Solar Neutrino Spectroscopy
(Before and After SuperKamiokande)∗

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Abstract

Results of solar neutrino spectroscopy based on data from four experiments are presented. Perspectives related to forthcoming experiments are discussed. Implications of the results for neutrino properties are considered.
1 Introduction

More than 30 years ago Bahcall \cite{1} and independently Zatsepin and Kuzmin \cite{2} had put forward the program of the complete solar neutrino spectroscopy. The idea was to perform several (radiochemical) experiments with different absorbers and absorption thresholds which are sensitive to different parts of the $\nu_\odot$-spectrum. This allows one to find fluxes of different components of the spectrum. In turn, comparing the fluxes one can study both interior of the Sun and (as it was realized later) properties of neutrinos themselves. Along with this line the Gallium experiment was proposed \cite{3}. That time Reines \cite{4} started to use direct electronic methods to get the information about $\nu_\odot$-fluxes.

In eighties, an analysis of the Homestake data had risen the question about time variations of $\nu_\odot$ - fluxes. Searches for possible time variations give very important and complementary information both about physics of the Sun and on neutrinos.

Great success of helioseismology in studies of interior of the Sun shifted the interests to the second aspect of solar neutrino spectroscopy: properties of neutrinos.

Today there are the data from five experiments: Homestake \cite{5}, Kamiokande \cite{6}, SAGE \cite{7}, GALLEX \cite{8} and first preliminary results from SuperKamiokande \cite{9}. Already with these data we can perform tentative spectroscopy of solar neutrinos. Forthcoming and future experiments will open a possibility to realize the program completely.

2 Neutrino Spectroscopy

2.1 Original fluxes and conversion probabilities

Fluxes of the electron neutrinos in the Earth detectors, $F_i(E), (i = pp, Be, pep, N, O, B)$, can be written as

$$F_i(E) = P(E) \cdot f_i \cdot F_i^0(E),$$

where $F_i^0$ are the fluxes in the reference standard solar model (RSSM). And in what follows we will use the model \cite{10} as the reference one.

Factors $P(E)$ are the electron neutrino survival probabilities: $\nu_e \rightarrow \nu_e$. They describe possible effects of neutrino transformations and can be called the neutrino factors. These factors depend on neutrino parameters: $\Delta m^2, \theta, E, \mu$ as well as on characteristics of the Sun – density, magnetic fields etc., and satisfy the restriction $0 \leq P(E, \theta, \Delta m^2) \leq 1$.

On the contrary, $f_i$ are the solar model factors which describe deviation of true original neutrino fluxes from those predicted by the reference SSM. The product $f_i \cdot F_i^0$ is the original flux of $i$-component, so that $f_i$ can be considered as the flux in the units of the
The problem is to find separately $f_i$ and $P(E)$ from the solar neutrino data. Although in (1) they enter as the product, there are two features which allow one to distinguish their effects.

- In general, $P$ depends on neutrino energy, whereas $f_i = \text{const}$;
- In the case of the flavor conversion the fluxes of $\nu_\mu$ and $\nu_\tau$ appear. If there is no transition to sterile neutrinos, these fluxes equal

$$F_i(\nu_\mu, \nu_\tau) = (1 - P) f_i F_0^i.$$  

They depend not only on the product $f \cdot P$, as in (1), but also separately on $f_i$. Neutrinos $\nu_\mu$ and $\nu_\tau$ contribute to signals (e.g. $\nu - e$ scattering, $\nu d \rightarrow \nu np$) due to neutral current interactions.

If the Sun is “standard”, then $f_i = 1$. For “standard” neutrinos (massless, unmixed) we take $P = 1$.

### 2.2 Boron neutrinos: implications of Kamiokande and SuperKamiokande results

It is worthwhile to start the analysis by the Kamiokande data for two reasons: (i) The data are sensitive to one component of the spectrum – boron neutrinos only, and therefore have simple interpretation. (ii) In near future SK and SNO will perform precise measurements of the boron neutrino flux, and this flux will be the reference point in the spectrum.

Basic conclusions one can draw from the Kamiokande result are the following:

- Kamiokande has certainly detected the Boron neutrinos: The shape of energy spectrum of the recoil electrons corresponds to original spectrum of the boron neutrinos. In particular, maximal energies of the electrons correspond to the end point of neutrinos from the $^8B$ - decay.

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$^\dagger$If there is the transition to both active and sterile neutrinos, the flux of $\nu_\mu, \nu_\tau$ is proportional to $(1 - P - P_s)$, where $P_s$ is the probability of $\nu_e \rightarrow S$.

$^\ddagger$The contribution from Hep neutrinos is negligibly small.
• The boron neutrinos are produced in the reaction at the end of the pp-III - nuclear cycle. Therefore the Kamiokande result means that all previous reactions which lead to production of $^8B$ take place, i.e. the complete pp-III -cycle operates inside the Sun.

• Once the pp-cycle operates, we know the components of the neutrino spectrum as well as energy dependencies of these individual components which are determined essentially by known kinematics of the weak interaction processes.

However all this does not allow one to restore the absolute values of fluxes. Moreover, present solar neutrino data do not even prove that pp-cycle is the main source of energy of the Sun. In [12] it was shown that neutrino data themselves do not contradict hypothetical possibility that CNO-cycle dominates in the energy release. The gallium data can be explained by fluxes of $N-$ and $O-$ neutrinos. An agreement with observations can be achieved, if one suggests that there is a conversion of neutrinos (e.g. the resonance conversion) which strongly suppresses fluxes in the energy range 0.8 - 2 MeV.

**Spectroscopy with Kamiokande.** Kamiokande data (although being in agreement with undistorted neutrino spectrum) do not exclude an appreciable distortion of spectrum. Let us introduce the ratio of the observed energy spectrum of the recoil electrons $N_e$ and the one expected from the “standard” boron neutrinos, $N_e^{SSM}$:

$$R_e(E) \equiv \frac{N_e(E)}{N_e^{SSM}(E)}. \quad (3)$$

Obviously, $R_e(E) = 1$ in absence of distortion. Due to strong smoothing in the recoil electron spectrum, even strong distortion of the neutrino spectrum leads to approximately linear energy dependence of the ratio:

$$R_e(E) = R_e(E_0)[1 + s_e \cdot (E - E_0)]. \quad (4)$$

That is the distortion can be characterized by the slope parameter, $s_e$, which equals $s_e \equiv [(dR_e/dE)/R_e]_{E_0}$. We fix the slope in the middle of the detected interval at $E_0 = 10$ MeV. The $\chi^2$ fit of Kamiokande spectrum by (4) gives

$$s_e = 0.04 \pm 0.05 \text{ MeV}^{-1}, \quad (1\sigma). \quad (5)$$

The best fit point corresponds to $\chi^2 = 9$ for 6 d.o.f..

Obviously $s_e$ does not depend on $f_B$, and if nonzero, determines parameters of neutrinos: $s_e = s_e(\Delta m^2, \sin^2 2\theta, \ldots)$. For a given solution of the solar neutrino problem $s_e$ is
the function of the average conversion probability: $s_e = s_e(\langle P_B \rangle)$. Thus measurements of the slope determine $\langle P_B \rangle$ independently of $f_B$:

$$\langle P_B \rangle = \langle P_B(s_e) \rangle$$

Once $\langle P_B \rangle$ is known, from (I) one can find

$$f_B \approx \frac{R_e(E_0)}{\langle P_B \rangle}.$$ 

Therefore measurements of the slope and the absolute value of the recoil electrons counting rate allow one to find separately the original flux and the conversion probability.

SK will improve the sensitivity of measurements of $s_e$ by one order of magnitude, so that in future one may get the numbers like $s_e = 0.040 \pm 0.005$ (if e.g. the average value of the slope will be equal to present value). The accuracy in measurements of the absolute value of the flux will be mainly due to systematic uncertainties.

### 2.3 Spectroscopy in absence of $\nu_\mu$ and $\nu_\tau$ components

Results of spectroscopy depend crucially on whether neutrinos of other flavors are present in the solar flux. In future an admixture of the non-electronic flavor will be measured by the SNO-experiment. At present we should consider two possibilities.

Let us assume first that non-electron neutrino fluxes are absent or small. This corresponds to the astrophysical solutions or to conversion of $\nu_e$ into sterile neutrino. We also assume (as it is implied by models of the Sun) that $pp$-cycle gives dominant contribution to the energy release.

Present day analysis can be done in three steps:

**Step 1. Kamiokande.** If there is no $\nu_\mu$ - and $\nu_\tau$ - fluxes from the Sun, the Kamiokande signal is stipulated by $\nu_e$ -flux completely:

$$F_B = F_B^{kam} = (2.80 \pm 0.38) \cdot 10^6 \text{ cm}^2\text{s}^{-1}. \quad (6)$$

If the distortion of the spectrum is relatively weak we find:

$$f_B \cdot \langle P_B \rangle = R_{\nu e} = 0.42 \pm 0.06 \quad (7)$$

in the range $E = 7 - 14$ MeV. Here $\langle P_B \rangle$ is the average conversion probability.
Step 2. Kamiokande and Homestake [13] - [19]. The contribution of the boron neutrino flux measured by Kamiokande to the Ar - production rate is

\[ Q_{Ar}^B = \int dE \sigma_{Ar}(E) F_B(E) = (3.1 \pm 0.4) \text{ SNU} . \]  (8)

This contribution only weakly depends on possible distortion of spectrum. Indeed, it turns out that the Ar-production cross section and the integral of the product of the \( \nu - e \) differential cross section and the efficiency of registration in Kamiokande have approximately the same energy dependence [20]:

\[ \sigma_{Ar}(E) \sim \int dE_e dE'_e \frac{d\sigma_{\nu e}(E, E'_e)}{dE'_e} K(E_e, E'_e) . \]  (9)

Here \( K(E_e, E'_e) \) is the probability that the electron with energy \( E'_e \) is detected as having the energy \( E_e \). For this reason both experiments have approximately the same sensitivity to the neutrino spectrum.

Subtracting the contribution (8) from the experimental value of the Ar-production rate we get

\[ Q_{Ar}^{\text{exp}} - Q_{Ar}^B = Q_{Ar}^{Be} + Q_{Ar}^{pep} + Q_{Ar}^{NO} = -0.6 \pm 0.5 \text{ SNU} , \]  (10)

where \( Q_{Ar}^{i} \) (\( i = Be, pep, N, O \)) are the contributions of different components to the Ar-production rate. Putting \( Q_{Ar}^{pep} + Q_{Ar}^{NO} = 0 \), we get the upper bound on the product:

\[ f_{Be} P_{Be} = -0.44 \pm 0.36 . \]  (11)

If \( pep, N, O \) contributions equal to those in the reference model, then \( f_{Be} P_{Be} = -1.25 \pm 0.40 \).

The SK will strengthen this result. (After SK the bound will be determined essentially by systematics of SK and the error bars of the Homestake experiment).

The negative value of \( f_{Be} P_{Be} \) testifies for presence of neutrino flux which contributes to the Kamiokande signal and does not contribute to the Homestake signal. This can be the flux of the \( \nu_\mu \) and \( \nu_\tau \) (or \( \bar{\nu}_\mu \), \( \bar{\nu}_\tau \)) produced in the conversion \( \nu_e \rightarrow \nu_\mu \), \( \nu_\tau \). Note that due to equality (9) it is impossible to avoid the negative value of the product in (11) by distortion of spectrum. Thus precise measurements of the flux (being confronted with Homestake Chlorine as well as Iodine data) may give the proof of the neutrino flavor conversion, even without observations of distortion of the spectrum in SK.

Negative value of the product disfavors also the conversion into sterile neutrinos.

Step 3. Kamiokande, Homestake and Gallium [21, 22, 23]. The contribution of the boron neutrino flux (as measured in Kamiokande) to the Ge-production rate is \( Q_{Ge}^B = (7 \pm 1) \)
Table 1: The spectroscopy of the solar neutrinos

| # | E, MeV | $f_{pp}(P_{pp})$ | $f_{Be}(P_{Be})$ | $f_{B}(P_{B})$ | $f_{B}$ |
|---|-------|-----------------|-----------------|----------------|---------|
| 1 | 1     | 1.14 ± 0.31     | -0.44 ± 0.36    | 0.42 ± 0.06     | 0.73 ± 0.10 |
| 2 | 0.5   | 0.99 ± 0.31     | -0.16 ± 0.35    | 0.36 ± 0.05     | 0.88 ± 0.12 |
| 3 | 0.4   | 0.89 ± 0.30     | -0.03 ± 0.33    | 0.34 ± 0.05     | 1.05 ± 0.15 |
| 4 | 0.3   | 0.81 ± 0.30     | 0.16 ± 0.31     | 0.31 ± 0.045    | 1.31 ± 0.19 |
| 5 | 0.2   | 0.66 ± 0.28     | 0.45 ± 0.29     | 0.26 ± 0.04     | 1.80 ± 0.26 |
| 6 | 0.1   | 0.39 ± 0.28     | 0.95 ± 0.25     | 0.18 ± 0.03     | 1.80 ± 0.26 |

SNU. For $Q_{Ge}^{Be} = Q_{Ge}^{pep} = Q_{Ge}^{N,O} = 0$, as it is implied by Homestake and Kamiokande results, we get

$$Q_{Ge}^{pp} = Q_{Ge}^{exp} - Q_{Ge}^{B} = 63 \pm 8 \text{ SNU}, \quad (12)$$

and then

$$f_{pp}(P_{pp}(E)) = 0.90 \pm 0.11. \quad (13)$$

Here the error bars are due to experimental errors of the GALLEX only. At 2$\sigma$ level non zero flux of $^7$Be neutrinos is admitted: $f_{Be}P_{Be} = 0.28$. This gives the contribution $Q_{Ge}^{Be} = 11$ SNU. Now the $pp$-neutrino flux should be suppressed stronger to satisfy the Gallium results: $f_{pp}(P) = 0.74 \pm 0.11$. If one takes into account the negative value of $f_{Be}P_{Be}$, then $f_{pp}(P_{pp}) = 1.14 \pm 0.31$.

The results of the spectroscopy are summarized in the line #1 of Table I. Main features are: moderate suppression at high energies $E > 7$ MeV; strong suppression at the intermediate energies and weak (or absence of) suppression at low energies. The energy independent suppression is more than 5$\sigma$ out of the data.

2.4 Spectroscopy in presence of $\nu_\mu$, $\nu_\tau$ fluxes

The spectroscopy can lead to quite different picture of suppression if one admits an existence of the “non electron” neutrino components. Now the effective flux measured by Kamiokande can be written as

$$F_{B}^{kam} \approx f_{B} \cdot \langle P_{B}(E) \rangle \cdot \langle F_{B}^{0} \rangle \cdot \xi, \quad (14)$$
where
\[ \xi = 1 + r \frac{1 - \langle P_B \rangle}{\langle P_B \rangle}. \] (15)

The second term in the RH side is due to the neutral current scattering of non electron neutrinos, and \( r \approx 0.15 \) is the ratio of the differential cross sections of the \( \nu_\mu \) and the \( \nu_e \) scattering. (In fact, \( r \) rather weakly depends on the energy of electrons for \( E_e > 7 \) MeV.)

The averaged probability \( \langle P_B \rangle \) is unknown and in what follows we will perform the spectroscopy for different values \( \langle P_B \rangle \) (i.e. for different contributions of the \( \nu_\mu \), \( \nu_\tau \) neutrinos to Kamiokande signal) keeping in mind that forthcoming experiments will be able to fix \( \langle P_B \rangle \) along with the line described in sect. 2.2.

**Step 1. Kamiokande.** The role of the second term in (15) increases with diminishing \( \langle P_B \rangle \), so that the Kamiokande signal may be essentially due to \( \nu_\mu \)-, \( \nu_\tau \)- effect and in the limit \( P \rightarrow 0 \) one gets
\[ F_{B}^{kam} \approx f_B \cdot \langle F_B^0 \rangle \cdot r. \] (16)

This however implies that the original boron neutrino flux is much bigger than in the reference model: \( f_B \sim R_e r^{-1} \approx 2.8, \) \( ( R_e \equiv N_{e}^{kam}/N_{e}^0 ) \). As the consequence, one expects large double ratio \( (NC/CC)_{obs}/(NC/CC)_{SM} \gg 1 \) in the SNO experiment.

**Step 2. Homestake and Kamiokande.** The electron neutrino flux is smaller than the flux measured by Kamiokande, \( F_{B}^{kam} \). From (12) we get the suppression factor for the electron neutrinos can be estimated as
\[ f_B \langle P_B \rangle \approx R_e \xi^{-1}, \] (17)

Correspondingly, the contribution of boron neutrinos to the \( Ar \)-production rate equals
\[ Q_{Ar}^B \approx Q_{Ar,0}^B \xi^{-1}, \] (18)

where \( Q_{Ar,0}^B (= 3.1) \) SNU is defined in (8). For \( \langle P_B \rangle = 0.4, 0.3, 0.2 \) we find \( Q_{Ar}^B = 2.55, 2.30, 1.95 \) SNU respectively. The contribution of the boron neutrinos to \( Ar \) - production decreases with \( \langle P_B \rangle \), thus leaving the room for the beryllium neutrinos and other neutrinos of the intermediate energies. In particular, at \( \langle P_B \rangle \approx 0.4 \) we get \( Q_{Ar}^B \approx Q_{Ar,exp}^{exp} \). For \( \langle P_B \rangle = 0.1 \) the beryllium neutrinos may have (unsuppressed) RSSM - flux.

**Step 3. Homestake, Kamiokande and Gallium.** With decrease of \( \langle P_B \rangle \) and therefore the increase of \( f_{Be} P_{Be} \), the pp-neutrino flux should be suppressed to satisfy the gallium results.
The suppression factor equals
\[
f_{pp}\langle P_{pp} \rangle = \frac{1}{Q_{pp}^{SSM}} \left[ Q_{Ge}^{exp} - Q_{Ge}^{Be} - Q_{Ge}^B - Q_{Ge}^{pp} - Q_{Ge}^{NO} \right].
\] (19)

For completely suppressed beryllium flux, the Eq. (19) reproduces result (13). For unsuppressed beryllium flux we get
\[
f_{pp}\langle P_{pp} \rangle \leq 0.40 \pm 0.11.
\] (20)

The results of spectroscopy for different values of \( \langle P_B \rangle \) are shown in Table 2. With decrease of \( \langle P_B \rangle \), the fluxes of intermediate energies, and in particular, beryllium neutrino flux, allowed by data increase, whereas the suppression of the \( pp \)-neutrino flux becomes stronger. Strong decrease of \( \langle P_B \rangle \), implies big original boron neutrino flux.

\[
f_B = \frac{R_e}{\langle P_B \rangle \xi}.
\]

The energy independent suppression is disfavored: If e.g. \( \langle P_B \rangle \sim 0.3 \), then \( f \cdot P = const \) is out of the data for more than 2\( \sigma \).

### 2.5 Separate determination of \( f_i \) and \( P \)

As we discussed previously, further precise measurements of the distortion as well as the effects of \( \nu_\mu \) and \( \nu_\tau \) will give a possibility to determine \( f_i \) and \( \langle P_i \rangle \) separately. With present data one can get only some limits for \( f_i \).

1. Since \( P \leq 1 \), there is the lower bound \( f_i \geq f_i P_i \). In particular, from Kamiokande data we get
\[
f_B > 0.3 \ (2\sigma).
\] (21)

(Note that at \( P = 1 \) the fluxes of \( \nu_\mu \) and \( \nu_\tau \) are absent).

2. The upper bound on \( f_B \) can be obtained in assumption that the electron neutrinos convert into active neutrinos. For large \( f_i \), the contribution from \( \nu_\mu \) and \( \nu_\tau \) alone can explain the data. Therefore \( f_B < R_{\nu e} r^{-1} \approx 3.6 \). Stronger upper bound, \( f_B < 2.8 \ (2\sigma) \), can be obtained, if one takes into account also the Homestake result and suggests that the Argon is produced mainly by boron neutrinos. The limits can be further strengthen in the context of certain solution to the problem (see sect. 3).

3. The bounds on \( pp \)-neutrinos can be obtained from the solar luminosity normalization condition (in assumption of thermal equilibrium of the Sun) [24, 25, 26]:
\[
\sum_i \left( \frac{Q}{2} - E_i \right) F_i = L_\odot
\] (22)
(here $E_i$ is the average energy of $i$-neutrino component, and $Q$ is the the energy release in the hydrogen cycle), and from the nuclear condition [26]:

$$F_{pp} + F_{pep} \leq F_{Be} + F_B .$$

(23)

This gives [26]

$$0.5 < f_{pp} < 1.1.$$ 

The lower bound follows from (23). The restriction is even more tight if one admits that $pp$-flux dominates over other fluxes. This gives essentially $f_{pp} = 1.00 \pm 0.05$, and consequently, $\langle P(E) \rangle \approx 0.9 \pm 0.11$. To improve this number one should continue the Gallium experiments and perform new experiments like HELLAZ.

4. The $f_{Be}$ is restricted very weakly. The model independent upper bound follows from the solar luminosity normalization condition: $f_{Be} < 6.35$ [26].

### 3 Implications

Let us confront the suppression profiles obtained from solar neutrino spectroscopy (Table 1.) with energy dependencies of different effects.

#### 3.1 Astrophysical solutions

In this case $P_i(E) = 1$. Majority of solutions is based on (or effectively reduced to) diminishing of the central temperature of the Sun. This gives the suppression profile with $f_{pp} : f_{Be} : f_B = (T/T_{SSM})^{-1.2} : (T/T_{SSM})^{8-11} : (T/T_{SSM})^{18-25} = 1.05 : 0.7 : 0.4$ which should be compared with the profile #1 in Table I. The intermediate energies are suppressed weaker than the high energies and this is the basis of statements that the astrophysical solutions are very strongly disfavored. To explain the data one needs more sophisticated and more selective modification of the solar model (see e.g. [27]). However helioseismology gives very strong restrictions: The modifications which are consistent with helioseismology give only small changes of neutrino fluxes [28, 29]. These aspects have been discussed lively in the talks by John Bahcall [30] and Arnon Dar [31].

#### 3.2 Vacuum oscillations solution

Typical energy dependence of the survival probability [32] fits reasonably well the profiles #4,5 in the Table I. Rather big contribution from $\nu_\mu$ and $\nu_\tau$ is inferred. Basic features of the solution are [33] - [36]:

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(i) For $\Delta m^2 > 3 \cdot 10^{-11}$ eV$^2$ the boron neutrinos ($E > 7$ MeV) are in the first high energy minimum of the suppression curve, $P(E)$. The slope parameter, $s_e$, can be bigger than in the MSW solution.

(ii) The beryllium neutrinos are in the rapidly oscillating part of the $P(E)$, so that one expects an appreciable seasonal variations of the Be-neutrino flux due to annual change of distance between the Sun and the Earth. The strongest suppression is $P_{Be,min} = 1 - \sin^2 2\theta$. Large contribution of $\nu_\mu$ and $\nu_\tau$ to the Kamiokande signal resolves the problem of negative $f_{Be}P_{Be}$.

(iii) The $pp$-neutrino flux is in the region of averaged oscillations, where $P \sim 1 - 0.5\sin^2 2\theta$. Lower experimental value of $Q_{Ge}^{exp}$ makes the fit better. It is easy to reach the inequality $P_{pp} > P_B > B_{Be}$ implied by spectroscopy. However, there is an obvious relation between maximal suppression of the Be-line and suppression of $pp$-neutrinos: $P_{Be,min} = 2P_{pp} - 1$, and due to this the best fit configurations are not realized for $f_i = 1$. The best fit ($\chi^2 \sim 2$ for 2 d.o.f.) corresponds to $\Delta m^2 = 0.6 \cdot 10^{-10}$ eV$^2$, $\sin^2 2\theta = 0.9$ (for $f_i = 1$). Such a large mixing is disfavored by the data from SN1987A [37]. Good fit can be obtained also for moderate ($\sim 0.5$) suppression of the Be-line and $\sim 0.6$ suppression of the $pp$-neutrino flux.

The fit becomes better for values of $f_B$ bigger than 1 [33]. In this case the contribution of non-electron neutrinos to Kamiokande is large. On the contrary, with diminishing $f_B$ a suppression of the boron neutrino flux due to oscillations should be weaker. Therefore for fixed values of $\Delta m^2$ the allowed regions of parameters shift to smaller $\sin^2 2\theta$ [33, 34]. In particular, for $f_B = 0.7$, the region is at $\sin^2 2\theta < 0.7$, thus satisfying the bound from SN87A. For $f_B \sim 0.4$ mixing can be as small as $\sin^2 2\theta < 0.5 - 0.6$.

For $f_B \sim 0.5$ the allowed region appears at $\Delta m^2 \sim 5 \cdot 10^{-12}$ eV$^2$ [34] which corresponds to the $Be$-neutrino line in the first high energy minimum of $P(E)$, the $pp$-neutrinos are in the first maximum of the $P$ and high energy part of the boron neutrino spectrum is out of suppression pit. No appreciable time variations are expected. Distortion of the $pp$-neutrino spectrum is the signature of this solution [34].

Depending on neutrino parameters as well as on $f_B$, $f_{Be}$ ... one can get variety of distortions of the boron neutrino energy spectrum [34].

3.3 Resonance flavor conversion

Small mixing solution [38] can precisely reproduce the desired energy dependence – the profiles #3, 4 from the Table 1 [39, 40, 41, 42]. In the region of small mixing angles
one has
\[ P_{pp} \sim 1, \quad P_{Be} \sim 0, \quad P_B \sim e^{E_{na}/E}, \] (24)
where \( E_{na} = \Delta m^2 (\rho/\dot{\rho}) \sin^2 2\theta \) and \( \rho \) is the density profile.

Let us sketch main properties of the solution.

(i) The dependence of the slope parameter on mixing angle for \( \Delta m^2 = 6 \cdot 10^{-6} \text{ eV}^2 \) is shown in the Table 2. The best fit slope corresponds to \( \sin^2 2\theta = 4 \cdot 10^{-3} \). At \( 2\sigma \) level the values \( \sin^2 2\theta \sim 10^{-2} \) are allowed.

(ii) Additional contribution to Kamiokande, \( \Delta f_B \approx 0.09 \), follows from scattering of the converted \( \nu_\mu (\nu_\tau) \) on electrons. This solves the negative \( f_{Be} P_{Be} \) problem.

(iii) The \( pp \)-neutrino flux can be suppressed, if needed, by diminishing \( \Delta m^2 \). In this case the high energy part of the spectrum will be in the suppression pit, and one expects distortion of spectrum of the \( pp \)-neutrinos.

With diminishing \( f_B \) the suppression due to conversion should be weakened, and therefore \( \sin^2 2\theta \) decreases according to \( [24, 31, 11] \). At \( \Delta m^2 = 6 \cdot 10^{-6} \text{ eV}^2 \) the best fit of the data corresponds to the pairs of parameters \( [11] \): \( (f_B, \sin^2 2\theta) = (0.4, 1.0 \cdot 10^{-3}), (0.75, 4.3 \cdot 10^{-3}), (1.0, 6.2 \cdot 10^{-3}), (1.5, 9 \cdot 10^{-3}), (2.0, 10^{-2}) \). The decrease of \( f_{Be} \) gives an additional small shift of the allowed region to smaller values of \( \sin^2 2\theta \). A consistent description of the data has been found for \( [11] \)
\[ f_B \sim 0.4 - 2.5. \]

Other fluxes are restricted rather weakly. At \( 2\sigma \) level: \( f_{Be} < 6.35 \), and \( f_{pp} = 0.55 - 1.08 \) \( [20] \).

For \textit{very small mixing solution}: \( f_B \sim 0.5, \sin^2 2\theta \sim 10^{-3} \), all the effects of the conversion become very small in the high energy part of the boron neutrino spectrum \( (E > 5 - 6 \text{ MeV}) \). In particular, a distortion of the energy spectrum disappears, and the ratio \( \langle CC/NC \rangle^\text{exp}/\langle CC/NC \rangle^\text{th} \) approaches 1. Thus studying just this part of spectrum, it will be difficult to identify the solution (e.g., to distinguish the conversion and the astrophysical effects) \( [11] \).

\textit{Large mixing MSW} solution. The energy dependence of the effect gives reasonable approximation to the profiles \#4,5.

(i) Boron neutrinos are in the bottom of the suppression pit, the slope parameter is very small and has negative sign. The day - night effect can be observed.
Table 2: The dependence of the slope parameter on mixing angle for $\Delta m^2 = 6 \cdot 10^{-6}$ eV$^2$.

| $\sin^2 2\theta \times 10^3$ | 2    | 4    | 6    | 8    | 1.0  |
|-----------------------------|------|------|------|------|------|
| $s_e$, MeV$^{-1}$           | 0.017| 0.034| 0.046| 0.055| 0.065|

(ii) Due to contribution from $\nu_\mu$ and/or $\nu_\tau$ leads the inequality $P_{pp} > P_B > P_{Be}$ can be realized.

(iii) For pp-neutrinos: $P \leq P^{vac} = 1 - \sin^2 2\theta/2$. Therefore small experimental values of $Q^{exp}_{Ge}$ lead to better fit.

The range of neutrino parameters is $\Delta m^2 = (6 \cdot 10^{-6} - 10^{-4})$ eV$^2$, $\sin^2 2\theta = 0.65 - 0.85$. With increase of $f_B$ a suppression of the boron neutrino flux should increase and the fit of the data becomes better \[40\]. Now the Kamiokande signal can be explained essentially by NC effect and mixing angle can be relatively small. Beryllium neutrino flux is sufficiently suppressed and suppression of the pp-neutrinos is rather weak. For $f_B \sim 2$, values $\sin^2 2\theta = 0.2 - 0.3$ become allowed.

Recently the effect of possible density fluctuations on the resonance conversion has been estimated \[43\]. The fluctuations could be related to the gravity modes of oscillations of the Sun. The effect leads to dumping of the neutrino conversion: a steady depart of neutrino state from the coherent mixture. As the result the probability of conversion approaches 1/2, if the time of evolution is enough. In the case of varying average density the effects is mainly collected in the region of resonance layer. Small $\Delta \rho/\rho$ leads to the biggest effect, when the vacuum mixing angle is small \textit{i.e.} the resonance occurs in the central parts of the Sun, and correspondingly $P(E)$ is modified most strongly near the adiabatic edge. It can reach $\Delta P \sim 0.1$ for $\Delta \rho/\rho \sim 2\%$. The effect can be important for $^7Be$ as well as pp-neutrinos. Effectively the adiabatic edge is shifted to higher $E$ which is equivalent to diminishing $\Delta m^2$. Apprecciable shift of the allowed region of neutrino parameters implies $\Delta \rho/\rho \sim 5\%$.

Note however, the density fluctuations with typical scale $L \sim 10^2$ km and $\Delta \rho/\rho \sim 2 - 5\%$ may contradict helioseismological data.

\textit{Conversion into sterile neutrino $\nu_e \rightarrow S$}. This solution differs from the flavor case in two points: (i). Effective matter density which determines the refraction effect is now
\[ N = N_e - N_N/2, \text{ i.e., it depends also on density of neutrons. Since the concentration on neutrons is not big even in the center of the Sun the effect is also quite small. } \]

(ii). There is no contribution from S in the Kamiokande detector, and the fit of the data is worse than in the flavor case. Therefore confirmation of negative value of \( f_{Be} P_{Be} \) will disfavor this solution.

The regions of neutrino parameters are approximately the same as in flavor case if one restricts the original boron neutrino flux by \( f_B \leq 2 \).

\textit{Three neutrino mixing.} The analysis of data in terms of two neutrino mixing is quite realistic, since in the most interesting cases (simultaneous solution of the solar and hot dark matter problems, or solar and atmospheric neutrino problems) the third neutrino has a large mass, so that its \( \Delta m^2 \) is beyond the solar resonance triangle region and its mixing to the electron neutrino is rather small. This reduces the three neutrino task to the case of two neutrino mixing. However, there is an interesting example, where third neutrino could influence solutions of the solar neutrino problem. It was considered previously [44, 45, 46] and reanalyzed recently in [17, 18]. The third neutrino is in the region of a solution of the atmospheric neutrino problem: \( m_3 \sim 0.1 \text{ eV} \), and it has an appreciable admixture to the electron neutrino state: \( \nu_e = \cos \phi \, \nu' + \sin \phi \, \nu_3 \), where \( \nu' = \cos \theta \, \nu_1 + \sin \theta \, \nu_2 \) and \( \phi \) is not small. The third neutrino \( \nu_3 \) “decouples” from the system (as far as we deal with the Sun) and its effect is reduced just to the averaged vacuum oscillations. In turn, \( \nu' \) converts resonantly into its orthogonal state. Therefore the survival probability can be written as \[45\]

\[ P = \cos^4 \phi \, P_2 + \sin^4 \phi, \quad (25) \]

where \( P_2 \) is the two neutrino survival probability. Additional regions of the neutrino parameters \( \Delta m^2 = (10^{-5} - 10^{-6}) \text{ eV}^2 \) and \( \sin^2 2\theta = 3 \cdot 10^{-4} - 3 \cdot 10^{-3} \) are allowed for \( \cos^4 \phi \sim 0.5 - 0.7 \). Both \( pp \)- and \( Be \)- neutrinos can be outside the \( 2\nu \)- suppression pit \[13\], where \( P_2 \approx 1 \) and according to \( (25) \) the suppression factor for them is \( (\cos^4 \phi + \sin^4 \phi) \). This allows one to get about 1/2 suppression of the gallium production rate, and to reconcile the Homestake and Kamiokande results at \( 2\sigma \) level. Moreover, now the boron neutrinos can be on the adiabatic edge of the suppression pit because of distortion of the spectrum is weakened by factor \( \cos^4 \phi \) and the slope parameter equals \( s \approx \cos^4 \phi \cdot s_0 \), \( s_0 \) is the slope in the \( 2\nu \) case). The SK will certainly be able to check this scenario.

A number of new possibilities appears, if both \( \Delta m^2 \) are in the resonance triangle of the Sun \[45\], or if one of the resonances is in the resonance triangle whereas another one is in the region of “just-so” solutions.
3.4 Resonance spin-flavor precession

As is well known the RSFP [49] leads to suppression factor which can perfectly reproduce the configuration of Fig. 2a [50], and which is very similar to that of small mixing MSW solution. There are however three important differences.

(i) Asymptotic at $E \to \infty$:

$$P \to \begin{cases} 1 & \text{for the MSW} \\ P_0(B, \Delta m^2, \theta ... ) & \text{for the RSFP} \end{cases} \quad (26)$$

In the case of the RSFP the asymptotic value can be any number in the interval $0 \leq P_0 \leq 1$ depending on the magnetic field profile, possible twist of the field, neutrino magnetic moment etc.. If the $^7$Be-line is strongly suppressed then the boron neutrino flux ($E > 7$ MeV) can be near the asymptotic region where the slope parameter is rather small.

(ii). Correlation of NC/CC and distortion. In contrast to the MSW $2\nu$-solution a distortion of spectrum is not correlated with absolute value of the probability. Strong suppression can be accompanied by weak distortion. One may observe large anomalous ratio NC/CC in SNO and in the same time weak distortion of energy spectrum of the boron neutrinos. Note however that in the case of $3\nu$-resonance flavor conversion the correlation can also be lost.

(iii). Time variations. There are strong time variations of the magnetic phenomena at the surface of the Sun and therefore it is difficult to expect that magnetic field profile is constant on the way of neutrinos inside the Sun or that changes are such that the integral effect is always the same. Therefore time variations are generic consequences of this solution. Although a relation of the neutrino fluxes with surface activity can be rather complicated (neither simple correlations nor anticorrelations).

It is assumed that the field which influences neutrino propagation is the toroidal one. It has different polarities in the northern and southern semispheres of the Sun, so that in the equatorial plane the field has zero strength. The size of this equatorial gap is about $5 \rightarrow 7^0$ and one predicts seasonal variations of the neutrino flux due to presence of the gap and inclination of the Earth orbit with respect to the equatorial plane [51]. No gap effect has been found by Kamiokande [9].

Reality of time variations of the solar neutrino fluxes is still open question. An analysis shows that all experimental data are statistically compatible with constant neutrino fluxes [52]. However even strong time variations, and in particular anti correlations with solar activity, are not excluded.

Suggested anticorrelations weakened during last 5 years. According to the analysis [53] the confidence level of the anticorrelation with sunspot number (as well as with some
other characteristics of solar activity) declined very quickly since 1990 to 1994 from 0.9997 to 0.9.

In [54] the anticorrelations of the Homestake data with surface magnetic flux have been studied. Strong anticorrelation (probability $\sim 99\%$) is found for the magnetic flux within central band ($\pm 5^\circ$ of solar latitude) that has been delayed by 0.3 - 1.4 years with respect to neutrino signal. Still the results are not conclusive and the question of whether this anticorrelations are statistical or physical in nature requires data that span several solar cycles.

4 Conclusion

1. We performed the solar neutrino spectroscopy using data from four experiments. The suppression profiles $f_i P$ as the functions of the neutrino energy are found. The results depend strongly on whether the $\nu_\mu$, $\nu_\tau$ components are present in the solar neutrino flux.

2. The data give some indication (“negative” beryllium neutrino flux) of presence of the non-electronic neutrinos in the neutrino flux. This result will be checked by
   (i) more precise measurements of signal in SK;
   (ii) measurements of the CC events in SNO and comparison with SK data;
   (iii) measurements of the (NC)/(CC) ratio by SNO.

3. Present data agree within $1\sigma$ with zero slope parameter, although rather strong distortion of the boron neutrino spectrum is not excluded. Measurements of the slope parameter of the recoil electron energy spectrum by SK, and directly the slope of neutrino energy spectrum by SNO will allow one to find independently $f_B$ and $\langle P_B \rangle$, that is to restore original boron neutrino flux and to get within given solution of the solar neutrino problem the bounds on neutrino parameters. These measurements will give certain discrimination among solutions.

4. If there is no $\nu_\mu$ and $\nu_\tau$ neutrino flux, then the data lead to energy profile $\neq 1$ from the Table 1 with very strong suppression at the intermediate energies.

5. If $\nu_\mu$, $\nu_\tau$ - components exist, then depending on $\langle P_B \rangle$ one may get rather diverse dependences of $f \cdot P$ on energy. With decrease of $\langle P_B \rangle$ the fluxes of the intermediate energies, and in particular, the beryllium neutrino flux, allowed by data, increase, whereas the suppression of the $pp$ - neutrino flux becomes stronger. Strong decrease of $\langle P_B \rangle$ implies,
however, big original boron neutrino flux.

6. What can we expect after SK and SNO? Suppose the SK will not find neither
distortion of the recoil electron spectrum nor the day-night effect. This may be explained
by: (i) very small mixing MSW, (ii) large mixing MSW, (iii) RSFP, (iv) very small $\Delta m^2$
vacuum oscillation solution, (v) averaged vacuum oscillations, and (vi) still we will discuss
astrophysical possibilities.

Further discrimination can be done by SNO. If strong anomaly in ratio NC/CC will
be found, then solutions (i), (iv) and (vi) will be excluded. If SNO will not find NC/CC
anomaly, only large mixing MSW conversion to the active neutrinos will be excluded and
BOREXINO results will be decisive.

If SK will find distortions, then one should discriminate among small mixing MSW,
vacuum oscillations and RSFP. If SK will find large day-night effect, it will be the proof
of the large mixing MSW solution.

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