Dynamics of the Peccei Quinn Scale

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Abstract

Invoking the Peccei-Quinn (PQ) solution to the strong CP problem substitutes the puzzle of why $\theta_{\text{qcd}}$ is so small with the puzzle of why the PQ symmetry is of such high quality. Cosmological and astrophysical considerations raise further puzzles. This paper explores this issues in several contexts: string theory and field theory, and theories without and with low energy supersymmetry. Among the questions studied are whether requiring axion dark matter can account for the quality of the PQ symmetry, to which the answer is sometimes yes. In non-supersymmetric theories, we find $f_a = 10^{12}$ GeV is quite plausible. In gauge mediation, cosmological constraints on pseudomoduli place $f_a$ in this range, and require that the gravitino mass be of order an MeV.
1 Axions: Their Virtues and Deficiencies

Nuclear physics is almost indifferent to the QCD angle[1], yet for some reason $\theta_{qcd}$ is incredibly small. In thirty years, only three persuasive solutions of this puzzle have been put forward.

1. $m_u = 0$: This could result as an accident of discrete flavor symmetries[2], or a result of “anomalous” discrete symmetries as in string theory[3].

2. Spontaneously broken CP: Here one postulates that CP is an exact symmetry of the microscopic theory, which is spontaneously broken. $\theta$ is then calculable, in principle, and under certain circumstances, might be small[4, 5]. In critical string theories, CP is an exact (gauge) symmetry[6, 7], spontaneously broken at generic points in typical moduli spaces. So this would seem a plausible framework.

3. Axions: in the presence of an approximate, global symmetry (Peccei-Quinn (PQ) symmetry) with a QCD anomaly, the pseudo-Goldstone boson which arises from symmetry breaking (the axion) adjusts to yield $\theta \approx 0$. This raises two puzzles. First, the symmetry must be extremely good if it is to solve the strong CP problem, and second, with simple, but strong, cosmological assumptions, the decay constant of the axion must be small compared to scales such as the unification scale and the Planck scale. Critical string theories typically exhibit an extremely good (but approximate) global symmetry; these symmetries are exact in perturbation theory, broken only by non-perturbative effects[8]. Moreover, in the string theory framework, the standard cosmological assumptions do not hold[9]. So the axion solution, also, has a certain plausibility.

Each of these solutions, however, poses problems.

1. $m_u = 0$. While the work of [2] suggests that this possibility is often realized in models of flavor, lattice computations appear to rule out $m_u = 0[10]$.

2. Spontaneous CP: To assess the plausibility of this idea, it is necessary to consider some sort of underlying structure. If nature is described by theories which resemble critical string theory, one needs to consider fixing of moduli. If we take, as a model, moduli fixing in flux vacua, one often speaks of $10^{500}$ states as arising from turning on many different fluxes. But typically only half of the fluxes are invariant under CP. This means that, say $10^{500} \rightarrow 10^{250}$ states. So only an extremely tiny fraction of states preserve CP. It is not
clear that these are otherwise singled out (e.g. that most states with some other low energy feature preserve CP microscopically, or by cosmological or anthropic considerations). So, at least in this framework, the CP solution does not appear particularly natural.

3. Axions: If nature is described by a nearly critical string theory, as we will review, it is not clear, when moduli are fixed, why axions should survive to low energies. If the Peccei-Quinn symmetry breaking can be seen within low energy, four dimensional field theory, one cannot address the quality of this symmetry without discussing the ultraviolet structure in which it is embedded\[11\]. By itself, this argument does not rule out the axion solution, but it makes its status more uncertain.

Given these remarks, were it not for the lattice results, it would be tempting to view the massless u quark as the most plausible solution of the strong CP problem. Assuming that the lattice results are verified by other groups, this will not be an option, however. While conceptually elegant, the landscape framework, at least, calls the spontaneous CP violation solution into question. Our goal, then, in this paper is to examine the axion solution in various settings, and to consider carefully what is required for its successful implementation. By successful, we mean that the axion not be in conflict with basic facts of particle physics (the smallness of $\theta_{qcd}$) and cosmology and astrophysics (nucleosynthesis, dark matter energy density). By natural, we mean that there should be a sense in which the value of $f_a$ and the quality of the axion potential should be generic.

To sharpen the notion of naturalness, we will distinguish two theoretical frameworks for the axion:

1. PQ symmetry broken by stringy or higher dimensional effects: This case is characterized by the possibility of exponential suppression of PQ symmetry violating operators.

2. PQ symmetry broken in a low energy effective field theory: Here, one needs something like a large $Z_N$ symmetry to account for an accidental PQ symmetry[7].

Within these categories, we will consider three cases:

1. No low energy supersymmetry: Under certain circumstances, we will see that the existence of a low value of $f_a$ is generic. Under the same circumstances, the requirements of a high quality axion are not as onerous as in other settings.
2. Supersymmetry broken at an intermediate scale ("gravity mediation")

3. Supersymmetry broken at a low scale (gauge mediation) We will touch on the main issues here, leaving a more extensive discussion, and explicit model building, to a subsequent publication[12].

In all of these cases, one needs to ask: what might account for the existence of a PQ symmetry? Why should it be such a good symmetry that it can account for the small value of $\theta_{qcd}$? As in any discussion of naturalness, one has in mind here the notion that there is some underlying distribution of possible theories, or states within theories, and one views as natural choices of parameters and other features which are typical of this distribution, consistent with some set of facts (priors)$^1$. One does not have to include the existence of observers among these facts, even if some of the constraints one imposes are essential for, say, the existence of galaxies, or chemistry. One possible prior which might account for high quality axions – perhaps the only one – is that the axion is a generic way to account for the dark matter. Imposing axion dark matter as a constraint requires that the PQ symmetry be quite good, sometimes (but not always) good enough to account for the smallness of $\theta_{qcd}$. Our remarks about the likelihood, say, of low $f_a$, are also to be viewed in this context.

We will see that, imposing the requirement that the axion constitute the dark matter, can, in non-supersymmetric settings, potentially account for the value of $f_a$, and possibly for the quality of the PQ symmetry. In intermediate scale supersymmetry breaking, we are uneasy about imposing this condition, as there are other, possibly more plausible, dark matter candidates; still, in a string/higher dimension setting, this condition can readily account for the small value of $\theta$ (the axion quality). Low scale scale breaking (gauge mediation) provides a more plausible setting for the dark matter condition. Successfully implementing the axion solution in this setting places stringent requirements on the mechanism of PQ breaking, and turns out to require a relatively high scale of supersymmetry breaking (a Goldstino decay constant $F \sim 10^{16}$ GeV$^2$), while forbidding $f_a$ much greater than $10^{13}$ GeV.

The rest of this paper is organized as follows. In section 2, we review the question of axion quality, and explain its possible connection to dark matter. In section 3, we discuss the question of axions in string theory or higher dimensional settings, with and without supersymmetry. In section 4, we discuss Peccei-Quinn symmetry breaking in non-supersymmetric field theories;

$^1$As an example, attempts to construct measures of fine-tuning of the weak scale presuppose a distribution of possible theories, characterized by some parameters, and ask, say, the probability of finding the observed gauge boson masses within this distribution; theories are discarded if the probability is deemed too small.
this is followed by a discussion of supersymmetric field theories in section 5. The implications of our observations are considered in the concluding section.

2 Axion Quality

The PQ solution to the strong CP problem raises at least two serious issues.

1. Astrophysics and cosmology seem to constrain the axion decay constant to a rather narrow range, $10^9$ to $10^{13}$ GeV[13, 14]. If the axion is to be dark matter, and if the initial axion misalignment is of order 1, then $f_a \sim 10^{12}$ GeV. Except, possibly, for the scale intermediate between the weak and the Planck scale, this number does not correspond to other scales we suspect to be relevant to physics, such as the GUT scale or the scale associated with neutrino masses.

2. The PQ symmetry is a global symmetry, so it is presumably an accident. It needs to be an extremely good symmetry if it is to solve the strong CP problem[11].

We can easily quantify the latter problem. The contribution to the axion potential from QCD has roughly the form:

$$V_{qcd} \approx -m_a^2 f_a^2 \cos \left( \frac{a}{f_a} \right).$$  \hspace{1cm} (1)

On the other hand, the natural value of axion potential is:

$$V_a = Q f_a^4 \cos \left( \frac{a}{f_a} - \theta_0 \right).$$  \hspace{1cm} (2)

where $Q$ is a constant which we will call the axion quality. We see that if the axion is to solve the strong CP problem, we require a suppression of the potential below this natural value by 62 orders of magnitude, i.e.

$$Q < 10^{-62} \left( \frac{10^{12} \text{GeV}}{f_a} \right)^4.$$  \hspace{1cm} (3)

Things need not be as extreme as this. In the case of low energy supersymmetry, the natural scale of the potential might be much smaller than $f_a^4$. If the Goldstino decay constant, $F$, is smaller than $f_a$, one finds that the potential is naturally suppressed by $F/f_a^2[12]$. 

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2.1 Accounting for A Very Good Global Symmetry

The nature of the problem is different if the Peccei-Quinn symmetry is broken at the level of
four dimensional effective field theory, or if it is broken in a higher dimensional theory or string
theory. The issues in the field theory are indicated by a simple model, with a complex scalar
field, $\phi$, on which the PQ symmetry acts as $\phi \rightarrow e^{i\alpha} \phi$, and $\langle \phi \rangle = f_a$. An operator of the form
\[
\frac{\phi^{n+4}}{M_p^N}
\]
breaks the PQ symmetry. The “quality factor” is given by
\[
Q = \left( \frac{f_a}{M_p} \right)^n.
\]
So, if $f_a = 10^{12}$, we require $n > 10$; if $f_a = 10^{15}$, we require $n > 20$! In the optimal super-
symmetric case, as explained in[12] we still require $n \geq 10$. The situation is somewhat better,
again, if we give up the dark matter constraint and allow for lower $f_a$. Probably the simplest
way in which one might try to account for a suppression is through a discrete symmetry. The
symmetry would need to be quite large ($Z_{14}$ in our single field model with smaller $f_a$, $Z_{11}$
in the supersymmetric case). We will see cases which are not so extreme shortly. Still, such
symmetries may require additional structure.

In critical string theory, the appearance of PQ symmetries in perturbation theory is a
familiar phenomenon[8]. One might hope that in a setting where moduli are fixed, the breaking
of PQ symmetries would then be governed by (powers of) a small number, such as $e^{-8\pi^2/g^2}$,
with $g^2$ some suitable (generalized) coupling constant. We will discuss the precise requirements,
and say a little about their plausibility, later. However, lacking anything like a complete theory
of moduli stabilization, we will not be able to make definitive statements.

2.2 Axion as Dark Matter

Axions have long been considered a plausible dark matter candidate. They are produced co-
herently in the early universe, by misalignment$^2$[15, 16]. The energy density of axions is pro-
portional to the square of the misalignment angle, $\theta_0$.
\[
\Omega_a h^2 \approx 0.7 \left( \frac{\theta_0}{\pi} \right)^2 \left( \frac{f_a}{10^{12}} \right)^{7/6}.
\]

$^2$We will assume that the reheat temperature after inflation is below $f_a$. 

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This gives an upper bound on $f_a$, if $\theta_0 \sim \pi$, of order $10^{12}$ GeV. If the bound is saturated, the dark matter is accounted for. There is a lower bound coming from more conventional astrophysics of about $10^9$ GeV.

Several mechanisms have been suggested to relax the upper bound on $f_a$. These include:

1. Late decays of particles (e.g., moduli in string theory) can allow $f_a$ up to $10^{14} - 10^{15}$ [16, 17, 9].

2. Luck (or the lack of it)[18]: If the PQ transition occurs after inflation, different regions have different $\theta_0$.

3. Anthropic[19, 20, 21]: An elaboration on the idea above is the possibility that anthropic considerations related to the density of dark matter select for small $\theta_0$. Existing studies make plausible that hospitable universes lie in a narrow range of $\Omega_{dm}$, though they hardly demonstrate this conclusively. Note the assumption of inflation means that, if there is some peaking in the distribution, some (anthropic) selection is inevitable.

Having introduced in a rather non-controversial way, anthropic selection for $\theta_0$, it is tempting to consider anthropic selection for

1. The existence of axions

2. Other parameters, such as $f_a$.

The first point requires that, in some theoretical framework, axions be a particularly “generic” type of dark matter. As to the second, in an underlying landscape, one might expect that $f_a$ varies. This might be interesting if requiring an axion to be the dark matter simultaneously explains the smallness of the observed $\theta$.

**2.3 Pseudogoldstone Boson as Accidental Dark Matter**

A light pseudogoldstone boson could serve as dark matter, independent of whether it solves the strong CP problem. For example, in the simple model of eqn. 4, the axion has a cosine potential and very weak coupling. Misalignment of this field with the stationary point of its potential will give rise to “axion” cold dark matter. How light does this axion have to be in order to serve as dark matter? The basic requirement is that the axion not dominate the energy
density for temperatures above about 1 eV. If the Peccei-Quinn symmetry is violated by some higher dimension operator, scaled by $M_p$, such as

$$\delta = \left( \frac{h}{M_p^n} \phi^{n+4} + \text{c.c.} \right)$$  \hspace{1cm} (7)

then the axion mass is:

$$m_a^2 = h f_a^2 \left( \frac{f_a}{M_p} \right)^n.$$  \hspace{1cm} (8)

On the other hand, the initial axion energy density is of order $f_a^2/M_p^2 = 10^{-12} \left( \frac{f_a}{10^{12}} \right)^2$. So we require:

$$10^{27} \left( \frac{f_a}{M_p} \right)^{n+12} < 1$$

For $f_a = 10^{12}$, this indeed requires $n > 8$, but the requirement of small enough $\theta$ means $n > 10$. So this condition is strong, but it is not, by itself, quite sufficient to account for $\theta_{\text{qcd}}$. It is hard to assess the relative likelihood of these two cases; e.g. if the suppression is due to a discrete symmetry, one requires a large discrete symmetry in each case, but one might worry that a larger symmetry ($Z_{14}$) is exponentially less likely than a smaller one ($Z_{12}$)[22]. We will comment on this issue further when we discuss all of this in the context of the landscape, but we will not provide a definitive answer.

Interestingly, for larger $f_a$ there is a crossover; the requirement of dark matter insures small enough $\theta$ for $f_a \sim 10^{14}$. However, the required $n$’s are huge, more than 20! In string theory, within our present, limited understanding, the problem looks different, as we will discuss in the next section

### 3 PQ Symmetry Broken In String Theory/Higher Dimensions

As we have remarked, critical string theories seem to implement all of the known solutions to the strong CP problem. Such theories always have moduli, however, and the issue is whether these phenomena – unbroken CP microscopically, axions, and discrete symmetries (or simple accidents) which might account for a vanishing $m_u$, survive in quantum gravity theories without moduli. At present, the only framework in which we can formulate these questions is in (hypothetical) nearly critical theories, in which moduli are “fixed.” It is not clear that any model of this kind exists in which systematic analysis is possible; the most complete scenario for such moduli fixing is that of KKLT[23]. In any picture of moduli fixing, the problem is
to understand why the non-perturbative effects which break the PQ symmetries are extremely small (i.e. why the relevant couplings are small). In the simplest version of the KKLT scenario, all moduli fixed at high scales. The lightest is a Kahler modulus. It’s superpotential,

\[ W = W_0 + e^{-\rho}, \tag{9} \]

explicitly breaks the would-be Peccei-Quinn symmetry. There is a distribution of values of \( W_0 \), and the small parameter arises simply because there are many possible states, some with small \( W_0 \). In the scenario, which relies heavily on approximate supersymmetry, \( W_0 \gg m_{3/2} \), so the would-be axion lies in a massive chiral supermultiplet; the axion does not solve the strong CP problem\[24, 25\]. One might speculate that in some cases where there are multiple Kahler moduli, a subset would not appear in the superpotential, or appear suppressed by \( e^{-n\rho} \), for some \( n \), or suppressed by some other small quantity entirely. In the KKLT scenario, the small parameter is tied with the scale of supersymmetry breaking (up to powers of coupling constants).

\[ e^{-\rho} = \frac{m^2_{3/2}}{M_p^2} \tag{10} \]

So in order to sufficiently suppress PQ violating effects for some other modulus and solve the strong CP problem, one needs \( n > 3 \). We simply do not know enough about these theories to determine whether such a suppression might be generic. As for the field theory models, we can ask whether dark matter might select for it. The problem takes a different for depending on whether or not the low energy theory is supersymmetric.

### 3.1 String Theory: A picture without supersymmetry

Consider, first, the possibility that there is no low energy supersymmetry, and that there is a small parameter, \( e^{-8\pi^2/g^2} = \epsilon \). If \( \epsilon \) is the strength of PQ breaking, and if \( f_a = 10^{15} \), then we require

\[ Q = \epsilon = 10^{-74} \tag{11} \]

to account for \( \theta \).

We can now ask whether the condition to obtain suitable dark matter is equally strong. Again, we will take \( f_a = 10^{15} \), a plausible scale for string theory; we expect that as the axion starts to oscillate, it represents a fraction \( f_a^2/M_p^2 \) of the energy density. Requiring that the axion not dominate the energy density before \( T = 1 \) eV, gives \( \epsilon < 10^{-78} \). So under such circumstances, the requirement of dark matter might explain the small value of \( \theta_{qcd} \).
3.2 String Theory With Low Energy Supersymmetry

If supersymmetry is broken at low energies, there is at least one small parameter, $\epsilon = m^2_3/2M_p^2$. Assuming that there is a PQ symmetry violated only by terms of order $\epsilon^n$, and again taking $f_a = 10^{15}$ GeV, the requirement that the axion yield the dark matter yields $n \geq 3$. Again, this is enough to explain $\theta_{qcd}$.

Alternatively, there might be some other small quantity, $\epsilon' \ll \epsilon$. But we are clearly on shakier ground in imposing the requirement that the axion is the dark matter in the framework of supersymmetry; there are other plausible candidates, which might well arise in more generic ways (i.e. through a conserved $R$ parity).

4 PQ Breaking Within Non-Supersymmetric Effective Field Theories

In non-supersymmetric field theories, in addition to the question of axion quality, the small value of the axion decay constant is a puzzle. We can ask, along the lines of Aguirre et al, whether not only the initial value value of $\theta$, $\theta_0$, might be selected to account for a narrow range of dark matter densities, but similarly $f_a$. In a non-supersymmetric theory, we would expect small $f_a$ to be much more improbable than small $\theta_0$; if $M_p$ is the fundamental scale, and assuming a uniform distribution of $f_a^2$, $f_a = 10^{12}$ would be extremely improbable (unnatural); $\theta_0 < 10^{-3}$ would seem far more reasonable.

4.1 Dynamical Breaking of PQ Symmetry

The situation is different the Peccei-Quinn symmetry is dynamically broken. Consider, for example, an $SU(N)$ gauge theory, with a set of fields with $SU(N) \times SU(5)$ quantum numbers:

\begin{equation}
Q = (N, 5) \quad \overline{Q} = (\overline{N}, 5) \quad q = (N, 1) \quad \overline{q} = (\overline{N}, 1).
\end{equation}

This model has a PQ symmetry with a QCD anomaly. This symmetry is broken by

\begin{equation}
\langle Q Q \rangle \approx \Lambda^3 \quad \langle \overline{q} q \rangle \approx \Lambda^3
\end{equation}

with $f_a \approx \Lambda$. Now, again in a landscape context, one might expect

\begin{equation}
\Lambda = M_p e^{-\frac{\alpha e^2}{\sqrt{\phi^2}}}. \quad (14)
\end{equation}
If $g^2$ is uniformly distributed, small $f_a$ is favored over small $\theta_0$.

With the assumption that $\theta_0$ and $g^2$ are uniformly distributed, one can even quantify the relative likelihood of small $f_a$ vs. small $\theta_0$. From equation 6 we have seen that one either needs $\theta_0 \sim 3 \times 10^{-4}$, or $f_a \approx 10^{12}$, or some combination of the two. Defining $d\theta_0 df_a P(f_a, \theta_0)$ as the fraction of the $\theta_0, f_a$ space allowed by the dark matter constraint with $f_a, \theta_0$ in the volume $d\theta_0 df_a$, we have:

$$
\int d\theta_0 df_a P(f_a, \theta_0) \delta \left( 1 - 0.7 \left( \frac{\theta_0}{\pi} \right)^2 \left( \frac{f_a}{10^{12}} \right)^{7/6} \right) = 1
$$

then

$$
F(f_a) = \int_0^{f_a} df'_a \int_0^{2\pi} d\theta_0 P(f'_a, \theta_0) \delta \left( 1 - 0.7 \left( \frac{\theta_0}{\pi} \right)^2 \left( \frac{f_a}{10^{12}} \right)^{7/6} \right)
$$

has most of its support at $f_a \sim 10^{12}$ GeV. This is a naturalness argument that axions might be observable in cavity experiments.

![Figure 1](image_url)

Figure 1: The function $F(f_a)$, defined in the text. $F = 0.8$ means that 80% of the allowed range of parameters has smaller $f_a$.

In this dynamical context, a smaller discrete symmetry might account for the quality of the PQ symmetry. Operators allowed by the gauge symmetries, such as

$$
\left( \frac{1}{M_p} \right)^{3n-4} ((\bar{Q}Q)^n, (\bar{q}q)^n, \text{etc.})
$$

(17)
break the $U(1)$ explicitly, giving rise to an axion potential. If $f_a = 10^{12}$ GeV, one requires $n \geq 4$. So a $Z_5$ symmetry might be needed to adequately suppress $\theta$. On the other hand, a $Z_4$ symmetry is more than adequate to yield the axion as a suitable dark matter candidate. Arguably, the difference between $Z_4$ and $Z_5$ is not so great; moreover, the $Z_4$ symmetry is already borderline.

5  Peccei-Quinn Breaking in Supersymmetric Field Theories

Supersymmetric theories raise new issues. Most important, the axion decay constant is large compared to scales usually contemplated for low energy supersymmetry breaking. As a result, $f_a$ is determined by a (pseudo) modulus. This particle is not necessarily the saxion; indeed, once supersymmetry is broken, the superpartner of the axion need not be a mass eigenstate[12]. The lightest modulus typically will couple to $F^2_{\mu\nu}$ with strength comparable to that of the axion to $F \tilde{F}$, and similarly for the axino.

5.1 Simple Models

Before committing to a particular scale of supersymmetry breaking, we consider two renormalizable models which illustrate the inevitable role of a pseudomodulus in determining the scale $f_a$. Both possess a continuous $R$ symmetry. Start, first, with three fields, $\chi, S_{\pm}$, where $\chi$ possesses $R$ charge 2 and vanishing PQ charge, while $S_{\pm}$ carry $R$ charge 0 and PQ charge $\pm 1$. The superpotential takes the form:

$$W = \chi(S_+ S_- - \mu^2).$$

(18)

Supersymmetry is unbroken; there is a moduli space of vacua, which we can describe by writing:

$$S_{\pm} = (\mu + \rho(x))e^{\pm A}$$

(19)

Here $\rho$ and $\chi$ are massive, and $A$ is the axion supermultiplet. The axion decay constant is

$$f_a^2 = \mu^2 \cosh 2\text{Re}A$$

(20)

To break supersymmetry, we can add fields, $X, Y$, etc, neutral under the Peccei-Quinn symmetry, along the lines of [26], which break supersymmetry and the $R$ symmetry. Now if we add some “messengers”, $M, \bar{M}$, carrying PQ charges and coupled both to $S_{\pm}$ and $X$, one loop effects will give rise to a potential on the pseudomoduli space, $A[12]$. 

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Alternatively, consider an O’Raifeartaigh-like model in which fields carrying $R$ charge also transform under the Peccei-Quinn symmetry. Below, $X, Z_\pm$ carry $R$ charge 2 and PQ charges $0, \pm 1$:

$$W = \lambda X(S_+ S_- - \mu^2) + m_1 Z_+ S_- + m_2 Z_- S_+.$$  \hspace{1cm} (21)

For large $\mu$, the PQ symmetry is broken at tree level: $\langle S_\pm \rangle \neq 0$. Classically, there is a moduli space with

$$S_+ X + m_1 Z_+ = 0 \quad S_- X + m_2 Z_- = 0.$$  \hspace{1cm} (22)

On this moduli space, for large $X$, the axion decay constant is given by:

$$f_a^2 = |Z_+|^2 + |Z_-|^2$$  \hspace{1cm} (23)

The modulus, responsible for PQ breaking, is not the saxion in this model. Indeed, it is the partner of the $r$-axion, the “$r$-saxion”:

$$\tilde{r} = \text{Re}(Z_+ + Z_-).$$  \hspace{1cm} (24)

While the axion and saxion arise from the orthogonal linear combination:

$$a = \text{Im}(Z_+ - Z_-) \quad s = \text{Re}(Z_+ + Z_-)$$  \hspace{1cm} (25)

which has mass of order the supersymmetry breaking scale.

At one loop, the modulus is fixed, and vanishes; the $R$ symmetry is unbroken But more intricate versions of this model, following ideas of ref. [26], yield $R$ symmetry breaking and potentially large PQ breaking[12].

5.2 Intermediate scale SUSY breaking

We have earlier remarked that a scale of order $10^{12}$ GeV does not correspond, in a natural way, to other high energy scales which have been discussed in particle physics, but this is not quite true: in “gravity mediation”, this is the natural scale of supersymmetry breaking. One might then ask whether PQ breaking might be correlated with supersymmetry breaking in such a framework. At first sight, this would seem appealing. This would require the axion would emerge from the hidden sector dynamics, tying $f_a$ to the scale of supersymmetry breaking. In that case, the axion need not lie in an identifiable supermultiplet; there need be no saxion or axino, nor any light modulus responsible for determining $f_a$. The problem, however, is that in
order to generate the coupling of the axion to the Standard Model, the axion must couple to fields with mass of order this intermediate scale. As a result, the sparticles of the MSSM fields would be very massive, and any connection of supersymmetry to the hierarchy problem would be lost.

The alternative, which has generally been considered in this context, is that the axion couples only through Planck (or other large scale) suppressed operators to the hidden sector\[27\]. The axion then necessarily is accompanied by a pseudomodulus, whose value determines \( f_a \). It is natural to call this modulus the saxion, but if there are multiple moduli, this identification may be ambiguous. If it has a TeV scale mass, it is cosmologically problematic\[28\]. At about 30 Tev, it decays before nucleosynthesis\[29\]. The situation is somewhat better if the relevant scale is lower (e.g. \( M_{\text{gut}} \)). It is necessary to produce the baryon asymmetry in these decays. The problem of higher dimension operators is only slightly ameliorated in intermediate scale models, as the natural scale of the potential is \( M_{\text{int}}^4 \). As in the case of low energy supersymmetry in string theory, we are on shaky grounds in selecting for axions as dark matter; stable neutralinos resulting from a conserved \( R \) parity seem at least as likely to play the role of dark matter in this framework.

5.3 Low Scale Supersymmetry Breaking (Gauge Mediation)

In gauge mediated models which implement a Peccei-Quinn symmetry, the scale of supersymmetry breaking is necessarily well below the scale \( f_a \). Calling \( F \) the Goldstino decay constant, one has roughly \( 10^5 \text{GeV} < \sqrt{|F|} < 10^9 \text{GeV} \). Suppose that the saxion couples to the messengers/susy breaking sector through Planck or Gut suppressed operators. Even in the latter case, and for \( \sqrt{|F|} = 10^9 \), \( m_s \sim 1 \text{ GeV} \). Its lifetime is of order

\[
\Gamma \approx \frac{m_s^3}{M^2} \approx 10^{-32}
\]

long after nucleosynthesis. This suggests that the axion multiplet should couple directly to messengers. Strategies for model constructions are suggested by our discussion above, and implemented in [12]. This work constructs models in which either the saxion or the \( r \)-saxion is responsible for breaking the Peccei-Quinn symmetry. In these models, the saxion is not necessarily a mass eigenstate. What is most interesting about this analysis is that it constrains \textit{both} the scales of supersymmetry breaking and PQ breaking, as we discuss below.
5.4 Cosmology and the scales $F$ and $f_a$ in Gauge Mediation

Without working through models in detail, it is easy to see that cosmological considerations tend to require a large scale for the breaking of supersymmetry – towards the high end of what is allowed for gauge mediation. The issue is the mass of the light pseudomodulus, $P$.

$$m_P^2 = \text{loop factor} \times \frac{|F|^2}{f_a^2} \sim 10^{-4}\text{GeV}^2 \left( \frac{\sqrt{|F|}}{10^5} \right)^4 \left( \frac{10^{12}}{f_a} \right)^2$$ (26)

and its lifetime is of order

$$\Gamma = \frac{1}{4\pi} \frac{\alpha_s^2 m_P^3}{f_a} \sim 10^{-35}\text{GeV} \left( \frac{\sqrt{|F|}}{10^5} \right)^6 \left( \frac{10^{12}}{f_a} \right)^5$$ (27)

So of $f_a = 10^{12}$, we require $\sqrt{|F|} \sim 10^{8.5}$ if $P$ is to decay before nucleosynthesis. In this case,

$$m_s \sim 10^4\text{GeV} \quad \Gamma \sim 10^{-17},$$ (28)

well before nucleosynthesis. In this case, the messengers have mass of order $f_a$. If one does not require that the axion be the dark matter, lower values of $F$ are possible. These issues will be discussed in [12].

6 Conclusions

There are a few general lessons which we take from this discussion.

1. Spontaneous CP violation is not a likely solution of the strong CP problem. In that case, assuming that the lattice result that $m_u \neq 0$ is confirmed, the axion solution is the only viable solution. The axion solution itself has deficiencies, and the possible mechanisms for their resolution points to interesting physics.

2. The existence of a PQ symmetry, of good enough quality to solve strong CP, might be correlated with the existence of dark matter.

3. In non-supersymmetric theories, low $f_a$ is natural if PQ breaking is dynamical.

4. In string theory, the existence of suitable axions is likely correlated with the existence of a very small parameter as well as with dark matter.

5. Implementing the PQ solution of the strong CP problem in supersymmetry points towards gauge mediation, with supersymmetry broken at a rather high scale ($> 10^8$ GeV).
6. Implementing the PQ solution of the strong CP problem in supersymmetry points towards gauge mediation, with supersymmetry broken at a rather high scale (> $10^8$ GeV). Assuming that the axion constitutes the dark matter, it also points towards PQ breaking scales at the conventional upper limit of $10^{12}$ GeV. It also points towards PQ breaking scales at the conventional upper limit of $10^{12}$ GeV.

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