RESEARCH OF FERR-RESONANCE OSCILLATIONS AT THE FREQUENCY OF SUBHARMONICS IN THREE-PHASE NON-LINEAR ELECTRIC CIRCUITS AND SYSTEMS.

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Annotation. It is known that the occurrence and existence of ferroresonant oscillations at the subharmonic frequency (SHC) in power transmission lines (TL) and in power supply systems is extremely undesirable, since they cause ferroresonant overvoltages at different frequencies. At the same time, there is a wide class of nonlinear electrical circuits, in which the excitation of autoparametric oscillations (AIC) at the frequency of the SHC forms the basis of frequency converting devices serving as secondary power sources. It is shown that three-phase nonlinear systems are in one way or another equivalent circuits for power transmission lines, the main elements of which are: longitudinal compensation capacitors, transverse compensation reactors, and transformers with a nonlinear characteristic. To study the regularities of the excitation and maintenance of SHC at a frequency in three-phase electro-ferromagnetic circuits (EFMC), theoretical and experimental studies of an equivalent model of a three-phase circuit with nonlinear inductance were carried out. For the analysis of the steady-state mode of the SHC at the frequency, the method of a small parameter (averaging) was applied. A shortened differential equation of motion for a three-phase nonlinear circuit is obtained. By solving them, the regions of existence of the SHC and the critical parameters of the chain were determined. The obtained results of theoretical research are confirmed by experimental studies.

Key words: ferroresonance, self-oscillation, subharmonic, approximation, lowest harmonic, small parameters, ferromagnetic element.

Introduction

Electrical systems contain a large number of elements with significant inductance (generators, transformers, reactors, etc.). On the other hand, power lines have capacities to ground and between phases. Often, to regulate the voltage and increase the stability of parallel operation, additional capacitances relative to ground are included in the line cut.

Combinations of such inductances and capacitances create a number of complex oscillatory circuits in the circuit of the electrical system.

In the normal operating mode of the system, the capacitances and inductances of these circuits are shunted by the load or connected directly to the terminals of a powerful source so that free oscillations cannot develop in them.

With various commutations in the system, part of the oscillatory circuits can be caused and energetic oscillations develop in them, leading to significant overvoltages [1,2,3,4,7,8,9,10,11,12].

A number of works [2,3,4,8,10,11,12] are devoted to experimental and analytical studies of the physics of the phenomenon of subharmonic oscillations in three-phase circuits, its quantitative and qualitative assessment depending on the parameters of the circuit and the applied influence. operation and design of power transmission lines in order to reduce their accident rate, as well as in the development of alternating current switches.

Analysis of the conditions for the excitation of subharmonic modes of three-phase nonlinear systems, depending on the parameters of the circuit, and the applied impact, allows us to identify the main patterns of overvoltage in power lines and possible measures to prevent or reduce them to permissible values.

Despite the abundance of publications illuminated by subharmonic oscillations in three-phase circuits, the processes in them are illuminated on the basis of purely experimental data, the processes in them are illuminated on the basis of purely experimental data, a theoretical analysis was carried out for a single-phase analogue of a three-phase circuit, which distorts the quantitative and qualitative side of the process. This is due to the solution of nonlinear systems of inhomogeneous differential equations, the right side of which has fixed phase shifts.

The excitation conditions and the nature of the AIC processes in the ferroresonant circuit depend...
mainly on the circuit parameters, on the degree of inductance nonlinearity, initial conditions, amplitude and phase of the input action. [1,9,10,11,12]

In three and multiphase electroferromagnetic circuits (EFMC), the conditions for the excitation and maintenance of the APC also depend on the structure of the circuit and the methods of connecting its elements, as well as the nonlinear interaction of ferromagnetic elements (FE) in phases.

Analysis of the conditions for the excitation of subgammonic oscillations in three-phase nonlinear systems, taking into account the parameters of the circuit and the applied voltage, makes it possible to identify the main patterns of overvoltage in power lines and, if possible, take measures to prevent such anomalous modes or mitigate their negative consequences.

Theoretical analysis of the excitation of subharmonic oscillations at frequency \( \frac{\omega}{3} \).

To study the regularities of the excitation of subharmonic oscillations \( \frac{\omega}{3} \) in three-phase nonlinear circuits, theoretical and experimental studies of an equivalent model of a three-phase circuit with series-connected elements with isolated neutral were carried out (Fig. 1).

The diagram shown in Fig. 1 according to Kirchhoff's laws is described by the following system of differential equations:

\[
\begin{align*}
\frac{d^2 \Phi_1}{dt^2} + R_0 \frac{d\Phi_1}{dt} + R \frac{d\Phi_1}{dt} &= \omega U_m \cos \omega t, \\
\frac{d^2 \Phi_2}{dt^2} + R_0 \frac{d\Phi_2}{dt} + R \frac{d\Phi_2}{dt} &= \omega U_m \cos[\omega t - 120^\circ], \\
\frac{d^2 \Phi_3}{dt^2} + R_0 \frac{d\Phi_3}{dt} + R \frac{d\Phi_3}{dt} &= \omega U_m \cos[\omega t + 120^\circ], \\
\frac{d\Phi_1}{dt} &= \frac{d\Phi_1}{dt} + \frac{d\Phi_2}{dt} + \frac{d\Phi_3}{dt},
\end{align*}
\]

(1)

where \( \Phi_1, \Phi_2, \Phi_3 \) are the fluxes of ferromagnetic elements.

\( i_1, i_2, i_3 \) - phase currents.

\( R_0 \) - active resistance of neutrals.

\( R, C \) - active resistance and capacitance of each phase.

Approximating the nonlinear Weber-ampere characteristic of ferromagnetic elements in the form:

\[ i_v = \alpha \Phi_v + \beta \Phi_v^3 \]  

(2)

Here \( \alpha \) and \( \beta \) are the coefficients of the approximating function

\[ v = 1, 2, 3 \] - phase order

From (1), taking into account (2), after transformation, we obtain a system of second-order nonlinear differential equations with a zero wire:
\[
\frac{d^2 \Phi_v}{dt^2} + R_0 \alpha \sum_{n=1}^{3} \frac{d \Phi_n}{dt} + \frac{\alpha}{C} \Phi_v + \frac{\beta}{C} \Phi_v^3 + 3 \beta R_0 \sum_{n=1}^{3} \Phi_n^2 \frac{d \Phi_n}{dt} + 3 \beta R \Phi_v^3 \frac{d \Phi_v}{dt} + R \alpha \frac{d \Phi_v}{dt} = \omega U_{mv} \cos \left( \omega t + \varphi - \frac{2\pi(v-1)}{3} \right)
\]

Derivation of truncated equations.

Let us consider the process of excitation and the existence of SGC \( \left( \frac{\omega}{3} \right) \) in circuits with series-connected elements with isolated neutral (Fig. 1)

\[
\frac{d^2 \Phi_v}{dt^2} + \delta_0 \sum_{n=1}^{3} \frac{d \Phi_n}{dt} + h_0 \sum_{n=1}^{3} \Phi_n^2 \frac{d \Phi_n}{dt} + \mu h \Phi_v^2 \frac{d \Phi_v}{dt} + \delta \frac{d \Phi_v}{dt} + k_0 \Phi_v + k \Phi_v^3 =
\]

where: \( \mu \) - small parameter \((0 < \mu < 1)\),

\[
\delta_0 = \frac{3R_0 \alpha}{\omega}; \quad \delta = \frac{3R \alpha}{\omega}; \quad h_0 = \frac{3R_0 \beta}{\omega}; \quad h = \frac{3R \beta}{\omega}.
\]

Using the method of Bogolyubov N.N. Metropol'skiy Yu.A. [5] and taking into account the phase relations \( \varphi = \frac{(v-1)2\pi}{3} \) for three variants

\[
\Phi_v = \Phi_{v,1} e^{-j \left[ \tau \left( \frac{(v-1)2\pi}{3} \right) \right]} + \Phi_{v,2} e^{-j \left[ 3\tau \left( \frac{(v-1)2\pi}{3} \right) \right]} + kC + \mu \Phi_{v,3}
\]

where: \( kC \) - complex conjugation is the value of complex numbers;

\( q \) - the order of the phases \((q = 1, 2)\).

\( \Phi_v \) and \( \Phi_{v,q} \) - slowly varying complex amplitudes of fundamental and subharmonic oscillations:

\( W_0 \) is a limited function of orders and

Substituting (6) into (4), neglecting the order terms\( \mu^2 \), for we \( W_0 \) obtain:

\[
\mu \left( \hat{W}_v + W_v \right) = H_v
\]

Using the following restrictions:

\[
\frac{1}{2\pi} \int_0^{2\pi} H_v e^{j \left[ \tau \left( \frac{(v-1)2\pi}{3} \right) \right]} d\tau = 0
\]

\[
\frac{1}{2\pi} \int_0^{2\pi} H_v e^{j \left[ \tau \left( \frac{(v-1)2\pi}{3} \right) \right]} d\tau = 0
\]

Taking into account the phase relations for the three-phase circuit of the subharmonic regime \( \left( \frac{\omega}{3} \right) \) by the averaging method [5], shortened equations describing the dynamics of the system were obtained.

Nonlinear differential equations of a three-phase EFMC with a zero wire (3) after the transition to a new time \( t = 3t/\omega \) can be represented as:

\[
\frac{d^2 \Phi_v}{dt^2} + \delta \sum_{n=1}^{3} \frac{d \Phi_n}{dt} + h \sum_{n=1}^{3} \Phi_n^2 \frac{d \Phi_n}{dt} + \mu h \Phi_v^2 \frac{d \Phi_v}{dt} + \delta \frac{d \Phi_v}{dt} + k_0 \Phi_v + k \Phi_v^3 =
\]

where: \( \varphi \) is the phase shift in the subharmonic, the solution of equations (4) has the form:

\[
\Phi_v = \Phi_{v,1} e^{-j \left[ \tau \left( \frac{(v-1)2\pi}{3} \right) \right]} + \Phi_{v,2} e^{-j \left[ 3\tau \left( \frac{(v-1)2\pi}{3} \right) \right]} + kC + \mu \Phi_{v,3}
\]

Depending on \( q \) i.e. from the order of the phases of the SHC, the phase shifts between the adjacent phases of the three-phase system of the SHC are in different combinations. In particular, at \( q = 1 \), these shifts are \( 0^0, 40^0, 80^0 \), which corresponds to the direct sequence of phases in the third-order SHC. At \( q = 2 \) are equal \( 0^0, 80^0, 160^0 \) and \( 0^0, 160^0, 360^0 \) \((0^0, 40^0, 160^0)\), which corresponds to the reverse phase sequence. From (6) for \( q = 2 \) we obtain the following truncated equations for each phase of a three-phase circuit with a neutral wire in complex amplitudes:
Passing from complex quantities to real amplitudes and phases, we use the expressions:

\[
\begin{align*}
\Phi_1 &= \Phi_1 \cdot e^{j\phi_1}, \\
\Phi_2 &= \Phi_2 \cdot e^{j\phi_2}, \\
\Phi_3 &= \Phi_3 \cdot e^{j\phi_3}, \\
\overline{\Phi}_1 &= \Phi_1 \cdot e^{-j\phi_1}, \\
\overline{\Phi}_2 &= \Phi_2 \cdot e^{-j\phi_2}, \\
\overline{\Phi}_3 &= \Phi_3 \cdot e^{-j\phi_3}.
\end{align*}
\]

From (9) and (10) we obtain the following equations:

\[
\begin{align*}
3\Phi_1 &= -3\xi_2 \Phi_1 \sin(\phi_1 - 3\phi_3), \\
3\Phi_2 &= -3\xi_2 \Phi_1 \sin(\phi_2 - 3\phi_3), \\
3\Phi_3 &= -3\xi_2 \Phi_1 \sin(\phi_3 - 3\phi_3), \\
3\Phi_1 &= -3\xi_2 \Phi_1 \sin(\phi_1 - 3\phi_3), \\
3\Phi_2 &= -3\xi_2 \Phi_1 \sin(\phi_2 - 3\phi_3), \\
3\Phi_3 &= -3\xi_2 \Phi_1 \sin(\phi_3 - 3\phi_3).
\end{align*}
\]

As a result, we obtain equations for the amplitude-frequency characteristics of the system. The regime is now determined from equations (11). In the case of a three-phase EFMC with a neutral wire:

\[
\begin{align*}
\Phi_{\alpha} &= \Phi_{\alpha} \cdot e^{j\phi_{\alpha}}, \\
\Phi_{\beta} &= \Phi_{\beta} \cdot e^{j\phi_{\beta}}, \\
\Phi_{\gamma} &= \Phi_{\gamma} \cdot e^{j\phi_{\gamma}}.
\end{align*}
\]

**Analysis of the steady state SHC \( \omega_3 \).**

The steady-state mode of the third-order SHC is determined from equations (11), equating to zero the derivatives:

\[
\begin{align*}
\dot{\Phi}_1 &= 0, \\
\dot{\Phi}_2 &= 0, \\
\dot{\Phi}_3 &= 0, \\
\dot{\phi}_1 &= 0.
\end{align*}
\]

\[
\begin{align*}
\delta + 0.82\delta_0 &= 3k\Phi_{13} \Phi_1 \sin(3\phi_3 - \phi_1), \\
-\xi + 0.3\delta_0 &= -k(3\Phi_{13}^2 + 6\Phi_1^2) = 3k\Phi_{13} \Phi_1 \cos(3\phi_3 - \phi_1), \\
\delta + 0.82\delta_0 &= 3k\Phi_{23} \Phi_2 \sin(3\phi_2 - \phi_2), \\
-\xi + 0.3\delta_0 &= -k(3\Phi_{23}^2 + 6\Phi_2^2) = 3k\Phi_{23} \Phi_2 \cos(3\phi_2 - \phi_2), \\
\delta + 0.82\delta_0 &= 3k\Phi_{33} \Phi_3 \sin(3\phi_3 - \phi_3), \\
-\xi + 0.3\delta_0 &= -k(3\Phi_{33}^2 + 6\Phi_3^2) = 3k\Phi_{33} \Phi_3 \cos(3\phi_3 - \phi_3).
\end{align*}
\]

To determine the amplitude of the SHC, depending on the parameters of the system and the applied action, we square equations (12) and add them. Replacing, \( \Phi_{\alpha}^2 = y, \Phi_{\beta}^2 = x \), we obtain the equations of curves of the second order.
\[
36x^2 + 9y^2 + 27xy + 12 - \frac{\delta - 0.3\delta_0}{k} x + 6\frac{\xi - 0.3\delta_0}{k} y + \left(\frac{\delta + 0.82\delta_0}{k^2}\right)^2 + \left(\frac{\xi - 0.3\delta_0}{k^2}\right)^2 = 0
\]

\[
36x^2 + 9y^2 + 27xy + 12 - \frac{\delta + 0.3\delta_0}{k} x + 6\frac{\xi + 0.3\delta_0}{k} y + \left(\frac{\delta + 0.82\delta_0}{k^2}\right)^2 + \left(\frac{\xi + 0.3\delta_0}{k^2}\right)^2 = 0
\]

\[
36x^2 + 9y^2 + 27xy + 12 - \frac{\xi}{k} x + 6\frac{\xi}{k} y + \left(\frac{\delta + 0.867\delta_0}{k^2}\right)^2 + \left(\frac{\xi}{k^2}\right)^2 = 0
\]

Invariants for (13) are equal [8]:

\[
\Delta_1 = 142 \left[ \left(\frac{0.82\delta_0}{k^2}\right)^2 - 0.14 \left(\frac{\xi - 0.3\delta_0}{k^2}\right)^2 \right]
\]

\[
\Delta_2 = 142 \left[ \left(\frac{0.82\delta_0}{k^2}\right)^2 - 0.14 \left(\frac{\xi + 0.3\delta_0}{k^2}\right)^2 \right]
\]

\[
\Delta_3 = 142 \left[ \left(\frac{0.867\delta_0}{k^2}\right)^2 - 0.14 \left(\frac{\xi}{k^2}\right)^2 \right]
\]

\[
f_v = 142, \quad s_v = 45 \quad (15)
\]

where \( f_v \) - and \( s_v \) - are constant coefficients.

If in (15) \( \Delta_v/f_v < 0 \) then equations (14) describe real ellipses (Fig. 2), then its positive value of the squares of the amplitude of the fundamental harmonic (\( \Phi_v^2 \)) corresponds to the positive value of the squares of the amplitude of the SHC (\( \Phi_{v3}^2 \)), i.e. ellipses are in the first square: \( X_Y > 0 \), \( Y_Y > 0 \).

From equations (13), the coordinates of the centers of the ellipses will be:

\[
x_{10} = \frac{13.5}{142k} (\xi - 0.3\delta_0) > 0 \quad y_{10} = 2x_1 > 0
\]

\[
x_{20} = \frac{13.5}{142k} (\xi + 0.3\delta_0) > 0 \quad y_{20} = 2x_2 > 0
\]

\[
x_{30} = \frac{13.5}{142k} \xi > 0 \quad y_{30} = 2x_3 > 0
\]

From (16) it follows that the coordinates of the center of the ellipse will shift with a change in the parameters of the chain, and the ellipses are rotated relative to the coordinate axes by an angle \( \operatorname{tg}2\alpha = 2b/\alpha c = 1 \), independent of the parameters of the chain and equal to 22°30′ for all three phases. The length of the semi-axes ellipses is determined by the expression:

\[
a_1 = \left\lfloor \frac{1}{3.5} \left( \frac{\delta + 0.82\delta_0}{k} \right)^2 - \left( \frac{\xi - 0.82\delta_0}{k} \right)^2 \cdot 0.14 \right\rfloor, \quad a_1^2 = 0.082a_1^2
\]

\[
a_2 = \left\lfloor \frac{1}{3.5} \left( \frac{\delta + 0.82\delta_0}{k} \right)^2 - \left( \frac{\xi + 0.82\delta_0}{k} \right)^2 \cdot 0.14 \right\rfloor, \quad a_2^2 = 0.082a_2^2
\]

\[
a_3 = \left\lfloor \frac{1}{3.5} \left( \frac{\delta + 0.867\delta_0}{k} \right)^2 + \left( \frac{\xi}{k} \right)^2 \cdot 0.14 \right\rfloor, \quad a_3^2 = 0.082a_3^2
\]

The ratio of the semi-axes of the ellipses is constant, so when changing the parameters of the chain, the ellipses remain similar.

The region of existence of the SHC increases.

\[
X_Y > 0, \quad Y_Y > 0
\]

\[
\Delta_v = \frac{13.5}{142k} (\xi - 0.3\delta_0) > 0, \quad \Delta_2 = \frac{13.5}{142k} (\xi + 0.3\delta_0) > 0, \quad \Delta_3 = \frac{13.5}{142k} \xi > 0
\]

\[
\Delta_v < 0
\]

It can be seen that with an increase in the detuning (\( \xi \)), the coordinates of the centers and the length of the semi-axes of the ellipses increase, i.e., the region of existence \( \xi \) of the SHC increases.

**Determination of the critical parameters of the SHC\( (\alpha_{v3}) \).**

It follows from (15) that for the existence of the SHC in the system, it is necessary that the coordinates of the centers and parameters of the ellipses be positive and greater than zero:

\[
(\xi - 0.3\delta_0)^2 - 7(\delta + 0.82\delta_0)^2 \geq 0
\]

\[
(\xi + 0.3\delta_0)^2 - 7(\delta + 0.82\delta_0)^2 \geq 0
\]

\[
\xi^2 - 7(\delta + 0.867\delta_0)^2 \geq 0
\]
The critical values of the pattern in the system, at which third-order SHC s are excited, will be,

\[
\begin{align*}
\text{Phase A:} & \quad \xi_{1,2} = 0.3\delta_0 \pm \sqrt{4.54\delta_0^2 + 7.12\delta_0^2 + 11.48\delta_0^2} \\
\text{Phase B:} & \quad \xi_{1,2} = -0.3\delta_0 \pm \sqrt{4.54\delta_0^2 + 7.12\delta_0^2 + 11.48\delta_0^2} \\
\text{Phase C:} & \quad \xi_{1,2} = 2.63(\delta + 0.867\delta_0)
\end{align*}
\]

(19)

respectively:

\[
\begin{align*}
\text{Phase B:} & \quad \xi_{12} = 1.82\delta_0; \\
\text{Phase C:} & \quad \xi_{12} = 2.3\delta_0.
\end{align*}
\]

(21)

As follows from (Fig. 2) and expressions (16), (17) with increasing (the subharmonic components of the FE flux increase and the region of their existence becomes wider, and when the critical value (\(\xi_{cp}\)) is reached, the oscillations are disrupted. From (19) it follows that the region of existence of the SHC is limited by some maximum and minimum values of the capacitance (detuning) (Fig. 2) If the resistance of the neutral wire is \(R_0=0\) (\(\delta_0=0\)), then the amplitudes of the SHC in three phases are the same, and with increasing \(R_0\) they become different.

In the first phase, with an increase in the resistance of the neutral wire, the subharmonic components of the magnetic flux decrease, while in the other two phases they increase. When \(R_0\) reaches a certain critical value, the coordinates of the center of the ellipses also contract to zero, which corresponds to the breakdown of the SGC. (Family of curves of ellipses at \(R = 5\) Ohm, \(C = 120\) \(\mu\)F, \(R_0 = 0 + 15\) Ohm). With an increase in the \(R\) chain, the region of existence of SHC narrows, while the amplitude of SHC decreases in one phase and increases in the other two. When the resistance value in the first, second and third phases reaches a certain critical value (\(R_{cp} = 15\) Ohm), then the coordinates of the centers of the ellipses become zero (\(x,y = 0\)), which corresponds to the breakdown of oscillations in all three phases. Here, a family of curves is plotted at \(R_0 = 2\) Ohm, \(C = 120\) \(\mu\)F, \(R = 0 + 15\) Ohm.

**Experimental studies of SHC excitation** (\(H_3\)) in three-phase ferroresonant circuits.

Theoretical analysis and the results of mathematical modeling on a computer have shown that third-order
SHC in symmetric three-phase EFMC can arise at certain ratios of the circuit parameters and applied voltage and are limited to a certain region of existence. In most cases, third-order SHC s are excited “rigidly” after switching processes and are accompanied by an abrupt change in currents, voltages and violation of the symmetry of the system.

Studies show that in circuits with a neutral wire, third-order SHC s can exist in one, two or simultaneously in three phases, and in circuits without a neutral wire, in most cases, simultaneously in three phases with different options for phase shifts between adjacent phases. The likelihood of third-order SHC occurrence in phases with different variants of phase shifts mainly depends on the initial conditions, circuit parameters, applied voltage, the degree of nonlinearity of ferromagnetic elements, as well as on the switching conditions.

To obtain reliable results, all experiments were carried out repeatedly at various values of the parameters and the magnitude of the mains voltage that affect the excitation and maintenance of the SHC.

In (Fig. 3,4), the regions of existence of the SHC \( \frac{\pi}{7} \) in a three-phase circuit are shown when the parameters of the circuit and the applied voltage change and the failure of the SHC when the active resistance of the circuit changes. The oscillogram of the stress curves on the SHC \( \frac{\pi}{7} \) capacitance is shown in (Fig. 5).

The obtained experimental characteristics correspond to the theoretical calculated data and the critical values of the circuit parameters

Fig. 3 Input-output characteristics of SHC current and voltage in the container at frequency \( \frac{\pi}{7} \).

Figure: 4 Influence of active resistance on the area of existence of SHC \( \frac{\pi}{7} \).

Fig. 5. Oscillograms of the voltage in the capacitor of the SHC \( \frac{\pi}{7} \) in a three-phase EFMC.

**Conclusion**

1. SHC in three-phase nonlinear circuits and systems are excited at certain ratios of circuit parameters, input voltage and nonlinearities of ferromagnetic elements, both in “soft” and “hard” modes and is limited to a certain region of existence.

2. In a three-phase ferroresonant circuit, the excitation of SHC and the nature of the transient process depend on the structure and methods of connecting the circuit elements and on the initial conditions, as well as the amplitude-phase interactions of frequencies and nonlinear interactions of individual phases.
3. The excitation of self-oscillations in one phase causes the corresponding reactions of other phases and, accordingly, the breakdown of oscillations in any phase is reflected in all other phases.

4. When self-oscillations are excited at higher and lower frequencies, the symmetry of the system is violated and significant currents and voltages appear in the phases, linear and neutral wires, as well as on individual circuit elements.

5. SHC in three-phase ferroresonant circuits are excited with different variants of phase shifts and form asymmetric systems of direct, reverse, zero sequence.

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