STRANGENESS AS A QGP SIGNAL IN AN ISENTROPIC QUARK-HADRON PHASE TRANSITION

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Abstract

Lattice QCD results reveal that the critical parameters and the order of the quark-hadron phase transition are quite sensitive to the number of dynamical flavours and their masses included in the theory. Motivated by this result we develop a phenomenological equation of state for the quark-gluon plasma consisting of \( n_f \) flavours retaining the entropy per baryon ratio continuous across the quark-hadron phase boundary. We thus obtain a generalised expression for the temperature and baryon chemical potential dependent bag constant. The results are shown for the realistic case, i.e., involving u, d and s quarks only. We then obtain a phase boundary for an isentropic quark-hadron phase transition using Gibbs’ criteria. Similarly another phase boundary is obtained for the transition to an ideal QGP from the solution of the condition \( B(\mu, T) = 0 \). The variation of critical temperature \( T_c \) with the number of flavours included in the theory. Also the variation of \( (\varepsilon - 4P)/T^4 \) with temperature are studied and compared with lattice results. Finally the strange particle ratios \( \frac{\Lambda}{\bar{\Lambda}}, \frac{\Xi}{\bar{\Xi}} \) and \( \frac{K^+}{\bar{K}^0} \) are obtained at both phase boundaries. We propose that their variations with the temperature and baryon chemical potential can be used in identifying the quark-gluon plasma in the recent as well as in future heavy-ion experiments.

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1 Introduction

Recent lattice gauge simulations reveal interesting results [1-3] for the number of flavours and their masses included in the theory, in determining the order and the critical parameters of the phase transition. In realistic cases, i.e., for three or more flavours, the transition is found to be a first order. However, for two light flavours, one finds a continuous transition. As the mass of the strange quark is increased from zero, the transition changes from first-order to a continuous one. Similarly the critical parameters are also sensitive to the number of flavours $n_f$ and it is shown that the critical temperature $T_c$ drops with increasing flavours ($T_c \propto n_f^{-1/2}$). The lattice approach, however, is still not a suitable tool for studying the strongly interacting matter in the baryon rich environment. Therefore, the critical behaviour of such a matter with finite baryon chemical potential can reliably be studied in the framework of a QCD-motivated phenomenological models. However, within the framework of such models, the specific entropy per baryon ($S/B$) is found to be discontinuous across the phase boundary [4-6]. Leonidov et al. [7] have recently proposed a modified equation of state (EOS) for the quark-gluon plasma (QGP) with a temperature $T$ and baryon chemical potential $\mu$ dependent bag constant in order to ensure that the $S/B$ ratio is continuous along the phase boundary. In our earlier works [8-9], we have modified the $\mu$ and $T$ dependence of $B(\mu,T)$ by incorporating the QCD perturbative corrections in the EOS for QGP and we have also explored in detail the consequences of such a bag constant on the deconfining phase transition. The purpose of this paper is to generalise the expression of $B(\mu,T)$ for $n_f$ flavours in the low baryon density region and then we investigate the effect of a massless as well as a massive $s$-quark on the critical behaviour of the phase transition. We compare our results with those of lattice in the high $T$ region qualitatively. It should be noted that when $S/B$ ratio is made continuous across the phase boundary, the quark-hadron phase transition is still first-order and it occurs at a common temperature and chemical potential.

We have then determined the critical parameters from the two phase boundaries - one obtained from the Gibbs criteria for an isentropic, equilibrium phase transition and the
other for the transition to an ideal, noninteracting QGP. Finally we study the variations of the ratios $\frac{\Lambda}{\Lambda}$, $\frac{\Xi}{\Xi}$ and $\frac{K^+}{K^-}$ either with the temperature or with baryon chemical potential at both phase boundaries. Here we mainly concentrate on the particle production from the QGP in the midrapidity region so that it can be used for studying the properties of QGP produced at future RHIC and LHC experiments.

2 Formulation of $B(\mu, T)$ for $n_f$ Flavours

The chemical potential ($\mu$) and temperature dependence ($T$) of the bag constant can be derived by constructing an isentropic, equilibrium phase transition from a quark-gluon plasma (QGP) to a hadron gas (HG) at a fixed $T$ and $\mu$, and we put a constraint that entropy per baryon ratio is continuous across the phase boundary, i.e.,

$$\frac{S}{B} \bigg|_{QGP} = \frac{S}{B} \bigg|_{HG} \quad (1)$$

The above condition can be achieved by assigning a $T$ and $\mu$ dependence to the bag constant explicitly $[7]$. The resulting correction factors $-\frac{\partial B(\mu, T)}{\partial T}$ and $-\frac{\partial B(\mu, T)}{\partial \mu}$ to the entropy density and the baryon density, respectively, in the QGP phase will modify eq.(1) as follows:

$$\frac{S_{QGP}^0 - \frac{\partial B(\mu, T)}{\partial T}}{n_{QGP}^0 - \frac{\partial B(\mu, T)}{\partial \mu}} = \frac{S_{HG}}{n_{HG}} \quad (2)$$

The above partial differential equation can be solved iteratively by exploiting the symmetry property of the bag pressure, i.e., $B(\mu, T) = B(-\mu, -T)$.

We use simple models to describe the EOS in each of the two phases and then perform a Maxwell construction to determine their common phase boundary. The QGP phase consists of a perturbatively interacting gas of quarks and gluons. Hence the thermodynamical potential for a QGP can be written in terms of the grand partition function as$[10]$

$$\Omega(T, V, \mu) = -\frac{T}{V} \ln Z_{QGP} \quad (3)$$
and \( \ln Z_{QGP} = \ln Z^o_{QGP} + \ln Z^{\text{Int}}_{QGP} + \ln Z^{\text{Vac}}_{QGP} \), where \( Z^o \) is the zeroth order contribution, \( Z^{\text{Int}} \) arises due to perturbative corrections and the term \( Z^{\text{Vac}} \) represents the non-perturbative vacuum contribution in the form of a \( \mu \) and \( T \) dependent bag constant \(- B(\mu, T)\).

The zeroth order contribution to the partition function is:

\[
\ln Z^o_{QGP} = -\frac{V}{T} \sum_f \frac{g_f}{6\pi^2} \int_0^\infty \frac{p^4 dp}{\sqrt{(p^2 + m_f)^2}} F_f[p; T, \mu_f]
\]  

(4)

and the perturbative interaction part up to second order in strong coupling constant \( g_s \) (\( \alpha_s = \frac{g_s^2}{4\pi} \)) can be written as follows by using finite temperature field theory:

\[
\ln Z^{\text{Int}}_{QGP} = -\frac{V}{T} \left[ \frac{1}{3} \pi \alpha_s N_g T^2 \sum_f \int_0^\infty \frac{d^3 p}{(2\pi)^3} \frac{1}{E_p} F_f[p; T, \mu_f] \right] \\
+ \pi \alpha_s N_g \sum_f \int_0^\infty \frac{d^3 p}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \left[ \frac{F_f^-(q) F_f^- (p) + F_f^+(q) F_f^+ (p)}{E_p E_q} \right] \left( \frac{2m_f^2}{(E_p - E_q)^2 - \omega^2} + 1 \right) \\
+ \pi \alpha_s N_g \sum_f \int_0^\infty \frac{d^3 p}{(2\pi)^3} \frac{d^3 q}{(2\pi)^3} \left[ F_f^-(q) F_f^+ (p) + F_f^+(q) F_f^- (p) \right] \left( \frac{2m_f^2}{(E_p + E_q)^2 - \omega^2} + 1 \right) \\\n+ \frac{1}{36} \pi \alpha_s N_c N_g T^4 \]

(5)

where \( F_f = F_f^+ + F_f^- \); \( F_f^+ (F_f^-) \) are the fermionic (antifermionic) equilibrium distribution functions of \( f \)th flavour, respectively:

\[
F_f^+ = \left[ e^{\beta(E_f - \mu_f)} + 1 \right]^{-1}
\]  

(6)

\[
F_f^- = \left[ e^{\beta(E_f + \mu_f)} + 1 \right]^{-1}
\]  

(7)

Here \( E_f \) is the energy of \( f \)-flavour. Considering massless quarks the total thermodynamic potential for the QGP phase reduces to the following expression [11]:

\[
\Omega = -\frac{\pi^2}{45} T^4 \left[ 8 + \frac{21}{4} n_f + \frac{45}{2\pi^2} \sum_f g(\mu_f, T) \right] \\
+ \frac{2\pi}{9} \alpha_s T^4 \left[ 3 + \frac{5}{4} n_f + \frac{9}{2\pi^2} \sum_f g(\mu_f, T) \right] + B(\mu_f, T)
\]  

(8)
where $g(\mu_f, T) = \left[ \frac{\mu_f^2}{T^2} + \frac{\mu_f^4}{2\pi^2 T^4} \right]$

If we further make use of the various conservation conditions such as strangeness ($\mu_s = 0$) and charm conservations, and also put $\mu_u = \mu_d = \mu_q = \frac{\mu}{3}$, we get the expression for the pressure in QGP:

$$P_{QGP} = \frac{\pi^2}{45} T^4 \left[ 8 + \frac{21}{4} n_f + \frac{45}{2\pi^2} \sum_q g(\mu_q, T) \right] - \frac{2\pi}{9} \alpha_s T^4 \left[ 3 + \frac{5}{4} n_f + \frac{9}{2\pi^2} \sum_q g(\mu_q, T) \right] - B(\mu_q, T) \quad (9)$$

In these calculations, $\mu$ and $T$ dependence of $\alpha_s$ is taken as follows [12] :

$$\alpha_s(\mu_f, T) = \frac{4\pi}{(11 - \frac{3}{2} n_f) \ln \frac{M^2}{\Lambda^2}} \quad (10)$$

where

$$M^2 = \frac{4}{3} \left[ 16 \int_0^\infty dp \, p^4 f_G + \sum_f g_f \int_0^\infty dp \, p^4 F_f \right] \left[ 16 \int_0^\infty dp \, p^4 f_G + \sum_f g_f \int_0^\infty dp \, p^4 F_f \right]^{-1}$$

Here $f_G$ is the gluonic and $F_f$ is the quark distribution functions, $g_f$ is the quark degeneracy factor for the f-th flavour.

Hadron gas consists of all strange as well as nonstrange hadrons. The partition function for baryons (B) then becomes

$$\frac{T}{V} \ln Z_B(T, \mu) = \left[ \sum_{i=1}^{n_1} \frac{g_i m_i^2 T^2}{\pi^2} K_2 \left( \frac{m_i}{T} \right) \right] \cosh \left( \frac{\mu}{T} \right) + \left[ \sum_{j=1}^{n_2} \frac{g_j m_j^2 T^2}{\pi^2} K_2 \left( \frac{m_j}{T} \right) \right] \cosh \left( \frac{c \mu}{T} \right) + \left[ \sum_{k=1}^{n_3} \frac{g_k m_k^2 T^2}{\pi^2} K_2 \left( \frac{m_k}{T} \right) \right] \cosh \left( \frac{d \mu}{T} \right) \quad (11)$$

here $c = 1 - f_s$ and $d = 1 - 2f_s$ and $f_s (\equiv \frac{\mu_s}{\mu})$ is a factor arising from the strangeness conservation condition $n_s - n_{\pi} = 0$ in the hadronic phase and $n_1$, $n_2$ and $n_3$ are the number of nonstrange, singly and doubly strange baryons, respectively. The summation arises due to quantum statistics used for the distribution functions. Similarly the partition
function for the mesonic (M) sector is

\[
\frac{T}{\mathcal{V}} \ln Z_M(T, \mu) = \frac{\pi^2}{30} T^4 + \sum_{l=1}^{r_1} \frac{g_l m_l^2 T^2}{\pi^2} K_2 \left( \frac{m_l}{T} \right) + \sum_{n=1}^{r_2} \frac{g_n m_n^2 T^2}{\pi^2} K_2 \left( \frac{m_n}{T} \right) \cosh \left( \frac{f_s}{T} \right)
\]

(12)

Here the first term is for massless pions, second and third term is due to nonstrange and strange mesons, respectively. In order to incorporate repulsive interactions among the hadrons in the HG phase within the excluded-volume approach, we multiply all pointlike quantities by a volume correction factor [13] of the type \[ 1 + V_{o_{HG}}(T, \mu) \] where \( n_{o_{HG}}(T, \mu) \) is the net baryon density for pointlike baryons.

The requirement of conserved entropy per baryon across the phase boundary leads to the following expression in the low baryon density and high temperature limit:

\[
\frac{\partial B}{\partial \mu} \approx n_{o_{QGP}} - \frac{S_{QGP}}{S_M} n_{HG}
\]

(13)

We have neglected the terms proportional to \( \mu^2 \) and \( \mu^3 \) as well as \( \frac{\partial B}{\partial T} \) term. We have also assumed that the entropy in the hadron gas at a large T and very small \( \mu \) is mainly contributed by mesons and thus \( S_{HG} \approx S_M \).

In the low baryon density limit, we can approximate the entropy density \( S_{QGP} \) as almost independent of \( \mu \) and hence we get,

\[
B(\mu, T) = B_o + \frac{1}{9} \mu^2 T^2 - \frac{S_{QGP}}{S_M} \left[ P_B(T, \mu) - P_B(T, \mu \approx 0) \right]
\]

(14)

Here \( P_B \) is the baryonic pressure in the HG phase. Although \( S_{QGP} \) may not be \( \mu \)-independent but it involves \( \mu^2 \) and higher order terms, which we ignore in the \( \mu \rightarrow 0 \) limit. Finally after second iteration, we get the expression for \( B(\mu, T) \) at high T and small \( \mu \) limit as:

\[
B(\mu, T) = B_o + \frac{1}{9} \mu^2 T^2 + \frac{\mu^4}{162\pi^2} - \frac{S_{QGP}}{S_M} \left[ P_B(T, \mu) - P_B(T, \mu \approx 0) \right] - \frac{1}{2} \left( \frac{S_{QGP}'}{S_M} - \frac{S_{QGP}'}{S_M} \right) \left( P_B(T, \mu) - P_B(T, \mu \approx 0) \right)^2
\]

(15)
Here

\[ S'_{QGP} = \frac{\partial S_{QGP}}{\partial T} \quad \text{and} \quad S'_M = \frac{\partial S_M}{\partial T} \]

Using these equations of state (EOS) for the QGP and HG, one can construct a first-order isentropic and equilibrium phase transition with the help of Gibbs criteria i.e., \( P_{QGP} = P_{HG} \) and thus determine the critical parameters \( T_c, \mu_c \) for the phase boundary. Similarly one can also determine the other phase boundary by determining \( T'_{c}, \mu'_{c} \) for the phase transition from a hadron gas to an ideal QGP from the condition \( B(\mu,T) = 0 \) in the eq.(15). We can then calculate the strange particle ratios along these boundaries. We choose \( T'_{c} \) as the approximate temperature where \( (\varepsilon - 3P)/T^4 = 0 \), signifying the onset of the ideal QGP behaviour (\( \varepsilon \) and \( P \) being the energy density and pressure in the QGP phase, respectively). Thus we get two sets of critical parameters \( (T_c, \mu_c) \) and \( (T'_{c}, \mu'_{c}) \) along which we will calculate strange particle ratios. The region under the curve defined by \( (T_c, \mu_c) \) is considered effectively as the hadron gas phase, while the ideal QGP phase resides above the curve defined by \( (T'_{c}, \mu'_{c}) \). The region between the two curves may be considered as the transition region from a deconfined phase consisting of massive quarks into a chiral symmetry restored ideal QGP phase. The strange particle ratios along \( (T_c, \mu_c) \) curve predict quantitatively the minimum QGP values because they represent the deconfining phase boundary. Similarly \( (T'_{c}, \mu'_{c}) \) curve ensures that, if measurements of the observables yield an experimental value for temperature and quark chemical potential above this curve, an ideal quark-gluon plasma has been produced in the heavy-ion collisions.

3 Calculation of Strange particle Ratios

The idea that the strangeness abundance can provide a useful signature for quark-gluon plasma formation originates from the enhancement of the strange-antistrange quark \( (s\bar{s}) \) pairs in a thermally equilibrated and baryon dense QGP as compared to the production of u, d light quark flavours[14]. This is possible only when the Fermi energy of the u, d quarks
become larger than the strange quark mass and hence the Pauli blocking stops further creation of light quark pairs. For a $q\bar{q}$ symmetric and baryonless QGP, the abundance of $s\bar{s}$ can still occur provided the temperature is much larger than the strange quark mass. Furthermore, the strangeness abundance in QGP is also related with the lower mass threshold involved in the creation of an $s\bar{s}$ pair in QGP rather than $K\bar{K}$ pair in the hadron gas. Thus it seems worthwhile to investigate in detail the utility of strangeness as a QGP signal in the context of future LHC and RHIC experiments where the production of baryonless QGP is expected. However, in this context, we will be searching for the strange particle ratios $\frac{\Lambda}{\Xi}$, $\frac{\Xi}{\Xi}$ and $\frac{K^+}{K^-}$ as a QGP signal.

In calculating the above ratios we entirely dwell on the formalism in the quark-gluon plasma phase formed at a high temperature and a small baryon density. Thus the composition of the QGP mainly involves the gluons and hence the glue-based processes dominantly contribute to the strangeness production. The probability for the creation of quarks of different flavours is calculated in the particle production mechanism [15,16]

$$f_q = f_o \, e^{-\frac{m_q^2}{\kappa}}$$

(16)

where $m_u = 5$ MeV, $m_d = 9$ MeV, $m_s = 170$ MeV, and the QCD string tension $\kappa = 1$ (GeV/fm). The normalisation constant $f_o$ is given by

$$f_o = \frac{1}{2 + e^{-\frac{m_s^2}{\kappa}}}$$

(17)

The hadronisation of QGP is still not understood properly. We assume that once QGP has been produced, the resulting hadrons do not get sufficient time to achieve thermal and chemical equilibrium. Thus the chemical composition existing during the QGP stage does not change and hence the strangeness abundance achieved in the QGP phase still survives during the process of hadronisation. Thus the probability for the creation of a primordial hadron with $q$ number of quarks per unit of phase space volume is obtained

$$P = \prod_q f_q \, \lambda_q \, g_q \, e^{-\frac{E_q}{T}}$$

(18)

8
where \( f_q \) is the gluon fragmentation probability, and involves the quark mass dependence, the quark fugacity \( \lambda_q = e^{\mu_q/T} \), \( g_q \) is the statistical degeneracy factor, \( \gamma_q \) is the relative equilibration factor and \( E_q \) is the energy of the \( q \)-th quark. We assume complete equilibration for u, d, s quarks in QGP, i.e, \( \gamma_u = \gamma_d = \gamma_s = 1 \). The energy of the emitted particle is \( E = \sum_q E_q \). This factor is integrated out in eq.(18) when we carry out the phase space integration and take the particle ratios in the same \( m_T \) range : \( E = m_T \cosh(y_{pr} - y_{mr}) \), where \( m_T = \sqrt{m^2 + p_T^2} \) [18], with \( y_{pr} \) (\( y_{mr} \)) being the projectile (mid) rapidity. Thus for ratios like \( \Lambda/\bar{\Lambda} \), \( \Xi/\bar{\Xi} \) and \( K^+/K^- \), the numerator as well as denominator gives similar kinematical factor after integrations and hence they are cancelled out. However, these ratios depend upon the product of \( f_q \lambda_q \) and hence on the mass of the quark as well as on the temperature and quark chemical potential. We introduce \( \lambda_i = e^{\mu_i/T} \) as the fugacity of i-th hadron species in HG and is simply the product of the fugacities of the constituent quarks so that \( \lambda_N = \lambda_q^3 \), \( \lambda_K = \lambda_q \lambda_s \), etc. Since the u, d, s flavours are separately conserved in the time scale of hadronic collisions, the production (or annihilation) can only occur in pairs. So the chemical potentials for particle and antiparticle are opposite to each other, we get \( \lambda_q = \lambda_q^{-1} \). Finally, particle ratios in the same \( m_T \) range, take the form [17] :

\[
\frac{\Lambda}{\bar{\Lambda}} = e^{-2(\mu_u + \mu_d)/T} e^{-2\mu_s/T} \\
\frac{\Xi}{\bar{\Xi}} = e^{-2\mu_u/T} e^{-4\mu_s/T} \\
\frac{K^+}{K^-} = e^{-2\mu_u/T} e^{-2\mu_s/T}
\]

All the above ratios depend on the factor \( \exp(\pm \mu_q/T) \) and strange quark chemical potential \( (\mu_s) \) explicitly. In the pure QGP phase, however, \( \mu_s \) is identical to zero because of the exact strangeness conservation. The factor \( \gamma_s \) accounts for much of our ignorance about the dynamics of strangeness formation and the approach to equilibration of the strange quarks in the QGP phase. A value \( \gamma_s = 1 \) is believed to favour QGP interpretation of the data. It is convenient to denote \( \mu_q = (\mu_u + \mu_d)/2 \); \( \delta \mu = \mu_d - \mu_u \), \( \mu_B = 3 \mu_q \); where \( \mu_q \) is quark chemical potential, \( \mu_B \) is the baryochemical potential and \( \delta \mu \) describes the (small) asymmetry in the number of up and down quarks due to neutron excess in heavy-ion
collisions, i.e., \( \frac{\mu}{T} \neq 0.5 \). In practice we find \( \delta \mu \) to be very small [19], and, therefore, we neglect it here. Finally we can calculate these ratios on the phase boundaries using the values of the critical parameters \((T_c, \mu_c)\) and \((T_c', \mu_c')\).

4 Results and Discussions

In Fig.1 we have shown the variation of \( B(\mu, T) \) with temperature \( T \) as obtained from eq.(15) in the low \( \mu \) and high \( T \) limit. We find that the values for \( B(\mu, T) \) differ much from the case of u, d massless quarks and decreases faster as \( T \) increases. Moreover, the incorporation of interactions into the calculation modifies the result significantly. We find that \( B(\mu, T) \) becomes zero for \( T = 210 \) MeV if the interactions among quarks and gluons are incorporated and it is far less than the value \( T = 440 \) MeV obtained in the case for three massless quarks included in the QGP without any interactions as shown by curve B. Vanishing of \( B(\mu, T) \) at a certain value of \( \mu \) and \( T \) signifies that the quarks and gluons are almost free. We infer that the ideal gas limit for a QGP consisting of three flavours is reached at a value \( T \gg T_c \) with the critical temperature \( T_c \simeq 160 \) MeV for the deconfining phase transition. However, we find that the calculation shows very little change when we take u, d massless quarks and s as a massive quark \( (m_s = 150 \) MeV) if compared with the case of three massless quarks.

In order to illustrate the above point more clearly, we have plotted in Fig.2 the variation of the quantity \( (\varepsilon - 3P)/T^4 \) with temperature \( T \) for a QGP consisting of u, d, s massless quarks. Here \( \varepsilon \) represents the energy density and \( P \) is the pressure of the QGP. The quantity \( (\varepsilon - 3P) \) represents a measure of the ideal gas behaviour since it vanishes for an ideal gas. We find that \( (\varepsilon - 3P)/T^4 \) asymptotically vanishes. This curve agrees with that obtained in the lattice gauge results. Thus it gives us confidence in the QCD-motivated calculations for the EOS of a QGP.

We conclude that the inclusion of \( n_f \) flavours in the temperature and baryon chemical
potential dependent bag constant yields results in agreement with those obtained in the lattice gauge results [3]. However, we find that there is very little difference in the results whether the strange quark is massless or massive. This result differs from the lattice finding. Moreover, our phenomenological model is valid for a QGP with a finite but small baryon chemical potential. The \( \mathcal{B}(\mu,T) \) thus obtained guarantees the phase transition between QGP and hadron gas at the same value of temperature and chemical potential and hence the process of reheating is not required during the mixed phase of hadronisation. It also yields continuity of entropy per baryon ratio across the phase boundary. We thus hope that the phenomenological form for \( \mathcal{B}(\mu,T) \) thus obtained will be of considerable use in deriving the properties and the signals of QGP to be produced at proposed RHIC and LHC experiments because we expect the net baryon density in these experiments to be low enough as used in the above calculation. However, the baryon density will never vanish precisely and, therefore, the lattice calculations can still not be used as such here. We, therefore, hope that the description of a deconfinement phase transition achieved in a QCD-motivated model by making the bag constant dependent on \( \mu \) and \( T \) will be a more realistic one in the future LHC and RHIC experimental situations.

In Fig.3, we have separately shown the variations of \( \frac{\Lambda}{\tilde{\Lambda}} \) with the baryon chemical potential and also with the temperature. We have separated the regions of HG phase, interacting QGP phase and the ideal or non-interacting QGP phase. Our results are valid in the low baryon density limit and hence cannot be extended beyond \( \mu > 300 \) MeV which corresponds to \( \mu_q = 100 \) MeV. In future colliders like RHIC and LHC, we expect the formation of an ideal quark matter at \( \mu \leq 100 \) MeV. It means that the value of our calculated ratio \( \frac{\Lambda}{\tilde{\Lambda}} \geq 0.56 \) should indicate the formation of such a matter. However, the variation of \( \frac{\Lambda}{\tilde{\Lambda}} \) with the temperature does not show any similar constraint on the temperature. It simply tells us that an ideal QGP can be formed at temperatures \( T < 180 \) MeV. The present experimental value [20,21] of \( \frac{\Lambda}{\tilde{\Lambda}} \) is \( 0.22 \pm 0.01 \) and it signifies a hot and dense HG rather than a QGP unless \( \mu > 300 \) MeV for which our predictions are rather unreliable and inconclusive.
In Fig. 4, we have shown the similar variations for the ratio $\Xi$ with $\mu$ and $T$ separately. We again find that an extraordinary large value for this ratio ( $> 0.74$ ) at $\mu < 100$ MeV will indicate the formation of QGP at RHIC or LHC experiments. In Fig. 5, we have also shown the variations of $K^+ / K^-$ either with $\mu$ or with $T$. In this case, we find that the formation of an ideal QGP at $\mu < 100$ MeV requires the value of the ratio $K^+ / K^- \leq 1.3$.

Recently Asprouli and Panagiotou [17] performed an identical analysis. However, they fixed the phase boundaries by choosing $T_c, \mu_c$ etc. quite arbitrarily. Thus their comparison with the experimental data revealed a faulty conclusion that the present CERN and AGS heavy-ion experimental data [20, 21] indicate QGP formation. Furthermore, they conclude that Sulphur induced reactions at midrapidity region reveal that the ideal QGP has been produced to within 60 % possibility. Our results, on the contrary, suggest that the CERN experimental datas correspond to an ideal thermalised HG picture. However, the results obtained here clearly indicate that strange particle ratios can provide a signal in the midrapidity and baryon-free region. The experimental results do not correspond to this region. Nevertheless, the EOS employed here for the QGP phase breaks down strictly at $n_B = 0$ region because entropy per baryon ratio becomes meaningless. So our analysis is applicable only in the low baryon density ($n_B$) region.

It has been claimed that thermal gluon decay into a quark-antiquark pair dominates for a wide range of quark masses in the midrapidity region and thus provides the most important source for the quark or the antiquark density [22, 23]. Normally, the gluon cannot decay into a strange quark-antiquark pair because its thermal mass is below the threshold required for such a pair creation. For a temperature ($T$) around 200 MeV and the coupling constant $g = 2$ for a QGP consisting of two massless flavours, one gets the thermal gluon mass $m_g = \frac{2}{3} g T = 267$ MeV in the lowest order perturbation theory and threshold for $s \bar{s}$ pair creation corresponds to $\approx 300$ MeV. However, it has been suggested that in addition to acquiring a thermal mass of the order of $g^2 T$, gluons also acquire a width determined by the large damping rate [24, 25]. Thus $g \rightarrow s \bar{s}$ decay is allowed and thus can account for the strangeness enhancement in the midrapidity region.
In summary, we have examined the strange particle production in the midrapidity region within a QGP formalism. We assume that the gluon fragmentation into $q\bar{q}$ pairs is the main source of the quark and antiquark density in the plasma. However, we have calculated the values of the critical parameters for the transition from the HG to an ideal QGP using our EOS for the quark matter with a $\mu$ and $T$ dependent bag constant obtained at a low baryon density. The lower limit to the critical parameters corresponds to Gibbs criteria for an equilibrium and isentropic phase transition and upper limit is obtained by putting $B(\mu, T) = 0$. We have plotted the strange particle ratios as a function of $T$ and $\mu$, separately along the two phase curves. From the values of these strange particle ratios one can infer an approximate values of temperature and the baryon chemical potential reached in the heavy-ion experiments. Our studies reveal that the strangeness enhancement can still be regarded as the signature for a QGP formed in the central rapidity or at almost baryon-free region. However, in the absence of a reliable lattice result for $n_B \neq 0$ region, our approach is justified. We hope that our results will provide a basis for future studies regarding the signals of QGP and their detection in the future collider experiments.
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Fig. 1. Variation of bag constant $B(\mu, T)$ with temperature $T$ at a baryon chemical potential $\mu = 50$ MeV. Curve A represents the free QGP EOS consisting of u, d massless flavours. Curve B is for the free u, d, s massless flavours. Curve C stands for u, d massless and interacting flavours with QCD scale parameter $\Lambda = 100$ MeV. Curve D represents interacting QGP EOS but s-quark is massive ($m_s = 150$ MeV). Curve E is same as D but s-quark is massless. In all these curves, we have used $B_o^{1/4} = 235$ MeV.

Fig. 2. Variation of $(\varepsilon - 3P)/T^4$ which is a measure of an ideal plasma behaviour with temperature $T$ for $\mu = 50$ MeV and $B_o^{1/4} = 235$ MeV. Here $\varepsilon$ is the energy density and $P$ is the pressure of the QGP phase of three massless flavours.

Fig. 3. Variations of $\frac{\Lambda}{\Lambda}$ with $\mu$ and $T$ are shown separately. The dashed line represents the ratio on the phase boundary determined by Gibbs criteria for equilibrium phase transition between interacting QGP and HG whereas the solid line denotes the ratio on the phase boundary between HG and ideal QGP phases.

Fig. 4. The notations are the same as Fig. 3 but for the ratio $\Xi$.

Fig. 5. The notations are the same as Fig. 3 but for the ratio $\frac{K^+}{K^-}$. 

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