Spin effects in neutrino gravitational scattering

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Abstract

We study spin oscillations of neutrinos gravitationally scattered off a nonrotating black hole (BH). We derive the transition and survival probabilities of spin oscillations in quadratures when neutrinos interact with BH only. The dependence of the probabilities on the impact parameter is analyzed. Then, we obtain the effective Schrödinger equation for neutrino spin oscillations in neutrino scattering off BH surrounded by background matter. This equation is solved numerically in the case of a supermassive BH with a realistic accretion disk. We find that the observed neutrino fluxes can be reduced almost 20% because of spin oscillations when neutrinos experience gravitational scattering. The neutrino interaction with an accretion disk results in the additional asymmetry in the intensities of outgoing fluxes depending on the neutrino trajectory.

1 Introduction

The experimental observation of neutrino oscillations, reported, e.g., in Ref. [1], confirmed that neutrinos are massive particles having nonzero mixing between different flavors. Among various types of neutrino oscillations, we distinguish neutrino spin oscillations [2], which are the transitions between different helicity states within one neutrino type. If a left polarized neutrino changes its polarization, it cannot be observed since right neutrinos are sterile in the standard model. It will result in the effective reduction of the initial neutrino flux.

It is known that external backgrounds, e.g., the neutrino interaction with matter [3], can modify the process of neutrino oscillations. The gravitational interaction was found in Ref. [4] to influence flavor oscillations of neutrinos. Neutrino spin oscillations in various external fields in curved spacetime were studied in Refs. [5–7]. We considered both static metrics and a time dependent backgrounds, like a gravitational wave. Note that the evolution of the fermion spin in curved spacetime was analyzed in Refs. [8, 9] using both quasiclassical and quantum approaches.

The gravity induced neutrino spin oscillations, studied in Refs. [5,6,10,12], were analyzed for neutrinos orbiting a massive object, e.g., a black hole (BH). However, in this situation, even if neutrino spin oscillations can be quite intense, it is rather difficult to understand what kind of observational effects one can expect since a particle is gravitationally captured by BH, or a neutrino falls to the BH surface. That is why it is interesting to study spin effects, or neutrino spin oscillations, e.g., in the neutrino gravitational scattering, when one can control the helicities of both incoming and outgoing particles.

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This research is inspired by the recent observation of the shadow of a supermassive BH (SMBH) [13], which provides the unique test of the general relativity in the strong field limit. A bright ring around a BH shadow is formed by photons, which are emitted by an accretion disk and then experience strong lensing in the gravitational field of BH [14]. However, besides photons, a significant flux of neutrinos was found in Ref. [15] to be emitted by an accretion disk. These particles are subject to neutrino oscillations. In this work, we shall examine how a strong gravitational field of BH and the neutrino interaction with an accretion disk can modify the helicity of scattered particles.

The neutrino gravitational scattering was studied recently [16], mainly in connection with the determination of the BH shadow produced by these particles [17]. In our work, we shall focus on the analysis of spin oscillations in the neutrino gravitational scattering, which effectively reduce the flux of neutrinos measured in a detector.

Photons, which form the ring around the BH shadow, interact both with its gravitational field and with plasma which surrounds BH. The interaction with plasma can modify the size and the form of the BH shadow (see Ref. [18] for a review). In the present work, we shall study how the neutrino interaction with background matter, e.g., with an accretion disk, can influence the observed flux of gravitationally scattered neutrinos.

In this our work, we continue our studies of neutrino spin oscillations in Refs. [5,7]. We start in Sec. 2 with the analysis of the neutrino spin evolution when a particle gravitationally scatters off a nonrotating BH. We find the expressions in quadratures for the transition and survival probabilities for ultrarelativistic neutrinos and analyze them for different impact parameters. Then, in Sec. 3, we formulate the effective Schrödinger equation for neutrino spin oscillations in the scattering off BH surrounded by background matter. We study astrophysical applications in Sec. 4. In particular, we consider the effect of spin oscillations on the measured neutrino fluxes when particles scatter off SMBH with a realistic accretion disk. Finally, in Sec. 5, we discuss our results. We remind how a scalar particle moves in the Schwarzschild metric in Appendix A.

2 Neutrino spin evolution in scattering off BH

In this section, we study how the spin of a neutrino evolves when a particle scatters off a Schwarzschild BH. We solve the spin evolution equation in quadratures and analyze the solution for ultrarelativistic neutrinos. The transition and survival probabilities for neutrino spin oscillations are derived.

We study the neutrino motion in the vicinity of a nonrotating BH. Using the spherical coordinates \((r, \theta, \phi)\), the interval in this case has the form [19] p. 284,

\[
d\tau^2 = A^2 dt^2 - A^{-2} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2),
\]

where \(A = \sqrt{1 - r_g/r}, r_g = 2M\) is the gravitational radius, and \(M\) is the BH mass. Since the Schwarzschild metric in Eq. (1) is spherically symmetric, we can take that a neutrino moves in the equatorial plane with \(\theta = \pi/2\), i.e. \(d\theta = 0\).

In Refs. [5,6], we found that the neutrino invariant spin \(\zeta\), defined in a locally Minkowskian frame, evolves as

\[
\frac{d\zeta}{dt} = \frac{2}{\gamma}(\zeta \times \Omega_g),
\]
where $\gamma = \frac{dt}{d\tau}$. If a neutrino interacts with a Schwarzschild BH, the vector $\Omega_g$ in Eq. (2) has only one nonzero component \[5\],

$$\Omega_g = (0, \Omega_2, 0), \quad \Omega_2 = \frac{1}{2} \frac{d\phi}{dt} \left(-A + \frac{\gamma}{(1 + \gamma A)} \frac{r_g}{2r} \right), \quad (3)$$

where $d\phi/dt = L A^2 / Er^2$ is the angular velocity, which can be obtained using Eq. (18), $L$ is the conserved angular momentum of a neutrino, and $E$ is the neutrino energy. The parameter $\gamma$ in Eqs. (2) and (3) has the form, $\gamma = E/mA^2$.

We are interested in neutrino spin oscillations, i.e. in the change of the neutrino helicity, $h = (\zeta u)/|u|$, where $u$ is the spatial part of the neutrino four velocity in the locally Minkowskian frame. Therefore, besides the study of the neutrino spin in Eq. (2), we should account for the evolution of $u$.

The expression for $u$ in the Schwarzschild metric has the form \[5\],

$$u = \left( \frac{dr}{d\tau} A^{-1}, 0, r \frac{d\phi}{d\tau} \right) = \left( \pm \frac{1}{m} \left[ E^2 - m^2 A^2 \left( 1 + \frac{L^2}{m^2 r^2} \right) \right]^{1/2}, 0, L \right), \quad (4)$$

where the signs $\pm$ stay for outgoing and incoming neutrinos respectively (see Eq. (18)). At $r \rightarrow \infty$, $u \rightarrow u_{\pm \infty} = \left( \pm \left[ E^2 - m^2 \right]^{1/2} / m, 0, 0 \right)$, i.e. the asymptotic neutrino motion happens along the first axis in the locally Minkowskian frame. In this frame, an incoming neutrino propagates oppositely the first axis. An outgoing particle moves along this axis.

Since only $\Omega_2 \neq 0$, the nonzero neutrino spin components are $\zeta_{1,3} \neq 0$, and $\zeta_2 = 0$. It is convenient to represent

$$\zeta_1 = \cos \alpha, \quad \zeta_3 = \sin \alpha, \quad (5)$$

where $\alpha$ is the rotation angle of the spin from its initial direction.

Now we have to specify the initial condition for Eq. (2). We suppose that, initially, at $r \rightarrow \infty$ and $\phi \rightarrow 0$, an incoming neutrino is left polarized, i.e. the helicity is negative, $h_{-\infty} = (\zeta_{-\infty} u_{-\infty})/|u_{-\infty}| = -1$. Accounting for the expression for $u_{-\infty}$ above, we get that $\zeta_{-\infty1} = 1$ and $\zeta_{-\infty3} = 0$, or $\alpha_{-\infty} = 0$ in Eq. (5).

The helicity of an outgoing neutrino has the form, $h_{+\infty} = (\zeta_{+\infty} u_{+\infty})/|u_{+\infty}|$, where $\zeta_{+\infty} = (\cos \alpha_{+\infty}, 0, \sin \alpha_{+\infty})$ and $u_{+\infty}$ is given above. Using Eq. (5), we get that $h_{+\infty} = \cos \alpha_{+\infty}$. The transition $P_{LR}$ and survival $P_{LL}$ probabilities for neutrino spin oscillations are

$$P = \frac{1}{2} (1 \pm h_{+\infty}), \quad (6)$$

where the upper sign stays for $P_{LR}$ and the lower one for $P_{LL}$.

The angle $\alpha$ corresponds to the spin projection on the $x$-axis in the neutrino rest frame. This projection is in $m/E$ times shorter for a nonmoving observer, which measures the neutrino polarization, because of the Lorentz contraction. It means that the observed angle should be rescaled by the factor $E/m$: $\alpha \rightarrow \alpha E/m$. It is equivalent to the replacement of $\Omega_2$: $\Omega_2 \rightarrow \Omega_2 E/m$. 


Now we should find $\alpha_{+x}$. Using Eqs. (2), (3), (5) and (18), we get that the angle $\alpha$ obeys the equation,

$$\frac{d\alpha}{dr} = F, \quad F(r) = \pm \frac{AL}{mr^2} \left( \frac{3r^2}{2} - 1 \right) - \frac{A^3}{m^2} \left[ \frac{E^2}{m^2} - A^2 \left( 1 + \frac{L^2}{m^2r^2} \right) \right]^{-1/2},$$  

where the signs $\pm$ stay for outgoing and incoming neutrinos. Then we should account for the initial condition $\alpha_{-x} = 0$ and the fact that $\alpha_{+x}$ is twice the angle corresponding to the minimal distance between a neutrino and BH. We express the final result for ultrarelativistic neutrinos, with $E \gg m$, as

$$\alpha_{+x} = y \int_{x_m}^{\infty} \frac{dx}{x^2} \frac{(3 - 2x)\sqrt{x - 1}}{\sqrt{x^2 - y^2(x - 1)}}$$  

where $y = b/r_g$, $b = L/E$ is the impact parameter, and $x_m$ is the maximal root of the equation

$$x^3 - y^2(x - 1) = 0.$$  

Note that $y > y_0 = 3\sqrt{3}/2$ for a neutrino not to fall to BH (see Appendix A).

The expression for roots $x_{1,2,3}$ of Eq. (9) for the arbitrary $y$ has the form,

$$x_k = \frac{2y}{\sqrt{3}} \cos \left[ \frac{1}{3} \arccos \left( -\frac{3\sqrt{3}}{2y} \right) - \frac{2\pi}{3}(k - 1) \right], \quad k = 1, 2, 3,$$

where $x_1 \equiv x_m$ is the maximal root. First, let us analyze Eq. (9) in the case $y \gg y_0$. Using Eq. (10) and keeping only the leading terms, we get that the roots have the form, $x_1 = y - \frac{1}{2} - \frac{3}{8y} + O(y^{-3})$, $x_2 = 1 + O(y^{-4})$, and $x_3 = -y - \frac{1}{2} + \frac{3}{8y} + O(y^{-3})$. In this case, we get that

$$\alpha_{+x} = 8y \int_{\alpha}^{\infty} \frac{dx}{(2x - 1)^2} \frac{(1 - x)}{\sqrt{x^2 - a^2}} \approx -\pi - \frac{\pi}{4y^2},$$

where $a = y - \frac{3}{8y}$. The transition and survival probabilities in Eq. (5) take the form,

$$P_{LR} = \frac{1}{2} \left[ 1 - \cos \frac{\pi}{4y^2} \right] \approx \frac{\pi^2}{64y^4}, \quad P_{LL} = \frac{1}{2} \left[ 1 + \cos \frac{\pi}{4y^2} \right] \approx 1 - \frac{\pi^2}{64y^4}.$$  

One can see that $P_{LR} \to 0$ (and $P_{LL} \to 1$) if $y \gg y_0$. This is expected since, at $y \gg y_0$, a neutrino propagates far away from BH. The gravitational interaction, which causes the spin flip, is weak. Thus the neutrino polarization is practically unchanged.

Now we discuss the situation when $y \to y_0$. Then, Eq. (9) has the following roots: $x_1 = x_2 = 3/2$ and $x_3 = -3$. The spin rotation angle takes the value

$$\alpha_{+x} = -3\sqrt{3} \int_{3/2}^{\infty} \frac{dx}{x^2} \frac{\sqrt{x - 1}}{\sqrt{x^2 + 3}} = -\frac{2\pi}{3},$$

which is finite even if a neutrino asymptotically approaches BH. The corresponding probabilities are $P_{LR} = 0.25$ and $P_{LL} = 0.75$ for such neutrinos. We shall present the transition and survival probabilities for arbitrary $y$ in Sec. (4), when we study some possible astrophysical applications.
3 Neutrino gravitational scattering accounting for the matter interaction

In this section, we formulate the neutrino spin evolution equations in background matter under the influence of a gravitational field when a neutrino scatters off BH. Then, we derive the effective Schrödinger equation for scattered neutrinos.

Using the forward scattering approximation, one gets that the neutrino interaction with background matter is described by the following effective Lagrangian in Minkowsky space-time [20]:

\[ \mathcal{L}_m = -\sqrt{2} G_F \bar{\nu} \gamma^\mu (1 - \gamma^5) \nu G^\mu, \] (14)

where \( \nu \) is the neutrino bispinor, \( \gamma^\mu \) and \( \gamma^5 \) are the Dirac matrices, and \( G_F = 1.17 \times 10^{-5} \text{GeV}^{-2} \) is the Fermi constant. The four vector \( G^\mu \) is the linear combination of the hydrodynamic currents and polarizations of background fermions. It depends on the chemical composition of matter and the type of the neutrino. The explicit form of \( G^\mu \) can be found in Ref. [21].

Basing on Eq. 14, the influence of the neutrino interaction with background matter on its spin evolution in curved spacetime was studied in Refs. [6, 7]. It results in the appearance of the additional components of the vector \( \Omega_g \) in Eq. (3):

\[ \Omega_g \to \Omega = \Omega_g + \Omega_m. \]

If we study the neutrino interaction with nonmoving and unpolarized background fermions in curved spacetime, the vector \( \Omega_m \) has the form,

\[ \Omega_m = \frac{G_F g^0}{\sqrt{2}} \frac{\gamma}{\gamma} = \frac{G_F}{\sqrt{2}} n_{\text{eff}} \left( \frac{dr}{d\tau}, 0, Ar \frac{d\phi}{d\tau} \right), \] (15)

where \( g^0 = e^0_\mu G^\mu = AG^0, e^0_\mu = (A, 0, 0, 0) \) is the vierbein vector in the Schwarzschild metric (see Ref. [5]), and \( G^0 = n_{\text{eff}} \) is the invariant effective density of background matter. We use Eq. (4) to derive Eq. (15).

If we study spin oscillations of electron neutrinos in the electrically neutral hydrogen plasma then \( n_{\text{eff}} = n_e \), where \( n_e \) is the electron number density. The expressions for \( n_{\text{eff}} \) for other neutrino oscillations channels and various types background fermions can be found in Ref. [21].

Instead of dealing with Eq. (2) for the spin precession it is convenient to study the neutrino polarization density matrix, \( \rho = \frac{1}{2} \left[ 1 + (\sigma \zeta) \right] \), which obeys the equation, \( i \dot{\rho} = [H, \rho] \), where \( H = -(\sigma \Omega) \) and \( \Omega \) includes both the gravity and matter contributions in Eqs. (3) and (15).

Since the Liouville–von Neumann equation for the density matrix is rather complicated for the analysis, we can use the Schrödinger equation, \( i \dot{\psi} = H \psi. \) As we mentioned in Sec. (2), neutrinos move along the first axis in the locally Minkowskian frame at \( r \to \infty \). Hence, it is convenient to use this axis for the spin quantization. It mean that we should transform the Hamiltonian \( H \to U_2 H U_2^\dagger \), where \( U_2 = \exp(i \pi \sigma_2/4) \). This procedure brings the meaning to the effective wave function \( \psi \).

As in Eq. (7), it is convenient to rewrite the Schrödinger equation using the radial coordinate \( r \),

\[ i \frac{d\psi}{dr} = H_r \psi, \quad H_r = -U_2 (\sigma \Omega_r) U_2^\dagger, \] (16)

where

\[ \Omega_r = \frac{dt}{dr} \Omega = \left( \frac{G_F}{\sqrt{2}} n_{\text{eff}}, \frac{F}{2}, Ar \frac{d\phi}{dr} \frac{G_F}{\sqrt{2}} n_{\text{eff}} \right). \] (17)
Here $F$ is given in Eq. (7).

Equation (16) should be supplied with the initial condition $\psi_{+\infty}^T = (1, 0)$, which means that all incoming neutrinos are left polarized. Since the neutrino velocity changes the direction at $t \to +\infty$, the transition probability reads $P_{LR} = |\psi_{-\infty}^{(1)}|^2$, and, correspondingly, the survival probability is $P_{LL} = |\psi_{+\infty}^{(2)}|^2$, where $\psi_{+\infty}^T = (\psi_{+\infty}^{(1)}, \psi_{+\infty}^{(2)})$ is the asymptotic solution of Eq. (16).

The solution of Eqs. (16) and (17) can be found only numerically because of the nontrivial dependence of $\Omega_r$ on $r$. Moreover, in Sec. 4 we discuss the situation when $n_{\text{eff}} = n_{\text{eff}}(r)$, which make the analysis more complicated.

We also mention, that we cannot integrate Eqs. (16) and (17) to the turn point $r_m$ and then automatically reconstruct $\psi_{+\infty}$, as we made in Sec. (2). In the presence of the background matter, the dynamics of the neutrino polarization is nonabelian. Moreover, we assume that the neutrino beam scatters off a SMBH surrounded by an accretion disk. We discuss different orientations of neutrino trajectories with respect to the disk plane. Measurable neutrino fluxes are obtained.

First we notice, that standard model neutrinos are produced as left polarized particles. If they gravitationally interact with BH, some incoming left neutrinos become right polarized after scattering. A neutrino detector can observe only left neutrinos. Hence, the observed flux of neutrinos is $F_\nu = P_{LL}F_0$, where $F_0$ is the flux of scalar particles. The value of $F_0$ is proportional to the differential cross section, $F_0 \sim d\sigma/d\Omega$, which is studied in Appendix A.

We assume that the neutrino beam scatters off a SMBH surrounded by an accretion disk. For example, we can suppose that such a SMBH is in the center of a Seyfert galaxy. We take that the mass of SMBH in question is $M = 10^8M_\odot$, the plasma density in the vicinity of SMBH can be up to $n_e \sim 10^{18}\text{ cm}^{-3}$ [23]. Thus, the dimensionless effective potential $V(r) = GM_n e(r) r_g / \sqrt{2}$, reads $V(x) = V_{\text{max}} x^{-\beta}$, where $x = r / r_g$.

One can consider various neutrino trajectories with respect to an accretion disk. However, to highlight the effect of the neutrino interaction with matter we study two extreme cases: the neutrino motion in the plane perpendicular to an accretion disk, marked by the symbol $\perp$, and the neutrino propagation in the plane of an accretion disk, labeled by the symbol $\parallel$. The effect of the neutrino matter interaction is maximal in the latter situation since we assume that a disk is slim.

First we discuss the case $\perp$, when only the gravity contributes the neutrino scattering off BH. The transition and survival probabilities, as the functions of the dimensionless impact parameter $y = b / r_g$, are shown in Figs. (a) and (b). Despite we show the probabilities for $y_0 < y < 11y_0$ (see also Figs. (2) and (3) below), we take that $y < 30y_0$ in our simulations. One can see in Figs. (a) and (b) that $P_{LR}^{(\perp)} \to 0.25$ ($P_{LL}^{(\perp)} \to 0.75$) at $y \to y_0$ and $P_{LR}^{(\perp)} \to 0$ ($P_{LL}^{(\perp)} \to 1$) at $y > y_0$, which is in agreement with the results in Sec. 2.
Figure 1: (a) The transition probability of spin oscillations $P_{LR}^{(\perp)}$, when a neutrino moves perpendicularly to an accretion disk, versus the dimensionless impact parameter $y$. (b) The survival probability of spin oscillations $P_{LL}^{(\perp)}$, when a neutrino moves perpendicularly to an accretion disk, as a function the dimensionless impact parameter $y$. (c) The ratio of the measured fluxes of neutrinos and scalar particles, when they move perpendicularly to an accretion disk, versus the scattering angle $\chi$ normalized by $\pi$. 
The probabilities as functions of the impact parameter, shown in Figs. 1(a) and 1(b), are not the measurable quantities. In Fig. 1(c), we show the measured flux of neutrinos, moving perpendicularly to an accretion disk, \( F^\perp_{\nu} \), normalized by the flux of scalar particles, versus the scattering angle \( \chi \). These fluxes are proportional to the differential cross section, \( F \sim d\sigma/d\Omega \), where \( d\Omega = 2\pi \sin \chi d\chi \). The flux of scalar particles (or the cross section), \( F_0 \), is given in Appendix (see also Ref. [24]).

One can see in Fig. 1(c) that spin effects in the neutrino gravitational scattering off BH significantly reduce the observed flux of neutrinos compared to the case of scalar particles. The influence of spin oscillations is maximal for neutrinos scattered backwards. The reduction of the flux can be more than 20% in this situation.

Now we turn to the discussion of the neutrino interaction with both the gravity and an accretion disk, i.e. we discuss the case \( \parallel \). The transition and survival probabilities are shown on Figs. 2 and 3 for various \( V_{\text{max}} \), or maximal plasma density \( n_e \), and \( \beta \).

One can see that the best coincidence between \( \perp \) and \( \parallel \) cases is implemented when \( V_{\text{max}} = 0.1 \) and \( \beta = 0.5 \); cf. Figs. 1(a) and 2(a), as well as Figs. 1(b) and 3(a). Indeed, this situation corresponds to a low density accretion disk with \( n_e = 10^{18} \text{ cm}^{-3} \), which has relatively rapid density decrease (great \( \beta = 0.5 \)), i.e. the influence of the neutrino matter interaction is minimal. The opposite case is presented in Figs. 2(d) and 3(d), where the influence of matter on neutrino spin oscillations is maximal since matter density is higher, \( n_e = 2 \times 10^{18} \text{ cm}^{-3} \). Moreover, the density profile is less steep (small \( \beta = 0.2 \)), i.e. a neutrino stays longer inside such a disk.

We also mention that the analysis of neutrino spin oscillations in accretion disks with a constant density, studied in Ref. [23], is problematic because of difficulties in the numerical solution of Eqs. (16) and (17) in the limit \( \beta \to 0 \).

We have already mentioned above that the probabilities of spin oscillations versus the impact parameter cannot be measured in an experiment. In Fig. 4, we show the fluxes of neutrinos \( F^\parallel_{\nu} \) scattered off BH and interacting with an accretion disk. These fluxes are normalized to the flux of scalar particles. As mentioned above, the best coincidence between \( \perp \) and \( \parallel \) cases is implemented for \( V_{\text{max}} = 0.1 \) and \( \beta = 0.5 \); cf. Figs. 4(a) and 2(c).

Now we explicitly compare \( \perp \) and \( \parallel \) cases by plotting the ratios of the corresponding fluxes in Fig. 5. First, we mention that \( F^\perp_{\nu} < F^\parallel_{\nu} \). Indeed, if a neutrino interacts only with gravity, spin oscillations are in the resonance (see Refs. 5, 6). The interaction with matter makes the survival probability greater. This fact explains the observed feature.

The difference between \( F^\perp_{\nu} \) and \( F^\parallel_{\nu} \) can reach almost 20%; see Fig. 5(d). It means that, if high energy astrophysical neutrinos experience gravitational lensing by BH surrounded by an accretion disk, the observed flux depends on the orientation of the neutrino trajectory with respect to the disk plane. This maximal difference between \( F^\perp_{\nu} \) and \( F^\parallel_{\nu} \) is for backwardly scattered neutrinos.

5 Discussion

In the present work, we have considered spin effects in the neutrino scattering off a nonrotating BH. The neutrino spin evolution in curved spacetime has been accounted for quasiclassically basing on the approach developed in Refs. 5, 6. We have studied the neutrino scattering off SMBH surrounded by an accretion disk and considered some astrophysical applications.
Figure 2: The transition probability of spin oscillations $P_{LR}^{(\parallel)}$ when a neutrino interacts with an accretion disk, versus the dimensionless impact parameter $y$. (a) $V_{\text{max}} = 0.1 \ (n_e = 10^{18} \text{ cm}^{-3})$ and $\beta = 0.5$; (b) $V_{\text{max}} = 0.2 \ (n_e = 2 \times 10^{18} \text{ cm}^{-3})$ and $\beta = 0.5$; (c) $V_{\text{max}} = 0.1$ and $\beta = 0.2$; (d) $V_{\text{max}} = 0.2$ and $\beta = 0.2$. 
Figure 3: The survival probability of spin oscillations $P_{\parallel\perp}$, when a neutrino interacts with an accretion disk, versus the dimensionless impact parameter $y$ for different $V_{\text{max}}$ and $\beta$. (a) $V_{\text{max}} = 0.1$ ($n_e = 10^{18} \text{ cm}^{-3}$) and $\beta = 0.5$; (b) $V_{\text{max}} = 0.2$ ($n_e = 2 \times 10^{18} \text{ cm}^{-3}$) and $\beta = 0.5$; (c) $V_{\text{max}} = 0.1$ and $\beta = 0.2$; (d) $V_{\text{max}} = 0.2$ and $\beta = 0.2$. 
Figure 4: The fluxes of neutrinos, normalized by the flux of scalar particles, for particles interacting with background matter of an accretion disk. Here we represent the dependence of $F_{\nu}^{(\parallel)}$ on $\chi$ for different $V_{\text{max}}$ and $\beta$. (a) $V_{\text{max}} = 0.1$ ($n_e = 10^{18}$ cm$^{-3}$) and $\beta = 0.5$; (b) $V_{\text{max}} = 0.2$ ($n_e = 2 \times 10^{18}$ cm$^{-3}$) and $\beta = 0.5$; (c) $V_{\text{max}} = 0.1$ and $\beta = 0.2$; (d) $V_{\text{max}} = 0.2$ and $\beta = 0.2$. 
Figure 5: The ratio $F_{V}^{(\perp)}/F_{V}^{(\parallel)}$ versus the scattering angle $\chi$ for different $V_{\text{max}}$ and $\beta$. (a) $V_{\text{max}} = 0.1 \ (n_{e} = 10^{18} \text{ cm}^{-3})$ and $\beta = 0.5$; (b) $V_{\text{max}} = 0.2 \ (n_{e} = 2 \times 10^{18} \text{ cm}^{-3})$ and $\beta = 0.5$; (c) $V_{\text{max}} = 0.1$ and $\beta = 0.2$; (d) $V_{\text{max}} = 0.2$ and $\beta = 0.2$. 
In Sec. 2, we have studied the neutrino spin evolution in a gravitational scattering in the Schwarzschild metric. Supposing that all incoming neutrinos are ultrarelativistic and left polarized, we have obtained that the transition probability $P_{LR}$ for outgoing particles can reach $25\%$ if the impact parameter is close to the critical one $b \approx b_0 = 3\sqrt{3}r_g/2$. Note that the fact that the helicity of ultrarelativistic (massless) particles can be changed under the influence of a gravitational field was mentioned earlier in Ref. [25]. We also mention that our calculation of the probabilities in the limit $y \gg y_0$ in Eq. (12) is consistent with the result of Ref. [26], where the neutrino helicity flip in the idealized gravitational field was studied.

Then, in Sec. 3, we have derived the effective Schrödinger equation for a neutrino scattering off BH surrounded by background matter with a nonuniform density. In the case of only gravitational scattering, studied in Sec. 2 it was possible to obtain analytically the transition and survival probabilities for some impact parameters. If, besides gravity, a neutrino interacts with a background matter, the probabilities can be derived only in the numerical solution of Eqs. (16) and (17).

In Sec. 4, we have considered the astrophysical applications of our results. In particular, we have studied the effect of spin oscillations on the neutrino scattering off SMBH surrounded by the accretion disk. We have taken the parameters of the accretion disk, such as the number density and the mass distribution, close to the values resulting from observations and hydrodynamics simulations. Using the numerical solution of Eqs. (16) and (17), we have found the transition and survival probabilities, as well as the observed fluxes of outgoing neutrinos for different orientations of the particles trajectories with respect to the accretion disk.

As one can see in Figs. 1(c) and 4, there is no deviation of the fluxes for the forward neutrino scattering at $\chi = 0$ if one compares them with the fluxes of scalar particles. It means that neutrino spin oscillations do not affect the size of the BH shadow. The major effect of spin oscillations is for the backward neutrino scattering at $\chi = \pi$. Thus the intensity of the glory flux for neutrinos is almost $20\%$ less than for scalar particles; cf. Fig. 4(a).

The influence of the plasma interaction on the gravitational scattering of scalar particles (photons) was extensively studied (see, e.g., Ref. [18] for a review). For example, the photons propagation in plasma surrounding a nonrotating BH was examined in Ref. [27]. The form of the BH shadow was found to be unchanged, but its size can be magnified. In Fig. 5, we predict the asymmetry in the observed neutrino fluxes depending on the orientation of the neutrino trajectory with respect to the accretion disk. This asymmetry is maximal for the backward neutrino scattering. Although neutrinos interact with plasma much weaker than photons, the asymmetry can reach almost $20\%$ for the realistic accretion disk; cf. Fig. 5(d).

Since the effects of spin oscillations on the neutrino gravitational scattering are valid for ultrarelativistic particles, the results obtained in the present work are of importance for the rapidly developing area of the neutrino astronomy [28], where a significant success was achieved in the detection of the ultrahigh energy (UHE) cosmic neutrinos. Neutrinos with energies in the PeV range were reported in Ref. [29] to be detected. Moreover there are sizable efforts in the identification of the sources of UHE neutrinos with astronomical objects such as active galactic nuclei [30]. In our work we have demonstrated that, if the incoming flux of cosmic neutrinos experience the gravitational lensing, the observed flux can be reduced up to $20\%$, compared to its initial value, because of neutrino spin oscillations.
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A Particle motion in the Schwarzschild metric

In this Appendix, we briefly remind how to describe the motion of a scalar particle interacting with a nonrotating BH, as well as how to calculate the differential cross section of the gravitational scattering. These problems were studied in details in Refs. [19, pp. 287-290] and [24].

The energy $E$ and the angular momentum $L$ are conserved quantities for a particle with the mass $m$ moving in the Schwarzschild metric in Eq. (1). The equation of motion and the trajectory are defined by

$$\frac{dr}{dt} = \pm \frac{mA^2}{E} \left[ \frac{E^2}{m^2} - A^2 \left( 1 + \frac{L^2}{m^2 r^2} \right) \right]^{1/2}, \quad \frac{d\phi}{dr} = \pm \frac{L}{mr^2} \left[ \frac{E^2}{m^2} - A^2 \left( 1 + \frac{L^2}{m^2 r^2} \right) \right]^{-1/2},$$

for a particle moving in the equatorial plane. In Eq. (18), the minus signs stay for incoming particles and the plus ones for outgoing particles.

If we study ultrarelativistic particles, the angle corresponding to the minimal distance between a particle and BH is

$$\phi_m = y \int_{x_m}^{x_0} \frac{dx}{\sqrt{x^3 - y^2(x - 1)}},$$

where $y = b/r_g$, $b = L/E$ is the impact parameter, and $x_m$ is the maximal root of Eq. (9). The parameter $y > y_0 = 3\sqrt{3}/2$. Otherwise a particle falls to BH.

While computing the differential cross section, $d\sigma/d\Omega$, where $d\Omega = 2\pi \sin \chi d\chi$, we should take into account that a particle, before being scattered off, can make multiple revolutions around BH, both clockwisely and anticlockwisely. One should account for this fact in the determination of the angle $\chi$, fixing the position of a detector, which is in the range $0 < \chi < \pi$.

In Fig. we present the result of the numerical computation of the cross section. While building this plot, we take that $y_0 < y < 30y_0$ and account for up to two revolutions of a particle around BH in both directions. This our result is used in Sec. 4 when we study the neutrino scattering off a realistic BH surrounded by an accretion disk.

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Figure 6: The differential cross section of the gravitational scattering of scalar particles off a nonrotating BH, normalized by $r_g^2$, versus the angle $\chi$.

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