Nonlinear Pulsations

J. R. Buchler

Physics Department, University of Florida, Gainesville, FL 32611, USA

Abstract. We review some of the recent advances in nonlinear pulsation theory, but also insist on some of the major extant shortcomings.

1. Introduction

It is a great pleasure for me to dedicate this lecture to Art Cox whose contributions to nonlinear pulsations go back to the very pioneering years of numerical hydrodynamics some 35 years ago. It is fair to say that there isn’t a type of star that Art hasn’t attempted to model.

It is of course impossible to cover in this short review all the topics of interest to nonlinear stellar pulsations and I will have to make a selection that necessary reflects my biases. There has been a huge amount of work done on stellar pulsations and I also apologize upfront for many important omissions. Extensive references are provided in the excellent reviews of Gautschy and Saio (1995, hereafter GS).

There are basically two approaches to nonlinear stellar pulsations that are complementary in some ways. The first, and oldest, is numerical hydrodynamics. While this is a 'brute force' approach, it has the advantage that with state-of-the-art physical input and numerical methods it can yield accurate information about the nonlinear pulsations of individual stellar models. The second approach is the amplitude equation formalism, and it is perhaps of a more fundamental nature (e.g. Buchler 1988). It gives a broad overview of the possible behavior, such as modal selection (bifurcations in modern language) and the effects of resonances (Buchler 1993). We note that currently this is the only tool with which we can understand nonlinear nonradial pulsations.

About 25 years ago John Cox (1975) felt that "overall, pulsation theory and its applications are in a fairly satisfactory state, except for a few disturbing problems". In the intervening time, of course, a good deal of progress has been made. However, there remain some "disturbing problems", and in fact some new ones have appeared recently.

Perhaps the most important progress came from outside pulsation theory, namely from a revision of the stellar opacities (Iglesias & Rogers 1992, Seaton, Kwan, Mihalas & Pradhan 1994), and it essentially solved two longstanding Cepheid problems. First, the so-called Cepheid bump mass problem (e.g. Art Cox 1980) essentially disappeared and the agreement with observation appears now to be quite satisfactory for the Galactic Cepheids. (Moskalik, Buchler &
Second, the beta cephei models finally became linearly unstable (cf. GS).

The very first numerical Lagrangean hydrodynamical computations of Cepheid variables were already apologetic about the poor resolution of the partial hydrogen ionization front during the pulsation. In the late 70s Castor, Davis & Davison (1977) developed the first code that could track the moving sharp features and Aikawa & Simon (1983) started to use it systematically. More recently, taking advantage of new developments in computational physics and of faster computers, several groups have developed more flexible adaptive codes (Gehmeyr & Winkler 1992, Dorfi & Feuchtinger 1991, Buchler, Kollath & Marom 1997). With these codes it is now possible to obtain a good spatial resolution that satisfactorily resolves all shocks and ionization fronts and achieves a much enhanced numerical accuracy of the pulsation. The most striking improvement is in the smoothness of the lightcurves and radial velocity curves. Fortunately, though, we do have to discard the Lagrangean results since quantities such as Fourier decomposition parameters are not substantially different from those obtained with Lagrangean codes.

However, instead of congratulating ourselves on these and other successes, it is perhaps more useful to dwell on the "disturbing problems".

2. 'Disturbing Problems'

2.1. Low metallicity Cepheids:

The microlensing projects have provided us with a large treasure trove on variable stars in the Magellanic Clouds, and since these galaxies have been found to be metal deficient compared to the Galaxy the new observations have considerably enlarged our data base, especially for the Cepheids since they should be strongly affected by metallicity content.

The Fourier decomposition parameters of the fundamental Cepheid lightcurves in the SMC (Beaulieu & Sasselov 1997) and the LMC (Beaulieu et al. 1995, Welch et al. 1995) indicate that the $\phi_{21}$ phase progression is very similar to that of the Galaxy, although it may be shifted by $\pm 1$ or perhaps 2 days in period. However, the size of the excursion in $\phi_{21}$ in the resonance region is essentially the same as in the Galaxy.

A comparison of the linear bump Cepheid models with mass–luminosity relations derived from evolutionary computations show up an irreconcilable difference with the observations (Buchler, Kollát, Beaulieu & Goupil 1996). Furthermore the nonlinear calculation of low Z Cepheid model pulsations give $\phi_{21}$ in which the size of the excursion in the 10 day resonance region almost vanishes as one goes to Z values of 0.005 (Fig. 1) (see also poster by Goupil).

As far as beat Cepheids are concerned, the hope had been that the observed period ratios could give a powerful constraint on the stellar model parameters. While globally, it might appear that the observed period ratios of the Galactic, LMC and SMC beat Cepheids are in agreement with the linear models obtained with the new opacities, when looked at in detail, i.e. by considering individual stars, the agreement is no longer as good. More seriously, (unknown) nonlinear period shift corrections of as little as 0.1% can give substantially different or uncertain mass assignments (Buchler et al. 1996).
2.2. Overtone Cepheids (s Cepheids)

The Fourier decomposition parameter $\phi_{21}$ for the first overtone Cepheids display a "Z" shape around a period of 3 days for the Galaxy (e.g. Antonello et al. 1990) and around $\approx 3.5$ and 4.0 days for the LMC (Beaulieu et al. 1995, Welch et al. 1997) and SMC (Beaulieu & Sasselov 1997), respectively. Hydrodynamical models of overtone Cepheids do not agree with the observations (Antonello & Aikawa 1993). Similar results were obtained by Schaller & Buchler in a fairly extensive survey of s Cepheids (unpublished preprint, 1994).

2.3. RR Lyrae:

Overall, the modelling of RR Lyrae pulsation gives decent agreement (except for 'double–mode' RRd pulsations) (cf GS for references), but when a more detailed comparison with observation is carried out in a systematic fashion, serious discrepancies pop up as Kovács & Kanbur (1997) show.

2.4. Population II Cepheids

BL Her stars: The modelling of these low period stars (Hodson, Cox & King 1982, Buchler & Buchler 1994; cf. also GS) shows overall agreement for the lightcurve data, but the $\phi_{21}$ are considerably smaller than the observational data indicate.

W Vir and RV Tau stars: It is now well known that the W Vir and RV Tau stars belong to the same group (Wallerstein & Cox 1984) and that the properties vary gradually from the low period, low luminosity W Vir stars to the long period, high luminosity RV Tau stars. Although the observations are not very extensive they indicate that the W Vir stars are periodic up to $\approx 15$ days from whence they start showing alternations in the cycles, alternations that become increasingly irregular with 'period'. The mechanism for this irregular behavior remained a mystery until relatively recently.

Numerical hydrodynamical modelling of sequences of W Vir models (Buchler & Kovács 1987) uncovered very characteristic nonlinear behavior that goes under the name of low dimensional chaos. Since the concept of low dimensional chaos is still very new in Astronomy we stress that this behavior is very different from a static multi-periodic, and also different from an evolving multiperiodic system. (For the reader familiar with chaos we mention that the presence of
period doubling along sequences of models quite clearly shows the presence of a horseshoe dynamics with almost one-dimensional return maps. The chaos in these models is of the stretch–and–fold type, very similar to the one that occurs in the Rössler system of 3 ODEs, e.g. Thompson & Stewart (1986).

There is however a discordance with observations, that is the onset of period doublings and chaos occurs already at 7–10 days, rather than at the ≈ 15 days indicated by the observations.

The numerical modelling of RV Tau behavior is much harder because the ratio of growth-rate/pulsation frequency is much greater. The pulsations are thus much more violent and result in sudden loss of the whole envelope in our calculations (see however Fadeyev & Fokin 1985, Takeuti & Tanaka 1995). We think that a physical dissipation mechanism is missing from our modelling, even though we solve the radiation hydrodynamics equations. Most likely turbulent dissipation plays a role in taming the pulsations of these stellar models.

The occurrence of chaos in hydrodynamical models is quite robust as we have already indicated, but could it be an artifact of the theoretical modelling, even though it was confirmed with a totally different code (Aikawa 1990). Clearly it needed to be challenged by observation. A recent nonlinear analysis that goes under the name of 'global flow reconstruction' rather conclusively shows that the irregular lightcurve of R Sct, a star of the RV Tau type, is the result of a low-dimensional chaotic dynamics (For details we refer the reader to Buchler et al. 1995, or to a didactic review Buchler 1997). More specifically, the analysis establishes that the dimension is as low as 4. In other words, the lightcurve is generated by 4 coupled ordinary differential equations. Put differently, if \( s(t) \) denotes the magnitude of the star, then at any time \( t_n \) the lightcurve is a function of only four preceding times, 

\[
s(t_n) = F[s(t_{n-1}), s(t_{n-2}), s(t_{n-3}), s(t_{n-4})]
\]

This result is quite remarkable since the pulsations of this star are quite violent (factors of 40 changes in luminosity!) with shocks and ionization fronts running about. As a physicist, though, we are not satisfied merely with this result, but would like to know what more physics we can learn about this star. A four dimensional dynamics indicates that probably two vibrational modes are involved in the dynamics. This is strongly corroborated by a linearization of the dynamics about the equilibrium that tells us that two spiral stability roots are involved, one unstable with frequency \( f_0 = 0.0068 \, \text{d}^{-1} \), the other stable with frequency \( f_1 = 0.014 \, \text{d}^{-1} \gtrsim 2 f_0 \). The physical picture that emerges is then that the irregular lightcurve of R Sct is the result of the nonlinear interaction between an unstable, lower frequency mode and a linearly stable overtone with approximately twice the frequency.

A recent nonlinear analysis of the AAVSO lightcurve of AC Her similarly shows low dimensional chaos (Kolláth et al. 1997).

To summarize, the predictions of the nonlinear hydrodynamics, viz. that the irregular behavior of these stars is due to low dimensional chaos, are thus confirmed, but better numerical modelling is necessary to achieve closer agreement with observations.

Could there be a common cause for most of the "disturbing problems"? It is very unlikely that the opacities or the equation of state are still at fault. A better treatment of radiative transfer (e.g. poster by C. G. Davis) is also not likely to fix most of the discrepancies. However, as we have already pointed
out, we seem to be missing a physical dissipation mechanism in our radiative codes. In Lagrangean codes it is necessary to include pseudo-viscosity (à la von Neumann-Richtmyer) in order to handle shocks. While this viscosity is ok for many explosive shock problems, such as supernova explosions, it unfortunately also provides artificial and unphysical dissipation elsewhere. Kovács (1990) found that when he reduced the artificial dissipation to a minimum the nonlinear pulsation amplitudes kept increasing to unrealistically large values. Similarly, when we increase the spatial resolution of the models the pulsation amplitudes also increase (Fig. 2). In fact it turns out that no combination of linear and quadratic viscosity parameters give satisfactory fundamental and first overtone models. (We hasten to add though that the Fourier decomposition parameters and the Floquet stability coefficients of the limit cycles, fortunately, are reasonably independent of these changes, so that we do not have to throw away everything we have done so far!)

3. Turbulent diffusion and convection

I had always hoped that, at least near the blue edge of the instability, the major effect of convection was static and would thus merely cause a small systematic change in the structure of the models, so that we could get away with purely radiative hydro models. (Of course it is the important dynamic effect of convection which gives rise to the red edge.) The problems and tests that we have described above however indicate that we have to include turbulent convection in the hydrocodes in order to provide a missing powerful dissipation mechanism.

Turbulent convection is of course a 3D phenomenon and at present, and for some time to come, it is not possible to run realistic 3D pulsation models. Progress has been made with relatively idealized 3D convection modelling, but it is slow and these calculations do not yet allow us to extract the 1D recipes that we need for our radial pulsation codes. In the meantime we have to rely on ad hoc 1D recipes with ad hoc parameters.

The earliest models for convection were local both in time and in space and were found to be inadequate for stellar pulsations. Today we have a family of
of time-dependent turbulent diffusion models that go back to Spiegel (1963), Unno (1967) and Castor (1968). A simplified version was implemented in a hydrodynamics code by Stellingwerf (1982). Recent applications have been made by several groups, viz. Bono et al. (1997), Gehmeyr (1992), Feuchtinger & Dorfi (1996) and by Yecko, Kolláth & Buchler (1997). The strategy has been and remains to compare the predictions of the models with observations and from thence calibrate the unknown parameters.

Our own numerical testing shows that with a turbulent diffusion model the saturation amplitude of the pulsations becomes largely independent of the zoning (Fig. 2, kindly prepared by Phil Yecko). Another positive point is that with the reduced pulsation amplitudes the shocks are absent or much weaker. This in turn allows one to reduce the artificial dissipation to a very small value.

As a word of caution, we note though that these models may still be too local in space (they only have a diffusion operator for the turbulent energy) and the existence of plumes may have to be taken into account as suggested by Rieutord & Zahn (1995).

4. Amplitude Equations

We have already pointed out that the amplitude equation formalism offers an alternative to 'brute force' numerical modelling. We would like to stress here that contrary to the claim of GS this formalism is not an Ansatz, but is a mathematically rigorous approach, namely normal form theory. Essentially, the only restriction is that the formalism applies to weakly nonadiabatic pulsators. Many of the interesting stars, viz. the classical Cepheids, the RR Lyrae, the delta Scuti and the white dwarfs definitely fall into that category. For details of the formalism as applied to stellar pulsations we refer the reader to reviews (Buchler 1988, 1993). We note also that the formalism has recently been extended to nonradial pulsations, in an Eulerian formulation by Goupil & Buchler (1994) and in a Lagrangean one by van Hoolst (1994).

In a nutshell, the formalism reduces the PDEs of hydrodynamics and radiation transfer to a small set of ODEs for the amplitudes of the excited modes. The structure of the equations is uniquely determined by the types of resonances that occur among the linear modes of oscillation; the remaining physics is all contained in the values of the nonlinear coefficients. The amplitude equations are generic and capture the essence of the behavior of the system; it is not astonishing thus that they pop up in many different areas of physics, chemistry, biology, etc..

The solutions of the amplitude equations (usually the fixed points) tell us then about the possible types of behavior for a sequence or array of stellar models. They also explain the effect of resonances on the morphology of the lightcurves and radial velocity curves. Perhaps the most useful and best known application of the formalism has been to describe the behavior of the Fourier decomposition parameters through the Hertzsprung progression of the bump Cepheids (e.g. Buchler 1993).

5. Potpourri
5.1. ‘Double-mode’ behavior

As an application of the formalism to beat (double-mode) behavior we have plotted in Fig. 3 the predictions of the amplitude equation formalism in which it is assumed that two nonresonant modes (e.g. the fundamental and first overtone) interact and can give rise to a double-mode pulsation along a single parameter sequence of models (scenario ’AB’ in Fig. 1 of Buchler & Kovács 1986). Here we denote by $A_0$ both the Fourier amplitude of the fundamental mode for the fundamental pulsators, and also the amplitude of the fundamental component in the case of double mode pulsators. The right figure shows the corresponding first overtone amplitudes. Note that the transition from single mode to double-mode occurs smoothly for both amplitudes. In a realistic sample of models one would of course get a dispersion both vertically and horizontally, but the conclusion remains unaffected.

Let us compare this now to the observations, first to the RR Lyrae in M15 of Sandage et al. as reproduced in Buchler & Kovács 1986. In their fig. 8 the first overtone amplitudes are displayed on the left as dots for the RRc and as
crosses for the RRd, whereas the fundamental amplitudes are the dots on the right for RRab and open circles for the RRd. The $A_1$ amplitudes of the RRc and RRd stars indeed vary continuously, but the $A_0$ amplitudes of the RRd are considerably smaller than those of the RRab. We are forced to conclude that a nonresonant scenario is not in agreement with the observations. To explain the jump in the fundamental amplitudes from the RRab to the RRd is is necessary to invoke the presence of a resonance. (A jump in the amplitudes might also be brought about by higher order, viz. quintic nonlinearities, but in no studies so far have such nonlinearities ever been found to play a role, and furthermore the coefficient of the cubic nonlinearity would have to have a sign opposite to its usual one.) However, no low order resonances are present in the stellar parameter range of the RR Lyrae, and it therefore has to be a higher order resonance that is involved. We note in passing that therefore these stars were better called beat RR Lyrae because more than 2 modes are involved.

Let us now turn to the Cepheids. J.-P. Beaulieu has kindly provided me with his SMC Fourier decomposition data that are displayed in Fig. 4. The first overtone amplitudes of the beat Cepheids fall right into the range of the s Cepheids, but the fundamental amplitudes again are much smaller for the beats than for the fundamental Cepheids. We are forced to interpret this to mean that a resonance must also be involved in the beat Cepheids.

5.2. The Blazhko effect

Several mechanisms for the Blazhko effect have been proposed (cf. GS), but a fully satisfactory understanding has so far defied us. In the following we want to present an observational constraint that to our knowledge has not yet been discussed. Let us define the Fourier decomposition as

$$m(t) = m_o + a \cos(\omega t + \phi_1) + b \cos(2\omega t + \phi_2) + \ldots$$

and as usual $\phi_{21} = \phi_2 - 2\phi_1$.

The lightcurves over a whole Blazhko cycle have been published by Walraven (1949) for RR Lyr and by Balázs & Detre (1939) for AR Her. In Fig. 5 we show
the variation of the pairs of Fourier parameters, a, b, and φ<sub>21</sub>. All quantities are seen to oscillate about some center, which is of particular interest for the phase φ<sub>21</sub>. The fact that the phase does not run through 2π over a cycle imposes a severe constraint that can be used to eliminate some models.

5.3. The dip in the Galactic Cepheid period histogram

In 1977 Becker, Iben & Tuggle published Cepheid period histograms for several galaxies. The histogram for the Galaxy and for M31 showed a pronounced dip in the 8–10 day period range, whereas the corresponding histograms for the LMC and SMC were devoid of such a deficiency. Trying to explain the dip on the basis of their stellar evolution calculations Becker et al. had to invoke an ad hoc double-humped birthrate function. Nonlinear calculations show that this is no longer necessary (Buchler, Goupil & Piciullo 1997). Indeed, a perhaps unexpected side effect of the new opacities is that the fundamental limit cycle of the Cepheid variables can be unstable. This instability is found to occur in the 8–10 day period range for metallicity parameters 0.013 < Z < 0.035. Note that this is consistent with a dip in the Galaxy and M31 and with the absence of a dip in LMC and SMC.

5.4. Strange Cepheids and RR Lyrae

It has recently been found that strange modes can occur even in weakly nonadiabatic stars such as Cepheids and RR Lyrae. A thorough study of the phenomenon has shown that these modes are surface modes that can be self-excited to the hot side of the blue edge of the normal Cepheid instability strip (Buchler, Yecko & Kolláth 1997). The strange modes are recurrent at higher order wave-vectors, but the lowest ones have typical periods 1/4 to 1/5 that of the fundamental pulsational mode, i.e. they have periods ranging from ≈0.2 days to ≈10 days, depending in their luminosity. Their locations in schematic HR and PL diagrams are shown in Fig. 6.

What does one expect the pulsations to look like? We have computed some nonlinear (radiative) models and find limit cycles with amplitudes in the millimag and 10–100 m/s ranges, respectively. It might be feared that, because the strange modes are surface modes their driving could be destroyed by convection.
Preliminary computations with the turbulent diffusion hydro code however indicate otherwise.

Finally we note that the same trapping and driving mechanisms also work in RR Lyrae models and that therefore strange RR Lyrae should also exist on the hot side of the RR Lyrae instability range.

6. Conclusions

The theoretical study of stellar pulsations is still faced with many challenges. We have seen that radiative hydrocodes, while giving decent agreement with many observations, are not fully satisfactory. We hope that a proper inclusion of the important dissipative effects of turbulent convection will help resolve many of the extant difficulties and discrepancies.

Acknowledgments. I wish to congratulate Joyce Guzik and Paul Bradley for the smooth organization of this excellent meeting, and I also would like to thank them for their kind financial support. This research has been supported by NSF (AST95–18068, INT94–15868) and an RCI account at the NER Data Center at UF.

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