Maintenance Policy in Public-Transport Involving Government Subsidy

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Abstract. A public transport with government subsidy is considered to encourage the sustainability of the transportation. The transportation revenue is determined by the maximum of the uptimes of the vehicle. In this paper, we study a one-dimensional maintenance policy for new vehicle which is characterized by age parameter. We consider that the degradation of the vehicle is affected by the age of the vehicle, and modelled by using a one-dimensional approach. The owner performs both preventive and corrective maintenance actions, and the preventive maintenance action will reduce the vehicle failure rate and hence it will decrease the corrective maintenance cost during the life time of the vehicle. The decision problem for the owner is to find the optimal preventive maintenance time of the vehicle of each subsidy option offered by maximizing the expected profit for each subsidy.

1. Introduction

As a capital city, Jakarta faces a problem with the increase of air pollution. Transportation contributes 70% pollutant in Jakarta [1]. In order to cope with this condition government tends to promote the use of public transport. Nowadays, to increase the passenger of public transport government introduces subsidy scheme. Since 2016, Indonesia’s transportations ministry give subsidy for Damri to supply buses in 11 big cities, that is Medan, Padang, Palembang, Bandar Lampung, Bandung, Yogyakarta, Surakarta, Surabaya, Denpasar, Mataram, and Makassar. With this subsidy people can have other alternative for they transport way.

This paper deals with government subsidy model for Damri. There are two subsidy models that will be studied, that is subsidy for buying buses and for reducing ticket price. We consider a non-cooperative game theory solutions to maximize the expected profit. In non-cooperative solution, manufacturer act as a leader and take policy to maximize profit first. Damri, as a follower, maximize profit based on manufacturer’s policy.

This paper is organized as follows. Section 2 gives model formulation and section 3 gives solution for the model. Section 4 gives case study to see which model is better and finally conclude in section 5.

2. Model Formulation

The following notations will be used in model formulation
• \( q \) = Number of passengers per year
• \( Q \) = Total number of passengers
• \( n \) = Bus demand
• \( PM = \) Preventive Maintenance
• \( CM = \) Corrective Maintenance
• \( K \) = Bus operating year
• \( N \) = Number of PM
• \( \delta \) = Degree of repair
• \( \lambda_0(t) \) = Failure rate without PM
• \( \lambda(t) \) = Failure rate after PM
• \( p \) = Ticket price
• \( C_m \) = Expectation total cost CM
• \( C_r \) = Cost per CM
• \( C_p \) = Expectation total cost PM
• \( C_{pr} \) = Production cost of manufacture
• \( \tau \) = PM’s period
• \( u \) = Subsidy amount per year
• \( w \) = Bus price
• \( \Psi_d \) = Damri’s profit
• \( \Psi_m \) = Manufacture’s profit
• \( \Psi \) = Total profit

In this model formulation, assumed that wholesale price of bus is determined by manufacturer and ticket price is determined by Damri. Otherwise, assumed that every failure item of the bus only need minimal repair so that time between failures is negligible [2].

2.1. Damri’s Revenue
In ceteris paribus condition the demand’s law said that if the product price increase then demand will decrease and if the product price decrease then demand will increase. In this case, if the number of passengers per year is \( q \) and ticket price is \( p \) have a linear relation, then the demand function is

\[
q(p) = \gamma_0 + \gamma_1 p \quad \text{with} \quad \gamma_1 < 0 \quad \text{and} \quad \gamma_0 > 0
\]  

(1)

The number of buses per year \( n \) can be obtained by dividing \( q \) with bus capacity \( m \) so that

\[ n = \frac{q(p)}{m} ; \quad n, m > 0. \]

If Damri operate in \( K \) year and the number of passengers per year constant, then the total number of passengers is

\[
Q(p, K) = q(p)K ; \quad K > 0
\]  

(2)

Damri’s revenue, \( R_d \), is obtained from total passengers multiplied by ticket price

\[
R_d(p, K) = Q(p, K)p
\]  

(3)

In this paper, we will use two government subsidy models. First, subsidy for buying bus from manufacturer and the second one is subsidy for reducing ticket price. For simplicity, we use index 1 for first model and 2 for second model function.

For the first model, subsidy doesn’t influence demand function so \( q(p_1) = \gamma_0 + \gamma_1 p_1 \). For the second model, subsidy amount \( u \) influence demand function so \( q(p_2, u) = \gamma_0 + \gamma_1 p_2 + \gamma_2 u \) where \( u, \gamma_2 > 0 \). Thus, based on (3) we have two revenue functions \( R_d \)

\[
R_d = \begin{cases} 
(\gamma_0 + \gamma_1 p_1) p_1 K & \text{for first subsidy model} \\
(\gamma_0 + \gamma_1 p_2 + \gamma_2 u) p_2 K & \text{for second subsidy model}
\end{cases}
\]  

(4)

And we have two number of bus functions
2.2. Damri’s Expense
The main Damri’s expense is preventive maintenance (PM), corrective maintenance (CM), and buying buses. Maintenance is purposed to reduce failure intensity.

2.2.1. Preventive Maintenance. Let Damri do $N$ times PM in $K$-year period. Then, the interval time between two PM is formulated by $\tau = (K/N + 1)$ year. According to Kijima-Type 1 model, PM turns age of bus $t$ into virtual age $v(t) < t$. Assumed every PM has the same degree of repair $0 \leq \delta \leq 1$ where $\delta = 1$ means minimal repair and $\delta = 0$ means perfect repair [3].

\[
\lambda(t) = \lambda_0(v(t)) = \theta(i\delta t + t - i\tau)
\]

If every cost PM is $C_p = a + (1 - \delta) \tau b$ then total cost for $N$ times PM is

\[
C_p = Na + N(1 - \delta) \tau b = Na + \left[ NbK(1 - \delta)/(N + 1) \right]
\]

2.2.2. Corrective Maintenance. While bus may have failure at a random time. When failures occur, bus need to be repaired. In this paper, every failure is assumed minimally repaired so that the failure intensity just the same as that just before the failure. Without any PM, failure is Non-Homogenous Poisson Process (NHPP) with failure intensity $\dot{\lambda}_0(t)$ [4][5]. After PM, failure process in interval $[i\tau, (i+1)\tau]$ for $i = 0, 1, 2, ..., N$ is still NHPP with intensity function $\dot{\lambda}(t) = \dot{\lambda}_0(v(t))$ (Kim, et al.,2004). The expected total number of failure is

\[
\sum_{i=0}^{N} \int_{i\tau}^{(i+1)\tau} \lambda_0(v(t))dt = \sum_{i=0}^{N} \int_{i\tau}^{(i+1)\tau} \theta(i\delta t + t - i\tau)dt = \frac{K^2\theta(N\delta + 1)}{2(N + 1)}
\]

If cost for every PM is $C_f$, then the expected total amount of PM $C_m$ for $K$-year operating is

\[
C_m = \left[ C_f K^2\theta(N\delta + 1)/(N + 1) \right]
\]
2.2.3. Bus Price. Another expense for Damri is bus price $w$. For first subsidy model Damri has government subsidy for buying bus, so that Damri must pay $w_1 < w$. For the second model, Damri must pay $w_2 = w$ for buying bus from manufacturer.

2.2.4. Damri’s Profit Function. Profit function is the difference between revenue (4) and expense for PM (5), CM (6), and bus price. Damri’s profit function for first subsidy model is given by

$$\Psi_{d_1} (p_1, w_1, K, N, \delta) = R_{d_1} (p_1, K) - (w_1 + C_p + C_m) n_1$$

$$= (\gamma_0 + \gamma_1 p_1) p_1 K - (w_1 + C_p + C_m) \left( \frac{\gamma_0 + \gamma_1 p_1}{m} \right)$$

(7)

And for the second model is

$$\Psi_{d_2} (p_2, w_2, K, u, N, \delta) = R_{d_2} (p_2, K) - (w_2 + C_p + C_m) n_2$$

$$= (\gamma_0 + \gamma_1 p_2 + \gamma_2 u) K p_2 - (w_2 + C_p + C_m) \left( \frac{\gamma_0 + \gamma_1 p_2 + \gamma_2 u}{m} \right)$$

(8)

2.2.5. Manufacture’s Profit Function. If the production cost for every is $c_m$ then manufacture’s profit function for the first model is

$$\Psi_{m_1} (p_1, u, w_1, K) = uK + (w_1 - C_r) n = uK + (w_1 - C_r) \left( \frac{\gamma_0 + \gamma_1 p_1}{m} \right)$$

(9)

And for second model will be

$$\Psi_{m_2} (p_2, w_2, u) = (w_2 - C_r) n = (w_2 - C_r) \left( \frac{\gamma_0 + \gamma_1 p_2 + \gamma_2 u}{m} \right)$$

(10)

3. Non-cooperative Model Solution

In the non-cooperative solution, manufacturer will act as a leader and make profit policy first. Damri will act as a follower and make profit policy based on manufacturer’s policy [6][7]. In first subsidy model, we determine ticket price $p_1$ that maximize profit function (7) by differentiating $\frac{\partial \Psi_{d_1}}{\partial p_1} = 0$ and $\frac{\partial^2 \Psi_{d_1}}{\partial p_1^2} < 0$, yields

$$p_1 = \frac{1}{2mK} (w_1 + C_p + C_m) - \frac{\gamma_0}{2\gamma_1}$$

(11)

Substitute (11) into (8), we have

$$\Psi_{m_1} (p_1, u, w_1, K) = uK + \frac{\gamma_1}{2m^2K} (w_1 - C_r) \left( w_1 + C_p + C_m + \frac{\gamma_0 mK}{\gamma_1} \right)$$

(12)

To get manufacturer’s maximum profit, determine $w_1$ so that $\frac{\partial \Psi_{m_1}}{\partial w_1} = 0$ and $\frac{\partial^2 \Psi_{m_1}}{\partial w_1^2} < 0$, yields

$$w_1 = \frac{1}{2} \left( C_r - C_p - C_m - \frac{\gamma_0 mK}{\gamma_1} \right)$$

(13)

Substitute (13) to (12), then we have manufacturer maximum profit is $\Psi_{m_1} (\text{max}) = uK - \left( \frac{\gamma_0}{8m^2K} \right) A^2$; $\gamma_1 < 0$ with $A = \left[ C_r + C_p + C_m + \left( \frac{\gamma_0 mK}{\gamma_1} \right) \right]$. For Damri’s profit, substitute (13) to (11) we have
\[ p_i = \frac{1}{4mK} \left( C_r + C_p + C_m - \frac{3\gamma_0 mK}{\gamma_1} \right) \]  

(14)

Substitute (14) to (7), we have

\[ \Psi_{d_1}^{(\text{max})} = -\left( \frac{\gamma_1}{16m^2K} \right) A^2. \]

With the same way, we can find maximum profit for Damri and manufacturer for the second model. We have manufacturer’s maximum profit is \( \Psi_{m_2}^{(\text{max})} = -\left( \frac{\gamma_1}{8m^2K} \right) B^2 \) and Damri’s maximum profit is \( \Psi_{d_2}^{(\text{max})} = -\left( \frac{\gamma_1}{16m^2K} \right) B^2 \) with

\[ B = \left[ C_r + C_p + C_m + \left( \frac{\gamma_0 mK}{\gamma_1} \right) + \left( \frac{\gamma_2 umK}{\gamma_1} \right) \right]. \]

3.1. **Proposition 1**: For the second model, Damri’s profit is better than the first model \( \Psi_{d_1} < \Psi_{d_2} \).

3.2. **Proposition 2**: Degree of repair and the number of PM that make optimum profit are \( \delta = 0 \) and

\[ N = \left( K \left( C_p + K \theta - 2b \right) / 2a \right)^{1/2} - 1 \]

with \( N \) is integer. The value \( \delta = 0 \) means Damri do perfect repair PM.

4. **Case Study**

We use bus first failure data which is collected from Damri’s maintenance workshop (see. Fig 2) and fit data with Weibull \((\alpha, \beta, \gamma)\) distribution. We apply MLE to find the estimation parameter values. By using Minitab 18, the estimation values be as follow \( \hat{\alpha} = 91.49388 \), \( \hat{\beta} = 1.31733 \), and \( \hat{\gamma} = 84.87987 \).

![Figure 2. First Failure Data with 3-parameters Weibull Distribution.](image)

Note: Levenne test, KruskallWalliis test, MIL HDBK189 test are conducted before using Goodness of fit test [6]

The failure intensity function of Weibull is \( \lambda(t) = \left( \frac{\beta}{\alpha} \right) \left( t - \gamma / \alpha \right)^{\beta-1} \). After PM, the failure intensity function become \( \lambda(v(t)) \) and the total expected failure is

\[ C_m = \alpha^{-\beta} C_f \left[ \frac{N \delta K}{2} + \frac{K}{N+1} - \gamma \right]^{-\beta} \left[ \frac{N \delta K}{2} - \gamma \right]^{-\beta}. \]

We have a linear model for the number of passengers per year \( q(p,u) = \gamma_0 + \gamma_1 p + \gamma_2 u \), where \( \gamma_0 = 1975000 \), \( \gamma_1 = -208 \) and \( \gamma_2 = 0.00006 \). If bus capacity for one year is \( m = 54000 \) passengers then we have number of buses is \( n = q(p,u) / 54000 \). Let failure intensity function of bus is Weibull with \( \alpha = \sqrt{0.5} \) and \( \beta = 2 \) so the failure intensity function become \( \lambda_0(t) = 4t \). Cost for every CM is
\[ C_f = Rp700,000, \quad \text{and for every PM is} \quad C^*_p = a + (1 - \delta) \tau b \quad \text{with} \quad a = Rp300,000, \quad \text{and} \quad b = Rp200,000. \] 

According to proposition 2, we have \( \delta = 0 \) and \( N = \left[ K(C_fK\theta - 2b) / 2a \right]^{-1/2} - 1 \) to maximize the profit. Here is results profit of Damri and manufacture for \( K = 3 \) years.

### Table 1. Expected profit for Model 1 and Model 2

| Subsidy   | Model 1      | Model 2      |
|-----------|--------------|--------------|
|           | \( \Psi_{d1} \) | \( \Psi_{m1} \) | \( \Psi_{d2} \) | \( \Psi_{m2} \) |
| Rp 150,000,000.00 | Rp 1,384,930,704.20 | Rp 3,219,861,408.39 | Rp 1,614,838,227.29 | Rp 3,229,676,454.57 |
| Rp 200,000,000.00 | Rp 1,384,930,704.20 | Rp 3,369,861,408.39 | Rp 1,622,085,442.83 | Rp 3,244,170,885.66 |
| Rp 250,000,000.00 | Rp 1,384,930,704.20 | Rp 3,519,861,408.39 | Rp 1,629,348,884.34 | Rp 3,258,697,769.68 |
| Rp 300,000,000.00 | Rp 1,384,930,704.20 | Rp 3,669,861,408.39 | Rp 1,636,628,551.81 | Rp 3,273,257,103.61 |
| Rp 350,000,000.00 | Rp 1,384,930,704.20 | Rp 3,819,861,408.39 | Rp 1,643,924,445.24 | Rp 3,287,848,890.47 |
| Rp 400,000,000.00 | Rp 1,384,930,704.20 | Rp 3,969,861,408.39 | Rp 1,651,236,564.63 | Rp 3,302,473,129.25 |
| Rp 450,000,000.00 | Rp 1,384,930,704.20 | Rp 4,119,861,408.39 | Rp 1,658,564,909.98 | Rp 3,317,129,819.96 |
| Rp 500,000,000.00 | Rp 1,384,930,704.20 | Rp 4,269,861,408.39 | Rp 1,665,909,481.29 | Rp 3,331,818,962.59 |
| Rp 550,000,000.00 | Rp 1,384,930,704.20 | Rp 4,419,861,408.39 | Rp 1,673,270,278.57 | Rp 3,346,540,557.14 |
| Rp 600,000,000.00 | Rp 1,384,930,704.20 | Rp 4,569,861,408.39 | Rp 1,680,647,301.81 | Rp 3,361,294,603.61 |

From the examples above, clearly, we can see that profit for Damri from second model is higher than first model.

### 5. Conclusion

We have studied two model government subsidies for one of the public transportations, that is bus. This subsidy is purposed to increase people’s interest in using bus. By this way, government is hoped it can decrease the traffic jam and reduce air pollutions in big city. After analyze both model, we have conclusion as follow.

a) PM can reduce failure rate and make bus operating time longer  
b) For manufacturer, government subsidy model for buying bus is better  
c) For Damri, government subsidy model for reducing ticket price is better

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