Small-Angle Measurement of Laser Beam Steering Based on Total Internal-Reflection Effect

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Abstract. A precision small-angle measurement system is introduced based on optical total internal reflection effect. When the incident angle of a polarized beam is larger than the critical angle, and the beam has a small steering angle in the vicinity of the critical angle, the total internal reflection will take place and a relative phased shift between the s- and the p-polarized components is produced. By utilizing the characteristics we could measure the small angle displacement of the incident beam. A differential optical system including four right-angle prisms is set up based on above, and we take the difference of the interference intensity of s- and p- components divided by their sum as the output signal. A good linearity between the output signal and the angular displacement is proved by the theoretical simulation. The system result and error analysis are given as well. The experiment shows that the measurement range of this system is ±2°, and a resolution of 10” is achieved. This method has many advantages, such as simple configuration, high sensitivity, good linearity and high antinoise ability.

1. Introduction
Liquid Crystal Optical Phased Array (LCOPA) is recently developing fast, and has become the practical technology in agile beam steering because of its prominent characteristics, such as wide birefringence range, wideband, low-voltage driving, and its fabricate process is similar to that of microelectronics. To develop LCOPA, the relationship between the controlling voltage and the beam steering angle should be known exactly, so it is necessary to find out the angle of beam steering in precision.

Angle measurement based on the internal-reflection effect (AMIRE) is a new developed method for small-angle measurement, which is proposed by P.S. Huang in 1992 [1]. It is based on the different phase shift between different polarized components when the internal-reflection occurs. This method has many merits, such as low cost, simple configuration, small bulk, high sensitivity and better linearity compared to traditional optical small-angle measurement instruments, because it does not need to use bi-frequency laser. Since this method is presented, some scholars provide new different schemes [2–6]. According to present publications, a few domestic authors involve this research field; in fact, there is still useful work to do for the practical application.

This paper plans to measure the small angle of laser beam steering based on total internal-reflection effect. It aims at measuring the steering characteristics of the LCOPA to obtain the relationship of the control voltage versus optical beam steering angle. Besides small-angle measurement, these methods
can be applied to machine tool calibration, surface profile measurement of mirrors, calibration of polygon mirrors, and the development of an atomic force microscope and a nanoradian angle sensor.

In Section 2 we describe the principle of the measurement method. In Section 3 we present the experimental setup and the experimental data, and the conclusions are given in Section 4.

2. Principle
When a light beam propagates from the optically denser medium to the optically less dense medium, if the incident angle is greater than the critical angle, the light will totally reflect to the optically denser medium, without any transmitted to the optically less dense medium. This is called total internal reflection effect. Figure 1 shows how the total internal reflection occurs when light passes through a right-angle prism. The incident light is 45° polarized from the incident plane and the incident angle is greater than the critical angle at the hypotenuse face. According to Fresnel’s law of reflection and refraction, the s- and p-polarized components have different phase shift under the condition of total internal reflection. The relative phase shift between them is expressed as

\[
\phi = 2\tan^{-1}\left(\sin^{2}\theta_i - \frac{1}{n^2}\right)^{1/2} \tan \theta_i \sin \theta_i
\]

Where \(\theta_i\) is the incident angle at hypotenuse face and \(n\) is the relative refraction index.

Figure 2 shows the \(\phi - \theta_i\) curve when \(n=1.51509\). It can be seen that, in the vicinity of the critical angle (41.3°), \(\phi\) is highly sensitive to changes of \(\theta_i\). It is a good idea to receive the phase shift directly by the phase meter; therefore we could know the incident angle \(\theta_i\). Whereas the price of the phase meter is expensive, the linearity of the measurement data versus angular displacement is not satisfied. We let the s- and the p-polarized components interfere with each other, and detect the interference intensity by an optical detector. To increase the sensitivity and linearity of the measurement, we use two right-angle prisms to double the reflectance times. To decrease the influence of the ambient lights, we design a differential optical paths, see figure 3.

![Figure 1. Laser beam transmitting through a right-angle prism.](image1)

![Figure 2. Relative phase shift between the s- and the p-polarized components.](image2)

A linear polarized light oriented at 45° is used as the incident beam, which could be considered as the combination of s- and p-polarized beams with equal intensity and phase. When the incident beam emits perpendicularly (\(\theta_i=0^\circ\)) at the surface of the non-polarizing beam splitter (NPBS), as the solid line shows, the output beam is split to two beams with equal intensity and polarized state. We denote the transmitted path and the reflected path as optical path 1 and 2, and distinguish them by 1 and 2 subscript, respectively. Optical path 1 and 2 are all composed of a rhombic prism, an analyzer oriented at 45° direction and an optical detector. Rhombic prisms P1 and P2 are all agglutinated by two right-
angle prisms, which result in two total internal reflections occur in each optical path, therefore a higher sensitivity is achieved. There is phase shift between s- and p-polarized components due to internal reflection, so the interference will occur by locating analyzers AN1 and AN2 in 45° direction, and the interference intensity is detected by PD1 and PD2 respectively. The value of the intensity could be read directly by the dual-channel power meter connected to them, so it is a real-time measurement.

![Diagram of optical path](image)

**Figure 3.** Differential optical path of angle measurement.

We take optical path 1 as the example to discuss the interference intensity. Assume that the intensity of the incident light is $I_0$, and then the intensity of the transmitted beam should be $I_0/2$. As the s- and p-components have equal intensity, so we have $I_S=I_p=I_0/4$. When the optics is collimated, shown as the solid line, the initial incident angle at the hypotenuse face of the first right-angle prism $\theta_{i1}$ is 45°, after two internal reflections the phase shift between s- and p-polarized components according to Eq. (1) should be:

$$\phi_1 = 4 \tan^{-1} \left( \frac{\left( \sin^2 \theta_{i1} - 1/n^2 \right)^{1/2}}{\tan \theta_{i1} \sin \theta_{i1}} \right)$$  \hspace{1cm} (2)

When s- and p- components reach AN1, they will interfere with each other in 45°direction, and the interference intensity detected by PD1 could be expressed as:

$$I_1 = (I_0/4)(1 + \cos \phi_1)$$  \hspace{1cm} (3)

In the same way, assume that the incident angle at the hypotenuse face of the optical path 2 is $\theta_{i2}$, the corresponding phase shift $\phi_2$ and the interference intensity $I_2$ could be expressed as follows:

$$\phi_2 = 4 \tan^{-1} \left( \frac{\left( \sin^2 \theta_{i2} - 1/n^2 \right)^{1/2}}{\tan \theta_{i2} \sin \theta_{i2}} \right)$$  \hspace{1cm} (4)

$$I_2 = (I_0/4)(1 + \cos \phi_2)$$  \hspace{1cm} (5)

With angular displacement $\Delta \theta=0$, the optical path 1 and 2 are the same, so $\theta_{i1}=\theta_{i2}=45°$, $\phi_1=\phi_2$, $I_1=I_2$.

Once the incident beam has an angular displacement $\Delta \theta$, shown as the dashed line in Figure 3, the two output beams from NPBS will change. The incident angle $\theta_{i1}$ at the entrance face of P1 increases from 0 to $\Delta \theta$, and the incident angle $\theta_{i2}$ at the entrance face of P2 changes from 0 to $\Delta \theta$, and we could have the incident angles at the hypotenuse face respectively by Snell’s law:

$$\theta_{i1} = 45° + \sin^{-1} \left( \frac{\sin \Delta \theta}{n} \right)$$  \hspace{1cm} (6)
\theta_2 = 45^\circ - \sin^{-1}\left(\frac{\sin \Delta \theta}{n}\right) \tag{7}

Substitute Eq. (6) and Eq. (7) into Eq. (2)~(5), we could obtain the interference intensity with the angular displacement \(\Delta \theta\). We intend to find a system output which is not only insensitive to the ambient light but also linear to the angle measurement. Finally, the signal \(S\) expressed below is the satisfied output:

\[ S = \frac{I_1 - I_2}{I_1 + I_2} = \frac{\cos \phi_1 - \cos \phi_2}{2 \cos \phi_1 + \cos \phi_2} \tag{8} \]

From this equation we build a mathematical relationship between the output \(S\) and angular displacement \(\Delta \theta\) indirectly. The simulation in MATLAB shows that a good linearity is obtained at a measurement range of \(\pm 5^\circ\).

3. Experiments

3.1. Experimental setup. The theoretical diagram of the experimental setup is shown in Figure 4 [9]. It can be divided into three sections: light source, beam steering optics and signal processing. Light source is composed of He-Ne laser, polarized beam splitter (PBS), aperture and polarizer. The type of He-Ne laser is HNK250, wavelength is 632.8nm, output power is about 3mW, and the power stability is less than \(\pm 0.19\%\). The PBS is oriented at 45° from the X axis, which turns the natural light to 45° direction polarized light. The extinction ratio is up to 10:1, which can be considered appropriate. The aperture is used to observe the return light and to collimate the system. The polarizer is used to adjust the output intensity of the two paths. Beam steering section is carried out by putting the differential optical path mentioned above onto a rotary table, whose resolution is 0.012°. PL1 and PL2 are inserted by the intention of compensating the fabrication error of the NPBS, and they are all fixed at 45° from the X axis. The last section is signal processing. The intensities \(I_1\) and \(I_2\) detected by PD1 and PD2 are sent to dual-channel power meter 2832-C. 2832-C can not only display the two intensity synchronously, but also send these data to a computer for further processing through a serial cable. After receiving the measurement data, we could display them in a GUI program and analyze them with other assistant tools. According to the optical path described above, we build a corresponding experimental system, and design a practical data-acquisition interface using virtual instrument graphic software Labview. The objective diagram of the system is shown in Figure 5.

Figure 4. Theoretical schematics of the experimental setup. Figure 5. Objective diagram.

3.2. Experimental data. Figure 6 shows the measurement data obtained under the condition of dark environment and a total incident intensity of \(I_0=95.72\mu\text{W}\) is used. The dotted line is all acquired by the measurement system, and the un-dotted line is the simulated result by computer software.
From subfigure (a) we could see that $I_1$ and $I_2$ decreases and increases with the step of $\Delta \theta$. Their difference decreases with the step of $\Delta \theta$, and their sum keeps almost a constant. Because a whole measurement would take about 15 minutes, the sum and difference signal appear periodic fluctuation overall the measurement range. Subfigure (b) indicates that the output signal $S$ has the similar waveform with the difference signal, by adding the fluctuation of the sum signal, $S$ is more flexuous. The slope of the fitting curve of $S$ is -0.058177, and the theoretic slope is -0.058, the error is 0.305%. If the output power of the laser source has higher stability, the fluctuation of the output should be restrained effectively and the measurement solution is improved dramatically.

![Graph](image)

(a) The two intensities and their sum and difference.  
(b) Output $S$ and the linear fit $S_{fit}$.

**Figure 6.** Comparison between measurement data and theoretical data.

According to the steering characteristics of the LCOPA, the 256×256 model manufactured by BNS Co. steers at $\pm 1.5^\circ$, so the measurement range and precision of the system meet our needs.

### 4. Conclusion

This paper describes a beam steering angle measurement method based on total internal reflection. It includes the principle derivation and experimental setup. Experimental data is given and validates the theoretical analysis, which confirms the reliability of the system. The measuring resolution is $10^\circ$, and measurement range is proved to be $\pm 2^\circ$. It can be encapsulated to portable angle-measurement instrument after carrying out successfully on the optical base. It will be simple to operate and convenient to use.

### References

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