Dynamic Analysis and Simulation of Six-Axis Cooperative Robot Based on Screw Theory

Chengpu Zheng1, *, Kang Han1, Jialei Bao1, Weigang Wen1, Kai Sun2

1 School of Mechanical, Electronic and Control Engineering, Beijing Jiaotong University, Beijing, China
2 School of Materials Science and Mechanical Engineering, North China University of Technology, Beijing, China

*Corresponding author e-mail: 17121256@bjtu.edu.cn

Abstract. In order to realize the precise control of the cooperative robot, it is necessary to establish an accurate dynamic model of the robot. Compared with traditional industrial robots, cooperative robots have joint flexibility effect, so joint flexibility should be taken into account when building dynamic model. In this paper, the six-axis cooperative robot is taken as the research object. The dynamic model of the robot is modeled and analyzed by Newton-Euler method based on screw theory, and the dynamic simulation of the robot is carried out based on Simulink and SimMechanics simulation platform, and the relevant performance curves are obtained. The simulation results verify the validity of the dynamic theoretical model.

1. Introduction

With the continuous development of industry, traditional robots have been unable to keep up with the development of market and production, and it is difficult to meet the needs of small and medium-sized enterprises and emerging collaborative market. In this context, cooperative robots are favored by the majority of small and medium-sized enterprises because of their characteristics of light and fast, low configuration cost. It can be said that collaborative robots [1] have gradually replaced traditional industrial robots and become an unstoppable trend.

However, there are flexible elements such as harmonic reducer in the joint of cooperative robot, which makes the cooperative robot have joint flexibility effect [2]. In addition, the robot itself is lighter and less rigid, which makes the robot dynamics change greatly.

The traditional dynamic modeling of robot regards robot as a whole rigid body system, and uses dynamic analysis method to establish the dynamic model of rigid robot. However, the cooperative robot system is not a rigid system. The joint flexibility and link flexibility will have a significant impact on the end error and other dynamic performance of the robot. Therefore, flexibility must be considered in the dynamic analysis of cooperative robot.

At present, the commonly used dynamic analysis methods are Newton-Euler method, Lagrange method, screw theory method [3], Kane method [4], etc. The more complex the robot structure is, the more complex the joints will be. At this time, Newton Euler method [5] is not conducive to the overall modeling of the robot; Lagrangian method [6] has a large amount of calculation, which is not conducive to the real-time control of the robot; while spinor theory method can not only get the overall dynamic
model of the robot according to $s$, but also get the complete machine according to Lagrangian method. The dynamic model of human system, with small calculation, is suitable for computer real-time data processing. The six axis industrial robot studied in this paper not only has a high demand for real-time control, but also has a large number of rigid bodies connected, so it is difficult to calculate the dynamic equation. Therefore, after comprehensive consideration, Newton Euler method based on spinor theory is adopted to analyze the dynamics of the robot. This method is suitable for the real-time control of robot.

2. **Robot structure and coordinate system**

In this paper, the small six axis cooperative robot EC66 of Beijing Elite Technology Co., Ltd. is taken as the research object. The whole robot is composed of a base, a large arm, a small arm and other mechanisms, with six degrees of freedom of rotation. The three-dimensional model established by SolidWorks is shown in Figure 1: in order to facilitate the derivation of robot dynamics equation, the Post coordinate system method is adopted in this paper. Take the end of each joint axis of EC66 robot as the coordinate origin to establish the linkage coordinate system, as shown in Figure 2.

![Figure 1. Three dimensional model of robot.](image1)
![Figure 2. Link Coordinate System.](image2)

| Link | $q_i$ | $d_i$ | $a_i$ | $\alpha_i$ |
|------|-------|-------|-------|-------------|
| 1    | $q_1$ | 0.096 | 0     | $\pi/2$    |
| 2    | $q_2$ | 0.138 | 0.418 | 0           |
| 3    | $q_3$ | -0.114| 0.398 | 0           |
| 4    | $q_4$ | 0.098 | 0     | $-\pi/2$   |
| 5    | $q_5$ | 0.098 | 0     | $\pi/2$    |
| 6    | $q_6$ | 0.089 | 0     | 0           |

3. **Newton Euler method based on screw theory**

3.1. **Screw theory**

The motion of any rigid body can be equivalent to the rotation around a certain axis plus the movement parallel to that axis. This motion is called screw motion, and its infinitesimal amount is called motion spinor [7-8].

Suppose the rotation axis of the rigid body is $\omega \in R^3$, and $\|\omega\| = 1$, $P \in R^3$ is a point on the axis. Introducing matrix $\hat{\xi}$:

\[
\hat{\xi} = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{bmatrix}
\]
\[ \hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{4 \times 4} \]  

(1)

where, \( \hat{\xi} \) is motion spinor; Linear velocity vector is: \( v = -\omega \times P \); \& express antisymmetric matrix, if \( \omega = [\omega_1 \omega_2 \omega_3]^T \), then:

\[ \hat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \]  

(2)

Introducing operators \( \vee \) (Vee), Define:

\[ \xi = \begin{bmatrix} \hat{\omega} \\ v \end{bmatrix}^\vee = \begin{bmatrix} v \\ \omega \end{bmatrix} \in \mathbb{R}^{6 \times 1} \]  

(3)

If \( S \) is the basic coordinate system of the robot and \( T \) is the tool coordinate system, then the forward kinematics mapping of the open chain robot can be expressed as

\[ g_{ST}(q) = e^{\hat{\xi}_1 q_1} \cdot e^{\hat{\xi}_2 q_2} \cdots e^{\hat{\xi}_n q_n} g_{ST}(0) = \begin{bmatrix} R(q) & p(q) \\ 0 & 1 \end{bmatrix} \]  

(4)

\( R(q) \) represents the rotation matrix of coordinate system \( T \) relative to \( S \); \( p(q) \) represents the position vector. Among them:

\[ e^{\hat{\xi} q} = e^{\hat{\omega} q} (I - e^{\hat{\omega} q})(\omega \times v) + q\omega \omega^T v \]  

(5)

\( \hat{\xi} \) is the rotational coordinate of the motion of \( \hat{\xi} \), The \( 4 \times 4 \) transformation matrix \( ad_\xi \) of the rotation coordinate is defined as:

\[ ad_\xi = \begin{bmatrix} \hat{\omega} \\ 0 \\ \hat{\omega} \end{bmatrix} \]  

(6)

The spinor theory can also describe the forces acting on the rigid body. The forces on the rigid body include the rotating component and the moving component, so it can be expressed by six dimensional matrix [9]:

\[ F = \begin{bmatrix} f \\ \tau \end{bmatrix} \in \mathbb{R}^{6 \times 6} \]  

(7)

where, \( f \) is the force vector and \( \tau \) is the moment vector.

The space velocity of rigid body \( c \) in reference system \( A \) and the object velocity of rigid body \( c \) in reference system \( B \) can be expressed as:

\[ V^A_c = \begin{bmatrix} v^A_c \\ \omega^A_c \end{bmatrix} \quad V^B_c = \begin{bmatrix} v^B_c \\ \omega^B_c \end{bmatrix} \]  

(8)

Then the relationship between formula (5) and formula (6) can be expressed as:
\[ V_c^A = \begin{bmatrix} R_{BA} & \hat{p}_{BA} R_{BA} \\ 0 & R_{BA} \end{bmatrix} V_c^B \]  

(9)

Defining six dimensional adjoint matrix:

\[ Ad_{gBA} = \begin{bmatrix} R_{BA} & \hat{p}_{BA} R_{BA} \\ 0 & R_{BA} \end{bmatrix} \]  

(10)

The requirements are as follows:

\[ Ad^{-1}_{gBA} = Ad^{-1}_{gBA} = \begin{bmatrix} R_{BA}^T & -R_{BA} \hat{p}_{BA}^T \\ 0 & R_{BA}^T \end{bmatrix} \]  

(11)

3.2. Newton Euler method based on screw theory

Newton Euler method based on spinor theory is used to model the dynamics of a six axis cooperative robot, which consists of two parts.

(1) In the first part, the velocity and acceleration of the connecting rod are calculated outward:

\[ V_i = Ad_{g_{i-1,i}} V_{i+1} + \xi_i \dot{q}_i \quad \dot{V}_i = \xi_i \dot{q}_i + Ad_{g_{i-1,i}} \dot{V}_{i-1} - ad \xi_i \left( Ad_{g_{i-1,i}} V_{i-1} \right) \]  

(12)

Where, \( q_i, \dot{q}_i, \ddot{q}_i \) represents the angle, angular velocity and angular acceleration of the robot joint \( i \); \( V_i, \dot{V}_i \) represents the generalized velocity and acceleration of the robot link \( i \).

(2) In the second part, the interaction force and moment between the connecting rods and the joint driving moment are calculated by iteration inward:

\[ F_i = Ad_{g_{i+1,i}}^T F_{i+1} + M_i \dot{V}_i - ad \xi_i M_i V_i \quad \tau_i = \xi_i^T F_i \]  

(13)

Where, \( M_i \) is the inertia matrix of the robot link \( i \), \( F_i \) is the force rotation of the robot link \( i \), and \( \tau_i \) is the torque of the robot joint \( i \).

4. Dynamic model

4.1. Dynamics model of rigid robot

According to the theorem of spinor, the kinematic helix of each joint can be calculated:

\[ \xi_1 = [0 \ 0 \ 0 \ 0 \ 0 \ 1]^T \quad \xi_2 = [d_i \ 0 \ 0 \ 0 \ -1 \ 0]^T \quad \xi_3 = [d_i + a_2 \ 0 \ 0 \ 0 \ -1 \ 0]^T \]  

(14)

\[ \xi_4 = [d_i + a_2 + a_3 \ 0 \ 0 \ 0 \ -1 \ 0]^T \quad \xi_5 = [-(d_2 + d_3 + d_4) \ 0 \ 0 \ 0 \ 0 \ 1]^T \]  

(15)

\[ \xi_6 = [d_i + a_2 + a_3 + d_3 \ 0 \ 0 \ 0 \ -1 \ 0]^T \]  

(16)

At that time \( q=0 \), the initial pose of the robot end relative to the basic coordinate system is as follows:

\[ g_{se}(0) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -(d_2 + d_3 + d_4 + d_6) \\ 0 & 0 & 1 & d_1 + d_5 + a_2 + a_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]  

(17)
From formula (5) and (6), we can get \( e^{\hat{\theta}h} - e^{\hat{\theta}h_n} \), and then calculate \( g_n(q) \) according to formula (4), so we can deduce \( Ad_{x_1} \).

(1) Extrapolation iteration formula:

\[
\begin{align*}
V_1 &= Ad_{g_0,1}V_0 + \xi_1\dot{q}_1 \\
V_2 &= Ad_{g_1,2}V_1 + \xi_2\dot{q}_2 \\
V_3 &= Ad_{g_2,3}V_2 + \xi_3\dot{q}_3 \\
V_6 &= Ad_{g_5,6}V_5 + \xi_6\dot{q}_6
\end{align*}
\]

\[
\begin{align*}
\dot{V}_1 &= \xi_1\ddot{q}_1 + Ad_{g_0,1}\dot{V}_0 - ad\xi_1\left(Ad_{g_0,1}V_0\right) \\
\dot{V}_2 &= Ad_{g_1,2}\dot{V}_1 - ad\xi_2\left(Ad_{g_1,2}V_1\right) \\
\dot{V}_3 &= Ad_{g_2,3}\dot{V}_2 - ad\psi_3\left(Ad_{g_2,3}V_2\right) \\
\dot{V}_6 &= Ad_{g_5,6}\dot{V}_5 - ad\xi_6\left(Ad_{g_5,6}V_5\right)
\end{align*}
\]  

(18)

(2) Extrapolation iteration formula:

\[
\begin{align*}
F_6 &= Ad_{g_6,1}^T F_7 + M_6\dot{V}_6 - ad\dot{V}_6 (M_6 V_6) \\
F_5 &= Ad_{g_5,6}^T F_6 + M_5\dot{V}_5 - ad\dot{V}_5 (M_5 V_5) \\
F_4 &= Ad_{g_4,7}^T F_5 + M_4\dot{V}_4 - ad\dot{V}_4 (M_4 V_4)
\end{align*}
\]  

(19)

The initial value of dynamic analysis is:

\[
V_0 = 0, \dot{V}_0 = \begin{bmatrix} q_0 \\ \dot{q}_0 \\ \ddot{q}_0 \\ \dddot{q}_0 \\ \cdots \\ \dddot{q}_0 \end{bmatrix}, F_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}
\]  

(20)

Define the following matrix:

\[
\begin{align*}
q &= \begin{bmatrix} q_1 \\ \vdots \\ q_6 \end{bmatrix}, \xi &= \begin{bmatrix} \xi_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \xi_6 \end{bmatrix}, p_0 &= \begin{bmatrix} Ad_{g_0,1} \\ 0 \\ \vdots \\ 0 \end{bmatrix}, V &= \begin{bmatrix} V_1 \\ \vdots \\ V_6 \end{bmatrix}
\end{align*}
\]  

(21)

\[
\dot{V} = \begin{bmatrix} \dot{V}_1 \\ \vdots \\ \dot{V}_6 \end{bmatrix}, F = \begin{bmatrix} F_1 \\ \vdots \\ F_6 \end{bmatrix}, M = \begin{bmatrix} M_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & M_6 \end{bmatrix}, \tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_6 \end{bmatrix}
\]  

(22)

\[
G = \begin{bmatrix} I & 0 & 0 & \cdots & 0 \\ Ad_{g_1,2} & I & 0 & \cdots & 0 \\ Ad_{g_2,3} & Ad_{g_1,2} & I & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ Ad_{g_6,1} & Ad_{g_5,6} & \cdots & Ad_{g_6,1} & I \end{bmatrix}, \Gamma = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ Ad_{g_6,1} & 0 & \ddots & \vdots \\ 0 & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & Ad_{g_5,6} \end{bmatrix}
\]  

(23)

When the friction and external force are both 0, the dynamic equations can be derived from equations (13), (18), and (19):

\[
\tau = M(q)\ddot{q} + C(q, \dot{q})\ddot{q} + G(q)
\]  

(24)
\[
\begin{aligned}
M(q) &= \xi^T G^T MG \xi \\
C(q, \dot{q}) &= \xi^T G^T \left( MGa_\xi \Gamma + ad_\xi^T M \right) G \xi \\
G(q) &= \xi^T G^T MGPV_0
\end{aligned}
\] (25)

Where \( M(q)\dot{q} \) is the inertial force term; \( C(q, \dot{q})\dot{q} \) represents the centripetal force and Coriolis force term; \( G(q) \) represents the gravity term.

4.2. Dynamics model of cooperative robot

Taking joint flexibility into account, the simplified model of flexible joint is shown in Figure 3. Joint flexibility can be regarded as a linear spring, that is, the flexible deformation of the joint is linearly related to the applied force or torque, and the elastic coefficient of the spring is the stiffness of the flexible joint system [10]. According to this assumption, the joint moment from the reducer end to the joint end generated by joint flexibility can be expressed as \( \tau_i = K_i (\theta_i - q_i) \). Where \( K_i \) is the elastic coefficient of the equivalent spring, and \( q_i \) is the stiffness of the rotation system of the next connecting rod connected with the ith joint.

\[
\begin{aligned}
M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) &= K(\theta - q) \\
J\ddot{q} + K(\theta - q) &= \tau
\end{aligned}
\] (26)

Where, \( J \) is the inertia matrix of the robot and the rotating inertia matrix of the rotor after the deceleration ratio; \( \theta \) is the motor angle and joint angle after the deceleration ratio;

Figure 3. Simplified model of single flexible joint manipulator.

5. Dynamics simulation

5.1. Dynamics simulation of rigid robot

In this paper, the joint simulation platform of SolidWorks and MATLAB is built. The dynamic simulation of the manipulator is realized by the powerful modeling function of SolidWorks and the powerful calculation function of MATLAB.

The dynamic simulation system of the manipulator is shown in Figure 4. The system mainly consists of three parts: trajectory planning module, dynamics theoretical model module and SimMechanics model of the manipulator.

Among them, the theoretical model of manipulator dynamics is derived from the formula in the theoretical analysis and converted into Matlab programming language, and the theoretical joint moment is calculated by the powerful calculation ability of MATLAB, while the SimMechanics model of manipulator is obtained from the actual physical model of manipulator. Input the same joint motion function to the two models, and judge whether the dynamic model of the manipulator is correct by
comparing the difference between the moment calculated by the theory of the manipulator and the output moment of the actual SimMechanics physical model of the manipulator.

Figure 4. Dynamics simulation system of rigid robot.

Figure 5. Simulation program of robot SimMechanics model.

Define the same joint variables in the track planning module. In order to verify the inertial force term, the centripetal force term and the Coriolis force term of the rigid robot in the theoretical analysis, the joint velocity and the joint acceleration of the robot cannot be 0. In this paper, the quadratic function[11] 28-29 is used to plan the joint angular displacement of the rigid robot.

The simulation results are shown in Figure 6-11. The abscissa represents time, and the ordinate represents the moment difference between the theoretical model of rigid robot dynamics and the SimMechanics model.
According to figure 6-11, the moment difference of each joint of the rigid robot is less than $4\times10^4 \text{N} \cdot \text{m}$, which is approximately 0. The simulation results show that the moment output by the theoretical model of each joint of the six axis rigid robot and the SimMechanics model is equal within the allowable error range. So far, it is verified that the dynamic model of rigid robot established by Newton-Euler method based on screw theory is correct.

5.2. Dynamics simulation of cooperative robot

After the dynamic model of rigid robot is verified, joint flexibility is introduced. As shown in Figure 12, a subsystem of flexible joint is built in SimMechanics. The simulation system model of flexible joint is similar to that of rigid joint. In the simulation process, the equivalent stiffness coefficient and moment of inertia of the motor end of the flexible joint are respectively [12]:

$$K_i = K_2 = K_3 = K_4 = K_5 = 5500, J_1 = J_2 = J_3 = J_4 = J_5 = J_6 = 0.2.$$ Assume that the joint torque on the loading joint is respectively: $\tau_1 = \sin t, \tau_2 = 0.5\sin t, \tau_3 = 0.5\sin t, \tau_4 = 0.5\sin t, \tau_5 = 0.5\sin t, \tau_6 = 0.5\sin t$. 
the angular displacement simulation results of each joint can be obtained as shown in Figure 13-18 by the variable step integrator ode23s.

As shown in Figure 13-18, the black solid line represents the change process of the angular displacement of the rigid joint, and the red dotted line represents the change process of the angular displacement of the flexible joint. It can be seen from the figure that the angular displacement of the flexible joint is basically the same as that of the rigid joint within the error range, and in the process of change, the angular displacement of the flexible joint fluctuates in a small range around the angular displacement of the rigid joint, thus verifying the correctness of the dynamics of the flexible joint.
6. Conclusion
To sum up, this paper uses Newton Euler method based on spinor theory to analyze the dynamics theory of six degree of freedom cooperative robot, and uses Simulink simulation platform of MATLAB software to simulate the dynamics of rigid robot and flexible robot successively, and the simulation results verify the correctness of the dynamics model. In addition, the research content of this paper also provides a theoretical basis for the follow-up robot motion control.

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