Defect formation in inhomogeneous 2-nd order phase transition: theory and experiment.

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The status quo in our understanding of defect formation during a rapid transition into the broken symmetry state in condensed matter and cosmology is discussed. An observation of vortex nucleation in neutron absorption experiments in superfluid $^3$He-B is interpreted in terms of defect formation during inhomogeneous cooling through $T_c$. Due to the temperature gradient in the locally heated region the superfluid phase transition occurs as a propagating front. The theoretical considerations of vortex formation at the propagating front are based on work by Kibble-Volovik, Kopnin-Thuneberg, and Aranson-Kopnin-Vinokur (AKV).

I. INTRODUCTION

To produce a new vortex line in the vortex-free state of superfluid liquid is not an easy job. If the container is devoid of the remnant vorticity, which can be pinned by rough surface, the vortices are created only when a threshold $v_c$ for the hydrodynamic instability of the superflow is reached. The thermal activation or quantum tunneling can assist the nucleation only in the narrow vicinity of the instability threshold, where the external perturbations, however, are more effective. In superfluid $^3$He-B, because of the large size $r_c$ of the vortex core, the region near the threshold, where thermal activation or quantum tunneling can be important, is particularly small, $v_c - v_n \sim 10^{-6}v_c$. In a typical cylindrical container with radius $R = 2.5$ mm and height $L = 7$ mm, rotating with angular velocity $\Omega = 3$ rad/s, the vortex-free state stores a huge amount of kinetic energy $(1/2) \int dV \rho_s(v_s - v_n)^2 \sim 10$ GeV. This energy cannot be released, since the intrinsic half period of the decay of this metastable state is essentially larger than the proton life time.

That cosmic rays can assist in releasing this energy by producing vortex rings, I first heard from my supervisor, professor Iordanskii, in 1972. The natural scenario for that was thought as depicted in Fig. 1. The energetic particle heats a region above the superfluid transition temperature $T_c$. During the cooling the normal liquid in this region can continuously evolve to form the core of a vortex loop, which starts growing if the radius $R_b$ of the heated region is larger than the radius of the ring sustained by counterflow, i.e. $|v_s - v_n| > v_{c1} = (\kappa/4\pi R_b) \ln (R_b/r_c)$, where $\kappa$ is the quantum of circulation around the vortex.

If the counterflow essentially exceeds this threshold, the evolution, which is most favourable for vortex production, leads to the closely packed vortex rings of the critical size, which can further develop. This gives the following estimation for maximal number of vortex loops, which can grow further: $N \sim (|v_s - v_n|/v_{c1})^3$.

Experiments with irradiated superfluid $^3$He were started in 1992 in Stanford, where it was found that the irradiation assists the transition of supercooled $^3$He-A to $^3$He-B. In 1994 the neutron irradiation of $^3$He-B was found to produce a shower of quasiparticles in Lancaster and vortices in rotating $^3$He-B in Helsinki. Energy deficit found in low-$T$ Grenoble experiments indicated possible formation of vortices in $^3$He-B even without rotation. In Helsinki the observed number of vortices produced per one event showed both the threshold behavior and the cubic dependence at large rotation velocity: Above the threshold it was well approximated by
$N \sim (v_s - v_n/v_{c1})^2 - 1$. This indicated that the nature has chosen some scenario, which produces the maximal possible number of vortices. What is the reason for that?

The decay products from the neutron absorption reaction generate ionization tracks, the details of which are not well known in liquid $^3$He. At the moment we have two working scenarios of thermalization of the energetic particles:

(i) The mean free path is long and increases with decreasing of the energy. This can lead to a “Baked Alaska” effect, as has been described by Leggett \[9\]. A thin shell of the radiated high energy particles expands with the Fermi velocity $v_F$, leaving behind a region at reduced $T$. In this region, which is isolated from the outside world by a warmer shell, a new phase can be formed. Such Baked Alaska mechanism for generation of new phase has also been discussed in high energy physics, where it describes the result of a hadron-hadron collision. In this relativistic case the thin shell of energetic particles expands with the speed of light. In the region separated from the exterior vacuum by the hot shell a false vacuum with a chiral condensate can be formed \[9\]. This scenario provides possible explanation of formation of the B-phase in the supercooled A-phase \[9\].

(ii) During thermalization the mean free path is less than the dimension of the region where the energy is deposited and the temperature is well determined during the phase transition through $T_c$. In this case there is no Baked Alaska effect: no hot shell separating the interior region from the exterior. So the exterior region can effectively fix the phase in the cooled bubble, suppressing the formation of the vacuum states, which would be different from that in the bulk liquid. Due to this proximity effect the formation of vortices can be also suppressed.

In both cases of monotonic and nonmonotonic temperature profile, two mechanisms of the vortex formation are important:

(a) The Kibble-Zurek (KZ) mechanism of the defect formation during the quench. For the scenario (ii), where the interior region is not separated from the exterior by the warmer shell, the KZ mechanism is to be modified to include spatial inhomegeneity, which leads to the moving transition front. The proximity effect of the exterior region is not effective if the phase transition front moves sufficiently rapidly \[8\]. The modified KZ mechanism is not sensitive to the existence of the external counterflow, which only role is to extract the formed vortices from the bubble. The same KZ mechanism could be responsible for the formation of the A-B interfaces, which provides another scenario of the B-phase nucleation in the supercooled A-phase \[8\].

(b) Instability of the normal-superfluid interface, which occurs in the presence of the counterflow \[12\]. Here we discuss these two mechanisms (a) and (b) of vortex formation during inhomogeneous quench as manifested in numerical simulations \[12\].

II. KZ SCENARIO IN PESENCE OF PLANAR FRONT.

For a rough understanding of the KZ scenario of vortex formation let us consider the time-dependent Ginzburg-Landau (TDGL) equation for the one-component order parameter (OP) $\Psi = \Delta / \Delta_0$:

$$\tau_0 \frac{\partial \Psi}{\partial t} = \left(1 - \frac{T(r, t)}{T_c}\right) \Psi - \frac{\Psi}{\tau} + \xi_0^2 \nabla^2 \Psi . \quad (1)$$

Here $\tau_0 \sim 1/\Delta_0$ and $\xi_0$ are correspondingly the relaxation time of the OP and the coherence length far from $T_c$.

If the quench occurs homogeneously in the whole space $r$, the temperature depends only on one parameter, the quench time $\tau_Q$:

$$T(t) \approx \left(1 - \frac{t}{\tau_Q}\right) T_c . \quad (2)$$

In the presence of a temperature gradient, say, along $x$, a new parameter appears:

$$T(x - ut) \approx \left(1 - \frac{t - x/u}{\tau_Q}\right) T_c . \quad (3)$$

Here $u$ is the velocity of the temperature front which is related to the temperature gradient

$$\nabla_x T = \frac{T_c}{u \tau_Q} . \quad (4)$$

There exists a characteristic critical velocity $u_c$ of the propagating temperature front. At $u \geq u_c$ the vortices are formed, while at $u < u_c$ the defect formation is either strongly suppressed \[13\] or completely stops \[7\].

At slow velocities, $u \to 0$, the order parameter almost follows the transition temperature front:

$$|\Psi(x, t)|^2 = \left(1 - \frac{T(x - ut)}{T_c}\right) , \quad T < T_c . \quad (5)$$

In this case the phase coherence is preserved behind the transition front and thus no defect formation is possible.

The extreme case of large velocity of the temperature front, $u \to \infty$, corresponds to the homogeneous quench. As was found by Kopnin and Thuneberg \[8\], if $u$ is large enough, the phase transition front cannot follow the temperature front: it lags behind (see Fig. 2). In the space between these two boundaries the temperature is already below the phase transition temperature, $T < T_c$, but the phase transition did not yet happen, and the OP is still not formed, $\Psi = 0$. This situation is unstable towards the formation of bubbles of the new phase with $\Psi \neq 0$. This occurs independently in different regions of the space, leading to vortex formation according to the KZ mechanism. At a given point of space $r$ the development of the instability can be found from the linearized TDGL
order parameter in moving normal/superfluid interface

![Diagram](image)

FIG. 2. The OP distribution from Ref. [7] at nonzero velocity $u$ of the planar temperature front in the reference frame of the moving front. The higher $u$ the larger is the lag between the temperature front at $x = 0$, where $T = T_c$, and the OP front, which bounds the region with $\Psi = 0$. At $u > u_c$, the exponentially growing OP fluctuations in the space between these two boundaries are not washed out by the moving front and lead to the vortex formation according to KZ scenario. Dashed line is the Eq. (6).

equation, since during the initial growth of the OP $\Psi$ the cubic term can be neglected:

$$\tau_0 \frac{\partial \Psi}{\partial t} = \frac{t}{\tau_Q} \Psi .$$

This gives an exponentially growing OP, which starts from some seed $\Psi_{\text{fluc}}$, caused by fluctuations:

$$\Psi(\mathbf{r}, t) = \Psi_{\text{fluc}}(\mathbf{r}) \exp \left( \frac{t^2}{2\tau_Q \tau_0} \right) .$$

Because of the exponential growth, even if the seed is small, the modulus of the OP reaches its equilibrium value $|\Psi_{\text{eq}}| = \sqrt{1 - T/T_c}$ after the Zurek time $t_Z$

$$t_Z = \sqrt{\tau_Q \tau_0} .$$

This occurs independently in different regions of space and thus the phases of the OP in each bubble are not correlated. The spatial correlation between the phases becomes important at distances $\xi_v$ where the gradient term in Eq. (4) becomes comparable to the other terms at $t = t_Z$. Equating the gradient term $\xi_v^3 \nabla^2 \Psi \approx (\xi_v^2/\xi_c^2) \Psi$ to, say, the term $\tau_0 \partial \Psi/\partial t |_{\text{Zurek}} = \sqrt{\tau_0/\tau_Q} \Psi$, one obtains the characteristic Zurek length scale which determines the initial distance between the defects in homogeneous quench:

$$\xi_v = \xi_c \left( \frac{\tau_Q}{\tau_0} \right)^{1/4} .$$

We can estimate the lower limit of the characteristic value of the fluctuations $\Psi_{\text{fluc}} = \Delta_{\text{fluc}}/\Delta_0$, which serve as a seed for the vortex formation. If there is no other source of fluctuations, caused, say, by external noise, the initial seed is provided by thermal fluctuations of the order parameter in the volume $\xi_c^3$. The energy of such fluctuation is $\xi_v^4 \Delta_{\text{fluc}}^2 N_F/E_F$, where $E_F$ is the Fermi energy and $N_F$ the fermionic density of states in the normal Fermi liquid. Equating this energy to the temperature $T \approx T_c$ one obtains the magnitude of the thermal fluctuations of the OP

$$\frac{|\Psi_{\text{fluc}}|}{|\Psi_{\text{eq}}|} \sim \left( \frac{\tau_0}{\tau_Q} \right)^{1/8} \frac{T_c}{E_F} .$$

Since the fluctuations are initially rather small their growth time exceeds the Zurek time by the factor $\sqrt{\ln |\Psi_{\text{eq}}|/|\Psi_{\text{fluc}}|}$.

The criterion for the defect formation is that the time of growth of fluctuations, $\sim t_Z = \sqrt{\tau_Q \tau_0}$, is shorter than the time $t_{\text{sw}} = x_0(u)/u$ in which the transition front sweeps the space between the two boundaries. Here $x_0(u)$ is the lag between the transition temperature front and the OP front (see Fig. 2). Thus the equation $t_Z = x_0(u_c)/u_c$ gives an estimate for the critical value $u_c$ of the velocity of the temperature front, at which the laminar propagation becomes unstable. At large $u$ one has $x_0(u) \sim u^3 \tau_Q \tau_0^2/4 \xi_0^2$ and thus

$$u_c \sim \frac{\xi_0}{\tau_0} \left( \frac{\tau_0}{\tau_Q} \right)^{1/4} ,$$

which agrees with estimation $u_c = \xi_v/t_Z$ in [8].

In the case of the neutron bubble the velocity of the temperature front is $u \sim R_0/\tau_Q$, which makes $u \sim 10$ m/s. The critical velocity $u_c$ we can estimate to possess the same order of magnitude value. This estimation suggests that the thermal gradient should be sufficiently steep in the neutron bubble such that defect formation can be expected. The further fate of the vortex tangle formed under the KZ mechanism is the phase ordering process: the intervortex distance continuously increases until it reaches the critical size, when the vortex loops are expanded by the counterflow. This reproduces the most favourable scenario of the vortex formation with the cubic law.
FIG. 3. Rough scenario of instability of the superfluid-normal interface in the presence of external superflow.

III. INSTABILITY OF NORMAL/SUPERFLUID INTERFACE.

Another mechanism of the vortex formation has been recently found in 3D numerical simulations in Ref. [12]. It is related to the instability of the normal-superfluid interface in the presence of the superflow. Let us consider a simple hand-waving interpretation of such an instability. The process can be roughly split into two stages (see Fig. 3).

At first stage the heated region of the normal liquid surrounded by the superflow undergoes a superfluid transition. The transition should occur into the state with the lowest energy, which corresponds to the superfluid at rest, i.e. with $v_s = 0$. Thus there appears the superfluid-superfluid interface, which separates the state with superflow (outside) from the state without superflow (inside). Such a superfluid-superfluid interface with tangential discontinuity of the superfluid velocity represents a vortex sheet by definition. Such vortex sheet, on which the phase of the OP is not determined, was suggested by Landau and Lifshitz for He-II to describe the superfluid state of $^4$He under rotation [13] (see also [14] and [15]).

The vortex sheet is unstable towards breaking up into a chain of quantized vortex lines. The development of this instability represents the second stage of the process. In numerical simulation the resulting chain of vortices is clearly seen (see Figs. 4 and 5).

The evolution in Fig. 3 is thus caused by the hydrodynamic instability of the normal/superfluid interface in the presence of the tangential flow. Since vorticity is quantized, such instability leads to the formation of the vortex chain only above the threshold required to achieve the circulation quantum from the tangential superflow. If the counterflow is large the number of vortices in this chain $N \approx |v_s - v_n| R_b/\kappa$, i.e. one has a linear law instead of cubic.

Nucleation of the KZ vortices due to the motion of the superfluid/normal interface is also observed in numerical simulations in Ref. [12]. It occurs during shrinking of the interior region with normal fluid.

IV. DISCUSSION.

Two mechanisms of the vortex formation have been identified in numerical simulations [12]: (a) vortices are formed behind the propagating front due to KZ mechanism, as discussed in Refs. [6] and [7]; and in addition (b) vor-
Numerical simulation of vortex formation in neutron experiment
I.S. Aranson, N.B. Kopnin, V.M. Vinokur, cond-mat/9905286.

Chain of vortices (vortex sheet)

Kibble-Zurek vortices
decay because superflow is screened by vortex sheet

Vortices from vortex sheet grow due to Magnus force from external superflow

FIG. 5. 2D simulations, which assume an axisymmetric development (from Ref. [12]).

Vortices are formed due to the corrugation instability (vortex sheet instability) of the front in the presence of external superflow. Each of these mechanisms can be derived either analytically for a simple geometry, or understood qualitatively with simple physical picture in mind. The AKV calculations actually showed that each mechanism is fundamental: it does not depend much on the geometry and on parameters of the TDGL equation. Probably both mechanisms hold even if TDGL theory cannot be applied.

The interplay of the two mechanisms must depend on details of the microscopic physics. In their calculations based on TDGL model, AKV found that the chain of vortices formed in the process (b) screens the external superflow very effectively. The KZ vortices formed in the process (a) cannot grow: they decay before the screening chain escapes to the bulk liquid. Thus in the AKV scenario only the chain of vortices survives. This gives the linear dependence of the vortex number \( N \) on the counterflow \( v_s - v_n \) instead of the observed cubic law.

This does not exclude the possibility of another regime, where KZ vortices have enough time to escape to the bulk. This is probably what the cubic law found in Helsinki experiments tells us. Maybe the latter regime cannot be obtained in the TDGL scheme and one must discuss the combined dynamics of the OP and quasiparticles.

In conclusion, in the period between LT-21 and LT-22 the principles of defect formation in inhomogeneous phase transition have been developed.

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