Loan risk assessment based on Pythagorean fuzzy analytic hierarchy process

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Abstract. Risk assessments are increasingly being applied to financial institutions and banks, especially on loan risk assessments. When deciding to approve the applicant's loan or not, financial institutions or banks need to assess whether the lender has the loan conditions. Usually this is a very difficult decision. In this paper, a new integrated approach Fuzzy Pythagoras Scoring (FPS) model, including Pythagorean fuzzy analytic hierarchy process (PFAHP) and fuzzy comprehensive evaluation (FCE), is used for personal loan risk assessment. The risks of decision analysis in loans are assessed by the proposed method. Comparing the results of the traditional analytic hierarchy process (AHP) and PFAHP, it shows that the method can obtain reliable evaluation results and evaluate personal loans more accurately.

1. Introduction
Risk assessment is used to assess risks of all aspects of life, such as health, production operations, social issues and so on. Risk assessment is an important step toward identifying potential hazards and evaluating the risks associated with the hazards[1]. The purpose of risk assessment is to better reduce, avoid or otherwise manage them[2].

Loan risk assessment is a multi-attribute decision-making (MADM) problem. We need to consider the various conditions of the lender to assess the quality of the loan. As the most popular and widely used method to solve the MADM problem, the AHP and FAHP (Fuzzy AHP) methods can handle the uncertainty problem very well[3][4][5]. AHP method was first proposed to solve the MADM problems in Satty[6]. The AHP method refers to decomposing element that is always related to decision-making in goals, criteria, programs, etc. The methods of quantitative and qualitative decision analysis are combined based on the above work. The weight value of each element is calculated by the AHP method, and the importance of the element is obtained. Vahidnia[7] proposed a framework to determine the optimum site for a new hospital in the urban area by using a method combining Geographical Information System (GIS) and FAHP. In this method FAHP was used to evaluate the decision factors and their impacts on alternative sites. Tarik Aouam[8] introduced the AHP method to the bank credit risk assessment and determined if the lender has the ability to repay on time. Z.H. Che[9] applied the FAHP and Data Envelopment Analysis (DEA) methods to the loan decision of small and medium banks. Yager[10] has developed Pythagorean fuzzy sets in multicriteria decision making. Ilbahar[11] used the PFAHP method to conduct a risk assessment of occupational health and safety. Yucesan[12] adopted PFAHP to access the safety of hydropower station operation safety. GulM[13] utilized PFAHP and
VIKOR methods in occupational health and safety in manufacturing risk assessment, and the comparison with the intuitionistic fuzzy set proves the effectiveness of the method.

Different from the literature, we integrate PFHAP and fuzzy comprehensive evaluation into personal loan risk assessment. In this paper, PFAHP method is employed to loan risk assessment, and FCE method is used to get the score of the lender. First, we use the PFAHP method to calculate the severity of the metrics we choose to evaluate the loan. The weights obtained by the PFAHP method are compared with the weights obtained by the AHP method. In addition, the FCE method is used to obtain the membership matrix, and the fuzzy operation is performed with the previous weight matrix, and the result is converted into a risk score. Finally compare the risk scores obtained using the FPS and the AHP&FCE.

The rest of this paper structured as follows: the PFAHP steps, the FCE method and the FPS method are described in Section 2. The experimental procedures and results are shown in Sections 3 and 4, respectively. Section 5 is the conclusion of the experiment.

2. Method

2.1 PFAHP

As a generalization of fuzzy set, the Pythagorean fuzzy set proposed by Yager[10] considers both membership and non-membership. The following steps are implemented by PFAHP method.

- Let a set X be a universe of discourse. A Pythagorean fuzzy set P is an object having the form[14]

\[ P = \{ (x, P(\mu_p(x), v_p(x))) | x \in X \} \]

- The attribute paired comparison matrices are constructed, and the importance of each attribute is evaluated by experts according to the attribute comment table proposed by Ilbahar[11]. The value table is as shown in Table 1 below.

| Linguistic terms | Membership Degree | Non-membership Degree |
|------------------|-------------------|-----------------------|
| CL1              | 0                 | 0                     |
| VL1              | 0.1               | 0.2                   |
| LI               | 0.2               | 0.35                  |
| BAI              | 0.35              | 0.45                  |
| AI               | 0.45              | 0.55                  |
| AAI              | 0.55              | 0.65                  |
| HI               | 0.65              | 0.8                   |
| VHI              | 0.8               | 0.9                   |
| CHI              | 0.9               | 1                     |
| EE               | 0.1965            | 0.1965                |

- The difference matrix \( A = (a_{ij})_{n \times n} \) of the upper and lower values of the membership function and the non-membership function is calculated by the following equations (1) and (2)[12]:

\[ a_{ijL} = \mu_{ijL}^2 - v_{ijU}^2 \]  (1)
\[ a_{ijU} = \mu_{ijU}^2 - v_{ijL}^2 \]  (2)

- Use the equations (3) and (4) to find the interval multiplication matrix \( M = (m_{ij})_{n \times n} \):

\[ m_{ijL} = \sqrt{1000a_{ijL}} \]  (3)
\[ m_{ijU} = \sqrt{1000a_{ijU}} \]  (4)

- Calculate the determinacy value \( d = (d_{ij})_{n \times n} \) using equation (5)[12]:

\[ d_{ij} = 1 - (\mu_{ij}^2 - \mu_{ijL}^2) - (v_{ij}^2 - v_{ijL}^2) \]  (5)

- Multiply the determinacy degrees with the interval matrix \( M = (m_{ij})_{n \times n} \) yields a weight matrix \( S = (s_{ij})_{n \times n} \), which is then normalized by equation (6). Calculate each normalized priority weight \( W_i \) using equation (7):

\[ s_{ij} = \left( \frac{m_{ijL} + m_{ijU}}{2} \right) d_{ij} \]  (6)
\[ W_i = \frac{\sum_{j=1}^{n} s_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} s_{ij}} \]  (7)
2.2 Fuzzy comprehensive evaluation method

- Experts can only give qualitative assessments such as “good”, “general” and “poor”[15].
  - We construct the membership matrix by using several experts to make similar qualitative evaluation of each index[16].

2.3 Fuzzy Pythagorean scoring model

- By constructing a judgment matrix, the weight of the attribute is obtained using the PFAHP method.
  - Fuzzy operation is performed between weight matrix and membership matrix.
  - Through the set of score evaluation sets, the point multiplication operation is performed to obtain the final score.

3. Process

3.1 Datasets

We randomly selected 200 samples from the data set of Home credit default score as experimental data. And listened to the advice of the credit experts, selected 21 representative attributes from the hundreds of attributes in the data set as the attributes of the experimental data. These attributes are NAME_EDUCATION_TYPE(ORT), DAYS_BIRTH(DB), OCCUPATION_TYPE(OCT), ORGANIZATION_TYPE(ORT), DAYS_BIRTH, OBS_30_CNT_SOCIAL_CIRCLE(OC30), DEF_30_CNT_SOCIAL_CIRCLE(DC30), OBS_60_CNT_SOCIAL_CIRCLE(OC60), DEF_60_CNT_SOCIAL_CIRCLE(DC60), DAYS_EMPLOYED(DE), AMT_CREDIT(AC), AMT_ANNUITY(AA), FLAG_OWN_CAR(FOC), FLAG_OWN_REALTY(FOR), CNT_CHILDREN(CCF), NAME_FAMILY_STATUS(NFS), NAME_FAMILY_STATUS(NFS), AMT_INCOME_TOTAL(AIT), CNT_FAM_MEMBERS(CFM), NAME_HOUSING_TYPE(NHT), EXT_SOURCE_1(ES1), EXT_SOURCE_2(ES2), EXT_SOURCE_3(ES3).The data set uses 0 and 1 to represent the final loan result, 0 is the loan approved, and 1 is the loan rejected.

3.2 Hierarchical model

Known by the AHP method, we must first establish a hierarchical relationship. The 21 attributes of the data set were divided into four levels of Personal Credit (PC), Repayment Ability (RA), Loan Characteristics (LC) and Third-Party Evaluation (TPE), and a loan risk level model was established. The 21 attributes are divided into four level indicators to become the second level indicators of the loan risk assessment level model. Secondary indicators are divided as shown in the Table 2.

| Primary indicator | Secondary indicators |
|-------------------|----------------------|
| PC                | NET, OCT, ORT, DB, OC30, DC30, OC60, DC60, DE; |
| RA                | FOC, FOR, CC, NFS, AIT, CFM, NHT; |
| LC                | AC, AA; |
| TPE               | ES1, ES2, ES3. |

Table 2. Tier 1 indicator of PFAHP loan risk assessment

The judgment matrices could be constructed according to Table 1. The Table 3 is the importance of the four first-level indicators; The judgment matrices obtained by personal credit, loan characteristics, third-party evaluation and repayment ability are shown in Table 4 to Table 7, respectively.

| Primary indicator | Secondary indicators |
|-------------------|----------------------|
| PC                | EE, AAI, HI, VHI |
| RA                | BAI, EE, AAI, HI |
| LC                | LI, BAI, EE, HI |
| TPE               | VLI, LI, LI, EE |

Table 3. Judgment matrix of 1-level index

According to the method in Section 2.1, the weight vectors corresponding to the five judgment matrices are obtained. The weights of the first-level indicators, personal credit, repayment ability, loan characteristics and third-party evaluation indicators are [0.567, 0.246, 0.127, 0.060], [0.070, 0.043, 0.053, 0.059, 0.077, 0.110, 0.173, 0.339, 0.076], [0.147, 0.147, 0.107, 0.077, 0.253, 0.122, 0.147], [0.566, 0.434], [0.164, 0.561, 0.275], respectively.
### Table 4. Personal credit

| NET  | OCT  | ORT  | DB   | OC30 | DC30 | OC60 | DC60 | DE  |
|------|------|------|------|------|------|------|------|-----|
| EE   | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI |
| BAI  | EE   | AI   | AI   | AAI  | AAI  | AAI  | AAI  | AAI |
| AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI |
| AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI |
| AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI |
| AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI |
| BAI  | EE   | AI   | AI   | AAI  | AAI  | AAI  | AAI  | AAI |
| AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI |
| AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI |
| AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI |
| BAI  | EE   | AI   | AI   | AAI  | AAI  | AAI  | AAI  | AAI |
| BAI  | EE   | AI   | AI   | AAI  | AAI  | AAI  | AAI  | AAI |
| AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI |
| AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI |
| AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI  | AAI |

### Table 5. Loan Characteristics

| AC | AA |
|----|----|
| EE | AAI|
| BAI| EE |

### Table 6. Third-party assessment

| ES1 | ES | ES3 |
|-----|----|-----|
| EE  | LI | AI  |
| HI  | EE | AAI |
| AI  | BAI| EE  |

### Table 7. Repay capability

| FOC | FOR | CC | NFS | AIT | CFM | NHT |
|-----|-----|----|-----|-----|-----|-----|
| EE  | AAI | AAI| AAI | AAI | AAI | AAI |
| AAI | AAI | AAI| AAI | AAI | AAI | AAI |
| AAI | AAI | AAI| AAI | AAI | AAI | AAI |
| AAI | AAI | AAI| AAI | AAI | AAI | AAI |

After obtaining the weight of the indicator, firstly, according to the method of how to obtain the membership matrix mentioned in the previous section, 10 experts were asked to evaluate the attributes in 4 dimensions to obtain the membership matrix. Under the first-level indicator of personal credit, there are 9 secondary indicators, and the experts evaluate them according to the applicant's data, and obtain the following membership matrix.

\[
M_1 = \begin{bmatrix}
0.1 & 0.1 & 0.1 & 0.1 \\
0.2 & 0.2 & 0.2 & 0.2 \\
0.3 & 0.3 & 0.3 & 0.3 \\
0.4 & 0.4 & 0.4 & 0.4
\end{bmatrix}, \quad J = \begin{bmatrix}
0.1 & 0.1 & 0.1 & 0.1 \\
0.2 & 0.2 & 0.2 & 0.2 \\
0.3 & 0.3 & 0.3 & 0.3 \\
0.4 & 0.4 & 0.4 & 0.4
\end{bmatrix}
\]

Then, the membership matrix of personal credit and the weight of personal credit are fuzzy, that is, the 4*9-dimensional matrix is multiplied by the 9*1-dimensional matrix to obtain a new matrix.

\[
J_1 = M_1 \times [0.070, 0.043, 0.053, 0.059, 0.077, 0.110, 0.173, 0.339, 0.076]^T
\]

This \(J_1\) is a 4*1-dimensional matrix, and then multiplies \(M_2, M_3, \) and \(M_4\) by their respective weight matrices to obtain \(J_2, J_3,\) and \(J_4.\) \(J_1, J_2, J_3,\) and \(J_4\) form a 4*4-dimensional matrix \(J.\) Multiply the matrix \(J\) by the weight of the first-order indicator to obtain a matrix \(F\) of 4*1.

\[
F = J \times [0.567, 0.246, 0.127, 0.060]^T = [0.1, 0.2, 0.3, 0.4]
\]

Finally, calculate the applicant loan evaluation score, score= 0.1*100 + 0.2*80 + 0.3 *60 + 0.4*40=60. In this way, we can conduct a loan risk score for each applicant.

### 4. Experimental result

#### 4.1 Weight of indicator

The weight of each index can be obtained through the above methods. The weights obtained by AHP method are compared with that obtained by PFAHP, as shown in Figure 1. The information expressed in the figure above shows that the historical data of the lender is the dominant factor. 60 days of default, 60 days of overdue and 30 days of default, the number of overdue 30 days greatly affected the decision whether to approve the loan. And the lender’s income and the lender’s loan amount also have a great relationship with whether the loan is passed.
4.2 AHP&FCE and FPS model scores

In this experiment, we compared the results of the AHP & FCE (AHPFCE) method with the FPS method. 4 evaluation levels were set, with the values of 100(good), 80(relatively good), 60(general) and 40(poor) respectively from high to low. All samples scored between 40 and 100. The prediction labels of all samples are added in the sample sets. If the assessment agrees to the loan, the prediction label of the sample is 0; if the assessment result in a loan rejection, the prediction label of the sample is 1. Calculate the number of samples with the same prediction label and the real label, and divide it by the total number of samples to get the accuracy. The comparison is shown in the table 8 below.

Table 8. Accuracy of different thresholds under two methods

| Threshold | 72   | 73   | 74   | 75   | 76   | 77   | 78   | 79   | 80   |
|-----------|------|------|------|------|------|------|------|------|------|
| AHPFCE Accuracy | 0.950 | 0.940 | 0.920 | 0.895 | 0.885 | 0.850 | 0.810 | 0.780 | 0.740 |
| FPS Accuracy   | 0.960 | 0.950 | 0.935 | 0.915 | 0.900 | 0.875 | 0.835 | 0.800 | 0.755 |

So, the scores are between 80 and 100, the rating is good or better; Scores ranged from 60 to 80, and the evaluation is average or general; Scores ranged from 40 to 60, and the evaluation is very poor or below. Therefore, the threshold range should be set between 60 and 80 for the decision to approve the loan or reject the loan. However, in practical application, it still depends on the distribution of the final scores of data and the judgment of relevant experts. In our experiment, we set the threshold from 72 to 80, comparing the two methods from accuracy and robustness. According to the experimental results, we found that the FPS method has a maximum accuracy of 96% when the threshold is set to 72, which is higher than the AHPFCE method. Moreover, the accuracy of FPS is higher than the AHPFCE method in the set dynamic threshold interval, which shows that its robustness is better than that of the AHP method.

5. Conclusion

Conduct a risk assessment of the loan business of a bank or other financial institution to avoid economic losses caused by the inability to recover the loan. This paper combines Pythagorean fuzzy analytic hierarchy process with fuzzy comprehensive evaluation method and applies it to personal loan risk assessment. Refer to the advice of experts, the loan applicants are evaluated. At the same time, compared with the traditional AHPFCE method from the accuracy and robustness, the FPS model can obtain more reliable and accurate evaluation results. The model can well solve the problem of quantification and trend prediction of risk assessment, and is suitable for risk assessment and provides a new way for risk assessment in other fields in the future.

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