Next-to-leading order corrections
to gauge-mediated gaugino masses

Marco Picariello and Alessandro Strumia

Dipartimento di Fisica, Università di Pisa and
INFN, sezione di Pisa, I-56126 Pisa, Italia

Abstract

We compute the next-to-leading order corrections to the gaugino masses \( M_i \) in gauge-mediated models for generic values of the messenger masses \( M \) and discuss the predictions of unified messenger models. If \( M < 100 \text{ TeV} \) there can be up to 10\% corrections to the leading order relations \( M_i \propto \alpha_i \). If the messengers are heavier there are only few \% corrections. We also study the messenger corrections to gauge coupling unification: as a result of cancellations dictated by supersymmetry, the predicted value of the strong coupling constant is typically only negligibly increased.

1 Introduction

The "gauge mediation" scenario for supersymmetric particle masses [1] can be realized in reasonable models [2, 3]. Furthermore, with a unified spectrum of messenger fields, it gives rise to some stable and acceptable prediction for the spectrum of supersymmetric particles. One of these predictions is the 'unification prediction' for the gaugino masses \( M_i \), \( i = 1, 2, 3 \): at one-loop order the RGE-invariant ratio \( \rho_i = M_i/\alpha_i \) is the same for all the three factors of the SM gauge group \( G_{\text{SM}} = \bigotimes_i G_i = U(1)_Y \otimes SU(2)_L \otimes SU(3)_C \). This prediction is sufficiently stable that it is interesting to compute it with more accuracy.

A detailed computation allows a comparison with the gaugino spectrum predicted by the alternative scenario known as "unified supergravity" [4]. Unification relations for the \( \rho_i(E) \) are infact the more stable prediction of this second scenario: gauge couplings and gaugino masses could receive sizeable GUT threshold corrections; but these corrections largely cancel out in the ratios \( \rho_i(M_{\text{GUT}}) \), since \( \alpha_i \) and \( M_i \) have the same one-loop RGE evolution. The testable predictions for the low-energy running \( \rho_i \) ratios in the \( \overline{\text{DR}} \) scheme [4] are (including NLO RGE corrections [4], but neglecting possible unknown \( O(\%) \) GUT-scale threshold effects [5])

\[
\frac{\rho_1(M_Z)}{\rho_2(M_Z)} \approx 1.02, \quad \frac{\rho_2(M_Z)}{\rho_3(M_Z)} \approx 0.97.
\]

This should be compared with the corresponding prediction in gauge-mediation models for the ratios \( \rho_i/\rho_j \), plotted in figs 2 and 3.

The computation of gaugino masses with NLO precision is done in sections 2 and 3 for generic values of the messenger mass \( M \). It requires the following main steps:

1. compute the renormalized running gaugino masses, \( M_i(E_H) \) at \( E_H \sim M \), in the effective theory without messengers.
2. compute the running gaugino masses, $M_i(E_L)$ at $E_L \sim M_Z$, evolving $M_i(E_H)$ down to the Fermi scale with 2 loop RGE equations.

3. use $M_i(E_L)$ to compute directly measurable quantities, like the gaugino pole masses, $M_i^{\text{pole}}$, including all 1 loop effects at the electroweak scale.

The computation necessary for step 1 is done in section 2. We will employ supersymmetric dimensional regularization, so that the renormalization scale $E$ will be the $\overline{\text{DR}}$ scale, $\bar{\mu}$. The RGE necessary for step 2 (recalled in appendix B) can be read from the literature. The one-loop expressions for pole gaugino masses in terms of running parameters are also well known. Since various unmeasured and unpredicted parameters (like the so-called $\mu$-term) would enter the final step 3, we prefer to show our final predictions for the running MSSM gaugino masses renormalized at $\bar{\mu}_L = M_Z$, without including the gauge corrections at the electroweak scale.

We will compute these predictions in unified messenger models. One more step is necessary to impose the unification constraints on the messenger spectrum, namely

0. compute the messenger spectrum, $M_n(E_H)$, evolving the unified $M_n(M_{\text{GUT}})$ down to the messenger scale $E_H \sim M_n$ with 2 loop RGE equations.

The necessary RGE equations are given in appendix B. In sec. 3 we study the predictions of gauge mediation models with an unified messenger spectrum. If the messenger spectrum is only negligibly splitted by supersymmetry breaking effects, the NLO corrections to the LO unification-like relations $M_i \propto \alpha_i$ are around few % and numerically not much different from the ones present in unified supergravity models. Larger effects (up to 10%) can be present if the messengers are very light, $M \lesssim 50 \text{ TeV}$.

We also study the corrections to gauge coupling unification due to the presence of messenger fields below the unification scale. Messenger threshold effects largely cancel messenger corrections to two loop RGE running, as dictated by supersymmetry.

2 Computation

In this section we do the computations necessary for step 1. We assume that the messenger fields sit in real representations $R = \bigoplus_n R_n$ of the SM gauge group and have a supersymmetric mass term $M_n$ together with a non-supersymmetric mass term $F_n$. For the moment we assume that the messengers lie in self-conjugate complex representations, $X_n \equiv X_n \oplus \bar{X}_n$. See appendix A for a more detailed discussions of the notations, of the model, of its spectrum, and of the relevant Lagrangian.

In order to compute the running gaugino masses $M_i(\bar{\mu})$ at $\bar{\mu} \lesssim M_n$ in the effective theory we first compute the gauge-independent pole gaugino masses in the two versions of the theory. The computation in the full theory (with messengers) is done in section 2.1 — the computation in the effective theory (with messengers integrated out) is done in section 2.2. Requiring that the two theories describe the same gaugino masses up to second order in $\alpha_i$ we get, in section 2.3, the gaugino mass terms in the effective theory.

We employ the Feynman-Wess-Zumino gauge and the supersymmetric $\overline{\text{DR}}$ regularization in both versions of the theory. The final $\overline{\text{DR}}$ values of the gauge couplings $\alpha_i$ and of the gaugino masses $M_i$ can be converted into the $\overline{\text{MS}}$ ones (i.e. the ones obtained with naive dimensional regularization) using

$$
\alpha_i^{\overline{\text{MS}}} = \alpha_i^{\overline{\text{DR}}}(1 - \frac{C(G)}{3} \frac{\alpha_i}{4\pi}), \quad M_i^{\overline{\text{MS}}} = M_i^{\overline{\text{DR}}}(1 + C(G) \frac{\alpha_i}{4\pi})
$$

where the group factors are defined as follows. For each representation $R$ of a gauge group $G = \bigotimes_i G_i$ we define the "Dinkin index" $T_i(R)$ and the "quadratic Casimir" $C_i(R)$ in terms of the generators $T_{Ri}^a$ as

$$
\sum_a T_{Ri}^a \cdot T_{Rj}^a = C_i(R) \mathbb{1}, \quad \text{Tr} T_{Ri}^a T_{Rj}^a = C_i(R) \delta_{ij} \delta^{ab}.
$$

With generators canonically normalized so that $T(n) = T(\bar{n}) = 1/2$ for the fundamental $n$ representation of a SU(n) group, the quadratic Casimir of the adjoint representation of a SU(n) group is $C(G) = T(G) = n$, while
The NLO correction to the pole gaugino masses are given by the ten two-loop diagrams shown in figure 1 and some one-loop renormalization factor. It is convenient to separate the renormalization factors due to the light external gaugino momentum to zero, except in the infrared divergent graphs. We write the various contributions to the pole gaugino masses as

\[ M_i^{\text{full pole}} = \hat{M}_i^{(1)} + \sum_{\Gamma} \delta M_i^{\Gamma}, \quad \hat{M}_i^{(1)} \equiv \sum_n \hat{M}_i^{(1,n)} = \frac{\alpha_i}{4\pi} \sum_n \frac{F_n}{M_n} T_i(R_n) \hat{g}_1(x_n) = M_i^{(1)} + O(\varepsilon) \]  

where \( x_n = F_n/M_n^2 \) and the sum \( \sum_n \) extends over all the messengers. The one-loop function \( \hat{g}_1(x) \equiv g_1(x) + \varepsilon \delta g_1(x) + O(\varepsilon^2) \) is

\[ \hat{g}_1(x) = \frac{1}{x} + \frac{x}{x^2} \ln(1+x) \left( 1 + \varepsilon \left[ 1 - \frac{\ln(1+x)}{2} \right] \right) \left( 1 + \varepsilon \ln \frac{\bar{\mu}^2}{M^2} \right) + O(\varepsilon^2) = 1 + x^2 + O(x^4, \varepsilon) \]

All parameters are unrenormalized ('bare'). Here we list all the contributions to the pole gaugino masses.

- The contribution given by the sum of the two loop diagrams of fig. 1. They give

\[ \delta M_i^{\text{loop}} = \frac{\alpha_i}{4\pi} \sum_n \frac{F_n}{M_n} T_i(R_n) \left[ \frac{\alpha_i}{4\pi} C_i(G) \left( \hat{g}_1(x_n) \left( \frac{4}{\varepsilon_{\text{uv}}} - \frac{4}{\varepsilon_{\text{ir}}} + g_{2n} + 2g_2(x_n) \right) \right) + \sum_{j=1}^{3} \frac{\alpha_j}{4\pi} C_j(X_n) \left[ g_C(x_n) - 4g_1(x_n) - \frac{2x_n \hat{g}_1'(x_n)}{\varepsilon_{\text{uv}}} \right] \right] \left( \frac{\bar{\mu}^2}{M^2} \right)^\varepsilon \]

where \( \hat{g}_1'(x) \) is the derivative of \( \hat{g}_1(x) \) with respect to \( x \), and the functions \( g_2 \) and \( g_C \) are

\[ g_2(x) = \frac{\ln(1+x)}{x^2} - \left[ 2(1+x) + \ln(1-x) + (2x + 2 + 1/x) \ln(1+x) + \ln(1+x) \right] + \frac{1}{x} \left[ \log_2 \left( \frac{2x}{1-x} \right) - 2\log_2(x) \right] + (x \to -x) \]

\[ g_C(x) = \frac{\ln(1+x)}{x^2} \left[ 8 + 6x + \ln(1-x) + (1 - x + 2/x) \ln(1+x) \right] + (x \to -x) \]

In the limit \( F \ll M^2 \) \( (x \to 0) \) \( g_1(0) = g_2(0) = 1 \) and \( g_C(0) = 0 \). We have denoted as \( 1/\varepsilon_{\text{uv}} \) an \( 1/\varepsilon \) ultraviolet (UV) pole, and as \( 1/\varepsilon_{\text{ir}} \) a pole of infrared (IR) origin. In all the graphs it is possible to set the external gaugino momentum to zero, except in the infrared divergent \( \lambda \alpha \) graph of fig. 1. It is convenient to split it into a part computed with zero external momentum, that contributes to \( g_2 \), plus the remainder, that gives the \( g_{\text{IR}} \) term in eq. 3. The value of \( g_{\text{IR}} \) coincides with the ‘non naïve part’ of the ‘asymptotic
The diagrams that contribute to the NLO gauge corrections to gauge mediated gaugino masses. The thick continuous (dashed) lines represent the fermionic (bosonic) messengers. The thin wavy (continuous) lines represent the gauge bosons (gauginos).

expansion\footnote{The general technique of asymptotic expansions of Feynman diagrams is described in \cite{12}; a much simpler discussion, sufficient for the purposes of this computation, can be found in \cite{13}, where an accurate distinction between UV and IR divergences is made.} in the external gaugino momentum, and can be seen as the contribution of the $\lambda\lambda$ graph with the heavy messenger loop contracted to a point. This technical detail is useful, because a corresponding one-loop diagram gives the same contribution to the effective theory, so that we do not need to compute $g_{\text{IR}}$.\footnote{The general technique of asymptotic expansions of Feynman diagrams is described in \cite{12}; a much simpler discussion, sufficient for the purposes of this computation, can be found in \cite{13}, where an accurate distinction between UV and IR divergences is made.}

- **On-shell renormalization of the gaugino wave function:** the renormalized gaugino field is $\lambda_i = (1 + z_i + z_i')^{1/2} \cdot \lambda_i |_{\text{bare}}$ with

\[
z_i' = \frac{\alpha_i}{4\pi} \sum_n T_l(R_n) \left\{ \frac{1}{\varepsilon} + \ln \frac{\mu^2}{M_{\text{Pl}}^2} + \frac{x_n^2 + (1 - x_n^2)[\ln(1 - x_n) + \ln(1 + x_n)]}{2x_n^2} \right\}.
\]

We have separated this correction into two parts: $z$ due to all the MSSM particles, and $z'$ due to messenger loops only. The corresponding corrections to the gaugino masses, $\delta M_i^2 + \delta M_i'^2$, are obtained expressing...
As an aside remark, it could be of interest to know that the NLO squared pole mass of the lightest scalar messenger, \( M_{\text{n-}}^2 \), is

\[
\frac{M_{\text{n-}}^2}{M_i^2 - F_n} = 1 - \sum_j \frac{\alpha_j}{4\pi} C_j(X_n) \left\{ 2(x_n - 2)(2 + \ln \frac{\mu^2}{M_n^2}) + \frac{2x_n - 3 - 3x_n^2}{x_n - 1} \ln(1 - x_n) + \frac{2x_n}{x_n - 1} \ln x_n - (1 + x_n) \ln(1 + x_n) \right\}
\]

when expressed in terms of \( \overline{\text{DR}} \) parameters.

| rep. | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( T_1 \) | \( T_2 \) | \( T_3 \) |
|------|-----------|-----------|-----------|-----------|-----------|-----------|
| \( d \oplus \bar{d} \) | 2/15 | 0 | 8/3 | 2/5 | 0 | 1 |
| \( L \oplus \bar{L} \) | 3/10 | 3/2 | 0 | 3/5 | 1 | 0 |
| \( Q \oplus \bar{Q} \) | 1/30 | 3/2 | 8/3 | 1/5 | 3 | 2 |
| \( u \oplus \bar{u} \) | 8/15 | 0 | 8/3 | 8/5 | 0 | 1 |
| \( e \oplus \bar{e} \) | 6/5 | 0 | 0 | 6/5 | 0 | 0 |

Table 2: Values of the group factors for the \( G_{\text{SM}} \) fragments of the \( 5 \oplus 5 \) and \( 10 \oplus 
\overline{10} \) SU(5) representations.
2.2 Computation in the effective theory

In the effective theory we have to compute the pole gaugino masses in terms of the coefficients of the running gaugino mass term operator, $-M_i(\lambda, \bar{\lambda} + \text{h.c.})$, expanded as a series in the gauge couplings:

$$ M_i = \frac{\alpha_i}{4\pi} M_i^{(1)} + \frac{\alpha_i}{4\pi} \frac{\alpha_j}{4\pi} M_{ij}^{(2)} + \cdots $$

(8)

where $M_i^{(1)}$ are the known LO coefficients, and $M_{ij}^{(2)}$ are the NLO coefficients that we want ultimately to extract. The pole gaugino masses at $\mathcal{O}(\alpha^2)$ order are simply given by the full theory result, omitting those quantum corrections that are present also in the full theory.

The matching procedure is particularly simple: the running gaugino masses in the effective theory at NLO order are

- the contribution from the renormalization of the gaugino wave function. This coincides with the $\delta M^Z_i$ correction present also in the full theory.
- the contribution from the renormalization of the gauge couplings. This coincides with the $\delta M^{\alpha\alpha}_i$ correction present also in the full theory.
- a one loop diagram (gauge correction to the gaugino propagator). As said, it is not difficult to see that it gives the same contribution of the asymptotic expansion in the external gaugino momentum of the two-loop $\lambda\lambda$ diagram of fig. [1].

More in detail the pole gaugino masses in the effective theory are

$$ M_i^{\text{eff}} = \frac{\alpha_i}{4\pi} M_i^{(1)} \left\{ 1 + \frac{\alpha_i}{4\pi} C_i(G)(\frac{4}{\varepsilon_{\text{uv}}} - \frac{4}{\varepsilon_{\text{ir}}} + g_{1R}) + \frac{\alpha_i}{4\pi} \frac{\alpha_i}{4\pi} M_{ij}^{(2)} + \delta M^Z_i + \delta M^{\alpha\alpha}_i. \right\} $$(9)

A further simplification occurs. The sum of the three effective-theory quantum corrections, all proportional to $M_i^{(1)}$, is both infrared and ultraviolet convergent (because the combination $M_i/\alpha_i$ is RGE-invariant at one loop). For this reason we do not need to worry about the $\mathcal{O}(\varepsilon)$ terms that distinguish $M_i^{(1)}$ from $M_i^{(1)}$.

2.3 Matching

The matching procedure is particularly simple: the running gaugino masses in the effective theory at NLO order are simply given by the full theory result, omitting those quantum corrections that are present also in the effective theory result, eq. (8). The MSSM running gaugino masses at NLO order are

$$ M_i = \frac{\alpha_i}{4\pi} \sum_n \frac{F_n T_i(R_n)}{M_n} \left\{ g_1(x_n) + \frac{\alpha_i}{4\pi} \left[ 2C_i(G)g_2(x_n) + \sum_m T_i(R_m)g_T(x_m) \right] + \sum_j \frac{\alpha_i}{4\pi} C_j(X_n) \right\} \left[ g_C(x_n) - 2x_n g_1'(x_n) \ln \frac{\bar{\mu}^2}{M_n^2} + \frac{\lambda^2_n}{(4\pi)^2} g_\lambda(x_n) \right] + \lambda^2_n (11a) $$

(10)

The parameters are renormalized as discussed in sec. 2.2. The last term is the effect of a possible Yukawa coupling $\lambda_n S X_n \bar{X}_n$ in the superpotential, where $S$ is a gauge singlet. The functions $g_1$, $g_2$ and $g_C$ have been defined in eqs. (4) and (5), while $g_T$ and $g_\lambda$ are

$$ g_T(x) = \frac{2x^2 - 3}{6x^2} \ln(1 - x^2) - \frac{1}{2}, $$

(11a)

$$ g_\lambda(x) = -\frac{\ln(1 + x)}{2x^3} \left[ 2x(3 + 2x) - x \ln(1 - x) + (5 + 3x) \ln(1 + x) \right] - \frac{x^4}{2} \ln 2 + \ln(2^{-1}) = \frac{x^4}{2} \ln 2 + (x \to -x). $$

(11b)

The functions $g_1$ and $g_2$ are normalized such that $g_1(0) = g_2(0) = 1$, while $g_C(0) = g_T(0) = g_\lambda(0) = 0$. So far we have assumed that $R_n = X_n \oplus \bar{X}_n$. If there are also messenger fields $\Sigma$ in a real representation $R_\Sigma$, the appropriate group factors are $T(R_\Sigma) = T(\Sigma)$ and $C(X) \to C(\Sigma)$.

\[2\] The computation of Yukawa corrections, that we do not present, involves one new feature: a renormalization of $F_n/M_n$. For simplicity, we have given the Yukawa correction assuming that all the $F_n$ and $M_n$ are produced by a vacuum expectation value of the superfield $S$. In this case the new feature becomes an irrelevant common renormalization of $F_n/M_n$. The corresponding effect in the final result, eq. (10), has been absorbed in an appropriate (non RGE) renormalization of $F_n$. 


Figure 2: NLO predictions for the measurable ratios $\rho_i(M_Z)/\rho_j(M_Z)$ ($\rho_i \equiv M_i/\alpha_i$) in the minimal gauge mediated model ($n_5 = 1$, $n_{10} = 0$) for generic values of the unified messenger mass. For comparison, we also show the prediction of unified supergravity models (neglecting possible small GUT-scale effects).

In the limit $x_n \to 0$ the NLO prediction for the running gaugino masses does not depend on the messenger spectrum

$$M_i(\bar{\mu}) \left[ \frac{F_n}{M_n} \right] \frac{\alpha_i(\bar{\mu})}{4\pi} \sum_n \frac{F_n}{M_n} T_i(R_n) \left[ 1 + \frac{\alpha_i}{4\pi} 2C_i(G) \right] \quad \text{at } \bar{\mu} \sim M_n.$$  (12)

We remember that $C_i(G) = \{0, 2, 3\}$. This prediction for the three gaugino masses, without knowing the values of the $F_n/M_n$ parameters, is of interest only if their number is less than three. This happens, for example, if the messengers lie in a $5 \oplus \bar{5}$ representation of SU(5).

3 Predictions of unified messenger models

We will show the NLO predictions in models where the messenger spectrum satisfies unification relations. These models are not only more predictive but also more appealing: the successful unification of the gauge couplings is not destroyed and a unified messenger spectrum helps in avoiding undesired one-loop contributions to sfermion masses. To be more specific we assume that the messengers fill $n_5$ copies of $5 \oplus \bar{5}$ and $n_{10}$ copies of $10 \oplus \bar{10}$ representations of the unified group SU(5), so that the messenger contribution to the one loop coefficient of the gauge $\beta$ functions is $T_i(R) = n_5 + 3n_{10}$. We assume that the messenger mass parameters $M_n$ and $F_n$ arise from the vacuum expectation value of one SU(5)-singlet field $S$ coupled to the messengers via Yukawa interactions $\lambda_n S X_n \bar{X}_n$. Imposing the unification relations the running DR mass parameters at $\bar{\mu} \sim M_n$ are thus obtained with NLO precision via two-loop RGE evolution from $M_{\text{GUT}}$ down to $\bar{\mu}$:

$$x_n(\bar{\mu}) = \frac{F_n}{M_n^2} = U_n(M_{\text{GUT}} \to \bar{\mu})[x_N(M_{\text{GUT}}) + \delta_n x_N] \quad \text{for messengers } R_n \text{ unified in } R_N$$

where $\delta_n x_N$ represent unknown one-loop threshold effects at the unified scale, that we will neglect. ‘Reasonable’ threshold effects give small corrections also when $x \sim 1$. We also neglect NLO Yukawa corrections, possibly
Measurable ratios $\rho_i/\rho_j$ relevant when $x \sim 1$. The NLO RGE equations for $M_n$ and $F_n$ are given in appendix B (the combination $F_n/M_n$ is RGE-invariant, and is not corrected by threshold effects). Already at LO, the RGE evolution of the messenger spectrum gives corrections of relative order $O(x^2 \alpha \ln M_{\text{GUT}}/M)$ to the relations $M_i \propto \alpha_i$: for light messengers the leptonic messengers are approximately 2 times lighter than the hadronic ones. This explains the larger effect present for light messengers ($x_N \sim 1$).

Fig. 2 shows the prediction of the minimal model with $(n_5, n_{10}) = (1, 0)$ for the measurable ratios $\rho_i/\rho_j$ (the experimental errors on the gauge couplings negligibly affect the predictions for the $\rho_i(M_Z) \equiv M_i(M_Z)/\alpha_i(M_Z)$). In this figure we have considered the whole range of possible messenger masses, distinguishing the smaller values of the messenger mass for which the lightest supersymmetric particle (LSP) decays before escaping detection, from the higher values for which the LSP decay is so slow (in absence of R-parity breaking) that destroys the nucleosynthesis products.

In fig. 3 we consider models with more than a single unified messenger. For the sake of illustration we have combined their contributions assuming that all the messengers have the same unified $M_n$ and $F_n$. For values of $M$ higher than the ones considered in fig. 3 ($x < 1$) all neglected NLO terms are completely irrelevant, the prediction does not depend on the messenger content, and is the same as in fig. 2. In all plots we have fixed the $F$-terms requiring that the running gluino mass be $M_3(M_Z) = 500$ GeV. Any other reasonable value of the gluino mass gives the same prediction for $\rho_i$. We remember that we have not included the one-loop corrections at the electroweak scale to $M_i$, that depend on unmeasured (but measurable) and unpredicted parameters (mainly the $\mu$-term). The error on these predictions, due to remaining NNLO effects, is estimated to be at the per-mille level, much smaller than the expected experimental error on the gaugino masses.

In figs. 2, 3 we have also plotted the corresponding NLO prediction of unified supergravity models, without including unknown possible GUT-scale corrections. The unification relation $\rho_i \propto M_i$ is in fact only corrected at the % level by GUT-scale threshold and gravitational effects, that could instead give much larger corrections to the unification relations for the $\alpha_i$ and for the $M_i$. The RGE contribution from the top $A$ term, values of $x_N > 1$ (negative squared tree-level masses for scalar messengers) give an acceptable spectrum of physical messenger masses ($x_n(\mu) < 1$), leaving only charge and/or colour breaking (CCB) minima at very large field values. This situation is not forbidden. On the contrary, non-dangerous CCB minima are quite generic in gauge mediated models.
driven towards its IR fixed point value, \( A_t (M_Z) \approx 2 M_2 \), is also numerically negligible. The same can be said for the bottom and \( \tau \) contributions, that remain negligible also if \( \tan \beta \) is large.

At this point it is also interesting to discuss the NLO correction that the presence of messenger fields gives to the unification prediction for \( \alpha_3(M_Z) \). The percentage correction to \( \alpha_3(M_Z) \) is plotted in fig. 4 for models with different messenger content, assuming that all different messengers have a common unified value of the \( M \) and \( F \) parameters (if \( F \ll M^2 \) it is only necessary to assume that the various \( M_n \) have the same order of magnitude). If \( n_5 + 3 n_{10} > 5 \) too light messengers give a non-perturbative value of the unified gauge coupling.

It is interesting to see more in detail why this correction is much smaller than its naive expectation and why it exhibits some curious property. For given values of the unification scale and of the unified gauge coupling, the low energy gauge couplings receive two contributions due to the presence of messengers: \( \delta \alpha_3^{-1}(M_Z) = \delta_{\text{th}} + \delta_{\text{RGE}} \).

The first contribution, \( \delta_{\text{th}} \), is due to messenger thresholds (gauge corrections distort the unified messenger spectrum); the second one is due to messenger corrections to the running of the gauge couplings (we can reabsorb the one-loop contribution in the definition of the unified gauge coupling, and consider only the two-loop contribution). In a sufficiently accurate approximation the two corrections are

\[
\delta_{\text{th}} = \frac{1}{4\pi} \sum_{j} \left\{ b_{ij}^{(2)} \ln r_j - b_{ij}^{(2)\text{MSSM}} \ln r_j^{\text{MSSM}} \right\} \\
\delta_{\text{RGE}} = \frac{1}{4\pi} \sum_{p \in R_n} b_{ip} \ln \frac{\mu^2}{M_p^2} + \frac{1}{4\pi} \sum_{i} \sum_{j} 4T_i (R_n) C_j (X_n) \ln r_j \tag{13b}
\]

where \( \sum_p \) extends over all the fermionic and scalar messengers, \( b_{ip}^p \) is the contribution of a given messenger to \( b_{i}^{(1)} = b_{i}^{\text{MSSM}} + \sum_p b_{ip}^p \); and \( b_{i}^{(1)} \) and \( b_{ij}^{(2)} \) are the one and two-loop coefficients of the gauge \( \beta \)-functions in presence of messengers (explicitly given in appendix B). We define \( r_i \equiv (1 - b_i \alpha_3(\mu))/(4\pi) \ln M_{\text{GUT}}^2/\mu^2 \) while \( r_i^{\text{MSSM}} \) is its value without messengers. The overall correction to the unification prediction for the strong coupling constant is

\[
\delta_{\text{mess}} \alpha_3(M_Z) = \delta_{\text{th}} \alpha_3 + \delta_{\text{RGE}} \alpha_3 = -\alpha_3^2(M_Z) (\delta_{\text{th}} + \delta_{\text{RGE}}) I_i \tag{14}
\]
where $\Pi_i = \{5/7, -12/7, 1\}$. The two corrections, $\delta_{i}^{\text{th}}\alpha_3$ and $\delta_{RGE}^{\text{th}}\alpha_3$ can be quite large ($\pm O(0 \div 20)\%$) and depend separately on $n_5$ and on $n_{10}$. However the sum of the two contributions, plotted in fig. 4 is much smaller, typically positive, $\delta_{RGE}^{\text{th}}\alpha_3 = (0 \div 3)\%$, and depends only on the correction to the one-loop $\beta$-function coefficient $\delta^{\text{th mess}}_{i} = n_5 + 3n_{10}$ (this is not true in the limit $x \sim 1$, where supersymmetry-breaking effects become relevant). This cancellation can be seen summing the expression for $\delta_{i}^{\text{th}}$ in the limit $F \ll M$, eq. (13d) (in which we have inserted the messenger masses $M_n$ obtained via one-loop RGE evolution), with the RGE correction (in which we insert the general values of the two-loop $\beta$-function coefficients, written in eq. (5.2)):

$$\delta^{\text{RGE}}_{i} = \frac{\delta^{\text{th}}_{i}}{4\pi} \left\{ 2C(G_i) b^{\text{mess}}_{i} \ln r_i + b^{(2)}\text{MSSM}_{i} \ln \frac{r_j}{r_{j}\text{MSSM}} \right\}.$$  

(15)

Corrections due to possible messenger Yukawa couplings would cancel out. These cancellations reproduce the exact result found by J. Hisano and M. Shifman in [3, 4] working in toy models with the holomorphic supersymmetric gauge couplings. As shown in [4] this same reason is at the basis of the analogous cancellation between RGE and threshold effects encountered in our NLO computation of gaugino masses at $F \ll M$.

4 Conclusion

We have computed the next-to-leading order corrections to gaugino masses in gauge-mediated models for generic values of the messenger masses $M$. In unified messenger models there are up to 10% corrections to the unification-like relations $M_i(\bar{\mu}) \propto \alpha_i(\bar{\mu})$ between the running gaugino masses and the gauge couplings, but only if the messengers are strongly split by supersymmetry breaking. If instead $M > 100$ TeV there are only small (few %) corrections to the leading-order approximation $M_i \propto \alpha_i$, as shown in figs. 2 and 3.

We have also studied the messenger corrections to gauge coupling unification. As a result of cancellations, dictated by supersymmetry, between large RGE and threshold corrections the predicted value of the strong coupling constant is typically only negligibly increased, as shown in fig. 4.

In the limit $M^2 \gg F$ (heavy messengers) the same NLO prediction for gaugino masses, together with NLO predictions for sfermion masses, can be obtained [16] combining the techniques described in [17] and [9, 10]. If $F \sim M^2$ it is more difficult to obtain a NLO prediction of sfermion masses; however the LO results [11] show that the effects of large supersymmetry breaking in the messenger spectrum are much less relevant in the sfermion sector than in the gaugino sector.

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A Relevant Lagrangian in quadri-spinor notation

We consider a theory with messenger chiral superfields $\Phi_L$ and $\Phi_R$ in self-conjugate complex representations of the SM gauge group (with generators $T$ and $-T^T$). In presence of the superpotential

$$W = \int d^2\theta \lambda S \Phi_L \Phi_R, \quad \lambda(\bar{S}) = M + \theta F.$$  

The messenger superfields $\Phi = A + \sqrt{2}\theta \psi + \cdots$ contain the following mass eigenstates: messenger fermions $\psi_L, \psi_R$ with a Dirac mass $M$, and pseudoscalar (scalar) messengers $A_{\pm} \equiv (A_L \pm A_R)/\sqrt{2}$ with mass $M_3^2 = M^2 \pm F^2 \equiv M^2(1 \pm x)$. The supersymmetric gaugino Lagrangian is

$$\mathcal{L}_\chi = \bar{\lambda} \gamma \mu D_{\mu} \lambda - \frac{m_2}{2}(\lambda \lambda + \bar{\lambda}\bar{\lambda}) - \sqrt{2}g(A^L_{\mu} T \psi_L \lambda + \bar{\lambda}\bar{\psi}_L T A_L - \lambda \psi_L T A_R - A^R_{\mu} T \bar{\psi}_R \bar{\lambda})$$  

where $D_{\mu}$ is the standard gauge-covariant derivative and $\lambda, \psi_L$ and $\psi_R$ are Weyl fermions with the same chirality.
We want to rewrite the Lagrangian in terms of mass eigenstates. In order to employ our Mathematica code for analytic computation of Feynman graphs, we need to write the messenger fermions as Dirac quadri-spinors $\Psi$ and the gauginos $\lambda$ as Majorana spinors $\Lambda$:

$$
\Psi \equiv \left( \begin{array}{c} \psi_L \\ \psi_R \end{array} \right), \quad \Lambda = \left( \begin{array}{c} \lambda \\ \lambda \end{array} \right) = \Lambda C \Lambda^T
$$

where $C$ is the charge-conjugation matrix. The gaugino Lagrangian becomes

$$
\mathcal{L}_\lambda = \frac{1}{2} \lambda \left( i \partial - m \right) \lambda - g(A_\lambda^\dagger T \lambda \Psi + \Psi \lambda T A - A_\lambda^\dagger \lambda \gamma_5 \Psi + \Psi \gamma_5 \lambda T A)
$$

and the $D$ terms become

$$
D^{\alpha} = A_\lambda^\dagger T^\alpha A - A_R T^\alpha A_R + \cdots = A_\lambda^\dagger T^\alpha A - A^\dagger T^\alpha A + \cdots
$$

The gaugino $\Lambda \Lambda$ propagator is a standard fermion propagator. It is possible to show that graphs with $\Lambda \Lambda$, $\bar{\Lambda} \Lambda$ and $\bar{\Lambda} \bar{\Lambda}$ propagators have the same value of the standard $'\Lambda \bar{\Lambda}'$, by appropriately rewriting the vertices in terms of charge conjugated fields. This shows that the gaugino can be treated as an ordinary fermion field, but with symmetry factors computed like the ones of a real field.

In a general theory there will be several messenger pairs, each one with its $M_n$, $F_n$ and $x_n \equiv F_n/M_n^2$.

## B RGE evolution

The necessary RGE can be read from the literature. The RGE equations for gauge couplings and gaugino masses in the DM scheme are

$$
\frac{d}{dt} \frac{1}{g_i^2} = b_i^{(1)} + \frac{1}{(4\pi)^2} \left[ \sum_j b_{ij}^{(2)} g_j^2 - \sum_a b_{ia}^{(2)} \lambda_a^2 \right]
$$

$$
\frac{d}{dt} M_i = g_i^2 b_i^{(1)} M_i + \frac{g_i^2}{(4\pi)^2} \left[ - \sum_j b_{ij}^{(2)} g_j^2 (M_i + M_j) + \sum_a b_{ia}^{(2)} \lambda_a^2 (M_i - A_a) \right]
$$

where $t(E) \equiv (4\pi)^{-2} \ln M_{GUT}^2/E^2$ and $a$ runs over the third generation particles, $a = \{t, b, \tau\}$. In a general supersymmetric model with $\{\Phi\}$ matter superfields the coefficients are

$$
b_i^{(1)} = -3C(G_i) + \sum \Phi T_i(\Phi), \quad b_i^{(2)} = 2C(G_i) b_i^{(1)} \delta_{ij} + 4 \sum \Phi T_i(\Phi) C_j(\Phi).
$$

In the models under consideration the field content is given by the MSSM fields plus $n_5$ copies of $5 \oplus 5$ and $n_{10}$ copies of $10 \oplus \overline{10}$ SU(5) messenger multiplets (in the effective theory without messengers, the coefficients are obtained taking $n_5 = n_{10} = 0$). The values of the coefficients are

$$
b_i^{(1)} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} + (n_5 + 3n_{10}) \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad b_i^{(2)} = \begin{pmatrix} 26/5 \\ 6 \\ 2 \\ 18/5 \end{pmatrix}
$$

$$
b_{ij}^{(2)} = \begin{pmatrix} 199/15 + 7n_5 + 23n_{10} \\ 98/15 + 7n_5 + 6n_{10} \\ 27/15 + 9n_5 + 3n_{10} \\ 88/15 + 48n_{10} \end{pmatrix}.
$$

In numerical computations it is useful to employ the RGE for $\rho_i \equiv M_i/\alpha_i$, since it starts at two-loop order

$$
\frac{d}{dx} \rho_i = -\frac{g_i^2}{4\pi} \left[ \sum_j b_{ij}^{(2)} g_j^2 M_j + \sum_a b_{ia}^{(2)} \lambda_a^2 A_a \right].
$$

Only the $X \bar{X}$ diagram of fig. that needs a Majorana gaugino propagator to be non-zero, requires a more detailed treatment of the charge conjugation factors.
When $F \sim M^2$ supersymmetry is hardly broken in the effective theory below the messenger scale: the couplings at the supergauge gaugino vertices differ (by a numerically negligible amount) from the corresponding gauge couplings. However in this case the messengers are light so that it is sufficient to employ the one loop RGE equations below the messenger scale.

Finally the gauge contribution to the 2 loop $d\Omega$ RGE equations for the supersymmetric messenger masses $M_n$ and for the ‘$F$-terms’ $F_n$ is

$$\frac{d}{dt} \ln M_n = \frac{d}{dt} \ln F_n = 2C_i(X_n)g_i^2 - \frac{1}{(4\pi)^2} \left\{ g_i^2 g_j^2 2C_i(X_n) 2C_j(X_n) + g_i^4 2C_i(X_n) b_i^{(1)} \right\}.$$ 

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