Folding model analysis of the elastic and inelastic scattering of K$^+$ from $^{12}$C

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Optical potentials for the scattering of K$^+$ from the $^{12}$C nucleus are calculated using the folding model. Angular distributions of the elastic and inelastic scattering differential cross sections at 635, 715, and 800 MeV/c are successfully described using these potentials. Good fits with data are obtained without modifying any of the potential parameters. The extracted deformation parameters and reaction and total cross sections are also considered.

Subject Index D20, D21, D22, D23

1. Introduction

The reasons for interest in the kaon-nucleus problem are mostly due to the differences between the basic KN and $\pi N$ amplitudes. These differences are fundamental and arise in large measure because of the nonzero strangeness of kaons. We are dealing with a situation quite different from the one presently existing for proton or pion scattering. The K$^+$-nucleus cross section should be fairly weak and the amplitude is reasonably simple because of the simple K$^+$-nucleon interactions. We expect the K$^+$-nucleus interaction to be the weakest of any strongly interaction probe, and the resulting mean free path to be large (in the order of 7 fm) [1]. There is no true absorption of K$^+$ to form complicated things, as distinct from the case for low energy $\pi$-nucleus scattering, where the mean free path is also large.

Recently, the elastic and inelastic scattering of K$^+$ from $^6$Li and $^{12}$C at 635, 715, and 800 MeV/c were analyzed [2,3], using the Watanabe superposition model with a Woods–Saxon shape together with the coupled-channel Born approximation (CCBA) [4]. It is well known that there are ambiguities in the phenomenological optical potentials, where it is often found that several potentials fit the same experimental elastic scattering data equally well. This suggests that a comparison between theoretical and phenomenological potentials is not sufficient to test the reliability of theoretical models, since they may differ significantly from each other and yet be consistent with experiment.

In the present work we study the K$^+$ optical potentials for $^{12}$C at 635, 715, and 800 MeV/c. The optical potential has been obtained using the Watanabe superposition model, in terms of the alpha-particle optical potential, taking into account the 3$\alpha$-cluster structure of $^{12}$C. It is, therefore, preferable to test theoretical potentials by comparing them directly with experimental data without modifying any of the parameters. The present manuscript is organized as follows: in the next section the theoretical formalism is presented, results and discussion are given in Sect. 3, and the conclusions are presented in Sect. 4.
2. Formalism

On the basis of the Watanabe superposition model [5], the optical potential of the $^{12}$C nucleus has been investigated in terms of the $\alpha$-particle optical potential, using the $3\alpha$-cluster structure to describe the $^{12}$C nucleus:

$$V_{\text{Watanabe}}^{12\text{C}}(r) = \int |\phi(R, \rho)|^2 \left\{ V_1 \left( \sqrt{\frac{1}{3}} \bar{\rho} - \frac{1}{2} \bar{R} \right) + V_2 \left( \sqrt{\frac{1}{3}} \bar{\rho} + \frac{1}{2} \bar{R} \right) + V_3 \left( \sqrt{\frac{2}{3}} \bar{\rho} \right) \right\} d\bar{R} d\bar{\rho},$$

(1)

where $\bar{r}$ is the separation vector between the centers of the two scattered particles, $V_1$, $V_2$, and $V_3$ are the optical potentials of the alpha clusters constituting $^{12}$C and $\phi(R, \rho)$ is the internal wave function of $^{12}$C. The internal position vectors $\bar{R}$ and $\bar{\rho}$ are defined by the position vectors of the three alpha particles constituting $^{12}$C as

$$\bar{R} = \bar{R}_1 - \bar{R}_2 \quad \text{and} \quad \bar{\rho} = \bar{R}_3 - \frac{1}{2} (\bar{R}_1 + \bar{R}_2).$$

Neglecting the internal structure of the $\alpha$ clusters in $^{12}$C, the internal wave function reduces to [6]

$$|\phi(R, \rho)|^2 = \frac{24\sqrt{3}\gamma^3}{\pi^3} e^{-\gamma(4\rho^2+3R^2)}.$$  

(2)

The form of $V_{\text{Watanabe}}^{12\text{C}}$ was obtained by Taylor expansions of $V_1(|\bar{r} + \frac{1}{3} \bar{\rho} - \frac{1}{2} \bar{R}|)$, $V_2(|\bar{r} + \frac{1}{3} \bar{\rho} + \frac{1}{2} \bar{R}|)$, and $V_3(|\bar{r} - \frac{2}{3} \bar{\rho}|)$ about $\bar{R} = \bar{\rho} = 0$, up to second order in $\bar{R}$ and $\bar{\rho}$. Such expansions reduce $V_{\text{Watanabe}}^{12\text{C}}$ of expression (1) to the simple form [7]

$$V_{\text{Watanabe}}^{12\text{C}}(r) = 3 V_{\alpha-\text{folded}}(r),$$

(3)

where

$$V_{\alpha-\text{folded}}(r) = V_{\alpha}(r) + \frac{1}{36\gamma} \left\{ \frac{d^2 V_{\alpha}(r)}{dr^2} + \frac{2}{r} \frac{d V_{\alpha}(r)}{dr} \right\},$$

with

$$V_{\alpha}(r) = (-V_o - i W_I) \left( 1 + \exp \left( \frac{r - R_{w} w}{d_{w}} \right) \right)^{-1}. \quad \text{(4)}$$

Here, $V_o$, $R_o$, $r_w A^{1/3}$, and $d_w$ are, respectively, the depth, radius, and diffuseness of the real part of the potential and $W$, $R_w$ $r_w A^{1/3}$, and $d_w$ are the corresponding quantities for the imaginary part, $A = 4$. The six parameters $V_o$, $r_o$, $d_o$, $W_I$, $r_w$, and $d_w$ of the WS potential are taken to be the average values given in Table 2 in Ref. [2]; these values are $V_o = 28 \text{ MeV}$, $W_I = 55 \text{ MeV}$, $r_v = r_w = 1.6 \text{ fm}$, and $d_v = d_w = 0.7 \text{ fm}$.

On the other hand, since $\phi(R, \rho)$ of expression (2) depends explicitly on the spatial coordinates $R$ and $\rho$, $\phi(R, \rho)$ is symmetric with respect to the exchange of the spatial coordinates of the alpha clusters constituting $^{12}$C. This allows Eq. (1) to be written as

$$V_{\text{Watanabe}}^{12\text{C}}(r) = 3 \int |\phi(R, \rho)|^2 V_{\alpha} \left( \sqrt{\frac{2}{3}} \bar{\rho} \right) d\bar{R} d\bar{\rho}. \quad \text{(5)}$$
Taking the $z$ axis along the direction of the vector $\vec{r}$ and putting (2) into (5), one finds

$$V_{12C}(r) = \frac{48\gamma^{3/2}}{\sqrt{\pi}} \int_0^\infty \rho^2 \exp(-4\gamma\rho^2) \int_{-1}^1 V_\alpha \left( \left| \vec{r} - \frac{2}{3} \vec{\rho} \right| \right) d\mu d\vec{\rho},$$

(6)

where $\mu = \cos \theta = \frac{\vec{r} \cdot \vec{\rho}}{r \rho}$, and $\theta$ is the angle between the vectors $\vec{r}$ and $\vec{\rho}$.

The potentials given by Eqs. (3) and (6) depend on the parameter $\gamma$ of the internal wave function of $^{12}$C. This parameter may be adjusted to give the root mean square (RMS) radius of the ground state of the $^{12}$C nucleus. Adopting a definition for the RMS radius of the $^{12}$C nucleus and neglecting the antisymmetrization between the three constituent alpha clusters of $^{12}$C [8], one may write

$$\langle r^2 \rangle = \int \phi(R, \rho)^* O_{op} \phi(R, \rho) \, d\vec{R} \, d\vec{\rho}$$

(7)

where $\langle r^2 \rangle$ is the mean square radius, the operator $O_{op}$ is given by

$$O_{op} = \frac{1}{12} \sum_{i=1}^{12} \left( \vec{R}_i - \frac{1}{12} \sum_{i=1}^{12} \vec{R}_i \right)^2,$$

(8)

and $\vec{R}_i$ is the position vector of the $i$th nucleon of the $^{12}$C nucleus. As a first approximation one neglects the internal structure of the constituent $\alpha$ clusters, hence the operator $O_{op}$ reduces to

$$O_{op} = \frac{2}{9} \rho^2 + \frac{1}{6} R^2.$$

(9)

The substitution of (2) and (9) into (7) yields

$$\langle r^2 \rangle = \frac{1}{6\gamma}.$$  

(10)

One may include the internal structure of the $\alpha$ particle by adding a term representing the RMS radius of the $\alpha$ particle

$$\langle r^2 \rangle_{\text{exp}} = \frac{1}{6\gamma} + \langle r^2 \rangle_{\alpha \text{ exp}}.$$  

(11)

where $\langle r^2 \rangle_{\text{exp}}^{1/2} = 2.37$ fm [9] and the experimental RMS radius of the $\alpha$ particle, $\langle r^2 \rangle_{\alpha \text{ exp}}^{1/2} = 1.43$ fm [10]. Neglecting the internal structure of the $\alpha$ particle (Eq. (10)), we get $\gamma = 0.02967$ fm$^{-2}$. On the other hand, if we include the structure of the $\alpha$ particle (Eq. (11)), we get $\gamma = 0.04667$ fm$^{-2}$. Using Eq. (11), Kermode’s value of $\gamma = 0.0884$ fm$^{-2}$ [6] gives an RMS radius for $^{12}$C equal to 2.116 fm.

In the present work, we adopt the value $\gamma = 0.04667$ fm$^{-2}$ in order to calculate potentials (3) and (6). The potentials obtained are used to analyze the elastic scattering process.

Inelastic-scattering measurements are usually analyzed using a deformed optical model potential [11]. This provides a transition potential whose radial dependence is

$$F_l(r) = -\delta_l \frac{dV(r)}{dr},$$

(12)

where $V(r)$ is the optical potential found to fit the corresponding elastic scattering (Eqs. (3) and (6)), the deformation length $\delta_l$ determines the strength of the interaction, and $l$ is the multipolarity; it denotes the corresponding deformation length for the transition to the $2^+$ and $3^-$ states in $^{12}$C.
3. Results and discussion

Based upon the Watanabe superposition model, both expressions (3) and (6) fold the alpha-particle optical potential into the internal wave function of $^{12}$C. Expression (3) is an approximate relation for $V_{^{12}C}$ in terms of $V_\alpha$, while expression (6) is an exact one. The potential $V_\alpha$ used in expressions (3) and (6) is taken, as mentioned in the previous section, from the average potential parameters listed in Table 2 in Ref. [2]. They are kept fixed during the calculations.

The resulting real and imaginary optical potentials for K$^+$ scattered from $^{12}$C from expressions (3) and (6) are shown in Fig. 1. As shown in this figure, the real parts of both potentials (left panel), calculated using $\gamma = 0.04667 \text{ fm}^{-2}$, have very similar shapes. The full folding potential, however, seems to be slightly shallower than the other one over the total plotted radial range ($R = 0–9 \text{ fm}$). Similar behavior is noticed for the imaginary parts of both potentials (right panel). Very similar results are also found considering the other values of the $\gamma$ parameter, 0.02967 and 0.0884 fm$^{-2}$. So, there is no need to plot the potentials derived using these values. Hence we can note that the Taylor expansion used here is an appropriate simplification to calculate the optical potentials of the K$^+–^{12}$C system. The potentials of expression (3) and (6) are used to generate the angular distribution of differential cross sections for the elastic and inelastic scattering of K$^+$ from $^{12}$C using the value of the parameter $\gamma$, which equals 0.04667 fm$^{-2}$. An analysis of the experimental data of K$^+$ elastically and inelastically scattered from $^{12}$C at different momenta has been carried out using the computing program CHUCK3 [4], fed with the values obtained using expressions (3) and (6). The radial integrations have been carried out to a maximum radius of 40 fm in steps of 0.1 fm to account properly for Coulomb excitations. The calculations have been done by individually coupling each state to the ground state for all cases under consideration. The Coulomb potential used here is due to a uniformly charged sphere of radius $R_C = r_{OC} A^{1/3} \text{ fm}$, where $r_{OC} = 1.2 \text{ fm}$ for $^{12}$C [12], $A = 12$. The resulting angular distributions are shown in Figs. (2–4) compared with the corresponding experimental data taken from Refs. [13,14].

Successful reproduction of the elastic scattering of kaons from $^{12}$C is obtained by the present calculations of both types of potentials. This may suggest that the effects of break-up and stripping
Fig. 2. Differential elastic scattering cross sections for 635, 715, and 800 MeV/c K\(^+\) on \(^{12}\)C. The solid curves are the optical potential predictions from the approximate expression (3) and the broken curves are from the exact expression (6), with \(\gamma = 0.046 \, 67 \, \text{fm}^{-2}\). The solid circles represent the experimental data taken from Refs. [13,14].

Fig. 3. Differential inelastic scattering cross sections of 635, 715, and 800 MeV/c K\(^+\) leading to the first excited 2\(^+\) state in \(^{12}\)C with the present optical potentials with \(\gamma = 0.046 \, 67 \, \text{fm}^{-2}\). The experimental data are taken from Refs. [13,14].

corrections are negligible here due to the large binding energy of \(^{12}\)C against three-alpha break-up. It may also be useful to mention here that the considered experimental data were previously analyzed using the nuclear matter densities and a Kisslinger local potential for positive kaons elastically and inelastically scattered from \(^6\)Li and \(^{12}\)C at different energies by simply taking into account relativistic kinematics [15]. It could be noted that the present calculations fit the data as well as these previous calculations. It is noted, however, that the comparison with the data is made over a limited angular range (\(\leq 50^\circ\)); additional measurements at angles larger than 50\(^\circ\) are required in order to investigate
the reality of the considered potentials. Therefore, the calculated differential cross sections depend
consecutively on the parameter $\gamma$ such that the cross sections increase as $\gamma$ increases, and the fitting
with the experimental data when $\gamma = 0.04667 \text{ fm}^{-2}$ is more reasonable than that for the other two
values of $\gamma$ at all energies considered here. The diffractive maxima and minima increase as the beam
energy increases.

Since inelastic scattering in the collective model is driven by the derivative of the optical potential,
these calculations are carried out for inelastic scattering to the $2^+$ and $3^-$ states of $^{12}\text{C}$. The present
calculations are made for $K^+$ at 635, 715, and 800 MeV/c using the average potential parameters
mentioned above. The CCBA fits to the inelastic angular distribution for the excitation of the $2^+$
and $3^-$ states in $^{12}\text{C}$ are shown in Figs. 3 and 4. The present values of deformation length $\delta_l$
are adjusted to obtain reasonable agreement with the data. To calculate the deformation parameter from
the deformation length, we use the relation

$$\delta_l = \beta_l r_o A^{1/3},$$  \hspace{1cm} (13)

where $\beta_l$ is the deformation parameter and $r_o = 1.6 \text{ fm}$. The resulting deformation parameters for
both the real and imaginary potentials are listed in Table 1. As shown in Figs. 3 and 4, the inelastic
scattering data for the $2^+$ and $3^-$ collective states in $^{12}\text{C}$ are well reproduced by the deformed poten-
tials using the same parameters employed in the elastic scattering analysis. The obtained deformation
parameters $\beta_l$ agree well with the corresponding values extracted by others [13,16,17]. The Watanabe
superposition model is thus a good alternative formalism to other sophisticated formalisms.

The CHUCK3 code, using either of the two forms of potential considered here, also calculates
the reaction and total cross sections, $\sigma_R$ and $\sigma_T$, respectively, of $K^+$ scattering from $^{12}\text{C}$ at 488,
531, 656, and 714 MeV/c kaon laboratory momenta. These energies are chosen because there are
corresponding calculations of $\sigma_R$ and $\sigma_T$. Figure 5 shows the predicted values of $\sigma_R$ and $\sigma_T$
from the present work compared with those estimated by Friedman et al. [18]. The values of $\sigma_R$ and $\sigma_T$
predicted by the two potentials are in good agreement with those of Ref. [18]. However, it is noticed
that, for all considered energies, the predictions of the exact expression (6) are the nearest to the
corresponding cross sections estimated in Ref. [18] for both the $\sigma_R$ and $\sigma_T$ results.
### Table 1. Deformation parameters $\beta_r$ and $\beta_i$ obtained from $K^+$ inelastic scattering from $^{12}$C compared with those of previous studies.

| $P_{\text{lab}}$ (MeV/c) | State | Present | Other work | Refs. |
|--------------------------|-------|---------|------------|-------|
| 635                      | $2^+$ | 0.60    | 0.51       | 0.58  | [13] |
| 715                      |       | 0.62    | 0.50       | 0.40–0.61 | [16] |
| 800                      |       | 0.61    | 0.53       | 0.55–0.59 ± 0.05 | [17] |
| 635                      | $3^-$ | 0.50    | 0.46       | 0.48  | [13] |
| 715                      |       | 0.50    | 0.45       | 0.37–0.41 | [16] |
| 800                      |       | 0.51    | 0.44       |       | |

### 4. Conclusions

Considering the $3\alpha$-cluster structure of the $^{12}$C nucleus, the $K^+–^{12}$C nuclear potential is derived using the Watanabe superposition model. Two procedures in the calculation are adopted. Firstly, we use the Taylor expansion approximation and, secondly, an exact calculation is performed using the folding model formalism. Almost identical potentials are obtained using both the considered procedures. Angular distributions of the differential cross section for $K^+$ elastically and inelastically scattered from $^{12}$C at 635, 715, and 800 MeV/c kaon laboratory momenta are calculated, using both the extracted real and imaginary potentials. Both potentials produce a good reproduction of the data. However, the calculation of the second folding procedure is more successful in describing the data than that based upon the approximated Taylor expansion, particularly for the inelastic scattering. The extracted reaction and total reaction cross sections, as well as the deformation parameters for both considered excited states, are quite consistent with the corresponding values deduced in previous studies using more sophisticated calculations. Finally, it is worth concluding that the $3\alpha$ representation of the $^{12}$C nucleus seems to be a fruitful tool for constructing single folding optical potentials to perform successful analysis of scattering reactions involving the $^{12}$C nucleus.

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