Collective excitations, instabilities, and ground state in dense quark matter

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We study the spectrum of light plasmons in the (gapped and gapless) two-flavor color superconducting phases and its connection with the chromomagnetic instabilities and the structure of the ground state. It is revealed that the chromomagnetic instabilities in the 4-7th and 8th gluonic channels correspond to two very different plasmon spectra. These spectra lead us to the unequivocal conclusion about the existence of gluonic condensates (some of which can be spatially inhomogeneous) in the ground state. We also argue that spatially inhomogeneous gluonic condensates should exist in the three-flavor quark matter with the values of the mass of strange quark corresponding to the gapless color-flavor locked state.

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It is natural to expect that cold quark matter may exist in the interior of compact stars. This fact motivated intensive studies of this system which revealed its remarkably rich phase structure consisting of many different phases (for a review, see Ref. [1]). The question which phase is picked up by nature is still open.

This problem is intimately connected with another one: Recently, it has been revealed that the $\beta$-equilibrated gapped (2SC) and gapless (g2SC) two-flavor color superconducting phases suffer from a chromomagnetic instability connected with the presence of imaginary (tachyonic) Meissner masses of gluons [2]. Later a chromomagnetic instability has been also found in the gapless three-flavor quark matter [3, 4].

In this Rapid Communication, we will address the problem of the origin of the chromomagnetic instability and the structure of the ground state in cold dense quark matter. The basic idea underlying the present analysis is the following: While for calculating (screening) Meissner masses of gluons, it is sufficient to study the gluon polarization operator only at one point $[(\rho_0, \vec{p}) = (0, \vec{p} \to 0)]$ in momentum space, we will extend this analysis to nonzero energy and momenta and study the spectrum of light plasmons. As will be shown, the plasmon spectrum yields an important information about the structure of the genuine ground state in this system. Although here only the two-flavor quark matter will be considered, as will be argued below, the present analysis should be relevant also for the three-flavor case.

Recall that in the two-flavor case, the manifestations of the chromomagnetic instability are quite different in the regimes with $\delta \mu < \Delta < \sqrt{2} \delta \mu$ and $\Delta < \delta \mu$, where $\delta \mu$ yields a mismatch between chemical potentials for the up and down quarks, and $\Delta$ is a diquark gap. The (strong coupling) regime with $\delta \mu < \Delta$ corresponds to the 2SC solution, and the (intermediate coupling) regime with $\Delta < \delta \mu$ corresponds to the gapless g2SC one [5]. While in the g2SC solution both the 4-7th gluons and the 8th one have tachyonic Meissner masses, in the 2SC solution, with $\delta \mu < \Delta < \sqrt{2} \delta \mu$, only the Meissner masses of the 4-7th gluons are tachyonic.

Our analysis reveals that these two chromomagnetic instabilities correspond to very different spectra of excitations in the 4-7th and 8th channels. In the 4-7th channels, the chromomagnetic instability reflects the typical Bose-Einstein condensation phenomenon: While at subcritical values of $\delta \mu < \Delta/\sqrt{2}$, there are light plasmons with the gap (mass) squared $0 < M^2 \lesssim \Delta^2 \ll \mu^2$ ($\mu$ is the quark chemical potential), at supercritical values $\delta \mu > \Delta/\sqrt{2}$, plasmons become tachyons with $M^2 < 0$. This picture corresponds to the conventional continuous (second order) phase transition and leads us to the unequivocal conclusion about the existence of gluonic condensates in the ground state in the two-flavor case, like those revealed and described recently in Ref. [6].

The spectrum of plasmons in the 8th channel is quite different. There are no light plasmons at all in the 2SC phase in that channel. On the other hand, in the g2SC phase, there is a gapless tachyonic plasmon with the dispersion relation $p_0^2 = v^2|\vec{p}|^2$ for small momenta, where the velocity squared $v^2$ is negative. This collective excitation occurs because of the existence of gapless modes in the g2SC phase. The wrong sign of the velocity $v^2$ implies a wrong sign for the derivative term $\delta \mu \bar{A}(8) \partial_\mu \bar{A}(8)$ in the effective action for gluons. The latter indicates on a possibility of the existence of a spatially inhomogeneous gluon condensate $\langle \bar{A}(8)(x) \rangle$ in the genuine ground state for the values $\delta \mu > \Delta$. As we will discuss below, similar gapless tachyonic plasmons with the quantum numbers of $A^{(1)}$, $A^{(2)}$ gluons and a linear combination of diagonal $A^{(3)}$, $A^{(8)}$ gluons and photon $A^{(1)}$ should exist in the...
gapless color-flavor locked (gCFL) phase\(^\text{7}\) in the three-flavor case, thus signalizing the existence of a spatially inhomogeneous gluon condensates in the ground state in that phase. Note that because the single plane-wave Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) state\(^\text{8,9}\) can be characterized by a homogeneous condensate \(\langle \mathcal{A}^{(8)} \rangle\), the ground state with spatially inhomogeneous gluon condensates is an essentially more complicated medium than the single plane-wave LOFF one\(^\text{10}\). On the other hand, the dynamics with a spatially inhomogeneous gluon condensate \(\langle \mathcal{A}^{(8)}(\vec{x}) \rangle\) is not necessarily inconsistent with the multiple plane-wave LOFF state\(^\text{11}\).

Let us consider the polarization tensor for the 4-7th gluons in the dense two-flavor quark matter in \(\beta\)-equilibrium. It is convenient to introduce the fields \(\phi^+_{\mu} \equiv \frac{1}{\sqrt{2}}(A^{(4)}_{\mu} + iA^{(5)}_{\mu}, A^{(6)}_{\mu} + iA^{(7)}_{\mu})\) and \(\phi^-_{\mu} \equiv \frac{1}{\sqrt{2}}(A^{(4)}_{\mu} - iA^{(5)}_{\mu}, A^{(6)}_{\mu} - iA^{(7)}_{\mu})\), which are color doublets of the gauge group \(SU(2)_c\) in the 2SC/g2SC phase. The polarization tensors \(\Pi^{\mu\nu}(p_0, \vec{p})\) and \(\Pi^{\mu\nu}(p_0, 0)\) for these fields were studied in Ref.\(^\text{2}\) and we will use those results.

These polarization tensors are decomposed as

\[
\Pi^{\mu\nu}(p_0, \vec{p}) = (g^{\mu\nu} - u^{\mu}u^{\nu} + \frac{p^{\mu}p^{\nu}}{p^2})H_{\mp} + u^{\mu}u^{\nu}K_{\pm} - \frac{p^{\mu}p^{\nu}}{p^2}L_{\pm} + \left(\frac{u^{\mu}p^{\nu}}{p} + \frac{u^{\nu}p^{\mu}}{p}\right)M_{\pm},
\]

(1)

where \(p \equiv |\vec{p}|\), \(p^{\mu} \equiv (0, \vec{p})\), and \(u^{\mu} \equiv (1, 0, 0, 0)\). The dispersion relations for plasmons are given by\(^\text{12}\)

(magnetic mode): \(p_0^2 - p^2 + H_{\pm} = 0\),

(2)

(electric mode): \(p_0^2K_{\pm} - p^2L_{\pm} - 2p_0KM_{\pm} + K_{\pm}L_{\pm} + M_{\pm}^2 = 0\). \(\text{(3)}\)

Note that there are two magnetic modes corresponding to two transverse components of plasmons, and one electric mode corresponding to their longitudinal component.

We are interested in deriving the gap (mass) spectrum \(\mathcal{M}_{\pm}\) of light plasmon excitations, i.e., with gaps \(|\mathcal{M}_{\pm}|^2 \ll \mu^2\). For this purpose, we consider the long wavelength limit \(|\vec{p}| \to 0\) with \(p_0\) being finite. Because the color neutrality condition yields \(\muS \sim O\left(\frac{\Delta^2}{\mu}\right)\)\(^\text{13}\), where \(\muS\) is the color chemical potential, the approximation with \(\muS = 0\) is well justified and will be used here (the role of a nonzero \(\muS\) will be clarified below). It is easy to check that the following relations are valid in this limit and in this approximation: \(L_{\pm}(p_0, |\vec{p}| \to 0) = H_{\pm}(p_0, |\vec{p}| \to 0)\), \(M_{\pm}(p_0, |\vec{p}| \to 0) = 0\), and \(K_{\pm}(p_0, |\vec{p}| \to 0) \neq 0\). Besides that, the functions \(H_{\pm}, L_{\pm}\), and \(K_{\pm}\) are of order \(\mu^2\). Therefore, in the hard dense loop (HDL) approximation that we utilize, both dispersion relations\(^\text{4,14}\) for \(|\vec{p}| \to 0\) are reduced to the equation \(H_{\pm}(p_0, |\vec{p}| \to 0) = 0\). In particular, the gaps for magnetic and electric modes coincide.

![FIG. 1: The plasmon gap squared for 4-7th gluons (solid curve).](image)

For \(\muS = 0\), the calculations of the function \(H_{\pm}(p_0, 0)\) become straightforward and we obtain:

\[
H_{\pm}(p_0, 0) = H_-(p_0, 0) = -\frac{g^2\bar{\mu}^2}{12\pi^2} \left[4 + \frac{\Delta^2}{p_0^2} \ln \left(\frac{1 - \frac{p_0^2}{\Delta^2}}{4\frac{\delta\mu^2}{\Delta^2}}\right) - 4\frac{\delta\mu^2}{\Delta^2} p_0^2\right] + \theta(\delta\mu - \Delta) \frac{\Delta^2}{p_0^2} \ln \frac{\Delta^4 - p_0^2(\delta\mu - \sqrt{\delta\mu^2 - \Delta^2})^2}{\Delta^4 - p_0^2(\delta\mu + \sqrt{\delta\mu^2 - \Delta^2})^2}
\]

(4)

where \(\bar{\mu} \equiv \mu - \delta\mu/3\) and \(g\) is the QCD coupling constant.

For a general value of \(p_0\), the equation \(H_{\pm}(p_0, 0) = 0\) can be solved only numerically. However, near the critical point \(\delta\mu_{cr} = \Delta/\sqrt{2}\), we can expand Eq. (4) with respect
1. Magnetic mode

In the HDL approximation, we studied these dispersion relations for the magnetic and electric modes. As one can see, the dispersion relations for plasmons in this channel were found numerically for all $0 < \mu^2 < \mu^2$. The form of dispersion relations in the $g$SC phase (dashed lines) is only slightly different from those in the 2SC one. Note that to make the comparison more convenient, the gaps corresponding to the two dashed lines were chosen to coincide with the gaps of the two lower bold solid lines.

Let us turn to the spectrum of light plasmons in the 8th channel. The polarization tensor $\Pi_{\mu\nu}(p, \bar{p})$ for the $A^{(8)}$ gluon can be decomposed as in Eq. (1) with the functions $H_{\pm}, K_{\pm}, L_{\pm},$ and $M_{\pm}$ replaced by $H_{88}, K_{88}, L_{88},$ and $M_{88}$. In the HDL approximation, the dispersion relations for plasmons in this channel were found numerically. The results are the following. There are no light plasmons in the 8th channel at all in the 2SC phase (with $\mu < \Delta$). On the other hand, in the g2SC phase (with $\mu > \Delta$), there exist both magnetic and electric gapless tachyonic plasmons with the dispersion relation $p_0^2 = v^2 p^2$ as $p \to 0$, where the velocity squared $v^2$ is negative. These collective excitations occur because of the existence of gapless modes in the g2SC phase. We emphasize that there are no light gapped plasmons (i.e., with nonzero $p_0$ as $p \to 0$) in this channel.

In Fig. 4, we depict the velocities squared of the gapless magnetic and electric tachyonic plasmons with quantum numbers of the 8th gluon. Near the critical point $\delta \mu = \Delta + 0$, both velocities have the form $v^2 \sim -\sqrt{1 - (\Delta/\delta \mu)^2}$.

\begin{equation}
M_{\pm} \simeq \Delta \sqrt{\frac{2}{7} \left(1 - 2 \frac{\delta \mu^2}{\Delta^2}\right)}.
\end{equation}

Only when the mismatch $\delta \mu$ is less than the critical value $\delta \mu_{cr} = \Delta/\sqrt{2}$, the plasmon gap is real. For $\delta \mu > \delta \mu_{cr}$, the plasmon gap becomes purely imaginary (tachyonic), thus signalizing a Bose-Einstein (BE) instability leading to a gluon (plasmon) condensation, as that described in Ref. [6]. We also analyzed the case with nonzero $\mu_8$. Its main effect is in splitting the $M_{\pm}$ and $M_{\pm}$ gaps.

It is noticeable that the BE instability occurs both for the magnetic and electric modes. Recall that unlike the Meissner mass, the (screening) Debye mass for the electric mode remains real for all values of $\delta \mu$ both in the 2SC and g2SC phases. Therefore the BE instability, connected with the spectrum of plasmons, essentially differs from the chromomagnetic instability in this respect.

As was mentioned above, in this approximation, the values of the gaps for the magnetic and electric modes are equal. As a function of $\delta \mu/\Delta$, their gap is shown in Fig. 1. For nonzero momenta, the dispersion relations for the magnetic and electric modes are different. In the HDL approximation, we studied these dispersion relations numerically for all $0 < p^2 < \mu^2$. Due to limited space, here we will describe the results only for the magnetic modes. Their dispersion relations are plotted in Fig. 2 for several fixed values of $\delta \mu/\Delta$ both in the 2SC and g2SC phases. As one can see, the dispersion relations in the 2SC phase (solid bold lines) have a qualitative form $p_0^2 = m^2 + v^2 p^2$ with $m^2 > 0$ for $\delta \mu/\Delta < 1/\sqrt{2}$ and $m^2 < 0$ for $\delta \mu/\Delta > 1/\sqrt{2}$. The velocity parameter $v$ is real and less than 1. The form of dispersion relations in the g2SC phase (dashed lines) is only slightly different from those in the 2SC one. Note that to make the comparison more convenient, the gaps corresponding to the two dashed lines were chosen to coincide with the gaps of the two lower bold solid lines.

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In Fig. 3 we depict the velocities squared of the gapless magnetic and electric tachyonic plasmons with quantum numbers of the 8th gluon. Near the critical point $\delta \mu = \Delta + 0$, both velocities have the form $v^2 \sim -\sqrt{1 - (\Delta/\delta \mu)^2}$.

\begin{equation}
\delta \mu > \delta \mu_{cr}, \quad \text{with} \quad \delta \mu_{cr} = \Delta + 0.
\end{equation}

Only when the mismatch $\delta \mu$ is less than the critical value $\delta \mu_{cr} = \Delta/\sqrt{2}$, the plasmon gap is real. For $\delta \mu > \delta \mu_{cr}$, the plasmon gap becomes purely imaginary (tachyonic), thus signalizing a Bose-Einstein (BE) instability leading to a gluon (plasmon) condensation, as that described in Ref. [6]. We also analyzed the case with nonzero $\mu_8$. Its main effect is in splitting the $M_{\pm}$ and $M_{\pm}$ gaps.

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8th channel in the g2SC phase, the existence of the gapless tachyonic plasmon seems to be counter-intuitive. What is the mathematical origin of its appearance? The answer to this question is connected with a peculiar structure of the function $H_{ss}(p_0, p)$. In the kinematic region of interest, with $|p_0|, |p| \ll \Delta$, the leading terms in this function have the following form:

$$H_{ss}(p_0, p) = -\frac{g^2}{3\pi^2} S^2 \left[ 1 + \left( 1 - \frac{p_0^2}{p^2} \right) Q \left( \frac{p_0}{p} \right) \right]$$

where $Q(x)$ is

$$Q(x) = -\frac{1}{2} \int_0^1 \frac{dx}{\xi} \left( \frac{\xi}{\xi + x - i\varepsilon} + \frac{\xi}{\xi - x - i\varepsilon} \right)$$

$$= \frac{1}{2} \ln \left( \frac{x + 1}{x - 1} \right) - 1 - \frac{\pi}{2} x \theta(1 - x) \quad \text{for real } x,$$

$$= y \arctan \frac{1}{y} - 1 \quad \text{for imaginary } x \equiv iy \quad (7)$$

(subleading terms, such as those of order $\bar{m}^2(p_0/\Delta)^2$, $\bar{m}^2(p/\Delta)^2$, etc., were omitted here). Note that the function $H_{ss}$ depends on the ratio $p_0/p$. This fact implies that the point $(p_0, p) = (0, 0)$ is singular: At $(p_0, p) = (0, 0)$ this function is multivalued and its value depends on the ratio $p_0/p$ as $p_0, p \to 0$. Therefore this singularity has many “faces” and its manifestations are different in different dynamical regimes. While the screening Meissner mass, related to a static regime with $p_0/p = 0$, is negative in the g2SC phase, there also exists the gapless tachyon with the ratio $p_0/p = v$ being purely imaginary. The physics underlying this behavior of the function $H_{ss}$ is of course connected with the presence of gapless fermions in the spectrum.

Let us turn to a more detailed discussion of the results obtained above. The spectrum of light plasmons in the 4-7th channels leads us to the conclusion about the existence of gluonic condensates in the ground state in the two-flavor case. These condensates correspond to the conventional Bose-Einstein condensation phenomenon which can be described in the framework of the Ginzburg-Landau (GL) approach [6].

While the instability in the 4-7th channels is connected with a wrong sign of the gap squared $M^2$ and, therefore, with the potential part of the effective action, the origin of the instability in the 8th channel is quite different. Indeed, this instability is connected with a wrong sign of a velocity squared $v^2$ of a gapless tachyonic plasmon, i.e., with the wrong sign of the term $\partial_i A^{(8)} \partial_i \bar{A}^{(8)}$ in the derivative part of the effective action. This point is a signal of the existence of a spatially inhomogeneous gluon condensate in the dynamics with $\Delta < \delta \mu$ connected with the g2SC state.

A note of caution is in order. As was shown above, there exist homogeneous gluonic condensates in the 4-7th channels both for $\delta \mu < \Delta$ and $\delta \mu > \Delta$. Because the condensates break spontaneously most of the initial symmetries in the 2SC state [6], their dynamics is quite rich and complex. In particular, one cannot exclude that these gluonic condensates themselves could remove the gapless tachyonic plasmon in the 8th channel and lead to a consistent spectrum of excitations in the system.

In fact, the manifestations of a gapless tachyonic instability could be cleaner in the three-flavor case. It is known that the lightest value of the mass $M$, of strange quark at which the chromomagnetic instability occurs is that corresponding to the border between the CFL and gCFL phases, with $M^2/\mu \approx 2\Delta$. The instability is generated in the channels with the quantum numbers of $A^{(1)}, A^{(2)}$ gluons and a linear combination of diagonal $A^{(3)}, A^{(8)}$ gluons and photon $A^{(7)}$. This instability is similar to the chromomagnetic $A^{(8)}$ instability in the g2SC state. Therefore, one should expect that a gapless tachyonic plasmon exists in these channels in the gCFL state. It is important that for this value of the strange quark mass, no homogeneous gluonic condensates are generated (they could occur in the 4-7th channels at larger values of $M^2/\mu$ [1]). Therefore, the phase transition to a spatially inhomogeneous state in the three-flavor case can be clearer than in the two-flavor one.3

The following remark is in order. Unlike the case of the $A^{(8)}$ gluon in the g2SC phase, the $A^{(1)}, A^{(2)}$ and $A^{(3)}, A^{(8)}$ gluons in the gCFL phase couple to gapless quarks with an almost quadratic dispersion relation [7]. However, it is important that in the vicinity of the transition point to the gapless phase, from the side of the gapless state, there are hardly any qualitative differences between these two systems. The reason is that it is precisely the region where all gapless modes have an approximately quadratic dispersion law. Because of that, one should expect that the transition to the gapless phase is similar for the cases of 2- and 3-flavor quark matter. Only deeply in the gapless regime, the analysis in these two cases may develop qualitative differences.

Although the present analysis does not yield a complete answer to the question what kind of a transition is connected with the gapless tachyonic instability, it is clear that it is not the conventional second order phase transition described by the GL effective action. The point is that a characteristic feature of this gapless

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1 This dynamics admits a dual, gauge invariant, description as a dynamics with a condensation of exotic vector mesons [6].
2 In connection with that, we checked that a gapless tachyonic plasmon exists in a toy model of the gCFL phase considered in the second paper in Ref. [6].
3 Due to shortage of space, here we do not consider an influence of the kaon condensation on instabilities in the gCFL phase [1].
tachyon is that it occurs from “nothing”: As we emphasized above, there is no plasmon in the 8th channel in the 2SC state which is transformed into the gapless tachyonic plasmon in the g2SC state. In other words, there is an abrupt change of the spectrum of light excitations at the point $\delta \mu = \Delta$ dividing the 2SC and g2SC phases.

Does it imply that the phase transition connected with this instability is a first order one? We believe that this is not necessarily the case: Although the first order phase transition is a viable option [and it could for example be similar to that in the model with a p-wave K-meson condensate [14]], there are known examples of a continuous phase transition with an order parameter going smoothly to zero at the critical point but with an abrupt change of the spectrum of light excitations at that same point [15]. Such phase transitions are characterized by an essential singularity in the order parameter at the critical point. In this regard, it is noticeable that the velocity squared $v^2$ of gapless tachyonic plasmon goes smoothly to zero as $\delta \mu / \Delta \to 1 + 0$ (see Fig. 2). On the other hand, the free energy and Green’s functions of gluons in the 2SC/g2SC state at zero temperature include the step function $\theta(\delta \mu - \Delta)$ (see Eq. (4)). This function introduces sort of an essential singularity at the border between the 2SC and g2SC phases and cuts off the dynamics responsible for light collective excitations in the 8th channel from the region with $\delta \mu < \Delta$. This issue deserves further study.

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