On discrete symmetry for spin 1/2 and spin 1 particles in external monopole field and quantum-mechanical property of self-conjugacy

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Abstract

Particles of spin 1/2 and 1 in external Abelian monopole field are considered. \(P\)-inversion-like operators \(\hat{N}\), commuting with the respective Hamiltonians, are constructed: \(\hat{N}\)\textsubscript{bisp.} is diagonalized onto the relevant wave functions, whereas \(\hat{N}\)\textsubscript{vect.} does not. Such a paradox is rationalized through noting that both these operators are not self-conjugate. It is shown that any \(N\)-parity selection rules cannot be produced. Non-Abelian problems for doublets of spin 1/2 and 1 particles are considered; corresponding discrete operators are self-conjugate and selection rules are available.
1. Introduction

An investigation of the quantum mechanical particles in the external Dirac monopole’s field has been carried out by many authors (see, for example, in [1-6]). Particularly, a special interest was given to properties of these systems with respect to the operation of spatial $P$-inversion [7-12]. As known, in virtue of the monopole-based $P$-violation, the usual particle’s $P$-inversion operator $\hat{\Pi}_{\text{particle}} \otimes \hat{P}$ does not commute with the Hamiltonian $\hat{H}$. The way of how to obtain a certain formal covariance of the monopole-containing system with respect to $P$-symmetry there has been a subject of special interest in the literature.

For instance: (a) those possibilities were discussed ([9]) in the context of the generalized (allowing for the monopole presence) CPT theorem ($CPT \rightarrow CMPT \equiv CNT$); (b) in a number of works (for example, see references [13-17] it was claimed that this operator plays a role in hierarchy of the established selection rules with respect to the relevant generalized quantum number $j$.

All the suggestions represent, in the essence, a single one: the magnetic charge is to be considered as a pseudo scalar quantity. For the subject under consideration, this assumption implies that one ought to accompany the ordinary $P$-transformation with a formal operator $\hat{\pi}$ changing the parameter $g$ into $-g$. Correspondingly, the composite discrete operator $\hat{N} = \hat{\pi} \otimes \hat{\Pi}_{\text{particle}} \otimes \hat{P}$ will commute with the relevant Hamiltonian.

Analysis of certain aspects of that monopole $P$-asymmetry constitutes a basic goal of the present paper. Technical innovation of the exposition below is the use of wave equations in the frame of the tetrad formalism of Tetrode-Weyl-Fock-Ivanenko [18,19]. At this, the Dirac ($S = 1/2$) and Duffin-Kemmer ($S = 1$) equations are referred to a basis of diagonal spherical tetrad; correspondingly, we will use explicit forms of wave functions referring to the same tetrad basis (a monopole potential is taken in Schwinger’s form; we adhere designations used in [20,21]).

In Sec.2, several facts on properties of spin $S = 1/2$ particle’s wave functions affected by external monopole field are briefly remembered. Particularly, it is noted that there exists a discrete operator replacing the ordinary $P$-reflection: $\hat{N}_{\text{bisp.}} = \hat{\pi} \otimes \hat{\Pi}_{\text{bisp.}} \otimes \hat{P}$ which commutes with the Hamiltonian and can be diagonalized on the wave functions $\Psi_{\epsilon jm}^{eg \cdot S = 1/2}(x)$. In Sec.3 the case of $S = 1$ is considered; here also there is an operator $\hat{N}_{\text{vect.}}$: $\hat{N}_{\text{vect.}} = \hat{\pi} \otimes \hat{\Pi}_{\text{vect.}} \otimes \hat{P}$; but, in contrast to the $S = 1/2$ case, the $\hat{N}_{\text{vect.}}$ cannot be diagonalized on the functions $\Psi_{\epsilon jm}^{eg \cdot S = 1}(x)$. So, these two systems exhibit sharp distinction. In Sec.4, two questions are analyzed. The first one is the property of non-self-conjugacy for the discrete operators constructed for those $eg$-systems. The second is the non-existence of any $N$-parity selection rules, though the $\hat{N}_{\text{bisp.}}$ can be diagonalized on the relevant wave functions. As evidenced in Sec.4, this operator $\hat{N}_{\text{bisp.}}$ does not result in a basic structural condition

$$\Psi(t, -\vec{r}) = (4 \times 4 - \text{matrix}) \Psi(t, \vec{r})$$ (1a)

which would guarantee indeed the existence of certain selection rules with respect to

\footnote{One should take into account that this, as it is, applies only to the Schwinger basis; the use of the Dirac gauge or any other, except Wu-Yang’s, implies quite definite modifications in representation of the $P$-operation on the monopole 4-potential.}
the discrete quantum number. Instead, there arises only the following one:

\[ \Psi^{+\text{eg}}(t, -\vec{r}) = (4 \times 4 - \text{matrix}) \Psi^{-\text{eg}}(t, \vec{r}) \]  

(1b)

take notice of a change at \( \text{eg} \) parametre: this minor alteration is completely detrimental to the possibility of producing any selection rules. Else one added fact is emphasized: the radial system of equations at \( S = 1/2 \) case only depends on the modulus of the parametre \( \text{eg} \), whereas in the \( S = 1 \) case it depends on the sign of the \( \text{eg} \) too. Evidently, it may be thought as indication that the formal diagonalizing of the \( \hat{N}_{\text{bisp.}} \) (and non-diagonalizing of \( \hat{N}_{\text{vect.}} \)) correlates just with the latter circumstance.

Sec. 5 treats briefly some facts on discrete symmetry in the non-Abelian model: an isotopic doublet of Dirac fermions is discussed. Here, the relevant discrete operator (containing \( P \)-inversion) is self-conjugated, and correspondingly selection rules on a composite (isotopic-Lorentzian) parity are available. In Sec. 6, the case of isotopic doublet of vector particles in the external t’Hooft-Polyakov potential is considered. The account is given of how the discrete operator simplifies corresponding wave functions and how the system of radial equations fits well with limitations imposed on the functions by diagonalization of this operator. It may be noticed that just those mathematical relations which have supplied “bad” peculiarities in the Abelian theory have produced, in another background, “good” characteristics of the corresponding non-Abelian problems. So, the paper reveal the interplay between Abelian and non-Abelian models regarding their properties under discrete symmetry. Sec. 7 provides some more discussion on possible implications of monopole-based \( P \)-(a)symmetry.

2. \textit{eg}-system at \( S = 1/2 \)

The generally relativistic Dirac equation in the chosen basis has the form [20]

\[
\left[ i\gamma^0 \partial_t + i\gamma^3 \left( \partial_r + \frac{1}{r} \right) + \frac{1}{r} \Sigma^\lambda_{\theta,\phi} - \frac{mc}{\hbar} \right] \Psi(x) = 0
\]

(2a)

where

\[
\Sigma^\lambda_{\theta,\phi} = i\gamma^1 \partial_\theta + \gamma^2 \frac{i\partial_\phi + (ij^{12} - \lambda) \cos \theta}{\sin \theta}
\]

(2b)

and \( \lambda = \text{eg}/hc \). The wave function with quantum numbers \( \epsilon, \ j, \ m \) (See all details in [20]) is

\[
\Psi_{\epsilon jm}(t, r, \theta, \phi) = \frac{e^{-ict}}{r} \begin{pmatrix} f_1 D_{\lambda - 1/2} \\ f_2 D_{\lambda + 1/2} \\ f_3 D_{\lambda - 1/2} \\ f_4 D_{\lambda + 1/2} \end{pmatrix}
\]

(3)

the symbol \( D_\sigma \) denotes the Wigner functions: \( D_\sigma \equiv D^j_{m,\sigma}(\phi, \theta, 0) \). For \( \lambda \) and \( j \), only the following values are allowed:

\[
\lambda = \text{eg}/hc = \pm 1/2, \pm 1, \pm 3/2, \ldots \quad \text{and} \quad j = | \lambda | - 1/2, | \lambda | + 1/2, | \lambda | + 3/2, \ldots
\]

(4)
correspondingly the substitution (3) is valid only for \( j > j_{\text{min.}} = | \lambda | -1/2 \). The case of minimal allowable value \( j_{\text{min.}} = | k | -1/2 \) must be separated out and looked into in a special way. For example, let \( \lambda = \pm 1/2 \), then to the minimal value \( j = 0 \) there correspond the wave functions

\[
\Psi_{\lambda=+1/2}^{(j=0)}(x) = \frac{e^{-\mathrm{i}t}}{r} \begin{pmatrix} f_1(r) & \frac{v}{r} f_3(r) & 0 \end{pmatrix}, \quad \Psi_{\lambda=-1/2}^{(j=0)}(x) = \frac{e^{-\mathrm{i}t}}{r} \begin{pmatrix} 0 & f_2(r) & \frac{v}{r} f_4(r) \end{pmatrix}.
\] (5)

Thus, if \( \lambda = \pm 1/2 \), then to the minimal allowed values \( J_{\text{min}} \) there correspond the function substitutions which do not depend at all on the angular variables \((\theta, \phi)\); at this point there exists some formal analogy between these electron-monopole states and \( S \)-states (with \( l = 0 \)) for a boson field of spin zero: \( \Phi_{t=0} = \Phi(r, t) \). However, it would be unwise to attach too much significance to this formal coincidence because such \((\theta, \phi)\)-independence of \((e-g)\)-states is not a fact invariant under tetrad gauge transformations. In contrast, the relation \( \Sigma_{\theta, \phi}^{\pm 1/2} \Psi_{\lambda=\pm 1/2}^{(j=0)}(x) \equiv 0 \) is gauge invariant. Correspondingly, the matter equation above takes on the form

\[
\left[ i \gamma^0 \partial_t + i \gamma^3 \left( \partial_r + \frac{1}{r} \right) - mc/\hbar \right] \Psi^{(j=0)} = 0.
\] (6)

It is readily verified that both functions in (5) are directly extended to \((e-g)\)-states with \( j = j_{\text{min.}} \) at all the other \( \lambda = \pm 1, \pm 3/2, \ldots \). Indeed,

\[
\Psi_{j_{\text{min.}} > 0}^{(j=0)}(x) = \frac{e^{-\mathrm{i}t}}{r} \begin{pmatrix} f_1(r) D_{\lambda-1/2} & f_4(r) D_{\lambda-1/2} \end{pmatrix}, \quad \Psi_{j_{\text{min.}} < 0}^{(j=0)}(x) = \frac{e^{-\mathrm{i}t}}{r} \begin{pmatrix} 0 & f_2(r) D_{\lambda+1/2} & f_4(r) D_{\lambda+1/2} \end{pmatrix}.
\] (7)

and, as can be shown, the relation \( \Sigma_{\theta, \phi}^{\lambda} \Psi_{j_{\text{min.}}} \equiv 0 \) still holds.

After separating the variables, the radial system is \( (\nu = \sqrt{(j+1/2)^2 - \lambda^2}; \) for simplicity, here let us restrict ourselves to the non-minimal \( j \) states

\[
\begin{align*}
\epsilon f_3 - i \frac{d}{dr} f_3 - i \frac{\nu}{r} f_4 - m f_1 &= 0, & \epsilon f_4 + i \frac{d}{dr} f_4 + i \frac{\nu}{r} f_3 - m f_2 &= 0, \\
\epsilon f_1 + i \frac{d}{dr} f_1 + i \frac{\nu}{r} f_2 - m f_3 &= 0, & \epsilon f_2 - i \frac{d}{dr} f_2 - i \frac{\nu}{r} f_1 - m f_4 &= 0.
\end{align*}
\] (8)

As can be readily shown, on the functions (3) it is possible to diagonalize a discrete operator constructed on the base of the usual bispinor \( P \)-reflection. This \( P \)-reflection in the Cartesian tetrad basis is

\[
\hat{P}^{\text{Cart.}} = \hat{P}^{\text{Cart.}} \otimes \hat{P}, \quad \hat{P}^{\text{Cart.}} = \begin{pmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ i & 0 & 0 & 0 \\ 0 & i & 0 & 0 \end{pmatrix}, \quad \hat{P}(\theta, \phi) = (\pi - \theta, \phi + \pi).
\]
being subjected to translation into the spherical one, \( \hat{P}^{sph.} = S(\theta, \phi) \hat{P}^{Cart.} S^{-1}(\theta, \phi) \) gives us

\[
\hat{P}^{sph.} = \hat{\Pi}^{sph.}_{bisp.} \otimes \hat{P}, \quad \hat{\Pi}^{sph.}_{bisp.} = \begin{pmatrix}
0 & 0 & 0 & -1 \\
0 & 0 & -1 & 0 \\
0 & -1 & 0 & 0 \\
-1 & 0 & 0 & 0
\end{pmatrix}.
\]

A required operator is of the form \( \hat{N}^{bisp.} = \hat{\pi} \otimes \hat{\Pi}^{sph.}_{bisp.} \otimes \hat{P} \); here, \( \hat{\pi} \) is a special formal operation changing \( +eg \) into \( -eg \), and conversely: \( \hat{\pi} F(\lambda) = F(-\lambda) \). From the equation on proper values \( \hat{N}^{bisp.} \Psi_{\epsilon jm}^\lambda = N \Psi_{\epsilon jm}^\lambda \) it follows \( (\delta = \pm 1) \)

\[
N = \delta(-1)^{j+1} : \quad f_4 = \delta f_1, \quad f_3 = \delta f_2
\]

these limitations are compatible with the radial system (8). It should be emphasized that some unexpected peculiarities with that procedure, in reality, occur as we turn to the states of minimal values of \( j \). Actually, let \( \lambda = +1/2 \) or \(-1/2 \) \( (j = 0) \); then from the equation on proper values \( \hat{N} \Psi_{(j=0)} = N \Psi_{(j=0)} \) it follows

\[
\begin{pmatrix}
0 \\
-f_3 \\
0 \\
-f_4
\end{pmatrix} = N \begin{pmatrix}
f_1 \\
0 \\
f_3 \\
0
\end{pmatrix}, \quad \text{or} \quad \begin{pmatrix}
-f_4 \\
0 \\
-f_2 \\
0
\end{pmatrix} = N \begin{pmatrix}
0 \\
f_2 \\
0 \\
f_4
\end{pmatrix}.
\]

Evidently, they both have no solutions, excluding trivially null ones (and therefore being of no interest). Moreover, as may be easily seen, in both cases the function \( \Phi(x) \), defined by \( \hat{N} \Psi_{(j=0)} \equiv \Phi(x) \), lies outside a fixed totality of states that are only valid as allowed quantum states of the system under consideration. At greater values of this \( \lambda \), we come to analogous relations.

It should be useful to notice that the above simplification \( (\Psi_{\epsilon jm} \rightarrow \Psi_{\epsilon jm\delta}) \) can also be obtained through the diagonalization of the so-called generalized Dirac operator \( \hat{K}^{\lambda} \)

\[
\hat{K}^{\lambda} = -\gamma^0 \gamma^\lambda \Sigma_{\theta, \phi}^{\lambda}.
\]

Actually, from \( \hat{K}^{\lambda} \Psi_{\epsilon jm}(x) = K \Psi_{\epsilon jm} \) we produce \( (\delta = \pm 1) \)

\[
K = -\delta \left( j + 1/2 \right) : \quad f_4 = \delta f_1, \quad f_3 = \delta f_2.
\]

In turn, as regards the operator \( \hat{K}^{\lambda} \) for the \( j_{\text{min}} \) states we get \( \hat{K}^{\lambda} \Psi_{j_{\text{min}}} = 0 \); that is, this state represents the proper function of the \( \hat{K} \) with the null proper value. So, application of this \( \hat{K} \) instead of the \( \hat{N} \) has an advantage of avoiding the paradoxic and puzzling situation when \( \hat{N} \Psi_{j_{\text{min}}} \not\in \{ \Psi \} \). In a sense, this second alternative (the use of \( \hat{K}^{\lambda} \) instead of \( \hat{N} \) at separating the variables and constructing the complete set of mutually commuting operators) gives us a possibility not to attach great significance to the monopole discrete operator \( \hat{N} \) but to focus our attention solely on the operator \( \hat{K}^{\lambda} \).
3. *eg*-system at $S = 1$

The basic Duffin-Kemmer equation is [21]

$$\left[ i\beta^0 \partial_t + i (\beta^3 \partial_r + \frac{1}{r} (\beta^1 j^{31} + \beta^2 j^{32}) ) + \frac{1}{r} \Sigma^\lambda_{\theta,\phi} - \frac{mc}{\hbar} \right] \Phi(x) = 0 ; \quad (11)$$

$$\Sigma^\lambda_{\theta,\phi} = \left[ i\beta^1 \partial_\theta + \beta^2 i\partial_\phi + \left( ij^{12} - \lambda \right) \cos \theta \right] \sin \theta . \quad (12)$$

The wave functions with quantum numbers $(\epsilon,j,m)$ can be taken in the form

$$\Phi_{\epsilon jm}(x) = e^{-i\epsilon t} \left[ f_1(r) D_{\lambda} , f_2(r) D_{\lambda-1} , f_3(r) D_{\lambda} , f_4(r) D_{\lambda+1} , f_5(r) D_{\lambda-1} , \right.$$

$$\left. f_6(r) D_{\lambda} , f_7(r) D_{\lambda+1} , f_8(r) D_{\lambda-1} , f_9(r) D_{\lambda} , f_{10}(r) D_{\lambda+1} \right]$$

(13)

here, as above, $Da = D_{j-m,\sigma}(\phi,\theta,0)$. For quantities $\lambda$ and $j$, the values are allowed

1. \textit{if} $\lambda = \pm 1/2$, \textit{then} $j = |\lambda|, |\lambda| + 1, \ldots$

2. \textit{if} $\lambda = \pm 1, \pm 3/2, \ldots$, \textit{then} $j = |\lambda| - 1, |\lambda|, |\lambda| + 1, \ldots$

Correspondingly, the substitution (13) is applied only to the non-minimal $j$ values; for simplicity, let us consider just those states. After separation of variables we get

$$-(\frac{d}{dr} + \frac{2}{r}) f_6 - \sqrt{2}\frac{1}{r} \left( c f_5 + d f_7 \right) - mf_1 = 0 ,$$

$$i\epsilon f_5 + i(\frac{d}{dr} + \frac{1}{r}) f_8 + i\sqrt{2}\frac{c}{r} f_9 - mf_2 = 0 ,$$

$$i\epsilon f_6 + \frac{2i}{r} (-c f_8 + d f_{10}) - mf_3 = 0 ,$$

$$i\epsilon f_7 - i(\frac{d}{dr} + \frac{1}{r}) f_{10} - i\sqrt{2}\frac{d}{r} f_9 - mf_4 = 0 ,$$

$$i\epsilon f_2 + \sqrt{2}\frac{c}{r} f_1 - mf_5 = 0 ,$$

$$-i\epsilon f_3 - \frac{d}{dr} f_1 - mf_6 = 0 ,$$

$$i\epsilon f_4 + \sqrt{2}\frac{d}{r} f_1 - mf_7 = 0 ,$$

$$-i(\frac{d}{dr} + \frac{1}{r}) f_2 - i\sqrt{2}\frac{c}{r} f_3 - mf_8 = 0 ,$$

$$i\sqrt{2}\frac{1}{r} \left( c f_2 - d f_4 \right) - mf_9 = 0 ,$$

$$i(\frac{d}{dr} + \frac{1}{r}) f_4 + i\sqrt{2}\frac{d}{r} f_3 - mf_{10} = 0 \quad (14)$$
where \( c = \frac{1}{2} \sqrt{(j + \lambda)(j - \lambda + 1)} \), \( d = \frac{1}{2} \sqrt{(j - \lambda)(j + \lambda + 1)} \).

As in case of a fermion field above, here we try to use a generalized operator \( \hat{N}_{\text{vect.}} \), commuting with the wave operator in (11). The vector ordinary \( P \)-reflection in Cartesian tetrad, is

\[
\hat{P}^{\text{Cart.}} = \hat{N}_{\text{vect.}} \otimes \hat{P}, \quad \hat{N}_{\text{vect.}} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -I & 0 & 0 \\
0 & 0 & -I & 0 \\
0 & 0 & 0 & +I
\end{pmatrix}
\]

(15a)

where a symbol "I" denotes a unit 3\( \times \)3 matrix. After translating this \( \hat{P}^{\text{Cart.}} \) into the spherical tetrad's basis according to \( \hat{P}^{\text{sph.}} = O(\theta, \phi) \hat{P}^{\text{Cart.}} O^{-1}(\theta, \phi) \), where \( O(\theta, \phi) \) is a 10-dimension rotational matrix associated with taking the Cartesian gauge into the spherical one, it takes on the form (the standard cyclic basis in the vector space is used)

\[
\hat{P}^{\text{sph.}} = \hat{N}^{\text{sph.}} \otimes \hat{P}, \quad \hat{N}^{\text{sph.}} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & +E & 0 & 0 \\
0 & 0 & +E & 0 \\
0 & 0 & 0 & -E
\end{pmatrix}, \quad E \equiv \begin{pmatrix} 0 & 0 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \end{pmatrix}
\]

(15b)

A required operator (in the spherical basis) is

\[
\hat{N}^{\text{sph.}} = \hat{\pi} \otimes \hat{N}_{\text{vect.}} \otimes \hat{P}
\]

(15c)

From the equation \( \hat{N}_{\text{vect.}} \Psi_{\epsilon j m}^\lambda = N \Psi_{\epsilon j m}^\lambda \) we get

\[
N = (-1)^{j+1} : \quad f_1 = f_3 = f_6 = 0, \quad f_4 = -f_2, \quad f_7 = -f_5, \quad f_{10} = +f_8; \quad (16a)
\]

\[
N = (-1)^j : \quad f_9 = 0, \quad f_4 = +f_2, \quad f_7 = +f_5, \quad f_{10} = -f_8. \quad (16b)
\]

In contrast to the fermion case above, here the relations (16a,b) are readily shown not to be compatible with the radial system (14). However, as can be easily verified, this operator indeed commutes with the wave operator in (11). Thus, apparently there exists a contradiction. So different properties of particles with spin 1/2 and 1 in external monopole field, while one notes their complete origin similarity, seem to be rather surprising and puzzled.

4. \( N \)-operator and property of self-conjugacy

So, in both cases \( S = 1/2 \) and \( S = 1 \), the respective \( N \)-operators are constructed in accordance with the same pattern:

\[
\hat{N} = [ \hat{\pi} \otimes \hat{\Pi}_{\text{particle}} \otimes \hat{P} ], \quad [\hat{N}, \hat{H}^{\text{eg}}]_\pm = 0
\]

(17)

where \( \hat{\Pi}_{\text{particle}} = \hat{\Pi}_{\text{bisp.}} \) or \( \hat{\Pi}_{\text{vect.}} \), and \( \hat{H}^{\text{eg}} = \hat{H}^{\text{eg}}_{\text{bisp.}} \) or \( \hat{H}^{\text{eg}}_{\text{vect.}} \), respectively. However, as was just noted, there are some essential distinctions between these two situations, and this deserves special consideration. At a glance, the situation at \( S = 1 \) looks as very
contrasting with all generally accepted concepts of the conventional quantum mechanics. Indeed, the commutation relation required \([\hat{N}_{\text{vect.}}, \hat{H}_{\text{vect.}}] = 0\) holds, but this \(\hat{N}_{\text{vect.}}\) is not diagonalized onto \(H_{\text{vect.}}\)'s eigenfunctions \(\Phi_{ejm}\). As regards to \(S = 1/2\) situation, that (as would be seemed) entirely comes under the common and familiar requirements of quantum theory. However, on more closing consideration, it will be clear that, first, \(S = 1\) situation does not turn out to contradict the commonly acknowledged requirements of quantum mechanics; second, the \(S = 1/2\) situation does not provide us just else one trivial illustration to the familiar interrelation of the commutation rule \([\hat{A}, \hat{H}] = 0\) and the possibility to measure simultaneously those quantities \(\hat{A}\) and \(\hat{H}\).

All above, as a correcting and revealing remark, it must be stressed that the quantum mechanics, when dealing with some specific operator \(\hat{A}\), implies essentially its self-conjugacy property: \(<\Psi \mid \hat{A} \Phi> = <\hat{A} \Psi \mid \Phi>\). For example, the usual bispinor \(P\)-reflection presents evidently a self-conjugate one, since one has

\[
<\Psi(\vec{r}) \mid \gamma^0 \hat{P} \Phi(\vec{r})> = \int \bar{\Psi}^*(\vec{r}) \Phi(-\vec{r}) \, dV ,
\]

\[
<\gamma^0 \hat{P} \Psi(\vec{r}) \mid \Phi(\vec{r})> = \int \bar{\Psi}^*(-\vec{r}) \Phi(\vec{r}) \, dV .
\]

The \(\Psi\) with over symbol \(\sim\) denotes a transposed column-function, that is, a row-function; and the asterisk \(*\) designates the operation of complex conjugation. In the presence of the external monopole field, the whole situation is completely different from the above, namely, the \(\hat{N}\) used here does not possess the required self-conjugacy property. Indeed,

\[
<\psi^{+eg}(\vec{r}) \mid \hat{N} \Phi^{+eg}(\vec{r})> = \int (\bar{\Psi}^{+eg}(\vec{r}))^* \Phi^{-eg}(-\vec{r}) \, dV ,
\]

\[
<\hat{N} \psi^{eg}(\vec{r}) \mid \Phi^{+eg}(\vec{r})> = \int (\bar{\Psi}^{-eg}(\vec{r}))^* \Phi^{+eg}(-\vec{r}) \, dV .
\]

It is evident at a glance that right hand sides of these two equalities vary in sign at \(eg\) parametre; thereby it follows that the discrete operator \(\hat{N}\) does not possess the self-conjugacy property. As regards to such a property of \(\hat{N}\), the case of \(S = 1\) looks completely alike. This peculiarity of \(\hat{N}_{\text{bisp.}}\) and \(\hat{N}_{\text{vect.}}\) may be interpreted as follows: those \(\hat{N}\) do not afford any physical observables which could be measured by any physical apparatus. In other words, the features of \(S = 1\) case mentioned above do not go into contradiction with proper principles of the quantum theory. On the other hands, one could acknowledge oneself puzzled when only specializing to \(S = 1/2\) system. In the latter case, as it would seems, the familiar connection between commutation relations and measuring the \(\hat{N}\) is realized. But such a natural reference to this familiar arrangement is not valid here because of already mentioned arguments of non-self-conjugacy; and what is more, the existence of contrasting situations at \(S = 1/2\) and \(S = 1\) directly suggests that one must attach more significance to the latter (of non-self-conjugacy) requirement. In this connection, one must take notice of the manner in which the \(eg\) parametre enters the radial system for \(f_1, \ldots, f_4\): it occurs through \(\nu = \sqrt{(j + 1/2)^2 - \lambda^2}\). The latter leads
to independence on \( \lambda \)'s sign. Therefore, the two distinct systems with the characteristics \(+eg\) and \(-eg\) respectively have their radial systems exactly identical:

\[
F_{s=1/2}^{+eg}(f_1, \ldots, f_4) = F_{s=1/2}^{-eg}(f_1, \ldots, f_4).
\]  
(18)

In contrast to this, the \( S = 1 \) affords an essentially different case: here, the parametre \( eg \) enters the relevant radial system through \( c \) and \( d \), that is, two radial systems marked by \(+eg\) and \(-eg\) respectively, though can easily be inverted into each other by simple formal procedure, vary in their explicit form:

\[
F_{s=1}^{+eg}(f_1, \ldots, f_{10}) \neq F_{s=1}^{-eg}(f_1, \ldots, f_{10}).
\]  
(19)

As an illustration to manifestations of the non-self-conjugacy property of the \( N \)-operator, let us consider a question concerning \( P \)-parity selection rules in presence of the Hamiltonian:

\[
\hat{N} = \hat{\pi} \otimes \hat{\Pi}_{\text{bisp.}} \otimes \hat{P}, \quad \hat{N} \Psi_{\epsilon j m \mu}^g(\vec{r}) = \mu (-1)^{j+1} \Psi_{\epsilon j m \mu}^g(\vec{r})
\]  
(20)

(\( \hat{\pi} \Psi_{\epsilon j m \mu}^g(\vec{r}) = \Psi_{\epsilon j m \mu}^{-g}(\vec{r}) \)), but this does not allow us to obtain any \( N \)-parity selection rules. Let us consider this question in more detail. A matrix element for some physical observable \( \hat{G}_0(x) \) is to be

\[
\int \bar{\Psi}_{\epsilon j m \mu}^g(\vec{r}) \hat{G}_0(\vec{r}) \Psi_{\epsilon' j' m' \mu'}^g(\vec{r}) \, dV \equiv \int r^2 dr \int f(\vec{r}) \, d\Omega.
\]  
(21)

First we examine the case \( eg = 0 \), in order to compare it with the situation at \( eg \neq 0 \). Let us relate \( f(\vec{r}) \) with \( f(\vec{r}) \). Considering the equality (and the same with \( j' m' \delta' \))

\[
\Psi_{\epsilon j m b}^0(\vec{r}) = \hat{\Pi}_{\text{bisp.}} \delta (-1)^{j+1} \Psi_{\epsilon j m b}^0(\vec{r})
\]  
(22a)

we get

\[
f^0(\vec{r}) = \delta \delta' (-1)^{j+j'+1} \bar{\Psi}_{\epsilon j m b}^0(\vec{r}) \left[ \hat{\Pi}_{\text{bisp.}} \hat{G}_0(\vec{r}) \hat{\Pi}_{\text{bisp.}} \right] \Psi_{\epsilon' j' m' b'}^0(\vec{r}).
\]

If \( \hat{G}_0(\vec{r}) \) obeys the equation

\[
[ \hat{\Pi}_{\text{bisp.}} \hat{G}_0(\vec{r}) \hat{\Pi}_{\text{bisp.}} ] = \omega^0 \hat{G}_0^0(\vec{r})
\]  
(22b)

here \( \omega^0 \) defined to be \( +1 \) or \( -1 \) relates to the scalar and pseudo scalar, respectively, then \( f(\vec{r}) \) can be brought to \( f^0(\vec{r}) = \omega \delta \delta' (-1)^{j+j'+1} f^0(\vec{r}) \). The latter can generate the wellknown \( P \)-parity selection rules:

\[
\int \bar{\Psi}_{\epsilon j m \mu}^0(\vec{r}) \hat{G}_0^0(\vec{r}) \Psi_{\epsilon' j' m' \mu'}^0(\vec{r}) \, dV = \left[ 1 + \omega \delta \delta' (-1)^{j+j'+1} \right] \int r^2 dr \int_{1/2} f^0(\vec{r}) \, d\Omega
\]  
(22c)

where the \( \theta, \phi \)-integration is performed on a half-sphere. In contrast to everything just said, the situation at \( eg \neq 0 \) is completely different since any equality in the form (22a)
does not appear here. In other words, in virtue of the absence any correlation between \( f^{eg}(\vec{r}) \) and \( f^{eg}(-\vec{r}) \), there is no selection rules on discrete quantum number \( N \). In accordance with this, for instance, an expectation value for the usual operator of space coordinates \( \vec{x} \) need not equal zero and one follows this (see in [13-17]).

In the same time, from the above it follows that there exist quite definite correlations between \( \Psi^\pm_{eg}(\vec{r}) \) as well as between \( f^\pm_{eg}(\vec{r}) \) (supposedly, the relation (22b) still holds):

\[
\Psi^\pm_{eg}(-\vec{r}) = \delta(-1)^{j+1} \hat{N}_{bisp.} \Psi^\mp_{eg}(\vec{r}) \quad , \quad f^\pm_{eg}(-\vec{r}) = \omega \delta \delta' (-1)^{j+j'+1} f^\mp_{eg}(\vec{r}) .
\]

Those latter provide certain indications that in a non-Abelian (monopole-contained) model no problems with discrete \( P \)-inversion-like symmetry might occur. In confirmation to this let us consider some facts on particle-monopole systems in the non-Abelian situation.

5. Doublet of fermions

It can be shown that the wave functions for the doublet of Dirac particles in the external monopole (t’Hooft-Polyakov’s) potential can be constructed in the form (for more detailed see [22])

\[
\Psi_{ejm\delta}(t, r, \theta, \phi) = e^{-i\delta} \left[ T_{+1/2} \otimes \begin{pmatrix} f_1 D_{-1} \\ f_2 D_0 \\ f_3 D_{-1} \\ f_4 D_0 \end{pmatrix} + \delta \delta' T_{-1/2} \otimes \begin{pmatrix} f_4 D_0 \\ f_3 D_{+1} \\ f_2 D_0 \\ f_1 D_{+1} \end{pmatrix} \right] \tag{23}
\]

they represent eigenfunctions of operators \( \vec{J}^2, J_3, \hat{N} = \sigma_1 \otimes \hat{N}_{bisp.} \otimes \hat{P} \). Here, the discrete operator \( \hat{N} \) provides a self-conjugated quantity. In addition, the wave functions obey the condition \( \delta = \pm 1 \)

\[
N = \delta(-1)^{j+1} : \quad \Psi_{ejm\delta}(-\vec{r}) = \delta(-1)^{j+1}(\sigma_1 \otimes \hat{N}_{bisp.}) \Psi_{ejm\delta}(\vec{r})
\]

in virtue of that the corresponding selection rules are available\(^2\). In particular, these selection rules predict that the expectation value of the spatial coordinates will be equated to zero

\[
< \Psi_{ejm\delta} | \vec{r} | \Psi_{ejm\delta} > \sim \left[ 1 - \delta^2(-1)^{2j} \right] \equiv 0 . \tag{24a}
\]

That vanishing can be readily understood from the following expansions

\[
\Psi_{ejm\delta} = \left[ T_{+1/2} \otimes \Psi^+ + T_{-1/2} \otimes \Psi^- \right],
\]

\[
< \Psi_{ejm\delta} | \vec{r} | \Psi_{ejm\delta} >= \left[ < \Psi^+ | \vec{r} | \Psi^+ > + < \Psi^- | \vec{r} | \Psi^- > \right] \tag{24b}
\]

\(^2\)These questions and a number of other ones will be analyzed with much more details in a separate paper of the author.
and the fitting relationship
\[
\bar{\Psi}^\pm (-\vec{r}) (-\vec{r}) \Psi^\pm (-\vec{r}) = -\bar{\Psi}^\mp (\vec{r}) (\vec{r}) \bar{\Psi}^\mp (\vec{r}) .
\] (24c)

That is, the “bad” mathematical relations (24c) in the Abelian model turn out to be “good” ones in the non-Abelian theory background.

6. Doublet of vector particles

Now, let us consider briefly how the problem of discrete symmetry looks in the situation of the vector particles doublet in the 'tHooft-Polyakov potential. Here, the matter equation is (the spherical tetrad basis and the Shwinger unitary gauge in isotropic space are used)

\[
\left[ i \beta^0 \partial_t + i( \beta^3 \partial_r + \frac{1}{r} (\beta^1 J^{31} + \beta^2 J^{32}) + \frac{1}{r} \Sigma_{\theta,\phi} + \frac{er^2 K(r) + 1}{r} (t^2 \otimes \beta^1 - t^1 \otimes \beta^2) - \frac{mc}{\hbar} \right] \Phi(x) = 0
\] (25a)

where
\[
\Sigma_{\theta,\phi} = i \beta^1 \partial_\theta + \beta^2 \frac{i \partial_\phi + (ij^{12} + t^3) \cos \theta}{\sin \theta} .
\] (25b)

The function \( K(r) \) enters the non-Abelian monopole solution \( W_i^{(a)} = \epsilon_{iab} x^b K(r); t^i = \frac{1}{2} \sigma^i \).

The composite wave function is to be [22]

\[
\Phi_{\epsilon,jm} = e^{-it} \begin{pmatrix} T_{+1/2} \otimes \begin{pmatrix} f_1 D_{-1/2} \\ f_2 D_{-3/2} \\ f_3 D_{-1/2} \\ f_4 D_{+1/2} \\ f_5 D_{-3/2} \\ f_6 D_{-1/2} \\ f_7 D_{+1/2} \\ f_8 D_{-3/2} \\ f_9 D_{-1/2} \\ f_{10} D_{+1/2} \end{pmatrix} \\ + T_{-1/2} \otimes \begin{pmatrix} g_1 D_{+1/2} \\ g_2 D_{-1/2} \\ g_3 D_{+1/2} \\ g_4 D_{+3/2} \\ g_5 D_{-1/2} \\ g_6 D_{+1/2} \\ g_7 D_{+3/2} \\ g_8 D_{-1/2} \\ g_9 D_{+1/2} \\ g_{10} D_{+3/2} \end{pmatrix} \end{pmatrix}
\] (26)

where \( D_\sigma \equiv D_{\sigma,m,\sigma}(\phi, \theta, 0) \); the quantum number \( j \) takes values 1/2, 3/2, ... To separate the variables, actually new calculations (required in addition to the above Abelian case) concern only the term proportional to \([er^2 K(r) + 1]/r\) \(\equiv W \) (just it mixes up two isotopic components):

\[
(t^2 \otimes \beta^1 - t^1 \otimes \beta^2) \Phi_{\epsilon,jm} = e^{-it} \times
\]
After separation the variables, we produce the equations on twenty functions; we rearranged them in couple as convenient):

\[
\begin{pmatrix}
T_{-1/2} \otimes \begin{pmatrix}
-f_7 D_{1/2} \\
+i f_9 D_{-1/2} \\
+i f_{10} D_{+1/2} \\
0 \\
+f_1 D_{-1/2} \\
0 \\
0 \\
-i f_3 D_{-1/2} \\
-i f_4 D_{+1/2} \\
0
\end{pmatrix}
+ T_{+1/2} \otimes \begin{pmatrix}
-g_5 D_{-1/2} \\
0 \\
-i g_8 D_{-1/2} \\
-i g_9 D_{+1/2} \\
0 \\
0 \\
0 \\
+g_1 D_{+1/2} \\
0 \\
+ig_2 D_{-1/2} \\
+ig_3 D_{+1/2}
\end{pmatrix}
\end{pmatrix}.
\]

\[
-(\frac{d}{dr} + \frac{2}{r}) f_6 - \sqrt{2} \frac{1}{r} (c^+ f_5 + d^+ f_7) - m f_1 - W g_5 = 0,
\]

\[
-(\frac{d}{dr} + \frac{2}{r}) g_6 - \sqrt{2} \frac{1}{r} (c^- g_5 + d^- g_7) - m g_1 - W f_7 = 0;
\]

\[
i \epsilon f_5 + i(\frac{d}{dr} + \frac{1}{r}) f_8 + i\sqrt{2} \frac{c^+}{r} f_9 - m f_2 = 0,
\]

\[
i \epsilon g_7 - i(\frac{d}{dr} - \frac{1}{r}) f_{10} - i\sqrt{2} \frac{d^-}{r} g_9 - m g_4 = 0;
\]

\[
i \epsilon f_6 + \frac{2i}{r} (c^+ f_8 + d^+ f_{10}) - mf_3 - iW g_8 = 0,
\]

\[
i \epsilon g_6 + \frac{2i}{r} (c^- g_8 + d^- g_{10}) - mg_3 + iW f_{10} = 0;
\]

\[
i \epsilon f_7 - i(\frac{d}{dr} + \frac{1}{r}) f_{10} - i\sqrt{2} \frac{d^+}{r} f_9 - m f_4 - iW g_9 = 0,
\]

\[
i \epsilon g_5 + i(\frac{d}{dr} + \frac{1}{r}) g_8 + i\sqrt{2} \frac{c^-}{r} g_9 - m g_2 + iW f_0 = 0;
\]

\[
i \epsilon f_2 + \sqrt{2} \frac{c^+}{r} f_1 - m f_5 = 0,
\]

\[
i \epsilon g_4 + \sqrt{2} \frac{d^-}{r} g_1 - m g_7 = 0;
\]

\[
-\epsilon f_3 - \frac{d}{dr} f_1 - m f_6 = 0,
\]

\[
-\epsilon g_3 - \frac{d}{dr} g_1 - m g_6 = 0;
\]

\[
i \epsilon f_4 + \sqrt{2} \frac{d^+}{r} f_1 - m f_7 + W g_1 = 0,
\]

\[
i \epsilon g_2 + \sqrt{2} \frac{c^-}{r} g_1 - m g_5 + W f_1 = 0;
\]
\[-i \left( \frac{d}{dr} + \frac{1}{r} \right) f_2 - i\sqrt{2} \frac{c^+}{r} f_3 - m f_8 = 0 ,
+ i \left( \frac{d}{dr} + \frac{1}{r} \right) g_4 + i\sqrt{2} \frac{d^-}{r} g_3 - m g_{10} = 0 ;
\]
\[i\sqrt{2} \frac{1}{r} \left( c^+ f_2 - d^+ f_4 \right) - m f_9 + iW g_2 = 0 ,
\]
\[i\sqrt{2} \frac{1}{r} \left( c^- g_2 - d^- g_4 \right) - g_9 + iW f_4 = 0 ;
\]
\[i \left( \frac{d}{dr} + \frac{1}{r} \right) f_4 + i\sqrt{2} \frac{d^+}{r} f_3 - m f_{10} + iW g_3 = 0 ,
- i \left( \frac{d}{dr} + \frac{1}{r} \right) g_2 - i\sqrt{2} \frac{c^-}{r} g_3 - m g_8 - iW f_3 = 0 . \quad (27)\]

where (see Sec. 3) \(c = \frac{1}{2}\sqrt{/(j+\lambda)(j-\lambda+1)}\), \(d = \frac{1}{2}\sqrt{/(j-\lambda)(j+\lambda+1)}\) and the signs + (plus) and − (minus) relate to the \(\lambda = -1/2\) and \(\lambda = +1/2\) respectively. It is easily verified that the composite discrete operator \(\hat{N} = (\sigma_1 \otimes \hat{\Pi}_{\text{vect}} \otimes \hat{P})\) commutes with the wave operator in (25a). Further, from the equation on proper values \(\hat{N} \Phi_{\epsilon jm} = N \Phi_{\epsilon jm}\) it follows

\[N = \delta (-1)^{j+1} : \quad g_1 = \delta f_1 , \quad g_2 = \delta f_4 , \quad g_3 = \delta f_3 , \quad g_4 = \delta f_2 ,
\]
\[g_5 = \delta f_7 , \quad g_6 = \delta f_6 , \quad g_7 = \delta f_5 , \quad g_8 = -\delta f_{10} , \quad g_9 = -\delta f_9 , \quad g_{10} = -\delta f_8 . \quad (28)\]

Finally, it is readily verified that those limitations (28) are consistent with the above system (27); so we get 10 equations (one ought to take into account the relation \(c^\pm = d^\mp\))

\[-(\frac{d}{dr} + \frac{2}{r}) f_6 - \sqrt{2} \frac{1}{r} \left( c^+ f_5 + d^+ f_7 \right) - m f_1 - \delta W f_7 = 0 ,
\]
\[i \epsilon f_5 + i(\frac{d}{dr} + \frac{1}{r}) f_8 + i\sqrt{2} \frac{c^+}{r} f_9 - m f_2 = 0 ,
\]
\[i \epsilon f_6 + \frac{2i}{r} \left( -c^+ f_8 + d^+ f_{10} \right) - m f_3 - i\delta W f_{10} = 0 ,
\]
\[i \epsilon f_7 - i(\frac{d}{dr} + \frac{1}{r}) f_{10} - i\sqrt{2} \frac{d^+}{r} f_9 - m f_4 + i\delta W f_9 = 0 ,
\]
\[i \epsilon g_5 + i(\frac{d}{dr} + \frac{1}{r}) g_8 + i\sqrt{2} \frac{c^-}{r} g_9 - m g_2 + iW f_9 = 0 ,
\]
\[i \epsilon f_2 + \sqrt{2} \frac{c^+}{r} f_1 - m f_5 = 0 ,
\]
\[-i \epsilon f_3 - \frac{d}{dr} f_1 - m f_6 = 0 ,
\]
\[i \epsilon f_4 + \sqrt{2} \frac{d^+}{r} f_1 - m f_7 + \delta W f_1 = 0 ,
\]

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\[ -i \left( \frac{d}{dr} + \frac{1}{r} \right) f_2 - i\sqrt{2} \frac{e^+}{r} f_3 - m f_8 = 0, \]
\[ i\sqrt{2} \frac{1}{r} (c^+ f_2 - d^+ f_4) - m f_9 + i\delta W f_4 = 0, \]
\[ i \left( \frac{d}{dr} + \frac{1}{r} \right) f_4 + i\sqrt{2} \frac{d^+}{r} f_3 - m f_{10} + i\delta W f_3 = 0. \] (29)

It is no difficulty to see that this discrete operator is self-conjugated one, and the relevant selection rules on the composite \(N\)-parity are quite available.

### 7. Discussion

In author’s opinion, analysis of all unusual selection rules with respect to the quantum number of the generalised momentum \(j\) on the monopole background (studied in the literature), which certainly exhibit definite traces and accompanying features of the monopole-based \(P\)-violation, accomplishes almost nothing about quite symmetrical character of that \(P\)-violation:

\[ \Psi^{\pm eg}(\vec{r}) = \text{Matrix} \ \Psi^{\mp eg}(\vec{r}). \] (30)

Instead, those selection rules rather agree passively with the absence of \(P\)-symmetry in presence of the Abelian monopole. In that context, the task was to clarify the all significance and implications of the relation (30) and also to find the points where it will play a part (really substantial in the sense of its experimental and theoretical manifestations).

The present study has shown that the general outlook on this matter which prescribes to consider a magnetic charge as pseudo-scalar under \(P\)-reflection seem hardly effective one as we turn to the most reliable matter — relevant selection rules. In author’s opinion, the assertion that the magnetic charge \(g\) is a pseudo-scalar provides rather accidental (though reasonable at first glance) interpretation of the information carried by the relation (30).

In that context, the task was to clarify the all significance and implications of the relation (30) and also to find the points where it will play a part (really substantial in the sense of its experimental and theoretical manifestations).

In any case, the non-existence of the relevant selection rules needs to be understood and rationalised in term of firmly established and reliable principles. In that sense, the main suggestion of the paper — to formulate some weak points of this (pseudo scalar) line of arguments in terms of the property of non-self-conjugacy seemingly supplies a firm mathematical base for their discussion. Because of that non-self-conjugacy, the pseudo scalar nature of the magnetic charge should be used in theoretical constructions with extreme caution so as not to lead us to quite speculative results.

The analysis above also has shown a contrasting relationship between Abelian and non-Abelian models regarding the monopole \(P\)-(a)symmetry. It may be noticed that just those mathematical relations which supply “bad” peculiarities in the Abelian theory produce, under other circumstanes, “good” characteristics of the corresponding non-Abelian problems.

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References

[1] Tamm I.E. // Zeit. Phys. Bd. 71. 1931. P. 141.
[2] Harish-Chandra // Phys. Rev. 1948. V. 74. P. 883.
[3] Goldhaber A.S. // Phys. Rev. 1965. V. B140. P. 1407.
[4] Kazama Y., Yang C.N., Goldhaber A.S. // Phys. Rev. * 1977. V. D15. P. 2287.
[5] Wu T.T., Yang C.N. // Nucl. Phys. 1976. V. B107. P. 365.
[6] Frenkel A., Hraskò P. // Ann. Phys.(NY). 1977. V.105. P. 288.
[7] Ramsey N.F.// Phys. Rev. 1958. V. 109. P. 225.
[8] Tomil’chik L.M. // ZhETF. 1963. V. 44. P. 160 (in Russian).
[9] Zwanziger D. // Phys. Rev. 1972. V. D6. P. 458.
[10] Tomil’chik L.M. // Phys. Lett. 1976. V. B61. P. 50.
[11] Goddard P., Olive D. // Phys. Rep. 1978. V. 44.P. 1358.
[12] Tolkachev E.A., Tomil’chik L.M. // Phys. Lett. 1979. V. B81. P. 173.
[13] Tolkachev E.A., Tomil’chik L.M., Shnir Ya.M. // Yad. Phys. 1983. V. 38. P. 541 (in Russian).
[14] Tolkachev E.A., Tomil’chik L.M., Shnir Ya.M. // Yad. Phys. 1989. V. 50. P. 442 (in Russian).
[15] Barut A.O., Shnir Ya.M., Tolkachev E.A. // J. Phys. A.: Math. and Gen. 1993. V. 26. L101.
[16] Savinkov A.G., Shapiro I.S. // ZhETF Pisma. 1988. V. 47. P. 292 (in Russian).
[17] Frampton P.H., Jian-Zu Z., Yong-Chang G. // Phys. Rev. 1989. V. D40. P. 3533.
[18] Sokolov A., Ivanenko D. Quantum field theory. Moskow. GITTL. 1952 (in Russian).
[19] Mitskevich M.V. Physical fields in general relativity. Moskow. Nauka. 1969 (in Russian).
[20] Red’kov V.M. Generally relativistical Tetrode-Weyl-Fock-Ivanenko formalism and behaviour of quantum-mechanical particles of spin 1/2 in the Abelian monopole field. 25 pages; quant-ph/9812002
[21] Red’kov V.M. Generally relativistical Daffin-Kemmer formalism and behaviour of quantum-mechanical particle of spin 1 in the Abelian monopole field. 17 pages; quant-ph/9812007

[22] Red’kov V.M. The Dirac fermions in the field of the non-Abelian monopole and discrete symmetry. in Nonlinear Phenomena in Complex Systems. Minsk, 1998.