Supplementary Information: Nanomechanical Spectroscopy of 2D Materials

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1. **Finite element method (FEM) simulations**

For the FEM modelling we use the structural mechanics module of COMSOL Multiphysics Version 5.5. We build a model around the suspended area partially including the silicon support (Fig. S1a) and use a swept mesh for the thin layers with high density around for TMD flake and SiN window (Fig. S1b). To determine the resonance frequency and mode shape (Fig. S1c) we conduct a prestressed eigenfrequency study. To include the effect of laser heating, we add a study step to implement a Gaussian heat source (Fig. S1d, 30.4% absorption and 30 µW laser power – comp. Fig. 3a main paper) and calculate the heat profile upon laser heating of the center of the suspended TMD (Fig. S1e). This allows us to determine the conversion factor, which captures the tuning of the fundamental mode with laser power, following Eq. 1 from the main paper. The conversion factor slightly depends on wavelength of the heating laser because the laser spot size varies with wavelength, what results in a slightly different heat profile in the suspended TMD. To account for this, we measure the spot size of heating laser at different wavelengths and use this as input for our simulations. In Fig. S1f we plot the conversion factor for device #1 (3L WSe$_2$). For device #2 (4L MoS$_2$), we obtain a conversion factor in the range of 461 to 476 Hz/µW showing comparable scaling with wavelength as device #1. The difference between devices here is due to different thermal conductivities, hole sizes and layer thickness between devices.
In order to optimize the dimensions of the SiN window, we simulate the driven mechanical resonances in the frequency domain. We start by simulating a circular TMD-only drum resonator (diameter 10 µm) as reference and adjust the isotropic damping to match the experimental Q for such resonators (~100). We then simulate the entire hybrid device (including SiN and gold). In Fig. S2a we plot the simulated displacement vs. frequency for the hybrid device probed on the SiN area, 2 µm away from the suspended TMD area. Again, we adjust the isotropic damping in SiN and gold to match experimental values. We now vary the SiN window size and extract the amplitude of motion at constant drive (signal strength, plotted in Fig. S2b). As expected, larger devices oscillate at large amplitudes providing more signal. Nevertheless, while oscillating at a higher amplitude, larger devices are less responsive to heating. Indeed, in Fig. S2c we plot the relative responsivity (change of resonance frequency for a constant laser heating) vs. window size. Combining the insights from Fig. S2b,c we choose a window size of 20 µm as a reasonable compromise between high vibrational amplitude and high responsivity.
Figure S2 Finding ideal device parameters a) Simulated mechanical motion of the hydride with Q matching experimental results b) Simulated amplitude (signal strength) vs. SiN window size b) Relative responsivity to laser heating vs. SiN window size. We choose a window size of 20 µm (orange spot) as a compromise between high responsivity and sufficient amplitude amplification.

All material properties used in our simulations are summarized in table 1. For quantities that show a large spread in the literature values (values for the TMD materials in particular) we used average values. In general, we preferably choose experimental references for suspended samples of the suitable layer thickness.

| Material  | Quantity                      | Value          | Reference                      |
|-----------|-------------------------------|----------------|-------------------------------|
| MoS\(_2\) | Young’s modulus \(E\)         | 330 GPa        | 1 In agreement with AFM force-indentation measurements (see below) |
|           | Poisson’s ratio \(\nu\)      | 0.125          | 1                             |
|           | Density \(\rho\)             | 5060 kg/m\(^3\) | 2                             |
|           | Thermal conductivity \(\kappa\) | 60.3 W/(m·K)  | 3–6                           |
|           | Thermal expansion coefficient \(\alpha\) | \(7.6 \cdot 10^{-6}\) 1/K | 7                             |
|           | Heat capacity at constant pressure \(c_p\) | 397 J/(kg·K)) | 9,10                          |
|           | Built-in stress (tension) \(\sigma_0\) | 44.7 MPa \((\sigma_{2D} = 0.11 \text{ N/m})\) | Force-indentation AFM |
|           | Layer thickness \(d\)        | 0.615 nm       | 11                            |
| Material | | Young’s modulus $E$ | | Poisson’s ratio $\nu$ | | Density $\rho$ | | Thermal conductivity $\kappa$ | | Thermal expansion coefficient $\alpha$ | | Heat capacity at constant pressure $c_p$ | | Built-in stress (tension) $\sigma_0$ | | Layer thickness $d$ |
|---------|---------|-------------|---------|-------------|---------|-------------|---------|-------------|---------|-------------|---------|---------|
| WSe$_2$ | | 167.3 GPa  | | 0.19      | | 9320 kg/m$^3$ | | 26.5 W/(m·K) | | $7 \cdot 10^{-6}$ 1/K | | 188 J/(kg·K) | | 46.2 MPa $(\sigma_{2D} = 0.09 \text{ N/m})$ | | 0.651 nm |
| Au     | | 78.5 GPa   | | 0.42      | | 19300 kg/m$^3$ | | 312 W/(m·K)  | | $14 \cdot 10^{-6}$ 1/K | | 130 J/(kg·K) | | 160 MPa  | |   |
| SiN    | | 232 GPa    | | 0.23      | | 2810 kg/m$^3$ | | 31 W/(m·K)   | | $2.55 \cdot 10^{-6}$ 1/K | | 887 J/(kg·K) | | 240 MPa  | | Norcada (manufacturer) |
| Material | Property                        | Value  | Notes   |
|----------|---------------------------------|--------|---------|
| Si       | Young’s modulus $E$             | 160 GPa|         |
|          | Poisson’s ratio $\nu$           | 0.27   |         |
|          | Density $\rho$                  | 2330 kg/m³ |         |
|          | Thermal conductivity $\kappa$   | 160 W/(m·K) |         |
|          | Thermal expansion coefficient $\alpha$ | $3 \cdot 10^{-6}$ 1/K |         |
|          | Heat capacity at constant pressure $c_p$ | 692 J/(kg·K) |         |
|          | Pretension                      | 0      | Irrelevant for simulations |

2. **AFM force indentation**

One crucial parameter, which is known to vary from device to device is the built-in tension and 2D elastic modulus. To eliminate this uncertainty in our simulations, we perform force indentation measurements in the centre of the membrane (following Ref. 21) and extract the built-in tension and 2D elastic modulus for each sample. We use cantilevers of intermediate stiffness ($k \approx 3$ N/m) and only apply small loads (150 nN) to avoid damaging the sample. In Fig. S3a we show a force-displacement-curve for device #1. We account for cantilever bending and deformation of the SiN membrane. We fit a curve following:

$$F = \pi \sigma 2d d + q^3 \frac{E_{2D}}{\pi^2} d^3$$  \hspace{1cm} (S1)

Here $a$ is the radius of the drum, and $q = \frac{1}{1.05 - 0.15\nu - 0.16\nu^2}$ is a dimensional factor dependent on the Poisson’s ratio $\nu$ ($q = 0.98$ for WSe$_2$ and $q = 0.97$ for MoS$_2$). For a range of samples, we find a linear dependence on tension with layer thickness (Fig. S3b). We attribute the observed homogeneity to the cleanliness and uniformity in our samples after annealing (comp. Fig. S4).
Figure S3 AFM force indentation to determine pre-tension. a) Force vs. displacement as well as a fit to Eq. S1. We extract a pretension of roughly 0.1 N/m for most our devices b) Statistics on pre-tension vs. thickness. We find a linear relation between pre-tension and layer thickness in our devices.

3. Sample fabrication and overview

To fabricate our hybrid devices, we transfer TMDs onto a circular hole using the all-dry PDMS method. The SiN chip is beforehand covered with a thin layer of gold (30 nm) to electrically contact the TMD and to increase its reflectivity. After transfer we perform an annealing step (3 h, 200 °C) in vacuum to remove residues and assure good adhesion to the substrate. We fabricate and measure multiple samples. Microscope images and AFM topography scans for device #1-3 are shown in Fig. S4a,b,d,e,g,h. For all samples (Fig. S4c,f,i), we find a high Q fundamental mode of almost constant (except device #3, which has a thicker gold layer). There are some variations in frequency, because the hole size and gold thickness are different for the devices. Our simulations (grey dashed lines) describe the measured frequencies well (Fig. S4c,f,i).
Figure S4 Sample overview. Microscope images (a,d,g) and AFM topography (b,e,h) of device #1-3. The samples are uniform and well attached to the substrate c,f,i) Displacement (amplitude) vs. frequency for device #1-3. We find a dominant high-Q fundamental mode for all samples. The resonance frequencies match with simulated values (grey dashed lines).

4. Details on interferometric motion detection

The sample is placed upside down in a vacuum chamber of $<10^{-5}$ mbar. By applying a DC+AC voltage relative to the non-reflective gate electrode, we mechanically actuate the suspended area of the chip. The motion of the TMD is detected by a Michelson interferometer. We focus a 632.8 nm HeNe laser ($<1 \mu$W, ~1.5 µm spot size) on the SiN area of the sample and the reflected light is superimposed with the light coming from the reference arm and guided into an avalanche photodetector. The resulting interference signal is highly sensitive to relative displacements and allows us to detect the motion of suspended samples. We actively stabilize the relative position of the reference arm via a mirror on a piezo and thereby ensure constant interference conditions and good signal strength over a large period of time. In addition to the probe laser, we implement an excitation laser of tunable colour (1.2 – 3.1 eV, blue in Fig. 1c). We use a band pass filter (BP) to block the excitation laser from reaching the detector and overloading it. The large separation (40 µm) of the non-reflective gate and sample negates all
cavity-related optomechanical backaction effects and allows us to measure purely static heating effects in our sample over a very large range of photon energies.

The interferometric setup is shown in detail in Fig. S5a. Along the beam path of the probe laser, we first implement an optical isolator to avoid back reflected light into the laser, which can cause instabilities and power fluctuations. The beam is then expanded to completely fill the objective (40x 0.6NA). In a first beam splitter we add light from the excitation laser and in a second beam splitter, we guide half the light towards the reference arm and half through the objective onto the sample in a vacuum chamber. The relative position of the reference arm to the sample determines the amplitude of the interferometric signal. We use a piezo electric element to control this distance and stabilize the system using a PID-loop locked to a small reference signal at 941 Hz sourced by Lock-In amplifier (Zurich Instruments MLFI). The sample in the vacuum chamber is clamped upside down onto our sample holder and with a spacing of roughly 40 µm, we place our grounded gate electrode. Electrical driving is realized by mixing a DC voltage (210 V, supplied by a Keithley source meter) with an AC component (typically -5 dBm) from our vector network analyzer (VNA, Agilent E5071C) in a high voltage Bias T (Particulars BT-01) and applying it to the gold layer of the sample, which contacts the TMD flake. For smaller frequency ranges and phase-locked-loop (PLL) measurement, we use a lock-in amplifier (Zurich Instruments MFLI). In Fig. S5b we show the power spectra of our excitation laser source (measured at the sample position) with different neutral density filters (ND) implemented, which are used to calculate the relative frequency shifts \( \Delta f \). We perform a small linear correction (order of Hz) to account for temperature changes in the room during measurements of the maps (Fig. 2 a,c). In the PLL-configuration (25 kHz bandwidth) we can measure the heating induced frequency shifts \( \Delta f \) quickly and with high sensitivity even at low laser powers (raw data for ND 1.5 in Fig. S5c).
**5. Consideration of dynamical back-action effects**

In nanomechanical resonators also dynamic optomechanical back-action (in contrast to static heating) effects can alter the resonance frequency \( (f) \) and its FWHM \( (f_{FWHM}) \) especially at large laser powers.\(^{23-25}\) This occurs e.g. in cavity interferometers, where the laser power, which the oscillating membrane is exposed to, varies significantly over a short spatial distance.\(^{23,24}\) For this a reflective surface close to the moving membrane is needed.\(^{23,24}\) The effects furthermore only occur when the spatial symmetry is broken due to deforming the membrane out of plane.\(^{23,24}\) In our system the gate is non-reflective and far away from the membrane (~ 40 µm). Additionally, the applied electrostatic pressure by the gate voltage is relatively small and SiN-TMD hybrid system rather stiff, so there is no breaking of symmetry in out of plane direction. Considering all the points above, we can exclude cavity related back-action effects in our system.

Also, strain-induced shifts in absorption in the material itself can cause dynamic back-action effects.\(^{25}\) Here again a breaking of symmetry, large laser powers and soft systems (small spring constant) are needed. We therefore also exclude material related back-action effects.
To verify this experimentally we extract $f_{FWHM}$, whilst illuminating the sample at different wavelengths (Fig. S6a,b). If there were any dynamic back-action effects influencing the system, the $f_{FWHM}$ should show significant variations.\textsuperscript{23–25} We do not observe such variations and thereby experimentally confirm the absence of dynamic back-action effects.

![Figure S6](image_url)

**Figure S6 Reference measurements check for dynamic back-action effects. a,b) FWHM vs. wavelength for device #1 (WSe$_2$) and #2 (MoS$_2$) and photoluminescence measurements as reference for the excitonic resonances. We observe a constant FWHM over the entire wavelength range and thereby experimentally exclude dynamic optomechanical back action effects.**

6. **Springs in parallel model (derivation of Eq.1)**

The goal of equation 1 from the main text is to intuitively relate the heating-induced change of the overall resonance frequency of our resonator to tension/frequency changes of its components, i.e. the TMD alone and the SiN alone. That expression is important for developing a qualitative understanding of our system, whilst we use FEM simulations to capture the complex device geometry for all quantitative evaluations and results shown in the main text. We express frequencies ($f$) and frequency changes ($\Delta f$) of each resonator upon illumination/heating via effective spring constants defined as $k = 4\pi^2 f^2 m_{eff}$, where $m_{eff}$ is the numerically determined effective mass\textsuperscript{4} (see Fig. S7).

So, the question we would like to answer: is there a simple expression relating effective spring constants of the SiN ($k_{SiN}$), the TMD ($k_{TMD}$), and the compound system ($k_{total}$)?
As a first step towards obtaining a simple model, we numerically obtain the frequency and frequency changes upon illumination (power 30 µW; 30.4% absorption) of our 3 resonators using detailed FEM-simulations of the experimental geometry (see Fig. S7 a-c). From the resonance frequencies and effective masses, we find: $k_{TM D} \approx 0.52 \, N/m$, $k_{SiN} \approx 50.43 \, N/m$ and $k_{total} \approx 65.48 \, N/m$. For the heating-induced frequency changes upon laser illumination with 30 µW laser power and 30.4% absorption (corresponds to the measurement in Fig. 3a @ 2.92 eV), we find: $\Delta k_{TM D} \approx -0.16 \frac{N}{m}$, $\Delta k_{SiN} \approx -0.01 \frac{N}{m}$ and $\Delta k_{total} \approx -0.20 \frac{N}{m}$. These simulations match experiments: from the experimentally measured heating-induced frequency shift, we extract:

$$\Delta k_{Exp}^{total} = - k_{Exp}^{total} \left( 1 - \frac{(f - \Delta f)^2}{f_0^2} \right) = -64.45 \frac{N}{m} (1 - 0.9970) \approx -0.19 \, N/m$$

(with $f_0 = 4.67 \, MHz; \Delta f = 7.2 \, kHz$). Moreover, $\Delta k_{total}$ linearly depends on $\Delta k_{TM D}$. We see that our numerical results can be described with reasonable precision by a simple formula, $k_{total} = k_{TM D} + k_{SiN}$.

This formula corresponds to effective springs of the TMD and the SiN connected “in parallel”. In fact, this expression can be derived analytically in a 1D toy model of the combined TMD/SiN resonator (see Fig. S7d). We start by assuming that the hole in the SiN does not affect the effective elastic constants of SiN (good approximation when the hole is small) and simplify the membrane profile for ease of estimates (Fig. S7d). In this geometry we can approximate the extension of the central point of the membrane as:

$$\delta x \approx \frac{\chi^2}{2L} \quad (S2)$$
Approximating that both TMD and SiN have the same strain ($\epsilon$), we can now define the potential and kinetic energy as following:

$$E_{pot} = 2\sigma \delta x = 2\epsilon (h_1 E_{2D}^{SiN} + h_2 E_{2D}^{TMD}) \delta x = \frac{\epsilon}{E} (h_1 E_{2D}^{SiN} + h_2 E_{2D}^{TMD}) x^2$$  \hspace{1cm} (S3)$$

$$E_{kin} = \frac{m(\delta x)^2}{2} \approx \frac{m(x)^2}{2}$$  \hspace{1cm} (S4)$$

From the conservation of energy, we obtain the angular frequency of the harmonic motion of the combined system:

$$\omega^2 = \frac{2\epsilon (h_1 E_{2D}^{SiN} + h_2 E_{2D}^{TMD})}{L m_{eff}}$$  \hspace{1cm} (S7)$$

Defining an effective spring constants for the resonator, with $k_{SiN/TMD} = \frac{2\epsilon}{E} (h_1 E_{2D}^{SiN} + h_2 E_{2D}^{TMD})$, we see that the resonance frequency of the combined system can be expressed as:

$$f_0 = \frac{1}{2\pi \sqrt{L m_{eff}}} \sqrt{k_{SiN} + k_{TMD}}$$  \hspace{1cm} (S8)$$

This is exactly the resonance frequency of the resonator with TMD and SiN springs “in parallel”.

Next, we will look at changes in frequency ($\Delta f = f(T) - f_0$) caused by laser heating. With the heating laser turned on, light is absorbed, and the TMD resonator heats up and overall softens. The SiN is well heat sunk via the gold layer. Our simulations show that its temperature and stress remain almost constant ($\Delta T \approx 0.03$ K, $\frac{\Delta \sigma}{\sigma_0} \approx 0.02\%$, for 30.4% absorption and 30 $\mu$W incident laser power).

Therefore, we assume that $k_{SiN}$ is temperature-independent and express the resonances frequency with laser heating as:

$$\Delta f = \frac{1}{2\pi \sqrt{L m_{eff}}} \sqrt{k_{SiN} + k_{TMD}} - f_0 - \Delta k_{TMD} - f_0$$  \hspace{1cm} (S9)$$

We now expand the term to first order for $\frac{\Delta k_{TMD}}{k_{TMD} + k_{SiN}} \ll 1$ and obtain
\[ \Delta f = f_0 \left( 1 - \frac{\Delta k_{TMD}}{k_{TMD} + k_{SIL}} - 1 \right) \approx f_0 \frac{\Delta k_{TMD}}{2(k_{TMD} + k_{SIL})} \]  

(S10)

For the “TMD-spring”, we can relate the change in spring constant to a change in built-in tension:

\[ \frac{\Delta k_{TMD}}{k_{TMD}} = \frac{\Delta \sigma}{\sigma_0} \]  

(S11)

The change in tension due to thermal expansion is given by:

\[ \Delta \sigma = \frac{a E_{2D}}{1 - \nu} \Delta T, \]  

(S12)

Where \( \alpha \) is the thermal expansion coefficient, \( E_{2D} \) is the 2D elastic modulus and \( \nu \) is the Poisson’s ratio of the TMD. The change in temperature \( \Delta T \) is proportional to the amount of absorbed laser power:

\[ \Delta T = \frac{\beta \text{Abs}(\lambda)}{\kappa h} \Delta P, \]  

(S13)

where \( \kappa \) is the thermal conductivity, \( h \) is the thickness of the membrane and \( \beta \) is a pre-factor determined by the temperature profile in the membrane. Combining Eq. S11-13 we obtain Eq. 1 from the main text:

\[ \Delta f \approx f_0 \frac{k_{TMD}}{2(k_{TMD} + k_{SIL})} - \sigma_{TMD} \frac{a E_{2D} \beta \text{Abs}(\lambda)}{h \kappa} \Delta P \]  

(S14)
Figure S7 Numerical evaluation of spring constants and springs in parallel model. a-c) Mode shape of the individual resonances of the substrate (SiN+gold), the TMD resonator and the combined system. d) Sketch for simplified model describing the springs in parallel.

7. Transmission measurement as benchmark

In order to validate our nanomechanical measurement approach and backup our simulations, we perform optical transmission measurements with an objective below and above the sample (Fig. S8a). We use a broadband white light laser and measure the transmission through the sample (Fig. S8b), deduct the dark counts and normalize to an empty hole without any TMD material to obtain the transmission:

\[ T = \frac{T_{\text{sample}} - T_{\text{dark}}}{T_{\text{hole}} - T_{\text{dark}}}. \]  

(S11)

To calculate the amount of absorbed light we also measure reflection

\[ R = \frac{R_{\text{sample}} - R_{\text{hole}}}{R_{\text{mirror}} - R_{\text{dark}}}. \]  

(S12)
and use $\textit{Abs} = 1 - R - T$ to calculate the amount of absorbed light (Fig. S8c). Overall we find very good agreement between this new transmission measurements (blue) and previously obtained data from nanomechanical spectroscopy (red).

8. Reflection measurements

The setup presented in the main text also allows us to perform reflection measurements. We block the reference arm, turn off the probe laser and then and use our tunable excitation light source to sweep the wavelength whilst recording the reflected signal off our sample using a chopper (920Hz) and the lock-in amplifier (Fig. S9a, green). We then subtract spectra from that from an empty hole as shown in Fig. S8a (Fig. S9a, blue) and normalize the data by dividing by a “100% reflection reference”, which we obtain measuring reflection of a silver mirror (Fig. S9a, red) with known reflection properties (Thorlabs PF10-03-P01). The resulting reflection data is shown in Fig. S9b.
9. Obtaining the dielectric function

Reflection and transmission of electromagnetic waves was computed with the transfer matrix formalism. Two types of matrices are required: a propagation matrix \( P \) and a boundary matrix \( T \). The propagation matrix contains elements responsible for phase change inside a material

\[
P(\lambda, n, d) = \begin{pmatrix} e^{2\pi ind/\lambda} & 0 \\ 0 & e^{-2\pi ind/\lambda} \end{pmatrix},
\]

where \( n \) is complex refractive index of the material, \( \lambda \) wavelength of light, \( d \) is the thickness of the material. Whereas the boundary matrix depends on the refractive indices on both sides of the boundary \( n_1 \) and \( n_2 \):

\[
T(n_1, n_2) = \frac{1}{t_{12}} \begin{pmatrix} 1 & r_{12} \\ r_{12} & 1 \end{pmatrix}.
\]

The \( t_{12} = \frac{2n_1}{n_1 + n_2} \) is a Frensel transmission coefficient for oblique incidence and the \( r_{12} \) is a reflection coefficient \( \frac{n_1 - n_2}{n_1 + n_2} \). The overall transfer matrix \( M \) of vacuum suspended TMDC yields

\[
M = T(n_{\text{vacuum}}, n_{\text{TMDC}}).P(\lambda, n_{\text{TMDC}}, d).T(n_{\text{TMDC}}, n_{\text{vacuum}}).
\]
For each wavelength, we compute the refractive index \( n_{TMDC} = n + ik \) using matrix elements. The system of two equations is solved for two variables \( n,k \).

\[
\begin{aligned}
\text{Trans} &= 1 - \text{Abs} - \text{Refl} = 1 - |M_{11}|^2 \\
\text{Refl} &= |M_{21}|^2 / |M_{11}|^2,
\end{aligned}
\]  
(S16)

\( \text{Abs}, \text{Refl} \) are experimentally obtained absorption and reflection, respectively. The dielectric function \( \varepsilon \) is obtained using relation \( \varepsilon = n^2 \).

10. RPA and BSE calculations

To determine the theoretical response function the ground-state of the material was first calculated using density functional theory (DFT). Within DFT, the exchange-correlation energy was approximated by the local density approximation (LDA), which is well known for underestimating the band gap of insulators and semiconductors. In order to estimate the experimental direct band gap \( G_0 W_0 \) calculations were performed and the DFT band-structure was then corrected by the scissor operator to obtain the correct direct band gap.

This corrected band-structure was then used to determine the response function of the material. In order to account for excitonic effects the Bethe Salpeter equation (BSE) was solved. Solving the BSE is computationally very demanding and hence the BSE Hamiltonian was diagonalized in a restricted active space of a few bands around the Fermi level. However, the consequence of this restriction is that the response function is only determined in a limited low energy window around the band-gap. In order to obtain the response function at higher energies, where excitonic effects are negligible, we use the so-called random-phase approximation (RPA) within linear response time-dependent density functional theory (TDDFT). This procedure does not account for excitonic effects, but bands up to 100 eV above the Fermi energy are included and is an accurate method for determination of response function away from the band-gap energies.

Computational parameters: Spin-orbit coupling was included for all calculations. For the DFT calculations the in-plane lattice parameter for WSe\(_2\) (MoS\(_2\)) was 3.28 Å (3.16 Å) with an interlayer
spacing of 6.48 Å (6.15 Å), a distance of 3.34 Å (3.17 Å) between the chalcogens in each layer, and vacuum spacing between top and bottom layers of at least 12 Å for both the tri- and tetra-layer calculations. A k-point grid of 30x30x1 was used in all cases. The BSE hamiltonian was diagonalized in the restricted active space of 8 valence and 8 conduction states around the Fermi level. In order to account for many-body effects we have performed a single shot, finite temperature (a temperature of 500 K was used), all electron, spin-polarized GW calculations, where the spectral function on the real axis is constructed using a Pade approximation. Spin-orbit coupling was included in the GW calculations and a Matsubara cut-off of 12 Ha was used. All calculations were performed using state-of-the-art, all-electron, full-potential code Elk.29

11. Determination of sensitivity via Allan deviation:

The Allan deviation is defined as:31

\[ \sigma_A^2(t) = \frac{1}{2(N-1)f_0^2} \sum_{i=2}^{N} (f_i - f_{i-1})^2 \]  

(S15)

where \( f_i \) is the average frequency measured over the \( i \)th time interval of length \( t \). We perform time stability measurements (Fig. 5b, main paper) of the resonance frequency with the heating laser turned off using a PLL with a bandwidth \( BW = 2.5 \text{ kHz} \). We extract \( \sigma_A \) and find \( \sigma_A < 5 \cdot 10^{-7} \) over a broad range (Fig. 5c, main paper). Plugging \( \frac{\Delta f}{\Delta P} = 792 \text{ Hz/µW}, f_0 = 4.6702 \text{ MHz} \) and an optimal \( \sigma_A = 2.426 \cdot 10^{-7} \) at a sampling period of \( t = 4 \text{ ms} \) into equation S16, we calculate \( = 90 \frac{P_W}{\sqrt{Hz}} \). The measurement fulfills the condition of \( t \gg \frac{1}{BW} \).

\[ \eta = \frac{\sigma_f}{f_0(\Delta f/\Delta P)} = \frac{\sigma_A f_0}{\Delta f} \]  

(S16)

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