Instabilities, solitons and rogue waves in $\mathcal{PT}$-coupled nonlinear waveguides

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Abstract
We considered the modulational instability of continuous-wave backgrounds, and the related generation and evolution of deterministic rogue waves in the recently introduced parity–time ($\mathcal{PT}$)-symmetric system of linearly coupled nonlinear Schrödinger equations, which describes a Kerr-nonlinear optical coupler with mutually balanced gain and loss in its cores. Besides the linear coupling, the overlapping cores are coupled through the cross-phase-modulation term too. While the rogue waves, built according to the pattern of the Peregrine soliton, are (quite naturally) unstable, we demonstrate that the focusing cross-phase-modulation interaction results in their partial stabilization. For $\mathcal{PT}$-symmetric and antisymmetric bright solitons, the stability region is found too, in an exact analytical form, and verified by means of direct simulations.

Keywords: parity–time symmetry, instabilities, rogue wave, nonlinear Schrödinger equation

(Some figures may appear in colour only in the online journal)

1. Introduction

It is a generally recognized fact that, independently of the underlying physics, an instability of the background is a prerequisite for the emergence of regular or random rogue waves (see, e.g., the discussion in [1]). In its turn, the instability is determined, on the one hand, by the interplay between the dispersion and the nonlinearity, and, on the other hand, by the competition between losses and gain, if an open system is considered. In this latter case, one can speak about dissipative rogue waves [2], which are identified by an enhanced probability of generating high-amplitude pulses.

In addition to the above-mentioned generic situations, there exist special dissipative systems obeying the so-called parity–time ($\mathcal{PT}$) symmetry, i.e., featuring spatially separated and exactly balanced gain and loss. These systems are described by non-Hermitian Hamiltonians, which may have purely real spectra of eigenvalues, provided that the strength of the anti-Hermitian part of the Hamiltonian (which accounts for the balanced gain and loss) does not exceed a certain critical value [3, 4].

Optics represents a unifying framework for a variety of wave phenomena. In particular, the $\mathcal{PT}$-symmetry was experimentally implemented in coupled optical waveguides [5]. Moreover principles of its implementation in plasmonic waveguides [6] and in gaseous mixtures of resonant atoms [7] were recently proposed. On the other hand, optical rogue waves have also been observed in some settings [8, 9] and predicted in others, such as periodic arrays of waveguides [10]. While the original ideas of the use of the $\mathcal{PT}$ symmetry in quantum mechanics imply complex potentials obeying the condition $V(x) = V(-\bar{x})$ [4] (hereafter the overbar stands for complex conjugation), in the experimental realization [5] and numerous theoretical studies nonlinear dual-core waveguides (couplers), with one core carrying the gain and the other one being lossy, were
explored as optical implementations of the \( PT \)-symmetric systems. The dual-core systems are described by systems of coupled nonlinear Schrödinger equations (NLSEs), one with the gain and the other with the loss. These models and their generalizations in the form of sequences of couplers give rise to bright \([11–13, 15]\) and dark \([16]\) solitons, vortices \([14]\), breathers \([17]\), and describe a switch for solitons between the cores \([18]\).

As concerns optical rogue waves, there are two major directions of the work in this field. The first relates rogue events to the well-known process of supercontinuum generation \([19–23]\) in optical fibers. The soliton dynamics affects the supercontinuum generation process at a very early stage, namely, the fission of higher-order solitons \([24, 20, 25]\), which is followed by multiple interactions of solitons with dispersive waves at advanced stages \([22, 26, 27]\). In particular, the strongest-Raman-shifted solitons \([28]\) were proposed as possible candidates for rogue waves \([8]\). Crests of soliton collisions were proposed too, as alternative candidates \([29, 30]\). Recently, ‘long-lasting’ accelerating optical rogue waves with an oblong shape, resembling the shape of their oceanic counterparts, were reported \([31, 32]\). Another approach \([33–35]\) is based on solutions for Akhmediev breathers \([36]\), and, in particular, on the single-peak solution often referred to as the Peregrine soliton (or Peregrine rogue wave) \([37]\), which represents a deterministic rogue wave \([38]\) generated by the NLSE that was recently observed experimentally \([9]\). These works reveal waves that arise from the modulational instability (MI) and subsequently disappear, which is consistent with the behavior of the famous ship killers in the ocean \([39]\).

The main objective of this paper is to study rogue waves in \( PT \)-symmetric optical models based on dual-core couplers. One of our goals is to introduce an analog of the Peregrine soliton in this setting. More specifically, we are interested in how the presence of the balanced dissipation and gain, i.e., the \( PT \) symmetry, affects the MI of the background and the possibility of creation of waves localized in space and time in such systems. In this context, it is relevant to mention a number of previous studies of deterministic rogue waves carried out in the framework of the coupled NLSEs describing two-component matter waves in Bose–Einstein condensates \([40]\), multi-parametric vector solitons, and, in particular, bright–dark rogue waves \([41]\).

The rest of the paper is organized as follows. The model is introduced in section 2, which is followed by the analysis of the MI of the continuous-wave (CW) solutions in section 3, and the study of rogue-wave solutions, following the pattern of the Peregrine soliton, in section 4. Exact analytical results, verified by direct simulations, for the stability of \( PT \)-symmetric and -antisymmetric solitons in the same system are reported in section 5, and the paper is concluded by section 6.

2. The model

We consider a system of linearly coupled NLSEs for field variables \( \psi_1 \) and \( \psi_2 \),

\[
i \frac{\partial \psi_1}{\partial z} = -\frac{\partial^2 \psi_1}{\partial x^2} + (\chi_1 |\psi_1|^2 + \chi_2 |\psi_2|^2)\psi_1 + i\gamma \psi_1 - \psi_2, \quad (1)
\]

\[
i \frac{\partial \psi_2}{\partial z} = -\frac{\partial^2 \psi_2}{\partial x^2} + (\chi_1 |\psi_1|^2 + \chi_2 |\psi_2|^2)\psi_2 - i\gamma \psi_2 - \psi_1, \quad (2)
\]

which describes a set of two parallel planar waveguides, with \( z \) and \( x \) being dimensionless propagation and transverse coordinates. Accordingly, the initial-value problem corresponds to an optical beam shone into the waveguides’ input at given \( z = z_i \). Alternatively, the model describes a dual-core fiber coupler, where \( x \) plays the role of the temporal variable \([11, 12, 18, 13]\). Equations (1) and (2) are coupled nonlinearly by the cross-phase-modulation (XPM) \( \sim \chi \), and linearly by the last terms with the respective coupling constant scaled to be 1. Lastly, the constant \( \gamma > 0 \) describes the \( PT \)-balanced gain in equation (1) and dissipation in equation (2). In optics, this setting can be realized using a system of two lossy parallel-coupled waveguides, doped by gain-providing atoms, in which only one waveguide is pumped by the external source of light which supplies the gain.

Although the first core carries the gain, its linear coupling to its lossy mate makes the zero state in the system neutrally stable, allowing for propagation of linear waves. This is the well-known situation, which takes place if the gain/loss term is small enough, compared to linear coupling through which the energy is transferred from the core with gain to the lossy one, or, more specifically, when \( \gamma \leq 1 \) \([42]\). In such a situation, modes can be excited in the system by input beams but do not arise spontaneously. Below, without loss of generality, we restrict the consideration to this case, and therefore introduce a convenient parametrization,

\[
\gamma = \sin \delta, \quad 0 < \delta < \pi/2. \quad (3)
\]

Following \([11]\), we look for \( PT \)-symmetric (+) and -antisymmetric (−) solutions to equations (1) and (2) as

\[
\psi_2(x, z) = \pm e^{i\delta} \psi_1(x, z), \quad (4)
\]

with the function \( \psi_1 \) obeying the single equation

\[
i \frac{\partial \psi_1}{\partial z} = -\frac{\partial^2 \psi_1}{\partial x^2} + (\chi_1 + i\gamma |\psi_1|^2 \mp (\cos \delta) \psi_1. \quad (5)
\]

An observation particularly relevant to solutions having the form of equation (4) is that the dissipation and gain break the conventional symmetry of the coupler. This symmetry is now substituted by the following reduction: if \( (\psi_1(x, z), \psi_2(x, z)) \) is a solution of equations (1) and (2), then the pair \( (\psi_2(x, -z), \psi_1(x, -z)) \) is a solution too. This reduction corresponds to the change \( \delta \rightarrow \pi - \delta \). Therefore, below we consider the domain of variation of \( \delta \) to be \([0, \pi]\), where the values \( \delta \) and \( \pi - \delta \) correspond to the two different solutions at the same dissipation and gain. In other words, the intervals \( 0 \leq \delta \leq \pi/2 \) and \( \pi/2 \leq \delta \leq \pi \) correspond to the \( PT \)-symmetric and \( PT \)-antisymmetric solutions.
3. Modulational instability

Up to a trivial phase shift, the CW solutions to equations (1) and (2) are

\[ \psi_j^{(cw)} = \rho \exp \left[ i k x - i b z + i(-1)^j \delta/2 \right], \]

where \( k \) represents a background current, and \( b = k^2 + \rho^2 (\chi_1 + \chi) - \cos \delta \) (see, e.g., [16] for more details), i.e., the amplitudes of the fields are equal in both cores, which is natural in view of the necessity to ensure the balance between gain and loss. To study the MI of the CW states, we use the standard ansatz,

\[ \psi_j = \rho e^{i(-1)^j \delta/2} + \eta_j e^{-i(\beta - k x)} + \bar{\eta}_j e^{i(\beta - k x)} e^{ikx - ibz}, \]

\( j = 1, 2 \), with \( |\eta_j| |\bar{\eta}_j| \ll 1 \). Then, two branches \( \beta = \beta_{1,2}(k) \) of the dispersion relation for the stability eigenvalues are given by

\[ \beta_1(k) = 2k + \sqrt{k^2 + 2 \rho^2 (\chi_1 + \chi)}, \]

\[ \beta_2(k) = 2k - \sqrt{k^2 + 2 \cos \delta \left[ k^2 + 2 \cos \delta + 2 \rho^2 (\chi_1 - \chi) \right]}. \]

We aim to identify parametric domains where the background is subject to the MI. Due to the Galilean invariance of underlying equations (1) and (2), the instability is not affected by the boost \( k \). Next, we observe from equations (7) and (8) that there are three different sources of the MI. Firstly, the instability occurs at

\[ \chi_1 + \chi < 0. \]

This is the ‘standard’ (i.e., observed also for the conservative system of nonlinearly coupled NLSEs, without linear coupling) instability stemming from equation (7) due to the long-wavelength excitations; this domain of the parameters is not influenced by gain/dissipation.

Another instability domain,

\[ \cos \delta < \max \{0, \rho^2 (\chi_1 - \chi_1)\}, \]

ensues from equation (8), and linear coupling between NLSEs gives rise to the appearance of this instability domain. Nevertheless, here the presence of gain/dissipation (\( \delta \neq 0, \pi \)) makes the situation significantly different from that in the conservative system (\( \delta = 0 \) or \( \pi \)) [43], as distinct from the previous case. The largest instability growth rate, \( \nu = \max_k [\text{Im} \beta(k)] \), is

\[ \nu \equiv \text{Im} \left[ \beta(\kappa_m) \right] = \rho^2 |\chi_1 - \chi|, \]

\[ \kappa_m^2 = \rho^2 (\chi_1 - \chi) - 2 \cos \delta, \]

in the case of

\[ 2 \cos \delta < \rho^2 (\chi_1 - \chi), \]

and

\[ \nu = 2 \sqrt{\left[ \cos \delta + \rho^2 (\chi_1 - \chi) \right] \cos \delta}, \]

\[ \kappa_m^2 = 0 \]

at

\[ \rho^2 (\chi_1 - \chi) < 2 \cos \delta < 0 \quad \text{and} \quad 0 < \rho^2 (\chi_1 - \chi) < 2 \cos \delta < 2 \rho^2 (\chi_1 - \chi). \]
by a single NLS equation (1) with fast growing peaks. Obviously, such peaks can be described in the lossy waveguide to the one with the gain, accompanied by in figure 2(b) we observe a rather fast power transfer from and (d) correspond to points A, B, C and D in figure 1, respectively.

Figure 2. The evolution of field components $|\psi_1(x, z)|^2$ and $|\psi_2(x, z)|^2$ (left and right columns) of the plane-wave solution with parameters $k = 0$, $\delta = \pi/4$, $\rho = 1.604$, $\chi_1 = 0.5$, $\chi = -1$ (a), $\rho = 0.76$, $\chi_1 = -1.5$, $\chi = 1$ (b), $\rho = 0.79$, $\chi_1 = -0.5$, $\chi = 1$ (c) and $\rho = 0.98$, $\chi_1 = 0.25$, $\chi = 1$ (d). The parameters of (a), (b), (c) and (d) correspond to points A, B, C and D in figure 1, respectively.

in figure 2(b) we observe a rather fast power transfer from the lossy waveguide to the one with the gain, accompanied by fast growing peaks. Obviously, such peaks can be described by a single NLS equation (1) with $\psi_2 = 0$. The observed behavior is due to the focusing SPM, $\chi_1$, and therefore is not significantly altered even when one passes from the domain of parameters (9) (figure 2(b)) to the one defined by equation (10), as shown in figure 2(c). A significant change, i.e., the third scenario of the evolution of the MI, appears when the SPM is defocusing too (figure 2(d)). This is the case where the MI occurs only due to the imbalance of the gain and loss, resulting in nearly homogeneous growth (decay) of the field in the waveguide with gain (dissipation), respectively.

Examples of the modulational instability and stability for the CW solution with nonzero wavevector $k$ (current) are presented in figure 3. Here, we restrict our consideration to the MI with the focusing SPM, $\chi_1 < 0$, but when $\chi_1 + \chi < 0$ (point C in figure 1(b)). Thus, the evolution of the MI in this case occurs according to the same scenario as for $k = 0$, cf figures 2(c) and 3(a). At the same time, the respective MI peak is shifted in the positive direction of the $x$-axis, which coincides with the direction of the current. Meanwhile, in the domain where the CW state is predicted to be stable (above the green line in figure 1(b)—e.g., at point C’), the stability is confirmed by the numerical simulations, see figure 3(b).

4. The Peregrine soliton in the $\mathcal{PT}$-symmetric system: the case of $\chi_1 + \chi < 0$

Turning now towards studying the Peregrine soliton (rogue wave) propagating against an unstable background we start with the case (9). This readily allows one to write down the Peregrine soliton of equations (1) and (2) in the form (j = 1, 2) [40, 45]

$$\psi_j(x, z) = \rho e^{(-1)ib/2+iks-iz} \times \left[ 1 - 4 \frac{1 - 2i(\chi_1 + \chi)\rho^2 z}{1 - 2(\chi_1 + \chi)\rho^2 (x - 2kz)^2 + 4(\chi_1 + \chi)^2 \rho^4 z^2} \right].$$

(13)

Notice that, when $|z| \to \infty$, or, equivalently, $|x| \to \infty$, solution (13) merges into the background given by equation (6). Below, we separately consider two cases: the Peregrine soliton based on the background without the current ($k = 0$), and the current-based Peregrine soliton, with $k \neq 0$.

Examples of Peregrine solitons whose backgrounds (without the current, $k = 0$) correspond to points A and B in figure 1 are depicted in figure 4. The spatial evolution of $|\psi_j(x, z)|^2$ was obtained by numerical simulations of equations (1) and (2) with the initial condition corresponding to the Peregrine soliton (13) at $z = z_i = -4$ (figures 4(a) and (b)), or $z = z_i = -2$ (figures 4(c) and (d)). In the case of the defocusing SPM and focusing XPM (figures 4(a) and (b)), the central peak, corresponding to the Peregrine solution, appears at $x = z = 0$, before the MI peaks. At the same time, in the case of the focusing SPM and defocusing XPM (figures 4(a) and (d)), the appearance of the Peregrine-soliton peak at $x = z = 0$ causes further growth of the peak in the first (gain-pumped) component, and decrease in the second (lossy) core. Notice that the structure of the rogue-wave evolution in this case
Figure 4. Peregrine solitons in the $PT$-limit for $ho = 1.604, \chi_1 = 0.5, \chi = -1, \delta = \pi/4$ (a), or $\delta = 3\pi/4$ (b); $\rho = 0.76, \chi_1 = -1.5, \chi = 1, \delta = \pi/4$ (c), or $\delta = 3\pi/4$ (d). The parameters of (a) and (b) correspond to point A, while those of (c) and (d) correspond to point B in figure 1.

In the case when the background carries the current ($k \neq 0$), the central peak of the Peregrine soliton moves with group velocity $2k$ in the positive direction of the $x$-axis, as seen in equation (13) and confirmed by figures 5(a) and (b). Also, for the focusing XPM ($\chi < 0$) case, the $PT$-symmetric ($\delta < \pi/2$) rogue wave is more ‘stable’ (in the sense that the MI peaks appear at a longer propagation distance after the principal rogue-wave peak), see figure 5(a), if compared to the $PT$-antisymmetric wave with $\delta > \pi/2$, see figure 5(b).

In order to describe this rogue wave ‘stability’ quantitatively, we will use one of the principal properties of the Peregrine soliton, which follows from equation (13), namely $\psi_j(-x, -z) = \psi_{j-1}(x, z)$. If the phase of the solution is not taken into account, this property turns into $|\psi_j(-x, -z)|^2 = |\psi_j(x, z)|^2$. Thus, we introduce the discrepancy as

$$S = \int_{-\infty}^{\infty} \left[ |\psi_1(-x, -z)|^2 + |\psi_2(-x, -z)|^2 - |\psi_1(x, z)|^2 - |\psi_2(x, z)|^2 \right] dx,$$

in order to eliminate phase effects. In the ideal case, where the shape of the rogue wave coincides with the Peregrine soliton (13), the discrepancy is zero, $S \equiv 0$. Thus, $S$ serves to measure how much the numerically obtained solution differs from the Peregrine soliton, or in other words, how much the chaotic nature of MI influences the Peregrine soliton. The results are depicted in figures 5(c) and (d). For the focusing XPM (figure 5(c)) and for $\delta \leq \pi/2$ the discrepancy abruptly grows at $\rho \gtrsim 1$. At the same time, in this range of $\delta$ the discrepancy almost does not depend on $\delta$ (the lines for $\delta = \pi/4, 9\pi/20$, and $11\pi/20$ are indistinguishable on the scale of figure 5(c)). Meanwhile, for $\delta > \pi/2$ the situation is opposite: the discrepancy increases with $\delta$ (compare the lines for $\delta = 11\pi/20$ and $3\pi/4$). For the defocusing XPM (figure 5(d)), the discrepancy $S$ decreases with the increase of $\delta$ in the whole range of $0 \leq \delta \leq \pi$, while an abrupt growth occurs at $\rho \gtrsim 0.4$. As a result, for the focusing XPM, the $PT$-symmetric rogue wave with $\delta < \pi/2$ is more stable than its antisymmetric counterpart, while for the defocusing XPM the situation is opposite.

5. Bright solitons

Obvious bright-soliton solutions of equation (5) with arbitrary amplitude $\eta$ are available too, for $\chi_1 + \chi < 0$,

$$\psi_j = \frac{\eta}{\sqrt{\chi_1 + \chi}} \cosh \left( \eta x / \sqrt{2} \right) \exp \left[ i \left( \frac{\delta}{2} + \cos \delta + \frac{1}{2} \eta^2 \right) z \right],$$

(14)
boundary for antisymmetric solitons, depicted by the dashed green curve (A2), pertains to dotted black curve labeled S1(an) displays the analytical counterpart of the S1 boundary, as predicted by equation (15). The stability boundary for antisymmetric solitons, depicted by the dashed green curve (A2), pertains to \( \chi = -1 \) and \( \chi_1 = 0.5 \). (b) The stability boundaries in the case of identical signs of the SPM and XPM coefficients, \( \chi = -1 \) and \( \chi_1 = -3 \). The boundaries (S3) and (A3) for symmetric and antisymmetric solitons are shown by solid blue (S) and dashed red curves (A), respectively. The dotted black curve labeled S3(an) is the analytical counterpart of the latter boundary, as predicted by equation (15).

\[ \eta^2_{\text{cr}} = \frac{16(-\chi_1 - \gamma) \cos \delta}{(\sqrt{-2\gamma \chi_1 + 7\chi - 3\sqrt{-\chi_1 - \gamma}})(\sqrt{-2\gamma \chi_1 + 7\chi + \sqrt{-\chi_1 - \gamma}})}, \]

(15)

Figure 6. (a) Stability boundaries for \(^\mathcal{PT}\)-symmetric and -antisymmetric solitons (14) are shown in the plane of the gain–loss coefficient \( \gamma \) (recall that \( \gamma \equiv \sin \delta \)) and soliton amplitude \( \eta \), in the case of opposite signs of the SPM and XPM coefficients. The boundaries for the \(^\mathcal{PT}\)-symmetric (S1) and -antisymmetric (A1) solitons are shown by solid blue and red lines, respectively, for \( \chi = 1 \) and \( \chi_1 = -1.5 \). The dotted black curve labeled S1(an) displays the analytical counterpart of the S1 boundary, as predicted by equation (15). The stability boundary for antisymmetric solitons, depicted by the dashed green curve (A2), pertains to \( \chi = -1 \) and \( \chi_1 = 0.5 \). (b) The stability boundaries in the case of identical signs of the SPM and XPM coefficients, \( \chi = -1 \) and \( \chi_1 = -3 \). The boundaries (S3) and (A3) for symmetric and antisymmetric solitons are shown by solid blue (S) and dashed red curves (A), respectively. The dotted black curve labeled S3(an) is the analytical counterpart of the latter boundary, as predicted by equation (15).

\[ \eta^2 < \eta^2_{\text{cr}} \]

The result makes sense when equation (15) yields a positive value, otherwise the \(^\mathcal{PT}\)-symmetry-breaking bifurcation does not occur, and the stability may only be studied numerically (in addition to the instability mode represented by equation (15), other instabilities are possible too). In particular, condition (15) cannot simultaneously hold for the \(^\mathcal{PT}\)-symmetric and -antisymmetric solitons. Further, because existence of solitons of either type requires \( \chi_1 + \chi < 0 \), the condition of \( \eta^2_{\text{cr}} > 0 \) actually holds for the \(^\mathcal{PT}\)-symmetric solitons at \( -\chi_1 > -\chi \), and for the \(^\mathcal{PT}\)-antisymmetric solitons in the opposite case, at \( -\chi_1 < -\chi \).

We have performed direct simulations of the evolution of perturbed solitons within the framework of equations (1) and (2), aiming to identify stability borders for the \(^\mathcal{PT}\)-symmetric and -antisymmetric solitons, and, in particular, to verify the analytical prediction (15). Perturbations were introduced by adding 2% to the amplitude of one component, and subtracting 2% from the other. Figures 6(a) and (b) display the thus identified stability boundaries in the cases of opposite and identical signs of \( \chi \) and \( \chi_1 \), respectively. For the sake of comparison with [11, 12], we demonstrate these borders as a function of \( \gamma \), rather than \( \delta \) (see equation (3)).

The numerically found stability boundaries are close to their analytical counterparts. Some discrepancy between them is explained by the fact that some solitons, which are stable against infinitesimal perturbations, may be destabilized by finite-amplitude excitations.

Typical examples of the unstable and stable evolution of antisymmetric solitons, taken on both sides of the stability boundary, are demonstrated in figure 7, for \( \chi = -1 \) and \( \chi_1 = 0.5 \). The quick stabilization of the symmetric soliton in the same case is demonstrated in figure 8 for a large amplitude, \( \eta = 3 \). Actually, the \(^\mathcal{PT}\)-symmetric solitons are stable for all \( \eta \) in this case, the stability border being relevant for the antisymmetric ones.

It is also relevant to note that, in the Manakov limit, \( \chi_1 = \chi \) [44], the stability boundary predicted by equation (15)
diverges. Indeed, direct simulations demonstrate that all the solitons are stable in this case.

6. Conclusion

In this work we have considered the MI (modulational instability) of CW backgrounds and the emergence and evolution of rogue waves in the system of linearly coupled \( P\overline{T}\) symmetric coupled NLSEs. We have shown that the focusing XPM nonlinear interactions extend the effective stability region for rogue waves of the Peregrine type. The system can support non-dissipative rogue waves too. The stability region for \( P\overline{T}\)-symmetric and -antisymmetric solitons was found in the exact analytical form and verified by direct simulations. It may be interesting to extend the analysis for 2D versions of the system, which may have realizations in nonlinear optics, cf [48] and references therein.

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