\pi^2)^2 \times \delta$, which is about $10^{-9} \times \delta$ where $\delta$ are typical phases in the $\Delta$ terms. This resultant strong CP phase is consistent with current experimental bounds, and at the same time is reasonably exciting for the neutron electric dipole moment searches.

Even if the Planck scale universality conditions on the gaugino phases are not exact, what the above estimate implies is that the wino phases must be within $(1/16\pi^2)^2$ at that scale or they will induce too large a strong CP phase.

The only exception will be if $M_R < M_{SUSY}$ because in this case we have a low energy SUSY left-right symmetric theory, and the left and right handed wino mass term phase contributions to the Gluino phases will cancel. Note that the illustrative models of case 1, 2, and 3 can have $M_R \leq M_{SUSY}$

**Result 4: Without Universality, $M_R \leq M_{SUSY}$** - Even if the wino phases are greater than $10^{-4.5}$ (and the soft SUSY breaking terms have their most general form), this solution to the strong CP problem works. This is because both left and right winos are present at low energies if $M_R < M_{SUSY}$, and their effects will cancel. However since $M_R > M_W$ there will be a mixing between the right wino and the right handed Higgsinos, due to the VEVs $\langle \Delta^c \rangle$ and $\langle \bar{\Delta}^c \rangle$. However both expectation values are required to induce a phase, and this phase will therefore be of the order $\delta \times \langle \Delta^c \rangle \langle \bar{\Delta}^c \rangle / M_{SUSY}^2$, where $\delta$ are the typical phases in $\Delta^c$ terms. This must be less than $10^{-4.5}$ to be consistent with the neutron edm. This bound does not necessarily imply a high $M_{SUSY}$, because it is possible to choose soft SUSY breaking parameters such that one of the $SU(2)_R$ triplet fields picks up a much smaller VEV than the other $SU(2)_R$ triplet. However these choices of parameters may not be consistent with some of the universality conditions. Only in such cases (and if we are working in supergravity theory which gives the universality condition) this implies the bound $\delta \times M_R^2/M_{SUSY}^2 \leq 10^{-4.5}$.

Another very interesting possibility for low-energy left-right symmetry is with $\chi, \chi^c, \bar{\chi}, \bar{\chi}^c$ fields instead of the triplets, and with universal $A$ and $B$ terms. Once we impose R-parity, and noting that terms like $\chi^T \phi \chi^c$ must have a real coupling constant (due to parity), we can see that the phase in the $\chi^c T \bar{\chi}^c$ can be rotated away. Universal $A$ and $B$ terms must be real due to parity. Hence we once again have real vacuum expectation values for the Higgs fields. There is no problem with having to worry about $Q_{em}$ breaking minima as the result of Reference \[10\] does not apply to doublet fields. In this model the majorana mass term for the right-handed neutrino can arise in loops (or the other option is to introduce extra R-parity odd singlets, "neutrinos", to be able to give a large mass to right-handed neutrinos). But this solution requires $M_R \leq M_{SUSY}$, since introducing any R-parity even singlets to increase the scale of $M_R$ will lead to additional complex couplings. The only way to allow the singlets would be to prevent the coupling term $\chi^T \phi \chi^c$ by a symmetry. Once we do this the rest of the analysis for $\chi^c$ fields is exactly similar to that for breaking $SU(2)_R$ via the $\Delta^c$ fields. But now the Majorana mass cannot be generated by loops, and
we need to introduce extra neutrinos.

Non-supersymmetric left-right model

It is worth noting that the term $\lambda(\Delta^c_t \Delta^c_c \, Tr \left( \tau_2 \phi^T \tau_2 \phi \right))$ that was the source of the strong CP problem in non-supersymmetric left-right models can be eliminated by discrete symmetries like $\phi_1 \rightarrow i\phi_1$, $Q_3 \rightarrow -iQ_3$, etc. We need to introduce enough bidoublets and have enough non-trivial symmetries that transform the quark fields in a family number dependent way, so as to prevent all such troublesome terms, while at the same time obtaining a consistent quark mass matrix. These symmetries are in the low-energy theory and hence will lead to the domain wall problem, unless soft breaking terms (that break these symmetries, but not Parity) of dimension 2 are allowed. In this case we can have solutions to the strong CP problem in the non-supersymmetric version, without having to introduce CP as a good symmetry of the Lagrangian.

Conclusions

We have shown that supersymmetry with parity can solve the strong CP problem in many cases including the interesting cases of having the MSSM or some of its extensions below the Planck/GUT/intermediate scales, as well as for the case where we have a low-energy SUSY left-right model.

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