Ion-acoustic solitary waves and shocks in a collisional dusty negative ion plasma

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We study the effects of ion-dust collisions and ion kinematic viscosities on the linear ion-acoustic instability as well as the nonlinear propagation of small amplitude solitary waves and shocks (SWS) in a negative ion plasma with immobile charged dusts. The existence of two linear ion modes, namely the ‘fast’ and ‘slow’ waves is shown, and their properties are analyzed in the collisional negative ion plasma. Using the standard reductive perturbation technique, we derive a modified Korteweg-de Vries-Burger (KdVB) equation which describes the evolution of small amplitude SWS. The profiles of the latter are numerically examined with parameters relevant for laboratory and space plasmas where charged dusts may be positively or negatively charged. It is found that negative ion plasmas containing positively charged dusts support the propagation of SWS with negative potential. However, the perturbations with both positive and negative potentials may exist when dusts are negatively charged. The results may be useful for the excitation of SWS in laboratory negative ion plasmas as well as for observation in space plasmas where charged dusts may be positively or negatively charged.

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I. INTRODUCTION

The nonlinear propagation of solitary waves and shocks (SWS) in dusty plasmas have been widely studied for understanding the electrostatic disturbances in space plasma environments [1, 2] as well as in laboratory plasma devices [3, 4]. Many researchers have pointed out that charged dust grains can drastically modify the existing response of electrostatic wave spectra in plasmas depending upon whether the charged dusts are considered to be static or mobile [3, 12].

The existence of dust ion-acoustic (DIA) solitons was first predicted by Shukla and Silin [5], and was later observed experimentally by Barkan et al. [6]. The phase velocity of these DIA waves increases when the density of electrons decreases. The nonlinear evolution of such small amplitude waves is described by the Korteweg-de-Vries (KdV) equation, which was first derived by Washimi and Tanui [7] in a two-component plasma. It was also reported that the KdV equation can have both compressive and rarefactive solitary wave solutions when sufficient amount of negative ions are present in multi-component plasmas [8]. Furthermore, it was also observed that when the negative ion density exceeds a critical value, only negative or rarefactive solitons can be excited [15].

One of the nonlinear phenomena of DIA waves is the generation of shocks, which have already been observed experimentally [10]. The shock waves are described with an additional Burger term due to dissipation in the KdV equation, known as the KdV-Burger (KdVB) equation [9, 10]. The nonlinear properties of dust ion-acoustic shocks and holes have been studied [11] with the consideration of ion kinematic viscosity in a dusty plasma. Furthermore, the nonlinear properties of small amplitude ion-acoustic shocks in a collisional dusty plasma has been investigated by Ghosh et al. [18]. In their analysis, they have ignored the frictional force due to ion-dust collision and considered the effects of ion kinematic viscosity in dusty plasmas with positive ions. Mamun et al. [19] investigated the existence of dust electron-acoustic shocks in a negative ion plasma with dust charge fluctuation. They reported the formation of DIA shocks with negative potentials in the plasma. In a recent work of Adhikary et al. [12], the effect of kinematic viscosity has been considered to study the propagation of DIA shocks in multi-ion plasmas, and it has been stressed that the viscosity in dusty plasmas plays a key role in the formation of DIA shocks. Thus, both the viscosity and collision of ions with dusts may play crucial roles in dissipation on the propagation of SWS in dusty multi-ion plasmas.

Since the charging rates of dusts by the positive and negative ions are nearly balanced, the existence and the formation of dusty plasmas with both positive and negative ions may be possible under some circumstances. Recently, Kim et al. [10] have investigated that dust particles injected in laboratory negative ion plasmas can become positively charged when the number density of negative ions greatly exceeds (≳ 500) that of the electrons. In
space environments, the possible role of negative ions has been discussed, and it has been found that dusts can be positively charged if there is a sufficient number density of heavy negative ions (with mass \( \geq 300 \text{ amu} \)) \[20\].

In this work, we investigate the ion-acoustic instability as well as the nonlinear propagation of DIA SWS in an unmagnetized collisional dusty plasma containing both positive and negative ions. The effects of ion thermal pressures, ion-dust collisions as well as the kinematic viscosity of the ion fluids are taken into account to maintain the equilibrium of ions. The present paper generalizes and extends the previous work of Adhikary \[12\] to include collisional effects of ions with dusts, different mass and thermal pressures of ions as well as different expression for ion kinematic viscosities. We show that when the negative ion concentration is much larger than the density of electrons or when plasma is the admixture of positively charged (static) dusts, SWS with negative potential exist. However, when the concentration of negative ions is smaller than a certain value or plasma contains negatively charged dusts, both the compressive and rarefactive SWS may exist.

II. BASIC EQUATIONS

We consider the one-dimensional propagation of DIA waves in an unmagnetized collisional dusty plasma, which consists of singly charged adiabatic positive and negative ions, Boltzmann distributed electrons and immobile charged dusts. Since the dusts are too heavy to move on the time scale of the ion-acoustic waves, we do not consider the dynamics of charged dusts. The latter can, however, affect the collision rates with ions as well as the wave dispersion and nonlinearity. We have neglected the electron inertia, since the electron thermal speed is much larger than that of ions. We also assume that the negative ions are heavier than the positive ions. The immobile dust particles carry some charges so as to maintain the overall charge neutrality condition given by

\[
n_{e0} + n_{n0} = n_{p0} \pm z_d n_{d0}, \tag{1}
\]

where \( n_{j0} \) is the unperturbed number density of charged species \( j \) \((j = e, p, n, d, \text{respectively, stand for electrons, positive ions, negative ions and static dusts}), z_d \ (> 0) \) is the dust charge state. The upper (lower) sign in Eq. (1) corresponds to positively (negatively) charged dusts. The condition (1) can also be written as

\[
\mu_e = \mu_i + \zeta \mu_d - 1, \tag{2}
\]

where \( \mu_e = n_{e0}/n_{n0}, \mu_i = n_{i0}/n_{n0}, \mu_d = z_d n_{d0}/n_{n0} \) are the density ratios and \( \zeta = 1 (-1) \) correspond to positively (negatively) charged dusts. The basic equations in one space dimension are

\[
\left( \frac{d}{dt} + \nu_{jd} \right) v_j = \frac{-q_j}{m_j} \partial \phi - \frac{3k_B T_j}{2m_j n_{j0}^2} \frac{\partial n_j^2}{\partial x} + \eta_j \left( \frac{\partial^2 v_j}{\partial x^2} \right), \tag{4}
\]

\[
\frac{\partial^2 \phi}{\partial x^2} = 4 \pi e \left( n_e - n_p + n_n - \zeta z_d n_{d0} \right), \tag{5}
\]

and the Boltzmann distribution for electrons

\[
n_e = n_{e0} \exp \left( e \phi / k_B T_e \right), \tag{6}
\]

where \( d/dt = \partial / \partial t + v_j \partial / \partial x \) is the convective derivative. The physical quantities \( n_j, v_j \) and \( m_j \) respectively denote the number density, velocity and mass of \( j \)-species particles. Furthermore, \( q_j = e \) and \( q_n = -e \), where \( e \) is the elementary charge. Also, \( \phi \) is the electrostatic potential, \( k_B \) is the Boltzmann constant and \( T_j \) is the particle’s thermodynamic temperature. In Eq. (4), \( \nu_{jd} \sim \sigma_{jd} n_{d0} v_{ij} \) denotes the collision rate in which \( \sigma_{jd} \) is the collision cross section of \( j \)-species ions with dusts and \( v_{ij} = \sqrt{k_B T_j/m_j} \) is the thermal velocity of \( j \)-species ions. Moreover, \( \eta_j \) is the kinematic viscosity of \( j \)-species ions which arises mainly due to the ion-dust collisions. We here mention that the exact expression for \( \eta_j \) in terms of \( \nu_{jd} \) or \( n_{d0} \) has not yet fully investigated so far. So, we will consider arbitrary values of it in order to see the damping of DIA waves numerically \[21\]. We have used the adiabatic equation of state in Eq. (4), namely \( P_j / P_{j0} = (n_{j0}/n_j)^3 \) with \( P_{j0} = n_{j0} k_B T_j \) for each of the ion species. The adiabatic index \( \gamma = 3 \) \([= (2 + D)/D, D \text{ being the number of degrees of freedom}] \) is due to the one-dimensional geometry of the system. Furthermore, in the ion continuity equations the electron-ion recombination effects, being smaller at low-pressure limit \((\sim 4 - 5 \times 10^{-4} \text{ Torr})\), have been neglected \[16\]. However, the contribution from the frictional force \( \nu_{jd} v_j \) may not be neglected compared to that from the viscous term \( \eta_j v_j \). It has been found that in laboratory experiments \[21\], the nondimensional viscosity parameter \( \eta_j = n_j / \lambda_j^2 \omega_m \) may be about 10 times of the collisional frequency \( \nu_{jd} v_j / \omega_m \), where \( \omega_m = \sqrt{4 \pi n_{n0} e^2 / m_n} \) is the negative ion plasma frequency and \( \lambda_j = \sqrt{k_B T_e} / 4 \pi n_{n0} e^2 \) is the Debye length. Also, \( \tilde{\eta}_j \) and \( \tilde{\nu}_{jd} \) are almost linearly proportional to the dust number density \( n_{d0} \).

Next, we normalize the physical quantities according to \( \phi \rightarrow e \phi / k_B T_e, n_j \rightarrow n_j / n_{j0}, v_j \rightarrow v_j / c_s, \) where \( c_s = \sqrt{k_B T_e / m_n} \) is the ion-acoustic speed. The space and time variables are normalized by the Debye length \( \lambda_D \) and the inverse of the negative ion plasma frequency \( \omega_m \) respectively. Thus, from Eqs. (3)-(5) we obtain the following normalized set of equations

\[
\left( \frac{d}{dt} + \tilde{\nu}_{jd} \right) v_j = \beta_j \left[ \frac{\partial \phi}{\partial x} - \frac{3 \sigma_{jd} n_{d0}^2}{2m_j} \frac{\partial n_j^2}{\partial x} \right] + \tilde{\eta}_j \left( \frac{\partial^2 v_j}{\partial x^2} \right), \tag{8}
\]
\[ \frac{\partial^2 \phi}{\partial x^2} = \mu_e \phi - \mu_i n_p + n_n - \zeta \mu_d, \]  

where the upper (lower) sign in \( \mp \) on the right-hand side of Eq. 6 corresponds to positive (negative) ions, \( \sigma_j = T_j / T_e \), \( \beta_j = m_n / m_j \) with \( \beta_p \equiv \beta \), say.

### III. Dispersion Relation: Ion-Acoustic Instability

In order to identify the different wave eigenmodes and to study their stability/instability under the dissipation due to ion-dust collisions and ion kinematic viscosities, we perform a linear analysis in which the perturbations of the physical quantities vary as \( f \sim \exp(ikx - i\omega t) \), where \( \omega (k) \) is the wave frequency (number). Thus, Fourier analyzing the Eqs. (7)–(9) we obtain the following dispersion relation [6]

\[ D(\omega, k) \equiv 1 + \sum_{j=e, p, n} \chi_j = 0, \]  

where \( \chi_e = \mu_e / k^2 \) and

\[ \chi_p = \mu_i \beta \left[ 3 \beta \sigma_p k^2 - \omega \left( \omega + i \nu_{pd} + i \nu_{p} k^2 \right) \right]^{-1}, \]

\[ \chi_n = \left[ 3 \beta \sigma_n k^2 - \omega \left( \omega + i \nu_{nd} + i \nu_{n} k^2 \right) \right]^{-1}. \]

We note that the damping terms \( i(\nu_{j, d} + \nu_{j} k^2) \) for \( j = p \) and \( n \) arise due to the ion-dust collisions and the viscosity effects, and thereby cause the DIA instability in the collisional plasma. When \( k^2 \ll 1 \), \( \nu_{j} k^2 \) term is negligible compared to \( \nu_{j, d} \). Thus, for long wavelength modes, the damping effect is mainly due to the ion-dust collisions. In particular, in absence of these dissipative effects, the dispersion relation Eq. (10) reduces to

\[ 1 + \frac{\mu_e}{k^2} = \frac{\mu_i \beta}{\omega^2 - \beta \sigma_p} + \frac{1}{\omega^2 - \beta \sigma_n}. \]  

The first and second terms on the right-hand side of Eq. 13 are, respectively, the contributions from the positive and negative ion species, whereas the term \( \propto \mu_e \) is from the electron species in plasmas and the constant term 1 is from the effect of dispersion due to charge separation (deviation from quasineutrality) of the species. If the phase speed of the wave satisfies \( \omega / k \gg c_s > v_{j, d} \), then from Eq. 13 the fast ion-acoustic wave mode can be recovered as

\[ \omega^2 \simeq \frac{(1 + \mu_i \beta)k^2}{\mu_e + k^2}. \]  

In absence of negative ions in plasmas one obtains from Eq. 13 the following dispersion relation [6].

\[ \omega^2 = 3 \beta \sigma_p k^2 + \frac{\mu_i \beta k^2}{\mu_e + k^2}. \]  

From Eqs. (14) and (15) we find that for a fixed \( \beta, \mu_i \) and/or \( \sigma_p \), the frequency of the ion wave modes increases with the wave number. However, as the wave number increases beyond a certain value, e.g., \( k \gtrsim 0.2 \), the fast mode in the limit may approach more or less a constant value. Also, as the dust number density increases, i.e., \( \mu_e \) increases (decreases) for positively (negatively) charged dusts, the phase velocity of the fast mode decreases (increases). Physically, this decrease (increase) of the phase velocity is attributed to the addition (removal) of electrons by the dust grains in order to maintain the charge neutrality condition 1 of the plasma.

We numerically analyze the dispersion relation Eq. (10) by considering parameters that may be representative of laboratory negative ion plasmas with positively and negatively charged dusts. As in Ref. [22], we consider a plasma in which positive ions are singly ionized \( K^+ \) and the heavy negative ions are \( SF_6 \), such that \( \beta \equiv m_n / m_p = 146/39 \simeq 3.74 \). The temperature and density are considered as \( T_e \sim T_p \sim 0.2 \) eV, \( T_d \sim T_e / 8 \) with \( n_{e0} \sim 2 \times 10^9 \) cm\(^{-3} \), \( n_{c0} = n_{e0} / 700 \), \( n_{p0} = 500 n_{c0} \) for positively charged dusts (and \( n_{e0} = n_{e0} / 100 \), \( n_{p0} = 150 n_{c0} \) for negatively charged dusts) to a surface potential \( \phi_s \sim 0.1 \) V and grain radius \( R = 5 \) \( \mu m \), so that \( z_{d} \sim R \phi_s / \epsilon \sim 350 \). In this case, \( n_{d0} \sim 1.7 \times 10^6 \) cm\(^{-3} \) for which the rate of ion collisions with dusts would be higher than that with neutrals. We identify two ion wave modes, namely the ‘fast’ and ‘slow’ modes as exhibited in Figs. 1-3.

Figure 1 shows the plots of the real wave frequency (upper panel) and the growth rate (lower panel) of the DIA instability versus the wave number \( k \) obtained by numerically solving Eq. (10) for positively charged dusts. The thick (black) line corresponds to the case when no dust is present in the plasma, whereas the thin (blue) line is for the case when dusts (positively charged) are present. In absence of ion thermal pressures, the real wave frequency and the growth rate assume almost a constant value for \( k \gtrsim 0.2 \) as shown in Fig. 1 (red lines). This is evident from the analytic expression (14). It is seen that the growth occurs, however, the frequency is reduced in presence of dusts. Thus, when dust is introduced into the plasma, the enhanced ion-dust collision frequency damps the instability rate. From Fig. 2, we observe that when the dusts are negatively charged, the real wave frequency and the growth rate become larger than that of the case of no dust. This is in contrast to the case of positively charged dusts. The behaviors of these modes with or without charged dusts in plasmas suggest a possible diagnostic for the roles of positively and negatively charged dusts as well as the effects of ion-dust collisions.

Recently, it has been suggested by Rapp et al. [20] that the presence of sufficient amount \( (n_{\text{d0}} > 40n_{\text{e0}}) \) of heavy negative ions (with mass \( > 300 \) amu) may play important roles in the observation of positive dusts in the Earth’s mesosphere between about 80 to 90 km. They also remarked that the presence of negative ions corresponding to sub-nm size \((0.3 - 0.4 \) nm\) negatively charged dusts...
may lead to the positive dusts with larger size (~2 nm) grains. In order to observe some features of the DIA modes in space plasmas, we consider plasma parameters that are representative of a dusty region at an altitude of about 95 km where \( n_e/n_p = 300/28 \approx 10.7 \), \( T_e \sim T_p \sim T_n \sim 200 \text{ K} \), \( n_{e0} \sim n_{p0} \sim 2 \times 10^4 \text{ cm}^{-3} \), \( n_{e0} = n_{a0}/700 \), \( n_{p0} = 500n_{e0} \), \( n_{d0} \sim 1.7 \times 10^6 \text{ cm}^{-3} \), \( \tilde{\eta}_p = 0.2 \) and \( \tilde{\eta}_n = 0.1 \).

So, in the nonlinear regime, the fast mode may propagate as DIA solitary waves due to nice balance of the dispersive and nonlinear effects. However, as the dissipation effects due to ion-dust collisions and ion kinematic viscosities, are entered into the dynamics, the system’s evolution shows shock-like perturbations. These features will be studied in the next section.

IV. DERIVATION OF THE EVOLUTION EQUATION

We consider the nonlinear propagation of small but finite amplitude DIA waves in a collisional dusty negative ion plasma. In order to observe the evolution of the waves in a different frame of reference which is moving with a speed \( v_0 \), we stretch the coordinates as \( \xi = \epsilon^{1/2}(x - v_0 t) \) and \( \tau = \epsilon^{3/2}t \), where \( \epsilon \) is a small parameter measuring the weakness of perturbations. We also assume that \( \tilde{\nu}_{jd} = \epsilon^{3/2}\nu_{j0} \) and \( \tilde{\eta}_j = \epsilon^{1/2}\eta_{j0} \), where \( \nu_{j0} \) and \( \eta_{j0} \) are of the order of unity or less. Such a consideration of smallness of \( \eta_j \) and \( \nu_{jd} \) can be found in the literature \[13\], and is valid in many experimental situations (see e.g., Ref.
The dynamical variables are expanded as
\[ n_j = 1 + \epsilon n_j^{(1)} + \epsilon^2 n_j^{(2)} + \cdots, \]
\[ v_j = \epsilon v_j^{(1)} + \epsilon^2 v_j^{(2)} + \cdots, \]
\[ \phi = \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots. \]  

We then substitute these expansions and the stretched coordinates into Eqs. (7)–(9), and equate different powers of \( \epsilon \). In the lowest order of \( \epsilon \) (i.e., \( \epsilon^{3/2} \)) we obtain the following relations for the first order perturbations
\[ n_j^{(1)} = \alpha_j \phi^{(1)}, \quad v_j^{(1)} = \alpha_j v_0 \phi^{(1)}, \]

(16)

(17)

together with the expression for the wave speed in the moving frame of reference as
\[ v_0^2 = \frac{1}{2\mu_c} \left[ S \pm \sqrt{S^2 - 12\beta \mu_c [\sigma_p + \sigma_n (\mu_i + 3\mu_p \sigma_p)]} \right], \]

(19)

where \( S = 1 + \mu_i \beta + 3(\sigma_p + \beta \sigma_p) \mu_c \) and \( \alpha_j = \pm \beta_j / (v_0^2 - 3\beta_j \sigma_j) \). Here \( \pm \) denotes the quantities corresponding to positive \( (j = p) \) and negative \( (j = n) \) ions respectively. This \( \pm \) sign appears due to the consideration of different mass and temperatures of positive and negative ions.

However, in a limited situation where \( m_p = m_n \) and \( T_p = T_n \), we can recover the expression (here one should consider the plus sign before the square root in \( v_0^2 \)) as in Ref. 12. However, in the next section, we will see that only the minus sign \( (\pm) \) favors the formation of SWS in laboratory and space plasmas, since for the plus sign, the nonlinear term becomes much larger than the dispersive term in the evolution equation to be shown shortly.

Proceeding to the next order of \( \epsilon \) (i.e., \( \epsilon^{5/2} \)) we obtain the following set of equations for the second order perturbed quantities
\[ -v_0 \frac{\partial n_j^{(2)}}{\partial \xi} + \alpha_j \frac{\partial \phi^{(1)}}{\partial \tau} + \alpha_j^2 v_0 \frac{\partial (\phi^{(1)})^2}{\partial \xi} + \frac{\partial v_j^{(2)}}{\partial \xi} = 0, \]

(20)

\[ = v_0 \left( \frac{\partial v_j^{(2)}}{\partial \xi} + \eta_j \alpha_j \right) \frac{\partial^2 \phi^{(1)}}{\partial \xi^2} + \beta_j \left( 3\sigma_j \frac{\partial n_j^{(2)}}{\partial \xi} \pm \frac{\partial \phi^{(2)}}{\partial \xi} \right), \]

(21)
After few steps, the following evolution equation of the Korteweg-de-Vries-Burger (KdVB) equation of the Korteweg de-Vries-Burger (KdVB) (20)-(22), we obtain, after few steps, the following evolution equation of the Korteweg de-Vries-Burger (KdVB) type.

\[
\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = \mu_e \left( \frac{1}{2} \frac{\partial (\phi^{(1)})^2}{\partial \xi} + \frac{\partial \phi^{(2)}}{\partial \xi} \right) + \frac{\partial n_a^{(2)}}{\partial \xi} - \mu_i \frac{\partial n_p^{(2)}}{\partial \xi},
\]

where \( \pm \) sign in Eq. (21) corresponds to positive \((j = p)\) and negative \((j = n)\) ions respectively.

Eliminating the second order quantities from Eqs. (20), (21), we obtain, after few steps, the following evolution equation of the Korteweg de-Vries-Burger (KdVB) type.

\[
\frac{\partial \Phi}{\partial \tau} + A \frac{\partial \Phi}{\partial \xi} + B \frac{\partial^3 \Phi}{\partial \xi^3} = \eta \frac{\partial^2 \Phi}{\partial \xi^2} - \nu \Phi,
\]

where \( \Phi \equiv \phi^{(1)} \). The coefficients of nonlinearity and dispersion as well as of dissipation due to the ion kinematic viscosities and the ion-dust collisions are, respectively, given by

\[
A = 3 \alpha_p \mu_i (\nu_0^2 + \beta \sigma_p) + 3 \beta \sigma_n (\nu_0^2 + \sigma_n) + \beta (\alpha_n - \mu_i \alpha_p),
\]

\[
B = \frac{\beta}{2 \nu_0 (\mu_i \alpha_p^2 + \beta \alpha_n^2)},
\]

\[
\eta = \frac{\mu_i \nu_0 \alpha_p^2 + \beta \nu_0 \alpha_n^2}{2 (\mu_i \alpha_p^2 + \beta \alpha_n^2)}, \quad \nu = \frac{\mu_i \nu_0 \alpha_p^2 + \beta \nu_0 \alpha_n^2}{2 (\mu_i \alpha_p^2 + \beta \alpha_n^2)}. \tag{26}
\]

In particular, for \( m_n = m_p, T_n = T_p \) and \( \eta_n = \eta_p \), we recover the expressions as in Ref. [12]. When the dissipation terms proportional to \( \eta \) and \( \nu \) are ignored, i.e., in absence of frictional force and viscous stress, the resultant equation is the KdV equation discussed in Ref. [13]. Thus, the Burger term in Eq. (23) gives rise to the generation of shocks and the frictional force term provides damping of the wave due to ion-dust collision. The latter increases with the increase of dust number density.

**V. RESULTS AND DISCUSSION**

Typically, the existence of SWS with positive or negative potential depends on the sign of the nonlinear coefficient \( A \). So, we numerically examine the behaviors of the nonlinear and the dispersion coefficients \( A \) and \( B \) with the variations of the positive to negative ion density ratio \( \mu_i \). The parameters are considered for both positively (Fig. 4) and negatively (Fig. 5) charged dusts as in Fig. 1. The solid and dashed lines correspond to \( T_n = T_e / 8 \), \( T_p \sim T_e \) and \( T_n = T_e / 10 \), \( T_p \sim T_e / 2 \) respectively. We have considered, respectively, the negative and positive...
signs in the expression for $v_0^2$ [Eq. (19)] for the subplots [(a), (c)] and [(b), (d)] of Figs. 4 and 5 to compare the corresponding values of $A$ and $B$. From Figs. 4(b) and (d) we find that for multi-ion plasmas with positively charged dusts, the numerical values of the dispersion coefficient $B$ are much larger than those of the nonlinear coefficient $A$. However, for plasmas with negatively charged dusts, the coefficient $A$ can become larger than $B$ with an increasing value of $\mu_i$ [See subplots 5(a) and (c)]. So, in both these cases, the balance between the nonlinear and the dispersive effects may not occur for the formation of KdV solitons in collisional dusty negative ion plasmas. Furthermore, when the dissipation overwhelms the wave dispersion and it is in nice balance with the nonlinearity arising from the nonlinear mode coupling of finite amplitude waves, the monotonic shocks may be generated. It turns out that the values of $A$ and $B$ in the subplots [(a), (c)] of Fig. 4 may favor the formation of SWS with negative potentials (Since $A$ maintains only the negative sign), whereas those in the subplots [(b), (d)] of Fig. 5 may favor the formation of SWS with both positive and negative potentials (Since $A$ can have both the positive and negative signs). Thus, we may conclude that plasmas with positively charged dusts may support the propagation of SWS with negative potentials, however, SWS with both positive and negative potential may exist when dusts are negatively charged. In the former case, the number density of negative ions greatly exceeds that of electrons and positive ions, whereas, positive ion density is to be larger than the density of negative ions in the latter case. In our numerical solution of Eq. (26) below, we present the characteristic features of those SWS which have negative potential (as the qualitative features of SWS with positive potential will be similar).

Next, we numerically solve Eq. (26) by Runge-Kutta scheme with an initial condition of the form $\Phi(\xi) = -0.03sech^2(\xi/15)$. The development of this waveform at different times and for different parameters are shown in Figs. 6-8. The parameter values are considered as in Figs. 1-3 for both positively and negatively charged dusts in electronegative plasmas. Figure 6 shows the case of KdV soliton, i.e., in absence of any dissipation for different times: (a) $\tau = 0$, (b) $\tau = 200$, (c) $\tau = 300$ and (d) $\tau = 500$. We see that the leading part of the initial pulse steepens due to positive nonlinearity. As the time progresses, the pulse separates into solitons and a residue due to the wave dispersion. It is clear from Fig. 6(d) that once the solitons are formed and separated, they propagate without changing their shape due to the nice balance of the nonlinear and dispersion terms. This subplot also shows that (see the dashed line) when charged dusts are introduced into the plasma, the magnitude of the soliton height is reduced, but the width is increased. However, as the number density of negative ion increases (see the dotted line), both the amplitude and width of the soliton decrease. In each of these cases the soliton is seen to be up-shifted.

Figure 7 shows the profiles of the solution of KdVB equation for plasmas with positively charged dusts. In this case, the values of $\eta_j$, $\nu_j$ are considered arbitrarily in order to observe the damping of solitary waves. From Fig. 7(b), it is clear that as the dissipation due to ion kinematic viscosity enters into the dynamics, the solitary pattern breaks up and a shock profile is formed. Also, as the term $\propto \eta_j$ increases, the height of the shock profile decreases. Physically, the increase of $\eta_j$ corresponds to the increase of the dust number density, which, in turn, increases the ion-dust collision frequency. While compared with Fig. 7 (a), Figures 7(c) and (d) show the wave damping due to the presence of ion-dust collisions. The latter are shown to decrease the wave amplitude significantly as in Fig. 7(d) when some lower values of them are considered. From Figure 7(c) we find that the presence of charged dusts as well as the increase of the negative ion density decrease the amplitude of shocks. This is expected as the introduction of charged dusts causes dissipation via the collisional term and the viscosity term. Also, as the negative ion density increases, more electrons will be removed by the dust grains in order to maintain the charge neutrality. This also results to the increase of the positively charged dust density and hence may cause the reduction (in magnitude) of the wave amplitude.

Figure 8 shows the developments of solitary waves into shocks for plasmas with negatively charged dusts. In absence of the dissipative effects [Fig. 8(a)], a different wave pattern with a train of solitons is seen to form due to the dominant role of wave dispersion over the nonlinearity. As the dissipation due to viscosity effects starts playing a role, the number of wave trains reduces and the wave steepening begins to occur at a value of $\eta_j < 1$ [See Fig. 8(b)]. Further increasing the value of $\eta_j$ ($\eta_j \sim 1$) results into the fact that the train of solitons disappear leaving only one wave front with a lower value of $|\Phi|$ [See Fig. 8(c)]. From Fig. 8(d) it is seen that due to the effects of ion-dust collisions, the wave gets damped [Similar to the case of positively charged dusts (Fig. 7)] and the wave amplitude reduces in its magnitude. We note that the damping seen in this case is qualitatively different from the case of positively charged dusts (Fig. 7). From Fig. 8(d) we also find that in contrast to Fig. 7(c) where the negative ion density decreases the wave amplitude, as the number density of positive ions increases, the magnitude of $\Phi$ increases. Here the increase of the positive ion density causes the addition of more electrons into the dust grain surface, which, in turn, increases the dust charge state in order to maintain the charge neutrality. As a result, the dissipative effects due to dust density enhancement become more pronounced and the wave steepening occurs with an increased $|\Phi|$.

Another interesting feature may be the formation of double-layer solutions of Eq. (26). Such solutions consist of two layers with positive and negative space charge and support localized electric fields. Also, double-layers (DLs) can play important roles in providing a mechanism for supporting electric fields in collisionless plasmas with
FIG. 4. (Color online) The plots of the nonlinear coefficient $A$ (upper panels) and the dispersion coefficient $B$ (lower panels) versus the positive to negative ion density ratio $\mu$ are shown for plasmas with positively charged dusts. The other parameters are as in Fig. 1. The solid and dashed lines correspond to $T_n = T_e/8$, $T_p \sim T_e$ and $T_n = T_e/10$, $T_p \sim T_e/2$ respectively. In the subplots [(a), (c)] and [(b), (d)], the values of $v_0^2$ [Eq. (19)] corresponding respectively to the negative and positive signs have been considered to compare the results as well as to show that the numerical values of $A$ and $B$ in the subplots [(a), (c)] may favor (while the others may not) the formation of solitary waves and shocks with negative potentials.

nearly zero resistivity. Furthermore, DLs have relevance to charge particle acceleration in cosmic plasmas and in plasma thrusters. However, there are several mechanisms by which DLs can be formed (See, e.g., Ref. [24] for some recent review works on DLs). Nevertheless, such DLs are more difficult to generate in a laboratory and require a fine tuning of the plasma parameters. However, the detail discussion on the formation and the properties of DLs is limited to the present study as we have mainly focused on the formation and the characteristics features of solitons, and shocks so generated due to dissipative effects.

In what follows, Eq. (23) can have a travelling wave solution, in absence of the collisional effects ($\nu = 0$), given by

$\Phi(\xi, \tau) = C_1 - \frac{12\eta^2}{25AB (1 + C_2 e^\psi)^2},$

$\psi = -\frac{\eta}{5B} \xi + \left( \frac{\eta AC_1}{5B} - \frac{6\eta^3}{125AB^2} \right) \tau,$  \hspace{1cm} (27)

where $C_1$ and $C_2$ are arbitrary constants. The profiles of Eq. (27) for different plasma parameters are shown in Fig. 9. Figures 9(a)-(c) exhibit some shock profiles corresponding to plasma parameters relevant for negatively charged dusts, whereas Fig. 9(d) is for negative ion plasmas with positively charged dusts. We find that the DLs with both the compressive and rarefactive potentials may exist [Figs. 9(a) and (b)] for a certain range of values of $\eta_{j0}$, beyond which DLs with only negative potential (rarefactive) may be formed [Fig. 9(c)]. From Figs. 9(a) and (b), it is also seen that for a fixed negative ion density as the density of positive ions increases, the value of $|\Phi|$ increases by the same physical reason as in Fig. 8(d). The opposite trend may be seen to occur by increasing the negative ion density (in case of positively charged dusts) or by decreasing the values of $\eta_{j0}$. The effects of the latter are shown in Fig. 9(c) exhibiting DLs with only negative potential. It turns out that the wave front with two layers may not be formed by gradually decreasing the values of $\eta_{j0}$ or by reducing the charged dust state or density in negative ion plasmas with negatively charged dusts, rather we can see the shock wave fronts with only negative potentials. Figure 9(d) also shows that if the dusts are positively charged or electrons are almost absorbed by the dust grains in pair-ion plasmas, the formation of DLs may not be possible. However, a shock profile with negative potential may exist with a significant drop of the wave amplitude $|\Phi|$ compared to the case of negatively charged dusts.
FIG. 5. (Color online) The plots of the nonlinear coefficient $A$ (upper panels) and the dispersion coefficient $B$ (lower panels) versus the positive to negative ion density ratio $\mu_1$ are shown for plasmas with negatively charged dusts. The other parameters are as in Fig. 1. The solid and dashed lines correspond to $T_n = T_e/8$, $T_p \sim T_e$ and $T_n = T_e/10$, $T_p \sim T_e/2$ respectively. In the subplots [(a), (c)] and [(b), (d)] the values of $v^2_{i0}$ [Eq. (19)] corresponding respectively to the negative and positive signs have been considered to compare as well as to show that the numerical values of $A$ and $B$ in the subplots [(b), (d)] may favor (while the others may not) the formation of solitary waves and shocks with both positive and negative potentials.

To summarize, we have investigated the propagation characteristics of DIA waves in a collisional negative ion plasma with immobile charged dusts. The latter may be positively charged when the number density of negative ions exceeds that of positive ions and is much larger than that of electrons. In the linear regime, we have studied numerically the ion-acoustic instability with plasma parameters relevant for both laboratory [19, 22] and space plasmas [20]. We find that the two modes, namely ‘fast’ and ‘slow’ waves exist in dusty negative ion plasmas. Due to the higher values of the phase velocity of the fast ion wave than the ion thermal speeds, the Landau damping effect on dissipation is negligible, however, the damping of the solitary waves is mainly caused by the ion-dust collisions and the ion kinematic viscosities. We find that the slow modes propagate without any instability and with frequency below the negative ion plasma frequency. However, for long wavelength fast modes, the damping effect is mainly due to the ion-dust collisions. For laboratory negative ion plasmas when the electron flow is sufficient to drive the ion-acoustic instability, the inclusion of micron seized dust grains may damp the instability due to their collision with ions. The frequency and the growth rates are reduced when dusts are positively charged. However, the opposite trend occurs when dusts are negatively charged. Thus, the phase velocity of the waves may decrease or increase with the dust density according to when the dusts are positively or negatively charged. These results suggest a possible diagnostic for the presence of positively or negative charged dusts as well as for the effects of ion-dust collisions in dusty negative ion plasmas [19, 22]. For the space situation, though the behaviors remain similar, however, the wave frequency and the instability rate increase faster with the wave number than the case of laboratory plasmas.

We have also studied the nonlinear propagation of small amplitude fast modes as DIA SWS in dusty negative ion plasmas. We show that the evolution of such waves can be described by a modified KdVB equation. The latter generalizes and modifies the previous investigation in negative ion plasmas [12]. The KdVB equation is numerically solved to show that the perturbations with negative potential may propagate as SWS in plasmas with positively charged dusts, whereas SWS with both positive and negative potential may exist when dusts are negatively charged. The profiles of these SWS with only negative potential are shown graphically (as those with positive potential are similar), and analyzed with pa-
FIG. 6. (Color online) Development of an initial pulse of the form $\Phi(\xi) = -0.03\text{sech}^{2}(\xi/15)$ into solitary waves at different times for the parameters as in Fig. 1, and when $\nu_{jd} = \bar{\eta}_{jd} = 0$. In the subplot (d), the dashed (red) and dotted (black) lines correspond to the cases of no dust and when negative ion density is different, i.e., $n_{n0} = 1000n_{e0}$ in the plasma respectively.

rameters relevant for laboratory and space plasmas. We find that, in contrast to the effects of negatively charged dusts, the presence of positively charged dusts reduces the wave amplitude, but enhances the width of solitary waves. The theoretical results may be useful for the observation of dust ion-acoustic waves in space plasmas, e.g., a dusty meteor trail region in the upper atmosphere as well as the experimental verification of the excitation of ion-acoustic instability and the nonlinear propagation of ion-acoustic solitary and shock waves in dusty multi-ion plasmas.

There may be some open issues, e.g., the theoretical deduction of ion kinematic viscosity and the effects of ion drag forces due to positive and negative ions on the dust particles could be problem of interest but beyond the scope of the present investigation. Such forces, which are opposite in direction, may also be comparable in magnitude when the positive and negative ion densities are comparable. Since the outward ion drag force is known to be responsible for the formation of a void under microgravity conditions, the presence of both the positive and negative ions could forbid the formation of voids.

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[1] E. Grün, G.E. Morfill and D.A. Mendis, Planetary Rings (eds R. Greenberg, and A. Brahic), (Univ. of Arizona Press, Tucson, 1984).
[2] C. K. Goertz, Rev. Geophys. 27, 271 (1989).
[3] A. Barkan, N. D. Angelo, and R. L. Merlino, Phys. Rev. Lett. 73, 3093 (1994).
[4] N. C. Adhikary, M. K. Deka and H. Bailung, Phys. Plasmas 16, 063701 (2009).
[5] N. N. Rao, P.K. Shukla, M. Y. Yu, Planet. Space Sci. 38, 543 (1990).
[6] P. K. Shukla and V. P. Silin, Phys. Scr. 45, 508 (1992).
[7] A. P. Misra, A. R. Chowdhury and K. R. Chowdhury, Phys. Lett. A 323, 110 (2004).
[8] A. P. Misra, K. R. Chowdhury and A. R. Chowdhury, Phys. Plasmas 14, 012110 (2007).
[9] N. S. Saini and I. Kourakis, Phys. Plasmas 15, 123701 (2008).
FIG. 7. (Color online) Development of an initial pulse of the form $\Phi(\xi) = -0.03\text{sech}^2(\xi/15)$ into shocks at $\tau = 500$ for different values of $\nu_j0$ and $\eta_j0$ as in the figure. The parameters values are as in Fig. 1. In the subplot (c), the dashed (black) line corresponds to the case of different negative ion density $n_{n0} = 900n_{e0}$ with the same other parameters as the solid line.

[10] A. A. Mamun, Phys Lett. A 372, 4610 (2008).
[11] A. A. Mamun, R. A. Cairns and P. K. Shukla, Phys Lett. A 373, 2355 (2009).
[12] N. C. Adhikary, Physics Letters A 376, 1460 (2012).
[13] H. Washimi and T. Taniuti, Phys. Rev. Lett. 17, 996 (1966).
[14] G. C. Das and S. G. Tagare, Plasma Phys. 17, 1025 (1975).
[15] Y. Nakamura, J. L. Ferreira, and G. O. Ludwig, J. Plasma Phys. 33, 237 (1985).
[16] Y. Nakamura, H. Bailung, P.K. Shukla, Phys. Rev. Lett. 83, 1602 (1999).
[17] P. K. Shukla, Phys. Plasmas 7, 1044 (2000).
[18] S. Ghosh, S. Sarkar, M. Khan, and M. R. Gupta, Phys. Plasmas 9, 378 (2002).
[19] S. -H. Kim and R. L. Merlino, Phys. Plasmas 13, 052118 (2006).
[20] M. Rapp, J. Hedin, I. Strelnikova et al, Geophys. Res. Lett. 32, L23821 (2005).
[21] Y. Nakamura and A. Sarma, Phys. Plasmas 8, 3921 (2001).
[22] M. Rosenberg and R. L. Merlino, Planet Space Sci. 55, 1464 (2007).
[23] A. Y. Wong, Introduction to experimental plasma physics, vol. 1, Springer (1977).
[24] N. Singh, Phys. Plasmas 18, 122105 (2011).
[25] A. D. Polyanin and V. F. Zaitsev, Handbook of nonlinear partial differential equations, Second edition, CRC Press, pp. 885 (2012).
FIG. 8. (Color online) Development of an initial pulse of the form $\Phi(\xi) = -0.03\text{sech}^2(\xi/15)$ into shocks at $\tau = 500$ for different values of $\nu_j0$ and $\eta_j0$ as in the figure. The parameters are for plasmas with negatively charged dusts where $n_e0 = n_m0 / 10$, $n_p0 = 20n_e0$. Others parameters are as in Fig. 1. In the subplot (d), the dashed (black) line corresponds to different value of the positive ion density, i.e., $n_p0 = 40n_e0$ with the same other parameters as the solid line.
FIG. 9. (Color online) Analytic solution of Eq. (27) for different plasmas with negatively charged dusts [subplot (a)-(c)] and positively charged dusts [subplot (d)]: (a) $n_{e0} = n_{n0}/2$, $n_{p0} = 5n_{e0}$, $\eta_{p0} = 1.2$, $\eta_{n0} = 1.1$; (b) $n_{e0} = n_{n0}/2$, $n_{p0} = 10n_{e0}$, $\eta_{p0} = 1.3$, $\eta_{n0} = 1.2$; (c) $n_{e0} = n_{n0}/2$, $n_{p0} = 5n_{e0}$, $\eta_{p0} = 0.7$, $\eta_{n0} = 0.6$; (d) $n_{e0} = n_{n0}/700$, $n_{p0} = 500n_{e0}$, $\eta_{p0} = 1.2$ and $\eta_{n0} = 1.1$. 