The influence of diffraction gratings relief noise on the intensity distribution in diffraction orders

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Abstract. The method of security hologram quality inspection, in which the relief phase parameters are determined by registering results of the intensity distribution in diffraction orders, is known. The profile of relief as a harmonious distribution is represented. Deviation of the real relief profile from the ideal profile, another words phase relief noise, influence on the accuracy of this method. In the paper, on the assumption of homogeneity of the phase relief noise, the mathematical expressions for evaluating the influence of the phase relief noise on the intensity distribution in the diffraction orders are represented. Parameters of the correlation functions approximation describing the phase relief noise are determined. The dependence of the intensity values from the standard deviation of the phase relief noise is represented.

1. Introduction
The quality inspection method of security hologram based on indirect measurements of diffraction grating parameters is described in the paper [1]. The principle of the method is to estimate the parameters of the diffraction grating by measuring the intensity distribution values in the diffraction orders, the orientation of the plane and the angular size of the diffraction orders. In the paper [1] we accepted the profile has a perfect sinusoidal shape. But the real profile is always different from the ideal, and these differences are random. Therefore, there is the important task of estimating the influence of diffraction grating profile random distortion on the intensity distribution in the diffraction orders. The aim of the paper is to develop the mathematical expressions that describe the intensity distribution in diffraction orders with considering the phase relief noise that characterizes deviation of the real relief profile from the ideal profile. This is necessary to analyze the phase relief noise influence.

2. The intensity distribution in diffraction orders with considering the phase relief noise
We accepted, the ideal relief profile of diffraction grating is describes by the following expression:

\[ Z(x) = A \left[ 1 - \cos \left( \frac{2\pi x}{T} \right) \right] = 2A\sin^2 \left( \frac{\pi x}{T} \right) . \]

(1)

where \( A \) – amplitude of the sinusoidal profile, \( T \) – period of the sinusoidal profile.
As shown in [1, 2], the dependence of the phase shift from the coordinate \( x \) of wavefront is determined as
\[
\Delta_1(x) = r_0(x) \cdot \left[ \sin(\alpha + \varphi) + \sin(\beta + \varphi) \right].
\]  \hspace{1cm} (2)

where \( \alpha \) and \( \beta \) - the angle of the incidence plane and the diffraction angle respectively,

\[
r_0(x) = \sqrt{x^2 + \left[ 2A \sin^{2}\left(\frac{\pi x}{T}\right) \right]^2}. \hspace{1cm} (3)
\]

In the particular case when \( \alpha = 0 \)

\[
tg \varphi = \frac{2A \sin^{2}\left(\frac{\pi x}{T}\right)}{x}. \hspace{1cm} (4)
\]

We assumed that the ideal sinusoidal profile of the diffraction grating is distorted by the noise \( \xi(x) \) which is a uniform random field, described by the Gauss's law \( w_\xi(\xi) \) with a zero expectation value \( \langle \xi(x) \rangle = 0 \) and a variance \( \sigma^2_\xi = \langle \xi^2(\xi) \rangle \).

To calculate the wave amplitude in diffraction orders we need to find an expression for the phase shift \( \Delta_\xi(x) \) with considering the random field \( \xi(x) \) of phase relief noise. The phase shift from analogy with (2) can be determined by the expression

\[
\Delta_\xi(x) = r_0(x) \left[ \sin(\varphi') + \sin(\beta + \varphi') \right]. \hspace{1cm} (5)
\]

where

\[
r_0'(x) = \sqrt{x^2 + \left[ 2A \sin^{2}\left(\frac{\pi x}{T}\right) + \xi(x) \right]^2}; \hspace{1cm} (6)
\]

\[
tg \varphi' = \frac{2A \sin^{2}\left(\frac{\pi x}{T}\right) + \xi(x)}{x}. \hspace{1cm} (7)
\]

We assumed the standard deviation \( \sigma_\xi << A \). Then the expression (5) can be expanded in a Maclaurin series by \( \xi \):

\[
\Delta_\xi(x, \xi) = \Delta_\xi(x, 0) + \frac{\partial \Delta_1(x, \xi)}{\partial \xi} \bigg|_{\xi=0} \cdot \xi + \frac{\partial^2 \Delta_1(x, \xi)}{\partial \xi^2} \bigg|_{\xi=0} \cdot \xi^2 + ... \hspace{1cm} (8)
\]

Using only two first terms of the series, we get an approximate expression for calculation of the phase shift spatial distribution when the noise of sinusoidal relief profile is appeared.

\[
\Delta_\xi(x) = \Delta_1(x) + \Delta_1\xi(x), \hspace{1cm} (9)
\]

where

\[
\Delta_1(x) = \sqrt{x^2 + \left[ 2A \sin^{2}\left(\frac{\pi x}{T}\right) \right]^2} \cdot \left[ \sin(\varphi) + \sin(\beta + \varphi) \right], \hspace{1cm} (10)
\]

\[
\Delta_1\xi(x) = \frac{\partial \Delta_1(x, \xi)}{\partial \xi} \bigg|_{\xi=0} \cdot \xi(x), \hspace{1cm} (11)
\]
Next, we get an expression for the intensity distribution in diffraction orders with the influence of aforementioned phase relief noise $\xi(x)$ described by the Gauss’s law with a zero expectation value and a variance $\sigma_\xi^2$.

As observed in [2] the complex amplitude value for diffraction angle $\beta$ and number of periods $N$ in diffraction grating is determined as

$$U = U_0 \cdot U_1 \cdot U_2 \quad (12)$$

where $U_0$ — amplitude of plane wave;

$$U_2 = \sum_{j=0}^{N} e^{ikj\Delta_0} = \frac{1 - e^{ikN\Delta_0}}{1 - e^{ik\Delta_0}}; \quad (13)$$

$$U_1 = \int_0^T e^{ik\Delta_0 \xi(x)} \, dx; \quad (14)$$

$$\Delta_0 = T \cdot (\sin \alpha + \sin \beta). \quad (15)$$

Respectively, the intensity value is determined as

$$I = I_0 \cdot I_1 \cdot I_2, \quad (16)$$

where $I_0 = |U_0|^2$;

$$I_2 = |U_2|^2 = \frac{\sin^2 \left( \frac{\pi N \Delta_0}{\lambda} \right)}{\sin^2 \left( \frac{\pi \Delta_0}{\lambda} \right)}. \quad (17)$$

Since the component $U_1$ of the wave amplitude is a random variable, then the intensity is determined by averaging over the ensemble of realizations.

$$I_1 = \langle U_1 \cdot U_1^* \rangle = \int_0^T \int_0^T \exp \left\{ ik \left[ \Delta_1(x_1) + \Delta_1\xi(x_1) \right] \right\} dx_1 \cdot \exp \left\{ -ik \left[ \Delta_1(x_2) + \Delta_1\xi(x_2) \right] \right\} dx_2. \quad (18)$$

Using the property of the linearity for expectation operation, equation (18) can be written as

$$I_1 = \left\langle U_1 \cdot U_1^* \right\rangle = \int_0^T \int_0^T \exp \left\{ ik \left[ \Delta_1(x_1) - \Delta_1(x_2) \right] \right\} \exp \left\{ -ik \left[ \Delta_1\xi(x_1) - \Delta_1\xi(x_2) \right] \right\} dx_1 dx_2. \quad (19)$$

After calculating mathematical expectation, part of the expression (19), we obtain the expression for the intensity of a particular diffraction order.

$$I_1 = \int_0^T \int_0^T \exp \left\{ ik \left[ \Delta_1(x_1) - \Delta_1(x_2) \right] \right\} \exp \left\{ -2\pi^2 \sigma_\Delta^2 \frac{x_1 - x_2}{\lambda^2} \right\} dx_1 dx_2. \quad (20)$$

The analysis of the expression (20) shows that the random relief noise of the grating reduces diffraction efficiency. To quantify the influence of the phase relief noise we should know the variance $\sigma_\xi^2$ and the normalized correlation function $\rho_\xi(x_1 - x_2)$ of the noise random field. These characteristics are determined using processed measuring results of the profile realizations.
3. Quantitative estimation of the phase relief noise

To quantify the influence of the random relief noise using (20) intensity for the first-order diffraction was calculated with the following initial data:

- grating period \( T = 1 \mu m \);
- wavelength длина волны \( \lambda = 0.55 \mu m \);
- incident angle \( \alpha = 0 \);
- first-order diffraction angle \( \beta \approx 33^\circ \);
- amplitude of the phase relief \( A = 0.1 \mu m \).

Root-mean-square (RMS) deviation \( \sigma_q \) of the phase relief noise is varied during the research.

Normalized correlation function is approximated by the following function:

\[
\rho(x_1 - x_2) = \exp \left( \frac{x_1 - x_2}{k_p} \right) \cos \left[ 2\pi \left( \frac{x_1 - x_2}{\gamma T} \right) \right],
\]

where \( k_p \) and \( \gamma \) — approximation parameters.

Figure 1 shows the section of correlation function for the case \( k_p = 0.2 \) and \( \gamma = 0.5 \).

![Figure 1. Normalized correlation function of phase relief noise](image)

The phase relief noise influence is estimated by intensity percentage change \( \delta_I \) in the first diffraction order according to following expression:

\[
\delta_I = \left( \frac{I_{I \xi_1} - I_I}{I_I} \right) \times 100\%.
\]

Figure 2 presents functional relation intensity percentage change versus RMS deviation of the phase relief noise in the setting of:

- \( k_p = 0.2 \) и \( \gamma = 0.5 \) (solid line, \( \delta_1 \));
- \( k_p = 0.1 \), \( \gamma = 0.5 \) (dotted line, \( \delta_2 \));
- \( k_p = 0.2 \), \( \gamma = 0.25 \) (cross-hatching line, \( \delta_3 \)).
Obtained results led us to conclusion that RMS deviation of the phase relief noise affects to intensity value as well as correlation function signature. It is worth mentioning that during research the method error of (20) is quantified for first-order intensity calculations in the presence of random relief noise. For RMS deviation approaching to zero ($\sigma_\xi \rightarrow 0$) the percentage error of (20) doesn’t exceed 3%.

4. Conclusions
The phase relief grating noise of security holograms has a significant impact to intensity distribution in diffraction orders. Therefore it is necessary to consider the phase relief noise influence inspecting quality of security holograms.

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