Fast Constraint Propagation for Image Segmentation

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Abstract
This paper presents a novel selective constraint propagation method for constrained image segmentation. In the literature, many pairwise constraint propagation methods have been developed to exploit pairwise constraints for cluster analysis. However, since most of these methods have a polynomial time complexity, they are not much suitable for segmentation of images even with a moderate size, which is actually equivalent to cluster analysis with a large data size. Considering the local homogeneity of a natural image, we choose to perform pairwise constraint propagation only over a selected subset of pixels, but not over the whole image. Such a selective constraint propagation problem is then solved by an efficient graph-based learning algorithm. To further speed up our selective constraint propagation, we also discard those less important propagated constraints during graph-based learning. Finally, the selectively propagated constraints are exploited based on $L_1$-minimization for normalized cuts over the whole image. The experimental results demonstrate the promising performance of the proposed method for segmentation with selectively propagated constraints.

1 Introduction
Image segmentation is a fundamental problem in computer vision. Despite many years of research (Shi and Malik, 2000; Martin et al., 2001; Carson et al., 2002; Unnikrishnan et al., 2007; Arbelaez et al., 2011), general-purpose image segmentation is still a very challenging task because segmentation is inherently ill-posed. To improve the results of image segmentation, much attention has been paid to constrained image segmentation (Yu and Shi, 2004; Eriksson et al., 2007; Xu et al., 2009; Ghanem and Ahuja, 2010), where certain constraints are initially provided for image segmentation. In this paper, we focus on constrained image segmentation using pairwise constraints, which can be derived from the initial labels of selected pixels. In general, there exist two types of pairwise constraints: must-link constraint denotes a pair of pixels belonging to the same image region, while cannot-link constraint denotes a pair of pixels belonging to two different image regions. In previous work (Wagstaff et al., 2001; Klein et al., 2002; Xing et al., 2003; Hoi et al., 2006), such weak supervisory information has been widely used to improve the performance of machine learning and pattern recognition (Lu, 2006; Lu et al., 2008; Lu and Ip, 2009; Lu and Peng, 2011) in many challenging tasks.

The main challenge in constrained image segmentation is how to effectively exploit a limited number of pairwise constraints for image segmentation. A sound solution is to perform pairwise constraint propagation to generate more pairwise constraints. Although many pairwise constraint propagation methods (Lu and Carreira-Perpinan, 2008; Li et al., 2008; Lu and Ip, 2010; Lu and Peng, 2013) have been developed for constrained clustering (Kamvar et al., 2003; Kulis et al., 2005), they mostly have a polynomial time complexity and thus are not much suitable for segmentation of images even with a moderate size (e.g., $200 \times 200$ pixels), which is actually equivalent to clustering with a large data size (i.e. $N = 40,000$). For constrained image segmentation, we need to develop more efficient pairwise constraint propagation methods, instead of directly utilizing the existing methods like (Lu and Carreira-Perpinan, 2008; Li et al., 2008; Lu and Ip, 2010; Lu and Peng, 2013). Here, it is worth noting that even the simple assignment operation incurs a large time cost of $O(N^2)$ if we perform pairwise constraint propagation over all the pixels, since the number of all possible pairwise constraints is $N(N - 1)/2$. The unique choice is to propagate the initial pairwise constraints only to a selected subset of pixels, but not across the whole image.

Fortunately, the local homogeneity of a natural image provides kind of supports for this choice, i.e., a selected subset of pixels may approximate the whole image (such downsampling is widely used in image processing). More importantly, the selectively propagated constraints over a selected subset of pixels are enough for achieving a good quality of image segmentation, as verified by our later experimental results. Hence, in this paper, we develop a selective constraint propagation (SCP) method for constrained image segmentation, which propagates the initial pairwise constraints only to a selected subset of pixels (the first meaning of our selective constraint propagation). Although there exist different sam-
pling methods in statistics, we only consider random sampling for efficiency purposes, i.e., the subset of pixels used for selective constraint propagation are selected randomly from the whole image. In this paper, we formulate our selective constraint propagation as a graph-based learning problem which can be efficiently solved based on the label propagation technique [Zhou et al., 2004]. To further speed up our algorithm, we also discard those less important propagated constraints during iteration of graph-based learning, which is the second meaning of our selective constraint propagation.

To the best of our knowledge, we have made the first attempt to develop a selective constraint propagation method for constrained image segmentation.

Finally, the selectively propagated constraints obtained by our selective constraint propagation are exploited to adjust the original weight matrix based on optimization techniques, in order to ensure that the new weight matrix is as consistent as possible with the selectively propagated constraints. In this paper, we formulate such weight adjustment as an L1-minimization problem [Elad and Aharon, 2006; Mairal et al., 2008; Wright et al., 2009; Xiao et al., 2011], which can be solved efficiently due to the limited number of selectively propagated constraints. The obtained new weight matrix is then applied to normalized cuts [Shi and Malik, 2000] for image segmentation. The flowchart of our selective constraint propagation for constrained image segmentation is illustrated in Figure 1. Although our selective constraint propagation method is originally designed for constrained image segmentation, it can be readily extended to other challenging tasks (e.g., semantic image segmentation and multi-face tracking) where only a limited number of pairwise constraints are provided initially.

It should be noted that the present work is distinctly different from previous work on constrained image segmentation [Yu and Shi, 2004; Eriksson et al., 2007; Xu et al., 2009; Ghanem and Ahuja, 2010]. In [Yu and Shi, 2004], only linear equality constraints (analogous to must-link constraints) are exploited for image segmentation based on normalized cuts. In [Eriksson et al., 2007; Xu et al., 2009; Ghanem and Ahuja, 2010], although more types of constraints are exploited for image segmentation, the linear inequality constraints analogous to cannot-link constraints are completely ignored just as [Yu and Shi, 2004]. In contrast, our selective constraint propagation method exploits both must-link and cannot-link constraints for normalized cuts. More notably, as shown in our later experiments, our method obviously outperforms [Yu and Shi, 2004] due to extra consideration of cannot-link constraints for image segmentation.

The remainder of this paper is organized as follows. In Section 2, we develop a selective constraint propagation method which propagates the initial pairwise constraints only to a selected subset of pixels. In Section 3, the selectively propagated constraints are further exploited to adjust the original weight matrix based on L1-minimization for normalized cuts. Finally, Sections 4 and 5 give the experimental results and conclusions, respectively.

2 Selective Constraint Propagation

This section presents our selective constraint propagation (SCP) in detail. We first give our problem formulation for propagating the initial pairwise constraints only to a selected subset of pixels from a graph-based learning viewpoint, and then develop an efficient SCP algorithm based on the label propagation technique [Zhou et al., 2004].

2.1 Problem Formulation

In this paper, our goal is to propagate the initial pairwise constraints to a selected subset of pixels for constrained image segmentation. To this end, we need to first select a subset of pixels for our selective constraint propagation. Although there exist different sampling methods in statistics, we only consider random sampling for efficiency purposes, i.e., the subset of pixels are selected randomly from the whole image. In the following, the problem formulation for selective constraint propagation over the selected subset of pixels is elaborated from a graph-based learning viewpoint.

Let $\mathcal{M} = \{(i, j) : l_i = l_j, 1 \leq i, j \leq N\}$ denote the set of initial must-link constraints and $\mathcal{C} = \{(i, j) : l_i \neq l_j, 1 \leq i, j \leq N\}$ denote the set of initial cannot-link constraints, where $l_i$ is the region label assigned to pixel $i$ and $N$ is the total number of pixels within an image. The set of constrained pixels is thus denoted as $\mathcal{P}_c = \{i : (i, j) \in \mathcal{M} \cup \mathcal{C}, 1 \leq j \leq N\} \cup \{i : (j, i) \in \mathcal{M} \cup \mathcal{C}, 1 \leq i \leq N\}$ with $N_c = |\mathcal{P}_c|$. Moreover, we randomly select a subset of pixels $\mathcal{P}_s \subset \{1, 2, \ldots, N\}$ with $N_s = |\mathcal{P}_s|$, and then form the final selected subset of pixels used for our selective constraint propagation as $\mathcal{P}_u = \mathcal{P}_s \cup \mathcal{P}_c$ with $N_u = |\mathcal{P}_u|$. 

![Flowchart of our selective constraint propagation (SCP) for constrained image segmentation, where the selectively propagated constraints are exploited based on L1-minimization for normalized cuts (NCuts).](image-url)
In this paper, we construct a \( k \)-nearest neighbors (\( k \)-NN) graph over all the pixels so that the normalized cuts for image segmentation can be performed efficiently over this \( k \)-NN graph. Let its weight matrix be \( W = \{w(i, j)\}_{N \times N} \). We define the weight matrix \( W_u = \{w_u(i, j)\}_{N_u \times N_u} \) over the selected subset of pixels \( P_u \) as:
\[
w_u(i, j) = w(P_u(i), P_u(j)),
\]
where \( P_u(i) \) denotes the \( i \)-th member of \( P_u \). The normalized Laplacian matrix is then given by
\[
L_u = I - D^{-1/2} W_u D^{-1/2},
\]
where \( I \) is an identity matrix and \( D \) is a diagonal matrix with its \( i \)-th diagonal entry being the sum of the \( i \)-th row of \( W_u \). Moreover, for convenience, we represent the two sets of initial pairwise constraints \( \mathcal{M} \) and \( \mathcal{C} \) using a single matrix \( Z_u = \{z_u(i, j)\}_{N \times N} \) as follows:
\[
z_u(i, j) = \begin{cases} 
+1, & (P_u(i), P_u(j)) \in \mathcal{M}; \\
-1, & (P_u(i), P_u(j)) \in \mathcal{C}; \\
0, & \text{otherwise}.
\end{cases}
\]

Based on the above notations, the problem of selective constraint propagation over the selected subset of pixels \( P_u \) can be formulated from a graph-based learning viewpoint:
\[
\min_{F_v, F_h} \| F_v - Z_u \|_F^2 + \mu \text{tr}(F_v^T L_u F_v) + \| F_h - Z_u \|_F^2 + \mu \text{tr}(F_h L_u F_h^T) + \gamma \| F_v - F_h \|_F^2,
\]

where \( \mu \) and \( \gamma \) denote the positive regularization parameters, \( \| \cdot \|_F \) denotes the Frobenius norm of a matrix, and \( \text{tr}(\cdot) \) denotes the trace of a matrix. Here, it is worth noting that the above problem formulation actually imposes both \textit{vertical} and \textit{horizontal} constraint propagation upon the initial matrix \( Z_u \). That is, each column (or row) of \( Z_u \) can be regraded as the initial configuration of a \textit{two-class label propagation} problem, which is formulated just the same as [Zhou et al., 2004]. Moreover, in this paper, we assume that the vertical and horizontal constraint propagation have the same importance for constrained image segmentation.

The objective function given by Eq. (4) is further discussed as follows. The first and second terms are related to the \textit{vertical} constraint propagation, while the third and fourth terms are related to the \textit{horizontal} constraint propagation. The fifth term then ensures that the solutions of these types of constraint propagation are as approximate as possible. Let \( F_v^* \) and \( F_h^* \) be the best solutions of vertical and horizontal constraint propagation, respectively. The best solution of our selective constraint propagation is defined as:
\[
F_u^* = (F_v^* + F_h^*)/2.
\]

As for the second and fourth terms, they are known as Laplacian regularization [Zhu et al., 2003; Zhou et al., 2004; Belkin et al., 2006], which means that \( F_v \) and \( F_h \) should not change too much between similar pixels. Such Laplacian regularization has been widely used for different graph-based learning problems in the literature.

To apply our selective constraint propagation (SCP) to constrained image segmentation, we have to concern the following key problem: \textit{how to solve Eq. (2) efficiently}.

**2.2 Efficient SCP Algorithm**

Let \( Q(F_v, F_h) \) denote the objective function in Eq. (4). The alternate optimization technique can be adopted to solve \( \min_{F_v, F_h} Q(F_v, F_h) \) as follows: 1) Fix \( F_h = F_h^* \), and perform the vertical propagation by \( F_v^* = \arg \min_{F_v} Q(F_v, F_h^*) \); 2) Fix \( F_v = F_v^* \), and perform the horizontal propagation by \( F_h^* = \arg \min_{F_h} Q(F_v^*, F_h) \).

**Vertical Propagation:** When \( F_h \) is fixed at \( F_h^* \), the solution of \( \min_{F_v} Q(F_v, F_h^*) \) can be found by solving
\[
\frac{\partial Q(F_v, F_h^*)}{\partial F_v} = 2(F_v - Z_u) + 2 \mu L_u F_v + 2 \gamma (F_v - F_h^*) = 0,
\]
which is actually equivalent to
\[
(I + \mu L_u) F_v = (1 - \beta) Z_u + \beta F_h^*,
\]
where \( \mu = \mu/(1 + \gamma) \) and \( \beta = \gamma/(1 + \gamma) \). Since \( I + \mu L_u \) is positive definite, the above linear equation has a solution:
\[
F_v^* = (I + \mu L_u)^{-1}((1 - \beta) Z_u + \beta F_h^*).
\]

However, this analytical solution is not efficient at all for constrained image segmentation, since the matrix inverse incurs a large time cost of \( O(N_u^3) \). In fact, this solution can also be \textit{efficiently found using the label propagation technique} [Zhou et al., 2004] based on \( k \)-NN graph over \( P_u \) (see the SCP algorithm outlined below).

**Horizontal Propagation:** When \( F_v \) is fixed at \( F_v^* \), the solution of \( \min_{F_h} Q(F_v^*, F_h) \) can be found by solving
\[
\frac{\partial Q(F_v^*, F_h)}{\partial F_h} = 2(F_h - Z_u) + 2 \mu F_h L_u + 2 \gamma (F_v^* - F_h) = 0,
\]
which is actually equivalent to
\[
F_h(I + \mu L_u) = (1 - \beta) Z_u + \beta F_v^*.
\]

This linear equation can also be efficiently solved using the label propagation technique [Zhou et al., 2004] based on \( k \)-NN graph, similar to what we do for Eq. (6).

Since the weight matrix \( W_u \) over \( P_u \) is derived from the original weight matrix \( W \) of the \( k \)-NN graph constructed over all the pixels according to Eq. (1), \( W_u \) can be regarded as the weight matrix of a \( k \)-NN graph constructed over \( P_u \). Hence, we can adopt the label propagation technique [Zhou et al., 2004] to efficiently solve both Eq. (6) and Eq. (8). Moreover, to speed up our selective constraint propagation, we also discard those less important constrained constraints during both vertical and horizontal propagation. That is, the two matrices \( F_v \) and \( F_h \) are forced to become sparser and thus less computation load is needed during iteration.

The complete SCP algorithm is outlined as follows:

1. Compute \( S_u = I - L_u \), where \( L_u \) is given by Eq. (2).
2. Initialize \( F_v(0) = 0, F_h(0) = 0, \) and \( F_h(0) = 0; \)
Discard those less important propagated constraints with $F_u(t) < \epsilon$ during the vertical propagation, where we set $\epsilon = 10^{-7}$ in this paper;

$F_v(t+1) = \alpha S_u F_v(t) + (1- \alpha) ((1-\beta)Z_u + \beta F_u^\alpha)$, where $\alpha = \mu/(1+\mu)$ and $\beta = \gamma/(1+\gamma)$ in (0, 1);

(5) Iterate Steps (3)–(4) for the vertical propagation until convergence at $F_v^*$;

(6) Discard those less important propagated constraints with $F_h(t) < \epsilon$ during the horizontal propagation;

$F_h(t+1) = \alpha F_h(t) S_u + (1- \alpha) ((1-\beta)Z_u + \beta F_u^\alpha)$;

(8) Iterate Steps (6)–(7) for the horizontal propagation until convergence at $F_h^*$;

(9) Iterate Steps (3)–(8) until the stopping condition is satisfied, and output the solution $F_u^* = (F_v^* + F_h^*)/2$.

Similar to [Zhou et al., 2004], the iteration in Step (4) converges to $F_u^* = (1- \alpha)(I- \alpha S_u)^{-1} ((1-\beta)Z_u + \beta F_u^\alpha)$, which is equal to the solution (7) given that $\alpha = \mu/(1+\mu)$ and $S_u = I - L_u$. Moreover, in our later experiments, we find that the iterations in Steps (5), (8) and (9) generally converge in very limited steps (<10). Finally, based on $k$-NN graph, our SCP algorithm has a time cost of $O(kN_u^2)$ ($N_u \ll N$).

Hence, it can be considered to provide an efficient solution (note that even a simple assignment operation on $F_u^*$ incurs a time cost of $O(N_u^2)$).

3 Constrained Image Segmentation

In this section, we discuss how to exploit the selectively propagated constraints stored in the output $F_u^*$ of our SCP algorithm for image segmentation based on normalized cuts. The basic idea is to adjust the original weight matrix $W_u$ over the selected subset of pixels $P_u$ using these selectively propagated constraints. The problem of such weight adjustment for normalized cuts can be formulated as:

$$\min_{\hat{W}_u \geq 0} \frac{1}{2} ||\hat{W}_u - F_u^*||_F^2 + \lambda ||\hat{W}_u - W_u||_1,$$

where $\hat{W}_u \in \mathbb{R}^{N_u \times N_u}$ is the new weight matrix over $P_u$, and $\lambda$ is the regularization parameter. It is worth noting that the new weight matrix $\hat{W}_u$ is actually derived as a nonnegative fusion of $F_u^*$ and $W_u$ by solving the above $L_1$-minimization problem. More notably, the $L_1$-norm regularization term $||\hat{W}_u - W_u||_1$ can force the new weight matrix $\hat{W}_u$ not only to approach $W_u$ but also to become as sparse as $W_u$, given that $\hat{W}_u$ can be regarded as the weight matrix (thus sparse) of a $k$-NN graph constructed over $P_u$.

The problem given by Eq. (9) can be solved based on some basic $L_1$-minimization techniques [Elad and Aharon, 2006; Mairal et al., 2008; Wright et al., 2009; Xiao et al., 2011]. In fact, it has an explicit solution:

$$\hat{W}_u^* = \text{soft}_{\lambda}(F_u^*, W_u, \lambda),$$

where $\text{soft}_{\lambda}(\cdot, \cdot, \lambda)$ is a soft-thresholding function. Here, we directly define $z = \text{soft}_{\lambda}(x, y, \lambda)$ as:

$$z = \begin{cases} z_1 = \max(x - \lambda, 0), & f_1 \leq f_2 \\ z_2 = \max(0, \min(x + \lambda, 0)), & f_1 > f_2 \end{cases},$$

where $f_1 = (z_1 - x)^2 + 2\lambda |z_1 - y|$ and $f_2 = (z_2 - x)^2 + 2\lambda |z_2 - y|$. Since the $L_1$-minimization problem given by Eq. (9) is limited to $P_u$, finding the best new weight matrix $\hat{W}_u$ incurs a time cost of $O(N_u^2)$ ($N_u \ll N$).

Once we have found the best new weight matrix $\hat{W}_u^* = \{\hat{w}(i, j)\}_{N_u \times N_u}$ over the selected subset of pixels $P_u$, we can derive the new weight matrix $\hat{W} = \{\hat{w}(i, j)\}_{N \times N}$ over all the pixels from the original weight matrix $W = \{w(i, j)\}_{N \times N}$ of the $k$-NN graph as:

$$\hat{w}(i, j) = \begin{cases} \hat{w}(i', j'), & i, j \in P_u, P_u(i') = i, P_u(j') = j \\ w(i, j), & \text{otherwise} \end{cases},$$

where $P_u(i')$ denotes the $i'$-th member of $P_u$. This new weight matrix $\hat{W}$ over all the pixels is then applied to normalized cuts for image segmentation.

The full algorithm for constrained image segmentation can be summarized as follows:

(1) Generate the selectively propagated constraints using our SCP algorithm proposed in Section 2;

(2) Adjust the original weight matrix by exploiting the selectively propagated constraints according to Eq. (10);

(3) Perform normalized cuts with the adjusted new weight matrix for image segmentation.

As we have mentioned, Steps (1) and (2) incur a time cost of $O(kN_u^2)$ and $O(N_u^2)$ ($N_u \ll N$), respectively. Moreover, since the adjusted new weight matrix $\hat{W}$ is very sparse, Step (3) can be performed efficiently. Here, it is worth noting that the most time-consuming component of normalized cuts (i.e., eigenvalue decomposition) has a linear time cost when the weight matrix is very sparse. In summary, our algorithm runs very efficiently for constrained image segmentation.

4 Experimental Results

In this section, our algorithm is evaluated in the task of constrained image segmentation. We first describe the experimental setup, including information of the feature extraction and the implementation details. Moreover, we compare our algorithm with other closely related methods.

4.1 Experimental Setup

For segmentation evaluation, we select 50 images from the Berkeley segmentation database [Martin et al., 2001] (along with ground truth segmentations), and some sample images are shown in Figures 3 and 4. It can be observed that these selected images generally have confusing backgrounds, such as the penguin and kangaroo images. Furthermore, we consider a 6-dimensional vector of color and texture features for each pixel of an image just as [Carson et al., 2002]. The three color features are the coordinates in the L*a*b* color space, which are smoothed to avoid over-segmentation arising from local color variations due to texture. The three texture features are contrast, anisotropy, and polarity, which are extracted at an automatically selected scale.

The segmentation results are measured by the adjusted Rand (AR) index [Hubert and Arabie, 1985] which takes values between -1 and 1, and a higher AR score indicates
that a higher percentage of pixel pairs in a test segmentation have the same relationship (joined or separated) as in each ground truth segmentation. We do not consider the original Rand index [Rand, 1971; Unnikrishnan et al., 2007] for segmentation evaluation, since there exists a problem with this measure [Hubert and Arabie, 1985]. In the following, our normalized cuts with selective constraint propagation (NCuts_{SCP}) is compared with three closely related methods: normalized cuts with linear equality constraints (NCuts_{LEC}) [Yu and Shi, 2004], normalized cuts with spectral learning (NCuts_{SL}) [Kamvar et al., 2003], and standard normalized cuts (NCuts) [Shi and Malik, 2000]. Here, we do not make comparison to other constraint propagation methods [Lu and Carreira-Perpinän, 2008; Li et al., 2008; Lu and Ip, 2010; Lu and Peng, 2013], since they have a polynomial time complexity and are not suitable for image segmentation.

We randomly select a small set of labeled pixels to infer the initial set of linear equality constraints (for NCuts_{LEC}) and pairwise constraints (for NCuts_{SCP} and NCuts_{SL}). Moreover, we set the parameters of our NCuts_{SCP} algorithm as: $k = 60$, $\alpha = 0.9$, $\beta = 0.1$, $\epsilon = 10^{-7}$, and $\lambda = 0.001$. The parameters of other closely related methods are also set their respective optimal values.

### 4.2 Segmentation Results

We first show the effect of $N_s$ (i.e. the number of randomly selected pixels) on the performance of our NCuts_{SCP} algorithm in Table 1. Here, we measure the performance of our NCuts_{SCP} algorithm for constrained image segmentation by both AR index and running time averaged over all the images. In particular, we collect the running time by running our NCuts_{SCP} algorithm (Matlab code) on a computer.
Figure 4: The results of constrained image segmentation (cont.). A small set of labeled pixels (object and background denoted by markers ‘o’ and ‘x’, respectively) are provided to infer the initial set of linear equality constraints and pairwise constraints.

Table 2: The average segmentation results achieved by different image segmentation methods.

| Methods       | NCuts | NCuts_SL | NCuts_LEC | NCutsSCP |
|---------------|-------|----------|-----------|----------|
| AR index      | 0.36  | 0.39     | 0.40      | 0.50     |
| Time (sec.)   | 28    | 31       | 25        | 35       |

with 3GHz CPU and 32GB RAM. From Table 1, it can be clearly observed that our NCutsSCP algorithm requires more running time but leads to higher AR index when $N_s$ takes a larger value. Considering the tradeoff between the effectiveness and efficiency of constrained image segmentation, we select $N_s = 2,500$ for our NCutsSCP algorithm. This setting is used throughout the following experiments.

We further illustrate the selectively propagated constraints obtained by our NCutsSCP algorithm with $N_s = 2,500$ in Figure 3. Here, to explicitly represent the selectively propagated constraints, we need to infer the labels of $N_s$ randomly selected pixels from them. In fact, this inference can be done by simple voting according to the output of our SCP with the initial set of labeled pixels being regarded as the voters. Once we have inferred the labels of $N_s$ randomly selected pixels, we can show them out by marking the pixels within object and background by blue ‘o’ and yellow ‘x’, respectively. From Figure 3, we find that the selectively propagated constraints obtained by our NCutsSCP algorithm are mostly consistent with the ground truth segmentation.

The comparison between different image segmentation methods is listed Table 2. Meanwhile, this comparison is also illustrated in Figures 2 and 4. Here, the segmentation results are evaluated by both AR index and running time averaged over all the images. In particular, we collect the running time by running all the algorithms (Matlab code) on a computer with 3GHz CPU and 32GB RAM. The immediate observation is that our NCutsSCP algorithm significantly outperforms the other three methods in terms of AR index. Since our NCutsSCP algorithm incurs a time cost comparable to closely related methods, it is preferred for constrained image segmentation by an overall consideration. In addition, it can be clearly observed that NCutsSCP, NCutsLEC, and NCutsSL lead to better results than the standard NCuts due to the use of constraints for image segmentation.

5 Conclusions

In this paper, we have investigated the challenging problem of pairwise constraint propagation in constrained image segmentation. Considering the local homogeneity of a natural image, we choose to perform pairwise constraint propagation only over a selected subset of pixels. Moreover, we solve such selective constraint propagation problem by developing an efficient graph-based learning algorithm. Finally, the selectively propagated constraints are used to adjust the weight matrix based on $L_1$-minimization for image segmentation. The experimental results have shown the promising performance of the proposed algorithm for constrained image segmentation. For future work, we will extend the pro-
posed algorithm to other challenging tasks such as semantic segmentation and multi-face tracking.

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