Kaon Condensation in High Density Quark Matter

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Abstract

We point out that the problem of kaon condensation in dense hadronic matter can be addressed in perturbative QCD. Indeed, perturbative calculations suggest that negative kaons are condensed in high density quark matter if the electroweak interaction is taken into account. This observation sheds new light on the proposal that the low density hyperon and high density quark matter phases of QCD are continuously connected.
The behavior of hadronic matter at very high baryon density has been a fascinating subject for quite some time. In the early 1970's Migdal, Sawyer, and Scalapino suggested that nuclear matter might exhibit pion condensation at densities near the saturation point of nuclear matter [1–3]. In 1986 Kaplan and Nelson pointed out that kaons might condense at densities several times the saturation density [4–7]. More recently, work on QCD at finite baryon density has mostly focussed on the behavior of quark matter at extremely high baryon density. This work goes back to the basic observation by Frautschi that asymptotic freedom combined with the presence of a Fermi surface implies that cold quark matter is a color superconductor [8–10]. This idea was revived in [11,12] where it was emphasized that the corresponding gaps could be quite large, on the order of 100 MeV at densities 5-10 times larger than the nuclear saturation density.

The next important step was taken by Alford, Rajagopal, and Wilczek who realized that in quark matter with three flavors the dominant order parameter involves the coupling of color and flavor degrees of freedom, “color-flavor-locking” [13]

\[
\langle \psi^a_i C \gamma_5 \psi^b_j \rangle = \phi \left( \delta^a_i \delta^b_j - \delta^b_i \delta^a_j \right). \numberwithin{equation}{section}
\]

Here, \(a, b\) are color indices and \(i, j\) are flavor indices. This particular order parameter is distinguished by the fact that it has the largest residual symmetry group [13], and therefore leads to the largest condensation energy [14]. The color-flavor-locked condensate breaks both the original \(SU(3)\) color symmetry and the \(SU(3)_L \times SU(3)_R\) flavor symmetry, but leaves a diagonal \(SU(3)\) symmetry unbroken. This implies that chiral symmetry is spontaneously broken, and that the spectrum contains almost massless pseudoscalar Goldstone bosons. Indeed, the excitation spectrum of high density quark matter bears a remarkable resemblance to the spectrum of low density hyperon matter. This has lead to the suggestion that the low and high density phases of three flavor QCD might be continuously connected [14].

The systematic study of the low energy effective lagrangian of three flavor QCD at high density was started by Casalbuoni and Gatto [15]. The effective lagrangian for the pseudoscalar Goldstone bosons takes the form [15,16]
\[ \mathcal{L}_{\text{eff}} = \frac{f_\pi^2}{4} \text{Tr} \left[ \partial_0 \Sigma \partial_0 \Sigma^\dagger - v_\pi^2 \partial_i \Sigma \partial_i \Sigma^\dagger \right] - c \left[ \det(M) \text{Tr}(M^{-1} \Sigma) + h.c. \right]. \]  

(2)

Here, \( \Sigma \in SU(3) \) is the Goldstone boson field, \( v_\pi \) is the velocity of the Goldstone modes and \( M = \text{diag}(m_u, m_d, m_s) \) is the quark mass matrix. The effective description is valid for energies and momenta below the scale set by the gap, \( \omega, q \ll \Delta \). At very high density, \( \mu \gg \Lambda_{\text{QCD}} \), asymptotic freedom implies that the coupling is weak and the coefficients in the low energy lagrangian can be determined in perturbation theory. The first step is the calculation of the gap. The result for \( N_f = 3 \) is  \[ \Delta = 512 \pi^4 2^{-1/3} (2/3)^{5/2} \mu g^{-5} \exp \left( -\frac{3 \pi^2}{\sqrt{2} g} \right). \]  

(3)

Here, the factor \( (2/3)^{5/2} \) reflects the larger amount of screening in three flavor QCD as compared to the two flavor case, and the factor \( 2^{-1/3} \) is related to the structure of the color-flavor locked state.

The low energy constants in (2) were determined by Son and Stephanov [16], see also [23, 24]. They find \( v_\pi^2 = 1/3 \) and

\[
\begin{align*}
    f_\pi^2 &= \frac{21 - 8 \log(2)}{18} \frac{\mu^2}{2 \pi^2}, \quad &\text{(4)} \\
    c &= \frac{3 \Delta^2}{2 \pi^2}. \quad &\text{(5)}
\end{align*}
\]

We can now determine the masses of the Goldstone bosons

\[
m_{\pi \pm}^2 = C(m_u + m_d) m_s, \quad m_{K \pm}^2 = C m_s (m_u + m_s),
\]

where \( C = 2c/f_\pi^2 \). This result shows that the kaon is lighter than the pion. This can be understood from the fact that, at high density, it is more appropriate to think of the interpolating field \( \Sigma \) as

\[
\Sigma_{ij} \sim \epsilon_{ikl} \epsilon_{jmn} \epsilon^{abc} \epsilon^{def} \bar{\psi}_{L,k} \bar{\psi}_{L,l} \psi_{R,m} \psi_{R,n}
\]

rather than the more familiar \( \Sigma_{ij} \sim \bar{\psi}_{L,i} \psi_{R,j} \) [15]. Using (7) we observe that the negative pion field has the flavor structure \( \bar{d} \bar{s} u s \) and therefore has mass proportional to \( (m_u + m_d) m_s \).
Putting in numerical values we find that the kaon mass is very small, \( m_{K^-} \simeq 5 \text{ MeV} \) at \( \mu = 500 \text{ MeV} \) and \( m_{K^-} \simeq 2.5 \text{ MeV} \) at \( \mu = 1000 \text{ MeV} \).

We would like to remind the reader why this is so. First of all, the Goldstone boson masses in the color-flavor-locked phase are proportional to the quark masses squared rather than linear in the quark mass, as they are at zero density. This is due to an approximate \( Z_2 \) chiral symmetry in the color-flavor-locked phase [13]. The diquark condensate is invariant under the transformations \( \psi_{L,R} \to -\psi_{L,R} \), but a linear Goldstone boson mass term is not. In addition to that, the Goldstone boson masses are suppressed by a factor \( \Delta/\mu \). This is a consequence of the fact that the Goldstone modes are collective excitations of particles and holes near the Fermi surface, whereas the quark mass term connects particles and antiparticles, far away from the Fermi surface [23].

The fact that the meson spectrum is inverted, and that the kaon mass is exceptionally small opens the possibility that in dense quark matter electrons decay into kaons, and a kaon condensate is formed. Consider a kaon condensate \( \langle K^- \rangle = v_K e^{-i \mu_e t} \) where \( \mu_e \) is the chemical potential for negative charge. The thermodynamic potential \( \mathcal{H} - \mu_e Q \) for this state is given by

\[
\epsilon(\rho_q, x, y, \mu_e) = \frac{3 \pi^2}{4} \rho_q^{4/3} \left\{ x^{4/3} + y^{4/3} + (1 - x - y)^{4/3} + \pi^{-4/3} \rho_q^{-2/3} m_s^2 (1 - x - y)^{2/3} \right\} \\
- \frac{1}{2} \left( \mu_e^2 - m_{K^-}^2 \right) v_K^2 + O(v_K^3) + \mu_e \rho_q \left( x - \frac{1}{3} \right) - \frac{1}{12 \pi^2} \mu_e^4
\]

(8)

Here, \( \rho_q = 3 \rho_B \) is the quark density, and \( x = \rho_u/\rho_q \) and \( y = \rho_d/\rho_q \) are the up and down quark fractions. For simplicity, we have dropped higher order terms in the strange quark mass and neglected the electron mass. These corrections are included in the results shown below. We have also assumed that neutrinos can leave the system. This assumption is appropriate in the case of neutron stars. In order to determine the ground state we have to make (8) stationary with respect to \( x, y, \mu_e \) and \( v_K \). Minimization with respect to \( x \) and \( y \) enforces \( \beta \) equilibrium, while minimization with respect to \( \mu_e \) ensures charge neutrality. Below the onset for kaon condensation we have \( v_K = 0 \) and there is no kaon contribution to the charge density. Neglecting \( m_e \) and higher order corrections in \( m_s \) we find
\[ \mu_e \simeq \frac{m_s^2}{4p_F} = \frac{m_s^2}{4\pi^{2/3} \rho_B^{1/3}}. \]

In the absence of kaon condensation, the electron chemical potential will level off at the value of the electron mass for very high baryon density. The onset of kaon condensation is determined by the condition \( \mu_e = m_K \). At this point it becomes favorable to convert electrons into negatively charged kaons. Once the amplitude of the kaon condensate starts to grow, nonlinear terms in the effective lagrangian have to be taken into account.

Results for the electron chemical potential and the kaon mass as a function of the light quark Fermi momentum are shown in Fig. 1. In order to assess some of the uncertainties we have varied the quark masses in the range \( m_u = (3 - 5) \) MeV, \( m_d = (6 - 8) \) MeV, and \( m_s = (120 - 150) \) MeV. We have used the one loop result for the running coupling constant at two different scales \( q = \mu \) and \( q = \mu/2 \). The scale parameter was set to \( \Lambda_{QCD} = 238 \) MeV, corresponding to \( \alpha_s(m_\tau) = 0.35 \) \cite{27}. An important constraint is provided by the condition \( m_s < \sqrt{2p_F \Delta} \) which ensures that flavor symmetry breaking does not destroy color-flavor-locking \cite{28,29}. We have checked that this condition is always satisfied for \( p_F > 500 \) MeV. Figure 1 shows that there is significant uncertainty in the relative magnitude of the chemical potential and the kaon mass. Nevertheless, the band of kaon mass predictions lies systematically below the predicted chemical potentials. We therefore conclude that kaon condensation appears likely even for moderate Fermi momenta \( p_F \simeq 500 \) MeV. For very large baryon density\footnote{This is not entirely correct. If \( p_F > m_s^2/(4m_e) \) and kaons are not yet condensed then the system can no longer maintain \( \beta \) equilibrium and \( \mu_e \) goes to zero.} \( \mu_e \rightarrow m_e \) while \( m_{K^-} \rightarrow 0 \) and kaon condensation seems inevitable.

In the regime which is of physical interest the numerical values of \( m_K \) and \( \mu_e \) are very close, and it is important to address the uncertainties.

1. The most important uncertainty is related to the value of the gap. The leading terms in the perturbative expansion are...
\[
\log \left( \frac{\Delta}{\mu} \right) = -\frac{3\pi^2}{\sqrt{2}g} - 5 \log(g) + \log \left( b'_0 512 \pi^4 (2/N_f)^{5/2} \right) + \ldots.
\]  

(10)

While there is general agreement on the \(O(g^{-1})\) and \(O(\log(g))\) terms, the constant \(b'_0\) in the \(O(g^0)\) term has not been completely determined yet. Brown et al. calculated the critical temperature up to order \(O(g^0)\) and find \(b'_0 = \exp(-\pi^2 + 4)(N_f - 1)/16) \sim 0.17\) [21], which corresponds to a substantial reduction of the gap. The origin of this correction is a reduction of the strength of the quasiparticle pole in the dense medium. Manuel studied the effect of quasiparticle damping and obtained a reduction of the gap by a factor \(\sim 2\) at Fermi momenta \(p_F \simeq 500\) MeV [30]. This effect, however, appears to be a true higher order \(O(g)\) correction.

The modification of the gap due to color-flavor-locking was considered in [22,31,32]. This leads to a correction factor \(2^{-1/3}\) which we have already included in (3). All these effects reduce the gap and increase the likelihood of kaon condensation, at least as long as the magnitude of the gap exceeds the critical value for color-flavor-locking.

2. A related question is the problem of determining the scale \(\Lambda\) at which the running coupling constant is evaluated. This problem cannot really be addressed without performing a higher order calculation. Beane et al. suggested to carry out a leading log resummation of the gap equation [33]. This calculation results in a substantial enhancement of the gap, corresponding to \(b'_0 = \exp((33/16)(\pi^2/4-1)) \sim 20\) in (10). This may be serious overestimate, however, because the authors use the perturbative beta function for momenta well below the screening scale \(g\mu\).

3. At moderate densities QCD may generate a dynamical strange quark mass which is significantly bigger than the current quark mass \(m_s \simeq 150\) MeV. This effect helps kaon condensation because the electron chemical potential grows as \(m_s^2\), whereas the kaon mass behaves as \(m_s^{1/2}\). If the strange quark mass becomes too large, then color is no longer locked to flavor and the kaon disappears.

4. Several authors have suggested that the Goldstone boson masses in the color-flavor-locked phase are of the form \(m_{GB}^2 \sim m_q^2 (\Delta \bar{\Delta}/\mu^2) \log(\mu/\Delta)\) [24,26], where \(\bar{\Delta}\) is the “gap” for anti-particles. If \(\bar{\Delta} = \Delta\) then the extra logarithm would lead to a modest increase of the
kaon mass. Beane et al. also find a value of $f_\pi$ which is smaller, by a factor of 2, than the value quoted above.

5. Manuel and Tytgat considered the contribution of a small instanton-generated quark condensate to the Goldstone boson masses [23]. They conclude that this effect dominates over the perturbative result (6) for chemical potentials of physical interest, $\mu < 3$ GeV. We believe that this result is based on a mistake in the calculation of $\langle \bar{\psi}\psi \rangle$ reported in [22]. In that work we calculated the effective quark mass generated by instantons in the color-flavor-locked phase. Using this result we extracted the quark condensate. In this context we made a mistake similar to the one in the first version of [16]: Contrary to the result given in [22] there is no contribution to $\langle \bar{\psi}\psi \rangle$ which is proportional to the density of states on the Fermi surface. The correct result is suppressed by an additional factor of $(\Delta/\mu)$. As a result, terms linear in the quark mass may give an important contribution to the Goldstone boson masses in the case of moderate densities, $p_F \simeq 500$ MeV, but probably not for larger densities.

6. If the negative kaon mass becomes very small, the electromagnetic contribution to the mass may start to play a role. This effect was considered by Hong who finds $\delta m_{GB\pm}^2|_{em} \sim (e/g)^2 \Delta^2$, where the overall coefficient is of order 100 [34]. This result seems surprisingly large. A different estimate of the electromagnetic contribution to the charged pion and kaon masses can be obtained using a current algebra sum rule, which relates $\delta m_{GB\pm}^2|_{em}$ to the difference between the vector and axial-vector spectral functions [35]. The excitation spectrum in the vector channels was studied by Rho et al. [36], who find that the vector and axial-vector are degenerate to leading order in perturbation theory. Using their results we conclude that $\delta m_{GB\pm}^2|_{em} < O(e^2 \Delta^2 e^{-c/g})$. This estimate is oversimplified, because the derivation of the sum rule relies on Lorentz invariance. This question clearly deserves further study.

Conclusions: We find that perturbative calculations suggest that negative kaons are condensed in high density baryonic matter. The mechanism for kaon condensation considered in this work is very different from the mechanism originally proposed by Kaplan and Nel-
son. In the original work, kaon condensation was driven by the attractive kaon-nucleon interaction implied by the kaon-nucleon sigma term $\Sigma_{K,N}$. In the color-flavor locked phase all fermionic excitations have a large gap and the interaction between Goldstone bosons and quarks (baryons) is very weak. Kaon condensation is possible because color-flavor locking inverts the Goldstone boson spectrum and the kinematics of the Fermi surface suppresses Goldstone boson masses. The mechanism considered here shares an important feature with the hadronic scenario of Brown et al. [7] in that the presence of electrons is essential.

It is quite remarkable that the problem of kaon condensation in dense hadronic matter can be addressed in perturbative QCD. This observation also sheds new light on the idea of quark-hadron continuity in QCD at finite baryon density. Even though low and high density QCD are qualitatively similar, there are important quantitative differences. Low density QCD is characterized by large fermion masses and small Fermi surface gaps, whereas high density QCD has small masses and large gaps. Also, while $f_\pi$ and $\langle(\bar{\psi}\psi)^2\rangle$ do not seem to change much between $\mu = 0$ and $\mu = 500$ MeV, the quark condensate $\langle\bar{\psi}\psi\rangle$ is reduced by a large factor. We suggest that the low and high density phase are continuously connected, and that the density above which it becomes more useful to describe matter in terms of quarks and gluons rather than hadrons coincides with the point where the kaon and pion become degenerate.

We should note that a simple and robust mechanism for pion condensation in QCD with zero baryon but finite isospin density, $\mu_u = -\mu_d$, was recently discussed by Son and Stephanov [37].

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FIG. 1. Electron chemical potential (solid lines) and kaon mass (dashed lines) in the color-flavor-locked quark phase. The two curves for both quantities represent a simple estimate of the uncertainties due to the value of the strange quark mass and the scale setting procedure.