Metacognitions of Senior High Students in Solving Mathematics Problems

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Abstract. This is an ethnographic qualitative research of which three main objectives are to describe the 1) advanced, 2) intermediate, and 3) low metacognition levels in solving mathematics problem. In conducting the research, purposive sampling technique was used to determine the research subjects that were the students (i.e. the students with advanced, intermediate, and low metacognition levels) of X IPA 3 class of SMA Negeri 2 Surakarta. The data were collected through observations, documentations, and interviews in which the data validity was measured using triangulation between the sources and the methods used. Afterwards, the data were analyzed inductively. The results of this research suggested that: 1) students with advanced metacognition level are able to perform all the characteristics of awareness aspect in indicator A1, three characteristics of evaluation aspect of indicator E1, and two characteristics of regulation aspect in indicator R1, 2) students with intermediate metacognition level are able to perform four characteristics of awareness aspect in A1, two characteristics of evaluation aspects in E1 indicator, and two characteristics of regulation aspect in R1, 3) students with low metacognition level are able to perform two characteristics of awareness aspect in A1, two characteristics of evaluation aspacts in E1, and one characteristic of regulation aspect in R1.

1. Introduction

Education is one of the most important factors in life. Therefore, people with access to education will have different mindsets compared to those do not. [1] stated that education can be seen from two perspectives; education as a process and a result. Education as a process is the interaction of an individual with his environment, while education as a result is the product of that interaction which manifests in behavioral changes. To add on that, [2] explained that education is a voluntary process in achieving a desired result based on some expected intentions. Furthermore, [2] added that the purpose of education is ideal in nature whereas the product is actual. Educational performance is the manifestation of an accomplished educational purpose, thus, the performance is aligned with the prearranged educational purpose.

Compared to other subjects, mathematics is more implemented because it is used not only within the education world, but also implemented in real life. [3] stated that mathematics is a language to depict events that occur in the day-to-day life as well as complex matters in businesses, science, and
technology. In addition, Gauss [4] proclaimed mathematics as the king of knowledge. It is independent, while the developments of other subjects are based on mathematics. Hence, mathematics is essential to education as it helps develop logical, analytical, systematic, critical, and creative thinking since mathematics itself is constructed by human reasonings.

Reasoning and cognitive skills are required in mathematics problem solving. Reasoning is related to metacognition whose role is vital to mathematics problem solving. Those who intend to become a good problem solver must have a good metacognition skill [5]. [6] stated that metacognition is to think about what one’s thought process which refers to a higher order of thinking. [7] explained that someone who has an outstanding skill will also have an outstanding metacognition performance. Besides, [8] also stated that metacognition is connected to what someone already knows about himself as an individual and how he can adjust his behaviors. It means that when an individual is in awareness when he is using metacognition, he is training himself to put forward the best strategy to solve any problems.

In problem solving, metacognition can be observed and measured from several aspects and each aspect consists of some indicators. [9] explained that the research about metacognition in mathematics problem solving focused on studying behaviors that could be identified using three aspects; awareness, regulation and evaluation. Furthermore, [10] explained that awareness aspect is related to the awareness during the process of problem solving, the knowledge of specific contents, and the problem solving strategies; that includes the awareness of what someone needs to do, has done, and might do to solve the problem. Meanwhile, regulation aspect covers the cumulative knowledge that are obtained from competence and current knowledge of the on-going mental process. Lastly, evaluation aspect is related to the assessment of the thinking process, capacity, and limitation within a situation as self-attributes.

In terms of this research, those indicators were used and described as follow: A1 is the aspect of awareness as an indicator to describe what one knows about the problem; E1 is the aspect of evaluation to describe the means used in solving the problem; and R1 is the aspect of regulation to show the plans needed to take to solve the problem. These are supported by [11] who elaborated the aspects and indicators of metacognition; awareness and evaluation consist of five indicators, while regulation only has four. They are further described in Table 1.

| Number | Aspects | Indicators |
|--------|---------|------------|
| 1      | Awareness | A1. Rethinking what someone has already known about the problem.  
A2. Rethinking the questions within the problem and associate them with similar past problem.  
A3. Rethinking any unsolved problems.  
A4. Rethinking the next required steps.  
A5. Rethinking the analysis of the answers of the problem |
| 2      | Evaluation | E1. Rethinking the means used in solving the problems  
E2. Rethinking the steps taken in solving the problems.  
E3. Rethinking the answers to the problems.  
E4. Rethinking the truth about the answers.  
E5. Rethinking the flaws in setting up the answers. |
| 3      | Regulation | R1. Rethinking how to solve the problems.  
R2. Rethinking the difference in answering the problems.  
R3. Rethinking the next steps to answer the problems.  
R4. Rethinking how to change in order to solve the problems. |

Metacognition plays an important role in mathematics. It is supported by [6] who stated that metacognition is essential to ensure successful learning process. Metacognition skill encourages individuals to acknowledge their strengths and weaknesses to help them plan their studies, choose the
most suitable learning strategies and styles, and evaluate their learning performances. Additionally, [7] suggested that another importance of metacognition in achieving successful learning experience is by enabling individuals to process their cognitive skills and minimize their weak cognitive skills, which will develop a new and better set of cognitive skill. [12] concluded that the learning performances of students whose metacognition is low are below those who have advanced metacognitive skill. Hence, it can be said that better learning performances are linear to a higher level of metacognition.

The students of X IPA 3 class of SMA Negeri 2 Surakarta have different levels of metacognition in answering mathematics questions. [7] divided students’ metacognition levels into three categories as presented in Table 2.

Table 2. Level of Metacognition Skill

| Criteria | Scale |
|----------|-------|
| High     | >75   |
| Middle   | 61-75 |
| Low      | <60   |

Students’ metacognition levels describe the students’ success rates in solving mathematics problems. Students have diverse traits and skills based on the levels of their metacognition. In the process of solving a question, students with advanced metacognition level are able to write down what they already know and what the questions are asking for. Moreover, they are also able to describe and understand the problems within the questions as well as map out a correct plan to solve the problem and find the most effective ways to execute it. Finally, they can evaluate the process and draw a conclusion. This helps them obtain high scores in the learning process. Students whose initial skills are higher have more advanced metacognitive skill compared to those whose initial skills are lower. Based on those brief explanations from some experts, it can be said that metacognition plays such an immense role in mathematics problem solving. The objectives of this research were to describe the 1) advanced, 2) intermediate, and 3) lower level of metacognition in mathematics problem solving.

2. Method
This is a qualitative ethnography research in which the researcher explores and depicts the social situation [13] related to metacognition in solving mathematics problem of the students of X IPA 3 class of SMA Negeri 2 Surakarta during 2019/2020 academic year. The research subjects were 28 students from XI IPA 3 class. The subjects were drawn using purposive sampling and classified into three categories of students with advanced, intermediate, and low metacognition.

The data were collected using document analyses, participative observations, and in-depth interviews. Document analyses were used to gather the data related to mathematics scores. These data were used to categorize the students’ metacognition skills to advanced, intermediate, and low level. On the other hand, participative observations were carried out during the learning process at the school. They were used to observe the activities of the students and teachers in mathematics class. The observations were related to students’ ability to solve mathematics problems as well as teachers’ skills to get the students to make use of their metacognition.

In addition to that data collection technique, in-depth interviews were also carried out towards the teachers and the students. The students were interviewed to examine any difficulties that the students might have had in solving mathematics problems. Meanwhile, the teachers were interviewed to obtain more information related to the strategies used to get the students accustomed to using their metacognition in critical thinking process. Data were then validated using triangulation between the sources and the methods used. It was carried out inductively by collecting the data, carrying out data reduction, displaying the data, and drawing conclusion [14]. Figure 1 describes the flow of activities taken in the process of data collection to data analysis.
3. Result and Discussion

Document analysis was done to the students’ works related to quadric equation. Following is the question asked:

“A piece of rectangular cardboard will be used to make a box without a lid by cutting 3x3 cm² square of each corner of the cardboard. If the length of the base of the box is 2 cm longer that its width and the volume of the box is 105 cm³, find the length and width of the box!”

From the document analysis, the lowest score was 50 while the highest was 100. The scores from solving the quadratic equation were classified into different levels of metacognition. From the categorization, seven students (25%) were within the advanced level of metacognition, fourteen students (50%) were within the intermediate level, and seven students (25%) belong to the low level of metacognition.

3.1. Advanced level of student’s metacognition in mathematics problem solving

Figure 2 displays one of the students’ works whose metacognition level is advanced in solving a given mathematics problem.

Based on the student’s answer displayed in Figure 2 and the result of the interview, the aspect of awareness (i.e. rethinking what they have already known about the problem) of the students with advanced metacognition matches all the characteristics of that aspect indicators. The
characteristics include reading the questions multiple times, underlining the keywords, taking notes of the important clues, stating what is known and asked using symbols, making important notes, and drawing conclusion of what is known as the prerequisites to map out the plan to solve the question. This is in alignment with the research conducted by [15] in which the result showed that research subjects who had advanced metacognition level could write what was known and asked for in a question; they could also convert those information to the language of mathematics.

Another relevant study was conducted by [16] in which the result displayed that students with advanced metacognition level were able to identify provided information and the question asked; they were able to visualize it using variable and symbols as well as determining the requirements provided and needed in order to solve the problem. These students could write down the provided information using their own simpler, clear, and correct language. The teacher involved in that research suggested that students with advance level of metacognition could master every characteristics of the awareness indicators that are related to their ability to rethink what information were provided in order to solve the problem well. It was further claimed that the dominant factor was the culture of respecting others. This is in line with the research carried out by [17] in which she stated that religious culture at school could develop more positive ways of thinking, respects, and conducive learning environment which will lead to satisfactory learning process and outcome.

3.1.2. Evaluation

One of the indicators of evaluation aspect is to rethink the means used to answer a question. Students with advanced metacognition master three characteristics within that indicator. Those three characteristics are highlighting the means and steps used to solve the problem, paying attention to important clues and cases, and making correlation between familiar cases and means used to solve the problem. [18] concluded that students with advanced skill could accurately map out plans to solve the problem. They could determine formula that would suit a problem best.

To minimize obstacles in solving mathematics problems within the evaluation aspect, students need guiding to acknowledge the benefits from solving the problem. [19] supported this as he explained that students need to understand the benefits of the process, outputs or outcomes in order to get themselves used to doing it respectfully. It is important that students feel comfortable, happy, curious, respected as well as respectful. Following is an excerpt of the interview with the student with advanced metacognition; this one in particular is related to the indicators of evaluation aspect about rethinking the means used to answer the problem. (R = Researcher, S = Student).

R = “How did you answer the question?”
S = “I carefully examine the question first”
R = “And then?”
S = “Then I was looking for the known information and what was asked. Only then did I determine the ways to solve it.”

This excerpt shows that students with advanced metacognition rethink the methods they used to solve the problem by considering the important cases used in solving the problem and making connection between the information and the methods needed to take. This indicates that they master the characteristics of evaluation aspect. It is supported by [20] who said that subjects with advanced skill integrated information from the questions to their prior knowledge. Based on that research, students with advanced metacognition are able to master three characteristics within the indicator of evaluation by rethinking the means used to solve the problem well. These three characteristics include highlighting important information and steps used to solve the problem, examining important cases used to solve the problem, and making connection between known cases and means required to solve the problem.
3.1.3. Regulation

Students with advanced metacognition meet the characteristics of regulation aspect which is related to how they rethink the plans to solve the problem. These two characteristics make students check their answers multiple times before drawing conclusion and determining a proper and easy way to solve the problem. It is relevant to the research conducted by Yani et al., (2016) in which they concluded that subjects with high skills were able to integrate information within their thinking schemes to help them solve the problem. Following is another excerpt from the interview with one of the students with advanced metacognition in relation to the indicators of regulation aspect of rethinking the plans in solving a problem. (R = Researcher, S = Student).

R = “Are you sure that it is correct?”
S = “Insyaallah it is”
R = “Have you checked it?”
S = “Yes, I have”

The excerpt suggested that the student checked the answer before drawing conclusion; which is in line with the study conducted by [21] in which they stated that students with advanced skill were able to determine a proper way to solve a problem. They also proposed that students with advanced metacognition were able to meet two characteristics within the indicator of regulation aspect related to how they rethink their plans to solve the problem. These two characteristics demand students to check their answers multiple times before drawing conclusion and determining a proper and easy way to solve the problem.

3.2. Students with intermediate level of metacognition in solving mathematics problem

Figure 3 displays one of the works of the students with intermediate metacognition in solving mathematics problem.

![Figure 3](image)

**Figure 3.** One of the works of the students with intermediate metacognition

3.2.1. Awareness

The answers of students with intermediate metacognition were analyzed in relevance to the indicators of awareness aspect of rethinking what they already know about the problem. The analysis result suggested that the students met four characteristics of the indicator. Those four characteristics include reading the given question multiple times, taking notes of the problem as the known case, stating what was known and asked using symbols, and noting down as well as drawing conclusion as the prerequisite to finding ways to solve the question. This is supported by the research conducted by [22] which showed that students with intermediate skill were able to articulate what was known using their own words. Besides, [23] explained that their research subjects with intermediate skill were able to determine what was known and asked as well as model the information to solve the problem.
3.2.2. Evaluation

The result of the analysis showed that students with intermediate metacognition were able to meet two characteristics of the indicator of evaluation aspect that is related to rethinking the means used to answer the question. Following is an excerpt of the interview with one of the students with intermediate metacognition. (R = research, S = student).

R = “Does that way of problem solving match the question?”
S = “Yes, because the volume and height are provided”
R = “Alright, what’s next?”
S = “It means that I have to use the formula for the volume of a cube (p.l.t=..), so I get x.y.3=105”

This is in alignment with the research conducted by [15] in which they explained that students with intermediate level of metacognition were able to make a connection between the presented case and the steps to solve the problem.

3.2.3. Regulation

Students with intermediate metacognition level were able to meet two characteristics of regulation indicator related to how they rethink their plans to solve the problem. Those characteristics make students recheck the answers multiple times before drawing a conclusion and determining an easy and proper way to answer the question. It is supported by the results of a study conducted by [24]; the results showed that research subjects with intermediate metacognition were able to choose and determine proper plans to solve the problem.

3.3. Students with low metacognition level in solving mathematics problem

Figure 4 displays one of the works of the students with low metacognition level in solving mathematics problem.

![Figure 4](image)

3.3.1. Awareness

All the works of the students with low metacognition were analyzed and the result of the analysis suggested that they met the characteristics of the indicators of awareness related to the rethinking what was provided in that question. Those two characteristics require students to take notes of the case and express the provided information and the question using symbols. This fact is supported by the research conducted by [25]; the result indicated that students with low metacognition were able to mention what information was provided and express them using mathematics models.

3.3.2. Evaluation

From the result of students’ work analysis, it was found that students with low metacognition met two characteristics of the indicators of evaluation aspect that is related to the process of
rethinking the means used to solve the problem. Those two characteristics help students to pay attention to important cases used to solve the problem and make connection between the identified case and the means used. This is relevant with the results of a research conducted by [20] which suggested that subjects with low metacognition were able to state what were provided in the question.

3.3.3. Regulation
The results of analysis indicated that students with low metacognition met only one characteristic of the indicator of regulation that is related to rethinking the plans to solve a problem [26]. This characteristic makes students determine an easy and proper way to solve a problem. It is supported by the result of the interview between the researcher and one of the students with low metacognition level. Following is the excerpt of the interview. (R = researcher, S = student).

R = “How did you get the answer?”
S = “(smiling)”
R = “You have used the correct formula, but why didn’t you finish answering it?”
S = “I am too lazy to factor it”
R = “How come?”
S = “Because I already know that the length is 7 and the width is 5 (holding up the answer sheet)”
R = “How could you be so sure that your answer is correct?”
S = “(smiling) I got it from my friend”

From the excerpt, it was clear that the student was only able to meet one characteristic of the indicators within regulation aspect (i.e. determining an easy and proper way to answer the question). [25] supported this finding by stating that students with low metacognition skill could map out plan in solving a problem.

4. Conclusion
Students with advanced metacognition were able to meet all the characteristics of awareness aspect (i.e. rethinking what was provided in the question), three characteristics of evaluation aspect (i.e. rethinking the ways used to solve the problem) and two characteristics of regulation aspect (i.e. rethinking the plans in solving the problem). Meanwhile, students with intermediate metacognition level in solving mathematics problems were able to meet four characteristics of awareness (i.e. rethinking what was provided within the question), two characteristics of evaluation aspect (i.e. rethinking the means used to solve the problem) and two characteristics of regulation aspect (i.e. rethinking the plans needed to solve the problem). On the other hand, students with low metacognition level could only meet two characteristics of awareness aspect (i.e. rethinking what was provided within the question), two characteristics of evaluation aspect aspects (i.e. rethinking the means used to solve the problem) and one characteristic of regulation aspect (i.e. rethinking the plans needed to solve the problem).

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