STOCHASTIC PIONISATION IN HOT QUARK-GLUON MATTER

Jan-e Alam$^{1,a}$, Abhijit Bhattacharyya$^{1,b}$, Sanjay K. Ghosh$^{2,c}$, Sibaji Raha$^{2,d}$ and Bikash Sinha$^{1,3,e}$

$^1$ Variable Energy Cyclotron Centre, 1/AF, Bidhannagar, Calcutta- 700 064, India

$^2$ Department of Physics, Bose Institute, 93/1, A.P.C. Road, Calcutta 700 009, India

$^3$ Saha Institute of Nuclear Physics, 1/AF, Bidhannagar, Calcutta- 700 064, India.

We present a microscopic approach to dynamical pionisation of the hot quark-gluon matter formed in ultrarelativistic heavy ion collisions. The time evolution of the system is described assuming that quarks undergo Brownian motion in a thermal bath provided by the gluons. The rate of hadronisation as well as the time dependence of the temperature of the system are seen to be quite sensitive to the QCD Λ parameter. Even in a non-equilibrium scenario, we find that there appears a clear hint of a first order phase transition.

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Recently there have been some attempts to study the formation of hadrons in quark matter using different semi-microscopic approaches. These studies can be characterized either as model dependent calculations [8], or the computer codes based on the string phenomenology [9], or other phenomenological description of hadronization [1]. None of these approaches account for the essential lack of equilibrium in the quark-gluon phase. Some efforts have also been made to estimate hadronization within the parton cascade model by introducing a cut-off to mimic the non-perturbative effects [10]. To the best of our knowledge, the first study aimed at investigating the dynamical process of hadronization in a non-equilibrated quark-gluon system from a physically transparent approach was in [11]. In this letter, we present the details of this scenario describing the evolution of the non-equilibrated hot quark-gluon matter towards hadronization. For simplicity, we have restricted ourselves to the case of pions only in the present work; since the pions account for the bulk of the multiplicity, the conclusions should remain valid not only qualitatively and even semi-quantitatively.

We have earlier shown [11,12] that perturbative estimates of the gluon-gluon, quark-gluon and quark-quark cross sections allow us to study the evolution of the quark-gluon matter formed in ultrarelativistic heavy ion collisions by visualizing the quarks as Brownian particles in a hot gluonic thermal bath. In this work, we start with such a premise, which, in our opinion, describes the non-equilibrium aspects of the evolution in a physically transparent manner.

Hadronization in such a system can be studied in the light of Smoluchowski’s theory of coagulation in colloids, which was elaborated further by Chandrasekhar [13]. This theory suggests that coagulation results as a consequence of each colloidal particle being surrounded by a sphere of influence of a certain radius $R$ such that the Brownian motion of a particle proceeds unaffected only so long as no other particle comes within its sphere of influence. When another Brownian particle does come within a distance $R$ of the test particle, they form a two body cluster. This cluster also describes a Brownian motion but at a reduced rate due to its increased size/mass. The process continues further till a single cluster of all the particles is formed.
In order for this stochastic scenario of cluster formation to apply to a system of Brownian quarks in the hot gluon bath, it is essential that each quark has an appropriate sphere of influence of radius $r$. Obviously, this radius $r$ will depend on the spin-isospin combination of the final cluster (whether the final cluster is scalar, pseudoscalar, vector or axial vector meson or even a baryon). Mesons are formed when one quark and one antiquark with proper quantum numbers come within the spheres of influence of each other. (Clusters with greater numbers of quarks and antiquarks (e.g. baryons) can also be formed by imposing the conditions of colour neutrality and charge balance properly.) This implies that the radius of the sphere of influence corresponds to the correlation length between the quarks in the proper hadronic channel. In other words, this is the screening length of the corresponding hadrons in the hot quark-gluon matter. For the present purpose, the radius corresponds to the following equation:

$$r = \frac{2\pi}{\sqrt{2\pi}} \equiv \frac{2\pi}{\sqrt{2\pi}}$$

The running coupling constant $\alpha_s$, as well as $\gamma$, is given by

$$\alpha_s = 12\pi \left( \frac{33 - 2n_f}{33 - 2n_f} \right)$$

In eqs. (1), the first term is the rate of pion formation (both $a$ and $b$); the second term is the rate of pions decaying back to quarks and the third term is due to Bjorken (longitudinal) expansion of the system. The $\epsilon_i$ stands for $u$ or $d$ (we ignore the other heavier flavours). The average relative velocity in the radial direction is calculated using the Juttner distribution,

$$f(x, p) = e^{-\beta p \cdot u(x)}$$

There would also be a corresponding rate equation for the antiquarks, which looks exactly like eq. (2) and hence not explicitly written. In eq. (3) the $\Gamma_{\pi \to q \bar{q}}$ and $\Gamma_{\pi \to gg}$ stand for the corresponding net quantities.

As mentioned earlier, we are considering a non-equilibrated quark matter and hence the pions formed will also be out of equilibrium. This is taken into account by multiplying the relevant distribution functions with the ratios $\frac{r_q}{n_q}$ as $\frac{n_\pi}{n_{\pi}}$ where $n_q$ and $n_\pi$ are non-equilibrium and equilibrium densities of quarks and pions. The details are given in (3).

In all these expressions, the appropriate masses are the effective masses including the current as well as thermal contributions, whose importance in determining the dynamics of the hot quark matter has been well established. For quarks (antiquarks), this is

$$m_{\text{eff}} = \sqrt{m_q (\text{curr})^2 + m_q (\text{thermal})^2}$$

where $m_q (\text{thermal}) = (1 + \frac{r_q}{2}) \frac{g_v T}{3}^2$ (5)

and $m_q (\text{curr})$ is taken to be 10 MeV. For gluons the thermal mass is,

$$m_g (\text{thermal}) = \frac{2}{3} g_v T$$

The running coupling constant $\alpha_s$ as a function of temperature is given by

$$\alpha_s = \frac{12\pi}{(33 - 2n_f) \ln \left( \frac{Q^2}{\Lambda^2} \right)}$$

with $Q^2 = m_{\text{eff}}^2 (T) + 9T^2$.

Simultaneously, we must take account of energy momentum conservation which, for a Bjorken flow, corresponds to the following equation

$$\frac{\partial \epsilon}{\partial t} = - \epsilon + P$$

where $\epsilon \equiv \epsilon_{\text{total}} = \epsilon_q + \epsilon_g + \epsilon_\pi$. We also include the one loop correction to $\epsilon_g$ (3). $\epsilon$ and $P$ are related through the
velocity of sound, as in [5]. For a complete description of the system, eqs. (1), (2), (3) and (8) must be solved self-consistently. The initial conditions are taken from [5] for RHIC energies. The initial time \( t_g \) is the time when gluons thermalise (=0.3 fm), where \( r_q = 0.15, r_g = 1 \) and \( r_\pi \) is taken to be 0. The temperature at this time is 500 MeV. The pion decay width, dynamical mass and screening mass are taken, as already mentioned, from the NJL model [12,14]. Note that we are working at \( y = 0 \) so that \( t \) and \( \tau \) are the same and the baryon chemical potential is zero.

Figure 1 : Time evolution of quark and pion number densities for various values of QCD parameter \( \Lambda \).

Figure 1 shows the variation of pion and quark number densities with time, for various values of the QCD parameter \( \Lambda \). In all three cases, we find the same qualitative feature that pions start appearing in the system quite early on but they become appreciable in number only after some time. At late times the system is dominated by pions. This cross over occurs at \( t \geq 4 \) fm for \( \Lambda = 0.2 \) or 0.3 GeV while for \( \Lambda = 0.4 \) GeV this happens at \( t \sim 6 \) fm.

Figure 2 shows the variation of temperature with time. Obviously, there is a dramatic effect of the QCD parameter \( \Lambda \). In all the cases, there is a change at \( T \sim 215 \) MeV, corresponding to \( t \sim 3.5 \) fm; the variation of temperature with time becomes slower, as is expected in the mixed phase of a first order phase transition. At \( \Lambda = 0.2 \) GeV, this occurs for a very short period of time, before the system starts cooling again. The duration of the constant temperature configuration increases with \( \Lambda \), and for \( \Lambda = 0.4 \) GeV, it persist upto 9 fm before the temperature of the system starts falling again.

Figure 2 : Time evolution of the Temperature for various values of QCD parameter \( \Lambda \).

Obviously, this is a clear indication of an apparent first order transition. Microscopically, the appearance of the mixed phase at a temperature of \( \sim 215 \) MeV can be understood from the fact that the pion decay width goes to zero at such a temperature [14]. All the pions that were formed earlier in the system tended to decay back to quarks and antiquarks on a fast time scale. Only after the pion decay width becomes small would the formed pions become stable.

The role of the QCD parameter \( \Lambda \) is better understood from figure 2. The higher the value of \( \Lambda \), the higher the quark thermal masses. As a result, the lower the relative velocity, which would lead to a lower rate of pion production as well as a slower depletion of the quark number density. Thus the mixed phase, \( i.e. \) the domain where quarks and pions have comparable densities, would not only occur later in time but also persist for a longer duration. It should however be mentioned that there are also other effects associated with the QCD \( \Lambda \) (like the quark production rate from gluon fusion and/or decay, the gluon energy density and so on) which have com-
peting roles in determining the actual number densities. This may be the reason why one does not notice a drastic difference between \( \Lambda = 0.2 \) and 0.3 GeV in figure 1, while at \( \Lambda = 0.4 \) GeV, the change is more noticeable even in figure 1. A detailed analysis of all these different effects is in progress now.

We must stress the fact that the first order phase transition is only an approximate one. There is no unique demarcation between the different phases. High temperature domains are dominated by quarks with a few pions present in the system, the intermediate region (\( T \sim 215 \) MeV) having comparable numbers of quarks and pions and the low temperature domain being mostly pions with a few quarks. Nonetheless, it can be fairly concluded from these results that the concept of a first order confining phase transition is not a bad approximation for the process of hadronization in QGP. It should also be noted that the so called critical temperature of \( \sim 215 \) MeV, which corresponds to the temperature where the pion decay width vanishes, is a model dependent quantity. In our case this derives from the NJL model, but the qualitative features should of course be model independent.

In conclusion, we have studied, in a physically transparent picture, the dynamical process of hadronization (pion formation) in a non-equilibrated quark-gluon system formed in ultrarelativistic heavy ion collision. Our results show that even in a microscopic analysis, there is a clear indication of an apparent first order phase transition in the system. Nonetheless, the persistence of non-perturbative modes in the high temperature phase seems to be a real possibility, as also the existence of some colour degrees of freedom till lower temperatures. These issues deserve urgent attention in the context of QGP diagnostics.

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