Phenomenological optics with self-made liquid lenses in the physics classroom

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Abstract
Why does a raindrop on a window pane show an image of the environment that is turned upside-down? And why does vision go blurry underwater, but is perfectly clear with diving goggles? Our everyday life is rich in optical phenomena. Unfortunately, these phenomena often play a subordinate role in Optics teaching, compared to ray constructions or mechanistic light models. In our new teaching-learning sequence designed for introductory physics courses at secondary schools, the observation of the phenomena assumes a more prominent position and the observer’s sense of sight becomes the starting point of learning about Optics. The centrepiece of our concept is the use of students’ self-made Optics inventory including liquid lenses in various experiments.

Keywords: optics, phenomena, experiment, self-made, teaching-learning sequence

1. Optics in introductory physics education
Optics is an integral topic of introductory physics courses at secondary schools which provides teachers with a multitude of possibilities for designing phenomenon- and experiment-oriented physics lessons. However, teaching introductory courses on Optics is a demanding task: optical phenomena are omnipresent and hence, students come up with explanations for their observations. This leads to ’an abundance of knowledge’ which ‘is spontaneously formed before any formal learning takes place’ [1, p 58]. To explain optical phenomena, students primarily draw on their hands-on experience. Amongst others, common Optics teaching even tends to support this habit since mechanical analogies are often overemphasised [2]. As a consequence, the model character of light rays may not necessarily become obvious to the students [3], and later, as the wave or particle
models of light are introduced, students could easily mix up the different ideas and develop new hybrid models [4]. Consequently, teachers are confronted with various students’ pre-conceptions when it comes to teaching Optics, and over the last decades extensive research has been conducted to investigate these students’ preconceptions on (geometrical) Optics topics, see [5–8]. For instance, the concept of vision does not become clear to many students. Students might think that light is necessary to illuminate an object, but they often do not fully understand that light needs to travel to the eye to make it see a given object. In fact, many students believe that the eye actively sends out some substance in order to scan the object instead [9]. Besides, the concept of image formation poses a hurdle for students when it comes to learning Optics: students correctly observe that the orientations top and bottom are swapped in the real image. However, the change of left and right often remains undetected [10]. Students might also conjecture that the image travels through the lens as a whole. Accordingly, an aperture smaller than the diameter of the lens must shrink the image or cut off parts of it [11].

2. Design principles for the development of a new teaching-learning sequence

In this article, we regard the difficulties and problems in introductory courses on Optics at secondary schools outlined in section 1 as an opportunity: we report on a new teaching-learning sequence—referred to as Erlangen teaching-learning sequence of Optics—that takes these issues into account. For the development of our teaching-learning sequence, we set up three design criteria, each matching one of the problems described in the previous section:

(a) Optical phenomena should be described without the use of mechanistic models of light. For this purpose, we have adapted ideas of the Optics of visual experience, a phenomenological concept of Maier [12]. Models of light are almost entirely replaced by the precise observation of images, the optical phenomena themselves.

(b) The topic of the visual process ought to accompany the whole curriculum. The sender-emission-receiver concept as described in [3, 9] is consequently used to include the topic of vision in our teaching-learning sequence.

(c) Students’ conceptions of image formation are dealt with pre-emptively. The students fabricate their own liquid lenses and a corresponding self-made inventory of optical components. They are then guided through a set of experiments to clarify the principles of image formation. The use of self-made optical components for students’ experiments has a known precedent, presented in [14] as early as 1965: ‘a small glass baby-food jar makes a good cylindrical lens’ [14, p 516].

In the following subsections, we provide explanations as to how these criteria have concretely been taken into account in the development of our Erlangen teaching-learning sequence of Optics. For a summary, see table 1. Based on that, we provide a detailed overview of our teaching-learning sequence in section 3.

2.1. An optics of visual experience

To match the first design criterion and avoid mechanistic models of light in our introductory Optics course, we have applied some principles of Georg Maier’s phenomenological Optics. The following methods, amongst others, are characteristic for the Optics of visual experience.

- Learning Optics along the images: Proponents of a phenomenological Optics question the traditional way of teaching Optics. The traditional way of teaching Optics is referred to as Optics ‘for the blindman’ (translated from [16, p 2]), and thus, misses the use of great potentials for teaching: the beauty and fascination of an optical phenomenon itself is replaced by ray constructions and abstract models of light. The critics request that the phenomena should be at

\[ \text{His book describing phenomenological Optics has also been published in the English language [13].} \]
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Table 1. Design principles of the Erlangen teaching-learning sequence of introductory optics and how we addressed these design principles in the development of our teaching-learning sequence.

| Design principle                        | Consideration in the Erlangen teaching-learning sequence of optics |
|-----------------------------------------|---------------------------------------------------------------------|
| Avoidance of mechanistic models of light | Implementation of ideas from the *Optics of visual experience* [12, 15] |
| Consistent thematisation of the vision process | Focus on the sender-emission-receiver-concept [3, 9] |
| Prevention of students’ conceptions of image formation | Experiments with self-made optical components including liquid lenses |

Figure 1. Apparent depth of the coin for different positions of the observer.

the centre of Optics learning, and the condition of their appearance should be identified by investigating the phenomenon carefully [16].

- *The observer becomes part of the experiment:* For example, the bottom of a water basin is seen to be more lifted by the observer, as his glance meets the water surface at a flatter angle (cf figure 1). Thus strongly involved, the individual student is enabled to understand that one’s own vision is part of every optical issue.

- *Visual and haptic experiences are treated as separate sensations:* The students learn to distinguish between the properties of their visual sense and their sense of touch. As soon as they realise that the two senses conform to different rules, they are empowered to formulate the relations between both kinds of sensual experience [2].

In close collaboration with Georg Maier, the physics teacher Manfred von Mackensen developed a teaching concept for introductory Optics and adjusted Maier’s ideas to the school context [15].

2.2. The Sender-emission-receiver-concept

Several authors have pointed out that consequent thematisation of the visual process is essential for effective Optics teaching [3, 9]. This second design criterion of our teaching-learning sequence is addressed by the adaptation of the sender-emission-receiver concept. In this context, light propagation within the vision process is described as follows: light is emitted into all directions by the emitting sender. Thereby, some light might reach a re-emitting sender. As a result of scattering, the re-emitting sender in turn emits light in all directions. Subsequently, light arrives at the receiver, our eye. The propagation of light suits Fermat’s principle and therefore occurs straight-lined in the case of a homogeneous surrounding medium.

This concept is introduced at the very beginning of our Optics curriculum and is applied to each of the further steps (cf figure 7). A pictographic representation is used to make the process of vision more obvious (cf figure 2): conical arrows grossly indicate the paths of light in accordance with its colour [3].
Figure 2. The light propagation within the vision process is described in the sender-emission-receiver-concept: light is emitted by an emitting sender in direction of a re-emitting sender. Due to scattering, the re-emitting sender emits light inter alia in direction of the receiver, namely the eye of the observer.

2.3. Experiments with liquid lenses

With our teaching-learning sequence we aim to avoid typically widespread students’ difficulties with the concept of image formation (cf third design principle in table 1): the students are guided through a set of experiments, which either imply a new conception immediately or at least make it obvious that the former conception cannot be consistent (for details cf section 3).

To enable the students to experience their own handling of the optical experiments, they are guided to fabricate their own self-made Optics inventory, including liquid lenses (cf figure 3), at the beginning of the series of experiments: the components can be moved on a cable duct as a slide [17]. For each component, a small segment is cut out of the upper part of the duct and a fold-back clip is attached to it with glue. The clip can then either hold a liquid lens, an aperture cut out of paper, or a translucent screen. A nine-LED-torch serves as a sender. To achieve an asymmetrical image, the single LEDs can be coloured with different impermanent markers, as suggested in [18].

The plano-convex liquid lenses are made out of a plane overhead film and a piece of the curved surface of a plastic bottle, and may be filled with any clear liquid (cf figure 4).

In the Optics of visual experience [12], the topic refraction is introduced by the observation of the apparent depth of an object behind a water surface. Hence, our choice of introducing image formation via liquid lenses as a special case of this refraction fits in well with the concept of Optics of visual experience [12]. As a further optical component, all students fabricate their own prisms: a folded over-head film is glued to a microscope slide. The prism is then filled with water (cf figure 5). We describe the usage of these water prisms within our teaching sequence in the next section 3.

Finally, we use self-made lenses with variable curvatures (cf figure 6): the lens corpus is made of two lids for food preservation that are glued together. A tube connects the inside of the lens with a syringe. One can then vary the curvature of the lens by regulating the volume of water inside the lens with the syringe [19].

As the students are challenged to create certain images with their self-made inventory on their own, they become more involved in finding the general conditions of image formation. We are aware of the fact that several authors have pointed out that students might interpret their observations in line with their pre-conceptions and, for instance, report that the image became smaller as they were using an aperture [11]. This is why we enable students to make observations as objectively as possible, e.g. by asking students to measure the concrete size of the images. Details on our teaching-learning sequence are outlined in the following section 3.

3. The Erlangen teaching-learning sequence of optics

The Erlangen teaching-learning sequence is intended as an introductory Optics course for 7th grade students at secondary schools (12/13 years of age). A minimal time frame of ten 45 min lessons (or accordingly five 90 min sessions) is required to implement it in physics classrooms.

In our teaching-learning sequence (cf figure 7), we guide the students starting from the process of vision. After that, refraction is introduced as a case of change of light propagation at an interface. Subsequently, we make a transition to the image formation by looking through
3.1. Vision and brightness

At the beginning of the course, the students work out that visual and haptic sensations differ and might not go along with each other: there are observations that are purely visual, e.g. mirror images, and others which we can only sense by touch, e.g. the roughness of a scarf. Based on this distinction, the sender-emission-receiver-concept is the first topic of our Optics curriculum, as suggested in [3]. Therefore, numerous trials adapted from the Optics of visual experience [12] are included in our teaching-learning sequence: we start by asking the students to close their eyes and face the window, then turn their faces towards the wall, and finally, to cover their eyes with their hands. Hence, we support the idea that vision is basically a reception of light but not an active operation of the eye, except for the accommodation to light. The students sense that brightness varies as they move through the room, which makes them capable of orienting themselves in space [15]. This is followed by a discussion of the topic of contrast and in a further step, we focus on the origin of the varying levels of brightness in a room: the students discuss different emitters of light, e.g. the Sun or a torch. Some students might
The liquid lenses are made of the curved surface of a plastic bottle, which is glued onto a plane overhead film. The liquid is filled in through a tiny hole in the film, using a syringe. The resulting images are the images of a converging lens (at the top right).

Figure 4.

Figure 5. Self-made water prism.

bring up the moon, which may serve as a prime-example for objects that can only re-emit light: it makes a difference if an object is capable of glowing by itself or not. Together with the students, the conditions that make a surface appear bright and in a certain colour to us are discussed [15]. In this chapter’s final step, the students vary the visual contact between each other and conclude that light always travels in a straight direction—at least, this turns out to be the case for only one surrounding medium.

3.2. Refraction and apparent depth

In the case of light passing the interface between two media, the straight paths are bent—this is the
key item of the second chapter of our teaching sequence. As a corresponding visual experience, the image of an object which is located beneath the surface of water undergoes a displacement compared to the actual position of the object: viewed from an arbitrary angle, the object appears in an apparent depth beneath the water surface and this apparent depth is smaller than the objects’ actual distance from the water surface (cf figure 8), as has previously been discussed in [20, 21].

The students explore the displacement of the image in group work: an object is fixed at the bottom of a water basin. To make the shift become visible to all students, the observer targets the image by looking through a drinking straw that is placed on the edge of the basin. The position of the straw is fixed and a skewer is slipped into the straw, representing the direction of the student’s glance (cf figure 9) [11]. In their groups, the students discuss how the amount of image...
displacement changes as the perspective of the observer is varied. Lastly, the underlying principle of refraction, namely the change of light’s direction, is demonstrated as the kink of a shadow edge (see [22]). In this way, the phenomenon and the theory are brought together.

In our teaching-learning sequence, we limit the topic of refraction to the case of light passing from a thick medium to a thinner medium, as it is described in the previous experiment. Still, the opposite case can also be realised in a vision-based experiment by using a glass basin: the students look through the water surface from beneath it, observe the shift of images for different angles of view and explore the phenomenon of total internal refraction\textsuperscript{2}. All cases of refraction are now available to explain the curved propagation of light in the phenomenon of inferior mirage, which the students might know from driving on a hot road in summer. Inferior mirage can be realised in class with layered liquids of different density [23]. This is in line with the phenomenological approach and, unlike in the case of air, makes the interfaces between the layers become visible. If one wants to emphasise the temperature

\textsuperscript{2}To enable students to view through the floor of the basin, put the basin on an acrylic glass plate, which bridges the gap between two tables.
dependency of the refraction index of air, suggestions for demonstrations can be found in [24, 25].

3.3. The look through a prism

In the third chapter of our teaching-learning-sequence, the students get to know an object, the surface of which is always turned towards the observer in a tilted way: the prism. Each student fabricates his own prism corpus, which is then filled with water. As the students are looking through their water prisms, they observe that the image of the environment is shifted towards one of the edges of the prism, the towing edge of the prism (cf figure 10).

The look through the water prisms allows for a smooth transition from the topic of refraction to the concept of image formation, as suggested in [15]. The students observe how the image changes as they vary the conditions of the experiment: the shift of the image becomes larger both if one moves the object backwards and away from the prism (cf figure 11), or if one enlarges the angle at the towing edge by turning the prism (cf figure 12).

Lastly, the students join two water prisms together to form a double prism, which may serve as a first approximation of a plano-convex lens (cf figure 13). As the observer enlarges the distance of an object behind the prism, the image of the object is pulled apart and travels towards the edges of the double prism. A similar image can be observed as one looks through a converging lens: the virtual image extends as one enlarges the distance of an object behind the lens.

3.4. The images of a converging lens

At the beginning of the course’s last chapter, students fabricate their own optical components (cf figure 3), including liquid lenses (cf figure 4). For this purpose, we provide extensive craft instructions. As the crafting takes some time with students, we recommend teachers planning on at least four lessons of 45 min each (or two lessons of 90 min) for the whole chapter. The time frame can of course be extended to cover topics beyond our course: for example, using the self-made optical components to set up and investigate a model for a telescope might add value to the course [17].

The students begin to explore their self-made lenses by taking a look through them: gradually, the object distance is extended until the image fills out the lens completely. The image size has become infinite. Then, the image transforms and one can see the object turned upside-down and magnified. As the distance is still increased, the image shrinks gradually—an anticipation of the principles of real image formation (cf figure 14).

As a nice link to everyday experience, the images of the self-made lenses can be compared to common water lenses such as a water drop on a window pane [26] (cf figure 15).

In the following, the students get the opportunity to autonomously explore the image formation on a screen, using their self-made Optics inventory in an experimental series: the students are asked to map a sharp image of the sender that is (a) in original size, (b) enlarged and, lastly, (c) downsized (cf figure 16). From their observations, the students recognise that an object width $g$ is always coupled with a certain image width $b$ in order to get a sharp image. These findings may lead to the introduction of the thin lens formula, which, however, we do not focus on in our

3 In this case, the object width is almost equal to the focal length of the lens, which can be introduced to the students as a characteristic magnitude of the lens.
teaching-learning sequence on phenomenological Optics.

Within this experimenting series, typical students’ pre-conceptions of image formation (cf section 1) are taken into account (cf design principles summarised in table 1). In this respect, the students are set the task to experimentally check the orientations of the image: therefore, following the suggestion in [18], the students colour two single LEDs on their nine-LED torch with impermanent markers of different colours. We recommend using a transparent screen so that the students can compare the image with the object looking through the screen (cf figure 17).

A second task deals with the widespread students’ conception that images will go through the lens as a whole, and might therefore be cut off by using a small aperture. In order to deal with this conception, the students use a circular aperture made of cardboard, which can be varied: the aperture has a small lug, through which you can slide different inserts e.g. a cardboard with a very small hole or the profile of a butterfly (cf figure 18). Contrary to some students’ expectations, the image on the screen does not have an outline in the form of the aperture. However, as they change from the largest circular aperture to a small pinhole, the students may recognise that the image becomes less bright.

At this point, the sequence may optionally be expanded by introducing light ray considerations: we believe that following the multi-view approach presented in [27] could add value to the lessons in this phase. Thereby, ‘lens imaging is seen as a superposition of sharp images from different viewpoints, so-called elemental images’ [27]. In particular, the concept takes into account the students’ conception that the image will pass the lens as a whole. At any rate, teachers should mention, that optical phenomena can be described with the help of geometrical and mathematical models that will be presented later in the secondary school.

In our teaching-learning sequence, however, we stick with the principle of not using the model of light rays, at least not at this stage, as it is recommended for introductory Optics by [3]. To still emphasise that image formation is based on the principle of refraction, we added two further experiments to our teaching-learning sequence. These experiments link to the students’ knowledge from chapter 2, namely that refraction takes place at the interface between two media. The first of these experiments is shown in figure 19. This experiment can be assigned to students as a homework, which they have to use everyday components and one of the self-made lenses filled with water for: the lens is inserted into a glass basin (e.g. a casserole) which is empty at first. Image formation is realised with the torch as a sender and a screen behind the basin. The walls of the basin are (nearly) vertical to the optical axis and not curved, so these interfaces can be ignored for...
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Figure 13. Upper row, left: two self-made prisms are joined together (double prism). Upper row, right: similarity between the form of the double prism and a converging lens. Lower row: analogous to the magnified virtual image of a converging lens, the image of the double prism is pulled apart as the object width is increased (on the right).

Figure 14. The students observe the virtual images of their self-made lenses as the object distance is increased successively (adapted from [15]).

the formation. Later, the basin is slowly filled with water. As soon as the lens is surrounded by water entirely, the image disappears and cannot be found at a different image width either. Then, the water lens is replaced by a glycerine lens and an image appears again, at this stage of course for a different image width. The conclusion should be that the image formation is caused by the difference of the media or, to be more concrete, at their interface [28].

We know this sensation from swimming: when we open our eyes underwater, images appear blurred. The reason is that the refraction index of the eye lens is close to the index of water, hence, the eyes do not focus. However, as soon as we put on diving goggles, the interface air-eye lens is recovered, while the interface water-air of the goggles is either plain or slightly concave, which does not change the image’s orientation [29].

Using their self-made inventory, the students conduct a second experiment. In this experiment, the students switch between self-made lenses that are filled with e.g. water, soap and glycerine (cf figure 20). The object width stays fixed throughout. By producing a sharp image and measuring the image width or image size for each lens, the students see that their refractive properties differ.

We conclude our teaching-learning sequence with an experiment that creates a reference to the human vision, the topic of the first chapter of our teaching-learning sequence (cf section 3.1). The students explore their eyes’ ability of focussing...
Figure 15. The image of a raindrop on a window pane is compared with the image of the self-made lens.

Figure 16. Real images of different sizes are produced by the students using their inventory.

Figure 17. The students colour single LEDs and check the orientation of the image at a translucent screen.
The students observe that the brightness of the image changes as they switch between different apertures (upper row), while neither the form nor the size of the image are affected.

![Figure 18](image1.png)

**Figure 18.** The students observe that the brightness of the image changes as they switch between different apertures (upper row), while neither the form nor the size of the image are affected.

A self-made lens which is filled with water is inserted into a glass basin. At first, the basin is empty, and an image can be produced on a screen behind the basin (left). The image becomes less bright as water is filled into the basin, and finally disappears, as the water-lens is surrounded by water entirely (right).

![Figure 19](image2.png)

**Figure 19.** A self-made lens which is filled with water is inserted into a glass basin. At first, the basin is empty, and an image can be produced on a screen behind the basin (left). The image becomes less bright as water is filled into the basin, and finally disappears, as the water-lens is surrounded by water entirely (right).

objects at different distances (i.e. different object widths). This is referred to as accommodation. The self-made liquid lens with variable curvature (cf figure 6) is then used as a model for the image formation of the eye according to [19]. Therefore, the image width remains constant, and the students use the syringe to change the curvature of the lens and focus upon objects that are positioned at various distances (cf figure 21). The students compare their eyes’ accommodation to the model and, for instance, register a differentiated feeling of tenseness, as the ciliary muscles change the shape of the eye lens to focus a certain object distance. In [14, p 514], a simple experiment to investigate the pupil’s dilation and contraction called *Exercising pupils* is presented which we believe may complement this lesson in a valuable way.
Figure 20. Image width and size of the image change for a fixed object distance, as the students switch between lenses that are filled with different clear liquids (e.g. water, soap, glycerine).

Figure 21. By changing the curvature of the self-made water-lens, an object positioned at various distances can be focused (accommodation).
4. Conclusion and outlook
We outlined hurdles and widespread students’ pre-conceptions in introductory Optics teaching discussed in the literature. Proceeding from this starting point, we derived design principles for the development of a new teaching-learning sequence for introductory Optics at secondary schools. In the development of the Erlangen teaching-learning sequence of Optics, we have taken into account these design principles. This procedure resulted in an innovative teaching proposal that combines ideas from phenomenological Optics with a focus on the learners’ own activity: the production of one’s own inventory of optical components is an essential element of our course, and we believe that this allows learners to engage independently with Optics experiments far beyond classroom physics and, thus, also far beyond our teaching-learning sequence.

In the future, our current version of the Erlangen teaching-learning sequence of introductory Optics will be used in laboratory studies: we will conduct acceptance surveys with individual learners since this method has previously shown to be fruitful in an early stage of concept development in different thematic fields (see [30]). From such laboratory studies we expect detailed feedback on the individual instructional elements within our teaching-learning sequence. In the sense of the Design-Based-Research paradigm, we will use these insights to revise the teaching-learning sequence from both content and didactical points of view. Subsequently, the transition from laboratory to field studies is envisaged in order to evaluate the teaching-learning sequence in real teaching scenarios at secondary schools. In this phase, the teachers’ views on our teaching material (see [31]) will be collected in order to generate further ideas for refinement.

Data availability statement
No new data were created or analysed in this study.

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Ethical statement
The study was conducted in accordance with IOP Publishing’s Ethical policy. Authors acknowledge that the research was conducted anonymously, that consent was obtained from all participants, and that all participants are informed about the publication of the results of this study.

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