Categorization of problems to assess and improve proficiency as teachers and learners

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Abstract

We describe how graduate students categorize introductory mechanics problems based on the similarity of their solutions. Graduate students were asked at the end of a teaching assistant training class to categorize problems from their own perspective and from the perspective of typical introductory physics students whom they were teaching. We compare their categorizations with the categorizations by introductory physics students and physics faculty who categorized the same problems. The utility of categorization as a tool for teaching assistant training and faculty development workshops is discussed.
I. INTRODUCTION

To help students learn effectively instructors should become familiar with their students’ level of expertise at the beginning of a course. Instruction should be designed to build on what students already know to ensure that they acquire the desired expertise as determined by the goals of a course.\(^\text{1-8}\)

Physics experts often take for granted that introductory students will be able to distill the underlying physics principles of a problem as readily as experts can. However, beginning physics students are usually much more sensitive to the context and surface features of a physics problem than experts. If an instructor teaches the principle of conservation of angular momentum with the example of a spinning skater and gives an examination problem requiring the use of the same principle in the context of a collapsing neutron star under its own gravitational force, students may wonder what this astrophysics problem involving a neutron star has to do with introductory mechanics. Without appropriate guidance, the spinning skater problem may look nothing like a neutron star problem to a beginning student even though both problems can be solved using the same physics principle. The difference between what instructors and students “see” in the skater and neutron star problems is due to the fact that physics experts view physical situations at a much more abstract level than beginning students who often are sidetracked by context-dependent features.\(^\text{9-16}\)

A crucial difference between the problem solving strategies used by experts in physics and beginning students lies in the interplay between how their knowledge is organized and how it is retrieved to solve problems. Categorizing various problems based on similarity of their solutions can be a useful tool for teaching and learning.\(^\text{14-16}\) In a classic study by Chi et al.\(^\text{14}\) a categorization task was used to assess introductory physics students’ level of expertise in physics. Introductory physics students were asked to group mechanics problems into categories based on the similarity of their solutions. They were also asked to explain the reasons for their groupings. Unlike experts who categorize them based on the physical principles involved to solve them, introductory students categorized problems involving inclined planes in one category and pulleys in a separate category.\(^\text{14}\)

In Chi et al.’s out-of-classroom study, 24 problems from introductory mechanics were given to eight introductory physics student volunteers (novices) and eight physics graduate student volunteers (experts).\(^\text{14}\) There were no differences in the number of categories
produced (approximately 8.5 categories by each group on average) and four of the largest categories produced by each student from both groups captured the majority of the problems (80% for experts and 74% for novices). Immediately after the first categorization, each student was asked to re-categorize the same problems. The second categorization matched the first categorization very closely. It was concluded that both experts and novices were able to categorize problems into groups that were meaningful to them.\textsuperscript{\textcolor{red}{14}}

Further analysis of the data in Ref. \textsuperscript{14} showed that experts and novices group their problems in different categories based on their knowledge associated with the categories. Physics graduate students (experts) were able to distill physics principles applicable in a situation and categorize the problems based on those principles. In contrast, novices based their categorization on the problem’s literal features. For example, 75%, 50%, 50%, and 38% of the novices had springs, inclined plane, kinetic energy, and pulleys as one of their categories, respectively; 25% of the experts used springs as a category but inclined plane, kinetic energy, and pulleys, were not chosen as categories by any of the experts.

A categorization task can also be used as a tool to help students learn effective problem-solving strategies and to organize their knowledge hierarchically, because such tasks can guide students to focus on the similarity of problems based on the underlying principles rather than on the specific contexts. For example, introductory physics students with different levels of expertise can be given categorization tasks in small groups, and students can be asked to categorize problems and discuss why different problems should be placed in the same group without asking them to solve the problems explicitly. Then there can be a class discussion about why some categorizations are better than others, and students can be given a follow-up categorization task to ensure individual accountability. One advantage of such an activity is that it focuses on conceptual analysis and planning stages of problem solving and discourages the plug and chug approach. Without guidance, students often implement a problem solution without thinking whether a particular principle is applicable.\textsuperscript{\textcolor{red}{2}}

In this paper we report about the results of our study on the nature and level of understanding of physics graduate students about the initial physics knowledge of introductory students. We asked graduate students at the end of a course for teaching assistants to categorize problems based on the similarity of their solutions, both from their own perspective and from the perspective of an introductory physics students. We compared their categorizations with those performed by physics professors and introductory physics students.
One surprising finding is the resistance of graduate students to categorizing problems from a typical introductory physics student’s perspective with the claim that such a task is “useless”, “impossible”, and has “no bearing” on their teaching assistant (TA) duties. Based on our finding, we suggest that inclusion of such tasks can improve the effectiveness of TA training courses and faculty development workshops and help TAs and instructors focus on issues related to teaching and learning.

II. GRADUATE STUDENTS IN A TA TRAINING COURSE

We will discuss the process and outcome of the categorization of 25 introductory mechanics problems by 21 physics graduate students enrolled in a TA training course at the end of the course. Graduate students first performed the categorizations from their own perspective and later from the perspective of a typical introductory student. The goals of the study were to investigate the following issues:

- How do graduate students enrolled in a TA training course categorize introductory physics problems from their own perspective?
- How do graduate students categorize the same problems from the perspective of a typical introductory physics student? Do they have an understanding of the differences between their physics knowledge structure and those of the introductory physics students?
- How does the categorization by the graduate students from their own perspective compare with the categorization by introductory physics students and physics faculty from their own perspective?
- How do introductory physics students in an in-class study categorize the introductory mechanics problems after instruction compared to the eight introductory student volunteers studied in Ref. 14? Does the ability to categorize introductory mechanics problems by introductory physics students depend strongly on the nature and context of the questions that are asked?

The issues involved in a detailed comparison with Ref. 14 will be discussed elsewhere. All those who performed the categorization were provided the following instructions given
at the beginning of the questions:\textsuperscript{17}

- Your task is to group the 25 problems below based upon similarity of solution into various groups on the sheet of paper provided. Problems that you consider to be similar should be placed in the same group. You can create as many groups as you wish. The grouping of problems should NOT be in terms of “easy problems”, “medium difficulty problems” and “difficult problems” but rather it should be based upon the features and characteristics of the problems that make them similar. A problem can be placed in more than one group created by you. Please provide a brief explanation for why you placed a set of questions in a particular group. You need NOT solve any problems.
- Ignore the retarding effects of friction and air resistance unless otherwise stated.

The sheet on which individuals were asked to perform the categorization of problems had three columns. The first column asked them to use their own category name for each of their categories, the second column asked them for a description of the category that explains why those problems may be grouped together, and the third column asked them to list the problem numbers for the questions that should be placed in a category. Apart from these directions, students were not given any other hints about the category names they should choose.

III. TYPES OF QUESTIONS AND RATING OF CATEGORIES

We were unable to obtain the questions in Chi et al.’s study except for a few that have been published. We therefore chose our own mechanics questions on sub-topics similar to those chosen in Ref.\textsuperscript{14}. The context of the 25 mechanics problems varied and the topics included one- and two-dimensional kinematics, dynamics, work-energy, and impulse-momentum.\textsuperscript{17}

Many questions related to work-energy and impulse-momentum concepts were adapted from an earlier study\textsuperscript{18} and many questions on kinematics and dynamics were chosen from other earlier studies\textsuperscript{19,20} because the development of these questions and their wording had gone through rigorous testing by students and faculty members. Some questions could be solved using one physics principle for example, conservation of mechanical energy, Newton’s second law, conservation of momentum.\textsuperscript{17} The first two columns of Table 1 show the question numbers and examples of primary categories in which each question can be placed (based
Questions 4, 5, 8, 24 and 25 are examples of problems that involve the use of two principles for different parts of the problem. Questions 4, 8, and 24 below can be grouped together in one category because they require the use of conservation of mechanical energy and momentum:

• (4) Two small spheres of putty, A and B, of equal mass, hang from the ceiling on massless strings of equal length. Sphere A is raised to a height $h_0$ as shown below and released. It collides with sphere B (which is initially at rest); they stick and swing together to a maximum height $h_f$. Find the height $h_f$ in terms of $h_0$.

• (8) Your friend Dan, who is in a ski resort, competes with his twin brother Sam on who can glide higher with the snowboard. Sam, whose mass is 60 kg, puts his 15 kg snowboard on a level section of the track, 5 meters from a slope (inclined plane). Then, Sam takes a running start and jumps onto the stationary snowboard. Sam and the snowboard glide together till they come to rest at a height of 1.8 m above the starting level. What is the minimum speed at which Dan should run to glide higher than his brother to win the competition? Dan has the same weight as Sam and his snowboard weighs the same as Sam’s snowboard.

• (24) You are standing at the top of an incline with your skateboard. After you skate down the incline, you decide to “abort”, kicking the skateboard out in front of you such that you remain stationary afterwards. How fast is the skateboard travelling with respect to the ground after you have kicked it? Assume that your mass is 60 kg, the mass of the skateboard is 10 kg, and the height of the incline is 10 cm.

Questions 5 and 25 below can be grouped together because they can be solved using conservation of mechanical energy and Newton’s second law:

• (5) A family decides to create a tire swing in their backyard for their son Ryan. They tie a nylon rope to a branch that is located 16 m above the earth, and adjust it so that the tire swings 1 meter above the ground. To make the ride more exciting, they construct a launch point that is 13 m above the ground, so that they don’t have to push Ryan all the time. You are their neighbor, and you are concerned that the ride
might not be safe, so you calculate the maximum tension in the rope to see if it will hold. Calculate the maximum tension in the rope, assuming that Ryan (mass 30 kg) starts from rest from his launch pad. Is it greater than the maximum rated value of 2500 N?

• (25) A friend told a girl that he had heard that if you sit on a scale while riding a roller coaster, the dial on the scale changes all the time. The girl decides to check the story and takes a bathroom scale to the amusement park. There she receives an illustration (see below), depicting the riding track of a roller coaster car along with information on the track (the illustration scale is not accurate). The operator of the ride informs her that the rail track is smooth, the mass of the car is 120 kg, and that the car sets in motion from a rest position at the height of 15 m. He adds that point B is at 5m height and that close to point B the track is part of a circle with a radius of 30 m. Before leaving the house, the girl stepped on the scale which indicated 55 kg (the scale is designed to be used on earth and displays the mass of the object placed on it). In the rollercoaster car the girl sits on the scale. According to your calculation, what will the scale show at point B?

Although we had an idea about which categories created by individuals should be considered good or poor, we validated our assumptions with other experts. We randomly selected the categorizations performed by twenty introductory physics students and gave it to three physics faculty who had taught introductory physics recently and asked them to decide whether each of the categories created by individual students should be considered good, moderate, or poor. We asked them to mark each row which had a category name created by a student and a description of why it was the appropriate category for the questions that were placed in that category. If a faculty member rated a category created by an introductory student as good, we asked that he/she cross out the questions that did not belong to that category. The agreement between the ratings of different faculty members was better than 95%. We used their ratings as a guide to rate the categories created by everybody as good, moderate, or poor. A category was considered “good” only if it was based on the underlying physics principles. We typically rated both conservation of energy or conservation of mechanical energy as good categories. Kinetic energy as a category name was considered a moderate category if students did not explain that the questions placed in that category
can be solved using mechanical energy conservation or the work energy theorem. We rated a category such as energy as good if students explained the rationale for placing a problem in that category. If a secondary category such as friction or tension was the only category in which a problem was placed and the description of the category did not explain the primary physics principles involved, it was considered a moderate category. Table 1 shows examples of the primary and secondary categories and one commonly occurring poor/moderate category for each question given in the categorization task.

More than one principle or concept may be useful for solving a problem. The instruction for the categorizations told students that they could place a problem in more than one category. Because a given problem can be solved using more than one approach, categorizations based on different methods of solution that are appropriate was considered good (see Table 1). For some questions, conservation of mechanical energy may be more efficient, but the questions can also be solved using one- or two-dimensional kinematics for constant acceleration. In this paper, we will only discuss categories that were rated good. If a graph shows that 60% of the questions were placed in a good category by a particular group (introductory students, graduate students, or faculty), it means that the other 40% of the questions were placed in moderate or poor categories.

For questions that required the use of two major principles, those who categorized them in good categories either made a category which included both principles such as the conservation of mechanical energy and the conservation of momentum or placed such questions in two categories created by them – one corresponding to the conservation of mechanical energy and the other corresponding to the conservation of momentum. If such questions were placed only in one of the two categories, it was not considered a good categorization.

IV. CATEGORIZATION BY GRADUATE STUDENTS FROM THEIR OWN PERSPECTIVE

A histogram of the percentage of questions placed in good categories (not moderate or poor) is given in Fig. 1. This figure compares the average performance of 21 graduate students at the end of a TA training course when they were asked to categorize questions from their own perspective with 7 physics faculty and 180 introductory students who were given the same task. Although this categorization by the graduate students is not on par
with the categorization by physics faculty, the graduate students displayed a higher level of expertise in introductory mechanics than the introductory students and were more likely to group the questions based on physical principles. We note that in Ref. [14] the experts were graduate students and not physics professors.[14]

Physics professors (and sometimes graduate students) pointed out multiple methods for solving a problem and specified multiple categories for a particular problem more often than the introductory students. Introductory students mostly placed one question in only one category. Professors (and sometimes graduate students) created secondary categories in which they placed a problem that were more like the introductory students’ primary categories. For example, in the questions involving tension in a rope or frictional force[17], many faculty and some graduate students created these secondary categories called tension or friction, but also placed those questions in a primary category, based on a fundamental principle of physics. Introductory physics students were much more likely to place questions in inappropriate categories than the faculty or graduate students, for example, placing a problem that was based on the impulse-momentum theorem or conservation of momentum in the conservation of energy category. For questions involving two major physics principles, for example, question 4 related to the ballistic pendulum, most faculty and some graduate students categorized it in both the conservation of mechanical energy and conservation of momentum categories in contrast to the introductory students who either categorized it as an energy problem or as a momentum problem. The fact that introductory students only focused on one of the principles involved to solve question 4 is consistent with an earlier study in which students either noted that this problem can be solved using conservation of mechanical energy or conservation of momentum but not both.[18]

Many of the categories generated by the three groups were the same, but there was a major difference in the fraction of questions that were placed in good categories by each group. What introductory students chose as their primary categories were often secondary categories created by the faculty. Rarely were there secondary categories made by the faculty, for example, apparent weight, that were not created by students. There were some categories such as ramps, and pulleys, that were made by introductory physics students but not by physics faculty or graduate students. The percentage of introductory students who selected ramps, pulleys or even springs as categories (based mainly upon the surface features of the problem rather than based upon the physics principle required to solve the
problem) is significantly less (less than 15% for each of these categories) than in the study of Ref. 14. This difference could be due to the fact that ours was an in-class study with a large number of students and the categorization task was given a few weeks after instruction in all relevant concepts. In contrast, in Ref. 14 there were only eight student volunteers and they might not have taken introductory mechanics recently. Another reason for the difference could be due to the difference in questions that were given to students in the two studies. In our study introductory students sometimes categorized questions 3, 6, 8, 12, 15, 17, 18, 22, 24, and 25 as ramp problems, questions 6 and 21 as spring problems (question 21 was categorized as a spring problem by introductory students who associated the bouncing of the rubber ball with a spring-like behavior) and question 17 as a pulley problem. The lower number of introductory students making spring or pulley as a category in our study could be due to the fact that there are fewer questions than in Ref. 14 that involve springs and pulleys. However, ramp was a much less popular category for introductory students in our study than in Ref. 14 in which 50% of the students created this category and placed at least one problem in that category (although may questions can potentially be categorized as ramp problems even in our study).

Some introductory physics students created the categories speed and kinetic energy if the question asked them explicitly to calculate those physical quantities. The explanations provided by the students as to why a particular category name, for example, speed, is most suitable for a particular problem were not adequate; they wrote that they created this category because the question asked for the speed. Graduate students were less likely than introductory students to create such categories and were more likely to classify questions based on physical principles, for example, conservation of mechanical energy (or conservation of energy which was taken to be a good category with proper explanation) or kinematics in one dimension. Even if a problem did not explicitly ask for the “work done” by a force on an object, faculty and graduate students were more likely to create and place such questions which could be solved using work-energy theorem or conservation of mechanical energy in categories related to these principles. This task was much more challenging for the introductory physics students who had learned these concepts recently. For example, it was easy to place question 3 in a category related to work because the question asked students to find the work done on an object.  Placing question 7 in the work-energy category was more difficult because students were asked to find the speed.
Figures 2–5 show histograms for questions 15, 21, 23, and 24 respectively of some common categories created by different percentages of introductory students, graduate students, and physics faculty. (The categorization by graduate students from a typical introductory physics student’s point of view will be discussed in Sec. V.) As expected, physics faculty performed most expert-like categorization for each of the problem based upon the physics principles required to solve it followed by graduate students. For question 21 (see Figure 3) all faculty created an impulse category while only 5 out of 7 faculty also categorized it as a question related to momentum. For this question categorization by all faculty was considered good. On the other hand, some graduate students and introductory students placed this problem only in energy or force categories that were not considered good.

V. CATEGORIZATION BY GRADUATE STUDENTS FROM INTRODUCTORY STUDENTS’ PERSPECTIVE

After the graduate students had submitted their own categorizations, they were asked to categorize the same questions from the perspective of a typical introductory physics student. A majority of the graduate students had not only served as TAs for recitations, grading, or laboratories, but had also worked during their office hours with students one-on-one and in the Physics Resource Room at the University of Pittsburgh. The goal of this task was to assess whether the graduate students were familiar with the level of expertise of the introductory students whom they were teaching and whether they realized that most introductory students do not necessarily see the same underlying principles in the questions that they do. The graduate students were told that they were not expected to remember how they used to think 4–5 years ago when they were introductory students. We wanted them to think about their experience as TAs in introductory physics courses while grouping the questions from an introductory students’ perspective. They were also asked to specify whether they were recitation TAs, graders, or laboratory TAs that semester.

The categorization of questions from the perspective of an introductory physics student met with widespread resistance. Many graduate students noted that the task was useless or meaningless and had no relevance to their TA duties. Although we did not tape record the discussion with the graduate students, we took notes immediately following the discussion. The graduate students often asserted that it is not their job to “get into their students’
heads.” Other graduate students stated that the task was “impossible” and “cannot be accomplished.” They often noted that they did not see the utility of understanding the perspective of the students. Some graduate students explicitly noted that the task was “silly” because it required them to be able to read their students’ minds and had no bearing on their TA duties. Not a single graduate student stated that they saw merit in the task or said anything in favor of why the task may be relevant for a TA training course. The discussions with graduate students also suggest that many of them believed that effective teaching merely involves knowing the content well and delivering it lucidly. Many of them had never thought about the importance of knowing what their students think for teaching to be effective.

It is surprising that most graduate students enrolled in the TA training course were so reluctant or opposed to attempting the categorization task from a typical introductory student’s perspective. This resistance is intriguing especially because the graduate students were given the task at the end of a TA training course and most of them were TAs for introductory physics all term. It is true that it is very difficult for the TAs (and instructors in general) to imagine themselves as novices. However, it is possible for TAs (and instructors) to familiarize themselves with students’ level of expertise by giving them pre-tests at the beginning of a course, listening to them carefully, and by reading literature about student difficulties, for example, as part of the TA training course.

After 15–20 minutes of discussion we made the task more concrete and told graduate students that they could consider categorizing from the perspective of a relative whom they knew well after he/she took only one introductory mechanics course if that was the only exposure to the material they had. We also told them that they had to make a good faith effort even if they felt the task was meaningless or impossible. Figure 6 shows the histogram of how the graduate students categorized questions from their own perspective and from the perspective of a typical introductory student/relative who has taken only one physics course. Figure 7 shows the histogram of how the graduate students categorized questions from the perspective of a typical introductory student/relative in comparison to the categorization by introductory students. Figure 6 shows that the graduate students recognized that the introductory physics students do not understand physics as well as graduate students and hence they re-categorized the questions in worse categories when performing the categorization from the perspective of a typical introductory physics student (also see Figs. 2–5). However,
if we look at questions placed in each category, for example, conservation of momentum, there are sometimes significant differences between the categorization by graduate students from an introductory students’ perspective and by introductory students from their own perspective. This implies that while graduate students may have realized that a typical introductory student/relative who has taken only one physics course may not perform as well as a physics graduate student on the categorization task, overall they were not able to anticipate the frequency with which introductory students categorized each problem in the common less-expert-like categories.

VI. DISCUSSION

The reluctance of TAs to re-categorize the questions from introductory students’ perspective raises the question of what should the graduate students learn in a TA training class. In a typical TA training class, a significant amount of time is devoted to writing clearly on the blackboard, speaking clearly and looking into students’ eyes, and grading students’ work fairly. There is a lack of discussion about the fact that teaching requires not only knowing the content but understanding how students think and implementing strategies that are commensurate with students’ prior knowledge and expertise.

After the graduate students had completed both sets of categorization tasks, we discussed the pedagogical aspects of perceiving and evaluating the difficulty of the questions from the introductory students’ perspective. We discussed that pedagogical content knowledge, which is critical for effective teaching, depends not only on the content knowledge of the instructor, but also on the knowledge of what the students are thinking. The discussions were useful and many students explicitly noted that they had not pondered why accounting for the level of expertise and thinking of their students was important for devising strategies to facilitate learning. Some graduate students noted that they will listen to the introductory students and read their written responses more carefully in the future.

One graduate student noted that after this discussion he felt that, similar to the difficulty of the introductory students in categorizing the introductory physics questions, he has difficulty in categorizing questions in the advanced courses he has been taking. He added that when he is assigned homework/exam questions, for example, in the graduate level electricity and magnetism course in which they were using the classic book by Jackson,

he
often does not know how the questions relate to the material discussed in the class even when he carefully goes through his class notes. The student noted that if he goes to his graduate course instructor for hints, the instructor seems to have no difficulty making those connections to the homework. The spontaneity of the instructor’s connection to the lecture material and the insights into those questions suggested to the student that the instructor can categorize those graduate-level questions and explain the method for solving them without much effort. This facility is due in part because the instructor has already worked out the questions and hence they have become an exercise. Other graduate students agreed with his comments saying they too had similar experiences and found it difficult to figure out how the concepts learned in the graduate courses were applicable to homework problems assigned in the courses. These comments are consistent with the fact that a graduate student may be an expert in the introductory physics material related to electricity and magnetism but not necessarily an expert in the material at the Jackson level course. Such difficulty is not surprising considering that a handful of fundamental physics principles are applied in diverse contexts. Solving questions with different contexts involves transferring relevant knowledge from the context in which it was learned to new contexts. The mathematical tools required to solve the questions in advanced problems may increase the mental load while solving questions and make it more difficult to discern the underlying physics principle involved.

VII. SUMMARY

We found that graduate students perform better at categorizing introductory mechanics questions than introductory students but not as well as physics faculty. When asked to categorize questions from a typical introductory physics student’s perspective, graduate students were very reluctant and many explicitly claimed that the task was useless. This study raises important issues regarding the content of TA training courses and faculty professional development workshops and the extent to which these courses should allocate time to help participants learn about pedagogical content knowledge in addition to the usual discussions of logistical issues related to teaching. Asking the graduate students and faculty to categorize questions from the perspective of students may be one way to draw instructor’s attention to these important issues in the TA training courses and faculty professional development workshops.
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| Question | Examples of Primary Categories | Examples of Secondary Categories | Poor/Moderate Categories |
|----------|--------------------------------|---------------------------------|--------------------------|
| 1        | (a) momentum conservation or (b) completely inelastic collision | speed                           |                          |
| 2        | (a) mechanical energy conservation or (b) 1D kinematics         | speed                           |                          |
| 3        | work by conservative force/definition of work                   | ramp                            |                          |
| 4        | mechanical energy conservation and momentum conservation        | only energy or momentum          |                          |
| 5        | mechanical energy conservation and Newton’s second law           | centripetal acceleration, circular motion/tension | only tension or only force |
| 6        | mechanical energy conservation                                    | only spring                      |                          |
| 7        | work-energy theorem/definition of work or Newton’s second law/1D kinematics | relation between kinetic energy | speed                    |
| 8        | momentum conservation or completely inelastic collision and mechanical energy conservation | only energy or momentum          |                          |
| 9        | 2D kinematics                                                    | cliff                           |                          |
| 10       | Newton’s second law                                             | circular motion/friction         | only friction            |
| 11       | linear momentum conservation or completely inelastic collision  | speed                           |                          |
| 12       | mechanical energy conservation and work-energy theorem/definition of work | friction | only friction |
| 13       | Newton’s second law                                             | Newton’s third law               | force                    |
| 14       | 2D kinematics                                                    | force/ cliff                    |                          |
| 15       | mechanical energy conservation                                   | speed                           |                          |
| 16       | mechanical energy conservation or 2D kinematics                  | speed                           |                          |
| 17       | Newton’s second law                                             | Newton’s third law/tension       | only tension             |
| 18       | mechanical energy conservation or 2D kinematics                  | speed                           |                          |
| 19       | Impulse-momentum theorem                                         | force                           |                          |
| 20       | mechanical energy conservation or 2D kinematics                  | speed                           |                          |
| 21       | impulse-momentum theorem                                         | force                           |                          |
| 22       | 2D kinematics                                                    | force                           |                          |
| 23       | Newton’s second law/1D kinematics or Work-energy theorem/definition of work | kinematic variables | force                    |
| 24       | mechanical energy conservation or momentum conservation or completely inelastic collision | speed                           |                          |
| 25       | mechanical energy conservation and Newton’s second law           | centripetal acceleration, circular motion/normal force | ramp/force |

**TABLE I:** Examples of the primary and secondary categories and one commonly occurring poor/moderate category for each of the 25 questions
Figure Captions

FIG. 1: Histogram of introductory physics students, graduate students, and physics faculty who categorized various percentages of the 25 problems in “good” categories when asked to categorize them based on similarity of solution from their own point of view (SelfPoV). Good categories were determined in consultation with three faculty members as discussed in Sec III. Physics faculty performed best in the categorization task followed by graduate students and then introductory physics students.

FIG. 2: Histogram of introductory physics students, graduate students, and physics faculty who categorized various percentages of question 15 in different categories when asked to group them based on similarity of solution from their own point of view. The categorization by graduate students from a typical introductory physics student’s point of view is also shown. Physics faculty performed best in categorization from their own point of view followed by graduate students and then introductory physics students. When graduate students re-categorized problems from a typical introductory physics students’ point of view, they grouped them in worse categories (closer to the categories made by introductory students from their own point of view) than when they categorized from their own perspective. Some category names have been abbreviated, e.g., the category “energy” includes conservation of energy or conservation of mechanical energy.

FIG. 3: Histogram of introductory physics students, graduate students, and physics faculty who categorized various percentages of question 21 in different categories when asked to categorize them based on similarity of solution from their own point of view. Categorization by graduate students from a typical introductory physics student’s point of view is also shown.

FIG. 4: Histogram of introductory physics students, graduate students, and physics faculty who categorized various percentages of problem 23 in different categories when asked to categorize them based on similarity of solution from their own point of view. Categorization by graduate students from a typical introductory physics student’s point of view is also shown.
FIG. 5: Histogram of introductory physics students, graduate students, and physics faculty who categorized various percentages of problem 24 in different categories when asked to categorize them based on similarity of solution from their own point of view (Self PoV). Categorization by graduate students from a typical introductory physics student’s point of view is also shown.

FIG. 6: Histogram of percentages of graduate students who categorized various percentages of the 25 problems in “good” categories when asked to categorize them based on similarity of solution from their own point of view and when asked to categorize from the perspective of a typical introductory physics students (As noted in the text, the task was made more concrete later by asking graduate students to consider a relative who had taken only one mechanics course.).

FIG. 7: Histogram of introductory physics students and graduate students who categorized various percentages of the 25 problems in good categories when asked to categorize them based on similarity of solution from their own point of view and when asked to categorize from the perspective of a typical introductory physics student respectively.
Percent of People who Placed Problem 15 in Select Categories

- Faculty Self PoV
- Grad Self PoV
- Grad-Introductory Student PoV
- Introductory Student Self PoV

| Category      | Percent of People |
|---------------|-------------------|
| Energy        | 90                |
| Kinematics    | 10                |
| Momentum      | 0                 |
| Ramp          | 0                 |
| Work          | 0                 |
Percent of People who Placed Problem 24 in Select Categories

Percent of Problems Placed in "Good" Categories