Accurate redshift determination of standard sirens by the luminosity distance space-redshift space large scale structure cross correlation

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We point out a new possibility to determine the average redshift distribution of a large sample of gravitational wave standard sirens, without spectroscopic follow-ups. It is based on the cross correlation between the luminosity-distance space large scale structure (LSS) traced by standard sirens, and the redshift space LSS traced by galaxies in preexisting electromagnetic wave observations. We construct an unbiased and model independent estimator $E_z$ to realize this possibility. We demonstrate with BBO and Euclid that, 0.1% accuracy in redshift determination can be achieved. This method can significantly alleviate the need of spectroscopic follow-up of standard sirens, and enhance their cosmological applications.

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Introduction.— Gravitational wave (GW) events of black hole (BH)/neutron star (NS)-BH/NS mergers have been detected in the nearby universe \cite{2017PRep...10a3009T, 2016PhRvL.116i1103A, 2017PASJ...69L...1T, 2017MNRAS.465.3503B} and will be detected in the distant universe by future experiments. A unique and powerful application of these GW events is to measure cosmological distance as standard sirens \cite{2017PhRvD..96d4060W}. Such measurement is based on first principles and therefore avoid various systematics associated with traditional methods of electromagnetic (EM) wave observations. They will then have profound impact on cosmology. However, to fully realize this potential, usually it requires spectroscopic follow-ups to determine redshifts of their host galaxies or electromagnetic counterparts. This will be challenging for several reasons. First, some events such as BH-BH mergers may not have EM counterparts. Second, future GW experiments are capable of detecting millions of standard sirens and the majority of them will be at $z > 1$. EM follow-ups to determine their spectroscopic redshifts will be highly challenging. Various alternatives have been proposed to circumvent this stringent need of spectroscopic follow-ups \cite{2017arXiv170704324M, 2017PhRvD..96b4036B, 2018arXiv180508920K, 2018arXiv180504287F}.

In \cite{2017PhRvD..96b4036B} we point out a new possibility. Analogous to the large scale structure (LSS) traced by galaxies in the redshift space (RS), standard sirens map the LSS in the luminosity-distance space (LDS). The LSS in this new space by itself provides the desired redshift information, through the encoded baryon acoustic oscillation (BAO) and the Alcock-Paczynski test. We estimate that 1\% level accuracy in redshift determination may be achieved for BBO (the Big Bang Observer, \cite{2013PhRvD..88h3501K, 2013arXiv1307.7145L}) or experiments of comparable capability. In the current paper, we point out that the LDS-RS LSS cross correlation can improve the redshift determination accuracy to the level of $\sim 0.1\%$, yet model independently. We design an estimator $E_z$, based on a basic property of LSS. When two LSS (overlapping in sky area) match better in their redshift distribution, their cross correlations are tighter. By design, $E_z$ reaches its global maximum only when the galaxy redshift distribution matches that of standard sirens. Therefore the determined redshift distribution is both model independent and unbiased. Finding the maximum is essentially a differential process. Therefore there is a build-in effect of cancellation of bulk statistical fluctuations, resulting into S/N higher than conventional estimations. $E_z$ then differs in the above aspects from existing proposals using cross correlation with galaxies \cite{2017PhRvD..96b4036B, 2017PhRvD..96b4036B, 2018arXiv180508920K, 2018arXiv180504287F, 2017PhRvD..96b4036B}. The time of the third generation GW experiments, there will exist galaxy surveys of $10^6$-$10^9$ spectroscopic redshifts to $z \sim 1-2$ (e.g. DESI, Euclid, PFS, WFIRST and SKA). 21cm intensity mapping \cite{2017arXiv170704324M} may probe the even more distant universe. In combination with them, the average redshift of standard sirens and its derivative $dz/d\ln D_L$ in many narrow luminosity-distance bins can be determined to $\sim 0.1\%$ and $\sim 1\%$ accuracy respectively.

The method.— Our goal is to determine the true redshift distribution $\tilde{\sigma}_{\text{GW}}(z)$ of standard sirens within a luminosity-distance bin $(D_1 \leq D_{\text{obs}} \leq D_2)$. $D_{\text{obs}}$ is the measured luminosity-distance. It has measurement error of r.m.s. $\sigma_{\ln D}$, corresponding to r.m.s. redshift error $\sigma_z$. The average distance is $\bar{D} \equiv \langle D_1 + D_2 \rangle / 2$ and the bin width is $\Delta D \equiv D_2 - D_1$. The true redshifts corresponding to $D_{1,2}$ are $z_{1,2}$ and the true mean redshift $\bar{z} \equiv (z_1 + z_2) / 2$. Due to $\sigma_{\ln D} \neq 0$ ($\sigma_z \neq 0$), the true redshift distribution is wider than $\Delta z \equiv z_2 - z_1$.

For a given galaxy redshift survey, we can apply an arbitrary weighting function in redshift ($W_{\tilde{g}}(z)$) to form a weighted galaxy sample. The following $E_z$ estimator measures the mismatch between the galaxy redshifts and
standard siren redshifts,

\[ E_z(\ell|W_g) = \frac{C_{GW-g}(\ell|W_g)}{\sqrt{C_g(\ell|W_g)}}. \tag{1} \]

Here \( C_{GW} \), \( C_{GW-g} \) and \( C_g \) are the corresponding angular power spectra respectively. Notice that the cross correlation is measured only using standard sirens overlapping in sky with the galaxy survey. The expectation value of \( E_z \) is \( r \sqrt{C_{GW-g}} \). \( r \) is the cross correlation coefficient between the two LSS. Since \( C_{GW} \) is a fixed quantity, better match in redshift distribution of the two LSS results into larger \( r \) and larger \( E_z \). Therefore the redshift distribution of the weighted galaxy sample maximizing \( E_z \) tells us the true redshift distribution of standard sirens. Fig. \[ \] shows the dependence of \( E_z \) on the galaxy redshift distribution. Indeed, when the galaxy distribution has the same \( \bar{z} \) and \( \Delta z \) as the standard sirens, its derivatives become zero and \( E_z \) reaches maximum. Notice that \( C_{GW-g} \) does not have this desired property.

The above argument can be proved more rigorously. The surface number overdensity of standard sirens and galaxies are

\[ \delta^2 \Sigma_{GW}(\bar{z}) = \Sigma_{GW}^{-1} \int_0^\infty \delta_{GW}(z, \bar{z}) \hat{n}_{true}^{GW}(z) dz \]
\[ \delta^2 \Sigma_g(W_g) = \Sigma_g^{-1}(W_g) \int_0^\infty \delta_g(z, \bar{z}) n_g(z) W_g(z) dz \tag{2} \]

Here \( \Sigma_{GW}^{-1} = \int_0^\infty \hat{n}_{true}^{GW}(z) dz = \int D_L^2 \hat{n}_{obs}^{true}(P_L^{'}) dD_L^{obs} \) is the mean surface number density of standard sirens. \( \hat{n}_g \) is the mean galaxy number density distribution, fixed by the given spectroscopic redshift survey. The weighted galaxy sample has mean surface number density \( \Sigma_g = \int_0^\infty \hat{n}_g(z) W_g(z) dz \). The angular power spectra are

\[ C_{GW-g} = \int P_{GW-g}(k = 4 \frac{\ell}{\chi(z)}, z) \hat{n}_{true}^{GW} n_g W_g \chi^{-2} \frac{d\chi}{\chi} dz \]
\[ C_g = \int P_g(k = 4 \frac{\ell}{\chi(z)}, z) n_g^2 W_g \chi^{-2} \frac{d\chi}{\chi} dz \tag{3} \]

\( \chi \) is the comoving radial distance. \( P_g \) and \( P_{GW-g} \) are the 3D galaxy and galaxy-GW host galaxy power spectrum respectively. The above expressions adopt a flat universe and the Limber approximation. But the proof holds otherwise. Varying \( E_z \) with respect to \( W_g \), we obtain

\[ \delta E_z = \int n_g \chi^{-2} \frac{d\chi}{\chi} \times \delta W_g(z) \]
\[ \left( P_{GW-g} \hat{n}_{true}^{GW} - P_g \frac{C_{GW-g}}{C_g} \hat{n}_g W_g \right). \tag{4} \]

The solution to maximize \( E_z \) (\( \delta E_z/\delta W_g = 0 \)) is

\[ W_{g}^{max}(z|\ell) \propto b_{GW/g}(z) \left( \frac{\hat{n}_{GW}(z)}{\hat{n}_{true}^{GW}(z)} \right). \tag{5} \]

Therefore for each multipole \( \ell \), we have an estimation of the true redshift distribution,

\[ \hat{n}_{GW}(z) \propto n_g(z) W_g^{max}(z) \propto b_{GW/g}(z) \hat{n}_{true}^{GW}(z). \tag{6} \]

Here \( b_{GW/g}^{GW}(z) \equiv P_{GW-g}(k, z)/P_g(k, z) \) and \( k = \ell/\chi(z) \). In the above expressions, we have ignored several normalization factors, since the overall normalization is fixed by the total number of observed standard sirens. For the same reason, the overall amplitude of \( b_{GW/g}(z) \) is irrelevant. But its redshift variation does matter. It biases the estimated average redshift by \( \delta \bar{z} = b' [\Delta z^2/12 + \sigma^2] \). Here \( b' = d \ln b_{GW/g}/dz \) at \( z = \bar{z} \). We have verified the excellent agreement between this prediction and the numerical result from the maximum likelihood fitting described later. BBO is able to achieve \( \sigma_{ln D} \sim 0.02 \) (\( \sigma_{z} = 0.8 d_{ln D} \) at \( z = 1 \)). This allows us to choose narrow luminosity distance bin with \( \Delta z \sim 0.04 \). Therefore \( \delta \bar{z} \sim 4 \times 10^{-4} b' \). If the host galaxies of standard sirens and EM galaxies are of the same population, \( b' = 0 \). Otherwise, we expect \( |b'| \lesssim 1 \) since it may only vary over cosmic time scale. Therefore this systematic bias is statistically insignificant, and will be neglected hereafter.

Physically, we do not need to vary \( W_g \) as a completely free function. The reason is that there are only limited degrees of freedom in \( \hat{n}_{GW}(z) \). It is fixed by the known PDF \( p(D_L|D_L^{obs}) \) of distance measurement error and the
$D_L$-$z$ relation to be determined,

$$\tilde{n}_{GW}(z) = \int_{D_L}^{D_2} \frac{dD_L}{dz} p(D_L|D_L^{\text{obs}}) \tilde{n}_{\text{obs}}(D_L^{\text{obs}}) dD_L^{\text{obs}}. \quad (7)$$

Since the $D_L$-$z$ relation is smooth, it is naturally described by the Taylor expansion around $D_L z(D_L) = \tilde{z} + z'(D_L - \bar{D})/\bar{D} + \cdots$. Here $z' \equiv dz/d\ln D_L$. Given the Taylor expansion coefficients $\lambda = (\tilde{z}, \tilde{z}', \cdots)$, we obtain $\tilde{n}_{GW}(z|\lambda)$ using Eq. (7). Correspondingly,

$$W_6(z|\lambda) = \frac{\tilde{n}_{GW}(z|\lambda)}{\bar{n}_g(z)} , \quad E_z = E_z(W_6(z|\lambda)) . \quad (8)$$

Therefore instead of varying a free function $W_6$, we only need to vary a few parameters in $\lambda$. For narrow bins of $\Delta D/\bar{D} \sim 0.05$ ($\Delta z \sim 0.04$ at $z = 1$) that we consider, the Taylor expansion to the linear order is accurate to $\sim 0.01\%$. Therefore we adopt $\lambda = (\tilde{z}, \tilde{z}')$, namely the mean redshift and the slope of the redshift-distance relation.

**The constraints.**— To avoid model dependence on LSS of standard sirens and galaxies, we do not fit $E_z$. Instead we only use the model independent condition that $\partial E_z / \partial \lambda = 0$ if the galaxy redshift distribution matches that of standard sirens. Therefore the (post-processed) data set we will fit is $D \equiv \partial E_z / \partial \lambda$. The corresponding likelihood is

$$p(\lambda|D) \propto p(D|\lambda)p(\lambda) \propto \exp \left( -\frac{1}{2} \Delta \chi^2 \right) , \quad \Delta \chi^2 = D C^{-1} D^\tau = \sum_{\ell} \left( \frac{\partial E_z}{\partial \lambda_\alpha} C_{\alpha\beta}^{-1} \frac{\partial E_z}{\partial \lambda_\beta} \right) \ell . \quad (9)$$

We choose a flat prior on $\lambda$. Usually the galaxy number density is orders of magnitude higher than that of standard sirens. So the covariance matrix $C$ is dominated by statistical fluctuations in $C_{GW-g}$. It is determined by both statistical fluctuations in the RS LSS and in the LDS LSS. The former may have comparable contribution from both shot noise and cosmic variance in the galaxy distribution. Therefore we have to keep both. But the later is dominated by shot noise, due to sparse standard siren distribution. Taking this approximation, we obtain

$$C_{\alpha\beta} = \frac{1}{2\ell f_{\text{sky}}} \left( \frac{4\pi f_{\text{sky}}}{N_{GW}} \right)^2 C_{\alpha\beta}^{-1} \eta_{\alpha\beta} ,$$

$$\eta_{\alpha\beta} = N_{GW} C_{\bar{g}} \int_0^\infty \tilde{W}_\alpha \tilde{W}_\beta \bar{n}_g dz \times \left( 1 + \frac{P_\ell(k = \ell/\chi(z), z)}{4\pi f_{\text{sky}}/\bar{n}_g} \chi - \frac{dz}{d\chi} \right) . \quad (10)$$

$\tilde{W} \equiv W_6/(\sqrt{C_{GW-g}})$, and $\rho_\alpha \equiv \partial / \partial \lambda_\alpha$. $\bar{n}_g$ has a specific normalization such that $\bar{n}_g(z) \equiv dN_g(<z)/dz$ is the number of galaxies per redshift interval. $\eta_{\alpha\beta} \lambda_\alpha \lambda_\beta$ is dimensionless. The second term in the parentheses quantifies the ratio of cosmic variance and shot noise.

We adopt a flat $\Lambda$CDM cosmology with $\Omega_m = 0.268$, $\Omega_\Lambda = 1 - \Omega_m$, $\Omega_b = 0.044$, $h = 0.71$, $\sigma_8 = 0.83$ and $n_s = 0.96$. We show the forecast on BBO [12, 13] and Euclid [15], as an example. We follow [13] to estimate $\tilde{n}_{GW}(z)$, but update the local NS-NS merger rate to a higher value, $R_0 = 1540$Gpc$^{-3}$year$^{-1}$ [4]. The total number of standard sirens per year is 0.33, 1.07, 1.77 at $z < 1, 2, 5$ respectively. The survey duration is adopted as 3 years. BBO has a positioning accuracy better than 1 arc-minute [13]. Therefore we will neglect the angular smoothing of LDS LSS, whose major contribution comes from $\ell \sim 100$. For $\sigma_{in,D}$, we adopt 0.02 [13] as the fiducial value. But we will also consider the cases of $\sigma_{in,D} = 0.01, 0.03$. For Euclid, we adopt the galaxy number density as 1.68(0.11) $\times 10^{-3}$(Mpc/h)$^{-3}$ at $z = 1(2)$ [15]. The sky coverage is 15000 deg$^2$ ($f_{\text{sky}} = 0.36$).

For standard sirens in the bin with $\tilde{z} = 1$ and $\Delta z = 0.04$, $\sigma_\Delta = 6 \times 10^{-4}$, and $\sigma_\Delta = 3 \times 10^{-2}$ are achievable (Fig. 2). This high accuracy in $\tilde{z}$ is surprising, given that $C_{GW-g}$ can only be measured with $\sim 100\sigma$. The reason is that, statistical fluctuations in $C_{GW-g}(\lambda + \delta \lambda)$ and $C_{GW-g}(\lambda)$ are basically the same in their common redshift range. Therefore most statistical fluctuations cancel each in $\partial E_z / \partial \lambda$. This cancellation effect is fully captured by $\tilde{W}_\alpha$ in Eq. (10) which vanishes near $\tilde{z}$. This is an intrinsic property of the $E_z$ estimator, since finding the maximum is essentially a differential process.

Around $z = 1$, cosmic variance in the RS LSS mapped by Euclid is comparable to that of shot noise fluctuations.
Therefore including other galaxy surveys helps, but not much. The errors then scale as \( b^{-1}_{\text{GW}} \bar{n}_{\text{GW}}^{-1/2} \), where \( b_{\text{GW}} \) is the density bias of standard siren host galaxies. These errors decrease with decreasing \( \sigma_{\text{in}} D \) (Fig. 3). They also depend on the bin width \( \Delta z \), or the equivalent \( \Delta z \) (Fig. 3). \( \sigma_z \) decreases with increasing \( \Delta z \), for the obvious reason that wider bin size provides better constraint on the variation of \( z \) with respect to \( D_L \). In contrast, \( \sigma_z \) first decreases with increasing \( \Delta z \) until \( \Delta z \sim 2.5 \sigma_z \), and then begins to increase with \( \Delta z \). This is caused by the competition of two effects, that larger \( \Delta z \) suppresses LSS information along the radial direction while reducing shot noise.

Combining BBO and Euclid, the mean redshift of standard sirens can be determined in many narrow luminosity distance bins over \( 0.7 < z < 2 \). At \( z > 1 \), the shot noise fluctuation gradually dominates over the cosmic variance in the galaxy distribution, due to decreasing galaxy number density. Nevertheless, \( \sigma_z = 4 \times 10^{-3} \) and \( \sigma_{z'} = 0.2 \) can still be achieved, for the bin at \( \bar{z} = 2 \) and \( \Delta z = 0.08 \). The errors now scale as \( \bar{n}_{\text{GW}}^{-1/2} \). Therefore they can be significantly reduced by including other surveys such as PFS \[21\], the billion galaxy survey of SKA \[10\] and WFIRST \[17\]. The proposed HI intensity mapping by SKA \[18\] will not only improve the redshift determination at \( z \sim 2 \), but also push it to \( z \sim 3 - 4 \).

**Discussions.**— The above proof of concept study neglects several complexities. One is the lensing magnification on \( D_L \). Its direct impact on \( \delta_{GW} \) is negligible since lensing lacks variation along the radial separation. However, it increases the effective distance measurement error \( \sigma_{\text{in}}^2 D / \sigma_{\delta z} \). Since \( \sigma_{\delta z} \sim 0.02 \) at \( z = 1 \), it may increase the error budget by 50% (Fig. 3). Since delensing with cosmic shear surveys is inefficient \[19\], this may set a lower limit on \( \sigma_{\text{in}}^2 D / \sigma_{\delta z} \) and we may only consider \( \Delta z \gtrsim 0.06 \). Nevertheless, 0.1% accuracy in \( \bar{z} \) is still feasible. Another effect neglected is the enhancement of \( \delta_{GW} \) and \( \delta_{\text{GW}} \) by coherent peculiar velocity. It enhances the effective \( b_{\text{GW}} \) by \( \sim 10\% \) \[11\], and results into a \( \sim 10\% \) reduction in the redshift errors.

The redshift determination achieved by the \( E_z \) method will allow for many cosmological applications, beyond the dark energy constraint using the \( D_L - z \) measurements. (1) With the accurately determined mean redshift, the distance duality can be determined to higher accuracy than the joint LDS and RS LSS analysis without cross correlation \[11\]. The error will be dominated by BAO measurement in the RS LSS. In term of the distance duality violation parameter \( \epsilon \), BBO and Euclid/SKA are capable of constraining \( \epsilon \) to better than 1% over a dozen bins. This will distinguish between modified gravity models such as the RR model and GR, with high significance. (2) \( \bar{z}' \) is analogous to \( H(z) \) measured by the radial BAO of galaxy surveys. It directly tells us the expansion rate at \( z' \). It is also a key quantity to break the dark energy-curvature degeneracy. (3) With the redshift determined, LDS-RS auto and cross correlations can be combined together to reduce cosmic variance in constraints of primordial non-Gaussianity and relativistic effects \[20\].

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[1] B. P. Abbott, R. Abbott, T. D. Abbott, M. R. Abernathy, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, et al., Physical Review Letters 119, 141101 (2017), 1709.00660.
[2] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, et al., Physical Review Letters 116, 061102 (2016), 1602.03837.
[3] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, et al., Physical Review Letters 118, 221101 (2017), 1706.01812.
[4] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, et al., Physical Review Letters 119, 161101 (2017), 1710.05832.
[5] B. F. Schutz, Nature (London) 323, 310 (1986).
[6] B. P. Abbott, R. Abbott, T. D. Abbott, F. Acernese, K. Ackley, C. Adams, T. Adams, P. Addesso, R. X. Adhikari, V. B. Adya, et al., Nature (London) 551, 85
[7] T. Namikawa, A. Nishizawa, and A. Taruya, Physical Review Letters 116, 121302 (2016), 1511.04638.
[8] M. Oguri, Phys. Rev. D 93, 083511 (2016), 1603.02356.
[9] R. Nair, S. Bose, and T. D. Saini, Phys. Rev. D 98, 023502 (2018), 1804.06885.
[10] S. Mukherjee and B. D. Wandelt, ArXiv e-prints (2018), 1808.06615.
[11] P. Zhang, ArXiv e-prints (2018), 1810.11915.
[12] C. Cutler and J. Harms, Phys. Rev. D 73, 042001 (2006), gr-qc/0511092.
[13] C. Cutler and D. E. Holz, Phys. Rev. D 80, 104009 (2009), 0906.3752.
[14] T.-C. Chang, U.-L. Pen, J. B. Peterson, and P. McDonald, Physical Review Letters 100, 091303 (2008), 0709.3672.
[15] L. Amendola, S. Appleby, A. Avgoustidis, D. Bacon, T. Baker, M. Baldi, N. Bartolo, A. Blanchard, C. Bonvin, S. Borgani, et al., ArXiv e-prints (2016), 1606.00180.
[16] F. B. Abdalla, P. Bull, S. Camera, A. Benoit-Lévy, B. Joachimi, D. Kirk, H. R. Kloeckner, R. Maartens, A. Raccanelli, M. G. Santos, et al., Advancing Astrophysics with the Square Kilometre Array (AASKA14) 17 (2015), 1501.04035.
[17] D. Spergel, N. Gehrels, J. Breckinridge, M. Donahue, A. Dressler, B. S. Gaudi, T. Greene, O. Guyon, C. Hirata, J. Kalirai, et al., ArXiv e-prints (2013), 1305.5422.
[18] M. Santos, P. Bull, D. Alonso, S. Camera, P. Ferreira, G. Bernardi, R. Maartens, M. Viel, F. Villaescusa-Navarro, F. B. Abdalla, et al., Advancing Astrophysics with the Square Kilometre Array (AASKA14) 19 (2015), 1501.03989.
[19] N. Dalal, D. E. Holz, X. Chen, and J. A. Frieman, ApJL 585, L11 (2003), astro-ph/0206339.
[20] J. Yoo, N. Hamaus, U. Seljak, and M. Zaldarriaga, Phys. Rev. D 86, 063514 (2012), 1109.0998.
[21] https://pfs.ipmu.jp/cosmology.html