On stabilizing hydraulic motion experiencing Stribeck friction via PI controller: Circle criterion approach

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Abstract. A friction is inevitable in a hydraulic actuator due to the presence of seal. Unfortunately, the existence of friction makes the system prone to limit cycle oscillation as PI controller is applied. This research investigates the maximum integrator constant of PI controller that can be introduced to the system while maintaining global asymptotic stability in Lyapunov sense. This condition is achieved as the system complies with circle criterion. In this research, a simplified hydraulic system which behaves linearly is adopted. Thus, the only source of nonlinearity is the Stribeck friction. The main contribution of this research is a formulation of stable PI strategy that is free from the effect of inertia. The PI strategy is suitable for a hydraulic testing machine of which the mass of device under the test is varying.

Keywords: Stribeck friction, PI controller, hydraulic, limit cycle oscillation, circle criterion

1. Introduction
Hydraulic based position control is applied in the various industries for its capability to handle high load. To ensure the stability and performance of the control system, the old fashioned PID is still employed. Unfortunately, the expected performance cannot be achieved effortlessly as friction is inevitable in hydraulic actuator. This friction is generated by the seal of hydraulic actuator to reduce leakage during the energy transformation from fluid to mechanical. The friction makes the system liable to limit cycle oscillation and hunt the final position. This condition diminishes the accuracy of hydraulic positioning control.

There are a lot of friction models that have been developed to capture various friction behaviors. The comparison of this model is well investigated by Liu [1]. However, based on the experiment led by Bistray [2], Stribeck friction fit to describe the friction of Hydraulic actuator sealing system. Regardless the cause of friction, Marton [3] and Qu [4] also found the Stribeck behavior on the hydraulic cylinder actuator. The Stribeck friction model is characterized by strong nonlinearities due to the presence of sign function in pre-sliding regime. The more relax nonlinearities during the pre-sliding regime is given by Sin [5] who employed what so called Stribeck friction with threshold. On this model, the pre-sliding regime is characterized by a finite slope but not hysterical.

A breakthrough analysis about friction that causes limit cycle in mechanical position control scheme is developed by Hensen [6],[7]. The result of Hensen demonstrated the significance of integrator gain of PID that gives rise limit cycle of the system. In accordance to the result of Hensen, Maneetham [8] applied PD to stabilize position control of high speed servo system. Moreover, advance control algorithm such as sliding mode control was also applied to deal with limit cycle due to friction [9]. Unfortunately, the application of PD controller causes a steady state error [10] while the advanced
controller is not applicable in industrial world. Thus, some researcher devoted their research to understand the behavior of limit cycle due to friction in position control scheme.

The behavior of limit cycle in hydraulic position control is investigated by Chin [11] and Yanjay [12]. Both Chin and Yanjay investigated the existence of limit cycle via describing function method. While Chin only focus on the effect of Stribeck friction to the existence of limit cycle, Yanjay analyzed more in depth to the interaction between transport lag and Stribeck friction. However, both Chin and Yanjay do not give any criterion to avoid the limit cycle oscillation.

The limit cycle problem due to friction can also be addressed by dissipating all of the energy of the system by means fulfilling global asymptotic stability in Lyapunov sense. This approach is done by Lee [13] who developed Lyapunov energy like function of the system and found the PID criterion that will stabilize it. A more straightforward method to reach global asymptotic stability is to ensure the system comply with absolute stability. The application of absolute stability is suggested by Golestani [14]. Golestani developed a stable PID strategy based on circle criterion for a simple mechanical system.

This research is intended to maintain the research path of Chin [11] and Yanjay [12]. In this research, a PI criterion that will stabilize hydraulic position control in the presence of Stribeck friction is well developed. The Stribeck model applied on this research is Stribeck friction with the threshold applied by Sin [7]. The PI strategy is found as the system conforms to circle criterion that will make all of the energy of the system be dissipated. Having this approach, the hydraulic system needs to be simplified so that it behaves as a linear system. Hence, the only non-linearity present on the system is the Stribeck friction. The analysis shows that the maximum integrator gain of PI controller depends on the Stribeck properties during the pre-sliding behavior (stick behaviours). In addition, the developed PI strategy is independent of inertia. Thus, the PI guideline is very convenient to be applied to universal hydraulic testing of which the mass of device under the test is varying.

2. Hydraulic System and Stribeck Friction

Hydraulic system is a highly nonlinear system. This nonlinearity is generated by pressure-volume formulation of the fluid system. In order to simplify this formulation, the pressure-volume mapping is linearized. This research follows the linearization of symmetric hydraulic cylinder developed by other [15] who simplified the hydraulic system into second order system. The final block diagram of the symmetric hydraulic cylinder with PI controller is shown in Figure 1 (a). In the figure 1, \( \xi_{Ref}, \xi_{Act}, K_P, \) and \( K_I \) represents reference position \( (\xi_{Ref}) \), actual position \( (\xi_{Act}) \), proportional gain \( (K_P) \) and integrator gain \( (K_I) \) of the controller. Moreover, \( M, A_P, B, K_O, K_C, \) and \( K_{cb} \) represent mass \((M)\), area of the cylinder \((A_P)\), viscous friction \((B)\), gain coefficient \((K_O)\), flow pressure coefficient \((K_C)\), and compress flow coefficient \((K_{cb})\) respectively. The compress flow coefficient is determined by bulk modulus \((\beta)\) of the fluid and the volume of the chamber \((V_T)\) as it is shown in Eq. 1.

\[
K_{cb} = \frac{V_T}{4\beta}
\]

In order to make the system can be subjected to the circle criterion, it has to be represented in a Lure form. In the Lure representation, the system which behaves linear is separated from its nonlinear feedback. Moreover, the Lure system has no input and thus it will reveal the true behavior of the system. The lure system of the system above is shown in Figure 1 (b). In this Lure representation, the linear system is denoted by \( G(s) \) of which the feedback is Stribeck nonlinear friction. Having an elaboration from the Lure representation above, the formulation of \( G(s) \) can be restated as it is shown in Eq. 2. In the light of Hurwitz stability, \( G(s) \) will be stable if \( K_I \) is less than \( K_{I,\text{Lin}} \) and \( K_P \) is less than \( K_{P,\text{Lin}} \). The requirement of Hurwitz stability for the following system is shown in equation (3).

\[
G(s) = \frac{s^2(K_C + K_{vb})}{MK_{vb} s^2 + \left(\frac{BK_{vb} + MK_C}{s^3} + \frac{BK_C + A_P^2}{s^2} + \frac{A_P K_Q K_P}{s} + \frac{A_P K_Q K_I}{s}\right)}
\]

\[
K_1 < K_{1,\text{Lin}} = \text{max} \left(\frac{BK_C + A_P^2}{BK_{vb} + MK_C} \cdot \frac{K_P A_P}{MK_{vb}} \cdot \frac{MK_{vb}}{BK_{vb} + MK_C} \right) \quad \text{and} \quad K_P < K_{P,\text{Lin}} = \frac{BK_{vb} + MK_C}{A_P K_Q MK_{vb}} (BK_C + A_P^2)
\]

The formulation of Stribeck friction is the combination of coulomb friction, viscous friction and Stribeck phenomenon inherently. However, as the viscous friction possess linear behavior, it is covered
by the linear system \( G(s) \). For that reason, the Stribeck formulation employed in this research is only the combination of coulomb friction and the Stribeck phenomenon as it is shown in Eq. 4. There are two regimes defined in this Stribeck formulation. Those are the pre-sliding regime where \( |v|<\beta \) and slipping regime where \( |v|\geq\varepsilon \). In this formulation, \( F_s \), \( F_c \), and \( F_f \) represent friction force \((F_f)\), static friction force \((F_s)\), and coulomb friction \((F_c)\) respectively. Meanwhile, \( v \), \( v_s \), \( \varepsilon \), and \( n \) stand for velocity \((v)\), Stribeck velocity coefficient \((v_s)\), and Stribeck threshold \((\varepsilon)\), Stribeck power coefficient \((n)\) respectively.

\[
F_f(v) = \begin{cases} 
\frac{F_s}{\varepsilon} & \text{for } |v|<\varepsilon \\
F_c + (F_s - F_c) \exp \left( -\left( \frac{|v|}{v_s} \right)^n \right) \text{sign}(v) & \text{for } |v|\geq\varepsilon 
\end{cases}
\]

Figure 1. Position control scheme of symmetric hydraulic cylinder via PID. (a) Simplified block diagram (reproduced from [15]). The symmetric hydraulic cylinder system is simplified into the second order system. (b) The Lure representation of the corresponding blocking diagram. In this Lure representation, the linear part of the system is denoted by \( G(s) \) which is fed-back by a friction nonlinear function.

3. Circle Stability

Circle stability is an absolute stability criterion built under frequency domain analysis and sector nonlinearity. Observing Eq. 4, the Stribeck nonlinearity sector is bounded between \( m_1 \) and \( m_2 \) where \( m_1 = F_s/\varepsilon \) and \( m_2 = 0 \). Based on circle criterion, the stability would be achieved as \( G(s) \) fulfils Hurwitz and the Nyquist plot of the linear system \( G(s) \) does not cross the line \( \text{Real} = -\varepsilon/F_s \). The mathematical formulation of the circle criterion for this case is shown in Eq. 5. In this formulation, imaginary unit is denoted by \( i \).
\[ \text{Real}[G(j\omega)] = \text{Real} \left[ \frac{-\omega^2 \left( K_C + iK_{Vp} \omega \right)} {MK_{Vp} \omega^4 - i(BK_{Vp} + MK_C) \omega^3 - (BK_{C} + A_p^2) \omega^2 + iA_p K_Q K_p \omega + A_p K_Q K_1} \right] > -\frac{e}{F_s} \forall \omega \in \text{Real} \quad (5) \]

Having some algebra, the above equation leads to

\[ \frac{-\omega^2 \left( K_C \left( MK_{Vp} \omega^4 - (BK_{C} + A_p^2) \omega^2 + A_p K_Q K_1 \right) - K_{Vp} \omega \left( BK_{Vp} + MK_C \right) \omega^3 - A_p K_Q K_p \omega^2 \right)} {MK_{Vp} \omega^4 - (BK_{C} + A_p^2) \omega^2 + A_p K_Q K_1} > -\frac{e}{F_s} \forall \omega \in \text{Real} \quad (6) \]

To simplify equation (5), a new variable, \( p \), is defined as \( p = \omega^2 \) so that equation (6) can be represented as follow:

\[ C_4 p^4 + C_3 p^3 + C_2 p^2 + C_1 p + C_0 > 0 \quad \forall \quad p \in \text{Re} ; \quad p > 0 \quad (7) \]

Where

\[
\begin{align*}
C_4 &= \frac{e}{F_s} \left( MK_{Vp} \right)^2 \\
C_3 &= \frac{e}{F_s} \left[ MK_{Vp} \left( BK_{C} + A_p^2 \right) + \left( BK_{Vp} + BK_{C} \right) \right] + BK_{Vp}^2 \\
C_2 &= \frac{e}{F_s} \left[ BK_{C} + A_p^2 \right] + MK_{Vp} A_p K_Q K_1 - 2 \left( BK_{Vp} + BK_{C} \right) A_p K_Q K_1 + K_C \left( BK_{C} + A_p^2 \right) - K_{Vp} A_p K_Q K_p \\
C_1 &= \frac{e}{F_s} \left[ BK_{C} + A_p^2 \right] A_p K_Q K_1 + \left( A_p K_Q K_1 \right)^2 - K_C A_p K_Q K_1 \\
C_0 &= \frac{e}{F_s} \left( A_p K_Q K_i \right)^2
\end{align*}
\]

The easiest solution of inequality shown in Eq. (6) is \( C_k > 0 \); \( k = 0, 1, \ldots, 4 \). By the observation, the value of \( C_1, C_3, \) and \( C_4 \) are positive. Hence, the boundary of Eq. 6 lays on the value of \( C_2 \) and \( C_1 \). The simplification of \( C_2 > 0 \) leads to Eq. (8) while the simplification of \( C_1 > 0 \) is shown in Eq. (9)

\[ K_p < K_{p,\text{Max}} = \frac{K_C \left( BK_{C} + A_p^2 \right)} {MK_{Vp} A_p K_Q} \quad (8) \]

\[ K_i < K_{i,\text{Max}} = \frac{e} {F_s} \left( \frac{A_p K_Q} {K_C} \right) K_p^2 \quad (9) \]

Based on the derivation above, there is a maximum value of \( K_p \) and \( K_i \) to guarantee the system stability. The maximum value of \( K_p \) is independent of friction. It only depends on the properties of the hydraulic actuator and fluid properties. In contrast, the maximum value of \( K_i \) depends on the properties of friction around the pre-sliding regime (\( e/F_s \)) and the properties of hydraulic actuator itself. This analysis also shows that the maximum \( K_p \) and \( K_i \) are independent from the inertia of the system (\( M \)). It has to be noted that equation (8) and equation (9) only work given that \( G(s) \) is stable in linear manner.

### 4. Illustrative example

To give a practical understanding about the strength of PI strategy above, an illustrative example is made. The actuator hydraulic as well as its friction characteristic is taken from reference [3]. Meanwhile, the valve property is extracted from Moog D765 series specification. These parameters are shown in Table 1. It has to be noted that the output of controller is assumed to be electrical current that will directly drive the torque motor of hydraulic servo. For this reason, the unit of proportional gain (\( K_p \)) is mA/m and the unit of integrator gain (\( K_i \)) is mA/(m.s).

There are six cases which will be evaluated on this section. Case 1-3 shows the application of equation (9) which claims that the maximum integrator gain can be applied to the system is proportional to the quadratic of \( K_p \). In case 1-3, the value of \( K_p \) varies from 50 to 500 and the mass is 20 kg. In these systems, the \( K_i \) is set to the value of \( K_{i,\text{Max}} \) per equation (9). Thus, those systems are predicted to be stable. Case 3-4 shows the practical aspect of the analysis that also claims that the maximum integrator gain is independent on the inertia of the system. In case 3-4 the mass of system varies from 100 kg to 400 kg. Meanwhile, the value of \( K_p \) is set to 500 and \( K_i \) is set to \( K_{i,\text{Max}} \) which is about 675.34. Thus,
those systems will be stable. On the contrary in the last case, the value of $K_I$ is set to $0.8K_{I_{Lin}}$ which is computed without considering the friction nonlinearity. As the system violates the circle criterion, the system is predicted to be unstable (presented in Table 1 and Table 2).

| Table 1. Simulation Parameter | Parameter | Unit | Value |
|-------------------------------|-----------|------|-------|
| $A_P$                         | cm$^2$    |      | 6.6   |
| $V_T$                         | cm$^3$    |      | 177.4 |
| $\beta$                       | MPa       |      | 689.0 |
| $B$                           | N/(m/s)   |      | 1,219.1 |
| $K_Q$                         | cm$^3$/s/mA |   | 29.5   |
| $K_C$                         | cm$^3$/s/MPa |  | 47.2   |
| $F_s$                         | N         |      | 1,015.4 |
| $F_c$                         | N         |      | 513.2 |
| $v_s$                         | mm/s      |      | 69.6   |
| $E$                           | mm/s      |      | 6.9    |
| $N$                           |           |      | 1.000  |

The simulation result of those cases are shown in Fig. 2. Figure 2 (a) shows the result of cases 1-3. The simulation result shows that those systems are stable with different rate of rise time. These results exhibit that a high value of $K_I$ can be employed when the system has a high value of $K_P$. Figure 2 (b) presents the result of cases 3-4. As it is shown in figure 2 (b) those systems are stable. These results demonstrate that the developed PI strategy is independent on the mass of the system. Figure 2 (c) give an example when the $K_{I_{set}}$ is higher than the proposed $K_{I_{Max}}$. Thus, the system is trapped in limit cycle oscillation as it is presented in figure 2(c).

![Simulation result](image)

**Figure 2.** Simulation result (a) Simulation result for $M=100$kg with different value of $K_P$. In these cases, the $K_I$ is set to $K_{I_{Max}}$ such that the system fulfills circle stability and achieves stability. (b) Simulation result for $K_P = 500$ and $K_I = 675.34$ with different value of $M$. These cases show that the PI strategy is independent on the inertia of the system. (c) Simulation result for $K_P=160$; $M=100$ and $K_I$ is not higher than it should be so that the system is trapped in limit cycle oscillation.

| Table 2. Simulation Parameter | Case | $K_P$ (mA/m) | Mass (kg) | $K_{I_{set}}$ [mA/(m/s)] | Prediction |
|-------------------------------|------|--------------|-----------|---------------------------|------------|
| 1                             | 50   | 100          | 6.75      | Stable                    |
| 2                             | 160  | 100          | 69.15     | Stable                    |
| 3                             | 500  | 100          | 675.34    | Stable                    |
| 4                             | 500  | 200          | 675.34    | Stable                    |
| 5                             | 500  | 400          | 675.34    | Stable                    |
| 6                             | 160  | 100          | 7930.93   | Unstable                  |

5. Conclusion
The PI strategy has been developed to make hydraulic system achieve stability in the presence of Stribeck friction. The strategy is built under the rule of circle criterion. The analysis shows that maximum value of $K_I$ relies upon the properties of friction in the pre-sliding regime. Moreover, the
maximum $K_I$ can be employed in the system that is proportional to the quadat of $K_P$ applied. The benefit of the developed PI strategy is that it is independent on the inertia of the system. The simulation confirms this phenomenon. Unfortunately the PI strategy developed here is not the limit of instability. Thus, violating the proposed PI strategy doesn’t necessarily mean that the system would be unstable. This fact is the weakness point of the approach applied in this research and might be an interesting topic to study for the future research. Moreover, this research can be followed by having an optimal control of PID for DC motor system.

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