Instability of the de Sitter Reissner–Nordstrom black hole in the 1/D expansion

Kentaro Tanabe

Theory Center, Institute of Particles and Nuclear Studies, KEK, Tsukuba, Ibaraki, 305-0801, Japan

E-mail: ktanabe@post.kek.jp

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Abstract

We study the large D effective theory for D dimensional charged (Anti) de Sitter black holes. Then we show that the de Sitter Reissner–Nordstrom black hole becomes unstable against gravitational perturbations at larger charge than a critical charge in a higher dimension. Furthermore, we find that there is a nontrivial zero-mode static perturbation at the critical charge. The existence of static perturbations suggests the appearance of non-spherical symmetric solution branches of static charged de Sitter black holes. This expectation is confirmed by constructing the non-spherical symmetric static solutions of large D effective equations.

Keywords: classical gravity, black hole physics, perturbation analysis

1. Introduction

The effect of a cosmological constant on black hole properties is quite nontrivial. As one example, Konoplya and Zhidenko [1] found that the de Sitter Reissner–Nordstrom black hole becomes unstable at enough large charge in $D > 6$ dimensions (see also [2, 3]). This instability of the de Sitter Reissner–Nordstrom black hole is a remarkable property of black hole solution with a cosmological constant (for stability of other static black holes, see [6, 7]). Furthermore, the paper [1] found that there is a nontrivial zero-mode perturbation at the edge of the instability, and they constructed static deformed solutions of the zero-mode perturbation. This perturbative deformed static solution suggests the existence of a deformed static charged de Sitter black hole solution in contrast to the uniqueness of the static charged asymptotically flat black hole [8].

1 The instability and static deformed solution of the de Sitter Reissner–Nordstrom black hole were also observed independently in the Freund–Rubin solution, which is the Nariai limit of the de Sitter Reissner–Nordstrom black hole [4, 5].
The purpose of this paper is to study some properties of charged (Anti) de Sitter black holes such as quasinormal modes and non-linear deformations of static solutions by using the large $D$ expansion method \cite{9}. In particular, we derive large $D$ effective equations for charged (Anti) de Sitter black holes in a similar way to \cite{10, 11} in section 2. These effective equations describe $O(1/D)$ non-linear dynamical deformations of charged black holes. Then we obtain an analytic formula for quasinormal mode frequencies of the (Anti) de Sitter Reissner–Nordstrom black hole by perturbation analysis of the effective equations in section 3. From this analytic formula the instability modes and their thresholds of the de Sitter Reissner–Nordstrom black hole are identified. As found in \cite{1} the threshold modes are static zero-mode perturbations, which suggest the existence of deformed non-spherical symmetrically charged de Sitter black hole solutions in higher dimensions. In this paper, the existence of deformed static charged black hole solutions will be shown explicitly by constructing the solutions of large $D$ effective equations in a non-linear way in section 4. These results show that the large $D$ expansion method is also a powerful tool to investigate black hole properties for charged and non-asymptotically flat solutions.

2. Large $D$ effective equations

We derive effective equations for charged (Anti) de Sitter black holes by a $1/D$ expansion method. We solve the Einstein–Maxwell equation with a cosmological constant and the Maxwell equation

\[
R_{\mu\nu} - \frac{1}{2} R\delta_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{2} \left( F_{\mu\rho} F^\rho_{\nu} - \frac{1}{4} F_{\mu\rho} F^{\rho\sigma} g_{\nu\sigma} \right), \quad \nabla^\mu F_{\mu\nu} = 0, \tag{2.1}
\]

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The metric ansatz for $D = n + 3$ dimensional charged black holes in ingoing Eddington–Finkelstein coordinates is

\[
ds^2 = -Adv^2 + 2(u, dv + u, dz)dr + 2C, dv dz + r^2 G dz^2 + r^2 H^2 d\Omega_n^2. \tag{2.2}
\]

The gauge field ansatz is

\[
A_\mu dv^\mu = A_t dv + A, dz. \tag{2.3}
\]

Note that we do not consider $A_t, dr$ in the gauge field because it does not contribute to effective equations at the leading order in $1/D$ expansion. We introduce a curvature scale, $L$, of a cosmological constant given by \cite{3}

\[
\Lambda = \frac{(n + 1)(n + 2)}{2L^2}. \tag{2.4}
\]

In this coordinate, the de Sitter Reissner–Nordstrom black hole is given by

\[
ds^2 = -\left( 1 - \frac{r^2}{L^2} \right)^n - \left( \frac{r_+}{r} \right)^n + \tilde{Q}^2 \left( \frac{r_+}{r} \right)^{2n} dr^2 + 2dv dr + r^2 (dz^2 + \sin^2 z d\Omega_n^2), \tag{2.5}
\]

\[\text{2 The extension of the large } D \text{ effective theory to asymptotically flat charged black holes has been considered recently in \cite{12}.}
\[\text{3 Here we set a cosmological constant to be positive. The following analysis can also be applied to Anti-de Sitter black holes just by replacing } L \rightarrow -L.\]
and
\[ A_d d\nu^\mu = \sqrt{\frac{2(n+1)}{n} \tilde{Q} \left( \frac{r_v}{r} \right)^n} d\nu. \]  
(2.6)

\( r_v \) is a horizon radius parameter, and \( \tilde{Q} \) is a charge parameter of the solution. We construct large \( D \) effective theory for dynamical deformations of this exact solution. To obtain the effective theory, we assume that the radial gradient becomes dominant at large \( D \) as
\[ \partial_v = O(1), \partial_r = O(1) \text{ and } \partial_z = O(n). \]
Note that we use \( n \) as a large parameter instead of \( D \) in the following. It is useful to introduce new radial coordinate \( R \) defined by
\[ R = \left( \frac{r}{r_0} \right)^n, \]
(2.7)
where \( r_0 \) is a constant parameter of the solution. We set \( r_0 = 1 \). Then \( \partial_v = O(n) \) corresponds to \( \partial_R = O(1) \). The large \( n \) scalings of metric and gauge field functions are
\[ A = O(1), \quad u_v = O(1), \quad u_z = O(n^{-1}), \quad C_z = O(n^{-1}), \]
\[ A_v = O(1), \quad A_z = O(n^{-1}). \]
(2.8)

The metric functions, \( G \) and \( H \), are assumed to be
\[ G = 1 + O(n^{-1}), \quad H = H(z). \]
(2.9)

\( u_v \) is a shift vector on \( r = \text{const.} \) surface, and we choose \( u_v = u_v(v, z) \) as a gauge choice. Other metric functions and gauge field functions depend on \( vr \). The Reissner–Nordstrom black hole satisfies these large \( n \) scaling rules. We assume that the deviation from the Reissner–Nordstrom black hole starts at \( O(n) \) in the metric. For example, the Reissner–Nordstrom black hole has \( C_z = 0 \), but our solutions considered here have \( C_z = O(n^{-1}) \).

The leading orders of the field equations (2.1) contain only \( R \)-derivative, and we can solve them easily. As a result, we obtain the following leading-order solutions of the field equations in \( 1/n \) expansion
\[ A = A_0^2 \left( 1 - \frac{m}{R} + \frac{q^2}{R^2} \right), \quad u_v = \frac{\sqrt{2} q}{R}, \quad u_z = \frac{A_0 L H(z)}{\sqrt{L^2 (1 - H'(z)^2)} - H(z)^2}, \]
(2.10)

and
\[ u_v = \frac{u_v^{(0)}(z)}{n}, \quad C_z = \frac{1}{n} \left[ \frac{p_z}{R} - \frac{p_q q^2}{m R^2} \right], \quad A_z = \frac{1}{n} \sqrt{2} p_q q, \quad G = 1 + O(n^{-2}). \]
(2.11)

Here we omit terms of \( O(1/n) \) for simplicity. \( m(v, z), q(v, z) \) and \( p_z(v, z) \) are integration functions of leading-order solutions. \( m(v, z) \) is a mass density, and \( q(v, z) \) is a charge density of the solution. \( p_z(v, z) \) is a momentum density along \( z \) direction. \( u_v^{(0)}(z) \) is an arbitrary function of \( z \). \( A_0 \) is a constant describing the redshift factor of the background geometry. Although \( A_0 \) can be a function of \( z \) in general, we set \( A_0 \) to be constant\(^4\). We can see that \( g_{\nu\nu} \) vanishes at \( R = R_\pm \) given by
\[ R_\pm = \frac{m \pm \sqrt{m^2 - 4q^2}}{2}. \]
(2.12)

\(^4\) In [13] some properties of \( O(1) \) deformed static solutions with \( A_0(z) \) have been studied. Since we consider small deformation with \( O(1/n) \) amplitude in this paper, we can take \( A_0 \) to be constant. At the next-to-leading order we have the \( z \)-dependent redshift factor.
in the leading-order solution. These surfaces can be regarded as an outer and inner horizon by analogy with the Reissner–Nordstrom black hole. The extremal condition is \( m(v, z) = 2q(v, z) \), and we assume \( m(v, z) > 2q(v, z) \geq 0 \) in this paper. As boundary conditions, we imposed the regularity condition of metric functions at \( R = R_+ \) and the asymptotic condition at \( R \gg 1 \)

\[
A_v = O(R^{-1}), \quad A_z = O(R^{-1}), \quad C_z = O(R^{-1}).
\]

The momentum constraint of the Einstein–Maxwell equation on \( r = \text{const.} \) surface at the leading order gives

\[
\frac{d}{dz} \sqrt{H(z)} = 0.
\]

Using this constancy condition we define a constant \( \hat{k} \) by

\[
\hat{k} = \frac{A_0 \sqrt{H'(z)^2 - H(z)^2}}{2LH(z)}.
\]

The constant \( \hat{k} \) has a relation with the surface gravity of static solutions as we will see below.

Note that \( O(n^0) \) parts of the leading-order solutions in equations (2.10) and (2.11) have the same form as one of the Reissner–Nordstrom black holes except for \( (v, z) \) dependences in \( m \) and \( q \). This reminds us of the Birkhoff theorem for spherically symmetric solutions in the Einstein–Maxwell system. Actually, the radial dynamics becomes dominant at large \( n \), and, as a result, the leading-order solutions at large \( n \) have the same structure in the radial dynamics with spherically symmetric spacetimes. This feature can also be observed in the rotating black holes [14], and the paper [12] used this feature to construct the effective theory of general charged black holes at large \( D \).

At the next-to-leading order of the field equation (2.1), we obtain conditions for \( m(v, z) \), \( q(v, z) \) and \( p_z(v, z) \) as the effective equations for the charged (Anti) de Sitter black hole. The effective equations are

\[
\partial_z q = \frac{A_0^2 H'(z)}{2\hat{k}H(z)} \partial_z q + \frac{H'(z)}{H(z)} \frac{p_z}{m} = 0,
\]

\[
\partial_z m = \frac{A_0^2 H'(z)}{2\hat{k}H(z)} \partial_z m + \frac{H'(z)}{H(z)} \frac{p_z}{2} = 0,
\]

and

\[
\partial_z p_z = \frac{A_0^2 H'(z)}{2\hat{k}H(z)} \frac{m^2 - 4q^2}{m} \partial_z p_z + \left[ \frac{A_0^2 H'(z)}{2\hat{k}H(z)} \frac{m - \sqrt{m^2 - 4q^2}}{m} \right] \partial_z m
\]

\[
- \left[ \frac{A_0^2 (1 - H'(z)^2)}{2\hat{k}H(z)^2} - \frac{A_0^2 H'(z)^2}{2\hat{k}H(z)^2} \frac{m^2 - 4q^2}{m} - \frac{H'(z)}{H(z)} \frac{p_z}{H(z)} \right] p_z = 0.
\]

The contribution of a cosmological constant enters only through \( \hat{k} \) and \( A_0 \). We can find a static solution to effective equations by assuming \( m(v, z) = m(z) \), \( q(v, z) = q(z) \) and \( p_z(v, z) = p_z(z) \). From equations (2.16) and (2.17), \( p_z(z) \) and \( q(z) \) are found to be

\[
p_z(z) = \frac{A_0^2 m'(z)}{2\hat{k}}, \quad q(z) = \hat{Q}m(z).
\]
\( \hat{Q} \) is an integration constant, and \( q \) should be proportional to \( m \). Equation (2.18) gives a simple equation for \( m(z) = e^{P(z)} \) as

\[
P''(z) = \left[ \frac{H'(z)}{H(z)} - \frac{H(z)}{L^2 \sqrt{1 - 4\hat{Q}^2}} \right] P'(z) = 0. \tag{2.20}
\]

The static solution has the Killing vector \( \partial_\nu \). The associated surface gravity, \( \kappa \), of this static solution at the outer horizon \( R = R_+ \) is calculated as

\[
\kappa = n \hat{\kappa} \sqrt{1 - 4\hat{Q}^2} - (1 - 4\hat{Q}^2) [1 + O(n^{-1})]. \tag{2.21}
\]

Hence, the static solution has a constant surface gravity, as expected. To solve equation (2.20), we need the explicit form of \( H(z) \). The function \( H(z) \) can be specified if we consider an embedding of the leading-order solution into a background geometry. As a condition for the embedding, \( H(z) \) should satisfy the condition (2.14).

Note that we have considered static solutions of effective equations. This static solution of effective equations also describes the static spacetime. As seen in equation (2.19) the momentum \( p_z \) is given by the total derivative, so there is no global momentum along \( z \) direction. This means that the spacetime is also static.

3. Instability of the de Sitter Reissner–Nordstrom black hole

We apply effective equations to the stability analysis of the (Anti) de Sitter Reissner–Nordstrom black hole. The (Anti) de Sitter Reissner–Nordstrom black hole is realized as a static solution of effective equations in the embedding into de Sitter spacetime in the spherical coordinate. The embedding in the spherical coordinate is given by

\[
H(z) = \sin z. \tag{3.1}
\]

This embedding satisfies the condition for \( H(z) \) in equation (2.14). The redshift factor \( A_0 \) is set to be

\[
A_0 = \sqrt{1 - \frac{1}{L^2}}, \tag{3.2}
\]

and we consider the small de Sitter black hole \( L > 1 \). In this embedding, the constant \( \hat{\kappa} \) becomes

\[
\hat{\kappa} = \frac{L^2 - 1}{2L^2}. \tag{3.3}
\]

Then the de Sitter Reissner–Nordstrom black hole is given by a static solution

\[
m(v, z) = 1 + Q^2, \quad q(v, z) = Q, \quad p_z(v, z) = 0. \tag{3.4}
\]

\( Q \) is a charge parameter of the de Sitter Reissner–Nordstrom black hole. In this solution, we set the horizon radius to be unit as \( R_+ = 1 \). In this unit, the extremal limit is \( Q = 1 \). The relation of \( Q \) and \( \hat{Q} \) in equation (2.19) is

\[
\hat{Q} = \frac{Q}{1 + Q^2}. \tag{3.5}
\]
By perturbing effective equations (2.16), (2.17) and (2.18) around the static solution (3.4) as

\[ m(v, z) = (1 + Q^2)(1 + \epsilon e^{i\omega t} F_m(z)), \quad q(v, z) = Q(1 + \epsilon e^{-i\omega t} F_q(z)), \]

and

\[ p_z(v, z) = \epsilon e^{-i\omega t} F_z, \]

we obtain quasinormal modes of the de Sitter Reissner–Nordstrom black hole given by

\[ \omega_{\pm} = \pm \frac{i(\ell - 1)}{1 + Q^2} \pm \frac{1}{2} \sqrt{\ell(\ell^2(1 + Q^2) - (1 + Q^2)^2) - L^2(1 + Q^2)}} \]

for the gravitational perturbation \((F_m(z) = F_q(z))\), and

\[ \omega = -i\ell, \]

for the gauge field perturbation \((F_m(z) \neq F_q(z))\). \(\ell = 0, 1, 2, \ldots\) is the quantum number on \(S^{t+1}\). As the boundary condition we demanded \(F_m(z) \propto (\cos z)^{\ell}\) at \(z = \pi/2^5\). The modes with \(\ell > 0\) (\(\ell > 0\)) describe dynamical perturbation of gravitational perturbations (gauge field perturbations). So we consider the gravitational perturbations with \(\ell > 1\) and gauge field perturbations with \(\ell > 0\). At the zero cosmological constant limit \(L \to \infty\), these results reproduce the quasinormal modes of the Reissner–Nordstrom black hole obtained in [12]. At \(Q = 0\), \(\omega_{\pm}\) reduces to quasinormal modes of scalar type perturbation of the (Anti) de Sitter Schwarzschild black hole [15]. At both limits, the perturbations are stable. On the other hand, we can see that, as a striking effect of a positive cosmological constant, the gravitational perturbation becomes unstable at \(Q > Q_{lt}\) where

\[ Q_{lt} = \frac{L^2(\ell - 1) - 1}{L^2(\ell - 1) + 1}. \]

We can see \(Q_t > Q_{lt-2}\). Thus, the most unstable mode is \(\ell = 2\). Furthermore, \(\omega_{\pm}\) is a purely imaginary mode at \(Q > Q_t\), where

\[ Q_t \geq Q_{lt-2}. \]

The boundary condition comes from the behavior of the spherical harmonics at large \(D\) [17].
So the instability mode is always a purely imaginary mode, and there is a nontrivial zero-mode static perturbation at $Q = Q_r$. Then we expect that there is a non-spherical symmetrically static solution branch at $Q = Q_r$. In Figure 1 we show plots of gravitational quasinormal modes $\omega_n$ of the de Sitter Reissner–Nordstrom black hole. In Figure 2 we show the unstable region of the de Sitter Reissner–Nordstrom black hole in $(L,Q)$ plane. The thick line describes $Q_r$ of $\ell = 2$, and the dashed line is of $\ell = 4$. The region above each line is an unstable region for each mode perturbation. The unstable region seems to extend to $Q = 0$ at $L = 1$. However, since our solution breaks down at the extremal limit $L = 1$ due to $\tilde{\kappa} = 0$ (see equation (3.3)), our result does not immediately imply that the de Sitter Schwarzschild black hole is unstable at the Nariai limit $L = 1$. The stability at the Nariai limit was studied in [4, 5].

In the paper [3] an analytic formula for the critical charge in arbitrary dimensions was given. However the formula is not the same as our formula (3.10) even at large $D$. The analytic formula in [3] was obtained by extrapolating their numerical results in $D = 6 \sim 11$. However, there are nonperturbative contributions in $1/D$ expansions with $O(e^{-D})$ [15], and such nonperturbative corrections cannot be ignored in not much higher dimensions such as $D = 6$. So, if one uses numerical results in such dimensions to predict the $1/D$ expanded formula, the correct formula in $1/D$ expansion would not be obtained.

**AdS Reissner–Nordstrom black hole.** Replacing $L$ by $iL$, we can obtain effective equations for charged Anti-de Sitter black holes. In particular, we can obtain quasinormal modes of the Anti-de Sitter Reissner–Nordstrom black hole from equations (3.8) and (3.9) by the replacing of $L \rightarrow iL$. These quasinormal modes do not show any instabilities as can be seen in equation (3.10). In fact, we have $Q_r > 1$ for the Anti-de Sitter case. So the black hole is stable unless we consider overcharged solutions. This is consistent with numerical results [16].
4. Deformed solution branches

The perturbation analysis suggests an appearance of a non-spherical symmetrically static charged de Sitter black hole solution branch at $Q = Q_\ell$. This expectation can be confirmed directly by solving effective equations for static solutions. The equation for the static solution is given in equation (2.20). In the spherical coordinate embedding (3.1), the solution is obtained as

\[ P(z) = m_0 + b(\cos z)^{k(Q)}, \tag{4.1} \]

where

\[ k(Q) = 1 + \frac{1 + Q^2}{L^2(1 - Q^2)}. \tag{4.2} \]

$b$ is an integration constant, and it describes an $O(1/n)$ deformation amplitude from spherical symmetry. $m_0$ is an $O(1/n)$ redefinition of the horizon radius $r_0$. For general $Q$, $k(Q)$ takes a non-integer value, and the solution is not analytic at $z = \pi/2$. However, we obtain $k(Q) = \ell$ at $Q = Q_\ell$ with $\ell = 0, 1, 2, \ldots$. Then the solution becomes regular at $z = \pi/2$ for $Q = Q_\ell$, $\ell = 0$ and $\ell = 1$ are just gauge modes in the spherical coordinate embedding, so they do not describe physical deformations. For $\ell > 1$, the solution gives a static solution branch of a non-spherical symmetrically charged de Sitter black hole. Note that this deformation is similar to deformations of the Myers–Perry black hole observed in [17], that is, the bumpy black holes. So it would be interesting to study their relations to clarify the nature of the instabilities of the de Sitter Reissner–Nordstrom black hole.

5. Summary

We solved the Einstein–Maxwell equation with a cosmological constant and Maxwell equations for charged black holes by using the large $D$ expansion method. As a result, we obtained effective equations for charged (Anti) de Sitter black holes. The effective equations describe the non-linear dynamical deformations of the de Sitter Reissner–Nordstrom black holes. The perturbation analysis gives the analytic formula for quasinormal modes of the (Anti) de Sitter Reissner–Nordstrom black hole. From this formula we found that the gravitational perturbation of the de Sitter Reissner–Nordstrom black hole becomes unstable, and the quasinormal mode is purely imaginary in an unstable region. At the onset of instabilities there is a non-trivial zero-mode static perturbation. We obtained a corresponding non-linear deformed solution at the leading order in $1/D$ expansion. The non-linear solutions give solution branches of non-spherical symmetrically static charged de Sitter black holes in higher dimensions.

There are various interesting developments and extensions of this work such as the construction of a phase diagram of deformed solutions obtained in this paper, including $1/D$ corrections to compare numerical results more accurately or rotation to study the instabilities of a charged rotating de Sitter black hole. The charged rotating black hole has not been found analytically even in the asymptotically flat case, so such studies should give fruitful results. The $1/D$ corrections introduce the dimensional dependences to the results, so we may be able to observe a critical dimension, where the instabilities of the de Sitter Reissner–Nordstrom black hole disappear as they did in the analysis for the critical dimensions of a non-uniform black string [18]. It would also be quite interesting to solve effective equations for charged de Sitter black holes numerically. The effective equations can describe non-linear dynamical
evolution of the instabilities of the de Sitter Reissner–Nordstrom black hole. The numerical solutions of effective equations will give deeper understanding of the instability and its endpoint.

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