Field-Induced SDW and Butterfly Spectrum in Three Dimensions

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I. INTRODUCTION

Rich electronic states arising from nesting of Fermi surfaces continue to provide fascination in various classes of materials. Organic crystals provide particularly versatile Fermi surfaces, and it has indeed been shown that a curious series of spin density wave (SDW) states emerge in strong magnetic fields in a family of quasi-two-dimensional organic conductors (TMTSF)$_2$X (X=PF$_5$ etc.), called the Bechgaard salt. The field-induced spin density wave (FISDW) occurs when the nesting of the Fermi surface is incomplete. The Landau quantization in the pockets formed as a result of an incompletely nesting then causes a series of gaps to appear around the main SDW gap. Since $E_F$ always lies in the largest Landau gap, an integer quantum Hall effect arises. When the magnetic field is increased, successive phase transitions take place because the energetically favorable SDW nesting vector jumps along the way, which results in discontinuous changes of the Hall conductivity. This has been considered for the TMTSF compound, which happens to have very anisotropic transfer energies between molecules with $t_x : t_y : t_z \sim 1 : 0.1 : 0.003$, so that the system is almost perfectly two-dimensional (2D).

So a challenging problem we address here is: (i) can we have such Landau-quantization-assisted FISDW states in three-dimensional (3D) systems, not as a remnant of the 2D FISDW but as 3D-specific, energetically favorable states, and if so, (ii) how and why do the successive phase transitions arise in 3D? Lebed introduced the third direction allowing to FISDW for the first time, and several authors studied the quantum Hall effect in 3D FISDW, where Hall conductivities $\sigma_{xy}$ and $\sigma_{xz}$ are predicted to be quantized respectively. However, the condition for the emergence of 3D FISDW phase itself has not been worked out except for a limited case of (TMTSF)$_2$X where three dimensionality is very small. So it has remained to be clarified whether and how FISDW phases really do exist in 3D.

This is exactly the purpose of the present paper. We consider a possibility of FISDW phases in 3D systems in magnetic fields, where we shall show that the favorable situation is anisotropic 3D systems with an anisotropy such that the transfer energies satisfy $t_x \gg t_y \sim t_z$ (as contracted with $t_x \gg t_y \gg t_z$ in (TMTSF)$_2$X). With varied magnitude and orientation of the magnetic field $B = (0, B_y, B_z)$, we have optimized the SDW nesting vector to show that a series of 3D FISDW phases do indeed exist, which is best expressed as a phase diagram against $(B_y, B_z)$. The phases comprise rich families, where they are characterized by quantized Hall conductivities $\sigma_{xy}$ and $\sigma_{xz}$ as one hallmark of the 3D-nature. On the energy axis, the FISDW is seen to reside on a fractal energy spectrum like Hofstadter’s butterfly, which, curiously, also indicates the 3D-specific nature of the 3D FISDW. In fact this can be regarded as one realization, through a density-wave formation, of the butterfly and the quantum Hall effect in 3D we have proposed on a general mathematical basis. An intuitive reason why the butterfly spectrum arise in the 3D FISDW is discussed in terms of the topology of the incompletely nested Fermi surface in 3D in the final section.

II. FORMULATION FOR THE 3D FISDW

We consider a simple orthorhombic metal with an energy dispersion

$$\epsilon(k) = -t_x \cos k_x a - t_y \cos k_y b - t_z \cos k_z c,$$

(1)

where $a, b, c$ are lattice constants and the transfer energies are assumed to satisfy $t_x \gg t_y, t_z$ (i.e., quasi-1D). The dispersion along $k_z$ around the Fermi energy can be approximated as a linear function $v_F(k_z - k_F)$ (with $\hbar = 1$ and $\epsilon(k)$ measured from $E_F$), while the three-dimensionality (warping of the Fermi surface) can be described by the leading-order expansion in $t_y$ and $t_z$ as

$$\epsilon(k) = v_F(|k_z| - k_F) + \epsilon_\perp(k_\perp),$$

(2)

$$\epsilon_\perp(k_\perp) = -t_y \cos k_y b - t_z \cos k_z c - t'_y \cos 2k_y b - t'_z \cos 2k_z c - t''_{yz} \cos (k_y b + k_z c) + \cos (k_y b - k_z c),$$

(3)

where $k_\perp \equiv (k_y, k_z)$, and
$t'_{y} = \alpha t_{x}^{2}/t_{x}$,

$t'_{z} = \alpha t_{x}^{2}/t_{x}, t'_{yz} = 2\alpha t_{y}t_{z}/t_{x}$

(4)

with $\alpha = -(\cos k_{F}a)/(4\sin^{2}k_{F}a)$.

Let us apply a magnetic field $(0, B_{y}, B_{z})$ normal to the conductive axis $z$. We take the spin quantization axis parallel to $z$. We assume that an SDW is the most likely instability as in the Bechgaard salts[2] and look at the mean-field equation for the wave function with the 3D nesting vector $q = (q_{x}, q_{y}, q_{z})$ specifies the spin order direction on the two order parameters.

The phase difference between the $q$ single-mode function $\Delta(x)$, while $\Delta$ is the band energy measured from the Fermi surface $(q_{x},q_{y})$, $\Psi$ is the corresponding wave function for an up-spin electron on the right Fermi surface (down-spin on the left). $\Delta(x)$ represents the mean-field electron interaction, which can be approximately written as a single-mode function $\Delta(x) \sim e^{i\epsilon_{x}x}$. We determine $\Delta$ and $\epsilon_{x}$ self-consistently so as to minimize the free energy at $T = 0$ (i.e., the ground state energy). The SDW also mixes down-spin states around the right Fermi surface and up-spins around left Fermi surface, which defines another order parameter. The phase difference between the two order parameters specifies the spin order direction on the $xy$-plane.

If we separate out the $\epsilon_{x}$-dependent phase as

$u(x) = \tilde{u}(x)\exp \left[-\frac{i}{v_{F}} \int_{0}^{x} \epsilon_{x}(k_{z} = q_{z} - \epsilon A_{z})dx \right]$, $v(x) = \tilde{v}(x)\exp \left[+\frac{i}{v_{F}} \int_{0}^{x} \epsilon_{x}(k_{z} = q_{z} - \epsilon A_{z})dx \right]$, 

$\Delta(x) = \tilde{\Delta}(x)\exp \left[-\frac{i}{v_{F}} \int_{0}^{x} \epsilon_{x}(k_{z} = q_{z} - \epsilon A_{z})dx \right] + \epsilon_{x}(k_{z} = q_{z} - \epsilon A_{z})dx \right]$ (6)

Eq.(8) reads

$\left( E + i v_{F}\partial_{x} \Delta \right) \left( \partial_{x} \right) \left( E - i v_{F}\partial_{x} \Delta \right) \left( \tilde{u}(x) \right) \left( \tilde{v}(x) \right) = 0$, (7)

where the effect of the magnetic field is included in the off-diagonal part, $\Delta$. When we plug Eq.(8) into $\Delta$, we obtain

$\tilde{\Delta}(x) = e^{i\epsilon_{x}x} \sum_{m_{x},m_{y}} J_{n_{x}}(z_{1})J_{n_{y}}(z_{2}) \times ... \times J_{n_{x}}(z_{6}) \times e^{-i(n_{1}+2n_{3}+n_{5}+n_{6})G_{x}x-i(n_{2}+n_{4}+n_{5}+n_{6})G_{y}x+i\delta} \Delta \left( \epsilon_{x} \right) \left( E - \epsilon_{x} \right) \left( E - \epsilon_{x} \right) \left( \epsilon_{x} \right)$ (8)

with

$z_{1} = 2t_{y}/(G_{b}v_{F})\cos(q_{x}/2), z_{2} = 2t_{z}/(G_{c}v_{F})\cos(q_{x}/2), z_{3} = t_{y}'/(G_{b}v_{F})\cos q_{y}, z_{4} = t_{z}'/(G_{c}v_{F})\cos q_{z}, z_{5} = t_{y}'/[G_{b} + G_{c}]v_{F}\cos[(q_{y} + q_{z})/2], z_{6} = t_{z}'/[G_{b} - G_{c}]v_{F}\cos[(q_{y} - q_{z})/2]$.

(9)

where $J_{n}$ is the Bessel function,

$G_{b} = eB_{b}G_{c} = eB_{c}c$, and $\delta(q_{y},q_{z})$ is a phase factor independent of $x$. The summation in Eq.(8) can be rearranged into

$\tilde{\Delta}(x) = \Delta \sum_{m_{x},m_{y}} I_{m_{x}}e^{i(q_{x}-mG_{b}-nG_{c})x+i\delta}$, (10)

where $I_{m_{x}}$ is a sum of products of $J_{n}$'s. We can see that the energy gaps of width $|\Delta I_{m_{x}}|$ open at $k_x = \pm 1/(q_x - mG_b - nG_c)$. Since the Fermi energy (at $k_x = \pm k_F$) must lie in the largest gap to minimize the energy, we obtain

$1/2(q_x - MG_b - NG_c) = k_F$, (11)

where $M,N$ are $m,n$ that give the largest $I_{m_{x}}$. Thus the $x$ component of the SDW nesting vector becomes $q_{x} = 2k_F + MG_b + NG_c$. Here we assume $k_F \gg G_b, G_c$, which is reasonable as long as typically $B < 10^{4}$T.

To be precise, the gaps other than one at $E_F$ can affect the stability of the FISDW, but in the weak-coupling regime at $T = 0$ we can show that the stability of the FISDW phase is determined by the width of the gap in which $E_F$ resides. Suppose $G_b/G_c$ is rational with

$G_b = pG, G_c = qG$, where $p,q$ are mutually prime integers. Equation (10) can then be rewritten as

$\tilde{\Delta}(x) = \Delta \sum_{l} I_{l}e^{i(q_{x}-lG)x+i\delta}$, (12)

where $I_{l}$ is the summation of $I_{m_{x}}$ over those $(m,n)$ satisfying $mp = nq = l$. The energy spectrum has a gap at $k_x = \pm 1/(q_x - lG)$ for each integer $l$. We consider the situation where the gap widths are smaller than the gap intervals. We can then express the energy dispersion along $x$ in the extended zone (shown in Fig. 2) as

$E^{\pm}(k_x) = \xi^{\pm} + \sum_{l} \left[ \text{sgn}(\xi^{\pm} - l\epsilon)\sqrt{(\xi^{\pm} - l\epsilon)^{2} + |\Delta I_{l}|^{2}} - (\xi^{\pm} - l\epsilon) \right]$, (13)

where $\xi^{\pm}(k_x) = \pm \hbar v_{F}(k_x \pm 1/2q_x)$ are the dispersions for $\Delta = 0$ measured from the gap at $l = 0$ for the right
(\(\xi^+\)) and left (\(\xi^-\)) Fermi surfaces with \(\varepsilon = \hbar v_F G/2\). The energy gained by opening the gap in the metallic state is

\[
F = \frac{|\Delta|^2}{v_0} + \sum_{k_\pm} [E^\pm(k_x) - \xi^\pm(k_x)].
\]

(14)

Here \(v_0(>0)\) is a molecular-field constant, and the summation taken over \(E_F - \xi < E^\pm(k_x) < E_F\), where \(\xi_c\) is a cutoff. If we insert Eq. (13) into this equation, we have

\[
F = \frac{|\Delta|^2}{v_0} - D_0 \frac{|\Delta I|}{v_0^2} \left( \frac{1}{2} \log \frac{4\varepsilon^2}{|\Delta I|^2} \right)
\]

\[+ D_0 \sum_{l \neq 0} |\Delta I_{L+l}|^2 \log \frac{l\varepsilon}{|\xi_c + l\varepsilon|},
\]

(15)

where \(L\) is the index of the gap that contains \(E_F\), and \(D_0\) is the density of states for \(\Delta = 0\) which is assumed to be a constant. From the gap equation, \(\partial F/\partial |\Delta|^2 = 0\), we obtain

\[
|\Delta I_L| = 2\xi_c \exp \left( \frac{-1}{|I_L|^2 v_0 D_0} + \sum_{l \neq 0} \frac{|I_{L+l}|^2}{I_L^2} \log \frac{l\varepsilon}{|\xi_c + l\varepsilon|} \right),
\]

(16)

\[
F = -D_0 \frac{|\Delta I_L|^2}{2}.
\]

(17)

Thus \(\Delta\) in general depends not only on the width of the gap at \(E_F\) (\(\propto I_L\)) but also those of other gaps. In the weak-coupling limit \(v_0 \to 0\), however, \(\Delta\) is mainly determined by the factor \(\exp[-1/|I_L|^2 v_0 D_0]\). So larger \(I_L\) gives larger \(\Delta\) in (16), which gives smaller \(F\) in (17). Therefore, we only have to maximize \(I\) in order to minimize the free energy.

III. PHASE DIAGRAM AND HALL CONDUCTIVITY

We have obtained the phase diagram against \((B_y, B_z)\) by maximizing \(I_{mn}(q_y, q_z)\) for mesh points on \((B_y, B_z)\) and \((q_y, q_z)\) around \((\pi, \pi)\). Fig. 2 shows the result for \(t_y = t_z\) (a) and \(t_y > t_z\) (b). In both cases we do have a series of phases that are characterized by \((M, N)\) defined in Eq. 13. An essential finding here is that there are FISDW phases specific to 3D, which exist only when both \(t_y\) and \(t_z\) are nonzero. We can see this by comparing Figs. 2(a) and (b), where the 3D-specific phases (shaded) are seen to shrink as \(t_z/t_y \to 0\). The 3D-specific phases are classified into several families: \((M, N) = (N, -N)\) phases lying along \(\theta \equiv \tan^{-1}(B_y/B_z) = 45^\circ\), and \((-2N, 0)\) phases around \((B_y, B_z) \approx (0.1, 0)\), etc., and their mirror images \((B_y \leftrightarrow B_z)\). Sun and Maki have shown that a small \(t_z\) in (TMTSF)\(_2\)X (i.e., \(t_z^2 \ll t_x^2\) neglected) can give rise to a phase with nonzero \(M, N\) just at a particular angle of \(B\) (Lebed’s angle, corresponding to 45\(^\circ\) in our model for \(b = c\)). The Sun-Maki phase is possibly related to the present 3D phases, although it does not belong to the \((N, -N)\) family here.

The integers \((M, N)\) have an important physical meaning — the Hall conductivity. Following Yakovenko’s formulation for 2D, Sun and Maki have predicted that the FISDW phase having \((M, N)\) should have Hall conductivities \(\sigma_{xy} = 2\pi\varepsilon/M, N\) (2: spin factor). In our previous paper that demonstrated a realization of Hofstadter’s butterfly in non-interacting 3D systems, we have obtained the quantum Hall integers residing on the fractal spectrum by making use of Streda’s formula following Halperin-Kohmoto-Wu, where these integers are identified to be topological invariants assigned to each gap in the butterfly. If we apply this general argument to the FISDW problem treated here, the result coincides with Sun-Maki’s. What is interesting about the FISDW states considered here \((t_y \sim t_z)\) is that the wild variation of \((M, N)\) with the magnetic field accompanies a wild variation in the quantum Hall conductivities.

The mathematical origin of the 3D phases can be traced back to the basic equations above (while we discuss the intuitive reason later). For \(\theta \to 45^\circ\), \(G_b = G_c\) vanishes and the argument of one of the Bessel functions, \(J_n(z_0)\), diverges. Since \(J_n(z)\) has the maximum at \(z = n\), \(\Delta(x)\) has a large Fourier component \(e^{-inx}(G_c-G_b)\) with a nonzero \(n_6\). If we assume other \(z\)’s are small, \(I_{mn}\) has a maximum at \((m, n) = (n_6, -n_6)\), which corresponds to the \((N, -N)\) phases. Similarly, \((-2N, 0)\) phases correspond to the divergence of \(z_3 \propto 1/G_b\).

Now we come to the stability of the 3D phases. When we go from the 3D systems over to 2D (\(t_z \to 0\)), the 3D phases vanish and we are left with the 2D phases with \((N, 0, 0)\) that depend only on \(B_z\) (\(B_y\)), as seen from Fig. 2(b). These phases are known for (TMTSF)\(_2\)X, while the 3D phases are new. The nesting vector \((q_y, q_z)\) is pinned to \((\pi, \pi)\) in the \((N, -N)\) and \((-2N, 0)\) phases, while in the 2D \((N, 0)\) phases and some of 3D phases the nesting starts to deviate from \((\pi, \pi)\) with \(N\). We also notice that the 3D phases do not require very large magnetic fields. In fact, when \(B_y\) or \(B_z\) becomes too large the 3D phases give way to 2D ones even when \(t_y \sim t_z\) as seen in Fig. 2(a). This is because a large in-plane component of \(B\) tends to confines the electron motion within each layer so that the system becomes 2D-like.

The 3D FISDW phases with larger integers are less stable since \(I_{mn}\) (width of the energy gap) generally decreases with increasing \(m, n\). Hence the FISDW should become unstable when the magnetic field is too close to \(\theta = 0, 45^\circ\), or \(90^\circ\), where the Hall integers diverge. In this region, some metallic phase may become stable, or some FISDW with \((q_y, q_z)\) far from \((\pi, \pi)\) may appear, while we have studied the range \(0.9\pi \leq q_x, q_y \leq \pi\) here.
IV. ENERGY SPECTRUM

The second key result in this paper is the quasi-particle spectrum, which is plotted against $B_z/B_y$ in Fig. 3(a). A structure reminiscent of Hofstadter’s butterfly are conspicuous around the Fermi energy. A closer examination reveals that the whole spectrum, consisting of various butterflies pieced together, is much more delicately constructed than a single butterfly. This is exactly because the optimized nesting vector (which jumps from one optimal $(M,N)$ to another as $B$ is varied) makes the spectrum pieced together in such a way that the Fermi energy always lies in the largest gap. For comparison we display in Fig. 3(b) the energy spectrum when the optimization of the nesting vector is neglected with a fixed $\Delta$. A zigzag trajectory of the position at which the largest gap occurs corresponds to the gap at $E=0$ in (a).

We can also trace back the mathematical reason why we have a butterfly. Namely, the quasi-particle equation for the present system happens to coincide to that for the 3D butterfly in non-interacting systems previously studied in that the two periods $G_b, G_c$ (arising from uniform $B_z, B_y$) compete with each other, where a difference is that the amplitude of the periodicity is here related to the order parameter $\Delta$. So the spectrum plotted against $B_z/B_y$ is in fact expected to have the same structure as Hofstadter’s butterfly revealed in Fig. 3(a). An important distinction from the non-interacting case, however, is that the FISDW phase adjust itself in such a way that the largest gap in the butterfly has the Fermi energy in it.

So, while in the non-interacting case the butterfly structure is observed only around the bottom (or top) of the entire band, now we have the butterfly precisely around the Fermi level by construction, so the situation should be easier to realize experimentally.

V. DISCUSSIONS

Intuitive Picture — To help understand the butterfly intuitively, we can look at the topology of the Fermi surface. If we first look at the case of the 3D butterfly in non-interacting systems, a typical Fermi surface around the band bottom consists of nearly parallel planes with a set of holes connecting them as shown in Fig. 3(a). So we end up with, topologically, a coexistence of a bunch of pipes $\parallel y$ and another bunch $\parallel z$, and this induces a competition between the Landau quantizations due to $B_y$ and $B_z$, which causes the 3D butterfly. If we go back to the present FISDW, we can see that the incompletely nested Fermi surface has a similar structure after the SDW gap formation, as typically shown in Fig. 3(b). There we display a warped Fermi surface in 3D, where the Fermi surface shifted by the nesting vector $q$ is superposed to show that how they are interwoven. When the SDW gap opens in this incompletely nested Fermi surface in 3D, we have a multiply-connected Fermi surface (i.e., a network of pipes) reminiscent of Fig. 3(a) as well as isolated pockets.

The situation sharply contrasts with the incompletely nested Fermi surface in 2D, where we end up with isolated pipes after the SDW formation. Thus the multiply-connected Fermi surface explains how the butterfly-like spectrum appears, although, to be more precise, there is magnetic breakthrough across the pockets and multiply-connected Fermi surface. So we expect that the 3D butterfly tends to appear in systems having multiply-connected Fermi surfaces.

Figure 3(b) also explains intuitively why SDW gaps are not formed for magnetic fields having $\theta \sim 0, 45^\circ, 90^\circ$, since the semiclassical orbits on the multiply-connected Fermi surface are open in this case, so that the SDW formation is not energetically favorable. Mathematically, the divergence of the arguments in Bessel functions mentioned above is related to the configuration of the Fermi ‘pipes’.

Experimental possibilities — Experimentally, a best region to probe in the phase diagram, Fig. 2 to observe the 3D FISDW and the 3D butterfly should be where the 3D phase is observed for the entire tilting angle $(0 < |\theta| < 45^\circ)$ of the magnetic field with a fixed $|B|$. This corresponds to a situation,

$$t'_y, t'_z \gtrsim eBb v_F. \quad (18)$$

Why this should be the criterion may be understood as follows. The basic equation is written in terms of $z_1...z_6$. As discussed above, the 3D butterfly is a result of a competition between the periods $G_b, G_c$. In other words, we need to have $z_3, ..., z_6 \gtrsim O(1)$, since $z_3, ..., z_6$ contributes to the Fourier component of $G_b$ or $G_c$ through $J_n(z)$. We can exclude $z_1, z_2$ from our analysis, since they are always small when $(q_y, q_z) \sim (\pi, \pi)$. So we end up with the criterion, $t'_y, t'_z \gtrsim eBb v_F$ from the definition of $z_3, ..., z_6$ for $b \approx c$. We do not have to add a condition $t''_{yz} (= 2\sqrt{t'_y t'_z}) \gtrsim eBb v_F$, since this condition is already included in the above one.

We can give a rough idea how we can realize the above condition. If we have a material with, say, $t'_y, t'_z \sim 10K$ (cf. $t'_y \sim 10K \gg t'_z$ in (TMTSF)$_2X$) with the values of $v_F, b, c$ similar to those in (TMTSF)$_2X$, then the butterfly and the peculiar quantum Hall effect should be observed for a moderate $B \lesssim 10T$. The energy scale of the butterfly will be $t'_y$ or $t'_z$ as seen in Fig. 3. To have a large FISDW gap energy scale, on the other hand, larger the $|B|$ the better, since for a small magnetic field (for which $z$’s become large) $I_{mx}$, has a spreaded distribution against $m, n$ and the gaps become smaller.

M.K. would like to acknowledge a Research Fellowship of the Japan Society for the Promotion of Science for Young Scientists for a financial support. He also wishes to thank Prof. B.I. Halperin and his hospitality at Harvard University where the manuscript is completed.
FIG. 1. The structure of the energy spectrum representing Eq. (13) in the text.

FIG. 2. The phase diagram for the FISDW in 3D at $T = 0$ in the weak-coupling regime is shown against $(B_y, B_z)$ for $t_z/t_y = 1$ (a) or 0.7 (b) [i.e., $t'_z/t'_y = 0.1$ (a) or 0.49 (b) in eq. (3)]. The phases are labeled by the quantum Hall integers $(M, N) = \sigma_{xy}, \sigma_{xz}$ in units of $(\hbar/2e^2)$], and those having $(q_y, q_z) \neq (\pi, \pi)$ are underlined. We assume $b = c, t_y/t_x = 0.1$ and $\alpha = 0.4$. The 3D-natured phases are shaded.

FIG. 3. (a) The quasi-particle energy spectrum against $B_z$ for $t_z/t_y = 1$ with $B_y$ fixed to 2.5 (dashed line in Fig. 1). We assume a coupling constant $\nu_0D_0 = 0.34$ and the cut-off energy $E_c = 12.5t'_y$. Vertical lines indicate boundaries between different FISDW phases labeled by $(M, N)$. (b) Similar spectrum when we do not optimize the nesting vector (i.e., $q = (2k_F, \pi, \pi)$) with a fixed $\Delta(= 0.5t'_y$ here) for comparison. The positions of the gaps having the largest $I_{mn}$ are indicated by a solid line.

FIG. 4. (a) A typical Fermi surface for a non-interacting quasi-1D system with $t_z \gg t_y \sim t_x$ and $E_{F} \sim t_y, t_z$ from the band bottom. (b) A typical Fermi surface (mesh) superposed with the nested one (gray) translated by $q$ for the 3D FISDW case. After the SDW gap opening the Fermi surface consists of pockets and a multiply-connected network of pipes. Solid lines exemplify open orbits for $\theta = 0, 45^\circ$.

1. L. P. Gor’kov and A. G. Lebed, J. Physique Lett. 45, L433 (1984).
2. D. Poilblanc, M. Héritier, G. Montambaux, and P. Lederer, J. Phys. C 19, L321 (1986).
3. K. Maki, Phys Rev. B 33, 4826 (1986).
4. T. Ishiguro, K. Yamaji and G. Saito Organic Superconductors 2nd ed. (Springer, 1998).
5. A. G. Lebed, JETP Lett. 43, 174 (1986).
6. G. Montambaux and P. B. Littlewood, Phys. Rev. Lett. 62, 953 (1989).
7. In principle, superconducting states are also possible, since magnetic fields, which generally suppress superconductivity, can possibly, when extremely strong, help a reentrance of superconductivity in quasi-1D conductors as suggested by, e.g., N. Dupuis, G. Montambaux and C. A. R. Sá de Melo, Phys. Rev. Lett. 70, 2613 (1993). In our system, however, the required magnetic field for that would be much stronger than those for the FISDW.
8. Y. Sun and K. Maki, Phys. Rev. B 49, 15356 (1994).
9. Y. Hasegawa, Phys. Rev. B 51, 4306 (1995).
10. D. R. Hofstadter, Phys. Rev. B 14, 2239 (1976).
11. M. Koshino, H. Aoki, K. Kuroki, S. Kagoshima, and T. Osada, Phys. Rev. Lett. 86, 1062 (2001).
12. M. Kohmoto, B. I. Halperin, and Y. Wu, Phys. Rev. B 45, 13488 (1992).
13. V. M. Yakovenko, Phys. Rev. B 43, 11353 (1991).
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