HEAVY-TO-HEAVY AND HEAVY-TO-LIGHT WEAK DECAY FORM FACTORS IN THE LIGHT-FRONT APPROACH: THE EXCLUSIVE $0^-$ TO $0^-$ CASE

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Abstract

Weak transition form factors among heavy pseudoscalar mesons are investigated within a relativistic quark model formulated on the light-front. It is shown that the light-front result derived in the time-like region for the matrix elements of the plus component of the weak vector current coincides with the spectator pole term of the quark triangle diagram. For the first time, the dependence of the form factors on the squared four-momentum transfer $q^2$ is calculated in the whole accessible kinematical region $0 \leq q^2 \leq q_{\text{max}}^2$. For the numerical investigations of the semileptonic $B \to D \ell \nu_\ell$, $B \to \pi \ell \nu_\ell$, $D \to K \ell \nu_\ell$ and $D \to \pi \ell \nu_\ell$, the equal-time wave functions corresponding to the updated version of the ISGW model are adopted. Our results for the form factors and the decay rates are presented and compared with available experimental data and predictions of different approaches. Moreover, the $K^0_L \to \pi^\pm e^\mp \nu_e$ decay is briefly discussed. Our approach is consistent with experimental data.

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1 Introduction

Semileptonic heavy-quark decays play an important role for the understanding of weak and strong interactions. These decays proceed via the spectator mechanism, in which the heavy-quark decays into another heavy or a light quark by emitting a W-boson, which materializes into a lepton pair. The decay amplitudes are given by the product of the leptonic and hadronic $V - A$ currents. Since only two quarks are in the final state, no interfering diagrams and no effects from the final state interactions should be taken into account, at variance with the case of non-leptonic decays. The matrix elements of the hadronic current are determined by the bound state properties of the initial and final hadrons and, therefore, they can provide relevant information on the internal structure of heavy hadrons. The knowledge of the weak hadron form factors, combined with measurements, allows to determine some fundamental quantities of the Standard Model, the Cabibbo-Kobayashi-Maskawa (CKM) parameters. The knowledge of the hadron matrix elements is also of interest for estimating the non-leptonic decays of heavy hadrons. A theoretical challenge consists in the appropriate description of the non-perturbative aspects of the strong interaction. The non-perturbative approaches, commonly applied in this field, include the Heavy Quark Effective Theory (HQET), lattice QCD calculations, the QCD sum rules (SR) and constituent quark models (CQM).

In order to calculate the decay rates and to determine the CKM parameters from experiment, it is necessary to know both the normalization and the dependence of the hadron form factors upon the squared four-momentum transfer $q^2$. There are few lattice QCD calculations of the $B \to D$ transitions in the limit of infinite $b$ and $c$ quark masses [1], while there are various QCD SR predictions [2-5], which, however, do not cover the full range of $q^2$. It should be stressed that even within the CQM the evaluation of the $q^2$-dependence of the form factors in the whole kinematical region accessible in semileptonic decays have not yet been performed. Usually, the form factors are calculated at a fixed value of $q^2$ appropriate for the specific model and, then, extrapolated to the full range of $q^2$. In the original BSW approach [6] the form factors are calculated at $q^2 = 0$ and extrapolated to $q^2 > 0$ using a monopole form factor $(1 - q^2/M^2_{pole})^{-1}$, where $M_{pole}$ is the mass of the lowest-lying meson resonance relevant in the given decay channel. Within the ISGW model [7,8] the form factors are calculated at the point of zero recoil (i.e., $q^2 = q^2_{max}$) and, then, extrapolated to $q^2 = 0$ assuming an exponential-like behaviour of the form factor. A relativistic CQM model due to Jaus [9] uses the light-front (LF) formalism to compute the form factors for space-like values of the momentum transfer (i.e., $q^2 \leq 0$). Then, the form factors are extrapolated to the time-like region $q^2 > 0$ assuming a particular two-parameter formula, which reproduces the values of the form factors and their first two derivatives at $q^2 = 0$.

In this paper, we adopt a relativistic CQM and, for the first time, we present the results of the calculation of the $q^2$ dependence of weak transition form factors among pseudoscalar mesons in the whole accessible kinematical region. As in ref. [9], our approach is based on the LF formalism, which allows a proper treatment of the effects of both relativistic composition of quark spins and center-of-mass recoil. However, instead of the Breit frame, which is appropriate only for space-like values of $q^2$, our calculations are performed in a
reference frame where the momentum transfer is purely longitudinal, which is appropriate for time-like values of \( q^2 \). The relevance of such a frame for the calculation of the Isgur-Wise function has been stressed in ref. [10].

The plan of the paper is as follows. In Section 2 we describe our approach for the calculation of the weak pseudoscalar meson form factors. We evaluate the triangle diagram for the weak vector current, where the integration over the light-front energy for the plus component of the current is the essential step. Then, we compare this result with the one obtained within the Hamiltonian light-front dynamics. We show that the latter coincides with the contribution of the spectator pole in the quark triangle diagram. In Section 3 our results for the form factors and the exclusive decay rates of the \( B \to D \ell \nu \ell \), \( B \to \pi \ell \nu \ell \), \( D \to K \ell \nu \ell \) and \( D \to \pi \ell \nu \ell \) weak decays, are presented and compared with the experimental data as well as with results of different approaches. Moreover, the \( K^0_L \to \pi^\pm e^\mp \nu e \) decay is briefly discussed. Finally, our conclusions are summarized in Sect. 4.

## 2 Weak decay vector form factor

### 2.1 Kinematics

Here below, we denote by \( P_1, P_2 \) and \( M_1, M_2 \) the 4-momenta and masses of the parent and daughter mesons, respectively. The four-momentum transfer \( q \) is given by \( q = P_1 - P_2 \) and the invariant \( y \) is defined as

\[
y = \frac{P_2^+}{P_1^+} = 1 - \frac{q^+}{P_1^+}
\]  

We work in a frame where the momentum transfer is such that \( q_\perp = 0 \). Thus, it can be easily checked that

\[
q^2 = (1 - y) \left( M_1^2 - \frac{M_2^2}{y} \right)
\]

yielding two solutions for the variable \( y \), viz.

\[
y_{1,2} = \zeta \left( \eta \pm \sqrt{\eta^2 - 1} \right)
\]

where \( \zeta \equiv M_2/M_1 \) and the "velocity transfer" \( \eta \) is defined as \( \eta = U_1 \cdot U_2 = P_1 \cdot P_2/(M_1 M_2) \), with \( U_1 \) and \( U_2 \) being the four-velocities of the parent and daughter mesons. The two signs in eq. (3) correspond to whether one chooses the 3-momentum \( p_2 \) of the final meson to be in the positive or negative direction of the 3-axis. As is well known, the relation between the kinematical variables \( \eta \) and \( q^2 \) is given by

\[
q^2 = M_1^2(1 + \zeta^2 - 2\zeta \eta)
\]

At the point of zero recoil, i.e. \( q^2 = q^2_{\text{max}} = (M_1 - M_2)^2 \) corresponding to \( \eta = 1 \), one has \( y_{1,2}(q^2_{\text{max}}) = \zeta \), while at \( q^2 = 0 \), corresponding to \( \eta = \eta_{\text{max}} \equiv \frac{1}{2}(\zeta + \frac{1}{\zeta}) \), one gets \( y_1(0) = 1 \) and \( y_2(0) = \zeta^2 \).
In this paper we are interested in the weak decay of a pseudoscalar meson into another pseudoscalar meson. We define the form factors of the $P_1(Q_1q) \to P_2(Q_2q)$ transition between two pseudoscalar mesons in the usual way, viz.

\[
\langle P_2 | \bar{Q}_2 \gamma_\mu Q_1 | P_1 \rangle = \sqrt{M_1 M_2} \left[ h_+(q^2) (U_1 + U_2)_\mu + h_-(q^2) (U_1 - U_2)_\mu \right]
\]

\[
= f_+(q^2) (P_1 + P_2)_\mu + f_-(q^2) (P_1 - P_2)_\mu
\]

where $J_\mu = \bar{Q}_2 \gamma_\mu Q_1$ is the weak vector current. Let us remind that the term containing the form factor $f_-(q^2)$ in eq. (5) yields a contribution to the semileptonic decay rate proportional to the lepton mass and, therefore, it does not contribute to the transition amplitude, except in case of the heavy $\tau$ lepton and $K_{\mu3}$ decay. The relationship between the two sets of form factors is given by

\[
f_\pm(q^2) = \frac{1}{2\sqrt{M_1 M_2}} \left( (M_1 + M_2) h_\pm(q^2) - (M_1 - M_2) h_\mp(q^2) \right)
\]

(7)

Another set of form factors can be introduced, viz.

\[
\langle P_2 | \bar{Q}_2 \gamma_\mu Q_1 | P_1 \rangle = F_1(q^2) \left( P_1 + P_2 - \frac{M_1^2 - M_2^2}{q^2} q_\mu \right) + F_0(q^2) \frac{M_1^2 - M_2^2}{q^2} q_\mu
\]

(8)

with

\[
F_1(q^2) = f_+(q^2)
\]

(9)

\[
F_0(q^2) = f_+(q^2) + \frac{q^2}{M_1^2 - M_2^2} f_-(q^2)
\]

The physical meaning of these new form factors is clear in the helicity basis, where the form factors $F_1$ and $F_0$ are associated with the transition amplitudes corresponding to the exchange of a vector $(1^-)$ and a scalar $(0^+)$ boson in the t-channel. However, the masses of heavy scalar mesons are poorly known. Therefore, in what follows, the form factors $f_\pm(q^2)$ within the assumption of pole dominance are constructed in a much simpler way as follows

\[
f_\pm(q^2) = \frac{f_\pm(0)}{1 - q^2/M_{1^-}^2}
\]

(10)

where $M_{1^-}$ denotes the mass of the lowest-lying vector $\bar{Q}_2 Q_1$ meson. This simple picture has clear limitations that will be addressed in Section 3.

Neglecting the lepton masses, the rate $\Gamma$ for semileptonic decays reads as

\[
\Gamma = \frac{16}{3} \Gamma_0 |V_{Q_1Q_2}|^2 \zeta^4 \int_1^{\zeta + \frac{1}{4}} d\eta (\eta^2 - 1)^{3/2} f_\mp^2(\eta)
\]

(11)

where $\Gamma_0 = G_F^2 M_1^3 / (4\pi^3)$ and $V_{Q_1Q_2}$ is the relevant CKM matrix element.
2.2 The Feynman triangle diagram

The LF wave function is defined on the plane \(x^0 + x^3 = 0\) and can be obtained from the Bethe-Salpeter amplitude by performing the integration over the LF “energy” \(k^- = k_0 - k_3\) (see, e.g., ref. [9]). As a matter of fact, in ref. [11] the pion charge form factor has been evaluated starting from the quark triangle diagram in a Breit frame where \(q^+ \equiv q_0 + q_3 = 0\), and the integration over \(k^-\) in the loop integral allowed the identification of the LF wave function. Moreover, it is known [12,13] that, when \(q^+ = 0\), the use of the so-called “good” component of the current \(J^+ \equiv J^0 + J^3\) allows to suppress the effects of quark pair creation from the vacuum. In this way the quark triangle diagram is equivalent to the result which can be obtained within the Hamiltonian LF dynamics [14]. In this subsection we evaluate the quark-triangle diagram for the weak vector current, using a frame where the momentum transfer is purely longitudinal, i.e., \(q_{\perp} = 0\) and \(q^+ > 0\), as it is appropriate for time-like values of \(q^2\). We find two contributions, which will be referred to as the partonic and non-partonic ones. In the next subsection we will show that the partonic term coincides with the result obtained using the Hamiltonian LF approach. The non-partonic contribution corresponds to the quark pair creation diagram and its calculation will not be addressed in this paper. An estimate made in ref. [10] shows that the non-partonic contribution may be expected to be negligible for heavy-quark decays. However, its contribution might become more relevant for kaon decays, as it will be highlighted in Section 3.

Assuming a constant vertex function \(\Lambda\), the quark triangle diagram, depicted in Fig. 1, yields (cf. ref. [9])

\[
< P_2 | \bar{Q}_2 \gamma_\mu Q_1 | P_1 > = iN_c \Lambda_1 \Lambda_2 \int \frac{d^4k}{(2\pi)^4} \cdot 
\]

\[
Sp \left[ \frac{-\hat{k} + m}{k^2 - m^2 + i\varepsilon \gamma_5 (P_2 - k)^2 - m_2^2 + i\varepsilon \gamma_\mu (P_1 - k)^2 - m_1^2 + i\varepsilon \gamma_5} \right] \]

where \(p_1 \equiv P_1 - k\) and \(p_2 \equiv P_2 - k\) are the four-momenta of the active quarks, with \(k\) being the four-momentum of the spectator quark. In eq. (12) \(m_1\) and \(m_2\) are the masses of the active quarks, whereas \(m\) is the mass of the spectator quark; finally, \(N_c\) is the number of colours. After changing the variables from \((k_0, k)\) to \((k^-, k^+, k_{\perp})\) with \(k^\pm \equiv k_0 \pm k_3\), one obtains

\[
< P_2 | \bar{Q}_2 \gamma_\mu Q_1 | P_1 > = iN_c \Lambda_1 \Lambda_2 \frac{d^3k_{\perp}}{2(2\pi)^4} \int \frac{dk^-dk^+d^2k_{\perp}}{k^+(k^+ - P_2^+)(k^+ - P_1^+)} I_\mu \cdot
\]

\[
\left\{ (k^- - \frac{m_{\perp}^2 - i\varepsilon}{k^+})(k^- - P_2^- - \frac{m_{\perp}^2 - i\varepsilon}{k^+ - P_2^+})(k^- - P_1^- - \frac{m_{\perp}^2 - i\varepsilon}{k^+ - P_1^+}) \right\}^{-1}
\]

where \(m_{\perp}^2 = k_{\perp}^2 + m_i^2\) \((i = 1, 2)\), \(m_{\perp}^2 = k_{\perp}^2 + m^2\) and \(I_\mu = Sp((\hat{k} + m)(\hat{P}_2 - \hat{k} + m_2)\gamma_\mu(\hat{P}_1 - \hat{k} + m_1))\). For sake of simplicity, in eq. (13) and in what follows, \(P_{1\perp} = P_{2\perp} = 0\) is assumed.

Let us now calculate eq. (14) for the plus component of the weak vector current, because for this component the \(k^-\)-integration is convergent. Applying the Cauchy theorem,
four different cases should be analyzed: \( k^+ < 0 \), \( 0 < k^+ < P_2^+ \), \( P_2^+ < k^+ < P_1^+ \) and \( k^+ > P_1^+ \). The first and fourth cases do not contribute to the integral over \( k^- \), because the three poles in eq. (13) have imaginary parts with the same sign. It can be easily seen that the only surviving contributions come from the regions \( 0 < k^+ < P_2^+ \) (i.e., \( 0 < x < y \)) and \( P_2^+ < k^+ < P_1^+ \) (i.e., \( y < x < 1 \)), where \( x \equiv k^+/P_1^+ \) is the LF momentum fraction of the spectator quark in the parent meson. The first region corresponds to the so-called spectator pole, i.e. to the situation in which the spectator quark is on its mass shell. Therefore, its plus component of the momentum cannot exceed that of \( P_2 \) (i.e., \( x/y < 1 \)). The second region corresponds to the final active quark on its mass shell. We will refer to these two contributions as the partonic and the non-partonic ones \(^b\). Hereafter, the partonic and the non-partonic terms will be denoted as \( J_A^+ \) and \( J_B^+ \), respectively. Performing all the traces, one obtains

\[
J_A^+ = \frac{N_c \lambda_1 \lambda_2}{2(2\pi)^3} \int_0^y \frac{dx}{x(1-x)(1-x')} \int d^2k_\perp (M_1^2 - M_{10}^2)(M_2^2 - M_{20}^2) \frac{I_A^+}{I_B^+} \tag{14}
\]

\[
J_B^+ = \frac{N_c \lambda_1 \lambda_2}{2(2\pi)^3} \int_1^y \frac{dx}{x(1-x)(1-x')} \int d^2k_\perp (M_1^2 - M_{10}^2)(M_2^2 - q^2 - q_{10}^2) \frac{y}{1-y} \tag{15}
\]

with \( x' \equiv x/y \) and

\[
M_{10}^2 \equiv M_{10}^2(x, k_1^2) = \frac{m^2 + k_1^2}{x} + \frac{m_2^2 + k_1^2}{1-x} \tag{16}
\]

\[
M_{20}^2 \equiv M_{20}^2(x', k_2^2) = \frac{m^2 + k_2^2}{x'} + \frac{m_2^2 + k_2^2}{1-x'} \tag{17}
\]

\[
M_{12}^2 \equiv M_{12}^2(x, x', k_1^2) = \left[ \frac{m_1^2 + k_1^2}{1-x} - \frac{m_2^2 + k_2^2}{y-x} \right] (1-y) \tag{18}
\]

\[
I_A^+ \equiv I_A^+ \bigg|_{k^- = \frac{m^2 + k_1^2}{x v_1}} = 4 m m_1 m_2 \left[ (1 + v \cdot v_2) v_1^+ + (1 + v \cdot v_1) v_2^+ + (1 - v_1 \cdot v_2) v_1^+ \right] \tag{19}
\]

\[
I_B^+ \equiv I_B^+ \bigg|_{k^- = p_1^+ - \frac{m_1^2 + k_1^2}{(1-x) p_1^+}} = 4 M_1 m_1 m_2 \left[ (U_1 \cdot v_2 - \xi) v_1^+ + (U_1 \cdot v_1 - \xi) v_2^+ \right] \tag{20}
\]

where \( \xi \equiv (m_1 - m)/M_1, v_1 \equiv p_i/m_i \) are the four-velocities of the active quarks and \( v \equiv k/m \) is the four-velocity of the spectator quark.

One can easily recognize that the expressions \( J_A^+ \) (eq. (14)) and \( J_B^+ \) (eq. (15)) are the contributions of the LF diagrams (A) and (B), depicted in Fig. 1, respectively. It should be stressed that the term \( J_B^+ \) corresponds to the creation of a virtual \( Q_2 Q_2 \) pair from the vacuum and to the subsequent conversion of the \( Q_2 Q_1 \) pair into a \( W \)-boson. Because of the integration limits in eq. (15), \( J_B^+ \) is relevant only for kinematics corresponding to a
For a finite range vertex the following substitutions should be performed in eqs. (14-15) (cf. ref. [9])

$$\chi_{1}(x, k_{\perp}^{2}) \quad (21)$$

$$\chi_{2}(x', k_{\perp}^{2}) \quad (22)$$

where the normalization of the functions $\chi_{i}$ will be fixed later on (see eq. (26)). It can be easily checked that $J_{A}^{+}$ involves the wave functions $\chi_{i}$ of both the initial and final mesons, whereas the wave function $\chi_{2}$ of the final meson cannot be reconstructed in $J_{A}^{+}$, because in the non-partonic diagram the LF fraction $x/y$ of the spectator quark in the final state would exceed 1.

### 2.3 The Hamiltonian light-front approach

Within the LF formalism the parent pseudoscalar meson $Q_{1}\bar{q}$ is represented by the following state vector

$$|P_{1}> = \int \frac{d^{2}k_{\perp}}{\sqrt{16\pi^{3}}} \frac{dx}{x} [M_{10}^{2} - (m_{1} - m)^{2}]^{1/2} \sqrt{N_{c}} \chi_{1}(x, k_{\perp}^{2}) Q_{1}^{\dagger}|0> \quad (23)$$

where $Q_{1}^{\dagger} = Q_{1}^{\dagger}(\vec{p}_{1}, \lambda)$ and $q^{\dagger} = \bar{q}^{\dagger}(\vec{k}, \bar{\lambda})$ are the creation operators for a heavy quark and a spectator (anti-) quark with the following anticommutation rules

$$\{Q_{1}^{\dagger}(\vec{p}_{1}, \lambda), Q_{1}(\vec{p}_{1}', \lambda')\} = (2\pi)^{3} 2p_{1}^{+} \delta_{\lambda\lambda'} \delta(\vec{p}_{1} - \vec{p}_{1}') \quad (24)$$

$$\{q^{\dagger}(\vec{k}, \bar{\lambda}), \bar{q}(\vec{k}', \bar{\lambda}')\} = (2\pi)^{3} 2k^{+} \delta_{\bar{\lambda}\bar{\lambda}'} \delta(\vec{k} - \vec{k}')$$

where $\vec{k} = (k^{+}, k_{\perp})$ and $\vec{p}_{1} = \vec{P}_{1} - \vec{k} = (P_{1}^{+} - k^{+}, -k_{\perp})$ are LF momenta. The spectator quark carries the fraction $x$ of the plus component of the meson momentum, while the heavy quark carries the fraction $1 - x$. In eq. (23) $R_{00}^{(1)}(x, k_{\perp})$ is a momentum-dependent quantity arising from the Melosh rotations of the quark spins (see below eq. (30)), and $M_{10}^{2}$ has been already defined in eq. (13). The state vector $|\vec{P}_{2}>$ of the daughter meson $Q_{2}\bar{q}$ has a form analogous to eq. (23) with the obvious replacements $m_{1} \rightarrow m_{2}$, $M_{10}^{2} \rightarrow M_{20}^{2}$, $R_{00}^{(1)} \rightarrow R_{00}^{(2)}$, $\chi_{1} \rightarrow \chi_{2}$ and $Q_{1}^{\dagger} \rightarrow Q_{2}^{\dagger}$. The state vectors $|P_{i}> \quad (i = 1, 2)$ are normalized as

$$< P_{i}'|P_{i} > = (2\pi)^{3} 2P_{i}^{+} \delta(\vec{P}_{i} - \vec{P}_{i}') \quad (25)$$

so that the normalization of the wave functions $\chi_{i}(x, k_{\perp}^{2})$ reads as

$$\int_{0}^{1} dx \frac{1 - x}{x} \int d^{2}k_{\perp} [M_{i0}^{2} - (m_{i} - m)^{2}] \chi_{i}^{2}(x, k_{\perp}^{2}) = 1 \quad (26)$$
Following the Brodsky-Huang-Lepage prescription [12] the function \( \chi_i(x, k^2_\perp) \) can be related to the equal-time wave function \( w_i(k^2) \) normalized according to

\[
\int_0^\infty dk \, k^2 \, w_i^2(k^2) = 1
\]

provided that the fraction \( x \) is replaced by the longitudinal momentum \( k_3^{(i)} \) defined as

\[
k_3^{(i)} = \left( x - \frac{1}{2} \right) M_{i0} + \frac{m_i^2 - m^2}{2M_{i0}}
\]

Explicitly, one has

\[
\chi_i(x, k^2_\perp) = \frac{1}{2(1-x)} \frac{\sqrt{M_{i0}[1 - (m_i^2 - m^2)^2/M_{i0}^4]}}{\sqrt{M_{i0}^2 - (m_i - m)^2}} \frac{w_i(k^2)}{\sqrt{4\pi}}
\]

with \( k^2 \equiv k_1^2 + (k_3^{(i)})^2 \). Finally, using the chiral representation of the Dirac spinors (see, e.g., ref. [9]), one has

\[
[R_{00}^{(i)}(x, k_\perp)]_{\lambda \bar{\lambda}} = \frac{1}{\sqrt{2}} \frac{\sqrt{M_{i0}^2 - (m_i - m)^2}}{\sqrt{M_{i0}^2 - (m_i - m)^2}} \bar{u}(\vec{p}_i, \lambda) \gamma_5 v(\vec{k}, \bar{\lambda})
\]

Thus, the matrix element of the "good" current \( J^+ = J_0 + J_5 \) reads as

\[
J^+(y) = \int_0^y dx \int d^2k_\perp \chi_1(x, k^2_\perp) \chi_2 \left( \frac{x}{y}, k^2_\perp \right) I^+_{LF}(x, y, k^2_\perp)
\]

where \( y = y_1 \) or \( y = y_2 \) (see eq. (3)) and \( I^+_{LF} \) is the spin contribution

\[
I^+_{LF} = 4m_1m_2 \left\{ (1 + v \cdot v_2)v_1^+ + (1 + v_1 \cdot v)v_2^+ + (1 - v_1 \cdot v_2)v^+ \right\}
\]

It can be seen that eq. (33) coincides with eq. (19). Moreover, it can be easily checked that, using eqs. (21-22) the partonic term \( J^+_A \) coincides with eq. (31). In other words, the result (31), obtained within the Hamiltonian LF dynamics, is equivalent to the spectator pole term of the quark triangle diagram. This result generalizes to the time-like region \( (q^2 > 0) \) the result obtained in ref. [11] in the Breit frame \( (q^2 \leq 0) \). In terms of the LF variables \( x \) and \( k^2_\perp \) one gets

\[
I^+_{LF} = \frac{4P^+_1}{x^d} \left\{ [m(1 - x) + xm_1][m(1 - x') + x'm_2] + k^2_\perp \right\}
\]

The form factors \( h_{\pm} \), or \( f_{\pm} \), can be evaluated using only the matrix elements of the "good" component of the current \( J^+ = Q_2 \bar{\gamma}_+ Q_1 \). In order to invert eq. (9) we calculate the matrix
element of $J^+$ in two reference frames having the 3-axis parallel and anti-parallel to the 3-momentum of the daughter meson. In this way two different matrix elements, denoted hereafter as $J_1^+(q^2) \equiv 2P_1^+ H_1(q^2)$ and $J_2^+(q^2) \equiv 2P_1^+ H_2(q^2)$, are obtained. Then, $h_\pm$ and $f_\pm$ are given by

$$
h_+(q^2) = \frac{(\zeta - y_2)H_1(q^2) - (\zeta - y_1)H_2(q^2)}{\sqrt{\zeta (y_1 - y_2)}} \quad (35)
$$
$$
h_-(q^2) = \frac{(\zeta + y_2)H_1(q^2) - (\zeta + y_1)H_2(q^2)}{\sqrt{\zeta (y_2 - y_1)}} \quad (36)
$$
$$
f_+(q^2) = \frac{(1 - y_2)H_1(q^2) - (1 - y_1)H_2(q^2)}{y_1 - y_2} \quad (37)
$$
$$
f_-(q^2) = \frac{(1 + y_2)H_1(q^2) - (1 + y_1)H_2(q^2)}{y_2 - y_1} \quad (38)
$$

Note that at the point of zero recoil, i.e. $q^2 = q_{\text{max}}^2$, where $y_1 = y_2 = \zeta$, the two reference frames coincide, so that $H_1(q_{\text{max}}^2) = H_2(q_{\text{max}}^2)$. This implies that the form factors have no singularity at $q^2 = q_{\text{max}}^2$. At the point of maximum recoil, i.e. $q^2 = 0$, the expression of $f_+(0) = H_1(0)$ simplifies to

$$
f_+(0) = \int_0^1 dx \int d^2k_\perp \frac{dk_3^{(1)}}{dx} \frac{dk_3^{(2)}}{dx} \sqrt{\frac{4\pi}{\sqrt{m(1-x) + xm_1]|m(1-x) + xm_2| + k_\perp^2}}
$$
$$
\frac{w_1(k^2) w_2(k^2)}{x(1-x)[M_{10}^2(x) - (m_1 - m)^2] \sqrt{x(1-x)[M_{20}^2(x) - (m_2 - m)^2]}} \quad (39)
$$

where

$$
\frac{dk_3^{(i)}}{dx} = \frac{M_{i0}}{4x(1-x)} \left[1 - \left(\frac{m_i^2 - m^2}{M_{i0}^2}\right)^2\right] \quad (40)
$$

Eq. (39) can be rewritten as

$$
f_+(0) = \int_0^1 dx \int d^2k_\perp \Phi_1(x, k_\perp^2) \Phi_2(x, k_\perp^2) \frac{A_1A_2 + k_\perp^2}{\sqrt{(A_1^2 + k_\perp^2)(A_2^2 + k_\perp^2)}} \quad (41)
$$

where $A_i \equiv x m_i + (1-x)m$ and $\Phi_i(x, k^2) = \int \frac{dk_3^{(i)}}{dx} \frac{w_i(k^2)}{\sqrt{4\pi}}$. It can be seen that eq. (41) coincides with the result of refs. [9] and [15] obtained using the Hamiltonian light-front formalism in the Breit frame.

Note also that in the non-relativistic limit ($k^2 \ll m^2$, $m_i^2$ and $k'^2 \ll m^2, m_i^2$) eq. (41) reduces to

$$
J_{1,2}^+(q^2) = 2 \sqrt{M_1M_2} \int d^3k w_1(k^2) w_2(k'^2)(1 \mp \frac{k_3}{2m_1} \pm \frac{k_3 + p}{2m_2}) \quad (42)
$$
where $k'^2 = k_1'^2 + k_2'^2$ and $k_3' = k_3 - \frac{m}{M} p$, with $p$ being the 3-momentum of the daughter meson in the rest frame of the parent meson. In the non-relativistic limit one has $p = M_2(y_1/\zeta - 1)$. Eq. (12) written for the "currents" $J_0 = \frac{1}{2}(J_1^+ + J_2^+)$ and $J_3 = \frac{1}{2}(J_1^+ - J_2^+)$ coincide with the non-relativistic result derived in the ISGW model [7].

3 Results

The calculation of the semileptonic form factors $f_{\pm}(q^2)$ (eqs. (37-38)) has been performed using our LF result (eqs. (31) and (34)) for computing the matrix elements $J_{1,2}^+$ and eq. (29) for the meson wave functions $\chi_i$. As for the radial wave function $w_i(k^2)$ appearing in eq. (29), the Gaussian ansatz of the ISGW model has been adopted; the values of the parameters (the constituent quark masses and the harmonic oscillator (HO) lengths) are taken from the updated version of this model [16].

The $q^2$ behaviour of the form factors $f_{\pm}$ for the $b \to c\ell\nu_\ell$, $b \to u\ell\nu_\ell$, $c \to s\ell\nu_\ell$, $c \to u\ell\nu_\ell$ and $s \to u\ell\nu_\ell$ quark decays are shown in Figs. 2-6, respectively. The solid lines are the results of our LF calculations obtained using the ISGW2 values for the quark masses and HO parameters, but adopting the physical values for the meson masses taken from PDG '94 [17]. For comparison, the results obtained using all the ISGW2 parameters, i.e. adopting also the meson masses used in [16], are shown by the dashed lines. The dotted lines are the monopole approximations for the form factors (eq. (10)), obtained using the value of the form factors at $q^2 = 0$ and the pole masses given by the lowest-lying vector meson mass for the given channel [6]. It can be seen that: i) our simple pole approximation yields a $q^2$-dependence which sharply differs from the one obtained within our LF approach, particularly in case of heavy-to-light decays, like $B \to \pi$ and $D \to \pi$; for these decays the form factors do not obey at all the pattern of pole dominance near the zero-recoil point ($\eta = 1$); ii) for the heavy-to-light decays $B \to \pi$ and $D \to \pi$, as well as for the $K_{e3}$ decay, the form factors are strongly sensitive to the choice of the values of the meson masses (compare dashed and solid lines); it is worth noting that this happens not only near the kinematical end-point of maximum recoil, but in the whole accessible kinematical region.

The results for the semileptonic decay rate $\Gamma$ (eq. (14)) are collected in Table 1. It can be seen that: i) the monopole approximation of ref. [6] underestimates the rates, ranging from $\sim 5\%$ for $K \to \pi$ to $\sim 20\%$ for $B \to D$ and to $\sim 40\%$ for $B \to \pi$; ii) the choice of the values of the meson masses is crucial for the $K_{e3}$ decay.

We now present our results, organized by the underlying quark decay and arranged in order of decreasing active quark mass. We will compare our results to experimental data and to predictions of different models.

\textsuperscript{a}In what follows this version will be referred to as the ISGW2 model.
3.1 Decay $B \to D\ell\nu_\ell$

The $q^2$ behaviour of the form factor $f_\pm$ for the semileptonic $B \to D$ transition is shown in Fig. 2. The slope $\hat{\rho}^2$ of the form factor $h_+$ (eq. (25)) at the point of zero recoil, defined as $h_+(\eta) \approx h_+(1) \left(1 - \hat{\rho}^2(\eta - 1)\right)$, turns out to be $\hat{\rho}^2 = 1.3$ within our LF approach. As is known, the slope parameter $\hat{\rho}^2$ of the physical form factor $h_+$ differs from the slope parameter $\rho^2$ of the universal Isgur-Wise function by corrections that violate the heavy-quark symmetry. Using an approximate relation between the two quantities, namely $\hat{\rho}^2 \approx \rho^2 + 0.2$ [18], we obtain $\rho^2 = 1.1$. This value compares well with the quark-model prediction of ref. [19] and it is only slightly higher than QCD sum rule predictions, which typically range from 0.7 to 1.0 (see, e.g., ref. [20]).

Performing the calculations with and without the effects of the Melosh composition of quark spins (i.e., assuming $R^{(1)}_{00} \neq 1$ or $R^{(1)}_{00} = 1$ in eq. (23)), it turns out that the effects of the Melosh rotations increase $\rho^2$ by $\sim 20\%$; this result agrees with the conclusion of ref. [19] obtained in the zero-binding approximation.

From the measured rate
\[
\Gamma(B^0 \to D^-\ell^+\nu_\ell) = (1.27 \pm 0.43) \cdot 10^{10} \text{ s}^{-1}
\] (43)
and our predicted rate from Table 1, we obtain
\[
|V_{bc}| = 0.036 \pm 0.004
\] (44)

Our LF prediction is only $\sim 10\%$ larger than the corresponding ISGW2 prediction ($|V_{bc}| = 0.033 \pm 0.004$ [16]) and in agreement with the updated ”experimental” determinations of $|V_{bc}|$ [21] obtained from exclusive and inclusive semileptonic decays of the B-meson ($|V_{bc}|_{exc} = 0.0373 \pm 0.0045_{exp} \pm 0.0065_{th}$ and $|V_{bc}|_{incl} = 0.0398 \pm 0.0008_{exp} \pm 0.0040_{th}$).

Within an accuracy of few percent our form factor $f_+(q^2)$ is reproduced by a monopole approximation with $M_{pole} = 4.82$ GeV. As a matter of fact, at a level of accuracy of $\sim 10\%$ for the decay rate, the details of the $q^2$ dependence of $f_+$ do not matter as long as $q^2 \ll M_{pole}^2$ and the form factor varies slowly with $q^2$, as it is the case for the semileptonic $B \to D$ decay.

3.2 Decay $B \to \pi\ell\nu_\ell$

We now consider the semileptonic B-meson decay corresponding to the quark process $b \to u\ell\nu_\ell$. The investigation of this decay is very important for the determination of the $|V_{ub}|$ CKM matrix element, which plays an important role for the CP violation in the Standard Model. The experimental studies of such a decay have begun and, very recently, the CLEO Collaboration [22] has reported the first signal for exclusive semileptonic decays of the B meson into charmless final states, in particular for the decay mode $B \to \pi\ell\nu_\ell$. However, there is a significant model dependence in the simulation of the reconstruction efficiencies.

\[d\]This value has been derived by combining the branching ratio $Br(B^0 \to D^-\ell^+\nu_\ell l) = (1.9 \pm 0.5)\%$ with the world average of the $B^0$ lifetime $\tau_{B^0} = 1.50 \pm 0.11$ ps [17].
The observed branching ratios, extracted adopting the ISGW [8] and WSB [6] models, is

\[
Br(B \rightarrow \pi \ell \nu_\ell) = (1.34 \pm 0.45) \cdot 10^{-4} \quad ISGW
\]
\[
Br(B \rightarrow \pi \ell \nu_\ell) = (1.63 \pm 0.57) \cdot 10^{-4} \quad WSB
\]

Combining the average of these results with the world average value of the \(B^0\) lifetime, one gets

\[
\Gamma(B^0 \rightarrow \pi^- \ell^+ \nu_\ell) = (0.99 \pm 0.25) \cdot 10^{-4} \quad ps^{-1}
\]

From our predicted rate \(\Gamma(B^0 \rightarrow \pi^- \ell^+ \nu_\ell) = 9.62 \, |V_{ba}|^2 \quad ps^{-1}\) (see Table 1), we obtain

\[
|V_{ba}| = 0.0032 \pm 0.0004
\]

Using our previous result (44) for \(\frac{|V_{bu}|}{|V_{cb}|}\) we get

\[
\frac{|V_{bu}|}{|V_{cb}|} = 0.088 \pm 0.015
\]

which is in nice agreement with the value derived from measurements of the end-point region of the lepton spectrum in inclusive semileptonic decays [23, 24], viz.

\[
\left|\frac{V_{bu}}{V_{cb}}\right|_{\text{incl}} = 0.08 \pm 0.01_{\text{exp}} \pm 0.02_{\text{th}}
\]

A large variety of calculations of the ratio \(\frac{|V_{bu}|}{|V_{cb}|}\), based on various non-perturbative approaches, exists in the literature; the results typically lie in the range from 0.06 to 0.11.

The \(q^2\) behaviour of the form factors \(f_{B \rightarrow \pi}^\pm\) for the semileptonic \(B \rightarrow \pi\) transition is shown in Fig. 3. Our result for \(f_{B \rightarrow \pi}^\pm(q^2 = 0)\) (see Table 2) nicely agrees with the one obtained from a recent analysis of the \(B \rightarrow \pi\) form factors using the light-front QCD sum rule [5]. Other model predictions for \(f_{B \rightarrow \pi}^\pm(0)\) and the \(B \rightarrow \pi \ell \nu_\ell\) decay rate are collected in Tables 2 and 3.

### 3.3 Decay \(D \rightarrow K \ell \nu_\ell\)

The value of \(|V_{cs}|\) can be extracted from measurements of charmed hadron production in neutrino experiments. However, such a procedure depends crucially on the assumption about the strange quark density in the partonic sea. The most conservative assumption (i.e., an \(SU(3)\) symmetric sea) leads to the bound \(|V_{cs}| > 0.59\). Therefore, it is better to proceed in a way analogous to the method used for extracting \(|V_{bc}|\) and \(|V_{bu}|\) from \(B\)-meson decays. By combining the experimental data on the branching ratios for the semileptonic \(D^0\) decay \(Br(D^0 \rightarrow K^- e^+ \nu_e) = 3.68 \pm 0.21\%\) [17]) with the accurate value of the \(D^0\) lifetime \((\tau_{D^0} = 0.415 \pm 0.004 \quad ps\) [17]), one has

\[
\Gamma(D^0 \rightarrow K^- e^+ \nu_e) = (0.089 \pm 0.005) \quad ps^{-1}
\]
From our predicted rate $\Gamma(D^0 \to K^- e^+ \nu_e) = 0.113 \cdot |V_{cs}|^2 \text{ ps}^{-1}$ it follows

$$|V_{cs}| = 0.89 \pm 0.03$$  \hspace{1cm} (52)

The constraint of unitarity of the CKM matrix with three generations of leptons gives a much higher value, viz. $|V_{cs}| = 0.974$ [17].

The $q^2$ behaviour of the form factors $f_\pm$ for the $D \to K$ transition is shown in Fig. 4. At $q^2 = 0$ we have obtained: $f_{D \to K}^+(0) = 0.78$ and $f_{D \to K}^+(q_{\text{max}}^2) = 1.56$ (see Table 1). Our value for $f_{D \to K}^+(0)$ compares favourably with the experimental average $0.75 \pm 0.03$ [34], as well as with the recent experimental results [35] $f_{D \to K}^+(0) = 0.77 \pm 0.01 \pm 0.04$ and $f_{D \to K}^+(q_{\text{max}}^2) = 1.42 \pm 0.25$. For comparison, the ISGW2 predictions are $f_{D \to K}^+(0) = 0.85$ and $f_{D \to K}^+(q_{\text{max}}^2) = 1.23$, respectively [8]. Very recently, the E687 collaboration has investigated the $D^0 \to K^- \mu^+ \nu_\mu$ and $D^0 \to K^- \pi^+$ decays and has reported [36] the following values: $f_{D \to K}^+(0) = 0.71 \pm 0.03 \pm 0.03$ and $f_{D \to K}^+(q_{\text{max}}^2) = -1.3 \pm 0.36 \pm 0.6$. The latter result is reproduced by our calculations, yielding the value $-1.06$. It is worth noting that the lattice QCD simulations of ref. [32] gives the value $-1.2 \pm 0.5$, whereas both the WSB [6] and ISGW [7] quark model predictions are much lower, namely $-0.46$ and $-0.60$, respectively.

### 3.4 Decay $D \to \pi \ell \nu_\ell$

This is the only heavy-to-light decay where a comparison with experiment is possible. Moreover, there exist several model calculations in the literature, using QCD sum rules [3-5, 30], quark models [6-9, 16], and few lattice QCD calculations [31, 32]. The results of ref. [29] are based on the HQET, adopting the experimental results [33] as input for the values of the form factors of the $B$ based on the HQET, adopting the experimental results [33] as input for the values of the quark models [6-9, 16], and few lattice QCD calculations [31, 32]. The results of ref. [29] are based on the HQET, adopting the experimental results [33] as input for the values of the form factors of the $B$ based on the HQET, adopting the experimental results [33] as input for the values of the quark models [6-9, 16], and few lattice QCD calculations [31, 32]. The results of ref. [29] are based on the HQET, adopting the experimental results [33] as input for the values of the form factors of the $B$ based on the HQET, adopting the experimental results [33] as input for the values of the quark models [6-9, 16], and few lattice QCD calculations [31, 32].

Our form factor $f_+(q^2)$ turns out to be flat near $q^2 = 0$ with a negative slope. Assuming $|V_{cd}| = 0.221 \pm 0.003$ [17], which is inferred from the unitarity of the CKM matrix, we predict for the semileptonic decay rate the following value

$$\Gamma(D^0 \to \pi^- e^+ \nu_e) = 7.8 \cdot 10^{-3} \text{ ps}^{-1}$$  \hspace{1cm} (53)

This prediction should be compared with the experimental result

$$\Gamma_{\exp}(D^0 \to \pi^- e^+ \nu_e) = (9.4^{+5.5}_{-2.9}) \cdot 10^{-3} \text{ ps}^{-1}$$  \hspace{1cm} (54)

and the recent light-front QCD sum rule prediction [5]

$$\Gamma_{\text{SR}}(D^0 \to \pi^- e^+ \nu_e) = (7.6 \pm 0.2) \cdot 10^{-3} \text{ ps}^{-1}$$  \hspace{1cm} (55)

The comparison of our values for $f_{D \to \pi}^+(0)$ and the decay rate $\Gamma$ with other theoretical results is presented in Tables 2 and 3.

For this decay CLEO [35] has obtained for the pole mass of the $f_+$ form factor the value $|M_\text{pole}| = 2.00 \pm 0.12 \pm 0.18 \text{ GeV}$, assuming a monopole shape. Using the measured lepton spectrum, the decay rate can be transformed into a measurement of $f_+(q^2 = 0)$. 

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The ratio of the branching ratio of the Cabibbo suppressed decay $D \rightarrow \pi \ell \nu$ to that of the Cabibbo favoured decay $D \rightarrow K \ell \nu$ is found to be $1.426 \ |V_{cd}/V_{cs}|^2 = 0.051 \pm 0.001$. Using the unitarity constraint on the CKM matrix, yielding $|V_{cd}/V_{cs}|^2 = 0.051 \pm 0.001$, we obtain the value
\begin{equation}
R_0 = \frac{Br(D \rightarrow \pi \ell \nu)}{Br(D \rightarrow K \ell \nu)} = 7.3\% \tag{56}
\end{equation}
which is slightly lower than the predictions from various theoretical models, namely the CQM ($R_0 = 8.8\%$ [6], $R_0 = 5.3\%$ [8]), the QCD sum rules ($R_0 = 9.3\%$ [30], $R_0 = 8.3\%$ [37]), and lattice QCD calculations ($R_0 = 8.6\%$ [31]).

The $D \rightarrow K \ell \nu$ and $D \rightarrow \pi \ell \nu$ decays have been recently investigated by the CLEO-II collaboration and both charged [38] and neutral [39] $D$-meson decays have been measured. The results are
\begin{align}
\frac{Br(D^+ \rightarrow \pi^0 e^+ \nu_e)}{Br(D^+ \rightarrow K^0 e^+ \nu_e)} &= (8.5 \pm 2.7 \pm 1.4)\% \tag{57} \\
\frac{Br(D^0 \rightarrow \pi^- e^+ \nu_e)}{Br(D^0 \rightarrow K^- e^+ \nu_e)} &= (10.3 \pm 3.9 \pm 1.3)\% \tag{58}
\end{align}
Assuming isospin invariance (i.e., $Br(D^+ \rightarrow \pi^0 e^+ \nu_e)/Br(D^0 \rightarrow \pi^- e^+ \nu_e) = 1/2$) and the pole dominance for the $q^2$-dependence of the form factors, with the mass of the vector resonance given by the mass of the $D^*(D_s^*)$ meson for the $\pi e\nu_e(K e\nu_e)$ decay, these results can be translated into the following values
\begin{align}
\left| \frac{f_{D^+ \rightarrow \pi^0}(0)}{f_{D^+ \rightarrow K}(0)} \right| &= 1.29 \pm 0.21 \pm 0.11 \tag{59} \\
\left| \frac{f_{D^+ \rightarrow \pi^-}(0)}{f_{D^+ \rightarrow K}(0)} \right| &= 1.01 \pm 0.20 \pm 0.07 \tag{60}
\end{align}
With respect to the average of these results we predict a slightly lower ratio, viz.
\begin{equation}
\left| \frac{f_{D^+ \rightarrow \pi}(0)}{f_{D^+ \rightarrow K}(0)} \right| = 0.87 \tag{61}
\end{equation}
The ISGW2 value for this ratio is $0.71$ and other model predictions typically range from $0.7$ to $1.4$.

### 3.5 $K \rightarrow \pi e\nu_e$

The non-partonic contribution $J^+_{\overline{B}}$ to the matrix element of the weak vector current might be more important for light meson decays than for the heavy-to-heavy and heavy-to-light transitions. However, the corrections arising from the non-partonic diagram are expected to be relevant mainly for the form factor $f_-(q^2)$. Therefore, it is of interest to consider the $K e3$ decay, whose differential decay rate is governed by $f_+(q^2)$ only. The form factors
for $K_{e3}$ decays are usually referred at the $SU(3)$ normalization point $q^2 = 0$. Our result $f_+(0) = 0.976$ is in nice agreement with the "standard" value $f_+(0) = 0.97 \pm 0.01$ [40]. Our result for $f_+(q_{\text{max}}^2) = 1.120$ is roughly consistent with the Ademollo-Gatto theorem [41], which protects $f_+(q_{\text{max}}^2)$ from substantial deviations from unity. The $q^2$ behaviour of the form factor $f_+$ for the $K \to \pi$ transitions is shown in Fig. 6. Using $|V_{us}| = 0.2205 \pm 0.0018$ [17], we obtain $\Gamma(K_{e3}) = (7.58 \pm 0.13) \cdot 10^6 \text{ s}^{-1}$ in agreement with the experimental average value $(7.7 \pm 0.5) \cdot 10^6 \text{ s}^{-1}$ [17].

4 Conclusions

The weak transition form factors, which govern the heavy-to-heavy, heavy-to-light and $K_{e3}$ semileptonic decays of pseudoscalar mesons, have been investigated within a relativistic constituent quark model based on the light-front formalism. Using the "good" component of the weak vector current, it has been shown that the partonic term of the quark triangle diagram is equivalent to the result which can be obtained within the Hamiltonian light-front dynamics, generalizing in this way to the time-like region a previous result [11] derived only for space-like values of the momentum transfer. For the numerical investigations, the equal-time wave function of the ISGW model has been adopted, so that, for the first time, the transition form factors have been calculated in the whole kinematical region accessible in semileptonic decays. We have calculated the form factors and the decay rate for the $B \to D \ell \nu_\ell$, $B \to \pi \ell \nu_\ell$, $D \to K \ell \nu_\ell$, $D \to \pi \ell \nu_\ell$ and $K \to \pi \ell \nu_\ell$ weak decays. The relevance of the use of the physical values of the meson masses, as well as the possible limitations of the pole dominance approximation, have been illustrated. Our results have been successfully compared with available experimental data and predictions from different approaches. In particular, using the available experimental information on the semileptonic decay rates, the CKM parameters have been estimated. With the only exception of $|V_{cs}|$ our results are in good agreement with existing determinations of these parameters. Before closing, it should be reminded that in our calculations the contribution of the pair creation from the vacuum has been neglected; therefore, an estimate of such contribution, particularly in case of the heavy-to-light and $K_{e3}$ transitions, is mandatory for a complete comparison with experimental data.

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Table 1

Form factor $f_+(q^2)$ (eq. (37)), evaluated at $q^2 = 0$ and $q^2 = q_{\text{max}}^2$, and the decay rate $\Gamma$ (eq. (11)) for various semileptonic decays (in units $ps^{-1}$). $\Gamma'$ is the same as $\Gamma$, but using the meson masses from ref. [16] instead of their experimental values. $\Gamma_{\text{pole}}$ denotes the decay rate calculated using the pole approximation for $f_+(q^2)$ (eq. (10)) with the pole masses taken from [6].

| weak transition | $f_+(0)$ | $f_+(q_{\text{max}}^2)$ | $\Gamma$ | $\Gamma'$ | $\Gamma_{\text{pole}}$ |
|-----------------|---------|-------------------------|----------|---------|-------------------|
| $B \to D$       | 0.684   | 1.365                   | 9.78$|V_{bc}|^2$ | 9.78$|V_{bc}|^2$ | 7.89$|V_{bc}|^2$ |
| $B \to \pi$     | 0.293   | 1.658                   | 9.62$|V_{bu}|^2$ | 9.05$|V_{bu}|^2$ | 5.80$|V_{bu}|^2$ |
| $D \to K$       | 0.780   | 1.560                   | 0.113$|V_{cs}|^2$ | 0.094$|V_{cs}|^2$ | 0.095$|V_{cs}|^2$ |
| $D \to \pi$     | 0.681   | 1.289                   | 0.160$|V_{cu}|^2$ | 0.121$|V_{cu}|^2$ | 0.142$|V_{cu}|^2$ |
| $K_{e3}$        | 0.976   | 1.119                   | 1.56-10^{-4}$|V_{su}|^2$ | 0.20-10^{-4}$|V_{su}|^2$ | 1.48-10^{-4}$|V_{su}|^2$ |
Table 2
The form factor $f_+ (eq. (6))$ for the $b \rightarrow u$ and $c \rightarrow d$ transitions at $q^2 = 0$ in different models.

| Reference | $f_+^{B \rightarrow \pi}(0)$ | Reference | $f_+^{D \rightarrow \pi} N(0)$ |
|-----------|-----------------------------|-----------|-------------------------------|
| This paper | 0.293                       | This paper | 0.684                        |
| [5]$^{a}$ | 0.29 ± 0.01                 | [5]$^{a}$ | 0.66 ± 0.03                   |
| [3]$^{a}$ | 0.26 ± 0.02                 | [3]$^{a}$ | 0.5 ± 0.1                     |
| [25]$^{a}$ | 0.26 ± 0.01                 | [30]$^{a}$ | 0.75 ± 0.05                   |
| [26]$^{a}$ | 0.23 ± 0.02                 | [6]$^{b}$ | 0.69                          |
| [27]$^{a}$ | 0.4 ± 0.1                   | [8]$^{b}$ | 0.51                          |
| [6]$^{b}$ | 0.33                        | [29]$^{c}$ | 0.79                          |
| [8]$^{b}$ | 0.09                        | [31]$^{d}$ | 0.58 ± 0.09                   |
| [28]$^{b}$ | 0.21 ± 0.02                 | [32]$^{d}$ | 0.84 ± 0.12 ± 0.35            |
| [29]$^{c}$ | 0.89                        | [33]$^{e}$ | 0.80±0.21|0.14               |

$^{a}$ QCD sum rules.

$^{b}$ Quark model.

$^{c}$ HQET and chiral perturbation theory.

$^{d}$ Lattice QCD calculations.

$^{e}$ Experimental value.
Table 3

The semileptonic decay rate $\Gamma$ (eq. (11)) for the $b \rightarrow u$ and $c \rightarrow d$ transitions in units $|V_{ub}|^2 \cdot 10^{13} s^{-1}$ and $|V_{cd}|^2 \cdot 10^{11} s^{-1}$, respectively.

| Reference | $\Gamma(B^0 \rightarrow \pi^+e^−\bar{\nu})$ | Reference | $\Gamma(D^0 \rightarrow \pi^-e^+\nu)$ |
|-----------|-----------------------------------------|-----------|--------------------------------------|
| This paper$^a$ | 0.962 (0.79) | This paper$^a$ | 1.60 (1.42) |
| [5]$^b$ | 0.81 | [5]$^b$ | 1.56 |
| [3]$^b$ | 0.51 ± 0.11 | [3]$^b$ | 0.80 ± 0.17 |
| [25]$^b$ | 0.68 ± 0.23 | [30]$^b$ | 1.66$^{+0.23}_{-0.21}$ |
| [26]$^b$ | 0.302 ± 0.005 | [6]$^c$ | 1.41 |
| [27]$^b$ | 1.45 ± 0.59 | [8]$^c$ | 0.77 |
| [6]$^c$ | 0.74 | [31]$^e$ | 0.99$^{+0.34}_{-0.28}$ |
| [8]$^c$ | 0.21 | [32]$^e$ | 2.09$^{+2.24}_{-1.44}$ |
| [28]$^c$ | 0.31 ± 0.06 | [33]$^f$ | $|0.22/V_{cd}|^2 \cdot (1.9^{+1.1}_{-0.6})$ |
| [29]$^d$ | 5.4 |

$^a$ In the parentheses the rates obtained assuming the monopole approximation for the form factor $f_+$ (eq. (10)) and the pole masses from ref. [6], are shown.

$^b$ QCD sum rules.

$^c$ Quark model.

$^d$ HQET and chiral perturbation theory.

$^e$ Lattice QCD calculations.

$^f$ Experimental value.
Figure Captions

Fig. 1 The Feynman triangle diagram for the form factor and the corresponding light-front diagrams.

Fig. 2 The form factors $f_{\pm}(\eta)$ (eq. (37-38)) for the $B \rightarrow D$ transition. The relation between the kinematical variables $\eta$ and $q^2$ is given by eq. (3). The solid lines are the results of our LF calculations obtained using the ISGW2 parameters, but adopting the experimental values for the meson masses. The results obtained with all the ISGW2 parameters, including the meson masses from ref. [16] are shown by the dashed lines. The dotted lines are the monopole approximation for the form factors (eq. (10)), calculated using the values of the form factor at $q^2 = 0$ and the pole masses given in ref. [6].

Fig. 3 The form factors $f_{\pm}(\eta)$ for the $B \rightarrow \pi$ transition. The notations are the same as in Fig. 2.

Fig. 4 The form factors $f_{\pm}(\eta)$ for the $D \rightarrow K$ transition. The notations are the same as in Fig. 2.

Fig. 5 The form factors $f_{\pm}(\eta)$ for the $D \rightarrow \pi$ transition. The notations are the same as in Fig. 2.

Fig. 6 The form factor $f_+(\eta)$ for the $K \rightarrow \pi$ transition. The notations are the same as in Fig. 2.
\[ P_1 - k = (A) + (B) \]

N. Demchuk et al., Sov. J. Nucl. Phys.: fig. 1
$B \rightarrow D \ell \nu_\ell$

N. Demchuk et al., Sov. J. Nucl. Phys.: fig. 2
\[ B \rightarrow \pi \ell \nu_\ell \]

N. Demchuk et al., Sov. J. Nucl. Phys.: fig. 3
$D \rightarrow K \ell \nu_\ell$

N. Demchuk et al., Sov. J. Nucl. Phys.: fig. 4
\[ D \rightarrow \pi \ell \nu_\ell \]

N. Demchuk et al., Sov. J. Nucl. Phys.: fig. 5
$K \rightarrow \pi \ell \nu_\ell$

N. Demchuk et al., Sov. J. Nucl. Phys.: fig. 6