Study on Tire-attached Energy Harvester for Low-speed Actual Vehicle Driving

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Abstract. This study reports a tire-attached energy harvester, in which a cantilever beam pasted piezoelectric film and magnets with the same polarity are fabricated as a bistable vibrating system, for low-speed actual-vehicle driving. As the wheel rotates, the energy harvester is subjected to the noise produced from the interaction between the paved road and the rotating tire, and tangentially gravitational force as a periodic input can be applied to achieve the occurrence of stochastic resonance. Stochastic resonance can significantly stimulate the response of the bistable vibrating system, and therefore enhance the energy harvesting efficiency.

1. Introduction

Several researchers have developed bistable energy harvesters, which performs a single degree of freedom behavior in a certain direction to exploit stochastic resonance on a vibrator [1]. In this approach, a novel system presented can be excited to vibrate under the condition of double degrees of freedom by the rotating motion; thereby, the periodic force is autonomously self-excited from a combination of gravitational effects and the rotational degree of freedom without consuming additional energy.

Moreover, this energy harvesting model is exposed under low-speed actual noise instead of typical Gaussian noise [2], ensuring that this novel bistable device has the ability to perform a real behaviour for the realization of occurrence of stochastic resonance. Furthermore, comparison with other researches related to nonlinear bistable systems applied on the tire environment [3], with consideration of the theory of stochastic resonance in this research [4], the vibrating responses become significantly strong to generate more available energy, where the energy captured from not only the road ambient, but also its gravitational acting, especially at the lower frequency filed when one potential well motion of nonlinear bistable system is dominant under the only periodical force excitation.

Based on experiment results, when the angular frequency of tire revolution is 34.5 rad/s, which is close to the value theoretically predicted for the stimulation of stochastic resonance, the system can maintain the inter-well motion going all the time, and the displacement responses maintain the highest
level. Hence, it is demonstrated that the prototype of the proposed energy harvester can be efficiently utilized under the low-speed actual-vehicle driving circumstances.

2. **Mathematic model**

A new energy harvester model is presented for adjusting Kramers rate [5] to the corresponding angular velocity of automobile tires by tuning the distance $d$ between two magnets, when tire wheel revolves at different speeds, as shown in Figure 1. In order to eradicate all passive centrifugal force effect on the stiffness of the cantilever beam, the tip mass of the proposed energy harvester is mounted on the center axis of the tire wheel. By using a dipole mode [6], the analytic expression of the resultant restoring force of magnetic mass and cantilever beam in the tangential direction is derived and can be expressed as

$$F(x) = \frac{\mu_0 v^2}{4\pi} \left[ \frac{3(M_x - xM_y) + M_z}{(d^2 + x^2)^{3/2}} - \frac{15x(xM_x + dM_y)(xM_z + dM_y)}{(d^2 + x^2)^{5/2}} \right] - kx,$$

where $\mu_0$ is the permeability of free space, $v$ is the volume of the magnets, and $M = (M_x, M_y, M_z)$ are the magnetization amplitudes of permanent magnet attached on frame and the magnetic tip mass, $k$ is the linear coefficient of the beam, and $x$ is the deflection of the tip magnet.

![Figure 1. Schematic illustration of proposed energy harvester mounted on a rotating tire.](image)

The equation (1) can be expanded into a Taylor series, calculated around $x = 0$ and truncated as

$$F(x) = \frac{\mu_0 v^2}{4\pi d^3} \left[ \frac{9M_x M_y - 12M_y M_z}{8\pi d^4} - k \right] x - \frac{\mu_0 v^2}{8\pi d^3} \left( 75M_x M_y - 90M_y M_z \right) x^3.$$

From the reference [3], the duffing equation of tire rotation system can be acquired as

$$m\ddot{x} + c\dot{x} + kx = N(t) + G\sin\omega t,$$

where $m$ is the mass of the magnetic tip mass, $c$ is the viscous damping coefficient of the beam, and $N(t) = \sqrt{E(t)}$ is the noise excitation from the road. By means of the rotation of tire the periodic force $G$ will be produced from the effect of gravity on the end magnet, $a$ is the combinational linear coefficient of the beam and
magnets, and \( b \) is the nonlinear coefficient between two magnets. Thereby, the nonlinear restoring force can be given as

\[
F_{ax}^{'} = -ax + bx^3.
\]

Therefore, corresponding to Eq. (2), Eq. (5) and (6) are obtained as follows:

\[
a = \frac{\mu \nu^2}{4\pi} \left( \frac{3(3M_{\alpha}M_{\beta} + M_{\alpha}M_{\beta})}{d^3} - \frac{15M_{\alpha}M_{\beta}d^2}{d^3} \right) - 4\pi k,
\]

\[
b = \frac{\mu \nu^2 (75M_{\alpha}M_{\beta} - 90M_{\alpha}M_{\beta})}{8\pi d^7}.
\]

Substituting Eq. (5) and (6) into Eq. (7) Kramers rate equation in the case of weak friction as

\[
f = \left[ \frac{\mu \nu^2}{4\pi} \left( \frac{3(3M_{\alpha}M_{\beta} + M_{\alpha}M_{\beta})}{d^3} - \frac{15M_{\alpha}M_{\beta}d^2}{d^3} \right) - 4\pi k \right] \exp \left( -\frac{4\pi k - \mu \nu^2 (3M_{\alpha}M_{\beta} + M_{\alpha}M_{\beta})}{8\pi D\mu \nu^2 (75M_{\alpha}M_{\beta} - 90M_{\alpha}M_{\beta})} \right),
\]

Hence, Kramers rate formula is derived as a function of the distance between two magnets.

3. Road noise measurement and simulation study

The smooth paved road and EV car (Coms, ZAD-TAK30-DS, TOYOTA AUTO BODY Co., Ltd) are taken into consideration as the experimental environments (see Figure 2). A wireless acceleration sensor (MVP-RF3-J, MicroStone Corporation) is installed on the inside of the front-wheel in where the road noise can be gauged vertically (see Figure 3). When the vehicle travels on the smooth paved road at the low speeds, the density of the accelerations almost keep the same level by the analyses of the measured signals. Hence, the acceleration measured around the speed of 20 km/h can be considered as the simulation condition. Based on Kramers rate, the stochastic resonance is easy to occur at the angular frequency which is smaller than 43 rad/s.

![Figure 2. Test car travelling on paved road.](image)

![Figure 3. The road noise measurement on front-wheel.](image)

From the implemented simulation results as depicted in Figure 4, under the input signals of gravitational acceleration and measured road noise, the responses of cantilever beam become strong owing to the inter-well motion between two stable positions, when the angular frequency of tire revolution is 38 rad/s, which is close to the value theoretically predicted for the stimulation of stochastic resonance, the system can maintain the inter-well motion going all the time, and the displacement and velocity responses maintain the highest level, among the five cases as shown in Figure 4(c). The avially estimated power reaches maximum of 3.5 mW with the effective value of
1.02 mW as depicted in Figure 5. Hence, it is demonstrated that the probability of the proposed energy harvester is verified under the low-speed actual-vehicle driving circumstances.

![Displacement and velocity responses under paved road noise excitation and self-provided rotational force at different rolling angular frequencies](image)

**Figure 4.** Displacement and velocity responses under paved road noise excitation and self-provided rotational force at different rolling angular frequencies: (a) $\omega = 33$ rad/s, (b) $\omega = 35$ rad/s, (c) $\omega = 38$ rad/s, (d) $\omega = 40$ rad/s, and (e) $\omega = 42$ rad/s.

![Instantaneous power (mW) vs Time (s)](image)

**Figure 5.** Availably estimated power under stochastic resonance.

4. **Experimental investigation**

A shaker (F-08000BDH/SLS16/Z02, EMIC Corporation) that can generate periodic vibration and noise vibration under the range of amplitude 100 mm$_{p-p}$ and frequency 0 Hz – 2000 Hz is utilized to reappear the measured real-world road noise excitation as shown in Figure 6. The energy harvester installed on the shaker consists of a cantilever beam that has dimensions of 42 mm × 10 mm × 4 mm, and an adjustable screw rod that enables it to be optimal application for the automotive field through adjusting the distance between two magnets as described in figure 7.

By means of analysing the power spectral density of the measured road acceleration, it shows that the energy of paved road noise excitation is trapped in the frequency bandwidth of 0 Hz – 200 Hz, and the simulated road noise from the shaker is consistent with the real-world road noise by using a composite filter as compared in figure 8. Finally, as shown in figure 9, under both the gravitational
effect and real-world road noise condition, the response of cantilever beam can be enhanced due to the occurrence of stochastic resonance at the angular frequency of 34.5rad/s.

Figure 6. Overview of the experimental setting.

Figure 7. Fabricated energy harvester.

Figure 8. Regeneration of real-world road noise.

Figure 9. Responses under three conditions.

5. Conclusion
In this study, a miniature energy harvester is proposed and fabricated to apply to low-speed vehicle tires. The simulation study is investigated under the case of real-world road noise excitation and gravitational effect, and it is verified that the feasibility of stochastic resonance can enhance the vibration response. Meanwhile, the effective power that can be generated is of the order of 1.02 mW. By using a shaker to accurately regenerate the vibrational noise from tire rotation, the occurrence of stochastic resonance is validated at the low-speed actual-vehicle driving circumstances.

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