Mhd flow of nanofluids in the presence of porous media, radiation and heat generation through a vertical channel

R Mehta\(^1\), V S Chouhan\(^2\) and T Mehta\(^3\)

\(^1\)Dept. of Mathematics & Statistics, Manipal University Jaipur, Jaipur (Raj.), India
\(^2\)Dept. of Mathematics, S. S. Jain Subodh College, Jaipur (Raj.), India
\(^3\)ruchika.mehta@jaipur.manipal.edu
virendrasingh.chouhan@jaipur.manipal.edu (Corresponding Author)
m.tripti24@gmail.com

Abstract:

Aim of present study is to identify the effects of radiation and heat generation in unsteady MHD mixed convective flow of nanofluids along a upstanding channel through porous medium. Four types of nanofluids were studied (i) Water based – Ag (ii) Water based – Al\(_2\)O\(_3\). Water is taken as convectional base fluid and Ag and Al\(_2\)O\(_3\) particles are incorporated in this base fluid. Keeping in mind the congregating and random motion of nanoparticles, “Koo and Kleinstreuer model” is used. The main characteristics of the nanofluids have finer thermo physical properties such as high thermal conductivity, minimal blocking in flow passage, long term solidity and uniformity. The governing boundary layer equations are formulated and reduced to a set of ordinary differential equations using perturbation technique. Solutions are obtained for motion and temperature, discussed and pertinent results are shown through graphs.

Key words: Radiation, Heat generation, Mixed convection, Nanofluids, Brownian motion.

1. Introduction:

Nanofluids are a different type of heat transfer fluids which contain a base fluid and nanoparticles. This term nanofluid is defined as a mixture of nanoparticles of Solid metallic materials, such as Ag, Cu and Fe, and non-metallic materials, such as Al, CuO and carbon nanotubes attached in a base fluid like Water, toluene, C\(_2\)H\(_5\)O\(_2\) or oil. The term nanofluid was introduced by Liao (1992) in 1995, he found that the thermal conductivity will be increased if some amount of nanoparticles is added to the base fluid. After that many researchers tried to investigate for heat transfer improvement in different thermal applications of nanofluids. Today, nanofluids have wide range of applications in Biomedical (drug
Cooling Applications, Smart Fluid, Nuclear-

It u anofluid in presence of Uniform Heat.

2. Results and Discussion

Steady two-dimensional Steady flow through a very porous medium in the presence of free convection and radiation bounded by a standing infinite porous plate Raptis [17]. Xuan et al. [22] examined CuO nanofluid with laminar forced convection. Das [4] for evaluated temperature with increment of four nano fluids with H2O as base fluid and particles of Alumina or Copper Oxide as suspension material. Xuan et al. [22] described their result by studied the effect of random motion, measurements and congregating of nanoparticles.

Koo and Kleinreuter [11] generate a model to study the result of random motion of nanoparticles. The effect of the nanoparticle/base-fluid relative velocity more significantly with compare to dispersion models Buongiorno [3]. MHD oscillatory fluid flow between parallel plates with porosity Makinde, O. D., Mhone, P. Y. [13]. Temperature distribution in upstanding plates partway filled with porous medium of nanofluids Hajipour M. and Dehkordi , A. M. [9]. Flow of a fluid with natural convection flow which includes heat generation / absorption and ramped wall temperature Jha, B. K., et al [10]. In a lid driven square cavity filled with nanofluids Laminar mixed convection heat transfer is investigated by Z. Said, H.A. Mohammed, R. Saidur [23]. MHD nanofluid flow of stretchable sheet as well as moving permeable flat plate with various effects Hail E. and Shankar B. [7,8]. MHD flow and heat transfer of Ferrofluid along a stretching cylinder in presence flux of heat Qasim, M., et al. [16]. Heat Transfer of Nano fluid flow past a moving flat plate Turkyilmazoglu, M. [19].Flow of nano fluids with free convection including boost wall temperature Asma, K., et al [2].MHD nanofluid flow and temperature distribution past a moving standing plate Das, S., Jana, R. N. [5]. Basics of Nanofluids and applications Uddin M. J., Al Kalbani K. S., Rahman1 M. M., Alam M. S., Al-Salti N. and Eltayeb I. A. [20]. Nano fluid flow with temperature and concentration profile including chemical reaction effect Abbas [1].MHD flow of nanofluid through upstanding plates with heat creation Prakash, D., Suriyakumar, P. [15]. MHD flow of nanofluids with mixed convection including effect of radiation and heat generation Gul, A., Khan, I. and Shafie, S. [6]. Study of temperature distribution in Casson Nanofluid in presence of Uniform Heat Source Sink and Convective Condition Kumar, K. G., Gireesha, B. J., Krishnamurthy, M. R. and Prasannakumara, B. C. [12]. Nanofluid flow and heat transfer with immunization through enlarging and shrinking porous parallel plates Olawale, O. J. [14].

Inside a absorbent enclosure via non-equipoise model which includes flow of H2O based nanofluid flow Sheikholeslami, M. and Shehzad, S. A. [18]. Nanofluid flow through parallel plates in presence of slip Xu H., Cui J. [21].

Now a days scientists all over the world are attempting to reach at an agreemen-}

2. Formulation of the problem:

In present study, behavior of water based nanofluid which is containing Silver and Alumina nano particles is investigated. In the direction of the flow oscillatory type of pressure gradient is applied. Effect of heat generation parameter and radiation parameter is also taken into account. The width of vertical channel is taken as d and the channel is assumed to be filled with porous medium. Due to polarization, external electric field and electric field is assumed null. Induced magnetic field is also neglected, by taking magnetic Reynolds number very small. One partition of the channel is maintained at sustained temperature \( T_w \) while other boundary has time dependent temperature \( T_0 + \varepsilon(T_0 - T_w)e^{\alpha t} \)

where \( \varepsilon \) (<<1) is a positive quantity. In the normal direction of the flow y- axis is taken while along the channel x-axis is taken. A transversal magnetic field of constant strength \( B_0 \) is applied in the direction of the flow, due to this fluid has low electrical conductivity. So now, when electrical conductivity which
the fluid have is very small and the produced electromagnetic force is also small. Then, assuming Boussinesq fluid model, the equations governing fluid motion are given as:

\[
\rho_{nf} \frac{\partial u}{\partial t} = - \frac{\partial p}{\partial x} - \sigma_{nf} B_0^2 u + (\rho\beta)_{nf} g (T - T_0) + \mu_{nf} \frac{\partial^2 u}{\partial y^2} - \frac{\mu_{nf}}{K_0} u \tag{1}
\]

\[
\left(\rho c_p\right)_{nf} \frac{\partial T}{\partial t} = k_{nf} \frac{\partial^2 T}{\partial y^2} + Q_0 (T - T_0) - \frac{\partial q}{\partial y} \tag{2}
\]

along with boundary conditions

\[
u(0,t) = 0, \quad T(0,t) = T_0 + \epsilon (T_0 - T_w) e^{i\omega t} \tag{3}
\]

\[
u(d,t) = 0, \quad T(d,t) = T_w \tag{4}
\]

where the subscript \(nf\) is used for nanofluids

| \( u = u(y,t) \) in x- direction it shows velocity of fluid | \( (\rho\beta)_{nf} \) -- the thermal expansion of the nanofluid |
| \( T = T(y,t) \) is the nanofluid temperature | \( g \) -- the acceleration generates through gravity |
| \( (c_p)_{nf} \) -- the nanofluid’s specific heat of at constant pressure | \( \rho_{nf} \) -- the nanofluid’s density |
| \( \mu_{nf} \) -- the dynamic viscosity of the nanofluid | \( k_{nf} \) -- the thermal conductivity of the nanofluid |
| \( \sigma_{nf} \) -- the electrical conductivity of the nanofluid | \( q \) -- the radiative heat flux in x-direction |

Now using the reference [16, 2, 19, 5] the expression for \( \rho_{nf} \), \( (\rho\beta)_{nf} \), \( (\rho c_p)_{nf} \), \( \sigma_{nf} \), \( k_{nf} \) and \( \sigma \) are derived and given by

\[
\rho_{nf} = \phi \rho_f + (1 - \phi) \rho_s + \rho_f \tag{5}
\]

\[
(\rho\beta)_{nf} = \phi (\rho\beta)_f + (1 - \phi) (\rho\beta)_s \tag{5}
\]

\[
(\rho c_p)_{nf} = \phi (\rho c_p)_s + (1 - \phi) (\rho c_p)_f + \rho_f \tag{5}
\]

\[
\sigma_{nf} = \sigma_f \left[ 1 + \frac{3(\sigma - 1)\phi}{(\sigma + 2) - (\sigma - 1)\phi} \right] \tag{5}
\]

\[
k_{nf} = \alpha_{nf} (\rho c_p)_{nf} \tag{5}
\]

\[
\sigma = \frac{\sigma_s}{\sigma_f} \tag{5}
\]

where subscript \( f \) is used for base fluid, subscript \( s \) is used for solid nanoparticles, \( \phi \) is the volume fraction of the nanoparticles and heat capacitance is shown by total term \( \rho c_p \).

Some substantial properties of nanoparticles and base fluid are given in table 1 [16, 2, 19, 5]. These values will be helpful in numeric calculation of this study.

Table-1 : Properties of basefluids and nanoparticles

| Model | \( c_p \) | \( \rho \) | \( k \) | \( \beta \cdot 10^3 \) | \( \sigma \) |
|-------|----------|----------|--------|-----------------|---------|
| Water \( (H_2O) \) | 4179 kg\(^{-1}\)K\(^{-1}\) | 997.1 kgm\(^{-3}\) | 0.613 Wm\(^{-1}\)K\(^{-1}\) | 21 K\(^{-1}\) | 5.5 \cdot 10\(^{8}\) Sm\(^{-1}\) |
| Silver \( (Ag) \) | 235 kg\(^{-1}\)K\(^{-1}\) | 10500 kgm\(^{-3}\) | 429 Wm\(^{-1}\)K\(^{-1}\) | 1.89 K\(^{-1}\) | 6.30 \cdot 10\(^{7}\) Sm\(^{-1}\) |
The generalized model of Koo and Kleinstreuer is used for effective thermal conductivity and viscosity of nanofluid:

\[ k_{\text{nanofluid}} = k_{\text{static}} + k_{\text{Brownian}} \]

where

\[ k_{\text{static}} = k_f \frac{(k_f + 2k_f) - 2\Phi(k_f - k_s)}{(k_f + 2k_f) + \Phi(k_f - k_s)} \]

and

\[ k_{\text{Brownian}} = 5 \times 10^4 \beta \rho_f c_p \sqrt{\frac{k_b T}{\rho_s d_p}} f(T, \Phi, \text{etc.)} \]  

and

\[ \mu_{\text{nanofluid}} = \mu_{\text{static}} + \mu_{\text{Brownian}} \]

where

\[ \mu_{\text{static}} = \frac{\mu_f}{(1 - \Phi)^5} \]

and

\[ \mu_{\text{Brownian}} = 5 \times 10^4 \beta \rho_f \sqrt{\frac{k_b T}{\rho_s d_p}} f(T, \Phi, \text{etc.)} \]  

\[ k_b = \text{The Constant of Boltzmann} = 1.3807.10^{-23} \text{ J/K} \]

\[ T = \text{Nanofluids’s Temperature} \quad 300K > T > 325K. \]

\[ d_p = \text{Solid particles Diameter} \]

\[ \beta = \text{Denotes the fraction of the liquid volume, which travels with a particle and includes particle motion} \]

\[ f(t, \Phi, \text{etc.}) = \text{It depends on particle interactions. The values of } \beta \text{ and } f(t, \Phi, \text{etc.}) \]  

are mentioned in table [2] and [3], using the generalized Koo and Kleinstreuer model

| Particle | \( \beta \) | Note |
|----------|-------------|------|
| Ag       | 0.0137(100\%)^{0.8229} | \( \Phi < 1\% \) |
| Al\(_2\)O\(_3\) | 0.0017(100\%)^{0.0841} | \( \Phi > 1\% \) |

Table 2. for \( \beta \)

| Particle | \( f(t, \Phi, \text{etc.}) \) |
|----------|-----------------------------|
| Ag       | 1                           |
| Al\(_2\)O\(_3\) | (-64\(\Phi+0.4705\))T+(1722.3\(\Phi-134.63\)) |

The heat flux radiation is given by:

\[ -\frac{\partial q}{\partial y} = 4\alpha_0^2(T - T_0) \]  

and pressure gradient is given by

\[ -\frac{\partial p}{\partial x} = \lambda \exp(i\omega_1 t) \]  

where \( \alpha_0 \) the mean radiation absorption coefficient, \( \lambda \) constant, \( \omega_1 \) the oscillation frequency.

### 3. Mathematical Analysis:

Introducing the following undimensionl variables:

\[ x^* = \frac{x}{d}, y^* = \frac{y}{d}, u^* = \frac{u}{u_0}, t^* = \frac{t u_0}{d}, P^* = \frac{P}{\mu_f u_0 p}, T^* = \frac{T - T_0}{T_w - T_0}, \omega^* = \frac{d \omega_1}{u_0}, \]  

The system of equations (1) to (4) reduces to:

\[ a_0 \frac{du}{dt} = \lambda \varepsilon \exp(i\omega t) + \phi_2 \frac{d^2 u}{dy^2} - a_3 u + a_4 T \]
\[ u(0, t) = 0, \quad u(1, t) = 0, \quad t > 0 \]  
\[ b_0^2 \frac{dr}{dt} = \frac{d^2r}{dy^2} + b_1^2 T \]  
\[ T(0, t) = e^{i\omega t}, \quad T(1, t) = 1, \quad t > 0 \]

where

\[ a_0 = \phi_1 Re, \quad \phi_1 = (1 - \phi) + \phi \frac{\rho_s}{\rho_f}, \quad \phi_2 = \frac{1}{(1 - \phi)^2} + 5.10^4 \beta \phi \frac{\rho_f}{\mu_f} \sqrt{\frac{K_b T}{\rho_s \mu_i}} f(t, \phi, \text{etc.}), \]

\[ \phi_3 = (1 - \phi) + \phi \left( \frac{\rho \beta \rho_f}{\rho f} \right), \quad \phi_5 = 1 + \frac{3(\sigma - 1)\phi}{(\sigma + 2) - (\sigma - 1)\phi}, \quad Re = \frac{U_0 d}{v}, \]

\[ M^2 = \frac{\sigma_f}{\mu_f} B^2 d^2, \quad m_0^2 = \phi_2 M^2, \quad \frac{1}{K} = \frac{d^2}{K_0}, \quad Gr = \frac{\beta_f d^2}{v_f U_0} g(T_w - T_0), \]

\[ a_1 = \phi_3 \left( \frac{\rho \beta \rho_f}{\rho f} \right) g \frac{d^2}{U_0} (T_w - T_0) = \phi_3 Gr, \quad a_3 = m_0^2 + \frac{\phi_2}{K}, \quad a_4 = (1 - \phi) + \phi \left( \frac{\rho \beta \rho_f}{\rho f} \right) \]

\[ Pe = \left( \frac{\rho C_p}{\mu f} \right) \frac{U_0 d}{K_f}, \quad \lambda_n = \frac{((K_s + 2K_f) - 2\phi(K_f - K_s))}{((K_s + 2K_f) + \phi(K_f - K_s))} + 5.10^4 \beta \phi \frac{\rho \beta \rho_f}{K_f} \left( C_p \right) \frac{K_b T}{\rho_s \mu_i} f(t, \phi, \text{etc.}) \]

\[ Q = \frac{d^2Q_0}{K_f}, \quad N^2 = \frac{4a^2 d^2}{K_f}, \quad b_0^2 = \frac{\phi \beta e}{\lambda_n}, \quad b_1^2 = \frac{Q + N^2}{\lambda_n}, \quad m_1^2 = \frac{a_2}{\phi_2}, \quad a_2 = \frac{a_1}{\phi_2}, \quad \lambda_1 = \frac{\lambda}{\phi_2} \]

Re Reynolds Number, M magnetic parameter, Gr thermal Grashof Number, Pe Peclet Number, Q heat generation Parameter and N Radiation Parameter.

4. Method of Solution:

Equations (10) to (13) show that their solutions are not easily traceable. Therefore, perturbation method is a global approach to find the solution of such differential equations. Evidently, the parameter \( \varepsilon \) is assumed to be small such that \( 0 < \varepsilon < 1 \). In order to solve equations (10) to (13) assuming

\[ u(y, t) = u_0(y) + \varepsilon u_1(y) e^{i\omega t}, \]  
\[ T(y, t) = T_0(y) + \varepsilon T_1(y) e^{i\omega t}. \]

where \( u_0, T_0 \) and \( u_1, T_1 \) denote steady and unsteady parts of velocity and temperature distribution respectively.

Through straight forward calculations, the solutions of the equations (11) to (14) with the help of equations (15) and (16) are known and given by

\[ T(y, t) = \frac{\sin b_1 y}{\sin b_1} + \varepsilon \frac{\sin(m_1 - m_3 y)}{\sin m_3} - \sin(b_1 y) e^{i\omega t} \]  
\[ u(y, t) = -\frac{a_1}{b_1^2 + m_1^2} \sinh m_1 y + \frac{a_2}{b_1^2 + m_1^2} \sinh b_1 y + \varepsilon \left( J e^{m_1 y} + J e^{-m_1 y} + \frac{a_2}{m_1^2 + m_2^2} \left( \sin(m_1 - m_3 y) + \frac{\lambda_1}{m_2^2} \right) \right) e^{i\omega t} \]  

(18)
where
\[ m_2 = \sqrt{\frac{a_1 + a_2 \omega}{\phi}} , \quad m_3 = \sqrt{b_1^2 - b_2^2 \omega} , \quad J_1 = \frac{a_2 m_3}{m_3^2 + m_2^2} \left[ \frac{e^{-m_2}}{e^{m_2} - e^{-m_2}} \right] - \frac{\lambda_2}{m_2} \left[ \frac{1 - e^{-m_2}}{e^{m_2} - e^{-m_2}} + 1 \right], \]
\[ J_2 = -\frac{a_2 m_3}{m_3^2 + m_2^2} \left[ \frac{e^{m_2}}{e^{m_2} - e^{-m_2}} \right] + \frac{\lambda_2}{m_2} \left[ \frac{1 - e^{m_2}}{e^{m_2} - e^{-m_2}} + 1 \right]. \]

Rate of change of velocity and heat transfer are calculated from equations (17) and (18)
\[
(Nu)_{y=0} = \frac{b_1}{\sin b_1} - \varepsilon m_3 \frac{\cos m_3}{\sin m_3} e^{i\omega t} \tag{19}
\]
\[
(r)_{y=0} = -\frac{a_2 m_3}{(b_1^2 + m_2^2) \sinh m_3} + \frac{a_2 b_1}{(b_1^2 + m_2^2) \sin b_1} + \varepsilon \left( J_1 m_2 - J_2 m_2 - \frac{a_2 m_3}{m_3^2 + m_2^2} \frac{\cos m_3}{\sin m_3} \right) e^{i\omega t} \tag{20}
\]

5. Results and Discussion:

‘Figure 1’: Velocity profile for varying size of Ag particles in water based nanofluid when Gr = 0.2 , Re = 2 , Pe = 2 , M = 2 , \( \omega = 0.2 , \ t = 0.1 , \ \varepsilon = 0.01 , \ Q = 5 , \ \lambda = 1 , \ N = 2 , \ k_0 = 2 , \ \phi = 0.04. \)
'Figure 2': Velocity profile for varying size of Al$_2$O$_3$ particles in water based nanofluid when Gr = 0.2 , Re = 2 , Pe = 2 , M = 2 , $\omega = 0.2$ , $t = 0.1$ , $\varepsilon = 0.01$ , $Q = 5$ , $\lambda = 1$ , N = 2 , $k_0 = 2$ , $\phi = 0.04$.

'Figure 3': Velocity profile for varying $\phi$ of Ag particles in water based nanofluid when Gr = 0.2 , Re = 2 , Pe = 2 , M = 2 , $\omega = 0.2$ , $t = 0.1$ , $\varepsilon = 0.01$ , $Q = 5$ , $\lambda = 1$ , N = 2 , $k_0 = 2$ , $d = 100$ nm.
Figure 4: Velocity profile for varying $\phi$ of $\text{Al}_2\text{O}_3$ particles in water based nanofluid when $Gr = 0.2$, $Re = 2$, $Pe = 2$, $M = 2$, $\omega = 0.2$, $t = 0.1$, $\varepsilon = 0.01$, $Q = 5$, $\lambda = 1$, $N = 2$, $k_0 = 2$, $d = 100$ nm.

Figure 5: Velocity profile for varying $Q$ of Ag particles in water based nanofluid when $Gr = 0.2$, $Re = 2$, $Pe = 2$, $M = 2$, $\omega = 0.2$, $t = 0.1$, $\varepsilon = 0.01$, $\phi = 0.04$, $\lambda = 1$, $N = 2$, $k_0 = 2$, $d = 100$ nm.
Figure 6: Temperature profile for varying $\phi$ of Ag particles in water based nanofluid when $Gr = 0.2$, $Re = 2$, $Pe = 2$, $M = 2$, $\omega = 0.2$, $t = 0.1$, $\varepsilon = 0.01$, $\lambda = 1$, $Q = 5$, $N = 2$, $k_0 = 2$, $d = 100$ nm.

Figure 7: Temperature profile for varying $Q$ of Ag particles in water based nanofluid when $Gr = 0.2$, $Re = 2$, $Pe = 2$, $M = 2$, $\omega = 0.2$, $t = 0.1$, $\varepsilon = 0.01$, $\lambda = 1$, $\phi = 0.04$, $N = 2$, $k_0 = 2$, $d = 100$ nm.
6. Discussion:
Graphical results with discussion are included in this section. Koo and Kleinstreuer model is used to derive various results for silver and alumina – water based nano fluids. Using the thermophoretic properties water, silver and alumina, Charts 1-8 are plotted. ‘Figures 1-5’ are plotted for motion and ‘Figures 6-8’ are plotted for temperature distribution.
‘Figure 1’ depicts motion profile of silver nano particles of various size in H₂O based nanofluids. From this figure easy to understand that fluid velocity increases due to increasing in size of silver nano particles in water based nanofluids. The rising of motion with size of nanoparticles means that viscosity and thermal conductivity of nanofluids are decreasing. Which implies that in comparison to small size particles, large size particles has good possibility of generating clumps. ‘Figure 2’ is plotted to describe motion of alumina in water based nanofluids. This figure shows that dissemination of alumina nanoparticles in H₂O gives exactly same outcome as in study of silver nano particles in H₂O. One more point is for noting that we can adjoin nanoparticles in base fluids within a limit. ‘Figure 3’ shows with increase of volume fraction of nano particles in case of silver particles the velocity of nano fluid decreases and the reverse behavior is noted in case of alumina as plotted in ‘Figure 4’ although the base fluid is same in both the cases. This means in case of silver particles when we rising volume fraction of particles viscosity of fluid also rising and the velocity of fluid reduce due to this. Behavior of alumina particles is just opposite to silver, in case of increasing of volume fraction. ‘Figure 5’ is drawn to explain the variation of fluid velocity with respect to Q . Velocity profile in this case directly proportional to Q, as Q increases velocity of nano fluid is also rising. ‘Figure 6’ evaluate temperature profile of nano fluid with varying volume fraction. It is obvious from the graph that temperature of nanofluids increases with increase of volume fraction. ‘Figure 7’ is drawn to explain the variation of fluid temperature with respect to Q . Temperature profile in this case inversely proportional to Q, as Q increases temperature of nano fluid reduces. ‘Figure 8’ depicts the variation of radiation parameter with respect to temperature profile.
Logically, the rate of heat transfer of the fluid increases on rising radiation parameter, hence the temperature increases.

7. References:
[1] Abbas W 2017 Homotopy Perturbation Method for Solving MHD Nanofluid Flow with Heat and Mass Transfer Considering Chemical Reaction Effect Current Science International 6(1) pp 12-22.
[2] Asma K, Gul A, Khan I, Shafie S and Khan A 2015 Exact Solutions for Free Convection Flow of Nanofluids, with Ramped Wall Temperature The European Physical Journal Plus 130 pp 57-71.
[3] Buongiorno J 2005 Convective transport in nanofluids Journal of Heat Transfer: American Society of Mechanical Engineers 128 pp 240-250.
[4] Das S K 2003 Temperature Dependence of Thermal Conductivity Enhancement for Nanofluids Journal of Heat Transfer 125(4) pp 567-574.
[5] Das S and Jana R N 2015 Natural Convective Magneto-Nanofluid Flow and Radiative Heat Transfer Past a Moving Vertical Plate Alexandria Engineering Journal, 54(1) pp 55-64.
[6] Gul A, Khan I and Shafie S 2018 Radiation and heat generation effects in magnetohydrodynamic mixed convection flow of nanofluids Thermal Science 22 (1A) pp 51-62.
[7] Hail E and Shankar B 2014 Magnetohydrodynamic nanofluid flow of a stretching sheet with thermal radiation, viscous dissipation, chemical reaction and ohmic effects Journal of Nanofluids, 3 pp 227-237.
[8] Hail E and Shankar B 2015 A Steady Mhd boundary-layer flow of water-based nanofluids over a moving permeable flat plate International Journal of Mathematical Research 4 (1) pp 27-41.
[9] Hajipour M and Dehkordi A M 2012 Analysis of nanofluid heat transfer in parallel-plate vertical channels partially filled with porous medium International Journal of Thermal Sciences 55 pp 1-11.
[10] Jha B K, Samaila A K and Ajibade A O 2012 Natural Convection Flow of Heat Generating/Absorbing Fluid near a Vertical Plate with Ramped Temperature Journal of Encapsulation and Adsorption Sciences 2(4) pp 61-68.
[11] Koo J and Kleinstreuer C 2004 A New Thermal Conductivity Model for Nanofluids Journal of Nanoparticle Research 6(6) pp 577-588.
[12] Kumar K G, Gireesha B J, Krishnamurthy M R and Prasannakumara B C 2018 Impact of Convective Condition on Marangoni Convection Flow and Heat Transfer in Casson Nanofluid with Uniform Heat Source Sink Journal of Nanofluids 7(1) pp 108–114.
[13] Makinde O D and Mhone P Y 2005 Heat Transfer to MHD Oscillatory Flow in a Channel Filled with Porous Medium Romanian Journal of Physics 50 pp 931-938.
[14] Olawale O J 2018 On the Effect of Nanofluid Flow and Heat Transfer with Injection through an Expanding or Contracting Porous Channel Journal of Computational Applied Mechanics 49(1) pp 1-8.
[15] Prakash D and Suriyakumar P 2017 Transient hydromagnetic convective flow of nanofluid between asymmetric vertical plates with heat generation International Journal of Pure and Applied Mathematics 113(12) pp 1-10.
[16] Qasim M, Khan Z H, Khan W A and Shah I A 2014 Mhd Boundary Layer Slip Flow and Heat Transfer of Ferrofluid along a Stretching Cylinder with Prescribed Heat Flux PloS One 9 https://doi.org/10.1371/journal.pone.0083930
[17] Raptis A 1998 Radiation and free convection flow through a porous medium Int. Comm. In Heat and Mass Tran. 25 pp 289-295.
[18] Sheikholeslami M and Shehzad S A 2018 Simulation of water based nanofluid convective flow inside a porous enclosure via non-equilibrium model International Journal of Heat and Mass Transfer 120 pp 1200–1212.
[19] Turkyilmazoglu M 2014 Unsteady Convection Flow of Some Nanofluids Past a Moving Vertical Flat Plate with Heat Transfer *Journal of Heat Transfer* **136** (3) ID 031704.

[20] Uddin M J, Kalbani K S, Rahman M M, Alam M S, Al-Salti N and Eltayeb I A 2016 Fundamentals of Nanofluids: Evolution, Applications and New Theory *International Journal of Biomathematics and Systems Biology* **2**(1) pp 1-32.

[21] Xu H and Cui J 2018 Mixed convection flow in a channel with slip in a porous medium saturated with a nanofluid containing both nanoparticles and microorganisms *International Journal of Heat and Mass Transfer* **125** pp 1043–1053.

[22] Xuan Y, Li Q and Hi W 2003 Aggregation Structure and Thermal Conductivity of Nanofluids *AIChE Journal* **49**(4) pp. 1038-1043.

[23] Said Z, Mohammed H A and Saidur R 2013 Mixed convection heat transfer of nanofluids in a lid driven square cavity: a parametric study *International Journal of Mechanical and Materials Engineering* **8**(1) pp 48-57.