Abstract

The axial $N \rightarrow \Delta (1232)$ transition form factors are calculated within the light cone QCD sum rules method. A comparison of our results with the predictions of lattice theory and quark model is presented.

Key words: Axial Form Factors, Nucleon Structure, Light Cone QCD Sum Rules

PACS: 14.20.Dh, 13.75.Ev, 11.55.Hx

1 Introduction

Form factors have great importance in the investigation of the internal structure of baryons. The inner structure of the nucleon is encoded in several form factors. The electromagnetic form factors of the nucleon, which parameterize...
the matrix element of the electromagnetic current operators, are measured in a wide range of $q^2$ values \([1,2,3]\). The interest in nucleon form factor has been renewed by recent progress in experimental physics, where it became possible to get polarized beams and polarized targets. This possibility opened up two new methods [4] to measure the ratio of the electric and magnetic form factors, namely polarization transfer and beam-target asymmetry. New experimental data obtained from $e + p \rightarrow e + p$ reaction, which is performed at JLAB [5], gave the result that the ratio $\mu G_E/G_M$ decreases from unity substantially for high $Q^2$ values. Not only form factors relevant for the diagonal transitions, but also the electromagnetic transition form factors for electro-production of the $\Delta$ has also been the subject of recent experimental [5,6,7] and theoretical studies [8].

The main advantage of the nucleon to $\Delta$ transition is that, the $\Delta$ is a dominant nucleon resonance and its identification is easy since the spin-parity selection rules provide us information about the wave function of these baryons. The $N - \Delta$ transition form factors due to the weak axial current can give valuable information about the structure of the baryons, complementary to that obtained from electromagnetic transition. For example, measurement of axial form factors for $N - \Delta$ transition, allows us to check off diagonal Goldberger-Treiman relation, order of conservation of the axial current, etc. Weak axial $N - \Delta$ transition form factors are investigated in neutrino (or charged lepton) scattering on deuterium or hydrogen in the $\Delta$ region experiments. New information on the weak axial form factors is expected from parity-violating electron scattering experiments planned at Jefferson Laboratory [9].

In the present work, we calculate the axial $N \rightarrow \Delta$ transition form factors in light cone QCD sum rules (LCQSR). In the light cone QCD sum rules
method, operator product expansion is carried out near the light cone \( x^2 \approx 0 \), and non-perturbative dynamics is parameterized by the light cone distribution amplitudes that determine the matrix elements of the nonlocal operators between vacuum and one particle states. The expansion near the light cone is an expansion in the twist of the operators rather than the dimension as in the traditional QCD sum rules (for more about this method see e.g. [10,11] and references therein). The light cone distribution amplitudes of the proton is calculated in [24,25,26]. Using the light cone distribution amplitudes of the proton semileptonic \( \Lambda_b \to p\ell\nu \) decay, scalar form factor of the proton [16,17], axial and induced pseudo scalar form factors of the nucleon [18,19], \( \Sigma - n \) form factors [20] are studied. In [21], the distribution amplitudes of \( \Lambda \) in leading conformal spin is calculated and the obtained amplitudes are used to study the \( \Lambda_c \to \Lambda\ell\nu \) decay. Note that these form factors are calculated in lattice QCD in [12,13], and in chiral constituent quark model in [14].

The plan of this work is as follows: In section 2 we consider the generic correlator function and present the LCQSR formalism. In this section we obtain the LCQSR for the axial \( N \) to \( \Delta \) transition form factors. The numerical analysis and discussion is presented in section 3.

2 Sum Rules for the Axial Nucleon to Delta Transition Form Factors

In the present section, light cone QCD sum rules for the axial \( N \) to \( \Delta \) transition form factors are derived. These form factors are defined by the matrix element
of the axial current (in our case isovector part of the axial current), i.e.
\[ J_\nu^3(x) = \bar{\psi}(x)\gamma_\nu\gamma_5\tau^3\psi(x) \]  

(1)

between the nucleon state with momentum \( p \) and the \( \Delta \)-state with momentum \( p' = p - q \), where \( \tau^3 \) is the third Pauli matrix, and \( q \) is the transferred momentum. Hence, the axial \( N \to \Delta \) transition form factors are defined by the matrix element \( \langle \Delta(p',s') | J_\nu^3 | N(p,s) \rangle \). This \( N \) to \( \Delta \) weak matrix element can be expressed in terms of four invariant transition form factors as follows \[22,23]\:

\[ \langle \Delta(p',s') | J_\nu^3 | N(p,s) \rangle = i u_\lambda(p',s') \left\{ \left( \frac{C^A_3(q^2)}{m_N} \gamma_\mu \right) + \frac{C^A_4(q^2)}{m^2_N} p'_\mu \right\}
\]
\[ g_{\lambda\rho}g_{\mu\nu}q^\rho + C^A_5(q^2)g_{\lambda\nu} + \frac{C^A_6(q^2)}{m^2_N} q_{\lambda}q_{\nu} \} u(p,s) \]  

(2)

where \( u_\lambda(p',s') \) is the Rarita-Schwinger spinor for \( \Delta \) and \( u(p,s) \) is the nucleon spinor. Note that partial conservation of axial current (PCAC) and pion meson dominance leads to the relation \[13]\:

\[ C^A_6(Q^2) = \frac{m^2_N}{Q^2 + m^2_\pi} C^A_5(Q^2) \]  

(3)

For the calculation of the above mentioned form factors within the light cone QCD sum rule method we consider the matrix element in which one of hadrons is described by an interpolating current with quantum numbers of this hadron and another one is represented by the state vector. For the construction of the light cone sum rules, the distribution amplitudes (DA’s) of state vector hadron is needed. The nucleon distribution amplitudes for all three quark operators are calculated in \[24,25,26]\ and DA’s for \( \Delta \) isobar is not yet calculated. For
this reason, we consider the following correlator function where nucleon is described by state vector

$$\Pi_{\mu\nu}(p, q) = i \int d^4 x e^{iqx} < 0 | T\{\eta_\mu(0), J^3_\nu(x)\} | N(p) > \quad (4)$$

The interpolating current for the $\Delta^+$ isobar is chosen in the form [27]

$$\eta_\mu(0) = \frac{1}{\sqrt{3}} \varepsilon^{abc}[2(u^a T(0) C \gamma_\mu d^b(0)) u^c(0) + (u^a T(0) C \gamma_\mu u^b(0)) d^c(0)] \quad (5)$$

and the axial current is:

$$J^3_\nu(x) = \frac{1}{2} [\bar{\pi}(x) \gamma_\nu \gamma_5 u(x) - \bar{d}(x) \gamma_\nu \gamma_5 d(x)] \quad (6)$$

In order to construct the sum rules for the form factors, the correlation function will be represented in two different forms, i.e. in terms of hadron parameters and in terms of the quark-gluon parameters. Let us first consider the representation of the correlation function in terms of hadron parameters (phenomenological part). Phenomenological part of the correlation function can be obtained by inserting the complete set of hadrons with the same quantum numbers of $\eta_\mu(0)$ inside the correlation function. Saturating the correlator function with these hadrons and isolating the contribution coming from ground state hadron we get

$$\Pi_{\mu\nu}(p, q) = \sum_{s'} < 0 | \eta_\mu | \Delta^+(p', s') > < \Delta^+(p', s') | J_\nu | N(p, s) > \frac{m_\Delta^2 - p'^2}{m_\Delta^2 - \mu_2^2} + \cdots \quad (7)$$

where $m_\Delta$ is $\Delta^+$ mass and $\cdots$ represents contributions from higher states and the continuum. The matrix element $< \Delta^+ | J_\nu(x) | N >$ entering in Eq. (7) is given by Eq. (2) and the matrix element $< 0 | \eta_\mu(0) | \Delta^+ >$ is determined as

$$< 0 | \eta_\mu(0) | \Delta^+ > = \lambda_\Delta w^\mu(p', s') \quad (8)$$
where $\lambda_\Delta$ is the residue of $\Delta^+$ isobar and the value of the residue $\lambda_\Delta$ is determined from the two point sum rules [27,28,30,31]. Performing summation over spin of Rarita-Schwinger spinor using the formula

$$u_\mu(p',s)u_\nu(p',s) = -(\not{p'} + m_\Delta) \left\{ g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2p'_\mu p'_\nu}{3m_\Delta} + \frac{p'_\mu \gamma_\nu - p'_\nu \gamma_\mu}{3m_\Delta} \right\} \tag{9}$$

and using Eqs. (2), (7), (8) and (9) the contribution of $\Delta^+$ to the correlation function can be written as

$$\Pi_{\mu\nu}(p,q) = \frac{-i\lambda_\Delta}{m_\Delta^2 - q^2} (\not{p'} + m_\Delta) \left\{ g_{\mu\lambda} - \frac{1}{3} \gamma_\mu \gamma_\lambda - \frac{2p'_\mu p'_\lambda}{3m_\Delta} + \frac{p'_\mu \gamma_\lambda - p'_\lambda \gamma_\mu}{3m_\Delta} \right\} \left\{ \left[ C_3^A(q^2) \gamma_\alpha + \frac{C_4^A(q^2)}{m_N^2} p'_\alpha \right] (g_{\lambda\nu} q_\alpha - q_\lambda g_{\nu\alpha}) + C_5^A(q^2) g_{\lambda\nu} \right. \right.$$  

$$\left. + \frac{C_6^A(q^2)}{m_N^2} q_\lambda q_\nu \left\} u(p) \right. \tag{10}$$

From Eq. (10) it follows that the correlator function contains numerous tensor structures and not all of them are independent. The dependence can be removed by ordering gamma matrices in a specific order. In this work the ordering $\gamma_\mu$, $\not{p'}\gamma_\nu$, $\not{q}$ is chosen. With this ordering the correlation function becomes

$$\Pi_{\mu\nu}(p,q) = \frac{-i\lambda_\Delta}{m_\Delta^2 - q^2} (\not{p'} + m_\Delta) \left\{ g_{\mu\nu} \gamma^\alpha C_3^A(q^2) + g_{\mu\nu} \not{p'} C_4^A(q^2) \left[ C_3^A(q^2) \gamma_\alpha + \frac{C_4^A(q^2)}{m_N^2} p'_\alpha \right] \right. \right.$$  

$$\left. \times \left( C_5^A(q^2) + C_4^A(q^2) \frac{p'_\nu}{m_N^2} \right) - q_\mu (\not{p'} + m_\Delta) \gamma_\nu \frac{C_3^A(q^2)}{m_N^2} \right. \right.$$  

$$\left. - q_\mu (\not{p'} + m_\Delta) p'_\nu \frac{C_4^A(q^2)}{m_N^2} + q_\mu q_\nu (\not{p'} + m_\Delta) \frac{C_5^A(q^2)}{m_N^2} \right\} u(p) \right. \right.$$  

$$+ \text{other structures with } \gamma_\mu \text{ at the beginning or which are proportional to } p'_\mu. \tag{11}$$

The reason why the structures $\sim p'_\mu$ and structures with $\gamma_\mu$ at the beginning is not considered is follows. The interpolating current $\eta_\mu$ couples not only to spin-parity $(3/2)^+$ states, but also to spin-parity $(1/2)^-$ states. In other words
\( \eta_\mu \) has nonzero overlap with spin 1/2 states. This coupling can be written in the most general form as:

\[
\langle 0 | \eta_\mu | \frac{1}{2}(p') \rangle = (A p'_\mu + B \gamma^\mu) u(p)
\]  

(12)

Using the condition \( \gamma^\mu \eta_\mu = 0 \) one can easily obtain that \( B = -\frac{A m_{1/2}}{4} \). Therefore, in general case spin 1/2 states give also contribution to the considered correlation function but they contribute only to the structures which contain a \( \gamma_\mu \) at the beginning or which are proportional to \( p'_\mu \). By choosing the ordering \( \gamma_\mu p'_\nu \not\gamma_\nu \) and ignoring the structures proportional to \( p'_\mu \) and the structures that contain a \( \gamma_\mu \) at the beginning, the contributions to the correlation function from states that have spin 1/2 are eliminated.

Now we turn our attention to the calculation of the correlator function from the QCD side for the large Euclidean virtuality \( p'^2 = (p-q)^2 << 0 \) in terms of the nucleon distribution amplitudes. Using the explicit expression for the \( \Delta^+ \) isobar interpolating current Eq. (5) and axial current Eq. (6), and carrying out all contractions for the correlation function, Eq. (4), in \( x \) representation we get the following result

\[
\Pi_{\mu\nu}(p, q) = \frac{-1}{16\pi^2\sqrt{3}} \int \frac{d^4x e^{iqx}}{x^4} \left\{ (C \gamma_\mu)^{\alpha\beta} (\gamma_\nu \gamma_5)_{\rho\sigma} \varepsilon^{abc} < 0 | [4u^a_\eta(0)u^b_\theta(x)d^c_\phi(0) - 4u^a_\eta(0)u^b_\theta(0)d^c_\phi(x)(2g_\alpha g_\sigma g_\lambda \phi(\not{x})\lambda_\rho + g_\alpha g_\sigma g_\lambda \phi(\not{x})\rho_\sigma + g_\alpha g_\sigma g_\lambda \phi(\not{x})\alpha_\rho)] | N(p) > \right\}
\]  

(13)

where we have used the light cone expanded light quark propagator as [30]:

\[
S(x) = \frac{i}{2\pi^2 x^4} - <qq> (1 + \frac{m_0^2 x^2}{16}) - ig_s \int_0^1 dv \frac{x}{16\pi^2 x^2} G_{\mu\nu} \sigma^{\mu\nu}
\]

\[
- v x^\mu G_{\mu\nu} \gamma^\nu \frac{i}{4\pi^2 x^2}
\]  

(14)
The terms proportional to the gluon strength tensor can give contribution to four and five particle distribution functions but they are expected to be small \cite{24,25,26} and for this reason we will neglect these amplitudes in further analysis. The terms proportional to $<qq>$ can also be omitted because Borel transformation eliminates these terms and hence only first term in Eq. (14) is relevant for our discussion. From Eq. (13) it follows that for the calculation of $\Pi_{\mu\nu}(p, q)$ we need to know the matrix element

$$< 0 | [4u^a_\eta(0)u^b_\nu(x)d^c_\rho(0) | N(p) > .$$

This three quark matrix element between vacuum and the proton state is given in \cite{24,25,26} as:

$$4\langle 0 |\epsilon^{abc}u^a_\alpha(a_1 x)u^b_\beta(a_2 x)\bar{d}^c_\rho(3 a_3 x)| P \rangle = S_1 m_N C_{\alpha\beta}(\gamma_5 N)_{\gamma} + S_2 m_N^2 C_{\alpha\beta}(\gamma \gamma_5 N)_{\gamma}$$

$$+ P_1 m_N (\gamma_5 C)_{\alpha\beta} N_{\gamma} + P_2 m_N^2 (\gamma_5 C)_{\alpha\beta} (\gamma \gamma_5 N)_{\gamma} + (V_1 + \frac{x^2 m_N^2}{4}) (\bar{C})_{\alpha\beta}(\gamma_5 N)_{\gamma}$$

$$+ V_2 m_N (\bar{C})_{\alpha\beta}(\gamma_5 N)_{\gamma} + V_3 m_N (\gamma_5 C)_{\alpha\beta}(\gamma_\mu \gamma_5 N)_{\gamma} + V_4 m_N^2 (\gamma_5 C)_{\alpha\beta}(\gamma_5 N)_{\gamma}$$

$$+ V_5 m_N^2 (\gamma_\mu C)_{\alpha\beta}(i \gamma_\mu x_\rho \gamma_5 N)_{\gamma} + V_6 m_N^3 (\gamma_5 C)_{\alpha\beta}(\gamma_5 N)_{\gamma} + (A_1$$

$$+ \frac{x^2 m_N^2}{4} A_1^M) (\gamma_5 C)_{\alpha\beta} N_{\gamma} + A_2 m_N (\bar{C})_{\alpha\beta}(\gamma_5 N)_{\gamma} + A_3 m_N (\gamma_5 C)_{\alpha\beta}(\gamma_\mu N)_{\gamma}$$

$$+ A_4 m_N^2 (\gamma_5 C)_{\alpha\beta} N_{\gamma} + A_5 m_N^2 (\gamma_\mu C)_{\alpha\beta}(i \gamma_\mu x_\rho N)_{\gamma} + A_6 m_N^3 (\gamma_5 C)_{\alpha\beta}(\gamma_5 N)_{\gamma}$$

$$+ (T_1 + \frac{x^2 m_N^2}{4} T_1^M) (\gamma_5 C)_{\alpha\beta}(\gamma_\mu \gamma_5 N)_{\gamma} + T_2 m_N (\gamma_5^\mu p^\nu i \gamma_\nu C)_{\alpha\beta}(\gamma_5 N)_{\gamma}$$

$$+ T_3 m_N (\gamma_\mu C)_{\alpha\beta}(\gamma_\mu \gamma_5 N)_{\gamma} + T_4 m_N (\gamma_\mu p^\nu i \gamma_\nu C)_{\alpha\beta}(\gamma_5 N)_{\gamma}$$

$$+ T_5 m_N^2 (\gamma_\mu C)_{\alpha\beta}(\gamma_\mu \gamma_5 N)_{\gamma} + T_6 m_N^2 (\gamma_\mu p^\nu i \gamma_\nu C)_{\alpha\beta}(\gamma_5 N)_{\gamma}$$

$$+ T_7 m_N^3 (\gamma_\mu C)_{\alpha\beta}(\gamma_\mu \gamma_5 N)_{\gamma} + T_8 m_N^3 (\gamma_\mu p^\nu i \gamma_\nu C)_{\alpha\beta}(\gamma_5 N)_{\gamma} .$$

where the calligraphic functions are functions of the scalar product ($px$) and of the parameters $a_i$, $i = 1, 2, 3$. These functions can be expressed in terms of the nucleon distribution amplitudes with increasing twist. Explicit expressions of distribution amplitudes with definite twist are (see also \cite{8,24,25,26}):

$$S_1 = S_1,$$

$$2px S_2 = S_1 - S_2,$$
\[ P_2 = P_1 - P_2 \]  

\[ V_2 = V_1 - V_2 - V_3, \]
\[ 2V_3 = V_3, \]
\[ 4pxV_4 = -2V_1 + V_3 + 4V_4 + 2V_5, \]
\[ 4pxV_5 = V_4 - V_3, \]
\[ 4(px)^2V_6 = -V_1 + V_2 + V_3 + 4V_4 + V_5 - V_6 \]  

\[ A_2 = -A_1 + A_2 - A_3, \]
\[ 2A_3 = A_3, \]
\[ 4pxA_4 = -2A_1 - A_3 - A_4 + 2A_5, \]
\[ 4pxA_5 = A_3 - A_4 \]
\[ 4(px)^2A_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6 \]  

\[ T_2 = T_1 + T_2 - 2T_3, \]
\[ 2T_3 = T_7, \]
\[ 2pxT_4 = T_1 - T_2 - 2T_7, \]
\[ 2pxT_5 = -T_1 + T_5 + 2T_8, \]
\[ 4(px)^2T_6 = 2T_2 - 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8, \]
\[ 4pxT_7 = T_7 - T_8, \]
\[ 4(px)^2T_8 = -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8 \]  

where Eqs. (16), (17), (18) and (19) are for scalar, pseudo scalar, vector, axial vector and tensor distribution amplitudes respectively. The distribution amplitudes \( F = S_i, P_i, V_i, A_i, T_i \) can be written as:

\[ F(a_i px) = \int dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1)e^{-ip\Sigma_i x_i a_i} F(x_i), \]  

where \( x_i \) with \( i = 1, 2, 3 \) corresponds to the longitudinal momentum fractions carried by the quarks.
Using Eqs. (13)-(20) and after carrying out the fourier transformation, the correlator function is obtained in the momentum representation. Its explicit expression is given in Appendix B.

In order to construct sum rules for axial $N - \Delta$ transition form factors we need to choose structures. From Eq. (11) it follows that in principle different tensor structures can be used for obtaining sum rules for axial form factors. We have chosen the structures proportional to $g_{\mu\nu} p' q', q_{\mu} p' q', g_{\mu\nu} p'$ to obtain sum rules for the form factors $C_3^A, C_4^A, C_5^A + C_4^A \frac{m_\Delta}{m_B}$ and $C_6^A$, respectively.

Choosing the coefficients of the structures $g_{\mu\nu} p' q', q_{\mu} p' q', g_{\mu\nu} p'$ in Eqs. (11) and (B.1) and applying the Borel transformation with respect to the variable $p'^2 = (p - q)^2$ which suppress the contributions of the higher states and continuum we get desired sum rules for the form factors $C_3^A, C_4^A, C_5^A$ and $C_6^A$ as:

\[
C_3(Q^2) = \frac{m_N}{\sqrt{3} \lambda_\Delta} \frac{m_\Delta^2}{e^{\frac{m_\Delta^2}{M_B^2}}} \left\{ \int_{t_0}^{1} dx_2 \int_{0}^{1-x_2} dx_1 \frac{e^{-\frac{s(x_2, Q^2)}{M_B^2}}}{x_2} \left[ 2V_1 - T_1 \right](x_i) + \int_{t_0}^{1} dx_3 \int_{0}^{1-x_3} dx_1 \int_{0}^{1-x_3} dx_2 \int_{t_0}^{x_3} dt_1 \frac{e^{-\frac{s(t_1, Q^2)}{M_B^2}} m_2^2}{M_B^2} \left( x_3 - t_1 \right) T_{234578}(x_i') \right\} \tag{21}
\]

\[
C_4(Q^2) = \frac{m_N^2}{\sqrt{3} \lambda_\Delta} \frac{m_\Delta^2}{e^{\frac{m_\Delta^2}{M_B^2}}} \left\{ \int_{t_0}^{1} dx_2 \int_{0}^{1-x_2} dx_1 \int_{t_0}^{x_2} dt_1 \frac{e^{-\frac{s(t_1, Q^2)}{M_B^2}}}{M_B^2} \left[ \frac{2m_N}{M_B t_1} V_{123} - T_{123} \right](x_i) + \frac{2m_N}{M_B t_1} (1 - 2t_1) \frac{2m_N}{M_B t_1} \left( -A_{123} + T_{127} \right)(x_i) \right\}
\]
\[ + \frac{4m_N^3}{M_B^4t_1}(-1 + t_1)(t_1 - x_2) \mathcal{A}_{123456}(x_i) + \frac{4m_N^3}{M_B^4t_1}(-1 + t_1)(t_1 - x_2) \mathcal{T}_{125678}(x_i) \]

\[ + \int_{t_0}^{1-x_2} dx_2 \int_{t_0}^{1-x_2} dx_1 \frac{m_0}{m_B} \left[ \left( \frac{2m_Nt_0}{Q^2 + m_N^2t_0^2} \right) (V_{123} - \mathcal{T}_{123})(x_i) \right] \]

\[ - \frac{2m_Nt_0(-1 + 2t_0)}{Q^2 + m_N^2t_0^2} (-\mathcal{A}_{123} + \mathcal{T}_{127})(x_i) \]

\[ + \frac{4m_N^2t_0}{Q^2 + m_N^2t_0^2} \left\{ (-1 + t_0)(-x_2 + t_0) \left( \frac{2m_N^2t_0^5}{(Q^2 + m_N^2t_0^2)^2} + \frac{1}{M_B^2} \right) - \frac{t_0^3(4x_2 - 5(1 + x_2)t_0 + 6t_0^2)}{(Q^2 + m_N^2t_0^2)^3} \right\} \left( \mathcal{A}_{123456} + \mathcal{T}_{125678}(x_i) \right) \]

\[ + \int_{t_0}^{1-x_3} dx_3 \int_{t_0}^{1-x_3} dx_1 \int_{t_0}^{1-x_3} dt_1 \frac{m_0}{m_B} \left[ -\frac{m_N^3}{M_B^2} (2V_{123} + \mathcal{A}_{123})(x'_i) - \frac{2m_N(-1 + t_1)}{M_B^2t_1} \mathcal{T}_{127}(x'_i) \right] \]

\[ + \int_{t_0}^{1-x_3} dx_3 \int_{t_0}^{1-x_3} dx_1 \frac{m_0}{m_B} \left[ -\frac{m_N^2t_0^2}{Q^2 + m_N^2t_0^2} (2V_{123} + \mathcal{A}_{123})(x'_i) - \frac{2m_Nt_0(-1 + t_0)}{Q^2 + m_N^2t_0^2} \mathcal{T}_{127}(x'_i) \right] \]

\[ + \frac{2m_N^5(-1 + t_0)t_0^5(-x_3 + t_0)}{(Q^2 + m_N^2t_0^2)^3} - \frac{m_N^2t_0^3(4x_3 - 5(1 + x_3)t_0 + 6t_0^2)}{(Q^2 + m_N^2t_0^2)^2} \]

\[ + \frac{m_N^3(-1 + t_0)t_0(-x_3 + t_0)}{M_B^2(Q^2 + m_N^2t_0^2)} \left( 2V_{123456} - \mathcal{A}_{123456} + 2\mathcal{T}_{125678}(x'_i) \right) \]

\[ (22) \]

\[ C_5(Q^2) = \frac{1}{\sqrt{3} \lambda_\Delta} e^{\frac{m_B^2}{m_B^2}} \left\{ \int_{t_0}^{1-x_2} dx_2 \int_{t_0}^{1-x_2} dx_1 e^{-\frac{s(x_2,Q^2)}{m_B^2}} m_N(-S_1 + P_1 - V_3 + A_3)(x_i) \right\} \]

\[ + \int_{t_0}^{1-x_3} dx_3 \int_{t_0}^{1-x_3} dx_1 e^{-\frac{s(x_3,Q^2)}{m_B^2}} m_N \left( -V_3 + \frac{A_3}{2} \right)(x'_i) \]

\[ + \int_{t_0}^{1-x_2} dx_2 \int_{t_0}^{1-x_2} dx_1 \int_{t_0}^{1-x_2} dt_1 e^{-\frac{s(t_1,Q^2)}{m_B^2}} \left[ \frac{m_N}{t_1} \right] \]

\[ \times \left[ \left( 2 - \frac{Q^2 + m_N^2(-1 + 2t_1) + s(t_1,Q^2)}{M_B^2} \right) V_{123}(x_i) + \frac{m_N^2(t_1 - x_2)}{M_B^2} \mathcal{T}_{125678}(x_i) \right] \]

\[ + \frac{Q^2 + m_N^2(-1 + 2t_1) + s(t_1,Q^2)}{2M_B^2} \left( -2\mathcal{A}_{123} + \mathcal{A}_{123} + \mathcal{T}_{127}(x_i) \right) \]

\[ + \int_{t_0}^{1-x_2} dx_2 \int_{t_0}^{1-x_2} dx_1 \frac{m_0}{Q^2 + m_N^2t_0^2} \left[ -\frac{m_N^2t_0}{Q^2 + m_N^2t_0^2} + m_N^2t_0(-2t_2) \mathcal{T}_{125678}(x_i) \right] \]

\[ 11 \]
\[ (-m_N^2 + Q^2 + s(t_0, Q^2) + 2m_N^2 t_0)(-\mathcal{V}_{123} - \mathcal{A}_{123} + \mathcal{T}_{123} + \mathcal{T}_{127})(x_i) \]

\[ + \int_{t_0}^{1} dx_3 \int_{0}^{1-x_3} dx_1 \int_{0}^{x_3} dt_1 e^{-s(t_1, Q^2)} \frac{m_N^3(t_1 - x_3)}{2M_B^2 t_1} (2\mathcal{V}_{123456}(x_i') - \mathcal{A}_{123456}(x_i')) \]

\[ + \int_{t_0}^{1} dx_3 \int_{0}^{1-x_3} dx_1 e^{-\frac{m_N}{M_B}} \frac{m_N^3 (t_0 - x_3) t_0}{2(Q^2 + 2M_B^2 t_0^2)} (2\mathcal{V}_{123456}(x_i') - \mathcal{A}_{123456}(x_i')) \}

\[ - \frac{(m_N^2 - m_D^2 + Q^2)}{2m_N^2} C_4(Q^2, M_B^2) \]  

(23)

\[ C_6(Q^2) = \frac{m_N^2}{\sqrt{3\lambda}} e^{\frac{m_N^2}{M_B^2}} \left\{ \int_{t_0}^{1} dx_2 \int_{0}^{1-x_2} dx_1 \int_{0}^{x_2} dt_1 e^{-\frac{s(t_1, Q^2)}{M_B^2}} \right. \]

\[ \left[ -2m_N(-1 + t_1) \frac{(-2\mathcal{A}_{123} + \mathcal{T}_{123})(x_i) + (-2m_N(-1 + t_1)(-1 + 2t_1)) \mathcal{T}_{127}(x_i)}{M_B^2 t_1^2} \right] \]

\[ + \int_{t_0}^{1} dx_2 \int_{0}^{1-x_2} dx_1 e^{-\frac{m_N}{M_B}} \left[ \frac{4m_N(-1 + t_0) t_0}{Q^2 + m_N^2 t_0^2} \right] \mathcal{A}_{123}(x_i) \]

\[ - \frac{2m_N(-1 + t_0)}{Q^2 + m_N^2 t_0^2} \mathcal{T}_{123}(x_i) - \frac{2m_N(-1 + t_0)(-1 + 2t_1)}{Q^2 + m_N^2 t_0^2} \mathcal{T}_{127}(x_i) \]

\[ + \left( \frac{8m_N^3(-1 + t_0)^2 t_0^2 (-x_2 + t_0)}{Q^2 + m_N^2 t_0^2} \right) - \frac{4m_N^3(-1 - t_0) t_0^2 (3x_2 + t_0(-4 - 5x_2 + 6t_0))}{(Q^2 + m_N^2 t_0^2)^2} \]

\[ \frac{4m_N^3(-1 + t_0)^2(-x_2 + t_0)}{M_B^2 (Q^2 + m_N^2 t_0^2)} \right) \mathcal{A}_{123456} + \mathcal{T}_{125678}(x_i) \]

\[ \int_{t_0}^{1} dx_3 \int_{0}^{1-x_3} dx_1 \int_{0}^{x_3} dt_1 e^{-\frac{s(t_1, Q^2)}{M_B^2}} \frac{m_N(1 - t_1)}{M_B^2 t_1} \]

\[ \left[ \mathcal{A}_{123}(x_i') + \frac{1}{t_1} \mathcal{T}_{123}(x_i') + \frac{(-1 + 2t_1)}{t_1} \mathcal{T}_{127}(x_i') \right] \]

\[ + \frac{2m_N^2(-1 - t_1)(t_1 - x_3)}{M_B^2 t_1} (\mathcal{V}_{123456} - \frac{1}{2} \mathcal{A}_{123456} + \mathcal{T}_{125678})(x_i') \]

\[ + \int_{t_0}^{1} dx_3 \int_{0}^{1-x_3} dx_1 e^{-\frac{m_N}{M_B}} \frac{m_N(-1 + t_0)}{Q^2 + m_N^2 t_0^2} \left[ t_0 \mathcal{A}_{123}(x_i') + \mathcal{T}_{123}(x_i') - (-1 + 2t_0) \mathcal{T}_{127}(x_i') \right] \]

\[ + \frac{2m_N^2 (-1 + t_0) t_0^2 (-x_3 + t_0)}{Q^2 + m_N^2 t_0^2} - \frac{t_0^2 (3x_3 + t_0(-4 - 5x_3 + 6t_0))}{Q^2 + m_N^2 t_0^2} \]

\[ + \frac{(-1 + t_0)(-x_3 + t_0)}{M_B^2} \right) (\mathcal{V}_{123456} - \frac{1}{2} \mathcal{A}_{123456} + \mathcal{T}_{125678})(x_i') \]  

(24)
where following [25], the following shorthand notations for various combinations of the DA’s are employed:

\[
\begin{align*}
T_{234578} &= T_2 - T_3 - T_4 + T_5 + T_7 + T_8 \\
T_{123} &= T_1 + T_2 - 2T_3 \\
T_{127} &= T_1 - T_2 - 2T_7 \\
T_{125678} &= -T_1 + T_2 + T_5 - T_6 + 2T_7 + 2T_8 \\
V_{123} &= V_1 - V_2 - V_3 \\
V_{123456} &= -V_1 + V_2 + V_3 + V_4 + V_5 - V_6 \\
A_{123} &= A_1 - A_2 + A_3 \\
A_{123456} &= A_1 - A_2 + A_3 + A_4 - A_5 + A_6
\end{align*}
\]

(25)

and

\[
\begin{align*}
\mathcal{F}(x_i) &= \mathcal{F}(x_1, x_2, 1 - x_1 - x_2) \\
\mathcal{F}(x'_i) &= \mathcal{F}(x_1, 1 - x_1 - x_3, x_3)
\end{align*}
\]

(26)

The Borel transformation and the continuum subtraction are performed using the following substitutions (see [24][25]):

\[
\begin{align*}
\int dt \frac{\rho(t)}{(q - tp)^2} &= \frac{1}{t} \int_0^1 \frac{dt}{s-p^2} \Rightarrow \frac{1}{t_0} \int_0^1 \frac{dt}{t} \rho(t) e^{-\frac{s}{M_B^2}} , \\
\int dt \frac{\rho(t)}{(q - tp)^4} &= \frac{1}{t^2} \int_0^1 \frac{dt}{(s-p^2)^2} \Rightarrow \frac{1}{M_B^2} \int_0^1 \frac{dt}{t^2} \rho(t) e^{-\frac{s}{M_B^2}} + \frac{\rho(t_0) e^{-\frac{s_0}{M_B^2}}}{Q^2 + t_0^2 m_N^2} , \\
\int dt \frac{\rho(t)}{(q - tp)^6} &= \frac{1}{t^3} \int_0^1 \frac{dt}{(s-p^2)^3} \Rightarrow \frac{1}{2M_B^4} \int_0^1 \frac{dt}{t^3} \rho(t) e^{-\frac{s}{M_B^2}} \\
&+ \frac{\rho(t_0) e^{-\frac{s_0}{M_B^2}}}{2t_0(Q^2 + t_0^2 m_N^2)M_B^2} - \frac{t_0^2}{2(Q^2 + t_0^2 m_N^2)} \left[ \frac{d}{dt_0} \frac{\rho(t_0)}{t_0(Q^2 + t_0^2 m_N^2)} \right] e^{-\frac{s_0}{M_B^2}} ,
\end{align*}
\]

\[
\begin{align*}
s(t, Q^2) &= (1 - t) M^2 + \frac{(1 - t)}{t} Q^2 , \\
t_0(s_0, Q^2) &= \sqrt{(Q^2 + s_0 - m_N^2)^2 + 4m_N^2 Q^2} - (Q^2 + s_0 - m_N^2) .
\end{align*}
\]

(27)

where \( Q^2 = -q^2 \) and \( t_0 \) is the solution of the \( s(t_0, Q^2) = s_0 \). The terms \( e^{-\frac{s_0}{M_B^2}} \) are so-called surface terms which appear in successive partial integrations.
to reduce the power of denominators. In the hadronic representation for the correlator functions the Borel transformation is performed by substituting
\[
\frac{1}{m_{\Delta}^2 - p'^2} \to e^{-\frac{m_{\Delta}^2}{M_B^2}}.
\]

3 Numerical analysis

From explicit expressions of the sum rules for the axial $N - \Delta$ transition form factors it follows that the main input parameters are the nucleon distribution amplitudes (DA’s). In general these distribution amplitudes contain hadronic parameters which should be determined by some means. Various methods to determine these parameters give different results. In this work, we considered all three different determinations of these parameters: a) QCD sum rules based DA’s, where corrections to the DA’s are taken into account and the parameters in DA’s are determined from QCD sum rules, b) A model for nucleon DA’s where parameters are chosen in a such way that the nucleon electromagnetic and axial form factors are described well within LCSR and c) Asymptotic forms of DA’s of all twists (see for example [24]).

Explicit expressions of corresponding DA’s can be found in [24] and for completeness we present their expressions in Appendix. In the appendix, the values of the hadronic parameters obtained from three different methods is also presented.

For the numerical evaluation of the sum rules for the $N - \Delta$ transition form factors we need also specify the values of the residue of $\Delta$ baryon $\lambda_{\Delta}$, the continuum threshold $s_0$ and Borel parameter $M_B^2$. The residue $\lambda_{\Delta}$ and $s_0$ are determined from analysis of the mass sum rules: $\lambda_{\Delta} = 0.038 \ GeV^3$ and $s_0 =$
2.6 − 3 GeV$^2$ [27,28,29,30,31] which we have used in numerical calculations.

The Borel mass parameter $M^2_B$ is the auxiliary parameter of sum rules. Therefore we need to find a suitable region of $M^2_B$, where physical results are independent of this parameter. A suitable region of $M^2_B$ is determined in the following way. From one side, $M^2_B$ has to be small enough in order to guarantee suppression of higher resonances and the continuum contributions to the correlation function and from other side it should be large enough in order to guarantee convergence of the light cone expansion with increasing twist in QCD calculation.

In Figs. 1, 2, 3, and 4, we present the dependence of the form factors $C_3(Q^2)$, $C_4(Q^2)$, $C_5(Q^2)$ and $C_6(Q^2)$ on $M^2_B$ for first set of DA’s with two fixed values of $s_0$ and three fixed values of $Q^2$ respectively. From these figures it follows that all considered form factors exhibit good stability with respect to variation of $M^2_B$ in the region $1.2 \ GeV^2 \leq M^2_B \leq 2 \ GeV^2$, so this region of $M^2_B$ can be considered as a working region where form factors are practically independent of $M^2_B$. We performed similar analysis for the two other sets of DA’s and obtained that within the above mentioned region of $M^2_B$, the form factors are rather stable to variation of $M^2_B$.

In Figs. 5, 6, 7 and 8 we present the dependence of the form factors $C_i(Q^2)$, $i = 3, 4, 5, 6$ on $Q^2$ at fixed values of $M^2_B$ and $s_0$ for all three sets of DA’s. From these figures we see that form factors are very sensitive to the choice of DA’s. For form factor $C_3(Q^2)$ sets 2 and 3 leads to the same result, for $C_4(Q^2)$ and $C_5(Q^2)$ form factors sets 1 and 2 give results which are very close to each other and for $C_6(Q^2)$ up to $Q^2 = 4 \ GeV^2$ all sets of DA’s leads to the different results and when $Q^2 \geq 4 \ GeV^2$ all three sets of DA’s leads to
indistinguishable predictions. Our results on form factors $C_4(Q^2)$ and $C_5(Q^2)$ for three sets of DA’s satisfy the relation $C_4(Q^2) = C_5(Q^2)/4$ assumed in the experimental analysis.

Figs. (7) and (8) contain also the predictions of lattice calculations from [13]. The points correspond to the values obtained using a hybrid action and assuming $Z_A = 1.1$ (see [13] for details of the notation). Note that, due to different conventions used, the lattice results has been multiplied with $\sqrt{2}/3$. From the figures, it is seen that there is a good agreement between the lattice results and our predictions within error bars. Note also that, the biggest difference between sum rules predictions and lattice calculations is seen in $C_5(Q^2)$ when the asymptotic distributions are used, i.e. when the hadronic parameters from Set 3 are used.

And finally, in Fig. (9), $R(Q^2)$ is plotted as a function of $Q^2$. The function $R(Q^2)$ is defined as:

$$R(Q^2) = \frac{C_6^A(Q^2) Q^2 + m_N^2}{C_5^A(Q^2) m_N^2}$$

From Eq. (3), it is seen that, assuming PCAC and pion dominance, $R(Q^2) = 1$. From Fig. (9), it is seen that at $Q^2 \simeq 1.5 \text{ GeV}$, PCAC and pion dominance approximations are valid. But for larger values of $Q^2$, $R(Q^2)$ deviates considerable from unity, and hence we may conclude that PCAC and pion dominance assumptions break down.

As we have already noted, the axial form factors for the $\Delta \rightarrow N$ transition have also been calculated in the framework of the constituent quark model in [14]. Our predictions on these form factors differ from the results of [14].
In summary, we have calculated axial $N - \Delta$ transition form factors which play important role for understanding the dynamics of weak $N - \Delta$ transition and compare our results with existing lattice and quark model predictions.

4 Acknowledgment

This work is partially supported by TUBITAK under the project number 106T333. One of the authors, A.O., would like to thank TUBA for the funds provided under the GEBIP program. Also, K. Azizi would like to thank TUBITAK for their partially support.
A Appendix A

In Eqs. (16), (17), (18) and (19) the distribution amplitudes depend on scale and can be expanded with the conformal operators, to the next-to-leading conformal spin accuracy, they are obtained in [24][25][26]:

\[
V_1(x_i, \mu) = 120 x_1 x_2 x_3 [\phi_3^0(\mu) + \phi_3^+(\mu)(1 - 3 x_3)],
\]
\[
V_2(x_i, \mu) = 24 x_1 x_2 [\phi_4^0(\mu) + \phi_4^+(\mu)(1 - 5 x_3)],
\]
\[
V_3(x_i, \mu) = 12 x_3 \{ \psi_4^0(\mu)(1 - x_3) + \psi_4^-(\mu)[x_1^2 + x_2^2 - x_3(1 - x_3)]
\]
\[
+ \psi_4^+(\mu)(1 - x_3 - 10 x_1 x_2) \},
\]
\[
V_4(x_i, \mu) = 3 \{ \psi_5^0(\mu)(1 - x_3) + \psi_5^- (\mu)[2 x_1 x_2 - x_3(1 - x_3)]
\]
\[
+ \psi_5^+(\mu)(1 - x_3 - 2(x_1^2 + x_2^2)) \},
\]
\[
V_5(x_i, \mu) = 6 x_3 [\phi_5^0(\mu) + \phi_5^+(\mu)(1 - 2 x_3)],
\]
\[
V_6(x_i, \mu) = 2[\phi_6^0(\mu) + \phi_6^+(\mu)(1 - 3 x_3)],
\]
\[
A_1(x_i, \mu) = 120 x_1 x_2 x_3 \phi_3^-(\mu)(x_2 - x_1),
\]
\[
A_2(x_i, \mu) = 24 x_1 x_2 \phi_4^-(\mu)(x_2 - x_1),
\]
\[
A_3(x_i, \mu) = 12 x_3 (x_2 - x_1) \{ (\psi_4^0(\mu) + \psi_4^+(\mu)) + \psi_4^- (\mu)(1 - 2 x_3) \},
\]
\[
A_4(x_i, \mu) = 3 (x_2 - x_1) \{ - \psi_5^0(\mu) + \psi_5^- (\mu)x_3 + \psi_5^+(\mu)(1 - 2 x_3) \},
\]
\[
A_5(x_i, \mu) = 6 x_3 (x_2 - x_1) \phi_5^-(\mu)
\]
\[
A_6(x_i, \mu) = 2(x_2 - x_1) \phi_6^- (\mu),
\]
\[
T_1(x_i, \mu) = 120 x_1 x_2 x_3 [\phi_3^0(\mu) + \frac{1}{2}(\phi_3^- - \phi_3^+)(\mu)(1 - 3 x_3)],
\]
\[
T_2(x_i, \mu) = 24 x_1 x_2 [\xi_4^0(\mu) + \xi_4^+(\mu)(1 - 5 x_3)],
\]
\[
T_3(x_i, \mu) = 6 x_3 \{ (\xi_4^0 + \phi_4^0 + \psi_4^0)(\mu)(1 - x_3) + (\xi_4^- + \phi_4^- + \psi_4^-)(\mu)[x_1^2 + x_2^2 - x_3(1 - x_3)]
\]
\[
+ (\xi_4^+ + \phi_4^+ + \psi_4^+)(\mu)(1 - x_3 - 10 x_1 x_2) \},
\]
\[
T_4(x_i, \mu) = 3 \{ (\xi_5^0 + \phi_5^0 + \psi_5^0)(\mu)(1 - x_3) + (\xi_5^- + \phi_5^- + \psi_5^-)(\mu)[2 x_1 x_2 - x_3(1 - x_3)]
\]
\[
+ (\xi_5^+ + \phi_5^+ + \psi_5^+)(\mu)(1 - x_3 - 2(x_1^2 + x_2^2)) \},
\]
\[
T_5(x_i, \mu) = 6 x_3 [\xi_5^0(\mu) + \xi_5^+(\mu)(1 - 2 x_3)],
\]
\[
T_6(x_i, \mu) = 2[\phi_6^0(\mu) + \frac{1}{2}(\phi_6^- - \phi_6^+)(\mu)(1 - 3 x_3)],
\]
\[
T_7(x_i, \mu) = 6 x_3 \{ (-\xi_4^0 + \phi_4^0 + \psi_4^0)(\mu)(1 - x_3) + (-\xi_4^- + \phi_4^- + \psi_4^-)(\mu)[x_1^2 + x_2^2 - x_3(1 - x_3)]
\]
\[
+ (-\xi_4^+ + \phi_4^+ + \psi_4^+)(\mu)(1 - x_3 - 10 x_1 x_2) \},
\]
\[
T_8(x_i, \mu) = 3 \{ (-\xi_5^0 + \phi_5^0 + \psi_5^0)(\mu)(1 - x_3) + (-\xi_5^- + \phi_5^- + \psi_5^-)(\mu)[2 x_1 x_2 - x_3(1 - x_3)]
\]
\[
+ (-\xi_5^+ + \phi_5^+ + \psi_5^+)(\mu)(1 - x_3 - 2(x_1^2 + x_2^2)) \},
\]
\[
S_1(x_i, \mu) = 6 x_3 (x_2 - x_1) \{ (\xi_4^0 + \phi_4^0 + \xi_4^+ + \phi_4^+ + \psi_4^+)(\mu) + (\xi_4^- + \phi_4^- + \psi_4^-)(\mu)(1 - 2 x_3) \}
\]
\[ S_2(x_i, \mu) = \frac{3}{2} (x_2 - x_1) \left[ - \left( \psi_5^0 + \phi_5^0 + \xi_5^0 \right) (\mu) + \left( \xi_5^- + \phi_5^- - \psi_5^0 \right) (\mu) x_3 \\
+ \left( \xi_5^+ + \phi_5^+ + \psi_5^0 \right) (\mu) (1 - 2x_3) \right] \]
\[ P_1(x_i, \mu) = 6x_3(x_2 - x_1) \left[ \left( \xi_4^0 - \phi_4^0 - \psi_4^0 + \xi_4^+ - \phi_4^+ - \psi_4^+ \right) (\mu) + \left( \xi_4^- - \phi_4^- + \psi_4^- \right) (\mu) (1 - 2x_3) \right] \]
\[ P_2(x_i, \mu) = \frac{3}{2} (x_2 - x_1) \left[ \left( \psi_5^0 + \phi_5^0 + \xi_5^0 \right) (\mu) + \left( \xi_5^- - \phi_5^- + \psi_5^0 \right) (\mu) x_3 \\
+ \left( \xi_5^+ - \phi_5^+ - \psi_5^0 \right) (\mu) (1 - 2x_3) \right] . \] (A.1)

the parameters used above are defined in terms of the following eight independent parameters \( f_N, \lambda_1, \lambda_2, V_1^d, A_1^u, f_d^1, f_d^2 \) and \( f_u^1 \) as

\[
\begin{align*}
\phi_3^0 &= \phi_6^0 = f_N \\
\phi_4^0 &= \phi_8^0 = \frac{1}{2} (\lambda_1 + f_N) \\
\xi_4^0 &= \xi_4^0 = \frac{1}{6} \lambda_2 \\
\psi_4^0 &= \psi_8^0 = \frac{1}{2} (f_N - \lambda_1) \\
\phi_3^- &= \frac{21}{2} A_1^u, \\
\phi_3^+ &= \frac{7}{2} (1 - 3V_1^d), \\
\phi_4^- &= \frac{5}{4} \left( \lambda_1 (1 - 2f_1^d - 4f_1^u) + f_N (2A_1^u - 1) \right), \\
\phi_4^+ &= \frac{1}{4} \left( \lambda_1 (3 - 10f_1^d) - f_N (10V_1^d - 3) \right), \\
\psi_4^- &= -\frac{5}{4} \left( \lambda_1 (2 - 7f_1^d + f_1^u) + f_N (A_1^u + 3V_1^d - 2) \right), \\
\psi_4^+ &= -\frac{1}{4} \left( \lambda_1 (-2 + 5f_1^d + 5f_1^u) + f_N (2 + 5A_1^u - 5V_1^d) \right), \\
\xi_4^- &= \frac{5}{16} \lambda_2 (4 - 15f_2^d), \\
\xi_4^+ &= \frac{1}{16} \lambda_2 (4 - 15f_2^d), \\
\phi_5^- &= \frac{5}{3} \left( \lambda_1 (f_1^d - f_1^u) + f_N (2A_1^u - 1) \right), \\
\phi_5^+ &= -\frac{5}{6} \left( \lambda_1 (4f_1^d - 1) + f_N (3 + 4V_1^d) \right), \\
\psi_5^- &= \frac{5}{3} \left( \lambda_1 (f_1^d - f_1^u) + f_N (2 - A_1^u - 3V_1^d) \right), \\
\psi_5^+ &= -\frac{5}{6} \left( \lambda_1 (-1 + 2f_1^d + 2f_1^u) + f_N (5 + 2A_1^u - 2V_1^d) \right),
\end{align*}
\]
\[ \xi^-_5 = -\frac{5}{4} \lambda_2 f_d^d, \]
\[ \xi^+_5 = \frac{5}{36} \lambda_2 (2 - 9 f_d^d), \]
\[ \phi^-_6 = \frac{1}{2} \left( \lambda_1 (1 - 4 f_d^d - 2 f_u^u) + f_N (1 + 4 A_1^u) \right), \]
\[ \phi^+_6 = -\frac{1}{2} \left( \lambda_1 (1 - 2 f_d^d) + f_N (4 V_d^d - 1) \right) \quad (A.2) \]

Our numerical values are obtained using:

\[ f_N = (5.0 \pm 0.5) \times 10^{-3} \text{ GeV}^2, \]
\[ \lambda_1 = -(2.7 \pm 0.9) \times 10^{-2} \text{ GeV}^2, \]
\[ \lambda_2 = (5.4 \pm 1.9) \times 10^{-2} \text{ GeV}^2 \quad (A.3) \]

And for other five independent parameter we have used three sets as:

Set 1
\[ A_1^u = 0.38 \pm 0.15, \]
\[ V_1^d = 0.23 \pm 0.03, \]
\[ f_1^d = 0.40 \pm 0.05, \]
\[ f_2^d = 0.22 \pm 0.05, \]
\[ f_1^u = 0.07 \pm 0.05 \quad (A.4) \]

Set 2
\[ A_1^u = \frac{1}{14}, \]
\[ V_1^d = \frac{13}{42}, \]
\[ f_1^d = 0.40 \pm 0.05, \]
\[ f_2^d = 0.22 \pm 0.05, \]
\[ f_1^u = 0.07 \pm 0.05 \quad (A.5) \]

Set 3
\[ A_1^u = 0, \]
\[ V_1^d = \frac{1}{3}, \]
\[ f_1^d = \frac{3}{10}, \]
\[ f_2^d = \frac{4}{15}, \]
\[ f_1^\mu = \frac{1}{10} \] (A.6)

Note that the asymptotic forms of DA’s can be obtained from Eqs. (A.1) using the values given in the 3\textsuperscript{rd} set.

\section*{B Appendix B}

In this appendix, we present the explicit form of the correlation function containing all the Dirac structures.

\[
\Pi_{\mu\nu}(p, q) = \frac{i}{\sqrt{3}} \left\{ \int_0^{x_1} dx_1 \int_0^{x_2} dx_2 \frac{1}{(q - x_2p)^2} \right. \\
\left. \begin{array}{l}
 m_N [\gamma' g_{\mu\nu} x_2 + (\gamma' q_\mu - \gamma g_{\mu\nu})(1 - x_2)] (S_1 - P_1 + V_3 - A_3)(x_i) \\
 + 2 \left[ - \gamma' \gamma g_{\mu\nu} + \gamma' \gamma q_\mu + g_{\mu\nu} p' q - p'_\nu q_\mu \right] V_1(x_i) \\
 + \left[ 2q_\mu (p'_\nu + 2q_\nu)(-1 + x_2) + g_{\mu\nu} \left( p' q + p (q - x_2p) + q^2 - (p' + q)^2 \right) x_2 + 2p'_\nu q_\mu x_2 \right] A_1(x_i) \\
 + \left\{ \gamma' \gamma g_{\mu\nu} + \gamma' \gamma q_\mu(-1 + x_2) - \gamma' \gamma q_\mu(1 + x_2) - g_{\mu\nu} x_2(2p' q + q^2) \\
 - 2q_\mu (p'_\nu + 2q_\nu)(-1 + x_2)x_2 + g_{\mu\nu}(p' + q)^2 x_2 \right\} T_1(x_i) \right. \\
+ \left. \frac{1}{\sqrt{3}} \left\{ \int_0^{x_1} dx_1 \int_0^{x_2} dx_3 \frac{1}{(q - x_3p)^2} \right. \\
\left. m_N [\gamma' g_{\mu\nu} x_3 + (\gamma' q_\mu - \gamma g_{\mu\nu})(1 - x_3)] \left( V_3 - \frac{A_3}{2} \right) (x'_i) \\
+ q_\mu \left[ (\gamma' q - 2q_\nu)(-1 + x_3) - \gamma' q_\nu x_3 \right] \left( V_1 + \frac{A_1}{2} \right) (x'_i) \\
+ \left[ (\gamma' q_\nu + 2q_\nu) q_\mu - g_{\mu\nu} (\gamma' q + q^2) - 2q_\mu (p'_\nu + q_\nu)x_3 + g_{\mu\nu}(p' + q)^2 x_3 \right] T_1(x'_i) \right. \\
+ \left. \frac{1}{\sqrt{3}} \left\{ \int_0^{x_1} dx_1 \int_0^{x_2} dx_3 \int_0^{x_2} dt_1 \left( \frac{1}{(q - t_1p)^6} \right. \\
\left. - 8m_N^3(-1 + t_1)(t_1 - x_2)q_\mu \left[ t_1 p'_\nu + (-1 + t_1)q_\nu \right] t_1 p' + (-1 + t_1) q_\nu A_{123456}(x_i) \\
+ 4m_N^3(-1 + t_1)(t_1 - x_2)q_\mu \\
\times \left\{ p.(q - t_1p) \left[ t_1 \gamma' \gamma_\nu - (-1 + t_1) \gamma_\nu q_\nu \right] \\
+ t_1 p'_\nu \left( 2 \gamma' q + p.(q - t_1p) + t_1 p'^2 + (-3 + 2t_1)p'.q + (-1 + t_1)q^2 \right) \\
- (1 - t_1)q_\nu \left( 2 \gamma' q + 3p.(q - t_1p) + t_1 p'^2 + (-3 + 2t_1)p'.q + (-1 + t_1)q^2 \right) \right\} T_{234578}(x'_i) \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right.
\[-8m_N^3(-1 + t_1)(t_1 - x_2)q_\mu [t_1 p'_\nu -(1-t_1)q_\nu] [t_1 p' - (1-t_1) g_1 T_{125678}(x'_i)\]

\[+ \frac{1}{(q - t_1 p)^4} \left[ 2m_N^2(t_1 - x_2) q_\mu \gamma_\nu A_{123456}(x_i) \right. \]

\[+ m_N^2(t_1 - x_2) \left[g_{\mu \nu}(p.(q - t_1 p) + t_1 p'^2 + (-1 + t_1)p'.q + (-1 + t_1)q^2) \right. \]

\[+ q_\mu \left\{ (3 - 2t_1) p'_\nu (\gamma_\nu \not{q} - 2q_\nu) - 2q_\nu \right\} T_{234578}(x_i) \]

\[-m_N^2(t_1 - x_2) \left[g_{\mu \nu} p'_\nu + (-1 + t_1)(g_{\mu \nu} \not{q} + q_\mu \gamma_\nu) \right] T_{125678}(x_i) \]

\[+ m_N^2q_\mu(-1 + t_1) \left[ \gamma_\nu \not{q}(1 - t_1) - p'_\nu t_1 + 2p'_i t_1 \right] \left(S_1 - S_2 + P_1 - P_2\right)(x_i) \]

\[+ m_N \left[ 2p'_\nu \gamma_\nu \not{q}_{\mu} - 2 \not{g}(g_{\mu \nu}(q - t_1 p) - p'_i q_\mu)(-1 + t_1) \right. \]

\[-2 \left( p'_\nu g_{\mu \nu}(q - t_1 p) + \not{p}'_\nu \gamma_\nu \not{q}_\mu + \not{p}'_i q_\mu \right) t_1 \]

\[+ \gamma_\nu q_\mu(-1 + t_1)(-3p'.q + 3p.(q - t_1 p) - q^2 + (p' + q)^2t_1) \right] V_{123}(x_i) \]

\[-\frac{1}{2}m_N^2 \left\{ q_\mu(1 - t_1) \left[ 3\gamma_\nu \not{q}(1 - t_1) + \gamma_\nu q(1 - t_1) + 2 p'_\nu t_1 \right] \right. \]

\[+ \left[ -2q_\mu(1 - t_1) (q_\mu(1 - t_1) + 2p'_i t_1) + g_{\mu \nu} \left\{ q^2 - 2(p'.q + q^2)t_1 + (p' + q)^2t_1 \right\} \right] \right\} \left(V_4 - V_3\right)(x_i) \]

\[-m_N \left\{ 2 \not{g}(1 - t_1) + 2 p'_i t_1 \right\} \left\{ g_{\mu \nu}(q - t_1 p) - q_\mu(p'_i + 2q_\nu) + 2q_\mu(p'_i + q_\nu)t_1 \right\} \]

\[-\gamma_\nu q_\mu(-1 + t_1) \left\{ -p'.q + p.(q - t_1 p) - q^2 + (p' + q)^2t_1 \right\} \right\} A_{123}(x_i) \]

\[-2q_\mu M^2(-1 + t_1) \left[ q_\mu(-1 + t_1) + p'_i t_1 \right] A_{1345}(x_i) \]

\[-q_\mu M^2(-1 + t_1) \left[ \gamma_\nu \not{q}(2q_\nu)(1 - t_1) - p'_\nu t_1 \right] A_{3 - A_4}(x_i) \]

\[+ m_N \left( \not{g}g_{\mu \nu} - \gamma_\nu q_\mu \right) p.(q - t_1 p)(-1 + t_1) \]

\[+ p' \left( -2q_\mu q_\nu + g_{\mu \nu}(q - t_1 p)t_1 + 2q_\mu(p'_i + q_\nu)t_1 \right) T_{123}(x_i) \]

\[+ M \left[ -p'_\nu \gamma_\nu \not{q}_\mu(1 - t_1) + 2 p'_i q_\mu q_\nu + \not{p}'_\nu \left( g_{\mu \nu}(q - t_1 p) - 2q_\mu(p'_i + 3q_\nu) \right) t_1 \right. \]

\[+ 4 p'_i q_\mu(p'_i + q_\nu)t_1^2 + q(-1 + t_1) \left( g_{\mu \nu}(q - t_1 p) - 2q_\mu(p'_i + 2q_\nu) + 4q_\mu(p'_i + q_\nu)t_1 \right) \]

\[-\gamma_\nu q_\mu(-1 + t_1) \left( p.(q - t_1 p) - q^2 + 2p'.q + (p' + q)^2t_1 \right) \right\} T_{127}(x_i) \]

\[+ q_\mu m_N^2(-1 + t_1) \left[ \gamma_\nu \not{q}(4q_\nu)(-1 + t_1) - p'_\nu t_1 - 2p'_i t_1 \right] T_{158}(x_i) \]

\[+ \frac{m_N}{2(q - t_1 p)^2} \left[ \left( -p' + \not{q}\right)g_{\mu \nu} - \gamma_\nu q_\mu \right] \left( -2A_{123} + T_{123} + T_{127}\right)(x_i) \]

\[+ 3m_N g_{\mu \nu}(V_4 - V_3)(x_i) - 2((p' + \not{q})g_{\mu \nu} - \gamma_\nu q_\mu) V_{123}(x_i) \]

\[+ \int_0^1 dx_1 \int_0^{1-x_1} dx_3 \int_0^{x_3} dt_1 \left( \frac{1}{(q - t_1 p)^6} \right) \right\]
\[
2q_{\mu}m_N^2(1-t_1)[q_{\nu}-(p'_{\nu}+q_{\nu})t_1]\{2[p' q - p' q + (p' + q)(q - t_1 p)]T_{234578}(x'_i) + m_N[g - (p' + q)t_1] (2\mathcal{V}_{123456} - A_{123456} + 2T_{125678})(x'_i)\} (t_1 - x_3)
\]
\[+ \frac{1}{(q - t_1 p)^4}\[\frac{1}{2}m_N^3g_{\mu\nu}(g(-1 + t_1) - p' t_1)(t_1 - x_3)A_{123456}(x'_i) + m_N^2(t_1 - x_3)[g_{\mu\nu}(p' q - p' q + (p' + q)(q - t_1 p))]
- q_{\mu}(\gamma_{\nu} g - 2q_{\nu})(1 - t_1) - q_{\mu}\gamma_{\nu} p' t_1]T_{234578}(x'_i) - m_N^3(t_1 - x_3)(-1 + t_1)q_{\nu}\gamma_{\nu}T_{125678}(x'_i)
\]
\[+ 2m_Nq_{\mu}(g(-1 + t_1) + p' t_1)(q_{\nu}(-1 + t_1) + p'_{\nu} t_1)\mathcal{V}_{123}(x'_i)
\]
\[+ \frac{1}{2}m_N^2(-1 + t_1)\{[g_{\mu\nu} + 2q_{\nu}](1 - t_1) - p'_{\gamma\nu} t_1\}V_{1345}(x'_i)
\]
\[- \frac{1}{2}m_N^2(1 - t_1)\{[g_{\mu\nu} + 2q_{\nu}](1 - t_1) - p'_{\gamma\nu} t_1\}V_{123456}(x'_i)
\]- m_Nq_{\mu}[(g(-1 + t_1) + p' t_1)(t_1 - x_3)\mathcal{V}_{123456}(x'_i)
\]
\[- \frac{1}{4}m_N^2(1 - t_1)\{[(\gamma_{\nu} g - 2q_{\nu})(1 - t_1) - p'_{\gamma\nu} t_1]\}A_{1345}(x'_i)
\]
\[- \frac{1}{4}m_N^2(1 - t_1)\{[(\gamma_{\nu} g + 2q_{\nu})(1 - t_1) - p'_{\gamma\nu} t_1]\}(A_3 - A_4)(x'_i)
\]
\[- \frac{1}{2}m_Nq_{\mu}(-1 + t_1)\{[p'_{\mu} g - 2p' q_{\nu} + \gamma_{\nu}[p' q - (p' + q)(q - t_1 p)]\}T_{123}(x'_i)
\]
\[+ \frac{1}{2}m_Nq_{\mu}(-1 + t_1)\{[- p'_{\gamma\nu} g - 2(p' + q)]q_{\nu} + \gamma_{\nu}[p' q - (p' + q)(q - t_1 p)]
\]
\[+ 4(p' + q)(p'_{\nu} + q_{\nu})t_1\}T_{127}(x'_i) - 2q_{\mu}m_N^2(-1 + t_1)(q_{\nu}(-1 + t_1) + p'_{\nu} t_1)T_{158}(x'_i)\}\right\} N(p)
\]

\[\text{(B.1)}\]

where \(p' = p - q\). The short hand notations for the functions \(A_X\), \(T_X\) and \(\mathcal{V}_X\) are defined within the main body of the text. The shorthand notations that do not appear within the main text are defined as:

\[
A_{1345} = 2A_1 + A_3 + A_4 - 2A_5
\]
\[
T_{158} = -T_1 + T_5 + 2T_8
\]
\[
\mathcal{V}_{1345} = -2V_1 + V_3 + V_4 + 2V_5
\]

\[\text{(B.2)}\]
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Figure Captions

Fig. 1 The dependence of the form factor $C_3(Q^2)$ on the Borel parameter squared $M_B^2$ for the values of the continuum threshold $s_0 = 2.6 \text{ GeV}^2$ and $s_0 = 3.0 \text{ GeV}^2$ and at the values of $Q^2 = 3 \pm 1 \text{ GeV}^2$.

Fig. 2 The same as Fig. (1) but for the form factor $C_4(Q^2)$.

Fig. 3 The same as Fig. (1) but for the form factor $C_5(Q^2)$.

Fig. 4 The same as Fig. (1) but for the form factor $C_6(Q^2)$.

Fig. 5 The dependence of the form factor $C_3(Q^2)$ for three different sets of distribution amplitudes at the continuum threshold $s_0 = 2.6 \text{ GeV}^2$ and the Borel parameter $M_B^2 = 1.5 \text{ GeV}^2$.

Fig. 6 The same as Fig. (5) but for the form factor $C_4(Q^2)$.

Fig. 7 The same as Fig. (5) but for the form factor $C_5(Q^2)$. Results from the lattice are also shown.

Fig. 8 The same as Fig. (7) but for the form factor $C_6(Q^2)$.

Fig. 9 The dependence of $R(Q^2)$ as a function of $Q^2$ for the three sets of DA’s.
Fig. 1. The dependence of the form factor $C_3(Q^2)$ on the Borel parameter squared $M_B^2$ for the values of the continuum threshold $s_0 = 2.6 \text{ GeV}^2$ and $s_0 = 3.0 \text{ GeV}^2$ and at the values of $Q^2 = 3 \pm 1 \text{ GeV}^2$.

Fig. 2. The same as Fig. (1) but for the form factor $C_4(Q^2)$. 
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Fig. 4. The same as Fig. (1) but for the form factor $C_6(Q^2)$. 

28
Fig. 5. The dependence of the form factor $C_3(Q^2)$ for three different sets of distribution amplitudes at the continuum threshold $s_0 = 2.6$ GeV$^2$ and the Borel parameter $M_B^2 = 1.5$ GeV$^2$

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