Effect of Anomalous Cross-Field Diffusion on the Field-Aligned Current Generation

Takashi Yamamoto¹, S. Inoue², and C.-I. Meng³

¹Department of Earth and Planetary Physics, University of Tokyo, Tokyo 113, Japan
²Aichi College of Technology, Gamagohri-shi, Aichi 443, Japan
³The Johns Hopkins University, Applied Physics Laboratory, Laurel, Maryland 20723-6099, U.S.A.

(Received June 12, 1996; Revised April 3, 1997; Accepted April 7, 1997)

First, on the basis of the satellite observations of the broadband electrostatic noise and the Alfvén wave noise in the auroral magnetosphere, within a frame of the quasi-linear theory we show that the proton (anomalous cross-field) diffusion coefficient averaged over the flux tube is likely to reach a significant fraction of the Bohm rate at least in the disturbed periods. Second, as an extension of recently proposed model that a pair of the region 1 and region 2 field-aligned currents (FACs) can be generated as a result of “natural distortion” of the hot plasma torus (HPT) in the magnetosphere under the influence of the usually prevailing convection with twin vortex cells (when the interplanetary magnetic field (IMF) is southward), we study the possible effect of the anomalous cross-field diffusion on the large-scale FACs resulting from the HPT polarization. In the magnetosphere, the hot (≥1 keV) plasma particles are primarily distributed in a magnetic shell which is connected to two ovals of diffuse auroras on the northern and southern polar ionospheres. Such hot plasma population is called the hot plasma torus (HPT). The numerical calculations specifically show that primary modification of the FAC pattern by this effect is the appearance of the triple FAC structure (i.e., upward FAC zone sandwiched between two downward FAC zones) in the midnight sector, which can be identified from the satellite observations at least during substorm periods. As a direct consequence of formation of such a triple FAC structure, the Harang discontinuity appears in the electric potential pattern. Thus the formation of the Harang discontinuity can be understood as a result of the HPT polarization by both effects of magnetic drift and anomalous cross-field diffusion.

1. Introduction

There have been a number of authors who pointed out the importance of anomalous cross-field diffusion of plasma particles in the magnetosphere. For example, from observations of the magnetic-field fluctuations with near the proton cyclotron frequency in the vicinity of the magnetopause boundary layer Eviatar and Wolf (1968) inferred that protons in the boundary layer can diffuse across field lines at a rate of one proton gyroradius every few cyclotron periods, i.e., roughly the Bohm rate (kT/16eB, where T is the plasma temperature, B is the magnetic field intensity, and e is the electronic charge). Furthermore, Tsurutani and Thorne (1982) argued that anomalous cross-field diffusion of magnetosheath ions is a direct consequence of cyclotron-resonant scattering by electrostatic emissions which are continuously present within the magnetopause boundary layer. It was shown that resonant electron cross-field diffusion is always insignificant for the typical intensity of observed waves. However, magnetosheath ions, resonant with low-frequency electrostatic waves, may be transported inward at a rate approaching one tenth the Bohm rate. Their estimated rate of ion diffusion is comparable to the value required to account for the typical thickness of the boundary layer. Also Reiff et al. (1977) proposed cross-field diffusion as one of the mechanisms for solar wind particle injection at the dayside magnetospheric cusps. The observed energy dispersion of cusp protons in some case is fitted quite well by the
dispersion curve that is expected on the basis of a simple gyroresonant diffusion model coupled with poleward convection. The cross-field (proton) diffusion rate in this event was estimated to be larger than the Bohm rate.

As a possible mechanism for the generation of field-aligned currents, the anomalous particle diffusion has recently been invoked by Yamamoto et al. (1993a, 1994), in the context of the formation of large amplitude undulations on the evening diffuse auroral boundary. In their model proton diffusion is assumed to proceed at the Bohm diffusion rate $kT/16eB$ (Bohm et al., 1949). The Bohm diffusion formula agrees with many results of the laboratory experiments to within a factor of two or three (see, e.g., Chen (1984)). By numerical simulations, Okuda et al. (1981) have shown that anomalous cross-field diffusion comparable to Bohm diffusion takes place in the presence of electrostatic proton cyclotron instabilities.

Recently, Yamamoto and Ozaki (1993) and Yamamoto et al. (1996) have proposed that a pair of the region 1 and region 2 field-aligned currents (FACs) can be generated as a result of natural distortion of the hot plasma torus (HPT) under the influence of a two-cell global convection when the interplanetary magnetic field (IMF) is southward. In the magnetosphere, the hot ($\geq 1$ keV) plasma particles are assumed to be distributed in a magnetic shell which is connected to two ovals of diffuse auroras on the northern and southern polar ionospheres. Such hot plasma population is called the hot plasma torus (HPT). Precisely the HPT is defined as the plasma concentration in terms of the flux tube content ($N$), not in the “local” number density ($n$). The flux tube content $N$ is the total number of particles in a flux tube with unit (ionospheric) cross section:

$$N = \int_{s_i}^{s_e} n(s) \frac{B_i}{B(s)} ds$$

where $s$ is the field-aligned distance, $s_e$ and $s_i$ are at the equator and the ionospheric height; $B(s)$ and $B_i$ are the magnetic field intensities at the distance $s$ and $s_i$, respectively, and $n(s)$ is the number density of plasma particles. A possible mechanism for the formation of the HPT has been discussed by Yamamoto et al. (1997). Specifically, the HPT is primarily the combined regions of the boundary plasma sheet (BPS) and the central plasma sheet (CPS). The most poleward part of the nightside HPT is regarded as the plasma sheet boundary layer (PSBL). Notably, the ionosphere to magnetosphere mapping analysis based on the satellite observations of particle precipitation (Newell and Meng, 1992) has shown that the CPS nearly completely encircles the Earth and that the BPS extends considerably into the dayside.

The numerical simulations by Yamamoto et al. (1997) have also shown that the pattern of region 1 and region 2 field-aligned currents (FACs) which are produced on the HPT is in good agreement with the FAC pattern identified observationally (e.g., Iijima and Potemra, 1976, 1978). In the present paper, as an extension of our model, we numerically study how the pattern of the paired region 1 and region 2 FACs is modified by the effect of anomalous cross-field diffusion of the plasma particles. It is specifically shown that primary modification of the FAC pattern by this effect is the appearance of the triple FAC structure (i.e., upward FAC zone sandwiched between two downward FAC zones) in the midnight sector, which can be identified from the satellite observations at least during substorm periods. As a direct consequence of the formation of the triple FAC structure, we find the Harang discontinuity in the potential distribution. Now it can be understood that the cause of the Harang discontinuity is the HPT polarization by both effects of magnetic drift and anomalous cross-field diffusion.

In the next section, on the basis of the satellite observations of low-frequency electrostatic and electromagnetic fluctuations in the auroral magnetosphere we estimate the cross-field diffusion rate for the HPT particles in a frame of the quasi-linear theory. It will be shown that the proton diffusion coefficient averaged over the flux tube can reach a significant fraction of the Bohm rate at least in disturbed periods. In Section 3 we discuss the possible effects on field-aligned current generation in the auroral zone.
2. Evaluation of Cross-Field Diffusion

The cross-field diffusion rate of the electrons with keV energies, as constituting the HPT, is small, compared with that of the HPT protons, in the interaction with the low-frequency waves (e.g., Tsurutani and Thorne, 1982). (In the present discussion the low-frequency waves are defined as those having frequencies comparable to or less than the proton cyclotron frequency and phase velocities on the order of the thermal velocity of protons with keV energies.) This is because most of the hot electrons cannot resonate with such waves and the time for an electron to transit over one wavelength is short. (Both effects may shorten the self-correlation time which is defined in Eq. (A4) in Appendix A.) Therefore, the following discussion will be concentrated on the proton diffusion. For estimating the diffusion rate along a magnetic flux tube, we assume the axisymmetric background magnetic field with mirror symmetry. Our coordinate system taken here is based on the background field \( B \), choosing unit vectors

\[
\hat{s} = B/B, \quad \hat{X} = R_c \hat{s} \cdot \nabla \hat{s} \quad \text{and} \quad \hat{Y} = \hat{s} \times \hat{X}
\]

where \( R_c \) is the field curvature radius. The corresponding (local) coordinates \( X \) and \( Y \) increase inward and westward, respectively. In terms of the longitude \( \phi \) and the distance from the symmetry axis, denoted by \( r \), the flux conservation yields

\[
\hat{Y} \cdot \nabla \phi = 1/r \quad \text{and} \quad \hat{X} \cdot \nabla L = -r B/L R_c^2 B_e (2)
\]

(e.g., Southwood, 1976), where \( \phi \) increases westward, \( L \) is the field line equatorial distance from the Earth center, which is measured in units of the Earth radius \( R_E \), and \( B_e \) is the equatorial field.

So far as the azimuthally aligned structures of large-scale field-aligned currents are concerned, radial diffusion of the particles is more important than azimuthal diffusion. Our study is then focused on radial diffusion, while extension of the result to azimuthal diffusion is straightforward. Anomalous diffusion results from the cumulative effect of a random time series of field perturbation (Southwood, 1972). The particle motion in \( L \) contributing to radial diffusion is then given by the drift of a particle

\[
\dot{L}(t) = \frac{d L}{d t} = \left( \frac{\delta E \times B}{B^2} + \frac{v_\parallel \delta B}{B} + \frac{1}{q B^2} \frac{B}{\nabla B_\parallel} \right) \cdot \nabla L (3)
\]

where \( v_\parallel \) is a field-aligned velocity of the particle, \( \mu \) is the magnetic moment, \( \delta E \) and \( \delta B \) are the disturbance electric and magnetic fields, respectively, and \( \delta B_\parallel \) is the field-aligned component of \( \delta B \). The above equation was originally derived for low-frequency large-scale disturbances (e.g., Southwood, 1976). However, this may generally be applied to the drift motion of resonant particles which experience almost the constant field of a wave, even if the wave frequency is comparable to the cyclotron frequency or the wavelength is comparable to the cyclotron radius (Okuda et al., 1981). The second term in Eq. (3) represents the drift due to the tilt in magnetic field produced by \( \delta B \) (Southwood and Kivelson, 1981). The third term is the gradient drift due to \( \delta B_\parallel \), but it is not considered in estimation of the diffusion rate here, because there has been no available satellite data on \( \delta B_\parallel \) for the electromagnetic disturbances which will be taken up.

For practical calculation of the diffusion coefficient, we formally expand the disturbance field in \( (s, L, \phi) \) coordinates in Fourier series

\[
\delta E = \sum_k E_k \exp i(lL + m\phi + k_\parallel s - \omega_k t), \quad (4)
\]

\[
\delta B = \sum_k B_k \exp i(lL + m\phi + k_\parallel s - \omega_k t)
\]
where \( \omega_k \) is the wave frequency and \( k \) represents the wavenumber vector specified by \((l, m, k_{ll})\): the “local” wavenumber \((k_X, k_Y)\) in the \((X,Y)\) space is obtained from Eq. (2)

\[
k_X = -\left( r B / LR_E^2 B_e \right) l \quad \text{and} \quad k_Y = m / r.
\] (5)

In our paper we derive the radial diffusion coefficients \( \bar{D} \) for two cases: 1. electrostatic waves, and 2. electromagnetic waves in which only Landau resonance with \( \omega - k_{ll} v_{ll} = 0 \) is important. Their derivation is given in Appendix A. The result is as follows.

\[
\bar{D} = \sqrt{8 \pi \left( \frac{r}{r_e} \right)^2} \sum_{n} \sum_{k} \frac{(\mid \mathbf{E}_{Y_k} \mid^2)}{B_e^2} \times \frac{1}{k_{ll} \mid v_{ll} \mid} \exp\left[-\frac{\left\{ (\omega_k - n \Omega_p) / k_{ll} - U \right\}^2}{2 v_{ll}^2} \right] \Gamma_n(k_{ll}r_L) \quad \text{(case 1),} \quad (6)
\]

\[
\bar{D} = \sqrt{8 \pi \left( \frac{r}{r_e} \right)^2} \sum_{k} \left( \frac{m}{k_{ll}} \right)^2 \frac{(\mid \mathbf{E}_{||} \mid^2)}{B_e^2} \times \frac{1}{k_{ll} \mid v_{ll} \mid} \exp\left[-\frac{(\omega_k / k_{ll})^2}{2 v_{ll}^2} \right] \Gamma_0(k_{ll}r_L) \quad \text{(case 2).} \quad (7)
\]

Here, for the hot protons, we assume a Maxwellian velocity distribution with a temperature \( T_p \) and a field-aligned streaming velocity \( U \) (but \( U = 0 \) is assumed for case 2). (For definitions of the other parameters, see Appendix A.) Note that \( \bar{D} \) is the radial diffusion coefficient at an arbitrary point, in which the diffusion spread is expressed in terms of the equatorial distance \( r_e \) (from the Earth center), i.e., \( r_e = LR_E \), using the field line mapping.

We now proceed to estimate the proton diffusion rate \( \bar{D} \) for the wave disturbances which has commonly been observed in the auroral flux tube. Practically we take up the broadband electrostatic noise and the Alfven wave noise, to which diffusion Eqs. (6) and (7) can be applied, respectively. Previous observational and theoretical works on these waves are briefly reviewed in Appendix B.

**Case 1: Broadband electrostatic noise.** Here we numerically estimate the proton diffusion rate for the loss cone driven and beam driven proton cyclotron waves, which are assumed to be responsible for the low-frequency \((\approx \Omega_p, \text{the proton cyclotron frequency})\) part of the broadband electrostatic noise (BEN).

For the loss cone driven proton cyclotron waves, our estimate of the diffusion rate is entirely based on the linear analysis by Ashour-Abdalla and Thorne (1978). As an example, we choose the case shown in Fig. 7 (left panel) of their paper: The first three proton cyclotron harmonic bands having frequencies centered around \( 1.3 \Omega_p, 2.35 \Omega_p \) and \( 3.35 \Omega_p \), respectively, are expected to grow in the equatorial auroral region at \( L = 10 \), where the ambient magnetic field is 30 nT, the proton and electron temperatures are 1 keV, the temperature ratio between the cold and hot ions is \( 2 \times 10^{-2} \), and the density ratio between them is 0.1. Accordingly, the proton cyclotron frequency is \( 2.87 \text{ s}^{-1} \), and the typical Larmor radius \( r_L \) of the hot protons is about 150 km. The unstable wavenumber vectors \((k_{ll}, k_{ll})\) for the first three harmonics are centered around \((k_{ll}r_L, k_{ll}r_L) = (0.7, 10), (0.7, 16) \) and \((0.9, 20)\), respectively. For obtaining an approximate value of the diffusion coefficient \( \bar{D} \) in Eq. (6), the functions of \( k \), except for \((\mid \mathbf{E}_{Y_k} \mid^2)\), are evaluated for the most unstable wavenumbers and they are taken out of the summation over \( k \). Using the parameter values specified above, \( \bar{D} \) in the equatorial region, i.e., \( r \sim r_e \) is given by

\[
\bar{D} \sim (1.9 \sum_{1st-k} (\mid \mathbf{E}_{Y_k} \mid^2) + 1.1 \sum_{2nd-k} (\mid \mathbf{E}_{Y_k} \mid^2) + 0.8 \sum_{3rd-k} (\mid \mathbf{E}_{Y_k} \mid^2)) \times 10^{14} \quad \text{(in MKS)}
\]
where the proton bulk velocity \( U \) is taken to be zero, \( \sum_{\text{th-k}} \) denotes the summation over the wavenumber range associated with the \( i \)th cyclotron harmonic waves, and largest several terms in the summation over \( n \) are added up for each harmonics. If the electric field intensity \( \delta E \) of each harmonic wave turbulence is equally taken as \( \delta E \sim 2 \text{ mV/m} \), we have

\[
\bar{D} \sim 1.5 \times 10^9 \text{ m}^2/\text{s}
\]

which is about 70% of the Bohm diffusion rate \( kT_p/16eB \) for 1 keV protons. For a typical wave amplitude of the BEN observed in the auroral flux tubes, see Gurnett and Frank (1977), which is reviewed in Appendix B. (In Eq. (6) for \( \bar{D} \), we assume a Maxwellian velocity distribution. This is different from a loss cone distribution which is capable of driving the proton cyclotron waves. In spite of this, substitution of the isotropic (nonstreaming) Maxwellian may give a reasonable approximation to the diffusion coefficient \( \bar{D} \) (averaged over the velocity space). This is because \( \bar{D} \) is insensitive to the velocity space derivatives of a distribution function at resonant velocities, and it depends only on a percentage of the resonant particles and a thermal energy of the particles when the characteristics of wave turbulence are specified.)

For the beam driven proton cyclotron waves, our estimate of \( \bar{D} \) is based on the paper by Ashour-Abdalla and Okuda (1986): the cyclotron waves at \( \omega_k \lesssim n\Omega_p \) can be destabilized by field-aligned streaming protons with a temperature comparable to that of the hot plasma sheet particles. This proton beam is then thought to originate from the plasma sheet boundary layer. (In this respect, we note the observational fact that hot ion distributions in the plasma sheet boundary layer to the central plasma sheet commonly show an evolution from highly anisotropic earthward-streaming or counter-streaming beams into the more isotropic distributions (Eastman et al., 1984).) Since the wave excitation was seen predominantly at the first harmonic frequency, i.e., \( |\omega_k| \sim \Omega_p \) in the simulation by Ashour-Abdalla and Okuda (1986), we only consider the proton diffusion due to the first cyclotron harmonic waves. The unstable wavenumber was found to be \( (k_{||}r_L, k_{\perp}r_L) = (0.15, 1.5) \), where \( r_L \) is again the typical Larmor radius of the hot (streaming) protons. Assuming \( U = (\omega_k + \Omega_p)/k_{||} \approx v_{||0} \) (thermal velocity parallel to \( B \)) with \( \omega_k \lesssim -\Omega_p \) and \( k_{||} > 0 \), \( \bar{D} \) in the equatorial region, where \( B \sim 30 \text{ nT} \), becomes

\[
\bar{D} \sim 1.2 \times 10^{15} \sum_k |E_{k\perp}|^2 \quad \text{(in MKS)}.
\]

If the electric field intensity is taken as 1.3 mV/m, we have

\[
\bar{D} \sim 2.0 \times 10^9 \text{ m}^2/\text{s}
\]

which is nearly equal to the Bohm rate for 1 keV protons.

**Case 2: Alfvén wave noise.** For evaluation of the proton diffusion rate in the presence of the Alfvén wave noise, we use Eq. (7) which is applicable to Landau-resonant interaction with low-frequency electromagnetic waves. Bearing in mind the observational features (described in Appendix B) of the Alfvén wave noise, for simple calculation we assume the followings: Basically, \( \bar{D} \) is estimated on the dipole field line at \( L = 7 \). The temperature \( (kT_p) \) of hot protons in the plasma sheet is constant and it is taken as \( kT_p = 6 \text{ keV} \). The dispersion relation for the Alfvén wave noise is simplified as \( \omega/k_{||} \sim V_A \) (Alfvén speed) where the effect of finite Larmor radius is neglected. The Alfvén speed \( V_A \) along the field line decreases with altitude, except for the altitudes below 1000 km, as was shown in Fig. 1 of Mallinckrodt and Carlson (1978). It is convenient to specify values of \( V_A \) at various altitudes: \( V_A \sim 2500 \text{ km/s} \) at 1.8 \( R_E \) altitude, \( V_A \sim 1300 \text{ km/s} \) at 2.4 \( R_E \) altitude, and \( V_A \sim v_{\parallel0} \sim 760 \text{ km/s} \) at the equator, where the background density of oxygen ions (massive ions are more important for determination of \( V_A \)) at
1.8 and 2.4 $R_E$ altitudes is taken as 25 cm$^{-3}$, considering the density profiles given by Maeda (1975). The typical frequency of the Alfvén wave noise is 0.3 Hz, namely the power spectral density (PSD) is peaked at this frequency, and the typical azimuthal wavenumber $k_Y$ is 0.25 km$^{-1}$ at 850 km altitude where the ICB 1300 measurements (see Appendix B) were done. The corresponding angular wavenumber $m$ is calculated as $m = r/L k_Y \sim 730$ where $r_L$ is the distance from the dipole axis to the point at 850 km altitude on the $L = 7$ field line, and $k_Y$ at any altitude is given by $k_Y = m/r$. The wavenumber $k_X$ is taken as $k_X = k_Y$.

Using the parameters specified above, the diffusion coefficient $D$ in Eq. (7) can be expressed as

$$D \sim 1.4 \times 10^{16} G(V_A/v||) \sum_k (|E_{||k}|^2) \Gamma_0(k_L r_L)$$

where $G(\xi)$ is defined as $G(\xi) = \xi^3 \exp(-\xi^2/2)$. Note that even if the wave intensity is constant along the field line, $D$ varies with altitude, through altitude-dependent factors of $G(V_A/v||)$ and $\Gamma_0(k_L r_L)$. As assumed above, the ratio of $V_A/v||$ increases with decreasing altitude, from unity at the equator to the order of ten at 1000 km altitude, while $G(\xi)$ is maximized at $\xi = \sqrt{3}$ and it approaches zero as $\xi$ becomes larger. On the other hand, $\Gamma_0(k_L r_L)$ increases with decreasing altitude, due to a decrease in $k_L r_L$. From such variations of $G(V_A/v||)$ and $\Gamma_0(k_L r_L)$ along the field line, we find that $D$ is maximized around 2.4 $R_E$ altitude in case of a constant wave intensity. Taking the amplitude of a parallel electric field as $\delta E_{||} \sim 0.8$ mV/m, we have the maximum value of $D$

$$D_{\text{max}} \sim 4.8 \times 10^9 \text{ m}^2/\text{s}$$

which is about 16% greater than the corresponding value of the Bohm diffusion rate. Note that $\delta E_{||} \sim 1$ mV/m is measured at midaltitudes of $\lesssim 2 R_E$ by the Viking satellite and $\delta E_{||} \sim 10$ mV/m is measured at low altitudes of $\sim 850$ km by the ICB 1300 satellite. (For reviews of these observations, see Appendix B.) For $\delta E_{||} \sim 0.8$ mV/m, at the equator $D$ is $\sim 21\%$ of the Bohm rate, and at 1.8 $R_E$ altitude $D$ is $\sim 16\%$ of that. Below this altitude, the diffusion rate becomes insignificant as the Alfvén speed is much increased. (This reduction of $D$ is primarily due to a lowering percentage of the resonant particles.)

Although more precise evaluation of the particle diffusion rate would require observations of wave intensities in all altitude range, our inference from the observational facts so far obtained is summarized as follows: due to the intense Alfvén wave noise and broadband electrostatic noise (low-frequency ($\approx \Omega_p$) part), the diffusion coefficient for the plasma sheet protons with keV energies, averaged over the flux tube, is likely to reach a significant fraction of the Bohm rate at least during magnetically active periods. Here it is worthy of noting the ISEE 1 observations (Maynard et al., 1982) of turbulent electric fields on auroral $L$ shells, which appear to comprise both the broadband electrostatic noise and the Alfvén wave noise. They showed that the turbulence is remarkably enhanced during periods of magnetic activity and that it penetrates to very low $L$ values down to about 4. From this observation it should be emphasized that the anomalous cross-field diffusion can be enhanced particularly during disturbed periods, reacting to high intensity levels of wave turbulence.

Finally we need to know some limitations on the present evaluation of anomalous cross-field diffusion in the magnetosphere as well as possible ways of improving it:

1. Basically it is impossible to separate spatial and temporal variations on the measurements aboard one satellite. For this difficulty, there is uncertainty in determination of the wave characteristics such as frequency, wavenumber and polarization, using the satellite data. In such a situation, for more reliable evaluation of the particle diffusion rate, we have to develop a sophisticated theory precisely describing the generation of waves and their propagation in the auroral magnetosphere.
2. The power spectral density (PSD) of observed waves sometimes shows a relatively sharp peak around some frequency. In this case, the quasi-linear theory may not provide a good approximation to the actual diffusion rate. However, this does not mean insignificance of the anomalous particle diffusion in the interaction with quasi-monochromatic waves. Rather, coherent interaction between such waves and particles could extend the self-correlation time (see Eq. (A4)), possibly resulting in a diffusion rate greater than that predicted by the quasi-linear diffusion theory. (A similar condition of persistent wave-particle correlation in coherent wave turbulence was studied by e.g., Lysak et al. (1980), but for particle diffusion in the velocity space.) Hence, in such cases, a nonlinear diffusion theory or a numerical simulation is important as a future work to improve evaluation of the anomalous particle diffusion rate.

3. Field-Aligned Current Generation by Cross-Field Diffusion

As is studied in the preceding section, in the magnetosphere the anomalous cross-field diffusion of the hot protons is likely to be enhanced compared with the electron diffusion. (Recall that most of the hot electrons cannot resonate with the low-frequency waves and the time for an electron to transit over one wavelength is short.) This may directly lead to the occurrence of charge separation in the HPT and the field-aligned current generation because the magnetospheric Pedersen mobility is usually negligible, in contrast with that in the collisional ionosphere, and the ion inertial effect does not effectively act to discharge in large-scale and slow (in time) processes (see Appendix A of Yamamoto et al. (1996)). Hereafter we calculate the production rate of space charges in a flux tube with unit cross-sectional area at the ionospheric height. First consider the plasma sheet protons contained in the volume element of \( (B_i/B(s))ds \) at a field-aligned distance \( s \). Using the field line mapping, the random walk steps of these particles are mapped onto the ionospheric plane, which are represented by \( \Delta x \) and \( \Delta y \) in the latitudinal and longitudinal directions. Thus the diffusion rates on the ionospheric plane (of the “virtual” particles as described below) are written as \( D_{xx} \sim \langle (\Delta x)^2 \rangle / \tau \) and \( D_{yy} \sim \langle (\Delta y)^2 \rangle / \tau \) (see Eq. (A15)). (The population of virtual particles confined on the ionospheric plane is assumed as representing the cross-field dynamics of the plasma particles in the magnetosphere. The positions of individual virtual particles are given by the ionospheric projections of the real particles. Taking the projections of all the particles in the flux tube, the “surface” number density of the virtual particles can be taken as the flux tube content. For more explanation, see Yamamoto et al. (1997, Section 2)). Here it is assumed that \( D_{xx} \) and \( D_{yy} \) are averaged over the velocity space, and that \( D_{xy} \) is negligible. (Even if \( D_{xy} \) is not negligible, this does not affect the diffusion effect on the azimuthally aligned structure which we will consider.) Noting that the surface density of those virtual particles (in the volume element \( (B_i/B(s))ds \)) is given by \( n(B_i/B(s))ds \), the associated space charge production rate, \( \partial \sigma_c / \partial t \), can be expressed as

\[
\frac{\partial \sigma_c}{\partial t} = e \left( \frac{\partial}{\partial x} D_{xx} \frac{\partial}{\partial x} + \frac{\partial}{\partial y} D_{yy} \frac{\partial}{\partial y} \right) \left( n \frac{B_i}{B(s)} \right) ds
\]

where the electron diffusion is assumed negligible. For simplicity, we assume the “isotropy” of \( D_{xx} \sim D_{yy} \), which are denoted by \( D_i \). Neglecting a spatial variation of \( D_i(s) \) in a plane perpendicular to the magnetic field, Eq. (8) becomes

\[
\frac{\partial \sigma_c}{\partial t} = e \nabla^2 \left( D_i(s) \frac{B_i}{B(s)} \right) nds.
\]

The diffusion-induced FAC density \( J_{||} \) at the ionospheric height is given by the charge production rate of all the particles in the flux tube

\[
J_{||} = -e \nabla^2 \int_{s_i}^{s_e} ds D_i(s) \frac{B_i}{B(s)} n.
\]
Fig. 1. HPT particle distribution for example 1. The particles are distributed uniformly in the azimuthal direction. (This HPT distribution is also assumed as the initial condition for the simulation run 1 by Yamamoto et al. (1997, Fig. 1).) (a) Equicontours of the number density \( n_i \) are indicated, where \( n_i \) is proportional to the flux tube content \( N \), as \( n_i = N/(R_B B_i)_0 \), and \( (R_B B_i)_0 \) is the value of \( R_B B_i \) at the center line of the HPT belt. In all the plots in this paper, the invariant latitude and MLT coordinates are used, and the magnetic pole is indicated by the cross. The center of the magnetic drift circle (assumed in the simulation) is indicated by the dot. This is also the center of the circular isodensity contours. (b) Latitudinal profile \( n_i(x) \), where the origin of the latitudinal coordinate \( x \) is set at the center of the HPT.
Defining the diffusion coefficient, $\bar{D}$, averaged over the flux tube volume as

$$\bar{D} = \frac{1}{R_B} \int_{s_i}^{s_f} \frac{1}{B(s)} ds$$

and

$$R_B = \int_{s_i}^{s_f} \frac{1}{B(s)} ds,$$  \hspace{1cm} (9)

$J_{||}$ can be written as

$$J_{||} = -e\nabla_{i||} \cdot (\bar{D} N) \sim -e\bar{D} \nabla_{i||} N$$  \hspace{1cm} (10)

where the plasma density $n$ is assumed uniform along the field lines, and $N$ is the flux tube content defined in Eq. (1). Equation (10) leads us to calculate the FAC density $J_{||}$ from the charge separation rate of the virtual particles with the surface density $N$, which are assumed to diffuse at a rate of $\bar{D}$ on the ionospheric plane. In the following calculations of $J_{||}$ we assume that the proton diffusion coefficient $\bar{D}$ is equal to one third of the Bohm diffusion rate, i.e., $\bar{D} \sim kT_p/48eB_i$. (Note that the Bohm rate at any altitude is converted to the same ionospheric value of $kT_p/16eB_i$, assuming the isotropic field line mapping and a constant temperature.) Numerical examples (1-3) of the diffusion-induced FACs are shown for the undistorted and distorted HPTs which are treated in the numerical simulations of Yamamoto et al. (1997). Such HPT distributions as well as the associated field-aligned current and potential profiles from their simulations are reproduced, in the present paper, as Figs. 1 and 3, where the convection-distorted HPT is responsible for the paired region 1/region 2 FACs; the potential distribution is calculated assuming that the FACs are closed by Pedersen currents on the uniform ionosphere and that the field lines are equipotential. In the present analysis, we assume the same parameter values as used in the simulations: e.g., $kT_p = 6$ keV, $kT_e$ (electron temperature) = 2 keV, $B_i = 6 \times 10^4$ nT, and $\Sigma_p$ (height-integrated Pedersen conductivity) = 8 mho.

**Example 1** The first example of the FAC distribution due to the anomalous diffusion is calculated for the density ($n_i$) profile of the (undistorted) HPT which is shown in Figs. 1(a) and 1(b). (The number density $n_i$ is defined as $n_i = N/(R_B B_i)_0$, where $(R_B B_i)_0$ is the value of $R_B B_i$ at the center line of the HPT belt.) Since this density distribution is uniform in the azimuthal ($y$) direction, the resulting FAC density profile has only the $x$ dependence, which is shown in Fig. 2. The central part of the HPT is negatively polarized because the protons diffuse upward.

![Fig. 2. Example 1. Latitudinal profile of the field-aligned current density ($J_{||}$), at the ionospheric height, which results from the cross-field particle diffusion in the HPT distribution specified in Figs. 1(a) and 1(b). The HPT density ($n_i$) profile is also shown by the dashed line.](image-url)
Fig. 3. (a) HPT distribution for example 2. (This HPT is deformed by the impression of the solar wind convection, and it is assumed at $t = 0$ in run 4 by Yamamoto et al. (1997, Fig. 12).) (b) Field-aligned current density resulting from polarization of the deformed HPT in (a). The polarization is due to the effect of particle magnetic drift only. Dashed equicontours are for upward FAC density $J_{\parallel}(>0)$ at the ionospheric height and solid contours are for downward FAC density $J_{\parallel}(<0)$. For the region 1 FACs, the contours begin at $\pm 0.2 \mu A/m^2$ and are incremented thereafter by $\pm 0.4 \mu A/m^2$. For the region 2 FACs, the contour interval is $0.2 \mu A/m^2$. (c) Electric potential distribution corresponding to the FAC distribution in (b). Solid and dashed contours are for positive and negative potentials, respectively, and the contour interval is 1 kV.

out of that region faster than the electrons. For charge neutrality, some of the HPT electrons in that region are precipitated into the ionosphere, carrying an upward FAC. The peripheral parts of the HPT is positively polarized so that the ionospheric electrons flow up to those regions carrying downward FACs. The estimated FAC densities in Fig. 2 exceed $1 \mu A/m^2$, which is then expected to significantly modify the pattern of the paired region 1/region 2 FACs.

Example 2 The second example is calculated for the HPT profile in Fig. 3(a), which is used in simulation run 4 of Yamamoto et al. (1997). Note that this HPT is already deformed by the impression of the solar wind convection, being responsible for the generation of the paired region 1/region 2 FACs. Figure 4(a) shows the two-dimensional pattern of the FACs due to the anomalous cross-field diffusion of the HPT particles. As can be seen from the first (one-dimensional) example, the central part of the HPT corresponds to the region of upward FAC while the peripheral parts correspond to the downward FAC regions. The FAC densities are greater on the nightside where the HPT density is assumed to be larger. Figure 4(b) shows a superposition of the FAC system due to the magnetic drift, which is shown in Fig. 3(b), and the FAC system due to the anomalous diffusion (in Fig. 4(a)). The resulting FAC pattern remarkably agrees with the observations by Iijima and Potemra (1978): besides the triple structure centered
(b) FIELD-ALIGNED CURRENT DENSITY (magnetic drift)

(c) INTERNAL-POTENTIAL

Fig. 3. (continued).
Fig. 4. Example 2. (a) Field-aligned current density resulting from the cross-field particle diffusion in the HPT distribution in Fig. 3(a). The contours begin at ±0.1 µA/m² and are incremented thereafter by ±0.2 µA/m². (b) Superposition of the FAC system in (a) and the FAC system (in Fig. 3(b)) due to the HPT polarization by the magnetic drift. For the downward region 2 FAC, the contour interval is 0.2 µA/m², and for the other FACs the contours begin at ±0.2 µA/m² and are incremented thereafter by ±0.4 µA/m². (c) Potential distribution corresponding to the FAC distribution in (b). The contour interval is 1 kV.

near local midnight, the locations of the postnoon region 2 FAC at lower latitude and the prenoon region 2 FAC at higher latitude are reproduced in Fig. 4(b), which are the effects of overlapping of these two FAC systems. In addition, the triple structure tends to shift toward the duskside as the time proceeds. (The result is not shown.) This is because the region of vanishing FACs (in the paired region 1/region 2 FAC system) shifts westward in accordance with the westward drift of the HPT protons. In the local time dependence of the FAC densities, there is a difference between the observation by Iijima and Potemra (1978, Fig. 14) and our calculation in Fig. 4(b). Namely, in the observation the region 1 FAC densities are peaked (in magnitude) in the prenoon and postnoon sectors, but such a signature is not found in Fig. 4(b). This difference can be explained by the presence of the region 1 FACs in the low-latitude boundary layer, which are not included in our model.

A recent analysis of the Defense Meteorological Satellite Program (DMSP) satellite data (M. Watanabe, private communication, 1994) has revealed that the triple FAC structure centered around local midnight is clearly identified only during the periods of substorms. (The previous statistical summary of the FAC pattern in the case of |AL|< 100 nT, by Iijima and Potemra (1978, Fig. 13) might include data during substorm periods.) Notably, this observational feature is consistent with our theoretical prediction that highly intense turbulence of the broadband electrostatic noise and/or the Alfvén wave noise, as observed during disturbed periods, is responsible
(b) FIELD-ALIGNED CURRENT DENSITY (magnetic drift + diffusion)

(c) POTENTIAL (HPT polarization due to magnetic drift + diffusion)

Fig. 4. (continued).
for the formation of triple FAC structure, through the enhanced anomalous cross-field diffusion of the HPT protons.

It should be noted here that the diffusion-induced FACs are not self-consistently calculated in the meaning that the HPT deformation is obtained neglecting the cross-field diffusion of the particles. For this reason, we need to point out that there is a physical process counterbalancing the cross-field diffusion of the HPT particles so that they do not indefinitely diffuse out and the HPT polarization continues to be effective for the FAC production. On the nightside, such a counterbalancing effect is brought about by the $E \times B$ flows inherent in the two-cell convection (see Fig. 3(c)): in the region of the HPT on the nightside, the latitudinal convection flow (toward lower latitudes) slows down with decreasing latitudes, which leads to contraction of the HPT in the latitudinal direction. Narrowing of the HPT width, on the nightside, is manifested in the course of simulation runs 1 and 2 of Yamamoto et al. (1997). Moreover, it is found that the narrowing is more significant nearer to local midnight. The condition required for the continuous polarization of the HPT by cross-field particle diffusion is that the decrement of the latitudinal HPT width, $\delta c x$, due to the convection should be comparable to or greater than the increment, $\delta d x$, due to the diffusion. In what follows, it is shown that this condition can be satisfied, at least, in the midnight sector, where the triple FAC structure is expected to emerge. From the simulation results (see Fig. 7 of Yamamoto et al. (1997)), around local midnight $\delta c x$ during the time period of $\sim 41$ min is found to be about $360$ km (on the ionospheric plane). On the other hand, the diffusion spread $\delta d x$ during a time period of $\delta t$ is estimated as $\delta d x \sim (\frac{\delta k T_p}{48 e B_i})^{1/2}$ (see Eq. (A15)). For $\delta t \sim 41$ min and $k T_p \sim 6$ keV (as assumed above), we find the diffusion spread $\delta d x \sim 72$ km, which is even less than the decrement $\delta c x$ of the HPT width due to the convection. Consequently, at least in the midnight sector, the deterioration of the diffusion-induced HPT polarization by diffusion itself does not occur, owing to the latitudinal contraction of the HPT by the two-cell convection. Now we may contend that the charge separation induced by the anomalous cross-field diffusion of the HPT particles is responsible for the formation of the triple FAC structure as observed in the midnight or premidnight sector during disturbed periods.

The potential distribution corresponding to the FAC pattern in Fig. 4(b) is shown in Fig. 4(c), where the coupling between FACs and Pedersen currents with $\Sigma_P = 8$ mho and the equipotential field lines are assumed as before. Compared with the potential without diffusion, which is shown in Fig. 3(c), the potential pattern with the diffusion effect has a more pronounced asymmetry about the midnight meridian: the equipotentials are deformed into a tongue shape extending from dusk to dawn. The thus deformed equipotential contours can form the so-called Harang discontinuity (e.g., Heppner, 1977; Kunkel et al., 1986), which is defined as the line separating westward from eastward convection flow in the evening-midnight sector within or near to the auroral oval. In our model the polarization of the HPT due to the anomalous cross-field diffusion plays a crucial role in the formation of the Harang discontinuity.

**Example 3** Finally, as the third example, we consider the generation of small-scale FACs near the open-close boundary, which is assumed to be primarily due to the anomalous cross-field diffusion of the plasma sheet particles. The ratio between the FAC density ($J_{||}^D$) by diffusion and that ($J_{||}^M$) by magnetic drifts (i.e., paired region 1/region 2 FAC density) is approximately given by

$$\frac{J_{||}^D}{J_{||}^M} \sim \frac{e \bar{D} \nabla_i^2 N}{e (\nabla_{m,i}^{p} - \nabla_{m,i}^{e}) \cdot \nabla_i N} \sim \frac{\bar{D}}{L_x (\nabla_{m,i}^{p} + \nabla_{m,i}^{e}) \sin \theta}.$$  

(For explanation of the current density $J_{||}^M$ and the HPT distortion angle $\theta$, see Yamamoto et al. (1996).) Taking the proton diffusion coefficient as one third of the Bohm rate, i.e., $\bar{D} \sim k T_p / 48 e B_i \sim 2.1$ (km$^2$/s), the average magnetic drift velocity of $\bar{V}_{m,i}^{p} + \bar{V}_{m,i}^{e} \sim 0.32$ km/s (for $k T_p = 6$ keV and $k T_e = 2$ keV), and the distortion angle $\theta \sim 3^\circ$ (which corresponds to moderate
distortion, judging from the simulation results), the ratio of \( \frac{J^D}{J^M} \) is expressed as a function of the latitudinal scale length \( L_x \) in kilometers (at the ionospheric height)

\[
J^D / J^M \sim 125/L_x.
\]

This means that the diffusion-induced FAC is more important when the scale length \( L_x \) is less than about 100 km, particularly near local midnight with a small distortion angle \( \theta \) (where the paired region 1/region 2 FAC density is small). If the density of a hot plasma falls off sharply at the open-close boundary, a pair of upward (at low latitude) and downward (at high latitude) FACs is expected to arise from the charge separation induced by the anomalous diffusion. This situation is illustrated in Fig. 5: the profile \( n_i(x) \) (of the hot plasma) as a function of the latitudinal coordinate \( x \) is assumed proportional to \( 1 + \tanh(-4x/L_x) \) and it is shown in panel (a), and the associated FAC density profile resulting from the anomalous diffusion is shown in panel (b). Such a pair of FACs will be distinguished near the poleward edge of the auroral oval when the scale

![Diagram](image)

**Fig. 5.** Example 3. (a) Assumed latitudinal profile of the hot plasma density \( n_i(x) \) near the poleward edge of the auroral oval. (b) The associated field-aligned current density caused by the anomalous particle diffusion. The current density \( J_\parallel (x) \) is proportional to \( d^2n_i(x)/dx^2 \).
length $L_x$ is less than 100 km, which may often occur as inferred from the particle precipitation flux observed from the satellites (e.g., Meng et al., 1978). The recent observations from the EXOS D satellite (e.g., Yamamoto et al., 1993b) have revealed the frequent occurrence of that kind of a FAC pair (so-called region 0 FAC) along the poleward edge of the nightside auroral oval.

If the magnetospheric electrons carrying the upward FAC are accelerated by a field-aligned potential drop in excess of a few kilovolts, the region 0 upward FACs can be responsible for the discrete auroras which delineate the poleward boundary of the auroral oval (see DMSP auroral images presented by e.g., Meng et al. (1977)). Here it is worthy of noting the commonly observed auroral feature that the poleward edge of the diffuse auroral region has relatively bright auroral emissions which can be distinguished from the background diffuse aurora. For example, in the auroral images from the Dynamic Explorer (DE) 1 satellite, Hones et al. (1989) have identified the “bars” of brighter illumination bordering the horse-collar aura. In summary, it is suggested that the anomalous cross-field diffusion of, particularly, ions in the magnetosphere plays a crucial role in the formation of discrete auroras aligned along the poleward edge of the closed region.

4. Conclusions

On the basis of the satellite observations of the broadband electrostatic noise and the Alfvén wave noise in the auroral magnetosphere, we have shown that the proton (anomalous cross-field) diffusion coefficient averaged over the flux tube is likely to reach a significant fraction of the Bohm rate at least in disturbed periods. As an extension of recently proposed model of the paired region 1 and region 2 field-aligned current generation, we have demonstrated that the observationally identified FAC pattern with the triple structure in the midnight sector can be reproduced by including the effect of the anomalous cross-field diffusion on the HPT polarization. At the same time, this mechanism can account for the formation of the Harang discontinuity in the potential distribution.

The author (T. Y.) is indebted to M. Watanabe and T. Iijima at University of Tokyo, for useful discussion on the DMSP F7 particle data. The work of T. Yamamoto was supported in part by the Ministry of Education, Japan, grant 07804029 as well as the joint research programs of Radio Atmospheric Science Center, Kyoto University, Uji, Kyoto and the Institute of Space and Astronautical Science, Sagamihara, Kanagawa.

Appendix A

Here, from Eq. (3) in text we derive the radial diffusion coefficient of hot protons with keV energies. The diffusion (in $L$) coefficient is defined as (e.g., Drummond and Rosenbluth, 1962; Okuda and Dawson, 1973)

$$D_{LL} = \lim_{\tau \to \infty} \langle (\Delta L(\tau))^2 \rangle / \tau$$

(A1)

where the change in $L$, $\Delta L$, is given by

$$\Delta L(\tau) = \int_0^\tau \dot{L}(t)dt$$

(A2)

and the angle bracket $\langle \rangle$ indicates an ensemble average for random perturbations. The essence of quasi-linear diffusion theory is to integrate $dL/dt$ along the particle’s unperturbed orbit to obtain $\Delta L$, using the random phase approximation (Drummond and Rosenbluth, 1962). The mean square value of $\Delta L$, after a time interval $\tau$ averaged over all initial times, is calculated as
The diffusion coefficient is now expressed as

\[ D_{LL} = 2 \int_0^\infty \langle \dot{L}(t') \dot{L}(t' + t) \rangle dt. \]  

(A3)

Introducing the self-correlation time \( \tau_s \) which is defined as (Spitzer, 1960)

\[ \tau_s = \int_0^\infty \langle \dot{L}(t') \dot{L}(t' + t) \rangle dt / \langle (\dot{L}(t'))^2 \rangle, \]  

(A4)

\( D_{LL} \) is formally written as

\[ D_{LL} = 2 \langle (\dot{L}(t'))^2 \rangle \tau_s. \]

As will be shown later, the correlation time \( \tau_s \) is long enough to produce a significant diffusion rate when the particles can resonate with some waves. For each Fourier component of the disturbance field defined in Eq. (4), addition of the first two terms in Eq. (3) reduces to (see, e.g., Southwood (1976))

\[ \dot{L}_k = \frac{1}{i \omega_k} \left( -i \omega_k + v_{||} \frac{\partial}{\partial s} \right) \left( \frac{r EY_k}{LR_e^2 B_e} - \frac{m v_{||}}{\omega_k} \left( \frac{E_{||}}{LR_e^2 B_e} \right) \right) \]  

(A5)

where we use Faraday's law, i.e., \( i \omega_k B_k = \text{rot} E_k \) along with Eq. (2). For purely electrostatic disturbances with \( \text{rot} \delta E = 0 \), it may be more appropriate to express the electrostatic potential \( \delta \Phi \) as a Fourier series

\[ \delta \Phi = \sum_k \Phi_k \exp \left[ i (L + m \phi + k_{||} s - \omega_k t) \right]. \]  

(A6)

In this case one can immediately find that

\[ \dot{L}_k = \frac{im \Phi_k}{LR_e^2 B_e}. \]  

(A7)

For the case of Landau resonance with electromagnetic waves (case 2 defined in text), the dominant contribution to \( \dot{L}_k \) is the last term in Eq. (A5)

\[ \dot{L}_k = - \frac{m v_{||}}{\omega_k} \left( \frac{E_{||}}{LR_e^2 B_e} \right). \]  

(A8)

Using the random phase approximation with the relation of \( \omega_{-k} = -\omega_k \), substitution of Eq. (A7) into Eq. (A3) yields, for the case of electrostatic waves (case 1 defined in text)

\[ D_{LL} = 2 \left( \frac{1}{LR_e^2 B_e} \right)^2 \sum_k m^2 \langle \Phi_k \rangle^2 \int_0^\infty dt \exp \left[ i \{ L \Delta L(t) + m \Delta \phi(t) + k_{||} \Delta s(t) - \omega_k t \} \right] \]  

(A9)

where \( \Delta L(t) = L(t + t') - L(t') \), \( \Delta \phi(t) = \phi(t + t') - \phi(t') \), and \( \Delta s(t) = s(t + t') - s(t') \), which should be evaluated along the unperturbed particle orbit. Since the self-correlation time \( \tau_s \) defined in Eq. (A4) is supposed to be on the order of wave period, namely, at most a few seconds for the wave disturbances to be studied here, the field-aligned traveling distance of a hot proton with keV energy, during the time period of \( \tau_s \), is limited to about 0.5 \( R_E \). This means that as a first approximation, \( v_{||} \) (t) can be assumed constant in the time integration in Eq. (A9), resulting in \( \Delta s(t) \sim v_{||} t \). (In other words the particle bouncing between magnetic mirror points is not
considered.) For the same reason, the exponential argument, \( l\Delta L(t) + m\Delta \phi(t) \), is approximated by \( \mathbf{k}_\perp \cdot \mathbf{\rho}(t) \), using the "local" wavenumber \( \mathbf{k}_\perp = (k_X, k_Y) \) (see Eq. (5)) and the displacement \( \mathbf{\rho}(t) \) due to the cyclotron motion in the local coordinates \((X, Y)\). (For simplicity, the magnetic drift is neglected.) Equation (A9) is then written as

\[
D_{LL} = 2(\frac{1}{LR_E^2 B_e})^2 \sum_k m^2 |\Phi_k|^2 \int_0^\infty dt \exp\{i(k_\perp \cdot \mathbf{\rho}(t) + (k||v|| - \omega_k)t)\]. \tag{A10}
\]

For case 2 (electromagnetic waves), we similarly obtain from Eqs. (A3) and (A8)

\[
D_{LL} = 2(\frac{1}{LR_E^2 B_e})^2 \sum_k \frac{m v||}{\omega_k} \langle |E||k||^2 \rangle \int_0^\infty dt \exp\{i(k_\perp \cdot \mathbf{\rho}(t) + (k||v|| - \omega_k)t)\}. \tag{A11}
\]

The displacement \( \mathbf{\rho}(t) \) is analytically expressed as (e.g., Ichimaru, 1973)

\[
\mathbf{\rho}(t) = \frac{1}{\Omega_p} \begin{pmatrix} \sin \Omega_p t & 1 - \cos \Omega_p t \\ -(1 - \cos \Omega_p t) & \sin \Omega_p t \end{pmatrix} \begin{pmatrix} v_X \\ v_Y \end{pmatrix}
\]

where \( \mathbf{v}_\perp = (v_X, v_Y) \) is the initial velocity and \( \Omega_p \) is the proton cyclotron frequency, i.e., \( \Omega_p = eB/M \). Supposing that \( \mathbf{v}_\perp \) has an angle \( \psi \) with respect to \( \mathbf{k}_\perp \), \( k_\perp \cdot \mathbf{\rho}(t) \) is calculated as

\[
k_\perp \cdot \mathbf{\rho}(t) = \zeta\{\sin(\Omega_p t - \psi) + \sin \psi\}
\]

where \( \zeta \equiv k_\perp v_\perp /\Omega_p \). Expanding in a series of the Bessel function \( J_n \), we have

\[
\exp\{i k_\perp \cdot \mathbf{\rho}(t)\} = \sum_{n=-\infty}^{\infty} \sum_{n'}^{\infty} J_n(\zeta) J_{n'}(\zeta) \exp\{i(n(\Omega_p t - \psi) + n'\psi)\}.
\]

Averaging over the phase angle \( \psi \) yields

\[
\frac{1}{2\pi} \int d\psi \exp\{i k_\perp \cdot \mathbf{\rho}(t)\} = \sum_{n=-\infty}^{\infty} J_n^2(\zeta) \exp(in\Omega_p t).
\]

Conventionally assuming an infinitesimally small imaginary part of \( \omega_k \) for the time integration in Eqs. (A10) and (A11), the diffusion rates of resonant particles are obtained as

\[
D_{LL} = 2\pi(\frac{1}{LR_E^2 B_e})^2 \sum_k m^2 |\Phi_k|^2 \sum_n J_n^2(\zeta) \delta(k||v|| - \omega_k + n\Omega_p) \quad \text{(case 1)}, \tag{A12}
\]

\[
D_{LL} = 2\pi(\frac{1}{LR_E^2 B_e})^2 \sum_k \frac{m v||}{\omega_k} \langle |E||k||^2 \rangle J_n^2(\zeta) \delta(k||v|| - \omega_k) \quad \text{(case 2)} \tag{A13}
\]

where only the \( n = 0 \) term is retained for case 2 as is prescribed in text.

The radial diffusion coefficient \( D \) in the equatorial distance \( r_e \) (from the Earth center), i.e., \( r_e = LR_E \), is obtained by multiplying \( D_{LL} \) by \( R_E^2 \)

\[
D = R_E^2 D_{LL}.
\]

Denoting \( -ik_Y \Phi_k = -i(m/r)\Phi_k \) by \( E_{Yk} \), the diffusion coefficient \( D \) for case 1 is written as

\[
D = 2\pi(\frac{r_e}{r})^2 \sum_k \frac{|E_{Yk}|^2}{B_e^2} \sum_n J_n^2(\zeta) \delta(k||v|| - \omega_k + n\Omega_p). \tag{A14}
\]
Also the "local" diffusion coefficient \( D_{XX} \), which is defined as

\[
D_{XX} = \lim_{\tau \to \infty} \langle (\Delta X(\tau))^2 \rangle / \tau
\]

is obtained by multiplying \( D_{LL} \) by \((\Delta X/\Delta L)^2 = (LR^2_B e/rB)^2 \) (see Eq. 2))

\[
D_{XX} = 2\pi \sum_k \left( \frac{E_{Y_k}}{B^2} \right)^2 \sum_n J_n^2(\zeta) \delta(k \parallel v_\parallel - \omega_k + n\Omega_p).
\]

Note that the \( n = 0 \) term in \( D_{XX} \) is essentially the same as the Landau-resonant diffusion rate in Drummond and Rosenbluth (1962), who derived the diffusion coefficient by time-integrating the drift velocity to obtain \( \Delta X \) (not integrating the velocity correlation (see Eq. A3)) as is done in our paper. For case 2, the diffusion coefficient \( D \) in \( r_e \) is found from Eq. (A13)

\[
D = 2\pi \sum_k \left( \frac{mv_\parallel}{\omega_k} \right)^2 \left( \frac{E_{Y_k}}{B_e^2} \right)^2 J_0^2(\zeta) \delta(k \parallel v_\parallel - \omega_k).
\]

Finally, the thus obtained diffusion coefficients in Eqs. (A14) and (A16) are averaged over the velocity distribution of the particles. Suppose that the proton distribution function \( f(v) \) is Maxwellian

\[
f(v) = \left( \frac{M}{2\pi k T_p} \right)^{3/2} \exp\left[ -\frac{Mv^2}{2k T_p} \right].
\]

where \( T_p \) is the proton temperature, \( k \) is the Boltzmann's constant, and \( U \) is the field-aligned streaming velocity. (Here we assume the proton bulk flow which could be free energy source for the broadband electrostatic noise (e.g., Dusenbery and Lyons, 1985), although the self-consistent relationship between particle distribution and wave excitation is beyond the scope of the present study.) After some straightforward algebra we obtain the averaged diffusion coefficients

\[
\bar{D} = \int D f(v) dv
\]

\[
= \sqrt{8\pi} \left( \frac{r_e}{r_h} \right)^2 \sum_n \sum_k \left( \frac{E_{Y_k}}{B_e^2} \right)^2 \times \frac{1}{|k\parallel|v_{\parallel 0}} \exp\left[ -\frac{\{(\omega_k - n\Omega_p)/k\parallel - U\}^2}{2v_{\parallel 0}^2} \right] \Gamma_n(k\parallel r_L) \quad \text{(case 1)},
\]

\[
\bar{D} = \sqrt{8\pi} \left( \frac{r_e}{r_h} \right)^2 \sum_k \left( \frac{m}{k\parallel} \right)^2 \left( \frac{E_{Y_k}}{B_e^2} \right)^2 \times \frac{1}{|k\parallel|v_{\parallel 0}} \exp\left[ -\frac{(\omega_k/k\parallel)^2}{2v_{\parallel 0}^2} \right] \Gamma_0(k\parallel r_L) \quad \text{(case 2)}
\]

where \( U = 0 \) is assumed for case 2, \( v_{\parallel 0} \) and \( v_{\perp 0} \) are the thermal velocities parallel and perpendicular to the ambient magnetic field, i.e., \( v_{\parallel 0} = \sqrt{k T_p / M} \) and \( v_{\perp 0} = \sqrt{2k T_p / M} \), respectively; the typical Larmor radius of the protons, \( r_L \), is given by \( r_L = v_{\perp 0} / \Omega_p \); \( \Gamma_n(\zeta) \) is defined as \( \Gamma_n(\zeta) = 0.5 \exp[-\zeta^2/2]I_{|n|}(\zeta^2/2) \), and \( I_n \) is the modified Bessel function of the \( n \)th order.

Appendix B

In this appendix we briefly review previous observational and theoretical works on the broadband electrostatic noise and the Alfvén wave noise in the Earth magnetosphere.
B.1 Broadband electrostatic noise

The broadband electrostatic noise (BEN) was first observed in the boundary layer and geomagnetic tail regions. Early observations of BEN were made by Scarf et al. (1974) and Gurnett et al. (1976) using IMP 7 and 8 satellites, respectively. Gurnett et al. (1976) found a broad range of the wave frequency, from 40 Hz to several kHz, and an average rms electric field amplitude of about 1 mV/m, at radial distances ranging from about 23.1 to 46.3 \( R_E \). A typical power (electric field) spectral density (PSD) of BEN was shown to strongly increase toward lower frequencies (down to the lowest detectable frequency, 40 Hz). (See Fig. 5 of Gurnett et al.) Thereafter, using Hawkeye 1 and Imp 6 data Gurnett and Frank (1977) showed the frequent occurrence of similar broadband emissions on the auroral zone field lines. The BEN was detected on essentially every transit of the auroral flux tubes at all local times; it occurs over a relative wide region, in the altitude range between a few thousand kilometers and many Earth radii, and several degrees in magnetic latitude, at \( L \) values of typically 8–12. This turbulence is most intense during periods of auroral activity and typical maximum electric field strengths are about 10 mV/m. The PSD of BEN peaks just above the local proton cyclotron frequency \( \Omega_p \) and exhibits a pronounced decrease below \( \Omega_p \) and above the lower hybrid frequency.

Recently, Matsumoto et al. (1994) have shown that the BEN observed in the distant magnetotail exhibits an electrostatic solitary pulse structure, from the GEOTAIL Wave Form Capture covering the frequency range between 10 Hz and 4 kHz. As is discussed below, however, such relatively high-frequency (\( \gg \Omega_p \)) part of the BEN is not important for the proton anomalous cross-field diffusion. Instead, the low-frequency (\( \approx \Omega_p \)) part (associated with proton cyclotron waves) of the BEN, which has commonly been detected on the auroral flux tubes, is shown to cause significant proton diffusion. For this reason, we do not study here the solitary pulse BEN observed from GEOTAIL, as a possible cause of the anomalous cross-field diffusion. (In addition, as is addressed in text, coherent nature of waves, as such, may not necessarily result in short self-correlation time nor insignificant particle diffusion.)

Theoretical research efforts have been made to understand the excitation mechanism of BEN. Grabbe and Eastman (1984) first proposed that the BEN can be driven by the ion beam instability. They only considered the stability of cold beams. Dusenbery and Lyons (1985) examined wave generation for a plasma population including both hot and cold beams. They argued that streaming ionospheric (cold) ions can generate electrostatic broadband waves propagating in the beam-acoustic mode while the wave growth rates are enhanced when hot streaming boundary layer ions are present. Ashour-Abdalla and Okuda (1986) carried out a particle simulation of the generation of BEN: The plasma sheet particle population was modelled by counter-streaming ion beams as well as by hot ions and electrons. Both beam-acoustic and ion-ion two-stream instabilities was shown to grow, but only when the ion beam temperature is much lower than the electron temperature in the plasma sheet. This condition is likely to be met if the cold ionospheric ion beams are accelerated toward the plasma sheet. When the beam ion temperature is comparable to the temperature of the hot plasma sheet electrons, electrostatic ion cyclotron instabilities become unstable, giving rise to low-frequency noise at \( \omega \lesssim n\Omega_p \). Grabbe (1987) also showed that the beam-acoustic and ion-ion two-stream instabilities are likely to be responsible for the generation of BEN at frequencies significantly above the ion cyclotron frequency. Besides the above-mentioned ion cyclotron instability as a possible excitation mechanism for low-frequency (\( \approx \Omega_p \)) part of BEN, Ashour-Abdalla and Thorne (1978) earlier proposed the loss cone instability on the hot plasma sheet protons, destabilizing \((n + 1/2)\) proton cyclotron waves. Since unstable waves associated with the beam-acoustic and ion-ion two-stream instabilities have short wavelengths of the order of ten times the electron Debye length (Ashour-Abdalla and Okuda, 1986), the rate of cross-field particle diffusion caused by these waves is quite small due to short self-
correlation time. Consequently, in text we numerically estimate the proton diffusion rate for the loss cone driven and beam driven proton cyclotron waves, which are assumed to be responsible for the low-frequency part of BEN.

B.2 Alfvén wave noise

Besides the broadband electrostatic noise, the low-frequency electromagnetic noise has commonly been observed on the auroral flux tubes. On the basis of DE 1 measurements, Gurnett et al. (1984) investigated the distribution and properties of the low-frequency auroral zone noise down to 1.78 Hz (lowest detectable frequency). They found that 1. the noise is basically electromagnetic, and it becomes increasingly electromagnetic with decreasing altitude; 2. the PSD (power spectral density) strongly increases toward lower frequencies (down to 1.78 Hz); 3. the noise always occurs in regions of low-energy, 100 eV to 10 keV, auroral electron precipitation and field-aligned currents; 4. the electric field is randomly polarized in a plane perpendicular to the ambient magnetic field; 5. the Poynting flux is always directed downward. More recent measurements of even lower frequency (down to 0.05 Hz) electric fields from the Viking satellite covering the altitude range of 2000–13500 km (Block and Falthammar, 1990; Lundin et al., 1990) showed that 1. the PSD peak occurs in the frequency range of 0.3–0.6 Hz and it reaches $10^3$ (mV/m)$^2$/Hz; 2. the PSD for parallel electric fields exceeds 1 (mV/m)$^2$/Hz at frequencies between 0.2 and 2.0 Hz; 3. the temperature of the upflowing ions is well correlated with the PSD. The electromagnetic disturbances identified from the Intercosmos Bulgaria (ICB) 1300 satellite measurements at low altitudes (~850 km) (Chmyrev et al., 1985; Dubinin et al., 1985, 1988) have the following characteristics: 1. The amplitudes in the electric and magnetic fields reach ~100 mV/m and 100 nT, respectively. 2. The radial (from the Earth center) component of the magnetic field, which is roughly parallel to the ambient field, is very small, but the electric radial component reaches up to ~30 mV/m. 3. The PSD increases toward lower frequencies down to 0.3 Hz and it also increases with decreasing perpendicular wavenumber down to 0.25 km$^{-1}$ (see Dubinin et al. (1988, Fig. 8)). 4. The electric fields are elliptically or randomly polarized in a plane perpendicular to the ambient magnetic field.

Theoretical research efforts have been made to understand the generation and propagation of these low-frequency electromagnetic disturbances. According to the DE 2 satellite data analysis of correlated perpendicular components of electric and magnetic fields by Ishii et al. (1992), the perturbations with perpendicular scale lengths less than 32 km are interpreted as a consequence of Alfvén wave propagation, while the larger scale (>64 km) perturbations are interpreted as being static spatial variations due to field-aligned currents. Consequently the aforementioned electromagnetic disturbances, except for their low-frequency (≤0.2 Hz) part, may be regarded as being associated with Alfvén waves. In fact, for the electromagnetic noise (frequency ≥1.78 Hz) observed from the DE 1 satellite, Gurnett et al. (1984) showed that the Alfvén wave model is in general agreement with the altitude dependence of the ratio between measured magnetic and electric fields and that this altitude dependence is in strong disagreement with the static model if the field line is equipotential. Also, for the wave disturbances observed from the ICB 1300 satellite, Dubinin et al. (1988) argued that the power spectra of electric and magnetic fields show the characteristics of Alfvén wave turbulence. Thus, in the present paper the low-frequency (>0.3 Hz) electromagnetic fluctuations which are commonly observed on the auroral zone flux tubes are called the Alfvén wave noise. More specifically, due to the occurrence of parallel electric fields in the waves, they may be identified with kinetic Alfvén waves as was suggested by Gurnett et al. (1984) and Chmyrev et al. (1988). (The kinetic Alfvén wave was first proposed by Hasegawa (1977): when the wave normal angle of the shear Alfvén mode is sufficiently large, the wave can develop an electric field component parallel to the magnetic field.) Notably, considering the downward Poynting flux observed from the DE 1 satellite (Gurnett et al., 1984) the Alfvén wave noise is presumed to originate from a magnetospheric region at high (say, >3 $R_E$) altitude.
(However, energy source for the Alfvén wave generation has not yet been identified clearly (see, e.g., Lundin et al. (1990)).)

REFERENCES

Ashour-Abdalla, M. and H. Okuda, Theory and simulations of broadband electrostatic noise in the geomagnetic tail, *J. Geophys. Res.*, 91, 6833–6844, 1986.

Ashour-Abdalla, M. and R. M. Thorne, Toward a unified view of diffuse aurora precipitation, *J. Geophys. Res.*, 83, 4755–4766, 1978.

Block, L. P. and C.-G. Fälthammar, The role of magnetic-field-aligned electric fields in auroral acceleration, *J. Geophys. Res.*, 95, 5877–5888, 1990.

Bohm, D., E. H. S. Burhop, and H. S. W. Massey, The use of probes for plasma exploration in strong magnetic fields, in *Characteristics of Electrical Discharges in Magnetic Fields*, edited by A. Guthrie and R. K. Wakerling, p. 13, McGraw-Hill, New York, 1949.

Chen, F. F., *Introduction to Plasma Physics and Controlled Fusion*, vol. 1, *Plasma Physics*, p.190, Plenum, New York and London, 1984.

Chmyrev, V. M., V. N. Oraevsky, S. V. Bilichenko, N. V. Isaev, G. A. Stanev, D. K. Teodosiev, and S. I. Shkolnikova, The fine structure of intense small-scale electric and magnetic fields in the high-latitude ionosphere as observed by Intercosmos Bulgaria 1300 satellite, *Planet. Space Sci.*, 33, 1383–1388, 1985.

Chmyrev, V. M., S. V. Bilichenko, O. A. Pokhotelov, V. A. Marchenko, V. I. Lazarev, A. V. Streltsov, and L. Stenflo, Alfvén vortices and related phenomena in the ionosphere and the magnetosphere, *Phys. Scr.*, 38, 841, 1988.

Drummond, W. E. and M. N. Rosenbluth, Anomalous diffusion arising from microinstabilities in a plasma, *Phys. Fluids.*, 5, 1507–1513, 1962.

Dubinin, E. M., P. L. Isaievich, N. S. Nikolaeva, I. Kutiev, and I. M. Podgorny, Localized auroral disturbances in the morning sector of topside ionosphere as a standing electromagnetic wave, *Planet. Space Sci.*, 33, 597–606, 1985.

Dubinin, E. M., A. S. Volokitin, P. L. Isaievich, and N. S. Nikolaeva, Auroral electromagnetic disturbances at altitudes 900 km: Alfvén wave turbulence, *Planet. Space Sci.*, 36, 949–962, 1988.

Dusenbery, P. B. and L. R. Lyons, The generation of electrostatic noise in the plasma sheet boundary layer, *J. Geophys. Res.*, 90, 10935–10943, 1985.

Eastman, T. E., L. A. Frank, W. K. Peterson, and W. Lennartsson, The plasma sheet boundary layer, *J. Geophys. Res.*, 89, 1553–1572, 1984.

Eviatar, A. and R. A. Wolf, Transfer processes in the magnetopause, *J. Geophys. Res.*, 73, 5561–5576, 1968.

Grabbe, C. L., Numerical study of the spectrum of broadband electrostatic noise in the magnetotail, *J. Geophys. Res.*, 92, 1185–1192, 1987.

Grabbe, C. L. and T. E. Eastman, Generation of broadband electrostatic noise by ion beam instabilities in the magnetotail, *J. Geophys. Res.*, 89, 3865–3872, 1984.

Gurnett, D. A. and L. A. Frank, A region of intense plasma wave turbulence on auroral field lines, *J. Geophys. Res.*, 82, 1031–1050, 1977.

Gurnett, D. A., L. A. Frank, and R. P. Lepping, Plasma waves in the distant magnetotail, *J. Geophys. Res.*, 81, 6059–6071, 1976.

Gurnett, D. A., R. L. Huff, J. D. Menietti, J. L. Burch, J. D. Winningham, and S. D. Shawhan, Correlated low-frequency electric and magnetic noise along the auroral field lines, *J. Geophys. Res.*, 89, 8971–8985, 1984.

Hasegawa, A., Kinetic properties of Alfvén waves, *Proc. Indian Acad. Sci.*, 86, 151, 1977.

Heppner, J. P., Empirical models of high-latitude electric fields, *J. Geophys. Res.*, 82, 1115–1125, 1977.

Hones, E. W., Jr., J. D. Craven, L. A. Frank, D. S. Evans, and P. T. Newell, The horse-collar aurora: a frequent pattern of the aurora in quiet times, *Geophys. Res. Lett.*, 16, 37–40, 1989.

Ichimaru, S., *Basic Principles of Plasma Physics*, p. 47, W. A. Benjamin, INC., Reading, Mass., 1973.

Iijima, T. and T. A. Potemra, The amplitude distribution of field-aligned currents at northern high latitudes observed by Triad, *J. Geophys. Res.*, 81, 2165–2174, 1976.

Iijima, T. and T. A. Potemra, Large-scale characteristics of field-aligned currents associated with substorms, *J. Geophys. Res.*, 83, 599–615, 1978.

Ishii, M., M. Sugiuira, T. Iyemori, and J. A. Slavin, Correlation between magnetic and electric field perturbations in the field-aligned current regions deduced from DE 2 observations, *J. Geophys. Res.*, 97, 13877–13887, 1992.

Kunkel, T., W. Baumjohann, J. Untiedt, and R. A. Greenwald, Electric fields and currents at the Harang discontinuity: a case study, *J. Geophys. Res.*, 59, 73–86, 1986.

Lundin, R., G. Gustafsson, A. I. Eriksson, and G. Marklund, On the importance of high-altitude low-frequency electric fluctuations for the escape of ionospheric ions, *J. Geophys. Res.*, 95, 5905–5919, 1990.

Lysak, R. L., M. K. Hudson, and M. Temerin, Ion heating by strong electrostatic ion cyclotron turbulence, *J. Geophys. Res.*, 85, 678–686, 1980.

Maeda, K., A calculation of auroral hiss with improved models for geoplasma and magnetic field, *Planet. Space
Effect of Anomalous Cross-Field Diffusion

Sci., 23, 843–865, 1975.
Mallinckrodt, A. J. and C. W. Carlson, Relations between transverse electric fields and field-aligned currents, J. Geophys. Res., 83, 1426–1432, 1978.
Matsumoto, H., H. Kojima, T. Miyatake, Y. Omura, M. Okada, I. Nagano, and M. Tsutsui, Electrostatic solitary waves (ESW) in the magnetotail: BEN wave forms observed by GEOTAIL, Geophys. Res. Lett., 21, 2915–2918, 1994.
Maynard, N. C., J. P. Heppner, and T. L. Aggson, Turbulent electric fields in the nightside magnetosphere, 87, 1445–1454, 1982.
Meng, C.-I., R. H. Holzworth, and S.-I. Akasofu, Auroral circle-delineating the poleward boundary of the quiet auroral belt, J. Geophys. Res., 82, 164–172, 1977.
Meng, C.-I., A. L. Snyder, Jr., and H. W. Kroehl, Observations of auroral westward traveling surges and electron precipitations, J. Geophys. Res., 83, 575–585, 1978.
Newell, P. T. and C.-I. Meng, Mapping the dayside ionosphere to the magnetosphere according to particle precipitation characteristics, Geophys. Res. Lett., 19, 609–612, 1992.
Okuda, H. and J. M. Dawson, Theory and numerical simulation on plasma diffusion across a magnetic field, Phys. Fluids, 16, 408–426, 1973.
Okuda, H., C. Z. Cheng, and W. W. Lee, Anomalous diffusion and ion heating in the presence of electrostatic hydrogen cyclotron instabilities, Phys. Rev. Lett., 46, 427–430, 1981.
Reiff, P. H., T. W. Hill, and J. L. Burch, Solar wind plasma injection at the dayside magnetospheric cusp, J. Geophys. Res., 82, 479–491, 1977.
Scarf, F., L. Frank, K. Ackerson, and R. Lepping, Plasma wave turbulence at distant crossings of the plasma sheet boundaries and neutral sheet, Geophys. Res. Lett., 1, 189–192, 1974.
Southwood, D. J., Preservation of the second adiabatic invariant during cross L diffusion, J. Geophys. Res., 77, 1123–1127, 1972.
Southwood, D. J., A general approach to low-frequency instability in the ring current plasma, J. Geophys. Res., 81, 3340–3348, 1976.
Southwood, D. J. and M. G. Kivelson, Charged particle behavior in low-frequency geomagnetic pulsations 1. transverse waves, J. Geophys. Res., 86, 5643–5655, 1981.
Spitzer, L. Jr., Particle diffusion across a magnetic field, Phys. Fluids, 3, 659–661, 1960.
Tsurutani, B. T. and R. M. Thorne, Diffusion processes in the magnetopause boundary layer, Geophys. Res. Lett., 9, 1247–1250, 1982.
Yamamoto, T. and M. Ozaki, A theory of current generator in the magnetosphere-ionosphere coupling, Proc. of the NIPR Symposium on Upper Atmosphere Phys., 6, 62–69, 1993.
Yamamoto, T., K. Makita, and C.-I. Meng, A particle simulation of “giant” undulations on the evening diffuse auroral boundary, J. Geophys. Res., 98, 5785–5800, 1993a.
Yamamoto, T., E. Kaneda, H. Hayakawa, T. Mukai, A. Matsuoka, S. Machida, H. Fukunishi, N. Kaya, K. Tsuruda, and A. Nishida, Meridional structures of electric potentials relevant to premidnight discrete auroras: a case study from Akebono measurements, J. Geophys. Res., 98, 11135–11151, 1993b.
Yamamoto, T., M. Ozaki, S. Inoue, K. Makita, and C.-I. Meng, Convective generation of “giant” undulations on the evening diffuse auroral boundary, J. Geophys. Res., 99, 19499–19512, 1994.
Yamamoto, T., S. Inoue, N. Nishitan, M. Ozaki, and C.-I. Meng, A theory for generation of the paired region 1 and region 2 field-aligned currents, J. Geophys. Res., 101, 27199–27222, 1996.
Yamamoto, T., S. Inoue, and C.-I. Meng, Numerical study on dynamics and polarization of the hot plasma torus in the magnetosphere: cause of generation of the paired region 1 and region 2 field-aligned currents, J. Geomag. Geoelectr., 49, this issue, 879–922, 1997.