Bounded Orbits and Multiple Scroll Coexisting Attractors in a Dual System of Chua System

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ABSTRACT A special three-dimensional chaotic system was proposed in 2016, as a dual system of Chua system, which is satisfied \(a_{12}a_{21} < 0\). The dynamics characteristics are different from the Jerk system \(a_{12}a_{21} = 0\) and Chua system \(a_{12}a_{21} > 0\). In this paper, a method for generating \(M \times N \times L\) grid multiple scroll attractors is presented for this system. Also, in order to ensure the rigor of the theoretical results, we prove existence of the complex scenario of bounded orbits, such as homoclinic and heteroclinic orbits, and illustrate concurrent created and annihilated of symmetric orbits. Then, Shilnikov bifurcation and the possible relationship between the birth and death of the scroll attractors are studied. Furthermore, two theorems are demonstrated for these bounded orbits. Finally, the Lyapunov exponents, bifurcation diagrams, and multiple scroll coexisting attractors are displayed, which are related to the parameters and initial condition.

INDEX TERMS Grid multiple scroll attractors, homoclinic and heteroclinic orbits, Shilnikov bifurcation, coexisting attractors.

I. INTRODUCTION
By and large, the research on chaos, various types of strange attractors, coexisting attractors, synchronization control and their applications have become the hotspots [1]–[5]. Their interesting and complex dynamical phenomena are applied in various fields, for example chaos prediction in nonlinear viscoelastic plates [6], nonlinear resonances and multi-stability in simple neural circuits [7], synchronization control [8], [33], grid multi-scroll chaotic attractors [4], [5], [9], [30], multi-wing hyper chaotic system [10, 30], electronic circuits [11], [13], [22], [24], image encryption application [25, 34], oscillator of memory elements [14], [35], [36], [41], embedded implementation for digital systems [37]–[40] et al. As well as we known, the chaotic application in electronic circuits began Chua circuit and generation of multiple scroll attractors started from Suykens and Vandewalle, beyond the double scroll [1]. Up to now, the main work and research on chaos theory, analysis methods and application fields are as follows: (i) chaotic systems and scroll attractors seem to have reached their maturity, i.e. some scholars and their co-workers proposed plenty of methods for designing the grid multi-scroll and multi-wing chaotic and hyper chaotic attractors [4], [5], [9], [29]–[31], L. Zhou and his fellows proposed a novel no-equilibrium hyper chaotic system [10], J. Kengne et al. presented dynamics for Jerk or Jerk-like system with multiple attractors [11], [12], [19], Q. Lai and S. Chen proposed a polynomial function method for generating multiple strange attractors from the Sprott B system [31], just to name a few; (ii) all kinds of analysis methods have been described on unusual and striking dynamics, such as the, self-excited antimonotonicity [11], [14], [21] and hidden attractors [15], coexisting attractors [16]–[19], [22]–[24], without equilibrium chaotic system [10], [18], [19], and multiple stability [20] and synchronization [8], [33] as new research directions, are still in their infancy. Especially, in \(n\)-scroll chaotic system, the occurrence of two or more asymptotically stable or attracting basin have been analyzed, symmetry and broken symmetry have been reported. For example, J. Kengne and co-workers proposed a novel memristor-based oscillator and analysis chaos, periodic windows, antimonotonicity and crises [11]; Subsequently, the fellows of the this research group, Z.T. Njitacke and co-workers found a autonomous Jerk circuit...
and similar prosperities [12]; L. Zhou and his coworkers observed coexisting attractors and antimonotonicity (named the full Feigenbaum tree) in a flux-controlled memristor [14]; Lai Q et al. proposed a new 4D chaotic system [17], and represented coexisting attractors [22]–[24], dynamic analysis and circuit realization [31], [32]; In coupled systems, multi-stability of infinitely many attractors was also recently reported [7], [20], and so on; (iii) chaotic applications also became interesting and rich. For example, chaos prediction in nonlinear system subjected to subsonic flow and external load was demonstrated by D. Younesian and H. Norouzi [6], chaotic system synchronization and message recovery scheme was proposed by V. Sharma, B.B. Sharma, and R. Nath [8], [33], the new chaotic systems and their electronic implementation were proposed [13], [21]–[24], image encryption based on some chaotic systems were explored [23]–[25], design and implementation based on the ARM-embedded and FPGA for secure video communication system were presented [37]–[40], and so on; (iv) the proof for existence on the complex scenario of bounded orbits [26]–[29] for chaotic and hyper-chaotic systems are in its infancy. It has become a serious attempt and gained a wide attention in academic circles.

In three-dimensional chaotic systems, for the linear part, $A = \{a_{ij}\}_{i,j=1,2,3}$, the most famous Chua system satisfies the condition $a_{12}a_{21} > 0$ while Jerk system satisfies $a_{12}a_{21} = 0$. There are many literatures on analysis and applications for some classical chaotic systems, i.e. Chua system, Jerk circuit, Van der Pol-Duffing circuit and Sprott B system [3]–[5], [12], [15], [16], [19], [21], et al. However, some essential works have been neglected, such as the dual system of Chua system, which is also a special type of a canonical form and proposed in 2016. This system satisfies the condition $a_{12}a_{21} < 0$. Compared with Chua system and Jerk circuit, it has similar but topologically not equivalent dynamic characteristics [5]. As already noted above, for the sake of the completeness and ensure the characteristics for this type of chaotic system, our motivation is focus on exploring a novel method for generating $M \times N \times L$ multiple grid scroll attractors. Also, the proof on existence of the complex scenario of bounded orbits (such as homoclinic and heteroclinic orbits). Furthermore, analysis the more complex and unusual dynamics for the proposed chaotic system. By a new perspective discusses the relationship between the birth and death of the attractors and the mechanism of chaos (i.e. bifurcation, grid scroll attractors and coexisting attractors). This paper is developed in the following sections: the background on the proposed dual system of Chua system is introduced and a novel method for generating $M \times N \times L$ grid multiple scroll attractors is proposed in Section II; the existence of the complex scenario of bounded orbits (i.e. homoclinic and heteroclinic orbits) according to Shilnikov Theorem are proved and two theorems are presented for these bounded orbits in Section III; in Section IV, we also display the domains of parameters space by bifurcation and coexisting scroll attractors; finally, the paper is summarized in Section V.

II. THE THREE-DIMENSIONAL CHAOTIC SYSTEM AND $M \times N \times L$ GRID MULTIPLE SCROLL ATTRACTORS

A. BACKGROUND

In 2016, we proposed a dual system of Chua system, which can be thought a systematic methodology for creating multiple scroll chaotic attractors from a simple three-dimensional system. One of the simplest and dimensionless form is described as follows:

$$\begin{align*}
\frac{dx}{dt} &= -ay \\
\frac{dy}{dt} &= x + by - f(x + z) \\
\frac{dz}{dt} &= dx - cz - (d - c)f(x + z)
\end{align*}$$

(1)

where $x, y, z$ are state variables; $a, b, c, d$ are real parameters and $f(x + z)$ is the nonlinear function, which is the method for constructing $2N+1$-and $2(N+1)$-scrolls, as shown at the bottom of the next page, where $N$, an integer, is the sum of the upper bound. The grid scroll attractors can be displayed simultaneously, as shown in Fig.1.

![FIGURE 1. The grid multiple scroll chaotic attractors: (a) $a = 4$, $b = 0.2$, $c = 5$, $d = 0$, $N = 2$ and initial conditions $(0.1, 0.1, 0.1)$, 5-scroll; (b) $a = 9$, $b = 0.32$, $c = 5.5$, $d = 0$, $N = 2$ and initial conditions $(0.9, 0.9, 0.9)$, 6-scroll.](image)

It can be seen that generation of scroll attractors are only along a certain direction, that is $\{x+z\} = 1$. It is an essential question that whether there are some other methods can obtain multiple scroll attractors along the different directions. In the next subsection, a new way to introduce $M \times N \times L$ grid multiple attractors.

As a simple case of three-dimensional chaotic system (1), which has some insufficient, such as: (i) although the emergence of chaos can be very convincing from computer simulations in this system, there are remain legitimate objections from some critics who demand no less than a rigorous mathematical proof; (ii) according to these parameters, they are not enough to reflect all the complex dynamic characteristics and more details should be required for this type of chaotic systems ($a_{12}a_{21} < 0$); (iii) especially, the parameter $d$ represents the different feature from the other parameters, that is the horizontal direction of the attractors: if $d = 0$, which means they parallel to the $X$-axis in horizontal direction; if $d < 0$, which means the horizontal direction and $X$-axis at obtuse angle; and if $d > 0$, which means the horizontal direction and $X$-axis at acute angle [5]; (iv) the $f(x + z)$ is the key factor to influence the distribution of grid multiple
scroll attractors. In system (1), the attractors are only in one direction, the others directions cannot be realized; (v) as the dual model of Chua system and a new type of canonical direction, the others directions cannot be realized; (v) as the scroll attractors. In system (1), the attractors are only in one direction, the others directions cannot be realized; (v) as the scroll attractors.

### B. THE MODEL AND NOVEL METHOD FOR M x N x L SCROLL ATTRACTIONS

Here, the general mathematical model \( a_{12} \cdot a_{21} < 0 \) for generating \( M \times N \times L \) grid multiple attractors is described as following

\[
\begin{align*}
\frac{dx}{dt} &= -e \left[ x - f(x) \right] - a \left[ y - f(y) \right] \\
\frac{dy}{dt} &= h \left[ x - f(x) \right] + b \left[ y - f(y) \right] \\
\frac{dz}{dt} &= d \left[ x - f(x) \right] - c \left[ z - f(z) \right]
\end{align*}
\]

(3a)

where \( x, y, z \) are state variables; \( a, b, c, d, e, h \) are real parameters and \( f(x), f(y), f(z) \) are the nonlinear \( \text{sign} \) function. It is notice that the \( \text{sign} \) function and five parameters \( (a, b, c, d, e, h) \) are necessary conditions for influencing the location and numbers of scroll attractors, the parameters \( d \) also represents the horizontal direction of the attractors. Next, the five main bifurcation parameters are analyzed in detail.

When \( MeR, NeR, LeR \) are three integers, the nonlinear function \( f(x)/f(y)/f(z) \) represents the novel methods and \( M \times N \times L \) (named three directions) grid multiple scroll attractors, which can be given:

\[
f(x) = \begin{cases} 
2N + 1 - \text{Scroll} & \text{if} \ A \sum_{n=1}^{N} \left[ \text{sgn} \left( x + 2nA \right) \right] \\
2(N + 1) - \text{Scroll} & \text{if} \ A \sum_{n=1}^{N} \left[ \text{sgn} \left( x + (2n-1)A \right) + \text{sgn} \left( x - (2n-1)A \right) \right]
\end{cases}
\]

(3b)

When \( A = 1, a = 2, b = 0.85, c = 0.6, d = 1.5, e = 0.6, h = 0.1 \), initial conditions \((0.12, 0.12, 0.12)\), we rewritten the system (3) and \( 6 \times 5 \times 6 \) scrolls can be shown in Fig.2.

\[
\begin{align*}
\frac{dx}{dt} &= -0.6 \times \left[ x - \text{sgn} \left( x + 1 \right) - \text{sgn} \left( x - 1 \right) \right] -\text{sgn} \left( x + 3 \right) - \text{sgn} \left( x - 3 \right) \\
&\quad -2 \times \left[ y - \text{sgn} \left( y + 1 \right) - \text{sgn} \left( y - 1 \right) \right] -\text{sgn} \left( y + 4 \right) - \text{sgn} \left( y - 4 \right) \\
\frac{dy}{dt} &= 0.1 \times \left[ x - \text{sgn} \left( x + 1 \right) - \text{sgn} \left( x - 1 \right) \right] -\text{sgn} \left( x + 3 \right) - \text{sgn} \left( x - 3 \right) \\
&\quad +0.85 \times \left[ y - \text{sgn} \left( y + 1 \right) - \text{sgn} \left( y - 1 \right) \right] -\text{sgn} \left( y + 4 \right) - \text{sgn} \left( y - 4 \right) \\
\frac{dz}{dt} &= 1.5 \times \left[ x - \text{sgn} \left( x + 1 \right) - \text{sgn} \left( x - 1 \right) \right] -\text{sgn} \left( x + 3 \right) - \text{sgn} \left( x - 3 \right) \\
&\quad -0.6 \times \left[ z - \text{sgn} \left( z + 1 \right) - \text{sgn} \left( z - 1 \right) \right] -\text{sgn} \left( z + 3 \right) - \text{sgn} \left( z - 3 \right)
\end{align*}
\]

(4)

Notably that it is difficult to obtain the same results as the proposed method (i.e. nonlinear \( \text{sign} \) function) due to the particularity of system (3), when we choose the other
methods (i.e. sine function, piecewise function, hyperbolic function et al.). Also, this method can be easily implemented by the circuit. Then, it is enough to reflect all the complex dynamic characteristics of this type of chaotic systems and more details can be represented. We have done a lot of experiments to verify it. The most important is that the proposed method is related to prove existence of the complex scenario of bounded orbits and the geometric structure.

It can be obviously observed that the symmetry about the origin \((P_0(0,0,0))\) is shown in system (3). It means that if \((x, y, z)\) is a solution and \((-x, -y, -z)\) is also a solution. Also, the equilibrium points are including \(P_0(0,0,0)\), which is a trivial symmetric static solution, and many symmetric nonzero equilibriums \(P_{\pm}\). Thus, multiple scroll attractors in phase space are symmetric and occur in pairs. This symmetry may be explain the existence of coexisting scroll attractors in state space. The symmetric equilibrium points, eigenvalues and eigenvectors are shown in Table1.

### Table 1. Numbers, equilibrium, eigenvalues and corresponding eigenvectors of scroll attractors.

| M/N/L | \(f(x)\) | Number | Equilibriums | Eigenvalues and corresponding eigenvectors |
|-------|-----------|--------|-------------|-------------------------------------------|
| 0/0/0 | -         | 1      | (0,0,0)     | \(\lambda_1=-0.6, \lambda_2=-0.125 + 0.156i, \lambda_3=-0.125 + 0.156i; \) |
|       |           |        | (-1,-1,-1), | \(\zeta_1=[0,0,0]^T, \) |
|       |           |        | (-1,0,0),   | \(0.1245i, 0.1176 - 0.0125i, 0.7976)^T, \) |
| 1/1/1 | \(f_1, f_2, f_3\) | 3x3x3 | (1,1,0),    | \(\lambda_1=0.5782 + 0.075, \lambda_2=0.075, \lambda_3=-0.075 + 0.075i; \) |
|       | \(f_1, f_2, f_3\) |       | (1,1,1)     | \(\zeta_1=[0,0,0]^T, \) |
|       | \(f_1, f_2, f_3\) |       | (...)       | \(0.1245i, 0.1176 + 0.0125i, 0.7976)^T, \) |
| 1/2/1 | \(f_1, f_2, f_3\) | 3x4x5 | (1,1,1),    | \(\lambda_1=0.125 - 0.4737i, \lambda_2=-0.4, \lambda_3=-0.22 + 0.2358, \lambda_4=-0.22 - 0.2358, \) |
|       | \(f_1, f_2, f_3\) |       | (1,2,1)     | \(\zeta_1=[0,0,0]^T, \) |
|       | \(f_1, f_2, f_3\) |       | (1,1,1),    | \(0.2854i, 0.1371 + 0.0285i, -0.9039)^T, \) |
|       | \(f_1, f_2, f_3\) |       | (...)       | \(0.2854i, 0.1371 - 0.0285i, -0.9039)^T, \) |
| 2/2/2 | \(f_1, f_2, f_3\) | 6x5x6 | (0,0,0),    | \(\lambda_1=0.03765 + 0.1432i, \lambda_2=-0.0884 + 0.0172i, \lambda_3=0.9108)^T, \) |
|       | \(f_1, f_2, f_3\) |       | (2,2,2),    | \(\zeta_1=[0,0,0]^T, \) |
|       | \(f_1, f_2, f_3\) |       | (2,2,3)     | \(0.1432i, -0.0884 + 0.0172i, 0.9108)^T, \) |

Actually, the dynamics of the system is classified via its Lyapunov Exponents (LEs). The related detail are given in Table2. Also, the sign of the largest \(LE_{\text{max}}\) determines the relationship between all small perturbations and the state of the system.

When \(A = 1, a = 2.5, b = 1.8, c = 0.8, d = 1.5, e = 0.6, h = 0.4, N = M = L = 1\), initial conditions \((0.06, 0.06, 0.06)\), the system generates \(4 \times 3 \times 3\) scroll attractors and the time domain graph can be shown in Fig.3., the system can be given as follows:

\[
\begin{aligned}
\frac{dx}{dt} &= -0.6 \times [x - \text{sgn}(x) - \text{sgn}(x + 2) - \text{sgn}(x - 2)] \\
&\quad -2.5 \times [y - \text{sgn}(y + 1) - \text{sgn}(y - 1)] \\
\frac{dy}{dt} &= 0.4 \times [x - \text{sgn}(x + 2) - \text{sgn}(x - 2)] \\
&\quad + 1.8 \times [y - \text{sgn}(y + 1) - \text{sgn}(y - 1)] \\
\frac{dz}{dt} &= 1.5 \times [x - \text{sgn}(x + 2) - \text{sgn}(x - 2)] \\
&\quad - 0.8 \times [z - \text{sgn}(z + 1) - \text{sgn}(z - 1)]
\end{aligned}
\]

It can be seen from the above figure that when the parameters and initial conditions are set, if a limited range is given, the phase trajectory and time domain conditions can be clearly observed. The LEs spectrum is shown in Fig.4.

The LEs of system (3) are \(LE_1 = 2.17, LE_2 = 0, LE_3 = -0.01\). Based on the LEs, the Hausdorff dimension is also calculated \(DL = 1.39\). Within the time interval, this system shows the chaotic behavior quickly.

### III. HOMOCLINIC BIFURCATION AND HETEROLINIC ORBITS

#### A. THE GEOMETRIC STRUCTURE

Firstly, we extract the essential properties of the vector field associated with system (3) to define a generalized family of vector fields \(U\), which is the separatrix plane for each region.
but not parallel to each eigenspace and does not through the origin. Let vectors in $\mathbb{R}^3$ be denoted by $X = (x, y, z)^T \in \mathbb{R}^3$. Real and imaginary parts of a complex eigenvalue are given by $\sigma$ and $\omega$, respectively. Real eigenvalues is denoted by $\gamma$. Take the direction of scroll attractors is $|x + z| = 1$ as an example, the following conclusion can be obtained.

(i) nonlinear vector fields ($f(x)$) are denoted by $S: \mathbb{R}^3 \rightarrow \mathbb{R}^3$. That means the $S(X)$ can be denoted the vector field evaluated at $X$ and emanating always from the origin $P_0$, and symmetric with respect to $P_0$;

(ii) when the system presents three equilibrium points $(P_{+1}, P_0, P_{-1})$, the planes $U_{\pm 1} = \{(x, y, z) \mid |(x, z)| = \pm 1\}$ are used to define three domains, as shown in Fig. 4:

\[
\begin{align*}
P_+ &= (1, 0, 1) \in D_1 = \{(x, y, z) \mid (x, z) > 1\} \\
P_0 &= (0, 0, 0) \in D_0 = \{-1 \leq (x, y, z) \mid (x, z) \leq 1\} \\
P_- &= (-1, 0, -1) \in D_{-1} = \{(x, y, z) \mid (x, z) < -1\}
\end{align*}
\]

(iii) three equilibrium points, one is located at the origin $P_0$, one in the interior of $D_{+1}$ (labeled $P_+$) and the other in the interior of $D_{-1}$ (labeled $P_-$);

(iv) the eigenspace associated with either the real or the complex eigenvalue at each equilibrium point is not parallel to $U_{+1}$ or $U_{-1}$. For each eigenspace, and define:

$E_r(0) \triangleq 2 - D$ eigenspace corresponding to complex eigenvalue at the origin, and the attractor is located in the plane;

$E_r(0) \triangleq 1 - D$ eigenspace corresponding to real eigenvalue at the origin;

$E_r(P_+) \triangleq 2 - D$ eigenspace corresponding to complex eigenvalue at $P_+$, and the attractor is located in the plane;

$E_r(P_+) \triangleq 1 - D$ eigenspace corresponding to real eigenvalue at $P_+$;

$E_r(P_-) \triangleq 2 - D$ eigenspace corresponding to complex eigenvalue at $P_-$, and the attractor is located in the plane;

$E_r(P_-) \triangleq 1 - D$ eigenspace corresponding to real eigenvalue at $P_-;

\]

In Fig.5, we have defined the following important points:

\[
\begin{align*}
A_{\pm} &= L_0 \cap L_2; B_{\pm} = L_1 \cap L_2; C_{\pm} = E_r(0) \cap U_{\pm 1} \cap S_{\pm 1}; \\
F_{\pm} &= \{x \in L_2 \mid D_{\pm} = E_r(P_{\pm}) \cap U_{\pm 1} : P_{\pm} = E_r(P_{\pm}) \cap S_{\pm 1}\}.
\end{align*}
\]

They can prove that no two lines among $L_0, L_1, L_2, L_3, L_4$ are parallel to each other, except $L_0//L_1$. It is different from the Chua circuit [26]. The existence and significance of relevant important points and straight lines have been proved in [27]. Because of the dual relationship between the Chua system and system (3). We do not need to discuss the common properties between them but the differences via the saddle focus point. Our next goal is to study the possible relationship between the birth (and death) of the attractors and homoclinic bifurcations (homoclinic orbits).

B. HOMOCLINIC ORBIT

According to Shilnikov theorem, it is assumed that there are some homoclinic orbits from $P_0$ to a saddle focus point. The presence of a saddle-focus separatrix (i.e. a homoclinic curve in Poincare’s terminology) plays a fundamental role in the onset and disappears of chaotic behavior. To observe this type of trajectory, two conditions should be sufficient: a separatrix loop and Shilnikov’s saddle-focus.

The following conditions can be obtained: $a > 1.3 & 0 < b < e + c & c \geq 0.5 & e \geq 0.5 & 0 < h < 0.5$, according to the dissipativity and Jacobin matrix of system (3) in order to ensure that an equilibrium point is a hyperbolic saddle focus (or simple, saddle focus). Next, some results can be established and proved:

**Theorem 1.** As a three-dimensional chaotic system, which satisfies the condition $a_{1,2}a_{21} < 0$ and can generate multiple scroll attractors at equilibrium points, $P_n \ (n = 0, 1, 2, \ldots, N, N$ is integer), there are a real eigenvalue $\lambda_3 = \gamma > 0$ and a pair of complex eigenvalues.

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**FIGURE 4.** Lyapunov exponents spectrum.

**FIGURE 5.** Eigenspaces of the equilibria.
of complex eigenvalues $\lambda_{1,2} = \sigma \pm iw$ with $\sigma < 0$, $\omega \neq 0$ at each equilibrium. Suppose the following:

(i) The equilibrium point $P_0$ is a hyperbolic saddle focus;

(ii) At the $P_0$, the system has a homoclinic orbit $H$, and if $|\sigma|/\gamma < 1$, the flow can be perturbed into a flow with a countable set of horseshoes.

Proof: Let $p(t) = (x(t), y(t), z(t))$ be the solution of system (3), that

$$\lim_{t \to +\infty} p(t) = p_+ \quad \text{and} \quad \lim_{t \to -\infty} p(t) = p_- \quad (7)$$

where $p_+ = p_- = \{p_+, p_-\}$ or $p_+ = p_- = \{p_+, p_0, p_-\}$.

Assume that the system (3) has none of homoclinic orbits joining $P_+$ and $P_-$, that is $P_+ \neq P_- \neq P_0$ as $t \in \{+\infty, -\infty\}$.

Denote $p_{0+} R^3$ is the initial point and $p(t; p_0) = \{(x(t; p_0), y(t; p_0), z(t; p_0))\}$ is the solution. When $t = 0$, $p(t; p_0) = \{x(t; p_0), y(t; p_0), z(t; p_0)\}$, the following equation can be given

$$DL = \frac{dL(x(t), y(t), z(t))}{dt} \big|_{t=0} = \left[(x(t) \cdot \dot{x}(t) + y(t) \cdot \dot{y}(t) + z(t) \cdot \dot{z}(t)) \right]_{t=0} = \left[-ex^2(t) + by^2(t) - cz^2(t)\right]_{t=0} \leq c\left[y^2(t) - z^2(t)\right]_{t=0} + \left[ex^2(t)\right]_{t=0} = 0 \quad (8)$$

When $c \neq 0$ and $e \neq 0$ and $t = 0$, we notice that $DL = 0$. That is $P_+ = P_- = P_0$, it is a contradiction! Therefore, system (3) has the homoclinic orbits. Then, Theorem 1 is proved.

There are three kinds of motion states at each equilibrium, which are the stable state, the critical state, and the unstable state. Taking $N = 1$, 3-scroll attractors in $x$-direction as an example, which is depicted in Fig.6. when the equilibrium point is the saddle focus, the unstable state will evolve into chaos (see Theorem 1). When $a = 2.2$, $c = 3.1$, $d = 2$, $e = 0$, $h = 1$, by changing the value of $b$: (1) $b < 0$, the only stable regimes are the equilibrium $P_0$ (Fig.6.(a) and (b)); (2) as $b$ increases, the equilibrium lost stability at the critical, until $b = 0$, and the periodic orbits appear (Fig.6.(c) and (d)); (3) $b > 0$ increases further, the system undergoes a saddle-node bifurcation, which leads to the appearance of unstable symmetric periodic orbits (Fig.6.(e) and (f)). Therefore, the bifurcation would occur when there are a pair of purely imaginary complex eigenvalues $\lambda_{1,2} = \sigma \pm iw$ with $\sigma = 0$, $\omega \neq 0$ at the origin $P_0$ in the system (3), that is

$$\lambda_1 + \lambda_2 = 0 \quad (9)$$

This is a only critical condition in combination with a homoclinic bifurcation. Also, $b > 0$ is the bifurcation condition.

Consider changing other parameters, which do not appear to be the chaotic transition like Fig.6. (i.e. “stable — critical — unstable”, see Fig.7~12). Because of that the others ($P_{\pm}$) are not a saddle focus, we don’t need to analyze in detail. According to Theorem 1, the equilibrium $P_0$ of a vector field is thought a homoclinic point if there exists a trajectory which tends to $P_0$ as $t \to +\infty$ and as $t \to -\infty$ corresponding to the positive engenvalue $\lambda_3 > 0.5$. When the eigenvalues $\lambda_{1,2}$ are complex conjugate, the homoclinic trajectory spirals along the eigenplane corresponding to $\lambda_{1,2}$. If there are some parameters to make the trajectory along the unstable real eigenvector from $P_0$ across the stable eigenspace and return to the origin, and symmetry, it also along the other unstable real eigenvector would behave in the same way. This process is called a homoclinic bifurcation associated with the saddle-focus $P_0$. Also, this trajectory is called a homoclinic orbit. The existence of a homoclinic orbit is critical to analyze the sensitivity of initial conditions. As a saddle-focus fixed point, the meaning of the Shilnikov homoclinic orbit is that a joint of the stable and unstable manifolds, and implies that there is a horseshoe in the neighborhood of this orbit. Therefore, this class of system is chaos.

The simulation (see Fig.6) consistently show that as $b$ increases while fixing other parameters, the scroll attractors collide with each other and eventually emerges after $b > 0$. This collision process leads to the “birth of the scroll attractors”. Based on Shilnikov Theorem, the keys are to localize the homoclinic orbits in the parameter space and determine the structure of this chaotic system via these orbits. It indicates that the behavior of chaos and bifurcations may
help us to understand the relationship between Shilnikov homoclinic orbits and scroll attractors. However, homoclinic orbits are not structurally stable: they disappear under small perturbations of the vector field. Also, the saddle cycle itself does not undergo any bifurcations.

C. HETEROCLINIC ORBIT

The heteroclinic orbit is the other kind of trajectory that could connect an equilibrium point or a closed orbit to another equilibrium point or closed orbit. Also, the existence of them is also assumed in Shilnikov theorem. This research is not only an essential but also a difficult task for this paper. In this subsection we give a new insight for the existence of this orbit in system (3). Summarizing the above analysis and discussion the following results could be given:

**Theorem 2:** As a three-dimensional chaotic system, which satisfies $a_1^2 a_2 < 0$ and the equilibriums are as in Theorem 1. Let $P_{2m}$ and $P_{2m+1}$ $(m = 1, 2, \ldots, N)$ be two distinct equilibriums, which are origin-symmetric. Both $P_{2m}$ and $P_{2m+1}$ are saddle focus that satisfy the Shilnikov inequality: $|\gamma_m| > |\sigma_m| > 0 (i = 1, 2)$ with the further constraint $\gamma_m \sigma_n > 0 or \sigma_m \gamma_n > 0$. This system exists a heteroclinic loop, $H_1$, which joins $P_{2m}$ to $P_{2m+1}$ and makes up of two heteroclinic orbits $H_{mi}$ $(i = 1, 2)$. Both the system and its perturbed would exhibit the Smale horseshoe chaos.

_Proof:_ Similar Theorem 1, there is a function $p(t)$ and assume that the system (3) has none of heteroclinic orbits to joining $P_+$ and $P_-$, that is $P_+ = P_- = O$ as $t \to +\infty$ and as $t \to -\infty$, and $DL|_{t \neq 0} = 0$.

Let $p_+(t) = \{x_+(t), y_+(t), z_+(t)\}$ in $E_C(P_+)$ and $p_-(t) = \{x_-(t), y_-(t), z_-(t)\}$ in $E_C(P_-)$. They are symmetrical to each other. The following equation can be obtained

$$DL = \left. \frac{dL(x(t), y(t), z(t))}{dt} \right|_{t \neq 0} = x(t) \dot{x}(t) + y(t) \dot{y}(t) + z(t) \dot{z}(t) = -ex^2 (t) + by^2 (t) - cz^2 (t) \leq \left| c \left( y^2 (t) - z^2 (t) \right) \right| + \left| ex^2 (t) \right|$$

\[ (10) \]
When \( c \neq 0 \) \& \( e \neq 0 \) \& \( t \neq 0 \), we get \( DL \neq 0 \) also means \( P_+ \neq P_- \neq 0 \). It is a contradiction! Therefore, there are heteroclinic orbits for system (3). Then, Theorem 2 is thus proved.

According to Theorem 2, we can observe that bifurcation of suitable heteroclinic orbits would get the same features as do bifurcations of homoclinic orbits. It means that a homoclinic orbit is forwards and backwards asymptotic to the same equilibrium \( P_0 \), while a heteroclinic orbit is a generalization of a saddle loop.

IV. BIFURCATION AND MULTIPLE SCROLL COEXISTING ATTRACTIONS

To investigate the sensitivity of initial condition \((x_0, y_0, z_0)\) with respect to a single control parameter, we fix \( A \), only vary one parameter in the certain range and setting initial conditions \((0.1, 0.1, 0.1)\) (blue solid curve) and \((-0.1, -0.1, -0.1)\) (red solid curve), respectively. Bifurcation diagrams can be obtained in Fig. 7~12. Let \( M = 3, N = 2, L = 3 \).

Case 1: \( A = 1, b = 0.5, c = 0.85, e = 0.6, h = 0.1, d = 1.5 \) and \( ae[0.7] \), the bifurcation diagrams are given in Fig. 7. Parameters \( a \) would be considered in the following:

(1) \( ae[0.7] \), blue solid curve

Along with increase of \( a \) in the interval \([0, 0.1581]\), there is no chaotic behavior and scroll attractors. Then, the orbits from stable to chaos, and to multiple scroll attractors can be observed successively, in the interval \([0.1581, 7]\). Furthermore, three main periodic windows with coexisting scroll-1 to scroll-6 \((1 < n < 6)\) and to scroll-6 are obtained, in the intervals \([0.581, 2.07), [2.07, 2.585)\) and \([2.585, 7]\), respectively.

(2) \( ae[0.7] \), red solid curve

When \( a \) is increasing in the interval \([0, 0.1581]\), there is the similar state. Also, in the interval \([0.1581, 7]\), the whole movement process can be described from stable to chaos, and to multiple scroll attractors. Three main periodic windows with coexisting are scroll-1 to scroll-6 \((1 < n < 6)\) and to scroll-6, in the intervals \([0.581, 2.861), [2.861, 4.498)\) and \([4.498, 7]\), respectively.

We can also obtain the following results from below bifurcation diagrams: (i) when the initial values and parameters are determined, after a period of time, the final scroll attractors state conforms to the simulation analysis (such as Fig. 2); (ii) with the same conditions but the shorter time, multi-scroll attractors can also be obtained, but the number of number is less than the analysis results, which is not a final state and dangerous. Once it is applied to the engineering, it will lead to the wrong effective or immeasurable losses; (iii) the different number scrolls could be presented and identified hysteretic dynamics (imply multiple stability), which have previously observed in some chaotic system [21], [26].
When $a = 1$, $b = 0.5$, $c = 0.85$, $d = 1.5$, $h = 0.1$, $e = 0.6$, the coexisting scroll-1 attractors are given in Fig.8. Others coexisting scrolls can be shown later (see Fig 13).

It can be seen from above graph that coexisting scroll attractor and the relationship between the number of attractors and initial condition are described clearly. Also, the equilibrium point is also related to the proposed method (i.e. sign function) and initial condition, but nothing to do with the parameters.

Case 2: $A = 1$, $a = 3$, $c = 0.85$, $e = 0.6$, $h = 0.1$, $d = 1.5$ when $be[0,8]$, the bifurcation diagrams are given in Fig.9:

(1) $be[0,8]$, blue solid curve

The orbits occurrence chaotic phenomenon and grid scroll attractors. Three main periodic windows with coexisting are scroll-1 to scroll-2 and to scroll-5, in the intervals $[0,0.5)$, $[0.5,0.5953)$ and $[0.5953, 8)$, respectively.

(2) $be[0,8]$, red solid curve

Along with increase of $b$ in the interval $[0,1.5]$ by local enlargement, the orbits occurrence chaos and multiple scroll attractors can be observed. Then, three main periodic windows with coexisting scroll-1 to scroll-3 and to scroll-5 can be depicted, in the intervals $[0, 0.3344)$, $[0.3344, 0.388)$ and $[0.388, 0.8)$, respectively.

Case 3: $A = 1$, $a = 6$, $b = 0.35$, $e = 0.6$, $h = 0.1$, $d = 1.5$ when $ce[0,0.8]$, the bifurcation diagrams are given in Fig.10.

Parameters $c$ will be considered in the following:

(1) $ce[0,0.8]$, blue solid curve

Three main periodic windows with multiple coexisting are scroll-6 to scroll-1 and to scroll-5, in the intervals $[0, 0.5458)$, $[0.5458, 0.5619)$ and $[0.5619, 0.8)$, respectively.

(2) $ce[0,0.8]$, red solid curve

when $c$ is increasing in the interval $[0,0.4]$ by local enlargement, three main periodic windows with coexisting are scroll-6 to scroll-3 and to scroll-1, in the intervals $[0, 0.3023)$, $[0.3023, 0.3157)$ and $[0.3157, 0.8)$, respectively.

Case 4: $A = 1$, $a = 2$, $b = 0.5$, $c = 0.75$, $e = 0.2$, $d = 0.5$ when $ee[0,0.5]$, the plots are shown in Fig.11.

(1) $ee[0,0.5]$, blue solid curve

The orbits and attractors would be generated. There are three main periodic windows with coexisting scroll-6 to scroll-1 and to scroll-5, in the intervals $[0, 0.202)$, $[0.202, 0.214)$ and $[0.214, 0.5)$, respectively.

(2) $ee[0,0.5]$, red solid curve

The local enlargement can be seen in the interval $[0,0.4]$ Three main periodic windows with coexisting scroll-6 to scroll-1 can be observed, in the intervals $[0, 0.1753)$ and $[0.1753, 0.4)$, respectively.

Case 5: $A = 1$, $a = 2$, $b = 0.5$, $c = 0.75$, $e = 0.6$, $d = 0.5$ when $he[0,1.4]$, the bifurcation diagrams are depicted in Fig.12. Parameters $e$ will be considered in the following:

(1) $he[0, 1]$, blue solid curve

The orbits occurrence chaotic behaviors, multiple scroll attractors and three main periodic windows with coexisting scroll-6 to scroll-5 to scroll-3 and to scroll-1 are described in the intervals $[0, 0.7391)$, $[0.7391, 0.8227)$, $[0.8227, 0.8662)$ and $[0.8662, 1]$, respectively.

(2) $he[0,1]$, red solid curve

When $h$ in the interval $[0,0.6]$, we can observe two main periodic windows.
From bifurcation diagrams [see Fig.7(b,e), Fig.9(c, f), Fig.10(c, f), Fig.11(b, c, e, f), and Fig.12(b, c, e, f)], we further analysis carried out above could occur different number scrolls and identify hysteretic dynamics (imply multiple stability), when monitoring five parameters. In this paper, when parameters of system (3) within a certain range, the long term behavior depends crucially on initial conditions \((x_0, y_0, z_0)\) and parameters. Thus, the interesting and complex dynamics behavior of chaotic attractors can be clearly demonstrated. It ought to be mentioned that, the phenomena of multiple stability have previously observed in Chua system, Jerk circuit, Lorenz-like system [27], etc. These systems are also applied in many engineering fields [28]. However, few research on multiple scroll chaotic system before, except [21].

Next, when the parameters \((a, b, c, d, e, h) = (a, 0.5, 0.85, 1.5, 0.6, 0.1)\), the multiple scroll coexisting attractors are list in Fig.13. It can be seen that the distribution of scroll attractors in below diagram completely conform to the description of the bifurcation plots in Fig.7.

It is well known that as one of the most fundamental bifurcation processes and applications in nonlinear systems, periodic orbits for a chaotic systems can be created and annihilated, this phenomenon can be named antimonotonicity [21]. It has been observed in many chaotic system, i.e. Jerk circuits, Chua system, Lorenz-like system, and so on. In this paper, it is necessary to form periodic islands in the parameter space, for developing forward and reversed period-doubling bifurcations (see Fig.7~12). The system (3) appears the chaotic transition “stable-critical-unstable” or “unstable-critical-stable”. A stable symmetric periodic orbit possible around each equilibrium (i.e. P + and P−) and persist in a certain (rather small and unstable) interval corresponding to the “periodic-n window” . That means when the parameters vary in a certain range, some discrete values and scrolls can be obtained. Also, antimonotonicity and reverse period-doubling scenarios can occur [11].

According to the above analysis on the Chua system’s dual system, we find that it has more complexity than other systems (no matter that the three-dimensional first-order or second-order differential equations.). It means that this type of system has wide potential application value, for example, the memristive system, chaotic secure communication and synchronization control, image encryption algorithm and so on, which are the focus of our next step, including the entropy.

V. CONCLUSION
The research on analysis and application for Chua system \((a_{12}a_{21}) > 0\) seem to have reached their maturity. However, as a dual chaotic system and a special canonical model, these studies have been neglected to the system (3), which satisfies \(a_{12}a_{21} < 0\) and has similar but topologically not equivalent dynamic characteristics. Therefore, it is important to supplement them. Also, in most nonlinear dynamical systems, although the rich and interesting varieties of chaotic phenomena can be observed and convincing from computer simulations. It is remain necessary to find a rigorous mathematical method to prove them in order to deal with the legitimate objections from their critics. By exploiting classical nonlinear analysis tools such as Silnikov theorem, the saddle focus point, bifurcation diagrams, LEs, and phase space trajectories, the existence of chaotic behaviors and dynamics of proposed system has been characterized with respect to its parameters.

As the major results of this work, a method for generating \(M \times N \times L\) grid multiple scroll attractors is proposed, two theorems are demonstrated for the bounded orbits (i.e. homoclinic and heteroclinic orbits), bifurcation and multiple scroll are also given. The analyzed chaotic system in this paper represents one of the important three-dimensional nonlinear systems that few reported to date and had been not found in other literature before. Furthermore, a very good agreement is observed between theoretical and simulations and it would play a potential role in chaotic dynamic analysis and some engineering applications of chaos theory in future.

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