Bounded time-delay control performance of two-degree-of-freedom nonlinear vehicle system under random road excitation using MR damper

Guangfeng Yan, Zuguang Ying¹ and Ronghua Huan
Department of Mechanics, School of Aeronautics and Astronautics, Zhejiang University, Hangzhou 310027, China

¹ E-mail: yingzg@zju.edu.cn

Abstract. The stochastic vibration control of nonlinear vehicle system under random road excitation is an important research subject. In this paper, based on a more practical and complicated nonlinear vehicle system with torsion-bar-support suspension, the vertical coupling motion between suspension and wheel is considered, and the MR damper is used for the vibration control of the vehicle system under random road excitation. The dynamic differential equations of the two-DOF vehicle system with MR damper are established. Then the HJB equation is established according to the stochastic dynamic programming principle. The feedback control law is designed based on the equation and control boundedness of the MR damper. The control effectiveness is evaluated by comparing the stochastic responses of semi-actively and passively controlled systems. The bounded time-delay control characteristics and the improvement of the bounded time-delay control for the nonlinear stochastic vibration of the two-DOF vehicle system are discussed.

1. Introduction
The random vibration of a vehicle under rough road has a great impact on the normal operation of vehicle devices, and therefore it needs to be controlled. At present, the intelligent suspension control is an important research subject in the field of vehicle vibration mitigation.

Rough road as an excitation to vehicle is a random process. Therefore, the stochastic optimal control strategy is commonly used for the vehicle vibration mitigation. The linear stochastic optimal control of suspension and car model has been studied [1-3]. The sky-hook control design has been used for the semi-active damping suspension [4, 5]. The adaptive control, fuzzy logic control and neural networks control have also been used for the active suspension systems with uncertainty [6-8]. However, most of these researches on the vehicle vibration control were based on a simplified quarter vehicle model and considered a non-large-motion nonlinearity of the suspension.

Because of low cost, simple and compact structure, the torsion-bar-support suspension is used for special kinds of vehicles. However, the torsion-bar-support suspension leads to the strong nonlinear vibration due to bar rotation. The nonlinear stochastic vibration control of the torsion-bar suspension vehicles is very different from that of ordinary vehicles. The research on the structure optimization of torsion-bar suspension systems has been given [9, 10]. The nonlinear stochastic vibration control of the torsion-bar suspension systems needs to be studied.

The magneto-rheological (MR) damper as a smart semi-active control device has been studied extensively and applied to structural vibration control [11-15]. However, the control force produced by
MR damper is bounded due to magnetic saturation, and then the control boundedness needs to be considered. On the other hand, the time delay from state measurement to control execution is unavoidable for feedback control, and it can make the control performance degenerate. Therefore, the control boundedness and time delay are necessary to be considered.

In this paper, the MR damper is used for the vibration control of a vehicle with torsion-bar-support suspension under random road excitation. The vertical coupling motion between the suspension and wheel is considered. The differential equations of motion of the two-degree-of-freedom (DOF) vehicle system with MR damper are established and then transformed into strongly nonlinear coupled vibration equations. The random excitation is produced based on road spectrums. According to the stochastic dynamic programming principle, the Hamilton-Jacobi-Bellman (HJB) equation is established. Based on the equation and control boundedness of the MR damper, the feedback control law is designed by combining linear and bang-bang control [16,17]. The control time delay is considered, and the bounded time-delay control characteristics of the two-DOF vehicle system are discussed. The control effectiveness is evaluated by comparing the stochastic responses of semi-actively and passively controlled systems. The improvement of the bounded time-delay control for the nonlinear stochastic vibration of the two-DOF vehicle system is discussed.

2. Optimal vibration control equations of torsion-bar suspension system

![Figure 1. Simplified model of torsion-bar suspension vehicle system.](Image)

The simplified model of the torsion-bar suspension vehicle system is shown in figure 1. The $oxy$ is the global inertial coordinate system, where the $ox$ axis coincides with the pavement datum line and the $oy$ axis along vertical direction. The $o_1x_1y_1$ is the vehicle motion coordinate system, and the direction is parallel to the global inertial coordinate system. The origin $o_1$ is at the torsion shaft, and $o_1y_1$ is collinear with $oy$. The body is assumed to be rigid, which vertical motion is considered. The wheel is hinged with the lower end of the balance elbow and the damper, and the upper end of the balance elbow and the damper is hinged with the vehicle body. The system has two degrees of freedom: the first degree of freedom is move in the vertical direction of the vehicle body, which is described by the absolute coordinate $y_c$ of the coordinate system $oxy$, and the second degree of freedom is the balance elbow rotation, which is represented by the angle $\theta_z$ relative to the horizontal line. $l_z$ is the length of the balance elbow, $\theta_{z0}$ is the pre-set angle, and $k_z$ is the torsion stiffness of the torsion shaft. $k_s$ is the stiffness of the spring contacted with the road surface, and $r_{w0}$ is the original length of the spring. $m_c$ is the mass of the vehicle body, and $m_w$ is the wheel mass. The coordinates of the MR damper at the
body end in $o_1x_1y_1$ are $(x_{id}, y_{id})$. The $\theta$ is the angle of the damper relative to the horizontal line, and $l_d$ is the distance between the two ends. The acceleration of gravity is $g$, and $y(t)$ is the rough road displacement relative to $ox$ axis. The dynamic equations of the system are obtained by the Lagrangian equations as

$$
(m_c + m_w)\ddot{y}_c - m_w l_c \cos \theta \dot{\theta} \dot{z} + m_w l_c \sin \theta \dot{z}^2 + (m_c + m_w)g + k_y (y_c - l_z \sin \theta - r_{wo}) = k_y y_r
$$  

(1)

$$
m_w l_z^2 \ddot{\theta} - m_w l_z \cos \theta \dot{y}_c + k_y (\dot{\theta} - \dot{x}_d) - m_w gl_z \cos \theta - k_y (y_c - l_z \sin \theta - r_{wo}) \cos \theta

- l_z F_d \sin (\theta_d - \theta) = - k_z l_z \cos \theta y_r
$$  

(2)

where $F_d$ is the force of the MR damper, which is according to the Bingham model [11],

$$
F_d = -C_0 \dot{\theta} - f_y \text{sgn}[\dot{\theta}_d(\theta_z)] = -C_0 \dot{\theta} - U
$$  

(3)

where $C_0$ is the viscous damping coefficient, and $U$ is the semi-active control force. The $f_y$ is the yield force of the damper, which can be changed by applied external voltage. Its value is bounded due to magnetic saturation, i.e., $f_y \in [0, U_{sat}]$. The distance between two ends of the damper is

$$
l_d(\theta_z) = \sqrt{(x_{id} - l_z \cos \theta_z)^2 + (y_{id} + l_z \sin \theta_z)^2}
$$  

(4)

Substituting equations (3) and (4) into equations (1) and (2) and using the equilibrium equations yield the vibration equations

$$
\ddot{\alpha} + \frac{m_c l_z \sin(\alpha + \alpha)}{m_c + m_w \sin^2(\theta_0 + \alpha)} \dot{\alpha}^2 + \frac{k_y [u + y_0 - l_z \sin(\theta_0 + \alpha) - r_{wo}] \sin^2(\theta_0 + \alpha)}{m_c + m_w \sin^2(\theta_0 + \alpha)} = - \frac{k_y (\dot{x}_d - \dot{\theta})}{l_z}
$$  

(5)

$$
\dot{\alpha} + \frac{m_c \cos(\theta_0 + \alpha) \sin(\theta_0 + \alpha)}{m_c + m_w \sin^2(\theta_0 + \alpha)} \dot{\alpha}^2 + \frac{k_y (m_c + m_w)}{m_c + m_w \sin^2(\theta_0 + \alpha) m_w l_z} \dot{\alpha}

- \frac{k_y m_c [u + y_0 - l_z \sin(\theta_0 + \alpha) - r_{wo}] \cos(\theta_0 + \alpha) - k_y m_c (y_0 - l_z \sin \theta_0 - r_{wo}) \cos \theta_0}{m_c + m_w \sin^2(\theta_0 + \alpha) m_w l_z}

\frac{C_0 (m_c + m_w) [y_{id} \cos(\theta_0 + \alpha) + x_{id} \sin(\theta_0 + \alpha)]^2}{[m_c m_w + m_w^2 \sin^2(\theta_0 + \alpha)] [2l_z [y_{id} \sin(\theta_0 + \alpha) - x_{id} \cos(\theta_0 + \alpha)] + x_{id}^2 + y_{id}^2 + l_z^2]}

- \frac{U (m_c + m_w) [y_{id} \cos(\theta_0 + \alpha) + x_{id} \sin(\theta_0 + \alpha)]}{[m_c + m_w \sin^2(\theta_0 + \alpha) m_w l_z]}

\frac{2l_z [y_{id} \sin(\theta_0 + \alpha) - x_{id} \cos(\theta_0 + \alpha)] + x_{id}^2 + y_{id}^2 + l_z^2} = - \frac{k_y m_c \cos(\theta_0 + \alpha)}{m_c + m_w \sin^2(\theta_0 + \alpha) m_w l_z}
$$  

(6)

The distance between two ends of the damper is

$$
l_d(\theta_z) = \sqrt{(x_{id} - l_z \cos \theta_z)^2 + (y_{id} + l_z \sin \theta_z)^2}
$$  

(4)
where line displacement \( u = y_c - y_0 \), angular displacement \( \alpha = \theta_c - \theta_0 \), \( y_0 \) and \( \theta_0 \) are the absolute line and angular coordinates in static equilibrium, respectively.

Let the system state vector be \( Z = [u, \alpha, a, \dot{a}]^T \). The state equation of the system is obtained as

\[
\dot{Z} = A(Z) + B(Z)U + F(t)
\]

where vectors \( A(Z), B(Z) \) and \( F(t) \) are determined by equations (5) and (6), and \( U \) is the control. The control aim of system (7) is to minimize the body response. The performance index of the stochastic optimal control is

\[
J = E\left[ \int_0^T f\{Z(t)\} dt + \psi(Z(t_T)) \right]
\]

where \( E[\cdot] \) is the expectation operator, \( f(Z) \geq 0 \), \( t_f \) is the terminal time, \( \psi \) is the terminal value of the control. Equations (7) and (8) construct the optimal parametric control problem on the nonlinear stochastic vibration of the vehicle system.

3. Bounded time-delay vibration control law

For the optimal parametric control of the nonlinear stochastic vibration of the vehicle system, the vibration control law can be determined based on the stochastic dynamic programming principle and bang-bang strategy. According to the stochastic dynamic programming principle, establish the HJB equation for system (7) and index (8)

\[
\frac{\partial V}{\partial t} + \min_u \left\{ \frac{1}{2} \text{tr}(ee^T \frac{\partial^2 V}{\partial Z^2}) + [A(Z) + B(Z)U]^T \frac{\partial V}{\partial Z} + f(Z) \right\} = 0
\]

where \( V \) is the value function, \( \text{tr}[\cdot] \) is the trace operator, \( e \) is the amplitude vector of white noises in excitation \( F \). With considering the control force \( U \) produced by the MR damper as bounded, the vibration control law is obtained by minimizing the second term on the left side of equation (9) as

\[
U^* = \begin{cases} U_1 & \frac{B^T \frac{\partial V}{\partial Z}}{\frac{\partial V}{\partial Z}} > 0 \\ U_2 & \frac{B^T \frac{\partial V}{\partial Z}}{\frac{\partial V}{\partial Z}} \leq 0 \end{cases}
\]

where \( U_1 \) is the lower limit of the control force, \( U_2 \) is the upper limit of the control force. When the vibration is small, the system is approximately linear. Then the control force in \([-u_1, u_1]\) is proportional to the velocity, and \( U \) is the upper bound of the linear part of the control law. Based on expression (3), the control force \( U \in [0, U_a] \) when \( \dot{\theta}_k(\theta_k) > 0 \), and \( U \in [-U_a, 0] \) when \( \dot{\theta}_k(\theta_k) < 0 \).

Substitute equation (10) into equation (9) to obtain the value function equation, and \( V \) is obtained by solving the equation. Then the optimal control force is determined by the expression (10). For ordinary control, \( U \) is determined by system parameters in real time. For time-delay control, \( U = U(t - \tau) \) where \( \tau \) is a delay time. The time-delay control force lags behind the system state, which makes the control performance degenerate. Under certain control time delay, the control effectiveness needs to be improved by optimizing control parameters such as the control bound.

4. Numerical results and discussions

The parameter values of the vehicle system with torsion-bar suspension are as follows: \( m_c = 3000kg \), \( m_s = 105kg \), \( l = 0.35m \), \( k_s = 12500N\cdot m^{-1} \), \( k = 2 \times 10^7N\cdot m^{-1} \), \( \theta_0 = 0.85rad \), \( r_{m} = 0.295m \), \( x_{ms} = 0.621m \), \( y_{ms} = 0.312m \), \( g = 9.8m\cdot s^{-2} \), \( C_0 = 1500kg\cdot s^{-1} \), \( U_0 = 4000N \), \( u_0 = 0.01m\cdot s^{-1} \). The control force \( U \) is determined by equation (10), and the controlled system responses are obtained using the Runge-Kutta method.
The rough road excitation $y_r(t)$ is generated by the filtered Gaussian white noise, and its differential equation is

$$\dot{y}_r(t) + 2\pi n_0^3 y_r(t) = 2\pi n_0 \sqrt{G_q(n_0)} \nu W(t)$$  \quad (11)

where $n_0=0.01$ m$^{-1}$ is the spatial cut off frequency, $n_0=0.1$ m$^{-1}$ is the reference space frequency, $\nu$ is the vehicle speed, $W(t)$ is the Gaussian white noise with unit power spectral density. $G_q(n_0)$ is the power spectral density of road surface under the reference space frequency $n_0$, which is called the road roughness coefficient. Based on different values of $G_q(n_0)$, the road roughness is classified into several grades.

![Figure 2. Rough road excitation $y_r(t)$.

Figure 2. Rough road excitation $y_r(t)$.](image1)

![Figure 3. Spectrum of rough road excitation.

Figure 3. Spectrum of rough road excitation.](image2)

![Figure 4. Body displacement.

Figure 4. Body displacement.](image3)

![Figure 5. Body acceleration.

Figure 5. Body acceleration.](image4)

Figures 2 and 3 are respectively the rough road excitation $y_r(t)$ and its power spectral density in the case of B-grade road and vehicle speed of 50 km h$^{-1}$. Figures 4-6 show the control performances of body displacement and acceleration and the power spectral density of body displacement, respectively. The semi-active control force is shown in figure 7. Based on the numerical results, the standard deviation of the passively controlled body displacement is 0.0396 m and the standard deviation of the semi-actively controlled body displacement is 0.0110 m. The displacement standard deviation is reduced by 72.35% using the semi-active bounded control. The standard deviation of the passively controlled body acceleration is 2.0766 m s$^{-2}$ and the standard deviation of the semi-actively controlled body acceleration is 1.2910 m s$^{-2}$. The acceleration standard deviation is reduced by 37.83% using the semi-active bounded control. Then it is obtained that the proposed control strategy can achieve a good control effectiveness of vehicle system vibration.

However, the control time delay can make the performance degenerate. As shown in figure 8, with the increase of delay time, the relative reductions in the standard deviations of the body displacement and acceleration decrease. The effect of small control delay time on the control effectiveness is slight. For certain control delay time, the parameters of the bounded time-delay semi-active control need to be optimized. Figure 9 shows that when the delay time $\tau=0.1$ s, the relative reductions in the standard
deviations of the body displacement and acceleration vary with the control bound. The bounded time-delay semi-active control can be improved by optimizing the control force bound of the MR damper.

5. Conclusions
The stochastic optimal semi-active control problem of strongly nonlinear vibration of a vehicle system with torsion-bar-support suspension and MR damper under random road excitation has been investigated. The nonlinear coupling vibration equations of suspension system have been established. According to the stochastic dynamic programming principle, the bounded time-delay feedback control law has been designed by combining linear and bang-bang control. The control effectiveness has been evaluated by comparing the stochastic responses of semi-actively and passively controlled systems. The bounded time-delay control characteristics and the improvement of the bounded time-delay control for the nonlinear stochastic vibration of the two-DOF vehicle system are discussed. Numerical results show that: (1) the proposed control strategy has good control effectiveness on the strong nonlinear stochastic vibration of the two-DOF vehicle system; (2) the control time delay will reduce the control effectiveness, and the control effectiveness decreases with the increase of delay time; (3) the time-delay control effectiveness can be improved by optimizing the control force bound. In a word, the proposed optimal semi-active control can effectively control the strong nonlinear stochastic vibration of the vehicle system under random road excitation.

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