A CMB lensing-galaxy intrinsic alignment contaminant to gravitational lensing cross-correlated probes of the universe and a proposal for calibration

M. A. Troxel* and Mustapha Ishak†

Department of Physics, The University of Texas at Dallas, Richardson, TX 75080, USA

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We introduce here a cross-correlation term between CMB lensing and galaxy intrinsic alignment, noted hereby as $\phi_I$. This effect acts as a contaminant to the cross-correlation between CMB lensing and galaxy lensing. The latter cross-correlation has recently been detected for the first time, and measurements will greatly improve as the area of overlap between galaxy and CMB surveys increases and measurements of the CMB polarization become more significant. This will constitute a powerful probe for studying the structure and evolution of the universe. The magnitude of the $\phi_I$ term is found to be about 15% of the pure CMB lensing-galaxy lensing component and acts to reduce the magnitude of its measured spectrum. This offset in the spectrum will strongly impact its use for precision cosmological study if left unmitigated. We also propose here a method to calibrate this $\phi_I$ contamination through use of a scaling relation that allows one to reduce the impact of $\phi_I$ by a factor of 20 or more in all redshift bins, which would reduce its magnitude down to detection limits in almost all cases. This will allow the full use of this probe for precision cosmology.

Introduction. Gravitational lensing of galaxy shapes due to the intervening large scale structure of the universe (cosmic shear) is a powerful probe for studying the structure and evolution of the universe. It constrains, for example, the properties of dark energy and the amplitude of matter density fluctuations $\sigma_8$. Similarly, surveys of the cosmic microwave background (CMB) are measuring for the first time the imprint of the gravitational lensing signature in the temperature fluctuations and polarization of the CMB $\Theta_2$, This allows us to utilize the CMB as a lensing source of known and fixed redshift to map the total intervening structure in the universe. These measurements will rapidly improve with better CMB polarization measurements, e.g., see $\Theta_3$, and can be used to calibrate systematics in shear measurements $\Theta_4$.

These probes are complementary, measuring the same information on large-scale gravitational potentials, and thus allow us to directly compare information about mass distributions in the universe across large redshifts through two independent methods, each with their own sources of error and systematics. Since these two probes measure the same lensing signal, they should be correlated. This cross-correlation has been detected by $\Theta_5$ with a significance of $3.2\sigma$, which provides constraints on the amplitude of density fluctuations at intermediate redshifts, where the efficiency of the probes overlap.

The most serious physical systematic in cosmic shear surveys is the intrinsic alignment of galaxies (IA), where correlations involving the intrinsic shape of the galaxy before lensing contaminate the lensing signal $\Theta_6$, and bias cosmological information. For the power spectrum, there exist two correlations involving the intrinsic alignment of the galaxies. First, there is a direct correlation $\Theta_7$ between the shapes of pairs of galaxies which reside or evolve within the same dark matter halo, and thus are tidally aligned causing a positive correlation in their shapes. The second correlation $\Theta_8$, first identified by $\Theta_9$, is instead a correlation between the intrinsic shape of a foreground galaxy and the shape of a background galaxy that is lensed by the same structure which tidally aligns the first. This $GI$ correlation is an anti-correlation, and thus competes with the $II$ correlation in its impact on the lensing signal. While the $II$ correlation greatly decreases in magnitude as the physical separation of the two galaxies increases, and thus can be rendered negligible by utilizing cross-correlations between large redshift bins, the $GI$ correlation can increase with separation and must be dealt with in more sophisticated ways.

There exists no intrinsic alignment contamination in CMB lensing, since there are no intrinsic galaxy shapes. However, the cross-correlation between CMB lensing and galaxy lensing will be contaminated by an intrinsic alignment correlation like $GI$, which we label $\phi_I$. In the same way the intrinsic shape of a foreground galaxy can be correlated with the lensing of a background galaxy, a foreground galaxy can be correlated with the lensing deflection induced in the CMB temperature fluctuations or the polarization signal. This can be a potential source of bias, and we explore the expected magnitude of the $\phi_I$ correlation as a fraction of the CMB lensing-galaxy lensing cross-correlation. We also extend the previously developed self-calibration methods of $\Theta_{10}$ to propose a method for calibrating the $\phi_I$ contamination by using information gained from galaxy weak lensing surveys. This work can be generalized to higher-order correlations, where there exist analogous contaminants such as $\phi_{II}$.
\(\phi GI, \text{ and } \phi \phi I\) in the bispectrum.

**Galaxy and CMB gravitational lensing formalism.** Under the Born approximation, the convergence \(\kappa\) of a source galaxy at comoving distance \(\chi_G\) and direction \(\hat{\theta}\) in a flat, \(\Lambda CDM\) universe is related to the matter density contrast \(\delta\) through the lensing kernel \(W_L(\chi_L, \chi_G) = \frac{2}{3}\Omega_m(1 + z_L)\chi_L(1 - \frac{\chi_L}{\chi_G})\) by \(\kappa(\hat{\theta}) = \int_0^{\chi_G} \delta(\chi_L, \hat{\theta})W_L(\chi_L, \chi_G)d\chi_L\), when \(\chi_L < \chi_G\) and zero otherwise, and where \(\Omega_m\) is the current day Hubble constant.

The comoving distance \(\chi\) is given in units of \(c/H_0\), where \(H_0\) is the current day Hubble constant. The 3D matter power spectrum is then defined from the convergence as

\[
\langle \kappa(\ell_1)\kappa(\ell_2) \rangle = (2\pi)^2\delta^D(\ell_1 + \ell_2)P(\ell_1),
\]

where \(\langle \cdots \rangle\) denotes the ensemble average and \(\delta^D(\ell)\) is the Dirac delta function. Under the Limber approximation, we can relate the 2D angular power spectrum to the 3D matter power spectrum as

\[
C_{\kappa^2}(\ell) = \int_0^\chi W^G_0(\chi')W^\phi_0(\chi')\chi'^2P(\ell; \chi')d\chi',
\]

where \(\alpha, \beta \in G, \phi\), where \(G\) and \(\phi\) represent galaxy and CMB lensing, respectively. For galaxy lensing, the weighting function \(W^G_0\) is given by

\[
W^G_0(\chi) = \int_0^\chi W_L(\chi', \chi)f_i(\chi')d\chi',
\]

where \(f_i(\chi)\) is the comoving distribution of galaxies in the \(i\)-th redshift bin. For a CMB source, \(f_i(\chi) \approx \delta^D(\chi - \chi^*)\), where \(\chi^*\) is the comoving distance to the surface of last scattering. This simplifies Eq. 3 to be

\[
W^\phi(\chi) = \frac{3}{2}\Omega_m(1 + z)\chi(1 - \frac{\chi}{\chi^*}),
\]

where we have suppressed the redshift bin denotation, since the source is at a single redshift.

**CMB lensing-galaxy lensing (\(\phi I\)) contaminant.** In cosmic shear studies using galaxy shapes, the impact of the GI intrinsic alignment correlation on lensing information is well-known, and significant work has gone into measuring or mitigating it (e.g. [13, 24, 22, 25-28]). While there is no intrinsic alignment to impact CMB lensing-auto-correlations, there should be a similar correlation between the lensing information encoded in the deflection and polarization of CMB photons and the intrinsic alignment of foreground galaxies, as well, through the CMB lensing-galaxy lensing cross-correlation. This becomes obvious from the following argument, where we build the correlation \(\phi I\).

First, consider that the foreground galaxy shape is composed of both an intrinsic shape component (I) and a component due to gravitational lensing (G). In the case of CMB lensing-galaxy cross-correlations, this foreground galaxy can, in fact, be at very high redshift. The intrinsic component (I) is influenced (or aligned) by the tidal action of the surrounding matter halo. This alignment is then also correlated with the gravitational lensing of background photons (CMB) by this structure. This can be expressed analytically as a measured galaxy shape, \(\kappa_{\text{obs}}^G = \kappa^G + \kappa^I\), being composed of both a lensing and intrinsic alignment contribution. The observed CMB lensing-galaxy lensing cross-correlation is then

\[
\langle \kappa^G \kappa_{\text{obs}}^G \rangle = \langle \kappa^G (\kappa^G + \kappa^I) \rangle = \langle \kappa^G \kappa^G \rangle + \langle \kappa^G \kappa^I \rangle.
\]

We will label these two terms \(\phi G\) and \(\phi I\), respectively.

The cross-correlation of CMB lensing and galaxy lensing has been proposed as a method to constrain cosmology at intermediate redshifts, and thus we should consider the impact of such an intrinsic alignment contamination to the pure \(\phi G\) lensing signal. In cosmic shear studies using galaxy shapes, the intrinsic alignment contamination GI has been shown to contaminate the signal by up to 10%. We compare the resulting GG, GI, and II signals for a single-bin Stage IV weak lensing survey in Figure 1 with both \(\phi G\) and \(\phi I\). The various spectra are calculated as discussed in detail below. Both lensing signals have similar levels of contaminations due to the intrinsic alignment correlations on the order of 10-15%. However, \(\phi I\) (like \(\phi G\)) is slightly larger in magnitude than GI (GG), and is fractionally about a 50% stronger contaminant. We thus expect a similar or stronger level of bias in cosmological measurements due to the intrinsic alignment correlation in the CMB lensing-galaxy lensing cross-correlation as for the galaxy lensing auto-correlation, where it impacts constraints of the matter fluctuation amplitude at the 10% level.

**Proposal to isolate and remove the intrinsic alignment \(\phi I\) contamination.** In previous methods developed to self-calibrate the GI correlation, complementary information in the form of the galaxy-ellipticity spectrum was used to isolate and remove the GI contamination. Here we extend this process to calibrate the galaxy intrinsic alignment contamination to the CMB lensing-galaxy lensing cross-correlation. This presumes, of course, that one has successfully applied some method, like self-calibration (e.g. [22]), nulling (e.g. [26]), or direct detection (e.g. [27]), to measure or isolate the GI galaxy ellipticity-IA cross-correlation, and requires overlapping measurements of the lensing of galaxy shapes and the CMB. A first detection of this CMB lensing-galaxy lensing cross-correlation has been made [14], however, and we anticipate that much stronger detections will be possible with the design of overlapping fields in ongoing and future galaxy and CMB surveys.

To calibrate the \(\phi I\) correlation, we proceed in a way that is analogous to the 2-point GI self-calibration [22], where we first build a scaling relationship between the intrinsic alignment information in each observable. The two observable spectra of interest are the ellipticity-ellipticity correlation from cosmic shear surveys and the
under the Limber approximation as

We will ignore the \( II \) term, as it is not a necessary part of calibrating the \( \phi I \) lensing signal. This effect is stronger than the corresponding negative \( GI \) contamination of the galaxy lensing signal, which is about 10% of \( G \). The total magnitude of both \( \phi G \) and \( \phi I \) is larger than that of \( GG \) and \( GI \), and is consistent with the detection in [14].

CMB lensing-galaxy lensing cross-correlation:

\[
C_{ij}^{(1)}(\ell) = C_{ij}^{GG}(\ell) + C_{ij}^{GI}(\ell) + C_{ij}^{II}
\]

\[
C_{ij}^{(2)}(\ell) = C_{ij}^{\phi G}(\ell) + C_{ij}^{\phi I}(\ell).
\]

(6)

(7)

We will ignore the \( II \) correlation in what follows, as it is not a necessary part of calibrating the \( \phi I \) cross-correlation. The \( GI \) and \( \phi I \) spectra can be expressed under the Limber approximation as

\[
C_{ij}^{IG}(\ell) = \int_0^\chi W_G^G(\chi') f_G(\chi') P_{SI}(\ell; \chi') d\chi',
\]

(8)

\[
C_{ij}^{\phi G}(\ell) = \int_0^\chi W_G^\phi(\chi') f_G(\chi') P_{SI}(\ell; \chi') d\chi'.
\]

Assuming a sufficiently narrow comoving distribution of galaxies in each photo-z bin, these can be approximated

\[
C_{ij}^{IG}(\ell) \approx W_G^G P_{SI}(\ell; \chi_i),
\]

(9)

\[
C_{ij}^{\phi G}(\ell) \approx W_G^\phi P_{SI}(\ell; \chi_i),
\]

(10)

where \( W_G^G = \int_0^\chi W_G^{GG}(\chi') f_G(\chi') d\chi' \) and \( W_G^\phi = \int_0^\chi W_G^{\phi G}(\chi') f_G(\chi') d\chi' \). This is identical to the factor derived as part of the GI self-calibration [22]. Combined, we can express \( C_{ij}^{\phi G} \) as a scaling of \( C_{ij}^{IG} \)

\[
C_{ij}^{\phi G}(\ell) \approx W_G^\phi W_G^G C_{ij}^{IG}(\ell).
\]

(11)

Unlike the scaling relation in the GI self-calibration, this scaling factor \( W_G^\phi/W_G^G \) can be simplified as

\[
W_G^\phi = \int_0^\chi (1 + z_G) \chi_G (1 - \frac{\chi_G}{\chi}) f_G(\chi_G) d\chi_G
\]

\[
W_G^G = \int_0^\chi (1 + z_G) \chi_G (1 - \frac{\chi_G}{\chi}) f_G(\chi_G) d\chi_G d\chi_L.
\]

(13)

Though this process requires overlapping measurements in both CMB lensing and galaxy lensing surveys, we can see from this simplification that, unlike the GI self-calibration, there is no need for priors on \( \Omega_m \) and \( H_0 \) to evaluate the scaling relation, and it is instead dependent only on the redshift (or comoving) distribution of galaxies. It also doesn’t require information about the galaxy bias, which reduces the impact of measurement errors in the calibration process compared to the GI self-calibration.

**Accuracy of the scaling relationship between \( \phi I \) and \( GI \).** In order to probe the performance of such a calibration, we assume a Stage IV photometric weak lensing survey covering half the sky with a fully overlapping CMB lensing map. The redshift distribution of such a survey is given by \( f(z) = \frac{z^2}{2 \sigma_z^2} e^{-z^2/\sigma_z^2} \), where \( \sigma_z = 0.27 \), the mean galaxy number density is 40 per arcminute², and the photo-z PDF is given by \( p(z|z^p) = \frac{1}{\sigma_z} e^{-\frac{(z-z^p)^2}{2\sigma_z^2}} \), with \( \sigma_z = 0.05(1+z) \). In order to compute the non-linear matter power spectrum \( P_h(k; \chi) \), we use the fitting formula of [29]. The intrinsic alignment spectra, \( P_{SI}(k; \chi) \) and \( P_{I}(k; \chi) \), are calculated using the non-linear alignment model of [12], which modifies the linear alignment model of [21] on small scales by using the non-linear matter power spectrum. We estimate the spectrum amplitude \( C_1 \) through comparison to Fig. 2 of Hirata & Seljak [21]. The galaxies are split into redshift bins of width \( \Delta z^p = 0.2 \), centered at redshifts \( z^p = 0.1 + 0.2i \), where \( i = 1 \ldots 12 \).

There will be some systematic error induced in the CMB lensing-galaxy lensing cross-correlation measurement (\( \phi G \)) due to inaccuracies of the approximations used in Eqs. [12]. We parameterize this uncertainty as

\[
\epsilon_{ij}^{(1)} = \left( \frac{W_G^\phi C_{ij}^{IG}(\ell)}{W_G^G C_{ij}^{IG}(\ell)} \right)^{-1} - 1.
\]

(14)

This inaccuracy is shown in Fig. [2] for \( i = j \). We find that the calibration of \( \phi I \) from \( GI \) performs with an inaccuracy of 5-30%. This corresponds to a reduction of a factor of 3 in the lowest photo-z bin, though it is more typically a factor of 10 or more in higher redshift bins. The actual systematic error induced in the lensing measurement is then given by \( \delta f_{ij} = \epsilon_{ij} f_{ij}^{(1)} \), where \( f_{ij}^{(1)} = C_{ij}^{\phi G}/C_{ij}^{GG} \) is the fractional contamination of the lensing cross-spectrum. Typically, \( f_{ij}^{(1)} \ll 1 \). Thus, the systematic error is typically significantly less than \( \epsilon_{ij} \).

**Calibration of \( \phi I \) from observables through the GI self-calibration technique.** While the scaling relation in Eq. [12] requires no information on the galaxy bias, it
is possible to improve on the previous systematic estimates by considering the calibration of $\phi I$ as part of the $GI$ self-calibration process. In the $GI$ self-calibration, the galaxy density-intrinsic ellipticity spectrum ($gI$) can be isolated from the galaxy density-ellipticity spectrum, $C_{ij}^{(3)} = C_{ij}^{gG} + C_{ij}^{gI}$, using the estimator derived in [22]. $C_{ij}^{G I}$ is then calculated from the scaling relation

$$C_{ij}^{G I}(\ell) \approx \frac{W_{ij}^{G}}{b_{i} \Pi_{ii}} C_{i}^{gI}(\ell),$$

where $\Pi_{ii}$ is the average galaxy bias in the $i$-th redshift bin and $\Pi_{ii} = \int_{0}^{\infty} f_{i}^{2}(x) d\lambda$. We can combine Eqs. [12] & [15] into a single scaling relationship, which simplifies to

$$C_{ij}^{G I}(\ell) \approx \frac{W_{ij}^{G}}{b_{i} \Pi_{ii}} C_{i}^{gI}(\ell).$$

We show the resulting accuracy of this relationship,

$$\epsilon_{i}^{(2)} = \left( \frac{W_{i}^{\phi} C_{i}^{gI}(\ell)}{b_{i} \Pi_{ii} C_{i}^{gI}(\ell)} \right)^{-1} - 1,$$

in Fig. 3. This is more accurate than Eq. [12] since the scaling information is directly mapped from the $i$-th photo-z bin to itself, and performs significantly better than the $GI$ self-calibration for the same reason. For low photo-z bins, we have only a 5% inaccuracy, which reduces to effectively zero for high photo-z bins. This allows for an almost perfect mitigation of $\phi I$, with reductions down to detection limits in all photo-z bins.

**Residual statistical errors in the $\phi I$ calibration.** Any residual statistical errors in the estimate of $C^{gI}$ will propagate from the estimate of $C^{GI}$ as

$$\Delta C_{i}^{G I}(\ell) \approx \frac{W_{i}^{\phi}}{W_{ij}^{G}} \Delta C_{i}^{G}(\ell).$$

The relative fractional error will thus be unchanged between $GI$ and $\phi I$, given by the ratio of Eqs. [18] & [12].

If we use the self-calibration of $GI$ as an example, then

$$\Delta C_{i}^{G I}(\ell) \approx \frac{W_{i}^{\phi}}{b_{i} \Pi_{ii}} \Delta C_{ii}^{gI}(\ell),$$

where $\Delta C_{ii}^{gI}(\ell)$ is given in [22]. The residual statistical error is typically less than the minimum survey error for the $GI$ self-calibration, and thus it follows that the calibration of $\phi I$ is also safe from residual statistical errors due to the calibration process. Similarly, other conclusions regarding errors in the measurement of $b_{i}$, cosmological uncertainties, stochasticity, etc... hold for each, and we refer the reader to [22] for more.

**Conclusion.** Measurements of CMB lensing and galaxy lensing are rapidly improving. Substantial overlap in coverage and increased statistical power in surveys which measure these quantities will allow us to explore cross-correlations between them, namely the CMB lensing-galaxy lensing signal, which provides us with an additional probe of structure at intermediate redshifts. This cross-correlation has already been detected by [14]. One of the benefits to using CMB lensing is that the primary physical systematic which contaminates cosmic shear measurements from galaxy ellipticities, the intrinsic alignment of galaxies, is absent. However, we have shown here that this contamination is present in the CMB lensing-galaxy lensing cross-correlation at the level of $C^{GI}/C^{gG} \approx 15\%$, about 50% stronger than the equivalent $GI$ contamination. We expect this to produce strong biases to any derived cosmological information, and must
be accounted for in any precision cosmological study.

We also proposed a method to calibrate the φI contamination by relating it to the GI cross-correlation. This relationship is made more accurate when the φI calibration is incorporated as part of the GI self-calibration process that allows one to estimate C_φI from C_gI, which can in principle be measured in a weak lensing survey. This calibration is more accurate than the estimation of GI, allowing for a nearly complete reduction of φI by greater than a factor of 20 for all photo-z bins. This process could totally alleviate the impact of bias due to the φI contamination on cosmological information. It also isolates a much deeper cross-correlation signal between the intrinsic alignment of galaxies, which encodes information about formation and evolution within local dark matter halos, and the CMB lensing signal, which fully maps the mass distribution in the universe back to the surface of last scattering. This φI signal thus will allow us to better study the evolution of the intrinsic alignment strength of galaxies within their dark matter halos over a much wider range of structure formation history.

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[1] Bacon D.J., Refregier A.R., Ellis R.S., 2000, MNRAS, 318, 625