Coleman-Weinberg Mechanism
and Interaction of D3-Branes in Type 0 String Theory

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Abstract

The low-energy theory on the world volume of parallel static D3-branes of type 0 strings is the Yang-Mills theory with six scalar fields in the adjoint representation. One-loop corrections in this theory induce Coleman-Weinberg effective potential, which can be interpreted as an interaction energy of D3-branes. The potential is repulsive at short distances and attractive at large ones. In the equilibrium, a large number of D3-branes forms a spherical shell with the radius proportional to the characteristic energy scale of the world-volume theory.
1 Introduction

The duality of $\mathcal{N} = 4$, $D = 4$ supersymmetric Yang-Mills theory to the type IIB string theory on the near-horizon geometry of the three-brane [1, 2, 3] have led to considerable progress in understanding of its large-$N$ limit. Along the same lines, type 0B theory on the background of RR charged three-brane solution was proposed to give a dual description of non-supersymmetric $D = 4$ gauge theory coupled to adjoint bosonic matter [4]. Perturbatively unstable tachyon of type 0 theory was argued to be stabilized in the presence of the three-brane through interaction with background RR flux.

The low-energy theory on the world volume of $N$ coincident type 0B D3-branes [3] is $U(N)$ gauge theory with six scalar fields in the adjoint representation. According to [4], this theory has a dual description in terms of type 0B strings on the background of the three-brane. The classical gravity approximation to the dual picture already involves qualitative features expected from the gauge theory, such as logarithmic dependence of the coupling on a scale with UV fixed point at zero coupling [3, 6]. This duality also predicts IR fixed point at infinity [3].

The gauge theory considered in [4, 6, 7] has the same tree-level bosonic action as $\mathcal{N} = 4$ SYM theory. So, the scalar potentials in both theories have the same flat directions. These flat directions correspond to transverse coordinates of D3-branes. But, unlike in $\mathcal{N} = 4$ theory, in the non-supersymmetric case the flat directions are not protected from being lifted by quantum corrections, which reflects the fact that parallel D3-branes of type 0 strings interact with one another, while type II D-branes are BPS states and they can be moved apart at no energy cost. Qualitative arguments based on the string calculation of the interaction potential suggest that type 0 branes attract at large distances [4].

We study the interaction between type 0 D3-branes computing the one-loop effective potential in the world-volume field theory. Similar calculations for $\mathcal{N} = 4$ SYM theory with the supersymmetry broken by the finite temperature [8] were done in [9]. We find that the potential has a maximum at zero separation between branes (at zero expectation values of scalar fields) and gains a minimum at finite separation due to Coleman-Weinberg mechanism [10].

2 Interaction potential

The tree-level action of the low-energy theory on the world volume of $N$ parallel D3-branes of the type 0 string theory is

$$S = \frac{1}{g_{\text{YM}}^2} \int d^4 x \, \text{tr} \left\{ -\frac{1}{2} F_{\mu\nu}^2 + \left( D_{\mu} \Phi^i \right)^2 - \frac{1}{2} [\Phi^i, \Phi^j]^2 \right\}. \tag{2.1}$$

We consider the theory in the Euclidean space from the very beginning. Strictly speaking, the field theory with the action (2.1) is not renormalizable and require counterterms quadratic and quartic in scalar fields. In what follows we imply that all necessary counterterms are added to the action.

The scalar potential in (2.1) has a degenerate set of minima:

$$\Phi^i_{\text{cl}} = \text{diag}(y^i_a), \quad a = 1, \ldots, N. \tag{2.2}$$
The coordinates $y^i_a, i = 1, \ldots, 6$ describe positions of $N$ parallel static three-branes in nine-dimensional space. Since the potential in (2.1) does not depend on $y^i_a$, D-branes do not interact at the classical level.

However, one-loop corrections induce interaction between branes via the Coleman-Weinberg mechanism. We calculate the interaction potential expanding scalar fields around the classical background (2.2):

$$\Phi^i = \Phi^i_{cl} + \phi^i$$  \hspace{1cm} (2.3)

and integrating out quantum fluctuations. To integrate over the gauge fields, we add to the action the gauge fixing term:

$$S_{gf} = -\frac{1}{g^2_{YM}} \int d^4x \, \text{tr} \left( \partial_\mu A_\mu - \alpha [\Phi^i_{cl}, \Phi^i] \right)^2,$$  \hspace{1cm} (2.4)

where $\alpha$ is a gauge fixing parameter. The action for ghosts in the chosen gauge is

$$S_{gh} = \frac{1}{g^2_{YM}} \int d^4x \, \text{tr} \left( \partial_\mu \bar{c} D_\mu c - \alpha [\Phi^i_{cl}, \bar{c}][\Phi^i, c] \right).$$  \hspace{1cm} (2.5)

Expanding the action to the second order in fluctuations and integrating them out we get the one-loop effective potential:

$$\Gamma = \frac{1}{2} \text{Tr} \ln \left[ \left(-\partial^2 + Y^2\right) \delta_{\mu\nu} + \left(1 - \frac{1}{\alpha}\right) \partial_\mu \partial_\nu \right] - \text{Tr} \ln \left(-\partial^2 + \alpha Y^2\right) + \frac{1}{2} \text{Tr} \ln \left[\left(-\partial^2 + Y^2\right) \delta^{ij} - (1 - \alpha)Y^i Y^j \right].$$  \hspace{1cm} (2.6)

The first term is the contribution of the gauge fields, the second is that of the ghosts, and the third of the scalars. By $Y^i$ we denote the following matrix in the adjoint representation of $U(N)$:

$$Y^i = [\Phi^i_{cl}, \cdot].$$  \hspace{1cm} (2.7)

Taking into account that $[Y^i, Y^j] = 0$, we find:

$$\Gamma = 4 \text{Tr} \ln \left(-\partial^2 + Y^2\right) = \text{Vol} \int \frac{d^4p}{(2\pi)^4} \, \text{tr} \ln \left(p^2 + Y^2\right) = \text{quadratically divergent term} + \text{Vol} \frac{1}{8\pi^2} \text{tr} Y^4 \ln \frac{Y^2}{M^2},$$  \hspace{1cm} (2.8)

where $M$ is an UV cutoff. The quadratic and the logarithmic divergencies in the effective action should be canceled by appropriate counterterms.

The matrix $Y^2$ has eigenvalues $(y_a - y_b)^2$, so the one-loop corrections induce only two-body interactions of D-branes – the interaction potential is

$$\Gamma = \text{Vol} \frac{1}{4\pi^2} \sum_{a < b} |y_a - y_b| \ln \frac{|y_a - y_b|^2}{\Lambda^2},$$  \hspace{1cm} (2.9)

where $\Lambda$ is a non-perturbative mass scale of the world-volume theory.
3 Equilibrium configuration of D-branes.

The potential of interaction between two D-branes (fig. 1):

\[
V(r) = \frac{1}{4\pi^2} r^4 \ln \frac{r^2}{\Lambda^2},
\]

is such that D-branes repulse at short distances. Thus, a stack of D-branes put on top of each other is unstable. The D-branes will tend to separate by distances of order \(\Lambda\). In the equilibrium, transverse coordinates of D-branes satisfy the equations:

\[
\sum_b (y_a^b - y_a^b)|y_a - y_b|^2 \ln \frac{|y_a - y_b|^2}{\Lambda^2 e^{-1/2}} = 0.
\]

We are interested in the case when the number of D-branes, \(N\), is large. In the \(N \to \infty\) limit, D-branes form continuous spherically symmetric distribution and the equation (3.2) takes the form:

\[
y_a^b f(|y_a|^2) = 0.
\]

The function \(f\), in principle, can have several zeros, but we adopt rather natural assumption that for the configuration of D-branes which has a minimal energy the equation (3.3) has only one root, \(|y|^2 = R^2\). So, the D-branes in the equilibrium will form a spherical shell of radius \(R\) in the six-dimensional transverse space with a surface RR charge density \(\rho = N/\pi^3 R^5\).

From eqs. (3.2), (3.3) we find:

\[
R = \Lambda e^{-\frac{\ln 8}{240}},
\]

and the interaction energy per unit volume, which shifts the tension of a D-brane, is

\[
\Delta T = -\frac{7N^2\Lambda^4}{48\pi^2} e^{-\frac{\ln 8}{240}}.
\]

*We use the string units, \(\alpha' = 1\).
4 Discussion

We have calculated the interaction potential between D3-branes of type 0B string theory at weak coupling in the world-volume field theory. As long as the one-loop approximation can be trusted, the field theory predicts the repulsion of type 0 D3-branes at short distances and the attraction at large ones. As a result, D3-branes tend to spread over the distances that are determined by a characteristic scale of the low-energy world-volume theory. Such behavior is not expected in the case of dyonic branes discussed in [12], because the field theory on their world volume is conformal.

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†The definition of the YM coupling in terms of VEVs of the dilaton and the tachyon is discussed in [7, 11].