COSMOLOGICAL CONSTRAINTS FROM GRAVITATIONAL LENS TIME DELAYS

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ABSTRACT

Future large ensembles of time delay lenses have the potential to provide interesting cosmological constraints complementary to those of other methods. In a flat universe with constant \( w \) including a Planck prior, LSST time delay measurements for \( \sim 4,000 \) lenses should constrain \( h \) to \( \sim 0.007 \) (\( \sim 1\% \)), \( \Omega_{\text{de}} \) to \( \sim 0.005 \), and \( w \) to \( \sim 0.026 \) (all 1-\( \sigma \) precisions). Similar constraints could be obtained by a dedicated gravitational lens observatory (OMEGA) which would obtain precise time delay and mass model measurements for \( \sim 100 \) lenses with spectroscopic redshifts. We compare these constraints (as well as those for a general cosmology) to the "optimistic Stage IV" constraints expected from weak lensing, supernovae, baryon acoustic oscillations, and cluster counts, as calculated by the Dark Energy Task Force. Time delays yield a modest constraint on a time-varying \( w(z) \), with the best constraint on \( w(z) \) at the "pivot redshift" of \( z \approx 0.31 \). Our Fisher matrix calculation is provided to allow time delay constraints to be easily compared to and combined with constraints from other experiments. We also show how cosmological constraining power varies as a function of numbers of lenses, lens model uncertainty, time delay precision, redshift precision, and the ratio of four-image to two-image lenses.

Subject headings: cosmological parameters — dark matter — distance scale — galaxies: halos — gravitational lensing — quasars: general

1. INTRODUCTION

The HST Key Project relied on 40 Cepheids to constrain Hubble’s constant \( H_0 \) to 11% (Freedman et al. 2001). The first convincing measurements of the accelerating expansion rate of the universe (suggesting the existence of dark energy) by Riess et al. (1998) and Perlmutter et al. (1999) required 50 and 60 supernovae, respectively. So far, time delays have only been reliably measured for \( \sim 16 \) gravitational lenses, thanks to dedicated lens monitoring from campaigns such as COSMOGRAIL (Eigenbrod et al. 2005). Yet recent analyses of 10–16 time delay lenses already claim to match or surpass the Key Project’s 11% precision on \( H_0 \) (Saha et al. 2006; Oguri 2007; Cols. 2008). Future surveys promise to yield hundreds or even thousands of lenses with well-measured time delays, which will enable us to obtain much tighter constraints on \( H_0 \) as well as constraints on other cosmological parameters.

To date, most efforts have focused on studies of individual time delay lenses. In theory, one might be able to control all systematics and constrain \( H_0 \) unambiguously given a single "golden lens". Such a lens would have a sufficiently simple and well-measured geometry. The closest to a golden lens may be B1608+656. In Suyu et al. (2004), the authors claim all systematics have been controlled to 5%. A new estimate for Suyu et al. (2009), the authors claim all systematics have been overcome.

Historically, analyses of individual lenses have yielded varying answers for \( H_0 \) (see the Appendix of Jackson 2007 for a recent review). This can be attributed to two factors, both of which, it appears, are now being overcome.

The first factor is simple intrinsic variation in lens properties (especially mass slope) and environment (lensing contributions from neighboring galaxies). Consider the following estimate from a simple empirical argument. If statistical uncertainties on \( H_0 \) decrease as \( 1/\sqrt{N} \) (assuming systematics can be controlled), and the current uncertainty from 16 lenses is \( \sim 10\% \), then the uncertainty on a single lens might be \( \sim 40\% \). Thus, assuming \( h = 0.7 \) (where \( H_0 = 100h \) km s\(^{-1}\) Mpc\(^{-1}\)), individual lenses may be expected to yield a wide range of \( h = 0.42 \pm 0.98 \) (1-\( \sigma \)). (We will revisit these assumptions in this work.)

The second factor in the wide range of reported \( H_0 \) values is that different analyses have assumed different mass profiles to model the lenses, including isothermal, de Vaucouleurs, and mass follows light, and mass follows light. There is substantial weight of evidence that galaxy lenses are roughly isothermal on average, at least within approximately the scale radius (e.g., Koo et al. 2004). Theoretical work supports this idea, showing that a wide range of plausible luminous plus dark matter profiles all combine to yield roughly an isothermal profile at the Einstein radius, though the profile may deviate significantly from isothermal beyond that radius (Mandelbaum et al. 2008).

In recent years we have witnessed a steady increase in the number of strong lenses discovered by searches such as CLASS (Meyers et al. 2003), SLACS (Bolton et al. 2006), SL2S (Cabanac et al. 2007), SQuaLS (Inada et al. 2008), HAGGLEs (Marshall et al. 2009b), and searches of AEGIS (Moustakas et al. 2007) and COSMOS (Faure et al. 2008). Based on this experience, we can expect that future surveys such as Pan-STARRS\(^1\) (Kaiser 2004), LSST\(^2\) (Ivezic et al. 2008), JDEM / IDECS\(^3\), and SKA\(^4\) (Lazio 2008) will yield an explosion in the number of strong lenses known (e.g., Koo et al. 2004; Fassnacht et al. 2008).

1 The Panoramic Survey Telescope & Rapid Response System, http://pan-starrs.ifa.hawaii.edu
2 The Large Synoptic Survey Telescope, http://www.lsst.org
3 The Joint Dark Energy Mission, http://jdem.gsfc.nasa.gov
4 The Square Kilometer Array, http://www.skatelescope.org
lenses are elegant geometric consequences of how light travels through the universe while grazing massive galaxies. When the line of sight alignment is very close, light takes multiple paths around the curved space of the lens. These paths form multiple images, and the light takes a different amount of time to travel each path. Light passing closer to the lens is deflected by a larger angle (increasing its path length) and experiences a greater relativistic time dilation, further delaying its arrival. If the source flares up, or otherwise varies in intensity (e.g., if it is an active galactic nucleus, or AGN), we can observe these “time delays” between or among the images. These time delays are functions of the angular diameter distances between the source, lens, and observer, as well as the properties of the lens itself.

The ability of time delays to constrain other cosmological parameters has been explored only recently. Mörtsell & Sunesson (2006) and Dobke et al. (2009) examined the constraints that large ensembles of lenses might place on $H_0$ and $\Omega_A = 1 - \Omega_m$ (assuming a flat universe). Below we present the first full treatment of the cosmological constraints expected on $(h, \Omega_m, \Omega_{de}, \Omega_k, w_0, w_a)$ from ensembles of time delay lenses.

Lens statistics from well-controlled searches for strongly-lensed sources have also been used to constrain cosmology (e.g., Chad 2007; Oguri et al. 2008). If time delays can be obtained for the lenses in such a sample, the lens statistics and time delays might combine to yield tighter cosmological constraints. This potential is not explored in this work.

The reader is invited to skip ahead to our results in §4 where cosmological constraints expected from time delays (according to our calculations) are compared to those expected from other methods (weak lensing, supernovae, baryon acoustic oscillations, and cluster counts). Table 5 summarizes the assumed priors including a guide to specific sections and figures.

The remainder of our paper is organized as follows. In §2 we provide the time delay equations and discuss how cosmology is derived from observed time delays. We define the quantity $T_C(h, \Omega_m, \Omega_{de}, \Omega_k, w_0, w_a; z_L, z_S)$ which time delays are capable of constraining. In §3 we briefly describe simulations of time delay lens ensembles performed in a companion paper (Coe & Moustakas 2009a, hereafter Paper I). In §4 we give constraints on $T_C$ expected as a function of number of lenses, lens model uncertainties, redshift precision, time delay precision, and the ratio of lenses which produce four images (quads) to those that produce two images (doubles). In §5 we illustrate the dependence of $T_C$ on cosmological parameters $(h, \Omega_m, \Omega_{de}, w_0, w_a)$. In §6 as highlighted above, we give projections for time delay constraints on $(h, \Omega_{de}, \Omega_k, w_0, w_a)$ and compare to other methods. Systematic biases are discussed in §7 and their impact on our ability to constrain cosmology is analyzed in another companion paper (Coe & Moustakas 2009d, hereafter Paper III). Finally we present our conclusions in §8.

We assume all constraints to be centered on the concordance cosmology $h = 0.7, \Omega_m = 0.3, \Omega_{de} = 0.7, \Omega_k = 0$, $w_0 = -1.2, w_a = 0.12$ (assuming constant $w$).
w_0 = -1, and w_a = 0, where \( H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \).

2. COsmological Constraints from Time Delays

2.1. Time Delay Equations

A galaxy at redshift \( z_L \) strongly lenses a background galaxy at redshift \( z_S \) to produce multiple images. The lensing effect delays each image in reaching our telescope by a different amount of time, given by

\[
\Delta \tau = \frac{1}{c} D \left[ \frac{1}{2} \left( \beta - \beta' \right)^2 - \phi \right]
\]

(e.g., Blandford & Narayan 1986) with terms defined below. The factors in the time delay equation can be grouped into a product of two terms:

\[
\Delta \tau = T_C T_L.
\]

The first factor,

\[
T_C \equiv \frac{1}{c} \frac{D_L D_S}{D_{LS}},
\]

is a function of cosmology and the lens and source redshifts, \( z_L \) and \( z_S \). The second factor,

\[
T_L \equiv \left[ \frac{1}{2} \left( \beta - \beta' \right)^2 - \phi \right],
\]

is a function of the projected lens potential \( \phi \), the source galaxy’s position on the sky \( \beta \), and the image positions \( \beta' \).

We concentrate on the cosmological dependence of \( T_C \). The factor

\[
D \equiv \frac{D_L D_S}{D_{LS}}
\]

is a ratio of the angular-diameter distances from observer to lens \( D_L = D_A(0,z_L) \), observer to source \( D_S = D_A(0,z_S) \), and lens to source \( D_{LS} = D_A(z_L,z_S) \). Angular-diameter distances are calculated as follows (Fukugita et al. 1992), filled beam approximation; see also Hogg 1999:

\[
D_A(z_1,z_2) = \frac{c}{H_0} E_A(z_1,z_2),
\]

\[
E_A = \frac{\sinh \sqrt{\left| \Omega_k \right| E_A^*}}{\left| \Omega_k \right|},
\]

where \( \sinh(u) = \sin(u), u, \text{ or } \sinh(u) \) for an open, flat, or closed universe respectively (\( \Omega_k < 0, \Omega_k = 0, \text{ or } \Omega_k > 0 \)). The curvature is given by \( \Omega_k \equiv 1 - (\Omega_m + \Omega_A) \) and

\[
E_A^* = \frac{dz}{E(z')}. \quad \text{(8)}
\]

The normalized Hubble parameter \( E(z) \) can have different expressions depending on the cosmology assumed:

\[
E(z) \equiv \frac{H(z)}{H_0} = \sqrt{\Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_A} = \sqrt{\Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_{de} (1+z)^{3(1+w)}}
\]

\[
= \sqrt{\cdots + \Omega_{de} (1+z)^{3(1+w_0+w_a)}} \exp \left( \frac{-3w_a z}{1+z} \right)
\]

Here we have progressed from a universe with a cosmological constant \( \Lambda \) to one with dark energy with an equation of state \( p = w \rho \). In the last line, the last term has been rewritten in terms of an evolving dark energy equation of state

\[
w = w_0 + w_a (1 - a)
\]

\[
w = w_0 + w_a \left( \frac{z}{1 + z} \right),
\]

a common parametrization first introduced by Chevallier & Polarski (2001) and Linder (2003). The universe scale factor \( a = (1 + z)^{-1} \).

We next define the dimensionless ratio

\[
\mathcal{E} = \frac{E_L E_S}{E_{LS}}
\]

with factors defined similarly to those above for \( D_A: E_L = E_A(0,z_L), E_S = E_A(0,z_S), E_{LS} = E_A(z_L,z_S) \). We find that many factors cancel, and \( T_C \) simplifies to:

\[
T_C = \frac{\mathcal{E}}{H_0}
\]

We see here clearly that time delays (\( \Delta \tau = T_C T_L \)) scale inversely with \( H_0 \). There is also a complex though weaker dependence on the other cosmological parameters (\( \Omega_m, \Omega_{de}, \Omega_k, w_0, w_a \)) as embedded in \( \mathcal{E} \).

2.2. Deriving Cosmology from Time Delays

Given observed time delays \( \Delta \tau \) and assuming a lens model (and thus \( T_C \), one can obtain measures of \( T_C \). These measures will have some scatter due to both observational uncertainties and deviations of the lens from the assumed model.

Recent studies suggest that galaxy lenses, on average, have roughly isothermal profiles within the Einstein radius (see Fig. 11). Deviations from this simple description include variation in lens slope, external shear, mass sheets, and substructure. Oguri (2005) parametrized the deviations as the reduced time delay, the ratio of the observed time delay to that expected due to an isothermal potential in a given lens:

\[
\Xi \equiv \frac{\Delta \tau}{\Delta \tau_{iso}}.
\]

In our notation, these observed deviations are due to deviations in the lens model:

\[
\Xi_L \equiv \frac{T_C}{T_{L,iso}}.
\]

\[
E(z) \equiv \frac{H(z)}{H_0} = \sqrt{\Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_A} = \sqrt{\Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_{de} (1+z)^{3(1+w)}}
\]

Here we have progressed from a universe with a cosmological constant \( \Lambda \) to one with dark energy with an equation of state \( p = w \rho \). In the last line, the last term has been rewritten in terms of an evolving dark energy equation of state

\[
w = w_0 + w_a (1 - a)
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We next define the dimensionless ratio

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We see here clearly that time delays (\( \Delta \tau = T_C T_L \)) scale inversely with \( H_0 \). There is also a complex though weaker dependence on the other cosmological parameters (\( \Omega_m, \Omega_{de}, \Omega_k, w_0, w_a \)) as embedded in \( \mathcal{E} \).
By assuming an isothermal model ($T_L = T_{L, \text{iso}}$), these deviations get absorbed into the derived cosmology:

$$\Xi_C = \frac{T_C}{T_{C,0}},$$

where $T_{C,0}$ is the true cosmology. For example, a lens which is steeper than isothermal yields $\Xi > 1$; thus when assuming an isothermal model ($\Xi = 1$), we derive $\Xi > 1$ (since $\Xi = \Xi_C \Xi_L$). In traditional analyses assuming fixed $\epsilon$, $\Xi > 1$ would simply yield a low $\Xi$. This approximation is adequate for small samples of lenses but not for the large samples to come in the near future.

Similarly, observational uncertainties affecting $\Delta \tau$ are absorbed into the derived cosmology. In this paper, we study how observational and intrinsic (lens model) uncertainties combine to yield scatter in the observed $\Delta \tau$. We will assume these measurements yield $T_C$ with the correct mean but a simple Gaussian scatter and explore how this propagates to Gaussian uncertainties on cosmological parameters for different numbers of lenses, quad-to-double ratios, lens model precisions, and observational precisions.

In practice we do not expect measurements of $\Delta \tau$ to have Gaussian scatter, but this serves as a useful approximation. The true expected $P(\Xi)$ from time delay measurements and methods for handling these distributions are studied in Oguri (2007) and Paper I.

3. LENS ENSEMBLE SIMULATIONS

Lensing simulations are required to quantify the expected uncertainties resulting from both the lens modeling and the time delay measurements themselves. We present the full details of our simulations in Paper I but highlight the key details here.

We conduct a mock survey of over a million lensed galaxies using the GRAVLENS software v1.99k (Keeton 2001). Ten thousand galaxies lens one hundred background sources each. All lenses are modeled as power-law ellipsoidal mass distributions with surface density $\kappa \propto r^{-\alpha-1}$, where $r$ is radius in elliptical coordinates. Their properties are drawn from the following Gaussian distributions, following Oguri (2007) and consistent with observations: slope $\alpha = 1 \pm 0.15$ (where $\alpha = 1$ corresponds to isothermal) and ellipticity $\epsilon = 1 - b/a = 0.3 \pm 0.16$. To these models we add the environmental effects of external shear $\gamma_{\text{ext}} = 0.03 \pm 0.35$ dex, and external convergence, or a mass sheet, $\kappa_{\text{ext}} = 0.03 \pm 0.6$ dex. Line of sight convergence is also added, drawn from a distribution observed in the Millennium simulation Hilbert et al. (2007).

The choices made above will affect the final scatter in the reduced time delay $\Xi$. Most significantly, the $15\%$ scatter in lens slope ($\alpha = 1 \pm 0.15$) results in a $15\%$ scatter in $\Xi$. The choice of $15\%$ scatter in slope is motivated by observations but in fact is meant to be a conservative spread (though see §4. If the exact spread expected in a large lens ensemble is uncertain.

In our simulations, care was taken to account for various selection effects: lens cross section (as a function of its slope and other parameters) and observational bias (highly magnified sources are more likely to be detected in a given survey). Again, full details can be found in Paper I.

For simplicity, we consider the Gaussian redshift distributions $z_L = 0.5 \pm 0.15$ and $z_S = 2.0 \pm 0.75$ with $z_S > z_L$ (Fig. 1). These distributions were used by Dobke et al. (2009) as reasonable approximate assumptions for near-future missions including LSST.

4. CONSTRAINTS ON $T_C$ FROM FUTURE EXPERIMENTS

Recent studies have constrained $T_C$ to $\sim 10\%$ using time delays, where $T_C$ encodes all of the cosmological dependencies ($\Xi C$). Constraints on $T_C$ have generally been interpreted to be equivalent to direct constraints on $h$. This assumption is reasonable for current sample sizes, but will need to be revised in the future ($\Xi C$). Using 16 lenses, Oguri (2007) obtain $h = 0.70 \pm 0.06$ (stat.). Similar studies by Saha et al. (2006) and Cole (2008) using a different method obtain similar constraints using 10 and 11 lenses, respectively. The latter finds $h = 0.71^{+0.08}_{-0.06}$.

We can improve on these constraints in three ways: obtaining larger samples of lenses, better constraining our lens models, and obtaining more precise time delay measurements. Spectroscopic redshifts have been obtained for all lenses and sources in the current ensemble, but future large samples will likely rely on photometric redshifts adding to the uncertainties on $T_C$. In this section we present expectations as a function of number of lenses, lens model uncertainty, redshift precision, time delay precision, and quad-to-double ratio.

The reader is invited to skip ahead to the results in §4.7 where $\delta T_C$ is given for Pan-STARRS 1, LSST, OMEGA, and LSST + OMEGA combined.

Here we consider statistical uncertainties only, with systematics to be discussed in §7. We will assume that all other things being equal, increasing our sample size beats down our errors by $\sqrt{N}$ for $N$ lenses. This assumption is borne out well by our detailed simulations (Paper I), for the case of no systematic uncertainties.

Statistical uncertainties in the reduced time delay $\Xi$ (Eq. 13) have two broad sources. The numerator, the observed time delay $\Delta \tau$, is subject to observational measurement uncertainties $\Delta(\Delta \tau)$. The denominator,
the expected time delay $\Delta t_{\text{iso}}$, suffers from all the lens model uncertainties (including environmental factors). Fractional uncertainties of the lens model ($\delta \Xi_{\text{L}}$) and observations ($\delta \Xi_{\text{O}}$) add in quadrature:

$$\delta \Xi^2 = \delta \Xi_{\text{L}}^2 + \delta \Xi_{\text{O}}^2. \quad (17)$$

Once we assume a lens model, this uncertainty is entirely subsumed by the cosmology such that $\delta T_{\text{C}} = \delta \Xi.$

Below, we evaluate $\delta \Xi_{\text{L}}$ and $\delta \Xi_{\text{O}}$ separately for quads and doubles to be recombined later. We find quads yield higher uncertainties in both the lens model and observational measurements.

(We adopt a notation in which “$\Delta$” refers to uncertainties with units and “$\delta$” to fractional uncertainties. Thus a time delay of 20 days measured to 2-day precision has $\Delta(\Delta \tau) = 2$ days and $\delta(\Delta \tau) = 0.1.$)

## 4.1. Time Delay Observational Precision

Fractional uncertainties $\delta(\Delta \tau)$ would be trivial to calculate if a survey measured all time delays to a fixed fractional uncertainty (e.g., 10%). However, it is unlikely that the same observing strategy would measure both the ~hour and ~month-long time delays to 10% (Fig. 2 left), without being tailored to do so as in COSMORAIL (Eigenbrod et al. 2005).

A given survey is more likely to measure all time delays to a fixed duration (e.g., 2 days). This precision depends on such things as photometric accuracy and survey cadence (Eigenbrod et al. 2005). For example, if measurements are only obtained every 2 days, then much shorter time delays will be difficult to determine.

The right panel of Fig. 2 converts the time delays to fractional precision $\delta(\Delta \tau_i) = (2 \text{ days}) / \Delta \tau_i$ assuming measurement precisions of $\Delta(\Delta \tau) = 2 \text{ days}.$ These are added in quadrature to obtain the effective fractional uncertainty of the ensemble: $\delta(\Delta \tau) = \sqrt{\sum_i \left(\delta(\Delta \tau_i)^2 \right) / N}.$ This summation is performed separately for doubles and quads to obtain $\delta(\Delta \tau_2)$ and $\delta(\Delta \tau_4).$

For a time delay uncertainty of $\Delta(\Delta \tau) = 2 \text{ days},$ we find $\delta(\Delta \tau_2) = 0.085$ and $\delta(\Delta \tau_4) = 0.391.$ Quads yield much higher fractional time delay uncertainties because they produce smaller time delays on average. This is mitigated somewhat by the fact that each quad yields multiple time delay measurements.

Each double contributes one observed time delay, but each quad contributes up to 6 time delays, a number we will define as $6\zeta,$ with $0 < \zeta < 1.$ Note that we consider each time delay measurement to be an independent constraint, as in Oguri (2007). That is, we assume that no attempt is made to model each quad lens to fit all 6 time delays simultaneously. Instead, the time delay measurements will be combined statistically such that the effective uncertainty from each quad will be $\delta \Xi_{\text{r4}} = \delta(\Delta \tau_4) / \sqrt{6\zeta}.$ Improving a survey’s time delay precision from, say, 5 days to 2 days has two direct consequences. Not only does it make $\delta(\Delta \tau)$ lower, it also allows shorter time delays to be measured. We assume time delays can only be measured if they are longer than the measurement precision, $\Delta \tau > \Delta(\Delta \tau).$ For example, a time delay measurement of $\Delta \tau = 1 \pm 2 \text{ days}$ is discarded.

| $\Delta(\Delta \tau)$ | $\delta \Xi_{\text{r2}}$ | $\delta(\Delta \tau_2)$ | $\delta(\Delta \tau_4)$ | $\delta \Xi_{\text{r4}} / \sqrt{6\zeta}$ | $6\zeta$ |
|---------------------|----------------|-----------------|-----------------|-----------------------|-------|
| 5                   | 0.204          | 0.540           | 0.311           | 3.007                 |       |
| 2                   | 0.085          | 0.391           | 0.191           | 4.189                 |       |
| 1.5                 | 0.064          | 0.354           | 0.168           | 4.450                 |       |
| 1                   | 0.043          | 0.306           | 0.141           | 4.741                 |       |
| 0.5                 | 0.022          | 0.245           | 0.108           | 5.120                 |       |
| 0.2                 | 0.009          | 0.184           | 0.079           | 5.435                 |       |
| 0.1b                | 0.005          | 0.152           | 0.064           | 5.597                 |       |
| 0.01                | 0.001          | 0.100           | 0.042           | 5.814                 |       |
| 0                  | 0              | 0               | 0               | 6                     |       |

**Note.** For variable time delay uncertainties $\Delta(\Delta \tau),$ we provide the effective fractional time delay uncertainty across the ensemble individually for doubles ($\delta(\Delta \tau_2)$) and quads ($\delta(\Delta \tau_4)$). These estimates are based on the distributions of time delays in our simulations (Fig. 5). Each quad yields $6\zeta$ time delay observations reducing the effective observational uncertainty per quad to $\delta \Xi_{\text{r4}} = \delta(\Delta \tau_4) / \sqrt{6\zeta}.$ We assume time delays can only be measured when they are longer than the uncertainty, or $\Delta \tau > \Delta(\Delta \tau).$

- Assumed for most current and future ensembles.\(^a\) Goal for OMEGA.

### 4.2. Photometric Redshift Uncertainties

Currently all lenses which have reliable time delay measurements also have spectroscopic redshifts measured for both lenses and sources (e.g., Oguri 2007). The telescope time required to obtain spectroscopic redshifts is generally a small fraction of that required to obtain accurate time delays, so the extra investment is worthwhile.

Future surveys which repeatedly scan the sky, however, will yield time delays “for free” for many more lenses than can be efficiently followed up spectroscopically. For these lenses we will have to rely on photometric redshift measurements. The uncertainties will affect measurements of cosmological parameters.

Photometric redshift uncertainties for the lenses (typically elliptical galaxies at $z_{\text{L}} \sim 0.5$) are expected to be $\Delta z_{\text{L}} \sim 0.04(1 + z_{\text{L}}),$ similar to that found in the CFHT Legacy Survey (Ilbert et al. 2006). Redshift uncertainties for the lensed sources (quasars) are expected to be somewhat higher. We will adopt $\Delta z_{\text{S}} \sim 0.10(1 + z_{\text{S}}),$ roughly that found in the analysis of one million SDSS quasars (Richards et al. 2009).

Now we will determine how such redshift uncertainties...
ties translate into uncertainties on $T_C$. For simplicity, let us assume that redshift uncertainties are Gaussian. Let us further assume that uncertainty in $x$ scales linearly with redshift uncertainty. (This is approximately true for reasonable uncertainty levels $\Delta z \lesssim 0.2$.)

Using equations (7) – (13), we find for a typical lens-source combination with $(z_L, z_S) = (0.5, 2.0)$, that lens and source redshift uncertainties translate to $\Delta \Xi_L \sim 2.75 \Delta z_L$ and $\Delta \Xi_S \sim -0.16 \Delta z_S$, respectively. These relations are strong functions of redshift and become catastrophic for sources very close to the lens. We plot this behavior in Fig. 3. If accurate and precise redshifts are available, we must concentrate our analysis on systems with high separation in redshift between the lens and source.

For a lens ensemble with Gaussian redshift distributions $z_L = 0.5 \pm 0.15$ and $z_S = 2.0 \pm 0.75$, we find $\Delta \Xi_L \sim 4.09 \Delta z_L$ and $\Delta \Xi_S \sim 1.31 \Delta z_S$. Given the photometric redshift uncertainties quoted above ($\Delta z_L \approx 0.04(1+z_L)$ and $\Delta z_S \approx 0.10(1+z_S)$) with the Gaussian redshift distributions we have adopted, these translate to $\Delta \Xi_L \sim 24\%$ and $\Delta \Xi_S \sim 22\%$, respectively, for a single lens-source combination. Added in quadrature these combine to yield a $\Delta \Xi \sim 33\%$ uncertainty. This is clearly significant, on the same order as the intrinsic lens model uncertainties $\Delta \Xi_{L2}$ and $\Delta \Xi_{L4}$.

As shown in Table 4, these photometric redshift uncertainties increase the total uncertainty from $\sim 30\%$ to $\sim 45\%$ per lens in future ensembles (with $\Delta(\Delta \tau) = 2$ days assumed throughout).

Of course, these are just estimates for large ensembles. In practice, redshift probability distributions $P(z)$ will be properly folded into the $P(T_C)$ determinations. Biased redshifts would yield biased $T_C$ at the levels given above. For example, if all source redshifts were biased by $\Delta z_S = 0.10(1+z_S)$ the result would be a $22\%$ bias in $T_C$. It is unlikely for all redshifts to be biased so badly, but some fraction may have such bias if photometry has been miscalibrated. The effects of biased $T_C$ are studied in Paper III.

4.3. Positional Uncertainties

We must also consider observational uncertainties in the measured positions of the lens and multiple images. Based on our simulations (Paper I), we find that positional uncertainties $\Delta x$ introduce cosmological uncertainties $\delta \Xi_x \approx \Delta x/R_E$, where $R_E$ is the lens Einstein radius. Uncertainties in the lens and image positions add in quadrature.

These uncertainties are generally small compared to the other uncertainties discussed above. For example, galaxies in the CASTLES survey (Falco et al. 2001) have typical Einstein radii of $R_E \approx 1''$ per lens positions measured to $\sim 0.01''$ precision and multiple image positions measured to $\sim 0.003''$. These translate to a total of $\sim 1\%$ uncertainties on $T_C$ per lens, much smaller than the $\sim 30\%$ uncertainties from lens modelling plus time delay measurements. LSST is expected to achieve a similar astrometric uncertainty of $0.01''$ (Ivezic et al. 2008).

Note that the lens position has an additional uncertainty in that the dark matter halo centroid may not align perfectly with the observed luminous centroid. This uncertainty too is expected to be random and the total positional uncertainty will still be very small compared to other uncertainties.

4.4. Intrinsic Lens Model Uncertainties

Intrinsic uncertainties will depend on several lens model variables, including the mass density slope, the mass sheet, and the external shear. These all produce deviations from the simple lens model assumed (e.g., isothermal), yielding $\Xi_L \neq 1$. For now let us assume that all the systematics can be controlled, yielding $\Xi_L$ with a Gaussian distribution and the correct mean of 1. Systematic biases will be revisited in Paper III.

Doubles (two-image systems) and quads (four-image systems) yield $\Xi_L$ with different levels of scatter. Quads yield larger $\Delta \Xi_L$ than doubles (e.g., Oguri 2007, Paper 1). ²

In our simulations (Paper I) given our inputs (summarized in [4]), we find $\delta \Xi_{L2} \approx 13.6\%$ while $\delta \Xi_{L4} \approx 30.3\%$. The quad distribution is highly non-Gaussian, with a second peak at $\Xi_{L4} \sim 0$ (the primary peak being at $\Xi_{L4} \sim 1$), but we consider a Gaussian approximation here centered on $\Xi_{L4} = 1$.

In fact, the $\Delta \Xi_L$ uncertainties are highly dependent on our input assumptions. They are strong functions

²These scatters also vary significantly with the observed image configuration as studied in Oguri (2007) and Paper I. In this paper we only consider the two populations of doubles and quads.
of the scatter in lens properties, especially the distribution of mass slopes. The spread in mass slopes has been measured but not to high precision, with various studies giving a range of values for $\delta \Xi_L$ (Treu & Koopmans 2004; Hamana et al. 2005; Rusin & Kochanek 2005; Koopmans et al. 2006, 2009). Below, we renormalize our $\delta \Xi_L$, doubling them, based on empirical evidence from Oguri (2007).

4.5. Total Uncertainties from the Current Ensemble

The lens model and observational uncertainties are added in quadrature for doubles and quads separately:

$$\delta \Xi_{L2}^2 = \delta \Xi_{L2}^2 + \delta \Xi_{O2}^2,$$

$$\delta \Xi_{L4}^2 = \delta \Xi_{L4}^2 + \delta \Xi_{O4}^2,$$

where the observational uncertainties include (most significantly) time delay and redshift uncertainties:

$$\delta \Xi_{O2}^2 = \delta \Xi_{L2}^2 + \delta \Xi_{O2}^2,$$

$$\delta \Xi_{O4}^2 = \delta \Xi_{L4}^2 + \delta \Xi_{O4}^2,$$

Finally, the uncertainties from doubles and quads are combined:

$$\frac{1}{\delta \Xi^2} = \frac{N_2}{\delta \Xi_{L2}^2} + \frac{N_4}{\delta \Xi_{L4}^2}.$$  \hspace{1cm} (22)

The current sample (Oguri 2007) consists of 16 lenses with $N_2 = 10$ doubles and $N_4 = 6$ quads with time delays measured to roughly $\Delta(\Delta \tau) \approx 2$ days. Spectroscopic redshifts are measured for all lenses ($\delta \Xi_Z \approx 0$), so the only observational uncertainties come from the time delay measurements; thus $\delta \Xi_Z = \delta \Xi$. Given our calculated values for $\delta \Xi_L$ and $\delta \Xi_Z$ (Table 2), we find $\delta \Xi \approx 0.048$ for the 16-lens ensemble. For this relatively large uncertainty, $\delta h \approx \delta T_C = \delta \Xi$ (see §4.4.1). A 4.8% uncertainty on Hubble’s constant corresponds to $h = 0.70 \pm 0.034$.

This is similar to the $h = 0.70^{+0.03}_{-0.02}$ constraint found by Oguri (2007) in his initial analysis. Oguri (2007) then reanalyzed the data using jackknife resampling to find $h = 0.70 \pm 0.06$. This discrepancy is most likely due to underestimates of the lens model uncertainties. If we double our lens model uncertainties $\delta \Xi_{L2}$ and $\delta \Xi_{L4}$ and recalculate the total expected uncertainty, we recover the observed uncertainty $\delta h = 0.06$. Thus we conservatively adopt this renormalization throughout our paper as we extrapolate to predict future constraints on $\delta T_C$ from large lens ensembles.

The final lens model uncertainties we adopt are $\delta \Xi_{L2} = 0.271$ and $\delta \Xi_{L4} = 0.606$, for doubles and quads, respectively, twice the values quoted previously. Summarizing our time delay uncertainties, assuming $\Delta(\Delta \tau) = 2$ day precision for all time delay measurements, we find $\delta \Xi_{\tau2} = \delta(\Delta \tau_2) = 0.085$ and $\delta(\Delta \tau_4) = 0.391$ per time delay measurement. We further find $\delta \zeta = 4.2$ time delay measurement per quad, such that each quad yields an uncertainty $\delta \Xi_{\tau 4} = \delta(\Delta \tau_4)/\sqrt{\delta \zeta} = 0.191$.

Our uncertainty budget is summarized in Table 2. These uncertainties all add in quadrature using equations (18 – 22) to yield the final scatter $\delta \Xi$, which equals that subsumed by our cosmological constraints $\delta T_C = \delta \Xi$.

Note that uncertainties on lens models and redshifts dominate the error budget. Reducing these in any large sample should be the first priority. Until that is accomplished, little can be gained by obtaining more precise time delays than the $\Delta(\Delta \tau) = 2$ days assumed here.

4.6. Obtaining Tight Constraints on a Fraction of the Sample

OMEGA plans to obtain tight model and observational constraints on ($\sim$ 100) lenses, reducing both $\delta \Xi_L$ and $\delta \Xi_Z$. It will not be feasible to obtain such tight constraints on the factor of 10 or more time delay lenses expected from Pan-STARRS and LSST. However, we can propose obtaining tight constraints on some fraction of
these large ensembles.

Measurements including that of the lens velocity dispersion may reduce \( \delta \Sigma_L \) to 10% or perhaps even 5% as claimed for B1608+656 (see discussion in §7.1). Time delays may be measured more precisely and spectroscopic redshifts may be obtained for some or all of the ensemble.

Assuming all Gaussian uncertainties, constraints from the “tight” sample can be combined with constraints from the rest of the ensemble (the “loosely” constrained lenses) as follows:

\[
\frac{1}{\delta \Sigma_L^2} = \frac{N_{\text{loose}}}{\delta \Sigma_L^2_{\text{loose}}} + \frac{N_{\text{tight}}}{\delta \Sigma_L^2_{\text{tight}}}
\]

This strategy will be explored below.

4.7. Future Experiments

Pan-STARRS and LSST will both survey the sky repeatedly, opening the time domain window over vast solid angles for astronomical study. Pan-STARRS 1 (PS1) has recently begun its 3\( \pi \) survey, repeatedly observing the entire visible sky to \( \sim \)23rd magnitude every week over a 3-year period. LSST promises similar coverage and depth every 3 days with first light scheduled for 2014.

These surveys will reveal many time-variable sources, among them gravitationally-lensed quasars. The persistent monitoring over many years should yield time delays for many strongly-lensed quasars. Simulations (M. Oguri 2009, private communication) show that Pan-STARRS 1 and LSST are expected to yield \( \sim 1,000 \) and \( \sim 4,000 \) strongly-lensed quasars with quad fractions of 19% and 14%, respectively.

We may expect time delays to be measured to about 2-day precision, or similar to that currently achieved today. This is consistent with predictions based on detailed simulations by Eigenbrod et al. (2005) which study factors including survey cadence, object visibility, and the complicating effects of microlensing. We note this estimate may be a bit optimistic for PS1 with its slower sampling rate compared to LSST.

Alternatively, we can obtain tighter constraints on relatively fewer lenses. As mentioned above, OMEGA would monitor 100 time delay lenses to achieve precise and accurate \( \sim 0.1 \) day time delay measurements. Supporting measurements would aim to reduce the model uncertainty of each lens to 5% (\( \delta \Sigma_L = 0.05 \)), or that claimed recently for B1608+656 (Suyu et al. 2009). These measurements, including velocity dispersion in the lens and characterization of the group environment (see discussion in §7.1), would be carried out either with OMEGA itself or through coordinated efforts by ground-based telescopes and JWST.

Lenses targeted by OMEGA would likely be all quads. This strategic decision might somewhat hamper OMEGA’s ability to constrain \( T_C \), but would yield measurements of time delay ratios within quads which provide constraints on the dark matter substructure mass function (Keeton & Moustakas 2008).

Table 3 summarizes the progress we can expect to make in “Stages” corresponding to those defined by the Dark Energy Task Force (DETF; Albrecht et al. 2006, 2009): “Stage I” = current, “II” = ongoing, “III” = currently proposed, “IV” = large new mission. Again, we stress these are estimates of statistical uncertainties only.

Current constraints on \( T_C \) are \( \sim 8.5\% \) given 16 lenses all with spectroscopic redshifts. Pan-STARRS 1 could improve this constraint to \( \delta T_C = 1.4\% \) if time delays can be reliably measured for all \( \sim 1,000 \) strongly-lensed quasars it reveals. LSST will obtain better sampled light curves for \( \sim 4,000 \) lensed quasars and could yield \( T_C \) to 0.7%. We study these Stage IV constraints (\( \delta T_C = 0.7\% \)) in detail in §6 and §7. OMEGA could obtain similar constraints (\( \delta T_C = 0.8\% \)) by obtaining strong constraints on 100 lenses. The combined power of constraints from LSST and OMEGA would reduce \( \delta T_C \) to 0.54%.

Table 4 and Figs. 4 and 5 provide \( \delta T_C \) as a function of various ensemble properties. Various observational strategies can be compared. The standard “FUTURE” ensemble is based on expectations for LSST (and roughly Pan-STARRS): photometric redshifts for all objects, time delays measured to \( \Delta (\Delta \tau) = 2 \) day precision, and a quad fraction of 14%. Photometric redshift uncertainties are assumed to be \( \Delta z_L = 0.04(1 + z_L) \) and \( \Delta z_S = 0.10(1 + z_S) \) (see §4.2).

If spectroscopic redshifts can be obtained for all lenses and sources, constraints on \( T_C \) improve by a factor of \( \sim 1.5 \). Spectroscopic redshifts have been obtained for all of our current time delay lenses, but the higher quad fraction brings the improvement on \( T_C \) down to a factor of \( \sim 1.3 \).

We consider several cases in which a fraction of the lenses are selected for detailed study and are “tightly” constrained. For this fraction, spectroscopic redshifts are obtained for both lenses and sources, and the lens models are constrained to \( \delta \Sigma_L = 10\% \). The statistical power of this “tight” subsample is combined with that of the
remaining less-well-constrained lenses to yield the final constraints on $T_C$ (1.0).

Obtaining such tight constraints on $10\%$, $50\%$, and $100\%$ of the ensemble improves the $T_C$ constraints by factors of $\approx 1.4$, $2.4$, and $3.3$, respectively, over our standard “FUTURE” ensemble. We also consider a case in which all lenses are constrained to $\delta \Xi = 5\%$ (in addition to the spectroscopic redshifts); In this case, the $T_C$ constraints are $\approx 4.3$ times tighter than with the standard ensemble.

Next we explore ensembles for which a fraction of lenses are constrained to $\delta \Xi = 10\%$, but spectroscopic redshifts are obtained for the entire sample. For example, given tight constraints on $10\%$ of the lens sample, spectroscopic redshifts obtained for the remaining $90\%$ loose lenses make the $T_C$ constraints $\approx 1.2$ times tighter (a modest improvement).

More precise time delays of $\Delta(\Delta \tau) = 0.1$ day are also considered. Normally tighter time delay constraints help little, as the uncertainties are dominated by lens model uncertainties (Table 2). When lens model uncertainties $\delta \Xi$ are better controlled, the time delay uncertainties dominate, and higher precision time delays become much more valuable. Combining $\Delta(\Delta \tau) = 0.1$ day with $10\% \Xi$ measurements and spectroscopic redshifts yields 4.4 times tighter $T_C$ than the “standard” ensemble. Further constraining the lens models to $\delta \Xi = 5\%$ measurements yields 8.5 times tighter $T_C$. OMEGA would aim to obtain such tight constraints for 100 lenses, but as they would all be quads, the constraints would be somewhat weaker: $\approx 5.5$ times as tight as the standard future ensemble, the equivalent of observing $\sim 30$ times more lenses. In this way, OMEGA nearly matches LSST in precision on $T_C$ though it would only observe and monitor 100 lenses versus $\sim 4,000$ for LSST.

5. Dependence of $T_C$ on Cosmology

We expect LSST time delay lenses to constrain $T_C$ to $\sim 0.7\%$. In this section we begin to explore how this “Stage IV” constraint translates to constraints on cosmological parameters. We study the dependence of $T_C$ on $(\Omega_m, \Omega_{de}, \Omega_k, w_0, w_a)$ for several cosmologies as outlined in Table 5.

5.1. Flat universe with a cosmological constant ($h, \Omega_\Lambda = 1 - \Omega_m$)

First, we add a single free parameter $\Omega_\Lambda$ (in addition to $h$) in considering a flat universe with a cosmological constant ($w = -1$). Given $\delta T_C = 0.7\%$ from an ensemble with all lenses at $z_L = 0.5$ and all sources at $z_S = 2.0$, the factors of $\approx 1.5, 2.5$, and $3.3$, respectively, over our standard “FUTURE” ensemble. By default, spectroscopic redshifts are only obtained for the tightly constrained lenses. In the FUTURE sample, no lenses are tightly constrained so all have photometric redshifts for which we adopt uncertainties $\Delta z_L = 0.04(1 + z_L)$, $\Delta z_S = 0.10(1 + z_S)$. Current constraints are better than standard “FUTURE” constraints for the same number of lenses when we assume that spectroscopic redshifts are not available for future lenses.

Table 3: Estimated Current and Future Constraints on $T_C$

| Stage   | Experiment         | $N_L$ | quad | $\Delta z^a$ | $\Delta(\Delta \tau)$ | $\delta \Xi$ | $\delta T_C$ |
|---------|-------------------|------|------|-------------|------------------------|-------------|-------------|
| I       | current           | 16   | spec | 2            | 2 days                 | 8.5%        | 1.00        |
| II      | Pan-STARRS 1      | 1,000| spec | 2            | 2 days                 | 1.4%        | 1.05        |
| IV      | LSST              | 4,000| phot | 7.0%        | 2 days                 | 0.7%        | 1.48        |
| ...     | OMEGA             | 100  | spec | 5            | 0.1 day               | 5%          | 0.8%        |
| ...     | LSST + OMEGA      | 420  | spec | 5.48        | 0.54%                  | 0.54%        |

$^a$ Spectroscopic or photometric redshift measurements. For the latter we assume $\Delta z_L = 0.04(1 + z_L)$ and $\Delta z_S = 0.10(1 + z_S)$.

Table 4: Average uncertainty on $T_C$ for a single lens given various observational uncertainties

| Ensemble | Quad fraction | Tight fraction | Tight $\delta \Xi$ | Spec-z $\Delta(\Delta \tau)$ | Improvement factor |
|----------|---------------|---------------|------------------|-------------------------------|--------------------|
| FUTURE   | 14%           | 0%            | 10%              | 2 days                        | 1.00               |
| all doubles | 0%           | -             | 10%              | 2 days                        | 1.05               |
| spec-z   | 37%           | -             | 10%              | 2 days                        | 1.48               |
| current  | 40%           | -             | 10%              | 2 days                        | 1.32               |
| 10% tight| -             | 10%           | 10%              | 2 days                        | 1.32               |
| 50% tight| -             | 50%           | 10%              | 2 days                        | 1.32               |
| tight    | -             | 50%           | 10%              | 2 days                        | 1.32               |
| tighter  | -             | 50%           | 10%              | 2 days                        | 1.32               |
| 10% tight, all spec-z | - | 50% | 10% | 2 days | 1.32 |
| 50% tight, all spec-z | - | 50% | 10% | 2 days | 1.32 |
| precise $\Delta \tau$ | - | 50% | 10% | 2 days | 1.32 |
| tight, precise $\Delta \tau$ | - | 50% | 10% | 2 days | 1.32 |
| OMEGA    | 100%          | all           | 5%               | 0.1 day                       | 5.48               |

$^a$ For an ensemble of $N$ lenses, the $\delta T_C$ values listed here should be divided by $\sqrt{N}$. $^b$ FUTURE ensembles are those to be provided by large surveys such as Pan-STARRS and LSST. Values for this ensemble are defaults for the remaining ensembles. These are adopted wherever entries are left blank. $^c$ We give $\delta \Xi = 10\%$ as the default even though no lenses are constrained tightly in the FUTURE ensemble. $^d$ By default, spectroscopic redshifts are only obtained for the tightly constrained lenses. In the FUTURE sample, no lenses are tightly constrained so all have photometric redshifts for which we adopt uncertainties $\Delta z_L = 0.04(1 + z_L)$, $\Delta z_S = 0.10(1 + z_S)$. $^e$ Current constraints are better than standard “FUTURE” constraints for the same number of lenses when we assume that spectroscopic redshifts are not available for future lenses.
TABLE 5

| Cosmology                                               | $h$ | $\Omega_m$ | $\Omega_{de} / \Omega_\Lambda$ | $\Omega_k$ | $w_0$ | $w_a$ | Sections | Figures |
|---------------------------------------------------------|-----|-------------|---------------------------------|------------|-------|-------|----------|---------|
| Flat universe with cosmological constant                | Free| 1 - $\Omega_\Lambda$ | Free ($\Omega_\Lambda$) | 0 | -1 | 0 | $\sigma_L$ | $\sigma_S$ |
| Curved universe with cosmological constant              | Free| 1 - ($\Omega_\Lambda + \Omega_k$) | Free ($\Omega_\Lambda$) | Free | -1 | 0 | $\sigma_L$ | $\sigma_S$ |
| Flat universe with constant $w$                         | Free| 1 - $\Omega_\Lambda$ | Free | 0 | Free | 0 | $\sigma_L$ | $\sigma_S$ |
| Flat universe with time-variable $w$                    | Free| 1 - $\Omega_{de}$ | Free | 0 | Free | 0 | $\sigma_L$ | $\sigma_S$ |
| General (curved with time-variable $w$)                 | Free| 1 - ($\Omega_{de} + \Omega_k$) | Free | Free | Free | Free | $\sigma_L$ | $\sigma_S$ |

Note. — We consider six cosmological parameters of which five are independent since $\Omega_m + \Omega_{de} + \Omega_k = 1$.

When $w_0 = -1$ and $w_a = 0$, $\Omega_{de} = \Omega_\Lambda$, the cosmological constant. In the $T_\Sigma$ dependencies are explored. In additional priors are assumed and time delay constraints are compared to those from other methods. Given this cosmology, we assume a Planck prior in $\Omega_\Lambda$.

Given a general cosmology, in we assume a prior of Planck + “Stage II” WL+SN+CL (see that section for details).

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**Fig. 5.**— Constraints on $\delta T_\Sigma$ for a single lens as a function of ensemble properties. These $\delta T_\Sigma$ should be divided by $\sqrt{N}$ for a sample of $N$ lenses. Solid lines show how constraints improve from left to right by tightening lens model constraints for some fraction of the ensemble. Our base ensemble for FUTURE surveys relies on photometric redshifts, $\Delta(\Delta \tau) = 2$ days, and has 14% quads (vertical dashed line). For each point along the thin black solid line, a fraction of the ensemble has the following “tight” constraints: lens model uncertainties reduced to $\delta \Sigma_L = 10\%$ and spectroscopic redshifts. For the thick black solid line, the tight lens model constraints are “tighter”: $\delta \Sigma_L = 5\%$. For the thick black solid line with circles, the tighter fraction includes precise time delay measurements ($\Delta(\Delta \tau) = 0.1$ day). The dashed lines plot functions of quad fraction rather than “tight fraction”. For the thick black dashed line with circles, all lenses are “tighter” ($\delta \Sigma_L = 5\%$ with spec-z) but a varying quad fraction. At the right end of this line (all quads), OMEGA is plotted. Also plotted in red are samples for which spectroscopic redshifts have been measured for all lenses and sources. For the solid red line, all lenses (14% quads) have spec-z but only a fraction have “tight” $\delta \Sigma_L = 10\%$ constraints. For the dashed red line, no lenses have tight constraints, but all have spec-z with a variable quad fraction. Along this dashed line, at a quad fraction of 3/8, is plotted the current constraint.

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we would obtain confidence contours shown in Fig. 6.

The shape of these curves shifts somewhat as a function of $z_L$ and $z_S$. Given an ensemble of lenses and sources with Gaussian redshift distributions $z_L = 0.5 \pm 0.15$ and $z_S = 2.0 \pm 0.75$ as discussed above, we begin to break the $(h, \Omega_\Lambda)$ degeneracy (Table 7). Assuming a flat universe, Stage IV time delays could provide independent evidence for $\Omega_\Lambda > 0$. Whether this remains interesting by Stage IV remains to be seen. The constraints on $h$ are certainly tighter and would be improved by the introduction of a prior on $\Omega_\Lambda$, which we defer until Fig.

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**Fig. 6.**— Confidence contours (1- and 2-$\sigma$ colored bands) for $(h, \Omega_\Lambda = 1 - \Omega_m)$ given “Stage IV” $\delta T_\Sigma = 0.7\%$ obtained from an ensemble with all lenses and sources at $z_L, z_S = (0.5, 2.0)$. Here we assume a flat universe with a cosmological constant $(w = -1)$. Also plotted are contours of constant $\Sigma_L = \Sigma_L / T_\Sigma$, where $T_\Sigma = 0.7\%$ for the input redshifts and cosmology. The input cosmology $(h, \Omega_m, \Omega_\Lambda) = (0.7, 0.3, 0.7)$ is marked with dotted lines and a white dot.

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**Fig. 7.**— Confidence contours (1- and 2-$\sigma$ colored bands) for $(h, \Omega_\Lambda = 1 - \Omega_m)$ given $\delta T_\Sigma = 0.7\%$ and assuming a flat universe with a cosmological constant $(w = -1)$. Each of the three fainter curves corresponds to all lenses and sources at the same pair of redshifts: $z_L, z_S = (0.65, 2.75), (0.5, 2.0), (0.35, 1.25)$, as marked. Next we consider an ensemble of lenses and sources with Gaussian redshift distributions: $z_L, z_S = (0.5 \pm 0.15, 2.0 \pm 0.75)$. These yield the tighter constraints (marked “ensemble”). The input cosmology $(h, \Omega_m, \Omega_\Lambda) = (0.7, 0.3, 0.7)$ is marked with a white dot.

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**5.2. Curved universe with cosmological constant**

$(h, \Omega_m, \Omega_\Lambda, \Omega_k)$

If we relax the flatness parameter, adding another free parameter $\Omega_m$ (where curvature is determined by $\Omega_k = 1 - (\Omega_m + \Omega_\Lambda)$), we run into the degeneracy in Fig.
5.3. Flat universe with constant dark energy EOS

\((h, \Omega_{de} = 1 - \Omega_m, w)\)

Current cosmological constraints are consistent with a flat universe with a cosmological constant (as explored in §5.1). As a first perturbation to this model, it is common to explore constraints on \(w \neq -1\) while maintaining constant \(w\) in a flat universe. This cosmology has three free parameters \((h, \Omega_m, \Omega_{de}, w)\) with \(\Omega_m = 1 - \Omega_{de}\).

Given enough data and appropriate priors, time delay lenses could place strong constraints on the dark energy equation of state parameter \(w\) (see §5.2). Figs. 10 and 11 explore the dependence of \(T_C\) on \((w, \Omega_{de})\) assuming a flat universe and constant \(w\).

5.4. Flat universe with time-variable dark energy EOS

\((h, \Omega_{de} = 1 - \Omega_m, w_0, w_a)\)

The most interesting constraints we can hope to place on dark energy are to verify or falsify the following: \(w = -1\) (cosmological constant) and \(w_0 = 0\) (constant \(w\)). In Fig. 12 we explore the dependence of \(T_C\) on \((w_0, w_a)\) (see Eq. 10). The colored bands are the constraints we could obtain given perfect knowledge of \((h, \Omega_m, \Omega_{de})\). The solid lines on the left show the curves’ migration as a function of \(h\). On the right, we also explore dependence on \(\Omega_{de}\) for a flat universe (\(\Omega_m + \Omega_{de} = 1\)).
delay constraints are compared to those expected from other experiments as estimated by the Dark Energy Task Force (Albrecht et al. 2006, 2004). To efficiently explore this parameter space, we perform Fisher matrix analyses.

6.1. Fisher Matrix Analysis

The Fisher matrix formalism provides a simple way to study uncertainties of many correlated parameters. Constraints from various experiments and/or specific priors may be combined with ease. A “quick-start” instructional guide and software are provided in a companion paper Coe (2009). Fisher matrices approximate all uncertainties as Gaussians. The true uncertainties may be somewhat higher and non-Gaussian. The full information of the dependencies as shown in §5 is not retained. Yet as cosmological parameters are constrained close to their true values, these approximations should suffice.

As above we consider a “Stage IV” ensemble of time delays which constrains } \text{T} \text{C} \text{ to 0.7\% with Gaussian distributions of lens and source redshifts (} \text{z}_{L} = 0.5 \pm 0.15; \text{z}_{S} = 2.0 \pm 0.75). Assuming such a Gaussian distribution for } \text{T} \text{C} \text{ and the aforementioned redshift ensemble, we calculate (numerically) the Fisher matrix for cosmological parameters of interest. The Fisher matrix consists of partial derivatives of } \chi^2 \text{ with respect to the parameters. For parameters } (p_{i}, p_{j}) \text{, element } (i, j) \text{ in the Fisher matrix is given by}

\[ F_{ij} = \frac{1}{2} \frac{\partial \chi^2}{\partial p_{i} \partial p_{j}}. \]

The Stage IV (\( \delta T_{C} = 0.7\% \)) time delay Fisher matrix is given in Table 6 for the cosmological parameters \((h, \Omega_{de}, \Omega_{k}, w_{0}, w_{a})\). The Fisher matrix may be easily scaled to other } \text{T} \text{C values. For example, to scale from LSST (\( \delta T_{C} = 0.7\% \)) to Pan-STARRS 1 (\( \delta T_{C} = 1.4\% \)), simply divide all the values in the Fisher matrix by \( \sqrt{2} \). Or multiply them by 1.14 to explore the LSST + OMEGA constraints (\( \delta T_{C} = 0.54\% \)). To scale from 4,000 lenses (LSST) to 1,000 lenses (PS1), divide the Fisher matrix by 4. If one is interested in constraints on \( \Omega_{m} = 1 - (\Omega_{de} + \Omega_{k}) \), \( \omega_{m} \equiv \Omega_{m} h^2 \), or any other related variable, a transformation of variables can be performed as outlined in Coe (2009).

In Fig. 13 we show the time delay constraints possible on all parameters and pairs of parameters assuming perfect knowledge of all the other parameters. These plots can be compared to those presented in §5. Such perfect priors are unrealistic, but they help to demonstrate the parameter dependencies and degeneracies.

6.2. Flat universe with constant } w

We first consider the simple case of a flat universe with constant } w. This is a common perturbation to the concordance cosmology. The goal is to detect deviation from } w = -1, equivalent to the cosmological constant } \Lambda. This 3-parameter cosmology \((h, \Omega_{de}, w_{0}, w_{a})\), with } \Omega_{m} = 1 - (\Omega_{de}) \text{ was explored above in Fig. 13.}

The top row of Fig. 14 shows Stage IV time delay constraints with a Planck prior in a flat universe with constant } w. Given these priors, we estimate that time delays will constrain } h to 0.007 (\sim 1\%), } \Omega_{de} to 0.005, and } w to 0.026 (all 1-\sigma precisions).

In the bottom row of Fig. 14 we compare these time delay constraints (TD) to those expected from other methods: weak lensing (WL), baryon acoustic oscillations (BAO), supernovae (SN), and cluster counts (CL). We consider “optimistic Stage IV” expectations from these methods as calculated by the Dark Energy Task Force (DETF; Albrecht et al. 2006, 2004) and made available in the software DETFast\(^8\). A Planck prior (also calculated by the DETF) is again assumed for all experiments.

\(^8\)http://www.physics.ucdavis.edu/DETFast/
In manipulating the DETF Fisher matrices we adopt their cosmology \((\Omega_m, \Omega_{de}, h) = (0.27, 0.73, 0.72)\), but we revert to our chosen cosmology \((\Omega_m, \Omega_{de}, h) = (0.3, 0.7, 0.7)\) for the rest of our analysis. These differences have negligible impact on our results.

6.3. General Cosmology

We now assume a general cosmology allowing for curvature and a time-varying \(w\). To help constrain this larger parameter space \((h, \Omega_{de}, \Omega_k, w_0, w_a)\), with \(\Omega_m = 1 - (\Omega_{de} + \Omega_k)\), we add additional priors. In addition to the Planck prior, we adopt “Stage II” (near-future) constraints from weak lensing (WL) + supernovae (SN) + cluster counts (CL), all as calculated by the DETF. The DETF uses this prior (in addition to Planck) in many of their calculations comparing the performance of Stage III – IV techniques.

The Stage II DETF WL + SN + CL prior yields the following uncertainties: \(\Delta h = 0.031\ (4.4\%)\), \(\Delta \Omega_{de} = 0.023\), \(\Delta \Omega_k = 0.010\), \(\Delta w_0 = 0.128\), \(\Delta w_a = 0.767\) (along with various covariances between parameters). The addition of the Planck prior reduces these to: \(\Delta h = 0.017\ (2.4\%)\), \(\Delta \Omega_{de} = 0.012\), \(\Delta \Omega_k = 0.003\), \(\Delta w_0 = 0.115\), \(\Delta w_a = 0.525\). Note that Stage II WL+SN+CL constrains \(h\) well enough (to 4.4\%) that an HST Key Project prior \((h = 0.72 \pm 0.08)\) appears to be unnecessary. Even SHOES \((h = 0.742 \pm 0.036\), or 4.9\%) provides a weaker constraint on \(h\). However, as noted in the introduction, these combined WL+SN+CL experiments yield a prediction of \(h\) based on an assumed cosmological model and are no substitute for local measurements of \(h\) (Riess et al. 2009).

These Stage II constraints are also rather optimistically combined, assuming that all experiments have converged on the same best fit cosmology without systematic offsets among them. The true Stage II constraints should be somewhat weaker.

Plotted in Fig. 13 are time delay constraints assuming a prior of Planck + Stage II WL+SN+CL. A progression is shown from Stage I (present) time delay constraints \(\delta T_C = 8.6\%\) through Stage II \(\delta T_C = 1.4\%\) and on to Stage IV \(\delta T_C = 0.7\%\). The current constraints barely improve upon this aggressive prior. While the Stage II – IV constraints certainly improve upon the prior, note that the outer bounds of the time delay and prior ellipses nearly intersect. This indicates that the size of the time delay ellipse is controlled by that of the prior, at least for these constraints and prior. Were the prior significantly weaker or the time delay constraints significantly stronger, we have verified that the time delay ellipses would shrink well within the prior ellipses.

In Fig. 16 we compare Stage IV time delay constraints to those expected from other methods for various parameters of interest. Plotted are constraints on \((h, \Omega_k)\), \((h, w_0)\), and \((w_0, \Omega_k)\), and \((w_0, w_a)\). An example of how these constraints combine is given in §6.4.2.

We give extra attention to constraints on the dark en-

![Figure 13](image_url)
energy parameters \((w_0, w_a)\). The DETF figure of merit (FOM) for a given experiment is defined as the inverse of the area of the ellipse in the \((w_0, w_a)\) plane. In Fig. 17 we plot FOM for various experiments versus the "pivot redshift", defined as follows. For a time-varying \(w(z)\), time delays constrain \(w\) best at \(z \approx 0.31\). This redshift is known as the pivot redshift (Huterer & Turner 2001; Hu & Jain 2004) and can also be calculated simply from the \((w_0, w_a)\) constraints (Coe 2009). As in the previous plot, we assume a prior of Planck + Stage II (WL+SN+CL).

6.4. Time delays do not simply constrain \(h\)

6.4.1. Relaxing the “perfect prior” on \((\Omega_m, \Omega_k, w_0, w_a)\)

To date, analyses of time delay lenses have quoted uncertainties on \(T_C\) as uncertainties on \(h\), assuming \(\delta h = \delta T_C\). This assumption has been valid to date, but future constraints on \(h\) will be weaker than the constraints on \(T_C\), that is \(\delta h > \delta T_C\).

This is demonstrated in Fig. 18 left. The dashed line shows \(\delta h = \delta T_C\), or the "perfect prior" on \((\Omega_m, \Omega_k, w_0, w_a)\) generally assumed in analyses. For future samples (at the left side of the plot), as this prior is loosened, we find \(\delta h > \delta T_C\). In Fig. 18 right, we plot \(\delta h / \delta T_C\). For example, given a "Stage II" prior on WL+SN+CL, and LSST constraints on time delays \((\delta T_C = 0.7\%)\), we find \(\delta h / \delta T_C \approx 2.25\% \sim 1.5\%.\) Alternatively, assuming a Planck prior in a flat universe with constant \(w\), we would find \(\delta h / \delta T_C \approx 1.48\% \sim 0.98\%\).

(Note that the Stage II WL+SN+CL prior claims a constraint of \(\delta h = 0.03\), such that it outperforms current constraints from time delays \(\delta h = \delta T_C\).)

6.4.2. Time delays provide more than constraints on \(h\)

In the Introduction we commented on the ability of any experiment to improve constraints on \(w\) and \(\Omega_k\) simply by tightening the constraints on \(h\). Several methods have the potential to further improve the constraints on \(h\) (Olling 2007). Do time delays offer more than a simple constraint on \(h\) for the purposes of constraining the dark energy equation of state?

In Fig. 19 we compare Stage IV time delays (left) to a simple \(h\) constraint (right) in ability to constrain dark energy. Each is combined with Stage IV supernova constraints plus a prior of Planck + Stage II WL+SN+CL. We find time delays are more powerful than the simple \(h\) constraint. The \((SN + TD + prior)\) figure of merit (FOM) on \((w_0, w_a)\) is \(\approx 19\%\) higher than that from \((SN + H + prior)\).

The "H" constraint \(\delta h = 0.009\) was chosen such that when combined with the prior, the resulting \(\delta h\) would equal that from TD + prior. Both \(H + prior\) and TD + prior yield \(\delta h = 0.008\). However we find TD outperforms even a perfect \(H\) prior \((\delta h \sim 0)\) by \(13\%\). Simply put, the time delay constraints on \((\Omega_m, \Omega_k, \Omega_k, w_0, w_a)\) are clearly making contributions.

When combined with experiments other than SN, TD offers less marked improvements over \(H\) constraints. Replacing SN with BAO, WL, and CL, we find TD outperforms \(H\) by \(7\%, 5\%,\) and \(3\%,\) respectively.

6.5. Lens and Source Redshift Distribution

\(^9\) Strictly speaking we have not taken the proper care in combining constraints from the Stage II and Stage IV supernova experiments, as their nuisance parameters have been marginalized over in the DETF Fisher matrices. But this analysis will suffice for illustrative purposes here.
There are many potential sources of systematic bias, as alluded to throughout this work. At the risk of putting the cart before the horse, we have presented systematic-free projections for time delay cosmological constraints. These should serve to motivate a more considered look at systematics, in the context of the behavior of random uncertainties in these studies. Ideally, efforts should be undertaken to reduce systematics on a timescale comparable to that presented here (e. g., 0.7% by “Stage IV”). If this cannot be accomplished, we study prospects for estimating cosmological parameters in spite of large residual systematic biases in Paper III (Coe & Moustakas 2009d).

Here we discuss briefly the greatest potential sources of systematic bias. We should consider which of our main sources of statistical uncertainty (lens modelling, redshift measurements, and time delay measurements) could also contribute significant systematic bias. Time delay uncertainties are generally not expected to be biased in any preferred direction. Redshift biases are somewhat worrisome but will not be discussed further here. Most daunting are potential biases due to imprecise lens modeling.

7. SYSTEMATICS

As with any measurement, there are many potential sources of systematic bias, as alluded to throughout this work. At the risk of putting the cart before the horse, we have presented systematic-free projections for time delay cosmological constraints. These should serve to
Fig. 16.— Comparisons of “Stage IV” constraints possible from time delays (TD), weak lensing (WL), supernovae (SN), baryon acoustic oscillations (BAO), and cluster counts (CL) in a general cosmology (allowing for curvature and a time-variable $w$). For TD, we assume an ensemble which constrains $T_C$ to 0.7% (see text for details). For the rest we use “optimistic Stage IV” expectations calculated from Fisher matrices provided by the Dark Energy Task Force (DETF). A prior of Planck + Stage II (WL+SN+CL) is assumed for all five experiments and is plotted in gray. For each parameter pair, experiments are plotted in order of $\text{FOM} \propto (\text{Ellipse Area})^{-1}$, with the best experiment on top.

Yet a recent analysis of 58 SLACS lenses finds a slightly higher average slope of $\gamma = 2.085^{+0.025}_{-0.018} (\text{stat.}) \pm 0.1 \, (\text{syst.})$ (Koopmans et al. 2009b). If the average proved to be exactly $\gamma = 2.085$, this would result in an 8.5% bias in $T_C$: $\delta T_C = \delta \gamma / 2 = \delta \alpha$ were we to assume an average of $\gamma = 2$ instead.

Mass profile slopes for individual lenses are determined by measuring mass within two radii: the Einstein radius (from the positions of multiple images) and a smaller radius (from velocity dispersions). The latter require detailed spectroscopy (e.g., Koopmans 2004). Velocity dispersion measurements have yet to be obtained for all currently known time delay lenses. As time delays are measured in future large ensembles of lenses, significant investments of telescope time would be required to keep pace and obtain velocity dispersion measurements for any appreciable fraction of the lenses. If this were to be attempted, what would be gained?

Velocity dispersions are typically measured to 10% precision, translating to 10% uncertainty in lens mass slope (e.g., Koopmans 2004). In § 4.7 we determined the benefits gained by obtaining 10% constraints on the lens models for various fractions of the lens ensemble (Table 4). Obtaining such constraints on 10%, 50%, and 100% of the ensemble improves the $\delta T_C$ constraints by factors of…

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10 We use two definitions common in the literature regarding lens slope: two-dimensional mass surface density $\kappa \propto r^{-\alpha}$, and three-dimensional mass surface density $\rho \propto r^{-\alpha - \gamma}$. These parameters are related by $\alpha + \gamma \approx 3$ (see discussion in van de Ven et al. 2006).
Relative constraints on $\Omega_m$ and a “Stage II” prior from (WL+SN+CL). This Stage II prior constrains $\Omega_m$ which estimate the effects of nearby neighbors (e.g., Momcheva et al. 2006; Auger 2008) and simulations via spectroscopic (and photometric) studies (e.g., Keeton & Zabludoff 2004; Dalal 2005). Similar studies also attempt to identify groups along the line of sight and estimate their mass sheet contributions (e.g., Fassnacht et al. 2006).

The alternative is a statistical approach. Measurements of $\kappa_{\text{env}}$ or $\kappa_{\text{los}}$ would not be required for individual lenses if we had knowledge of the distributions $P(\kappa_{\text{env}})$ and $P(\kappa_{\text{los}})$. These could be obtained from simulations, though one might wonder whether they would prove accurate to the percent level. Any errors would yield residual systematics in our estimation of $T_C$.

To aid such a statistical approach, lenses in obvious groups can be excluded from the analysis leaving only those systems with low $\kappa_{\text{env}}$. Such low mass systems would introduce smaller biases, though a detailed exploration of this approach will await future work.

8. CONCLUSIONS

We have presented the first analysis of the potential of gravitational lens time delays to constrain a broad range of cosmological parameters. The cosmological constrain-
ing power $\delta T_C$ was calculated for Pan-STARRS 1, LSST, and OMEGA based on expected numbers of lenses (including the quad-to-double ratio) as well as the expected uncertainties in lens models, photometric redshifts, and time delays. Our Fisher matrix results are provided to allow time delay constraints to be easily combined with and compared to constraints from other methods.

We concentrate on “Stage IV” constraints from LSST. In a flat universe with constant $w$ including a Planck prior, LSST time delay measurements for $\sim 4,000$ lenses should constrain $h$ to $\sim 0.007$ ($\sim 1\%$), $\Omega_{de}$ to $\sim 0.005$, and $w$ to $\sim 0.026$ (all 1-$\sigma$ precisions). We compare these results as well as those for a general cosmology to other “optimistic Stage IV” constraints expected from these results as well as those for a general cosmology to other “optimistic Stage IV” constraints expected from weak lensing, supernovae, baryon acoustic oscillations, and cluster counts, as calculated by the Dark Energy Task Force (DETF).

Combined with appropriate priors (those adopted by the DETF), time delays provide modest constraints on a time-varying $w(z)$ that complement the constraints expected from other methods. Time delays constrain $w$ best at $z \approx 0.31$, the “pivot redshift” for this method.

We find that LSST and OMEGA represent an even trade in “quantity versus quality” in terms of constraining cosmology with time delays. LSST could yield $\delta T_C \sim 0.7\%$ by measuring time delays for $4,000$ lenses, while OMEGA could yield $\delta T_C \sim 0.8\%$ by obtaining high-precision time delay measurements and lens model constraints for $100$ lenses with spectroscopic redshifts. The combined statistical power of these two missions would further improve the cosmological constraints to $\delta T_C \sim 0.54\%$.

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