A dielectric microcylinder makes a nanocylindrical trap for atoms and ions

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In the diffraction of visible light by a dielectric microcylinder, packages of evanescent waves always arise. However, a single-wave incidence corresponds to rather small impact of evanescent waves outside the cylinder. In this paper, we theoretically show that a symmetric pair of plane waves impinging a glass microcylinder corresponds to much higher impact of the evanescent waves. Namely, the interference of the evanescent waves with the propagating ones results in the suppression of the electromagnetic field in an area with very small cross section. This area is located in free space at a substantial distance from the rear side of the microcylinder and along its axis. It may serve a linear optical trap for cold atoms and ions.

I. INTRODUCTION

The most known near-field effect of the visible light diffraction by a dielectric microcylinder, is the waist of the called photonic nanojet [1]. This nanojet is a wave beam whose waist is centered at the rear edge of the particle [1–4]. In this area, the field spatial spectrum comprises a noticeable evanescent-wave component [2–4] that implies a nonzero longitudinal component of the electric field in the case of the TM-incidence [2–4]. However, the effective width of this waist for a cylinder is not very subwavelength (of the order of 0.5λ) and the local enhancement of the electric intensity is modest (3−4). Such near-field effect can be called slightly subwavelength − the contribution of evanescent waves into the field in the region of the waist is not dominant [4]. It is dominant at the frequencies of well-known resonances - those of whispering gallery modes and at Mie resonances of the microcylinder [5–7]. However, the spatial regions where evanescent waves dominate at these resonances are located inside the cylinder and this domination implies high values of the local electric intensity compared to that of the incident wave. Briefly, for a dielectric microparticle (cylinder or sphere) known near-field effects are effects of subwavelength field concentration [7–9].

However, there are no theoretical restrictions for pronounced near-field effects that arise outside a microsphere or a microcylinder. If a cylinder is made of a dielectric material with the refractive index, say, n = 1.4−1.5 the spatial variation of the induced eigenmodes inside and outside the cylinder have the same scale. Inside a microcylinder, the fields are expressed via Bessel’s functions Jm(kr) (here kr = kn and k is the wave number in free space) and outside − via Hankel’s functions Hm(1)(kr) which comprise the Bessel component as well. Though it is usually thought that all practically important near-field effects for a glass microparticle (cylinder or sphere) are effects of subwavelength concentration of the electromagnetic fields, we will show that it is not so. We will report an amazing near-field effect which arises for a microcylinder and has nothing to do with the field concentration in it. It arises at the frequencies slightly shifted from those of high-order Mie resonances.

II. SPATIAL FANO RESONANCE BEHIND THE MICROCYLINDER

Consider a microcylinder of radius R ≫ λ symmetrically impinged by a wave beam consisting of two TM-polarized plane waves with the same electric field amplitude E0 = H00η (η is free-space impedance, H0 is the magnetic field amplitude) and opposite phases. Let the wave vectors k+ be tilted to the axis y by a small angle β ≪ π/2 as it is depicted in Fig. 1(a). Expanding these plane waves into cylindrical functions we deduce after some algebra the following expression for the magnetic field of the incident beam:

Hib = a2H0∞ m=−∞ im Jm(kzr)eimφ sin mβ, (1)

where H0 = −2iH1 and a2 is the unit vector of the axis z. The corresponding electric field of the incident beam can be easily obtained by differentiation of every series term in (1):

Eib = 1 jωc ∇ × Hib = 1 jωc [aρ, 1 ρ ∂ Hib ∂ρ − aφ ∂ Hib ∂φ ],

where aρ and aφ are two other unit vectors of the coordinate system.

For the total magnetic field ˆH outside the cylinder we further obtain

H = a2H0∞ m=−∞ im [Jm(kr) + TmHm(1)(kr)] sin mφ sin mβ, (2)
where

\[ T_m = \frac{k_c J_m'(kR) J_m(k_c R) - k J_m'(k_c R) J_m(k R)}{k J_m'(k_c R) H_m^{(1)}(kR) - k_c J_m(k_c R) [H_m^{(1)}(kR)]'}. \]

The factor \( \sin m \beta \) in every term of series (2) nullifies \( H, E, \) and \( \partial H/\partial \phi \) on the axis \( y, \) and drastically changes the field spatial distribution everywhere.

Near the high-order Mie resonance one of the series terms with number \( m = M \gg 1 \) describing the \( M \)-th quasi-mode (leaky mode) of the cylinder dominates over any other modes. In the case of a single-wave incidence (that is equivalent to letting \( \sin m \beta = 1 \) in the right-hand side of (2)), the almost-resonant \( M \)-th mode weakly interferes with the spatial quasi-continuum of non-resonant terms. Numerical analysis shows that in this case the sum of all non-resonant terms has the smaller magnitude than the magnitude of the almost-resonant mode. Outside the cylinder the evanescent part of the \( M \)-th mode rapidly decays, whereas its leaky part (cylindrical wave corresponding to the large-argument asymptotic of \( H_m^{(1)}(k \rho) \)) varies in sync with the quasi-continuum. Meanwhile, in our case, when the factor \( \sin m \beta \) is present and \( \beta \) is small enough, the situation changes drastically. First, the convergence of the series improves – all terms with \( m > M \) give a negligibly small contribution into the series sum. Second, the magnitude of the quasi-continuum of lower-order modes \( m < M \) has the magnitude of the same order as that of the \( M \)-th mode in the whole region of our interest – near the back side of the illuminated cylinder.

This is the prerequisite of the pronounced interference. Behind the cylinder on the axis \( y, \) a spatial minimum of the electric field \( E(\phi = 0, \rho) = E_\rho = (\omega \varepsilon_0 \mu_0)^{-1} \partial H/\partial \phi \) arises where the electric field of the \( M \)-th mode and that of the quasi-continuum have the same amplitudes and opposite phases. Inside the cylinder near its back edge their interference is constructive and \( E(\phi = 0, \rho) = E_\rho \) has a local maximum. A pair of maximum and minimum, adjacent to the coordinate axis \( y, \) can be treated as the spatial Fano resonance. When a Fano resonance occurs, over the frequency axis, its minimum is deep and narrow. Is it possible to observe a similarly sharp minimum in our spatial Fano resonance? Yes, our calculations have shown that for sufficiently large \( kR (kR > 8-10) \) and sufficiently small \( \beta (\beta < \pi/kR) \), some values of the cylinder permittivity grant an ultimately sharp field minimum. Alternatively, we may find this regime for any fixed \( \varepsilon \) (at least within the interval \( 2 < \varepsilon < 4 \)) varying \( kR \) and \( \beta \). For certainty, below we report our results corresponding to the fixed size parameter \( kR = 10 \).

Since the incident beam intensity varies versus \( x, \) it is reasonable to normalize the intensity of the total electric field \( I = E_\rho^2 + E_\phi^2 \) to the intensity of the incident beam \( |E_0|^2, \) averaged over the relevant interval \(-R < x < R \). The integration is simplified by the condition \( \beta \ll 1, \) and it is easy to derive for \( I_0 \) the formula

\[ I_0 = (\eta H_0)^2 \sin^2(kR \sin \beta)/\sin^2 \beta. \]

Since on the axis \( y \) the incident and total electric fields are polarized longitudinally \( E(\phi = 0, \rho) = [E_\rho(\phi = 0)]^2 \), normalized intensity on this axis is equal \( I(\phi = 0, \rho)/I_0 = E_\rho^2/I_0 \). If \( \beta < \pi/2kR \), the whole cylinder is located between two maxima of the incident beam intensity. If \( \pi/2kR \leq \beta \leq \pi/kR \), the cylinder covers these two maxima located between two minima of the incident beam intensity. First, we report the results for the case \( \beta < \pi/2kR \).

The logarithmic plot of the normalized intensity in the coordinate interval \( ky = k \rho = 10 - 20 \) is presented in Fig. 1(b). The cylinder permittivity \( \varepsilon = 2.4445 \) (corresponds to \( m = 12 \) Mie resonance) offers a weak minimum to the normalized intensity at the distance of the order of \( \lambda \) behind the cylinder. Gradually increasing \( \varepsilon \) from this value we tune the system to the regime of the Fano resonance. We found several values of \( \varepsilon \) corresponding to ultimately narrow and deep minima of the electric field behind the cylinder. Two of them are shown in Fig. 1(b). Permittivities \( \varepsilon = 2.908 \) (corresponds to \( m = 13 \) Mie resonance) and \( \varepsilon = 3.4975 \) (near \( m = 14 \) Mie resonance), offer intensity in minimum smaller than \( I_0 \) by 10 and...
14 orders of magnitude, respectively. Since the magnetic field on the axis $y$ is identically zero, in these minima the electromagnetic field practically vanishes.

The distribution of the normalized electric intensity in the plane $(x-y)$ confirms our insight that these minima are namely those of spatial Fano resonances. Fig. 2(a) presents the plot of $\ln (I/I_0)$ in the plane $(x-y)$ for $\beta = 0.01$ and $\varepsilon = 3.4975$. Internal maxima corresponding to the almost-resonant mode $M = 14$ are located around the cylinder near its surface. One of these maxima is higher than the others and is located at the axis $y$. It forms together with our minimum a typical Fano resonance. Meanwhile, in the color map Fig. 2(b) we see that the electric field distribution is very different from the typical picture of a mode $M \gg 1$ in the range of its resonance excited by a single plane wave [6]. The distorted modal distribution with sharp interference minima is explained by the interference of the $M$-th almost resonant mode and the quasi-continuum of lower modes which have the same magnitudes in the region of our interest.

When $\beta < \pi/2kR$ both real and imaginary parts of the electric field phasor change the sign at different points of the axis $y$, and our Fano minimum lies between these points being distant from the cylinder by $\rho - R \approx 0.7\lambda$. In the vicinity of our minimum the phase of the electric and magnetic fields differ by nearly $\pi/2$, that clearly links the effect to evanescent waves generated on the back surface of the cylinder. Conventionally, near-field effects cannot be observed far from the scattering object. However, conventional near-field effects imply the local field enhancement. Our near-field effect is opposite – it is cancellation of the small longitudinal component of a wave beam by the evanescent waves and it may nicely occur at distances about $\lambda$.

When $\pi/2kR < \beta < \pi/kR \text{ Re}(E_\rho)$ and $\text{Im}(E_\rho)$ may nullify at the same point on the axis $y$. This result is not a numerical artefact – account of $N$ high-order ($m > M$) terms in the series (1) does not change this result for any $N$. In other words, for a given value of $\beta$, a small variation of the permittivity may locate a point in which $\text{Re}(E_\rho) = \text{Im}(E_\rho) = 0$. Thus, behind the cylinder there is an amazing point where the electromagnetic field vanishes. This situation is shown in Fig. 3(a) for the case $\beta = 0.15$, $\varepsilon = 3.360595$. In this case the zero point is distanced from the cylinder still by $0.7\lambda$. For $\beta = 0.1$ we found $\varepsilon = 3.4617233453067$ granting the similar zero at the distance nearly equal $\lambda$. The normalized intensity versus $k_\rho$ behind the cylinder for these two cases are shown in Fig. 3(b) in the log scale.

In Fig. 4(a) we depict the color map of the normalized intensity in the plane $(x-y)$ for the case $\beta = 0.15$, $\varepsilon = 3.360595$. This distribution resembles the picture of the $TM_{14}$ resonance excited by a single plane wave. However, there is a drastic difference – a set of sharp minima. Four of them are located on the axis $y$ outside the cylinder. Only at the minimum located behind the cylinder, the exact zero is achieved. Three minima located in front of it are much weaker – in them $I/I_0 \sim 10^{-3} - 10^{-4}$). Fig 4(b) represents the same color map as in Fig. 4(a) as a contour plot shown around the main Fano minimum. This plot lets one see that the shape of the minimum is not circular, it is elongated in the axial direction.

III. OPTICAL TRAP AT THE FANO MINIMUM

The revealed effect is, to our opinion, very promising for trapping the atoms and ions. An atom with polarizability $\alpha$, experiences in the monochromatic light of non-uniform intensity $I(x,y)$ the so-called gradient force $F_g(x,y) = 0.5\text{Re}(\alpha)\nabla I(x,y)$. This formula, initially derived in [10] for dielectric nanoparticles, was generalized for atoms in the laser light field in [11][12]. In the case of...
blue detuning (from the main excited state of the atom) \( \text{Re}(\alpha) < 0 \) and \( F_g \) stretches towards the minimum of the electric intensity. Since our minima in Figs. 2 and 3 are ultimately sharp, a trapped atom will be centered at the corresponding point of the plane \((x - y)\) being movable only along \(z\). With a simple glass microcylinder we may prepare a unique object – a straight linear chain of atoms in free space. Linear density of the atoms in this chain can be controlled.

Our singular Fano minimums can serve as a trap also for ions. In accordance to [14], the paraxial region of a Bessel beam with radial polarization and nonzero order can serve a subwavelength thin trap for charged particles. It is possible to show that this trapping property keeps in our 2D case when the Bessel beam is replaced by our two-wave beam. Cold ions gathering at the plane \((y - z)\), due to their charges, will be then moved to our light intensity minimums by the gradient force.

When the incident light has the same wavelength as that of the resonant optical absorption \(\lambda_A\) is pumped and the atom turns out to be cooled [15]. When the flux density is of the order of one kW per square cm, optical potential of an atom expressed in Kelvins is of the order of dozens of milliK. If the frequency detuning \(\Delta = \omega - \omega_A\) of the incident light with respect to the transition frequency is positive, the polarizability of an atom has the negative real part and the trapping effect arises in the minimums of the electric intensity where the atom optical potential drops from dozens of milliK to one milliK or less [15–17].

Figs 5 and 6 depict the optical potential \(U(x, y)\) in our optical trap for the case when the trapped atoms are those of Cs. They experience at the wavelength \(\lambda_A = 852\) nm the transition of type \(6S_{1/2} - 6P_{3/2} D2\). \(U\) is calculated using the formula (see e.g. in [16])

\[
U = \mu^2 E^2 / 2 k_B h \Delta,
\]

where \(\mu = 2.67 \cdot 10^{-29}\) SI is the dipole moment (matrix element) of the optical transition, \(E^2 = I\) is the electric intensity, corresponding to the power flux 1 kW/cm\(^2\) in the intensity maximum of our beam, \(k_B\) and \(h\) are Boltzmann and Planck constants, respectively. In these calculations, we adopted \(\Delta = 10^5 \Gamma\), where \(\Gamma = 3.07 \cdot 10^7\) s\(^{-1}\) is the Lorentzian damping fre-
FIG. 5. (a) The optical potentials of an atom of Cs varying along the beam axis for different dielectric constants of the cylinder material. The choice $\epsilon = 3.36$ corresponds to the absolute minimum of the potential. The black dotted rectangles show the width of the trap that can be reduced to 230 nm adjusting $\epsilon$. (b) The same potentials depicted over both $x$ and $y$ axes allow one to compare the trap width and height (both width and height are subwavelength). Transition in Cs is enabled by laser radiation at wavelength $\lambda = 852\text{nm}$. Poynting vector magnitude at maximum of the incident beam intensity is equal to 1 kW/cm$^2$.

FIG. 6. Contour plots of optical potential: (a) $\epsilon = 3.36$, (b) $\epsilon = 3.359$, and (c) $\epsilon = 3.358$. Dashed contours correspond to the optical potential at 1 mK distance from the minimum (effective perimeter of the trap).

Frequency of the Cs atom [18]. Approaching the minimum of the optical potential to zero offers the long-time confinement of atoms, because the atoms located at the minimum practically do not heat up. As to the trap transverse sizes, they noticeably shrink with a small deviation of the minimal potential from the exact zero as we can see in Fig. 5(a). The effective dimensions of our optical trap in the $x$ and $y$ directions were estimated in accordance to the criteria formulated in [18] (1 mK distance from the potential minimal value). In Fig. 5, we see that these sizes in the longitudinal and transverse directions can be as small as 230 and 180 nm, respectively if the permittivity of the cylinder is finely adjusted. The laser pumping with the flux density 1 kW/cm$^2$ and even much higher one has been used for optical trapping since 1980s [19]. Contour plots of optical potential for different permittivities of the cylinder are shown in Fig. 6. We can conclude from our calculations that the maximal trap size can be done smaller than $\lambda/4$. If we do not target the minimal possible size of the trap, the optical potential at its center can be engineered nearly equal to zero (on the level of microKelvins).

Thus, a simple glass microcylinder illuminated by an intensive cosine wave beam of laser light creates in free space a long optical trap with subwavelength cross section. This idea is illustrated by Fig. 7. Cold atoms or ions can be guided along this trap e.g. by a static electric field, and it can operate as an atomic waveguide. The creation of such atomic waveguides is an actual problem of modern physics. Therefore, similar traps has been recently developed and corresponding works forms a body of literature (see e.g. in [20–22]). However, all these traps, to our knowledge, have been formed only inside the diffraction-free light beams. In our opinion, this approach to the creation of the cylindrical optical trap has an inherent drawback – complexity of its implementation.

No one realistic wave beam is ideally diffraction-free. In focused Bessel beams of zero order [21], where the atoms are trapped in the area of the maximal intensity centered by the beam axis, the effective length of the cylindrical trap (whose cross section has the diameter close to $\lambda$) does not exceed a dozen of microns. There
are other problems with the trapping of atoms in these light needles where are studied in work \[21\]. In this paper it was shown that the tightly focused diffraction-free beams have no practical advantages compared to the usual Gaussian beams. Only a hollow Bessel beam with radial polarization which keeps diffraction-free up to hundreds of microns (from its birthplace at the apex of an axicon lens) grants a really thin and long cylindrical atom trap. Such the trap is demonstrated in work \[22\]. However, in order to obtain such a magnificent light beam as in work \[22\], one needs a very expensive optical equipment. Meanwhile, our wave beam is a simple superposition of two plane waves. All we need is splitting the parallel laser beam onto two beams with the small angle between them and a diaphragm to get rid of the side-lobes. A glass microcylinder is simply a piece of an optical microfiber aligned along straight line without cladding. It can be prepared of arbitrary length.

IV. CONCLUSIONS

In this work we have theoretically studied an unusual near-field effect – spatial Fano resonance arising when the wave beam formed by two plane waves with the small angle between their wave vectors impinges a dielectric microcylinder. The physics of this phenomenon is interference of the evanescent field of the nearly-resonant cavity mode with the quasi-continuum of all other modes and with the leaky field of the same mode. We have shown that the Fano minimum is located at a substantial distance from the rear edge of the microcylinder, this is very unusual for a near-field effect. Our main result is the singularity of this minimum – the electromagnetic field in it decreases very sharply and may even utterly nullify. Practically, it results in the possibility of creation a long optical trap that seems to be promising for cold atoms and ions whose cross section is subwavelength. Such a trap can be called a particle waveguide. The evident advantage of our optical trap compared to its known analogues is the simplicity of its implementation. We hope that our finding will be interesting for physicists developing optical traps, especially particle waveguides for quantum computing \[23, 24\].

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