Non-minimal couplings in Randall-Sundrum Scenarios

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In this paper we propose a new model to solve the problem of Yang-Mills localization in Randall-Sundrum scenarios without the introduction of other fields or new degrees of freedom. The model is based only in non-minimal couplings with the gravity field. We show that two non-minimal couplings are necessary, one with the field strength and the other with a mass term. Despite the loosing of five dimensional gauge invariance by the mass term a massless gauge field is obtained over the brane. To obtain this, we need of a fine tuning of the two parameters introduced through the couplings. The fine tuning is obtained by imposing the boundary conditions and to guarantee non-abelian gauge invariance in four dimensions. With this we are left with no free parameters and the model is completely determined. The model also provides analytical solutions to the linearized equations for the zero mode and for a general warp factor.

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The Randall Sundrum (RS) model appeared in the Physics of higher dimensions as an alternative to compactification that included the possibility of solving the Hierarchy problem.\textsuperscript{1,2} To solve the physical problem of dimensional reduction, the RS model should obtain fields with zero mode confined to the brane in order to recover the Physical models when the fields are properly integrated over the extra dimension. In the case of the gravity field, localization is attained, however gauge fields as simple as $U(1)$ minimally coupled to gravity are not localized and this was a problem to the theory.\textsuperscript{3,4} A number of extensions to RS were proposed in order to provide localized gauge fields. A smooth warp factor was investigated?\textsuperscript{4,5}, but also did not yield localized gauge fields. Some models obtained the localization by the addition of new degrees of freedom such as a scalar or the dilaton fields, but a more natural approach would be to obtain an extension of minimal couplings that would localize the gauge field without introducing other fields. A step in this direction was the introduction of a boundary interaction with the field strength,\textsuperscript{11} but it is known that this produces only a quasi-localized zero mode.

Other idea is the breaking of the $U(1)$ gauge invariance in five dimensions by the addition of a mass term. It was found that the only consistent way of getting a localized zero mode is again the introduction of a boundary term,\textsuperscript{12} this time with the mass term.\textsuperscript{12} Soon it was show that the boundary mass term was not enough to provide a consistent model in the case of Yang-Mills (YM) fields because non-abelian gauge invariance is lost in the membrane.\textsuperscript{13} The work\textsuperscript{13} shows that a combination of the above boundary terms are needed to solve the problem of YM localization. In the track of understanding the origin of the mass boundary term introduced in\textsuperscript{12}, some of us identified that a non-minimal coupling of the Ricci scalar with the mass term could generate it.\textsuperscript{14} More then this, it was discovered that this kind of coupling solve the localization of any $U(1)$ $p$–form gauge field in co-dimension one brane worlds, valid for any warp factor.\textsuperscript{15} However the mechanism does not work for Yang-Mills fields by the same reasons of Ref.\textsuperscript{13}. Other non-minimal couplings of gravity with the field strength seems to be need also in this case. These kinds of couplings of gauge fields with gravity has been proposed and studied in 4D\textsuperscript{16} and recently it was shown that they generate the boundary term for the field strength in Ref.\textsuperscript{17}. However the model keeps the property of being only quasi-localized. Here we show that specific non-minimal couplings with gravity are enough to localize YM fields over the brane, moreover, the confined fields displays the expected properties, that is, are massless and therefore gauge invariant. Our approach also has the advantage of being valid for any extension of the RS model which recovers it asymptotically. This is obtained when we merge the coupling as proposed in\textsuperscript{15} with the non-minimal coupling proposed by us in Ref.\textsuperscript{14}. Importantly, localization and gauge invariance depend on the tuning of two otherwise free parameters of the model.

The background metric of the RS model in its conformal form is given by $ds^2 = e^{2A(z)}g_{MN}dx^Mdx^N$, where $z = x^5$. After considering a delta-like brane and an AdS vacuum a stable background solution to the Einstein equation is found, which is given by $A = -\ln(k|z| + 1)$. The question of localizability of fields is resumed to find a square integrable solution($\psi$) of a Schroedinger-like mass equation with potential $U$ which emerges from the process of dimensional reduction. At the end this provides a finite four dimensional action after the integration in the extra dimension $\int \psi^2 dz$ is performed. The above warp factor generates effective potentials with Dirac delta functions\textsuperscript{1,2}. These singularities can be smoothened out through the introduction of kink-like membranes such as in Refs.\textsuperscript{1,2} that recovers the RS case only asymptotically. Therefore the better approach is to consider a...
general warp factor $A$ and to look for a solution to the localization problem that does not depends on it. We call such solution warp independent for obvious reasons. For a minimally coupled $U(1)$ gauge field the effective potential obtained is $U = A''/2 + A'^2/4$. Analyzing more carefully, for a thin brane, the solution has a convergent and a divergent component as $z \to \infty$. However this is not the only boundary condition to be imposed since $A$ depends on $|z|$ and we get a delta function in the potential. When we impose the proper boundary condition in $z = 0$ the solution is completely fixed and we discover that it is not localized. The same conclusion is obtained for arbitrary $A$ and is warp independent. The form of the above potential provides a general solution given by $\psi = e^{-A/2}$ and this is not square integrable for asymptotic RS models.

As said before, a model was constructed which introduced a boundary term through an interaction with a delta function $|z|$, that is, besides the standard gauge action a contribution given by $\frac{\delta(z)}{m^2} F_{\mu \nu}^2$ was introduced, where $\mu$ are four dimensional indices and $m$ is a mass parameter. Although the origin of this term is not explained, the authors argue that it is needed to guarantee that there is no current in the extra dimension. Then a quasi-localized zero mode is obtained. Some time latter it was shown in [13] that this in not enough when we consider YM fields. The reason is that now we have quartic terms in the action and even if the zero mode is integrable we generally have $\int \psi^3 dz \neq \int \psi^4 dz$ causing the lost of gauge invariance after the integration is performed. However Batell et al showed that this can be solved by te use of both boundary terms of Refs. [11, 12] and the final action which solves the problem was found to be

$$S = -\frac{1}{4e_5^2} \int d^5x \sqrt{-g} \left[ Tr F_{MN}^2 + \beta \delta(z) Tr F_{\mu \nu}^a F^{\mu \nu a} \right] - \frac{1}{2} \int d^5x \sqrt{-g} M^2(z) Tr A_{\mu}^a A^a \mu$$

(2)

where $e_5$ is the five dimensional gauge coupling and $F_{MN} = \partial_M A_N^a - \partial_N A_M^a + f^{abc} A_M^c A_N^b$. Since $M(z) = (a + b \delta(z))$ the model posses three parameters: $a$ and $b$ to guarantee the localization and a third parameter $\beta$ is introduced to guarantee the gauge invariance in four dimensions. However, we show here that this action must be corrected to preserve the expected symmetries of the system.

Searching for the origin of the above action, some of us discovered in a series of previous papers that a non-minimal coupling with the Ricci scalar given by $\gamma RA_N^2$ consistently provides a solution to the localization of $U(1)$ gauge field in co-dimension one brane worlds [14, 15, 19]. This term modifies the potential that now is given by $U = p A'' + p^2 A'^2$ where $p$ is the order of the $p-$form considered. The zero mode for this potential has solution $\psi = e^{pA}$ which is square integrable for any warp factor that asymptotically recovers RS, being therefore a warp independent solution. Another advantage is that the existence of general covariance determines one of the parameters leaving only $\gamma$ which is fixed by boundary conditions, and at the end we get a model with no free parameters. However the problem for YM fields is not solved and other non-minimal coupling seems to be needed.

With the above ingredients we propose a model defined by the following action

$$S = -\frac{1}{4e_5^2} \int d^5x \sqrt{-g} \left[ Tr F_{MN}^2 + \gamma_1 \Delta_{AB}^C Tr F_{AB} F^{CD} \right] - \frac{\gamma_2}{2} \int d^5x \sqrt{-g} R Tr A_{\mu}^a A^a \mu$$

(3)

where $\gamma_1$ and $\gamma_2$ are parameters that will be fixed by the boundary condition and the demand of gauge invariance in $4D$. From now on we will not write explicitly the group indices. The linearized equations of motion are

$$\partial_M (\sqrt{g} F^{MN} ) + \gamma_1 \partial_M (\Delta^{MNP} \sqrt{g} F_{OP} ) = -\gamma_2 R \sqrt{g} g^{MN} A_M$$

(4)
and from the above we get the five dimensional divergenceless condition $\partial_M(\sqrt{g}Rg^{MN}A_N) = 0$, or in components

$$\partial_5(Re^{3A}\Phi) + Re^{3A}\partial_\mu A^\mu = 0,$$

(5)

where we have defined $\Phi \equiv A^5$. By computing $\Delta A_{\mu}$ explicitly we obtain the equations of motions in components. For $N = \nu$ we get

$$e^A(1 + \gamma_1 f)\partial_\mu F^{\mu\nu} + \partial(e^A(1 + \gamma_1 h)F^{5\nu}) = -\gamma_2 Re^{3A} A^\nu$$

and for $N = 5$

$$e^A(1 + \gamma_1 h(z))\partial_\mu F^{5\mu} = -\gamma_2 Re^{3A}\Phi$$

(6)

where $e^{2Ah} = -3A'^2/4$ and $e^{2Af} = -A''/2 - A'^2/4$. Therefore our system is defined by the three equations above, which includes a vector and a scalar field in four dimensions. From Eq. 16 we see that our four dimensional vector field does not satisfy the four dimensional divergenceless condition. The strategy, as explained before in Ref. 14 is to split the field in longitudinal and transverse parts $A^\mu_L = (\delta^\mu_\nu - \frac{\partial^\mu A^\nu}{\partial z}) A^\nu_L$, $A^\mu_T = \frac{\partial^\mu}{\partial z} A^\nu$ and to interpret 5 as an equation relating the longitudinal part of $A^\mu$ and the scalar field. The main question posed at this point is if we can decouple the transverse component in order to obtain a well defined YM field in four dimensions. For the $U(1)$ case and for $\gamma_1 = 0$, with only the coupling to the mass term, we have shown that this is the case. Now we have the additional complication of the non-minimal coupling with the field strength, however we will see that it is also valid for the present case. For this end first we define $F^{5\mu}_L = \partial_\nu A^\mu_L - \partial^\mu \Phi$, form where we get the identity $F^{5\mu}_L = -\frac{\partial^\mu}{\partial z} F^{5\nu}$. Beyond this also we have $\partial_\mu F^{\mu\nu} = \Box A^\nu_T$ and Eq. 16 becomes

$$e^A(1 + \gamma_1 f)\Box A^\nu_L + \partial(e^A(1 + h \gamma_1)A^\nu_T) + \gamma_2 Re^{3A} A^\nu_T + \partial(e^A(1 + h \gamma_1) F^{5\nu}_T) + \gamma_2 Re^{3A} A^\nu_T = 0.$$

(8)

Now by using Eq. 16, the definition of $A^\mu_L$ and our identity for $F^{5\mu}_L$ we can show that

$$\partial(e^A(1 + \gamma_1 h) F^{5\nu}_T) = -\gamma_2 Re^{3A} A^\nu_L$$

where we have used the fact $\partial_5 h = 0$ from the tracelessness of $\Delta$. Therefore we successfully get our decoupled equation for the transverse YM field given by

$$e^A(1 + \gamma_1 f)\Box A^\nu_T + \partial(e^A(1 + h \gamma_1) \partial A^\nu_T) + \gamma_2 Re^{3A} A^\nu_T = 0.$$

Now performing the standard separations of variables $A^\nu_T = \tilde{A}^\nu_T (z^{\mu}) \theta - \frac{2}{3}(z) \psi(z)$, with $\theta = e^A(1 + h \gamma_1)$ and by considering the zero mode $\Box \tilde{A}^\nu_T = 0$ we get the Schrödinger equation $\psi'' - U(z) \psi = 0$ with

$$U(z) = \frac{1}{2} A'' + \frac{1}{4} A'^2 - \gamma_2 Re^{3A}.$$
there are other couplings which can solve the problem. If this is true we should also answer what kind of restriction must be imposed to get a desirable theory in four dimensions. It is also important to understand the reason why this kind of coupling generates solutions which are localized independently of a specific form of the warp factor. Another question is if this kind of couplings works for other fields. In this direction some of us has shown that the non-minimal coupling with the mass term can be used to localize ELKO spinors and $p$-form fields in a warp independent way \[14, 15, 20, 21\]. In particular, spin 1/2 fields are specifically interesting, but this seems to be an yet more difficult problem since basically all the models and extensions of RS gives only one chirality of the field localized on the brane. However, since our model provides a localized gauge field we must also have a localized current, what seems to imply that the fermion field is also localized.

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