REGULARIZATION AND THE POTENTIAL OF EFFECTIVE FIELD THEORY IN NUCLEON-NUCLEON SCATTERING

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This paper examines the role that regularization plays in the definition of the potential used in effective field theory (EFT) treatments of the nucleon-nucleon interaction. I consider NN scattering in S-wave channels at momenta well below the pion mass. In these channels (quasi-)bound states are present at energies well below the scale $m_\pi^2/M$ expected from naturalness arguments. I ask whether, in the presence of such a shallow bound state, there is a regularization scheme which leads to an EFT potential that is both useful and systematic. In general, if a low-lying bound state is present then cutoff regularization leads to an EFT potential which is useful but not systematic, and dimensional regularization with minimal subtraction leads to one which is systematic but not useful. The recently-proposed technique of dimensional regularization with power-law divergence subtraction allows the definition of an EFT potential which is both useful and systematic.

1 The potential of effective field theory

As described elsewhere in these proceedings, the rise of effective field theory as a technique in nuclear physics began with the seminal papers of Weinberg. These papers proposed implementing the EFT program in nuclear physics by applying the power-counting arguments of chiral perturbation theory to an n-nucleon effective potential rather than directly to the n-nucleon S-matrix. Only n-nucleon irreducible graphs should be included in the n-nucleon effective potential. The potential obtained in this way is then to be inserted into a Lippmann-Schwinger or Schrödinger equation and iterated to all orders. Of course, unknown coefficients appear in this effective potential, but these can be fit to experimental data as in ordinary chiral perturbation theory.

Such an EFT treatment of the $NN$ interaction differs in a fundamental way from conventional EFT applications like $\pi\pi$ scattering in chiral perturbation theory. In both cases operators are ordered in an effective Lagrangian in the same way. However, in $\pi\pi$ scattering the operator expansion in the effective Lagrangian maps to a power series in $k/M$ in the scattering amplitude. It is straightforward to see that EFT treatments where there is a direct mapping from the Lagrangian to the S-matrix are systematic. On the other hand,

\footnote{A more detailed description of much of the work discussed here was given in Ref.}
when the mapping is from the Lagrangian to an effective potential which is subsequently iterated to all orders, many issues arise which lead one to question the existence of a systematic power counting in the potential.

In order to discuss some of these issues I consider $NN$ scattering in the $^1S_0$ channel at momentum scales $k \ll m_\pi$. The EFT at these scales involves only nucleons since the pion is heavy and may therefore be “integrated out”. The effective Lagrangian thus consists of contact operators of increasing dimensionality constrained by spin and isospin. In any $S$-wave channel the operators which contribute take the form:

$$\mathcal{L} = N^\dagger \partial_t N - N^\dagger \frac{\nabla^2}{2M} N - \frac{1}{2} C(N^\dagger N)^2 - \frac{1}{2} C_2(N^\dagger \nabla^2 N)(N^\dagger N) + h.c. + \ldots \quad (1)$$

I do not intend that this EFT should provide a quantitative description of the $NN$ phase shifts. Instead, I study it because the scattering amplitudes can be calculated analytically. It therefore allows the elucidation of issues of principle in EFT for $NN$ scattering.

Such a Lagrangian leads to the following expansion of the potential in $S$-wave channels:

$$V(p', p) = C + C_2(p^2 + p'^2) + C_4(p^4 + p'^4) + C'_4 p^2 p'^2 + \ldots \quad (2)$$

The prejudice of effective field theory is that the coefficients in the potential should be “natural”, i.e. $C_{2n} \sim \frac{1}{m_\pi^{2n}}$. The potential (2) is then to be iterated via the Lippmann-Schwinger (LS) equation:

$$T(p', p; E) = V(p', p) + M \int \frac{d^3q}{(2\pi)^3} V(p', q) \frac{1}{EM - q^2 + i\epsilon} T(q, p; E). \quad (3)$$

The hope is that by adopting this procedure one can generate two-nucleon (quasi-)bound states at the experimentally-observed energies, which are “unnaturally” low, while maintaining “natural” coefficients in the potential. In doing this the expansion (2) must be truncated at some finite order in the quantities $p/m_\pi$ and $p'/m_\pi$. Provided $p, p' \ll m_\pi$ the neglected terms will be small.

Upon iteration of any truncation of (2) ultraviolet divergences arise. Hence, non-perturbative regularization and renormalization are required when iterating to all orders using the LS equation. Divergences arise because in solving the LS equation one integrates the potential over all momenta. Of course, the expansion (2) is not a truthful representation of the physics of the $NN$ potential for $p, p' > m_\pi$. Thus, a key question is whether all divergences can be...
regularized and renormalized in such a way that the momentum scales probed inside loops are ultimately well below $m_\pi$. If this cannot be done then it follows that the expansion (2) will be being used outside its domain of validity, and the $NN$ potential derived from effective field theory will not be any more systematic than the many phenomenological $NN$ potentials on the market.

So, the key question which I address in this paper is whether there is a regularization technique which leads to an expansion for the $NN$ potential that is both:

- systematic, i.e. can be truncated at a finite order and is then used only within the domain of validity of this truncation; and

- useful, i.e. when put in the regularized Lippmann-Schwinger equation, provides a reasonable description of the $NN$ phase shifts in what we would expect to be the domain of validity of the EFT, $k < m_\pi$.

Here this question is addressed using three different regularization methods. In Section 2 I discuss the regularization of the interaction by a momentum-space cutoff. I show how one can renormalize the coefficients in the potential, and how such an approach can provide a valid description of the $NN$ scattering data in the $^1S_0$ channel. This description can be progressively improved by adding more coefficients in the effective potential. However, in spite of this success, I will show that the typical momentum inside loops in the Lippmann-Schwinger equation is such that the potential is being used in a region where to truncate it at any finite order is not a justified procedure. (Similar conclusions are discussed elsewhere in this volume and in Ref. 14.) In Section 3 I review the failure of dimensional regularization with minimal subtraction as a candidate for the role of regulator in the Lippmann-Schwinger equation. In particular, I will explain why this approach leads to an expansion which is systematic but not useful. In Section 4 the use of the so-called PDS modification of dimensional regularization in this problem will be discussed. In particular, this scheme satisfies both of the above criteria, although the sense in which one is learning something about the underlying $NN$ potential remains somewhat unclear. Finally, in Section 5 I will offer some conclusions.

2 Effective Field Theory with Cutoffs

In this section I investigate the possibility of an EFT for $NN$ scattering in the presence of a finite cutoff. This physically intuitive approach has been advocated by Lepage 9,19. The idea is that one takes an underlying theory of $NN$ interactions and introduces a (sharp or smooth) momentum cutoff $\beta$ representing the scale at which the first new physics becomes important. All
loops now only include momenta $p < \beta$. Of course, one must compensate for the
effects of the neglected modes. However, Lepage argues that since these
modes are highly virtual, one may approximate their effects by a sequence of
local contact interactions. Furthermore, if the cutoff $\beta$ is placed well below the
mass $\Lambda$ of some exchanged quantum, then, for momenta $p$ and $p'$ below the
cutoff, the exchange of this quantum:

$$V_{\Lambda}(p', p) \sim \frac{1}{(p' - p)^2 + \Lambda^2};$$

may be replaced by a contact interaction, since $p', p < \beta < \Lambda$. Therefore the
effects coming from exchanges of quanta with masses well above the cutoff
scale $\beta$ may also be approximated by contact interactions. For the numerical
application of these ideas to the $NN$ problem see [6, 8, 9, 10, 11, 12].

Now, all that has been said in the previous paragraph still applies if the
cutoff $\beta$ is set below the scale $m_{\pi}$. Then the only explicit degrees of freedom in
the problem are nucleon modes with momenta below $\beta$. All higher-momentum
nucleon modes and all exchanged mesons are integrated out. This cutoff ef-
fective field theory of the $NN$ interaction is of little practical use, but can be
investigated analytically in a way that raises issues of principle. The effective
Lagrangian is that of Eq. (1). The effective potential which corres-
ponds to this Lagrangian now includes theta functions which introduce a sharp cutoff:

$$V(p', p) = [C + C_2(p^2 + p'^2) + \ldots] \theta(\beta - p) \theta(\beta - p'),$$

so all integrals (not just the divergent ones) will be cut off sharply at mo-
momentum $\beta$. After renormalization the coefficients $C$, $C_2$, etc., will, of course,
depend on the cutoff scale $\beta$, as well as on physical scales in the problem.

Of course, the expression (5) is an infinite series, and for practical compu-
tation some method of truncating it must be found. The fundamental philosophy
of cutoff EFT provides a rationale for this as follows. If we work to any finite
order in the effective potential, cutoff-dependent terms in the scattering am-
plitude will appear. These must be in correspondence with neglected higher-order
operators in $V$. If such terms are progressively added to $V$, one may remove
the cutoff dependence order-by-order. Below we will see that one can indeed
define such a “systematic” procedure in this problem. However, in order that
it really make sense to truncate the effective potential (5) at some finite order
it must be that the operators which are neglected are in some sense small.
2.1 “Second order” cutoff EFT calculation

I now investigate whether this is indeed the case by looking at the amplitude and renormalization conditions which arise when one takes the “second-order” effective potential:

\[ V^{(2)}(p', p) = [C + C_2(p^2 + p'^2)] \theta(\beta - p) \theta(\beta - p'), \]

and iterates it via the Lippmann-Schwinger equation. Standard methods lead to the amplitude:

\[ \frac{1}{T_{\text{on}}(k)} = \frac{(C_2 I_3 - 1)^2}{C + C_2^2 I_5 + k^2 C_2 (2 - C_2 I_3)} - \mathcal{I}, \]

where

\[ I_n \equiv -M \int \frac{d^3 q}{(2\pi)^3} q^n, \]

and

\[ \mathcal{I} \equiv -M \int \frac{d^3 q}{(2\pi)^3} \frac{1}{k^2 - q^2 + i\eta} = I_1 - \frac{iMk}{4\pi} + Mk^2 \mathcal{P} \int \frac{dq}{2\pi^2} \frac{1}{k^2 - q^2}. \]

In this calculation all integrals are understood to be sharply cutoff at \( q = \beta \), and \( k = \sqrt{ME} \) is the on-shell momentum. The \( \mathcal{P} \) in Eq. (10) indicates a principal value integral.

I now renormalize by demanding that, up to terms of \( O(k^4) \), Eq. (6) reproduce the inverse amplitude obtained when only the first two terms in the effective range expansion are retained

\[ \frac{1}{T_{\text{on}}(k)} = \frac{M}{4\pi a} \left[ -\frac{1}{a} + \frac{1}{2} \frac{r_e k^2 - ik}{r_e k^2 - ik} \right]. \]

This yields the following equations for \( C \) and \( C_2 \):

\[ \frac{M}{4\pi a} = \frac{(C_2 I_3 - 1)^2}{C + C_2^2 I_5} - I_1; \]

\[ \frac{Mr_e}{8\pi} = \left( \frac{M}{4\pi a} + I_1 \right)^2 \frac{C_2 (2 - C_2 I_3)}{(C_2 I_3 - 1)^2} + \frac{M}{2\pi^2 \beta}, \]

where the last term in Eq. (13) arises because the presence of the cutoff generates additional energy dependence when the integral \( \mathcal{I} \) is evaluated.
Once $\beta$ is fixed these equations can be solved for $C$ and $C_2$. Of course, as $\beta$ is varied the $C$ and $C_2$ that satisfy Eqs. (12) and (13) will change significantly. However, because one is fitting to low-energy scattering data different values of $\beta$ will not lead to any fundamental differences in the low-energy T-matrix. Since $C$ and $C_2$ are fit to the first two terms in the effective-range expansion sensitivity to the cutoff appears in the on-shell inverse amplitude at order $(k/\beta)^4$. The results of this procedure for $NN$ phase shifts in the $^1S_0$ channel are shown in Fig. 1, for a range of cutoff values, from $\beta = 150$ MeV to $\beta = \infty$. Also shown are the $NN$ phase shifts in the Nijmegen group’s phase shift analysis, and the amplitude (11). Note that this amplitude yields an excellent fit to the experimental data up to momenta $k$ of order $m_\pi$. The amplitudes obtained in the cutoff EFT also do a reasonable job of describing the data, especially as the cutoff is increased. These results, and the similar in spirit though much more thorough studies of Refs. [9,10,11], show that the potential in cutoff effective field theory satisfies the second of our two conditions: it is useful.

Furthermore, working to higher order in the potential will progressively improve these fits. If I constructed the “next-order” effective potential, $V^{(4)}$ and refitted the (four) coefficients appearing in it to the first four terms in the effective range expansion, then, by construction, sensitivity to the cutoff will appear in $k \cot \delta$ at $O((k/\beta)^8)$. In this sense the introduction of higher-dimension operators does allow for the “systematic” removal of cutoff dependence in the amplitude. This is the systematicity seen in the work of Scaldeferri et al., Lepage[1], and Steele and Furnstahl[10,11]. Nevertheless, this does not yield a systematic EFT potential in the sense I have defined it here. After all, any parameterization of a potential which is rich enough to successively fit the terms in the effective-range expansion will be similarly improved as one adds parameters to it. The question I address here is whether one can use the power counting to argue a priori that certain contributions to the potential of Eq. (5) will be systematically small and hence can be neglected at some specified level of accuracy.

### 2.2 Power counting in this effective field theory

To answer this question I must examine the values of $C$ and $C_2$ which are required to solve the equations above. Assuming that the cutoff obeys $1/a \ll \beta$ it follows that the second of these two equations becomes

$$\frac{M}{8\pi} \left( r_e - \frac{4}{\pi\beta} \right) \approx \frac{I_2^2}{I_3} \frac{C_2(2 - C_2)}{(C_2 - 1)^2},$$

(14)
where \( \overline{C}_2 \equiv C_2 I_3 \). This leads to a quadratic equation for \( \overline{C}_2 \), which for values of \( \beta \) up to some \( \beta_{\text{max}} = \frac{16 \pi}{\beta} \) has real solutions. For \( \beta > \beta_{\text{max}} \) the renormalization condition for \( C_2 \) has no real solution if \( r_e > 0 \). This is related to the fact that a Hermitian potential of range \( R \) which is to be used in the Schrödinger equation can only yield an effective range \( r_e \) consistent with \( [22,23,24] \).

\[
\frac{r_e}{\alpha} \leq 2 \left( R - \frac{R^2}{\alpha} + \frac{R^3}{\alpha^2} \right). \tag{15}
\]

As I take \( \beta \to \infty \) I force the range of our EFT potential to zero. So, if \( r_e > 0 \) is to be obtained, the potential must become non-Hermitian. However, Fig. 1 shows that one still obtains perfectly sensible phase shifts even if this is the case. Another way to evade the bound (15) and so produce positive effective ranges with a zero-range force is via the introduction of a dibaryon field \( [25,26,27] \). This idea is discussed further in other contributions to these proceedings \([28,29,30]\).
As for the scaling of the coefficients $C$ and $C_2$, Eqs. (14) and (12) lead to
\[
C_2 \sim \frac{1}{M^3} \Rightarrow C \sim \frac{1}{M^3}.
\]

Note that this behavior does not arise if $a$ is natural, i.e. of order $1/m_\pi$, and $\beta$ is chosen to be less than $m_\pi$; then the leading order behavior of the coefficients $C$ and $C_2$ is very different. In fact,
\[
C \sim \frac{1}{Mm_\pi}; \quad C_2 \sim \frac{1}{Mm_\pi^2 \beta}.
\]

In this case all loop effects coming from $C_2$ are suppressed by a factor of at least $(\beta/m_\pi)^2$. Indeed, all loop effects are suppressed by a factor of at least $\beta/m_\pi$. Therefore, if $\beta < m_\pi$ a non-perturbative calculation is not necessary. In other words, if the experimental parameters are natural then cutoff field theory with $\beta < m_\pi$ gives a perturbative EFT in which loop graphs are consistently suppressed. However, if a perturbative calculation is performed then the regularization scheme chosen becomes immaterial, as the short-distance physics may be renormalized away.

On the other hand, for unnaturally long scattering lengths Eq. (16) shows that:
\[
\frac{C_2}{C} \sim \frac{1}{\beta^2}.
\]

Consequently the condition for us to be able to truncate the expansion of the potential at zeroth order would be $\tilde{p}^2 \ll \beta^2$. It is easy to give a heuristic justification of why the behavior (18) arises in a non-perturbative cutoff EFT calculation, and why to expect similar behavior to all orders in the effective potential. After all, the choice of a theta function to regulate the momentum-space integrals as in Eq. (5) is entirely arbitrary. All that has been said above could be reformulated with a smooth cutoff. This would result in an effective potential of the form
\[
V(p',p) = [\tilde{C} + \tilde{C}_2(\tilde{p}^2 + p'^2) + \ldots]g(\tilde{p}^2/\beta^2, p'^2/\beta^2)
\]
where $g(x,y)$ obeys $g(0,0) = 1$, $g(x,y) = g(y,x)$ and $g(x,y) \to 0$ faster than any power of $x$ as $x \to \infty$ with $y$ held fixed. In a non-perturbative calculation the effective potential should be essentially unaltered by this change in the form of the cutoff. However, this necessarily means that the ratios $\tilde{C}_{2n}/\tilde{C}$ differ from those $C_{2n}/C$ by terms of order $1/\beta^{2n}$. Therefore for a generic cutoff function $g$ the ratio $C_{2n}/C$ must be of order $1/\beta^{2n}$.

Now, if the ratio $C_{2n}/C$ goes like $1/\beta^{2n}$, then in order to justify a systematic truncation of the effective potential we require $\tilde{p}^{2n} \ll \beta^{2n}$. However, the
effective potential is to be used in a momentum regime which extends up to \( \beta \), and at the upper end of this momentum regime it is clear that all terms in the expansion for \( V \) are equally important.

Of course, if internal loops were dominated by the external momentum, \( k \), and so \( \hat{p} \approx k \), then this behavior of the coefficients would not be cause for concern, since \( k \ll \beta \) can be maintained. However, virtual momenta up to \( \beta \) flow through all internal loops. Therefore I believe it makes sense to consider a quantum average in testing to see if \( \hat{p}^{2n} \ll \beta^{2n} \), since such an average is sensitive to these virtual effects.

All arguments about the size of operators in the effective action would apply equally well if there was a low-energy bound state in the channel under consideration. So, let us evaluate quantum averages of the operator \( \hat{p}^{2n} \) using the bound-state wave function obtained from the zeroth-order EFT potential. The zeroth-order potential yields a wave function for the bound state of energy \( E = -B \),

\[
\psi^{(0)}(p) = N \frac{M}{MB + p^2} \theta(\beta - p),
\]

where \( N \) is some normalization constant. For \( MB \ll \beta^2 \) this gives

\[
\langle \hat{p}^{2n} \rangle \frac{1}{\beta^{2n}} \equiv \frac{\langle \psi^{(0)}(p) \hat{p}^{2n} | \psi^{(0)}(p) \rangle}{\beta^{2n}} = \frac{4}{(2n-1)\pi} \frac{\sqrt{MB}}{\beta}, \quad n = 1, 2, \ldots .
\]

Thus, \( \hat{p}^{2n} \ll \beta^{2n} \) is apparently satisfied. However, Eq. (21) shows that if \( \langle V \rangle \) is calculated with the wave function \( \psi^{(0)} \) there is no reason to truncate the expansion at any finite order, since all terms beyond zeroth order contribute with equal strength to the quantum average.

If there was systematic power counting for the \( NN \) potential in cutoff field theory then the contribution of these “higher-order” terms in the potential should get systematically smaller as the “order” is increased. However, it is clear that this does not happen—rather, all terms beyond zeroth order contribute to the potential at the same order. Therefore one cannot justify a truncation of Eqs. (5) and (19) at some finite order in \( p \) and \( p' \). Such a truncation may result in a good fit to the experimental data for on-shell momenta \( k \ll \beta \), but it is not based on a systematic expansion of the \( NN \) potential in powers of momentum.

3 Dimensional Regularization with Minimal Subtraction

In ordinary perturbative EFT calculations the existence of a consistent power-counting scheme in the S-matrix relies on removing the short-distance physics
that arises in loop graphs via the renormalization procedure. Since the short-distance physics is removed in a manner insensitive to choice of regularization scheme it is economical to use dimensional regularization (DR) to regularize and renormalize, because DR respects chiral and gauge symmetries. When considering the relevance of EFT methods in nuclear physics one might choose to extrapolate intuition gained from perturbation theory and regulate the divergent loops in our Lippmann-Schwinger equation using DR. It is important to realize that the use of DR implicitly assumes that the short-distance physics buried in loop graphs does not contribute to low-energy physics.

In this section I compare dimensional regularization with minimal subtraction (DR with $\overline{\text{MS}}$), as implemented by Kaplan et al. in their 1996 paper, and cutoff schemes. There are two fundamental points I wish to make

- The conclusions which one reaches about both the underlying potential and the usefulness of the effective field theory are different in the two different regularization schemes.
- When low-lying (quasi-)bound states are present, a necessary (but not sufficient) condition for a workable EFT treatment of $NN$ scattering is that some short-distance effects from loops contribute to the physical scattering amplitude.

The expression displayed above is in fact true in any regularization scheme. In regularization schemes other than the cutoff method the integrals will be defined in different ways. We now use DR with $\overline{\text{MS}}$ to regulate the integrals $I_1$, $I_3$, and $I_5$ which appear in Eq. (7). This is a convenient way to implement an idea which is central to the success of perturbative EFT: the power-law divergent pieces of integrals over internal loop momenta should not affect the final physical scattering amplitude. In DR with $\overline{\text{MS}}$ all power-law divergences vanish, therefore $I_1 = I_3 = I_5 = 0$. Consequently, the on-shell amplitude takes the form

$$
\frac{1}{T^\text{on}(k)} = \frac{1}{C^\overline{\text{MS}} + 2C_2^\overline{\text{MS}}k^2} + \frac{iMk}{4\pi}.
$$

(22)

Renormalizing so as to reproduce the effective-range expansion to second order leads to the values

$$
C^\overline{\text{MS}} = \frac{4\pi a}{M}; \quad C_2^\overline{\text{MS}} = \frac{\pi a^2 r_e}{M}.
$$

(23)

Hence, in this instance we conclude that the expansion of the EFT potential has a domain of validity $k^2 \ll 2/(ar_e)$, as that is the point at which the
second term in the expansion of the EFT potential becomes as large as the first. This is a natural region if both \( a \) and \( r_e \) are natural, as then the EFT is valid for \( k^2 \ll \Lambda^2 \). However, if the scattering length is unnaturally large the momentum domain over which the EFT obtained when DR with \( \overline{\text{MS}} \) is used is small. In fact, as noted by Kaplan et al., and shown in Fig. 2, Eq. (23) reproduces the data in the \( ^1S_0 \) extremely poorly, since it only agrees with the phenomenologically efficacious amplitude (11) at very small \( k \). In order to restore the agreement higher-dimensional operators, all containing powers of the low-energy scale \( 1/a \), would have to be added to the theory. Thus, in contrast to the results obtained with cutoff regularization, where we concluded that the effective theory was valid for \( k^2 \ll \beta^2 \leq m^2_\pi \), here we conclude that the domain of validity of the EFT is \( k^2 \ll 1/(ar_e) \). So, different conclusions about the effectiveness of the EFT are reached when different regularization schemes are chosen.

In fact, since there are no logarithmic divergences in this problem and the loop graphs have no finite real part, it is straightforward to show that DR with \( \overline{\text{MS}} \) gives an amplitude in which only the absorptive parts of the loop graphs are retained. In other words, using DR with \( \overline{\text{MS}} \) is equivalent to making a power-series expansion in the momentum \( k \) for the on-shell K-matrix, and then unitarizing the result.

Note also that from Eq. (23) we would infer that the \( NN \) EFT potential was real, in sharp distinction to the conclusions implied by the Wigner bound. However, this distinction occurs because \( C \) and \( C_2 \) do not represent coefficients in an expansion of a quantum mechanical potential. Indeed, as explained above, they really represent coefficients in an expansion of the on-shell K-matrix. Thus, the connection of the amplitude \( C + C_2(p^2 + p'^2) \) to the underlying dynamics is less transparent if DR with \( \overline{\text{MS}} \) is used than in the cutoff approach of Section 2.

The use of DR with \( \overline{\text{MS}} \) does lead to an effective field theory which is systematic. Since only the on-shell part of internal loops is retained the EFT “potential” is used at all times only within the domain where the expansion is valid, provided that on-shell momenta \( k \ll 1/a \) are considered. However, as discussed above, this is a small energy domain, and so the resulting EFT is not particularly useful. This occurs because when there are low-lying bound states the low-energy scale \( 1/a \) still sets the scale of the coefficients in the “potential” of the DR with \( \overline{\text{MS}} \) calculation. This was the problem which iterating the “potential” via the LS equation was supposed to avoid.

However, this failure of dimensional regularization with minimal subtraction is not particularly surprising when one considers that in quantum mechanics an unnaturally large scattering length can occur via the cancelation
between a natural “range” and a natural “strength” of a potential. The implementation of this general cancelation between range and strength must be an element of any useful EFT description of NN scattering. DR with \( \overline{\text{MS}} \) discards all short-distance physics that comes from loops. This follows necessarily from its being a scale-independent regularization scheme. In general, information about the range of the potential enters through these power-law divergences, and so DR with \( \overline{\text{MS}} \)'s neglect of all power-law divergences means it does not retain this information on the range of the interaction. Hence, in DR with \( \overline{\text{MS}} \) the coefficients in the Lagrangian are forced to take on unnatural sizes, and although the resulting effective field theory is systematic, it is not useful.

4 Dimensional regularization with power-law divergence subtraction

This raises the question of whether we can modify the minimal subtraction scheme in a way that avoids these problems. Recently Kaplan et al. have proposed a different subtraction scheme, power-law divergence subtraction (PDS), which attempts to do precisely this. This scheme is described more fully elsewhere in these proceedings. Its success is based on its modification of the usual DR prescription of ignoring all power-law divergences. In PDS a term corresponding to a linear divergence is included in the definition of the divergent integral \( \mathcal{I} \), where

\[
\mathcal{I}(k) = \left( \frac{\mu}{2} \right)^{4-D} M \int \frac{d^{(D-1)}p}{(2\pi)^{D-1}} \frac{1}{k^2 - p^2 + i\eta},
\]

is the integral of Eq. (9) evaluated in \( D \) space-time dimensions, and \( \mu \) is an arbitrary scale. This is achieved by adding a piece to \( \mathcal{I} \) which cancels its logarithmic divergence in \( D = 3 \). When the resulting subtracted integral is continued back to \( D = 4 \) we get:

\[
\mathcal{I}^{\text{PDS}} = -\frac{M}{4\pi} (ik + \mu). \tag{25}
\]

In terms of the integrals \( I_1, I_3, \) and \( I_5 \) discussed above we still have \( I_3 = I_5 = 0 \), but now \( I_1 = -\frac{M\mu}{4\pi} \). The additional piece added to cancel the logarithmic divergence in \( D = 3 \) corresponds to terms linear in \( \beta \) in the cutoff EFT approach discussed in Section 2. Note that PDS reduces to the result obtained in the \( \overline{\text{MS}} \) scheme if \( \mu = 0 \).

From Eq. (25) it is trivial to see that when we solve the effective field theory by taking the second-order tree-level amplitude, iterating it via the Lippmann-Schwinger equation, and using DR with PDS to regulate the loops, we get an
amplitude

\[ \frac{1}{T^{\text{on}}(k)} = \frac{1}{C^{\text{PDS}}(\mu) + 2C^{\text{PDS}}_2(\mu)k^2} + \frac{M\mu}{4\pi} + \frac{iMk}{4\pi}. \]  

(A similar amplitude has recently been written down independently.) In the language used in the previous section the amplitude \((26)\) may be obtained from that found in the MS scheme by the addition of a constant dispersive part to all loops.

As is discussed in Refs.\[16,17,18\] it is now a simple matter to show that, with a suitable choice of \(\mu\), this approach is both systematic and useful. The coefficients \(C(\mu)\) and \(C_2(\mu)\) may be adjusted to fit the scattering length and effective range. This leads to

\[ C^{\text{PDS}}(\mu) = \frac{4\pi}{M} \frac{1}{-\mu + 1/a}; \quad C^{\text{PDS}}_2(\mu) = \frac{4\pi}{M} \left( \frac{1}{-\mu + 1/a} \right)^2 \frac{r_e}{4}. \]  

The PDS amplitude with various choices of \(\mu\) is compared with the effective range expansion in Fig. 2. Note that a good result is obtained when \(\mu\) is chosen to be of order, but larger, than \(k\). Thus, with a suitably large choice for \(\mu\), PDS gives a useful EFT for \(NN\) scattering in the \(^1S_0\) channel. The EFT thus obtained is also systematic, since in a theory with an unnatural scattering length and a natural effective range the scaling of the coefficients for \(\Lambda \gg \mu \gg 1/a\) is

\[ C_{2n} \sim \frac{1}{M\mu^{n+1}A^n}. \]  

Now we see that there will be a valid reason to truncate the expansion for \(V\):

\[ V = \sum_{n=0}^{\infty} C_{2n} \hat{p}^{2n}, \]  

since terms beyond zeroth order will be suppressed by powers of the on-shell momentum over \(\Lambda\).

Thus, in PDS Kaplan et al. have proposed a definite solution to the difficulties which have beset the effective field theory in the \(NN\) interaction. They

\[ ^{b} \text{Note that in using PDS in this way I am not following the approach of its inventors. They use the derived scaling of the coefficients } C_{\text{PDS}}(\mu) \text{ and } C^{\text{PDS}}_2(\mu) \text{ to infer that all operators beyond the lowest-order one } C(N\dagger N)^2 \text{ should be treated perturbatively. Here I iterate all operators in the tree-level amplitude to all orders. If the higher-dimensional operators are truly suppressed then the difference between this approach and that advocated by Kaplan et al. should be small.} \]
I have modified the usual DR with \( \overline{\text{MS}} \) prescription just enough to implement the cancelation between range and strength which allows quantum mechanical potentials with natural strengths and sizes to lead to unnaturally shallow bound states. In doing so they have generated an expansion for the \( NN \) interaction which is both useful and systematic. However, it should be noted that the interaction thus obtained is not really a usual quantum mechanical “potential”, since PDS, by construction, does not include all of the effects of virtual momenta inside the loops.

I will conclude this section by raising two questions about PDS which intrigue me:

1. Are other modifications of DR with \( \overline{\text{MS}} \) possible, and what do they do to the power counting? In particular, why doesn’t the introduction of a term which mimics the effects of terms linear in a cutoff parameter
lead to the scaling of the coefficients found in Section 2. If PDS was completely equivalent to a cutoff theory, and \( \mu \) played the role of the cutoff then we would expect the scaling

\[
C_{2n} \sim \frac{1}{M\mu^{2n+1}}. \tag{30}
\]

If such scaling behavior were found we would be forced to conclude that power-counting had broken down again, and so PDS’s avoiding this behavior is a key ingredient to its success. Consequently, it would be interesting to know why PDS does not lead to the behavior \( (30) \).

2. The demand that all power-law divergences vanish in DR with \( \overline{\text{MS}} \) can be traced to DR’s requirement that loop integrals such as

\[
\int d^d q \frac{1}{q^n} \tag{31}
\]

should be scale invariant. This is an extremely useful property of DR, since it makes the preservation of chiral and gauge symmetries much easier than it is in a scale-dependent regularization such as Pauli-Villars. However, PDS also leads to a scale-dependent definition for integrals such as \( (31) \), since they acquire a dependence on \( \mu \). It remains to be seen how this presence of an artificial scale in such integrals affects gauge and chiral invariance.

5 Conclusion

In summary, neither of the two obvious regularization methods, DR with \( \overline{\text{MS}} \), or a cutoff, lead to an expansion for the \( NN \) interaction which is both systematic and useful. The expansion obtained using a cutoff gives a reasonable fit to the phase shift data, but from it we learn nothing about the underlying short-range potential, since operators neglected in that potential are generally not smaller than those included. By contrast, we come to different conclusions about both the domain of validity of the effective theory and the systematicity of the approach when DR with \( \overline{\text{MS}} \) is used. DR with \( \overline{\text{MS}} \) fails to give a useful description of the \( NN \) amplitude, because it does not implement the cancelation between “range” and “strength” which allows potentials with hadronic-scale ranges and depths to produce bound states with nuclear energies. By modifying DR with \( \overline{\text{MS}} \) to include part of the effects of the range of the potential, namely the linear divergences, Kaplan et al. have constructed a useful and systematic approach to the effective field theory of the \( NN \) interaction.
However, it is not clear how the resulting coefficients $C_{2n}$ which are defined in the PDS scheme would be related to any underlying NN potential. So, it seems that regardless of the regularization scheme used one actually learns little about the short-range NN potential. Indeed the meaning of the operator $C + C_2(p^2 + p'^2)$ is completely different in the three different regularization schemes considered. This is not necessarily a problem. After all, the quantum mechanical potential is not an observable. However, if we wish to systematically use chiral expansions of the sort proposed by Weinberg in order to describe the physics of the NN potential in a controlled way we need to be reassured that the momenta for which we are expanding the potential are small. This is not obviously true inside loops in the Lippmann-Schwinger equation, and that casts doubt on the validity of such a program.

Salvation for Weinberg’s arguments occurs because nuclear wave functions are typically dominated by momenta of scale $m_\pi$ or below. Once wave functions with those scales are generated, whether by fair means or foul, Weinberg’s power-counting may enable the inclusion of the physics of pion exchanges in a way that respects chiral symmetry and is systematically improvable. Therefore, while the potential of effective field theory is not observable, the potential of effective field theory in nuclear physics remains large and attractive.

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