Tunable 0–π transition by interband coupling in iron-based superconductor Josephson junctions

Y C Tao1, S Y Liu1, N Bu1, J Wang1 and Y S Di1,2
1 Department of Physics and Institute of Theoretical Physics, Nanjing Normal University, Nanjing 210023, People’s Republic of China
2 Department of Physics, Southeast University, Nanjing 210096, People’s Republic of China
E-mail: yctao88@163.com and jwang@seu.edu.cn

Keywords: tunable 0–π transition, interband coupling, iron-based superconductor Josephson junctions

Abstract
An extended four-component Bogoliubov–de Gennes equation is applied to study the Josephson effect in ballistic limit between either two iron-based superconductors (SCs) or an iron-based SC and a conventional s-wave SC, separated by a normal metal. A 0–π transition as a function of interband coupling strength α is always exhibited, arising from the tuning of mixing between the two trajectories with opposite phases. The novel property can be experimentally used to discriminate the s±-wave pairing symmetry in the iron-based SCs from the s++-wave one in MgB2. The effect of interface transparency on the 0–π transition is also presented. The 0–π transition as a function of α is wholly distinct from that as a function of barrier strength or temperature in recent theories (Linder et al 2009 Phys. Rev. B 80 020503(R)). The possible experimental probe of the phase-shift effect in iron-based SC Josephson junctions is commented on as well.

1. Introduction

The discovery of iron-based superconductors (SCs) [1] has kindled much interest in recent years among members of the scientific community [2–12]. The identification of pair potential symmetry is a crucial issue for the iron-based SC. The pairing symmetry in MgB2, having two s-wave energy gaps with the same phase in σ and π bands, known as s± pairing symmetry, is well understood [13]. However, for the iron-based SCs, though a number of works have contributed to this issue, the controversies still remain unsolved [14, 15]. Many possible pairing states, such as p-wave, [16] d-wave, [17] extended s-wave or s±-wave, have been proposed. Most researchers favor the so-called s±-wave pairing state with the pair potentials for the hole and electron bands being of isotropic s-wave state but of opposite sign [18]. Therefore, it is important to verify the s±-wave pairing symmetry in the iron-based SCs experimentally [19]. An experimental setup was considered to distinguish between s± and s++ types of pairing in the iron-based SC [20]. The experiment is based on the existence of a phase soliton, a topological defect formed in the relative phase of superconducting gaps the SC with s± type of pairing. A great many proposals to search for a definite signature have been implemented, e.g., tunneling spectroscopy, [21, 22] corner junctions, [23] observation of half-inter flux-quantum jump, [24] scanning tunnel microscopy, [25] and so forth. In addition, phase-sensitive measurements, [26] a direct and unambiguous evidence, were performed for cuprate high-temperature SC. Such experiments were also proposed to be available for identifying the s±-wave pairing symmetry in the iron-based SC, [27–29] however, they have not been practically realized yet. It is widely agreed that a decisive experiment should comprise a phase sensitive probe such as the Josephson effect, [30] sensitively reflecting superconducting states of each electrode in Josephson junctions [19, 23, 24, 28].

The Josephson effect between two SCs, a macroscopic quantum effect, is associated with superconductivity. As a result of the quantum character of the superflow, a phase difference φ between the SCs emerges and the current–phase relation of the link is periodic function of φ. The Josephson effect with the phase of the superconductivity involved is sensitively dependent on the underlying pairing symmetry of SCs, and offers an invaluable tool to pin down the pairing symmetry of SCs [31]. The current–phase relation is not only in favor of...
broadening our general understanding on the Josephson effect but also points to a new possible direction for applications [32]. The maximum Josephson current or critical current \( I_c \) can change sign when the SCs are coupled by a thin ferromagnet (FM) layer [33]. This change is corresponding to a \( \pi \)-phase shift of \( \phi \) and thus the Josephson junctions are usually called \( \pi \) ones.

Soon after the discoveries of the iron-based SCs, several Josephson junctions [20, 34, 35] have been theoretically proposed, which are composed of either two iron-based SCs or an iron-based SC and a conventional \( s \)-wave SC. Some results may be used to discriminate between \( s_\uparrow \) and \( s_\downarrow \) symmetries in SCs [20]. Experimentally, Josephson junctions have been fabricated between a conventional SC and these multiband SCs including MgB\(_2\) [36]. None has found any indication that a pure \( p \)-wave or \( d \)-wave pairing symmetry is likely realized in the iron-based SCs, while strong evidence for \( s_\pm \)-wave pairing symmetry was displayed.

The possibility of the characteristic \( 0-\pi \) transitions in the situation of iron-based SC, another possible experimental signature, has been suggested previously in some theoretical work [19, 24]. Linder \textit{et al} first put forward the \( 0-\pi \) transitions as a function of temperature or barrier strength in a diffusive \( s \)-wave/normal metal (NM)/\( s_\pm \)-wave junction [37]. Shortly afterwards, they also perform a complementary, more comprehensive study of these effects in the ballistic limit [38]. So far, however, there have been no reports on the \( 0-\pi \) transition as a function of interband coupling strength in the previous works, which will be detailedly studied in this work. For superconducting heterojunctions, the barrier strength is usually introduced to describe the barrier at interface. It reflects the tuning of mixing between the two trajectories with the same bands through interface scattering. But the interband coupling strength is a characteristic parameter in superconducting heterojunctions with a two-band SC, describing the tuning of mixing between the two trajectories with different bands through interface scattering. The two bands in the SC with \( s_\pm \)-wave pairing symmetry have opposite phases, while the same phases in the SC with \( s_{\uparrow\downarrow} \)-wave pairing symmetry. Therefore, the study of the \( 0-\pi \) transitions as a function of interband coupling can uncover some novel properties and help to directly get insight into the essential features of the multiband SCs with \( s_\pm \)-wave and \( s_{\uparrow\downarrow} \) pairing symmetry. It is expected that the study will provide a powerful tool for probing and identifying the \( s_\pm \)-wave pairing symmetry in the iron-based SCs.

In this work, we use an extended four-component Bogoliubov–de Gennes (BDG) equation to study the Josephson junction, in which a NM layer is sandwiched in between either two iron-based SCs or an iron-based SC and a conventional \( s \)-wave SC. A \( 0-\pi \) transition is always shown as a function of interband coupling strength, which can be utilized to discriminate the pairing symmetry in the iron-based SC from the \( s_{\uparrow\downarrow} \)-wave one in MgB\(_2\). The mechanism is attributed to the tuning of mixing between the two trajectories with opposite phases, leading to the modulation between twofold Andreev reflections. Since the interband coupling strength between two bands depends on doping of the material, the mechanism is referred to as ‘extrinsic’. However, in \( s \)-wave SC/FM/\( s \)-wave SC junction, the \( 0-\pi \) transition induced by varying FM thickness is originated from the inner nature of the trajectory with different spin, which can be therefore considered ‘intrinsic’. The \( 0-\pi \) transition as a function of interband coupling strength also differs from that as a function of barrier strength or temperature in Linder \textit{et al} theory [37, 38]. The influence of interfacial transparency on the Josephson effect is also presented. The corresponding mechanism is analyzed as well. Finally, the experimental feasibility of the results is assessed.

2. Model

The model that we consider is an \( s \)-wave SC/NM/\( s_\pm \)-wave SC hybrid structure with thickness \( d \) of the NM, shown schematically in figure 1. The layers are assumed to be the \( x-y \) plane and to be stacked along the \( z \)-direction. The left SC is single-band \( s \)-wave with the temperature dependence of gap energy \( \Delta_x \equiv \Delta_x(T) = \Delta_x(0) \tanh(1.74 \sqrt{T_c/T - 1}) \) and corresponding phase \( \phi_x \). The right SC is \( s_\pm \)-wave with two superconducting gap energies \( \Delta_{\lambda} \) (\( \lambda = 1, 2 \)) and relative superconducting phases \( \phi_{\lambda} \). The temperature dependence of \( \Delta_{\lambda} \) can be given by \( \Delta_{\lambda} \propto s \rightarrow \lambda \). The internal phase shift is characterized by \( e^{i \phi_{\lambda} - i \phi_{\lambda-1}} = \zeta \). For the \( s_\pm \)-wave pairing state with unequal \( s \)-wave gaps of opposite sign, we have \( \phi_1 - \phi_2 = \pi \), and then \( \zeta = -1 \);
while for the two-band $s$-wave pairing state, we have $\phi_1 = \phi_2$, and then $\zeta = 1$. The macroscopic phase difference of the two SC electrodes is defined as $\phi = \phi_1 - \phi_2$.

The four-component function for the two-band $s$-wave SC $\Psi(r) = [\psi_1(r), \psi_2(r)]^T$ with $\psi_k(r)$ given by $[u_k(r), v_k(r)e^{-i\phi_k}]^T$ is applied for the structure. The BDG equation [39] is given by

$$ H\Psi = \epsilon \Psi $$

with $\epsilon$ the positive quasiparticle energy considered here and

$$ H = \begin{pmatrix} \epsilon_{k,1} + V(z) & \Delta(z) & \alpha(z) & 0 \\ \Delta^*(z) - \epsilon_{k,1} - V(z) & 0 & -\alpha(z) & 0 \\ \alpha(z) & 0 & \epsilon_{k,2} + V(z) & \Delta_2(z) \\ 0 & -\alpha(z) & \Delta_2^*(z) - \epsilon_{k,2} - V_2(z) & 0 \end{pmatrix}. $$

Here $\epsilon_{k,1,2}$ are dispersions for the two-band SCs measured from the Fermi level $\epsilon_F$, $\alpha(z) = \alpha_0 \delta(z - d/2)$ with $\alpha_0$ the interband hopping parameter describing the coupling of the two bands through the interface scattering, $\Delta(z)$ are given by $\Delta, \Theta(-z - d/2)$ and $\Delta_2, \Theta(z - d/2)$, and $V(z)$ the barrier potentials at the two interfaces are respectively described by $V_0 \delta(z + d/2)$ and $V_2 \delta(z - d/2)$.

Consider the injection of an electronlike quasiparticle (ELQ) with energy $\epsilon$ and incident angle $\theta$ to the interface normal. With the general solution of equation (2), the wave function in the left $s$-wave SC is given by

$$ \Psi_L(z) = \begin{pmatrix} e^{i\epsilon z} + b_1 e^{-i\epsilon z} \nu(z) & \nu(z) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^T = \begin{pmatrix} f_1 e^{i\epsilon z} + f_2 e^{-i\epsilon z} \nu(z) & \nu(z) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^T $$

for $z \leq -d/2$, where coefficients $b_{1,2}$ and $a_{i,2}$ correspond to the inherent (novel) normal reflections and the inherent (novel) Andreev reflections (ARS), respectively. The wave vectors for the electrons and holes in the $s$-wave SC are given by $k^\pm = \sqrt{(2m/\hbar^2)(\epsilon_F \pm \Omega) - k_{F1}^2}$ with $k_{F1} = \sqrt{(2m/\hbar^2)(\epsilon_F + \epsilon) \sin \theta}$ the parallel component of the wave vector and $\alpha^2 = 1 - v^2 = (1 + \Omega/\epsilon)/2$ with $\Omega = \sqrt{\epsilon^2 - |\Delta|^2}$. In the middle NM and right $s_s$-wave SC regions, we have the wave functions

$$ \Psi_M(z) = \begin{pmatrix} f_1 e^{i\epsilon z} + f_2 e^{-i\epsilon z} \nu(z) & \nu(z) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^T $$

for $-d/2 \leq z \leq d/2$ and

$$ \Psi_R(z) = \begin{pmatrix} c_1 e^{i\epsilon z} \nu(z) & e^{i\epsilon z} \nu(z) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^T = \begin{pmatrix} c_2 e^{i\epsilon z} \nu(z) & e^{i\epsilon z} \nu(z) & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^T $$

for $z \geq d/2$. In equations (4) and (5), $\nu = 1 - v^2 = (1 + \Omega/\epsilon)/2$ with $\Omega = \sqrt{\epsilon^2 - |\Delta|^2}$, $k_{F2} = \sqrt{(2m/\hbar^2)(\epsilon_F \pm \Omega) - k_{F1}^2}$, and $q^\pm = \sqrt{(2m/\hbar^2)(\epsilon_F \pm \epsilon) - k_{F1}^2}$.

All the coefficients $a_{i,2}, b_{1,2}, c_{i,2}, d_{i,2}$, and $f_i(i = 1 - 8)$ are determined by the following matching conditions of the wave functions:

$$ \frac{\partial \Psi_M(z)}{\partial z} \bigg|_{z=-d/2} - \frac{\partial \Psi_L(z)}{\partial z} \bigg|_{z=-d/2} = \frac{2mV_0}{\hbar^2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, $$

$$ \Psi_M(z = -d/2) = \Psi_L(z = -d/2), $$

$$ \frac{\partial \Psi_R(z)}{\partial z} \bigg|_{z=d/2} - \frac{\partial \Psi_M(z)}{\partial z} \bigg|_{z=d/2} = \frac{2mV_2}{\hbar^2} \begin{bmatrix} V_1 & 0 & 0 & 0 \\ 0 & V_1 & 0 & 0 \\ 0 & 0 & V_2 & 0 \\ 0 & 0 & 0 & V_2 \end{bmatrix} + \alpha_0 \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, $$

$$ \Psi_M(z = d/2) = \Psi_R(z = d/2), $$

where two-dimensionless parameters $Z_{\alpha,0} = mV_{\alpha,0}/(\hbar^2k_F)$ and $\alpha = m\alpha_0/(\hbar^2k_F)$ with $k_F = \sqrt{2m\epsilon_F}/\hbar$ are respectively introduced to describe the barrier and interband coupling strengths. Analogously, one can easily obtain the ARs $d_R(z)$ as an ELQ and normal reflections $b_{1,2}$ as a holelike quasiparticle (HLQ) for the injection of a HLQ into the NM from the left $s$-wave SC. In the presence of the interband coupling, the quasiparticle trajectories in the $s_s$-wave SC for different bands will be mixed, specifically, one quasiparticle in a trajectory for one band can enter into the trajectory for the other band. Boundary conditions are important in the theories of charge transport, which are briefly discussed here. By using phenomenological approach based on the analogy...
between quantum waveguide theory and interband scattering, Araújo and Sacramento [40] obtained the boundary conditions between a single-band NM and a multiband SC. However, the basis of this theory is not fully microscopic since interband and intervalley scattering effects are not taken into account properly. Burmistrova et al. [41] likewise acquired the boundary conditions in multi-band superconducting systems by extending the tight-binding approach beyond the effective mass approximation and taking into account complex nonparabolic and anisotropic energy spectra, as well as unconventional pairing symmetry. The theory of dc Josephson effect in contacts between iron-based and spin-singlet s-wave SCs is also presented, [42] based on the calculation of temperature Green’s function in the junction within the tight-binding mode. The corresponding boundary conditions in the quasiclassical approximation at SC/insulator boundary are also obtained. In our work, $\alpha_0$ of the boundary condition given by equation (9) essentially indicates that two bands mix through the interface scattering.

The dc Josephson current can be expressed in terms of the AR amplitudes by using the temperature Green’s function formalism, [43] as shown in [44]

$$I = \frac{e}{4\hbar} \sum_{k} T \sum_{\omega_n} \left[ \frac{4 \alpha}{\Omega_{1n}} \left( k_{in}^+ + k_{in}^- \right) \left( \frac{a_{1n}}{k_{in}^+} - \frac{a_{1n}'}{k_{in}^-} \right) + \frac{4 \alpha}{\Omega_{2n}} \left( k_{2n}^+ + k_{2n}^- \right) \left( \frac{a_{2n}}{k_{2n}^+} - \frac{a_{2n}'}{k_{2n}^-} \right) \right],$$

(10)

where $k_{1(2)n}^+$, $k_{1(2)n}'$, $a_{1(2)n}$, and $a_{1(2)n}'$ are respectively obtained from $k_{1(2)n}^+$, $k_{1(2)n}'$, $a_{1(2)n}$, and $a_{1(2)n}'$ by the analytic continuation $\epsilon \rightarrow i \omega_n$ and $\Omega_{1(2)n} = \sqrt{\omega_{1(2)n}^2 + \Delta_{1(2)}^2}$. The Matsubara frequencies $\omega_n$ are given by $\pi k T (2n + 1)$ with $n = \pm 1, \pm 2, \ldots$.

The dc Josephson current can be acquired by another formalism [32]. As in [38], four Andreev bound states being the simple physical analysis for the 0 s-wave SC transition in [46], the relevant quantity measured experimentally. In figures 2(a) and (b), the maximum Josephson

### 3. Results and discussions

Before presenting the calculated results and discussions for the s-wave SC/NM/s-s-wave SC junctions, we first give the simple physical analysis for the 0-π transition in s-wave SC/FM/s-wave SC junctions [45]. The directions of currents which the Andreev bound states with opposite spin channels carry are contrary in the structure. As a result of the competition between different spin channels, for instance, 0-π oscillations in the Josephson current as a function of the FM thickness can arise. More specifically, the AR magnitude for the incidence of an ELQ with a certain spin $\sigma$ from the left SC is usually bigger or smaller than that for a HLQ with the same spin, whereas the situation with the spin $\sigma$ opposite to the spin $\sigma$ is exactly contrary.

On the basis of equation (10), we calculate the maximum Josephson current $I_c$, taken to be its absolute value here as in [46], the relevant quantity measured experimentally. In figures 2(a) and (b), the maximum Josephson...
Figure 2. Maximum currents $I$ as a function of $\alpha$ for $s$-wave SC/NM/$s_z$-wave SC (a) and $s$-wave SC/NM/$s_{z+}$-wave SC (b) junctions with high (the left column) and low (the right column) transparencies. Here, $T/T_c = 0.01$ with $T$ of $s$-wave SC, $\Delta_0 = \Delta_0 = 1.4$ meV, $\Delta(0)/\epsilon_F = 10^{-3}$, $Z_0 = Z_1$, and the various $r_\alpha$ are indicated.

currents $I$, in $s$-wave SC/NM/$s_z$-wave SC and $s$-wave SC/NM/$s_{z+}$-wave SC junctions are respectively illustrated as a function of $\alpha$ at different $r_\alpha$ and $r_\alpha = Z_2/Z_1$. It is shown that at high transparency $Z_1 = Z_2 = 0$, for $s$-wave SC/NM/$s_z$-wave SC junction, $I$ first decreases along one direction, then increases along the opposite direction, and last gradually decreases to zero. The position of the dip is found to shift toward smaller $\alpha$ with enhancing $r_\alpha$, and the dip corresponds to a $0-\pi$ transition. Since there is no difference between the two spins in the $s$-wave SC/NM/$s_z$-wave SC junction, the $0-\pi$ transition should be just originated in the twofold ARs. At the low transparency $Z_1 = 0.5$ with $r_\alpha = 2$, the features are found to keep the same except that the values of dips become much smaller and the positions of the dips slightly shift toward smaller $\alpha$ with increasing $r_\alpha$. However, for the $s$-wave SC/NM/$s_{z+}$-wave SC junction, it is found that the features are thoroughly different from those for $s$-wave SC/NM/$s_z$-wave SC one. Specifically, at high transparency, $I$ all the time decreases up to zero and does more rapidly for smaller $r_\alpha$. This indicates that there is no $0-\pi$ transition and the mechanism just stems from that the phases for the two bands in $s_{z+}$-wave SC are the same, which is different from the situation for the $s_z$-wave SC with opposite phases. The features at low transparency is also similar to those at the high transparency except for some slight differences. For instance, especially at bigger $r_\alpha = 2$, $I$ decreases slowly.

In order to understand how the twofold ARs determine the $0-\pi$ transition, we plot in figure 3(a), the twofold ARs $a_1$ and $a_2$ as a function of $\alpha$ at high transparency in the $s$-wave SC/NM/$s_z$-wave SC junction. For the incidence of an ELQ, with the increase of $\alpha$, the inherent AR $a_1$ first increases from positive value, then decreases to negative value, and finally increases with a trend toward zero. However the situation for the novel AR $a_2$ is just contrary and it first decreases from zero. Hence, the currents the inherent and novel ARs carry are of opposite direction. For the incidence of a HLQ, the lines of the inherent ARs $a_1'$ and $a_1$ are symmetrical about the $\alpha$ axis, while the novel AR $a_2'$ is almost the same as $a_2$. As a result of the competitions of the twofold ARs and types of incident quasiparticles, damped oscillations of $I$ with $\alpha$ are generated, leading to $0-\pi$ and $\pi-0$ transitions. For comparison, the corresponding results of the $s$-wave SC/NM/$s_{z+}$-wave SC junction are also presented in figure 3(b). For the incidence of an ELQ, with increasing $\alpha$, the inherent AR $a_1$ decreases from positive value with a trend toward zero and the novel AR $a_2$ is about zero. But for the incidence of an HLQ, the lines of the inherent ARs $a_1'$ and $a_1$ are also symmetrical about $\alpha$ axis, and the novel AR $a_2'$ is about 0 as well. Therefore, the twofold ARs are predominated by the inherent AR, and the junction is similar to a conventional $s$-wave SC/NM/$s$-wave SC junction, indicating there is no $0-\pi$ transition. Physically, the interband coupling acts as the tuning of mixing between the two trajectories of different bands, bringing about the new interference, and resulting Andreev
bound states are determined by the interband coupling strength. Accordingly, the mechanism of the $0-\pi$ transition in the $s$-wave SC/NM/$s_\pm$-wave SC junction induced by the interband coupling can be considered extrinsic. It is thoroughly different from the mechanism caused by the inner nature of the channels in $s$-wave SC/FM/$s$-wave SC junction, the spin nature which is therefore intrinsic.

Moreover, the barrier strength merely adjusts the mixing degree between different trajectories with the same bands, and hence leading to no variation of features of the twofold ARs as in figure 3. Obviously, variation of $r_\Delta$ can only alter the magnitude of $I_c$ but not the features, and the competition between $r_\Delta$ and $r_a$ determines the shift direction of the dip features of $I_c$ in $s$-wave SC/NM/$s_\pm$-wave SC junctions.

Next, we investigate effect of interband coupling on the current–phase relation $I(\phi)$. In figure 4(a) are illustrated the characteristic variations of $I(\phi)$ in the vicinity of crossover between 0 and $\pi$ states for the $s$-wave SC/NM/$s_\pm$-wave SC junction, corresponding to the dips of dotted lines in figure 2(a). It is shown that there are only 0 and $\pi$ states but also the transitions from 0 to $\pi$ state irrespective of the transparency at the two interfaces. The region of coexisting 0 and $\pi$ states at high transparency is 0.405 $< \alpha < 1.490$ with the corresponding dip $\alpha_c = 1.022$, which is narrower than the region 0.830 $< \alpha < 1.717$ at low transparency with corresponding dip $\alpha_c = 1.294$. These mean that the $0-\pi$ transition is more easily achieved at low transparency and large $\alpha$, which can be used to design the quantum interference devices. In figure 4(b), the only 0 states are shown for $I(\phi)$ of $s$-wave SC/NM/$s_{\pm\pm}$-wave SC junction, since there is no dip in figure 2(b). In addition, the features of $I(\phi)$ at low transparency keep the same as those at high transparency except for the slight difference in the magnitude.

In what follows, we turn attention to the Josephson effect in $s_\pm$-wave SC/NM/$s_\pm$-wave SC and $s_{\pm\pm}$-wave SC/NM/$s_{\pm\pm}$-wave SC junctions. In figures 5(a) and (b), the maximum Josephson currents $I_c$ in the two structures are shown as a function of $\alpha$, respectively. One can find that the features are similar to those for the previous corresponding junctions with a conventional $s$-wave SC except for some differences. For instance, the values of $\alpha_c$ corresponding to the dips all become much smaller irrespective of the high and low transparencies.

In figure 6 (a), the corresponding twofold ARs as a function of $\alpha$ are also presented for the $s_\pm$-wave SC/NM/$s_\pm$-wave SC junction. With the increase of $\alpha$ from 0, the signs of the inherent and novel ARs $a_{1(2)}$ for the incidence of an ELQ first are opposite and then begin to become the same at $\alpha = 1.8$. However, for the incidence of a HLQ, the situation is just contrary. More specifically, with the increasing $\alpha$ from 0, the signs of the inherent and novel ARs $a_{1(2)}$ first are the same and then begin to become opposite at $\alpha = 1.7$. The features are
Figure 4. Current–phase relation $I(\phi)$ for $s$-wave SC/NM/$s_{\pm}$-wave SC (a) and $s$-wave SC/NM/$s_{++}$-wave (b) structures with high (the left column) and low (the right column) transparencies. Here, the parameters are the same as in figure 2 except that $r_2 = D$ and the various $\alpha$ are indicated.

Figure 5. The same as in figure 2 except that the left $s$-wave SCs of (a) and (b) are respectively replaced by $s_{\pm}$ and $s_{++}$-wave SCs.
thoroughly different from those for the previous corresponding structure with a conventional $s$-wave SC. This indicates that, it is the left $s_-$-wave SC that leads to the difference, which can be explained as follows. For the present structures, the incoming trajectories with different phases from different bands of $s_-$-wave SC are also mixed through the first interface. However, for the previous structure with a conventional $s$-wave SC, the incoming trajectories with the same phases from the band of $s$-wave SC are mixed through the first interface. It is therefore obvious that the twofold ARs or twofold Andreev bound states are different. Similarly, for comparison, the corresponding results for the $s^+-$wave SC/NM/$s^-$wave SC junction are also shown in figure 6(b). For the incidence of an ELQ, with the enhancement of $\alpha$, the inherent AR $a_1$ tends to decrease from the positive value to 0 and the novel AR $a_2$ first increases slightly from 0 and then decreases up to 0. Nevertheless, for the incidence of an HLQ, the lines of the inherent (novel) ARs $a_{1(2)}$ and $a'_{1(2)}$ are symmetrical about the $\alpha$ axis. These are ascribed to the incoming trajectories of different bands with the same phases in $s^+-$wave SC. The mixing of the trajectories through the two interfaces leads to twofold Andreev bound states with unchanged sign of energy or the twofold ARs with the same signs of the inherent and novel ARs. As a result, there exists no transition of $0-\pi$ state.

The current–phase relations $I(\phi)$ for the two structures are also given. Similarly, for the $s^-$-wave SC/NM/$s_-$-wave SC structure, there are not only 0 and $\pi$ states but also the transition from 0 to $\pi$ state as shown in figure 7(a). Only the transition region of coexisting 0 and $\pi$ states for both the high and low transparencies, is narrower than that in the $s$-wave SC/NM/$s_-$-wave SC structure. For the SCs with $s^+_+$-wave pairing symmetry in the SC/NM/SC junction, there also exists only 0 state as shown in figure 7(b), since no dip in figure 5(b) appears. These indicate that the variation from $s$- to $s^+_+$-wave pairing symmetry of the left SC does not result in change in feature of the two junctions.

Finally, the experimental feasibility of our results is evaluated. The length of the junction $d = 30k_F^{-1}$ is much less than the BCS coherence length at zero temperature $\xi_0 = \hbar v_F/2\Delta_0(0)$ with $v_F$ Fermi velocity. As a result, the requirement of constructing Josephson junctions that the length of the middle layer or coupled zone is less than the coherence length of the two SC electrodes is satisfied. More importantly, the dependence of observable $0-\pi$ transition on $\alpha$ is considered here, therefore, a more thorough rational for this parameterization should be discussed. Tailoring the experimental setup can be achieved by applying the material with appropriate Fermi surfaces through changing the doping level in the $s^-$-wave region of the junction, where $\alpha$ is correspondingly adjusted. So, it is believable that a $0-\pi$ transition can be observed by tuning the doping level, analogously as to how $0-\pi$ transitions are observable in conventional SC/FM/SC junctions via varying interlayer FM thickness.

Figure 6. The same as in figure 3 except for the two structures corresponding to figure 5.
4. Summary

In summary, we make use of an extended four-component BDG equation to study the Josephson junction with a NM layer, sandwiched in between either two iron-based SCs or an iron-based SC and a conventional $s$-wave SC. A $0-\pi$ transition as a function of interband coupling strength is always exhibited. It can be experimentally applied to probe and identify the $s_{\pm}$-wave pairing symmetry in the iron-based SC different from the $s^{++}$-wave one in MgB$_2$. The mechanism originates from the tuning of mixing between the two trajectories with opposite phases and consequent twofold ARs, which is extrinsic. And it differs from that induced by the inner nature of the trajectory with different spin in $s$-wave SC/FM/$s$-wave SC junction, and is intrinsic. The $0-\pi$ transition as a function of interband coupling strength is different from that as a function of interfacial barrier strength or temperature in Linder et al [32, 33] theory. The influence of interfacial transparency on the Josephson effect is also displayed and then the corresponding mechanism is analyzed as well. With the experimental techniques grown to maturity, the $0-\pi$ transition is easily controlled by tuning the doping level in the $s_{\pm}$-wave SCs. It is expected that experimental measurements of $0-\pi$ transition in the present structures will be able to confirm the $s_{\pm}$-wave pairing symmetry in the iron-based SC distinct from the $s_{+-}$-wave one in MgB$_2$ and find potential applications in quantum information. In addition, the effect of twofold ARs or bound states on the charge transport in $s_{\pm}$-wave SC junction is briefly commented on. As in the Josephson junction of the work, the twofold Andreev bound states also have significant influences on the tunneling conductance of NM/$s_{\pm}$-wave SC junction, which is determined by not only twofold normal reflections but also twofold ARs. However, we find that the twofold Andreev bound states cannot give rise to the zero-bias conductance peak according to our previous calculation, which is thoroughly different from that in NM/$d$-wave SC junction [47]. In the junction, the Andreev bound states significantly influence the tunneling spectroscopy and there exists a zero-bias conductance peak [48].

Acknowledgments

This work was supported by the National Science Foundation of China under Grant Nos 10947005, 10974170, 11070403, and 11274059, the Natural Science Foundation of Jiangsu Province under Grant No BK2009399, and the Natural Science Foundation of Jiangsu Education Department of China under Grant No 2008102TSJ0083.
References

[1] Kamihara Y, Watanabe T, Hirano M and Hosono H 2008 J. Am. Chem. Soc. 130 3296
[2] Raghu S, Qi X L, Liu C X, Scalapino D J and Zhang S C 2008 Phys. Rev. B 77 220503(R)
[3] Liu R H et al 2008 Phys. Rev. Lett. 101 087001
[4] Chen X H, Wu T, Wu G, Liu R H, Chen H and Fang D F 2008 Nature 453 761
[5] Mazin I I and Schmalian J 2009 Physica C 469 614
[6] Dong J K, Zhou S Y, Guan T Y, Zhang H, Dai Y F, Qiu X, Wang X F, He Y, Chen X H and Li S Y 2010 Phys. Rev. Lett. 110 017006
[7] Putzke C et al 2012 Phys. Rev. Lett. 108 047002
[8] Hänke T, Sykora S, Schlegel R, Baumann D, Zabolotnyy V B, Harnagea L, Wurmehl S, van den Brink J and Büchner B 2013 Phys. Rev. Lett. 110 127001
[9] Umezawa K et al 2012 Phys. Rev. Lett. 108 037002
[10] Allan M P, Rost A W, Mackenzie A P, Xie Y, Davis J C, Kihou K, Lee C H, Iyo A, Eisaki H and Chuang T-M 2012 Science 336 563
[11] Dong J K, Zhou S Y, Guan T Y, Zhang H, Dai Y F, Qiu X, Wang X F, He Y, Chen X H and Li S Y 2010 Phys. Rev. Lett. 104 087005
[12] Drew A J et al 2008 Phys. Rev. Lett. 101 097010
[13] Xi X X 2008 Rep. Prog. Phys. 71 116450
[14] Lin S Z 2009 Phys. Rev. B 86 014510
[15] Fernandes R M andMillis A J 2013 Phys. Rev. Lett. 111 127001
[16] Hirschfeld P J, Korshunov M M and Mazin I I 2011 Rep. Prog. Phys. 74 124508
[17] Golubov A A and Mazin I I 2013 Appl. Phys. Lett. 102 032601
[18] Lee P A and Wen X G 2009 Phys. Rev. B 78 144517
[19] Yao Z J, Li X and Wang D Z 2009 New. J. Phys. 11 025009
[20] Si Q andAbrahams E 2008 Phys. Rev. Lett. 101 076401
[21] Mazin I I, Singh D J, Johannes M D and Du M H 2008 Phys. Rev. Lett. 101 057003
[22] Wang F, Zhai H, Ren Y, Vishwanath A and Lee D H 2009 Phys. Rev. Lett. 102 047005
[23] Chen W Q, Ma F J, Lu Z Y and Zhang F C 2009 Phys. Rev. Lett. 103 207001
[24] Huang W M and Lin H H 2010 Phys. Rev. B 81 052504
[25] Vakaryuk V, Stanev V, Lee W C and Levchenko A 2012 Phys. Rev. Lett. 109 227003
[26] Nagai Y and Hayashi N 2009 Phys. Rev. B 79 224508
[27] Golubov A A, Brinkman A, Tanaka Y, Mazin I I and Dolgov O V 2009 Phys. Rev. Lett. 103 077003
[28] Onari S and Tanaka Y 2009 Phys. Rev. B 79 174526
[29] Tsai W F, Yao D X, Bernevig B A and Hu J P 2009 Phys. Rev. B 80 012511
[30] Chen C T, Tsuei C C, Ketchen M B, Ren Z A and Zhao Z X 2010 Nat. Phys. 6 260
[31] Hanaguri T, Niiaka S, Kuroki K and Takagi H 2010 Science 328 474
[32] Tsuei C C andKirtley J R 2000 Rev. Mod. Phys. 72 969
[33] Wu J andPhillips P 2009 Phys. Rev. B 79 092502
[34] Parker D and Mazin I I 2009 Phys. Rev. Lett. 102 227007
[35] Golubov A A and Mazin I I 2013 Appl. Phys. Lett. 102 032601
[36] Apostolov S and Levchenko A 2012 Phys. Rev. B 86 224501
[37] Li S Z 2012 Phys. Rev. B 86 041450
[38] Golubov A A, Kupriyanov M Y and Illich E 2004 Rev. Mod. Phys. 76 411
[39] Ryazanov V V, Oboznov V A, Rusanov A Y, Veretennikov A V, Golubov A A and Aarts J 2001 Phys. Rev. Lett. 86 2427
[40] Ryazanov V V, Oboznov V A, Veretennikov A V and Rusanov A Y 2001 Phys. Rev. B 65 020501
[41] Kontos T, Aprili M, Lesieur J and Grison X 2001 Phys. Rev. Lett. 86 304
[42] Ng T K and Nagaosa N 2009 Europhys. Lett. 87 17003
[43] Ota Y, Machida M, Koyama T andMatsumoto H 2009 Phys. Rev. Lett. 102 237003
[44] Zhang X H, Oh Y S, Liu Y, Yan L Q, Kim K H, Greene R L and Takeuchi I 2009 Phys. Rev. Lett. 102 147002
[45] Linder J, Sperstadel B and Sudbo A 2009 Phys. Rev. B 80 020503(R)
[46] de Gennes P G 1966 Superconductivity of Metals and Alloys (New York: Benjamin)
[47] Araújo M A N and Santiago M A 2009 Phys. Rev. B 79 174529
[48] Burmistrova A V, Devyatov I A, Golubov A A, Yada K and Tanaka Y 2013 Phys. Soc. Japan 82 034716
[49] Burmistrova A V, Devyatov I A, Golubov A A, Yada K, Tanaka Y, Tortello M, Gonnelli R S, Stepanov V A, Ding X X and Wen H H 2015 Phys. Rev. B 91 214501
[50] Fujisaki A and Tsukada M 1991 Solid State Commun. 78 29
[51] Kashiwaya M and Tanaka Y 2000 Rep. Prog. Phys. 63 1641
[52] Barash Y S, Bobkova I V and Koppp T 2002 Phys. Rev. B 66 140503(R)
[53] Barash Y S and Bobkova I V 2002 Phys. Rev. B 65 144502
[54] Radovic Z, Lazarides N andFlytzanis N 2003 Phys. Rev. B 68 014501
[55] Hu C R 1994 Phys. Rev. Lett. 72 1526
[56] Tanaka Y and Kashiwaya S 1995 Phys. Rev. Lett. 74 3451