Stringy gravity at short distances

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ABSTRACT

Analysis of string interactions indicates a weakening of gravity at the string length scale, thus avoiding black holes and their singularities.
1. Introduction

Physical singularities generally indicate the breakdown of a field theory. In electromagnetism a timelike singularity is the result of approximating a source as a point particle rather than a field configuration or fluid, but in general relativity there are spacelike singularities, which have no physical interpretation even as an approximation. The result is a black hole, as delineated by an event horizon, the border of the region where even light can’t escape the singularity. Recent attention to black holes has focused on the resultant information loss, but the resolution of the true problem, the existence of the singularity, would also eliminate the accompanying event horizon, and with it the symptom of information loss.

One proposal to eliminate this problem is quantum modifications of general relativity. (For a concrete application to gravitational collapse, see [1].) However, non-renormalizability makes such modifications ambiguous. A more predictive approach to the problem is string theory, which can be considered as incorporating at the classical level effects that might be considered quantum in some “dual” formulation. In this paper we will consider such stringy effects to black hole formation. In particular, we find that gravity weakens at short distances, so that no singularity is created. (An alternative approach is to work with the Euclidean solution, where the Schwarzschild solution has no singularity or black hole [2], but then spacetime loses its classical interpretation even at macroscopic distances.)

2. Vertices

The usual method for obtaining the Schwarzschild solution is difficult to apply in string theory for various reasons: (1) An explicit form for string field theory in terms of a metric tensor or analog, or the complete effective gravitational theory, is not known. (However, there is a perturbative construction of the closed-string field theory action [3].) (2) There is no analog to a “point source”, as strings have an intrinsic length, and it is exactly this distance scale at which we seek to probe its modifications of general relativity.

Consequently the first approximation we will make is to consider only single graviton exchange. This is equivalent to lowest order in the gravitational coupling, or examining the gravitational “potential”. This is sufficient in ordinary general relativity to find the singularity, since there exist gauges (e.g., Eddington-Finkelstein coordinates) where the exact solution to the field equations for the Schwarzschild metric is given at lowest order of perturbation. Since we do not know how to construct
the metric from this linearized solution, even though it probably can be done, the analysis of the event horizon will not be obvious, but since the singularity itself will disappear, an event horizon is no longer expected.

Our first attack on the problem is based on the explicit form of the Witten vertex [4] for open-string field theory. We should really use a closed string to describe the graviton, but the feature we use is a property common even to the lightcone 3-string vertex [5]. Specifically, we examine the higher-derivative nature of the 3-string coupling, which is known to be the source of various effects otherwise thought to be unique to string theory [6]. To simplify the analysis, we consider the simpler example of a “scalar” graviton, since stringy effects are expected to eliminate singularities for all forces.

The relevant action is then of the form

\[ L \sim \phi \Box \phi + \phi e^{\Box/4}T \]

for “graviton” field \( \phi \) and source \( T \). (In Witten’s vertex the \( \frac{1}{4} \Box \) in the exponent has a coefficient of \( \ln(27/16) \approx .52 \) times \( \alpha' \).) This is equivalent after \( \phi \) field redefinition (at least for this calculation) to

\[ L' \sim \phi \Box e^{-\Box/2} \phi + \phi T \]

which simply moves the higher-derivative factor from the vertex to the propagator. Note that this modification produces no extra poles in the propagator, and in fact regularizes it. By thus trivializing the coupling, the effective gravitational potential can be seen directly from the propagator (as in ordinary gravity). Note that naively taking \( T = 0 \) in the former action and solving \( \Box \phi = 0 \) outside a certain region does not give a physical solution (e.g., as from gravitational collapse), since localizing \( T \) to a \( \delta \) function yields a smeared source \( e^{\Box/4}T \), while choosing \( e^{\Box/4}T \) to be a \( \delta \) function yields an unphysical source

\[ T \sim e^{-\Box/4} \delta^3(\vec{x}) \sim e^{r^2} \]

that blows up as \( r^2 \equiv \vec{x}^2 \to \infty \). (We consider generic 4D strings, or set compactification scales to infinitesimal.)

The gravitational potential is thus of the form

\[
\frac{1}{\Box} e^{\Box/2} \delta^3(\vec{x}) \sim \int d^3 p \ e^{ip \cdot x} \int_0^\infty d\tau \ e^{-\tau p^2/2} \sim \int_0^\infty d\tau \ \tau^{-3/2} e^{-r^2/2\tau} \\
\sim \frac{1}{r} \int_0^r d\sigma \ e^{-\sigma^2/2} \sim \frac{1}{r} \text{erf}\left(\frac{r}{\sqrt{2}}\right)
\]
in terms of the error function (where we used the change of variables \( \tau = r^2/\sigma^2 \)). The result is that the long-distance \( 1/r \) potential is smoothed off to a parabola at short distances.

### 3. Scattering

The previous calculation neglected contributions from higher-spin fields, which might couple in a similar way, and thus affect a measurement meant for only gravity. (Compare, e.g., the effect of a scalar in a scalar-tensor theory of gravity.) Thus we now look at a complete 4-point amplitude, and try to pick out the relevant part. Since we are really interested in scattering off an external potential (no back reaction), we choose one of the incoming and outgoing particles to be a string state with mass comparable to a star, and thus relevant for gravitational collapse. (A possible alternative might be to consider the star as a statistical ensemble of string states; then the phase transition when the collapsing star reaches the Hagedorn temperature should mark the end of existence of the graviton and its force.) In terms of the Mandelstam variables, we thus use the approximation

\[
s \approx u \approx M^2 \gg t, \frac{1}{\alpha'}
\]

(We do not specify whether the string scale \( \sqrt{\alpha'} \) is similar to the Planck length \( \kappa \) or, e.g., some hadronic scale.) This approximation will also allow us to neglect absorption and emission of the particle by the star. In this limit, the behavior of any 4-point string amplitude is dominated by the leading Regge trajectory. (Alternatively, since Regge behavior is characteristic of any bound state, we can also apply this analysis to any bound-state formulation of gravity.) We thus have the amplitude

\[
A_4 \approx g^2 \Gamma(-\alpha't)(\alpha's)^{2+\alpha't} \approx \kappa^2 M^4 \alpha' \Gamma(\alpha'p^2)(\alpha'M^2)^{-\alpha'p^2}
\]

in terms of the string coupling \( g \) and gravitational coupling \( \kappa \), where we have assumed the usual linear (closed-string) trajectory with slope \( \alpha' \), and no tachyon. We have also written \( t = -p^2 \) for comparison with the previous section.

The qualitative behavior is similar to the previous analysis: At distances much longer than \( \sqrt{\alpha'} \), we have the usual \( 1/p^2 \) behavior. The last factor does not have a significant effect until the length scale (from rewriting that factor as an exponential)

\[
\sqrt{\alpha' \ln(\alpha'M^2)}
\]

which is only an order of magnitude or so greater than the string length scale. At distances between \( \sqrt{\alpha'} \) and \( 1/M \), using the Stirling approximation for \( \Gamma \), we see
the amplitude still decreases exponentially. At shorter distances the $\Gamma$ factor wins out, but the Regge approximation is not valid in that region, and back reaction and absorption/emission become important. In any case that distance $1/M$ is the Compton radius of the star, almost 40 orders of magnitude shorter than the Planck length, and we expect many of our assumptions to break down long before reaching that scale.

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Added note

Some time ago Tseytlin [7] considered the same modification to gravity as in section 2, as an example of a possible string correction, and reached similar conclusions.

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