Application of the compress sensing theory for improvement of the TOF resolution in a novel J-PET instrument

Abstract. Nowadays, in positron emission tomography (PET) systems, a time of flight (TOF) information is used to improve the image reconstruction process. In TOF-PET, fast detectors are able to measure the difference in the arrival time of the two gamma rays, with the precision enabling to shorten significantly a range along the line-of-response (LOR) where the annihilation occurred. In the new concept, called J-PET scanner, gamma rays are detected in plastic scintillators. In a single strip of J-PET system, time values are obtained by probing signals in the amplitude domain. Owing to compressive sensing (CS) theory, information about the shape and amplitude of the signals is recovered. In this paper, we demonstrate that based on the acquired signals parameters, a better signal normalization may be provided in order to improve the TOF resolution. The procedure was tested using large sample of data registered by a dedicated detection setup enabling sampling of signals with 50-ps intervals. Experimental setup provided irradiation of a chosen position in the plastic scintillator strip with annihilation gamma quanta.

Key words: compressed sensing • positron emission tomography • time of flight

Introduction

Positron emission tomography (PET) [1] is currently one of the most perspective techniques in the field of medical imaging. PET is based on the fact that the electron and positron annihilate and their mass is converted to energy in the form of two gamma quanta flying in the opposite directions. The two gamma quanta registered in coincidence define a line referred to as line of response (LOR). The image of distribution of the radionuclide is obtained from the high statistics sample of reconstructed LORs. In time of flight (TOF) PET systems [2, 3], the applied detectors measure the difference in the arrival time of the two gamma rays that enables to shorten significantly a range along the LOR used for the reconstruction of the image.

Currently, all commercial PET devices use inorganic scintillator materials, usually lutetium oxyorthosilicate (LSO) or lutetium-yttrium oxyorthosilicate (LYSO) crystals, as radiation detectors. These are characterized by relatively long rise and decay times of the order of tens of nanoseconds. The J-PET collaboration investigates a possibility of construction of a PET scanner from plastic scintillators, which would allow for simultaneous imaging of the whole human body. The J-PET chamber is built out from long strips forming the cylinder [4, 5]. Light signals from each strip are converted into electrical...
signals by two photomultipliers (PM) placed at opposite edges. It should be noted that the better the time resolutions of the detection system, the better is the quality of a reconstructed images.

In this paper, we investigate the TOF resolution of a novel J-PET scanner and show a simple method to improve the results by applying the compressive sensing (CS) theory. In the following, we define the time resolution and present shortly the method of signal normalization based on CS theory. Then we describe an experimental setup used for signal registration and present results of improving TOF resolution in 30-cm-long plastic scintillator strip, read-out on both sides by the Hamamatsu R4998 PMs. Signals from the PMs were sampled in 50-ps steps using the Lecroy Signal Data Analyzer 6000A.

Materials and methods

Definition of time resolution

In the following, we are interested in the determination of moment of interaction of gamma quantum with a strip. The interaction moment \( t_{\text{hit}} \) is given by

\[
\text{hit} = \frac{t_1 + t_2}{2} - \frac{D}{2v_{\text{eff}}}
\]

where \( t_1 \) and \( t_2 \) are the arrival times to the PM1 and PM2, respectively; \( D \) is the length of whole strip; and \( v_{\text{eff}} \) is the effective speed of the light in used scintillator. In the recent work [6], the speed of the light in the scintillator was estimated to be 12.6 cm/ns.

In order to determine the resolution (standard deviation) of \( t_{\text{hit}} \) determination, an indirect method based on the estimation of the resolution of time difference \( \Delta t = t_2 - t_1 \) will be provided. We assume for the sake of simplicity that \( v_{\text{eff}} \) in Eq. (1) is known exactly. As the time difference \( \Delta t = t_2 - t_1 \), we have

\[
\sigma^2(\Delta t) = \sigma^2(t_1) + \sigma^2(t_2)
\]

and the resolution of \( t_{\text{hit}} \) based on Eq. (1) may be expressed as

\[
\sigma^2(t_{\text{hit}}) = \frac{\sigma^2(t_1) + \sigma^2(t_2)}{4} = \frac{\sigma^2(\Delta t)}{4}
\]

which implies that the resolution of the determination of interaction moment \( t_{\text{hit}} \) is twice better than the resolution of the time difference \( \Delta t \).

Experimental setup

A necessary data to carry out the research has been acquired by a single module of the J-PET detector [6]. The scheme of experimental setup is presented in Fig. 1. The 30-cm-long strip was connected on two sides to the PMs. The radioactive \(^{22}\text{Na}\) source was moved from the first to the second end in steps of 6 mm. At each position, about 10 000 pairs of signals from PM1 and PM2 were registered in coincidence with reference detector. The signals were sampled using the scope with a probing interval of 50 ps.

Compressive sensing framework

As the shape and amplitude of signals are predominantly related with the hit position, further improvement of the resolution may be provided by
the analysis of the full-time signals. According to the theory of CS [8, 9], a signal that is sparse in some domain can be recovered based on far fewer samples than required by the Nyquist sampling theorem. In the recent articles [10–12], we have proposed a novel signal recovery scheme based on the CS method and the statistical analysis that fits to the signal processing scenario in J-PET devices. Under this theory, only a recovery of sparse or compressible signals is possible. In articles [10, 11], the sparse representation of signals was provided by the principal component analysis (PCA) decomposition. We will not describe all the steps of signal processing here, but we just state that the recovery of full-time signals based on eight samples is very accurate. For further details about signal recovery scheme in the J-PET framework, the interested reader is referred to Refs. [10–12].

The application of CS theory enables to take an advantage from fully sampled signals and opens an area for completely new algorithms for the estimation of the values of times \(t_1\) and \(t_2\) (see Fig. 2). In the following, we use the recovered signals to provide the signal normalization.

**Method of signal normalization**

Owing to the low detection efficiency of the plastic scintillators (and interaction of gamma quanta predominantly via Compton effect), low number of photons reach the PMs and the charge, as well as amplitude of signals is subject to a large variations. However, the shapes of the signals are highly related with the position of the interaction. In order to improve the time resolution, a method of signal normalization that permits to decrease the smearing of signals charge is proposed. The procedure of signal normalization is as follows.

Consider \(L\) data sets representing charges of signals at PM1 and PM2 gathered for \(L\) positions. The mean values of the charges at positions along the strip \(i = 1, \ldots, L\) at PM1 and PM2 will be denoted by \(Q_{m1}(i)\) and \(Q_{m2}(i)\), respectively. Furthermore, the standard deviation of charges at each position along the strip \(i = 1, \ldots, L\) at PM1 and PM2 will be denoted by \(Q_{s1}(i)\) and \(Q_{s2}(i)\), respectively.

Suppose that new pair of signals \(s_1\) and \(s_2\) have been recovered based on time samples registered at four amplitudes levels at PM1 and PM2, respectively. The charges of the signals, \(Q_1\) and \(Q_2\), are calculated as an integrations of the \(s_1\) and \(s_2\) functions, respectively. The proposed normalization procedure qualifies a new measurement, represented by \(Q_1\) and \(Q_2\), to the \(j\)-th position:

\[
(4) \quad j = \arg \min \left( (Q_{m1}(i) - Q_1)^2 + (Q_{m2}(i) - Q_2)^2 \right)
\]

Next, the recovered signals \(s_1\) and \(s_2\) are normalized according to the formula:

\[
(5) \quad \begin{align*}
    s_{n1} &= s_1 \cdot \frac{Q_{m1}(i)}{Q_1} \\
    s_{n2} &= s_2 \cdot \frac{Q_{m2}(i)}{Q_2}
\end{align*}
\]

where \(s_{n1}\) and \(s_{n2}\) denote the normalized signals.

**Experimental results**

In the first step of the analysis, the charge distributions of signals were investigated. Experimental results based on the signals registered along the scintillator strip are presented in Fig. 3. Mean values of charges at PM1 and PM2 are marked with solid blue and red curves, respectively. As expected, the curves are symmetrical with respect to the centre of the scintillator strip (position of 15 cm). The distributions of \(Q_{m1} \pm Q_{s1}\) and \(Q_{m2} \pm Q_{s2}\) at PM1 and PM2 along the strip are marked with dashed blue and red curves, respectively. From Fig. 3, one may observe that \(Q_{m1}\) and \(Q_{m2}\) have the same trend as \(Q_{m1}\) and \(Q_{m2}\), respectively, and are in the range from about 6 to 20 pC.

**An example of the signal normalization**

Figure 4 shows an example of the normalization of signals registered at PM1 at fixed position 5 cm from PM1. The three corresponding signals registered at second PM (PM2) are not shown here but were used during the normalization process to estimate the position of irradiation (see Eq. (4)). Figure 4a shows a three randomly selected raw signals registered via scope. Figure 4b presents the same signals after the normalization procedure provided according to the description given in section ‘Method of signal normalization’. The same colors of the signals in Fig. 4a and 4b indicate a corresponding pair of signals before and after normalization. As it is seen from Fig. 4a, the shapes of the three signals are similar but the charges differ. However, an estimated positions according to Eq. (4) were found to be very close, and therefore, the charges of the normalized signals were also very similar (see Fig. 4a).

**Time resolution of the event time reconstruction**

The normalization method was verified using signals from all the irradiation positions. Each pair of signals was recovered via CS method based on the information from four amplitude levels (see Fig. 2) and then normalized according to the description given in section ‘Method of signal normalization’. In order to compare the time resolutions before and after normalization process, the normalized signals, \(s_{n1}\) and \(s_{n2}\), were sampled in the voltage domain at four levels from 50 to 300 mV (see Fig. 2). For each position and at each level, the distribution of time difference was calculated.

In Figs. 5 and 6, the resulting resolutions (\(\sigma(\Delta t)\)) are presented as a function of irradiated position, determined when applying the lowest threshold (50 mV) and the highest threshold (300 mV), respectively. In Figs. 5 and 6, the resolutions obtained based on the raw and normalized signals are indicated with blue and red squares, respectively. As expected, because of the time walk effect, the resolution determined when applying threshold at 50 mV is better with respect to the resolution ob-
obtained at 300 mV. However, the influence of time walk effect is more visible in the case of raw signals. From Figs. 5 and 6, one can infer that the time resolution is almost independent of the position of irradiation. An average resolution of the time difference along the strip at the lowest tested amplitude level (Fig. 5) was determined to be 172 and 160 ps for the raw and normalized signals, respectively. This corresponds to the improvement of the resolution of the moment of the interaction from 86 ps to about 80 ps.

Conclusions

In this paper, the concept of signal normalization in a novel J-PET scanner was introduced. J-PET device is based on plastic scintillators and, therefore, is a promising solution in view of the TOF resolution. In the related works [10, 11], it was shown that CS theory can be successfully applied to the problem of signal recovery in a J-PET scanner. The information from fully recovered signals was used in order to provide the normalization of the signals. It was shown that with fully recovered signals, a better time resolution of J-PET scanner is achieved; the resolution of the moment of the interaction was improved from about 86 to 80 ps. It should be stressed that different approaches for using the recovered information from CS theory may be considered and the studies are in progress.

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