Forecasts on dark energy from the X-ray cluster survey with eROSITA: constraints from counts and clustering

Annalisa Pillepich1,2,3*, Thomas H. Reiprich4, Cristiano Porciani4, Katharina Borm4, and Andrea Merloni5

1Max-Planck-Institut für Astronomie, Königstuhl 17, 69117 Heidelberg, Germany
2Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138
3Department of Astronomy and Astrophysics, University of California, Santa Cruz, CA 95064, USA
4Argelander-Institut für Astronomie, Auf dem Hügel 71, D-53121 Bonn, Germany
5Max-Planck-Institut für Extraterrestrische Physik, Postfach 1312, 85741 Garching bei München, Germany

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ABSTRACT
We forecast the potential of the forthcoming X-ray galaxy-cluster survey with eROSITA to constrain dark-energy models. We focus on spatially-flat cosmological scenarios with either constant or time-dependent dark-energy equation-of-state parameters. Fisher information is extracted from the number density and spatial clustering of a photon-count-limited sample of clusters of galaxies up to \( z \sim 2 \). We consider different scenarios for the availability of (i) X-ray follow-up observations, (ii) photometric and spectroscopic redshifts, and (iii) accurate knowledge of the observable – mass relation down to the scale of galaxy groups. With about 125,000 clusters (detected with more than 50 photons and with mass \( M_{500c} \gtrsim 10^{13} h^{-1} M_\odot \)) from an average all-sky exposure of 1.6 ks, eROSITA will give marginalized, one-dimensional, 1σ errors of \( \Delta w = 0.006 \) (1% per cent), \( \Delta \Omega_m = 0.006 \) (2.2 per cent), \( \Delta \omega_k = 0.07 \) (7 per cent), and \( \Delta w_a = 0.25 \) (optimistic scenario) in combination with (and largely improving upon) current constraints from various cosmological probes (cosmic microwave background, BAOs, Type Ia SNe). Our findings correspond to a dark-energy figure of merit in the range of \( 1.6 \) (after the four years of all-sky survey), making eROSITA one of the first Stage IV experiments to come on line according to the classification of the Dark Energy Task Force. To secure improved mass calibrations and to include high-redshift clusters (\( z \gtrsim 0.5 \)) as well as objects at the group-mass scale (\( M_{500c} \lesssim 5 \times 10^{13} h^{-1} M_\odot \)) will be vital to reach such accuracies.

Key words: cosmology: cosmological parameters, large-scale structure, early Universe – galaxies: clusters – X-rays: galaxies: clusters.

1 INTRODUCTION

Dark energy (DE) is invoked to explain the late-time accelerated expansion of the Universe (Riess et al. 1998; Perlmutter et al. 1999). In the last decade, ambitious observational programs have been designed with the ultimate goal of determining its physical properties. Experiments are generally used to constrain a number of phenomenological parameters that characterize dark energy and differentiate it from the cosmological constant. This is done notwithstanding the caveat that many DE parameterizations, however, cannot easily be associated with physical dark-energy models (Scherrer 2015). Yet, different parameterizations of DE leave different signatures in the time evolution of the expansion rate, the growth rate of structure, and the angular-diameter distance vs. redshift relation (e.g. Albrecht et al. 2006), which, in turn, can be inferred via the measurement of galaxies and galaxy clusters statistics (see e.g. the reviews by Frieman et al. 2008; Allen et al. 2011; Huterer & Shafer 2017).

Samples of massive galaxy clusters for cosmological studies have been assembled in the last decade across wavelengths and redshift ranges, e.g. in the X-rays (e.g. REFLEX, NORAS, HIFLUGCS, 400d, MACS, XCS, XXL, X-CLASS), in the optical bands (e.g. maxBCG, RCS2, DES, PanSTARSS) and at submm wavelengths (e.g. with SPT, ACT, Planck), bearing counts from just a handful to a few thousands. Cosmological constraints on DE have been obtained from the evolution of their number density across redshift (aka cluster counts; Henry et al. 2009; Vikhlinin et al. 2009b; Mantz et al. 2010; Rozo et al. 2010; Tinker et al. 2012; Benson et al. 2013; Mantz et al. 2015; de Haan et al. 2016), from their spatial clustering properties (Schuecker et al. 2003a,b), from the measurement of their gas fractions...
We also investigate the impact of high-redshift clusters and unveil the cosmic and its implications on the X-ray scaling-relation parameters are briefly discussed in Sections 4 and 5. The main results of the paper are given in Section 6, where we present the constraints which will be obtained with our forecast is based and also examine the constraints that the forthcoming X-ray all-sky galaxy-cluster survey after four years of observations as well as those derived from half-sky cov- eries. The optimal choice of the redshift and photon-count bins and (b) augmenting availability and accuracy of redshift estimates for the constraints that the forthcoming X-ray all-sky galaxy-cluster survey after four years of observations as well as those derived from half-sky coverage only. We critically discuss our methodology and assumptions on the abundance and spatial clustering of its photon-count limited, all-sky, galaxy-cluster sample. Our calculations are therefore meticulously tailored around the characteristics of the eROSITA telescope and the actual way observations will be taken. Here we expand on the methods and results of Pillepich et al. (2012) where special attention was devoted to the possibility of detecting primordial non-Gaussianity and to simultaneously constraining cosmological and intra-cluster medium (ICM) parameters (the so-called, self-calibration technique). In what follows, we quantify the benefits of (a) including X-ray follow-up observations to tighten the available priors on the adopted observable-mass relation (luminosity–mass); (b) augmenting availability and accuracy of redshift estimates for the eROSITA-detected clusters (photometric vs spectroscopic redshifts); and (c) extending the sample to lower-mass objects yet at a fixed detection limit of 50 photons per cluster.

The aim of this paper is to provide the community with baseline expectations for the eROSITA constraints on DE, by focusing on the abundance and spatial clustering of its photon-count limited, all-sky, galaxy-cluster sample. Our calculations are therefore meticulously tailored around the characteristics of the eROSITA telescope and the actual way observations will be taken. Here we expand on the methods and results of Pillepich et al. (2012) where special attention was devoted to the possibility of detecting primordial non-Gaussianity and to simultaneously constraining cosmological and intra-cluster medium (ICM) parameters (the so-called, self-calibration technique). In what follows, we quantify the benefits of (a) including X-ray follow-up observations to tighten the available priors on the adopted observable-mass relation (luminosity–mass); (b) augmenting availability and accuracy of redshift estimates for the eROSITA-detected clusters (photometric vs spectroscopic redshifts); and (c) extending the sample to lower-mass objects yet at a fixed detection limit of 50 photons per cluster.

The paper is organized as follows. In Section 2, we introduce the cosmological models used in our study, while, in Section 3, we summarize our analysis setup. After giving some basic definitions, here, we present the statistical tools and observables upon which our forecast is based and also examine the eROSITA survey strategies. The optimal choice of the redshift and photon-count bins and the necessity of follow-up observations to improve the knowledge on the X-ray scaling-relation parameters are briefly discussed in Sections 4 and 5. The main results of the paper are given in Section 6, where we present the constraints which will be obtained with the eROSITA all-sky cluster survey after four years of observations (by itself and in combination with other leading cosmological probes). We also investigate the impact of high-redshift clusters and provide ‘early’ eROSITA constraints that will be available after one year of observations as well as those derived from half-sky coverage only. We critically discuss our methodology and assumptions in Section 7, where we also compare some of our Fisher forecasts with a full likelihood study. We conclude in Section 8.
Dark-energy forecasts with eROSITA

Table 1. Model parameters adopted in the analysis: fiducial values and priors used throughout unless otherwise stated. Values for the cosmology sector and X-ray luminosity–mass and temperature–mass relations are taken, respectively, from Komatsu et al. (2009) and Vikhlinin et al. (2009a). $\mathcal{N}(\sigma)$ represents the normal distribution with mean given by the fiducial value and variance $\sigma^2$; $U(x_1, x_2)$ denotes the uniform distribution with endpoints $x_1$ and $x_2$; and $\Delta_D(x)$ signifies that the parameter is kept fixed at the fiducial value $x$. Throughout the paper, we assume a flat cosmology, with $\Omega_{DE} = 1 - \Omega_m$; Gaussian initial conditions, $f_{NL} = 0$; and temperature–mass relation parameters by Vikhlinin et al. (2009), coupled to the luminosity–mass relation via a bivariate lognormal distribution with null correlation coefficient between luminosity and temperature (see Pillepich et al. (2012) for a discussion). Standard priors are taken from Riess et al. (2011) for the Hubble Parameter ($\Omega_m$); and temperature–mass relation parameters by Vikhlinin et al. (2009), coupled to the luminosity–mass relation via a bivariate lognormal distribution with null correlation coefficient between luminosity and temperature.

| Description | Fiducial Value | Adopted Priors |
|-------------|---------------|---------------|
| Cosmology   |               |               |
| $\sigma_8$  | Normalization of $P(k)$ | 0.817 | $U(-\infty, +\infty)$ |
| $\Omega_m$  | Total Matter Fraction | 0.279 | $U(-\infty, +\infty)$ |
| $n_s$       | Spectral index | 0.96 | $U(-\infty, +\infty)$ |
| $h$         | Hubble Constant | 0.701 | $\mathcal{N}(0.022)$ |
| $\Omega_b$  | Baryon Fraction | 0.0462 | see Cooke et al. (2014) |
| $\omega_0$  | DE Eos Parameter: present value | $-1$ | $U(-\infty, +\infty)$ |
| $\omega_a$  | DE Eos Parameter: rate of change | 0 | $U(-\infty, +\infty)$ |
| X-ray Scaling-Relations |
| $\alpha_{LM}$ | LM relation: Slope | 1.61 | $\mathcal{N}(0.14)$ |
| $\gamma_{LM}$ | LM relation: $z$-dependent Factor | 1.85 | $\mathcal{N}(0.42)$ |
| $\beta_{LM}$ | LM relation: Normalization | 101.483 | $\mathcal{N}(0.085)$ |
| $\sigma_{LM}$ | LM relation: Logarithmic Scatter | 0.396 | $\mathcal{N}(0.039)$ |
| $\alpha_{TM}$ | TM relation: Slope | 0.65 | $\Delta_D(0.65)$ |
| $\beta_{TM}$ | TM relation: Normalization | $3.02 \times 10^{14} M_{\odot} h^{-1}$ | $\Delta_D(3.02 \times 10^{14} M_{\odot} h^{-1})$ |
| $\sigma_{TM}$ | TM relation: Logarithmic Scatter | 0.119 | $\Delta_D(0.119)$ |

Table 2. Cosmological models studied in this paper and cosmological parameters adopted for the marginalization in addition to the four X-ray luminosity–mass scaling-relation parameters, that are always allowed to vary in the statistical analysis, yet with various prior strategies (see Table 1 and Section 5). Throughout the paper, we assume a flat cosmology, with $\Omega_{DE} = 1 - \Omega_m$, and external priors on $h$ and $\Omega_b$ described in Section 2.

| Model                  | Parameter Set (+ LM Sector) |
|------------------------|-------------------------------|
| Vanilla Model          | $\{\sigma_8, \Omega_m, n_s, h, \Omega_b\}$ |
| Constant DE            | $\omega_0$CDM $\{\sigma_8, \Omega_m, n_s, h, \Omega_b, \omega_0\}$ |
| Evolving Models        | $\omega$CDM $\{\sigma_8, \Omega_\Delta, n_s, h, \Omega_b, \omega_0, \omega_a\}$ |

We calculate galaxy-cluster number counts and power spectra by following the method introduced in Pillepich et al. (2012) and briefly summarized below. eROSITA galaxy clusters are selected in terms of the raw photon counts\(^1\) ($n$) that will be collected at the detectors. To do so, we follow a series of steps. First, we compute the mass function and the linear bias coefficient of dark-matter haloes following Tinker et al. (2008) and Tinker et al. (2010). Subsequently, we associate each halo with an X-ray flux by adopting observationally motivated scaling relations, namely the luminosity–mass–temperature–mass relations from Vikhlinin et al. (2009a) (see Table 1). We allow the four parameters of the luminosity–mass relation to vary throughout the statistical analysis, unless otherwise stated. We therefore take into account: (i) the spectral energy distribution of the ICM emission (spec in XSPEC: Smith et al. 2001; Arnaud 1996) by assuming an average intracluster metallicity of 0.3 $Z_\odot$ (Anders & Grevesse 1989); (ii) the photoelectric absorption suffered by the photons along the line sight by assuming an average hydrogen column density of $3 \times 10^{20}$ atoms cm$^{-2}$ (Kalberla et al. 2005); (iii) the expected telescope response (by Frank Haberl: erosita_4v_Tlength_ff.rsp). Eventually, we are able to evaluate one- and two-point clustering statistics as a function of the raw photon counts.

It is important to mention that the linear matter transfer function has been computed using the CAMB\(^2\) code (Lewis et al. 2000) and that, throughout the text, cluster masses are defined in terms of the spherical-overdensity criterion with respect to the critical density of the Universe, i.e. $M_{500c} = (4\pi/3) \Delta_{crit} \rho_{crit} (z) R_{500c}^3$, with $\Delta_{crit} \equiv 500$ and $R_{500c}$ being the radius encompassing a sphere that either select clusters based on their un-observable virial mass or assume a generic mass-observable relation.

\(^1\) This improves upon common but unrealistic approaches in the literature

\(^2\) http://camb.info/, with default choices but for the adjustments to our fiducial cosmological parameters and selecting ‘High precision: Yes’, ‘Matter/Power: Calculated Values’, ‘k per logint’ = 10.

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with average matter density equal to 500 times the critical density of the Universe $\rho_{\text{crit}}(z)$.

3.1 Survey strategies and parameters

*eROSITA* will perform an X-ray all-sky survey in about 4 years, after reaching a L2 orbit. The entire celestial sphere will be scanned in 8 successive passages, with overlap of all great circles at the ecliptic poles and a uniformly complete coverage of the entire sky at progressively deeper exposures. In the following, eRASS stands for *eROSITA* All-Sky Survey and the numbers 1-8 denote increasing exposure from the first to the eighth passage.

In Table 3, the survey parameters relevant for the analysis are summarized in terms of energy band, sky fraction, exposure time, photon count detection threshold, and an additional mass cut: this is a redshift-dependent photon-count threshold aimed at removing too-low-mass objects that would pass the photon threshold at low redshifts (see Pillepich et al., 2012, for details). Although cluster detectability depends on a host of factors, including e.g. the extent of the source, the adopted 50-photons limit as detection threshold has been demonstrated to be a reliable zeroth-order choice (see Clerc et al., 2018). With the adopted fiducial cosmological parameters and the values reported in Table 3, we expect eROSITA to detect $\sim 8.9 \times 10^4$ clusters of galaxies ($\sim 9.3 \times 10^4$, including Poisson errors in the photon counts) with a median redshift of $\sim 0.35$, when objects below $5 \times 10^{13} h^{-1} M_\odot$ ($\sim 7 \times 10^{13} M_\odot$) are removed. Such numbers correspond to a maximum redshift of $z \sim 2.5$, while only about $10^5$ clusters are expected to reside beyond $z \gtrsim 1$. The all-sky data will be split into two equal, non-overlapping parts between the German and Russian Consortia, respectively: however, in this paper, we will consider a sky fraction of $\sim 66$ per cent as in Pillepich et al. (2012) but provide results for half the sky in Section 6.3 for completeness.

After the completion of the all-sky survey, pointed observations will be undertaken for other 3.5 years. Spectroscopic redshifts will be readily available thanks to the detection of iron lines shifts will be undertaken for other 3.5 years. Spectroscopic redshifts for clusters can be measured with an accuracy of $0.0025(1+z)$, with SPIDERS aiming at 0.001(1+z) out to $z \sim 0.6$ (Clerc et al. 2016). In parallel, a host of multi-band photometric data will be used to identify cluster and AGN counterparts, and to estimate photometric redshifts. Overlap of the *eROSITA* sky with completed or ongoing programs like SDSS, PanSTARRS, and DES will provide cluster redshifts with accuracies ranging from 0.018(1+z) at $z \lesssim 0.6$ to 0.025(1+z) up to at $z \sim 1.1$ (Merloni et al. 2012) or even better (e.g. $\lesssim 0.02(1+z)$ all the way up to $z \lesssim 1.3$, Drlica-Wagner et al. 2018).

### Table 3. eROSITA All-Sky Survey (eRASS) parameters, for 65.8 per cent sky coverage excising $\pm 20$ deg around the Galactic plane. eROSITA will scan the sky eight times in four years, with an average integrated exposure time of 1.6 ks.

| Choices/Description | eRASS: $\alpha$ |
|---------------------|----------------|
| X-ray Energy Band [keV] | 0.5-2.0 |
| Detection limit (min raw photon count): $\eta_{\text{min}}$ | 50 |
| Minimum cluster mass [$h^{-1} M_\odot$] | $1 \times 10^{13}$ |
| Sky coverage: $f_{\text{sky}}$ [deg$^2$] | 27,145 |
| Exposure Time: $T_{\text{exp}}$ [s] | $n \times 200$ |

3.2 Probes

We consider two experimental probes: the number counts of galaxy clusters as a function of redshift and their angular clustering that we quantify in terms of the power spectrum. Unless otherwise stated, throughout the paper we will quote cosmological constraints obtained combining the two probes. We refer the reader to Pillepich et al. (2012) for a detailed description of our method while here we only briefly summarize its most important features and how we model the different observables.

3.2.1 Number counts

The total number of clusters detected above a certain photon-count threshold and within a given redshift bin can be calculated by integrating the redshift distribution of the clusters:

$$\frac{dN}{dz} \times \eta_{\text{min}}(z) = 4 \pi f_{\text{sky}} \int_{\eta_{\text{min}}}^{\infty} \frac{dA}{d\eta} \frac{d\eta}{dz} \frac{dN}{d\eta} (\eta, z) d\eta,$$  

where $c$ is the speed of light, $D_A$ is the comoving angular diameter distance, and $\frac{dA}{d\eta}(\eta, z)$ denotes the DM halo mass function properly mapped into a raw-count function for the eROSITA clusters.

3.2.2 Angular clustering

After splitting the clusters in bins of *eROSITA* photon counts and redshift, we adopt the Limber approximation to compute their angular (auto and cross) power spectra,

$$C_l(i, j) = 4 \pi \int_0^{\infty} dz \frac{dV}{dz} P_{\text{lin}}(\ell + 1/2, D_A, z) W_i(z) W_j(z).$$

Here, $V$ is the comoving volume and $P_{\text{lin}}(k, z)$ denotes the linear matter power spectrum evaluated at wavenumber $k$ and redshift $z$. The weight function for the $i$-th bin is

$$W_i(z) = \frac{1}{N_i} \frac{dN}{dz}(b_i(z)),$$

where $N_i$ gives the number counts of the clusters and $b_i$ is their linear bias parameter. We consider spherical harmonics with $5 < \ell < \ell_{\text{max}}$. The minimum value has been set considering the wide-area extension of the survey. On the other hand, we fix $\ell_{\text{max}}$ based on the angular separation which corresponds to a wavenumber of $k_{\text{max}} = 0.1 \ h\ Mpc^{-1}$, in order to alleviate potential issues related to cluster exclusion effects as well as non-linearities in the dynamics of the density perturbations and in the cluster bias (see Section

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3 4-m Multi-Object Spectroscopic Telescope for ESO
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5 Panoramic Survey Telescope and Rapid Response Systems
6 Dark Energy Survey

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3.3 Figure of merit

The relative performance of different experiments in constraining a subset of model parameters can be conveniently quantified by introducing a figure of merit (FoM) which is inversely proportional to the volume of their 68.3 or 95.4 per cent joint credible region. The higher the FoM, the more suitable an experiment is to constrain the selected parameter set. We define a FoM for the DE parameters $w_0$ and $w_a$ in terms of their (marginalized) covariance matrix as

$$\text{FoM}^{DE} = \left[ \det \text{Cov}(w_0, w_a) \right]^{-1/2}. \quad (7)$$

In order to compute $\text{Cov}(w_0, w_a)$ we first invert the full Fisher-information matrix and then extract the $2 \times 2$ submatrix corresponding to the DE parameters. Since the Fisher-matrix formalism assumes a Gaussian posterior distribution, our FoM is a factor 6.17 π larger than that introduced by the DETF (Albrecht et al. 2006) but exactly matches the entries $[\sigma(w_a) \times \sigma(w_p)]^{-1}$ in the tables of their Section IX (here $w_p$ denotes the value of $w$ at a pivot redshift chosen so that errors in $w_a$ and $w_p$ are uncorrelated).7

Throughout the paper, $\sigma(p)$ indicates the forecast uncertainties on the parameter $p$ and gives the rms error obtained after marginalizing over all the other model parameters. Similarly, we visualize constraints for variable pairs by plotting the boundary of their joint 68.3 per cent credible region (after marginalizing over the remaining parameters). We present results and quote FoM values for both eROSITA alone as well as eROSITA in combination with other data. In fact, all values in the tables of Section IX of Albrecht et al. (2006) give forecasts for future probes combined with Planck. Hence, only analog combinations of eROSITA data with Planck priors can be compared with them.

4 BINNING STRATEGY

We split the eROSITA cluster sample into bins both in photon counts and redshifts. We explore various options to choose the optimal binning strategy that maximizes the return of the data set.

For the photon counts, we consider either equal-sized logarithmic bins in the range $50 \lesssim \eta \lesssim 5 \times 10^4$ or variable-sized bins approximately containing the same number of clusters. For the redshift, we take bins of size $\Delta z = (1 + z_{\text{bin}})$, where $\Delta z$ is a parameter we vary and $z_{\text{bin}}$ is the central value in a bin. We consider $\Delta z = 0.2, 0.1, 0.05, 0.02, 0.01$, corresponding to $5, 10, 21, 54, \text{and } 108$ slices in the range $0.01 < z \lesssim 2$. In what follows, we refer to the cases with $\Delta z = 0.05$ and $\Delta z = 0.01$ as `photometric' and `spectroscopic' accuracies, respectively. This conservative designation refers to the fact that such bin widths correspond to the typical $3 - 4\sigma$ redshift errors expected for the eROSITA clusters (see Section 3).

We find that, in order to break the strong degeneracies among the model parameters, it is necessary to finely sample the data both in redshift and in photon counts, $\eta$. In general, for a fixed total number of bins, binning in redshift is more efficient than binning in the photon counts. However, the latter comes at no cost and should always be applied.

The results we quote in the paper for the cluster counts are well converged by considering 20 bins in $\eta$ (logarithmic or not: the forecast does not depend on this) and assuming either photometric or spectroscopic redshift accuracies. On the other hand, for the clustering measurement, we adopt a different binning strategy. In fact, while the angular power-spectrum grows by considering thinner and thinner redshift slices, the number of clusters per bin rapidly drops and the systematic shot-noise correction overcomes the actual clustering signal. If shot noise was exactly Poissonian, it would not be a problem to subtract its contribution out before fitting the power spectra with a model. However, in reality, it is not obvious how to accurately model discreteness and exclusion effects. Therefore, we prefer not to work in a regime where shot noise dominates, which happens already whenever we consider about 30 cells in total. For the clustering study, we therefore first slice the data into four redshift bins spanning the range $0.1 < z \lesssim 2$ and further partition them into five $\eta$ bins chosen so that $\sim 4,000$ clusters are assigned to each of them. Note that this binning strategy is not particularly demanding in terms of redshift-measurement errors as it roughly corresponds to setting $\Delta z \sim 0.1$. We have checked that our results are well converged once clustering and cluster counts are combined together.

5 BEYOND SELF-CALIBRATION: EXTERNAL PRIORS ON THE X-RAY SECTOR

In principle, we could use the eROSITA data to simultaneously constrain cosmology, selection effects, and the cluster mass-observable relation. As in Pillepich et al. (2012), this could be achieved by adopting very broad non-informative priors on the slope, normalization, time-evolution and scatter of the X-ray luminosity–mass relation ($\alpha_{LM}, \beta_{LM}, \gamma_{LM}$ and $\sigma_{LM}$). However, such self-calibration scheme would provide conservative results as, in effect, some information on the X-ray scaling relation is already available. Even more importantly, follow-up observations are already being planned to provide subsets of eROSITA clusters with more stringent mass estimates.

In this Section, we quantify the impact of adopting informative priors on the parameters of the $L_X - M_{500}$ relation. We use the functional form given by Vikhlinin et al. (2009a), who also measured $1\sigma$ uncertainties on the best-fit parameters from two sets of Chandra clusters with median redshifts of about 0.05 and 0.5. The corresponding relative errors on $\alpha_{LM}, \beta_{LM}, \gamma_{LM}$ and $\sigma_{LM}$ are 8.6, 23, 0.1, 9.8 per cent, respectively, with negligible covariances for the purposes at hand (if the scaling relations are written as in equation 18 of Pillepich et al. (2012); Vikhlinin, private communication). In Fig. 1, we show how the constraints on the cosmological parameters set by eROSITA change when the current uncertainties on the scaling-relation parameters will be improved by a factor.
of \( N \gtrsim 1 \) thanks to the synergy of X-ray follow up with Chandra, XMM-Newton, Suzaku, possibly NuSTAR\(^8\), and eROSITA itself. It is apparent that considering external information on the scaling relations is key to get tighter cosmological constraints. This remains true also when data from other probes are considered (e.g. combining eROSITA with Planck, not shown in the figure). In particular, we find that the improvements due to the refinement of the LM sector are much more pronounced for \( \sigma_8 \) and \( \Omega_m \) than for the DE parameters (since the correlations between the LM and the DE sectors are weaker). For instance, considering an evolving DE model (Fig. 1, right panel) and comparing the results obtained assuming \( N = 4 \) with those of the self-calibration, we find that the uncertainties on \( \sigma_8 \) and \( \Omega_m \) shrink by a factor of 4 and 2, respectively, while those on \( w_0 \) and \( w_a \), in comparison, only reduce by 20–30 per cent. Further discussion about the dependence of our forecast on the adopted priors for the LM relation will be presented in the next Section.

6 RESULTS

We present here the results of our forecast. We refer to the full eROSITA sample, obtained after four years of observations (eRASS:8). Unless otherwise stated, we quote marginalized 68.3 per cent credible intervals derived from the combination of number counts and angular clustering.

As summarized in Table 4, we focus on two scenarios. We call ‘pessimistic’ the case where (i) photometric redshifts are available for all the clusters, (ii) the size of the adopted priors for the LM sector coincides with the measurement errors in Vikhlinin et al. (2009a), and (iii) the functional form and accuracy of the LM scaling-relation can be trusted down to cluster masses of \( M_{500c} = 5 \times 10^{13} h^{-1} M_{\odot} \). On the other hand, we call ‘optimistic’\(^9\) the case with (i) spectroscopic redshifts, (ii) priors on the LM relation which are improved by a factor of \( N = 4 \) compared to Vikhlinin et al. (2009a), and (iii) the inclusion in the analysis of all 125,300 clusters with \( \eta \gtrsim 50 \) and \( M_{500c} \gtrsim 1 \times 10^{13} h^{-1} M_{\odot} \). In this second option, we extrapolate the functional form (and fiducial values) of the LM relation to galaxy groups. In fact, X-ray studies which extend scaling relations towards masses of \( \sim 10^{13} h^{-1} M_{\odot} \) are available (Sun et al. 2009; Eckmiller et al. 2011; Lovisari et al. 2015; Bharadwaj et al. 2015), and have shown both an increase in the scatter (Eckmiller et al. 2011) as well as a steepening of the LM relation (Lovisari et al. 2015). However, our goal here is to emphasize the impact of low-mass objects and, for this reason, we opt for a simplified approach.

Our baseline cluster analysis includes, in addition to the eRASS:8 dataset and mass proxies from X-ray follow-up data, Gaussian priors on the Hubble parameter (with \( \Delta h = \pm 0.022 \), Riess et al. 2011) and on the cosmic baryon density (with \( \Delta(\Omega_b h^2) = \pm 0.0045 \), Cooke et al. 2014).

6.1 Constraints from eRASS:8

The constraints that eROSITA will set on the cosmological parameters with its 4-yr all-sky survey are reported for a subset of relevant parameters in Table 5 and Fig. 2 considering three different cosmological models: \( \Lambda \)CDM, \( w_0 \)CDM, and \( \omega \)CDM (see Table 2)\(^{10}\).

In the upper panel of Fig. 2, the ability of eROSITA to constrain \( \sigma_8 \) and \( \Omega_m \) within a \( \Lambda \)CDM model is compared to the results of a variety of complementary cluster measurements from the literature (solid thin colored contours), via optical (Rozo et al. 2010; Tinker et al. 2012), X-ray (Henry et al. 2009; Vikhlinin et al. 2009b; Mantz et al. 2015; Schellenberger & Reiprich 2017), and Sunyaev-Zel’dovich observations (Benson et al. 2013; Bocquet et al. 2015; both StageIII and StageIV experiments, the DETF assumed surveys with a much smaller number of detected clusters than the actual eROSITA cluster count.

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\(^{8}\) The Nuclear Spectroscopic Telescope Array (Harrison et al. 2013)

\(^{9}\) Note that our strategy is somewhat more conservative than that adopted by the DETF for cluster data: there, an improvement by a factor of 7 is assumed in the mass-observable relation parameterization between their pessimistic and optimistic scenarios, with an optimistic relative uncertainty on the mean and variance per redshift bin as small as 1.6 per cent. However, in

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Table 4. Scenarios for the analysis of the eROSITA data. ‘Self calibration’ denotes the configuration adopted for the main results of Pillepich et al. 2012, i.e. when no priors are applied to the X-ray observables — mass relation parameters. ‘Photo-z’ and ‘spectro-z’ redshift accuracies are implemented here in terms of the width of the redshift slices the data can be binned into, i.e. of width $\Delta z \ (1 + z)$ with $\Delta z = 0.05$ and $\Delta z = 0.01$, respectively. This distinction is to be intended only for the counts experiment, as for the clustering the slicing in redshift-space is limited by shot-noise to a handful of bins in redshift space.

| Scenario       | Priors on LM parameters | Redshift Bins Accuracy | Minimum Cluster Mass (M$_{200c}$, [M$_{\odot}$/h]) | # Clusters |
|----------------|-------------------------|------------------------|--------------------------------------------------|------------|
| Self calibration | $\mathcal{U}(-\infty, +\infty)$ | photo-z: $\Delta z = 0.05$ | $5 \times 10^{13}$ | 88,900     |
| Pessimistic    | current knowledge       | photo-z: $\Delta z = 0.05$ | $5 \times 10^{13}$ | 88,900     |
| Optimistic     | x 4 better than current knowledge | spectro-z: $\Delta z = 0.01$ | $1 \times 10^{13}$ | 125,300    |

Figure 2. eROSITA forecasts after 4 years of all-sky survey. Joint 68.3 per cent credible regions are shown in shades of blue for a selection of parameter pairs, from the combination of cluster abundances and clustering, obtained by marginalizing over all the other model parameters (Table 2), in a LCDM cosmological model (top), in a cosmological model with constant (bottom left) and evolving (bottom right) DE equation-of-state parameters, respectively. Thin color curves denote analog constraints from other cluster samples, in combination or not with additional data. Black thin contours display the results from the combination of Planck, BAO and supernova data – see text for details.
All combined but for three cases to CMB results available at that time\(^\text{11}\). Owing to its \(10^5\) clusters, \(e\text{ROSITA alone}\) will largely improve upon any currently available cluster dataset, with constraints on \(\sigma_8\) and \(\Omega_m\), of the order of 1 per cent and 3 per cent, respectively, even when a constant DE equation-of-state parameter is included.

In the case of spatially flat, constant-\(w\) models, constraints on \(\sigma_8\) and \(\Omega_m\), derived from \(e\text{ROSITA alone}\) are identical to the flat \(\Lambda\text{CDM}\) ones, with \(\Delta\sigma_8\) as good as \(\pm 0.037\). For models with an evolving equation of state, \(e\text{ROSITA alone}\) will be able to provide marginalized, one-dimensional, 1\(\sigma\) errors as good as \(\Delta\sigma_8 = \pm 0.015\), \(\Delta\Omega_m = \pm 0.010\), \(\Delta\sigma_8 = \pm 0.098\), and \(\Delta\sigma_8 = \pm 0.39\) (optimistic scenario). These results correspond to a DE FoM of about 69 (\(e\text{ROSITA alone}\)).

Differently from the case where also primordial non-Gaussianity of the local type is included (as in Pillepich et al. 2012), the constraints in Fig. 2 are dominated by the abundance of the clusters rather than by their clustering signal contribution: see Fig. 3, upper panel. Specifically, the combination of counts and clustering improves upon the sole clustering experiment by a factor of about 4 and 30 in \(\Delta\sigma_8\) and in the DE FoM, respectively, while the improvements wrt to the abundance experiment alone read about 70 per cent and 60 per cent, respectively.

We also find that what mostly drives the tightening of the constraints from the pessimistic to the optimistic scenario is (i) the better knowledge of the LM relation, especially for \(\sigma_8\) and \(\Omega_m\), and (ii) the lower cluster-mass threshold, i.e. larger number of objects, particularly for the DE sector. Interestingly, when also group-size objects are included in the analysis and pessimistic priors on the LM sector are adopted, errors on \(\sigma_8\) and \(\omega_w\) shrink by about an additional 20-30 per cent in comparison to the case when only high-mass objects are included in the analysis. The resulting improvement on the DE FoM (Fig. 3, top panel) is similar to the case where the uncertainties on the LM relation are reduced by a factor of four at the cluster-mass scale (as described in Section 5).

Within our framework, results do not seem to depend on the accuracy of the redshift measurements, once reasonable priors are adopted on the LM relation and at least a few bins in redshift are considered. This implies that the results of our optimistic scenario will not sensibly deteriorate if spectroscopic redshifts will not be available. However, they will still be of pivotal importance for point-source identification, for the measurement of redshift-space distortions, and for the identification of (admittedly rare) structures in projection.

\(\text{Table 5. } e\text{ROSITA forecasts after 4 years of all-sky survey: one-dimensional, 1-}\sigma\text{ errors and Figure of Merit corresponding to the contours of Fig. 2 and obtained by marginalizing over the parameters listed in Table 2. The dark-energy figure of merit is defined in Eq. 7.}\)

| Data                          | Scenario | Model | \(\Delta\sigma_8\) | \(\Delta\Omega_m\) | \(\Delta\sigma_w\) | \(\Delta\omega_w\) | FoM\(^{DE}\) |
|-------------------------------|----------|-------|-------------------|-------------------|-----------------|-----------------|-------------|
| \(e\text{ROSITA} + H_0 + BBN\) | Pessimistic | \(\Lambda\text{CDM}\) | 0.016 | 0.014 | - | - | - |
| \(e\text{ROSITA} + H_0 + BBN\) | Optimistic | \(\Lambda\text{CDM}\) | 0.011 | 0.008 | - | - | - |
| \(e\text{ROSITA} + H_0 + BBN + Planck + BAO + JLA\) | Pessimistic | \(\Lambda\text{CDM}\) | 0.009 | 0.005 | - | - | - |
| \(e\text{ROSITA} + H_0 + BBN + Planck + BAO + JLA\) | Optimistic | \(\Lambda\text{CDM}\) | 0.007 | 0.004 | - | - | - |
| \(e\text{ROSITA} + H_0 + BBN\) | Pessimistic | \(\omega_0\text{CDM}\) | 0.017 | 0.014 | 0.059 | - | - |
| \(e\text{ROSITA} + H_0 + BBN\) | Optimistic | \(\omega_0\text{CDM}\) | 0.011 | 0.008 | 0.037 | - | - |
| \(e\text{ROSITA} + H_0 + BBN + Planck + BAO + JLA\) | Pessimistic | \(\omega_0\text{CDM}\) | 0.010 | 0.007 | 0.030 | - | - |
| \(e\text{ROSITA} + H_0 + BBN + Planck + BAO + JLA\) | Optimistic | \(\omega_0\text{CDM}\) | 0.007 | 0.005 | 0.024 | - | - |

\(\text{\(e\text{ROSITA alone}\)}\)

\(\Delta\Omega_m\) and \(\Delta\omega_w\) shrink by about an additional 20-30 per cent in comparison to the case when only high-mass objects are included in the analysis. The resulting improvement on the DE FoM (Fig. 3, top panel) is similar to the case where the uncertainties on the LM relation are reduced by a factor of four at the cluster-mass scale (as described in Section 5).

\(\text{In Table 5 and Fig. 2, we combine the results above with the analysis of the CMB anisotropies performed by the Planck collaboration (PlanckCollaboration et al. 2016). Specifically, we adopt the 2015 constraints from the all-sky CMB temperature and polarization and BAO and SNe data (thin, black curve in the top panel of Fig. 2). In the case of DE models, the optimistic case the combination of Planck} + \text{BAO} + \text{JLA} \text{data. Finally, in the most general model we consider, the combination with Planck will further reduce \(e\text{ROSITA alone}\) optimistic constraints to marginalized, one-dimensional, 1\(\sigma\) errors as good as \(\Delta\sigma_8 = \pm 0.008\), \(\Delta\Omega_m = \pm 0.006\), \(\Delta\sigma_w = \pm 0.008\), and \(\Delta\sigma_w = \pm 0.25\).

\(\text{In the optimistic scenario and within the \(\Lambda\text{CDM}\) model, \(e\text{ROSITA alone}\) return comparable constraints on the derived parameters \(\sigma_8\) and \(\Omega_m\) to the combination of CMB temperature and polarization and BAO and SNe data (thin, black curve in the top panel of Fig. 2). In the case of DE models, the optimistic} e\text{ROSITA dataset alone will outperform Planck+BAO+JLA data. Finally, in the most general model we consider, the combination with Planck will further reduce \(e\text{ROSITA}\) optimistic constraints to marginalized, one-dimensional, 1\(\sigma\) errors as good as \(\Delta\sigma_8 = \pm 0.008\), \(\Delta\Omega_m = \pm 0.006\), \(\Delta\sigma_w = \pm 0.008\), and \(\Delta\sigma_w = \pm 0.25\).

\(\text{We use the base-w-wa}\text{plikHM_TT+EE+lowTEB_BAO_H070p6_JLA}\text{ chain results, here simply denoted as ‘Planck + BAO + JLA’, according to the nomenclature of the Planck data release:}\text{http://wiki.cosmos.esa.int/planckpla/index.php/Cosmological_Parameters.}\)

\(\text{11\ The combination with CMB data is manifest in the inclination of the elliptical contours: cluster data alone provide anti-correlated estimates for}\ \sigma_8\ \text{and}\ \Omega_m\ \text{while WMAP data give correlated measurements. The cluster samples quoted from the literature are made by a few tens to a few hundreds of clusters, and indeed their CMB-combined cosmological constraints are often strongly dominated and determined by CMB data. Note that Schellenberger & Reiprich (2017) combined the constraints from the cluster mass function with constraints from the cluster gas mass fraction, which shifted the best fit of \(\Omega_m\) to a larger value, within the uncertainties of the cluster-count only results (see their Fig. 1).}\)
Dark-energy forecasts with eROSITA

Figure 3. The DE FoM from eROSITA for different survey assumptions and configurations (top) and in comparison to the DETF expectations (bottom). In the top panel, we highlight the impact of individual choices in tightening cosmological constraints from the pessimistic to the optimistic scenario (see Table 4) and the separate contributions from cluster counts and spatial clustering. In the bottom section, we focus on the optimistic scenario, from eRASS:8 alone or in combination with other leading probes. See Section 3.3 for the operational definition of FoM that is adopted throughout: note that all the FoM values quoted by the DETF refer to forecasts of current or future probes in combination with Planck priors.

Table 6. Comparison among eROSITA (optimistic case), Planck with and without additional data, and the DES results, for cosmological models with constant or evolving DE (see Section 6.2 for details). The results for the 1st-year constraints from the DES are from Troxel et al. 2017. As a future, specific example of StageIV experiment, we give the predictions for the Euclid mission according to Giannantonio et al. 2012 (weak-lensing + 2D spectroscopic clustering of galaxies) and Sartoris et al. 2016 (cluster counts and 3D power spectrum but assuming perfect knowledge of the cluster mass-observable scaling relations). Notes: (1) Planck = base_plikHM_TTTEEE_JowTEB; Planck + BAO + JLA = base-w-wa_plikHM_TTTEEE_JowTEB_BAO_H070p6_JLA. (2) Here, perfect knowledge of the cluster scaling relations is assumed which leads to very optimistic constraints.

| Data                        | $\Delta \sigma_8$ | $\Delta \Omega_m$ | $\Delta w_0$ | $\Delta w_a$ |
|-----------------------------|-------------------|-------------------|--------------|--------------|
| Planck                      | 0.013             | 0.009             | -            | -            |
| Planck + BAO + JLA          | 0.017             | 0.010             | 0.11         | 0.41         |
| eRASS:8                     | 0.015             | 0.010             | 0.098        | 0.39         |
| eRASS:8 + Planck(1)         | 0.008             | 0.006             | 0.082        | 0.33         |
| eRASS:8 + Planck + BAO + JLA| 0.008             | 0.006             | 0.068        | 0.25         |
| DES 1st year                | -                 | +0.074            | +0.26        | -            |
| Euclid + Planck (w1+gc)     | 0.005             | 0.004             | 0.035        | 0.15         |
| Euclid + Planck (clusters)(2)| 0.001             | 0.001             | 0.034        | 0.16         |

These results correspond to a DE FoM of about 162 (116) in the optimistic (pessimistic) scenario, placing eROSITA at the level of the StageIV DE experiments identified by the DETF. As we show in Fig. 3, according to the recommendations of Albrecht et al. (2006), StageIII and StageIV experiments in combination with Planck-like data should be able to constrain DE models with a FoM in the range [8, 43] and [27, 645], respectively. These values bracket their pessimistic and optimistic assumptions on systematics control and survey characteristics, and combine results from all types of DE techniques (BAO, clusters, supernovae, and weak lensing). Thanks to its 10^5 clusters, the constraints set by eROSITA will exceed the DETF expectations from cluster data (FoM in the range [6, 39], both at the StageIII and StageIV levels), even with conservative assumptions on the LM relation and on redshift availability. In fact, eRASS:8 will provide StageIV-level results also without Planck priors.

To better put eROSITA into context, in Table 6 we contrast our forecasts (optimistic case) with Planck constraints (alone and in combination with BAO and SN probes) and the recent results from the 1st-year analysis of the Dark Energy Survey (Troxel et al. 2017). The Planck only case (from the base_plikHM_TTTEEE_JowTEB chains) is meant to highlight the complementary contribution on the DE sector of eROSITA alone when the latter is combined with other cosmological probes, e.g. BAO and Type Ia SNe. As a future example of StageIV experiment, we show some expectations for the Euclid mission (Laureijs et al. 2011), a galaxy and galaxy cluster survey. In particular, we compare to the predictions by Giannantonio et al. (2012) and Sartoris et al. (2016), who had studied the constraining power of the
weak-lensing and the 2D spectroscopic clustering of galaxies and of the cluster counts and 3D power spectrum, respectively, the latter assuming perfect knowledge of the cluster mass-observable scaling relations (see Amendola et al. 2018 for a recent review on cosmology with Euclid).

6.3 From eRASS:1 to eRASS:8
So far, we have provided constraints that eROSITA will be able to achieve at its final survey stage, namely after eight successive scans of the entire sky. These will be available after four years of observations, i.e. sometime in 2023. It is, however, interesting to investigate what we can expect from early and intermediate data releases.

In Fig. 4, the progression of the eROSITA constraints during the four years of its all-sky survey is obtained assuming gradually deeper average exposures. Here and in Table 7, where we provide results for eRASS:2 and compare them to the final ones, we only consider our pessimistic set-up. Given that the data will be equally split between the German and the Russian consortia and considering the unavailability of follow-up campaigns covering the whole sky before late 2019, in Table 7 we also list constraints obtained with only half-sky coverage.

Within the ΛCDM model, eROSITA will return per-cent level statistical constraints on σ8 and Ωm from its very first year of observations (Fig. 4, top left; all-sky case). For a DE model with constant equation of state, eRASS:2 will be able to provide constraints on w0 better than 15 or 4 per cent alone or in combination with Planck data, respectively (for both sky coverages, not tabulated). For evolving DE models, eROSITA will qualify as a StageIV experiment already after two years of observations (Table 7, top sections) and eRASS:8 will be able to place competitive constraints on DE (Δw0 = ±0.09, Δw0 = ±0.31) even in the case where data from only half the sky will be included in the analysis (Table 7, fourth and eighth row, without and with additional data).

The promising outcome of eRASS:2 displayed in the top panel of Fig. 4 in combination with Planck data and even adopting pessimistic assumptions should not be mistaken as a reason to stop the all-sky survey after one year only of observations. In fact, the deeper sky scans will be vital to shed new light on the ongoing debate regarding possible tensions between cluster results and Planck constraints.

6.4 The impact of high-redshift clusters
Finally, we quantify the impact of high-redshift clusters on our forecast. Although the median redshift of the eROSITA clusters is z ∼ 0.35, we expect to find a fourth of the sample (with M500k ≤ 5 × 10^14h⁻²M⊙) at z ≥ 0.5 and about 1,100 clusters with z ≥ 1. As reported in Table 7 for wCDM models, while the bulk of the information for σ8 and Ωm gets saturated with the low-redshift clusters (especially at z ≤ 1), the DE sector largely benefits from the leverage of the objects at higher redshifts. For example, within the pessimistic assumptions and for eROSITA alone, the constraints on the EOS parameters would degrade by a factor of ∼ 3 if we were in the unfortunate conditions to exclude objects above redshift 0.5, because of identification issues, lack of redshift confirmation or poor knowledge of the scaling relations. Even more interestingly, the exclusion of the thousand clusters beyond redshift 1 would provoke a deterioration of the error bars on w0 and w1 of about 50 per cent. This result highlights the vital importance of high-redshift clusters for the DE sector, for which good observable-mass calibrations will be needed.

7 DISCUSSION

The scope of this paper is to provide the community with baseline expectations for the eROSITA constraints on DE, by focusing on the abundance and spatial clustering of its photon-count limited, all-sky, galaxy-cluster sample. So far, we have focused on the statistical errorbars of the parameters of interest. Here, we comment on possible systematic uncertainties.

7.1 On the fiducial cosmology and observable–mass relation
Fisher forecasts require assuming an underlying cosmological model. Here, we have used the cosmological constraints from Komatsu et al. 2009 (5-year WMAP + BAO and SN data). This choice was dictated primarily by consistency with the X-ray LM scaling relation derived in Vikhlinin et al. (2009a,b). We have tested the effects of changing the fiducial cosmology to the most recent Planck results. In this case, however, we have artificially decreased the cluster luminosities predicted by the Vikhlinin et al. (2009a) relation by 40 per cent in order to keep the overall number of eROSITA clusters unchanged with respect to our reference case. The forecasts for the cosmological parameters are consistent in the two scenarios.

Another possible source of systematics in our predictions lies in the adopted observable–mass relation(s). We have replicated our predictions by assuming the X-ray scaling relations by Reichert et al. (2011). For the wCDM scenario but keeping the observable–mass scaling relation parameters frozen (e.g. as in the ‘LM:fixed’ case of Fig. 1), we recover the same degeneracies among parameters as with the Vikhlinin et al. (2009a) relations: the discrepancies in the statistical forecasts for the two sets of assumptions are smaller than about 10 per cent, for all cosmological parameters but for the spectral index and the baryonic cosmic fraction. However, for this comparison we have assumed the same LM intrinsic scatter, namely σLM = 0.396, as per Vikhlinin et al. (2009a) findings. A larger scatter as the one found by Andreon et al. 2016 (σLM = 0.47) would actually translate into somewhat tighter statistical constraints, by no more than 10 per cent in our parameters of interest in the cluster abundance only calculation.

7.2 On the underlying theoretical models
Another aspect of a Fisher forecast is that it does not require extremely accurate models of the observables. Basically, the forecast relies on the assumption that accurate models will be available when the actual data will be analyzed. It is only in this case that the potential of the survey is fully realized in practice. However, even in a forecast, it is necessary to make sure that the underlying model grasps the key features, trends, and parameter degeneracies that characterize the survey, as, in our case, the cluster number counts as a function of redshift and η.

In this respect, we remark that the fitting functions we have used for the halo mass function (Tinker et al. 2008) have not been calibrated for DE models different from a cosmological constant (but see Bhattacharya et al. 2010; Cui et al. 2012b). The Tinker et al. (2008) model is accurate at percent level at fixed cosmology and shows about 5 per cent deviations within the range of ΛCDM.
Figure 4. Top: eROSITA constraining power as a function of time i.e. at subsequent stages of its all-sky survey, shown here at annual intervals. Bottom: eROSITA constraining power as a function of maximum available cluster redshift. Results are given adopting the pessimistic assumptions described in Table 4, standard priors on \( h \) and \( \Omega_b \), and without and in combination with Planck CMB data: blue and gray curve sets, respectively. Black thin contours denote results from the combination of Planck, BAO and supernova data, for reference.

Table 7. eROSITA constraining power on the \( w_{CDM} \) models for different survey strategy configurations. The expectations in the top eight rows correspond to two stages of the all-sky survey: the early case eRASS:2 (after one year of observations) and the final one eRASS:8 (after four years), for two different assumptions on the sky coverage, all vs. half sky. Results of eRASS:2 are in both cases given adopting pessimistic assumptions and are dominated by the Planck priors; results from eRASS:8 are obtained with optimistic assumptions in both the all-sky and half-sky cases. On the bottom section of the table, different scenarios for the availability of high-redshift clusters are quantified, with pessimistic assumptions.

| Data | \( f_{\text{sky}} \) [deg\(^2\)] | \( T_{\text{exp}} \) [s] | # Clusters | \( \Delta \sigma_8 \) | \( \Delta \Omega_m \) | \( \Delta w_0 \) | \( \Delta w_a \) | FoM\(^{DE} \) |
|------|-----------------|--------|-------------|---------|-------------|---------|---------|-------------|
| eRASS:2 + \( H_0 \) + BBN | 27,145 | 400 | 10,016 | 0.037 | 0.026 | 0.28 | 1.11 | 7 |
| eRASS:8 + \( H_0 \) + BBN | 27,145 | 1600 | 125,300 | 0.015 | 0.010 | 0.10 | 0.39 | 69 |
| eRASS:2 + \( H_0 \) + BBN | 13,573 | 400 | 5,009 | 0.042 | 0.028 | 0.37 | 1.47 | 5 |
| eRASS:8 + \( H_0 \) + BBN | 13,573 | 1600 | 62,650 | 0.024 | 0.015 | 0.20 | 0.68 | 26 |
| eRASS:2 + \( H_0 \) + BBN + Planck + BAO + JLA | 27,145 | 400 | 10,016 | 0.012 | 0.008 | 0.10 | 0.37 | 74 |
| eRASS:8 + \( H_0 \) + BBN + Planck + BAO + JLA | 27,145 | 1600 | 125,300 | 0.008 | 0.006 | 0.07 | 0.25 | 162 |
| eRASS:2 + \( H_0 \) + BBN + Planck + BAO + JLA | 13,573 | 400 | 5,009 | 0.013 | 0.008 | 0.10 | 0.37 | 69 |
| eRASS:8 + \( H_0 \) + BBN + Planck + BAO + JLA | 13,573 | 1600 | 62,650 | 0.010 | 0.007 | 0.09 | 0.31 | 98 |
| \( z \lesssim 0.5 \) eRASS:8 + \( H_0 \) + BBN | 27,145 | 1600 | 67,500 | 0.040 | 0.027 | 0.39 | 1.89 | 4 |
| \( z \lesssim 1.0 \) eRASS:8 + \( H_0 \) + BBN | 27,145 | 1600 | 87,800 | 0.030 | 0.022 | 0.23 | 0.86 | 18 |
| \( z \lesssim 2.5 \) eRASS:8 + \( H_0 \) + BBN | 27,145 | 1600 | 88,900 | 0.029 | 0.022 | 0.15 | 0.58 | 29 |
cosmologies but possibly worse accuracies for evolving-DE cosmological models. Moreover, additional uncertainties of a few per cent are introduced by the numerical interpolation which is needed to convert the Tinker’s formulas into functions of $\Delta_{\text{crit}}$. Therefore, we cannot exclude that our results are affected by uncertainties of this size or slightly larger.

Even more importantly, our approach neglects the effects of baryonic physics (as we use the DM halo mass function and bias by Tinker et al. 2008, 2010). Over the last years, a number of studies have investigated the impact of cluster gas physics on the functional form of the halo mass function via hydrodynamical cosmological simulations (Cui et al. 2012a; Sawala et al. 2013; Cusworth et al. 2014; Vogelsberger et al. 2014; Bocquet et al. 2016) and thus on the number of expected galaxy clusters and associated cosmological constraints (Martizzi et al. 2012; Balaguera-Antolínez & Porciani 2013; Bocquet et al. 2016). AGN feedback, galactic winds, and complex ICM physics have been shown to modify the expectations from dark-matter only simulations on a host of observables, including the DM and total-matter-density profiles within clusters, the meaning of cluster mass, the clustering of dark and luminous matter across spatial scales and times, and the DM halo and galaxy bias (see e.g. Vogelsberger et al. 2014; Schuller et al. 2015; Chisari et al. 2018; Springel et al. 2018, for Illustris, EAGLE, Horizon-AGN, and IllustrisTNG results). Owing to the complexity of the mechanisms at work, no consensus has yet been reached across simulation campaigns and subgrid physics prescriptions as to how and by how much the N-body fitting halo mass function and halo bias formulas shall be modified to take into account the effects of baryons. Yet, the majority of the researchers conducting hydrodynamical numerical campaigns agree that such modifications far exceed the accuracies required to fulfill the promises of the era of precision cosmology.

In fact, because of the enormous computational cost that is needed to simulate very massive structures or very large cosmological volumes by simultaneously resolving the details of the intra-cluster plasma, of the cluster galaxies, and hence of the interplay of cluster plasma, of the cluster galaxies, and hence of the interplay mechanisms at work, no consensus has yet been reached across simulation campaigns and subgrid physics prescriptions as to how and by how much the N-body fitting halo mass function and halo bias formulas shall be modified to take into account the effects of baryons. Yet, the majority of the researchers conducting hydrodynamical numerical campaigns agree that such modifications far exceed the accuracies required to fulfill the promises of the era of precision cosmology.

In fact, because of the enormous computational cost that is needed to simulate very massive structures or very large cosmological volumes by simultaneously resolving the details of the intra-cluster plasma, of the cluster galaxies, and hence of the interplay between galaxies and feedback, currently no baryonic halo mass function predictions extending to a few $10^{15} \, M_\odot$ can be considered statistically sound. We therefore conclude that, while our results are most certainly affected by a non negligible systematic uncertainty, the landscape of baryonic predictions is still not fully mature for the problem at hand and a thorough quantitative assessment has to be postponed to future studies.

### 7.3 Fisher matrix vs sampling the posterior distribution

The Fisher information matrix is a precious statistical tool for assessing the constraining power of planned experiments. However, this method has limitations and its results do not always agree with those obtained computing the posterior distribution based on Bayes theorem. The main assumption of the Fisher formalism is that the likelihood function (or the posterior distribution, depending on the application) averaged over all possible datasets is a multi-variate Gaussian. This is a strong hypothesis and examples have been provided showing sets of parameters for which the Fisher-matrix constraints do not match the actual contour levels of the likelihood function (e.g. Wolz et al. 2012; Khedekar & Majumdar 2013). Typically, problems arise for highly degenerate parameters with large uncertainties. In these cases, it is recommended to reparameterize the model using combinations of the degenerate variables before computing a Fisher forecast (e.g. Albrecht et al. 2006).

In order to validate our Fisher forecasts, in Fig. 5 we compare them with those obtained by sampling the full posterior distribution with a Monte Carlo Markov Chain (MCMC). Specifically, we use the Metropolis sampler included in the COSMOMC software (Lewis & Bridle 2002, version from 06/2015) together with our own likelihood function for the $e$ROSITA clusters. For this test, we only consider the cluster number counts that, as shown above, dominate the cosmological constraints. We find that the marginalized 68 per cent credible intervals are consistent to better than 17 per cent for all the parameters and cosmological models we have considered. In particular, the marginalized Fisher errors on $w_0$ and $w_\alpha$ in the evolving DE scenario are slightly overestimated. In most cases, the Gaussian approximation for the likelihood function is excellent, especially close to the peak. However, occasionally, mild asymmetries are present (see e.g. the 95 per cent credible region in the bottom-left panel of Fig. 5). Based on this analysis, we conclude that the Fisher-matrix formalism is a reliable forecasting technique for the purposes of our paper.

### 8 SUMMARY AND CONCLUSIONS

We have forecast the potential of the upcoming X-ray telescope $e$ROSITA to constrain cosmological parameters, with particular attention to DE models. $e$ROSITA will be launched in 2019 and will provide the first all-sky, X-ray selected galaxy-cluster sample after ROSAT, at an average exposure of 1.6 ks in the (0.5-2) keV energy band. The $e$ROSITA catalog will consist of about 89,000 clusters more massive than $5 \times 10^{13} h^{-1} M_\odot$ with a median redshift of $z \sim 0.35$; or of more than 125,000 objects when also groups of galaxies down to a mass limit of $10^{12} h^{-1} M_\odot$ are included (assuming a detection limit of 50 photons).

Our Fisher-matrix analysis is based on the measurement of the abundance and angular clustering of $e$ROSITA clusters and considers spatially-flat cosmological models with either constant (ΛCDM and $w_0$CDM) or time-dependent ($w$CDM) DE equation-of-state parameters. Our most general case considers eleven model parameters, seven of which characterize the cosmological model, while the remaining four describe the luminosity–mass relation for X-ray selected clusters. We find that, after four years of all-sky survey, $e$ROSITA will give marginalized, one-dimensional, 1σ errors of $\Delta \sigma_8 \pm 0.011$, $\Delta \Omega_m = \pm 0.007$, $\Delta w_0 = \pm 0.08$, and $\Delta w_\alpha = \pm 0.29$ in a pessimistic scenario; and of $\Delta \sigma_8 = \pm 0.008$, $\Delta \Omega_m = \pm 0.006$, $\Delta w_0 = \pm 0.07$, and $\Delta w_\alpha = \pm 0.25$ in an optimistic scenario, in both cases in combination with and improving upon currently existing Planck, BAO and SNIa data (see Table 5 and Fig. 2).

For the sets of parameters considered here, the constraining power of the $e$ROSITA observations is dominated by the cluster number counts (see Fig. 3). In fact, shot noise and strong degeneracies among different parameters limit the information that can be retrieved from spatial-clustering studies. We also find that what mostly drives the tightening of the constraints from the pessimistic to the optimistic scenarios is the more accurate knowledge of the luminosity–mass relation (particularly for the $\sigma_8$ and $\Omega_m$ sector) and the possibility of extending it to galaxy groups with lower masses while keeping the same accuracy (particularly for the DE sector, see Figs. 1 and 3). With pessimistic (i.e. current) knowledge of the luminosity–mass relation for massive clusters, the leverage on the DE sector of high-redshift $e$ROSITA clusters will be vital, with constraints on $w_0$ and $w_\alpha$ degrading by about a factor of 1.5 (3) when clusters above $z \sim 1$ ($z \sim 0.5$) are excluded from the analysis or cannot be reliably placed in redshift space. This result
Figure 5. Test of the cosmological constraints obtained with the Fisher-matrix method against a full likelihood analysis. We consider the cluster abundance only for eRASS8 in a pessimistic scenario. The ellipses denote the 68.3 and 95.4 per cent credible regions obtained with the Fisher formalism and are similar to those reported in Fig. 2 – where, however, also the clustering is considered. The shaded areas mark the 68.3 and 95.4 per cent credible regions obtained by sampling the corresponding posterior distribution with a MCMC.

The forecasts presented in this work agree to better than 20 per cent with those obtained from a full study of the likelihood function using an MCMC sampler (see Fig. 5). This shows that the Fisher-matrix formalism provides a reliable tool for the problem at hand.

We can therefore conclude that, with an expected DE FoM ranging between 116 and 162, eROSITA will be one of the first StageIV experiments to come online according to the classification of the Dark Energy Task Force. It will improve upon current error bars on $\sigma_8$ and $\Omega_m$ after its very first year of observations, at which point the DE equation-of-state parameter $w_0$ (in the $w_0$CDM model) will be known to better than 14 per cent (4 per cent) without (with) including Planck data, even if only half sky is considered (see Fig. 4, top panels, and Table 7).

However, our findings call for ambitious, synergistic follow-up programs in order to (i) reduce uncertainties in the X-ray luminosity–mass relations by at least a factor of four with current constraints; (ii) extend their characterizations at similar levels of accuracy down to halo masses of $10^{13} h^{-1} M_\odot$; and (iii) equip eROSITA clusters with redshift estimates characterized by statistical errors below $0.05(1+z)$ also at redshifts larger than $z \gtrsim 0.5$. Simultaneously, these results call for a similarly ambitious and robust program of numerical calculations aimed at providing statistically-sound theoretical models with which to fit the forthcoming data. Such models will have to take into account a wide range of physical processes that may affect the distribution of (dark) matter within clusters, in order to improve upon (or confirm) gravity-only predictions.
