Power corrections in Drell-Yan production beyond the leading order

G. P. Korchemsky

Laboratoire de Physique Théorique et Hautes Energies
Université de Paris XI, Centre d’Orsay, bût. 211
91405 Orsay Cédex, France

and

Laboratory of Theoretical Physics
Joint Institute for Nuclear Research
141980 Dubna, Russia

Abstract

We study the power corrections to the cross-section of Drell-Yan production within the Wilson line approach and apply the methods of QCD asymptotic dynamics to identify the leading renormalon contribution.

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*E-mail: korchems@qcd.th.u-psud.fr
†Laboratoire associé au Centre National de la Recherche Scientifique (URA D063)
POWER CORRECTIONS IN DRELL-YAN PRODUCTION
BEYOND THE LEADING ORDER

G.P. KORCHEMSKY

LPTHE, Université de Paris XI, 91405 Orsay, France and LTP, JINR, 141980 Dubna, Russia

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1 Introduction

To control the accuracy of perturbative QCD predictions for physical observables it becomes important to have a regular way of estimating corrections suppressed by powers of large momentum scale, $\Lambda_p/Q$, and if not calculate explicitly the nonperturbative scale $\Lambda_p$ then find the leading exponent $p$ at least. At present, the understanding of power corrections has a solid theoretical status only for the special class of processes like $e^+e^- \to$ hadrons and deeply inelastic scattering (DIS) for which the analysis based on the operator product expansion (OPE) is applicable. The OPE fixes the structure of power corrections (as $1/Q^4$ for the total cross-section of $e^+e^-$ annihilation and $1/Q^2$ for the structure functions of DIS) and allows to identify the corresponding scales $\Lambda_p$ as matrix elements of higher twist composite operators in QCD. The understanding of power corrections in the processes which do not admit the OPE is the subject of intensive discussions (see review [1] and references therein).

An important example of the process, which does not admit the OPE is the Drell-Yan (DY) process $h_A + h_B \to \mu^+\mu^- + X$. Here, the lepton pair with invariant mass $Q^2$ is created in the partonic subprocess consists of the lepton pair with the energy $Q$ accompanied by a soft gluon radiation with the total energy $(1 - \tau)Q/2$. Due to the presence of two different scales the partonic cross-section gets large perturbative (Sudakov logs in $\ln(1 - \tau)$) and nonperturbative (as inverse powers of $(1 - \tau)Q$) corrections induced by soft gluons [2]. Calculating the moments $\sigma_N = \int_0^1 d\tau\tau^N d\sigma_{DY}/dQ^2$ at large $N \sim 1/\tau$ and subtracting collinear divergences in the DIS-scheme one can represent the leading term of the expansion of $\sigma_N$ in powers of $1/Q$ as

$$\ln \sigma_N = \sum_k \alpha_s^k(Q/N)C_k(\ln N) + (\Lambda_{DY}N/Q)^p,$$

(1)

where the coupling constant is defined at the characteristic soft gluon scale, $C_k$ contains the powers of Sudakov logs plus finite, $\ln^0N$, contributions and the second term represents the leading power correction. As $N$ increases, the energy of soft gluons in the final state decreases as $Q/N$ toward the infrared region in which perturbative expressions should fail. Indeed, it is well-known that the perturbative expansion in (3) is not well-defined due to factorial growth of coefficients $C_k \sim k!p^{-k}$ at higher orders in $\alpha_s$. This makes the perturbative series non Borel summable and induces an IR renormalon ambiguity into perturbative contribution to (3) at the level of power corrections, $\delta_{IR}\sigma_N \sim \exp(-\ln C_k/\ln N) = (\Lambda_{QCD}N/Q)^p$. Then, IR renormalon ambiguity of perturbative expression is compensated in (3) by ambiguity in the definition of the scale $\Lambda_{DY}$. Thus, examining the large order behaviour of the perturbative series one can determine the level $p$ at which the leading power correction appear in (3). However, since it is impossible to calculate the coefficients $C_k$ exactly to higher orders in $\alpha_s$ one has to find a reasonable approximation to $C_k$ which would allow to identify the $k!$ factorial growth of the coefficients. To this end different schemes were proposed [4,5,6]. Being applied to the Drell-Yan cross-section they predict that there are no $N/Q$ corrections to (3) and the leading renormalon contribution has the following form

$$\delta_{IR}\sigma_N = 0 \cdot (N/Q) + (\Lambda_{DY}N/Q)^2.$$

(2)

We would like to stress that these schemes are ap-
proximate and although they were tested in the case of $e^+e^-$ annihilation and DIS, it is not clear yet whether they are applicable in the Drell-Yan process and whether they really predict the leading power correction to the cross-section.

2 Wilson line approach

A different approach to the analysis of power corrections has been proposed in Ref.\textsuperscript{[3]}. It is based on the remarkable property of soft gluons that their contribution to the Drell-Yan cross-section can be factorized with a power accuracy into universal factor having the form of a Wilson line. Similar to the OPE approach, the power corrections can be identified with the matrix elements of certain operators built from the Wilson line.

The Wilson line operator appears in the Drell-Yan partonic subprocess as an eikonal phase of quarks interacting with soft gluon radiation. Combining the eikonal phases of quark and antiquark together one gets the expression for partonic cross-section as

$$\frac{d\sigma_{\text{DY}}}{dQ^2} = \sum_n |\langle n|TU_{\text{DY}}(0)|0\rangle|^2 \delta(E_n - (1 - \tau)Q/2)$$

where summation is performed over all possible final states consisting of an arbitrary number, $n$, of soft gluons with the total energy $E_n = (1 - \tau)Q/2$ and $T$ stands for time-ordering of gauge fields. Here, $U_{\text{DY}}(0) = P \exp(i \int C(x) dx \cdot A(x))$ is the eikonal phase of quark + antiquark and $A_\mu(x)$ is the gauge field operator describing soft gluons. The contour $C(0)$ in Minkowski space-time corresponds to the classical trajectory of quark and antiquark. It goes from $0$ to the annihilation point $0$ along the quark momentum and then returns to $\infty$ in the direction opposite to the antiquark momentum. Performing summation over final states in the last relation one can obtain the representation for the Drell-Yan cross section for $\tau \to 1$ as a Fourier transformed Wilson line expectation value\textsuperscript{[4]}

$$\frac{d\sigma_{\text{DY}}}{dQ^2} = \frac{Q}{2} \int_{-\infty}^{\infty} \frac{dy_0}{2\pi} e^{-iy_0(1-\tau)Q/2} \langle 0|W(y_0)|0\rangle$$

$$W(y_0) = \frac{\mathcal{T}U_{\text{DY}}(0)}{U_{\text{DY}}(y_0)}$$

where $U_{\text{DY}}(y_0)$ is defined at the point $y = (y_0, 0)$. Calculating the moments of this expression one obtains the following relation\textsuperscript{[5]}

$$\sigma_N = \frac{1}{N_c} \text{tr} \left(0|W(y_0)|0\right), \quad \text{for } y_0 = iN/Q,$$  \hspace{1em} (4)

which is valid up to corrections vanishing as $N \to \infty$ and which takes into account all Sudakov logs and power corrections in $N/Q$. The scale $Q/N$ enters into (4) just as a parameter of the integration contour in $U_{\text{DY}}(y_0)$. Eq. (4) states that the asymptotic expansion of the moments of the Drell-Yan cross section is related to the behaviour of the Wilson line expectation value at small $y_0 = i\frac{N}{Q}$.

Perturbative expansion of (4) involves standard “short-distance” logarithms $\alpha_s \ln^n(|y_0|\mu)$, which are transformed into Sudakov logs for $y_0 = iN/Q$. The factorization scale $\mu \approx Q$ has a meaning of the maximal energy of soft gluons contributing to the partonic cross-section and after factorization of soft gluon emission it appears in (3) and (4) as a ultraviolet renormalization scale of the Wilson line operators. As a result, all perturbative Sudakov corrections to the Drell-Yan cross-section can be effectively resummed using the remarkable renormalization properties of Wilson lines. They are summarized by the following RG equation

$$\frac{d\ln\sigma_N}{d\ln\mu^2} = 2\Gamma_{\text{cusp}}(\alpha_s)\ln(N\mu/Q) + \Gamma_{\text{DY}}(\alpha_s),$$

with $\alpha_s = \alpha_s(\mu)$ and $\Gamma_{\text{cusp/DY}}$ being some anomalous dimensions. The evolution of $\sigma_N$ from $\mu = Q/N$ up to $\mu = Q$ generates Sudakov logs while the initial condition for $\sigma_N$ at $\mu = N/Q$ takes into account nonperturbative power corrections in $N/Q$. To understand their structure we perform the operator expansion of the Wilson line $W(y_0)$ in powers of $y_0$

$$(W(y_0)) = (W(0)) + y_0\langle \partial W(0) \rangle + ....$$  \hspace{1em} (6)

Substituting this expansion into (4) we find that the power corrections to the cross-section are described by matrix elements of the operators defined as derivatives of the Wilson line.\textsuperscript{[6]} Although these operators are nonlocal in QCD, one can represent them as local operators in an effective non-abelian Bloch-Nordsieck theory in which Wilson line appears as a result of integration over effective quark fields. Solving the RG equation (5)\textsuperscript{[7]}

\textsuperscript{[3]}This identity is an analog of the relation between the moments of the structure functions of DIS and matrix elements of composite operators in the OPE.
and using (3) as a boundary condition we obtain that the leading power correction to the Drell-Yan cross-section appears as

$$\sigma^\text{power}_N = \langle i\partial W(0) \rangle N/Q + \mathcal{O}(N^2/Q^2),$$

(7)

provided that there is no any hidden symmetry which would prohibit the appearance of the linear $\sim y_0$ term in the expansion of the Wilson line (3) and would enforce the matrix element $\langle i\partial W(0) \rangle$ to vanish.

3 IR renormalon analysis

One possibility to test the presence of the linear term in (3) is to apply the IR renormalon approach and identify the source of linear terms inside perturbative series for the Wilson line. At the leading order of renormalon calculus (4) one can perform the explicit one-loop calculation of $\langle W(y_0) \rangle$ with a gluon mass $\lambda$ and identify the power corrections as nonanalytical terms in the small $(y_0 \lambda)$ expansion. One finds that $\alpha_s \ln(y_0 \lambda)$ term is absent in agreement with the Bloch-Nordsieck cancellation, the linear term $\alpha_s(\lambda y_0)$ also vanishes and the leading term is $\sim \alpha_s(\lambda y_0)^2$. This means, that at the leading order the contribution of IR renormalons has the form (3). The same result can be obtained in the limit of large number of light flavours in which one performs 1-loop calculation of $\langle W(y_0) \rangle$ with the gluon propagator dressed by a chain of quark loops. We notice that in both cases one calculates essentially abelian diagrams containing the color factors $\alpha_s C_F$ and $\alpha_s C_F(\alpha_s N_f)^n$, respectively. It is well-known that the contribution of abelian diagrams to the Wilson line exponentiates to higher orders in $\alpha_s$ and one can generalize the above statement about the absence of linear $\sim y_0$ term to a much larger class of abelian diagrams beyond the leading order of the renormalon calculus.

However, the natural question rises whether the abelian diagrams provide a meaningful approximation to the exact expression for the Wilson line or may be there is the leading contribution to $\langle W(y_0) \rangle$ coming only from nonabelian diagrams. To give an example where the second possibility is realized we mention the breakdown of the Bloch-Nordsieck cancellation of IR logs in the Drell-Yan cross-section at twist $1/Q^4$. The IR divergent part of the cross-section has the form $\sim \alpha_s^2 C_A C_F Q^{-2} \ln \lambda^2$ with $C_A(C_F)$ being gluon (quark) Casimir operators. It does not appear at order $\alpha_s$ and at order $\alpha_s^2$ it comes only from diagrams with nonabelian color structure. One might expect that similar phenomenon may happen when one considers linear in $y_0$ contributions to the Wilson line beyond the leading order.

The straightforward way to check this possibility would be to perform 2-loop calculation of the Wilson line with a power accuracy in $\lambda$. However, even without going through this complicated calculation one can employ the operator methods of the QCD asymptotic dynamics in order to analyse the properties of the matrix element

$$\langle \partial W(0) \rangle = \frac{1}{N_c} \text{tr} (\mathcal{T} U_{\text{DY}}(0|A) \partial T U_{\text{DY}}(0|A)),$$

(8)

parameterizing the leading $N/Q$ power correction to the Drell-Yan cross-section. The operator $U_{\text{DY}}(y_0|A)$ describes the eikonal phase of fast quark and antiquark annihilating at the space-time point $y_0$ and $T$—ordering of gauge field operators $A^\mu(x)$ is needed in (3) and (5) to describe properly the possibility for quark and antiquark to exchange by a virtual soft gluon in the initial state. As a result, due to effects of the initial state interaction

$$T U_{\text{DY}}(y_0|A) \neq U_{\text{DY}}(y_0|A).$$

This does not happen however for single-parton initiated process like DIS and it is this phenomenon that makes different the properties of power corrections in DY and DIS. Indeed, if we omit the $T$—ordering in (8) then it is easy to show by direct calculation that the derivative $\mathcal{T} U_{\text{DY}} \partial U_{\text{DY}}$ is proportional to the generators of the $SU(N_c)$ group and it does not contribute to (3). To work out the effects of quark-antiquark correlations in $\langle W(y_0) \rangle$ one defines two gauge field operators $A^\mu_\pm(x) = \frac{1}{2} (A^\mu(x) \pm A^\mu(-x))$ and notices that the field $A^\mu_+$ describes the Coulomb gluons while the field $A^\mu_-$ is associated with radiative gluons forming quark and antiquark coherent states. Since $[A_{\mu}^{\pm}(x), A_{\nu}^{\pm}(y)] = 0$ and $[A_{\mu}^{\pm}(x), A_{\nu}^{\mp}(y)] \neq 0$, the expansion of the matrix element $\langle n|T U_{\text{DY}}(y_0|A^+ + A^-) |0 \rangle$ takes the following form

$$\langle n|T U_{\text{DY}}^{ij}(y_0|A^+ + A^-) |0 \rangle = [e^{i\Phi}]^{ij}_{ij},$$

(9)

$$\times \langle n|U_{\text{DY}}^{i,j'}(y_0|A^-) V_{i,j'}^{j,i'}(y_0|A^+, A^-) |0 \rangle,$$

where the flow of quark color is indicated explicitly. Here, in the r.h.s. the first factor is the nonabelian unitary Coulomb phase matrix and the
eikonal phase $U_{\text{DY}}$ depends only on commutative operators $A^-$. The residual factor $V$ is unitary, $V^\dagger V = \mathbb{1}$, and it describes the correlations between Coulomb and radiative gluons. The perturbative expansion of $V$ looks like
\[
V = \mathbb{1} + ig^3 f^{abc} t^a \otimes t^b \int d^4 x G(x, y_0) A^+_\mu(x) + \mathcal{O}(g^4)
\]

where effective nonlocal coupling $G(x, y_0)$ resembles the Fadin-Kuraev-Lipatov vertex. In aubelian theory $V = \mathbb{1}$ to all orders, but in QCD it gets nonabelian corrections starting from order $g^3$ in the strong coupling constant.

Substituting (8) into (9) one observes that the contribution of the residue to (8) comes only from higher order terms in $V$. It immediately follows from the explicit form of $V$ that the corresponding Feynman diagrams should have a nonabelian color structure and they arise starting from $\alpha_s^2$ order. The set of relevant 2-loop Feynman diagrams was identified in Ref. 4. This also explains why linear term did not appear in 1-loop calculation of the Wilson line performed in Ref. 4.

The fact that the contribution of the residual factor $V$ to the Wilson line $W(y_0)$ is nonzero was demonstrated by a direct calculation in Ref. 4 in which a small gluon mass $\lambda$ was used as an IR cutoff and only logarithmic terms, $\sim \ln^k \lambda$, were kept. It was shown that the leading term $W(0)$ of the expansion of the Wilson line in Eq. (8) gets a higher twist correction $\sim \alpha_s^2 C_F C_A Q^{-2} \ln \lambda$, which leads to the breakdown of Bloch-Nordsieck theorem for the Drell-Yan cross-section but is nevertheless consistent with the Kinoshita-Lee-Nauenberg (KLN) theorem. To find the linear term in (8) one has to perform the calculation of the same diagrams with a power accuracy in $\lambda$, although interpretation of the linear term $\sim \lambda^2$ to order $\alpha_s^2$ as a “true” power correction is not obvious. One may dress instead the gluon propagators by a single chain of quark loops, resum all 2-loop diagrams of order $\alpha_s^2 C_F C_A (\alpha_s N_f)^k$ in the large $N_f$ limit and identify the contribution of the leading IR renormalon.

4 Conclusions

We conclude that the effects of the initial state interaction, that is correlations between Coulomb and radiative gluons, provide a new source of $N/Q$ power corrections in the Drell-Yan process. In the Wilson line approach they are described by the matrix element of the derivative of the Wilson line operator in Eq. (6). In the IR renormalon analysis these effects could manifest themselves only in nonabelian Feynman diagrams at higher orders in $\alpha_s$. The presence of $N/Q$ corrections in the Drell-Yan cross-section is consistent with the KLN theorem. Summation over degenerate initial partonic states in (8) and (9) leads to the expression for the KLN cross-section, $\sigma_{\text{KLN}}^N = \sum_k \langle k | W(iN/Q) | k \rangle$, where sum goes over an arbitrary number of incoming soft gluons. As was shown in Ref. 4, the initial state interaction effects can be analysed in $\sigma_{\text{KLN}}^N$ on the same footing as the final state effects thus resulting in the cancellation of $N/Q$ corrections in $\sigma_{\text{KLN}}^N$.

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