I. INTRODUCTION

In the theory of multifractal time \( [1] - [10] \) for the case when the fractional dimensions of time \( d_t \) almost coincide with integer value (equal unit) \( (d_t = 1 + \sum \beta_i L_i \tau(t), \tau = 1 + \varepsilon, |\varepsilon| \ll 1) \) was shown (see [3]) that for modifying Lorentz transformations in the theory of fractal time it is necessary to change factor \( \beta = \sqrt{1 - v^2/c^2} \) by factor \( \beta^* = \sqrt[4]{1 + 4a_0^2} \). Then relativistic energy has form

\[
E = mc^2
\]  
(1)

\[
m = \beta^{-1}m_0, \beta^2 = 1 - \frac{v^2}{c^2}, v \leq c
\]  
(2)

\[
m = \beta^*-1m_0\sqrt{1 + \beta^*2 + \beta^2}, \beta^2 = \frac{v^2}{c^2} - 1, v \geq c
\]  
(3)

\[
p = \beta^{-1}m_0v
\]  
(4)

\[
\beta^* = \sqrt[4]{1 + 4a_0^2(t)}
\]  
(5)

\[
a_0 = \sum_i \beta_i F_{0,i} \frac{v}{c} \tau_i, \sum_i F_{0,i} = \sum_i \frac{\partial}{\partial t} L_i
\]  
(6)

On the base of fractal theory in this paper we presented the new equations. These equations are analogues of relativistic equations (scalar, vector, tensor, spinor) and valid in the domain of arbitrary velocities including speed of light if external physical fields don’t equal zero (but so small that fractal dimensions of time is near unit). These equations in the case of absence of physical fields coincide with the usual relativistic equations, but give the new fields with imaginary energies (including the case of rest energies). If the external fields are not equal zero, the received equations give also a new spin characteristics.

II. EQUATIONS FOR FIELDS ON THE BASE OF MODIFIED RELATIONS FOR ENERGY

Let us write formulas \( [1] - [2] \) in the form (using the hypothesis about approximate conservation of the energy-momentum vector in the space with fractal time \( [1] \))

\[
E^2 = \frac{E_0^2}{\beta^2} = \frac{P^2 c^2}{\beta^2} + E_0^2
\]  
(7)

We introduce now the new designation for relativistic energy and momentum similar to those that have been used in SR \( (\beta^2 = E^2E_0^{-2}, E = m_0c^2\beta^{-2}) \) (see \( [1] \)). Then equation without roots has form

\[
\frac{(E^2 - P^2c^2)^2}{(1 + 4a_0^2E^4E_0^{-4})} = E_0^4
\]  
(8)

\[
E = m_0\beta^{-1}, p = m_0v\beta^{-1}
\]  
(9)

The equation \( (8) \) is the base equation for describing the energy in the space with multifractal time. For \( E \) we receive in the absence of fields and the momentum equal zero the four solutions: \( E = \pm E_0, E = \pm iE_0 \). Thus, there are particles and anti-particles with real \( (E = \pm E_0) \) and imaginary \( (E = \pm iE_0) \) masses. The last are new sort of particles with imaginary masses. These particles are not taxions because they exist in the domain of velocities \( v \geq 0 \) (including velocity \( v = c \)). For receiving equations for fields and particles we use the ordinary method replacing the energy and the momentum by derivatives. This method consists in changing the energy \( E = E_0/(1 - v^2/c^2)^{1/2} \) and the momentum \( P = m_0v/(1 - v^2/c^2)^{1/2} \) by derivatives with respect to time and space coordinates that used in quantum mechanics: \( \hbar = c = 1, \frac{\partial}{\partial t} \rightarrow i\frac{\partial}{\partial \tau} = \hat{E}, \frac{\partial}{\partial \tau} \rightarrow -i\nabla_j = \hat{p}, (j = 1, 2, 3) \). So we obtain \( ( \text{in case } v \leq c ) \) for function \( \Phi (r, t) \) the integral -differential equation

\[
\frac{(\hat{E}^2 - \hat{P}^2)^2}{(1 + 4a_0^2E^4E_0^{-4})} \Phi (r, t) = E_0^4 \Phi (r, t)
\]  
(10)
where \( \hat{E} \) and \( \hat{p} \) are differential operators and was determined early. For simplifying this equation we multiply it by operator \( (1 + 4a_0^2) \frac{\partial^4}{\partial t^4} \) (thou it may introduce non-physical solutions but it is makes the equation (11) only differential). So we obtain

\[
(\Box^2 - 4a_0^2 \frac{\partial^4}{\partial t^4}) \Phi(r, t) = E_0^4 \Phi(r, t)
\]

where \( \Box \) is D’Alamber operator \( (\Box = \Delta - \frac{\partial^2}{\partial t^2}) \), \( \Delta \) is Laplasian). \( \Phi \) are functions describing particles or fields. For scalar \( \Phi \) equation (10) describes the scalar field in the space with fractal dimensions that originated by the presence of the external physical fields \( (a_0 \neq 0) \). The corrections in (11) to the usual D’Alamber equation are the result of modifying the Lorentz transformation. The last is consequences of fractal nature of time. For receiving the equations in which taken into account the influence of multifractal structure of time on using the derivatives and receive more correct equation it is necessary to use the generalized Riemann-Liouville fractional derivatives (GFD) \[7, 1\]. In that case equation (11) take the form

\[
(D^4_{+, t} - \Delta^2) \Phi(r, t) = [E_0^4 + 4a_0^2(D^4_{+, t})^2] \Phi(r, t)
\]

where

\[
D^4_{+, t} f(t) = \left( \frac{d}{dt} \right)^n \int_a^t \frac{f(t')dt'}{\Gamma(n - d(t'))(t - t')^{d(t') - n + 1}}
\]

and

\[
D^4_{-, t} f(t) = \left( \frac{d}{dt} \right)^n \int_t^b \frac{(-1)^n f(t')dt'}{\Gamma(n - d(t'))(t' - t)^{d(t') - n + 1}}
\]

where \( \Gamma(x) \) is Euler’s gamma function, and \( a \) and \( b \) are some constants from \([0, \infty)\). In these definitions, as usually, \( n = \{ d \} + 1 \), where \( \{ d \} \) is the integer part of \( d \) if \( d \geq 0 \) (i.e. \( n - 1 \leq d < n \)) and \( n = 0 \) for \( d < 0 \). In this paper we don’t consider equations with fractal derivatives and restrict consideration only by calculation of corrections from alterations of Lorentz transformation

III. EQUATIONS OF FOUR AND SECOND ORDER IN DERIVATIVES

It is useful rewrite (11) in the form

\[
[\Box^2 - 4a_0^2 \frac{\partial^2}{\partial t^2}] \Phi = \alpha_1 E_0^4 \Phi
\]

where \( \Phi \) is a four element bispinor column

\[
\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \\ \Phi_3 \\ \Phi_4 \end{pmatrix}
\]

So, we have four equations for \( \Phi_1, \ldots, \Phi_4 \) (17).

\[
\Box \Phi_1 - 2a_0^2 \frac{\partial^2}{\partial t^2} \Phi_4 = E_0^2 \Phi_1
\]

\[
\Box \Phi_2 - 2a_0^2 \frac{\partial^2}{\partial t^2} \Phi_3 = E_0^2 \Phi_2
\]

\[
\Box \Phi_3 - 2a_0^2 \frac{\partial^2}{\partial t^2} \Phi_2 = -E_0^2 \Phi_3
\]

\[
\Box \Phi_4 - 2a_0^2 \frac{\partial^2}{\partial t^2} \Phi_1 = -E_0^2 \Phi_4
\]

In the equations (18)-(21) the first two equations describe the particles or fields with real energies, the last two equations describe the new particles or fields with imaginary energies. The energies of particles with real energies depends on behavior of fields with imaginary energies and vice versa. The new spin characteristics consequences of these equations.

IV. EQUATIONS OF FIRST ORDER IN DERIVATIVES

For receiving of first order equations in derivatives it is necessary to introduce the Dirac matrices \( \gamma_i \) \( (i = 0, 1, 2, 3) \) and Dirac type matrices \( \sigma_j \) \( (j = 0, 1, 2, 3) \). These matrices may be used for splitting of the left-hand side and the right-hand side of each of the equations (18)-(21). We write the function \( a_0 \) in the form

\[
a_0 = a_g + a_e + a_n = \beta_3 L_3^3(r, t) + \beta_n L_n(r, t)
\]

where \( L_3^3, L_n \) are Lagrangians density of energies for gravitational, electro-weak and strong fields. Let a module of complex function \( \Phi_i \) is the probability to find the particle in a moment \( t \) in a point \( r \). In that case the square root \( \sqrt{\Phi_i} = \psi_i(r, t) \) gives the function with characteristics: \( \psi \bar{\psi} \) has sense of probability. Now if change in equations (18)-(21) the differential operators by \( E \) and \( P \), translate in the right-hand side of equations all members with \( a_0 \) and than extract a square root from left-hand side and right-hand side of these equations we obtain (after splitting we change \( E \) and \( p \) by...
ordinary differential operators and use the designations \( \psi_j(i) \) where indexes \( i \) corresponds to the indexes \( i \) at \( \Phi_i \) and indexes \( j \) corresponds to each splitting component of \( \Phi_i \)

\[
i\gamma_i \partial_i \sqrt{\Phi} = E_0 \sigma_1 \sqrt{\Phi} + \sqrt{2a_i \sigma_2 i \partial_i \sqrt{\Phi}} + \sqrt{2a_i \sigma_3 i \partial_i \sqrt{\Phi}} + \sqrt{2a_i \sigma_4 i \partial_i \sqrt{\Phi}} \tag{23}\]

\[
i\gamma_i \partial_i \sqrt{\Phi} = E_0 \sigma_1 \sqrt{-\Phi} + \sqrt{2a_i \sigma_2 i \partial_i \sqrt{-\Phi}} + \sqrt{2a_i \sigma_3 i \partial_i \sqrt{-\Phi}} + \sqrt{2a_i \sigma_4 i \partial_i \sqrt{-\Phi}} \tag{24}\]

\[
i\gamma_i \partial_i \sqrt{\Phi} = E_0 \sigma_1 \sqrt{-\Phi} + \sqrt{2a_i \sigma_2 i \partial_i \sqrt{\Phi}} + \sqrt{2a_i \sigma_3 i \partial_i \sqrt{\Phi}} + \sqrt{2a_i \sigma_4 i \partial_i \sqrt{\Phi}} \tag{25}\]

\[
i\gamma_i \partial_i \sqrt{\Phi} = E_0 \sigma_1 \sqrt{\Phi} + \sqrt{2a_i \sigma_2 i \partial_i \sqrt{-\Phi}} + \sqrt{2a_i \sigma_3 i \partial_i \sqrt{-\Phi}} + \sqrt{2a_i \sigma_4 i \partial_i \sqrt{-\Phi}} \tag{26}\]

\[
\sqrt{\Phi}_j = \begin{pmatrix} \psi_1(j) \\ \psi_2(j) \\ \psi_3(j) \\ \psi_4(j) \end{pmatrix}, \quad (j = 1, 2, 3, 4) \tag{27}\]

The equations \ref{23}-\ref{26} are generalized Dirac equations based on taking into account only the modified by fractal nature of time Lorentz transformations (and possibility of moving with speed of light as the consequences of it). The eight equations \ref{23} and \ref{24} describe the particles (or fields) with spin \( \frac{1}{2} \) (\( \Phi_i \) are bispinors), real energy and new characteristics "quasi-spin" originated by influence of the fields with imaginary energy. The last described by the equations \ref{25} and \ref{26}. Thus there are two sorts of particles described by equations \ref{23}-\ref{26}: with real energies and with imaginary energies. Each sort of particles have own anti-particles and "quasi-spin". In these equations taken into account the influences of all known fields (gravitational, electro-weak, strong).

V. ELECTROMAGNETIC FIELDS WITH A REST MASS (EQUATIONS PROCA)

Let us consider the case when each of functions \( \Phi_i \) is four-vector of electro-magnetic field \( \Phi_i(0), \Phi_i(1), \Phi_i(2), \Phi_i(3) \) with a rest energy \( E_0 \). The equations \ref{11} take form

\[
\Box \Phi_\mu(i) = [\alpha_1 E_0^2 + 2a_0 \partial_\mu \partial_\nu \alpha_2] \Phi_\mu(i), \quad (\mu = 0, 1, 2, 3) \tag{28}\]

So there are four sorts of electro-magnetic Proca fields: fields of Proca photons and Proca anti-photons with real and imaginary energies and different "quasi-spins". For \( a_0 = 0 \) all the equations coincide with Proca equations.

VI. MAXWELL EQUATIONS

If the Maxwell equations of electro-magnetic field may be considered as the equations of the Proca in the limits of a rest mass equal zero, in that case the new sorts of electro-magnetic fields (anti-photons fields and fields with imaginary energies) as consequences of the Proca fields appear. Let us write equations for 4-vector electromagnetic potentials \( A_\mu(i) \) (which are consequences of equations \ref{11} and \ref{12}-\ref{21}) for electromagnetic fields where role \( E_0^2 \) plays 4-vector of electric current \( j_\mu, (\mu = 0, 1, 2, 3) \)

\[
\Box A_\mu(i) = [\alpha_1 j_\mu(i) + 2a_0 \partial_\mu \alpha_2] A_\mu(i), \quad (\mu = 0, 1, 2, 3) \tag{29}\]

In equation \ref{29} \( A_\mu(i) \) is a 4-column with respect to \( i \). The equations \ref{29} are generalized Maxwell equations for photon and anti-photon fields, with usual and "quasi" spins and coincide for \( a_0 = 0 \) with usual Maxwell equations. The matrices \( \alpha_1 \) and \( \alpha_2 \) may be chosen for example as

\[
\alpha_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \alpha_2 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \tag{30}\]

The last eight equations \ref{29} describe anti-photon fields and its energy (if electrical charges for anti-photon fields are real) may be imaginary. In these fields the role of minus-sign charges will play plus-sign charges and vice versa. There is a difference between speed of photons light and anti-photons light. The speeds of photons light and anti-photons light are equal only if \( a_0 = 0 \).

VII. CONCLUSION

There are new consequences of equations based on the modified Lorentz transformations in the multifractal time theory and now we stress it: a) the existence of two imaginary solutions for the rest energy \( (iE_0 \) and \( -iE_0) \) and thus the existence of new class of particles with imaginary mass. Because its velocities are arbitrary \( (0 \leq v \leq \infty \) they are not taxions); b) the appearance of new "quasi-spin" characteristics. The cause of it lays in the more high order (four order) differential equations then usual equations and in additional splitting of square roots and taking into account the existence of external fields \( a_0 \). The physical sense and physical nature of new spin characteristics are not clear. The nature of the additional "spins" originated by additional decompositions of square roots for equations of four order and physical sense of it needs in special investigation; c) all equations coincide with known physical equation if fractal dimension of time is integer (so in that sense the theory is not contradicts known physical theories). In the fractal
theory of time the last case corresponds to vanishing of physical fields (the last originates the fractional dimensions of time); d) the theory use the improved classical relations for relativistic energy which take into attention the fractional dimensions of time and allow motions with arbitrary velocities (including velocities equal the speed of light); e) it may be shown that the equations for case \( v \geq c \) some differs at equations used in the paper but main results the theory based on them coincide qualitative with the results used in this paper(if use the equations of this paper for case \( v \geq c \)).

The presented in this paper the theory based on a relative motions in almost inertial systems which in turn based on the multifractal time theory [1] and gives the new describing for characteristics of moving bodies (energy, momentum, mass and so on). The main results of this theory used in this paper are: a) the possibility of moving with arbitrary velocities without appearance of infinitum energy and imaginary mass; b) existence of maximum energy if \( v = c \); c) possibility of experimental verification the main results of the theory.

The theory [2, 11] describes the Universe as an open systems (the theory of open systems see in [11]). This theory coincides with SR after transition to inertial systems (if neglect by the fractional dimensions of time) or almost coincides (the differences are non-essential) for velocities \( v < c \). The movement of bodies with velocities that exceed the speed of light is accompanied by a series of physical effect’s which can be found by experiments (these effects was considered in the separate papers (3, 5, 9)) in more details). It is shown in these papers the necessity to receive the particles with energies \( \sim E_0 \frac{1}{\sqrt{t}} \) and \( \frac{\partial}{\partial x} a_e < \frac{\partial}{\partial x} a_g \) for verification of the theory. If accelerate the particles by electric fields then

\[
E_0 [2E(Mc_2)^{-1}]^{-\frac{1}{2}} < E_0 (\sqrt{2a_g t})^{-1}
\]

In this formula \( E \) is the electric field strength, \( M \) is the mass of electric charges originated the \( E \). It is useful to pay attention to the problem of receiving the particles with such energies of the physicians of known accelerate centers. If the possibilities for organizing such works will be found, the results are useful for development of our views on the nature of an energy, the time and the space.

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