A comment on ‘on inflation expectations in the NKPC model’

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Abstract
Franses (Empir Econ, 2018. https://doi.org/10.1007/s00181-018-1417-8) criticised the practice in the empirical literature of replacing expected inflation by the sum of realised future inflation and an error in estimating the parameters of the new Keynesian Phillips curve (NKPC). In particular, he argued that this assumption goes against the Wold decomposition theorem and makes the error term in the hybrid NKPC equation correlated with future inflation, invalidating the maximum likelihood (ML) estimator of Lanne and Luoto (J Econ Dyn Control 37:561–570, 2013). We argue that despite the correlation, the Wold theorem is not violated, and the ML estimator is consistent.

Keywords Inflation · New Keynesian Phillips curve · Non-causal time series · Non-Gaussian time series

JEL Classification C22 · E31

According to the hybrid new Keynesian Phillips curve (NKPC),

\[ \pi_t = \mu + \alpha E_t \pi_{t+1} + \beta \pi_{t-1} + \gamma x_t, \]

(1)
current inflation \( \pi_t \) in period \( t \) depends on expected future inflation (conditional on current information) \( E_t \pi_{t+1} \), past inflation \( \pi_{t-1} \), and the marginal costs \( x_t \). The empirical literature concentrates on measuring the relative importance of past and expected future inflation in determining current inflation, i.e., on the significance and relative magnitudes of \( \alpha \) and \( \beta \).

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Expected inflation is not observable, and therefore, to take the model to data, it is typically assumed that $E_t \pi_{t+1} = \pi_{t+1} + \omega_{t+1}$, where $\omega_t$ is an independently and identically distributed (iid) error term. Under this assumption, Eq. (1) becomes

$$\pi_t = \mu + \alpha \pi_{t+1} + \beta \pi_{t-1} + \gamma x_t + \varepsilon_t,$$

where $\varepsilon_t = \alpha \omega_{t+1}$. The error term $\varepsilon_t$ is correlated with $\pi_{t+1}$, but given valid instruments, the parameters can be consistently estimated by instrumental variables methods, and this approach has been used in much of the previous literature (see Lanne and Luoto 2013 and the references therein). Alternatively, as suggested by Lanne and Luoto (2013), consistent maximum likelihood (ML) estimates of the parameters of interest, $\alpha$ and $\beta$, can be obtained by rewriting the model as a non-causal autoregressive (AR) model for inflation. In contrast to the conventional causal AR model containing only lags, its non-causal counterpart also contains leads of inflation.

Incorporating also $x_t$ into the error term, the hybrid NKPC (1) can now be written as

$$\pi_t = \mu + \alpha \pi_{t+1} + \beta \pi_{t-1} + \eta_t,$$

where $\eta_t = \alpha \omega_{t+1} + \gamma x_t$. As Lanne and Luoto (2013) show, under different assumptions concerning $x_t$, or, the error term $\eta_t$, model (3) has various AR representations. In particular, if $\eta_t$ is iid, ignoring the intercept term, model (3) can be written as the non-causal AR(1, 1) model of Lanne and Saikkonen (2011),

$$(1 - \phi B) (1 - \varphi B^{-1}) \pi_t = \varepsilon_t,$$

where $B$ is the usual backshift operator, and $\varepsilon_t \equiv (\varphi / \alpha) \eta_{t+1}$ [for details, see Lanne and Luoto (2013, Section 3)]. The intercept term can be omitted without loss of generality if the model is estimated on demeaned data. Consistent estimators of $\alpha$ and $\beta$ are obtained as functions of the estimated roots of the polynomials $1 - \phi B$ and $1 - \varphi B^{-1}$.

If $\pi_t$ is weakly stationary, there is a causal AR(2) process corresponding to the non-causal process (4), with the same mean, variance and autocovariances. This AR(2) process has a moving average representation in accordance with Wold’s decomposition theorem, implying that, contrary to Franses’s (2018) claims, the hybrid NKPC (3) does not violate Wold’s decomposition theorem. This follows from the fact that the Wold theorem only gives equality of any (purely non-deterministic) time series and a weighted sum of current and past errors in the mean square sense, and up to the second moments, the non-causal AR process and its causal counterpart are equivalent. They can only be distinguished when the error term is non-Gaussian. However, the Wold decomposition is not really relevant in the context of non-causal processes, where its natural counterpart is a two-sided infinite-order moving average representation. Utilising that representation, consistency of the ML estimator of the parameters of

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1 The approach can be generalised in a straightforward manner to the case where the marginal cost variable included in the error term $\eta_t$ is autocorrelated (for details, see Lanne and Luoto 2013). In this case, a higher-order AR process for inflation is specified.
the non-causal model, and thus, also those of the hybrid NKPC can be shown, even though the error term in (3) is correlated with \( \pi_{t+1} \).

As an alternative solution to the endogeneity problem, Franses (2018) proposed a MIDAS type approach to estimate the parameters of the NKPC, based on information in data observed at two frequencies. Specifically, he regressed the annual US CPI inflation (1956–2016) in year \( t \) on inflation in year \( t - 1 \), and the logarithmic change in consumer prices in the same month between years \( t \) and \( t - 1 \). Franses’s and Lanne and Luoto’s (2013) approaches are similar in that neither relies on observations of the marginal cost variable, but both assume it to be weakly stationary. The main difference between them is that the former does not require the error term to be non-Gaussian. The estimates of \( \alpha \) and \( \beta \) in Franses’s Table 3 vary quite a lot depending on the month used in computing the latter regressor, which makes it difficult to draw general conclusions, whereas Lanne and Luoto’s approach yields unique estimates of these parameters.\(^2\)

Compliance with ethical standards

Conflict of interest The authors declare that there is no conflict of interest.

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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\(^2\) Franses’s (2018) estimates of \( \alpha \) vary between 0.676 and 1.104, while his maximum estimate of \( \beta \) equals 0.337, and in a number of cases a negative estimate of \( \beta \) is obtained that is difficult to reconcile with macroeconomic theory. The corresponding estimates of \( \alpha \) and \( \beta \) based on a non-causal AR(3, 1) model estimated on the quarterly US CPI inflation from the same period equal 0.740 and 0.204, respectively. In contrast to Franses’s MIDAS type model, the latter approach uses information at one frequency only, so annual data are too sparse for reliable inference. The estimated degree-of-freedom parameter of the \( t \) distribution assumed for the error term equals 5.06, clearly indicating non-Gaussianity.