Vibration-based damage detection in a concrete beam under temperature variations using AR models and state-space approaches

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Abstract.
The Structural Health Monitoring of civil structures subjected to ambient vibrations is very challenging. Indeed, the variations of environmental conditions and the difficulty to characterize the excitation make the damage detection a hard task. Auto-regressive (AR) models coefficients are often used as damage sensitive feature. The presented work proposes a comparison of the AR approach with a state-space feature formed by the Jacobian matrix of the dynamical process. Since the detection of damage can be formulated as a novelty detection problem, Mahalanobis distance is applied to track new points from an undamaged reference collection of feature vectors. Data from a concrete beam subjected to temperature variations and damaged by several static loading are analyzed. It is observed that the damage sensitive features are effectively sensitive to temperature variations. However, the use of the Mahalanobis distance makes possible the detection of cracking with both of them. Early damage (before cracking) is only revealed by the AR coefficients with a good sensibility.

1. Introduction
The early damage detection in civil structures is a key goal to ensure user security and to reduce maintenance costs. The more convenient approach to achieve this aim is the analysis of ambient vibration data, since it is a passive approach (no artificial excitation) and it does not necessitate to close the structure. But the lack of excitation knowledge and the variations of environmental conditions imposed to the structure make it also more challenging.

The most spread approach is the use of auto-regressive (AR) models fitted to vibrations time series to track damage occurrence [1–5]. On the other hand, damage sensitive features (DSF) based on state space representation of time series have received an increasing interest recently [6–9]. However, a comparative study of the two cited methods is rarely carried out.

This work proposes a DSF based on the Jacobian matrix of the observed dynamics, estimated in the state space of the system. A concrete beam subjected to temperature variations and progressively damaged by a cyclic static loading is used to compare the sensitivity to damage of the proposed approach and the AR coefficients.
2. Theoretical background

2.1. State-space Damage sensitive feature

The proposed Damage Sensitive Feature (DSF) is the Jacobian Matrix of the dynamics in the state space. Any dynamical system can be represented in its state space where each dimension is a degree of freedom. When it is impossible to measure all dof, as in instrumented civil structures, one can reconstruct qualitatively the state-space based on the measure of only one scalar time series by using the delayed coordinates method [10].

The estimation of the appropriate time delay, $\tau$, is commonly associated with the first zero of the autocorrelation function or the first minimum of the mutual information function [11]. Then the best choice of the embedding dimension is indicated by the false nearest neighbors method [12]. A brief description of the reconstruction process can be found in [13].

The scalar time series ($x$) = $x(1), \ldots, x(N)$ is transformed into a collection of $n$-dimensional vectors:

$$X(k) = \{ x(k), x(k+\tau), \ldots, x(k+(n-1)\tau) \}$$

(1)

The dynamics of the system can be represented by the evolution operator $F$ which links one point to the next one in a trajectory (Eq.2).

$$X(k+1) = F(X(k))$$

(2)

If $Y(k)$ is a close neighbor of $X(k)$, and noting $\delta X_i = X(k+i) - Y(k+i)$, the first order Taylor expansion of $F$ introduces the Jacobian matrix calculated at the point $X(k)$, $J [X(k)]$

$$\delta X_1 = J [X(k)] \delta X_0 + o \| \delta X_0 \|$$

(3)

and

$$\delta X_2 = J [X(k+1)] J [X(k)] \delta X_0 + o \| \delta X_0 \|$$

(4)

To evaluate the Jacobian matrix with experimental data, the first steps of the method used to estimate Lyapunov exponents are employed [14].

A fiducial point $X(k)$ is chosen in the reconstructed state-space, and its $r$, nearest neighbors, noted $X_{nn}(k), i = 1 \ldots r$, are selected to form a neighborhood. The difference vectors are constructed as follows:

$$\delta X_{nn}(k) = \{ X(k) - X_{nn}^i(k) \mid i = 1 \ldots r \}$$

(5)

To estimate the Jacobian matrix, the best linear mapping between the neighborhoods $k$ and $k+1$ (Eq.6 and 7) is estimated using a least-square method.

$$\delta X_{nn}(k+1) = J_1 [\delta X_{nn}(k)]$$

(6)

$$\delta X_{nn}(k+2) = J_2 [\delta X_{nn}(k+1)]$$

(7)

A preliminary study has shown that the sensitivity to damage is improved if the mapping does not include the initial neighborhood [15]. Hence the components of $J_2$ (Eq.7) are used to form a feature vector which will be referred to as the Jacobian Feature Vector (JFV).

$$JFV = J_2(\cdot)$$

(8)

This process is repeated for 100 fiducial points across the state-space. The number of neighbors, $r$, is set to twice the number of parameters to be estimated in the Jacobian matrix. For more details, a complete presentation can be found in [15].
2.2. Autoregressive Modeling

Autoregressive (AR) modeling is commonly used to study linear stochastic process. It can be written as

\[ x(k) = \sum_{i=1}^{p} \alpha_i x(k-i) + \epsilon_k \]  

(9)

where \( \epsilon_k \) is the random error at value \( k \), \( \alpha_i \) is the \( i^{th} \) AR coefficient and \( p \) the order of the model.

To determine the best model order, partial autocorrelation (PAC) function or Akaike’s information criterion (AIC) are often used [16]. The estimation of the AR coefficients is performed by a least-square method. A DSF is formed by these coefficients.

2.3. Novelty Detection

The detection of damage is done by comparison between unknown data and a reference baseline. This problem is referred to as novelty detection and can be managed by using the Mahalanobis distance (MD). It computes the distance of the current DSF vector to the reference baseline, taking into account the correlations between coordinates. The multivariate DSF is transformed into a scalar value. If the reference baseline is characterized by a mean vector \( \overline{DSF} \) and a covariance matrix \( C_{DSF} \), the square MD to the \( i^{th} \) DSF vector is

\[ MD_i^2 = (DSF_i - \overline{DSF})^T (C_{DSF})^{-1} (DSF_i - \overline{DSF}) \]  

(10)

To classify a DSF vector as damaged or undamaged, the whole database is split in three parts :

- **the reference baseline**, composed of half of the undamaged database. It is used to estimate \( C_{DSF} \) and \( \overline{DSF} \).
- **the classification base**, composed of a quarter of the undamaged database. This base allows the determination of classification threshold. In a primary approach, it is set to three standard deviations of the MD computed on the classification base with respect to the reference baseline. Assuming a normal distribution of the MD, it allows 1% of false alarms.
- **the test base**, composed of the last quarter of the undamaged database and all the damaged time series. It is used to perform a blind test. Both false alarms and non-detection can be quantified.

To ensure the efficiency of this method, the reference baseline has to cover all the various normal conditions encountered by the structure.

3. Case Study

3.1. Setup description

The case study is a 200 × 8.5 × 12 cm fiber reinforced concrete beam which is heated by six infra-red lights located in front of one side (Fig1). Beam displacements are constrained at both ends by a steel plate. Two thermocouples embedded in the beam allows collecting temperature variations. Four accelerometers (Nominal sensitivity 10 mV.g\(^{-1}\)) placed at the top surface, measure vertical accelerations. The excitation, a 800 Hz band-limited noise, is performed by an electrodynamic shaker of 18 N peak force. Fig.1 presents the positions of the different elements of the setup.

Time series are composed of 8192 points sampled at 2048 Hz. Each set of measures is composed of five time series.

Since damage detection is achieved by comparison between a reference healthy state and an unknown state, data are collected in two stages :
(i) heating and cooling of the beam while measures are recorded. The results is an undamaged database of 410 time series related to various temperature conditions.

(ii) progressive damage induced by four-points bending cycles until beam collapse. After each damage process, the beam vibrations are recorded. Meanwhile, temperature is randomly varied. The result is 410 time series with increasing damage related to random temperature variations.

3.2. Results and discussion

The preparatory study of the data has shown that the optimal AR model order is about 45. For the embedding parameters, a time delay of 3 and a reconstruction dimension of 7 are selected.

The Fig.2 presents the MD of the test base. Before the 44th set of measures, time series are from the undamaged state. In spite a few false alarms, these time series are correctly classified with both the DSF. For the damaged state, AR model coefficients are very sensitive since even the first step of loading is detected. Moreover, it is observed that the MD is slightly increasing with the progression of the cyclic loading until set #61. Regarding the JFV, all time series keep
being classified as undamaged until set #61. This indicates that the increase of micro cracking induced by the first loading cycles in well detected by AR coefficients but not by the JFV.

At the 62\textsuperscript{th} set, a high increase of MD is noted for both the DSF, followed by significant variations which remain above the classification threshold. This seems to indicate that a major event takes place at this point. Actually, the test report reveals that the first visible crack was observed at the set #66. It is likely that macro-cracking has initiated at the 62\textsuperscript{th} set. Therefore, both the DSF appear very sensitive to macro-cracking.

![Figure 3. AR coefficient #20 and temperature for the undamaged database](image1)

The MD appears to be an efficient tool to filter temperature effects before cracking. Indeed, Fig.3 shows a nonlinear correlation between AR coefficient and temperature and yet, before set #61, there is no variation of MD correlated with the temperature for both the DSF. On the other hand, after macro-cracking, strong variations of MD are observed for the two DSF. The sensitivity of the DSF to temperature seems to be increased by the cracking. The Fig.4 plots on the same graph temperature variations during the experiment and MD of the two DSF for the damage part of the test base. No correlation is observed between MD and temperature before cracking. After cracking, an increase in the temperature seems to lead to an increase in the Mahalanobis distance for AR coefficients and JFV.

![Figure 4. Mahalanobis distance and temperature variations for the damaged part of the test-base, – – – threshold, • damaged time series, — — Temperature](image2)
This sensitivity to temperature could be explained by the fact that thermal dilatations may generate a bending moment if the steel plates, which are supposed to constrain the beam, does not press at the center of the cross-section. This moment could be responsible for an increase of the crack opening, and hence for more variations in DSF with respect to temperature.

Despite the increased sensitivity to temperature resulting from macro-cracking, both the DSF clearly detect damage since, after cracking, MD values quite remain over the classification threshold. Moreover, after cracking, a good correlation is noted between the MD of the two DSF and damage evolution (crack progression) to collapse. It means that the two DSF are able to detect cracking, but also to monitor crack progression.

4. Conclusion
The aim of the present work is to compare two approaches for vibration-based damage detection. The first damage sensitive feature is composed of the coefficients of AR models fitted on time series, the second one is the Jacobian matrix calculated in the reconstructed state-space. Mahalanobis distance provides a tool to convert multi-dimensional information to scalar information and to filter variations resulting from environmental conditions.

To test the damage sensitive features, a concrete beam, subjected to temperature variations, is damaged with cyclic loading.

The DSF are directly sensitive to temperature. Indeed, the AR shows a correlation with temperature. However, the Mahalanobis distance filter efficiently these variations. Both DSF are able to detect the macro-cracking. But only the AR coefficients are sensible to micro-cracking induced by the first step of loading.

In future application, the joint use of both damage sensitive features with appropriate data fusion algorithm can provide a complementary information since the Jacobian matrix detects damage progression only after cracking while AR coefficients seem to be able to monitor micro-damaging. The integration of temperature measurement in the classification process could also improve the environmental variation filtering.

Several other questions need also to be addressed as the definition of a robust classification threshold, the potential of other classifiers as auto-associative neural network or support vector machine and the use of information provided by all the sensors.

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