Nucleon knockout experiments and configuration mixing in nuclei

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Abstract. The importance of nucleon knockout experiments for a deeper understanding of nucleon properties in the nuclear medium is clarified. Results of \((e, e'p)\) experiments yield a clear picture of the importance of configuration mixing [1] at all energy scales. A comparison of experimental data with Green’s function calculations indicates that a quantitative understanding of the properties of protons in the nuclear ground state is within reach. The latter feature makes nuclear physics the first branch of physics where such an understanding of the in-medium properties of strongly interacting constituents has been accomplished.

1. Introduction

The \((e, e'p)\) reaction has provided detailed information in the last 20 years that delimits the boundaries of the relevance of the conventional mean-field (mf) or shell-model description of nuclei. The \((e, e'p)\) reaction [2] maps the properties of protons in the ground state of the target nucleus. The physical picture associated with the \((e, e'p)\) reaction involves the intimate connection of this reaction with the probability of removing a nucleon with momentum \(p\) while keeping the binding energy of this nucleon (or the missing energy) fixed. The resulting cross sections therefore sample the square of the corresponding momentum-space wave functions of these nucleons when distortion and absorption effects of the outgoing proton have been properly accounted for. The results from the NIKHEF facility are shown for four different nuclei in Fig. 1 [2]. The shapes of the wave functions in momentum space correspond closely to those expected on the basis of standard Woods-Saxon potential wells. This is an important observation, since the \((e, e'p)\) reaction probes the interior of the nucleus.

While the shapes of the valence nucleon wave functions correspond to the basic ingredients of nuclear structure physics, there is a significant departure with regard to their normalization. This quantity is usually referred to as the spectroscopic factor and is shown in Fig. 2 for the data obtained at NIKHEF [2]. These results indicate that there is a global reduction of the sp strength of about 35 %. The \((e, e'p)\) reaction also exhibits the fragmentation of the sp strength associated with more deeply bound orbitals. Isolated peaks are obtained in the vicinity of the Fermi energy, whereas for more deeply bound states a stronger fragmentation of the strength is obtained with larger distance from \(\varepsilon_F\). In Sec. 2 some theoretical ingredients of Green’s function calculations are presented. The Green’s function method is suitable for tackling the calculation of the sp strength, since it does not require the use of correlated wave functions, but instead, determines transition amplitudes directly, leading to a reduced numerical effort. In Sec. 3 the results of several calculations employing this many-body technique [3] are discussed. The role of
short- and long-range correlations will be addressed and the consequences of the experimental strength distribution for the admixture of high-momentum components in the ground state will be explored. Some more recent data covering a larger missing energy domain will also be discussed. Finally, conclusions are drawn in Sec. 4.

2. Theoretical concepts

The relevant theoretical quantity for comparison with the \((e, e'p)\) data is the spectral function associated with the removal of particles from the ground state of the target nucleus. This spectral function is part of the sp propagator describing the properties of a nucleon in the nuclear medium. The latter quantity is given in its Lehmann representation by

\[
G(\alpha, \beta; E) = \sum_m \left| \langle \Psi_{m}^{A-1} | a_{\alpha} | \Psi_{0}^{A} \rangle \right|^2 \delta(E - (E_{0}^{A-1} - E_{m}^{A-1})),
\]

where \(a_{\alpha}\) and \(a_{\beta}^{\dagger}\) are creation and annihilation operators for particles with quantum numbers \(\alpha\) and \(\beta\), respectively. The quantum numbers \(\alpha\) and \(\beta\) refer to an appropriately chosen set of sp quantum numbers relevant for the problem under study. The first contribution to Eq. (1) involves the propagation of a particle, or, equivalently, is associated with adding a particle to the target nucleus. The second term of the propagator in Eq. (1) provides information relevant for the \((e, e'p)\) reaction. This can be understood by taking the imaginary part of Eq. (1) for \(\alpha = \beta\)

\[
S_{h}(\alpha, E) = \frac{1}{\pi} \text{Im} G(\alpha, \alpha; E) = \sum_n \left| \langle \Psi_{n}^{A-1} | a_{\alpha} | \Psi_{0}^{A} \rangle \right|^2 \delta(E - (E_{0}^{A-1} - E_{n}^{A-1})).
\]

This definition of the (hole) spectral function incorporates the simple notion of representing the (energy) probability density for the removal of a particle with quantum numbers \(\alpha\) from
Figure 2. Spectroscopic factors from the \((e,e'p)\) reaction as a function of target mass. The dotted line with a height of 1, illustrates the prediction of the independent-particle model. Data have been obtained at the NIKHEF accelerator in Amsterdam [2].

the ground state of the target nucleus, while leaving the system with the hole at an energy \(E\). In the domain of valence hole states in nuclei the corresponding energies refer only to discrete states and the spectral function factors into an energy conserving \(\delta\)-function and the removal probability at that energy for an orbital with quantum numbers \(\alpha\). Another useful quantity to gauge the strength of correlations is given by the summed (integrated) strength below the Fermi energy. This occupation number for quantum numbers \(\alpha\) is thus given by

\[ n(\alpha) = \int_{-\infty}^{E_F} dE \, S_h(\alpha, E). \]  

(3)

The perturbation expansion of the sp propagator [3] permits the introduction of the nucleon self-energy \(\Sigma\) that determines the propagator from the Dyson equation and the noninteracting (or mf) propagator \(G^{(0)}\)

\[ G(\alpha, \beta; E) = G^{(0)}(\alpha, \beta; E) + \sum_{\gamma, \delta} G^{(0)}(\alpha, \gamma; E)\Sigma(\gamma, \delta; E)G(\delta, \beta; E). \]  

(4)

Several relevant diagrammatic choices for the nucleon self-energy must be considered for nuclear systems, as discussed in the next section.

A clearer understanding of the physical significance of the self-energy and the importance of the Dyson equation can be gleaned by converting the latter to an equivalent Schrödinger-like equation for the transition amplitudes \(z_n^\alpha = \langle \Psi^A_n | a\alpha | \Psi^A_0 \rangle\) in the case of discrete states \(n\) in the \(A - 1\) system. Choosing the coordinate system representation (and spin), one can show [3] that these amplitudes obey the following equation

\[ -\nabla^2 \frac{z_n^\alpha}{2m} + \sum_{m'} \int d^3r'\Sigma(rm, r'm'; \varepsilon_n)z_{r'm'}^n = \varepsilon_n z_{rm}^n. \]  

(5)

The nonlocal potential is thus represented by the self-energy. Note that the eigenvalue \(\varepsilon_n^\alpha\) must coincide with the energy argument of the self-energy. An important difference with the ordinary Schrödinger equation involves the normalization of the quasihole wave functions \(z_{rm}^n\). This result is most conveniently expressed in terms of the sp state that corresponds to the quasihole wave
Figure 3. Proton occupation probabilities calculated by including TDA correlations in the self-energy [4] and multiplied by a factor 0.9 below $\varepsilon_F$ to simulate the effect of SRC.

function $z_{\alpha}^{n_{\alpha}}$. Using the eigenstate that diagonalizes Eq. (5), denoted by notation $\alpha_{qh}$, the normalization condition then reads

$$|z_{\alpha_{qh}}^n|^2 = \left(1 - \frac{\partial \Sigma(\alpha_{qh}, \alpha_{qh}; E)}{\partial E}_{\varepsilon_n}ight)^{-1}.$$  \hspace{1cm} (6)

The subscript $qh$ refers to the quasihole nature of this state and the fact that for valence hole states very near to the Fermi energy the normalization yields a number of order 1. In the analysis of an ($e, e'p$) experiment a local potential well of Woods-Saxon type is employed that generates a sp state at the removal energy for the transition under study. The well is adjusted to provide the best possible fit to the experimental momentum dependence of the cross section (with proper inclusion of complications due to electron and proton distortion) [2]. The overall factor necessary to bring the resulting calculated cross section into agreement with the experimental data, can then be interpreted as the absolute spectroscopic factor corresponding to the experimental wave function according to Eq.(6). Although the well is local, it provides important information on the self-energy of a nucleon at energies below $\varepsilon_F$. From the theoretical perspective it is clear that in order to obtain spectroscopic factors in qualitative agreement with the experimental data (i.e. $< 1$), one must include an energy-dependent self-energy in Eqs. (5) and (6).

3. Theoretical results

The qualitative features of the sp strength distribution can be understood by realizing that a considerable mixing occurs between hole states and two-hole one-particle (2h1p) states. This mixing leads to different strength patterns depending on the location of the orbital under consideration. An orbital in the immediate vicinity of the Fermi energy will still yield a large fragment near its original position, since the 2h1p states are quite far in energy. For the same reason, only small components of this orbit appear at 2h1p energies. An orbit which is surrounded by many 2h1p states will yield a strongly fragmented pattern, the width depending on the strength of the mixing interaction. These features are all observed experimentally and can be easily understood. The theoretical description of this mixing requires an energy-dependent self-energy as discussed in the previous section. For medium-heavy nuclei like $^{48}$Ca and $^{90}$Zr a fairly good description of the fragmentation pattern of valence hole states can be obtained by including the coupling to low-lying collective states in the self-energy. This typically requires these particle-hole states to be correlated at the level of the Tamm-Dancoff (TDA) or random
phase approximation (RPA) [4]. Diagrammatic contributions to the self-energy that represent these correlations involve the so-called ring diagrams.

For quantitative results one also requires the inclusion of short-range and tensor correlations. These contributions are represented diagrammatically by ladder diagrams that must be inserted into the nucleon self-energy. On the one hand, the inclusion of short-range correlations (SRC) leads to a global depletion of mf orbitals which ranges from 10% in light nuclei to about 15% in heavy nuclei and nuclear matter [3]. The combination of these results leads to the typical pattern of occupation numbers that is illustrated in Fig. 3 for protons in $^{48}$Ca [4]. The coupling to collective surface vibrations, reflecting the influence of long-range correlations, has a distinctively different pattern. This pattern is characterized by long-range correlations having almost no effect on occupation numbers of orbits very far from the Fermi energy but increasing their effect for orbits as they approach $\varepsilon_F$. The mixing of nearby particle states with 2h1p states leads to a small occupation of these orbits of a few percent as illustrated in Fig. 3.

Recent experimental data from NIKHEF [5] for $^{208}$Pb confirm the characteristics of these occupation numbers and their link to different types of correlations. These data demonstrate that all deeply bound proton orbits are depleted by about the amount expected from nuclear-matter calculations that include SRC [6]. The reduction of the sp strength of mf orbits below the Fermi energy, on the other hand, must be accompanied by the admixture of high-momentum components in the ground state. These high-momentum nucleons have now been unambiguously identified experimentally using the $(e,e'p)$ reaction [7]. While the results for medium-heavy nuclei are satisfactory, there is a considerable discrepancy for the spectroscopic factors in $^{16}$O [8]. Nevertheless, it is possible to summarize general and quantitative conclusions about

Figure 4. The middle column of the figure illustrates the mf picture. The right column identifies the location of the sp strength of the orbit just below the Fermi energy when correlations are included. The physical mechanisms responsible for the strength distribution are also identified. In the left column the diagram that is responsible for the admixture of high-momentum components in the ground state is depicted leading to high-momentum nucleons at large missing energies.
the whereabouts of protons in the ground states of heavier closed-shell nuclei, as illustrated in Fig. 4.

The identification of high-momentum nucleons \[7\] in addition to locating all the sp strength associated with the mf orbits \[5\] completes the identification of the properties of protons in the ground state of the nucleus. The latter understanding is illustrated in Fig. 4. Several generic diagrams are identified in this figure, that have unique physical consequences for the redistribution of the sp strength when they are taken into account in the solution of the Dyson equation. The middle column of the figure characterizes the mf picture that is used as a starting point of the theoretical description. The right column identifies the location of the sp strength of the orbits just below the Fermi energy when correlations are included. One may apply this picture for example to the \(3s_{1/2}\) proton orbit in \(^{208}\)Pb. The physical mechanisms responsible for the correlated strength distribution are also identified. The strength of this orbit remaining at the quasihole energy is about 65\%. Long-range correlations are responsible for the loss of 20\% of the strength due to the coupling to nearby 2p1h and 2h1p states. This loss is symmetrically distributed above and below the Fermi energy and is physically represented by the coupling to low-lying surface modes and higher-lying giant resonances. The resulting occupation number of this orbit therefore corresponds to 75\% as inferred from experiment \[9\]. More deeply bound nucleons have higher occupation numbers corresponding to 85\%. This is true for all these deep-lying orbits and is consistent with a global depletion due to SRC of 15\%. The corresponding location of this strength is identified at very high energy in the particle domain and is due to the short-range and tensor correlations induced by a realistic nucleon-nucleon interaction. In the left column of Fig. 4 the generic diagram that is responsible for the admixture of high-momentum components in the ground state is depicted that have now been confirmed experimentally at high missing energy \[7\].

4. Conclusions

An overview of the properties of protons as present in the nuclear ground state has been presented. Essential ingredient is a complete experimental picture based on the analysis of the \((e, e'p)\) reaction in a wide domain of missing energy and momentum. These results identify the location of all the relevant sp strength. The mf strength is depleted by about 15\% and replaced by high-momentum protons. This depletion effect uniquely points to SRC as being responsible, confirming an older prediction based on nuclear matter results. The distribution of the sp strength can be adequately described by Green’s function calculations. From these results it becomes clear that configuration mixing in nuclei must be incorporated to the highest energy scale associated with the core of the NN interaction \[1\]. It appears that only in nuclear physics it is possible to identify the detailed properties of strongly interacting constituent particles.

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