A modified inverse method for determining spectral radiative properties of participating medium from normal-hemispherical transmittance and reflectance

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Abstract. This paper presents a modified inverse method for retrieving the spectral absorption and transport scattering coefficients of participating medium, the method combines (i) several analytical expressions obtained from the modified two-flux (TF) approximation, (ii) the Monte Carlo (MC) method for predicting normal-hemispherical transmittance $T_{nh}$ and reflectance $R_{nh}$ generally obtained from experiments, as well as (iii) an optimization method employing the genetic algorithm (GA). Three types of typical participating medium, the green glass, the low-iron glass, and the silica aerogel with known radiative properties taken form the literatures, were used to illustrate the accuracy and robustness of the modified method by retrieving the input properties from the inverse identification in which the $T_{nh}$ and $R_{nh}$ 'experimental data’ were obtained from the MC predictions. The results show that the modified inverse method was able to retrieve accurately the absorption and transport scattering coefficients of participating medium.

1. Introduction

The spectral absorption and transport scattering coefficients are essential radiative properties for modeling radiative transfer in various participating medium for solar energy usage. These properties can be predicted theoretically from the optical properties of the bulk materials [1-2], however, the method cannot be used for many practical applications due to the lack of optical properties data. Another way to obtain the radiative properties is to perform experiments, the transmittance and/or reflectance experimental data were measured, then, the extinction coefficient [3-5], or the absorption and scattering coefficients [6, 7] were determined by solving an inverse problem, and the method was widely employed for practical applications as the optical properties are not required. The inverse method generally require solve radiative transfer equation (RTE) for many times to retrieve acceptable radiative properties, thus the solution efficiency of the forward method is important. Moreover, as the inverse methods are generally ill-posed, the accuracy of the forward method should also be guaranteed.

The Monte Carlo (MC) method is generally employed to provide benchmark solutions for radiative transfer problems [8, 9], however, the method is time consuming, especially for medium with large optical thickness [10, 11]. Some approximate methods, such as the two-flux (TF) approximation, was widely employed in engineering calculations due to their high efficiency and acceptable accuracy for some specific problems [12], however, the employment of the TF approximation for inverse problems may lead
to systematic errors of the retrieved parameters due to its intrinsic low solution accuracy for some participating medium.

This paper presents an efficient inverse method to determine the absorption and transport scattering coefficients of participating medium, in the inverse method, the forward radiative transfer problem was solved by TF approximation and was corrected by corresponding MC solutions, thus the accuracy of the solutions were improved, while the time cost was not increased obviously. Three types of typical participating medium, the green glass, the low-iron glass, and the silica aerogel with known radiative properties taken form the literatures, were used to illustrate the efficiency and accuracy of the modified inverse method by retrieving the input properties, where the $T_{nh}$ and $R_{nh}$ ‘experimental data’ were obtained from the forward MC predictions.

2. Theory and analysis

2.1. Physical model and governing equation

Figure 1(a) shows a schematic of one-dimensional radiative transfer in a refracting, absorbing, scattering, and non-emitting plane parallel slab of thickness $L$. Assuming that the slab was illuminated with collimated, unpolarized, and normally incident light, thus radiative transfer in the plane parallel slab can be governed by the 1-D radiative transfer equation (RTE) expressed as [1, 2]

$$\mu \frac{\partial I_\lambda (\mu, z)}{\partial z} = -(\kappa_\lambda + \sigma_{s,\lambda}) I_\lambda (\mu, z) + \frac{\sigma_{s,\lambda}}{2} \int_{-1}^{1} \Phi_\lambda (\mu', \mu) I_\lambda (\mu', z) d \mu'$$

(1)

where, $\kappa_\lambda$ and $\sigma_{s,\lambda}$ are the absorption and scattering coefficients, $\Phi_\lambda (\mu', \mu)$ is the scattering phase function. It is usually too involved to use the scattering phase function in many applications, thus, for a simpler analysis of the directional scattering behavior, the asymmetry factor $g_\lambda = \frac{1}{2} \int_{-1}^{1} \Phi_\lambda (\mu_0) \mu_0 d \mu_0$ is generally employed for some approximate scattering phase functions. The Dirac-delta function based on the transport approximation is one of the simple scattering phase function approximated by the sum of an isotropic phase function and a $\delta$-peak in the forward direction [12]

$$\Phi_\lambda (\mu_0) = 1 - g_\lambda + 2 g_\lambda \delta(1 - \mu_0)$$

(2)

Here, $\delta$ is the Dirac function used to model forward scattering peak.

2.2. Normal-hemispherical reflectance and transmittance

By employing the TF approximation to model radiative transfer in plane parallel slab, the $R_{nh,\lambda}$ and $T_{nh,\lambda}$ of the slab can be predicted from [12]

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Figure 1 Schematic of one-dimensional radiative transfer.
\[ R_{\text{sh,}0} = R_{\text{sh,}0}^0 + \frac{D_2}{2} \left[ \frac{1 + B_2}{\rho_{\text{sh,}0}} + C_{\text{sh,}0} \right] \], and \[ T_{\text{sh,}0} = T_{\text{sh,}0}^0 + \frac{D_2}{2} \left[ \frac{1 + \rho_{\text{sh,}0}}{\rho_{\text{sh,}0}} \right] \exp(-\tau_{\text{sh,}0}) \]

where, \( \tau_{\text{sh,}0} = \beta_{\text{sh,}0} L \) \[ \left[ \kappa_{\lambda} + (1-g_{\lambda}) \sigma_{\lambda,0} \right] L \) is the transport optical thickness, the parameters \( R_{\text{sh,}0}^0 \) and \( T_{\text{sh,}0}^0 \) are expressed as [1]

\[ R_{\text{sh,}0}^0 = \rho_{\lambda,0} + \left( 1 - \rho_{\lambda,0} \right)^2 C_{\lambda,0} \]

\[ T_{\text{sh,}0}^0 = \rho_{\lambda,0} + \left( 1 - \rho_{\lambda,0} \right)^2 C_{\lambda,0} \exp(-\tau_{\text{sh,}0}) \]

where, \( \rho_{\lambda,0} \) is the normal-normal reflectivity of the optically smooth interface given by Fresnel’s equations and is expressed as \( \rho_{\lambda,0} = \left[ \left( n_{\lambda,m} - 1 \right)^2 + k_{\lambda,m}^2 \right] \left[ \left( n_{\lambda,m} + 1 \right)^2 + k_{\lambda,m}^2 \right] \), here, \( n_{\lambda,m} \) is the refractive index, and \( k_{\lambda,m} \) is the absorption index of the medium. If \( k_{\lambda,m} \) of the medium can be neglected compared with \( n_{\lambda,m} \), then, \( \rho_{\lambda,0} = \left( n_{\lambda,m} - 1 \right)^2 \left( n_{\lambda,m} + 1 \right)^2 \). The parameters \( A_{\lambda} \), \( C_{\lambda,0} \), \( B_{\lambda} \), and \( D_{\lambda} \) are defined as [12]

\[ A_{\lambda} = \left( \gamma_{\lambda,0} - \gamma_{\lambda,2} \rho_{\lambda,0} \right) \left( \phi_{\lambda} s_{\lambda} + c_{\lambda} \right) \exp(-\tau_{\text{sh,}0}) + \left( \gamma_{\lambda,2} - \gamma_{\lambda,0} \right) C_{\lambda,0} \]

\[ 1 + \phi_{\lambda}^2 s_{\lambda} + 2 \phi_{\lambda} c_{\lambda} \]

\[ B_{\lambda} = \left( \gamma_{\lambda,0} - \gamma_{\lambda,2} \rho_{\lambda,0} \right) \left( \phi_{\lambda} s_{\lambda} + c_{\lambda} \right) \exp(-\tau_{\text{sh,}0}) + \left( \gamma_{\lambda,2} - \gamma_{\lambda,0} \right) C_{\lambda,0} \]

\[ 1 + \phi_{\lambda}^2 s_{\lambda} + 2 \phi_{\lambda} c_{\lambda} \]

\[ D_{\lambda} = \gamma_{\lambda} \left( 1 - \mu_{\lambda,0}^2 \right) / \zeta_{\lambda}^2 \]

where, the parameters \( \gamma_{\lambda,0} = \gamma_{\lambda,2} \rho_{\lambda,0} / \gamma_{\lambda,1} \left( 1 + \rho_{\lambda,1} \right) \mu_{\lambda,0} = \left( n_{\lambda,m} - 1 \right)^2 \left( n_{\lambda,m} + 1 \right)^2 \left( 1 - \omega_{\text{sh,}0} \right) \left( 1 - \omega_{\text{sh,}0} \right) \left( 1 - \omega_{\text{sh,}0} \right) \right) \), \( \gamma_{\lambda,1} = 1 - 2 \gamma_{\lambda} \), \( \gamma_{\lambda,2} = 1 + 2 \gamma_{\lambda} \), \( \phi_{\lambda} = 2 \gamma_{\lambda} / \zeta_{\lambda} \), \( s_{\lambda} = \sinh(\zeta_{\lambda} \tau_{\text{sh,}0}) \), and \( c_{\lambda} = \cosh(\zeta_{\lambda} \tau_{\text{sh,}0}) \), respectively. The transport single scattering albedo \( \omega_{\text{sh,}0} \) was defined as \( \omega_{\text{sh,}0} = (1-g_{\lambda}) \sigma_{\lambda,0} / \left[ \kappa_{\lambda} + (1-g_{\lambda}) \sigma_{\lambda,0} \right] \). To predict \( R_{\text{sh,}0} \) and \( T_{\text{sh,}0} \), the slab thickness \( L \), the refractive index \( n_{\lambda,m} \), the absorption index \( k_{\lambda,m} \), and the transport scattering coefficient \( \sigma_{\lambda,\text{sh}} = \sigma_{\lambda,0} \left( 1 - g_{\lambda} \right) \) are needed.

The normal-hemispherical reflectance and transmittance of the slab can also be predicted from the MC simulations, more details about the MC simulation can be seen in the literature [8].

2.3. Inverse method

Figure 2 shows the block diagram of a classical inverse method for retrieving the absorption and transport scattering coefficients. The “experimental data” \( R_{\text{sh,}0,\text{exp}} \) and \( T_{\text{sh,}0,\text{exp}} \) are generated form the MC simulations. The objective of the inverse method was to find \( \kappa_{\lambda} \) and \( (1-g_{\lambda}) \sigma_{\lambda,0} \) that minimize the difference between the numerical predictions (by MC or TF) and the “experimental data”, this was achieved by finding the minimize value of the objective function defined as

\[ F = \sqrt{\left( R_{\text{sh,}0,\text{exp}} - R_{\text{sh,}0,\text{pred}} \right)^2 + \left( T_{\text{sh,}0,\text{exp}} - T_{\text{sh,}0,\text{pred}} \right)^2} \]

Genetic algorithm (GA) is employed as the optimization method, it was approved to be robust thus can be used to find the global minimum of the objective function [13]. The number of individuals of P = 500 and a maximum generations of G = 100 was used in the present GA optimization. The convergence criteria was such that \( F < 10^{-6} \) or the generations reached the maximum. In the present classical inverse method, the forward problem was solved by the TF approximation, while the MC method was used to generate only the \( R_{\text{sh,}0,\text{exp}} \) and \( T_{\text{sh,}0,\text{exp}} \) “experimental data”. 

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Figure 2 Block diagram of the traditional inverse method.

Figure 3 presents the block diagram of the modified inverse method. Unlike the traditional inverse method presented in figure 2, the predicted normal-hemispherical reflectance and transmittance were firstly predicted from TF approximation, and then corrected by the MC solution predicted from the best individual of the latest generation.

\[
R_{\text{nh},i,j} = R_{\text{nh},i,j}^\text{TF} + \Delta R_{\text{nh},i,j} \quad \text{and} \quad T_{\text{nh},i,j} = T_{\text{nh},i,j}^\text{TF} + \Delta T_{\text{nh},i,j} \tag{8}
\]

Where, \( R_{\text{nh},i,j} \) and \( T_{\text{nh},i,j} \) are the spectral normal-hemispherical reflectance and transmittance predicted from the TF approximation for individual \( i \) and generation \( j \). While \( \Delta R_{\text{nh},i,j} \) and \( \Delta T_{\text{nh},i,j} \) are the corrected spectral normal-hemispherical reflectance and transmittance for individual \( i \), generation \( j \), respectively, and can be calculated from

\[
\Delta R_{\text{nh},i,j} = R_{\text{nh},i,j}^\text{MC} - R_{\text{nh},i,j}^\text{TF} \quad \text{and} \quad \Delta T_{\text{nh},i,j} = T_{\text{nh},i,j}^\text{MC} - T_{\text{nh},i,j}^\text{TF} \tag{9}
\]

Where, \( R_{\text{nh},i,j}^\text{MC} \) and \( T_{\text{nh},i,j}^\text{MC} \) are the spectral normal-hemispherical reflectance and transmittance predicted from the MC method employing the best individual of generation \( j-1 \).

In the modified inverse method, the optimization method and parameter data sets are remains the same with those for the traditional inverse method.

Figure 3 Block diagram of the modified inverse method.

3. Results and discussion

3.1. Validation of the Monte Carlo code

The MC code was validated by comparing the MC predictions with those reported in the literatures [10, 11], the results were shown in Table 1. The MC predictions agree well with the data reported in the
literatures for various optical thickness $\tau_L$ and single scattering albedo $\omega$, it indicates that our MC code can predict accurately the $T_{nh}$ and $R_{nh}$, thus can be used to generate ‘experimental data’, and can be used as benchmark solutions for correcting the TF predictions presented in figure 3.

### Table 1 Validation of the $T_{nh}$ and $R_{nh}$ predicted form the present MC simulations.

| $\tau_L$ | $\omega$ | $R_{nh}$ | $T_{nh}$ |
|----------|----------|----------|----------|
| 0.1      | 0.1      | 0.01196  | 0.01196  |
| 0.5      | 0.5      | 0.07312  | 0.07285  |
| 0.9      | 0.9      | 0.16976  | 0.16913  |
| 2.0      | 0.5      | 0.11400  | 0.11279  |
| 0.9      | 0.9      | 0.36491  | 0.36144  |

3.2. Retrieved radiative properties of two types of glass and silica aerogel

The main purpose of the present study was to examine the accuracy and efficiency of the modified inverse method, therefore, we did not perform experiments, instead, we used the radiative properties reported in the literatures [16] and [7] to generate numerically the normal-hemispherical reflectance and transmittance as the input “experimental data”.

Figure 4(a) reports the retrieved $k_\lambda$ of green and low-iron glass by the two inverse methods, and Figure 4(b) presents the relative error of the retrieved properties defined as $\Delta = (k_{\text{exact}} - k_{\text{retrieved}})/k_{\text{exact}} \times 100\%$. The retrieved properties agree very well with the exact values for most wavelengths regardless of the inverse method employed, it indicates that the TF formulae based traditional inverse method can be used to retrieve absorption coefficient of glass like absorbing participating medium.

Figure 5(a) shows the retrieved $k_\lambda$ and $\sigma_s,\lambda, tr$ of silica aerogel, note that the silica aerogel was regarded as isotropic scattering medium [7], thus $\sigma_s,\lambda, tr = \sigma_s,\lambda$, figure 5(b) presents the relative error of the retrieved parameters.
Figure 5 retrieved (a) $k_\lambda$ and $\sigma_{s,\lambda}$ of silica aerogel, (b) relative error of the retrieve absorption coefficient.

It can be seen that the two methods can retrieve accurately the absorption coefficient, the relative error of the retrieved absorption coefficient fall within 2% for the whole wavelength range investigated. However, the scattering coefficient retrieved from the traditional inverse method deviate systematically from the true values, the relative error exceed 6% for wavelength larger than 6μm, it indicate that the traditional inverse method cannot retrieve accurately the scattering coefficient of silica aerogel. On the other hand, the scattering coefficient retrieved form the modified inverse method agree well with the true values (the relative error fall within 1.5%), thus the method can be used to retrieve scattering coefficient of silica aerogel. Only an increase of about 10% time cost was observed for a single inverse optimization (the generations reach the maximum) employing the modified inverse method compared with the traditional inverse method, thus proved the modified method is efficient.

4. Conclusions

This paper presents a modified inverse method for retrieving absorption and scattering coefficients of participating medium. The MC method and the TF approximation were employed to solve RTE in the inverse method. The modified inverse method was used to retrieve radiative properties of glass and silica aerogel. Both the traditional and modified inverse methods can be used to retrieve absorption coefficient of green glass and low-iron glass. The absorption coefficient of silica aerogel retrieved from the two inverse methods go well with the exact values, however, the scattering coefficient retrieved from the traditional inverse method deviate from the true values, thus the traditional inverse method cannot be used to retrieve accurately the scattering coefficient. The modified inverse method can be used to retrieve accurately the absorption and scattering coefficients of silica aerogel like materials.

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