Nuclear symmetry energy at subnormal densities from measured nuclear masses

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Abstract

The symmetry energy coefficients for nuclei with mass number $A = 20 \sim 250$ are extracted from more than 2000 measured nuclear masses. With the semi-empirical connection between the symmetry energy coefficients of finite nuclei and the nuclear symmetry energy at reference densities, we investigate the density dependence of symmetry energy of nuclear matter at subnormal densities. The obtained results are compared with those extracted from other methods.

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I. INTRODUCTION

The nuclear symmetry energy $e_{\text{sym}}(\rho)$, which represents the energy cost per nucleon to convert all the protons to neutrons in symmetric nuclear matter, has attracted lots of attention in astrophysics and nuclear physics, because it intimately relates to a wealth of astrophysical phenomena, the structure character of nuclei and the dynamical process of nuclear reactions. The density dependence is a key point in the study of symmetry energy. Many theoretical and experimental efforts have been paid to constrain the density dependence of the symmetry energy [1–3]. Presently, some constraints on the symmetry energy at subnormal densities have been made from the double n/p ratio and isospin diffusion in intermediate energy heavy ion collisions of isospin asymmetric nuclei [1, 2, 4] and from the nuclear properties such as the thickness of neutron skin and the binding energy of finite nuclei [5–8]. The uncertainty of the symmetry energy coefficient and the density dependence of symmetry energy at subnormal densities is still large, and more study is still needed.

We try in this work to constrain the symmetry energy from more than 2000 precisely measured nuclear masses. By directly fitting the measured nuclear masses with the liquid drop mass formula, one can obtain the symmetry energy coefficients of nuclei in which both volume and surface term are included [9–11]. It is known that the symmetry energy coefficients of finite nuclei $a_{\text{sym}}$ are considerably smaller than that of the infinite nuclear matter due to the influence of the surface region of nuclei. A semi-empirical connection between the symmetry energy of nuclear matter at reference density and the properties of finite nuclei was proposed in [12]. More recently, a relation $a_{\text{sym}}(A) = e_{\text{sym}}(\rho_A)$ between the symmetry energy coefficients of finite nuclei and the symmetry energy of nuclear matter at reference density was proposed in [5]. For $^{208}\text{Pb}$, the reference density has a value of about $\rho_{^{208}\text{Pb}} \simeq 0.1 \text{ fm}^{-3}$. The relation provides a possible way to explore the property of nuclear matter from the property of finite nuclei. Combining this relation and the symmetry energy coefficients of finite nuclei extracted from the measured nuclear masses, we investigate the density dependence of nuclear symmetry energy at subnormal densities.

The paper is organized as follows: In Sec.II, we determine the symmetry energy coefficients for nuclei with mass number $A = 20 \sim 250$ by analyzing the measured nuclear masses [13]. In Sec.III, we constrain the nuclear symmetry energy at subnormal density with the relation of $a_{\text{sym}}(A) = e_{\text{sym}}(\rho_A)$, and compare our results with those obtained by using other
approaches. The reference density as a function of nuclear mass number is also deduced. A short summary is given in Sec.IV.

II. SYMMETRY ENERGY COEFFICIENTS OF FINITE NUCLEI

In Ref. [14], the symmetry energy coefficients of finite nuclei were studied. The energy per particle $e(A, I)$ of a nucleus can be expressed as a function of mass number $A$ and isospin-asymmetry $I = (N - Z)/A$ according to the Weizsäcker nuclear energy formula:

$$e(A, I) = a_v + a_s A^{-1/3} + e_{\text{Coul}}(A, I) + a_{\text{sym}}(A) I^2 + a_p A^{-3/2} \Delta_{np} + e_w,$$

(1)

with

$$\Delta_{np} = \begin{cases} 1 & \text{for even-even nuclei,} \\ 0 & \text{for odd-}A \text{ nuclei,} \\ -1 & \text{for odd-odd nuclei.} \end{cases}$$

(2)

Here a small correction term, i.e., the Wigner term $e_w$ is introduced for a better description of the systematic behavior in the symmetry energy coefficients of nuclei. The $a_v$, $a_s$ and $a_p$ denote the coefficients of the volume, the surface, and the paring term, respectively. Subtracting the Coulomb term and the Wigner term from the energy per particle, one obtains

$$e_m(A, I) = e(A, I) - e_{\text{Coul}}(A, I) - e_w$$

$$= e_0(A) + a_{\text{sym}}(A) I^2.$$  

(3)

Here we assume that the $a_v$ and $a_s$ terms are independent of isospin asymmetry and can be rewritten as $e_0(A)$. For the Coulomb energy, we take the same form as in Ref. [14]. For the Wigner term $e_w = E_w/A$, we take the form as in [15]

$$E_w = -C_0 \exp(-W|I|/C_0)$$

(4)

with two parameters $C_0 = -10$ MeV and $W = 42$ MeV, which is a direct consequence of the independent-particle model. We have checked that the obtained symmetry energy coefficients of finite nuclei do not change appreciately by varying the parameter $W$ in a reasonable region of $42 \sim 47$ MeV [16].
FIG. 1: (Color online) (a) Value of $e_m$ as a function of isospin asymmetry for selected series of isobaric nuclei. The filled circles and the solid curves denote the experimental data and the results by fitting the experimental data, respectively. (b) Symmetry energy coefficients of nuclei as a function of mass number. The filled circles denote the extracted results from the measured nuclear masses. The shades denote the results of Danielewicz et al [9]. The vertical short dashes denote the results when the shell corrections of nuclei from Ref. [10] are removed from measured energy per particle. The red curves denote the results by fitting the circles.

Now we can take $e_0(A)$ and $a_{sym}(A)$ as parameters and perform a two-parameter parabola fitting to the $e_m(A, I)$ for each series nuclei with the same mass number $A$. The experimental $e(A, I)$ data are taken from the mass table AME2003 [13]. For nuclei with even mass number, only even-even nuclei are taken into account in our calculations to consider the pairing effects. All available isobaric nuclei with mass number $A = 20 \sim 250$ are considered in the calculations. Fig.1 (a) shows the values of $e_m$ as a function of isospin asymmetry for series of isobaric nuclei with $A = 21, 40, 77, 116, 208$, as examples. The filled circles and the solid curves denote the experimental data and the results with two-parameter parabola fitting, respectively. The curvature of each curve gives the corresponding symmetry energy coefficient $a_{sym}(A)$ of the nuclei with mass number $A$. The extracted symmetry energy coefficients of finite nuclei as a function of mass number are shown in Fig.1(b). The filled circles denote the extracted $a_{sym}(A)$ from the measured nuclear masses. The shades denote
the results of Danielewicz et al [9]. The vertical short dashes denote the results when the shell corrections of nuclei [10] are removed from the measured values of energy per particle. In the region $A < 120$, the $a_{\text{sym}}(A)$ obtained in our approach show some oscillations and fluctuations. For heavy nuclei, our results of $a_{\text{sym}}(A)$ are comparable with those of Danielewicz et al. When the shell corrections are taken into account, the fluctuations in the extracted $a_{\text{sym}}(A)$ are reduced effectively. The mass dependence of the symmetry energy coefficients of nuclei is written by Danielewicz et al. [9] as

$$a_{\text{sym}}(A) = S_0(1 + \kappa A^{-1/3})^{-1},$$

(5)

where $S_0$ is the volume symmetry energy coefficient of nuclei, i.e. the nuclear symmetry energy at normal density $e_{\text{sym}}(\rho_0)$, and $\kappa$ is the ratio of the surface symmetry coefficient to the volume symmetry coefficient. By performing a two-parameter fitting to the $a_{\text{sym}}(A)$ obtained previously, i.e. the filled circles in Fig.1(b), we can obtain the values of $S_0$ and $\kappa$. With 95% confidence intervals of $S_0$ and $\kappa$, we obtain $S_0 = 31.1 \pm 1.7$ MeV and $\kappa = 2.31 \pm 0.38$, respectively. The results by fitting the circles are shown in Fig.1(b) with the red curves. The obtained value of $S_0$ is in good agreement with the range of $S_0 = 30.2 \sim 33.8$ MeV given by the pygmy dipole resonance (PDR) data [6].

III. NUCLEAR SYMMETRY ENERGY AT SUBNORMAL DENSITY

Now let us turn to study the density dependence of nuclear symmetry energy at subnormal densities. In experiment, recent research in intermediate-energy heavy-ion collisions (HIC) is consistent with a dependence at subnormal densities [2, 4, 17]

$$e_{\text{sym}}(\rho) = S_0(\rho/\rho_0)^\gamma,$$

(6)

With the relation

$$e_{\text{sym}}(\rho_A) = a_{\text{sym}}(A),$$

(7)

proposed by Centelles et al. [5], one can obtain the symmetry energy for nuclear matter from the symmetry energy coefficients of finite nuclei. The $\rho_A$ is the reference density in nuclear matter to make the equation hold [5, 12], which is significantly smaller than the saturation density because of the surface region of nuclear density profile. It is known that
the relation of Eq.(7) is hold at $\rho_A \simeq 0.1 \text{ fm}^{-3}$ for $^{208}\text{Pb}$ from various effective interactions [5, 18]. Now inserting Eq.(5), (6) into Eq.(7), one obtain

$$\left(\frac{\rho_A}{\rho_0}\right)^\gamma = (1 + \kappa A^{-1/3})^{-1}.$$  \hspace{1cm} (8)

Applying $\rho_{208} = 0.1 \text{ fm}^{-3}$ [5, 12, 18] and the confidence interval $\kappa = 2.31 \pm 0.38$ determined by $a_{\text{sym}}(A)$ in the previous section, we obtain the range of $\gamma$

$$\gamma = 0.7 \pm 0.1.$$  \hspace{1cm} (9)

With the relation of Eq.(8), one can obtain the expression of the reference density $\rho_A$ for a nucleus with mass $A$, i.e.

$$\rho_A = \rho_0 / \left(1 + \kappa A^{-1/3}\right)^{1/\gamma}.$$  \hspace{1cm} (10)

The value of $\rho_A$ decreases with the decrease of nuclear size because of the enhanced surface effects in light nuclei. Moreover, Eq.(10) indicates that the reference density $\rho_A$ depends on the ratio $\kappa$ and the parameter $\gamma$ which describes the stiffness of symmetry energy. Fig.2 shows the reference densities $\rho_A$ as a function of nuclear mass number. The black solid
FIG. 3: (Color online) Density dependence of the nuclear symmetry energy with different methods. The two vertical dashed lines give the corresponding density region of nuclei with $A = 20 \sim 250$. The gray shades denote the region with $S_0 = 31.1 \pm 1.7$ MeV and $\gamma = 0.7 \pm 0.1$, using the form $e_{\text{sym}}(\rho) = S_0 (\rho/\rho_0)^\gamma$.

curve denotes the results of Eq.(10) with $\rho_0 = 0.16$ fm$^{-3}$, $\kappa = 2.31$ and $\gamma = 0.7$. From the figure one sees that the reference densities for finite nuclei with $A = 20 \sim 250$ cover the densities in the range of $0.42 \rho_0 \leq \rho \leq 0.64 \rho_0$. The results from the parameterized expression of $\rho_A = \rho_0 - \rho_0/(1 + cA^{1/3})$ proposed by Centelles et al. in [5] with various effective interactions, are also shown in the figure with the red dashed curve for comparison.

One can see that the reference densities obtained from the two different approaches are in good agreement with each other. The validity of the parameterized expression proposed by Centelles et al. has been tested in the mass region $40 \leq A \leq 208$ [5]. For mass region $A < 40$ and $A > 208$, the two models give similar extrapolation. From Fig.2 one sees that for heavy nuclei the reference densities change slowly with the mass number and are close to 0.1 fm$^{-3}$, while for light nuclei the reference densities fall very fast with decrease of mass. Therefore it is better to apply the relation (7) by choosing the symmetry energy coefficient of heavy nuclei to relate the symmetry energy for nuclear matter at the reference density.
Fig. 3 shows the symmetry energy of nuclear matter as a function of $\rho/\rho_0$ obtained in this work and the comparison with the results obtained by other methods. The green solid curve corresponds to the symmetry energy calculated with $e_{\text{sym}}(\rho) = 31.1(\rho/\rho_0)^{0.7}$, in which $S_0 = 31.1$ MeV is the favorite value from $a_{\text{sym}}(A)$ and $\gamma = 0.7$ is obtained from $\rho_A$. The two vertical dashed lines show the corresponding density region $0.42\rho_0 \leq \rho \leq 0.64\rho_0$ of nuclei with $A = 20 \sim 250$. One should note that the results in this work for density regions ($\rho < 0.5\rho_0$ and $\rho > 0.63\rho_0$, corresponding to $A < 40$ and $A > 208$, respectively) represent an extrapolation. The orange dashed curve denotes the symmetry energy constrained by comparing the measurements of the isospin diffusion and the neutron to proton double ratio in $^{124,112}\text{Sn} + ^{124,112}\text{Sn}$ reactions with the calculations of improved quantum molecular dynamics model (ImQMD) where $e_{\text{sym}}(\rho) = 12.5(\rho/\rho_0)^{2/3} + 17.6(\rho/\rho_0)^{\gamma/2}$ with $\gamma = 0.7$ [2]. The dot-dashed curve denotes the symmetry energy as $e_{\text{sym}}(\rho) = 31.6(\rho/\rho_0)^{\gamma}$ with $\gamma = 0.69$, obtained by the comparison between NSCL-MSU isospin diffusion data and the IBUU04 [4]. The blue solid squares give the mapped $e_{\text{sym}}(\rho)$ from the correlation between temperature, excitation energy, density and the isoscaling parameter [19, 20]. The red filled circle gives the constrained $e_{\text{sym}}(0.1)$ from the excitation energy of giant dipole resonance (GDR) in $^{208}\text{Pb}$ [18]. The gray shades in Fig. 3 are bounded by $S_0 = 31.1\pm 1.7$ MeV and $\gamma = 0.7\pm 0.1$. One can see from Fig. 3 that the obtained nuclear symmetry energy in this work locate between the results of ImQMD and IBUU04. The gray area is consistent with the results from ImQMD, IBUU04, GDR and the mapped data from isoscaling parameter in [19, 20] at subnormal densities.

Based on the extracted value of $S_0$ and the extrapolated nuclear symmetry energies at densities around the normal density, we further study the slope parameter $L = 3\rho_0 \frac{\partial e_{\text{sym}}(\rho)}{\partial \rho}|_{\rho_0}$, which is an effective quantity to characterize the density dependence of symmetry energy. From previously obtained range of $\gamma = 0.7\pm 0.1$ and the value of $S_0$, the value of $L$ can be obtained directly. The gray shades in Fig. 4(a) show the area of $L$ determined by $S_0 = 31.1\pm 1.7$ MeV and $\gamma = 0.7\pm 0.1$. The range of $L$ for $S_0 = 31.1$ MeV is shown with the solid line. The dashed and the dashed-dot-dot line correspond to the case of $S_0 = 29.4$ and 32.8 MeV, respectively, which are the boundary of the confidence interval of $S_0$. The values of $L$ for the three cases are listed in Table I. From the calculations, we obtain the largest range of the slope parameter, $53 \lesssim L \lesssim 79$ MeV for $S_0 = 31.1\pm 1.7$ MeV. To compare with the values of $L$ obtained from other methods, we show the range of slope parameter $L$ determined in
FIG. 4: (Color online) (a) Range of slope parameter $L$ determined by $S_0 = 31.1 \pm 1.7$ MeV and $\gamma = 0.7 \pm 0.1$ in this work. (b) Range of slope parameter $L$ determined from different observables. The results are taken from [21] with PDR, from [22] with neutron skin thickness and from [2] with isospin diffusion, respectively.

### TABLE I: Range of slope parameter $L$ for different cases of $S_0$.

| $S_0$(MeV) | $L_{\text{min}}$(MeV) | $L_{\text{max}}$(MeV) | $L$(MeV)     |
|-----------|------------------------|------------------------|--------------|
| 29.4      | 52.9                   | 70.6                   | 61.7 ± 8.8   |
| 31.1      | 56                     | 74.6                   | 65.3 ± 9.3   |
| 32.8      | 59.1                   | 78.7                   | 68.9 ± 9.8   |

this work and those from other recent analyzing with different observables in Fig.4(b). The black solid line is the result from the measured nuclear masses in this work. The result from isospin diffusion is obtained from [2] with $S_0 = 30.1$ MeV and $\gamma = 0.4 \sim 1.05$. The result from PDR is taken from [21] with $L = 64.8 \pm 15.7$ MeV and $S_0 = 32.3 \pm 1.3$ MeV. The result from neutron skin thickness is taken from [22]. The range of the values of $L$ obtained in this work is in consistent with those obtained from other analyzing.

### IV. SUMMARY

The symmetry energy coefficients $a_{\text{sym}}(A)$ for nuclei with mass numbers $A = 20 \sim 250$ have been determined from more than 2000 precisely measured nuclear masses based on
the liquid drop mass formula with the contribution of Wigner term being involved. Taking the form of \( a_{\text{sym}}(A) = S_0(1 + \kappa A^{-1/3})^{-1} \) we obtain the 95% confidence intervals of the volume symmetry energy coefficient \( S_0 = 31.1 \pm 1.7 \) MeV and the symmetry parameter ratio \( \kappa = 2.31 \pm 0.38 \) from the determined \( a_{\text{sym}}(A) \).

Based on the relation between the nuclear symmetry energy at reference density and the symmetry energy coefficients of nuclei \( e_{\text{sym}}(\rho_A) = a_{\text{sym}}(A) \), we investigate the nuclear symmetry energy at a narrow subnormal density range, \( 0.42\rho_0 \leq \rho \leq 0.64\rho_0 \). Applying the reference density for \(^{208}\text{Pb}, \rho_{208} \simeq 0.1 \text{ fm}^{-3} \), and the symmetry parameter ratio \( \kappa = 2.31 \pm 0.38 \), we determine the range of \( \gamma \), i.e. \( \gamma = 0.7 \pm 0.1 \), by inserting the nuclear symmetry energy \( e_{\text{sym}}(\rho) = S_0(\rho/\rho_0)^\gamma \) and the symmetry energy coefficients of nuclei \( a_{\text{sym}}(A) = S_0(1 + \kappa A^{-1/3})^{1/\gamma} \) into the relation \( e_{\text{sym}}(\rho_A) = a_{\text{sym}}(A) \). The obtained range of \( \gamma \) in this way is independent on the nuclear symmetry energy coefficient \( S_0 \). Simultaneously, we deduce the mass dependence of the reference density \( \rho_A \), which explicitly depends on \( \kappa \) and \( \gamma \). Finally, the range of the slope parameter \( L \) of nuclear symmetry energy at normal density is determined to be \( 53 \lesssim L \lesssim 79 \) MeV, based on the extracted value of \( S_0 \) and the extrapolated nuclear symmetry energies at densities around the normal density. The constraint on the nuclear symmetry energy at subnormal density provided in this work is in good agreement with the results from other recent analyses.

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