Investigation on the Modal Strain Energy for Dynamic Analysis of Steel-Concrete Vertically Mixed Structures

Jiang Qian¹, Zhi Zhou*² and Wei Huang²

¹ Professor, State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, China
² Doctoral Candidate, State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, China

Abstract

For a vertically mixed structure composed of two parts, a lower part made of concrete and an upper part made of steel, the damping matrix of the structure is non-proportional. This paper investigates the consequences of using the modal strain energy method to estimate equivalent viscous damping ratios for such an irregular structure. First, a steel-concrete vertically mixed structure is simulated by an equivalent SDOF-SDOF oscillator using the first modal characteristics of each subsystem. The equivalent modal damping ratios are plotted as functions of the mass ratios and frequency ratios of the two subsystems. The equivalent modal damping ratios predicted by the MSE method are confirmed by exact time history analysis under various seismic ground motions. The analysis indicates that the MSE method can properly represent the displacement responses of the subsystems and that the displacement error is mainly concentrated around a frequency ratio of Rω = 0.5-1.5. Finally, a formula is derived to obtain the relative steady-state error of the structural response under harmonic excitation. It is concluded that higher modes may generate greater response errors in modal superposition when the seismic spectrum provides intense amplification at high frequencies.

Keywords: vertically mixed structure; subsystem; modal strain energy; equivalent viscous damping ratio; harmonic load

1. Introduction

In recent years, the vertically mixed structure has become an accepted approach for high-rise buildings to meet the demands for varied functions and novel configuration. It consists of at least two parts, of which the lower part is usually made of concrete and the upper part is made of steel. Additionally, its lateral stiffness, which varies along its vertical direction, can satisfy the requirements of structural deformation under wind and earthquake loads. Several applications of this type of structure can be found in practice. For example, the Wuhan International Security Building in China comprises a reinforced concrete structure below the 6th story and a steel braced frame structure above, as shown in Fig.1.a. Another common application is stadiums, in which the structural part of spectator seats is made of concrete frames or concrete dual wall-frame systems and is used to house the auxiliary facilities. The cover over the seats which rests on top of the concrete part is often constructed using steel trusses, as shown in Fig.1.b.

In a vertically mixed structure, components of different materials over the height of the structure have distinct energy dissipation mechanisms, which makes the structural damping force different from the inertial force and elastic force. The damping matrix is non-proportional and is difficult to determine. Consequently, a classical modal analysis, which would be very convenient for practical design, does not yield a diagonal normalized damping matrix and complex eigenmodes are thus needed to avoid a full time history analysis (Ikago, K. et al., 2012). In the first attempt to combine complex eigenvalues with an error estimation procedure, Papageorgiou and Gantes (2010) applied the equivalent modal damping ratio for irregularly damped structures to real-valued analyses. Although the exact solution can be computed by the complex method, this approach is rarely used in analysis and design processes involving commercial software by engineers because it is difficult to understand from physical viewpoints in practical engineering.

However, at the initial stage of structural design, especially with commercial software such as ETABS, SAP2000 and PKPM, the modal damping ratio as a fundamental input parameter must be determined for response spectrum analysis. Therefore, the equivalent modal damping ratio is needed for the design of a vertically mixed structure. One of the most popular approaches for computing the effective damping ratio
is the modal strain energy (MSE) method. This method, which utilizes undamped mode shapes, is provided in many commercial and academic software packages, such as ABAQUS, MIDAS and PERFORM-3D. This method is also used in building code documents, such as Guide Specifications for Seismic Isolation Design in the USA (2010) and the Manual for Menshin Design of Highway Bridges in Japan (1994). Shen et al. (1995) performed shake table tests of reinforced concrete frames, and added viscoelastic dampers after they were damaged. In order to predict the response of the damaged structures after the implementation of the dampers, they proceeded with a modification of the strain energy method. The MSE method was used to compute equivalent modal damping ratios of structures with added viscous damping by Charney and McNamara (2008). The results of this study showed that the MSE method consistently produced increasingly effective damping with increased damper capacity, which could significantly overestimate the damping ratios for certain systems.

Although the MSE method can represent the energy dissipation appropriately, it is still an approximation because it is based on an assumption that the real part of the complex modal vector is zero and the imaginary part corresponds to the modal vector of the undamped system (Lu et al., 2012). The application of this method to steel-concrete vertically mixed structures has been rarely studied. In this work, the vertically mixed structure is first substituted by a representative SDOF-SDOF (single degree of freedom) oscillator model, of which each part is modeled as SDOF with the dynamic characteristics of the subsystem’s fundamental mode, as proposed by Huang et al. (1996) and Papageorgiou and Gantes (2011). The MSE method is used to estimate the equivalent modal damping ratios provided by the 2-DOF oscillator model with different dynamic properties in each part. Then, the equivalent modal damping ratios predicted by the MSE method are confirmed by exact time history analysis under various seismic ground motions. Eventually, a formula is derived to obtain the relative steady-state error of the structural response under harmonic excitation by using the MSE method to study the error of each subsystem from the mode number and frequency.

2. Damping Matrix of Vertically Mixed Structures

The damping matrix of vertically mixed structures is proposed theoretically by Clough and Penzien (2003) and Chopra (2007). A damping matrix is directly assembled from those of component subsystems. For each subsystem, the Rayleigh type-damping matrix is adopted, which is well established in practical engineering applications for systems composed of unitary materials.

The stiffness and mass matrices of the combined system are shown in Fig.2.a and 2.b respectively, with the contributions of the steel frame located in the upper left corner of the combined matrices and that of the concrete frame in the lower right corner. The common degrees of freedom at the interface between the two subsystems (designated as areas "X" in the figure) include contributions of both the steel

![Steel trusses](image1)

(a) Wuhan International Securities Building

![Concrete spectator seats](image2)

(b) Stadium with the spectator seats and the covering

![Fig.1. Engineering Project](image3)

![Fig.2. Assembly of Combined System's Property Matrices](image4)

computed from Eq. 2. Eventually, the damping matrix of the combined system is obtained by Eq. 3.

\[
\begin{bmatrix}
\alpha_i \\
\beta_i
\end{bmatrix} = \frac{2\xi}{\omega_m + \omega_n} \begin{bmatrix} \omega_m & \omega_n \\ \omega_n & 1 \end{bmatrix}, i = c, s
\]

(1)

\[
C' = \alpha_c M' + \beta_c K', i = c, s
\]

(2)

\[
C = \sum_i C', i = c, s
\]

(3)

### 3. Modal Strain Energy Method

The modal strain energy method is often used to estimate equivalent viscous damping ratio for a vertically mixed structure. The damping ratio of SDOF is defined as the ratio of the dissipated energy to the total energy storage of the structure under harmonic load (Chopra, 2007) by

\[
\xi = \frac{E_D}{4\pi E_s}
\]

(4)

where \(\xi\) = the modal damping ratio; \(E_D\) = the dissipated damping energy; and \(E_s\) = the strain energy. Both \(E_D\) and \(E_s\) are shown in Fig. 3.

\[\text{Fig. 3. Definition of Energy Loss } E_D \text{ in a Cycle of Harmonic Vibration and Maximum Strain Energy } E_s\]

As the viscous damping theory illustrates, the generalized eigenvalue problem can be solved to obtain the corresponding eigenvalue and eigenvector for a multi-degree of freedom (MDOF) structure. Based on the one-circle vibration energy of the \(j^{th}\) mode (Eq. 5) and the maximum deformation energy (Eq. 6), the equivalent damping ratio can be achieved for each mode \(j\) as shown in Eq. 7

\[
E_D = \int_0^{2\pi} \frac{1}{2} C \dot{\mathbf{x}} \dot{\mathbf{x}}\, dt = \pi \omega_j \mathbf{\Psi}_j^T C \mathbf{\Psi}_j
\]

(5)

\[
E_s = \frac{1}{2} \mathbf{\Psi}_j^T K \mathbf{\Psi}_j
\]

(6)

\[
\xi_j = \frac{\mathbf{\Psi}_j^T C \mathbf{\Psi}_j}{2\omega_j \mathbf{\Psi}_j^T M \mathbf{\Psi}_j}
\]

(7)

where \(\xi_j\) = the modal damping ratio of the \(j^{th}\) mode; \(C\) = the damping matrix of the structure from Eq. 3; \(\mathbf{\Psi}_j\) = the undamped modal shape of the \(j^{th}\) mode; \(M\) = the mass matrix of the structure; and \(K\) = the stiffness matrix of the structure.

### 4. Modal Damping Ratio

An MDOF vertically mixed structure as shown in Fig. 4. features a steel frame structure (denoted by S) standing on top of a reinforced concrete frame structure (denoted by C). Because of the different energy dissipation mechanisms of the two subsystems, the vertically mixed structure is represented by an equivalent SDOF-SDOF oscillator according to Huang et al. (2011). The dynamic properties of each DOF are determined by the fundamental frequencies of each subsystem. Thus, the Rayleigh coefficients of this irregular damping model are implemented by only two natural frequencies. The damping ratios of the equivalent structure, which is \(\xi_s = 0.05\) and \(\xi_s = 0.02\), are relevant to the parts of the complete structure.

To characterize the response of the system relevant to the properties of the two subsystems, the frequency ratio \(R_\omega\) and mass ratio \(R_m\) of the vertically mixed structure are defined as Chen and Wu (1999) proposed:

\[
R_\omega = \frac{\omega_s}{\omega_c}, R_m = \frac{M_s}{M_c}
\]

(8)

where \(\omega_s\) and \(\omega_c\) are the fundamental frequencies of its substructure and superstructure respectively, and \(M_s\) and \(M_c\) are the first mode masses of its substructure and superstructure respectively, which are achieved by modal normalization of the largest element equal to 1.

\[\text{Fig. 4. MDOF Vertically Mixed Structure and Equivalent 2-DOF Structure}\]
First, the distribution of the modal damping ratios of different structural parameters is analyzed. For the generality of the analysis and practical needs for functional diversity, the frequency ratio $R_m$ ranges from 0 to 3 and the mass ratio $R_s$ ranges from 0 to 1. The parameters of the system are chosen as $M = 30000\text{Kg}$, $\omega_s = 20\text{rad/s}$, $\xi_s = 0.05$, and $\xi_R = 0.02$. And then the complete 2-DOF structure can be formed after assigning the properties to the system.

The equivalent modal damping ratios are calculated by the MSE method based on the viscous damping theory, as shown in Fig.5. Fig.6. shows the modal damping ratios with the complex eigenvalues presented by Papageorgiou and Gantes (2010). These two methods have similar results, and the difference between the two approaches results from the diagonal normalized matrix assumption used in the procedure for estimating the damping ratio of the MES method. As the results show in Fig.5., when frequency ratios are small ($R_m < 0.75$), the first mode damping ratio of the entire structure is near the damping ratio of its superstructure, while the second mode damping ratio of the structure is near that of its substructure. It indicates that the dynamic amplification factor is small, with very flexible superstructure relative to the substructure. In other words, the internal distortion of the relatively rigid substructure has no effect on the superstructure under the first mode, which corresponds with the results obtained by Biggs (1971). When frequency ratios are larger ($R_m > 2.0$), the first modal damping ratio is hardly relevant to $R_s$ and the second modal damping ratio is near that of its superstructure. It can be demonstrated that the first order vibration is dominated by the vibration of its superstructure and the second order vibration by its substructure when the frequency ratio $R_m < 0.75$. When the frequency ratio $R_m > 2.0$, it ends in the opposite way.

5. Performance of MSE Under Seismic Loads

To evaluate the response error of the vertically mixed structures when the modal damping ratio is calculated by the MSE method, 15 earthquake accelerograms from PEER (acceleration response spectra are shown in Fig.7. and seismic parameters are listed in Table 1.) are selected from Site type III for time-history analysis, according to the Chinese Code (2010). The records are normalized to have a peak ground acceleration equal to 1 m/s$^2$. For each pair of $R_m$ and $R_m$, a complete time history analysis is performed, as described by

$$M \ddot{y} + C \dot{y} + K y = -M \ddot{\xi}_g$$

where $M$, $C$ and $K$ are the mass, damping, and stiffness matrices of the structure, respectively; $y$ is the vector of the relative displacements of the structure's DOF with respect to its base; and $\xi = [1 \ 1]^T$ is the spatial
### Table 1. Seismic Record and the Ground Motion Parameter

| No. | Name                        | Station             | Component | Predominant period (s) | PGA (g) | PGV (cm/s) | PGD (cm) |
|-----|-----------------------------|---------------------|-----------|------------------------|---------|------------|---------|
| 1   | Cape Mendocino 1992/04/25   | 89156 Petrolia      | PET-UP    | 0.44                   | 0.16    | 24.50      | 31.78   |
| 2   | Chi-Chi, Taiwan 1999/09/20  | CHY006              | CHY006-N  | 0.46                   | 0.35    | 42.80      | 15.18   |
| 3   | Chi-Chi, Taiwan 1999/09/20  | CHY028              | CHY028-N  | 0.28                   | 0.82    | 67.00      | 23.28   |
| 4   | Coyote Lake 1979/08/06      | 57382 Gilroy Array#4| G04270    | 0.28                   | 0.25    | 23.10      | 2.60    |
| 5   | Duzce, Turkey 1999/11/12    | Bolu                | BOL000    | 0.32                   | 0.73    | 26.40      | 23.07   |
| 6   | Imperial Valley 1940/05/19  | 117 El Centro Array#9| I-ELC180  | 0.46                   | 0.31    | 29.80      | 13.32   |
| 7   | Imperial Valley 1979/10/15  | 5054 Bonds Corner   | H-BCR230  | 0.38                   | 0.78    | 45.90      | 14.89   |
| 8   | Kocaeli, Turkey 1999/08/17  | Duzce               | DZC180    | 0.38                   | 0.31    | 58.80      | 44.11   |
| 9   | Loma Prieta 1989/10/18      | 1028 Hollister City Hall | HCH099  | 0.82                   | 0.25    | 38.50      | 17.83   |
| 10  | Northridge 1944/01/17       | 90055 Valley-Katherine Rd | KAT090 | 0.32                   | 0.64    | 37.80      | 5.09    |
| 11  | Supersitin Hill (B) 1987/11/24 | 11369 Westmorland Fire Sta | B-WSM090 | 0.40                   | 0.17    | 23.50      | 13.00   |
| 12  | Supersitin Hill (B) 1987/11/24 | 1335 El Centro Imp.Co.Cent | B-ICC000 | 0.22                   | 0.36    | 46.40      | 17.50   |
| 13  | San Fernando 1971/02/09     | 135 LA-Hollywood Stor Lot | PELO90  | 0.24                   | 0.21    | 18.90      | 12.40   |
| 14  | Parkfield 1966/06/28        | 1013 Cholame#2      | C02065    | 0.62                   | 0.48    | 75.10      | 22.49   |
| 15  | Erzincan, Turkey 1992/03/13 | 95 Erzincan         | ERZ-EW    | 0.30                   | 0.50    | 64.30      | 22.78   |

distribution of the force. These results are considered to accurately represent the response of the 2-DOF structure because the exact damping distribution, i.e. 5% of the substructure and 2% of the superstructure, is considered.

With the equivalent modal damping ratio $\xi$ having been evaluated by the MSE method, the modal damping matrix and the equation of motion can be written as

$$D_d = \begin{bmatrix} \xi_1 & 0 \\ 0 & \xi_2 \end{bmatrix}$$

(10)

$$C_d = (\Phi^T)^{-1} D_d \Phi^{-1}$$

(11)

$$M \ddot{y} + C_d \dot{y} + K y = -M \ddot{x}_g$$

(12)

where $M$, $C_d$ and $K$ are the mass, damping, and stiffness matrices, respectively; and $y_g$ is the approximation of the response of the 2-DOF structure.

The maximum displacements of the exact and approximate procedures are obtained respectively, and the relative displacement response error of each subsystem under the $i^{th}$ earthquake wave can be acquired by

$$e_{\text{disp}}^{p,i}(t) = \frac{\max[y_p^{p,i}] - \max[y_d^{p,i}]}{\max[y_d^{p,i}]}. p = c,s$$

(13)

To obtain an overview of the error distribution over the $R_c$-$R_m$ plane, the mean absolute values of the errors for the 15 earthquake records are extracted by Eq. 14 for each pair of the eigenfrequencies and mass ratios. The distribution of the overall displacement error over the $R_c$-$R_m$ plane is shown in Fig.8.

$$e_{\text{disp}}^{p} = \frac{1}{N} \sum_{i=1}^{N} e_{\text{disp}}^{p,i} . p = c,s$$

(14)

Under earthquake loads, the maximum error of the substructure is about 8% and that of the superstructure is about 2%. The mean error distribution is likely concentrated in a limited area of the $R_c$-$R_m$ plane, especially in the range of $R_c = 0.5$~1.5 and more obviously in the substructure with greater errors (Fig.8.a). The result could occur because the modal periods of the structure are close to each other under these combinations of frequency ratios and mass ratios. Moreover, some of the seismic records may cause intense amplification in short periods that correspond with the second mode of the 2-DOF system. Thus, the higher modes may generate greater response errors in the modal superposition of the equivalent modal damping ratios calculated by the MSE method. Nevertheless, when the 2-DOF structure is subjected to earthquake loads, the errors obtained by the MSE method are considered sufficiently small for design.

### 6. Steady-State Error Under Harmonic Excitation

The MSE method can be used to determine the damping ratio of each mode of a vertically mixed structure. These damping ratios could be used directly in a modal response history analysis of the structure. In the third set of the analyses, in order to obtain the relative steady-state error of the structure under harmonic excitation using the MSE method, a formula is derived to analyze the error from mode number and frequency.
The general equation for the forced vibration motion of an MDOF system is

$$M \ddot{x} + C \dot{x} + K x = F(t)$$  \hspace{1cm} (15)$$

with the initial conditions \(x(0) = x_0\) and \(\dot{x}(0) = \dot{x}_0\).

Then the modal coordinate transformation of Eq. 15 can be accomplished by multiplying the eigenvector matrix \(\Phi\) at its left and \(f(t) = \Phi^T F(t)\) as

$$x = \Phi q$$  \hspace{1cm} (16)$$

$$\ddot{q} + D \dot{q} + \Omega q = f(t)$$  \hspace{1cm} (17)$$

$$D = \Phi^T C \Phi$$  \hspace{1cm} (18)$$

where \(D = \text{modal damping matrix; } \Phi = [\Psi_1, \Psi_2, \ldots, \Psi_n]\) is the modal-shape matrix; and \(\Omega = \text{diag}[\omega_1, \omega_2, \ldots, \omega_n]\) is the spectral matrix.

After the equivalent modal damping ratio is determined by Eq. 7, the modal damping matrix and the equation of motion can be written as

$$D_{\text{eq}} = \begin{bmatrix} \xi_1 & 0 & \ldots & 0 \\ 0 & \xi_2 & \ldots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \ldots & \xi_n \end{bmatrix}$$  \hspace{1cm} (19)$$

$$\ddot{q}_{\text{eq}} + D_{\text{eq}} \dot{q}_{\text{eq}} + \Omega q_{\text{eq}} = f(t)$$  \hspace{1cm} (20)$$

where \(q_{\text{eq}}\) is the approximate solution of the equivalent damping ratio calculated by the MSE method. Since \(D_{\text{eq}}\) is a diagonal matrix, Eq. 20 is composed of \(n\) linearly independent equations and a modal response history analysis can be used to calculate the response of the structure under external loads.

The displacement error is defined as

$$e(t) = q(t) - q_{\text{eq}}(t)$$  \hspace{1cm} (21)$$

The error response equation can be obtained by subtracting Eq. 20 from Eq. 17 as follows:

$$\ddot{e}(t) + D \dot{e}(t) + \Omega e(t) = -(D - D_{\text{eq}}) \ddot{q}_{\text{eq}}(t)$$  \hspace{1cm} (22)$$

Complex modal analysis of the vibration of damped system by Ma et al. (2010), Woodhouse (1998) and Li and Li (2002) indicates that eigenvalues and eigenvectors of the viscous damped system exist in conjugate pairs. Under the harmonic load of a specific frequency, the damped system will have modal damping vibration at that frequency, and then transfer function of the damped system can be derived. Likewise, if Eq. 15 is under harmonic load, it is still subject to a harmonic load at the same frequency after modal coordinate transformation. This assumes that the load after modal coordinate transformation is

$$f(t) = F(t)e^{j\omega t}$$  \hspace{1cm} (23)$$

where \(F(t)\) = the load amplitude; the spatial distribution of the force \(i = [1, 1, \ldots, 1]^T\); and \(i = \sqrt{-1}\).

The steady-state displacement response of the original equation of motion (Eq. 15) is

$$Q(\omega) = H(\omega) F(t)$$  \hspace{1cm} (24)$$

\(H(\omega)\) is the frequency-response function:

$$H(\omega) = (\Omega - \omega^2 I + i\omega D)^{-1}$$  \hspace{1cm} (25)$$

The steady-state displacement response of Eq. 20 is

$$\tilde{Q}(\omega) = H_{\text{eq}}(\omega) F(t)$$  \hspace{1cm} (26)$$

\(H_{\text{eq}}(\omega)\) is the frequency-response function of Eq. 20:

$$H_{\text{eq}}(\omega) = (\Omega - \omega^2 I + i\omega D_{\text{eq}})^{-1}$$  \hspace{1cm} (27)$$

The steady-state displacement response of Eq. 22 is

$$E_{\text{eq}}(\omega) = -i\omega H_{\text{eq}}(\omega)(D - D_{\text{eq}}) Q(\omega) = -i\omega H_{\text{eq}}(\omega)(D - D_{\text{eq}}) H(\omega) F(t)$$  \hspace{1cm} (28)$$

The steady-state displacement response of each DOF of the MDOF structure can be determined by

$$E(\omega) = -i\omega \Phi H_{\text{eq}}(\omega)(D - D_{\text{eq}}) H(\omega) F(t)$$  \hspace{1cm} (29)$$

The relative displacement error of the \(j^{th}\) DOF of the structure under harmonic load of frequency \(\omega\) can be measured by

$$\gamma_{\text{L}}(\omega) = \frac{\|\tilde{E}(\omega)\|_2}{\|\Phi Q(\omega)\|_2} = \frac{\|i\omega \Phi H_{\text{eq}}(\omega)(D - D_{\text{eq}}) H(\omega) F(t)\|_2}{\|\Phi Q(\omega)\|_2}$$  \hspace{1cm} (30)$$
Fig. 9. shows the relative displacement error of each subsystem of the 2-DOF structure under harmonic load with various circular frequencies. It indicates that the relative error of the subsystems has remarkable amplification when the excitation frequency is near the natural frequency of the structure, owing to the effect of resonance. Meanwhile, the error is much smaller under other excitation frequencies. As exhibited in Fig. 9., when the excitation frequency is around the second frequency, the error may be greater than that under excitation at the first frequency. It can be explained that the MSE method may significantly overestimate the damping ratios for certain systems. Moreover, when the seismic spectrum provides intense amplification at high frequencies, the higher modes may generate greater response errors in the modal superposition, which is consistent with the results above.

7. Conclusion

The response behavior of a vertically mixed structure can be very irregular and complex when it is subjected to dynamic loads. This irregularity may arise from damping properties that are difficult to identify due to different energy dissipation mechanisms of the materials distributed over the height of the structure. This paper investigates the efficiency of an analysis procedure using the modal strain energy method to estimate the elastic dynamic response of a vertically mixed structure. At first, the MSE method is adopted to estimate the equivalent modal damping ratios of a simple SDOF-SDOF oscillator model with different dynamic properties in each part. Then the damping ratios obtained from the MSE method are validated by time history analysis with several seismic ground motions. The results show that the displacement errors are concentrated in a limited area of the $R_\omega$-$R_m$ plane, especially in the range of $R_\omega = 0.5$–1.5. Moreover, they are sufficiently small, thus encouraging their application.

To study the error of each subsystem from mode number and frequency, a formula is derived to obtain the relative steady-state error under harmonic excitation. It indicates that the relevant error of the subsystems has remarkable amplification when the excitation frequency is near the natural frequency of the structure. And when the seismic spectrum provides intense amplification in high frequencies, the higher modes may generate greater response errors in the modal superposition.

Acknowledgement

The authors acknowledge with thanks support from (a) the National Key Technology R&D Program (grant No. 2012BAJ13B02); (b) the Ministry of Science and Technology of China (grant No. SLDRCE15-B-06).
References

1) American Association of State Highway. (2010) Guide Specifications for Seismic Isolation Design. USA: Transportation Officials.

2) Biggs, J. M. (1971) Seismic response spectra for equipment design in nuclear power plants. Proceedings First International Conference on Structural Mechanics in Reactor Technology, Berlin, West Germany, pp.329-343.

3) Charney, F. A. and McNamara, R. J. (2008) Comparison of methods for computing equivalent viscous damping ratios of structures with added viscous damping. Journal of structural engineering, 134(1), pp.32-44.

4) Chen, G. and Wu, J. (1999) Transfer-function-based criteria for decoupling of secondary systems. Journal of engineering mechanics, 125(3), pp.340-346.

5) Chopra, A. K. (2007) Dynamics of structures: theory and applications to earthquake engineering. 3rd Edition. USA: Prentice Hall International.

6) CMC (China Ministry of Construction). (2010) Code for Seismic Design of Buildings (GB 50011-2010). Beijing: China Architecture and Building Press. (In Chinese)

7) Clough, R. W. and Penzien, J. (2003) Dynamics of structures. Berkeley, California, USA: Computers and Structures, Inc.

8) Huang, B. C. and Leung, A. et al. (1996) Analytical determination of equivalent modal damping ratios of a composite tower in wind-induced vibrations. Computers and Structures, 59(2), pp.311-316.

9) Ikago, K. et al. (2012) Modal response characteristics of a multiple-degree-of-freedom structure incorporated with tuned viscous mass dampers. Journal of Asian Architecture and Building Engineering, 11(2), pp.375-382.

10) Li, G. Q. and Li, J. (2002) Dynamic test theory and application of engineering structure. Beijing: Science Press. (In Chinese)

11) Lu, X. L. (2007) Seismic theory and application of complex high-rise structures. Beijing: Science Press. (In Chinese)

12) Lu, X. L. and Zhang, J. (2012) Damping behavior of vertical structures with upper steel and lower concrete components. China Civil Engineering Journal, 45(3), pp.10-16. (In Chinese)

13) Ma, F., Morzfeld, M. and Imam, A. (2010) The decoupling of damped linear systems in free or forced vibration. Journal of Sound and Vibration, 329(15), pp.3182-3202.

14) Papageorgiou, A. V. and Gantes, C. J. (2010) Equivalent modal damping ratios for concrete/steel mixed structures. Computers and structures, 88(19), pp.1124-1136.

15) Papageorgiou, A. V. and Gantes C. J. (2011) Equivalent uniform damping ratios for linear irregularly damped concrete/steel mixed structures. Soil Dynamics and Earthquake Engineering, 31(3), pp.418-430.

16) PEER Ground Motion Data base. The Pacific Earthquake Engineering Research Center, Berkeley, USA. http://ngawest2.berkeley.edu/.

17) Shen, K. L., Soong, T. T., and Chang, K. C. et al. (1995) Seismic behavior of reinforced concrete frame with added viscoelastic dampers. Engineering Structures, 17(5), pp.372-380.

18) Sugita, H. and Mahin, S. A. (1994) Manual for Menshin design of highway bridges: Ministry of Construction, Japan. EERC Report No. 94/10, Earthquake Engineering Research Center, University of California at Berkeley, CA.

19) Woodhouse, J. (1998) Linear damping models for structural vibration. Journal of Sound and Vibration, 215(3), pp.547-569.