Understanding Dilepton Production in Heavy Ion Collisions by Vector Mesons of Different Varieties

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A simple schematic model anchored on the notion of hadronic freedom inferred from hidden local symmetry in the vector manifestation, the infinite tower of vector mesons in holographic QCD and “stickiness” of \( \pi \pi \) interactions inferred from dispersion relations is used to describe the dileptons produced in relativistic heavy ion collisions at PHENIX/RHIC. It is shown that due to the near “blindness” of dileptons to Brown-Rho scaling, those dileptons with invariant mass less than \( m_\rho = 770 \text{ MeV} \) come mostly from pion-composites that we interpret as “transient \( \rho \)'s.”

\[
m_\rho^* \simeq m_\rho / \sqrt{2}.
\]

The way in which these \( \rho^* \)'s of mass less than \( m_\rho \) give rise to dileptons resembles the way in which \( ^{12}\text{C} \) is formed by 3 \( \alpha \) particles in stars. In the case of stars, no bound state of \( ^8\text{Be} \) is available to hold two \( \alpha \) particles together, but rather by what we call “stickiness” a 0\(^+\) excited state at 0.092 MeV holds the two \( \alpha \) particles together long enough for a third one to come along and carry the 2\( \alpha \) system to the 7 MeV 0\(^+\) excited state in \( ^{12}\text{C} \). This mechanism is amazingly effective, the presence of only one excited state in \( ^8\text{Be}^* \) being necessary for 10\(^9\) \( \alpha \)'s in order to produce the required amount of \( ^{12}\text{C} \).

II. Blindness of Dilepton Production to BR-Scaling ρ-Mesons and “Stickiness” in the \( \pi^+ \pi^- \) Collisions:— We mention that for all practical purposes, dileptons are “blind” to BR-scaling \( \rho \) mesons, these entering at most at the level of an order of magnitude less than those from on-shell \( \rho \)'s and \( \rho^* \)'s. This has been established in detail by BHHRS [1].

The “stickiness” in \( \pi^+ \pi^- \) collisions was used in many papers on the Paris potential; see e.g. [2]. A lucid review of the Paris potential can be found in §3.10.3 in [2].

In dispersion theory the \( J = 1 \) (\(^3P_0\)) state of \( \pi^+ \) and \( \pi^- \) experiences a velocity-velocity dependent attraction,

\[\pi^+\pi^-\]

FIG. 1: The dispersion theory mechanism for binding the \( \rho \), and for obtaining the lower mass pion-composite.

In the Paris potential [3,4], for instance, the two-pion exchange potential in the \( \rho \) channel (see Fig.1) with a helicity amplitude \(^3P_0\) can be characterized by an exchange of a “pion-composite” with a distributed mass which shows not only the resonance peak at 770 MeV but also a non-resonant background. We will refer to this pion-composite as “transient \( \rho \)’s” or \( \rho^* \) for short. Note that the “mass” of the \( \rho^* \), denoted \( m_\rho^* \) (equivalent to \( \mu \) in [5]), can decrease to zero in the chiral limit but this has nothing to do with BR scaling. It will be argued that the largest number of these composites are formed at an energy

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In the case of the \( \rho^* \)’s formed by \( \pi^+ \pi^- \) collisions, a “stickiness” is furnished by the attractive p-wave interaction, which holds them together long enough so that the \( \rho^* \)” can emit dileptons with \( \pi^+ \pi^- \) free-space parameters (in particular with \( a = 2 \) and vector dominance). Note the similarity of astrophysical and heavy ion situations, both of them having a background thermal field \( T \) so that the \( ^8\text{Be} \) in the first place and the \( \rho^* \) in the second place form in thermal templates and break up continuously.

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In dispersion theory the \( J = 1 \) (\(^3P_0\)) state of \( \pi^+ \) and \( \pi^- \) experiences a velocity-velocity dependent attraction,
which we take to be linear in the relative velocity. The velocity-velocity interaction is known to be attractive from the analysis of the \( p + p \rightarrow p + p + \pi^+ + \pi^- \) interaction in which the interaction between \( \pi^+ \) and \( \pi^- \) can be measured. For the \( \rho \) meson used in the Paris potential the phase shift of the \( \pi^+ \pi^- \) bound state is \( \delta = \pi/2 \). (In principle, the phase shift could be greater than \( \pi/2 \), but in practice this is close to maximum because the weighting of the attractive velocity-velocity interaction runs out here.) In the years of the Mandelstam representation leading towards the Paris potential the calculations of the helicity amplitudes, the \( 3^P_0 \) one furnishing the properties of the \( \rho \), were developed. These were forgotten, as the quark model took over and the \( \rho \) as discussed was made up out of a \( q\bar{q} \) pair. However, the pion mass would be zero in the chiral limit, and it is clear that the lowest mass \( \rho^* \) is made up chiefly out of \( \pi^+ \) and \( \pi^- \), or \( q^2\bar{q}^2 \) in quark language. The hidden local symmetry is a field theory with hadronic, not quark, variables so it is clear that it contains the \( \rho^* \) of composite \( \pi^+ \pi^- \) nature.

The most numerous \( \pi^+ \pi^- \) collisions just following the flash point where the hadrons go on shell are those with relative angle of 90° because of the large solid angle. These would give a lower \( \delta_{\pi\pi} \approx \pi/8 \). This is rather far from a bound state \( \rho \) for which \( \delta_{\pi\pi} = \pi/2 \) is necessary, but indicates a “stickiness” with the \( \pi^+ \) and \( \pi^- \) pushing each other with their thermal velocities. The pions only need to be pushed in the right direction so as to stick together for several fm/c.

In any case, neither in the case of pions colliding in heavy ion collisions nor in \( ^{12}\text{C} \) production by an additional \( \alpha \) attaching itself to a \( 2\alpha \) state are we dealing with the bound states of the two particles, but rather with temperature dependent templates because the two-body systems form and reform as they are jostled about, due to the background temperature. This feature will be exploited below for dilepton production.

**III. Making a Quantitative Theory for the PHENIX Dileptons from the Hidden Local Symmetry Vector Manifestation:**— Let us consider what happens to one batch of \( SU(4) \) hadrons, initially at (nearly) zero mass at \( T_c \). From \[1\] we have at the flash point 66 pions, among which there are 18 reconstructed \( \rho^* \)'s, 6 of the latter being \( \rho^0 \) (half of these result from \( a_1 \) decay). These 6 \( \rho^0 \) mesons can give rise to dileptons at the dilepton invariant mass of 770 MeV. From \( \text{STAR} \), we estimate \[1\] that there are 6 to 7 \( \pi^- \) per \( \rho^0 \) meson \[1\], also 6 to 7 \( \pi^+ \) (corresponding to the lower and upper limits of number of pions involved in the \( \text{STAR} \) ratio); therefore 36 to 49 possible \( \pi^+ \pi^- \) collisions. We will take the mean value \( \sim 42 \) in the numerical estimate made below. These pions have been given off mostly—among other mesons—from the heavy particles, \( \rho \) and \( a_1 \), going on-shell at the flash point that is reached after a free flow from the hadronic free regime and are therefore more or less equally distributed over angle. This spherically symmetrical distribution at the flash point is the basic structure of the system that we associate with hidden local symmetry in the vector manifold and this will play a key role in what follows. Thus, there are between 36 and 49 \( \pi^+ \pi^- \) collisions per \( SU(4) \) batch which can make \( \rho^* \)'s with the \( \rho^0 \)'s with the mass 770 MeV identified as the on-shell \( \rho^* \).

The temperature at the end of free flow is between 135 MeV and the flash temperature, i.e., 120 MeV. Equilibration is assumed to take place during this interval of temperatures. Ignoring the pion mass that comes from tiny quark masses, the invariant mass of the colliding \( \pi^+ \) and \( \pi^- \) is given in terms of the relative solid angle \( \theta \) as

\[
\mathcal{M}(\theta, \epsilon) = 2\epsilon \sin \frac{\theta}{2}
\]

where \( \epsilon \) is the pion energy (or momentum with zero mass). Since we are neglecting the pion mass, the pion energy is just the thermal energy \( \epsilon \approx 3T_{\text{flash}} \), so that a \( \pi^+ \) and \( \pi^- \) colliding head-on will produce a \( \rho^* \) which is nearly on-shell, with mass \( \mathcal{M}(\pi, 3T_{\text{flash}}) \approx 720 \text{ MeV} \). We will call this object \( \rho^*_{720} \). By “head-on collision,” we mean that in which the \( \pi^- \) collides within 180° ± 45° with the \( \pi^+ \), for which the relative solid angle is

\[
\frac{\Delta \Omega}{\Omega} = 2\pi \frac{\int_{180^\circ}^{360^\circ} \sin \theta d\theta}{4\pi} \approx 0.15.
\]

Since the pions are assumed to get distributed spherically symmetrically coming from the “heavy” mesons, the total number of collisions leading to near on-shell \( \rho^* \)'s can be simply estimated as \( \sim 0.15 \times 42 \sim 6 \). We note that the number of these “near on-shell” \( \rho^* \)'s is essentially the same as the on-shell \( \rho^0 \)'s that would be detected from one set of the \( SU(4) \) hadrons at the flash point, 3 of the \( \rho^0 \)'s coming directly from the set and 3 more from the \( a_1 \) decaying into \( \rho^0 \) plus \( \pi \). This implies that the number of \( \rho^0 \)'s just below the on-shell \( \rho \) peak should be roughly continuous going down in mass through 770 MeV, the on-shell \( \rho \) mass \[12\].

This is, however, not what is seen in the PHENIX data \[10\]. In it the dileptons emitted by the \( \rho \) as well as \( \omega \) mesons are shown as a cocktail contribution. This means that the on-shell \( \rho^0 \) did not go through the fireball. In our theory the 3 of the \( \rho^0 \)'s from the \( a_1 \) decay will have joined the other \( \rho^0 \)'s in flow and should also be included in the cocktail peak. This can be done within given errors.

Going down from the cocktail peak to 720 MeV, we position our \( \sim 720 \text{ MeV} \) \( \rho^* \)'s at \( \sim 1.2 \times 10^{-4} \) resulting from 6 \( \rho^0 \)'s. In doing this, we are equating the number of \( \rho^* \)'s to that of on-shell \( \rho^0 \)'s at the point where they join. This defines our normalization.

Now the relative velocity of \( \pi^+ \) and \( \pi^- \) decreases by \( 1/\sqrt{2} \) in moving from \( \theta = 2\pi \) to \( \pi \). Therefore from Eq. \[2\], we find that at \( 90^\circ \), the mass of \( \rho^* \) would be \( \approx (720/\sqrt{2}) \approx 500 \text{ MeV} \). Given our principal assumption of the equal distribution over angle, there would be

\[
f_1 \sim 2.4
\]

times more \( \pi^+ \pi^- \) pairs between 135° and 90° than between 180° and 135° at which \( \rho^0 \)'s are more or less the
on-shell $\rho$’s, and similarly for 90° and 45° and for 45° and 0°. Note that the larger number of collisions at $\sim 90°$ results from the larger relative solid angle, and may be characterized as a continuous function proportional to $\sin \theta$ with relative velocity $\sin 2\theta$ assumed to give the drop in $m_{\rho^*}$.

The above reasoning gives the number of $\rho^*$’s as a function of the solid angle $\theta$ as well as a relation between $m_{\rho^*}$ and the solid angle. All we need for addressing the PHENIX dileptons is then the dependence of the dileptons on $m_{\rho^*}$.

IV. Calculating the dileptons from $\rho^*$’s:— In order to compute the dilepton production from $\pi^+\pi^-$ collisions, we need to know how the photon couples to the transient $\rho$ (i.e., $\rho^*$). As noted earlier, this was parameterized from a knowledge of the attraction between the two pions determined experimentally from $p + p \rightarrow p + p + \pi^+ + \pi^-$ reactions. We should note that we are constructing the $\rho$ and $\rho^*$ as sort of “energy fluctuations”; i.e., as templates in a heat bath to be weighted thermally. We are in fact constructing the entire process of $\pi^+\pi^- \rightarrow \rho^+$ with the maximum $m_{\rho^*} = m_{\rho^*} = 770$ MeV; the $\rho^*$’s make the transition to the observed dileptons.

In a generalized hidden local symmetry theory, the $\rho^*$ coupling to the virtual photon is vector-dominated by the infinite tower of vector mesons as is suggested in holographic dual QCD (hQCD) [4, 5]. When the $\pi^+$ and $\pi^-$ have exactly opposite momenta, the on-shell $\rho$, here identified at $\sim 720$ MeV corresponding to the flash temperature, will be produced. This corresponds to the lowest-lying vector meson $\rho(770)$ with width of 150 MeV in the tower to which the dileptons with invariant mass peaked at 770 MeV will couple. These dileptons are counted as cocktails in PHENIX. The $\rho^*$’s with mass less than 770 MeV that will be produced at lower angles will couple to dileptons through the higher-lying members of the infinite tower. We can assume using closure approximation that the photon couples point-like to the latter since the closure energy $E$ should be much greater than the mass of the $\rho^*$’s lying below 720 MeV. This point-like photon coupling to $\rho^*$’s via the massive members of the tower will be the key ingredient for allowing the simple numerical estimates to be made below.

Now in the $e^+e^-$ CM frame, the photon propagator in the dilepton coupling to the $\rho^*$ is

$$D_\gamma \sim 1/M(\theta)^2$$

where $M$ is defined in Eq. (2). $\epsilon$ will be taken to be $3T_{\text{flash}}$, so will be omitted in the expression in what follows. A simple calculation shows that the propagator gives in the dilepton cross section a scaling factor

$$f_2 \equiv \left(\frac{M(\pi)}{M(\theta)}\right)^2 \approx \left(\frac{m_{\pi^+\pi^-}}{M(\theta)}\right)^2$$

Thus, given the assumption of the direct coupling explained above, we would expect, in going from $m_{\rho^*} \approx 720$ to $\sim 509$ MeV, an increase in $\rho^*$’s of a factor

$$f_1f_2 \sim 2.4 \times 2 \sim 4.8.$$  

This increase in number of $\rho^*$’s from 720 MeV to 500 MeV – which is expected to be more or less linear – is approximated by a straight line in Fig. 2.

Now in going from $\sim 500$ to $\sim 200$ MeV, one would expect the decrease in the number of $\pi^+\pi^-$ collisions to be roughly canceled by the factor $(M(\pi)/M(\theta))^2$ so that the number of experimental points to be more or less flat. This is indicated as a rough approximation by a straight horizontal line in Fig. 2. Note that our model does not include the fact that the $\rho^*$ mass cannot drop below $2m_\pi$.

To compare with experiments, the cocktail contributions need to be included. Even without the latter, the agreement with experiments is quite satisfactory. We expect that the same mechanism will be operative for the NA60 dileptons except that much fewer pions will be involved here.

![FIG. 2: PHENIX data $(1/N_{\text{evt}})dN/dM_{ee}(e^2/\text{GeV})$ in PHENIX acceptance vs. $M_{ee}$.

We suggest that the rapid rise at very low masses other than that from $\pi^0$ decay comes from the pions going into $\sigma$-mesons. (See Fig. 5 of Shuryak and Brown [13].) The pions that result from the decay of the $\sigma$ have non-thermal, very low kinetic energies. This matter will be considered in a separate paper.

Conclusions— Hidden local symmetry in the vector manifestation which gives rise to BR scaling suggests hadronic freedom between the critical point and the flash point. The way in which the 32 SU(4) hadrons massless at $T_c$ according to the calculations of Park, Lee and Brown starting from unquenched QCD [14] go on-shell at the flash point leads to the “blindness” of the theory to BR-scaling $\rho$ mesons.

We have reconstructed the $\pi^+\pi^-$ pairs which experience attraction and which are identified with the usual on-shell $\rho$ mesons when their phase shift $\delta_\pi\pi$ is equal to $\pi/2$ as in the construction of the Paris potential for NN scattering. When a bound state is not formed, there is still enough attraction in the lower-mass fluctuation $\rho^*$

![Image](image.png)
in order to hold the $\pi^+$ and $\pi^-$ together long enough to produce dileptons. These $\rho$'s coupled to the infinite tower of $\rho$ mesons are sufficient to explain the low-mass dileptons in the PHENIX experiment.

Our principal conclusion is that the PHENIX data can be successfully analyzed in terms of hidden local symmetry in the vector manifestation together with the role of the infinite tower suggested in hQCD [4, 5]. This result follows from the blindness of the dileptons to BR-scaling vector mesons, the concept of the flash point and hadronic freedom, all of which are an essential ingredient of HLS/VM. Although not observable directly, BR scaling plays a crucial role in understanding the observed spectrum. The BR-scaling vector mesons that carry signals for partial as well as full chiral restoration should in principle be present below the $\rho$ peak but are highly suppressed by the vector manifestation of hidden local symmetry, rendered invisible in the background of the pion-composites.

If the proposed scenario is correct as suggested by our analysis, then the standard phase diagram may have to undergo a major revamping. There have been suggestions in the literature that the presently “predicted” phase structure at high density and low temperature could be wrong. In fact there can be a plethora of other phases that could replace or render obsolete the ones figuring in the present phase diagram: Kaon condensation at a density as low as 3 times normal nuclear matter which would send compact stars into black holes at higher density [15], a half-skyrmion quantum critical phase related to hadronic freedom in density below the chiral critical density [10], a quarkyonic phase with quark confinement but restored chiral symmetry [17], an anomaly-induced spectral continuity with no phase transition [18] etc. At present, there is no model-independent theoretical method to determine which are viable and which are not. On the contrary, it has been generally accepted, based on available lattice QCD and experimental data, that the phase structure at high temperature and low-density is more or less known modulo some details. The present study, however, suggests that this understanding is not entirely correct. The dilepton experiments indicate that there is the hitherto unsuspected “Hadronic Freedom” phase shown in Fig.3 which makes a drastically different picture of how phase transitions occur in temperature (as suspected in density).

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