N-policy for a multi-component machining system with imperfect coverage, reboot and unreliable server

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This investigation aims at predicting the transient performance measures of a machining system comprising of $M$ operating units and multi types of warm standbys under the care of a single unreliable server. To deal with more realistic situations, the concepts of imperfect recovery and reboot delay are also incorporated. The queue size distribution is obtained by using the matrix method. The explicit expressions for various performance measures such as reliability, throughput, expected number of failed units in the system, failure frequency of the server, etc. are obtained. A numerical illustration is presented to demonstrate the practical application of the proposed model in the flexible manufacturing systems.

Keywords: N-policy; queue length; imperfect coverage; reboot delay; unreliable server; matrix method; flexible manufacturing systems

1. Introduction

In practical industrial scenario, the unexpected failure of the deployed machines results into a very significant loss of production and throughput which brings an undesirable loss of revenue as well as goodwill in the market. This situation can be controlled up to some extent by providing an efficient support of standby units and repair crews. The concerned multi-component machining system involves the redundancy in terms of multi types of warm spares. In some cases, operating units may fail, but the system remains operative and continues to perform its assigned job due to the spare part support. The concept of standby support has attracted the attention of several researchers working in the area of queuing theory (cf. Ke & Wang, 2007; Sivazlian & Wang, 1989). Reliability and availability issues of a standby system were studied by Hajeeh (2011). Ke and Wu (2012) developed a multi-server machine repair model with standby support. Recently, Jain (2013) presented the transient analysis of a machining system with mixed standbys incorporating the concepts of service interruption and priority.

The computer controlled manufacturing technology has brought tremendous changes in the machine design to control the risk of machine failure. Nowadays, the machines are equipped with an in-built fault-handling mechanism which automatically detects the failure of a component and recovers the system by replacing the failed operating unit with a standby unit, if available. In many software embedded systems, the machines can also reconfigure itself temporarily by reboot process, in the case when the fault handling

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mechanism fails to detect and recover the faults. But in some practical situations, the fault-handling device may prove inadequate to recover a fault perfectly. This situation is known as imperfect coverage. Only a few researchers have contributed to the literature on the machine repair problem incorporating the concepts of imperfect coverage and reboot in different frameworks (cf. Amari, Dugan, & Mishra, 1999; Amari, Pham, & Dill, 2004; Moustafa, 1997; Trivedi, 2002). The reliability measures of a repairable system with imperfect fault coverage and reboot were obtained by Hsu, Lee, and Ke (2008). The availability of three systems incorporating reboot and switching failures was compared by Wang and Chen (2009). Ke, Su, Wang, and Hsu (2010) applied simulation inferences for repairable system with imperfect fault coverage. A statistical model for a standby system involving reboot, switch failure, and unreliable repair was presented by Hsu, Ke, and Liu (2011). Jain, Agrawal, and Preeti Naresh (2012) evaluated fuzzy reliability for a repairable system with imperfect coverage by incorporating the concepts of reboot and shock failure. Wang, Liou, and Lin (2013) provided a comparative analysis of machine repair problem including imperfect coverage along with service pressure condition.

When the system is not successfully recovered by the reboot or recovery process, the completely failed units can be repaired by a skilled repairman. Generally, while considering a classical machine repair problem, it is assumed that the repair facility is perfect. But the practical scenario may be quite far away from this ideal situation as the repairman is also subject to breakdown randomly while providing the repair to a failed unit. The practical situation of an unreliable repairman has been discussed by the queue theorists in different frameworks (cf. Hsieh & Hsieh, 2003; Ke, 2006). Kalyanaraman, Thillaigovindan, and Kannadasom (2010) discussed a single server queue with an unreliable server. Jain, Sharma, and Sharma (2012) examined a batch arrival retrial queuing system with server breakdown and Bernoullii vacation. A very few research work is available on machine repair problems incorporating the concepts of reboot and imperfect coverage along with an unreliable server. However, Ke, Hsu, Liu, and Zhang (2013) used the concept of imperfect coverage along with unreliable multi-repairmen to provide computational analysis of a machine repair problem.

In most of the studies on machine repair problems, it is assumed that a failed unit is immediately repaired if failure occurs. But the situation is most often different in the real scenario as medium and small industries do not afford to recruit a full-time server and in large industries, a server has to take care of a number of equipment. Thus, a maintenance staff may not be available all the times to repair the failed units immediately after their failure. Also, due to the cost constraints, it is not feasible to call the server as and when a unit breaks down. In such cases, the failed units can accumulate and the repair is provided when the number of failed units reaches a certain threshold level, say N, which is pre-decided by the production/system engineers. This situation is well known as N-policy in queuing literature. A noticeable work on N-policy can be found in the studies of Wang, Wang, and Pearn (2007), Parthasarathy and Sudhesh (2008) and many more. Jain and Chauhan (2010) proposed an optimal control policy for a queuing model with service interruptions, vacation, and setup. Single server queuing model under N-policy and removable server was developed by Singh and Jain (2011). Jain and Kumar (2013) suggested N-policy, switching failure, and vacations while analyzing (M,m) degraded machining system with heterogeneous servers.

In the present study, we study a machine repair problem with reboot and imperfect coverage under the care of a single unreliable server. The research is inspired by the real-time smart machining system equipped with in-built fault-handling devices. The noble and innovative feature of our study is the incorporation of N-policy. N-policy
proves helpful in reducing the cost incurred on the system due to the maintenance staff. The rest of the paper is organized as follows. In section 2, model description along with the underlying assumptions to develop the proposed model, are provided. Section 3 is devoted to the matrix method for facilitating the transient solution of the queue size distribution of the number of failed units. Various performance indices in terms of probabilities and cost elements are presented in sections 4 and 5, respectively. By taking an illustration of flexible manufacturing system (FMS), numerical results are provided in section 6. Sensitivity analysis for the performance indices is given in sections 7. Finally, section 8 outlines the concluding remarks to highlight the innovative features and some future prospects of the present research work.

2. Model description

Consider a multi-component machining system comprising of $M$ operating units and $S_j$ ($1 \leq j \leq k$) $j$th type of warm standby units. The system is supported by a single unreliable server who repairs the failed units following the exponential distribution. An operating (or standby) unit is prone to failure, independent of others; the life time of an operating (standby) unit is exponentially distributed with a rate $\lambda (\alpha, 0 < \alpha < \lambda)$. A failed operating unit is immediately detected, located, and replaced by a standby unit, if available, with a coverage probability $C$. The recovery time follows an exponential distribution with rate $\sigma$. An uncovered fault is cleared with a complimentary probability $1 - C$, by reboot operation with a rate $\beta$ ($\beta > \lambda > \alpha$) following exponential distribution.

The failed units are repaired by a single server operating under N-policy i.e. the server does not render repair to the failed units until N failed units are not accumulated in the queue. The FCFS queue discipline is followed by the server to provide the repair to the failed units. Before providing repair to the failed units, the server takes a setup time, following exponential distribution with mean $1/\psi$. While providing repair to the failed units, the server may break down randomly, following an exponential distribution with mean $1/\theta$. The failed server returns back to perform his assigned job with rate $\eta$, according to exponential distribution. When the server is unavailable, the failed units accumulate and wait for the repair until the server becomes available. The system fails completely if there is less than $m$ number of operating units. The transition diagram of

![Figure 1. Transition diagram.](image-url)
system states is shown in Figure 1. Depending upon the states of the server, the system states at time \( t \) are denoted as:

\[
\zeta(t) = \begin{cases} 
1; & 0 \leq n \leq L; \text{ the system is in accumulation state} \\
2; & 1 \leq n \leq L; \text{ the server is busy in rendering the repair} \\
3; & 1 \leq n \leq L; \text{ the server is broken down and under repair}
\end{cases}
\]

Initially, when \( t = 0 \), the system starts with no failed unit in it. We define the transient state probabilities at time \( t \) for different states as follows:

- \( P_{1,0}(t) \) The probability that the system is in accumulation state and there is no failed unit in the system at time \( t \)
- \( P_{i,n}(t) \) The probability that the system is in \( i \)th (\( 1 \leq i \leq 3 \)) state and there are \( n \) (\( 1 \leq n \leq L \)) failed units in the system at time \( t \)
- \( Q_{1,1}(t) \) The probability that the system is in accumulation state and first unit is taking reboot at time \( t \)
- \( Q_{i,n}(t) \) The probability that the system is in \( i \)th (\( 1 \leq i \leq 3 \)) state and \( n \)th (\( 2 \leq n \leq L \)) failed unit is taking reboot at time \( t \)
- \( R_{1,1}(t) \) The probability that the system is in accumulation state and first unit is under recovery at time \( t \)
- \( R_{i,n}(t) \) The probability that the system is in \( i \)th (\( 1 \leq i \leq 3 \)) state and \( n \)th (\( 2 \leq n \leq L \)) unit is under recovery at time \( t \)

The state-dependent mean failure rate for the proposed model is defined as follows:

\[
\Lambda(n) = \begin{cases} 
M \lambda + \left( \sum_{i=2}^{k} S_i x_i + (S_1 - n) x_1 \right); & 0 \leq n < S_1 \\
M \lambda + \left( \sum_{i=1}^{j} S_i - n \right) x_j + \sum_{i=j+1}^{k} S_i x_i; & \sum_{i=1}^{j-1} S_i \leq n < \sum_{i=1}^{j} S_i; \quad j = 2, 3, 4, \ldots, k \\
(M + \sum_{i=1}^{k} S_i - n) \lambda; & \sum_{i=1}^{k} S_i \leq n < \sum_{i=1}^{L} S_i = M + \sum_{i=1}^{k} S_i - (m - 1)
\end{cases}
\]

For brevity of notation, we denote:

\[
a_n = \begin{cases} 
\left( \sum_{i=2}^{k} S_i x_i + (S_1 - n) x_1 \right); & 0 \leq n < S_1 \\
\left( \sum_{i=1}^{j} S_i - n \right) x_j + \sum_{i=j+1}^{k} S_i x_i; & \sum_{i=1}^{j-1} S_i \leq n < \sum_{i=1}^{j} S_i; \quad j = 2, 3, 4, \ldots, k
\end{cases}
\]

Now, we construct Chapman–Kolmogorov equations for the system states as follows:

\[
\frac{dP_{1,0}(t)}{dt} = -(M \lambda + a_0)P_{1,0}(t) + \mu P_{2,1}(t)
\]

\[
\frac{dP_{1,n}(t)}{dt} = -(M \lambda + a_n)P_{1,n}(t) + a_{n-1}P_{1,n-1}(t) + \sigma R_{1,n}(t) + \beta Q_{1,n}(t); \quad n = 1, 2, \ldots, N - 1
\]
\[
\frac{dP_{1,N}(t)}{dt} = -(M\lambda + \psi)P_{1,N}(t) + a_{N-1}P_{1,N-1}(t) + \sigma R_{1,N}(t) + \beta Q_{1,N}(t) 
\]

(3)

\[
\frac{dP_{1,n}(t)}{dt} = -((M + N - n)\lambda + \psi)P_{1,n}(t) + \sigma R_{1,n}(t) + \beta Q_{1,n}(t); \quad n = N + 1, N + 2, \ldots, L - 1 
\]

(4)

\[
\frac{dP_{1,L}(t)}{dt} = -\psi P_{1,L}(t) + \sigma R_{1,L}(t) + \beta Q_{1,L}(t). 
\]

(5)

\[
\frac{dP_{2,1}(t)}{dt} = -(M\lambda + a_1 + \theta + \mu)P_{2,1}(t) + \mu P_{2,2}(t) + \eta P_{3,1}(t) 
\]

(6)

\[
\frac{dP_{2,n}(t)}{dt} = -(M\lambda + a_n + \theta + \mu)P_{2,n}(t) + a_{n-1}P_{2,n-1}(t) + \mu P_{2,n+1}(t) + \eta P_{3,n}(t) + \sigma R_{2,n}(t) + \beta Q_{2,n}(t); \quad n = 2, 3, \ldots, N - 1 
\]

(7)

\[
\frac{dP_{2,N}(t)}{dt} = -(M\lambda + \theta + \mu)P_{2,N}(t) + a_{N-1}P_{2,N-1}(t) + \mu P_{2,N+1}(t) + \eta P_{3,N}(t) + \psi P_{1,N}(t) + \sigma R_{2,N}(t) + \beta Q_{2,N}(t) 
\]

(8)

\[
\frac{dP_{2,n}(t)}{dt} = -((M + N - n)\lambda + \theta + \mu)P_{2,n}(t) + \mu P_{2,n+1}(t) + \eta P_{3,n}(t) + \psi P_{1,n}(t) + \sigma R_{2,n}(t) + \beta Q_{2,n}(t); \quad n = N + 1, N + 2, \ldots, L - 1 
\]

(9)

\[
\frac{dP_{2,L}(t)}{dt} = -\theta P_{2,L}(t) + \eta P_{3,L}(t) + \psi P_{1,L}(t) + \sigma R_{2,L}(t) + \beta Q_{2,L}(t). 
\]

(10)

\[
\frac{dP_{3,1}(t)}{dt} = -(M\lambda + a_1 + \eta)P_{3,1}(t) + \theta P_{2,1}(t) 
\]

(11)

\[
\frac{dP_{3,n}(t)}{dt} = -(M\lambda + a_n + \eta)P_{3,n}(t) + a_{n-1}P_{3,n-1}(t) + \theta P_{2,n}(t) + \sigma R_{3,n}(t) + \beta Q_{3,n}(t); \quad n = 2, 3, \ldots, N - 1 
\]

(12)

\[
\frac{dP_{3,N}(t)}{dt} = -(M\lambda + \eta)P_{3,N}(t) + a_{N-1}P_{3,N-1}(t) + \theta P_{2,N}(t) + \sigma R_{3,N}(t) + \beta Q_{3,N}(t) 
\]

(13)

\[
\frac{dP_{3,n}(t)}{dt} = -((M + N - n)\lambda + \eta)P_{3,n}(t) + \theta P_{2,n}(t) + \sigma R_{3,n}(t) + \beta Q_{3,n}(t); \quad n = N + 1, N + 2, \ldots, L - 1 
\]

(14)
In this section, we describe the process of obtaining the transient solution for the model of multi-component machine repair problem formulated in previous section. The set of differential equations (1)–(19) is converted into the set of simultaneous algebraic equations by taking its Laplace transform. The probability vector is evaluated by taking the eigen values of the coefficient matrix, representing the set of simultaneous algebraic equations.

After taking Laplace transforms of the Equations (1)–(19), the matrix equation can be formulated as:

\[
\frac{dP_{i,n}(t)}{dt} = -\eta P_{i,n}(t) + \theta P_{i,1}(t) + \sigma R_{i,n}(t) + \beta Q_{i,n}(t)
\]

\[
\frac{dR_{i,n}(t)}{dt} = -\sigma R_{i,n}(t) + M \lambda C_{i,n-1}(t); \quad i = 1, 1 \leq n \leq N; \quad i = 2, 3; \quad 2 \leq n \leq N
\]

\[
\frac{dR_{i,n}(t)}{dt} = -\sigma R_{i,n}(t) + (M + N - n + 1) \lambda C_{i,n-1}(t); \quad i = 1, 2, 3;
\]

\[
\frac{dQ_{i,n}(t)}{dt} = -\beta Q_{i,n}(t) + (M + N - n + 1) \lambda C_{i,n-1}(t); \quad i = 1, 2, 3;
\]

\[
n = N + 1, N + 2, \ldots, L.
\]

For notational convenience to evaluate the probabilities, we replace the probabilities \( P_{i,n}(t), Q_{i,n}(t), \) and \( R_{i,n}(t) \) by defining the probability vector having single suffix elements as follows:

\[
\Pi(t) = [\Pi_1(t), \Pi_2(t), \Pi_3(t)]
\]

where \( \Pi_1(t) = [P_{1,0}(t), P_{1,1}(t), \ldots, P_{1,1}(t), P_{2,1}(t), P_{2,2}(t), \ldots, P_{2,1}(t), P_{3,1}(t), P_{3,2}(t), \ldots, P_{3,1}(t)] \);

\[
\Pi_2(t) = [Q_{1,1}(t), Q_{1,2}(t), \ldots, Q_{1,1}(t), Q_{2,2}(t), Q_{2,3}(t), \ldots, Q_{2,1}(t), Q_{3,2}(t), \ldots, Q_{3,1}(t)]
\]

\[
\Pi_3(t) = [R_{1,1}(t), R_{1,2}(t), \ldots, R_{1,1}(t), R_{2,2}(t), R_{2,3}(t), \ldots, R_{2,1}(t), R_{3,2}(t), \ldots, R_{3,1}(t)]
\]

Let \( \Pi^*(s) \) denote the Laplace transform of \( \Pi(t) \). The Laplace transform of probability \( \pi_n(t) \) is given by

\[
\pi_n(s) = \int_0^\infty e^{-st} \pi_n(t) dt; \quad 0 \leq n \leq 9L - 4
\]

The initial conditions are given as:

\[
\pi_0(0) = 1; \quad \pi_n(0) = 0; \quad 1 \leq n \leq 9L - 4.
\]
where $\Pi^*(s)$ is a column vector of order $(9L-3)$ given as:

$$\Pi^*(s) = [\Pi_1^*(s), \Pi_2^*(s), \Pi_3^*(s)] = [\pi_n^*(s)]; \quad 0 \leq n \leq 9L - 4$$

and $\Pi(0) = [1, 0, 0, \ldots, 0, 0, 0, 0, 0, 0]^T|_{(9L-3)}$ is the initial vector. Here, $D(s)$ denotes the transition rate matrix of order $(9L - 3) \times (9L - 3)$ and is given as

$$D(s) = [M_{lj}]_{3x3}$$

Here, all $M_{ij}$ (1 $\leq i \leq 3$, 1 $\leq j \leq 3$) are functions of $s$ and are given as

$$M_{11} = \begin{bmatrix} A_0 & 0 & 0 \\ B_0 & B_1 & B_2 \\ 0 & C_1 & C_2 \end{bmatrix}_{(3L+1) \times (3L+1)}$$

$M_{13} = \text{Diag.}[A_6, B_7, C_8]_{(3L+1) \times (3L+2)}$

$M_{22} = \text{Diag.}[D_4, E_5, F_6]_{(3L-2) \times (3L-2)}$

$M_{33} = \text{Diag.}[G_4, H_5, J_6]_{(3L-2) \times (3L-2)}$

and $M_{23} = M_{32} = [0]_{(3L-2) \times (3L-2)}$

Sub-matrices of matrix $M_{11}$ are given as

$$A_0 = \text{Diag.}[X_1, X_2, X_3]_{(L+1) \times (L+1)}$$

$$B_0 = \begin{bmatrix} 0 & 0 \\ 0 & Y_1 \end{bmatrix}_{(L) \times (L+1)}$$

$$B_1 = \begin{bmatrix} Y_2 \\ 0 \end{bmatrix}_{(L) \times (L)}$$

where

$$X_1 = [a_{ii}, a_{ij}]_{(N+1) \times (N+1)} \quad \text{where} \quad a_{ii} = (s + M \lambda + a_{i-1}); \quad i = 1, 2, \ldots, N; \quad \text{and}$$

$$a_{ij} = a_{j-1}; \quad i = j + 1; \quad j = 1, 2, \ldots, N.$$  

and $a(N+1)(N+1) = (s + M \lambda + \psi)$, $N = S_1 + S_2 + \cdots + S_k$.

$$X_2 = \text{Diag.}[(s + M - i)\lambda - \psi, (s + m \lambda + \psi)]_{(L-N-1) \times (L-N-1)};$$

$$i = 1, 2, \ldots, (L - N - 2).$$

$$X_3 = [-\delta]$$

$$Y_1 = \text{Diag.}[\psi]_{(L-N+1) \times (L-N+1)}$$

$$Y_2 = [a_{ii}, a_{ij}]_{(N) \times (N)} \quad \text{where} \quad a_{ii} = (s + M \lambda + \theta + \mu + a); \quad i = 1, 2, \ldots N - 1; \quad \text{and}$$

$$a_{ij} = a_j; \quad i = j + 1; \quad j = 1, 2, \ldots, N - 1.$$  

and $a_{NN} = (s + M \lambda + \theta + \mu)$,

$$Y_3 = [0 \ 0 \ 0 \ \cdots \ \mu]^T_{N \times 1}$$

$$Y_4 = [a_{ii}, a_{ij}]_{(L-N) \times (L-N)} \quad \text{where} \quad a_{ii} = ((s + M - i)\lambda + \theta + \mu); \quad a_{ij} = \mu;$$

$$j = i + 1; \quad i = 1, 2, \ldots, (L - N - 1), a_{(L-N) \times (L-N)} = [(s + \theta + \mu)].$$
\[ Z_1 = \text{Diag}.[\theta]_{(L)\times(L)} \]

\[ Z_2 = [a_{ij}]_{(N)\times(N)} \] where \( a_{ii} = (s + M \lambda + \eta + a_i); \quad i = 1, 2, \ldots, N - 1, \) and \( a_{jj} = a_j; \quad i = j + 1; \quad j = 1, 2, \ldots, N - 1. \)

\[ \text{and } a_{NN} = (s + M \lambda + \eta), \]

\[ Z_3 = \text{Diag}.[(s + M - i)\lambda - \eta]_{(L-N-1)\times(L-N-1)}; \quad i = 1, 2, 3, \ldots, (L - N - 1). \quad Z_4 = [-\eta] \]

Sub-matrices of matrix \( M_{12} \) are given as

\[ A_3 = [0, X_4]^{T}_{(L+1)\times(L)} \]

\[ B_4 = C_5 = [0, Y_6]^{T}_{(L)\times(L-1)}; \]

\[ X_4 = \text{Diag}.[\sigma]_{(L)\times(L)} \quad Y_6 = \text{Diag}.[\sigma]_{(L-1)\times(L-1)} \]

Sub-matrices of matrix \( M_{13} \) are given as

\[ A_6 = [0, X_3]^{T}_{(L+1)\times(L)} \]

\[ B_7 = C_8 = [0, Y_7]^{T}_{(L)\times(L-1)}; \]

\[ X_5 = \text{Diag}.[\beta]_{(L)\times(L)} \quad Y_7 = \text{Diag}.[\beta]_{(L-1)\times(L-1)} \]

Sub-matrices of matrix \( M_{21} \) are given as

\[ D_1 = \text{Diag}.[X'_1, X'_2]_{(L+1)\times(L+1)} \quad E_2 = F_3 = \text{Diag}.[X'_3, X'_4]_{(L-1)\times(L)} \]

\[ X'_1 = \text{Diag}.[(M - i)\lambda C, 0]_{(L-N-1)\times(L-N)}; \quad i = 1, 2, \ldots, (L - N - 1) \]

\[ X'_3 = \text{Diag}.[(M + i)\lambda C]_{(N)\times(N)} \]

\[ X'_4 = \text{Diag}.[(M - i)\lambda C]_{(L-N-1)\times(L-N-1)}; \quad i = 1, 2, \ldots, (L - N - 1). \]

Sub-matrices of matrix \( M_{22} \) are given as

\[ D_4 = \text{Diag}.[s + \sigma]_{(L)\times(L)} \quad E_5 = F_6 = \text{Diag}.[s + \sigma]_{(L-1)\times(L-1)} \]

Sub-matrices of matrix \( M_{31} \) are given as

\[ G_1 = \text{Diag}.[Y'_1, Y'_2]_{(L+1)\times(L+1)} \quad H_2 = J_3 = G_1 = \text{Diag}.[Y'_3, Y'_4]_{(L-1)\times(L)} \]

where

\[ Y'_1 = \text{Diag}.[(M + i)\lambda C]_{(N+1)\times(N+1)} \]

\[ Y'_2 = \text{Diag}.[(M - i)\lambda C, 0]_{(L-N-1)\times(L-N-1)}; \quad i = 1, 2, \ldots, (L - N - 1) \]

\[ Y'_3 = \text{Diag}.[\lambda C]_{(N)\times(N)} \]

\[ Y'_4 = \text{Diag}.[(M - i)\lambda C]_{(L-N-1)\times(L-N-1)}; \quad i = 1, 2, \ldots, (L - N - 1). \]

Sub-matrices of matrix \( M_{33} \) are given as

\[ G_4 = \text{Diag}.[s + \beta]_{(L)\times(L)} \quad E_5 = F_6 = \text{Diag}.[s + \beta]_{(L-1)\times(L-1)} \]

**Computation of probability vector**

In order to compute the \( \pi_n^*(s) \), following Cramer’s rule, we obtain

\[ \pi_n^*(s) = \frac{|D_{n+1}(s)|}{|D(s)|}; \quad 0 \leq n \leq 9L - 4 \] (24)
where $D_{n+1}(s)$ is obtained by replacing the $(n + 1)$th column of $D(s)$ with initial vector $\Pi(0)$. Now, we obtain the characteristic roots of the matrix $D(s)$. It is clear that $s = 0$ is one of the roots of $|D(s)| = 0$. Substituting $s = -r$, $D(s)$ yields

\[
0 = (D - rI)
\]

where $D = D(0)$ is a square matrix of order $(9L - 3)$ and $I_{(9L - 3)}$ denotes the identity matrix. Now, Equation (21) can be rewritten as

\[
D(-r)\Pi^{*}(s) = (D - rI)\Pi^{*}(s) = \Pi(0)
\]  

(26)

Now, in order to obtain the characteristic roots (excluding 0) of the matrix $D(s)$, we put $|D - rI| = 0$ which provides us distinct roots, say out of which some may be real and some may be complex. Let $x$ be the number of real distinct roots excluding zero and $y$ be the number of complex distinct roots occurring in pairs, denoted by $r_1, r_2, \ldots, r_x$ and $(r_{x+1}, \bar{r}_{x+1}), (r_{x+2}, \bar{r}_{x+2}), \ldots, (r_{x+y}, \bar{r}_{x+y})$ respectively; here $x$ and $y$ satisfy the condition that $x + 2y = 9L - 4$. Thus, we get

\[
|D(s)| = s \left( \prod_{h=1}^{x} (s + r_h) \right) \left( \prod_{h=1}^{y} (s + r_{x+h} + \bar{r}_{x+h}) \right)
\]

(27)

Using result of Equation (27), Equation (24) becomes

\[
\pi_{n}^{*}(s) = \frac{a_0}{s} + \sum_{h=1}^{x} \frac{a_h}{s + r_h} + \sum_{h=1}^{y} \frac{sb_h + c_h}{s^2 + (r_{x+h} + \bar{r}_{x+h})s + r_{x+h} \bar{r}_{x+h}},
\]

(28)

where $a_0, a_h, b_h$ and $c_h$ ($h = 1, 2, \ldots, 9L - 4$) are unknown real numbers and are obtained as:

\[
a_0 = \frac{|D_{n+1}(0)|}{\left( \prod_{h=1}^{x} r_h \right) \left( \prod_{h=1}^{y} r_{x+h} \bar{r}_{x+h} \right)}
\]

(29)

\[
a_h = \frac{|D_{n+1}(-r_h)|}{(-r_h) \left( \prod_{d=1}^{x} (r_d - r_h) \right) \left( \prod_{d=1}^{y} (-r_h + r_{x+d}) (-r_h + \bar{r}_{x+d}) \right)}; \quad h = 1, 2, 3, \ldots, x.
\]

(30)

Let the complex eigenvalue $r_{x+h}$ be a combination of real part $u_h$ and imaginary part $v_h$. Then

\[
b_h(-r_{x+h}) + c_h = \frac{|D_{n+1}(-r_{x+h})|}{\left( -r_{x+h} \right) \left( \prod_{d=1}^{x} (r_d - r_{x+h}) \right) \left( \prod_{d=1}^{y} (-r_{x+h} + r_{x+d}) (-r_{x+h} + \bar{r}_{x+d}) \right)}; \quad h = 1, 2, 3, \ldots, y
\]

(31)
Now, we assume that the complex eigenvalue $r_x + h$ can be represented as a combination of $u_h$ and $v_h$ which are real and imaginary parts, respectively. Taking the inverse Laplace transform of Equation (28), we obtain

$$\pi_h(t) = a_0 + \sum_{h=1}^{X} a_h \exp(-r_h t) + \sum_{h=1}^{Y} \left\{ b_h \exp(-u_h t) \cos(v_h t) + \frac{c_h - b_h u_h}{v_h} \exp(-u_h t) \sin(v_h t) \right\}$$ (32)

4. Performance measures

In this section, we establish various performance indices which are required to predict the queuing and reliability characteristics of the multi-component machining system. These performance measures will enable us to provide some meaningful interpretation of the behavior of the machining system.

- The system reliability at time $t$ is given by
  $$R_Y(t) = 1 - \sum_{i=1}^{3} P_{i,L}(t)$$ (33)

- Mean time to failure
  $$MTTF = \int_{t=0}^{\infty} R_Y(t) dt = \int_{t=0}^{\infty} \left( 1 - \sum_{i=1}^{3} P_{i,L}(t) \right) dt = \lim_{s \to 0} \left[ \int_{t=0}^{\infty} \left( 1 - \sum_{i=1}^{3} P_{i,L}(t) \right) e^{-st} dt \right]$$
  $$= \lim_{s \to 0} \left[ \frac{1}{s} - \sum_{i=1}^{3} P^{*}_{i,L}(s) \right]$$ (34)

- Failure frequency of the server at time $t$ is
  $$f(t) = \theta \sum_{n=1}^{L} P_{2,n}(t)$$ (35)

- The probability that the system is in build-up state
  $$P_N(t) = \sum_{n=0}^{N-1} P_{1,n}(t) + \sum_{n=1}^{N-1} \{ Q_{1,n}(t) + R_{1,n}(t) \}$$ (36)

- The probability that the server is in setup state for providing the repair to the failed units at time $t$
  $$P_{SU}(t) = \sum_{n=N}^{L} P_{1,n}(t) + \sum_{n=N}^{L} \{ Q_{1,n}(t) + R_{1,n}(t) \}$$ (37)
The probability that the server is busy in providing the repair to the failed units at time $t$

$$P_B(t) = \sum_{n=1}^{L} P_{2,n}(t)$$  \hspace{1cm} (38)

The probability that the server is broken down and is under repair at time $t$

$$P_D(t) = \sum_{n=1}^{L} P_{3,n}(t) + \sum_{n=2}^{L} \{Q_{3,n}(t) + R_{3,n}(t)\}$$  \hspace{1cm} (39)

The probability that the system is in reboot state at time $t$

$$P_R(t) = \sum_{n=1}^{L} Q_{1,n}(t) + \sum_{n=2}^{L} \{Q_{2,n}(t) + Q_{3,n}(t)\}$$  \hspace{1cm} (40)

The probability that the system is in recovery state at time $t$

$$P_{RC}(t) = \sum_{n=1}^{L} R_{1,n}(t) + \sum_{n=2}^{L} \{R_{2,n}(t) + R_{3,n}(t)\}$$  \hspace{1cm} (41)

The probability that the server is idle at time $t$

$$P_I(t) = \sum_{n=N}^{L} \{Q_{2,n}(t) + R_{2,n}(t)\}$$  \hspace{1cm} (42)

The probability that the system is unavailable due to reboot state or failed state at time $t$

$$U_{RF}(t) = P_R(t) + \sum_{i=1}^{3} P_{i,L}(t)$$  \hspace{1cm} (43)

Throughput

$$\tau(t) = \mu \sum_{n=1}^{L} P_{2,n}(t)$$  \hspace{1cm} (44)

Expected number of failed units at time $t$

$$E\{N(t)\} = \sum_{i=1}^{3} \sum_{n=2}^{L} n\{P_{i,n}(t) + Q_{i,n}(t) + R_{i,n}(t)\} + \sum_{i=1}^{3} P_{i,1}(t)$$  \hspace{1cm} (45)

5. Cost function

The transient cost function is established in terms of different cost elements by considering repair rate $\mu$ as decision variable. The main objective to construct the cost function is to determine the optimal repair rate $\mu$, say $\mu^*$ so as to minimize the total cost $TC(\mu,t)$.

We select the following cost elements associated with different activities:

- $C_H$ Holding cost per unit time of failed units present in the system;
- $C_B$ Cost per unit time of the server while rendering service to the failed units;
Using the above-listed cost elements, the transient cost function is constructed in terms of appropriate performance measures and corresponding cost elements as follows:

\[
TC(\mu, t) = CH E\{N(t)\} + CB P_B(t) + CD P_D(t) + CI P_I(t) + CN P_N(t) + CSU P_{SU}(t) + CU \mu
\]

(46)

Since the cost function is highly nonlinear, it would be an extremely difficult task to determine the optimum values of discrete parameters. However, heuristic search technique based on discrete allocation scheme can be used to obtain optimum values of discrete system parameters. In order to facilitate numerical results for optimal service rate (\(\mu^*\)), the optimization problem can be solved numerically using MATLAB software.

6. Numerical illustration

In order to explore the practical applicability of our model in real-time machining system, we cite an illustration from FMS where the robots are used for the packaging purpose. Consider a pharmaceutical company working with three types of robots, six advanced (high technology level) robots are kept as operating units and may fail at a rate of .2 robots per day. There are three medium technology-level robots, which may fail at a rate of .05 robots per day, and two low technology-level robots which may fail at a rate of .07 per day; these robots are kept in reserve as first and second types of standby units, respectively. Whenever a robot fails, its failure is detected, diagnosed, and recovered with a .9 probability rate and recovery rate is assumed to be 100 per day. If the fault is not detected due to imperfect coverage, it is cleared by the reboot or reset operation at a rate 500 per day. The repair to the failed robots is provided only when all the five spare robots are used, that is on complete failure, the robot is reprogrammed by a server at a rate of 15 robots per day. Before starting the repair, the server takes a setup time of one day. During operation i.e. in active state, the server can also break down randomly, and may not be available for repair with a rate of .01 per day and becomes available after repair for reprogramming at a rate of 200 per day. The system can work till there are at least three robots in operation.

6.1. Performance measures for the FMS

As mentioned in illustration, for the computation purpose we set the default parameters as \(M = 6, N = 5, S_1 = 3, S_2 = 2, m = 3, \lambda = .2, \alpha_1 = .05, \alpha_2 = .07, \mu = 15, C = .9, \sigma = 100, \beta = 500, \psi = 1, \theta = .01, \eta = 200\). On the basis of numerical experiments, it is observed that the system becomes stable in three days. So we obtain various system performance measures such as expected number of failed robots, throughput of the server, reliability of the system, mean time to failure and the probability of the system to be in different states, by computing the probabilities at \(t = 3\) days. The numerical results for different indices are evaluated using Equations (31)–(45) and are displayed in Table 1.
6.2. Cost optimization for the FMS

In order to obtain an optimal repair rate $\mu^*$ so as to minimize the total cost of FMS, the necessary condition is that the cost function should be convex. We consider the set of cost elements as $C_H = $85, $C_B = $100, $C_D = $200, $C_I = $300, $C_N = $40, $C_{SU} = $50, $C_U = $6, $C_R = $7. By taking the default parameters same as for other performance indices, we obtain the total costs with the help of Equation (46) at $t = 3$ for varying values of $\mu$ and $M$, as shown in Figure 2.

The optimal repair rate ($\mu^*$) and corresponding minimum cost $TC(\mu^*)$ for $M = 6,7,8$ are summarized in Table 2.

6.3. Direct search method for cost optimization

Considering $\mu$ as a discrete value, we fix $M = 6$ and vary $\mu$ from 10 to 25 with an increment of 1. Firstly, the cost function shows a decreasing trend for $10 \leq \mu \leq 15$, then an increasing trend for $16 \leq \mu \leq 25$. Thus, the optimal cost exist when $15 \leq \mu \leq 16$. Now, keeping all other values same, we vary $\mu$ from 15 to 16 with an increment of .1 and

![Figure 2. Total cost vs. $\mu$ on varying M.](image-url)
notice that the optimal value of the cost function exist when 15.1 ≤ μ ≤ 15.2. Further, μ is varied with an increment of .01 between 15.1 and 15.2 and the optimal cost, for M = 6, is obtained at μ = 15.13. Similarly, The optimal cost for M = 7 and 8 is obtained when μ = 15.47 and 20.0, respectively.

7. Sensitivity analysis

The key elements involved in the designing and developing of a real-time machining system are the knowledge of various performance measures. In this section, numerical experiment is performed to demonstrate the applicability and tractability of the suggested computational approach based on matrix method. We facilitate the numerical results for various performance measures such as total cost function, probabilities of the server being idle, busy and under repair, expected number of failed units in the system, throughput, reliability, mean time to failure, failure frequency of the server, etc. The sensitivity of the performance measures with respect to different system descriptors is presented by varying time. The transient results by developing a program in ‘MATLAB’ software are computed and are summarized in Tables 3–9 and displayed in Figures 3–5.

To perform numerical simulations, we fix the default parameters as M = 6, N = 5, S1 = 3, S2 = 2, m = 2, λ = 2, α1 = .1, α2 = .3, μ = 20, σ = 100, β = 500, γ = 1, θ = .01, η = 200. First of all, we examine the effects of key parameters on the cost function in Tables 3 and 4. The cost elements are set as for the cost optimization problem of FMS described in previous section. The effects of different parameters are examined as follows:

1. **Cost function**: Tables 3 and 4 show that the total cost increases on increasing the number of operating units (M) which is same as we expect. As the failure rates of the server (θ) and standby units (α) increase, the total cost of the system also increases. It is also seen that the total cost of the system increases as time grows which matches with our expectation.

2. **Effect of time (t)**: Figures 3–5 exhibit the effect of time on different performance indices. It is observed from Figures 3(a) and (b) that as time increases, the probabilities of the system to be in reboot and recovery states increase. It is clear from the Figures 3(c)–(f) that the probabilities of the server to be in busy, setup, and repair states increase but on the contrary, probability of the server being in idle state decreases. Figures 4(a)–(b) show that the unavailability of the system increases but reliability decreases as time grows. In Figures 5(a) and (b), we notice that the throughput and failure frequency decrease on increasing time.

3. **Effect of number (M) of operating machines**: From Tables 5–9, it is noticed that an increment in M causes a decrement in the unavailability of the system. On the contrary, mean time to failure, reliability of the system, throughput, and expected number of failed units in the system increase. A very slight increment in the failure frequency of the server is also seen on increasing M. It is observed...
in Tables 5–9 that the trend of reliability and unavailability are opposite in nature as such more operating units cause more reliability and less unavailability. But more operating units cause a longer queue of failed units which results into increased expected number of failed units, failure frequency, and throughput.

(4) Effect of service rate (μ): Table 5 reveals that the unavailability of the system and expected number of failed units in the system seem to decrease as we increase the repair rate (μ). On the other hand, the reliability of the system, mean time to failure, failure frequency of the server, and throughput increase due to faster repair rate.

(5) Effect of breakdown rate (θ) and repair rate (η) of the server: Table 6 shows that an increment in the breakdown rate of the server (θ), results into increments
in the unavailability of the system, expected number of failed units, and failure frequency, but on the contrary, mean time to failure, system reliability, and throughput decrease. The results appropriately match with our expectation as with more frequent failures, the failure frequency of the server increases which also results in the case of less reliability and more unavailability. Table 7 displays the effect of the repair rate of the server on different performance indices. The trend of all the performance indices for increasing $\eta$ is opposite to that of the trend of performance indices for increasing $\theta$ shown in Table 6. This is due to the fact that breakdown and repair rates have reverse impact on the system behavior.

Table 3. Effect of $M$ and $\theta$ on total cost $TC(t)$.

| $t$ | $\theta = .01$ | $\theta = .05$ | $\theta = .01$ | $\theta = .05$ | $\theta = .01$ | $\theta = .05$ |
|-----|----------------|----------------|----------------|----------------|----------------|----------------|
| .5  | 2124.23        | 2128.70        | 2141.17        | 2144.37        | 2163.56        | 2164.15        |
| 1.0 | 2129.70        | 2133.03        | 2143.64        | 2146.43        | 2166.70        | 2167.70        |
| 1.5 | 2134.31        | 2138.13        | 2147.11        | 2148.32        | 2169.31        | 2173.64        |
| 2.0 | 2137.84        | 2141.89        | 2149.66        | 2151.59        | 2173.82        | 2176.82        |
| 2.5 | 2143.90        | 2146.19        | 2151.90        | 2154.29        | 2178.31        | 2179.36        |
| 3.0 | 2152.84        | 2158.84        | 2153.84        | 2156.39        | 2182.68        | 2185.84        |

Table 4. Effect of $M$ and $\alpha$ on total cost $TC(t)$.

| $t$ | $\alpha = .01$ | $\alpha = .05$ | $\alpha = .01$ | $\alpha = .05$ | $\alpha = .01$ | $\alpha = .05$ |
|-----|----------------|----------------|----------------|----------------|----------------|----------------|
| .5  | 2124.23        | 2138.32        | 2141.17        | 2148.31        | 2163.56        | 2169.65        |
| 1   | 2129.33        | 2146.46        | 2143.64        | 2153.36        | 2166.70        | 2171.73        |
| 1.5 | 2134.31        | 2153.76        | 2147.11        | 2156.35        | 2169.31        | 2176.65        |
| 2   | 2137.68        | 2159.82        | 2149.66        | 2163.66        | 2173.82        | 2181.90        |
| 2.5 | 2143.90        | 2166.19        | 2151.90        | 2168.12        | 2178.31        | 2187.11        |
| 3   | 2152.21        | 2173.82        | 2153.84        | 2175.38        | 2182.68        | 2191.31        |

Figure 4. (a) Unavailability and (b) reliability of the system vs. time on varying $\lambda$. 

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Figure 5. (a) Throughput and (b) failure frequency of the server vs. time on varying $\lambda$.

Table 5. Effect of $M$ and $\mu$ on performance measures.

| $M$ | $t$ | $\mu$ | $U_{RF}(t)$ | $R_{\lambda}(t)$ | MTTF | $f(t)$ | $\tau(t)$ | $E(N(t))$ |
|-----|-----|-------|-------------|------------------|------|--------|----------|-----------|
| 15  | 15  | .4329 | .568        | .032             | .004 | 6.2692 | 8.3062   |
| 20  | 25  | .3229 | .879        | .121             | .004 | 9.8584 | 6.1649   |
| 25  | 25  | .2985 | .903        | .297             | .005 | 10.481 | 6.0923   |
| 3   | 15  | .2017 | .800        | .100             | .007 | 10.1800 | 6.8607  |
| 20  | 25  | .1901 | .811        | .189             | .008 | 10.3510 | 6.0830  |
| 25  | 25  | .1829 | .858        | .242             | .009 | 10.6513 | 5.3676  |
| 15  | 15  | .3156 | .317        | .317             | .006 | 2.2350 | 9.3420   |
| 20  | 20  | .0702 | .532        | .068             | .007 | 9.8594 | 7.6188   |
| 8   | 25  | .0095 | .892        | .108             | .008 | 11.849 | 7.0257   |
| 3   | 20  | .0176 | .884        | .116             | .008 | 12.540 | 6.6353   |
| 25  | 25  | .0093 | .923        | .177             | .008 | 12.742 | 6.0412   |

Table 6. Effect of $M$ and $\theta$ on performance measures.

| $M$ | $t$ | $\theta$ | $U_{RF}(t)$ | $R_{\lambda}(t)$ | MTTF | $f(t)$ | $\tau(t)$ | $E(N(t))$ |
|-----|-----|----------|-------------|------------------|------|--------|----------|-----------|
| .01 | .1229 | .8786    | .1214       | .0049            | 9.8584 | 6.1649 |
| 1   | .03  | .1229    | .8786       | .1214            | .0148 | 9.8578 | 6.1650   |
| .05 | .1230 | .8785    | .1213       | .0246            | 9.8571 | 6.1651 |
| .01 | .1901 | .8114    | .1886       | .0052            | 10.3506 | 6.0830 |
| 3   | .03  | .1901    | .8114       | .1886            | .0155 | 10.3504 | 6.0830   |
| .05 | .1902 | .8113    | .1885       | .0259            | 10.3501 | 6.0831 |
| .01 | .0702 | .9316    | .0684       | .0049            | 9.8594 | 7.6188 |
| 1   | .03  | .0703    | .9315       | .0684            | .0148 | 9.8580 | 7.6196   |
| .05 | .0704 | .9314    | .0683       | .0246            | 9.8567 | 7.6204 |
| .01 | .1176 | .8844    | .1156       | .0063            | 12.5403 | 6.6353 |
| 3   | .03  | .1176    | .8842       | .1155            | .0188 | 12.5401 | 6.6354   |
| .05 | .1177 | .8842    | .1155       | .0314            | 12.5400 | 6.6355 |
Effect of recovery rate (C) and reboot rate (β): Tables 8 and 9 present the effects of coverage factor (C) and reboot rate (β), respectively, on different performance measures. It is clear from Table 8 that for the increasing values of C, the unavailability of the system, failure frequency, throughput, and expected number of failed units in the system decrease while reliability of the system and mean time to failure increase. Table 9 exhibits the same trend of performance indices for increasing values of β as noticed in Table 8 for increasing C; this trend is quite reasonable as coverage factor and reboot behave alike.

Effect of failure rate (λ) of the operating units: Figures 3–5 reveal the effect of λ on different performance measures. Figures 3(a)–(f) show that on increasing λ, the probabilities of the system to be in different states such as reboot and recovery increase. The probabilities of the server to be in busy, broken down, and setup states also increase. On the other hand, the probability of the server to be in idle state decreases which is quite reasonable. Figures 4(a)–(b) show that the unavailability (reliability) of the system increases (decreases) as λ increases. It is seen in Figures 5(a)–(b) that throughput and the failure frequency of the server increase on increasing λ. This is because of the pressure of providing service to the failed units at a faster rate in order to avoid congestion.

| M   | t   | η   | URF(t) | RT(t) | MTTF | f(t) | τ (t) | E(N(t)) |
|-----|-----|-----|--------|-------|------|------|------|--------|
| 150 | .12294 | .8786 | .12014 | .0049 | 9.8584 | 6.16489 |
| 1   | 200 | .10782 | .8938 | .1062 | .0045 | 11.1590 | 5.79421 |
| 250 | .09636 | .9053 | .0947 | .0040 | 12.1100 | 5.53639 |
| 6   | 150 | .19008 | .8114 | .1886 | .0051 | 10.3510 | 6.08298 |
| 3   | 200 | .14345 | .8581 | .1419 | .0045 | 11.0370 | 5.81215 |
| 250 | .11134 | .8903 | .1097 | .0037 | 11.5600 | 5.60648 |
| 1   | 150 | .06856 | .9331 | .0669 | .0050 | 10.0010 | 6.75639 |
| 200 | .04953 | .9523 | .0477 | .0047 | 11.4260 | 6.26273 |
| 8   | 250 | .0413 | .9606 | .0394 | .0042 | 12.4500 | 5.93995 |
| 150 | .11761 | .8844 | .1156 | .0062 | 12.5400 | 6.63534 |
| 3   | 200 | .08616 | .916 | .084 | .0055 | 13.0840 | 6.39739 |
| 250 | .06487 | .9373 | .0627 | .0047 | 13.4690 | 6.22206 |

Table 7. Effect of M and η on performance measures.

| M   | t   | C   | URF(t) | RT(t) | MTTF | f(t) | τ (t) | E(N(t)) |
|-----|-----|-----|--------|-------|------|------|------|--------|
| 1   | .9  | .12294 | .8786 | .12014 | .0049 | 9.8584 | 6.16489 |
| 1   | .91 | .12245 | .8789 | .1211 | .0043 | 9.8512 | 6.1646 |
| 6   | .92 | .12196 | .8792 | .1214 | .0042 | 9.8442 | 6.1644 |
| 3   | .91 | .19008 | .8114 | .1884 | .0052 | 10.3510 | 6.0829 |
| 1   | .92 | .18982 | .8115 | .1885 | .0052 | 10.3450 | 6.0826 |
| 8   | .91 | .18957 | .8116 | .1886 | .0051 | 10.3390 | 6.0823 |
| 3   | .9  | .07021 | .9316 | .0676 | .0049 | 9.8594 | 7.6188 |
| 1   | .91 | .06961 | .932 | .0680 | .0049 | 9.8473 | 7.6189 |
| 8   | .92 | .069 | .9324 | .0684 | .0048 | 9.8353 | 7.6191 |
| 3   | .91 | .11761 | .8844 | .1154 | .0062 | 12.5400 | 6.6353 |
| 1   | .92 | .11731 | .8845 | .1155 | .0061 | 12.5310 | 6.6347 |
| 8   | .91 | .117 | .8846 | .1156 | .0060 | 12.5210 | 6.6342 |
The numerical simulations presented in this section show the tractability of model and can be implemented for the quantitative assessment of many real-life problems, not only for some specific type of problems.

8. Conclusion

In the present study, we have investigated the transient performance measures for a multi-component machining system with standbys by incorporating the concepts of (i) reboot delay, (ii) imperfect coverage, (iii) unreliable server, (iv) N-policy, and (v) setup time. It is concluded that it is economic to adopt N-policy for appointing a repairman inspite of appointing a full-time repairman. The developed model can be implemented to predict the performance of various systems such as production systems, telecommunication systems, FMS, etc. which deal with automatic machining systems equipped with fault-handling mechanism. The cost analysis done may be helpful to the decision-makers and the system analysts to provide insight for the improvement and future design of the system at an optimal cost. Further, the research work can be extended for multi-repairmen problem.

References

Amari, S. V., Dugan, J. B., & Misra, R. B. (1999). Optimal reliability of systems subject to imperfect fault-coverage. *IEEE Transactions on Reliability, 48*, 275–284.

Amari, S. V., Pham, H., & Dill, G. (2004). Optimal design of K-out-of-N: G subsystems subject to imperfect fault coverage. *IEEE Transactions on Reliability, 53*, 567–575.

Hajeeh, M. A. (2011). Reliability and availability of a standby system with common cause failure. *International Journal of Operational Research, 11*, 343–363.

Hsieh, C. G., & Hsieh, Y. C. (2003). Reliability and cost optimization in distributed computing systems. *Computers and Operations Research, 30*, 1103–1119.

Hsu, Y. L., Lee, S. L., & Ke, J. C. (2008). A repairable system with imperfect fault coverage and reboot: Bayesian and asymptotic estimation. *Mathematics and Computers in Simulation, 79*, 2227–2239.

Hsu, Y. L., Ke, J. C., & Liu, T. H. (2011). Standby system with general repair, reboot delay, switching failure and unreliable repair facility – A statistical standpoint. *Mathematics and Computers in Simulation, 81*, 2400–2413.

Jain, M. (2013). Transient analysis of machining systems with service interruption, mixed standbys and priority. *International Journal of Mathematics in Operational Research, 5*, 604–625.

| $M$ | $t$ | $\beta$ | $U_{R_3}(t)$ | $R_Y(t)$ | MTTF | $f(t)$ | $\tau(t)$ | $E(N(t))$ |
|-----|-----|------|------------|--------|------|------|--------|----------|
| 1   | 500 | .1229 | .8786      | .1214  | .0049 | 9.8684| 6.1648  |
| 6   | 600 | .1228 | .8787      | .1216  | .0048 | 9.8614| 6.1648  |
| 3   | 700 | .1227 | .8788      | .1217  | .0048 | 9.8605| 6.1648  |
| 1   | 500 | .1900 | .8114      | .1886  | .0051 | 10.351| 6.0832  |
| 8   | 600 | .1898 | .8114      | .1886  | .0050 | 10.350| 6.0831  |
| 700 | .1897| .8115  | .1887      | .0050  | 10.348| 6.0829|
| 1   | 600 | .0700 | .9316      | .0684  | .0049 | 9.8594| 7.6186  |
| 6   | 700 | .0700 | .9318      | .0687  | .0047 | 9.8699| 7.6188  |
| 1   | 500 | .1176 | .8844      | .1156  | .0068 | 12.547| 6.6353  |
| 3   | 600 | .1173 | .8845      | .1156  | .0067 | 12.544| 6.6355  |
| 700 | .1171| .8846  | .1156      | .0069  | 12.541| 6.6357|
Jain, M., & Chauhan, D. (2010). Optimal control policy for state dependent queueing model with service interruption, setup and vacation. *Journal of Informatics and Mathematical Sciences, 2*, 171–181.

Jain, M., & Kumar, K. (2013). Threshold N-policy for (M, m) degraded machining system with heterogeneous servers, standby switching failure and multiple vacation. *International Journal of Mathematics in Operational Research, 5*, 423–445.

Jain, M., Agrawal, S. C., & Naresh, Preeti (2012). Fuzzy reliability evaluation of a repairable system with imperfect coverage, reboot and common-cause shock failure. *International Journal of Engineering, 25*, 231–238.

Jain, M., Sharma, G. C., & Sharma, R. (2012). A batch arrival retrial queuing system for essential and optional services with server breakdown and Bernoulli vacation. *International Journal of Internet and Enterprise Management, 8*, 16–45.

Kalyanaraman, R., Thillaigovindan, N., & Kannadasom, G. (2010). A single server fuzzy queue with unreliable server. *International Journal of Computational Cognitions, 8*, 35–47.

Ke, J. C. (2006). Vacation policy for machine interference problem with an unreliable server and state dependent rates. *Journal of the Chinese Institute of Industrial Engineers, 23*, 100–114.

Ke, J. C., & Wang, K. H. (2007). Vacation policies for machine repair problem with two type spares. *Applied Mathematical Modelling, 31*, 880–894.

Ke, J. C., & Wu, C. H. (2012). Multi-server machine repair model with standbys and synchronous multiple vacation. *Computers & Industrial Engineering, 62*, 296–305.

Ke, J. C., Su, Z. L., Wang, K. H., & Hsu, Y. L. (2010). Simulation inferences for an available system with general repair distribution and imperfect fault coverage. *Simulation Modeling, Practice and Theory, 18*, 336–347.

Ke, C., Hsu, Y. L., Liu, T. H., & Zhang, Z. G. (2013). Computational analysis of machine repair problem with unreliable multi-repairmen. *Computers and Industrial Engineering, 40*, 848–855.

Moustafa, M. S. (1997). Reliability analysis of K-out-of-N: G systems with dependent failures and imperfect coverage. *Reliability Engineering and System Safety, 58*, 15–17.

Parthasarathy, P. R., & Sudhesh, R. (2008). Transient solution of multi server Poisson queue with N policy. *Computers and Mathematics with Applications, 55*, 550–562.

Singh, C. J., & Jain, M. (2011). Single server queueing model with N-policy and removable server. *Canadian Applied Mathematics Quarterly (CAMQ), 19*, 113–123.

Sivazlian, B. D., & Wang, K. H. (1989). Economic analysis of the M/M/R machine repair problem with warm standbys. *Microelectronics Reliability, 29*, 25–35.

Trivedi, K. S. (2002). *Probability and statistics with reliability, queueing and computer science applications* (2nd ed.). New York, NY: Wiley.

Wang, K. H., & Chen, Y. J. (2009). Comparative analysis of availability between three systems with general repair times, reboot delay and switching failures. *Applied Mathematics and Computation, 215*, 384–394.

Wang, K. H., Wang, T. Y., & Pearn, W. L. (2007). Optimal control of the N policy M/G/1 queueing system with server breakdowns and general startup times. *Applied Mathematical Modelling, 31*, 2199–2212.

Wang, K. H., Liou, C. D., & Lin, Y. H. (2013). Comparative analysis of the machine repair problem with imperfect coverage and service pressure condition. *Applied Mathematical Modelling, 37*, 2870–2880.