The reversibility of the Goos-Hänchen shift near the band-crossing structure of one-dimensional photonic crystals containing left-handed metamaterials

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We perform a theoretical investigation on the Goos-Hänchen (GH) shift in one-dimensional photonic crystals (1DPCs) containing left-handed metamaterials (LHMs). We find an unusual effect of the GH shift near the photonic band-crossing structure, which is located at the condition, $-k_z^{(A)}d_A = k_z^{(B)}d_B = m\pi (m = 1, 2, 3 \ldots)$, under the inclined incident angle, here A denotes the LHM layer and B denotes the dielectric layer. Above the frequency of the band-crossing point (BCP), the GH shift changes from negative to positive as the incident angle increases, while the GH shift changes reversely below the BCP frequency. This effect is explained in terms of the phase property of the band-crossing structure.

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It is well known that the Goos-Hänchen (GH) shift refers to the lateral shift of a well-collimated light beam totally reflected from the interface of two different media\textsuperscript{[1, 2]}. Recently the GH shifts have extensively studied in various situations, for example, dielectric slab systems\textsuperscript{[3, 4]}, left-handed materials (LHMs)\textsuperscript{[5, 6]}, Otto configuration\textsuperscript{[7]}, one-dimensional photonic crystals (1DPCs)\textsuperscript{[8, 9]}, metal surfaces\textsuperscript{[10]}, 1DPCs with a nonlinear metamaterials\textsuperscript{[11]}, coherent driving atomic systems\textsuperscript{[12]}, and electro-optic crystals\textsuperscript{[13]} and so on. Meanwhile, Li et al.\textsuperscript{[14]} have discovered that multilayered structures containing LHMs possess zero averaged refractive index gap (zero-$\pi$ gap)\textsuperscript{[14]}, which is quite different from a traditional Bragg gap. Shadrivov et al.\textsuperscript{[15]} have further found that in such structures there exists unusual angular dependencies of the beam transmission in which two photonic bands may touch under a certain condition. In this report, we consider the GH shifts near the band-crossing structure of the 1DPCs containing the LHMs, and show that the GH shifts have unusual properties: Above the frequency of the band-crossing point (BCP), the GH shift changes from negative to positive as the incident angle increases, however the GH shift has the opposite behavior below the BCP frequency.

For the simplicity, we only consider a TE-polarized light beam incident from vacuum into the 1DPC at an inclined angle $\theta$, as shown in Fig. 1. The results are similar for TM-polarized light fields with frequencies near the band-crossing structure. The 1DPC is consisted of alternative layers A and B, with the structure $(AB)^N$, where $N$ is period. $d_A$ and $d_B$ are widths of layers A and B, respectively. The layer A is a lossless LHM with the effective dielectric permittivity and magnetic permeability

$$\begin{align*}
\epsilon_A(\omega) = 1 - \frac{\omega_p^2}{\omega^2}, \quad \mu_A(\omega) = 1 - \frac{\omega_{mp}^2}{\omega^2},
\end{align*}$$

where $\omega_p/2\pi = 10$ GHz and $\omega_{mp}/2\pi = 10.5$ GHz. The layer B, without loss of generality, is vacuum ($\epsilon_B = 1$ and $\mu_B = 1$) in this brief report. In general, the electric and magnetic fields at any two positions $z$ and $z + \Delta z$ in the same layer can be related via a transfer matrix\textsuperscript{[10]}

$$M_j(\Delta z, \omega, \theta) = \begin{pmatrix}
\cos[k_j^z \Delta z] & i\frac{q_j}{\omega} \sin[k_j^z \Delta z] & 0 \\
-iq_j \sin[k_j^z \Delta z] & \cos[k_j^z \Delta z] & 0 \\
0 & 0 & 1
\end{pmatrix},$$

where $k_j^z = \frac{\omega}{c} \sqrt{\epsilon_j \mu_j} \sqrt{1 - \sin^2 \theta/\epsilon_j \mu_j}$ is the z component of wave vector $k_j$ in the $j$th layer $(j = A, B)$, $q_j = \sqrt{\epsilon_j / \mu_j} \sqrt{1 - \sin^2 \theta/\epsilon_j \mu_j}$, and $c$ is the light speed in vacuum. Then the reflection and transmission coefficients $[r(\omega, \theta)$ and $t(\omega, \theta)]$ can be readily obtained from the transfer matrix method\textsuperscript{[16]}

$$\begin{align*}
r(\omega, \theta) &= \frac{q_0(x_{22} - x_{11}) - (q_0^2 x_{12} - x_{21})}{q_0(x_{22} + x_{11}) - (q_0^2 x_{12} + x_{21})}, \\
t(\omega, \theta) &= \frac{2q_0}{q_0(x_{22} + x_{11}) - (q_0^2 x_{12} + x_{21})},
\end{align*}$$

where $q_0 = \cos \theta$ for the vacuum of the space $z < 0$ before the incident end and the space $z > L$ after the exit end.
(L is the total length of the 1DPC, and \(x_{ij} \ (i, j = 1, 2)\) are the matrix elements of \(X_N(\omega, \theta) = \prod_{j=1}^{2N} M_j(d_j, \omega, \theta)\) which represents the total transfer matrix for the finite 1DPC. For obtaining the GH shift of the incident beam with a sufficiently large beam waist (i.e., the beam with a very narrow angular spectrum, \(\Delta k << k\)), the GH shifts of both the reflected and transmitted beams can be expressed explicitly as \([4]\)

\[
\Delta_{rt,t} = -\frac{\lambda}{2\pi} \frac{d\phi_{r,t}}{d\theta} = -\frac{\lambda}{2\pi} \frac{1}{|\Theta_{r,t}|^2} \left[ \text{Re}(\Theta_{r,t}) \frac{d\text{Im}(\Theta_{r,t})}{d\theta} - \text{Im}(\Theta_{r,t}) \frac{d\text{Re}(\Theta_{r,t})}{d\theta} \right],
\]

where \(\phi_{r,t}\) are the phases of the reflection and transmission coefficients, \(\Theta_{r,t} = r(\omega, \theta)\) and \(\Theta_{t} = t(\omega, \theta)\), and \(\lambda\) correspond to the wavelength of the incident beam with angular frequency \(\omega\).

In order to know the information of the photonic band gap for an infinite periodic structure \((N \rightarrow \infty)\), according to Bloch’s theorem, the dispersion at any incident angle follows the relation \([17]\)

\[
\cos[\beta_z D] = \cos[k_z(A) d_A + k_z(B) d_B] = \frac{1}{2} \left( \frac{q_B}{q_A} + \frac{q_A}{q_B} - 2 \right) \sin[k_z(A) d_A] \sin[k_z(B) d_B],
\]

where \(\beta_z\) is the \(z\) component of Bloch wave vector, and \(D = d_A + d_B\). The condition of Eq. \((6)\) having no real solution for \(\beta_z\) is \(|\cos[\beta_z D]| > 1\), which is well-known as the Bragg condition of the photonic band gap. In the LHMs (i.e., layers A) \(k_z(A)\) is negative, while in layers B \(k_z(B)\) is positive. Therefore Eq. \((5)\) becomes

\[
\cos[\beta_z D] = \cos[k_z(B) d_B - k_z(A) d_A] + \frac{1}{2} \left( \frac{q_B}{q_A} + \frac{q_A}{q_B} - 2 \right) \sin[k_z(A) d_A] \sin[k_z(B) d_B],
\]

when \(-k_z(A) d_A = k_z(B) d_B \neq m\pi \ (m = 1, 2, 3, \cdots)\), Eq. \((5)\) has no real solution. Therefore there exists a distinctive band gap, the so-called zero-averaged refractive index gap (zero-\(\pi\) gap) \([14]\). However, when

\[
-k_z(A) d_A = k_z(B) d_B = m\pi \ (m = 1, 2, 3, \cdots),
\]

the upper and lower photonic bands may touch together. This phenomena has been pointed by Shadrivov et al. in Ref. \([15]\). Here we would like to emphasize that, in the case of normal incidence, Eq. \((5)\) leads to the touch effect of both the upper and lower photonic bands, however such an effect does not belong to the band-crossing effect because the upper and lower bands are still parabolic shape. In the case of inclined incidence, Eq. \((5)\) leads to the band-crossing effect, which refers to that both the upper and lower photonic bands touch together and the dispersion near the touch point is linear. In the following discussion, we focus ourselves to investigate the GH shift near the band-crossing structure when the 1DPC’s structure satisfies the condition of Eq. \((5)\) in the case of inclined incidence.

In Fig. 2, we plot the photonic band structure in the parameter plane \((\omega, k_y)\), where the transverse wave number \(k_y = \frac{\sin \theta}{\cos \theta} \) is relative to incident angle \(\theta\) for a fixed \(\omega\). From Fig. 2(a) and (b), it is seen that the band-crossing effect occurs at the angles where Eq. \((5)\) is satisfied for the inclined incidences. Close to the BCP, the dispersion relation between \(\omega\) and \((k_y - k_{ym})\) is linear, where \(k_{ym}\) is the transverse wave number for the \(m\)th band-crossing structure. We find that such band-crossing structures are similar to the Dirac band structure in some two-dimensional photonic crystals, which is consisted of triangular or honeycomb lattices of the dielectric cylinders \([18, 19, 20, 21, 22]\). Recently, we also found that it is possible to realize such a band-crossing structure in homogenous negative-zero-positive index media \([18]\). Near the band-crossing structure, one has demonstrated some unusual properties, such as conical diffraction \([13]\), static diffusive property of the light fields \([21, 22]\) and Zitterbewegung of optical pulses \([22, 24]\). For comparison, when \(-k_z(A) d_A = k_z(B) d_B < \pi\), we plot Fig. 2(c) to demonstrate a zero-\(\pi\) gap, which is an omnidirectional gap and is almost independency of incident angle \([25]\).

In Fig. 3(a), we plot the typical GH shifts of trans-
structures are very unusual: it can change from negative, which is similar to that in the conventional 1DPCs, but is very different from the present cases. Therefore, we conclude that the GH shifts inside the band-crossing following cases. Fig. 3(a) shows that, above the BCP frequency located below or above the BCP frequency. For or negative or from negative to positive, depending on light

Transits from the gap region into the passing band region, the GH shift for the light with frequencies above the BCP

it corresponds to the frequency with a zero-averaged refractive index of the total 1DPC. The relative phase is always positive above the BCP frequency; are similar GH effects near the band-crossing structures with \( m = 3 \) and \( m = 2 \), see Fig. 4(a) and 4(b), which correspond to the cases in Fig. 2(b). From Fig. 3(a), and Figs. 4(a) and 4(b), it is further found that the GH shift changes almost linearly from positive to negative (or from negative to positive) inside the passing bands of such band-crossing structures.

To gain a deeper insight into the physical mechanism for such reversed GH shifts near the band-crossing structures, we plot Fig. 5 to show the relative phase change as a function of \( \theta \) at different frequencies. From Fig. 5, the relative phase change of the transmitted light beam, respect to the phase of the incident light beam, varies from negative to positive as the light frequency crosses over the BCP frequency. Here we have to point out that the BCP frequency corresponds to the frequency with a zero-averaged refractive index of the total 1DPC. The relative phase is always positive above the BCP frequency;

\[ \Delta \approx 2 \pi \]

| \( \omega > \omega_{BCP} \) | [8] and \( \omega < \omega_{BCP} \), in our cases \( \omega_{BCP} \approx 2 \pi \times 7.24137 \) GHz, there is a negative GH shift as incident angle \( \theta \) transits from the gap region into the passing band region, however it becomes a positive GH shift as \( \theta \) increases from the passing band into the gap region. Therefore the GH shift for the light with frequencies above the BCP frequency can change from negative to positive with the increasing of \( \theta \). Conversely, below the BCP frequency \( (\omega < \omega_{BCP}) \), the GH shift changes from positive to negative with the increasing of \( \theta \). Figure 3(b) is a contour plot of Fig. 3(a). It is clear that, near the band-crossing structure, the GH shift can be changed from positive to negative or from negative to positive, depending on light frequency located below or above the BCP frequency. For comparison, we plot Figs. 3(c) and 3(d) to show the GH shifts in the case of Fig. 2(c). It is seen that the GH shifts near the band edges of the zero-\( \pi \) gap are always positive, which is similar to that in the conventional 1DPCs [8] but is very different from the present cases. Therefore we conclude that the GH shifts inside the band-crossing structures are very unusual: it can change from negative to positive above the BCP frequency and change conversely below that frequency. We can expect that there

FIG. 4: (Color online). The GH shift \( \Delta \) near the band-crossing structures with (a) \( m = 3 \) and (b) \( m = 2 \) as in Fig. 2(b).

FIG. 5: (Color online). The relative phase shift as a function of angle \( \theta \) at both sides of the band-crossing structure with \( m = 1 \). On each curve, the range between two solid squared points denotes the range of the passing band.
meanwhile it varies initially from a small to large positive value when $\theta$ moves from the gap region into the passing band and then decreases from the maximal phase change to a small one when $\theta$ moves out of the passing band again. Thus the GH shifts for the light beams with frequencies above the BCP frequency change from negative to positive, and the GH shifts can be very large negative (or large positive) near the transition regions from the gap region (or the passing band) to the passing band (or the gap region). From Fig. 5, it is clear that below the BCP frequency, the relative phase is completely opposite with that above the BCP frequency. Therefore the GH shift is reversed near the band-crossing structure when the light frequency is located below the BCP frequency. Such unusual GH shifts are helpful for understanding the special properties of the band-crossing structures. Finally we have explained this effect in terms of the phase properties of the upper and lower passing band of the band-crossing structure.

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