ADVANCED TENSOR APPROACH TO FAST REROUTE WITH QUALITY OF SERVICE PROTECTION UNDER MULTIPLE PARAMETERS

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Background. The paper proposes a solution to such an urgent problem today, as to ensure the fault tolerance of infocommunication networks with the support of the required level of quality of service. The proposed solution is based on the implementation of an advanced tensor approach to fast reroute with the protection of the level of quality of service under multiple parameters.

Objective. The aim of the article is to improve the flow-based fast rerouting model in the infocommunication network, which is based on updated conditions for ensuring quality of service in terms of bandwidth, average end-to-end delay, and probability of packet loss. It was possible to obtain updated conditions for ensuring the quality of service through the use of a tensor approach to modeling infocommunication networks.

Methods. As research methods, graph theory, tensor theory, queuing systems were used. For mathematical modeling and experimental studies, the MatLab simulation package was used.

Results. As a result of the study, under the conditions of implementing fast rerouting, it was possible to provide the required level of quality of service in the infocommunication network. At the same time, with an increase in QoS requirements, thanks to an improved tensor approach, the updated conditions for ensuring the quality of service while implementing fast rerouting were adequate, which, as a result, contributed to a more efficient use of the available network resource.

Conclusions. Implementation of an improved tensor approach to solving the problem of fast rerouting will ensure the fault tolerance of the infocommunication network with the protection of the level of quality of service in terms of bandwidth, average end-to-end delay, and the probability of packet loss.

Keywords: infocommunication network; fast rerouting; bandwidth; average end-to-end packet delay; probability of packet loss; tensor; space; coordinate system.

Introduction

Today, modern infocommunication networks (ICN) are characterized by a rapid increase in the intensity and heterogeneity of the transmitted traffic. In addition, requirements for the level of their availability, fault tolerance and quality of service (QoS) are constantly increasing for modern network services [1-4]. However, in existing networks in a limited network resource, the noted requirements often contradict each other. This is due to the fact that in order to increase the fault tolerance of the network as a whole, it is necessary to introduce structural and/or functional resource redundancy. The reserve can be network devices, to which the load is redistributed in the event of a failure of a network element (node, link, path). The introduction of such a reserve does not allow it to be used under normal conditions of ICN operation, which may adversely affect network performance.

At the same time, for multiservice ICN, the task of ensuring reservation (protection) of not only the network elements, but also the level of the quality of service provided, when the QoS requirements would be fulfilled along both the primary and the backup set of paths, for example, allowable values allocated bandwidth, average end-to-end delay, jitter, packet loss probability.

Analysis of existing solutions

One of the effective means of increasing ICN reliability at the network level of the OSI model is the use of fault tolerant routing protocols. Such protocols can provide fault tolerance of network solutions both at the level of network access, namely, the protection of the default gateway, and within the network, implementing Fast ReRoute policies with the protection of nodes, links and routes in the ICN [5-8].

The main disadvantage of these solutions is the limited consideration of the characteristics of the transmitted traffic, as well as unbalanced use of available network resource. Therefore, the main direction of development in this area is the revision of mathematical models and methods that form the basis of fault tolerant protocols, with the aim of further
accounting and balancing the available network resource to ensure the required QoS in the network.

So, in paper [9-10], results were obtained that, under the conditions of fast rerouting, are oriented towards providing and protecting the required level of quality of service in terms of bandwidth, as one of the key indicators of QoS. However, these solutions are narrowly focused and are not able to provide the required level of providing most multimedia services, for example, video conferencing or VoIP. This required the maintenance of the accounting traffic of the extended list of QoS indicators.

In this connection, solutions were proposed in [11-16] that ensure the quality of service based on two QoS indicators, for example, bandwidth and packet loss probability [11, 12].

Several solutions [13-16], based on the tensor representation of ICN, provide protection simultaneously in terms of bandwidth and average end-to-end delay.

To expand the functionality of solutions for fast routing to all three main QoS-indicators, it is proposed to use a tensor approach to modelling and calculating ICN, taking into account possible network losses. This will allow obtaining, in an analytical form, mutually complementary conditions for ensuring the QoS both in terms of bandwidth, and the average end-to-end delay, and the probability of packet loss. That seems especially important in the conditions of the network functioning, close to congestion. Thus, the problem with the design of the advanced tensor approach to fast reroute with the protection of the level of quality of service under multiple parameters is relevant.

Tensor model for obtaining conditions of quality of service protection under multiple parameters

As shown by the analysis of scientific papers on the formation and development of the theory and methodology of tensor modelling of the infocommunication network (ICN) [13-15, 17-21], the first step in building a tensor model of a multiservice ICN is the geometrization of its structure. In accordance with the approach proposed by G. Kron [17] and developed in [18-21], the structure of the ICN will be modeled by a one-dimensional network \( S = (U, V) \). Then the ICN routers will be described by a set of network nodes \( U = \{u_i, i = 1, m\} \), where \( m \) is the total number of nodes in the network \( S \). A set of network edges \( V = \{v_z, z = 1, n\} \) simulate ICN communication links, where \( n \) is the total number of edges in the network \( S \). Thus, to describe the network elements, their end-to-end numbering is used.

The nodes of the network \( S \) are the nodes, which simulate routers through which a particular flow of packets arrives or leaves the ICN. Further research will also be used, such structural characteristics of the network \( S \), such as: \( \kappa(S) \) is the number of basic interpolar paths in the network \( S \); \( \theta(S) \) is the number of basic internal node pairs in the network \( S \), where the set of internal node pairs includes all node pairs except the pole.

In the case of ICN modelling by a connected one-dimensional network \( S \), the given structural characteristics are interconnected by such dependencies:

\[
\kappa(S) = n - m + 2; \quad \theta(S) = m - 2. \tag{1}
\]

A discrete \( n \)-dimensional geometric space is introduced on the structure of a telecommunication network, that is, its size is determined by the number of communication links in the ICN. Depending on the aspect of ICN consideration in the introduced discrete \( n \)-dimensional space can be determined by a number of coordinate systems (CS), in which the coordinate axes are different types of basic paths [18-21]: edges, contours, node pairs, cuts, etc. In the framework of this work, such orthogonal coordinate systems will be adopted, in which the projections of tensors of the main functional parameters of ICN will be further interconnected [19, 20]:

- coordinate system of network edges \( \{v_z, z = 1, n\} \), projections of tensors in which will be denoted by an index \( v \);
- coordinate system of interpolar paths \( \{\gamma_i, i = 1, \kappa\} \) and internal node pairs \( \{e_j, j = 1, 9\} \) of the network \( S \), projections of the tensor in which will be denoted by an index \( \gamma e \). The orthogonality of these coordinate systems is justified by the fact that according to expression (1) the condition \( n = \kappa(S) + \theta(S) \) is satisfied.

In Fig. 1 shows an example of determining for a network \( S \) of basic interpolar paths and internal node pairs, when the nodes \( u_i \) and \( u_0 \) were the poles, and the main structural characteristics took the following values: \( n = 12, \kappa(S) = 5, \theta(S) = 7 \).
In the introduced \( n \)-dimensional space, the telecommunication network relative to each separately selected packet flow, for which it is necessary to obtain conditions for ensuring the quality of service, can be described using a mixed divalent tensor [17-20]:

\[
Q = T \otimes \Lambda,
\]

where \( \otimes \) is the tensor multiplication operator; \( T \) is the univalent covariant tensor of average packet delays; \( \Lambda \) is the univalent contravariant flow intensity tensor in the coordinate paths of the network.

Expression (2) is sometimes appropriate to present also in index form:

\[
q_{ij} = \tau_j \lambda^i, \quad (i, j = 1, n),
\]

where \( \tau_j \) is average packet delay along the \( j \)-th coordinate path (s); \( \lambda^i \) is the average intensity of the packets flow that are transmitted along the \( i \)-th coordinate path (1/s). In the general case, the components of the mixed divalent tensor \( Q \) (2) are interconnected using the corresponding metric tensors [17-20]:

\[
T = EA \quad \text{and} \quad \Lambda = GT
\]

where \( E \) is the two-dimensional metric tensor; \( G \) is the doubly contravariant metric tensor.

In the index form, expressions (4) take the following form:

\[
\tau_j = e_i \lambda^i \quad \text{and} \quad \lambda^i = g_{ji} \tau_j, \quad (i, j = 1, n).
\]

The tensor equations (4) in one or another coordinate system take the corresponding vector-matrix form. For example, in the coordinate system of network edges, tensor equations (4) will take the following form:

\[
T_e = E_v \Lambda_v \quad \text{and} \quad \Lambda_v = G_v T_v,
\]

where \( \Lambda_v \) and \( T_v \) are the projections of tensors \( \Lambda \) and \( T \) in the coordinate system of the edges, respectively, which are represented by \( n \)-dimensional vectors of flow intensity and average packet delay in the ICN communication links; \( E_v = \| e_i \| \) is the projection of the double covariant metric tensor \( E \) in the coordinate system of edges, which is represented by the diagonal matrix of the size \( n \times n \); \( G_v = \| g_{ij} \| \) is the projection of the double contravariant metric tensor \( G \), which is also represented by the corresponding diagonal matrix of the size \( n \times n \).

In this case, the following rule holds:

\[
E_v = [G_v]^{-1},
\]

where \([ \ ]^{-1}\) is matrix transposition operation.

Similarly, in the coordinate system of the interpolar paths and internal node pairs of the network, tensor equations (4) have the following form:

\[
T_{\gamma e} = E_{\gamma e} \Lambda_{\gamma e} \quad \text{and} \quad \Lambda_{\gamma e} = G_{\gamma e} T_{\gamma e},
\]

where \( \Lambda_{\gamma e} \) and \( T_{\gamma e} \) are the projections of tensors \( \Lambda \) and \( T \) in the coordinate system of the interpolar paths and internal node pairs, which are represented by \( n \)-dimensional vectors of flow intensity and average packet delay in the corresponding interpolar paths and internal node pairs of ICN; \( E_{\gamma e} = \| e_{\gamma i} \| \) is the projection of the double covariant metric tensor \( E \) in the coordinate system of the interpolar paths and internal node pairs, which is represented by the diagonal matrix of the size \( n \times n \); \( G_{\gamma e} = \| g_{\gamma ij} \| \) is the projection of the double contravariant metric tensor \( G \) in the coordinate system of the interpolar paths and internal node pairs, which is represented by the corresponding diagonal matrix of the size \( n \times n \).

By analogy with (7), the following rule holds:
\[ E_{\gamma\varepsilon} = [G]_{\gamma\varepsilon}^{-1}. \] (9)

As is known [17-20], the projections of tensors in various coordinate systems are interconnected using linear transformation laws. Then, the transformation of the projection coordinates of the covariant tensor \( T \) during the transition from the coordinate system of the interpolar paths and internal node pairs to the coordinate system of the network edges is carried out as follows:

\[ T^\varepsilon = A^\varepsilon_{\gamma\varepsilon} T_{\gamma\varepsilon}, \] (10)

where \( A^\varepsilon_{\gamma\varepsilon} \) is covariant transformation matrix of the size \( n \times n \) for these bases.

Tensor projection of average packet delays in the basis of interpolar paths and internal node pairs, which is represented by an \( n \)-dimensional vector \( T_{\gamma\varepsilon} \), has the following structure [18, 19]:

\[
T_{\gamma\varepsilon} = \begin{bmatrix}
\tau^\gamma \\
M & M \\
\tau^\nu \\
M & M
\end{bmatrix} ;
T^\gamma = \begin{bmatrix}
\tau^\gamma \\
M & M \\
\tau^\nu \\
M & M
\end{bmatrix},
\] (11)

where \( \tau^\gamma \) is a \( \kappa \)-dimensional vector of average packet delays packets along the basic interpolar network paths of the network; \( T^\nu \) is a \( \Theta \)-dimensional vector of average packet delays packets between nodes that form the basic internal node pairs; \( \tau^\nu_p \) is a average packet delay along the \( j \)-th base pole path (\( \gamma_j \)); \( \tau^\nu_p \) is a average packet delay between the nodes that form the \( p \)-th basic internal node pair (\( \varepsilon_p \)).

The law of contravariant coordinate transformation when changing the considered coordinate systems can be described by a nonsingular matrix \( C^\nu_{\gamma\varepsilon} \) of the size \( n \times n \) [19, 20]:

\[ \Lambda_{\gamma\varepsilon} = C^\nu_{\gamma\varepsilon} \Lambda_{\gamma\varepsilon}, \] (12)

where \( \Lambda_{\gamma\varepsilon} \) is a \( \kappa \)-dimensional vector of flow intensities along the basic interpolar network paths; \( \Lambda_{\varepsilon\varepsilon} \) is a \( \Theta \)-dimensional vector of flow intensities between nodes that form the basic internal node pairs; \( \lambda^\nu_j \) is the intensity of the flow along the \( j \)-th basic pole path (\( \gamma_j \)); \( \lambda^\nu_p (t) \) is the intensity of the flow that enters the network and leaves the network through nodes that create the \( p \)-th basic internal node pair (\( \varepsilon_p \)).

Matrices of covariant and contravariant coordinate transformations when changing these bases are related by orthogonality conditions

\[ C^\nu_{\gamma\varepsilon} (A^\varepsilon_{\gamma\varepsilon})^T = I, \] (14)

where \( I \) is the identity matrix of size \( n \times n \); \([ \cdot ]^T \) – operation of matrix transposition. The rules for the formation of covariant and contravariant coordinate transformation matrices of the introduced tensors when changing the described coordinate systems are described in detail in [17-20].

The metric properties of the geometric space introduced in the ICN structure directly depend on the following main functional parameters of the network and its elements:

- characteristics of network traffic: the number of flows, their intensity (packet rate), packet length, etc.
- router interface settings: bandwidth, the degree of congestion, the maximum size (\( N \)) and the congestion of the queue buffer, etc.
- considerations of network traffic:

Consider an example where the functioning of the interfaces of network routers is modeled by the queuing system (QS) \( M/M/1/N \), which is adequate in the conditions of overload of ICN. Then the average packet delay in the \( i \)-th communication link, which is the corresponding coordinate of the projection of the tensor \( T \) in the coordinate system of the network edges (\( T_e \)), can be calculated using the formula [13-15, 18-20]:

\[ \tau_i = \frac{\rho_i - \rho_i^{N+2} - (N+1)\rho_i^{N+1}(1-\rho_i)}{\lambda_i(1-\rho_i^{N+1})(1-\rho_i)}, \] (15)
where \( \varphi_i \) and \( \rho_i \) are the bandwidth and the utilization coefficient of the \( i \)th communication link, respectively:

\[
\rho_i = \frac{\lambda_i}{\varphi_i}, \quad \rho_i < 1,
\]

where \( \lambda_i \) is the total intensity of all packet flows that are sent to the \( i \)th ICN communication link. If expression (15) is generalized to the entire set of communication links and reduced to a vector-matrix form (6), then the projection coordinates of the double-contravariant metric tensor \( G \) in the coordinate system of the network edges can be represented by the values of the diagonal elements of the matrix \( v_G \) [17-21]:

\[
g^{ij} = \frac{\lambda_i (1 - \rho_i^{N+1}) (1 - \rho_i)}{\rho_i - \rho_i^{N+2} - (N + 1)\rho_i^{N+1} (1 - \rho_i)}, \quad \text{where} \quad \lambda_i \text{ is the intensity of the packets flow, which is considered from the point of view of constructing the tensor model (3), in the } i \text{th ICN communication link.}
\]

In [19, 20], examples of metrization of the tensor TCM model were demonstrated in the case of simulating the operation of the QS interface SS/M/1/N, where the SS symbol indicates self-similarity of the incoming flow.

For internal (non-polar) nodes, which are transit in relation to the analyzed packet flow, all coordinates \( \lambda_j \) of vector \( \lambda_\epsilon \) (13) determine for each \( j \)th node the total intensity of the stream of lost packets over all its interfaces. Then the condition for ensuring quality of service in terms of the probability of packet loss takes the form

\[
\sum_{j=1}^{a} \lambda_j \leq \lambda_{\text{(req)}} \cdot P_{\text{(req)}},
\]

where \( \lambda_{\text{(req)}} \) is the average intensity of the packet flow (1/s) at the entrance to the ICN, in the interests of which a tensor model is constructed (3); \( P_{\text{(req)}} \) is the admissible probability of packet loss of this flow in ICN. In fact, the values \( \lambda_{\text{(req)}} \) and \( P_{\text{(req)}} \) determine the QoS requirements for the level of bandwidth and reliability that ICN should provide for this flow.

Further studies will be based on the fact that the average end-to-end delay of packets that are transmitted between a given pair of routers (network poles) using set of routes \( P \), is calculated by the formula

\[
\tau_{MP} = \sum_{p=1}^{|P|} x_p \tau_p,
\]

where \( x_p \) is the fraction of the packet flow that was successfully delivered to the destination router using the \( p \)th path; \( \tau_p \) is the average delay of packets that were transmitted along the \( p \)th path in ICN; \( |P| \) is the power of the set \( P \), the value of which determines the total number of paths available for routing.

In general, expression can be used for calculation \( x_p \)

\[
x_p = \frac{\lambda_p}{\lambda^*},
\]

where \( \lambda_p \) is the intensity of the flow of packets, which were successfully delivered to the destination router using the \( p \)-th path; \( \lambda^* \) is the intensity of the flow of packets, which were successfully delivered to the destination router using all available paths from set \( P \).

If no packet loss in ICN, then \( \lambda^* = \lambda_{\text{(req)}} \). At the same time, the condition for ensuring QoS requirements for the average end-to-end delay is as follows:

\[
\tau_{MP} \leq \tau_{\text{(req)}}.
\]

According to expressions (8)–(10), (12), (14) it is possible to write down the law of transformation of the projections of the double covariant tensor \( E \) when the coordinate systems change from the basis of edges to the basis of interpollar paths and internal node pairs:

\[
E_{\text{ICN}} = (C_{\text{ICN}}^\dagger)^\text{T} E_{\text{ICN}} C_{\text{ICN}}^\text{T}.
\]

Then, taking into account (11) and (13), equation (8) can be represented in such a vector-matrix form:

\[
\begin{bmatrix}
T_T \\
T_E
\end{bmatrix}
= \begin{bmatrix}
E^{(1)}_{\text{ICN}} & E^{(2)}_{\text{ICN}} \\
E^{(3)}_{\text{ICN}} & E^{(4)}_{\text{ICN}}
\end{bmatrix}
\begin{bmatrix}
\Lambda_T \\
\Lambda_E
\end{bmatrix}
\]

at

\[
\begin{bmatrix}
E^{(1)}_{\text{ICN}} & E^{(2)}_{\text{ICN}} \\
E^{(3)}_{\text{ICN}} & E^{(4)}_{\text{ICN}}
\end{bmatrix}
= E_{\text{ICN}},
\]
where \( E^{(1)}_{\gamma \epsilon} \) is the square sub-matrix of the size \( \kappa \times \kappa \); 
\( E^{(2)}_{\gamma \epsilon} \) is the square sub-matrix of the size \( 9 \times 9 \); 
\( E^{(3)}_{\gamma \epsilon} \) is the sub-matrix of the size \( 9 \times \kappa \). From expression (23) we can obtain the formula

\[
T_{\gamma} = E^{(1)}_{\gamma \epsilon} \Lambda_{\gamma} + E^{(2)}_{\gamma \epsilon} \Lambda_{\epsilon}.
\]

To obtain an expression equivalent to (19), for first multiply the left and right sides of formula (24) by \( \lambda_\gamma \), 
and then divide them by \( \lambda^* \). As a result of the transformations, the following equality is true:

\[
\tau_{MP} = \frac{1}{\lambda^*} \left( \Lambda_{\gamma} E^{(1)}_{\gamma \epsilon} \Lambda_{\gamma} + \Lambda_{\epsilon} E^{(2)}_{\gamma \epsilon} \Lambda_{\epsilon} \right). \tag{25}
\]

Taking into account inequality (21), the condition for ensuring quality of service by the average packet delay and bandwidth under conditions of probable network congestion it will take the form:

\[
\tau_{\{\text{req}\}, \epsilon} \geq \Lambda_{\gamma} E^{(1)}_{\gamma \epsilon} \Lambda_{\gamma} + \Lambda_{\epsilon} E^{(2)}_{\gamma \epsilon} \Lambda_{\epsilon}. \tag{26}
\]

Inequalities (18) and (26) determine the conditions of protection for ensuring the quality of service when transmitting packets along the primary path or multipath, which can include any network elements – links and routers in the ICN.

The advantages of the conditions (18) and (26) are as follows:

- interconnected coverage of simultaneously many different types of QoS-indicators: packet transmission rate, average end-to-end delay (26) and probability of packet loss (18);
- taking into account the basic structural and functional parameters of the network, traffic characteristics and packet service disciplines;
- an analytical form of conditions that allows their use in optimizing traffic management processes, such as routing, distribution and reservation of channel and buffer resources, etc.;
- invariance of the type of conditions obtained, which are valid for a wide class of initial data on the structural and functional construction of the network, traffic characteristics and packet service disciplines used on routers, including in conditions of congestion and possible packet loss when implementing both singlepath and multipath routing strategies, algorithms for static and/or dynamic distribution of network resources.

Then to implement a fast rerouting strategy, it is necessary to introduce conditions (18) and (26) for backup multipath in the network.

In general, the method of obtaining the desired conditions for ensuring the Quality of Service for a set of the mentioned indicators, but already along the backup multipath, is similar to the one described above, however, has several important features.

To do this, it is also necessary to build a tensor model of an infocommunication network \( S \), in which the network structure already defines the discrete \( \pi \)-dimensional space, where \( \pi \) is the number of communication links in the network except for those links that are to be protected.

Let’s agree that all other parameters introduced in the tensor model of the ICN (1)-(26) and referred to the calculation of the set of backup multipath will be denoted by the same variables, but with an overline.

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In this case, similar to expressions (1)-(26), the condition for ensuring quality of service in terms of the probability of packet loss, when using a backup route, takes the form:

\[
\sum_{j=1}^{\overline{n}} \overline{\tau}_{\{\text{req}\}}^{(j)} \leq \lambda_\gamma^{\{\text{req}\}} P_{\{\text{req}\}}, \tag{27}
\]

where \( \overline{n} \) is the number of internal node pairs in the network \( S \), which, in the general case, can be less than \( n \) (1) if the nodes of the network are protected during FRR.

The conditions for ensuring QoS in a set of indicators for each \( k \) th flow of packets, but already now the backup multipath, can be represented as follows:

\[
\tau_{\{\text{req}\}}^{(k)} \geq \overline{\Lambda}_{\gamma} E^{(1)}_{\gamma \epsilon} \overline{\Lambda}_{\gamma} + \overline{\Lambda}_{\epsilon} E^{(2)}_{\gamma \epsilon} \overline{\Lambda}_{\epsilon}, \tag{28}
\]

where \( \overline{\lambda} \) is the intensity of the flow of packets, which were successfully delivered to the destination router using all available backup paths; 
\( \overline{E}^{(1)}_{\gamma \epsilon} \) and \( \overline{E}^{(2)}_{\gamma \epsilon} \) are components of the projections of the metric tensor \( \overline{E}_{\gamma \epsilon} \) in the interpolar paths and internal node pairs coordinate system; 
\( \overline{\Lambda}_{\gamma} \) is the vector of flow intensities along the basic interpolar network paths; 
\( \overline{\Lambda}_{\epsilon} \) is the vector of flow intensities between nodes that form the basic internal node pairs, but introduced on the network structure \( \overline{S} \).
Flow-based Fast ReRoute model in the infocommunication network

In the general case, as shown in [11, 15, 16], the structure of the mathematical flow-based Fast Rerouting (FRR) model in ICN contains the following conditions, which, when formulating the corresponding optimization problems, act as restrictions or optimality criteria. These conditions include:

- conditions of flow conservation on separate routers, which are part of the primary and backup path and in ICN as a whole;
- conditions for preventing overload of network communication links;
- lack of loops in the calculated routes; conditions of load balancing for available network resources;
- conditions of the implementation of a single or multipath routing;
- conditions of the protection of network elements and its bandwidth;
- conditions for ensuring guaranteed quality of service in ICN.

As the basic flow-based routing model in ICN, a model was selected, which is described in detail in paper [11, 15, 16]. Therefore, let the ICN structure be modelled, which is described in detail in [11, 15, 16], by a directed graph \( \Gamma = (U,W) \), where \( U = \{u_i, i = 1,m \} \) is a set of vertices (nodes) that simulate network routers, and \( W = \{w_{i,j}, i,j = 1,m; \ i \neq j \} \) is a set of arcs of a graph \( \Gamma \), each of arc \( w_{i,j} \) which simulates the communication link connecting the \( i \)th and \( j \)th routers, and \( W = \{w_{i,j}, i,j = 1,m; \ i \neq j \} \) is a set of arcs of a graph \( \Gamma \), each of arc \( w_{i,j} \) which simulates the communication link connecting the \( i \)th and \( j \)th ICN routers. Thus, the basic modeling model uses non-end-to-end numbering of communication links (as in previous chapters), and double – according to the numbers of adjacent routers. That is, each edge \( v_{i,j} \) of the network \( S \) can be associated with an arc \( w_{i,j} \) of the graph \( \Gamma \). Also let’s denoted \( w_{i,j} \) as the bandwidth (1/s) of the link \( w_{i,j} \), which is actually determined by the bandwidth of the \( j \)th network interface on the \( i \)th router of the ICN.

Let a lot of flows of packet \( K \) circulate in a multiservice network generated by the corresponding network applications. Then, in the process of solving fast rerouting problems in the network, it is necessary to calculate a set of route variables \( x^{i,j}_{k} \) and \( \pi^{i,j}_{k} \), each of which quantitatively determines the fraction of the packet flow that is sent from the \( i \)th to the \( j \)th router via the link \( (i,j) \) that comprises the primary or backup path (multipath), respectively. However, not all packets arriving on this or that interface will be transmitted by the corresponding link. Some of these packets may be lost, dropped from the queue due to its overflow. Therefore, we denote by \( p^{i,j}_{k} \) and \( \bar{p}^{i,j}_{k} \) the probability of packet loss of the \( k \)th flow on the \( j \)th interface of the \( i \)th router when it is used by the primary or backup path respectively. Then the intensity of the \( k \)th flow of packets that are dropped (lost) on the \( j \)th interface of the \( i \)th router, which are part of the primary or backup path respectively, can be calculated using the following formula

\[
\varrho^{i,j}_{k} = \lambda^{\text{loss}}_{k} x^{i,j}_{k} p^{i,j}_{k} \quad \text{and} \quad \bar{\varrho}^{i,j}_{k} = \lambda^{\text{loss}}_{k} x^{i,j}_{k} \bar{p}^{i,j}_{k}.
\]

(29)

Accordingly, the intensities of successfully transmitted (i.e., lossless) packets of the \( k \)th flow in the communication links, that comprises the primary or backup path (multipath), and which is modelled by arc \( w_{i,j} \), are determined as follows

\[
\lambda^{i,j}_{k} = \lambda^{\text{req}}_{k} x^{i,j}_{k} (1 - p^{i,j}_{k}) \quad \text{and} \quad \bar{\lambda}^{i,j}_{k} = \lambda^{\text{req}}_{k} x^{i,j}_{k} (1 - \bar{p}^{i,j}_{k}).
\]

(30)

Depending on the routing strategy that is supported on the network, on the route variables \( x^{i,j}_{k} \) and \( \pi^{i,j}_{k} \) imposed conditions:

- when implementing singlepath routing:

\[
x^{i,j}_{k} \in [0,1] \quad \text{and} \quad \pi^{i,j}_{k} \in [0,1],
\]

(31)

- when implementing multipath routing:

\[
0 \leq x^{i,j}_{k} \leq 1 \quad \text{and} \quad 0 \leq \pi^{i,j}_{k} \leq 1.
\]

(32)

To ensure the connectivity of the calculated routes, the conditions of the flow conservation on a network routers, which are part of the primary path should be fulfilled [11, 15, 16]:

\[
\sum_{j:w_{i,j} \in W} x^{k}_{i,j} = 1, \quad k \in K, \quad u_{i} = s_{k},
\]

\[
\sum_{j:w_{i,j} \in W} x^{k}_{i,j} - \sum_{j:w_{i,j} \in W} x^{k}_{j,i}(1 - p^{k}_{j,i}) = 0, \quad k \in K, \quad u_{i} \neq s_{k}, d_{k},
\]

\[
\sum_{j:w_{i,j} \in W} x^{k}_{j,i}(1 - p^{k}_{j,i}) = b^{k}, \quad k \in K, \quad u_{i} = d_{k},
\]

where \( s_{k} \) is the router-source (sender); \( d_{k} \) is the router-destination of packets of the \( k \)th flow; \( b^{k} \) is the
fraction of the $k$ th flow of packet that was successfully transmitted (delivered) by the network from the routersource to the router-destination using the primary path (multipath).

Constraints similar to conditions (33) are also imposed on routing variables of the backup path:

\[
\begin{align*}
\sum_{j \in \mathcal{M}_j, e \in \mathcal{W}} \bar{x}_{i,j} & = 1, \\
& \text{for all } k \in \mathcal{K}, \quad u_i = s_k, \\
\sum_{j \in \mathcal{M}_j, e \in \mathcal{W}} \bar{x}_{i,j} & - \sum_{j \in \mathcal{M}_j, e \in \mathcal{W}} \bar{x}_{j,i}(1 - \bar{p}_{j,i}) = 0, \\
& \text{for all } k \in \mathcal{K}, \quad u_i \neq s_k, d_k, \\
\sum_{j \in \mathcal{M}_j, e \in \mathcal{W}} \bar{x}_{j,i}(1 - \bar{p}_{j,i}) & = \bar{b}^k, \\
& \text{for all } k \in \mathcal{K}, \quad u_i = d_k,
\end{align*}
\]

where $\bar{b}^k$ is the fraction of the $k$ th flow of packet that was successfully transmitted (delivered) by the network from the router-source to the router-destination using the backup path (multipath).

Conditions of flow conversation on network routers (33) and (34) have non-linear character since in the general case the probability of packet loss is a nonlinear function of both the characteristics of the traffic and the parameters of the interface itself. The mathematical formalization of expressions for calculating the probability of packet loss is usually determined by the implemented discipline of their service on the interface.

For example, if the operation of the $j$ th interface of the $i$ th node is modeled by a queuing system with failures of the form $M/M/1/N$, then the packet loss probability at the nodes interfaces of the primary ($p_{i,j}$) and backup ($\bar{p}_{i,j}$) routes can be calculated as:

\[
\begin{align*}
\bar{p}_{i,j} &= \frac{(1 - p_{i,j})(\bar{p}_{i,j})^N}{1 - (\bar{p}_{i,j})^{N+1}}, \\
p_{i,j} &= \frac{(1 - \bar{p}_{i,j})(\bar{p}_{i,j})^N}{1 - (\bar{p}_{i,j})^{N+1}}.
\end{align*}
\]

where $p_{i,j}$ and $\bar{p}_{i,j}$ the utilization coefficients of the $j$ th interface on the $i$ th router, which are part of the primary or backup path respectively, is determined by the formula

\[
\begin{align*}
\bar{p}_{i,j} &= \frac{\sum_{k \in \mathcal{K}} \lambda_{k}^{(req)} x_{i,j}^{k}}{\varphi_{i,j}}, \\
p_{i,j} &= \frac{\sum_{k \in \mathcal{K}} \lambda_{k}^{(req)} x_{i,j}^{k}}{\varphi_{i,j}}.
\end{align*}
\]

In order to prevent overloading the communication links of the network, when it is used in the primary or backup path, the following restrictions are imposed on the route variables $x_{i,j}^k$ and $\bar{x}_{i,j}^k$:

\[
\sum_{k \in \mathcal{K}} \lambda_{k}^{(req)} x_{i,j}^k < \varphi_{i,j} \quad \text{and} \quad \sum_{k \in \mathcal{K}} \lambda_{k}^{(req)} \bar{x}_{i,j}^k < \varphi_{i,j},
\]

where $\lambda_{k}^{(req)}$ is the average intensity of the $k$ th flow of packets (1/s) at the input to the ICN, the value of which directly determines the bandwidth requirements required for this flow.

Actually during implementation of restrictions (37) the fulfillment of previously introduced conditions (15) is ensured. Given the limited maximum queue length and random nature of modern network traffic, the implementation of restrictions (37) does not guarantee that the queue itself is not overloaded. In the best case, under conditions of probability of the packet loss, execution (37) actually provides only controllability of the process of dealing with overloading in ICN. Thus, in terms of the described basic model, the condition for ensuring the quality of service with the probability of packet loss (18), (27) in relation to the $k$ th flow, which is transmitted along the primary or backup multipath can be represented as follows:

\[
\begin{align*}
p_{k}^{\text{req}} & \geq p_{r}^{k} \text{ at } p_{r}^{k} = 1 - b^{k}, \\
p_{k}^{\text{req}} & \geq \bar{p}_{r}^{k} \text{ at } \bar{p}_{r}^{k} = 1 - \bar{b}^{k},
\end{align*}
\]

where $p_{k}^{\text{req}}$ is valid value of the probability of packet loss of the $k$ th flow in ICN; $p_{r}^{k}$ and $\bar{p}_{r}^{k}$ is limited value of the probability of packet loss of the $k$ th flow along the primary or backup multipath in ICN, respectively.

QoS-conditions (26), (28) and (38), (39) act as additional restrictions on route variables $x_{i,j}^k$ and $\bar{x}_{i,j}^k$.

The projections of the metric tensors $E$ and $G$ depend on the values of the route variables as follows

\[
\begin{align*}
\lambda_{z} &= \sum_{k \in \mathcal{K}} \lambda_{k}^{(req)} x_{i,j}^k \quad \text{and} \quad \lambda_{z}^{\bar{c}} = \sum_{k \in \mathcal{K}} \lambda_{k}^{(req)} x_{i,j}^k (1 - p_{i,j}^k). \\
\overline{\lambda}_{z} &= \sum_{k \in \mathcal{K}} \lambda_{k}^{(req)} \bar{x}_{i,j}^k \quad \text{and} \quad \overline{\lambda}_{z}^{\bar{c}} = \sum_{k \in \mathcal{K}} \lambda_{k}^{(req)} \bar{x}_{i,j}^k (1 - \bar{p}_{i,j}^k).
\end{align*}
\]

In expressions (40) and (41), the intensities of the flows (aggregated and separately of the $k$ th) in the same communication link in ICN that is modeled by a edge $v_z$ within the end-to-end numbering, and with double numbering – an arc $w_{i,j}$.
It is proposed to present the fast rerouting with QoS protection in an optimization form. Then, as a criterion for the optimality of the resulting routing solutions, it is advisable to choose a condition related to maximizing the overall performance of the infocommunication network:

\[ J = \sum_{k} (c^k b^k + \bar{c}^k \bar{b}^k) \rightarrow \max , \]  

(42)

where \( c^k \) and \( \bar{c}^k \) are weighting coefficients characterizing the importance (priority) of the \( k \)th flow. In this case, the condition \( c^k > \bar{c}^k \) must be satisfied so that the QoS level for any flow along the primary path is not worse than the QoS level for the same flow along the backup path.

**Calculation Example of Solving the Problem of Fast ReRouting with QoS Level Protection Based on Implementation of the Proposed Tensor Model**

To assess the adequacy of the proposed model of fast rerouting (1)-(42) and the demonstrativeness of the obtained calculation results, we will solve this problem for a fragment of the infocommunication network, which is shown on Fig. 2. Let the network under investigation consist of 9 routers and 12 communication links, indicating their capacity (1/s) in the gaps of the links.

![Fig. 2. The structure of the investigated infocommunication network](image)

The flow of packets is transmitted between \( R_1 \) and \( R_9 \) routers with the following QoS requirements:

\[ \lambda^{(req)} = 500 \text{ (1/s)}; \quad p^{(req)} = 0.02; \quad \tau^{(req)} = 150 \text{ (ms)}. \]  

(43)

In the course of solving the problem of fast rerouting, it is needed to implement the protection scheme of the router \( R_7 \) in case of its failure.

For example, the operation of each of the interfaces of the network routers was simulated by the M/M/1/N queuing system, and the buffer capacity was \( N = 50 \).

To calculate the primary multipath in the course of tensor formalization, the proposed model was used.

The choice of coordinate systems on the given fragment of the network structure (Fig. 2), shown in Fig. 1.

During the implementation of the node \( R_7 \) protection scheme, when calculating the set of backup paths, the number of available communication channels in the network will decrease to twelve, i.e. \( \pi = 10 \) with \( \beta = 6 \) and \( \pi = 4 \).

Then Fig. 3 shows the solution to the problem of calculating the set of primary multipath. The following QoS indicators were provided:

\[ \bar{p}_r = 0.0008 \text{ and } \bar{\tau}_{MP} = 64.8 \text{ ms}, \]

that satisfy the requirements (43).

![Fig. 3. The routing order of a flow of packets that is transmitted along the primary multipath](image)
\[ R^r = 0.0197 \text{ и } R^rb = 141.1 \text{ ms}, \]

which fully complies with the requirements of (43) and confirms the fact of protecting the level of quality of service in the network when applying the model proposed in the work.

The numerical example demonstrates the performance of the proposed advanced tensor approach to fast reroute with detailed geometrization of the network structure: the choice of space, coordinate systems (network edges, interpolar paths and internal node pairs), basic interpolar paths, basic internal node pairs. An example was the case of a possible failure of an arbitrary network router and (or) incident communication links. The results of studies showed that the use of the optimality criterion (42) contributed to the minimization of the primary and backup multipath used.

As a result of the solution of the optimization problem, the primary and backup multipath were obtained, along which a given level of Quality of Service was provided for the parameters of the bandwidth, the average end-to-end delay and the probability of packet loss.

**Conclusion**

The article proposed an advanced tensor approach to fast reroute with the protection of the level of quality of service under multiple parameters – bandwidth, average end-to-end delay, and packet loss probability.

Thus, the presented calculation example confirmed the adequacy of the proposed the advanced tensor approach to fast reroute (1)-(42) in terms of the implementation of protection schemes for network elements (links, nodes) and the level of Quality of Service under multiple parameters – bandwidth, average end-to-end delay, and packet loss probability.

**References**

[1] R. White, E. Banks. Computer Networking Problems and Solutions: An innovative approach to building resilient, modern networks // Addison-Wesley Professional. – 2017. – 832 p.

[2] A. S. Monge, K. G. Szarkowicz. MPLS in the SDN Era: Interoperable Scenarios to Make Networks Scale to New Services // O’Reilly Media, Inc. – 2016. – 920 p.

[3] J. Rak. Resilient routing in communication networks // Switzerland: Springer. – 2015. – 181 p. https://doi.org/10.1007/978-3-319-22333-9

[4] J. Rak, D. Papadimitriou, H. Niedermayer, P. Romer. Information-driven network resilience: Research challenges and perspectives // Optical Switching and Networking. – 2017. – Vol.23, №2. – P.156-178. https://doi.org/10.1016/j.osn.2016.06.002

[5] O. Lemeshko, O. Yeremenko, N. Tariki. Solution for the Default Gateway Protection within Fault-Tolerant Routing in an IP Network // International journal of electrical and computer engineering systems. – 2017. – Vol. 8(1). – P.19-26. https://doi.org/10.32985/ijpecs.8.1.3

[6] H. Hasan, J. Cosmas, Z. Zaharis, P. Lazaridis, S. Khwandah. Development of FRR mechanism by adopting SDN notion // 2016 24th International Conference on Software, Telecommunications and Computer Networks (SoftCOM) Proceedings. – 2016. – P.1-72.

[7] M. Pióro, A. Tomaszewski, C. Żukowski, D. Hock, M. Hartmann, M. Menth. Optimized IP-based vs. explicit paths for one-to-one backup in MPLS fast reroute // 14th International Telecommunications Network Strategy and Planning Symposium (NETWORKS) Proceedings. – 2010. – P.1-6. https://doi.org/10.1109/NETWORKS.2010.5624923

[8] A. Alashaikh, D. Tipper, T. Gomes. Supporting differentiated resilience classes in multilayer networks // 12th International Conference on the Design of Reliable
Protection Scheme for Bandwidth and Probability of Packet Loss in Software-Defined WAN // 15th International Conference (CCNC). – Las Vegas, NV. – 2011. – P.685-686.

IEEE Consumer Communications and Networking Conference (CCNC). – Las Vegas, NV. – 2011. – P.685-686. https://doi.org/10.1109/CCNC.2011.5766571

[13] O. Lemeshko, O. Yeremenko, M. Yevdokymenko, A.M. Hailan. Tensor Based Load Balancing under Self-Similar Traffic Properties with Guaranteed QoS // Advanced Trends in Radioelectronics, Telecommunications and Computer Engineering (TCSET): Proceedings of the 15th International Conference. – 25–29 February, 2020. – Lviv-Slavsk, Ukraine. – P.293-297, https://doi.org/10.1109/TCSET49122.2020.235442.

[14] O. Lemeshko, M. Yevdokymenko, A. Alsaleem. Development of the tensor model of multipath QoE-routing in an infocommunication network with providing the required Quality Rating // Eastern-European Journal of Enterprise Technologies. – 2018. – №2(95). – P.40-46. https://doi.org/10.15587/1729-4061.2018.141989

[15] O. Lemeshko, O. Yeremenko, M. Yevdokymenko. Tensor Model of Fault-Tolerant QoS Routing with Support of Bandwidth and Delay Protection // XIIIth International Scientific and Technical Conference Computer Sciences and Information Technologies (CSIT) Proceedings. – 2018. – P. 135-138. https://doi.org/10.1109/stc-csit.2018.8526707

[16] O. Lemeshko, M. Yevdokymenko, O. Yeremenko, O. Nevzorova, A. Snihurov, T. Kovalenko. Fast ReRoute Model with VoIP Quality of Experience Protection // 3rd IEEE International Conference Advanced Information and Communication Technologies (AICT). – July 2–6, 2019. – Lviv, Ukraine. – P. 1-6.

[17] G. Kron. Tensor analysis of networks // J. Wiley and Sons. – 1949. – 635 p.

[18] O. Yeremenko. Development of the dynamic tensor model for traffic management in a telecommunication network with the support of different classes of service // Eastern-European Journal of Enterprise Technologies. – 2016. – Vol. 6, Issue 9 (84). – P.12–19. DOI: 10.15587/1729-4061.2016.85602

[19] O.V. Lemeshko, O.S. Yeremenko, A.M. Hailan. QoS solution of traffic management based on the dynamic tensor model in the coordinate system of interpolyer paths and internal node pairs // Radio Electronics & Info Communications (UkrMiCo): Proceedings of the International Conference. – 2016. – Kiev, Ukraine. – P.1–6. DOI: 10.1109/UkrMiCo.2016.7739625

[20] O. Lemeshko, O. Yeremenko. Routing Tensor Model Presented in the Basis of Interpoler Paths and Internal Node Pairs // Problems of Infocommunications Science and Technology (PIC S&T): Proceedings of the Third International Scientific-Practical Conference. – 2016. – Kharkiv, Ukraine. – P.201–204. DOI: 10.1109/INFOCOMMST.2016.7905381

[21] O. Lemeshko, M. Yevdokymenko, Z. Hu, O. Yeremenko. Inter-domain routing method under normalized Quality of Service based on hierarchical coordination // CEUR Workshop Proceedings of The Third International Workshop on Computer Modeling and Intelligent Systems (CMIS). – 2020. – Vol. 2608. – P. 394-408.

Лемешко О.В., Єводокоменко М.О.

Вдосконалені тензорні підхід до швидкої перемаршрутизації із захистом рівня якості обслуговування за кількома показниками

Проблематика: В роботі запропоновано вирішення такої актуальної на сьогоднішній день проблеми, як забезпечення відмовостійкості інфокомунікаційних мереж з підтримкою необхідного рівня якості обслуговування. Запропоноване рішення засноване на реалізації вдосконаленого тензорного підходу до швидкої перемаршрутизації із захистом рівня якості обслуговування за кількома показниками.

Завдання. Метою статті є вдосконалення потокової моделі швидкої перемаршрутизації в інфокомунікаційній мережі, яка грунтується на оновлених умовах забезпечення якості обслуговування за показниками пропускної здатності, середньої міжмісцевої затримки та ймовірності втрат пакетів. Оновлені умови забезпечення якості обслуговування вдалося отримати завдяки використанню тензорного підходу до моделювання інфокомунікаційних мереж.

Методи. В якості методів дослідження використовувалися теорія графів, теорія тензорів, системи масового обслуговування. Для проведення математичного моделювання та експериментальних досліджень використовувався пакет імітаційного моделювання MatLab.
Результати. В результаті проведеного дослідження в умовах реалізації швидкої перемаршрутизації вдалося забезпечити заданий рівень якості обслуговування в інфокомунікаційній мережі. При цьому, з підвищенням QoS-вимог, завдяки вдосконаленому тензорному підходу, обновлені умови забезпечення якості обслуговування при реалізації швидкої перемаршрутизації були адекватними, що, як результат, сприяло більш ефективному використанню доступного мережного ресурсу.

Висновки. Реалізація та впровадження вдосконаленого тензорного підходу при вирішенні задачі швидкої перемаршрутизації дозволить забезпечити відповідність інфокомунікаційної мережі з захистом рівня якості обслуговування за такими показниками, як пропускна здатність, середня межкінцева затримка та ймовірність втрат пакетів.

Ключові слова: інфокомунікаційна мережа; швидка перемаршрутизація; пропускна здатність; середня межкінцева затримка пакетів; ймовірність втрат пакетів; тензор; простір; система координат.

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Усовершенствованный тензорный подход к быстрой перемаршрутизации с защитой уровня качества обслуживания по нескольким показателям

Проблематика: В работе предложено решение такой актуальной на сегодняшний день проблемы, как обеспечение отказоустойчивости инфокоммуникационных сетей с поддержкой требуемого уровня качества обслуживания. Предложенное решение основано на реализации усовершенствованного тензорного подхода к быстрой перемаршрутизации с защитой уровня качества обслуживания по нескольким показателям.

Задачи. Целью статьи является усовершенствование потоковой модели быстрой перемаршрутизации в инфокоммуникационной сети, которая основывается на обновленных условиях обеспечения качества обслуживания по показателям пропускной способности, средней межкінцевой задержки и вероятности потерь пакетов. Обновленные условия обеспечения качества обслуживания удалось получить благодаря использованию тензорного подхода к моделированию инфокоммуникационных сетей.

Методы. В качестве методов исследования использовались теория графов, теория тензоров, системы массового обслуживания. Для проведения математического моделирования и экспериментальных исследований использовался пакет имитационного моделирования MatLab.

Результаты. В результате проведенного исследования в условиях реализации быстрой перемаршрутизации удалось обеспечить заданный уровень качества обслуживания в инфокоммуникационной сети. При этом, с повышением QoS-требований, благодаря усовершенствованному тензорному подходу, обновленные условия обеспечения качества обслуживания при реализации быстрой перемаршрутизации были адекватными, что, как результат, способствовало более эффективному использованию доступного сетевого ресурса.

Выводы. Реализация и внедрение усовершенствованного тензорного подхода при решении задачи быстрой перемаршрутизации позволит обеспечить отказоустойчивость инфокоммуникационной сети с защитой уровня качества обслуживания по таким показателям, как пропускная способность, средняя межкінцевая задержка и вероятность потерь пакетов.

Ключевые слова: инфокоммуникационная сеть; быстрая перемаршрутизация; пропускная способность; средняя межкінцевая задержка пакетов; вероятность потерь пакетов; тензор; пространство; система координат.