Generation of strong magnetic fields in hybrid and quark stars driven by the electroweak interaction of quarks

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Abstract. We study the generation of strong large scale magnetic fields in compact stars containing degenerate quark matter with unbroken chiral symmetry. The magnetic field growth is owing to the magnetic field instability driven by the electroweak interaction of quarks. In this system we predict the enhancement of the seed magnetic field $10^{12}$ G to the strengths $(10^{14} - 10^{15})$ G. In our analysis we use the typical parameters of the quark matter in the core of a hybrid star or in a quark star. We also apply of the obtained results to model the generation of magnetic fields in magnetars.

The origin of strong magnetic fields $B \sim 10^{15}$ G in magnetars [1] is a puzzle for modern astrophysics. Despite the existence of multiple models describing the generation of such magnetic fields, which are based on magnetohydrodynamics of stellar plasmas, none of them can satisfactory describe the observational data. Recently, in [2], we proposed the new approach to generate strong magnetic fields in quark matter owing to the instability of the magnetic field driven by the electroweak interaction of quarks. In the present work we summarize the results of [2] and discuss the applicability of this model for the generation of magnetic fields in magnetars.

Let us consider a dense quark matter consisting of $u$ and $d$ quarks. The density of this matter is supposed to be high enough for the chiral symmetry to be restored. In this case we can take that quarks are effectively massless. Therefore we can decompose the quark wave functions into left and right chiral components, which evolve independently, and attribute different chemical potentials $\mu_{qL,R}$, where $q = u, d$, for each chiral component.

Generalizing the results of [3,4], we get that, in the external magnetic field $B$, there is the induced electric current

$$J = \Pi B, \quad \Pi = \frac{1}{2\pi^2} \sum_{q=u,d} e^2 e_q (\mu_{5q} + V_{5q}), \quad (1)$$
where \( e_u = 2e/3 \) and \( e_d = -e/3 \) are the electric charges of quarks, \( e > 0 \) is the elementary charge, \( \mu_{5q} = (\mu_{qR} - \mu_{qL})/2 \) is the chiral imbalance, \( V_{5q} = (V_{qL} - V_{qR})/2 \), and \( V_{qL,R} \) are the effective potentials of the electroweak interaction of left and right quarks with background fermions. The potentials \( V_{qL,R} \) were found in [5],

\[
\begin{align*}
V_{qL} &= -\frac{G_F}{\sqrt{2}} n_d \left( 1 - \frac{8}{3}\xi + \frac{16}{9}\xi^2 - 2|V_{ud}|^2 \right), \\
V_{qR} &= \frac{G_F}{\sqrt{2}} n_d \left( \frac{4}{3}\xi - \frac{16}{9}\xi^2 \right),
\end{align*}
\]

where \( G_F = 1.17 \times 10^{-5} \text{GeV}^{-2} \) is the Fermi constant, \( \xi = \sin^2 \theta_W = 0.23 \) is the Weinberg parameter, \( n_{u,d} \) are the number densities of \( u \) and \( d \) quarks, and \( V_{ud} = 0.97 \) is the element of the Cabbibo-Kobayashi-Maskawa matrix. The matter of the star is supposed to be electrically neutral. Thus we should have \( n_u = n_0/3 \) and \( n_d = 2n_0/3 \), where \( n_0 = n_u + n_d \) is the total number density of quarks in the star. Basing on equation (2) and assuming that \( n_0 = 1.8 \times 10^{38} \text{cm}^{-3} \), we get that \( V_{5u} = 4.5 \text{eV} \) and \( V_{5d} = 2.9 \text{eV} \).

Using equation (1) and the results of [4], we can obtain the system of kinetic equations for the spectra of the density of the magnetic helicity \( h(k,t) \) and of the magnetic energy density \( \rho_B(k,t) \), as well as the chiral imbalances \( \mu_{5u}(t) \) and \( \mu_{5d}(t) \), in the form,

\[
\begin{align*}
\frac{\partial h(k,t)}{\partial t} &= -\frac{2k^2}{\sigma_{\text{cond}}} h(k,t) + \frac{8\alpha_{\text{em}}}{\pi\sigma_{\text{cond}}} \left\{ \frac{4}{9} \left[ \mu_{5u}(t) + V_{5u} \right] + \frac{1}{9} \left[ \mu_{5d}(t) + V_{5d} \right] \right\} \rho_B(k,t), \\
\frac{\partial \rho_B(k,t)}{\partial t} &= -\frac{2k^2}{\sigma_{\text{cond}}} \rho_B(k,t) + \frac{2\alpha_{\text{em}}}{\pi\sigma_{\text{cond}}} \left\{ \frac{4}{9} \left[ \mu_{5u}(t) + V_{5u} \right] + \frac{1}{9} \left[ \mu_{5d}(t) + V_{5d} \right] \right\} k^2 h(k,t), \\
\frac{d\mu_{5u}(t)}{dt} &= \frac{4\pi\alpha_{\text{em}}}{\mu_{5u}^2\sigma_{\text{cond}}/9} \int dk \left[ k^2 h(k,t) \right. \\
&\quad \left. - \frac{4\alpha_{\text{em}}}{\pi} \left\{ \frac{4}{9} \left[ \mu_{5u}(t) + V_{5u} \right] + \frac{1}{9} \left[ \mu_{5d}(t) + V_{5d} \right] \right\} \rho_B(k,t) \right] - \Gamma_u \mu_{5u}(t), \\
\frac{d\mu_{5d}(t)}{dt} &= -\frac{4\alpha_{\text{em}}}{\mu_{5d}^2\sigma_{\text{cond}}/9} \int dk \left[ k^2 h(k,t) \right. \\
&\quad \left. - \frac{4\alpha_{\text{em}}}{\pi} \left\{ \frac{4}{9} \left[ \mu_{5u}(t) + V_{5u} \right] + \frac{1}{9} \left[ \mu_{5d}(t) + V_{5d} \right] \right\} \rho_B(k,t) \right] - \Gamma_d \mu_{5d}(t),
\end{align*}
\]

where \( \Gamma_u = 2.98 \times 10^{-10} \mu_0 \) and \( \Gamma_d = 5.88 \times 10^{-12} \mu_0 \) are the rates for the helicity flip in \( ud \) plasma [2], \( \alpha_{\text{em}} = e^2/4\pi = 7.3 \times 10^{-3} \) is the QED fine structure constant, \( \sigma_{\text{cond}} \) is the electric conductivity of \( ud \) quark matter, \( \mu_u = 0.69\mu_0 \) and \( \mu_d = 0.87\mu_0 \) are the mean chemical potentials of \( u \) and \( d \) quarks in the electroneutral \( ud \) plasma, and \( \mu_0 = 346 \text{MeV} \). The wave number \( k \) in equation (3) is in the range: \( k_{\text{min}} < k < k_{\text{max}} \), where \( k_{\text{min}} = 1/R = 2 \times 10^{-11} \text{eV} \), \( R = 10 \text{km} \) is the stellar radius, \( k_{\text{max}} = 1/A_B^{(\text{min})} \), and \( A_B^{(\text{min})} \) is the minimal scale of the magnetic field, which is a free parameter.

In our model for the magnetic field generation in magnetars, we suggest that background fermions are degenerate. Nevertheless there is a nonzero temperature \( T \) of the quark matter, which is much less than the chemical potentials: \( T \ll \mu_q \). The conductivity of the degenerate quark matter was estimated in [6]. It can be rewritten in the form [2],

\[
\sigma_{\text{cond}} = \sigma_0 \frac{T_0^{5/3}}{T_{5/3}}, \quad \sigma_0 = 3.15 \times 10^{22} \text{s}^{-1},
\]
where \( T_0 = (10^8 - 10^9) \text{K} \) is the initial temperature corresponding to the time \( t_0 \sim 10^2 \text{yr} \), when the star is already in a thermal equilibrium. To derive equation [4] we take that the QCD fine structure constant \( \alpha_s \sim 0.1 \). Note that \( \sigma_{\text{cond}} \) in quark matter is several orders of magnitude less than the conductivity of electrons in the nuclear matter in a neutron star (NS) [7]. Basing on the energy conservation in the system consisting of background fermions and the magnetic field as well as accounting for equation [4], one gets that the factor

\[
F = \left(1 - \frac{B_0^2}{B_{\text{eq}}^2}\right)^{5/6}, \quad B_{\text{eq}}^2 = 1.23 \mu_0^2 T_0^2.
\]

should be introduced in rhs of equation [3]. It should be noted that the quenching in equation [3] allows one to avoid the excessive growth of the magnetic field when \( B \rightarrow B_{\text{eq}} \).

While solving of equation [3] numerically, we use the initial Kolmogorov spectrum of the magnetic energy density, \( \rho_B(k, t_0) = Ck^{-5/3} \), where the constant \( C \) depends on the seed magnetic field \( B_0 \) [4]. The initial spectrum of the magnetic helicity density is \( h(k, t_0) = 2r \rho_B(k, t_0)/k \), where the parameter \( 0 \leq r \leq 1 \), corresponds to initially nonhelical, \( r = 0 \), and maximally helical, \( r = 1 \), fields. In our simulations we shall take that \( \mu_{\text{s}_{\text{d}}}(t_0) = \mu_{\text{s}_{\text{d}}}(t_0) = 1 \text{MeV} \). These initial conditions are quite possible in a dense quark matter in a hybrid star (HS) or in a quark star (QS) [8].

In figure 1 we show the amplification of the initial magnetic field \( B_0 = 10^{12} \text{G} \) by two or three orders of magnitude. One can see in figure 1 that the magnetic field reaches the saturated strength \( B_{\text{sat}} \). This result is analogous to the findings of [9–11]. For \( T_0 = 10^8 \text{K} \) in figures 1(a) and 1(b) \( B_{\text{sat}} \approx 1.1 \times 10^{14} \text{G} \); and for \( T_0 = 10^9 \text{K} \) in figures 1(c) and 1(d) \( B_{\text{sat}} \approx 1.1 \times 10^{15} \text{G} \). However, unlike [9–11], \( B_{\text{sat}} \) in figure 1 is defined entirely by \( T_0 \). The obtained \( B_{\text{sat}} \) is close to the magnetic field strength predicted in magnetars [1], especially if \( T_0 = 10^9 \text{K} \).

The time of the magnetic field growth to \( B_{\text{sat}} \) is several orders of magnitude shorter than in [9–11]. This fact is due to the smaller value of the electric conductivity \( \sigma_{\text{cond}} \) in quark matter in equation [4] compared to \( \sigma_{\text{cond}} \) for electrons in nuclear matter which we used in [9–11]. This fact can be understood on the basis of equation [3]. see also [2]. Moreover, we can see that short scale magnetic field should reach \( B_{\text{sat}} \) faster. The later fact, which was also established in [4,9–11], is confirmed by the comparison of figures 1(a) and 1(b) as well as figures 1(c) and 1(d).

In our model of the magnetic field generation, the thermal energy of background fermions is converted to the magnetic energy. One can say that a star cools down magnetically. The typical values of \( t_{\text{sat}} \) are \( \lesssim 10 \text{h} \) in figures 1(a) and 1(b) and \( \lesssim 10^2 \text{min} \) in figures 1(c) and 1(d). At such short time scales, other cooling channels, such as that due to the neutrino emission, do not contribute to the temperature evolution significantly. Therefore, unlike [4,9–11], we omit them in the present simulations.

In figure 1 we can see that, although the initial magnetic helicity can be different (see solid and dashed lines there), the subsequent evolution of such magnetic fields is almost indistinguishable, especially at \( t \sim t_{\text{sat}} \). It means that, besides the generation of a strong magnetic field, we also generate the magnetic helicity in quark matter. This result is in the agreement with [4,9–11].

In the present work we have applied the mechanism for the magnetic field generation, proposed in [4,4,9], to create strong large scale magnetic fields in dense quark matter. This mechanism is based on the magnetic field instability driven by a parity violating electroweak
Figure 1. The magnetic field versus time for different initial temperatures $T_0$ and minimal length scales $\Lambda_{B}^{(\text{min})}$. The solid lines correspond to initially nonhelical fields with $r = 0$ and dashed ones to the fields having maximal initial helicity, $r = 1$. (a) $T_0 = 10^8$ K and $\Lambda_{B}^{(\text{min})} = 1$ km. (b) $T_0 = 10^8$ K and $\Lambda_{B}^{(\text{min})} = 100$ m. (c) $T_0 = 10^9$ K and $\Lambda_{B}^{(\text{min})} = 1$ km. (d) $T_0 = 10^9$ K and $\Lambda_{B}^{(\text{min})} = 100$ m.

interaction between particles in the system. We have established the system of kinetic equations for the spectra of the magnetic helicity density and the magnetic energy density, as well as for the chiral imbalances, and have solved it numerically.

Although there is a one-to-one correspondence between the mechanisms for the magnetic field generation in [3,4,9–11], where we studied the case of NS, and in the present work, the scenario described here is likely to be more realistic. As mentioned in [12] the generation of the anomalous current in equation (1) is impossible for massive particles. Electrons in NS are ultrarelativistic but have a nonzero mass. The chiral symmetry can be restored only at densities $n \sim M_W^3 \sim 10^{46}$ cm$^{-3}$, that is much higher than one can expect in NS. Therefore the chiral magnetic effect for electrons as well as the results of Refs. [3,4,9,11] are unlikely to be applied in NS. Recently this fact was also mentioned in [12].

On the contrary, the chiral symmetry was found in [13] to be restored for lightest $u$ and $d$ quarks even at densities corresponding to a core of HS or in QS. Accounting for the existence
of the electroweak parity violating interaction between $u$ and $d$ quarks, we can conclude that the application of the methods of [3, 4, 9–11] to the quark matter in a compact star is quite plausible.

We have obtained that, in quark matter, the seed magnetic field $B_0 = 10^{12}$ G, which is typical in a young pulsar, is amplified up to $B_{\text{sat}} \sim (10^{14} – 10^{15})$ G, depending on the initial temperature. Such magnetic fields are predicted in magnetars [1]. Therefore HS/QS can become a magnetar. The obtained growth time of the magnetic field to $B_{\text{sat}}$ is much less than that in electron-nucleon case studied in [3, 4, 9–11]. It means that, in our model, strong magnetic fields are generated quite rapidly with $t_{\text{sat}} \sim$ several hours after a star is in a thermal equilibrium.

Note that, in the present work, instead of the quenching of the parameter $\Pi$ in equation [1] suggested in [9] to avoid the excessive growth of the magnetic field, we used the conservation of the total energy and the dependence of the electric conductivity on the temperature in equation [4]; cf. [10, 11]. It results in a more explicit saturation of the magnetic field, see equation [5] and figure [1].

Summarizing, we have described the generation of strong large scale magnetic fields in dense quark matter driven by the magnetic field instability owing to the electroweak interaction of quarks. The described phenomenon may well exist in the core of HS or in QS. We suggest that the obtained results can have implication to the problem of magnetars since the generated magnetic fields have strength close to that predicted in these highly magnetized compact stars.

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