Optimal concentrating arbitrary partially entangled W states with linear optics

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We proposed two optimal entanglement concentration protocols (ECPs) for arbitrary single-photon multi-mode W state and multi-photon polarization W state, respectively. In both ECPs, we only require one pair of partially entangled W state, and do not consume any auxiliary photon. Both ECPs are based on the linear optics which can be easily realized. On the other hand, the concentrated maximally entangled states can be remained, which are quite different from the previous ECPs. Moreover, for the concentration of the arbitrary single-photon N-mode W state or N-mode polarization W state, the total success probability is equal to Nth the modulus square of the Schmidt coefficient of the smallest magnitude. It makes both ECPs optimal than all the previous ECPs. Our ECPs may be useful in current quantum communication fields.

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I. INTRODUCTION

In the recent years, quantum information processes have developed rapidly\cite{1}, among which the most important branches are the long-distance quantum communication and quantum computation. Entanglement, which is a uniquely quantum mechanical feature, is considered to be an essential resource for both the two branches. In practical applications, entanglement is usually produced locally and can be distributed to the remote parties. It not only can hold the power for the quantum nonlocality\cite{2}, but also can provide wide applications in the quantum information processing (QIP)\cite{3}. For example, many popular research areas such as the quantum teleportation\cite{4,5}, quantum denescoding\cite{6}, quantum secret sharing\cite{7,8,9}, quantum state sharing\cite{10,11,12}, and quantum secure direct communication\cite{13,14}, all require entanglement to set up the quantum entanglement channels.

Among various entanglement forms, the multi-mode and multi-particle W states have quite important applications. The perfect entangled W states are the maximally entangled W state, which can be written as

\[
|W\rangle_{\text{multi-mode}} = \frac{1}{\sqrt{N}}(|100\cdots0\rangle + |010\cdots0\rangle + \cdots + |000\cdots1\rangle),
\]

\[
|W\rangle_{\text{multi-photon}} = \frac{1}{\sqrt{N}}(|HV\cdots V\rangle + |VH\cdots V\rangle + |VVH\cdots V\rangle + \cdots + |VVV\cdots VH\rangle),
\]

(1)

where the $|H\rangle$ and $|V\rangle$ represent the horizontal and vertical polarization of the photon state, while $|1\rangle$ and $|0\rangle$ represent one photon and no photon, respectively. It has been proved that the W states are highly robust against the loss of one or two qubits\cite{15,16}. There are many works have been done based on both multi-particle W state and single-photon multi-mode W state, such as the protocols of perfect teleportation and superdense coding with W states\cite{17}, the generation of the W state\cite{18,19}, entanglement transformation\cite{20}, distillation\cite{21,22} and concentration\cite{23,24} of the W states. Interestingly, Gottesman et al. proposed a protocol for building an interferometric telescope based on the single-photon multi-mode W state\cite{25}. The protocol has the potential to eliminate the baseline length limit, and allows in principle the interferometers with arbitrarily long baselines.

However, in practical applications, the signals will inevitably interact with the environment during the storage and transmission process. In this way, the perfect entangled W states also may be degraded to a mixed state or a pure partially entangled states because of the environmental noise. During the applications, such partially entangled state

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may further decrease and cannot ultimately set up the high quality quantum entanglement channel\cite{45}. Therefore, we need to recover the mixed state or pure partially entangled W state into the maximally entangled W state.

Here, we focus on recovering the pure partially entangled W state into the maximally entangled W state. The entanglement concentration is a powerful method to distill the maximally entangled state from the partially entangled state\cite{33, 43, 46, 59}. In 1996, Bennett et al. proposed the first entanglement concentration protocol (ECP) which is known as the Schmidt projection method\cite{46}. Since then, various ECPs have been put forward successively, such as the ECP based on the entanglement swapping\cite{47} and the ECP based on unitary transformation\cite{48}. In 2001, Zhao et al. and Yamamoto et al. proposed two similar concentration protocol independently with linear optical elements, and later realized them in experiments, respectively\cite{49, 50}. In 2008, Sheng et al. developed their protocols with the help of the cross-Kerr nonlinearity\cite{51}. However, most ECPs described above are focused on the two-particles entanglement, which can not be used to concentrate the pure partially entangled W state. In 2003, Cao and Yang firstly proposed an ECP for W state with the joint unitary transformation\cite{36}. In 2007, Zhang et al. proposed an ECP for the W state with the help of the collective Bell-state measurement\cite{37}. In 2010, Wang et al. proposed an ECP for a special W state as $\alpha |HHV\rangle + \beta (|VHV\rangle + |VVH\rangle)$ with linear optics\cite{38}. In 2012, Gu et al. and Du et al. improved the ECP for the special W state with the help of the cross-Kerr nonlinearity\cite{39, 40}. Later, Ren et al. proposed an ECP for multipartite electron-spin states with CNOT gates\cite{52}. The concentration protocols for both arbitrary multi-photon partially entangled W state and single-photon multi-mode W state were proposed\cite{41, 42, 54}. Unfortunately, all the previous ECPs for partially entangled W state are not optimal. Some of the ECPs are focused on the special types of the W states, and some ECPs need the cross-Kerr nonlinearity medium to complete the task, which cannot be realized in current experimental conditional. Moreover, Most ECPs cannot reach a high success probability.

In this paper, we will present two optimal ECPs for multi-mode single-photon W state and multi-photon polarization W state, respectively, inspired by the recent excellent concentration work for two-photon system proposed by the group of Deng\cite{60}. Both of our two ECPs do not require any auxiliary photon, and only resort to the linear optical elements. Therefore, they can be easily realized under current experimental condition. Meanwhile, our ECPs only require local operations, which can simplify the operations largely. Moreover, our ECP only need to be operated for one time, and its success probability is higher than all the previous ECPs for W states\cite{41, 42, 43, 54}. Based on the features above, our ECPs may be useful in current quantum communications.

The paper is organized as follows. In Sec. 2, we first briefly explain the ECP for the single-photon multi-mode partially entangled W state. In Sec. 3, we explain the ECP for the multi-photon polarization partially entangled W state. In Sec. 4, we make a discussion and summary.

II. THE EFFICIENT ECP FOR THE SINGLE-PHOTON MULTI-MODE W STATE

Now we first start to explain our ECP for the single photon three-mode W state and then extend this method to the case of single-photon multi-mode partially entangled W state. The basic principle of our ECP is shown in Fig. 1. Suppose a single photon source S emits a single photon, and sends it to three parties, say Alice, Bob and Charlie. In this way, it can create a single photon multi-mode W state in the spatial mode $a_1, b_1$ and $c_1$ as

$$|\Phi_1\rangle_{a_1b_1c_1} = \alpha |100\rangle_{a_1b_1c_1} + \beta |010\rangle_{a_1b_1c_1} + \gamma |001\rangle_{a_1b_1c_1}. \tag{2}$$
Here, $\alpha$, $\beta$, and $\gamma$ are the initial entanglement coefficients and $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$. Meanwhile, we suppose $|\alpha| > |\beta| > |\gamma|$.

Our ECP can be divided into two steps. In the first step, Alice makes the photon in the $a_1$ mode pass through a variable beam splitter (VBS1) with the transmittance of $t_1$. After VBS1, the photon state in the $a_1$ mode evolves to

$$|\phi\rangle_{a_1} = \alpha\sqrt{t_1}|1\rangle_{a_2} + \alpha\sqrt{1-t_1}|1\rangle_{a_3}. \quad (3)$$

After passing through the VBS1, the initial state becomes

$$|\Phi_1\rangle_{a_1b_1c_1} = \alpha|100\rangle_{a_1b_1c_1} + \beta|010\rangle_{a_1b_1c_1} + \gamma|001\rangle_{a_1b_1c_1} \rightarrow (\alpha\sqrt{t_1}|100\rangle_{a_2b_1c_1} + \alpha\sqrt{1-t_1}|100\rangle_{a_3b_1c_1}) + \beta|010\rangle_{a_2b_1c_1} + \gamma|001\rangle_{a_2b_1c_1}. \quad (4)$$

Then, Alice detects the photon in the $a_3$ mode by the single photon detector $D_1$. When $D_1$ detects no photon, the single photon state in the three parties becomes

$$|\Phi_2\rangle_{a_2b_1c_1} = \alpha\sqrt{t_1}|100\rangle_{a_2b_1c_1} + \beta|010\rangle_{a_2b_1c_1} + \gamma|001\rangle_{a_2b_1c_1}, \quad (5)$$

with the success probability of $|\alpha|^2 t_1 + |\beta|^2 + |\gamma|^2$.

It can be found that if Alice can find a suitable VBS1 with $t_1 = \frac{|\gamma|^2}{|\alpha|^2}$, Eq. (5) can be rewritten as

$$|\Phi_2\rangle_{a_2b_1c_1} = \gamma|100\rangle_{a_2b_1c_1} + \beta|010\rangle_{a_2b_1c_1} + \gamma|001\rangle_{a_2b_1c_1}, \quad (6)$$

which only has two different entanglement coefficients $\gamma$ and $\beta$.

Until now, the first concentration step is completed. In the first step, by selecting the suitable VBS with $t_1 = \frac{|\gamma|^2}{|\alpha|^2}$ and the case that the photon detector $D_1$ detects no photon, Alice successfully convert Eq. (2) to Eq. (6) with the success probability of

$$P_1 = 2|\gamma|^2 + |\beta|^2, \quad (7)$$

where the subscript "1" means in the first concentration step.

The second concentration step is operated by Bob and the whole operation process is quite similar with the first step. Firstly, Bob makes the photon in the $b_1$ mode pass through the VBS2 with the transmittance of $t_2$. After the VBS2, the photon state in the $b_1$ mode can evolve to

$$|\phi'\rangle_{b_1} = \beta\sqrt{t_2}|1\rangle_{b_2} + \gamma\sqrt{1-t_2}|1\rangle_{b_3}. \quad (8)$$

Bob also detects the photon in the $b_3$ mode by the single photon detector $D_2$. When $D_2$ detects no photon, the single photon state in the three parties can evolve to

$$|\Phi_3\rangle_{a_2b_2c_1} = \gamma|100\rangle_{a_2b_2c_1} + \beta\sqrt{t_2}|010\rangle_{a_2b_2c_1} + \gamma|001\rangle_{a_2b_2c_1}, \quad (9)$$

with the success probability of $\frac{2|\gamma|^2 + |\beta|^2}{2|\gamma|^2 + |\beta|^2}$.

Similarly, if Bob can select a suitable VBS2 with $t_2 = \frac{|\alpha|^2}{|\gamma|^2}$, Eq. (9) can finally evolve to

$$|\Phi\rangle_{a_2b_2c_1} = \gamma|100\rangle_{a_2b_2c_1} + \gamma|010\rangle_{a_2b_2c_1} + \gamma|001\rangle_{a_2b_2c_1} \rightarrow \frac{1}{\sqrt{3}}(|100\rangle_{a_2b_2c_1} + |010\rangle_{a_2b_2c_1} + |001\rangle_{a_2b_2c_1}), \quad (10)$$

which is the maximally entangled single photon W state. When $t_2 = \frac{|\gamma|^2}{|\beta|^2}$, the success probability of the second concentration step is

$$P_2 = \frac{3|\gamma|^2}{2|\gamma|^2 + |\beta|^2}, \quad (11)$$

where the subscript "2" means in the second concentration step.

So far, the whole ECP is completed and the three parties can finally share a maximally entangled W state from the partially entangled single photon W state. In the practical experiment, the two concentration steps are absolutely
independent, which can be completed by Alice and Bob alone, respectively. The total success probability equals to the product of the success probability in each concentration step, which can be written as

$$P_{\text{total}} = P_1 P_2 = (2|\gamma|^2 + |\beta|^2)^3 |\gamma|^2 = 3|\gamma|^2.$$  \hspace{1cm} (12) $$

Similarly, it is obvious that our ECP can be extended to concentrate single photon N-mode partially entangled W state. Suppose the N-mode single photon W state is shared by N parties, which can be written as

$$|\Phi_N\rangle = a_1|100\cdots0\rangle + a_2|010\cdots0\rangle + a_3|001\cdots0\rangle + \cdots + a_N|000\cdots01\rangle,$$  \hspace{1cm} (13) $$

where $|a_1|^2 + |a_2|^2 + |a_3|^2 + \cdots + |a_N|^2 = 1$, and $|a_1| > |a_2| > |a_3| > \cdots > |a_N|$. Under this case, N-1 parties need to perform the concentration step, respectively. In each concentration step, a suitable VBS with the transmittance of $t_i = \frac{|a_i|^2}{|a_1|^2}$ should be provided. After the N-1 concentration steps, Eq. (13) can be finally converted to the maximally entangled W state as

$$|\Phi'_N\rangle = \frac{1}{\sqrt{N}}(|100\cdots0\rangle + |010\cdots0\rangle + |001\cdots0\rangle + \cdots + |000\cdots01\rangle),$$  \hspace{1cm} (14) $$

with the success probability of

$$P_{\text{N-total}} = N|a_N|^2.$$  \hspace{1cm} (15) $$

III. THE ECP FOR THE MULTI-PHOTON POLARIZATION W STATE

![Diagram](image)

FIG. 2: The schematic drawing of the ECP for the three-photon polarization W state. The ECP also can be divided into two independent steps, which only requires local operations from Alice and Bob, respectively. In each step, the polarization beam splitters (PBSs) are used to transmit the polarization photon state and reflect the polarization photon state. The VBSs are used to adjust the entanglement coefficients.

Interestingly, with the basic principle in Sec. 2, we can still propose an efficient ECP for concentrating the partially entangled multi-photon polarization state. We take the three-photon W state as an example. The basic principle of the ECP is shown in Fig. 2. Suppose a single photon source S emits three photons and sends them to Alice, Bob and Charlie, respectively, which creates a partially entangled three-photon W state in a1, b1 and c1 modes as

$$|\Psi_{1}\rangle_{a1b1c1} = \alpha|HVV\rangle_{a1b1c1} + \beta|VHV\rangle_{a1b1c1} + \gamma|VVH\rangle_{a1b1c1}.$$  \hspace{1cm} (16) $$

Here, $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$ and we also suppose $|\alpha| > |\beta| > |\gamma|$. The ECP also can be divided into two steps. In the first step, Alice firstly makes the photon in the a1 mode pass through a polarization beam splitter (PBS), here named PBS1, which can transfer a |V| polarization photon and reflect a |H| polarization photon. After PBS1, $|\Psi_{1}\rangle_{a1b1c1}$ can evolve to

$$|\Psi_{2}\rangle = \alpha|HVV\rangle_{a2b1c1} + \beta|VHV\rangle_{a3b1c1} + \gamma|VVH\rangle_{a3b1c1}.$$  \hspace{1cm} (17) $$

Then Alice makes the photon in the a2 mode pass through a variable beam splitter (VBS1) with the transmittance of $t_1$. In this way, Eq. (17) can evolve to

$$|\Psi_{2}\rangle = \alpha\sqrt{t_1}|HVV\rangle_{a4b1c1} + \alpha\sqrt{1-t_1}|HVV\rangle_{a5b1c1} + \beta|VHV\rangle_{a3b1c1} + \gamma|VVH\rangle_{a3b1c1}.$$  \hspace{1cm} (18) $$
After that, Alice detects the photon in the a5 mode by the single photon detector D1. If D1 detects no photon, Eq. (15) will collapse to

$$|\Psi_2\rangle = \alpha \sqrt{t_1}|HVV\rangle_{a\overline{b}c1} + \beta |VHV\rangle_{a\overline{b}c1} + \gamma |VVH\rangle_{a\overline{b}c1},$$  \hspace{1cm} (19)

with the possibility of $|\alpha|^2 t_1 + |\beta|^2 + |\gamma|^2$. Similar with Sec. 2, if $t_1 = |\gamma|^2 / |\alpha|^2$, Eq. (19) can be written as

$$|\Psi_2\rangle \rightarrow \gamma |HVV\rangle_{a\overline{b}c1} + \beta |VHV\rangle_{a\overline{b}c1} + \gamma |VVH\rangle_{a\overline{b}c1},$$  \hspace{1cm} (20)

which only has two different entanglement coefficients $\beta$ and $\gamma$.

Finally, Alice makes the photon in the a3 and a4 mode pass through another PBS, here named PBS2. After PBS2, Eq. (20) evolves to

$$|\Psi_3\rangle_{a\overline{b}c1} = \gamma |HVV\rangle_{a\overline{b}c1} + \beta |VHV\rangle_{a\overline{b}c1} + \gamma |VVH\rangle_{a\overline{b}c1}.$$  \hspace{1cm} (21)

Until now, the first concentration step is completed and we successfully obtain the three-photon W state with only two different entanglement coefficients, with the success probability of $P_1 = |\beta|^2 + 2|\gamma|^2$.

The second concentration step is operated by Bob alone, which is quite similar with the first concentration step. As shown in Fig. 2, by making the photon in the b1 mode pass through PBS3 and the photon in b2 mode pass through the VBS2 with the transmittance of $t_2$, Eq. (21) can ultimately evolve to

$$|\Psi_4\rangle = \gamma |HVV\rangle_{a\overline{b}c3} + \beta \sqrt{t_2} |VHV\rangle_{a\overline{b}c4} + \gamma \sqrt{1-t_2} |VVH\rangle_{a\overline{b}c3} + \gamma |VVH\rangle_{a\overline{b}c3}.$$  \hspace{1cm} (22)

Then, the photon in the b5 mode is detected by the single photon detector D2. Under the case that D2 detects no photon, Eq. (22) will collapse to

$$|\Psi_5\rangle = \gamma |HVV\rangle_{a\overline{b}c3} + \beta \sqrt{t_2} |VHV\rangle_{a\overline{b}c4} + \gamma |VVH\rangle_{a\overline{b}c3},$$  \hspace{1cm} (23)

with the probability of $t_2$. If $t_2 = |\beta|^2$, Eq. (23) can finally be written as

$$|\Psi_5\rangle = \frac{1}{\sqrt{3}}(|HVV\rangle_{a\overline{b}c3} + |VHV\rangle_{a\overline{b}c4} + |VVH\rangle_{a\overline{b}c3}).$$  \hspace{1cm} (24)

Finally, Bob makes the photon in the b3 and b4 modes pass through the PBS4. After the PBS4, the three parties can share a maximally entangled polarization W state as

$$|\Psi_6\rangle = \frac{1}{\sqrt{3}}(|HVV\rangle_{a\overline{b}c6} + |VHV\rangle_{a\overline{b}c6} + |VVH\rangle_{a\overline{b}c6}).$$  \hspace{1cm} (25)

The total success probability of the ECP also equals the product of the success probability in each concentration round, which is the same as that in Eq. (12).

Similarly, by performing N-1 concentration steps described above, our ECP can also be extended to concentrate the partially entangled N-photon polarization W state as

$$|\Psi_N\rangle = a_1 |HVV \cdots V\rangle + a_2 |VHV \cdots V\rangle + a_3 |VVH \cdots V\rangle + \ldots + a_N |VVV \cdots VH\rangle,$$  \hspace{1cm} (26)

where $|a_1|^2 + |a_2|^2 + |a_3|^2 + \cdots + |a_N|^2 = 1$, and $|a_1| > |a_2| > |a_3| > \cdots > |a_N|$. With the help of the PBSs and suitable VBS in each concentration step, Eq. (27) can be finally recovered to the maximally entangled N-photon polarization W state as

$$|\Psi_N\rangle = \frac{1}{\sqrt{N}}(|HVV \cdots V\rangle + |VHV \cdots V\rangle + |VVH \cdots V\rangle + \cdots + |VVV \cdots VH\rangle),$$  \hspace{1cm} (27)

with the same success probability in Eq. (13).

**IV. DISCUSSION AND SUMMARY**

In the paper, we propose two efficient ECPs for partially entangled multi-mode single photon W state and multi-photon polarization W state. Our ECPs only require the linear optical elements, among which the VBS is the key elements. We require the VBSs with suitable transmittance to adjust the entanglement coefficients and finally obtain
In the current ECP, we suppose \(|\alpha| > |\beta| > |\gamma|\), so that we make \(\alpha \in (\sqrt{\frac{3}{7}}, \sqrt{\frac{7}{3}})\). As the ECPs in Ref \([41–43, 54]\) can be used repeatedly to further concentrate the partially entangled W state, we make curve A represents both the two steps are operated for one time, curve B represents both two steps are operated for three times, and curve C represents both the two steps are operated for five times. Curve D represents the ECP in our current paper.

Moreover, although our current ECPs can not be recycled, their success probability is higher than the previous ECPs for W state \([41]–[43, 54]\). Now, we will compare the success probability of our current ECPs with our previous ECPs for the W state \([41–43, 54]\). In the current paper, we suppose \(|\alpha| > |\beta| > |\gamma|\), so that we make

\[
\begin{align*}
\sqrt{\frac{3}{7}} \leq t_1 & \leq \sqrt{\frac{7}{3}}, \\
\sqrt{\frac{3}{7}} \leq t_2 & \leq \sqrt{\frac{7}{3}}, \\
\sqrt{\frac{3}{7}} \leq t_3 & \leq \sqrt{\frac{7}{3}}.
\end{align*}
\]

The maximally entangled W state will not be destroyed, and can be used in other applications. Meanwhile, in our ECPs, each concentration step only requires local operation, which can simplify the experimental operation largely. On the other hand, in linear optics, when the photon is detected by the detector, it will be destroyed, which is well known as the post selection principle. In our ECPs, as each party only selects the case that the photon detector measures no photon, the generated maximally entangled W state will not be destroyed, and can be used in other applications.

Therefore, by repeating both steps, the total success probability is

\[
\begin{align*}
P_{\text{total}} &= P_1^1 (P_1^2 + P_2^2 + \cdots + P_M^2) + P_2^1 (P_1^2 + P_2^2 + \cdots + P_M^2) + \cdots + P_M^1 (P_1^2 + P_2^2 + \cdots + P_M^2) \\
&= \sum_{N=1}^{\infty} P_N^1 \sum_{M=1}^{\infty} P_M^2.
\end{align*}
\]

Here, we calculate the total success probability of both our current ECPs in Eq. (12) and the previous ECP in Eq. (29) in Fig. 3. Here, we choose \(\beta = \frac{1}{\sqrt{2}}\). In the current paper, we suppose \(|\alpha| > |\beta| > |\gamma|\), so that we make

\[
\begin{align*}
P_N^1 &= \frac{|\alpha|^{2N} (|\beta|^{2N-2} |\gamma|^2 + 2 |\beta|^{2N})}{(|\alpha|^{2N} + |\beta|^{2N})(|\alpha|^{2N-1} + |\beta|^{2N-1})\cdots(|\alpha|^2 + |\beta|^2)}, \\
P_M^2 &= \frac{3 |\beta|^{2M} |\gamma|^{2M}}{(\gamma^{2M} + |\beta|^{2M})(\gamma^{2M-1} + |\beta|^{2M-1})\cdots(\gamma^2 + |\beta|^2)} \cdot \frac{1}{(\gamma^2 + 2 |\beta|^2)}.
\end{align*}
\]

where the superscript "1" and "2" mean in the first and second concentration step, respectively. The subscripts "N" and "M" mean in the Nth and Mth concentration round.
In Fig. 3, curves A, B, and C represent the total success probability of the ECP in Ref. [41]. Curve A represents that both the two steps are operated for one time. Curve B represents that both two steps are operated for three times. Curve C represents that both the two steps are operated for five times. Curve D represents the ECP in the current paper. It can be found that in both two ECPs, the success probability is largely altered with the initial entanglement coefficient $\alpha$. The higher initial entanglement can obtain the higher success probability. Moreover, although the success probability of the ECP in Ref. [41] increases with the cycle times, it is still lower than that of our current ECPs. Especially, when $\alpha = \frac{1}{\sqrt{3}}$, the success probability of our current ECP is 1, while that of the ECP in Ref. [41] can only obtain about 0.93, when both two concentration steps are operated for five times. Certainly, we can further increase its success probability by increasing its cycle times. However, by mathematical calculation, we can get when the ECP in Ref. [41] is repeated indefinitely, its success probability curve (which will not be presented in Fig. 3) will be coincided with curve D. During our ECPs, the total success probability essentially is decided by the smallest coefficient of the initial state.

Actually, in the early theoretical work of concentration of the two-particle Bell state, Lo and Popescu showed that the maximum probability with which a Bell state can be obtained by purifying a single entangled pair is twice the modulus square of the Schmidt coefficient of smaller magnitude [62]. The result of the recent work of Deng’s group is consist with Lo and Popescu [63]. It reveals that the total entanglement is a conserved quantity. Interestingly, our ECPs can be regarded as the extension of the result from the previous work of two-particle case, which can be concluded as the maximum probability of concentrating a N-particle partially entangled W state or single-photon N-mode partially entangled state is Nth the modulus square of the Schmidt coefficient of the smallest magnitude.

In summary, we proposed two optimal ECPs for concentrating the single-photon multi-mode W state and N-photon polarization W state. In both ECPs, we only require one pair of partially entangled W state, and do not consume any auxiliary photon. In each concentration step, we mainly require the VBS to adjust the entanglement coefficients. Our ECPs have some obvious advantages. First, they only require the linear optical elements, which makes them can be easily realized under current experimental condition. Second, the generated maximally entangled W state will not be destroyed, and can be used in other applications. Third, our ECPs only need to be operated for one time, but they can obtain higher success probability than previous ECPs. Based on the advantages above, our ECPs may be useful in current quantum communication fields.

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[1] M. A. Nielsen, and I. L. Chuang, "Quantum computation and quantum information" (Cambridge University Press, Cambridge, England, 2000).
[2] A. Einstein, B. Podolsky, and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete?" Phys. Rev. 47, 777-780 (1935).
[3] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, "Quantum cryptography," Rev. Mod. Phys. 74, 145-195 (2002).
[4] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, "Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels," Phys. Rev. Lett. 70, 1895-1899 (1993).
[5] A. Karlsson and M. Bourennane, "Quantum teleportation using three-particle entanglement," Phys. Rev. A 58, 4394 (1998).
[6] F. G. Deng, C. Y. Li, Y. S. Li, H. Y. Zhou, and Y. Wang, "Symmetric multiparty-controlled teleportation of an arbitrary two-particle entanglement," Phys. Rev. A 72, 022338 (2005).
[7] C. H. Bennett and S. J. Wiesner, "Communication via one- and two-particle operators on Einstein-Podolsky-Rosen states," Phys. Rev. Lett. 69, 2881-2884 (1992).
[8] M. Hillery, V. Bužek, A. Berthiaume, "Quantum secret sharing," Phys. Rev. A. 59, 1829-1834 (1999).
[9] A. Karlsson, M. Koashi, N. Imoto, "Quantum entanglement for secret sharing and secret splitting," Phys. Rev. A 59, 162-168 (1999).
[10] L. Xiao, G. L. Long, F. G. Deng, J. W. Pan, "Efficient multiparty quantum-secret-sharing schemes," Phys. Rev. A, 69, 052307 (2004).
L. M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, "Long-distance quantum communication with atomic ensembles and
D. Gottesman, T. Jennewein, S. Croke, "Longer-Baseline Telescopes Using Quantum Repeaters," Phys. Rev. Lett. 92, 177903 (2004).
L. Zhou, "Theoretically efficient high-capacity quantum-key-distribution scheme," Phys. Rev. A 65, 032302 (2002).
F. G. Deng, G. L. Long, and X. S. Liu, "Two-step quantum direct communication protocol using the Einstein-Podolsky-Rosen pair block," Phys. Rev. A 68, 042317 (2003).
C. Wang, F. G. Deng, Y. S. Li, X. S. Liu, and G. L. Long, "Quantum secure direct communication with high-dimension quantum superdense coding," Phys. Rev. A 71, 044305 (2005).
A. Sen(De), U. Sen, M. Wiesniak, D. Kaszlikowski, and M. Żukowski, "Multiqubit W states lead to stronger nonclassicality than Greenberger-Horne-Zeilinger states," Phys. Rev. A 68, 062306 (2003).
W. Dür, G. Vidal, and J. I. Cirac, "Three qubits can be entangled in two inequivalent ways," Phys. Rev. A 62, 062314 (2000).
R. Chaves, and L. Davidovich, "Robustness of entanglement as a resource," Phys. Rev. A 82, 052308 (2010).
P. Agrawal, and A. Pati, "Perfect teleportation and superdense coding with W states," Phys. Rev. A, 74, 062320 (2006).
J. Song, Y. Xia, and H. S. Song, "Quantum nodes for W-state generation in noisy channels," Phys. Rev. A 78, 024302 (2008).
H. Nha, and J. Kim, "Demonstrating multipartite entanglement of single-particle W states: Linear optical schemes," Phys. Rev. A 75, 012326 (2007).
H. Nha, "Linear optical scheme to demonstrate genuine multipartite entanglement for single-particle W states," Phys. Rev. A 78, 062328 (2008).
S. Bugc, C. Yesilyurt, and F. Ozaydin, "Enhancing the W-state quantum-network-fusion process with a single Fredkin gate," Phys. Rev. A 87, 022331 (2013).
S. K. Özdemir, E. Matsunaga, T. Tashima, T. Yamamoto, M. Koashi, and N. Imoto, "An optical fusion gate for W-states," New J. Phys. 13, 103003 (2011).
T. Tashima, T. Kitano, S. K. Özdemir, T. Yamamoto, M. Koashi, and N. Imoto, "Demonstration of local expansion toward large-scale entangled webs," Phys. Rev. Lett. 105, 210503 (2010).
T. Tashima, S. K. Özdemir, T. Yamamoto, M. Koashi, and N. Imoto, "Local expansion of photonic W state using a polarization dependent beam splitter," New J. Phys. 11, 032024 (2009).
T. Tashima, S. K. Özdemir, T. Yamamoto, M. Koashi, and N. Imoto, "Elementary optical gate for expanding an entanglement web," Phys. Rev. A 77, 030302(R) (2008).
T. Yamamoto, M. Koashi, S. K. Özdemir, and N. Imoto, "Experimental extraction of an entangled photon pair from two identically decohered pairs," Nature 421, 343 (2003).
T. Yamamoto, K. Hayashi, S. K. Özdemir, M. Koashi, and N. Imoto, "Robust photonic entanglement distribution by state-independent encoding onto decoherence-free subspace," Nature Photonics 2, 488 (2008).
T. Tashima, T. Wakatsuki, S. K. Özdemir, T. Yamamoto, M. Koashi, and N. Imoto, "Local transformation of two Einstein-Podolsky-Rosen photon pairs into a three-photon W-state," Phys. Rev. Lett. 102, 130502 (2009).
W. Cui, E. Chitambar, and H. K. Lo, Phys. Rev. A 82, 062314(2010).
A. Yildiz, "Optimal distillation of three-qubit W states," Phys. Rev. A 82, 012317 (2010).
W. Cui, E. Chitambar, and H. K. Lo, "Randomly distilling W-class states into general configurations of two-party entanglement," Phys. Rev. A 84, 052301 (2011).
E. Chitambar, and H. K. Lo, "Entanglement monotones for W-type states," Phys. Rev. A 85, 062316 (2012).
Z. L. Cao and M. Yang, "Entanglement distillation for three-particle W class states," J. Phys. B 36, 4245 (2003).
L. H. Zhang, M. Yang, and Z. L. Cao, "Entanglement concentration for unknown W class states," Phys. A 374, 611 (2007).
H. F. Wang, S. Zhang, and K. H. Yeon, "Linear optical scheme for entanglement concentration of two partially entangled threephoton W states," Eur. Phys. J. D 56, 271 (2010).
B. Gu, "Single-photon-assisted entanglement concentration of partially entangled multiphoton W states with linear optics," J. Opt. Soc. Am. B 7, 1685-1689 (2012).
F. F. Du, T. Li, B. C. Ren, H. R. Wei, and F. G. Deng, "Single-photon-assisted entanglement concentration of a multiphoton system in a partially entangled W state with weak cross-Kerr nonlinearity," J. Opt. Soc. Am. B 29, 1399-1405 (2012).
Y. B. Sheng, L. Zhou, S. M. Zhao, "Efficient two-step entanglement concentration for arbitrary W states," Phys. Rev. A 85, 042302 (2012).
L. Zhou, Y. B. Sheng, W. W. Cheng, L. Y. Gong, S. M. Zhao, "Efficient entanglement concentration for arbitrary single-photon multimode W state," J. Opt. Soc. Am. B 30, 71-78 (2013).
L. Zhou, "Efficient entanglement concentration for electron-spin W state with the charge detection," Quant. Inf. Process. 12, 2087-2101 (2013).
D. Gottesman, T. Jennewein, S. Croke, "Longer-Baseline Telescopes Using Quantum Repeaters," Phys. Rev. Lett. 109, 070503 (2012).
L. M. Duan, M. D. Lukin, J. I. Cirac, and P. Zoller, "Long-distance quantum communication with atomic ensembles and linear optics," Nature 414, 413-418. (2001).
[46] C. H. Bennett, H. J. Bernstein, S. Popescu, and B. Schumacher, "Concentrating partial entanglement by local operations," Phys. Rev. A 53, 2046 (1996).
[47] S. Bose, V. Vedral, and P. L. Knight, "Purification via entanglement swapping and conserved entanglement," Phys. Rev. A 60, 194 (1999).
[48] B. S. Shi, Y. K. Jiang, and G. C. Guo, "Optimal entanglement purification via entanglement swapping," Phys. Rev. A 62, 054301 (2000).
[49] Z. Zhao, J. W. Pan, and M. S. Zhan, "Practical scheme for entanglement concentration," Phys. Rev. A 64, 014301 (2001).
[50] T. Yamamoto, M. Koashi, and N. Imoto, "Concentration and purification scheme for two partially entangled photon pairs," Phys. Rev. A 64, 012304 (2001).
[51] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, "Nonlocal entanglement concentration scheme for partially entangled multipartite systems with nonlinear optics," Phys. Rev. A 77, 062325 (2008).
[52] Y. B. Sheng, F. G. Deng, and H. Y. Zhou, "Single-photon entanglement concentration for long-distance quantum communication," Quant. Inf. & Comput. 10, 272-281 (2010).
[53] B. C. Ren, M. Hua, T. Li, F. F. Du, F. G. Deng, "Multipartite entanglement concentration of electron-spin states with CNOT gates," Chin. Phys. B 21, 090303 (2012).
[54] Y. B. Sheng, L. Zhou, L. Wang, S. M. Zhao, "Efficient entanglement concentration for quantum dot and optical microcavities systems," Quant. Inf. Process, 12, 1885-1895 (2013).
[55] L. Zhou, Y. B. Sheng, W. W. Cheng, L. Y. Gong, S. M. Zhao, "Efficient entanglement concentration for arbitrary less-entangled NOON states," Quant. Inf. Process. 12, 1307-1320 (2013).
[56] Y. B. Sheng, L. Zhou, S. M. Zhao, B. Y. Zheng, "Efficient single-photon-assisted entanglement concentration for partially entangled photon pairs," Phys. Rev. A 85, 012307 (2012).
[57] F. G. Deng, "Optimal nonlocal multipartite entanglement concentration based on projection measurements," Phys. Rev. A 85, 022311 (2012).
[58] C. Wang, Y. Zhang, G.S. Jin, "Entanglement purification and concentration of electron-spin entangled states using quantum-dot spins in optical microcavities," Phys. Rev. A 84, 032307 (2011).
[59] C. Wang, "Efficient entanglement concentration for partially entangled electrons using a quantum-dot and microcavity coupled system," Phys. Rev. A 86, 012323 (2012).
[60] B. C. Ren, F. F. Du, F. G. Deng, "Practical hyperentanglement concentration for two-photon four-qubit systems with linear optics," arXiv: 1306.6050v1 (2013).
[61] C. I. Osorio, N. Bruno, N. Sangouard, H. Zbinden, N. Gisin, R. T. Thew, "Heralded photon amplification for quantum communication," Phys. Rev. A 86, 023815 (2012).
[62] H. K. Lo, and S. Popescu, "Concentrating entanglement by local actions—beyond mean values," arXiv:quant-ph/9707038 (1997).