Design of adaptive sliding mode controllers for perturbed nonlinear systems with partial unmeasurable states and state constraints

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Abstract A sliding mode control (SMC) strategy is proposed in this paper for a class of perturbed nonlinear systems with unmeasurable states and state constraints to deal with the state tracking problems. First of all, a partial states observer is designed for solving the problems due to unmeasurable states. The estimation errors will approach zero in a finite time. Secondly, based on a designed barrier Lyapunov function, one designs the sliding surface function and an adaptive sliding mode tracking controller to ensure that the states have the ability to track the desired signals. Moreover, the tracking error is capable of converging to zero in a finite time without violating the given state’s constraints. Perturbation estimator and adaptive mechanisms are also utilized so that there is no need to know the upper bounds of perturbations and perturbation estimation errors in advance. Finally, a numerical example and a practical application are provided to demonstrate the effectiveness and applicability of the proposed control strategy.

Keywords Barrier Lyapunov function · State observer · Sliding mode control · Adaptive control · Perturbation estimator

1 Introduction

In order to increase the applicability of the designed control systems, there are two important factors one has to consider besides the stability issue. The first one is the availability of the state variables, the second is the magnitudes of states that the controlled system can sustain. These two factors generally increase the difficulty of designing the controllers. Therefore, studying systems that have unmeasurable states and also have state constraints has become a very important issue.

In general, there are two ways to deal with the problems caused by unmeasurable states. One of which is to design the output feedback controller. This includes [1–5]. The other way is to employ an observer. For controlling nonlinear systems with utilization of observer, Li et al. [6, 7], Park et al. [8], Wang et al. [9], Choi and Yoo [10] and Huang [11] all made the estimation errors uniformly ultimate bounded (UUB). For solving the problems of regulation, Lei [12] utilized a high-gain predictor for a single-input single-output (SISO) nonlinear system with state delay and finally made the estimation error asymptotically stable (A.S.). Laila [13] proposed an observer for a combustion engine test bench, and also made the estimation error A.S. Although the control methods mentioned above can handle the systems with unmeasurable states, several disadvantages are found in these systems: I. The dynamic equations of plants in [4–10, 12] have to be in strict feedback form. II. The methods proposed in [6–11] can only achieve...
the property of UUB. III. They cannot solve the problems of state constraints. IV. The methods proposed in [3, 4, 7–9, 12, 13] need to meet Lipschitz condition.

As for solving the state constraint problems, many methods mainly relied on numerical methods or used complex algorithms. For example, invariant set method [14, 15], model predictive control [16–18], and reference governor [19]. The disadvantage of these methods is their huge amount of computation, which leads to longer control time and may cause instability in the system. Therefore, lots of researches on state constraints have been developed based on establishing barrier Lyapunov functions due to its simplicity. When utilizing a barrier Lyapunov function methodology [20–35], one needs to create a barrier Lyapunov function first, and then design the controller through the function such that not only the barrier Lyapunov function is bounded, but also the controlled system is stable.

In order to deal with unmeasurable states and state constraint problems, backstepping control which utilizing barrier Lyapunov function were proposed in [21–27] for perturbed nonlinear systems, these methods all achieved the property of UUB. Intelligent control schemes were also developed for solving unmeasurable states and state constraint problems, and they also achieved the property of UUB [28–35]. Although the aforementioned works [21–35] can handle the problems of unmeasurable states and state constraints at the same time, several disadvantages are found in these control systems: I. The dynamic equations with match or unmatched perturbations considered in [21–28, 30–35] have to be in strict feedback form. II. The methods proposed in [21–35] can only achieve the property of UUB. III. The methods proposed in [21, 24, 26, 28–32] may be only applied to SISO systems.

Recently Chiang and Cheng [36] proposed an adaptive output feedback sliding mode control scheme for a class of nonlinear systems with matched and unmatched perturbations to solve regulation problems. The advantages of this control scheme include (a) the dynamic equations of the system do not need to be in strict feedback form; (b) there is no need to know the upper bounds of both match and unmatched perturbations in advance; (c) the state variables can converge to zero within a finite time; (d) it can be applied to multi-input multi-output (MIMO) systems.

In this paper, we extend the research of [36] and propose an adaptive SMC scheme as well as a partial state observer for a class of perturbed systems with state constraints, so that both the estimation errors of states and tracking errors can converge to zero in a finite time. The contributions of this paper are as follows. (1) The proposed control scheme not only eliminates the disadvantages I to IV mentioned above, but also has the advantages (a) to (d) addressed previously. (2) Some state variables do not have to be measurable. (3) The proposed controller has the ability to constrain the magnitudes of full states; (4) The proposed control scheme can solve tracking problems, and all the state-tracking errors can converge to zero within a finite time and stay thereafter if the desired tracking signals are properly assigned.

The organization of this paper is as follows. The system description and problem formulation is introduced in Sect. 2. In Sect. 3, the design methodology of partial states observer is demonstrated. Section 4 presents the stability analysis of designed partial states observer. The proposed adaptive sliding mode controller is given in Sect. 5. In Sect. 6, the stability analysis of the overall controlled system is addressed in detail. In order to show the effectiveness and applicability of the proposed control scheme, a numerical example and a practical application are illustrated in Sects. 8 and 9 respectively. Finally, the conclusions and future works are stated in Sect. 10.

**2 System descriptions and problem formulations**

Consider a class of MIMO nonlinear systems with matched and unmatched perturbations, whose dynamic equations are governed by

\[
\dot{x}(t) = f(x) + Bu(t) + \Delta p(t, x, u) \\
y(t) = Cx(t),
\]

(1)

where \(x(t) \in \mathbb{R}^n\) represents unmeasurable state of the system, \(u(t) \in \mathbb{R}^m\) is the control input, \(y(t) \in \mathbb{R}^p\) is the measurable output. The nonlinear vector \(f(x)\) is known and \(f(x) = 0\) if \(x = 0\). The constant matrices \(B\) and \(C\) with approximate dimensions are also known, and \(\text{rank}(B) = m, \text{rank}(C) = p\). The unknown vector \(\Delta p(t, x, u)\) can be viewed as model uncertainty, nonlinearity or external disturbance. In addition, the number of outputs is greater than or equal to the number of inputs, i.e., \(p \geq m\).
The aim of this paper is to design an adaptive sliding mode tracking controller and a partial state’s observer such that the following objectives are achieved:

(a) The estimation error \( \tilde{z} = z - \hat{z} \) will converge to zero in a finite time, where the states \( z \) and \( \hat{z} \) will be introduced in Sect. 3.

(b) The state tracking error \( e(t) = x - x_d(t) \) will also approach zero in a finite time, and satisfies the requirement of \( |e(t)| < 2k_b + ||Wz|| \), \( \forall t \); where \( x_d(t) \) is the desired tracking signal, \( k_b > 0 \) is a preassigned real constant, and \( W \) is a matrix which will be introduced in Sect. 3. The notation \( ||\cdot|| \) stands for the Euclidean norm of a vector or the induced two-norm of a matrix.

3 Design of partial states observer

For achieving the control objectives, we first design a new state variable as

\[
s(t) = Fy(t) \in R^n,
\]

where \( F \) is a designed constant matrix which fulfills the matrix equation \( B^S = FC \), and \( B^S = (B^T B)^{-1}B^T \) denotes the Moore–Penrose pseudoinverse of \( B \). Note that if the columns of \( B^S \) are in the \( R(C^T) \), then matrix \( F \) exists, where \( R(\cdot) \) symbolizes the range space of \( \cdot \).

According to the method addressed in [36], one is able to transform (1) as

\[
\dot{z}(t) = W^S \Phi_1(s(t)) + \Delta p_1(t, x, u),
\]

\[
\dot{s}(t) = B^S \Phi_1(s(t)) + B^S \Phi_2(z, s) + u(t) + B^S \Delta p,
\]

where \( M[z^T(t), s^T(t)]^T = x(t) \), \( M \triangleq [W, B] \), \( W \in R^{n \times (n-m)} \) is a full rank matrix, \( R(W) \cap R(B) = \{0\} \), \( \Delta p_1 \triangleq W^S \Phi_2 z + W^S \Delta p \). The matrix \( W^S = (W^T W)^{-1} W^T \) is the Moore–Penrose pseudoinverse of \( W \), the two matrices \( \Phi_1, \Phi_2 \) are given by [36]

\[
\Phi_1(s) = \int_0^1 \frac{\partial f(0, A_1)}{\partial A_1} \bigg|_{A_1 = s} ds \in R^{n \times m},
\]

\[
\Phi_2(z, s) = \int_0^1 \frac{\partial f(A_2, s)}{\partial A_2} \bigg|_{A_2 = z} ds \in R^{n \times (n-m)}
\]

Noted that the new state variable \( z(t) \in R^{n-m} \) is unmeasurable.

Now, one designs a partial states observer for estimating \( z(t) \) as

\[
\dot{\hat{z}}(t) = W^S \Phi_1(s(t)) - \frac{e_z}{||e_z||^2} \sin \theta_1(e_z) \cos \theta_1(e_z) + \eta_1(t) + W^S \dot{x}_d,
\]

where

\[
e_z = \hat{z} - W^S x_d,
\]

\[
\theta_1 = \frac{||W||^2 ||e_z||^2 \pi}{2k_b^2}.
\]

The function \( \eta_1 \in R^{n-m} \) in (4) is generated from

\[
\dot{\eta}_1(t) = \begin{cases} 
-\rho_1 + h_1 \zeta(u) + \dot{c}_0(t) + \dot{c}_1(t) \beta(\hat{\xi}) & \text{if } \eta_1(t) \neq 0 \\
\Phi_1 e^{-f} & \text{if } \eta_1(t) = 0 
\end{cases}
\]

where \( \rho_1 \) is a designed positive constant, \( \Phi_1 \) is a designed vector with positive constant entries, \( \eta_1 = ||\eta_1|| + ||W^S \hat{x}_d|| + ||\sin \theta_1 \cos \theta_1||/||e_z|| \). The adaptive gains \( \dot{c}_k(t), k = 0, 1, \) are computed from the following adaptive laws as

\[
\dot{c}_0(t) = \begin{cases} 0 & \text{if } \eta_1 = 0 \\
1, & \text{otherwise} \end{cases}
\]

\[
\dot{c}_1(t) = \begin{cases} 0 & \text{if } \eta_1 = 0 \\
\beta(\hat{\xi}), & \text{otherwise} \end{cases}
\]

where \( \beta(\hat{\xi}) \) in (9), \( h_1 \) and the function \( \zeta(u) \) in (7) are introduced in the next section. By using the Squeeze Theorem [37] and L’Hospital’s rule, one is able to verify that

\[
\lim_{||e_z|| \to 0} \frac{e_z}{||e_z||^2} \sin \theta_1(e_z) \cos \theta_1(e_z) = 0.
\]

4 Stability analysis of partial states observer

Let \( \tilde{z}(t) = z(t) - \hat{z}(t) \) be the estimation error of partial states observer. Then from (3) and (4), one can obtain

\[
\dot{\tilde{z}}(t) = \Delta p_1 + \frac{e_z}{||e_z||^2} \sin \theta_1 \cos \theta_1 - \eta_1(t) - W^S \hat{x}_d.
\]

The stability analysis of estimation error \( \tilde{z}(t) \) is addressed in the following theorem.
Theorem 1 Consider the dynamic equation (10). Suppose that the designed vector $\Phi_1(s) \neq \mathbf{0}$ for some $t$, and there exist unknown positive constants $c_0$, $c_1$ satisfying the following inequality
\[ \|\Delta p(t, x, u)\| \leq c_0 + c_1 \beta(\hat{x}) + h_1 \zeta(u) \] (11)
in the domain of interest, where $\beta(\hat{x})$ and $\zeta(u)$ are known nonnegative functions, $h_1 > 0$ is also a known constant. If the partial states observer (4) with adaptive laws (8) and (9) are employed, then (a) the estimation error $\hat{z}$ will approach zero within a finite time;
(b) the state variable $\eta_1$ is bounded for all time;
(c) the adaptive gains $\hat{c}_i$, $i = 0, 1$ are all bounded, and $\hat{c}_i$ will converge to a finite limit respectively as $t \to \infty$.

Proof Define a Lyapunov function candidate as
\[ V_1 = \|\hat{z}(t)\| + \|\eta_1(t)\| + \frac{1}{2} \sum_{k=0}^{1} \zeta_k^2(t), \] (12)
where $\zeta_k(t) = \hat{c}_k(t) - c_k$ are adaptive errors of unknown positive constants $c_k$ respectively, $k = 0, 1$.

All the possible cases that may occur when computing the time derivative of $V_1$ are analyzed as follows.

Case 1: $\hat{z} \neq \mathbf{0}$, and $\eta_1 \neq \mathbf{0}$

Differentiate (12) with respect to time, we obtain
\[ \dot{V}_1 = \frac{\hat{z}^T(t)}{\|\hat{z}(t)\|} \hat{z}(t) + \frac{\eta_1^T(t)}{\|\eta_1(t)\|} \dot{\eta}_1 + \sum_{k=0}^{1} \hat{c}_k(t) \dot{\hat{c}}_k(t). \] (13)

Substituting (10) into (13) and using (11) yields
\[ \dot{V}_1 \leq h_1 \zeta(u) + c_0 + c_1 \beta(\hat{x}) + \frac{1}{\|e_\zeta\|} \sin \theta_1(e_\zeta) \cos \theta_1(e_\bar{z}) + \|\eta_1\| + \|W^\phi \dot{x}_d\| \frac{\eta_1^T(t)}{\|\eta_1(t)\|} \dot{\eta}_1 + \sum_{k=0}^{1} \hat{c}_k(t) \dot{\hat{c}}_k(t). \] (14)

By using (7), (8) and (9), one is able to obtain
\[ V_1 \leq -\rho_1 < 0. \] (15)

In this case, the magnitude of $V_1$ is bounded, which also indicates that the states $\hat{z}$, $\eta_1$, and adaptive errors $\hat{c}_0$, $\hat{c}_1$ are all bounded.

Case 2: $\hat{z} \neq \mathbf{0}$, and $\eta_1 = \mathbf{0}$

In this case, one can see that $V_1 = \|\hat{z}\| + \frac{1}{2} \sum_{k=0}^{1} \hat{c}_k^2$. From (8), (9) and (14), we can know that
\[ \dot{V}_1 \leq h_1 \zeta(u) + c_0 + c_1 \beta(\hat{x}) + \frac{1}{\|e_\zeta\|} \sin \theta_1 \cos \theta_1 + \|W^\phi \dot{x}_d\|. \]

In this case, $\dot{V}_1$ may be greater or smaller than zero. According to (7), $\dot{\eta}_1 \neq \mathbf{0}$ in this case. This implies that the trajectory of $\eta_1$ will cross the surface $\eta_1 = 0$ immediately during a very small time interval, that is, $\eta_1(t + \delta) \neq 0$ ($\delta$ is small enough such that $\dot{\eta}_1(t)$ does not change sign in the interval $[t, t + \delta]$). Hence, the trajectory of $\eta_1$ will cross the surface $\eta_1 = 0$ immediately, and the status of the system will switch to another case where $\eta_1 \neq 0$. Nevertheless, the states $\hat{z}$, $\eta_1$, and adaptive errors $\hat{c}_0$, $\hat{c}_1$ are all bounded due to the continuity of the controlled state variables.

Case 3: $\hat{z} = \mathbf{0}$ and $\eta_1 \neq \mathbf{0}$

From (12), it is seen that the Lyapunov function becomes $V_1 = \|\eta_1\| + \frac{1}{2} \sum_{k=0}^{1} \hat{c}_k^2(t)/2$. Differentiate it with respect to time and using (7), (8) and (9), one obtains
\[ \dot{V}_1 = -\rho_1 - h_1 \zeta(u) - c_0 - c_1 \beta(\hat{x}) - \Psi \leq -\rho_1 < 0. \]

Case 4: $\hat{z} = \mathbf{0}$ and $\eta_1 = \mathbf{0}$

In this case $V_1 = \frac{1}{2} \sum_{k=0}^{1} \hat{c}_k^2$. Differentiating it with respect to time and using (8), (9), one obtains
\[ \dot{V}_1 = \hat{c}_0 \dot{\hat{c}}_0 + \hat{c}_1 \dot{\hat{c}}_1 = 0. \]

The previous analysis shows that $V_1$ is a bounded function even if the occurrence of case 2. Hence one is able to see that
(a) the estimation error $\hat{z}$ will approach zero in a finite time as long as $\eta_1 \neq \mathbf{0}$;
(b) the state variable $\eta_1$ is bounded since $\eta_1^T \dot{\eta}_1 \leq 0$;
(c) the adaptive gains $\hat{c}_i$, $i = 0, 1$ are monotonically increasing and are all bounded in accordance with (8), (9) and the above analysis. Therefore, according to proposition 2.14 ([38], p. 83), there exit finite constants $c_{i, \infty}$, such that $\lim_{t \to \infty} \hat{c}_i = c_{i, \infty}$, $i = 0, 1$. \square

Remark 1 For avoiding the occurrence of case 2, one can modify (7) as
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\[ \dot{\eta}_1(t) = \begin{cases} 
-\left[ \rho_1 + h_1 \zeta(u) + \tilde{c}_0(t) + \tilde{c}_1(t) \beta(\tilde{x}) \right. & \text{if } \|\eta_1(t)\| \geq \alpha \\
\left. + \psi_I \frac{\eta_1(t)}{\|\eta_1(t)\|} \right] & \text{if } \|\eta_1(t)\| < \alpha \\
-\varphi_1 \text{sgn}[\eta_1(t)] & \text{if } \eta_1(t) = 0 \\
0 & \text{if } \eta_1(t) = 0 
\end{cases} \] 

(16)

where \( \alpha \) is a designed small positive constant. Equation (16) indicates that when \( \|\eta_1(t)\| < \alpha \), the magnitude of \( \eta_1(t) \) will slowly decrease if one chooses small value of \( \varphi_1 \).

5 Design of adaptive sliding mode tracking controller

To solve the tracking problem, one firstly designs the sliding surface function \( \sigma(t) \in \mathbb{R}^n \) as

\[ \sigma(t) = F(y - y_d), \]

(17)

where \( y_d = Cx_d \). By using (2), (3), (17) and (4), one can compute the time derivative of \( e_z \) and \( \sigma \) as

\[ \dot{e}_z(t) = W^z \Phi_1(s) s(t) - \frac{e_z}{\|e_z\|^2} \sin \theta_1 \cos \theta_1 + \eta_1 \]

(18)

\[ \dot{\sigma}(t) = F \dot{y} - F \dot{y}_d = \dot{s} - FCx_d \\
= B^z \Phi_1 s(t) + B^z \Phi_2 z + u(t) + B^z \Delta p - B^z \dot{x}_d. \]

(19)

Now, we propose an adaptive sliding mode tracking controller as

\[ u(t) = u_f(t) + u_s(t), \]

(20)

where

\[ u_f(t) = -\varepsilon \sigma(t) - B^z \Phi_1(s) s(t) - \eta_2(t) - p_{est} + B^z \Delta \dot{x}_d. \]

(21)

\[ u_s = \begin{cases} 
-\frac{\sigma(t)}{\|\sigma(t)\|} \left[ \kappa \cos^2 \theta_2(\sigma) + \sin \theta_2(\sigma) \cos \theta_2(\sigma) \right. & \text{if } \sigma(t) \neq 0 \\
\left. + \psi_2(e_z, s) \cos^2 \theta_2(\sigma) \right] & \text{if } \sigma(t) = 0 \\
0 & \text{if } \sigma(t) = 0 
\end{cases} \]

(22)

and the state \( \eta_2 \) is generated from

\[ \dot{\eta}_2(t) = \begin{cases} 
-\gamma - \psi_2(e_z, s) \frac{\eta_2(t)}{\|\eta_2(t)\|} & \text{if } \eta_2(t) \neq 0 \\
\varphi_2 e^{-\gamma} & \text{if } \eta_2(t) = 0, e_z \neq 0, \sigma = 0 \\
0 & \text{otherwise} 
\end{cases} \]

(23)

where the parameters \( \varepsilon, \kappa, \gamma \) are designed positive constants, and

\[ \varphi_2(\sigma) = \frac{\|B\| \|s\| \pi}{2k_b}, \]

\[ \psi_2(e_z, s) = (\|\eta_1\| + \|W^z \Phi_1(s)\|) ||s|| \|e_z\| \sec^2 \theta_1(e_z). \]

(24)

The adaptive law is designed as

\[ \ddot{c}_2(t) = \begin{cases} 
\sec^2 \theta_2(\sigma) & \text{if } \sigma \neq 0 \\
0 & \text{if } \sigma = 0 
\end{cases} \]

(25)

where \( \varphi_2 \) is a designed vector with positive constant entries.

The perturbation estimation scheme proposed in [36] is also utilized in this research. Following the similar design procedure as addressed in [36], one designs a nominal signal \( \sigma_{\text{nom}}(t) \) and the perturbation estimation signal, respectively, as

\[ \dot{s}_{\text{nom}}(t) = -\varepsilon \sigma_{\text{nom}}(t) + u_s - p_{est}, \]

(26)

\[ p_{est}(t) = \dot{p}(t) + \varepsilon [\sigma - \sigma_{\text{nom}}]. \]

(27)

where \( \dot{p}(t) \) is the output of the derivative estimator developed in [39]. If the perturbation estimation error is defined as

\[ \Delta \dot{p}_2(t, x, u) = \dot{B}^z \Phi_2(z, s) z(t) + \dot{B}^z \Delta p - \eta_2 - p_{est}, \]

(28)

then one can verify that

\[ \Delta \dot{p}_2(t, x, u) = -\delta(t), \]

(29)

where \( \delta(t) \) stands for the derivative estimation error. Equation (29) indicates that causing the perturbation estimation error is only due to the derivative estimation error signal \( \delta(t) \). Note that if the derivative estimator is designed properly, the signal \( \delta(t) \) is not only bounded, but also asymptotically stable in accordance with the analysis in [39].

6 Stability analysis

The stability of the proposed control system is addressed in the following theorem.

\[ \square \] Springer
**Theorem 2** Consider the dynamic system (1) and the observer (4). Suppose that

\[
\| \Delta \hat{p}_2(t, x, u) \| \leq c_2, \tag{30}
\]

where \( c_2 \) is an unknown positive constant; and the initial conditions \( e(0) \) and \( \hat{z}(0) \) satisfy the following inequalities:

\[
\| B^e x(0) \| \leq \frac{k_b}{\| B \|},
\]

\[
\| e_2(0) \| = \| z(0) - W^e x_0(0) \| < \frac{k_b}{\| W \|},
\]

respectively. If the controller (20) with adaptive law (25) is employed, then

(a) the states \( z \) and \( x \) are all bounded; both \( \sigma \) and \( e_2 \) will reach zero within a finite time. Moreover, not only \( \| e(t) \| < 2k_b + \| W z \| \) for all time, but also \( e(t) \) will approach zero in a finite time;

(b) the state variable \( \eta_2 \), adaptive gain \( \hat{c}_2 \), the nominal signal \( \sigma_{\text{nom}} \), and the perturbation estimation signal \( \tilde{p}_{\text{est}}(t) \) are all bounded for all time.

**Proof** Substituting (21) into (19) yields

\[
\dot{\sigma}(t) = -\varepsilon \sigma(t) + B^e \Phi_2 z(t) + B^e \Delta p + u_s - \eta_2 - \tilde{p}_{\text{est}}. \tag{31}
\]

Define a Barrier Lyapunov function candidate as

\[
V = \frac{k_b^2}{\| W \|^2 \pi} \tan \theta_1(e_2) + \frac{2k_b}{\| B \| \pi} \tan \theta_2(\sigma) + \frac{1}{2} \hat{c}_2(t)^2
+ \| \eta_2(t) \|, \tag{32}
\]

where \( \hat{c}_2(t) \triangleq \hat{c}_2(t) - c_2 \) is adaptive error of unknown positive constant \( c_2 \). All the possible cases that may occur when computing the time derivative of \( V \) are analyzed as follows.

**Case 1:** \( e_2 \neq 0, \sigma \neq 0 \), and \( \eta_2 \neq 0 

By using (28), one can compute the time derivative of (32) along the trajectory of (18) and (31) as

\[
\dot{V} \leq \| e_2 \| \| W^e \Phi_1 \| \| s(t) \| \sec^2 \theta_1 + \| e_2 \| \| \eta_1 \| \sec^2 \theta_1 - \tan \theta_1
+ \frac{\sigma^T(t)}{\| \sigma(t) \|} - \varepsilon \sigma + u_s + \Delta p_2 \right) \sec^2 \theta_2 + \| \Delta \hat{p}_2(t) \| \sec^2 \theta_2
+ \hat{c}_2(t)^2 + \frac{\eta_2^2(t)}{\| \eta_2(t) \|} \hat{\eta}_2. \tag{33}
\]

By using (22) and (30), from (33) one is able to obtain

\[
\dot{V} \leq \| e_2 \| \| W^e \Phi_1 \| \| s(t) \| \sec^2 \theta_1 + \| e_2 \| \| \eta_1 \| \sec^2 \theta_1 - \tan \theta_1
- \varepsilon \| \sigma \| \sec^2 \theta_2 - \kappa - \tan \theta_2 - \hat{c}_2 \sec^2 \theta_2 - \varphi_2
+ c_2 \sec^2 \theta_2 + \hat{c}_2(t)^2 + \frac{\eta_2^2(t)}{\| \eta_2(t) \|} \hat{\eta}_2
= -\varepsilon \| \sigma \| \sec^2 \theta_2 - \kappa - \hat{c}_2(t) \sec^2 \theta_2 - \tan \theta_1 - \tan \theta_2
+ \hat{c}_2(t)^2 + \frac{\eta_2^2(t)}{\| \eta_2(t) \|} \hat{\eta}_2. \tag{34}
\]

Substituting (25) and (23) into (34) further simplifies (34) as

\[
\dot{V} \leq -\varepsilon \| \sigma \| \sec^2 \theta_2(\sigma) - \tan \theta_1(e_2) - \tan \theta_2(\sigma) - \gamma
< -\gamma < 0. \tag{35}
\]

According to (35), it can be seen that \( V \) is bounded and its magnitude decreases. It also implies that \( \sigma, e_2, \hat{c}_2, \) and \( \eta_2 \) are all bounded in this case.

**Case 2:** \( e_2 \neq 0, \sigma = 0 \), and \( \eta_2 \neq 0 

In this case, \( V = k_b^2 \tan \theta_1(e_2) / (\| W^e \| \pi) + \| \eta_2(t) \| + \hat{c}_2^2(t) / 2 \). By using (18) and (23), one computes the time derivative of \( V \) as

\[
\dot{V} \leq \| e_2 \| \| W^e \Phi_1 \| \| s(t) \| \sec^2 \theta_1 + \| e_2 \| \| \eta_1 \| \sec^2 \theta_1
- \tan \theta_1 - \gamma - \psi_2(e_2, s)
= -\tan \theta_1 - \gamma < 0. \tag{36}
\]

Hence, \( V \) is bounded and the magnitude of \( V \) will decrease, which also indicates that the magnitude of \( e_2, \eta_2, \) and \( \hat{c}_2 \) are all bounded in this case.

**Case 3:** \( e_2 = 0, \sigma \neq 0 \), and \( \eta_2 \neq 0 

In this case, \( V = 2k_b \tan \theta_2(\sigma) / (\| W \| \pi) + \hat{c}_2^2 / 2 + \| \eta_2 \| \). By using (28) and (30) one is able to compute the time derivative of \( V \) along the trajectory of (31) as

\[
\dot{V} \leq \frac{\sigma^T(t)}{\| \sigma(t) \|} - \varepsilon \sigma + u_s + \Delta p_2 \right) \sec^2 \theta_2 + \hat{c}_2(t)^2
+ \frac{\eta_2^2(t)}{\| \eta_2(t) \|} \hat{\eta}_2
\leq \frac{\sigma^T(t)}{\| \sigma(t) \|} - \varepsilon \sigma + u_s \right) \sec^2 \theta_2 + c_2 \sec^2 \theta_2 + \hat{c}_2(t)^2 \hat{\eta}_2(t)
+ \frac{\eta_2^2(t)}{\| \eta_2(t) \|} \hat{\eta}_2. \tag{37}
\]
Substituting (22), (25) and (23) into (37) yields
\[
\dot{V} \leq -\varepsilon \|\sigma\| \sec^2 \theta_2(\sigma) - \kappa - \tan \theta_2(\sigma) - \gamma - 2\Psi_2(\varepsilon_t, s) \\
\leq -\tan \theta_2(\sigma) - \gamma \leq -\gamma < 0.
\]  
\tag{38}
\]

Equation (38) also implies that $V$ is bounded and will continuously decrease. Therefore, $\sigma$, $\tilde{c}_2$ and $\eta_2$ are all bounded in this case.

**Case 4: $e_\varepsilon \neq 0$, $\sigma \neq 0$, and $\eta_2 = 0$**

In this case, $V = k_b^2 \tan \theta_1(e_\varepsilon) / (\|W\|^2 \pi) + 2k_b \tan \theta_2(\sigma) / (\|B\|\pi) + \tilde{c}_2(t)^2 / 2$. By using (25), from (34), one can obtain
\[
\dot{V} \leq -\varepsilon \|\sigma\| \sec^2 \theta_2(\sigma) - \kappa - \tan \theta_1(e_\varepsilon) - \tan \theta_2(\sigma) \\
\leq -\kappa - \tan \theta_1(e_\varepsilon) - \tan \theta_2(\sigma) \leq -\kappa < 0.
\]  
\tag{39}
\]

Therefore, $V$ is bounded and its magnitude decreases, which means that the magnitude of $e_\varepsilon$, $\sigma$ and $\tilde{c}_2$ are all bounded in this case.

**Case 5: $e_\varepsilon = 0$, $\sigma \neq 0$, and $\eta_2 = 0$**

In this case, $V = 2k_b \tan \theta_2(\sigma) / (\|W\|^2 \pi) + \tilde{c}_2(t)^2 / 2$. By using Eqs. (31), (28) and (30), one computes the time derivative of $V$ as
\[
\dot{V} \leq \frac{\sigma^T(t)}{\|\sigma(t)\|} \left[ -\varepsilon \sigma + u_\sigma \right] \sec^2 \theta_2 + c_2 \sec^2 \theta_2 + \tilde{c}_2(t) \tilde{c}_2(t).
\]  
\tag{40}
\]

Substituting (22) and (25) into (40) yields
\[
\dot{V} \leq -\tan \theta_2 - \kappa \leq -\kappa < 0.
\]  
\tag{41}
\]

Equation (41) clearly indicates that $V$ is bounded, and the value of $V$ will decrease. Hence, $\sigma$ and adaptive error $\tilde{c}_2(t)$ are bounded in this case.

**Case 6: $e_\varepsilon \neq 0$, $\sigma = 0$, and $\eta_2 = 0$**

In this case, $V = k_b^2 \tan \theta_1(e_\varepsilon) / (\|W\|^2 \pi) + \tilde{c}_2(t)^2 / 2$. By using (25), one computes the time derivative of $V$ along the trajectory of (18) as
\[
\dot{V} \leq \|e_\varepsilon\| \|Wx_\sigma\| \|\sigma_1\| \|s\| \sec^2 \theta_1 + \|e_\varepsilon\| \|\eta_1\| \sec^2 \theta_1 - \tan \theta_1.
\]  
\tag{42}
\]

Equation (42) indicates that $\dot{V}$ may be greater or smaller than zero. Although in this case $\eta_2 = 0$, $\dot{\eta}_2 \neq 0$ in accordance with (23). This implies that the trajectory of $\eta_2$ will cross the surface $\eta_2 = 0$ immediately during a very small time interval, that is, $\eta_2(t + \delta) \neq 0$ ($\delta$ is small enough such that $\dot{\eta}_2(t)$ does not change sign in the interval $[t, t + \delta]$). Then, the status of system will switch to another case where $\eta_2 \neq 0$. Noted that the state $e_\varepsilon$ is still bounded in this case due to the continuity of its trajectory in accordance with (18).

**Case 7: $e_\varepsilon = 0$, $\sigma = 0$, and $\eta_2 \neq 0$**

In this case the Lyapunov function becomes $V = \|\eta_2(t)\|^2 + \tilde{c}_2(t)^2 / 2$. By using (25) and (23), one computes the time derivative of $V$ as
\[
\dot{V} = -\gamma - \Psi_2(e_\varepsilon, s) \leq -\gamma < 0.
\]  
\tag{43}
\]

According to (43), one can see that $V$ is a bounded function and the magnitude of $V$ will decrease. Hence, one can conclude that $\eta_2$ and $\tilde{c}_2$ are bounded in this case.

**Case 8: $e_\varepsilon = 0$, $\sigma = 0$, and $\eta_2 = 0$**

In this case, $V = \tilde{c}_2^2 / 2$. Then, $\dot{V} = 0$ in accordance with (25). It indicates that the value of $V$ will stop decreasing in this case.

From the preceding analysis of cases 1–8, it can be seen that
(a) $\dot{V} < 0$ except in the cases 6 and 8. Due to the quick switching ability of the proposed controller and the continuity of the functions in case 6, the value of $V$ will still be bounded even if the occurrence of case 6. The preceding analysis also reveals that the function $e_\varepsilon$ and $\sigma$ will reach zero within a finite time. Hence, the state variables $z$, $s$ and $x$ are all bounded for all time in accordance with (5), (17) and (2).

On the other hand, from (24) and (6), it is seen that if $\|\sigma\|$ approaches the boundary $k_b / \|B\|$ or $\|e_\varepsilon\|$ approaches the boundary $k_b / \|W\|$, then both $\theta_1$ and $\theta_2$ approach $\pi / 2$, it also implies that $V \to \infty$, which will not happen in accordance with the previous analysis. Hence, if $\sigma(0)$ and $e_\varepsilon(0)$ fulfill $\|\sigma(0)\| < k_b / \|B\|$ and $\|e_\varepsilon(0)\| < k_b / \|W\|$, respectively, then $\|B x_\sigma(t)\| < k_b / \|B\|$, $\|e_\varepsilon(t)\| < k_b / \|W\|$ for all time, and the Lyapunov function $V$ will not approach infinity. By using the fact that $z - W x_\sigma = \tilde{z} + e_\varepsilon$, from (2) one can obtain
\[
M^{-1} e(t) = \left[ \tilde{z} + e_\varepsilon(t) \right] / \sigma(t)
\]  
\tag{44}
\]

in accordance with (17). Equation (44) also implies that $e(t) = W(e_\varepsilon + \tilde{z}) + B \sigma$, which further implies $\|e(t)\| \leq \|W\| \|e_\varepsilon\| + \|W\| \|z\| + \|B\| \|\sigma\| < 2k_b / \|W\|$.  

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∀t. Since \( \dot{z}, e_2 \) and \( \sigma \) will tend to zero in a finite time, \( e(t) \) will also reach zero within a finite time.

(b) The adaptive gain \( \hat{c}_2 \) is monotonically increasing and bounded above in accordance with (25) and the analysis in part (a). Therefore, according to the proposition 2.14 ([38], p. 83), there exits a finite constant \( c_{2\infty} \) such that \( \lim_{t \to \infty} \dot{c}_2 = c_{2\infty} \). Since \( \eta_2^T \hat{y}_2 \leq 0 \) in (23), the signal \( \eta_2 \) is bounded.

According to the previous analysis, one can see that the states \( z, s \) and \( x \) are all bounded. From (31) and (26), one can obtain

\[
\dot{\sigma} - \dot{\sigma}_{\text{nom}} = -\varepsilon [\sigma - \sigma_{\text{nom}}] + B^g \Phi_2 z + B^g \Delta p - \eta_2(t). \tag{45}
\]

It is seen that the term \( B^g \Phi_2 z(t) + B^g \Delta p - \eta_2 \) in (45) is bounded due to the previous analysis. Hence, according to (45), one can see that \( \sigma - \sigma_{\text{nom}} \) is bounded since \( \varepsilon > 0 \). Therefore, \( \sigma_{\text{nom}} \) must be bounded, and from (27) the perturbation estimation signal \( p_{\text{est}} \) must be bounded due to \( \dot{p}(t) \) is bounded.

**Remark 2** The features of the proposed barrier Lyapunov function (32) are as follows.

1. If \( \|e_2\| \) approaches the boundary \( k_b/\|W\| \), then \( \theta_1(e_2) \) approaches \( \pi/2 \) in accordance with (6). It implies that \( V \to \infty \). This will not happen in accordance with the previous stability analysis. Hence, \( \|e_2\| \leq k_b/\|W\|, \forall t \).

2. According to (24), the Lyapunov function \( V \) will go to infinity if \( \|\sigma\| \) reaches the boundary \( k_b/\|B\| \). This will not happen either in accordance with the previous stability analysis. Hence \( \|\sigma\| < k_b/\|B\|, \forall t \).

3. The state variable \( \eta_2 \) plays an important role in the proposed controller. It helps to suppress the perturbations encountered in the system. If there is no \( \eta_2 \), then \( \dot{V} \) may not be smaller than zero in case 2 and 7.

The establishments of the inequalities \( \|e_2\| \leq k_b/\|W\| \) and \( \|\sigma\| < k_b/\|B\| \) are very important since they lead into the result of \( \|e(t)\| < 2k_b + \|Wz\|, \forall t \).

**Remark 3** In order to avoid the occurrence of case 6, one can modify (23) similarly as that in (16).

### 7 The effects of designed parameters on system’s performance

It is very important to know the impact of each designed parameter on system’s performance, this is analyzed as follows.

(a) \( \rho_1 \): According to (15), one is able to see that choosing larger value of \( \rho_1 \) will cause \( V_1 \) decreasing rapidly. It also implies that the estimation error \( \dot{z} \) will approach zero more faster.

(b) \( \varepsilon \): If larger value of \( \varepsilon \) is chosen, the convergent rate of \( \sigma_{\text{nom}} \) and \( \sigma \) will become faster in accordance with (26) and (35) respectively, but it may increase the control input energy in accordance with (21).

(c) \( \kappa \): From (39), one can see that larger value of \( \kappa \) may drive the trajectory of \( \sigma \) to zero more faster. However, it may also increase the magnitude of control input \( u \) in accordance with (22).

(d) \( \gamma \): According to (35), (36), (38) and (43), choosing larger value of \( \gamma \) will make the values of \( V, \sigma, e_2 \), and \( \eta_2 \) decreasing rapidly, but it may result in larger control effort in accordance with (23).

(e) \( \varphi_1 \) and \( \varphi_2 \): The purpose of designing \( \varphi_1, i = 1, 2 \) in (7) and (23) is to force the trajectories of \( \eta_i, i = 1, 2 \) to leave the surface \( \eta_1 = 0 \). Thus, only a \( \varphi_i \) with small entries is enough to achieve this goal.

### 8 Numerical example

Consider a perturbed nonlinear system with dynamic equation given by (1), where the known vector and matrices \( f(x), B, C \), are

\[
f(x) = \begin{bmatrix}
  x_3 x_2 \\
  x_4 \cos(x_3) \\
  x_3 \sin(x_2) \\
  x_1^2 x_3
\end{bmatrix}, \quad B = \begin{bmatrix}
  0 & 0 \\
  1 & 0 \\
  0 & 1 \\
  0 & 0
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
  0 & 1 & 1 \\
  1 & 0 & 1 \\
  1 & 1 & 0
\end{bmatrix},
\]

and \( x \triangleq [x_1, x_2, x_3, x_4]^T \). The vector \( u \triangleq [u_1, u_2]^T \) represents the control input. For demonstrating the robustness and effectiveness of the proposed control scheme by using computer simulation, we assume that the unknown perturbation \( \Delta p(t, x, u) \) is...
as partial state observer are designed in accordance with (20) and (4), respectively. The designed parameters are chosen to be \((\mu_1, \phi_1, \phi_2, h_1, \varepsilon, k, \gamma) = (1, [2, 2]^T, [2, 2]^T, 0.2, 4, 2, 1)\) respectively. Figures 1, 2, 3, 4, 5, 6 and 7 show the results of computer simulation with step time 0.1 ms, the unknown initial condition is assumed to be \(x(0) = [0.1, -0.2, 0.5, -0.3]^T\). Figure 1 indicates that the trajectory of estimation error \(\tilde{z}\) approaches zero in 5.8 s. All the state tracking errors, as shown in Fig. 2, converge to zero before 6 s. The trajectory of \(\|e(t)\|\) is shown in Fig. 3, it converges to zero before 6 s, and satisfies the required constraint \(\|e(t)\| < 4 + \|W\tilde{z}\|\) for all time. The trajectories of adaptive gains \(\hat{c}_0, \hat{c}_1\) and \(\hat{c}_2\) are shown in Fig. 4, and they are all bounded and reach a finite limit, respectively. The state variables \(\eta_1\) and \(\eta_2\), as illustrated from Figs. 5 and 6, are all bounded. Noted that if \(\eta_{ij} = 0\), then \(\hat{\eta}_{ij} \neq 0\) in the first three seconds, \(i, j = 1, 2\) for avoiding the occurrence of case 6. The control input \(u\), as shown in Fig. 7, is bounded, but it reveals the chattering phenomenon. Noted that the perturbation estimation signal \(p_{est}(t)\) and the nominal signal \(\sigma_{nom}(t)\) are all bounded for all time.

If the requirement of \(\|e\|\) changes to \(\|e(t)\| < 2 + \|W\tilde{z}\|\), then \(k_b = 1\), which will affect the control input in accordance with (24) and (22). The design parameters \((\rho_1, \phi_1, \phi_2, h_1, \varepsilon, k, \gamma)\) are not affected by changing the value of \(k_b\). The computer simulations with \(k_b = 1\) are illustrated from Figs. 8 and 9. Figure 8 confirms that the requirement \(\|e(t)\| < 2 + \|W\tilde{z}\|\) is not violated for all time, and \(\|e(t)\|\) reaches zero in 6.3 s. Figure 9 reveals that the control energy increases when smaller \(k_b\) is given.

9 Practical application

We consider controlling the joints of a polar robot manipulator with two-degree of freedom [40], which is shown in Fig. 10. The dynamic equations of this polar robot manipulator can be expressed in the form of (1), where \(x \triangleq [x_1, x_2, x_3, x_4]^T\), and

\[
f(x) = \left[ \begin{array}{c} \frac{x_2}{(\mu + M)x_1 + Ma}x_2^2/(\mu + M) \\ x_4 \\ -2[(\mu + M)x_1 + Ma]x_2x_4 \\ 0/(\mu + M) \\ 0 \\ 0 \end{array} \right].
\]

The state \(x_1\) and \(x_2\) are the position of sliding joint and its velocity, respectively; \(x_3\) and \(x_4\) denote the angle of rotational joint and the angular velocity, respectively. The coefficient \(A \triangleq 1/(J_1 + J_2 + \mu a^2 + M(x_1 + a)^2)\), where \(\mu = 1\) kg denotes the mass of motional link, \(M = 1.5\) kg stands for the payload, \(J_1 = J_2 = 1\) kg m² are moments of inertia of the motional link with respect to the vertical axis through \(c\) and \(o\), and \(a = 1\) m.

In order to demonstrate the applicability of the proposed control scheme, we assume that the known matrix \(C\) is

\[
C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.
\]

The matrix \(W\) is designed as

\[
W = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}^T,
\]

and obtain \(B^s\), \(F\), respectively, as

\[
B^s = \begin{bmatrix} 0 & \mu + M & 0 & 0 \\ 0 & 0 & 0 & A \end{bmatrix}, \quad F = \begin{bmatrix} \mu + M & 0 \\ 0 & A \end{bmatrix}.
\]
Noted that $s_1 = (\mu + M)x_2, s_2 = (\mu + M)x_4$ in accordance with (2), which are measurable, whereas the state variables $x_1 = z_1$ and $x_3 = z_2$ are unmeasurable in this application. We assume that the state vector $e(t)$ has to fulfill $\|e(t)\| < 2 + \|W\tilde{z}\|$ for all time, which implies $k_b = 1$. The desired tracking signal is $x_d = [0.5, 0, \pi/3 \text{ rad}, 0]^T$, and the nonlinear positive function $\beta(\hat{x})$ and $\zeta(u)$ in Theorem 1 are given by $\beta(\hat{x}) = 2||\hat{x}||$, $\zeta(u) = ||u||$.

The designed parameters are chosen to be $(\rho_1, \varphi_1, \varphi_2, \theta_1, \varepsilon, \kappa, \gamma) = (2, [2 \ 2]^T, [2 \ 2]^T, 0.2, 2, 0.01, 1)$, respectively. Figures 11, 12, 13 and 14 show the results of computer simulation with step time 0.1 ms, the unknown initial condition is assumed to be $x(0) = [0.54, -0.25, 0.98, 0.32]^T$. Figure 11 indicates that the trajectory of estimation error $\tilde{z}$ approaches zero in 3.5 s. All the tracking errors, as shown in Fig. 12, converge to zero around 0.5 s. The trajectory of $\|e(t)\|$ is shown in Fig. 13, it converges to zero within 0.5 s, and satisfies the constraint $\|e(t)\| < 2 + \|W\tilde{z}\|$ for all time. Figure 14 displays the control input $u$, which is bounded, but it also reveals the chattering phenomenon.

Although the proposed control scheme is able to track the states $x_1$ to $x_4$ theoretically, the desired tracking signals cannot be arbitrarily assigned in this practical application. This often happens in the underactuated systems. In order to highlight this point, we assume that there is no perturbation in the system, and assign $dx_{d1}(t)/dt = x_{d2}(t), dx_{d3}(t)/dt = x_{d4}(t)$ since $dx_1(t) = x_2(t)$ and $dx_3(t) = x_4(t)$ in (46). When there are conflicts between the desired tracking signals, then the tracking errors $e_2$ and $e_4$ generally converge to zero within a finite time, whereas $e_1$ and $e_3$ are bounded for all time.
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Fig. 6  Trajectory of $\eta_2$

Fig. 7  Trajectory of control input $u$ with $k_b = 2$, $\kappa = 2$

Fig. 8  Trajectory of $\|e\|$, $k_b = 1$

Fig. 9  Control energy with different $k_b$ and $\kappa$

Fig. 10  A two-degree of freedom polar manipulator [40]

Fig. 11  Trajectory of estimation error $\tilde{z}$ (practical example)

Fig. 12  Trajectory of tracking error $e$ (practical example)

Fig. 13  Trajectory of $\|e(t)\|$ (practical example)
10 Conclusions

An adaptive sliding mode tracking controller and a partial states observer are successfully proposed in this paper for a class of perturbed nonlinear systems with state constraints. This research results clearly shows that the proposed control strategy has the following advantages: (1) Some state variables do not need to be measurable. (2) The number of the state-tracking handled by the proposed control scheme is \( n \), which is the dimension of the system to be controlled. If the desired tracking signals are properly assigned, then all the state tracking errors can converge to zero within a finite time. (3) It has the ability to constrain the maximum magnitude of tracking error. (4) The dynamic equations of the plant do not need to be in strict feedback form. (5) There is no need to know the upper bounds of both match and unmatched perturbations in advance. (6) It can be applied to MIMO systems.

For future study, it is worth extending this research to the cases where 1. there are constraints on the control inputs. 2. super twisting control algorithm is utilized so that the chattering phenomenon can be further reduced effectively. 3. the fixed-time control problems can be handled.

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Declarations

Conflict of interest The authors declare that they have no conflict of interest.

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