On Dominant Interference in Random Networks and Communication Reliability

Dong Liu, Baptiste Cavarec, Lars K. Rasmussen, and Jing Yue

Division of Information Science and Engineering
KTH Royal Institute of Technology
Stockholm, Sweden
E-mail: \{doli, cavarec, lkra, jyue\}@kth.se

Abstract— In this paper, we study the characteristics of dominant interference power with directional reception in a random network modelled by a Poisson Point Process. Additionally, the Laplace functional of cumulative interference excluding the \( n \) dominant interferers is also derived, which turns out to be a generalization of omni-directional reception and complete accumulative interference. As an application of these results, we study the impact of directional receivers in random networks in terms of outage probability and error probability with queue length constraint.

I. INTRODUCTION

The studies of communication technologies, services and applications for massive Machine Type Communication (MTC) have significantly increased in recent years. Emerging MTC requires significant performance improvements for a communication link, which is either an increase of capacity for vehicular technology, or an increase in high reliability while keeping low latency constraints for tactile internet and factory automation purposes. A key point of these challenges is to accommodate the increasing number of devices with reliable services. Multiple access schemes, especially non-orthogonal multiple access, have been identified as potential solutions to meet future massive access requirements in wireless networks under various applications. However, fading is modeled as path loss, and hence only the Euclidean distance was taken into account to derive the \( n^{th} \) \((n = 1, 2, \cdots)\) dominant interference power in these works. A closed form of the Euclidean distance distribution to the \( n^{th} \) nearest neighbor in networks modeled by a Poisson Point Process (PPP) was derived in [5], [6].

Though path loss is one of the major factors causing wireless signal fading, channel fading is also significantly influencing the strength of wireless signals. Due to the dynamic nature of such fading, the signal power received from a nearer transmitter is possibly smaller than that from a farther transmitter. Thus we want to study characteristics of the \( n^{th} \) dominant interference power as well as accumulative interference in terms of power itself rather than Euclidean distance. It is also of interests to see how these affect communication reliability in wireless networks. In addition, directional reception induces sectorization in cellular networks, which is a typical method for interference management. Thus, directional reception angle also affects the characteristics of dominant and accumulative interference power.

In this paper, we model stationary random networks by homogeneous PPP. We give probability distribution of the \( n^{th} \) dominant interference power in stationary random networks where receivers have directional reception. In addition, the partial accumulative interference excluding some dominant interferers is studied, which is a generalization of the omni-directional case and also complete accumulative interference. The outage probability and transmission error probability with queue length constraint in Nakagami-m fading are studied.

The remainder of this paper is organized as follows. In Section II we derive the closed-form distribution of the \( n^{th} \) dominant interference power and Laplace functional of partial accumulative interference power. In Section III we discuss the communication outage probability and transmission error probability with queue length constraint. In Section IV numerical results are given to verify the derived results. Finally, we conclude this paper in Section V.

II. DOMINANT AND ACCUMULATIVE INTERFERENCE

We model a random network by a PPP \( \Phi = \{ x_i \}, x_i \in \mathbb{R}^2 \), with intensity \( \lambda \), where \( x_i \) denotes the coordinates of node \( i \). The intensity measure of \( \Phi \) is denoted by \( \Lambda \). For a receiver located at \( x \in \mathbb{R}^2 \) in the random network, the interference power \( I_i \) from node \( i \) is

\[
I_i = h_i |x - x_i|^{-\alpha},
\]

where \( h_i \) is the channel gain from \( x_i \) to \( x \), \( |\cdot| \) the Euclidean norm, and \( \alpha > 2 \) the path-loss parameter. We arrange \( \{ I_i \} \) into a descending-ordered sequence \( \{ I_n \}_{n=1,2,...} \) such that \( I_n \) is the \( n^{th} \) dominant interference power \( (I_1 > I_2 > \cdots) \).

To study the distribution of \( I_n \), point process mapping and displacement [6] are be applied on \( \Phi \). We assume directional reception and the reception is within deterministic angle \( \phi \), where \( 0 < \phi \leq 2\pi \). Since \( \Phi \) is homogeneous, without losing generality, we define the interferers to node \( x \) by

\[
\Phi' = \{ x' = x_i \mid \angle xx_i \leq \phi, x_i \in \Phi \},
\]

1Term “node” and “point” are alternatively used depending on the context of network or point process.
where \( \angle \overrightarrow{x'x} \) gives the angle of vector \( \overrightarrow{x'x} \) regarding the horizontal axis on \( \mathbb{R}^2 \). Then we map \( \Phi' \) to \( \Phi_1 \) as

\[
\Phi_1 = \{ y_i = \|x'_i - x\|^α \mid x'_i \in \Phi', α > 2 \}.
\]

We further define the point process \( \Phi_2 \) that takes values from \( \mathbb{R}^+ \) as a displacement of \( \Phi_1 \) as

\[
\Phi_2 = \left\{ z_i = \frac{y_i}{h_i} \mid y_i \in \Phi_1 \right\}.
\]

We arrange the points of \( \Phi_2 \) into an ascending-ordered sequence \( \{z_n\}_{n=1,2,\ldots} \) along the power axis \( \mathbb{R}^+ \). Hence, \( z_n \) is the \( n \)-th closest point of \( \Phi_2 \) to origin on the power axis \( \mathbb{R}^+ \) leading the \( n \)-th dominant interference power to be \( I_n = z_n^{-1} \).

**Theorem 1 (The \( n \)-th Dominant Interference Power):** In a PPP on \( \mathbb{R}^2 \) with intensity \( λ, z_n \), the inverse of the \( n \)-th dominant interference power \( I_n \), to a receiver with reception angle \( φ \) has the Probability Density Function (PDF)

\[
f_{z_n}(z) = \frac{2}{αz(α-1)!} \exp\left(-\frac{\lambda}{z}\right),
\]

where \( h_{2/α} = E_h[h^{2/α}] \) denotes expectation supported by PDF of channel gain \( h \).

**Proof:** Combining the definition in (2) and (3) gives

\[
\Phi_1 = \{ y_i = \|x'_i - x\|^α \mid \angle \overrightarrow{x'x} ≤ φ, φ, α > 2, x_i \in \Phi \}.
\]

Denote the intensity measure of \( \Phi_1 \) by \( Λ_1 \) and its intensity function by \( λ_1 \). Since \( Φ_1 \) is obtained by thinning and mapping independently from \( Φ, Φ_1 \) is also a PPP \([6],[9]\). The intensity \( Λ_1 \) is

\[
Λ_1(y) = E[Φ_1([0,y])] = \int_{B_x(y)} \left( \frac{\lambda}{2} \right) dx' = \frac{\lambda}{2} y^{2/α},
\]

where \( B_x(r) = \{ x' \mid \|x - x\| < r, r \in \mathbb{R}^2, r \in \mathbb{R}^+ \} \) is a disc centered at \( x \) with radius \( r \), and \( ι(\cdot) \) is the indicator function. Thus, the intensity function of \( Φ_1 \) is

\[
λ_1(y) = \frac{∂Λ_1(y)}{∂y} = \frac{\lambda}{2} y^{2/α-1} \frac{1}{α}, y > 0.
\]

Since the point process \( Φ_2 \) is actually a displacement of \( Φ_1, \Phi_2 \) is also a PPP according to the Displacement Theorem \([9]\). The intensity measure \( Λ_2 \) of \( Φ_2 \) can be obtained according to the Displacement Theorem. We have

\[
P\left(\frac{y}{h} < z\right) = 1 - F_h\left(\frac{y}{z}\right),
\]

where \( F_h(\cdot) \) is the Cumulative Distribution Function (CDF) of \( h \). The probability kernel \( ρ(y, z) \) of the displacement is

\[
ρ(y, z) = \frac{∂}{∂z} \left[ 1 - F_h\left(\frac{y}{z}\right) \right] = \frac{y}{z^2} f_h\left(\frac{y}{z}\right).
\]

Hence, the intensity function \( λ_2(z) \) of \( Φ_2 \) is obtained as

\[
λ_2(z) = \int_0^∞ λ_1(y) ρ(y, z) dy = \frac{φλ}{α} z^{2/α-1} E_h[h^{2/α}],
\]

where \( E_h[h^{α}] = \int_0^∞ h^{α} f_h(h) dh \). Then

\[
Λ_2(z) = \int_0^z λ_2(z) dz = \frac{φλ}{α} E_h[h^{2/α}] z^{2/α}, z \in \mathbb{R}^+.
\]

We arrange points of \( Φ_2 = \{z_n\} \) into an ascending-ordered sequence \( \{z_n\}_{n=1,2,\ldots} \). By definition, \( I_n = z_n^{-1} \) is the \( n \)-th largest interference power from points of \( Φ_2 \). Denote the number of points of \( Φ_2 \) within \([0,z]\) on the power axis \( \mathbb{R}^+ \) by \( N_z \). Then \( N_z \) is a Poisson random variable with intensity \( Λ_2(z) \) and we have

\[
P(N_z = k) = \frac{(Λ_2(z))^k}{k!} \exp(-Λ_2(z)), k = 0, 1, 2, \ldots.
\]

Let \( F_{z_n}^φ(z) \) be the CDF of \( z_n \), we have

\[
F_{z_n}^φ(z) = 1 - \sum_{k=0}^{n-1} P(N_z = k) = \frac{γ(n, Λ_2(z))}{Γ(n)},
\]

where \( Γ(α) = \int_0^∞ x^{α-1} \exp(-x) dx \) is gamma function and \( γ(a,b) = \int_0^b x^{α-1} \exp(-x) dx \) is lower incomplete gamma function. By taking the derivative of \( F_{z_n}^φ \) the PDF of \( z_n \) is

\[
f_{z_n}^φ(z) = \frac{2}{αn!} \exp\left(-\frac{Λ_2(z)}{n}\right).
\]

**Corollary 1.1:** By Theorem 1, the \( n \)-th dominant interference power is expected to be

\[
E[I_n] = \frac{γ(n, Λ_2(z_n))}{Γ(n)} = \frac{Γ(n - α / 2)}{Γ(n)}, n > α / 2.
\]

As expected, the \( n \)-th dominant interference power expectation decreases with path-loss parameter \( α \). \( E[I_n] \) increases with the increasing reception angle \( φ \) and node intensity \( λ \). As a function of \( n, E[I_n] \) decreases with \( Γ(n - α / 2) / Γ(n) \), whereas the expectation of the \( n \)-th nearest node’s distance increases with \( Γ(n + 1/2) / Γ(n) \).

**Corollary 1.2:** The CDF of the \( n \)-th dominant interference power is obtained as

\[
F_{I_n}^φ(z) = \frac{Γ(n, Λ_2(z^{-1}))}{Γ(n)},
\]

where \( Γ(a, b) = \int_0^b x^{α-1} \exp(-x) dx \) is the upper incomplete gamma function.

Let us define the partial accumulative interference power as

\[
I(n) = \sum_{k=n+1}^∞ I_k = \sum_{k=n+1}^∞ z_k^{-1}, n = 1, 2, \ldots.
\]

**Theorem 2:** [Partial Accumulative Interference Power] In a PPP on \( \mathbb{R}^2 \) with intensity \( λ \), the partial accumulative interference \( I(n) \) excluding the first \( n \) dominant interferers to a receiver with reception angle \( φ \) is characterized by its Laplace functional

\[
L_I(n)(s|z_n) = \exp\left\{ \frac{φλ}{α} h_{2/α} q_{z_n}(s) \right\},
\]
where \( q_z(t) = s^{\frac{1}{2}} \gamma \left(-\frac{2}{\alpha}, \frac{1}{z} \right) + \frac{\alpha}{2} s^{\frac{2}{\alpha}}. \)

**Proof:**

\[
L_{\mathcal{I}(n)}(z) = E \left[ \exp \left\{ -s \sum_{k=n+1}^{\infty} z_k^{-1} \right\} \right] = E \left[ \prod_{k=n+1}^{\infty} \exp \left\{ -sz_k^{-1} \right\} \right] = \exp \left\{ \int_0^\infty \exp \left\{ -s \Lambda_2(dz) \right\} \right\}.
\]

\[
(a) = \exp \left\{ \int_0^\infty \exp \left\{ -s \Lambda_2(dz) \right\} \right\} = \exp \left\{ \Gamma(n+1) \Lambda_2(dz) \right\}
\]

\[
(b) = \exp \left\{ \frac{\alpha}{2} h_{2/\alpha} \left( \frac{\alpha}{2} \right)^{2/\alpha} \right\} = \frac{1}{\Gamma(n)} \frac{\Gamma(n+1)}{\Gamma(n+1/2)}.
\]

Thus the derivative of \( L_{\mathcal{I}(n)}(s) \) can be directly calculated as

\[
\frac{\partial L_{\mathcal{I}(n)}(s)}{\partial s} = \frac{\partial E[\mathcal{I}(n)]}{\partial s} = - \frac{E[\mathcal{I}(n)]}{s}.
\]

Then we have

\[
\frac{\partial L_{\mathcal{I}(n)}(s)}{\partial s} = - \frac{E[\mathcal{I}(n)]}{s} = - \frac{\gamma \left(1 - 2/\alpha, s/z \right)}{\alpha - 2}.
\]

Thus we can calculate the average of accumulate interference as follows

\[
\bar{\mathcal{I}}(n) = E[\mathcal{I}(n)] = \int_0^\infty E[\mathcal{I}(n)|z_n] f_{\mathcal{I}}(z_n) dz_n.
\]

Applying Theorem 1 and substituting Eq. (22) gives us the expectation of \( \mathcal{I}(n) \)

\[
\bar{\mathcal{I}}(n) = \frac{2}{\alpha - 2} \left( \frac{\alpha}{2} \right)^{1/2} \Gamma(n+1) \Gamma(n+1/2).
\]

**Corollary 2.2 (Scaling from Omni-directional Reception Case):** In a Poisson random network with density \( \lambda \), the partial accumulative interference taken in angle \( \phi \) directional reception averagely can be equivalent to the accumulative taken in omni-directional reception, if the interference within \( B_0(R) \) is avoided, where

\[
R = \left( \frac{\phi h_{2/\alpha}}{2} \right) \frac{\pi}{\gamma} \frac{1}{s} \frac{\alpha}{\alpha - 2}.
\]

**Proof:** The accumulative interference without the nodes within \( B_0(R) \) is formulated as

\[
\mathcal{I}_{\Phi \setminus B_0(R)} = \sum_{x_i \in \Phi \setminus B_0(R)} h_{2/\alpha} \left| |x_i| - \gamma \right| \alpha.
\]

The Laplace functional of \( \mathcal{I}_{\Phi \setminus B_0(R)} \) is

\[
\mathcal{L}_{\mathcal{I}_{\Phi \setminus B_0(R)}}(s) = E \left[ \exp \left\{ -s \mathcal{I}_{\Phi \setminus B_0(R)} \right\} \right] = E \left[ \prod_{x_i \in \Phi \setminus B_0(R)} E_h \left[ \exp \left\{ -s h_{2/\alpha} \left| |x_i| - \gamma \right| \alpha \right\} \right] \right] = \exp \left\{ \int_{\Phi \setminus B_0(R)} E_h \left[ \exp \left\{ -s h_{2/\alpha} \left| |x| - \gamma \right| \alpha \right\} \right] dx \right\}.
\]

Thus the derivative of \( \mathcal{L}_{\mathcal{I}_{\Phi \setminus B_0(R)}}(s) \) is

\[
\frac{\partial}{\partial s} \mathcal{L}_{\mathcal{I}_{\Phi \setminus B_0(R)}}(s) = \left\{ -\lambda \int_{\Phi \setminus B_0(R)} E_h \left[ \exp \left\{ -s h_{2/\alpha} \left| |x| - \gamma \right| \alpha \right\} \right] dx \right\}.
\]

Then we have

\[
E[\mathcal{I}_{\Phi \setminus B_0(R)}] = - \lim_{s \to 0} \frac{\partial}{\partial s} \mathcal{L}_{\mathcal{I}_{\Phi \setminus B_0(R)}}(s) = 2 \Phi \left( R^2, \frac{\alpha}{\alpha - 2} \right).
\]

Then comparing \( E[\mathcal{I}_{\Phi \setminus B_0(R)}] \) with \( \bar{\mathcal{I}}(n) \) gives the equivalent condition.

**Corollary 2.3 (Lower bound of accumulative interference):** A lower bound on the Laplace functional of partial accumulative interference \( \mathcal{I}(n) \) is

\[
\mathcal{L}_{\mathcal{I}(n)}(s) = \exp \left\{ n + \frac{\phi h_{2/\alpha}}{\alpha} E_{\gamma \left(2/\alpha, s/z \right)} \right\}.
\]

where \( E_{\gamma \left(2/\alpha, s/z \right)} \) is the expectation of \( \gamma \left(2/\alpha, x/s \right) \) with the support of probability density function of \( z_n \), i.e. \( f_{\gamma \left(2/\alpha, s \right)} \).

**Proof:** The lower bound is obtained straightforwardly by applying Jensen’s inequality. Since exponential function \( \exp(x) \) is a convex function regarding \( x \), thus we have

\[
\int_0^\infty \mathcal{L}_{\mathcal{I}(n)}(s) f_{\mathcal{I}}(z_n) dz_n \geq \exp \left\{ \int_0^\infty \frac{\phi h_{2/\alpha}}{\alpha} s^{\gamma \left(2/\alpha, s \right)} f_{\mathcal{I}}(z_n) dz_n \right\} = \exp \left\{ n + \frac{\phi h_{2/\alpha}}{\alpha} E_{\gamma \left(2/\alpha, s \right)} \right\}.
\]
**Corollary 2.4 (Upper bound of accumulative interference):**

An upper bound on the Laplace functional of partial accumulative interference \( I(n) \) is

\[
L^\phi_{I(n)}(s) = \exp \left\{ \frac{\phi \lambda \bar{h}_{2/\alpha}}{\alpha} \gamma(s, \bar{z}_n) \right\},
\]

where \( \bar{z}_n = E[|z_n|] = \left( \frac{h_{n,0} \bar{h}_{1/\alpha}}{2} \right)^{-\alpha/2} \frac{\Gamma(\alpha+\alpha/2)}{\Gamma(n+\alpha/2)} \).

**Proof:** See Appendix A.

### III. APPLICATIONS TO COMMUNICATION RELIABILITY

In this section, communication reliability is studied with Nakagami-m fading model, the channel gain \( h \) has for PDF

\[
f_h(x) = \frac{m^m x^{m-1}}{\Omega^m \Gamma(m)} \exp \left\{ -\frac{mx}{\Omega} \right\}, \quad m > 1/2,
\]

where \( m \) is the fading parameter and \( \Omega \) the fading power.

#### A. Outage Probability

Outage of communication occurs when the Signal to Interference Ratio (SIR) drops below a threshold \( \eta \). The outage probability can be calculated by evaluating the CDF of the SIR at the threshold \( \eta \)

\[
F_{SIR}^\phi(\eta) = \mathbb{P} \left( \eta > \frac{h u^{-\alpha}}{I(n)} \right) = E_{I(n)} \left[ F_h(u^{\alpha} \eta I(n)) \right],
\]

where \( u \) is the Euclidean distance between transmitter and receiver, \( E_{I(n)}[\cdot] \) denotes expectation regarding \( I(n) \) and

\[
F_h(x) = \frac{\gamma(m, mx/\Omega)}{\Gamma(m)} = 1 - \sum_{k=0}^{m-1} \frac{(mx/\Omega)^k}{k!} \exp \left\{ -mx/\Omega \right\},
\]

where \( (a) \) uses PDF \( (35) \) and \( (b) \) achieves when \( m \) is positive integer. Then applying Eq. (5) and (19) gives

\[
F_{SIR}^\phi(\eta) = E_{I(n)} \left[ 1 - \sum_{k=0}^{m-1} \frac{(mu^{\alpha} \eta I(n))^k}{\Omega^k k!} e^{-\mu^{\alpha} \eta I(n)/\Omega} \right]
\]

\[
= 1 - \sum_{k=0}^{m-1} \frac{(mu^{\alpha} \eta)^k}{\Omega^k k!} E_{I(n)} \left[ \frac{I(k)^k}{\Gamma(k)} e^{-\frac{mu^{\alpha} \eta I(n)}{\Omega}} \right]
\]

\[
= 1 - \sum_{k=0}^{m-1} \frac{(-mu^{\alpha} \eta)^k}{\Omega^k k!} E_{I(n)}\left[ \frac{I(k)}{\Gamma(k)} \frac{(mu^{\alpha} \eta)^{\bar{z}_n}}{\Omega} \right],
\]

where \( I(k)^k(\cdot|z_n) \) is the \( k \)-th derivative of \( I(n)^k(\cdot|z_n) \).

In the Nakagami-m fading case,

\[
\bar{h}_{2/\alpha} = E_h[h^{2/\alpha}] = \left( \frac{m}{\Omega} \right)^{\alpha} \frac{\Gamma(m + \frac{2}{\alpha})}{\Gamma(m)}.
\]

#### B. Error probability under QoS Constraint

QoS (Quality of Service) is defined by parameter pair \((\epsilon_q, Q_{max})\) and used to measure communication link quality, where \( Q_{max} \) is the tolerable queue length for service data at transmitter and \( \epsilon_q \) is the violation probability of constraint \( Q_{max} \). (10), (11) give the approximation of \( \epsilon_q \) as

\[
\epsilon_q \approx \exp \left\{ -\theta Q_{max} \right\},
\]

where \( \theta \) is QoS exponent. For any required QoS, corresponding effective bandwidth \( a(\theta) \) (10), (11) gives the minimal data rate to meet the QoS requirement \((Q_{max}, \epsilon_q)\), defined as

\[
a(\theta) = \lim_{t \to \infty} \frac{\log E[\exp \{\theta A(t)\}]}{t \theta},
\]

where \( A(t) \) is the cumulative source data over time interval \([0, t] \). If transmitter can send data out with guaranteed rate \( r = a(\theta) \), violation error probability can be bounded by \( \epsilon_q \). However, data rate over wireless channel is dynamic and unreliable. The selected rate \( r \) by transmitter could be failed due to poor SIR. With derived result in Section III-A, the error probability that wireless channel can not provide rate \( r \) is

\[
\epsilon_r = \mathbb{P}[\log(1 + \text{SIR}) < a(\theta)] = F_{SIR}^\phi(\exp\{a(\theta)\} - 1).
\]

The overall error probability \( \epsilon \) is due to either queue violation either channel fading and can be approximated as

\[
\epsilon \approx 1 - (1 - \epsilon_q)(1 - \epsilon_r) = \epsilon_q + \epsilon_r - \epsilon_q \epsilon_r.
\]

Hence for a given queue length constraint \( Q_{max} \) and chosen transmission \( r \), the total error can be approximated by Eq. (43).

#### C. Relationship between \( r \) and \( \epsilon \)

The following theorem shows the whether a given QoS specification is possible:

**Theorem 3:** Assume a wireless link with error probability \( \epsilon_r(r) \) for corresponding link achievable rate \( r \). Denote the target QoS specification by \((\epsilon', Q_{max})\). The target QoS is possible to be met by rate adaptation (increasing \( r \)), if the condition

\[
\epsilon_r(r^*) \leq 1 - \sqrt{1 - \epsilon'}
\]

is met. Here \( r^* \) is the root to equation

\[
\epsilon_q(r) = \epsilon_r(r),
\]

where \( \epsilon_q(r) \) is the queue violation probability with service rate \( r \) and maximum tolerable queue length at transmitter \( Q_{max} \). Note that the equation \( \epsilon_q(r) = \epsilon_r(r) \) has at most one root.

**Proof:** As stated the total error \( \epsilon(r) \) is actually a function of the selection rate \( r \), which can be formed as

\[
\epsilon(r) = \epsilon_q(r) + \epsilon_r(r) - \epsilon_q(r) \epsilon_r(r).
\]

For an error probability \( \epsilon' \), the selected rate must satisfy

\[
r > a(\theta'),
\]

**2**Larger \( \theta \) stands for higher QoS requirement, i.e. smaller \( Q_{max} \) or violation probability bound \( \epsilon_q \).
where
\[ \theta' = -\log \epsilon' / Q_{max}, \] (48)
otherwise the packet queue at transmitter would not be stable and there would be no bound on the queue violation error.

We rewrite \( \epsilon(r) \) and have
\[ \epsilon(r) = 1 - \left(1 - \epsilon_r(r)\right) \left(1 - \epsilon_q(r)\right) \geq 1 - \left(1 - \epsilon_r(r) + \epsilon_q(r)\right)^2, \] (49)
where \((a)\) uses that arithmetic mean of non-negative numbers is greater than or equal to their geometric mean. The equality achieves when the \( \epsilon_r(r) = \epsilon_q(r) \). By setting \( a(\theta) = r \), we have
\[ \frac{\partial \epsilon_q(r)}{\partial \theta} = -Q_{max} e^{-\theta} \frac{\partial Q_{max}}{\partial \theta}. \] (50)

According to [11], effective bandwidth is an increasing function of \( \theta \), thus we have
\[ \frac{\partial r}{\partial \theta} > 0. \] (51)

Combining Eq. (50) and (51) gives
\[ \frac{\partial \epsilon_q(r)}{\partial r} < 0. \] (52)

Thus, we concludes that \( \epsilon_q(r) \) is monotonically decreasing function of selected rate \( r \), i.e. selecting larger rate \( r (r \geq a(\theta')) \) leads smaller queue violation error \( \epsilon_q(r) \).

On the other hand, we would like to show that communication link failure error \( \epsilon_r(r) \) is an increasing function of selected rate \( r \). Assuming that the probability density function of SINR is \( f (f > 0 \text{ in its domain}) \) and using Shannon capacity, we have
\[ \frac{\partial \epsilon_r(r)}{\partial r} = \frac{\partial}{\partial r} \int_0^{e^{-1}} f(x)dx \\
= f(e^{-1}) e^r \\
> 0. \] (53)

Since \( \epsilon_r(r) \) is monotonically increasing and \( \epsilon_q \) is monotonically decreasing when we choosing larger \( r \), there is one and only one root to \( \epsilon_r(r) = \epsilon_q(r) \) if the condition
\[ \epsilon_r(a(\theta')) \leq \epsilon_q(a(\theta')) \] (54)
is met.

Assume that the condition (54) is met and \( r' \) is the unique root to \( \epsilon_r(r) = \epsilon_q(r) \), according to (49), we get
\[ \inf \epsilon = 1 - \left(1 - \epsilon_r(r^*) + \epsilon_q(r^*)\right)^2 \\
= 1 - (1 - \epsilon_r(r^*)). \] (55)

We have the QoS requirement that error is no larger than \( \epsilon' \). Thus, this is possibly be met by choosing better communication rate if
\[ \inf \epsilon \leq \epsilon'. \] (56)

Otherwise, we can not guarantee that the initial QoS specification \((\epsilon', Q_{max})\) would be met by choosing larger \( r \).

Substituting Eq. (55) into (56) gives the condition
\[ \epsilon_r(r^*) \leq 1 - \sqrt{1 - \epsilon'}, \] (57)
where \( \epsilon_r(r^*) = \epsilon_q(r^*) \).

Theorem [3] gives the sufficient condition to evaluate if a proposed QoS specification can be reasonably fulfilled with a certain communication link condition. However, it is also possible that we can find a root \( r^* \) for Eq. (45) that can not fulfill the condition of inequality (44). In this case, we claim that it is possible to find a rate \( r^* \) that brings lowest total error \( \epsilon \) but without meeting the QoS requirement \((\epsilon', Q_{max})\). Still, this rate selection \( r^* \) gives the smallest error, i.e. \( \epsilon(r^*) \).

There is worse situation where \( \epsilon_r(a(\theta')) > \epsilon'_q \). In this case, there is no way for QoS \((\epsilon', Q_{max})\) to be met. Due to the monotonic property of \( \epsilon_r \) and \( \epsilon_q \) regarding \( r \), Eq. (45) does not has root. But it is still possible find rate \( r \) such that \( \epsilon(r) < \epsilon(a(\theta')) \). This could be done by solving the first derivative equation
\[ \frac{\partial \epsilon(r)}{\partial r} = 0. \] (58)
subject to
\[ r > a(\theta'). \] (59)

IV. NUMERICAL RESULTS

This section shows some numerical results (“Sim.”) and their comparisons with analytic results (“Ana.”). We set the parameters as node intensity \( \lambda = 10^{-4} \), path-loss exponent \( \alpha = 3 \), \( m = 2 \), \( \phi = \pi/4 \) and \( \Omega = 1 \), unless stated otherwise.

As shown in Fig. 1 curves of CDF for different \( n \)th dominant interference power \( I_n \) are given numerically and analytically. Since \( I_n \) is directly sorted by the interference power, we compare it with the \( n \)th nearest interferer’s power (“Nearest Sim.”) under same fading context. As shown, the difference between \( F_{I_n}^\phi \) and distribution of “Nearest Sim. n” is larger as \( n \) increases. In small range of \( I_n \), \( F_{I_n}^\phi \) is smaller than CDF sorted by distance. But in large range of \( I_n \), \( F_{I_n}^\phi \) is larger. That means that distance-based approximation can be overestimation or underestimation depending on \( I_n \). For larger \( n \), the Euclidean-distance-based approximation has larger bias.

Fig. 2 shows the outage probability against the reception angle \( \phi \), which matches well with \( F_{IR}^\phi \) in Eq. (33). Here \( \eta = 1 \). As expected, reception with larger angle \( \phi \) is subject to heavier interference and thus the outage probability increases along with the rising \( \phi \). Additionally, \( F_{IR}^\phi \) decreases obviously for increasing \( n \), when excluding more dominant interferers. The changes of outage probability vary with network setting such as \( \phi \). The Rayleigh fading \( (m = 1) \) is simulated for comparison. In small range of \( \phi \), the outage probability of Rayleigh fading is larger than that of \( m = 2 \), since signal fading of interest due to absence of direct line of sight (LOS) dominates. But, in large value of \( \phi \), outage of Nakagami-m \( (m = 2) \) is larger.

3Here \( n \)th nearest node is sorted out by Euclidean distance. Thus the \( n \)th nearest interferer is the \( n \)th closest neighbor by distance.
The bias of Euclidean-distance-based approximation was studied. To reduce communication link reliability by metrics of outage probability and partial accumulative interference power were studied. We could lead smaller total error even in larger range of $\varepsilon$. This means smaller error can be expected by loosening constraint on $Q_{\text{max}}$ when initial $Q_{\text{max}}$ is not large. Otherwise $\varepsilon$ flattens around $\varepsilon_\gamma$. Choosing larger $r$ is an effective way to get lower error before $\varepsilon$ flattens but it could bring larger error in larger range of $Q_{\text{max}}$. However, larger $n$ could lead smaller total error even in larger range of $Q_{\text{max}}$.

**V. CONCLUSION**

In this paper, we studied the dominant interference power in random networks modeled by PPP. Both the $n^{th}$ dominant and partial accumulative interference power were studied. We showed the bias of Euclidean-distance-based approximation by the $n^{th}$ nearest interferer numerically. This bias could be large for large $n$. Then, the obtained results were used to evaluate communication link reliability by metrics of outage probability and error probability with consideration of queue length violation. The possible way to decrease outage probability and total error was simulated and discussed.

**APPENDIX A**

**Upper Bound of $L_{\mathcal{I}(n)}(s|z_n)$**

**Proof:** For the upper bound of $L_{\mathcal{I}(n)}(s)$, we will show that $L_{\mathcal{I}(n)}(s|z_n)$ is concave regarding $z_n$ asymptotically for growing $z_n$ (decreasing $n^{th}$ dominant power), i.e. the concavity of $L_{\mathcal{I}(n)}(s|z_n)$ as a function of $z_n$ is also preserved for non-dense networks (small $\lambda$). If this condition satisfied, the conditional upper bound is as follows

$$
\int_0^\infty L_{\mathcal{I}(n)}(s|z_n) f_{z_n}(z_n) d z_n \leq \exp \left\{ \frac{\phi \bar{h}_2 / \alpha}{\alpha} \gamma(s, z_n) \right\},
$$

where

$$
\bar{z}_n = E[z_n] = \left( \frac{\bar{h}_2 / \alpha \phi}{\lambda} \right)^{-\alpha/2} \Gamma(n + \alpha/2) \Gamma(n).
$$

According to Theorem 2 the Laplace functional for partial accumulative interference is

$$
L_{\mathcal{I}(n)}(s|z_n) = \exp \left\{ \frac{\phi \bar{h}_2 / \alpha}{\alpha} \gamma(s, z_n) \right\}
$$

The second derivative of $L_{\mathcal{I}(n)}(s|z_n)$ regarding to $z_n$ is

$$
\frac{\partial^2 L_{\mathcal{I}(n)}(s|z_n)}{\partial z_n^2} = L_{\mathcal{I}(n)}(s|z_n) \left( \frac{\phi \bar{h}_2 / \alpha}{\alpha} \right)^2 \gamma(s, z_n) + \left( \frac{\phi \bar{h}_2 / \alpha}{\alpha} \right)^2 \gamma(s, z_n) + \left( \frac{\phi \bar{h}_2 / \alpha}{\alpha} \right)^2 \gamma(s, z_n)
$$

It is obvious that $L_{\mathcal{I}(n)}(s|z_n) \frac{\phi \bar{h}_2 / \alpha}{\alpha}$ is positive. Then it is the formula inside the parenthesis that decides the sign of
second derivative of $\mathcal{L}_{I(n)}(s|z_n)$ in Eq. (63). Thus we need to analyze the two terms inside the parenthesis to see its sign. The $\frac{\partial h_{2s,\alpha}}{\partial s}(\frac{\partial q_{z_n}(s)}{\partial z_n})^2$ is positive. Since

$$q_{z_n}(s) = s^{2/\alpha} \gamma \left( -2/\alpha, \frac{s}{z_n} \right) + \frac{\alpha z_n^{2/\alpha}}{2},$$

we have the first derivative of $q_{z_n}(s)$ regarding $z_n$ as

$$\frac{\partial q_{z_n}(s)}{\partial z_n} = \left( 1 - e^{-s/z_n} \right) z_n^{-1+2/\alpha},$$

and the second derivative as

$$\frac{\partial^2 q_{z_n}(s)}{\partial z_n^2} = z_n^{-2-2+4/\alpha} e^{-s/z_n} \left[ - \left( e^{s/z_n} - 1 \right) z_n^{\alpha - 2} - s \alpha \right].$$

Since $\alpha > 2$, and both $s$ and $z_n$ are positive, it is straightforward that $\frac{\partial^2 q_{z_n}(s)}{\partial z_n^2}$ is always negative. On the other hand, as we mentioned that the $\left( \frac{\partial q_{z_n}(s)}{\partial z_n} \right)^2$ is positive, we could hardly decide if Eq. (63) is positive or negative directly. Calculating the the derivative of Eq. (63) would make the analysis more complicated since the root of it is not be analytically obtained. Thus we use the power series for its asymptotic analysis:

$$\exp(z) = \sum_{k=0}^{\infty} \frac{z^k}{k!}.$$ (67)

Applying this, we have

$$\left( \frac{\partial q_{z_n}(s)}{\partial z_n} \right)^2 = \left( 1 - e^{-s/z_n} \right)^2 z_n^{-2-4/\alpha} = \left( 1 - 2e^{-s/z_n} + e^{-2s/z_n} \right) z_n^{-2+4/\alpha} = 1 - 2 \sum_{k=0}^{\infty} \frac{(-s)^k}{k!} z_n^{-k} + \sum_{k=0}^{\infty} \frac{(-s)^k}{k!} z_n^{-k} \left( e^{-s/z_n} - 1 \right) z_n^{-2+4/\alpha} = \sum_{k=1}^{\infty} \frac{(-2s)^k - 2(-s)^k}{k!} z_n^{-k} z_n^{-2+4/\alpha}.$$ (68)

Similarly, the second derivative of $q_{z_n}(s)$ can be formed as

$$\frac{\partial^2 q_{z_n}(s)}{\partial z_n^2} = \sum_{k=2}^{\infty} \frac{(-2s)^k - 2(-s)^k}{k!} \frac{z_n^{-k} z_n^{-2+4/\alpha}}{k!} = z_n^{2/\alpha - 2} \mathcal{O}(z_n^{2/\alpha - 2}).$$

Substituting Eq. (68) and (69) into (63) gives

$$\frac{\partial^2 \mathcal{L}_{I(n)}(s|z_n)}{\partial z_n^2} = \mathcal{L}_{I(n)}(s|z_n) \frac{\phi \partial \mathcal{L}_{I(n)}(s|z_n)}{\partial z_n} \frac{\alpha/z_n}{\alpha} = \mathcal{O}(z_n^{2/\alpha - 2}) + \mathcal{O}(z_n^{-1}).$$

Since $\alpha > 2$, $2 - \frac{2}{\alpha} > 1$. Then $\mathcal{O}(z_n^{-1})$ fades slower than $\mathcal{O}(z_n^{2/\alpha - 2})$ as $z_n$ increases. Also

$$\lim_{z_n \to 0} \frac{\phi \partial \mathcal{L}_{I(n)}(s|z_n)}{\partial z_n} \frac{\alpha/z_n}{\alpha} = 0,$$

$$\lim_{z_n \to 0} \frac{\partial^2 q_{z_n}(s)}{\partial z_n^2} = 0.$$ (71)

Thus, the positive term in Eq. (70) approaches to 0 faster than the negative term. We can say when $z_n$ is larger than a certain value $z$, $\frac{\partial^2 \mathcal{L}_{I(n)}(s|z_n)}{\partial z_n^2}$ remains negative. In addition, for non-dense networks, i.e. $\lambda$ is small, positive term of $\frac{\partial^2 \mathcal{L}_{I(n)}(s|z_n)}{\partial z_n^2}$ is relatively small and then second derivative of $\mathcal{L}_{I(n)}(s|z_n)$ is negative throughout positive axis of $\mathbb{R}$.

REFERENCES

[1] M. Shirvamutoghadam, M. Dohler, and S. J. Johnson, “Massive non-orthogonal multiple access for cellular IoT: Potentials and limitations,” IEEE Communications Magazine, vol. 55, no. 9, pp. 55–61, 2017.

[2] Z. Ding, M. Peng, and H. V. Poor, “Cooperative non-orthogonal multiple access in 5G systems;” IEEE Communications Letters, vol. 19, no. 8, pp. 1462–1465, 2015.

[3] C. Ma, W. Wu, Y. Cui et al., “On the performance of successive interference cancellation in D2D-enabled cellular networks,” in IEEE Conference on Computer Communications, 2015, pp. 37–45.

[4] M. Wildemeersch, T. Q. Quek, M. Kontoulis et al., “Successive interference cancellation in uplink cellular networks,” in IEEE 14th Workshop on Signal Processing Advances in Wireless Communications, 2013, pp. 310–314.

[5] M. Haenggi, “On distances in uniformly random networks,” IEEE Transactions on Information Theory, vol. 51, no. 10, pp. 3584–3586, 2005.

[6] ———, Stochastic geometry for wireless networks. Cambridge University Press, 2012.

[7] C. Ptosas, M. Mohammadi, I. Krikidis, and H. A. Suraweera, “Impact of directionality on interference mitigation in full-duplex cellular networks,” IEEE Transactions on Wireless Communications, vol. 16, no. 1, pp. 487–502, 2017.

[8] S. Yi, Y. Pei, and S. Kalyanaraman, “On the capacity improvement of ad hoc wireless networks using directional antennas,” in Proceedings of the 4th ACM international symposium on Mobile ad hoc networking & computing, 2003, pp. 108–116.

[9] F. Baccelli and B. Blaszczyszyn, “Stochastic geometry and wireless networks: Volume I Theory,” Foundations and Trends in Networking, vol. 3, no. 34, pp. 249–449, 2010. [Online]. Available: http://dx.doi.org/10.1561/1700000006.

[10] D. Wu and R. Negi, “Effective capacity: a wireless link model for support of quality of service,” IEEE Transactions on Wireless Communications, vol. 2, no. 4, pp. 630–643, 2003.

[11] C. Chang and J. A. Thomas, “Effective bandwidth in high-speed digital networks,” IEEE Journal on Selected areas in Communications, vol. 13, no. 6, pp. 1091–1100, 1995.