A new three flavor oscillation solution of the solar neutrino deficit in $R$-parity violating supersymmetry

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Abstract

We present a solution of the solar neutrino deficit using three flavors of neutrinos within the $R$-parity non-conserving supersymmetric model. In vacuum, mass and mixing is restricted to the $\nu_{\mu}$-$\nu_{\tau}$ sector only, which we choose in consistency with the requirements of the atmospheric neutrino anomaly. The $\nu_e$ is massless and unmixed. The flavor changing and flavor diagonal neutral currents present in the model and an energy-dependent resonance-induced $\nu_e$-$\nu_{\mu}$ mixing in the sun result in the new solution to the solar neutrino problem. The best fit to the solar neutrino rates and spectrum (1258-day $SK$ data) requires a mass square difference $\sim 10^{-5}$ eV$^2$ in vacuum between the two lightest neutrinos. This solution cannot accommodate a significant day-night effect for solar neutrinos.

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Neutrino oscillation is the most popular solution of the solar neutrino problem $^[4]$, $^[5]$ and the atmospheric neutrino anomaly $^[6]$. Oscillation in vacuum or in matter, through the MSW resonance mechanism, posits that neutrinos have non-vanishing, non-degenerate masses and that the basis defined by these eigenstates does not coincide with the flavor basis.

Supersymmetry (SUSY) with $R$-parity non-conservation is an extension of the Standard Model (SM) which is consistent with all particle physics experiments and is phenomenologically rich $^[7]$. It carries within it new interactions between leptons and quarks which violate baryon ($B$) and lepton ($L$) number. In this work we show that the flavor changing neutral currents (FCNC) and flavor diagonal neutral currents (FDNC) due to the $L$-violating interactions induce mixing amongst neutrinos in matter, the key feature in this alternative solution of the solar neutrino discrepancy, even though, in vacuum, the $\nu_e$ state is massless and does not mix with the other neutrinos. We also indicate how in this model the parameters can be chosen to address the atmospheric neutrino anomaly, solving the solar neutrino problem at the same time.

Origins of neutrino oscillation other than mass-mixing, notable among them being non-standard interactions of neutrinos with matter, like FCNC, were examined by Wolfenstein $^[8]$.

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It is noteworthy that FCNC and FDNC interactions can drive neutrino oscillations even for massless neutrinos without any vacuum mixing through an energy-independent resonance effect \(^3\). However, these solutions require large \(L\)-violating couplings near their present experimental upper bounds \(^4\). This has been examined earlier in connection with the solar \(^5\) and atmospheric neutrino data \(^5\) \(^6\) in the two flavor oscillation framework.

The new explanation of the solar neutrino deficit that we propose, in contrast, relies on two solar \(^6\) \(^7\) \(^9\) and atmospheric neutrino data \(^8\) \(^10\) in the experimental upper bounds \(^7\) \(^8\). This has been examined earlier in connection with the twenty-seven rotation matrix) and write the effective mass squared matrix, \(\hat{m}^2\), assuming that bilinear terms have been rotated away with appropriate redefinition of superfields. Here, \(i,j\), and \(k\) are generation indices, \(L\) and \(Q\) are chiral superfields containing left-handed lepton and quark doublets and \(E\) and \(D\) are chiral superfields containing right-handed charged-lepton and \(d\)-quark singlets. There are nine \(\lambda\) (antisymmetric in \((ij)\)) and twenty-seven \(\lambda'\) couplings, only a few of which will be relevant for this analysis.

The interaction of neutrinos with the electrons and \(d\)-quarks in matter induces transitions (i) \(\nu_i + e \rightarrow \nu_j + e\), and (ii) \(\nu_i + d \rightarrow \nu_j + d\). (i) can proceed via \(W\) and \(Z\) exchange for \(i = j\), as well as \(\lambda\) couplings for all \(i,j\), while process (ii) is possible through \(\lambda'\) couplings and \(s\)-quark exchange. Here we concentrate only on the \(\lambda\)-induced contributions.

The time evolution of the neutrino flavor eigenstates \((\nu_i, i = e, \mu, \tau)\) is governed by

\[
H = H^0 + h^{\text{matter}}
= \begin{pmatrix}
E & 0 & 0 \\
0 & E + S_+ - T_1 & T_2 \\
0 & T_2 & E + S_+ + T_1
\end{pmatrix} + \begin{pmatrix}
R_{11} + A_1 - A_2 & 0 & R_{13} \\
0 & -A_2 & 0 \\
R_{13} & 0 & R_{33} - A_2
\end{pmatrix},
\]

where \(S_\pm = (m_1^2 \pm m_2^2)/4E\), \(T_1 = S_- \cos 2\theta_{23}\), \(T_2 = S_- \sin 2\theta_{23}\), \(A_1 = \sqrt{2}G_F n_e\), \(A_2 = G_F n_N/\sqrt{2}\), and \(R_{ij} = \lambda_{ijk}\lambda_{jk}n_e/4\tilde{m}^2\). \(E\) is the neutrino energy and \(\theta_{23}\) the vacuum mixing angle in the \(\nu_\mu - \nu_\tau\) sector. \(n_N\) and \(n_e\) are the neutron and electron number densities in matter and \(\tilde{m}\) is the slepton mass. \(A_1\) and \(A_2\) in \(h^{\text{matter}}\) arise from SM charged and neutral current interactions, respectively. In vacuum, \(h^{\text{matter}} = 0\) and \(H\) contains mixing only in the \(\nu_\mu - \nu_\tau\) sector. In \(h^{\text{matter}}\), we choose \(^{11}\) \(k = 2\) in the matter-induced contributions \(R_{ij}\). For anti-neutrinos, the time evolution is determined by a similar total hamiltonian \(\hat{H} = H^0 - h^{\text{matter}}\).

To obtain the mass eigenstates, first we rotate by \(U' = U_{23} U_{13}\) (where \(U_{ij}\) is the standard rotation matrix) and write the effective mass squared matrix, \(\frac{\hat{M}^2}{2E} = H - E - A_1 - A_2\), in the new basis as

\[
\frac{\hat{M}^2}{2E} \approx \begin{pmatrix}
R_{11}c_{13}^2 - 2R_{13}c_{23}s_{13}c_{13} + \Lambda + s_{13}^2 & -R_{13}s_{23}c_{13} & 0 \\
-R_{13}s_{23}c_{13} & -R_{13}s_{23}s_{13} & 0 \\
0 & R_{13}s_{23}s_{13} & R_{11}s_{13}^2 + 2R_{13}c_{23}s_{13}c_{13} + \Lambda + s_{13}^2
\end{pmatrix},
\]

\(^{11}\)In view of the antisymmetry of \(\lambda_{ijk}\) in \((i,j)\), in order to generate the mixing of the \(\nu_e\) with the other neutrinos we have to choose \(k = 2\) or 3. For the latter choice, mixings due to \(R\) interactions are very small; for example, \(\lambda_{131}\lambda_{231}\) is highly constrained from \(\mu \rightarrow 3e\) decay \(^4\).
where
\[ \Lambda_\pm = \left[ S_+ - A_1 + \frac{R_{33}}{2} \right] \pm \left[ S_- \cos 2(\theta_{23\nu} - \theta_{23}) + \frac{R_{33}}{2} \cos 2\theta_{23} \right] \] (4)
and \( c_{ij} \equiv \cos \theta_{ij} \) and \( s_{ij} \equiv \sin \theta_{ij} \). Furthermore,
\[ \tan 2\theta_{23} = 2T_2/(2T_1 + R_{33}); \quad \tan 2\theta_{13} = 2R_{13}/D_1; \quad D_1 = \Lambda_+ - R_{11}. \] (5)

Note that \( \theta_{23} \approx \theta_{23\nu} \) while \( \theta_{13} \approx 0 \) except near a possible resonance, when \( D_1 = 0 \). We show below that this resonance condition cannot be achieved in the sun. Consequently, to a good approximation, the third state in this basis decouples in eq. (3). The upper left \( 2 \times 2 \) block is readily diagonalised, resulting in three effective masses \( \tilde{m}_i \) as:
\[ \tilde{m}_1^2/(2E) = c_{12}^2 \left( R_{11}c_{13}^2 - R_{13}c_{13} \sin 2\theta_{13} + \Lambda_+ s_{13}^2 \right) + R_{13}s_{23}c_{13} \sin 2\theta_{12} + \Lambda_- s_{12}^2 \] \[ \tilde{m}_2^2/(2E) = s_{12}^2 \left( R_{11}c_{13}^2 - R_{13}c_{13} \sin 2\theta_{13} + \Lambda_+ s_{13}^2 \right) - R_{13}s_{23}c_{13} \sin 2\theta_{12} + \Lambda_- c_{12}^2 \] \[ \tilde{m}_3^2/(2E) = R_{11}s_{13}^2 + R_{13}c_{23} \sin 2\theta_{13} + \Lambda_+ c_{13}^2. \] (6)

where
\[ \tan 2\theta_{12} = -\frac{2R_{13}s_{23}c_{13}}{D_2}; \quad D_2 = \Lambda_- - R_{11}c_{13}^2 + R_{13}c_{23} \sin 2\theta_{13} - \Lambda_+ s_{13}^2. \] (7)

A resonant enhancement of \( \theta_{12} \) occurs when \( D_2 = 0 \). The neutrino flavor eigenstates \( \nu_\alpha = \nu_{e,\mu,\tau} \) are related to the mass eigenstates \( \nu_i = \nu_{1,2,3} \) by
\[ \nu_\alpha = \sum_i U_{\alpha i} \nu_i, \] (8)

where \( U_{\alpha i} \) are elements of the unitary mixing matrix
\[ U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & 0 \\ s_{13}s_{23} - c_{12}s_{23}s_{13} & -c_{12}s_{23}s_{13} & c_{23}c_{13} \end{pmatrix}. \] (9)

We have chosen real \( L \) violating couplings and as such there is no \( CP \) violating phase in the above mixing matrix. Further, in order to satisfy \( 0 \leq \theta_{12} \leq \pi/2 \) in eq. (4) for convenience, we take \( \lambda_{121}\lambda_{321} < 0 \).

As noted above, level crossings and resonance behavior, which are energy dependent due to neutrino masses, can occur in two situations, namely, (a) when \( D_1 = 0 \), and (b) when \( D_2 = 0 \). Of these, only the latter can be satisfied inside the sun, as we now discuss. The sub-GeV and multi-GeV zenith angle dependence of atmospheric neutrinos as well as the energy dependence of the up-down asymmetry require \( \Delta m_{32} \approx m_{31}^2 \approx 10^{-3} \text{ eV}^2 \) with maximal vacuum mixing in the \( \nu_\mu - \nu_\tau \) sector \([3]\). The presence of \( L \) violating interactions does not alter this significantly (see later). On the other hand, \( n_e \) at the core of the sun is about \( 1.13 \times 10^{12} \text{ eV}^3 \). Thus even for \( E \) as high as 20 MeV, it is not possible to satisfy the (a) resonance condition and hence we consider only the (b) resonance in the subsequent

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\[ \text{footnote: This follows as } \Lambda_+ - R_{11} \approx m_{32}^2/(2E) \text{ is very large with respect to } R_{13} \text{ in the sun.} \]
the SSM, are 0.56 and 0.44, respectively. In fact, one can see from eqs. (2) and (7) that only \( \nu_{SM} \) where only \( \theta \) interactions, appears in tan \( 2 \theta_{12} \) if \( U_{13} \) in vacuum is very small then solar neutrinos will be almost unaffected by the mass of \( \nu_e \) and analysis with three neutrino flavors may not be essential. However, unlike in the SM, where only \( \nu_e \) interactions with matter are relevant for neutrino oscillation, in the \( R \) supersymmetric Model, FCNC and FDNC interactions of all three flavors of neutrinos turn out to be important. In fact, one can see from eqs. (2) and (7) that \( R_{13} \), arising from FCNC interactions, appears in tan \( 2 \theta_{12} \) and plays a pivotal role.

We now turn to the oscillation of solar neutrinos due to their interaction with matter inside the sun. As already discussed, \( \nu_e \) in the sun can experience only one of the two resonances. \( s_{13} \) in eq. (9) is very small as noted earlier and we use the survival probability of \( \nu_e \) valid for a two flavor analysis:

\[
P_{\nu_e \rightarrow \nu_e} = \frac{1}{2} + \left( \frac{1}{2} - P_{\text{jump}} \right) \cos 2\theta_{12}(x_1) \cos 2\theta_{12}(x_0),
\]

where \( x_0 \) is the production point inside the sun and \( x_1 \) the detection point at earth. The jump probability is \( P_{\text{jump}} \approx \exp \left[ -\pi \gamma_{\text{res}} F/2 \right] \), \( \gamma_{\text{res}} \) being the adiabaticity parameter. \( F = 1 \) for the exponential density profile since the vacuum mixing angle is zero and

\[
\gamma_{\text{res}} = \frac{\tilde{m}_2^2 - \tilde{m}_1^2}{4E\theta_{12}} \approx \frac{m_2^2}{E} \left( \frac{p}{\kappa} \right)^2 \left( \frac{n_e}{n_{\nu_e}} \right)_{\text{res}},
\]

where \( \kappa = \frac{(2\lambda_{121}^2 - \lambda_{321}^2)}{(8\tilde{m}_\mu^2) + \sqrt{2}G_F} \sim \sqrt{2}G_F \) and \( p = |\frac{\lambda_{121}\lambda_{321}}{4m_\mu^2}| \).

In order to obtain the best-fit values of \( \Delta m_{12} \approx m_2^2 \) and \( p \), we have performed a \( \chi^2 \) analysis using the Standard Solar Model (SSM) and the solar neutrino rates from the Homestake (Cl), Gallex, Sage, and Kamiokande (K) experiments. We have also used the latest SK rates and spectrum data for 1258 days. Taking into account the production point distributions of neutrinos from the different reactions (e.g., pp, pep, \( 7 \)Be, \( 8 \)B etc.), we have calculated the averaged survival probabilities using eq. (10). Here, we include a parameter \( X_B \) to take into account a possible deviation of the overall normalization of the \( 8 \)B flux from its SSM value. We set \( \theta_{23v} = \pi/4 \). The best-fit values of the parameters are presented in Table 1 along with \( \chi^2_{\text{min}} \), the goodness of fit (gof), and the calculated rates using these values of the parameters. Case (1) is a fit to the total rates. Note that the best-fit parameters result in an unusually good fit to the Cl rate and the Ga prediction is right near the average of the Sage and Gallex data. We have found that the fit improves even more if the K-rate is excluded. In case (2) we have fitted the SK spectrum while (3) is a fit to the total rates and the SK spectrum. In Fig. 1 is shown the calculated spectrum for SK for the best-fit parameters along with the experimental data. Also shown is the charged current spectrum expected at SNO for one sample case, the best-fit values in case (3).

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3Notice that \( \cos 2\theta_{12}(x_1) = 1 \), corresponding to \( \theta_{12} = 0 \) in vacuum.

4\( \lambda_{121} \) and \( \lambda_{321} \) are tightly constrained. Besides significant cancellation between these terms is possible if they are of same order.

5We have dropped a small contribution from the hep process.

6We have checked that the fit (3) is essentially unchanged if the SK rate is excluded from the fit.

7For these best-fit values, the prediction for the neutral and charged current rates at SNO, normalized to the SSM, are 0.56 and 0.44, respectively.
Table 1: The best-fit values of the parameters, $p = |\frac{\lambda_{121} \lambda_{321}}{4m_\tilde{b}^2}|$, $m_2^2$, and $X_B$ from fits to (1) all rates, (2) the $SK$ spectrum, and (3) rates and $SK$ spectrum. The rates for the different experiments obtained using these best-fit parameters are also shown.

The best-fit values of $R$ couplings in Table 1 are consistent with the existing constraints. For example, in case (3), choosing $m_{\tilde{b}} \sim 100$ GeV, we get $\lambda_{121} \lambda_{321} \approx 0.0144$. $\lambda_{121}$ is constrained from $\mu \rightarrow e\bar{\nu}_e \nu_\mu$ decay (with selectron exchange tree level diagram apart from the SM $W$ exchange diagram). The bound on $\lambda_{321}$ is from $R \equiv \Gamma(\tau \rightarrow e\nu\bar{\nu})/\Gamma(\tau \rightarrow \mu\nu\bar{\nu})$ which gets a contribution from a selectron exchange diagram. For $m_\xi \sim 200$ GeV or more, the requirements are easily satisfied.

Turning now to atmospheric neutrinos, for a simple-minded analysis we can consider the earth to be a slab of a single density. $n_e$ in earth lies in the range $(3 - 6)N_A$ cm$^{-3}$. So the resonance condition, $D_2 = 0$, cannot be met for atmospheric neutrinos having energy near the GeV range. In order to explain the observed zenith angle dependence, we must choose $\Delta m_{32} \sim 10^{-3}$ eV$^2$. This precludes the occurrence of the other resonance, $D_1 = 0$. Since neither resonance condition can be satisfied, there will be almost no effect on atmospheric neutrino oscillation due to the $L$ violating interactions as the associated couplings are very small. So one can consider the mixing matrix in eq. (9) valid for vacuum for which only $\theta_{33b}$ is non-zero. Thus the solution to the atmospheric neutrino anomaly is just the standard two neutrino mass-mixing one.

The neutrino masses and mixing pattern in vacuum required in this solution can naturally arise in many models. For example, the trilinear couplings in eq. (11) contribute to the neutrino mass matrix at the one-loop level through slepton or squark exchange diagrams. In particular, from the $\lambda'$ couplings one obtains:

$$m_{ij}^{\text{loop}} = \frac{3m_b^2}{8\pi^2 \bar{m}_{\tilde{b}}^2} \frac{(A_b + \mu \tan \beta)}{\lambda'} \lambda'_{333} \lambda'_{333}. \quad (12)$$

where $A_b$ and $\mu$ are soft SUSY-breaking parameters, $\bar{m}_{\tilde{b}}$ is the $b$-squark mass and $\tan \beta$ is the ratio of two Higgs vacuum expectation values. The last two generation indices in $\lambda'$ have been chosen as 3 for which the loop contributions are enhanced via the $b$–quark mass. We remark that $m_{ij}$ is very small when $i = 1$ and/or $j = 1$ because of the more stringent constraint on $\lambda'_{333}$. Notice that this mass matrix can correspond to almost maximal mixing for $\nu_\mu$ and $\nu_\tau$ if $\lambda'_{233} \approx \lambda'_{333}$, with two neutrino masses very small and one neutrino having significantly higher mass $m_3 \approx 2 m_{33}^{\text{loop}}$, which can be suitably chosen by taking appropriate values of the different parameters in (12). It should be borne in mind that $m_2$ depends on the difference
of $\lambda'_{233}$ and $\lambda'_{333}$ and can be several orders less than the mass of the heavier neutrino while there will be almost maximal mixing. The remaining neutrino mass is $m_1 \approx 0$. Thus masses and vacuum mixings can be as required in the model under consideration.

This neutrino mixing pattern also satisfies the bound $U_{13}^2 \leq 0.04$ in vacuum from the CHOOZ reactor experiment [14]. In fact, in vacuum $U_{13}^2 = 0$.

A comment about the earth regeneration effect for solar neutrinos is pertinent. The $\nu_e$ is unmixed with the other neutrinos in vacuum. As $n_e$ in earth is about two orders less than that near the core of the sun, no resonance condition will be satisfied. Hence, there will not be an earth effect for solar neutrinos. In comparison with the small angle MSW fits [15], the somewhat larger best-fit $\Delta m_{12}$ and the zero value of $\theta_{12}$ in vacuum here, result in a smaller day-night effect.

Though our discussion has been within the framework of $R$-parity violating SUSY, there are other models [16] where FCNC and FDNC interactions are present. Our results can be adapted to these scenarios in a straight-forward manner.

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Figure 1: The calculated SK solar neutrino spectrum for the best-fit parameters $\Delta m_{12}$, $p$, and $X_B$ from (1) rates, (2) SK spectrum, and (3) rates and spectrum. The SK 1258-day data [12] and the predicted SNO charged current spectrum for the best-fit (3) are also shown.