Seismic diagnostics on stellar convection treatment from oscillation amplitudes of \( p \)-modes.

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Abstract. The excitation rate \( P \) of solar \( p \)-modes is computed with a model of stochastic excitation which involves constraints on the averaged properties of the solar turbulence. These constraints are obtained from a 3D simulation. Resulting values for \( P \) are found \( \sim 9 \) times larger than when the calculation assumes properties of turbulent convection which are derived from an 1D solar model based on Gough (1977)’s formulation of the mixing-length theory (GMLT). This difference is mainly due to the assumed values for the mean anisotropy of the velocity field in each case. Calculations based on 3D constraints bring the \( P \) maximum closer to the observational one.

We also compute \( P \) for several models of intermediate mass stars \( 1 \lesssim M \lesssim 2 \, M_\odot \). Differences in the values of \( P_{\text{max}} \) between models computed with the classical mixing-length theory and GMLT models are found large enough for main sequence stars to suggest that measurements of \( P \) in this mass range will be able to discriminate between different models of turbulent convection.

Keywords: convection, turbulence, oscillations, Sun

1. Introduction

Excitation of solar-type oscillations is attributed to turbulent movements in the outer convective zone of intermediate mass stars.

Accurate measurement of the rate \( P \) at which acoustic energy is injected into such oscillations will be possible with the future seismic missions (e.g. COROT and EDDINGTON). Comparison between measured and theoretical values of \( P \) obtained with different models of turbulent convection will then provide valuable information about the properties of stellar convection zones.

Models for stochastic excitation have been proposed by several authors (e.g. Goldreich and Keeley, 1977; Balmforth, 1992; Samadi and Goupil, 2001). In the present work, we consider the formulation of Samadi and Goupil (2001, Paper I hereafter). Constraints on the time averaged properties of the solar turbulent medium are obtained from a 3D simulation. They allow us to compute \( P \) and compare it with solar seismic observations and results obtained with Gough (1977)’s formulation of the mixing-length theory (GMLT hereafter).
2. The model of stochastic excitation

According to Paper I, the rate at which a given mode with frequency $\omega_0$ is excited can be written as:

$$P(\omega_0) \propto \int_0^M \rho_0 w^4 \left\{ \frac{16}{15} \frac{\Phi}{3} \left( \frac{d\xi_r}{dr} \right)^2 S_R + \frac{4}{3} \frac{\alpha_s \bar{s}^2}{\rho_0 w} \frac{g_r}{\omega_0^2} S_S \right\}$$

In Eq. (1), $\rho_0$ is the mean density, $\xi_r$ is the radial component of the fluid displacement adiabatic eigenfunction $\xi$, $\alpha_s = (\partial p/\partial s)_p$ where $p$ denotes the pressure and $s$ the entropy, $\bar{s}^2$ is the rms value of the entropy fluctuations, $g_r(\xi_r, m)$ involves the first and the second derivatives of $\xi$ with respect to $r$, $S_R(\omega_0, m)$ and $S_S(\omega_0, m)$ are driving sources inferred from the Reynolds and the entropy fluctuations respectively, $\Phi$ is Gough (1977)'s mean anisotropy factor defined as $\Phi(m) \equiv <u^2> - <u>^2/w^2$ where $u$ is the velocity field, $< . >$ denotes time and horizontal average and $w$ is the mean vertical velocity ($w^2 \equiv <u^2_z> - <u_z>^2$). Expressions for $S_R(\omega_0, m)$, $S_S(\omega_0, m)$ and $g_r(\xi_r, m)$ are given in Paper I.

The driving sources $S_R$ and $S_S$ include the turbulent kinetic energy spectrum $E(k, m)$, the turbulent spectrum of the entropy fluctuations $E_s(k, m)$ and $\chi_k(\omega)$ the frequency-dependent part of the turbulent spectra which model the correlation time-scale of an eddy with wavenumber $k$. $\chi_k(\omega)$ is modeled here with a non-gaussian function constraint from the 3D simulation.

3. Numerical constraints and computation in the solar case

We consider a 3D simulation of the upper part of the solar convective zone as obtained by Stein and Nordlund (1998). The simulated domain is 3.2 Mm deep and its surface is 6 x 6 Mm$^2$. The grid of mesh points is 256 x 256 x 163, the total duration 27 mn and the sampling time 30s.

The simulation data are used to determine the quantities $E(k, m)$, $E_s(k, m)$, $w$, $\bar{s}$ and $\Phi$ involved in the theoretical expressions for $S_R$, $S_S$ and $P$. More details will be given in forthcoming papers.

We compute $P$ according to Eq. (1): the eigenfunctions ($\xi$) and their frequencies ($\nu = \omega_0/2\pi$) are calculated with Balmforth (1992)'s non-adiabatic code for a solar 1D model built with the GMLT approach. The $k$-dependency of $E(k, z)$, is modeled as $(k/k_0)^{+1}$ for $k < k_0$ and as $(k/k_0)^{-5/3}$ for $k > k_0$ where $k_0 = 2\pi/\beta\Lambda$, $\Lambda = \alpha H_p$ is the mixing-length, $H_p$ the pressure scale height. The value of the mixing-length parameter, $\alpha$, is imposed by a solar calibration of the 1D GMLT model.
The value of $k_0$ - hence of $\beta$ - is obtained from the 3D simulation. The above analytical $k$-dependency of $E$ and $E_s(k, z)$ reproduce the global features of $E$ and $E_s$ derived from the 3D simulation.

Results of $P$ computations are presented in Fig. 1 (solid curve) and compared with values of $P$ (filled dots) derived from solar seismic measurement by Chaplin et al. (1998).

We also compute $P$ in the case when the quantities involved in Eq. (1) are all obtained from the GMLT model. In the mixing-length approach, the anisotropy factor, $\Phi$, is a free parameter. We then assume two different values for $\Phi$: $\Phi = 1.3745$ (dot dashed curves) and $\Phi = 2$ (dashed curves). The value $\Phi = 1.3745$ used in the GMLT model provides the best fit between computed solar damping rates and the measured ones by Chaplin et al. (1998).

![Figure 1.](image.png)

Figure 1. Solar $p$-modes excitation rate $P(\nu)$ (See text for details).

Without any adjustment of scaling parameters and instead using all the constraints inferred from the 3D simulation, we find a maximum of $P$ much larger ($\sim 9$ times larger) than is obtained using a 1D GMLT solar model with $\Phi = 1.37$. This difference is mainly due to the assumed value for $\Phi$ in the excitation region. Indeed, analysis of the 3D simulation suggests that $\Phi \sim 2$ in the excitation region. When $\Phi = 2$ is used to compute the excitation rate with the GMLT model, $P$ comes close to $P$ calculated with the 3D simulation constraints.

Our result then shows that the values of $\Phi$ found for the solar GMLT model when adjusted on the damping rates is not compatible with the actual properties of turbulent medium in the excitation region. An improvement could come from a consistent calculation which would
assume a depth dependent $\Phi(m)$, as suggested by the simulation, in both damping and excitation rates computation.

Our calculations using the 3D constraints bring $P_{\text{max}}$, the $P$ maximum, closer to the solar seismic measurements but still under-estimates them by a factor $\sim 2$. More sophisticated assumptions for $k_0(z)$ and $\lambda$ will likely lead to a better agreement with the observations.

4. Scanning the HR diagram

We consider two sets of stellar models previously investigated by Samadi et al. (2001): Models in the first set are computed with the classical MLT whereas those belonging to the the second set are computed with GMLT. We find that the maximum of $P$, $P_{\text{max}}$, scales as $3.2 \log(L/L_\odot \times M_\odot/M)$ for the first set and scales as $3.5 \log(L/L_\odot \times M_\odot/M)$ for the second one ($L$, $L_\odot$, $M$ and $M_\odot$ have their usual meaning). This result suggests that measurements of $P$ in several different intermediate mass stars ($1 \lesssim M \lesssim 2 M_\odot$) will enable one to discriminate between different models of turbulent convection.

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References

Balmforth, N. J.: 1992, ‘Solar pulsational stability - Part three - Acoustical excitation by turbulent convection’. MNRAS 255, 639.

Chaplin, W. J., Y. Elsworth, G. R. Isaak, R. Lines, C. P. McLeod, B. A. Miller, and R. New: 1998, ”Solar p-mode excitation: further insight from recent low-l BiSON helioseismological data“. MNRAS 298, L7–L12.

Goldreich, P. and D. A. Keeley: 1977, ‘Solar seismology. II - The stochastic excitation of the solar p-modes by turbulent convection’. ApJ 212, 243–251.

Gough, D. O.: 1977, ‘Mixing-length theory for pulsating stars’. ApJ 214, 196–213.

Samadi, R. and M. . Goupil: 2001, ‘Excitation of stellar p-modes by turbulent convection. I. Theoretical formulation’. A&A 370, 136–146 (Paper I).

Samadi, R., G. Houdek, M.-J. Goupil, Y. Lebreton, and A. Baglin: 2001, ‘Oscillation power across the HR diagram : sensitivity to the convection treatment’. In: 1st Eddington Workshop: Stellar Structure and Habitable Planet Finding, Vol. ESA SP-485. pp. 87–94 (astro-ph/0109174).

Stein, R. F. and A. Nordlund: 1998, ‘Simulations of Solar Granulation. I. General Properties’. ApJ 499, 914.