Renormalization Group Relations and searching for Abelian $Z'$ Boson in the four-fermionic processes

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Abstract

The method of searching for signals of heavy virtual particles is developed. It is based on a renormalization group equation for the low energy effective Lagrangian and the decoupling theorem. As an application, the model independent search for Abelian $Z'$ boson in four-fermion processes is analyzed. The basic one-loop renormalization group relation for the parameters of the effective Lagrangian is derived which gives possibility to reduce the problem to the scattering of the Standard Model particles in the “external field” substituting heavy virtual $Z'$ state. From the set of derived relations it is determined that the absolute value of the $Z'$ coupling to the axial currents has to be the same for all fermions and is strongly correlated to the $Z'$ coupling to the scalar fields. The corresponding dependences between the parameters of the effective Lagrangians are derived.

1 Introduction

An important problem of nowadays high energy physics is searching for deviations from the Standard Model (SM) of elementary particles. At energies of the present day accelerators, the deviations may appear due to virtual states of new heavy particles entering the extended models and having the masses $\Lambda_i$ much greater than the $W$-boson mass: $\Lambda_i \gg m_W$. One of approaches to describe contributions of these heavy states is construction of the effective Lagrangians (EL).
It is generally believed that the EL result from the decoupling of contributions from virtual states of heavy particles to the amplitudes of scattering processes featuring SM particles. Below, we employ a traditional definition of the EL as the sum of scattering amplitudes of SM particles at external momenta $p_{\text{ext}} \ll \Lambda_i$. In the case of this definition, the contributions of virtual heavy particles to the amplitudes decouple in the form of local operators involving SM fields, their derivatives, and masses. These operators, $O_i$, by construction respect gauge symmetries of the SM, have the dimension $d > 4$, and are suppressed by the appropriate powers of heavy masses:

$$L_{\text{eff}} = L_{\text{SM eff}}^{\text{eff}} + \sum_i \frac{\alpha_i O_i}{\Lambda^2} + O\left(\frac{m_{W}^{4}}{\Lambda^4}\right). \quad (1)$$

The dimensionless parameters $\alpha_i$ characterize physics beyond the SM. They are model-dependent and expressed in terms of dimensionless coupling constants.

In this way, one can write down the EL of type (1) for any model supposed as the basis low-energy theory. For instance, the minimal super-symmetric SM can be considered. In what follows, we choose the minimal SM as the basis low-energy theory. All the types of operators $O_i$ that may arise in specific renormalizable gauge theories, containing the SM as a subgroup, are investigated in Refs. [3]. Then, the EL is constructed as the sum of the type (1) with the coefficients $\alpha_i$ treated as independent parameters to be determined from experimental data. This meaning of $\alpha_i$ is peculiar to the approaches used in Refs. [1]. In contrast, the parameters $\alpha_i$ entering Eq. (1) are not independent in general. For such a definition, a set of relations between $\alpha_i$ has been derived and called the renormalization group (RG) relations [4]. It is a consequence of the RG equations for $S$-matrix elements and the decoupling theorem [5]. In what follows, we extend the analysis to the case of searching for heavy $Z'$ boson in four-fermionic processes. We suppose that the effective (or basis) low-energy theory (the minimal SM in our case) is specified. That means we choose the appropriate set of operators $O_i$ in Eq. (1). We discriminate between the Abelian and the non-Abelian $Z'$ bosons, assuming the different types of their interactions with light particles.

The low-energy model is the remnant of some unknown high-energy theory predicting the Abelian $Z'$ boson. The constraints on the $Z'$ parameters can be derived with or without knowledge of the underlying theory. In this regard, we will call them the model dependent and the model independent ones [6]. The RG relations being the consequence of renormalizability are the model independent constraints.

Let us summarize the main steps in deriving the RG relations [4]. First, as the decoupling theorem guarantees, the operators originating from interactions of the renormalizable types have to be preserved in the EL in leading order in $1/\Lambda$. Since the underlying theory is not specified, the corresponding $\alpha_i$ must...
be considered as arbitrary parameters. Hence, one is able at least to relate different parameters describing different scattering processes. Second, a heavy virtual state can be treated as an external field scattering SM light particles. The transition amplitude is splitted in two scattering vertices with arbitrary vertex coefficients. The renormalizability of the theory can be used in a way to incorporate information about the $\beta$ and $\gamma$ functions entering the RG operator at low energies in the framework of the external field problem. The key point of the renormalizability of the vertices exploited is that the algebraic structure of the divergent part of a scattering amplitude calculated in higher orders of perturbation theory coincides with that of the tree-level vertex. Hence, the relation between the vertex coefficient and the $\beta$ and $\gamma$ functions follows. If the underlying theory is specified, the relation is just an identity. But if the theory is not assumed and the number of $\beta$ and $\gamma$ functions is less than the number of RG relations, a non-trivial system of algebraic equations for vertex coefficients and, hence, for parameters $\alpha_i$ in Eq. (1) can be obtained.

Although the underlying theory predicting the Abelian $Z'$ boson is not specified, it is possible to parameterize the $Z'$ interactions with the SM particles assuming the effective gauge group to be $\text{SU}(2)_L \times \text{U}(1)_Y \times \tilde{\text{U}}(1)$ as it is often discussed in literature \cite{7, 8}. This parameterization is natural, in regard of the decoupling theorem, when the leading order operators $\sim 1/\Lambda^2$ are investigated. In general, there are not reasons to assume a priori that the complete Lagrangian of the model is the $\tilde{\text{U}}(1)$ invariant. However, for our approach the low-energy parameterization is important. As it will be seen, the derived RG relations distinguish the Abelian $Z'$ and the most general parameterizations of the $Z'$ interactions with the SM fields. Moreover, in the latter case the subgroup of $Z'$ gauge group is fixed (due to the renormalizability).

In what follows, we will formulate the low-energy RG relations for four-fermion amplitudes in order $\sim 1/\Lambda^2$. As it will be shown, the relations hold only in the case if the absolute value of the $Z'$ coupling to the axial-vector currents is the same for all the SM fermions. Hence, some of the parameters $\alpha_i$ are not independent. Moreover, being a consequence of the renormalizability, the RG relations ensure that the Yukawa terms of the SM Lagrangian respect the $\tilde{\text{U}}(1)$ gauge symmetry associated with the $Z'$ boson.

The content is as follows. In Sec. 2 the model investigated is described. In Sec. 3 the general RG relation for a $S$-matrix element at low energies is derived. In Sec. 4 it is applied to four-fermion scattering processes to obtain correlations between the vertex parameters describing new physics at low energies. In Sec. 5 the corresponding relations for the parameters $\alpha_i$ of the EL are derived. The results of the investigation as well as the prospects are discussed in Sec. 6.
2 The Model

The model under consideration is assumed to be the low energy remnant of some unknown underlying theory. It involves the SM and the additional Abelian vector boson \( Z' \) which is supposed to be a massive particle with the mass \( \Lambda_0 \) much heavier than the \( W \)-boson mass. In the present paper the mechanism of heavy mass generation is not considered since the underlying theory describing interactions at energies \( E \sim \Lambda_0 \) is unspecified. So, the Lagrangian includes the explicit mass term for the \( Z' \) boson, and the mass \( \Lambda_0 \) is treated as a parameter of the model.

The most general parameterization of the interactions between the Abelian \( Z' \) and the SM fields can be performed on the base of the effective gauge group \( SU(2)_L \times U(1)_Y \times \tilde{U}(1) \) [7, 8]. Then, the corresponding Lagrangian is

\[
\mathcal{L} = \frac{1}{2} \left( \partial_\mu - i g A_\mu^a - i g' B_\mu - i \tilde{g} \phi Z'_{0\mu} \right) \phi^2 + \sum_{f_L} \sum_{i,j=1,2} \bar{f}_L i \left( i \partial_\mu I_{ij} + g \frac{\sigma^a}{2} A_\mu^a \right)
\]

\[
+ \sum_{f_R} \bar{f}_R \left( i \partial_\mu + g' Q f B_\mu + \tilde{g} \tilde{Y}_f Z'_{0\mu} \right) \gamma^\mu f_R,
\]

where \( \phi \) is the SM scalar doublet, and the summation over all the SM left-handed doublets \( f_L \) and the SM right-handed singlets \( f_R \) is understood. The charges \( g, g', \tilde{g} \) and the fields \( A_\mu^a, B_\mu, Z'_{0\mu} \) correspond to the gauge groups \( SU(2)_L, U(1)_Y, \tilde{U}(1) \), respectively, \( Q_f \) is the fermion charge in the positron charge units, \( \sigma^a \) are the Pauli matrices, \( I_{ij} \) is a \( 2 \times 2 \) unity matrix, and \( y_L = -1 \) for leptons and \( y_L = 1/3 \) for quarks. The values of the dimensionless constants \( Y_f, \tilde{Y}_f, \tilde{Y}_o \) depend on the particular higher-energy theory including \( Z' \) boson. Here, they are to be considered as arbitrary parameters. This parameterization, for instance, accounts for the most general \( Z' \) interactions generated in string theories [8].

Up to this point the electroweak symmetry \( SU(2)_L \times U(1)_Y \) was supposed to be unbroken, and all the SM particles were massless. The masses of the SM particles are generated by the spontaneous breakdown of the symmetry \( SU(2)_L \times U(1)_Y \rightarrow U(1)_{em} \) originated due to the non-zero vacuum expectation value of the scalar doublet. However, the mass eigenstates are appeared to be shifted away from the original fields \( A_\mu^a, B_\mu, Z'_{0\mu} \), since the corresponding mass terms become non-diagonal. The physical fields \( A_\mu, Z_\mu, Z'_\mu \) are obtained by the orthogonal transformation:

\[
\begin{align*}
A_\mu &= A_\mu^3 \sin \theta_W + B_\mu \cos \theta_W, \\
Z_\mu &= Z'_{0\mu} \sin \theta_0 + \left( A_\mu^3 \cos \theta_W - B_\mu \sin \theta_W \right) \cos \theta_0, \\
Z'_\mu &= Z'_{0\mu} \cos \theta_0 - \left( A_\mu^3 \cos \theta_W - B_\mu \sin \theta_W \right) \sin \theta_0,
\end{align*}
\]

(3)
\[
\begin{align*}
B_\mu &= A_\mu \cos \theta_W - (Z_\mu \cos \theta_0 - Z'_\mu \sin \theta_0) \sin \theta_W, \\
A'_\mu &= A_\mu \sin \theta_W + (Z_\mu \cos \theta_0 - Z'_\mu \sin \theta_0) \cos \theta_W, \\
Z'_{\mu0} &= Z_\mu \sin \theta_0 + Z'_\mu \cos \theta_0,
\end{align*}
\]

where \( \theta_W \) is the SM value of the Weinberg angle (\( \tan \theta_W = g'/g \)), and \( \theta_0 \) denotes the mixing angle relating physical states \( Z_\mu, Z'_\mu \) to massive neutral components of the \( SU(2)_L \times U(1)_Y \times U(1) \) gauge fields. The value of the angle \( \theta_0 \) can be determined from the relation

\[
\tan^2 \theta_0 = \frac{m_W^2 / \cos^2 \theta_W - m_Z^2}{\Lambda^2 - m_W^2 / \cos^2 \theta_W},
\]

confirming Ref. [7].

The masses of the physical fields are given by the expressions:

\[
m_A^2 = 0, \\
m_Z^2 = \frac{m_W^2}{\cos^2 \theta_W} \left( 1 - \frac{4\tilde{g}^2\tilde{Y}_\phi^2}{g^2} \frac{m_W^2}{\Lambda^2 - m_W^2 / \cos^2 \theta_W} \right), \\
\Lambda^2 = \Lambda_0^2 + \left( \frac{m_Z^2}{\cos^2 \theta_W} - m_Z^2 \right) + \frac{4\tilde{g}^2\tilde{Y}_\phi^2}{g^2} m_W^2.
\]

As is seen, the mass of the physical \( Z \)-boson differs from the SM value \( m_W / \cos \theta_W \) by the small quantity of order \( m_Z^2 / \Lambda^2 \). So, the mixing angle \( \theta_0 \) is also small \( \theta_0 \approx \tan \theta_0 \approx \sin \theta_0 \approx m_Z^2 / \Lambda^2 \). The difference \( m_Z^2 - m_W^2 / \cos^2 \theta_W \) is negative and completely determined by the \( Z' \) coupling to the scalar doublet. Thus, constraints on the \( Z' \) interaction with scalar fields can be derived by means of experimental detecting this observable:

\[
\frac{\tilde{g}^2\tilde{Y}_\phi^2}{\Lambda^2} = \left( 1 - \frac{m_Z^2 \cos^2 \theta_W}{m_W^2} \right) \frac{g^2}{4m_W^2} + \mathcal{O} \left( \frac{m_W^4}{\Lambda^4} \right).
\]

Using Eqs. (4) the Lagrangian of the model can be expressed in terms of the physical fields. The dependence on the mixing \( \theta_0 \) in Eqs. (4) causes new interactions \( \sim m_W^2 / \Lambda^2 \) originally absent in the Lagrangian (2).

In the present paper we will deal with four-fermion amplitudes of order \( \sim \Lambda^{-2} \) produced by virtual \( Z' \) boson in the \( s \) channel. The corresponding tree-level diagram is shown in Fig. 1. At low energies the effective four-fermion vertex is generated due to the heavy mass in the \( Z' \)-boson propagator: \( (p^2 - \Lambda^2)^{-1} \rightarrow -\Lambda^{-2}[1 + \mathcal{O}(p^2 / \Lambda^2)] \). So, expressions involving such amplitudes can be computed neglecting the terms proportional to \( \sim m_W^2 / \Lambda^2 \) in the Lagrangian. Only linear in \( Z' \) terms of the Lagrangian are needed, which have the following form in the 't Hooft-Feynman gauge:

\[
\mathcal{L}' = \tilde{g}Z'_\mu \sum \bar{f} \gamma^\mu \left( \tilde{Y}_f^L \omega_L + \tilde{Y}_f^R \omega_R \right) f
\]
\[ +g\tilde{Y}_\phi Z'_\mu \left[ H \partial^\mu \chi_3 + i\chi^- \partial^\mu \chi^+ + 2\epsilon A_\mu \chi^- \chi^+ ight. \\
- \frac{g}{2\cos\theta_W} \left( H^2 + 2vH - 2\chi^- \chi^+ \cos 2\theta_W + \chi_3^2 \right) Z'_\mu \\
- ig (H + v) \left( W^\mu_+ \chi^- - W^-_\mu \chi^+ \right) \\
- g\chi_3 \left( W^\mu_+ \chi^- + W^-_\mu \chi^+ \right) \right] + O \left( \frac{m^2_W}{\Lambda^2} \right), \]

where \( \phi \partial^\mu \chi \equiv \phi \partial^\mu \chi - \chi \partial^\mu \phi \), \( \omega_{L,R} = (1 \mp \gamma^5)/2 \), \( \epsilon = g\sin\theta_W \) is the positron charge, \( v \) denotes the vacuum expectation value of the scalar doublet, \( H \) is the Higgs scalar particle, and \( \chi^\pm, \chi_3 \) are the Goldstone fields.

### 3 RG relations in a theory with decoupling

As we mentioned before, the model under consideration is supposed to be the low-energy limit of some unspecified renormalizable theory describing interactions at energies \( E \geq \Lambda \). As is known, due to renormalizability the \( S \)-matrix elements are invariant with respect to the RG transformations which express independence of the amplitudes of the normalization point \( \kappa \). In a theory with different mass scales the RG flow and the decoupling of heavy loop contributions at the thresholds of heavy masses give possibility to determine all the parameters of scattering amplitudes. The decoupling is resulted in an important property of the low energy amplitudes: the running of the proper functions is regulated by loops of light particles. Therefore, the \( \beta \) and \( \gamma \) functions at low energies, \( E \ll \Lambda \), are determined by the SM particles, only. The latter fact allows to formulate a series of the RG relations for scattering amplitudes. We will derive them for the case of heavy \( Z' \) boson which, as well as other heavy particles, is decoupled.

We will use the notations \( \lambda_a = g^2, g'^2, g_s^2, \tilde{g}^2, G_f, \lambda \) (where \( g_s, G_f = m_f/v \) and \( \lambda \) denote the QCD charge, the coupling of the Yukawa interaction between \( f \) and \( \phi \), and the scalar self-coupling, respectively) and \( X = \Phi, m_i^2, \Lambda^2 \) (where \( \Phi \) represents all the fields of the theory, and \( m_i \) are particle masses) to refer to the charges, the fields, and the masses. Then the RG equation reads:

\[ \frac{d}{d \ln \kappa} \hat{S} = 0, \]
\[ \frac{d}{d \ln \kappa} \hat{D} = \frac{\partial}{\partial \ln \kappa} + \sum_a \hat{\beta}_a \frac{\partial}{\partial \lambda_a} - \sum_X \hat{\gamma}_X \frac{\partial}{\partial \ln \kappa}, \]

where renormalized (running) parameters \( \hat{\lambda}_a, \hat{X} \) are the solutions to the equations:

\[ \frac{d\hat{\lambda}_a}{d \ln \kappa} = \hat{\beta}_a \left( \hat{\lambda}_a, \hat{X} \right), \quad \frac{d \ln \hat{X}}{d \ln \kappa} = -\hat{\gamma}_X \left( \hat{\lambda}_a, \hat{X} \right). \]
Hats over quantities mark the parameters of the underlying theory. They include the contributions of loops containing both the SM particles and the heavy ones.

Below, in carrying out calculations the dimensional regularization and the $\overline{\text{MS}}$ renormalization scheme \cite{9} will be used. The $\overline{\text{MS}}$ scheme is mass independent one, so the RG operator $\hat{D}$ has the same form at high, $E \geq \Lambda$, and low, $E \ll \Lambda$, energies.

In the energy range $E \ll \Lambda$ all the heavy fields are decoupled. So, one can introduce the running parameters $\lambda_a, X$ completely defined by the low-energy effective theory \cite{10}:

$$d\lambda_a/d\ln \kappa = \beta_a (\lambda_a, X), \quad d \ln X / d \ln \kappa = -\gamma_X (\lambda_a, X),$$

where $\beta$ and $\gamma$ functions contain no contributions from loops with heavy particles. These parameters can be expressed in terms of the original ones:

$$\lambda_a = \tilde{\lambda}_a + a_{\lambda_a} \ln \frac{\hat{\Lambda}^2}{\kappa^2} + b_{\lambda_a} \ln^2 \frac{\hat{\Lambda}^2}{\kappa^2} + ...$$

$$X = \tilde{X} \left(1 + a_X \ln \frac{\hat{\Lambda}^2}{\kappa^2} + b_X \ln^2 \frac{\hat{\Lambda}^2}{\kappa^2} + ... \right),$$

where the matching between the both sets of parameters $(\lambda_a, X$ and $\tilde{\lambda}_a, \tilde{X})$ is chosen to be done at the normalization point $\kappa \sim \Lambda$:

$$\lambda_a |_{\kappa=\Lambda} = \tilde{\lambda}_a |_{\kappa=\Lambda}, \quad X |_{\kappa=\Lambda} = \tilde{X} |_{\kappa=\Lambda}.$$ (13)

Actually, Eq. (12) is the redefinition of the parameters of the theory. As it has been shown in Refs. \cite{10, 11}, all the heavy particle loop contributions proportional to $\ln \kappa$ are completely removed from the RG equation (9) by such a redefinition. This fact is a consequence of the decoupling theorem \cite{5}.

Then, the RG equation (14) for the $\hat{S}$ matrix expressed in terms of the low-energy quantities $\lambda_a, X$ becomes:

$$\hat{D}S = \left(\frac{\partial}{\partial \ln \kappa} + \sum_a \beta_a \frac{\partial}{\partial \lambda_a} - \sum_X \gamma_X \frac{\partial}{\partial \ln X}\right) S = 0,$$ (14)

where $\hat{D}$ and $S$ denote, respectively, the RG operator and the $S$-matrix element of the low-energy effective theory, calculated without the heavy particle loops.

The familiar usage of Eq. (14) is to improve scattering amplitudes calculated in perturbation theory. In contrast, in what follows we will apply Eq. (14) to obtain algebraic relations between the parameters $\hat{Y}_f^L$, $\hat{Y}_f^R$, $\hat{Y}_\phi$, which are to be considered as arbitrary numbers, if the underlying theory is not assumed. The reasons for that are as follow. In the case when the underlying theory is specified ($\hat{Y}_f^L$, $\hat{Y}_f^R$, $\hat{Y}_\phi$ are to be computed) and the $\beta$ and $\gamma$ functions as well as the $S$-matrix elements are calculated in a fixed order of perturbation theory, Eq. (14)
is just the identity. In a sense, it is a necessary condition of renormalizability. If the underlying theory is not specified, whereas the $\beta$, $\gamma$ functions and the $S$-matrix elements are computed in a fixed order of perturbation theory, then, as we will show for four-fermion processes at one-loop level, the equality (14) holds only if the parameters $\tilde{Y}_L^f$, $\tilde{Y}_R^f$, $\tilde{Y}_\phi$ satisfy some specific correlations. The latter result in the dependences between the appropriate parameters $\alpha_i$ in the EL (1).

Let $S^{(1)}$ and $S^{(0)}$ represent the one-loop and the tree parts of the $S$-matrix element, respectively. Introducing the one-loop RG operator:

$$D^{(1)} \equiv \sum_a \beta^{(1)}_a \frac{\partial}{\partial \lambda_a} - \sum_X \gamma^{(1)}(X) \frac{\partial}{\partial \ln X},$$

where $\beta$ and $\gamma$ functions are computed in one-loop order, one can derive from Eq. (14) the following identity:

$$\frac{\partial}{\partial \ln \kappa} S^{(1)} + D^{(1)} S^{(0)} = 0.$$  

(16)

Since the parameters $\lambda_a$, $X$ and $\tilde{X}_a$, $\tilde{X}$ differ at the one-loop level, one could freely substitute one set of them by another in Eq. (16).

Equation (16) is the starting point to analyze the RG relations for amplitudes at low energies. The relation (16) ensures the leading logarithmic terms of the one-loop $S$-matrix element to reproduce the appropriate tree-level structure that is a simple consequence of renormalizability. When the couplings are represented by arbitrary unknown parameters, as it takes place in the considered model and, in general, in the discussed EL approach, the relation (16) is the one-loop equation for the parameters which must be determined with the computed $\beta$ and $\gamma$ functions. If due to a symmetry the number of $\beta$ and $\gamma$ functions is less than the number of RG relations, the non-trivial system of equations correlating the originally independent parameters may occur. We will use Eq. (16) to derive dependences between the parameters $\tilde{Y}_L^f$, $\tilde{Y}_R^f$, and $\tilde{Y}_\phi$.

4 RG relations for four-fermion amplitudes

In this section the one-loop RG relation (16) is applied to the four-fermion processes $f_1 f_1 \to Z'_{\nu*} \to f_2 f_2$, containing $Z'$ boson as the virtual state. In lower order in $\Lambda^{-2}$ such amplitudes produce the effective contact interactions of the type current $\times$ current described by the EL (1) [1, 3].

Let the renormalized fields, masses, and charges are defined as follows:

$$f = Z_f^{1/2} f^{\text{bare}}, \quad \left(\begin{array}{c} A^\mu \\ Z^\mu \\ Z'_\mu \end{array}\right) = Z^{1/2}_V \left(\begin{array}{c} A^{\text{bare}}^\mu \\ Z^{\text{bare}}^\mu \\ Z'^{\text{bare}}_\mu \end{array}\right),$$

$$m_i^2 = m_i^{2,\text{bare}} - \delta m_i^2, \quad \Gamma_{Vf} = \sum_{V_1} (Z^{-1}_g)_{V_1 V} \Gamma_{V_1 f}^{\text{bare}},$$

(17)
where $\Gamma_{Vf}$ is the vertex describing interaction between the fermion $f$ and the neutral vector boson $V$ and $(Z_V^{-1/2})_{V_1V_2}$ and $(Z_g^{-1})_{V_1V_2}$ are $3 \times 3$ matrices with $(V_i = A, Z, Z')$. The fermion renormalization constant $Z_f^{-1/2}$ is the SM one. Its value is known. Quantities $Z_V^{-1/2}$ and $\delta m_V$ are represented at the one-loop level by the divergent terms ($\sim \varepsilon^{-1}$) of the gauge-independent part of the polarization operator of neutral vector bosons $\Pi^{(1)}_{V_1V_2}(p^2)$ being evaluated from the diagrams in Fig. 2:

$$
\left(Z_V^{1/2}\right)_{V_1V_2} = \delta_{V_1V_2} + \frac{\delta_{V_1V_2}}{2} \frac{\partial}{\partial p^2} \Pi^{(1)\text{div}}_{V_1V_2}(p^2)
$$

$$
- \frac{1 - \delta_{V_1V_2}}{m_{V_1}^2 - m_{V_2}^2} \Pi^{(1)\text{div}}_{V_1V_2}(m_{V_1}^2),
$$

$$
\delta m_V^2 = - \Pi^{(1)\text{div}}_{V_1V_2}(m_{V_1}^2),
$$

where $\delta_{V_1V_2}$ is the Kronecker symbol. In order to find the constant $Z_g$ in one-loop order one has to calculate the divergent ($\sim \varepsilon^{-1}$) part of the vertex function $\Gamma^{(1)}_{\psi V}$ (the diagrams in Fig. 3):

$$
Z_g = Z_f^{-1}\left(Z_V^{-1/2}\right)^T \left(1 - \Gamma^{(1)\text{div}}_{fV}/\Gamma^{(0)}_{fV}\right),
$$

where $\Gamma^{(0)}_{fV}$ is the vertex $\Gamma_{\psi V}$ at the tree level.

The $S$-matrix element expressed in terms of the renormalized quantities is finite. It can be evaluated using the diagrams in Figs. 3 and 4 where shaded blobs stand for the finite parts of the vertex function $\Gamma^{(1)\text{fin}}_{fV} = \Gamma^{(1)}_{fV} - \Gamma^{(1)\text{div}}_{fV}$ and the polarization operator $\Pi^{(1)\text{fin}}_{V_1V_2} = \Pi^{(1)}_{V_1V_2} - \Pi^{(1)\text{div}}_{V_1V_2}$. On the mass-shell of fermions the gauge-dependent part of $\Pi_{V_1V_2}$ ($\sim p_\mu p_\nu$) gives no contribution to the physical amplitude due to transversality of the vertex $\Gamma_{fV}$.

Because of the one-loop mixing between neutral bosons, Eq. (16) for the renormalized $S$-matrix element cannot be reduced to relations for vertices. To derive these relations one has to avoid the mixing. The corresponding procedure was described in Ref. [11].

One has to introduce the quantities $\tilde{V}$ and $\tilde{\Gamma}_{fV}$ instead of the fields $V = A_\mu$, $Z_\mu$, $Z'_\mu$ and vertices $\Gamma_{fV}$ according to the following relations:

$$
V_1 = \sum_{V_2} \zeta_{V_1V_2}^{1/2} \tilde{V}_2, \quad \Gamma_{fV_1} = \sum_{V_2} \zeta_{V_2V_1}^{-1/2} \tilde{\Gamma}_{fV_2},
$$

$$
\zeta_{V_1V_2}^{\pm1/2} = \delta_{V_1V_2} \mp \frac{1 - \delta_{V_1V_2}}{m_{V_1}^2 - m_{V_2}^2} \ln \frac{\kappa}{\Lambda} \lim_{\varepsilon \to 0} \varepsilon \Pi^{(1)}_{V_1V_2}(m_{V_2}^2). \quad (20)
$$

After such a redefinition the mixing between neutral bosons in Eq. (11) vanishes. In what follows, we will use the quantities $\tilde{V}$ and $\tilde{\Gamma}_{fV}$, therefore only.
the diagonal terms of the polarization operator are needed in computing of the
$S$-matrix element $S^{(1)}$. Since the difference between the two sets of parameters
is of order $\sim g^2/16\pi^2$, one can substitute the tilded parameters by the other set
in the amplitudes in lower order.
When the diagonalization of the leading logarithmic terms of the vector
boson polarization operator is fulfilled, Eq. (16) can be reduced to relations for
vertices describing scattering of fermions in the external field $\Lambda^{-1}$ substituting
the heavy $Z'$ boson:
\[ \frac{\partial \Gamma^{(1)}_{fV}}{\partial \ln \kappa} + D^{(1)} \left( \frac{\Gamma^{(0)}_{fV}}{\Lambda} \right) = 0. \]  
(21)

The quantities $\Gamma^{(0)}_{fV}$ and $\Gamma^{(1)}_{fV}$ in Eq. (21) have been calculated in the 't
Hooft-Feynman gauge using diagrams in Figs. [1, 2, 3]. The computation gives the
following relations for the left-handed and the right-handed fermions, respectively:
\[ \frac{1}{8\pi^2} \left[ g^2 \left( \frac{1}{4\cos^2 \theta_W} + (Q_f^2 - |Q_f|) \tan^2 \theta_W + \frac{\tilde{Y}_L}{2Y_L} \right) \right. 
+ 4g_s^2 - G_f^2 \frac{\tilde{Y}_R}{Y_R} + G_f^2 \frac{\tilde{Y}_R}{Y_R} 
+ \left. \frac{\beta^{(1)}_g}{2g^2} + \frac{1}{2} \gamma^{(1)} - 2\gamma^{(1)}_{1L} = 0, \right] \]  
\[ \frac{1}{8\pi^2} \left[ g^2 Q_f^2 \tan^2 \theta_W + \frac{4g_s^2}{3} + G_f^2 \frac{\tilde{Y}_L}{Y_L} + \frac{\tilde{Y}_R}{Y_R} + 2t_f \tilde{Y}_R \right] 
+ \frac{\beta^{(1)}_g}{2g^2} + \frac{1}{2} \gamma^{(1)} + 2\gamma^{(1)}_{1R} = 0, \]  
(22)
where
\[ \beta_g = \frac{d\tilde{g}^2}{d\ln \kappa}. \]  
(23)
The notation $f'$ stands for the isopartner of $f$ ($l' = \nu_l, \nu_{l'}, q_{d'} \equiv q_d, g_{s,f}$
equals to $g_s$ for quarks and zero for leptons, $q_{d', u'} = q_u$),
\[ t_f = 2T_3^f = \left\{ \begin{array}{c} +1, f = \nu_l, u, c, t \vspace{1mm} \\
-1, f = l, d, s, b \end{array} \right., \]  
(24)
where $l = e, \mu, \tau$ denotes leptons, and $T_3^f$ is the third component of the weak
isospin.
The SM values of the one-loop fermion anomalous dimensions are known [12]:

$$\gamma^{(1)}_{fL} = \frac{1}{16\pi^2} \left[ g^2 \left( \frac{1}{4\cos^2 \theta_W} + (Q_f^2 - |Q_f|)\tan^2 \theta_W + \frac{1}{2} \right) 
+ \frac{4}{3} g^2_{s,f} + G_f^2 + G'_f \right],$$

$$\gamma^{(1)}_{fR} = \frac{1}{16\pi^2} \left[ g^2 Q_f^2 \tan^2 \theta_W + \frac{4}{3} g^2_{s,f} + 2 G_f^2 \right].$$

(25)

By employing Eqs. (18)-(20), it is easy to compute $\beta_{\tilde{g}}$ and $\gamma_{\Lambda^2}$ in one-loop order:

$$\left(\tilde{Y}_f^L\right)^2 \beta^{(1)}_{\tilde{g}} = \frac{g^2 \tilde{Y}_f^L}{24\pi^2} \left[ 2g^2 \tilde{Y}_f^L \Sigma + 3g^2 \left( \tilde{Y}_f^L - \tilde{Y}_f^R \right) 
+ 6G_f^2 \left( \tilde{Y}_\phi t_f + \tilde{Y}_f^L - \tilde{Y}_f^R \right) 
- 6G'_f \left( \tilde{Y}_\phi t_f - \tilde{Y}_f^L + \tilde{Y}_f^R \right) \right] + O \left( \frac{m_W^2}{\Lambda^2} \right),$$

$$\left(\tilde{Y}_f^R\right)^2 \beta^{(1)}_{\tilde{g}} = \frac{g^2 \tilde{Y}_f^R}{12\pi^2} \left[ -3g^2 \left( 2\tilde{Y}_\phi t_f + \tilde{Y}_f^L + \tilde{Y}_f^L - 2\tilde{Y}_f^R \right) 
+ g^2 \tilde{Y}_f^R \Sigma \right] + O \left( \frac{m_W^2}{\Lambda^2} \right),$$

(26)

$$\gamma^{(1)}_{\Lambda^2} = - \frac{g^2 \Sigma}{12\pi^2}.$$

(27)

where $\Sigma = \tilde{Y}_\phi^2 + \sum_f \left( \left( \tilde{Y}_f^L \right)^2 + \left( \tilde{Y}_f^R \right)^2 \right)$.

Since the $\beta_{\tilde{g}}$ function is the same for both Eqs. (24), the following relations can be derived:

$$\tilde{Y}_f^L = \tilde{Y}_f^L, \quad \tilde{Y}_f^R = \tilde{Y}_f^L + t_f \tilde{Y}_\phi.$$  

(28)

Hence, one can see that originally independent parameters $\tilde{Y}_f^L$, $\tilde{Y}_f^R$, and $\tilde{Y}_\phi$ appear to be the connected ones. Finally, one can also check that the RG relations (22) are fulfilled only if Eq. (28) holds.

In a sense, the relations (28) mean that the $Z'$ boson couplings to the SM axial-vector currents have the universal absolute value, if a single light scalar doublet exists. So, among the four constants $\tilde{Y}_f^L$, $\tilde{Y}_f^R$, $\tilde{Y}_f^L$, and $\tilde{Y}_f^R$ parameterizing the interaction between the $Z'$ boson and the SU(2) fermion isodoublet only one is really arbitrary. The rest ones can be expressed through it and the hypercharge $\tilde{Y}_\phi$ of the $Z'$ coupling to the SM scalar doublet. Thus, the fermion and the scalar sectors of new physics are to be correlated.

Being derived from the condition of renormalizability of a scattering amplitude in the external field of heavy particle, the relations (28) respect gauge
invariance and have a transparent interpretation. First of them means that the upper and the lower components of a left-handed doublet transform in the same way under $\tilde{U}(1)$ gauge group. If it does not hold, SU(2)$_L$ gauge invariance would violate. The second of relations guarantees, that the SM Yukawa interaction terms have to be invariant under $\tilde{U}(1)$ gauge transformations.

5 Dependences between the parameters of the effective Lagrangian

The relations (28) result in dependences between the parameters of the EL (1). In Ref. [3] all the effective operators $O_i$ describing deviations from the SM and appearing at the tree level due to heavy virtual states have been derived. The heavy $Z'$ boson produces the following effective vertices:

\[ -\frac{\tilde{g}^2\tilde{Y}_\phi^2}{8\Lambda^2} \left( (D^e_{\mu} \phi)^\dagger \phi - \phi^\dagger D^e_{\mu} \phi \right)^2 = -\frac{\tilde{g}^2\tilde{Y}_\phi^2}{4\Lambda^2} \left( O_{\partial\phi} - 2O^{(3)}_{\phi} \right), \]

\[ -\frac{i\tilde{g}^2\tilde{Y}_\phi}{2\Lambda^2} \left( (D^e_{\mu} \phi)^\dagger \phi - \phi^\dagger D^e_{\mu} \phi \right) \tilde{f} \gamma^\mu \left( \tilde{Y}^L_f \omega_L + \tilde{Y}^R_f \omega_R \right) f = -\frac{\tilde{g}^2\tilde{Y}_\phi}{2\Lambda^2} \left( O^{(1)}_{\phi f} + O_{\phi f} + h.c. \right), \]

\[ \frac{\tilde{g}^2}{(1 + \delta_{f_1 f_2}) \Lambda^2} \tilde{f}_1 \gamma^\mu \left( \tilde{Y}^L_{f_1} \omega_L + \tilde{Y}^R_{f_1} \omega_R \right) f_1 \]

\[ \times \tilde{f}_2 \gamma^\mu \left( \tilde{Y}^L_{f_2} \omega_L + \tilde{Y}^R_{f_2} \omega_R \right) f_2 = \frac{2\tilde{g}^2}{\Lambda^2} \left( \tilde{Y}^L_{f} \tilde{Y}^L_{f} O^{(1)}_{q f} + \tilde{Y}^R_{f} \tilde{Y}^R_{f} O_{q f} \right) \]

\[ + \frac{\tilde{Y}^L_{q_1} \tilde{Y}^L_{q_2} O^{(1)}_{q_1 q_2} + \tilde{Y}^R_{q_1} \tilde{Y}^R_{q_2} O^{(1)}_{q_1 q_2}}{1 + \delta_{q_1 q_2}} \], \tag{29} \]

where $l$ and $q$ stand for leptons and quarks; $D^e_{\mu}$ is the electroweak covariant derivative, and the definitions of $O_i$ correspond to Ref. [3]:

\[ O_{\partial\phi} = \frac{1}{2} \partial_\mu \left( \phi^\dagger \phi \right) \partial^\mu \left( \phi^\dagger \phi \right), \quad O^{(3)}_{\phi} = \left| \phi^\dagger D^e_{\mu} \phi \right|^2, \]

\[ O^{(1)}_{\phi f} = i \left( \phi^\dagger D^e_{\mu} \phi \right) \tilde{f}_L \gamma^\mu f_L, \quad O_{\phi f} = i \left( \phi^\dagger D^e_{\mu} \phi \right) \tilde{f}_R \gamma^\mu f_R, \]

\[ O^{(1,1)}_{q_1 q_2} = \frac{1}{2} \left( \tilde{q}_1 L \gamma^\mu q_1 L \right) \left( \tilde{q}_2 L \gamma^\mu q_2 L \right), \]

\[ O^{(1)}_{q_1 q_2} = \frac{1}{2} \left( \tilde{q}_1 R \gamma^\mu q_1 R \right) \left( \tilde{q}_2 R \gamma^\mu q_2 R \right), \]
\[ \mathcal{O}_{lf}^{(1)} = \frac{1}{2} (\bar{l}_L \gamma^\mu l_L) (\bar{f}_L \gamma^\mu f_L), \]
\[ \mathcal{O}_{lf} = \frac{1}{2} (\bar{l}_R \gamma^\mu l_R) (\bar{f}_R \gamma^\mu f_R). \]

Thus, the relations (28) give rise to the following dependences between the parameters \( \alpha_i \):

\[ \alpha^{(3)}_\phi = -2 \alpha_{\phi \phi} = \frac{2 \alpha^{(1)}_{\phi \phi}}{\alpha_{ee}}, \quad \alpha_{\phi f} = \alpha^{(1)}_{\phi f} + \frac{2 t_f \alpha^{(1)}_{\phi e}}{\alpha_{ee}}, \]
\[ \alpha^{(1)}_{q_1 q_2} = \alpha^{(1,1)}_{q_1 q_2} + \frac{4 t_f t_{q_2}}{1 + \delta_{q_1 q_2}} \left( \frac{\alpha^{(1)}_{\phi q_2}}{t_{q_2}} + \frac{\alpha^{(1)}_{\phi q_1}}{t_{q_1}} + 2 \frac{\alpha^{(1)}_{\phi e}}{\alpha_{ee}} \right), \]
\[ \alpha_{lf} = \alpha^{(1)}_{lf} + \frac{4 t_f}{1 + \delta_{lf}} \left( \frac{\alpha^{(1)}_{\phi l}}{t_f} - \frac{\alpha^{(1)}_{\phi e}}{\alpha_{ee}} \right). \]

Hence it follows that the arbitrary constants are to be the ones parameterizing the processes \( \bar{f}_L f_L \to \bar{f}_L f_L \) and \( \bar{e}_L e_L \to e_L e_L \).

The relations (31) hold if the only additional heavy particle extending the SM is the Abelian \( Z' \) boson. Other models may lead to other type of correlations between the parameters \( \alpha_i \). The advantage of the expressions (31) is their independence of the mechanism of arising of the Abelian \( Z' \) in a specific theory describing physics at energy scale \( E \geq \Lambda \).

6 Discussion

Let us discuss the main points of our analysis and further perspectives. Usually, the EL (1) describing at low energies physics beyond the SM contains a set of the operators with parameters \( \alpha_i \) assumed to be arbitrary numbers which must be fixed in experiments. However, as we have demonstrated above considering the additional \( Z' \) boson as extension of the SM, the renormalizability of the underlying theory may result in some correlations between \( \alpha_i \) which have to be of interest in searching for new physics effects. Most important is that the derived relations (28), being a consequence of renormalizability formulated in the framework of scattering in the external field, are independent of the specific underlying (GUT) model.

The latter statement requires to be discussed in more detail. The Lagrangian (2) respects \( \text{U}(1) \) gauge invariance but includes only the interactions of renormalizable type. The terms of the form \( \sim (\partial_{\mu} Z'_{\nu} - \partial_{\nu} Z'_{\mu}) \bar{f}_\mu f \) are omitted because they, being generated at GUT (or some intermediate \( \Lambda_{\text{GUT}} > \Lambda' > \Lambda \)) mass scale, are suppressed by the factors \( 1/\Lambda_{\text{GUT}}, 1/\Lambda' \). Hence, one can conclude that at low energies, \( E \ll \Lambda \), the renormalizable interactions are to be dominant. Therefore, the only model dependent information in Eq. (2) is the
specific values of the parameters $\tilde{Y}^f_L$, $\tilde{Y}^f_R$ and $\tilde{Y}_\phi$. The relations (28) hold for arbitrary values of them, as it was demonstrated in Sec. 4. As a consequence, the relations (31) also hold for arbitrary $\tilde{Y}^f_L$, $\tilde{Y}^f_R$, $\tilde{Y}_\phi$ and are independent of them explicitly.

Here, it will be useful to stress once again the main features of our approach as compared to the ones, where the $\alpha_i$ parameters are assumed to be arbitrary numbers. In the latter case, the origin of the composed operators is completely hidden. In the former one, we first came to the one step back in the analysis and have taken into account the most important consequences of the renormalizability: 1) at low energies, the interactions of renormalizable type are dominant whereas the nonrenormalizable interaction terms are suppressed by the additional degree of $\Lambda^{-1}$; 2) for any renormalizable interactions at low energies one can interrelate scattering processes due to the renormalizability of scattering amplitudes in the external field of heavy particle.

Now, let us consider a more general parameterization in Eq. (2). In principle, one could employ the most general form of the generator $\tilde{\phi} \rightarrow \text{diag}(\tilde{\phi}, 1, \tilde{\phi})$ which could be just the part of the appropriate generator of the underlying theory. Then, the RG relations can be used to obtain all possible solutions for the numbers $\tilde{\phi}_i$, $\tilde{Y}^f_L$, and $\tilde{Y}^f_R$. As one can check, the two sets of correlations are allowed. The first one,

$$\tilde{Y}^R_f = 0, \quad \tilde{\phi}_1 + \tilde{\phi}_2 = 0, \quad \tilde{Y}^L_f + \tilde{Y}^L_f = 0,$$

(32)

describes the vector boson analogous to the SM field $A_\mu$. Such a field may appear as a component of the non-Abelian gauge field. The second solution leads to the relations derived in this paper:

$$\tilde{Y}^R_f = \tilde{Y}_f = \tilde{Y}^L_f, \quad \tilde{Y}^R_f = \tilde{Y}^L_f + t_f \tilde{Y}_\phi.$$

(33)

They correspond to the Abelian vector boson considered. Thus, the proposed method to relate the parameters $\alpha_i$ could be applied to interactions of light SM particles with other heavy particles (leptoquarks, for example) treated as the external fields.

In Ref. [7] a general renormalization framework for the SU(2)$_L \times$ U(1)$_Y \times$ U(1) theory was presented, and, in particular, the set of relevant counterterms has been derived. Owing to U(1) symmetry the same counterterms were postulated for different $Z'$ couplings to fermions. However, as it was shown above, this fact requires the specific relations between the hypercharges $\tilde{Y}^L_f$, $\tilde{Y}^R_f$, $\tilde{Y}_\phi$. Naturally, the relations hold automatically for a specific renormalizable higher-energy theory. However, if the hypercharges are treated as unknown parameters these relations are to be taken into account in order to respect the gauge symmetry.

In fact, the obtained relations (28) demonstrate that the fermion and the scalar sectors of new physics are strongly connected. The couplings of the $Z'$
boson to the SM axial-vector currents are completely determined by its interaction with the scalar fields, if a single light scalar doublet is assumed. When the $Z'$ coupling to the scalar doublet is switched off ($\tilde{Y}_\phi = 0$), the $Z'$ boson interacts at the tree level with the SM vector currents, only ($\tilde{Y}_f^R - \tilde{Y}_f^L = 0$). But even in this case the couplings to the axial currents are to be produced by loops being suppressed by the additional small factor $\sim g^2/16\pi^2$.

As it follows from Eq. (28), the RG relations respect the $U(1)$ symmetry of the Yukawa terms responsible for generating the masses of the SM fermions. Therefore, assuming the basis low-energy theory to be, for example, the two-Higgs-doublet SM, one could expect that the relations identical to Eq. (28) can be derived for the Abelian $Z'$ couplings to each of the scalar doublets. However, in general, the RG relations are not determined by the specific gauge invariance of the Lagrangian. For instance, consider the non-Abelian $Z'$ couplings described by the solution (32). As one can check, the relations (32) require no specific gauge invariance of the Yukawa terms.

As it was mentioned in Sec. 2, the $Z'$ interaction with the scalar doublet is completely determined by the observable $m_Z^2 - m_W^2/\cos^2\theta_W$ (see Eq. (4)). Therefore, one is able to predict the coupling of the $Z'$ to the SM axial-vector currents. When $Z'$ boson does not interact with the scalar doublet, the physical mass of $Z$ boson is to be identical to its SM value. It is worth noticing that the experimental value of $\cos^2\theta_W$ in Eq. (4) must be properly chosen in order to satisfy the definition $\tan\theta_W = g'/g$. The $Z'$ existence has not to affect the quantity $\cos^2\theta_W$ in order $\sim m_W^2/\Lambda^2$. So, the determination of the Weinberg angle in terms of the QED coupling constant $\alpha$ and the Fermi constant $G$ measured from the muon decay is preferable, since the tree level values of these parameters $\alpha = g^2 \sin^2\theta_W/4\pi$, $G = \sqrt{2}g^2/8m_W^2$ include the contributions of $W$ bosons and the QED sector, only.

The proposed method can be applied to find the relations of the derived type for different effective four-fermion operators generated by the $Z'$ (for instance, discussed in Refs. [13]). In general, it gives possibility to search for other heavy virtual states that is the problem for future.

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Figure 1: The tree-level amplitude of the process $\bar{f}_1 f_1 \rightarrow Z' \rightarrow \bar{f}_2 f_2$.

Figure 2: The one-loop level contributions to the polarization operator of neutral vector bosons $\Pi^{(1)}_{V_1 V_2}(p^2)$. 
Figure 3: The one-loop level contributions to the vertex function $\Gamma^{(1)}_{\psi V}$.

Figure 4: The $\ln \kappa$-dependent contributions to the one-loop $S$-matrix element describing scattering $f_1 f_1 \to V^* \to f_2 f_2$. 