Abstract

A common problem in structural optimization is the formation of checkerboard patterns due to numerical instabilities. One way to mitigate it is to consider regularization techniques such as the application of a smoothing filter during the optimization procedure. This approach leads to what is now called sensitivity filtering, which has become a popular method in the engineering community due to easy implementation and applicability to large-scale optimization problems. However, the method suffers from a lack of mathematical foundation and theoretical justification [1].

In an attempt to merge sensitivity filtering into the standard optimization technology, a framework for sensitivity filtering in conjunction with mesh-free methods and node-based shape optimization was developed in [2]. The resulting method is referred to as vertex morphing. A remarkable feature of vertex morphing is that intermediate design iterations exhibit certain desirable properties, depending on the choice of filter radius, such as the attenuation of high-frequency modes and the preservation of tiny details in the initial design. Since they can be seen as valid design choices, the method is usually stopped before convergence.

In this contribution, we provide a mathematical foundation for the vertex morphing method and establish a connection with known regularization techniques. To analyze the behavior of the method, we regard the linear least squares problem and show that the intermediate results correspond to some regularized solution to the problem. A comparison between the vertex morphing method and other regularization techniques is made for the least squares problem and the compliance minimization of a thin shell structure, based on the assumed natural deviatoric strain formulation.

Framework for sensitivity filtering

We are concerned with general optimization problems of the form

\[
\min_J(q,u), \quad \text{s.t.} \quad \mathcal{E}(q,u) = 0,
\]

where \( J \) is the objective function, \( E \) is the state equation operator, \( \mathcal{Q}_u \) is the set of admissible design variables, and \( \mathcal{U}_u \) is the space of admissible state variables. Based on [2], the main idea is to consider design variables as fields defined on some domain \( D \subset \mathbb{R}^n \) that can be written as

\[
\phi(x) = \mathcal{K}p(x) = \int_D k(x,y) p(y) \, dy, \quad x \in D,
\]

with the integral kernel \( k \in L^2(D \times D) \) and control variable \( p \in L^2(D) \). The mapping \( \mathcal{K} \) defines a Hilbert-Schmidt integral operator, which is a compact linear operator on \( L^2(D) \). We choose \( \mathcal{Q}_u \subset L^2(K) \).

Using the above formulation, we can derive the so-called filtering rules, which form the basis for the design update rules:

\[
q(x) = \mathcal{K}p(x) = \int_D k(x,y) p(y) \, dy, \quad x \in D.
\]

Here, \( \nabla \mathcal{K} \) is the adjoint of \( \mathcal{K} \), and \( f = g \circ \mathcal{K} \) is the pullback of a real-valued function \( g \) defined on \( \mathcal{Q}_u \) by \( \mathcal{K} \).

If we apply the steepest descent method with constant step size satisfying \( 0 < \alpha < 2/\|K\|^2 \) to the linear least squares problem

\[
\min_p \|Kp - b\|^2,
\]

with data \( b \in \mathbb{R}^m \), then

\[
q = T^* q + 1 - T^* b, \quad \text{with} \quad T = (1 - \alpha \mathcal{K}^* \mathcal{K})^{-1},
\]

where \( T \) defines the orthogonal projection onto the image of \( K \). Small singular values cause slow convergence of the method. Early stopping yields solution to

\[
\min_p \|Kp - b\|^2 + \|Pp\|^2,
\]

with Tikhonov matrix given by

\[
P = \sum_{\ell=1}^{\infty} \gamma_{\ell}(v_\ell, p) u_\ell
\]

Empirically, the larger we choose the filter radius, the more singular values are close to zero. We see that high-frequency modes of the data are attenuated, while tiny characteristics of the initial design are preserved.

Numerical experiments

On the top, we minimize the strain energy of a half-cylindrical structure under load and illustrate the effect of sensitivity filtering. The simulation was done using Kratos Multiphysics. In the right figure, we fit a curve to some data points and compare the vertex morphing method to the suggested regularization as well as two other methods using \( L^2 \) and reproducing kernel Hilbert space (RKHS) regularizer, respectively.

Discussion and conclusion

1. Vertex morphing provides a way to reduce checkerboard patterns and perform shape optimization with many design variables.
2. Regularization effect of sensitivity filtering is an artifact of early stopping when using first-order iterative optimization methods.
3. Same solution can be obtained by applying a fast converging method to an equivalent regularized problem.

References

[1] Ole Sigards and Kurt Maute. Sensitivity filtering from a continuum mechanics perspective. Structural and Multidisciplinary Optimization, 48:471-495, 2013.
[2] Kai-Uwe Bletzinger. A consistent frame for sensitivity filtering and the vertex-assigned morphing of optimal shape. Structural and Multidisciplinary Optimization, 48:673-695, 2014.