Design of Sliding Mode Controller for Tilting Quadrotor UAV Based on Predetermined Performance

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Abstract. Aiming at the influence of modeling error and uncertain interference such as external sidewind in the control system of tilting quadrotor UAV, a non-singular fast terminal sliding mode controller based on predetermined performance is designed for its vertical take-off and landing mode. Firstly, based on the classical mechanics theory, the nonlinear dynamics model of tilt-rotor UAV in vertical take-off and landing mode was established. Then, on the basis of the non-singular fast terminal sliding mode controller, the predetermined performance function was introduced to make the system error within the specified performance range, and the final bounded convergence of the tracking error in the closed-loop system was proved based on the Lyapunov stability theory. Finally, the simulation experiment was carried out in MATLAB/Simulink. The simulation results show that the designed method can still achieve the stable tracking of the predetermined trajectory under the circumstance of uncertain external interference, and it has good control performance and anti-interference ability.

1. Introduction

In recent years, the newly designed tilt-rotor UAV that integrates fixed-wing and rotary-wing structures has attracted great attention from governments and military forces. This new type of UAV not only inherits the advantages of rotary-wing UAV vertical take-off and landing, and hovering in the air, but also inherits the advantages of fixed-wing UAV fast cruise and long-range flight. It has three flight modes: rotor mode, fixed wing mode and transition mode, and with high flight efficiency and large load capacity, it has broad application prospects in military and civilian fields. But its unique structural design not only improves its flight performance, it also brings many new problems and technical difficulties to it.

Advanced motion control technology is the key to completing the scheduled flight mission, and experts and scholars have proposed many trajectory tracking control algorithms, typically including PID control, linear quadratic regulation (LQR), backstepping control, sliding model control (SMC) and adaptive control. Most of them are to study how to reduce the error of trajectory tracking and improve its robustness and anti-interference ability. Literature[1] designed a quad-rotor drone's PID controller for quad-rotor. Literature[2] proposed a four-rotor aircraft control method based on fuzzy PID control. And compared with the traditional PID control method, a backstepping-based rotorcraft control method was proposed to improve its control effect[3]. For rotorcraft with uncertain effects such as non-modeling dynamics and external interference, a sliding mode-based control method was proposed[4]. With the gradual application of these advanced control theories in the control of rotorcraft, they have shown great advantages in fault-tolerant control and anti-interference. However, the above methods cannot guarantee that the dynamic and steady-state performance of the system always remain within the predetermined...
range during the entire convergence process. In order to solve the problem, Bechlioulis\cite{5} proposed a predetermined performance control method in 2008. After continuous improvement and development, it has been applied to many fields.

The tilting quadrotor UAV was taken as the research object in this paper, and aiming at the trajectory tracking control problem in the vertical take-off and landing mode, a non-singular fast terminal sliding mode controller based on predetermined performance was designed. By introducing a predetermined performance function, the system error is always kept within the specified performance range to suppress or eliminate the influence of uncertain factors, thereby improving the control performance of the system. And the validity and reliability of the designed control algorithm were verified by simulation experiments.

2. Dynamic model

The small tilting quad-rotor UAV studied in this paper adopts a layout scheme that combines quad-rotor and fixed wing, as shown in Fig.1. The two sets of wings are installed on the front and back sides of the UAV, and the areas of the front and back wings are not equal. The rotor nacelle is installed at the end of the wing, and the nacelle is controlled to rotate through the steering gear to complete the mutual switching between flight modes.

![Figure.1 The structure of the tilting quadrotor UAV](image)

In the vertical take-off and landing mode, the lift of the UAV is controlled by controlling the synchronous rotation of four motors, and the pitch, yaw, and roll motion of the UAV are controlled by controlling the speed difference of different motors. The flight principle is the same as that of a quadrotor drone. And in order to simplify the modeling process, the following assumptions are made:

1. The tilt-rotor UAV is regarded as a rigid body model, with uniform weight and symmetrical geometry.
2. The acceleration of gravity remains unchanged.
3. The influence of the rotor downwash on the aerodynamic force of the wing is not considered.
4. The four rotor nacelles tilt synchronously, so the inclination angles of the four rotor nacelles are the same at any time.

Let \( \Omega = [p \ q \ r]^T \) and \( V_g = [\dot{x} \ \dot{y} \ \dot{z}]^T \), and by adopting Newton-Euler equation to model it overall, the dynamic equation can be obtained as

\[
\begin{bmatrix}
ml_{b_{33}} & 0 \\
0 & I_b
\end{bmatrix}
\begin{bmatrix}
\dot{\Omega}_b \\
\Omega_b \times (I_b \Omega_b)
\end{bmatrix}
+
\begin{bmatrix}
0 \\
\Omega_b \times (I_b \Omega_b)
\end{bmatrix}
= \begin{bmatrix}
F_g \\
M_b
\end{bmatrix}
\]

where \( m \) represents the mass, \( F_g \) represents the resultant external force in the ground coordinate system, \( I_b \) and \( M_b \) respectively represent the moment of inertia and the resultant external moment in the airframe coordinate system, \( I_{b_{33}} \) represents the unit matrix of \( 3 \times 3 \). The external forces that tilt-rotor UAVs receive mainly include rotor pull \( F_r \), wing aerodynamics \( F_w \), and gravity \( G \). The external torques are mainly the torque produced by the rotor \( M_r \), the aerodynamic torque produced by the wing \( M_w \), and the gyroscopic effect torque \( M_{gyro} \) and anti-torque produced by the rotor \( M_q \).

Define the input of the tilt-rotor UAV as
On the basis of mechanics and mathematical analysis, a mathematical model of the UAV is established, which is expressed as follows:

\[
\begin{align*}
    u_x &= -(c_x s_x c_y + s_x s_y)u_i \\
    u_y &= -(c_x s_x s_y - s_x c_y)u_i \\
    u_z &= -c_x c_y u_i \\
    u_2 &= k_f (\omega_x^2 - \omega_y^2 + \omega_z^2 - \omega_i^2) \\
    u_3 &= k_d (\omega_x^2 + \omega_y^2 - \omega_z^2) \\
    u_4 &= k_q (\omega_x^2 - \omega_y^2 + \omega_z^2 - \omega_i^2)
\end{align*}
\]  

(2)

where \( l \) is the lateral distance from the rotor to the center of mass, \( d \) is the longitudinal distance from the rotor to the center of mass, and \( F_{wx}, F_{wy}, F_{wz} \) are the three-axis components of the total aerodynamic force generated by the wing. \( k_q \) is the anti-torque coefficient, and \( k_q \) is the pulling force coefficient of the rotor.

3. Predetermined performance function

Predetermined performance function is often used to constrain the transient and steady-state performance of the system, which usually is selected as the following exponential form[6]:

\[
\lambda(t) = (\lambda_0 - \lambda_\infty)e^{-\gamma t} + \lambda_\infty
\]

(4)

where the parameters are all positive and satisfy \( \lambda_0 = \lambda(0), \lambda_\infty = \lambda(\infty), \lambda_0 > \lambda_\infty, |\varepsilon(0)| < \lambda(0) \), thus \( \lambda(t) > 0 \) and according to the exponential law converges to \( \lambda_\infty \). The predetermined performance function is shown in Figure 2.

![Figure 2: The predetermined performance function](image)
Convert the constrained tracking error into an unconstrained tracking error, expressed as

\[ e(t) = \lambda(t)S(\varepsilon(t)) \]  

where \( e(t) \) is the conversion error, \( S(\varepsilon(t)) \) is the hyperbolic tangent function:

\[ S(\varepsilon(t)) = \frac{e^{\varepsilon(t)} - e^{-\varepsilon(t)}}{e^{\varepsilon(t)} + e^{-\varepsilon(t)}} \]  

According to the properties of the hyperbolic tangent function, the inverse function can be obtained:

\[ \varepsilon(t) = S^{-1}\left(\frac{e(t)}{\lambda(t)}\right) = \frac{1}{2} \ln \left(\frac{\lambda(t) + e(t)}{\lambda(t) - e(t)}\right) \]  

If the conversion error \( \varepsilon(t) \) is bounded, the original tracking error \( e(t) \) is also bounded, so the output range can be limited and the preset performance index requirements can be met.

### 4. Controller design

**4.1. Model unification**

In order to simplify the design of the controller, ignoring the gyroscopic effect torque generated by the rotor, and considering modeling errors and external disturbances, after sorting out can be obtain:

\[
\begin{align*}
\dot{x} &= \frac{u}{m} + \frac{F_{1x}}{m} + \Delta(\dot{x}) + d_x(t) \\
\dot{y} &= \frac{u}{m} + \frac{F_{1y}}{m} + \Delta(\dot{y}) + d_y(t) \\
\dot{z} &= \frac{u}{m} - g + \frac{F_{1z}}{m} + \Delta(\dot{z}) + d_z(t) \\
\dot{p} &= \frac{I_x - I_y}{I_z}qr + \frac{u_2}{I_z} + \Delta\dot{p} + d_p(t) \\
\dot{q} &= \frac{I_y - I_z}{I_x}pr + \frac{u_3}{I_x} + \Delta\dot{q} + d_q(t) \\
\dot{r} &= \frac{I_z - I_x}{I_y}pq + \frac{u_4}{I_y} + \Delta\dot{r} + d_r(t)
\end{align*}
\]  

The six subsystems in Eq.(8) can be unified into the following form:

\[
\begin{align*}
\dot{x}_i &= x_2 \\
x_2 &= f(x) + g_uu + d_w(t) \\
y &= x_1
\end{align*}
\]  

Where \( g_u \neq 0 \), \( d_w(t) = \Delta f(x) + d(t) \) is the composite total interference, which is the sum of the uncertainty of the system model \( \Delta f(x) \) and the external disturbance \( d(t) \), \( |d_w(t)| \leq |\Delta f(x)| + |d(t)| \leq \tilde{d}_w \) and \( \tilde{d}_w \) is a normal number.

**4.2. Non-singular terminal sliding mode control law based on predetermined performance**

Assuming that the expected output to be tracked by each subsystem is \( y_j \), the system output error can be defined as \( \varepsilon = x_i - y_j \). According to Eq.(5), the conversion error can be obtained:
\[ \varepsilon = S^{-1}\left(\frac{e}{\lambda}\right) = \frac{1}{2} \ln \left(\frac{\lambda + e}{\lambda - e}\right) = \frac{1}{2} (\ln(\lambda + e) - \ln(\lambda - e)) \quad (10) \]

then

\[ \dot{\varepsilon} = M_1 + M_2 \dot{\varepsilon} = M_1 + M_2 (f + g_0 u + d_m - \ddot{x}_d) \quad (11) \]

where

\[ M_1 = \frac{(\lambda + e) - (\dot{\lambda} + \dot{e})^2}{2(\lambda + e)^2}, \quad M_2 = \frac{(\lambda - e) - (\dot{\lambda} - \dot{e})^2}{2(\lambda - e)^2} \]

The non-singular fast terminal sliding surface is selected as follows\[7\]

\[ s = \varepsilon + \frac{1}{c_1} |\varepsilon|^{p_1/h} \text{sgn}(\varepsilon) + \frac{1}{c_2} |\dot{\varepsilon}|^{p_2} \text{sgn}(\dot{\varepsilon}) \quad (12) \]

where \( c_1 > 0, c_2 > 0 \), and \( p, q, g, h \) are all odd and \( 1 < p / q < 2, g / h > p / q \).

Take the derivation of Eq. (12), it can be obtained:

\[ \dot{s} = \dot{\varepsilon} + \frac{1}{c_1} \frac{1}{h} |\varepsilon|^{p_1/h - 1} \dot{\varepsilon} + \frac{1}{c_2} \frac{1}{q} |\dot{\varepsilon}|^{p_2 - 1} \left( M_1 + M_2 f + M_2 g_0 u + M_2 d_m - M_2 \ddot{x}_d \right) \quad (13) \]

Let \( u_c = M_2 g_0 u \), then the equivalent control law and the nonlinear switching control law are respectively designed as

\[ u_{eq} = -c_2 \frac{q}{p} |\varepsilon|^{p_2} \left( 1 + \frac{1}{c_1} |\varepsilon|^{p_1/h} \right) - M_1 - M_2 f + M_2 \ddot{x}_d \quad (14) \]

\[ u_{sw} = c_1 \frac{q}{p} \left( -k s - h_1 |s|^{\alpha} \text{sgn}(s) - h_2 |s|^{\beta} \text{sgn}(s) \right) \quad (15) \]

where \( k > 0, h_1 > 0, h_2 > 0, \alpha > 1, 0 < \beta < 1 \). Therefore, the non-singular fast terminal sliding mode control law based on predetermined performance can be obtained as:

\[ u = \frac{u_c}{M_2 g_0} = \frac{u_{eq} + u_{sw}}{M_2 g_0} \quad (16) \]

Select the Lyapunov function as \( V = \frac{1}{2} s^2 \), then it can be obtained:

\[ \dot{V} = s \dot{s} \leq |\varepsilon|^{p_1 - 1} \left( -k s^2 - h_1 |s|^{\alpha+1} - h_2 |s|^{\beta+1} + b |s| \right) \quad (17) \]

where \( b = \frac{1}{c_2} \frac{q}{p} M_2 |\bar{d}_m| > 0 \).

When the system state is far away from the sliding surface, that \( |s| \geq 1 \), thus

\[ V \leq |\varepsilon|^{p_1 - 1} \left( -k s^2 - h_2 |s|^{\beta+1} - |s| \left( h_1 |s|^{\alpha} - b \right) \right) \quad (18) \]

If \( h_1 |s|^{\alpha} - b \geq 0 \), then according to the literature\[8\], the sliding surface \( s \) converges to the area \( |s| \leq \left( \frac{b}{h_1} \right)^{\frac{1}{\alpha}} \) in a finite time.

When the system state is close to the sliding surface, that \( |s| < 1 \), thus

\[ V \leq |\varepsilon|^{p_1 - 1} \left( -k s^2 - h_2 |s|^{\beta+1} - |s| \left( h_1 |s|^{\alpha} - b \right) \right) \quad (19) \]
If \( h_1 | s |^b - b \geq \sigma \), where \( \sigma \) is an arbitrarily small positive number, the sliding surface can converge to the area \( | s | \leq \left( \frac{b + \sigma}{h_1} \right)^{\frac{1}{b}} \) in a finite time similarly.

In summary, the sliding surface can converge to a small area, the specific form is as follows:

\[
| s | \leq \Omega = \min \left\{ \frac{b}{h_1}, \left( \frac{b + \sigma}{h_1} \right)^{\frac{1}{b}} \right\}
\]

After the sliding mode surface converges to the disturbance stability region, the system state will keep sliding motion in this small neighborhood and gradually approach the equilibrium point. Then there is \( \Delta \in \Omega \), so that

\[
s = \varepsilon + \frac{1}{c_1} \left| \varepsilon \right|^{\nu/k} + \frac{1}{c_2} \left| \dot{\varepsilon} \right|^{\mu/q} = \Delta
\]

The following Lyapunov function is defined as \( V_1 = \frac{1}{2} \varepsilon^2 \), and then it can be obtained

\[
\dot{V}_1 \leq -\varepsilon^2 \left( \frac{1}{c_1} - \frac{g}{\varepsilon^k} \right) \frac{g}{2^{k/p} \varepsilon^{1/k}} V_1 \leq c_i \frac{g^{1+\nu/q}}{2^{k/p} \varepsilon^{1/k}} V_1^{2/p}
\]

Select appropriate parameters \( c_i \) to make \( \frac{1}{c_i} \geq \left( \frac{g}{\varepsilon^k} \right)^{1/k} > 0 \), then the conversion error \( \varepsilon \) can reach a small neighborhood near zero within a finite time:

\[
| \varepsilon | \leq \left( \frac{\Omega}{c_i} \right)^{\frac{1}{k}}
\]

It can be obtained that the tracking error is bounded and meets the performance requirements set by the predetermined performance function.

5. Simulation analysis

The simulation model parameters of the tilt-rotor UAV are listed in Tab.1:

| Parameter | Value       | Parameter | Value       |
|-----------|-------------|-----------|-------------|
| \( m \)   | 0.5 kg      | \( g \)   | 9.8 N·kg⁻¹  |
| \( d \)   | 0.35 m      | \( l \)   | 0.5 m       |
| \( J_p \) | \( 6 \times 10^4 \) kg·m² | \( I_y \) | 0.0634 kg·m² |
| \( I_z \) | 0.0634 kg·m² | \( I_x \) | 0.0852 kg·m² |
| \( k_x \) | \( 4.079 \times 10^5 \) N·s² | \( k_y \) | \( 7.5 \times 10^7 \) N·s² |

The initial state and target state of the UAV are set as follows:

Initial state: \((x_0, v_y, z_0) = (2, 1, 0)\) and \((\dot{\phi}_0, \dot{\theta}_0, \psi_0) = (0, 0, 0)\).

Target state: \((x_d, y_d, z_d) = (0, 0, 10)\) and \(\phi_d = \pi / 3\).

To verify the robustness of the designed control algorithm, the model uncertainty and external interference of each channel as well as the predetermined performance function are set as follows:
X: $\Delta \dot{x} = 0.02 \sin(x), \quad d_1(t) = 0.04 \sin(t), \quad \lambda_1(t) = (3.1 - 0.1)e^{-2t} + 0.1$

Y: $\Delta \dot{y} = 0.03 \sin(y), \quad d_2(t) = 0.03 \sin(t), \quad \lambda_2(t) = (2.1 - 0.1)e^{-2t} + 0.1$

Z: $\Delta \dot{z} = 0.02 \sin(z), \quad d_3(t) = 0.06 \sin(t), \quad \lambda_3(t) = (12.1 - 0.1)e^{-2t} + 0.1$

Roll: $\Delta \dot{\rho} = 0.02 \sin(\rho), \quad d_4(t) = 0.07 \sin(t), \quad \lambda_4(t) = (\pi - 0.1)e^{-2.5t} + 0.1$

Pitch: $\Delta \dot{\eta} = 0.04 \sin(\eta), \quad d_5(t) = 0.05 \sin(t), \quad \lambda_5(t) = (\pi - 0.1)e^{-2.5t} + 0.1$

Yaw: $\Delta \dot{\gamma} = 0.04 \sin(\gamma), \quad d_6(t) = 0.04 \sin(t), \quad \lambda_6(t) = (\pi - 0.1)e^{-2.5t} + 0.1$

All six channels use the same controller parameters: $c_1 = 1.5, c_2 = 2.0, g = 7, h = 3, p = 7, q = 5$ and $k_1 = 3.0, \alpha = 5 / 3, \beta = 3 / 5$. Simulation comparison curves of tracking trajectory and tracking error of each channel are shown in Fig.3-Fig.8, where “PPF” denotes the predetermined performance function, “TNFTSM” denotes the traditional non-singular fast terminal sliding mode, “PNFTSM” denotes the designed non-singular fast terminal sliding mode controller based on predetermined performance.

And the control input curves are shown in Fig.9.
From the above position and attitude angle tracking simulation, it can be seen that in the case of model uncertainty and external interference, the two control algorithms can achieve stable tracking of the target trajectory. The non-singular fast terminal sliding mode control algorithm based on predetermined performance makes the system error converge quickly within the specified performance range through the predetermined performance function. However, the traditional non-singular fast terminal sliding mode control algorithm without a predetermined performance function has a systematic error beyond the preset performance range during the control process. In addition, the non-singular fast terminal sliding mode control algorithm based on predetermined performance has a faster convergence speed. Because of the faster convergence speed, this also leads to an increase in the control input amplitude correspondingly. Therefore, the controller designed in this paper has better control performance under the allowed control input.

6. Conclusion
Aimed at the trajectory tracking control of the vertical take-off and landing mode of a small tilting quad-rotor UAV, a non-singular fast terminal sliding mode control method based on predetermined performance was proposed in this paper. By introducing a predetermined performance function into the traditional sliding mode control, the system error was always kept within the predetermined performance range. The tracking control simulation experiment was carried out in MATLAB/Simulink, and the simulation results show that the designed non-singular fast terminal sliding mode control based on the predetermined performance not only has small convergence error and fast convergence speed, but also can make the error convergence curve always within the preset performance range. It has good transient and steady-state performance, thus verifying the effectiveness and superiority of the method.

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References
[1] Badr Sherif, Mehrez Omar, Kabeel A E . A design modification for a quadrotor UAV: modeling, control and implementation[J]. Advanced Robotics, 2018:1-20.
[2] Wang Yixuan, Shi Yan, Cai Maolin, et al. Optimization of Air-fuel Ratio Control of Fuel-powered UAV Engine Using Adaptive Fuzzy-PID[J]. Journal of the Franklin Institute, 2018, 355(17):8554-8575.
[3] Zuo Zongyu, Mallikarjuna Srinath. L1 Adaptive Backstepping for Robust Trajectory Tracking of UAVs[J]. Industrial Electronics, IEEE Transactions on, 2017, 64(4):2944-2954.
[4] Chen Fuyang, Jiang Rongqiang, Zhang Kangkang, et al. Robust Backstepping Sliding-Mode Control and Observer-Based Fault Estimation for a Quadrotor UAV[J]. IEEE Transactions on Industrial Electronics, 2016, 63(8):5044-5056.
[5] Bechlioulis C.P., Rovithakis G.A. Robust Adaptive Control of Feedback Linearizable MIMO Nonlinear Systems With Prescribed Performance[J]. IEEE Transactions on Automatic Control, 2008, 53(9):2090-2099.
[6] Bechlioulis C.P., Rovithakis G.A. Adaptive control with guaranteed transient and steady state tracking error bounds for strict feedback systems[J]. Automatica, 2009, 45 (2): 532-538.
[7] Li Shengbo, Li Keqiang, Wang Jianqiang, et al. Nonsingular fast terminal-sliding-mode control method and its application on vehicular following system[J]. Control Theory and Applications, 2010(05):543-550(in Chinese).
[8] Si Yujie, Song Shenmin. Continuous reaching law based three-dimensional finite-time guidance law against maneuvering targets[J]. Transactions of the Institute of Measurement and Control, 2019,41(2):321-339.