CS-based processing for high-resolution passive radar with incoherent dictionary

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Abstract: Passive radar (PR) systems are attractive for various military and commercial applications, where illuminators of opportunity are used. Using the fact that most target scenes are very sparsely populated makes it possible to apply compressive sensing (CS) for PR. CS provides higher resolution than that obtained by using traditional ambiguous function, but suffers from increasing coherence associated with the finer grids. A novel incoherent CS dictionary construction method is presented here to deal with the case of finer grids. Experimental results demonstrate that the proposed CS-based processing scheme with incoherent dictionary can work well for high-resolution PRs.

1 Introduction

Passive radar (PR) [1] is the novel radar that does not emit electromagnetic energy and exploits some other radiations (e.g. FM, GSM, DTV etc). PR systems have attracted attention in military and commercial applications, mainly because of their covertness, low-cost implementation, and immune to jamming etc. Meanwhile, a multistatic configuration is easy to be built for PR, which can provide more information.

However, PR has some disadvantages compared with active radar. For example, the resolution of PR is limited because of the narrow signal bandwidth (typically several to tens of megahertz) [2]. Moreover, to improve the detection performance of small targets, increasing the integration time is a useful method. However, the range migration and Doppler migration are unfavourable to processing coherent accumulation [3]. Thus, novel signal processing schemes are expected to obtain high resolution in short integration time of PR.

Compressive sensing (CS) [4, 5] is a sparse signal processing technique, which can recover either sparse or compressible signals with lower rates than their Nyquist rates. This technique involves sparse representation (dictionary), design of measurement matrix, and recovery of sparse coefficient vector. CS-based radar is able to provide better resolution [6, 7]. Recently, CS is used in monostatic or multistatic radar system to estimate the range and Doppler frequency of targets [8–10].

The form of dictionary will affect reconstruction quality based on the CS theory. In PR, the dictionary is normally equal to the set of the template signals with discrete delays and Doppler shifts. Fine-gridding is used to achieve high resolution of the observation scene. However, the coherence of dictionary is deteriorated significantly, which will degrade the reconstruction performance. In this paper, we present a CS-based PR processing scheme with incoherent dictionary to deal with the case of fine-gridding and verify its performance by simulation experiments.

The remainder of this work is organised as follows. The targets scene model and radar signal model of CS are presented in Section 2. Section 3 introduces the incoherent dictionary formulation. In Section 4, simulation results are given and the performances are analysed. Finally, we make our conclusion in Section 5.

2 CS model of passive radar

2.1 Signal model for monostatic PR

We assume that the observation consists of a small number of point-like reflectors (moving targets). Thus, the impulse response $h(t)$ is modelled as

$$h(t) = \sum_{k=1}^{K} A_k \delta(t - \tau_k) \exp(j2\pi f_d t)$$

(1)

where $A_k$ is a complex reflection factor (to describe amplitude and phase of the reflection from $k$th point reflector), $\tau_k$ is the bistatic delay associated with the position, and $f_d$ is the Doppler frequency associated with the velocity.

The received signal of $K$-point scatterer can be described as

$$y(t) = x(t) \ast h(t) + n(t)$$

$$= \sum_{k=1}^{K} A_k (x(t - \tau_k) \exp(j2\pi f_d t) + n(t))$$

(2)

where $x(t)$ and $y(t)$ are the transmitted and received signals, respectively, $\ast$ is a convolution operator, and $n(t)$ denotes a white noise. It must be noted that in a PR receiver, the signal $x(t)$ is practically obtained by radio reception from the reference antenna.

In the PR application, $y(t)$ can be written as a linear combination of $\mathbf{L}$ basis vectors

$$y_i = y(t) = \sum_{l=1}^{K} \alpha_l \varphi_l(t) + n(t) = \mathbf{w}_i \mathbf{a}_l + n(t)$$

(3)

The set $\mathbf{w}_i = \{\varphi_1, \varphi_2, \ldots, \varphi_L\}$ is called dictionary. To construct the columns of $\mathbf{w}$, it is necessary to consider many delay-Doppler pairs arising from differential delays associated with targets’ spatial positions as well as finite number of possible velocities [8]. The dictionary in the $i$th column (also called as atom) (see Fig. 1) is

$$\varphi_l(t) = x(t - \tau_l) \exp(j2\pi f_d t)$$

(4)

where $n = 1, 2, \ldots, N$, $m = 1, 2, \ldots, M$. Thus, the target-scene coefficient vector $\mathbf{a}_l = [a_{11}, a_{12}, \ldots, a_{1M}]^T$ is a sparse vector with length of $L = MN$, which has $K$ non-zero coefficients.

2.2 Signal model for multistatic PR

To simplify the system, we assume that the multistatic PR consists of a transmitter (of opportunity) plus a co-located array of receiving antennas. The emitted signals are reflected by target scenes on the position and Doppler frequency of targets. For example, the resolution of PR is limited because of the

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\[ y_i = \text{vec}(y_1, \ldots, y_I) \]  

(7)

To apply CS for a multistatic radar system, we rearrange it in the following relation

\[ y_{ij} = \psi_{ij}\alpha_{ij} + \tilde{n} \]  

(8)

where the dictionary is a set, \( \psi_{ij} = \{\psi_1, \psi_2, \ldots, \psi_I\}_{IP \times Ii} \) (see Fig. 3). The dictionary in the \( i \)th block is

\[ \psi_i = \text{diag}(Y_1, \ldots, Y_I) \]  

(9)

\[ Y_i = x(t - c_i)\exp(j2\pi f_{at}t) \]  

(10)

where \( n = 1, 2, \ldots, N \), \( m = 1, 2, \ldots, M \), \( \tilde{n} = \text{vec}[n_1, \ldots, n^M] \), and \( \alpha_{ij} = \text{vec}(a_1^T, \ldots, a_L^T) \) is block sparse coefficient vector with length of \( I \times L \), which have \( I \times K \) non-zero coefficients.

### 2.3 CS-based recovery

Orthogonal matching pursuit (OMP) [11] can be used to solve (3). It estimates the support of the sparse signal iteratively. Advantages of this algorithm are its accuracy and computational efficiency. To reconstruct the sparse vector, this algorithm first tries to find the column of dictionary, which best matches the observation vector, and then find the next one to best fit the residual. When the support is estimated, the corresponding coefficients of sparse signal can be computed by least square method.

Several algorithms for block-sparse vectors have been described in the literature. A block version of OMP (BOMP) has been proposed in [12]. The difference between BOMP and OMP is the selection stage. Instead of selecting only a single atom, BOMP selects a group containing the highest correlation with residual. BOMP is an effective solution to block sparse signal reconstruction and have better performance. Here, we utilise BOMP to solve (8).

### 3 Incoherent dictionary formulation

#### 3.1 Gram matrix of dictionary

The Gram matrix \( G = \psi^H\psi \) is a metric of qualitatively representing the degree of dictionary correlation. Fig. 4 is the Gram matrix of the block sparse dictionary.

The block sparse dictionary is divided by the different Doppler shift. Thus, the Gram matrix of block sparse dictionary can be regarded as the combination of \( G_{ij}^L = \psi_{ij}^TR_{ij}^L[\psi_{ij}^L]^T \). On the basis, we define the total Doppler inter-block coherence as

\[ \mu_B = \sum_{j=1}^{b} \sum_{m=1}^{\frac{B}{m}} \| G_{ij}^L \|^2_F \]  

(11)

and the total Doppler sub-block coherence as

\[ v = \sum_{j=1}^{b} \| G_{ij}^L \|^2_F - \sum_{m=1}^{\frac{B}{m}} (G_{ij}^L)^2 \]  

(12)

where \( B \) is the number of interest Doppler interval. Then, \( G_{ij}^L \) is the diagonal entries of \( G_{ij}^{Ic} \). It can be seen directly from Fig. 4, the total Doppler sub-block coherence is mainly dominated by the grid of time delay, and the total Doppler inter-block coherence is mainly influenced by the grid of Doppler shift. Similarly, the interior of...
\[ G(15) = m \cdot G(16) \]

\[ h = \cdot \]

\[ \eta = \cdot \]

\[ \mu = \cdot \]

\[ h(\cdot) = \left\{ \begin{array}{ll} 1 & \quad i = j, m = n \\ 0 & \quad i \neq j, m = n \\ \eta h(\cdot) & \quad i = j, m \neq n \\ \mu h(\cdot) & \quad i \neq j, m \neq n \end{array} \right. \quad (13) \]

\[ G^0_{i,j}(m,n) = \left\{ \begin{array}{ll} 1 & \quad i = j, m = n \\ 0 & \quad i \neq j, m = n \\ G^0_{i,j}[m,n] & \quad i = j, m \neq n \\ 0 & \quad i \neq j, m \neq n \end{array} \right. \quad (14) \]

Fig. 5 Grid refinement

Fig. 6 Comparison of Gram matrices for different grids
(a) \( \Delta t_0 = \Delta t, \Delta f_0 = \Delta f_0, \) (b) \( \Delta t_1 = \Delta t, \Delta f_1 = \Delta f_0/4, \) (c) \( \Delta t_2 = \Delta t/2, \Delta f_2 = \Delta f_0/4 \)

Input: dictionary \( \psi \)

Initialize: iteration count \( n = 0 \)

1. Eigenvalue decomposition \( \psi^\top \psi = Q \Sigma Q^\top \)
2. Transform matrix \( \Phi = \frac{1}{2} Q \Sigma^\frac{1}{2} \)

While repeat until convergence do
1. Set \( G_{f_0}^{(\psi)} = \psi^\top \Phi^0 \psi \)
2. Calculate \( h(G_{f_0}^{(\psi)}) \) as in (13).
3. Obtain eigenvectors \( V \) of \( \Sigma^\frac{1}{2} Q^\top \psi \)
4. Set \( \Phi^{(n+1)} = \Delta v \cdot \Sigma^\frac{1}{2} Q^\top \)
5. \( n = n + 1 \)

Output: \( \psi = \Phi^{(n)} \psi \)

Fig. 7 Construction of incoherent dictionary

\[ G^0_{i,j} \] is dominated by the number of time delay grids and the number of stations in multistatic radar. For monostatic radar system, the difference of the Gram matrix is the red block, which can be seen as integration. However, we only discuss the incoherent dictionary optimisation method can deal with it. The present method has general applicability to monostatic and multistatic radar systems.

Refining the grids of dictionary is a general way to obtain high resolution (see Fig. 5). The gram matrix will become deteriorate with fining grid. The comparison of the grids of time and Doppler, we present a novel incoherent dictionary design method inspired by [13].

According to refining direction, an appropriate weight of \( \sigma \) can be chosen to improve the reconstruction performance. The incoherent dictionary optimisation method can deal with it. The details of the algorithm are in Fig. 7, where \( G_{f_0} \) is a function of the
concluded that incoherence dictionary could be constructed by choosing small $\sigma$.

Increasing the grids of time to $\Delta \tau = \Delta \tau_d/2$ while keeping the Doppler grid fining, the Gram matrix of the dictionary and the corresponding RD result are shown in Figs. 9a and b, respectively. Fig. 10a exhibits the Gram matrix results of the dictionary constructed by the proposed incoherent method with $\sigma = 0.01$, from which, it can be seen that the Gram matrix become more ideal. In Fig. 10b, an RD image of the moving target with no many artefacts nearby illustrates that the presented method can guarantee good quality of image with high-resolution under the condition of fining grid.

An output SNR gain is defined as the ratio of output SNR of the decorrelated method and the traditional method. It is employed here for the quantitative measure of the difference of reconstruct process accuracy between decorrelation processing and classical method. In the case of suffering from the Doppler inter-block coherence and the incoherent dictionary is designed by $\sigma = 0.01$, Fig. 11 shows the effect of finer grid on reconstruction. It shows that decorrelation processing can exhibit better reconstruction performance than classical method in fine-gridding.

Under the condition of low SNR, the performance of monostatic radar system deteriorates. The range-Doppler (RD) results of monostatic radar system and multistatic radar system are shown in Fig. 12. The black patch represent real target, and the grey patches represent the ghost target. It can be seen that the multistatic radar system improves the detection accuracy significantly.

Similar to the situation in monostatic radar system, the Gram matrix of refining the grid in multistatic radar system is shown in Fig. 13a. In Fig. 13b, the ambiguous of target can be seen obviously. The Gram matrix and RD result exploiting the incoherent construction method with $\sigma = 0.01$ are shown in Fig. 14. In Fig. 14b, it shows that the incoherent dictionary construction method efficiently reduces the ambiguity.

5 Conclusion

In this paper, we present a construction method of incoherent dictionary, in which a weight can be chosen to suppress the Doppler inter-block coherence and Doppler sub-block coherence of Gram matrix. The presented method reduces the correlation of dictionary caused by fine-gridding. We demonstrate the output SNR gain under different refinement degree and the simulation shows that the decorrelation processing gets high SNR result while the traditional one does not. Finally, the simulation indicates that the incoherent dictionary processing scheme guarantees good reconstruction quality of the range-Doppler map of PR system based on high-resolution CS.

Fig. 9 Finer dictionary before decorrelation in the monostatic simulation
(a) Gram matrix, (b) RD result

Fig. 10 Finer dictionary after decorrelation with $\sigma = 0.01$
(a) Gram matrix, (b) RD result

Fig. 11 Output SNR gain under fine-gridding
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