On the radius of strong interaction of hadrons at asymptotically high energies

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Abstract

The uncertainty relation is established for systems developing by the cascade decays. On its basis, it is shown that the diffusion of partons in the impact parameter plane is possible only simultaneously with similar diffusion in the transverse-momentum space. The assumption that partons transverse momenta are bounded on average, leads to a logarithmic growth of the radius of strong interaction of hadrons at asymptotically high energies.

1 Introduction

The Froissart bound, which establishes the maximum possible increase in the total cross sections for strong interactions of hadrons,

\[ \sigma_t(s) \leq \frac{4\pi}{t_0} \ln^2 s, \quad s \to \infty, \]

is derived [1, 2, 3] from general principles of local field theory, such as unitarity and analyticity \((t_0\) is the rightmost point on the Martin ellipse). The mode of saturation of this bound corresponds to a physical picture of a black disk with an exponentially falling border and a logarithmically growing radius, \(R \sim \ln s\) as \(s \to \infty\) (see e.g. [4]). However, it is still not known whether a saturation of the bound (1) occurs. In particular, the dynamic reasons leading to the mentioned saturation mode are unknown. In this regard, it is of interest to identify independent conditions leading to the above physical picture or any part of it.

In this letter, we show that the logarithmic increase in the radius of interaction occurs as a consequence of a restriction of the transverse momenta of partons in softly colliding fast hadrons. The justification is very general in nature and is based on the Heisenberg uncertainty relation. However, we use this fundamental relation in a specific form corresponding to the problem under consideration. Namely, we find that in systems developing by the cascade decays, the product of variance of the coordinates of final particles (counted from the geometric center) and the corresponding variance of momenta is proportional to the number \(N\) of the cascade decays. As a result, it
is possible a symmetry between the evolution patterns in configuration and momentum spaces, i.e. the growth of both variances proportionally to $\sqrt{N}$ with increasing number of decays. However, if the variance in one of the spaces is limited due to some dynamic reasons, this entails an increased growth of another variance so that the product of the variances remains proportional to $N$.

In relation to the evolution of partons in a fast-moving hadron, we first note that the maximum number of decays in cascades increases with energy as $\ln s$ [5, 6]. In turn, partons during decays in general propagate in perpendicular directions with respect to the hadron motion, which is described by the impact parameter $\rho$ and transverse momenta $k$. So, as long as partons are independent of each other, this propagation is unlimited and the root mean square (rms) values of $\rho$ and $k$ actually evolve as $\sqrt{\ln s}$. Hence, the hadron interaction radius increases as $\sqrt{\ln s}$ (the picture of pomeron exchange). However, if at some stage the transverse momenta become bounded, then further $\Delta \rho$ increases as $\ln s$. This means asymptotic behavior $R \sim \ln s$ as $s \to \infty$ for the hadron interaction radius.

In the next section, we give formal derivation of the relation $\Delta \rho \Delta k \geq N$ in the case of systems developing by the cascade decays, and discuss some consequences of this relation. The appropriate scenarios for the evolution of partons in a fast-moving hadron are considered in Section 3. In conclusion, we discuss the results.

## 2 Uncertainty relation for cascade processes

First of all, we recall that the uncertainty relation resulting from commutation relations

$$[\hat{x}_i, \hat{k}_j] = i\delta_{ij},$$

(2)

takes the well-known form $\Delta x_i \Delta k_i \geq 1/2$ for one-dimensional motion only. In the case of arbitrary particle motion in $n$-dimensional space the uncertainty relation is written as

$$\Delta x \Delta k \geq n/2,$$

(3)

where $(\Delta x)^2 = \langle \sum \hat{x}_i^2 \rangle - \langle \hat{x}_i \rangle^2$ and similarly for $\Delta k$. The actual value of the product of variances depends on the state in which the system is found. The vacuum-like state with Gaussian distribution has the least uncertainty. The corresponding wave functions in the coordinate and momentum representations have the form:

$$<\vec{x}|\psi> \equiv \psi(\vec{x}) = (\mu^2/\pi)^{n/4} e^{-\vec{x}^2/\mu^2},$$

(4)

$$<\vec{k}|\psi> \equiv \tilde{\psi}(\vec{k}) = (4\pi/\mu^2)^{n/4} e^{-\vec{k}^2/(2\mu^2)}.$$

(5)

Here, it is assumed that the particle is on average at the origin and at rest, $\mu^2$ is a single parameter of variance,

$$<\psi|\hat{x}^2|\psi> = \frac{n}{2}\mu^{-2}, \quad <\psi|\hat{k}^2|\psi> = \frac{n}{2}\mu^2.$$

(6)

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1Defined as a normalized solution to the equation $\hat{a}^- \psi = 0$, where $\hat{a}^- = (\hat{k} - i\mu^2\hat{x})/\sqrt{2\mu^2}.$
If a particle is under the influence of external forces, its wave function may be non-Gaussian. In this case a strict inequality is realized in (3).

Next, we consider a speculative model that describes a system of particles in two-dimensional space, developing by sequential splitting. This system will simulate the behavior of partons in the perpendicular plane relative to the direction of motion of the fast hadron. We assume that each particle of the system emits once a similar particle, as it occurs in the case of development of a multi-peripheral comb [6]. However, we do not turn to any model of field theory and limit ourselves to non-relativistic quantum mechanics, taking advantage of the fact that for a fast hadron the motion in the perpendicular plane is non-relativistic. In accordance with this, we characterize the position of each particle \( i \) by the radius vector \( \vec{r}_i \) counted from the position of the parent particle. Herewith, the radius vector of the first particle \((i = 1)\) is counted from the geometric center of the system. Each particle is described by the wave function of \( \vec{r}_i \). Assuming that all particles are independent and distributed relative to parent particles with a variance \( \mu^{-2} \), we write the wave function of a system with \( N \) splittings as

\[
\Psi_N(\{\vec{r}_i\}) = \prod_{i=1}^{N} \psi(\vec{r}_i) = \left(\frac{\mu^2}{\pi}\right)^{N/2} \exp \left\{ -\frac{\mu^2}{2} \sum_{i=1}^{N} \vec{r}_i^2 \right\}.
\]

It is worth noting a translational non-invariance of the given method of specifying the system, which is the pay for taking into account the quantum trembling of each particle of the system. We also emphasize the introduction of sequentially relative coordinates that allows both factorization and separation of variables. When describing, for example, in terms of absolute distances from the geometric center, the factorization is preserved but the variables are mixed up due to the contributions of scalar products of the vectors. Simultaneously, each particle in this parametrization is found effectively in its own two-dimensional space. Hence it is evident an analogy of the introduced system of \( N \) particles with one particle placed in \( 2N \)-dimensional space.

In the momentum representation, the wave function (7) has the form

\[
\tilde{\Psi}_N(\{\vec{k}_i\}) = \prod_{i=1}^{N} \tilde{\psi}(\vec{k}_i) = \left(\frac{4\pi/\mu^2}{\mu^2}\right)^{N/2} \exp \left\{ -\frac{1}{2\mu^2} \sum_{i=1}^{N} \vec{k}_i^2 \right\}.
\]

Here \( \vec{k}_i \) are local momenta of the particles. Their averages are zero, and each momentum is distributed with a variance \( \mu^2 \). In the context of the parton model \( \mu^2 \) is determined by the scale of the transverse momenta arising at the parton splittings. From the viewpoint of the considered speculative model \( \mu^2 \) should be connected with the effective mass \( m \) of the particles of the system. Since the system is non-relativistic, the connection has the form \( \mu^2 = m\omega \), where \( \omega \) is an auxiliary parameter such that to ensure relation \( m \gg \mu \). One may imagine the occurrence of \( \omega \) as the result of action of a weak \( (\omega/\mu \ll 1) \) effective oscillatory potential emanating from parent particles. Such a potential does not change the Gaussian structure of the wave function and provides non-relativistic properties of the system. Both of these conditions is what we need from the system \( (m \text{ and } \omega \text{ are not used further}) \).
Consider now the last particle formed as a result of the $N$-th decay. Its position relative to the geometric center is described by the vector

$$\vec{R} = \sum_{i=1}^{N} \vec{r}_i. \quad (9)$$

Actually $\vec{R}$ has the meaning of a collective variable describing development of the system as a whole. The average of $\vec{R}$ is, obviously, zero (the system does not shift relative to the geometric center). So its rms value determines the variance:

$$(\Delta R)^2 = \langle \vec{R}^2 \rangle = \langle \Psi | \sum_{i=1}^{N} \hat{r}_i^2 + \sum_{i \neq j}^{N} \hat{r}_i \hat{r}_j | \Psi \rangle. \quad (10)$$

The first sum in (10) contains $N$ terms, the second one about $N^2$. So $\Delta R$ can not grow faster than $N$. If the particles are not correlated with each other, then contribution of the second sum vanishes, and $(\Delta R)^2 \sim N$. In the particular case of wave function (7), we have

$$(\Delta R)^2 = N \mu^{-2}. \quad (11)$$

Since the system is non-relativistic, the momentum collective variable is the dual to (9) vector,

$$\vec{K} = \sum_{i=1}^{N} \vec{k}_i. \quad (12)$$

$\vec{K}$ has the meaning of the momentum in the center-of-mass frame of the last particle formed at the $N$-th splitting, provided that the effective masses of the particles of the system are equal each other. Since $\vec{K}$ is distributed around zero (the momentum is conserved), its variance is determined by the rms value,

$$(\Delta K)^2 = \langle \vec{K}^2 \rangle = \langle \Psi | \sum_{i=1}^{N} \hat{k}_i^2 + \sum_{i \neq j}^{N} \hat{k}_i \hat{k}_j | \Psi \rangle. \quad (13)$$

If the particles are not correlated, then $(\Delta K)^2 \sim N$, and in the case of wave function (8),

$$(\Delta K)^2 = N \mu^2. \quad (14)$$

So, $\Delta K$ increases as $\sqrt{N}$. This increasing means an accumulation of uncertainty in the value of momenta during successive splitting, and the $\Delta K$ itself characterizes the size of the system in the two-dimensional momentum space.

From (11) and (14) it follows the relation

$$\Delta R \Delta K = N. \quad (15)$$

If the distribution of particles were not Gaussian, then instead of equality (15) we would get the inequality, similar to (3). This result can be obtained in the most general case by noting that the fundamental commutation relation (2) implies

$$[\hat{R}_\alpha, \hat{K}_\beta] = iN \delta_{\alpha \beta}. \quad (16)$$
where $\alpha$, $\beta$ are components of vectors in two-dimensional space. Acting further by standard scheme and taking into account the two-dimensionality of space, we obtain

$$\Delta \mathbf{R} \Delta \mathbf{K} \geq N. \quad (17)$$

This result has far-reaching consequences. For instance, if in the r.h.s. (10) and (13) in one of the expressions we get negative correlations, which reduce the growth rate of the corresponding variance, then the variance in the other expression must undergo enhanced growth. For example, if at large $N$ the behavior $\Delta \mathbf{K} \sim 1$ is established, then (17) requires that there should be positive correlations in (10) resulting in the behavior $\Delta \mathbf{R} \sim N$. Of course, the wave function of the system in this case can not be Gaussian.

To describe the behavior of the system as a whole, it is convenient to introduce probability density distributions over collective variables. According to the definitions of $\mathbf{R}$ and $\mathbf{K}$, they would describe the distributions of the last particle formed in the cascade decays. In the configuration space the corresponding density (normalized to unity) is

$$F_N(\mathbf{R}) = \int \left( \prod_i d\mathbf{r}_i \right) \delta(\mathbf{R} - \sum_i \mathbf{r}_i) |\Psi_N(\{\mathbf{r}_i\})|^2. \quad (18)$$

In the case of Gaussian behavior (7), direct calculations yield

$$F_N(\mathbf{R}) = \mu^2 \pi^N \exp \left\{ - \frac{\mathbf{R}^2 \mu^2}{N} \right\}. \quad (19)$$

From this, in full accordance with (11), it follows

$$\langle \Delta \mathbf{R} \rangle^2 = \langle \mathbf{R}^2 \rangle = N \mu^{-2}. \quad (20)$$

In momentum space, a similar distribution density is defined as

$$\mathcal{F}_N(\mathbf{K}) = \int \left[ \prod_i \frac{d\mathbf{k}_i}{(2\pi)^2} \right] (2\pi)^2 \delta(\mathbf{K} - \sum_i \mathbf{k}_i) |\tilde{\Psi}_N(\{\mathbf{k}_i\})|^2. \quad (21)$$

Substitution of (8) gives

$$\mathcal{F}_N(\mathbf{K}) = \frac{4\pi}{N \mu^2} \exp \left\{ - \frac{\mathbf{K}^2 \mu^2}{N \mu^2} \right\}, \quad (22)$$

and

$$\langle \Delta \mathbf{K} \rangle^2 = \langle \mathbf{K}^2 \rangle = N \mu^2. \quad (23)$$

Now we discuss the results. First of all, we recall that Gaussian character of the dependence on collective variables in (19) and (22) is a consequence of the conditions set out above formula (7). A remarkable property of a system obeying such conditions is that its development may be described as a random Wiener process, i.e. a diffusion-like process. This is reflected in the fact that both distribution (19) and (22) are the
fundamental solutions to the two-dimensional diffusion equation in which \( N \) plays the role of diffusion time. Earlier [6] this property was the basis for determining the parton distribution in the impact parameter plane, where the parton rapidity played the role of the time. Additionally [6] suggested that in the transverse momenta space the diffusion does not occur. However, as we have seen above, such an assumption is incompatible with the property of diffuse propagation in the configuration space. A diffusion process can occur only simultaneously in both spaces. If there is no diffusion expansion in the transverse momentum space, i.e. \( \Delta K \sim 1 \) with the growth of \( N \), then by virtue of (17) the variance \( \Delta R \) should increase as \( N \), but not \( \sqrt{N} \) as at the diffusion. In this case, the wave function of the system must be radically rearranged and the distribution density can not be Gaussian. This scenario can be realized if the dependence on \( N \) disappears in \( F_N \) at \( N \to \infty \). Then \( \Delta K \sim 1 \) is set, and correspondingly \( \Delta R \sim N \).

\section{Parton distributions in the transverse plane}

According to modern concepts, parton distributions in fast-moving hadrons are established due to cascade decays, during which fast moving partons give rise to slow ones. There is an extensive literature devoted to study distribution of partons over longitudinal momenta. In the transverse directions the distribution was studied less thoroughly. The main results obtained in this area in the traditional parton model are presented in the lectures of Gribov [6] and in the monographs [4, 7]. Here we supplement these studies with the results of our analysis.

However, we must first make an important clarification related to the fact that in a real hadron the number of partons is not fixed. Correspondingly, the hadron is characterized not by a single wave function, but by their set with a different number of partons. Given this fact, the probability distribution over collective variables characterizing the hadron must be written as

\[
F(\vec{R}) = \sum_{N=1}^{N_{\text{max}}} |c_N|^2 F_N(\vec{R}),
\]

and similarly for \( F(\vec{K}) \). Here \( N_{\text{max}} \) is a maximum number of decays or partons in a cascade. If simultaneously several cascades are developing in the hadron, we consider them combined by means of obvious generalization of (18). We also neglect the differences in the positions of the centers of different cascades and between the initial values of their transverse momenta. These simplifications are inessential for our analysis since ultimately we are interested in asymptotic properties of distributions.

Coefficients \( |c_N|^2 \) in (24) determine the probabilities that fast hadron consists of \( N \) partons, and satisfy the relation

\[
\sum_{N=1}^{N_{\text{max}}} |c_N|^2 = 1.
\]

\section*{Footnote}

\footnotetext{2}{Below we reproduce this distribution in (29).}
A priori, $|c_N|^2$ are unknown and so the distributions $F(\mathcal{R})$ and $\mathcal{F}(\mathcal{K})$ are unknown, too, whatever the partial distributions $F_N$ may be. However, (24) provides some information about the behavior of $F(\mathcal{R})$ and $\mathcal{F}(\mathcal{K})$. Namely, since at large $\mathcal{R}$ and $\mathcal{K}$ the main contributions in the sum (24), and in the similar sum for $\mathcal{F}$, are made by densities with widest possible distributions, the $F$ and $\mathcal{F}$ for asymptotically large $\mathcal{R}$ and $\mathcal{K}$ are approximated by partial distributions $F_{N_{\text{max}}}$ and $\mathcal{F}_{N_{\text{max}}}$. Simultaneously, the inner areas are dominated by contributions of relatively narrow distributions. Therefore, the distributions $F$ and $\mathcal{F}$ should be narrowed compared to the uttermost distributions $F_{N_{\text{max}}}$ and $\mathcal{F}_{N_{\text{max}}}$. 

In general, the variances $\Delta \mathcal{R}$ and $\Delta \mathcal{K}$ of the $F$ and $\mathcal{F}$ are the averages of the partial variances. So, if $F_N(\mathcal{R})$ and $F_N(\mathcal{K})$ are Gaussian, we can calculate $\Delta \mathcal{R}$ and $\Delta \mathcal{K}$. Really, by virtue of (24) and taking into account (20), we obtain

$$ (\Delta \mathcal{R})^2 = \sum_{N=1}^{N_{\text{max}}} |c_N|^2 N \mu^{-2} = \bar{N} \mu^{-2}. $$

Here $\bar{N}$ is the average number of partons. Its value, obviously, lies in the interval $1 < \bar{N} < N_{\text{max}}$. So in the general case $\bar{N} = \kappa N_{\text{max}}$, where $\kappa < 1$. From here

$$ (\Delta \mathcal{R})^2 = \kappa \mu^{-2} N_{\text{max}}. $$

Similarly, taking into account (23), we get

$$ (\Delta \mathcal{K})^2 = \kappa \mu^2 N_{\text{max}}. $$

For a more detailed determination of the variances it is necessary to estimate $N_{\text{max}}$. With this purpose, we use the result about the number of decays $N$ in a cascade [6, 7]. The essence is that the parton rapidities in the direction of hadron motion each time change during the decays by a value of the order of unity. Therefore, the number of decays in the cascade is proportional to the rapidity shift, $N = \gamma (\eta_P - \eta)$, where $\eta_P \approx \ln 2P/\mu$ is the maximal rapidity of the parton moving with a longitudinal momentum $P$, $\eta$ is the final parton rapidity, $\gamma$ is a dimensionless coefficient, and $\mu$ is a scale of strong interactions in hadron.

The obtained estimate is useful in many ways. In particular, it allows one to determine the distribution of partons depending on their rapidity. Really, by leaving in (24) only the contributions of cascades with partons that have reached the rapidity $\eta$ and, moreover, by taking these contributions with truncation on the partons with rapidity $\eta$, we obtain distribution

$$ F_\eta^* (\rho) = \frac{C(\eta) \mu^2}{\pi \gamma (\eta_P - \eta)} \exp \left\{ -\frac{\rho^2 \mu^2}{\gamma (\eta_P - \eta)} \right\}. $$

Here $\rho$ is a collective variable defined by (9) where the sum is taken over partons with rapidity at least $\eta$. Formula (29) is derived taking into account the fact that the partial truncated contributions are defined by (18) with $\mathcal{R}$ replaced by $\rho$, and all of them are equal to (19) with $N = \gamma (\eta_P - \eta)$. The above result (29) reproduces formula...
(12) of [6], obtained by analysing the front of the “diffusion wave”. From this formula it follows that partons with the minimal rapidity $\eta \approx 0$ have the widest distribution. Such partons complete evolution in cascades. In this limiting case $\rho$ coincides with $R$ and (29) becomes $F_{N_{\text{max}}}(R)$.

From the above we have $N_{\text{max}} = \gamma \ln 2P/\mu$. By virtue of (27) this implies an estimate of the transverse size of fast-moving hadron, $\Delta R = \mu^{-1} \sqrt{\kappa \gamma \ln 2P/\mu}$, and further a known result for the radius of interaction of hadrons at pomeron exchange,

$$R \sim \sqrt{\ln s}, \quad s \to \infty.$$  \hspace{1cm} (30)

From (27) there also follows another remarkable result on independence from the energy of the density of number of partons in the transverse plane for fast hadron. We can get a quantitative estimate of this density. Really, the hadron cross-sectional area is estimated as $S = \pi (\Delta R)^2$. In turn, $\Delta R$ can be related to the slope of the diffraction cone of elastic hadron scattering by well-known formula $B = (\Delta R)^2/2$. On the other hand, $B = B_0 + 2\alpha'(0) \ln s/s_0$. Collecting formulas, at $P \to \infty$ we get

$$\varrho \equiv \frac{N_{\text{max}}}{S} = \frac{\gamma}{8\pi \alpha'(0)}.$$ \hspace{1cm} (31)

Assuming that partons carry on average half the momentum of the parton-parent, we have $\gamma = (\ln 2)^{-1} [4]$. From the analysis of elastic $pp$ and $p\bar{p}$ scattering data, including those from the TOTEM experiment, $\alpha'(0) = 0.165$ GeV$^{-2}$ [8]. A substitution of these estimates to (31) gives $\varrho \approx (0.3 \text{ fm})^{-2}$. This means that in a fast-moving hadron each parton occupies an area with a diameter of approximately 0.3 fm, and this value remains constant with increasing energy. This gives rise to a very simple physical picture: new partons that produced with increasing energy push the existing ones apart, and this leads to swelling the transverse size of the hadron with preserving the average transverse density.

At the end of this discussion, we note that by virtue of (27) and (31) we have $\mu \approx \kappa^{1/2} \times 1$ GeV, i.e. $\mu < 1$ GeV. This seems quite natural for the characteristic scale of the transverse momenta at the parton decays, see (8). Taking into account this result, formula (27) can be rewritten as

$$(\Delta R)^2 \approx 1 \text{ GeV}^{-2} \times N_{\text{max}} \approx (0.2 \text{fm})^2 \times N_{\text{max}}.$$ \hspace{1cm} (32)

Recall that this estimate is asymptotic, i.e. it is valid only for large hadron momenta (large $N_{\text{max}}$). We also emphasize that this estimate was obtained under the assumption of the diffusive nature of the propagation of partons in the transverse plane.

Now we turn to a discussion of the parton distribution by transverse momenta. As noted above, our conclusions at this point diverge from [6, 7] where it was assumed that partons are subject to diffusion in the impact parameter plane, but their transverse momenta are limited. The rationale for the latter assumption was the observation of the limited transverse momenta of secondary hadrons formed in soft hadron collisions. Unfortunately, at current energies the experimental situation at this point is not entirely clear. However, regardless of the experimental situation the assumption [6, 7] is not compatible with the diffusion nature of the development of
the parton cascades. Really, if the variance in the transverse momenta $\Delta K$ does not depend on the number of decays, then the variance $\Delta R$ should increase according to (17) in proportion to the number of decays, i.e. as $\ln s$. This behavior is in direct contradiction with conclusions of [6, 7].

The contradiction is removed if we assume that both distributions are diffusion, in the impact parameter plane and in the transverse momenta space. In this case by virtue of (28) we have

$$\langle \Delta K^2 \rangle \approx \rho^2 \times 1 \text{ GeV}^2 \times N_{\text{max}}. \quad (33)$$

We emphasize the presence of factor $\rho^2$ in the r.h.s. in (33). For finite $N_{\text{max}}$, it allows one to parametrically reduce the estimate of $\Delta K$ to almost any value below 1 GeV.

Nevertheless, if $N_{\text{max}} \to \infty$ and $\rho$ is finite, estimate (33) leads to unlimited growth of $\Delta K$. From our point of view, after exceeding a certain threshold this would contradict the integrity of the hadron. To overcome the absurd one must assume that for some dynamic reason $\Delta K$ can not exceed a certain bound $\Delta K_0$. This means that when $\Delta K$ approaches this bound, the wave function of the system must be radically rearranged so that the dependence on $N_{\text{max}}$ in $F$ disappears. This case was considered at the end of the previous section. The result for $\Delta R$ is to change from (32) to

$$\Delta R \sim (\Delta K_0)^{-1} N_{\text{max}}. \quad (34)$$

In turn, this leads to a logarithmic growth of the radius of hadron interactions,

$$R \sim \ln s, \quad s \to \infty. \quad (35)$$

The behavior of (34) in view of (10) means the mode of maximum correlations when moving in the transverse plane, whereas (32) corresponds to uncorrelated movement. We emphasize that after the mode change the wave function of the system can not be represented in the factorized form, as was assumed in (7) and (8), as each parton begins to "feel" the presence of other partons. The reason for this mode change is unclear. Perhaps it may be related to the increasing with the energy of the longitudinal part of the space occupied by the hadron [9, 10, 11] (effect of uncertainty in the localization). As a results, the distances between partons might effectively increase. If so, this should lead to an increase in coupling, which may be the cause of correlated motion of the partons. However, further research is required to clarify this possibility.

### 4 Conclusion

We investigated parton distributions in the transverse plane with respect to direction of motion of a fast hadron. To identify the main patterns, we turned to a speculative quantum-mechanical model that imitates the behavior of partons in the transverse projection. From its viewpoint partons are represented by quasi-particles that spontaneously multiply by means of successive cascade decays, remaining at rest on average.
The effective mass of the quasi-particles is determined by the scale of transverse momenta arising from decays of real partons, and thus the decays of quasi-particles occur without loss of masses. Simultaneously, the propagation of quasi-particles during decays takes place due to inclusion of new regions in accordance with the uncertainty relation, of the position and momentum counted from the points of the decay of parent particles. As a whole, the propagation is controlled by a specific uncertainty relation relevant for systems developing by the cascade decays.

Based on this relation, we show that in the absence of correlations between partons their rms transverse coordinates and momenta both increase as $\sqrt{\ln P}$ with increasing momentum $P$ of the hadron. If the transverse momenta are limited, this means appearance of correlations between partons. For transverse coordinates this leads to the growth as $\ln P$. In turn, this means a logarithmic growth (35) of the radius of strong interaction of hadrons at asymptotically high energies.

From the viewpoint of Froissart bound, the latter result implies a detection of an independent dynamic condition resulting in fulfillment the major provision for its saturation. If it is found that the transverse parton momenta are limited on average at high-energy soft hadron interactions, then the results of this work coupled with the results [1, 2, 3, 4] will be a strong argument in favor of saturation of the Froissart bound at asymptotically high energies.

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