Finite size effects at phase transition in compact U(1) gauge theory

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We present and discuss the results of a Monte-Carlo simulation of the phase transition in pure compact U(1) lattice gauge theory with Wilson action on a hypercubic lattice with periodic boundary conditions. The statistics are large enough to make a thorough analysis of the size dependence of the gap. In particular we find a non-zero latent heat in the infinite volume limit. We also find that the critical exponents $\nu$ and $\alpha$ are consistent with the hyperscaling relation but confirm that the critical behavior is different from a conventional first-order transition.

1. INTRODUCTION

The interest about the nature of the phase transition in compact 4D U(1) lattice gauge theory has been revived by the recent development of two new line of results. On the one hand, Kerler, Rebbi and Weber [1] studied the critical properties of the model by adding to the standard Wilson action a coupling $\lambda$ controlling the density of monopoles. They concluded to the existence of a non-gaussian second-order critical point in the $(\beta, \lambda)$ plane. Damm and Kerler [2] are also investigating whether the critical exponent of this transition is universal or changes with $\lambda$.

On the other hand, Jersák, Lang and Neuhaus [3] studied the compact U(1) gauge theory on lattices with sphere-like topology with a Wilson action extended by a coupling $\gamma$ of charge 2

$$S = \beta \sum_P \cos \Theta_P + \gamma \sum_P \cos 2 \Theta_P$$

They found no gap on these lattices for $\gamma \leq 0$. They also made a thorough finite-size size scaling analysis of their data and concluded to the existence, for $\gamma \leq 0$, of a second-order transition with a non-gaussian continuum limit.

Finally there is an investigation [4] of the scaling behaviour of gauge-ball masses and of the static potential, which seems to confirm the second-order nature of the transition also on lattices with periodic boundary conditions at $\gamma = -0.2$ and $\gamma = -0.5$.

However we must note that the critical exponents of these two approaches are different. Moreover there is always an apparent contradiction between the simulations on lattices with spherical topology and on lattices with periodic boundary conditions since one observes a gap on the latter even when $\gamma < 0$. One can fairly state that some confusion about the nature of the transition still persists.

Therefore it is useful to reconsider the simulation of compact pure 4D U(1) lattice gauge theory with the standard Wilson action ($\gamma = 0$) and periodic boundary conditions provided that such a simulation fulfills two goals not met previously [5–7]:

- give an estimation of the infinite volume limit of the gap which is observed on finite-size lattices with periodic boundary conditions.

- make a careful modern finite-size scaling analysis of the bulk critical behavior (as has been done on lattices with sphere-like topology [3]).

In order to estimate the infinite volume limit of the gap, one needs extrapolation formulas which require at least 3 parameters. Also the asymptotic scaling formulas depend in general upon 3
parameters. Determining these parameters from the measurement of one observable only requires at least 6 to 7 data points. One needs more data points if one wants to have some chance of estimating the subleading correction terms.

Therefore to be as systematic as possible the simulation has been done on 9 lattice sizes from \( L = 4 \) to \( L = 16 \). One could argue that these linear sizes are too small to reach the asymptotic scaling regime. If this turns out to be indeed the case, then these smaller lattice sizes are anyhow needed to determine the corrections to scaling which will be required even when data on larger lattices become available.

2. SIMULATION 1

Since accurate data were lacking for many of the above lattice sizes, a first simulation has been done to determine their pseudo-critical coupling to about 1 part in \( 10^{-4} \). In order to reach this accuracy a scanning of the pseudo-critical regions was done with a step \( \Delta \beta = 10^{-4} \) and \( 10^{5} \) iterations at each coupling constant \( \beta \).

We used standard histogramming techniques to locate the double-peak structure found in these simulations which is usually characteristic of a first-order transition. A pseudo-critical coupling was defined as the gap between the two peaks at this first-order transition. A pseudo-critical coupling

\[
\Delta \beta = \beta_{c}\left( \beta_{0} \right) + aL^{-b}
\]

was defined as the coupling for which peaks have equal statistical weight. The latent heat was defined as the gap between the two peaks at this pseudo-critical coupling.

The details of this simulation are described in \[5\] and we only give here a short summary of the results.

A three-paramater fit to the gap \( \Delta e(L) \) of the form

\[
\Delta e(L) = \Delta e(\infty) + aL^{-b}
\]

(2)
gives \( \Delta e(\infty) = 0.014(5) \) and \( b = 1.03(16) \). We note that the exponent \( b \) is not consistent with the first-order prediction (at least in Potts models) \( b = D = 4 \). Fixing \( b \) to 1 reproduces the data quite well but with an infinite volume limit of the latent heat which is definitely different from zero. However it should be stressed that it is very difficult to constrain the functional form of a three-parameter fit to the gap data. Indeed an exponential fit

\[
\Delta e(L) = \Delta e(\infty) + a \exp(-bL)
\]

reproduces the data as well with \( \Delta e(\infty) = 0.0278(15) \).

An asymptotic finite-size scaling analysis of the pseudo-critical couplings

\[
\beta_{c}(L) = \beta_{c}(\infty) + aL^{-b}
\]

yields the results \( \beta_{c}(\infty) = 1.01132(10) \) and \( \nu = 0.326(8) \).

We have also checked the scaling of the maxima of the specific heat for lattice sizes in the range \( L = 4 - 12 \). A two-parameter ansatz

\[
C_{V, \text{max}}(L) = aL^{\frac{\nu}{b} - D}
\]

(5)
gives the independent determination \( \nu = 0.330(2) \) but with a rather high \( \chi^{2} \approx 3 \) which hints at the need of correction terms to the asymptotic formula.

All these results taken together confirm the rather paradoxical nature of the U(1) phase transition. The critical exponents are completely consistent with a second-order phase transition but with an index \( \nu \approx 0.33 \) which is different from the value, \( \nu \approx 0.36 \), quoted in \[3,4\]. This discrepancy raises the suspicion about universality at different values of \( \gamma \). On the other hand any reasonable fitting ansatz to the gap data yields a non-zero value of the latent heat in the infinite volume limit. But again the approach to this limit is different from the asymptotic formula expected within the description of first-order transitions in the double gaussian approximation \[10\]. This disagreement might mean that the asymptotic regime is not yet reached with lattice sizes up to \( L = 16 \).

3. SIMULATION 2

Going to larger lattices is impracticable with local algorithms since already we could not overcome the hysteresis on the \( 16^{4} \) lattice with \( 10^{5} \) iterations. However it is possible to attack the problem indirectly by increasing the statistics on the smaller lattices so as to make a full finite size analysis including corrections to scaling. The comparison of the finite size scaling of several cumulants can unravel the systematic errors in the
critical exponents induced by the corrections to scaling. Combined fits can reveal whether the critical exponents vary when excluding the smallest lattice sizes.

Therefore we have made a second simulation at 3 to 5 coupling constants selected in the pseudo-critical interval determined in simulation 1 at each lattice size, except $L = 16$. $10^6$ iterations have been done at each coupling constant. These $10^6$ iterations were divided in two independent runs, $5 \times 10^5$ sweeps each, respectively from a hot start and a cold start, using different random generators.

The data analysis, which is not yet completed, makes an extensive use of the reweighting technique [11]. All independent runs on the same lattice size are used as independent samples at the same $\beta$. The total amount of statistics that we get is quite comparable to many of the Monte-Carlo simulations of $3D$ spin models.

We are making a finite-size scaling analysis of 3 cumulants: the specific heat per plaquette $C_v$, the Binder Cumulant $U_4$ and the second cumulant $U_2$. We are also adding an analysis of their derivatives $dC_v/d\beta, dU_4/d\beta, dU_2/d\beta$. These 6 cumulants are algebraically independent and the position of their extrema defines a pseudo-critical coupling if located in the scaling region. The value of each pseudo-critical coupling is determined independently for every run by minimizing the corresponding reweighted histogram with respect to $\beta$. Finally we take the statistical average over all runs at each lattice size. The caveat of the method, and the limiting factor of its applicability, is to ensure that the Ferrenberg-Swendsen technique remains valid throughout the minimization process.

4. CONCLUSION

Preliminary results from independent asymptotic finite size scaling fits to the various definitions of the pseudo-critical couplings show deviations $\approx 2\%$ among the values of the critical exponent $\nu$. Even if these deviations are much larger than the statistical errors, it will be very difficult to extract the corrections to scaling.

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