Calculating optimum gear ratios of a two-stage helical reducer with first stage double gear sets

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Abstract. This paper presents a study on the determination of the optimum gear ratios of a two-stage helical reducer with first stage double gear sets. In the study, an optimization problem was built in order to find the optimum gear ratios of the reducer. In the optimization problem, the objective function was the reducer cross section area. Also, the influence of the input factors including the total reducer ratio, the wheel face width coefficient, the allowable contact stress and the output torque were evaluated. In order to investigate the effects of these factors on the optimum gear ratios, a simulation experiment was designed and conducted. In addition, equations for calculating the optimum gear ratios were introduced. By using these equations, the gear ratios can be determined accurately in a simple way.

Keywords. Transmission ratio, optimum reducer design, helical reducer.

1. Introduction

Until now, many studies have been done for the determination of optimal gear ratios of the reducer of a mechanical driven system. This is because the gear ratios powerfully affect the size, the mass and therefore the cost of the reducer.

In previous studies, the optimal gear ratios have been found for two-step reducers [1-4], three-step reducers [4-8], and four-step reducers [4, 8-10]. Besides, the gear ratios have been determined by one of three basically methods including the graph method, the “practical method” and the model method. The graph method has used in many studies, such as in [1, 2 and 4]. An example of it is shown in Figure 1. In the figure, the gear ratios of a three-step helical reducer are determined graphically.

The “practical method” was first introduced in [2]. In this method, the gear ratios were determined based on the practical data. For example, the weight of a two-step helical reducer is minimal when the ratio of center distance of the second step to that of the first step is from 1.4 to 1.6 [2]. From that, the optimal gear ratios can be proposed.

The model method is the most common and the best method for calculating the optimal gear ratios. This is because it can give the values of the gear ratios directly and speedily. In the model method, the objective can be the minimal volume of gears [4], the minimal acreage of the cross section of the reducer [5], the minimal length of the reducer [7], the minimal total mass of the reducer [8] or the minimal mass of gears [9, 10].
This paper introduces a study for the determination of the optimum gear ratios of a two-stage helical reducer with second stage double gear sets. In this study, an optimization problem in which the objective is the minimum reducer cross section area was carried out. In addition, the effect of the input factors on the optimum gear ratios was investigated.

2. Optimization problem

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For a two stage helical reducer with first stage double gear sets, its cross section area is calculated by the following equation (see figure 2):

$$A = L \cdot h$$

In which, $L$ and $h$ are determined by:

$$L = \frac{d_{w1}}{2} + a_{w1} + a_{w2} + \frac{d_{w2}}{2}$$

$$h = \max (d_{w1}, d_{w2})$$

In above equations, $a_{w1}$ and $a_{w2}$ are the center distances of the first and the second stage; $d_{w1}$, $d_{w21}$ and $d_{w22}$ are pitch diameters (mm) of the first and the second stage, respectively. The diameters $d_{w1}$, $d_{w21}$ and $d_{w22}$ can be calculated as [11]:

![Figure 1. Gear ratios versus total ratio [1].](image1.png)

![Figure 2. Calculation schema.](image2.png)
\[ d_{w11} = 2 \cdot a_{w1} \cdot \left( u_1 + 1 \right) \]  
\[ d_{w21} = 2 \cdot a_{w1} \cdot u_1 \cdot \left( u_1 + 1 \right) \]  
\[ d_{w22} = 2 \cdot a_{w2} \cdot u_2 \cdot \left( u_2 + 1 \right) \]  

In which, \( u_1, u_2 \) are the gear ratios of the first and the second stages; \( u_1 \cdot u_2 = u_g \) with \( u_g \) is the total reducer ratio; Consequently, the optimization problem can be expressed as

\[
\text{minimize} \quad A = L \cdot h
\]  

With the following constraints

\[ 1 \leq u_1 \leq 9 \]  
\[ 1 \leq u_2 \leq 9 \]  

From above equations, for solving the optimization problem we need to determine the center distances of the first stage \( a_{w1} \) and the second stage \( a_{w2} \).

### 2.1. Determining the center distance of the first stage

\[
a_{w1} = K_a \cdot \left( u_1 + 1 \right) \cdot \left( \frac{T_{11} \cdot k_{H\beta}}{\sigma_H^2 \cdot u_1 \cdot \psi_{ba1}} \right)^{1/3}
\]  

Where, \( k_{H\beta} \) is the contact load ratio for pitting resistance; For the first stage of the reducer \( k_{H\beta} = 1.02 \pm 1.28 \) [11] and we can choose \( k_{H\beta} = 1.1 \); \( \sigma_H \) is the allowable contact stress (MPa); In practice, \( \sigma_H = 350 \ldots 420 \) (MPa); \( k_a \) is the material coefficient; For steel helical gear we have \( k_a = 43 \) [11]; \( \psi_{ba} \) is the coefficient of wheel face width; for the for the first stage of the reducer \( \psi_{ba1} = 0.3 \ldots 0.35 \). From the moment equilibrium condition of the reducer we have:

\[
T_{out} = 2 \cdot T_{11} \cdot \eta_{hg}^2 \cdot \eta_{be}^3 \cdot u_g
\]

In which, \( \eta_{hg} \) is the helical gear transmission efficiency (\( \eta_{hg} = 0.96 \ldots 0.98 \) [11]); \( \eta_{be} \) is the transmission efficiency of a pair of rolling bearing (\( \eta_{be} = 0.99 \ldots 0.995 \) [11]). Choosing \( \eta_{be} = 0.97 \) and \( \eta_{be} = 0.992 \) and substituting them into (10) gets:

\[
T_{11} = 0.5444 \cdot T_{out} / u_g
\]

Substituting (11) and \( k_{H\beta} = 1.1 \) into (9) we have:

\[
a_{w1} = 36.2442 \cdot \left( u_1 + 1 \right) \cdot \left( \frac{T_{out}}{\sigma_H^2 \cdot u_1 \cdot u_g \cdot \psi_{ba1}} \right)^{1/3}
\]

### 2.2. Calculating the center distance of the second stage

The center distance of the second stage \( a_{w2} \) can be determined by [11]:

\[
a_{w2} = K_a \cdot \left( u_2 + 1 \right) \cdot \left( \frac{T_{12} \cdot k_{H\beta}}{\sigma_H^2 \cdot u_2 \cdot \psi_{ba2}} \right)^{1/3}
\]

In addition, for the second stage we have

\[
T_{out} = T_{12} \cdot \eta_{hg} \cdot \eta_{be}^2 \cdot u_2
\]

Choosing \( \eta_{hg} = 0.97 \) and \( \eta_{be} = 0.992 \) as in section 2.1 and substituting them into (14) gives

\[
T_{12} = 1.0476 \cdot T_{out} / u_2
\]
Substituting (15), $k_a = 43$ and $k_{Hb} = 1.1$ (as in section 2.1) into (13) gets:

$$a_{w2} = 45.0814 \cdot (u_2 + 1) \cdot \left( \frac{T_{out}}{[\sigma_H]^2 \cdot u_2^2 \cdot \psi_{ba2}} \right)^{1/3}$$

(16)

2.3. Experimental work

For exploring the effect of the input factors on the optimum gear ratios, a simulation experiment was designed and conducted. In this case, a 2-level full factorial design was carefully chosen. Table 1 presents 5 input factors which were selected for the investigation. Consequently, there are $2^5 = 32$ number of tests for conducting. From equation (7) and (8), a computer program was built for performing the experiment. The various levels of input factors and the results of the output of the program (the optimum gear ratio of the first stage $u_1$) are shown in Table 2.

Table 1. Input parameters.

| Factor                        | Code | Unit | Low  | High |
|-------------------------------|------|------|------|------|
| Total reducer ratio           | $u_g$| -    | 5    | 30   |
| Coefficient of wheel face width of stage 1 | $x_{ba1}$ | -    | 0.3  | 0.35 |
| Coefficient of wheel face width of stage 2 | $x_{ba2}$ | -    | 0.35 | 0.4  |
| Allowable contact stress      | AS   | MPa  | 350  | 420  |
| Output torque                 | $T_{out}$ | Nmm | $10^5$ | $10^7$ |

3. Results and discussions

Figure 3 shows the effect of input factors on the response and the relative strength of the effect. From the figure, it was found that the optimum gear ratio of the first stage $u_1$ depends strongly on the total reducer ratio $u_g$. It increases significantly with the increase of the total reducer ratio. In addition, it is effected by the wheel face width coefficients of the first and the second stages ($\psi_{ba1}$ and $\psi_{ba2}$). Moreover, the optimum gear ratio is not effected by the allowable contact stress of the helical gear set AS, and the output torque $T_{out}$.

Table 2. Experimental plans and output response.

| StdOrder | RunOrder | CenterPt | Blocks | $u_g$ | Xba1 | Xba2 | AS (MPa) | $T_{out}$ (Nm) | $u_1$ |
|----------|----------|----------|--------|-------|------|------|----------|----------------|-------|
| 25       | 1        | 1        | 1      | 5     | 0.3  | 0.35 | 420      | 10000         | 3.45  |
| 17       | 2        | 1        | 1      | 5     | 0.3  | 0.35 | 350      | 10000         | 3.45  |
| 29       | 3        | 1        | 1      | 5     | 0.3  | 0.4  | 420      | 10000         | 3.3   |
| 6        | 4        | 1        | 1      | 30    | 0.3  | 0.4  | 350      | 100           | 9     |
| 15       | 5        | 1        | 1      | 5     | 0.35 | 0.4  | 420      | 100           | 3.47  |
| 19       | 6        | 1        | 1      | 5     | 0.35 | 0.35 | 350      | 10000         | 3.63  |
| 31       | 7        | 1        | 1      | 5     | 0.35 | 0.35 | 420      | 10000         | 3.47  |
| ...      |          |          |        |       |      |      |          |                |       |
| 2        | 31       | 1        | 1      | 30    | 0.3  | 0.35 | 350      | 100           | 9     |
| 28       | 32       | 1        | 1      | 30    | 0.35 | 0.35 | 420      | 10000         | 9     |

The Pareto chart of the standardized effects is shown in figure 4. It was found from this graph, the bars that represent the total reducer ratio (factor A), the coefficients of wheel face width of the first and the second stages (factors B and C) and the interactions between them (AB and AC) cross the reference
line. Therefore, these input factors are statistically significant at the 0.05 level with the response model (the gear ratio of the first stage $u_1$). Also, because the bars which represent the allowable contact stress of the helical gear set (factor D), and the output torque (factor E) do not cross the reference line, these factors (D and E) do not affect the response model.

Figure 3. Main effects plot for the optimum gear ratio of the first stage.

Figure 4. Pareto Chart of the Standardized Effects.

Figure 5. Normal Plot for the optimum gear ratio of the first stage.
Figure 5 presents the Normal Plot of the standardized effects. This graph is used to describe which effects increase or decrease the response. From the Figure, it was found that the total reducer ratio (factor A) is the most significant factor for the optimum gear ratio. In addition, the total reducer ratio and the coefficient of wheel face width of the first stage (factor B) have a positive standardized effect. If they change from the low level to the high level of the factors, the optimum gear ratio of the first stage increases. Also, the wheel face width coefficient of the second stage (factor C) has a negative standardized effect. The optimum gear ratio decreases when it increases.

Figure 6 presents the estimated effects and coefficients for the optimum gear ratio. From the figure, the total reducer ratio \( u_g \), the wheel face width coefficient of the first and the second stage (\( \psi_{ba1} \) and \( \psi_{ba2} \)) and their interactions have P-values lower than 0.05. Therefore, these factors are significant to the optimum gear ratio. In addition, the relation between the optimum gear ratio and these significant effect factors can be described as follows:

\[
 u_1 = 2.1413 + 0.2205 \cdot u_g + 4.95 \cdot \psi_{ba1} - 3.07 \cdot \psi_{ba2} - 0.14 \cdot u_g \cdot \psi_{ba1} + 0.124 \cdot u_g \cdot \psi_{ba2} - 2 \cdot \psi_{ba1} \cdot \psi_{ba2} \quad (17)
\]

As the adj-R\(^2\) and pred-R\(^2\) are in the high values (figure 6), the above equation fit the data very well.

Equation (17) is used for determining the optimum gear ratio of the first stage \( u_1 \). After getting \( u_1 \), the gear ratio of the second stage can be found by \( u_2 = u_g / u_1 \).

4. Conclusions
The minimum cross section area of a two-stage helical reducer with second stage double gear sets can be obtained by determination of optimum gear ratios of the reducer.
Equations for calculating the optimum gear ratios of a two-stage helical reducer with second stage double gear sets were introduced for getting the minimum reducer cross section area.
The optimum gear ratios of the reducer can be calculated accurately and simple as the estimated models are explicit.

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