An Integrated Multi-Product Process and Maintenance Planning (IPPMP) Modeling Approach to Optimize the Total Production Cost

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Abstract
Nowadays competitive manufacturing environment has forced modern companies to further decrease their costs. Finding appropriate solutions in production and maintenance planning are of two critical aspects of manufacturing systems success. In this study, we consider the problem of integrated multi-product process and maintenance planning (IPPMP) on a capacitated machine that prone to random breakdown. Maintenance procedures involve general perfect repair (noncyclical) as preventive maintenance (PM) at the beginning of periods and minimal repair as corrective maintenance at machine failure. We have considered a realistic assumption that the cost and time of PM are related to the interval between the previous perfect repair and current PM. The aim is to minimize the production, PM and the expected corrective repair costs. A mixed integer linear programming (MILP) model is presented. The model is compared to an independency hypothesis (ignoring machine age effects on the PM parameters). Results show the assumption of machine age effect decreases the total cost and results in a more realistic solution. While the dependency level goes higher, the improvement increases. The proposed model extended to the cyclical preventative maintenance by adding linear periodic constraints.

Keywords: Production Planning, lot size, Maintenance scheduling, Noncyclical Preventive maintenance

Introduction
Manufacturing industries continually deal with two imperative subjects: production, and maintenance planning. The Production department seeks to minimize production costs typically including production, holding, backlog, and setup cost. Production managers look for maximum machinery capacity, and key equipment to meet customers’ demand within promised level of quality [1]. On the other hand, the maintenance department has to perform preventive maintenance, and adopts maintenance strategies (for instance, dynamic sampling strategy [2], semi-dynamic maintenance [3], dynamic maintenance [4], and pro-active maintenance scheduling [5]) to keep machines functionality well and prevent the machines failure due to breakdown. Both production and maintenance departments’ activities have been done on the same equipment and have to use equipment capacity to promote productivity and reliability of them. Although carrying out PM may prevent failure, because of the differences in the objectives of these two departments, the conflict arises [6]. One would like to produce non-stop and the other is likely to put more weight on longevity, reliability, and required service level of the equipment [7]. Notwithstanding the trade-off between the activities of production scheduling and maintenance planning, they are classically planned and executed individually in a manufacturing system even if industrial productivity can be improved by optimization of both production scheduling and maintenance planning decision concurrently [8].

Therefore, it is necessary to find an optimal balance of production scheduling and maintenance activities for the equipment. So, for industrial factories, a coordination between production and maintenance departments is essential to prevent production interruption and unplanned repair (corrective maintenance).

This paper is organized as follows: in the next section we fold literature review dividing into three groups, lot size, maintenance, integrated production and maintenance planning. In section 2, we specify the innovation that makes the paper distinguished. Then the problem statement and mathematical formulation are described in sections 3 and 4 respectively. The methodology and software used to solve the problem is mentioned in section 5. Section 6 dedicated to numerical examples and the results. Finally, conclusion and potential future work are presented in section 7.

1. Literature Review

1.1. Lot Size

There are numerous papers on lot sizing. Jans and Degraeve [9] reviewed many studies in this case and classified papers based on setup, production process, inventory, demand, planning horizon and also with regard to tactical and strategic lot sizing problems. Demand fluctuations in production matters have led to many researches. In this context, Robinson et al [10] covered different approaches of production problems in terms of single or multi products, setup cost, capacity and heuristic solving methods. Buschkuhl [11] presented literature review about capacitated lot-sizing problems, modeling and their solution methods. The majority of researches are about multi-level capacitated lot-sizing problem such as [12] and [13]. All economic lot-sizing problems are seeking for minimizing costs while satisfying demand. Andriolo et al [14] did comprehensive surveys from 1913 to 2014. Lot size has direct effect on scheduling. For this reason, [14] and [15] considered the problem of production planning and scheduling simultaneously.

1.2. Maintenance

Wang [16] categorized papers about maintenance. The author has used several maintenance policies to segment maintenance studies of deteriorating systems. He compared both single and multi-unit systems and classified maintenance restoration levels in three groups: minimal, imperfect and perfect. All these papers aim to reduce maintenance cost and increase machine time availability. Grigoriev et al [17] did a literature review on the periodic preventive maintenance. Periodic preventative maintenance planning means finding the optimal interval between two consecutive preventive maintenance. There aren't abundant literature reviews on noncyclical preventive maintenance. Among the factors related to manufacturing, maintenance plays a critical role. Faccio et al [18] covered a variety of maintenance strategies to reduce spare parts cost, human resources, missing production capacity and other indirect costs. Kader et al [19] studied the problem of spare parts in maintenance planning and considered the used and new spare parts to be replaced during corrective and preventive maintenance.

1.3. Integrated production and maintenance planning

Budai et al [20] surveyed the papers considering the relation between production and maintenance planning. In this paper, three relationships were mentioned: Production planning which is maintenance based, maintenance planning which is production based and integrated production and maintenance planning.
Ashayeri et al. [21] proposed an integrated model for the parallel production lines in which minimal corrective maintenance is considered and repair cost depends on the machine age. Aghezzaf et al. [22] proposed a multi-item production model with periodic preventive maintenance in which shortage is unacceptable over planning horizon. Aghezzaf and Najid [23] proposed two models for the parallel manufacturing system. On failure, maintenance policy consists of minimal repair to restore the machine to the previous working condition and the perfect repair is done as preventive maintenance to return the machine as good as a new one. The model of periodic PM is mixed integer nonlinear programming and the model of non-periodic PM planning is mixed integer linear programming. A Lagrange-based heuristic method was used to solve the problems. Yalaoui et al. [24] extended the solution method by using fix and relax method to reduce the time of solving wide range of large problems in an acceptable time period. Aghezzaf et al. [25] presented an integrated production and maintenance planning model considering imperfect preventive maintenance. The imperfect repair reduces the machine age and does not restore it to a new one condition. A mixed integer linear programming was proposed for this model. Noureflah and Chatelot [26] considered a manufacturing system with parallel components in which multi-component PM has less cost than single component PM. The aim is reducing the sum of PM cost, corrective repair cost, production cost, holding, shortage and setup cost. Fakher et al. [27] used hybrid genetic algorithm to solve the problem and hybrid GA with Tabu Search algorithm to manage initial GA population. Noureflah et al. [28] involved quality in production and maintenance planning. The problem is considered as multi-product and multi-period lot-sizing in which machines have two states of under control and out of control. In out of control state, some of products are not acceptable. The purpose is minimizing the total cost while satisfying customers' demand. Results show that if PM cost is not too high, carrying out PM can reduce the quality cost. Also, general (not necessarily periodic) PM reduces total costs. Likewise, Bouslah et al. [29] considered interactions between production, holding, maintenance and quality by continuous sampling. Bouslah et al. [30] also considered three aspects of integrated production planning, maintenance planning and quality control. Dahane et al. [31] focused on the relation between machines failure and production rate. They considered two product types. Type a demand must be satisfied without any shortage over planning horizon and satisfying demand of type b product is optional though its profit is high. The aim is maximizing the total expected profit. Sarper [32] proposed a production and maintenance planning model for single machine and single product that their demand depend on time and advertisement. A similar paper was proposed by Cavory et al. [33] to maximize manufacturing line potential. They used GA to solve the problem. Kenne et al. [34] presented multi-production and multi-stage manufacturing modeling using Markov Chain to find optimal production and repair rate. Ben-Daya [35] proposed economic lot-sizing problem over infinite planning horizon and assumed PM as an imperfect repair since each PM reduces the machine age based on the repair level. System states include under control and out of control states. Sheu and Chen [36] developed the model and considered two states for the under control situation. If under control 1 occurs, minimal repair should be carried out. If the under control 2, machine is stopped and the imperfect repair is done. Machine age is closely related to repair level. Chebbi et al. [37] combined lot-sizing and maintenance problem regarding the defective items. The goal is finding optimal PM length and reducing the cost of finished goods.

In general, noncyclical PM has better performance than periodic one. Gustavsson et al. [39] proposed a binary linear programming model for multi component maintenance planning regarding the dependency of PM cost on the interval. This paper proved that considering the dependency improves maintenance costs.

2. Innovation

Aghezzaf and Najid [23] modeled production and maintenance planning with noncyclical PM. Although the model is mixed integer linear programming and includes setup cost, this model has not considered that cost of corrective repair is zero if machine does not run and machine age remains constant. The same lack is seen in [24] and [6]. In practice, a significant percentage of machine capacity is allocated to setup activities. Industries need a more realistic and comprehensive model to plan [9]. In this study, in addition to covering the issues stated above, time and cost of PM are considered based on the interval between the previous PM and the current PM. By increasing this length, time and cost of PM rise up [39]. The following model has developed the model of [6] with assumption in [39] that was proposed in form of mixed integer linear programming. It enables us to reach a global optimal solution even for large scale problems. The model is non-cyclic and is flexible enough to become periodic by adding a few simple mathematical equations.

3. Problem Statement

Assume a capacitated machine in manufacturing system that faces random failure [40]. Scheduling department including production and maintenance department seeks plans for optimal production and Maintenance. Maintenance strategy considers corrective maintenance (unplanned repair) which is minimal and restores the machine’s previous working condition and doesn’t change its age. If no PM is performed and machine is used for production process in a period, the machine age will increase for one period (time unit is period) and failure rate goes up [4], [41]. Maintenance policies focus on noncyclical PM, though it can be periodic. If a perfect PM (replacement maintenance) is conducted, the machine age status is transformed to a new machine meaning the machine age is zero. It is similar to substitution of the machine with a new one [42].

Cost and time of PM depend on the machine age [43]. It is obvious that the cost and time required to perform PM ascend as the machine age increases because it needs more spare parts to be replaced. Maintenance cost includes the sum of PM cost and expected corrective maintenance cost. The goal of maintenance department is to minimize total maintenance cost.

The production department must satisfy the demand of P product(s) over planning horizon H. To meet the demand, production department uses the machine capacity to advance the production progress. Production of each period is in a lot. Shortage is unacceptable but the demand can be satisfied after due date with penalty cost. Machines have to be prepared for manufacturing products, so setup cost and time must be considered. The aim of the production department is to meet all demand over planning horizon H while minimizing production cost, holding cost, backorder and setup cost. Scheduling department mission is combining production and maintenance purposes in form of single goal to decrease the overall cost. Each department uses the machine capacity to advance the production and maintenance activities with regard to their limitations. Scheduling department should integrate planning to minimize the total production and maintenance costs.

4. Mathematical Formulation

Maintenance department deals with two types of activities including planned repair or preventive maintenance that would be done at the beginning of a period and unplanned repair or corrective maintenance that is done during a period when facing machine failure. Due to the random nature of corrective maintenance, we use expected time and cost of unplanned repair. Assume planning
horizon \( (H) \) consists of \( T \) periods with length \( (\tau) \). Failure rate remains unchanged with performing the corrective maintenance, because it is a minimal repair. So, the expected number of failure \( E \) during \( [a, b] \) is calculated as follow:

\[
E = \int_a^b r(u)du
\]

In which, \( r \) is the machine failure rate function obtaining as equation from density \( f \) and cumulative probability function \( F \) (equation 2).

4.1. Model formulation

Indices

- \( t \) Period index \( t \in \{1, 2, ..., T\} \)
- \( l \) Length index (a period) \( l \in \{1, 2, ..., T\} \)
- \( p \) Product index \( p \in \{1, 2, ..., P\} \)

Parameters

- \( d_{pt} \) Demand of product \( p \) in period \( t \)
- \( h_p \) A period holding cost of product \( p \)
- \( b_p \) A period backorder cost of product \( p \)
- \( sc_p \) Machine setup cost for product \( p \)
- \( st_p \) Machine setup time for product \( p \)
- \( \pi_p \) Process cost of product \( p \)
- \( \phi_p \) Required capacity to process product \( p \)
- \( PMC^1 \) Preventive maintenance cost when the machine age is \( l \) at the beginning of period.
- \( PMT^1 \) Preventive maintenance time when the machine age is \( l \) at the beginning of period.
- \( RC \) Corrective maintenance cost
- \( RT \) Corrective maintenance time
- \( e^1 \) Expected number of the machine failure during the period when the machine age is \( l \) at the beginning of period.
- \( L \) Nominal machine capacity
- \( M \) A big enough positive number

Variables

- \( a_t \) Machine age at the beginning of period \( t \), Integer
- \( z_t^l \) If PM set to be done in period \( t \) while the pervious PM has been done \( l \) period before \( t \), the value is 1 otherwise 0., Binary
- \( Z_t \) If PM set to be done in period \( t \), the value is 1 otherwise 0., Binary
- \( CPM_t \) PM cost in period \( t \), Continuous
- \( CRM_t \) Expected corrective maintenance cost in period \( t \), Continuous
- \( TPMT_t \) PM time in period \( t \), Continuous
- \( TRM_t \) Expected corrective maintenance time in period \( t \), Continuous
- \( x_{pt} \) Production amount of product \( p \) in period \( t \), Integer
- \( I_{pt} \) Inventory level of product \( p \) in period \( t \), Integer
- \( B_{pt} \) Backorder level of product \( p \) in period \( t \), Integer
- \( y_{pt} \) If machine runs to produce product \( p \) in period \( t \), the value is 1 otherwise 0., binary
- \( Y_t \) If machine runs to produce in period \( t \), the value is 1 otherwise 0., Binary
- \( w_t^l \) If \( l = a_t + 1 \), then the value is 1 otherwise 0., Binary

\[
r(u) = \frac{f(u)}{1 - F(u)}
\]
4.2. Problem formulation

\[
\begin{align*}
\text{Min} & \sum_{p=1}^P \sum_{t=1}^T (h_p l_{pt} + b_p B_{pt} + \pi_p x_{pt} + sc_p y_{pt}) + \sum_{t=1}^T (CPM_t + CRM_t) \\
& \text{subject to:} \\
& l_{pt} - B_{pt} = l_{p(t-1)} - B_{p(t-1)} + x_{pt} - d_{pt} \\
& x_{pt} \leq My_{pt} \\
& \sum_{p=1}^P \phi_p x_{pt} \leq L - TPM_t - TRM_t = \sum_{p=1}^P s_t y_{pt} \\
& \sum_{t=1}^T z_t^i \leq 1 \\
& Z_t = \sum_{j=1}^J Z_t^j \\
& \sum_{j=t}^T \sum_{i=1}^J l_z^j = t - 1 \text{ if } Z_t = 1 \\
& \alpha_t = (a_{t-1} + Y_t)(1 - Z_t) \\
& MY_t \geq \sum_{p=1}^P y_{pt} \\
& \sum_{t=1}^T hw_t^i = \alpha_t + 1 \\
& \sum_{t=1}^T w_t^i = 1 \\
& E_t = \left(\sum_{t=1}^T e^t w_t^i\right) Y_t \\
& CPM_t = \sum_{t=1}^T PMC_t z_t^i \\
& CRM_t = E_t RC \\
& TPM_t = \sum_{t=1}^T PMT_t z_t^i \\
& TRM_t = E_t RT
\end{align*}
\]

Equation 5 is the Objective function with two terms that should be minimized. The first term relates to manufacturing cost including the production cost, holding cost, backorder cost and setup cost. The second term relates to the maintenance cost including the PM cost and expected corrective cost. Equation 6 considers the relation between the amount of production, demand, inventory and backorder level. Constraint 7 ensures that the production is enabled while the machine is set up to produce. Capacity limitation is defined as equation 8. Available production capacity is achieved after deducting the capacity requiring for PM activities, downtime corrective maintenance and setup time from nominal capacity. Equation 6-8 are about production part. Equation 9 ensures that not more than 1 PM is considered for the machine. Equation 10 defines in which periods PMs are planned. Equation 11 ensures that sum of the intervals between PMs before planned PM in period t must be t-1 periods. The machine age calculation is based on machine run variable Y_t by equation 12. In this equation, the machine age increases if Y_t = 1. Equation 13 force Y_t = 1 if the machine runs to produce at least one product. Equation 14-16 allocate the expected number of machine failure in period t based on the machine age. Equation 17 and 19 calculate cost and time of PM for period t. Equation 18 and 20 calculate the expected cost and time of corrective maintenance for period t.

4.3. Linearization

Equation 11, 12 and 16 are nonlinear. They can be linear by replacing linear substitution equations. Equation 11 is determined based on Z_t value. The model has this constraint when Z_t = 1. Replacing following equations makes it linear.

\[
\begin{align*}
\sum_{t=1}^T \sum_{i=1}^J l_z_t^i & \leq t - 1 + (1 - Z_t)M & t = 2, ..., T \\
\sum_{t=1}^T \sum_{i=1}^J l_z_t^i & \geq t - 1 - (1 - Z_t)M & t = 2, ..., T
\end{align*}
\]

Equation 12 is the product of two terms. The term 1 - Z_t is binary, so the equation can be replaced with three following linear equations.

\[
\begin{align*}
\alpha_t & \leq (1 - Z_t)M \\
\alpha_t & \leq (a_{t-1} + Y_t) & t = 1, ..., T \\
\alpha_t & \geq (a_{t-1} + Y_t) - Z_t M & t = 1, ..., T
\end{align*}
\]

Equation 16 is also the product of a term in binary variable (Y_t). Similarly, it can be replaced by the following linear equations.
Majority of parameters were taken from \cite{1}. Consider household appliances production factory with a machine which is prone to random failure over an 8-month horizon. The machine is brand new and it’s unused at the beginning of the horizon. This condition is similar to carrying out PM at the first period. The machine probability function is Weibull(2,2) and failure rate, expected cost and time of PM are three parameters that are obtained from the probability function. Table 1 shows the expected numbers of failure, cost and time of PM based on the machine age. Table 2 contains information about the nominal machine capacity and cost and time of corrective maintenance. The demand is assumed certain and it is shown in Table 3 for two products. The production cost, production time, holding cost, backorder cost and setup cost and time are arranged in Table 4.

### 4.4. Periodic Maintenance

Mathematical programming of the model is flexible enough that it can be periodic by adding a few constraints. We define a binary variable \(PL_l\) that is 1 if the interval between consecutive PMs is \(l\). The constraints are formulated as follow:

\[
\sum_{l=1}^{T} z_l^1 = PL^1(T-1) \quad 29
\]

\[
\sum_{l=1}^{T} z_l^l = PL^l \left( \frac{T}{l} - 1 \right) \quad l = 2, ..., T \quad 30
\]

\[
\sum_{l=1}^{T} PL^l = 1 \quad 31
\]

Equation 29 and 30 ensure possibility of PMs with interval 1 (\(l = 1\) cell). Equation 29 is somewhat different from 30 because of the assumption that in all circumstances in the first period, PM is carried out and we suppose that the machine is as a new one. Equation 31 ensures that the interval between preventive maintenances is unique. By amending equation 29-31, periodic assumption would be added to the model.

### 5. Methodology

It has been tried to obtain a linear model, albeit with integer and binary variables. The most advantage of the linear model in comparison to other similar nonlinear models is the certainty exists for achieving an optimal solution through branch and bound (B&B) procedure by software such as CPLEX and GAMS. But solving nonlinear models and ensuring their global optimality are difficult to achieve. However, non-linear models allow the modeling process to add easier and more realistic assumptions. In this study, the model transforms to linear. For solving the model, GAMS software is used on a laptop with a dual core 2.5 MHz processor and 4GB of RAM.

### 6. Numerical Example

Table 5: Cost and Time of PM in dependency and independency

|          | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Average |
|----------|---|---|---|---|---|---|---|---|---------|
| Dependency (Model A) |   |   |   |   |   |   |   |   | 4000     |
| \(PMC^1(\$)\) | 1613 | 2016 | 2520 | 3150 | 3937 | 4922 | 6152 | 7690 | 4       |
| \(PMT^1(h)\) | 1.6 | 2.0 | 2.5 | 3.2 | 3.9 | 4.9 | 6.2 | 7.7 | 4       |
| Independency (Model B) |   |   |   |   |   |   |   |   | 4000     |
| \(PMC^1(\$)\) | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4000 | 4       |
| \(PMT^1(h)\) | 4  | 4  | 4  | 4  | 4  | 4  | 4  | 4  | 4       |

Table 6: Optimal solution results for model A and B

|          | Period (\(t\)) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | \(CPM^t\) | Cost(\$) |
|----------|----------------|---|---|---|---|---|---|---|---|------------|----------|
| Model A  |                | 0 | 1613 | 0 | 2016 | 0 | 2016 | 0 | 0 | 5645 | 4500 | 48230 | 58375 |
| Model B  |                | 0 | 4000 | 0 | 4000 | 0 | 4000 | 0 | 0 | 8000 | 6000 | 49630 | 63630 |

Assume two models, A and B. Model A (dependency) considers that cost and time of PM are related to the machine age. In model B (independency), cost and time of PM are considered independent and are set on average.
All parameters are mutual for model A and B, except information about cost and time of PM that is shown in Table 5. We made model A and B to assess and compare dependency of cost and time of PM to machine age. In model B, we assume that cost and time of PM are constant and determined with averaging the same parameters in dependency model A. On the other hand, means of cost and time of PM over machine age (index l) are equal for models A and B (see last column of Table 5). In a practical problem, cost and time of PM are dependent on machine age. After solving models A and B, the following result was obtained (Table 6). Using mean value and ignoring dependency assumption in model B causes less planned PM and it causes corrective cost to go up. Also, production cost is more than what obtained from model A with a dependency assumption. Considering an independency assumption in model B makes it more unrealistic. To clarify this issue, the following definition is necessary.

G is given as a model objective function for solution X and parameter P, \( g^* \) is a value of objective function for optimal solution \( X^* \) and parameter P, then:

\[
\begin{align*}
    g^*_A &= G(X^*_A, P_A) \\
    g^*_B &= G(X^*_B, P_B) \\
    \hat{g}_B &= G(X^*_B, P_B)
\end{align*}
\]

According to Table 6, \( g^*_A = 58375 \) and \( g^*_B = 6363 \). If we calculate objective function of model B by using dependency parameter of model A, which is more realistic, \( \hat{g}_B = G(X^*_B, P_A) \) and is equal to 59662 (values in parentheses of Table 6). In fact, the Model B calculates \( \hat{g}_B = 63650 \) for a decision maker at the beginning of a planning horizon, whereas this calculation is unrealistic. True value is obtained by considering the dependency of the time and cost of preventative maintenance as \( \hat{g}_B \). In this condition, all costs except the cost of preventative maintenance remain unchanged. In the provided numerical example, this amount has changed from 8000 to 4000. Although, the actual measured \( \hat{g}_B \) value is less than \( g^*_B \), it is more than \( g^*_A (g^*_A < \hat{g}_B < g^*_B) \).

Comparing dependency and independency through model A and B shows that assuming the dependency would decrease the total cost. The question is, “How much does dependency effect on decreasing the total cost?” To answer this question, we defined three levels of High, Medium and Low dependency severity. Each level will be compared to a corresponding case without dependency assumption like what we have done to compare model A and B. Cost and time of PM are presented in Table 7. Differences of each level can be seen in Figure 1.

| Table 7: Cost and time of PM for High, Medium and Low level of dependency |
|-----------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| l                           | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | Average |
| "PMC" ($ )                 |     |     |     |     |     |     |     |     | 4000    |
| High                       | 234 | 422 | 760 | 1367| 2461| 4430| 7974| 14352|         |
| Medium                     | 650 | 974 | 1462| 2193| 3289| 4933| 7400| 11100|         |
| Low                        | 1940| 2327| 2793| 3351| 4022| 4826| 5791| 6950 |         |
| "PMT" (hours)              |     |     |     |     |     |     |     |     | 4       |
| High                       | 0.2 | 0.4 | 0.8 | 1.4 | 2.5 | 4.4 | 8   | 14.4  |         |
| Medium                     | 0.6 | 1.0 | 1.5 | 2.2 | 3.3 | 4.9 | 7.4 | 11.1  |         |
| Low                        | 1.9 | 2.3 | 2.8 | 3.3 | 4   | 4.8 | 5.7 | 6.9   |         |

Figure 1: Left: Variety of PM Cost based on machine age. Right: Variety of PM Time based on machine age.
According to the results shown in Table 8, by increasing the level of dependency, improvement rises up. Improvement is defined as $g_A^2/g_B$. Reduction of the total cost for High level is 8% and it drops to 0.5% for Low level dependency. Increasing the severity of dependency has led to a greater improvement in the achieved total cost.

Table 8: Total cost results for different level of dependency

| Total Cost | Low  | Medium | High |
|------------|------|--------|------|
| $g_A^2$    | 60004| 54524  | 52042|
| $g_B$      | 63630| 63630  | 63630|
| $g_A^2/g_B$| 100  | 99.5%  | 94.7%|
|            | 92.0%|        |      |

We amend periodic constraints to consider periodic PM assumption in model A. It is definitely clear that adding periodic constraint does not improve the total cost as shown in Table 9.

Table 9: Cost obtained for cyclical and periodic model A

|                  | PM    | Corrective | Production | Total |
|------------------|-------|------------|------------|-------|
| Cyclical         | 5645  | 4500       | 48230      | 58375 |
| Periodic         | 6048  | 4000       | 49630      | 59678 |

Totally, maintenance cost in noncyclical model is more than periodic model, but much more reduction in production cost causes the total cost of cyclical model to get lower than the periodic model.

Conclusion

The proposed model is a combination of model [6] and [39] which leads to noncyclical preventive maintenance and a production model that considers cost and time of PM based on the machine age to make a more realistic model. It is tried to make a linear model even with binary and integer variables that results the MILP model. The dependency assumption was compared to independency and the obtained results show that considering dependency improves the total cost. Also, by increasing severity of dependency, the total cost reduces more. The presented model can be periodic by amending periodic constraints. In summary, the mathematical model can cope with cyclical and periodic preventive maintenance and dependency and independency of cost and time of PM. The model is developed for a capacitated machine and the model is sought to be expanded for a multi stage manufacturing system and present heuristic methods to solve the large-scale problems approximately.

Finally, authors would like to have some recommendation for the interested scholars to expand this research. To prove the practicality of the model, a specific type of manufacturing and machine could be chosen to apply the model e.g., steel converter plant [5], industrial evaporation network [44], and automotive industries [45]. Furthermore, Including different maintenance strategies adds valuable intuitions to the mathematical model [2], [46], and [47].

References

[1] Y. Ao, H. Zhang, and C. Wang, “Research of an integrated decision model for production scheduling and maintenance planning with economic objective,” Comput. Ind. Eng., vol. 137, p. 106092, Nov. 2019.

[2] H. Rivera-Gómez, A. Gharbi, J.-P. Kenm, O. Montaño-Arango, and J. R. Cazono-Amezcúa, “Joint optimization of production and maintenance strategies considering a dynamic sampling strategy for a deteriorating system,” Comput. Ind. Eng., vol. 140, p. 106273, Feb. 2020.

[3] X. Zhou and M. Yu, “Semi-dynamic maintenance scheduling for multi-station series systems in multi-specification and small-batch production,” Reliab. Eng. Syst. Saf., vol. 195, p. 106753, Mar. 2020.

[4] K. Kang and V. Subramanium, “Joint control of dynamic maintenance and production in a failure-prone manufacturing system subjected to deterioration,” Comput. Ind. Eng., vol. 119, pp. 309–320, May 2018.

[5] T. Wu, X. Ma, L. Yang, and Y. Zhao, “Proactive maintenance scheduling in consideration of imperfect repairs and production wait time,” J. Manuf. Syst., vol. 53, pp. 183–194, Oct. 2019.

[6] M.-C. Filoosi and M. Nourbakhsh, “Integrating noncyclical preventive maintenance scheduling and production planning for a single machine,” Int. J. Prod. Econ., vol. 136, no. 2, pp. 344–351, Apr. 2012.

[7] S. Dellagi, A. Chebbi, and W. Tabeti, “Joint integrated production–maintenance policy with production plan smoothing through production rate control,” J. Manuf. Syst., vol. 42, pp. 262–270, Jan. 2017.

[8] Q. Liu, M. Dong, and F. F. Chen, “Single-machine-based joint optimization of predictive maintenance planning and production scheduling,” Robot. Comput. Integr. Manuf., vol. 51, pp. 238–247, Jun. 2018.

[9] R. Jans and Z. Degreve, “Modelling industrial lot sizing problems: a review,” Int. J. Prod. Res., vol. 46, no. 6, pp. 1619–1643, Mar. 2008.

[10] P. ROBINSON, A. NARAYANAN, AND F. SAHIN, “Coordinated deterministic dynamic demand lot-sizing problem: A review of models and algorithms,” Omega, vol. 37, no. 1, pp. 3–15, Feb. 2009.

[11] L. Buchhöhl, F. Sahlings, S. Helber, and H. Tempelmeier, “Dynamic capacitated lot-sizing problems: a classification and review of solution approaches,” OR Spektrum, vol. 32, no. 2, pp. 231–261, Apr. 2010.

[12] H. Chen, “Fix-and-optimize and variable neighborhood search approaches for multi-level capacitated lot sizing problems,” Omega, vol. 56, pp. 23–36, Oct. 2015.

[13] A. Boonmee and K. Sethanam, “A GILNPSo for multi-level capacitated lot-sizing and scheduling problem in the poultry industry,” Eur. J. Oper. Res., vol. 250, no. 2, pp. 652–665, Apr. 2016.

[14] A. Andriolo, D. Battini, R. W. Grubbström, A. Persona, and F. Sgarbossa, “A century of research on production scheduling in industrial systems,” Eur. J. Oper. Res., vol. 275, no. 2, pp. 596–78, Apr. 2019.

[15] C. Woloszewicz, S. Dauzé-Pèrez, and R. Aggouné, “A Lagrangian heuristic for an integrated lot-sizing and fixed scheduling problem,” Eur. J. Oper. Res., vol. 244, no. 1, pp. 3–12, Jul. 2015.

[16] H. Wang, “A survey of maintenance policies of deteriorating systems,” Eur. J. Oper. Res., vol. 139, no. 3, pp. 469–489, Jun. 2002.

[17] M. Faccio, A. Persson, F. Sgarbossa, and G. Zanin, “Industrial maintenance policy development: A quantitative framework,” Int. J. Prod. Econ., vol. 147, pp. 85–93, Jan. 2014.

[18] B. Kader, D. Sefiane, R. Nidhal, and E. Walid, “Ecological and joint optimization of preventive maintenance and spare parts inventories for an optimal production plan,” IFAC-PAPERSONLINE, vol. 48, no. 3, pp. 2139–2144, 2015.

[19] L. A. Hadidi, U. M. Al Turki, and A. Rahim, “Integrated models in production planning and scheduling, maintenance and quality: a review,” Int. J. Ind. Syst. Eng., vol. 10, no. 1, p. 21, 2012.

[20] J. ASHAYERI, A. TEELEN, AND W. SELINI, “A production and maintenance planning model for the process industry,” Int. J. Prod. Res., vol. 34, no. 12, pp. 3311–3326, Dec. 1996.

[21] E. H. Aghezzaf, M. A. Jamali, and D. An-Kadi, “An integrated production and preventive maintenance planning model,” Eur. J. Oper. Res., vol. 181, no. 2, pp. 679–685, Sep. 2007.

[22] J. Ashayeri, A. Teelen, and W. Selini, “A production and preventive maintenance planning model for the process industry,” Int. J. Prod. Res., vol. 34, no. 12, pp. 3311–3326, Dec. 1996.

[23] E.-H. Aghezzaf and M. N. Najid, “Integrated production planning and preventive maintenance in deteriorating production systems,” Inf. Sci. (Ny), vol. 178, no. 17, pp. 3352–3392, Sep. 2008.

[24] A. Yalaoui, K. Chaabi, and F. Yalaoui, “Integrated production planning and preventive maintenance in deteriorating production systems,” Inf. Sci. (Ny), vol. 278, pp. 841–861, Sep. 2014.

[25] E.-H. Aghezzaf, A. Khatib, and P. Le Tam, “Optimizing production and
imperfect preventive maintenance planning’s integration in failure-prone manufacturing systems,” Reliab. Eng. Syst. Saf., vol. 145, pp. 190–198, Jan. 2016.

M. Nourelfath and E. Châtelet, “Integrating production, inventory and maintenance planning for a parallel system with dependent components,” Reliab. Eng. Syst. Saf., vol. 191, pp. 59–66, May 2022.

H. B. Faher, M. Nourelfath, and M. Gendreau, “Hybrid genetic algorithm to solve a joint production maintenance model,” IFAC-PapersOnLine, vol. 48, no. 3, pp. 747–754, 2015.

M. Nourelfath, N. Nahas, and M. Ben-Daya, “Integrated preventive maintenance and production decisions for imperfect processes,” Reliab. Eng. Syst. Saf., vol. 148, pp. 21–31, Apr. 2016.

B. Bouslah, A. Gharbi, and R. Pellerin, “Joint economic design of production, continuous sampling inspection and preventive maintenance of a deteriorating production system,” Int. J. Prod. Econ., vol. 173, pp. 184–198, Mar. 2016.

B. Bouslah, A. Gharbi, and R. Pellerin, “Joint optimal lot sizing and production control policy in an unreliable and imperfect manufacturing system,” Int. J. Prod. Econ., vol. 144, no. 1, pp. 143–156, Jul. 2013.

M. Dahane, N. Rezg, and A. Chelbi, “Optimal production plan for a multi-products manufacturing system with production rate dependent failure rate,” Int. J. Prod. Res., vol. 50, no. 13, pp. 3517–3528, Jul. 2012.

H. Sarper, “Scheduling for the maintenance of completely processed low-demand large items,” Appl. Math. Model., vol. 17, no. 6, pp. 321–328, Jun. 1993.

G. Cavory, R. Dupas, and G. Goncalves, “A genetic approach to the scheduling of preventive maintenance tasks on a single product manufacturing production line,” Int. J. Prod. Econ., vol. 74, no. 1–3, pp. 135–146, Dec. 2001.

J. P. Kenne, E. K. Bouskas, and A. Gharbi, “Control of production and corrective maintenance rates in a multiple-machine, multiple-product manufacturing system,” Math. Comput. Model., vol. 38, no. 3–4, pp. 351–365, Aug. 2003.

M. Ben-Daya, “The economic production lot-sizing problem with imperfect production processes and imperfect maintenance,” Int. J. Prod. Econ., vol. 76, no. 3, pp. 257–264, Apr. 2002.

S.-H. Sheu and J.-A. Chen, “Optimal lot-sizing problem with imperfect maintenance and imperfect production,” Int. J. Syst. Sci., vol. 35, no. 1, pp. 69–77, Jan. 2004.

A. Chelbi, N. Rezg, and M. Radhoui, “Simultaneous determination of production lot size and preventive maintenance schedule for unreliable production system,” J. Qual. Maint. Eng., vol. 14, no. 2, pp. 161–176, May 2008.

Z.-L. Lin, Y.-S. Huang, and C.-C. Fang, “Non-periodic preventive maintenance with reliability thresholds for complex repairable systems,” Reliab. Eng. Syst. Saf., vol. 136, pp. 145–156, Apr. 2015.

E. Gustavsson, M. Patriksson, A.-B. Strömberg, A. Wojciechowski, and M. Önnheim, “Preventive maintenance scheduling of multi-component systems with interval costs,” Comput. Ind. Eng., vol. 76, pp. 390–400, Oct. 2014.

B. Zhou, Y. Qi, and Y. Liu, “Proactive preventive maintenance policy for buffered serial production systems based on energy saving opportunistic windows,” J. Clean. Prod., vol. 253, p. 119791, Apr. 2020.

H. Rivera-Gómez, A. Gharbi, J.-P. Kenne, O. Montaño-Arango, and E. S. Hernández-Green, “Subcontracting strategies with production and maintenance policies for a manufacturing system subject to progressive deterioration,” Int. J. Prod. Econ., vol. 200, pp. 103–118, Jun. 2018.

E. Pan, W. Liao, and L. Xi, “Single-machine-based production scheduling model integrated preventive maintenance planning,” Int. J. Adv. Manuf. Technol., vol. 50, no. 1–4, pp. 365–370, Sep. 2010.

V. Polotzki, J.-P. Kenne, and A. Gharbi, “Joint production and maintenance optimization in flexible hybrid Manufacturing-Remanufacturing systems under age-dependent deterioration,” Int. J. Prod. Econ., vol. 216, pp. 239–254, Oct. 2019.

C. G. Palacin, J. L. Pitarch, C. Jasch, C. A. Méndez, and C. de Prada, “Robust integrated production-maintenance scheduling for an evaporation network,” Comput. Chem. Eng., vol. 110, pp. 140–151, Feb. 2018.

L. Gury, N. H. C. Anos, and A. Tsehese, “Towards strategic development of maintenance and its effects on production performance by using system dynamics in the automotive industry,” Int. J. Prod. Econ., vol. 200, pp. 151–169, Jan. 2018.

R. Gössinger, H. Helmke, and M. Kuhl, “Condition-based release of maintenance jobs in a decentralised production–maintenance system – An analysis of alternative stochastic approaches,” Int. J. Prod. Econ., vol. 193, pp. 528–537, Nov. 2017.

G. Si, T. Xia, Y. Zhu, S. Du, and L. Xi, “Triple-level opportunistic maintenance policy for leasehold service network of multi-location production lines,” Reliab. Eng. Syst. Saf., vol. 190, p. 106519, Oct. 2019.