Polarization–orbital angular momentum duality assisted entanglement observation for indistinguishable photons

Nijil Lal1 · Sarika Mishra1,2 · Anju Rani1,2 · Anindya Banerji3 · Chithrabhanu Perumangatt3 · R. P. Singh1

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Abstract
Duality in the entanglement of identical particles manifests that entanglement in only one variable can be revealed at a time. We demonstrate this using polarization and orbital angular momentum (OAM) variables of indistinguishable photons generated from parametric down-conversion. We show polarization entanglement by sorting photons in even and odd OAM basis, while sorting them in two orthogonal polarization modes reveals the OAM entanglement. Indistinguishable photons entangled in two variables could find applications in robust quantum communication, remote entanglement generation and distributed quantum sensing.

Keywords Entanglement · Quantum cryptography · Identical particles · Photon indistinguishability

1 Introduction

Complementarity is a unique manifestation of quantum mechanics, like entanglement. Introduced as a concept by Niels Bohr [1] and developed through further rigorous scientific discussion, complementarity broadly states that objects possess mutually exclusive properties such that the full knowledge of one property precludes full knowledge of the conjugate one. The wave–particle duality [2] and Heisenberg’s uncertainty principle [3] are closely associated with the complementarity principle.
Welcher Weg experiments demonstrate the complementarity between distinguishability and interference visibility, implying that quantum interference will take place only if the measurement does not distinguish between the interfering pathways [4–7]. In other words, indistinguishability leads to quantum interference. It is interesting to note that interference is related to coherence, which is a wave property, while distinguishability is associated with localized variables, which is a particle-like property [8, 9]. Indistinguishability of photons evokes great interest in many quantum information protocols [10–14]. In fact, many studies have aimed toward reducing the distinguishability in entangled systems [15–17] to achieve higher visibility of quantum interference.

Indistinguishability requires perfect overlap in spatiotemporal position, energy, polarization, etc. which can be demonstrated through two-photon interference experiments. Indistinguishable photons could be generated in the output of a Hong–Ou–Mandel interferometer [18]. A source of highly indistinguishable and entangled photon pairs is crucial to various quantum information applications [19–21]. However, it is not possible to observe the entanglement between such indistinguishable photons unless we sort and separate them in terms of a physical variable, such as their position, momentum, polarization and orbital angular momentum (OAM). For a general case of degenerate, non-collinear type II spontaneous parametric down-conversion (SPDC) output, we can write the joint state of the two identical photons (idler \((i)\) and signal \((s)\)) in terms of their different degrees of freedom as,

\[
|\Psi\rangle = \frac{1}{\sqrt{2}} (|H, k_i\rangle|V, k_s\rangle + |V, k_i\rangle|H, k_s\rangle)
\]

(1)

where the state is written in terms of polarization \((H, V)\) and linear momentum \((k_i, k_s)\). This can also be written as a polarization entangled state,

\[
|\Psi\rangle = \frac{1}{\sqrt{2}} (|H\rangle_{k_i}|V\rangle_{k_s} + |V\rangle_{k_i}|H\rangle_{k_s})
\]

(2)

using their linear momentum as a label to differentiate the subsystems. It is also possible to express the same state as,

\[
|\Psi\rangle = \frac{1}{\sqrt{2}} (|k_i\rangle_H|k_s\rangle_V + |k_s\rangle_H|k_i\rangle_V)
\]

(3)

which is entangled in linear momentum and the individual subsystems are labeled by their polarization. This complementary behavior of two independent degrees of freedom of identical particles is called entanglement duality. The introduction of duality in entanglement [22] as an implication of quantum indistinguishability was followed by experiments exploring the dualism between momentum and polarization [23] and path and polarization [24], schemes for entanglement sorting between OAM and polarization [25] as well as studies exploiting this property in a variety of systems [10, 26, 27]. To demonstrate entanglement duality, we use the even–odd basis of orbital angular momentum, along with polarization. In addition to polarization and OAM being two widely explored candidates of photon degrees of freedom in quantum information
processing, the choice of an even–odd basis for OAM allows utilizing all available photons generated in SPDC without having to project them into a reduced basis.

2 Methodology

Consider a type II SPDC process where the generated photon pairs are independently entangled in polarization and OAM, following the birefringence properties of the crystal and OAM conservation ($l_{\text{pump}} = l_1 + l_2$) [28–30]. The corresponding state can be expressed as,

$$|\Psi\rangle = \frac{1}{2} (|H, l_1, k_i\rangle |V, l_2, k_s\rangle + |H, l_2, k_i\rangle |V, l_1, k_s\rangle + |V, l_1, k_i\rangle |H, l_2, k_s\rangle + |V, l_2, k_i\rangle |H, l_1, k_s\rangle). \quad (4)$$

For a collinear output, the linear momentum labeling given in Eq. (4) becomes unavailable since $k_i = k_s = k$ and it becomes impossible to observe the entanglement. However, one can write,

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|H\rangle_{l_1} |V\rangle_{l_2} + |V\rangle_{l_1} |H\rangle_{l_2}) \otimes |k\rangle = \frac{1}{\sqrt{2}} (|l_1\rangle_H |l_2\rangle_V + |l_2\rangle_H |l_1\rangle_V) \otimes |k\rangle. \quad (5)$$

In most protocols involving the entanglement of orbital angular momentum of photons, the infinite-dimensional OAM spectrum in the output of SPDC is restricted to a two-dimensional basis by the post-selection of the twin photons. Due to this post-selection, a large amount of generated photons which belong to the other states in the infinite-dimensional OAM basis is lost. A method to avoid this loss is to use an alternate basis defined by the even ($E$) and odd ($O$) states of OAM [31, 32].

In this work, we propose that the OAM of twisted photons defined in their even–odd basis can be used to separate the otherwise completely indistinguishable photons in the collinear output. For a pump beam carrying an odd OAM value, the SPDC photons will be generated in pairs of even and odd OAM states, following the conservation of OAM. For pump OAM, $l_p = 1$, in a collinear type II SPDC process where the idler-signal pairs are generated in orthogonal polarization states, the output OAM state can be written as,

$$|\Psi\rangle_{\text{SPDC}} = \sum_{m=-\infty}^{+\infty} c_{m,1-m} |m\rangle_H |1-m\rangle_V$$

$$= c_{0,1} |0\rangle_H |1\rangle_V + c_{1,0} |1\rangle_H |0\rangle_V$$

$$+ c_{2,-1} |2\rangle_H (-1)\rangle_V + c_{-1,2} (-1)\rangle_H |2\rangle_V + ...$$

$$= \frac{1}{\sqrt{2}} (|E\rangle_H |O\rangle_V + |O\rangle_H |E\rangle_V). \quad (6)$$
Here, the twin photons are indistinguishable in every other degree of freedom, including their spatial position, except for their polarization and OAM. However, it is not possible to make individual measurements on these photons unless we separate them under some label. We use their polarization state to label these individual photons and observe the entanglement in the even–odd basis of the OAM. In the same way, the state in Eq. (6) can be written by labeling them as even and odd OAM states,

\[
|\Psi_{\text{SPDC}}\rangle = \frac{1}{\sqrt{2}} (|H\rangle_E |V\rangle_O + |V\rangle_E |H\rangle_O).
\] (7)

Hence, depending upon whether the polarization or OAM has been used for the labeling, we can observe the entanglement in the other degree of freedom. It can be seen that the distinguishability of the associated particles reveals the entanglement. Experimentally, this means that the method that we use to distinguish signal and idler photons dictates the degree of freedom in which the photons are entangled.

Sorting of photons based on the polarization can be achieved using a simple polarizing beam splitter which separates H and V, revealing the entanglement in even–odd OAM states. To observe the polarization entanglement, we use an even–odd OAM sorter. The conventional and commonly used method for selecting and measuring the OAM component of photons is the phase-flattening technique. The transverse phase profile corresponding to a desired OAM state is flattened using spiral phase plates or holograms displayed on spatial light modulators. The resulting fundamental Gaussian mode is then coupled to a single-mode fiber for measurement. However, such projective measurements based on phase flattening have shortcomings in terms of efficiency and dependence on pump characteristics [33]. Moreover, phase flattening projects the photon into one of the OAM states and hence cannot be used in even–odd sorting.

Using the two-dimensional even–odd basis for the twin-photon OAM states, the efficiency of entanglement protocols could be increased. Figure 1a illustrates a basic setup for an even–odd OAM sorter that involves a Mach–Zehnder interferometer with a Dove prism in each arm [34, 35]. A Dove prism flips the beam along one transverse direction and leaves it unchanged along the other during the internal reflection. Thus, a
beam carrying OAM $+l$ changes to $-l$ upon exiting the Dove prism [36]. In addition, upon rotation of the Dove prism, the output image rotates twice the angular rotation of the prism. When two Dove prisms are kept in the two arms of an interferometer, it introduces an OAM dependent relative phase $2l\alpha$ where $\alpha$ is the relative rotation of the Dove prisms. The two Dove prisms are oriented perpendicular to one another, and hence, $\alpha = \pi/2$. This introduces a phase $l\pi$ between the two arms of the interferometer. The relative phase difference would turn out to be odd multiples of $\pi$ for all odd OAM orders and even multiples of $\pi$ for even OAM orders. As a result, the constructive interference will take place in different output ports for even and odd OAM values. To overcome the stability concerns, the Mach–Zehnder arrangement in Fig. 1a could have been reconfigured as a more robust polarizing Sagnac interferometer [37, 38]. However, a polarizing interferometer destroys the indistinguishability in polarization. In this work, we adopt a folded Mach–Zehnder arrangement (Fig. 1b) to set up a robust interferometer without affecting the indistinguishability as well as the duality. A double Mach–Zehnder-type interferometer could be understood as a normal Mach–Zehnder interferometer, folded back such that the input and output beam splitters become the same. Such a configuration will have the stability of a common path interferometer, since both the arms see same optical components, and the ease of inserting independent components in the interfering arms as in a Mach–Zehnder interferometer.

We take a type II periodically poled potassium titanyl phosphate (ppKTP) crystal to observe the duality in entanglement of polarization and OAM of twin photons generated in SPDC. In type II SPDC, the idler and signal photons will have perpendicular polarizations along ordinary and extraordinary axes of the crystal. The output polarization state can be written as,

$$|\Psi\rangle = \frac{1}{\sqrt{2}} |H\rangle_i |V\rangle_s \pm |V\rangle_i |H\rangle_s. \quad (8)$$

The characteristics of non-collinear down-converted photon pairs are discussed in Appendix. The non-collinear photon pairs are labeled by their spatial positions and hence distinguishable. A collinear output can instead give indistinguishable photon pairs since they are not separated in spatial modes.
For a periodically poled crystal, the phase matching will be governed by the temperature of the crystal \cite{39,40}, and hence, the crystal is mounted on a temperature controlling oven. By varying the temperature, one can achieve collinear phase matching condition as shown in Fig. 2. For our crystal, the collinear output is obtained at 40 °C.

### 2.1 Observation of polarization entanglement through OAM sorting

The experimental schematic is given in Fig. 3. We use a Toptica TopMode (405 nm, 1 mW) laser as the pump which is loosely focused within the crystal using the lens, $L_1$ ($f = 50$ cm) such that the paraxial approximation for OAM conservation is valid. A band-pass filter (BF, 810 ± 5 nm) blocks the residual pump while transmitting the down-converted output. With this spectral filtering, we limit our source to be giving degenerate photon pairs in the setups discussed in this work. The down-converted photons are collimated using lens, $L_2$ ($f = 10$ cm). As shown in Eq. 6, the pump should carry an odd OAM in order to generate even–odd OAM pairs in the SPDC. For this, a spiral phase plate (SPP) corresponding to a topological charge, 1, is introduced in the pump beam. The non-collinear SPDC source (given in Appendix as Fig. 8) is modified into a collinear source by adjusting the temperature of the crystal-mounting oven as shown in Fig. 2 in order to introduce indistinguishability in terms of momentum vectors.

We use a double Mach–Zehnder OAM sorting interferometer to separate the photons on the basis of their OAM and reveal the entanglement in polarization. The collinear correlated pairs of photons having even and odd OAM orders are sent to a double Mach–Zehnder interferometer containing two Dove prisms which are kept in the individual paths as given in Fig. 3. The Dove prisms are kept such that their relative orientation is perpendicular to each other.

The individual photons undergo single-photon interference within the interferometric sorter. Photons carrying an odd OAM will constructively interfere in the odd port ($O$), whereas photons having even OAM will show up in the even port ($E$). Photons bearing odd or even OAM coming through the respective output ports of the sorter are then coupled to multi mode fibers (MMF) through fiber coupling systems, FC (Thorlabs CFC-5X-B), consisting of a fiber launcher and an aspheric lens (L$_{FC}$, $f = 4.6$ mm). These photons are detected at single-photon counting modules, SPCM (Excelitas AQRH-16-FC, efficiency = 0.6 at 810 nm, dark counts $\sim 25$ cps), whose output is given to a coincidence counter, CC (ID Quantique ID800 TDC), to obtain the coincidence counts. The polarization projections are done using the combination of a half-wave plate and a polarizer kept in each output port. At first, an alignment laser beam (810 nm, Thorlabs) is used to verify the sorting of even and odd OAM modes. The collinear output is then sent along the same path. The two-photon state after the sorter is,

$$|\Psi\rangle_{SPDC} = \frac{1}{\sqrt{2}}(|H\rangle_E|V\rangle_O + |V\rangle_E|H\rangle_O).$$
Fig. 3  Schematic to sort the even–odd states of OAM from a collinear SPDC with pump carrying OAM \( (l_p = 1) \). The Dove prisms within the double Mach–Zehnder interferometer is kept orthogonal to each other. Half-wave plate (HWP) along with polarizer (P) corresponds to the polarization projectors. \( O \) refers to the constructive port for odd OAM and \( E \) labels the constructive port for even OAM.

Coincidences are maximized in the detectors kept in ports \( E \) and \( O \), and polarization projection measurements are taken to observe the entanglement visibility.

### 2.2 Observation of OAM entanglement through polarization sorting

The entanglement in the even–odd basis of orbital angular momentum can be observed by separating the indistinguishable photons with their polarization as a label.

A polarizing beam splitter (PBS) separates the \( H \) and \( V \) polarized photons in the transmitted and reflected ports, respectively, as given in Fig. 4. The state after the polarizing beam splitter is given as,

\[
|\Psi_{\text{SPDC}}\rangle = \frac{1}{\sqrt{2}} (|E\rangle_H |O\rangle_V + |O\rangle_H |E\rangle_V).
\]

The entanglement in the even–odd basis can be characterized by making projective OAM measurements in the two output ports of the PBS. While a desired measurement setting for practical applications would demand that the OAM projections in the even–odd basis are carried out using interferometric sorters [31], we have performed demonstrative measurements using spatial light modulators (SLMs). SLMs
are widely used for the generation as well as measurement of light carrying orbital angular momentum. The computer-generated phase profiles are duplicated as voltage gradients along the pixel electrodes in the CMOS chip on the SLM head. This alters the refractive index of the liquid crystals sandwiched in between the electrodes and allows manipulation of the phase of light falling on each pixel. We carry out OAM projections by incidenting incoming photons on an SLM (Hamamatsu LCOS-SLM) and then coupling the phase-flattened photons to single-mode fibers. Additional lenses are used to image the crystal plane to the SLM, as well as the SLM plane to the coupling fiber tip for efficient mode projection and coupling. The combination of two lenses, $L_2$ and $L_3$ in each arm, images the modes generated in the crystal plane onto the SLM planes. The lens after the SLM, $L_4$ along with the aspheric lens within the fiber coupler, $L_{FC}$, images the modes generated after phase flattening at the SLM onto the fiber tip. The lenses are chosen such that the spatial mode sizes match the mode field diameter of the fiber coupling system. OAM projections in the even–odd basis are carried out with the help of identical spatial light modulators kept in the two arms. The holograms corresponding to superpositions of even and odd OAM orders act as the counterparts of $D$ and $A$ projections in the $HV$ basis.

### 3 Results and discussion

While passing through the OAM sorting setup given in Fig. 3, the down-converted photons alternately choose between the even and odd ports depending upon their OAM value. Before making polarization entanglement measurements on these photons, the action of even–odd sorting within our setup needs to be verified. The OAM state of the photons in the output ports of the sorter is measured using the standard technique involving phase flattening through SLM and coupling to single-mode fibers. The sin-
Fig. 5 Verification of sorting of even and odd OAM states in the folded Mach–Zehnder sorter. The top two rows correspond to the singles output in the even and odd ports when pumped with a Gaussian ($l_p = 0$) and the bottom two rows correspond to that for a pump carrying $l_p = 1$ OAM. The gray scale is normalized with respect to the maximum counts which is obtained in the even port as $l = 0$ mode for a Gaussian pump beam (white cell in top row).

The values are normalized with respect to the number photons in the Gaussian mode when pumped with a Gaussian beam, being the largest among all. When pumped with a Gaussian beam, photons are down-converted in pairs of odd–odd or even–even pairs, following the conservation of OAM, and thus, the photon pairs end up in the same port. It can be easily seen from the chart that photons carrying even and odd OAM values line up in the corresponding ports and their intensities are defined by the OAM spectrum of the SPDC output. For a pump carrying OAM, $l_p = 1$, the pairs are generated in even–odd pairs and they go to different ports. This is evident from how the corresponding intensity values are distributed between the two ports. For example, 0 in even port and 1 in odd port show similar intensity since they are generated together and so on. Moreover, the stark complementary behavior in the intensity corresponding to even and odd ports for a particular OAM value shows the effective sorting in the setup.

Figure 6 shows the polarization correlations between the even and odd output ports of the sorter. The indistinguishable photons are efficiently sorted under the label of their orbital angular momentum and polarization correlations are observed in both $HV$ and $DA$ basis. The normalized coincidences are plotted by varying the polarization
Fig. 6  Polarization correlations corresponding to projections in the output ports of the even–odd sorted collinear SPDC output. Visibility curves are plotted for odd port polarization projection angles, $\theta_1 = 0$ (H—green dot), $\theta_1 = \pi/4$ (D—red dot), $\theta_1 = \pi/2$ (V—blue dot) and $\theta_1 = 3\pi/4$ (A—purple dot). $\theta_2$ corresponds to even port projections. Solid curves are respective cosine fits. Error bars indicate statistical uncertainty of one standard deviation (Color figure online).

Projection angle set in the even port of the sorter, $\theta_2$, for fixed values of odd port projection angles, $\theta_1 = 0$ (H—green dot), $\theta_1 = \pi/4$ (D—red dot), $\theta_1 = \pi/2$ (V—blue dot) and $\theta_1 = 3\pi/4$ (A—purple dot). Here, $\theta_1$ and $\theta_2$ are twice the angle of rotation of HWP$_1$ and HWP$_2$, respectively, from Fig. 3. Solid curves represent cosine fit for the individual data set. The singles are $\sim 33000$ cps and maximum coincidences are $\sim 1300$ cps.

The observed visibilities are $V_1 = 77.5 \pm 0.3\%$ (HV basis) and $V_2 = 71.6 \pm 0.3\%$ (DA basis). The mentioned visibilities are the average values of H and V (D and A) plots. The Bell parameter is determined from the measured visibilities as per the relation, $S = 2\sqrt{2}(\frac{V_1+V_2}{2})$. Bell-CHSH inequality is violated if visibility of the coincidence plots, $V \geq 0.707$. The Bell parameter is estimated to be, $S = 2.11 \pm 0.03$. It can be seen in the plot that the minima corresponding to different visibility profiles are not going completely to zero. This could be attributed to the possible leakage of even OAM modes into the odd port (and vice versa). The reduced visibility can be understood as a manifestation of the imperfections in the sorting interferometer. Hence, with an improved interferometric sorter, it is possible to obtain near unity visibility.
The OAM correlations between the $H$ and $V$ output ports of the polarizing beam splitter (Fig. 4) in the even–odd basis of OAM are given in Fig. 7. The indistinguishable photons are efficiently sorted under the label of their polarization, and OAM visibility is observed in both $EO$ and $DEO\ AEO$ basis. The normalized coincidences are plotted against the angle of OAM projection in the $H$ port of the PBS, $\theta_2$, corresponding to fixed values of $V$ port projection angle, $\theta_1 = 0\ (E—green\ dot)$, $\theta_1 = \pi/4\ (DEO—red\ dot)$, $\theta_1 = \pi/2\ (O—blue\ dot)$ and $\theta_1 = 3\pi/4\ (AEO—purple\ dot)$. The cosine fits for individual data sets are given by the solid curves.

Here, $\theta_1$ and $\theta_2$ are the angle of rotation of holograms in SLM$_1$ and SLM$_2$, respectively, from Fig. 4. The cosine fits for individual data sets are given by the solid curves. The singles are $\sim 6500$ cps and maximum coincidences are $\sim 120$ cps. A calculation of visibility gives $92.7 \pm 0.3\%\ (EO\ basis)$ and $80.9 \pm 0.3\%\ (DEO\ AEO\ basis)$. The Bell parameter is estimated to be, $S = 2.46 \pm 0.08$.

While one is required to lay out multiple interferometers along with sorters to undertake a general set of projective measurements in the true linear and diagonal even–odd basis [31], we have taken the measurements through OAM projections using SLM. The use of SLM, however, introduces efficiency constraints and limitations in exploiting all the available OAM modes. Since our aim is to demonstrate the duality...
of entanglement in our setup, we have taken the measurements in a basis defined by \( l \in \{1, 2\} \) where 1 and 2 demonstrate the odd and even OAM states, respectively. This is why the count rates are low for the OAM projections that were carried out. However, for practical applications, the projective measurements in the even–odd basis need to be taken as given in Ref. [31] in order to explore all the available photons. The difference between the visibility of \( E \) and \( O \) curves in Fig. 7 can be understood as the result of our choice of a reduced OAM basis consisting of only \( l = 1 \) and \( l = 2 \). More equal visibilities could be achieved by taking the measurements in the full even—odd basis as mentioned above.

4 Conclusion

In this paper, we demonstrate the duality in entanglement of a collinear, indistinguishable pair of photons generated in a spontaneous parametric down-conversion process. We show polarization entanglement for indistinguishable photons by sorting the photon OAM using a double Mach–Zehnder even–odd sorter. This method can increase the availability of entangled photons since we are not eliminating any photon from the generated output in contrast to the case of limiting them to two-dimensional OAM bases such as \((+l, -l)\) or \((0, l)\). All the down-converted photons are sorted using an even–odd sorter in order to observe the polarization entanglement of otherwise indistinguishable collinear photons. Similarly, we demonstrate OAM entanglement by sorting photons using a simple polarizing beam splitter and executing OAM projections on the photon pairs in the even–odd basis.

The entanglement studies of systems that display duality must give identical results in both the variables but not observed here. We think that by improving the efficiency of the sorter and incorporating all available OAM modes in even–odd projections, the entanglement measures estimated in the present work could be improved and shown to be more identical than obtained. This kind of systems will be interesting to study the entanglement unaffected by the mutual interaction of particles involved. In addition, duality assisted observation of entanglement can be used as a test for verification of indistinguishability of photons in quantum information processing. The indistinguishable entangled photons may also find applications in distributed quantum sensing through phase estimation as well as remote entanglement generation in a quantum network.

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Data availability The data sets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.
Appendix

Consider the non-collinear generation of photon pairs where the two photons are emitted along the directions determined by the phase matching condition,

\[ \mathbf{k}_i + \mathbf{k}_s \approx \mathbf{k}_p \]

where \( \mathbf{k} \) is the linear momentum vector. The down-converted output forms a cone of correlated signal-idler photon pairs following the non-collinear phase matching condition. The experimental schematic is given in Fig. 8. We use the same experimental conditions as in Sects. 2.1 and 2.2 for the case of non-collinear scenario, a Toptica TopMode (405 nm) laser as the pump which is loosely focused within the crystal using the lens, \( L_1 (f = 50 \text{ cm}) \), such that the paraxial approximation for OAM conservation is valid.

A band-pass filter (BF, 810 ± 5 nm) blocks the residual pump while transmitting the down-converted output. With this spectral filtering, we consider our source to be giving degenerate photon pairs in the setups discussed in this work. The diverging cone of SPDC photons is collimated using lens, \( L_2 (f = 10 \text{ cm}) \), for the overall length of the experiment (Fig. 9). The low-intensity photon distributions are imaged using an

Fig. 8 Experimental setup to observe the entanglement in a non-collinear type II SPDC from a ppKTP crystal (\( \chi^{(2)} \)). Polarization measurements are taken using a combination of half-wave plate (HWP) and a polarizer (P) in each arm. L FC—Aspheric lens associated with the fiber coupler (FC), SMF—single-mode fiber, SPCM—single-photon counting module, CC—coincidence counter

Fig. 9 Collimated SPDC output images obtained using an EMCCD kept at different distances from the crystal plane. A 10 cm lens placed after the SPDC output collimates the diverging cone of photon pairs
EMCCD (Andor iXon3). *Signal* and *idler* photons are then coupled to single-mode fibers (SMF) through fiber coupling systems, FC (Thorlabs CFC-5X-B), consisting of a fiber launcher and an aspheric lens (LFC, \( f = 4.6 \text{ mm} \)).

These photons are detected at single-photon counting modules, SPCM (Excelitas AQRH-16-FC), whose output is given to a coincidence counter, CC (ID Quantique).

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**Fig. 10** The crystal position and tilt are adjusted such that the SPDC output corresponding to both the polarizations are spatially overlapping. Diametrically opposite regions (in red circles) correspond to the entangled photons (Color figure online).

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**Fig. 11** Polarization correlations corresponding to the spatially separated photons in the non-collinear down-conversion pair. Visibility curves are plotted for *idler* polarization projection angles, \( \theta_1 = 0 \) (H—green dot), \( \theta_1 = \pi/4 \) (D—red dot), \( \theta_1 = \pi/2 \) (V—blue dot) and \( \theta_1 = 3\pi/4 \) (A—purple dot). \( \theta_2 \) is the *signal* polarization projection angle. Solid curves are respective cosine fits. Error bars indicate statistical uncertainty of one standard deviation (Color figure online).
ID800 TDC), to obtain the coincidence counts. To observe maximum entanglement, maximum overlap between the two polarization modes has to be ensured. The crystal position and tilt for pump incidence are adjusted such that good spatial overlap between the $H$-polarized cone and $V$-polarized cone is achieved (Fig. 10). The correlated photons will be falling along diametrically opposite points following the phase matching conditions. Two regions, marked in red circles, are selected and coupled to the detector system.

The polarization projections are done using the combination of a half-wave plate (HWP) and a polarizer (P) kept in each arm of the SPDC output (Fig. 8). The half-wave plate, HWP$_1$, in the idler arm is kept at fixed angles, thereby selecting the idler photon in $H$, $D$, $V$ and $A$ polarization, respectively. To observe the polarization correlation corresponding to each of these settings, the half-wave plate (HWP$_2$) in the signal arm is rotated.

The experimentally observed polarization correlations, corresponding to the projections in HV basis as well as DA basis, are given as the visibility curves in Fig. 11. The normalized coincidences are plotted with the variation of signal polarization projection angle, $\theta_2$, by keeping idler polarization projection angles fixed at, $\theta_1 = 0$ ($H$, green dot), $\theta_1 = \pi/4$ ($D$, red dot), $\theta_1 = \pi/2$ ($V$, blue dot) and $\theta_1 = 3\pi/4$ ($A$, purple dot). Here, $\theta_1$ and $\theta_2$ are twice the angle of rotation of the half-wave plates, HWP$_1$ and HWP$_2$, respectively. Solid curves in the same color are respective fits into a cosine function. The observed visibilities in the two bases are $96.4 \pm 0.2 \%$ (HV basis) and $94.1 \pm 0.3 \%$ (DA basis), respectively. The Bell parameter is estimated to be $S = 2.69 \pm 0.03$.

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