Monitoring of dynamical phase-space trajectories with sub-Heisenberg indeterminacy

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The Heisenberg uncertainty relation describes the fact that the phase-space trajectory of a single quantum system cannot be precisely determined with respect to semi-classical reference values to better than twice the quantum system’s ground state uncertainty. Einstein, Podolsky and Rosen pointed out that according to quantum theory, however, there are pairs of quantum systems whose properties with respect to each other can be arbitrarily precisely determined. Here we report the experimental proof that even the dynamics of a quantum system, i.e. its phase-space trajectory, can be precisely determined with respect to another quantum system. We present measurements with a remaining indeterminacy of canonical conjugate variables ten times smaller than the lowest possible for a semi-classical reference. Our result may trigger research on the foundations of quantum mechanics and supports quantum technology for entanglement-enhanced metrology and secure communication.

INTRODUCTION

The Heisenberg uncertainty relation is one of the most distinctive features of quantum mechanics. The idea of a characteristic ‘uncertainty’ that sets a lower bound to the precision of simultaneous position and momentum measurements performed on the same physical system was introduced by W. Heisenberg in an article in 1927, in which he concludes, “the more precisely the position is determined, the less precisely the momentum is known, and vise versa” [1]. Shortly after, the mathematical foundation of the Heisenberg uncertainty relation was given on the grounds of quantum theory by E. H. Kennard [2], H. Weyl [3], and H. P. Robertson [4]. Today, it is widely accepted that two canonical conjugate quantities of a physical system cannot be precisely determined simultaneously with respect to semi-classical references, i.e. measurement devices. It is less well known, however, that two such quantities can be precisely determined with respect to another quantum system, and that this property indeed allows for arbitrarily precise measurement values simultaneously performed on conjugate quantities. This was correctly pointed out by A. Einstein, B. Podolsky and N. Rosen (EPR) in a gedanken experiment in 1935, which was in fact intended to support their incorrect conclusion quantum theory be incomplete [5]. Two (sub-)systems having conjugate quantities that are simultaneously precisely determined with respect to each other are called ‘EPR entangled states’ [6–8]. Various types of well-engineered quantum experiments took advantage of such EPR-entangled states, ranging from quantum teleportation [9–12] to high-precision quantum measurements [13–15].

Here, we show for the first time a dynamical phase-space trajectory measured with sub-Heisenberg indeterminacy. The observed effect is strong. We are able to track the dynamics of a (sub-)system with a precision ten-times higher than possible for any quantum mechanical system without quantum correlations. Our experiment uses optical, continuous-variable EPR-entangled states with Gaussian quantum statistics, and achieves continuous and unconditional monitoring of the phase-space trajectory.

We consider quantities that are used in optical communication, namely phase and amplitude modulation depths carried by quasi-monochromatic laser light, see Fig. 1. The depth (modulation index) of the amplitude modulations in the frequency band $f \pm \Delta f/2$ is quantified by the dimensionless operator $\hat{X}_{f,\Delta f}$ [16]. This opera-
Uncertainty Relation, as pointed out by E. Schrödinger [14]. This operator is also known as ‘amplitude quadrature amplitude’. The corresponding depth of phase modulations is (in the limit of weak phase modulations) quantified by the operator $\hat{Y}_{f,\Delta f}$. This operator is also known as ‘phase quadrature amplitude’. $\hat{X}_{f,\Delta f}$ and $\hat{Y}_{f,\Delta f}$ do not commute ($[\hat{X}_{f,\Delta f}, \hat{Y}_{f,\Delta f}] = i/2$), i.e. there is a Heisenberg-uncertainty-relation describing the fact that there are no simultaneously and precisely determined amplitude and phase quadrature excitations for the same modulation frequency $f$

$$\Delta \hat{X}_{f,\Delta f} \Delta \hat{Y}_{f,\Delta f} \geq 1/4. \tag{1}$$

Here $\Delta$ is the standard deviation of the eigenvalues of the respective operator. $\hat{X}_{f,\Delta f}$ and $\hat{Y}_{f,\Delta f}$ span a so-called phase-space, in which the area of uncertainty is bounded from below according to Eq. (1).

The lower bound in Eq. (1) can experimentally be achieved, if an ensemble of identical systems in the same pure states is available and measurements of $\hat{X}_{f,\Delta f}$ and $\hat{Y}_{f,\Delta f}$ are ‘ideal’, i.e. sequentially performed without splitting the states. To monitor the phase and amplitude quadrature in order to track changes of the modulations, it is necessary to perform simultaneous measurements at times $t_i$. The physical system needs to be divided in two halves, on which the conjugate observables are simultaneously measured, respectively, utilizing two independent measurement devices. The splitting reduces the signal-to-noise ratio. The splitting can be described as opening a new port through which another unit of vacuum uncertainty enters the detection. Tracking the time evolution of a modulation with simultaneous measurements needs to cope with doubled quantum uncertainties [17], which increases standard deviations by the factor $\sqrt{2}$, yielding

$$\Delta(\hat{X}_{f,\Delta f}(t_i)) \Delta(\hat{Y}_{f,\Delta f}(t_i)) \geq 1/2. \tag{2}$$

The above inequality represents the fundamental precision limit when amplitude and phase modulations of a beam of light are measured simultaneously with respect to reference values of a semi-classical measurement device. Similar inequalities limit the simultaneous measurement of position and momentum of a particle. In the following we omit subscript $(f, \Delta f)$.

In the first instance, it may come at a surprise that quantum uncertainties in sensing of phase-space displacements can in principle be fully avoided. This puzzled A. Einstein, B. Podolsky, and N. Rosen in 1935 [15] and led them to the wrong conclusion quantum theory be incomplete. Indeed, it is a well-known fact that the commutator of a difference and sum of non-commuting observables of two quantum systems $A$ and $B$ is zero ($[\hat{X}_A \pm \hat{X}_B, \hat{Y}_A \mp \hat{Y}_B] = 0$), from which follows that such a sum and difference (or vice versa) are simultaneously determined precisely without a limitation by a Heisenberg-Uncertainty-Relation, as pointed out by E. Schrödinger [15]. To employ the lack of indeterminism of phase-space sensing, however, requires entanglement between $A$ and $B$ [18]. This was theoretically reformulated in the framework of quantum estimation theory in [19] and used for the measurement of a stationary interferometer signal in [14].

Since positions of two quantum systems (here $\hat{X}_A$ and $\hat{X}_B$) as well as their momenta (here $\hat{Y}_A$ and $\hat{Y}_B$) can exist precisely determined with respect to each other at any instance of time $t_i \pm \Delta t/2$ simultaneously, it is possible to continuously monitor (track) the dynamics of system $A$ in the phase-space spanned by $X(t_i)$ and $Y(t_i)$ without any quantum noise if the two measurements are
performed relative to the dynamics of system $B$. If the quantum uncertainties of $A$ and $B$ are entangled in a stationary way, they cancel out, at least partly if the entanglement is not maximal.

Fig. 2 shows the schematic of our experiment. A commercial erbium-doped fibre laser provided 1 W of quasi-monochromatic light at the wavelength of 1550 nm. About half of the light was frequency doubled to generate the pump light for two squeezed light resonators. The latter used resonator-enhanced degenerate type 0 optical-parametric amplification (OPA) in periodically poled potassium titanyl phosphate (PPKTP). The two squeezed fields carried a broadband sideband spectrum of modulations including 5 MHz in squeezed vacuum states. A detailed description of the electro-optical phase control of the overlapped squeezed fields and the stable production of strong stationary entanglement in terms of 10 dB of two-mode squeezing is given in [8]. The balanced homodyne detectors (BHDs) used optical local oscillators (LO’s) of about 10 mW at 1550 nm from the same commercial fibre laser. The LO’s, as parts of the BHD’s, were overlapped with the measurement field on balanced beam splitters with a stably controlled relative phase of either zero or ninety degrees. At zero degrees, values of the amplitude quadrature amplitude were measured. At ninety degrees, values of the phase quadrature amplitude were measured. The voltages from the BHD’s were recorded via a data acquisition card (DAQ) with a sampling frequency of 200 MHz for each channel. We avoided aliasing by using an analog lowpass-filter with a corner frequency of $f_{-3dB} = 50$ MHz in each channel. Post processing was done with a self-written python script, which was used to digitally demodulate the data of the signal frequency at 5 MHz and subsequent finite impulse response (FIR)-lowpass-filtering with a cut off frequency of 1 kHz. To avoid correlation in the data set after lowpass-filtering, every second thousandth data point was used.

Two continuous-wave fields $A$ and $B$ that carried entangled quantum noise of the modulations at frequency of $f \pm \Delta f/2 = 5$ MHz $\pm 50$ kHz were produced from squeezed vacuum states. One of the entangled fields was overlapped at a beam splitter of reflectivity $R = 99.99\%$ with another field that carried a displaced coherent modulation state, i.e. a classical modulation at 5 MHz. The signal that we tracked in phase-space was the 0.01% transmitted from this semi-classical resource. The depth of the (generic) modulation signal $\langle X_A \cos \phi + Y_A \sin \phi \rangle$ was continuously varied in time by varying the voltage to the electro-optical modulator (EOM) shown in Fig. 2. The type of modulation (amplitude or phase quadrature modulation) was continuously varied in time by varying the relative phase angle $\phi$ at which the fields combined at the 99.99% beam splitter. This was achieved via the voltage applied to the piezo-electric actuator (PZT) that changed the position of a steering mirror. Using a rather high reflectivity beam splitter minimized the optical loss, FIG. 3. Measured trajectories with sub-Heisenberg indeterminacy. The rather narrow (green) measuring points represent example phase-space trajectories of system A measured with respect to a quantum reference. The small dots were measured at subsequent times $t_i$ with $t_{i+1} - t_i = 5$ ns. The two coordinates $X_A(t_i)$ and $Y_A(t_i)$ of every dot were measured simultaneously. Corresponding measurements on system A’s ground state are shown in the centre for comparison. The trajectory in 3(a) started at a phase of about $\phi_0 = -130^\circ$ and rotated clockwise to about $220^\circ$ with a constant amplitude and back to a phase of about $\phi = -30^\circ$. Thereby the modulation type changed twice across the period from a pure phase to a pure amplitude modulation. In 3(b) the trajectory showed a continuously decreasing quadrature amplitude. Both trajectories showed a product $\Delta X_A \Delta Y_A$ a factor of ten smaller than the product for the ground state, which represents the lower bound in Eq. 2. Our setup is able to track arbitrary, including random, phase-space trajectories with strong reduction of the quantum noise. Other noise, such as electronic noise from detectors was negligible.
i.e. the decoherence on the entanglement. In front of the optical component BS1, two stationary entangled systems $A$ and $B$ existed. System $A$’s expectation value $\langle X_A \cos \phi + Y_A \sin \phi \rangle$ was time dependent. System $B$’s expectation value was zero for all phases at all times.

To simultaneously track displacement values that correspond to those of $X_A(t_i)$ and $Y_A(t_i)$, system $A$ was split on a balanced beam splitter (BS1). To enable measurements with respect to system $B$, the latter was overlapped at the same beam splitter with system $A$. On one joint output, a balanced homodyne detector (BHD) continuously measured the eigenvalue $\hat{X}_A(t_i) - \hat{X}_B(t_i)/\sqrt{2}$. On the second output, a second BHD continuously measured the eigenvalue $\hat{Y}_A(t_i) + \hat{Y}_B(t_i)/\sqrt{2}$. We call these eigenvalues $\hat{X}_A(t_i)$ and $\hat{Y}_A(t_i)$, respectively. This is justified because any measurement implies a relative measurements to some reference (semi-classical or quantum), which is in our case system $B$. We chose the BHD read out phases in such a way that both time series showed squeezed (reduced) quantum noise. Combining the data at times $t_i$ resulted in highly resolved phase-space trajectories as shown in Fig. 3. Plotted are individual measuring points $(X_A(t_i); Y_A(t_i))$ taken every 5 ns with respect to our quantum reference system $B$. The dynamics of the signal was slow compared to the acquisition rate. The standard deviations in $\hat{X}_A(t_i)$ and $\hat{Y}_A(t_i)$ were reduced by more than $\sqrt{10}$. The time dependence of $\langle X_A \cos \phi + Y_A \sin \phi \rangle(t)$ was thus tracked with an uncertainty beyond the Heisenberg-Uncertainty-Relation. System $B$ did not have any influence on the measured expectation value since $\langle X_B \cos \phi + Y_B \sin \phi \rangle = 0$ for all phases and times. Inequality [2] was violated by more than a factor of 10, as shown in Fig. 3(a) and (b). Inequality [1] was violated by a factor of 5. The factor by which Heisenberg’s uncertainty limit was surpassed directly corresponded to the strength of the entanglement. Our entanglement source is based on two squeezed light sources and described in detail in [8]. Increasing the entanglement strength requires further reduction of optical loss, including further increase of photo detection efficiency.

Figure 3(a) shows a trajectory with constant amplitude but with phase changed from about $\phi_0 = -130^\circ$ to about $220^\circ$ and back to about $-30^\circ$. The region of less density is due to higher speed of the trajectory. To demonstrate the stability of our setup we collected data over fifteen cycles and got stable reduction of the uncertainty product by about 0.1. The trajectory is compared with data in the centre, which is taken from system $A$ without displacement and being in the ground state, i.e. without entanglement. The port for system $B$ was empty in this case, i.e. the corresponding mode also in its ground state. The data in the centre correspond to the minimal quantum uncertainty in inequality [2], i.e. the ground state reference of a semi-classical measurement device. In Fig. 3(b), we additionally varied the modulation index, resulting in a phase and amplitude dependent trajectory. The examples show that the setup is able to track arbitrary trajectories including random walks with the same reduction in quantum uncertainty.

In conclusion, we proved the principle that a dynamical quantum trajectory in phase-space can be monitored with a precision strongly surpassing Heisenberg’s uncertainty relation. We reduced the product of the standard deviations of the indices of amplitude and phase modulations of a beam of light by the factor of ten, which certainly is of practical significance and thus supports the emergent field of quantum sensing. On the fundamental side, our work demonstrates that the often quoted interpretation of Heisenberg’s uncertainty relation “two non-commuting observables of a quantum system are not simultaneously determined with arbitrary precision” has no relevance to the limitation of the accuracy of displacements measurements. A displacement, that is associated to the measured observables, can be determined simultaneous in a single measurement with arbitrary precision. Related to that a more precise statement is: ‘Heisenberg’s uncertainty relation sets a lower bound to the indeterminacy of two observables with respect to semi-classical reference values.’ In the case system $A$ is maximally entangled with system $B$, the precision in every pair of simultaneous measurements at any subsequent time $t_i$ is unlimited, providing a perfectly determined phase-space trajectory as in classical physics. In practice, limitation occurs due to decoherence in terms of photon loss, which reduces the strength of the entanglement.

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[1] W. Heisenberg, “Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik,” Zeitschrift für Physik, vol. 43, pp. 172–198, mar 1927.
[2] E. H. Kennard, “Zur Quantenmechanik einfacher Bewegungstypen,” Zeitschrift für Physik, vol. 44, pp. 326–352, apr 1927.
[3] H. Weyl, “Quantenmechanik und Gruppentheorie,” Zeitschrift für Physik, vol. 46, pp. 1–46, nov 1927.
[4] H. P. Robertson, “The Uncertainty Principle,” Physical Review, vol. 34, pp. 163–164, jul 1929.
[5] A. Einstein, B. Podolsky, and N. Rosen, “Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?,” Physical Review, vol. 47, pp. 777–780, may 1935.
[6] M. Reid, “Demonstration of the Einstein-Podolsky-Rosen paradox using nondegenerate parametric amplification,” Physical Review A, vol. 40, pp. 913–923, jul 1989.
[7] Z. Y. Ou, S. F. Pereira, H. J. Kimble, and K. C. Peng, “Realization of the Einstein-Podolsky-Rosen paradox for continuous variables,” Physical Review Letters, vol. 68,
[8] T. Eberle, V. Händchen, and R. Schnabel, “Stable Control of 10 dB Two-Mode Squeezed Vacuum States of Light,” *Optics Express*, 2013.

[9] D. Bouwmeester, J.-W. J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, H. Weinfurter, and A. Zeilinger, “Experimental quantum teleportation,” *Nature*, vol. 390, p. 575, 1997.

[10] A. Furusawa, J. L. Sorensen, S. L. Braunstein, C. A. Fuchs, H. J. Kimble, and E. S. Polzik, “Unconditional quantum teleportation,” *Science*, vol. 282, pp. 706–9, oct 1998.

[11] W. P. Bowen, N. Treps, B. C. Buchler, R. Schnabel, T. C. Ralph, H.-A. Bachor, T. Symul, and P. K. Lam, “Experimental investigation of continuous-variable quantum teleportation,” *Physical Review A*, vol. 67, p. 032302, mar 2003.

[12] W. Bowen, N. Treps, B. Buchler, R. Schnabel, T. Ralph, and T. Symul, “Unity gain and nonunity gain quantum teleportation,” *IEEE Journal of Selected Topics in Quantum Electronics*, vol. 9, pp. 1519–1532, nov 2003.

[13] M. Ozawa, “Universally Valid Reformulation of the Heisenberg Uncertainty Principle on Noise and Disturbance in Measurements,” *Physical Review A*, vol. 67, 07 2002.

[14] S. Steinlechner, J. Bauchrowitz, M. Meinders, H. Müller-Ebhardt, K. Danzmann, and R. Schnabel, “Quantum-dense metrology,” *Nature Photonics*, vol. 7, pp. 626–630, jun 2013.

[15] E. S. Polzik and K. Hammerer, “Trajectories without quantum uncertainties,” *Annalen der Physik*, vol. 527, pp. A15–A20, jan 2015.

[16] R. Schnabel, “Squeezed states of light and their applications in laser interferometers,” *Physics Reports*, vol. 684, pp. 1–51, apr 2017.

[17] E. Arthurs and J. L. Kelly, “On the Simultaneous Measurement of a Pair of Conjugate Observables,” *Bell System Technical Journal*, vol. 44, pp. 725–729, apr 1965.

[18] E. Schrödinger, “Die gegenwärtige Situation in der Quantenmechanik,” *Die Naturwissenschaften*, vol. 23, pp. 844–849, dec 1935.

[19] G. M. D’Ariano, P. Lo Presti, and M. G. A. Paris, “Using Entanglement Improves the Precision of Quantum Measurements,” *Physical Review Letters*, vol. 87, p. 270404, dec 2001.