Čerenkov radiation and scalar stars

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We explore the possibility that a charged particle moving in the gravitational field generated by a scalar star could radiate energy via a recently proposed gravitational Čerenkov mechanism. We numerically prove that this is not possible for stable boson stars. We also show that soliton stars could have Čerenkov radiation for particular values of the boson mass, although diluteness of the star grows and actual observational possibility decreases for the more usually discussed boson masses. These conclusions diminish, although do not completely rule out, the observational possibility of actually detecting scalar stars using this mechanism, and lead us to consider other forms, like gravitational lensing.

PACS no.: 04.40.Dg, 04.40.-b

I. INTRODUCTION

One of the first attempts to incorporate a scalar field in the theory of gravity was done by Brans andDicke\textsuperscript{[1]}, who extended Einstein’s theory in order to accommodate Mach’s principle and Dirac’s large number hypothesis. In early universe scenarios, a scalar field (inflaton) could drive inflation through its potential energy\textsuperscript{[2]}, and it could give rise to the formation of topological defects when some fundamental symmetry is broken\textsuperscript{[3]}. Furthermore, the presence of a scalar field (dilaton) naturally appears in the limit of low energy (with respect to the Planck one) of superstring theory, and it is responsible for the so-called super-inflation\textsuperscript{[4]}

All these examples, though in different contexts, strongly suggest the need to introduce a scalar field in cosmological models. Owing to these facts, one could investigate if such scalar fields may be the seed of astrophysical structures or for observable phenomena that could signal their existence. Objects made up of scalar massive particles were introduced by Kaup\textsuperscript{[5]} and Ruffini and Bonazzola\textsuperscript{[6]} as a new type of star model, the so-called boson star. Such objects are macroscopic quantum states that are gravitationally prevented from collapsing (in opposition to black holes) by the Heisenberg uncertainty principle, it keeps scalar particles from being localized to within their Compton wavelength.

In a recent work\textsuperscript{[7]}, Liddle and Schunck studied the gravitational redshift of the radiation emitted within a boson star potential and the rotational curves of accreted particles to assess the possible detectability of these scalar stars. They conclude that very massive boson stars could look very similar to an active galactic nucleus with a black hole at its center. Suggestions in that sense were also given before, by Tkachev\textsuperscript{[8]}. This is indeed supported by the fact that the solution far away from the boson star center mimics the Schwarzschild space-time. Later, moreover, Schunck and Torres\textsuperscript{[9]} explored to what extent these properties are exclusive of the boson star potential used in Ref.\textsuperscript{[7]}, finding that in fact they are even common to more generic Lagrangian densities.

In our own galactic center, while the existence of a single large mass has been favored as the upper bound on its size tighten and stability criteria rules out complex clusters (see for instance Ref.\textsuperscript{[12]}), it is not established that this central mass has to be a black hole. If Sgr\textsuperscript{A*} (the super-massive compact object name) is a black hole, its luminosity should be three order of magnitude bigger than what actually is. This discrepancy is called “the blackness problem”, and led to the concept of a black hole on starvation. However, observational data comes from regions \(4 \times 10^4\) Schwarzschild radius away from a black hole of mass \(2.6 \times 10^6\)\(M_\odot\) -the inferred mass of the central object-: proofs of the existence of a super-massive black hole in the center of the galaxy are not conclusive by now.

We have constructed a model of a boson star galactic center and studied some of its properties elsewhere\textsuperscript{[10]}.

With the aim to search for some new physical effect able to reveal the presence of scalar stars in the universe, other than the already considered gravitational lensing\textsuperscript{[11]}, we analyze the possibility that a charged particle, propagating in the gravitational field generated by such a compact object, could emit radiation via Čerenkov process.

A. Čerenkov gravitational radiation

Čerenkov radiation occurs when a fast particle moves through a medium at a constant velocity \(v\), which is
greater than the velocity of light in that medium. Because of the superluminal motion of the particle, a shock wave is created and this yields to a loss of energy. The wavefront of the radiation propagates at a fixed angle

$$\cos \theta = \frac{v_{\text{phase}}}{v} = \frac{c/n(\nu)}{v}$$

(1)

where $\nu$ is the photon frequency and $n$ is the refractive index. Only in this direction do the wavefronts add up coherently. The value $\cos \theta = 1$ corresponds to the threshold for emission. It is clear that $\cos \theta < 1$ cannot be satisfied (and then will be no radiation) if $n < 1$ (because $v$ always is less than $c$). See, for example, Refs. [13][14].

As is well known, charged particles with acceleration $a$ emit electromagnetic radiation according to the Larmour’s relation $dE/dt \sim a^2$. However, acceleration due only to a gravitational field does not produce any electromagnetic radiation, in agreement with the equivalence principle [13]. This follows because the acceleration induced by gravity disappears in the local inertial frame, so that the particle moves in that frame with constant velocity, without radiating. Nevertheless, a particle moving with constant velocity through an external medium (even a gravitational field) can emit Čerenkov radiation. This possibility, i.e. that an external gravitational field acts as an effective refractive index for light, has been recently analyzed by Gupta, Mohanty and Samal [19]. They showed that the background gravitational field has an effective refractive index given by

$$n_\gamma^2(k_0) = |\eta^{00}| \left( 1 - \frac{R_i^1}{|\eta^{00}|\kappa_0^2} \right),$$

(2)

where $R_i^1$ is understood as the sum on the spatial indices of the Ricci tensor $R^\mu\nu$, i.e. $R_i^1 = \sum_{i=1}^3 R_i^1$, $k_0$ is the frequency of the emitted photon $\gamma$, and $\eta^{00}$ is the 00-component of the metric tensor in the inertial frame, $\eta_{\mu\nu} = (-1,1,1,1)$. The crucial point, in order that the Čerenkov radiation be kinematically allowed, is that $R_i^1 < 0$, so that $n_\gamma^2(k_0) > 1$.

In the paper [19], the scattering process $f(p) \rightarrow f(p') + \gamma(k)$, responsible for the Čerenkov radiation, is analyzed in the local inertial frame of the incoming fermion $f(p)$ with momentum $p$, while $f(p')$ and $\gamma(k)$ are the outgoing fermion with momentum $p'$ and the emitted photon with momentum $k$. As one can immediately realize, Čerenkov emission is not vanishing in the inertial frame, unlike that produced by curvature radiation, since the refractive index turns out to be proportional to the spatial parts of the Ricci tensor.

The energy radiated by Čerenkov process by a charged particle moving in a background gravitational field is given by (for details, see [14])

$$\frac{dE}{dt} = \frac{Q^2\alpha_{\text{em}}}{4\pi g_0^2} \int_{k_{01}}^{k_{02}} \! dk_0 \left[ (p_0 - k_0) - \frac{1}{2} k_0^2 \right] n_\gamma^2 - 1, \quad n_\gamma^2 = \frac{1}{2} \left( \frac{p_0}{k_0} - \alpha_{\text{em}} \right),$$

(3)

where $Q$ is the charge of the fermion emitting the photon, $\alpha_{\text{em}}$ is the electromagnetic coupling constant, $p_0$ is the energy of the fermion, and $k_{01}, k_{02}$ stands for the allowed range of frequencies where radiation can occur.

The interesting result is obtained for $n_\gamma^2 \gg 1$, since the spectrum of energy radiated by a charged particle, Eq. (4), assumes the form

$$\frac{d}{dk_0} \left( \frac{dE}{dt} \right) = \frac{Q^2\alpha_{\text{em}}}{4\pi} \left[ 1 - \frac{k_0}{p_0} - \frac{k_0^2}{2p_0^2} \right],$$

(4)

which differs in a substantial way from thermal or synchrotron emission. We show this spectrum for a monochromatic proton energy $p_0 = 10$ GeV in Fig. 1. If this result holds for boson stars, it strongly suggest the possibility to reveal their presence via their output in Čerenkov radiation, in addition to gravitational lensing effects [1]. Note that in most astrophysical situations, the dense media that could give rise to Čerenkov radiation are also optically thick to the emitted radiation, making it not easily observed. However, a transparent - uncharged- boson star offer a better environment for such a detection since absorption within the star will be inexistent.

However, our main result is proof of just the opposite: no Čerenkov radiation can be produced by particles traversing a boson star. In Sect. II we shortly review the theory underlying boson and soliton stars, recalling the main ideas and equations for evaluating the refractive index, which will be the argument of Sect. III. Conclusions are drawn in Sect. IV.

II. BOSON STARS PHYSICS–BRIEF REVIEW

We shall now introduce the formalism which gives rise to mini-boson, boson, and soliton stars. To do this we study the Lagrangian density of a massive complex self-gravitating scalar field, which is (taking $\hbar = c = 1$)

$$\mathcal{L} = \frac{1}{2} \sqrt{-g} \left[ m_{\psi}^2 R + \partial_{\mu} \psi^* \partial^{\mu} \psi - U(|\psi|^2) \right],$$

(5)
where $R$ is the scalar of curvature, $g$ the determinant of the metric $g_{\mu \nu}$, and $\psi$ is a complex scalar field with potential $U$. Using this Lagrangian as the generator of the matter sector of the theory, we get the standard field equations

$$R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = -\frac{8\pi}{m_{\text{Pl}}^2} T_{\mu \nu}(\psi),$$  

(6)

$$\Box \psi + \frac{dU}{d|\psi|^2} \psi = 0,$$

(7)

where the stress energy tensor is given by,

$$T_{\mu \nu} = (\partial_\mu \psi^* ) (\partial_\nu \psi ) - \frac{1}{2} g_{\mu \nu} \left[ g^{\alpha \beta} (\partial_\alpha \psi^*) (\partial_\beta \psi) - U(|\psi|^2) \right],$$

(8)

and $\Box = \partial_\mu \left[ \sqrt{|g|} \ g^{\mu \nu} \partial_\nu \right] / \sqrt{|g|}$ is the covariant d’Alembertian. Because of the fact that the potential is a function of the square of the modulus of the field, we obtain a global $U(1)$ symmetry. This symmetry is related with the conserved number of particles. The particular form of the potential, however, is what makes the difference between mini-boson, boson, and soliton stars. Conventionally, when the potential is given by

$$U(|\psi|^2) = m^2 |\psi|^2 + \frac{\lambda}{2} |\psi|^4,$$

(9)

where $m$ is the scalar mass and $\lambda$ a dimensionless constant measuring the self-interaction strength, mini-boson stars are those spherically symmetric equilibrium configurations with $\lambda = 0$. Boson stars, on the contrary, have a non-null value of $\lambda$. The previous potential with $\lambda \neq 0$ was introduced by Colpi et al. [20], who numerically found that the masses and radius of the configurations were deeply enlarged in comparison to the mini-boson case, even in the case of extremely small $\lambda$.

Soliton (also called non-topological soliton) stars are different in the sense that, apart from the requirement that the Lagrangian must be invariant under a global $U(1)$ transformation, it is required that –in the absence of gravity– the theory must have non-topological solutions; i.e. solutions with a finite mass, confined to a finite region of space, and non-dispersive. An example of these kind of potentials is the one introduced by Lee and his coworkers in a serie of 1987 papers [21].

We shall now briefly explain how these boson configurations can be obtained (see Refs. [22] for details). We adopt a spherically symmetric line element

$$ds^2 = e^{\nu(r)} dt^2 - e^{\mu(r)} dr^2 - r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2),$$

(10)

with a scalar field time dependence ansatz consistent with this metric:

$$\psi(r, t) = \sigma(r) e^{-i\omega t},$$

(11)

where $\omega$ is the (eigen-)frequency. This form of the field -when the scalar has no nodes- ensures us to be working in the configurations of minimal energy (see appendix of Ref. [21]).

The non-vanishing components of the energy-momentum tensor are

$$T_{0}^{0} = \rho = \frac{1}{2} [\omega^2 \sigma^2 (r)e^{-\nu} + \sigma'^2 (r)e^{-\mu} + U],$$

(12)

$$T_{1}^{1} = p_r = \frac{1}{2} [\omega^2 \sigma^2 (r)e^{-\nu} + \sigma'^2 (r)e^{-\mu} - U],$$

(13)

$$T_{2}^{2} = T_{3}^{3} = p_\perp = \frac{1}{2} [\omega^2 \sigma^2 (r)e^{-\nu} - \sigma'^2 (r)e^{-\mu} - U],$$

(14)

where $' = d/dr$. One interesting characteristic of this system is that the pressure is anisotropic; thus, there are two equations of state $p_r = \rho - U$ and $p_\perp = \rho - U - \sigma'^2 (r)e^{-\mu}$. The non-vanishing independent components of the Einstein equation are

$$\nu' + \mu' = \frac{8\pi}{m_{\text{Pl}}^2} (\rho + p_r) e^\mu,$$

(15)

$$\mu' = \frac{8\pi}{m_{\text{Pl}}^2} \rho e^\mu - \frac{1}{r} (e^\mu - 1).$$

(16)

Finally, the scalar field equation is

$$\sigma'' + \left( \frac{\nu' - \mu'}{2} + \frac{2}{r} \right) \sigma' + e^{\mu - \nu} \omega^2 \sigma - e^\mu \frac{dU}{d\sigma^2} \sigma = 0.$$  

(17)

To do numerical computations and order of magnitude estimations, it is useful to have a new set of dimensionless variables. The usual ones are: $x = mr$ for the radial distance, $\Omega = \omega/m$ for the eigenvalue, we redefine the radial part of the boson field as $\sigma = \sqrt{4\pi} \sigma/m_{\text{Pl}}$, and introduce $\Lambda = \lambda m_{\text{Pl}}^2/4\pi m^2$. In order to obtain solutions which are regular at the origin, we must impose the following boundary conditions $\sigma'(0) = 0$ and $\mu(0) = 0$. These solutions have two fundamental parameters: the self-interaction and the central density (represented by the value of the scalar field at the centre of the star). The mass of the scalar field fixes the scale of the problem. Boundary conditions representing asymptotic flatness must be applied upon the metric potentials, these determine -which is actually accomplished via a numerical shooting method- the initial value of $\nu = \nu(0)$. Then, having defined the value of the self interaction, or alternatively, the form of the soliton potential, the equilibrium configurations are parameterized by the central value of the boson field. As this central value increases, so does the mass and radius of the the star. This happens until a maximum value is reached in which the star looses its stability and disperses away (the binding energy being positive). Up to this value of $\sigma(0)$, catastrophe theory can be used to show that these equilibrium configurations are stable [23]. The numerical code used in this paper is a modification of the scalar-tensor boson star program developed in Ref. [24].
III. ČERENKOV RADIATION AND SCALAR STARS

In this Section we calculate if Čerenkov radiation (in the sense of Gupta et al.’s paper) can be emitted by a charged particle which feels the gravitational field generated by a boson star. As already explained in the Introduction, in order to allow for the emission of radiation by Čerenkov process, the sum on the spatial components of the Ricci tensor has to be negative. To calculate it, let us rewrite the Einstein equation (11) in the following form

\[ R_{\mu\nu} = -\frac{8\pi}{m_{Pl}^2} \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right), \]  

(18)

where \( T \) is the trace of the stress–energy tensor \( [3] \). The scalar curvature is then

\[ R = \frac{8\pi}{m_{Pl}^2} T = \frac{8\pi}{m_{Pl}^2} \left| -|\partial_\mu \psi|^2 + 2U(|\psi|^2) \right| \]  

(19)

where the potential \( U \) is defined in \( [3] \). From (18) one can calculate the component \( R_{00} \), getting

\[ R_{00} = \frac{8\pi}{m_{Pl}^2} \left| -|\partial_0 \psi|^2 + \frac{1}{2} U(|\psi|^2) \right|, \]  

(20)

so that the sum of the spatial components of the Ricci tensor are given by

\[ R^i = R - R_{00} = \frac{8\pi}{m_{Pl}^2} \left| -|\partial_0 \psi|^2 + \frac{3}{2} U(|\psi|^2) \right|. \]  

(21)

Inserting Eq. (21) into (2), one infers the effective refractive index

\[ n_2^2(k_0) = |\eta|\left(1 - \frac{8\pi}{|\eta|\eta_0 m_{Pl}^2 k_0^5} (-|\partial_0 \psi|^2 + \frac{3}{2} U(|\psi|^2))\right). \]  

(22)

If \( R^i \) in Eq. (21) is negative, then the emission of Čerenkov radiation is a kinematically allowed process. The condition \( R^i < 0 \) (so that \( n_2^2 > 1 \)) can be satisfied if one invokes the condition that the spatial variation of the scalar field \( \psi \) is greater than three half its potential energy, i.e.

\[ |\partial_\mu \psi|^2 > \frac{3}{2} U(|\psi|^2). \]  

(23)

A. Boson stars: Colpi et al. potential

In order to evaluate the order of magnitude of this requirement for boson stars let us turn into dimensionless variables the expression:

\[ \frac{8\pi}{m_{Pl}^2 k_0^5} \left| -|\partial_\mu \psi|^2 + \frac{3}{2} U(|\psi|^2) \right|. \]  

(24)

Using the adopted ansatz for the field and the previously quoted dimensionless variables we obtain:

\[ \frac{2 m_0^2}{k_0^5} f(x) = \frac{2 m_0^2}{k_0^5} \left[ -\left( \frac{d\sigma}{dx} \right)^2 + \frac{3}{2} \left( \sigma \right)^2 + \frac{\Lambda}{2} \sigma \right]. \]  

(25)

Thus, \( n_2^2 = 1 - (m^2/2k_0^5)f(x) \). If, and when, \( f(x) \) is negative, Čerenkov radiation might happen. However, in Fig. 2 we show several numerical integrations for different boson stars models, i.e. with different central densities and self-interaction. It is clear that the square of the derivative of \( \sigma \) is never bigger than the terms involving the square and the fourth power of \( \sigma \) itself, and then that there is no possible radiation.

Starting form the dimensionless expression of \( f(x) \), and as in principle \( \Lambda \) could assume any value, even negative ones, we can think of doing the following trick. Let us take those shells very near the center of the star. There, because of the boundary condition, \( \sigma \sim 0 \), and \( \sigma \) itself adopts its maximum value. Then, if \( \Lambda < 0 \), \( f(x) \) will be negative for all values which fulfill \( |\Lambda| > 2/\sigma(0)^2 \). But interestingly enough, this inequality can not be accomplished by stable stars. For instance, if \( \sigma(0) = 0.19 \), \( |\Lambda| > 55 \). But as can be confirmed by Table 1 of Ref. [6], for values of \( \Lambda < -20 \), central densities above \( \sigma(0) = 0.067 \) already represent unstable solutions. Because this happens for all values of \( \sigma(0) \), this is why no physical (stable) boson star, under Colpi et al.’s usual potential can generate Čerenkov radiation.

B. Soliton stars

It is clear that the requirement expressed by Eq. (23) depends on the form of the potential. As an example of this we study here the Lee et al.’s soliton star potential given by

\[ U = m^2 |\psi|^2 \left(1 - \frac{|\psi|^2}{\chi_0^2}\right)^2, \]  

(26)

where \( \chi_0 \) is a constant. As we already mentioned, compared with the usual boson star case, non-topological soliton stars have to fulfill two characteristics: 1. The Lagrangian must be invariant under a global \( U(1) \) transformation. 2. In the absence of gravity, the theory must have non-topological solutions. In general, boson stars accomplish the requirement 1, but not 2. Invariance under \( U(1) \) only requires that the potential be a function of \( \psi^* \psi \), but in order to accomplish condition 2, \( U \) must contain attractive terms. This is why the coefficient of \( (\psi^* \psi)^2 \) of Lee’s potential has a negative sign.
Finally, when $|\psi| \to \infty$, $U$ must be positive, which leads, minimally, to a sixth order function of $\psi$ for the self-interaction.

As we did above, we shall obtain the dimensionless form for expression (24), but in this case for the soliton potential. It is given by (without multiplying factors)

$$g(x) = -\left(\frac{d\sigma}{dx}\right)^2 + 3 \left(\sigma(x)^2 - 2\alpha^2\sigma(x)^4 + \alpha^4\sigma\right), \quad (27)$$

where $\alpha = m_k^2/4\pi\chi_0^2$. Note that the parameter $\alpha$ does not directly depend on the mass of the boson, but just on the particular value of the $\chi_0$ parameter. It is usually assumed, however, that this parameter is of the order of the boson mass.

We can now think of the following situation: as $\sigma$ depends on $x$, for values of $\sigma = 1/\alpha$, the second term is exactly zero, and then $g(x)$ will be negative if the derivative of the field is not null. This can be easily obtained starting from a central value of $\sigma$ slightly higher than $1/\alpha$ (so as to pass through $\sigma = 1/\alpha$ well inside the structure of the star, where the derivative is not zero). In Fig. 3 we show different models fulfilling these constraints. We may see that the higher the value of $\alpha$, the more dilute the center of the star is. It is also possible to see that for bigger values of $\alpha$, $|g(x)|$ takes even smaller values. Note too that a particular choice for the parameters (basically, for the central density) determine the spherical shell in which $g(x)$ is negative, and thus where Čerenkov radiation might occur. For instance, if $\alpha$ is very big, but $\sigma(0)$ is also very large, we could still find Čerenkov radiation in the outer “crust” of the star.

This shows, in principle, interesting observable difference between boson and non-topological stars, which is caused by the form of the potential. The actual prospects for really observing this are to be determined particularly by the value of the boson mass involved. If $m$ is about 30 GeV, and $\chi_0$ is of the order of the boson mass, the parameter $\alpha$ is quite large, and the observational possibility diminishes, since Čerenkov radiation would happen only for extremely dilute stars (the concept of star itself looses sense in this situation). We have checked, however, that the negativeness of $g(x)$ is numerically preserved up to values of $\alpha = 10^4$. But is so small as $10^{-10}$. Observing radiation generated in these conditions would be quite a different story. However, we may note that there could be a sort of fine tuning in the values of $\chi_0$ and $m$ for which $n^2 = 1 - (m^2/2k_0)g(x)$ be large. If this fine tuning has any physical motivation should be decided on particle physics grounds. Additionally, we mention that for values of $m_{pl}/m \sim \alpha^{17}$, it is needed a different numerical technique: basically we need to solve the equations in three separate parts, making adequate expansions (see the papers by Lee et al.’s for details [21]). In those cases, the mass and radius of the stars are enlarged up to galaxy and galaxy cluster masses within some light years of linear spread. We have not taken these cases here into account.

**IV. FINAL REMARKS**

In this paper we have analyzed the possibility that a charged particle, moving in a gravitational field generated by a scalar star, could emit radiation through the gravitational Čerenkov process recently introduced by Gupta et al. in Ref. [10]. We have numerically shown that the usual boson star model, based on the potential introduced in this context by Colpi et al., is not able to generate a refractive index bigger than 1, and thus, that Čerenkov radiation can not proceed. We have also shown that, on the contrary, soliton stars could have Čerenkov radiation for particular values of the boson mass. However, diluteness of the star grows and actual observational possibility decreases for the more usually assumed boson masses and $\chi_0$ parameters. For these cases, $(n^2 - 1) \sim 0$ and Čerenkov process might be considered as experimentally insignificant within current observational constraints, on a par with what Gupta et al. have concluded for the galactic gravitational field [13]. We may think of a possibility to overcome the smallness of $(n^2 - 1)$ if a soliton star is alligned with a strong proton source. A more, maybe unexpected form, would be to find soliton stars with the right combination of $\chi_0$ and $m$, as to have a large effective refractive index. Overall, it appears that other ways -apart from the already mentioned gravitational lensing effects [11]- have to be devised in order to look for scalar stars.

**Acknowledgments**

We thank Drs. L. A. Anchordoqui, F. E. Schunck, S. Mohanty, as well as two anonymous referers, for criticism and valuable suggestions. S. C. and G. L.’s research was supported by MURST fund (40%) and art. 65 D.P.R. 382/80 (60%). G.L. further thanks UE (P.O.M. 1994/1999). D.F.T. was supported by CONICET as well as by funds provided by Fundación Antorchas, and acknowledges the hospitality provided by the ICTP (Italy) during the latest stages of this work.

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FIG. 1. Typical spectrum for the radiation emitted by monochromatic charged particles with $|p| = 10$ GeV in a gravitational field with effective refractive index bigger than 1, as was first obtained by Gupta et al. in Ref. [19].

FIG. 2. Numerical integration of the boson star structure equations for different central densities and self-interaction. All of them represent stable configurations. As $f(x)$ is always positive, Čerenkov radiation can not be present.
FIG. 3. Numerical integration of soliton star structure equations for different central densities and self-interaction parameters. We show here the function \( g(x) \) involved in the computation of the refractive index: where \( g(x) \) is negative, Čerenkov radiation might occur. The difference in the form of the function between the middle and the bottom and top plots occurs when the departure of \( \sigma(0) \) from the critical value \( 1/\alpha \) is bigger.