Measurement of the Transverse-Longitudinal Cross Sections in the $p(\vec{e}, e'p)\pi^0$ Reaction in the $\Delta$ Region

C. Kunz $^a$, N.I. Kaloskamis $^b$, M.O. Distler $^{a,1}$, Z.-L. Zhou $^a$, R. Alarcon $^c$, D. Barkhuff $^a$, A. M. Bernstein $^{a,*}$, W. Bertozzi $^a$, J. Calarco $^i$, F. Casagrande $^a$, J. Chen $^a$, J. Comfort $^c$, G. Dodson $^{a,2}$, A. Dooley $^e$, K. Dow $^a$, M. Farkhondeh $^a$, S. Georgakopoulos $^b$, S. Gilad $^a$, R. Hicks $^d$, A. Hotta $^d$, X. Jiang $^{d,3}$, A. Karabarbounis $^b$, S. Kowalski $^a$, D.J. Margaziotis $^f$, C. Mertz $^{c,b}$, R. Miskimen $^d$, I. Nakagawa $^a$, C.N. Papanicolas $^b$, M.M. Pavan $^{a,4}$, G. Peterson $^d$, A. Ramirez $^e$, D. Rowntree $^a$, A.J. Sarty $^{e,5}$, J. Shaw $^d$, E. Six $^c$, N. Sparveris $^b$, S.-B. Soong $^a$, S. Stiliaris $^b$, T. Tamae $^g$, D. Tieger $^a$, C. Tschalaer $^a$, G. Tsentalovich $^a$, W. Turchinetz $^a$, C.E. Vellidis $^b$, G.A. Warren $^{a,6}$, S. Williamson $^h$, A. Young $^c$, J. Zhao $^a$, and T. Zwart $^a$

(The MIT-Bates OOPS Collaboration)

$^a$Physics Department and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, MA 02139, USA
$^b$Institute of Accelerating Systems and Applications and Department of Physics, University of Athens, Athens, Greece
$^c$Department of Physics and Astronomy, Arizona State University, Tempe, AZ 85287, USA
$^d$Department of Physics, University of Massachusetts at Amherst, Amherst, MA 01003, USA
$^e$Department of Physics, Florida State University, Tallahassee, FL 32306, USA
$^f$Department of Physics and Astronomy, California State University at Los Angeles, Los Angeles, CA 90032, USA
$^g$Laboratory of Nuclear Science, Tohoku University, Sendai 982-0826, Japan
$^h$Nuclear Physics Laboratory, University of Illinois, Urbana, IL 61820, USA
$^i$Department of Physics, University of New Hampshire, Durham, New Hampshire 03824, USA

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Abstract

Accurate measurements of the $p(\vec{e},\vec{e}p)\pi^0$ reaction were performed at $Q^2 = 0.127 \ (GeV/c)^2$ in the $\Delta$ resonance energy region. The experiments at the MIT-Bates Linear Accelerator used an 820 MeV polarized electron beam with the out of plane magnetic spectrometer system (OOPS). In this paper we report the first simultaneous determination of both the TL and TL' ("fifth" or polarized) cross sections at low $Q^2$ where the pion cloud contribution is predicted to dominate the quadrupole amplitudes (E2 and C2). The real and imaginary parts of the transverse-longitudinal cross section provide both a sensitive determination of the Coulomb quadrupole amplitude and a test of reaction calculations. Comparisons with model calculations are presented. The empirical MAID calculation gives the best overall agreement with this accurate data. The parameters of this model for the values of the resonant multipoles are $|M_{1+}(I = 3/2)| = (40.9 \pm 0.3) \times 10^{-3}/m_\pi, CMR = C2/M1 = -6.5 \pm 0.3\%, EMR = E2/M1 = -2.2 \pm 0.9\%$, where the errors are due to the experimental uncertainties.

Key words: EMR, CMR, electro-pion production, out-of-plane

Experimental confirmation of the deviation of the nucleon shape from spherical symmetry is of fundamental significance and has been the subject of intense investigation [1] since this possibility was originally raised by Glashow [2]. For the $J = 1/2$ nucleon, this determination has focused on the determination of the electric and Coulomb quadrupole amplitudes (E2, C2) in the predominantly M1(magnetic dipole-quark spin flip) $\gamma^* N \rightarrow \Delta$ transition. Thus measurements of the E2 and C2 amplitudes represent deviations from spherical symmetry of the $N, \Delta$ system and not the nucleon alone. The experimental difficulty is that the E2/M1 and C2/M1 ratios are small (typically $\simeq -2$ to $-8\%$ at low four momentum transfered, $Q^2$). In this case the non-resonant (background) and resonant quadrupole amplitudes are the same order of magnitude. Therefore experiments have to be designed to attain the required precision to separate the signal and background contributions. This has been accomplished for photo-pion reactions using polarized photon beams [3,4].

Observation of the deviation from spherical symmetry in pion electroproduction is more pronounced than in photoproduction. This is due to the interfer-
ence between the longitudinal C2 and the dominant M1 amplitudes in the $\sigma_{TL}$ cross section [5]. On the other hand the presence of the additional longitudinal multipoles means that there are more multipoles to determine and therefore more extensive data must be taken. The experiments for an extensive database that would allow a model independent analysis have just begun [1,6,7,8]. At the present time one must rely on reaction models to extract the resonant M1, E2, and C2 amplitudes of interest from the data. As has been pointed out in previous publications, the model error can be much larger than the experimental error [5,6,9]. Therefore it is important to test model calculations for a range of center of mass (CM) energies $W$ in the region of 1232 MeV, the $\Delta$ resonant energy, which provides a range of background and resonant amplitudes. It is also important to determine specific cross sections (e.g. $\sigma_{TL}$, $\sigma_{TL}'$) which are primarily sensitive to the C2 and background amplitudes respectively. In this letter we present the first measurement that provides such information at and below the resonance energy at low $Q^2$. This requires a polarized electron beam and out of plane hadron detection.

In the constituent quark model the d state admixtures in the nucleon and $\Delta$ wave functions are caused by the hyperfine tensor interaction between quarks [11]. The pion cloud contribution to the nucleon and $\Delta$ structure is primarily in the p wave. This is due to the spontaneously broken chiral symmetry of QCD in which the pion, an almost Goldstone Boson, interacts with hadrons via gradient coupling [12]. Therefore it is not surprising [7,13,14] that model calculations have shown that at low $Q^2$ the pion cloud contributes significantly to the M1 amplitude and dominates the E2 and C2 contributions to the $\gamma^* N \rightarrow \Delta$ transition. The present experiment was performed at $Q^2 = 0.127$ (GeV/c)$^2$ which is close to the predicted maximum of the pion cloud contribution [13,14,15]. Thus this experiment is ideally suited to test these calculations.

The coincident $p(\vec{e}, e' \pi)$ cross section in the one-photon-exchange-approximation can be written as [16]

$$\frac{d\sigma}{d\omega d\Omega_e d\Omega_{\pi}} = \Gamma_v \sigma_h(\theta, \phi), \quad (1)$$

where

$$\sigma_h(\theta, \phi) = \sigma_T + \varepsilon \sigma_L + \sqrt{2\varepsilon(1 + \varepsilon)} \sigma_{TL} \cos \phi \nonumber$$

$$+ \varepsilon \sigma_{TT} \cos 2\phi + h p_e \sqrt{2\varepsilon(1 - \varepsilon)} \sigma_{TL}' \sin \phi,$$

$\Gamma_v$ is the virtual photon flux, $h = \pm 1$ is the electron helicity, $p_e$ is the magnitude of the longitudinal electron polarization, $\varepsilon$ is the virtual photon polariz-
tion, $\theta$ and $\phi$ are the pion CM polar and azimuthal angles relative to the momentum transfer $\vec{q}$, and $\sigma_L$, $\sigma_T$, $\sigma_{TL}$, and $\sigma_{TT}$ are the longitudinal, transverse, transverse-longitudinal, and transverse-transverse interference cross sections, respectively [16].

The TL and the TL’ (transverse-longitudinal) cross sections are the real and imaginary parts of the same combination of interference multipole amplitudes. Approximate expressions for these are:

\[
\begin{align*}
\sigma_{TL}(\theta) &= -\sin \theta \text{Re}[A_{TL} + B_{TL} \cos \theta], \\
\sigma_{TL}'(\theta) &= \sin \theta \text{Im}[A_{TL} + B_{TL} \cos \theta], \\
A_{TL} &\approx -\frac{q}{k} L_{0+}^* M_{1+}, \\
B_{TL} &\approx -6 \frac{q}{k} L_{1+}^* M_{1+},
\end{align*}
\]

where the pion production multipole amplitudes are denoted by $M_{l\pm}$, $E_{l\pm}$, and $L_{l\pm}$, indicating their character (magnetic, electric, or longitudinal), their total angular momentum ($J=l \pm 1/2$), $q$ and $k$ are the pion and photon center of mass momenta. In the first two lines of Eq.2 it has been assumed that the pions are produced in s and p waves only. In the next two lines an additional truncated multipole approximation is made, namely only terms which interfere with the dominant magnetic dipole amplitude $M_{1+}$ are kept. The exact formulas without this approximation can be found in [16]. In model calculations [13,14,17,18] this approximation is not made and significant deviations from the truncated multipole approximation occur.

As has been previously demonstrated $\sigma_{TL}$ is sensitive to the magnitude of the longitudinal quadrupole amplitude $C_2$ [5]. Adding a measurement of $\sigma_{TL}'$ to this provides a stringent test of the background magnitudes and phases of the reaction calculations. It should be pointed out that a determination of the background amplitudes is an important part of the physics of the $\gamma\pi N$ system.

To precisely determine the resonant quadrupole amplitude in the $\gamma^* N \to \Delta$ transition at low $Q^2$ and to address the issue of background contributions, a program has been developed at the MIT-Bates Linear Accelerator. For this purpose we have developed a special out-of-plane magnetic spectrometer system (OOPS) in which the spectrometers are deployed symmetrically about the momentum transfer $\vec{q}$ [10]. We observed the TL’ cross section using a polarized electron beam of 0.85% duty factor at an energy of 820 MeV. A typical polarization and an average current were 37% and 6 $\mu$A, respectively. A liquid H$_2$ target was used in a cylindrical cell of 1.6 cm diameter with a 4.3 $\mu$m-thick Havar wall. The scattered electrons were detected in the One Hundred Inch Proton Spectrometer (OHIPS) and the coincident protons in two out-of-plane spectrometers deployed at a fixed laboratory angle relative to $\vec{q}$ and
with out-of-plane angles $\phi = 225^\circ, 315^\circ$. The focal plane instrumentation of each spectrometer consisted of three horizontal drift chambers for track reconstruction and scintillators for triggering. Detailed optics studies were done for each spectrometer, and the detection efficiencies were measured as functions of all independent reaction coordinates. The total efficiency of the system was calibrated by using elastic electron scattering data from the liquid H$_2$ target. Boiling effects in the target were studied by varying the beam current and they were negligible. The phase-space normalization of the cross section and various corrections applied to the data, including radiative corrections, were calculated with a Monte Carlo simulation. The cross sections were obtained from the part of the phase space of the two spectrometers which were matched in four dimensions ($W, Q^2, \theta, \phi$) [19], where $W$ is a central invariant mass.

The experiment was performed at $Q^2 = 0.127$ (GeV/c)$^2$, $W = 1232$ MeV and 1170 MeV. The results and kinematic settings are presented in Table 1 and in Figs. 1, 2, and 3. The helicity asymmetry $A_{TL'}(\theta, \phi)$ is

$$A_{TL'}(\theta, \phi) = \frac{\sigma_{h=1}(\theta, \phi) - \sigma_{h=-1}(\theta, \phi)}{\sigma_{h=1}(\theta, \phi) + \sigma_{h=-1}(\theta, \phi)},$$

$$= \frac{\sqrt{2\epsilon(1-\epsilon)}\sigma_{TL'}(\theta)\sin(\phi)}{\sigma_{unpol}(\theta, \phi)}$$

where the quantities were defined in Eq. 1 and $\sigma_{unpol}$ is the electron helicity independent part (the first four terms) of $\sigma_h$. To first approximation this quantity can be extracted from the data without detailed Monte Carlo calculations of the phase space acceptance of the apparatus, and therefore has a smaller error. There are only small corrections due to the finite acceptances of the spectrometers. In the quoted results the quantities have been referred to the central spectrometer settings [19]. The absolute values of the TL' cross sections $\sigma_{TL'}$ have also been extracted from the data [19] and are presented here.

We note that the sign of $A_{TL'}$ is negative. In the $\pi^0$ channel the recoil protons were detected with the protons being emitted in the forward direction relative to $\vec{q}$, the momentum transfer with out of plane angles $\phi_{pq} = 45^\circ, 135^\circ$. In Eq. 1 the angles of the pion are involved, and $\phi = 225^\circ, 315^\circ$. Therefore a negative sign for $A_{TL'}$ means that the sign of $\sigma_{TL'}$ is positive.

The experimental results are compared to calculations [13,14,17,18] in Figs. 1, 2, and 3. The most ambitious calculations are the Sato-Lee model [13] which calculates all of the multipoles and $\pi - N$ scattering from dynamical equations. It is in agreement with the photoproduction data (some of the model parameters were fit to these data). The Sato-Lee model also agrees with our data for the unpolarized cross sections $\sigma_0 = \sigma_T + \epsilon\sigma_L$ but unfortunately is
in strong disagreement with our measurements of $\sigma_{\text{TL}}$ and $\sigma_{\text{TL}'}$. The dispersion relations calculation [18] agrees with some of our data but disagrees with our measurement of $\sigma_0$ at $W = 1170$ MeV and with our $\sigma_{\text{TL}}$ measurements. On the other hand dispersion relation calculations provide good agreement with photo-pion production data [20]. The Mainz Unitary Model (MAID) is a flexible way to fit observed cross sections as a function of $Q^2$ [17]. It incorporates Breit-Wigner resonant terms, Born terms, higher $N^*$ resonances, and is unitarized using empirical $\pi-N$ phase shifts. The parameters of the model have been previously fit to a range of data, including our previous results [5], and are in reasonable agreement with our data [17] with the exception of $\sigma_{\text{TL}}(\theta = 119^\circ, W = 1170$ MeV) which is in disagreement with all calculations. The Dubna - Mainz - Taipei (DMT) model [14] includes dynamics for the resonant channels and uses the background amplitudes of the MAID model. This model is in reasonable agreement with our data at resonance ($W = 1232$ MeV) but not with $\sigma_0$ and $\sigma_{\text{TL}}$ below resonance ($W = 1170$ MeV).

The Sato-Lee and DMT dynamical models [13,14] predict that the pion cloud is the dominant contribution to the quadrupole amplitudes at low values of $Q^2$. This behavior is an expected consequence of the spontaneous breaking of chiral symmetry in QCD [7]. Unfortunately these models are not in overall agreement with our data. In contrast the Sato-Lee model showed much better predictions of the recently reported JLab Hall B result for the $p(\vec{e}, e'\vec{p})\pi^0$ reaction in the $\Delta$ region from $Q^2$ from 0.4 to 1.8 (GeV/c)$^2$ [21]. This seems to indicate that the dominant meson cloud contribution, which is predicted to be a maximum near our values of $Q^2$, is not quantitatively correct.

Recently a measurement of $A_{\text{TL}'}$ for the $p(\vec{e}, e'\vec{p})\pi^0$ reaction in the $\Delta$ region was performed at Mainz [22]. The kinematics include a range of $Q^2$ values from 0.17 to 0.26 (GeV/c)$^2$ and backward $\theta$ angles. These data were only compared to the MAID model with which they disagreed.

It is of interest to compare the TL and TL' results presented here with those of the recoil polarizations which are proportional to the real and imaginary parts of interference multipole amplitudes. For the $p(\vec{e}, e'\vec{p})\pi^0$ channel the outgoing proton polarizations have been observed in parallel kinematics (the protons emitted along $\vec{q}$ or $\theta = 180^\circ$) [23,24]. For this case the observables in the truncated multipole approximation are:

\begin{align*}
\sigma_0 p_x &\propto Re[A_{\text{TL}}^x], \\
\sigma_0 p_y &\propto Im[B_{\text{TL}}^y], \\
\sigma_0 p_z &\propto Re[C_{\text{TT}}^z], \\
A_{\text{TL}}^x &\approx B_{\text{TL}}^y \approx (4L_{1+}^* - L_{0+}^* + L_{1-}^*)M_{1+}, \\
C_{\text{TT}}^z &\approx |M_{1+}|^2 + Re[(6E_{1+}^* - 2E_{0+}^*)M_{1+}],
\end{align*}

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where $\sigma_0$ is the unpolarized cross section, $p_x$, $p_y$, and $p_z$ are defined in [17], and the constants of proportionality contain only kinematic factors (for the full expressions see [16]). This shows both the similarity and detailed difference between a measurement of TL and TL$'$ and the recoil polarizations. In the published papers [23,24] the data were compared to the MAID model which is found not to be in good agreement with the data. At the present time we do not have sufficient data to pin down the multipoles that are responsible for this difference (a discussion of the data requirements for model independent analyses is presented in [7]). On the other hand there is a possible experimental problem since the data do not agree with a model independent sum rule [25].

The empirical MAID calculation gives the best overall agreement with the accurate data presented here but also with the overall set of the data obtained by our collaboration. The parameters of this model for the values of the resonant multipoles are $|M_{1+}(I = 3/2)| = (40.9 \pm 0.3) \times 10^{-3}/m_\pi$, $CMR = C2/M1 = -6.5 \pm 0.3\%$, $EMR = E2/M1 = -2.2 \pm 0.9\%$. The errors are experimental and were obtained by varying the magnitudes in MAID of the resonant amplitudes by one $\sigma$ in a $\chi^2$ fit to our data. In a previous paper [5] we showed that the dominant error was due to model uncertainties, which was estimated by taking the differences between the multipoles of the different models. We argue here that only one model is in reasonable agreement with our data, and so this approach can no longer be used. Therefore the model errors come from uncertainties in the non-resonant multipole amplitudes of the MAID calculation, which at the present time are not known. We plan to address this issue in a future publication.

In conclusion, we have performed the first simultaneous measurement of both the real and imaginary parts of the transverse-longitudinal interference (TL) cross section for the $p' (\vec{e}, e' p) \pi^0$ reaction in the $\Delta$ region at a low $Q^2$ where the meson cloud is predicted to be the leading cause of deformation. It is found that only the more empirical MAID model [17] is in reasonable agreement with the accurate data obtained for $\sigma_0 = \sigma_T + \epsilon \sigma_L$ and $\sigma_{TL'}$ and also with our previous data [5]. On the other hand, for recoil polarization data obtained at the same $Q^2$ [23,24], and for $\sigma_{TL'}$ taken at slightly higher values of $Q^2$ [22], there are possible problems with the MAID model (results for other models were not presented in these publications). At the present time there are not sufficient data to ascertain which multipoles might be responsible for this situation. We are presently analyzing new data taken at $Q^2 = 0.127 \ (GeV/c)^2$ which should provide a more accurate determination of these values [26] in conjunction with the new generation, double polarization experiments [27].

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Table 1
Results of the present $p(\bar{e},e'p)\pi^0$ experiment at $Q^2 = 0.127$ (GeV/c)$^2$.

| $W$ (MeV) | $\theta$ | $\sigma_0$ (µb/sr) | $\sigma_{TL}$ (µb/sr) | $\sigma_{TL'}$ (µb/sr) |
|-----------|-----------|---------------------|------------------------|------------------------|
| 1170      | 119°      | 17.31 ± 0.90        | 0.91 ± 0.18            | 1.65 ± 0.55            |
| 1232      | 129°      | 26.39 ± 0.47        | 2.83 ± 0.20            | 3.11 ± 0.55            |
Fig. 1. The helicity asymmetry $A_{TL}$ for the $p(\vec{e}, e'p)\pi^0$ reaction at $Q^2 = 0.127$ (GeV/c)$^2$ plotted versus $\theta$ the CM angle between the outgoing pion and the momentum transfer $\vec{q}$. The curves are MAID [17] (solid), Sato-Lee [13] (dashed), DMT [14] (dotted), dispersion theory [18] (dot-dashed).

Fig. 2. Cross sections for the $p(\vec{e}, e'p)\pi^0$ reaction for $W = 1170$ MeV, $Q^2 = 0.127$ (GeV/c)$^2$ plotted versus $\theta$. Panel (a) is for $\sigma_0 = \sigma_T + \epsilon \sigma_L$. Panel (b) is for $\sigma_{TL}$ and panel (c) is for $\sigma_{TL'}$. See the Fig. 1 captions for an explanation of the curves.
Fig. 3. Cross sections for the $p(\vec{e}, e'p)\pi^0$ reaction for $W = 1232$ MeV, $Q^2 = 0.127$ (GeV/c)$^2$ plotted versus $\theta$. Panel (a) is for $\sigma_0 = \sigma_T + \epsilon \sigma_L$. Panel (b) is for $\sigma_{TL}$ and panel (c) is for $\sigma_{TL'}$. See the Fig. 1 captions for an explanation of the curves.