The method of displacements calculating in the joints of timber structures

A S Markovich*, M Abu Mahadi, S A Gazizova, D A Miloserdova
Peoples' Friendship University of Russia (RUDN University), Moscow, Russia
Email: markovich-as@rudn.ru

Abstract. This article is devoted to the peculiarities of calculating compliance modifications of the timber elements on mechanical linkages. Here we analyze in detail the method of C M Kochenov using in the design standards of the Russian Federation. This method allows to determine accurately the number of links that are able to resist the shear force in the linkage, however, this method does not include analyzing of shear deformation in modifications. And it is a disadvantage. In this article the author considers another approach, based on the idea of determining the shear deformation of the mechanical linkage, installed in the connection. To calculate the shear deformations of connections in the linkage it is necessary to conduct a preliminary analysis of the construction, to establish the type of connection and thickness of the connected elements, to determine the number of shear planes, the number of established joints and the level load on the joint. After determining these values, structural analysis design is performed. This article describes in detail the theoretical aspects of this method, used assumptions and limitations. It is considered a test case for validation of the considered methods. The FEM joints model is investigated. The forces and displacements of the bonds obtained by the FEM model are compared with the result of the theoretical calculation.

1. Introduction
According to the current norms of design of timber structures [1, 2], calculation of the dowel joints of wooden elements perform according to experimental-theoretical method developed in 1953 by Professor, Doctor of technical Sciences V.M. Kochenov [3]. Russian scientists have developed other practical methods of calculation in the assumption of elastic work of the dowel pin in the joint, which are given in the special literature [4-8].

The objective of current study was analyzing and comparing the result of the theoretical calculation with the result of FEM model.

2. Materials and Methods
The method of V M Kochenov is based on the idea of the elastic-plastic work of dowel pin in the joint with the following assumptions:

1. the dowel pin is considered as a thin bar of small bending stiffness, contacting with elastic base - wood of a dowel pin gainer; thus the dowel pin itself is endowed with properties of the ideal elastic-plastic material with strongly expressed areas of elastic 1-2 and plastic 2-3 behavior (figure 1), tensions on a site of plasticity are accepted for a steel dowel pin - a yield strength of steel, and for a dowel pin gainer - the design bearing resistance of timber;

2. shearing breaking strain of dowel joint is defined as the sum of elastic and plastic deformations: $u_l + u_{pl} = 2u_l$;

3. at deformation of a connection it is assumed, that the central axis of a dowel pin before reaching of a yield strength of steel remains a rectilinear and orthogonal with plane of shear, and in view of possible formation as a result of lumber shrinkage of gaps between connecting surfaces the friction between them are neglected;
4. Connection destruction is considered to be the moment of exceeding the breaking strain, therefore the breaking strain of dowel joints is limited by the norms of design and is accepted equal 1.5... 2 mm.

Guided by these assumptions, the calculation of the two shear wood dowel pin joints is to determine the minimum bearing capacity of the dowel pin \([T]\) on one plane of shear, based on three possible variants of destruction:

- crushed of the dowel pin gainer in the edge elements
  \[T_1] = 0.8cd\]

- crushed of the dowel pin gainer in the middle elements
  \[T_2] = 0.5cd\]

- and excessive deformations due to bending of the dowel pin itself
  \[T_3] = 1.8d^2 + 0.02a^2 \leq 2.5d^2\]

where \(a\) and \(c\) – the thickness of the edge and middle elements, \(d\) – the diameter of the dowel pin.

After finding the minimum bearing capacity of the dowel pin \([T]\), it is enough to simply determine the amount of dower pins, that is necessary for the perception of shearing force \(N\) by the formula:

\[n_u = \frac{N}{[T]n_s}\]  

The use of this method allows to reliably design the dowel pin joints in the case of known beforehand load.

However, in practice, especially in the structural survey and evaluation of the bearing capacity of existing timber structures, there is a need to determine the compliance of connections.

For these purposes, the following technique can be used.

The basis of this technique are the assumptions of elastoplastic behaviour and breaking strain of the joint, laid in the basis of the method of V.M. Kochenov.

To evaluate the stiffness of the whole joint, it is necessary to know the relative stiffness of each brace established in the joint. The numerical relative stiffness of \(i\)-th brace equals shear force at displacement of \(i\)-th brace on 1 mm:

\[K_{n,i} = \frac{N_i}{u_i}\]  

In view of absence of numerical description of this characteristic in Russian code, the linear stiffness should be defined according to the corresponding experimental diagrams of «\(N-u\)» tests of separate joints, or to be guided reference materials of foreign standards [10-19].

For example, according to the annex to EuroCode 5 [10], the first branch stiffness of the connection, corresponding to the elastic work of the joint, can be determined by the semi-empirical formula depending on the diameter of the dowel \(d_i\) and the density of the wood of the joined elements \(\rho\):
\[ K_{n,i} = \frac{d\sqrt{\rho^3}}{23}, \quad (6) \]

On the basis of the accepted assumption about the breaking strain of the dowel pin joint taking into account that \( u = u_i + u_{pl} = 2u_i \), the average stiffness of the connection beyond the proportional elastic limit will be equal to:

\[ K_{m,i} = 0.5K_{n,i}. \quad (7) \]

In this case the average linear stiffness of joint at installation of dowels of the same diameter will be:

\[ K_m = K_i n_u n_t, \quad (8) \]

where \( n_u \) and \( n_t \) – the number of dowel pins and plane of shear in the joint.

Then displacement of a brace in a plane of one shear fracture at uniform redistribution of shearing force \( N \) between dowels will be equal:

\[ u_i = \frac{N}{K_{m,i} n_u}. \quad (9) \]

2.1. Test example

As a test example, let's consider a simple double-shear joint (figure 2), elements of which are connected to each other by means of wooden mending plates on both sides. The joint is stretched by force \( N = 15.35 \) kN. Thickness of the connected elements \( c = 50 \) mm, overlays – \( a = 25 \) mm, the density of wood pine \( \rho = 500 \) kg/m\(^3\).

In this case the smallest bearing capacity of the joint on one shear will be determined from the bending condition of the dowel by the formula (3):

\[ [T] = 1.277 \text{ kN}. \]

To perceive the connection of tensile force \( N \) requires the installation of 6 pieces of steel dowel diameter 8 mm.

![Figure 2. Double-shear stretched joint.](image)

Now let's consider the inverse problem – we shall define a deformation capacity of the same joint on the described method.

The initial linear stiffness of the brace (dowel) according to the formula (6) is:

\[ K_{n,i} = 8 \cdot 500^{1.5}/23 \cdot 1000 = 3.888 \text{ kN/mm}. \]

The average stiffness of the brace according to the formula (7) is equal to:

\[ K_{m,i} = 0.5 \cdot 3.888 = 1.944 \text{ kN/mm}. \]

Then displacement the brace in a plane of one shear by the formula (9) will be:

\[ u_i = 15.35/1.944 \cdot 6 = 1.316 \text{ mm}. \]

As the calculation showed, the result is well correlated with the breaking strain of the dowel joint – 1.5...2 mm, accepted as a criterion of reliability in the method of V.M. Kochenov.

It should be noted that the calculation of the deformation capacity of timber joints by the current norms is not regulated.

In view of this, it is recommended to define stiffness of joints for each characteristic case on the basis of experimental researches in accordance with recommendations [9], also at determination of stiffness it is possible to be guided by reference materials foreign standards [11-23].
2.2. FEM model
Consider the FEM model of the joint shown in Fig. 2. For modeling of wood were used the massive finite elements (spatial problem of elasticity theory). Steel dowel pins with a small bending stiffness, modeled spatial bars. The calculation was performed in the program complex Lira-Sapir.

Geometry of the joint: \( a = 25 \) mm; \( c = 50 \) mm; \( h = 125 \) mm. The distance between the dowel pins \( d = 8 \) mm) was \( s_1 = s_3 = 31.25 \) mm; \( s_2 = 62.5 \) mm.

Tensile force \( N = 15.35 \) kN was exerted on the joint.

Timber was considered as an orthotropic material with the following physical and mechanical properties:
\[
\begin{align*}
E_x &= 11700000 \text{ kN/m}^2; \\
E_y &= 510000 \text{ kN/m}^2; \\
E_z &= 430000 \text{ kN/m}^2; \\
G_{xy} &= 500000 \text{ kN/m}^2; \\
G_{yz} &= 710000 \text{ kN/m}^2; \\
G_{xz} &= 1140000 \text{ kN/m}^2; \\
\nu_{xy} &= 0.03; \\
\nu_{yx} &= 0.49; \\
\nu_{xz} &= 0.41; \\
\nu_{zx} &= 0.037.
\end{align*}
\]

Deformations in the massive finite elements were calculated by formulas:
\[
\begin{align*}
\varepsilon_x &= \left( \frac{\sigma_x - \nu_{yx}\sigma_y - \nu_{zx}\sigma_z}{E_x} \right) / \varepsilon_y = \left( \frac{\sigma_y - \nu_{xy}\sigma_x}{E_y} \right) / \varepsilon_z = \frac{\sigma_z - \nu_{zx}\sigma_x}{E_z} / \varepsilon_y = \left( \frac{\sigma_y - \nu_{xy}\sigma_x}{G_{xy}} \right) / \varepsilon_z = \frac{\sigma_z - \nu_{zx}\sigma_x}{G_{xz}} \right) ; \\
\varepsilon_y &= \left( \frac{\sigma_y - \nu_{yx}\sigma_x - \nu_{zy}\sigma_y}{E_y} \right) / \varepsilon_x = \left( \frac{\sigma_z - \nu_{zy}\sigma_y}{G_{zy}} \right) / \varepsilon_z = \frac{\sigma_x - \nu_{xz}\sigma_z}{E_x} / \varepsilon_y = \left( \frac{\sigma_y - \nu_{zy}\sigma_z}{G_{zy}} \right) / \varepsilon_z = \frac{\sigma_x - \nu_{xz}\sigma_z}{G_{xz}} \right) .
\end{align*}
\] (10)

The FEM model (figure 3) included 47629 nodes, 38592 elements. The number of nodes unknown was 139779.

3. Results and discussion
As a result of FEM analysis the special features of the joint deformation were investigated (figure 4).
The maximum value of the brace displacement in the joint (figure 5) was 1.26 mm.

![Figure 5. Brace displacement in the joint.](image)

Into consideration, the value of the brace displacement on the considered theory was 1.316 mm (discrepancies in comparison with the FEM calculation – 4%).

FEM analysis showed that in the deformation of elements shear forces between braces are distributed almost evenly. The maximum value of the shear force in the dowel pin on one plane of the shear fracture was 1.27 kN (figure 6), which coincides with the analytical calculation according to the formula (3).

![Figure 6. Shear forces distribution in dowel pin.](image)

4. Conclusion
The forces and displacements of the bonds obtained by the FEM model have been compared with the result of the theoretical calculation. The results of the FEM analysis confirm the reliability of the proposed method of determining the deformation capacity of joints of timber elements on cylindrical dowel pins.

References
[1] Design Code SP 64.13330.2017 2017 Timber structures (Moscow: Standartinform)
[2] Guidelines on Designing Timber Structures to SNiP II-25-80 CNIISK im. V A Kucherenko 1986 (Moscow: Strojizdat Publ.) p 215
[3] Durability of Elements of Joints of Timber Constructions 1953 (Moscow: Gosstrojizdat Publ.) p 320
[4] Timber and Plastic constructions 1986 (Moscow: Strojizdat Publ.) p 543
[5] Rasha I K, Pertseva O N, Lazareva A Yu, Martynov G V 2017 Computational modelling of random distribution of stresses for wooden structures Magazine of Civil Engineering 69 (1) 23-33.
[6] Dmitriev P A 1973 Research of Long Durability of Joints of Timber Elements on Steel Nails Construction and Architecture 5 28-35
[7] Dmitriev P A, Strizhakov Ju D, Shvedov V N 1999 About Calculation Asymmetrical Nails Joints of Timber Elements with Steel Linings Construction (Novosibirsk: News of higher educational institutions) 5 10-15
[8] Shvedov V N, Grohotov A B 2000 Effective Method of Joints of Timber Constructions Increase of Efficiency of Rural Construction The International Collection of Works of NGAU 61-63
[9] Recommendations on Test of Joints of Timber Constructions 1980 CNIISK im. V A Kucherenko (Moscow: Strojizdat Publ.) p 40
[10] Designers’ Guide to Eurocode 5 2012 Design of Timber Buildings. EN 1995-1-1 A.J. Porteous and P. Ross. 978-0-7277-3162-3 (London: Ice publishing) p 220
[11] British Standard BS 5268-2:2002 Structural use of timber Code of practice for permissible stress design, materials and workmanship (London: BSI) p 170
[12] British Standard BS EN 408:2010+A1:2012 2012 Timber structures Structural timber and glued laminated timber Determination of some physical and mechanical properties (London: BSI) p 42
[13] Calgaro J-A, Gulvanessian H, Holicky M 2002 Basis of structural design. Designer’s guide to EN 1990 (London: Thomas Telford) p 192
[14] British Standard BS EN 912:2011 2011 Timber fasteners. Specifications for connectors for timbers (London: BSI) p 52
[15] Dias A M P G, Cruz H, Lopes S M R, Van de Kuilen J-W 2010 Stiffness of dowel-type fasteners in timber–concrete joints Proceedings of the ICE-Structures and Buildings 163 (584) 257-266
[16] British Standard BS EN 13271:2002 Timber fasteners Characteristic load-carrying capacities and slip-moduli for connector joints (London: BSI) p 18
[17] British Standard BS EN 1383:1999 Timber Structures Test Methods Pull-Through Resistance of Timber Fasteners (London: BSI) p 8
[18] Technical standard DIN 1052-2008 2008 Design of timber structures - General rules and rules for buildings (Berlin: Beuth Verlag GmbH) p 239
[19] Technical standard DIN 1052-10-2012 2012 Design of timber structures Part 10 Additional provisions (Berlin: Beuth Verlag GmbH) p 19
[20] Varenik A S Varenik K A 2014 Long bearing capacity of wooden structures Structural mechanics of engineering constructions and buildings 2 23-30
[21] Svinitsov V P, Liytov L V 2015 Experimental research on thin steel plate-to-timber joints Structural mechanics of engineering constructions and buildings 6 16-20
[22] Markovich A S 2016 To the question of stress state of joints of wooden elements with the frontal cuttings Structural mechanics of engineering constructions and buildings 4 21-25
[23] Razzakov S J, Juraev B G, Juraev E S 2018 Sustainability of walls of individual residential houses with a wooden frame RUDN Journal of engineering researches 14(5) 427-435

| Name           | Prefix    | Research Field                        | Email                  | Personal website       |
|---------------|-----------|---------------------------------------|------------------------|------------------------|
| A S Markovich | Ph.D. in  | Calculation and design of buildings and structures | markovich-as@rudn.ru   | http://engr.rudn.ru    |
| M Abu Mahadi  | Ph.D. in  | Calculation and                       | moham_d@mail.ru        | http://engr.rudn.ru    |
| Name            | Position                  | Field                          | Email            | Website                       |
|-----------------|---------------------------|-------------------------------|------------------|-------------------------------|
| S A Gazizova    | Engineering Science, Associate Professor | design of buildings and structures | s.a.gazizova@gmail.com | http://engr.rudn.ru |
| D A Miloserdova | Ph.D. student             | Calculation and design of buildings and structures | milos-dasha@yandex.ru | http://engr.rudn.ru |