Anisotropic Inflation from Extra Dimensions

Marco Litterio\textsuperscript{1,}\textsuperscript{*}, Leszek M. Sokołowski\textsuperscript{2}, Zdzisław A. Golda\textsuperscript{2}
Luca Amendola\textsuperscript{3} and Andrzej Dyrek\textsuperscript{4}

\textsuperscript{1}Istituto Astronomico, Università di Roma “La Sapienza”, Via Lancisi 29, I-00161 Roma, Italy
\textsuperscript{2}Astronomical Observatory, Jagellonian University, Orla 171, 30-244 Kraków, Poland
\textsuperscript{3}Osservatorio Astronomico di Roma, Viale del Parco Mellini 84, I-00136 Roma, Italy
\textsuperscript{4}Institute of Physics, Jagellonian University, Reymonta 4, 30-059 Kraków, Poland

PACS: 98.80 Cq, 04.50+h
KEYWORDS: inflationary cosmology, multidimensional Kaluza-Klein theory

Abstract

Vacuum multidimensional cosmological models with internal spaces being compact $n$-dimensional Lie group manifolds are considered. Products of 3-spheres and $SU(3)$ manifold (a novelty in cosmology) are studied. It turns out that the dynamical evolution of the internal space drives an accelerated expansion of the external world (power law inflation). This generic solution (attractor in a phase space) is determined by the Lie group space without any fine tuning or arbitrary inflaton potentials. Matter in the four dimensions appears in the form of a number of scalar fields representing anisotropic scale factors for the internal space. Along the attractor solution the volume of the internal space grows logarithmically in time. This simple and natural model should be completed by mechanisms terminating the inflationary evolution and transforming the geometric scalar fields into ordinary particles.

* Corresponding author; e-mail address: litterio@astrom.astro.it; fax: 39-6-4403673
The cosmological evolution of the universe must be driven by some kind of matter. If the large scale geometry of our spacetime is described by isotropic Friedmann-Lemaitre-Robertson-Walker (FLRW) models, as they best fit to the observational data, the galactic matter (and other, yet undetected, forms of matter) cannot be viewed as test particles but as a self-gravitating fluid. In fact, the vacuum FLRW models are either inconsistent (spatially closed models) with Einstein field equations without cosmological constant or merely describe flat Minkowski space (open and flat models). In the early universe its self-gravitating contents form a structureless plasma of all elementary particles present in the standard model (and possible exotic particles beyond the model) and the evolution rate and physical processes occurring in a given era depend on which particle species dominates.

It is widely believed that the standard physical cosmology should be modified in the earliest eras by the concept of inflationary evolution [1]. Inflationary (i.e. accelerated) expansion is possible due to the existence of large negative pressure in the gravitating source, therefore it cannot be driven by a relativistic gas of the particles in thermal equilibrium. Only scalar fields can provide negative pressure. However the Higgs scalar of the standard model is inappropriate to this aim. It is necessary to introduce ad hoc a special inflaton field playing otherwise no role in particle physics and this is why there have been proposed so many mechanisms of inflation, none of which is well grounded in fundamental physics.

Scalar fields appear in a natural way in higher-dimensional physics. Yet it is rather little known what particles live in higher dimensions. It is therefore interesting to assume that the primordial universe was $d = 4 + n$-dimensional anisotropic and empty. All its spatial dimensions evolved as being dynamical degrees of freedom under influence of vacuum gravity. The total spacetime was split into a product of four-dimensional spacetime $\mathcal{M}$ named “external space” (which has then evolved into the world we see around us) and an $n$-dimensional closed space $\mathcal{B}$ (“internal space”) with a Riemannian metric. The splitting allows for performing dynamical reduction of the full theory to the low-energy physics occurring in the external space $\mathcal{M}$; the extra dimensions forming the internal space provide then a number of scalar fields in $\mathcal{M}$ and this “matter” of geometrical origin acts as a source for 4-dimensional gravity. We find this model as natural, simple and elegant. It is conceivable that the geometrical matter is later turned in some process into ordinary particles of the standard model. We shall not address here the latter problem.

In this letter we present results of qualitative analysis of cosmological evolution of the external space which is driven by $n$ scalar fields representing anisotropic evolution of $n$ internal dimensions. We are interested both in isotropization process of these dimensions and
in possible inflation of the ordinary 3 external dimensions. We find as a general rule that initial anisotropy of the internal space diminishes in time and for some cases vanishes at all. Inflationary solutions are also generic (attractor solutions) and are usually associated with a mild, acceptable growth of the internal space. Yet the graceful exit problem remains open.

The total spacetime $\mathcal{P}$ is taken as a “warped” metric product $\mathcal{P} = \mathcal{M} \times \mathcal{B}$, $\dim \mathcal{M} = 4$ and $\dim \mathcal{B} = n$. In general the internal space $\mathcal{B}$ can evolve anisotropically since its metric can depend on the external coordinates $x^\mu$, $\mu = 0, \ldots, 3$. The components of the metric thus generate $\frac{1}{2} n(n + 1)$ scalar fields in $\mathcal{M}$. As $\mathcal{B}$ we choose a compact Lie group manifold with a diagonal metric (giving rise to $n$ scalars).

The metric tensor for $\mathcal{P}$ is then

$$\hat{g}_{AB}(x, y) = \begin{pmatrix} e^{-nF(x)}g_{\alpha\beta}(x) & 0 \\ 0 & L^2e^{2(F(x) + \phi_a(x))}h_{ab} \end{pmatrix},$$

where $h_{ab} = L^2e^{2(F + \phi)}\delta_{ab}$, $a, b = 1, \ldots, n$, is a metric on $\mathcal{B}$ in a right-invariant vector basis. $h_{ab}$ must be independent of internal coordinates $y^a$. Here $\phi_a$ are the relative anisotropic deformation factors subject to the linear constraint $\sum_{a=1}^{n} \phi_a = 0$; $L e^F$ is the mean radius of the internal space and $F$ is the dilaton field. Note that the physical metric field for $\mathcal{M}$ is $g_{\alpha\beta}$ and not $\hat{g}_{\alpha\beta} = e^{-nF}g_{\alpha\beta}$, as required by the fact that we have used the warped metric product (see for details, [3], cf. also [1, 4, 5]).

Since we assume Einstein gravity in the vacuum total spacetime, the action is

$$S = \int_{\mathcal{B}} d^n y \int d^4 x \sqrt{-\hat{g}} \left[ \hat{R}(\hat{g}) - 2\Lambda \right],$$

the presence of a (positive) cosmological constant is essential for our analysis. Although the fields $F$ and $\phi_a$ have direct geometrical meaning, after dimensional reduction they give rise to non-standard kinetic terms in the effective action. In order to give the action the standard form one makes linear transformations to new fields, $P \equiv \alpha F - q$, $\phi_a \equiv \sum_{i=1}^{n-1} A_{ai}\psi_i$ with $\alpha, q$ and $A_{ai}$ appropriately chosen constants. Then

$$S = \int d^4 x \sqrt{-g} \left[ R(g) - 2\Lambda_4 - P_{,\mu}P^{,\mu} - \sum_{i=1}^{n-1} \psi_{i,\mu}\psi_{i}^{,\mu} - 2V(P, \psi) \right],$$

where $R(g)$ is the curvature scalar for $g_{\alpha\beta}(x)$, $\Lambda_4 \equiv 2\Lambda/(n + 2) > 0$ and

$$V(P, \psi) = \Lambda \left[ e^{-\beta P} - \mu e^{-\gamma P} R_B - \frac{2}{n + 2} \right]$$

is an effective potential generated by the curvature scalar $R_B(\psi)$ of the internal space. Here $\beta \equiv \sqrt{2n/(n + 2)} = n/\alpha = 2/\gamma$ and $\mu \equiv \beta^2/2R_0$ with $R_0 \equiv R(\phi_a = 0)$. The physical system
in four dimensions consists of the metric $g_{\mu\nu}$ and the “geometric matter”: the dilaton $P$ and $n - 1$ independent deformation factors $\psi_i$, $i = 1, \ldots, n - 1$. The field equations take on then the standard form

$$G_{\mu\nu} + \Lambda_4 g_{\mu\nu} = T_{\mu\nu}(P) + \sum_{i=1}^{n-1} T_{\mu\nu}(\psi_i) - V(P, \psi) g_{\mu\nu},$$

(5)

$$\Box P = \frac{\partial V}{\partial P} \text{ and } \Box \psi_i = \frac{\partial V}{\partial \psi_i},$$

(6)

with

$$T_{\mu\nu}(f) \equiv f_{,\mu} f_{,\nu} - \frac{1}{2} g_{\mu\nu} f_{,\alpha} f^{,\alpha}.$$  

(7)

It is clear that the effective potential $V(P, \psi)$ fully characterizes the dynamics of the system and in the following we will discuss qualitative behaviour of the system employing the potential instead of searching for exact solutions eqs. (5)-(6).

It turns out that Einstein field equations in $d$ dimensions are compatible with the metric (1) only if the Ricci tensor for $h_{ab}$ on $B$ is diagonal. This condition for compact non-abelian Lie groups with diagonal metrics holds e.g. for the $SU(3)$ space and products of 3-spaces (and is trivially satisfied for the abelian flat group space, $n$-torus).

Consider first the $SU(3)$ group manifold. Although it has no direct geometrical interpretation, its significance for particle physics makes it an interesting model for the higher-dimensional theory. As far as we know this space has never been investigated in this context nor in cosmology. Requiring the Ricci tensor for $SU(3)$ to be diagonal and taking into account the constraint $\sum_a \phi_a = 0$ one finds that there are at most five independent deformation factors $\psi_i$ ($n = 8$). For the external spacetime metric we take the simplest case — flat FLRW model. Even then generic dynamical system with the six scalar fields is intractable. Fortunately, additional symmetries of the system can be used to simplify the problem and it turns out that the most symmetric case already contains all the relevant features of the whole system and thus there is no loss of generality in studying it.

The most symmetric $SU(3)$ case, compatible with the field equations, is $\phi_1 = \phi_2 = \phi_3$ and $\phi_4 = \phi_5 = \phi_6 = \phi_7$. In terms of $\psi$’s these read: $\psi_1 = \psi_2 = 0$, $\psi_4 = \sqrt{3/5} \psi_3$, $\psi_5 = \sqrt{2/5} \psi_3$ and $\psi_6 = \sqrt{2/7} \psi_3$. There are only three independent scalar fields: $P$, $\psi_3$ and $\psi_7$. The system (5)-(6) has a unique stationary point $P = \psi_3 = \psi_7 = 0$ ($= \phi_a$) around which the linearized eqs. (6) are

$$\Box P \simeq -\frac{2}{5} \Lambda P,$$  

(8)
\[ \Box \psi_3 \simeq \frac{2}{105\sqrt{2}} \Lambda (8\sqrt{2} \psi_3 - 3\sqrt{21} \psi_7), \]  
(9)

\[ \Box \psi_7 \simeq \frac{6}{35\sqrt{21}} \Lambda (-8\sqrt{2} \psi_3 + 3\sqrt{21} \psi_7). \]  
(10)

From the trace of the Einstein equations it follows that in full generality \( \dot{H} < 0 \); thus, if initially \( H < 0 \), the system can only evolve towards the physical singularity \( H \to -\infty \).

Remembering that in the FLRW metric \( \Box P = -(\dot{P} + 3H \dot{P}) \), a negative, and large in modulus, \( H \) is equivalent to the presence of an anti-friction force acting on the fields, that makes unstable any equilibrium point.

The phase space \((P, \dot{P})\) is illustrated in Fig. 1 in Poincaré projection. Along trajectories pointing to North (N), \( P \to +\infty, \dot{P} \to +\infty \) or to South (S), \( P \to -\infty, \dot{P} \to -\infty \), the Hubble parameter is negative, \( H < 0 \). Since \( H \) has two opposite values, at each point of the phase space there are two trajectories, but, for simplicity we have shown only one of them in the figure. Note also that the points N and S are reached not only when the initial value of \( H \) is negative; since \( \dot{H} < 0 \) for scalar field dynamics, also starting with \( H > 0 \) many solutions will turn to \( H < 0 \). In all these cases the Universe reaches a physical singularity \((H, \dot{H} \to -\infty)\) in a finite time interval. The remaining possibility (note that the West point (W) lies in the forbidden region of the phase-space) is given by the trajectories pointing to East (E), \( P \to +\infty, \dot{P} \to 0 \). In this case \( H \to 0^+ \) for \( t \to +\infty \). An example of such a solution is shown in Fig. 2.

It is of relevant importance that also the attractor trajectory in Fig. 1 has the same properties. Its analytic expression can be found by looking for solution of the form \( \dot{P} = A e^{mP} \) in the limit \( P \to +\infty \). Under condition \( \psi_3 = \psi_7 = \dot{\psi}_3 = \dot{\psi}_7 = 0 \) (the condition is always met asymptotically when \( H > 0 \) since it turns out to be a stable focus point of the linear system (9)-(11)) and inserting the trial solution in the field equation (5)-(6) one finds that the evolution of the scale factor \( a(t) \) for flat FLRW universe is

\[
\begin{align*}
\frac{\dot{a}}{a} & = a_0 \left[ 1 - m A e^{mP_0}(t - t_0) \right]^p, \quad \text{and} \\
P & = P_0 - \frac{1}{m} \ln \left[ 1 - m A e^{mP_0}(t - t_0) \right], \quad \text{with} \\
p & = \frac{2}{\beta^2}, \quad m = -\frac{\beta}{2}, \quad A = \left( \frac{2\beta^2 \Lambda}{6 - \beta^2} \right)^{1/2},
\end{align*}
\]  
(11)

where \( t_0 \) is an integration constant, and \( a_0 = a(t_0), P_0 = P(t_0) \). In the present case \((n = 8)\), along the attractor, the physical evolution of the system is described by the leading terms (at
$t \to \infty$ in (11),

$$a \approx \left( \frac{t}{t_i} \right)^{5/4} \text{ and } P \approx \sqrt{\frac{5}{2}} \ln t,$$

(12)

where $t_i = (-mAe^{mP})^{-1}$ and $a_0 = 1$. This is an inflationary stage, though of a rather mild kind (power law inflation), see Fig. 3.

To summarize the $SU(3)$ case, it is possible to have power law inflation with the scalars $\psi_3$ and $\psi_7$ dissipating their kinetic energy on the plateau of their effective potential, but then the mean volume (proportional to $e^{nF}$) of the internal space increases. In fact, there exists in the phase space an attractor solution along which the leading-term evolution is given by (12). The logarithmic growth of $P$ (or $F$) seems rather slow but actually it is not slow enough to have a negligible effect on physics in our external world. Primordial nucleosynthesis is very stringent in this respect [1]: the ratio of the values of the radius $b(t)$ of the internal space at the nucleosynthesis epoch ($b = b_N$ at $t = t_N$) and today ($b = b_0$ at $t = t_{\text{now}}$) is constrained by $0.99 \leq b_N/b_0 \leq 1.01$. Since $b = L \exp F = L \exp [(P + q)/\alpha]$ this means, for $P$ given in (12), that

$$-0.01005 \leq \frac{2}{\alpha \beta} \ln \frac{t_N}{t_{\text{now}}} \leq 0.00995.$$  

(13)

One finds $\ln(t_N/t_{\text{now}}) = -34.54$, for $t_N \approx 10^2$ sec and $t_{\text{now}} \approx 10^{17}$ sec, thus the solution does not meet the constraint, and the internal space evolves far too rapidly. Furthermore, the fact that the power law inflation corresponds to the asymptotic attractor of the phase space means that the universe never stops inflating and never enters the hadron dominated era. Thus the geometrical inflation from the extra dimensions encounters the graceful exit problem and the latter should be solved by an independent physical mechanism (e.g. inserting a counterterm in the dilaton potential).

Although the nucleosynthesis constraint excludes the solution (12), it is worth noticing that the growth of the radius of the internal space needs not make it observable at the end of inflationary epoch. Assuming traditionally [7] that satisfactory inflationary evolution corresponds to $N = 60$ $e$-foldings, $N = \ln(a_{\text{final}}/a_{\text{initial}})$, one finds from (12) $N = p \ln (t_{\text{final}}/t_{\text{initial}})$ and the growth of the extra dimensions is

$$\frac{b_{\text{final}}}{b_{\text{initial}}} = e^{\beta N/\alpha} = e^{N/5} \approx 1.6 \cdot 10^5.$$  

(14)

Unless $b_{\text{initial}}$ is many orders of magnitude larger than the Planck length, it remains unobservable after 60 $e$-foldings of the inflationary evolution. The duration of the inflationary era
is \( t_{\text{final}}/t_{\text{initial}} = e^{48} \approx 10^{21} \), an acceptable value provided inflation starts soon after the Planck era.

As the second case one takes the internal space as being either isometric to a “squashed” 3-sphere or the product of two spheres, \( B = S^3 \times S^3 \).

The common feature of all the models we have considered is that all the subspaces whose product composes the internal space isotropize separately. For instance, if \( B \) is the product of \( N \) three-spheres, \( n = 3N \), then each sphere isotropizes while the scale factors for different spheres will diverge in general. The divergence is due to the constraint \( \sum a \phi_a = 0 \); it is easily seen in the case of \( N - 1 \) spheres having equal scale factors, \( \phi_a = \phi \) for \( a = 1, \ldots, n - 3 \) and \( \phi_a = -(N - 1)\phi \) for \( a = n - 2, n - 1, n \), then the last sphere evolves in the apposite direction to the others. Thus the internal space remains anisotropic although the degree of anisotropy decreases.

The effective potential \( V(P, \phi) \) is now unbounded from below and has a global maximum at \( P = \phi_a = 0 \), the extremum point belongs to the constraint hypersurface. The qualitative features of the evolution can be summarized as follows: a) for a single sphere, the axially symmetric (oblate) and isotropic configurations are energetically favourable; however the process of symmetrization and the dilaton expansion or contraction occurs with similar time scales; b) if the initial conditions for \( P \) are such that first an inflationary phase occurs, i.e. if \( P \) starts near \( P = 0 \) with negligible velocity, then the duration of the phase is not increased by anisotropic initial conditions; c) for the product of two or more spheres each of them evolves towards the “round” sphere with different radius (see above); d) in addition to the singular solutions \( H \rightarrow -\infty \), there are always generic solutions exhibiting the power law inflation in four dimensions, \( H = p/t \), \( p > 1 \), corresponding to an attractor trajectory in the phase space \( (P, \dot{P}) \).

Since any initial anisotropy of each 3-sphere tends to vanish (Fig. 4), let us consider the case of a single isotropic sphere. Using the same method as in the \( SU(3) \) case one finds an attractor solution in the limit \( P \rightarrow +\infty \) (Fig. 4). The leading terms in the solution (11) are, now,

\[
a(t) \sim t^{5/3} \quad \text{and} \quad P \approx \sqrt{\frac{10}{3}} \ln t.
\]  

(15)

The external world expands even faster than in the case of \( SU(3) \) space, therefore the sufficient inflation, \( N = 60 \) e-foldings, is achieved in a shorter time, \( t_{\text{final}}/t_{\text{initial}} \approx e^{N/p} \approx e^{36} \approx 10^{15} \).
Also the internal space expands faster and in this inflationary period its growth is

\[ \frac{b_{\text{final}}}{b_{\text{initial}}} \approx \exp \left( \frac{2N}{3p} \right) \approx e^{24} \approx 10^{10}, \quad (16) \]
an acceptable factor. Once again the nucleosynthesis constraint excludes this attractor solution as incompatible with observations: the constraint (13) is replaced by

\[ -0.01005 \leq \frac{2}{3} \ln \frac{t_N}{t_{\text{now}}} \leq 0.00995 \quad (17) \]
and the latter cannot be satisfied, unless an exit mechanism is provided.

Finally, we compare our results with a recent work by Levin [8]. The difference is that Levin does not use the conformal factor \( e^{-nF} \) to construct the physical metric field in four dimensions and the internal space in that work is a multidimensional torus. The comparison clearly shows the impact of the factor and of the curvature of \( B \) on the evolution of the external world. While we assume that \( (\mathcal{M}, g) \) is the physical spacetime, Levin takes \( (\mathcal{M}, \hat{g}) \) with \( \hat{g}_{\alpha\beta} = e^{-nF} g_{\alpha\beta} \). The radius of the internal space is defined in the same way here and in [8], \( b = \hat{b} = e^F \) \((L = 1\) for simplicity). If \( g_{\alpha\beta} \) is the metric of flat FLRW spacetime than the cosmic scale factors are related by

\[ \hat{a} = \exp \left( -\frac{n}{2} F \right) a = b^{-\frac{n}{2}} a. \quad (18) \]
In the case of flat \( n \)-torus there is no effective potential and the evolution of \( \hat{a} \) is driven solely by the time derivatives of the scalar field \( b \) (“kinetic inflation”). Assuming \( \Lambda = 0 \) it was found that [8]

\[ \hat{a} = c_1 (1 - \tau/\tau_0)^{-k}, \quad k \equiv \frac{-3 + \sqrt{3n^2 + 6n}}{3(n + 3)} \quad \text{ and } \]
\[ \hat{b} = b = c_2 (1 - \tau/\tau_0)^w, \quad w \equiv \frac{n + \sqrt{3n^2 + 6n}}{n(n + 3)}, \quad (19) \]
where \( c_1, c_2 \) and \( \tau_0 \) are constants and \( \tau \) is the proper time for the metric \( \hat{g}_{\alpha\beta} \) related by a conformal transformation to the proper time \( t \) for \( g_{\alpha\beta} \),

\[ d\tau = \exp \left( -\frac{n}{2} F \right) dt = b^{-n/2} dt. \quad (20) \]
This is an accelerating (“inflationary”) solution, starting from a regular state at \( \tau = 0 \) and evolving towards a future singularity in \( b(\tau) \) at \( \tau_0 \). In terms of \( (\mathcal{M}, g) \) viewed as the physical spacetime, this solution has a quite different interpretation. It can be obtained as a solution to [3] and [6] for \( \Lambda = V(P, \psi) = \dot{\psi}_i = 0, \ i = 1, \ldots, n \). Then it reads

\[ a = c_3 (\bar{t} - t)^{1/3} \quad \text{ and } \quad P = \frac{2}{3} \ln(\bar{t} - t) \quad (21) \]
with constant $c_3$ and $\tilde{t}$. One can easily check that (19) and (21) are related via transformations (18) and (20). Now expansion has turned into contraction and a future singularity appears also in four dimensions. There exists also a slowly growing solution $a \propto \tau^r$ with $0 < r < 1$; in the metric $g_{\alpha\beta}$ it corresponds to $a \propto t^q$ with $0 < q < 1$. Since in our opinion the physical metric is $g_{\alpha\beta}$ and not the conformally related $\hat{g}_{\alpha\beta}$ [3, 4, 5, 6], we conclude that in the model with flat internal space and vanishing cosmological constant, an inflationary evolution of the external world is not present.

It is worth noticing, however, that the contracting solution (21) is accelerating ($\dot{a} < 0$ and $\ddot{a} < 0$), i.e., it represents deflation. Recently it was shown [9] that deflation may also solve the horizon, flatness and amplification of perturbation problems of the big-bang cosmology. In this sense, also the toroidal internal space can be considered among the viable models of the primordial universe.

In conclusion, anisotropic or isotropic cosmological evolution driven by the curvature in the higher internal dimensions turns out to be generically of the form of power law inflation. The essential ingredients are a curved internal space (this was shown for the case of $SU(3)$ group manifold and 3-spheres) and a positive cosmological constant. There is no matter in the multidimensional world and gravity is the only interaction. Matter appears only after dimensional reduction in the form of self-interacting scalar fields of geometrical origin. This approach allows one to avoid introducing ad hoc matter and an inflaton field. The dynamics of the four-dimensional world is entirely determined by geometry of the internal space. The inflationary epoch (attractor solution) is long enough to solve the horizon and flatness problems of the standard cosmology. The power-law inflation must be terminated by an unknown mechanism (the graceful exit problem). Furthermore, this or another mechanism should stabilize the internal space at sufficiently small size (the growth of the space should be stopped approximately simultaneously with the end of the inflationary epoch) and turn the scalar fields into ordinary elementary particles. These mechanisms are beyond our model. Nevertheless it is interesting to find purely geometrical inflation without matter, fine-tuning and special potentials.

Details of all the calculations will be published elsewhere [10].

Acknowledgement

We are grateful to Prof. Andrzej Staruszkiewicz for helpful comments and discussions. The work of Z.A. G., L.M. S. and A. D. was partially supported by grants of the Polish Committee for Scientific Research.
References

[1] E. W. Kolb and M. S. Turner, *The Early Universe*, Addison-Wesley Publishing Company (1990).

[2] B. Mc Innes, *Class. Quantum Grav.* 2 (1985) 661.

[3] L.M. Sokołowski, *Class. Quantum Grav.*, 6 (1989) 59.

[4] K. Maeda, *Class. Quantum Grav.* 3 (1986) 651.

[5] L. Sokołowski and B. Carr, *Phys. Lett.* B176 (1986) 334.

[6] Y. M. Cho, *Phys. Rev.* D35 (1987) 2628; *Phys. Rev. Lett.* 68 (1992) 3133.

[7] A. H. Guth, *Phys. Rev.* D23 (1981) 347.

[8] J. J. Levin, *Phys. Lett.* B343 (1995) 69.

[9] M. Gasperini and G. Veneziano, *Phys. Rev.* D50, 2519 (1994).

[10] Z. A. Golda, M. Litterio, L. M. Sokołowski, L. Amendola and A. Dyrek, submitted to *Ann. Phys.*, N. Y.

Figures

1. The phase-space (in Poincaré projection) of the dilaton $P$, in the $SU(3)$ maximally symmetric case. $P$ is proportional to the logarithm of the mean radius of the internal space. The most interesting feature is the attractor trajectory pointing to E, with $P \to +\infty$, $\dot{P} \to 0$.

2. The whole dynamics of the universe along the attractor trajectory shown in Fig. 1. (2a) The Hubble parameter $H$ goes asymptotically to zero. (2b) The expected behaviour along the attractor $P \to +\infty$, $\dot{P} \to 0$. (2c) All the other fields of the model go to the stationary point $\phi_1 = \phi_2 = \phi_3 = 0$, $\phi_4 = \phi_5 = \phi_6 = \phi_7 = 0$.

3. Power law inflation in the external spacetime of scale factor $a(t)$, is naturally present in the $SU(3)$ internal space case. In fact, for a large set of initial conditions, once the attractor trajectory has been reached, it happens that: (3a) $H \to 0$, (3b) $a(t) \propto t^{5/4}$. Of course, different solutions do not reach the attractor at the same time.
4. In the $S^3$ internal space, complete isotropy is quickly achieved. (4a) The initial degree of anisotropy, measured by $\psi_2 \neq 0$, in a comparison to $P$, is quickly swept away. (4b) The corresponding runs of $H$ overlap for most of the inflationary stage, i.e. for large $a$ and $P$, when $H$ is almost constant. Note also the plateau at $a \geq 1$; a larger initial anisotropy does not improve the expansion that results to be of only a couple of $e$-foldings.

5. The phase-space (in Poincaré projection) of the dilaton $P$, in the $S^3$ case. In this case also there is an attractor trajectory $P \to +\infty$, $\dot{P} \to 0$. Along it, $a \propto t^p$, with $p \approx 1.67$, and again we are in presence of a power law inflation.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/gr-qc/9510067v1
This figure "fig2-1.png" is available in "png" format from:

http://arxiv.org/ps/gr-qc/9510067v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/gr-qc/9510067v1
This figure "fig2-2.png" is available in "png" format from:

http://arxiv.org/ps/gr-qc/9510067v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/gr-qc/9510067v1