Impact of generalized uncertainty principle on the accretion process within the asymptotically safe ambiance

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We investigate the impact of quantum gravity on accretion onto a modified Schwarzschild black hole within the context of the generalized uncertainty principle (GUP). The minimal measurable length connected to GUP modifies the Schwarzschild black hole, giving it the capacity to accommodate the correction due to quantum gravity. We look at potential critical point locations and calculate the critical speed of the matter accreting. We determine the temperature and total integrated flux correction at the event horizon for the polytropic matter using the least measurable length conjecture offered by the GUP. We note that quantum gravity has a significant impact on the accretion process. Additionally, the quantum gravity regime also maintains an asymptotically safe ambiance.

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I. INTRODUCTION

The investigations on the properties of black holes have been leading to many interesting physical phenomena. The accretion phenomenon onto the black hole is one of them. Accretion is a process by which a massive astrophysical object such as a black hole or a star captures particles from the fluid in its vicinity which leads to an increase in mass of the accreting body. It is one of the most fascinating and ubiquitous processes which is going on happening in the Universe. The existence of supermassive black holes at the centers of the active galaxies suggests that black holes would have evolved through the accretion process. Accretion of dust or matter from the surrounding regions for a sufficiently long period would seem to be a suitable model for the formation of massive giant black holes. However through the accretion process mass of the compact source does not always get enhanced, sometimes the in-falling matter is thrown away in the form of jets or cosmic rays. The probable scenario of the accretion process is that it may not be static in the true sense. The velocity of free-fall and the energy density of the fluid is likely to vary from one spacetime point to another. Bondi in his seminal work formulated the problem of accretion of matter onto the compact object \cite{1} using the Newtonian theory of gravity. After two decades, Michel \cite{2} extends the description of the accession process in Einstein’s theory of gravity with the formulation of accretion onto a spherically symmetric Schwarzschild black hole. Gradually physics related to accretion acquired huge attention. In the article \cite{3}, phantom energy accretion onto a black hole in the cyclic universe has been carried out. An attempt has been made in \cite{4} to investigate accretion of phantom-like modified variable Chaplygin gas onto static Schwarzschild black hole and phantom energy accretion on the stringy charged black hole was described in \cite{5}. Accretion of dark matter onto the Schwarzschild black hole has been reported by Kim et al. in \cite{6} and the accretion of dark energy onto the Kerr-Newman black hole has been studied by Madrid et al. in \cite{7}. Few other kinds of literature on the accretion of various components onto different black holes are available in the articles \cite{8,10}. In the articles \cite{10,17}, Babichev et al. formulated the accretion of phantom dark energy onto a static Schwarzschild black hole and have shown that static Schwarzschild black hole mass will gradually decrease due to the strong negative pressure of phantom energy and finally all the masses tend to zero near the big rip.

The aforesaid investigations are grounded with purely classical theories of gravity which are well known to be non-renormalizable. Therefore how quantum correction be physically sensible throughout the galactic accretion process is a possible concern. A well-reasoned response to that is as follows. If the gravitational constant acquires \( r \)-dependence, the Fourier transform of that makes it \( k \) dependent. Therefore the Renormalization Group (RG) prescription \cite{12} is a helpful and elegant method of describing quantum phenomena on a low energy scale. An effective theory can be developed by integrating out the quantum fluctuations associated with the higher energy scales which are higher than a specific cutoff scale. It includes a number of parameters known as RG flow that move along with the cutoff scale. Then, using the effective action generated from the classical equations of motion, one can obtain the quantum effects. The primary hurdle that appears here is that we do not know quantum gravity theory in its matured form, which is
accepted as it is capable of describing physics at the Planck scale. However according to the fascinating concept of asymptotically safe gravity proposed by Weinberg in [19, 20], quantum gravity can be characterized by a finite set of parameters that lead to nontrivial fixed points in the ultraviolet (UV) scale limit. This remarkable proposal has been applied to investigate the existence of the UV fixed point in various theories, like Einstein’s theory of gravity, f(R) gravity, scalar-tensor theory, etc. [21, 22].

At this juncture, it has been hypothesized that there is also the possibility of an infrared (IR) fixed point in quantum gravity, which has been examined in cosmology [23, 24] when the cosmological late time effects are applied to the renormalization group flow [25]. However, the concept that was introduced in [24] is different from how asymptotic safety requirements applied in the UV scale [24]. In fact, it was developed with an emphasis on the IR behavior of the field, and its implications for dark energy and cosmology have been studied as well. The cosmological fixed points which are expected to result from the complicated coupled dynamical equations may or may not be able to correlate to fixed points of the formal RG flow of UV limit. However, it is believed that in a theory, the RG equation can be used to derive a low-energy effective action. Regarding the RG scale [26], this is, in fact, a challenging task. In [28] it has been attempted to establish that quantum effects continue even after the metric leaves the classical singularity, despite the fact that it is severely modified close to it. This has been demonstrated through the precise computation of the impact on the dynamics of the test particles near RGI (RG improved)-Schwarzchild and Kerr black holes in asymptotic safety with higher derivatives in the IR limit.

With this in mind, the fixed point may occur in IR limit the RGI-metric of rotating black hole has been described in [29] and has been constrained in [30] using X-ray reflector spectroscopy of a Novikov-Page-Thorne type accretion disc. Similarly, a study in [31] examined the iron line form anticipated in the reflection spectrum of an accretion disc surrounding black holes in the IR limit of the asymptotic safe gravity with larger derivatives. In recent research, [32], investigated the impact of quantum gravity on the radiation characteristics of a relativistic Novikov-Page-Thorne model of a thin accretion disc surrounding an RGI-Schwarzchild black hole in the setting of the IR limit of an asymptotically safe theory. In the context of asymptotically safe gravity, the quantum gravity correction has been addressed for the accretion process onto black holes in [33]. In the article [34], the author analyzes quantum gravity corrections to the accretion onto black holes in the context of asymptotically safe gravity. Considering steady, spherical accretion onto a static and a spherically symmetric black hole the author has determined the critical point and computed the mass accretion rate, and subsequently observed the total integrated flux. In the same vein, we consider a static and spherically symmetric Schwarzchild black hole. The quantum gravity corrections to the Schwarzchild black hole metrics are accounted for by the use of the generalized uncertainty principle. The issue, therefore, enters into the realm of quantum spacetime with a running gravitational coupling. Analyzing the gravity-induced quantum interference pattern and the Gedanken experiment for measuring the weight of a photon, it has been found that the running Newton constant can be stimulated by the generalized uncertainty principles and that leads to quantum gravity corrections to Schwarzchild black hole metrics [34]. This improved metric is considered here to study the accretion onto the black hole.

The article is organized as follows. Sec.2 is devoted to the description of GUP associated with the minimum measurable length. In Sec. 3 we have discussed the general formulation of the accretion phenomena onto the black holes. In Sec. 4 we consider a polytropic equation of the state of the surrounding medium of the black hole and compute the correction due to the quantum gravity effect to the temperature and to the total integrated flux at the event horizon. The final Sec.5 contains a brief discussion and conclusion.

II. INSERTION OF QUANTUM GRAVITY EFFECT INTO THE SCHWARZSCHILD METRIC

Before jumping into the formulation of having a modified black hole endowed with quantum gravity effect through GUP perspective it would be beneficial to give a brief description of GUP. Let us turn into that.

A. Description GUP with minimum measurable length

Heisenberg uncertainty principle is undoubtedly the keystone of the quantum theory which puts a fundamental limit on the precision of measuring the position and momentum. However various approaches to quantum gravity including string theory [35, 42], noncommutative geometry [41], loop quantum gravity [43] and black hole physics [44] predict the existence of a minimum measurable length of the order of Planck length. It leads to different generalizations of uncertainty relation in the context of quantum gravity. [39, 43, 45, 47]. In the usual standard Heisenberg uncertainty principle, $\Delta x \Delta p \geq \hbar$ goes to zero in the high momentum limit therefore the standard Heisenberg uncertainty relation $\Delta x \Delta p \geq \hbar$ becomes inadequate to explain the existence of a minimum measurable length. Hence, to incorporate the concept of minimum measurable length, the ordinary Heisenberg uncertainty principle should be
replaced by the Generalized Uncertainty Principle (GUP) [48]. He showed that a generalized uncertainty relation could be defined as
\[
\Delta x_i \Delta p_j \geq \hbar \delta_{ij} \left(1 + \alpha \left[\langle \Delta p \rangle^2 + \langle p \rangle^2\right]\right), \tag{1}
\]
where \(\alpha\) is GUP parameter and it is defined as \(\alpha = \alpha_0 / (M_{Pl} c)^2\). Here \(M_{Pl}\) is the Planck mass and \(\alpha_0\) is of the order of unity which indeed has the ability to accommodate the minimum measurable length within the revised principle. It transpires that the above GUP [11], leads to a minimum non-zero length:
\[
(\Delta x)_{min} = \hbar \sqrt{\alpha} \sqrt{1 + \alpha \langle p \rangle^2}. \tag{2}
\]
thereafter setting \(\langle p \rangle = 0\) results in the absolute minimal measurable length:
\[
(\Delta x)_{min} = \hbar \sqrt{\alpha} = \sqrt{\alpha_0 l_{Pl}}, \tag{3}
\]
where \(l_{Pl} = \sqrt{\frac{G \hbar}{c^3}} \approx 10^{-35}\text{m} [48, 69, 70]\) is the Planck length. This generalized uncertainty relation [11] leads to the following deformed commutation relation between position and momentum [48]
\[
[x_i, p_j] = i\hbar \delta_{ij} \left(1 + \alpha p^2 \right), \tag{4}
\]

What follows next is an attempt to have a modification of the relation [11] and [48] in one dimension which is given by
\[
\Delta x \Delta p \geq \hbar \left(1 + \alpha (\Delta p)^2\right), \tag{5}
\]
\[
[x, p] = i\hbar \left(1 + \alpha p^2\right) = i\hbar z, \tag{6}
\]
where \(z = 1 + \alpha p^2\). Hence, Eq. (5) can be rewritten down as
\[
\Delta x \Delta p \geq \hbar z. \tag{7}
\]
There are other type of modification with the frame work of GUP. Few recent investigations with diffract type of GUP proposal are found in the articles [71–76].

### III. SCHWARZSCHILD METRIC ENDOVED WITH QUANTUM GRAVITY CORRECTION

Let us now formulate a Schwartz-like spacetime metric where quantum gravity correction gets induced through the generalized uncertainty principle keeping in view the Gedanken-experiment initially proposed by Einstein. Einstein made an attempt to exhibit the violation of the uncertainty principle through a Gedanken-experiment which was supposed to measure the weight of photons [34–36]. He assumed a box containing photon gas with a fully reflective wall was suspended by a spring scale. There was a mechanical system inside the box that caused the shutter to open and close at moment \(t\) for the time interval \(\Delta t\), which allowed to pass out just a single photon. A clock capable of showing extremely high precision measurement of time could be used to measure the time interval \(\Delta t\) and at the same time, the mass difference of the box would determine the energy of the emitted photon. According to Einstein’s assumption, the time interval required for photon radiation is exactly \(\Delta t \to 0\) which can lead to the violation of the uncertainty relation for energy and time, i.e. \(\Delta E \Delta t \to 0\). However, Bohr argued that [35], Einstein’s deduction was not flawless since he neglected the time-dilation effect which would play a vital role due to the difference in gravitational potential. Based on general relativity, when altitude changes, the rate of time flow also changes due to the change in their gravitational potential. Thus, for the clock in the box, the time uncertainty \(\Delta t\), in terms of the vertical position uncertainty \(\Delta x\), would be expressed as [35, 36]
\[
\Delta t = \frac{g \Delta x}{c^2} t, \tag{8}
\]
where \(t\) represents the time period of weighing the photon. As it is known, according to the quantum theorem, the uncertainty relation in energy and time of the photon is express as \(\Delta E \Delta t \geq \hbar\) which after substituting Eqn. (8) turns into
\[
\Delta E \geq \frac{hc^2}{g t \Delta x}. \tag{9}
\]
Now look at the relation between the weight of the photon in the Gedanken experiment and the corresponding quantities in quantum mechanics in the GUP framework (7). Now if we focus on the original position of the pointer on the box before opening the shutter we will find that after releasing the photon in the box, the pointer moves up with reference to its original position. To get back the pointer in its original position in a period of time $t$, some weights equal to the weight of the photon must be added to the box. If we now use Eqn. (7), having accuracy in measuring the position $\Delta x$ as marked by the indicator of the clock, the minimum uncertainty in momentum $\Delta p_{\text{min}}$ will be

$$\Delta p_{\text{min}} = \frac{zh}{\Delta x}. \tag{10}$$

Since the quantum weight limit of a photon is $g\Delta m$, in a period of time $t$ the smallest photon weight will be equal to $zh/t/\Delta x = \Delta p_{\text{min}}/t \leq g\Delta m$. Now, using Eqn. (10) we also find that

$$zh = \Delta x\Delta p_{\text{min}} \leq gt\Delta x\Delta m. \tag{11}$$

If the relation $\Delta E = c^2\Delta m$, is used for $\Delta m$ the Eqn. (11) can be written down as

$$\Delta E \geq \frac{zhc^2}{gt\Delta x}. \tag{12}$$

where $z$ refers to the the GUP effects. Note that in the absence of GUP framework, i.e. when $\alpha \to 0$, the standard energy-time uncertainty relation Eqn. (9) is reobtained.

A careful look on the standard uncertainty relation between energy and time (9) and the generalized uncertainty relation (7) gives rise to an interpretation that gravitational field strength $g$ is modified to $\bar{g}$. Then, replacing $g$ by $\bar{g}$ we have

$$\bar{g} = \frac{g}{z} = \frac{G_0M}{zR^2}. \tag{13}$$

Hence, using (13), the modified Schwarzschild metric turns into

$$ds^2 = - \left( 1 - \frac{2G_0M}{zc^2r} \right) c^2 dt^2 + \left( 1 - \frac{2G_0M}{zc^2r} \right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{14}$$

where $G_0$ stands for universal gravitational constant. On the other hand, as stated in some other literature [34, 60], when two virtual particles with energies $\Delta E$ are at a distance $\Delta S$ from each other, the tidal force between them is obtained by

$$F = \frac{2G_0M}{r^3} \frac{\Delta E}{c^2} \Delta S. \tag{15}$$

So, the uncertainty in momentum will be given by

$$\Delta p = F\Delta t = \frac{2G_0M}{r^3} \frac{\Delta E}{c^2} \Delta x\Delta t, \tag{16}$$

where $\Delta t$ represents the life time of the particle.

If virtual particles turns into real particles having the exposure of tidal force, the uncertainty relations $\Delta p\Delta x \geq \hbar$ and $\Delta E\Delta t \geq \hbar$ can be used with reasonably well justifiable manner. Therefore, using these uncertainty relations in (16), we find that

$$(\Delta p)^2 \geq \frac{2\hbar^2G_0M}{c^2r^3}. \tag{17}$$

Accordingly, it can be written that $p^2 \approx (\Delta p)^2 \approx \frac{2\hbar^2G_0M}{c^2r^3}$. Hence applying this modified uncertainty relation, we obtain the Schwarzschild metric in the presence of minimal measurable length as

$$ds^2 = - \left( 1 - \frac{2G_0Mr^2}{c^2\left( r^3 + \alpha \frac{2\hbar^2G_0M}{c^2r^3} \right) } \right) c^2 dt^2 + \left( 1 - \frac{2G_0Mr^2}{c^2\left( r^3 + \alpha \frac{2\hbar^2G_0M}{c^2r^3} \right) } \right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{18}$$
This metric specifies a set of spacetime that depends on different scales of momentum via the modified mass of the black hole. For dimensionless case, the modified Schwarzschild metric \(18\) reads

\[
    ds^2 = -\left(1 - \frac{2Mr^2}{r^3 + 2\alpha M}\right) dt^2 + \left(1 - \frac{2Mr^2}{r^3 + 2\alpha M}\right)^{-1} dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right). \tag{19}
\]

In the following sections, for the sake of simplicity, we use the modified Schwarzschild metric \(18\) in the form of

\[
    ds^2 = -F(r\alpha)c^2 dt^2 + \frac{1}{F(r\alpha)} dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right), \tag{20}
\]

where

\[
    F(r\alpha) = 1 - \frac{2G_0Mr^2}{c^2 \left( r^3 + \alpha \frac{2G_0M}{c^2} \right)} = 1 - \frac{2MG_0}{r^3 + 2\alpha h^2 \left( \frac{G_0}{c^2} \right)} = 1 - \frac{2M}{c^2 r} G(r\alpha), \tag{21}
\]

with

\[
    G(r\alpha) = \frac{Gr^3}{r^3 + 2\alpha h^2 \left( \frac{G_0}{c^2} \right)} \tag{22}
\]

So for \(r \rightarrow 0\), \(G(r\alpha) \rightarrow G_0\), and the quantum correction disappears and we get back to the standard Schwarzschild metric. If we consider the metric in equation \(11\) to find out the position of the Horizon we need to have the solution of the equation

\[
    1 - \frac{2Mr^2}{r^3 + 2\alpha M} = 0. \tag{23}
\]

We have set \(G_0 = 0\), and \(\hbar = 0\) in equation \(22\). The solution of the equation \(23\) is found out to be

\[
    r_H = \frac{2}{3} M + \frac{4}{3} M \cos \left[ \frac{1}{3} \cos^{-1} \left( 1 - \frac{27}{8} \frac{\alpha}{M^2} \right) \right], \tag{24}
\]

provided the mass of the black hole satisfy the condition \(M > M_c\), where \(M_c\) is called the critical mass: \(M_c = \frac{27}{16} \alpha\). When \(M \leq M_c\) it fails to describe any horizon in the spacetime geometry since equation \(23\) can not provide any positive solution in that situation.

This article is devoted to study the accretion phenomena onto a spherically symmetric Schwarzschild black hole using a modified uncertainty relation that admits a quantum gravity correction that finds its place holding the hand of the concept of minimal measurable length. Although, there are other models which are associated with the GUP which correspond to the idea of minimum measurable length and maximum measurable momentum length simultaneously. This type of problem is amenable to have a solution for all types of available generalization of uncertainty relation \([49, 50]\). We will consider only generalization associated with the existence of a minimal length. In this context, we consider steady, accretion onto a modified static and spherically symmetric Schwarzschild black hole. We obtain the critical point, critical fluid velocity, temperature, mass accretion rate, and observed total integrated flux with this GUP framework.

### IV. FORMULATION OF GENERAL ACCRETION PHENOMENA ONTO THE BLACK HOLE

Spherically symmetric Schwarzschild black hole is a solution of Einstein’s equation of General relativity. Giving consideration to the generalized uncertainty principle (GUP) one may have quantum gravity corrected improved spacetime metric since GUP is fairly accepted as one of the important ingredients of quantum gravity phenomenology. This improved Schwarzschild solution of the particular type of GUP described in Sec.2 reads

\[
    ds^2 = -F(r\alpha)c^2 dt^2 + \frac{1}{F(r\alpha)} dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right), \tag{25}
\]

where \(F(r\alpha) = 1 - \frac{2GMr^2}{c^2 \left( r^3 + 2\alpha h^2 \left( \frac{G_0}{c^2} \right) \right)}\). Note that this metric provides a black hole when \(F(r\alpha) = 0\). To investigate the accretion phenomena onto this modified black hole the steady-state radial inflow of gas onto this modified black hole
is needed to bring into consideration. The gas owing to its very weak intermolecular attraction can be approximated as a perfect fluid. The energy-momentum tensor of perfect liquid is given by

$$ T_{\mu\nu} = (\mu + P)u_\mu u_\nu + Pg_{\mu\nu}. $$

(26)

Internal energy of the system is designated by $\epsilon$, where $\mu = \rho + \epsilon$. The four velocity is defined by $u^\mu = \frac{dx^\mu}{ds} = (u^0, u^1, 0, 0)$ with the condition $u_\mu u^\mu = -1$ that leads to

$$ g_{00}u^0 u^0 + g_{11}u^1 u^1 = -1, $$

(27)

which gives

$$ u^0 = \frac{F(r) + u^2}{F(r)}. $$

(28)

Here $u^1 = u$ and $u_0 = g_{00}u^0 = \sqrt{u^2 + F}$ which for the sake of simplicity, we use $F = F(r)$. The component of energy momentum tensor $T^1_0 = (\mu + P)u_0 u^1 = (\mu + P)u_0u$. From conservation of energy momentum we can write

$$ T^\nu;_\nu = 0, $$

(29)

which gives

$$ \frac{d}{dr} (T^1_0 \sqrt{-g}) = 0. $$

(30)

Therefore we have

$$ \frac{d}{dr} ((\mu + P)u_0 u^2 r^2) = 0, $$

(31)

that results out to

$$ (\mu + P)u^2 r^2 \sqrt{u^2 + F} = C_1, $$

(32)

since $\sqrt{-g} = r^2 \sin \theta$. Now mater flux is defined by $J^\mu = \rho u^\mu$. Since flux is conserved we have

$$ J^\mu;_\mu = 0, $$

(33)

which gives

$$ \frac{d}{dr} (\rho u \sqrt{-g}) = 0. $$

(34)

Therefore we ultimately find

$$ u \rho r^2 = C_2. $$

(35)

Equation (32) and (35) then leads to

$$ \frac{P + \mu}{\rho} \sqrt{u^2 + F} = \frac{C_1}{C_2} \Rightarrow \left(\frac{P + \mu}{\rho}\right)^2 (u^2 + F) = C. $$

(36)

Now taking the differential of equations (35) and (36) and eliminating $d\rho$ from resulting differentials we have

$$ \frac{du}{u} (V^2(u^2 + F) - u^2) + \frac{dr}{r} (2V^2(u^2 + F) - \frac{1}{2}r F') = 0, $$

(37)

where $V^2 + 1 = \frac{d\mu(P + \mu)}{d\ln \rho}$, and over prime indicate the derivative with respect to $r$. 
A. Sonic point and velocity of the particle at sonic point

It is known that physical results are accompanied by a critical condition [1, 61, 62]. Consider that the critical condition refers to \( r = r_c \) where velocity of the fluid increases monotonically along its trajectory (at least until the event horizon is reached), and the flow is smooth at all points, which means that both the numerator and denominator in Eqn. (37) will vanish at the same point, namely, the critical or sonic point.

\[
V_c^2(u_c^2 + F(r_c \alpha)) - u_c^2 = 0, \tag{38}
\]

\[
2V_c^2(u_c^2 + F(r_c \alpha)) - \frac{1}{2}r_c F'(r_c \alpha) = 0. \tag{39}
\]

From equation (39), we get

\[
V_c^2 = \frac{u_c^2}{(u_c^2 + F(r_c \alpha))} = \frac{r_c F'(r_c \alpha)}{4(u_c^2 + F(r_c \alpha))}, \tag{40}
\]

and substituting \( V_c \) in (38) we have

\[
u_c^2 = \frac{1}{4}r_c F'(r_c \alpha). \tag{41}\]

The spacetime metric chosen here refers \( F(r \alpha) = 1 - \frac{2GMr^2}{c^2(r^3 + 2\alpha \hbar^2 r^3 MG)} \). With the \( F(r) \) we obtain from the definition in Eqn. (21), we gain

\[
u_c^2 = \frac{MG(r^3 - \alpha \hbar^2 MG)}{2r_c^4 - 3\frac{MG}{c^2}r_c^3 + 8\alpha \hbar^2 \frac{MG}{c^2}r_c - 12\alpha \hbar^2 \frac{MG}{c^2} r_c}. \tag{42}\]

Consequently, using the expression of \( u_c^2 \) in (41), we find \( V_c^2 \) as

\[
V_c^2 = \frac{MG(r^3 - \alpha \hbar^2 MG)}{2r_c^4 - 3\frac{MG}{c^2}r_c^3 + 8\alpha \hbar^2 \frac{MG}{c^2}r_c - 12\alpha \hbar^2 \frac{MG}{c^2} r_c}. \tag{43}\]

V. POLYTROPIC SOLUTION AND COMPUTATION OF TEMPERATURE AND THE TOTAL INTEGRATED FLUX

In this section, we only discuss the case where the critical points are outside the outer horizon of this improved Schwarzschild black hole. From the studies [23–28], and specifically in [28], it has demonstrated in a decent and faithful manner that there is enough possibility of having an IR fixed point in quantum gravity and quantum effect may even be extended outside the horizon. It has been more or less In this context, it is worth mentioning that like the metric made in use in [33] this GUP modified metric also provides a black hole having both outer and inner horizons. We are now in a position to calculate quantum corrections to the temperature and to the total integrated flux related to this accretion phenomenon owing to the GUP considered in Sec. II. to have an improved metric. We settle up with the polytropic equation of state which was employed in [2] in this regard. It reads

\[
p = K \rho^\gamma. \tag{44}\]

Here \( K \) is a constant and \( \gamma \) represents adiabatic index. The temperature is defined as \( T = \frac{\hbar}{\rho} \). So \( p \) and \( \rho \) can be written down in terms of \( T \) as

\[
\rho = \frac{1}{K^n} T^n_p, \quad p = \frac{1}{K^n} T^{n+1}_p. \tag{45}\]

Here \( n = \frac{1}{\gamma-1} \), \( \epsilon + p = (n+1)p \), and \( \mu = \epsilon + p \). Therefore

\[
\frac{\mu + p}{\rho} = (n+1)T_p + 1, \tag{46}\]
which leads to

$$V^2 = \frac{(n + 1)T_p}{n[1 + (n + 1)T_p]}$$  \hspace{1cm} (47)

From equation (36) (Improved Bernoulli equation), we can write

$$\left[\frac{\mu + \rho}{\rho} \right] [u_e^2 + F(r_e\alpha)] = [1 + (n + 1)T_\infty]^2 = C.$$  \hspace{1cm} (48)

After a little algebra we find

$$[1 + (n + 1)T_p]^2[u_e^2 + F(r_e\alpha)] = [1 + (n + 1)T_\infty]^2 = C.$$  \hspace{1cm} (49)

At infinity, $T_\infty$ is considered to be small. Therefore $C$ can be approximated to

$$C \approx 1 + 2(n + 1)T_\infty.$$  \hspace{1cm} (50)

After some algebra, we find

$$[1 + (n + 1)T_p]^2[u_e^2 + F(r_e\alpha)] = [1 + (n + 1)T_\infty]^2 = C.$$  \hspace{1cm} (51)

At infinity $T_\infty$ is considered to be small and $C$ is nearly unity [33]. Therefore we have

$$C \approx 1 + 2(n + 1)T_\infty.$$  \hspace{1cm} (52)

It critical point using Eqn. (46) and (47), we express $u_e$ in terms of $T_e$ as follows:

$$nu_e^2 \left[1 - \frac{2MG'(ra)}{rc^2} + u_e^2\right] = [1 + (n + 1)T_\infty]^2 = C.$$  \hspace{1cm} (53)

Therefore (53) renders

$$T_e = \frac{nu_e^2}{(n + 1)(1 - 3u_e^2 - 2MG'(r_e\alpha))},$$  \hspace{1cm} (54)

which can be approximated to

$$T_e \approx \frac{n}{n + 1}u_e^2.$$  \hspace{1cm} (55)

Substituting (55) in equation (49), we obtain $C$ in terms of the critical parameters as follows:

$$C = \frac{(n + 1)(n - 3)u_e^2 - \frac{2M}{c^2}G'(ra)^2(1 - 3u_e^2 - \frac{2M}{c^2}G'(ra))}{(1 - 3u_e^2 - \frac{2M}{c^2}G'(ra))^2} = 1 + (2n - 3)u_e^2 - \frac{2MG'(ra)}{c^2}.$$  \hspace{1cm} (56)

Here $G'(ra) = \frac{dG'(ra)}{dr}$. We now obtain the expression of $T_e$ in terms of $T_\infty$:

$$T_e = \frac{2n}{2n - 3} [T_\infty + \frac{1}{n + 1} \frac{M}{c^2} G'(ra)].$$  \hspace{1cm} (57)

From equation (55), with the help of equation (56), it can be seen that

$$T_e^u u_e r_e^2 = \tilde{C}.$$  \hspace{1cm} (58)

Here $\tilde{C}$ is a constant that can be defined in terms of $C$ and $K$. Now substituting the expression of $T_e$ in (55), the constant $\tilde{C}$ is found out to be

$$\tilde{C} = \frac{(2n)^n G_0^2 M^2}{4c^2 [2(n + 1)]^2 \left[\frac{T_\infty}{2n - 3}\right]^{2n - 3} \left[\frac{2n - 3}{2n + 1}\right] \frac{MG'(r_e\alpha)}{c^2 T_\infty}}.$$  \hspace{1cm} (59)
Then, using the expression $T_p = \frac{\rho}{u_p}$, Eqn. (59) leads us to evaluate the temperature and matter density respectively as follows:

$$T_p = \frac{\sqrt{2n}}{4[2(n + 1)]^{\frac{2n-3}{2n}} \left( \frac{T_\infty}{2(n-3)} \right)^{\frac{2n-3}{2n}} \left( \frac{G_0 M}{m c^2} \right)^{\frac{2n-3}{2n}} \left[ 1 + \frac{2n-3}{2(n+1)} \frac{MG'(r,\alpha)}{c^2 T_\infty} \right].}$$ (60)

Since $T_\infty$ is considered to be small Eqn. (49) allows us to make this approximation.

$$\rho = \frac{1}{K^n} \left( \frac{\sqrt{2n}}{4} \right)^n \left( \frac{2(n+1)}{2(n-3)} \right)^{\frac{2n-3}{2n}} \left( \frac{G_0 M}{m c^2} \right)^{\frac{2n-3}{2n}} \left[ 1 + \frac{2n-3}{2(n+1)} \frac{MG'(r,\alpha)}{c^2 T_\infty} \right],$$ (61)

where Eqn. (45) is used. Next, applying the above information for the critical points, improved Bondi acceleration rate can be determined:

$$\dot{M} = 4\pi y^2 u_c \rho_c = \pi \left( \frac{2n}{K} \right)^n \left( \frac{2(n+1)}{2(n-3)} \right)^{\frac{2n-3}{2n}} \left( \frac{G_0 M}{m c^2} \right)^{\frac{2n-3}{2n}} \left[ 1 + \frac{2n-3}{2(n+1)} \frac{MG'(r,\alpha)}{c^2 T_\infty} \right]$$ (62)

Note that from equation (49), we have $u^2(r) \approx \frac{2MG'(r,\alpha)}{r^2}$, since at first approximation we have considered that $T_\infty << 1$. Let us now proceed to calculate the flux of the accreted mass in the form of \[8, 33\]

$$F_\nu = \frac{\epsilon_\nu \dot{M}}{4\pi d_L^2},$$ (63)

where $\epsilon_\nu$ is a constant, $d_L$ is the luminosity distance, and $L_\nu = \epsilon_\nu \dot{M}$ is the surface luminosity measured at infinity. The extra flux due to the quantum gravity effect is now given by

$$\Gamma = \frac{F_\nu - F_0\nu}{F_0\nu} = \frac{2n-3}{2(n+1)} \left[ \frac{MG'(r,\alpha)}{c^2 T_\infty} - \frac{MG'(r,\alpha)}{c^2 T_\infty} |_{\alpha=0} \right] = \frac{2n-3}{2(n+1)} \frac{MG'(r,\alpha)}{c^2 T_\infty},$$ (64)

We know that $r = r_H \approx \frac{2MG}{c^2}$ at the outer event horizon, then the temperature of the accreted matter at event horizon reads

$$T_{pH} = \sqrt{\frac{2n}{4(n+1)}} \left( \frac{T_\infty}{2(n-3)} \right)^{\frac{2n-3}{2n}} \left[ 1 + \frac{2n-3}{2(n+1)} \frac{MG'(r,\alpha)}{c^2 T_\infty} \right].$$ (65)

Hence, for $\gamma = \frac{4}{3}$, which corresponds to $n = 3$, the extra integrated flux \[64\] obtains as

$$\Gamma = \frac{3}{8} \frac{MG'(r,\alpha)}{c^2 T_\infty}.$$ (66)

However to have a precise determination of $T_{pH}$ we have to use the expression $r_H$ from the equation \[24\] maintaining the condition $M \leq \frac{2GM}{c^2}$. It depends on $T_\infty$ and $\alpha$ which is expected to vanish at $\alpha \to 0$ and that becomes evident from the expression \[64\]. Therefore, the introduction of GUP shows prominent quantum gravity correction in the thermal flux associated with the accretion and it ceases if GUP is replaced by the usual Heisenberg uncertainty relation. In our investigation, we did not include the effect of back-reaction of the accreting fluid on the spacetime geometry.

VI. DISCUSSION AND CONCLUSION

In this work, we have considered a static and spherically symmetric Schwarzschild black hole. The quantum gravity corrections to the Schwarzschild black hole metrics have been accounted for by applying the generalized uncertainty principle. Analyzing the gravity-induced quantum interference pattern and the Gedanken-experiment for measuring the weight of a photon, it has been found that the modified metric gets induced by the generalized uncertainty principles compatible with the minimum measurable length and that led to quantum gravity corrections to Schwarzschild black hole metrics.

We have considered our modified metric to examine the accretion of matter (perfect fluid) onto the black. With this metric, we have investigated the basic equations of motion for the radial flow of matter onto this black hole. We have obtained the critical point, fluid velocity, and velocity of sound during accretion onto the modified black hole.
Following the article [33], we have determined the corrected temperature and the integrated flux at the event horizon resulting from quantum gravity effects associated with the specific GUP launched here, for the polytropic matter. We observe that the critical point depends crucially on the GUP parameter that induces the quantum gravity effect in our model. The fluid velocity and velocity of sound during accretion onto this modified black hole and the flux resulting from the quantum gravity effect all depend on the GUP parameter $\alpha$ in a significant manner. If we switch off the quantum effect by inserting $\alpha = 0$ correction due to quantum effect reduces to zero. We have already mentioned that Back-reaction effects on the black hole geometry have been ignored in our computation. Although it is reasonable to ignore the effect of back-reaction for a black hole with a very large mass, it cannot be ignored for a lower-mass black hole. If the size of the black hole reduces up to the extent where mass becomes less than a critical mass, the back reaction in that situation can not be neglected since the self-gravity effect of the in-falling flow of fluid might play an important role [63, 64]. To the best of my knowledge, the study of accretion phenomena through the GUP framework maintaining asymptotic safe scenarios has not yet been reported earlier. To include the effect of back-reaction requires much involved mathematical analysis [63–68]. However, the formulation we have adopted here to include the GUP effect can be extended with the effect of back-reaction indeed.

The modified metric taken into account here can be regarded as equivalent to the running coupling constant because of the fact that the gravitational constant has acquired position (r) dependency. On the hand from the studies [23–28], it has demonstrated in a decent and faithful manner that there is enough possibility of having an IR fixed point in quantum gravity. As a result, there should be a considerable quantum correction during the accretion process. It would not be a good idea to exclude it. However, it is fair to admit that the overall formulation of correlating UV fixed point with IR fixed point through RG prescription is challenging and considerably complicated [77]. The benevolent UV/IR mixing conjecture [78, 79] may be helpful in this regard.

**Data Availability Statement:** No data is associated with the manuscript.

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