WAVELET ANALYSES OF HIGH MULTIPLICITY EVENTS

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Pseudo-rapidity distributions of high multiplicity JACEE events (Ca-C, Si-AgBr and G27) and Texas Lone Star (TLS) event are analysed by the wavelet transform method. Using the Daubechies’ wavelets, D_4, D_6, D_8, and D_10, wavelet spectra of those events are calculated. Two JACEE events are compared with the simulation calculations. We discuss how to distinguish prominent peak from apparent peaks. The wavelet spectrum of Ca-C event seems to resemble that simulated with the Poisson random number distributions. The wavelet spectrum of TLS event shows an oscillatory behavior. It is found that the wavelet spectra of JACEE G27 event are different from those of Arizona group.

1 Introduction

In high energy nucleus-nucleus (AA) collisions number density of secondary particles in the rapidity space becomes very high and studies of number density fluctuations in the rapidity space is expected to reveal new features of multiparticle production mechanisms. The pseudo-rapidity distributions of the three JACEE events (Ca-C, Si-AgBr and G27) and the Texas Lone Star (TLS) event are analysed by the wavelet transform method. Any function (data) can be expanded into self-similar wavepackets in this scheme. Wavelet spectra of those four events are calculated and some of them are compared with simulation calculations. In § 2 the wavelet transform concept is introduced and in § 3 the Daubechies’ mother wavelets, D_4, D_6, D_8, and D_10, are prepared to be used in § 4. The wavelet spectra of the above mentioned four events are calculated and compared with the simulation calculations in § 4. The final section is devoted to the concluding remarks.

2 Wavelet transform

Wavelets are constructed from dilation and translation of a scaling function \( \phi(x) \), which is constructed by an iteration equation,

\[
\phi_i(x) = \sum_{k=0}^{N-1} c_k \phi_{i-1}(2x - k) \quad i = 1, 2, \cdots ,
\]

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from a primary scaling function \( \phi_0(x) \) (\( \phi_0(x) = 1 \) for \( 0 \leq x < 1 \) and \( \phi_0(x) = 0 \) otherwise). In Eq. (1), \( N \) is an even number and \( c_k \) \((k = 0, 1, \cdots, N-1)\) are constants. The iteration is continued until \( \phi_i(x) \) becomes indistinguishable from \( \phi_{i-1}(x) \) and defines then \( \phi(x) \).

The mother wavelet \( W(x) \) is given by
\[
W(x) = \sum_{k=0}^{N-1} (-1)^{k+1} c_{N-1-k} \phi(2x + k - N + 1),
\] (2)
and defines the \( j \)-th level wavelet \((j = 0, 1, \cdots)\)
\[
\psi_{j,k}(x) = 2^j W(2^j x - k), \quad k = 0, 1, \cdots, 2^j - 1.
\] (3)

Coefficients \( c_k \) \((k = 0, 1, \cdots, N-1)\) should be determined in such a way that wavelets and scaling function satisfy the following conditions,
\[
\int \phi(x) \phi(x) dx = 1, \quad \int \phi(x) \psi_{j,k}(x) dx = 0,
\]
\[
\int \psi_{j,k}(x) \psi_{r,s}(x) dx = \delta_{jr} \delta_{ks}, \quad \int x^k W(x) dx = 0, \quad k = 0, \cdots, N/2 - 1.
\] (4)

If \( N=4 \), we have
\[
c_0 = 1 + \frac{\sqrt{3}}{4}, \quad c_1 = \frac{3 + \sqrt{3}}{4}, \quad c_2 = \frac{3 - \sqrt{3}}{4}, \quad c_3 = \frac{1 - \sqrt{3}}{4}.
\]

Then arbitrary function \( f(x) \) defined in the region \( 0 \leq x < 1 \) can be expanded as
\[
f = a_0 \phi(x) + \sum_{j=0}^{\infty} \sum_{k=0}^{2^j-1} a_{j,k} \psi_{j,k}(x).
\]

We shall consider here the case in which function \( f(x) \) \((0 \leq x < 1)\) is given by its discrete values, \( f_i = f(\Delta/2 + \Delta \cdot (i - 1)), \quad (i = 1, 2, \cdots, 2^r)\), where \( \Delta \cdot 2^r = 1 \). By the use of column matrices,
\[
F = \begin{pmatrix}
f_1 \\
f_2 \\
\vdots \\
f_{2^r}
\end{pmatrix}, \quad A_j = \begin{pmatrix}
a_{j,0} \\
a_{j,1} \\
\vdots \\
a_{j,2^j-1}
\end{pmatrix}, \quad j = 0, 1, \cdots.
\]
the wavelet coefficients can be written as

\begin{align}
   a_0 &= 2^{- \frac{r}{2}} L_1 L_2 \cdots L_r F, \\
   A_{j-1} &= 2^{- \frac{r}{2}} H_j L_{j+1} \cdots L_r F, \quad j = 1, 2, \cdots, r - 1, \\
   A_{r-1} &= 2^{- \frac{r}{2}} H_r F. \\
\end{align}

(5)

where \( L_j \) and \( H_j \) are \( 2^j \times 2^j \) matrices. The \( i \)-th row of \( L_j \) is expressed as,

\[
\begin{pmatrix}
0 & \cdots & 0 & l_{2(i-1)+1} & \cdots & l_{2(i-1)+N} & 0 & \cdots & 0
\end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & \cdots & 0 & c_0 & c_1 & \cdots & c_{N-1} & 0 & \cdots & 0
\end{pmatrix}.
\]

If \( 2(i-1) + k > 2^j \), element \( l_{2(i-1)+k} \) is added to the \((2(i-1)+k-2^j)\)-th element in each row. Matrix \( H_j \) is obtained, if \( c_k \) \((k = 0, 1, \cdots, N-1)\) in \( L_j \) is replaced by \((-1)^{k+1} C_{N-1-k}\).

Matrices \( L_j \) and \( H_j \) satisfy the following conditions (coming from Eq. (4)),

\[
L_j^t L_j = H_j^t H_j = I, \quad L_j^t H_j = H_j^t L_j = 0,
\]

(6)

where \( L_j^t \) denotes the transpose of matrix \( L_j \), and \( I \) is the \( 2^j \)-th order unit matrix.

From Eqs. (5) and (6), one gets the inverse wavelet transform:

\[
\begin{align}
F &= F^\phi + \sum_{j=0}^{r-1} F^{(j)}, \\
\quad F^{(j)} &= 2^j a_0 L_1 L_2 \cdots L_r, \\
\quad F^{(j-1)} &= 2^j \quad a_{j-1} H_j L_{j+1} \cdots L_r, \quad j = 1, 2, \cdots, r - 1, \\
\quad F^{(r-1)} &= 2^j \quad a_{r-1} H_r,
\end{align}

(7)

Finally, Eqs. (3) and (4) lead to the following identity;

\[
\quad F F 2^{-r} = 2^{-r} \quad \sum_{j=1}^{2^r} f_j^2 = a_0^2 + \sum_{j=0}^{r-1} E_j,
\]

\[
E_j = \quad a_{j,k} = \quad A A = \quad a_{j,k}^2,
\]

(8)

where \( E_j \) denotes the \( j \)-th level wavelet spectrum.
3 Daubechies’ mother wavelets

We prepare now Daubechies’ mother wavelets, $D_4, D_6, D_8$, and $D_{10}$, cf. Fig. 1, to be used for wavelet analysis below. If results are independent on these mother wavelets, one can say that wavelet spectra bear some physical /or stochastic meaning to be further explored.

Fig. 1. Daubechies’ mother wavelets, $D_4, D_6, D_8$, and $D_{10}$.

4 Wavelet spectra of the data

We analyse now by the wavelet transform the pseudo-rapidity distributions of the three JACEE events and the TLS event.

1) In the case of Ca-C and Si-AgBr JACEE events (cf. Fig. 2), both original data and subtracted data (original data - its background (BG)) are used. The corresponding wavelet spectra are shown in Fig. 2.
Fig. 2. Ca-C and Si-AgBr JACEE events and their wavelet spectra. Results obtained by the Poisson random number distributions are shown with $\langle E_j \rangle + \sigma_j$ and $\langle E_j \rangle - \sigma_j$. Notice that "$\langle \ldots \rangle$" means an assemble average.

We shall propose now how to distinguish a prominent peak from apparent ones. To this end, we use the wavelet coefficients satisfying the following condition,

$$|\alpha_{jk} - \mu| > \alpha \sigma,$$

(9)

where $\mu$ is the average and $\sigma$ is the standard deviation of the wavelet coefficients. If $\alpha$ is chosen as 1.96 or 2.58, the absolute values of the coefficients become large. Our results show that peak present in Si-AgBr event does not depend on the Daubechies’ mother wavelets $D_4$, $D_6$, $D_8$, and $D_{10}$. Therefore we can examine further its physical meaning. On the other hand, the peak in Ca-C event depends on the Daubechies’ mother wavelets. Therefore we can say that it is an apparent one.
2) The TLS event is a photon distribution. Using the same procedure, we obtain its wavelet spectra shown in Fig. 4. At present it is difficult to elucidate physical meaning of their oscillatory behaviour.
3) In this case of JACEE event G27, Arizona group\textsuperscript{14} and Bjorken\textsuperscript{15} have conjectured that this event probably contains some signal of the disoriented chiral condensate (DCC). The ratio $r = n_0/(n_0 + n_{ch})$\textsuperscript{16} is shown in Fig. 5. Using the same procedure as mentioned, we obtain the wavelet spectra shown in Fig. 5.

![Fig. 5. JACEE G27 event and its wavelet spectra.](image)

It is found that our result is different from that of Arizona group\textsuperscript{14} (Notice that our method produces for the random distribution an increasing behaviour of wavelet spectrum when $j$ increases.)

5 Concluding remarks

We have prepared the Daubechies’ mother wavelets, $D_4$, $D_6$, $D_8$, and $D_{10}$, and using them we have analysed high multiplicity events. It is found that our wavelet spectra obtained from three JACEE events and TLS event show different behaviour from that of Poisson random number distribution\textsuperscript{11}. We conclude therefore that one can use the wavelet analysis to gain some physical information/or stochastic properties from the high multiplicity distributions.

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References

1. F. Takagi, Phys. Rev. Lett. 53(1984), 427
2. N. Suzuki, Y. Yasutani, M. Biyajima and T. Mizoguchi, Prog. Theor. Phys. 85(1991), 149; N. Suzuki and M. Biyajima, Prog. Theor. Phys. 86(1992), 73
3. M. Biyajima and N. Suzuki, Proceedings of the XXI Internal Symposium on Multiparticle Dynamics (Wuhan, 1991), World Scientific, 1992, p.486
4. N. Suzuki, M. Biyajima and A. Ohsawa, Prog. Theor. Phys. 53 (1995) 91.
5. T. H. Burnett et al., Phys. Rev. Lett. 50(1983), 2062
6. D. H. Perkins and P. H. Fowler, Proc. Roy. Soc. A278(1964) 401.
7. I. Daubechies, Comm. Pure and Appl. Math. (1988)909; Ten Lectures on Wavelets, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, 1992
8. G. Strang, SIAM Review, 31(1989), 614
9. D. E. Newland, An Introduction to Random Vibrations, Spectral and Wavelet Analysis, third edition, Longman, Harlow, 1993
10. A. Morimoto, Master Thesis, Math. Dept. of Kyoto Univ. (1988)
11. M. Yamada and K. Ohkitani, Prog. Theor. Phys. 86(1991), 799
12. C. Meneveau and K. R. Sreenivasan, Phys. Rev. Lett. 59(1987), 1424
13. M. Mori, Programming for Numeric Calculations, Iwanami, Tokyo, 1986 (in Japanese)
14. I. Sarcevic, in this Proceedings.
15. J. D. Bjorken, in this Proceedings.
16. J. Huang, private communication.
Fig. 1(a)
Fig. 1(b)
Fig. 1(c)

The graph shows the function $W(x)$ for different values of $x$, with peaks at certain intervals. The label $c) D_8$ indicates a specific case or parameter within the graph.
Fig. 1(d)
Fig. 2(a)
Si - AgBr

Fig. 2(b)
Fig. 2(c)
Fig. 2(d)

- \( E_j \) vs. \( j \)

- \( \text{Sub. of BG} \) (solid line)
- \( \text{No Sub.} \) (dashed line)

- \( \text{Si-AgBr} \)
Ca -C - BG \( (D_4) \)

Fig. 3(a)
Fig. 3(b)

$F^p_k$

$\eta$

Si - AgBr  ($D_4$)
Fig. 3(c)
Fig. 3(d)

$F_{pk}$

Si - AgBr ($D_6$)
Fig. 4(a)
Fig. 5(a)
