Weyl symmetry and its spontaneous breaking
in Standard Model and inflation

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Abstract

We discuss the Weyl gauge symmetry and its spontaneous breaking and apply it to model building beyond the Standard Model and inflation. In models with non-minimal couplings of the scalar fields to the Ricci scalar, that are conformal invariant, the spontaneous generation by a scalar field(s) vev (or combination thereof) of a positive Newton constant demands a negative kinetic term for the scalar field, or vice-versa. This is naturally avoided in models with additional Weyl gauge symmetry. The Weyl gauge field $\omega_\mu$, shown to couple only to the scalar sector of a SM-like Lagrangian, undergoes a Stueckelberg mechanism and becomes massive after “eating” the (radial mode) would-be-Goldstone field (dilaton $\rho$) in the scalar sector. Before the decoupling of $\omega_\mu$, the dilaton can act as UV regulator and maintain the Weyl symmetry at the quantum level, with relevance for solving the hierarchy problem. After the decoupling of $\omega_\mu$, the scalar potential depends only on the remaining (angular variables) scalar fields. A successful hilltop inflation is then possible with one of these scalar fields identified as the inflaton. While our approach is formulated in Riemannian geometry, the natural framework is that of Weyl geometry which is shown to lead to a similar Lagrangian, up to a total derivative.

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1 Motivation

In this letter we discuss the Weyl gauge symmetry and its spontaneous breaking together with its implications for model building beyond the Standard Model (SM) and for inflation.

One phenomenological motivation relates to the observation that the SM with a Higgs mass parameter set to zero has a classical scale symmetry [1]. If this symmetry is preserved at the quantum level by (a scale-invariant) UV regularisation as in [2] (see also [3–7]), and is broken spontaneously only, it can naturally protect at the quantum level a hierarchy of fields vev’s of the theory [3, 6, 8–10]. The hierarchy we refer to is that between the Higgs field vev (electroweak scale) and that of “new physics” represented by the vev of the flat direction (dilaton) associated with (global) scale symmetry breaking (such hierarchy of vev’s can be generated by a classical hierarchy of the dimensionless couplings [11, 12]).

A proper study of the hierarchy problem demands including gravity and generating the Planck scale ($M_p$) spontaneously. This can be done in Brans-Dicke-Jordan theories of gravity via a non-minimal coupling between a scalar field(s) and the scalar curvature ($R$), when this field(s) develops a non-zero vev. However, demanding the theory be conformal-anomaly-free and spontaneous-only breaking of the conformal symmetry, leads to a negative kinetic term(s) for the corresponding scalar field(s), a “nuisance” that is often quietly glided over. Understanding better this problem is another motivation for this work.

The problem of a negative kinetic term is automatically avoided in models with Weyl gauge symmetry. This symmetry is the natural extension of conformal invariant models [13–22]; the conformal transformation of the metric is extended by the associated gauge transformation of a Weyl gauge field ($\omega_\mu$) which is of geometric origin. We show that $\omega_\mu$ undergoes a Stueckelberg mechanism and becomes massive by “eating” the would-be-Goldstone field (dilaton $\rho$) which is the radial direction in the field space of scalar fields with non-minimal coupling to $R$. The Weyl gauge symmetry is then spontaneously broken and there are no negative kinetic terms in the theory. The vacuum expectation value $\langle \rho \rangle$ of the flat direction (dilaton) controls the mass of $\omega_\mu$ and $M_p$. We show how this mechanism works for multiple scalar fields ($\phi_j$) of different non-minimal couplings ($\xi_j$). After decoupling of the massive $\omega_\mu$, the potential depends on the remaining (angular variables) scalar fields only.

While our analysis is formulated in Riemannian geometry (RG) extended by the Weyl gauge symmetry, the natural framework for this study is conformal Weyl geometry (WG) [13, 14]. In the RG case, imposing the Weyl symmetry leads to a Lagrangian with the corresponding current $K_\mu = \partial_\mu K$ where $K_\mu$ interacts with the field $\omega_\mu$ and $K = \rho^2$. We show that this Lagrangian is identical, up to a total derivative term, to that obtained in WG where the Weyl symmetric Lagrangian is naturally built-in with the curvature scalar and tensors of WG. This equivalence follows from the relation between $R$ computed in Riemann geometry with the Levi-Civita connection and its counterpart $\tilde{R}$ computed in Weyl geometry. This gives a geometrical interpretation of the Stueckelberg mechanism for the field $\omega_\mu$.

For two scalar fields with non-minimal couplings, after $\omega_\mu$ decouples, the potential depends only on the angular field $\theta$ and becomes constant at a large $\tan \theta$, making possible a successful hilltop inflation. We also discuss how the dilaton enforces a scale-invariant UV regularization before the Stueckelberg mechanism, which is relevant for quantum scale invariant models.
2 Implications of Weyl gauge symmetry

Here we analyze how models invariant under conformal transformations become ghost-free while generating spontaneously a positive Newton constant, when an associated Weyl gauge transformation is added. The Lagrangian so obtained is equivalent to that derived in Weyl geometry, up to a total derivative; a SM-like model with this symmetry is also constructed.

2.1 Weyl symmetry or how to obtain a Lagrangian without ghosts

Consider a (local) conformal transformation of the metric and of a scalar field \( \phi \) and a fermion \( \psi \), as follows

\[
g_{\mu\nu} \rightarrow g'_{\mu\nu} = e^{2\alpha(x)} g_{\mu\nu}, \\
\phi \rightarrow \phi' = e^{-\alpha(x)} \Delta_s \phi, \\
\psi \rightarrow \psi' = e^{-\alpha(x)} \Delta_f \psi.
\]  

Then \( g^{\mu\nu} = e^{-2\alpha(x)} g^{\mu\nu} \) and \( \sqrt{g'} = e^{4\alpha(x)} \sqrt{g} \) with \( g = |\det g_{\mu\nu}|. \) Here \( \Delta_s = 1 \) and \( \Delta_f = 3/2 \).

We would like to generate the Planck scale spontaneously, from the vev of a scalar field \( \phi \). To this purpose one uses that the Lagrangian

\[
L^\pm_1 = \pm \sqrt{g} \frac{\xi}{2} \left\{ \frac{1}{6} \phi^2 R + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right\}
\]  

is invariant under transformation \( \text{(1)} \). \( \xi \) is the non-minimal coupling and we assume \( \xi > 0 \).

Then one is facing the following issue. To generate the Einstein term

\[
L_E = -\frac{1}{2} \sqrt{g} M_p^2 R
\]  

after spontaneous breaking of conformal symmetry from a vev of \( \phi \) from the first term in \( \text{(2)} \), one must take the minus sign in front of \( \text{(2)} \), that means a negative kinetic term for \( \phi \) (ghost) is present in the theory, which may not be acceptable. Alternatively a positive kinetic term leads to \( M_p^2 < 0 \). One usually sets \( M_p = \langle \phi \rangle \) ("gauge fixing" the Planck scale) and the ghost presence is then ignored. Yet, one cannot have the benefit of conformal symmetry but ignore this "side effect", therefore we would like to understand its meaning.

To avoid this problem, we associate to transformation \( \text{(1)} \) that of a (Weyl) vector field \( \omega_\mu \), which, in the light of \( \text{(1)} \), is of geometric origin

\[
\omega_\mu \rightarrow \omega'_\mu = \omega_\mu - \frac{2}{q} \partial_\mu \alpha(x),
\]  

then consider adding the kinetic term below, with a suitable normalization coefficient

\[
L_2 = \frac{1}{2} (1 + \xi) \sqrt{g} g^{\mu\nu} \tilde{D}_\mu \phi \tilde{D}_\nu \phi, \\
\tilde{D}_\mu \equiv \partial_\mu - \frac{q}{2} \omega_\mu.
\]  

\(^1\text{Conventions: metric } (+, -, -, -), \quad R^\lambda_{\mu\nu\rho} = \partial_{[\nu} \Gamma^\lambda_{\mu\rho]} - \partial_{[\rho} \Gamma^\lambda_{\mu\nu]} + \Gamma^\lambda_{\nu\sigma} \Gamma^\sigma_{\mu\rho} - \Gamma^\lambda_{\rho\sigma} \Gamma^\sigma_{\mu\nu}, \quad R_{\mu\nu} = R^\lambda_{\mu\lambda\nu}, \quad R = g^{\mu\nu} R_{\mu\nu}.\)

\(^2\text{To see this, one uses that under eq. } \text{(1)} \text{ } R \text{ transforms as } R \rightarrow R' = e^{-2\alpha(x)}(R - 6 e^{-2\alpha(x)} \Box \phi(x)) \)
$L_2$ is invariant under (1), (4) since $\tilde{D}_\mu \phi \rightarrow e^{-\alpha} \tilde{D}_\mu \phi$, due to the presence of $\omega_\mu$. Since $L_1^\pm$ is also invariant under (1), (4), the sum $L_1^\pm + L_2$ is also invariant. Hereafter we take $L_1^-$. One has $L_1^- + L_2 = (1/2) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - (1/12) \xi \phi^2 R + \cdots$, with a canonically normalized kinetic term for $\phi$. Thus, the Planck (mass)$^2$ generated by $\langle \phi^2 \rangle$ and the kinetic term of $\phi$ are simultaneously positive. This is made possible by the additional presence of the Weyl field $\omega_\mu$; this is a sufficient condition for the consistency of the theory (absence of ghosts).

2.2 SM Lagrangian with Weyl gauge symmetry

We use the above observation about $L_1^- + L_2$ to construct a Lagrangian without ghosts and invariant under eqs. (1), (4). For generality, consider a version of $L_1^- + L_2$ with more scalar fields $\phi_j$ of non-minimal couplings $\xi_j$, then a Weyl-invariant Lagrangian is

$$L = \sqrt{g} \left\{ - \frac{\xi_j}{2} \left[ \frac{1}{6} \phi_j^2 R + g^{\mu\nu} \partial_\mu \phi_j \partial_\nu \phi_j \right] + (1 + \xi_j) \frac{1}{2} g^{\mu\nu} \tilde{D}_\mu \phi_j \tilde{D}_\nu \phi_j - V(\phi_j) \right\}. \quad (6)$$

A summation is understood over repeated index $j = 1, 2, 3 \cdots$. We also added a potential $V(\phi_j)$ for the scalars $\phi_j$; given the conformal symmetry, $V$ is a homogeneous function

$$V(\phi_j) = \phi_k^2 V(\phi_j/\phi_k), \quad k = \text{fixed.} \quad (7)$$

$L$ can be re-written as

$$L = \sqrt{g} \left\{ - \frac{\xi_j}{12} \phi_j^2 R + \frac{g^{\mu\nu}}{2} (\partial_\mu \phi_j) (\partial_\nu \phi_j) - \frac{q}{4} g^{\mu\nu} \omega_\mu K_\nu + \frac{q^2}{8} K \omega_\mu \omega^\mu - V(\phi_j) \right\}, \quad (8)$$

where we introduced the current

$$K_\nu = \partial_\nu K, \quad K = (1 + \xi_j) \phi_j^2. \quad (9)$$

$L$ above is invariant under (1), (4), for all values of $\xi_j$, thanks to the $\omega_\mu$-dependent terms. $L$ has positive kinetic term for $\phi_j$ and $M_\phi^2 > 0$ when generated by the vev of $\langle \phi \rangle$ (assuming $\xi_j > 0$). In the absence of the $\omega_\mu$-dependent part, $L$ is not conformal (unless $\xi_j = -1$), but only global conformal. Note that unlike in gauge theories, $\omega_\mu$ is a Weyl vector under a real transformation of the fields $\phi_j$ (missing the $i$ factor). The associated current $K_\mu$ is non-zero for $\phi_j$ reals (unlike in QED case where it would vanish).

Further, we include a kinetic term for $\omega_\mu$ with the “usual” (pseudo)Riemannian definition

$$L_g = -\frac{\sqrt{g}}{4} g^{\rho\sigma} g^{\mu\nu} F_{\mu\rho} F_{\nu\sigma}, \quad F_{\mu\nu} = D_\mu \omega_\nu - D_\nu \omega_\mu, \quad D_\mu \omega_\nu = \partial_\mu \omega_\nu - \Gamma_\mu^\rho \omega_\nu, \quad (10)$$

$L_g$ is invariant under (1), (4), since the metric part is invariant and $F_{\mu\nu} = (\partial_\mu \omega_\nu - \partial_\nu \omega_\mu)$ is invariant, too. The Riemann connection $\Gamma_\mu^\rho$, symmetric in $\mu, \nu$, is not invariant under (1).

$^3$The Riemann affine connection used here is $\Gamma_\mu^\rho = (1/2) g^{\rho\sigma} [\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}].$
Finally, one can consider the Weyl-invariant Lagrangian $L_f$ for the massless fermions of the theory that transform under (1). $L_f$ has the usual form in (pseudo)Riemann space

$$L_f = \sqrt{g} \bar{\psi} i\gamma^\mu e^\mu_a D_\mu \psi, \quad D_\mu \psi = \left( \partial_\mu + \frac{1}{2} \omega^{\mu}_{ab} \sigma_{ab} \right) \psi$$

(11)

where $\omega^{\mu}_{ab} = e^b_\lambda (-\partial_\mu e^\lambda_a + e^\nu_{ab} \Gamma^\mu_{\lambda \nu})$ is the spin connection and $\sigma_{ab} = \frac{1}{4}[\gamma_a, \gamma_b]$. Note that $g_{\mu \nu} = e^b_\mu e^a_\nu \eta_{ab}$ and $e^\mu_{ab} e^a_\nu \sigma_{ab} = \delta^\mu_{\nu}$. Under a Weyl transformation of the metric, eq.(1), the vielbein $e^a_\mu$ transforms as $e^a_\mu' = e^{\alpha(x)} e^a_\mu$, while for the spin connection we have $\omega^{\mu}_{ab}' = \omega^{\mu}_{ab} + (e^a_\mu e^b_\nu - e^b_\mu e^a_\nu) \partial_\mu \alpha$. Then it can be shown that $L_f$ is invariant under a Weyl gauge transformation, eqs.(1), (4), and there is no coupling of fermions to the gauge field $\omega^\mu_\nu$.

Regarding the SM gauge fields kinetic terms ($L_G$), these are invariant under Weyl gauge symmetry. Indeed, the gauge fields presence under the covariant derivative that contains $\partial_\mu$ shows that these are invariant, since coordinates do not transform under (1). Therefore, there is no coupling between SM gauge fields and $\omega^\mu_\nu$. For example, for the $U(1)_Y$ gauge field $A_\mu$, the covariant derivative can be written as $D_\mu A_\nu = \partial_\mu A_\nu - \Gamma^\rho_{\mu \nu} A_\rho$. The gauge kinetic terms do not contain the Christoffel symbols because $F_{\mu \nu} = D_\mu A_\nu - D_\nu A_\mu = \partial_\mu A_\nu - \partial_\nu A_\mu$.

The sum, $\mathcal{L} = L + L_f + L_G$, is the total SM-like Lagrangian with Weyl gauge symmetry which is invariant under (1), (4). Here $L$ is immediately adapted to accommodate the Higgs doublet of the SM with one of the $\phi_j$ fields to account for the Higgs neutral scalar. In conclusion, we have a Lagrangian that is invariant under (1), (4), in which, remarkably, only the scalar sector actually couples directly to the field $\omega^\mu_\nu$.

### 2.3 From Riemann to Weyl conformal geometry

The presence of the Weyl gauge field in the conformal action of our model is intriguing in the context of the Riemannian geometry. Its place is familiar in Weyl’s conformal geometry [13], see [14] for a review and applications. In the following we explore the relation between these two formulations for the model we considered.

Weyl geometry is a scalar-vector-tensor theory of gravity and thus provides a generalization (to classes of equivalence) of Brans-Dicke-Jordan scalar-tensor theory and of other conformal invariant models [15]. It was used for model building [16] with renewed recent interest in [17–22]. If the Weyl field is set to zero, one obtains (Weyl integrable) models similar to Brans-Dicke-Jordan theory, see [21] and references therein.

In Weyl geometry the curvature scalars and tensors and the connection are different from the Riemannian case where they are induced by the metric alone. In Weyl geometry

$$\tilde{\Gamma}^\rho_{\mu \nu} = \Gamma^\rho_{\mu \nu} + \frac{q}{2} \left[ \delta^\rho_\mu \omega_\nu + \delta^\rho_\nu \omega_\mu - g_{\mu \nu} \omega^\rho \right],$$

(12)

where $\Gamma^\rho_{\mu \nu}$ are the connection coefficients in the Riemannian geometry. Under (1), (4) the coefficients $\tilde{\Gamma}^\rho_{\mu \nu}$ are invariant, as one can easily check. The system is torsion-free. The Riemann tensor in Weyl geometry is then generated by the “new” $\tilde{\Gamma}^\rho_{\mu \nu}$

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*One could also add a Weyl tensor term to the action, which is invariant under (1), but we do not do it. The Weyl tensor itself also brings a negative kinetic term for the graviton.*
\[ \tilde{R}^\lambda_{\mu\nu\sigma} = \partial_\nu \tilde{\Gamma}^\lambda_{\mu\sigma} - \partial_\sigma \tilde{\Gamma}^\lambda_{\mu\nu} + \tilde{\Gamma}^\lambda_{\nu\rho} \tilde{\Gamma}^\rho_{\mu\sigma} - \tilde{\Gamma}^\lambda_{\sigma\rho} \tilde{\Gamma}^\rho_{\mu\nu}, \]  
(13)

and then \( \tilde{R} = \tilde{R}^\lambda_{\mu\lambda\sigma} + \tilde{R}^\mu_{\nu\lambda\sigma} \). We can then compute \( \tilde{R} \) and find

\[ \tilde{R} = R - 3q \left[ \partial_\mu \omega^\mu + \frac{1}{2} \omega^\rho \partial_\rho g_{\lambda\beta} \partial_\mu g_{\lambda\beta} \right] - \frac{3}{2} q^2 \omega^\mu \omega_\mu. \]  
(14)

Then one checks that under transformations (1) and (4),

\[ \tilde{R} 
\rightarrow
\tilde{R}'
= e^{-2 \alpha (x) \tilde{R}}. \]  
(15)

As a result, the Lagrangian

\[ L_{1w} = -\sqrt{g} \frac{1}{12} \xi_j \phi_j^2 \tilde{R}, \]  
(16)

is invariant under combined transformations (1), (4). This is unlike in the Riemannian case of the previous section where the non-minimal coupling term in the action was not invariant.

Further, we can define a kinetic term for \( \phi \) in Weyl geometry, invariant under (1), (4)

\[ L_{2w} = \frac{1}{2} \sqrt{g} g^{\mu\nu} \tilde{D}_\mu \phi_j \tilde{D}_\nu \phi_j - \sqrt{g} V(\phi). \]  
(17)

We also have a gauge kinetic term \( L_{3w} \) for \( \omega_\mu \), now defined by new coefficients \( \tilde{\Gamma} \) of (12)

\[ L_{3w} = -\sqrt{g} \frac{1}{4} g^{\mu\rho} F_{\mu\nu} F_{\rho\sigma}, \quad F_{\mu\nu} = \tilde{D}_\mu \omega_\nu - \tilde{D}_\nu \omega_\mu, \quad \tilde{D}_\mu \omega_\nu = \partial_\mu \omega_\nu - \tilde{\Gamma}^\rho_{\mu\nu} \omega_\rho. \]  
(18)

Adding together \( L_{1w}, L_{2w}, L_{3w} \), and \( L_{4w} \), each of these invariant under (1), (4), we obtain a total Lagrangian for the case of Weyl geometry. It is interesting to see that this Lagrangian is equal to \( L + L_g + L_f \) of (8), (10) and (11), up to a total derivative term. This follows from the relation

\[ L_{1w} + L_{2w} = L + q^2 \xi_j \partial_\mu g \phi_j^2 \omega^\mu. \]  
(20)

To show eq.(20), one uses the relation between \( \tilde{R} \) and \( R \) of eq.(11) that relates Weyl and Riemann scalar curvatures and the following relation for the Riemann metric \( \partial_\lambda g = g \gamma^a \partial_\nu g_{a\mu}. \)
Eq. (20) shows that our model agrees (for two fields case) with that in [17] elegantly built within Weyl geometry from the onset. We thus obtained the same Lagrangian in Riemann and Weyl geometry, albeit with different initial motivations. Our motivation for a consistent, ghost-free conformal action, with this symmetry broken spontaneously, lead us to introduce a gauge transformation and Weyl gauge field associated to (1).

3 Spontaneous breaking of Weyl conformal symmetry

In this section we show how the Weyl conformal symmetry of our model is spontaneously broken for one or more scalar fields of non-minimal couplings $\xi_j$ to $R$. Then, we show that the (radial mode) would-be Goldstone boson (dilaton $\rho$) of the Weyl symmetry decouples from the angular variables fields due to a Stueckelberg mechanism for the Weyl gauge field which thus becomes massive. Before decoupling, the dilaton can provide a scale-invariant ultraviolet (UV) regularisation for models in which quantum scale invariance is important.

3.1 One scalar field and Stueckelberg mechanism for $\omega_\mu$

Let us first show how spontaneous breaking of Weyl symmetry happens for one scalar field $\phi$ with $\phi = \langle \phi \rangle + \delta \phi$ and $\delta \phi$ are fluctuations. Then $\hat{L}$ of eq. (8) simplifies (no sum over $j$) and

$$ K = (1 + \xi) \phi^2, \quad V = \frac{\lambda}{4!} \phi^4, \quad (21) $$

where $V$ is the only one possible. One can expand $L$ about $\langle \phi \rangle$ and fix the coefficient of $R$ to $M_p^2$. However, fluctuations $\delta \phi$ are still coupled to $R$. To decouple these from $R$, we rescale the metric to

$$ \hat{g}_{\mu \nu} = \Omega g_{\mu \nu}, \quad \Omega = \frac{\xi}{6} \frac{\phi^2}{\langle \phi \rangle^2}. \quad (22) $$

Hereafter a hat on a variable denotes the Einstein frame value of that variable. From eq. (8) for one field and eq. (22) we obtain the tensor-scalar part of Einstein-frame Lagrangian as

$$ \hat{L} = \sqrt{\hat{g}} \left\{ -\frac{1}{2} \langle \phi \rangle^2 \hat{R} + \frac{3}{4} \langle \phi \rangle^2 (\partial_\mu \ln \Omega)^2 + \frac{1}{11} \left[ \frac{1}{2} (\partial_\mu \phi)^2 + \frac{q^2}{8} K \omega_\mu \omega^\mu - \frac{q}{4} \omega_\mu K_\mu \right] - \frac{V}{\Omega^2} \right\} \quad (23) $$

giving

$$ \hat{L} = \sqrt{\hat{g}} \left[ -\frac{1}{2} \langle \phi \rangle^2 \hat{R} + 3 \langle \phi \rangle^2 \left( 1 + \frac{1}{\xi} \right) \left( \frac{\partial_\mu \phi}{\phi} \right)^2 + \frac{3}{4} q^2 \langle \phi \rangle^2 \left( 1 + \frac{1}{\xi} \right) \omega_\mu \omega^\mu \right. $$

$$ \left. -3 q \langle \phi \rangle^2 \left( 1 + \frac{1}{\xi} \right) \omega^\mu \partial_\mu \ln \phi - \frac{3 \lambda}{2 \xi^2} \langle \phi \rangle^4 \right]. \quad (24) $$

where all contractions are with the new metric $\hat{g}_{\mu \nu}$. Finally, we introduce

$$ \omega'_\mu = \omega_\mu - \frac{2}{q} \partial_\mu \ln \phi. \quad (25) $$
\[
\hat{L} = \sqrt{\hat{g}} \left[ -\frac{1}{2} (\phi)^2 \hat{R} + \frac{3}{4} q^2 (\phi)^2 \left( 1 + \frac{1}{\xi} \right) \omega_\mu \omega^\mu - \frac{3\lambda}{2} \xi^2 (\phi)^4 \right]
\]  
(26)

As a result, the scalar (dilaton) field \(\phi\) is “eaten” by the Weyl gauge boson \(\omega_\mu\). The mass of \(\omega_\mu\) is \(m_\omega^2 = (3q^2/2)(1 + 1/\xi)\langle \phi \rangle^2\). Therefore, conformal symmetry is broken spontaneously as in the Stueckelberg formulation for a massive \(U(1)\), without a corresponding Higgs mode. The number of degrees of freedom remains the same (three): in Jordan frame we had a real scalar and a massless vector, while in Einstein frame, after breaking there is no scalar field but a massive vector boson. Also note that the gauge kinetic term \(L_g\), see eq.(10), is invariant under (22), (25). The scalar potential becomes a cosmological constant, \(V_0 = 3\lambda(\phi)^4/(2\xi^2)\), in Einstein frame.

Transformation (25) may be seen as a Weyl gauge transformation \(\alpha = \ln \sqrt{\Omega}\) corresponding to (22). Then the scalar field \(\phi\) transforms according to eq.(1) into

\[
\phi' = e^{-\ln \sqrt{\Omega}} \phi = \sqrt{6/\xi} \langle \phi \rangle.
\]  
(27)

and it is not dynamical anymore. Therefore spontaneous breaking of conformal symmetry fixing the Planck scale (to \(M_p = \langle \phi \rangle\)) and Stueckelberg mechanism may be seen as the effect of a special Weyl transformation.

### 3.2 Two scalar fields and Stueckelberg mechanism for \(\omega_\mu\)

Let us consider now the more interesting case of two scalar fields in eq.(8), \(j=1,2\). Then

\[
K = (1 + \xi_1) \phi_1^2 + (1 + \xi_2) \phi_2^2, \quad V(\phi_1, \phi_2) = \frac{\lambda_1}{4!} \phi_1^4 + \frac{\lambda_2}{4!} \phi_2^4 + \frac{\lambda_{12}}{4} \phi_1^2 \phi_2^2.
\]  
(28)

\(V\) shown above can be more general e.g. can contain operators like \(\phi_1^6/\phi_2^2\), etc. The results below are for an arbitrary potential, homogeneous function of \(\phi_{1,2}\) (i.e. scale invariant).

As seen in the Jordan frame, after spontaneous breaking of Weyl symmetry, \(M_p\) is fixed to \(v\) (eq.(29)). But to decouple \(R\) from the fluctuations \(\delta \phi_{1,2}\) about \(\langle \phi_{1,2} \rangle\), consider this breaking in the Einstein frame. Let us perform a metric rescaling of \(L\) eq.(8), to

\[
\hat{g}_{\mu\nu} = \Omega g_{\mu\nu}, \quad \Omega = \frac{1}{6v^2} (\xi_1 \phi_1^2 + \xi_2 \phi_2^2), \quad v^2 \equiv \langle \xi_1 \phi_1^2 + \xi_2 \phi_2^2 \rangle.
\]  
(29)

Here \(v\) ensures that \(\Omega\) is dimensionless. From eq. (8) for two fields and with (29), we obtain the corresponding Einstein-frame Lagrangian as

\[
\hat{L} = \sqrt{\hat{g}} \left[ -\frac{1}{2} v^2 \hat{R} + \frac{3}{4} v^2 (\partial_\mu \ln \Omega)^2 
\right.
\]

\[
+ \frac{1}{\Omega} \left( \frac{1}{2} (\partial_\mu \phi_1)^2 + \frac{1}{2} (\partial_\mu \phi_2)^2 + \frac{q^2}{8} K \omega_\mu \omega^\mu - \frac{q}{4} \omega^\mu K_\mu \right) - \hat{V}
\]  
(30)
where all contractions are with the new metric $\hat{g}_{\mu\nu}$; $\Omega$, $K$ and $\hat{V}$ are functions of $\phi_{1,2}$ with

$$\hat{V}(\phi_1, \phi_2) = \frac{1}{\Omega^2} V(\phi_1, \phi_2).$$  \hfill (31)

Then

$$\hat{L} = \sqrt{\hat{g}} \left[ -\frac{1}{2} v^2 \hat{R} + \frac{1}{2} G_{ij} \partial_\mu \phi_i \partial^\mu \phi_j + \frac{q^2 K}{8 \Omega} \omega_\mu \omega^\mu - \frac{q}{\omega^\mu} K_\mu - \hat{V} \right]$$  \hfill (32)

where

$$G_{ij} = \frac{1}{6 v^2 \Omega^2} \left( \begin{array}{cc} \xi_1 (1 + \xi_1) \phi_1^2 + \xi_2 \phi_2^2 & \xi_1 \xi_2 \phi_1 \phi_2 \\ \xi_1 \xi_2 \phi_1 \phi_2 & \xi_2 (1 + \xi_2) \phi_2^2 + \xi_1 \phi_1^2 \end{array} \right), \quad i, j = 1, 2. \hfill (33)\n
The kinetic terms in $\hat{L}$ become diagonal (no mixing) in a new fields basis of $(\rho, \theta)$ where

$$\phi_1 = \frac{1}{\sqrt{1 + \xi_1}} \rho \sin \theta,$$

$$\phi_2 = \frac{1}{\sqrt{1 + \xi_2}} \rho \cos \theta.$$  \hfill (34)

It is more illustrative however to first bring the Weyl terms in $\hat{L}$ to a quadratic form using

$$\omega'_\mu = \omega_\mu - \frac{1}{q} \partial_\mu \ln K,$$  \hfill (35)

where notice that $K = \rho^2$. Adding $\hat{L}_g$ of eq. (10) with eq. (29) for the Weyl field $\omega_\mu$ then

$$\hat{L} + \hat{L}_g = \sqrt{\hat{g}} \left[ -\frac{1}{2} v^2 \hat{R} + \frac{1}{2} G_{ij} \partial_\mu \phi_i \partial^\mu \phi_j - \frac{1}{8 K \Omega} (\partial_\mu K)^2 - \frac{1}{4} F'^{\mu}_{\nu} F'^{\nu}_{\mu} + \frac{K}{8 \Omega} q^2 \omega'_\mu \omega'^\mu - \hat{V} \right]$$

$$= \sqrt{\hat{g}} \left[ -\frac{1}{2} v^2 \hat{R} + \frac{1}{2} T_{ij} \partial_\mu \phi_i \partial^\mu \phi_j - \frac{1}{4} F'^{\mu}_{\nu} F'^{\nu}_{\mu} + \frac{K}{8 \Omega} q^2 \omega'_\mu \omega'^\mu - \hat{V} \right]$$  \hfill (36)

where $F'^{\mu}_{\nu} = \tilde{D}_\mu \omega'_\nu - \tilde{D}_\nu \omega'_\mu$ is invariant under (33). Above we denoted $T_{ij} = G_{ij} + H_{ij}$, $(i, j = 1, 2)$, with:

$$H_{ij} = -\frac{1}{\Omega} \frac{1}{K} \left( \begin{array}{cc} (1 + \xi_1)^2 \phi_1^2 & (1 + \xi_1)(1 + \xi_2) \phi_1 \phi_2 \\ (1 + \xi_1)(1 + \xi_2) \phi_1 \phi_2 & (1 + \xi_2)^2 \phi_2^2 \end{array} \right). \hfill (37)\n
In the new basis (34) the scalar kinetic terms in eq. (36) are reduced to a single term and

$$\hat{L} + \hat{L}_g = \sqrt{\hat{g}} \left[ -\frac{1}{2} v^2 \hat{R} + \frac{1}{2} F(\theta) v^2 (\partial_\mu \tan \theta)^2 - \frac{1}{4} F'^{\mu}_{\nu} F'^{\nu}_{\mu} + \frac{1}{2} m^2(\theta) \tilde{g}^{\mu\nu} \omega'_\mu \omega'_\nu - \hat{V} \right]$$  \hfill (38)
with
\[
F(\theta) = \frac{6b}{\xi_2} \frac{\tan^2 \theta + \xi_2/\xi_1}{(1 + \tan^2 \theta)(\tan^2 \theta + b)^2}, \quad b = \frac{\xi_2(1 + \xi_1)}{\xi_1(1 + \xi_2)}. \tag{39}
\]

Therefore, we are left with the “angular” kinetic term for $\theta$ only. The kinetic term of the radial (Goldstone) coordinate $\rho$ (where $\rho^2 = K$) has disappeared, via Stueckelberg mechanism, as it was “eaten” by the Weyl gauge boson $\omega'$ in eq. (33). This is similar to the case with one scalar field in eq. (25). Thus, in the Einstein frame we have a massive vector boson and one (real) scalar field left ($\theta$), while in Jordan frame we had two (real) scalar fields and a massless $\omega_\mu$, so the number of degrees of freedom is again conserved.

Further, the function $m^2(\theta)$ in (38) is given by
\[
m^2(\theta) = \frac{q^2 K}{4\Omega} = \frac{3q^2}{2} \frac{v^2 (1 + \xi_1)(1 + \xi_2)(1 + \tan^2 \theta)}{\xi_1(1 + \xi_2) \tan \theta^2 + \xi_2 (1 + \xi_1)}, \tag{40}
\]
with
\[
v^2 = \langle \rho \rangle^2 \left[ \frac{\xi_1}{1 + \xi_1} \sin^2 \langle \theta \rangle + \frac{\xi_2}{1 + \xi_2} \cos^2 \langle \theta \rangle \right]. \tag{41}
\]

Notice that if $\xi_1 = \xi_2$, the function $m^2(\theta)$ is actually independent of $\theta$ and the Weyl gauge field $\omega_\mu$ completely decouples from the Lagrangian.

On the ground state $\theta = \langle \theta \rangle$ and the mass of $\omega_\mu$ is
\[
m^2(\langle \theta \rangle) = \frac{3}{2} q^2 \langle \rho \rangle^2. \tag{42}
\]
The mass of $\omega_\mu$ is thus determined by $\langle \rho \rangle$ alone; unlike $\theta$ whose vev is determined from $\hat{V}$ (see below), $\langle \rho \rangle$ cannot be predicted by the theory and is a free parameter (flat direction).

The Planck scale $M_p^2 = v^2$, eq. (11), depends in general on $\langle \theta \rangle$. This is not a problem since unlike $\rho$, the field variable $\theta$ does not change under a Weyl transformation, eq. (1). However, if the theory has an $O(2)$ symmetry, i.e. identical non-minimal couplings $\xi_1 = \xi_2$, then $M_p$ is determined by the vev of the dilaton alone $M_p^2 = v^2 = \xi_1 \langle \rho \rangle^2/(1 + \xi_1)$: in this case, the would-be Goldstone (dilaton) field $\rho$ “eaten” by $\omega_\mu$ and “fixing” its mass also fixes the Planck scale. The same is true in the limit of large $\tan \theta \to \infty$, when $M_p^2 = v^2 = \xi_1 \langle \rho \rangle^2/(1 + \xi_2)$.

Regarding the potential $\hat{V}$ in eq. (38), it is given by eq. (31) expressed in terms of the new field variables $\rho, \theta$. With eq. (7) and $V(\phi_1, \phi_2)$ the initial potential in Jordan frame, then
\[
\hat{V} = 36v^4 \frac{b^2}{\xi_2^2} \frac{V(c \tan \theta, 1)}{(\tan^2 \theta + b)^2}, \quad \text{where} \quad c = \frac{1 + \xi_2}{1 + \xi_1}. \tag{43}
\]
which depends on $\theta$ only. Finally, another mechanism to decouple the dilaton was studied in [22] using a global version of the Weyl symmetry studied here (see also [23]).

\[5\langle \rho \rangle\] may be fixed by quantum corrections; however, in quantum scale invariant theories only ratios of field vev’s (scales) can be determined (in terms of dimensionless couplings), so it remains a free parameter.
3.3 More fields and Stueckelberg mechanism

The Stueckelberg mechanism for $\omega_\mu$ can be extended for more scalar fields with non-minimal couplings, using general coordinates. For three fields $\phi_1 = (1/\sqrt{1+\xi_1})\rho \sin \theta \cos \zeta$, $\phi_2 = (1/\sqrt{1+\xi_2})\rho \sin \theta \sin \zeta$, $\phi_3 = (1/\sqrt{1+\xi_3})\rho \cos \theta$. As before, the kinetic term of radial field $\rho$ is the Goldstone eaten by the vector boson $\omega_\mu$ of mass $q^2 K/(4\Omega)|_{\theta = \langle \theta \rangle}$. One is left with kinetic terms for the angular-coordinates fields $\theta$, $\zeta$; similarly, the scalar potential will depend only on these fields. This generalization is useful in cases where one of the scalar fields left can be a Higgs field, while the other is a second Higgs-like scalar, inflaton, etc. The scalar potential is then

$$
\hat{V}(\theta, \phi) = \frac{1}{\Omega^2} V(\phi_1, \phi_2, \phi_3) = \frac{36 v^4 V(z_1, z_2, z_3)}{(\xi_1 z_1^2 + \xi_2 z_2^2 + \xi_3 z_3^2)^2}
$$

where $V(\phi_1, \phi_2, \phi_3)$ is the initial potential in the Jordan frame and $z_j = \phi_j/\rho$ are functions of $\theta$, $\zeta$ only. If $\xi_1 = \xi_2 = \xi_3$, then the Planck scale is also determined by the same $\rho$ field. The extension to more scalar fields with non-minimal coupling is straightforward.

3.4 Other implications: UV scale-invariant regularization

The above results have implications for models with (global) scale invariance at the quantum level. Such models are important since they can have a quantum stable hierarchy between two scalar fields vev’s (higgs and dilaton), which is relevant for the SM hierarchy problem, as we detail below.

Consider first a classical scale invariant model. The SM with a vanishing higgs mass parameter is an example. This symmetry can be preserved at the quantum level, by ensuring that the UV regularization respects it. This is done by replacing the subtraction scale $\mu$ by the dilaton field $\rho$ [2]. After spontaneous breaking of this symmetry, $\mu \sim \langle \rho \rangle$. In this way one obtains scale invariant results at the quantum level [3,6,10]. After the quantum calculation one can expand the result (e.g. the scalar potential) about the vev of the dilaton to recover standard results (e.g. Coleman-Weinberg potential) plus additional higher dimensional operators suppressed by the dilaton vev [6]. Such models have only spontaneous breaking of the scale symmetry, thus there is no dilatation anomaly [2,4,7].

The relation to the hierarchy problem is that the dilaton vev is fixing $M_p$ and so it must be much higher than the Higgs vev. Such hierarchy can be the result of one initial classical tuning of the (dimensionless) couplings. This tuning remains stable at the quantum level, due to quantum scale invariance and a shift symmetry of the dilaton (Goldstone mode) [9]. However, the dilaton remains in the spectrum as a flat direction, even at the quantum level.

The result of this paper elegantly solves this situation. As we saw, the dilaton is “eaten” by the Weyl field $\omega_\mu$ which becomes massive and decouples from the spectrum. Before the Stueckelberg mechanism, the dilaton can enforce a scale invariant UV regularisation, as described above. In this way one can construct a quantum scale invariant theory, dilatation anomaly-free, with a quantum stable hierarchy $m_Z$ versus $M_p \sim \langle \rho \rangle \sim m_{\omega_\mu}$ (for further details see [6] and references therein).
4 Inflation from Weyl conformal symmetry

In this section we study inflation in models with spontaneously broken Weyl gauge symmetry. We consider the case of two scalar fields of Section 3.2 and regard the potential for the angular-variable field $\theta$, obtained after the Stueckelberg mechanism, as being responsible for inflation. The potential becomes constant for large $\tan \theta$. From eq. (38), the action for the field $\theta$ can be written as

$$L_{\text{infl}} = \sqrt{g} \left[ -\frac{1}{2} v^2 \hat{R} + \frac{3b v^2}{\xi_2} \frac{\tau^2 + \xi_2/\xi_1}{(1 + \tau^2 + b)^2} (\partial_\mu \tau)^2 - \hat{V}(\tau) \right], \quad \tau \equiv \tan \theta \quad (45)$$

For $V$ of (28), the Einstein-frame potential can be rewritten in terms of $\tau$ as

$$\hat{V} = \frac{36 v^4}{(\xi_1 \phi_1^2 + \xi_2 \phi_2^2)^2} \left( \frac{\lambda_1}{4!} \phi_1^4 + \frac{\lambda_{12}}{4} \phi_1^2 \phi_2^2 + \frac{\lambda_2}{4!} \phi_2^4 \right)$$

$$= \frac{36 v^4 (1 + \xi_1)^2}{\xi_1^2 (\tau^2 + b)^2} \left( c_1 \tau^4 + c_{12} \tau^2 + c_2 \right), \quad (46)$$

with

$$c_1 = \frac{\lambda_1}{4!(1 + \xi_1)^2}, \quad c_{12} = \frac{\lambda_{12}}{4(1 + \xi_1)(1 + \xi_2)}, \quad c_2 = \frac{\lambda_2}{4!(1 + \xi_2)^2}. \quad (47)$$

The potential is similar to that in Higgs portal inflation [24, 25], but the angular field $\theta$ is dynamical being responsible for a slow-roll inflation in our case (instead of being frozen).

To have a canonical kinetic term, introduce a new field $\chi$ (up to corrections $O(1/\tau^5)$)

$$\chi = \frac{v}{\tau} \sqrt{\frac{6b}{\xi_2}} \left[ 1 + \frac{1}{6} \left( \frac{\xi_2}{\xi_1} - 1 - 2b \right) \frac{1}{\tau^2} \right], \quad (48)$$

or

$$\frac{1}{\tau} = \frac{v}{\chi} \sqrt{\frac{\xi_2}{6b}} \left[ 1 - \frac{\xi_2}{36b} \left( \frac{\xi_2}{\xi_1} - 1 - 2b \right) \frac{\chi^2}{v^2} \right], \quad (49)$$

up to corrections $O(\chi^5/v^5)$. Under the approximation $\tau \gg \max\{\sqrt{b}, 1, \xi_2/\xi_1\}$ which gives $\chi^2 \ll 6b v^2 \{1/\xi_2, 1/\xi_1, b/\xi_2\}$ the kinetic term is canonical: $1/2 (\partial_\mu \chi)^2$. Then $\hat{V}$ becomes

$$\hat{V}(\chi) = V_0 \left( 1 + s_1 \frac{\chi^2}{v^2} + s_2 \frac{\chi^4}{v^4} \right) + O\left( \frac{\chi^6}{v^6} \right), \quad (50)$$

with

$$V_0 = \frac{3 v^4 \lambda_1}{2 \xi_1^2}, \quad (51)$$

$$s_1 = \frac{\xi_1 \lambda_{12}}{\lambda_1} - \frac{\xi_2}{3}. \quad (52)$$
As a result, we have hilltop inflation [26].

For $s_1 \neq 0$, the quadratic term in $\hat{V}$ dominates and the slow-roll parameters during inflation are

$$
\epsilon = \frac{v^2}{2} \left( \frac{\hat{V}'}{\hat{V}} \right)^2 \approx 2 s_1^2 \frac{\chi^2}{v^2}, \quad \eta = v^2 \left( \frac{\hat{V}''}{\hat{V}} \right) \approx 2 s_1 - 2 (s_1^2 - 6 s_2) \frac{\chi^2}{v^2}.
$$

Further, the number of e-foldings during inflation is also given by

$$
N = v^{-1} \int_{\chi_{\text{end}}}^{\chi_*} \frac{\text{sign}(\hat{V}')}{\sqrt{2\epsilon(\chi)}} d\chi = \frac{1}{2 |s_1|} \ln \left( \frac{\chi_{\text{end}}}{\chi_*} \right),
$$

where $\chi_*$ is evaluated at the horizon exit and $\chi_{\text{end}}$ is the inflaton value at the end of inflation, given by $\chi_{\text{end}} = v/(\sqrt{2}|s_1|)$ from $\epsilon = 1$. Therefore, we obtain the scalar spectral index and the tensor-to-scalar ratio in terms of the number of e-foldings

$$
n_s = 1 + 2 \eta_s - 6 \epsilon = 1 + 4 s_1 + \frac{4}{s_1^2} (3 s_2 - 2 s_1^2) e^{-4|s_1|^N},
$$

$$
r = 16 \epsilon_s = 16 e^{-4|s_1|^N}.
$$
The case with $s_1 < 0$ is favoured for the observed $n_s$, so we must take

$$\frac{\lambda_{12}}{\lambda_1} < \frac{\xi_2}{3\xi_1}. \quad (58)$$

This is possible for a small enough $\lambda_{12}$ (recall $\xi_{1,2} > 0$) while $\xi_{1,2}$ remain arbitrary. Here $\lambda_{12}$ is the coupling of the two scalar fields in the Jordan frame.

The normalization of the CMB anisotropies, $V_0/(24\pi^2 v^4 \epsilon_*) = 2.1 \times 10^{-9}$ [27], constrains the quartic coupling $\lambda_1$ and the non-minimal couplings $\xi_{1,2}$ to satisfy

$$\frac{\lambda_1}{\xi_1} = 3.3 \times 10^{-7} e^{-4|s_1|N}. \quad (59)$$

This constraint is respected by choosing a suitable $\lambda_1$ (or a large $\xi_1$).

Planck 2018 results [27] show that $n_s = 0.9659 \pm 0.0041$ and $r < 0.10$ at the pivot scale of $k_*=0.002$ Mpc$^{-1}$ with 95% CL from Planck TT+lowE+lensing, and $n_s = 0.9653 \pm 0.0041$ and $r < 0.064$ at the pivot scale of $k_*=0.002$ Mpc$^{-1}$ with 95% CL from Planck TT, TE, EE+lowE+lensing+BK14. In Figure 1 we show the parameter space for $s_1$ versus $s_2$ in the green region for $N=50$ (60) on left (right), satisfying the spectral index observed by Planck within 2$\sigma$ errors. The purple region is excluded by the bound on the tensor-to-scalar ratio; we also illustrated several values, $r = 0.01, 0.001$, in blue dashed lines. Consequently, we find that in order to be consistent with Planck data, one must satisfy the constraints $s_2 \approx \frac{2}{3} s_1^2$, and $|s_1| \gtrsim 2.5 \times 10^{-2}$ for $s_2 \gtrsim 10^{-3}$. These conditions can easily be satisfied by tuning the remaining parameters ($\lambda_2, \xi_{1,2}$).

Finally, consider the special case of $s_1 = 0$ in eq.(50), when we have quartic hilltop inflation. Then $3\lambda_{12}\xi_1 = \lambda_1\xi_2$; solving this for $\lambda_{12}$, we obtain the inflaton quartic coupling:

$$s_2 = \frac{1}{36} \left( \frac{\xi_1^2}{\lambda_1} - \frac{\xi_2^2}{\lambda_1} \right). \quad (60)$$

In this case, $s_2 < 0$ is favoured by the observed $n_s$, so one has the constraint

$$\frac{\lambda_2}{\lambda_1} < \left( \frac{\xi_2}{\xi_1} \right)^2. \quad (61)$$

This is easily respected for suitable quartic couplings $\lambda_{1,2}$, with arbitrary $\xi_{1,2}$. For the quartic hilltop inflation, the slow-roll parameters are given by

$$\epsilon \approx 8s_2^2 \left( \frac{\Lambda}{v} \right)^6 \approx \frac{1}{64|s_2|} \frac{1}{N^3}, \quad (62)$$

$$\eta \approx 12s_2^2 \left[ 1 - s_2 \left( \frac{\Lambda}{v} \right)^4 \right] \approx -\frac{3}{2N} \left( 1 + \frac{1}{64|s_2|N^2} \right), \quad (63)$$

where we have expressed the inflaton field value in terms of the number of e-foldings $N$.\footnote{The case with $s_1 > 0$ would lead to eternal inflation, which might be avoided in a hybrid type inflation.}
Then the spectral index and the tensor-to-scalar ratio are

\[ n_s = 1 - \frac{3}{N} - \frac{9r}{16}, \]
\[ r = \frac{1}{4|s_2|} \frac{1}{N^3}. \] (64) (65)

The maximal value of \( n_s \approx 1 - 3/N \) for small \( r \), is \( n_s = 0.94 \) (0.95) for \( N = 50 \) (60), respectively; the case \( n_s = 0.95 \) is at the lower 3\( \sigma \) limit of the Planck result for \( n_s \). The special case of quartic hilltop inflation (\( s_1 = 0 \)) is then only marginally allowed (for smallest \( r \)).

### 5 Conclusions

In this work we discussed the Weyl conformal symmetry and its spontaneous breaking and some implications for model building beyond the SM and inflation.

In models with conformal symmetry (of the Brans-Dicke-Jordan type) with scalar fields with non-minimal couplings to the Ricci scalar, one can generate spontaneously the Planck scale from the vev of a scalar field (or a combination of them). However, a positive (negative) Newton constant is accompanied by a negative (positive) kinetic term for this field, respectively. This situation is naturally avoided in models with an additional (Weyl) gauge symmetry and a gauge field \( \omega_\mu \) which is of geometric origin, with a gauge transformation dictated by the conformal transformation of the metric.

We showed that the Weyl field \( \omega_\mu \) couples only to the scalar sector of a SM-like model in curved space-time. This fact is interesting for model building and is ultimately due to the fact that only scalar fields have (dimension-four) non-minimal couplings to \( R \). Further, the field \( \omega_\mu \) undergoes a Stueckelberg mechanism and becomes massive after “eating” the radial mode \( \rho \sim \sqrt{K} \) (in field space) scalar field and would-be-Goldstone mode (dilaton). Its vev \( \langle \rho \rangle \) also determines the mass of \( \omega_\mu \) and the Planck scale \( M_p \) (up to possible additional angular-variables field dependence). The mass of \( \omega_\mu \) can be larger or smaller than \( M_p \) depending on the charge of the scalar fields and non-minimal couplings. After decoupling of \( \omega_\mu \), the potential depends on the angular variables fields only. For two scalar fields of equal non-minimal couplings, the field \( \omega_\mu \) also decouples from the action even if it is light.

For the case with two scalar fields, the scalar potential generally depends only on the angular variable field \( \theta \), and it is nearly constant at a large \( \tan \theta \). Therefore, this is relevant for a single-field inflation. Investigating the details of the inflaton potential in the limit of a large \( \tan \theta \), we found that the inflaton potential is dominated by the vacuum energy with quadratic and quartic correction terms during inflation, realizing a successful hilltop inflation.

While this study was formulated in (pseudo)Riemannian geometry extended with a real Weyl field (undergoing a gauge transformation dictated by conformal transformation), the natural framework is that of Weyl conformal geometry where this symmetry is manifest. In the Riemannian case imposing this symmetry avoids the ghost kinetic term of conformal theory and leads to a Lagrangian with a current \( \partial_\mu K \) that interacts with the Weyl field. This Lagrangian was shown to be identical, up to a total derivative term, to that obtained.
in Weyl geometry (WG) where the Weyl symmetric Lagrangian is naturally built-in, with curvature scalar, tensors and affine connection of Weyl geometry. This equivalence is showed using the relation between $R$ computed in Riemann geometry with Levi-Civita connection and its counterpart in Weyl geometry, relation which is the source of $\partial_{\mu}K$. This gives a geometrical interpretation of the Stueckelberg mechanism for the (geometric) field $\omega_{\mu}$.

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