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An extension for dynamic lot-sizing heuristics

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This paper presents an efficient procedure to extend dynamic lot-sizing heuristics that has been overlooked by inventory management literature and practice. Its intention is to show that the extension improves the results of basic heuristics significantly. We first present a comprehensive description of the extension procedure and then test its performance in an extensive numerical study. Our analysis shows that the extension is an efficient tool to improve basic dynamic lot-sizing heuristics. The results of the paper may be used in inventory management to assist researchers in selecting dynamic lot-sizing heuristics and may be of help for practitioners as decision support.

Keywords: EOQ; dynamic lot-sizing; heuristics

1. Introduction

Changes in the competitive environment, such as high time pressure and variations in the demand pattern of the customers, induce companies to simultaneously reduce the costs and increase the service level and quality of production and manufacturing processes. For this reason, the management of inventories is essential as inventories directly influence product and raw material availability as well as production lead time. It has a high cost impact and is therefore one of the main operational activities in the manufacturing industry. The management of inventories requires determining economic batch sizes by balancing inventory holding and setup/order costs with the objective of providing a high service level at minimal total costs. For its relevance in industry, it is not surprising that the lot-sizing problem has lost none of its attention in research and practice since the publishing of the first decision model to determine the economic order quantity (EOQ) (Harris, 1913).

The EOQ model is a simple, robust and efficient tool for companies to resolve the conflict between ordering and inventory costs (Dobson, 1988). While a plethora of extensions of the EOQ model exists and continues to be developed with a wide pallet of theoretical orientations from multi-stage models to incentives and productivity issues (see, for reviews, Brahimi, Dauzere-Prese, Najid, & Nordli, 2006; Glock, 2012; Glock, Grosse, & Ries, 2014; Jans & Degraeve, 2008), the basic case with deterministic time-varying demand, also known as dynamic lot-sizing problem, was solved more than 50 years ago (Wagner & Whitin, 1958). A typical dynamic lot-sizing decision is to determine the amount and timing of replenishment of items, with known but varying
demand and fixed ordering and linear holding costs. Although the Wagner–Whitin (WW) algorithm derives optimal solutions for this problem, it is only very infrequently used in practice where heuristics are applied more commonly. One may ask why anyone would use a heuristic when a practical optimal algorithm is available. This is partly due to the fact that the algorithm is not commonly known in practice and that heuristics are simpler and faster to compute (Boe & Yilmaz, 1983). However, proper computer implementation, meanwhile, makes it possible that the WW algorithm runs in linear time (Karimi, Fatemi Ghomi, & Wilson, 2003; Saydam & Evans, 1990). Nevertheless, dynamic lot-sizing heuristics still provide value for practical applications, research and educational exercises (Simpson, 2001). For example, material requirements planning modules in enterprise resource planning (ERP) software, to the best of the authors’ knowledge, do not provide the WW method (Bahl & Bahl, 2009). Instead, some of the heuristic methods that may lead to poor results are covered. This is consistent with the findings that poorer rules are well known in educational texts, while other more efficient rules have not received attention in research and practice (Simpson, 2001).

The paper at hand builds on this line of thought and presents an efficient extension algorithm for dynamic lot-sizing heuristics published in 1997 but inexplicably overlooked by inventory management literature. We test the extension for two basic and popular heuristics and illustrate its performance and robustness in an extensive numerical study. Thus, the intention of this paper is to introduce the algorithm and to show that it improves existing heuristics significantly. In addition, we present a practical approach as we implement the simulation study using the environment of MS Excel. This paper may support researchers in selecting heuristics for dynamic lot-sizing problems, and may be of help for practitioners to support inventory management decisions.

The remainder of the paper is structured as follows: The next section reviews related literature. Subsequently, formulations for the dynamic lot-sizing models and algorithms under study are presented in Section 3. This is followed by an extensive numerical analysis in Section 4 to evaluate the performance of the heuristics, and managerial implications are deduced from its results. Section 5 concludes the paper and provides suggestions for future research.

2. Literature review

Reviewing the overall literature on dynamic lot sizing is not within the scope of this paper due to the sheer number of works available. The reader may refer to Robinson, Narayanan, and Sahin (2009) for a review and Andriolo, Battini, Grubbström, Persona, and Sgarbossa (2014) as well as Glock et al. (2014) for recent overviews and surveys on dynamic lot sizing. In this paper, we present an overview of works that concentrated on the comparison and evaluation of dynamic lot-sizing heuristics and algorithms.

To find an optimal solution in the dynamic lot-sizing problem, dynamic programing was used (Wagner & Whitin, 1958). However, several authors developed heuristic solution procedures to solve the dynamic lot-sizing problem. A description and classification scheme of popular heuristics, such as least unit cost (LUC) (Gorham, 1968), Silver–Meal (SM) (Silver & Meal, 1973) and part-period balancing (PP) (DeMatteis, 1968), was given in the review paper of De Bodt, Gelders, and Van Wassenhove (1984). The authors noted that choosing a suitable heuristic for a specific application is not straightforward as cost structure and variability of demand determine how heuristics perform. Bitran, Magnanti, and Yanasse (1984) analytically derived worst case error bounds for LUC and PP. Early comprehensive overviews and performance evaluations of dynamic
lot-sizing procedures, including PP, SM, and WW, were developed by Blackburn and Millen (1980), Axsäter (1982, 1985), Baker (1989) and Ritchie and Tsado (1986). Blackburn and Millen (1985) added other algorithms, such as Groff’s rule (GR) (Groff, 1979), to previous performance evaluations. Another early classification scheme of basic heuristics was developed by Maes and Van Wassenhove (1988). The authors confirmed that the selection of heuristics depends on environmental characteristics and that universal conclusions about the performance of an algorithm are difficult to draw. Another comparison of dynamic lot-sizing techniques (including LUC, PP, SM and GR) with extensive numerical evaluation can be found in Zoller and Robrade (1988). A worst case and performance analysis of dynamic lot-sizing heuristics, including LUC, PP, and SM, was studied in Vachani (1992). Gupta, Keung, and Gupta (1992) compared the performance of dynamic lot-sizing heuristics, such as LUC and GR, in a multi-stage system. A comprehensive sensitivity analysis of popular dynamic lot-sizing heuristics, among them LUC and SM, was conducted by Pan (1994). Ganas and Papachristos (1997) presented an analytical evaluation and comparison of SM and PP. Kazan, Nagi, and Rump (2000) extended and compared the performance of WW and SM to account for unexpected changes in previously set production and setup schedules. Jeunet and Jonard (2000) evaluated the robustness of lot-sizing techniques, including WW, PP and LUC, taking into account uncertain environments. Simpson (2001) extensively evaluated WW, GR, SM and LUC, among others, in a computational study and concluded that there exist some inconsistencies in dynamic lot-sizing heuristics and that these algorithms are equivalent in performance. The author noted that, curiously enough, some algorithms that are known to generate poorer results are more popular in research and practice than other algorithms that outperform the well-known heuristics. A comparison of dynamic lot-sizing techniques (among them WW and SM), taking into account the so-called end-effect, was presented by Van Den Heuvel and Wagelmans (2005). Gafaar (2006) used SM as benchmark to test the applicability of a genetic algorithm to the dynamic lot-sizing problem with the special case of batch ordering. More recently, Jans and Degrave (2007) presented a summary and comparison of meta-heuristic solution algorithms for the dynamic lot-sizing problem. Ho, Solis, and Chang (2007) compared the performance of SM, PP, and least total cost (LTC) for the special case of deteriorating inventory. Teunter, Bayindir, and Van Den Heuvel (2006) and Schulz (2011) extended some heuristics, such as SM and LUC, to take into account product returns and remanufacturing considerations. Another modification of dynamic lot-sizing heuristics, among them SM, LUC, and PP, was presented by Toy and Berk (2013), who studied special ‘warm/cold’ production processes. In a current study, Baciarello, D’Avino, Onori, and Schiraldi (2013) presented a comparison of basic lot-sizing heuristics (among them PP, SM, and GR) in a computational study using the results of WW as benchmark. A recent comparison of dynamic lot-sizing algorithms, including SM and WW, for the special case of convex production costs and setups, can be found in Kian, Gürler, and Berk (2014).

From the literature review, we can deduce that most studies compared and tested the same popular heuristics, such as LUC, SM, or GR, while other heuristics and possible extensions have mainly been overlooked (Simpson, 2001). One unrecognized algorithm that extends basic dynamic lot-sizing heuristics is addressed in this paper. Leinz, Bossert, and Habenicht (1997) introduced a procedure which improves the results of stop rules using a combination of stop rules and cost comparisons. Interestingly enough, this extension algorithm has not received, to the best of the authors’ knowledge, any attention in research and practice since the publishing of the working paper (in German).
in 1997. We present a formal description of the algorithm, in the following denoted as LBH and test its performance in an extensive numerical study.

### 3. Dynamic lot-sizing algorithms under study

In this chapter, we give a short description of the dynamic lot-sizing algorithms under study. For our comparative analysis, we choose WW, LUC and GR as the literature review showed that these algorithms are popular in dynamic lot sizing. Special attention is paid to the LBH extension algorithm and how it can improve basic heuristics (here: LUC and GR).

#### 3.1. Definitions

The following definitions are made for the single-level dynamic lot-sizing problem:

- discrete time periods of equal duration over a finite planning horizon;
- demand is deterministic with time-varying feature;
- full demand occurs at the beginning of each period;
- all period demands and costs are non-negative;
- lead time is constant (fixed and known);
- order costs are fixed for every order;
- replenishment rate is infinite;
- stock is empty at the beginning and the end of the planning horizon;
- lot-splitting is not allowed;
- no shortages are allowed;
- all products are considered separately;
- holding costs only have to be determined for a stock which is carried over from one period to another;
- holding costs for carrying inventory during the consumption period are not considered.

The following terminology will be used throughout the paper:

- $l$ last period of a preliminarily considered order cycle
- $l^*$ final resulting last period which is calculated by a heuristic; the related demand $d_{l^*}$ in period $l^*$ is added to the order in period $b$
- $b$ first period of an order cycle
- $\tau_{b,l}$ order cycle range including period $b$ to $l$
- $\Delta \tau$ difference in the average order cycle range between a heuristic and WW
- $y_t$ binary tag in period $t$
- $c_{b,l}$ total cost per unit
- $c_O$ fixed cost per order
- $c_H$ holding cost per unit and period
- $C_O$ total ordering costs
- $\Delta C_O$ approximated marginal savings in ordering costs
- $C_H$ total holding costs
- $\Delta C_H$ approximated marginal-added holding costs
- $d_t$ demand in period $t$
3.2. Wagner–Whitin (WW)

The WW algorithm (Wagner & Whitin, 1958) leads to optimal solutions for the problem under study and is used in this paper to calculate reference values to evaluate the heuristic algorithms. It can be denoted as follows:

\[
C = \sum_{t=1}^{T} C_t(x_t, I_t) = \sum_{t=1}^{T} (y_t(x_t) \cdot c_D + I_t \cdot c_H) \rightarrow \text{Min!}
\]

subject to

\[
I_t = I_{t-1} + x_t - d_t
\]

\[
I_t, x_t \geq 0, \quad 1 \leq t \leq T,
\]

where

\[
y_t(x_t) = \begin{cases} 0, & \text{if } x_t = 0 \\ 1, & \text{if } x_t > 0. \end{cases}
\]

Without loss of generality, we take \(I_0 = I_T = 0\) (Baker, 1989).

Because the WW algorithm is established in the literature, the reader may refer to Wagner and Whitin (1958), De Bodt et al. (1984) and Gupta and Keung (1990), among others, for a detailed description of the algorithm.

3.3. Least unit cost (LUC)

LUC (Gorham, 1968) is a heuristic algorithm that is based on one of the characteristics of the EOQ model, which says that the order quantity is equal, regardless of whether the unit costs or the total costs are minimized. A proof is given in Appendix 1. Because of that characteristic, the LUC heuristic calculates the unit costs \(c_{b,t}\) from the beginning period \(b = 1\) and adds order quantities as long as the unit costs are decreasing. When the costs are increasing, for the first time, the cumulated order size is ordered and the calculation of the unit costs starts again in the period \(b = t^* + 1\) which is the first one not considered in the order before. The heuristic ends when the planning horizon is reached. The algorithm is expressed as follows:
The stop rule \( c_{b,l} - 1 \geq c_{b,l'} < c_{b,l' + 1} \) selects the final resulting last period \( l^* \) which leads to the following order quantity in period \( b \):

\[
x_b = \sum_{l=b}^{l^*} d_l
\]

### 3.4. Groff’s rule (GR)

One of the heuristic algorithms that has proven to lead to good results for the dynamic lot-sizing problem is GR (Groff, 1979; Simpson, 2001). It uses another feature of the optimal order quantity in the EOQ-model. For the optimal order quantity, the marginal holding costs and the marginal ordering costs are equal according to amount. A proof is given in Appendix 2. The heuristic starts in period \( b = 1 \) and ends when the planning horizon is reached. The order quantity is increased as long as the approximated marginal savings in ordering costs \( \Delta C_O \) from adding the \( l \)-th period’s demand exceed the approximated marginal holding costs \( \Delta C_H \). These two marginal costs are calculated as follows:

\[
\Delta C_O = \frac{c_O}{(l - b - 1)} - \frac{c_O}{(l - b)} = \frac{c_O}{(l - b - 1) \cdot (l - b)}
\]

\[
\Delta C_H = \frac{d_l \cdot c_H}{2}
\]

As soon as the stop rule \( \frac{c_O}{(l - b - 1)} - \frac{d_l \cdot c_H}{2} \leq 0 \) is reached, the final resulting last period \( l^* \) is selected and the order quantity for period \( b \) can be computed. The related order quantity is \( x_b = \sum_{l=b}^{l^*} d_l \). The calculations are continued in period \( b = l^* + 1 \) until \( l = T \) and the stop rule is not reached.

### 3.5. Leinz–Bossert–Habenicht (LBH)

The LBH extension algorithm can be described in five steps (Leinz et al., 1997).

Let \( b = 1 \) and \( l = 1 \):

- **Step 1:** Determine the first two order cycle ranges, \( \tau_{b,l} \) and \( \tau_{b+1,l} \), by starting in period \( b \) and using a stop rule such as the ones of LUC or GR. These results are cumulated to a preliminary order cycle range \( \tau_{b,l} = \tau_{b,l} + \tau_{b+1,l} = l^* - b + 1 \).

- **Step 2:** Set \( \psi = \tau_{b,l^*} \). The associated costs can be calculated using

\[
C_{\psi} = c_O + c_H \cdot \sum_{l=b}^{l^*} (t - b) \cdot d_l
\]

An attempt is made to decrease these costs by dividing the preliminary order cycle range into two parts which leads to \( \psi - 1 \) possible combinations of two cycles. The resulting costs can be determined as follows:

\[
C_{\phi} = c_O + c_H \cdot \sum_{l=b}^{l^*} (t - b) \cdot d_l + c_O + c_H \cdot \sum_{l=b+\phi}^{l^*} (t - (b + \phi)) \cdot d_l
\]

with \( \phi = 1, 2, \ldots, \psi - 1 \).

After these calculations, choose \( C^* = \min_{\phi} \{ C_{\phi} \} \) with \( \phi = 1, 2, \ldots, \psi \) so that \( \tau_{b,b+\phi-1} = \phi^* \). The associated preliminary total costs for the first order can then be calculated with

\[
C^* = c_O + c_H \cdot \sum_{l=b}^{\phi^*} (t - b) \cdot d_l
\]

If \( \phi^* \neq \psi \) respectively \( \phi^* > 1 \) go to step 3 otherwise go to step 4.
- Step 3: Another attempt is made to decrease the total costs by dividing the first calculated order cycle range $\tau^b_{b, b+\phi^* - 1}$ into two orders. To this end, the following costs have to be calculated:

$$C_0 = c_O + c_H \cdot \sum_{t=b}^{b+\theta-1} [(t - b) \cdot d_t] + c_O + c_H \cdot \sum_{t=b+\theta}^{b+\phi^*-1} [(t - (b + \theta)) \cdot d_t]$$

with $\theta = 1, 2, \ldots, \phi^* - 1$.

Determine $C^{**} = \min\{C_\theta\}$ with $\theta = 1, 2, \ldots, \phi^* - 1$.

- Step 4: Another case differentiation has to be made in this step.

If $C^{**} \geq C^p$ or $\phi^* = \psi$ or $\phi^* = 1$, one gets the following order quantity:

$$x_b = \sum_{t=b}^{b+\phi^*-1} d_t.$$

If $C^{**} < C^p$, one gets two different order quantities:

$$x_b = \sum_{t=b}^{b+\phi^*-1} d_t$$

and

$$x_{b+\theta} = \sum_{t=b+\theta}^{b+\phi^*-1} d_t.$$

- Step 5: Set $b = b + \phi^*$ and $l = b + \phi^*$. Go back to step 1 and continue in this manner until the planning horizon is reached.

In the next section, an extensive simulation study is conducted to evaluate the performance of the algorithms.

4. Simulation study

4.1. Description

In the numerical simulation study, LUC, GR and the LBH extension based on LUC (LBH-LUC) as well as on GR (LBH-GR) are examined assuming different demand structures, i.e. constant, systematic and erratic demand (Leinz et al., 1997; Zoller & Robrade, 1988). The algorithms were programed using MS Excel and run on a laptop with Intel core i5 (2G) processor. The following assumptions for all tests are made:

- average demand is around 1000 [units] except all demands with trend structures (see I–VI below);
- $pp$-quotient ($pp = \frac{c_O}{c_H}$) can attain the following values:
  $$pp \in \{1000; 2500; 5000; 7500; 10,000; 15,000\};$$
- 100 periods of demand are considered ($T = 100$).

Using the assumptions above, six data-sets are created for the case of constant demand with $d_t = 1000$.

For the systematic demand, the trend and season are analyzed for eight systematic demand structures. The demand for each period will be calculated as follows:

| (I)          | linear increasing trend  | $d_t = 1000 + 7.5 \cdot (t - 1)$ |
|--------------|--------------------------|----------------------------------|
| (II)         | progressive trend         | $d_t = d_{t-1} \cdot \left[1 + \frac{5 + 5 \cdot \left(\frac{t}{100}\right)}{1500}\right]$ with $d_1 = 1000$ |
| (III)        | saturation                | $d_t = 2000 - \left[1000 \cdot e^{-\frac{t}{50}}\right]$ |
| (IV)         | linear decreasing trend   | $d_t = 2000 + 7.5 \cdot (t - 1)$ |
| (V)          | degressive increasing trend | $d_t = 1000 + \sqrt{5 \cdot d_{t-1} \cdot (t - 1)}$ with $d_1 = 1000$ |
| (VI)         | degressive decreasing trend | $d_t = 2000 - \sqrt{5 \cdot d_{t-1} \cdot (t - 1)}$ with $d_1 = 2000$ |
| (VII)        | additive constant season  | $d_t = 1000 + s_t$ |
| (VIII)       | additive dynamic season   | $d_t = 1000 + \frac{t}{33} \cdot s_t$ |
with \( s_i = s_{i-5}; s_1 = -100; s_2 = 50; s_3 = 210; s_4 = 0; s_5 = -120 \) for VII and VIII. In combination with the six different \( pp \)-values, there are 48 data sets to examine.

The erratic demand is simulated by a uniform and a modified normal distribution (MND). The uniformly distributed random variables have an expected value of \( \mu = 1000 \) and fluctuation ranges of \{±20%; ±40%; ±60%; ±80%; ±100%\}, which lead to the variances \( \sigma^2 \in \{13,333; 53,333; 120,000; 213,333; 333,333\} \). The modified normal distributed random variables \( Y \) are generated out of normal distributed random variables \( X \) with an expected value \( \mu = 1000 \) and a variance \( \sigma^2 \in \{10,000; 100,000; 500,000\} \). The latter assumptions make negative demand values in \( Y \) possible, so in case a negative demand occurs, this value is set to zero in \( X \). By this alteration, the initially normally distributed values are actually no longer normally distributed, which also means, that the distribution of \( X \) is no longer symmetrical. In fact, there is the following change to the distribution \( P_{\text{MND}}(X = 0) = F_{\text{MND}}(0) \), where \( F_{\text{MND}}(x) \) is the cumulative distribution function of the normal distribution (ND). The probability density function then shows a small rise at \( x = 0 \) and is no longer defined from \(-\infty\) to \(+\infty\) but is now defined from \( 0 \) to \(+\infty\). With respect to the law of large numbers, we calculate 100 data sets for each erratic demand structure.

### 4.2. Results

The costs of the results obtained with a heuristic are compared to the results of the reference procedure WW (Leinz et al., 1997). The total cost variance is measured by the relative difference: \( \kappa = \frac{C_{\text{heuristic}} - C_{\text{WW}}}{C_{\text{WW}}} \times 100\% \). Because of the law of large numbers, 100 relative differences \( \kappa_i \) with \( i = 1, \ldots, 100 \) for each erratic demand structure are created and the arithmetical mean is needed for further calculations. The arithmetical mean can be computed as follows: \( \bar{\kappa}_{\text{erratic}} = \frac{1}{100} \sum_{i=1}^{100} \kappa_i \).

In addition, the average order cycle range difference compared to WW can be calculated as follows: \( \Delta \tau = \frac{\tau_{\text{heuristic}}}{n_{\text{heuristic}}} - \frac{\tau_{\text{WW}}}{n_{\text{WW}}} \) [periods]. Note that results shown in Figures 1–8 are calculated using the arithmetical mean for all test sets with the specific indicated demand structures (in figure captions) and \( pp \)-values, i.e. \( \bar{\kappa} \) and \( \bar{\Delta} \tau \), respectively.

The results for the case of constant demand are summarized in Table 1. In this case, other values than \( pp = 5000 \) lead to no variances compared to WW and are thus not illustrated. However, our results show that \( pp \)-values of 5000 in combination with a fixed planning horizon lead to variance in total costs and order cycle range compared to WW. A rolling planning horizon, as common in practice, may be suitable to prevent this variance. As can be seen, LBH eliminates this variance as well.

Regarding systematic demand with trend (cf. Section 4.1, No. I to VI), our results show that LBH leads to substantial improvements in the basic heuristics as illustrated in Figure 1. As can be seen, GR generates the worst results in this case. However, even employing large \( pp \)-values, the cost variance is only slightly above 1%.

As for systematic seasonal demand (cf. Section 4.1, No. VII and VIII), we also observe that LBH performs well and, in this case, GR outperforms LUC. This relative advantage remains applying LBH as LBH-GR leads to better results than LBH-LUC.

| Heuristic   | \( \kappa \) | \( \Delta \tau \) | \( \bar{\kappa}_{\text{erratic}} \) | \( \bar{\Delta} \tau \) |
|-------------|--------------|-------------------|-------------------------------|-------------------|
| \( pp = 5000 \) | .749         | .749              | 0                             | 0                 |
| \( \Delta \tau \) | -.089       | -.089             | 0                             | 0                 |
As can be seen, higher cost variances occur compared to the results of systematic demand with trend. This implies that inventory management for seasonal demand is a more challenging task in practice than for systematic demand with trend. However, as our results show, this situation could be enhanced significantly applying the LBH extension algorithm, in particular, LBH-GR as illustrated in Figure 2.

In the case of erratic demand, the results of our study show that higher pp-values lead to higher cost variances, except employing LUC, compare therefore Figure 3. Regarding LBH, the results show that average pp-values lead to the highest cost variances. As LUC calculates poor results with cost variance more than 6%, LBH-GR outmatches LBH-LUC accordingly.

All tested algorithms generate shorter average order cycle ranges compared to WW, as can be seen in Figure 4. One can observe a tendency that high variance in order
cycle ranges may lead to high total costs variance. However, as can be seen for LUC ($pp \leq 2500$) in Figures 3 and 4, for example, a low order cycle range variance may result in high total cost variance. The results in Figure 4 also show that the LBH extension algorithm generates longer order cycle ranges that are then closer to the results of WW compared to the results of the basic heuristics.

In case of $\sigma^2 < 60,000$, all heuristics lead to small cost differences apart from LUC, which generates high cost variances. As can be seen in Figure 5, a significant improvement of LUC is possible applying LBH with $pp \leq 5000$. However, as the basic heuristic calculates poor results, the results of LBH-LUC are still quite far off the ones of WW. It is also notable that LBH-LUC leads to the same cost variances with different $pp$-values. Moreover, we can observe here, that LBH-GR generates the lowest cost

Figure 3. Results for erratic demand and varying $pp$-values.

Figure 4. Order cycle range differences for erratic demand and varying $pp$-values.
variances compared to WW with all $pp$-values. The additional costs of GR can be considerably reduced applying LBH even with higher $pp$-values.

Most procedures lead to shorter order cycle ranges compared to WW with low $pp$-values and $\sigma^2 < 60,000$. In turn, as illustrated in Figure 6, LBH-LUC generates longer order cycle ranges with high $pp$-values. We also observe the tendency that the closer the order cycle range generated by a basic heuristic and a heuristic extended with LBH gets to the order cycle range of WW, the better cost results are obtained (compare Figures 5 and 6). However, there are exceptions, such as LBH-GR with high $pp$-values. An explanation is probably that equal average order cycle ranges to WW are derived, but the order timing is different leading to cost variance of about .5%.

For $\sigma^2 > 200,000$, comparable results are obtained to the case of $\sigma^2 < 60,000$. However, cost variance is noticeably higher for $\sigma^2 > 200,000$, see therefore Figure 7. The only
algorithm that leads to cost variances significantly lower than 1% compared to WW is LBH-GR with small \( pp \)-values. LBH-LUC is about 1% deviation. While GR generates acceptable cost variances with low \( pp \)-values, high \( pp \)-values lead in turn to significant poorer results. Obviously, LUC leads to the worst cost variances. Even though cost variance decreases with high \( pp \)-values, it is still very high. Particularly, in this scenario, one can see the strength of the LBH extension, which leads to considerable improvements in the results.

For the case of \( \sigma^2 > 200,000 \), we can again observe that average order cycle ranges calculated with all tested procedures are too short compared to WW, see Figure 8. Applying LBH leads to extended order cycle ranges. A peculiarity is that the cost results of LUC are better for high \( pp \)-values than for low \( pp \)-values (see Figure 7),

![Figure 7. Cost variances for \( \sigma^2 > 200,000 \).](image)

![Figure 8. Order cycle ranges for \( \sigma^2 > 200,000 \).](image)
although $\Delta t$ is nearer to zero, when $pp$-values are low (see Figure 8). However, we note that a certain variance in average order cycle ranges does not imply a certain goodness of an algorithm regarding cost performance (compare LUC with low $pp$-values).

The results of our study can be summarized as follows:

- LBH eliminates end-of-horizon effects for constant demand.
- All tested algorithms generate average order cycle ranges that are too short compared to WW.
- Average order cycle ranges that are close to WW can only be seen as an indication for good cost performance.
- LUC generates the highest cost variances compared to WW in most tested cases, particularly in the case of $\sigma^2 > 200,000$.
- High $pp$-values lead to higher cost variances for almost all tested scenarios in comparison to low $pp$-values. Only LUC leads to partly better results for high $pp$-values.
- The results of all tested heuristics can be significantly improved applying the LBH extension algorithm, and order quantities approximated the results of WW. However, poor results of a basic heuristic can only relatively be improved with LBH.

5. Conclusion

The intention of this paper was to present a comprehensive description of an extension for dynamic lot-sizing heuristics that has been overlooked by literature and practice. For this purpose, we conducted an extensive numerical study to evaluate its performance. The results of our study show that LBH leads to significant cost savings compared to the tested heuristics and may support researchers in improving existing dynamic lot-sizing heuristics. In addition, this paper may be of help for practitioners to support inventory management decisions. As ERP systems, to the best of the authors’ knowledge, do not provide the WW algorithm, LBH represents a practical extension to improve the results of implemented procedures. As shown above for LUC and GR, LBH could be easy to integrate into ERP systems, as it may extend and improve any heuristic of the dynamic lot sizing problem significantly. The results of our numerical study showed, for example, that LBH-LUC reduced the cost variance of LUC compared to the optimal solution of WW from 11.4 to .79% (cf. Figure 3). We observed that LBH is adding about 20% computation time, whereas WW adds about 400% to the computation time of GR and LUC, which shows that LBH is suitable in practice. In addition, the results of this paper may be of help in education to advance existing heuristics in the literature.

This paper also has limitations. Generally, the improvements obtained with LBH depend on the results of the basic heuristic. The better the results of the heuristic, (i.e. near to the solution of WW) the better the result with LBH extension is. For example, the lot-for-lot heuristic (Orlicky, 1975) may be improved significantly applying LBH, but would still be far off the solution obtained with WW as LBH would solely consider two order periods, which means $\tau_{h,l} = 1$ and $\tau_{r+1,l} = 1$, for all order cycle ranges. In addition, we tested LBH solely within a range of possible heuristics. This and other limiting factors could be addressed in an extension of this paper.
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### Appendix 1

To show: The minimum of the unit costs is equal to the minimum of the total costs in an EOQ model.

**Proof:**

The total cost function for an EOQ-model is given by

\[
C(q) = C_H(q) + C_O(q) = c_H \cdot \frac{q}{2} \cdot T + c_O \cdot \frac{T \cdot d}{q} 
\]

The first derivation is set to zero to find the critical point:

\[
\frac{dC(q)}{dq} = c_H \cdot \frac{1}{2} \cdot T - c_O \cdot \frac{T \cdot d}{q^2} = 0
\]
This leads to the optimal value for the order quantity which can be expressed as follows:

\[ q = \sqrt{\frac{2 \cdot d \cdot c_O}{c_H}} \]

The unit costs can be denoted as follows:

\[ c(q) = \frac{C(q)}{T \cdot d} = c_H \cdot \frac{q}{2 \cdot d} + c_O \cdot \frac{1}{q} \rightarrow \text{Min!} \]

Here, the first derivation is also needed and has to be set to zero:

\[ \frac{dc(q)}{dq} = c_H \cdot \frac{1}{2 \cdot d} - c_O \cdot \frac{1}{q^2} = 0 \]

The resulting optimal order quantity is

\[ q = \sqrt{\frac{2 \cdot d \cdot c_O}{c_H}} \]

**Appendix 2**

To show: For the optimal order quantity, the marginal holding costs and the marginal ordering costs are equal according to amount.

Proof:

We need the total cost function of the EOQ-model again which is given by

\[ C(q) = C_H(q) + C_O(q) = c_H \cdot \frac{q}{2} \cdot T + c_O \cdot \frac{T \cdot d}{q} \rightarrow \text{Min!} \]

Differentiation and setting equal to zero can be mathematically expressed by

\[ \frac{dC(q)}{dq} = c_H \cdot \frac{1}{2} \cdot T - c_O \cdot \frac{T \cdot d}{q^2} = 0 \]

And the transposing of the equation leads to

\[ c_H \cdot \frac{1}{2} \cdot T = c_O \cdot \frac{T \cdot d}{q^2} \]

\[ \left| \frac{dC_H(q)}{dq} \right| = \left| \frac{dC_O(q)}{dq} \right| \]