Electrically controllable cyclotron resonance

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Cyclotron resonance (CR) is considered one of the fundamental phenomena in conducting systems. However, Kohn’s theorem excludes the possibility of electrically controlling the CR, which significantly limits the scope of research and does not allow to exploit the full application potential of this phenomenon. We demonstrate the breakdown of this theorem in the case of back-gated two-dimensional (2D) electron systems, where the 2D electron sheet is separated from the metallic steering electrode (“back-gate”) by a standard dielectric substrate. The resultant effect is attributed to the retardation of the interaction of 2D electrons and currents with their images in the back-gate. Consequently, we predict that a tremendous shift and narrowing of the CR line can be achieved by varying the electron density or the gate voltage, given the realistic parameters. The effective controllability of CR opens the door for exploring new physics and CR application possibilities.

In a classical plasma, an electron exposed to the magnetic field $B$ rotates in a closed orbit with the angular frequency $\omega_c = eB/mc$, where $-e$ and $m$ are the electron charge and effective mass, and $c$ is the speed of light in vacuum (we note that in this paper, we use the Gauss units). The absorption of an electromagnetic wave at this frequency, known as cyclotron resonance (CR), has long been used in fundamental studies and applications of both non-degenerate gaseous and degenerate condensed-matter plasmas. Initially, CR in two-dimensional electron systems (2DESs) was observed in silicon inversion layers [1, 2]. Nowadays, it is used extensively to characterize various kinds of 2DESs.

Depending on the presence of a steering electrode (the gate), all 2DESs can be divided into gated and ungated systems. Theoretically, the CR in infinite ungated 2DES was studied in Ref. [3], see also [4, 5]. It was found that for the 2DES situated between two half-spaces with dielectric permittivities $\varepsilon_1$ and $\varepsilon_2$, the half-width of the CR line is defined by the sum of collisional broadening $\gamma = 1/\tau$, with $\tau$ being the electron relaxation time in 2DES, and collective radiative broadening $\Gamma_{\gamma} = 2\Gamma/(\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2})$, where

$$\Gamma = \frac{2\pi e^2 n}{mc},$$

and $n$ is the 2D electron concentration. In ungated GaAs quantum wells, the collective nature of the CR decay rate $\Gamma_{\gamma}$ has been observed in time-domain experiments [6, 7], and interpreted as the effect of superradiance [8, 9].

However, to the best of our knowledge, collective contributions to the CR frequency and decay rate in gated 2DESs have not been previously studied in detail. The gated 2DES has a unique feature — a technical possibility to control the electron density over a wide range through the gate voltage. Thus, it enables, for example, fine-tuning of the electron-electron interaction parameter. However, according to the famous Kohn’s theorem [10], manifestations of many-body effects in the CR regime are strongly suppressed: the CR frequency in a standard homogeneous 2DES is independent of electron-electron interaction and, therefore, electron concentration as well. For this reason, Kohn’s theorem imposes a serious constraint on the fundamental and practical potential of CR physics. Nevertheless, there are special cases when the theorem can be invalidated. For instance, in Ref. [11] this theorem was violated by ultrasound to observe an analog of CR on composite fermions. Other examples include non-parabolic electron dispersion [12], polarons [13], non-equilibrium and dynamic effects [14, 15], etc. Although, as a rule, these effects have weak influence on the CR frequency.

In his work on CR, Kohn did not take into consideration electromagnetic retardation of electron-electron interaction. Consequently, CR in the regime of strong light-matter coupling is of particular interest, for review see Ref. [16]. Over the past decade, the interaction between the cyclotron motion of electrons and discrete photon modes in specially designed microwave or THz resonators has been investigated extensively in various independent studies [17–24]. It was found that the violation of Kohn’s theorem and the renormalization of the CR frequency occur indeed, though within a finite range of magnetic fields corresponding to the anti-crossing of the bare CR frequency and the photon frequency of the resonator.

In the present Letter, we report on significant and unexpected CR renormalization in a gated 2DES — namely, the CR frequency shift and the narrowing of its linewidth. We emphasize that, surprisingly, it takes place in the low-frequency regime, when the bare cyclotron frequency is much less than the characteristic frequency of the Fabry-Perot resonator formed by the 2DES and the metal gate, with a dielectric substrate in between, as illustrated in Fig. [1]. Furthermore, we assert that the given low-frequency CR renormalization can be large, even in conventional 2D electron structures with the back-gate. Therefore, it can be rather easily observed experimentally.

We establish the CR renormalization to be governed by
the retardation parameter $A$ defined as the ratio of the characteristic velocity in a gated 2DES ($V_p$) to the speed of light in the dielectric substrate separating the 2DES and the metal gate (c$/\sqrt{\varepsilon}$). Formally, the characteristic velocity equals to that of 2D plasma waves in the gated 2DES [26]:

$$V_p = \sqrt{\frac{4\pi n e^2 d}{\varepsilon m}},$$  \hspace{1cm} (2)

where $d$ is the distance between the gate and 2DES. The retardation parameter can be written in three equivalent forms, as follows:

$$A^2 = V_p^2 \varepsilon/c^2 = 2d\Gamma/c = 4\pi n e^2 d/(mc^2).$$  \hspace{1cm} (3)

We find that the renormalized CR frequency can be expressed in terms of $A$ through a simple relation:

$$\omega_{ren} = \omega_c/(1 + A^2).$$  \hspace{1cm} (4)

It should be stressed that the given retardation parameter can be easily modified by changing electron concentration by the chemical doping or with the gate voltage, which enables the electrical control of the renormalized cyclotron resonance frequency.

To determine the CR frequency and linewidth, we consider the absorption of the electromagnetic plane wave incident normally onto the gated 2DES, as depicted in Fig. 1. Given the 2DES sheet and the top surface of an ideal metal gate situated, respectively, at $z = 0$ and $z = -d$, the incident and reflected electromagnetic waves take the forms of $E_i \exp(-i\omega z/c - i\omega t)$ and $E_r \exp(i\omega z/c - i\omega t)$. To calculate the absorption, we follow the classic approach based on Maxwell’s equations and Ohm’s law $\mathbf{j} = \sigma \mathbf{E}$, with $\sigma$ denoting the 2DES conductivity tensor.

As a matter of convenience, we introduce the ”circular” variables: $E_{ix,\pm} = E_{ix} \pm iE_{iy}$, $E_{rx,\pm} = E_{rx} \pm iE_{ry}$, and $\sigma_{xx,\pm} = \sigma_{zz,\pm}$ + $i\sigma_{xy}$. Then, following a standard procedure Ref. [3], the amplitude of the reflection coefficient $r_{\pm} = E_{r\pm}/E_{i\pm}$ can be expressed as:

$$r_{\pm} = 1 - i\sqrt{\kappa} \cot(\omega\sqrt{\kappa}d/c) - 4\pi\sigma_{\pm}/c$$  \hspace{1cm} \bigg(1 + i\sqrt{\kappa} \cot(\omega\sqrt{\kappa}d/c) + 4\pi\sigma_{\pm}/c\bigg)^{1/2}.  \hspace{1cm} (5)

From the equation above, we can obtain an estimate of the resonance frequency and line width, considering that these properties are directly related to the poles of reflection (as well as absorption) coefficient. Given the Fabry-Perot frequency $\omega_{FP} = c/(\sqrt{\varepsilon}d)$, we introduce dimensionless frequencies $\Omega_c = \omega_c/\omega_{FP}$ and $\Omega = \omega/\omega_{FP}$. Then, using the Drude model for the conductivity tensor [27], we arrive at the following equation for the complex-valued frequency $\Omega$ corresponding to the poles of Eq. (5):

$$\left(\Omega \mp \Omega_c + i\gamma/\omega_{FP}\right)\left(\cot \Omega - i/\sqrt{\kappa}\right) = -A^2.$$  \hspace{1cm} (6)

As for the physical meaning of this relation, we note that parameter $A$ describes not only the electromagnetic retardation but also a collective contribution of the 2DES to the net response, since $A^2 \propto n$. In the zeroth approximation $A = 0$, and in the "clean" limit ($\gamma \to 0$), the sought frequency $\Omega$ is equal to the single-particle frequency of the cyclotron motion, $\Omega = \Omega_c$. Increasing $A$ results in the interaction of bare cyclotron motion with the photonic modes of the resonator, leading to the collective renormalization of CR.

Considering a high-quality resonance with $Im\Omega \ll Re\Omega$ and $\gamma \ll \omega_c$ in the low-frequency limit $|\Omega| \ll 1$, from Eq. (6) we obtain:

$$\Omega = \frac{\pm\Omega_c}{1 + A^2} - i\frac{A^2\Omega_c^2}{\sqrt{\kappa}(1 + A^2)} - i\frac{\gamma/\omega_{FP}}{1 + A^2}.$$  \hspace{1cm} (7)

The real and imaginary parts of the resultant frequency describe, respectively, the position and broadening of the renormalized CR line. It should be noted that in this case, the line-broadening term includes both radiative and collisional contributions. We also note that the sign notation in the numerator of the first term in Eq. (7) indicates the different sign of circular polarization.

Now, let us find the shape of the renormalized cyclotron line from the direct calculation of the energy absorption coefficient $P_\pm = 1 - |r_{\pm}|^2$. Thus, using the Drude model for the conductivity tensor, we derive the exact expression for $P_\pm$, as shown in the Supplemental Material [27]. Due to its cumbersome form, we do not include it in the main body of the paper. Importantly, of particular interest to us is the low-frequency regime $\Omega \ll 1$, i.e., $\omega \ll c/(d\sqrt{\varepsilon})$, where we can neglect all but the dominant terms in the full expression for $P_\pm$ to obtain the simplified relation:

$$P_\pm = \frac{4\gamma A^2 d/c}{(\omega_{\pm}\omega_c/\omega + A^2)^2} + \frac{\gamma^2}{c^2} + \frac{2\gamma A^2 d}{c} + \frac{d^4(\omega_{\pm}\omega_c)^2}{c^2}.$$  \hspace{1cm} (8)
for different values of $m \approx 0.128$, where solid lines denote the set of curves absorption in the gated 2DES, according to the exact for-
totic relation of the absorption in (8) calculated in the lim it of $\gamma/\omega = 1$. Clearly, the asymptotic
result is in excellent agreement with the exact calcula-
tion of (4) and (9) (blue dashed lines). We also note that when expressed in terms of $A$, the resonant frequency
is defined as follows:

$$\omega_{\text{ren}} = \sqrt{\frac{3\omega^2 d}{\sqrt{\frac{\sqrt{\frac{\omega^2 d^2}{\gamma^2 c^2}}}{\gamma c}} - 1 - \frac{\omega^2 d}{\gamma c}}}. \quad (10)$$

which can be used to find the resonant frequency $\omega_{\text{ren}}$ and linewidth $\Delta \omega$ analytically. We obtain that in clas-
sically strong magnetic field ($\gamma \ll \omega$), the resonant fre-
quency takes the form described in (4) — identical to the real part of Eq. (4), as expected.

Provided that $\gamma \ll \omega$, the linewidth of the resonance becomes:

$$\Delta \omega = \frac{2\gamma + A^2 \omega_{\text{ren}}^2 d/c}{1 + A^2}. \quad (9)$$

Here, the total linewidth $\Delta \omega$ is the sum of renor-
malized collisional broadening proportional to $\gamma$ and radi-
ative broadening. The half-width $\Delta \omega/2$ equals the imagi-
ary part of the frequency in Eq. (1), as expected.

In Fig. 2 we plot the frequency dependence of the ab-
sorption in the gated 2DES, according to the exact for-
mulation [27], where solid lines denote the set of curves for
different values of $A$. For comparison, the dashed curve indicates the line shape obtained from the asymptotic relation in (5) for $A = 1$. Clearly, the asymptotic result is in excellent agreement with the exact calculation.

In addition, Fig. 3 shows the resonance position and
linewidth as a function of the retardation parameter. Comparing the given numerical and analytical data likewise makes it evident that calculations based on the exact expression for $P_\perp$ [27] (green solid lines) perfectly agree with the asymptotic results from (4) and (5) (blue dashed lines). We also note that when expressed in terms of $A$, the linewidth $\Delta \omega$ reaches its maximum value $A_m^2$, defined by:

$$A_m^2 = \frac{\sqrt{3\omega^2 d}}{\gamma c} + \frac{\omega^2 d^2}{\sqrt{\frac{\gamma^2 c^2}}{\gamma c}} - 1 - \frac{\omega^2 d}{\gamma c}. \quad (10)$$

Next, let us analyze the absorption at the resonance
frequency, $P_\perp(\omega_{\text{ren}})$, as a function of the retardation pa-
rameter, $A \propto \sqrt{m}$. Considering the extreme cases, we recog-
ize that in the limit of $A \to 0$, the absorption ap-
proaches zero, as 2DES becomes virtually absent. Similarly, in the limit of $A \to \infty$, 2DES attains infinite con-
ductivity, which also leads to zero absorption due to full reflection of the incident radiation. Therefore, at finite values of $A$, the absorption is expected to have one or
more maxima. Furthermore, we find that under the con-
dition of "weak" magnetic field, i.e. $\omega^2 d/(\gamma c) < 4$, the absorption resonance $P_\perp(\omega_{\text{ren}})$ exhibits a single maxi-
num at $A^2 = 1$. On the other hand, given "strong"
magnetic field, i.e. $\omega^2 d/(\gamma c) > 4$, there occur two local maxima of $P_\perp(\omega_{\text{ren}})$ at $A^2_{1,2}$ defined as follows:

$$A_{1,2}^2 = -1 + \frac{\omega^2 d}{2\gamma c} \pm \frac{\sqrt{\frac{\omega^2 d^2}{4\gamma^2 c^2} - \frac{\omega^2 d}{\gamma c}}}{2}, \quad (11)$$
with the local minimum of $P_{\pm}(\omega_{\text{ren}})$ situated between the maxima, at $A^2 = 1$.

Now, let us compare the absorption of a circular polarized electromagnetic wave in gated and ungated 2DESs exposed to the magnetic field. In the case of the ungated 2DES in vacuum \cite{3}, the absorption maximum appears exactly at the cyclotron frequency $\omega_c$, while the half-linewidth of the resonance equals $\gamma + \Gamma$. By contrast, in the gated 2DES, the factor $(1 + A^2)^{-1}$ leads to the reduction in the linewidth, as well as the shift in the absorption peak away from $\omega_c$, as defined in Eq. (2). In addition, the radiative contribution to the linewidth becomes significantly suppressed due to the factor $d^2\omega^2_{\text{ren}}/c^2 \ll 1$, according to Eq. (3). Thus, in the gated 2DES, one can obtain that when the radiative contribution to the linewidth dominates that of collisional origin, the linewidth narrows with increasing electron concentration: $\Delta \omega \propto A^{-4} \propto n^{-2}$, which is very much unlike the case of the ungated 2DES.

Consequently, we demonstrate that Kohl’s theorem can be violated strongly, depending on the value of $A$. In contrast to the well-known effects of strong light-matter coupling \cite{10}, the theorem breaks down in the low-frequency regime $\omega \ll c/\sqrt{2d}$. We attribute this to the retarded interaction of electrons and currents in the 2DES with their images in the metal gate.

For practical purposes, we estimate the retardation parameter $A^2$ in 2DES based on a back-gated GaAs/AlGaAs quantum well, given the following characteristic parameters: $d = 0.4$ mm, $n = 5 \cdot 10^{11}$ cm$^{-2}$, and $m = 0.066m_0$, where $m_0$ is the free-electron mass. As a result, we find $A^2 \approx 1$, which in clean samples corresponds to the resonant frequency equal half the bare $\omega_c$. Therefore, CR renormalization in standard back-gated semiconductor structures can be far from negligible.

In summary, we have conducted the analytical and numerical investigation of the absorption of electromagnetic wave incident normally onto the gated or back-gated 2DES in the presence of a perpendicular magnetic field. Importantly, the study takes into account the effect of electromagnetic confinement in the natural resonator formed by the 2DES sheet and the metallic back-gate, with a dielectric substrate in between. Unexpectedly, we find the CR renormalization, i.e., the shift in the resonance frequency \cite{11} and narrowing of the linewidth \cite{9}, to occur in the low-frequency regime when the radiation frequency is much smaller than the Fabry-Perot frequency of the resonator. We establish that given renormalization is controlled by the retardation parameter $\kappa$, which depends on the electron concentration in the 2DES. We prove that this parameter can be large enough, even in standard back-gated samples. As a result, it can lead to a tremendous shift and narrowing of the CR line. Therefore, gated and especially back-gated 2DESs prove very promising for exploring new physical effects, for instance, the experimental studies of the extreme regimes of light-matter coupling.

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### Supplementary Material I. Conductivity tensor based on the Drude model

Consider a 2DES in a constant magnetic field $B$ applied perpendicular to the 2DES plane. In the framework of the Drude model, the dynamical longitudinal $\sigma_{xx}$ and transverse $\sigma_{xy}$ conductivities of the 2DES can be expressed as follows:

$$\sigma_{xx} = \frac{e^2 n}{m} \frac{1}{\tau - i\omega} \left( \frac{1}{\tau - i\omega} + \frac{\omega_c^2}{\omega_d^2} \right)$$

$$\sigma_{xy} = \frac{e^2 n}{m} \frac{-\omega_c}{\left( \frac{1}{\tau - i\omega} + \frac{\omega_c^2}{\omega_d^2} \right)}$$

where $n$ is the electron concentration in the 2DES, $-e$ and $m$ are the electron charge and effective mass, $\omega_c = |e|B/(mc)$ is the electron cyclotron frequency in the 2DES, and $\tau$ is the electron relaxation time.

### Supplementary Material II. The exact equation for the absorption coefficient

Using the Drude model for the conductivity tensor with the finite electron relaxation time $\tau$ \cite{12}, we find the exact expression for the energy absorption coefficient $P_\pm$, given the circular polarization of the incident wave:

$$P_\pm = \frac{8\Gamma \gamma}{\left( \sqrt{\gamma}(\omega + \omega_c) \cot \left( \frac{\omega_d \omega_c}{\gamma} \right) + 2\Gamma \right)^2 + \gamma^2 \left( \frac{\gamma}{\gamma \cot^2 \left( \frac{\omega_d \omega_c}{\gamma} \right) + 1} \right) + 4\Gamma \gamma + (\omega \pm \omega_c)^2}$$

where $\gamma = 1/\tau$ corresponds to the collisional broadening, and

$$\Gamma = \frac{2\pi e^2 n}{mc}$$

designates the radiative broadening of the absorption.
linewidth.

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[27] See Supplemental Material at http://link.aps.org/supplemental/xxx which includes the Drude model and the explicit relation for the absorption coefficient.