Classical Underpinnings of Gravitationally Induced Quantum Interference∗

Philip D. Mannheim
Department of Physics, University of Connecticut, Storrs, CT 06269
mannheim@uconnvm.uconn.edu

Abstract

We show that the gravitational modification of the phase of a neutron beam (the COW experiment) has a classical origin, being due to the time delay which classical particles experience in traversing a background gravitational field. Similarly, we show that classical light waves also undergo a phase shift in traversing a gravitational field. We show that the COW experiment respects the equivalence principle even in the presence of quantum mechanics.

In a landmark series of experiments [1], [2] Colella, Overhauser and Werner (COW) and their subsequent collaborators (see e.g. Refs. [3], [4] for overviews) detected the modification of the phase of a neutron beam as it traverses the earth’s gravitational field, to thus realize the first experiment which involved both quantum mechanics and gravity. A typical generic experimental set up is shown in the schematic Fig. (1) in which a neutron beam from a reactor is split by Bragg or Laue scattering at point A into a horizontal beam AB and a vertical beam AC (we take the Bragg angle to be 45° for simplicity and illustrative convenience in the following), with the subsequent scatterings at B and C then producing beams which Bragg scatter again at D, after which they are then detected. If the neutrons arrive at A with velocity $v_0$ (typically of order $2 \times 10^5$ cm sec$^{-1}$) and $ABCD$ is a square of side $H$, then the phase difference $\phi_{COW} = \phi_{ACD} - \phi_{ABD}$ is given by $-mgH^2/\hbar v_0$ to lowest order in the acceleration $g$ due to gravity [1], and is actually observable despite the weakness of gravity, since even though $\int \bar{p} \cdot d\bar{r}$ only differs by the very small amount $m(v_{CD} - v_{AB})H = -mgH^2/v_0$ between the $CD$ and $AB$ paths, nonetheless this quantity is not small compared to Planck’s constant, to thus give an observable fringe shift even for $H$ as small as a few centimeters.

The detected COW phase is extremely intriguing for two reasons. First, it shows that it is possible to distinguish between different paths which have common end points, with the explicit global ordering in which the horizontal and vertical sections are traversed leading to observable consequences. And second, it yields an answer which explicitly depends on the mass of the neutron even while the classical neutron trajectories (viz. the ones explicitly followed by the centers of the wave packets of the quantum mechanical neutron beam) of course do not. The COW result thus invites consideration of whether the detected ordering

∗UCONN 96-08, October 1996
is possibly a topological effect typical of quantum mechanics, and of whether quantum mechanics actually respects the equivalence principle\[1] As we shall see, the ordering effect is in fact already present in the motion of classical particles in gravitational fields, and even in the propagation of classical waves in the same background, with this latter feature enabling us to establish below that the mass dependence of the neutron beam COW phase is purely kinematic with the equivalence principle then not being affected.

To address these issues specifically we have found it convenient to carefully follow the neutron as it traverses the interferometer, to find that the two beams do not in fact arrive at the same point $D$ or even at the same time, with this spatial offset and time delay not only producing the interference effect, but also being present in the underlying classical theory. Quantum mechanics thus does not cause the time delay, rather it only serves to make it observable. Since gravity is a relativistic theory we shall need to introduce curvature (which we do below), but we have found it more instructive to consider the non-relativistic limit first. Since we can treat the neutron beams as rays, their motions round the $ABCD$ loop can be treated purely classically between the various scatterings. Moreover, the various scatterings themselves at $A$, $B$, $C$ and $D$ introduce no additional phases, are energy conserving, and give angles of reflection equal to the angles of incidence.\[2] Thus the entire motion of the neutron is the same as that of a classical macroscopic particle (the neutron spin plays no explicit role in the COW experiment, so we treat the neutrons as spinless) which undergoes classical mirror reflections.

The neutron which goes up vertically from $A$ to $C$ arrives at $C$ with a velocity $(0, v_0 - gH/v_0)$. The neutron $AC$ travel time is $t(AC)$ and the classical action $S = \int (p \cdot \, dr - E_0 dt)$ ($E_0 = mv_0^2/2$) undergoes a change $S(AC)$ where

$$
t(AC) = H/v_0 + gH^2/2v_0^3, \quad S(AC) = mv_0H - mgH^2/2v_0 - E_0 t(AC) \quad (1)
$$

On scattering at $C$ the neutron is then reflected so that it starts off toward $D$ with a velocity $(v_0 - gH/v_0, 0)$. On its flight it dips slightly to arrive at the next scattering surface at the point $D_1$ with coordinates $(H - gH^2/2v_0^2, H - gH^2/2v_0^2)$, so that there is a change in the end point of the motion which is first order in $g$ and thus relevant to our discussion. At $D_1$ the neutron has a velocity $(v_0 - gH/v_0, -gH/v_0)$, with the $CD_1$ segment taking a time $t(CD_1)$

---

1. Possible implications of the mass dependence of the COW phase on another related issue, viz. quantum mechanics and the action and reaction principle, are explored in Ref. \[\text{[1]}\].

2. This is not completely obvious since if the neutron slows down a little or is deflected a little, then at each subsequent scattering it is not quite incident with a wavelength and angle which satisfy a Bragg peak condition. However, consider a crystal for which Bragg scattering would occur for the pair of vectors $k_i$ and $k_f$, viz. equal length vectors which obey $k_i - k_f = G$ where $G$ is a reciprocal lattice vector which imparts a momentum transfer perpendicular to the scattering surface. Now instead allow $k_i$ to impinge upon a crystal with additional momentum $\Delta k_i$ with the outgoing $k_f$ beam then emerging with additional momentum $\Delta k_f$. Since each such scattering is coplanar the most general such momentum modification can conveniently be written as $\Delta k_i = \alpha_i k_i + \alpha_f k_f$, $\Delta k_f = \beta_i k_i + \beta_f k_f$, with energy conservation yielding $\beta_i = \alpha_f$, $\beta_f = \alpha_i$ since there is no energy transfer to the crystal. Consequently, the momentum transfer at a scattering is now given by $k_i + \Delta k_i - k_f - \Delta k_f = (1 + \alpha_i - \alpha_f)G$, and is thus still perpendicular to the scattering plane, but now no longer right on a diffraction maximum. The angle of reflection is thus still equal to the angle of incidence, but with a scattering intensity which is slightly reduced with respect to the $\Delta k = 0$ case. (The only way that we would actually be able to shift into a new Bragg peak condition would be to have $\alpha_i - \alpha_f = 0$, a condition which only occurs when each $\Delta k$ is parallel to the scattering surface.)
and contributing an amount $S(CD_1)$ to the classical action where

$$t(CD_1) = H/v_0 + gH^2/2v_0^3, \quad S(CD_1) = mv_0H - 3mgH^2/2v_0 - E_0t(CD_1)$$

(2)

The neutron which starts horizontally from $A$ arrives not at $B$ but at the point $B_1$ with coordinates $(H - gH^2/2v_0^2, -gH^2/2v_0^2)$ and with a velocity $(-gH/v_0)$. The $AB_1$ segment takes a time $t(AB_1)$ and the action changes by $S(AB_1)$ where

$$t(AB_1) = H/v_0 - gH^2/2v_0^3, \quad S(AB_1) = mv_0H - mgH^2/2v_0 - E_0t(AB_1)$$

(3)

After scattering at $B_1$ the neutron sets off toward $D$ with velocity $(-gH/v_0, v_0)$ and arrives not at $D$ or $D_1$ but rather at the point $D_2$ with coordinates $(H - 3gH^2/2v_0^2, H - 3gH^2/2v_0^2)$, and reaches there with velocity $(-gH/v_0, v_0 - gH/v_0)$. The $B_1D_2$ segment takes a time $t(B_1D_2)$ and the action changes by $S(B_1D_2)$ where

$$t(B_1D_2) = H/v_0 - gH^2/2v_0^3, \quad S(B_1D_2) = mv_0H - 3mgH^2/2v_0 - E_0t(B_1D_2)$$

(4)

As regards the neutron’s path around the loop, we see that the small vertical $gH^2/2v_0^2$ dip during each of the two horizontal legs causes each neutron to travel a distance $gH^2/2v_0^2$ less in the horizontal than it would have done in the absence of gravity, to thus provide a first order in $g$ modification to $\int \vec{p} \cdot d\vec{r}$ in each of these legs, even while these same vertical dips themselves only contribute to the action in second order. However, for the two horizontal sections, each leg is shortened by the same amount in the horizontal, so that the difference in $\int \vec{p} \cdot d\vec{r}$ between the $CD_1$ and $AB_1$ legs still takes the value $m(v_{CD} - v_{AB})H = -mgH^2/v_0$ quoted earlier. As regards the two vertical legs, we note that even though the $AC$ leg is completely in the vertical, since the neutron starts the $B_1D_2$ leg with a small horizontal velocity, during this leg the neutron changes its horizontal coordinate by an amount $gH^2/v_0^2$, thereby causing it to reach $D_2$ after having also traveled a distance $gH^2/v_0^2$ less in the vertical than it would travel in the $AC$ leg. Consequently, there is both a spatial offset $(gH^2/v_0^2, gH^2/v_0^2)$ between $D_1$ and $D_2$, and a time delay $t(AC) + t(CD_1) - t(AB_1) - t(B_1D_2) = 2gH^2/v_0^2$ between the arrival of the two beams, with the integral $\int \vec{p} \cdot d\vec{r}$ thus not taking the same value in each of the two vertical legs. However, our calculation shows that all the these modifications actually compensate in the overall loop with there being no difference in $\int \vec{p} \cdot d\vec{r}$ between the $ACD_1$ and $AB_1D_2$ paths. However, since there is a time delay, there is still a net change in the action, given by $S(AC) + S(CD_1) - S(AB_1) - S(B_1D_2) = -mgH^2/v_0$. However, we cannot identify this quantity with the COW phase $\hbar \Delta \phi_{COW}$ since the beams have not interfered due to the spatial offset between $D_1$ and $D_2$.

Before discussing the issue of this spatial offset, it is instructive to ask where the neutrons would have met had there been no third crystal at $D$ to get in the way. Explicit calculation shows that they would in fact have met at the asymmetric point $D_3$ with coordinates $(H - 3gH^2/2v_0^2, H - gH^2/2v_0^2)$ with the $CD_3$ and $B_1D_3$ segments taking times $t(CD_3)$ and $t(B_1D_3)$ while yielding action changes $S(CD_3)$ and $S(B_1D_3)$, where

$$t(CD_3) = H/v_0 - gH^2/2v_0^3, \quad S(CD_3) = mv_0H - 5mgH^2/2v_0 - E_0t(CD_3)$$

$$t(B_1D_3) = H/v_0 + gH^2/2v_0^3, \quad S(B_1D_3) = mv_0H - mgH^2/2v_0 - E_0t(B_1D_3)$$

(5)

The neutrons would thus meet at $D_3$ without any time delay, and with $S(AC) + S(CD_3) - S(AB_1) - S(B_1D_3) = -2mgH^2/v_0$. We thus see that for purely classical particles reflecting
off mirrors at \( B_3 \) and \( C \) the quantity \( \int \vec{p} \cdot d\vec{r} \) evaluates differently for the two paths \( ACD_3 \) and \( AB_1D_3 \). This is a thus a global, path dependent effect in purely classical mechanics in a background classical gravitational field which is completely independent of quantum mechanics.\(^3\) However, since the classical action is not observable in classical mechanics, it is only in the presence of quantum mechanics that phase differences become observable. (In classical mechanics what is observable is that the neutrons meet at \( D_3 \) rather than on the \( AD \) axis.)

Returning now to the COW experiment itself, in order to understand the implications of the time and spatial offsets between \( D_1 \) and \( D_2 \), it is instructive to consider the Young double slit experiment with purely classical light. As shown in Fig. (2), light from a source \( S \) goes through slits \( Q \) and \( R \) to form an interference pattern at points such as \( P \), with the distance \( \Delta x = QT \) representing the difference in path length between the two beams. Given this path difference, the phase difference between the two beams is usually identified as \( k\Delta x \), from which an interference pattern is then readily calculated. However, because of this path difference, the \( SQP \) ray takes the extra time \( \Delta t = \Delta x/c \) to get to \( P \), to thus give a net change in the phase of the \( SQP \) beam of \( k\Delta x - \omega\Delta t \) which actually vanishes for light rays. The relative phase of the two light rays in the double slit experiment thus does not change at all as the two beams traverse the interferometer. However, because of the time delay, the \( SRP \) beam actually interferes with an \( SQP \) beam which had left the source a time \( \Delta t \) earlier. Thus if the source is coherent over these time scales, the \( SQP \) beam carries an additional \( +\omega\Delta t \) phase from the very outset. This phase then cancels the \( -\omega\Delta t \) phase it acquires during the propagation to \( P \) (a cancellation which clearly also occurs for quantum mechanical matter waves moving with velocities less than the velocity of light), leaving just \( k\Delta x \) as the final observable phase difference, a quantity which is non-zero only if there is in fact a time delay. We thus see that the double slit device itself actually produces no phase change for light. Rather, the choice of point \( P \) on the screen is a choice which selects which time delays at the source are relevant at each \( P \), with the general interference pattern thus not only always involving the time delay at the source, but also in fact always requiring one.

With this in mind, we now see that we also need to monitor the time delay of the neutron in the COW experiment. However, since the total energy of the neutron does not change as it goes through the interferometer, the time delay contribution will still drop out of the final phase shift expression (explicitly but not implicitly). However, for the COW experiment we noted above that as well as a time delay between the \( ACD_1 \) and \( AB_1D_2 \) paths, there was also a spatial offset. Consequently, the \( AB_1D_2 \) path interferes not with the \( ACD_1 \) path, but

\[^3\]Since the momentum \( \vec{p} \) is the vector derivative \( \nabla S \) of the stationary classical action, for any trajectory which obeys the equations of motion the integral \( \int \vec{p} \cdot d\vec{r} \) then only depends on its endpoints (in a manner which in general depends on the explicit \( \nabla S \) direction in which each endpoint is approached.) Consequently, it is at first a little surprising that \( \int \vec{p} \cdot d\vec{r} \) could evaluate differently for the \( ACD_3 \) and \( AB_1D_3 \) paths, since these paths do share common endpoints. However, neither one of these paths is the one which is the stationary one for the motion which starts at \( A \) and finishes at \( D_3 \) (viz. a parabola close to the straight line \( AD_3 \)); and thus even while the \( ACD_3 \) path for instance is composed of the two subpaths \( AC \) and \( CD_3 \) each one of which is stationary between its own endpoints, nonetheless the sum of the two subpaths is not stationary between the overall \( A \) and \( D_3 \) endpoints, with \( \vec{p} \) then not being expressible as a total derivative for the whole \( ACD_3 \) (or \( AB_1D_3 \)) motion. In passing, we note that the fact that the sum of two stationary paths is not necessarily stationary between the overall endpoints is also the origin (see e.g. \( \text{[3]} \)) of the existence of the many paths of the Feynman path integral description of quantum mechanics.
rather with the indicated offset $A_1C_1D_2$ path, a very close by path which in fact is found to lie a distance $gH^2/v_0^2$ vertically below $AB$, an offset distance which is within the resolution of the beam. The evaluation of the phase shift is then exactly as before with $S(A_1C_1D_2)$ taking the exact same value as $S(ACD_1)$ to lowest order in $g$. Now we noted above that all the $\int \vec{p} \cdot d\vec{r}$ contributions actually cancel for this particular set of paths. However, because of the spatial offset between $D_1$ and $D_2$, the $AB_1D_2$ path beam has to travel an extra horizontal distance $A_2A = gH^2/v_0^2$ to first get to the interferometer (to therefore provide an analog to the distance $\Delta x = QT$ in the double slit experiment, with $A_1$ and $A_2$ acting just like the pair of slits $Q$ and $R$). Now in traveling this extra $A_2A$ distance this beam actually acquires yet another time delay $t(A_2A)$, to therefore impose yet another relative phase condition at the source which then identically cancels the associated $-E_0t(A_2A)$ change in the action. Moreover, in traveling this extra $A_2A$ the integral $\int \vec{p} \cdot d\vec{r}$ acquires yet one more contribution $mgH^2/v_0$, and this term then emerges as the only contribution in the entire circuit which is not canceled. Consequently, we obtain $\Delta \phi_{COW} = -mgH^2/hv_0$ as the final observable net COW phase shift. 

Turning now to a fully covariant analysis, we need to look at solutions to the Klein-Gordon equation $\phi^{\mu}_{\mu} - (mc/h)^2 \phi = 0$ ($\phi^{\mu}$ denotes $\partial \phi / \partial x^{\mu}$) in generic background fields of the form $d\tau^2 = B(r)c^2 dt^2 - dr^2/B(r) - r^2 d\Omega$ where $B(r) = 1 - 2MG/c^2r$. Firstly we note that the non-relativistic reduction of this Klein-Gordon equation is straightforward, with the substitution $\phi = \exp(-imc^2t/h)\psi$ then yielding

$$i\hbar \frac{\partial \psi}{\partial t} + \frac{\hbar^2}{2m} \nabla^2 \psi = \frac{mc^2}{2} \left( B(r) - 1 \right) = - \frac{mMG}{r} \tag{6}$$

for slowly moving particles. We thus see that the inertial mass $m$ which is defined via the starting Klein-Gordon equation thus also serves as the passive gravitational mass which serves to couple massive particles to gravity, so that the particle modes associated with the quantization of the Klein-Gordon field thus automatically obey the equivalence principle, precisely because of quantum mechanics in fact. 

As regards the covariant Klein-Gordon equation, it is convenient to make the substitution $\phi(x) = \exp(iS(x)/\hbar)$, so that the phase then obeys $S^{\mu}_{\mu} + m^2c^2 = i\hbar S^{\mu}_{\mu}$. In the eikonal or ray approximation the $i\hbar S^{\mu}_{\mu}$ term can be dropped, so that the phase $\hat{S}(x)$ is then seen to obey the classical Hamilton-Jacobi equation $S^{\mu}_{\mu} + m^2c^2 = 0$, an equation whose solution

\[\text{[\text{4\footnote{While we obtain the same answer for $\Delta \phi_{COW}$ as previous authors, nonetheless our derivation seems to be somewhat different from the previously published ones.}}] \text{[\text{5\footnote{We note that since the starting covariant Klein-Gordon equation only possesses one intrinsic mass scale, it would appear that the equivalence principle has to emerge. However, there is actually a hidden assumption in the use of the covariant Klein-Gordon equation (see \text{[\text{4}\footnote{While we obtain the same answer for $\Delta \phi_{COW}$ as previous authors, nonetheless our derivation seems to be somewhat different from the previously published ones.}}]}]) since it is not the most general equation in curved space which can reduce to the flat Klein-Gordon equation in flat space. Rather, the curved space equation could also possess additional explicitly curvature dependent terms, terms which would then vanish in the flat space limit, but which would modify the particle's coupling to gravity in curved space in a potentially equivalence principle violating manner. Now such possible terms (which would of course have to produce effects of order less than $10^{-31}$ so as not to spoil the Eotvos result) are simply ignored in standard gravity without any apparent justification as far as we can tell, with the standard gravitational phenomenology only in fact following in their assumed absence. Thus in passing it is of interest to note that any such possible additional curvature dependent terms are not in fact allowed to appear \text{[\text{4}\footnote{While we obtain the same answer for $\Delta \phi_{COW}$ as previous authors, nonetheless our derivation seems to be somewhat different from the previously published ones.}}]}, in the conformal invariant gravitational alternative currently being considered by Mannheim and Kazanas (see e.g. \text{[\text{4}\footnote{While we obtain the same answer for $\Delta \phi_{COW}$ as previous authors, nonetheless our derivation seems to be somewhat different from the previously published ones.}}])].}}\]
is the stationary classical action between relevant end points. We thus establish that in the eikonal approximation the phase of the wave function of a material particle is in fact the classical action just as required for the discussion of the COW experiment we gave earlier. In the eikonal approximation we can also identify $S^\mu$ as the momentum $p^\mu = mc dx^\mu/d\tau$, so that we can set $S(x) = \int p^\mu dx^\mu$; with the covariant differentiation of the Hamilton-Jacobi equation then yielding $\Box p^\mu p^\nu_{\mu\nu} = 0$, which we recognize as the massive particle geodesic equation.

In order to actually calculate the geodesics in the gravitational field of the earth it is convenient to rewrite the Schwarzschild metric in terms of a Cartesian coordinate system $x = rsin\theta cos\phi$, $y = rsin\theta sin\phi$, $z = rcos\theta - R$ erected at a point on the surface of the earth. With $z$ being normal to the earth's surface, to lowest order in $x/R$, $y/R$, $z/R$, $MG/c^2R$ (where $M$ is the mass of the earth and $R$ its radius) the Schwarzschild metric is then found \cite{3} to take the form

$$dr^2 = f(z)c^2dt^2 - dx^2 - dy^2 - dz^2/f(z)$$

where $f(z) = 1 - 2MG/c^2R + 2gz/c^2$ and where $g$ denotes $MG/R^2$. For this metric the non-relativistic geodesics for material particles are given by $\ddot{x} = 0$, $\ddot{y} = 0$, $\ddot{z} = -g$, to thus enable us to completely justify our earlier non-relativistic calculation.\footnote{Since the metric of Eq. (3) yields a uniform gravitational acceleration, it must therefore be flat, and indeed it can readily be transformed into a flat Cartesian metric $dr'^2 = c^2dt'^2 - dx'^2 - dy'^2 - dz'^2$ via the coordinate transformation $c't = c^2\sinh(gt/c)f^{1/2}(z)/g$, $z' = c^2(cosh(gt/c)f^{1/2}(z) - f^{1/2}(0))/g$ under which the small $(z, t)$ region transforms into the small $(z', t')$ region. However, under such a transformation the time delay in the COW experiment would not be removed (the time delay between the two beams at the source is a timelike vector which remains timelike under any coordinate transformation), with the fact of a time delay thus being a covariant indicator for the COW effect. Similarly, since the action is the covariant scalar $\int p_\mu dx^\mu$, its value in any trajectory must remain unchanged under this transformation to flat Cartesian coordinates. Now while there is no explicit reference to $g$ in flat coordinates, nonetheless, in an accelerated coordinate frame in flat space there is still an implicit dependence on $g$, since points such as $D$ will accelerate upward with acceleration $g$ causing the beam from $C$ to arrive at the $g$ dependent $D_1$, to thus expressly show the need for first order changes in the end points of the interferometer legs.}

As regards the purely classical, massless case, on defining $\phi(x) = \exp(itT(x))$, we can this time identify the eikonal phase derivative $\dot{T}^\mu$ with the wave number $k^\mu = dx^\mu/dq$ where $q$ is a convenient affine parameter which can be used to measure distances along trajectories. In the massless case the Hamilton-Jacobi equation takes the form

$$f(z)(k^0)^2 - (k_1)^2 - (k_2)^2 - (k_3)^2/f(z) = 0$$

and again yields the requisite massless particle geodesic equation $k^\mu k^\nu_{\mu\nu} = 0$ just as in the massive case. In the Schwarzschild geometry of interest these massless geodesic equations are found to take the form

$$k^0 = cdt/dq = \alpha_0/f(z) \quad k^1 = dx/dq = \alpha_1$$
$$k^2 = dy/dq = \alpha_2 \quad k^3 = dz/dq = (\alpha_0^2 - (\alpha_1^2 + \alpha_2^2)f(z))^{1/2}$$

where the $\alpha_i$ are integration constants. From these equations we recognize the existence of the gravitational frequency shift (since $k^0$ depends on $z$), the gravitational time delay ($dz/dt$ depends on $z$), and the gravitational bending of light ($dz/dx$ depends on $z$ if $\alpha_1 \neq 0$).
of these effects are found to be of relevance for the motion of a classical light wave around the $ABCD$ interferometer loop with explicit calculation (the details will be published elsewhere) then showing that the ensuing light rays again follow Fig. (1) around the interferometer. However, even while there is still a spatial offset $A_1A_2 = g H^2/c^2$ just as before, for light neither a time delay nor any net phase shift is found between the $A_1C_1D_2$ and $AB_1D_2$ paths. However, again as with the neutron case, the spatial offset itself leads to a time delay $A_2A/c$, so that there is still observable interference. Then, with $\alpha_0$ replacing $mc/\hbar$ in the normalization of the phase shift, we thus find that in traversing the interferometer the two beams acquire a final net relative phase shift $-\alpha_0 g H^2/c^2$ due entirely to the $A_2A$ segment alone. On recognizing that $\alpha_0$ is the value of $k^0$ at $z = 0$ we may set it equal to $2\pi/\lambda$ where $\lambda$ is the wavelength of the incident beam, to finally obtain for the phase shift $\Delta \phi_{CL} = -2\pi g H^2/\lambda c^2$ where $CL$ denotes classical light. Now while $H$ would have to be of the order of $10^5$ cm for $\Delta \phi_{CL}$ to actually be detectable in a Bragg scattering interferometer of the same sensitivity as the COW experiment, nonetheless we can still identify this phase shift as an in principle, completely classical effect which reveals the intrinsically global nature of classical gravity.

Now that we have obtained $\Delta \phi_{CL} = -2\pi g H^2/\lambda c^2$ it is instructive to compare it with $\Delta \phi_{COW}$. If we introduce the neutron de Broglie wavelength $\lambda_n = h/mv_0$ we may rewrite $\Delta \phi_{COW}$ in the form $-2\pi g H^2/\lambda_n v_n^2$. Comparison with $\Delta \phi_{CL}$ thus reveals a beautiful example of wave particle duality, with the quantum mechanical matter wave inheriting its interference aspects from the behavior of the underlying classical wave. Thus even while $\Delta \phi_{COW}$ does depend on the mass of the neutron, its dependence is strictly kinematic with gravity only.

As regards the Bragg scattering rules for light in curved space, it is shown in [10] that a wave undergoes no change in the magnitude of $k^0$ in a Bragg scattering, that it emerges with an angle of reflection equal to the angle of incidence, and that the magnitudes of the spatial components of the outgoing momentum take whatever values are needed to keep the outgoing wave on the light cone of Eq. (8). Thus, an upward going $k^3 = \alpha_0$ light wave, for instance, will reflect at $C$ into an initially horizontal light wave with momentum $k^1 = \alpha_0/f^{1/2}$. While this is still sizable for an interferometer, its interest lies in the fact that it allows us to detect, in principle at least, the gravitational bending of light using laboratory sized distance scales rather than solar system sized ones. Thus it would also be of interest to see what dimension interferometer might serve as a gravitational wave detector or be sensitive to any possible neutrino masses in neutrino beam interferometry.

Given our long experience with Newtonian potentials, we are used to the notion that gravity is local, with the most distant matter making the smallest contribution at a local point (and none at all if distributed spherically), with the non-vanishing of $\Delta \phi_{CL}$ thus exposing a fundamental difference between Newtonian and relativistic gravity. Thus it is of some interest to note that the potential of conformal gravity [8] takes the form $V(r) = -MG/r + \gamma c^2 r/2$ where $\gamma$ is a new parameter for sources, to show that there are theories of gravity in which even the potentials have global aspects, with the most important gravitational contributions at a given point then coming from the most distant objects, a very Machian situation. In fact, it has very recently been shown [11] that because of this global aspect of conformal gravity, the global Hubble flow then explicitly modifies the motion of matter within individual galaxies in a way which is then able to account for the observed systematics of galactic rotation curves without any apparent need for galactic dark matter.

The reason why $\Delta \phi_{COW}$ actually depends on $m$ at all is that even though the position of the minimum of the classical action is independent of $m$ (the equivalence principle) nonetheless the actual value of the classical action in this minimum does depend on $m$ (though only as a purely kinematic overall multiplying factor for $-mc \int dt$ which does not affect the position of the minimum); and even though the value of the classical action is not observable classically, nonetheless it is observable quantum mechanically as the phase of the wave function whose normalization then explicitly depends (kinematically) on $m$. 

---

7As regards the Bragg scattering rules for light in curved space, it is shown in [10] that a wave undergoes no change in the magnitude of $k^0$ in a Bragg scattering, that it emerges with an angle of reflection equal to the angle of incidence, and that the magnitudes of the spatial components of the outgoing momentum take whatever values are needed to keep the outgoing wave on the light cone of Eq. (8). Thus, an upward going $k^3 = \alpha_0$ light wave, for instance, will reflect at $C$ into an initially horizontal light wave with momentum $k^1 = \alpha_0/f^{1/2}$. While this is still sizable for an interferometer, its interest lies in the fact that it allows us to detect, in principle at least, the gravitational bending of light using laboratory sized distance scales rather than solar system sized ones. Thus it would also be of interest to see what dimension interferometer might serve as a gravitational wave detector or be sensitive to any possible neutrino masses in neutrino beam interferometry.

8Given our long experience with Newtonian potentials, we are used to the notion that gravity is local, with the most distant matter making the smallest contribution at a local point (and none at all if distributed spherically), with the non-vanishing of $\Delta \phi_{CL}$ thus exposing a fundamental difference between Newtonian and relativistic gravity. Thus it is of some interest to note that the potential of conformal gravity [8] takes the form $V(r) = -MG/r + \gamma c^2 r/2$ where $\gamma$ is a new parameter for sources, to show that there are theories of gravity in which even the potentials have global aspects, with the most important gravitational contributions at a given point then coming from the most distant objects, a very Machian situation. In fact, it has very recently been shown [11] that because of this global aspect of conformal gravity, the global Hubble flow then explicitly modifies the motion of matter within individual galaxies in a way which is then able to account for the observed systematics of galactic rotation curves without any apparent need for galactic dark matter.

9The reason why $\Delta \phi_{COW}$ actually depends on $m$ at all is that even though the position of the minimum of the classical action is independent of $m$ (the equivalence principle) nonetheless the actual value of the classical action in this minimum does depend on $m$ (though only as a purely kinematic overall multiplying factor for $-mc \int dt$ which does not affect the position of the minimum); and even though the value of the classical action is not observable classically, nonetheless it is observable quantum mechanically as the phase of the wave function whose normalization then explicitly depends (kinematically) on $m$. 

---

7As regards the Bragg scattering rules for light in curved space, it is shown in [10] that a wave undergoes no change in the magnitude of $k^0$ in a Bragg scattering, that it emerges with an angle of reflection equal to the angle of incidence, and that the magnitudes of the spatial components of the outgoing momentum take whatever values are needed to keep the outgoing wave on the light cone of Eq. (8). Thus, an upward going $k^3 = \alpha_0$ light wave, for instance, will reflect at $C$ into an initially horizontal light wave with momentum $k^1 = \alpha_0/f^{1/2}$. While this is still sizable for an interferometer, its interest lies in the fact that it allows us to detect, in principle at least, the gravitational bending of light using laboratory sized distance scales rather than solar system sized ones. Thus it would also be of interest to see what dimension interferometer might serve as a gravitational wave detector or be sensitive to any possible neutrino masses in neutrino beam interferometry.

8While this is still sizable for an interferometer, its interest lies in the fact that it allows us to detect, in principle at least, the gravitational bending of light using laboratory sized distance scales rather than solar system sized ones. Thus it would also be of interest to see what dimension interferometer might serve as a gravitational wave detector or be sensitive to any possible neutrino masses in neutrino beam interferometry.

9Given our long experience with Newtonian potentials, we are used to the notion that gravity is local, with the most distant matter making the smallest contribution at a local point (and none at all if distributed spherically), with the non-vanishing of $\Delta \phi_{CL}$ thus exposing a fundamental difference between Newtonian and relativistic gravity. Thus it is of some interest to note that the potential of conformal gravity [8] takes the form $V(r) = -MG/r + \gamma c^2 r/2$ where $\gamma$ is a new parameter for sources, to show that there are theories of gravity in which even the potentials have global aspects, with the most important gravitational contributions at a given point then coming from the most distant objects, a very Machian situation. In fact, it has very recently been shown [11] that because of this global aspect of conformal gravity, the global Hubble flow then explicitly modifies the motion of matter within individual galaxies in a way which is then able to account for the observed systematics of galactic rotation curves without any apparent need for galactic dark matter.

10The reason why $\Delta \phi_{COW}$ actually depends on $m$ at all is that even though the position of the minimum of the classical action is independent of $m$ (the equivalence principle) nonetheless the actual value of the classical action in this minimum does depend on $m$ (though only as a purely kinematic overall multiplying factor for $-mc \int dt$ which does not affect the position of the minimum); and even though the value of the classical action is not observable classically, nonetheless it is observable quantum mechanically as the phase of the wave function whose normalization then explicitly depends (kinematically) on $m$. 

---
coupling via the neutron’s de Broglie wavelength, with the COW experiment thus apparently being completely compatible with the equivalence principle.

The author is extremely indebted to Dr. H. Brown for introducing him to the field of gravitationally induced quantum interference, and would like to thank him, Dr. W. Moreau, Dr. R. Jones, and Dr. J. Javanainen for many helpful discussions. The author would like to thank the University of Canterbury in Christchurch, New Zealand for the award of an Erskine fellowship as well as for its kind hospitality while much of this work was performed. The author would also like to thank M. Mannheim for preparing the figures. This work has been supported in part by the Department of Energy under grant No. DE-FG02-92ER40716.00.

References

[1] A. W. Overhauser and R. Colella, Phys. Rev. Lett. 33, 1237 (1974).
[2] R. Colella, A. W. Overhauser and S. A. Werner, Phys. Rev. Lett. 34, 1472 (1975).
[3] D. M. Greenberger and A. W. Overhauser, Revs. Mod. Phys. 51, 43 (1979).
[4] S. A. Werner, Class. Quantum Grav. 11, A207 (1994).
[5] J. Anandan and H. R. Brown, Founds. Phys. 25, 349 (1995).
[6] P. D. Mannheim, Am. J. Phys. 51, 328 (1983).
[7] P. D. Mannheim, Gen. Rel. Grav. 25, 697 (1993).
[8] P. D. Mannheim, Founds. Phys. 24, 487 (1994).
[9] W. Moreau, R. Neutze and D. K. Ross, Am. J. Phys. 62, 1037 (1994).
[10] P. D. Mannheim, ”Local and Global Gravity”, preprint UCONN 96-09, November 1996.
[11] P. D. Mannheim, ”Are Galactic Rotation Curves Really Flat”, preprint UCONN 96-04, May 1996, Astrophysical Journal, in press.

Figure Captions

Figure (1). The paths followed by waves in a COW type interferometer.
Figure (2). The paths followed by waves in a double slit experiment.
