Study on flow instability in a diffuser with swirling flow under several conditions of pipe length and swirl intensity

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Abstract. A swirling flow in a diffuser such as a draft tube of a hydro turbine may induce the flow instabilities accompanied by pressure fluctuations known as vortex rope behaviour and cavitation surge. Cavitation surge is the self-excited oscillation, which induces the large flow rate fluctuation that results from the change of the cavity volume. In this research, the investigation of the effect of the pipe length and the swirl intensity on the flow instabilities in a diffuser was performed by experiments and numerical analyses using the draft tube component experimental facility. The length of the pipe was modified by up to about 25 times as long as the diameter of the throat in order to validate the one-dimensional analyses. In addition, the swirl intensity was changed by replacing another swirl generator. The frequency of cavitation surge was changed with regard to the swirl intensity as the one-dimensional analyses in the previous study has predicted it. Unsteady numerical simulations of the swirling flow with cavitation in the diffuser was performed. The results of experiments and numerical analyses correspond qualitatively with the result of the one-dimensional analyses, which suggested that the coupling with the experiments, CFD analyses and the one-dimensional analyses is the more effective way in order to predict the flow instabilities in the diffuser.

1. Introduction
The flow instability such as the cavitation surge and the vortex precession may occur due to a swirling flow in a draft tube when a water turbine is operated at off-design point. These flow instabilities, which can cause the fluctuation of the pressure and the flow rate, may become harmful to the stable operation of a water turbine. The vortex precession which is the forced oscillation, can be induced at partial load. According to the recent study by Nishi and Liu [1], these fluctuation may be amplified at a certain cavitation condition. In contrast, the cavitation surge, which is the self-excited oscillation, may occur when a water turbine is operated at full load. Chen et al [2] clarified using one-dimensional analyses that the diffuser effect of the draft tube leads to an instability of a hydraulic system which consists of a penstock, a runner, and a draft tube. In the previous study by the author [3], it was clarified that the cavitation surge may occur when the pressure recovery was positive at full load.

Meanwhile, computational approaches have been made to simulate the complex flow phenomenon in a diffuser with occurrence of cavitation using cavitation model of CFX by Ji et al [4] [5] [6]. In
addition, Muller et al [7] measured the void fraction in the draft tube at four different operating points using the Laplacian of Gaussian filter to detect the edge of the cavitation. The author [8] assessed the volume of the cavity calculated in a CFD analyses quantitatively by comparing it to the volume obtained from the binary image processing of shot images in the experiment. However, the effect of the factor such as the swirl intensity and the length of the diffuser has been investigated neither in an experiment nor a computational analysis. Thus, in this research, experiments and numerical analyses were carried out by changing the swirl generator and the length of the downstream pipe. In both case, the FFT analyses and the binary image processing were conducted to verify the one-dimensional analyses.

2. Experimental setup

2.1 Experiment facility

The experiment facility is a closed loop system, which consists of the diffuser, the swirler, the pipes, the tanks and the pump, as shown in Fig. 1. Each tank was located at upstream and downstream of the diffuser in order to prevent pressure fluctuations from propagating to the pump. Fig. 2 is the cross sectional view of the diffuser. The pressure transducers \((P_1, P_2, P_3, P_4, P_5, P_6, \text{and } P_7)\) are mounted to measure dynamic pressure of cavitation surge and vortex precession at the diffuser. At the same axial positions of \(P_1, P_5, \text{and } P_7\), another three transducers are flush-mounted 90 degrees from each position to detect the angular differences between two signals using cross-spectrum processing. In addition, high sensitive pressure transducers \((P_{u1}, P_{u2}, P_{d1}, \text{and } P_{d2})\) are set at the upstream and downstream pipe to calculate the dynamic flow rate according to the equation bellow. By neglecting the term of the resistance and compliance in the unsteady Bernoulli equation, the following equations are obtained for the upstream pipe.

\[
\hat{p}_{u1} - \hat{p}_{u2} = \frac{\rho l}{A} \frac{d\hat{Q}}{dt} \tag{1}
\]

where \(\rho\) is the water density, \(l\) is the distance between \(P_{u1}\) and \(P_{u2}\), and \(A\) is the cross sectional area of the pipe. Therefore, the flow fluctuation can be written as follows. For downstream pipe, the flow rate is similarly defined.

\[
\hat{Q} = \frac{A}{\rho l} \int (\hat{p}_{u1} - \hat{p}_{u2}) dt \tag{2}
\]

Two types of swirlers are set at the upstream of the diffuser to simulate the velocity distributions with different swirl intensity. Table 1 shows the specification of the swirlers. Swirl intensity is the ratio of the axial momentum and the circumferential momentum as shown in equation (3).

\[
SW = \frac{\int_{R}^{r} V_{z} V_{\theta} r^2 dr}{R \int_{r}^{R} V_{z}^2 r dr} \tag{3}
\]
where $r$, $R$, $V_r$, $V_z$, $V_\theta$ is radial distance, radius of diffuser throat, tangential velocity, axial velocity. In order to control the velocity distributions in the diffuser, the axial guide vane was installed upstream of the diffuser. Two types of the swirlers were prepared to change the swirl intensity. We assume a velocity distribution at off-design point is a Rankine combined vortex. Thus, the velocity gradually decrease toward the center of the diffuser, which forms the low pressure area in the diffuser. Fig.3 and Fig.4 illustrates the velocity distribution and the pressure distribution at the throat.

Table 1. The specification of the swirlers

|               | No.1 Swirler | No.2 Swirler |
|---------------|--------------|--------------|
| Number of blades | 6            | 8            |
| Angle of blades at shroud outlet | 70           | 30           |
| Angle of blades at hub outlet     | 80           | 70           |
| Swirl number          | 1.05         | 0.304        |

2.2 Experimental condition

In this study, we changed the downstream pipe length and the swirl intensity to investigate the effect of them on flow instability such as cavitation surge and vortex precession.

In both experiment, two types of pressure measurements were conducted under various conditions of cavitation number and Reynolds number. First, the cavitation number $\sigma$ was shifted from 1 to 20 for three flow rates conditions by changing the downstream tank pressure: 800 L/min ($Re = 3.60 \times 10^5$), 650 L/min ($Re = 2.92 \times 10^5$), and 500 L/min ($Re = 2.25 \times 10^5$). Second, the flow rate
was controlled from 300 L/min to 800 L/min for two cavitation number: \( \sigma = 3.6 \) and \( 8.0 \). Cavitation number and Reynolds number are defined as follows.

\[
\sigma = \frac{P_{tank} + \rho g h - p_v}{\rho u_{th}^2 / 2}
\]

(4)

\[
Re = \frac{u_{th} d_{th}}{\nu}
\]

(5)

where \( P_{tank}, h, p_v, u_{th} \) and \( d_{th} \) is the pressure of the downstream tank, the water level of the downstream tank from the center height of the diffuser, the vapor pressure, the area-averaged velocity and the diameter at the throat.

2.2.1 The downstream pipe length test

In the downstream pipe length test, we chose three downstream pipe lengths as parameters: 1758mm, 180mm and 3110mm by changing the layout of the test facility. The downstream pipe length is the distance from the diffuser outlet to the downstream tank inlet. That is, the standard downstream pipe length (1758mm) was made longer or shorter by moving the diffuser upstream or downstream. The total length of the pipe is constant because the test facility was closed-loop. However, the upstream tank and downstream tank are installed in order not to prevent pressure and flow fluctuation in the diffuser from transmitting over the tanks. Therefore, it is possible to investigate the effect of the downstream pipe length. The No.1 swirler was used in both measurements.

2.2.2 The swirl intensity test

In the swirl intensity test, as shown in Table 1, two types of swirlers were used to investigate the effect of the swirl intensity. No.2 swirler’s swirl intensity is approximately three times as small as No.1 swirler’s swirl intensity. Both swirlers are designed to produce a free vortex flow as shown in Fig. 3.

As seen in Fig.3, the gradient of the velocity of the No.1 swirler is much larger than that of the No.2 swirler. Therefore, the pressure distribution in the No.2 swirler is much smoother as seen in Fig.4. Pressure distributions influence the sensitivity of the occurrence of cavitation. Thus, it is assumed that the swirl intensity has the relation with the cavitation compliance as discussed below.

3. Numerical setup

The numerical analyses were performed to simulate the unsteady turbulent flow with cavitation using ANSYS CFX 15.0. Turbulence model is Reynolds-averaged Navier-Stokes (RANS) equation with the SST k-\( \omega \) model for the steady analysis and the SAS SST model for the unsteady analysis. Cavitation model is simplified Rayleigh-Plesset model. Details of numerical setup is shown in Table 2. Fig. 5 illustrates the computational domain which consists of upstream pipe, swirler, diffuser, downstream pipe, and tank. In this numerical simulation, we also consider both the length of the downstream pipe and the swirl intensity. In both simulations, to see the effect of the cavitation, the condition of cavitation was changed by changing the outlet boundary condition. The frequency of the unsteady phenomenon and the volume of the cavity in the diffuser was investigated.

| Table 2. Numerical setup |
|--------------------------|
| Turbulence model | SST k-\( \omega \) (Steady analysis) |
| Cavitation model | SAS SST (Unsteady analysis) |
| Total node number | Rayleigh-Plesset model |
| Boundary condition at inlet/outlet | Constant mass flow rate/ Constant static pressure |
| Time step | 0.001 |
4. The volume of the cavitation

Both in experiments and numerical analyses, the volume of the cavitation was calculated to obtain the cavitation compliance and investigate the effect of the parameter: the downstream pipe lengths and the swirl intensity. In the experiment, the commercial graphic software DIPP-MACRO was employed to obtain the image of the outline of the cavitation from the binary image shot by high-speed camera as shown in Fig. 6 [8]. Using this image, the calculation of the volume was performed as following equation.

\[ V_c = \int_{x_h}^{x_t} \pi D(x)^2 \, dx \]  

(6)

where \( D, x_t \) and \( x_h \) is the diameter of the cavitation and x-coordinates at the edge of the cavitation. In the numerical analysis, the volume of the cavitation is calculated by adding the vapor volume in each cell as follows.

\[ V_c = \sum_{i=1}^{N} \alpha_i V_i \]  

(7)

where \( N \) is the total number of control volumes in the computational domain, \( \alpha_i \) is the vapor volume fraction in each control volume and \( V_i \) is the volume of each cell.

5. Results and discussion

5.1 The calculation of unsteady flow rates by measurements of pressure

In one-dimensional analyses [2], the flow rate at the upstream of the draft tube does not fluctuate even when the cavitation surge occurs. In order to check whether this experimental facility satisfies the condition that one-dimensional analyses require, the unsteady flow rate was calculated by means of the pressure transducers. According to equation (2), the flow rates at the upstream and downstream of the diffuser was calculated. Then, there are small flow fluctuations at the upstream and the downstream of the diffuser in almost all condition. For instance, about 5 L/min flow fluctuation.
occurred when the flow rate was 650L/min and the cavitation coefficient was 3.6. No flow fluctuation condition requires too long upstream pipe. Thus, this experiment is slightly different from the ideal condition.

5.2 The effect of the length of the downstream pipe

According to the one-dimensional analyses by Chen et al [2], the following equations are obtained by combining the unsteady Bernoulli equation and the continuity equation with cavitating flow. The frequency of cavitation surge are determined by not only the cavitation compliance, but also the effective length of the diffuser as equation (8). Therefore, three types of the downstream pipe lengths are tested to investigate the effect of it both in experiments and numerical analyses.

\[ \omega_e = \sqrt{\frac{A_e}{\rho L_e C}} \]  

(8)

where \( A_e \) is the diffuser exit area, \( L_e \) is the effective length of the diffuser and \( C \) is the cavitation compliance defined as the following equation.

\[ C = -\frac{dV_c}{dP_c} \]  

(9)

where \( P_c \) is the core pressure in the diffuser. The larger compliance suggests that the volume of the cavity are more sensible to the change of the pressure in the diffuser.

Fig. 7 illustrates the results of spectrum analyses of the pressure fluctuations on the diffuser wall under original length of the downstream pipe with variable cavitation number and constant flow rate, 650L/min. In the previous study [8], pressure fluctuations are divided into four types: the type1 for vortex precession, the type2 for cavitation surge, the type3, and the type4. Though the causes of the type1 and the type2 have been identified by spectrum analyses and images shot by high-speed camera, the cause of the type3 and the type4 are not identified yet. There was little differences between (a) the original length of the pipe (1757mm), (b) the short length (180mm), and (c) the long length (3110mm). Only the pressure amplitude of the type3 in (b) the short length is smaller than that of the other length. In order to investigate the effect of the pipe length specifically, the relationships between the frequency of the cavitation surge and the cavitation coefficient for each pipe length were obtained as shown in Fig.8. Though there is a slight difference among the results of the pipe length test, we could not obtain the clear effect of the pipe length. In this experiment, degassing process was done before measuring the pressure to avoid the effect of the dissolved air. However, tiny amount of the air may affect the occurrence of the cavitation. Thus, we could not assume that there is the effect on the cavitation surge of the pipe length. The results of spectrum analyses with variable flow rate and constant cavitation number: 3.6 and 8.0 under the case of the short length of the pipe are shown in Fig. 9. The pressure amplitudes and the frequencies of each type of pressure fluctuations increase corresponding to the increase of the flow rate from this results. The same characteristics of the amplitude and the frequency of the pressure fluctuation were identified in the other cases of the pipe length.
The one-dimensional analysis suggests that the frequency of the cavitation surge decrease in inverse proportion to increase of the downstream pipe length. As a result, the frequency of the cavitation surge was not changed as one-dimensional analysis indicates, though the length of the downstream pipe was changed. In fact, this method might not be appropriate because the cavitation compliance could be also shifted accordingly when the length of the pipe was changed. Besides, the one-dimensional analysis is based on the assumption of no flow rate fluctuation at the upstream in order to neglect the effect of the length of the upstream pipe. Actually, the flow rate in the upstream pipe was fluctuating because of the pressure fluctuation in the diffuser. Therefore, more elaborate method might need to be devised to investigate the effect of the length of the pipe in detail.

5.3 The effect of the swirl intensity

5.3.1 Frequency characteristic of pressure fluctuation

Fig. 10 shows the spectrum analyses of pressure at $P_2$ under the condition that the flow rate is 650 L/min. The amplitudes of the pressure fluctuations of small swirl intensity is much smaller than that of large swirl intensity. The pressure fluctuation around 60 Hz in Fig.10 resulted from vortex precession because two signals at the same axial position had the angular differences of 90 degrees according to cross-spectrum analyses. In out of the range of cavitation number from 1 to 6, where the pressure recovery of the diffuser was positive, the frequency of the pressure fluctuation of low swirl intensity varied from 15 Hz to 60 Hz and had no differences of angle. Accordingly, it is assumed that this pressure fluctuation resulted from cavitation surge.
Mass flow gain factor $M = -dV_c/dQ$ as well as cavitation compliance $C$ is another key factor of cavitating flow. Fig.11 shows the results of spectrum analyses of pressure fluctuations of low swirl intensity with variable flow rate and constant cavitation number: 3.6 and 8.0. As is the results in the large swirl intensity, the amplitude and the frequency of the pressure fluctuation increased irrespective of the types of pressure fluctuations with an increase of flow rate. We calculated the volume of the cavity by binary image processing. Though the negative mass flow gain factor was obtained from the volume of the cavity, we could not have the clear relationships between mass flow gain factor and the frequency of the cavitation surge in both swirl intensity condition. Thus, the one-dimensional analyses which describe the mass flow gain factor explicitly should be derived because the mass flow gain factor is defined implicitly in this equation.

![Diagram](image_url)

(a) $\sigma = 3.6$
(b) $\sigma = 8.0$

**Figure 11. Pressure fluctuation at $P_2$ under variation of flow rate ($S_w = 0.304$)**

### 5.3.2 Evaluation of the effect of the swirl intensity

Fig.12 (a) shows the relationship between the cavitation number and the frequency of the cavitation surge in each swirl intensity. The frequency of the cavitation surge increases in proportion to the increase of the cavitation coefficient in both swirl intensity. The slope of the line of the No.2 swirler ($S_w = 0.304$) is larger than that of the No.1 swirler ($S_w = 1.05$).

In addition, the volume of cavitation is calculated for each swirler by means of the binary image processing of images shot by the high speed camera. In several cavitation conditions, the cavitation volume was calculated by computational analyses. Fig.12 (b) is the time-averaged cavitation volume in both swirl intensity. The volume of the cavitation increases more sharply in both cases when the cavitation number falls below a certain cavitation number ($\sigma = 1.0 \sim 3.0$). The less the cavitation number is, the more sharply the volume of cavitation increases in not only the experiment but also the calculation. In comparison between both swirler, the smaller swirl intensity has the sharper gradient of the cavitation volume. As is clear from the equation (4) and (9), the gradient of the cavitation volume implies the cavitation compliance. Therefore, it is assumed that the cavitation compliance increases when the swirl number becomes smaller.

![Diagram](image_url)

(a) The frequency of cavitation surge
(b) Cavitation volume

**Figure 12. The Effect of cavitation coefficient ($Q = 650$ L/min)**
As Fig.13 shows, the cavitation compliances were calculated from the slope of a line in Fig. 12(b). Over the entire range of the volume of the cavitation coefficient, the cavitation compliance of smaller swirl intensity is larger. The large cavitation compliance implies the low frequency of cavitation surge as the equation (8) shows. In addition, this graph suggests that the cavitation compliance is proportional to the volume of the cavity. That is, the following relation (10) is obtained.

\[ C = -\frac{dV_c}{dP_c} \propto V_c \]  \hspace{1cm} (10)

Combination of equation (8) and equation (10) leads to the following inversely proportional relation.

\[ \omega_e = \sqrt{A_e/\rho L_eC} \propto 1/\sqrt{V_c} \]  \hspace{1cm} (11)

![Figure 13. The Effect of the swirl intensity on the cavitation compliance (Q = 650 L/min)](image)

To investigate the equation (11), the square root of the volume of the cavity was calculated. Fig. 14 (a) shows the relationship between the square root of the volume of the cavitation and the frequency of the cavitation surge. As is the calculation using the experimental results, Fig.14 (b) was obtained from CFD analyses, which qualitatively agreed with the experimental results. Both in experiments and CFD cases, the frequency of the cavitation surge decreases with the increasing the cavitation volume as the equation (11) indicates. When it comes to compare the frequency between different swirl intensities under the same volume of the cavitation, the smaller the swirl intensity is, the lower the frequency of the cavitation surge is. As a result, the sensitivity of the cavitation volume for pressure change in the diffuser was enhanced by lowering the swirl intensity. Thus, it was revealed that an increase of cavitation compliance induces a decrease of the frequency of cavitation surge by both experiments and CFD analyses. The reason of this is related to the velocity distribution in Fig.3 and pressure distributions in Fig.4. Lowering the swirl intensity resulted in the gradual curve of velocity distributions in radial direction, which kept the gradient of the static pressure in the diffuser smooth. In case of the lower swirl intensity, due to the low gradient static distribution, the occurrence region of the cavitation expanded more widely corresponding to the change of the pressure. Therefore, it is assumed that the cavitation compliance, which is the rate of the change of the volume of the cavity with respect to pressure was enhanced.

![Figure 14. The effect of the swirl intensity on the frequency (Q = 650 L/min)](image)
6. Conclusions
In this study, we conclude as follows.

(1) The effect of the downstream pipe length was not clearly observed both in experiments and CFD analyses. It is possible that the pipe length may change the condition of the cavitation in the diffuser and affect the cavitation compliance. In this experiment, the degassing process was done well before the measurements of the pressure. However, the tiny amount of air affects the cavitation compliance and the amount of cavitation. In addition, there was fluctuation of the flow rate at the upstream pipe by means of equation (2). Therefore, better experimentally methods of investigation of the effect of the pipe length need to be considered.

(2) In previous research by Chen et al [2], the one-dimensional analyses derives the equation for the frequency of the cavitation surge. However, the validity of the analyses has not been investigated experimentally in terms of swirl intensity. In this research, it was revealed that the cavitation compliance increased by decreasing the swirl intensity, which resulted in the lower frequency of the cavitation surge as the one-dimensional analyses indicate.

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