Estimation of the boundaries of the region of stable functioning of elements of special-purpose information systems characteristic

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Abstract. In article the methods of imitating modeling realized in the conditions of ensuring unity and the required accuracy of results at assessment of stability of functioning of information structure of security system in the conditions of negative impacts are considered. Assessment of stability of functioning of information structure of security system in the conditions of negative impacts is initially made on the basis of development of model of information exchange of elements of information structures of security systems and negative impacts which is constructed on the basis of the set-theoretic approach allowing not only to describe features of creation of elements of the considered structures, but also to carry out formalization of process of functioning of models of security complexes in the form of creation of the discrete (matrix and vector) model differing in classifications of set of threats. On condition of the continuity of temporary parameters as information systems in the conditions of negative impacts it is possible to present process of functioning of information structures of security systems in the form of the integrated (dynamic) model constructed with use of the differential equations describing conditions of her elements that allows to estimate, in turn, borders of field of steady management of processes of interaction of the specified sets.

1. Introduction
Support of a reliable protection of objects is impossible without application of the modern information technologies [1-3].

In this regard the main tendency of development of the modern technical means and systems is their integration into a uniform hardware and software system – the integrated security arrangement [4-8].

2. Relevance
Within implementation of the integrated security system (ISS) it is necessary to mark relevance of implementation of actions for metrological support of process of integration of the hardware and software of end-to-end systems of safety of objects of a penal correction system into the task of the analysis of stability of information structures of the security systems S₁ (IS SS) in the conditions of negative impacts of S₂ (NI).

We will understand set as NI of both internal, and external security risks of an object.
In case of assessment of stability of functioning of the IS SS in the conditions of NI we will understand a complex of the actions directed to measurement of set of the quantitative and qualitative parameters specifying adequacy of simulation and also application of methods and means of ensuring of their unity and required accuracy [10] as metrological support.

3. Problem definition

We will consider imitating model of assessment of stability of functioning of IS SS in the conditions of NI on the example of the analysis of probabilistic characteristics of overcoming by the IS S_2 elements of the IS S_3 elements.

4. Theoretical part

We realize this approach to the analysis of behavior of IS SS on the basis of the experimental modeling (EM) of behavior of random variables according to expressions (1, 2) which characterize, respectively, distribution of probability of overcoming the IS S_3 elements in time and distributions of probability of overcoming the IS S_3 elements in the conditions of evolution ξ the IS SS elements [11].

\[
\gamma'_n(t) = \exp\left(-\frac{A}{B + t^2}\right) \\
h(\xi) = 1 - \exp\left(-\frac{\mu}{v + \xi^2}\right)
\]

where A, B and C – the parameters characterizing change of distribution of probability of overcoming the IS S_3 elements and the analyzed parameters which are formed on the basis of expert approach to ranging in intervals [1, 50] for parameter A as abilities of the IS S_2 elements to self-training to skills of overcoming the IS S_3 standard elements; [0, 10] – for the parameter B as the potential of the IS S_2 elements on overcoming the S_3 elements at the initial stage of the NI organization; and [0, 2] – for the parameter C as an indicator of dynamics of change of potential of the IS S_2 elements on overcoming S_3 with growth of time.

It is important to note that the parameters, μ_m, ν_m and δ_m equivalents within modeling of inverse processes of evolution and aging of the IS SS and NI elements.

We will consider EM on the basis of C_n of values of the stochastic sizes distributed according to not Gaussian law. For this purpose we will consider an algorithm of check of a hypothesis of accessory of the analyzed selection to the known law of distribution which is presented in the figure 1 in the form of the flowchart [12].

We will step by step consider an algorithm of check of a hypothesis of accessory of the analyzed selection to the known law of distribution.

1. Expert definition of set of cycles (n) of formation of stochastic sizes \(X_i[n]\).
2. Determination of values of parameters \(A, B, C, \mu, v, \delta\):

\[
A = A_m x_i, B = B_m x_2, C = C_m x_3, \\
\mu = \mu_m x_4, v = v_m x_5, \delta = \delta_m x_6,
\]

where \(A_m, B_m, C_m, \mu_m, v_m, \delta_m\) – maximum values of the corresponding parameters, \(x_i \in [0, 1]\), \(i = 0, ..., 8\) – the random variables which are formed on the basis of the uniform law of distribution with use of a random number generator.

3. Determination of values of delays \(t_0 = t_m x_7, \xi_0 = \xi_m x_8\) at stages of aging/evolution of the IS S_3 and S_2 elements. Delays are formed on the basis of use of set of the maximum temporary \(t_m\).
values characterizing change of complexity of $\xi_m$ of the considered IS in the course of evolution of their elements.

**Figure 1.** Flowchart of an algorithm of check of a hypothesis
As a result we will determine value by abscissa axis till the moment when dynamics of development of information process accepts nonlinear character. On ordinate axis we will take result of the solution of system of the equations for value (1), (2) at the following assumptions according to $t = 0$ and $\xi = 0$.

4. According to expressions (1) and (2), we will define the intervals satisfying to distribution of a random variable according to not Gaussian law. For this purpose for values $t_u = t_{\omega}x_0$ and $\xi_u = \xi_{\omega}x_0$ taking into account performance of conditions of inequalities $t_u < t_0$ and $\xi_u < \xi_0$ we will determine the corresponding stochastic sizes in a look: $X_1[n] = 1/t_0$ or $X_2[n] = 1/\xi_0$. In case conditions of inequalities aren’t satisfied, we will present stochastic sizes in the form (3).

$$X_1[n] = \frac{1}{t_0} + \frac{A \cdot C \cdot \chi_0^{c-1} e^{-\chi_0^2}}{B + x_0^c}$$

$$X_2[n] = \frac{1}{\xi_0} + \frac{\mu \cdot \delta \cdot x_0^{\delta-1} e^{-\mu \delta x_0^\delta}}{(v + x_0^\delta)^2}$$  \hspace{1cm} (3)

5. Formation of variation ranks and creation of histograms on the basis of information sets of the stochastic sizes $X_1[n]$ and $X_2[n]$ (fig. 2). From the drawing the illustration of splitting the histogram into $n$ of time intervals depending on the size of probability of overcoming the IS SS elements is visible.

6. Check about use of criterion of Kolmogorov-Smirnov of a hypothesis of $H_0$ of compliance of the created stochastic size to the above-stated law of distribution (check of simple hypotheses of accessory of the analyzed selection to some completely known law of distribution) [13].

Distinctive features of this statistical method of data processing are the possibility of the analysis of compliance of selection of random variables to any of the existing laws of distribution (Student’s $t$-criterion and Fischer’s $F$-criterion, for example, work only with normal laws of distribution) and also application of empirical functions of distribution $F_n(x)$.

Further, according to Kolmogorov-Smirnov’s criterion, we will correlate theoretical function of distribution $F(x)$ which is result of imitating modeling to her empirical representation of $F_n(x)$ (figure 3). The criterion statistics for empirical function of distribution $F_n(x)$ is defined as follows:

$$D_n = \sup_{-\infty < x < \infty} |F_n(x) - F(x)|$$ \hspace{1cm} (4)

where $\sup S$ – exact top side of a set

According to Kolmogorov’s theorem for the entered statistics taking into account $\forall t > 0$ correct:

$$\lim_{n \to \infty} P(\sqrt{n}D_n \leq t) = K(t),$$ \hspace{1cm} (5)

We will present the $K(t)$ function in the form:

$$K(t) = \sum_{j=-\infty}^{\infty} (-1)^j e^{-2jt^2}$$ \hspace{1cm} (6)

From the point of view of asymptotic behavior $K(t)$ will be function of distribution for the control size $\sqrt{n}D_n$. Then, on condition of excess by statistics of $\sqrt{n}D_n$ of a quantile of distribution of $K_\alpha$ of
the set significance value of \( \alpha \left( \sqrt[n]{D_n} \geq K_\alpha \right) \), we will consider \( H_0 \) hypothesis of compliance of distribution of information set of random variables to the X law \( F(x) \) received as a result of imitating modeling irrelevant.

Compliance of distributions of probability of overcoming elements of information structures of \( S_3 \) in their empirical \( F_n(t) \) and theoretical \( F(t) \) representations is reached, according to Kolmogorov-Smirnov's criterion, at the expense of the analysis of selection of parameters (figure 3). Finding of critical values of set of parameters is made based on tabular data [3].

a) \( A = 40, B = 8, C = 1.6; \)

b) \( A = 32, B = 6.4, C = 1.28; \)

c) \( A = 23.5, B = 4.7, C = 0.94; \)

Figure 2. Distribution of probability of overcoming the IS \( S_3 \) elements in the conditions of aging of the IS \( S_2 \) elements
Figure 3. Check of a hypothesis of accessory of the analyzed selection to some law of distribution

By use of inequality of Chebyshev [4] we will define quantity \( k \) and accuracy \( a \) of realization regarding the analysis of imitating model of assessment of stability of functioning of IS SS in the conditions of NI:

\[
P\left(\frac{x - M_x}{\sigma} \geq a\right) \leq \frac{D_x}{a^2},
\]

where \( x \) – the average value received on the basis of dependence

\[
\bar{x} = \frac{1}{C} \sum_{i=1}^{C} x_i
\]

where \( M_x \) – population mean of size \( x \); \( D_x \) – dispersion of size \( x \); \( a \) – accuracy.

We will define quantity \( k \) and accuracy \( a \) of realization for any law of distribution by an expression ratio for reliability assessment with Chebyshev’s inequality:

\[
\frac{D_x}{a^2} = 1 - \alpha,
\]

\[
a = \sqrt{\frac{D_x}{k}} \cdot \frac{1}{\sqrt{(1-\alpha)}} \quad \text{and} \quad k = \frac{D_x}{a^2} \cdot \frac{1}{(1-\alpha)}
\]

Calculations show that at \( \alpha = 0.5 \), \( k = 50 \), \( \sigma = 1 \) (a mean square deviation of the estimated parameter \( x \), \( D_x = \sigma^2 x \)), \( a = 0.2 \), at values \( \alpha = 0.5 \), \( a = 0.4 \), \( \sigma = 1 \), the number of realization of \( k = 13 \).

On the basis of experimental data with use of criterion of Kolmogorov-Smirnov it is shown that for the description of dependence of probability of \( P(\xi, t) \) overcoming the IS \( S_3 \) elements from the \( \xi \) and \( t \) parameters the ratio can be used (10):

\[
P(\xi, t) = \begin{cases} 
1, & \xi \leq 0 \\
\text{h}(\xi) + \gamma_m(t) \cdot \text{m}(\xi), & \xi > 0 
\end{cases}
\]

by \( \xi = 0, \xi > 0 \).

The ratio (10) is fair if the \( \text{h}(\xi) \) and \( \text{m}(\xi) \), variables as random variables of distribution of probability of overcoming the IS \( S_3 \) elements, are independent and form complete group of events. We will mark that as a result of evolution of the IS \( S_3 \) and \( S_2 \) elements as parallel the information processes proceeding in time there is a change and the probability of overcoming the IS ISS.
The illustration of the solution of a ratio (10), taking into account processes of evolution and aging of the IS ISS and NI elements, is provided in a figure 4. Spatial visualization of the decision (10) gives a better understanding of dynamics of change of $P(\xi,t)$. The flowchart of an algorithm «The Analysis of Probability of Overcoming the IS S₃ Elements in the conditions of Evolution and Ageing of Its Elements» is provided in a figure 5.

It is necessary to believe that the selections of random variables of overcoming the IS ISS elements which are result of imitating modeling and distributed according to not Gaussian law define dynamics of evolution and aging of the IS ISS and NI elements. On condition of the continuity of temporary parameters process of functioning of information structure of $S₃$ as information system (InS) in the conditions of negative impacts, it is possible to present in the form of the integrated (dynamic) model constructed with use of the differential equations describing conditions of her elements.

**Figure 4.** Probability of overcoming $P(\xi,t)$ the IS $S₃$ elements in the conditions of evolution and aging of her elements
As a result it should be noted that expressions are put in a basis of experimental development of imitating model of interaction of the IS $S_3$ and $S_2$ elements taking into account duration of the information processes proceeding at the same time (1, 2). Accuracy of determination of features of behavior of IS ISS in the conditions of NI as imitating model allows to use results of this modeling in numerical methods of the solution of system of the integro-differential equations (IDE) describing interaction of IS ISS and NI.

The analyzed InS dynamic model characterizes formalization of a task steady management of information processes of interaction of the IS ISS elements in the conditions of NI.

Transition of InS $S_3$ in the conditions of NI in a steady state is impossible without evolution of her components which, in turn, is characterized not only processes of concentration of information, but also her degeneration in connection with growth of the specification of the IS ISS and NI [14] elements.

In connection with complexity of implementation of the graphic description of the solution of system of the integro-differential equations describing interaction of IS ISS and NI his consideration on the example of the generalized approach to development of mathematical model of steady interaction of IS $S_2$ and $S_3$ [15] is expedient.

The common decision of the go system on the plane is crossing of their sets on coordinates $\xi, t, Y$.

Development and the solution of mathematical model of interaction of IS ISS and NI describes dynamics of evolution of their elements on the phase plane of space, and taking into account integrated representation of stability conditions and also the second method of Lyapunov represents mathematical model of steady management of information processes of interaction of IS ISS and NI [16].

![Flowchart of an algorithm «The Analysis of Probability of Overcoming the InS $S_3$ Elements in the conditions of Evolution and Ageing of Her Elements»](image)

**Figure 5.** Flowchart of an algorithm «The Analysis of Probability of Overcoming the InS $S_3$ Elements in the conditions of Evolution and Ageing of Her Elements»

Transition to the phase plane of space is interesting from the point of view of visualization of process of interaction of IS ISS and NI regarding assessment of limits of stability of IS $S_3$ (figure 6).
Figure 6. Definition of field of steady management of information process of interaction of IS ISS and NI in the conditions of evolution of their elements in this regard to another we will consider transition from one concentric circle from the point of view of acquisition by elements of this IS of qualitatively new characteristics.

5. The results of experimental studies

With use of criterion of Kolmogorov-Smirnov with the set degree of accuracy an inspection of a hypothesis of compliance of theoretical function of distribution of $F(x)$ probability of overcoming the IS SS elements received based on imitating modeling, to her empirical representation is carried out $F_n(x)$.

It has allowed to confirm the assumption that the nature of the not Gaussian laws of distribution describing features of behavior and dynamics of development of the IS $S_3$ and $S_2$ elements is formed on the basis of exponential distribution of probability of overcoming the IS SS elements in the conditions of change of an indicator exhibitors, according to the look which is adequately describing the frequency wording of the law of Tsipfa-Pareto in expressions (1, 2).

According to the frequency wording of the law of Tsipfa-Pareto, probabilistic characteristics of overcoming in time of IS ISS which are subordinated to not Gaussian law of distribution that corresponds to the results received in the empirical way are defined.

One of the main results of pilot studies is development of an algorithm of check of a hypothesis of accessory of the analyzed selection to the known law of distribution that has allowed to define finally some area (the phase vicinity) spaces (attractor) which presents itself set of concentric circles in the form of a set of the points which are attracting trajectories of evolution of the IS $S_3$ elements and indicating areas of her steady functioning [17].
Assessment of borders of an attractor has allowed revealing area in space of phase coordinates between the next circles, which is representable as a strange attractor, which, in turn, defines temporary instability of functioning of dynamic system [18].

As a result of the solution of system of the integro-differential equations by numerical methods has allowed to analyses not only specifics of information processes of interaction of IS ISS and NI, but also duration of their course on condition of variety of the operating parameters [19].

6. Conclusion

The analysis of the received results allows to draw a conclusion that the dependences presented in the figure 3 namely imitating and experimental models, have the general nature and adequately reflect features of distribution of probability of overcoming the IS S3 elements in the conditions of NI. It says about rationality of application of algorithms of imitating modeling in a problem of delimitation of area of steady functioning of IS SS in the conditions of NI.

It is important to note that it is possible to formalize the unformalized description of interaction of the IS S3 and S2 elements realized, for example, about use of set of operators of the relations by development of imitating model of assessment of stability of functioning of IS ISS in the conditions of NI taking into account that to each behavioral strategy of the IS S2 elements there corresponds the final information set of the IS S3 elements, available in respect of their overcoming.

Also, the relevance of definition on the basis of methods of imitating modeling necessary and enough the IS S3 elements taking into account continuous variation of values of parameters A, B, and with and also possibilities of the analysis of features of behavior of the information system S3 in the conditions of evolution of both own elements, and the IS NI elements doesn't raise doubts.

Experimental modeling of behavior of IS ISS in the conditions of NI and also development of imitating model of assessment of stability of her functioning have allowed to conduct a research of quantitative characteristics of the NI elements on the basis of ABC analysis of parameters of management.

It is necessary to conclude that at constancy of entry conditions not Gaussian distribution of probability of overcoming the IS ISS elements realized regarding the analysis of imitating model corresponds to the set modeling accuracy. As a result a conclusion about expediency of application of methods of imitating modeling in the analysis of features of functioning of the information systems S3 in the conditions of NI arises.

Within delimitation of some area (the phase vicinity) of space which is characterized by stability of behavior of IS ISS in the conditions of NI (attractor) it is important to consider, respectively, the attracting set of unstable trajectories in phase space of dissipative dynamic system, namely «a strange attractor» which is characterized by a number of properties [20]:

- consists of infinite number of unstable cycles of the different periods and an odd set of aperiodic points which trajectory doesn't become isolated;
- the mode of functioning is unstable, namely exponential increase in time of small indignations.

In more detailed analysis of the concentric circles presented a conclusion about implementation of IS S3 of self-regulation which is expressed by transition of a trajectory of evolution of IS ISS in space from coordinate «time» for coordinate "complexity" [21-27] arises upon figure 6.

The relevance of consideration of the phenomenon of a strange attractor is caused by the fact that with his help the area of phase space characterizing poorly governed behavior of IS S3 is formed.

Thus, truly to claim that IS S3 is steady in case the solution of system of the integro-differential equations describing information processes of interaction of elements of sets of IS ISS and NI belongs to internal area of a concentric circle of smaller radius (fig. 6,a). In case the solution of this system of the equations doesn't get to this area, to speak truly about instability of the IS S3 elements to NI (the figure 6,a).
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