CMB Polarization During the secondary Ionization of Matter

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The cosmic microwave background (CMB) polarization, whose experimental detection must become the next natural step in the study of the CMB radiation, depends on the dynamics of hydrogen recombination in the Universe, on the relation between the power of scalar perturbations and gravitational waves, and on the presence or absence of secondary ionization. The spectrum of the CMB anisotropy is currently known with an accuracy that is high enough to draw some conclusions about the cosmological parameters and about the presence of secondary ionization of matter. The reduction of observational data strongly suggests the presence of secondary ionization. We consider the effect of this ionization at redshifts $z < 100$ on the CMB polarization generated by the scalar mode on various angular scales.

1 Introduction

In recent years, measurements of the large- and intermediate-scale anisotropy in the CMB radiation have reached an accuracy that allows us to draw conclusions about the global parameters of the Universe and, in particular, about the presence or absence of secondary recombination. For example, de Bernardis et al. (1997) discussed the interpretation of the observed spectrum of CMB fluctuations over a wide spectral range (for $l \sim 2 - 600$). One of the possible conclusions of de Bernardis et al. is that the model for the spectrum of primordial perturbations must include secondary ionization. In this case, the observer who will measure the CMB polarization will encounter a picture that will differ from the standard scenario for the formation of polarization. We calculated the amplitude and angular dependence of polarization measurements in the model with secondary
ionizations.

The formation of CMB polarization in the standard model has been studied extensively (Polnarev 1986; Bond et al. 1994; Harrari and Zaldarriaga 1993; Sarzin and Benitez 1995). In addition to the standard models for the formation of the large-scale structure of the Universe, models with unstable particles, one of whose decay channels is photodecay with the release of photons that are hard enough to ionize the primordial matter, have been considered by Berezhiani et al. (1990), Sciama (1990), and Sakharov and Khlopov (1992). The parameters of these particles, such as the half-life for the photochannel and the energy of released photons, depend on a specific physical theory and vary over a wide range. In this case, the optical depth may become significant, and the secondary formation of polarization is possible.

2 Analysis of the kinetic equation

We assume the Friedmann model for the Universe with \( \Omega_0 = 1 \) and \( \Lambda = 0 \), \( \Omega_b = 0.05 \) and postulate a dust-dominated equation of state with \( p = 0 \).

The relation for the interval with metric fluctuation is \( ds^2 = a^2(\eta)(d\eta^2 - (\delta_{\alpha\beta} + h_{\alpha\beta})dx^\alpha dx^\beta) \), where the small quantity \( h_{\alpha\beta} \) satisfies the Einstein equation. Three independent types of metric perturbation are recognized: scalar, vector and tensor. In this paper we analyze the effect of only scalar perturbations or, more precisely, of growing modes of adiabatic perturbations on the CMB radiation. The perturbation spectrum is considered in Harrison-Zeldovich form with the spectral index \( n = 1 \) (Starobinsky 1984).

Below, we use the standard notation for scalar correction to the metric and their Fourier components:

\[
h_{\alpha\beta}(k, \eta) = h(k)(k\eta)^2 \gamma_\alpha \gamma_\beta,
\]

where \( h(k) \) are the stochastic variations that show a \( \delta \) correlation with the power spectrum

\[
\langle h(k)h^*(q) \rangle = \frac{P(k)}{k^3} \delta(k-q),
\]

Here, \( P(k) = P_0 k^{n-1} \).
and $n$ is the spectral index.

In order to calculate the degree of polarization, we solve the kinetic equation for the symbolic vector

$$\delta = \begin{pmatrix} \delta_l \\ \delta_r \\ \delta_u \end{pmatrix}.$$  

The form of the kinetic equation and its details can be found in Basko and Polnarev (1980) and in the book of Chandrasekhar (1960). As it was shown by Polnarev (1986), Sazhin and Benitez (1995), Harrari and Zaldarriaga (1993), and Crittenden et al. (1993), the solution of the kinetic equation for the function $\delta$ in the case of plane waves will suffice to determine the anisotropy and polarization of the CMB radiation for an arbitrary perturbation spectrum. The equation in Fourier components for one of the waves with the wave vector $k$ is

$$\frac{\partial \delta}{\partial \eta} + e^\alpha \frac{\partial \delta}{\partial x^\alpha} = \frac{1}{2} \frac{\partial h_{\alpha \beta}}{\partial \eta} e^\alpha e^\beta - a(\eta) N e \sigma(\delta - \int P(\Omega, \Omega') \delta(\Omega) d\Omega). \tag{1}$$

We choose a coordinate system in which the perturbations propagate along the $z$ axis. The angle between the axis of photon propagation and the $z$ axis is denoted by $\theta$. We then have the following relation for the driving force on the right side of the kinetic equation which describes the formation of polarization (Basko and Polnarev 1980, Polnarev 1986):

$$F = h k^2 \eta (\mu^2 - 1/3), \tag{2}$$

where $h$ is the amplitude of the scalar perturbations, and $\mu = \cos(\theta)$.

We eliminated the monopole anisotropy component to obtain the second Legendre polynomial. For convenience, we choose the following form for $\delta$:

$$\delta = \alpha \cdot (\mu^2 - 1/3) \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} (1 - \mu^2) \tag{3}$$

After substituting this vector into the kinetic equation for $\delta$ (see, e.g., Basko and Polnarev 1980; Polnarev 1986), we obtain a system of integrodifferential
equations. The solutions of this system are analyzed, in particular, by Gibilisco (1995, 1996) and Seljak and Zaldarriaga (1996) by numerical and analytical methods. Since the use of numerical methods always raises the question of generality of the solutions, we use the analytical method of expansion in terms of a small parameter — the optical depth $\tau$. This expansion yields estimates of the effect which are general enough and essentially independent of a particular scenario. The scenario with an optical depth that is small compared to unity produced by the secondary ionization is currently most popular (de Bernardis et al. 1997). The case of a large optical depth produced by the secondary ionization, where the approximation of instantaneous recombination can be used to obtain an analytical solution was considered by us in our previous paper (Sazhin and Toporensky 1995).

Substituting (3) into the kinetic equation, we obtain the system of equations

$$\frac{d\alpha}{d\eta} + ik\mu\alpha = F(k, \eta) - g(\eta)[\alpha - \frac{9}{16}I(k, \eta)],$$  \hspace{1cm} (4)

$$\frac{d\beta}{d\eta} + ik\mu\beta = -g(\eta)[\beta + \frac{9}{16}I(k, \eta)],$$  \hspace{1cm} (5)

$$I(k, \eta) = \int_{-1}^{1} d\mu[\alpha(k, \eta, \mu)(\mu^2 - \frac{1}{2})^2 - \beta(k, \eta, \mu)(1 - \mu^2)^2].$$  \hspace{1cm} (6)

We solve this system by expanding it in terms of $g(\eta)$. In the zeroth approximation (i.e., in the approximation in which the quantities $\alpha, \beta$ correspond to the anisotropy and polarization produced during the primary recombination) $\alpha$ is considerably larger in absolute value than $\beta$. As a result, when considering the first approximation, the effect of $\beta$ on $\alpha$ is considerably smaller than the inverse effect. The correction to $\beta$ that follows from the first term in the expansion of our system may exceed significantly (for a given value of the wave vector) the zero-approximation value for $\beta$.

In the zeroth approximation for $g(\eta)$, the solution for $\alpha$ is

$$\alpha_0 = \exp(-ik\mu\eta) \int F(k, \hat{\eta}, \mu) \exp(ik\mu\hat{\eta}) d\hat{\eta}.$$

The corresponding solution for $\beta$, as follows from the discussion above, can be taken in the form $\beta_0 = 0$. 4
The solution for $\beta$ in the first approximation is now
\[
\beta_1(k, \eta) = -\frac{9}{16} \exp(-ik\mu\eta) \int_{\eta_r}^{\eta} g(\eta)I_0(k, \hat{\eta}) \exp(i\mu\hat{\eta})d\hat{\eta}, \quad (8)
\]
where
\[
I_0(k, \eta) = \int_{-1}^{1} d\mu [\alpha(k, \eta, \mu)(\mu^2 - \frac{1}{2})^2] \quad (9)
\]

3 Secondary ionization and the degree of polarization

Let us consider the model of instantaneous secondary ionization in which the degree of ionization is equal to zero for redshifts $z > z_{sr}$ and unity for $z < z_{sr}$. In this case, the function $g(\eta)$ is described by a relation of the form
\[
g(\eta) = \sigma_T n_{sr} \left(\frac{\eta_{sr}}{\eta_0}\right)^6 \left(\frac{\eta_0}{\eta}\right)^4 \quad (10)
\]
Introducing the baryon density $\Omega_b$, we can write the preceding equation in the form
\[
g(\eta) = \frac{3H^2}{8\pi G m_p} \Omega_b \left(\frac{\eta_0}{\eta}\right)^4 \quad (11)
\]
Denoting $k(\eta - \eta_r)$ by $x$ and taking the integral in (9), we obtain
\[
I_0 = h_A \left(\frac{8}{45} - f(x) - k\eta_r \frac{\partial f(x)}{\partial x}\right),
\]
where the function $f(x)$ is given by
\[
f(x) = \sqrt{2\pi}[8J_{5/2}(x) - \frac{8}{3}J_{3/2}(x) + \frac{4}{9}J_{1/2}(x)].
\]
Thus, the degree of polarization $\beta$ is the product of the integral over $\eta$ and three constants ($H, \Omega_b, h_A$) that depend on the cosmological scenario:
\[
\beta(k, \mu, \eta) = -\frac{9}{16} \exp(-ik\mu\eta) \frac{3H^2}{8\pi G m_p} \Omega_b h_A \int_{\eta_r}^{\eta_0} d\eta \left(\frac{\eta_0}{\eta}\right)^4 f(x) \exp(i\mu\eta).
\]
Let us introduce a polarized anisotropy component given by
\[
p = \frac{\delta_l - \delta_r}{2} T
\]
Figure 1: The quantity $p$ versus the wave vector of scalar perturbations for $\eta_{sr} = 6$ (solid line) and $\eta_{sr} = 7$ (dashed line).

A plot of this quantity against the wave vector $k$ is shown in the figure. Note that the half-width at half-maximum (HWHM) of the spectral curve is essentially independent of the problem parameters and is determined only by the position of the maximum $k_{\text{max}}$ of the curve; it is equal to $\Delta k \approx 0.63k_{\text{max}}$.

Let us also define the angular scale in terms of $k_{\text{max}}$ as follows:

$$\theta_{\text{max}} = 2^\circ \frac{2\pi}{k_{\text{max}}}.$$

It should be emphasized that this angular scale is not optimal for choosing the parameters of the antenna that measured the polarization. This quantity characterizes the scale of fluctuations of the polarized component in the sky. To choose
an optimum angular scale requires optimization with allowance for the antenna beam and the method of measurements.

The data in the table were obtained for $\Omega_b = 0.05$, $H = 50$km/(s Mpc) and $h_A$ normalized to the COBE results (Smoot 1997). The first and second columns give the conformal time, which corresponds to the time of secondary ionization and the redshifts, respectively. The third column contains the values for the quantity $p_{\text{max}}$ defined as follows:

$$p_{\text{max}} = \sqrt{|\beta(k_{\text{max}})|^2 T(\mu K)}$$

The values of $\theta_{\text{max}}$ are given in the fourth column. Finally, the last column lists the rms values of the polarization in $\mu K$ as estimated from the relation

$$\delta T_p = \frac{1}{2} \beta_{\text{max}} \Delta k.$$

| $\eta_{sr}$ | $z_{sr}$ | $p_{\text{max}} (\mu K)$ | $\theta_{\text{max}}$ | $\delta T_p (\mu K)$ |
|------------|---------|----------------|-----------------|-----------------|
| 3.0        | 100     | 1.62           | 9°.6            | 0.70            |
| 3.5        | 72      | 0.85           | 12°.4           | 0.30            |
| 4.0        | 55      | 0.49           | 15°.2           | 0.14            |
| 5.0        | 35      | 0.20           | 21°.6           | 0.041           |
| 6.0        | 24      | 0.10           | 28°.5           | 0.016           |
| 7.0        | 17      | 0.05           | 36°.1           | 0.007           |

The existence of secondary ionization results in a change in the angular dependence of the amplitude of the polarized component. In particular, an additional peak appears on intermediate angular scales, which are most convenient, for example, from the standpoint of the SPORT experiment (Cortiglioni et al. 1997) and several other currently functioning or planned experiments.

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