Modular omnidirectional mobile robot with four driving and steering wheels

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Abstract. In this paper the authors present a modular omnidirectional mobile robot with four driving and steering robot. The paper describes the experimental prototype, the kinematic analysis in two configurations (four driving and steering wheels configuration and synchron-drive configuration), the electronic components, the testing process of the robot and future research directions.

1. Introduction

The attention for mobile robots is increasing mainly due to their many application. For a robot, the ability to move increases its flexibility and ability to perform new and more complex tasks.

An important issue regarding mobile robotics is the maneuverability of a robot. The maneuverability might be improved by making use of various locomotion systems. One such locomotion system is represented by the wheels. Beside maneuverability, the design of mobile robots has to provide viable solution to the traction, stability and controllability problems. These are influenced by design choice regarding the type of wheels, their configuration and actuating solutions [1]. Lately, in this regard, the specialised literature presents several achievements [1], [2], [3], [4].

In the following pages, the paper is structured as follows: the second section presents the omnidirectional robot and in the third section is presented the kinematic analysis of the robot. The components required for the control of the robot are presented in the fourth section, the fifth section deals with the testing process, the sixth section concludes the paper and lists future development objectives.

2. Architecture of the robot

The robot presented next is composed out of four identical modules, each containing the driving motors, driving train and the wheels. These modules are connected to a hexagonal platform which hosts four servomotors and the electronic drivers. Synchronism is achieved by simultaneously control of all four motors, each wheel of the robot possessing its dedicated driving motor.

2.1. The prototype description

The robot platform is symmetrical, hexagon shaped with a side length of 12.5 mm, made of PMMA (Poly(methyl methacrylate), Plexiglas) and the thickness is 4 mm.

Figure 1 a presented a photo robot developed in the Robotics Laboratory of the Mechatronics and Machine Dynamics Department at the Technical University of Cluj-Napoca [1]. [4]. The structural diagram of the entire robot is presented in figure 1 b. Figure 1 shows that for to steer the wheels, four servomotors (SM$_1$, SM$_2$, …) need to be synchronised. For driving the robot, another four motors need to be synchronously driven (Me$_1$, Me$_2$, …). The total height of the robot is of 22 mm and the weight of the robot, together with the power wiring, is of 1698 g. The robot is driven only on flat surfaces.
2.2. Wheeled Modules
The wheel modules attached to the hexagonal platform are four and are also made of PMMA with a thickness of 3 mm. These modules use a DC motor with a gear reducer and conical gear with straight teeth conical geared transmission \( z_1 = z_2 = 16 \) teeth) and steered by a servomotor (Fig. 2). The wheels are a diameter of 76 mm. The structural diagram and a photo of the wheel module are presented in figure 2 b, c [4].

3. Kinematics analysis of the robot
The kinematic analysis of the robot will be presented in two instances:
- when the robot has four driving and steering wheels
- when the robot has a synchro-drive configuration
The kinematic analysis presented next is for the instance when all the wheels of the robot are both driving and steering.

The proposed kinematic model is based on the geometrical constraints of the wheels. Thus, we are defining two coordinate systems \( X_{IOIYI} \) - the reference coordinate system and \( X_{RORYR} \) - the coordinate system attached to the robot’s center of gravity (Fig. 3 a).

In this case the methodology proposed in [5], [6], [7] can be used.

Thus, considering the speed of the robot's platform \( \dot{\xi}_r \) in the reference coordinate system \( X_{IOIYI} \), it is possible to determine the speed \( \dot{\xi}_r \) in the coordinate system that is attached to the robot \( X_{RORYR} \) by using the equations [5], [6]:

\[
\dot{\xi}_r = R(\theta) \dot{\xi}_j
\]  

(1)
with

\[ \dot{\mathbf{z}}_j = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \]  \hspace{1cm} (2) \\

\[ R(\theta) = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \]  \hspace{1cm} (3)

\( R(\theta) \) is the orthogonal rotation matrix expressing the orientation of the robot with respect to the base frame \( X_O Y_I \).

A steered standard wheel connected to the chassis and its parameters is shown in figure 3.b. The position of the wheel in the local reference frame is given in polar coordinates by the distance \( l \) and the angle \( \alpha \). The orientation of the plane of the wheel to with respect to \( l \) is represented by the angle \( \beta \). The rotation angle of the wheel about its (horizontal) axle is denoted by \( \varphi \) and the radius of the wheel by \( r \).

With \( \dot{\varphi}(t) \) is noted the rotation velocity of the wheel around their horizontal axes, and \( v = r \dot{\varphi}(t) \) the linear velocity of the wheel.

The wheel can spin over time, and is steerable, so \( \beta \) and \( \varphi \) are function of time, \( \beta = \beta(t), \varphi = \varphi(t) \).

The robot kinematics constraint can be obtained from the rolling and sliding constraint of wheels caused from the no-slip condition [5], [6], [7].

The rolling constraint for the wheel is given of the equation [5], [6], [7]:

\[ \begin{bmatrix} \sin(\alpha + \beta) & -\cos(\alpha + \beta) & -l \cos \beta \end{bmatrix} R(\theta) \dot{\mathbf{z}}_j - l \dot{\varphi} = 0 \]  \hspace{1cm} (4)

The sliding constraint for the wheel is given of the equation [5], [6], [7]:

\[ \begin{bmatrix} \cos(\alpha + \beta) & \sin(\alpha + \beta) \\ l \sin \beta \end{bmatrix} R(\theta) \dot{\mathbf{z}}_j = 0 \]  \hspace{1cm} (5)

The rolling and sliding constraints, equations (4), (5) are written down for the four wheels of the robot. From equation (5) we get:

\[ \begin{bmatrix} \cos(\alpha_i + \beta_i) & \sin(\alpha_i + \beta_i) & l \sin \beta_i \end{bmatrix} R(\theta) \dot{\mathbf{z}}_j = 0 \\
\cos(\alpha_i + \beta_i) & \sin(\alpha_i + \beta_i) & l \sin \beta_i \end{bmatrix} R(\theta) \dot{\mathbf{z}}_j = 0 \\
\cos(\alpha_i + \beta_i) & \sin(\alpha_i + \beta_i) & l \sin \beta_i \end{bmatrix} R(\theta) \dot{\mathbf{z}}_j = 0 \\
\cos(\alpha_i + \beta_i) & \sin(\alpha_i + \beta_i) & l \sin \beta_i \end{bmatrix} R(\theta) \dot{\mathbf{z}}_j = 0 \]  \hspace{1cm} (6)

This equation can be written as:

\[ \begin{bmatrix} \cos(\alpha_i + \beta_i) & \sin(\alpha_i + \beta_i) & l \sin \beta_i \end{bmatrix} R(\theta) \dot{\mathbf{z}}_j = 0 \\
\cos(\alpha_i + \beta_i) & \sin(\alpha_i + \beta_i) & l \sin \beta_i \end{bmatrix} R(\theta) \dot{\mathbf{z}}_j = 0 \\
\cos(\alpha_i + \beta_i) & \sin(\alpha_i + \beta_i) & l \sin \beta_i \end{bmatrix} R(\theta) \dot{\mathbf{z}}_j = 0 \\
\cos(\alpha_i + \beta_i) & \sin(\alpha_i + \beta_i) & l \sin \beta_i \end{bmatrix} R(\theta) \dot{\mathbf{z}}_j = 0 \]
where the matrix $C$ (4x3) depends on the steering angles $\beta_i(t)$, $i = 1, 2 \ldots 4$.

The mobility degree of the robot $\delta_m$ can be determined from equation (7) by knowing the number of independent constraints [5], [6], [7], [8].

$$\delta_m = 3 - \text{rank}(C).$$

Therefore, the two following situations can be identified:

- If the axes of the four wheels of the robot do not intersect in the same point, all the kinematic constraints are independent and rank $(C) = 3$, thus $\delta_m = 0$. In this situation the robot's chassis is not moving.
- If the axes of the four wheels are intersecting in the same point, three or four constraints become linear dependent, thus rank $(C) = 2$ which results in $\delta_m = 1$. The robot can rotate around the intersection point of the four wheel axes, point that is called instantaneous center of rotation (CIR). If the axes of the four wheels are parallel (the rotation center is infinite) the robot can move in a straight line.
- The kinematic analysis detailed below applies to the case when the platform of the robot is not being turned during travel, acting like a synchro-drive robot. In this situation, for the robot to function properly, it is needed that the wheels are well aligned and their speeds need to be identical. Also, all the wheels need to turn synchronously with the same angular displacement.

In figure 4 a, $X_WO_WY_W$ is the coordinate system attached to the robot's wheels, $X_OY_R$ is the coordinate system attached to the robot's center of gravity and $X_OY_I$ is the main coordinate system. In the following kinematic equations it is assumed that the robot is travelling on a plane surface, without slipping.

![Figure 4. Setup of coordinate systems on the robot and on its wheels for the synchro-drive configuration a) and the block diagram of the electronic circuit b)](image)

The kinematic analysis for this case is represented by the equations presented next.

### 3.1. Forward Kinematics

Next, $(x, y)$ represent the coordinates of the robot's platform center of gravity related to the $X_OY_I$ coordinate system and $\theta$ represents the orientation angle of the robot or the orientation of the robot's wheels related to an axis which is parallel to $O_X$. Also, $v$ and $\omega$ are the linear and angular speeds of the robot.

The evolution of $x$, $y$ and $\theta$ is given by the equations [9]:

$$\dot{\theta} = J(\theta)\dot{q}$$

$$\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
\cos \theta & 0 \\
\sin \theta & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
v \\
\omega
\end{bmatrix}.$$
Thus, if we wish for the robot to move in a straight line, we can assign a linear speed \( v \) and setting the angular speed \( \omega \) to zero. If the angular speed \( \omega \) is set to a value different than zero, the robot will then travel on a curved trajectory. The travel speed of the robot \( v \) varies depending on the rotation speed of the drive motor and it can be determined using the equation:

\[
v = \omega_R r = \frac{\pi n_m z_1}{30} r
\]

where:
- \( \omega_R \) is the angular speed of the wheels
- \( r \) is the radius of the wheels,
- \( n_m \) is the rotation speed of the motor \( Me \),
- \( n_R \) is the rotation speed of the wheels and it is given by the equation:

\[
n_R = n_m \frac{z_1}{z_2}
\]

By using equation (10), the position and orientation of the robot can be determined [9]:

\[
p = \int_0^t \dot{p} \, dt = \begin{bmatrix} x_0 \\ y_0 \\ \theta_0 \end{bmatrix} + \begin{bmatrix} \int_0^t v \cos \theta \, dt \\ \int_0^t v \sin \theta \, dt \\ \int_0^t \omega \, dt \end{bmatrix}
\]

\[
x = x_0 + \int_0^t v \cos \theta \, dt
\]

\[
y = y_0 + \int_0^t v \sin \theta \, dt
\]

\[
\theta = \theta_0 + \int_0^t \omega \, dt
\]

where \( x_0, y_0, \theta_0 \) is the initial positions of the robot.

With the help of this equation, the trajectory of the mobile robot can be estimated.

**Inverse Kinematics**

By using equation (10), the solution \( \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} \) could be easily found by determining the inverse matrix of the Jacobian \( J(q) \), if \( J(q) \) would be a square matrix. Due to the fact that \( J(q) \) is not a square matrix, \( \begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} \) can be found by using the pseudo-inverse matrix \( [J(q)]^+ \) of \( J(q) \), thus obtaining [9]:

\[
\dot{q} = [J(q)]^+ \dot{p} = ([J(q)^T][J(q)])^{-1}[J(q)^T] \cdot \dot{p}
\]

or

\[
\begin{bmatrix} \dot{v} \\ \dot{\omega} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix}
\]

The proposed method is simple, easy to use, can be easily implemented and is highly efficient when the robot travels onto flat surfaces.

**4. The control of the robot**

The robot is fitted with two microcontroller boards: Cerebot II [10] and SSC-32 [11] which communicate through a serial connection. The robot is controlled by the CEREBOT II board that is fitted with an Atmega64L microcontroller. The peripheral modules L293D and PmodHB5 are also used. The SSC-32 board, which is fitted with an AVR RISC structured ATmega8 microcontroller, generates the command impulses for the four servomotors. The block diagram of the electronic circuit is presented in figure 4b. The servomotors used are hi-tech HS-475HB and the geared drive motors are IG22 [12].

**5. The testing of the robot**
The robot only travels on a plane surface. During the testing process under laboratory conditions, the robot had a good performance. A few images from the testing process are shown in figure 5.

![Images from robot testing](image)

**Figure. 5** Images from robot testing

The robot is not energetically autonomous, being powered through wire from an external power source. The testing was done also for a synchro-drive configuration. After testing it was concluded that the main disadvantage of this robot is the large number of motors that have to be driven and controlled. Another important disadvantage is the lack of a suspension system which recommends the robot for use only on flat, even surfaces. The authors contributed to the design of the robot's modular structure.

### 6. Conclusions

The paper presents the design of an omnidirectional robot with four drive and steering wheels. For the robot to travel properly, the four wheels need to be synchronously driven and steered. This synchronisation is achieved by controlling the motors and servomotors from the robot's structure.

**Future research:** Next, the authors want to study the dynamics of the robot, to fit the robot with sensors and to use batteries for powering the robot in order to increase its autonomy. Improving the mechanical structure of this particular robot, in order for it to be able to travel over small obstacles, proves to be a challenge.

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