Operationalization of Basic Observables in Mechanics

Bruno Hartmann

Perimeter Institute, Waterloo, ON N2L 2Y5, Canada,
Humboldt University, D-12489 Berlin, Germany

April 15, 2015

Abstract: This novel approach to the foundation of the physical theory begins with thought experiments on measurement practice like Einstein for relativistic Kinematics \cite{8}. For a similar foundation of Dynamics one can start from Hermann von Helmholtz analysis of basic measurements \cite{4}. We define energy, momentum and mass from elemental ordering relations for “capability to execute work” and “impact” in a collision and apply Helmholtz program for quantification. From simple pre-theoretic (principle of inertia, impossibility of Perpetuum Mobile, relativity principle) and measurement methodical principles we derive all fundamental equations of Mechanics. We explain the mathematical formalism from the operationalization of basic observables.

1 Operationalization

The objective is a foundation of Physics from the operationalization of its basic observables. In earlier work \cite{19} we had applied Helmholtz fundamental analysis of basic measurements to relativistic motion. We define a spatiotemporal order by the practical comparison, whether one object or process covers the other. To express its value also numerically (how many times more) we introduce man-made tools and procedures. Without a word of mathematics one can manufacture uniform running light clocks $\mathcal{L}$ and place them literally one after the other or side by side and then count, how many building blocks it takes for assembling a regular grid which covers the measurement object. We define basic measures from physical operations. The interrelation of elementary measurement operations by different observers reveals a derivation of formal Lorentz transformation. That is a strictly physical approach to Physics where initially Mathematics must remain outside - and then every step where Mathematics is introduced requires extra justification.

A basic measurement requires knowledge of the method of comparison (of a particular attribute of two objects) ”$\sim_m$” and of the method of their physical concatenation ”$\ast_m$” \cite{4}. We are concerned with physical objects and the problem of determining physical operations really in a strictly physical way. We pursue a foundation of the theory from the active role
of physicists, their interventions in experiment and measurement. This does not presuppose one word of mathematics; but leads to the entire mathematical formulation of Mechanics. We demonstrate how this historic-genetic development occurs.

In absence of interactions one obtains the mathematical formalism of relativistic Kinematics from this operationalization of length and duration. Thus it can be taken as basis for a circularity free foundation of Dynamics. Next, we apply Helmholtz method for a basic measurement to interactions. According to Heinrich Hertz \[1\] introduction to theoretical physics the fundamental role of force must be avoided.\[1\] He outlines a novel treatment of mechanics based on the notion of energy for directly observable phenomena (independent from Newton’s equations of motion and broader). Though Hertz did face the difficulty to specify by which direct experiences we define the presence of energy and determine its quantity - without already anticipating the development of other formalisms for mechanics.

One can define basic observables from elementary comparison methods \(2\). We adhere to Leibniz principle (of measuring the cause by its effect) and to Helmholtz method, according to which a basic measurement consists in a reconstruction of the measurement object with a material model of concatenated units. Luce, Suppes \[18\] diagnosed until recently that a major hindrance to understanding basic measurements of energy (and momentum) had been the failure to uncover suitable empirical concatenation operations for properties ”effect potential” and ”impact”. We can provide them by constructing a calorimeter model. At first we construct a material model from pre-theoretic building blocks (by coupling elementary standard processes of irrelevant internal structure) \(3\) and guided by simple measurement methodical principles \(2\). Then we change to an abstract physical perspective and regard all co-acting elements therein solely as carriers of ”effect potential” and ”impact”. By counting these standardized measurement units we determine the magnitude of energy and momentum \(5\). From the layout of our building blocks in a collision model \(4.1\), calorimeter model \(4.2\) etc. we obtain a geometric proof for primary dynamical equations \(6\).

2 Basic observable

From daily work experience and in play (where the validity of our prognoses about natural processes pays off immediately) we can become conscious about the ”impact” of decelerating bodies and the associated ”capability to execute work” \(7\). We develop pre-theoretic comparison methods in words and by examples. Physicists fix the conventions for observer independent and reproducible procedures.

\[1\]The concept of ”force” - as it grew out of Newton’s axiomatic system - does not apply the category of causality (what is cause; what is effect) properly in mechanics. The reduction of all phenomena onto forces ties our thinking constantly to arbitrary and unsecured assumptions about individual atoms and molecules (their shape, cohesion, motion etc. is entirely concealed in most cases). These assumptions may have no influence on the final result and the latter may be correct; nonetheless details of those derivations are presumably in large part wrong - according to Hertz - the derivation is an illusory proof. Furthermore force is neither directly measurable nor can it be indirectly determined from Newton’s framework alone, without further implicit assumptions \[20\].
According to Principle of Inertia the state of motion is preserved unless some body is
effected by an external cause [2]. For collisions of irrelevant inner structure we define an
elementary ordering criterion.

**Definition 1** **Momentum** is the striking power, impact (Wucht) of a moving body \( a_v \) [6].
Object \( a_v \) has more impact than object \( b_v \)
\[
\xrightarrow{a_v} > P \xrightarrow{b_v} 
\] (1)
if in a head-on collision object \( a_v \) overruns object \( b_v \).

According to "impact" comparison two bodies are interchangeable \( a_v \sim b_v \) if in an
inelastic head-on collision \( a_v, b_v \Rightarrow a * b_0 \) the (joint) collision product moves neither
into the former direction of object \( a_v \) to the right nor with object \( b_v \) to the left.

**Definition 2** **Inertia** is the - passive - resistance against changes of their motion [14]. Ac-
cording to Galilei two bodies \( a, b \) have equal inertial mass
\[
\xrightarrow{a} \sim_{m_{(inert)}} \xrightarrow{b} 
\] (2)
if in inelastic head-on collision test \( a_v, b_v \Rightarrow a * b_0 \) with same initial velocity no one
overruns the other [12]. We conduct a special case of impulse comparison.

For the development of the concept of energy we follow the review of Schlaudt [16]. Mach
characterizes the everyday pre-scientific notion "driving force": Soon after Galilei one did
notice that behind the velocity of an object there is a certain capability to work. Something
which allows to overcome force. How to measure this "something" was the subject of the ”vis
viva” dispute [7]. It was initially a vague, pre-theoretic notion. It has the peculiar feature
- Schlaudt explains - that it cannot be quantified directly but solely by means of its effect.
This is not a mathematical problem but a practical, whose solution entails the mathematical
expression for force.

According to Leibniz **Equipollence** principle ”il faut avoir recours à l’equipollence de la
cause et de l’effect”. For quantification Leibniz further employs the principle of **Congruence**.
To measure the cause \( S \) (Ursache) by its effect requires: (i) providing a precise standard
action which successively consumes the source \( S \); (ii) the cumulative effect of formal repe-
titions reproduces the effect of \( S \) and (iii) guarantee that all copies of the standard action
are congruent with one another [16]. In a practical test ”\( \sim_E \)” they must generate the same
effect. Leibniz presents various candidates for reference actions, including the compression
of a standard spring by a fixed length. We measure the kinetic energy of a moving body
\( a_v \) by the number of obstacles it overcomes. We count how many standard springs can be
compressed (repeat congruent standard processes) before the body \( a_0 \) stops.

Consider a pair of springs \( S \) and \( \tilde{S} \). When they are compressed (and mechanically locked)
the charged springs \( S_E, \tilde{S}_E \) become possible causes of actions (energy sources). With their
charged state we associate a form of ”effect potential” (Wirkungsvermögen). We compare
it indirectly by measuring their (kinetic) effect against the same test system; whether the same test particles repulse with larger velocity $\Delta v_I$ than from a shot with the weaker source. Without restricting generality the charged springs $S_E\big|_0$, $\hat{S}_E\big|_0$ may initially be at rest and after pulling the trigger, after expending the associated capability to work (energy) both discharged springs $S$, $\hat{S}$ remain at the state of rest. This allows a clear separation. In return test particles begin to fly apart. With their motion we associate another form of effect potential (kinetic energy), which they can expend against third parties etc. According to Helmholtz measurement principle: the totality of all (transformable forms of) effect potential is conserved. In a measurement we consume one specific form of effect potential entirely (e.g. potential energy until a spring is entirely relaxed; kinetic energy until all projectiles come to rest etc.) in a transformation into other forms (preferably carried by separate elements of the system). Thus in our calorimeter model we will measure kinetic energy (of projectiles) by a transformation into potential energy (of the absorber material). If the latter comes in standard portions, which are congruent with one another, our quantification is complete.

Definition 3 Energy $E_S$ is the potential - of a separable source $S$ or of the entire system - to cause an action on a system $\{G\}$. A specific form is associated with exhausting a particular condition of the system (motion, configuration size, chemical bound etc.). According to Leibniz we compare (kinetic, potential, binding etc.) energy of two separate sources $S_E$ and $\hat{S}_E$ by their effect: Sources $S_E$ has more potential than source $\hat{S}_E$  

$$S_E >_E \hat{S}_E$$

if the effect of source $S_E$ on the same test system $\{G\}$ exceeds the effect of source $\hat{S}_E$.

We define an elementary ordering criterion for kinetic energy by comparing the effect against the same obstacle: Body $\bigodot v_a$ has same potential as body $\bigodot v_b$  

$$\bigodot v_a \sim_E \bigodot v_b$$

if the absorption effect of $\bigodot v_a$ in external calorimeter reservoir $\{\odot 0\}$ (until all motion stops) equals the effect of absorbing particle $\bigodot v_b$. If the absorption effect for particle $\bigodot v_a$ exceeds that of particle $\bigodot v_b$ the former has more kinetic energy $\bigodot v_a >_E \bigodot v_b$ and vice versa.

We define basic observables from elemental ordering relations. For standardization of experiment and measurement we want to express their value also numerically ("how many times" larger an absorption effect is, "how many times" more impetus one ball carries). We specify a reference process as sufficiently constant representative of "effect potential" and "impact", which is reproducible and available anywhere and anytime and in any number.

3 Reference standards

Huygens has studied symmetrical collisions between equivalent objects together with the relativity principle to derive the collision laws for Billiard balls. For the same reason Einstein
Figure 1: a) compression, reorientation and b) decompression $w$ of charged spring $S_E|_0$ kicks a particle pair into unit velocity and vice versa (neutral spring $S|_0$ remains at rest)

[9] and Feynman [13] examine interactions between objects which collide and stick together. Our reference for measurements in entire mechanics is an elementary standard process (of irrelevant internal structure).

We provide a reservoir with standard bodies $\odot$ with same inertial behavior $\odot \sim m_{\text{inert}} \oplus$. According to Galilei we can test pre-theoretically in a head-on collision if no one overruns the other. Similarly we can charge standard springs $S \sim E \bar{S}$ with same capability to execute work, which we can check according to Leibniz: They must catapult standard objects in the same way. In our reference process the energy source $S_1|_0$ (compressed spring)

$$w_1 : \quad S_1|_0, \odot_0, \odot_0 \Rightarrow \odot_{v_1}, \odot_{-v_1} \quad (4)$$

catapults two resting standard objects into diametrically opposed directions (see figure [1a]) or in the reverse course the particle pair with unit velocity $v_1$ can compress a neutral spring (see figure [1b]). In elementary standard process "$w_1"$ the standard spring $S_1$ turns standard particles $\odot$ into standard impulse carriers $\odot_{-v_1}$ and vice versa.

D’Alembert utilizes in his *Traité de dynamique* congruent actions of a spring. He discusses the compression of equivalent springs up to a fixed mark. This is a very instructive approach, Schlaudt remarks [16]. The action is quantified - not by the depth of compression (in one spring) but instead - by the number of springs which are compressed by a fixed distance. In this way one can disregard completely from the *inner dynamics* of the compression process.
4 Physical model

We outline the direction of our construction. The model building involves a steering task. If Alice couples compression and decompression of her spring; then the particle pair - which initially flew towards each other - will instantly be catapulted apart. Consecutive association of inelastic collisions $w_1^{-1} * w_1$ reproduces an eccentric elastic collision between equivalent bodies (same mass). If Bob drives by with same horizontal velocity, he will see the process as an elastic transversal collision. One particle kicks in from below and rebounds antiparallel. The other particle moves on with same velocity into a direction which is slightly deflected by a corresponding angle. The transversal standard kick $w_T$ becomes our elementary building block for collision models. We construct them - with intermediate steps (see figure 2) - from controlled association of our building blocks and by relativity principle (view from moving observer). When skillfully coupled they generate an elastic head-on collision $w_H$ between arbitrary (non equivalent!) bodies $\{1,1\}$. Ultimately with them we mediate an absorption process $W_{cal}$ for a generic particle $\oplus$ in the calorimeter reservoir $\{\oplus_0\}$.

We introduce a measurement method where a sequence of standard processes ”$w_1$” is set up and coupled. We concatenate ”*” them in a coinciding intermediate element, which between two interventions moves freely. Physicists steer initiation of each intervention timely and at suitable position, such that the desired effect is achieved (see figure 4). Therefore in every individual action it only matters which change in the state of motion is ultimately attained - irrespective of details in its spatiotemporal progression. By coupling units from
Figure 3: a) symmetric elastic collision with scattering angle set up by Alice b) appears as elastic transversal collision $w_T$ when Bob drives by with same horizontal velocity to left

an external reservoir $\{S_1|_0, \mathcal{O}_0\}$ we design an interplay of elementary standard processes

$$w_1 \ast \ldots \ast w_1 \sim_{E,p} w$$

which generate the same kinetic effect (element by element same changes in final state of motion $\Delta v_i$) like from a generic interaction $w$. First we model the elastic collision of two generic particles $\{4.1\}$ and then the absorption process in a calorimeter $\{4.2\}$.

We measure the associated energy-momentum gain $\{2\}$ of generic interaction $w$ by means of those models. They are built of (congruent) energy sources $\sharp \{S_1\}$ and momentum carriers $\sharp \{\mathcal{O}_{v_1}\}$; by counting them we find "how many times" more energy generic interaction $w$ transforms than our standard spring $S_1$ in reference process $w_1$.

### 4.1 Elastic collision model

**Lemma 1** Let in elastic transversal collision between equivalent objects (see figure 3b) $w_T$:

$$w_T: \quad \mathcal{O}_{v(i)}' , \mathcal{O}_{-v_1} \Rightarrow \mathcal{O}_{v'_i(i)} , \mathcal{O}_{-v_1}$$

reservoir particle $\mathcal{O}_{v_1}$ kick in from below with fixed velocity $\epsilon \cdot v_1$ and rebound antiparallel. Then incident object $\mathcal{O}_{v(i)}$ moves on with same velocity $v'_i = R_{\alpha_i} v_{(i)}$ deflected by angle $\alpha_i$

$$\sin \left( \frac{\alpha_i}{2} \right) = \frac{\epsilon}{v_{(i)}} .$$

**Proof:** The elastic collision of identically constituted bodies $\mathcal{O}_1$ is well-defined by symmetry and Galilei covariance. Let Alice prepare the initial velocities for an eccentric collision

$$v_{(i)} = \left( \frac{h_{(i)}}{-\epsilon} \right) \cdot v_{1(A)} , \quad \text{and} \quad v_{\mathcal{O}} = - \left( \frac{h_{(i)}}{-\epsilon} \right) \cdot v_{1(A)}$$

Alice can freely adjust the deflection $\tan(\tilde{\alpha}_i) = h_{(i)}$ by reorienting the spring between two standard processes $w_1^{-1} \ast w_1$ (in figure 1b) or with a suitable impact parameter in an eccentric collision of rigid balls.
with fixed horizontal and vertical components (see figure 3).

Let Alice move relative to Bob at constant velocity \( \mathbf{v}_A = \begin{pmatrix} h_{(i)} \\ 0 \end{pmatrix} \cdot v_{1(b)} \) in the horizontal direction. Measured values of motion transform by vectorial addition. For Bob incident body \( \overline{1} v_{(i)} \) has twice the horizontal velocity \( 2 \cdot h_{(i)} \cdot v_{1(b)} \) with same vertical component \( \epsilon \cdot v_{1(b)} \).

\[
\mathbf{v}_{(i)} = \begin{pmatrix} 2 \cdot h_{(i)} \\ \mp \epsilon \end{pmatrix} \cdot v_{1(b)} \\
\mathbf{v}_R = \begin{pmatrix} 0 \\ \pm \epsilon \end{pmatrix} \cdot v_{1(b)}
\]

while reservoir particle \( \overline{1} \epsilon \cdot v_1 \) moves up and down vertically with same velocity \( \epsilon \cdot v_{1(b)} \). For the same elastic collision Bob determines a scattering angle \( \alpha_i \) according to figure 3b.

For given initial velocity \( v_{(i)} \) and fixed transversal impact velocity \( \epsilon \cdot v_1 \) we can determine the deflection angle \( \alpha_i \) and vice versa provided the latter we find the admissible velocity \( v_{(i)} \).

By a series of transversal standard kicks \( w_T \) from reservoir particles we steer a reversion process for an incident particle \( \overline{1} v_{(i)} \) with velocity \( v_{(1)} \) (see figure 4); and similar for a faster particle \( \overline{1} v_{(2)} \) which requires twice the standard reservoir kicks, until its motion is exactly reversed. We align them in the depicted way (see figure 5a), so that all temporarily mobilized recoil particles can be captured again and recycled. In the total balance the reservoir particles do not appear. In the net result only the motion of all incident particles (from the left and right side) is exactly reversed. We determine their admissible velocities \( v_{(i)} \) from matching building blocks \( w_T \overline{1} v_{(1)} \) and \( w_T \overline{1} v_{(2)} \). By refinement of building blocks we construct similar models for the elastic collision of \( n+1 \) equivalent particles (see figure 5b) and in the refinement limit (where the spreading bundle narrows to a ray) for rigid composites of \( n+1 \) equivalent elements (see figure 5c).

We do not presuppose how velocities of two generic objects change in an elastic collision. The trick is to mediate their direct interaction by a steered replacement process with an external reservoir\(^4\). Our model solely consists of elastic collisions between standard elements which must behave in a symmetrical way. From their layout we derive the generic collision law. In the same way we will proceed for the absorption process in a calorimeter \( \{1.2\} \).

**Theorem 1** Consider a reservoir with identically constituted elements \( \{1\} \). Suppose we can tightly connect \( n \) of them \( \overline{1} \ast \ldots \ast \overline{1} =: \overline{\alpha} \) such that the composite acts like one rigid unit. Let in an elastic head-on collision two different composites of standard objects

\[
\begin{align*}
\begin{array}{l}
\overline{1} \mathbf{v} + \overline{\alpha} \mathbf{w} \\
\overline{1} \mathbf{- v} + \overline{\alpha} \mathbf{- w}
\end{array}
\end{align*}
\]

repulse from one another with reversed velocities. Then - in Galilei Kinematics - respective velocities must satisfy relation

\[
\mathbf{v} = - n \cdot \mathbf{w} .
\]

\(^4\)Steering the suitable coupling of standard processes is (like the placement of meter sticks along a straight line) an elementary operation in a measurement and does not require mathematics at all.
Proof: The proof follows from three auxiliary steps. We examine an elastic collision between two generic objects. Without restricting generality we assume they are (rigid) composites of unit objects \( R \). We know the collision law for \( 1 + 1 \) equivalent objects by symmetry and relativity principle (Lemma 1). Based on it we construct the collision model for \( 2 + 1 \) equivalent objects (step I) and for \( n + 1 \) equivalent objects (step II) and ultimately for composites of \( n + 1 \) equivalent objects (step III) to derive the collision law for two generic objects \( S \).

In step I we examine the elastic head-on collision between one unit object \( \overline{1}_v(2) \) from left with initial velocity \( v(2) \) and two unit objects \( \overline{1}_{R_{15^\circ}}v(1) \) and \( \overline{1}_{R_{-15^\circ}}v(1) \) from right with velocity \( v(1) \) under suitable orientation \( 15^\circ \) resp. \( -15^\circ \) (see figure 6a). We model the process by a series of elastic transversal collisions and derive the admissible velocities.

Let Alice and Bob have access to an external reservoir \( \{S|_0, \overline{1}_0\} \). They - temporarily - expend standard energy sources \( S \) (of strength \( \epsilon \)) against initially resting reservoir elements

\[
w_\epsilon : S|_0, \overline{1}_0, \overline{1}_0 \Rightarrow \overline{1}_\epsilon-v_1, \overline{1}_{-\epsilon-v_1}
\]

(9)

to prepare transversal impulse carriers \( \overline{1}_\epsilon-v_1 \) with velocity \( \epsilon \cdot v_1 \) into suitable direction \( \theta \) (see figure 6b). They fire them into the momentary way of incident particle \( \overline{1}_v(i) \) resp. \( \overline{1}_v(2) \) such that the former repulse antiparallel. Each transversal kick \( w_T \) successively deflects incident particle \( \overline{1}_v(i) \) \( i = 1, 2 \) by corresponding angle \( \alpha_i \) (see figure 6b). For fixed impact velocity \( \epsilon \cdot v_1 \) of the reservoir element \( \overline{1}_\epsilon-v_1 \) and matching deflection \( \alpha_1 = 60^\circ \) resp. \( \alpha_2 = 30^\circ \) we determine the admissible velocities \( v(i) := \sin^{-1}(\frac{\alpha_i}{2}) \cdot \epsilon \cdot v_1 \).

Alice steers the reversion process for incident object \( \overline{1}_v(1) \) from the left (see figure 1): Three assistants have to line up at the corners and initiate each steering kick - timely and at suitable position - such that its motion gets reversed. They couple a sequence of three transversal standard kicks

\[
W(1) := w_T^{(30^\circ)} * w_T^{(90^\circ)} * w_T^{(150^\circ)}
\]

at same object \( \overline{1}_v(1) \) which in each intermediate state moves freely with same velocity \( v(1) \). Similarly Bob’s team steers a separate reversion process for a faster incident particle \( \overline{1}_v(2) \) with admissible velocity \( v(2) > v(1) \) which requires twice the standard kicks (9). Six men line up at the corners and know how to fire reservoir elements \( \overline{1} \) into its way

\[
W(2) := w_T^{(15^\circ)} * w_T^{(45^\circ)} * \ldots * w_T^{(165^\circ)}
\]

After six successive kicks of the same strength its direction of motion is reversed too.

Alice and Bob align their reversion processes \( W(1) \) and \( W(2) \) for all incident objects. Alice rotates her reversion process for first incident particle \( \overline{1}_v(1) \)

\[
R_{\beta} \left[ w_T^{(30^\circ)} * w_T^{(90^\circ)} * w_T^{(150^\circ)} \right] = w_T^{(30^\circ+\beta)} * w_T^{(90^\circ+\beta)} * w_T^{(150^\circ+\beta)}
\]

\[\text{The detailed construction can be thought of as an appendix; the end is marked by the "□" symbol.}\]
Figure 4: In a coordinated effort Alice and Bob’s team of physicists steer a series of elastic transversal kicks $w_T$ in order to provide impulse reversion for particle $\mathbf{v}_{(1)}$ resp. $\mathbf{v}_{(2)}$ by $\beta = 195^\circ$ and similarly for the second incident particle $\mathbf{v}_{(1)}$ she rotates the entire configuration $R_{165^\circ}[W_{(1)}]$ by an angle $\beta = 165^\circ$. She instructs her assistants to rebuild the same model from same standard building blocks but with a modified orientation (symbolized by operator $R_{\beta}[\cdot]$). For every transversal steering kick $w_{T}^{(\theta)} := w_{T}^{(\theta)} * w_{T}$ they pick two resting unit objects $\mathbf{1}_0$ from the reservoir and generate two recoil particles $\mathbf{1}_{-\mathbf{v}_1^\epsilon}$ with same velocity $-\mathbf{v}_1^\epsilon$; one in the preparation $w_{T}^{(\theta)}$ (see figure 5a) and the other after the elastic kick $w_{T}$ (see figure 5b). In order to retrieve those resources Alice and Bob align their reversion processes

$$W_{(2)} * R_{165^\circ}[W_{(1)}] * R_{195^\circ}[W_{(1)}]$$

such that all transversal standard kicks pair up along dashed lines in diametrically opposed locations (see figure 5a)

$$\left(w_{T}^{(15^\circ)} * w_{T}^{(30^\circ+165^\circ)}\right) * \left(w_{T}^{(45^\circ)} * w_{T}^{(30^\circ+195^\circ)}\right) * \ldots * \left(w_{T}^{(165^\circ)} * w_{T}^{(150^\circ+195^\circ)}\right).$$

Each associated tuple of four antiparallel recoil particles $\mathbf{1}_{\mathbf{v}_1^\epsilon}$, $\mathbf{1}_{-\mathbf{v}_1^\epsilon}$, $\mathbf{1}_{-\mathbf{v}_1^\epsilon}$ and $\mathbf{1}_{-\mathbf{v}_1^\epsilon}$ reproduces the two - temporarily expended - standard energy sources $S_\epsilon|_0$ and returns four resting particles back into the external reservoir $\{|S_\epsilon|_0, \mathbf{1}_0\}$ (see figure 5c). In the end the reservoir remains unaltered. The net process (10) provides an elastic collision between three
Figure 5: model of elastic head-on collision between composites of standard objects
Figure 6: Alice and Bob at diametrically opposed positions set up and control process of 
a) expend energy source $S_0$ against $\Theta_0$ and $\Theta_0$ from their reservoir to provide a pair of
antiparallel impulse carriers for an b) elastic transversal collision with incident $\Theta_{1(1)}$ resp.
$\Theta_{1(2)}$ c) antiparallel recoil particles reproduce the two - temporarily expended - energy
units $S_0$ and return as resting particles back into the reservoir $\{\Theta_0\}$
equivalent objects:

\[ 3 \; v(2) \; ; \; 1 \; R_{15^\circ}v(1) \; ; \; 1 \; R_{-15^\circ}v(1) \; \Rightarrow \; 1 \; v(2) \; ; \; 1 \; R_{15^\circ}v(1) \; ; \; 1 \; R_{-15^\circ}v(1). \]

In the final state their motion is exactly reversed (see figure 5). Alice and Bob mediate their elastic repulsion by well-defined resources from an external reservoir. Those were temporarily expended but finally all recycled back. Every act of their procedure is reversible.

For step II we model the elastic head-on collision between one unit object \( v(n) \) from left and a spreading bundle of \( n \) unit objects \( 1 \; R_{\delta_1}v(1) \; , \; \ldots \; , \; 1 \; R_{\delta_n}v(1) \) from right. From the layout of our standard building blocks we determine the admissible velocities \( v(n) \) resp. \( v(1) \) and the suitable orientations \( \theta_k \) for \( k = 1, \ldots , n \) (see figure 5).

Alice and Bob refine the strength \( \epsilon \) of their transversal standard kicks \( w_{\tau} \). Each reservoir element \( 1 \; e \cdot v_1 \) deflects incident particle \( 1 \; v(i) \) with admissible velocity \( v(i) \) by corresponding angle \( \alpha_i \) \( i = 1, n \). Let Alice concatenate \( N(1) := \frac{\pi}{\alpha_1} \) transversal standard kicks

\[ W(1) := w_{\tau}^{-\alpha_1} * w_{\tau}^{-\alpha_2} * \ldots * w_{\tau}^{-2 \alpha_1} \]

(11)

to reverse the motion for each element \( 1 \; v(1) \) of the right incident bundle. Similarly Bob steers \( N(n) := \frac{\pi}{\alpha_n} \) transversal kicks of same strength \( \epsilon \) like Alice

\[ W(n) := w_{\tau}^{-\alpha_n} * w_{\tau}^{-\alpha_n} * \ldots * w_{\tau}^{-2 \alpha_n} \]

until the direction of motion for the left particle \( 1 \; v(n) \) with velocity \( v(n) \) is reversed too.

For alignment both reversion processes \( W(1) \) and \( W(n) \) must match with one another. We impose matching deflection angles

\[ \alpha_1 = n \cdot \alpha_n. \]

Then for fixed transversal impact velocity \( \epsilon \cdot v_1 \) of the reservoir element \( 1 \; e \cdot v_1 \) and deflection angle \( \alpha_i \) the admissible velocities \( v(i) \) for incident particle \( 1 \; v(i) \) \( i = 1, n \) are known [6].

Let Alice align the \( n \) bundle elements \( 1 \; R_{\delta_1}v(1) \; , \; \ldots \; , \; 1 \; R_{\delta_n}v(1) \) from right with velocity \( v(1) \)

- under orientations \( \theta_k := \frac{\mu + 1}{2} \cdot \alpha_n - k \cdot \alpha_n \) for \( k = 1, \ldots , n \)
- with equal spacing \( \Delta \theta = \alpha_n \) ranging between \( \theta_1 = + \frac{\alpha_n}{2} - \frac{\alpha_n}{2} \), \( \theta_n = - \frac{\alpha_n}{2} + \frac{\alpha_n}{2} \).

Then Alice turns the reversion process [11] for first bundle element \( 1 \; R_{\delta_1}v(1) \)

\[ R_{\beta_1} \left[ w_{\tau}^{(\delta_1)} * \ldots * w_{\tau}^{(\delta_n)} \right) = w_{\tau}^{(\delta_1 + \beta_1)} * \ldots * w_{\tau}^{(\delta_n + \beta_1)} \]

\[ R_{\beta_1} \left[ w_{\tau}^{(\delta_1)} * \ldots * w_{\tau}^{(\delta_n)} \right) = w_{\tau}^{(\delta_1 + \beta_1)} * \ldots * w_{\tau}^{(\delta_n + \beta_1)} \]

\[ R_{\beta_1} \left[ w_{\tau}^{(\delta_1)} * \ldots * w_{\tau}^{(\delta_n)} \right) = w_{\tau}^{(\delta_1 + \beta_1)} * \ldots * w_{\tau}^{(\delta_n + \beta_1)} \]

---

\[ \text{For step I with } n = 2, \alpha_1 = 60^\circ, \alpha_2 = 30^\circ \text{ we verify } \theta_1 := \frac{\alpha_1 - \alpha_2}{2} = \frac{1}{2} \cdot \alpha_2 \text{ and } \theta_2 := \frac{\alpha_1 + \alpha_2}{2} = -\frac{1}{2} \cdot \alpha_2 \text{ in accordance with figure 5}. \]
with \( \vartheta_j := -\frac{\alpha_1}{2} + j \cdot \alpha_1 \) for \( j = 1, \ldots, N_{(1)} \) around angle \( \beta_1 := \pi + \frac{\alpha_1}{2} - \frac{\alpha_n}{2} \) and similarly

she turns the reversion process \( R_{\beta_k}[W_{(1)}] \) for every other element \( _1 R_{\theta_k} v_{(1)} \) around angle \( \beta_k := \pi + \theta_k \) for \( k = 1, \ldots, n \). Similar to figure 3, Alice and Bob connect reversion processes

\begin{equation}
W_{(n)} \ast R_{\beta_{(1)}}[W_{(1)}] \ast \ldots \ast R_{\beta_{(n)}}[W_{(1)}]
\tag{13}
\end{equation}

such that all transversal steering kicks with \( \gamma_l := -\frac{\alpha_1}{2} + l \cdot \alpha_n \) for \( l = 1, \ldots, N_{(n)} \)

\[
\left\{ w_T^{(\gamma_1)} \ast \ldots \ast w_T^{(\gamma_{N_{(n)}})} \right\} \ast \left\{ w_T^{(\vartheta_1 + \beta_1)} \ast \ldots \ast w_T^{(\vartheta_{N_{(n)}} + \beta_1)} \right\} \\
\ast \ldots \ast \left\{ w_T^{(\vartheta_{1} + \beta_n)} \ast \ldots \ast w_T^{(\vartheta_{N_{(n)}} + \beta_n)} \right\}
\]

divide into antiparallel pairs where as before all byproducts can be retrieved

\[
\left( w_T^{(\gamma_1)} \ast w_T^{(\vartheta_1 + \beta_n)} \right) \ast \left( w_T^{(\gamma_2)} \ast w_T^{(\vartheta_2 + \beta_{n-1})} \right) \ast \ldots \ast \left( w_T^{(\gamma_{N_{(n)}})} \ast w_T^{(\vartheta_{N_{(n)}} + \beta_1)} \right).
\]

The net process (13) mediates the elastic collision of \( n + 1 \) equivalent objects

\( \textcircled{1} v_{(n)} ; \textcircled{1} R_{\theta_1} v_{(1)} ; \ldots ; \textcircled{1} R_{\theta_n} v_{(1)} \Rightarrow \textcircled{1} -v_{(n)} ; \textcircled{1} -R_{\theta_1} v_{(1)} ; \ldots ; \textcircled{1} -R_{\theta_n} v_{(1)} ; \)

their motion is exactly reversed.

In **step III** we refine the building blocks for collision model (13) to the limit \( \epsilon \to 0 \) where the impact of individual reservoir elements \( \textcircled{1}_e v_1 \) diminishes. Each transversal standard kick \( w_T \) deflects right bundle element \( \textcircled{1} v_1 \) with fixed velocity \( v_{(1)} = v_1 \) by angle \( \sin \frac{\alpha_1}{2} = \frac{\epsilon}{v_1} \) and left particle \( \textcircled{1} v_{(n)} \) by matching angle \( \alpha_n := \frac{1}{n} \cdot \alpha_1 \).

We integrate an increasing number \( N_{(1)} := \frac{\pi}{\alpha_1} \) resp. \( N_{(n)} := n \cdot N_{(1)} \) of refined standard kicks into the model until the motion of every particle from the right bundle and from the left is reversed. In return the spreading of the bundle \( \theta_1 - \theta_n := \lim_{\epsilon \to 0} (n - 1) \cdot \alpha_1 (v_1, \epsilon) = 0 \) narrows.

We rewrite the matching condition \( \alpha_1 \div n \cdot \alpha_n \) between Alice and Bob’s transversal

\[(\gamma_1 - (\delta_1 + \beta_n) := -\frac{\alpha_n}{2} + 1 \cdot \alpha_n - \left( -\frac{\alpha_1}{2} + 1 \cdot \alpha_1 + \pi + \frac{n + 1}{2} \cdot \alpha_n - n \cdot \alpha_n \right) \]

\[= \frac{\alpha_n}{2} - \frac{\alpha_1}{2} - \pi + \frac{n \cdot \alpha_n}{2} - \frac{\alpha_n}{2} \]

\[\text{Straight forward insertion confirms that first pair is aligned antiparallel and analogous for all the rest} \]

\[\gamma_1 - (\delta_1 + \beta_n) = -\pi. \]
standard kicks \( w_T \left[ \mathbb{1}_v(1) \cdot e^{-v_1} \right] \) resp. \( w_T \left[ \mathbb{1}_v(n) \cdot e^{-v_1} \right] \)

\[
\sin \left( \frac{\alpha_1}{2} \right) = \sin \left( n \cdot \frac{\alpha_n}{2} \right) = \sum_{k=0}^{n-1} \left( \begin{array}{c} n \\ k \end{array} \right) \cdot \cos^k \left( \frac{\alpha_n}{2} \right) \cdot \sin^{n-k} \left( \frac{\alpha_n}{2} \right) \cdot \sin \left( \frac{1}{2} (n - k) \cdot \pi \right)
\]

with trigonometric identity of multiple angles. With substitution \( \sin \frac{\alpha}{2} = \frac{\epsilon v}{v(1)} \) we obtain

\[
\frac{\epsilon}{v(1)} = \sum_{k=0}^{n-1} \left( \begin{array}{c} n \\ k \end{array} \right) \cdot \sqrt{1 - \frac{\epsilon^2}{v^2(n)}} \cdot \left( \frac{\epsilon}{v(n)} \right)^{n-k} \cdot \sin \left( \frac{1}{2} (n - k) \cdot \pi \right)
\]

\( \forall \epsilon > 0 \) and fixed \( v(1) = v_1 \) the admissible velocity \( v(n) := v(n)(v_1, \epsilon) \) for left particle \( \mathbb{1}_v(n) \). For \( \epsilon \ll v_1 < v(n) \) we neglect terms of higher order \( \mathcal{O} \left( \frac{\epsilon}{v(n)} \right)^2 \) and keep the dominant for \( k = n - 1 \)

\[
\frac{1}{v_1} = \lim_{\epsilon \to 0} n \cdot \sqrt{1 - \frac{\epsilon^2}{v^2(n)}} \cdot \frac{1}{v(n)} = n \cdot \frac{1}{\lim_{\epsilon \to 0} v(n)} .
\]

In the limit - of refined steering kicks \( \epsilon \to 0 \) by reservoir elements \( \mathbb{1}_v, e^{-v_1} \) - we model (13) the elastic head-on collision between one standard object \( \mathbb{1}_v \) and a parallel beam of \( n \) elements \( \{ \mathbb{1}_v, \ldots , \mathbb{1}_v \} \equiv \mathbb{1}_{v_1} \) (see figure 5c). Before and after the collision they fly with same velocity \( v_1 \) as if they were bound in a composite \( \mathbb{0}_v \)

\[
w_H : \mathbb{1}_v(n) \cdot \mathbb{0}_v \Rightarrow \mathbb{1}_v(n) \cdot \mathbb{0}_v .
\]

In the limit when the bundle becomes a ray admissible initial velocities \( v_1, v(n) \) satisfy relation

\[
\lim_{\epsilon \to 0} v(n) = -n \cdot v_1 .
\]

We learn something about elastic collisions which we did not presuppose before. Our model provides a physical derivation of fundamental (collision) equation \( m_1 \cdot \Delta v_1 = m_2 \cdot \Delta v_2 \) (including scope and limitations).

### 4.2 Calorimeter absorption model

Consider for example the generic elastic collision between one (fast) standard particle and a composite of 9 elements. For a drive-by observer the incident particle kicks a resting composite into motion \( 2 \cdot v \) and rebounds with reduced velocity to the left. This elastic head-on collision \( w_H \) provides the elementary building block for calorimeter model \( W_{\text{cal}} \)
Figure 7: incident particle successively comes to rest by means of elastic collisions with initially resting elements on the left resp. right side of the calorimeter reservoir \( \{ v=0 \} \)
(see figure\[\text{7}\]). On the left we can again place a suitable number of 7 resting reservoir elements into the way, such that they get kicked out with the same standard velocity $2 \cdot v$ etc. The incident particle successively rebounds with reduced velocity, until (in this example) after four right- and left-deceleration kicks $w_H$ it stops inside the calorimeter. For the controlled deceleration of a particle $\varpi_{10} \cdot v$ with velocity $10 \cdot v$ we activate in total 25 initially resting reservoir elements. We kick 10 pairs of antiparallel recoil particles $\{\varpi_{2} \cdot v, \varpi_{-2} \cdot v\}$ with same standard velocity $2 \cdot v$ out of both sides of the calorimeter and 5 single impulse carriers $\varpi_{2} \cdot v$.

$$W_{\text{cal}} : \varpi_{10} \cdot v , 25 \cdot \varpi_{0} \Rightarrow \varpi_{0} , 10 \cdot \{\varpi_{2} \cdot v, \varpi_{-2} \cdot v\} , 5 \cdot \varpi_{2} \cdot v .$$

We formulate this mechanical process as ”reaction equation” (based on the language use among chemists).\[\text{8}\] If we absorb the same standard particle $\varpi_{16} \cdot v$ with higher velocity $16 \cdot v$ in our calorimeter, we have to mobilize even 64 initially resting reservoir elements. Now in the same series of deceleration kicks

$$W_{\text{cal}} : \varpi_{16} \cdot v , 64 \cdot \varpi_{0} \Rightarrow \varpi_{0} , 28 \cdot \{\varpi_{2} \cdot v, \varpi_{-2} \cdot v\} , 8 \cdot \varpi_{2} \cdot v$$

we generate 28 congruent particle pairs and 8 standard impulse carriers etc.

This illustrates the essence of a basic measurement. The initially vague pre-theoretic comparison $\varpi_{16} \cdot v >_{E,P} \varpi_{10} \cdot v$ ”exceeding absorption effect” and ”overrunning in head-on collision” is now precisely determined by a number of equivalent reference elements (28 resp. 10 particle pairs $\{\varpi_{2} \cdot v, \varpi_{-2} \cdot v\}$ of equal capability to work and 8 resp. 5 recoil particles $\varpi_{2} \cdot v$ of equal impact). We will assess the physical meaning of extracted calorimeter elements as units of energy and momentum by pre-theoretic ordering relations in Proposition 2.

**Proposition 1** The calorimeter-deceleration-cascade $W_{\text{cal}}$ is a physical model for absorbing unit object $\varpi_{n} \cdot v_1$ with velocity $n \cdot v_1$ in a calorimeter where it comes to rest $\varpi_{0}$

$$W_{\text{cal}} : \varpi_{n} \cdot v_1 , \{\varpi_{0}\} \Rightarrow \varpi_{0} , \text{RB} [\varpi_{n} \cdot v_1 \Rightarrow \varpi_{0}] .$$

In return for its absorption we extract from an external reservoir with resting elements $\{\varpi_{0}\}$ the reservoir balance for absorption

$$\text{RB} [\varpi_{n} \cdot v_1 \Rightarrow \varpi_{0}] := \left\{ \frac{1}{2} \cdot n^2 - \frac{1}{2} \cdot n \right\} \cdot \{\varpi_{-v_1} , \varpi_{v_1}\} , n \cdot \varpi_{v_1}$$

a certain number of standard particle pairs $\{\varpi_{-v_1} , \varpi_{v_1}\}$ and impulse carriers $\varpi_{v_1}$.

**Proof:** We model the process from elastic head-on collisions and relativity principle. Alice successively decelerates incident particle $\varpi_{n} \cdot v_1$ by a cascade of elastic collisions with suitable packs of resting reservoir elements $\{\varpi_{0}\}$.

\[\text{8}\]Physicists formulate equations between measures (Maßgleichungen); chemists on the contrary transitions between their carriers (Maßträger) - we formulate both sides: carrier and its measure!
Consider a single step for given initial velocity $n \cdot v_1$. Let Bob prepare a composite $\bigodot_0 \ast \ldots \ast \bigodot_0 =: \bigodot_0$ of $n$ standard elements such that in an elastic head-on collision $w_H$:

$$\bigodot_{n \cdot v_1}, \bigodot_{-v_1} \Rightarrow \bigodot_{n \cdot v_1}, \bigodot_{v_1}$$

incident particle $\bigodot_{n \cdot v_1}$ with initial velocity $n \cdot v_1$ rebounds antiparallel from composite $\bigodot_{-v_1}$ with standard velocity $v_1$. Let Bob move relative to Alice with constant velocity $v_B = 1 \cdot v_1(4)$ to the left. Then Alice will see Bob’s head-on collision with different measured values of velocity (Galilei covariant transformation $v_i(4) = v_i(B) + v_B(4)$ for both objects $i = 1, n$). For Alice incident particle $\bigodot_{(n+1) \cdot v_1}$ kicks into the right side of the calorimeter with velocity $(n + 1) \cdot v_1$ and rebounds antiparallel with reduced velocity $(n - 1) \cdot v_1$ to the left

$$w_H^{(r)} : \bigodot_{(n+1) \cdot v_1}, \bigodot_0 \Rightarrow \bigodot_{-(n-1) \cdot v_1}, \bigodot_{2 \cdot v_1} \quad (15)$$

while composite $\bigodot_0$ of $n$ initially resting reservoir elements gets kicked into standard velocity $2 \cdot v_1$. On the left side of her calorimeter Alice places a new composite $\bigodot_0 \ast \ldots \ast \bigodot_0$ of $n - 2$ reservoir elements and generates the next deceleration kick

$$w_H^{(l)} : \bigodot_{-(n-2+1) \cdot v_1}, (n - 2) \cdot \bigodot_0 \Rightarrow \bigodot_{(n-3) \cdot v_1}, (n - 2) \cdot \bigodot_{-2 \cdot v_1} \quad (15)$$

After each round of right and left collisions $W := w_H^{(r)} \ast w_H^{(l)}$

$$W : \bigodot_{(n+1) \cdot v_1}, (n - 2) \cdot \bigodot_0, n \cdot \bigodot_0 \Rightarrow \bigodot_{(n-3) \cdot v_1}, (n - 2) \cdot \bigodot_{-2 \cdot v_1}, n \cdot \bigodot_{2 \cdot v_1} \quad (16)$$

we can add the extracted energy-momentum carriers $(n - 2) \cdot \bigodot_{-2 \cdot v_1}, n \cdot \bigodot_{2 \cdot v_1}$ on the left and right side of the calorimeter and the successive deceleration $\Delta v := -4 \cdot v_1$. Each particle pair $\{\bigodot_{-2 \cdot v_1}, \bigodot_{2 \cdot v_1}\}$ with same velocity $2 \cdot v_1$ can be recycled into energy source $S_2|_0$ by standard action $w_2^{-1} \ast \bigodot_{-2 \cdot v_1}, \bigodot_{2 \cdot v_1} \Rightarrow S_2|_0, \bigodot_0, \bigodot_0$. The two resting elements return back into calorimeter reservoir $\{\bigodot_0\}$. In every single deceleration step Alice extracts the reservoir balance

$$\text{RB} \left[ \bigodot_{(n+1) \cdot v_1} \Rightarrow \bigodot_{(n-3) \cdot v_1} \right] \quad \Rightarrow \quad (n - 2) \cdot S_2|_0, 2 \cdot \bigodot_{2 \cdot v_1} \quad (17)$$

$n - 2$ standard energy sources $S_2|_0$ and 2 congruent impulse carriers $\bigodot_{2 \cdot v_1}$.

Alice steers these (elastic) collisions into the left and right side of her calorimeter $N$ consecutive times $W_{\text{cal}} := W^{(l)} \ast \ldots \ast W^{(N)}$ to bring incident object $\bigodot_{(4N+2) \cdot v_1}$ to rest (let initial velocity be $(4N + 2) \cdot v_1$)

$$W_{\text{cal}} : \bigodot_{(4N+2) \cdot v_1}, \{\bigodot_0\} \Rightarrow \bigodot_0, \text{RB} \left[ \bigodot_{(4N+2) \cdot v_1} \Rightarrow \bigodot_0 \right] \quad (18)$$

Two consecutive deceleration rounds $W_{\text{cal}} := W \ast W$ bring incident particle $\bigodot_{10 \cdot v_1}$ with initial velocity $10 \cdot v_1$ to rest (see figure 7). Then Alice can count congruent measurement units: $7 + 3$ particle pairs $\{\bigodot_{2 \cdot v_1}, \bigodot_{-2 \cdot v_1}\}$ (representing energy unit $S_2|_0$) and $2 + 2 + 1$ congruent impulse units $\bigodot_{2 \cdot v_1}$.
On each step $W^{(i)}$, $i = 1, \ldots, N$ of deceleration cascade $W_{\text{cal}}$ Alice extracts $(4i - 1) \cdot S_2\vline_0$ congruent standard springs and $2 \cdot (1)_{2 \cdot v_1}$ individual recoil particles (see figure [4]). In total Alice accumulates the reservoir balance for absorption

$$\text{RB} \left[ (2N + 1)_{2 \cdot v_1} \Rightarrow (1)_{0} \right] = \frac{1}{2} (2N + 1)^2 - \frac{1}{2} (2N + 1) \quad (17)$$

$$\begin{aligned}
\sum_{i=1}^{N} \text{RB} \left[ (4i + 2)_{-v_1} \Rightarrow (4i - 2)_{-v_1} \right] + \text{RB} \left[ (1)_{2 \cdot v_1} \Rightarrow (1)_{0} \right] \\
\Rightarrow \sum_{i=1}^{N} ((4i - 1) \cdot S_2\vline_0, 2 \cdot (1)_{2 \cdot v_1}) + (0 \cdot S_2\vline_0, 1 \cdot (1)_{2 \cdot v_1}) \\
= \left( 2 \cdot (N^2 + N) \right) \cdot S_2\vline_0, (2N + 1) \cdot (1)_{2 \cdot v_1} \\
\Rightarrow \frac{1}{2} (2N + 1)^2 - \frac{1}{2} (2N + 1) \quad (18)
\end{aligned}$$

In new measurement units with velocity $v_{1,(A)} := 2 \cdot v_1$ and standard energy sources $S_{1,(A)}\vline_0 := S_2\vline_0$ and impulse carriers $(1)_{v_{1,(A)}}$ from reference action $w_{1(A)}$ Alice extracts with her calorimeter a reservoir balance with numbers according to (14).

\[ \square \]

### 5 Quantification

Now let us consider these models from an abstract physical perspective. From pre-theoretic energy-momentum comparison [2] we examine the physical meaning of our reference objects and of our calorimeter extract.

**Proposition 2** Standard energy source $S_1\vline_0$ represents unit energy $E_1$ and has no impulse.

$$\begin{aligned}
E \left[ S_1\vline_0 \right] &=: E_1 \\
\rightarrow \mathbf{p} \left[ S_1\vline_0 \right] &= 0
\end{aligned}$$

**Standard impulse carrier** $(1)_{v_1}$ represents unit momentum $\mathbf{p}_{1}$ and also has energy $\frac{1}{2} \cdot E_1$.

**Proof:** The two dimensions energy and impulse are inseparably intertwined in unit action $w_1 : S_1\vline_0, (1)_{0} \Rightarrow (1)_{-v_1}, (1)_{v_1}$ between our standard energy source and impulse carriers.

The resting energy source $S_1\vline_0$ can not overrun any moving object in a head-on collision test; if at all it will be overrun. Its impact $S_1\vline_0 \prec (1)_{v_{1,(A)}}$ is weaker than of any other moving object $\forall \epsilon > 0$. Its abstract momentum is zero $\mathbf{p} \left[ S_1\vline_0 \right] = 0$ (see Definition [14]).

We can convert two comoving elements $\left\{ (1)_{v_1}, (1)_{-v_1} \right\} \Rightarrow \left\{ (1)_{-v_1}, (1)_{v_1} \right\}$ into an antiparallel particle pair by letting, vividly spoken, one element repulse elastically $w_H : (1)_{v_1}, (1)_{v_{M}} \Rightarrow (1)_{-v_1}, (1)_{-v_{M}}$ from a much heavier ”reservoir block”. In the limit $m[\overline{1}] \ll [\overline{M}]$ one can show by refined calorimeter measurements that the ”bouncing block” $v_M \rightarrow 0$ is practically at rest and with negligible contribution to energy (for transitivity and details see [20]). Thus two impulse units generate the same absorption effect like one standard spring; by the equipollence of cause and effect (3) their energy is the same $2 \cdot E \left[ (1)_{v_1} \right] = E \left[ S_1\vline_0 \right]$.
The calorimeter extract has the same capability to execute work as the incident particle, because our calorimeter model is reversible. It also has the same impact, since otherwise one could construct a Perpetuum Mobile.

**Lemma 2** In our calorimeter model $W_{\text{cal}}$ the extracted impulse carriers $\mathfrak{1} * \ldots * \mathfrak{1} v_1$ have same impulse as incident particle $\mathfrak{a} v_a$. The transferred momentum is conserved.

**Proof:** Without restricting generality we consider the absorption \((14)\) of a standard particle $\mathfrak{a} \equiv \mathfrak{1}$ with velocity $v_a := -n \cdot v_1$, $n \in \mathbb{N}$ which extracts standard impulse carriers $\mathfrak{1} * \ldots * \mathfrak{1} v_1 =: \mathfrak{a} v_1$ with velocity $v_1$. In a generic inelastic head-on collision test incident particle $\mathfrak{a} v_a$ and its impulse model $\mathfrak{a} v_1$

$$w_{(d)} : \mathfrak{a} v_a \cup \mathfrak{a} v_1 \Rightarrow \mathfrak{a} * \mathfrak{a} v' . \quad (20)$$

form a bound aggregate $\mathfrak{a} * \mathfrak{a} v'$ with velocity $v'$ and bounding energy $E^*$. They have same momentum if they collide, stick together and come to rest (see physical \text{Definition} \[\text{1}\]). Hence, we have to show, that the bound aggregate $\mathfrak{a} * \mathfrak{a} v'$ must stop $v' \not= 0$.

Let us hypothetically assume the contrary. Then, like for any other moving body, we can absorb the bound aggregate $\mathfrak{a} * \mathfrak{a} v'$ in our calorimeter

$$W_{\text{cal}} : \mathfrak{a} * \mathfrak{a} v' \Rightarrow \mathfrak{a} * \mathfrak{a} 0 \cup \mathfrak{a} S_1|_0 \cup l \cdot \mathfrak{a} v_1$$

and extract an additional number $k$ energy units $S_1|_0$ and $l$ impulse carriers $\mathfrak{a} v_1$ into the direction of $v'$. But then we could set up a circular process (from reversible standard actions) which could kick impulse carriers $\mathfrak{a} v_1$ ”around the corner” $\mathfrak{a} R_0 v_1$ without effecting anything else (details see \[20\]).

This hypothetical process would violate Euler’s Principle of Sufficient Reason; that every change in the state motion requires an external cause (physical reason) \[2\] and Impossibility of a Perpetuum Mobile. A suitably moving observer (with velocity $v_1$) could set initially resting reservoir particles $\{\mathfrak{a} 0\}$ into motion with unit velocity $2 \cdot v_1$ but also into opposite direction with velocity $-2 \cdot v_1$ and thus generate particle pairs $\{\mathfrak{a} -2 v_1, \mathfrak{a} 2 v_1\} \sim_E S_2|_0$ resp. energy sources without any reaction from nothing. Hence $v' \not= 0$ must have been zero; in inelastic head-on collision test $w_{(d)}$ particle $\mathfrak{a} v_a$ and its impulse model $\mathfrak{a} v_1$ come to rest.

\[\square\]

\[10\] The absorption is a steered series of elastic head-on collisions in system $\mathfrak{a} n v_1 \cup \{\mathfrak{a} 0\}$ of incident particle and calorimeter reservoir. Every step of the deceleration cascade is reversible, because it is built up from solely congruent unit actions $w_1$ (see figure \[2\]) and because physicists can steer these processes both ways.
The kinetic energy of the incident particle $E[\bigcirc v_a]$ is completely transformed into potential energy of the absorber material; the latter comes in congruent portions $S_1|_0$. According to the Congruence principle

$$E[\bigcirc v_a] \overset{(\text{Equip.})}{=} E[S_1|_0 * \ldots * S_1|_0] \overset{(\text{Congr.})}{=} E \cdot E[S_1|_0]$$

we measure "how many times more" its kinetic energy is, than the potential energy of standard spring $S_1|_0$ in reference process $w_1$. By the number $E := \sharp \{S_1|_0\}$ (physical quantity) of extractable units $S_1|_0$ of standard energy $E[S_1|_0]$ (dimension) we quantify the basic observable energy. In the same way we conduct independent basic measurements of momentum, inertial mass and velocity.

Sommerfeld [11] defines the impulse of a moving body: "Impulse means (with regards to direction and magnitude) that kick, which is capable of generating a given state of motion from the initial state of rest." Another body transmits that kick which first moves and then stops itself. Our absorption model [14] provides a direct measurement

$$p[\bigcirc v_a] \overset{(\text{Perp. Mob.})}{=} p[\bigcirc v_1 * \ldots * \bigcirc v_1] \overset{(\text{Congr.})}{=} p \cdot p[\bigcirc v_1].$$

We generate this kick by a number $p := \sharp \{\bigcirc v_1\}$ of congruent standard kicks. Similarly we measure the inertia of body $\bigcirc$ with an (equally massive [2]) composite of standard elements and the latter according to the Congruence principle

$$m[\bigcirc] \overset{(\text{Galilei})}{=} m[\bigcirc v_1 * \ldots * \bigcirc v_1] \overset{(\text{Congr.})}{=} m \cdot m[\bigcirc v_1]$$

by the number $m := \sharp \{\bigcirc\}$ of standard elements $\bigcirc$ and their unit mass $m[\bigcirc]$.

6 Quantity equations

In the calorimeter model we can count individual standard elements and thus derive the relation between basic quantities.

Lemma 3 Let a composite of $m$ bound unit elements $\bigcirc v_1 * \ldots * \bigcirc \sim_{\text{inert}} \bigcirc$ have same inertia as generic particle $\bigcirc v_a$. Then the reservoir extract for absorbing it with velocity $v_a$

$$\text{RB}[igcirc v_a \Rightarrow \bigcirc 0] = m \cdot \text{RB}[igcirc v_1 \Rightarrow \bigcirc 0] \quad (21)$$

is $m$ times larger than for absorbing unit element $\bigcirc v_1$ with same velocity $v_a$.

Proof: Same inertia [2] implies, that in an elastic head-on collision test with same initial velocity $v'_a := \frac{1}{2} \cdot v_a$ composite $\bigcirc v_1 * \ldots * \bigcirc$ and generic particle $\bigcirc$ must repulse in the same

\[11\text{Sommerfeld's defining kick is associated with generating motion (from rest). We examine kicks which}
\text{annihilate motion (towards rest). In two-body collisions Sommerfeld regards the "recipient"; we the "giver".}\]
anti-symmetrical way \( w_H : \Box v'_a, \ 1 \cdots 1 - v'_a \Rightarrow \Box - v'_a, \ 1 \cdots 1 v'_a \). For an observer moving with relative velocity \(-v'_a\)
\[
w_H : \Box v_a, \ 1 \cdots 1 0 \Rightarrow \Box 0, \ 1 \cdots 1 v_a
\]
generic particle \( \Box v_a \) stops in return for kicking initially resting composite into same motion.

We neutralize this elastic head-on collision by absorbing the (spectator) composite in our calorimeter \( RB [1 \cdots 1 v_a \Rightarrow 1 \cdots 1 0] =: k_m S_1|_0, l_m \cdot 1 v_1 \) and by catapulting the (temporarily) resting particle in a reversed absorption \( -RB [\Box v_a \Rightarrow \Box 0] =: k_a S_1|_0, l_a \cdot 1 v_1 \)
back into its original state of motion. The net effect is a circular process. The corresponding reservoir balance (of extractable dynamical units)
\[
p [(k_a - k_m) \cdot S_1|_0, (l_a - l_m) \cdot 1 v_1] \overset{19}{=} p [(l_a - l_m) \cdot 1 v_1] \overset{22}{=} 0
\]
cannot have momentum (for conservation see Lemma 2). It also cannot have energy
\[
E [(k_a - k_m) \cdot S_1|_0, (l_a - l_m) \cdot 1 v_1] \overset{22}{=} 0.
\]

Thus for the absorption of particle \( \Box v_a \) we extract the same number \( l_a \overset{1}{=} l_m \) of impulse carriers \( 1 v_1 \) and the same number \( k_a \overset{1}{=} k_m \) of energy units \( S_1|_0 \)
\[
RB [\Box v_a \Rightarrow \Box 0] = RB [1 \cdots 1 v_a \Rightarrow 1 \cdots 1 0] = m \cdot RB [1 v_a \Rightarrow 1 0]
\]
like for the composite. We can absorb every element in a separate deceleration cascade. By the congruence of all extracted energy-momentum units their total number simply adds up. Thus we obtain \( m \) times the output as for the equally moving unit element \( 1 v_a \).

\[\square\]

**Theorem 2** Particle \( \Box v_a \) with inertial mass \( m_\Box = m \cdot m_\Box \) and velocity \( v_a = v \cdot v_1 \) has kinetic energy and momentum
\[
E [\Box v_a] = \left\{ \frac{1}{2} \cdot m \cdot v^2 \right\} \cdot E [S_1|_0] \quad (23)
\]
\[
p [\Box v_a] = \left\{ m \cdot v \right\} \cdot p [1 v_1].
\]

**Proof:** By reversibility and Equipollence principle the calorimeter absorption extract has same "effect potential" as incident particle \( \Box v_a \). Its kinetic energy is transformed
\[
E [\Box v_a] \overset{\text{Equip.}}{=} E [RB [\Box v_a \Rightarrow \Box 0]] \overset{21}{=} E [m \cdot RB [1 v_a \Rightarrow 1 0]] \overset{14}{=} m \cdot \left\{ \left( \frac{1}{2} \cdot v^2 - \frac{1}{2} \cdot v \right) \cdot E [S_1|_0] + v \cdot E [1 v_1] \right\} \overset{19}{=} \left\{ \frac{1}{2} \cdot m \cdot v^2 \right\} \cdot E [S_1|_0]
\]

\[22\]
into the potential energy of $\{ \frac{1}{2} \cdot m \cdot v^2 \}$ congruent energy units $S_1|_0$. The calorimeter extract has same impulse as incident particle $\mathcal{v}_a$ (see Lemma 2). Its impulse

$$p[\mathcal{v}_a] = p[R\mathcal{v}_a \Rightarrow \mathcal{v}_0]$$

is reproduced by $\{ m \cdot v \}$ congruent impulse units $\mathcal{v}_1$ from the calorimeter reservoir.

When we build the model in Galilei Kinematics we derive primary dynamical equations (23)

$$\left\{ \frac{E_a}{E_1} \right\} = \frac{1}{2} \cdot \left\{ \frac{m_a}{m_1} \right\} \cdot \left\{ \frac{v_a}{v_1} \right\}^2 \quad \left\{ \frac{p_a}{p_1} \right\} = \left\{ \frac{m_a}{m_1} \right\} \cdot \left\{ \frac{v_a}{v_1} \right\},$$

in which all numerical values for energy $E = \frac{E[\mathcal{v}_a]}{E[S_1|_0]}$, impulse $p = \frac{p[\mathcal{v}_a]}{p[\mathcal{v}_1]}$, mass $m = \frac{m[\mathcal{v}]}{m[\mathcal{v}]}$ and velocity $v = \frac{v_a}{v_1}$ occur in the form measure/unit measure. Each formal ratio symbolizes the result of a physical operation; counting congruent units in the calorimeter model. When we steer the same measurement process in Poincare Kinematics, then we will derive all equations of relativistic dynamics (21).

We measure the momentum from multi-partite systems by steering a separate absorption $W_{\text{cal}}^{(i)}$ for each individual element. We extract impulse carriers $\mathcal{v}_1$ and $\mathcal{v}_2$ - on the left and right side of the calorimeter-collision-cascade - in the direction of its motion $v_i$ (see figure 7). For a generic many-particle system the extracted impulse carriers $\{ \mathcal{v}_i \}_{i=1...N}$ head into arbitrary directions $v_i \neq v_j$.

**Theorem 3** Direction and magnitude of total momentum is calculable by vectorial addition

$$p[\mathcal{v}_1, \ldots, \mathcal{v}_N] = p[\mathcal{v}_1] + \ldots + p[\mathcal{v}_N]. \quad (24)$$

**Proof:** (In Galilei Kinematics) we construct a physical model $W$ for absorbing multiple standard elements $\mathcal{v}_i$ with velocities $v_i$ into various directions $v_i \parallel v_j$.

$$W : \{ \mathcal{v}_1, \ldots, \mathcal{v}_N \}, \mathcal{v}_0 \Rightarrow \{ \mathcal{v}_0, \ldots, \mathcal{v}_0 \}, \mathcal{v}_{(N)}.$$

All elements $i = 1, \ldots, N$ of the system $\{ \mathcal{v}_0, \ldots, \mathcal{v}_0 \}$ stop; while one initially resting absorber particle $\mathcal{v}_{(N)}$ gets kicked, as we will show, into velocity $v_{(N)} = v_1 + \ldots + v_N$.

We illustrate Alice complete momentum transfer from one moving particle $\mathcal{v}_1$ onto another particle $\mathcal{v}_2$, moving into perpendicular direction in figure 8, and similarly for a system of $N$ inequivalent impulse carriers $\{ \mathcal{v}_1, \mathcal{v}_2, \ldots, \mathcal{v}_N \}$ from relativity principle and Bob’s reversible standard actions $w^{(1)}$ by induction (details see [20]). At each step of his calorimeter-mediated process momentum is conserved by Lemma 2. We measure the total momentum of the system $p[\mathcal{v}_1, \ldots, \mathcal{v}_N] = p[\mathcal{v}_{(N)}]$ by one absorber particle and the latter in a
Figure 8: a) vectorial impulse addition by underlying b) isotropic unit actions

calorimeter by the number of equivalent impulse carriers $\mathcal{v}_1$ which now all point into the same direction of $\mathbf{v}(N) := \mathbf{v}_1 + \ldots + \mathbf{v}_N$. By linearity of the momentum-velocity relation (23) for a single particle the total impulse $p[\mathcal{v}_1, \ldots, \mathcal{v}_N] = \{m_1 \cdot \mathbf{v}_1 + \ldots + m_1 \cdot \mathbf{v}_N\} \cdot p[\mathcal{v}_1]$ is the vector sum over all elements $\{\mathcal{v}_i\}_{i=1\ldots N}$.

7 Origin

Commonly one ascribes astronomical observations the central role for the development of modern natural science. Following Lorenzen [5] we can agree insofar as "From the phenomenon of regular movement of celestial bodies humans conceived the idea of exact regularity in nature. (Though) despite precise astronomical observations, Babylonians made throughout the centuries (and despite their arithmetic rules for projections), one can not accredit to them the thought of a 'natural law' as we understand it today." It was a first step to the de-deification (Entgöttterung) of nature. We locate our initial assumptions in the historic-genetic development of everyday practical work.\(^{12}\)

\(^{12}\)According to the guiding principle of the historic school one can understand conceptual schemata of past epoches, if one recognizes "how historic problems originate from practical problems" in everyday life [5]. During industrial revolution steam engine and work machine are coupled together for the first time. That marks the transition from (personal interplay of) worker and work machine to (coupling of) engine drive and work machine. While traditional craftsmen learned to handle their tools (hammer, saw, needle etc.) intuitively, industrial revolution substitutes the former by an impersonal motor. Steering the latter became an unprecedented problem. Lorenzen [5] defines what we understand by physics today: "is designing
Already in the discussion on the principle of inertia we acknowledge, that Newton could draw on literature of practical mechanics on problems of machine construction and work economy. In this context of origin one can grasp inertial reference systems as isolated (from external disturbances) reproducible experimental prerequisites, which can be provided empirically. Then in practical experiment and measurement one examines primarily production processes, where the internal interactions between relevant parts (of a machine) exceed the effects of e.g. gravitation by far. In practice one can (separate from gravity and) regard Mechanics as science of constructing and steering (local) machines. For the foundation of mechanics one primarily focusses on technically controllable natural processes.

In this domain we define basic observables from elementary comparison methods. In a basic measurement - which is always a pair comparison between measurement object and material model of concatenated units - the latter have different functions. Physicists specify the measurement object (e.g. compare pre-theoretic "capability to execute work" and "impact" from a generic interaction) while they have to provide a measurement unit in a suitable way. For the derivation of relativistic Kinematics Einstein introduces rods and clocks as unstructured entities by the clock postulate (without having a theory of matter). In his Nobel Prize lecture he "once again tried to justify his provisional use of rods and clocks to give the geometrical statements of the theory empirical content"; though viewed it later as "a logical shortcoming of the theory of relativity in its present form to be forced to introduce measuring rods and clocks separately instead of being able to construct them as solutions to differential equations". Our calorimeter model illustrates another example in defense for the use of measurement instruments as primitive entity to the theory. One can manufacture standard springs and reference bodies as sufficiently constant representatives of "effect potential" and "impact". These provisory units can be replaces by other reference devices, more suitable for reproducible measurement practice. The protophysical test norm for manufacturing provisory or refined devices remains the same.

We can use the instrument for measuring e.g. gravitational or nuclear processes. We can couple a swarm of local calorimeters into a gravitational system for intrinsic measurements of energy and momentum associated with geodesic deviations (in global configurations). When
in an elementary particle collision new particle generations develop, we can capture a jet with the calorimeter and measure energy and momentum of separate decay products.

We regard three interactions: elementary standard process $w_1$, gravitational interaction $w_{\text{grav}}$ and nuclear interaction $w_{\text{QM}}$. Which is basic and which more complex? This question leads nowhere; even the internal structure of a compressed spring is unknown. We presuppose them solely as completed process (with unknown inner structure). We pick the compression of a standard spring as elementary building block for our calorimeter model because they are congruent and reproducible. Then we can measure the other two actions $w_{\text{grav}}$ and $w_{\text{QM}}$ with reference action $w_1$. This requires man-made tools and procedures. It is the task of the physicist to couple these congruent units. Our basic measurement of energy-momentum boils down to counting them in an organized way.

We construct the calorimeter model \{4,2\} from pre-theoretic building blocks \{3\} which are subject to physical principles (Causality, Inertia, Relativity, Impossibility of Perpetuum Mobile, Superposition) and methodical principles for the constructor (elementary comparison, Congruence, Equipollence) as well as a social condition: Physicists must cooperate to create material models. A team of assistants has to know, when, where and how to couple initially resting reservoir elements \( \odot \) into the deceleration process, to generate absorption. Their cooperation is community-building. Team work is crucial for the conduct of basic measurements. Basic physical quantities are a joint product and not generated individually. In all domains of experiment and measurement, where underlying physical and methodical principles are valid, one is entitled to postulate the associated primary dynamical equations \( (7),(23),(24) \) for basic observables \{2\} as fundamental for the mathematical framework.

We have demonstrated the foundation of physics as an empirical science (in contrast to pure mathematics). This approach is diametrically opposed to an axiomatic ansatz, insofar that our starting point lies in the definitions. From them one can prove the equations of motion and even the conservation laws, while in an axiomatic approach one postulates the latter (or corresponding symmetry laws) and ultimately derives the physical observables - though as a pure operand without empirical meaning. We begin from definitions which have a practical dimension. This operationalization builds on Helmholtz fundamental analysis of measurements along with Congruence principle, Equipollence principle etc. which are also being construed measurement theoretically. In this sense we are dealing with a foundation of Physics, but not with an explanation from mathematically formulated principles, nor from the empirically given - we explain neither from what one can say, nor from what one sees, but from what one does (a form of pragmatism). In this approach, which explains the mathematical formalism from the operationalization of basic quantities, one can address and understand scope and limitations of the formalism, with significance also for other formalisms in physics.

Acknowledgements Thank you to Bruno Hartmann sen. and Peter Ruben for introducing the research problem and essential suggestions. I also want to thank Thomas Thiemann for stimulating discussions and support and Oliver Schlaudt for clarification. This work was made possible initially by the German National Merit Foundation and finally with support
References

[1] Hertz H., Einleitung zur Mechanik, "Zur Grundlegung der theoretischen Physik", Rompe R., Treder H.-J. (Eds.), Berlin, Akademie Verlag, (1984)

[2] Euler L., Anleitung zur Naturlehre, Opera Omnia: III, Opera postuma 2 (1862)

[3] Helmholtz H. v., Über die Erhaltung der Kraft, aus: Philosophische Vorträge und Aufsätze, Hörz H. Wollgast S. (Eds.), Akademie Verlag, Berlin (1971)

[4] Helmholtz H. v., Zählen und Messen, erkenntnistheoretisch betrachtet, aus: Philosophische Vorträge und Aufsätze, Hörz H. Wollgast S. (Eds.), Akademie Verlag, Berlin (1971)

[5] Lorenzen P., Die Entstehung der exakten Wissenschaften, Springer (1960)

[6] Wolff M., Geschichte der Impetustheorie: Untersuchungen zum Ursprung der klassischen Mechanik, Suhrkamp, Frankfurt am Main (1978)

[7] Mach E., Die Mechanik in ihrer Entwicklung - Historisch-kritisch dargestellt, Wahsner R. Borzeszkowski H. H. v. (Eds.), Akademie Verlag, Berlin (1988)

[8] Einstein A., Zur Elektrodynamik bewegter Körper, Ann. Phys. 322: 891921 (1905)

[9] Einstein A., Elementary derivation of the equivalence of mass and energy, Bull. Amer. Math. Soc. Volume 41 (4): 223-230 (1935)

[10] Giovanelli M., But one must not legalize the mentioned sin: Phenomenological vs. dynamical treatments of rods and clocks in Einstein’s thought, Studies in History and Philosophy of Modern Physics 48: 20-44 (2014)

[11] Sommerfeld A., Mechanik, Verlag Harry Deutsch, Thun (1994)

[12] Weyl H., Philosophie der Mathematik und Naturwissenschaft, München Wien (1966)

[13] Feynman R. P., Leighton R. B., Sands M., The Feynman Lectures on Physics - Mainly Mechanics, Radiation and Heat, Addison-Wesley Publishing Company, (1977)

[14] Ruben P., Mechanik und Dialektik - Eine wissenschaftstheoretisch-philosophische Studie zum physikalischen Verhalten, Dissertation, Humboldt-University Berlin (1969)

[15] Janich P., Das Maß der Dinge: Protophysik von Raum, Zeit und Materie, Suhrkamp (1997)
[16] Schlaudt O., Messung als konkrete Handlung - Eine kritische Untersuchung über die Grundlagen der Bildung quantitativer Begriffe in den Naturwissenschaften, Verlag Königshausen & Neumann, Würzburg (2009)

[17] Hecht H., Leibniz‘ Kraftmaß - Veröffentlichtes und Unveröffentlichtes, ”Dialektik Arbeit Gesellschaft”, Festschrift für Peter Ruben, Crome E. Tietz U. (Eds.), Welt Trends, Berlin (2013)

[18] Luce R., Suppes P., Theory of Measurement, Encyclopaedia Britannica, Vol.11, 15th edition, 739-745 (1974)

[19] Hartmann B., Operationalization of Relativistic Motion (Kinematics), [http://arxiv.org/abs/1205.2680](http://arxiv.org/abs/1205.2680) [gr-qc] (2012)

[20] Hartmann B., Physical Determination of the Action, [http://arxiv.org/abs/1307.0499](http://arxiv.org/abs/1307.0499) [physics.hist-ph] (2013)

[21] Hartmann B., Operationalization of Relativistic Energy-Momentum, in preparation