Lepton Flavor Violation and the Tau Neutrino Mass

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Abstract

We point out that, in the left-right symmetric model of weak interactions, if $\nu_\tau$ mass is in the keV to MeV range, there is a strong correlation between rare decays such as $\tau \to 3\mu$, $\tau \to 3e$ and the $\nu_\tau$ mass. In particular, we point out that a large range of $\nu_\tau$ masses are forbidden by the cosmological constraints on $m_{\nu_e}$ in combination with the present upper limits on these processes.

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In the standard model of electroweak interactions, all lepton flavors $L_e$, $L_\mu$ and $L_\tau$ are conserved. On the other hand, in most extensions of the standard model, lepton flavor conservation is not maintained; therefore, it is hoped that the nature of lepton flavor violation can help to narrow the possibilities of new physics. Crucial tests of lepton flavor violation are provided by the rare decays of $\mu$ and $\tau$ such as $\mu \rightarrow 3e[1]$ and $\tau \rightarrow l_il_jl_k$ where $l_i$, $j$, $k$ go over $e$ and $\mu$. The present stringent limits on $\mu \rightarrow 3e$ already make it imperative that in all extensions of standard model, violation of $L_\mu + L_e$ be very weak. On the other hand, the present upper limits on the branching ratios for rare $\tau$ decays[2] allow for possible violation of $L_\tau + L_\mu$ or $L_\tau + L_e$ at a much higher level. One class of models, where the possibility of a significant lepton violation exists, is the left-right symmetric model with see-saw mechanism for neutrino masses[3]. In this note, we investigate the rare $\tau$ decays and their implications for violation of $L_\tau + L_\mu$ or $L_\tau + L_e$ quantum numbers in these models. We show that there is a strong correlation between the tau neutrino mass and $\tau \rightarrow 3\mu$ and $\tau \rightarrow 3e$ decays if $m_{\nu_\tau}$ is in the keV to MeV range[4], as is allowed by existing laboratory upper limits[2]. First we derive lower limits on $m_{\nu_\tau}$ for the case where $B(\tau \rightarrow 3\mu) = 0$; Once the flavor violating decay $\tau \rightarrow 3\mu$ or $\tau \rightarrow 3e$ is allowed we show that the present upper limits on their rates permit the lower bound on $m_{\nu_\tau}$ to be somewhat relaxed. Improvement of the present experimental upper limits on $m_{\nu_\tau}$ and the branching ratios for the above $\tau$-decay modes can therefore throw light on the nature of lepton flavor violation in the left-right symmetric models.

We consider the left-right symmetric model with a see-saw mechanism
for neutrino masses as described in ref.3. Let us display the leptonic and Higgs sector of the model. The three generations of lepton fields are $\Psi_a \equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_a$, where $a = 1, 2, 3$. Under the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, they transform as $\Psi_a L \equiv (1/2, 0, -1)$ and $\Psi_a R \equiv (0, 1/2, -1)$. Since our purpose is to study the possible degree of violation of $L_\mu + L_\tau$ or $L_e + L_\tau$ in the rare $\tau$-decay, we will impose one of these global symmetries on the model [F.1], for simplicity. We illustrate our idea for the model with $U(1)_{\tau + \mu}$ global symmetry. The Higgs sector then needs to be enlarged if we want the see-saw mechanism for all lepton flavors. We choose a single bi-doublet field $\phi \equiv (1/2, 1/2, 0)$ and two sets of triplet Higgs fields: 

$$\Delta_L(1, 0, +2) \oplus \Delta_R(0, 1, +2); \quad \text{with } L_\mu + L_\tau = -2 \quad (1.a)$$

$$\Delta'_L(1, 0, +2) \oplus \Delta'_R(0, 1, +2); \quad \text{with } L_\mu + L_\tau = 0 \quad (1.b)$$

The Yukawa coupling which are invariant under all symmetry can be written as:

$$L_Y = \bar{\Psi}_L h \phi \Psi_R + \bar{\Psi}_L \tilde{h} \phi \Psi_R$$

$$+ \Psi^{T}_L \tau_2 \tilde{f} \Delta_L C^{-1} \Psi_L + L \rightarrow R$$

$$+ h.c.$$ (2)

where $h$, $\tilde{h}$ and $f \Delta$ are the following matrices in generation space:

$$h \equiv \begin{pmatrix} h_{11} & 0 & 0 \\ 0 & h_{22} & h_{23} \\ 0 & h_{23} & h_{33} \end{pmatrix} \quad (3.a)$$

$$f \Delta \equiv \begin{pmatrix} f_{11} \Delta' & 0 & 0 \\ 0 & f_{22} \Delta & f_{23} \Delta \\ 0 & f_{23} \Delta & f_{33} \Delta \end{pmatrix} \quad (3.b)$$
and similarly for $\tilde{h}$.

The gauge symmetry is spontaneously broken by the vacuum expectation values:

$$< \Delta^0_R > = V_R \ ; \quad < \Delta^0_R' > = V'_R \ ; \quad (4.a)$$

$$< \Delta^0_L > = < \Delta^0_L' > = 0 \quad (4.b)$$

and

$$< \phi > = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \ . \quad (4.c)$$

As usual, $< \phi >$ gives masses to the charged fermions and Dirac masses to the neutrinos whereas $< \Delta^0_R >$ and $< \Delta^0_R' >$ lead to the see-saw mechanism for the neutrinos (this mechanism operates separately for $\nu_e$ and jointly for $\nu_\mu$ and $\nu_\tau$). These discussions are all standard and we do not repeat them here.

The physics we are interested in comes from the left-handed triplet sector of the theory. As indicated in eq.(4.b) these fields do not take part in the Higgs mechanism; therefore, if we ignore certain couplings in the Higgs potential, such as $\Delta_L \phi \Delta_R^{\dagger} \phi^{\dagger}$ etc., then $\Delta_L$ and $\Delta_R$ remain unmixed states. Of course there could be mixings between $\Delta_L$ and $\Delta_L'$; but we ignore these mixings here and comment later on their effect. For small $\Delta_L - \Delta_L'$ mixings, our main results do not change. In this limit, the electron generation separates for all practical purposes from the $\mu$ and $\tau$ generations. The $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$ are forbidden. We therefore focus on the $\mu - \tau$ sector.
Yukawa Lagrangian relevant to our discussions is given (in the basis where all the leptons are mass eigenstates) by

\[ \mathcal{L}_Y = \nu_L^T F'C^{-1}\nu_L \Delta^0 + \nu_L^T F''C^{-1}E_L \Delta^+_L \]
\[ + E_L^T FC^{-1}E_L \Delta^{++} + h.c , \quad (5) \]

where \( \nu = (\nu_\mu, \nu_\tau) \), \( E = (\mu, \tau) \); \( F, F' \) and \( F'' \) are 2 \( \times \) 2 matrices related to each other as follows:

\[ FK^T = F''; \quad KFK^T = F' , \quad (6) \]

where \( K \) is the leptonic Cabibbo matrix in the left-handed \( \mu - \tau \) sector. First we note that the off-diagonal element of \( K \) is the \( \nu_\mu - \nu_\tau \) mixing angle, which is directly measurable parameters, restricted to be, \( \theta_{\mu \tau} \leq 0.03 \) by existing accelerator experiments[5].

Now, we make the following observation. Suppose that the \( \nu_\tau \) mass is in the keV to MeV range and \( \nu_\mu \) and \( \nu_e \) masses are in the few electron volt range, a possibility consistent with present upper limit on neutrino masses from the accelerator data[5]. In this case, \( \nu_\tau \) must be unstable in order to be consistent with cosmological constraints on the mass density in the universe[6]. The mass and life time are then related by[7]

\[ \tau_{\nu_\tau} \leq (5.4 \times 10^{10} \text{ sec}) \left( \frac{100 \text{ keV}}{m_{\nu_\tau}} \right)^2 \quad (7.A) \]

A more stringent, but model dependent constraint can be derived from considerations of Galaxy formation[8]; it is given by
\[ \tau_{\nu_\tau} \leq 3 \times 10^7 \text{sec.} \] (7.B)

In the model under consideration, \( \nu_\tau \to \nu_\mu \bar{\nu}_\mu \nu_\mu \) occurs via \( \Delta^0_L \) exchange and can be used to satisfy the constraints in eqs.(7)[9]. The Hamiltonian for this process is given by

\[ H = \frac{G_{\nu_\tau} \bar{\nu}_\mu \gamma^\lambda (1 - \gamma_5) \nu_\mu \bar{\nu}_\mu \gamma_\lambda (1 - \gamma_5) \nu_\tau + h.c}{\sqrt{2}} \] (8.a)

where (we drop the subscript L from \( \Delta_L \) henceforth)

\[ G_{\nu_\tau} = \sqrt{2} \frac{F'_{\mu\mu} F'_{\mu\tau}}{4M^2_{\Delta^0}} \approx \sqrt{2} \frac{F_{\mu\mu}}{4M^2_{\Delta^0}} \times \left[ F_{\mu\tau} - \theta_{\mu\tau} (F_{\mu\mu} - F_{\tau\tau}) \right] \] (8.b)

The \( \nu_\tau \) lifetime is

\[ \tau_{\nu_\tau}^{-1} = \frac{2G^2_{\nu_\tau} m_{\nu_\tau}^5}{192\pi^3} \] (8.c)

From eq.(7.A) we get[7.2]

\[ G_{\nu_\tau} \geq (1.9 \times 10^{-12}) \left( \frac{\text{GeV}}{m_{\nu_\tau}} \right)^{3/2} \text{GeV}^{-2} \] (9.A)

To get a feeling for the order of magnitude of \( G_{\nu_\tau} \), note that for \( m_{\nu_\tau} = 10 \text{ MeV} \), \( G_{\nu_\tau} \geq 2 \times 10^{-4} G_F \) and \( m_{\nu_\tau} = 0.1 \text{ MeV} \), \( G_{\nu_\tau} \geq 0.2 \ G_F \). The corresponding constraint from Galaxy formation eq.(7.B) can be written as

\[ G_{\nu_\tau} \geq 8 \times 10^{-15} \left( \frac{\text{GeV}}{m_{\nu_\tau}} \right)^{5/2} \text{GeV}^{-2} \] (9.B)
Turning now to the $\tau$-lepton, we observe that exchange of $\Delta L^{++}$ contributes to the rare $\tau$-decay, $\tau^{-} \rightarrow \mu^{-} \mu^{-} \mu^{+}$ with a strength (defined analogously to the $\nu_{\tau}$ case)

$$G_{\tau} = \sqrt{2} \frac{F_{\mu \mu} F_{\mu \tau}}{4 M_{\Delta^{++}}^2}.$$  \hspace{1cm} (10)

Now, we first notice from eq.(8.b) that, even if $F_{\mu \tau} = 0$ (i.e there is no $\tau \rightarrow 3 \mu$ decay) the $\nu_{\tau}$ can decay. Since the decay rate depends on $\nu_{\tau}$ mass, let us see, if for the presently allowed range of $\theta_{\mu \tau}$ and $\nu_{\tau}$ masses, constraints in eqs.(7.A) and (7.B) are satisfied. To study this, we first note that vacuum stability requires all $F_{ab} \leq 1.2[11]$ and LEP data require that, $m_{\Delta_{L}^0} \geq 45$ GeV. Combining these and present upper limit of $\theta_{\mu \tau} \leq 3 \times 10^{-2}$ we find from eqs.(9) that, for case A and B, the $\nu_{\tau}$ mass must have the following lowest bounds:

Case A : \hspace{1cm} $m_{\nu_{\tau}} \geq 31$ keV \hspace{1cm} ; \hspace{1cm} (11.A)

Case B : \hspace{1cm} $m_{\nu_{\tau}} \geq 210$ keV \hspace{1cm} . \hspace{1cm} (11.B)

Once the $\nu_{\tau}$ masses go below the above limits, the LR model cannot satisfy the cosmological constraints without having $F_{\mu \tau} \neq 0$. We emphasize that, we have been extremely conservative in obtaining these lower bounds. (For instance, $F_{\tau \tau}$ is likely to be lower than its maximum allowed value and $m_{\Delta^0}$ is also likely to be heavier.) $F_{\mu \tau} \neq 0$ immediately leads to non-vanishing $\tau \rightarrow 3 \mu$ decay. We can therefore obtain a lower bound on the $B(\tau \rightarrow 3\mu)$ in these ranges of $m_{\nu_{\tau}}$. 

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In the presence of $F_{\mu\tau}$, we have

$$\Gamma(\tau \to 3\mu) = \frac{1}{4 m_{\Delta_0}^4} (F_{\mu\mu} F_{\mu\tau})^2 \frac{m_{\tau}^5}{192\pi^3} .$$  \hspace{1cm} (12)$$

Using the $\nu_\tau$ lifetime in eq.(8.c), we get,

$$B(\tau \to 3\mu) = \left( \frac{0.3 \times 10^{-12}}{\tau_{\nu_\tau} \text{insec}} \right) \left( \frac{m_{\Delta_0}}{m_{\Delta_{++}}} \right)^4 \left( \frac{m_{\tau}}{m_{\nu_\tau}} \right)^5 \left( \frac{F_{\mu\tau}}{F_{\mu\mu} - \theta_{\mu\tau}(F_{\mu\mu} - F_{\tau\tau})} \right)^2 .$$  \hspace{1cm} (13)$$

Using the cosmological upper bounds on $\tau_{\nu_\tau}$ in eqs.(7.A) and (7.B), we get,

Case A : $B(\tau \to 3\mu) \geq 9.5 \times 10^{-3} \left( \frac{100\text{keV}}{m_{\nu_\tau}} \right)^3 \left( \frac{m_{\Delta_0}}{m_{\Delta_{++}}} \right)^4 \epsilon_{\mu\tau}$ ; \hspace{1cm} (14.A)$$

Case B : $B(\tau \to 3\mu) \geq 16.8 \left( \frac{100\text{keV}}{m_{\nu_\tau}} \right)^5 \left( \frac{m_{\Delta_0}}{m_{\Delta_{++}}} \right)^4 \epsilon_{\mu\tau}$ . \hspace{1cm} (14.B)$$

where $\epsilon_{\mu\tau} = \left( \frac{F_{\mu\tau}}{F_{\mu\mu} - \theta_{\mu\tau}(F_{\mu\mu} - F_{\tau\tau})} \right)^2$. Note that once $m_{\nu_\tau}$ is below the lower bounds given in eqs.(11), $\epsilon_{\mu\tau}$ becomes a function of $m_{\nu_\tau}$; therefore for a given value of $m_{\nu_\tau}$, we can find a lower bound on $B(\tau \to 3\mu)$, (and vice-versa) if we have a lower bound on $(m_{\Delta_0}/m_{\Delta_{++}})^4$.

Let us therefore discuss the factor $(m_{\Delta_0}/m_{\Delta_{++}})^4$. We note that[10], the $\Delta_L$ multiplet contributes to the $\rho$-parameter as follows:

$$\rho_{\Delta} = \frac{G_F}{4\sqrt{2} \pi^2} [f_{(0,+)} + f_{(+,++)}] \equiv \frac{3G_F}{8\sqrt{2} \pi^2} \Delta m^2 ,$$  \hspace{1cm} (15)$$
where \( f_{a,b} = m^2_a + m^2_b - \frac{2m^2_a m^2_b}{m^2_b - m^2_a} \ln \frac{m^2_b}{m^2_a} \). Langacker[12] has given an upper bound on the new contribution to \( \rho \)-parameter from physics beyond the standard model as follows:

\[
m^2_t + \Delta m^2 \leq (194 \text{GeV})^2 .
\]

(16)

In the LR model, there exists the further relation:

\[
m^2_{\Delta^+} = m^2_{\Delta^0} (1 + 2\alpha); \quad m^2_{\Delta^0} = m^2_{\Delta^0} (1 + \alpha) ,
\]

(17)

where \( \alpha \) is a dimensionless parameter. Using these relations, we can obtain a lower bound on \( \alpha \) from \( \rho \)-parameter constraint (using the fact that \( m_{\Delta^0} \geq 45 \) GeV), which can then be converted to a lower bound on \( B(\tau \rightarrow 3\mu) \). We find that for \( m_t = 110 \) GeV , \( \alpha < 67 \) and for \( m_t = 150 \), \( \alpha < 40 \). Using this we obtain

**Case A:** \( B(\tau \rightarrow 3\mu) \geq \delta_A \left( \frac{100 \text{keV}}{m_{\nu \tau}} \right)^3 \epsilon_{\mu \tau}(m_{\nu \tau}) \); (18.A)

**Case B:** \( B(\tau \rightarrow 3\mu) \geq \delta_B \left( \frac{100 \text{keV}}{m_{\nu \tau}} \right)^5 \epsilon_{\mu \tau}(m_{\nu \tau}) \). (18.B)

In table I, we give the values of \( \delta_A \) and \( \delta_B \) for the two cases for two values of \( m_t \).

| \( m_t \) (GeV) | \( \delta_A \)       | \( \delta_B \)       |
|-----------------|--------------------|--------------------|
| 110             | \( 5.6 \times 10^{-7} \) | \( 1 \times 10^{-3} \) |
| 150             | \( 1.4 \times 10^{-6} \) | \( 2.5 \times 10^{-3} \) |

Table I. Values of \( \delta_A \) and \( \delta_B \)
To understand the implications of eqs.(18) further, let us first note that they depend on $F_{\mu \tau}$ explicitly. Clearly for values of $m_{\nu_\tau}$ far below the lower limits in eqs.(11), cosmology would require $F_{\mu \tau} \gg \theta_{\mu \tau} (F_{\mu \mu} - F_{\tau \tau})$ (e.g. $m_{\nu_\tau} = 100$ keV in case B would require $F_{\mu \tau} \simeq 0.2$, whereas $|\theta_{\mu \tau}(F_{\mu \mu} - F_{\tau \tau})|_{\max} \leq 0.06$). In such cases, $\epsilon_{\mu \tau} = 1$ so that, the lower bound is obtained by setting $\epsilon_{\mu \tau} = 1$ in the right-hand side of the inequalities (18.a) and (18.b). The present upper bound on $B(\tau \rightarrow 3\mu) \leq 4.8 \times 10^{-6}$[13]. Therefore, values of $m_{\nu_\tau}$ for which $F_{\mu \tau} \leq \frac{1}{3} |\theta_{\mu \tau}(F_{\mu \mu} - F_{\tau \tau})|_{\max} \simeq 0.02$ satisfies both the experimental upper bound on $B(\tau \rightarrow 3\mu)$ and the cosmological bound for case A ( $m_t = 150$ GeV ) leading to $m_{\nu_\tau} \geq 26$ keV, which is, then, the absolute lower bound on $m_{\nu_\tau}$ in this model. Turning to case B, we find that both constraints are satisfied for $F_{\mu \tau} \leq .02$ giving $m_{\nu_\tau} \geq 187$ keV. Thus, we see that allowing for $\tau \rightarrow 3\mu$ decay leads to slight relaxation of the lower bounds on $m_{\nu_\tau}$ allowed in the LR model. Further improvement of the upper limits on the $B(\tau \rightarrow 3\mu)$ as well as $\theta_{\mu \tau}$ will therefore help to further constrain the $m_{\nu_\tau}$ in these models.

Bounds on $m_{\nu_\tau}$ using only cosmological mass density constraints were discussed in ref.4, where two assumptions were made: a) $m_{\Delta^0} \simeq m_{\Delta^{++}}$ and b) there is no mixing between lepton generations. We do not make these assumptions here; further more, we point out the existence of a bound even if $B(\tau \rightarrow 3l) = 0$ unlike that in ref.4. Thus, our lower bounds are more rigorous than those of ref.4.

Let us close with a few comments:

a) For the case where $L_e + L_\tau$ symmetry is imposed on the theory, similar
results follow with $\mu$ replaced by $e$ everywhere in the final state. The lower bounds are now weaker since $\theta_{e\tau} \leq 0.17$. All result obtained for case A (i.e. mass density bound ) are lowered by a factor of 3 and for case B by a factor of 2. If $m_{\nu_\tau} > 1 \text{ MeV}$, the channel $\nu_\tau \rightarrow \nu_e + e^+ + e^-$ can also arise via $\Delta^+_L$ exchange, which is constrained by Supernova consideration[14], although the existing bounds do not yield any interesting constraint on the parameters under discussion.

b) In the model with $L_\mu + L_\tau$ symmetry, the existence of $\Delta_L - \Delta'_L$ mixing can lead to $L_\mu + L_\tau$ violating channels. We choose this mixing to be small. If however, this mixing were not negligible a new channel $\overline{\nu}_\tau \rightarrow \nu_\mu + \nu_e + \nu_e$ appears. This will weaken our bounds by a factor $(1 + y)^{3/2}$ in case A and $(1 + y)^{5/2}$ in case B, where $y = \Gamma(\nu_\tau \rightarrow \nu_\mu \nu_e \nu_e)/\Gamma(\nu_\tau \rightarrow 3\nu_\mu)$.

c) Specifically, our results will also apply to the minimal left-right symmetric model without any symmetry (and hence only a single set of $\Delta_L \oplus \Delta_R$ ) if we only chose either $F_{\mu\tau}$ or $F_{e\tau}$ to be zero, except that in this case there is always a second decay mode ( e.g. $\nu_\tau \rightarrow \overline{\nu}_\mu \nu_e \nu_e$ for $F_{e\tau} = 0$ ). Again, there will be a slight dilution of our lower limits.

d) Strictly speaking in order to avoid the existence of a Majoron in our model, we can add soft symmetry breaking terms of the form $(\Delta^+_L \Delta'_L + \Delta^+_R \Delta'_{R})$. In the absence of this, there exists a Majoron, but it does not provide any fast decay mode for $\nu_\tau$ (similar to the original singlet Majoron model). In either case, our results remain unchanged.
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Footnote

[F.1] Even though we have imposed the global symmetries $L_\tau + L_\mu$ to obtain these results, these are very likely to apply to the model without them. The reason is that, $\mu \to 3e$ requires the $\Delta_L$ coupling $F_{\mu e}$ to be nearly zero. Then the constraints of $\mu \to e\gamma$ imply that, $F_{\tau \mu}F_{\tau e} \leq 10^{-5}$. Therefore, if one of them is big (i.e. of order $10^{-1}$ or so), the second one is very small. In our case, if $F_{\mu \tau} \simeq 10^{-1}$, we expect $F_{\tau e} \leq 10^{-4}$. This is equivalent to approximate $L_\mu + L_\tau$ symmetry. Similarly if $F_{\mu \tau} \leq 10^{-4}$ and $F_{e \tau} \simeq 10^{-1}$, this is equivalent to imposing $L_e + L_\tau$ symmetry.

[F.2] Similar considerations are applied in the $e - \mu$ sector in ref.10.
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