Evaluating the angular power spectrum of cortical folding

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Abstract

The most defining feature of the cortex is its folding structure. While these peaks and valleys can be coarsely characterised using measures of cortical structure such as gyrification and fractal dimensionality, these are not directly sensitive to the different scales of folding that comprise the brain’s cortical structure. Here we developed an approach for characterising the angular power spectrum of cortical folding using spherical harmonics and informed by prior research investigating the cosmic microwave background. In this work, we ultimately yielded a single summary measure that is sensitive to minor folds along the cortical gyri and sulci and is sensitive to age-related differences in cortical structure.

Keywords: brain morphology; spherical harmonics; cortex; MRI; topological spatial frequency; gyrification

Introduction

The most defining feature of the cortex is its folding structure. An on-going challenge is to develop useful measures to characterise the folding pattern of the cortex and how individual brains may differ in their folding. Currently, the most established approach is a measure of gyrification (Armstrong, Schleicher, Omran, Curtis, & Zilles, 1995; Zilles, Armstrong, Schleicher, & Kretschmann, 1988), which is based on the ratio of cortical surface area relative to an estimated smooth surface that encloses the cortex. Sulci can also be identified and then quantified based on their width and depth (Madan, 2019a). Another approach is to treat the cortex as a complex natural structure and quantify its complexity using fractal geometry (Hofman, 1991; Kiselev, Hahn, & Auer, 2003). Recent findings have demonstrated that these approaches can be useful in characterising cortical structure and age-related differences in cortical structure (Madan & Kensinger, 2016, 2018).

Here we propose an alternative measure, the angular power spectrum of cortical folding. This approach relies on spherical harmonics, where the cortical structure is reconstructed using basis functions that vary in their topological spatial frequency of folding. This approach complements recent work to develop novel measures of brain morphology for subcortical structures (Madan, 2019b), with the overarching aim of improving our understanding of individual differences in brain structure associated with healthy aging, neurological disorders, cognitive abilities, and other inter-individual differences.

Methods

Calculation

As a Fourier approach can be used to approximate a complex natural linear function using weighted combinations of sinusoidal functions, spherical harmonics can be used to reconstruct complex structures through weighted summation of spherical harmonic bases that vary in topological frequency (degree, ℓ) and polarity (order, m). Figure 1 illustrates the spherical harmonics bases for positive orders.

Figure 1: Illustration of the spherical harmonic multipole components (i.e., bases), for positive orders. Components with negative orders appear analogous to the positive-order components, but with different polarity orientations.

Multipole components Spherical harmonics are based on an underlying spherical coordinate system, where θ ranges from one pole to another (i.e., [0, π] or 180°, akin to latitude) and φ wraps around the ‘equator’ (i.e., [0, 2π] or 360°, akin to longitude) following Euler angle conventions, as shown in the inset of Figure 1. Degree ℓ denotes the degree of the spherical harmonic multipole, with larger values corresponding to higher topological spatial frequencies. ℓ = 0 has no poles, ℓ = 1 has a singular polarity gradient (akin to a magnet). Higher degrees, corresponding to multiple poles, have more complicated arrangements but in combination can be used to...
approximate complex three-dimensional structures, including
the folding pattern of a human cortical surface, as in Figure 2.
Each degree $\ell$ has orders $m$ from $-\ell$ to $\ell$, yielding $2\ell + 1$
components for each degree. When constructing a com-
plex structure using spherical harmonics, each combination
of degree–order, which are individually referred to as multi-
pole components or bases, is multiplied with a correspond-
ing set of weights ($a_{\ell m}$), corresponding to the amplitudes for
each $\ell, m$ component. Due to the spherical system, unlike
Fourier bases, spherical harmonic multipole components can
present as distinct types of patterns, referred to as zonal (e.g.,
$\ell = 20, m = 0$ in Figure 1), sectorial (e.g., $\ell = 20, m = 20$), and
tesseral (e.g., $\ell = 20, m = 10$). The pattern of each multipole
component, $Y^m_\ell(\theta, \phi)$, is defined as:

$$
Y^m_\ell(\theta, \phi) = \begin{cases} 
  c_{\ell m} Y^{|m|}_\ell(\cos \theta) \sin(|m| \phi), & -\ell \leq m \leq -1 \\
  \frac{2m}{\ell \pi} Y^{|m|}_\ell(\cos \theta), & m = 0 \\
  c_{\ell m} Y^{|m|}_\ell(\cos \theta) \cos(|m| \phi), & 1 \leq m \leq \ell
\end{cases}
$$

where $Y^{|m|}_\ell(\theta)$ is the Legendre polynomial of degree
$\ell$ and order $m$, and $c_{\ell m}$ is a normalisation factor,
$\sqrt{(2\ell + 1)/(2\pi)(\ell - |m|)!/(\ell + |m|)!}$.

**Spherical harmonic cortical reconstruction** Cortical sur-
faces can be reconstructed using spherical harmonics, as
has been done in previous studies (e.g., Chung, Dalton,
Shen, Evans, & Davidson, 2007; Madan & Kensinger, 2017;
Williams, El-Baz, Nitzken, Switala, & Casanova, 2012). Here,
spherical harmonic multipole component amplitudes, $a_{\ell m}$,
were fit to the FreeSurfer cortical meshes using a weighted
Fourier series approach, following the method described in
Chung et al. (2007) and code from Chung (2014). This ap-
proach relies on a heat kernel smoothing method with a de-


$\ell_{max}$ available for approximating the cortical surface.

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**Angular power spectrum** The relationship between the
spherical harmonic degree $\ell$ and the angular scale of the
cortical folding (in degrees, $\lambda$) is defined by $180^\circ/\ell$. This cor-
borates the visual pattern evident in Figure 2: High power at
$\ell = 1$ (or $\lambda = 180^\circ$) corresponds to overall elongation of the
sphere in to a ellipsoid, while $\ell = 2$ (or $\lambda = 90^\circ$) adds an addi-
tional folding component in the orthogonal axis. The addition
of further degrees of spherical harmonics yields a structure
similar to an inflated brain surface at $\ell = 4$. Major gyri are
observable in the range of degrees $\ell = 10$ through 15, cor-
responding to angular folding of approximately $12 - 18^\circ$. Mi-
nor folds along the gyri are visible starting from approximately
$\ell = 30$ (or $\lambda = 6^\circ$).

The power for each spherical harmonic degree $\ell$, $C_\ell$, is de-

defined as the mean of the power across the $2\ell + 1$ multipole
components:

$$
C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2
$$

This equation is consistent with previous work on spherical
harmonics (e.g., Hinshaw et al., 2003). Note that the power
spectrum in studies of the cosmic microwave background is
often plotted re-scaled as $\ell(\ell+1)C_\ell/2\pi$ (Hinshaw et al., 2003;
Nolta et al., 2009; Souradeep, Saha, & Jain, 2006; Tegmark,
1997) or the square root of this value (Miller et al., 1999;
Tegmark, 1997), however, this is intended to normalise for the initial conditions and inflation of the universe (i.e., the Sachs-Wolfe plateau, $\ell \lesssim 100$) which is not relevant to the current investigation. The power spectrum of cortical folding is shown in the upper portion of Figure 3.

**Summarising the power spectrum** To make this power spectrum measure more straightforward to use as a gross measure of cortical structure for inter-individual difference analyses, we sought to determine a single summary measure. $R^2$ values between the power, $C_\ell$, and age were relatively consistent for degrees $\ell$ of 15 and higher, as shown in Figure 3 (middle). However, as untransformed values, power $C_\ell$ decreases drastically in relation to degree $\ell$. To make variations in angular power more comparable across degrees, power values were log-transformed, see upper portion of Figure 3 (right y-axis). This transformation had a negligible effect on the $R^2$ values for individual degrees, but would make averaging across degrees more consistent (rather than being more heavily weighted on the larger power on the lower topological spatial frequencies/higher angular scales. Taken together, we developed $\gamma$ as a summary measure of the angular power spectrum of cortical folding:

$$\gamma(\ell_{\text{min}}, \ell_{\text{max}}) = \frac{1}{\ell_{\text{max}} - \ell_{\text{min}} + 1} \sum_{\ell = \ell_{\text{min}}}^{\ell_{\text{max}}} \log(C_\ell)$$

A number of other summary statistics were examined, e.g., fitting a decreasing power function to the power spectrum values, but these were found to be less sensitive to inter-individual differences in cortical structure.

**Dataset**

Data consisted of 315 healthy adults (198 females), aged 20–89, from wave 1 of the Dallas Lifespan Brain Study (DLBS). Participants were screened for neurological and psychiatric issues. All participants scored 26 or above on the MMSE. T1 volumes were acquired using a Philips Achieva 3 T with an MPRAGE sequence. Scan parameters were: TR=8.1 ms; TE=3.7 ms; flip angle=12°; voxel size=1×1×1 mm. See Kennedy et al. (2015) for further details about the dataset.

**Preprocessing of MRI data**

The T1-weighted structural MRIs were processed using FreeSurfer v6.0 (Fischl, 2012). Surface meshes were estimated using the standard processing pipeline, i.e., recon-all, and no manual edits were made to the surfaces. 1 participant (female) was excluded from further analyses due to a failure to reconstruct the cortical surface.

**Results**

The overall angular power spectrum of cortical folding is shown in the upper portion of Figure 3. This approach to characterising cortical structure has never been done before.

As the $R^2$ values for individual degrees $\ell$ with age are relatively consistent for both $C_\ell$ and $\log(C_\ell)$ are highly consistent, the log measure appears preferable. The mean correlation of $C_\ell$ for degrees 15 through 50 is $-0.343$; the same measure for $\log(C_\ell)$ is $-0.338$. However, if the mean $C_\ell$ is calculated first and then correlated with age, as opposed to averaging across correlation statistics, is $-0.508$; again, the comparable statistic
for $\log(C_\ell)$ is $-0.458$. Despite this small decrease in correlation strength, the log transformation increases the sensitivity of the correlation to amplitudes of the higher degrees. Here we consider this more uniform influence of degree $\ell$ to be preferable as a summary statistic, even though power at higher degrees was slightly less related to age effects. Table 1 lists a series of correlations conducted to aid in evaluating the relationship between angular power spectrum and age, though further development is needed. As visible in the lower portion of Figure 3, there appear to be several distinct components to the relationship between power at different degrees.

| Correlation ($r$) | $\ell_{\text{min}}$ | $\ell_{\text{max}}$ | $C_\ell$ | $\log(C_\ell)$ |
|------------------|---------------------|---------------------|---------|----------------|
|                  | 15                  | 50                  | $-0.508$| $-0.458$       |
|                  | 15                  | 30                  | $-0.508$| $-0.499$       |
|                  | 30                  | 50                  | $-0.442$| $-0.410$       |
|                  | 0                   | 0                   | $+1.140$| $+1.159$       |
|                  | 1                   | 5                   | $-1.141$| $-1.150$       |
|                  | 8                   | 12                  | $-0.286$| $-0.321$       |

Table 1: Correlation values between mean angular power spectrum and age, for degrees between $\ell_{\text{min}}$ and $\ell_{\text{max}}$, both with and without log transformation.

**Conclusion**

While many features of cortical structure are known to differ in relation to inter-individual differences, here we developed a novel measure based on the angular power spectrum of cortical folding, $\gamma$. The primary aim is that additional measures of structure will capture unique sources of variance and allow us to better understand how inter-individual factors are reflected in brain structure (see Madan & Kensinger, 2018, and Madan, 2019b, for related investigations). Further work will be necessary to explore how this measure relates to other measures of cortical structure—such as cortical thickness, gyriﬁcation, sulcal morphology, and fractal dimensionality.

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