Propagation of surface plasmons through planar interface

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We analyze the scattering of the surface plasmon incident at a planar interface between two dielectrics. By using the scattering matrix technique, developed by Oulton et al. [Phys. Rev. B 76, 035408 (2007)], we calculate the transmission, reflection coefficients and radiative losses for oblique incident angles. We found that the transmission of a surface wave through a single interface between two dielectrics may be accompanied with radiation losses of 10-40 per cent of the plasmon energy.

I. INTRODUCTION

The excitation of a surface electromagnetic wave at the metallic interface - surface plasmon[1,2] - opens new ways in nanophotonics[3,4] and metamaterial physics[5]. One of the main constraints in using of surface plasmons is their short lifetime. It is well known[2,7–9,11] that a significant part of the plasmon energy is radiated when the plasmon is scattered at the surface impurity. A detailed quantitative analysis of the process of scattering and estimation of radiation losses is therefore important for understanding of propagation of surface plasmons.

The most simple scattering problem is the transmission and reflection of a surface plasmon at the planar permittivity step, created when the metallic surface is covered by two different dielectrics[7,12]. In the most simple scattering experiment, the metallic surface lies in the $z = 0$ plane, and two dielectrics fill the $z > 0$ half-space. The interface between two dielectrics is given by $x = 0$ plane, so that the dielectric permittivity is

$$\varepsilon_d = \begin{cases} \varepsilon_a & x < 0 \\ \varepsilon_b & x > 0. \end{cases}$$

In this paper, we study the propagation of the surface plasmon through the planar interface between two dielectrics which cover the metallic surface. A modified method of Oulton et al.[11] is used for the calculation of the transmission and reflection coefficient and analysis of the radiation losses accompanying the plasmon scattering. Our data confirm that significant part of the surface plasmon energy is radiated in the process of single scattering, and energy losses increase when the angle of incidence increases.

II. SURFACE PLASMON AT THE METAL - DIELECTRIC INTERFACE

The surface plasmon propagates along the metal dielectric interface located in the $z = 0$ plane. The intensity of the electric and magnetic field decays exponentially on both sides of the interface: $h \propto e^{-\kappa_d z}$ for $z > 0$ (dielectric) and $h \propto e^{+\kappa_m z}$ for $z < 0$ (metal). The parameters $\kappa_d$ and $\kappa_m$ are given by the dispersion relations[1,12]

$$\frac{\kappa_m}{\kappa_d} + \frac{\varepsilon_m}{\varepsilon_d} = 0$$

($\varepsilon_m$ is the metallic permittivity), and

$$k_m^2 - \kappa_m^2 = k_0^2 \varepsilon_d, z > 0$$
$$k_m^2 - \kappa_m^2 = k_0^2 \varepsilon_m, z < 0.$$  

Here, $k_{||} = \sqrt{k_0^2 + k_0^2}$ is the projection of the wave vector into the $xy$ plane, $k_0 = \omega/c$ and $c$ is the light velocity. From Eq. (2) we find explicit expressions for the components of the wave vector,

$$k_{||}^2 = k_0^2 \frac{\varepsilon_d \varepsilon_m}{\varepsilon_d + \varepsilon_m}, \quad k_d^2 = -k_0^2 \frac{\varepsilon_d^2}{\varepsilon_d + \varepsilon_m}, \quad \kappa_m^2 = -k_0^2 \frac{\varepsilon_m^2}{\varepsilon_d + \varepsilon_m}.$$  

These equations, together with the Drude expression for the metallic permittivity, $\varepsilon_m = 1 - \omega_p^2/\omega^2$, determines completely the frequency dependence of the wave vector of the surface plasmon.

The surface plasmon is TM polarized, with magnetic field parallel to the metal-dielectric interface. The intensity of magnetic and electric field is of the form

$$h = N_0(-\sin \theta, \cos \theta, 0) e^{i(k_xx + k_yy - \omega t)} \times \begin{cases} e^{-\kappa_d z} & z > 0 \\ e^{+\kappa_m z} & z < 0, \end{cases}$$

(5)
FIG. 1: Relation between two angles $\theta_b$ and $\theta_a$ for various frequencies $\omega$ of the surface plasmon and permittivity step $\varepsilon_b/\varepsilon_a = 5$. In the case of the transmission $a \rightarrow b$, the reflection angle $\theta_b$ is always larger than that for the plane wave. Consequently, in the case of the transmission $b \rightarrow a$ the critical angle $\theta_{c_P}$ is always smaller than the critical angle $\theta_c$ for plane waves.

and

$$ c = N_0 \frac{z_0}{k_0} e^{i(k_x + k_y y - \omega t)} \times \begin{cases} (+i\kappa_d \cos \theta, +i\kappa_d \sin \theta, -k_{\parallel}) e^{-\kappa_d z/\varepsilon_d} & z > 0 \\ (-i\kappa_m \cos \theta, -i\kappa_m \sin \theta, -k_{\parallel}) e^{+\kappa_m z/\varepsilon_m} & z < 0 \end{cases} $$

Here, $z_0 = \sqrt{\mu_0/\varepsilon_0}$ is the vacuum impedance, and $\theta$ determines the direction of propagation in the $xy$ plane: $\cos \theta = k_x/k_{\parallel}$, $\sin \theta = k_y/k_{\parallel}$. Normalization constant $N_0$ is specified in Appendix A.

A. Snell’s law for surface plasmon

In the scattering experiment, we consider the metal covered by two different dielectrics, $a$ and $b$, with permittivities $\varepsilon_a$ and $\varepsilon_b$. The interface between dielectrics lies in the $yz$ plane $x = 0$. From the continuity of the $y-$ component of the wave vector, $k_{ya} = k_{yb}$, we find the relation between the incident and the refractive angle,

$$ \frac{\sin \theta_b}{\sin \theta_a} = \sqrt{\frac{\varepsilon_a}{\varepsilon_b + \varepsilon_m}} \frac{\varepsilon_b + \varepsilon_m}{\varepsilon_a + \varepsilon_m}. $$

Figure II shows $\theta_b$ as a function of $\theta_a$ for various values of the plasmon frequency $\omega$. The most important consequence of the relation (7) is the existence of the critical angle $\theta_{c_P}$ for the surface plasmon incident from the media with higher permittivity. No transmission of surface plasmon is possible when the incident angle $\theta_b > \theta_{c_P}$.

III. TRANSMISSION AND REFLECTION COEFFICIENTS FOR THE SURFACE PLASMON

The transmission and reflection coefficients will be calculated from the requirement of the continuity of tangential components of both electric and magnetic fields at the interface $x = 0$ between two dielectrics $a$ and $b$,

$$ E_{za}(x \rightarrow 0^-) \equiv E_{zb}(x \rightarrow 0^+) \quad \text{and} \quad H_{ya}(x \rightarrow 0^-) \equiv H_{yb}(x \rightarrow 0^+). $$

Since the intensity of the surface plasmon decreases exponentially in the $z$ direction (Eqs. [6]), two plasmons on the opposite sides of the interface cannot satisfy the continuity relations (8) for all $z$. Therefore, we have to consider the full system of eigenwaves for the metal-dielectric interface. This system contains, besides the surface plasmon, an infinite number of plane waves. All plane wave have the same $y-$ component of the wave vector, but differ in the $z-$ component $k_z$. In our numerical analysis, we use $N$ plane waves with $k_{za} = (\alpha/N)k_{\text{max}}$ and $\alpha = 1, 2, \ldots, N$. The upper cutoff $k_{\text{max}}$ is specified in Appendix A.

With the use of the plane waves, we express the continuity equations for the electric and magnetic field given by Eq. (8) in the form

$$ \sum_{\alpha=0}^{N} (A_{\alpha} - \overline{A}_{\alpha}) E_{za\alpha} = \sum_{\alpha=0}^{N} (B_{\alpha} - \overline{B}_{\alpha}) E_{zb\alpha} $$

$$ \sum_{\alpha=0}^{N} (C_{\alpha} - \overline{C}_{\alpha}) H_{ya\alpha} = \sum_{\alpha=0}^{N} (D_{\alpha} - \overline{D}_{\alpha}) H_{yb\alpha} $$

where $A_{\alpha}$ and $B_{\alpha}$ are the coefficients of the plane waves $E_{za\alpha}$ and $E_{zb\alpha}$, respectively. The overline denotes the complex conjugate.
for the \( z \)-components of the electric field \( E_z \), and

\[
\sum_{\beta=0}^{N} (A_\beta + \overline{A}_\beta) H_{y\alpha\beta} = \sum_{\beta=0}^{N} (B_\beta + \overline{B}_\beta) H_{y\beta\alpha} \tag{10}
\]

for the \( y \) components of the magnetic field \( H_y \). Fields \( E_z \) and \( H_y \) are given in the Appendix A. Vectors \( A_\alpha = A(k_{\alpha\alpha}) \) and \( B_\beta = B(k_{\beta\beta}) \), \( \alpha, \beta = 0, 1, \ldots, N \) contain amplitudes of the surface plasmon \( (\alpha, \beta = 0) \) and \( N \) plane waves in the left and right media, respectively. \( A, B (\overline{A}, \overline{B}) \) represent fields propagating to the right and to the left, respectively.

All amplitudes can be calculated from the requirement that Eq. (3) must be fulfilled for any \( \varepsilon_{A,\beta} \). Another approach, which uses the coupling coefficients between electric and magnetic fields, suggested in Ref. 11, enables us to formulate the problem in terms of the \( 2(N+1) \times 2(N+1) \) scattering matrix \( S \)

\[
\begin{pmatrix}
B \\
A
\end{pmatrix} =
\begin{pmatrix}
S^{bb} & S^{ba} \\
S^{ab} & S^{aa}
\end{pmatrix}
\begin{pmatrix}
\overline{B} \\
A
\end{pmatrix},
\tag{11}
\]

which relates the amplitudes of the incoming waves \( A \) and \( \overline{B} \) with the outgoing waves \( \overline{A} \) and \( B \). Details of the calculation are given in Appendix A.

In the next Section, we analyze the case when the only incident wave is the surface plasmon propagating in media \( a \). Then \( A_\alpha = \delta_{\alpha a} \) and \( \overline{B} \equiv 0 \). From Eq. (11) we obtain the transmission and reflection coefficients for the surface plasmon,

\[
T_{a\rightarrow b} = |S^{ba}_{00}|^2 \quad \text{and} \quad R_{a\rightarrow a} = |S^{aa}_{00}|^2.
\tag{12}
\]

The components \( S^{aa}_{\alpha\alpha} \) and \( S^{ba}_{\beta\beta} \) determine radiation losses due to the scattering of the surface plasmon. Among all plane waves, only those with real \( k_x \) radiate the energy in the \( x \) direction. Since \( k_{ax\alpha}^2 = k_0^2 \varepsilon_a - k_y^2 - k_{az\alpha}^2 \), we have that \( k_{az\alpha} \) is real only for \( \alpha \) smaller than certain integer \( n_a \). Similarly, \( k_{bx\beta} \) is real only when \( \beta < n_b \). Total radiation losses are therefore obtained as \( S_a = S_{aa} + S_{ba} \), where

\[
S_{aa} = \sum_{\alpha}^{n_a} |S^{aa}_{\alpha\alpha}|^2 \quad \text{and} \quad S_{ba} = \sum_{\beta}^{n_b} |S^{ba}_{\beta\beta}|^2.
\tag{13}
\]

The conservation of the energy requires

\[
T_{a\rightarrow b} + R_{a\rightarrow a} + S_a = 1.
\tag{14}
\]

Physical meaning of other components of the scattering matrix is obvious. For instance, the element \( S^{ab}_{0\beta} \) gives the amplitude of a surface plasmon, excited in the media \( a \) by a plane wave \( \beta \) incident to the interface from the media \( b \).

### A. Normal incidence

Figures 2-4 show scattering parameters of the surface plasmon for the case of normal incidence. In numerical calculations, we use \( k_{\text{max}} \) given by Eq. (A3) and number of plane waves varies between \( N = 100 \) and \( N = 1577 \).

The transmission and reflection coefficients as well as radiative losses are given in Fig. 2 for various values of the permittivity steps \( \varepsilon_b/\varepsilon_a \). Our data agree with results of Ref. 12 and confirm that a significant part of the plasmon energy is radiated by plane waves. Radiation losses increase when the permittivity step increases. On the other hand, scattering coefficients depend only weakly on the plasmon frequency (data not shown).

As the test of numerical accuracy of the method, we used amplitudes \( \overrightarrow{A} \) and \( B \), obtained from the scattering matrix, and reconstruct the electric and magnetic fields on both sides of the \( x = 0 \) plane for the permittivity step \( \varepsilon_b/\varepsilon_a = 5 \). Figure 3 confirms that the tangential components of \( E_z \) and \( H_y \) are indeed continuous at the interface.

In Fig. 4 we present the amplitudes of radiated plane waves, \( |S^{ab}_{0\beta}|^2 \) for the plasmon incident from the media \( a \) and \( b \). The data confirm that the energy is mostly radiated in the direction of incoming plasmon. As shown in the right figure, the radiation possesses the sharp maximum in the direction of the critical angle for planar waves \( \theta_c \).
The surface plasmon is coming from the dielectric \( a \) (left) and from the dielectric \( b \) (right).

**B. Oblique incident angle**

Figures 5 and 6 show how the scattering coefficients depend on the incident angle. The plasmon is approaching the permittivity step either from the left or from the right side of the interface.

For the scattering from the side with lower permittivity, \( \varepsilon_a < \varepsilon_b \), all coefficients depend monotonously on the incident angle. More interesting is the case when the plasmon approaches the interface from the side with higher permittivity \( \varepsilon_b \). The transmission, \( T_{b\rightarrow a} \) decreases to zero when \( \theta \rightarrow \theta_{cP} \), but the reflection \( R \) does not increase to the unity. We explain this behavior by the presence of “evanescent plasmon” in the media \( a \). Although the \( x- \) component of the plasmon wave vector \( k_{ax} \) is imaginary, the intensity of the field on the left side of the interface is non-zero (even larger than for smaller incident angles). This field must be compensated by plane waves which radiate energy.

For higher permittivity contrast \( \varepsilon_b/\varepsilon_a \), we found that the reflection even decreases when the incident angle increases above the critical angle \( \theta_{cP} \). This decrease is accompanied by higher radiation losses. As shown in Fig. 6, entire plasmon energy can be radiated when \( \theta > \theta_{cP} \). Radiation losses have a maximum for the incident angle larger than the critical angle. The “total reflection” (\( R = 1 \)) takes place only for angles much larger than the critical angle for the surface plasmon.

**FIG. 3:** Test of the continuity of electric and magnetic field along the interface \( x = 0 \). Solid lines and symbols represent fields for \( x \rightarrow 0^- \) and \( x \rightarrow 0^+ \), respectively. Left figure show real part of both \( E \) and \( H \), and two other figures show imaginary parts of fields. Dielectric permittivities are \( \varepsilon_a = 1 \), \( \varepsilon_b = 5 \). \( N = 1577 \) plane waves were used with with maximal \( z- \) component of wave vector given by Eq. (A3). The maximal value of \( z \) is given as \( z_{\text{max}} = 32/k_{dx} \).
FIG. 4: Angle distribution of radiative losses $E(k_z)$ (in arbitrary units) given by scattering matrix elements $S_{ab}^{\omega_0}$ for the scattering of the surface plasmon with the frequency $\omega = 0.23\omega_p$ scattered at the interface between two dielectrics with permittivities $\varepsilon_a = 1$, $\varepsilon_b = 10$. $k_{0a} = k_0\sqrt{\varepsilon_a}$ and $k_{0b} = k_0\sqrt{\varepsilon_b}$. Left (right) figure shows radiation in the medium a (b), respectively. A and B determines the medium from which plasmon is coming. Sharp maximum in the right figure shows the scattering in the direction of the critical angle $\theta_c$ for plane waves.

IV. CONCLUSION

We analyzed quantitatively the scattering of the surface plasmon at the planar interface between two dielectrics which cover the metallic surface. The transmission, reflection coefficients and radiative losses were calculated for the normal and oblique incident angle. We confirm that the radiation of plane waves causes significant scattering losses: for normal incidence, the transmission through the single interface might cost 20-40% of the plasmon energy. The reduction of these losses represent the challenging problem for the theoretical research. One possible way how to avoid this problem is to cover the metallic surface by anisotropic metamaterial instead of a dielectric.

We analyzed how the transmission and reflection coefficients depend on the incident angle. While the transmission to the dielectrics with $\varepsilon_b > \varepsilon_a$ brings no surprising result, the transmission in the opposite direction exhibits non-monotonous dependence on the incident angle. The transmission coefficient decreases to zero when the incident angle increases to the critical angle $\theta_{cP}$ for the surface plasmon. However, the reflection does not reach unity for the critical angle, because the significant part of the energy is radiated.

In the present analysis, we used real (lossless) metallic permittivity. This is consistent with the formulation of the scattering experiment, in which the incident wave is coming from the infinity. Nevertheless, we verified that realistic...
losses, given by small imaginary part of the metallic permittivity in the Drude formula, do not influences the scattering coefficients.

**Appendix A: The method**

Since the single surface plasmon cannot satisfy the continuity relations, along the dielectric interface, a complete set of plane waves must be included into the scattering procedure. Each plane wave is given by a superposition of the wave incident to \((e^{-i \mathbf{k}_{dz}})\), reflected from \((re^{i \mathbf{k}_{dz}})\) and transmitted through the metal-dielectric interface \((te^{-i \mathbf{k}_{zm}})\). Here \(k_{dz}\) and \(k_{zm}\) are the \(z\)-components of the wave vector in dielectric and metal, respectively. The reflection amplitude \(r\) for the metal-dielectric interface is given by

\[
r = \frac{\varepsilon dk_{zm} - \varepsilon mk_{dz}}{\varepsilon dk_{zm} + \varepsilon mk_{dz}} \tag{A1}
\]

and \(t = 1 - r\). In numerical calculation, we consider \(N\) plane waves with different values of the \(z\)-component of the wave vector

\[
k_{dz\alpha} = \frac{k_{max}}{N} \times \alpha, \quad \alpha = 1, 2, \ldots, N. \tag{A2}
\]

Here, \(k_{max}\) is the largest allowed value of \(k_z\). We choose

\[
k_{max} = k_0 \sqrt{\varepsilon_d - \varepsilon_m} \tag{A3}
\]

where \(\varepsilon_d = \min(\varepsilon_a, \varepsilon_b)\). This choice guarantees that all waves transmitted from the dielectric to the metal decreases exponentially, so that no plane wave propagates inside the metal and the \(z\)-component of the wave vector in the metal,

\[
k_{zm} = \sqrt{k_{dz}^2 - k_{max}^2}, \tag{A4}
\]

is imaginary.

In what follows we need explicit form of the plane wave for the interface metal-dielectric \(a\) the fields \(E_z\) and \(H_y\). Neglecting the phase factor \(\exp[i(k_x x + k_y y - \omega t)]\), we have

\[
H_{yao}(\vec{r}) = N_{ao} \frac{k_{azo}}{k_0} \times \begin{cases} \left[ e^{-ik_{azo} z} + r_{azo} e^{ik_{azo} z} \right] & z > 0 \\ \left[ -t_{azo} e^{-ik_{azo} z} \right] & z < 0 \end{cases}, \tag{A5}
\]

and

\[
E_{zao}(\vec{r}) = -N_{ao} k_\parallel \frac{2 \alpha}{k_0} \times \begin{cases} \left[ e^{-ik_{azo} z} + r_{azo} e^{ik_{azo} z} \right] / \varepsilon_d & z > 0 \\ \left[ -t_{azo} e^{-ik_{azo} z} \right] / \varepsilon_m & z < 0 \end{cases}. \tag{A6}
\]
The requirement $C_{\alpha\beta} = \delta_{\alpha\beta}$ determines the norm $N_{aa}$:

$$N_{aa} = i \sqrt{\frac{k_0 k_{max}}{\varepsilon_0}} \frac{\varepsilon_a}{2\pi N r_{aa} k_{axa}}.$$  

The off-diagonal elements read

$$C_{\alpha\beta}^{ab} = -i N_{ab} N_{\beta\alpha} \frac{z_0 k_{a\parallel} k_{b\parallel}}{k_0 k_{b\parallel}} \frac{(1 - r_a)(1 - r_b)}{\varepsilon_m(k_m^{z\beta} - k_m^{z\alpha})} \left[ k_m^{z\beta} - \frac{\varepsilon_a}{\varepsilon_b} k_m^{z\alpha} - (k_m^{z\beta} - k_m^{z\alpha}) \right] \quad (\alpha, \beta > 0). \quad (A10)$$

Similarly, diagonal elements for two plasmons,

$$C_{00}^{ab} = \int_{-\infty}^{+\infty} dz e_{za} H_{yb} = N_{ab} N_{\beta\alpha} \frac{z_0 k_{a\parallel}}{k_0 k_{b\parallel}} k_{bx} \left[ \frac{1}{\varepsilon_a} \left( \frac{1}{\varepsilon_m \kappa_{ad} + \kappa_{bd}} + \frac{1}{\varepsilon_m \kappa_{am} + \kappa_{bm}} \right) \right], \quad (A11)$$

determines the normalization constant for the surface plasmon,

$$N_{a0} = \sqrt{\frac{k_0 \kappa_{da}}{z_0 k_{ax} \varepsilon_m^2 - \varepsilon_a^2}}.$$  

(A12)

Using the form of the electric and magnetic field of the surface plasmon, we obtain the coupling coefficients between the surface plasmon and the plane wave in the form

$$C_{0\beta}^{ab} = \int_{-\infty}^{+\infty} dz e_{za} H_{yb} = N_{ab} N_{\beta\alpha} \frac{z_0 k_{a\parallel}}{k_0 k_{b\parallel}} k_{bx} \left\{ \frac{1}{\varepsilon_a} \left[ \frac{r_{b\beta}}{\kappa_{ad} - i k_{b\beta}} - \frac{1}{\kappa_{ad} + i k_{b\beta}} \right] + \frac{1}{\varepsilon_m} \frac{r_{b\beta} - 1}{\kappa_{am} - i k_{m\beta}} \right\}. \quad (A13)$$

Finally, we obtain two sets of $N + 1$ linear equations for unknown amplitudes $A$, $B$, $\mathbf{A}$ and $\mathbf{B}$:

$$A - \mathbf{A} = C^T(B - \mathbf{B}), \quad B + \mathbf{B} = C \left( A + \mathbf{A} \right), \quad (A14)$$

which can be rearranged into the form

$$(B \ A) = \left( \begin{array}{cc} S_{bb} & S_{ba} \\ S_{ab} & S_{aa} \end{array} \right) \left( \begin{array}{c} \mathbf{B} \\ \mathbf{A} \end{array} \right),$$

(A15)

with

$$S_{bb} = -(1 + C C^T)^{-1}(1 - C C^T) \quad S_{ba} = (1 + C C^T)^{-1}2C$$

$$S_{ab} = (1 + C^T C)^{-1}2C \quad S_{aa} = (1 + C^T C)^{-1}(1 - C^T C).$$  

(A16)
Acknowledgments

This work was supported by grant APVV project No. 51-003505 and VEGA project No. 0633/09.

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