How to improve critical thinking skills and spatial reasoning with augmented reality in mathematics learning?

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Abstract. Critical thinking skills and spatial reasoning are important aspect in human life. On the other hand, currently, implementing augmented reality in many fields be a trend, including in education field. This study aims to explain how to improve critical thinking skills and spatial reasoning with augmented reality in mathematics learning. The method used in this research is literature review. The data obtained from scientific journals, conference proceedings, and other relevant sources. The results show that to improve spatial reasoning can be done by utilizing its features that can develop confidence of learners, facilitate student-centered learning, create problem solving, and create interactive learning. Besides that, spatial reasoning can be improved by designing dynamic objects with AR. Dynamic objects permit learners to view objects from different sides. Therefore, active learning can be promoted.

1. Introduction

Mathematics is one of important subjects, so it is taught throughout the school. The reason is that mathematics can be applied in all fields such as technology, production, automation, decision making, management, economic transactions, daily routines, security procedures communication, etc [1]. Moreover, the mathematics should be taught to children as an important element in communication, mathematics as a powerful tool that should be developed, to raise awareness of the appeal of mathematics, to foster self-confidence of children on their math skills, and fostering imagination, initiative, and flexibility of mind in mathematics [2]. Moreover, mathematics teaching has a goal to make the students have the ability to work in a systematic, independent and cooperative [2].

In the 21st century, every human being is required to have 21st century skills in order to compensate for the emergence of various new jobs in this modern era. These skills include: (1) critical thinking and problem solving (expert thinking), (2) communication and collaboration (communicating complex), and (3) creativity and innovation (applied imagination and invention) [3]. Furthermore, in critical thinking and problem solving skills, students should be able to reason effectively [3]. Critical thinking skills and reasoning, both of which are essential in everyday life [4]. Thus, critical thinking and reasoning ability are important abilities and need to be owned by every learner.

One of the necessary reasoning in mathematics is spatial reasoning. Spatial reasoning related to the concept of space or in mathematics included in the scope of geometry. Geometry modeling and spatial reasoning can be used to interpret and describe the physical environment, and is being an important tool in problem solving [5]. Özerem [6] argues that studying geometry is an important component in the
study of mathematics as geometry can help learners to analyse and interpret their environment and can be used as a preparation that can be applied in other topics.

Several studies have discussed about the difficulties of learners in learning geometry, such as Özerem [6]; Gal and Linchevski [7]; Komalasari [8]; and Karimah, Kusmayadi, and Pramudya [9]. The difficulties were found to be largely due to the low capacity of thinking and reasoning ability.

Geometry is a mathematical topic in the most common problems in teaching and learning activities [10]. Some researchers have suggested solutions that can be used to overcome the difficulties experienced by learners and teachers in learning activities in geometry. Learning mathematics should be done by utilizing instructional media [6], [8], [11] particularly visual learning media [6]. Further, the media on the topic of learning geometry should be presented in a more visual by using a computer [6].

This study discusses how to improve critical thinking skills and spatial reasoning with augmented reality (AR) in mathematics. Augmented reality is a software that can be used to design interactive learning media in geometry learning. By using augmented reality, geometry learning becomes more visual and interactive because AR has characteristics: (1) can combine the real and virtual world, (2) interactive in the real time, and (3) in the form of 3D [12].

1.1. Critical thinking skills
Mathematics learning is a subject that emphasizes problem solving. If students think critically, they can solve problem effectively [13]. Critical thinking is an exploration of the thinking process carefully to clarify our understanding so we can make smarter decisions [14]. The ability to think especially critical and creative thinking skills become the main focus in education [15], [16]. In addition, Arends and Kilcher [17] found that emotions, reasoning, problem solving, and critical thinking are important aspects in learning.

Permendikbud No. 36 of 2018 on Curriculum 2013 of Senior High School mentions that Curriculum 2013 was developed with the improvement mindset on strengthening the critical learning patterns [18]. Critical learning is applied in teaching aims to train critical thinking skills of learners. According to Chaffee [14], every human being has the ability to think critically. This is because man is given the natural ability to reflect or think back about what he was thinking, is being conducted, or are perceived [14].

Ennis [16] argues that critical thinking is reflective thinking that is sensible and focused on making decisions about what to believe or to do. Chaffee [14] in his book defines critical thinking as thinking about our minds so that we can clarify and deepen it. A simple definition given by Hapsari [19] which states that critical thinking is thinking effectively to make decisions.

On the other hand, some experts define critical thinking along with creative thinking. Bailin [15] argues that critical thinking is seen as analytical, which provides an assessment of the context is given and is still in the realm of that context. While creative thinking is seen as an imagination, construction, and generative which allows it to come out or go beyond that context. Budiman and Jailani [20] stated that the critical and creative thinking are fundamental skills in human because both skills could encourage someone to look at each issue critically and trying to find solutions creatively in order to obtain something newer, better and beneficial for human life. In addition, the ability to think creatively and think critically give direct effect to the decision-making styles and problem solving skills [21].

In connection with mathematics learning, Nugraha and Mahmudi [22] define critical thinking skills as the ability to identify, analyze, associate, and infer facts or relevant information in a situation. In addition, critical thinking skills are also skills to determine inconsistencies in the operation or product as well as the ability to assess the accuracy of any statement or operation based on the criteria or standards [22]. Based on the experts’ opinion, it can be concluded that critical thinking skills are skills that is think deeply to make inferences from relevant facts or informations.

According to ten Dam and Volman [23], learning characteristics that are believed to increase critical thinking skills are: (1) pay attention to develop the confidence of learners; (2) facilitate active learning; (3) problem-based curriculum; and (4) encourage learners to interact.
1.2. Spatial reasoning
Reasoning is necessary cognitive aspect of every human life. In mathematics, spatial reasoning is key to success in studying solid geometry concepts [24]. This is because spatial reasoning and geometry correlates strongly [25], [26]. Geometry learning is believed can improve spatial skills [27]. Geometry learning in school should enable learners to use visualization, spatial reasoning, and geometric modelling to solve problems [5].

According to Clements and Battista [25], spatial reasoning consists of a set of cognitive processes is a mental representation of spatial objects, relations, and transformations are constructed and manipulated. The two main components in spatial tasks is spatial orientation and spatial visualization. Spatial orientation that is understanding the relationship between the position of an object in the room with another notch, while the spatial visualization is the understanding and presentation of the movement of objects in images in 2D and 3D [25].

Mix and Cheng [26] define spatial reasoning with the analysis of factors including the spatial orientation, mental rotation and spatial visualization. Spatial orientation is defined as the ability to understand the position of a variety of objects in space, relative to each other and relative to the audience, especially across a change in orientation [26]. Mental rotation in question is the manipulation or mental rotation of the objects or elements that are remembered in a scene [26]. In short, mental rotation is the ability to imagine how the appearance of an object when the object is rotated [28]. Spatial visualization is defined as the ability to understand complex spatial patterns and understand the imaginary movement in space [26].

Based on the definitions from experts, it can be concluded that spatial reasoning is the ability to understand objects within a space that includes an understanding of the position of the object in space and understanding in imagining objects that change or movement.

Spatial reasoning can be improved through practice well-designed spatial [25], [26], [29]. In addition, the geometry of informal learning can also improve spatial ability, although it is not mentioned that spatial abilities cannot be improved through formal learning in general [25]. Cesaria and Herman [30] state that learning geometry by making use of ICT could improve the mathematical skills of students, such as the ability to understand mathematical concepts and reasoning.

1.3. Augmented reality (AR)
According to The National Council of Teachers of Mathematics [5], the use of technology in teaching and learning activities geometry provides an important role. Various technologies can be used for geometry learning, such as Adobe Flash [31]; GeoGebra [32], [33]; Cabri 3D [32] and Augmented Reality [34]. This paper will discuss how the use of Augmented Reality can improve critical thinking skills and spatial reasoning in mathematics learning in higher schools.

Augmented reality (AR) is the development of virtual reality (VR). With VR, users can not see the real world around him [12] because the concept of VR is to add a real object in a virtual object [35]. In contrast to the VR, using AR, the user can see the real world around him [12] because the concept of AR is to add virtual objects into real objects at the same time [35].

Azuma [12] defined AR as a system that has three characteristics, namely: (1) combining the virtual world with the real world, (2) can be used interactively at the same time, (3) can be used for three-dimensional objects. Simply, AR is defined as a state between real and virtual environments [36].

Based on Tzima, Styliaras, and Bassounas [36], applying augmented reality has two categories, namely image-based and location-based applications. For image-based applications, it has two categories: (1) marker-based, which require specific labels (e.g. Quick Respon Code) and (2) marker-less tracking, in which an image becomes the trigger or playback.

Mustaqim and Kurniawan [35] found AR has several advantages, namely (1) a more interactive, (2) effective in use, (3) can be implemented in a broad range of media, (4) modeling simple object, (5) more cost effective, and (6) easy to operate. In addition, Altinpulluk [37] stated that using AR in educational settings can (1) give the potential to provide objects in a three-dimensional perspective; (2) make collaborative and simultaneous learning; (3) enhances the cooperation among learners and the learners
with the educators; (4) make the invisible objects to visible objects; (5) be used as bridge in formal and informal learning; and (6) increase learners interest, motivation, imagination, and creativity. However, AR also has some drawbacks, namely (1) sensitive to the change point of view, (2) not many who design, and (3) require a lot of storage memory on the equipment installed [35].

2. Improving critical thinking skills and spatial reasoning with augmented reality

According to Mustaquin and Kurniawan [35], a good learning must contain interactive aspects, fun, challenging, motivating, and provide opportunities for students to develop their creativity and independence according to their talents and interests of learners. This learning model can be learning by using appropriate learning media. It has been many studies that show the advantages of AR in education. Utilization of augmented reality as a medium of learning developed in various forms, such as games [38], [39]; ARmatika [40]; and DiedricAR [41]. In this article, the researchers review studies about the use of AR in mathematics learning in order to improve learners’ critical thinking skills and spatial reasoning.

2.1. Improving critical thinking skills with augmented reality in mathematics learning

As one of the 21st century skills, critical thinking is a skill that should be trained in mathematics at this time. There are at least three dimensions that need to be considered in learning activities: (1) tasks or activities that students engage in and the problems that they solved; (2) tools representing mathematical ideas and problem situations; and (3) the normative practices [42]. Therefore, teachers need to select the appropriate dimensions in learning activities so that students' critical thinking skills can be improved.

One of tools that can be used for mathematics learning is augmented reality. Several studies have shown that learning with augmented reality can improve critical thinking skills. Qian and Clark [38] conducted a review of studies about game-based learning and 21st century skills. Game-based learning game which is used for learning the game. The game aims to improve knowledge and skills acquisition. In addition, the game involves problem solving activities. In this studies, augmented reality is a genre of game-based learning.

There are four 21st century skills items, namely critical thinking, creativity, communication, and collaboration. Based on the study of literature that was conducted by Qian and Clark [38], 70% of the assessed article focuses on the use of game-based learning to improve critical thinking skills compared with other 21st century skills. Furthermore, Qian and Clark [38] recommended to researchers and practitioners to use games-based learning in learning activities to improve students' 21st century skills.

Kaufmann and Papp [43] developed mathematics learning media called Construct3D. The media was used for learning 3D dynamic geometry. Construct3D was designed based on the constructivist theory. This theory allows students to construct their new knowledge by testing ideas and approaches based on their prior knowledge and experiences, then applying these to a new situation, and integrating the new knowledge gained with pre-existing intellectual constructs. This learning process involves problem-solving and critical thinking. Thus, to improve students critical thinking skills can be done by implementing constructivist theory in mathematics learning.

Literature review conducted by Saidin, Halim, and Yahaya [44] showed that AR makes active learning process because the interactivity of its applications. Thus, it encourages students critical and creative thinking skills. The researchers had proved that AR features can engage students in learning process and help improve their visualization skills. Besides that, AR features also help teachers to explain well and make student easily understand. AR features used in mathematics study is head-mounted display and personal interaction panel. The feature allows students to do active learning.

2.2. Improving spatial reasoning with augmented reality in mathematics learning

Spatial reasoning is a cognitive aspect that need to be possessed in human life. In geometry learning, spatial reasoning is very useful for learn about 3D concept.

According to Cheng and Mix [45], spatial ability has a strong relationship with the mathematical skills of learners. Their research showed that spatial training can improve mathematics skills of learners.
Spatial training can also be realized in the form of mathematical learning. The mathematics learning is executed by the media using augmented reality-based learning in order to improve spatial reasoning. Studies have shown that the use of augmented reality in learning can improve spatial reasoning learners.

Kaufmann and Schmalstieg [27] developed Construct3D which was based instructional media AR in the 3D geometry. It allowed students to work collaboratively. The media panel is equipped with a personal interaction so that students could work interactively with the media. Media which is developed by AR enables learners to view objects from different sides so that didactics learning by doing concept and training of spatial abilities can be implemented.

de Ravé, Jiménez-Hornero, Ariza-Villaverde and Taguas-Ruiz [41] developed DiedricAR which was based on descriptive geometry/dihedral system learning. DiedricAR is a 3D visualization tool that is designed for spatial abilities development. The interaction of DiedricAR is touch interface and markers by using Android/iOS as the operating system. The content and learning mode are autonomous learning approach (step by step exercise). Teacher assistance in DiedricAR is not required because there exist brief overview and application tutorial in the workbook. Figure 2 shows the example of DiedricAR.

![Figure 1. Example of Construct3D [43]](image1)

![Figure 2. Examples of DiedricAR [41]](image2)

Cesaria and Herman [30] did descriptive qualitative research to describe the students’ mathematical reasoning in geometry by using ICT. According to Cesaria and Herman [30], geometry learning consists of visual experience and spatial abstraction. Therefore, mathematical reasoning in geometry can be called spatial reasoning. ICT in this study refers to augmented reality. To improve spatial reasoning, augmented reality that is developed should be considered as an external dynamic visualization. By using augmented reality, students are helped in understanding a complex and abstract concept.

Lin, Chen, and Chang [46] developed learning tools marker-based augmented reality for 3D learning. The results showed that spatial reasoning and mathematics scores have a positive correlation, which means that the higher the scores imply higher spatial reasoning mathematics scores. While spatial reasoning and gender are correlated each other only at a very low level. In addition, the results also showed that the use of AR in mathematics effectively improve spatial reasoning of students with low academic achievement. This is because the developed learning media enable learners to rotate an object 360°.

Based on the studies, AR can improve critical thinking skills through its features that can make interactive learning. AR features can engage students in learning process, can help teacher to explain well, and make student easily understand. Learning process that combine constructivist theory and AR as instructional media, it also can improve critical thinking skills of students. The learning process involves student-centered and problem solving. Instructional media that is developed with AR will produce dynamic objects. These objects can be rotated 360°. Dynamic objects enable learner to view the objects from different sides. This feature facilitates learners to improve their spatial reasoning [30].
3. Conclusion

Based on studies that have been conducted, it has been many researches which state that the use of AR as an instructional media gives positive effects. Mathematics learning using AR as instructional media can improve critical thinking by utilizing its features that can develop confidence of learners, facilitate student-centered learning, create problem solving, and create interactive learning. Instructional media that is developed should implement the constructivist theory. Besides that, AR has feature to make dynamic objects. By using this feature, learners can understand the position of the object in space and understand in imagining objects that change or movement. Therefore, spatial reasoning of learners can be improved.

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