Does MOND follow from the CDM paradigm?

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ABSTRACT

In a recent paper, Kaplinghat and Turner (2001) (KT hereafter) claim that the CDM paradigm explains why there should appear in the baryon-dark-matter phenomenology an acceleration $a_0$ that is of order $cH_0$. They seem to imply that by this they have explained MOND. In fact, however, they only address one of several independent roles that $a_0 \sim cH_0$ plays in MOND phenomenology, and other predictions of MOND, not related to the value of $a_0$, that are not explainable in the KT scenario. The results of KT also disagree with those of CDM simulations, which, as they now stand, do not reproduce any aspect of MOND phenomenology.

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1. introduction

Kaplinghat & Turner (2001) (KT hereafter) claim in a recent paper that the CDM paradigm explains why there should appear in the baryon-dark-matter phenomenology an acceleration $a_0$ that is of order $cH_0$. They seem to imply that by this they have explained MOND. In fact, however, they only address one of several independent roles that $a_0 \sim cH_0$
plays in the phenomenology. They also do not explained predictions of MOND that are not related to the value of $a_0$. And, furthermore, their argument leads to predictions that are in clear conflict with observations, and, in fact, with those of CDM simulations based on the same physics. Such CDM simulations, which, obviously, are more reliable, do not reproduce any aspect of the MOND phenomenology; in particular, they do not point to any acceleration constant of special significance on galactic scales.

The KT argument, in paraphrase, is as follows: They argue that CDM halo formation produces a one-parameter family of halo density profiles of the form

$$\rho(r) = (A/4\pi)S(\ell/\ell_0)G^{-1}\ell^{-1}\hat{\rho}(r/\ell). \quad (1)$$

Here, $\ell$ is the present characteristic scale of the halo, related to the co-moving scale of the perturbation collapsing to the halo, $L$, by $\ell \propto L/(1 + z_c)$, $z_c$ being the red-shift of non-linearity for the halo. $\ell_0$ corresponds to $z_c = 0$, and $A$ is a constant with the dimensions of acceleration. Also, one can show that for their halos $S(x) = x^{2\theta/(3-\theta)}$, where $\theta = -2 - n_{eff}$, with $n_{eff}$ the logarithmic slope of the power spectrum of fluctuations at the relevant scale. (In KT, this factor appears in terms of $L$ as $S \propto L^\theta$.)

Their resulting halo acceleration profile (which KT do not give) is

$$a(r) = A S(\ell/\ell_0) \hat{a}(r/\ell), \quad (2)$$

where $\hat{a}(\lambda) \equiv \lambda^{-2} \int_0^\lambda x^2 \hat{\rho}(x)dx$. While $n_{eff}$ itself depends on $\ell$, KT take it very near $-2$, so that $\theta \ll 1$. Specifically, they take for galaxies $\theta \approx 0.2$, (this is what they call the first numerical coincidence) giving $S(x) = x^{0.14}$ for galaxies, which is practically constant over the range of values of $x$ corresponding to galaxies. Their basic and main, but unstated, result is then that CDM produces galactic halos that have a nearly universal acceleration profile, which differs from halo to halo, practically, only by scaling of the length.

Now, continues the argument, the baryon body in all galaxies has collapsed by a universal factor of $\alpha \approx 10$. From this they deduce that all galaxies should have a transition radius, $r_t = \ell/\alpha$, such that baryons dominate DM at radii smaller than $r_t$ and DM takes over at larger radii. The acceleration at that radius is [from eq.(2)] $S(\ell/\ell_0)A \hat{a}(1/\alpha)$. And, since $S(\ell/\ell_0)$ hardly varies among galaxies, they get a universal transition acceleration, which, by another numerical coincidence, happens to be of the order of $cH_0$.

2. The KT argument addresses only one of the many roles of $a_0$
If MOND succeeds in accounting for the mass discrepancy without dark matter (DM), it tells us that in the DM paradigm, $a_0 \sim cH_0$ appears in several independent roles in the phenomenology.

1. In galaxies whose central surface density is high (HSB galaxies), so that the acceleration in their inner parts becomes higher than $a_0$, there is a transition radius, $r_t$, which occurs where the acceleration equal to $a_0$, and which marks the transition from baryonic to DM dominance. So $a_0$ can be measured from the transition point.

2. $a_0$ determines the asymptotic acceleration fields in all galaxies (or, for that matter, in any isolated system). Outside the baryonic body, the acceleration at radius $r$ is given, asymptotically, by $\left(\frac{MGa_0}{r}\right)^{1/2}$ where $M$ is the total mass of the galaxy. So $a_0$ can be measured from the asymptotics.

3. $a_0$ defines the transitions from HSBs LSBs: Baryons dominate in the inner parts of galaxies whose central surface density is higher than some critical value of order $a_0 G^{-1}$, while in galaxies whose central surface density is much smaller, DM dominates everywhere.

4. $a_0$ tells exactly how much DM is needed inside the baryon body in LSBs: The lower the mean acceleration, $a$, is, the larger the DM-to-baryon ratio; specifically, this ratio is predicted to be $\approx a_0/a$.

5. $a_0$ control the dynamics of all galaxy system such as galaxy groups (Milgrom 1998), clusters (Sanders 1999), and large-scale filaments (Milgrom 1997), where MOND has been shown to explain away the need for DM. Since MOND has not yet explained away the need for DM in the inner part of x-ray galaxy clusters, the DM paradigm is not yet called upon to explain such a success. Clusters at large, however–say within two megaparsecs of the center are explained by MOND (Sanders 1999).

Strictly speaking, no two galaxies or galactic systems have had exactly the same history of formation-evolution-interaction. Inasmuch as MOND succeeds in obtaining the DM distribution from the baryon mass distribution in any such system, with only the aid of $a_0$—as it claims to be capable of—each such success would call for a separate explanation in the DM paradigm. The DM paradigm has, at best, tried to address general trends in the population behavior of galactic systems. This, indeed, is the most that can be asked from it, in light of the expected idiosyncrasies of individual objects. But why then should there be a simple theory that does account for these idiosyncrasies?

These appearances of $a_0$ in the DM phenomenology are independent because a-priori one could (easily) envisage baryon-plus-DM galaxies and galactic systems in which $a_0$ appears in any of the above roles but not in the others. For example, if halos are cut off
beyond some radius larger than $r_t$ we can have property 1 but not 2 (or 3 if the cutoff radius is small compared with the inter-galaxy distance). It follow, for example, that the appearance of $a_0$ in the TF relation is independent of its role as a transition acceleration. Another example, galaxies might have been such that they do not become more and more DM dominated as their central surface density decreases; so property 4 will not have been satisfied, but all the other MOND predictions could still be retained.

The KT galaxies are yet another example. They exhibit appearance 1 of $a_0$, but not the others. To explain 2 one would have to assume an asymptotic $r^{-2}$ halo density profile, which is not a consequence of CDM. Appearances 3 and 4 in the phenomenology actually fly in the face of the KT argument because all their galaxies have the same surface density (see details in the next section). The KT argument is said to apply on the scale of galaxies only, so does not pertain to appearance 5 of $a_0$. In fact, at larger scales $n_{eff} \to -3$ so $\theta \to 1$, and $S(\ell/\ell_0)$ becomes linear in $\ell$. By the KT picture we would then expect MOND phenomenology to miss by orders of magnitudes on large scales (hundreds of times larger than galactic scales). In fact, this is not so. And, even in galaxy clusters (say within 1-2 Mpc) where MOND does not yet explain away all the DM, the remaining gap is only of a factor of order 2, much smaller than what the KT scenario predicts.

We learn from all this that if one purports to explain MOND and the appearance of $cH_0$ in the phenomenology, within the DM paradigm, he should explain all the above appearances, because they do not follow from each other within this paradigm. One of the strengths of MOND is that it does tightly connect all those apparently different occurrences (MOND was constructed to account for RC asymptotics and all the others came as unavoidable predictions). In the DM paradigm this is still an unexplained miracle.

### 3. The KT predictions conflict with observations

An unavoidable prediction of the KT argument is that all galaxies must have a transition radius—occurring at a fixed fraction of the halo’s length scale—where the halo acceleration goes from $a > a_0$ to $a < a_0$. This follows from eq. (2), which implies that all halos have very nearly the same acceleration runs, so either no galactic halo has reaches an acceleration value of $a_0$ or they all do. In particular, there should be a range of radii, at least up to $r_t$, where the halo acceleration is above $a$ (adding the baryons changes the acceleration profile but not this fact). This (which is not a prediction of MOND) blatantly conflicts with observations: Many galaxies have accelerations measured directly from their
rotation curves—that are everywhere much smaller than \(a_0\) (generally classified as LSB galaxies). According to KT such galaxies are not produced in CDM.

This problem is yet augmented by the observation of pairs of galaxies whereby the two galaxies in the pair have the same total luminosity and the same asymptotic rotational velocity—so they lie on the same point of the TF relation—but very different acceleration profiles. In particular, there are such pairs with one galaxy being an HSB and the other an LSB. According to the KT scenario, the two galaxies in such a pair have the same halo, and, in particular, should have the same transition radius and acceleration. Yet, this is clearly not so. This point is demonstrated by le Blok & McGaugh (1998) with the pair NGC 2403 and UGC 128. The former is an HSB having a clear transition from a super-\(a_0\) region near the center to the sub-\(a_0\) region, occurring at about 10 kpc—several disc scale lengths, while the latter is an LSB with accelerations everywhere smaller that \(a_0\). Tully & Verheijen (1997) have further emphasized this point, showing that such pairs are quite common.

In the KT picture, baryons in all galaxies had undergone a collapse by the same factor. The resulting baryon-plus-DM systems all have the same surface-density distribution, again, differing only by their scale length. In particular, they all should have the same rotation curve. This is obviously not true observationally.

It need be emphasized, perhaps, that those features of the KT scenario that lead directly to clashes with observations—the near universality of the halo surface density, and of the baryon collapse—are indispensable; without them the argument itself collapses, and no universal transition acceleration emerges.

4. Is the argument valid at all?

Do the halos that KT required for their argumentation actually follow in the CDM paradigm? The best way to answer this, at present, is to consult the results of numerical CDM simulations, which start from the same physics. The fact is that such simulations do give approximately a one-parameter halo family, but not of the scaling deduced by KT—see e.g. Navarro Frenk & white (1997). In particular, no constant with the dimensions of acceleration emerges from the simulations on galactic scales, as it does in KT. This disparity is evident in the fact that the KT halos have a mass velocity relation \(M \propto v^{3.75}\), while the simulations of Navarro Frenk & white (1997), for example, give \(M \propto \delta^{3.2 - 3.3}\). A power of 3.3 corresponds to \(S(x) = x^{0.5}\), which gives a large variation of the transition acceleration among galaxies.
Perhaps this disparity can be traced back to the value of $n_{eff}$ that KT chose to suit their purpose, but which is not what the simulations dictate; perhaps it is due to details of the more exact simulations that are not captured by the argumentation of KT. If it is the former reason, we can note that to obtain an $M - v$ power of 3 one needs $\theta = 1$, and to get a power of 3.3 one needs $\theta = 0.66$, compared with the rather smaller value of 0.2 that KT took.

Beyond all this, there remains the fact that the KT scenario is very crude. It assumes a universal collapse factor for the baryons; it does not include the contribution of the baryons themselves to the acceleration; it does not allow for baryon accretion or expulsion; and, it neglects the response of the halo to baryon collapse. Moreover, the KT result depends crucially on their assumption that a large fraction of the original baryons in the halo had collapsed to form the observed galaxy. In contrast, it can be argued quite cogently that only a small fraction of the original baryons go into forming the observed galaxy. This follows from the high values of DM-to-observed-baryons mass ratios deduced for galaxies—e.g. in [Blok & McGaugh (1998), Zaritsky (1999), and McKay & al (2001)]—which are much higher than the cosmological value of this parameter, believed now to be of order 7-10.

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