Metastability of quantum droplet clusters

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We show that metastable ring-shaped clusters can be constructed from two-dimensional quantum droplets in systems described by the Gross-Pitaevskii equations augmented with Lee-Huang-Yang quantum corrections. The clusters exhibit dynamical behaviours ranging from contraction to rotation with simultaneous periodic pulsations, or expansion, depending on the initial radius of the necklace pattern and phase shift between adjacent quantum droplets. We show that, using an energy-minimization analysis, one can predict equilibrium values of the cluster radius that correspond to rotation without radial pulsations. In such a regime, the clusters evolve as metastable states, withstanding abrupt variations in the underlying scattering lengths and keeping their azimuthal symmetry in the course of evolution, even in the presence of considerable perturbations.

PhySH Subject Headings: Solitons; Superfluids; Mixtures of atomic and/or molecular quantum gases

Among the variety of possible self-trapped states in two-dimensional (2D) and three-dimensional (3D) nonlinear media [1-3], necklace patterns, built as clusters of solitons, pose a challenge due to their structural fragility. Such patterns have been analysed theoretically in diverse models [4-11] and experimentally in Kerr optical media [12]. An inherent feature of necklace clusters is orbital angular momentum, which can be imparted to them [6,9] to induce rotation of the cluster accompanying its radial evolution [8]. Various necklace patterns have been studied in two-component systems [7,9,10]. In most cases clusters are not steady states, as interactions between adjacent solitons induce forces which drive radial oscillations or the eventual self-destruction of the clusters. Indeed, being higher-order states, they are highly prone to instabilities that lead to their splitting into a set of fragments, often through fusion of the initial constituents. Therefore, exploration of physical mechanisms that can support and eventually stabilize such states, which occupy the intermediate niche between multipoles and vortex solitons, is of importance for different fields, including nonlinear optics [4-12] and atomic physics [13,14].

It has been theoretically shown that the decay of clusters is slowed down by competing nonlinearities, such as those represented by a combination of focusing quadratic and defocusing cubic terms, but such conditions are yet to be realized experimentally [10]. Necklaces carrying orbital angular momentum have been explored in models of nonlinear dissipative media based on complex Ginzburg-Landau equations. In that case, the clusters, instead of expansion, may undergo fusion into stable vortex rings [15]. Another possibility for the stabilization of clusters is provided by external potentials, including spatially periodic ones [16]. Such potentials determine the symmetry of the cluster and restrict its motion, by arresting, in particular, radial oscillations. Effective stabilization of necklace patterns was predicted and experimentally explored in nonlocal optical media [17], and also predicted in Bose-Einstein condensates (BECs) with long-range interactions [18]. However, with this exception, all stabilization schemes are awaiting experimental observation, in part due to the difficulty to identify systems, where complex multidimensional states may exist as robust objects.

Recently, a landmark advance in the field has been achieved with the advent of quantum droplets (QDs), which were predicted [19] as stable soliton-like states in two-component BECs, with the intercomponent attraction made slightly stronger than the intra-component repulsion. A key effect in the system is the Lee-Huang-Yang (LHY) correction to the mean-field approximation, induced by quantum fluctuations [20]. This is a remarkable example of the creation of stable nonlinear objects by competing nonlinearities: cubic mean-field attraction and LHY-induced quartic repulsion. The prediction was followed by the experimental creation of quasi-2D droplets, strongly compressed in one direction [21,22], and 3D isotropic ones [23]. The experiments made use of the Feshbach resonance (FR) to tune interactions between atoms in two hyperfine states [24], so as to make the inter- and intra-species interactions nearly equal in magnitude but opposite in sign. The remaining small imbalance in favor of the inter-species attraction may be used to adjust the relative strength of the LHY corrections and normal cubic terms [21-23]. QDs maintained by the interplay of the LHY terms and long-range attraction in dipolar BEC may be stable as well [25-27]. Formation of droplet crystals in dipolar condensates supported by a harmonic-oscillator trap was recently predicted too [28].

The experimentally relevant [21,22] reduction of the dimension from 3D to 2D changes the form of the LHY terms, replacing the above-mentioned cubic-quartic combination by the cubic terms multiplied by a logarithmic factor [29,30], which implies switching from self-attraction to repulsion with the increase of the density. Such nonlinearity supports stable 2D solitons, including ones with embedded vorticity, \( m \), which may be stable up to \( m = 5 \) [30], while 3D vortex solitons maintained by the cubic-quartic nonlinearity may be stable for \( m = 1, 2 \) [31]. All vortex solitons supported by the interplay of the long-range dipole interaction and LHY terms are unstable [32].

The goal of this Letter is to show that the 2D geometry allows formation of necklace clusters composed of fundamental droplets with properly selected phase differences, without confining potentials. This prediction opens the way to experimental creation of elusive complex self-sustained states in BEC, which, \( \textit{inter alia} \), exhibit stable oscillations and robust spiralling in free space.

We address the system of coupled Gross-Pitaevskii equations for wave functions \( \psi_{1,2}(x,y,t) \) of two components of a binary BEC that builds QDs in two dimensions [29,30]:
\[ \frac{i}{\hbar} \frac{\partial \psi_{1,2}}{\partial t} = -\frac{1}{2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \psi_{1,2} + \left( |\psi_{1,2}^1|^2 - |\psi_{1,2}^2|^2 \right) \psi_{1,2} + \alpha \left( |\psi_{1,2}^1|^2 + |\psi_{1,2}^2|^2 \right) \ln \left( |\psi_{1,2}^1|^2 + |\psi_{1,2}^2|^2 \right) \] (I)

\[
\psi_{1,2} = u_{1,2}(r) e^{i m \phi / 2 - i \phi \theta},
\]

where polar coordinates are defined around the centre of a given QD. We look for symmetric stationary states, with real \( u_1 = u_2 \equiv u \), vorticities \( \mu_1,2 \equiv \mu \), and real chemical potentials \( \mu_1,2 \equiv \mu \). The profiles \( u(r) \) of such states, found by means of the Newton method, are not affected by the difference cubic terms in Eq. (I), which allows to set \( \alpha = 1 \). However, when testing the stability analysis of perturbed states we keep the difference terms in Eq. (I), in order to test the stability of the stationary states against symmetry-breaking perturbations.

\[ E = \frac{1}{2} \int \left( |\nabla \psi_{1,2}^1|^2 + |\nabla \psi_{1,2}^2|^2 + |\psi_{1,2}^1|^4 + |\psi_{1,2}^2|^4 - 2 |\psi_{1,2}^1|^2 |\psi_{1,2}^2|^2 + \alpha (|\psi_{1,2}^1|^2 + |\psi_{1,2}^2|^2)^2 \ln (|\psi_{1,2}^1|^2 + |\psi_{1,2}^2|^2 - 1)^2 \right) dxdy, \]

\[ \mathcal{L} = \iint \left( \rho \times (\psi_{1,2}^1 \nabla \psi_{1,2}^1 - \psi_{1,2}^2 \nabla \psi_{1,2}^2 + \psi_{1,2}^1 \nabla \psi_{1,2}^1 + \psi_{1,2}^2 \nabla \psi_{1,2}^2) \right) dxdy, \]

where \( \rho = (x,y,z) \). We aim to construct ring-shaped clusters as sets of identical QDs. First we elucidate properties of the “building blocks”, i.e., fundamental and vortex QDs, generated by Eq. (I) as \( \psi_{1,2} = u_{1,2}(r) e^{i m \phi / 2 - i \phi \theta} \), where polar coordinates \( (r, \phi) \) are defined around the centre of a given QD. We look for symmetric stationary states, with real \( u_1 = u_2 \equiv u \), vorticities \( \mu_1,2 \equiv \mu \), and real chemical potentials \( \mu_1,2 \equiv \mu \). The profiles \( u(r) \) of such states, found by means of the Newton method, are not affected by the difference cubic terms in Eq. (I), which allows to set \( \alpha = 1 \). However, when testing the stability analysis of perturbed states we keep the difference terms in Eq. (I), in order to test the stability of the stationary states against symmetry-breaking perturbations.

}\[ \psi_{1,2} \mid_{t=0} = \sum_{k=-N}^{N} u_{1,2}(|r - r_k| e^{2\pi ik/N}), \] (3)

where \( r_k = \{ R \cos(2\pi k/N), R \sin(2\pi k/N) \} \) is the position of the centre of the \( k \)-th droplet, and \( 2\pi M / N \) is the phase difference between adjacent ones. While \( M \) may be considered as the cluster’s vorticity, the phase varies along the necklaces as a step-like function, making them different from ordinary vortex states. Actually, \( M \) is a key factor controlling the evolution of the cluster, as it determines interaction forces between adjacent QDs via phase difference \( \theta = 2 \pi M / N \) between them. Namely, for \( M \leq N/4 \) and \( M \geq 3N/4 \), the interaction between adjacent QDs is attractive, corresponding to \( \theta \leq \pi/2 \) or \( \theta \geq 3\pi/2 \), hence the net force, pointing at the centre, leads to an initial shrinkage of the cluster in the radial direction. For \( N/4 < M < 3N/4 \), the phase difference between adjacent QDs takes values \( \pi/2 < \theta < 3\pi/2 \), giving rise to a repulsive net force in the radial direction, that causes expansion of the cluster. Moreover, at \( M = 0 \) and \( M = N/2 \) the cluster carries nonzero angular momentum \( \mathcal{L} \), hence its radial contraction/ expansion is accompanied by rotation.

The input state \( (3) \) may be prepared by slicing the BEC in the trap by several narrow repulsive barriers, similar to how it was done for two droplets [35]. Phase-imprinting techniques, implemented by means of a far-detuned broad vortical beam [36], may be used to imprint the required phases onto the droplets forming the cluster.
One can estimate the expected evolution regimes of the clusters from the dependence of energy $E$ on the initial cluster’s radius, $R$, which is shown in Fig. 2 for different numbers $N$ of QDs in the cluster. These dependencies reveal that at $M \leq N/4$ an energy minimum exists at a specific initial radius $R_{\text{min}}$. The state realizing the energy minimum is a favorable one, hence it should exhibit minimal shape changes in the course of evolution. Clusters with $R > R_{\text{min}}$ perform nearly periodic oscillations [Figs. 3(a,b)], clearly seen in the time dependence of the cluster radius, $w = U^{-1} \int r(\psi_1^2 + \psi_2^2) dx dy$, in Fig. 3(b). For large $R$, corresponding to the region, where $E$ is nearly constant, the oscillation period is very large too. For $M = 0$, conspicuous emission of radiation occurs at the point of the maximal contraction of the cluster, gradually transforming it into a nearly flat-top and practically non-radiating pattern, that inherits the initial azimuthal structure [Fig. 3(a)]. At $\mathcal{L} = 0$, the radial oscillations are accompanied by rotation of the cluster. Clusters with $R < R_{\text{min}}$ initially expand and then also show oscillations (see an example in Fig. 5(a) at $t > 200$).

The central result of this Letter is the robustness of the quasistationary QD clusters in comparison with media with the usual cubic nonlinearity, where clusters are strongly unstable [12]. To elucidate their robustness, we added different random perturbations (up to 5% in amplitude) to the initial profiles of the two components. After performing several cycles of radial oscillations, the perturbed clusters with $R$ essentially different from $R_{\text{min}}$, split into several wide droplets, a phenomenon that is usually accompanied by a strong growth of the width, as shown by the red curve in the top row of Fig. 3(b). On the contrary, clusters with initial radii $R \approx R_{\text{min}}$ are robust against the perturbations, surviving up to $t > 2000$, as seen in the top panel of Fig. 3(c). Typical metastable clusters are shown in Fig. 4, with their radius increasing with $N$. We have verified that the minima in the $E(R)$ dependencies in Fig. 2 provide an accurate prediction of the clusters’ evolution, gradually transforming it into a nearly flat-top and practically non-radiating pattern, that inherits the initial azimuthal structure [Fig. 3(a)]. At $\mathcal{L} = 0$, the radial oscillations are accompanied by rotation of the cluster. Clusters with $R < R_{\text{min}}$ initially expand and then also show oscillations (see an example in Fig. 5(a) at $t > 200$).

The numerically found optimal values of the radius were used in all simulations of the evolution of the clusters displayed in Fig. 4. Most robust are ones with $N = 5$ [Fig. 4(a)] and $N = 6$ [Fig. 4(c)]. For other states, the evolution time over which the perturbed cluster survives usually decreases with $N$. All clusters in Fig. 4 exhibit rotation, which is slower than small radial pulsations in the $w(t)$ dependencies. A representative rotation period of the $N = 5$ cluster in Fig. 4(a) is $T \approx 245$. The present results persist in the presence of weak harmonic confinement in the $(x,y)$ plane, that may even enrich families of metastable clusters [37].

In addition to the stability of the clusters against initial perturbations, we have tested their robustness with respect to variation of parameters. In particular, this may be abrupt or smooth change of the nonlinearity strength, imposed by FR. To this end, we took a robust cluster with $N = 5$ and checked how a change of $\alpha$ in Eq. (1) at
\[ t \geq 200 \] affects the evolution. While for smooth variation of \( \alpha \) the cluster's radius adapts to it, an abrupt jump from \( \alpha = 1 \) to a smaller value entails noticeable radial oscillations, with the minimal radius being determined by the initial one [Fig. 5(a)]. If \( \alpha \) increases, the cluster shrinks instead, with the initial radius determining the maximal radius of subsequent oscillations [Fig. 5(b)]. The dependence of the maximal and minimal radii of the cluster on the final value of \( \alpha \) is displayed in Fig. 6(a). Note that the so perturbed clusters survive even if their radii undergo more than a double increase in the course of oscillations. There is a lower bound on the possible variation of \( \alpha \), since at \( \alpha < 0.2 \) the cluster expands indefinitely instead of performing radial oscillations, but no upper bound exists. The oscillation period monotonically decreases with \( \alpha \) [Fig. 6(b)].

We conclude that remarkably robust, albeit, rigorously speaking, metastable ring-shaped clusters built of QDs may be formed under suitable conditions in binary BEC in 2D geometries, where the LHY corrections to the mean-field dynamics provide the stabilization. The evolution of the clusters is determined by their initial radius and vorticity. The clusters selected by the energy-minimum principle withstand strong perturbations, such as the variation of the nonlinearity strength.

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