Incommensuration Effects and Dynamics in Vortex Chains

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(November 12, 2018)

We examine the motion of one-dimensional (1D) vortex matter embedded in a 2D vortex system with weak pinning using numerical simulations. We confirm the conjecture of Matsuda et al. (Science 294, 2136 (2001)) that the onset of the temperature induced motion of the chain is due to an incommensuration effect of the chain with the periodic potential created by the bulk vortices. In addition, under an applied driving force we find a two stage depinning transition, where the initial depinning of the vortex chain occurs through soliton like pulses. When an ac drive is added to the dc drive, we observe phase locking of the moving vortex chain.

PACS: 74.60.Ge, 74.60.Jg

The static and dynamical behavior of elastic media interacting with periodic or random substrates is a topic of intense study as it relates to a wide variety of physical systems, such as friction where atoms move over a periodic potential created by the underlying stationary atoms, sliding charge-density waves, and vortex motion in superconductors with random quenched disorder or periodic hole or dot arrays. A simplified case occurs when the motion is confined in 1D or quasi 1D systems. An example of this is vortex motion in superconductors with nano fabricated arrays of channels. There are also naturally occurring defects that can act as quasi-1D channels for vortex motion, such as grain and twin boundaries. The vortices in fabricated channels experience two types of pinning: the first from the intrinsic random disorder in the sample, and the second from the periodic potential created by the immoveable vortex lattice surrounding the channels. Experiments and simulations in these systems have observed commensuration effects and interesting dynamical states.

Another intriguing example of quasi 1D vortex matter is the vortex chain state which occurs in layered superconductors with tilted magnetic fields as first imaged by Bolle et al. Here a portion of the vortices align in a string with a smaller lattice spacing than that of the bulk vortices. Recently scanning Hall probe images and Lorentz microscopy images have shown that the vortex chain states exhibit a remarkably rich variety of behaviors and structures for various regimes. The recent Lorentz microscopy experiments found that for low temperatures, both the vortices along the chain state and the bulk vortices are stationary. For increasing temperatures the vortices along the chain disappear, which is speculated to be due to the onset of motion or oscillation of the vortex chains, so that the chain vortex images are blurred while the bulk vortices are still stationary. It was conjectured in that this effect is due to an incommensuration effect where vortices in the chain move in a periodic potential created by the bulk vortices. Since the vortices in the chain have a different spacing than the periodic potential, the chain is incommensurate and hence more weakly pinned than if the chain were commensurate. These images also showed that the blurring first occurs where there a mismatch accompanied by a topological defect in the chain. Additional evidence for this scenario is that as the field is increased, the periodicity of the potential decreases and the incommensuration effects are enhanced, causing the disappearance of the chains to shift to lower temperatures, as observed. Dodgson has proposed an alternative explanation in which the image disappears due the sudden formation of tilted chains. In addition to the intriguing physics of the vortex matter in confined geometries it is also of paramount importance to understand vortices in the chain state or along grain and twin boundaries, as these can act as weak lines where flux motion occurs, leading to the breakdown of superconductivity in the chains prior to the bulk.

In order to examine the conjecture that the disappearance of the vortex chain is due to an incommensuration effect, as well as to explore dynamical consequences of these incommensuration effects, we have performed numerical simulations of vortex chains coexisting with bulk vortices in the presence of weak quenched disorder and thermal noise. We also investigate the effects of applied dc and combined dc and ac external drives on the chain motion. Our results should be relevant for the vortex chain state where the vortices remain aligned in the z-direction so that much of the physics can be considered 2D. In addition our results are relevant for vortex motion in artificial channels and grain or twin boundaries where there are additional vortices outside the channels.

We consider a 2D sample with periodic boundary conditions in the x and y directions. We model the vortices as repulsive point particles which interact with weak random quenched disorder, applied driving forces, and thermal noise. A portion of the vortices are confined to move along a quasi-1D channel. The overdamped equation of motion for a vortex i is

\[
\eta \frac{dr}{dt} = f_i = f_{iv} + f_{rp} + f_d + f_T \tag{1}
\]

Here \(\eta\) is the damping constant that is set equal to unity. The vortex-vortex interaction force is
for the low-temperature in systems occurring along the chain but not in the bulk.

\[ f_i^w = - \sum_{j \neq i} N \nabla U_v(r_{ij}), \]

where the vortex interaction potential is \( U_v = -A_v \ln(r) \). The force from random quenched disorder \( f_i^{TP} \) comes from randomly placed attractive parabolic traps of range \( r_p \) with \( r_p/a = 0.1 \), where \( a \) is the vortex lattice constant and the traps have a maximum force of \( f_p = 1.0 \). The driving forces \( f_i^d \) include a dc force \( f_i^{DC} \) which is modeled as a uniform drive on the vortices in the \( x \)-direction. We start from zero driving and with small increments increase the dc driving force. At each increment we average the vortex velocity \( V_x \) over a fixed time interval. An ac drive can also be applied in the form \( f_i^{AC} = A \sin(\omega t) \hat{x} \), where \( A \) is the ac amplitude and \( \omega \) is the frequency. The forces from the thermal noise \( f_i^{\text{th}} \) come from random Langevin kicks with \( < f_i^{\text{th}}(t) > = 0 \) and \( < f_i^{\text{th}}(t) f_i^{\text{th}}(t') > = 2 \eta k_B T \delta(t-t') \). We measure temperature in units of \( T_m \), the temperature at which the bulk vortices melt. For high-\( T_c \) materials at fields where the chain state is observed, \( T_m \) is around 70 – 80K and for the low-\( T_c \) materials with nano-channels, \( T_m \) would correspond to \( T_c \). Lengths are measured in units of \( a \), and forces in \( A_v \), the vortex-vortex interaction prefactor.

We first consider the effects of temperature in systems with no external driving force to test the conjecture in [3]. We start the system at \( T/T_m = 0.0 \) where the bulk vortices form a triangular lattice with lattice constant \( a \). Additional vortices are added along the channel, giving a vortex spacing in the channel of \( a' < a \). We monitor the vortex displacements for fixed time intervals of \( 10^4 \) MD steps, and measure the vortex positions and trajectories.

For low \( T \) there is little motion of the vortices in the chain or the bulk as seen in Figure 1(a). In addition the incommensuration in the chain is stationary or pinned in a well defined location. For increasing temperatures there is a transition to a state in which the vortex chain exhibits a higher mobility than the bulk vortices, as seen in the vortex trajectory traces of Fig. 1(b) at \( T/T_m = 0.25 \).

To elucidate the exact nature of the motion along the chain, in Fig. 2(a) we plot the position of a single vortex in the chain for the states in Fig. 1(a) and Fig. 1(b). For the stationary chain state the vortex does not move continuously, but instead moves in discrete jumps of magnitude nearly \( a \). These jumps do not occur throughout the chain simultaneously, but run through the chain as a soliton like pulse with the vortices jumping sequentially. The discrete jumps occur due to the depinning of the incommensurations, which move randomly through the sample, causing each vortex to jump one lattice constant as the incommensuration passes over it. The discreteness of the jumps arises from the periodic

![FIG. 1. A portion of the sample with vortices (black dots) and trajectories (lines). The vortex chain is in the center. (a) \( T/T_m = 0.15 \). (b) \( T/T_m = 0.25 \), showing that motion is occurring along the chain but not in the bulk.](image)

![FIG. 2. (a) Time series for the relative \( x \)-position of a single vortex in the chain for \( T/T_m = 0.15 \) (light curve) and \( T/T_m = 0.25 \) (dark curve). For low \( T \), the vortex is stationary, while for higher \( T \), the vortex jumps back and forth by about one chain lattice constant \( a \). (b) The power spectra \( S(\nu) \) for the full time series of the vortex position. For \( T/T_m = 0.15 \) (lower light curve) the noise spectra is of low power and white. For \( T/T_m = 0.2 \) (upper dark curve) the noise power is much higher. The dashed line indicates a power law \( 1/f^\alpha \) with \( \alpha = 1.6 \).](image)
potential in which the chain vortices sit, created by the stationary ordered bulk vortices. Since the incommensurations are much more weakly pinned than the bulk vortices, they depin at a much lower temperature.

Figure 2(a) also shows that although there is motion of vortices along the chain, a single vortex does not diffuse but only moves back and forth by a single lattice constant. This can be understood by considering a single randomly moving incommensuration. When the incommensuration traveling in the positive \(x\)-direction passes over a vortex, the vortex moves by \(a\) in the positive \(x\)-direction. The same incommensuration cannot, however, later cause the same vortex to move by an additional distance in the positive \(x\)-direction, but only in the negative \(x\)-direction when the incommensuration returns. Thus a single randomly moving incommensuration produces no vortex diffusion along the chain but rather an oscillation. If a series of incommensurations all moving in the same direction pass over a vortex, the vortex could move several lattice constants in one direction. This is prevented because there is no net shear in the system. The moving incommensuration sets up a strain field that prevents the incommensuration from traveling large distances in the same direction. As a result, the incommensuration does not diffuse freely through the chain but tends to drift back and forth about its original starting position.

Using a Voronoi construction, we find that the incommensuration in the vortex chain coincides with a local defect composed of one 5-fold and two 7-fold coordinated vortices. It is the 1D motion of this defect that occurs in Fig. 1(b). The images in Ref. [3] also indicate that the initial blurring in the chain occurs where there are localized topological defects in the vortex lattice. This agrees well with our finding that it is the motion of defects that is occurring along the chains.

In Fig. 2(b) we show the power spectra for the mobile and immobile chain. For the immobile chain, the noise spectrum is white, while for the mobile chain, the low frequency noise power is several orders of magnitude higher, due to the correlated low frequency motions of the incommensuration, and there is now a \(1/f^\alpha\) signature, with \(\alpha = 1.6\). For \(T > T_m\), when all the vortices diffuse randomly, the noise spectra again becomes white.

In simulations where the chain is commensurate with the bulk vortex lattice, such that \(a = a'\), the chain and bulk vortex mobility are the same. Thus, the increased mobility of the vortices along the chain, as shown in Fig. 1(b), is due strictly to an incommensuration effect. When the vortex density along the chain is increased, lowering \(a'\) and increasing the number of incommensurations, the thermal depinning temperature of the chain decreases. The depinning and phase locking described below persist as the number of incommensurations increases.

A further probe of the relative pinning strength of the incommensurations along the chain, compared to the pinning of the bulk vortices, is the application of a dc drive. We consider the case with weak random pinning where, in the absence of a chain, there is a single well defined elastic depinning transition. In Fig. 3(a) the velocity-force curve for the incommensurate case shows two depinning transitions; the first is the depinning along the chain, and the second at \(f_d = 1.0\) is the bulk vortex depinning. A similar two stage depinning has been observed in artificial channels and periodic pinning arrays [8]. Imaging experiments in the vortex chain state also find that the chain depins first under an external driving force [9]. In Fig. 3(b) we plot the positions of the vortices in the chain as a function of time just above the depinning transition, showing that the motion is not continuous, but occurs in a pulse or stick-slip motion. Individual vortices remain stationary most of the time, and then jump one lattice constant in sequence. The disturbance or incommensuration moves much faster than the individual vortices. This is the same type of motion that occurs during the thermal depinning; however, in the driven case the incommensuration moves in only one direction. A similar type of 1D soliton motion has been
seen for vortex motion in 2D periodic pinning arrays in simulations [10] and experiments [6] where the vortices effectively channel along the pinning rows.

We have also investigated the effects of a combined dc and ac drive on the moving chain state. Since the motion along the chain occurs over an effective periodic substrate, interference effects should occur between the ac drive and the frequency induced by the dc drive. In this case, it is the incommensurations, rather than the individual vortices in the chain, that undergo phase locking. In Fig. 4(a) we show a series of velocity-force curves for the moving chain for different ac amplitudes and fixed ac frequency. Here clear phase locking effects can be seen as steps in the velocity curves. The widths of the critical depinning force and the higher order steps vary nonmonotonically with ac amplitude. If the phase locking steps are of the Shapiro step type, the nth step should vary as the Bessel function $J_n(A)$ [11]. In Fig. 4(b) and Fig. 4(c), the widths of the depinning force $\eta_0$ and the first step $\eta_1$ fit to $J_0$ and $J_1$, respectively, showing that the phase locking is best described as a Shapiro step similar to those seen in sliding CDW’s [2] as well as 2D vortex systems with periodic pinning [12]. The steps shown here occur below the bulk depinning transition, for $f_d/f_p < 1.0$, where only the chain vortices are depinned.

In conclusion, we have investigated vortices confined in 1D embedded in an ordered 2D vortex lattice with weak pinning. The vortices in the chain are incommensurate with the periodic potential created by the bulk vortices. We confirm the conjecture in Ref. [13] that the smearing of the vortex chain state is due to an incommensuration effect, where for increasing temperature the incommensurations thermally depin and become mobile before the bulk vortices do. The thermal motion along the chain occurs through a soliton like pulse which moves randomly through the sample. The vortices hop by one lattice constant in the positive or negative direction as the pulse moves through. The onset of motion along the chain can also be observed as an increase in the voltage noise with a $1/f^n$ characteristic spectra. Under application of an external dc drive, we observe a two stage depinning where the vortices in the chain depin first and form running solitons. When an ac drive is added to the dc drive, we observe phase locking of the moving incommensurations in the form of Shapiro steps in the velocity-force curves. Our predictions can be tested through noise measurements in the vortex chain state under application of external drives or temperature, and also apply to vortex motion in artificial channels.

**Note:** Just before submitting this work we became aware of recent experiments that find phase locking of dc and ac driven vortices in artificial channels [14].

This work was supported by the US Department of Energy under Contract No. W-7405-ENG-36. We thank P. Kes and A. Tonomura for useful discussions.

**References:**

1. B.N.J. Persson, *Sliding Friction: Physical Principles and Applications* (Springer, Heidelberg, 1998).
2. G. Gruner, Rev. Mod. Phys. 60, 1129 (1988); R.E. Thorne, Phys. Today 49(5), 42 (1996).
3. L. Balents, M.C. Marchetti, and L. Radzihovsky, Phys. Rev. B 57, 7705 (1998); P. LeDoussal and T. Giamarchi, Phys. Rev. B 57, 11356 (1998).
4. M. Baert et al., Phys. Rev. Lett. 74, 3260 (1995); K. Harada et al., Science 274, 1167 (1996); J.I. Martin et al., Phys. Rev. Lett. 79, 1929 (1997).
5. L. Van Look et al., Phys. Rev. B 60, R6998 (1999); C. Reichhardt et al., Phys. Rev. B 61, R19141 (2000).
6. R. Surdeanu et al., Europhys. Lett. 54, 682 (2001).
7. C. Reichhardt, C.J. Olson, and F. Nori, Phys. Rev. B 58, 6534 (1998).
8. A. Pruymboom et al., Phys. Rev. Lett. 60, 1430 (1988); M.H. Theunissen, E. Van der Drift, and P.H. Kes, *ibid.* 77, 159 (1996); R. Besseling, R. Niggebrugge, and P.H. Kes, *ibid.* 82, 3144 (1999); R. Besseling et al., cond-mat/0202485.
9. S. Anders et al., Physica C 332, 35 (2000); Phys. Rev. B 62, 15195 (2000).
10. M.J. Hogg et al., Appl. Phys. Lett. 78, 1433 (2001).
11. A. Gurevich et al., Phys. Rev. Lett. 88, 097001 (2002).
12. C.A. Duran et al., Nature (London) 357, 474 (1992); Phys. Rev. Lett. 74, 3712 (1995); U. Welp et al., *ibid.* 74, 3713 (1995); J. Groth et al., *ibid.* 77, 3625 (1996).
13. C.A. Bolle et al., Phys. Rev. Lett. 66, 112 (1991).
14. A. Grigorenko et al., Nature (London) 414, 728 (2001).
15. T. Matsuda et al., Science 294, 2136 (2001).
16. M.J.W. Dodgson, cond-mat/0201197.
17. A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York 1982).
18. N. Kokubo et al., cond-mat/0203353.