Profiles quantitative reasoning and students' generalization ability on topic of direct proportion

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Abstract. This research was qualitative that aims to described the quantitative reasoning and students' generalization ability in direct proportion. This research involved 35 grade VIII junior high schools; one of them with moderate math skills was chosen as the main subject. The data collected through the test, assignment, and task-based interviews. The results of this research showed that a generalization that was built by the subject begins with forming a relationship between two or more quantities by establishing ratio of the quantity which appears linking the two initial quantity, doing repetition to find the elements of common ground and further developed the pattern or relationship into a more general form of structure. The quantitative reasoning which belonged to the subject greatly affects the generalizations made. The results of this research are important as an input material for teachers, to develop a strong generalization in General, and then the student must construct the ratio of the quantity that appears with the linking between the quantities. In order to support the students' ability to perform a generalizing correctly.

1. Introduction

Quantitative reasoning has a very important role in processing generalization of students [1]. In everyday life we frequently encounter problems related to quantity. Quantity is defined as quality of something that is formed of a measurement process [2]. Developing relationships between students’ minds and real-life problems that can be solved mathematically is the main objective of many educators and teachers of mathematics worldwide [3]. In math teaching at schools, student are required to think hard to obtain results when performing calculations, hence, many people think mathematics is difficult. This is because of the lack of attention paid to the development of students' concept of quantity through calculation based on the nature of the object. There should be a way to look into the nature of the object to be calculated. Therefore this goal can be achieved largely by improving students' attitudes toward mathematics. Improvement will occur if students can use quantitative reasoning [3]. It has been defined as identification of quantities, constructing a new quantity, and representing relationships between quantities [4, 5]. Quantitative reasoning also refers to how students conceiving his or her situation which stresses on their cognitive developments [6, 7]. Quantitative reasoning focuses to the mental actions of students conceiving their situations, constructing quantities of their conceived situations, connecting, manipulating, and using that quantity to make their problem feasible.
Quantitative reasoning is an analysis of situations that changes into a structure in relations between quantities and quantitative relationships [8]. Those relations are conceptions of three quantities; two of them are determining the third quantities by using quantitative operations [9]. Like, one quantity related to another quantity, for example, speed is a quantity related to the quantity of distance and time. This aspect of generalization involves investigating various quantities and explanations of relationships that occur between cases in certain situations. In generalization activities, students think to change the quantity based on their activities [10]. It means that someone will try to find a rule that can change certain numbers and look back on whether the rule applies if it is used in the wider context.

The purpose of this study is how to profile quantitative reasoning and the ability to generalize students on a topic of direct proportion. In this project, the profile of quantitative reasoning is a natural and completed description of the method used by people to describe mental activity in analyzing quantity, linking the quantity from a situation, and determining the abilities and procedures which are used to resolve certain problems until taking a conclusion. Generalization ability with focusing on quantitative reasoning is shown by the characterization compiled by Ellis [11]. Three characterizations of generalization actions are: Type 1 Relating, Type 2 Searching, and Type 3 Extending. Relating type is outlined as forming a relationship between two or more problems or quantities, searching type is defined as repeating actions to find similar elements and extending type is defined as developing patterns or relations into a more general structure.

The profile of quantitative reasoning and the ability to generalize on related characteristics can be observed through aspects of how someone mentions similar problems that have been obtained previously with the problem given and mentions the relationship between quantities; searching characteristics are observed through aspects of how a person performs actions repeatedly to detect a fixed relationship between two or more quantities. Repeatedly, they perform procedures to detect relationships, detect the same pattern in the given problem/situation and how someone looks for the same solution or outcome; and the last characteristic of extending is observed through aspects explaining how one acquires equations so that they can be applied to all problems in a situation and mention new situations based on the given situation.

2. Method
This research is a qualitative research. The data are collected by giving a test to 35 students in SMPN 1 Pankep, South Sulawesi, Indonesia. The results of this test are used to categorize based on student’s level abilities Next, the researcher did a test by using a test quantitative reasoning to those 35 students. The quantitative reasoning test (QRT) given is a Mathematical problem that is related to direct proportions. The test of quantitative reasoning is showed like:

**Question QRT 1:** To produce 12 packs of Barongko which tasty and delicious cake, a traditional pastry in PANGKEP requires the following types of ingredients.

| No. | Ingredients       | Measurement  |
|-----|-------------------|--------------|
| 1.  | Kepok Banana      | 10           |
| 2.  | Egg               | 6            |
| 3.  | Sugar             | 300 grams    |
| 4.  | Coconut milk      | 250 ml       |

a. How much ingredients are needed if the pastry owner wants to make 18 packs of Barongko using the same quality and ingredients? Explain!
b. How many coconut milk are needed if the pastry owner wants to make 25 packs of Barongko?
c. How many cakes can be made if the pastry owner wants to make cakes with 10 eggs? Determine other ingredients needed?
d. How much Kepok bananas, eggs, sugar, coconut milk are needed if the pastry owner wants to make each of 24 packs of Barongko, 48 packs of Barongko, and 96 packs of Barongko?
e. How much Kepok bananas, eggs, sugar and condensed milk if you want to make n-Barongko? How much Kepok bananas, eggs, sugar, coconut milk and condensed milk are needed If you want to make 50 packs of Barongko?
Based on the result, the researcher chose one student with moderate level as the subject of this research. The researcher selection is based on their considerations that moderate level can give an overview to the two other groups. Next step, the subject was given a QRT 1 containing a case on how he utilizes his quantitative reasoning and his generalization ability. Besides that, to show subjects’ ability on using quantitative reasoning, the researcher gave the same test to other 34 students. The subjects’ result then analysed based on the theoretical framework of quantitative reasoning and the generalization steps that had been prepared. The researcher also did interviews based on the subjects’ result, while the other 34 students were analysed to get a general picture of this test. Furthermore, the researcher triangulated the data by giving another test (QRT II) similar to the QRT I. So, the researcher got a valid data about quantitative reasoning and generalization abilities from the students. The results then are analysed and used as a basis to interview the subject. The first and second analyses are compared to obtain the final data.

3. Results and discussions

3.1. The Description of Students’ Math Ability in Generalizing

The QRT 1 was completed by 35 students. QRT 1 consisted of 6 cases. Generally, those cases were given to look students’ ability in quantitative reasoning and generalizing. Based on the students’ answers, the researchers found sixteen students (45.7%) used generalizing method correctly; twelve students (34.3%) solved the test correctly, but did not use generalizing method correctly and had some mistakes in conceptual and procedure. And seven students (20%) had problems to solve some questions, they are constrained by the concept and strategy they can used. Most of them used the concept of comparison in quantitative reasoning. The most common mistakes were students failed to generalize or looked the general forms.

3.2. The Description of Students’ Quantitative Reasoning in the Step of Relating

Based on the result on QRT 1, the researchers found some activities relating by the subject. First, he built a similar relationship between two or more quantities and related that problem with his conceived situation previously, it’s similar with the situation on QRT 1, producing Barongko with the needed ingredients. Then the subject revealed his problem or situation that is similar to the problem he had, the problem is related to his daily life.

Based on the analysis and interviews above, the subject had ability to relate his understanding with the situation on making Barongko and the ingredients are needed by revealing situation that is similar to the given situation. Then, the subject reasoned the quantities of the cakes with the quantities of ingredients.
are needed and made relations between these quantities. Based on his understanding, the subject generalized this relationship, if production of the cakes increases, the more ingredients are needed.

3.3. The Description of Students’ Quantitative Reasoning in the step of Searching

Some aspects were done by the subject in this step. First, Subject repeated his action to find a stable relation between two quantities or more. He identified a stable relation between quantities for making 12 packs of Barongko. Next, he made the ratio for quantities reasoning that appeared by comparing them. The subject also revealed, to make 12 packs of Barongko the ratio between Kepok banana and eggs were 5:3, kepok banana and sugar were 1:30, etc. Then he emphasized the ratio will remain stable for amounts of cakes are made.

Researcher: You said there were relations between the ingredients, to make 12 packs of Barongko the ratio between Kepok banana and eggs were 10 to 6 or 5 to 3.
Subject: Yes, it was.
Researcher: How much the ratio between Kepok bananas and eggs if the owner wants to make 18, 20, 40 Barongko?
Subject: Same, it is 5 to 3.
Researcher: Why can it be the same?
Subject: To make a tasty and delicious cake, the ratio has to be the same.

The second activities were looking for the same procedure to analyse the relationship between quantities. Question “How many ingredients are needed by the owner to make 18 packs of Barongko with the same quality and ingredients? Explain!”.

First, the subject searched the quantities of cakes and ingredients to find their relationships. Second, the subject compared those quantities to find the ratio for making one pack of Barongko. Next, to find a solution, he used a procedural action by multiplying the quantity of cakes that are made (18 packs) with the quantity of ingredients. Based on analysis of the results, Subject used his quantitative reasoning to find a procedural action to solve his math problems.

The third activities were looking for patterns. The subject identified the relationship between the quantities of Barongko and the ingredients are needed, and then generalized them to find the same pattern. The pattern is known as a recursive pattern which cakes are added to 2-fold, 4-fold, and 8-fold.

So, the subject came with the argument, if the cakes are increased the quantities of ingredients will increase too and this pattern will remain stable. It showed the answer of subject in figure 2:

Figure 1: Subject answers to activity seeking patterns
3.4. The Description of Students’ Quantitative Reasoning in the step of Extending

Based on the analysis on extending step, the generalizations are built based on the activities of searching relationship. Such as the subject of reasoning with the ratio of cakes (18 barongko cake packs) and the ingredients needed to obtain the quantity of ingredients needed for making a barongko cake that would be produced. From this activity, the subject can expand its result, such as, to search how many ingredients are needed to make \( n \)-cake subject procedurally multiplied the \( n \)-cake with the ingredients that have been obtained to make a Barongko. It showed the answer of the subject in figure 2:

\[
\frac{n}{12} \times 10 \text{ bjr pisang kepek} = \left( \frac{5}{6} n \right) \text{ bjr pisang kepek} \\
\frac{n}{12} \times 6 \text{ bhar balar} = \left( \frac{1}{2} n \right) \text{ bhar balar} \\
\frac{n}{12} \times 300 \text{ gram gulai parir} = \left( 2.5 n \right) \text{ gram gulai parir} \\
\frac{n}{12} \times 150 \text{ m. tambun kental} = \left( 10 \frac{5}{6} n \right) \text{ m. tambun kental}
\]

**Figure 2:** Subject answers find a general form

Besides that, the researcher also found other activities from the subject. He not only focuses on its relation but also on the quantities of the pattern, it was shown on subjects’ answer in QRT 1 case d. When subject focused to the numbers, the generalizations that are formed will always related to the previous numbers. For example, subject was asked to determine the ingredients to make 24, 48 and 96 packs of Barongko, he found a recursive pattern. If the numbers of Barongko increased, for example 2-fold, 4-fold, and 8-fold, the ingredients will also increase in the same pattern.

Then, another question was given to the subject, “*how much ingredients are needed to make 192 packs of Barongko*?” To solve this question, Subject divided the quantities of Barongko (192 packs) with 12, then its result was multiplied with the known ingredients. This was a new way of generalizations focusing on the patterns of numbers.

On the step of applying new situations, the researchers gave a question “*Based on the test you have, can you make a question similar to this case?*” Subject knew that question is related to his experience in his daily life, so, he wrote some questions with the same concept and material, and then explained the similarity of those questions and how to solve them.

The results of this study, it highlights that giving a few quantitative situations (QRT 1), delivering generalizations that include connecting aspects, looking for the same relation, and expanding its findings for making general rules. The profile of the above research results is summarized as in table 1:

| General Type | Subject Activities |
|--------------|--------------------|
| **Relating** | • Subjects express problems or situations that are similar to the problems faced; the problem is related to problems in daily life.  
• The subject makes a relationship between the quantity of cakes (12 cake packs) and the quantity of ingredients. |
The results of the study above showed that students in developing mathematical concepts which he owns through quantitative reasoning were not through a procedure's algorithm [12], such as the ability to connect to one another for quantity, to produce problems solving faced then perform the procedure. This suggests that the giving issue/situation that rich quantitatively to the students showed that students can identify relationships, create relationships, viewing patterns, and make the generalization that all means quantitatively. In accordance, the results of Ellis research [13] states that if the students were given a problem that his focus is only on the pattern of numbers, then the students will perform the generalization related to patterns, rules, and procedures only. This is not in line with the opinion of [14,15] that put students in situations that quantitatively rich does not guarantee that they will create a good generalization. Therefore, according to Ellis [16] gives the kind of situations that used to explore one's quantitative reasoning is important, where we need to be careful and consider the kinds of issues/situations given. Although quantitative relationship with reasoning can support a better Math activity, students who fail to make new mathematical objects, such as a ratio that appears, may not get the added benefit though they already focus on quantity.

4. Conclusion
Research reveals that the profile of quantitative reasoning and the ability to generalize students on the type of relating is, the subject can reveal problems or situations that are similar to the problems faced and the subject uses quantitative reasoning such as making the relationship between the quantity of cakes (12 cake packs) with the quantity of ingredients and relationships between quantity of material. In the type of searching, the subject uses quantitative reasoning by finding a fixed relationship, namely the relationship between materials, looking for inter-quantity relationships and then carrying out calculations that focus on the quantity and the relationship between quantities to arrive at the solution or results obtained. The subject found a pattern that is a recursive pattern, and got a solution. In the last type, the extending type of the subject takes repeated actions to get general equations and can reveal new situations that are similar based on the given situation. Subjects who use quantitative reasoning prove evidence of generalization that is affected to more effectively, create the right conclusions, and produce the right justification assertion. Quantitative reasoning can help the students focus on quantity and quantitative relationships to produce generalizations which are true, strong, and connected with other knowledge.

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