Atomic diffraction by light gratings with very short wavelengths

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Abstract

Lasers with wavelengths of the order of the atomic size are becoming available. We explore the behavior of light-matter interactions in this emergent field by considering the atomic Kapitza-Dirac effect. We derive the diffraction patterns, which are in principle experimentally testable. From a fundamental point of view, our proposal provides an example of system where the periodicity of the diffraction grating is comparable to the size of the diffracted object.

1 Introduction

Lasers in the X-ray domain, with wavelengths of the order of the atomic size, have been reported in the literature [1, 2, 3]. We must study the underlying physics in that unexplored range of ultrashort wavelengths. In particular, we must analyze the behavior of the light-matter interaction. In order to carry out the analysis in a simple way, we shall consider a well-known scheme, the Kapitza-Dirac effect, which can be described with simple mathematical tools. In the Kapitza-Dirac effect a beam of atoms or electrons is diffracted or scattered by a standing light wave [4, 5, 6, 7, 8]. The effect can be extended to two-particle systems [9, 10].

First of all, we must derive the form of the light-atom interaction in this regime. We shall show that the dynamics is ruled by a lightshift potential in the dipole approximation. Being the light wavelength comparable to the atomic size, different parts of the atom feel different values of the field and the dipole approximation provides an incomplete description of the problem. We must consider higher multipole terms, in particular the quadrupole one. However, because of the high frequency of the light field we must average the interaction on time. The average of the higher permanent multipole terms is zero, reducing the total interaction to the usual lightshift potential in the dipole approximation. We shall also consider the quadrupole induced by the
electric field. The evaluation of its actual contribution to the problem is difficult because of the lack of reliable experimental data on quadrupole or higher order polarizabilities. Nevertheless we shall show that if these effects were of the same order of the lightshift potential they would not modify the fundamental results of our paper.

In the standard atomic Kapitza-Dirac arrangement the wavelength of the optical grating is close to an atomic transition, enhancing the strength of the interaction. The very short wavelengths we consider in this paper are fully detuned from the atomic transitions, making much more weak the light-atom interaction, which is only related to the atomic polarizability. However, considering a laser intensity of the order of $10^{14} \text{W/m}^2$ (the values used in the observation of the effect with electrons [7]) the strength of the light-atom interaction is comparable to that in the on resonance case and we can, in principle, observe the diffraction effects. We shall evaluate the diffraction patterns, obtaining the same form of the standard atomic Kapitza-Dirac arrangement (containing only even order peaks).

The high frequency values of the light can give rise, depending on the duration of the interaction, to the presence of large ionization rates which would make more difficult the observation of the diffraction effects. In these cases one must introduce some procedure to remove the ionized atoms from the experiment.

In addition to provide an example of light-matter interaction in the scale of very short wavelengths, our proposal is also interesting from a fundamental point of view. There has been an increasing interest in the study of quantum diffraction with large size objects, for instance, with $C_{60}$ [11] and $C_{70}$ [12] molecules in solid nanostructures or structures made of light [13], and $Na_2$ molecules whose de Broglie wavelength is smaller than their size [14]. However, there is yet a fundamental question that has not been addressed, the limiting size (relative to the spacing of the diffraction grating) of a quantum object to observe diffraction. Our proposal can be the basis to study quantum diffraction in the extreme regime where the periodicity of the diffraction optical grating is similar to the size of the diffracted system.

The plan of the paper is as follows. In Sect. 2 we present the arrangement and evaluate the lightshift potential relevant for the problem. In order to give a compact presentation of the main ideas involved in the evaluation, some more technical aspects of the treatment are discussed in Appendix 1. In Sect. 3 we derive the diffraction patterns and consider how to eliminate the ionized atoms. The values of the parameters involved in a realistic experimental implementation of the arrangement are estimated in Sect. 4. In the Discussion, we consider the potential impact of our proposal for other physical problems, in particular, the exploration of the limits of quantum diffraction with large size objects. Finally, the possible effects associated with a large quadrupole polarizability are presented in Appendix 2.
2 The arrangement

As usual in the Kapitza-Dirac effect our arrangement consists of a standing light wave generated by a laser. A beam of atoms interacts with that diffraction grating. Behind the grating we place detectors. From the data collected at the detectors we can infer the diffraction patterns (see Fig. 1).

Next, we consider the mathematical description of the interaction. First of all, we note that we shall deal with high laser intensities. Then we can resort to the semiclassical approximation, where the electromagnetic fields can be treated classically. In Appendix 1 we estimate the intensity values for which we can safely use this approximation in the framework of the Kapitza-Dirac effect.

![Figure 1: The wavelength of the laser (green) is similar to the size of the atom (blue). After the interaction we observe a diffraction pattern.](image)

The adiabatic condition plays an important role in this type of problem. When it is fulfilled, the time dependence of the effective optical potential experienced by the atoms is slow compared to the internal evolution. The center of mass (CM) dynamics can be decoupled from the internal one, and it can be described by an optical potential, generating a phase shift as a function of position [15]. As we shall see later (Sect. 3), the adiabatic condition is by far fulfilled in our case.

Once guaranteed adiabatic evolution, we can concentrate on the CM dynamics. The phase shift of the CM wavefunction is given by the dipole-type interaction between the electric field $E$ and the induced atomic dipole $d = \alpha E$, with $\alpha$ the polarizability. We describe this interaction by the lightshift potential $[5, 13]$

$$U_{LS}(R, t) = -\frac{1}{2} \alpha E^2(R, t)$$  \hspace{1cm} (1)

where $R$ denotes the CM coordinate of the atom. Being the CM timescale very different from the laser period, we must average over that period. If we take for the electric field of the standing wave the form

$$E(R, t) = E_0 \cos(k_L \cdot R) \cos(\omega_L t)$$  \hspace{1cm} (2)
with $k_L$ and $\omega_L$ the wavevector and frequency of the laser, the averaged lightshift potential takes the form

$$U(R) = \frac{1}{T_L} \int_0^{T_L} U_{LS} dt = -\frac{1}{4} \alpha E_0^2 \cos^2(k_L \cdot R)$$  \hspace{1cm} (3)$$

with $T_L = 2\pi/\omega_L$.

In the framework of the dipole approximation there is yet another interaction channel (see Appendix 1). In addition to the transitions between bound states that induce the atomic dipole, there can be also transitions to the continuum. As a matter of fact, due to the high energy of the involved photons we expect that absorption will lead in most cases to ionization, and the ionization rate in our problem can be large. This is not the case for the values of the parameters we shall propose to observe the effect (see Sect. 3). However, for other values of the parameters the ionization rate could be large. In these cases, as we are only interested into the diffraction properties of the non-ionized atoms, we must consider an scheme to eliminate the ionized atoms. In the next section we present such an scheme. Thus, due to the negligible value of the ionization rate in some cases and to the possibility of eliminate the ions in the rest of cases, we do not need to consider this interaction channel.

Up to now we have restricted our considerations to the leading dipole approximation. However, as remarked before, because the optical wavelength is comparable to the atomic size, the dipole approximation (which assumes no relevant variations of the field in distances of the order of the atomic size) does not provide a complete description of the problem. There are in the literature several examples of how to go beyond this approximation in the context of X-ray theory. For instance, quadrupole terms have been used to study X-ray spectroscopy [16], numerical non-dipole simulations of ionization by X-ray lasers have been presented in [17], and the Bloch equations without the dipole approximation have been derived in [18]. In this paper we follow the approach of considering higher multipole terms. Let us consider the next term in the perturbative series, the permanent quadrupole term, which has the form (see Appendix 1)

$$U_Q(R, t) = \frac{1}{2} Q_{ij} \frac{\partial E_i}{\partial R_j}$$  \hspace{1cm} (4)$$

Using the form of the electric field we have that $U_Q \sim \sin(k_L \cdot R) \cos(\omega_L t)$. Now, when performing the time average we have that the quadrupole interaction goes to zero. Thus, although there is a quadrupole interaction associated with the field variations along the atom size, its net effect vanishes. As discussed in Appendix 1 a similar conclusion holds for any higher order term. Thus, all the averaged multipole permanent contributions vanish and the light-atom interaction can be described via the dipole approximation. In Appendix 2 we shall discuss how induced quadrupole multipoles could be present in the problem.
In conclusion, the analysis of this section shows that the light-atom interaction can be expressed in the form

\[ U(R) = U_0 \cos^2(k_L \cdot R) \]  

(5)

with \( U_0 = -\alpha E_0^2 / 4 \). This is the usual lightshift potential used in the standard on resonance atomic Kapitza-Dirac effect.

### 3 Predicted diffraction pattern

We have shown in the previous section the existence of two interaction channels in our arrangement. On the one hand, the large energy of the photons in the range of frequencies considered can lead, depending on the duration of the interaction, to a high ionization rate. On the other hand, the atoms that are not ionized generate a diffraction pattern. We are only interested into the observation of these patterns. Then in the cases where the ionization is large we must remove the ionized atoms (and the electrons) in order to postselect the neutral ones.

One can easily design methods able to extract the ionized atoms and electrons from the experiment. For instance, an electric field perpendicular to the plane where the experiment takes place (perpendicular to the longitudinal and transversal directions) would deviate most electrons and ions from the detectors, whereas the evolution of the neutral atoms would be almost unaffected.

After the removal of ions and electrons we can focus on the dynamics of the neutral atoms interacting with the lightshift potential. The state of the CM at time \( t \) of the atoms with initial wavevector \( k_0 \) (at time \( t = 0 \)) can be expressed as \( e^{iU(X)t/\hbar} e^{ik_0X} \) in the Raman-Nath approximation. In this approximation we assume that the momentum of the atoms is large compared to that of the photons. Then the kinetic energy remains approximately constant and may be neglected. As it is well-known, this approximation is valid in the diffraction regime [5]. The coordinate \( X \) is that of the CM in the direction parallel to the grating. As usual in the Kapitza-Dirac effect we only consider the one-dimensional problem [5].

Using the relation \( \exp(i\xi \cos \varphi) = \sum_n i^n J_n(\xi) \exp(in\varphi) \), with \( J_n \) the n-th order Bessel function, we have

\[ e^{iU(X)\tau/\hbar} e^{ik_0X} = e^{iU_0\tau/2\hbar} \sum_{n=-\infty}^{\infty} i^n J_n \left( \frac{U_0 \tau}{2\hbar} \right) e^{i(2nk_L+k_0)X} \]  

(6)

with \( \tau \) the interaction time.

This pattern shows the standard form in Kapitza-Dirac diffraction (that with on resonance light-atom interaction). Only even diffraction orders are present. The intensity of the peaks is given by \( J_n(U_0\tau/2\hbar)^2 \).
4 Experimental parameters

We analyze in this section the values of the parameters of the problem that would lead to observable effects in an experimental realization of the above arrangement. We shall use a wavelength \( \lambda_L \approx 5 \times 10^{-10} \text{m} \). This wavelength differs in several orders of magnitude of those associated with atomic transitions, giving rise to a large detuning, which guarantees the adiabatic evolution in the problem.

First of all, in order to be in the diffraction regime the coefficient \( U/\epsilon \), with \( \epsilon = \hbar^2k^2_L/2m \) the recoil shift of the atom by absorption of a photon, must be larger than unity [5]. Taking the mass of the atom as ten to twenty times the proton mass, we have \( \epsilon \approx 10^{-4} \text{eV} \), and we must take \( U \approx 10^{-3} \text{eV} \).

From this value of the potential we can deduce the intensity of the laser beam. We use the approximate relation \( U \approx \alpha E^2_0 \) and the definition of intensity \( I = cE^2_0/8\pi \). The atomic polarizability values are in the range of \( 10^{-29} \text{m}^3 \) to \( 10^{-31} \text{m}^3 \) [20]. Taking an atom in the high part of the range, \( \alpha \approx 10^{-29} \text{m}^3 \), we would need a laser intensity of the order of \( 10^{14} \text{W/m}^2 \). This can seem a very high value when compared with the intensities in the standard atomic Kapitza-Dirac effect (\( I \approx 10^7 \text{W/m}^2 \)). However, a value of \( 5 \times 10^{14} \text{W/m}^2 \) has been used to demonstrate the effect with electrons [7]. The proposed value of \( \lambda_L = 5 \times 10^{-10} \text{m} \) has already been reached with beam intensities as high as \( 10^{17} \text{W/m}^2 \) [1, 2]. Note also that the proposed value, although very high, is yet a long way from the threshold of non-linear polarizabilities, around \( I = 10^{18} \text{W/m}^2 \).

The condition of high visibility of the interference pattern, \( U\tau/\hbar \approx 1 \) [5], implies a time of interaction of around \( 10^{-12} \text{s} \). As the duration of the pulses in [1, 2] is between \( 10^{-13} \) and \( 10^{-14} \text{s} \), a possibility to generate the standing wave is using counter-propagating pulses slightly enlarged on time (for instance, via dispersion of the ultrashort pulses). Another possibility, which does not modify the pulse duration, is to increase the velocity of the atoms. The smallest radius of the focused spots for this range of wavelengths are around 1\( \mu \text{m} \) [1]. Then \( \tau \approx 10^{-12} \text{s} \) requires atom velocities close to \( 10^6 \text{ms}^{-1} \), that is, an increase of three orders of magnitude with respect to the usual values. These velocities could be reached accelerating ions, which later would interact with free electrons generating neutral atoms (the remaining ions should be extracted from the beam by interaction with an electric field). In this second scheme one cannot use counter-propagating laser beams, which are shorter than \( \tau \), and should generate the standing wave with mirrors or other techniques.

We must also determine the fraction of atoms that are ionized during the interaction. If a photon is absorbed the probability of ionization is large in our arrangement. For \( \lambda_L \) the energy of one photon is \( E_{\text{ph}} = \hbar\omega_L \approx 3 \times 10^2 \text{eV} \). This energy is much larger than the ionization energy, giving rise to a large probability of ionization in the case of photon absorption. We estimate the number of non-ionized atoms, \( N \), in the usual way: \( dN = -\Gamma N dt \) with \( \Gamma \) the ionization rate, which is assumed to be time-independent. This gives \( N(\tau) = N_0 \exp(-\Gamma\tau) \) with...
$N_0$ the initial number of atoms in the beam. On the other hand, for the range of
intensities in our problem the ionization rate can be expressed as $\Gamma = \sigma I/\bar{h}\omega_L$
with $\sigma$ the photoionization cross section, which depends on $\omega_L$. Values of $\sigma$
can be found in several database. For instance for the Na atom and $\bar{h}\omega_L = 100$ eV we
have $\sigma = 5 \times 10^{-22} m^2$, which gives $\Gamma \tau \approx 10^{-3}$. Due to the very short duration
of the interaction the fraction of ionized atoms is negligible. For the values of
the parameters here proposed it is not necessary to include the procedure to
remove the ionized atoms.

5 Discussion

Some coherent light sources are reaching the scale where the optical wavelengths
are comparable to the atomic sizes. Our work is one of the first studies consid-
ering possible physical effects present in this unexplored regime. Contrarily to
a naive intuition, the effective interaction does not depend on multipole terms
beyond the dipole one, and the evolution can be described via the usual light-
shift potential. We have shown the existence of diffraction effects similar to
those present when the light is on resonance with atomic transitions. The main
obstacle to carry out the experiment is the short duration of the laser pulse. For
other possible experiments with much longer durations (for instance, Bragg’s
scattering with very short wavelengths) the ionization rates can be high and a
procedure to eliminate the ionized atoms should be added to the arrangement.

In addition to the interest of our proposal in atom optics, it could also
be relevant to other fields. We shall focus on three of them. The first one
concerns to fundamental physics, in particular, to the understanding of the wave
properties of quantum systems. Our scheme provides the first example where
the periodicity of the classical diffraction grating is similar to the size of the
quantum diffracted object. This is an extreme scenario for quantum diffraction
that extends the research presented in [11, 12, 13, 14]. Our analysis shows that
in the limit of objects of the size of the grating periodicity, diffraction survives.

Also, from a fundamental point of view, our proposal serves to test the
theory of light-matter interactions in the scale of very short wavelengths. This
knowledge would be necessary to study, for instance, the dispersion of this type
of light by atoms. It should be also in the basis of techniques aimed to provide
images of the spatial structure of atoms, a process in principle possible with
light of the same wavelength of the size of the illuminated object.

Finally, from a more practical point of view, the diffraction of atoms can
be used to measure the polarizability, by fitting the experimental data to the
detection distributions. This would be an alternative method of measurement
of this atomic property.

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Appendix 1

In this Appendix we present some technical points that complement the main developments in the paper.

Semiclassical approximation. We derive the values of the laser intensity that guarantee the use of the semiclassical approximation. The usual criterion for its validity is to have a large number of photons in the interaction region. In standard Kapitza-Dirac diffraction, where the approximation works well, the number of photons per unit volume is $I/\hbar \omega_L \approx 10^{18}$ photons/m$^3$, where we have used $\lambda_L = 500$nm and $I = 10^7$W/m$^2$. The typical volume of the interaction region is $10^{-12}$m$^3$ (10µm (aperture collimating the beam of atoms) × 1mm (height of the laser beam) × 100µm (width of the laser beam)). The number of photons in this characteristic volume is around $10^6$. For short wavelengths, the energy of each photon is around $10^2$eV and, in order to have a similar number of photons (assuming a similar characteristic volume), the intensity must be above $10^{11}$W/m$^2$. Then in our proposal we can safely use the semiclassical approximation.

Multipole expansion. For very short wavelengths different parts of the atom can feel different electric fields. We must go beyond the dipole approximation and consider higher multipole terms. The light-atom interaction potential can be expressed in the multipole form [19]:

$$D \cdot E(R, t) + \frac{1}{2} \sum_{i,j} Q_{ij} \left( \frac{\partial E_i}{\partial r_j} \right) (R, t) + \cdots$$

where $D$ and $Q_{ij}$ represent the dipole and $ij$-component of the quadrupole momenta of the atom. They can be rewritten using the second quantization of the atomic variables [19]. If we denote by $|h\rangle$ and $|l\rangle$ the eigenstates of the atom we have $D = \sum_{l,h} d_{lh} |l \rangle \langle h|$ with $D_{lh} = \sum_{e_n} <e_n | r_{e_n} | h\rangle$, and $Q_{ij} = \sum_{l,h} Q_{ij, lh} |l \rangle \langle h|$ with $(Q_{ij})_{lh} = \sum_{e_n} <e_n | (3(r_{e_n})_i (r_{e_n})_j - \delta_{ij} r_{e_n}^2) | l \rangle$. The sum, $\sum_{e_n}$, is over all the electrons of the atom. The diagonal elements (expectation values) $D_{hh}$ are null because the integrand has odd parity. The non-diagonal (transition) elements contribute to the atomic polarizability. In the static and spherically symmetric case the polarizability has the form

$$\alpha = \frac{2}{3} \sum_{l \neq g} \frac{|<g| \sum_{e_n} r_{e_n} |l>|^2}{\varepsilon_l - \varepsilon_g}$$

with $g$ denoting the ground state, and $\varepsilon_l$ he energy of the state [20].

The dipole term also describes the ionization of the atom. It is an alternative channel associated with ground-continuum transitions, with matrix elements in the form $<c|er|g>$, denoting $|c\rangle$ states of the continuum.

The next term in Eq. (7) represents the permanent quadrupole effects. In general, the values of the permanent quadrupole momenta can be expressed as $Q_{ij} \sim e r_0^2$, with $r_0$ Bohr’s radius and a proportionality coefficient of the order
of unity [21]. As signaled before, in order to be in the diffraction regime the potential must be of the order of \( U \approx 10^{-3} \text{eV} \). Then the quadrupole term must be similar, \( U_Q \approx 10^{-3} \text{eV} \). Taking for the field the standard form we can express the potential as \( U_Q \approx e r_0^2 k_L E_0 \approx e r_0 E_0 \). Using this expression and the relation between \( I \) and \( E_0 \) we have that the intensity of the laser must be around \( 10^8 \text{W/m}^2 \) in order to the quadrupolar effects to be relevant in the problem. This value is smaller than the intensity required to have a non-negligible atomic dipole induced by the polarizability. In Appendix 2 we discuss the role of induced quadrupole terms in our problem.

Note that if the n-th order permanent multipole coefficient follows a relation of the type \( Q_n \sim e r_0^n \) with a proportionality coefficient of the order of unity, we have \( U_Q \approx e r_0^n k_L^{-1} E_0 \approx e r_0 E_0 \) and that multipole term also has to be taken into account. However, as in the quadrupole case, its temporal dependence remains in the form \( \cos(\omega_L t) \), and after averaging it can be neglected.

Finally, we recall the fact that the electric quadrupole and the magnetic dipole energies have similar magnitudes. Thus, if the electric quadrupole term is relevant for the problem, the magnetic dipole one must also be taken into account. The magnetic dipole term reads as \( e \mathbf{D}_M \cdot \mathbf{B}(\mathbf{R}) \) with \( \mathbf{B}(\mathbf{R}) \) the magnetic field at the atomic CM position and \( \mathbf{D}_M \) the magnetic dipole moment, which can be expressed as \( \mathbf{D}_M = -(1/2m) \sum_i \mathbf{l}_i \) with \( \mathbf{l}_i \) the orbital angular momentum of the i-th electron in the atom. From these expressions and the form of the electromagnetic potential it is clear that the temporal dependence of this term is also proportional to \( \cos(\omega_L t) \), and it will vanish after time averaging just as in the electric quadrupole case.

Appendix 2 In this Appendix we analyze the atomic quadrupole induced by the electric field or its gradient. When the polarizability effects are taken into account the quadrupole momenta can be expressed as

\[
Q_{ij} = Q_{ij}^0 + \sum_k A_{kij} E_k + \sum_{kl} C_{ijkl} \frac{\partial E_k}{\partial x_l} \tag{9}
\]

where \( Q_{ij}^0 \) are the permanent momenta, \( A_{kij} \) is the dipole-quadrupole polarizability and \( C_{ijkl} \) the quadrupole-quadrupole one. The first term in the r. h. s. of the equation represents the quadrupole momenta in absence of external electric fields, the second the momenta induced by an electric field and the third these induced by the gradient of the field.

The order of magnitude of the coefficients is \( A \approx e^2 r_0^3 E_0^{-1} \) and \( C \approx e^2 r_0^4 E_0^{-1} \) with \( E_0 = 4 \times 10^{-18} \text{J} \). Introducing numerical values we obtain for the potentials associated with these two terms (\( U_A \approx AE_0 k_L E_0 \) and \( U_C \approx C(E_0 k_L)^2 \)):

\[
U_A \approx U_C \approx (e r_0 E_0)^2 E_0^{-1} \approx 10^{-4} - 10^{-5} \text{eV},
\]

where we have used \( r_0 k_L \approx 1 \). We expect the potentials to be ten to one hundred times smaller than the light-shift potential. However, to extract precise conclusions of the importance of the induced terms we should know the actual values of the coefficients. Unfortunately, very few experimental data are available, and in many cases there
are large uncertainties on their theoretical estimation (see, for instance, [22] for some recent work in this subject. Note that with the values suggested for Mg in this reference the induced potential would be comparable to the lightshift one).

In our particular problem, when the quadrupole polarizability is taken into account the full potential can be written as

\[
U(X) = U_0 \cos^2(k_L X) + U_A \cos^3(k_L X) \sin(k_L X) \\
+ U_C \cos^2(k_L X) \sin^2(k_L X)
\]  

(10)

Note that the temporal dependence in all the terms of the r. h. s. is \(\cos^2(\omega_L t)\). All the terms remain after the time averaging. The coefficient resulting from the integration is included in \(U_0, U_A, U_C\). Using simple trigonometric relations we obtain

\[
U(X) = \frac{U_0}{2} + \frac{U_C}{8} + \frac{U_0}{2} \cos(2k_L X) + \frac{U_A}{4} \sin(2k_L X) \\
+ \frac{U_A}{8} \sin(4k_L X) - \frac{U_C}{8} \cos(4k_L X)
\]  

(11)

The final detection pattern is

\[
e^{iU(X)/\hbar} e^{i k_0 X} = e^{i \left( \frac{U_0}{2} + \frac{U_C}{8} \right) \tau/\hbar} \sum_{n,m,l,r=-\infty}^{\infty} i^{n+r} J_n \left( \frac{U_0 \tau}{2\hbar} \right) \times \\
J_m \left( \frac{U_A \tau}{4\hbar} \right) J_l \left( \frac{U_A \tau}{8\hbar} \right) J_r \left( -\frac{U_C \tau}{8\hbar} \right) e^{i(2n+2m+4l+4r)k_L + k_0) X}
\]  

(12)

where we have used the relation \(\exp(i \xi \sin \varphi) = \sum_n J_n(\xi) \exp(in\varphi)\). The diffraction pattern shows the same analytical form found for the lightshift potential, the peaks correspond to even multiples of \(k_L\). The difference lies in the different intensities of the peaks, which in addition to \(U_0\) now also depend on \(U_A\) and \(U_C\).

In conclusion, in presence of quadrupole polarizability effects, we would obtain a diffraction pattern similar to that associated with dipole polarizability. Our main result remains valid, there are diffraction patterns in the regime of very short wavelengths. In addition, the diffraction pattern could be used to determine the \(U_A\) and \(U_C\) values and, in consequence, the quadrupole polarizabilities, which are very difficult to measure by other methods [22].

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