One-loop renormalization of heavy-light currents

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We calculate the mass dependent renormalization factors of heavy-light bilinears at one-loop order of perturbation theory, when the heavy quark is treated with the Fermilab formalism. We present numerical results for the Wilson and Sheikholeslami-Wohlert actions, with and without tree-level rotation. We find that in both cases our results smoothly interpolate from the static limit to the massless limit. We also calculate the mass dependent Brodsky-Lepage-Mackenzie scale $q^*$, with and without tadpole-improvement.

\section{INTRODUCTION}

Although lattice QCD offers a nonperturbative method of calculating weak matrix elements from first principles, in practice a perturbative renormalization is also required to extract the continuum quantities for heavy-light systems. In this talk we discuss the renormalization of heavy-light vector and axial vector currents. These currents are needed for heavy quark phenomenology, such as the calculation of the decay constants and semi-leptonic form factors of heavy-light mesons. Here we calculate explicitly the mass dependent renormalization factors of heavy-light currents at one-loop order, when the heavy quark is treated with the Fermilab formalism \cite{1}. Results for the Wilson action have been obtained first in Ref. \cite{2} and preliminary results for clover action have been reported in previous lattice conferences \cite{3}. For tree-level improvement at order $1/m_Q$, we include so-called rotation term here. Tadpole-improved renormalization factors are also presented. We also calculate mass dependent Brodsky-Lepage-Mackenzie scale $q^*$ \cite{3}, with and without tadpole-improvement. More details of this work will be given in Ref. \cite{4}.

\section{ONE-LOOP RESULTS}

The renormalization factors $Z_J$, of heavy-light currents are simply the ratio of the lattice and continuum radiative corrections:

$$Z_J = \frac{Z_1^{1/2} \Gamma_l Z_2^{1/2} \text{cont}}{Z_1^{1/2} \Gamma_l Z_2^{1/2} \text{lat}},$$  \hspace{1cm} (1)

where $Z_{2h}$ and $Z_{2l}$ are wave-function renormalization factors of the heavy and light quarks, and the vertex function $\Gamma_l$ is the sum of one-particle irreducible three-point diagrams. We calculate explicitly $Z_A$ and $Z_V$ at one-loop order of perturbation theory.

In view of the mass dependence, we write

$$e^{-m_Q^{[0]} a/2} Z_{J, c} = 1 + \sum_{i=1}^{\infty} \theta_0^{2i} Z_{J, c}^{[i]},$$  \hspace{1cm} (2)

so that the $Z_{J, c}^{[i]}$ are only mildly mass dependent. Fig.\textsuperscript{4} plots the full mass dependence of the renormalization factors for the axial vector current $Z_{A, c}^{[1]}$. These numerical results are for the SW action with and without rotation, and also for Wilson action without rotation. Our results agree with those previously obtained, for $c_{SW} = 0$ \textsuperscript{3}.

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Figure 1. One-loop renormalization coefficient $Z_{A_4}^{[1]}$ as a function of $am_0$.

and for $c_{SW} = 1, d_1 = 0$ we find that in both cases our results smoothly interpolate from the static to massless limit. The resulting analytical expressions are in Ref. [4]. Fig. 2 plots the tadpole-improved renormalization factor for $Z_{A_4}^{[1]}$. From this figure, we can see that tadpole-improvement significantly reduces the one-loop coefficients of renormalization factors. Results for $Z_{A_4}$ and $Z_{V_{4+i}}$ are given in Ref. [1].

The slope of our mass-dependent renormalization factors in the massless limit is related to the improvement coefficients $b_J$ and $c_J$. We find

$$b_V^{[1]} = 0.153239(14),$$
$$b_A^{[1]} = 0.152189(14),$$
$$c_V^{[1]} = -0.016332(7),$$
$$c_A^{[1]} = -0.0075741(15).$$

These results agree perfectly with Ref. [1]. We also obtain by subtracting the integrands first,

$$b_V^{[1]} - b_A^{[1]} = 0.0010444(16)$$

which is more accurate than the difference of the two numbers quoted above. We find our one-loop result of $b_V - b_A$ are far from nonperturbative calculations [5].

3. SETTING THE SCALE

The typical gluon momentum $q^*$ in the $V$-scheme, as suggested by Brodsky, Lepage and Mackenzie (BLM), is defined by

$$\ln(q^*^2) \equiv \frac{\int d^4q f(q) \ln(q^2)}{\int d^4q f(q)},$$

where $q$ is the momentum of gluon, and the form $\int d^4q f(q)$ is the one-loop integral for a particular renormalization constant, for example, $\int d^4q f(q) = Z_{J_4}^{[1]}$. Previously $q^*$ has been calculated for the light-light current [14] and the static-light current [11]. Here we calculate the mass dependent $q^*$ for the heavy-light current. Results are plotted in Fig. 3. For Wilson action case, our results agree with Ref. [1] in the massless limit. From Fig. 3 we can see that the mass dependence of $q^*$ is weak from massless limit to $m_0a \sim 1$, especially for clover with rotation case. The original BLM prescription of $q^*$ breaks down at larger masses, because $Z_{J_4}^{[1]}$ (denominator in Eq. 8) goes through zero at there. A prescription for $q^*$ in this case is given in Ref. [12]. We also calculate tadpole-improved $q^*$ and results are plotted in Fig. 4. We can see that plaquette tadpole-improvement significantly reduces $q^*$, on the other hand, the reduction is rather small for $\kappa_c$ tadpole-improvement. We summarize the results in the massless limit in Table 1.

We can also obtain the BLM scale for improvement coefficients $b_J$ and $c_J$. Then it is interesting to compare BLM perturbation theory with non-perturbative calculations of these coefficients [14]. We will present these results for $q^*$ and the mentioned comparison in another publication [14].

\footnotetext[3]{The coefficient $d_1$ is field rotation parameter. See Ref. [1].}
4. CONCLUSIONS

We have obtained one-loop results of $Z_A$ and $Z_V$ with tree-level rotation, which should be useful for lattice calculations of $f_B$ and of form factors for $B \to \pi \nu \bar{\nu}$. We have also obtained the BLM scale $q^*$ for arbitrary masses, which should reduce the uncertainty of one-loop calculations.

Acknowledgments

S.H. and T.O. are supported by the Grant-in-Aid of the Japanese Ministry of Education, (Nos.11740162, No.12640279). K.-I.I. and N.Y. are supported by the JSPS Research Fellowships. Fermilab is operated by Universities Research Association Inc., under contract with the U.S. Department of Energy.

Table 1

| Action | Massless Limit | With Rotation, No Improvement | With Rotation, Tadpole Improvement | Through $\kappa_c$ |
|--------|----------------|-------------------------------|-----------------------------------|-------------------|
| $Z_A$  | $-0.116457(2)$ | $-0.033124(2)$               | $-0.048938(2)$                    |                   |
| $Z_V$  | $-0.133375(2)$ | $-0.050042(2)$               | $-0.024803(4)$                    |                   |
| $q^*_Z$ | $2.839$         | $1.802$                       | $2.408$                           |                   |
|        | $2.533$         | $1.550$                       | $2.316$                           |                   |
| $q^*_Z$ | $2.845$         | $2.060$                       | $2.503$                           |                   |
|        | $2.370$         | $1.700$                       | $2.052$                           |                   |

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Figure 3. Brodsky-Lepage-Mackenzie scale $q^*$ for $Z_A^{[1]}$ as a function of $am_0$.

Figure 4. Tadpole-improved Brodsky-Lepage-Mackenzie scale $q^*$ for $Z_A^{[1]}$ as a function of $am_0$. 