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An optimization framework for COVID-19 vaccine allocation and inventory management: A case study

Jamal Nahofti Kohneh \textsuperscript{a}, Masoud Amirdadi \textsuperscript{b}, Ebrahim Teimoury \textsuperscript{c}\textsuperscript{,}\textsuperscript{*}

\textsuperscript{a} Glenn Department of Civil Engineering, Clemson University, 135 Lowry Hall, Clemson, SC 29634, United States
\textsuperscript{b} Department of Civil Engineering, Lassonde School of Engineering, York University, Toronto, Ontario, Canada
\textsuperscript{c} School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran

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\begin{abstract}
As the novel coronavirus pandemic wreaked havoc globally, governments have implemented massive vaccination programs to tackle it. However, since the pandemic’s emergence moves beyond the second year, some issues have stymied vaccination programs, including vaccine hesitancy, vaccine distribution inequality, new strains of the virus, and a possibility that the virus enters a stage of a requirement for cyclical vaccination. These challenges highlight the need for an appropriate mass COVID-19 vaccination program. Therefore, we attempt to address this problem by developing a bi-objective integrated vaccine allocation and inventory management framework. The goal is to minimize the system costs while maximizing the vaccination service level. Several important factors, such as multiple types of vaccines, the vaccines’ perishability concept, demand uncertainty, and motivational strategy, have been addressed using dynamic planning. Besides that, the model development mechanism is carried out to be compatible and applicable to the current general vaccination program policies, forcing few strategic changes. Then, a case study concerning the vaccination program of the city of Mashhad in Iran is applied to the model. The results demonstrated significant advantages in total cost, vaccine shortage, and wastage compared to the current policy. Finally, the Lagrangian relaxation method is implemented on the model to strengthen further its capacity to handle larger-scale problems.
\end{abstract}

\section{Introduction}

According to Our World Data (OWD) organization, more than 259 million people have contracted the novel coronavirus (SARS-CoV-2) or (COVID-19), and more than 5.7 million people have lost their lives by November 2021 [1]. On top of that, the pandemic has severely damaged many countries’ economic and social sectors. A few such consequences include the massive overload on national healthcare systems, disruption in the global supply chain, increased poverty and inequality, unemployment, and negative psychological impacts of lockdowns and isolations [2–4]. Consequently, governments attempted to curb the spread of the virus through initiatives such as social distancing guidelines, mask mandates in public places, temporary lockdowns, promoting companies to enable remote working, and mass vaccination programs [5,6].

Regarding vaccination programs, some public–private programs empowered by substantial funds and resources sought to accelerate the research and development of vaccines for the COVID-19 virus. Subsequently, a few pharmaceutical companies have accomplished this task and produced effective vaccines in a remarkable timeframe [7]. Two notable endeavors were the United States of America (USA) Wrap Operation program and the Vaccines Global Access (COVAX) program directed by the Global Alliance for Vaccines and Immunization (GAVI). Among the developed types of COVID-19 vaccines, some have received emergency authorization from organizations such as the World Health Organization (WHO), USA Food and Drug Administration (FDA), and European Medicines Agency (EMA), which are listed in Table 1 [8].

The vaccination program’s advantages include a sharp decrease in the number of infected people, less demand for hospitalization, fewer vulnerable groups of people requiring intensive care, decreased risk of transmissibility, and an overall lower number of deaths. They could be tracked in the data collected from countries such as the United Kingdom (UK), the USA, and Singapore [9,10].

However, several issues hinder mass vaccination programs in many countries. One challenge could be associated with the logistics and distribution of COVID-19 vaccines. A cold supply chain is often necessary for COVID-19 jabs transportation and distribution, especially for mRNA-developed types [11,12].
instance, the Singapore government had to build a makeshift air-
freight terminal called SATS Coolport to handle Pfizer-BioNTech
jab deliveries to the country. In this process, vaccines are shipped
from Brussel to Changi Airport in Singapore and stored in an
ultra-refrigerated warehouse with a capacity of 1.5 million doses.

Table 1: Different types of COVID-19 vaccine details.
Source: The official website of WHO, CDC, and the associated pharmaceutical companies.

| Producer company (vaccine name) | General efficacy | Recommended storage condition | Shelf-life | Recommended time to receive the second dose | Development type | Number of countries authorized the vaccine |
|---------------------------------|-----------------|------------------------------|------------|-------------------------------------------|-----------------|-------------------------------------------|
| Pfizer/BioNTech (BNT162b2)     | 95.30%          | −70 ± 10 °C                   | 5–31 days | 21 days                                   | mRNA           | 180                                       |
| Moderna (mRNA-1273)            | 94.10%          | −20 ± 5 °C                    | 30 days   | 28 days                                   | mRNA           | 102                                       |
| Oxford/AstraZeneca (AZD1222)   | 63.10%          | 2–8 °C                        | Up to 6 months | 4 to 12 weeks                             | Adenovirus-based | 182                                       |
| Johnson & Johnson (JNJ-78436735)| 72%             | 2–8 °C                        | 3 to 4.5 months | Not specified                             | Adenovirus-based | 113                                       |
| Sinovac (CoronaVac)            | 67%             | 2–8 °C                        | One year  | 2 to 4 weeks                              | Inactivated SARS-CoV-2 virus | 56                                       |
| Sinopharm (BBBP-CoV)           | 78.10%          | 2–8 °C                        | Two years | 21 days                                   | Inactivated SARS-CoV-2 virus | 85                                       |
| Gamaleya (Sputnik V)           | 91.6%           | −18 °C                        | One month | 21 days                                   | Adenovirus-based | 74                                       |
| Bharat Biotech (Covaxin)       | 78%             | 2–8 °C                        | Six months | 28 days                                   | Inactivated SARS-CoV-2 virus | 14                                       |
| Novavax (NVX-CoV2373)          | 90.4%           | 2–8 °C                        | 6–9 months | 21 days                                   | Protein Subunit  | 40                                       |

*This vaccine has limited production in a solid lyophilized powder form requiring regular refrigerator temperature.

Secondly, since the pandemic has left persistent and whipping
impacts on many departments of societal communities, govern-
ments are starting to move from a zero COVID-19 policy toward
a state of adaptation and co-existing with the virus. In December
2021, Australia, a country with one of the strictest COVID-19
policies, with six lockdowns lasting over 260 days, abandoned its
zero COVID-19 policy. When writing this article, only China re-
mains a major country still adopting the former policy. However,
a critical problem that threatens the policy of co-existing with the
virus is the sprawling vaccine hesitancy. This issue disrupts coun-
tries’ plans to reach herd immunity and slows down reversing
pandemic-era restrictions. For instance, as of November 2021, the
latest and highly transmissible Omicron variant has appeared in
South Africa, a country with persistent vaccine hesitancy among
its citizens.

Moreover, in Fig. 1, it can be noticed that many countries are
facing a stagnation phase in their immunization programs. For
example, in the USA, from May until December 2021, only an
additional 10% of the population got vaccinated, while in Japan,
the same measure was 68%. This phenomenon is primarily asso-
ciated with hesitancy and mistrust that severely hindered proper
and quick immunization programs in many countries, which has
morphed into a pandemic of unvaccinated people. The prevail-
sing sources of vaccine hesitancy, either unjustified or justified,
are associated with socio-demographic factors, comorbidity, risk
perception, lack of information, and misinformation [19,20].

Apart from that, given each country’s particular demographic,
types of available vaccines, information access and circulation,
vaccine distribution efficiency, and other factors, the degree of
people’s resistance toward vaccination differs immensely; nonethe-
less, addressing this problem is vital in overcoming the pan-
demic [21,22].

Hence, there is a growing call among scientists that the COVID-
19 virus cannot be eradicated, and in the future, at best, it will
become less dangerous but more persistent. Therefore, COVID-19
vaccination might become an endemicity (i.e., a cyclical require-
ment) similar to seasonal influenza shots.

Some motivational measures have been implemented in dif-
ferent countries in light of these challenges. For instance, a bonus
cart of 100$ was given to individuals who received their first
COVID-19 shot in New York City [24]. On the other hand, in
December 2021, the Austrian government put a lockdown on
unvaccinated citizens. In Singapore, although 95% of the popu-
lation has received their first shot, the government eliminated
subsidized social services for testing and hospitalization due to
the COVID-19 virus starting from November 2021 [25].

Finally, taking into account the unprecedented measures car-
ried out to finance and facilitate the vaccination programs, which
are evident in the form of free-of-charge vaccines available for
people in most countries, it is unlikely that the current generous
and accessible form of vaccination programs will last forever.
Thus, we argue that the overall demand for the COVID-19 vaccine gradually decreases as more people get their shots, but the vaccination systems do not become unnecessary and redundant any time soon. On the contrary, a steady and stable vaccination form should remain in place for years ahead.

Since the pandemic has had a widespread impact in most countries, a quick fix to the cold supply chain issues and infrastructure shortcomings might not be possible, at least for a short time. However, it does not eliminate the urgent need to develop a strategy to optimize immunization programs. This research proposes a bi-objective dynamic integrated allocation and inventory management model to address the COVID-19 vaccine distribution problem. Apart from optimizing the system’s vaccination expenses, the model also attempts to maximize vaccine service levels according to a vulnerability prioritization at medical centers to provide an effective and reliable framework. To the best of our knowledge, there is no approach available in the related literature that tackles the problem from the perspective undertaken by this study.

The following section encompasses an overview of the related literature concerning vaccine allocation and inventory management. Section 3 describes the detail of the problem and presents the proposed mathematical modeling. Section 4 explains the processes of model linearization and defuzzification. Section 5 applies the case study to the model and evaluates the results. In Section 6, the solution methodology of Lagrangian Relaxation is presented. Finally, Section 7 discusses the conclusion and recommendations for possible future endeavors.

2. Literature review

The previous coronavirus (SARS-CoV-1) appeared in 2003, and compared to SARS-CoV-2, it affected only a handful of countries and got under control relatively quickly. On the scale of the current pandemic, it requires us to go back to the 1930s when the influenza strain (i.e., the Spanish Flu) took the life of more than 20 million people [26]. This colossal time gap has led to limited and sparse attention on this topic. However, there are some notable articles in vaccine allocation and inventory management research areas in the last few years that we discuss here. Hovav and Tsadikovich [27] developed a framework for inventory and distribution management of the influenza vaccine. Lemmens et al. [28] reviewed integrated supply chain vaccine models comprehensively. Then, Lim et al. [29] sought to improve a vaccine inventory management network by enhancing efficiency and reducing costs. Foy et al. [30] analyzed different COVID-19 vaccine allocation strategies in India through a mathematical modeling framework. MacIntyre et al. [31] proposed a mathematical model for emergency response related to the COVID-19 vaccination program in New South Wales, Australia. They addressed the challenges of the limited supply of vaccines and a vaccination priority level set by the Australian government. Shim [32] investigated the initial vaccine supply in South Korea. The article develops a model for COVID-19 vaccine allocation to reduce the number of infections and deaths concerning different vulnerable groups.

Kumar et al. [33] published a thorough report on the COVID-19 vaccination program in India, encompassing the country's available types of vaccine, infrastructure capacity, and adopted policies. Bertsimas et al. [34] presented a data-driven model to locate vaccination centers adequately. They integrated a state-of-art DELPHI method to estimate the demand and optimize vaccine distribution networks. The information in this article was related to several states in the USA. Hosseini-Motlagh et al. [35] investigated a mechanism to prevent the spread of the COVID-19 virus. They developed a mathematical model to minimize the transmission rate among people and applied it to the case study related to the Tehran capital of Iran.

Concerning COVID-19 vaccination demand, a forecasting fuzzy time series model was developed by Kumar and Susan [36]. They solved the model using a particle swarm optimization method. Sun et al. [37] conducted a simulation-based analysis concerning vaccine distribution in Norway. Çakır et al. [38] presented a mobile facility location for COVID-19 vaccination clinics, which dealt with uncertainty using a spherical bipolar fuzzy method. Singh and Gupta [39] proposed a generalized SIR model for predictive monitoring of the COVID-19 pandemic.

The current pandemic dynamic has overlapping sections with research regarding post-disaster healthcare systems and blood supply chains. This connection derives from the paramount need to save lives. This phenomenon creates a condition where achieving both a reasonable service level and optimizing costs are necessary. These models often contain a perishable product with uncertain demand (e.g., blood unit, platelet, and COVID-19 vaccine). Thus, we decided to add some related disaster studies to this section. In this regard, a review article by Osorio et al. [40] demonstrated a comprehensive analysis of the blood supply chain. Ramirez-Nafarrate et al. [41] proposed a model and solution methodology for point of dispensing location and capacity problems during a public health emergency. Chintapalli [42] sought to incorporate the pricing and inventory management procedures of deteriorating perishable products. Dillon et al. [43] presented a framework for inventory management of a blood supply chain under an $(R, S)$ policy and stochastic demand. Loree and Aros-Vera [44] investigated a humanitarian logistics system for post-disaster concerning inventory management and distribution. Duong et al. [45] researched inventory control of perishable health products. For solution purposes, they implemented an integrated Discrete Event Simulation (DES), Analytic Hierarchy Process (AHP), and Data Envelopment Analysis (DEA). Gholami-Zanjani et al. [46] developed a robust location inventory under disruption for the food supply chain to elevate resiliency and competitiveness. Li et al. [47] designed a plan for emergency relief in the case of secondary disasters through a three-stage stochastic model. Also, an insightful article in this domain is presented by Song et al. [48].

As mentioned, the pandemic has caused catastrophic consequences in many countries. That is why an effective and inclusive government plan for a mass vaccination program is a pivotal decision. Although the vaccine supply chain domain is growing fast, it falls short of presenting a comprehensive, flexible, and resilient strategy to handle current vaccination programs’ challenges [49]. So far, the literature lacks an integrated allocation inventory management model for COVID-19 vaccines that deals with demand uncertainty, jabs perishability, dynamic planning, and motivational strategies. In the following paragraphs, the major contributions of this research are outlined, describing how each part of the proposed model addresses specific challenges of a COVID-19 vaccination program.

- The proposed model is an integrated allocation and inventory management taking advantage of a dynamic $(R, S)$ inventory management for COVID-19 vaccines, which is compatible with a general vaccination program in many countries.
- The model addresses multiple types of COVID-19 vaccines with different shelf-life. The vaccine’s demand is considered a fuzzy variable to deal with the inherent uncertainty. A credibility-based fuzzy method is adopted due to its solid performance in similar studies concerning inventory management of perishable products.
- The model aims to minimize total costs and maximize the service level of vaccination. This process elevates the ultimate rate of vaccination respective to different vulnerable groups of people. Also, the model encompasses a...
motivational strategy derived from governments’ policies. These incentives can be positive (e.g., gift cards) or negative (e.g., payment for regular COVID-19 tests), affecting the system’s vaccination rate.

- The model is applied to a real-world case study related to a vaccination program in the city of Mashhad. The results and analysis underscore the model’s superiority and utility compared to the current policy.
- Finally, the Lagrangian relaxation method is applied to the model to improve its capacity to deal with large-scale problems/case studies.

It is worth noting that in the proposed model, several techniques are used to enhance its applicability in real-world circumstances. Therefore, this study attempts to find a balance between developing a novel and efficient COVID-19 vaccine allocation and inventory management approach while refraining from over-complicating it to maintain a proper practicability gauge. In the next section, the problem details are discussed, and the mathematical formulation of the proposed model is presented.

3. Materials and methods

This article develops a mathematical model to address the current challenges of a COVID-19 vaccination program. Here, the assumption is that a vaccination program is underway, and the proposed model goal is to modify and enhance its performance, which would serve the ongoing circumstances in many countries. Thus, the proposed model does not seek to utterly overhaul or make wholesale changes to the current vaccination program but attempts to accommodate and improve the system to the best shape possible. A typical scheme of a vaccination program adopted by many countries such as Iran, Turkey, Singapore, and India is shown in Fig. 2. This article mainly investigated the program’s two lower echelons (i.e., medical centers and patient zones). Therefore, a dynamic Mixed Integer Non-Linear Programming (MINLP) model is developed for an integrated vaccine allocation and inventory management design.

Consequently, the proposed model determines the location and number of medical centers from a set of potential places, the vaccine inventory policy, and adequate vaccine allocation. We assume several medical centers are available for vaccination purposes. The model determines which and how many of them should operate during a predetermined planning horizon. Also, to elevate the convenience of the vaccination process, we seek to allocate people to their adjacent medical centers to receive their shots. However, this assumption does not eliminate the possibility of traveling to a far medical center to get vaccinated and only adds a penalty of inefficiency and inconvenience to the model. Besides that, the model consists of multiple types of vaccines, patient zones with specific demographic details, a vaccine priority level for different vulnerable groups, and potential incentive strategies. Also, in alignment with real-world circumstances, a vaccination satisfaction index associated with medical experts recommending a particular vaccine type to a vulnerable group or personal preference in getting a specific vaccine is considered.

Concerning vaccine demand in the model, since six months from vaccination rollout in many countries have passed, understandably, the demand rate gradually decreased and became steady. Nevertheless, it contains an inherent uncertainty due to unpredictable behavior from remaining eligible individuals and the emergence of new virus strains. In this article, we use fuzzy mathematical programming to tackle this problem. This method demonstrated effectiveness and reliability in coping with uncertainty surrounding logistics and inventory management problems [50–53].

Additionally, the adopted inventory management approach is an \((R, S)\) method in this article. In an \((R, S)\) inventory management strategy, the system periodically reviews the stock position \(s\) every \(r\) period and compares it to a predefined desirable threshold \(S\), deciding whether to place an order or not [54]. As we mentioned, COVID-19 jabs are a perishable product, meaning they have a limited shelf-life, which expires and should be discarded. Thus, responding to this issue is essential in reducing shortages and costs. The \((R, S)\) inventory method is compatible with necessary elements of a vaccination endeavor because it enables medical centers to put a periodic order to receive jabs and refrains from a Just In Time (JIT) policy that might be more efficient on the surface but given the nature of the problem is not a viable option. It is worth noting that this strategy is straightforward to implement and frequently has been employed in many contexts with solid performance, especially for perishable products [43].

Moreover, there is an assumption for a motivational strategy in the model. As a result of vaccine hesitancy, some governments worldwide introduced measures to incentivize citizens to get vaccinated. Although some strategies, such as gift cards, are easy to consider, others might prove difficult such as requiring unvaccinated individuals to test frequently, but we argue its mere existence in the model and acknowledging its effect on the demand rate is still more beneficial than ignoring it entirely. Therefore, the proposed model has two objectives: minimizing costs and maximizing service levels. Finally, we improve the model tractability by implementing modifications crucial for practitioners employing it.

There are some essential assumptions listed in the below paragraphs.

- The number of regions (i.e., patient zones), population, and demographics are known.
- The number of medical centers assigned for vaccination and their capacity are known.
- Lead time for an order by a medical center is predetermined.
- The shelf life of different types of vaccines is known.
- The priority index relating to different groups of people’s vulnerability to the virus is predefined.

3.1. Sets

\[ T \]  Time periods, \( t, k, u \in T \)

\[ R \]  Review intervals, \( r \in R \)

\[ P \]  Different vulnerable groups of people, \( p \in P \)
3.2. Parameters

- $C_{m_i}$: The operational cost of medical center $j$
- $C_0$: Order cost in period $t$
- $C_{s_{vt}}$: Shortage cost per unit type $v$ vaccine in period $t$
- $C_{w_{vt}}$: Wastage cost per unit type $v$ vaccine in period $t$
- $C_{h_{t}}$: Holding cost per unit type $v$ vaccine in period $t$
- $C$: The cost of motivational strategy for each individual
- $e_{ij}$: Distance between region $i$ and medical center $j$
- $D_{j}$: Fuzzy demand associated with a vulnerable group $p$ in region $i$ for vaccine type $v$ in period $t$
- $A$: Maximum number of potential medical centers
- $\beta_1$: A portion of eligible people who are likely to receive their vaccine without any special incentive
- $\beta_2$: A portion of eligible people who are likely to receive their vaccine if the government adopts a motivational strategy
- $\bar{S}_{jt}$: Maximum capacity to stock vaccines at medical center $j$ in period $t$
- $L$: Lead-time of an order
- $S_h$: Shelf-life of vaccine type $v$
- $\alpha_p$: Denoted priority of vulnerable group $p$
- $C_{p_{cv'}}$: Penalty cost for substituting vaccine type $v$ with type $v'$ ones, e.g., $C_{v,v} = 0$
- $C_o$: Commuting cost for people traveling to a medical center in period $t$
- $\theta_{jt}$: A binary parameter, if a new order by medical center $j$ occurs in period $t$ according to the periodicity of $r$ intervals, 0 otherwise
- $\sigma_{vt}$: Last period that vaccine type $v$ received in period $t$ is usable, i.e., min $\{t + S_h - 1, T\}$
- $\tau_{vt}$: Earliest period that vaccine type $v$ received in period $t$ is available to use, i.e., max $\{t - S_h + 1\}$
- $\kappa$: Maximum percentage of allowed wastage from ordered shots
- $M$: A large number

3.3. Variables

- $Y_j$: A Binary variable, equal 1 if medical center $j$ is operational; 0 otherwise
- $X_{vpijt}$: Number of individuals from vulnerability group $p$ in region $i$ allocated to get vaccine type $v$ at medical center $j$ in period $t$
- $W_{v'jt}$: Amount of vaccine type $v'$ used alternatively to satisfy the demand for vaccine type $v$ at medical center $j$ in period $t$
- $F_{vpijt}$: Amount of vaccine type $v$ received in period $k$ and injected to individuals from vulnerability group $p$ at medical center $j$ in period $t$ ($1 \leq k \leq t \leq T$)
- $G_{vjt}$: Amount of vaccine type $v$ wastage at medical center $j$ at the end of period $t$
- $Q_{vjt}$: Amount of vaccine type $v$ shortage at medical center $j$ in period $t$
- $IO_{vjt}$: Amount of on-hand inventory of vaccine type $v$ at medical center $j$ in period $t$
- $S_{vjt}$: Targeted stock of vaccine type $v$ at medical center $j$ in period $t$
- $IN_{vjt}$: Total inventory of vaccine type $v$ (on-hand and being delivered) at medical center $j$ in period $t$
- $O_{vjt}$: Amount of vaccine type $v$ ordered at medical center $j$ in period $t$
- $\lambda_{rj}$: An auxiliary variable denoting periodicity of $R$ at medical center $j$, i.e., if $\lambda_{rj} = 1$, then $r = R$
- $\psi_{jt}$: A binary variable equals 1 if a vaccine order at medical center $j$ in period $t$ is placed; 0 otherwise
- $\rho$: A binary variable equals 1 if the motivational strategy is adopted; 0 otherwise

The block diagram of the proposed model is shown in Fig. 3 [39].

3.4. Model formulation

\[
\begin{align*}
\text{Min } Z_1 &= \sum_j C_{m_j}Y_j + \sum_t \sum_i C_0\psi_{jt} \\
&+ \sum_j \sum_t \sum_i C_{s_{vt}}IO_{vjt} + C_{w_{vt}}G_{vjt} + C_{h_{t}}Q_{vjt} \\
&+ \sum_v \sum_v' \sum_j \sum_t C_{p_{cv'}}W_{v'jt} \\
&+ \sum_v \sum_j \sum_i \sum_t C_{e_{ij}}\psi_{jt}X_{vpijt} \\
&+ \sum_v \sum_j \sum_i \sum_t C_\rho D_{j} \left( \beta_2 - \beta_1 \right)
\end{align*}
\]

\[
\begin{align*}
\text{Max } Z_2 &= \sum_v \sum_p \sum_i \sum_t \left[ \alpha_p \sum_j X_{vpijt} D_{j} \right]
\end{align*}
\]

Subject to:

\[
1 \leq \sum_j Y_j \leq A
\]

\[
\sum_t \lambda_{rj} \leq Y_j \quad \forall j
\]

\[
\sum_t \theta_{jt} \lambda_{rj} = \psi_{jt} \quad \forall j, t
\]

\[
\sum_v S_{vjt} \leq \bar{S}_{jt} \quad \forall j, t
\]

\[
\sum_j X_{vpijt} \leq \beta_1 D_{j} + M \rho \quad \forall v, p, i, t
\]

\[
\sum_j X_{vpijt} > \beta_1 D_{j} + M (\rho - 1) \quad \forall v, p, i, t
\]

\[
\sum_j X_{vpijt} \leq \beta_2 D_{j} \quad \forall v, p, i, t
\]

\[
O_{vjt} = (S_{vjt} - IN_{vjt-1}) \psi_{jt} \quad \forall t \leq T - (L + 1), v, j
\]

\[
\sum_t \sum_{k=\tau_{rt}} F_{vpijt} = \sum_v W_{v'jt} \quad \forall v, j, t
\]

\[
O_{v,p,k} = \sum_t \sum_{t=k} X_{vpijt} + G_{vjt} \quad \forall k \geq L, v, j
\]
medical centers and the number of that vaccine type inoculated in period \( k | k > L \) (i.e., the last period of shelf-life) plus the amount of wastage at the same medical center.

Constraint (13) shows the relationship between the number of individuals allocated to each medical center to receive the assigned vaccine type and the number of vaccines inoculated, plus the shortage in the same center and period. Constraints (14) and (15) denote the on-hand stock and the total inventory level of a vaccine type at each medical center in each period, respectively. Constraint (16) warrants that the total amount of each vaccine type wastage in all the medical centers would not surpass a specific portion of orders. Finally, Constraints (17) and (18) indicate variables’ types.

### 4. Linearization and defuzzification

In this section, several properties of the proposed model are adjusted to reduce its complexity without losing accuracy. The model tractability is an integral part of a framework for vaccine allocation and inventory management, especially in the COVID-19 vaccination programs. Thus, in the first step, Constraint (10) that contains a multiplication of binary and continuous variables is modified as follows:

\[
O_{ijt} - (S_{ijt} - IN_{ijt-1}) \geq \bar{S}_{ij} (\varphi_{jt} - 1) \quad \forall t \leq T - (L + 1), v, j
\]  

(19)

\[
O_{ijt} - (S_{ijt} - IN_{ijt-1}) \leq \bar{S}_{ij} (1 - \varphi_{jt}) \quad \forall t \leq T - (L + 1), v, j
\]  

(20)

\[
O_{ijt} \leq M\varphi_{jt} \quad \forall t \leq T - (L + 1), v, j
\]  

(21)

Then, to transform the first objective function expression \( \varphi_{jt}X_{pijt} \), we need to introduce an auxiliary variable \( \omega_{pijt} \). Now, the model can be linearized using Constraints (22)–(25):

\[
\sum_{v} \sum_{p} \sum_{i} \omega_{pijt} \leq M\varphi_{jt} \quad \forall j, t
\]  

(22)

\[
\sum_{v} \sum_{p} \sum_{i} \omega_{pijt} \leq \sum_{v} \sum_{p} \sum_{i} X_{pijt} \quad \forall j, t
\]  

(23)

\[
\sum_{v} \sum_{p} \sum_{i} \omega_{pijt} \geq \sum_{v} \sum_{p} \sum_{i} X_{pijt} - (1 - \varphi_{jt}) \quad \forall j, t
\]  

(24)
Possibility (Pos) values of the fuzzy variable as follows:

Max \( Z \) (i.e., first objective function) can be written as follows:

\[
\eta_{\psi_{\text{pit}}} \geq 0 \quad \forall v, t
\]  

(25)

Next, the model uncertainty element concerning vaccine demand is addressed in light of governments’ aim to reach a desirable immunization rate in their vaccination programs. This study adopted the fuzzy credibility (Cr) method. This approach was introduced by Zhu and Zhang [55] to cope with the demand uncertainty. The essential advantage of the credibility-based fuzzy approach over the possibility and necessity methods is its self-duality. This feature means that if a credibility-based event’s membership function equals one, that event will occur, while this outcome is not guaranteed in the other two methods [56]. This method also ensures that the minimum satisfaction level of fuzzy constraints remains higher than 0.5 and does not add additional constraints to the model, making it a resilient and tractable approach. Then, if we consider \( \psi \) as a fuzzy variable with membership function \( f(x) \), and \( q \in \mathbb{R} \) then, according to Liu and Liu [57], the credibility measure can be written as:

\[
\text{Cr}\{\psi \leq q\} = \frac{1}{2} \left( \sup_{x} f(x|x \leq q) + 1 - \sup_{x} f(x|x > q) \right)
\]  

(26)

Also, this measure can be calculated using Necessity (Nec) and Possibility (Pos) values of the fuzzy variable as follows:

\[
\text{Cr}\{\psi \leq q\} = \frac{1}{2} \left( \text{Pos}\{\psi \leq q\} + \text{Nec}\{\psi \leq q\} \right)
\]  

(27)

Then, Liu and Liu [57] proved that the Expected Value (EV) of \( \psi \) can be written as follows:

\[
\text{EV}(\psi) = \int_{0}^{\infty} \text{Cr}\{\psi \geq q\} dq - \int_{-\infty}^{0} \text{Cr}\{\psi \leq q\} dq
\]  

(28)

Then, for \( \psi \) as a fuzzy trapezoidal measure with four prominent points of \( (\psi_1, \psi_2, \psi_3, \psi_4) \), the EV can be determined using the average of those four points. Therefore, for the fuzzy demand in the second objective function, if assuming \( D_{\text{pit}} \) as a trapezoidal fuzzy number, the crisp equivalent of it can be calculated as:

\[
\text{EV}(D_{\text{pit}}) = \left( \frac{D^1_{\text{pit}} + D^2_{\text{pit}} + D^3_{\text{pit}} + D^4_{\text{pit}}}{4} \right)
\]  

(29)

Now, the crisp correspondence of the last expression in Eq. (1) (i.e., first objective function) can be written as follows:

\[
\rho \sum_{v} \sum_{p} \sum_{i} \sum_{t} \left[ \frac{D^1_{\text{pit}} + D^2_{\text{pit}} + D^3_{\text{pit}} + D^4_{\text{pit}}}{4} \right] (\beta_2 - \beta_1)
\]  

(30)

According to the credibility technique, the following equations can be defuzzified for the fuzzy variable \( \psi \) as follows [55]:

\[
\text{Cr}\{\psi \leq q\} \geq \xi \iff q \geq (2 - 2\xi) \psi^3 + (2\xi - 1) \psi^4
\]  

(31)

\[
\text{Cr}\{\psi \geq q\} \geq \xi \iff q \leq (2\xi - 1) \psi^3 + (2 - 2\xi) \psi^4
\]  

(32)

Also, the nonlinear fraction of the second objective function can be linearized using a positive variable \( \eta_{\text{pit}} \) as follows:

\[
\eta_{\text{pit}} \leq \frac{\sum_{i} \alpha p X_{\text{pit}}}{D_{\text{pit}}} \quad \forall v, p, i, t
\]  

(33)

Next, the second objective function and the crisp equivalent of Eq. (33) can be written as follows:

\[
\text{Max } Z_2 = \sum_{v} \sum_{p} \sum_{i} \sum_{t} \alpha p \eta_{\text{pit}}
\]  

(34)

\[
\sum_{j} \alpha p X_{\text{pit}} \geq \eta_{\text{pit}} \left( (2 - 2\mu_i) D^1_{\text{pit}} + (2\mu_i - 1) D^4_{\text{pit}} \right) \quad \forall v, p, i, t
\]  

(35)

Then, the crisp correspondence of Constraints (8)–(10) can be written as Constraints (36)–(38), respectively.

\[
\sum_{j} X_{\text{pit}} \leq \beta_1 \left( (2\mu_i - 1) D^1_{\text{pit}} + (2 - 2\mu_i) D^2_{\text{pit}} \right) + M \rho \quad \forall v, p, i, t
\]  

(36)

\[
\sum_{j} X_{\text{pit}} > \beta_1 \left( (2 - 2\mu_i) D^3_{\text{pit}} + (2\mu_i - 1) D^4_{\text{pit}} \right) + M (\rho - 1) \quad \forall v, p, i, t
\]  

(37)

\[
\sum_{j} X_{\text{pit}} \geq \beta_2 \left( (2\mu_i - 1) D^1_{\text{pit}} + (2 - 2\mu_i) D^2_{\text{pit}} \right) \quad \forall v, p, i, t
\]  

(38)

\[
\mu_i \geq 0.5 \quad \forall v, p, i, t
\]  

(39)

Finally, the proposed model is a bi-objective problem that a commercial solver cannot solve directly. Hence, we complete the model transformation into a linear single objective problem by implementing an augmented \( \varepsilon \) constraint approach. This method contains the strength of the regular \( \varepsilon \) constraint, including the capability to produce non-extreme solutions and less dependence on decision-maker inputs than other techniques, such as weighting [58–60]. Besides that, the augmented \( \varepsilon \) constraint approach helps the decision-maker better control the number of generated solutions and intervals to search for a desirable optimal pareto solution [61,62]. Given the delicacy and short-term nature of the problem, these characteristics would be significantly valuable for the problem at hand. Next, we introduce this method in the following steps.

- We opted for the first objective function as the primary objective due to inclusiveness and the possibility of gauging the service level (i.e., the second objective function).
- The model is solved using the second objective function to calculate its worst and best values.
- The second objective function is transformed to Eq. (41). Then, \( \varepsilon_\tau \) demonstrates the lower bound of the second objective function and can be calculated using Eq. (42), where \( \tau \) denotes the specific interval among a set of \( \gamma \) intervals between the worst and best values of the second objective function in the previous step.
- Using the value of \( \varepsilon_\tau \), the model is solved for each \( \tau \) obtaining a set of optimal pareto solutions. Now, the decision-makers can choose an optimal solution according to their preference.

\[
\text{Min } Z_1
\]  

(40)

Subject to:

\[
Z_2 \geq \varepsilon_\tau
\]  

(41)

\[
\varepsilon_\tau = \text{Min}(Z_2) + \left[ \frac{\text{Max}(Z_2) - \text{Min}(Z_2)}{\gamma} \right] \tau
\]  

(42)

5. Case study

This section discusses the current vaccination program in Iran. Then, a comparative analysis between the proposed model and the current policy is conducted. The case study is related to a part of Iran’s national vaccination program, particularly in Mashhad, the second-largest city in the country with over 3 million population. As of December 2021, Iran medical centers have administered more than 102 million shots, from which approximately 44 million people were fully vaccinated, 12 million individuals got their first dose, and 750 thousand people got the booster shot.
The national vaccination program in Iran has followed a timeline shown in Table 2.

According to Iranian health authorities [63], five types of COVID-19 vaccine, AZD1222, BBIBP-CorV, Coviran Barakat, Pastu Covac, and Sputnik V, are available. The details of two domestically produced vaccines can be seen in Table 3. Also, as of November 2021, the official reports show imports of BNT162b2 and Covaxin, but their numbers were minimal. While the start of the vaccination program in Iran was hampered by many issues, in recent months, the vaccination speed has reached good momentum. Consequently, the numbers of deaths and people needing hospitalization show a downward trend compared to previous months, indicating overall good signs for a country that opted for a strategy of co-existent with the virus from early on of the pandemic emergence. Nonetheless, there are debates over limited portfolios of COVID-19 vaccines available in Iran and the transparency and lack of international approval for the domestically produced jabs, which is out of the scope of this research.

In this article, without loss of generality, we opted to analyze the COVID-19 vaccination program in one of the populated districts in Mashhad. The idea behind this decision is that it would enable us to more effectively gather data and information about daily vaccine demand in medical centers. Also, it would benefit us to thoroughly design and execute a problem with tractable and computable size since we deal with an uncertain dynamic inventory experiment. Given that the vaccination program in Iran is being run uniformly in all provinces and cities, the outcome should closely reflect the pros and cons of the national program.

As of November 2021, in Mashhad, all the medical centers assigned for the COVID-19 vaccination program are acquiring their jabs from the Mashhad University of Medical Science (i.e., the city's main storage facility), which itself receives the jabs from the central government in Tehran. Hence, the medical centers are independent and receive jabs based on their capacity. In these circumstances, the Iranian government, like other countries, aims to control vaccine demand and prevent massive shortages with a specific national vaccination schedule (see Table 2).

Mashhad has had 22 active vaccination centers since June 2021. They comprise 18 medical centers and four makeshift hubs used for vaccination purposes. In this article, only the vaccination program of district 10 of the city is investigated. As depicted in Fig. 4, District 10 consists of 3 areas with 12 precincts, which the latter is assumed as patient zones. Three vaccination centers are responsible for the majority of immunization in this region. One center is in the Ostad Yousefiprecinct, and the others are in the neighborhood districts. Also, people can travel to other medical centers in the city to get vaccinated. However, given the national online registration that assigns and encourages people to travel to adjacent medical centers and a high possibility of rejection in attending a far medical center without a scheduled appointment, such individuals should only account for a tiny percentage of the vaccination rate. Thus, the consideration of the three closest vaccination centers could be justified.

According to the national census, the population of district 10 was 296,823 in 2016. Hence, with the help of Iran’s average population growth rate, the estimated number of the current population would be close to 312,560 residents. As of November 2021, in Mashhad, 84.2% of individuals above 12 years old received at least one shot, and 63.8% have been fully vaccinated. Then, considering the city’s demographic details and the national vaccination program timeline, a preliminary approximation of the daily vaccine demand could be calculated. It should be noted that we expect daily demand would increase and then stabilize as more people get vaccinated. Thus, we took advantage of previous months’ national vaccination data and employed a linear regression function to estimate this trend (see Fig. 5). Then, to complement the accuracy of approximated demand, the opinions of registration operators in each medical center have also been taken into account (see Table 4). The final step considers the approximated daily demand in all four medical centers with a fuzzy description. This method might not provide an accurate estimation as the DELPHI method used by [34]; nonetheless, it should suffice for comparative analysis purposes in this article and be compatible with the (R, S) inventory policy.

According to Iranian health authorities [63], the number of vaccines distributed in the city fluctuates between 30000 to 400000 doses per day. Because the volume of available shots in the country has risen dramatically, we decided to consider this measure optimistically. The adopted inventory management of an
Table 4
The medical centers’ available vaccines and the approximated stock.

| Medical centers                           | Types of vaccines available | First dose | Second dose | Booster dose | The approximated stock of all types of vaccines |
|-------------------------------------------|----------------------------|------------|-------------|--------------|-------------------------------------------------|
| Located in the Ostad Yousefi precinct     | AZD1222                    | Available  | A           | Not available| 1000 to 2000 doses                               |
|                                           | BBIBP-CorV                 | A          | A           | NA           |                                                 |
|                                           | Sputnik V                  | NA         | A           | NA           |                                                 |
| Located south of the Frahangan precinct    | AZD1222                    | A          | A           | NA           | 2500 to 3500 doses                               |
|                                           | BBIBP-CorV                 | A          | A           | A            |                                                 |
|                                           | CovIran Barakat             | NA         | A           | A            |                                                 |
|                                           | Pastu Covac                | A          | NA          | NA           |                                                 |
| Located south of Imam Hadi precinct        | AZD1222                    | A          | A           | NA           | 1000 to 2000 doses                               |
|                                           | BBIBP-CorV                 | A          | A           | A            |                                                 |
|                                           | CovIran Barakat             | NA         | A           | A            |                                                 |
|                                           | Pastu Covac                | A          | NA          | NA           |                                                 |

Fig. 4. Map of district 10 of Mashhad city.

Fig. 5. The speed of national COVID-19 vaccination in Iran [63].

(R, S) policy accommodated two vital advantages to the medical centers: its relatively routine implementation process and decision makers’ familiarity due to its employment in handling blood units’ inventory management in many countries, including Iran, Canada, and Australia.

The planning horizon is assumed to be three months for the experiments. The period and lead time are both considered as a single day. The model’s review periods can be between 1–7 days. The national vaccination program is currently at phase 4 (above 12 years old and booster shots), and a three-month plan will coincide with the remaining eligible people from phases 1–3 and phase 4 itself. Hence, the experiment contains eight priority groups, from healthcare workers to those eligible for booster shots.

Furthermore, concerning vaccine perishability, most types of vaccines available in Iran have a shelf-life of more than six months (e.g., AZD1222, Sputnik V, BBIBP-CorV, and CovIran Barakat). However, the requirement for a cold supply chain still poses a significant challenge, and due to the existing financial sanctions on Iran, jabs procurement is a protracted process, according to government reports. This issue potentially reduces jabs’ shelf-life, roughly several weeks less than its specified duration for all types of vaccines.

Finally, we test the proposed model and the current policy in different circumstances, encompassing four values of fuzzy constraints confidence levels and three service levels. Notably, no limit for wastage is considered because, in reality, the national vaccination program does not impose this restriction. The experiments are solved by a commercial version of the CPLEX solver in the GAMS software. The configuration of the computer system is an Intel i7 CPU with a 2.6 GHz processor, 32 GB RAM, and Windows 10 operator. The experiment took under 3 h to be solved.

The results from Table 5 demonstrate that the proposed model can reduce operational costs from 5.1% to 44.8%, depending on the specified conditions. Besides that, the number of operational facilities fluctuates from a maximum of three to one. The second medical center (i.e., the largest one) is always operational, and then the first medical center (i.e., the closest one) has had the preferred priority by the model to operate.

Moreover, Fig. 6 encompasses four charts, each depicting the comparison between the proposed model results and the current policy regarding the number of vaccine shortages, wastages, and inventories. While the proposed model has performed better in all categories, the decreased volume of vaccine wastages has been more considerable, ranging from 50% to 59%, depending on the specified conditions. These improvements were around 30% for shortages and 20% for average inventories. Given the problem’s greater focus on a higher service level and a lower possible shortage, such outcomes could be justified. Finally, the results display an expected and logical trend, meaning that when the model constraints become more restricted, the costs increase and the negative indicators (i.e., shortage and wastage rate) become less desirable, indicating its validity in the process.
Table 5
The performance of the proposed model and the current policy.

| Confidence levels | Service levels | The optimal number of operational medical centers | The cost ratio of the proposed policy to the current policy |
|-------------------|----------------|--------------------------------------------------|----------------------------------------------------------|
| 0.5               | 90%            | 1                                                | 0.552                                                    |
|                   | 95%            | 2                                                | 0.708                                                    |
|                   | 99%            | 2                                                | 0.812                                                    |
| 0.7               | 90%            | 2                                                | 0.663                                                    |
|                   | 95%            | 2                                                | 0.760                                                    |
|                   | 99%            | 2                                                | 0.843                                                    |
| 0.9               | 90%            | 3                                                | 0.759                                                    |
|                   | 95%            | 3                                                | 0.841                                                    |
|                   | 99%            | 3                                                | 0.917                                                    |
| 0.95              | 90%            | 3                                                | 0.793                                                    |
|                   | 95%            | 3                                                | 0.884                                                    |
|                   | 99%            | 3                                                | 0.949                                                    |

Fig. 6. The comparative analysis between the proposed model and the current policy.

Fig. 7. A comparative analysis of the proposed model with and without motivational strategy.

Afterward, we seek to impose the wastage limit on the experiments and evaluate the model responses. While the Iranian government does not set this restriction on the vaccination program, we argue that soon, this would be a sensitive matter given the high cost of COVID-19 vaccine procurement and distribution in Iran. Fig. 7 depicts a comparative analysis between the proposed model (with motivational strategy) and the current policy (without motivational strategy). They explicitly indicate that the model responses dominate the current policy concerning the system’s total costs. The reduced costs advantage is more significant.
than the previous experiment, meaning for a confidence level of 0.90, the costs are 57% or 31%, depending on the specified conditions. Also, the model results are far better than the current policy regarding inventory and shortage.

5.1. Sensitivity analysis

This section seeks to draw insight from the model behavior by performing a series of assessments on its crucial features. As mentioned, the proposed model $(R, S)$ policy enabled medical centers to order vaccines in an optimal review period. However, in reality, medical centers need to check with the city’s main vaccine storage facility to accommodate their stock each day. Thus, we seek to evaluate the effect of such consideration in the model responses.

Without loss of generality, the information and performance of only one of the medical centers without any wastage threshold restriction are considered (The one located in the Ostad Yousefi precinct). The results in Table 6 highlight a decrease of 68% to 61% among vaccine wastages and 47% to 38% among jabs shortages depending on the specified system service level. Besides that, the medical center review period can be every two days, which is reduced to half compared to the current policy. It is worth mentioning that the model optimal vaccine stocks remain close to the stock kept by the current policy. This matter arises from the emphasis on maintaining a high service level in the vaccination program. Consequently, this outcome leads to a lower total cost and higher efficiency in the system. This experiment underscores the proposed model capability in scenarios where even if the medical center does not gain any strategic improvement (e.g., increased capacity, lower operating cost), it still produces superior performance.

Furthermore, we evaluate the effect of fuzzy credibility consideration in the model. To do so, we take advantage of a technique implemented by [51]. This method uses a series of Random Realizations (RR) of fuzzy numbers. For example, if $K$ is a fuzzy number with trapezoidal properties and four prominent points of $(K^1, K^2, K^3, K^4)$, any RR of $K$ has a value limited to the interval $[K^1, K^4]$. Then, if we generate several random realizations, we could compare their results with the fuzzy credibility model. In this method, each time we employ new RR values for fuzzy parameters, it is necessary not to alter the strategic decision values; thus, the number of medical centers in all scenarios is set as three.

The technique mentioned above is implemented on the model using ten different RR scenarios, and the results are depicted in Fig. 8. From the graph behavior, we point out the overall improvement in the model’s total cost in the proposed model compared to its crisp counterpart. Besides that, the deterministic approach Standard Deviation (SD) equals 307 compared to the fuzzy credibility methods SD(0.70) $= 62$, SD(0.90) $= 61$. This analysis explicitly shows the proposed model’s resiliency and consistency in coping with the uncertainty in the investigated problem.

Table 6

| Decision variables | The current policy | Proposed model with a confidence level of 0.90 |
|--------------------|--------------------|---------------------------------------------|
|                    | R                  | 1                                           | 2                                           |
| Total number of vaccine wastages | 8674.21           | 2785.97                                     | 3405.66                                     |
| Total number of vaccine shortages | 2851.23           | 1517.26                                     | 1780.35                                     |
| Total number of vaccine stocks (S) | 2000              | 1803                                        | 1925                                        |
| Percentage of people who got the recommended type of vaccine | NA                | 89.62%                                      | 95.17%                                      |

6. Solution methodology

Although the proposed model solution time using the commercial solver was reasonable for the case study, there might be conditions where solution time becomes much longer, which would not accommodate the urgency of a COVID-19 vaccination program. Now, if we decide to apply the model to a metropolitan area (e.g., the whole city of Mashhad), a province, or even a case using digitalized and online data that could change constantly, the solution time is likely to be problematic.

Therefore, an efficient solution algorithm could complement our model applicability and performance. This section introduces and implements a Lagrangian relaxation algorithm on the model. This algorithm has been used by many scholars in different fields, such as healthcare, logistics, and engineering [65–67].

The Lagrangian relaxation algorithm reduces solution time by dividing the problem into simpler sub-problems [68]. This algorithm initially finds the lower and upper bounds for the optimal solutions. Then, it attempts to achieve a desirable solution by updating obtained lower and upper bounds.

6.1. Lower bound

In general, some constraints could cause more complexity in problems than others. Thus, finding and relaxing these constraints can help us achieve a lower bound [68]. After several tests, we opted to relax Constraints (6) and (15) in the proposed model. This process provides the dual Lagrangian problem as follows:

$$\min L(\delta_1^1, \delta_2^1) = \sum_j C_m Y_j + \sum_j \sum_p \sum_i C_{e_i} \rho_{e_i} + \sum_j \sum_p \sum_t C_p e_i W_{e_i}$$

$$+ \sum_j \sum_p \sum_t \sum_i C_p e_i W_{e_i} X_{e_i}$$

$$+ \sum_j \sum_p \sum_t \sum_i C_p e_i W_{e_i} X_{e_i}$$

$$+ \sum_j \sum_p \sum_t \sum_i C_p e_i W_{e_i} X_{e_i}$$

$$\times (\beta_2 - \beta_1)$$

$$+ \sum_j \sum_t \sum_i \delta_1^1 (5 - \sum_i \delta_1^1)$$

$$+ \sum_j \sum_t \sum_i \delta_2^1$$

$$+ \sum_j \sum_t \sum_i \delta_2^1$$

$$- \sum_j \sum_t \sum_i \delta_2^1$$

$$- \sum_j \sum_t \sum_i \delta_2^1$$

$$- \sum_j \sum_t \sum_i \delta_2^1$$

$$- \sum_j \sum_t \sum_i \delta_2^1$$

$$- \sum_j \sum_t \sum_i \delta_2^1$$

Subject to: Constraints (3)-(5), (11)-(14), (16)-(25), (35)-(39), (41), and (42)

where $\delta_1^1$ and $\delta_2^1$ are non-negative and free Lagrangian coefficients, respectively. Eqs. (43) should be minimized using fixed values of the Lagrangian coefficients to find the lower bound for the primary model.
6.2. Upper bound

Relaxing constraints (6) and (15) could lead to infeasibility among outcomes of the dual Lagrangian problem (43) in some cases. Then, Eq. (40) (i.e., the model) should be solved subjected to Constraints (3)–(6), (11)–(25), (35)–(39), (41), and (42) where the obtained values of decision variables \( Y_j \) and \( \rho \) from solving the problem (43) are considered as parameters. This solution is an upper bound for the model (40).

6.3. Updating upper and lower bounds

In each iteration, the Lagrangian coefficients \( \delta_{jt}^1 \) and \( \delta_{jt}^2 \) are updated to calculate new upper and lower bounds. Eqs. (44) and (45) represent the assigned values given to each Lagrangian coefficient at iteration \( n+1 \) using the sub-gradient optimization approach proposed by Fisher [68].

\[
\delta_{jt}^{1,n+1} = \max \left\{ 0, \delta_{jt}^{1,n} + \text{stepsize}^1 \left( S_{jt} - \sum_v \delta_{vt} \right) \right\} \tag{44}
\]

\[
\delta_{jt}^{2,n+1} = \max \left\{ 0, \delta_{jt}^{2,n} + \text{stepsize}^1 \left( IN_{jt} - IO_{jt} - \sum_{k=1}^{t} O_{jk} \right) \right\} \tag{45}
\]

where \( n \) is the iteration number and stepsize \( 1^n \) and stepsize \( 1^n \) are as follows:

\[
\text{stepsize}^1 = \frac{\chi^n (UP - LB^n)}{\sum_j \sum_t \left( S_{jt} - \sum_v \delta_{vt} \right)^2} \tag{46}
\]

\[
\text{stepsize}^2 = \frac{\chi^n (UP - LB^n)}{\sum_j \sum_t \left( IN_{jt} - IO_{jt} - \sum_{k=1}^{t} O_{jk} \right)^2} \tag{47}
\]

where \( LB^n \) is the obtained lower bound at iteration \( n \) and \( UP \) is the best obtained upper bound. We set \( \chi = 2 \) at the first iteration. If the LB value does not improve after four consecutive iterations, \( \chi \) is halved. Once we obtain a feasible solution with an acceptable tolerance or reach the minimum value of the step size, the process is complete.

6.4. Large scale experiments

Using the previous case study data, we created a comparative analysis of solving the model with/without the Lagrangian relaxation algorithm. For this purpose, different problem sizes are randomly generated and evaluated, and their outcomes in two categories, solution time and GAP (i.e., differences between the objective function values in both approaches), have been assessed in Table 7. In Case 1 (the investigated case study), we can infer that the solution time of the model decreases significantly while the average GAP equals 3.3%, which is negligible. Also, as expected, both solution approaches require more time to achieve a response when the problem scale increases. However, the Lagrangian relaxation algorithm could solve large-scale experiments more efficiently than the CPLEX solver. Finally, in Case 4, the commercial solver could not provide a solution in less than 10 h, but the Lagrangian relaxation algorithm could achieve a solution in an acceptable time. The calculated gaps are negligible in all cases (less than 5%), proving that the Lagrangian relaxation algorithm effectively solves different problem sizes. All in all, sacrificing a bit of precision in exchange for quicker solution time is up to decision-makers, but using the Lagrangian relaxation algorithm indeed empowers practitioners’ performance, especially for larger-scale case studies.

7. Conclusions

On January 30, 2020, WHO announced the spread of the COVID-19 virus as a global pandemic, and yet until today, controlling and combating this disease still ranks a high priority worldwide. Many countries target achieving herd immunity as they roll out mass vaccination programs. However, due to global demand and lack of preparedness for such circumstances, acquiring COVID-19 vaccine shots has become challenging for many countries, particularly developing and lower-income ones. Besides that, unpredictability, advanced infrastructure, equipment required for the vaccine supply chain, and many other issues have complicated planning and operating a mass vaccination program. These matters highlight the need to design and run a national vaccination program with high efficiency and solid resiliency.

In this article, a bi-objective integrated vaccine allocation and inventory management have been developed. The essential model assumptions include vaccine perishability, demand uncertainty, different types of vaccines, people’s vulnerability groups, and motivational strategies. Notably, the model development process contained critical features to reduce unnecessary complexity and improve its applicability.

Afterward, the model was applied to a case study related to a vaccination program in district 10 of Mashhad city with a population of over a quarter of a million people. The following results and comparative analysis demonstrated the proposed model’s significant benefits over the current policy. We highlighted that the proposed model could enable the practitioners to improve the performance of the COVID-19 vaccination program in terms of the total cost, vaccine shortages, and wastages with the same infrastructure and equipment and only by optimizing some tactical decisions. Then, to complement our research, the Lagrangian Relaxation method was implemented on the model to improve its capability to solve larger-scale problems.

It should be noted that this study has some limitations. Firstly, the data presented by governments, including this research case study, are incomplete and lack necessary detail, such as the number of individuals who miss their vaccination appointment, the number of people who have shown side effects after injection, and in some cases, are inaccessible. Additionally, the vaccination programs in Iran and many other countries have frequently changed, making its planning difficult.

Moreover, the assumption of a potential cost for motivational strategies in the proposed model is not a straightforward task because only a few governments have employed financial incentives (e.g., fines or gift cards). The cost value depends on many factors, such as income, age, and living standards, which are hard to be translated and weighted purely financially.
This pandemic has changed the world, so the healthcare supply chain has been significantly affected. Consequently, there is an echo for developing efficient and digitized frameworks for the healthcare supply chain concerning lateral transshipment among the same facilities, online monitoring of inventories, and providing onsite services to elderly and disabled people are a few examples researchers have yet to address thoroughly.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Table 7

| Case number | Number of regions | Number of medical centers | Number of vaccine types | Total Cost (million Tomans) | GAP | Solution time (s) |
|-------------|------------------|--------------------------|-------------------------|-----------------------------|-----|------------------|
| 1           | 3                | 3                        | 5                      | 2051.4                      | 2121.3 | 3.3% 9606 2070 |
| 2           | 6                | 9                        | 7                      | 5873.9                      | 6147.5 | 4.5% 23100 4434 |
| 3           | 9                | 15                       | 8                      | 9102.1                      | 9490.2 | 4.1% 34692 6138 |
| 4           | 12               | 20                       | 9                      | N/A                         | 15,687.4 | – >36000 7752 |

This table compares the results with and without the Lagrangian Relaxation method for different problem sizes.
