Study of the tension of a heterogeneous hydroelastic system

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Abstract. The purpose of the article is to analyze seismic-acoustic wave fields and stress-strain state in a complex heterogeneous seismogenic structure of the contact area of the continental and oceanic crust, taking into account two types of impact: technogenic and pressure of the oceanic water column. The study uses the theory of block structures and the block element method, developed on the basis of a topological approach. It is shown that the contact stress can be considered as a superposition of stress waves occurring at the interface between the liquid and elastic media. The number of waves, their amplitude and frequency are determined by the physical and geometric characteristics of the volume of the liquid, the elastic semi-bounded layer, the elastic foundation, as well as the oscillation frequency of the external harmonic load. The model can be useful in the design of a transport network, production facilities on the coastal territory, since it allows one to assess seismic risks before the start of the process, as well as the risks of associated hazardous exogenous phenomena - landslides, tsunamis, and volcanism activation.

1. Introduction
The article presents new results that can be useful for studying dangerous endogenous processes in the densely populated and actively developed territory of the North Caucasus, in particular, the Azov-Black Sea region of the Krasnodar Territory. The manifestations of endogenous activity include the high seismic hazard of the region, Late Cenozoic volcanism and mud volcanism.

Natural disasters in the densely populated, nationally and socially problematic territory of the North Caucasus are fraught with serious environmental, political and economic consequences. In addition, the region is currently being intensively developed. Numerous critical structures (roads, pipelines, bridges, unique high-rise buildings, Olympic facilities) are under construction and have already been built [1]. Therefore, the study of the causes of the occurrence of strong earthquakes, other dangerous endogenous processes and accompanying exogenous catastrophic phenomena is an urgent scientific task.

The high seismic hazard in the North Caucasus is obvious [2, 3]. In addition, the region is characterized by a high population density and mainly low quality of civil buildings.

The study of catastrophic natural phenomena, primarily strong earthquakes, is an important part of a more general and very urgent problem of ensuring environmental safety and security of the population, especially in view of a noticeable increase in damage from strong earthquakes and associated landslides, tsunamis, etc. A reliable prediction of strong earthquakes in a practical aspect in the world has not yet been resolved, although these phenomena have become the objects of serious priority scientific research and protective measures, both in Russia and abroad.

There are several approaches to assessing seismicity [4-12].
Studies carried out by traditional seismic-tectonic methods are aimed at finding precursors, that is, geological criteria for seismicity. They are based on a comprehensive comparative analysis of the geological structure and the latest structure, using seismic statistics methods. The areas of increased seismic hazard identified in this case are located mainly where relatively strong earthquakes have already occurred.

The second approach, the extra-regional seismotectonic method, is the result of many years of research in this direction. The essence of the method is to conduct geodynamic zoning of the earth's crust, independent of seismological data, using features that characterize its current structure and state, using cluster analysis of a complex of geological and geophysical data.

The third approach is to analyze the tension of seismogenic structures responsible for the realization of the seismic potential of the region [13, 14]. According to the results of the analysis of the locations of earthquake foci and their intensity, one of such geological structures is the contact area of the continental and oceanic crust. Such structures make mutual slow horizontal movement. This process is accompanied by the gradual subsidence of the oceanic plate under the continental plate (plate subduction). It is obvious that the state of such a mechanical system is significantly influenced by a layer of oceanic or sea waters. Structures of this type are widespread in the Krasnodar Territory, the North Caucasus and Russia as a whole. Below, we propose a method for studying seismic-acoustic wave fields in a complex heterogeneous seismogenic structure of the contact area of the continental and oceanic crust, taking into account two types of impact: technogenic and pressure of the oceanic water column.

2. Method for studying the stresses of seismogenic structures of complex structure

Dynamic processes in the structures of the earth's crust, despite the large number of studies devoted to them, are far from complete description today. A number of approaches to predicting earthquakes requires studying the stress-strain state of lithospheric structures.

The complex structure of geological structures, the presence of inhomogeneities (faults of different thickness, inclusions of media of different rheology) significantly affect the propagation of wave fields generated by various sources.

The block structures of the geophysical medium, considered in the work, are distinguished by complex geometry, the materials of individual blocks have significantly different physical and mechanical properties. This makes it practically impossible to apply the methods of solid mechanics to describe wave processes.

In modern geophysics, direct numerical methods (finite difference, finite element method), the method of fundamental solutions, variants of the boundary element method, as well as methods based on variational approaches have become widely used.

None of these approaches can be considered universal. The classical schemes of the indicated methods are applicable only in limited regions. To estimate the energy carried away to infinity from the source, the questions arise of conjugation of the obtained solutions with the asymptotic behavior of the far zone. In addition, to estimate the energy carried by individual modes, it is necessary to use special signal processing methods.

The described problems, generated by the formulation of boundary value problems that simulate the behavior of wave fields in the block structures of the geophysical medium, are quite complex and require the development and application of a set of new methods. The topological approach used in this work - the block element method - is an effective tool for studying the dynamic behavior of geological structures modeled by a block-layered elastic medium in contact with blocks of a different rheology. The block element method has several advantages over the existing computational finite and boundary element methods. It serves as a qualitatively new addition to existing methods, allows one to study boundary value problems that simulate natural and man-made processes, for which direct numerical ones are ineffective [14, 15].

The use of the integral approach and its generalization - the differential factorization method - makes it possible to find the optimal representation of the solution, which is convenient for the analytical study
of the problem and numerical analysis, as well as to describe the behavior of solutions at infinity and to isolate the waves carrying away energy.

The method is implemented in three stages. At the first stage, the investigated geological fault-block structure is associated with the block structure as a set of separate blocks, the material of which has different mechanical, physical chemical properties and its own internal structure. Further, in each block, its own boundary problem is set, corresponding to the process under study, the connection of the blocks is carried out through interblock boundary conditions. Further, the boundary value problem is reduced to a scalar or matrix functional equation. At the second stage, the obtained functional equation is investigated, and an automorphism is realized on the block element, which leads to a system of pseudodifferential equations. Their solution gives us the opportunity to get a solution to the problem in each block. At the third stage, applying homeomorphisms and constructing factor-topological spaces, we sew the solutions for individual blocks into a solution for the block structure.

3. Mathematical formulation of the problem

The problem of assessing the stress-strain state and seismic-acoustic wave fields in a complex heterogeneous seismogenic structure of the contact area of the continental and oceanic crust is mathematically reduced to studying the distribution of contact stresses at the interface between an elastic foundation and a semi-bounded fluid basin. We consider a block structure (Figure 1).

![Figure 1. Seismogenic structure of the contact area of the continental and oceanic crust](image)

Block 1 is a semi-bounded elastic layer simulating a section of the continental crust.

Block 2 is a semi-bounded layer that models a fluid mass (ocean, sea).

Block 3 - non-deformable elastic foundation, simulates the oceanic crust. A homogeneous elastic half-space or a homogeneous elastic layer rigidly adhered to a non-deformable base can be considered as a base.

Time-harmonic oscillations in the system under study are modeled by a distributed load \( \{0, 0, q_0\} \) and correspond to a certain technogenic impact (production process, the impact of transport infrastructure, extraction of minerals). A steady-state oscillation process is considered, i.e. the dependence of all unknown and given functions on time is determined by a factor \( e^{-i\omega t} \).

The motion of points in the liquid, described by the velocity potential, satisfies the wave equation:

\[
\Delta \phi(x, z) = -\frac{\omega^2}{c^2} \phi(x, z).
\]

The displacements of the constituent elastic media satisfy the Lamé equations, represented for steady-state wave processes in the form:

\[
(\lambda + \mu) \text{grad} \text{div} \mathbf{u} + \mu \Delta \mathbf{u} + \rho \omega^2 \mathbf{u} = 0.
\]

The material of elastic blocks has the following characteristics: \( \rho \) – density; \( \lambda, \mu \) – Lame constants. \( c \) – the speed of sound in the liquid; \( \omega \) – frequency of harmonic vibrations.
On the vertical boundary of the liquid and the elastic semi-bounded layer, the impermeability conditions apply.

The conditions of continuity of displacements and stresses are taken as the conditions for conjugation at the boundary of the elastic semi-bounded layer and the base.

The interaction of the liquid medium and the elastic foundation is determined by the equality of the vertical components of the velocities of the points of the liquid and the elastic medium in the contact zone.

If an elastic half-space is considered as a base, the condition decreasing displacements during infinity and the radiation condition.

As a result of the implementation of the algorithm of the block element method [15] for the investigated system "Liquid - Elastic base", systems of pseudodifferential equations were obtained. Their transformation made it possible to construct and solve integral equations of the first kind with a kernel depending on the difference and the sum of the arguments, and to obtain expressions for determining the contact stresses at the interface between a liquid and an elastic medium. Then integral representations of solutions for displacements of points of an elastic foundation are calculated, as well as an explicit expression for the potential of velocities in a fluid. The inverse of the integral for the displacements of the points of the elastic foundation is performed numerically.

4. Results and Discussion

Numerical calculations were performed for a model structure. Water was chosen as a liquid. The following parameters were chosen as variable parameters: the thickness of the liquid layer, its density, the velocities of the longitudinal and transverse waves, and the frequency of oscillations of the external source. For various combinations of these parameters, the values of contact stresses at the interface between the liquid and elastic medium for the layer and half-space are obtained. Analysis is also possible for other combinations of geometric and mechanical parameters.

The study showed that in all cases of the initial variable parameters, the contact stress can be considered as a superposition of stress waves occurring at the block interface. The physical and mechanical characteristics of these waves, such as the amplitude, frequency, and their number, depend on the specified variable properties of the material in the sides of the structure under study and its geometric parameters.

After conducting the amplitude-frequency analysis of the system under study, it was concluded that, depending on the vibration frequency of the external source, it is possible to obtain different dependences for the blocks of an elastic semi-bounded layer and an elastic foundation in the form of a layer or half-space:

1) In the case of a half-space, the critical frequencies are those at which the wave number of the longitudinal vibrations of the elastic foundation coincides with the wave numbers of the natural vibrations of the fluid. In this case, there is a sharp, almost exponential increase in the amplitude of contact stresses.

2) There is a dependence of the nature of the growth of the amplitude of contact stresses on the vertical size of the liquid block. With an increase in the vertical dimension, a sharp, almost exponential increase in the amplitude of contact stresses corresponds to lower values of the frequencies of the external load.

3) There is a dependence on the stiffness of the base: a more rigid base corresponds to lower extreme values of the amplitude of contact stresses while maintaining other mechanical and geometric characteristics of the structure blocks and frequencies of external harmonic action.

4) In the case of a block of an elastic foundation in the form of a layer, a different dependence of the amplitude of contact stresses on the frequency of the external harmonic load is observed. Under harmonic influences of low intensity, the contact stresses are distributed uniformly over the interface between the liquid and elastic blocks. With an increase in the intensity of external influence, this pattern is violated.
5) The dependence of the nature of the growth of the amplitude of contact stresses on the vertical size of the liquid block was not noted.

5. Conclusion
Analysis of the results of computational experiments made it possible to establish the dependences of the stresses in the contact zone of the media of a block structure on the physical, geometric and frequency parameters of the problems under consideration for two models of an elastic foundation: a half-space and a layer with a restrained bottom edge.

The novelty of the presented study lies in the analysis of the dynamic behavior of a structure, including blocks of different types, characterized by different mechanical, physical and chemical properties, based on the use of the block element method, the solution of pseudodifferential equations in the form of integral equations, rather than using the averaged characteristics of contact stresses.

This approach to solving the problem makes it possible to increase the accuracy of the description of real processes and the reliability of assessments of the possible consequences of technogenic impact. This is especially important in regions where production, transport and mining infrastructure facilities are located near settlements or a coastal recreational zone. This study can be useful in the design of the transport network, production facilities on the coastal area, since it allows to assess seismic risks before the start of the process, as well as the risks of associated hazardous exogenous phenomena - landslides, tsunamis, and volcanism activation.

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