CLUSTERS IN VARIOUS COSMOLOGICAL MODELS:
ABUNDANCE AND EVOLUTION

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The combination of measurements of the local abundance of rich clusters of galaxies and its evolution to higher redshift offers the possibility of a direct measurement of $\Omega_0$ with little contribution from other cosmological parameters. We investigate the significance of recent claims that this evolution indicates that $\Omega_0$ must be small.

The most recent cluster velocity dispersion function from a compilation including the ESO Northern Abell Cluster Survey (ENACS) results in a significantly higher normalization for models, corresponding to $\sigma_8 \approx 0.6$ for $\Omega_0 = 1$, compared to the Eke, Cole, & Frenk result of $\sigma_8 = 0.52 \pm 0.04$. Using the ENACS data for a $z = 0$ calibration results in strong evolution in the abundance of clusters, and we find that the velocity dispersion function is consistent with $\Omega_0 = 1$. The results are dependent upon the choice and analysis of low-redshift and high-redshift data, so at present, the data is not good enough to determine $\Omega_0$ unambiguously.

1 Introduction

The present-day number density of rich clusters of galaxies and its evolution to high redshift can in principle provide a sensitive estimate of the value of $\Omega_0$. Several recent papers have attempted measurements of $\Omega_0$ from the evolution in cluster number density, as estimated from observations of X-ray temperatures, X-ray luminosities, and virial masses.

The Press-Schechter approximation accurately represents the number density of fairly massive clusters in $N$-body simulations. It has a Gaussian cutoff whose position is controlled roughly by $\delta_c/D_\sigma_8$ where $D$ is the linear fluctuation growth factor, so a small error in either $\delta_c$ or $\sigma_8$ can lead to a misestimate of the amount of evolution. For this reason, we perform a careful renormalization of models to $z = 0$ cluster data, including calibration of $\delta_c$ against high-resolution simulations.

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2 How to Use the Press-Schechter Approximation

The basic strategy is to relate a directly observable quantity to the virial mass, and then use the standard Press-Schechter formalism to obtain an abundance for that virial mass. For velocity dispersions observations, we assume that clusters are virialized and collapse spherically.

Girardi et al. (hereafter G97) calculated their preliminary mass function using virial radii that were estimated roughly assuming a low-$\Omega_0$ cosmology. Because of that, they need correction for other models. This requires knowledge of the mass (or $\sigma_{1D}$) profile, and we adopt the Navarro, Frenk, & White (NFW) $\sigma_{1D}$ profile with $c = 7$.

Once one has calculated a virial mass, one gets the abundance using the standard Press-Schechter formalism. This requires an assumption about the critical linear density for collapse ($\delta_c$). We calibrated that from simulations, where the masses were measured from the simulations using similar techniques to the real data — that is, for G97 masses, measuring the mass within the “virial” radius as G97 defines it, and $\delta_c$ is tuned so that the G97-mass Press-Schechter formalism agrees with simulations at $5.5 \times 10^{14} h^{-1} M_\odot$.

We used this formalism to find values of $\sigma_8$ and $n$ that produce a minimum $\chi^2$ subject to the Bunn & White $COBE$ normalization. The results are shown in Table 1.

Table 1: Model renormalizations to the G97 mass function and four year $COBE$ normalization, compared to Eke, Cole, & Frenk.

| Model  | $\Omega_c$ | $\Omega_b$ | $\Omega_\Lambda$ | $h$ | $\delta_c$ | $n$ | $\sigma_8$ | $\sigma_{8,EFC}$ |
|--------|------------|------------|------------------|-----|-------------|----|------------|------------------|
| CHDM-2v| 0.731      | 0.069      | 0.2              | 0.0 | 0.6         | 1.357 | 0.912      | 0.589            | 0.52±0.04        |
| ΛCDM   | 0.331      | 0.069      | 0.0              | 0.6 | 0.6         | 1.400 | 1.000      | 0.805            | 0.80±0.06        |
| OCDM   | 0.431      | 0.069      | 0.0              | 0.0 | 0.6         | 1.466 | 0.907      | 0.837            | 0.69±0.05        |
| TCDM   | 0.900      | 0.100      | 0.0              | 0.0 | 0.5         | 1.316 | 0.864      | 0.574            | 0.52±0.04        |

3 Evolution of Cluster Abundances and Cosmological Parameters

Now that we have refined the normalization parameters to correspond to $z = 0$ velocity dispersion data, we can use the calibrated Press-Schechter formalism to extrapolate each model’s mass function to higher redshift. Then, we can compare to high redshift velocity dispersion data from the Canadian Network for Observational Cosmology (CNOC), to discriminate between models.

Figure 1 shows the evolution in number density, where all assumptions of cosmology on the observations have been removed. The results favor a high-$\Omega_0$ cosmology, but there are many caveats that weaken that conclusion. These are discussed below.
Figure 1: Abundance of clusters with $\sigma_{1D} > 800$ km s$^{-1}$. All curves are Press-Schechter predictions normalized to agree with Fadda et al.[3] at $z = 0$. The low-$z$ data point is from G97 and the other two are from CNOC. We have removed assumptions of cosmology from the data by using velocity dispersions instead of masses, and by multiplying the number density by the cube of the ratio between the coordinate distance to $z$ and $cz/H_0$.

4 Caveats

In arriving at our normalization and evolution results, we had to make many assumptions. We assumed throughout all the analysis presented here that clusters were virialized objects. However, it is well established that clusters have remarkable substructures,[15] which represent the signature of a lack of virialization. We assumed clusters were spherical, but highly elongated clusters are observed both in simulations and in Abell clusters.[16] We made a very specific assumption about the mass profiles of clusters — that they were given by the NFW profile. Though this is a reasonable average case at $r \sim r_{\text{vir}}$, simulations show quite a lot of scatter around that distribution.

Small statistics are also a problem, as only eight CNOC clusters survive their $L_x$ cut. CNOC’s correction for $L_x$ selection is also uncertain due to the large observed scatter in the $\sigma_{1D}-L_x$ relation.[17]

The definition of CNOC clusters in terms of a physical radius, while the comoving virial radius of a cluster of a given mass is slowly varying with redshift, is a potential redshift dependent bias because a high redshift object will include more unbound material than low redshift object of the same mass.

There are further uncertainties in the Press-Schechter calibration, due to cosmic variance. Our choice of normalization at $M = 5.5 \times 10^{14} \, M_\odot$ is also arbitrary. Different masses will yield somewhat different normalizations.
5 Conclusions

Comparing cluster observations to simulations requires a great deal of massaging. It is preferable to perform most of the operations on the simulation data because the uncertainties are smaller. However, simulations are very expensive, which makes exhaustive searches in parameter space impossible. So, we must use semianalytic techniques such as the Press-Schechter approximation to “extrapolate” the models. With a modified Press-Schechter algorithm, we have renormalized all our models, and found that $\sigma_8$ is larger than Eke, Cole, & Frenk\footnote{Evrard et al., in preparation (1997).} if $\beta = 1$. Our result of $\sigma_8 \approx 0.6$ for $\Omega_0 = 1$ is consistent with $\beta = 1.15$, which is close to the value found by the Santa Barbara Cluster simulations.\footnote{Binggeli, Astron. Astrophys. 107, 338 (1982).}

Most importantly, using a plausible but non-unique set of assumptions, we have found a counterexample to strong recent statements that cluster number density evolution requires low $\Omega_0$. The data is not currently good enough to distinguish between reasonable values of $\Omega_0$.

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