Study on Pressure and Viscosity of Ferrofluid Film in a Spiral Groove Mechanical Seal

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Abstract. The viscosity relationship of ferrofluid in external magnetic field was deduced in a spiral groove mechanical seal. Based on Muijderman narrow groove theory, the pressure distribution of ferrofluid film was calculated with the trial method by trapezoid formula. It has been found that the pressure between the end faces increases at first and then decreases from outer diameter to inner diameter along the radial direction, and reaches the maximum value at the root of the spiral groove. It increases with the increase of the rotating speed, spiral groove angle, and groove depth, increases at first and then decreases with the increase of the ratio of groove length, and decreases with the increase of the film thickness. The pressure reaches the maximum value when the ratio of groove length is about 0.6. The film thickness has less influence on the film pressure when the film thickness is bigger than 6μm. The viscosity of ferrofluid changes with the change of the pressure.

1. Introduction
The ferrofluid generally is consisted of magnetite nano-sized particle of around 1 to 100nm and carrier liquid such as water, oils and hydrocarbons with the aid of surfactants in a continuous carrier phase[1]. The ferrofluid is really a new kind of lubricant[2,3]. This lubricant can prevent leakage and increase the film load capacity with an appropriate magnetic field[4]. Zhou et al. experimentally studied on the lubrication characteristic of the ferrofluid film in a spiral groove mechanical seal, and revealed that the dynamic pressure increases with the increase of the magnetic field intensity[5]. In this paper, the viscosity relationship of ferrofluid in external magnetic field was deduced, the pressure distribution was calculated with the trial method by trapezoid formula based on Muijderman narrow groove theory, and the effects of the operating parameters and geometrical parameters of spiral groove on the pressure and viscosity of ferrofluid film were analyzed in a spiral groove mechanical seal.

2. Viscosity of ferrofluid in magnetic field

2.1 Viscosity of the ferrofluid
The viscosity of the ferrofluid can be calculated by Roelands viscosity pressure relationship in isothermal condition without external magnetic field[6].

\[ \eta = \eta_0 \exp\left(\ln\eta_0 + 9.67\right) - 1 + \left(1 + 5.1 \times 10^{-9} p\right) \]

(1)

where \( \eta_0 \) is the initial viscosity of the base fluid of ferrofluid under the condition of \( T_0 \) and \( p=0, \text{ Pa} \cdot \text{s} \), \( z \) is a constant, \( z=a/[5.1 \times 10^{-9} (\ln\eta_0 + 9.67)] \), and \( a \) is the coefficient of viscosity pressure.
The Eq.(1) was revised by Rosensweig[7], and
\[ \frac{\eta_f}{\eta_c} = \frac{1}{1 - 1.25\phi + 1.55\phi^2} \]  
(2)
where \( \phi \) is the volume concentration of solid phase of the ferrofluid.

2.2 Viscosity of the ferrofluid in magnetic field
Chi, et al. proposed that the viscosity of the ferrofluid in magnetic field can be described as[8]
\[ \eta_{HI} = \eta_f + \Delta \eta_{HI} \]  
(3)
Chi, et al. also considered that the relationship between the viscosity augment in external magnetic field and that without magnetic field is
\[ \frac{\Delta \eta_{HI}}{\eta_f} = \frac{3}{2}\phi - 0.5\alpha L(\alpha) \sin^2 \beta \]  
(4)
where \( \beta \) is the angle between the external magnetic field vector and the vortex vector, \(^\circ\), and \( L(\alpha) \) is the Langevin function.

There are the following formulas according to Langevin law,
\[ M = \phi M_p L(\alpha), \quad \alpha = \frac{\rho d_p^3 \mu_0 H M_p}{6k_0 T}, \quad L(\alpha) = \coth \alpha - \frac{1}{\alpha} \]  
(5)
According to Eq. (5), there is the following relationship,
\[ 0.5\alpha L(\alpha) = \frac{\rho d_p^3 \mu_0 H M}{12\phi k_0 T} \]  
(6)
where \( H \) is the intensity of external magnetic field, \( A\cdot m^{-1} \), \( M \) is the intensity of magnetization, \( A\cdot m^{-1} \), \( \mu_0 \) is the vacuum permeability, \( \mu_0 = 4\pi \times 10^{-7} H\cdot m^{-1} \), \( d_p \) is the diameter of solid particle, m. \( k_0 \) is the Boltzmann's constant, and \( k_0 = 1.38 \times 10^{-23} J\cdot K^{-1} \).

The relationships among the intensity of magnetization, the intensity of external magnetic field and the intensity of magnetic induction are
\[ M = \frac{1}{\mu_r} B - H \quad \text{and} \quad H = \frac{1}{\mu_0 \mu_r} B \]  
(7)
where \( \mu_r \) is the relative permeability, \( H\cdot m^{-1} \), and \( B \) is the intensity of external magnetic induction, T.

Substituting \( \beta = 90^\circ \)[9], Eqs.(1), (2), (4), (6) and (7) into Eq.(3), and
\[ \eta_{HI} = \eta_0 \exp \left[ \left( \ln \eta_0 + 9.67 \right) \times \left( 1 + 5.1 \times 10^{-3} \rho \right) \right] \times \left[ \frac{1}{1 - 2.5\phi + 1.55\phi^2} + \frac{1.5}{1} \right] \times \frac{3k_0 T \mu_0 \mu_r^2}{2\mu_0^3 (\mu_r - 1) B^2} \]  
(8)
Eq.(8) is the ferrofluid viscosity relationship in external magnetic field.

3. Analytical calculation approach of lubricant film pressure between end faces in a spiral groove mechanical seal
3.1 Governing equations of lubricant film pressure
According to the narrow groove theory of Muiljderman[10], the governing equations of ferrofluid film pressure between the end faces along the radial of sealing ring are

In the region of sealing dam,
\[ \frac{dp}{dr} = \frac{6\eta_{HI} q_w}{\pi h_0^3} \frac{1}{\rho_{HI} r} \]  
(9)
In the region of spiral groove, 
\[
\frac{dp}{dr} = -\frac{6\eta_H \alpha g_1}{h_0} r + \frac{6\eta_H q_m s g_2}{m h_1 h_1} \rho_H r
\]
where \(g_1\) and \(g_2\) are the coefficients of spiral grooves respectively, and

\[
g_1 = \frac{\gamma H^3 \cot \alpha (1 - H_1) (1 - H_1)}{(1 + \gamma H_1^3) (\gamma + H_1^3) + H_1^3 \cot^2 \alpha (1 + \gamma)^2}
\]

\[
g_2 = \frac{(1 + \gamma) H_1 \cot \alpha (1 - H_1) (1 - H_1)}{(1 + \gamma H_1^3) (\gamma + H_1^3) + H_1^3 \cot^2 \alpha (1 + \gamma)^2}
\]

where \(q_m\) is the mass flux of ferrofluid, kg·s\(^{-1}\), \(\rho_H\) is the density of ferrofluid, kg·m\(^{-3}\), \(h_0\) is the film thickness of ferrofluid, m, \(h_1\) is the film thickness in the region of spiral groove, \(h_1 = h_0 + h_\gamma\), \(m\), \(h_\gamma\) is the depth of spiral groove, m, \(\omega\) is the angular velocity of rotation of seal ring, rad·s\(^{-1}\), \(\alpha\) is the angle of spiral groove, \(\gamma\) is the ratio of width of groove to weir, \(H_1\) is the ratio of the film thickness in the sealing dam to spiral groove, and \(H_1 = h_0/(h_0 + h_\gamma)\).

### 3.2 Solution of governing equations

The boundary conditions of Eqs.(9) and (10) are \(p|_{r=r_i}=p_i\) at the inner diameter and \(p|_{r=r_0}=p_0\) at the outer diameter of sealing ring.

According to Song, et al.[11], the solving approach is based on the law of mass conservation. The mass flux of ferrofluid flowing through the sealing dam is equal to the mass flux flowing through the region of spiral groove. Assuming the mass flux \(q_m\), the ferrofluid pressure, \(p_g\), at the root of spiral groove was calculated according to \(p|_{r=r_i}=p_i\) and Eqs.(8) and (9), and the ferrofluid pressure distribution, \(p(r)\), from \(r_i\) to \(r_\gamma\) can also be obtained. Substituting \(p_g\) into Eqs. (8) and (10), the ferrofluid film pressure distribution, \(p(r)\), from \(r_\gamma\) to \(r_0\) can be obtained, as well as \(p(r_0)\). If \(p(r_0)=p_0\), \(p(r)\) presents the pressure distribution between end faces. If not, the mass flux \(q_m\) should be valued as another value and the trial procedure be repeated until \(p(r_0)=p_0\).

### 4. Results analysis and discussion

#### 4.1. Example

The magnetic particle is Fe\(_3\)O\(_4\), \(r_p=5\)mm, and \(\varphi=10\%\). The base fluid is silicone oil, \(\eta_0=0.074\)Pa·s, \(a=22G·Pa^{-1}\), and \(\mu_r=2.93H·m^{-1}\). The parameters of mechanical seal are \(r_p=34\)mm, \(r_\gamma=43\)mm, \(r_0=52\)mm, \(h_0=5\)μm, \(\gamma=1\), and \(\alpha=15°\). The intensity of external magnetic induction, \(B\), is 0.036T[10], the pressure differential of seal chamber is 0.5MPa, \(T=293\)K, and \(\rho_H=1390\)kg·m\(^{-3}\).

#### 4.2. Analysis of the influence factors of the distributions of pressure and viscosity

4.2.1. Effects of the rotating speed on the distributions of pressure and viscosity

The pressure distribution of lubricant film between end faces along the radial direction is shown in Figure 1. It can be seen that there is a large pressure gradient along the radial direction between end faces. The ferrofluid flowing in the spiral groove is compressed gradually due to impeding of sealing weir, therefore the pressure of lubricant film is increased gradually from outer diameter to inner diameter of sealing ring, and the pressure of lubricant film reaches the maximum value at the root of the spiral groove where the leakage passage becomes narrow which makes the ferrofluid accumulate. When the ferrofluid flows into the region of the sealing dam, the dynamic effect of ferrofluid disappears, which makes the pressure of lubricant film decreases rapidly to the atmospheric pressure. It can also be seen that the pressure of lubricant film increases with the increase of rotating speed, because hydrodynamic force increases with the increase of rotating speed.

The viscosity of lubricant film between end faces along the radial direction is illustrated in Figure 2.
The change of the viscosity is consistent with the pressure of ferrofluid film, and the viscosity rises and falls with the pressure. Under the hydrodynamic pressure effect, the solid particles gather to the root of the spiral groove, and therefore the number of solid particles reaches the maximum value and viscosity of ferrofluid reach the maximum as well. With the increase of rotating speed, the movement of magnetic particle is more violent, and the viscosity of lubricant becomes larger.

4.2.2 Effects of the spiral groove angle on the distributions of pressure and viscosity

The spiral groove angle is a very important structural parameter in spiral groove mechanical seal. With the increase of the spiral groove angle, the pumping effect of spiral groove increases gradually, which makes the pressure between the end faces increases gradually and the accumulating effect of ferrofluid particles increases, and therefore the pressure and viscosity of ferrofluid film increases, as illustrated in Figures 3 and 4.

4.2.3 Effects of the groove depth on the distributions of pressure and viscosity

The pressure and viscosity of lubricant film increases with the increase of groove depth, as shown in Figures 5 and 6, because the increase of groove depth increases the number of solid particles, which makes the hydrodynamic pressure increase. The viscosity of ferrofluid changes with the change of the pressure.

4.2.4 Effects of the ratio of groove length on the distributions of pressure and viscosity

The groove length has a remarkable influence on the ferrofluid film, as shown in Figures 7 and 8. With the increase of the ratio of groove length, the pressure of ferrofluid increases at first and then decreases, and the pressure between the end faces reaches the maximum value when the ratio of groove length is 0.6. The viscosity of ferrofluid also changes with the change of the pressure.

4.2.5 Effects of the film thickness on the distributions of pressure and viscosity

Because the hydrodynamic pressure effect weakens with the increase of film thickness, the pressure of ferrofluid
film decreases with the increase of the film thickness, as can be seen in Figure 9. The smaller film thickness is, the bigger hydrodynamic pressure between end faces generates. The film thickness has less influence on the film pressure when the film thickness is bigger than 6μm. The tendency of viscosity of ferrofluid film is consistent with the pressure, as shown in Figure 10.

5. Conclusions

(1) The viscosity relationship of ferrofluid film in external magnetic field was deduced in a spiral groove mechanical seal. The pressure field of ferrofluid film was calculated based on Muijderman narrow groove theory, and the effects of the operating parameters and
geometrical parameters of spiral groove on the pressure and viscosity of ferrofluid film were analyzed.

2) The pressure between the end faces increases at first and then decreases from outer diameter to inner diameter along the radial direction, and it reaches the maximum value at the root of the spiral groove. It increases with the increase of the rotating speed, spiral groove angle, and groove depth, increases at first and then decreases with the increase of the ratio of groove length, and decreases with the increase of the film thickness. The pressure reaches the maximum value when the ratio of groove length is about 0.6. The film thickness has less influence on the film pressure when the film thickness is bigger than 6μm.

3) The viscosity of ferrofluid changes with the change of pressure between end faces.

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