Two-loop QED with External Magnetic Field

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Abstract

In two-loop effective Lagrangian, the low-temperature expansion of the $QED_{3+1}$ with a constant magnetic field and a finite chemical potential is performed. We then calculate the total fermion density, some components of polarization operator and de Hass-van Alphen oscillations. We find that there is a significant contribution from two-loop expansion to magnetization and fermion density for higher values of chemical potentials.

I. INTRODUCTION

In the last two decades, it has been argued that in the stellar objects such as surface of neutron stars [1,2], supernovae [3,4] and white magnetic dwarfs [2,5], the magnetic field strength and the fermion density is very high. Therefore, the corresponding quantum corrections are important [6] to such astrophysical objects. From the sample of more than 400 pulsars, the range of the surface magnetic field strength is in the range of $2 \times 10^{10} \text{ G} \leq H \leq 2 \times 10^{13} \text{ G}$ [7]. Though the magnetic fields permeate most astrophysical systems, but their origin is not known. There are two different physical mechanism leading to an amplification of some initial magnetic field in a collapsing stars. The one proposed by Bisnovatyi-Kogan [8] is due to differential rotation and other one is due to a dynamo mechanism proposed by Thompson and Duncan [9]. The magnetic fields as strong as $H \sim 10^{14} - 10^{16} \text{ G}$, or even more, might be generated in new born neutron stars. Recently, it has been argued that at the time of the primodial nucleosynthesis for the primodial Big-Bang
plasma, the amplitude of magnetic field fluctuations can be as large as \( H \simeq 10^{14} \text{ G} \) [10]. A magnetic fields upto the order \( H \simeq 10^{17} \text{ G} \) [11] also has been suggested for extra galactic gamma bursts in terms of mergers of massive binary stars. Even larger macroscopic magnetic fields \( (H \geq 10^{18} \text{ G}) \) can be contemplated are superconducting strings [12], which are more speculative systems. Very recently [13], it has been suggested that if the Higgs field possesses different electro-weak phases in neighboring regions may produce a magnetic field \( (H \sim 10^{21} \text{ G}) \). Since, the plasma of thermal equilibrium can sustain fluctuations of the electromagnetic fields, it is advisable to study the \( QED \) in such high magnetic fields.

In the present paper, we consider the constant and uniform magnetic fields \( (H) \) within \( QED \) with chemical potential \( \mu \), temperature \( T \). The one-loop effective \( QED \) Lagrangian \( L^{\text{eff}}(H, \mu, T) \) in the finite temperature and density is intensively discussed [14]. Also, it has been discussed and derived to obtain a low-temperature expansion of the one-loop effective Lagrangian for a wider range of parameters \( \mu \) and \( H \) [15]. The fermion density, the magnetization, the Hall conductivity and some components of the polarization operator in the static limit \( p_o = 0, \vec{p} \rightarrow 0 \) have also been derived [15]. Performing the expression upto two-loop effective Lagrangian, we shall obtain temperature corrections to the fermion density and de Haas-van Alphen oscillations for \( T^2 \ll eH, T^2 \ll \mu^2 - m^2 \). Then following the Ref. [15], we shall calculate some components upto the two-loop polarization operator in the static limit as well as Hall conductivity in the \( QED \). Here, we have considered only the temperature shifts in the energy of an electron or positron caused by the interaction with the plasma electrons and the plasma positrons in two-loop corrections. In the two-loop expressions for simplicity, we neglect the radiative shift of the energy of an electron or a positron in a constant magnetic field and shift in the energy of an electron or a positron caused by the interaction with equilibrium radiations.

**II. LOW-TEMPERATURE CORRECTIONS**

The \( QED \) Lagrangian with uniform magnetic field \( H \) at finite density, and at finite chemical potential \( \mu \) is:
\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\partial - eA - \gamma_0\mu - m)\psi.
\]

Here, we choose the external magnetic field to be parallel to the z-axis, \( F_{12} = -F_{21} = H \).

The effective Lagrangian at temperature \( T \), \( \mu \), \( H \neq 0 \), \( \mathcal{L}^{\text{eff}}(H, \mu, T) \) is written as \([14, 15]\):

\[
\mathcal{L}^{\text{eff}}(H, \mu, T) = \mathcal{L}^{\text{eff}}(H) + \tilde{\mathcal{L}}^{\text{eff}}(H, \mu, T),
\]

where \( \tilde{\mathcal{L}}^{\text{eff}}(H, \mu, T) = \mathcal{L}^{\text{eff}}_1(H, \mu, T) + \mathcal{L}^{\text{eff}}_2(H, \mu, T) \), correspond to one-loop and two-loop effective Lagrangian. The one -loop effective Lagrangian is given in Ref. \([15]\), which is:

\[
\mathcal{L}^{\text{eff}}_1(H, \mu, T) = \frac{eH}{2\pi^2} \sum_{n=0}^{\infty} b_n \int_{-\infty}^{\infty} dp_z \frac{p_z^2}{\varepsilon_n(p_z)} \left( f_+(T) + f_-(T) \right).
\]

Whereas, we follow the procedure as given in Ref. \([16]\) to derive the two - loop effective Lagrangian, which may be written as:

\[
\mathcal{L}^{\text{eff}}_2(H, \mu, T) = \frac{eH}{2\pi^2} \frac{\alpha}{2\pi} \sum_{n=0}^{\infty} b_n \int_{-\infty}^{\infty} dp_z \left[ \ln(\frac{p_z^2}{eH}) + 1 \right] (f_+(T) + f_-(T)).
\]

These expressions are the contribution due to the finite temperature and density. Here, \( p_z \) is the momentum parallel to the magnetic field and \( b_n \equiv 2 - \delta_{n0} \), for \( n = 0 \), \( b_0 = 1 \) is the lowest Landau level. In Eq. (2), \( \mathcal{L}^{\text{eff}}(H) \) is the Schwinger Lagrangian in the purely magnetic case \([17]\):

\[
\mathcal{L}^{\text{eff}}(H) = -\frac{1}{8\pi^2} \int_0^{\infty} \frac{ds}{s^3} \left[ eHS \coth(eHS) - 1 - \frac{1}{3}(eHS)^2 \right] \exp(-m^2s).
\]

In the expressions Eq.(3) and Eq.(4), \( f_{\pm}(T) \) represents the Fermi distribution,

\[
f_{\pm}(T) = \frac{1}{1 + e^{\beta(\epsilon_{\pm} - \mu)}}.
\]

Since we are considering the low-temperature limit of the \( QED_{3+1} \), in the zero temperature limit \( (T \to 0) \), we can replace the Fermi distribution to the step-function, \( \lim_{T \to 0} f_{\pm} = \theta(\pm \mu - \epsilon) \) and Eq. (3,4) may be written as:

\[
\mathcal{L}^{\text{eff}}_1(T = 0, H, \mu) = \\
\frac{eH}{2\pi^2} \sum_{n=0}^{\infty} b_n \left\{ \mu\sqrt{\mu^2 - m^2 - 2eHn} - (m^2 + 2eHn) \ln(\frac{\mu + \sqrt{\mu^2 - m^2 - 2eHn}}{\sqrt{m^2 + 2eHn}}) \right\},
\]

3
\[ L^{2\text{eff}}(T = 0, H, \mu) = \]  
\[ \frac{eH \alpha}{2\pi^2} \left( \frac{\mu^2}{2eH} \right) \sum_{n=0} \left( \mu^2 - m^2 - 2eHn \right) \ln \frac{\mu^2 - m^2 - 2eHn}{eH} \right) \right]. \]

Where the upper limit of \( n \) sum may be obtained from the relation \( \mu^2 - m^2 - 2eHn \geq 0 \).

To evaluate the low-temperature corrections to the effective Lagrangian (3, 4), we follow the procedure as given in Ref. [15]. First we take the derivative of \( \tilde{L}^{\text{eff}}(H, \mu, T) \) with respect to \( T \), with a fixed chemical potential. After some purely algebraic manipulations and integrating with respect to temperature, we obtain the temperature corrections to the zero temperature Lagrangian Eq. (7),

\[ \Delta L^{1\text{eff}}(T, H, \mu) = \frac{eH T^2}{2} \left( \frac{\mu^2}{2eH} \right) \sum_{n=0} \left( \mu^2 - m^2 - 2eHn \right) \frac{\mu}{(\mu^2 - m^2 - 2eHn)^{1/2}} + O(T^4), \]

\[ \Delta L^{2\text{eff}}(T, H, \mu) = \frac{eH T^2}{2} \left( \frac{\mu^2}{2eH} \right) \sum_{n=0} b_n \left[ 1 + \frac{2\mu^2}{\mu^2 - m^2 - 2eHn} + \ln \frac{\mu^2 - m^2 - 2eHn}{eH} \right] + O(T^4). \]

(8)

In the above expressions, we differ a factor of 1/2 from Ref. [15] for the one-loop corrections. The above expansions are valid for \( \frac{T}{\sqrt{m^2 + 2eHn}} \ll 1 \).

In figure 1, we present the ratio of total effective Lagrangian (including two-loop corrections) to one-loop effective Lagrangian as a function of magnetic field for fixed chemical potential. Where case (a), (b) and (c) are for \( \mu = 2, 5 \) and 10 MeV respectively. We noticed that the ratio is high for large value of chemical potential and it decreases with increase in magnetic fields. The ratios are quite different from each other for various values of chemical potentials with variation of magnetic fields.

Now, we can write a low-temperature expansions to the total effective Lagrangian including both the one-loop and two-loop effective Lagrangian:

\[ \tilde{L}^{\text{eff}}(T, H, \mu) = \]
III. FERMION DENSITY AND MAGNETIZATION

Using the total effective Lagrangian (9), one would calculate the fermion density \( \rho = \frac{\partial L_{\text{eff}}}{\partial \mu} \), the magnetization \( M = \frac{\partial L_{\text{eff}}}{\partial H} \), the Hall conductivity and some components of the polarization operator in the static limit \( p_0 = 0, \ p \to 0 \).

Thus the total fermion density is: \( \rho = \frac{\partial L_{\text{eff}}}{\partial \mu} \) one has:

\[
\rho(H, \mu, T) = \frac{eH}{2\pi^2} \sum_{n=0}^{\infty} b_n \left\{ \mu \sqrt{\mu^2 - m^2 - 2eHn} \left[ 1 - \frac{T^2\pi^2}{12} \frac{m^2 + 2eHn}{(\mu^2 - m^2 - 2eHn)^2} \right] \right\} + O(T^4).
\]

In Fig. 2, we show the fermion density as a function of the chemical potential for fixed magnetic field \( H \approx 10^{15} \text{ G} \) with (a) one - loop corrections and (b) one - loop and two - loop corrections. We noticed that the fermion density is showing an oscillating behavior as consecutive Landau levels are passing the Fermi level for all the cases. There is not much contribution from two - loop corrections to the fermion density in this case.

The fermion density as a function of magnetic field for fixed chemical potential \( \mu = 10 \text{ MeV} \) is shown in figure 3. The fermion density is showing an oscillating behavior as consecutive Landau
levels are passing the Fermi level. The fermion density is higher in case (b) due to the contribution of two-loop corrections. If one decreases the value of chemical potential, the two-loop corrections to fermion density reduces to one-loop corrections, case (a). In case of neutron stars, we would like to mention that the chemical potential of fermions are very high [14] and hence the two-loop corrections is significant and important.

The magnetization $M$ is derived from the total effective Lagrangian (9), which is:

$$M(H, \mu, T) = \frac{e}{2\pi^2} \sum_{n=0}^{\infty} b_n \left\{ \mu \sqrt{\mu^2 - m^2 - 2eHn} - (m^2 + 4eHn) \ln \frac{\mu + \sqrt{\mu^2 - m^2 - 2eHn}}{\sqrt{m^2 + 2eHn}} \right. $$

$$+ \frac{\pi^2 T^2}{6} \frac{\mu(\mu^2 - m^2 - eHn)}{(\mu^2 - m^2 - 2eHn)^{3/2}} + \frac{\alpha}{2\pi} \left[ (\mu^2 - m^2 - 4eHn) \ln \frac{\mu^2 - m^2 - 2eHn}{eH} - (\mu^2 - m^2) \right] $$

$$+ \frac{T^2 \pi^2}{6} \left( \frac{2\mu^2(\mu^2 - m^2) - 2eHn(\mu^2 - m^2 - 2eHn)}{(\mu^2 - m^2 - 2eHn)^2} + \ln \frac{\mu^2 - m^2 - 2eHn}{eH} \right) \right\}.$$ (11)

Since, our main aim is to focus the effect of two-loop contribution, for simplicity, we have neglected the vacuum magnetization in our calculations. The magnetic susceptibility $\chi$ is defined as $(\partial M/\partial H)$. The magnetization and magnetic susceptibility are useful to calculate the spatial components of polarization operators [18].

Figure 4 shows the magnetization as a function of magnetic field for fixed chemical potential ($\mu = 10$ MeV). We have not considered the vacuum magnetization in this figure, because the vacuum contribution is small [14]. We noticed that the fermion gas exhibits the de Hass - van Alphen effect without and with inclusion of two-loop corrections curve (a) and curve (b) respectively, which is in agreement with Ref. [14]. The magnetization is high at small values of magnetic fields and reaches to one-loop magnetization for large values of magnetic fields. So, the two-loop contribution to magnetization is significant at particular range of magnetic fields.

Next, we calculate the some components of the polarization operator in the static limit. In Ref. [15], it has been shown that the $\Pi_{00}$ -component of the polarization operator is nothing but the derivative of the total fermion density with respect to the chemical potential in the static limit,
e.g., $\Pi_{00}(p_0 = 0, \mathbf{p} \to 0) = e^2 \frac{\partial \rho}{\partial \mu}$,

$$\Pi_{00}(p_0 = 0, \mathbf{p} \to 0) =$$

$$e^2 \frac{eH}{\pi^2} \left[ \sum_{n=0}^{\infty} b_n \left\{ \mu (\mu^2 - m^2 - 2eHn)^{-1/2} + \frac{T^2 \pi^2}{4} \frac{\mu (m^2 + 2eHn)}{(\mu^2 - m^2 - 2eHn)^{5/2}} \right. \right]$$

$$+ \frac{\alpha}{2\pi} \left[ \left( 1 + \frac{2\mu^2}{\mu^2 - m^2 - 2eHn} + \ln \frac{\mu^2 - m^2 - 2eHn}{eH} \right) \right]$$

$$+ \frac{T^2 \pi^2}{6} \left[ \frac{3(\mu^2 - m^2 - 2eHn)^2 - 4\mu^2 (\mu^2 - 3m^2 - 6eHn)}{(\mu^2 - m^2 - 2eHn)^3} \right] \right\} \right].$$

(12)

In the above expression, we have included the two-loop corrections. At zero magnetic field the

Eq.(12) defines the Debye screening radius, $r_D^{-2} = \Pi_{00}(H = 0, p_0 = 0, \mathbf{p} \to 0)$ [19], but this is not

valid for $\mu$ and $H \neq 0$.

The other two components $\Pi_{01}$ and $\Pi_{02}$ and their conjugates may be evaluated by taking the

derivatives of the fermion density with respect to magnetic field in the static limit [15,19,20], which

has been discussed more explicitly in Ref. [15]:

$$\Pi_{0j}(p \to 0) = ie\varepsilon_{ij} p_i \frac{\partial \rho}{\partial H_j} \quad i, j = 1, 2.$$

(13)

It has been shown in Ref. [15] that the components $\Pi_{0j}$ describe a conductivity in the plane

orthogonal to the magnetic field which is Hall-like [15,21]:

$$\sigma_{ij} = i \frac{\partial \Pi_{0i}(p)}{\partial p_j} \bigg|_{p \to 0} = e\varepsilon_{ij} \frac{\partial \rho}{\partial H} \quad i, j = 1, 2.$$

(14)

Using Eq.(14) and the expression for the fermion density Eq.(10), one has

$$\Pi_{0j}(p_0, \mathbf{p} \to 0) = \frac{i\varepsilon_{ij} p_i}{\pi^2} \left[ \sum_{n=0}^{\infty} b_n \times \right]$$

$$\left\{ \frac{\mu^2 - m^2 - 3eHn}{(\mu^2 - m^2 - 2eHn)^{1/2}} - \frac{T^2 \pi^2}{12} \left( \frac{(\mu^2 - m^2 - 2eHn)(m^2 + 2eHn) + eHn(2\mu^2 + m^2 + 2eHn)}{(\mu^2 - m^2 - 2eHn)^{5/2}} \right) \right\}$$

$$+ \frac{\alpha}{2\pi} \mu \left[ \ln \frac{\mu^2 - m^2 - 2eHn}{eH} - \frac{2eHn}{\mu^2 - m^2 - 2eHn} \right].$$


\[ -\frac{T^2 \pi^2}{6} \left( \frac{2eHn(\mu^2 + 3m^2 + 6eHn) + (\mu^2 - m^2 - 2eHn)(3m^2 + 6eHn - \mu^2)}{\mu^2 - m^2 - 2eHn}^3 \right) \}. \tag{15} \]

It has been pointed out in Ref. [15] that the above expression is the Hall conductivity in the \( QED_{3+1} \) is an oscillating function of the chemical potential and the magnetic field in one-loop corrections, which are close to “giant oscillations”, well-known in condensed matter physics [22] and resonant effects in \( QED [6] \) and semiconductors [23]. Thus, with inclusion of two-loop corrections to one-loop corrections, we found from figure 3 that it enhances the oscillations further.

IV. WEAK MAGNETIC FIELD LIMIT

In the weak magnetic field limit, the state \( n \) can be replaced in the limit \( 2eHn = \theta \)

\[ \sum_{n=0}^{n^2-m^2} = \int_{0}^{\mu^2-m^2} d\theta n = \lim_{H \to 0} \frac{1}{2eH} \int_{0}^{2eHn} d\theta \] \tag{16}

Substituting Eq. (16) in Eq. (7), we have the effective Lagrangian as:

\[ \hat{L}_{\text{eff}}(T = 0, H_0 \ll (\mu^2 - m^2), \mu) \approx \frac{1}{(2\pi)^2} \left[ \frac{\mu(2\mu^2 - 5m^2)(\sqrt{\mu^2 - m^2})}{6} + \frac{m^4}{2} \ln \frac{\mu + \sqrt{\mu^2 - m^2}}{m} \right] \]

\[ + \frac{\alpha}{2\pi} \left( \frac{\mu^2 - m^2}{2} \right)^2 \left[ \ln \left| \frac{m^2 - \mu^2}{eH_0} \right| - \frac{1}{2} \right] \tag{17} \]

The fermion density is calculated from Eq. (11) for weak field limit at zero temperature by substituting \( 2eHn = \theta \), we get

\[ \rho(H_0 \ll (\mu^2 - m^2), \mu, T = 0) \approx \frac{1}{2\pi^2} \left\{ \frac{2}{3}(\mu^2 - m^2)^{3/2} \right. \]

\[ + \frac{\alpha}{2\pi} \left[ \mu(\mu^2 - m^2) \ln \left| \frac{\mu^2 - m^2}{eH_0} \right| \right] \} \tag{18} \]

In figure 5, we plot the ratio fermion density \( \rho(H, \mu, T = 0)/\rho(H_0 \ll (\mu^2 - m^2), \mu, T = 0) \) as a function of chemical potential with fixed magnetic field, (a) \( H \approx 5 \times 10^{15} \) G and (b) \( H \approx 10^{16} \) G. We choose the weak field limit \( H_0 \) to be \( 4.4 \times 10^{13} \) G. As we increase the chemical potential, the
ratio decreases and reaches to unity for both curve (a) and (b). We noticed that the magnetic field has significant contribution to the fermion density and hence to the total effective Lagrangian.

V. CONCLUSION

In conclusion, we discussed the effect of the two-loop QED effective Lagrangian at finite temperature and density in a constant external magnetic fields. In particularly following Ref. [15], we compute the low-temperature corrections to the effective Lagrangian. Also, we improved the calculation by including the two-loop corrections in the fermion density, magnetization and the some components of the polarization operators responsible for the Hall conductivity and Debye screening in a finite fermion density $QED_{3+1}$ with uniform magnetic fields. At the end, we derived the effective Lagrangian in the weak field limit at zero temperature. In this paper, we found that the two-loop corrections has a significant contribution to the fermion density and magnetization in variation with magnetic fields for higher chemical potentials. So, we do expect to the de Hass-van Alphen oscillations in the astrophysical objects like neutron stars and will be published elsewhere. Finally, it would be some interest to extend this work in $QED_{2+1}$. Also, one could treat the present calculation in slowly varying electric and magnetic fields.

VI. ACKNOWLEDGEMENT

I would like to thank R. Parwani for introducing this problem. It is great pleasure to thank S. Mohanty for useful discussions and Per Elmfors for comments.
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FIGURES

FIG. 1. The ratio of total effective Lagrangian \( \mathcal{L}_{\text{tot}} \) (including two-loop corrections) to one-loop effective Lagrangian \( \mathcal{L}_1 \) as a function of magnetic field for fixed chemical potential. Where curves (a), (b) and (c) are for \( \mu = 2, 5 \) and 10 MeV respectively.

FIG. 2. The fermion density as a function of the chemical potential for fixed magnetic field \( (H \approx 10^{15} \text{ G}) \) with (a) one-loop corrections and (b) one-loop and two-loop corrections.

FIG. 3. The fermion density vs magnetic field for fixed chemical potential \( (\mu = 10 \text{ MeV}) \). Curves (a) and (b) are same as figure 2.

FIG. 4. The magnetization as a function of magnetic field for fixed chemical potential \( (\mu = 10 \text{ MeV}) \). Curves (a) and (b) are same as figure 2.

FIG. 5. The ratio fermion density \( \rho(H, \mu, T = 0)/\rho(H_0 \ll (\mu^2 - m^2), \mu, T = 0) \) as a function of chemical potential with fixed magnetic field, (a) \( H \approx 5 \times 10^{15} \text{ G} \) and (b) \( H \approx 10^{16} \text{ G} \). Here the weak field limit \( H_0 \) to be \( 4.4 \times 10^{13} \text{ G} \).