Natural Inflation from 5D SUGRA 
and Low Reheat Temperature

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Abstract

Motivated by BICEP2’s recent observation of a possibly large primordial tensor component \( r \) of inflationary perturbations, we reanalyse in detail the 5D conformal SUGRA originated natural inflation model of Ref. [1]. The model is a supersymmetric variant of 5D extra natural inflation, also based on a shift symmetry, and leads to the potential of natural inflation. Analysis of the required number of e-foldings (from the CMB observations) points to the necessity of a very weak inflaton decay and low reheating temperature \( T_r \). We show that this can be naturally achieved within 5D gauge inflation giving \( T_r \lesssim \mathcal{O}(100) \) GeV. This is realized by coupling the bulk fields, generating the inflaton potential, with brane SM states. Some related theoretical issues of the construction, along with phenomenological and cosmological implications, are also discussed.

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1 Introduction

Inflation solves the problems of early cosmology in a natural way [2] and besides that produces a primordial fluctuation spectrum [3] which allows to discuss structure formation successfully. In detailed models (i) a sufficient number of e-folds for the inflationary phase has to be produced, (ii) guided by bounds presented by the Planck Collaboration [4, 5], the cosmic background radiation and a spectral index \( n_s = 0.9603 \pm 0.0073 \) should be generated. And (iii), the normalization of fluctuations has to be reproduced. Rather flat potentials for the inflaton field lead to the “slow roll” needed for (i). Such potentials appear naturally in (tree level) global supersymmetric models; higher loop corrections can be controlled, but the inclusion of supergravity easily produces an inflaton mass of the order of the Hubble scale.

In models with an extra dimension the fifth component of a U(1) gauge field entering in a Wilson loop operator can act as an inflaton field of pseudo Nambu-Goldstone type which is protected against gravity corrections and avoids a transplanckian scale [6], [7], present in the original model of “natural inflation” [8]. We have presented such a model [1] based on 5D conformal SUGRA on an orbifold \( S^1/\mathbb{Z}_2 \) with a predecessor based on global supersymmetry with a chiral “radion” multiplet on a circle in the fifth dimension [9]. We also made the interesting observation that a spectral index \( n_s \sim 0.96 \) as observed recently [different from a value very close to (1) usually obtained in straightforward SUSY hybrid inflation [11]], is obtained rather generically in gauge inflation. Actually, in the supersymmetric formulation we have a complex scalar field which besides the gauge inflaton \( A_5^1 \) contains a further “modulus” field \( M^1 \) which also might allow for successful inflation [1]. The main difference between the two inflation types is that gauge inflation leads to a large tensor to scalar ratio \( r(\sim 0.12 \text{ in } [1]) \) whereas modulus inflation leads to very small \( r(\lesssim 10^{-4} \text{ in } [1]) \). Recently the BICEP2 data [12] gave strong indication of a large ratio \( r = 0.2^{+0.07}_{-0.05} \) though there is a still ongoing discussion if this indeed has primordial origin [13]. We here therefore consider the gauge inflation of ref. [1] again with particular emphasis on the required length of inflation. The well known 62 e-folds solving the horizon problem will turn out to require a substantial expansion during the reheating period within the natural inflation scenario.

Let us present the organization of the paper and summarize some of the results. In Sec. 2 we perform a detailed analysis of natural inflation with cos-type potential. For the calculation of the spectral index \( n_s \) and the tensor to scalar ratio \( r \), we use a second order approximation with respect to slow roll (SR) parameters. Since these quantities \((n_s, r)\) are determined at the point where the SR parameters are tiny, this approximation is sufficient for all practical purposes. However, near the end of the inflation, when SR breaks down, we perform an accurate numerical determination of the point via the condition \( \epsilon_H = 1 \) on the Hubble slow roll parameter (see [14]- [16] for definitions). This is needed to compute, with desired precision, the number of \( e \)-foldings \( (N^\text{inf}_e) \) before the end of the inflation. Within 1\( \sigma \) deviation of the \( r \) (provided by BICEP2), we find that \( N^\text{inf}_e \lesssim 54 \), indicating that nearly \( \sim 10 \) e-foldings of the Universe expansion should occur during the reheating process. For this calculation (together with \( n_s \) and \( r \)) we use the value of the amplitude of curvature perturbations measured by the Planck collaboration [5]. All this allows to calculate the reheat temperature, which turns out to be low \((\lesssim O(10) \text{ TeV})\). We have also obtained results

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\(^5\)Genuine two field inflation was discussed in ref. [10]. The two basic inflation types depending on initial conditions turn out to be still like in [1]. Since inclusion of the \( M^1 \) modulus into the inflation process is fully legitimate, one can reserve this scenario as an alternative with a tiny tensor perturbations, if it should be.
for 3σ and 2σ deviations of \( r \) and \( n_s \) respectively.\(^6\)

In Sec. 3 and Appendix A we shortly review our model of ref. [1] in a more self-contained way and discuss how natural inflation emerges from 5D SUGRA. Using a superfield formulation, we do not need to go into the details of the component expressions in conformal 5D SUGRA of Fujita, Kugo and Ohashi (FKO) [17]. Indeed this emerged from our discussion [18, 19] (see also Ref. [21]) bringing the 5D conformal SUGRA formulation closer to the 4D global SUSY language [19]. We concentrate here on gauge inflation, i.e. on the case \( M^1 = 0 \) (stabilized moduli in the origin or a choice of initial conditions\(^7\)). In section 3, discussing the realization of natural inflation within 5D SUGRA, we present the mechanism for inflaton decay. We show that the inflaton’s slow decay is a natural consequence of the 5D construction, being realized by couplings of the heavy bulk supermultiplets (generating the inflaton potential through their gauge coupling) with brane SM states. Because the inflaton decay proceeds by 4-body decay and the decay width is additionally suppressed by the 2-nd power of the tiny \( U(1) \) gauge coupling constant (of the gauge inflaton-charged fields) and a relatively small inflaton mass, the proper suppression of the reheat temperature \( T_r \) also comes out naturally.

Our 5D SUGRA construction allows us to make an estimate

\[
T_r \sim 0.34 \rho_{reh}^{1/4} \sim |\lambda|^2 \times 100 \text{ GeV} \quad (\text{where } \lambda \lesssim 1 \text{ is a brane Yukawa coupling}).
\]

Appendix A discusses the Kaluza-Klein spectrum of the fields involved, as well as the SUSY breaking effects for brane fields. We also perform a derivation of higher dimensional operators involving the inflaton \( \phi_\Theta \) and light (MSSM) states relevant for the inflaton decay. As it turns out, the dominant decay channel is \( \phi_\Theta \rightarrow llhh \) (with \( l \) and \( h \) denoting SM lepton and Higgs doublets respectively). Sec. 4 includes a discussion and concluding remarks about some related issues.

## 2 Natural inflation

In this section we analyse inflation with the potential of natural inflation [8] given by:

\[
V = V_0 (1 + \cos(\alpha \phi_\Theta)),
\]

(1)

where \( \phi_\Theta \) is a canonically normalized real scalar field of inflation. In the concrete scenario of Ref. [1], we focus later on, the inflaton originates from a 5D gauge superfield, while the parameters/variables of (1) are derived through the underlying 5D SUGRA. See Eqs. (24), (25), (A.17) and also the comment underneath Eq. (A.17).

The slow roll parameters (“VSR”) are given by

\[
\epsilon = \frac{M_{Pl}^2}{2} \left( \frac{V'}{V} \right)^2 = \frac{(M_{Pl} \alpha)^2}{2} \tan^2 \frac{\alpha \phi_\Theta}{2}
\]

\[
\eta = M_{Pl}^2 \frac{V''}{V} = \frac{(M_{Pl} \alpha)^2}{2} \left( \tan^2 \frac{\alpha \phi_\Theta}{2} - 1 \right) = \epsilon - \frac{1}{2} (M_{Pl} \alpha)^2,
\]

\[
\xi = M_{Pl}^2 \frac{V'Y''}{V^2} = -(M_{Pl} \alpha)^4 \tan^2 \frac{\alpha \phi_\Theta}{2} = -2(M_{Pl} \alpha)^2 \epsilon,
\]

(2)

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\(^6\)Awaiting for more accurate measurements and a refined analysis, for this moment we still keep in mind possible changes corresponding to such deviations, and present appropriate numerical results.

\(^7\)For a discussion of moduli stabilization in the superfield formalism within 5D SUGRA see [22]. For a choice of initial conditions leading approximately to \( M^1 = 0 \) see Ref. [10].
where $M_{Pl} = 2.4 \cdot 10^{18}$ GeV is the reduced Planck mass. The number of e-foldings during inflation, i.e. during exponential expansion, denoted further by $N^\text{inf}_e$, is calculated as

$$N^\text{inf}_e = \frac{1}{\sqrt{2M_{Pl}}} \int_{\phi^i}^{\phi^e} \frac{1}{\sqrt{\epsilon_H}} d\phi \Theta.$$  

In this exact expression the HSR parameter $\epsilon_H$ (defined below), participates. The point $\phi^e$, at which inflation ends, is determined by the condition $\epsilon_H = 1$. The point $\phi^i$ corresponds to the begin of the inflation.

The observables $n_s$ and $r$ depend on the value of $\phi^i$ (the point at which scales cross the horizon). This allows to determine $\phi^i$ as follows. Via HSR parameters, the expressions for $n_s$ and $r$ are given by [14], [16], [15]:

$$n_s = 1 - 4\epsilon_{Hi} + 2\eta_{Hi} - 2(1 + C)\epsilon_{Hi}^2 - \frac{1}{2}(3 - 5C)\epsilon_{Hi}\eta_{Hi} + \frac{1}{2}(3 - C)\xi_{Hi},$$

$$r = 16\epsilon_{Hi}(1 + 2C(\epsilon_{Hi} - \eta_{Hi})),$$

where we have limited ourself with second order corrections. The HSR parameters $\epsilon_H, \eta_H, \xi_H$ are given by:

$$\epsilon_H = 2M_{Pl}^2 \left(\frac{H'}{H}\right)^2, \quad \eta_H = 2M_{Pl}^2 \frac{H''}{H}, \quad \xi_H = 4M_{Pl}^4 \frac{H'H'''}{H^2},$$

(5)

with the Hubble parameter $H$ and it’s derivative with respect to the inflaton field. The subscript ‘i’ in (4) indicates that the parameter is defined at the point at which scales cross the horizon. As it turns out, at this scale the slow roll parameters are small and second order corrections in $n_s$ and $r$ are small and the approximations made in (4) are pretty accurate. Exact relations between VSR ($\epsilon, \eta, \xi, \cdots$) and HSR parameters ($\epsilon_H, \eta_H, \xi_H, \cdots$) are given by [14], [16], [15]:

$$\epsilon = \epsilon_H \left(\frac{3 - \eta_H}{3 - \epsilon_H}\right)^2, \quad \eta = \frac{3(\epsilon_H + \eta_H) - \eta_H^2 - \xi_H}{3 - \epsilon_H},$$

$$\xi = 3\frac{3 - \eta_H}{(3 - \epsilon_H)^2} \left(3\epsilon_H\eta_H + \xi_H(1 - \eta_H) - \frac{1}{6}\sigma_H\right),$$

(6)

with $\sigma_H = 4M_{Pl}^4 \epsilon_H \frac{H^{(iv)}}{H}$. When the slow roll parameters are small, from (6), the HSR parameters to a good approximation can be expressed in terms of VSR parameters as

$$\epsilon_H \approx \epsilon - \frac{4}{3}\epsilon^2 + \frac{2}{3}\epsilon\eta, \quad \eta_H \approx \eta - \epsilon + \frac{8}{3}\epsilon^2 - \frac{8}{3}\epsilon\eta + \frac{1}{3}\eta^2 + \frac{1}{3}\xi,$$

$$\xi_H \approx 3\epsilon^2 - 3\epsilon\eta + \xi.$$  

(7)

Using these approximations in (4), we can write $n_s$ and $r$ in terms of VSR parameters:

$$n_s = 1 - 6\epsilon_i + 2\eta_i + \frac{2}{3}(22 - 9C)\epsilon_i^2 - (14 - 4C)\epsilon_i\eta_i + \frac{2}{3}\eta_i^2 + \frac{1}{6}(13 - 3C)\xi_i,$$

$$r = 16\epsilon_i \left(1 - \left(\frac{2}{3} - 2C\right)(2\epsilon_i - \eta_i)\right),$$

(8)
where we have still restricted the approximations up to the second order. Applying these expressions, for the model (determining $\epsilon, \eta$ and $\xi$ as given in Eq. (2)), we arrive at:

$$n_s = 1 - \left( 1 + 2 \tan^2 \frac{\alpha \phi^i}{2} \right) (M_{Pl}\alpha)^2 \left( \frac{1}{6} + \left( 1 - \frac{1}{2} C \right) \tan^2 \frac{\alpha \phi^i}{2} + \left( \frac{1}{3} - \frac{1}{2} C \right) \tan^4 \frac{\alpha \phi^i}{2} \right) (M_{Pl}\alpha)^4, \quad (9)$$

and

$$r = 8(M_{Pl}\alpha)^2 \left( 1 - \left( \frac{1}{3} - C \right) \left( 1 + \tan^2 \frac{\alpha \phi^i}{2} \right) (M_{Pl}\alpha)^2 \right) \tan^2 \frac{\alpha \phi^i}{2}. \quad (10)$$

From Eq. (10) we can express $\tan^2 \frac{\alpha \phi^i}{2}$ in terms of $r$ and $M_{Pl}\alpha$. As will turn out, the latter’s value is small, so to a good approximation we find:

$$\tan^2 \frac{\alpha \phi^i}{2} \approx \frac{r}{8(M_{Pl}\alpha)^2} \left( 1 + \left( \frac{1}{3} - C \right) (\frac{r}{8} + (M_{Pl}\alpha)^2) + \frac{1}{8} \left( \frac{1}{3} - C \right)^2 (M_{Pl}\alpha)^4 \right). \quad (11)$$

Plugging this into Eq. (9) for the spectral index we get:

$$n_s - 1 = -\frac{r}{4} (M_{Pl}\alpha)^2 \frac{1}{6} (M_{Pl}\alpha)^4 - \frac{r^2}{64} \left( \frac{3}{3} - \frac{3}{2} C \right) + \frac{r^3}{8} \left( \frac{3}{3} + \frac{3}{2} C \right) (M_{Pl}\alpha)^2 + \frac{r^2}{128} (\frac{10}{9} - C(\frac{13}{3} - 3C)) (M_{Pl}\alpha)^2. \quad (12)$$

Using current experimental values $n_s = 0.9603 \pm 0.0073$ [4] and $r = 0.2^{+0.07}_{-0.05}$ [12], relation (12) provides an upper bound for the value of $M_{Pl}\alpha$:

$$M_{Pl}\alpha \lesssim 0.17 \quad \text{(obtained via } 2\sigma \text{ variations of } n_s \text{ and } r). \quad (13)$$

This will be used as orientation for further analysis and various predictions.

So far, we have performed calculations in a regime of small slow roll parameters, determining the value of $\phi^i$ via Eq. (11). As was mentioned, the value of $\phi^i$ is determined from the condition $\epsilon_H = 1$. Near this point both $\epsilon$ and $\eta$ parameters turn out to be large and instead of an expansion we need to perform numerical calculations. This will be relevant upon the calculation of the number of e-foldings $N^{\text{inf}}$.

Since, within our model, via Eq. (2) VSR parameters are related to each other as

$$\eta = \epsilon - \frac{1}{2}(M_{Pl}\alpha)^2, \quad \xi = -2(M_{Pl}\alpha)^2 \epsilon, \quad (14)$$

the three equation in (6) can be rewritten as

$$\epsilon_H \left( \frac{3 - \eta_H}{3 - \epsilon_H} \right)^2 = \epsilon$$

$$\frac{3(\epsilon_H + \eta_H) - \eta_H^2 - \xi_H}{3 - \epsilon_H} = \epsilon - \frac{1}{2}(M_{Pl}\alpha)^2$$

$$3 \frac{3 - \eta_H}{(3 - \epsilon_H)^2} (3 \epsilon_H \eta_H + \xi_H (1 - \eta_H)) = -2(M_{Pl}\alpha)^2 \epsilon, \quad (15)$$

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8 Whether the light polarization effect is really due to the gravitational waves, or by the ordinary dust (as warned by Planck’s intermediate results [13]) still needs to be settled. While we are awaiting for joint results from the Planck and BICEP2 teams, at present we use the value of $r$ reported by BICEP2 [12].
where $\sigma_H$ has been dropped because of its smallness. From the system of (15), for a fixed value of $M_{Pl}\alpha$, the parameters $\epsilon_H, \eta_H$ and $\xi_H$ can be found in terms of the single parameter $\epsilon$. The dependence of these parameters on the value of $\epsilon$, for $M_{Pl}\alpha = 0.04$ are shown in Fig. 1 (for different values of $M_{Pl}\alpha$ shapes of the curves are similar). We see that $\epsilon_H = 1$ is achieved when $\epsilon = \epsilon_e \approx 2$ and thus, the expansion with respect to $\epsilon, \eta$ within this stage of inflation is invalid. On the other hand, the values of $\eta_H$ and $\xi_H$ remain relatively small. From the relation $2\epsilon = (M_{Pl}\alpha)^2 \tan^2 \theta$ one derives:

$$d\phi_{\Theta} = \frac{M_{Pl} \sqrt{2}}{\sqrt{\epsilon(2\epsilon + (M_{Pl}\alpha)^2)}} d\epsilon.$$  (16)

Using this, the integral in (3) can be rewritten as

$$N_{\text{inf}}^e = \int_{\epsilon_e}^{\epsilon_1} \frac{1}{2\epsilon + (M_{Pl}\alpha)^2} \frac{d\epsilon}{\sqrt{\epsilon\epsilon_H}}.$$  (17)

Having the numerical dependence $\epsilon_H = \epsilon_H(\epsilon)$ (depicted in Fig. 1), we can evaluate the integral in (17) and find $N_{\text{inf}}^e$ for various values of $M_{Pl}\alpha$. The results are given in Fig. 2. Curves in Fig. 2 demonstrate that, within our model, there is an upper bound on $N_{\text{inf}}^e$. Namely, within 1$\sigma$ error bars of $r$ and $n_s$ we have

$$N_{\text{inf}}^e \lesssim 54.$$  (18)

This, on the other hand, leads to another striking prediction and constraint.

As discussed in Refs. [23], [5], the $N_{\text{eff}}^e$, guaranteeing causality of fluctuations, should satisfy:

$$N_{\text{inf}}^e = 62 - \ln \frac{k}{a_0 H_0} - \ln \frac{10^{16}\text{GeV}}{V_1^{1/4}} + \ln \frac{V_3^{1/4}}{V_1^{1/4}} - \frac{4 - 3\gamma}{3\gamma} \ln \frac{V_1^{1/4}}{\rho_{\text{reh}}}.$$  (19)

where for the scale $k$ we take $k = 0.002\text{Mpc}^{-1}$ corresponding to the Planck’s data [4], [5], while the present horizon scale is $a_0 H_0 \approx 0.00033\text{Mpc}^{-1}$. The factor $\gamma$ accounts for the dynamics of the inflaton’s oscillations [24], [25] after inflation, and can be for our model approximated as $\gamma \simeq 1 - \frac{\sqrt{2}}{16} \frac{V_4}{V_0}$ (will turn out to be a pretty good approximation).
To reconcile the first two entries (62 − ln \(k_{a_0H_0}\) ≈ 60.2) of Eq. (19) with the bound of Eq. (18) (see also Fig. 2), the remaining entries of Eq. (19) should be significant enough to bring \(N_{\text{inf}}\) down (at least) to \(≈ 54\). The 3rd and 4th entries on the r.h.s. of Eq. (19) can be calculated with help of another observable - the amplitude of curvature perturbation \(A_s\), which according to the Planck measurements [5], should satisfy \(A_{s}^{1/2} = (4.686^{+0.056}_{-0.063}) \times 10^{-5}\) (this value, given in table 3 of Ref. [5], corresponds to the ΛCDM model). Generated by inflation, this parameter is given by:

\[
A_{s}^{1/2} = \frac{1}{\sqrt{12\pi}} \left| \frac{\mathcal{V}^{3/2}}{M_{Pl}^3 V'} \right|_{\phi_i} \approx \frac{4\sqrt{6} V_0^{1/2}}{3\pi M_{Pl}^2 r(1 + 8(M_{Pl}\alpha)^2/r)^{1/2}} .
\]

In order to obtain the observed value of \(A_{s}^{1/2}\), for \(r = 0.15, M_{Pl}\alpha = 0.04\) we need to have \(V_0^{1/4} \sim 10^{-2} M_{Pl}\). This, on the other hand, gives \(V_i^{1/4} \sim 0.01 M_{Pl}\) and \(V_e^{1/4} \sim 2 \cdot 10^{-3} M_{Pl}\). Using these values in (19) we see that the sum of the 3rd and 4th terms is\(\approx 3.4\). Thus, the last term should be responsible for a proper reduction of \(N_{\text{inf}}\). Namely, during the reheating process, the universe should expand by nearly 10 (or even more) e-foldings. This means that, for this case, the model should have a significant reheat history with \(\rho_{\text{reh}}^{1/4} \sim 400\ GeV. 9\) Within the scenario of natural inflation, this has not been appreciated before. For lower values of \(r\) (deviating from the central value by 2σ or more) the reheating temperature can be larger. For appropriate values of \(r\) and \(M_{Pl}\alpha\) (and \(n_s\)) the reheating temperature can be big. The numerical results are given in Table 1, where we considered cases with \(\rho_{\text{reh}}^{1/4}\) not smaller than \(10^{-3}\) GeV, and \(N_{\text{inf}}^e \leq 62\). The values of the spectral index running \(\frac{d n_s}{d \ln k} = 16\epsilon_i\eta_i - 24\epsilon_i^2 - 2\xi_i\) are also presented. The first three row-blocks correspond to the values of \((r, n_s)\) within 2σ ranges of the current experimental data. Central \((r = 0.2)\) and

\[\text{Figure 2: Number of e-foldings. Values of } n_s \text{ and } r \text{ are taken within 2σ range. Solid lines correspond to the values of } (r, n_s) \text{ which simultaneously fit with the current experimental data within 1σ error bars.}\]
In order to address the details of inflaton decay, related to the reheat temperature, we need to specify the underlying theory natural inflation emerged from. A very good candidate is a higher dimensional construction. In this context, the analysis of observations in the future will be relevant in case the larger values of $\sigma$ give $\theta$ less than $10^{-3}$ GeV and are refuted. That’s why we have started to look for the reheat temperature, which would be relevant in case the larger values of $r$ are not confirmed by the analysis of observations in the future.

Below we will show that within our scenario of natural inflation, a low reheat temperature is realized naturally and fits well with the proposed construction.

### 3 Natural inflation from 5D SUGRA

Lagrangian couplings, for the bulk $H = (H, H^c)$ hypermultiplets', components are:

\[
e^{-\frac{1}{4}}L(H) = \int d^4\theta (T + T^\dagger) (H^\dagger H + H^c H^c) + \int d^2\theta \left( 2H^c \partial_y H + g_i \Sigma_1 (e^{i \theta_1} H^2 - e^{-i \theta_1} H^c) \right) + h.c. \quad (21)
\]

For the component formalism of 5D conformal SUGRA see the pioneering work by Fujita, Kugo and Ohashi [17]. Note also, that the component off shell 5D SUGRA formulation, discussed by Zucker [20], was used in many phenomenologically oriented papers.

| $r$ | $M_{Pl} \alpha$ | $n_s$ | $10^4 \times \frac{dn_s}{d\ln k}$ | $N_{inf}$ | $V_1^{1/4}$ | $V_2^{1/4}$ | $V_3^{1/4}$ | $\rho_{reh}^{1/4}$ (GeV) |
|-----|-----------------|-------|-------------------------------|----------|------------|------------|------------|----------------------|
| 0.15 | 0.001           | 0.9624| -7.1                          | 53.62    | 19.8       | 2.01       | 0.53       | 3.14 x 10^4         |
|      | 0.04            | 0.9608| -7.71                         | 51.47    | 3.2        | 2.01       | 0.54       | 53.7                |
|      | 0.055           | 0.9594| -8.25                         | 49.73    | 2.77       | 2.01       | 0.54       | 0.31                |
|      | 0.065           | 0.9582| -8.71                         | 48.39    | 2.58       | 2.01       | 0.55       | 0.006               |
| 0.125| 0.04            | 0.9671| -5.43                         | 61.25    | 2.92       | 1.92       | 0.49       | 2.77 x 10^4         |
|      | 0.06            | 0.9651| -6.06                         | 57.92    | 2.45       | 1.92       | 0.5        | 1.39 x 10^10        |
|      | 0.08            | 0.9624| -6.95                         | 53.96    | 2.2        | 1.92       | 0.52       | 1.14 x 10^5         |
|      | 0.1             | 0.9588| -8.09                         | 49.79    | 2.04       | 1.92       | 0.5        | 0.5                 |
|      | 0.11            | 0.9567| -8.76                         | 47.72    | 2          | 1.92       | 0.55       | 0.001               |
| 0.1  | 0.097           | 0.9565| -5.53                         | 60       | 1.89       | 1.81       | 0.49       | 9.2 x 10^4          |
|      | 0.1             | 0.9651| -5.68                         | 59.14    | 1.87       | 1.81       | 0.5        | 7.13 x 10^11        |
|      | 0.12            | 0.9607| -6.79                         | 53.58    | 1.79       | 1.82       | 0.52       | 5 x 10^4            |
|      | 0.135           | 0.9569| -7.77                         | 49.7     | 1.74       | 1.82       | 0.5        | 0.52                |
|      | 0.143           | 0.9547| -8.33                         | 47.76    | 1.72       | 1.82       | 0.55       | 0.0017              |
| 0.05 | 0.172           | 0.9581| -4.53                         | 59.5     | 1.35       | 1.53       | 0.47       | 4.8 x 10^12         |
|      | 0.19            | 0.9517| -5.36                         | 53.48    | 1.34       | 1.53       | 0.49       | 8.7 x 10^4          |
|      | 0.2             | 0.9479| -5.86                         | 50.51    | 1.33       | 1.53       | 0.5        | 13.6               |
|      | 0.205           | 0.9459| -6.12                         | 49.12    | 1.33       | 1.53       | 0.5        | 0.225              |
|      | 0.21            | 0.9438| -6.39                         | 47.78    | 1.33       | 1.53       | 0.51       | 0.0044              |

Table 1: Numerical Results for different values of $r$ and $M_{Pl} \alpha$. For all cases $A_s^{1/2} = 4.686 \times 10^{-5}$.
where the odd fields $V_i$ are set to zero. $\Sigma_1$ is the $Z_2$ even $5^{th}$ component of the 5D $U(1)$ vector supermultiplet. With the parity assignments

$$Z_2 : \quad H \rightarrow H, \quad H^c \rightarrow -H^c,$$

(22)

the KK decomposition for $H$ and $H^c$ superfields is given by

$$H = \frac{1}{2\sqrt{\pi R}} H^{(0)} + \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{+\infty} H^{(n)} \cos \frac{ny}{R}, \quad H^c = \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{+\infty} \overline{H}^{(n)} \sin \frac{ny}{R}.$$  

(23)

With these decompositions, and steps given in Appendix A, we can calculate the mass spectrum of KK states, their couplings to the inflaton and with these, the one loop order inflation potential (dropping higher winding modes) having the form of (1) with

$$\alpha = \pi g_4 R, \quad V_0 = \frac{3}{16\pi^2 R^3} B \quad \text{and} \quad B = 1 - \cos(\pi R |F_T|).$$

(24)

The 4D inflaton field $\phi$ is related to the 5D $U(1)$ gauge field $A^1_5$ as:

$$\phi = \sqrt{2\pi R} A^1_5.$$  

(25)

Since the model is well defined, we also can write down the inflaton coupling with the components of $H$. The latter, having a coupling with the SM fields, would insure the inflaton decay and the reheating of the Universe. In our setup, we assume that all MSSM matter and scalar superfields are introduced at the $y = 0$ brane. Since $H$ is even under orbifold parity and a singlet under all SM gauge symmetries, it can couple to the MSSM states through the following brane superpotential couplings

$$\mathcal{L}_{H-br} = \sqrt{2\pi R} \int d^2\theta dy \delta(y) \lambda l h_u H + \text{h.c.}$$  

(26)

where $l$ and $h_u$ are 4D $N = 1$ SUSY superfields corresponding to lepton doublets and up type higgs doublet superfields respectively. In Eq. (26), without loss of generality, only one lepton doublet (out of three lepton families) is taken to couple with the $H$,

$$\mathcal{L}_{H-br} \supset -\lambda \left( \frac{1}{\sqrt{2}} \psi_H^{(0)} + \sum_{n=1}^{+\infty} \psi_H^{(n)} (lh_u + \bar{l}h_u) \right) + \left( \frac{1}{\sqrt{2}} H^{(0)} + \sum_{n=1}^{+\infty} H^{(n)} \right) \bar{l}h_u$$

$$- \left( \frac{1}{\sqrt{2}} F_H^{(0)} + \sum_{n=1}^{+\infty} F_H^{(n)} \right) \bar{l}h_u + \text{h.c.}$$

(27)

where $l$ now denotes the fermionic lepton doublet and $h_u$ an up-type higgs doublet. States $\bar{l}$ and $\bar{h}_u$ stand for their superpartners respectively. $H^{(n)}$ and $\psi_H^{(n)}$ in Eq. (27) indicate scalar and fermionic components of the superfield $H$.\(^{12}\)

---

\(^{11}\)The bulk hypermultiplet action of Eq. (21), derived from 5D off shell SUGRA construction [18], including coupling with a radion superfield $T$, in a rigid SUSY limit coincides with the one given in Ref. [27].

\(^{12}\)In Eq. (27) we have omitted $HF_l h_u$ and $H\bar{F}\bar{l}h_u$ type terms, which because of the smallness of the $\mu$ term ($\sim$ few $\times$ TeV) and suppressed lepton Yukawa couplings ($\lesssim 10^{-2}$) can be safely ignored in the inflaton decay process.
Upon eliminating all $F$-terms and heavy fermionic and scalar states (in the $H$ and $H^c$ superfields), we can derive effective operators containing the inflaton linearly. As it will turn out within the model considered (see discussion in Appendix A.1), the $\tilde{l}$ states are heavier than the inflaton and operators containing $\tilde{l}$ are irrelevant for the inflaton decay. Thus, the effective operators, needed to be considered, are

$$\phi_\Theta \left( C_0 (l \tilde{h}_u)^2 + C_1 (l \tilde{h}_u)^2 + \text{h.c.} \right) + C_2 \phi_\Theta (l \tilde{h}_u)(l \tilde{h}_u). \tag{28}$$

These terms should be responsible for the inflaton decay. Derivation and form of the $C$-coefficients are given in Appendix A.

### 3.1 Inflaton Decay and Reheating

As was mentioned above and shown in Appendix A.1, the slepton states $\tilde{l}$ have masses $\frac{1}{2} |F_T| \sim 1/(2R)$ and thus are heavier than the inflaton. Indeed, the latter’s mass, obtained from the potential, is:

$$M_{\phi_\Theta} = \frac{g_4 \sqrt{3} (1 - \cos(\pi R |F_T|))^{1/2}}{4\pi^2 R} \ll \frac{1}{R}. \tag{29}$$

($g_4 \ll 1$ for successful inflation). Thus the inflaton decay in channels containing $\tilde{l}$ is kinematically forbidden. Therefore, among operators generated via exchange of heavy fermionic $\chi_i^{(n)}$ and scalar $S_i^{(n)}$ states, only those given in Eq. (28) are relevant. For calculating the decay widths (in a pretty good approximation) it is enough to have the form of the $C_i$ coefficients.

As shown in Appendix A, within our model $C_2 = 0$ and the corresponding operator does not play any role. Moreover, according to Eqs. (A.26) and (A.30) we have $C_0 \sim R^2$ and $C_1 \sim R^3$ (with $|F_T| \sim 1/R$, dictated from the inflation). Thus, we get an estimate for the following branching ratio

$$\frac{\Gamma(\phi_\Theta \to ll\tilde{h}_u\tilde{h}_u)}{\Gamma(\phi_\Theta \to llh_uh_u)} \sim \frac{|C_1|^2 M_{\phi_\Theta}^5}{|C_0|^2 M_{\phi_\Theta}^3} \sim \frac{(RM_{\phi_\Theta})^2}{16\pi^4} \ll 1. \tag{30}$$

This means that the inflaton decay is mainly due to the $C_0$ operator, via the channel $\phi_\Theta \to llh_uh_u$ (the diagram in Fig. 3. Remember: $l$ denotes the SM lepton doublet and $h_u$ the scalar up type.

Figure 3: Diagram responsible for the inflaton’s dominant decay.
higgs doublet). For simplicity we assume that the state \( h_u \) includes the light SM higgs doublet \( h \) with weight nearly equal to one, i.e. \( h_u \gtrsim h \).

For the decay width we get:\(^{13}\)

\[
\Gamma(\phi_\Theta) \simeq \Gamma(\phi_\Theta \rightarrow llh_u h_u) = \frac{9}{9 \cdot 2(2\pi)^5} |C_0|^2 M_{\phi_{\Theta}}^5. \tag{31}
\]

The factor 9 in the numerator accounts for the multiplicity of final states. (The final \( llh_u h_u \) channel includes three combinations \( e^-e^-h^+h^+, \nu\nu h^0 h^0, \nu \bar{\nu} h^+ h^0 \) and for each pair of identical final states a factor 2 should be included.) The denominator factors in (31) come from the phase space integration. Using the form of \( C_0 \), given by Eq. (A.26), in expression (31), we get:

\[
\Gamma(\phi_\Theta) \simeq \frac{g_4^2|\lambda|^4}{2^{18\pi}} (RM_{\phi_{\Theta}})^4 M_{\phi_{\Theta}}. \tag{32}
\]

Expressing \( \rho_{reh} = \frac{\pi^2}{30} g_* T_r^4 \) through the reheat temperature [28]

\[
T_r = \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{M_{Pl} \Gamma(\phi_\Theta)} \tag{33}
\]

\( (g_\ast \) is the number of relativistic degrees at temperature \( T_r \) and using expressions (32) and (29), we get

\[
\rho_{reh}^{1/4} = 1.316 \left( M_{Pl} \Gamma(\phi_\Theta) \right)^{1/2} = 5.85 \cdot 10^{-7} M_{Pl} g_4^{7/2}|\lambda|^2 (RM_{\phi_{\Theta}})^{1/2} (1 - \cos(\pi R|F_T|))^{5/4}. \tag{34}
\]

From this, with \( RM_{Pl} \sim 10, \) \( R|F_T| \sim 1 \) and \( g_4 \sim 1.5 \cdot 10^{-3} \) we obtain \( \rho_{reh}^{1/4} \sim |\lambda|^2 \times 100 \) GeV.

Our 5D SUGRA construction allows more accurate estimates, because some of the parameters are related to each other. For instance, from (24) we have

\[
R \simeq \frac{0.118}{\sqrt{V_0^{1/4}} (1 - \cos(\pi R|F_T|))^{1/4} }, \tag{35}
\]

\[
g_4 = \frac{\alpha}{\pi R}. \tag{36}
\]

From (35) we see that in order to have \( RM_{Pl} \gtrsim 10 \) we need \( V_0^{1/4} \lesssim 3.4 \cdot 10^{16} \) GeV.\(^{14}\) The latter value suites well with most of the values of \( V_0^{1/4} \) given in Table 1 (calculated from the inflation potential). At the same time, we see from (35) that \( |F_T| \) can not be suppressed and should be \( |F_T| \sim 1/R \). Using Eqs. (35) and (36) in (34), we obtain

\[
\rho_{reh}^{1/4} = 5.45 \cdot 10^{-5} M_{Pl} (\alpha M_{Pl})^{7/2} \left( \frac{V_0^{1/4}}{M_{Pl}} \right)^4 |\lambda|^2 (1 - \cos(\pi R|F_T|))^{1/4}. \tag{37}
\]

This expression is useful to find the maximal value of \( \rho_{reh}^{1/4} \). Using the pairs of \( (\alpha M_{Pl}, V_0) \) given in Table 1, from Eq. (37) it turns out that \( \rho_{reh}^{1/4} \lesssim |\lambda|^2 \times 619 \) GeV. This is an upper bound on the

\(^{13}\)For 4-body phase space we have used an expression of [26] derived for the \( K \rightarrow \pi\pi ee \) decay, setting \( m_\pi, m_e \to 0 \) and replacing \( m_K \sim M_{\phi_{\Theta}}. \)

\(^{14}\)For adequate suppression of undesirable non local operators the large volume \( R \gtrsim 10/M_{Pl} \) is needed [6].
reheating energy density obtained within our 5D SUGRA scenario. In Table 2 we give the values of \( RM_{Pl}, g_4 \) and \( \rho_{reh}^{1/4} \) for different cases. Input values of \( V_0^{1/4} \) and \( M_{Pl} \alpha \) were taken from Table 1, which correspond to successful inflation. Also, we have selected the values of \( R|F_T| \) in such a way as to get \( RM_{Pl} \gtrsim 10 \). We see that within 2\( \sigma \) deviations of \( r \) and \( n_s \) we have \( \rho_{reh}^{1/4} \lesssim |\lambda|^2 \times 386 \) GeV, while a reduced value \( r = 0.05 \) gives \( \rho_{reh}^{1/4} \lesssim |\lambda|^2 \times 619 \) GeV. These correspond to reheat temperatures \( T_r \lesssim |\lambda|^2 \times 130 \) GeV and \( T_r \lesssim |\lambda|^2 \times 210 \) GeV respectively. These values can be easily reconciled with those low values of \( \rho_{reh}^{1/4} \), given in Table 1, by natural selection of the brane Yukawa coupling \( \lambda \) in a range \( 1/300 \lesssim \lambda \lesssim 1 \).

### Table 2: Values of \( RM_{Pl}, g_4 \) and \( \rho_{reh}^{1/4} \) for different cases of successful inflation.

| \( M_{Pl} \alpha \) | \( \frac{V_0^{1/4}}{10^{16} \text{GeV}} \) | \( R|F_T| \) | \( RM_{Pl} \) | \( g_4 \) | \( \frac{\rho_{reh}^{1/4}}{|\lambda|^2} \) (GeV) |
|-----------------|-----------------|----------|----------|----------|-----------------|
| 0.04            | 3.2             | 0.75     | 10.1     | 1.26 \times 10^{-3} | 60.5             |
|                 |                 | 0.9      | 10.5     | 1.22 \times 10^{-3} | 62.6             |
| 0.055           | 2.77            | 0.5      | 10.2     | 1.71 \times 10^{-3} | 90.6             |
|                 |                 | 0.9      | 12.1     | 1.45 \times 10^{-3} | 107              |
| 0.065           | 2.58            | 0.5      | 11       | 1.88 \times 10^{-4} | 122              |
|                 |                 | 0.9      | 13       | 1.6 \times 10^{-3}  | 145              |
| 0.1             | 2.04            | 0.5      | 13.9     | 2.29 \times 10^{-3} | 216              |
|                 |                 | 0.9      | 16.4     | 1.94 \times 10^{-3} | 255              |
| 0.135           | 1.74            | 0.5      | 16.3     | 2.64 \times 10^{-3} | 327              |
|                 |                 | 0.9      | 19.3     | 2.23 \times 10^{-3} | 386              |
| 0.2             | 1.33            | 0.5      | 21.3     | 3 \times 10^{-3}    | 442              |
|                 |                 | 0.9      | 25.2     | 2.53 \times 10^{-3} | 522              |
| 0.21            | 1.33            | 0.2      | 14.1     | 4.74 \times 10^{-3} | 346              |
|                 |                 | 0.9      | 25.2     | 2.65 \times 10^{-3} | 619              |

### 4 Discussion and Concluding Remarks

In the effective action of our 5D conformal SUGRA model the 5-th component (\( \Sigma_1 \)) of a \( U(1) \) vector supermultiplet couples to a charged hypermultiplet (\( H, H^c \)). This, due to a fixed compactification radius \( R \) leads to the potential of natural inflation for the CP odd part of \( \Sigma_1 \), neglecting the suppressed higher winding modes. We analysed this potential like in [8] putting emphasis on the potential of inflation and the number of e-folds of perturbations leaving the horizon. This we compared with the number of e-folds required by a causal connection between the observed universe background fluctuations and by the size of observed curvature perturbations. For a large tensor component \( r \), a small reheating temperature is needed for agreement. We inspected the decay of the gauge inflaton to the light MSSM fields living on a brane. These decays are mediated by the bulk hypermultiplet \( H \). The very same \( H \) hypermultiplet, together with it’s \( SU(2)_R \) partner \( H^c \), generates the inflation potential. The \( H \) is assumed to have superpotential Yukawa couplings to brane fields with a Yukawa strength \( \lambda \lesssim 1 \). This naturally led to a suppressed decay width and reheat temperature \( T_r \sim |\lambda|^2 \times 100 \) GeV. Within the considered scenario the dominant 4-body decays of the inflaton are mediated by fermionic components \( \psi_H \) (of \( H \)) with \( llhh \) final states (two lepton and two higgs doublets’ components). Other channels are either kinematically
forbidden due to heavy sleptons $\tilde{l}$ gaining large masses through the large $F_T$ term, a case of split SUSY, or are suppressed (due to the small inflaton mass $M_{\phi_0} \ll 1/R$) by an additional small factor $(RM_{\phi_0})^2$. Therefore, a similar mechanism can be realized also for extranatural inflation [6] without supersymmetry with a bulk fermionic $\psi_H$ generating the inflation potential and brane Yukawa coupling $\lambda h \psi_H$. Within our model (as shown in Appendix A), due to specific bulk couplings and degeneracy, the lepton number is conserved and neutrinos remain massless. The situation can be changed by introducing a brane Majorana mass term $\frac{1}{2}M_{\phi} HH$ and it is inviting to exploit such a possibility. Since this is not directly related to inflation, on one side, and trying to keep the calculus simple on the other side, we have not pursued this possibility in this paper and reserved it for future studies.

The model of [1], reanalyzed here in more detail, is by no means complete. A concrete mechanism for radion stabilization like in Ref. [22] has to be presented and the breaking of 4D SUSY has to be worked out in more detail. Here and in [1] we concentrated on the aspects that our model originates in a very straightforward way from 5D conformal SUGRA -which can be also interpreted as a result of M-theory [19] - and that the inflaton is related to a gauge field. If the new BICEP2 data, advertising large tensorial fluctuations, will turn out not to be mainly dust effects, then our gauge inflation is indeed a suitable and attractive candidate for inflationary model building. If further findings will discriminate the primordial origin, reveal dust effects and indicate a suppressed value of $r$, then as an alternative, the ‘modulus’ inflation of [1] should be pursued. This would mean that the inflaton is the real part of the $\Sigma_1$ chiral supermultiplet scalar component. Also, a more general two field inflation [10] from complex $\Sigma_1$ could get into focus again.

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A KK spectrum and the inflaton effective couplings

First let us discuss the emergence of the non zero $F_T$ term of the $T$ radion superfield. This can be easily understood by the effective 4D SUGRA description developed in [19]. The 4D supergravity action is given by [31]

\[
\mathcal{L}_D^{(4D)} + \mathcal{L}_F^{(4D)} \quad \text{with} \quad \mathcal{L}_D^{(4D)} = -3 \int d^4 \theta e^{-K/3} \phi^\dagger \phi, \quad \mathcal{L}_F^{(4D)} = \int d^2 \theta \phi^3 W + \frac{1}{4} \int d^2 \theta f_{IJ} W^I W^J + \text{h.c.} \quad (A.1)
\]

where $K$ and $W(\Phi)$ are the Kähler potential and the superpotential respectively, while $f_{IJ}(\Phi)$ is the gauge kinetic function. $\phi$ is the 4D compensator chiral superfield. Being a 4D effective theory, (A.1) would include zero modes of the 5D supermultiplets and the brane fields as well. Therefore, for the bulk states the form of (A.1) will be dictated by the 5D construction [19]. For instance, the 4D compensator $\phi$ is related with the 5D compensator as $\phi = \sqrt{2\pi R\kappa_5^{-1}} h^{\frac{1}{2}}$. From (A.1) we find the expressions for the F-terms:

\[
F^I = -M_P e^{K/3} \mathcal{K}^I J D_J \bar{W}, \quad F_\phi = M_P^2 e^{K/3} \left( \bar{W} - \frac{1}{3} \mathcal{K}^{IJ} \mathcal{K}_I D_J \bar{W} \right). \quad (A.2)
\]
where $I$ runs over all scalars. By plugging Eq. (A.2) back in to (A.1), one derives the $F$-term scalar potential (by setting $\phi = M_p$ and going to the 4D Einstein-frame, rescaling the metric $g_{\mu\nu} \rightarrow e^{K/3}g_{\mu\nu}$):

$$V_F = M_p^4 e^K \left( K^{ij} D_i W D_j \bar{W} - 3|W|^2 \right), \quad \text{with} \quad D_I \equiv \partial_I + K_I.$$  \hspace{1cm} (A.3)

For the $T$ modulus (the radion) the Kähler potential is $K = -3\ln(T + T^*)$. For the time being we take $W =$const. for the superpotential.  \hspace{1cm} ^{15}

With these, it is easy to check that we get a flat potential $V_F = 0$ with $F_\phi = 0$ and $F_T = M_p W^*$. Thus, we have fixed a non zero $F_T$ which plays a crucial role for the generation of the inflaton potential. This is enough for performing a calculation of the KK spectrum and the 1-loop inflaton potential. We will come back to the SUSY breaking at the end, upon discussion of the superpartners’ spectrum from the MSSM brane fields.

Any bulk state transforming non trivially under $SU(2)_R$ feels $F_T$ SUSY breaking. This happens of course with the bulk hypers described by the terms in (21). With the parametrization

$$F_T = -|F_T| e^{i\alpha}, \quad \text{with} \quad \alpha = \text{Arg}(F_T) + \pi,$$  \hspace{1cm} (A.4)

setting the scalar component of $T$ to one, and making a phase redefinition of the scalar components $H, H^c$:

$$H \rightarrow e^{-i(\hat{\theta} + \alpha)/2} H, \quad H^c \rightarrow e^{i(\hat{\theta} - \alpha)/2} H^c,$$  \hspace{1cm} (A.5)

the couplings in (21) give the potential:

$$V(H) = \frac{1}{2} \left( \partial_5 H - \frac{g_1}{2}(M^1 - \frac{i\Theta}{2\pi R})H^c - \frac{1}{2}|F_T| H^{c*} \right)^2 + 2 \left( \partial_5 H^c - \frac{g_1}{2}(M^1 - \frac{i\Theta}{2\pi R})H + \frac{1}{2}|F_T| H^c \right)^2 + \frac{1}{2}g_1 M^1 |F_T| (H^2 - H^{c2}) + \frac{1}{2}g_1 M^1 |F_T| (H^{c2} - H^{c*2}).$$  \hspace{1cm} (A.6)

With the decomposition of Eq. (23) and integrating along the fifth dimension $\int_0^{2\pi R} dy L^{(5)}$ we obtain

$$V(H^{(n)}) = \left| g_1 \Sigma_1 H^{(0)} - \frac{1}{2}|F_T| H^{(0)*} \right|^2 + \sum_{n=1}^{+\infty} \left| n R H^{(n)} + g_1 \Sigma_1 H^{(n)*} \right|^2 + \frac{1}{4} g_1 M^1 |F_T| \left( \sum_{n=0}^{+\infty} (H^{(n)})^2 - \sum_{n=1}^{+\infty} (\overline{H}^{(n)})^2 + \text{h.c.} \right)^2$$

$$\sum_{n=1}^{+\infty} \left| \frac{n}{R} \overline{H}^{(n)} - g_1 \Sigma_1 H^{(n)} + \frac{1}{2}|F_T| H^{(n)*} \right|^2 + \frac{1}{4} g_1 M^1 |F_T| \left( \sum_{n=0}^{+\infty} (H^{(n)})^2 - \sum_{n=1}^{+\infty} (\overline{H}^{(n)})^2 + \text{h.c.} \right)$$

$$H^{(0)} = \frac{1}{\sqrt{2}} (S_1^{(0)} + i S_2^{(0)})$$

for $n = 1, \ldots, +\infty$, \hspace{1cm} (A.7)

$$\begin{pmatrix} \text{Re} H^{(n)} \\ \text{Im} H^{(n)} \\ \text{Re} \overline{H}^{(n)} \\ \text{Im} \overline{H}^{(n)} \end{pmatrix} = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} S_1^{(n)} \\ S_2^{(n)} \\ S_3^{(n)} \\ S_4^{(n)} \end{pmatrix},$$  \hspace{1cm} (A.8)

\hspace{1cm} ^{15}\text{As shown in [1], this system (at zero mode level and for the purpose of discussing SUSY breaking) is equivalent to 5D SUGRA with gauged $U(1)_R$ and with suitable couplings of a linear supermultiplet.}
(where \( S_i^{(n)} \) are real scalars) and the potential mass terms will get diagonal and canonical forms:

\[
V(S^{(n)}) = \frac{1}{2} (m_1^{(0)})^2 (S_1^{(0)})^2 + \frac{1}{2} (m_2^{(0)})^2 (S_2^{(0)})^2 + \frac{1}{2} \sum_{i=1}^{4} \sum_{n=1}^{\infty} (m_i^{(n)})^2 (S_i^{(n)})^2 ,
\]

(A.9)

with

\[
(m_1^{(0)})^2 = \frac{1}{4} (g_1 A_5^1 + |F_T|)^2 + \frac{1}{4} (g_1 M^1)^2, \quad (m_2^{(0)})^2 = \frac{1}{4} (g_1 A_5^1 - |F_T|)^2 + \frac{1}{4} (g_1 M^1)^2 ,
\]

for \( n = 1, \ldots, +\infty \):

\[
(m_1^{(n)})^2 = \frac{1}{4} \left( \frac{2n}{R} + g_1 A_5^1 \pm |F_T| \right)^2 + \frac{1}{4} (g_1 M^1)^2 ,
\]

\[
(m_2^{(n)})^2 = \frac{1}{4} \left( \frac{2n}{R} - g_1 A_5^1 \mp |F_T| \right)^2 + \frac{1}{4} (g_1 M^1)^2 .
\]

(A.10)

As for the spectrum of the fermionic components of the \( H, H^c \) superfields, with the phase redefinition

\[
\psi_H \rightarrow e^{-i\hat{\theta}_1/2} \psi_H , \quad \psi_{H^c} \rightarrow e^{i\hat{\theta}_1/2} \psi_{H^c} ,
\]

(A.11)

from Eq. (21) we get the couplings

\[
\mathcal{L}^{(5)}_{\psi} \supset -2\psi_{H^c} \partial_\tau \psi_H - \frac{1}{2} g_1 (M^1 - iA_5^1) (\psi_H \psi_H - \psi_{H^c} \psi_{H^c}) + \text{h.c.} \quad (A.12)
\]

With the mode expansion of Eq. (23) and integration over the fifth dimension \( \int_0^{2\pi R} dy \mathcal{L}^{(5)}_{\psi} = \mathcal{L}^{(4)}_{\psi} \), from (A.12) we get terms

\[
\mathcal{L}^{(4)}_{\psi} \supset -\sum_{n=1}^{\infty} \frac{n}{R} \psi_H^{(n)} \psi_H^{(n)} - \frac{1}{4} g_1 (M^1 - iA_5^1) \left( \sum_{n=0}^{\infty} \psi_H^{(n)} \psi_H^{(n)} - \sum_{n=1}^{\infty} \psi_{H^c}^{(n)} \psi_{H^c}^{(n)} \right) + \text{h.c.} \quad (A.13)
\]

Now, with the substitution

\[
\psi_H^{(0)} = e^{i\omega_0} \chi_1^{(0)} , \quad \text{with} \quad \omega_0 = -\frac{1}{2} \text{Arg}(M^1 - iA_5^1) ,
\]

for \( n = 1, \ldots, +\infty \):

\[
\psi_H^{(n)} = \frac{1}{\sqrt{2}} (e^{i\omega_n} \chi_1^{(n)} + e^{-i\omega_n} \chi_2^{(n)}) , \quad \psi_{H^c}^{(n)} = \frac{i}{\sqrt{2}} (e^{i\omega_n} \chi_1^{(n)} + e^{-i\omega_n} \chi_2^{(n)}) ,
\]

with \( \omega_n = -\frac{1}{2} \text{Arg} \left( g_1 M^1 - i(g_1 A_5^1 + \frac{2n}{R}) \right) , \quad \bar{\omega}_n = -\frac{1}{2} \text{Arg} \left( g_1 M^1 - i(g_1 A_5^1 - \frac{2n}{R}) \right) \),

(A.14)

from Eq. (A.13) we will get diagonal and canonically normalized mass terms:

\[
\mathcal{L}^{(4)}_{\psi} \supset -\frac{1}{2} \sum_{n=0}^{\infty} m_{\chi_1}^{(n)} \chi_1^{(n)} - \frac{1}{2} \sum_{n=1}^{\infty} m_{\chi_2}^{(n)} \chi_2^{(n)} + \text{h.c.} \quad (A.15)
\]

with

\[
m_{\chi_1}^{(n)} = \frac{1}{2} \left| g_1 M^1 - i(g_1 A_5^1 + \frac{2n}{R}) \right| , \quad m_{\chi_2}^{(n)} = \frac{1}{2} \left| g_1 M^1 - i(g_1 A_5^1 - \frac{2n}{R}) \right| .
\]

(A.16)
With this spectrum, integrating out the corresponding KK states (including zero modes) leads to the 1-loop effective potential [1], [9]:

$$V_{\text{eff}}(\phi_\Theta) = \frac{3}{16\pi^6 R^4} \sum_{k=1}^{\infty} \frac{1}{k^5} (1 - \cos(\pi k R |F_T|)) \cdot \cos(\pi k g_4 R \phi_\Theta) \times$$

$$e^{-\pi k g_4 R |\phi_M|} \left( 1 + \pi k g_4 R |\phi_M| + \frac{1}{3} (\pi k g_4 R |\phi_M|)^2 \right),$$

(A.17)

written in terms of canonically normalized 4D scalar fields $\phi_\Theta = \sqrt{2\pi R} A_5^1$, $\phi_M = \sqrt{2\pi R} M^1$ and dimensionless 4D gauge coupling $g_4 = g_1/\sqrt{2\pi R}$. In (A.17) summation is performed with $k$ winding modes. The dominant contribution comes from $k = 1$ [29]. With this leading term, the minimum of the potential is achieved for $g_4 R \langle \phi_\Theta \rangle = g_1 R \langle A_5^1 \rangle = 1$ and $\langle \phi_M \rangle = 0$. Further, we assume that the modulus $\phi_M$ (i.e. $M^1$) is sitting in its minimum and study only the motion of $\phi_\Theta$’s quantum part as the inflaton. We add to the potential (A.17) a constant term in such a way as to set the ground state vacuum energy to be zero (usual fine tuning of 4D cosmological constant). With these, the inflaton potential (part with $k = 1$) gets the form of Eq. (1) with the parametrization given in Eq. (24).

Further, we work out the effective couplings of the inflaton with the MSSM states. For this purpose, in couplings (A.9), (A.16) (and in any relevant term) we make the substitution

$$g_1 A_5^1 \to g_1 \langle A_5^1 \rangle + g_1 A_5^1 = \frac{1}{R} + g_4 \phi_\Theta$$

(A.18)

and put $\langle M^1 \rangle = 0$. With this, we obtain the linear couplings of the inflaton with the heavy $S_i$ states:

$$\mathcal{L}_{\phi_\Theta SS} = -\frac{1}{2} g_4 \phi_\Theta \left( \sum_{n=0}^{+\infty} m_1^{(n)} (S_1^{(n)})^2 + m_2^{(n)} (S_2^{(n)})^2 - \sum_{n=1}^{+\infty} m_3^{(n)} (S_3^{(n)})^2 + m_4^{(n)} (S_4^{(n)})^2 \right),$$

with $m_1^{(n)} = \frac{1}{2} \left( 2n + 1 \pm |F_T| \right)$, $m_2^{(n)} = \frac{1}{2} \left( 2n + 1 \pm |F_T| \right)$.

(A.19)

At the same time, with (A.18) from (A.14) we have $\omega_n = -\pi_n = \pi/4$, and Eq. (A.15) gives inflaton couplings with heavy $\chi_i$ states:

$$\mathcal{L}_{\phi_\Theta \chi \chi} = \frac{1}{4} g_4 \phi_\Theta \left( \sum_{n=0}^{+\infty} \chi_1^{(n)} \chi_1^{(n)} - \sum_{n=1}^{+\infty} \chi_2^{(n)} \chi_2^{(n)} \right) + \text{h.c.}$$

(A.20)

Furthermore, we derive couplings of $S_i$ and $\chi_i$ states with the corresponding components of the brane superfields $l, h_u$. As shown in Appendix A.1, the $\tilde{l}$ states are heavy. Because of this, they will not be relevant for the inflaton decay and we will omit any term containing the $\tilde{l}$. From the part of Eq. (27) involving $\psi_H$ states we obtain

$$\mathcal{L}_{\chi_i h_u} \supset -\frac{\lambda_i \langle \pi - 2\theta_1 \rangle / 4}{\sqrt{2}} \left( \sum_{n=0}^{+\infty} \chi_1^{(n)} \chi_1^{(n)} - i \sum_{n=1}^{+\infty} \chi_2^{(n)} \chi_2^{(n)} \right) l h_u + \text{h.c.}$$

(A.21)
On the other hand, making (A.5) phase redefinitions, the part of Eq. (27) involving $H$ gives:

$$\mathcal{L}_{H\tilde{h}_u} \supset -\frac{\lambda e^{-i(\bar{\theta}_1 + \alpha)/2}}{\sqrt{2}} \left( H^{(0)} + \sqrt{2} \sum_{n=1}^{+\infty} H^{(n)} \right) \bar{h}_u + \text{h.c.} \quad (A.22)$$

From Eq. (A.22) we get $S_i\bar{h}_u$ type couplings:

$$\mathcal{L}_{S_i\bar{h}_u} \supset -\frac{\lambda}{2} e^{-i(\bar{\theta}_1 + \alpha)/2} \left( S_1^{(0)} + iS_2^{(0)} + e^{-\pi/4} \sum_{n=1}^{+\infty} (S_1^{(n)} + iS_2^{(n)} - S_3^{(n)} - iS_4^{(n)}) \right) \bar{h}_u + \text{h.c.} \quad (A.23)$$

Now, we integrate out the heavy $\chi_i$ and $S_i$ states, in order to obtain effective operators. Starting with the integration of the fermionic modes, at relatively low energies, we can ignore kinetic terms for the $\chi_i$ states. With this, via equations of motions $\frac{\delta \mathcal{L}}{\delta \chi_i^{(n)}} = \frac{\delta \mathcal{L}}{\delta \chi_{\bar{c}}^{(n)}} = 0$, we can solve $\chi_{1,2}^{(n)}$ and plug them back into the Lagrangian. Doing so [using couplings of Eqs. (A.15), (A.20) and (A.21)] and keeping terms up to the first power of $\phi_\Theta$, we obtain:

$$\frac{i\lambda^2}{4} e^{-i\bar{\theta}_1} \left( \sum_{n=0}^{+\infty} \left( \frac{1}{m^{(n)}_{\chi_1}} - \frac{1}{m^{(n+1)}_{\chi_2}} \right) + \frac{1}{2} g_4 \phi_\Theta \sum_{n=0}^{+\infty} \left( \frac{1}{(m^{(n)}_{\chi_1})^2} + \frac{1}{(m^{(n+1)}_{\chi_2})^2} \right) \right) (h_u)^2 + \text{h.c.} \quad (A.24)$$

With $\langle M^1 \rangle = 0$, $g_1\langle A_1^1 \rangle = 1/R$, from (A.16) we have $m_{\chi_1}^{(n)} = m_{\chi_2}^{(n+1)} = (2n + 1)/(2R)$. Using this in (A.24), we see that the first sum-term (coefficient in front of $d = 5$ operator) cancels out, i.e. no $d = 5$ lepton number violating operator emerges. This is understandable, because the whole theory has $U(1)$ gauge symmetry and the lepton number is a residual global symmetry (with $\langle M^1 \rangle = 0$) at $d = 5$ level. \(^{16}\) Thus, from Eq. (A.25) we obtain

$$\mathcal{L}_{d=6}^{(\chi)} = \frac{i\lambda^2}{4} e^{-i\bar{\theta}_1} \left( \sum_{n=0}^{+\infty} \frac{1}{(m^{(n)}_{\chi_1})^2} \right) g_4 \phi_\Theta (h_u)^2 + \text{h.c.} \quad (A.25)$$

where subscript $(\chi)$ indicates that this $d = 6$ operator is obtained through the integration of the heavy $\chi_i$ states. The sum in (A.25) is well convergent because $\sum_{n=0}^{+\infty} \frac{1}{(2n+1)^2} = \frac{\pi^2}{8}$. It turns out that a $\phi_\Theta (h_u)^2$ type operator emerges only via integration of the $\chi_i$ states. Taking into account these, comparing Eq. (A.25) and (28) we have

$$C_0 = \frac{1}{8} g_4 i\lambda^2 e^{-i\bar{\theta}_1} (\pi R)^2. \quad (A.26)$$

Next, by integrating out heavy $S_i$ states, the $\phi_\Theta (\bar{h}_u)^2$ and $\phi_\Theta (\bar{h}_u)(\bar{\tilde{h}}_u)$ type dimension 7 operators will be

$$C_1 \phi_\Theta (\bar{h}_u)^2 + \text{h.c.} + C_2 \phi_\Theta (\bar{h}_u)(\bar{\tilde{h}}_u),$$

with $C_1 = \frac{1}{8} g_4 i\lambda^2 e^{-i(\bar{\theta}_1 + \alpha)} \left( -\sum_{n=0}^{+\infty} \frac{e^{-i\pi/2}(1-\delta_{0n})}{(m_1^{(n)})^3} + \sum_{n=0}^{+\infty} \frac{e^{-i\pi/2}(1-\delta_{0n})}{(m_2^{(n)})^3} - i \sum_{n=1}^{+\infty} \frac{1}{(m_3^{(n)})^3} + i \sum_{n=1}^{+\infty} \frac{1}{(m_4^{(n)})^3} \right)$.

\(^{16}\) Different result would emerge if we have had included brane $H^2$ coupling which explicitly violates the lepton number. We do not consider such terms for the sake of simplicity.
Taking first derivatives on both sides, we get

\[ C_2 = \frac{1}{4} g_4 \lambda^2 \left( -\sum_{n=0}^{\infty} \frac{1}{(m_1^{(n)})^3} - \sum_{n=0}^{\infty} \frac{1}{(m_2^{(n)})^3} + \sum_{n=1}^{\infty} \frac{1}{(m_3^{(n)})^3} + \sum_{n=1}^{\infty} \frac{1}{(m_4^{(n)})^3} \right). \]  \hfill (A.27)

Taking into account \( m_3^{(n+1)} = m_2^{(n)} \) and \( m_4^{(n+1)} = m_1^{(n)} \), we see that the sums in \( C_2 \) precisely cancel out, while \( C_1(\sim R^3) \) remains non-zero. From the identity \[ \sec^2 \frac{\pi x}{2} = \frac{4}{\pi^2} \sum_{n=0}^{\infty} \left( \frac{1}{(2n+1+x)^2} + \frac{1}{(2n+1-x)^2} \right), \]  \hfill (A.28)

taking first derivatives on both sides, we get

\[ \sum_{n=0}^{\infty} \left( \frac{1}{(2n+1-x)^3} - \frac{1}{(2n+1+x)^3} \right) = \frac{\pi^2}{8} \left( 1 + \tan^2 \frac{\pi x}{2} \right) \tan \frac{\pi x}{2}. \]  \hfill (A.29)

Using Eq. (A.29), from (A.27) we finally obtain:

\[ C_1 = \frac{1}{4} g_4 \lambda^2 e^{-i(\theta_1 + \alpha)} \left( 8 \sqrt{2} e^{i\pi R^4 |F_T|} (1 + (R|F_T|)^2 (1 - (R|F_T|)^2) \tan \frac{\pi R|F_T|}{2} (1 + \tan^2 \frac{\pi R|F_T|}{2}) ) \right), \]  \hfill (A.30)

\[ C_2 = 0. \]

While the \( C_2 \) is precisely zero, the \( C_1 \) vanishes in the \( F_T \rightarrow 0 \) limit. However, with \( |F_T| \sim 1/R \), we have \( C_1 \sim R^3 \).

Remaining operators, as discussed in Sec. 3.1, will not have any relevance for the inflaton decay and we will not present them here.

**A.1 SUSY breaking on a brane**

We assume that all MSSM states, that are matter \{\( f \)\}, gauge \{\( V \)\} and higgs \( h_u, h_d \) superfields, live on a 4D brane. Matter superfields can be included in the \( \text{Kähler} \) potential as follows

\[ \mathcal{K} = -\ln(T + T^\dagger)^3 - \ln \left( 1 - \frac{2}{M_P^2} f^\dagger e^{-V} f \right) + \mathcal{K}(h_u) + \mathcal{K}(h_d), \]  \hfill (A.31)

where \( \mathcal{K}(h_{u,d}) \) account for part of the higgs superfields and will be specified below. With (A.31), from Eq. (A.3), for squark and slepton masses we get

\[ M_f^2 |\tilde{f}|^2, \quad \text{with} \quad M_f^2 = \frac{1}{4} M_P^2 |W|^2 = \frac{1}{4} |F_T|^2. \]  \hfill (A.32)

Due to the brane superpotential coupling of \( l, h_u \) with \( H \) state, there will be also a loop induced contribution to the soft mass\(^2\), which we do not display here. Thus, with the large \( F_T \)-term all squark and sleptons are heavier than the inflaton field and they play no role for the inflaton decay. On the other hand we need to keep at least one Higgs doublet to be light. Since the SUSY breaking scale is very high, this can be achieved only by price of fine tuning: assuming for instance that the light Higgs mainly resides in \( h_u \), and selecting its \( \text{Kähler} \) potential as

\[ \mathcal{K}(h_u) = (1 + \alpha (T + T^\dagger)^3) \frac{2 h_u^\dagger e^{-V} h_u}{(1 + 8\alpha) M_P^2}. \]  \hfill (A.33)
Note, that with this selection, the kinetic term for $h_u$ is canonically normalized for arbitrary values of $\alpha$. For the soft mass$^2$ of $h_u$ we obtain

$$M_{h_u}^2 = \frac{27 - (16\alpha + 5)^2}{8(1 + 8\alpha)^2} |F_T|^2.$$  \hspace{1cm} (A.34)

With the selection $16\alpha + 5 = \sqrt{27} - \mathcal{O}(\frac{M_W^2}{F_T^2})$, we obtain $M_{h_u} \sim \mathcal{O}(100 \text{ GeV})$ - the needed value.

As far as the gaugino masses are concerned, since the MSSM gauge supermultiplets are introduced on a brane they will not have direct couplings neither with the $T$ modulus nor with the compensator. By selecting, in Eq. (A.1), the gauge kinetic function $f_{IJ} = \delta_{IJ}$, the corresponding gauginos will remain light. By the same token, the higgsino mass - the $\mu$ parameter, can be around the TeV scale. Therefore, the lightest neutralino can be a dark matter candidate. This is the split SUSY scenario, which, as was shown [32], can have various remarkable phenomenological features and interesting implications.

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    (In Eq. (A16), of this paper, the second term $-(3-5C)\epsilon_H^2$ should be replaced by $-(5-3C)\epsilon_H^2$. Then, the net coefficient in front of $\epsilon_H^2$ will be correct.)

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