Comparative Analysis of Solution Methods for Swing Equation in Power System for Transient Stability Studies

Sanitha Michail C
CMR Institute of Technology, Bengaluru
E-mail: sanitha.c@cmrit.ac.in

Abstract. In power system swing equation plays a remarkable role in investigating the transient stability. Disturbances in system creates imbalance and it may lead to lose the synchronism. Solution of swing equation will help the operators to analyze the stability of the system during the disturbances and initiate the protective mechanism to save the system. The non linear differential swing equation is complex in nature. Commonly used numerical methods to solve swing equations like Point by Point, Modified Euler’s and Runge Kutta have explained in the paper. Along with numerical methods, two other methods, evolutionary algorithm based Particle Swarm Optimization method and analytical based Maximum Lyapunov Exponent method has also been investigated in this paper.

Keywords: Transient stability; numerical methods; analytical method; swing equation; stability of system

1. Introduction

Power system stability studies mainly focuses on the dynamics of power system during disturbances. If the system is able to come back to normal operation (without losing synchronism) when subjected to a disturbance then it is a stable system. In an interconnected power system, synchronous generators are working in synchronism with the rest of the system to generate power [1]. Power system stability can be divided into transient stability, dynamic stability and steady state stability. Dynamic stability investigates whether system can retain stability under small continuous disturbances. Transient stability focuses on stability of the system when it is subjected to major disturbances. Due to this large disturbance, power angle of the synchronous machine changes because of the deceleration/acceleration of the rotor shaft. Steady state stability deals with the response of the system for small and gradual changes in the power system operating conditions.

Due to the increasing demand of power and sustainable energy to load centers, the importance to the stability of power system is increasing. Transmission line outages, different types of faults, sudden loss of generators and other disturbances are creating instability in the system. The transient period of the system is expressed by swing equation and it is a differential equation with non linear nature. Swing curve is the graphical representation of this equation and stability of system can be determined. This will help to analyze machine is in equilibrium or not. Commonly used numerical methods to solve this equation are Point by Point, Modified Euler’s and Runge Kutta. These methods too have their own restrictions. Evolutionary algorithms can be applied to solve this equation. One such algorithm, Particle swarm Optimization, is discussed in the paper. Also Analytical solution methods using Lyapunov exponent has investigated in the paper.
2. Swing Equation
The relative motion of the rotor of synchronous machine and the magnetic field of stator as a function of time is expressed by swing equation. \( T_a = T_s - T_e \) where \( T_a \) is the accelerating torque, \( T_s \) is the shaft torque and \( T_e \) is the electromagnetic torque.

Accelerating power can be written as

\[
P_a = T_a \omega = I \alpha \omega = M \alpha
\]  \hspace{1cm} (1)

\( \omega \) is the angular velocity, \( \alpha \) is the angular acceleration and \( I \) is the moment of inertia in kg-m\(^2\).

\[
M = I \omega;
\]  \hspace{1cm} (2)

\( M \) is the angular momentum.

Angular acceleration \( \alpha \) can be written as

\[
\alpha = \frac{d^2 \theta}{dt^2}
\]  \hspace{1cm} (3)

Rotor angle (\( \theta \)) changes continuously with respect to time during disturbances in the system as follows

\[
\theta = \omega_s t + \delta;
\]  \hspace{1cm} (4)

\( \delta \) is the angular displacement. The relation between \( \theta \) and \( \delta \) is shown in fig.1.

![Fig. 1. Rotor position with respect to reference axis](image)

\[
\frac{d\theta}{dt} = \omega_s + \frac{d\delta}{dt}; \quad \frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2}
\]  \hspace{1cm} (5)

\[
M \frac{d^2 \delta}{dt^2} = P_a = P_s - P_e
\]  \hspace{1cm} (6)

is the swing equation.

3. Numerical methods
Commonly used numerical methods to solve the swing equation are (i) Point by point (ii) Modified Euler’s (iii) Runge Kutta

3.1. Point by Point method
Point by point method is used to calculate the rotor position during the disturbances for a small interval of time. It is based on certain assumptions (i) angular velocity calculated at the middle of the time period is constant throughout the period (ii) acceleration power calculated at the beginning of time period remains constant from the centre of previous time period to the centre of time period under consideration.

\[
P_a(0+) = P_s - P_e(0+)
\]  \hspace{1cm} (7)
\[
\frac{d^2 \delta}{dt^2} = \alpha_{(0+)} = \frac{P_{m(0+)}}{M}
\]
For the 1st time period, change in angular velocity

\[
\Delta \omega_1 = \alpha_{(0+)} \Delta t
\]
\[
\omega_1 = \omega_0 + \Delta \omega_1 = \omega_0 + \alpha_{(0+)} \Delta t
\]
here \( \omega_0 \) is the relative angular velocity at \( t=0 \).

Change in rotor angle

\[
\Delta \delta_1 = \Delta \omega_1 \Delta t
\]
\[
\delta_1 = \delta_0 + \Delta \delta_1 = \delta_0 + \Delta \omega_1 \Delta t = \delta_0 + \alpha_{(0+)} (\Delta t)^2
\]
There are three possibilities for the discontinuity that occur due to switching operation or removal of fault (i) discontinuity at the beginning of the time period (ii) discontinuity at the centre of the time period (iii) discontinuity at some time other than (i) and (ii)

For (i) use the average value of accelerating power before and after the discontinuity (ii) incremental value is taken same as at the beginning of the time period (iii) weighted average of the accelerating power before and after the discontinuity is considered.

3.2. Modified Euler’s method

Dependent variables are predicted by Modified Euler’s method at the end of the time period by using the derivatives at the beginning of the time period. Derivatives at the end of the time period are computed using these predicted values. Average of these derivatives is used to update the value of the variable. Numerical errors are expected as terms with higher orders of Taylor’s series are omitted.

Considering the initial values as \( \delta_0, \omega_0 \) and step size as \( \Delta t \)

\[
\frac{d \delta}{dt} = \omega
\]
\[
\frac{d \omega}{dt} = \frac{P_m - P_{\text{max}} \sin \delta}{M}
\]
\[
\left. \frac{d \delta}{dt} \right|_0 = D_1 = \omega_0
\]
\[
\left. \frac{d \omega}{dt} \right|_0 = D_2 = \frac{P_m - P_{\text{max}} \sin \delta}{M}
\]
\[
\delta^p = \delta_0 + D_1 \Delta t
\]
\[
\omega^p = \omega_0 + D_2 \Delta t
\]
\[
\frac{d\delta}{dt} = D_{1p} = \omega^p
\] (19)

\[
\frac{d\omega}{dt} = D_{2p} = \frac{P_m - P_{\text{max}} \sin \delta^p}{M}
\] (20)

\[
\delta_1 = \delta_0 + \left( \frac{D_1 + D_{1p}}{2} \right) \Delta t
\] (21)

\[
\omega_1 = \omega_0 + \left( \frac{D_2 + D_{2p}}{2} \right) \Delta t
\] (22)

3.3. Runge Kutta method

Runge Kutta method uses a set of formulae and approximations to truncate the Taylor series expansion

\[
k_1 = \omega_o \Delta t
\] (23)

\[
k_2 = \left( \omega_o + \frac{l_1}{2} \right) \Delta t
\] (24)

\[
k_3 = \left( \omega_o + \frac{l_2}{2} \right) \Delta t
\] (25)

\[
k_4 = \left( \omega_o + l_3 \right) \Delta t
\] (26)

\[
l_1 = \left[ \frac{P_m - P_{\text{max}} \sin \delta_0}{M} \right] \Delta t
\] (27)

\[
l_2 = \left[ \frac{P_m - P_{\text{max}} \sin \left( \delta_0 + \frac{k_1}{2} \right)}{M} \right] \Delta t
\] (28)

\[
l_3 = \left[ \frac{P_m - P_{\text{max}} \sin \left( \delta_0 + \frac{k_2}{2} \right)}{M} \right] \Delta t
\] (29)

\[
l_4 = \left[ \frac{P_m - P_{\text{max}} \sin \left( \delta_0 + k_3 \right)}{M} \right] \Delta t
\] (30)

\[
\delta_1 = \delta_0 + \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4]
\] (31)

\[
\omega_1 = \omega_0 + \frac{1}{6} [l_1 + 2l_2 + 2l_3 + l_4]
\] (32)

As per [2] for a 50Hz, synchronous generator with \( M = 5.77 \times 10^4 \) pu, \( |E_g| = 1.2 \) pu, \( x_d' = 0.3 \) pu, \( x_{T1} = 0.2 \) pu, \( x_{L1} = x_{L2} = 0.4 \) pu is connected to infinite bus bar as given in fig.2. At the center of one of the
transmission line a 3 phase fault has occurred and two situations are considered (i) fault is cleared in 2.5 cycles (ii) fault is cleared in 6.25 cycles. Fig 3 shows the plot of swing curve by point by point method for (i) sustained fault (ii) fault cleared in 0.05sec (iii) fault cleared in 0.125 sec using matlab. Fig 4 shows the plot of swing curve by all the three numerical methods for the fault cleared by 6.25 cycles.

Fig. 2. Synchronous machine connected to infinite bus bars

Fig. 3. Swing curve by Point by Point method

Fig. 4. Swing curve by Point by Point, Modified Euler’s & Runge Kutta

4. **Evolutionary Algorithm: Particle Swarm Optimization**
An evolutionary algorithm, Particle Swarm Optimization [3] can be used in approximation function to find best coefficients. Based on Fourier series swing equation can be approximated [3] as

\[ \delta(t) \approx \Delta_{\text{appx}}(t) = \sum_{m=1}^{n} a_m \cos \left( \frac{m \pi (t - t_o)}{L} \right) + b_m \sin \left( \frac{m \pi (t - t_o)}{L} \right) \]  

(33)
\[
\delta'(t) \approx \Delta_{appx}(t) = \\
\sum_{n=1}^{n} \left( \frac{m\pi}{L} \right)^2 a_n \cos \left( \frac{m\pi(t-t_o)}{L} \right) + \left( \frac{m\pi}{L} \right)^2 b_n \sin \left( \frac{m\pi(t-t_o)}{L} \right) \right]
\]
\[L = t_o - t_p\]  \hspace{1cm} (34)

Objective function is the weighted-residual functional (WRF) in the optimization model. It is minimized to
\[
\text{Minimize, } WRF = \int [W(t) \times |R(t)|] dt
\]  \hspace{1cm} (36)

\[R(t) = f(t, \Delta_{appx}(t), \Delta_{appx}''(t))\]

Best coefficients can be computed by [3]
\[V_i(t+1) = V_i(t) + c_1(P_{best,i} - X_i(t)) + c_2(G_{best} - X_i(t))\]  \hspace{1cm} (37)

\[X_i(t+1) = X_i(t) + V_i(t+1)\]  \hspace{1cm} (38)

\[c_1 \text{ & } c_2 : \text{Acceleration constants } \pm \text{positive}\]
\[G_{best} : \text{Global best position}\]
\[P_{best} : \text{Personal best position}\]
\[V_i : \text{Vector for velocity}\]
\[X_i : \text{Particle position}\]

As per the PSO algorithm given below [3], swing equation has been solved and the curve is plotted when there is a fault at the centre of one of the transmission line and fault cleared in 2.5 cycles and 6.5 cycles in fig.5

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{swing_curve.png}
\caption{Plot of swing curve by PSO [3]}
\end{figure}

\textbf{Algorithm 1:} PSO algorithm
**Input:** swarm size, initial particle positions, initial velocities  
**Output:** optimal $a_0, a_m, b_m$

while $t < max\_iterations$ do
update particle positions:
$$X_i(t) = X_i(t-1) + V_i(t-1);$$
evaluate objective function "weighted-residual functional" $WRF$ for each particle;
find the global best particle $G_{best}$ with minimum objective function value $J(G_{best});$
if $J(G_{best}) <$ tolerance then
STOP;
else
find the best position $P_{best,i}$ with minimum objective value for each particle $X_i;$
if $J(X_i) < J(P_{best,i})$ then
update the velocity $V_i$ for each particle:
$$V_i(t) = V_i(t-1) + c_1(P_{best,i}(t) - X_i(t)) + c_2G_{best} - X_i(t);$$

5. **Maximum Lyapunov Exponent**

For a system, the Maximum Lyapunov Exponent (MLE)[4] is estimated using phase angles at the generator buses by Phase Measurement units (PMU). Stability of the system is indicated by the magnitude and polarity of MLE. It is independent of system model.

$$\dot{\delta}_i^k = \omega_i \Delta \omega_i^k$$  \hspace{1cm} (40)

$$\Delta \omega_i = h_i(\delta^k, \Delta \omega_i^k)$$  \hspace{1cm} (41)

This is state space representation of $i^{th}$ synchronous machine of a power system with $m$ number of generators at $k^{th}$ instant of time and $\omega_i$ is the synchronous speed in rad/sec, $\delta_i^k$ is the internal voltage of $i^{th}$ generator in rad, $\Delta \omega_i^k$ is the per unit speed deviation from synchronous speed of the $i^{th}$ generator, $h_i$ is the dynamics function of the $i^{th}$ generator and $\Delta \omega_i = (\Delta \omega_1^k, ..., \Delta \omega_m^k)^T$. $X^k = (\delta_1^k, ..., \delta_m^k, \Delta \omega_1^k, ..., \Delta \omega_m^k)$ is the state vector of the power system at $k^{th}$ instant of time. The matrix $Y = (X^1, ..., X^N)^T$ represents time series data of state variables by phase measurements unite. Lyapunov exponents (LE) are the average exponential rate of divergence of nearby trajectories initially separated by small distance in phase plane. The $X^k$ is the nearest neighbour at $k^{th}$ point, $d^k(0) = \min_{k-1 \leq k' < k} \|X^k - X^{k'}\|$, $d^k(0)$ is the initial distance between $k^{th}$ point and nearest neighbour and $d^k(j) = \|X^{k+j} - X^{k'+j}\|$ is the distance after $j^{th}$ time step.
In MLE,  
\[
\langle \ln d^k(j) \rangle \approx \ln d^k(0) + \lambda_j(j \Delta t),
\]

(42)

\(\Delta t\) is the sampling time and \(\lambda_j\) is the maximum Lyapunov exponent.

Fig. 6 shows the one line diagram\[4\] of a 9 bus 3 machine system and a 3 phase fault has occurred in the transmission line connected between bus 6 and 7. The fault has occurred at 1 sec and compared the results of the system if the fault is cleared at 1.29 sec and 1.3 sec\[3\] as shown in fig. 7 and fig. 8.

![Fig. 6. Nine bus three machine system](image)

![Fig. 7. Plot of swing curve when fault is cleared in 1.29 sec](image)
MLE is negative when fault is cleared at 1.29 sec showing system regaining the stability and it is positive when fault is cleared at 1.3 sec making the system unstable as in fig.9.

6. Conclusions
Location and type of fault will greatly influence the transient stability of power system. Swing equation, a non linear differential equation plays a significant role in investigating the transient stability of power system. Solution of swing equation will help the power system operators to initiate the required emergency operations to save the system. Different methods to solve the swing equation which includes numerical methods like Point by Point, Modified Euler’s, Runge Kutta and evolutionary algorithm – Particle swarm optimization method and Lyapunov exponent based MLE method are explained in the paper. These methods will help the power system engineers to study and analyze the system under transient condition.

References
[1] C Sanitha Michail "A Review On Wind Forecasting Methods " Solid State Technology",Volume: 63 Issue: 4,Publication Year: 2020
[2] K Uma Rao, “Computer Techniques and Models in Power Systems”, I.K International Publishing House Private Limited,2007
[3] A. Zaidi and Q. Cheng, "An Approximation Solution of the Swing Equation Using Particle Swarm Optimization," 2018 IEEE Conference on Technologies for Sustainability (SusTech), Long Beach, CA, USA, 2018, pp. 1-5, doi: 10.1109/SusTech.2018.8671355
[4] P. Banerjee, S. C. Srivastava and K. N. Srivastava, "A Lyapunov exponent based method for online transient stability assessment," 2014 Eighteenth National Power Systems Conference (NPSC), Guwahati, India, 2014, pp. 1-6, doi: 10.1109/NPSC.2014.7103871.