THREE-DIMENSIONAL SIMULATIONS OF THE REORGANIZATION OF A QUARK STAR’S MAGNETIC FIELD AS INDUCED BY THE MEISSNER EFFECT

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Received 2005 October 20; accepted 2006 August 27

ABSTRACT

In a previous paper we presented a new model for soft gamma-ray repeaters (SGRs), based on the onset of color superconductivity in quark stars. In this model, a single burst results from the reorganization of the exterior magnetic field following the formation of vortices that confine the internal magnetic field (the Meissner effect). Here we present full three-dimensional simulations of the evolution of the inclined external magnetic field immediately following vortex formation. The simulations capture the violent reconnection events in the entangled surface magnetic field as it evolves into a smooth, more stable configuration that consists of a dipole field aligned with the star’s rotation axis. The total magnetic energy dissipated in this process is found to be of the order of $10^{44}$ ergs, and if it is emitted as synchrotron radiation, it peaks typically at 280 keV. The intensity decays with time in a way characteristic of SGR giant flares with a tail lasting from a few to a few hundred times the rotation period of the star, depending on the initial inclination between the rotation and dipole axes. One of the obvious consequences of our model’s final state (aligned rotator) is the suppression of persistent pulsed radio emission in SGRs and anomalous X-ray pulsars (AXPs) following their bursting era. We compare our model to observations and highlight our predictions.

Subject headings: elementary particles — gamma rays: bursts — stars: magnetic fields — stars: neutron — X-rays: stars

1. INTRODUCTION

Soft gamma-ray repeaters (SGRs) are sources of recurrent, short ($\tau \sim 0.1$ s), intense ($L \sim 10^{41} L_{\text{Edd}}$) bursts of gamma-ray emission with a soft energy spectrum. The normal pattern of SGR activity is intense activity periods that can last weeks or months, separated by quiescent phases lasting years or decades. The five known SGRs are located in our Galaxy or, in the case of SGR 0526–66, in the Large Magellanic Cloud. The three most intense SGR bursts ever recorded were the 1979 March 5 giant flare of SGR 0526–66 (Mazets et al. 1979), the similar 1998 August 28 giant flare of SGR 1900+14, and the 2004 December 27 burst (SGR 1806–20). The peak luminosities of these events ($\sim 10^{41} L_{\text{Edd}}$) exceeded the peak luminosities of “normal” SGR bursts by a factor of $>10^3$. Several SGRs have been found to be X-ray pulsars with an unusually high spin-down rate of $P' / P \sim 10^{-10}$ s$^{-1}$, usually attributed to magnetic braking caused by a superstrong magnetic field $B > 10^{14}$ G, which leads to the interpretation that SGRs are magnetars (Golenetskij et al. 1979; Duncan & Thompson 1992; Kouveliotou et al. 1998, 1999). In the magnetar model, the magnetic field is the likely provider of the burst energy. A common scenario assumes that magnetic stresses create a quake in the crust of the neutron star, which then ejects hot plasma Alfvén waves through its rigid magnetosphere (Thompson & Duncan 1995, 1996). The magnetic field of such a star would have grown to magnetar-scale strengths because of strong convection during the collapse of the proto–neutron star core (Duncan & Thompson 1992; Thompson & Duncan 1993).

1.1. Open Issues in the Magnetar Model of SGRs

In the magnetar model of SGRs, which is also that of anomalous X-ray pulsars (AXPs), the X-rays are ultimately powered by an internally decaying very strong magnetic field. However, there are still a few open questions that in our opinion leave room for new models to be explored.

1. Despite numerous attempts, no persistent radio emission has been detected from magnetars (Kris 

2. One might expect high- $B$ radio pulsars to be more X-ray bright than low- $B$ sources and to possibly exhibit AXp-like burst emission. However, X-ray observations of five high- $B$ radio pulsars reveal luminosities much smaller (by a few orders of magnitude) than those of AXPs (Pivovaroff et al. 2000; Gonzalez & Safi-Harb 2003; McLaughlin et al. 2003; Gonzalez et al. 2004; Kaspi & McLaughlin 2005). This has led to suggestions that high- $B$ radio pulsars may one day emit transient AXp-like emission and conversely that the transient AXPs may eventually exhibit radio pulsations (Kaspi & McLaughlin 2005)—a notion yet to be confirmed.

3. Hints of massive (>30–40 $M_\odot$) progenitors associated with AXPs and SGRs by recent observations (Gaensler et al. 2005) have led to the suggestion that pulsars and SGRs differ in their progenitor masses. It has also been suggested that massive progenitors could lead to neutron stars with millisecond periods (Heger et al. 2005), which would comply with the magnetar model for SGRs (Duncan & Thompson 1992). This, however, leaves open the question of why high- $B$ pulsars, formed from less massive progenitors (presumably with periods >15 ms), possess magnetar-like field strengths.

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4. All SGRs and AXPs known to date have spin periods between 6 and 12 s (Kaspi 2004). The period distribution can be described by models of field decay (Heyl & Kulkarni 1998; Colpi et al. 2000), but the underlying cause is not fully understood.

Here we explore an alternative model in which one assumes that AXPs and SGRs are quark stars, rather than magnetars. While quark stars have yet to be found in nature, formation scenarios have been suggested in the literature (see § 8.4 in Ouyed et al. 2004 and references therein). The qualitative idea is that the core of a neutron star eventually reaches deconfinement densities, leading to the conversion of the entire star to a quark star (Ouyed et al. 2002; Keränen et al. 2005). The new idea of this model is that the quark star enters a superconducting phase and subsequently experiences a “Meissner phase” that triggers the reorganization of the star’s magnetic field. Before going into more details we first describe properties of quark matter and the concept of color superconductivity.

1.2. Superconductivity and Meissner Effect in Quark Stars

The discovery of asymptotic freedom, leading to the formulation of quantum chromodynamics (QCD) as the theory of strong interactions, was soon followed by the suggestion that matter at sufficiently high densities consists of a deconfined phase of quarks (Collins & Perry 1975). Only shortly afterward it was pointed out (Barrois 1977) that the true ground state of cold dense quark matter exhibits color superconductivity (CSC), characterized by diquark condensation with an estimated energy gap \( \Delta \approx 1 \text{ MeV} \) between the highest occupied and the unoccupied quark state at the Fermi surface. Since this magnitude of the gap is rather small for phenomenological applications, CSC subsequently received little attention. The situation changed when reinvestigations (Alford et al. 1998; Rapp et al. 1998), using nonperturbative forces (e.g., instanton induced), showed that the gap can be substantially larger, \( \Delta \approx 100 \text{ MeV} \) for moderate quark chemical potentials, \( \mu_q \approx 350 \text{ MeV} \). Similarly large values are obtained from estimates based on perturbative calculations at asymptotically high densities (Pisarski & Rischke 1999; Son 1999). Thus, from the practical point of view, the existence of color superconductivity in compact stars has (re)emerged as an exciting possibility.

The detailed properties of CSC matter relevant to astrophysical applications depend on the interplay of the quark chemical potential, the quark-gluon interaction strength, and the bare masses of the (light) quarks \( u, d, \) and \( s \). In particular, for \( \mu_q \) below the strange quark mass, only \( u \) and \( d \) quarks are subject to Cooper pairing. The corresponding phase is known as two-flavor CSC (2SC). In the idealized case in which the quark chemical potential is much larger than the strange quark mass (\( m_s \)), the latter becomes negligible and all three flavors exhibit likewise pairing. The preferred symmetry-breaking pattern in this phase corresponds to the so-called color-flavor locking (CFL; Alford et al. 1999), since the underlying diquark condensate is invariant only under simultaneous color and flavor transformations. In the present work, we focus on the CFL phase (for a recent review and a more exhaustive list of references, see Schäfer 2003). Associated with CSC is the critical temperature, \( k_B T_c \approx 0.5\Delta \), above which pairing is washed out.

One of the most interesting properties of an ordinary superconductor is the Meissner effect, i.e., the expulsion of magnetic flux from the superconductor (Meissner & Ochsenfeld 1933). In the CFL phase, the gauge bosons connected with the broken generators obtain masses, which indicates the Meissner screening effect (Rischke 2000a, 2000b). This is at the heart of our model, which we describe next.

1.3. Our Model and Initial State

To turn into a quark star, a neutron star only has to reach a central density greater than the critical density for quark deconfinement. The transition from hadronic matter into quark matter in the core of a neutron star could happen immediately during or after the supernova explosion, but it could also happen much later than that. The factors contributing to this delay include the time needed for the neutron star to spin down to the quark deconfinement critical density (hadron to \( ud \) matter) and the time delay involved in nucleation (\( ud \) to uds matter), which leads to formation of the superconducting CFL star. This latter delay is discussed by Bombaci et al. (2004); it depends on currently unknown equation-of-state parameters and can be thousands of years. Such a transition could occur in a smooth stable manner (e.g., Bombaci & Datta 2000 and references therein) or in an explosive manner termed “quark nova” (Ouyed et al. 2002; Keränen et al. 2005).

We consider high-\( B (>10^{12} \text{ G}) \), high-mass (>1.5 \( M_\odot \)) neutron stars as the most likely candidates to experience a quark nova. When spinning down, such a star can reach deconfinement densities in the core at any time following its birth. The time to reach deconfinement density depends on the neutron star initial mass, magnetic field, and spin, and the equation of state of neutron matter adopted and is typically less than 1 yr (Staff et al. 2006). Thus, the delay is determined by the \( ud \) to uds nucleation process, which only depends on QCD parameters whose values have yet to be calculated accurately. We should also note that after deconfinement has been reached, QCD effects (\( ud \) to uds nucleation) produce a further delay that can easily last for thousands of years (Bombaci et al. 2004). The choice of adopted physics inputs (e.g., equation of state, deconfinement density, nucleation) will change the expected spin period distribution for giant SGR outbursts but will not change the fact that quark novae can be delayed by thousands of years from neutron star birth.

Our calculations start just after the quark nova has occurred (timescale \( \sim 10^{-4} \) s) with the superconductive phase transition ensuing immediately after (timescale of microseconds). Assume a quark star is born with a temperature \( T > T_c \) and enters the CFL phase as it cools rapidly by neutrino emission (Keränen et al. 2005). The CFL star quickly expands to the entire star followed by the formation of rotationally induced vortices, analogous to rotating superfluid \( ^3\text{He} \) (the vortex lines are parallel to the rotation axis; Tilley & Tilley 1990). Via the Meissner effect, the magnetic field is partially screened from the regions outside the vortex cores. The system now consists of alternating regions of superconducting material with a screened magnetic field and the vortices where most of the magnetic field resides. As discussed in Ouyed et al. (2004), this has interesting consequences on how

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5 We should note the existence of a modified electromagnetism that does not suffer the Meissner effect in the CFL phase. The effects of the component of the magnetic field that penetrates the color superconductor have been investigated in the literature (e.g., Ferrer et al. 2005 and references therein), where it was found that it enhances the quark condensates and changes the low-energy properties of the superconductor. This might have interesting consequences for the picture we present here.

6 Accretion from ISM or in a binary is less favored as a mechanism for the quark nova. For ISM the accretion rate is too low. For close binaries mass transfer is likely to limit the mass of the progenitors and results in low-mass neutron stars that will never undergo quark nova.
the surface magnetic field can adjust to the interior field that is pinned to the vortices. Figure 1 shows the starting point for this reorganization of the magnetic field, just after the Meissner effect has fully aligned the magnetic flux inside the star with the rotation axis. Within a transition region, the magnetic field switches from the vertical interior field to that of an inclined dipole outside. A conservative assumption (from the energetics point of view) is that the field in the transition region is a potential field (minimum-energy configuration). See Appendix A for more details.

In order to capture the complex nonlinear dynamics of the system we need the help of three-dimensional (3D) simulations. These simulations, performed in the corotating frame, allow us to follow the evolution and reorganization of the magnetic field induced by the vortices.

It is generally impossible to draw magnetic field lines of a three-dimensional field in a two-dimensional section, and what is shown in Figure 1 are not magnetic field lines but lines of constant $A_y$, where $A_y$ is the $y$-component of the magnetic vector potential. These isocurves are sufficient to show, for example, the propagation of well-defined Alfvén fronts. While for an axisymmetric magnetic field, poloidal field lines would be the isocurves of $sA_y$ (which would be identical to $\pm x A_x$ in the $x$-$y$ plane, and where $s$ is the field-line vector), this representation would not work for nonaxisymmetric fields such as that shown in Figure 1.

This paper is presented as follows: In §2 we describe and analyze the basic setup of our simulations. We calculate the synchrotron light curves that result from the simulations, wherein we find remarkable similarities to SGR light curves. Mechanisms for the observed subsequent bursts and the quiescent phase in SGRs are then briefly discussed in terms of our model in §3. In §4 we discuss the model predictions and how it can account, at least in its current stage, for the points listed in §1.1. Here we suggest a list of observations that could test our model. We conclude in §5.

2. SIMULATIONS

Our simulations begin after quark matter has made the transition into the CFL phase. Prior to the rather sudden conversion is a nucleation delay (Bombaci et al. 2004), required to form the initial critical-sized quark matter droplet. This transition is on the order of $R_{\text{QS}}/c < t_{\text{conv}} < R_{\text{NS}}/c$, or $t_{\text{conv}} \sim 10^{-13}$ s (Lugones & Benvenuto 1998), where $c$ is the speed of light and $c_s = c/\sqrt{3}$ is the sound speed; $R_{\text{NS}}$ is the neutron star radius. Since the transition is rapid, and considering that the interior flux expulsion timescale from the CFL matter into the vortices is on the order of microseconds and that the Alfvén crossing time in the magnetosphere is on the order of $1-100$ s, it is reasonable to assume the external magnetic field has not drastically changed from the dipole configuration.

Two fundamental parameters of our model are the inclination angle $\theta$ of the external dipole field relative to the rotation axis and $\beta$, the initial ratio of gas to magnetic pressure at the surface. The simulations start with the interior magnetic field confined in vertical vortices for $r < R_{\text{Q}}$ but with a still unperturbed inclined dipole field for $r > 1.3R_{\text{Q}}$. The transition region $R_{\text{Q}} < r < 1.3R_{\text{Q}}$ is filled with a potential field and is bounded by two current layers (see Fig. 1). The whole configuration was chosen by minimizing the total magnetic energy (see Appendix B).

We solve the following set of nonideal magnetohydrodynamic (MHD) equations, in the corotating frame, using the Pencil Code (see, e.g., Dobler et al. 2006).\footnote{See http://www.nordita.dk/software/pencil-code/. The Pencil Code is a high-order finite-difference code for solving the (classical) compressible hydromagnetic equations.}

$$\frac{D\ln \rho}{Dt} = -\nabla \cdot \mathbf{u},$$

$$\frac{Du}{Dt} = -c_s^2 \nabla \left( \frac{s}{c_p} + \ln \rho \right) - \nabla \Phi_{\text{grav}} + \frac{j \times B}{\rho} - 2\Omega \times \mathbf{u} + \nu \left( \nabla^2 \mathbf{u} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{u}) + 2 S \cdot \nabla \ln \rho \right),$$

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{u} \times \mathbf{B} - \eta \mu_0 j,$$

$$\rho \mathbf{T} \frac{DS}{Dt} = \nabla \cdot (K \nabla T) + \eta \mu_0 j^2 + 2 \rho \nu S^2.$$
the surface can be estimated for a given \( \beta_0 \) as \( B_0^2 = 8\pi P_0/\beta_0 \), where \( P_0 \) is the pressure at the surface of the star. Our simulation has the corona initially in magnetohydrostatic balance, which determines the initial density profile and also gives

\[
B_0 \approx \frac{5 \times 10^{13} \text{ G}}{\sqrt{\beta_0}} \left( \frac{\rho_0}{10^6 \text{ g cm}^{-3}} \right)^{1/2} \left( \frac{10 \text{ km}}{R_{QS}} \right)^{1/2} \left( \frac{M_{QS}}{M_0} \right)^{1/2}.
\]

(5)

In the equation above, \( \rho_0 \) is the average density of the gas close to the surface of the star. This corona is supplied by fallback material following the formation of the quark star (Keränen et al. 2005), similar to what has been suggested in the supernova case (Chevalier 1989). The density of the fallback matter, representative of the outermost layers of the crust material of the parent neutron star, is estimated to be of the order of \( 10^6 \text{ g cm}^{-3} \) (Datta et al. 1995).

Note that our MHD equations (1)–(4) are nonrelativistic and are thus applicable to regions well within the light cylinder. In our simulations, we treat the surface as a transition region with a minimum energy configuration. Furthermore, because our simulation is performed in the corotating frame, all bulk velocities remain low, so a full relativistic treatment is unnecessary. Table 1 gives the initial parameters used in our simulations with the resulting maximum Alfvén velocity, \( v_{\text{A, max}} \), and the characteristic decay time, \( t_{\text{decay}} \), roughly estimated from Figures 2 and 3.

| Initial Parameters | Results |
|-------------------|---------|
| \( \theta \) (deg) | \( \beta \) | \( v_{\text{A, max}} \) (km s\(^{-1}\)) | \( t_{\text{decay}} \) (s) |
| 15                | 0.1     | 150              | 1–2              |
|                   | 1.0     | 15               | 100–200          |
| 10                |         | 1.5              | >400             |
| 30                | 1.0     | 25               | 100–200          |
| 60                | 1.0     | 2.5              | >400             |

All figures shown here are for resolution 128\(^3\). We have run simulations at higher resolution but found that they differ little as far as energetics and evolution are concerned. We use effectively reflecting boundaries. The boundary conditions are impenetrable, free-slip conditions for the velocity and normal-field conditions for the magnetic field. For density and entropy, vanishing second derivative conditions were employed, which impose relatively little constraint on the variables. We have run simulations with different box sizes and found that the boundary effects are negligible for boxes 5 or more times the radius of the star.

2.1. Evolution and Reorganization of the Surface Magnetic Field

Figures 4 and 5 show the evolution of the exterior magnetic field as it adjusts to the vortex-confined field.

The complicated structure of the surface magnetic field is clearly seen in these panels, driven by the frequent magnetic reconnections as the surface field tries to align itself with the interior one (rotation axis). These random reconnection events would bear many similarities to the initial events (i.e., those near \( t = 0 \)), but we expect them to be less energetic as the magnetic field slowly decays and weakens. Eventually, the magnetic field evolves into a stable configuration (see Fig. 4) after which the star enters a quiescent phase.

The restructuring of the field in the transition region leads to an approximately spherical Alfvén wave traveling outward (see Fig. 4 for \( t = 1 \)) and is more prominent in simulations with a stronger magnetic field (i.e., smaller \( \beta \)). As the wave travels outward it creates steep gradients in the field, causing those regions to undergo reconnection, which both distorts the wave and eventually damps it out. Furthermore, the regions that underwent reconnection appear to show slow oscillatory motions between the reconnection site and the surrounding gas ("breathing"). This can be seen in the series of diminishing pulses in Figure 3, and the frequency of the pulses remains nearly constant (see Fig. 6).

We note that these pulses appear more prominently in simulations with lower \( \beta \) and do not arise in simulations with \( \beta > 1 \).

There are two pieces of physics that define the timescale for the decay (roughly estimated from Figs. 2 and 3 and shown in.

\[\text{FIG. 2. — Total (normalized) intensity emitted (}\beta_0^2 L)\text{ vs. time for } \beta = 0.3 \text{ and } \theta = 15^\circ, 30^\circ, \text{ and } 60^\circ (\text{angle between rotation and initial magnetic dipole axis}).\]
Table 1). First, the Alfvén crossing time that depends on $\beta$ defines how quickly the magnetosphere adjusts to the energy dissipation. Second, the dissipation through reconnection depends on the inclination angle, $\theta$, of the dipole (Fig. 2). Higher inclination angles imply more magnetic energy to tap from before alignment is reached. The amount of magnetic energy dissipated in the process is given in Appendix B (eq. $[B11]$). The overall decay timescale is not describable by a simple analytic formula, since reconnection is a nonlinear process.

2.2. Energetics and Emission

The magnetic energy released in the reorganization is shown in Figure 3 and can be cast into a simple equation,

$$E_M \sim \left(10^{44} \text{ ergs}\right) \left(\frac{\alpha}{0.5}\right)^2 \left(\frac{B}{5 \times 10^{13} \text{ G}}\right)^2 \left(\frac{R_{QS}}{10 \text{ km}}\right)^3,$$

where $\alpha$ is the fraction of the surface magnetic field that decayed via reconnection events. From Figure 7 we can infer the decay in magnetic energy to be roughly $\alpha = 0.4 - 0.6$.

As we have argued above, given the box size, most of the energy is dissipated into heat before the waves reach the boundaries; the amount of energy flowing off the grid is negligible. The dissipated energy goes into the heating of the plasma, and we assume that it is released as synchrotron emission. In the simulations, no radiative cooling or transfer is included. The synchrotron emission is calculated from the simulated results for plasma density and magnetic field. Furthermore, if we assume the electrons to have speeds of $\gamma_e \sim 1$, then the intensity emitted by synchrotron processes from the simulated region goes as $I_s \propto n_e B^2$. In addition, assuming peak emission is at $\nu_p = 0.29 \nu_c$, where $\nu_c$ is the critical frequency (see §§ 6.2 and 6.4 in Rybicki & Lightman 1979), we find this emission to be in the X-ray band,

$$h\nu_p \sim (335 \text{ keV}) \left(\frac{B}{5 \times 10^{13} \text{ G}}\right) \left[1 - 0.29 \left(\frac{M_{QS}}{M_\odot}\right) \left(\frac{10 \text{ km}}{R_{QS}}\right)\right]^{1/2},$$

where gravitational redshift is included in the last term. This intensity decays temporally in a way closely resembling SGR
Fig. 5.—Mock magnetic field lines in the x-y plane (perpendicular to the rotation axis) at $t = 1$ (top left), 25 (top right), 100 (bottom left), and 300 (bottom right). Shown are isocontours of the vector potential component $A_z$.

Fig. 6.—Time (in units of $1/\Omega$) evolution of the Fourier power spectrum (contours, right axis; where $P_{osc}$ is the period of oscillation in units of $1/\Omega$) for $\beta = 1$ and $\theta = 15'$ along with the normalized intensity (solid line, left axis). The subpulses in intensity are shown here to be modulated at ~3.25 (i.e., ~0.5P). In an observer’s frame, this modulation should be further modulated by the spin period.

Fig. 7.—Evolution of total magnetic energy. The change in magnetic energy ($\alpha$ in eq. [6]) can be estimated from these plots to be between 0.4 and 0.6, implying that energies of the order of $10^{44}$ ergs are released during the event.
bursts (see Fig. 3). The decay profiles of our simulated bursts depend on the initial plasma $\beta$ at the surface of the star and on the dipole inclination $\theta$. Varying these parameters in our model allows us to fit the exponential decay shape and duration of observed bursts.

2.3. Periodicity in Emission

The oscillations visible in Figure 3 (for different $\beta$) and emphasized in Figure 6 can be interpreted as magnetic “breathing” modes. In this situation, regions where magnetic reconnection occurs cause a disparity in the magnetic pressure between neighboring regions. This low-pressure region will draw plasma in from the surrounding regions causing either more reconnection or enhanced density where, in either case, there will be oscillations in the emission (i.e., $n_eB^2$).

Furthermore, since our simulation is performed in the corotating frame, our model is too simplistic to reproduce a spin-modulated pattern (e.g., the 8.0 s period observed in the March 5 event) superimposed on a smooth exponential decay in the light curves. However, as can be seen in Figures 4 and 5, there are hotter regions with more magnetic reconnection events than others, so spin modulation should occur naturally. We note that a few of these hot spots appear simultaneously at random locations, implying that spin modulation may consist of even smaller sub-pulses, therefore producing many harmonics in the light curve. In other words, if our model is a correct representation of SGRs, observations could constrain the number of hot spots.

3. SUBSEQUENT BURSTS AND QUIESCENT PHASE

We note that since the CFL phase transition occurs only once in a given star, there is only one giant burst in our model due to magnetic field alignment. Any heating would not likely raise the temperature above $T_c$ given the large heat capacity of superconducting quark matter (Iwamoto 1982). We estimate that about $10^{51}$ ergs of energy are required to reheat the star above $T_c$, which is unrealistic. Subsequent SGR bursts are not due to the realignment of the magnetic field induced by the Meissner effect. Instead, we argue that they are due to conversion of thermal energy into photon luminosity as described in Ouyed et al. (2004; see their eq. [11], in which their parameter $Q$ can be due to any heating mechanism, e.g., fallback material from the quark nova explosion or later solid body impact from the interstellar medium [ISM] as already noted by Usov 2001). We refer the interested reader to Figure 3 in Ouyed et al. (2004) for the subsequent burst luminosity.

Since SGR 1806–20 was active before the giant flare, it implies that the SGR 1806–20 giant flare was not a magnetic alignment event. Instead, we consider the possibility that it is a giant version of a normal SGR burst. For SGR 1806–20 a significantly larger source of matter is needed; a natural source for this matter is fallback from the ejected envelope from the quark nova explosion (Keränen et al. 2005). We are currently exploring delay mechanisms that allow a giant flare to occur decades after the quark nova, which would be consistent with the observed SGR activity of SGR 1806–20 prior to its giant flare.

The quiescent phase following bursting activity is due to vortex expulsion from spin-down and subsequent annihilation through magnetic reconnection near the surface. The number of vortices decreases slowly with spin-down leading to continuous, quiescent energy release (see Ouyed et al. 2004; Niebergal et al. 2006 for more details).

4. MODEL PREDICTIONS AND OBSERVATIONAL TESTS

In the light of the results presented above we now discuss our model predictions and offer our interpretation of the open issues listed in § 1.1.

1. Following the quark nova and associated reorganization of the outer magnetic field and its alignment with the rotation axis, our model naturally predicts the suppression of further radio pulsations. Our model explains why SGRs and AXPs show no radio pulsations, since they are aligned rotators. The current lack of detection of radio emission could also be a selection effect, since the long periods of SGRs and AXPs imply a small beaming angle. However, in our model, we predict that even with improved statistics (increased number of SGRs/AXPs), no radio detections would be made.

2. The high-$B$ (magnetar strength) radio pulsars that show no evidence of enhanced X-ray emission can be accounted for in our model if they are just neutron stars that have not experienced the Meissner effect. An alternative explanation is based on multipole moments for the difference in X-ray activity between the high-$B$ pulsars and the AXPs/SGRs. However, if future observations continue to fail to detect radio emission from AXPs/SGRs, then the multipole explanation would be challenged.

3. If observations do confirm that the progenitors of SGRs are very massive stars (Gaensler et al. 2005), this would strengthen our model, since massive progenitors are more likely to lead to massive neutron stars, for which it should be easier to reach deconfinement densities in the core following accretion or spin-down.

4. In our model, the fact that AXPs/SGRs show no subsecond periods has a natural explanation. That is, because neutron stars take some time to make the transition to quark stars, it gives the parent neutron star time to have spun down significantly. During most of the spin-down of the period from millisecond to seconds the object is a neutron star that does not expel the interior magnetic field. The high-$B$ neutron stars that experience quark nova after the supernova shell has dissipated have already reached periods greater than 100 ms. The majority of observable giant outbursts in our model occur for neutron stars with a period of few seconds, since they only spend a few hundred years at subsecond periods.

5. Colpi et al. (2000) explain the pileup of observed periods of SGRs/AXPs using a field decay formula (their eq. [2]) with coefficients based on physical models adjusted to fit the data. What is interesting to note about the Colpi et al. (2000) result is that the favored mechanism for field decay (their case C) seems to require a mechanism to expel the interior field. The Meissner effect is a natural mechanism to accomplish this. In a companion paper (Niebergal et al. 2006) we derive the magnetic field decay using equation (24) from Ouyed et al. (2004) to explain the period pileup in a self-consistent way.

Other predictions from our model are as follows.

6. The SGR source (the quark star) may be associated with a parent radio pulsar. In other words, in at least a few cases (if beaming is favorable) a parent radio pulsar should be detected in the same location in the sky as the SGR before the magnetic alignment burst.

7. If observations show quiescent X-ray emission before and after the burst, then this is a subsequent burst in our picture, meaning the SGR would have been a quark star for some time. In this case the association with a parent neutron star would be less obvious.

8. Assuming synchrotron-dominated emission, we should observe a peak at $\sim$(280 keV)$B/\left(5 \times 10^{13}\ G\right)$ for a solar mass quark star with a 10 km radius.
9. The total energy in a burst for a quark star with a moderate-strength magnetic field would be much weaker according to equation (6). We predict the burst energy from these stars to be of the order of $10^{40}$ ergs, but we would not expect to see bursts from $10^{12}$ G objects as their spin-down rate is insufficient to reach quark deconfinement densities in a reasonable amount of time.

10. Finally, we note that during the star’s phase transition from hadronic to quark matter, its radius decreases by a factor of 1.4—2 (Ouyed et al. 2002), amplifying the magnetic field at the surface by a factor of up to 4. Therefore, a quark star with a strong magnetic field is not necessarily the result of a parent star with a strong magnetic field. Thus, if quark stars undergoing the Meissner effect are indeed the origin of SGRs or AXPs, then stronger field strengths in these objects should be expected. The fossil field scenario (Ferrario & Wickramasinghe 2005) would provide an equally valid starting configuration: a high magnetic field neutron star spinning down with core density increasing.

5. CONCLUSION

We present 3D simulations of the reorganization of the magnetic field surrounding a newly born quark star. The reorganization is a consequence of the star entering the CFL phase, confining the interior field to vortices and leaving the exterior field in its tilted dipole configuration. In our model a quark nova is followed by a burst due to magnetic reconnection occurring while the exterior field aligns itself with the interior one. This magnetic alignment burst is a candidate mechanism for some SGR giant flares. We should emphasize that magnetic fields alone cannot be responsible for the properties of SGRs and AXPs, but rather a compact star experiencing the Meissner effect is required. While quark stars have not yet been observed in nature, our model seems to account for many observed features in SGRs and AXPs, thus warranting further investigation.

We thank R. Pudritz, C. Pethick, K. Mori, and C. Manuel for discussions. B. N. thanks the Canadian Institute for Theoretical Astrophysics (CITA) for hospitality and Sigma Xi for its Grant-in-Aid of research. The research of R. O. is supported by grants from the Natural Science and Engineering Research Council of Canada (NSERC) and the Alberta Ingenuity Fund (AIF).

APPENDIX A

MAGNETIC ENERGY OF A VERTICAL FIELD IN A SPHERE

Consider a strictly vertical magnetic field $B = B_z(x, y)\hat{z}$ in a sphere of radius $R$. Since the radial component of $B$ must be continuous at the surface, the field inside the sphere must be

$$B_z(r, \vartheta, \varphi) = \frac{B_z(R, \Theta, \varphi)}{\cos \Theta},$$

where $\Theta$ is the colatitude at the surface, which is related to $r$ and $\vartheta$ via

$$r \sin \vartheta = R \sin \Theta = s,$$

with $s$ the cylindrical radius. To calculate the magnetic energy inside the sphere, we integrate $B^2$ over $z$, then transform the $s$ integral in one over $\Theta$, using equation (A2). After a bit of algebra, we find the remarkable expression

$$E_{\text{mag}} = R^3 \int_0^{2\pi} d\varphi \int_0^\pi d\Theta \sin \Theta \frac{B_z^2(R, \Theta, \varphi)}{2\mu_0}.$$

APPENDIX B

INITIAL MAGNETIC FIELD CONFIGURATION

The initial magnetic field represents the phase in which the Meissner effect has forced the field inside the neutron star to be strictly vertical, but this reorganization has not yet had time to affect the external field.

In order to keep magnetic energy finite, we have to allow for a transition layer between the vertical internal field and the inclined dipolar external field. We are thus looking for a magnetic field configuration that minimizes total magnetic energy under the following requirements.

1. For $r > R_2$, the magnetic field is that of a dipole with dipole moment $m$, inclined by $\vartheta_2$ with respect to the vertical axis.
2. For $r < R_1$, the field is strictly vertical.

Minimizing the magnetic energy in the transition layer $R_1 < r < R_2$ implies that $B$ is a potential field in that region.

To find the minimum energy field satisfying these requirements, we represent the magnetic field through scalar potentials $S$ and $T$:

$$B = -\nabla \times (x \times \nabla S) - x \times \nabla T.$$
Neither the vertical field nor the inclined dipole field have a toroidal part $T$; thus, we set $T = 0$ everywhere. For the poloidal scalar potential $S$, we make the Ansatz

$$S = \sum_{m = -1}^{1} \left( a_{m}^{0} + \frac{b_{m}^{0}}{r} \right) Y_{1}^{m}(\theta, \varphi),$$

(B2)

where $Y_{m}^{m}(\theta, \varphi)$ are the spherical harmonics of order 1 and degree $m$. Equation (B2) is compatible with both the inner and the outer field; any spherical harmonics of higher order would only increase the total energy, so we do not include them.

Minimizing the sum of the energy equation (A3) (with $R = R_{1}$) and the energy of the potential field for $R_{1} < r < R_{2}$, we find the following coefficients for $S$ after a straightforward, but somewhat tedious, calculation:

$$a_{1}^{m} = \tilde{a}_{m} \beta_{1}^{m}, \quad b_{1}^{m} = \tilde{b}_{m} \beta_{1}^{m},$$

(B3)

where

$$\beta_{1}^{m} = \frac{\mu_{0} |m|}{4\pi} \sqrt{\frac{4\pi}{3}} \begin{cases} \cos \vartheta_{2}, & m = 0, \\ \sin \vartheta_{2} \sqrt{2}, & m = \pm 1, \end{cases}$$

(B4)

$$\tilde{a}_{0} = \begin{cases} \frac{1}{R_{2}}, & r < R_{1}, \\ \frac{1}{R_{2}}, & R_{1} < r < R_{2}, \\ 0, & r > R_{2}, \end{cases}$$

(B5)

$$\tilde{a}_{\pm 1} = \begin{cases} \frac{1}{R_{2} - R_{1}}, & R_{1} < r < R_{2}, \\ 0, & r > R_{1}, \end{cases}$$

(B6)

$$\tilde{b}_{0} = \begin{cases} 0, & r < R_{1}, \\ 0, & R_{1} < r < R_{2}, \\ 1, & r > R_{2}, \end{cases}$$

(B7)

$$\tilde{b}_{\pm 1} = \begin{cases} 0, & r < R_{1}, \\ -\frac{R_{1}^{3}}{R_{2}^{2} - R_{1}^{2}}, & R_{1} < r < R_{2}, \\ 1, & r > R_{2}. \end{cases}$$

(B8)

To obtain the magnetic vector potential (which is used by the Pencil Code), we use the formula

$$A = -x \times \nabla S.$$  

(B9)

For a thin transition layer of thickness $\epsilon \equiv R_{2} - R_{1} \ll R_{1}$, the contributions to the magnetic energy are

$$E(r > R_{2}) = E_{2} = \frac{\mu_{0} m^{2}}{12 \pi R_{1}^{3}},$$

(B10)

$$E(R_{1} < r < R_{2}) = \frac{\sin^{2} \vartheta_{2}}{\epsilon} E_{2} + O(1),$$

(B11)

$$E(r < R_{1}) = 2 \cos^{2} \vartheta_{2} E_{2} + O(\epsilon).$$

(B12)

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