Revisiting the sensitivity studies for leptonic CP-violation and mass hierarchy with T2K, NO\(\nu\)A and LBNE experiments

K N Deepthi, Soumya C and R Mohanta
School of Physics, University of Hyderabad, Hyderabad-500046, India
E-mail: rmsp@uohyd.ernet.in

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Abstract

Precision measurement of neutrino mixing parameters and the determination of mass hierarchy are the primary goals of present and upcoming neutrino experiments. In this work, we study the sensitivity of T2K, NO\(\nu\)A and LBNE experiments to discover leptonic charge parity (CP)-violation and the determination of neutrino mass hierarchy. We obtain the correlation between the CP-violating phase \(\delta_{CP}\) and the mixing angles \(\theta_{13}, \theta_{23}, \theta_{12}\), and the sensitivity to determine the octant of atmospheric mixing angle \(\theta_{23}\). The entire analysis is performed for a total of ten years (5\(\nu\) + 5 \(\bar{\nu}\)) of the running of T2K, NO\(\nu\)A and LBNE experiments. Furthermore, we also consider the impact of cross section uncertainties on the CP-violation sensitivity of the LBNE experiment.

1. Introduction

Experimental endeavour in the past decades has firmly established the phenomenon of neutrino oscillations, i.e. the composition of neutrino flavors change as they propagate. In the three-neutrino framework, the three-flavour eigenstates \((\nu_e, \nu_\mu, \nu_\tau)\) mix via the unitary lepton mixing matrix \(U_{\text{PMNS}}\) [1–3]; the analogue of the CKM matrix \(V_{\text{CKM}}\) that governs the mixing in the quark sector. This PMNS matrix can be parameterized in terms of three mixing angles \((\theta_{12}, \theta_{13} \text{ and } \theta_{23})\), which have all been measured experimentally and a charge parity (CP)-violating phase \(\delta_{CP}\) which is unknown. The likelihood of flavor oscillation also depends on the differences in the squared masses of the neutrinos, i.e., \(\Delta m_{21}^2\) \text{ and } \(\Delta m_{31}^2\), where \(\Delta m_{ij}^2 = m_i^2 - m_j^2\).

Neutrino oscillation data accumulated over many years allow us to determine the solar and atmospheric neutrino oscillation parameters with very high precision. The mixing angles \(\theta_{12}\) and \(\theta_{23}\), as well as the mass square differences, have been well constrained by various neutrino experiments. Recently, the reactor mixing angle \(\theta_{13}\) has been measured precisely [4–7] with a moderately large value. This provides a significant achievement in establishing the picture of three-flavor neutrino oscillations. The global analysis of the recent results of various neutrino oscillation experiments has been performed by several groups [8–11]. We have considered best-fit values and the 3\(\sigma\) ranges of the oscillation parameters from [11] throughout our simulations.

There are however many issues yet to be resolved. These include: (i) the value of the CP-violating phase \(\delta_{CP}\) is not yet constrained by any experiment. (ii) We still do not know the exact nature of neutrino mass hierarchy, i.e., whether the neutrino mass ordering is normal or inverted in nature. (iii) The possibility of observing CP-violation in the neutrino sector due to the presence of the Dirac-type CP-violating phase in the neutrino mixing matrix. (iv) Another interesting and crucial development in recent times is the indication of non-maximal atmospheric mixing angle by the MINOS [12] and T2K [13] experiments. The global analyses of all the available neutrino oscillation data [8–11] also prefer the deviation of the \(\theta_{31}\) value from maximal mixing, i.e., \(\sin^2 \theta_{23} \neq 0.5\). Thus, for the non-maximal value of \(\theta_{23}\), one can have two possible solutions, one with \(\theta_{23} < 45^\circ\) for which \((\sin^2 \theta_{23} - 0.5)\) is negative and the other with \(\theta_{23} > 45^\circ\) for which \((\sin^2 \theta_{23} - 0.5)\) is positive. The former case is known as a lower octant (LO) whereas the latter is known as a higher octant (HO) solution. This corresponds to the problem of octant degeneracy of \(\theta_{23}\). In this paper we aim to study the sensitivity of the current and future long-baseline experiments, i.e., T2K, NO\(\nu\)A and LBNE in addressing some of these issues. Although some of these aspects have recently been studied in detail by various authors [14–23], in this paper we...
have attempted to perform a complete analysis of all these issues in the context of the current generation and upcoming long-baseline super-beam experiments. Another important difference is that in most of the previous analyses the LBNE flux files used are either atmospheric or NOvA (which is an off-axis experiment) flux files, whereas we have considered the on-axis NuMI beam flux files for LBNE from [24]. In [19], the authors studied the sensitivities to the mass hierarchy, octant of $\theta_{13}$ and CP violation for LBNE. They have also included the data from T2K (5 + 0), NOvA (3 + 3) and atmospheric neutrinos. The differences between their work and ours are: (i) we have not taken into account the effect of atmospheric neutrinos; (ii) we have considered ten years of data for NOvA and T2K in the combinations (5 + 5) assuming that, by the time LBNE will start recording data, both NOvA and T2K will have completed ten years of operation and (iii) we have also studied various correlations between $\delta_{\text{CP}}$ and $\theta_{13}/\theta_{23}$, which will help us to constrain the value of $\delta_{\text{CP}}$. Furthermore, as discussed in [25], the uncertainties in cross sections play a crucial role in the determination of CP-violation sensitivities of various long-baseline super-beam experiments. Without considering any specific theoretical model, the errors on cross sections are expected to be in the range of (20–50)%. In this paper, we have studied the impact of these cross section uncertainties on the CP-violation sensitivity of the LBNE experiment.

The paper is organized as follows. In section 2, we discuss the $\delta_{\text{CP}}$ dependence of neutrino oscillation probabilities and also show how it is correlated with the octant of $\theta_{23}$ and neutrino mass ordering. The experimental details of the long-baseline experiments (NOvA, T2K and LBNE) are briefly discussed in section 3. The CP-violation sensitivity and the determination of mass hierarchy are outlined in sections 4 and 5. Section 6 contains the results on octant sensitivity determination of these experiments. The correlations between the CP-violating phase $\delta_{\text{CP}}$ and the mixing angles $\theta_{12}$ and $\theta_{23}$ are presented in section 7. Section 8 contains the summary and conclusions.

2. Effect of mass hierarchy and $\theta_{23}$ octant on $\delta_{\text{CP}}$ sensitivity

The three-flavor neutrino oscillation effects can be systematically demonstrated by considering oscillation channels $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$. The detailed study of these channels at the long-baseline experiments is capable of addressing almost all of the four major issues discussed in the previous section. In particular, the appearance channel, i.e., $\nu_\mu \rightarrow \nu_e$ is very sensitive in exploring the CP-violation effect in neutrino oscillation experiments which can be understood as follows. In matter of constant density, the appearance probability depends on $\delta_{\text{CP}}$ in its sub-leading term, can be expressed as [26–28]

$$P_{\mu e} \approx \sin^2 \theta_{13} \sin^2 2\theta_{12} \sin \left( \frac{1 - \hat{A}}{\hat{A}} \Delta \right)$$

$$+ \alpha \sin 2\theta_{13} \sin 2\theta_{12} \cos \theta_{13} \cos \left( \frac{\Delta + \delta_{\text{CP}}}{\hat{A}} \right) \sin \left[ \frac{(1 - \hat{A}) \Delta}{1 - \hat{A}} \right], \tag{1}$$

where $\alpha = \Delta m_{21}^2 / \Delta m_{31}^2$, $\Delta \equiv \Delta m_{21}^2 \sqrt{L}/4E$, $\hat{A} = 2\sqrt{2} G_F N_e L / \Delta m_{31}^2$. All six parameters governing neutrino oscillations ($\theta_{12}$, $\theta_{23}$, $\theta_{13}$, $\Delta m_{21}^2$, $\Delta m_{31}^2$ and $\delta_{\text{CP}}$) appear in this equation. It should be noted that the parameters $\alpha$, $\Delta$ and $\hat{A}$ are sensitive to the neutrino mass ordering, i.e., to the sign of $\Delta m_{31}^2$. Furthermore, the sign of $\hat{A}$ changes with the sign of $\Delta m_{31}^2$, which implies that the matter effect can be used to determine the mass hierarchy. Also $\hat{A}$ changes sign when going from neutrino to antineutrino mode, which indicates that it can mimic CP-violation and hence complicates the extraction of $\delta_{\text{CP}}$ by comparing the data from neutrino and antineutrino modes. Thus, for large $\theta_{13}$ from the dominant first term of equation (1), one can determine $\sin^2 \theta_{23}$ or in other words the octant of $\theta_{23}$. Secondly, as this term contains the large matter effect, the nature of mass ordering can also be extracted from it. The second sub-dominant term is sensitive for the determination of CP-violation as it contains both $\delta_{\text{CP}}$ and $\cos \delta_{\text{CP}}$ terms. As discussed in detail in [16], the following points can be inferred from equation (1).

- The CP-violation phase $\delta_{\text{CP}}$ appears in combination with the atmospheric mass-squared difference as $\cos (\Delta + \delta_{\text{CP}})$ and hence it suffers from the hierarchy-$\delta_{\text{CP}}$ degeneracy [22]. This in turn limits the CP-violation sensitivity which can be clearly understood from figure 1 where we have plotted the $P_{\mu e}$ energy spectrum for the LBNE experiment which has a baseline of 1300 km. In our analysis, we have used the atmospheric oscillation parameters $\Delta m_{\text{atm}}^2$ and $\theta_{13}$ extracted from the muon neutrino survival probability. For the muon neutrino disappearance experiment such as MINOS [29], T2K [30] and NOvA the survival probability is given by

$$P_{\mu \mu} = 1 - \sin^2 \theta_{13} \sin^2 \left( \frac{\Delta m_{\text{atm}}^2 L}{4E} \right) + O(\Delta m_{31}^2). \tag{2}$$

The relation between the atmospheric parameters measured and the standard three-flavour oscillation parameters are given as [31–33]
\[
\sin \theta_{23} = \frac{\sin \theta_{13}^\mu}{\cos \theta_{13}^\mu}
\]

(3)

\[
\Delta m_{23}^2 = \Delta m_{23}^2 \sin^2 \theta_{12} (\cos^2 \theta_{12} - \cos \delta_{CP} \sin \theta_{13} \sin 2 \theta_{12} \tan \theta_{23})
\]

(4)

where \(\Delta m_{23}^2\) is taken to be positive (negative) for normal hierarchy (inverted hierarchy). These relations become significant in light of the recently measured, moderately large value of \(\theta_{13}\). Therefore, to avoid erroneous results for the sensitivity studies, we use these corrected atmospheric parameters in our analysis.

We consider the true curves of \(\delta_{CP} = \pm 90^\circ\) and true hierarchy to be normal for both the panels. The test values for \(\delta_{CP} = 0^\circ\) and \(180^\circ\) and test NH are shown in the left panel and the same for test hierarchy as inverted is shown in the right panel. Thus, the left panel represents the separation between the CP-conserving test (\(\delta_{CP}^{test} = 0, \pi\)) and maximally CP-violation true (\(\delta_{CP}^{true} = -\pi/2\) or \(\pi/2\)) when the hierarchy is known, while the right panel represents the same when the hierarchy is unknown. Hence, one can see that the separation between the true cases, i.e., (NH, \(\delta_{CP} = \pm \pi/2\)) from the corresponding test CP-conserving cases (NH/IH, \(\delta_{CP} = 0\) or \(\pi\)) is hierarchy dependent, which will effectively introduce hindrance in the CP-sensitivity measurements.

- The probability \(P_{\mu e}\) is higher for NH than for IH due to matter effects as seen from the first term in equation (1).
- The second term of equation (1), which is sensitive to \(\delta_{CP}\), gives rise to intrinsic octant degeneracy [34], as it comes with the sin 2\(\theta_{23}\) term, i.e., \(P_{\mu e}(\theta_{23}) = P_{\mu e}(\pi/2 - \theta_{23})\). Furthermore, it has been shown in [35, 36] that the probability function for different values of \(\theta_{13}\) and \(\delta_{CP}\) may satisfy the relation \(P_{\mu e}(\theta_{23}^{true}, \theta_{13}, \delta_{CP}) = P(\theta_{23}^{true}, \theta_{13}, \delta_{CP})\) inducing the overall eight-fold degeneracy. This implies that there could be some probability that the test values of \(\theta_{23}\) occurring anywhere in the ‘wrong’ octant may give the same probability. The effect of octant degeneracy in distinguishing between the CP-conserving (\(\delta_{CP} = 0\) or \(\pi\)) and maximally CP-violating cases (\(\delta_{CP} = \pm \pi/2\)) for \(P_{\mu e}\) are depicted in figure 2 where the upper panel is for neutrinos and lower one for antineutrinos. The shaded bands correspond to the true value of \(\theta_{23}\) in the lower octant (LO). The figure in the top left panel shows that for the true LO and true \(\delta_{CP} = -\pi/2\), the true case cannot be distinguished, whereas for the plot in the right panel for \(\delta_{CP} = \pi/2\), there exists a clear distinction. For antineutrinos the behavior of \(\delta_{CP} = -\pi/2\) and \(\delta_{CP} = \pi/2\) is opposite. This fact implies that the combination of \(\nu_s\) and \(\bar{\nu}_s\) would be well suited for the removal of octant-\(\delta_{CP}\) degeneracy. Furthermore, it has recently been pointed out in [37] that the CP-violating phase \(\delta_{CP}\) and the mixing angle \(\theta_{23}\) can be measured precisely in an environment where there are strong correlations between them. This can be achieved by paying special attention to the appearance and the disappearance channels in long-baseline neutrino oscillation experiments.

Gathering information about these observational facts, we now proceed to study the sensitivities of various observables in the current and upcoming long-baseline experiments, e.g., NO\(\nu\)A, T2K and LBNE.
To determine the sensitivity of various observables in the currently running and upcoming long-baseline experiments, the simulation is performed using the GLoBES package [38, 39]. First we briefly describe the procedure that we have adopted for obtaining the numerical results. We calculate $\Delta \chi^2$ using the default definition in GLoBES. We then minimize the $\Delta \chi^2$ to compute the sensitivities on various parameters. The following are the experimental specifications for T2K, NO$\nu$A and LBNE setups that have been used in our analysis.

**T2K:** In the T2K experiment, a $\nu_\mu$ beam from J-PARK is directed towards the Super-Kamiokande detector which is a 22.5 kt (Water Cerenkov detector), 295 km away. It uses a 0.77 MW beam planned to run effectively for 5($\nu$) + 0($\nu$) or 3($\nu$) + 2($\bar{\nu}$) years. The initial plan of the T2K experiment was to run for five years with $10^{21}$ protons on target per year. In this paper we consider the option of T2K running for 5($\nu$) + 5($\bar{\nu}$) years and incorporate those results with NO$\nu$A and 10 years of LBNE runtime. The details of the T2K experiment can be found from [41]. We have considered the input files for T2K from the GLoBES package along with the inputs from [40–42].

**NO$\nu$A:** NO$\nu$A is a 14 kt totally active scintillator detector located at Ash River, a distance of 810 km from Fermilab [43, 44]. The beam power is assumed to be 0.7 MW NuMI beam with $6 \times 10^{20}$ protons on target per year. This experiment is scheduled to run for three years in neutrino mode at first and then three years in antineutrino mode. However, in our analysis we consider a runtime of 5($\nu$) + 5($\bar{\nu}$) years by 2024. The following are the signal and background efficiencies considered in our simulation:
- Signal efficiency: 45% for $\nu_\mu$ and $\bar{\nu}_\mu$ signal; 100% $\nu_\mu$ CC and $\bar{\nu}_\mu$ CC.
- Background efficiency:
  - (a) Mis-ID muons acceptance: 0.83% $\nu_\mu$ CC, 0.22% $\bar{\nu}_\mu$ CC;

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3. Experimental specifications for the simulation studies

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- Background efficiency:
  - (a) Mis-ID muons acceptance: 0.83% $\nu_\mu$ CC, 0.22% $\bar{\nu}_\mu$ CC;
(b) NC background acceptance: 2% (3%) $v_\mu (\bar{v}_\mu)$ NC;
(c) Intrinsic beam contamination: 26% (18%) $v_\mu (\bar{v}_\mu)$.

We consider 5% uncertainty on signal normalization and 10% on background normalization. The migration matrices for NC background smearing are taken from [45].

**LBNE:** For LBNE, we consider a 35 kt LAr detector at 1300 km baseline length [46]. The neutrino beam (0.5–8 GeV) is obtained from a proton beam of 700 KW beam power and 120 GeV beam energy resulting in $6 \times 10^{20}$ protons on target per year. We consider five years of data taken by detector in $v_\mu$ beam mode and five years in $\bar{v}_\mu$ beam mode. The GLoBES files and the detector parameter assumptions are taken from [47]. We consider 5% uncertainty on signal normalization and 10% on background normalization. Furthermore, we have not considered the effect of the near detector (ND) in our analysis. As discussed in [19], the presence of an ND will reduce the systematic uncertainties of $\nu_\mu$ signal (background) from 5% (10%) to 1% (1%) and this in turn will slightly enhance the various sensitivities.

Our primary objective is to perform the sensitivity studies with the LBNE setup. However, by the time LBNE starts collecting data, which is expected to be around 2022, both T2K and NO$\nu$A will have completed their scheduled run period. However, if they continue collecting data beyond their planned run periods, it would be interesting to study what additional information we can find by combining the data from different experiments. Therefore, we incorporate the T2K and NO$\nu$A data to the LBNE data set to perform the simulation. For all three experiments we consider the runs for five years in neutrino mode and five years in antineutrino mode.

### 4. CP-violation sensitivity with T2K, NO$\nu$A and LBNE

The determination of the CP-violating phase $\delta_{\text{CP}}$ is one of the most challenging problems in neutrino physics today. Since $\delta_{\text{CP}}$ is associated with the mixing angle $\theta_{13}$ in the PMNS matrix, the recent measurement of a non-zero and moderately large value of this angle by reactor and accelerator experiments is expected to be conducive to the measurement of $\delta_{\text{CP}}$. Since $\theta_{13}$ is found to be moderately large it is possible for NO$\nu$A and T2K to provide some hint on $\delta_{\text{CP}}$. In this section, we discuss the detection of CP-violation, i.e., the ability of an experiment to exclude the cases $\delta_{\text{CP}} = 0^\circ$ or $180^\circ$.

The sensitivity of the experiment in observing CP-violation is evaluated at a given value of $\delta_{\text{CP}}$ by minimizing $\chi^2$ at the fixed test values of $0$ and $\pi$. Thus, we determine two quantities:

$$\Delta \chi^2_0 = \chi^2 \left( \delta_{\text{CP}} = 0 \right) - \chi^2_{\text{true}}$$
$$\Delta \chi^2_\pi = \chi^2 \left( \delta_{\text{CP}} = \pi \right) - \chi^2_{\text{true}}$$

and then take

$$\Delta \chi^2 = \min \left( \Delta \chi^2_0, \Delta \chi^2_\pi \right)$$

(5)

The significance of CP-violation is obtained using $\sigma = \sqrt{\Delta \chi^2}$. The true values for different oscillation parameters considered in this analysis are:

$$\sin^2 \theta_{12} = 0.32, \quad \sin^2 \theta_{23} = 0.1, \quad \sin^2 \theta_{13} = 0.05, \quad \delta_{\text{CP}} = [\pi; \pi],$$
$$\Delta m^2_{\text{atm}} = 7.6 \times 10^{-3} \text{ eV}^2, \quad \Delta m^2_{\text{sol}} = \pm 2.4 \times 10^{-3} \text{ eV}^2 \text{ for NH/IH.}$$

(7)

Furthermore, we have performed marginalization over both hierarchies, $\sin^2 \theta_{23}, \sin^2 2 \theta_{13}$, within the following ranges: |$\Delta m^2_{\text{atm}}| [2.05; 2.75] \times 10^{-3} \text{ eV}^2, \sin^2 \theta_{23} [0.32, 0.68], \sin^2 2 \theta_{13} [0.07; 0.13]$. We also added a prior for $\sin^2 2 \theta_{13}$ with $\sigma (\sin^2 2 \theta_{13}) = 0.01$. We present our results as a function of $\delta_{\text{CP}}$ in figure 3. The $3\sigma$ ($5\sigma$) line corresponds to $\Delta \chi^2 = 9$ ($25$) (this correspondence is for one degree of freedom), which indicates 99.73% (99.999%) probability of determining the CP-violation. One can notice from the figure that NO$\nu$A and T2K suffer from hierarchy-$\delta_{\text{CP}}$ degeneracy, because of which their CP-detection potential is compromised for unfavorable values of $\delta_{\text{CP}}$. This degeneracy can be lifted by including information from LBNE, which excludes the wrong hierarchy solution. From figure 3 we can see that for both T2K and NO$\nu$A experiments the significance for determining the leptonic CP-violation phase is almost below $3\sigma$ for $5 + 5$ years of runtime. The CP-violation sensitivity for the LBNE experiment is above $3\sigma$ for nearly 40% of the $\delta_{\text{CP}}$ space. Once we combine all three experiments T2K + NO$\nu$A + LBNE, we can see that for almost 50% of the true values of $\delta_{\text{CP}}$ we can measure the leptonic CP-violation phase with $3\sigma$ confidence.

To understand the role of the cross section uncertainties in the determination of CP-violation for the LBNE experiment we consider two optimistic sets of errors of 10% and 20% on the individual cross sections of $v_\mu$ and $\bar{v}_\mu$.
in our analysis. The bands in the top panel of figure 4 represent the effect due to 10\% uncertainty on the individual cross sections, whereas the plots in the bottom panel show the 20\% cross-section uncertainty effects. Thus, as seen from these figures, the CP-violation sensitivity is significantly affected by the cross section uncertainties. Furthermore, it should also be noted that the region close to maximal CP-violation, (i.e., $\delta_{CP} = \pi/2$) is greatly affected due to these uncertainties. Also, as generally anticipated, there is an enhancement in these uncertainties due to the increase in detector volume as the possibility of neutrino–nucleus interactions occurring increases.
5. Determination of mass hierarchy

Long-baseline experiments such as NOνA, T2K and LBNE primarily use the $\nu_\mu \rightarrow \nu_e$ and the corresponding anti-neutrino oscillation channels to determine the neutrino mass hierarchy (MH). Using the approximate perturbative formula for the probability $P_{\mu e}$, it can be seen that there is a hierarchy-$\delta_{CP}$ degeneracy as discussed in section 2. As a result, the hierarchy sensitivity of these experiments is a strong function of the value of the CP-violating phase $\delta_{CP}$. The value of $\Delta \chi^2$ for the MH study can be obtained using the relation

$$ \Delta \chi^2 = \left| \chi^2_{MH_{true}} - \chi^2_{MH_{test}} \right|.$$  (8)

We have used equation (8) to obtain the MH significance and considered the following two cases. In the first, we consider true hierarchy to be normal hierarchy (NH) and obtain the values of $\Delta \chi^2$ by assuming inverted hierarchy as test hierarchy. In the second, we consider true hierarchy to be inverted hierarchy (IH) and assume NH to be the test hierarchy while obtaining the $\Delta \chi^2$ value. The true values for the oscillation parameters are used from equation (7) and the test values of $\Delta m^2_{atm}$, $\sin^2 \theta_{23}$, and $\sin^2 2 \theta_{13}$ are marginalised over their 3$\sigma$ ranges in both cases. We also added a prior for $\sin^2 2 \theta_{13}$ with $\sigma(\sin^2 2 \theta_{13}) = 0.01$. In figure 5, we present the resultant significance plots. The 3$\sigma$ and 5$\sigma$ lines correspond to $\Delta \chi^2 = 9$ and 25 (for one degree of freedom) which indicate approximately 99.73% and 99.99% probability of determining the correct MH respectively. The values of $\delta_{CP}$ for which the curve is above 3$\sigma$ (5$\sigma$) are the values for which hierarchy can be determined with 99.73% (99.99%) confidence level.

From figure 5, we can see that both T2K and NOνA experiments have significance to MH less than 5$\sigma$ when run for 10 years in normal and inverted hierarchies. LBNE, when run for (5 + 5) years, has more than 5$\sigma$ significance to measure the hierarchy for all the values of $\delta_{CP}$. When we combine the results from all three experiments T2K + NOνA + LBNE run for 10 years each, there will only be marginal improvement on the MH sensitivity over the LBNE result.

6. Octant sensitivity of $\theta_{23}$

In this section we aim to analyze the capabilities of current long-baseline neutrino oscillation experiments and LBNE to measure octant sensitivity. As in the case of MH determination, adding information from various experiments enhances the sensitivity. However, it is the precise knowledge of the value of $\theta_{13}$ that plays a crucial role in determining the octant correctly.

Using atmospheric neutrino oscillations, Super-Kamiokande has measured the value of $\sin^2 (2 \theta_{23})$ to be $>0.95$ at 90% confidence level [48]. This corresponds to a value of $\theta_{23}$ around 45° leaving an ambiguity of whether the value of $\theta_{23}$ is less than 45° i.e., in the LO or in the HO where $\theta_{23}$ is greater than 45°.

For the T2K and NOνA which are off-axis experiments with baselines 295 and 810 km, the beam energies peak at 0.6 and 2 GeV respectively. However, LBNE ($L = 1300$ km) which is an on-axis experiment has a broad band beam (0.5 GeV to 8 GeV) covering first and second oscillation maxima, with a minimal high energy tail above $\sim 5$ GeV. For these values of baseline lengths, the Earth matter density varies in the range 2.3–2.8 g cc$^{-1}$, and the corresponding matter resonance energies are above 10 GeV. Therefore the neutrino energies of these

Figure 5. Mass hierarchy significance as a function of true $\delta_{CP}$. Normal hierarchy (inverted hierarchy) is considered as true hierarchy and inverted (normal) is taken as test hierarchy for a runtime of (5 + 5) years of T2K, NOνA and LBNE in $\nu_\mu + \bar{\nu}_\mu$ mode in the left (right) panel.

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experiments lie well below the matter resonances and the oscillation probabilities will have only very small sub-leading matter effects. Thus, the expressions for the relevant oscillation probabilities (in a vacuum) can be obtained by assuming a one-mass scale dominance approximation [26] as

$$p_{\mu e} = \sin^2 \theta_{23} \sin^2 2 \theta_{13} \sin^2 \left(1.27 \frac{\Delta m^2_{31}}{E} \right).$$  \(9\)

This equation depends on the combination of mixing angles \(\sin^2 \theta_{23} \sin^2 2 \theta_{13}\). Thus, there exists a correlation between \(\sin^2 \theta_{23}\) and \(\sin^2 2 \theta_{13}\) which implies that for different values of \(\theta_{13}\) there can be values of \(\theta_{23}\) in opposite octants which give same value of oscillation probability.

The disappearance probability of the muon neutrino beam is given by

$$p_{\mu \nu} = 1 - \sin^2 2 \theta_{23} \sin^2 \left(1.27 \frac{\Delta m^2_{31}}{E} \right) + 4 \sin^2 \theta_{13} \sin^2 \theta_{23} \cos 2 \theta_{23} \sin^2 \left(1.27 \frac{\Delta m^2_{31}}{E} \right).$$ \(10\)

The leading order term in the above equation has its entire dependency on \(\sin^2 2 \theta_{23}\) giving rise to intrinsic octant degeneracy. The oscillation probability expressions (9) and (10) mentioned above are only given for illustration of octant degeneracy, and we have considered the full formula of oscillation probability in our calculations.

We first look into the bi-probability plots for LBNE experiment with (5 + 5) years of runtime, to estimate its capabilities in determining mass hierarchy and resolving octant degeneracy. The left panel of figure 6 shows \(\nu\) appearance events versus \(\bar{\nu}\) appearance events for all combinations of octant–hierarchy. Here the red curves are obtained by considering normal hierarchy mass ordering, LO (HO) i.e., \(\sin^2 \theta_{23} = 0.41\) (0.59) and the blue curves are obtained by considering inverted hierarchy mass ordering for LO (HO). We plot these ellipses by obtaining the event spectra for (5 + 5) years of runtime in \(\nu\) and \(\bar{\nu}\) mode for the LBNE experiment for all values of \(\delta_{CP}\).

For the analysis of octant determination of \(\theta_{23}\), we have used GLoBES to evaluate \(\Delta \chi^2\). In our simulation, we have kept true values of oscillation parameters as \(\sin^2 \theta_{13} = 0.32, \sin^2 2 \theta_{13} = 0.1, \Delta m^2_{31} = 7.6 \times 10^{-5} \text{eV}^2\), \(\delta_{CP} = 0\) and \(\Delta m^2_{\text{atm}} = 2.4 \times 10^{-3} \text{eV}^2\) (NH). Furthermore, we have performed marginalization over test values in the following ranges: for \(\sin^2 2 \theta_{13}\) and \(\Delta m^2_{\text{atm}}\) in their 3σ ranges, for \(\delta_{CP}\) in its full range and for \(\sin^2 \theta_{23}\) in LO for true higher octant and HO for true lower octant. We have also added priors for \(\sin^2 2 \theta_{13}\) and \(\sin^2 \theta_{23}\) with \(\sigma\left(\sin^2 2 \theta_{13}\right) = 0.01\) and \(\sigma\left(\sin^2 \theta_{23}\right) = 0.05\).

In the right panel of figure 6, we illustrate the ability of NOνA + T2K + LBNE to determine the octant as a function of the true value of \(\theta_{23}\). We see that with LBNE and NOνA + T2K + LBNE, the octant can be determined at >5σ CL when \(\sin^2 \theta_{23} = 0.41\). For values of \(\theta_{23}\) above 40°, (i.e, for \(0.5 \geq \sin^2 \theta_{23} > 0.41\)), the sensitivity in solving the octant degeneracy is reduced, well below 5σ.

6.1. Allowed regions in the test \(\delta_{CP}\) and test \(\sin^2 \theta_{23}\) plane

Next, we study the correlation between \(\delta_{CP}\) and \(\sin^2 \theta_{23}\) for different combinations of true hierarchy and true octant. For our study, we simulate data for 5 + 5 years of runtime of LBNE, T2K + NOνA and LBNE + T2K +
We have taken true $\delta^{CP} = 0$, assumed true hierarchy as NH and true octant as LO/HO. We have varied the test values of $\sin^2 \theta^{23}$ in the range [0.32:0.68] and that of $\delta^{CP}$ in its full range $[-\pi: \pi]$. We have performed marginalization over $\sin^2 \theta^{13}$ and $\Delta m^2_{31}$ and added a prior for $\sin^2 \theta^{13}$ with $\sigma(\sin^2 \theta^{13}) = 0.01$. Finally, we calculated the minimum $\chi^2$ over all of these test parameter combinations. The obtained result is then studied as a function of $\delta^{CP}$ (test) and $\sin^2 \theta^{23}$ (test). We have plotted the 2$\sigma$ contour in the space spanned by test values of $\delta^{CP}$ and $\sin^2 \theta^{23}$ for LBNE, T2K + NOvA and LBNE + T2K + NOvA. The top (bottom) plot corresponds to LO (HO) with known hierarchy as NH (IH) in the left (right) plot.

Figure 7 shows the allowed regions in $\delta^{CP}$ and $\sin^2 \theta^{23}$ plane at 2$\sigma$ C.L for true $\delta^{CP} = 0$ with combinations of 5 + 5 years of run of LBNE, T2K + NOvA and LBNE + T2K + NOvA. The top (bottom) plot corresponds to LO (HO) with known hierarchy as NH (IH) in the left (right) plot.

The allowed region is very tightly constrained which

![Figure 7. Allowed regions in $\delta^{CP}$ and $\sin^2 \theta^{23}$ plane at 2$\sigma$ C.L for true $\delta^{CP} = 0$ with combinations of 5 + 5 years of runtime of LBNE, T2K + NOvA and LBNE + T2K + NOvA. The top (bottom) plot corresponds to LO (HO) with known hierarchy as NH (IH) in the left (right) plot.](image)

8. Correlation between $\delta^{CP}$ and $\theta^{13}$

In this section we present the correlations between the CP-violating phase $\delta^{CP}$ and the mixing angles $\theta^{13}$/$\theta^{23}$. The correlation between $\delta^{CP}$ and $\theta^{13}$ is obtained by projecting $\chi^2$ onto the two-dimensional plane of $\delta^{CP}$ and $\theta^{13}$ by spanning over test values of $\delta^{CP} = [-\pi: \pi]$ and $\sin^2(2\theta^{13}) \in [0.07, 0.13]$. We have also marginalized over $\theta^{23}$ and $\Delta m^2_{31}$. We have obtained 1$\sigma$, 2$\sigma$, and 3$\sigma$ contours by considering three true values for $\delta^{CP} = 0, -\pi/2, +\pi/2$. We have set 10% error on each of the solar parameters and a 5% error for the matter density and assumed the hierarchy to be normal. The result is presented in the left panel of figure 8, where the blue/green/red curves correspond to 1/2/3$\sigma$ measurement contours for a total (5 + 5) years running of T2K + NOvA + LBNE.

The analogous plot between $\delta^{CP}$ and $\theta^{23}$ has been obtained following a similar procedure and the corresponding result is shown in the right panel of figure 8. The allowed region is very tightly constrained which
indicates that with the combined 10 years of LBNE + T2K + NOνA data, it is possible to constrain the value of \( \delta_{\text{CP}} \). The overlap in the $3\sigma$ contours in both the plots can be accounted for by the fact that we are considering three different true values for \( \delta_{\text{CP}} = 0, -\pi/2, +\pi/2 \).

8. Summary and conclusions

In this paper we have explored the possibility of determining the mass hierarchy, octant of the atmospheric mixing angle \( \theta_{23} \) and the CP-violation discovery potential in the current generation and upcoming long baseline experiments T2K, NOνA and LBNE and our findings are summarized below.

- For long-baseline experiments, it is well known that the measurement of the mass hierarchy is easier than a measurement of \( \delta_{\text{CP}} \) because matter effects enhance the separation between the oscillation spectra, and hence the event rates between normal and inverted hierarchies. The determination of mass hierarchy is defined as the ability to exclude any degenerate solution for the wrong (fit) hierarchy at a given confidence level. From our analysis, we find that if we combine the results from all three experiments for \((5 + 5)\) years of runtime we can determine the mass hierarchy of neutrinos above \(5\sigma\).

- The octant sensitivity of \( \theta_{23} \) also increases noticeably if we combine the results of the three experiments T2K, NOνA and LBNE. However, for values of \( \theta_{23} \) above \(40^\circ\), the sensitivity for solving the degeneracy is reduced, well below \(5\sigma\).

- The CP-violation discovery potential in the long-baseline experiments is also quite promising. A discovery of CP-violation, if it exists, basically means being able to exclude the CP-conserving values i.e., \( \delta_{\text{CP}} = 0^\circ \) or \(180^\circ\) at a given confidence level. From our analysis, we found that it is possible to measure the CP-violation phase above \(3\sigma\) CL for about 50% of the true \( \delta_{\text{CP}} \) range if we combine the data from all three experiments. Furthermore, it should also be noted that the CP-violation measurement becomes very difficult for the \( \delta_{\text{CP}} \) values which are closer to \(0^\circ\) or \(180^\circ\). Therefore, while it is possible to discover the mass hierarchy for all possible values of \( \delta_{\text{CP}} \), the same is not true for CP-violation.

- The cross-section uncertainties play a crucial role in determining the CP-violation sensitivity. These uncertainties significantly affect the region of maximal CP-violation.

- From the correlation plots between \( \delta_{\text{CP}} \) and \( \sin^2 \theta_{23} \) as well as from \( \delta_{\text{CP}} \) and \( \theta_{13}/\theta_{23} \) (figures 7 and 8), one can see that \( \delta_{\text{CP}} \) is severely constrained, implying a definitive measurement of \( \delta_{\text{CP}} \) could be possible with ten years of LBNE data collection.

In conclusion, we find that combining the data of \((5\nu + 5\bar{\nu})\) years of running T2K, NOνA and LBNE will help us to resolve most of the ambiguities associated with the neutrino sector.
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References

[1] Pontecorvo B 1957 Mesonium and antimesonium Sov. Phys. JETP 6 429
[2] Maki Z, Nakagawa M and Sakata S 1962 Remarks on the unified model of elementary particles Prog. Theor. Phys. 28 870
[3] Scheckter J and Valle J W F 1980 Neutrino masses in SU(2) ⊗ U(1) theories Phys. Rev. D 22 2227
[4] An F P et al (DAYA-BAY Collaboration) 2012 Observation of electron-antineutrino disappearance at Daya Bay Phys. Rev. Lett. 108 171803
[5] An F P et al (DAYA-BAY Collaboration) 2013 Improved measurement of electron antineutrino disappearance at daya bay Chin. Phys. C 37 011001
[6] Ahn I K et al (RENO Collaboration) 2012 Observation of reactor electron antineutrino disappearance in the RENO experiment Phys. Rev. Lett. 108 191802
[7] Abe K et al (T2K Collaboration) 2013 Evidence of electron neutrino appearance in a muon neutrino beam Phys. Rev. D 88 032002
[8] Forero D, Tortela M and Valle J 2012 Global status of neutrino oscillation parameters after Neutrino–2012–Phys. Rev. D 86 073012
[9] Fogli G L, Lisio E, Marrone A, Montanino D, Palazzo A and Rotunno A M 2012 Global analysis of neutrino masses, mixings and phases: entering the era of leptonic CP violation searches Phys. Rev. D 86 013012
[10] Gonzalez-Garcia M C, Maltoni M, Salvador J and Schwetz T 2012 Global fit to three neutrino mixing: critical look at present precision J. High Energy Phys. JHEP12(2012)123
[11] Forero D, Tortela M and Valle J 2014 Neutrino oscillations refitted Phys. Rev. D 90 093006
[12] Adamson P et al (MINOS Collaboration) 2013 Measurement of neutrino and antineutrino oscillations using beam and atmospheric data in MINOS Phys. Rev. Lett. 110 251801
[13] de Perio P et al (T2K Collaboration) 2014 Oscillation results from T2K (arXiv:1405.3871)
[14] Agarwalla S K, Prakash S and Sankar S U 2013 Resolving the octant of θ13 with T2K and NOvA J. High Energy Phys. JHEP07(2013)131
[15] Chatterjee A, Ghoshal P, Goswami S and Raut S K 2013 Octant sensitivity for large θ13 in atmospheric and long baseline neutrino experiments J. High Energy Phys. JHEP06(2013)101
[16] Ghosh M, Ghoshal P, Goswami S and Raut S K 2014 Evidence for leptonic CP phase from NOvA T2K and ICAL: a chronological progression Nucl. Phys. B 884 274
[17] Ghosh M, Ghoshal P, Goswami S and Raut S K 2014 Can atmospheric neutrino experiments provide the first hint of leptonic CP violation Phys. Rev. D 89 013001
[18] Barger V, Gandhi R, Ghoshal P, Goswami S, Marfatia D, Prakash S, Raut S K and Uma Sankar 2012 Neutrino mass hierarchy and octant determination with atmospheric neutrinos Phys. Rev. Lett. 109 019180
[19] Barger V, Bhattacharya A, Chatterjee A, Gandhi R, Marfatia D and Masud M 2014 Configurations of the long-baseline neutrino experiment (arXiv:1405.1064)
[20] Hiraide K et al 2006 Resolving θ13 degeneracy by accelerator and reactor neutrino oscillation experiments Phys. Rev. D 73 093008
[21] Bora K, Dutta D and Ghoshal P 2014 Determining the octant of θ13 at LBNE in conjunction with reactor experiments (arXiv:1405.7482)
[22] Minakata H and Nunokawa H 2001 Exploring Neutrino Mixing with Low Energy Superbeams J. High Energy Phys. JHEP10(2001)001
[23] Machado P A N, Minakata H, Nunokawa H and Funchal R Z 2014 What can we learn about the lepton CP phase in the next 10 years J. High Energy Phys. JHEP03(2014)109
[24] LBNE Collaboration http://lbne2-docdb.fnal.gov/cgi-bin/ShowDocument?docid=7487
[25] Huber P, Mezzetto M and Schwetz T 2008 On the impact of systematical uncertainties for the CP violation measurement in superbeam experiments J. High Energy Phys. JHEP03(2008)021
[26] Akhmedov E K, Johansson R, Lindner M, Ohlsson T and Schwetz T 2004 Series expansions for three-flavor neutrino mixing probabilities in matter J. High Energy Phys. JHEP04(2004)38
[27] Cervera A et al 2000 Golden measurements at a neutrino factory Nucl. Phys. B 579 17
[28] Freund M 2001 Analytic approximations for three neutrino oscillation parameters and probabilities in matter Phys. Rev. D 64 053003
[29] Adamson P et al (MINOS Collaboration) 2011 Phys. Rev. Lett. 106 181801
[30] Abe K et al (T2K Collaboration) 2014 Phys. Rev. Lett. 112 181801
[31] de Gouvea A, Jenkins J and Kayser B 2005 Neutrino mass hierarchy; vacuum oscillations and vanishingU21, Phys. Rev. D 71 113009
[32] Nunokawa H, Parke S and Funchal R Z 2005 Another possible way to determine the neutrino mass hierarchy Phys. Rev. D 72 013009
[33] Raut S K 2013 Effect of non-zero δ13 on the measurement of θ13 J. High Energy Phys. JHEP03(2013)101
[34] Fogli G L and Lisio E 1996 Tests of three-flavor mixing in long-baseline neutrino oscillation experiments Phys. Rev. D 54 35667
[35] Barger V, Marfatia D and Whisnant K 2002 Breaking eight-fold degeneracies in neutrino CP violation, mixing, and mass hierarchy Phys. Rev. D 65 073023
[36] Minakata H, Nunokawa H and Parke S J 2002 Parameter degeneracies in neutrino oscillation measurement of lepton CP and T violation Phys. Rev. D 66 093012
[37] Coloma P, Minakata H and Parke S J 2014 Interplay between appearance and disappearance channels for precision measurements of θ23 and δ CP Phys. Rev. D 90 093009
[38] Huber P, Lindner M and Winter W 2005 From parameter space constraints to the precision determination of the leptonic Dirac CP phase J. High Energy Phys. JHEP05(2005)020
[39] Huber P, Lindner M, Schwetz T and Winter W 2009 First hint for CP violation in neutrino oscillations from upcoming superbeam and reactor experiments J. High Energy Phys. JHEP11(2009)044
[40] Huber P, Lindner M and Winter W 2002 Superbeams versus neutrino factories Nucl. Phys. B 645 3
[41] Itow Y et al 2007 (T2K Collaboration) The JHF-Kamioka neutrino project (arXiv:hep-ex0106019)
[42] Ishitsuka M et al 2005 Resolving neutrino mass hierarchy and CP degeneracy by two identical detectors with different baselines Phys. Rev. D 72 033005

K N Deepthi et al
[43] Ayres D et al 2005 NOνA: proposal to build a 30 kiloton off-axis detector to study nu(mu) to nu(e) oscillations in the NuMI beamline (arXiv: hep-ex/0503053)

[44] Patterson R 2012 The NOνA experiment: status and outlook Neutrino 2012 Conf. (June 3-9, Kyoto, Japan) http://neu2012.kek.jp/

[45] Agarwalla S K et al 2012 Potential of optimized NOνA for large θ13 and combined performance with a LArTPC and T2K J. High Energy Phys. JHEP12(2012)075

[46] Akiri T et al (LBNE Collaboration) 2010 The 2010 interim report on the long-baseline neutrino experiment collaboration physics Working Groups (arXiv:1110.6249)

[47] LBNE Collaboration http://lbne2-docdb.fnal.gov/cgi-bin/ShowDocument?docid=5823

[48] Itow Y 2012 Atmospheric neutrinos: results from running experiments Neutrino 2012 Conf., (June 3-9, Kyoto, Japan) http://neu2012.kek.jp.