Energy Consumption Predication in China Based on the Modified Fractional Grey Prediction Model

Jiefang Liu¹ and Pumei Gao²

¹School of Management and Economics, Tianjin University of Technology and Education, Tianjin 300222, China
²Department of Economics and Management, Tianjin Electronic Information College, Tianjin 300350, China

Correspondence should be addressed to Jiefang Liu; liujf101@126.com

Received 10 May 2021; Revised 15 July 2021; Accepted 24 July 2021; Published 10 August 2021

Abstract
China’s increasing energy consumption poses challenges to economy and environment. How to predict the energy consumption accurately and regulate the future energy consumption production is a problem worth studying. In this paper, the fractional order accumulative linear time-varying parameter discrete grey prediction model (FTDGM(1, 1) model) is introduced. Firstly, the data are preprocessed by buffer operators, and then, the FTDGM (1, 1) model is established. In this paper, the parameter estimation method and the specific process of model establishment are presented. Finally, the models of energy consumption in China are built. The advantages and prediction accuracy of the model established in this paper are analyzed, and the data in the following years are effectively predicted, so as to provide theoretical support for the government to formulate reasonable energy policies.

1. Introduction

Grey system theory is a small sample modeling method proposed by Professor Deng Julong in 1982, which mainly includes grey prediction theory, grey correlation theory, grey decision theory, grey clustering theory, and grey game theory [1, 2]. Among them, the grey prediction theory is an important part of grey system theory, for the reason that the grey prediction theory does not have strict requirements on data. For example, the data do not need to meet certain distribution rule, and the high precision model can be established based on a small amount of data. Therefore, since the model is put forward, it has been widely used in the fields of agriculture, energy, economy, medicine, and so on and has achieved fruitful achievements [3]. Compared with other uncertainty methods, the advantage of grey system theory is that it treats the modeling object as a system and uses the idea of system theory to build the model.

In recent years, many scholars have carried out extensive research studies on the grey prediction model [4–17], whose achievements have played a positive role in improving the grey prediction theory. The GM (1, 1) model requires the transformation of differential equation and difference equation, which will lead to systematic errors. Based on this, Xie and Liu constructed the DGM (1, 1) model and its extended form. The DGM (1, 1) model does not require the conversion of differential equation and difference equation, but directly uses the difference equation for parameter estimation and model solution, so as to effectively reduce the error source within the model [18]. The GM (1, 1) and DGM (1, 1) models construct prediction models by accumulating quasi-exponential rules of sequences. However, accumulative sequences contain linear laws as well as quasi-exponential laws. Based on this, Zhang and Liu proposed a discrete grey prediction model with linear time-varying parameters and discussed the properties of the model and the parameter solving method, and the model has a completely error-free simulation accuracy for sequences with linear laws and exponential laws [19]. Wu et al. constructed time-varying parameters of the GM (1, 1) model by introducing a polynomial function of time [20]. Accumulation is the basis of grey prediction, through which the sequence regularity can be highlighted. Traditional accumulation is a kind of first-order accumulation, so the importance of data
has not been differentiated. In view of this, Wu Lifeng puts forward a fractional accumulative grey prediction model and discussed the modeling process and parameter-solving method [21, 22].

Time series are generally affected by the shock disturbance of the system. When the shock disturbance exists in the system, there will be relatively large modeling errors in the modeling of time series, and the inconsistency between the qualitative analysis and quantitative calculation will occur. This paper proposes a discrete grey prediction model based on the weakening buffer operator and fractional order accumulation of linear time-varying parameters. Firstly, the buffer operators are used to weaken the impact disturbance of the system, and then, the prediction model is constructed. In this paper, the structure is as follows. Section 2 introduces the differences between existing accumulative methods, the advantages of the fractional order accumulation and its calculation method, and the parameter-solving method. In Section 3, the modeling process of the fractional order accumulative linear time-varying discrete grey prediction model is described. In Section 4, two practical examples are given, and the feasibility and practicability of the proposed method is verified. The conclusion is drawn in Section 5.

2. Construction of Fractional Discrete Grey Models

The grey prediction model is established based on the accumulation sequence, which can effectively reduce the randomness of the system and show the development law of the system. For example, for an original data sequence \( X^{(0)} = (1, 4, 2, 5, 2, 3) \), its first-order accumulation sequence is \( X^{(1)} = (1, 5, 7, 12, 14, 17) \), as shown in Figure 1.

It can be seen from Figure 1 that the generation of first-order accumulation can transform the oscillation sequence with no obvious regularity into a monotonic increasing sequence, thus highlighting the change law of data and making a good preliminary preparation for the establishment of the grey prediction model.

The disadvantages of the integer order cumulative grey prediction model are analyzed by taking DGM (1, 1) as an example. For a sequence \( X^{(0)} = (1, 2, 6, 10, 14, 19) \), the first-order cumulative DGM (1, 1) model is established. And, we can calculate the parameters as follows: \( \beta_1 = 1.49 \) and \( \beta_2 = 3.85 \). The simulation sequence is \( X^{(0)} = (1.00, 4.34, 6.46, 9.61, 14.31, 21.30) \), and the average relative error is 23.78%. In comparison, if the 0.2 order cumulative DGM (1, 1) model is established, then we can get \( \beta_1 = 1.27 \) and \( \beta_2 = 2.51 \), the simulation sequence \( X^{(0)} = (3.00, 4.85, 7.38, 10.71, 15.11, 20.93, 28.64) \), and the average relative error is 3.07%.

Assume that the original value of the sequence is changed, and there is a sequence \( X^{(0)} = (3, 2, 6, 10, 14, 19) \); then, there are \( \beta_1 = 1.49 \) and \( \beta_2 = 2.87 \), the simulation sequence \( (3.00, 4.34, 6.46, 9.61, 14.31, 21.30) \), and the average relative error remains 23.78%. In comparison, when the 0.2 order cumulative DGM (1, 1) model is established, we can get \( \beta_1 = 1.27 \) and \( \beta_2 = 2.51 \), the simulation sequence \( X^{(0)} = (1.00, 3.58, 6.46, 9.94, 14.24, 19.61, 26.34) \), and the average relative error changes to 10.38%.

It can be seen from the above results that, for an integer order accumulation at the DGM (1, 1) model, the change of the original value will not influence the modeling results of the sequence, which means that the first value has not been effective utilized, which is a serious waste for small sample modeling. As to the fractional order accumulation at the DGM (1, 1) model, the change of the original value can lead to sequence change and the corresponding change of simulation precision, which means that fractional-order accumulation can effectively use the original value. Moreover, with the adjustment of the order, the fractional-order cumulative DGM (1, 1) model can obtain better modeling accuracy than the integer order cumulative DGM (1, 1) model. The same conclusion can be reached for other grey prediction models.

3. Construction of Novel Fractional Discrete Grey Model

For the series with disturbance information if the grey prediction model is established based on the original data, the simulation and prediction results obtained will be quite different from the actual trend of the system. This paper uses a classical weakening buffer operator to preprocess the data.

**Definition 1.** Assume a nonnegative sequence \( X^{(1)} = \{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\} \) is the first-order accumulative sequence of \( X^{(0)} \), where

\[
x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \ldots, n.
\]

Then,

\[
x^{(1)}(k + 1) = \beta_1 x^{(1)}(k) + \beta_2, \quad k = 1, 2, \ldots, n - 1,
\]

is called the discrete grey prediction model (DGM (1, 1) model).

**Theorem 1.** The parameters of the DGM (1, 1) model can be solved by using the following least squares estimation:

\[
\begin{bmatrix}
\beta_1 \\
\beta_2
\end{bmatrix} = (B^T B)^{-1} B^T Y.
\]

Among them
Definition 2. Assume the nonnegative sequence is
\[ X^{(0)} = \{ x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n) \} \]. \[ X^{(1)} = \{ x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n) \} \] is the first-order accumulative sequence of \[ X^{(0)}. \]

Among them

\[ B = \begin{bmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \vdots \\ x^{(1)}(n-2) & 1 \\ x^{(1)}(n-1) & 1 \end{bmatrix}, \]

\[ Y = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n-1) \\ x^{(1)}(n) \end{bmatrix}. \]

The time response formula is as follows:

\[ x^{(1)}(k+1) = \beta_1 x^{(0)}(1) + \frac{1 - \beta_1^k}{1 - \beta_1} \beta_2, \quad k = 1, 2, \ldots, n-1. \]

(5)

From the above modeling, it can be seen that the DGM (1, 1) model avoids the transformation from differential equation to check score equation, so it can reduce the system error. The DGM (1, 1) model is an exponential model, which reflects the exponential law generated by accumulation. After accumulating, the sequence has both a trend of index and a linear trend. The linear time-varying parameters’ discrete grey prediction model is inclusive for its complete linear specification. One of the problems for a linear time-varying discrete grey prediction model lies in ineffective use of the original value. Based on this, this paper carried out modeling by the fractional-order cumulative linear time-varying parameter discrete grey prediction model.

Theorem 2. The parameters of the TDGM (1, 1) model can be solved by using the following least squares estimation:

\[ \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix} = (C^T C)^{-1} C^T Y. \]  

(8)

Among them

\[ C = \begin{bmatrix} x^{(1)}(1) & x^{(1)}(1) & 1 & 1 \\ 2 x^{(1)}(2) & x^{(1)}(2) & 2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ (n-2) x^{(1)}(n-2) & x^{(1)}(n-2) & n-2 & 1 \\ (n-1) x^{(1)}(n-1) & x^{(1)}(n-1) & n-1 & 1 \end{bmatrix}. \]  

(9)

\[ Y = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n-1) \\ x^{(1)}(n) \end{bmatrix}. \]

The time response formula is

\[ x^{(1)}(k+1) = (\beta_1 x^{(1)}(k) + \beta_2) + \beta_3 k + \beta_4, \quad k = 1, 2, \ldots, n-1. \]  

(10)
Theorem 3. For sequence $y = ab^k$, parameters $\beta_2 = \beta_3 = 0$, and the TDGM (1, 1) model is degraded to the DGM (1, 1) model. For the sequence $y = ax + b$, parameter $\beta_1 = 0$, and the TDGM (1, 1) model was degraded to the NDGM (1, 1) model. For sequence $y = ab^k + c$, parameter $\beta_1 = 0$, and the TDGM (1, 1) model was degraded to the NDGM (1, 1) model.

**Theorem 4.** The parameters of the FTDGM (1, 1) model can be solved by using the following least squares estimation:

$$x^{(r)}(k) = \sum_{i=1}^{k} C_{r-1}^{k-i} x^{(0)}(i), C_{r-1}^{0} = 1, C_{r-1}^{k} = 0, \quad k = 1, 2, \ldots, n.$$  

**Definition 4** (see [23]). Assume the nonnegative sequence $X^{(0)}$, $X^{(r)}$ is defined as Definition 3. The equation

$$x^{(r)}(k + 1) = (\beta_1 k + \beta_2) x^{(r)}(k) + \beta_3 k + \beta_4, \quad k = 1, 2, \ldots, n - 1,$$

is called the fractional-order accumulative linear time-varying parameters’ discrete grey model FTDGM (1, 1) model.

It can be seen from Theorem 2 that the DGM (1, 1) model and NDGM (1, 1) model are uniform special cases of the TDGM (1, 1) model.

**Definition 3** (see [20–22]). Assume the nonnegative sequence is $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\}$. $X^{(r)} = \{x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n)\}$ is called the fractional-order accumulative sequence of $X^{(0)}$.

Among them

$$\hat{x}^{(0)}(k) = \tilde{x}^{(r)}(k) - \sum_{i=1}^{k} C_{r-1}^{k-i} \hat{x}^{(r)}(i), \quad k = 1, 2, \ldots, n.$$  

According to the calculation formula of fractional order accumulation, it is not difficult to calculate the reduced value of the predicted sequence as follows:

$$\hat{x}^{(0)}(k) = \bar{x}^{(r)}(k) - \sum_{i=1}^{k} C_{r-1}^{k-i} \hat{x}^{(r)}(i), \quad k = 1, 2, \ldots, n.$$  

It can be seen from the modeling process that the FTDGM (1, 1) model can better reflect the change rule of the system through the adjustment and optimization of the cumulative order. The model not only has a good modelling effect for practice series with exponential trend but also has a good trend tracking ability for time series with linear trend and nonlinear trend superposition. The optimal parameters can be determined by intelligent optimization algorithm, and genetic optimization algorithm is adopted in this paper.

For the time series with system impact disturbance, the modeling results of the model become very poor, and even the qualitative analysis and the theorem analysis are inconsistent. Professor Liu Sifeng deeply analyzed this phenomenon and believed that it was not due to the choice of model, but due to the impact disturbance of the system, which made the data series unable to show the real change rule of the system. Therefore, it was necessary to restore the real face of the system. Based on this, Professor Liu Sifeng put forward the concept of buffer operator, analyzed its properties, and carried out a lot of applications.

In reality, the system is relatively complex, and many systems can be regarded as complex systems with shock disturbance. For such a system, the first step of modeling is to present the real rules of the system and then build the model. In this paper, the weakening buffer operator proposed by Professor Liu Sifeng is selected for data preprocessing. The definition of the weakening buffer operator is as follows.
Definition 5. Assume $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$ is the original sequence and $X^{(0)}d_i$ is the buffered sequence and where

$$x^{(0)}(k)d_i = \frac{x^{(0)}(k) + x^{(0)}(k+1) + \cdots + x^{(0)}(n)}{n-k+1} \quad (17)$$

Then, $X^{(0)}d$ is called the average weakening buffer operator.

Definition 6. Assume $X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$ is the original sequence, $X^{(0)}d_1$ is the buffered sequence and $X^{(0)}d_2 = (x^{(0)}(1)d_1, x^{(0)}(2)d_1, \ldots, x^{(0)}(n)d_1)$ where

$$x^{(0)}(k)d_2 = \frac{(n-k+1)x^{(0)}(k)}{x^{(0)}k + x^{(0)}(k+1) + \cdots + x^{(0)}n}x^{(0)}k \quad (18)$$

Then, $X^{(0)}d_2$ is called the average strengthening buffer operator.

Weakening buffer operator can be used for real time series with fast change in the front and slow change in the back, while strengthening buffer operator can be used for real time series with slow change in the front and fast change in the back. $X^{(0)}d_1$ and $X^{(0)}d_2$ are the most commonly used buffer operators.

Therefore, the modeling ideas proposed in this paper are as follows. Firstly, the system data are buffered to restore the real law of the system. Then, the FTDGM (1, 1) model was established to calculate and optimize the parameters. In order to illustrate the practicability and effectiveness of the new model, two case studies are carried out as follows.

4. Case Study

Case 1: In recent years, with the development of society and economy, China’s total energy consumption grows rapidly, which poses higher requirements to energy production and natural environment. Accurate predictions for China’s energy consumption can help the government and enterprises take reasonable measures to cope with the situation. Therefore, energy consumption prediction in China is of great value. China’s total energy consumption between 2009 and 2019 is shown in Table 1.

We take the data from 2009 to 2015 as the modeling data and the data from 2016 to 2019 as the prediction data to test the prediction effect of different grey prediction models. Among them, genetic algorithm is used to determine the optimal order, and MATLAB software is used for programming calculation.

It can be seen from Table 2 that the prediction error of the B-FTDGM (1, 1) model has been effectively reduced. There is no obvious phenomenon that the error increases with the increase of step size. Thus, it can overcome the problem that the traditional grey prediction model can only be used for short-term prediction.

Case 2: In recent years, with the progress of China’s industrialization, electricity, as a very important energy source, become the foundation of economic development. Therefore, it become of great significance to forecast power consumption timely and accurately for economic and social development. The per capita consumption of domestic electricity in Beijing from 2011 to 2019 is shown in Table 3.

Data from 2011 to 2017 are used as modeling data, and data from 2018 to 2019 are used as prediction data to test the prediction effect of different grey prediction models. Calculation results of different grey prediction models are shown in Table 4.

As can be seen from Table 4, due to the impact disturbance of the system, the prediction accuracy of DGM
(1, 1) is not high, while the accuracy of the FTDGM (1, 1) model is improved to some extent. The 2-step average prediction error can be further reduced by using the sequence strengthened by the strengthening operator for modeling.

5. Conclusion

In this paper, a discrete grey prediction model of linear time-varying combination of buffering operator and fractional-order accumulation is constructed, and its properties, parameter-solving method, and modeling process are discussed. The model is applied to the prediction of energy consumption and per capita electricity consumption in China, and high prediction accuracies are obtained, showing the obvious superiority of the model. The discussion of other time-varying parameter grey prediction models and multivariable time-varying parameter grey prediction models is a direction that needs to be further studied in the future.

Data Availability

The data used to support the findings of this study have been deposited in http://www.stats.gov.cn/tjss/ndsj and http://tjj.beijing.gov.cn.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research was supported by the Project of Chinese Postdoctoral Science Foundation (2018M632777), Henan Provincial Department of Education the Generally Project of Humanities and Social Science (2017-ZZJH-180), Henan Province Backbone Young Teachers Program (2018GGJS115), Key Scientific Research Projects of Colleges and Universities in Henan province (20A790012), and General Program of Humanities and Social Sciences from Ministry of Education (20YJC630083).

References

[1] J. L. Deng, "Introduction to grey system theory," Journal of Grey System, vol. 1, no. 1, pp. 1–24, 1989.
[2] S. F. Liu and Y. Lin, Grey Information, Springer, London, UK, 2006.
[3] S. F. Liu, J. Forrest, and R. Valle, "Emergence and development of grey systems theory," Kybernetes, vol. 38, no. 39, pp. 1246–1256, 2009.
[4] X. Xiao, J. Yang, S. Mao et al., "An improved seasonal rolling grey forecasting model using a cycle truncation accumulated generating operation for traffic flow," Applied Mathematical Modelling, vol. 1, no. 51, pp. 386–404, 2017.
[5] X. Ma and Z. B. Liu, "Application of a novel time-delayed polynomial grey model to predict the natural gas consumption in China," Journal of Computational and Applied Mathematics, vol. 324, pp. 17–24, 2007.
[6] S. Ding, "A novel self-adapting intelligent grey model for forecasting China’s natural-gas demand," Energy, vol. 16, pp. 393–407, 2018.
[7] S. Ding, R. J. Li, S. Wu, and W. J. Zhou, "Application of a novel structure-adaptive grey model with adjustable time power item for nuclear energy consumption forecasting," Applied Energy, vol. 298, pp. 1–20, Article ID 117144, 2021.
[8] S. Ding, R. J. Li, and Z. Tao, "A novel adaptive discrete grey model with time-varying parameters for long-term photovoltaic power generation forecasting," Energy Conversion and Management, vol. 227, pp. 1–15, Article ID 113644, 2021.
[9] S. Ding and R. J. Li, "Forecasting the sales and stock of electric vehicles using a novel self-adaptive optimized grey model engineering," Applications of Artificial Intelligence, vol. 100, pp. 1–13, Article ID 104148, 2021.
[10] S. Ding, R. J. Li, S. Wu, and W. J. Zhou, "A novel composite forecasting framework by adaptive data preprocessing and optimized nonlinear grey bernoulli model for new energy vehicles sales," Communications in Nonlinear Science and Numerical Simulation, vol. 99, pp. 1–21, Article ID 105847, 2021.
[11] M. L. Cheng and G. J. Shi, "Modeling and application of grey model GM (2, 1) based on linear difference equation," Journal of Grey System, vol. 31, no. 2, pp. 37–50, 2019.
[12] X. J. Shen, M. H. Yue, P. F. Duan et al., "Application of grey prediction model to the prediction of medical consumables consumption," Grey Systems: Theory and Application, vol. 9, pp. 213–223, 2019.
[13] B. Zeng and C. Li, "Improved multi-variable grey forecasting model with a dynamic background-value coefficient and its application," Computers & Industrial Engineering, vol. 118, pp. 278–290, 2018.
[14] J. Ye, Y. G. Dang, and B. J. Li, "Grey-markov prediction model based on background value optimization and central-point triangular whiteningization weight function," Communications in Nonlinear Science and Numerical Simulation, vol. 54, pp. 320–330, 2018.
[15] J. Cui, S. F. Liu, B. Zeng, and N. M. Xie, "A novel grey forecasting model and its optimization," Applied Mathematical Modelling, vol. 37, pp. 4399–4406, 2013.
[16] Z. X. Wang and P. Hao, "An improved grey multivariable model for predicting industrial energy consumption in China," Applied Mathematical Modelling, vol. 40, pp. 5745–5758, 2016.
[17] M. L. Cheng and M. Y. Xiang, "Generalized GM (1, 1) model and its application," Journal of Grey System, vol. 29, no. 3, pp. 110–122, 2017.

Table 4: Predication results of different grey predication models.

| Original value | DGM (1, 1) predication | Predication error (%) | FTDGM (1, 1) predication (r = 1.21) | Predication error (%) | B-FTDGM (1, 1) predication (r = 0.43) | Predication error (%) |
|----------------|------------------------|-----------------------|--------------------------------------|-----------------------|---------------------------------------|-----------------------|
| 1185.5         | 1010.14                | 14.79                 | 1095.63                              | 7.58                  | 1087.62                              | 8.26                  |
| 1168.1         | 1065.65                | 8.77                  | 1226.11                              | 4.97                  | 1168.16                              | 0.00                  |
| MAPE           | 11.78                  |                       |                                      |                       |                                      | 4.13                  |
[18] N. M. Xie and S. F. Liu, “Discrete grey forecasting model and its optimization,” *Applied Mathematical Modelling*, vol. 33, pp. 173–1186, 2009.

[19] K. Zhang and S. F. Liu, “Linear time-varying parameters discrete grey forecasting model,” *Systems Engineering-Theory & Practice*, vol. 30, pp. 1650–1657, 2010.

[20] L. F. Wu, S. F. Liu, and L. G. Yao, “Discrete grey model based on fractional order accumulate,” *System Engineering-Theory & Practice*, vol. 34, pp. 1822–1827, 2014.

[21] L. P. Tu and Y. Chen, “An unequal adjacent grey forecasting air pollution urban model,” *Applied Mathematical Modelling*, vol. 99, pp. 260–275, 2021.

[22] L. F. Wu, S. F. Liu, W. Cui, D. L. Liu, and T. X. Yao, “Non-homogenous discrete grey model with fractional-order accumulation,” *Neural Computing and Applications*, vol. 25, no. 5, pp. 1215–1221, 2014.

[23] P. M. Gao, J. Zhan, and J. F. Liu, “Fractional-order accumulative linear time-varying parameters discrete grey forecasting model,” *Mathematical Problems in Engineering*, vol. 63, pp. 1–12, 2019.