The effect of reinforcement bridging on the elastic fracture energy of concrete

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Abstract. Failure of structures may be analyzed based on strength criterion or fracture mechanics criterion. Strength criterion deals with material resistance beyond the critical conditions such as yield, and ultimate strength. In fracture mechanics the critical condition is defined by toughness which can be stress intensity factor (K) or fracture energy (G). This study explores the critical point during loading up to the limit of elasticity based on linear elastic fracture mechanics – LEFM. However, nonlinearity frequently appears due to the existence of a relatively large fracture process zone – FPZ located at the crack-tip. The assumption of LEFM in quasi – brittle material such as concrete is therefore limited to large size structures only. The well-known approach to obtain the fracture energy Gf for infinite large structures is size effect law – SEL. Gf is defined as the specific energy, i. e. energy per unit crack plane area. This is elastic energy which is linear. To answer a question whether reinforcement affect the elastic fracture energy, a research was conducted following the principles of work-of-fracture – WOR using the RILEM Specification Test Method. Three-point-bend beam specimens were made of normal concrete, subjected to monotonic load P, until they failed. P–u relationship prevails the real work. The elastic fracture energy based on WOR that is \( G_{F\text{-et}}^{B} \) was found by divided the area under P–u curve of elastic range with ligament. The result is, \( G_{F\text{-et}}^{B} \) equal to 397.87 N/m for beam A1 (bending failure l/b=5.5), 133.6 N/m for beam B1 (Shear failure, l/b=5.0), 101.93 N/m for beam B2 (shear failure, l/b=5.0), 153.75 N/m for beam B3 (shear failure, l/b=5.0). The greater value \( G_{F\text{-et}}^{B} \) comparing to Gf implies that reinforcement affect significantly the elastic fracture energy.

1. Introduction

It is well known that failure behaviour of structures is greatly influenced by the behaviour of the material used. Concrete as a discrete material tends to be full of cracks even under service loads. According to fracture mechanics specialist, it appears natural that concrete structures should be designed based on fracture mechanics [1]. This approach is to find out the maximum critical conditions during the failure process. Fracture parameters which mainly affect the failure capacity are FPZ (contributed by 'bridging') and fracture toughness.

1.1. Fracture Process Zone - FPZ

Failure of quasi-brittle structures primarily is resulted from cracking of concrete, which leads to softening type-load deformation behaviour. Hence, it makes sense to accept that failure process of quasi-brittle mechanism is controlled by a propagation of the fracture zone.
Figure 1. Phenomenon of bridging mechanism

This zone exists at the tip of the actual crack and plays a role as crack arrester. Illustrated in Figure 1, one can see that this zone occurs as result of aggregate interlock. If these micro cracks solid significantly, then it may be having as yield or plastic zone under a constant stress – yield stress. According to the idea of Dugdale and Barentblatt [2][3], this yield stress will maintain as long as the forming of a plastic zone with a certain length, to meet the singularity condition.

The yield stress is defined as the lowest stress in which the asymptote of elastic stress distribution begins to infinite. Taken from [4] concrete has yield stress of 25 MPa. By applying the modified crack closure and the J-integral principle simultaneously, Patty (2004)[5] proposed a yield stress for fiber reinforced lightweight concrete with fiber volume fraction, of 0.5%, 1%, and 1.5% to be 14.051 MPa, 15.002 MPa and 17.104 MPa respectively.

1.2. Fracture Process Zone – FPZ

Fracture toughness can be clarified as both critical strain energy release rate denoted by parameter $G_c$, and critical stress intensity factor denoted by parameter $K_c$. These two parameters meet the following relationship

$$ K_c = \sqrt{EG_c} \tag{1} $$

One of the fundamental principles of fracture mechanics is that unstable crack growth (brittle fracture) occurs when the stress intensity factor $K$ reaches its critical value $K_c$. In case of reinforced concrete beams, the stress intensity factor due to applied load can be reduced by traction due to bridging effect between reinforcing steel and the concrete itself. So, to prevent structural materials from brittle fracture keep the calculated toughness below its critical value. Concrete is considered as a quasi-brittle material whose failure mechanism can be described based on energy principles as follow

$$ G_q = G_c + G_\sigma \tag{2} $$

where $G_q$ is energy release rate at the tip of the effective quasi-brittle crack. $G_c$ is the material surface energy, needed to create surface and $G_\sigma$ is a part of energy to overcome the cohesive pressure $\sigma(w)$ to separate the crack surfaces, $G_c$ is dealing with LEFM while $G_\sigma$ is dealing with NLFM

2. Literature review

Formula 2 describing a phenomenon that concrete may fail in two manners; brittle fracture, if $G_c$ is overcome, prevails
but if $G_\sigma$ is overcome it prevails

$$G_q = G_\sigma$$  (4)

The question in this case, does reinforcement-bridging affects $G_c$? In case of quasi-brittle concrete, bridging occurs when the crack has advanced beyond on aggregate that continues to transmit stresses across the crack until it ruptures or pulled out [6]. Bridging due reinforcement works as a composite action between steel bars and matrix, in case of reinforced concrete.

2.1. Researches of $G_c$ on plain concrete

As mentioned before, $G_c$ is energy consumed to create cracks. When the length of the cracks reaches their critical value $(a = a_c)$ then failure will occur rapidly. This condition happens, because of the no existence of micro cracks, consequently LEFM shall prevails. However, non-linearity frequently appears due to the existence of a relatively large fracture process zone located at the crack-tip. The assumption of LEFM in quasi-brittle materials is therefore limited to large size structures only. Figure 2 illustrates the load-deflection curves of geometrically similar structures [7]

![Figure 2. Load deflection diagram of geometrically similar structures of different size [7]](image)

$G_c$ is material property which has to be modeled for infinite large structures.

2.2. Size Effect Law (SEL)

The well-known approach to obtain the fracture energy $G_f$ for infinity large structures was proposed by Bazant (1987) based on size effect [8]. $G_f$ is defined as the specific energy, i.e., energy per unit crack plane area. For mode I fracture with three-point-bend loading (Figure 3) $G_f$ may be computed as follow

$$G_f = \frac{g(a_0)}{EA_B}$$  (5)

where

- $E$ = modulus of elasticity
- $a_0 = a_0/d$
- $a_0$ = length of the initial crack
- $d$ = height of specimen
- $A_B$ = constant to be determined

$g(a_0)$ is a geometric factor, given by

$$g(a_0) = \left(\frac{S}{b}\right)^2 \pi a_0[1.5 g_1(a_0)]^2$$  (6)
where \( S \) = span of beam specimen, \( a_0 = a_0/b \) and \( g_1(a_0) \) is the geometric function for stress intensity factor, as a function of \( a_0 \) for certain ratio of \( S/b \).

**Figure 3.** Series of geometrically similar structures (\( k_1 \) and \( k_2 \) are constant, two-dimensional similarity) [8]

2.3. Reported SEL-Based researches

Nallathambi and Karihaloo [9] used geometrically similar beam to determine \( G_f \) of normal concrete with maximum aggregate size \( d_a = 20 \) mm, that resulted \( G_f = 21.600 \) N/m.

Bazant and Pfeiffer [8][10] have reported \( G_f = 36.60 \) N/m for concrete with \( E = 27.7 \) GPa, and \( d_a = 13 \) mm. Concrete with \( E = 32.9 \) GPa and \( d_a = 5 \) mm, \( G_f = 23.67 \) N/m. Further, \( G_f = 38.35 \) N/m for concrete with \( d_a = 12.7 \) mm with \( E = 31.077 \) GPa.

Alexander [11] has found \( G_f = 71.06 \) N/m for concrete with \( d_a = 19 \) mm and \( E = 32.5 \) GPa.

Chan and Shieh [12] calculated \( G_f \) of lightweight concrete with \( E = 19.7 \) GPa; based on SEL, he found \( G_f = 34.42 \) N/m. Also reported for \( E = 22.3 \) GPa, \( G_f = 37.2 \) N/m.

The same approach used by Chan and Wang to determine fibre reinforced concrete with 1% fibre volume fraction resulting in \( G_f = 37.4 \) N/m.

2.4. \( G_c \) of reinforced concrete based on ‘Work-of-Fracture’

Work-of-fracture based on real work \( P–u \) which is obtained from a three-point-bend beam testing. Real work is the first energy method. The term ‘real work’ implies that the force and displacement considered are real [13]. Shown in Figure 4, \( P \) is external load increased monotonically and \( u \) is load line displacement ‘work’ is defined as

\[
w = \int_0^{u_1} Pd_u
\]

(7)
The relationship described by formula 7 represents the work needed to affect the considered point (point-c) to a value $u$

![Figure 4. Beam under elastic bending](image)

Further, the principle of real work applied as work-of-fracture is written as

$$W_f = \int_0^{u_1} P\,d\epsilon$$  \hspace{1cm} (8)

Where $u_1$ = final displacement at which the load is reduced to zero.

The fracture energy according to the RILEM definition, $G_f^R$, represents the average fracture energy in the ligament [14][15] obtained as

$$G_f^R = \frac{w_f}{bl}, l = (l - a_0)d$$  \hspace{1cm} (9)

where, $b$ = thickness of beam  
$l$ = length of ligament  
$d$ = depth of beam  
$a_0$ = $a_0/d$, $a_0$ = length of notch

as shown in Figure 6a, followed by P-u curve described by Figure 6b.

In this ‘case study’, the part A of P-u relationship which is the elastic energy, may be calculated using the principle of ‘work-of-fracture’ under the following basic consideration: The elastic peak load $P_c$ will drop to zero instantly at the moment $u=u_c$, if there is no bridging (due to reinforcing or aggregate interlock). This means that $u_c$ corresponding to $P_c$ is similar to $u_1$ corresponding to $P=0$. 

![Figure 5. Real work principle [13]](image)
Hence, the determination of elastic fracture energy for certain size of beams may be defined based on ‘work-of-fracture’ by using the formula 9 with $P=P_c$ and $u=u_c$.

![Figure 6. Three point bend beam (a) Specimen geometry, (b) $P-u$ curve](image)

3. A case of Work-of-Fracture – WOR

Three-point-bending reinforced concrete beams were tested as shown in Figure 6. There were four specimens made of normal concrete with $f'_c=30$ MPa and reinforced by 2 #12 mm just to avoid them from instantly failure, as shown Figure 7 and Figure 8. All of them, has (200x100) mm dimension with $l/h=5.5$ for beam A1; 5.0 for beam B1, B2, and B3. The dimension met the requirement of RILEM [16] regarding the maximum size of aggregate. Following are the result of loading where $P$ was increased monotonically until it fails.

4. Result and Discussion

Basically, concrete fails in two manners, brittle fracture which is LEFM based or quasi-brittle fracture which is nonlinear fracture mechanics – NLFM based. Brittle fracture deals with the energy consumed to create cracks; failure occurs at the moment when a critical length of the crack ($a_c$) is obtained. This study deals with brittle elastic energy performed by plain concrete and reinforced concrete as well. Plain concrete used SEL to find $G_f$ and reinforced concrete used WOR to find $G_{F-EL}^R$. The result is, $G_{F-EL}^R$ equal to 397.87 N/m for beam A1 (bending failure $l/h=5.5$), 133.6 N/m for beam B1 (Shear failure, $l/h=5.0$), 101.93 N/m for beam B2 (shear failure, $l/h=5.0$), 153.75 N/m for beam B3 (shear failure, $l/h=5.0$). The greater value of $G_{F-EL}^R$ comparing to $G_f$ implies that reinforcement significantly affect the elastic fracture energy. The result of the analysis shows that due reinforcement, the elastic energy for bending failure is greater than for shear failure. In addition, the critical length ($a_c$) for bending failure must be longer than that of shear failure. In bending failure plastic hinge might be developed due to wedge forces between reinforcement and concrete. On the contrary plastic hinge is not to be expected to occur in reinforced concrete with shear failure.
Figure 7. Results of three-point loading beam (A1 and B1).

(a) Reinforced Beam – 1, I/h = 5.5

(b) Reinforced Beam – 2, I/h = 5.0
Figure 8. Results of three-point loading beam (B2 and B3)
5. Conclusion

- Failure of structures catastrophically can be avoided by creating bridging effect, which is leading to perform micro crack at the tip of main crack.
- Slenderness of bending element may affect the mode of fracture, i.e. bending or shear.
- Presence of reinforcement roles in increasing elastic fracture energy, not only in energy dissipation (after the limit of elastic fracture energy is reached).

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