Sterile Plus Active Neutrinos and Neutrino Oscillations

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Abstract

Using a 3 + 1 neutrino model with one sterile and the three standard active neutrinos with a 4x4 unitary transformation matrix, $U$, relating flavor to mass neutrino states, the probability of $\nu_\mu$ to $\nu_e$ transition is estimated using sterile-active neutrino masses determined by MiniBooNE and other experiments and sterile-active neutrino angles in the 4x4 $U$ matrix.

1 Introduction

This is an extension of work on time reversal violation\cite{1} and CP violation\cite{2} via neutrino oscillations. That work used S-matrix theory with a 3x3 matrix to relate the standard three flavor neutrinos to three neutrinos with well-defined mass. In the present work we extend the standard model by including a fourth neutrino, a sterile neutrino. Recent experiments on neutrino oscillations\cite{3} (see Ref\cite{5} for references to earlier experiments) have suggested the existence of at least one sterile neutrino and the mass used in the present work. See Ref\cite{4} for a discussion of sterile neutrino oscillations and references to experimental and theoretical publications, and Ref\cite{5} for a 6x6 matrix model. Also, pulsar velocities have been estimated using sterile neutrino emission\cite{6} based on a recent estimate\cite{7} of the $\nu_e - \nu_\mu$ mixing angle.

Our present work is most closely related to Ref\cite{2} in which CPV was studied. $P(\nu_a \rightarrow \nu_b)$ is the transition probability for a neutrino of flavor $a$ to convert to a neutrino of flavor $b$; and similarly for antineutrinos $\bar{\nu}_a, \bar{\nu}_b$. The CPV probability difference for $\nu_\mu$ to $\nu_e$ oscillation is defined as

$$\Delta P^{CP}_{\mu e} = P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e).$$  

(1)
A main objective of Ref[2] was to find the dependence of $\Delta P_{\mu e}^{CP}$ on the parameter $\delta_{CP}$. The antineutrino oscillation probability is related to the neutrino oscillation probability by the neutrino matter potential $V \rightarrow -V$ and $\delta_{CP} \rightarrow -\delta_{CP}$. Since in the present work we are only investigating how $P(\nu_\mu \rightarrow \nu_e)$ is modified by the introduction of a sterile neutrino we set both $V$ and $\delta_{CP}$ equal to zero. $P(\nu_\mu \rightarrow \nu_e)$ is not very dependent on either quantity[2].

2 $P(\nu_\mu \rightarrow \nu_e)$ Derived Using the U Matrix

This is an extension of the method introduced by Sato and collaborators for three active neutrino oscillations[8, 9] to three active neutrinos plus one sterile neutrino. Active neutrinos with flavors $\nu_e, \nu_\mu, \nu_\tau$ and a sterile neutrino $\nu_s$ are related to neutrinos with definite mass by

$$\nu_f = U \nu_m ,$$

where $U$ is a 4x4 matrix and $\nu_f, \nu_m$ are 4x1 column vectors, which is an extension of the 3x3 matrix used in Refs. [8, 9] (with $s_{ij}, c_{ij} = \sin \theta_{ij}, \cos \theta_{ij}$),

$$U = O^{23} \phi O^{13} O^{12} O^{14} O^{24} O^{34}$$

with

$$O^{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{23} & s_{23} & 0 \\ 0 & -s_{23} & c_{23} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$O^{13} = \begin{bmatrix} c_{13} & 0 & s_{13} & 0 \\ 0 & 1 & 0 & 0 \\ -s_{13} & 0 & c_{13} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$O^{12} = \begin{bmatrix} c_{12} & s_{12} & 0 & 0 \\ -s_{12} & c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$O^{14} = \begin{bmatrix} c_\alpha & 0 & 0 & s_\alpha \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_\alpha & 0 & c_\alpha & 0 \end{bmatrix}$$

$$O^{24} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_\alpha & 0 & s_\alpha \\ 0 & 0 & 1 & 0 \\ 0 & 0 & s_\alpha & c_\alpha \end{bmatrix}$$

$$O^{34} = \begin{bmatrix} c_\alpha & s_\alpha & 0 & 0 \\ 0 & -s_\alpha & c_\alpha & 0 \\ 0 & 0 & -s_\alpha & c_\alpha \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\delta_{CP}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with $c_{12} = .83, s_{12} = .56, s_{23} = c_{23} = .7071$. We use $s_{13} = .15$ from the recent Daya Bay Collaboration[10]. In our present work we assume the angles $\theta_{j4} = \alpha$ for all three $j$, and $s_\alpha, c_\alpha = \sin \alpha, \cos \alpha$. An important aspect of our work is to find the dependence of neutrino oscillation probabilities on $s_\alpha, c_\alpha$.

From Eq[3] the 4x4 $U$ matrix is
by Eq. (5), is quite different, and as will be shown for the same $L$, $E$

magnitude of the $3 \times 3$ U-matrix and the $\delta m$ was the best fit parameter found via the 2013 MiniBooNE analysis, while use both $\delta m$ mass differences are $E$ or, since with $\delta m = m_i - m_j$ for a neutrino beam with energy $E$ and baseline $L$ by (8)

\[
\mathcal{P}(\nu_\mu \to \nu_e) = \sum_{i=1}^{4} \sum_{j=1}^{4} U_{1i} U^*_{1j} U_{2i} U^*_{2j} e^{-i(m^2_{ij}/E)L}, \tag{5}
\]

or, since with $\delta_{CP} = 0$ $U^*_{ij} = U_{ij}$,

\[
\mathcal{P}(\nu_\mu \to \nu_e) = U_{11} U_{21} [U_{11} U_{21} + U_{12} U_{22} e^{-i\delta L} + U_{13} U_{23} e^{-i\delta L} + U_{14} U_{24} e^{-i\gamma L}] + U_{12} U_{22} [U_{11} U_{21} e^{-i\delta L} + U_{12} U_{22} + U_{13} U_{23} e^{-i\delta L} + U_{14} U_{24} e^{-i\gamma L}] + U_{13} U_{23} [U_{11} U_{21} e^{-i\Delta L} + U_{12} U_{22} e^{-i\Delta L} + U_{13} U_{23} + U_{14} U_{24} e^{-i\gamma L}] + U_{14} U_{24} [U_{11} U_{21} + U_{12} U_{22} + U_{13} U_{23} e^{-i\gamma L} + U_{14} U_{24}], \tag{6}
\]

with $\delta = \delta m^2_{12}/2E$, $\Delta = \delta m^2_{13}/2E$, $\gamma = \delta m^2_{j4}/2E$ (j=1,2,3). The neutrino mass differences are $\delta m^2_{12} = 7.6 \times 10^{-5} (eV)^2$, $\delta m^2_{13} = 2.4 \times 10^{-3} (eV)^2$; and we use both $\delta m^2_{j4} = 0.9 (eV)^2$ and $\delta m^2_{j4} = 0.043 (eV)^2$, since $\delta m^2_{j4} = 0.043 (eV)^2$ was the best fit parameter found via the 2013 MiniBooNE analysis, while $\delta m^2_{j4} = 0.9 (eV)^2$ is the best fit using the 2013 MiniBooNE data and previous experimental fits\[3\].

Note that in Refs[11] $\mathcal{P}(\nu_\mu \to \nu_e) = |S_{12}|^2$, with $S_{12}$ obtained from the 3x3 U-matrix and the $\delta m_{ij}$ parameters. Therefore our formalism, given by Eq(5), is quite different, and as will be shown for the same $L, E$ the magnitude of $\mathcal{P}(\nu_\mu \to \nu_e)$ is also different.
From Eq(6),

\[
P(\nu_\mu \rightarrow \nu_e) = U_{11}^2 U_{21}^2 + U_{12}^2 U_{22}^2 + U_{13}^2 U_{23}^2 + U_{14}^2 U_{24}^2 + 2U_{11} U_{21} U_{12} U_{22} \cos L + 2(U_{11} U_{21} U_{13} U_{22} + U_{12} U_{22} U_{13} U_{23}) \cos \Delta L + 2U_{14} U_{24} (U_{11} U_{21} + U_{12} U_{22} + U_{13} U_{23}) \cos \gamma L .
\]

Using the parameters given above,

\[
U_{11} = .822 c_\alpha U_{12} = -.554 s_\alpha^2 + .084 c_\alpha \\
U_{13} = -.822 s_\alpha^2 c_\alpha - .554 s_\alpha^2 + .15 c_\alpha U_{14} = .822 s_\alpha c_\alpha^2 + .554 s_\alpha c_\alpha + .15 s_\alpha \\
U_{21} = -.484 c_\alpha U_{22} = .484 s_\alpha^2 + .527 c_\alpha \\
U_{23} = .484 c_\alpha - .527 s_\alpha^2 + .7 c_\alpha U_{24} = -.484 s_\alpha c_\alpha^2 + .527 s_\alpha c_\alpha + .7 s_\alpha .
\]

2.1 \( P(\nu_\mu \rightarrow \nu_e) \) for the 3x3 theory

First we give the results from using the 3x3 theory[2] for \( P(\nu_\mu \rightarrow \nu_e) \) for comparison with the 4x4 theory results given in the next subsection. In this previous work a main goal was to study the dependence of \( P(\nu_\mu \rightarrow \nu_e) \) on \( s_{13} \), but now it has been determined[10] to be approximately 0.15.

The results using the 3x3 theory from Ref[2] are shown in Figure 1.

2.2 \( P(\nu_\mu \rightarrow \nu_e) \) for the 4x4 theory

With the addition of a sterile neutrino, the 4th neutrino, there are three new angles, \( \theta_{14}, \theta_{24}, \) and \( \theta_{34} \). Our main assumption is that these three angles are the same, \( \theta_{34} = \alpha \). The angle \( \alpha \) is the main parameter that we are studying.

We also use two values for the sterile-active mass differences. The most widely accepted value for \( m_4^2 - m_1^2 \) is 0.9(eV)^2[3], but we also use \( m_4^2 - m_1^2 = .043(eV)^2 \) from the 2013 MiniBoonE result to test the sensitivity of \( P(\nu_\mu \rightarrow \nu_e) \) to the sterile neutrino-active neutrinos mass differences. Since \( m_4^2 - m_1^2 >> m_j^2 - m_i^2 \) for (i,j)\(=1,2,3 \), we assume that \( m_4^2 - m_j^2 = m_4^2 - m_i^2 \).

Figure 2 shows our results for \( P(\nu_\mu \rightarrow \nu_e) \) for the four experiments with \( m_4^2 - m_1^2 = 0.9(eV)^2 \) and \( \alpha = 45^\circ, 60^\circ, 30^\circ \). As one can see, \( P(\nu_\mu \rightarrow \nu_e) \) is very strongly dependent on \( \alpha \).

Next we use \( m_4^2 - m_1^2 = 0.043(eV)^2 \), as found in the recent MiniBooNE experiment, to study the effects of \( m_4^2 - m_1^2 \) on \( P(\nu_\mu \rightarrow \nu_e) \), with \( \alpha = 45^\circ, 30^\circ, 60^\circ \), as shown in Figure 3.

Note for \( \alpha = 0 \) (no sterile-active mixing) \( U_{14} = 0 \). Therefore, \( P(\nu_\mu \rightarrow \nu_e) \) is a 3x3 theory; however, we find that \( P(\nu_\mu \rightarrow \nu_e) \) is different with the model of Refs.[3][9], Eq(6), than the theory used in Ref.[2], shown in Figure 1.
Figure 1: The ordinate is $\mathcal{P}(\nu_\mu \to \nu_e)$ for MINOS ($L=735$ km), MiniBooNE ($L=500$ m), JHF-Kamioka ($L=295$ km), and CHOOZ ($L=1.03$ km) using the 3x3 U matrix. Solid curve for $s_{13}=0.19$ and dashed curve for $s_{13}=0.095$. The curves are almost independent of $\delta_{CP}$. 
Figure 2: The ordinate is $P(\nu_\mu \rightarrow \nu_e)$ for MINOS($L=735$ km), MiniBooNE($L=500$ m), JHF-Kamioka($L=295$ km), and CHOOZ($L=1.03$ km) using the 4x4 U matrix with $\delta m^2_{4j} = 0.9(eV)^2$ and (a),(b),(c) for $\alpha = 45^\circ, 60^\circ, 30^\circ$. The dashed curves are for $\alpha = 0$ (3x3)
Figure 3: The ordinate is \( P(\nu_\mu \rightarrow \nu_e) \) for MINOS (L=735 km), MiniBooNE (L=500 m), JHF-Kamioka (L=295 km), and CHOOZ (L=1.03 km) using the 4x4 U matrix with \( \delta m^2_{41} = 0.043 \text{(eV)}^2 \) and (a),(b),(c) for \( \alpha = 45^\circ, 60^\circ, 30^\circ \). The dashed curves are for \( \alpha = 0 \) (3x3).
3 Conclusions

As shown in the figures, the neutrino oscillation probability, $P(\nu_\mu \to \nu_e)$, is quite different for a model with four neutrinos, $\nu_e, \nu_\mu, \nu_\tau, \nu_s$. $P(\nu_\mu \to \nu_e)$ is also different for the sterile-active neutrino mass difference $m_4^2 - m_1^2 = 0.043(eV)^2$ vs $m_4^2 - m_1^2 = 0.9(eV)^2$, which is favored by experiment.

Our most important result is that $P(\nu_\mu \to \nu_e)$ is strongly dependent on the sterile-active neutrino mixing angles, with the oscillation probability very different for 30, 60 and 45 degrees for $m_4^2 - m_1^2 = 0.9(eV)^2$. Therefore, future experiments might be able to determine these sterile-active neutrino parameters. Note that Ref. [7] favors a small $\nu_e - \nu_s$ mixing angle.

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