Modeling of electrical impedance tomography to detect breast cancer by finite volume methods

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Abstract. The properties of the electrical impedance of tissue are an interesting study, because changes of the electrical impedance of organs are related to physiological and pathological. Both physiological and pathological properties are strongly associated with disease information. Several experiments shown that the breast cancer has a lower impedance than the normal breast tissue. Thus, the imaging based on impedance can be used as an alternative equipment to detect the breast cancer. This research carries out by modelling of Electrical Impedance Tomography to detect the breast cancer by finite volume methods. The research includes development of a mathematical model of the electric potential field by 2D Finite Volume Method, solving the forward problem and inverse problem by linear reconstruction method. The scanning is done by 16 channel electrode with neighbors method to collect data. The scanning is performed at a frequency of 10 kHz and 100 kHz with three objects numeric includes an anomaly at the surface, an anomaly at the depth and an anomaly at the surface and at depth. The simulation has been successfully to reconstruct image of functional anomalies of the breast cancer at the surface position, the depth position or a combination of surface and the depth.

1. Introduction

Based on data Globocan 2008, it was estimated that there were 12.7 million cancer cases and 7.6 million died of cancer in 2008 in the world. Breast cancer is one type of cancers that is most often diagnosed in women and at most cause of death in women, with the number of each about 23% of the total cancer cases and 14% of the total deaths from cancer [1]. Based on data from the American Cancer and society, data showed that the number of breast cancer increases with age. The more aged women, the higher the risk for breast cancer [2]. Therefore stepping 40 years should check the condition of her breasts regularly every year [3].

The earlier the detection and treatment, more and more women with breast cancer will be saved [4]. Therefore WHO running two programs to be against breast cancer, namely education and screening program. Education means helping women manually to be able to recognize the signs of breast cancer and immediately reported to the hospital if found. While screening means early identification using medical equipment before the signs appear.

Screening requires a medical device, which capable to detect the presence of breast cancer. Some equipment has been built to detect the presence of breast cancer, such as x-ray mammography, breast ultrasound and MRI mammography [5].

Mammography is a medical instrument that is commonly used to detect breast cancer, but this equipment has several weaknesses which are often provides negative or positive information false and
the dangers posed by radiation because it uses X-rays to the breast is exposed [6]. Early detection using mammography equipment will increase the risk almost doubled if it is conducted at the age of 40 to 50 years, and this risk will continue to increase until the age of 75 years [7].

The ability of ultrasound to detect breast cancer is still inferior compared to X-ray mammography, although ultrasound capability even increasing in recent years. However, because it is much more flexible and interactive, and users can scan the desired area repeatedly from different angles, the ultrasound is often used to check for suspicious anomalies near the surface of the skin or cyst. It is generally used to check the type of tumour with diagnostic purposes or as a monitor to assist the placement of a needle biopsy [8].

MRI mammography is rarely used for the detection of breast cancer because it is not economically. In general, MRI is used only to verify the diagnosis. MRI mammography is only used for early detection when X-ray mammography is not possible to use, for example to young women who generally have dense breasts or a woman is at high risk if exposed to X-ray radiation [8].

The electrical impedance of body tissue has long been an interesting study, because changes of the electrical impedance of organs are strongly associated with physiological and pathological properties. Both physiological and pathological properties are closely related to the medical application information [9]. Some experiments shown that breast cancer has a lower impedance compared to normal breast tissue, so that the imaging based on impedance can be used as an alternative to detect breast cancer [10]. It is also supported by Zoa and Caves studies that in future electrical impedance tomography (EIT) will replace the role of X-ray mammography as a medical device for the early detection of breast cancer [11].

2. Methods

Electrical impedance tomography is the scanning technique that utilizes the electrical parameters to image the object cross section. The image is electrical impedance distribution which is one of the very important electrical parameters. Therefore, to understand the electrical impedance tomography, it would need to understand the concept includes forward problems, methods of data collection and inverse problem.

Forward Problem is the process of electrical potential calculation based on a mathematical model. If there is no current source inside the object and the distribution of electrical conductivity is known, then the distribution of electrical potential within the object will satisfy Laplace equation. Forward problem solutions can be obtained through the concept of conservation of current flow in each element of the object.

Furthermore, the current will flow and distributed to elements of its neighbor to the whole element. Thus, the potential distribution will follow the shape of the object. Determination of electric potential is obtained through the solution of equation (1) through the discretization of the object resistivity. In this study, the discretization is conducted by dividing the object into small elements rectangular.

As well as the admittivity discretization, the potential is made to be discrete into a number of specific points in the domain \( \Omega \). In 2D case, value and potential coordinates points are obtained using finite volume method through an integral approach of the surface \( S \) of the Laplace equation, thus it obtain the equation (1).

\[
\iint_S (\nabla \cdot \gamma \nabla V) dS = 0
\]  

(1)

By using divergence theorem, then apply

\[
\iint_S (\nabla \cdot \gamma \nabla V) dS = \oint_{\partial \Omega} \gamma \nabla V \cdot dl
\]  

(2)

then the left side of equation (2) can be substituted into a closed integral along the trajectory \( l \) that surrounding elements, so it can be obtained the equation (6).
\[ \oint \gamma \nabla \cdot dl = 0 \tag{3} \]

Equation (3) states the sum of the current flux which penetrates normally to four sides that surround the rectangular surface. Equation (3) is a dot product of the current flux vector \( \gamma \) with a vector trajectory \( dl \). This segment is not equal to zero if the two vectors are parallel. In other words, the current flux must penetrate perpendicular to the side of the element. Flux current flows through a potential difference between two points that form a line and cut perpendicular to the side of the border of two mutually neighboring elements. This flux flows stream as possible if the potential distribution represented as a set of potential value at the midpoint of the element. Thus, the model solutions developed through such an approach would produce a potential distribution point located at any point of the central element.

The numerical solution of the equation (3) produces equation (4).

\[ \sum \gamma \nabla \Delta l = 0 \tag{4} \]

with the current boundary \( j_{e}(i) \) (current density on the element-e and i side) as follows

\[ j_{e}(i) = \begin{cases} \text{injection}, & \text{for element in injected boundary} \\ 0, & \text{for the element in the non injected boundary} \end{cases} \]

this is a Neumann boundary. Whereas, the injection of current is done with by grounding the polarity of the current source, thus the potential on injection becomes equal to zero. This condition is a Dirichlet boundary condition.

If an element is at the coordinates \((i, j)\), as Figure 1, the numerical solution of homogeneous equation (4) on the element produces equation (5).

\[ \gamma_{ij} \left[ (V_{i+\frac{1}{2}j} - V_{ij}) \Delta y - (V_{ij} - V_{i-\frac{1}{2}j}) \Delta y + (V_{ij+\frac{1}{2}} - V_{ij}) \Delta x - (V_{ij} - V_{ij-\frac{1}{2}}) \Delta x \right] = 0 \tag{5} \]

**Figure 1.** An element with 4 potentials point neighbor

boundary with a neighbor to the right, up, left and down:

\[ \gamma_{i+1j} \left( V_{i+1j} - V_{i+\frac{1}{2}j} \right) \Delta y + \gamma_{ij} \left( V_{ij} - V_{i+\frac{1}{2}j} \right) \Delta y = 0 \tag{6} \]

\[ \gamma_{ij+1} \left( V_{ij+1} - V_{ij+\frac{1}{2}} \right) \Delta x + \gamma_{ij} \left( V_{ij} - V_{ij+\frac{1}{2}} \right) \Delta x = 0 \tag{7} \]

\[ \gamma_{i-1j} \left( V_{i-1j} - V_{i-\frac{1}{2}j} \right) \Delta y + \gamma_{ij} \left( V_{ij} - V_{i-\frac{1}{2}j} \right) \Delta y = 0 \tag{8} \]

\[ \gamma_{ij-1} \left( V_{ij-1} - V_{ij-\frac{1}{2}} \right) \Delta x + \gamma_{ij} \left( V_{ij} - V_{ij-\frac{1}{2}} \right) \Delta x = 0 \tag{9} \]

Substituting any potential limit, i.e. \( V_{i+\frac{1}{2}j}, V_{ij+\frac{1}{2}}, V_{i-\frac{1}{2}j}, \) dan \( V_{ij-\frac{1}{2}} \) into the equation (6, 7, 8 and 9), then obtained :

\[ c_{ij} V_{i+1j} + d_{ij} V_{ij+1} + e_{ij} V_{i-1j} + f_{ij} V_{ij-1} + g_{ij} V_{ij} = h_{ij} \tag{10} \]

where,
\[ c_{ij} = \left( \frac{2y_{ij}y_{i+1,j}}{y_{ij}+y_{i+1,j}} \right) \frac{\Delta y}{\Delta x} \]
\[ d_{ij} = \left( \frac{2y_{ij}y_{i+1,j}}{y_{ij}+y_{i+1,j}} \right) \frac{\Delta y}{\Delta x} \]
\[ e_{ij} = \left( \frac{2y_{ij}y_{i-1,j}}{y_{ij}+y_{i-1,j}} \right) \frac{\Delta y}{\Delta x} \]
\[ f_{ij} = \left( \frac{2y_{ij}y_{i-1,j}}{y_{ij}+y_{i-1,j}} \right) \frac{\Delta y}{\Delta x} \]
\[ g_{ij} = -\left( c_{ij} + d_{ij} + e_{ij} + f_{ij} \right) \]

Then equation (10) is applied to all elements and produces a number of linear equations that connect the potential point of an element with all four elements of the potential value of its neighbors. Through merger and rearrangement of linear equations, the potential point of each element can be arranged in the form of the vector into the equation (11).

\[
\begin{bmatrix} G \\ V \end{bmatrix}_{(N \times N)} = \begin{bmatrix} 0 \end{bmatrix}_{(N \times 1)} (11)
\]

where, \([G]\) is admittance matrix, \([V]\) is potential matrix, \([C]\) is current matrix and \(N\) is number of element.

The electrical potential can be obtained when a current density is given on an element at the object boundary. There are several ways to put the position of the electrode as current source. Some ways to inject current and to capture potential data in the collection data are namely neighboring, opposite, multi-references, and adaptive methods. The study used neighboring data collection methods with 16 electrodes are placed on the surface of the object shown in Figure 2.

![Figure 2. The methods of neighbor collection data](image)

Data collection method was conducted by injecting an electric current into a pair of adjacent electrodes then measured potential difference between electrode pairs. For example, an electric current was injected into the electrode numbers 1 and 2, and then the potential difference was measured at electrode pair number 3 and 4, 4 and 5, and so on until the electrode pair number 15 and 16. The same procedure was repeated by injecting an electric current to the electrode pair number 2 and 3, 3 and 4, and so on until the electrode pair number 16 and 1.

Inverse Problem is the process to obtain the conductivity distribution of the object from limited voltage data. Several methods of different approaches have been proposed by some researchers that can generally be grouped into two categories, namely linear and non-linear [12]. The Linear reconstruction method will produce relative image that provides information about the relative differences to the reference image which to reflect changes of physiological function [13]. Nonlinear reconstruction methods will produce static image that gives information about the distribution of the absolute conductivity. The success of the non-linear method is determined by the match between the model geometry of forward problems and potential data from experiment [14]. Nonlinear Reconstruction consumes computing time because it requires iterative process, but it will produce a more accurate image reconstruction. Linearization method assumes that the potential change boundary is a linear function of the change in conductivity [14, 15]. The linearization reconstruction methods can be written as

\[
[\delta \sigma] = (\mathbf{S}^T \mathbf{S} + \alpha I)^{-1} \mathbf{S}^T [\delta V] \]

where \([\delta V]\) is change of boundary potential, \([\mathbf{S}]\) is sensitivity matrix, \(\alpha\) is regularisation parameter, \(I\) is identity matrix and \([\delta \sigma]\) is relative conductivity. The relative imaging methods require potential data.
as a reference data. The references can be obtained from the potential data from uniform conductivity or object conductivity from different frequency $\omega$.

Potential data from uniform conductivity is very difficult to obtain, but the potential data from different frequency is allowed to be obtained. Potential data when the conductivity changed because the frequency is called imaging of difference frequency. Imaging different frequency will produce functional image that can be used for monitoring physiological. The image is only image relative, not absolute image of value $\sigma_{t,\omega}$. In this way more real for imaging applications than absolute imaging $\sigma_{t,\omega}$ in different frequency imaging, data obtained from the two conditions of different frequencies to produce an image of change admittance. This way will effectively cancel the error together. Imaging of difference frequency has shown potential to produce a functional image in several medical applications.

Review comprehensively [16] and measurement [17] electrical parameters (electrical permittivity and conductivity) was conducted on normal breast and breast cancer based on different frequency. From the review shows that with different frequencies obtained different electrical parameter between normal breast and breast cancer significantly. There is information declare that the admittance of various biological tissues show a change with frequency [18, 19], so that imaging based on the difference frequency promises to generate an image of admittance distribution.

Therefore, the purpose of this research is to design, solve the mathematical model, namely forward and inverse problems and design 2D multi-frequency electrical impedance tomography which can be used to detect early breast cancer.

Electrical parameters of body tissue varies with frequency. This difference causes a change an electrical conductivity of body tissue. The breast is a tissue that change its electrical resistivity when measured at different frequencies. the study assume that the breast tissue is homogeneous, it contains anomalies, namely cancer. Here is the electrical conductivity of breast tissue used.

| Tabel 1. The conductivity of breast tissue [20] |
|-----------------------------------------------|
| Conductivity of breast tissue (mS/cm)         | 10 kHz | 100 kHz | 10 MHz |
| Tumor (inside)                                | 3.98   | 4.28    | 7.24   |
| Tumor (tissue surface)                       | 1.68   | 1.70    | 2.70   |
| Fat with tumor cells                          | 0.65   | 0.67    | 1.05   |
| The tissue that away from tumor               | 0.28   | 0.31    | 0.37   |
| Normal tissue                                 | 0.22   | 0.22    | 0.25   |

The numerical phantom is 2D numerical object that represents a rectangular shaped breasts in which there are anomalies with the size of the two elements. Each numerical objects represents a condition of one anomaly in a depth, one anomaly on surface and two anomalies in a depth and on surface with two different frequencies, namely 10 kHz and 100 kHz. The Breast numerical object is made based on the data Table 1, so that obtained the image 3.

![Figure 3. The breast numerical data with two element anomalies at a 10 kHz and 100 kHz](image-url)
The study includes a development of a mathematical model of the electric potential field method 2D Finite Volume Method (FVM), solve forward problem and solve inverse problem by linear methods, developing the algorithm and program code to solve forward and inverse problems.

3. Results

Simulation was conducted by making the program code to solve the forward problem to obtain voltage data on the numerical object. The simulation used a numerical object of breast model, it shaped rectangular with neighbore method to collect data. Based on equation (13) and (14) code program has been compiled to solve forward problem by the finite volume method. The complete scan data using 16 channels in the numerical object of normal breast without anomaly was shown in Figure 7. The Scanning data in Figure 7 if taken at the surface where the electrodes position in the normal breast, containing anomalies at surface, depth and surface and depth at a frequency 10 kHz and 100 kHz, it will be resulting the voltage data shown in Figure 8.

![Figure 4. The complete Solution of forward problem of volume element methods on the rectangular uniform object](image)

(a) (b)
Figure 5. The scanning data from 16 channels electrode by neighboring methods on uniform media and anomaly numerical object at 10 kHz and 100 kHz (a) surface anomaly (b) depth anomaly (c) surface and depth anomaly.

Visually, Figure 6 does not show differences in the data between uniform and anomalies medium at 10 kHz and 100 kHz, but numerically has RMSE 0.188% and 0.189% toward the reference.

Based on equation (20), has been compiled a back projection program code to reconstruct voltage data in Figure 5 to obtain the image reconstruction in Figure 6. During the reconstruction process, the regularization parameters need to be given to obtain stable results. Regularization parameter selected by trial and error, to obtain optimal image reconstruction. The simulation voltage data from forward problem solution at Figure 5 then is reconstructed using back projection method with $\alpha = 5 \times 10^{-5}$. By using data 10 kHz and a uniform object as a reference, so that obtained Figure 6.

Figure 6. Reconstruction image from data $V_f = 10$ kHz and $V_{ref} = \text{uniform object}$ by neighboring methods (a) surface anomaly (b) depth anomaly (c) surface and depth anomaly

The simulation voltage data from forward problem solution at Figure 5 then is reconstructed using back projection method with $\alpha = 5 \times 10^{-4}$. By using data 100 kHz and data 10 kHz as references, Figure 7 is obtained. In Figure 7, the simulations have been able to reconstruct the image of functional anomalies of breast cancer at the surface, depth or a combination of surface and depth.

Figure 7. Reconstruction image from data $V_f = 100$ kHz and $V_{ref} = 10$ kHz by neighboring methods (a) surface anomaly (b) depth anomaly (c) surface and depth anomaly

4. Conclusion
It has conducted a modelling of Electrical Impedance Tomography to detect breast cancer by Finite Volume Methods (FVM). The research includes the development of a mathematical model of the electrical potential field by 2D Finite Volume Method, solving forward problem and inverse problem by linear method. Scanning is done by using 16 channels electrode by neighbor data collection method. Scanning is conducted at 10 kHz and 100 kHz with three anomaly objects. They are an anomaly in the depths, an anomaly at the surface and an anomaly at depth and surface. In simulations have successfully reconstructed image of functional anomalies of breast cancer at the surface position, depth position, or a combination surface and depth position.

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