ON WD-WD MERGERS IN TRIPLE SYSTEMS: THE ROLE OF KOZAI RESONANCE WITH TIDAL FRICTION

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ABSTRACT

White dwarf–white dwarf (WD–WD) mergers may lead to type Ia supernovae events. Thompson (2011) suggested that many such binaries are produced in hierarchical triple systems. The tertiary induces eccentricity oscillations in the inner binary via the Kozai–Lidov mechanism, driving the binary to high eccentricities, and significantly reducing the gravitational wave merger timescale ($T_{GW}$) over a broad range of parameter space. Here, we investigate the role of tidal forces in these systems. We show that tidal effects are important in the regime of moderately high initial relative inclination between the inner binary and the outer tertiary. For $85° \leq i_t \leq 90°$ (prograde) and $97° \leq i_t \leq 102°$ (retrograde), tides combine with GW radiation to dramatically decrease $T_{GW}$. In the regime of high inclinations between $91° \leq i_t \leq 96°$, the inner binary likely suffers a direct collision, as in the work of Katz & Dong (2012) and tidal effects do not play an important role.

Subject headings: binaries: close — stellar dynamics — celestial mechanics — stars: supernovae: general — white dwarfs

1. INTRODUCTION

The merger of two white dwarfs (WDs) driven by gravitational wave (GW) radiation has been suggested as a possible mechanism leading to production of type Ia supernovae (Iben & Tutukov 1984; Webbink 1984; Howell 2011). For this mechanism to be observationally relevant, the merger rate has to be comparable to the rate of Ia supernovae events. Even though a large fraction of stars are born as binaries, only a small fraction are tight enough to merge via GW emission within a Hubble time. But if these binaries are in hierarchical triples, the presence of a tertiary on a highly inclined orbit can induce eccentricity oscillations in the inner binary via secular resonance (Kozai 1962; Lidov 1962). The GW radiation timescale ($T_{GW}$) and the time scale associated with dissipative tides are both a strong function of eccentricity; hence the presence of a third body on a highly inclined orbit pumping the eccentricity of the inner binary to high values can dramatically decrease the merger time scale of the binary (Blaes et al. 2002; Miller & Hamilton 2002; Wen 2003; Antonini & Perets 2012).

This mechanism was invoked by Thompson (2011) to enhance the rate of WD–WD, NS–WD and NS–NS mergers due to GW radiation. Such mergers are responsible for production of exotic objects such as Ia supernovae, γ-ray bursts (GRBs) and other transients. Thompson (2011) showed that the GW merger timescale for compact object binaries in triple systems was significantly decreased from that of the binary alone, which allowed for a larger range of the semimajor axes that could lead to a merger in a Hubble time, as well as an increased rate of prompt mergers ($< 10^8$ yr). For detailed discussion on how common these systems are, formation scenarios and rates we refer the reader to Thompson (2011).

In this paper we explore the role of tides in WD–WD merger events. We demonstrate that in the range of high inclinations ($91° \leq i_t \leq 96°$), the outcome of the evolution is a direct collision of the two WDs, in which tidal effects do not play a significant role. Tidal effects do play a significant role in the range of moderately high inclinations ($85° \leq i_t \leq 90°$ and $97° \leq i_t \leq 102°$) where they dramatically decrease $T_{GW}$.

In Section 2 we describing the Kozai–Lidov dynamics in the presence of additional forces and dissipation due to tides and gravitational wave radiation. We describe relevant timescales for our dynamical problem. In Section 3 we describe the results of numerical integrations of the equations of motion. We discuss our findings in Section 4.

2. UNDERSTANDING THE DYNAMICS

2.1. The Kozai–Lidov mechanism

The presence of a third body on a hierarchical orbit around the centre of mass of a binary will affect the orbital elements of the binary on a variety of timescales. The induced changes in the orbital elements of the binary will be particularly striking if the mutual inclination between the inner and the outer orbit is high. The two orbits will exchange angular momentum, causing both the eccentricity of the inner binary and the mutual inclination to undergo periodic oscillations known as Kozai cycles (Kozai 1962). Kozai (1962) showed that the mutual inclination required for having Kozai cycles is $i_{crit} \leq i \leq 180° - i_{crit}$, where $i_{crit} \approx 39.2°$ is a critical inclination. Kozai cycles result from a $1:1$ resonance between the longitude of the periastron $\varpi$ and the longitude of the ascending node $\Omega$ and therefore the condition for Kozai resonance, $\varpi - \Omega = 0$, is fulfilled only for inclinations in the Kozai regime ($i_{crit} \leq i \leq 180° - i_{crit}$). For inclinations outside of the Kozai regime, the apsidal line precesses in a prograde sense ($\varpi > 0$), while the line of nodes precesses in a retrograde sense ($\Omega < 0$). For prograde orbits ($i \leq 90°$) these cycles are out of phase, meaning that when the eccentricity reaches its maximum, the mutual inclination reaches its minimum and...
vice versa. On the other hand, for retrograde orbits ($i > 90^\circ$) these cycles are in phase: both the eccentricity and the mutual inclination reach maximum values simultaneously. The period of a Kozai cycle is significantly longer than either the orbital period of the inner or the outer binary, suggesting the use of the secular approximation. The secular approximation consists averaging the equations of motion over the orbital periods of the inner and the outer binary. Such averaged equations allow for exchange of angular momentum between the two orbits but not variations in the energy, so that the semimajor axes of the both orbits remain unchanged. The relevance of the secular approximation is discussed further in Section 4 The maximum eccentricity attained in Kozai cycles in the absence of additional forces is given by:

$$e_{\text{max}} = \left(1 - \frac{5}{3} \sin^2 i_0 \right)^{1/2}$$

where $i_0$ is initial mutual inclination between the inner and the outer orbit. Note that equation 1 is given in the test particle limit where the secondary is treated as a massless particle.

Kozai cycles can be suppressed by other dynamical effects that induce periapse precession in the inner binary. We take into account the following additional sources of periapse precession: apsidal precession due to tidal and rotational bulges, apsidal precession due to general relativity (GR) and the apsidal precession due to tidal dissipation, which is negligible in comparison to the other two. Tidal dissipation as well as gravitational wave radiation play a major role in driving the merger of the inner binary and is discussed in more details later in the paper.

We consider an inner WD–WD binary with semimajor axis $a_{\text{in}}$, eccentricity $e_{\text{in}}$, argument of a periapsis $\omega_{\text{in}}$, masses $m_1$ and $m_2$ and two equal radii $R$. The third body with mass $m_3$, semimajor axis $a_{\text{out}}$, eccentricity $e_{\text{out}}$ and argument of a periapsis $\omega_{\text{out}}$ is on a larger orbit around the center of mass of the inner binary. The mutual inclination between the two orbits is $i$. The mean motion of the inner binary is $\Omega = \sqrt{G(m_1 + m_2)}/a_{\text{in}}^3$. The precession due to rotational and tidal dissipation is captured in the tidal friction time scale $t_{\text{f}}$.

Table: System Parameters

| Symbol | Definition | Value |
|--------|------------|-------|
| $m_1$ | White dwarf (primary) mass | $0.8M_\odot$ |
| $m_2$ | White dwarf (secondary) mass | $0.6M_\odot$ |
| $m_3$ | Third companion mass | $1.0M_\odot$ |
| $a_{\text{in}}$ | Inner binary semimajor axis | $0.05AU$ |
| $a_{\text{out}}$ | Outer binary semimajor axis | $1AU$ |
| $e_{\text{in},0}$ | Inner binary initial eccentricity | $0.1$ |
| $e_{\text{out},0}$ | Outer binary eccentricity | $0.5$ |
| $i_0$ | Initial mutual inclination | $90^\circ$ |
| $\omega_{\text{in},0}$ | Initial argument of periastron | $0$ |
| $\Omega_\text{in}$ | Longitude of ascending node | $0$ |
| $R$ | White dwarf radius | $5 \times 10^9$ cm |
| $k_2$ | Tidal Love number | $0.1$ |
| $Q$ | Tidal dissipation factor | $10^7$ |

\[
\omega_{\text{Kozai}} = \frac{3}{4} \frac{GM_3}{a_{\text{in}}^3} \left(1 - e_{\text{in}}^2\right)^{3/2} \frac{1}{\sqrt{1 - e_{\text{in}}^2}} \times (2)
\]

\[
\omega_{\text{GR}} = \frac{3}{4} \frac{G(m_1 + m_2)}{a_{\text{in}}^3} (1 - e_{\text{in}}^2) \times (3)
\]

\[
\omega_{\text{TB}} = \frac{15}{16} \frac{G(m_1 + m_2)}{a_{\text{in}}^3} \left[8 + 12 e_{\text{in}}^2 + e_{\text{in}}^4 \right] \times (4)
\]

\[
\omega_{\text{RB}} = \frac{m_1 + m_2}{4G^2 a_{\text{in}}^3 (1 - e_{\text{in}}^2)^2} \times (5)
\]

\[
\frac{1}{m_1} \sum_{i=1,2} \left[ (2 \Omega_{\text{in}}^2 - \Omega_{\text{in}}^2 - \Omega_{\text{q}}^2) \right] + 2 \Omega_{\text{in}} \cot i \left( \Omega_{\text{in}} \sin \omega_{\text{in}} + \Omega_{\text{q}} \cos \omega_{\text{in}} \right) \times (6)
\]

where $t_{\text{f}} = \frac{1}{6} \left( \frac{a_{\text{in}}}{R} \right)^{5/2} \frac{1}{n} \left( \frac{m_2}{m_1} \right)$ is the tidal friction time scale for star with mass $m_1$. A similar expression with two indices 1 and 2 swapped holds for the tidal friction time scale $t_{\text{f}}$ for the star with mass $m_2$. The three components of the spin, $\Omega_i$, are the projection along the Laplace-Runge-Lenz vector, pointing along the apsidal line from the WD secondary at the apoapsis to where the WD primary, denoted by $\Omega_{\text{in}}$, along the total angular momentum vector, $\Omega_{\text{h}}$, and their cross product, denoted by $\Omega_{\text{q}}$.

As equation 3 shows, the term driving Kozai cycles can be either positive or negative depending on the value of $\sin i$. On the other hand, the terms driving the precession due to GR and tidal bulge are always positive and therefore tend to promote periapse precession. The effect of these two terms is to lower the maximum eccentricity attainable by the system, while the critical inclination increases (Eggleton & Kiseleva-Eggleton 2001; Miller & Hamilton 2002; Wu & Murray 2003; Fabrycky & Tremaine 2007, see their Figure 3). The term induced by the rotational bulge may have either positive or negative value. We assume that initially the system is tidally locked and that the spins are aligned, so this term tends to increase the rate of precession and hence suppress Kozai cycles. Both precession due to rotational and tidal bulge are parametrized by the tidal Love number $k_2$ which is a dimensionless constant that relates the mass of the multipole moment created by tidal forces on the spherical body to the gravitational tidal field in which that same body is immersed. Furthermore, $k_2$ encodes information on the internal structure of...
the body in question.4

Tidal dissipation in the stars in the inner binary, due to either an eccentric orbit or to asynchronous rotations, becomes important when the separation between the stars is of order of a few stellar radii (Mazeh & Shaham 1979; Eggleton & Kiseleva-Eggleton 2001). During the phases of high eccentricity the periapse distance may become sufficiently small as to lead to strong tidal dissipation. During these phases of high eccentricity the tidal dissipation will drain energy from the orbit, but not angular momentum. The energy loss results in a reduction of the semimajor axis and therefore enhances the rate of dissipation. Since the angular momentum remains conserved during this process, the eccentricity is damped as well until the orbit eventually circularizes and the system settles at a separation of only a few stellar radii. Tidal dissipation is parametrized by the tidal dissipation factor \( Q \), defined as the ratio of the energy stored in the tidal bulge to the energy dissipated per orbit.

Another source of dissipation in WD–WD binaries is gravitational wave radiation which drains both energy and angular momentum from the orbit. Like tidal dissipation, it tends to shrink and circularize the orbit. During the Kozai cycles, if the amplitude of the eccentricity oscillations is sufficiently large, the GW radiation becomes much stronger than in the circular case, leading to mergers on timescales much shorter than a single Hubble time, \( T_{\text{Hubble}} \) (Blaes et al. 2002; Miller & Hamilton 2002; Thompson 2011; Antonini & Perets 2012). As the inner eccentricity reaches values close to 1, the system toward merger would be completely dominated by the gravitational wave tides and GW radiation become comparable and neither can be neglected, as we discuss in more detail in next section (see figure[1]).

2.2. Timescales

The GW merger timescale in the limit of high eccentricity is (Peters 1964):

\[
T_{GW} = \frac{3}{85} \frac{a_{\text{in}}}{c} \left( \frac{a_{\text{in}}^3 c^6}{G^3 m_1 m_2 M} \right) (1 - e_{\text{in}}^2)^{\frac{5}{2}} \approx 5.4 \times 10^{12} \text{yr} \left[ \frac{0.672 M_\odot}{m_1 m_2 M} \right] \left( \frac{a_{\text{in}}}{0.05 \text{AU}} \right)^4 (1 - e_{\text{in}}^2)^{\frac{5}{2}},
\]

where \( M = m_1 + m_2 \). As seen from eqn[7], in the absence of a third body \( T_{GW} \) is greater than Hubble time, \( T_{\text{Hubble}} \approx 14 \text{Gyr} \), for \( a_{\text{in}} > 0.01 \text{AU} \).

As discussed in the previous section, GR precession tends to promote periapse precession and therefore suppress Kozai cycles. In general, GR precession decreases the maximum possible eccentricity attainable by the binary at a fixed initial inclination and increases the critical inclination required for undergoing Kozai cycles (Blaes et al. 2002; Fabrycky & Tremaine 2007; Prodan & Murray 2012). The timescale for GR precession is:

\[
T_{GR} = \frac{1}{3} \frac{a_{\text{in}}}{c} \left( \frac{a_{\text{in}}^3 c^6}{GM} \right)^{\frac{3}{2}} (1 - e_{\text{in}}^2)^{\frac{3}{2}} \approx 1.8 \times 10^3 \text{yr} \left[ \frac{0.672 M_\odot}{m_1 m_2 M} \right] \left( \frac{a_{\text{in}}}{0.05 \text{AU}} \right)^{\frac{3}{2}} (1 - e_{\text{in}}^2)^{\frac{3}{2}}.
\]

4 The apsidal precession constant, which is a factor of two smaller than the tidal Love number, but which we do not utilize, is often denoted by \( k_2 \) as well.

We consider the effects of tidal forces, where both rotational and tidal bulges tend to suppress Kozai cycles in similar manner as GR precession (Equation[8]). The timescale for precession induced by tidal bulge is given by (Prodan & Murray 2012; Fabrycky & Tremaine 2007):

\[
T_{TB} = \frac{16}{15k_2} \left( \frac{a_{\text{in}}}{R} \right)^{\frac{3}{2}} \left( \frac{a_{\text{in}}^3}{GM} \right)^{\frac{1}{2}} \frac{m_2^2 + m_1 m_2}{m_1} (1 - e_{\text{in}}^2)^{\frac{5}{2}} \frac{8 + 12 e_{\text{in}}^2 + e_{\text{in}}^4}{8 + 12 e_{\text{in}}^2 + e_{\text{in}}^4} \lesssim 5.9 \times 10^{13} \text{yr} \left[ \frac{a_{\text{in}}}{0.05 \text{AU}} \right]^{\frac{1}{2}} \left( \frac{5 \times 10^8 \text{cm}}{R} \right)^{\frac{5}{2}} \times \left( \frac{1.4 M_\odot}{M} \right)^{\frac{1}{2}} (1 - e_{\text{in}}^2)^{\frac{5}{2}} \lesssim 22 \text{yr} \left[ \frac{a_{\text{in}}}{0.05 \text{AU}} \right]^{\frac{1}{2}} \left( \frac{M}{1.4 M_\odot} \right)^{\frac{1}{2}} \left( \frac{a_{\text{out}}/a_{\text{in}}}{20} \right)^{\frac{1}{2}} (1 - e_{\text{out}}^2)^{\frac{1}{2}},
\]

The condition for the inner binary to undergo Kozai oscillation is that the Kozai timescale is shorter than the timescale of any of the suppressing effects. Furthermore, the Kozai timescale is strongly dependent on the ratio of the inner and the outer binary semimajor axis, which sets a limit on the maximum allowed \( a_{\text{out}} \), beyond which the Kozai oscillations are ineffective. During the evolution, depending on the type of stars in the inner binary (i.e. compact objects or main sequence stars) and tightness of its orbit, dissipative effects due to gravitational wave radiation or tides may be significant. Either of the two may lead to shrinkage and/or circularization of the inner orbit, which increases the ratio of the inner and outer semimajor axis and thus reduces the effectiveness of Kozai oscillations. Consequently, the Kozai timescale may become larger than either \( T_{GR} \) or \( T_{TB} \) and at this point the evolution of the system toward merger would be completely dominated by non-Kozai effects.

To emphasize the importance of Kozai oscillations for rapid mergers of compact objects and considering only the presence of a third body, [Thompson 2011] gives the following order-of-magnitude estimate of the merger time:

\[
T_{\text{merge}} \sim \frac{25}{153} \frac{a_{\text{in}}}{c} \left( \frac{a_{\text{in}}^3 c^6}{G^3 m_1 m_2 M} \right) \cos^2 i \approx 4.5 \times 10^4 \text{yr} \left[ \frac{0.672 M_\odot}{m_1 m_2 M} \right] \left( \frac{a_{\text{in}}}{0.05 \text{AU}} \right)^4 \left( \frac{\cos i}{\cos(88^\circ)} \right)^6,
\]

which shows a strong dependence on mutual inclination. Equation[11] is only valid in the Kozai regime, \( 39^\circ \leq i \leq 141^\circ \) and fails to account for additional sources of apsidal precession such as GR precession and precession due to the tidal and/or rotational bulge. As pointed out by [Thompson 2011], this expression significantly underestimates the merger time in certain regions of parameter space and hence should be used only to obtain rough estimate. For detailed discussion we refer the reader to the Section 4 of [Thompson 2011].
average over the orbital periods of both the inner binary and the outer companion and retain terms up to \((a_{in}/a_{out})^3\) to 3rd order. Beside the perturbations due to the presence of the third body via Kozai–Lidov mechanism, we include the following dynamical effects:

- periastron advance due to general relativity;
- periastron advance arising from quadrupole distortions of the white dwarf due to both tides and rotation;
- orbital decay due to tidal dissipation in the white dwarf;
- loss of binary orbital angular momentum due to gravitational radiation.

The equations used in our model are those of Blaes et al. (2002) for the octupole terms, which are based on those derived in Ford et al. (2000), combined with equations from Prodan & Murray (2012) for tidal effects. For the discussion of the relevance of the secular approximation we refer reader to Section 4.

Following Thompson (2011), as fiducial parameters we use: \(m_1 = 0.8M_{\odot}\), \(m_2 = 0.6M_{\odot}\), and \(m_3 = 1M_{\odot}\). The semimajor axis of the inner binary is \(a_{in} = 0.05AU\) and the semimajor axis of the outer binary is \(a_{out} = 1AU\). We use crude approximation that the radius of both white dwarfs in the inner binary is \(R = 5 \times 10^8cm\), while the fiducial Love number is \(k_2 = 0.1\) and tidal dissipation factor is \(Q = 10^7\). For the initial eccentricities and arguments of a periapse we take: \(e_{in,0} = 0.1\), \(e_{out,0} = 0.5\), \(\omega_{in,0} = \omega_{out,0} = 0\) throughout this paper. Initially we take the WDs to be tidally locked. We only evolve systems within the following range of initial mutual inclinations: \(85^\circ \leq i_{out} \leq 102^\circ\). We consider system merged when \(r_p \leq 2R_{\text{sum}} = 4R\). Next we discuss the results of our model for: high mutual inclination \(91^\circ \leq i_0 \leq 96^\circ\) and moderately high inclinations \(85^\circ \leq i \leq 90^\circ\) and \(97^\circ \leq i_0 \leq 102^\circ\).

### 3. NUMERICAL RESULTS

#### 3.1. Numerical model using the octupole approximation

In our numerical model we treat the gravitational effects of the third body in the octupole approximation, meaning we...
GW radiation as the only source of dissipation in the system. By design, the maximum eccentricity in the case of moderately high initial mutual inclinations is not high enough to lead to a collision. Instead, during the phases of high eccentricity the tidal dissipation and GW radiation are strong. As a consequence both the eccentricity and the semimajor axis of the inner binary decrease and eventually the periapse becomes comparable to the sum of the radii of the two white dwarfs. Figures 3 and 4 show the time evolution of the eccentricity, the semimajor axis and the periapse of the inner binary with and without tidal effects taken into account for $i_0 = 89^\circ$. The high eccentricity induced by the third body results in strong dissipation even with just GW radiation taken into account, which by itself shortens dramatically the merger timescale. However, comparing the merger timescale in the two cases clearly demonstrates that the merger timescale is at least an order of magnitude shorter when tidal dissipation is taken into account.

Figure 5 shows the merger timescale dependence on the initial mutual inclination for the cases with and without the tidal effects included. In the high inclination regime ($91^\circ \leq i_0 \leq 96^\circ$), the two white dwarfs collide at the first eccentricity maximum on a very short timescale (\sim 20 yr), so the suppressing effects due to additional sources of apsidal precession and dissipation are insignificant. On the other hand, in the moderately high inclination regime, tidal dissipation shortens the merger timescale for an additional order of magnitude as seen in figure 5. This implies that a combination of the perturbations due to the presence of a third body and tidal dissipation and GW radiation indeed dramatically shorten the merger timescale of WD–WD binaries in comparison to Hubble time, $T_{Hubble}$ when the system is in moderately high inclinations regime ($85^\circ \leq i_0 \leq 90^\circ$ and $97^\circ \leq i_0 \leq 102^\circ$).

4. DISCUSSION

In [Thompson (2011)], the relevance of WD–WD, NS–WD, and NS–NS mergers driven by GW radiation was explored for events like Ia supernovae, GRBs, and other transients. It was explicitly demonstrated that the GW merger timescale for these compact binaries becomes shorter by up to a few orders of magnitude due to the eccentricity oscillations induced by the presence of a third body at high inclination with respect to the inner orbit, compared to the case of an isolated binary. The triple scenario allows for a wider range of binary semimajor axes leading to a merger in a Hubble time, enhancing the population of compact object binaries capable of producing Ia supernovae and GRBs.

In this paper, we build on work of [Thompson (2011)] and explore the combined effect of tides and GW radiation on the merger timescale of WD–WD binaries in triple systems. We examine the evolution of the WD–WD binary in the presence of a third body at $a_{out}/a_{in} \sim 20$. When the mutual inclination is sufficiently high, the third body perturbs the inner binary orbit causing periodic oscillations in the eccentricity and mutual inclination via the Kozai–Lidov mechanism. As discussed in Section 2, tidal effects become important during the phases of high eccentricity where the inner binary periapse is of order of few stellar radii. In the case of high mutual inclination ($91^\circ \leq i_0 \leq 96^\circ$), the outcome of the evolution is a direct collision of the WDs at the first eccentricity maximum, where the Kozai torque is the dominant torque and all other dynamical effects are insignificant. In the other words,
none of the additional sources of precession such as GR and tidal and/or rotational bulges have timescale short enough to affect the maximum possible eccentricity or suppress Kozai cycles, contrary to what was expected in Thompson (2011). For the same reason the strong tidal dissipation or GW radiation do not affect the evolution of the system even though the eccentricity reaches values close to 1 as seen in Section 3.2. Such collisions have been discussed by several authors as possible channels for production of type Ia supernovae (Rosswog et al. 2009; Raskin et al. 2009, 2010; Katz & Dong 2012; Hamers et al. 2013). As shown here and in Thompson (2011), this scenario seems very promising for retrograde orbits with $i_0 \geq 90^\circ$. Globular clusters, where the third star can be captured in binary-binary interactions (Ivanova 2008; Ivanova et al. 2006) may produce such retrograde triples. Further study of such triples formed via binary-binary interactions is a subject of our next paper.

In Katz & Dong (2012), collisions of white dwarfs occur in moderately hierarchical triple systems ($3 \lesssim r_{p,\text{out}}/a_{in} \lesssim 10$) for wide range of $a_{in}$, where the inner binary reaches high eccentricities via Kozai–Lidov mechanism. A similar scenario is studied in Hamers et al. (2013). Katz & Dong (2012) use a symplectic three body integrator while Hamers et al. (2013) combine the secular three body dynamics with a detailed prescription for stellar, binary and triple evolution prior to the formation of the WD–WD binary. Considering the evolution on the main sequence is important because most of the systems that could be possible progenitors of WD–WD binaries (large $a_{in}$, small $a_{out}/a_{in}$) experience mergers while they are on main sequence (Hamers et al. 2013). The rate of direct collisions described in Katz & Dong (2012) is not known because the distribution of inclinations after stellar evolution and mass loss to form a WD-WD binary is not known. There is

Fig. 3.— The eccentricity as a function of time: upper panel shows the case where we take into account tidal effects and the lower panel shows the case where only GR precession and GW radiation are taken into account for our fiducial model with $i_0 = 89^\circ$. The tidal dissipation factor is $Q = 10^7$. We terminate the integrations when $r_p \leq 4R$. As the upper panel shows, including tidal dissipation leads to a merger timescale about an order of magnitude shorter than the merger time due to GW radiation alone.

Fig. 4.— The semimajor axis and the periapse of the inner binary as a function of time: upper panel shows the case where we take into account GR, GW and tidal effects, while the lower panel shows the case where only GR precession and GW radiation are taken into account. The plot represents our fiducial model, with $i_0 = 89^\circ$. The solid line represents the semimajor axis while the dashed line shows the periapse evolution. The dotted line represents the double sum of the radii of the two white dwarfs. As the upper panel shows, including tidal dissipation leads to a merger timescale about an order of magnitude shorter than the merger time due to GW radiation alone.
Katz & Dong (2012) find that the secular treatment of the Kozai–Lidov mechanism breaks down for most of their systems. The reason lies in the assumed ratio $R/\alpha_{in}$, for which their choice of parameters requires $1 - e_{in} > 10^{-6}$ in order for tides to be significant as for collisions to occur. This is not the case in our work since we are focused on very compact WD–WD binaries. For our choice of $R/\alpha_{in}$, the two WDs collide when $1 - e_{in} \sim 10^{-3}$. The secular treatment of the Kozai–Lidov mechanism gives an adequate description in this case (see equation 16 in Katz & Dong (2012)). Our choice of parameter space is relevant for close compact object binaries in the field, as well as for those in globular clusters. For example, Rosswo& et al. (2009) and Raskin et al. (2009) consider WDs that reside in the dense core of the globular clusters and similar dense stellar environments where the fact that stars are sufficiently close to each other makes collisions very likely. The authors propose WD–WD collision scenario with low impact parameter (close to head on collision) as a possible formation channel for type Ia supernovae in such environments. They carry out 3D hydrodynamical calculations of thermonuclear explosion of colliding WDs. Rosswo& et al. (2009) investigate the outcome of direct, head on collisions of several mass pairs, while Raskin et al. (2009) investigate the outcome of a collision of a single mass pair ($2 \times 0.6M_\odot$) with three different impact parameters. Both papers establish that a collision of moderately massive WD pair naturally leads to shock-triggered ignition and a synthesis of substantial amounts of $^{56}Ni$ for low impact parameter even if the total mass of the pair is below the Chandrasekhar limit. As emphasized by both groups, one should keep in mind that the outcome of the WD–WD collision hinge on several factors: masses and nuclear compositions, their relative speed and impact parameters. Results from Raskin et al. (2009) imply that the WD–WD collision with high impact parameters (i.e. grazing collision) lacks violent shocks seen in the cases with low impact parameters. Instead, there is negligible nuclear burning as well as a negligible amount of $^{56}Ni$ produced by the initial interaction. In this scenario both WDs become unbound and form a rotating disk of white dwarf debris that cools down and eventually collapses into a single compact object. In both studies the rates of collisions that may result in explosion are low and still subject to uncertainties (i.e. core-collapse evolution of the globular cluster). But, such rates indicate that WD–WD collision are not unlikely and hence they can contribute a modest fraction of type Ia events. The likelihood of low impact parameter collisions due to the eccentricity oscillations induced by the presence of a third body remains unknown. One should be cautious about claiming that these events can explain the SNe Ia rates, but as already pointed out these events definitely do contribute to the overall rate.

In the regime of moderately high inclinations previously described additional sources of precession affect the evolution of the system by suppressing the Kozai–Lidov mechanism as demonstrated in Section 3.3. We showed that including tidal dissipation together with GW radiation shortens the merger timescale by factor of a few to an order of magnitude compared to when only GW radiation is taken into account (see Figure 4). The fact that tidal dissipation can speed up the mergers driven by GW radiation implies that the very prompt merger rate ($< 10^8$ yr) may be higher than found in Thompson (2011). The tidal dissipation rate is determined by tidal dissipation factor $Q$. In this work we take $Q \sim 10^7$ as obtained in Prodan & Murray (2012), a value in a reasonable agreement with the findings of Piro (2011) and Fuller & Lai (2011).

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Fig. 5.— The merger time as a function of initial inclination. The open squares come from numerical integration of the equations of motion that include tidal dissipation and GW radiation while solid circles come from integration that includes only GW radiation. In the high inclination regime ($91^\circ \leq i_0 \leq 96^\circ$), the collision of the two white dwarfs occurs at the first eccentricity maximum on a very short timescale ($\sim 20$ yr) showing that the suppressing effects due to additional sources of apsidal precession and dissipation are insignificant. Including tidal dissipation in the moderately high inclinations regime ($85^\circ \leq i_0 \leq 90^\circ$ and $97^\circ \leq i_0 \leq 102^\circ$) leads to shorter merger timescale up to an order of magnitude comparing to the merger time for GW radiation alone.
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