Theoretical Analysis of Noise Figure for Modulated Wideband Converter

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Abstract—The Modulated Wideband Converter (MWC) is one of the promising sub-Nyquist sampling architectures for sparse wideband signal sensing, cognitive radio applications and so on. In order to design an MWC-based RF receiver that meets a target RF specification, noise figure (NF) of the MWC has to be well-defined by its design properties. In this paper, we investigate a comprehensive explanation for NF of MWC by an analytic approach based on a proposed notation of an average noise figure (ANF) of the MWC. Consequently, the analysis is proven with simulation results in order to demonstrate its feasibility.

Index Terms—Noise figure, modulated wideband converter, compressed sensing, sparse wideband signal spectrum sensing.

I. INTRODUCTION

WIDEBAND signal spectrum sensing has been successfully applied to cognitive radios and spectrum analyzers. Cognitive radio allows secondary users to opportunistically use the licensed spectrum when the corresponding band is vacant [1]–[3]. The traditional way for detecting such vacant bands in a wideband spectrum is channel-by-channel sequential scanning [4], which needs long sensing time. Another possible way is to use an RF front-end with a bank of narrow band-pass filters that is the inefficient way in terms of hardware complexity even if it solves the latency issue. An alternative method is to directly sense a wideband signal with a very high sampling rate ADC, which makes the method costly and infeasible [5]. Fortunately, human-made RF signals are often sparse in the allocated band. For instance, in the wireless standard IEEE-802.11, allocated wideband signal is centered at the 2.4 GHz with several numbers of active bands depending on the surrounding users.

The most common demodulation method of a single band is to mix the input wideband signal by a local oscillator with the desired frequency. Then, a low-pass filter is applied to eliminate unwanted adjacent bands to avoid aliasing-effect in the subsequent ADC device. Afterward, the ADC is expected to sample the output of the filter with Nyquist rate, which must be equal to the bandwidth of the single channel.

To overcome the sampling rate bottleneck, a sub-Nyquist sampling system named Modulated Wideband Converter (MWC) that uses reduced sampling rate in multi-band settings below Nyquist rate has been widely studied [6]–[16]. Compared to other sub-Nyquist samplers including periodic nonuniform sampler with time-interleaved ADCs [17], the MWC has several advantages such that a precise multi-phase clock is no longer needed and that carrier frequencies are unnecessary to be known in advance. In addition, a remarkable asset is a design flexibility that allows trade-off choice between each ADC sampling rate and a number of channels. The MWC can also be useful for an Automatic Test Equipment (ATE) to test devices with various frequency bands. An ATE typically has a lot of hardware resources that includes many ADCs available for analog testing. These ADCs have slow sampling-rate but high precision in many cases. Utilizing these hardware resources, we can realize multi-channel systems to achieve effectively high sampling-rate conversion based on the MWC.

The MWC system guarantees perfect reconstruction of a wideband sparse signal based on a theory of compressed sensing [7]. But in the presence of noise, such as thermal noise, the impact of the noise on the reconstruction result changes significantly depending on the target carrier frequency and other system design parameters. Defining clear explanation on noise figure (NF) of MWC greatly helps on designing complete RF receiver system that meets target system specification.

A prediction of any RF system performance demands each component’s intrinsic noise and nonlinearity properties. Accordingly, to implement practical RF receiver system that employs an MWC, it is essential to know each component’s NF, IIP3 and etc.

In [9], the prototype circuit for MWC was designed to have 15 dB SNDR, which they defined is sufficient level for successful detection of carrier frequencies from the input...
sparse wideband signal with subsequent digital processing under necessary condition. The total NF analysis of the system was made up to the output of the ADC. In some advanced applications such as high-precision test measurement for sparse wideband signals, much higher target SNDR would be required.

In [10] and [11], the prototype MWC system was designed for signal detection and blocker rejection simultaneously. The important idea for noise analysis was noise folding gain in the mixer of MWC. When the mixer aliases whole wideband signal into the baseband, noise floor will also be folded and accumulated. They also pointed out that this folding gain can be estimated by a ratio of the gain of a Fourier series coefficient that corresponds to the carrier frequency in the mixing function and sum of all the gains of the coefficients within the band of interest. In reality, when reconstructing the original wideband signal, matrix operation has to be performed to multiply a pseudo-inverse of reduced sensing matrix, which includes columns indexed by indices of recovered nonzero elements with received vector signal. This operation also affects the noise performance even though it is done in the digital domain.

In this paper, two common architectures of the MWC will be compared in terms of noise performance. First architecture is defined as a direct MWC that directly converts the input signal without any downconverter. It is also known as low-pass compressed sensing (CS) architectures that senses a frequency range from DC to $f_{\text{max}}$. This is also known as direct MWC. Second architecture is defined as an indirect MWC, which is also called as quadrature analog-to-information converter (QAIC) or time-segmented QAIC (TS-QAIC) as introduced in [18]–[20]. It converts the input signal using downconverter in front of the MWC and senses the spectrum from $f_{\text{min}}$ to $f_{\text{max}}$. This paper investigates a comprehensive analytical definition of the NF on MWC systems including the reconstruction process in the digital domain. The first contribution of this paper is a part-by-part definition of the NF on MWC systems and its dependence on design properties. Secondly, the paper shows that the MWC system can be directly used with RF signals based on the comparison between the direct MWC system and the indirect MWC system. The last contribution of this paper is the average noise figure to evaluate noise performance of MWC over all carrier frequencies based on its design parameters.

The proposed analysis will provide an analytical expression of the NF of the MWC system based on its design parameters, which can be utilized to predict and optimize the noise performance of the MWC considering the performance tradeoffs such as power consumption and hardware cost. It must also be useful to guide the appropriate choice for the building blocks of the entire receiver systems including analog front ends.

II. PRELIMINARIES

A. Wideband Sparse Signals

Frequency domain representation is useful to define a wideband sparse signal. The frequency domain representation of a multiband signal can be modeled as

$$X(f) = \sum_{l=-L_0}^{L_0} Z_l(f - l f_p),$$  \hspace{1cm} (1)

where $Z_l(f)$ is a separate spectrum slice placed at baseband and carrier frequencies are assumed to be aligned with integer multiples of $f_p$ here. As shown in Fig. 1, the frequency-shifted version of $Z_l(f)$ is sparsely placed in the frequency domain. The number of slots within the band of interest is counted as $L$ in (1).

If the signal is spectrally sparse, most of $Z_l(f)$ are zero. So (1) is rewritten as

$$X(f) = \sum_{l \in S} Z_l(f - l f_p),$$  \hspace{1cm} (2)

where $S$ is a support of $X(f)$, which is a set of $N$ indices that correspond to carrier frequencies (including conjugate frequencies) of active bands. As the signal is spectrally sparse, $N \ll L$. Furthermore, another important assumption is that each active band signal has a bandwidth $B$ which is narrower than $f_p$ so that any active band signals do not overlap with each other.

B. Modulated Wideband Converter (MWC)

Based on the MWC settings defined in [7], a basic and an advanced MWC are illustrated in Figs. 2(a) and 2(b), respectively. The main architecture of the MWC is divided into three parts to demonstrate its input and output dependence step by step. Part 1 includes a mixer that mixes the input signal with a mixing function $p_1(t)$ and an LPF that retains baseband part of the mixed signal. Part 2 indicates the conversion of the analog signal to the digital signal, while Part 3 shows digital processing such as digital modulations, support detection and reconstruction of the original signal. The design parameter notations are summarized in Table I.
A periodic sign function (PSF) $p_i(t)$ is a periodically constant sequence with $M$ length that switches the level between $-1$ and $+1$ for $T_p = 1/f_p$ interval. Formally,

$$p_i(t) = a_{i,j}, \quad j \frac{T_p}{M} \leq t \leq (j + 1) \frac{T_p}{M}, \quad 0 \leq j \leq M - 1$$

with a random $a_{i,j} \in \{+1, -1\}$ [7]. A frequency-domain illustration of a PSF $p_i(t)$ that is decomposed into Fourier series coefficients $c_{i,l}$ located at $l f_p$ is shown in Fig. 3(a). These components downconvert corresponding bands or spectrum slices of the input signal with individual amplitudes as in Fig. 3(b). From Fig. 3(b), $i$-th digital output signal $y_i[n]$ through Fourier transform can be expressed as

$$Y_i(e^{jwT}) = \sum_{l=-L_0}^{L_0} c_{i,-l} Z_l(f). \quad (4)$$

In the case of the advanced MWC, $q (> 1)$ digital channels are extracted from a single analog channel as shown in Fig. 2(b). For symmetry purpose, in the spectrum of sampled signal in Fig. 3(c), $q$ must be odd such that $q = 2q' + 1$ where $q'$ is an integer. Thus the general form of (4) for the advanced MWC can be rewritten as

$$\hat{Y}_i(e^{jwT}) = \sum_{k=-q'1}^{q'1} \sum_{l=-L_0}^{L_0} c_{i,-(t+k)} Z_{l+k} (f - kf_p). \quad (5)$$

In other words, the parameter $q$ of advanced MWC enables design flexibility between the number of analog channels and the complexity of digital signal processing.

If (4) is written as a vector signal $\mathbf{Y} = [Y_1(e^{jwT}), Y_2(e^{jwT}), ... Y_m(e^{jwT})]^T$ in matrix form, the generalized form can be expressed as

$$\mathbf{Y} = \mathbf{A} \mathbf{Z}, \quad (6)$$

where $\mathbf{Z}$ points out separate spectrum slices of the input discrete-time signal and $\mathbf{A}$ is a sensing matrix that is defined only by Fourier series coefficients of all PSF signals as

$$[\mathbf{A}]_{i,j} = c_{i,j}, \quad i \in [1, m], \quad j \in [-L_0, L_0]. \quad (7)$$

For the advanced MWC, vector form of sampled signal $\mathbf{Y}$ can be achieved by collapsing each analog signal $\hat{Y}_i(e^{jwT})$ into $q$ digital channels using digital filter and mixers as shown in Fig. 2(b). Thus $\mathbf{Y} = [\hat{Y}_1(e^{jwT}), \hat{Y}_2(e^{jwT}), ... \hat{Y}_m(e^{jwT})]^T$ is obtained and (6) is the same for the advanced MWC by arranging sensing matrix as

$$[\mathbf{A}]_{i',j} = c_{i,j+k}, \quad i \in [1, m], \quad k \in [-q', q'], \quad j \in [-L_0, L_0] \quad (8)$$

where $i' = (i - 1)q + k + q' + 1$. Once we have a digital vector signal, we have to recover support frequencies of active bands in the input wideband sparse signal using Greedy algorithms such as matching pursuit (MP), orthogonal matching pursuit (OMP), etc [21]. Recovered support frequency indicates nonzero signal indices of $\mathbf{Z} = [Z_{-L_0}(f), ..., Z_0(f), ..., Z_{L_0}(f)]^T$. These nonzero signals can be reconstructed using least-squares through the multiplication of the pseudo-inverse matrix as follows:

$$\hat{\mathbf{Z}}_S = \mathbf{A}_S^\dagger \mathbf{Y} \quad \text{s.t.} \quad \mathbf{S} = \text{supp}(\mathbf{Z}) = \{i : Z_i(f) \neq 0\}. \quad (9)$$

Here, subscript $\mathbf{S}$ denotes a set of row indices where $\mathbf{Z}$ takes nonzero values. In more detail, $\mathbf{A}_S^\dagger = (\mathbf{A}_S^H \mathbf{A}_S)^{-1}\mathbf{A}_S^H$ and $\mathbf{A}_S$ denotes a submatrix of $\mathbf{A}$ formed from column sets $\mathbf{S}$.

Theoretically, a necessary condition for perfect reconstruction is defined as follows: a number of channels $m$ is needed to be $m \geq 2N$ for blind detection of arbitrary support frequencies of the input wideband sparse signal. In other words, the number of rows of $\mathbf{A}$ must be at least two times larger than the number of nonzero rows within $\mathbf{Z}$ [7], because...
can be modeled as an active band may occupy two spectrum slices. In this paper, the carrier frequency is assumed to be an arbitrary frequency within a range \([0, L_0 f_p]\). These frequencies may lie at a boundary of two spectrum slots. Therefore, an active band can occupy up to two spectrum slices so that necessary condition can be determined as \(m \geq 2N\). Once support frequencies are detected by Greedy algorithms from the input signal, active bands will be reconstructed through (9). In order to have a unique solution, the number of rows \(m\) in \(A\) must be equal to or larger than the number of rows or unknown \(2N\) in \(Z\).

III. Noise Figure of MWC

In this section, the noise performance of MWC is investigated then its noise figure is defined analytically. The thermal noise, which has a uniform distribution in the frequency domain, is assumed as the input-referred noise to simplify the calculation of the whole system’s noise performance. Noise factor \(F\) of any systems is defined as [22]

\[
F = \frac{\text{SNR}_{\text{IN}}}{\text{SNR}_{\text{OUT}}}. \tag{10}
\]

Noise figure NF is a decibel form of the noise factor as given by

\[
\text{NF} = 10 \log_{10}(F) = \text{SNR}_{\text{IN}, \text{dB}} - \text{SNR}_{\text{OUT}, \text{dB}}, \tag{11}
\]

where \(\text{SNR}_{\text{OUT}}\) is a power ratio of the output signal and noise that is defined within the total effective bandwidth of \(NB\). Since the reconstruction process recovers only \(N\) active bands that are defined by \(S\) in (9), \(\text{SNR}_{\text{IN}}\) is also defined by a power ratio of signal and input-referred noise within the total effective bandwidth of the active bands \(NB\) as in the case for \(\text{SNR}_{\text{OUT}}\). The noisy sparse signal within the band of interest can be modeled as

\[
\tilde{X}(f) = \sum_{l \in S} Z_l(f - l f_p) + \sum_{l = -L_0}^{L_0} W_l(f - l f_p), \tag{12}
\]

where \(W_l(f)\) is a spectrum slice of the thermal noise as shown in Fig. 1. These Fourier transforms are assumed to be bandlimited baseband signal \(Z_l(f) = 0, |f| > f_p/2\). Let us assume that the average signal power in each active band is defined as

\[
P_l = \int_{-f_p/2}^{f_p/2} \|Z_l(f)\|^2 \, df \quad \text{where } l \in S, \tag{13}
\]

and the total power is expressed as

\[
NP_S = \sum_{l \in S} P_l \quad \text{where } l \in S. \tag{14}
\]

For the thermal noise, the average powers of each spectrum slice are expected to be

\[
P_n = \int_{-f_p/2}^{f_p/2} \|W_l(f)\|^2 \, df \quad \text{for all } l. \tag{15}
\]

Therefore, the input SNR is given by

\[
\text{SNR}_{\text{IN}} = \frac{\int_{-f_p/2}^{f_p/2} \|Z_l(f - l f_p)\|^2 \, df}{\int_{-f_p/2}^{f_p/2} \|W_l(f - l f_p)\|^2 \, df}
\]

\[
+ \sum_{l \in S - \{l\}} \|Z_l(f)\|^2 \, df = \frac{\sum_{l \in S} P_l}{\sum_{l \in S - \{l\}} \|W_l\|^2} = \frac{P_l}{P_n}, \tag{16}
\]

Here spectrum slices \(Z_l(f)\) are assumed to be disjoint with each other so it can be written as a sum of norms. Another way of stating (16) is to say

\[
\text{SNR}_{\text{IN}} = \frac{P_l}{P_n} = \frac{\|Z_S\|_F^2}{\|W_S\|_F^2}, \tag{17}
\]

where \(\|X\|_F\) is a Frobenius norm of the \(m \times n\) matrix \(X\) defined as

\[
\|X\|_F = \left( \sum_{i=1}^{m} \sum_{j=1}^{n} |x_{i,j}|^2 \right)^{1/2}. \tag{18}
\]

The matrix form (17) shows more practical definition rather than (16) because it is a discretized form. Substituting (12) into (6) yields

\[
\tilde{Y} = A(Z + W), \tag{19}
\]

where \(W\) is separate spectrum slices of discrete-time input-referred noise or discretized matrix form of the noise term in (12) similarly to \(Z\) in (6). Thus, the reconstructed signal (9) becomes

\[
\tilde{Z}_S = A^\dagger_S(AZ + AW). \tag{20}
\]

Here, the impact of the input-referred noise at the reconstructed signal can be clearly seen.

A. Noise Factor of a Mixer and LPF

First part of the basic MWC system consists of a mixer and an LPF as shown in Fig. 2(a). In this part, noise folding gain will be introduced in the mixer. The output signal of the LPF in \(i\)-th channel is given by

\[
\tilde{Y}_i(f) = \sum_{l \in S} c_{i,l} Z_l(f) + \sum_{l = -L_0}^{L_0} c_{i,l} W_l(f), \tag{21}
\]

where the first term indicates a linear combination of signal that aliased into baseband as shown in Fig. 3(b), while the second term shows a linear combination of noise that are also aliased into baseband and accumulated as illustrated by red rectangle in Fig. 3(b). Through the multiplication with the PSF, at the output of the mixer the signal power spreads
across \(kf_p\) frequencies for all integer \(k\) with the weight of \(c_{i,k-1}\) for \(Z_l(f)\). Then only the weighted copies at baseband are extracted with the LPF. On the other hand, the noise power, which is uniformly distributed throughout the original band of interest, is folded at baseband after the weighting. Then the output SNR of the mixer and LPF part is defined as

\[
\text{SNR}_{i\text{-th MIXLPF}} = \frac{\int_{-f_p/2}^{f_p/2} \left( \sum_{l=-L_0}^{L_0} c_{i,-l} Z_l(f) \right)^2 df}{\int_{-f_p/2}^{f_p/2} \left( \sum_{l=-L_0}^{L_0} c_{i,-l} W_l(f) \right)^2 df + \sum_{l=-L_0}^{L_0} \int_{-f_p/2}^{f_p/2} \|c_{i,-l} Z_l(f)\|^2 df}.
\]

(22)

Here, active band signals and noise spectrum slices are independent with each other. Thus,

\[
\int W_l(f) W_j^*(f) df = 0 \quad \text{and} \quad \int Z_l(f) Z_j^*(f) df = 0,
\]

(23)

(24)

for all \(i \neq j\). Using this property, (22) is rewritten as

\[
\text{SNR}_{i\text{-th MIXLPF}} = \frac{\sum_{l=-L_0}^{L_0} \|c_{i,-l}\|^2 \int_{-f_p/2}^{f_p/2} \|Z_l(f)\|^2 df}{\int_{-f_p/2}^{f_p/2} \left( \sum_{l=-L_0}^{L_0} c_{i,-l} W_l(f) \right)^2 df + \sum_{l=-L_0}^{L_0} \int_{-f_p/2}^{f_p/2} \|c_{i,-l} Z_l(f)\|^2 df}.
\]

(25)

Under an assumption that the power of noise spectrum slices are constant within the band, we substitute (13) and (15) into (25). Then we obtain

\[
\text{SNR}_{i\text{-th MIXLPF}} = \frac{\sum_{l=-L_0}^{L_0} \|c_{i,-l}\|^2 P_l}{\sum_{l=-L_0}^{L_0} \|c_{i,-l}\|^2 P_n}.
\]

(26)

The noise gain or noise folding gain can be seen clearly on this equation. If we assume the average signal power of each active band \(P_l\) is constant over all \(l\), noise factor \((F)\) of the first part at the \(i\)-th channel is simplified to

\[
F_{i\text{-th MIXLPF}} = \frac{\text{SNR}_{\text{IN}}}{\text{SNR}_{i\text{-th MIXLPF}}} = \frac{\sum_{l=-L_0}^{L_0} \|c_{i,-l}\|^2}{\sum_{l=-L_0}^{L_0} \|c_{i,-l}\|^2}.
\]

(27)

B. Noise Factor of ADC

For the second part of Fig. 2(a), which has ADC only, the noise figure of this part is determined as

\[
NF_{i\text{-th ADC}} = SNR_{i\text{-th MIXLPF}} - SNR_{i\text{-th ADC}},
\]

where \(SNR_{i\text{-th ADC}}\) is the SNR at the output of the ADC in the \(i\)-th channel. \(NF_{i\text{-th ADC}}\) may increase due to the downconversion noise of the ADC.

C. Noise Factor of the MWC System

In the last part of the basic MWC, reconstruction operation includes matrix multiplication and several modulations and additions in the digital domain as explained in Sect. II. The only effective block on noise is matrix multiplication. So it is sufficient to define a noise factor on the output of reconstruction multiplication. Beforehand, we define the output SNR of the reconstruction including the operation of the mixer in the first part as

\[
\text{SNR}_{\text{OUT}} = \frac{\|Z_S\|^2_F}{\|Z_S - \hat{Z}_F\|^2_F}.
\]

(29)

where \(Z_S\) is a submatrix of \(Z\) in (6) with rows indicated by \(S\) which is a set of row indices where \(Z\) takes nonzero values as defined by (9), while \(\hat{Z}_F\) specifies reconstructed version of \(Z_S\). By substituting (20) into the denominator of (29) under the assumption that we have the necessary condition for perfect reconstruction as discussed in Sect. II, we have

\[
\text{SNR}_{\text{OUT}} = \frac{\|Z_S\|^2_F}{\left(\|A_S^\dagger A_Z + A_S^\dagger A W F Z_S\|^2_F\right)} = \frac{\|Z_S\|^2_F}{\|A_S^\dagger A W F\|^2_F} = \frac{P_n}{\|A_S^\dagger A W F\|^2_F}.
\]

(30)

(31)

Combining (31), (17) and (10), the total noise factor of MWC on \(S(\xi)\), which is a \(\xi\)-th support out of all possible supports within \((L/2)^2\) slots in band of interest, can be written as

\[
F_{\text{TOTAL}}(\xi) = \frac{\text{SNR}_{\text{IN}}}{\text{SNR}_{\text{OUT}}} = \|A_S^\dagger A W F\|^2_F.
\]

(32)

The total noise factor of MWC is expressed in decibel form as \(NF_{\text{TOTAL}}(\xi) = 10 \log_{10}(F_{\text{TOTAL}}(\xi))\). The NF of the prior part (26) is implicitly included in (32) as the rows of the term \(A\) carries the denominator part of (26), while elements of \(A_S^\dagger\) contributes the noise factor of the reconstruction part.

D. Average Noise Figure (ANF)

As discussed in the introduction, the total NF of MWC strongly depends on the carrier frequency of the active band. Thus the choice of PSF and the location of the active bands within the band of interest are also very important to evaluate the total noise figure. For that reason, it is difficult to compare our analysis with currently implemented MWC systems based only on several measurement instances without knowing specific PSF waveforms. Therefore, in order to unify
all these noise figures of the MWC that is designed by the same parameters, we introduce an average noise figure (ANF) that is defined by

\[
\text{ANF}_{\text{TOTAL}} = 10 \log_{10} \left( \frac{1}{L_0} \sum_{\xi=1}^{L_0} F_{\text{TOTAL}}(\xi) \right)
\]  (33)

This ANF result shows only MWC’s noise performance. In practice, RF front-end circuits will be used in front of MWC, which may dominate the total NF, as will be discussed in Sect. IV-C.

### IV. SIMULATION RESULTS

In this section, the average NF defined in the previous section will be compared with MATLAB-based simulation results. The MATLAB code is implemented based on a demonstration by Mishali and Eldar [23]. We take into account the following assumption in this simulation for straightforward understanding.

- **Infinite precision ADC & Full-scale input on ADC:** For simplicity purpose, we set sufficiently large NOB on each ADC and set the peak-to-peak amplitude of the ADC input signal into full-scale of the ADC. This assumption makes the noise figure of the ADC 0 dB in Fig. 2(a).

#### A. Wideband Sparse Signal Generation

In this part, we will explain wideband sparse analog signal generation method. Prior MWC systems assume that the input signal exploits all carrier frequencies from DC to Nyquist frequency. But in practice, sparse wideband signal locates in specific bandpass region depending on the application. So, in the simulation part, we used the MWC system in two kinds of architectures for the passband sparse signal as depicted in Fig. 4. First architecture is the direct (w/o D.C.) MWC as presented in [6]–[11] that directly converts the wideband sparse signal after bandpass filter without any downconversion to IF. The second architecture, the indirect (w/ D.C.) MWC, which is also called as QAIC and TS-QAIC [18]–[20], includes prior downconversion step that shifts down the center frequency of passband sparse signal into IF band. Figs. 5 and 6 show the examples of the spectra of wideband sparse signals with only one active band out of 16 slots within the band of interest \((L = 32\) including the negative frequencies). Each active band includes a multi-tone signal that randomly lies within \(B = 1\) MHz. One of the reason to use the multi-tone signal in the simulation is that its sparsity in the frequency domain. In other words, it has no spectrum leakage to adjacent bands. In Figs. 5 and 6 there are two tones in the active band. To overcome peak to average power ratio (PAPR) related issues, phases of multi-tone are set optimally based on low-crest factor algorithm [24]. The input SNR is defined by the ratio of input signal power that is assumed to be 0 dBm and thermal noise power which is determined by \(-174 + 10 \log_{10} (\text{NOB})\) dBm in the experiment just for convenience. These numbers actually do not affect the NF analysis and the simulation results, because the sensing matrix \(A\) is not determined by input signal but by PSF only.

#### B. Reconstruction of a Sparse Signal in MWC and NF Calculation

Figs. 7 and 8 present the spectra of the input original and reconstructed signals. Here, the noise floor is gained by some amount due to aliased noise while the signal power retains at the same level on the reconstructed signal. Then \(\text{SNR}_N\) and \(\text{SNR}_{\text{OUT}}\) will be obtained within \(NB = 2\) MHz bands.
Fig. 7. Spectrum of sparse wideband signal for direct (w/o D.C.) MWC and reconstructed signal ($L = 32$, $N = 2$).

Fig. 8. Spectrum of sparse wideband signal for indirect (w/ D.C.) MWC and reconstructed signal ($L = 32$, $N = 2$).

Fig. 9. Time-domain waveform of sparse wideband signal for direct (w/o D.C.) MWC and reconstructed signal ($L = 32$, $N = 2$).

including conjugate band. Figs. 9 and 10 show time-domain waveforms of the original and the reconstructed signals. Since the input signal contains two tones, the time-domain signal is a sinewave with changing envelope in a narrow time span. Even though the noise floor is gained, it is still imperceptible in the time-domain. Thus the reconstructed signal perfectly matches with the original signal in Figs. 9 and 10. The NF is obtained from the difference between $\text{SNR}_{\text{IN},\text{dB}}$ and $\text{SNR}_{\text{OUT},\text{dB}}$ which are taken from the simulation. Then the simulation result of the NF will be compared with our analysis result that is calculated from (32). The calculation process based on (32) is as follows: First, we derive matrix $A$ based on the Fourier series coefficients of the used PSFs. Next, we extract the matrix $A^\dagger S$ through the pseudo-inverse operation of the reduced matrix $AS$. Consequently, the matrix multiplication is performed between $A^\dagger S$ and $A$ matrices as given by (32), then the matrix norm (18) of the product gives noise factor of the MWC system. When the design parameters for MWC and input signal parameters are respectively set with $m = 2$, $M = 161$, $f_p = 1$ MHz, $f_c = 161$ MHz and $L = 32$, $N = 2$, Fourier series coefficients of the PSF for direct (w/o D.C.) and indirect (w/ D.C. by 2410 MHz local oscillator) MWC can be found in Figs. 11 and 12, respectively. As illustrated in Fig. 11, PSF coefficients at high-frequencies within the band of interest $2411 - 2426$ MHz carry out the downconversion of the incoming signal by the mixer for the direct MWC. On the other hand, the indirect MWC downconverts the incoming signal prior to the MWC, and thus PSF coefficients at low-frequencies in the range of $1 - 16$ MHz are exploited as shown in Fig. 12. So even with the same PSF sequences the set of coefficients are different. The total NF from the simulation are plotted in Figs. 13 and 14. Here we find that the $\text{NF}_{\text{TOTAL}}$ is extremely high when a frequency index $j = 2415$ for...
w/o D.C. case as illustrated in Fig. 13. The reason is clearly seen in the zoom-in view of Fig. 11, which has nearly zero amplitude for the downconversion of the corresponding signal. That zero-crossing of PSF spectrum occurs at every integer multiple of \( M/T_P = f_c \). Hence, the number of symbols \( M \) of PSF must be carefully chosen such that these zero-crossings is not located within the band of interest, while it preserves necessary condition \( M \geq L \) [7]. In order to have the zero-crossing out of the band of interest, \( M \) can be increased to make the distance between two zero-crossings far-off. However, \( M \) can not be exceedingly large as it may increase the hardware complexity for the PSF generation. In the case of indirect (w/ D.C.) MWC or MWC with prior downconversion, the above condition is automatically satisfied through \( M \geq L \), the necessary condition from [7]. Therefore there is no such an issue in Fig. 14. To avoid the zero-crossing points in Fig. 11, \( M \) is changed into 159 so that the PSF spectrum does not have a zero amplitude within the band of interest (2410 – 2426 MHz) as shown in Fig. 15. Then NF on all frequencies become applicable as shown in Fig. 16. But still, these noise figures can be high when the number of channels \( m \) is just met on necessary condition. One of the straightforward ways to reduce the NF is to increase the number of channels \( m \) by using more hardware. In other words, the increment of \( m \) expands the matrix sizes of (32) to increase the number of measurements so that the noise is more averaged then the NF is decreased.

C. Average Noise Figure (ANF) of MWC

In order to compare different settings and cases, average noise figure notation is used in the following simulations that averages the F over every frequency within the band of interest. Also the more feasible ANF can be achieved with Monte Carlo simulations with different PSF for each simulation.
Fig. 17 illustrates ANF of direct (w/o D.C.) and indirect (w/ D.C.) MWC with one active band in the input sparse signal. We swept both \( m \) and \( L \) parameters in several combinations to plot the graph by sweeping \( m/L \) in this experiment. As we can see on the graph, the ANF is a function of the ratio \( m/L \). This is explained in the following way: As \( L \) is the number of slots within the band of interest, increasing \( L \) simply increases the total bandwidth that leads to a linear increase in thermal noise power accumulated at the baseband. Increasing the number of channels \( m \), on the other hand, is effectively the same as increasing the number of measurements. Thus after the matrix operation in the digital domain, the signal power, which is correlated among all the channels, increases with \( m^2 \) while the noise power, which is uncorrelated, increases linearly with \( m \). This results in a linear improvement in NF along with \( m \). Therefore, as demonstrated in Fig. 17, the ANF of the MWC depends on \( m/L \). There is actually no difference between the ANF results in two cases of MWC systems shown in Fig. 4 as expected by (32). In all cases, our analysis well predicts the simulated ANF of the MWC. The green dotted line shows the theoretical limitation, which has a \(-3 \)dB/octave slope [25] in \( m \leq L \) region while constant 0 dB in \( m \geq L \) region. In the example of \( L = 1, m = 1 \), the input signal that has only one active band including noise is located at baseband below \( B/2 \) bandwidth and only one ADC samples the input signal. In this case, if there is no quantization error on ADC, NF will be 0 dB. In general, a multi-band signal \((N > 2)\) is more realistic sparse signal for the MWC system. Fig. 18 summarizes the comparison in the cases with \( N = 4 \) and \( N = 6 \). It demonstrates that the ANF stays the same as long as MWC has necessary number of channels \( m \geq 2N \) even for a multi-band signal.

Fig. 19 shows the ANF of the overall systems including practical analog front-end components. Supposing the architectures shown in Fig. 4, we assumed to use a BPF with \(-0.9 \)dB gain and 0.9 dB NF, a downconverter with \(-7.2 \)dB conversion gain and 7.3 dB NF and an LPF with 7dB gain and 7.5 dB NF. As for the LNA, we assumed two different performance for comparison: LNA1 with 31dB gain and 0.8 dB NF and LNA2 with 16dB gain and 1.5 dB NF. The ANF of the overall system is calculated by Friis formula [22]. As in the case with LNA1, when the front-end LNA has sufficiently high gain and low NF, the ANF of overall system is almost dominated by the LNA performance only. However, with a moderate performance LNA, i.e. LNA2, depending on the design parameters of the MWC, the noise contribution of the MWC may come into the picture. We can also see the difference in the minimum achievable ANFs between the cases of w/ and w/o D.C. due to the contribution of the downconversion mixer. These results demonstrate that the proposed analysis on the NF of the MWC systems is useful to facilitate the system-level optimization of the MWC-based receiver architectures including the analog front-ends.

V. DESIGN GUIDELINE FOR MWC

According to the analysis and the simulation results that are demonstrated in the previous sections, this section discusses the way to optimize the MWC system in terms of NF. The dominant part of NF is presented in the mixer stage of the MWC that introduces noise-folding effect based on a
compressed sensing ratio $m/L$ [25]. In general, the ratio must be defined by

$$\frac{mq}{L} = \frac{mf_s}{Lf_p}$$

(34)

based on the design parameters in Table I. Thus the number of equivalent analog channels would be $mq$. Consequently, we can obtain similar ANF trends of the MWC system as in Figs. 17 and 18 using $mq/L$. From the result, it is easily seen that if the number of channels is increased $2x$, the noise figure of the MWC is improved by 3 dB in the region $m/L < 10^9$. In addition, we can further decrease NF by utilizing parameter $q$ by increasing the sampling rate $f_s$ of the ADCs. Similarly, for the 3 dB improvement in the NF, $q$ also needs to be increased $2x$. The region $10^9 - 10^2$ in Figs. 17 and 18 refers that the number of the total channel $mq$ is larger than the number of slots within the band of interest $L$, which is an inefficient design parameters for MWC even though the ANF of the MWC itself is close to 0 dB. On the other hand, the region $10^{-2} - 10^{-3}$ is realistic choice for practical purpose in terms of hardware efficiency as the NF of the whole system would be improved with appropriate front-end circuits as shown in Fig. 19. By using the proposed analysis, we can optimize the noise performance of the entire system with MWC considering the performance trade-offs such as power consumption and hardware cost. The proposed analysis is also useful to guide the appropriate choice for the building blocks of the MWC systems including analog front ends.

The choice of PSF also affects the ANF of the system. For the case of the MWC w/ D.C., all downconverted signals lie within $M/T_p$ as long as PSF have sufficient $M > L$ symbols in $T_p$ period so that the amplitudes of Fourier series coefficients of the PSF are guaranteed to be nonzeros. However, for the case of the MWC w/o D.C. $M$ must be chosen carefully because Fourier series coefficients of PSF have zero values on $km/T_p$ for any integer $k$, as explained in Sect. IV-B. The band of interest should not include the zero values otherwise a signal that corresponds to this zero cannot be reconstructed.

In the practical implementation of the MWC, the actual NF tends to be degraded from the theoretically estimated NF due to the non-ideality of the hardware components, which affects the actual PSF and whole system’s transfer function and leads to deviation on the sensing matrix $A$ compared to the theoretical definition. As we have demonstrated in the previous section, however, the analytical ways to predict the performance that guide the design choice are essential to achieve system-level optimization especially at the initial design stage.

VI. CONCLUSION

The NF of an MWC is analytically investigated in this paper. The simulation result shows that the analytic noise figure successfully estimates the simulated NF of MWC, which is created with different design parameters. The analysis and the simulation results show that MWC-based architectures in Fig. 4 have similar noise figures when all ADCs are assumed to be ideal. In the practical case, as the sampling rate and the number of channels are limited, most MWC-based applications such as spectrum analyzer and cognitive radio have great interest on the region $m \ll L$. Even though the noise figure of the MWC becomes considerably high when $m \ll L$, with an appropriate choice of low-NF, high-gain RF front-end circuits the overall noise figure can be reduced to target specification. The proposed theoretical analysis on NF of the MWC systems will facilitate the system-level optimization of the MWC-based receiver architectures in terms of noise performance.

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