Squeezed Thermal State Representation of the Inflaton and Particle Production in Bianchi Type-I Universe

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Abstract—We use a single-mode squeezed thermal vacuum state formalism and examine the nature of a massive homogeneous scalar field minimally coupled to gravity in the framework of semiclassical gravity in a Bianchi type-I universe. We have obtained an estimate leading solution to the semiclassical Einstein equation for the Bianchi type-I universe showing that each scale factor in its respective direction obeys $t^{2/3}$ power-law expansion. The mechanism of nonclassical thermal cosmological particle production is also analyzed in the Bianchi type-I universe.

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1. INTRODUCTION

The Standard Big Bang (SBB) model is an achievement of the 20th century that can confidently characterize the universe formation at large scale after the primordial nucleosynthesis. Even though it is a great success, the SBB model had several longstanding unresolved aspects, some of them are the Horizon problem, the Monopole problem and the Flatness problem. The cosmological inflationary model [1] provides satisfactory solutions to all these problems. At present, there are many versions [1–3] of the cosmological inflationary model, called inflationary universe scenarios. The concept behind the cosmological inflationary model is that a period of accelerated expansion of space lasted from $10^{-36}$ s after the Big Bang at an early stage of the universe. The inflationary era can be well understood by consideration of a slow-roll massive inflaton [1, 3, 4], where the energy density of the massive inflaton is entirely stored in the form of its potential energy density because the contribution of the kinetic energy is very small in the entire inflation period. The universe becomes cold after the inflationary era. Therefore, the temperature of the universe was too small to start the primordial nucleosynthesis, so the universe was empty in terms of various kinds of particles.

A particle production mechanism is needed to explain the decay of a massive inflaton into different types of particles, which is responsible for reheating of the universe through thermalization. Near the potential minimum, quasi-periodic motion of the massive inflaton with slowly decreasing amplitude can reveal its decay into a particle shower [5–7]. Hence, at the end of the inflationary era, a significant amount of particles was produced, which helped to repopulate the universe with radiation and matter. The created particles moved freely and start collisions among themselves, thus enhancing the temperature of the universe along with the massive inflaton decay, therefore, the universe became hot again, which was later known as the hot Big Bang model, and the primordial nucleosynthesis mechanism was triggered. Therefore, the oscillating phase of a massive inflaton and its related aspects play a key role in describing particle production and further formation of the early universe.

Many of the cosmological inflation paradigms are based on the Friedmann-Robertson-Walker (FRW) universe model [8–11] which is isotropic and homogeneous. The FRW models provide a satisfactory solution to the flatness problem, while it is not much clear about homogeneity and isotropy. Absolutely, the FRW cosmological models provide a necessary condition to solve the horizon problem but do not provide a sufficient condition to solve the homogeneity and isotropy problems. According to Rothman and Ellis [12], an anisotropic metric consideration can provide solutions to the isotropy problem, and they also demonstrated that such models can isotropize and inflate in many ways using some general conditions. Therefore, it was reported [13, 14] that anisotropies and inhomogeneities might have played a key role in the evolutionary history of the early universe. A satisfactory solution is provided by the isotropic model for the explanation of the universe evolution at later stages, but near the singularity or near the Planck scale this model does not provide a suitable description of the universe at a very early stage [15].
Einstein’s theory of relativity provides cosmological solutions near the Planck scale (a singularity) in which the expansion is anisotropic at initial stages but becomes isotropic at later stage, and near the singularity the gravitational collapse solution becomes anisotropic (locally) [16–18]. To remove the existence of a particle horizon, postulating specific initial conditions to investigate isotropic models, a variety of corrections and initial conditions have been provided by a rigorous study of anisotropic and inhomogeneous models of the universe.

The Bianchi type-I cosmological model is the simplest one among anisotropic cosmologies. In this model, the metric is assumed to be spatially homogeneous and anisotropic, and a mechanism of isotropization of the universe is examined through the path of time. Unlike the FRW metric, the Bianchi type-I metric has three cosmic scale factors that expand individually in their corresponding direction. Recently such models received much attention [13, 14, 17–21]. In the Bianchi’s type-I cosmology, Fujimori and Umezawa, based on temperature-related to the squeezing parameter. The inflationary cosmology was studied by Albrecht et al. [35] in the light of squeezed states etc. [36–38]. A variety of cosmological problems can be solved by applying the existing physical as well as mathematical information from these states [39–47]. In such a type of studies, both squeezed and coherent states are assumed at the absolute zero temperature. However, in modern cosmology, thermal properties of these states are an interesting research topic [48]. Thermal counterparts of coherent and squeezed states are recognized as a coherent thermal state [49–54] and a squeezed thermal state [50, 51, 55–58]. Thermal properties of nonclassical states play an important role to understand the thermal history and evolution of the early universe [59–65]. In a cosmological perspective, consideration of thermal effects in flat FRW universe has a large significance. Consequently, it motivates us to study the thermal and quantum effects of a massive inflaton in coherent and squeezed thermal states formalism in cosmology.

In this paper, we use the single-mode squeezed thermal vacuum state (STVS) formalism and examine the nature of an inflaton field minimally coupled to the gravity in an anisotropically expanding Bianchi type-I universe in the framework of the semiclassical theory of gravity (SCTG). We obtain an approximate leading solution to the semiclassical Einstein equations (SCEE) for a Bianchi type-I universe. The mechanism of nonclassical thermal cosmological particle production is also analyzed in a Bianchi type-I universe. We here use the natural unit system $G = c = \hbar = 1$.

### 2. REPRESENTATIONS OF A SINGLE-MODE SQUEEZED THERMAL VACUUM STATE

In 1975 thermo-field dynamics was developed by Takahashi and Umezawa, based on temperature-dependent vacuum states [66–71]. The thermo-field dynamics introduced that the expectation value of a mixed state at nonzero temperature is obtained by an equivalent calculation with a pure state. This is found by introducing an imaginary field which has
a similar appearance to the natural real field. Thus the most significant aspect of thermo-field dynamics consists in Bogoliubov canonical transformations that transforms a theory from zero to finite temperature. Hence, in this approach, a tilde space is needed in addition to usual Hilbert space. By use of the Hilbert and tilde space, we determine a direct product space. In Hilbert space, every operator and state has the corresponding equivalent part in the tilde space. Thermo-field dynamics generates a thermal operator $\Gamma(\rho)$ based on the Hilbert and tilde system, which is invariant under the tilde conjugation i.e., $\tilde{\Gamma}(\rho) = \Gamma(\rho)$ [66-70].

Hence, in thermo-field dynamics, a temperature-dependent vacuum state is described as [57, 72]

$$|\rho\rangle = \tilde{\Gamma}(\rho)(0, 0),$$

(1)

where $\rho$ is a thermal parameter, $|0, 0\rangle$ represents a temperature-independent two-mode vacuum state, and $\tilde{\Gamma}(\rho) \equiv \exp[\rho(\tilde{\hat{A}}^\dagger \tilde{\hat{A}} - \tilde{\hat{A}}^\dagger A)]$ stands for the thermal operator. Here, Hilbert space creation and annihilation introduced by $\hat{A}^\dagger$ and $\hat{A}$ and tilde space creation and annihilation operators are described by $\tilde{\hat{A}}^\dagger$ and $\tilde{\hat{A}}$. These operators obey the commutation relations $[\hat{A}, \hat{A}^\dagger] = [\tilde{\hat{A}}, \tilde{\hat{A}}^\dagger] = 1$ in Hilbert and tilde space, respectively. By appropriate action of the thermal operator on $\hat{A}$ and $\tilde{\hat{A}}^\dagger$, we obtain [57, 72]

$$\hat{\Gamma}(\rho) \hat{A} \hat{\Gamma}(\rho) = \hat{A} \cosh \rho + \hat{A}^\dagger \sinh \rho,$$

(2)

and

$$\hat{\Gamma}(\rho) \tilde{\hat{A}}^\dagger \hat{\Gamma}(\rho) = \tilde{\hat{A}}^\dagger \cosh \rho + \tilde{\hat{A}} \sinh \rho.$$

(3)

Now the mean number of produced particles in a thermal vacuum state can be examined as

$$\mathcal{N} = \langle \rho | \hat{A}^\dagger \hat{A} | \rho \rangle = \sinh^2 \rho.$$

(4)

Thermofield dynamics is an effective approach to governing the thermal properties of nonclassical states. Its main significant aspect consists in Bogoliubov canonical transformations from zero to finite temperature. Therefore, a squeezed vacuum state with a thermal effect is generated by applying the thermal operator to vacuum first, and then followed by the squeeze operator $\hat{S}(r_s, \theta)$ [73-75] as described by

$$|s, \rho\rangle_{\text{STVS}} = \hat{S}(r_s, \theta)|\rho\rangle,$$

(5)

from (1),

$$|s, \rho\rangle_{\text{STVS}} = \hat{S}(r_s, \theta)\tilde{\Gamma}(\rho)(0, 0),$$

(6)

where the squeezing operator $\hat{S}(r_s, \theta)$ [75] takes the following form:

$$\hat{S}(\vartheta) = \exp\left[\frac{1}{2} \left(\vartheta^* \hat{A}^2 - \vartheta \hat{A}^\dagger \hat{A}\right)\right],$$

(7)

and $\vartheta = r_s e^{i\theta}$. Its action on the creation and annihilation operators is described by the Bogoliubov canonical transformation

$$\hat{S}^\dagger \hat{A}^\dagger \hat{S} = \hat{A}^\dagger \cosh r_s - \hat{A} e^{-i\vartheta} \sinh r_s,$$

$$\hat{S}^\dagger \hat{A} \hat{S} = \hat{A} \cosh r_s - \hat{A}^\dagger e^{i\vartheta} \sinh r_s.$$

(8)

In the present paper, Caves’ method [39] is used to generate the STVS.

### 3. Inflaton Dynamics and SCEE in the Bianchi Type-I Metric

Let us consider a Bianchi type-I universe which is spatially homogeneous and anisotropic. The line element for such a universe is

$$ds^2 = -dt^2 + Z_1(t)dx^2 + Z_2(t)dy^2 + Z_3(t)dz^2,$$

(9)

where $Z_1(t)$, $Z_2(t)$, and $Z_3(t)$ are the cosmic scale factors describing the universe expansion in three spatial directions, and $t$ is physical time. The Bianchi type-I metric is a spatially anisotropic generalization of the FRW metric. All three cosmic scale factors $Z_1(t)$, $Z_2(t)$, $Z_3(t)$ are derived via Einstein’s field equations.

We consider minimal coupling of a massive inflaton to gravity, therefore, it obeys the equation

$$(g^{\alpha\beta} \nabla_\alpha \nabla_\beta - m^2)\Phi(x, t) = 0,$$

(10)

where $g^{\alpha\beta}$, $\nabla_\alpha$, $\Phi(x, t)$ and $m$ represent the metric tensor, a covariant derivative, a scalar field and its mass, respectively. The Lagrangian density of a massive inflaton can be expressed as

$$\mathcal{L} = -\frac{1}{2} \sqrt{-g} (g^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi + m^2 \Phi^2),$$

(11)

where $g = |g_{\alpha\beta}|$ is the metric tensor determinant. In the present study we consider the scalar field to be spatially homogeneous, $\Phi = \Phi(t)$, therefore, classically a massive inflaton follows the Klein-Gordon equation given by Eqs. (9) and (11) as

$$\ddot{\Phi}(t) + \sum_{i=1}^{3} \left(\frac{Z_i'(t)}{Z_i(t)}\right) \dot{\Phi}(t) + m^2 \Phi(t) = 0,$$

(12)

where dots stand for $d/dt$. The Hubble parameter $H = \frac{1}{2} \left(\frac{Z_1'(t)}{Z_1(t)} + \frac{Z_2'(t)}{Z_2(t)} + \frac{Z_3'(t)}{Z_3(t)}\right)$ plays the role of a friction term in field dynamics in the spatial directions $x, y, z$. Now, the massive inflaton can be quantized by the canonical quantization rules, in a way consistent with the equation of motion, the conjugate momenta corresponding to $\Phi$, are given as $-\Pi_\Phi = \partial \mathcal{L}/\partial \dot{\Phi}$.

A time-dependent harmonic oscillator model can be used to describe the evolution of the minimally
coupled massive inflaton to gravity in a Bianchi type-I universe, with a generalized Hamiltonian obtained by using Eq. (11) in the relation

$$H = \Pi_\phi \dot{\Phi} - \mathcal{L},$$

(13)

hence the Hamiltonian for the massive inflaton is determined as

$$H_m = \frac{\Pi_\phi^2}{2 \mathcal{Z}_1(t) \mathcal{Z}_2(t) \mathcal{Z}_3(t)} + \frac{1}{2} \mathcal{Z}_1(t) \mathcal{Z}_2(t) \mathcal{Z}_3(t) m^2 \Phi^2(t).$$

(14)

Here, we assumed minimal coupling between the massive inflaton and gravity. Therefore, the classical Friedmann equation can be written as

$$\frac{\dot{Z}_1(t) \dot{Z}_2(t)}{Z_1(t) Z_2(t)} + \frac{\dot{Z}_3(t) \dot{Z}_3(t)}{Z_2(t) Z_3(t)} + \frac{\dot{Z}_1(t) \dot{Z}_3(t)}{Z_1(t) Z_3(t)} = \frac{8\pi}{3} \frac{\mathcal{T}_{00}}{Z_1(t) Z_2(t) Z_3(t)},$$

(15)

where $\mathcal{T}_{00} = \mathcal{Z}_1(t) \mathcal{Z}_2(t) \mathcal{Z}_3(t) \left( \frac{1}{2} \Phi^2 + \frac{1}{2} m^2 \Phi^2 \right)$ is the time–time $(00)$ component of the stress tensor of the massive inflaton. In the cosmological context, the classical description of the Friedmann equation (15) means that the Hubble parameter $H_i = \dot{Z}_i(t)/Z_i(t)$ is calculated by the energy density of the dynamically evolving inflaton according to Eq. (12).

4. SCEE AND A MASSIVE INFLATION IN A NONCLASSICAL THERMAL STATE

Most of the inflationary universe models are established on classical gravity with the Einstein equations and a nonquantized (classical) inflaton field in the FRW metric. To understand the early universe at a deeper level, we need both classical Einstein equations and the inflaton field to be quantized. Despite much effort in the last few decades, there does not yet exist a consistent theory of quantum gravity. However, quantum effects and quantum properties of the inflaton play a very significant role solving many cosmological problems [39–47]. Therefore, for a proper explanation of a cosmological model, we apply an approximation to the Einstein equations known as the semiclassical approximation, or SCEE [76]. Such an approximation of the Einstein field equations is expected to be valid where quantum gravity effects are negligible [77, 78].

In the semiclassical approximation, the Einstein equations take the form

$$G_{\alpha \beta} = 8\pi \langle \Psi | \varepsilon_{\alpha \beta} | \Psi \rangle.$$

(16)

Equation (16) is known as the SCEE, where the left-hand side is a function of the metric ($G_{\alpha \beta}$) is the 4D Einstein tensor representing the curvature of spacetime, computed from the metric, $R_{\alpha \beta} - \frac{1}{2} g_{\alpha \beta} R$, and in the right-hand side the angular brackets mean the expectation value of the matter stress tensor operator in the suitable quantum state $|\Psi\rangle$, obeying the Schrödinger equation

$$i \frac{\partial}{\partial t} |\Psi\rangle = \hat{H}_m |\Psi\rangle,$$

(17)

where $\hat{H}_m$ is the Hamiltonian operator for the matter field.

In SCTG, the semiclassical Friedmann equation in the Bianchi type-I model can be written as

$$\frac{\dot{Z}_1(t) \dot{Z}_2(t)}{Z_1(t) Z_2(t)} + \frac{\dot{Z}_3(t) \dot{Z}_3(t)}{Z_2(t) Z_3(t)} + \frac{\dot{Z}_1(t) \dot{Z}_3(t)}{Z_1(t) Z_3(t)} = \frac{8\pi}{\mathcal{Z}_1(t) \mathcal{Z}_2(t) \mathcal{Z}_3(t)} \langle \hat{H}_m | \Psi \rangle,$$

(18)

where $H_m$ is given by Eq. (14).

The Fock space of the Hamiltonian (14) was constructed in [79],

$$\hat{A}^\dagger (t) \hat{A} (t) |n, \Phi, t\rangle = n |n, \Phi, t\rangle,$$

(19)

so, in the Bianchi type-I model, the annihilation and creation operators can be written in terms of fields operators as

$$\hat{A} (t) = \Phi^* (t) \hat{\Pi} - \mathcal{Z}_1(t) \mathcal{Z}_2(t) \mathcal{Z}_3(t) \Phi^* (t) \Phi,$$

$$\hat{A}^\dagger (t) = \Phi (t) \hat{\Pi} - \mathcal{Z}_1(t) \mathcal{Z}_2(t) \mathcal{Z}_3(t) \Phi^* (t) \Phi.$$  

(20)

From Eq. (20) we can calculate the field operators in the form

$$\hat{\Phi} = \frac{1}{i} (\Phi^* (t) \hat{A}^\dagger (t) - \Phi (t) \hat{A} (t)),$$

$$\hat{\Pi} = i \mathcal{Z}_1(t) \mathcal{Z}_2(t) \mathcal{Z}_3(t) (\Phi^* (t) \hat{A}^\dagger (t) - \Phi (t) \hat{A} (t)).$$

(21)

(22)

Next, we take the massive inflaton $\Phi$ and its conjugate $\Pi$ in single-mode STVS by substituting the number state $|n, \Phi, t\rangle = |\varsigma\rangle$ and calculate:

$$\langle \Phi^2 \rangle_{\text{STVS}} = 2 (\sinh^2 \rho \cos^2 \theta - 2A \cos \theta \cosh^2 \rho \sinh^2 \rho - A^2 \cos^2 \theta - \frac{1}{2} \Phi^2 - (K) \Phi^* \Phi - (L) \Phi^2),$$

(23)

and

$$\langle \Pi^2 \rangle_{\text{STVS}} = \mathcal{Z}_1^2(t) \mathcal{Z}_2^2(t) \mathcal{Z}_3^2(t) \left[ (2 \sinh^2 \rho \cos^2 \theta - 2A \cos \theta + \cosh^2 \rho \sinh^2 \rho) + 1 \right] \Phi^* \Phi - (K) \Phi^* \Phi - (L) \Phi^2,$$

(24)

where

$$K = B \cosh^2 \rho - C e^{-i\theta} \sinh^2 \rho - C e^{-i\theta} \cosh^2 \rho.$$


Now, we calculate the Hamiltonian in a single-mode STVS Eqs. (23) and (24) and obtain

\[
\langle \hat{H}_m \rangle_{\text{STVS}} = Z_1(t)Z_2(t)Z_3(t)
\]
\[
	imes \left[ \left( \sinh^2 \rho \cosh^2 r_s - 2A \cos \theta \right.ight.
\]
\[
+ \cosh^2 \rho \sinh^2 r_s + \frac{1}{2} \right) (\Phi^* \Phi + m^2 \Phi^* \Phi)
\]
\[
- \left. \frac{(K)}{2} (\Phi^* \Phi + m^2 \Phi^* \Phi) - \left( \frac{L}{2} (\Phi^* \Phi + m^2 \Phi^* \Phi) \right) \right],
\]
where $K$ and $L$ are given in Eqs. (25), (26), and the semiclassical Friedmann equation in a single-mode STVS takes the form

\[
\frac{\dot{Z}_1(t)}{Z_1(t)} + \frac{\dot{Z}_2(t)}{Z_2(t)} \frac{Z_3(t)}{Z_3(t)} + \frac{\dot{Z}_1(t)}{Z_1(t)} \frac{Z_3(t)}{Z_3(t)} = 8\pi \left[ \left( \sinh^2 \rho \cosh^2 r_s - 2A \cos \theta \right.ight.
\]
\[
+ \cosh^2 \rho \sinh^2 r_s + \frac{1}{2} \right) (\Phi^* \Phi + m^2 \Phi^* \Phi)
\]
\[
- \left. \frac{(K)}{2} (\Phi^* \Phi + m^2 \Phi^* \Phi) - \left( \frac{L}{2} (\Phi^* \Phi + m^2 \Phi^* \Phi) \right) \right].
\]

In Eq. (30), $\Phi^*$ and $\Phi$ obey the Wronskian boundary condition

\[
\Phi^*(t) \Phi(t) - \Phi^*(t) \Phi(t) = \frac{i}{2} \frac{Z_1(t)Z_2(t)Z_3(t)}{Z_1(t)Z_2(t)Z_3(t)}.
\]

Next, we solve analytically the self-consistent SCEE (30) corresponding to a single-mode STVS for which we can transform the solution of a massive inflaton in the following form:

\[
\Phi(t) = \frac{1}{[Z_1(t)Z_2(t)Z_3(t)]^{i/2}} \Omega(t),
\]

hence Eq. (12) takes the form

\[
\ddot{\Omega} + m^2 + \frac{1}{4} \sum_{i,j=1}^3 \left( \frac{\dot{Z}_i(t)}{Z_i(t)} \right)^2 \left( \frac{\dot{Z}_j(t)}{Z_j(t)} \right) + \frac{1}{2} \sum_{i,j=1}^3 \left( \frac{\dot{Z}_i(t)}{Z_i(t)} \right) \left( \frac{\dot{Z}_j(t)}{Z_j(t)} \right)
\]
\[
= \left( \frac{1}{2} \sum_{i=1}^3 \frac{\dot{Z}_i(t)}{Z_i(t)} \right) \Omega(t) = 0.
\]

In the present study, our main focus is the oscillatory phase of a massive inflaton in the matter-dominated era. Therefore in the parameter regime, the massive inflaton obeys the inequality

\[
m^2 > \frac{1}{4} \sum_{i=1}^3 \left( \frac{\dot{Z}_i(t)}{Z_i(t)} \right)^2 - \frac{1}{2} \sum_{i,j=1}^3 \left( \frac{\dot{Z}_i(t)}{Z_i(t)} \right) \left( \frac{\dot{Z}_j(t)}{Z_j(t)} \right)
\]
\[
- \frac{1}{2} \sum_{i=1}^3 \frac{\ddot{Z}_i(t)}{Z_i(t)}.
\]

Therefore, an oscillatory solution for the massive inflaton in the matter-dominated era can be written as

\[
\Omega(t) = \frac{i}{\sqrt{2\sigma(t)}} \exp(-i \int \sigma(t) dt),
\]

with

\[
\sigma(t) = m^2 - \frac{1}{4} \sum_{i=1}^3 \left( \frac{\dot{Z}_i(t)}{Z_i(t)} \right)^2
\]
\[
- \frac{1}{2} \sum_{i,j=1}^3 \left( \frac{\dot{Z}_i(t)}{Z_i(t)} \right) \left( \frac{\dot{Z}_j(t)}{Z_j(t)} \right)
\]
\[
- \frac{1}{2} \sum_{i=1}^3 \frac{\ddot{Z}_i(t)}{Z_i(t)} + 3 \left( \frac{\dot{\sigma}(t)}{2\sigma(t)} \right)^2 - \frac{3}{2\sigma(t)} \right)^{1/2}.
\]

Using Eqs. (32), (34), and (35) and calculating $\Phi$, $\Phi^*$, $\dot{\Phi}$, and $\ddot{\Phi}$, the semiclassical Friedmann equation (30) can be obtained as

\[
Z_1(t)Z_2(t)Z_3(t) = \frac{8\pi}{2\sigma \left( \frac{\dot{Z}_1(t)}{Z_1(t)} + \frac{\dot{Z}_2(t)}{Z_2(t)} + \frac{\dot{Z}_3(t)}{Z_3(t)} \right)}
\]
\[
\times \left[ \left( \sinh^2 \rho \cosh^2 r_s - 2A \cos \theta \right.ight.
\]
\[
+ \cosh^2 \rho \sinh^2 r_s + \frac{1}{2} \right) (\Phi^* \Phi + m^2 \Phi^* \Phi)
\]
\[
- \left. \frac{(K)}{2} (\Phi^* \Phi + m^2 \Phi^* \Phi) - \left( \frac{L}{2} (\Phi^* \Phi + m^2 \Phi^* \Phi) \right) \right].
\]

Next, we solved perturbatively Eq. (37) in a single-mode STVS, further on using the following approxi-
motions, which corresponds to the parametric resonance conditions:

\[ \sigma_0(t) = m, \quad Z_{10}(t) = Z_{10}t^{2/3}, \]
\[ Z_{20}(t) = Z_{20}t^{2/3}, \quad Z_{30}(t) = Z_{30}t^{2/3}, \]
\[ \dot{\sigma}_0(t) = 0, \quad \frac{\dot{Z}_{10}(t)}{Z_{10}(t)} = \frac{\dot{Z}_{20}(t)}{Z_{20}(t)} = \frac{2}{3t}. \]  

(38)

Now, using the parametric condition (38), the next order perturbative solution for the cosmic scale factor \( Z_1(t) \) is obtained as

\[
Z_{11}(t) = \frac{6\pi}{Z_{20}Z_{30}} \left[ \left( \sinh^2 \rho \cosh^2 r_s - 2A \cos \theta \right. \right.
\left. + \cosh^2 \rho \sinh^2 r_s + \frac{1}{2} \left( 1 + \frac{1}{2m^2t^2} \right) \right.
- \frac{1}{2m^2t^2} \left( B \cosh^2 r_s \cos(2mt) \right.
- C(\sinh^2 \rho + \cosh^2 \rho) \cos(\theta - 2mt)
\left. + B \sinh^2 r_s \cos(2(\theta - mt)) \right)
- \frac{1}{mt} \left( B \cosh^2 r_s \sin(2mt) \right.
- C(\sinh^2 \rho + \cosh^2 \rho) \sin(\theta - 2mt)
\left. + B \sinh^2 r_s \sin(2(\theta - mt)) \right] \right] m t^{2/3}. \]  

(39)

In the same way, we have obtained the next order perturbative solution for the cosmic scale factors \( Z_2(t) \) and \( Z_3(t) \) in a single-mode STVS: the expression for \( Z_2(t) \) is again (39) but with the replacement

\[
\frac{6\pi}{Z_{20}Z_{30}} \rightarrow \frac{6\pi}{Z_{10}Z_{20}}, \]  

(40)

and that for \( Z_3(t) \) is (39) with the replacement

\[
\frac{6\pi}{Z_{20}Z_{30}} \rightarrow \frac{6\pi}{Z_{10}Z_{20}}. \]  

(41)

These \( Z_{11}(t) \), \( Z_{21}(t) \), and \( Z_{31}(t) \) represent the next order perturbative solution for the cosmic scale factors in the \( x \), \( y \), and \( z \) directions, respectively. From the above expressions it follows that

\[
Z_{11}(t) \sim Z_{21}(t) \sim Z_{31}(t) \sim t^{2/3}. \]  

(42)

We observed that all three cosmic scale factors obey the same power-law expansion, \( t^{2/3} \).

5. THERMAL AND QUANTUM PARTICLE PRODUCTION IN A BIANCHI TYPE-I UNIVERSE

In this section, using the single-mode STVS formalism, we analyze the mechanism of particle production due to oscillations of the massive inflaton in the anisotropically expanding Bianchi type-I universe in the framework of SCTG. For this, first, we examine the Fock space which has a parameter based on the cosmological time \( t \). Then, at a later time \( t \), the number of produced particles from vacuum states at an initial time \( t_0 \) is given by

\[
N_0(t, t_0) = \langle 0, \Phi, t_0 \mid \hat{N}(t) | 0, \Phi, t_0 \rangle, \]  

(43)

where \( \hat{N}(t) = \hat{A}^\dagger(t) \hat{A}(t) \) is a number operator, and the vacuum expectation value of the right-hand side of Eq. (43) can be computed as

\[
\langle \hat{N}(t) \rangle = \Phi(t) \Phi^*(t) (\hat{\Pi}^2) = \frac{1}{4}\sigma(t)\sigma(t_0) \frac{Z_1(t)Z_2(t)Z_3(t)}{Z_1(t_0)Z_2(t_0)Z_3(t_0)} \]  

(44)

where \( \langle \hat{\Pi}^2 \rangle \), \( \langle \hat{\Pi} \hat{\Phi} \rangle \), \( \langle \hat{\Phi}^2 \rangle \), and \( \langle \hat{\Phi} \rangle \) are obtained as

\[
\langle \hat{\Pi}^2 \rangle = \frac{1}{4}\sigma(t)\sigma(t_0) \frac{Z_1(t)Z_2(t)Z_3(t)}{Z_1(t_0)Z_2(t_0)Z_3(t_0)} \Phi^* \Phi, \]
\[ \langle \hat{\Pi} \hat{\Phi} \rangle = \frac{1}{4}\sigma(t)\sigma(t_0) \frac{Z_1(t)Z_2(t)Z_3(t)}{Z_1(t_0)Z_2(t_0)Z_3(t_0)} \Phi \Phi^*, \]
\[ \langle \hat{\Phi}^2 \rangle = \frac{1}{4}\sigma(t)\sigma(t_0) \frac{Z_1(t)Z_2(t)Z_3(t)}{Z_1(t_0)Z_2(t_0)Z_3(t_0)} \Phi^* \Phi, \]
\[ \langle \hat{\Phi} \rangle = \frac{1}{4}\sigma(t)\sigma(t_0) \frac{Z_1(t)Z_2(t)Z_3(t)}{Z_1(t_0)Z_2(t_0)Z_3(t_0)} \Phi. \]  

(45)

Now, putting Eq. (45) in Eq. (44), we obtain

\[
N_0(t, t_0) = \left( Z_1(t)Z_2(t)Z_3(t) \right)^2 | \Phi(t) \Phi(t_0) \rangle \geq (\hat{\Phi}(t)\Phi(t_0))^2. \]  

(46)

Using the perturbative solution in the limit \( mt_0, mt > 1 \), the number of created particles in the vacuum states can be calculated as

\[
N_0(t, t_0) = \frac{1}{4\sigma(t)\sigma(t_0)} \frac{Z_1(t)Z_2(t)Z_3(t)}{Z_1(t_0)Z_2(t_0)Z_3(t_0)} \times \left[ \frac{1}{4} \sum_{i=j=1}^3 \left( \frac{\dot{Z}_i(t)\dot{Z}_j(t)}{Z_i(t)Z_j(t)} + \frac{\dot{Z}_i(t)\dot{Z}_j(t)}{Z_i(t)Z_j(t)} \right) \right] \]  

(47)

\[
- \frac{1}{2} \sum_{i=j=1}^3 \frac{\dot{Z}_i(t)\dot{Z}_j(t)}{Z_i(t)Z_j(t)} \sigma(t) - \frac{1}{2} \sum_{i=1}^3 \frac{\dot{Z}_i(t)}{Z_i(t)} \sigma(t) \]  

(48)

\[
+ \frac{\dot{\sigma}(t)}{\sigma(t)} + \frac{1}{2} \sum_{i=1}^3 \left( \frac{\dot{Z}_i(t)}{Z_i(t)} \right) \left( \frac{\dot{\sigma}(t)}{\sigma(t)} + \frac{\dot{\sigma}(t)}{\sigma(t)} \right) \]  

(49)

\[
+ \frac{1}{2} \left( \frac{\dot{\sigma}(t)}{\sigma(t)} - \frac{\dot{\sigma}(t)}{\sigma(t)} \right)^2 + (\sigma(t) - \sigma(t_0))^2 \right] \]  

(50)
\[ \langle \Pi \Phi \rangle_{\text{STVS}} = Z_1(t) Z_2(t) Z_3(t) \left[ \left( \cosh^2 \rho \cosh^2 r_s \right. \right. \\
- 2A(\cos \theta) + \sinh^2 \rho \sinh^2 r_s \Phi(t_0) \Phi^*(t_0) \\
- (K)\Phi^*(t_0)\Phi^*(t_0) - (L)\Phi(t_0)\Phi(t_0) \\
+ (\sinh^2 \rho \cosh^2 r_s - 2A(\cos \theta) \\
+ \cosh^2 \rho \sinh^2 r_s)\Phi^*(t_0)\phi(t_0) \right] , \tag{48} \]

and

\[ \langle \dot{\Pi} \dot{\Phi} \rangle_{\text{STVS}} = Z_1(t) Z_2(t) Z_3(t) \left[ \left( \cosh^2 \rho \cosh^2 r_s \right. \right. \\
- 2A(\cos \theta) + \sinh^2 \rho \sinh^2 r_s \Phi(t_0) \Phi^*(t_0) \\
- (K)\Phi^*(t_0)\Phi^*(t_0) - (L)\Phi(t_0)\Phi(t_0) \\
+ (\sinh^2 \rho \cosh^2 r_s - 2A(\cos \theta) \\
+ \cosh^2 \rho \sinh^2 r_s)\Phi^*(t_0)\phi(t_0) \right] . \tag{49} \]

Substituting Eqs. (23), (24), (48), and (49) in (44), we can calculate the number of particles produced in a single-mode STVS as

\[ \langle \dot{N} \rangle_{\text{STVS}} = \left( \sinh^2 \rho \cosh^2 r_s - A(2 \cos \theta) \right. \]
\[ + \cosh^2 \rho \sinh^2 r_s + 1 \right) N_0(t, t_0) \]
\[ + \sinh^2 \rho \cosh^2 r_s - A(2 \cos \theta) + \cosh^2 \rho \sinh^2 r_s \]
\[ - (Z_1(t)Z_2(t)Z_3(t))^2(K)E \]
\[ - (Z_1(t)Z_2(t)Z_3(t))^2(L)F, \tag{50} \]

where \( K \) and \( L \) are given by Eqs. (25), (26), and

\[ E = \frac{\exp(2i \int mt_0)}{4m^2(Z_1(t)Z_2(t)Z_3(t))^2 t^2 t_0^2} \]
\[ \times \left[ \frac{1}{t_0^2} + \frac{1}{t^2} - \frac{2}{tt_0} - \frac{2im}{t} + \frac{2im}{t_0} \right], \tag{51} \]

\[ F = \frac{\exp(-2i \int mt_0)}{4m^2(Z_1(t)Z_2(t)Z_3(t))^2 t^2 t_0^2} \]
\[ \times \left[ \frac{1}{t_0^2} - \frac{2}{tt_0} + \frac{1}{t^2} + \frac{2im}{t} - \frac{2im}{t_0} \right]. \tag{52} \]

In the same way, using the perturbative solution, the number of created particles in the limit \( mt_0, m t > 1 \) from a single-mode STVS can be calculated as

\[ N_{\text{STVS}}(t, t_0) \approx \left( \sinh^2 \rho \cosh^2 r_s - A(2 \cos \theta) \right. \]
\[ + \cosh^2 \rho \sinh^2 r_s + 1 \right) \frac{(t - t_0)^2}{4m^2 t_0^2} \]
\[ + \sinh^2 \rho \cosh^2 r_s - A(2 \cos \theta) + \cosh^2 \rho \sinh^2 r_s \]
\[ - 2 \left( b \cosh^2 r_s \cos(2mt) \right) - C(\sinh^2 \rho + \cosh^2 \rho) \]
\[ + b \sinh^2 r_s \cos(2mt) \left( \frac{t - t_0)^2}{4m^2 t_0^2} \right. . \tag{53} \]

6. CONCLUSIONS

In this paper, we have considered minimal coupling between a massive inflaton and gravity and analyzed the behavior of the inflaton field in an anisotropically expanding Bianchi type-I universe in the framework of SCTG. The quantum and finite thermal effects of the massive inflaton were studied by representing the inflaton in a nonclassical thermal state. We particularly addressed the oscillatory regime of the massive inflaton in a single-mode STVS formalism. In this state, we obtained an approximate leading solution to the SCEE for the cosmic scale factor. From Eq. (42) it follows that \( Z_1(t)_{\text{STVS}} \sim Z_{21}(t)_{\text{STVS}} \sim Z_{31}(t)_{\text{STVS}} \sim t^{2/3} \). Therefore, in the oscillating region of a massive inflaton in the single-mode STVS formalism an approximate dominating solution to the SCEE has an equivalent power-law expansion of a matter-dominated universe, \( t^{2/3} \), as with the classical Einstein equation.

Further, the solution for one of the cosmic scale factors depends on the initial values of the other cosmic scale factors of other directions. Hence it can be concluded that the evolution laws of the cosmic scale factors are related mutually. If we put \( Z_1(t) = Z_2(t) = Z_3(t) = Z(t) \), then the corresponding result reduces to the FRW model [62]. A damping process plays an important role in the transformation from anisotropic to isotropic model. One of the adequate damping processes could be noticed due to particle production in anisotropic models [19]. Hence, the particle reduction mechanism can bring isotropy into Bianchi type-I anisotropic models.

By the use of STVS, we also examined particle production of the oscillating massive inflaton in the framework of the SCTG in the context of the Bianchi type-I cosmological model. Using the single-mode STVS, we computed the production of particles in the limit \( mt_0, m t > 1 \); here, we observe that the produced particle number depends on the associated thermal parameter \( \rho \) and squeezing parameter \( (r) \). If we establish \( \rho = 0 \) in Eq. (53), then we obtain the number of produced particles in a squeezed vacuum state,
which totally matches with the results of J.K. Kim and S.P. Kim[81]. It is also noted that in the Bianchi type-I anisotropic model, a coherently oscillating massive inflaton in a single mode (STVS) suffers from particle production, and the produced particles show oscillations. After the inflation, such oscillation of the produced particles is necessary to reheat the universe to be hot again. Therefore, the single-mode STVS can be a possible quantum state with finite thermal effects in which our universe could have been present during the matter-dominated era.

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