The Bosonic Ancestor of Closed and Open Fermionic Strings

François Englert, a, Laurent Houart b,3 and Anne Taormina c,4

Abstract

We review the emergence of the ten-dimensional fermionic closed string theories from subspaces of the Hilbert space of the 26-dimensional bosonic closed string theory compactified on an $E_8 \times SO(16)$ lattice. They arise from a consistent truncation procedure which generates space-time fermions out of bosons. This procedure is extended to open string sectors. We prove, from bosonic considerations alone, that truncation of the unique tadpole-free $SO(2^{13})$ bosonic string theory compactified on the above lattice determines the anomaly free Chan-Paton group of the Type I theory. It also yields the Chan-Paton groups making Type O theories tadpole-free. These results establish a link between all M-theory strings and the bosonic string within the framework of conformal field theory. Its significance is discussed.

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2 E-mail : fenglert@ulb.ac.be
3 Research Associate of the FNRS, E-mail : lhouart@ulb.ac.be
4 E-mail : anne.taormina@durham.ac.uk
1 Introduction

It has been demonstrated previously that all the ten-dimensional closed superstring theories (Type IIA, Type IIB and the two distinct heterotic superstrings) are hidden in the Hilbert space of the 26-dimensional closed bosonic string theory \[1\]. The emergence of space-time fermions and of supersymmetry, anticipated by Freund \[2\], is an impressive property of the bosonic string. The generation of space-time fermions out of bosons appears in reference \[1\] as a stringy generalisation of the field theoretical mechanism by which non abelian monopoles become fermions in an appropriate environment \[3\].

The superstrings arise from a toroidal compactification of the bosonic strings on an \(E_8 \times E_8\) lattice where only the second \(E_8\) plays an active rôle. The superstring content of the bosonic string appears when all states pertaining to the first \(E_8\) are removed from the spectrum. We call this a “truncation”. The truncation required to reveal superstrings has in fact to be extended to all oscillator states in four of the eight compact dimensions spanned by the second \(E_8\) lattice of zero modes. However, some particular zero modes have to be retained in these dimensions. They play an essential rôle in the construction.

The \(E_8\) lattice is a sublattice of the \(SO(16)\) lattice and the theory is more elegantly formulated in terms of \(E_8 \times SO(16)\) \[4\]. This formulation was in fact a crucial step, because it led to uncover not only the superstrings, but also the non-supersymmetric fermionic strings \cite{4} Type OA and Type OB \cite{4} and all the consistent non-supersymmetric ten-dimensional heterotic strings discovered in reference \cite{1} emerged indeed from the bosonic string compactified on an \(E_8 \times SO(16)\) lattice, using the same truncation that generated the superstrings out of its \(E_8 \times E_8\) sublattice \cite{7, 8, 9}.

We extend the truncation process to open string sectors. We find that the Chan-Paton group \(SO(32)\) which is required to make the supersymmetric Type I superstring anomaly free, as well as the Chan-Paton groups eliminating tadpoles in Type O fermionic strings, are determined by the bosonic parent. The significance of these results will be discussed.

This report summarises the main results obtained in reference \cite{10}.

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\(^{1}\)We shall call fermionic strings all strings containing space-time fermions in separate left or right sectors whether or not they are projected out in the closed string spectrum.
2 Truncation of the closed bosonic string

2.1 Space-time fermions from bosons

To accommodate space-time fermions in the 26-dimensional bosonic string one must meet three requirements:

a) A continuum of bosonic zero modes must be removed. This can be achieved by compactifying \( d = 24 - s \) transverse dimensions on a \( d \)-dimensional torus. This leaves \( s + 2 \) non-compact dimensions with transverse group \( SO_{\text{trans}}(s) \).

b) Compactification must generate an internal group \( SO_{\text{int}}(s) \) admitting spinor representations. This can be achieved by toroidal compactification on the Lie lattice of a simply laced Lie group \( G \) of rank \( d \) containing a subgroup \( SO_{\text{int}}(s) \). The latter is then mapped onto \( SO_{\text{trans}}(s) \) in such a way that the diagonal algebra \( so_{\text{diag}}(s) = \text{diag}[so_{\text{trans}}(s) \times so_{\text{int}}(s)] \) becomes identified with a new transverse algebra. In this way, the spinor representations of \( SO_{\text{int}}(s) \) describe fermionic states because a rotation in space induces a half-angle rotation on these states.

c) The consistency of the above procedure relies on the possibility of extending the diagonal algebra \( so_{\text{diag}}(s) \) to the new full Lorentz algebra \( so_{\text{diag}}(s + 1, 1) \), a highly non trivial constraint. To break the original Lorentz group \( SO(25, 1) \) in favour of the new one, a truncation consistent with conformal invariance must be performed on the physical spectrum of the bosonic string. Actually, states described by 12 compactified bosonic fields must be truncated, except for zero modes \([1, 4]\).

This can be understood heuristically by counting the missing central charge, hidden in the light cone gauge, needed for the superghost and the longitudinal and time-like Majorana fermions. In units where each two-dimensional boson contributes 1 to the Virasoro central charge, the superghost contributes 11, while the longitudinal and time-like Majorana fermions contribute \( 2 \times 1/2 \). Therefore, one must truncate 12 compactified bosonic fields. The larger value of the transverse dimension is then \( s = 8 \), as the four remaining dimensions in the light cone gauge can then accomodate the internal group \( SO_{\text{int}}(8) \). The highest available space-time dimension accommodating fermions is therefore \( s + 2 = 10 \). The truncation should also remove the zero-point energies of the truncated oscillators. We shall show below that this amounts to retain some zero-modes of the 12 truncated bosonic fields.

\(^2\)We shall designate all locally isomorphic groups by the same notation.
2.2 Compactification and truncation

Consider the bosonic closed string compactified on a $d$-dimensional torus. In terms of the left and right compactified momenta, the mass spectrum is

$$\frac{\alpha' m_L^2}{4} = \alpha' p_L^2 + N_L - 1,$$

$$\frac{\alpha' m_R^2}{4} = \alpha' p_R^2 + N_R - 1,$$

and

$$m^2 = \frac{m_L^2}{2} + \frac{m_R^2}{2}, \quad m_L^2 = m_R^2. \quad (2)$$

In Eq. (1) $N_L$ and $N_R$ are the oscillator numbers in 26-dimensions and the zero modes $\sqrt{2\alpha'}p_L, \sqrt{2\alpha'}p_R$ span a 2$d$-dimensional even self-dual Lorentzian lattice with negative (resp. positive) signature for left (resp. right) momenta. This ensures modular invariance of the closed string spectrum [\ref{1}]. For generic toroidal compactifications, the massless vectors $\alpha_{-1,1}^\mu, \alpha_{-1,1}^i, \alpha_{1,1}^\mu, \alpha_{1,1}^i$ and are the eightfold degenerate vectors $\sqrt{2\alpha'}L_\mu, \sqrt{2\alpha'}L_i$ of the weight lattice of all even (for class $(c)$) states. Namely $\alpha_{-1,1}^\mu, \alpha_{-1,1}^i, \alpha_{1,1}^\mu, \alpha_{1,1}^i$ number of minus signs. The structure of the weight lattice is the same. In particular, this is the the root lattice, where the indices $\mu$ and $i$ respectively refer to non-compact and compact dimensions, generate a local symmetry $[U_L(1)]^d \times [U_R(1)]^d$. But more massless vectors arise when $\sqrt{2\alpha'}p_L$ and $\sqrt{2\alpha'}p_R$ are roots of simply laced groups $G_L$ and $G_R$ (with roots length $\sqrt{2}$). The gauge symmetry is enlarged to $G_L \times G_R$ and the theory is modular invariant provided the lattice of zero modes is self-dual Lorentzian even. In particular, this is the case for compactification on a $G \times G$ lattice where $G$ is any semi-simple simply laced group of rank $d$, if both $\sqrt{2\alpha'}p_L$ and $\sqrt{2\alpha'}p_R$ span the full weight lattice $\Lambda_{\text{root}}$ of $G$, but are constrained to be in the same conjugacy class. Namely $\sqrt{2\alpha'}(p_L - p_R)$ must be on the root lattice $\Lambda_{\text{root}}$ of $G$ [\ref{12}]. Hereafter, such lattice will be referred to as the EN lattice of $G$.

Now perform a toroidal compactification on a 16-dimensional $G_L \times G_R$ lattice and take $G_R \supset SO_{\text{min}}(8)$. Then truncate the spectrum of the right sector of the theory, by tentatively removing all states (i.e. oscillators and momenta) in 12 compactified dimensions keeping only the $SO_{\text{min}}(8)$.

The centre of the covering group of $SO(8)$ is $Z_2 \times Z_2$. Its four elements partition the weight lattice in four conjugacy classes $(o)_8, (v)_8, (s)_8, (c)_8$ isomorphic to the root lattice. The $(o)_8$ lattice is the root lattice itself and contains the element $\sqrt{2\alpha'}p_o = (0, 0, 0, 0)$. The $(v)_8$ lattice is the root lattice whose smallest weights are eight vectors of norm one; in an orthonormal basis, these are $\sqrt{2\alpha'}p_v = (\pm 1, 0, 0, 0) + \text{permutations}$. The $(s)_8$ and $(c)_8$ lattices are spinor lattices whose smallest weights also have norm one and are the eightfold degenerate vectors $\sqrt{2\alpha'}p_{s,c} = (\pm 1/2, \pm 1/2, \pm 1/2, \pm 1/2)$ with even (for class $(s)_8$) or odd (for class $(c)_8$) number of minus signs. The structure of the weight lattice of all $SO(4m)$ groups in a 2$m$-dimensional Cartesian basis is the same.

One must however keep zero modes in the 16 compact dimensions in such a way that

$$\alpha' p_R^2[\mathcal{G}_R] = \alpha' p_R^2[SO(8)] + \frac{1}{2}. \quad (3)$$
These zero modes are required to remove the zero-point energy (-12/24) contribution to the energy of the states taken out by the truncation.

As shown below, Eq.(3) can be satisfied by choosing
\[ G_R = E_8 \times SO(16) , \] (4)
and we shall prove at the end of section 2.3 that from this choice all known ten-dimensional closed fermionic strings emerge from the same truncation scheme. We decompose \( SO(16) \) in \( SO'(8) \times SO(8) \) and truncate all states created by oscillators in the 12 dimensions defined by the \( E_8 \times SO'(8) \) root lattice. To satisfy Eq.(3), we keep only \( SO(16) \) zero modes which are \( SO'(8) \) vectors of norm one. The latter are chosen as follows.

The decomposition of an \( SO(16) \) lattice in terms of \( SO'(8) \times SO(8) \) lattices yields
\[
\begin{align*}
(o)_{16} &= [(o)_{s'} \oplus (o)_s] + [(v)_s \oplus (v)_s] , \\
(v)_{16} &= [(v)_{s'} \oplus (o)_s] + [(o)_{s'} \oplus (v)_s] , \\
(s)_{16} &= [(s)_{s'} \oplus (s)_s] + [(c)_{s'} \oplus (c)_s] , \\
(c)_{16} &= [(s)_{s'} \oplus (c)_s] + [(c)_{s'} \oplus (s)_s] .
\end{align*}
\] (5)

The vectors of norm one in \( SO'(8) \) are the 4-vectors \( \sqrt{2}\alpha' p'\nu \), \( \sqrt{2}\alpha' p'\nu' \) and \( \sqrt{2}\alpha' p'c \) described above. We choose one vector \( \sqrt{2}\alpha' p'\nu \) and one vector \( \sqrt{2}\alpha' p'\nu' \) (or equivalently \( \sqrt{2}\alpha' p'c \)).

This gives the truncations
\[
\begin{align*}
(o)_{16} &\rightarrow (v)_s , & (v)_{16} &\rightarrow (o)_s , \\
(s)_{16} &\rightarrow (s)_s , & (c)_{16} &\rightarrow (c)_s ,
\end{align*}
\] (6)

or
\[
\begin{align*}
(o)_{16} &\rightarrow (v)_s , & (v)_{16} &\rightarrow (o)_s , \\
(s)_{16} &\rightarrow (c)_s , & (c)_{16} &\rightarrow (s)_s .
\end{align*}
\] (7)

It follows from the closure of the Lorentz algebra that states belonging to the lattices \( (v)_s \) or \( (o)_s \) are bosons while those belonging to the spinor lattices \( (s)_s \) and \( (c)_s \) are space-time fermions.

It is easily verified that the choice of zero modes \( \sqrt{2}\alpha' p'_\nu \) and \( \sqrt{2}\alpha' p'_\nu' \) (or \( \sqrt{2}\alpha' p'_c \)) preserve modular invariance in the truncation provided one flips the sign of the lattice partition functions of the lattices \( (s)_s \) and \( (c)_s \), in accordance with the spin-statistic theorem \[10\].

\[3\] For an alternate proof of the modular invariance of the truncated theory, see ref.[9]. Generalisation to multiloops is given in ref. [13].
2.3 Closed fermionic strings

We now explain how the consistent closed fermionic ten-dimensional strings are obtained by truncation from the 26-dimensional bosonic string.

First we consider the fermionic theory emerging from truncation in both left and right sectors. We thus examine a compactification on both sectors with \( G_L = G_R = E_8 \times SO(16) \). In the truncated theory, \( (E_8)_{L,R} \) merely disappear, and we therefore only discuss in detail the fate of the \( SO(16)_{L,R} \) representations under truncation. In the full bosonic string, the representations of \( SO(16) \) entering the lattice partition function are restricted by modular invariance. Typically, such partition function is a sum of products of left and right partition functions \( \tilde{\gamma}_{aL}, \gamma_{aR} \), \( [\alpha = (o), (v), (s), (c)] \).

Using the modular transformation properties, one finds that there are two distinct modular invariant partition functions. The first one is,

\[
\tilde{\gamma}(\mu)_{16, L} + \tilde{\gamma}(\mu)_{16, L} (\gamma(\nu)_{16, R} + \gamma(\nu)_{16, R}) = \gamma(\mu)_{16, L} \gamma(\nu)_{16, R} + \tilde{\gamma}(\mu)_{16, L} \gamma(\nu)_{16, R} + \tilde{\gamma}(\mu)_{16, L} \gamma(\nu)_{16, R}.
\]

and the second is,

\[
\tilde{\gamma}(\mu)_{16, L} \gamma(\nu)_{16, R} + \tilde{\gamma}(\nu)_{16, L} \gamma(\nu)_{16, R} + \tilde{\gamma}(\nu)_{16, L} \gamma(\nu)_{16, R} + \tilde{\gamma}(\nu)_{16, L} \gamma(\nu)_{16, R}.
\]

Eq. (8) can be rewritten as \( \tilde{\gamma}(\mu)_{E_8, L} \gamma(\nu)_{E_8, R} \) where the subscript \( (\mu) \) refers to the root lattice of \( E_8 \) which is a sublattice of the \( SO(16) \) weight lattice. Eqs. (8) and (9) describe compactifications on the EN lattices of \( E_8 \times E_8 \) and of \( E_8 \times SO(16) \).

To interpret the result of the truncation in conventional terms, we note that the \( SO(8) \) partition functions \( \gamma(\mu)_{8} \) and \( \gamma(\nu)_{8} \), divided by the Dedekind functions arising from the bosonic states in the eight non compact transverse dimensions are the Neveu-Schwarz partition functions with the ‘wrong’ and ‘right’ GSO projection \( (NS)_+ \) and \( (NS)_- \). The partition functions \( \gamma(\mu)_{8} \) and \( \gamma(\nu)_{8} \), divided by the same Dedekind functions, form the two Ramond partition functions of opposite chirality \( R_+ \) and \( R_- \).

Choosing the ghosts \( \sqrt{2\alpha^2} p'_{v} \) and \( \sqrt{2\alpha^2} p'_{s} \) in both sector, and truncating in accordance with Eq. (8), the first partition function Eq. (8) yields the supersymmetric chiral closed string

\[
\text{IIB} : \ (NS)_+ (NS)_+ + R_+ (NS)_+ + (NS)_+ R_+ + R_+ R_+ .
\]

Replacing \( p'_{s} \) by \( p'_{c} \) in, say, the right sector we get, using Eq. (7), the non chiral supersymmetric closed string

\[
\text{IIA} : \ (NS)_+ (NS)_+ + R_+ (NS)_+ + (NS)_+ R_- + R_+ R_- .
\]

The same choices of the ghosts in the second partition function yield the following non-supersymmetric strings

\[
\text{OB} : \ (NS)_+ (NS)_+ + (NS)_- (NS)_- + R_+ R_+ + R_- R_- ,
\]

\[\text{Strictly speaking the modular invariance does not fully determine the representations of } SO(16) \text{ at this level (because } \gamma(s) = \gamma(c)) \text{. However at the level of the amplitudes, this ambiguity is lifted.}\]
\begin{equation}
0A : \quad (NS)_+ (NS)_+ + (NS)_- (NS)_- + R_+ R_- + R_- R_+ .
\end{equation}

Heterotic strings are generically obtained from compactification on $G_L \times G_R$ by only truncating in the right sector with $G_R = E_8 \times SO(16)$. The partition function constructed on $G_L \times [E_8 \times SO(16)]_R$ must be modular invariant. We may replace the Lorentzian metric by a Euclidean one and drop the $E_8$ to preserve invariance under translation ($\tau \to \tau + 1$) in the Euclidean metric. This reduces the problem of finding all heterotic strings obtainable in this way to that of finding all 24-dimensional Euclidean even self-dual lattices containing a sublattice $\Lambda (SO(16))$. All 24-dimensional even self-dual Euclidean lattices have been classified: they are known as the Niemeier lattices. Heterotic strings are obtained from the relevant Niemeier lattices by the truncation Eq.(6) (or equivalently, by Eq.(7)) \cite{7}.

3 Brane fusion and the open string sectors

3.1 The bosonic open string ancestor

We first review the derivation of the existence of a 26-dimensional open bosonic string free of massless tadpole divergences \cite{14}.

The Chan-Paton group of the 26-dimensional unoriented, uncompactified, open string theory may be fully determined by the tadpole condition \cite{13, 16, 17}. The full one-loop vacuum amplitude of a theory with open and closed unoriented strings comprises the four loop amplitudes with vanishing Euler characteristic: the torus $T$, the Klein bottle $K$, the annulus $A$ and the Möbius strip $M$. The last three amplitudes contain ultraviolet divergences which are conveniently analysed in the transverse channel. This channel describes the tree level exchange of zero momentum closed string modes between holes and/or crosscaps. The divergences appear there in the infrared limit and are associated with the exchange of tachyonic and massless modes. One ensures the tadpole condition, that is the cancellation of the divergences due to the massless modes in the total amplitude, by fixing the Chan-Paton group. Here, the divergence is related to the dilaton tadpole, a non-zero one point function of a closed vertex operator on the disk or on the projective plane. If the tadpole condition is not imposed, the low-energy effective action acquires a dilaton potential. Let us stress that in the present case the tadpole condition defining the bosonic open string is not compulsory: the presence of the dilaton tadpole does not render the theory inconsistent if the vacuum is shifted by the Fishler-Susskind mechanism \cite{18}.

The tadpole condition for the 26-dimensional uncompactified bosonic string determines the Chan-Paton group to be $SO(2^{13})$. Let us discuss the derivation of this well-known result. Introducing a Chan-Paton multiplicity $n$ at both string ends, the four different one-loop amplitudes of the unoriented 26-dimensional bosonic string are given by (see
for example [19]):

\[ T = \int_{\mathcal{F}} \frac{d^2 \tau}{\tau_2^{14}} \frac{1}{\eta^{24}(\tau) \tilde{\eta}^{24}(\tau)}, \]  

(14)

\[ \mathcal{K} = \frac{1}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^{14} \eta^{24}(2i\tau_2)}, \]  

(15)

\[ \mathcal{A} = \frac{n^2}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^{14} \eta^{24}(i\tau_2)}, \]  

(16)

\[ \mathcal{M} = \frac{\epsilon n}{2} \int_0^\infty \frac{d\tau_2}{\tau_2^{14} \eta^{24}(i\tau_2/2 + 1/2)}, \]  

(17)

where \( \mathcal{F} \) is a fundamental domain of the modular group for the torus and \( \eta(\tau) \) is the Dedekind function:

\[ \eta(\tau) = q^{1/24} \prod_{m=1}^{\infty} (1 - q^m), \quad q = e^{2\pi i \tau}, \quad \tau = \tau_1 + i\tau_2. \]  

(18)

The ‘hatted’ Dedekind function in \( \mathcal{M} \) means that the overall phase is dropped in \( \eta(i\tau_2/2 + 1/2) \) ensuring that \( \tilde{\eta}(i\tau_2/2 + 1/2) \) is real. A similar notation will be used for a generic function \( f \) admitting an expansion \( f = q^a \sum_{i=0}^\infty a_i q^i \), namely \( \tilde{f}(\tau + 1/2) = e^{-i\pi a} f(\tau + 1/2) \). The world-sheet parity operator defining \( \mathcal{M} \) is \( \Omega = \epsilon (-1)^N \) where \( N \) is the open string oscillator number operator. The plus or minus sign \( \epsilon \) in Eq. (17) encodes the action of \( \Omega \) on the vacuum and the one-half shift in the argument of the Dedekind function in Eq. (17) encodes the action of the twist operator \((-1)^N\).

The amplitudes \( T/2 + K \) and \( A + M \) are respectively the partition function of the closed and open unoriented string sectors. The \( q \)-independent term in the expansion of the integrand of \( A + M \) gives the number of massless vectors and determines the nature of the Chan-Paton group. Using Eqs. (13) and (17) one finds \( n(n-\epsilon)/2 \) massless vectors: if \( \epsilon = +1 \) (resp. \( \epsilon = -1 \)), the Chan-Paton group is \( SO(n) \) (resp. \( USp(n) \)).

To impose the tadpole condition we interpret \( K \), \( A \) and \( M \) as amplitudes in the transverse (tree) channel. To this effect we first define \( t = 2\tau_2 \) (resp. \( t = \tau_2/2 \)) in \( K \) (resp. \( A \)) and express the modular form \( \eta^{24}(it) \) in the integrand of Eqs. (15) and (16) in terms of its \( S \)-transform. The change of variable \( l = 1/t \) yields

\[ K_{\text{tree}} = \frac{2^{13}}{2} \int_0^\infty \frac{dl}{\eta^{24}(il)} , \]  

(19)

and

\[ A_{\text{tree}} = \frac{n^2}{2} \frac{2^{-13}}{2} \int_0^\infty \frac{dl}{\eta^{24}(il)} . \]  

(20)

The subscript \( \text{tree} \) emphasises that the expressions Eqs. (13) and (20), although identical to the integrals Eqs. (15) and (16), are now rewritten in terms of tree level intermediate states. Throughout the paper, any amplitude formulated in the ‘direct’ channel as a one loop amplitude \( \mathcal{I} \), will be relabeled \( \mathcal{I}_{\text{tree}} \) when expressed in terms of transverse channel tree level intermediate states.
It is a little bit more tricky to go from the direct to the transverse channel for the Möbius amplitude. One expresses \( \hat{\eta}^{24}(i\tau/2 + 1/2) \) in terms of its \( P \)-transform [15] which combines the modular transformations \( S \) (i.e. \( \tau \to -1/\tau \)) and \( T \) (i.e. \( \tau \to \tau + 1 \)):

\[
P = T^{1/2}ST^2ST^{1/2}.
\]

(21)

One then performs the change of variable \( l = 1/(2\tau_2) \) to get

\[
\mathcal{M}_{\text{tree}} = 2\frac{\epsilon n}{2} \int_0^\infty dl \frac{1}{\hat{\eta}^{24}(il + 1/2)}.
\]

(22)

The tadpole condition can now be imposed by requiring the vanishing of the \( e^{-2\pi l} \)-independent term in the integrand of the total tree amplitude \( K_{\text{tree}} + A_{\text{tree}} + \mathcal{M}_{\text{tree}} \). One gets the following condition

\[
(2^{13} + 2^{-13}n^2 - 2\epsilon n) = 2^{-13}(2^{13} - \epsilon n)^2 = 0,
\]

(23)

which singles out \( \epsilon = +1 \) and the value \( n = 2^{13} \).

Therefore, one recovers that the uncompactified open bosonic string theory obeying the tadpole condition is unoriented and has an \( SO(2^{13}) \) Chan-Paton group [14].

### 3.2 Compactification of the open string ancestor on Lie algebra lattices and truncation

We now explain how the open string theories in 10 dimensions may be obtained by truncation from the compactified 26-dimensional bosonic string. We begin by discussing the construction of open string descendants from closed strings compactified on the two EN lattices of the rank sixteen groups \( \mathcal{G} = E_8 \times E_8 \) and \( \mathcal{G} = E_8 \times SO(16) \). We refer the reader to ref. [10] for a more general discussion on the open descendants of strings compactified on EN lattices of semi-simple Lie groups \( \mathcal{G} \). Having the amplitudes, we then perform the truncation.

#### 3.2.1 The \( E_8 \times E_8 \) compactification and Type I superstring

We consider here the compactification on the \( E_8 \times E_8 \) lattice. We write

\[
\gamma(0)_{E_8} \gamma(0)_{E_8} = (\gamma(0)_{16} + \gamma(s)_{16})(\gamma(0)_{16} + \gamma(s)_{16})_7.
\]

(24)

In order to get the amplitudes in this case, one first replaces the contribution of sixteen non-compact dimensions in the direct amplitudes eqs.([15], [16]) and ([17]) by the contribution of the lattice partition function and then using the modular transformations
one gets the tree amplitudes which read \[10\]

\[
K_{\text{tree}} = \frac{2^5}{2} \int_0^\infty \, dl \, \frac{1}{\eta^8(il)} \gamma_{(o)E_8}(il) \gamma_{(o)E_8}(il) , \\
A_{\text{tree}} = \frac{n^2}{2} \frac{2^{-5}}{2} \int_0^\infty \, dl \, \frac{1}{\eta^8(il)} \gamma_{(o)E_8}(il) \gamma_{(o)E_8}(il) , \\
M_{\text{tree}} = n \int_0^\infty \, dl \, \frac{1}{\eta^8(il + \frac{1}{2})} \hat{\gamma}_{(o)E_8}(il + 1/2) \hat{\gamma}_{(o)E_8}(il + 1/2) .
\]

We impose the vanishing of the dilaton tadpole. All the transverse amplitudes are proportional to the root lattice of \(E_8 \times E_8\). Therefore all divergences are eliminated by a single constraint

\[ n = 2^5 , \]

and \(\epsilon = +1\). The Chan-Paton group is then \(SO(2^5) = SO(32)\). This reduction of Chan-Paton group from \(SO(2^{13})\) by compactification is crucially related to the structure of the Lie algebra lattice. Indeed the reduction \(2^{13} \rightarrow 2^5\) in Eqs.\((25)\) and \((26)\) is a result of the reduction of the number of non-compact dimensions and of the modular properties of the lattice partition function of the \(E_8 \times E_8\) lattice. The latter is characterised by one conjugacy class and is thus invariant under the modular group. If we would compactify instead on a cartesian torus corresponding to \([U(1)]^{16}\) no reduction would occur. Indeed, in this case the S-transformation of the \([U(1)]^{16}\) lattice partition function would compensate the change due to the reduction of non-compact dimensions.

We are now in position to derive the truncated theory from the tadpole-free bosonic open string theory compactified on the EN lattice of \(E_8 \times E_8\). Eq.\((3)\) gives

\[ \gamma_{(o)E_8} \gamma_{(o)E_8} = \gamma_{(o)E_8} (\gamma_{(o)16} + \gamma_{(s)16}) \rightarrow \text{Truncation} \rightarrow \gamma_{(v)s} - \gamma_{(s)s} \]

and the transverse amplitudes Eqs.\((25)\), \((26)\) and \((27)\) become after truncation

\[
K'_{\text{tree}} = A'_{\text{tree}} = \frac{2^5}{2} \int_0^\infty \, dl \, \frac{1}{\eta^8(il)} (\gamma_{(v)s} - \gamma_{(s)s})(il) , \\
M'_{\text{tree}} = -2^5 \int_0^\infty \, dl \, \frac{1}{\eta^8(il + 1/2)} (\hat{\gamma}_{(v)s} - \hat{\gamma}_{(s)s})(il + 1/2) .
\]

The flip in sign in Eq.\((31)\) as compared to Eq.\((27)\) with \(\epsilon = +1\) is solely due to the definition of hatted functions. Truncating the direct amplitudes \(A\) and \(M\) we get

\[
A' = \frac{2^{10}}{2} \int_0^\infty \frac{d\tau^2}{\tau^2} \frac{1}{\eta^8(i\tau^2/2)} (\gamma_{(v)s} - \gamma_{(s)s})(i\tau^2/2) , \\
M' = -\frac{2^5}{2} \int_0^\infty \frac{d\tau^2}{\tau^2} \frac{1}{\eta^8(i\tau^2/2 + 1/2)} (\hat{\gamma}_{(v)s} - \hat{\gamma}_{(s)s})(i\tau^2/2 + 1/2) .
\]

The amplitudes in Eqs.\((32)\) and \((33)\) are equal to those in Eqs.\((30)\) and \((31)\) expressed in the transverse channel, a consequence of the fact that the truncation commutes with \(S\) and \(T\) transformations.
Massless modes in both open and closed string channels, created by bosonic oscillators in non-compact dimensions, become massive after truncation. New massless modes arise from the compact dimensions. In the closed string channel, these are massless spinors, NS-NS and R-R fields. The NS-NS and R-R tadpoles are eliminated by the condition Eq. (28) inherited from the bosonic string. This is easily checked in the truncated tree amplitude $K_{\text{tree}} + A_{\text{tree}} + M_{\text{tree}}$ given by Eqs. (30) and (31).

The Chan-Paton group $SO(32)$ is preserved under truncation. This can be checked by counting, in the open channel, the number of massless vectors arising from truncation. We see that the truncation of the unoriented tadpole-free bosonic open string theory, compactified on the $E_8 \times E_8$ lattice, results in the $SO(32)$ anomaly free Type I theory.

3.2.2 The $E_8 \times SO(16)$ compactification and Type 0 strings

We now consider the 26-dimensional unoriented bosonic open string theory compactified on the $E_8 \times SO(16)$ lattice. In this case there are four conjugacy classes $\mathcal{N}$ which give, compared to Eq. (19), a reduction $2^{13} \rightarrow 2^{5} = 2^6$. Taking into account the existence of these four classes and the possibility of introducing different Chan-Paton multiplicities, the tree amplitudes of this model are given by [10]

$$K_{\text{tree}} = \frac{2^6}{2} \int_0^\infty dl \frac{1}{\eta^8} \gamma(\sigma)_{16} \hat{\gamma}(\sigma)_{E_8}, \quad (34)$$

$$A_{\text{tree}} = \frac{2^{-6}}{2} \int_0^\infty dl \frac{1}{\eta^8} (a_1^2 \gamma(\sigma)_{16} + a_2^2 \gamma(\nu)_{16} + a_3^2 \gamma(s)_{16} + a_4^2 \gamma(c)_{16}) \hat{\gamma}(\sigma)_{E_8}, \quad (35)$$

$$M_{\text{tree}} = \epsilon a_1 \int_0^\infty dl \frac{1}{\eta^8} \hat{\gamma}(\sigma)_{16} \hat{\gamma}(\sigma)_{E_8}, \quad (36)$$

and

$$a_1 = n_o + n_v + n_s + n_c, \quad a_2 = n_o + n_v - n_s - n_c,$$
$$a_3 = n_o - n_v + n_s - n_c, \quad a_4 = n_o - n_v - n_s + n_c. \quad (37)$$

We enforce the tadpole conditions. In this case there are four tadpole conditions because there are four different types of massless modes giving rise to divergent contributions in the tree amplitudes: in addition to the graviton and dilaton encoded in the Dedekind function at level one, there are three types of massless modes arising from $\gamma(\sigma)_{16}$ at level 1 and from $\gamma(\sigma)_{16}$, $\gamma(s)_{16}$, $\gamma(c)_{16}$ at level zero. The tadpole conditions which eliminate the divergences at level one in the Dedekind function and in $\gamma(\sigma)_{16}$ fix $\epsilon = 1$, and the Chan-Paton group is $SO(n_o) \times SO(n_v) \times SO(n_s) \times SO(n_c)$, with $n_o + n_v + n_s + n_c = 2^6 = 64$. The pattern of symmetry breaking of $SO(64)$ is further determined by imposing the two remaining tadpole conditions corresponding to the
\( \gamma_{(s)16} \) and \( \gamma_{(c)16} \) divergences. One gets

\[
SO(n) \times SO(n) \times SO(32 - n) \times SO(32 - n) .
\]

We now derive the truncated theory. Using Eq.(6) we get

\[
K_{\text{tree}} = \frac{2^6}{2} \int_0^\infty \frac{dl}{\eta^8} \gamma_{(v)s} ,
\]

\[
A_{\text{tree}} = \frac{2^{-6}}{2} \int_0^\infty \frac{dl}{\eta^8} [(64)^2 \gamma_{(v)s} + 16(n - 16)^2 \gamma_{(o)s}] ,
\]

\[
M_{\text{tree}} = -64 \int_0^\infty \frac{dl}{\eta^8} \tilde{\gamma}_{(v)s} ,
\]

from which the direct channel amplitudes can be obtained. The Chan-Paton group is transferred from the untruncated bosonic theory to the truncated fermionic theory and we obtain the spectrum of the tadpole-free \([SO(32 - n) \times SO(n)]^2\) Type O theories discussed in ref.\[13, 21\]. Note that the Chan-Paton group has a higher rank than in Type I because \(E_8 \times SO(16)\) has more conjugacy classes than \(E_8 \times E_8\). We emphasize that this Chan-Paton gauge structure and the symmetry breaking pattern arise entirely from bosonic considerations.

### 3.3 Brane fusion

We shall relate the Chan-Paton groups of the tadpole free bosonic string compactified on the enhanced symmetry points considered in the previous section (and hence of the open fermionic strings) to the uncompactified \(SO(2^{13})\) unoriented bosonic string.

The latter has a clear geometrical interpretation \[22\]. The ends of the open strings live on D25-branes and the tension of a D25-brane can be derived from \(A_{\text{tree}}\) (see Eq.(20) with \(n = 1\)) by comparing with the field theory calculation \[22\].

The tension \(T_{D25}^{\text{bosonic}}\) is given by\[3\]

\[
T_{D25}^{\text{bosonic}} = \frac{\sqrt{\pi}}{2^4 \kappa_2 26} (2\pi\alpha'^{1/2})^{-14} ,
\]

where \(\kappa_2^{26} = 8\pi G_{26}\) and \(G_{26}\) is the Newtonian constant in 26 dimensions.

The action of the world-sheet parity operator on the 26-dimensional closed bosonic string introduces an orientifold 25-plane \(O25\). The tension of the \(O25\) can be derived from \(K_{\text{tree}}\) (and \(M_{\text{tree}}\) for the sign of this tension) again by comparing with the field theory calculation. The result is \[22\]:

\[
T_{O25}^{\text{bosonic}} = -2^{12} T_{D25}^{\text{bosonic}} .
\]

\[5\]We always give the Dp-brane tensions computed in the oriented closed theory.
Therefore the tadpole condition fixing the $SO(2^{13})$ gauge group means in this context that one has to introduce $n = 2^{13}$ D25-branes (2^{12}+ their images) to cancel the negative tension of the $O25$ [22].

We now derive the tension of the wrapped orientifold and D25-branes for compactifications on the EN lattices with Lie group $\mathcal{G} = E_8 \times E_8$ and $\mathcal{G} = E_8 \times SO(16)$. To this effect we describe the lattice in terms of constant background metric $g_{ab}$ and Neveu-Schwarz antisymmetric tensor $b_{ab}$ [20].

The squared wrapped orientifold tension is obtained from the coupling to gravity in the transverse Klein bottle amplitude $K_{\text{tree}}$. We recall that $K_{\text{tree}}$ is obtained from the direct amplitude $K$ by the S modular transformation, and that $K$ follows from the torus amplitude by inserting the world-sheet parity operator $\Omega$ which interchanges left and right sectors. This projects out the $b_{ab}$-field. For the above groups, the tension of the wrapped orientifold $T_{O25 \, \text{wr}}^{\text{bosonic}}$ after compactification is then given by

$$T_{O25 \, \text{wr}}^{\text{bosonic}} = T_{O25}^{\text{bosonic}}[(2\pi)^{16} \sqrt{g}] ,$$

where $g$ is the determinant of the metric. On the other hand the world-volume action of a D25-brane is given by the Born-Infeld action [23, 24, 16]. Accordingly, after compactification, the tension $T_{D25 \, \text{wr}}^{\text{bosonic}}$ of a wrapped D25-brane is given by [23]:

$$T_{D25 \, \text{wr}}^{\text{bosonic}} = T_{D25}^{\text{bosonic}}[(2\pi)^{16} \sqrt{e}] = n_f T_{D25}^{\text{bosonic}}[(2\pi)^{16} \sqrt{g}] ,$$

where $e$ is the determinant of $e_{ab} \equiv g_{ab} + b_{ab}$, and

$$n_f = \sqrt{e/g} .$$

The negative tension of the orientifold can be compensated by introducing $n_w$ wrapped D25 branes (including images) with:

$$n_w = 2^{13} \sqrt{g/e} = \frac{2^{13}}{n_f} .$$

The reduction factor $n_f$ defined in Eq.(46) can be computed [10] to give

$$n_f = \frac{2^{8}}{\sqrt{N}}$$

where $N$ is the number of conjugacy classes

Inserting this value in Eq.(47), we get the number of wrapped D25 branes required to ensure the dilaton tadpole condition in the compactified bosonic string. Thus the reduction factor $n_f$ coincides with the ratio of Chan-Paton multiplicities before and after compactification.

---

6One can use Eqs.(43), (48) and (42) to compute the tensions of branes in the fermionic string theories by comparing the zero mode contributions in $\mathcal{A}_{\text{tree}}$ and $\mathcal{A}_{\text{tree}}'$ both in the $E_8 \times E_8$ and $E_8 \times SO(16)$ compactifications [10]. The results obtained in this way agree with the known results.
For $E_8 \times E_8$ we have $\mathcal{N} = 1$ and Eq. (15) gives $n_f = 2^8$. Consequently we are left with $n_w = 2^5$ wrapped D25-branes. This is consistent with the fact that the Chan-Paton group of the tadpole-free $E_8 \times E_8$ compactification is $SO(32)$.

For $E_8 \times SO(16)$ we have $\mathcal{N} = 4$. Thus Eq. (15) gives $n_f = 2^7$. Consequently we are left with $n_w = 2^6$ wrapped D25-branes. This is consistent with the fact that the Chan-Paton group of the tadpole-free $E_8 \times SO(16)$ compactification is $[SO(32-n) \times SO(n)]^2$.

Note that, for the two Lie groups of rank $d = 16$ considered here one has

$$\frac{2^{d/2}}{\sqrt{\mathcal{N}}} = 2^{r/2},$$

(49)

where $r$ is the rank of the $b_{ab}$ matrix. This can be verified by direct inspection from the explicit form of $b_{ab}$. Such reduction, and its expression in terms of the rank of the $b_{ab}$ matrix, is in agreement with reference [26].

One may interpret the reduction of the total Chan-Paton multiplicity as a consequence of ‘brane fusion’. Eq. (15) shows that one D25-brane of the 26-dimensional bosonic string theory, compactified on the EN lattice, has the same tension as the integer number $n_f = \sqrt{e/g}$ of D25-branes when the theory is compactified on a generic Cartesian torus (for which $b_{ab} = 0$) with equal volume. This fact does not depend on the orientability of the string nor on the tadpole condition. In presence of the enhanced symmetry, the rôle of the quantised $b_{ab}$-field is to fuse $n_f$ component branes into one.

4 Discussions

One may always bosonise the world-sheet fermions to obtain, in the light-cone gauge, theories formally identical to those resulting from truncation[7]. The added information provided by truncation for closed strings is the encoding of all these theories in the 26-dimensional bosonic string theory, compactified on the EN lattice, has the same tension as the integer number $n_f = \sqrt{e/g}$ of D25-branes when the theory is compactified on a generic Cartesian torus (for which $b_{ab} = 0$) with equal volume. This fact does not depend on the orientability of the string nor on the tadpole condition. In presence of the enhanced symmetry, the rôle of the quantised $b_{ab}$-field is to fuse $n_f$ component branes into one.

The extension of the truncation to open string sectors reveals that this encoding explains, in terms only of properties of the bosonic strings, crucial properties of the fermionic strings, such as the cancellation of anomalies in Type I theory by the Chan-Paton group $SO(32)$, and the Chan-Paton groups which eliminate all tadpoles in Type O theories. Perhaps more surprising even is the fact that these results follow from specific properties of the $E_8 \times SO(16)$ lattice, singling out this lattice out of all possible toroidal compactifications.

At this stage, the link between all M-theory strings and the bosonic string results mainly from properties of conformal symmetry and of group theory. Namely, the preservation

7See [27] and references therein.
of modular invariance by truncation and the fusion mechanism on enhanced symmetry
points play a key role in deriving our results. One is tempted to inquire further into
the nature of this link and to search for its possible non perturbative dynamical origin.
The emergence of the space-time fermions from diagonal subgroups of direct pro-
ducts of space-time groups and internal groups is, as mentioned in the introduction,
suggestive of the generation of fermions through excitations from topological boson
backgrounds [3, 28]. In this context, the group $E_8$ (and/or $SO(16)$) might play a key rôle in attempting to construct some “bosonic M-theory” [29].

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