The hidden charm pentaquarks are the hidden color-octet uud baryons?

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Abstract

The $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$, $\frac{1}{2}(\frac{3}{2}^-)$, and $\frac{1}{2}(\frac{5}{2}^-)$ uud\bar{c} pentaquarks are investigated by the quark cluster model. This model, which reproduces the mass spectra of the color-singlet $S$-wave $q^2$ baryons and $q\bar{q}$ mesons, also enables us to evaluate the quark interaction in the color-octet uud configurations. It is shown that the color-octet isospin-$\frac{1}{2}$ spin-$\frac{5}{2}$ uud configuration gains an attraction. The uud\bar{c} states with this configuration have structures around the $\Sigma_c^+(1385)$ thresholds: one bound state, two resonances, and one large cusp are found. We argue that the negative parity pentaquark found by the LHCb experiments may be given by these structures.

Keywords: hidden-charm pentaquark; color-octet baryon; exotic hadron; multiquark hadron; baryon-meson scattering

1. Introduction

In 2015, LHCb collaboration reported that two candidates of the new exotic baryons, $P_c(4380)$ and $P_c(4450)$, had been observed in the $\Lambda_c^+ \rightarrow J/\psi pK^- \ decay$. The mass of the higher peak, $P_c(4450)$, is $4449.8\pm1.7\pm2.5$ MeV with a width of $39\pm5\pm19$ MeV, while the lower and broader peak, $P_c(4380)$, has a mass of $4380\pm8\pm29$ MeV and a width of $205\pm18\pm86$ MeV. The most favorable set of the spin-parity for the lower and the higher peaks is $J^P = \frac{1}{2}(\frac{1}{2}^-)$, though $(\frac{3}{2}^+, \frac{5}{2}^-)$ or $(\frac{5}{2}^+, \frac{3}{2}^-)$ are also acceptable \[1\]. Also, because the $J/\psi p$ contribution is necessary to describe the decay data \[2\], it is almost certain the peaks have the $\bar{c} p$ pair and are considered as the isospin-$\frac{1}{2}$ uud\bar{c} pentaquarks.

Beside of the predicting work \[3\], the LHCb observation of $P_c(4380)$ and $P_c(4450)$ has evoked many theoretical studies: the hadronic molecule with the meson exchange interaction, the chiral unitary approach with the hidden local gauge symmetry, the QCD sum rule, the chiral quark model, the diquark/triquark model as well as that of the kinematical effects \[4\]. At present, the theoretical and experimental knowledge is not enough and one cannot draw a definite picture of these peaks.

Here, we concentrate our attention on the short range part of the hidden-charm pentaquark structure, which is governed by the quark and gluon dynamics. For this purpose, we employ the quark cluster model, which successfully explained the short range part of the baryon-baryon interaction \[5\] and the structure of the light flavored pentaquark $\Lambda(1405)$ \[6\]. It is also shown that the baryon-meson interaction derived from the lattice QCD is found to be similar to that of the quark cluster model \[7\]. Since we are interested in the short range region, we have investigated the $S$-wave five-quark systems as a first step. They correspond to the negative-parity pentaquarks. In order to discuss the positive-parity pentaquark state, which has also been observed by the LHCb experiments, one has to investigate the $P$-wave five-quark systems, which is beyond the scope of the present paper.

Let us first discuss possible configurations of uud quarks in the uud\bar{c} systems. Since the whole system is the color-singlet and the $\bar{c} p$ pair is color-singlet or octet, the remaining three light quarks are also color-singlet or color-octet. So, when the orbital configuration is totally symmetric, the uud\bar{c} configuration in the uud\bar{c} systems is totally symmetric (56-plet) or mixed symmetric (70-plet) in the flavor-spin $SU_f(6)$ space. They are classified as:

\[ 56_{f\sigma} = 8_f \times 2_{\sigma} + 10_f \times 4_{\sigma} \]  
\[ 70_{f\sigma} = 1_f \times 2_{\sigma} + 8_f \times 2_{\sigma} + 8_f \times 4_{\sigma} + 10_f \times 2_{\sigma} \]  

Here the numbers are the dimension of the corresponding representations. The color-singlet uud systems correspond to the usual 56-plet baryons. The color-octet 70-plet systems can be decomposed into the flavor-singlet spin-$\frac{1}{2}$ ($1_f \times 2_{\sigma}$), the flavor-octet spin-$\frac{1}{2}$ ($8_f \times 2_{\sigma}$), the flavor-octet spin-$\frac{3}{2}$ states ($8_f \times 4_{\sigma}$), and the flavor-decuplet spin-$\frac{5}{2}$ states ($10_f \times 2_{\sigma}$). Since the present work concerns systems of the isospin $\frac{1}{2}$ and the strangeness zero, namely, flavor-octet systems, the configurations of the three light quarks correspond to one of the following three: (a) color-singlet spin-$\frac{1}{2}$, (b) color-octet spin-$\frac{1}{2}$, and (c) color-octet spin-$\frac{3}{2}$, each of them we denote by $[\eta^1\frac{1}{2}]$, $[\eta^2\frac{1}{2}]$, and $[\eta^3\frac{3}{2}]$ in...

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Table 1: The classification of the isospin-\(1/2\) negative parity \(qqq\bar{c}\) states. The \(uud\) spin \((s_q)\), color \((c)\), CMI of the five quark systems at the heavy quark limit \((\langle \mathcal{O}_{\text{CMI}} \rangle_{5q}^{(HQ)}\)) the \(\sigma\) spin \((\sigma_c)\), the total spin of \(uud\sigma\) \((J)\), the lowest \(S\)-wave threshold \((T)\) and the CMI contribution to the threshold energy \((\langle \mathcal{O}_{\text{CMI}} \rangle_{\text{T}}^{(HQ)}\)) are listed.

| \(s_q\) | \(c\) | \(\langle \mathcal{O}_{\text{CMI}} \rangle_{5q}^{(HQ)}\) | \(\sigma_c\) | \(J\) | \(T\) | \(\langle \mathcal{O}_{\text{CMI}} \rangle_{\text{T}}^{(HQ)}\) |
| --- | --- | --- | --- | --- | --- | --- |
| \([q^3^1/2]_{\frac{1}{2}}\) | \(\frac{1}{2}\) | 1 | -8 | 0 | \(\frac{1}{2}\) | \(N\eta_c\) | -8 |
| 1 | \(\frac{1}{2}\) | \(NJ\psi\) | -8 |
| 1 | \(\frac{3}{2}\) | \(N\bar{J}\psi\) | -8 |

| \([q^3^8^1/2]_{\frac{1}{2}}\) | \(\frac{1}{2}\) | 8 | -2 | 0 | \(\frac{1}{2}\) | \(\Lambda_c\bar{D}\) | -8 |
| 1 | \(\frac{1}{2}\) | \(\Lambda_c\bar{D}\) | -8 |
| 1 | \(\frac{3}{2}\) | \(\Lambda_c\bar{D}^*\) | -8 |

| \([q^3^8^1/2]_{\frac{1}{2}}\) | \(\frac{3}{2}\) | 8 | 2 | 0 | \(\frac{3}{2}\) | \(\Sigma_c\bar{D}\) | \(\frac{2}{3}\) |
| 1 | \(\frac{3}{2}\) | \(\Sigma_c\bar{D}\) | \(\frac{4}{3}\) |
| 1 | \(\frac{3}{2}\) | \(\Sigma_c\bar{D}^*\) | \(\frac{4}{3}\) |
| 1 | \(\frac{3}{2}\) | \(\Sigma_c\bar{D}^*\) | \(\frac{4}{3}\) |

By the following, respectively. Since the spin of the \(\sigma\) pair is either 0 or 1, the total spin of the \(uud\sigma\) systems is either \(1/2\) (5-fold), \(3/2\) (4-fold), or \(5/2\) (1-fold). (See Table 1)

In the short range part of the two-hadron interaction, the color-magnetic interaction (CMI) plays an important role. We evaluate the color flavor spin part of CMI,

\[
\mathcal{O}_{\text{CMI}} = - \sum_{ij} \frac{m_i^2}{m_im_j} \lambda_i \cdot \lambda_j \sigma_i \cdot \sigma_j , \tag{3}
\]

by the quark wave function. In eq. (3), \(m_i\), \(\lambda_i\), and \(\sigma_i\) are the constituent mass, the Gell-Mann matrix in the color space, and the Pauli spin matrix for the \(i\)th (anti)quark, respectively. Because of the factor \(m_i^2/m_{im_j}\), only the operators between the light quarks give non-zero contribution at the heavy quark limit. So, CMI estimated by the above three-light-quark configurations actually correspond to the estimates of the whole \(uud\sigma\) at that limit, \(\langle \mathcal{O}_{\text{CMI}} \rangle_{5q}^{(HQ)}\), which is listed in Table 1. There, we also show the lowest \(S\)-wave baryon-meson threshold for each state together with the contribution of \(\mathcal{O}_{\text{CMI}}\) to that threshold at the heavy quark limit:

\[
\langle \mathcal{O}_{\text{CMI}} \rangle_{\text{T}}^{(HQ)} = \langle \mathcal{O}_{\text{CMI}} \rangle_{H}^{(HQ)} + \langle \mathcal{O}_{\text{CMI}} \rangle_{M}^{(HQ)} , \tag{4}
\]

where \(\langle \mathcal{O}_{\text{CMI}} \rangle_{H}^{(HQ)}\) and \(\langle \mathcal{O}_{\text{CMI}} \rangle_{M}^{(HQ)}\) are \(\langle \mathcal{O}_{\text{CMI}} \rangle_{5q}^{(HQ)}\) evaluated by the baryon and the meson wave functions at the heavy quark limit, respectively. Suppose we estimate the baryon-meson potential arising from CMI by

\[
V_{\text{eni}(HQ)}^{\text{CMI}} \sim \langle \mathcal{O}_{\text{CMI}} \rangle_{5q}^{(HQ)} - \langle \mathcal{O}_{\text{CMI}} \rangle_{\text{T}}^{(HQ)} , \tag{5}
\]

then only those which include the \([q^3^8^1/2]\) configuration is attractive though its energy is actually the highest. As in Table 1 there are four states which gain such an attraction in the isospin-\(1/2\) \(uud\sigma\) systems. Since \(uud\sigma\) is color-singlet as a whole, the system of the color-octet \(uud\) with color-octet \(\sigma\) can be observed as \(\Lambda_c\bar{D}^{(*)}\) or \(\Sigma_c^{(*)}\bar{D}^{(*)}\) baryon meson states, where each of the hadrons is color-singlet. The attraction within the color-octet spin \(3/2\) \(uud\) is observed as the attraction between the \(\Sigma_c^{(*)}\) baryon and the \(\bar{D}^{(*)}\) meson. In this sense, when this interaction causes pentaquark states, one may call them ‘hidden color-octet \(uud\) baryons.’

As we will show in this letter, the above behavior remains visible even after we employ the realistic quark masses and perform dynamical calculations. We have found that there is a bound state in the \(J = \frac{5}{2}\) system, a resonance and a cusp in the \(\frac{5}{2}\) system, and a resonance in the \(\frac{3}{2}\) system; which exactly correspond to those of the \([q^3^8^1/2]\) configuration as listed in Table 1. Since the expectation values of CMI by the \(uud\) configurations can be calibrated by the observed hadron masses, and since they do not depend on the heavy quark mass, the above four structures are robust to change of the parameters. Though we use a rather complicated model Hamiltonian in the following in order to produce the threshold energies correctly, the results do not depend much on the model details; the situation is the same, for example, when the system goes to the bottom sector. Thus, we would like to argue that the negative parity peak of the LHCb pentaquark may consist of (some of) these structures caused by the color-octet \(uud\) configuration.

2. Method

We employ the coupled-channel quark cluster model to investigate the \(uud\sigma\) \(I(J^P) = \frac{1}{2} (\frac{1}{2}^- \frac{1}{2}^- \frac{1}{2}^- \frac{1}{2}^-)\) systems. This model becomes a \((0s)^5\) quark model in the baryon-meson short range region. In the long range region, this model becomes essentially a baryon-meson model. There, the interaction between the baryon and the meson arises from the quark degrees of freedom and from the interaction between quarks.

The model Hamiltonian, \(H_q\), consists of the central spin-independent term, \(H_c\), and the color spin term, \(V_{\text{CMI}}\). The \(H_c\) consists of the kinetic term, \(K\), the confinement term, \(V_{\text{conf}}\), and the color Coulomb term, \(V_{\text{Coul}}\):

\[
H_q = H_c + V_{\text{CMI}} \tag{6}
\]

\[
H_c = K + V_{\text{conf}} + V_{\text{Coul}} . \tag{7}
\]

Both of \(V_{\text{Coul}}\) and \(V_{\text{CMI}}\) come from the effective one-gluon exchange interaction between quarks. Each of the terms is taken to be slightly different from the conventional quark model \([6, 10]\). It is because we use a single Gaussian for the orbital wave function of each of the \(q\bar{q}\) and \(q^3\) hadrons in order to make it feasible to solve the five-quark systems.

The kinetic term is taken as nonrelativistic:

\[
K = \sum_i m_i + \frac{1}{2m_i} \left( p_i - \frac{m_i}{M_G} P_G \right)^2 , \tag{8}
\]
where \( p_i \) is the momentum of the \( i \)th (anti)quark, and \( M_G \) and \( P_G \) are the total mass and momentum of the five-quark system.

The linear confinement term is

\[
V_{\text{conf}} = \sum_{i<j} \lambda_i \cdot \lambda_j \left( -ac_{ij} r_{ij} + c_1 + \frac{c_2^2}{\mu_{ij}} + c_{q\Xi} \right). 
\] (9)

The value of the confinement strength, \( c_c \), is taken from the Lattice QCD calculation \( [8] \), whose value corresponds to 1.12 GeV\(^2\) for the \( q\Xi \) systems. The \( r_{ij} \) and \( \mu_{ij} \) are the relative distance and the reduced masses of the \( i \)th and the \( j \)th quarks, respectively. In the above equation, the \( c_1, c_2, \) and \( c_{q\Xi} \) are the constant parameters. The \( c_1 \) and \( c_2 \) express the constant mass shift which is expanded up to the \( \mu^{-1} \) term. The parameter \( c_{q\Xi} \) is nonzero only when this operates on quark-antiquark pairs. We use these constants as free parameters so that the model produces the observed masses of the relevant single hadrons, which have a quite wide energy range. The constant mass shift itself appears when the potential is constructed from the lattice QCD calculation, though the values are different \([8]\).

The color-Coulomb term is written as

\[
V_{\text{Col}} = \sum_{i<j} \lambda_i \cdot \lambda_j \frac{\alpha_s(r_{ij})}{r_{ij}} \] (10)

\[
\alpha_s(r) = \sum_{k=1} \alpha_k \text{erf}[\gamma_k r]. \] (11)

The strong coupling constant, \( \alpha_s \), is assumed to depend on the relative distance of the interacting quarks, following the manner of refs. \([8, 10]\). In this work, \( \alpha_s \) at small \( Q^2 \) is refitted so that it corresponds to the running coupling constant for \( Q^2 > 3 \) GeV in the momentum space, whereas it goes to 0.8 at \( Q^2 = 0: \) (4\( \pi \)) in GeV\(^2\), \( \alpha_s= (1.5, 0.45) \), (10, 0.15), (1000, 0.20).

The CMI term is

\[
V_{\text{CMI}} = -\sum_{i<j} \lambda_i \cdot \lambda_j \frac{2\pi}{3m_im_j} \sigma_i \cdot \sigma_j \delta^3(r_{ij}) \] (12)

\[
\alpha_{s}^{\ast sj} = \left\{ \begin{array}{ll}
\alpha_{s}^{\ast sj} m_u & \text{for a } q\bar{q} \text{ pair} \\
\alpha_{s}^{\ast sj} m_c & \text{for a } cc \text{ pair}.
\end{array} \right.
\] (13)

We use \( \alpha_{s}^{\ast sj} \) and \( \alpha_{s}^{\ast sj} \) as parameters to fit the contribution of the \( uu \) pairs in the baryons such as \( 2m_{u^2} + m_{\Sigma_0} - 3m_{\Lambda_0} \) and that of the \( uu \) pairs that of \( uu \) pairs such as \( m_{u^2} - m_{\Sigma_0}. \) The value of \( \alpha_{s}^{\ast sj} \) is determined by taking an average of the \( \bar{D}^s - \bar{D} \) and \( D^s - D \) mass differences. Again these mass dependent coupling constants have to be introduced so that the model gives the correct hyperfine splitting of the hadron masses. In this way, we calibrate the size of the CMI, which is the origin of the attraction focused in this work, from the observables. The parameters are summarized in Table 2.

The color flavor spin part of the \( q^3 \) or \( q\Xi \) wave functions is taken as a conventional way \([11, 12]\). The orbital wave function of the mesons, \( \phi_M \), and that of the baryons, \( \phi_B \), are written by Gaussian with a size parameter \( b, \phi(r, b): \)

\[
\phi_M(r_M) = \phi(r_{12}, \frac{x_0}{\sqrt{\mu_{12}}}) \] (14)

\[
\phi_B(r_B) = \phi(r_{12}, \frac{x_0}{\sqrt{\mu_{12}}}) \phi(r_{12-3}, \frac{x_0}{\sqrt{\mu_{12-3}}}), \] (15)

where the reduced masses, \( \mu_{12} \) and \( \mu_{12-3} \), correspond to the Jacobi coordinates, \( r_{12} \) and \( r_{12-3} \). We assume that the size parameter of the orbital motion can be approximated by \( b = x_0/\sqrt{\mu} \) and minimize the central part of the Hamiltonian, \( H_c \), against \( x_0 \) for each flavor set: \( uu, cc, uud, ud \). For the baryons, this means that the ratio of the size parameters is kept to a certain mass ratio; e.g., \( b_{uc}/b_{ud} \) in \( \Lambda_c \), \( \Sigma_c \) is equal to \( \sqrt{b_{uc}/b_{uc}} \).

The mesons of the relevant hadrons obtained by the models are summarized in Table 3 together with the observed masses. The obtained \( x_0 \)'s are listed in Table 4 with the corresponding size parameters. It is found that \( x_0 \) does not vary much while the difference between \( b_{uu} \), \( b_{uc} \) and \( b_{uc} \) is large. In order to investigate systems with more than one charm quark one needs to take into account the flavor dependence of the orbital motion.

We employ the resonating group method (RGM) to solve the five-quark systems. The wave function, \( \Psi \), can be expanded by the locally peaked Gaussians for each of the baryon-meson channel \( \nu \) as \([11, 12]\).

\[
\Psi = \sum_{\nu} c_{\nu}^{\nu} A_{\nu} \left\{ \psi_{\nu}^{\mu}(r_B) \psi_{\nu}^{\Xi}(r_M) \chi(R, S_i) \right\} \] (16)

\[
\chi(R, S_i) = i_{\nu} \left( 1 \over b \right) - \frac{1}{2b^2}(R^2 + S_i^2) \right\}, \] (17)

where \( A_{\nu} \) stands for the quark antisymmetrization, which operates on the four quarks, and \( i_{\nu}(z) \) is the modified

### Table 2: Model parameters. The quark masses, \( m_Q \), and the constant parameters, \( c_i \), are in MeV.

| \( m_u(= m_d) \) | \( m_c \) | \( c_1 \) | \( c_2 \) | \( c_{q\Xi} \) |
|----------------|--------|--------|--------|--------|
| 300            | 1741.5 | 86.4   | 113.9  | -5.65  |

### Table 3: Single hadrons masses obtained by the present model. The isospin-averaged masses are taken from \([12]\).

| Baryon | \( N \) | \( \Lambda_c \) | \( \Sigma_c \) | \( \Sigma_c^0 \) |
|--------|--------|----------------|----------------|--------------|
| Mass   | 922.3  | 2291.8        | 2453.6         | 2516.0       |
| Obs.   | 938.9  | 2286.5        | 2453.5         | 2518.1       |

| Meson | \( \eta_c \) | \( J/\psi \) | \( \bar{D} \) | \( \bar{D}^* \) |
|-------|-------------|-------------|---------------|--------------|
| Mass  | 2981.3      | 3100.9      | 1863.4        | 2004.9       |
| Obs.  | 2983.6      | 3096.9      | 1867.2        | 2008.6       |
spherical Bessel function. As for the scattering state, the wave function of the relative motion is connected smoothly to the spherical Hankel functions in the long range region.

By integrating out the internal wave function of the hadrons, the RGM equation can be obtained from the equation of motion for the quarks \((\hat{H}_q - E)\Psi = 0\), as

\[
\sum_{\nu
',j} (H_{\nu
',j}^{\nu,j'} - E N_{\nu
',j}^{\nu,j'}) c_{\nu
',j} = 0
\]

with the Hamiltonian and normalization kernels

\[
\begin{align*}
&\left\{ H_{\nu
',j}^{\nu,j} \middle| \begin{array}{l} (\nu_{ij}^\nu) \\ N_{\nu
',j}^{\nu,j'} \end{array} \right\} = \int d\mathbf{r} d\mathbf{r}' \psi_{\nu_{ij}^\nu}(\mathbf{r}_B)\psi_{\nu_{ij}^\nu}^*(\mathbf{r}_M)\chi(\mathbf{R}_i, \mathbf{S}_i) \\
&\times \left( \frac{1}{2} \right) \frac{1}{2} \{ A_{\nu_{ij}^\nu} \{ \psi_{\nu_{ij}^\nu}(\mathbf{r}_B)\psi_{\nu_{ij}^\nu}^*(\mathbf{r}_M)\chi(\mathbf{R}_i, \mathbf{S}_i) \} \}, \quad (19)
\end{align*}
\]

where \(d\mathbf{r}\) stands for the integration over all the Jacobi coordinates of the five-quark system. For the detail of the calculation, see, for example, appendix B of ref. \[6\]. We choose the parameters \(b\) and \(S\) in eqs. (18) and (19) so that the calculating results are stable against changing the parameters. Thus, after fitting single hadron masses, the model is essentially parameter-free.

Here we define a three-body operator, \(\mathcal{P}\), to extract the \(uud\) color-c, spin-s, orbital (0s)\(^3\) configuration from the resonance as well as from the bound states in order to evaluate its size. It is defined as

\[
\mathcal{P}_{cs} = \langle uud; c(0s)^3 | uud; c(0s)^3 \rangle
\]

\[
\mathcal{P} = \sum_{cs} \mathcal{P}_{cs}. \quad (21)
\]

We use the same value as that of \(b_{uu}\) in \(\Sigma_c\) for the size parameter of the (0s)\(^3\) component.

### 3. Results and Discussions

#### 3.1. \(uud\mathcal{P} I(J^P) = \frac{1}{2}(\frac{3}{2}^+\) state

Suppose the orbital part of the \(uud\mathcal{P}\) system is totally symmetric, such as the system in the orbital (0s)\(^5\) configuration, the color-spin-color part of the \(uud\mathcal{P}\) quarks should be totally antisymmetric. For the \(S\)-wave \(uud\mathcal{P}\)

\[
I(J^P) = \frac{1}{2}(\frac{3}{2}^+) \text{ channel}, \text{ this state is } \Sigma_c^* \mathcal{D}^* \text{ antisymmetrized over the quarks. The color flavor spin part of its normalization, } \langle \Sigma_c^* \mathcal{D}^* | A_{ij} | \Sigma_c^* \mathcal{D}^* \rangle, \text{ is } \frac{1}{2}. \]
In figure 1 we show phase shifts of the $S$-wave $uud\bar{c}$ $I(J^P)=\frac{1}{2}\left(\frac{3}{2}^+\right)$ system by the calculation where all five baryon-meson channels are coupled. The plotted phase shifts are the diagonal ones; namely, the initial and the final channels are taken to be the same. The bound state and the resonance found in the above three-channel calculation now become a sharp resonance and a cusp in the $\Lambda^*$ channel, respectively. The energy of the resonance or the cusp evaluated by the scattering wave function with the initial $N_J/\psi$ channel, the factor to find the $\Lambda^*$ bound state is $4317.0$ MeV, the proportion is $(0.05$ $0.19$ $0.76)$. Thus this bound state is a forbidden state when the system is totally symmetric in the orbital space. Also in this case, all the diagonal elements of the normalization are close to 1; no baryon meson state is affected strongly by the quark Pauli-blocking.

Figure 1: The diagonal scattering phase shifts of the $S$-wave $uud\bar{c}$ $I(J^P)=\frac{1}{2}\left(\frac{3}{2}^+\right)$ system. The solid line is that of the $N_J/\psi$ channel, the long-dashed line is for the $\Lambda^*$ channel, and the dotted, double-dashed, and triple-dot-dashed lines are for the $\Sigma^*_D\bar{D}$, $\Sigma^*_\rho\bar{D}^*$, and $\Sigma^*_c\bar{D}^*$ channels, respectively. (Color online.)

In Figure 2, we show the diagonal phase shifts of the $S$-wave $uud\bar{c}$ $I(J^P)=\frac{1}{2}\left(\frac{1}{2}^-\right)$ system. The solid and the dotted lines are those of the $N_J/\psi$ and the $N_{\eta_c}$ channels, the long-dashed and the long-dot-dashed lines are for $\Lambda_u\bar{D}$ and $\Lambda_d\bar{D}^*$ channels, and the dotted, double-dot-dashed, and triple-dot-dashed lines are for the $\Sigma_u\bar{D}$, $\Sigma_d\bar{D}^*$, and $\Sigma_c\bar{D}^*$ channels, respectively. (Color online.)

3.3. $uud\bar{c}$ $I(J^P)=\frac{1}{2}\left(\frac{1}{2}^-\right)$ states

The $uud\bar{c}$ $\frac{1}{2}\left(\frac{1}{2}^-\right)$ states consist of seven baryon-meson channels: $N_{\eta_c}$, $N_J/\psi$, $\Lambda_u\bar{D}$, $\Lambda_d\bar{D}^*$, $\Sigma_u\bar{D}$, $\Sigma_d\bar{D}^*$, and $\Sigma_c\bar{D}^*$, whereas there are five $(0s)^3$ states. So, there are two forbidden states when the system is totally symmetric in the orbital space. Also in this case, all the diagonal elements of the normalization are close to 1; no baryon meson state is affected strongly by the quark Pauli-blocking.

In Figure 2 we show the diagonal phase shifts of the $\frac{1}{2}\left(\frac{1}{2}^-\right)$ state, the proportion is (0.05 0.19 0.76). Thus this bound state is essentially an antisymmetrized $\Sigma_u\bar{D}$.

In Table 5, the bound state, resonances and cusp obtained by the present model. The four structures are identified by the letter A-D in the text. The identification with a dash (B’-D’) is used for the result of the $\Sigma^*_c\bar{D}^*$ three-channel calculation. The energies, $E_3$, are shown in MeV. The proportions of the factors to find each of $[q^11 \frac{1}{2}]$, $[q^38 \frac{1}{2}]$, and $[q^38 \frac{3}{2}]$, $(P_{cs})/(P)$, are listed under the entry $[q^3cs]$.

| id. initial channel $(J^P)$ | $E$ | $[q^11 \frac{1}{2}]$ | $[q^38 \frac{1}{2}]$ | $[q^38 \frac{3}{2}]$ |
|-----------------------------|-----|---------------------|---------------------|---------------------|
| A $\Sigma^*_c\bar{D}^*$ | 4519.9 | 0.00 | 0.00 | 1.00 |
| B $N_J/\psi$ | 4458.0 | 0.36 | 0.16 | 0.28 |
| C $N_J/\psi$ | 4379.3 | 0.24 | 0.16 | 0.60 |
| D $N_J/\psi$ | 4316.5 | 0.75 | 0.13 | 0.12 |
| B’ $\Sigma^*_c\bar{D}$ | 4457.8 | 0.02 | 0.08 | 0.90 |
| C’ $\Sigma^*_c\bar{D}$ | 4379.3 | 0.05 | 0.21 | 0.74 |
| D’ $\Sigma^*_c\bar{D}$ | 4317.0 | 0.05 | 0.22 | 0.73 |

Table 5: The bound state, resonances and cusp obtained by the present model. The four structures are identified by the letter A-D in the text. The identification with a dash (B’-D’) is used for the result of the $\Sigma^*_c\bar{D}^*$ three-channel calculation. The energies, $E_3$, are shown in MeV. The proportions of the factors to find each of $[q^11 \frac{1}{2}]$, $[q^38 \frac{1}{2}]$, and $[q^38 \frac{3}{2}]$, $(P_{cs})/(P)$, are listed under the entry $[q^3cs]$. component is the largest. The listed energy of each of the structures is that where the $\langle P \rangle$ becomes local maximum. All the resonance and cusp energies read from the phase shifts differ by less than 1 MeV from the listed ones except for the resonance C, where the phase shift increases up to above $\pi/2$ at by 4 MeV above the listed energy.

In this case, all the diagonal elements of the normalization are close to 1; no baryon meson state is affected strongly by the quark Pauli-blocking.

Again, we first discuss the results of the three-channel quark cluster model calculation: $\Sigma_u\bar{D}$, $\Sigma_d\bar{D}^*$, and $\Sigma_c\bar{D}^*$. There is one bound state with the binding energy 0.13 MeV, but no resonance is found. As is seen from the entry with an identification D’ in Table 5, major component of this bound state is $[q^38 \frac{3}{2}]$. In the antisymmetrized $\Sigma_u\bar{D}$ state, the proportion is (0.05 0.19 0.76). Thus this bound state is essentially an antisymmetrized $\Sigma_u\bar{D}$.

In this case, all the diagonal elements of the normalization are close to 1; no baryon meson state is affected strongly by the quark Pauli-blocking.

In Table 5 we list $(P_{cs})/(P)$ at the resonance energy under an identification D. The factor to find the $[q^38 \frac{3}{2}]$ configuration becomes small compared to that of the three-channel calculation; the $[q^38 \frac{1}{2}]$ configuration becomes the largest at the resonance. As we will show later, the coupling to the $N_J/\psi$ channel to the $\Lambda_u\bar{D}$ and $\Sigma_u\bar{D}^*$ channels is stronger in this $\frac{1}{2}\left(\frac{1}{2}^-\right)$ case. The existence of the attraction in the $\Sigma_u\bar{D}$ or $[q^38 \frac{3}{2}]$, however, is important to create a resonance.
3.4. uudπ pentaquarks

As summarized in Table 3 in the present model, one bound state is found in \( \frac{1}{2} (\frac{3}{2}^+) \), one resonance and a cusp in \( \frac{1}{2} (\frac{1}{2}^+) \), and one resonance in \( \frac{1}{2} (\frac{3}{2}^-) \). As seen in Table 1 these structures exactly correspond to the number of \([q^3\bar{s}\bar{u}]\) configurations in the negative parity baryon meson channels. The energy of the color-octet uud configuration itself is higher than that of the color-singlet uud configurations. CMI works as an attractive force in the \([q^3\bar{s}\bar{u}]\) configuration just because its energy is lower than that of the relevant baryon-meson threshold, \(\Sigma^{(*)}\). So, when the model space is enlarged from the \(\Sigma^{(*)}\) to all the relevant baryon-meson systems, for example, \(D\) to \(D^*\), the proportion of the \([q^3\bar{s}\bar{u}]\) configuration becomes smaller, from 0.73 to 0.12. The resonance, however, is still there. Situation is similar for the resonances B and C, though the reduction of the proportion by introducing the lower channels is less extreme. The \([q^3\bar{s}\bar{u}]\) configuration plays an important role even though the proportion becomes small. These structures are considered to be an appearance of the hidden color-octet uud baryon.

These resonances and cusp in \( \frac{1}{2} (\frac{3}{2}^-) \) exist in the energy range of the \( P_c(4380) \) peak, \( E \pm \Gamma/2 = 4278-4483 \) MeV. All the structures are very close to the baryon meson thresholds, and each of the resonances has a width of a few MeV, which is far smaller than the widths of the observed peaks. Including the light meson exchange effects between the \( Y_c \) baryon and the \( D \) meson may enlarge the width, which is an interesting topic and will be investigated in future works. We argue that these resonances and cusp may combine to form the broad peak of \( P_c(4380) \). Or, if the parity of \( P_c(4450) \) is found to be negative in future experiments, the cusp at 4458 MeV may correspond to that peak. The bound state in \( \frac{1}{2} (\frac{1}{2}^-) \), whose energy is higher than both of the observed pentaquark peaks, does not couple to the S-wave \( N/J/\psi \) channel; the higher partial wave mode is necessary to see this bound state from the \( N/J/\psi \) channel.

There are arguments that peaks which correspond to the pentaquarks should appear at \( \pi N \to J/\psi N \) or \( \gamma N \to J/\psi N \) reactions if the coupling between \( N/J/\psi \) and the pentaquarks is large. The diagonal elasticities of the present calculation, \( \eta \), which is the absolute value of the scattering matrix of the same initial and final channels, are plotted in Figure 3. In \( \frac{1}{2} (\frac{3}{2}^-) \), \( \eta \) of the \( N/J/\psi \) channel goes down to only 0.81 at the resonance C or to 0.70 at the cusp B. The elasticity of the \( \frac{1}{2} (\frac{1}{2}^-) \) goes down to 0.65 at the resonance D. In both of the channels, the mixing between \( N/J/\psi \) and the other channels is rather small except for the resonance energies, because the quark overlap is small and because the rearrangement of the charm quark is necessary. The average inelasticity of \( N/J/\psi \), \( 1 - \eta \), over the energy range of the broad \( P_c(4380) \) width is about 0.1–0.2. It should be checked whether this value is consistent with the above \( J/\psi \) production experiments.

The estimate of the cross section \( \sigma_{NJ/\psi} \) from the photo production experiment (\( \gamma N \to J/\psi N \)) is 3.5 ± 0.8 ± 0.5 mb. It corresponds to the scattering length \( |a_{NJ/\psi}| = 0.17^{+0.02}_{-0.03} \) fm by using \( \sigma = 4\pi a^2 \). This scattering length has been calculated by many theories. The QCD sum rule gives \( a_{NJ/\psi} = -0.10 \pm 0.02 \) fm. Two quenched Lattice QCD calculations were reported: the spin averaged scattering length was obtained as \(-0.71 \pm 0.48 \) fm or \(-0.39 \pm 0.14 \) fm, or about \(-0.35 \) fm (read from figure [20]). In the present calculation, the \( J/\psi N \) scattering length is \(-0.077 \) fm for \( J=\frac{3}{2} \) and \(-0.103 \) fm for \( J=\frac{1}{2} \). The spin averaged value is \( a_{NJ/\psi} = -0.085 \) fm. Since the quark interaction does not produce a direct interaction between \( N \) and the \( c\bar{s} \) mesons, this attraction here solely comes from the channel coupling between the \( N/J/\psi \) and the \( \Lambda_c D^{(*)} \) and \( \Sigma_c D^{(*)} \) channels.

Our results suggest that roughly half of the observed attraction between the \( N \) and \( J/\psi \) comes from the channel coupling.

In this work, we employ the quark interaction arising from the gluons. It is because we would like to investigate
the features of the color-octet $uud$ baryons, which newly appears in the hidden-charm pentaquarks. As for the $\Lambda_4\bar{D}$ or $\Sigma_7\bar{T}$ baryon meson channel, however, it is necessary to include the pion-exchange force in the long range region. As is reported in ref. [21], the meson-exchange models may give many bound states and resonances though the results seem to depend strongly on the cutoff value. It is very interesting to see whether the energies of the currently obtained resonances move or their widths become broader by introducing the meson-exchange in our model. Moreover, in order to discuss the $P_c(4450)$, which has an opposite parity to the $P_c(4380)$, simultaneously, the $P$-wave baryon-meson relative motion and the positive parity mesons should be introduced. Since the lowest orbital excitation is considered to be the $S$-wave $uud\bar{c}$ configuration with the $P$-wave $c\bar{s}$ pair, the color-octet $uuu$ configuration may again play an important role there. We take both of them as future problems.

It is also interesting to investigate the hidden-charm pentaquarks with the strangeness, where the flavor-octet $uds$ may play a similar role to the present work. As for the isospin-$\frac{1}{2}$ $uwd\bar{c}$ systems, the three light quarks are either color-octet spin-$\frac{1}{2}$ or color-singlet spin-$\frac{1}{2}$. Because CMI of these light quark configurations contributes repulsively, and because the Pauli-blocking gives strong repulsion there, all the five-quark systems seem to dissolve by the baryon-meson couplings, which we will also discuss elsewhere.

The isospin-$\frac{1}{2}$, or the flavor-octet hidden-$Q\bar{Q}$ baryons are a special place to look into for the color-octet $uud$ baryons. When we introduce the $b\bar{b}$ pairs instead of $c\bar{s}$ to the five-quark systems, the situation will become more manifest. The system is closer to the heavy quark limit, and the thresholds of the $\Sigma_4^{(*)}B^{(*)}$ are closer to each other. The four structures we find in the $uud\bar{c}$ states are also found to exist. The bound state in $J^P = \frac{3}{2}^-$ has a binding energy of more than 10 MeV. We discuss the system with the $b\bar{b}$ pairs as well as those with the $c\bar{s}$ pairs elsewhere.

4. Conclusions

The $I(J^P) = \frac{1}{2}(\frac{1}{2}^-), \frac{3}{2}(\frac{1}{2}^-)$, and $\frac{1}{2}(\frac{1}{2}^-)$ $uud\bar{c}$ systems are investigated by the quark cluster model. There is no strong repulsion due to the quark Pauli blocking in the relevant baryon meson systems. It is shown that the color-octet isospin-$\frac{1}{2}$ spin-$\frac{3}{2}$ $uud$ configuration gains an attraction from the color magnetic interaction. The $uud\bar{c}$ states with this configuration cause structures around the $\Sigma_4^{(*)}B^{(*)}$ thresholds. We have found one bound state in $\frac{1}{2}(\frac{1}{2}^-)$, one resonance and a cusp in $\frac{3}{2}(\frac{3}{2}^-)$, and one resonance in $\frac{1}{2}(\frac{3}{2}^-)$ in the negative parity channels. We argue that one of these structures may give $P_c(4450)$ if its parity is found to be negative, or these resonances and cusp may combine to form the broad peak of $P_c(4380)$.

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