Dynamic Domain Walls in Strongly Driven Ferromagnets

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(April 1995)

A multiple-time scaling analysis of the dissipative, transversely driven Landau-Lifshitz equation in presence of exchange, shape demagnetisation and weak anisotropy fields is performed for a dynamic domain state. Stationary solutions of the resulting equations explain the spatiotemporal structure of the walls and are in agreement with previous simulations.

I. INTRODUCTION

In the interdisciplinary, unifying work [1] on structure formation in driven dissipative systems ferromagnets are only treated in exceptional cases. This is surprising as ferromagnets exhibit already in static fields domain structures which are well understood (for reviews see [2]) and which even serve as textbook examples [3]. Dynamical questions have also been investigated. This has been done by extending the research [2] to weak driving fields (compare e.g. [4]). For stronger driving fields numerous work (reviewed in [5]) has been published to describe the ferromagnetic resonance instabilities [6].

Apart from single exceptions like [7], where in simulations dynamic domain states for a special model were reported, the existing investigations on dynamical questions are limited to the linear or to the weak nonlinear regime. An approach [8] to analyse the strongly nonlinear dynamics of driven ferromagnets in terms of structure formation has recently been proposed by the present author. In this approach a transversely driven model was investigated containing an exchange, an anisotropy and a shape demagnetisation field. Numerical simulations exhibited in the rotating frame a stationary domain structure with a precessing motion in the wall regimes. Some of the characteristic elements of this structure have been explained in [8] by analytical methods. The spatiotemporal dependence of the domain walls, however, remained unexplained.

It is the aim of the present investigations to work out analytically an explanation of these dynamic domain walls. In sec.II the basic ingredients of the present approach are given. After performing a transformation in sec.III a multiple-time scaling analysis is employed in sec.IV to deduce reduced equations of motion on the slow time scale. Representing the general result of this work these eqs. are in sec.V applied to determine the spatiotemporal dependence of the stationary walls. In sec.VI the results are compared with the simulations of [8]. Finally in sec.VII some conclusions are given.

II. THE MODEL AND BASIC ASSUMPTIONS

At a mesoscopic scale the dynamics of ferromagnets is governed by the Landau-Lifshitz equation, which takes the form

$$\partial_t \mathbf{m} = -(1 + \Gamma m^\times)(\mathbf{m} \times (h^\parallel \mathbf{e}_z + h^\perp \mathbf{e}_x) - \Delta m + A m^\times \mathbf{e}_z) + \omega \mathbf{m} \times \mathbf{e}_z$$

in the frame rotating with the driving frequency $\omega$ around the $\mathbf{e}_z$ direction. $\mathbf{m}(r,t)$ is the local magnetisation in the rotating frame being related to the magnetisation in the laboratory frame $\mathbf{m}_{lab}$ by $\mathbf{m}_{lab} = \exp(\omega t \mathbf{e}_z \times ) \mathbf{m}$. The amplitudes of the external static and the external circular driving rf field are denoted by $h^\parallel$ and by $h^\perp$, respectively. The term $\Delta m = V^{-1} \int m dV$ represents, in reduced units, the demagnetisation field of a sphere of volume $V$. The contribution $J \Delta m$ results from the isotropic ferromagnetic exchange interaction and the uniaxial anisotropy is described by $A m^\times \mathbf{e}_z$ where $A < 0$ is assumed in this work. The Landau-Lifshitz damping rate is represented by $\Gamma$. The gyromagnetic ratio and the magnitude of the magnetisation $m = |\mathbf{m}|$ are set equal to 1.

$A$ is assumed to be a small quantity which permits an perturbative treatment. Recall that eq.(1) results from a continuum approximation in space (compare [8]) which implies a slow spatial variation of $\mathbf{m}$. Thus the exchange field $J \Delta m$ is also treated as a perturbation. Introducing the expansion parameter $\epsilon = -A > 0$ and scaling the position as $r \to (J/A)^{1/2} r$ the perturbational fields of eq.(1) $J \Delta m + A m^\times \mathbf{e}_z$ take the form $\epsilon (\Delta m - m^\perp \mathbf{e}_z)$.

Due to the $\mathbf{m}(t)$ term, the problem described by eq.(1) is of the mean field type and the usual technique can be applied. This is to employ an ansatz for $\mathbf{m}(t)$, solve formally the problem for given $\mathbf{m}$ and in the last step examine the results for self-consistency.
Based of the findings of [8] as ansatz a stationary domain structure is assumed in which the magnetisation is constant nearly everywhere and takes only two values \( \mathbf{m}_+ \) or \( \mathbf{m}_- \) realised in the generally disconnected partial volumes \( V_+ = n_+ V \) and \( V_- = n_- V \), respectively. The wall regimes where the magnetisation changes from \( \mathbf{m}_- \) to \( \mathbf{m}_+ \) are assumed to be narrow. Then the volume of the wall regions can be neglected compared to the sample volume \( V \) which implies

\[
\mathbf{m}_{\text{dom}} = n_+ \mathbf{m}_+ + n_- \mathbf{m}_-; \quad \text{with} \quad n_+ + n_- = 1.
\]  

(2)

This work is focusing on the stationary domain solutions of eq.(1). To keep the analysis as simple as possible relaxation of \( \mathbf{m}(t) \) to \( \mathbf{m}_{\text{dom}} \) was found in [8].

In this previous work the zeroth order contributions to \( n_\pm \) and to \( \mathbf{m}_\pm \) have been calculated. The perturbation theory of this work makes it necessary to separate in notation between full quantities and their contributions to the \( \epsilon \) expansion. This is archived by generally setting \( a = \tilde{a} + \epsilon a^{(1)} + O(\epsilon^2) \) for an arbitrary quantity \( a \). This rather unconventional notation of the zeroth order contributions avoids complicated indices. Employing this notation the results of [8] take the form

\[
\tilde{n}_\pm = \frac{1}{2} \pm \frac{dw^2 + \omega w^2}{2uv}; \quad \tilde{\mathbf{m}}_\pm = u^{-1}(\mp v \tilde{e}_1 + \Gamma \tilde{h}_\perp \tilde{e}_3)
\]  

(3)

with

\[
d = h_\parallel - \omega; \quad u = (d^2 + \Gamma^2)^{1/2}; \quad v = (u^2 - \Gamma^2 h_\perp^2)^{1/2}; \quad w = (u^2 + h_\perp^2)^{1/2}
\]  

(4)

and where the transformation from the internal orthonormal system \((\tilde{e}_1, \tilde{e}_2, \tilde{e}_3)\) to \((e_x, e_y, e_z)\) is found to be given by

\[
\begin{pmatrix} e_x \\ e_y \\ e_z \end{pmatrix} = \frac{1}{uw} \begin{pmatrix} -dh_\perp & -du & \Gamma w \\ \Gamma h_\perp & \Gamma u & dw \\ -u^2 & uh_\perp & 0 \end{pmatrix} \begin{pmatrix} \tilde{e}_1 \\ \tilde{e}_2 \\ \tilde{e}_3 \end{pmatrix}.
\]  

(5)

With eqs.(3-5) and with the above replacements eq.(1) takes the form (compare [8])

\[
\partial_t \mathbf{m} = -\omega w^{-1} \mathbf{m} \times (u \mathbf{e}_1 + \Gamma h_\perp \mathbf{m} \times \mathbf{e}_2) - \epsilon \left(1 + \Gamma \mathbf{m} \times \mathbf{m} \times (\Delta \mathbf{m} - m_\perp \mathbf{e}_z - \mathbf{m}^{(1)}_{\text{dom}}) \right) + O(\epsilon^2)
\]  

(6)

where \( \mathbf{m}_{\text{dom}}^{(1)} \) denotes the first order contributions of \( \mathbf{m}_{\text{dom}} \) according to the convention in notation.

Eq.(6) is the starting point for the further analysis. Compared to eq.(1) this eq.(6) is constrained by \( \mathbf{m}(t) = \mathbf{m}_{\text{dom}} \). Note, however, that eq.(6) still describes in general nonlinear deviations from the domain state as long as these deviations are local.

The \( \mathbf{m}_\pm \) have to satisfy eq.(1) with \( \partial_t \mathbf{m} = \Delta \mathbf{m} = 0 \) from which

\[
\mathbf{m}_\pm^{(1)} = \mathbf{m}_\pm + \Gamma \left[ \mp \frac{w}{\omega w} \mathbf{m}_{\text{dom}}^{(1)} - \frac{u}{\omega w} \tilde{e}_1 + \frac{h_\perp}{\omega w} \tilde{e}_2 \right]
\]  

(7)

results. Eq.(4) leads in first order to

\[
\mathbf{m}_{\text{dom}}^{(1)} = n_+^{(1)} (\mathbf{m}_+ - \mathbf{m}_-) + n_-^{(1)} \mathbf{m}_+^{(1)} + n_+^{(1)} \mathbf{m}_-^{(1)}
\]  

(8)

where the \( \mathbf{m}_\pm \) and the \( n_\pm \) are given by eq.(3). Note that from eqs.(7) and (8) the dependence of \( \mathbf{m}_{\text{dom}}^{(1)}, \mathbf{m}_\pm^{(1)} \) and of \( n_\pm^{(1)} \) on the model parameters \( h_\parallel, h_\perp, \omega \) and \( \Gamma \) is not completely determined. For such a determination an additional relation between these quantities is needed. Thus at this stage of the calculation \( \mathbf{m}_{\text{dom}}^{(1)} \) and \( \mathbf{m}_\pm^{(1)} \) are treated as functions of \( n_\pm^{(1)} \) and of the model parameters.

### III. Transformations

In zeroth order the dynamics described by eq.(1) is hamiltonian [8]. Thus the dynamics should be formulated in variables, which in the \( \epsilon = 0 \) limit reduce to angle - action variables. The total transformation to these variables is divided into a sequence of single transformations. First an orthonormal system \((e_1, e_2, e_3)\) is defined by \( e_1 = \).
After that eq. (9) is employed to represent the rhs. of eq. (6) in terms of which completes the total transformation.

The quantities \( \tilde{b} \), \( \Phi \) with respect to the new \((3)\)-axis are introduced. These transformations are explicitly described by

\[
\begin{pmatrix}
\mathbf{m} \\
\mathbf{e}_\Theta \\
\mathbf{e}_\Phi
\end{pmatrix} =
\begin{pmatrix}
\sin \Theta \cos \Phi & \sin \Theta \sin \Phi & \cos \Theta \\
\cos \Theta \cos \Phi & \cos \Theta \sin \Phi & -\sin \Theta \\
-\sin \Phi & \cos \Phi & 0
\end{pmatrix}
\begin{pmatrix}
b_3 \cos \Theta & 0 & -b_3 p(\Theta) \\
0 & 1 & 0 \\
b_3 p(\Theta) & 0 & b_3 \cos \Theta
\end{pmatrix}
\begin{pmatrix}
\mathbf{e}_1 \\
\mathbf{e}_2 \\
\mathbf{e}_3
\end{pmatrix}
\]  

(9)

where \( p(\Theta) \) is given by \( p(\Theta) = \sqrt{b_3^2 - \cos^2 \Theta} \). Finally a change of variables from \( \Theta, \Phi \) to new independent variables \( \Theta, \varphi \) is performed by setting

\[
\tan \frac{\Phi(\Theta, \varphi)}{2} = \frac{\Omega}{p(\Theta) - \sin \Theta} \tan \frac{\varphi}{2} \quad \text{with} \quad \Omega = \sqrt{b_3^2 - 1}.
\]

(10)

which completes the total transformation.

Eq. (10) implies \( \text{d}\Phi = p^{-1}(\Theta) \cos \Theta \sin \Phi \text{d}\Theta + g(\Theta, \varphi) \text{d} \varphi \) where the function \( g(\Theta, \varphi) \) is defined as \( g(\Theta, \varphi) = p(\Theta) + \sin \Theta \cos \Phi \). With this expression for \( \text{d}\Phi \) eq. (\ref{eq:1}) leads to \( \partial_t \mathbf{m} = \rho \mathbf{p}_\varphi \partial_t \Theta + \sin \Theta \mathbf{e}_\Phi \partial_t \varphi \). To transform the rhs. of eq. (\ref{eq:1}) in a first step \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \) are expressed by the \( \mathbf{e}_i \) using eqs. (\ref{eq:3}) and the definition of the \( \mathbf{e}_i \). After that eq. (\ref{eq:3}) is employed to represent the rhs. of eq. (\ref{eq:3}) in terms of \( \mathbf{e}_0 \) and \( \mathbf{e}_\varphi \). Introducing a scaled time \( t' \) by \( t' = \omega u^2 b_3 v^{-1} t \) and setting \( \epsilon = \Gamma h_\perp \omega^{-1} \) the results of this basically straightforward calculation leads to

\[
\begin{align*}
\partial_t \Theta &= \epsilon \tilde{\rho}(\Theta) (X_{\Phi} + X_{\Phi}^{\text{ex}} + \Gamma X_{\Theta}^{\text{ex}} + \Gamma X_{\Theta}^{an} + Y_\Theta) + O(\epsilon^2) \\
\sin \Theta (\partial_t \varphi - 1) &= \epsilon (-X_{\Phi}^{an} - X_{\Phi}^{\text{ex}} + \Gamma X_{\Theta}^{\text{ex}} + \Gamma X_{\Theta}^{an} + Y_\Phi) + O(\epsilon^2).
\end{align*}
\]

(11)

The quantities \( \tilde{g} X_{\Phi}^{\text{ex}} = \mathbf{e}_0 \Delta \mathbf{m} \) and \( \tilde{g} X_{\Phi}^{an} = \mathbf{e}_\Phi \Delta \mathbf{m} \) are calculated to

\[
\begin{align*}
\tilde{g} X_{\Phi}^{\text{ex}} &= \nabla (\tilde{\rho} \cos \Theta (X_{\Phi} + X_{\Phi}^{\text{ex}} + \Gamma X_{\Theta}^{\text{ex}})) \sin \Theta \\
\tilde{g} X_{\Phi}^{an} &= \nabla (\tilde{\rho} \cos \Theta (X_{\Phi}^{an})) \sin \Theta
\end{align*}
\]

(12)

From the definitions \( \tilde{g} X_{\varphi}^{an} = -(\mathbf{m} \mathbf{e}_z) (\mathbf{e}_\Theta \mathbf{e}_z) \) and \( \tilde{g} X_{\Phi}^{an} = -(\mathbf{m} \mathbf{e}_z) (\mathbf{e}_\Phi \mathbf{e}_z) \)

\[
\begin{align*}
\tilde{g} X_{\Theta}^{an} &= h_\perp^2 \omega^{-2} (\sin \Theta \sin \Phi + \Gamma \tilde{g} \cos \Theta) \sin \Theta \\
\tilde{g} X_{\varphi}^{an} &= h_\perp^2 \omega^{-2} (\sin \Theta \sin \Phi + \Gamma \tilde{g} \cos \Theta) \cos \Theta.
\end{align*}
\]

(13)

is obtained and for \( Y_\Theta \) and \( Y_\Phi \)

\[
\begin{align*}
Y_\Theta &= -\Gamma h_\perp \omega^{-2} (\sin \Theta \mathbf{e}_1 \mathbf{m}_\text{dom}^{(1)} + \mathbf{e}_2 \mathbf{m}_\text{dom}^{(1)} | \sin \Theta \\
&\quad - \Gamma h_\perp^2 \omega^{-2} \tilde{g} \frac{(1 - \Gamma^2) \cos \Theta (\rho \cos \Phi + \sin \Theta) + 2 \Gamma \tilde{g} \sin \Phi)}{1 + \Gamma \tilde{g} \sin \Phi}
\end{align*}
\]

\[
\begin{align*}
Y_\Phi &= \Gamma h_\perp \omega^{-2} (\cos \Theta (\rho \cos \Phi + \sin \Theta) + (1 - \Gamma^2) \tilde{g} \sin \Phi)}{1 + \Gamma \tilde{g} \sin \Phi}
\end{align*}
\]

(14)

results with \( \Phi = \Phi(\Theta, \varphi) \) as defined by eq. (\ref{eq:1}). The quantities \( \tilde{p} \) and \( \tilde{g} \) stand for the zeroth order contributions to \( p \) and to \( g \) and are explicitly calculated to

\[
\tilde{p}(\Theta) = \sqrt{\Omega^2 + \sin^2 \Theta} \quad , \quad \tilde{g}(\Theta, \varphi) = \tilde{p}(\Theta) + \sin \Theta \cos \Phi \quad \text{with} \quad \Omega = \frac{u}{\Gamma h_\perp}. \]

(15)

In the zeroth order limit eqs. (\ref{eq:1}) reduce to \( \partial_t \Theta = 0 \) and to \( \partial_t \varphi = 1 \). Thus as required the variables \( \Theta, \varphi \) become angle - action variables in the \( \varepsilon = 0 \) limit. A second feature of the transformation should be pointed out. According to the definition of the \( \mathbf{e}_i \) and to eq. (\ref{eq:4}) \( \mathbf{m}(\Theta \to 0, \varphi) = \mathbf{m}_- \) and \( \mathbf{m}(\Theta \to \pi, \varphi) = \mathbf{m}_+ \) holds. Thus the values \( \Theta = 0 \) and \( \Theta = \pi \) characterise the interior of the domain regimes and a domain wall corresponds to a change from \( \Theta = 0 \) to \( \Theta = \pi \).
Next the standard multiple-time scaling method is applied to the eqs. Denoting the slow time scale as $T = \hat{t}$ the ansatz

$$\Theta(\hat{t}, r) = \tilde{\Theta}(T, r) + \hat{t} \Theta^{(1)}(\hat{t}, T, r) + O(\hat{t}^2)$$

$$\varphi(\hat{t}, r) = \hat{t} + \check{\varphi}(T, r) + \hat{t} \varphi^{(1)}(\hat{t}, T, r) + O(\hat{t}^2)$$

(16)

is employed which already satisfies the zeroth order of the perturbative expansion. In first order the $\hat{t}$-secular terms of $\Theta^{(1)}$ and of $\varphi^{(1)}$ are as usual eliminated. This procedure leads to

$$\partial_T \tilde{\Theta} = \hat{p}(\tilde{\Theta}) (X^{ex}_\Phi + \tilde{X}^{an}_\Phi) + \Gamma \langle X^{ex}_\Theta \rangle + \Gamma \langle X^{an}_\Theta \rangle + \langle Y_\Theta \rangle$$

$$\sin \tilde{\Theta} \partial_T \check{\varphi} = -\langle X^{ex}_\Theta \rangle - \langle X^{an}_\Theta \rangle + \Gamma \langle X^{ex}_\Phi \rangle + \langle Y_\Phi \rangle$$

(17)

which determines the $T$- dependence of $\tilde{\Theta}(T, r)$ and of $\check{\varphi}(T, r)$. In eqs. (17) $\langle f(\Theta, \varphi) \rangle = \tilde{\Omega}/(2\pi) \int_0^{2\pi} f(\tilde{\Theta}, \hat{t} + \check{\varphi}) \hat{t} d\hat{t}$ denotes an averaging of an arbitrary function $f(\Theta, \varphi)$ over the fast time variable $\hat{t}$. Using a substitution analogous to eq. (11) these averages can be performed and result in

$$\langle X^{ex}_\Theta \rangle = (\tilde{p} + \tilde{\Omega}) \nabla \frac{\nabla \tilde{\Theta}}{\tilde{p}} - \tilde{\Omega} \sin \tilde{\Theta} \cos \tilde{\Theta} (\nabla \check{\varphi})^2$$

$$\langle X^{ex}_\Phi \rangle = \sin \tilde{\Theta} \frac{\tilde{p} + \tilde{\Omega}}{\tilde{p} - \tilde{\Omega}} \nabla \check{\varphi}$$

$$\langle X^{an}_\Theta \rangle = h_\perp^2 \nu^{-2} (\tilde{\Omega} - \tilde{p}) [1 - \Gamma^2 (1 + \tilde{\Omega} \tilde{p} + \tilde{\Omega}^2)] \cot \tilde{\Theta}$$

$$\langle X^{an}_\Phi \rangle = 2 \Gamma h_\perp^2 \nu^{-2} (\tilde{\Omega} - \tilde{p}) \cot \tilde{\Theta}$$

$$\langle Y_\Theta \rangle = -\Gamma^2 h_\perp \nu^{-2} (u \mathbf{e}_{\text{dom}}^{(1)}) \sin \tilde{\Theta}$$

$$\langle Y_\Phi \rangle = \Gamma h_\perp \nu^{-2} u^{-1} [\nu^{-2} - \Gamma^2 h_\perp^2 + \mathbf{e}_{\text{dom}}^{(1)}] \sin \tilde{\Theta}.$$  }

(18)

where now $\tilde{p}$ stands for $\hat{p}(\tilde{\Theta})$.

Together with the expressions (14) the coupled set of eqs. (17) represents the general result of this work. On the slow time scale $T$ the temporal and the spatial evolution of the magnetisation is governed by these reduced equations of motion. It is expected that various, interesting questions can be analysed on the basis of these equations. One of these problems, the profile of a stationary domain wall is treated in the next section.

V. DOMAIN WALL

In this first approach to the domain wall problem I restrict myself to planar walls whose spatial dependence varies - locally and/or approximately - only in one direction, the normal direction $\mathbf{e}_\perp$ of the wall. The analysis is further restricted to stationary walls. Thus $\Theta(\xi)$ is assumed to be time independent. For the cyclic variable $\check{\varphi}$ the ansatz $\check{\varphi}(T, \xi) = \Omega^{(1)} \mathbf{e}_\perp$ is used where $\Omega^{(1)}$ describes a first order frequency contribution independent of $\xi$. Fixing the scale of $\xi$ by the replacement ($r \mathbf{e}_\perp$) $\rightarrow h_\perp \nu^{-1} \xi$ and introducing the ’local wave number variable’ by

$$Q(\xi) = \check{\varphi}'(T, \xi)$$

(19)

eqs. (17) lead to a system of ordinary differential eqs. for $\tilde{\Theta}(\xi)$ and for $Q(\xi)$

$$\left(\frac{\check{\varphi}'}{\tilde{p}}\right)' - \tilde{\Omega} \sin \tilde{\Theta} \cos \tilde{\Theta} Q^2 = \left(\frac{\tilde{p}}{\tilde{p} + \tilde{\Omega}}\right) \left[1 - \Gamma^2 (1 + \tilde{\Omega} \tilde{p} + \tilde{\Omega}^2)\right] \cot \tilde{\Theta} + C_\Theta \sin \tilde{\Theta}$$

$$\sin \tilde{\Theta} \left(\frac{\tilde{p} - \tilde{\Omega}}{\tilde{p} + \tilde{\Omega}}\right) Q' = 2 \Gamma \left(\frac{\tilde{p} - \tilde{\Omega}}{\tilde{p} + \tilde{\Omega}}\right) \cot \tilde{\Theta} + C_\Phi \sin \tilde{\Theta}$$

(20)

where $\tilde{p}$ still stands for $\hat{p}(\tilde{\Theta}) = \sqrt{\tilde{\Omega}^2 + \sin^2 \tilde{\Theta}}$ and where the constants $C_\Theta$ and $C_\Phi$ satisfy the linear system
\[ \Gamma C_\phi + C_\Phi = \Gamma^2 w^2 h_{\perp}^{-1} v^{-2} [u \hat{e}_1 \mathbf{m}_{\text{dom}}^{(1)} + h_{\perp} \hat{e}_2 \mathbf{m}_{\text{dom}}^{(1)}] \]

\[ - C_\Theta + \Gamma C_\Phi = w^2 h_{\perp}^{-2} \Omega^{(1)} - \Gamma w^2 h_{\perp}^{-1} v^{-2} u^{-1} [ (v^2 - \Gamma^2 h_{\perp}^2) \hat{e}_1 \mathbf{m}_{\text{dom}}^{(1)} + \Gamma^2 h_{\perp} u \hat{e}_2 \mathbf{m}_{\text{dom}}^{(1)}]. \]

(21)

(22)

Apart from solving eqs.(21) \( \tilde{\Theta}(\xi) \) and \( Q(\xi) \) have to satisfy at the boundaries

\[ \tilde{\Theta}(\xi \to -\infty) = 0; \quad \tilde{\Theta}(\xi \to \infty) = \pi; \quad \tilde{\Theta}'(\xi \to \pm\infty) = 0; \quad Q(\xi \to \pm\infty) = \text{const.} \]

(23)

where the range of \( \xi \) has been extended over the entire interval from \(-\infty\) to \(\infty\). Just by counting constants of integration it is obvious that the conditions (23) can generally not be satisfied, noting additionally that one constant is the center of the wall \( \xi_0 \) due to translation invariance of eqs.(21). For special values of \( C_\phi \) and \( C_\Phi \), however, solutions of this boundary eigenvalue problem may exist.

For \( C_\phi = C_\Phi = 0 \) a solution exists which exhibits the symmetry properties \( \tilde{\Theta}(\xi) = \pi - \tilde{\Theta}(\xi) \) and \( Q(\xi) = Q(-\xi) \). This will be demonstrated by construction and for this propose eqs.(21) are formally integrated

\[ (\tilde{\Theta}')^2 = 2 \tilde{p}^2 (\tilde{p} + \Omega)^2 \int_{\tilde{\Theta}}^{\pi/2} \frac{1 - \Gamma^2 (1 + p\tilde{\Omega} + \Omega^2)}{(p + \Omega)^3} + \frac{\tilde{\Omega} Q^2}{(p + \Omega)^2} \frac{dp}{\Omega^2} \]

\[ Q = 2 \Gamma \tilde{p} + \frac{\Omega}{\tilde{p} - \Omega} \int_{\tilde{\Theta}}^{\pi/2} \frac{\tilde{p}(p - \tilde{\Omega})^{1/2}}{(p + \Omega)^{5/2}} \frac{dp}{\tilde{\Theta}^2} \]

(24)

Consider both \( \tilde{\Theta}' \) and \( Q \) as functions of \( \tilde{\Theta} \) or according to the relation \( \sin \tilde{\Theta} = (\tilde{\Omega}^2 - \tilde{p}^2)^{1/2} \) as functions of \( \tilde{p} \). Then the eqs.(24) represent a system of integral eqs. for \( \tilde{\Theta}' \) and \( Q \) in dependence of \( \tilde{p} \). Denoting the explicit solutions of this system by \( F(\tilde{p}) = \tilde{\Theta}' \) and by \( G(\tilde{p}) = Q \) further integration leads to \( \xi = f(\tilde{\Theta}(\xi)) F^{-1}(\sqrt{\tilde{\Omega}^2 + \sin^2 \tilde{\Theta}}) \frac{d\tilde{\Theta}}{d\tilde{\Theta}} \) where the center of the wall has been fixed by \( \tilde{\Theta}(0) = \pi/2 \). Thus the \( \xi \) dependence of \( \tilde{\Theta}(\xi) \) is implicitly determined from which \( Q(\xi) \) and consequently \( \dot{\varphi}(T,\xi) \) can be calculated according to eq.(19).

In the limit \( \tilde{p} \to \tilde{\Omega} \) which corresponds to the limit \( \xi \to \pm\infty \) the eqs.(24) lead to \( F(\tilde{p}) \sim (\tilde{p} - \tilde{\Omega})^{1/2} \to 0 \) and to \( G(\tilde{p}) \to \text{const.} \). These findings are valid for all parameter values \( \tilde{\Omega} \) and of \( \Gamma \) and imply that all the boundary conditions (23) are satisfied.

Turning now to the question whether further solutions of the boundary eigenvalue problem exist eqs.(21) are again integrated formally. Due to \( C_\phi \neq 0 \) and \( C_\Phi \neq 0 \) additional contributions arise to eqs.(21). From these contributions the asymptotic behaviour for \( \xi \to \infty \) \( (\tilde{\Theta}'(-\xi))^2 - (\tilde{\Theta}'(\xi))^2 \sim C_\phi \) and \( Q(-\xi) - Q(\xi) \sim C_\Phi (\tilde{p} - \tilde{\Omega})^{-1/2} \) can be deduced. These results imply that the conditions (23) can not be satisfied with \( C_\phi \neq 0 \) or with \( C_\Phi \neq 0 \). Thus there are no further solutions and \( C_\phi = C_\Phi = 0 \) represents a compatibility condition.

In general the explicit solutions of eqs.(21) with \( C_\phi = C_\Phi = 0 \) have to be calculated numerically. Fig.(1) shows the results of such a calculation for the parameter values \( \tilde{\Omega} = \sqrt{3} \) and \( \Gamma = 0.1 \). Note that the values of \( Q \) are small compared to the variation of \( \tilde{\Theta} \). This findings imply that the spatial dependence of the wall is mainly determined by \( \tilde{\Theta}(\xi) \) and are attributed to the fact that \( Q(\xi) \) is proportional to \( \Gamma \). Thus alternatively an iteration procedure can be employed to find approximate solutions. Starting with \( Q(0) = 0 \) at the rhs. of eqs.(24) these eqs. determine first approximations. For the above parameter values the deviations from the exact solutions are found to be small. As usually \( \Gamma \ll 1 \) is satisfied this indicates that the first iteration of eqs.(24) - which can be calculated analytically - may be sufficient in many cases.

According to eq.(21) \( C_\phi = C_\Phi = 0 \) implies the relation \( u \hat{e}_1 \mathbf{m}_{\text{dom}}^{(1)} + h_{\perp} \hat{e}_2 \mathbf{m}_{\text{dom}}^{(1)} = 0 \). This additional relation to the eqs.(1) and (8) now determines in a unique way the dependence of \( n_{\perp}^{(1)} \) of \( \mathbf{m}_{\perp}^{(1)} \) and of \( \mathbf{m}_{\text{dom}}^{(1)} \) on the model parameter. With eq.(22) this also applies to \( \Omega^{(1)} \). Thus all quantities of the domain states including the spatiotemporal wall structure are uniquely determined. This result together with the fact that the wall width remains finite in the large \( V \) limit shows the self-consistency of the ansatz(4).

A further consequence of the additional relation is \( \langle Y_\phi \rangle = 0 \) and thus this term in eqs.(17) drops out. By a redefinition of the fast time scale in the first order term, it is possible to transfer the contribution \( \langle Y_\phi \rangle \) of eqs.(17) to the fast dynamics. Employing this redefinition both \( \langle Y_\phi \rangle \) as well as \( \langle Y_\delta \rangle \) drop out of eqs.(17). This findings considerably simplify the analysis of further problems based on the general eqs.(17).

Next the internal fields \( \mathbf{H}_1 \) and \( \mathbf{H}_2 \) are introduced by \( \mathbf{H}_2 = \mathbf{H}_1 + \omega \mathbf{e}_z = \hat{h}_{\parallel} \mathbf{e}_z + \hat{h}_{\perp} \mathbf{e}_z - \mathbf{m}_{\text{dom}} \). Employing again the above relation and eqs.(8) it is found that up to the first order

\[ \mathbf{H}_1 \mathbf{H}_2 = 0 \]

(25)
To conclude this section it is mentioned that within the domain regions a linear stability analysis has been performed with the result that all the in zeroth order undamped modes of \( m \) have now finite damping constants proportional to \( \epsilon \). This stability analysis does not include the wall regimes as for such a complete treatment numerical methods have to be applied. The complete zeroth order stability analysis, the present partial first order analysis and the results of the computer simulations yield considerable evidence for the stability of the dynamic domain states. In this context it is pointed out that in the weak coupling limit \( \Gamma \ll 1 \) a Lyapunov function can be constructed on the slow time scale which implies stability.

VI. COMPARISON WITH SIMULATIONS

With the numerical results of the last section and an additional integration the zeroth order contributions of eqs. (16) are determined. With these results for \( \Theta \) and \( \varphi \) the Cartesian components of \( m \) can be calculated according to eq. (9). This has been done for the parameter values \( h_\parallel = \omega = 2, \ h_\perp = 0.5 \) and \( \Gamma = 0.1 \) neglecting \( \Omega^{(1)} \) and all other first order corrections in eq. (3) which implies also \( e_i = e_i \). The results of this calculation are given in Fig. (2a) and Fig. (2b) where the spatiotemporal dependence of \( \cos \varphi = m_1 (m_1^2 + m_2^2)^{-1/2} \) and of \( m_3 \) are presented. For the time dependence anharmonic oscillations are found which are plotted on the \( \tau = \hat{\Omega} \hat{t} \) scale for one period.

Fig. (2c) and Fig. (2d) show the corresponding results of the computer simulations of eq. (1) performed with the same parameter values and using in addition \( J = 0.01 \) and \( A = -0.005 \). The original simulation results are rescaled to the theoretical scales. With the exception of the center of the wall and the time origin no fitting procedure has been used.

With deviations of usually less than one percent the analytic results agree with the simulations. Quantitative agreement for \( \bar{m}_{\text{dom}} \), for \( m_\pm \) and for \( n_+ \) was already found in [8] with a similar high accuracy. Based on these agreements it has been demonstrated that the employed methods are a suitable tool to explain the characteristic features of the dynamic domain states.

VII. CONCLUSIONS

In this work a rather realistic model for a ferromagnet was investigated. Depending on the parameter values this model can exhibit stationary dynamic domain states in the rotating frame. An important element of this states is the spatiotemporal structure of the domain walls. With the present investigation this structure is understood to a high extent.

The analysis of wall structure is based on the reduced equations of motion (17) which from the theoretical point of view represent the main result of this work. These eqs. which govern the temporal and the spatial evolution on the slow time scale should be applicable to other phenomena. Obvious extensions are investigations on drifting walls, on multi-dimensional walls and on the interaction between domain walls.

As a further propose the present approach should be extended to take stray fields into account. This would imply that the dipole interaction is completely included and the model will become very realistic for ferromagnetic materials. Moreover by analogy with the static case, it is expected that these dipolar stray fields contribute crucially to selection mechanism for the wall positions. In this context the formation of regular patterns in driven ferromagnets seems to be possible.

The author has benefited from discussions with W. Just and G. Sauermann. This work was performed within SFB 185.
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FIG. 1. Numerical solution of eq.(20) (with $C_{θ} = C_{φ} = 0$) for $Θ(ξ)$ and $Q(ξ)$ satisfying the boundary conditions. of eq.(23). The parameter values are $Ω = 3\frac{1}{2}$ and $Γ = .1$.

FIG. 2. Spatiotemporal dependence of a stationary domain wall in reduced units $ξ$ and $τ$. In Fig.(2a) and (2b) the results of the analytical treatment of this work are presented. For comparison Fig.(2c) and (2d) show the results of the numerical simulations of [8]. For both cases $cosφ = m_1(m_1^2 + m_2^2)^{-1/2}$ and $m_3$ are plotted, where $m_i$ are the components of the local magnetisation in the internal coordinates, defined by eqs.(4) and (5). The scaled position $ξ$ and scaled time $τ$ are given by $ξ = wh_{⊥}^{-1}(-A/J)_{1/2}eqr$ and by $τ = Ω t = ωvw^{-1}t$, respectively. Compare text for the parameter values.