Noncommutative gauge theory
with arbitrary U(1) charges

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Abstract

It is well-known that the charge of fermion is 0 or ±1 in the U(1) gauge theory on noncommutative spacetime. Since the deviation from the standard model in particle physics has not yet observed, and so there may be no room to incorporate the noncommutative U(1) gauge theory into the standard model because the quarks have fractional charges. However, it is shown in this article that there is the noncommutative gauge theory with arbitrary charges which symmetry is for example SU(3+1)*. This enveloping gauge group consists of elements \(\exp \{ \sum_{a=0}^{8} T^a x^a(\theta, \theta) + Q \beta(\theta, \theta) \} \) with \(Q = \text{diag}(e, e, e, e')\) and the restriction \(\lim_{\theta \to 0} \alpha^0(\theta, \theta) = 0\). This type of gauge theory is emergent from the spontaneous breakdown of the noncommutative SU(N)* or SO(N)* gauge theory in which the gauge field contains the 0 component \(A^0_{\mu}(x, \theta)\). \(A^0_{\mu}(x, \theta)\) can be eliminated by gauge transformation. Thus, the noncommutative gauge theory with arbitrary U(1) charges can not exist alone, but it must coexist with noncommutative nonabelian gauge theory. This suggests that the spacetime noncommutativity requires the grand unified theory which spontaneously breaks down to the noncommutative standard model with fractionally charged quarks.

1 Introduction

In the past several years, field theories on the noncommutative(NC) spacetime have been extensively studied from many different aspects. The motivation comes from the string theory which makes obvious that end points of the open strings trapped on the D-brane in the presence of two form B-field background turn out to be noncommutative[1] and then the noncommutative supersymmetric gauge theories appear as the low energy effective theory of such D-brane[2], [3]. Hayakawa[4] indicated that the matter field must have charge 0 or ±1 in U(1) NC gauge theory in order to keep the gauge invariance of the theory. Armoni[5] also indicated that U(N) gauge theory has the consistency in calculations of gluon propagator and three gluons vertex to one loop order, whereas SU(N) gauge theory is not consistent. These problems have been overcome by Wess et. al. [6] who constructed nonabelian gauge theory on the enveloping Lie algebra resulting from the Moyal star product. No extra fields other than fields in ordinary commutative gauge field theory appear in their formulation after performing the Seiberg-Witten map[3]. This approach has allowed them to construct the noncommutative standard model as well as SO(10) GUT[7]. NC standard model is also constructed by [8] in the different approach.

In this article, we will formulate the NC nonabelian gauge theory by use of the enveloping gauge group SU(N)* with enveloping Lie algebra which reduces to ordinary SU(N) gauge group in the limit of NC parameter \(\theta^{\mu\nu} \to 0\). Seiberg-Witten map[3] doesn’t play an important role as in Wess et.al.[6]. Thus, our NC non-abelian gauge theory is different from that of Wess et. al. [6]. Our nonabelian gauge theory has no \(A^0_{\mu}(x)\) component since it is only induced by gauge transformation and can be eliminated away. Thus, we don’t need to handle additional fields.

After the construction of standard model in particle physics more than decades ago, there have been several occasions that indicated the experimental deviations from the standard model. However, such deviations ultimately shrank to nothing and the correctness of the standard model has been confirmed. Thus, though there are many advanced theories beyond standard model, they must reduce to the standard model in their characteristic limits. This is the case also in the NC field theory. If not, it is branded not to be the qualified theory beyond standard model. The indication by Hayakawa[4] is serious to the NC gauge theory since quarks in standard model have fractional charges. There must be the story
other than Hayakawa’s indication if the noncommutativity on the spacetime is somewhat true in nature. We overcome this difficulty by considering nonabelian SU(N)* gauge theory in which the 0 component \( A_0(x, \theta) \) of gauge field is induced by gauge transformation, and the spontaneous symmetry breakdown of such gauge theory. Then, referring to this case, we propose the NC gauge theory with arbitrary U(1) charge. We conclude that such gauge theory can not exist alone, but it must coexist with NC nonabelian gauge theory.

This article consists of 5 sections. In second section, the NC nonabelian gauge theory is proposed. In third section, the spontaneous symmetry breakdown of SU(4)* gauge theory is discussed in order to show that quarks has the fractional \( B \)-charges whereas the lepton has charge \(-1\). In fourth section, the gauge theory with arbitrary charge is proposed referring to the example in third section. The last section is devoted to short conclusions.

## 2 Nonabelian gauge theory on noncommutative spacetime

Let us first consider the nonabelian gauge theory on the NC spacetime with the symmetry SU(N) given by the Lagrangian

\[
\mathcal{L} = -\frac{1}{2} \text{Tr} [F_{\mu\nu}(x) \ast F^{\mu\nu}(x)] + \bar{\Psi}(x) \ast \{ i\gamma^\mu (\partial_\mu - igA_\mu(x)) - m \} \ast \Psi(x)
\]

\[
+ \text{Tr} \left[ (D^\mu \varphi(x))^\dagger \ast D_\mu \varphi(x) + m^2 \varphi(x) \ast \varphi(x) - \lambda(\varphi(x))^\dagger \ast \varphi(x))^2 \right],
\]

where we omit the gauge fixing and FP ghost terms. The Moyal star product of functions \( f(x) \) and \( g(x) \) is defined as

\[
f(x) \ast g(x) = e^{\frac{1}{2} \theta^\mu \nu \partial_\mu \partial_\nu f(x) g(x)} \bigg| _{x_1 = x_2 = x},
\]

where \( \theta^{\mu\nu} \) is two rank tensor to characterize the noncommutativity of spacetime. Though \( \theta^{\mu\nu} \) is usually seemed to be a constant not to transform corresponding to Lorentz transformation, \( \theta^{\mu\nu} \) is regarded as two rank tensor in this letter as discussed in [9]. \( \Psi(x) \) is the fermion field with the fundamental representation. The quantity

\[
F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - ig[A_\mu(x), A_\nu(x)]_\ast
\]

is the field strength of gauge field with the configuration

\[
A_\mu(x) = \sum_{a=0}^{N^2-1} A_\mu^a(x) T^a.
\]

and the field \( \varphi(x) \) is the Higgs boson belonging to the adjoint representation of SU(4) which covariant derivative is

\[
\mathcal{D}^\mu \varphi(x) = \partial^\mu \varphi(x) - ig[A^\mu(x), \varphi(x)]_\ast.
\]

The gauge transformations of fields in (2.1) are defined as

\[
A_\mu^a(x) = U(x, \theta) \ast A_\mu(x) \ast U^{-1}(x, \theta),
\]

\[
\Psi^a(x) = U(x, \theta) \ast \Psi(x),
\]

\[
\varphi^a(x) = U(x, \theta) \ast \varphi(x) \ast U^{-1}(x, \theta)
\]

where the gauge transformation function \( U(x, \theta) \) is written as

\[
U(x, \theta) = e^{i \alpha(x, \theta) \ast} = \sum_{n=0}^{\infty} \frac{i^n}{n!} \alpha(x, \theta) \ast \alpha(x, \theta) \ast \alpha(x, \theta) \ast \cdots \ast \alpha(x, \theta)
\]

in terms of the Lie algebra valued function

\[
\alpha(x, \theta) = \sum_{a=0}^{N^2-1} \alpha^a(x, \theta) T^a
\]
with the condition that
\[
\lim_{\theta \to 0} a_0(x, \theta) = 0.
\] (2.9)

We call the ensemble of Lie algebra valued functions \(2.8\) keeping the condition \(2.9\) enveloping Lie algebra. In similar way, the ensemble of gauge function \(2.9\) is enveloping gauge group denoted by \(\text{SU}(N)^*\). The star commutator between two Lie algebra valued functions is calculated as

\[
[a(x, \theta), b(x, \theta)] = \sum_{a, b=0}^{N^2-1} \left( \alpha^a(x, \theta) \ast \beta^b(x, \theta) T^a T^b - \beta^b(x, \theta) \ast \alpha^a(x, \theta) T^b T^a \right)
\]
\[
= \sum_{c=0}^{N^2-1} \sum_{a, b=0}^{N^2-1} \left( i f^{abc} \frac{1}{2} \{ \alpha^a(x, \theta), \beta^b(x, \theta) \} \right) + d^{abc} \frac{1}{2} \{ \alpha^a(x, \theta), \beta^b(x, \theta) \} T^c,
\] (2.10)

where
\[
[T^a, T^b] = \sum_{c=0}^{N^2-1} i f^{abc} T^c, \quad \{T^a, T^b\} = \sum_{c=0}^{N^2-1} d^{abc} T^c, \quad \text{Tr}(T^a T^b) = \frac{1}{2} \delta^{ab}.
\] (2.11)

Thus, owing to the condition \(2.10\), the enveloping Lie algebra closes within itself for the star commutator \(2.10\). It should be noted that
\[
\lim_{\theta \to 0} U(x, \theta) = U(x, \theta) = e^{\sum_{a=1}^{N} \alpha^a(x) T^a} \in \text{SU}(N),
\] (2.12)

which indicates that the enveloping group \(\text{SU}(N)^*\) reduces to nonabelian group \(\text{SU}(N)\) when \(\theta^{\mu\nu}\) becomes to 0.

Under the gauge transformation in \(2.6\) the field strength \(F_{\mu\nu}(x)\) and the covariant derivative of \(D_\mu \varphi(x)\) are transformed covariantly
\[
F_{\mu\nu}^\varphi(x) = U(x, \theta) \ast F_{\mu\nu}(x) \ast U^{-1}(x, \theta),
\] (2.13)
\[
(D_\mu \varphi(x))^\varphi = U(x, \theta) \ast D_\mu \varphi(x) \ast U^{-1}(x, \theta).
\] (2.14)

Then, the gauge field term in \(2.1\) is transformed as in
\[
\text{Tr} \left[ F_{\mu\nu}^\varphi(x) \ast F^{\varphi\mu\nu}(x) \right] = \text{Tr} \left[ U(x, \theta) \ast F_{\mu\nu}(x) \ast F^{\mu\nu}(x) \ast U^{-1}(x, \theta) \right]
\] (2.15)

which shows the gauge term itself is not gauge invariant because of the Moyal \(*\)product but the action is invariant thanks to the rule
\[
\int d^4 x \, f(x) \ast g(x) = \int d^4 x \, g(x) \ast f(x).
\] (2.16)

We call this situation pre-invariance to gauge transformation. That is, the gauge boson term in Lagrangian is pre-invariant. That is the case for the Higgs boson term in \(2.1\) because of \(2.14\). On the other hand, the fermion term in Eq. \(2.1\) is invariant under gauge transformations \(2.6\).

In order to construct the nonabelian gauge theory on the NC spacetime, we start from \(2.1\) where the 0 component \(A_0^\mu(x, \theta)\) of the gauge field \(A_\mu(x)\) doesn’t get in. But, it is induced by the gauge transformation \(2.6\). Thus, the 0 component of \(A_\mu(x)\) in \(2.10\) depends on other components \(A_a^\mu(x)\) \((a = 1, 2, \cdots, 15)\) and so, it is not independent field. Moreover it vanishes owing to \(2.8\) when the NC parameter \(\theta^{\mu\nu}\) approaches to 0. Thus, we don’t need to quantize \(A_0^\mu(x, \theta)\). Thus, even if the 0 component of \(A^\mu(x)\) appears in \(2.6\), it is out of our considerations because we eliminate it away by the gauge transformation. The existence of the enveloping \(\text{SU}(N)^*\) gauge group insure the construction of nonabelian gauge theory on the NC spacetime.

Let us explain the difference between the nonabelian NC gauge theory in this paper and that of Wess et.al. \(6\). According to the paper of Wess et.al. \(6\) they construct the nonabelian gauge theory based on the enveloping Lie algebra restricted to satisfy
\[
(\delta_\alpha \delta_\beta - \delta_\beta \delta_\alpha) = \delta_{\alpha \times \beta},
\] (2.17)

where
\[
\delta_\alpha \psi(x) = i \Lambda(x) \ast \psi(x)
\]
\[
\delta_\alpha A_\mu(x) = \partial_\mu \Lambda(x) + i [\Lambda(x), A_\mu(x)]_*,
\] (2.18)
with $\Lambda(x)$ being an element of the enveloping Lie algebra. In order to satisfy the conditions, $\Lambda(x)$ must be the function of ordinary gauge field $a_\mu(x)$ and the Lie algebra $\alpha(x)$ satisfying

$$i\delta_\alpha \Lambda_\beta[a] - i\delta_\beta \Lambda_\alpha[a] + \Lambda_\alpha[a] \ast \Lambda_\beta[a] - \Lambda_\beta[a] \ast \Lambda_\alpha[a] = i\Lambda_\alpha \times \beta[a].$$  \hspace{1cm} (2.19)

The gauge field $A_\mu(x)$ and fermion field $\psi(x)$ must depend on $a_\mu(x)$. Then, they expand these function and fields in the order of $h \theta^{\mu\nu}$.

They call this process Seiberg-Witten map. According to their expansions, $\Lambda_\alpha[a] = \alpha(x) + h \Lambda_1^1[a] + h^2 \Lambda_2^2[a] + \cdots$, $\psi[a] = \psi^0[a] + h \psi^1[a] + h^2 \psi^2[a] + \cdots$, $A_\mu[a] = a_\mu(x) + h A_1^1[a] + h^2 A_2^2[a] + \cdots$, where terms higher than the order $h^1$ are the very complicated functions of $\alpha(x)$, $a_\mu(x)$ and their derivatives. Action which consists of gauge and fermion fields is also expanded in the series of $h \theta^{\mu\nu}$ as in Eqs.(5.4) and (5.5) in [6]. The 0-th order in the expansion is the ordinary action composed of ordinary fields $a_\mu(x)$, $\psi^0(x)$. Each term in expansions is invariant under ordinary gauge transformation. Terms higher than $h^1$ in the expansion of action are very complicated with $F_\mu^\nu$, $\psi^0$ and their derivatives. Their theory looks unrenormalizable because the couplings have the higher dimensions.

The gauge transformation function $U(x, \theta)$ in this paper is written in terms of the Lie algebra valued function $\alpha(x, \theta)$ with the condition (2.7). In this paper, we do not claim the restriction (2.17), and therefore, we do not need the Seiberg-Witten map. Gauge field $A_\mu(x)$ and fermion field $\psi(x)$ as well as the action are not expanded in $\theta^{\mu\nu}$. Thus, it is concluded that our non-abelian gauge field on NC spacetime is much different from that of Wess et.al. [6].

3 The spontaneous breakdown of SU(4)* gauge theory

SO(10) grand unified theory (GUT) including its supersymmetric version is most promising model in particle physics since it can incorporate the 15 existing fermions in addition to the right-handed neutrino and has possibilities to explain so many phenomenological puzzles. Pati-Salam symmetry $SU(4) \times SU(2)_L \times SU(2)_R$ is one of the intermediate symmetry of the spontaneous breakdown of SO(10) GUT. This symmetry spontaneously breaks down to the left-right symmetric gauge model with the symmetry $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_B$. In this stage, the spontaneous breakdown $SU(4) \rightarrow SU(3)_c \times U(1)_B$ occurs. $B$ charge of fermions is given by

$$Q_B \begin{pmatrix} q^r \\ q^g \\ q^b \\ l \end{pmatrix} = \begin{pmatrix} 1 \\ -1/3 \\ 2/3 \\ -1 \end{pmatrix} \begin{pmatrix} q^r \\ q^g \\ q^b \\ l \end{pmatrix}. \hspace{1cm} (3.1)$$

As an example, we pick up this process in order to investigate whether fermions have $B$ charge in the noncommutative version of SU(4) gauge theory.

The gauge boson in SU(4)* gauge theory is expressed in terms of 16 component gauge bosons by

$$A_\mu(x) = \sum_{a=0}^{15} A_\mu^a(x) T^a$$

$$= \frac{1}{2} \begin{pmatrix} A_{11}^\mu & C_{1}^\mu & C_{2}^\mu & X_{1}^\mu \\ G_{1}^\mu & A_{22}^\mu & G_{3}^\mu & X_{2}^\mu \\ G_{2}^\mu & G_{3}^\mu & A_{33}^\mu & X_{3}^\mu \\ X_{1}^\mu & X_{2}^\mu & X_{3}^\mu & A_{44}^\mu \end{pmatrix}. \hspace{1cm} (3.2)$$
explained so far as owing to the Moyal product. Thus, in the NC field theory, we should write the spontaneous breakdown.

However, we can’t do it in the same way because

\[ A_{\mu}^{11} = \frac{1}{\sqrt{2}} A_{\mu}^{0} + \frac{1}{\sqrt{3}} A_{\mu}^{3} + \frac{1}{\sqrt{6}} A_{\mu}^{15}, \]

\[ A_{\mu}^{22} = \frac{1}{\sqrt{2}} A_{\mu}^{0} - \frac{1}{\sqrt{3}} A_{\mu}^{3} + \frac{1}{\sqrt{6}} A_{\mu}^{15}, \]

\[ A_{\mu}^{33} = \frac{1}{\sqrt{2}} A_{\mu}^{0} - \frac{2}{\sqrt{3}} A_{\mu}^{8} + \frac{1}{\sqrt{6}} A_{\mu}^{15}, \]

\[ A_{\mu}^{44} = \frac{1}{\sqrt{2}} A_{\mu}^{0} - \frac{3}{\sqrt{6}} A_{\mu}^{15}. \] \tag{3.3}

The gauge field \( A_{\mu}(x) \) contains 8 color gluons, 6 gauge bosons causing proton decay, one extra boson \( A_{\mu}^{15}(x) \), and one 0 component boson \( A_{\mu}^{0}(x) \) dependent on other bosons. Here, we denote \( A_{\mu}^{15}(x) \) by \( B_{\mu}(x) \) and call it \( B \)-field.

The vacuum expectation value of Higgs boson \( \varphi(x) \) takes the form

\[ <\varphi(x)> = v \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -3 \end{pmatrix} \] \tag{3.6}

which yields the gauge boson mass term

\[ -i g [A_{\mu}(x), <\varphi(x)>]^2 = 8g^2 v^2 (X_{\mu}^{1} X_{\mu}^{1} + X_{\mu}^{2} X_{\mu}^{2} + X_{\mu}^{3} X_{\mu}^{3}). \] \tag{3.7}

Equation (3.7) shows that 6 proton decay causing gauge bosons acquire mass, so that symmetries relating to Lie algebra \( T^a \) \((a = 1, 2, \cdots, 8, 15) \) keep unbroken. Let us consider the gauge transformation specified by

\[ U^{cb}(x, \theta) = e^{i \alpha(x, \theta) \cdot \alpha^a(x, \theta) \cdot T^a} = \exp i \left\{ \sum_{a=0}^{8} T^a \alpha^a(x, \theta) \cdot +T^{15} \alpha^{15}(x, \theta) \cdot \right\}. \] \tag{3.8}

Under this gauge transformation, the part of the gauge field \( A_{\mu}(x) \) in (2.4)

\[ A_{\mu}^{cb}(x) = \sum_{a=0}^{8} A_{\mu}^{a}(x) T^a + B_{\mu}(x) T^{15} \] \tag{3.9}

and fermion field \( \Psi(x) \) and Higgs field transform in the similar way as in (2.6). Thus, it is easily shown that the Lagrangian (2.4) is still pre-invariant after the spontaneous breakdown coming from (3.6). This indicates that color symmetry yielding the strong interaction and \( B \)-symmetry due to the generator \( T^{15} \) remain unbroken. In the commutative field theory, this breakdown is written as

\[ \text{SU}(4) \rightarrow \text{SU}(3) \times \text{U}(1). \] \tag{3.10}

However, we can’t do it in the same way because

\[ U^{cb}(x, \theta) \neq \exp i \left\{ \sum_{a=1}^{8} \alpha^a(x, \theta) \cdot T^a \right\} \cdot \exp i \left\{ \alpha^{15}(x, \theta) \cdot T^{15} \right\}. \] \tag{3.11}

owing to the Moyal product. Thus, in the NC field theory, we should write the spontaneous breakdown explained so far as

\[ \text{SU}(4) \ast \rightarrow \text{SU}(3+1) \ast. \] \tag{3.12}
Interaction terms between fermion and $B$-gauge field extracted from the fermion term in (2.1) is given by

$$I_D = \bar{\Psi}(x) \ast \{ \gamma^\mu(gB_\mu(x)T^{15}) \} \ast \Psi(x)$$

$$= \frac{3}{2\sqrt{6}} g \bar{\Psi}(x) \ast \gamma^\mu \begin{pmatrix}
\frac{1}{3} & 0 & 0 & 0 \\
0 & \frac{1}{3} & 0 & 0 \\
0 & 0 & \frac{1}{3} & 0 \\
0 & 0 & 0 & -1
\end{pmatrix} B_\mu(x) \ast \Psi(x)$$

(3.13)

Then, if we define $B$-charge operator $Q_B$ and $B$-charge $e_B$

$$Q_B = \frac{2\sqrt{6}}{3} T^{15}, \quad e_B = \frac{3}{2\sqrt{6}} g$$

(3.14)

(3.1) is reproduced.

### 4 Arbitrary $B$-charge

In the previous section, we considered the spontaneous breakdown of SU(4)$^*$ gauge symmetry down to SU(3+1)$^*$ symmetry. Thus, charges of fermions are limited as shown in (3.1). However, apart from such construction, we can consider such a case that if the Lagrangian is pre-invariant under the gauge transformation function $U^s(x, \theta)$ given by

$$U^s(x, \theta) = \exp \{ Q \beta(x, \theta) \}$$

(4.1)

where

$$Q = \begin{pmatrix}
e & 0 & 0 & 0 \\
0 & e & 0 & 0 \\
0 & 0 & e & 0 \\
0 & 0 & 0 & e'
\end{pmatrix}$$

(4.2)

with arbitrary constants $e$ and $e'$, fermions may have arbitrary charges. This is because Interaction terms between fermion and $B$-gauge is given by

$$I_D = \bar{\Psi}(x) \ast \{ \gamma^\mu(gB_\mu(x)Q) \} \ast \Psi(x).$$

(4.3)

If there is only U(1)$^*$ gauge symmetry, the gauge transformation of gauge field $A_\mu(x) = QB_\mu(x)$ given by

$$U^c(x, \theta) = \exp i \{ Q \beta(x, \theta) \}.$$

(4.4)

leads to the inconsistency as indicated by Hayakawa [4]. But, in our case, there are two kinds of symmetry and therefore, the gauge transformation of gauge field $A_\mu(x) = \sum_{a=0}^{8} T^a A^a_\mu(x, \theta) + Q B_\mu(x)$ given by (4.4) has nothing to do with any contradiction because of $A^0_\mu(x, \theta)$ existence.

### 5 Conclusions

We have proposed nonabelian SU(N)$^*$ gauge theory on NC spacetime which doesn’t depend on the Seiberg-Witten map [3] as in Jurco et.al [6]. We considered the SU(4)$^*$ gauge theory which spontaneously breaks down to SU(3+1)$^*$ symmetry in order to obtain the gauge theory with plural U(1) charges. It is shown that such NC gauge theory with different charges more than two can not exist alone, but it must coexist with NC nonabelian gauge theory.

Let us discuss the present situations on NC gauge theory and show several undesirable results with respect to the quantized version of NC gauge theory. Martín and Sánchez-Ruiz [10] showed that U(1) NC gauge theory can be renormalizable but asymptotically free. Matusis, Susskind and Toumbas [11] found the unfamiliar IR/UV connection in NC gauge theory which involves non-analytic behavior in NC
parameter \( \theta \) making the limit \( \theta \to 0 \) singular. It is also pointed out by Armoni \cite{5} that the \( \theta \to 0 \) limit of the U(N) NC gauge theory does not converge to the ordinary SU(N)\( \times \)U(1) gauge theory. Finally, Ruiz Ruiz \cite{12} indicated tachyonic instability in U(1) NC gauge theory due to UV/IR mixing effects. In this paper, we do not investigate the quantum effects of the SU(N)* gauge theory. Then, it may display the same undesirable points which should be quickly addressed. However, we construct in this paper new formulation of non-abelian gauge theory on NC spacetime and propose how to incorporate fields with the arbitrary U(1) charges into NC gauge theory. These are main purposes in this paper which is the first step in order to achieve the true theory. As showed by Ruiz Ruiz \cite{12} and authors of \cite{11}, the defects stated above may be solved by considering the supersymmetric version of NC gauge theory. Then, we must construct the supersymmetric version of the present paper. However, there is a possibility to solve the problems within the present formalism since it has the different aspects from other NC gauge theories stated above. In this paper, \( A_{\mu}^0 \) component field is the dependent field on other components gauge fields and it can be eliminated away by gauge transformation. The charges in this paper are related to the \( \lambda^{15} \) in SU(4)* NC gauge theory with spontaneous breakdown which does not need to take the limit of \( \theta \to 0 \). These possibilities will be investigated in future works.

The deviation from the standard model in particle physics has not yet observed, and so any model beyond standard model must reduce to it in some approximation. NC gauge theory must also reproduce standard model in the limit of NC parameter \( \theta^{\mu\nu} \to 0 \). According to the considerations in section 3, the eligible enveloping gauge group to construct the standard model on NC spacetime seems to be SU(3c + 2L + 1Y)*. This work will appear in our forthcoming paper.

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