Backhaul-Aware Placement of a UAV-BS with Bandwidth Allocation for User-Centric Operation and Profit Maximization

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Abstract—Addressing the Quality-of-Service (QoS) requirements of users is crucial for service providers to improve the network performance. Furthermore, the transformation from the network-centric to user-centric service paradigm requires service providers to focus on improving the Quality-of-Experience (QoE) which is expected to become an important objective in Next Generation Cellular Networks (NGCNs). Managing QoE is not only a technical issue but also a marketing ability to improve profitability. An efficient strategy to improve the profitability is to apply price differentiation for different service levels. Unmanned Aerial Vehicle Base Stations (UAV-BSs) are envisioned to be an integral component of NGCNs and they create opportunities to enhance the capacity of the network by dynamically moving the supply towards the demand while facilitating services that cannot be provided via other means efficiently. However, building a reliable wireless backhaul link via optimized resource allocation is a key issue for the placement of UAV-BSs. In this paper, we consider a UAV-BS and a terrestrial network of Macro-cell Base Stations (MBSs) that the UAV-BS rely on for backhauling. The problem is to determine the 3D location of the UAV-BS and the bandwidth allocations to each user to maximize the profitability of service provided in terms of achievable data rate levels. We develop a Mixed Integer Non-Linear Programming (MINLP) formulation of the problem. To overcome the high complexity, we propose a novel search algorithm that is very efficient in terms of solution quality and time. The analysis performed through numerical evaluations reveal that offering multiple data rate options to users improves the QoE and at the same time allows the service providers to increase the total profit.

Index Terms—aerial base station, bandwidth allocation, backhaul capacity, price differentiation, unmanned aerial vehicle.

I. INTRODUCTION

Unmanned Aerial Vehicle Base Stations (UAV-BSs) are expected to be used in Next Generation Cellular Networks (NGCNs) to enhance the capacity of the network as well as expanding the coverage [1]. The rapid deployment and mobility advantage of a UAV-BS has the potential to enhance the Quality-of-Service (QoS) substantially provided that a reliable wireless backhaul link is available to sustain the agility of the network. UAV-BSs can be used in several scenarios, such as providing temporary connections in case of malfunctions in terrestrial networks or in case of natural disasters and enhancing the data rate when excessive demand exists [2].

Demand for diverse wireless services and massively large number of connected devices (i.e., Internet-of-Things – IoT) has been forcing the service providers to shift towards the user-centric services. Therefore, novel policies should be created and adopted to improve the Quality-of-Experience (QoE) in NGCNs. One way to achieve such an improvement is based on service pricing which is an important factor for user satisfaction and is a key component of marketing strategies together with product, place, and promotion [3]. Price differentiation is commonly used by suppliers in various markets to increase profitability. In fact, customers do not only seek the lowest price in the market, but they seek value for the money they invest as well. This perception can easily be applied to wireless services. For instance, serving with higher data rate to a user in return of higher fee can help suppliers to improve the QoE and increase, arguably, the long-lasting loyalty of the users.

QoS, in general, has a narrower scope, typically, focusing on network performance in comparison to QoE, which is affected by a wide-ranging set of factors. These factors can be grouped into three main categories: network, human, and context [4]. Network factors are technical characteristics of the network such as coverage, security, and privacy. Human factors are related to the users’ impression on the service and are affected by a wide range of constituents such as psychology and demographic profile. The context category is associated with features affecting the users’ perception of the service. Studies on UAV-BS utilization for communications have, mostly, been focused on the first group of the aforementioned factors so far. However, the overall quality can be improved by aligning all factors affecting both QoS and QoE [5]. Therefore, radical shifts from network-centric approaches to user-centric approaches have been forcing the service providers to create new mechanisms and solutions in the market not only in terms of QoS but also in terms of QoE. For wireless services, users expect to be served with ubiquitous and reliable services at affordable prices. This is an important challenge for a service provider to address since the performance of a service is likely to improve with high value for money while the capacity is limited. One way to overcome such a challenge is to match the expectation of the user with the available capacity.

In the service industry, users intuitively determine a max-
imum value to pay for a service and intend to procure the choice among all alternatives offered in the market by comparing their value for money. Although there are many factors that influence the user behavior, the price of a service is a crucial one for a user to switch to another service provider when the price of the existing service provider exceeds a threshold value. Such switches cause unpredictable profit fluctuations for the providers since most of the corporate decisions are made based on future estimations. Hence, the providers should determine the optimum price ranges for their services to gain the maximum profit. This optimization can be achieved when the price expectation of the user for a service matches the profit estimation of the service provider.

Both parties of a service agree on a contract to determine the condition of the service provision. The value of the contract is determined by two components: (i) the revenue gained when the service is received by the users according to the conditions defined in the contract and (ii) the penalty incurred when the provider is not able to provide the service agreed upon due to unforeseeable failures and unpredictable switches of the users. Then, the value of a contract can finally be determined by the difference between the revenue and the penalty. However, such a valuation scheme is unrealistic in telecommunication industry since the transaction time is very short (i.e., seconds) and the service level rapidly changes in relatively short time frames. Therefore, service providers rely on the long-term expected values of the revenue and the penalty to make their future plans instead of the actual rates of revenue and penalty incurred during the contract period.

The price paid for the achievable data rate is one of the important QoE metrics for users in evaluating different service providers in the industry. Most of the contracts between the providers and the users involve a minimum data rate level that is instantly provided to the user and it is defined as the Service Level Agreement (SLA) for the contract. For instance, an e-business firm can pay a higher price for higher data rates provided while an individual who enjoys surfing the internet for leisure may choose to settle for lower rates at a lower price.

In this paper, we search for an efficient deployment method of a single UAV-BS together with an efficient bandwidth allocation strategy among the users to increase the total profit of the network by applying a price differentiation strategy to improve the QoE of the users.

The rest of the paper is organized as follows. Section II presents the literature overview. The system model is given in Section III. Section IV gives the proposed solution approach. Section V presents the computational results. Section VI concludes the paper and provides future directions.

II. LITERATURE REVIEW

The opportunities brought by the UAV-BSs (such as enhancing capacity, improving QoE, extending coverage) are so promising that despite the relatively new appearance of the topic, it has found a considerable attention in both academia and industry. Especially, the location optimization problems have attracted significant interest since they have a significant impact on the network performance. In this section, we present a high-level overview of the UAV-BS literature and present the differences of our study from the existing literature.

In [2], the minimum number of UAV-BSs along with their positions are determined in areas with different user densities through a heuristic approach. In [3], a mathematical model is proposed for the single UAV-BS location problem. Instead of solving a 3D location problem, the authors used the ratio of the altitude of the UAV-BS to the radius of the area to be covered as a parameter. Then the problem is solved as a 2D location problem by searching over this parameter’s domain with a bisection algorithm. This approach, however, is not appropriate for our study since the data rate value is not only the function of the ratio but it is a function of the bandwidth allocated also.

In [9], the optimal positioning of an uncapacitated UAV-BS is investigated to satisfy various QoS requirements of the users. In [10], the optimal positioning of multiple UAV-BSs is explored over a predetermined area to enhance the coverage and lifetime of the UAV-BSs. The 3D location problem is converted to a circle packing problem. In [11], a user-demand-based cost function of an aerial relay is minimized for a given level of delay tolerance for several groups of users. In [12], the location of a fixed and mobile single UAV-BS is determined to maximize the coverage probability and sum-rate. It has been showed that there is an optimal altitude for the UAV-BS when the coverage probability and sum-rate are maximized, separately, and this location highly depends on the user density within the area to be covered. In [13], a joint optimization problem in which the locations of the multiple UAV-BSs and the association of the users and the UAV-BSs are found while the network is assumed to store caching information of the users. In [14], the minimum average data rate provided to the users in a finite time horizon is maximized by jointly determining the trajectories of the UAV-BSs and the user-UAV-BS associations such that the maximum hovering distance is not violated.

In [15], the location of a UAV-BS is determined to maximize the total number of users served while minimizing the UAV-BS’s transmit power. In [16], the placement problem of UAV-BSs in case of an emergency (i.e. natural disaster) is modeled and solved by a genetic algorithm. It is shown that UAV-BSs can improve the median and the fifth-percentile throughput of the network. In [17], the downlink coverage performance of a single UAV-BS is analyzed to maximize the coverage in case of both interference and no interference situations. In [18], the number of UAV-BSs is minimized to ensure that each ground terminal is served with at least one UAV-BS by reformulating the problem as a p-center problem. In [19], a model for determining UAV-BS locations and IoT devices to be matched with those UAV-BSs is presented where the objective is to minimize the total transmit power requirement for both the UAV-BSs and the IoT devices on the ground. In [20], placement of multiple UAV-BSs with respect to several demand areas is considered and a solution procedure is proposed to improve different performance metrics of the network such as the throughput and the fifth percentile efficiency.

One important drawback in most of the aforementioned studies is that the UAV-BSs are assumed to have unlimited reliable backhaul links to the MBSs (Macro-cell Base Stations).
located on the ground. There are only a few studies in the literature [2], [21], [22] that address the limited capacity of the backhaul links in the system design in the context of non-terrestrial communications. Since the backhaul links that the UAV-BSs rely on have limited capacity, the assumption of unlimited capacity can lead to a flawed design and analysis. Therefore, for designing reliable wireless services it is a necessity to include backhaul capacity constraints in the model. In [21], the placement of a single UAV-BS is considered subject to two limiting constraints, namely the bandwidth and the backhaul capacity. In [22], multiple UAV-BSs are located to provide wireless service to the users who have a predefined delay-tolerance parameter. The resource allocation to the users is also considered and an exhaustive search procedure is proposed to maximize a logarithmic utility function. However, in both studies, it is assumed that the users are identical in terms of SLAs. Although such an assumption can hold for certain systems, it cannot be applied to the general case because different user categories, naturally, have different SLAs and they play a crucial role in users’ decision on whether to continue with that particular service provider or not [23].

The instantaneous achievable data rate is one of the important SLA elements since the QoE is highly dependent on how the service is perceived by the user. In this paper, we assume that the users have different data rate demands and they also have different willingness values to pay for the rate provided by the service providers. Without loss of generality, for the ease of the exposition, the service providers are assumed to offer several data rate options to the users and the users are assumed to select only one option among the alternatives for a specific time.

The service providers are expected to satisfy the demand with respect to the backhaul capacity and the available bandwidth in the network, while the users are persuaded to pay for the service as long as the service is in line with the agreed SLA. To the best of our knowledge, this study is the first to combine the QoS and the QoE aspects of communication networks utilizing UAV-BSs. In this study, we first construct a novel Mixed Integer Non-Linear Programming (MINLP) formulation of the problem and then propose a novel search algorithm to solve it.

### III. The System Model

In our model, we consider a network of multiple MBSs and one UAV-BS that can have a dedicated backhaul link with one of the MBSs. Fig. 1 illustrates a sample representation of the considered system with two data rate levels. The users are offered multiple achievable data rate levels and are assumed to select only one option to be served. The willingness values of the users to pay for the service are shown by the number of $ icons in the figure. Although, the UAV-BS can rapidly be deployed anywhere in the air to cover as many users as possible, the availability of a backhaul link should be considered carefully to provide a reliable overall connection. We assume that each MBS has sufficient connection capacity to the communications infrastructure to transmit the data received from the UAV-BS to the core network, therefore, there is no congestion in the fixed infrastructure.

The presented model can be validated based on the idea of enhancing the capacity as well as providing rapid supply to difficult-to-predict situations such as crowded events and activities. For instance, the area enclosed within the ellipse in Fig. 1 is a potential use case of such a model, where some parts of the whole region can include more users than the rest of the region, and the UAV-BS can provide service to those users in the congested parts more efficiently than the MBSs. The presence of a UAV-BS also allows the service providers to exploit the idle capacity when the demand is not as high as the supply in some parts. Hence, the service providers can allocate the idle resources to the UAV-BS to create a relay connection for the users who cannot be served by the lightly loaded MBS itself (see Figure 1).

In the rest of the paper, the users are denoted by the set $I$, MBSs are denoted by the set $J$, and the data rate levels are denoted by the set $K$. We use $i \in I = \{1, \ldots, n\}$, $j \in J = \{1, \ldots, m\}$, and $k \in K = \{1, \ldots, s\}$ to index the users, the MBSs, and the service levels, respectively.

#### A. Channel Model

Let user $i$ is located at $(x_i, y_i)$ and the UAV-BS at $(x_d, y_d, h_d)$. Since all the users are on the ground, the corresponding height is taken as zero. Alternatively, $h_d$ can be interpreted as the height difference between the users and the UAV-BS. The air-to-ground channel model proposed by [24] is adopted, where there exist two propagation groups, one with users with Line-of-Sight (LoS) connections and the other with Non-Line-of-Sight (NLoS) connections. The latter group of users can still maintain their connections due to the mechanisms of electromagnetic wave propagation which can be used to convey information beyond the obstructions (e.g., reflection and diffraction). The path-loss between user $i$ and the UAV-BS (in dB), is modeled as

$$PL_i = FSPL + \mu_i,$$

(1)

Fig. 1: Illustration of a network with UAV-BS in NGCN.
where FSPL represents the Free Space Path-Loss according to Friis equation, $20\log\left(\frac{\sigma_f d_i}{c}\right)$, $f_c$ is the carrier frequency in Hz, $c$ is the speed of light in m/s, $\mu_i$ is the excess path-loss in dBi due to the LoS or NLoS channel between the user and the UAV-BS, and $d_i$ is the distance between the user and the UAV-BS. All distances are in meters and $d_i$ is equal to $\sqrt{(x_i - x_d)^2 + (y_i - y_d)^2 + h_d^2}$.

The probability of having a LoS link for user $i$ depends on the elevation angle, $\theta_i$ in degrees, between the user and the UAV-BS, which can be found by $\frac{\theta_i}{180} \arctan\left(\frac{h_i}{d_i}\right)$, where $l_i$ is the horizontal distance between user $i$ and the UAV-BS ($l_i = \sqrt{(x_i - x_d)^2 + (y_i - y_d)^2}$). This probability also differs with parameters, $\alpha$, $\beta$, $\varphi_{\text{LoS}}$, and $\varphi_{\text{NLoS}}$, which depend on the environment being suburban, urban, dense urban, or high-rise urban. Therefore, the probability of LoS can be modeled as:

$$P_{\text{LoS}}(h_d, l_i) = \frac{1}{1 + \alpha e^{-\beta\left(\frac{\theta_i}{180} \arctan\left(\frac{h_i}{d_i}\right)\right)}}.$$  

Path-loss for user $i$ can be expressed as follows:

$$\text{PL}_i = 20\log\left(\frac{\lambda f_c d_i}{c}\right) + P_{\text{LoS}}(h_d, l_i)\varphi_{\text{LoS}} + (1 - P_{\text{LoS}}(h_d, l_i))\varphi_{\text{NLoS}}.$$  

The FSPL model is used for the backhaul link between the UAV-BS and MBS $j$ as:

$$\text{PL}_j = 20\log\left(\frac{\lambda f_c d_j}{c}\right),$$

where $d_j$ denotes the distance between the MBS located at $(x_j, y_j)$ and the UAV-BS ($\sqrt{(x_j - x_d)^2 + (y_j - y_d)^2 + h_d^2}$).

### B. Problem Formulation

We consider point-to-point wireless connections between the UAV-BS and the MBSs. This connection is assumed not to interfere with user links of the UAV-BS. To facilitate such an assumption, reversed time-division duplexing is employed to avoid interference between the backhaul and user links, such that, during downlink of MBS, UAV-BS is in the uplink mode. However, when MBS is in the uplink mode, users may largely be affected by the MBS due to the self-interference. To avoid this, orthogonal frequency channels are used in the backhaul and user links [25]. It is also assumed that the coordinates of the users and the MBSs, and the willingness values of the users for each data rate level offered are known.

Our objective is to maximize the profit by determining the location of the UAV-BS subject to the backhaul capacity and available bandwidth. We also determine the allocation of the available bandwidth to the users.

Data rate provided to the users cannot exceed the capacity of the backhaul link between the UAV-BS and MBS $j$ that the UAV-BS is connected to, therefore,

$$\sum_{i \in I} r_i \tau_{ij} \leq C_j z_j, \quad \forall j \in J,$$  

where $\tau_{ij}$ is the binary decision variable that is equal to 1 if user $i$ is served by the UAV-BS which in turn has a backhaul connection to MBS $j$, and 0 otherwise. $z_j$ is the binary decision variable that equals to 1 if the UAV-BS has a backhaul connection to MBS $j$ and 0 otherwise. $r_i$ is the actual data rate provided to user $i$, which can be obtained as:

$$r_i = b_i \log_2(1 + 10^\frac{PL_i}{10}),$$

where $b_i$ is the bandwidth allocated to user $i$, and $\delta_j$ is the received signal-to-noise ratio (SNR) in dB of user $i$ associated with the UAV-BS. The SNR value of user $i$ can be calculated as $\delta_j = G_d - \text{PL}_i - \omega_i$, where $G_d$ is the transmit power of the UAV-BS in dBm, and $\omega_i$ is the thermal noise power at user $i$ which is equal to $10\log(b_i) - 174 + \sigma_\omega$ at $290^\circ K$ with a noise figure $\sigma_\omega$ [26].

Backhaul capacity, $C_j$, when the UAV-BS is receiving from MBS $j$ can be calculated in the same manner with the actual data rate of a user as

$$C_j = B_j \log_2(1 + 10^\frac{PL_j}{10}),$$

where $B_j$ is the total bandwidth allocated to the UAV-BS from MBS $j$ and $\delta_j$ is the received SNR at the UAV-BS from MBS $j$. $\delta_j$ is equal to $G_j - \text{PL}_j - \omega_j$, where $G_j$ is the transmit power of MBS $j$ in dBm, and $\omega_j$ is the noise power at MBS $j$ at $290^\circ K$, where $\omega_j = 10\log(B_j) - 174 + \sigma_\omega$.

On the other hand, the total bandwidth necessary to meet the SLA requirement of the users cannot exceed the total available bandwidth allocated to the UAV-BS, therefore

$$\sum_{i \in I} b_i \tau_{ij} \leq B_j z_j, \quad \forall j \in J.$$  

The UAV-BS can have a backhaul link with only one MBS, thus

$$\sum_{j \in J} z_j = 1.$$  

Similarly, a user can be served by at most one MBS which can be expressed, mathematically, as

$$\sum_{j \in J} \tau_{ij} \leq 1, \quad \forall i \in I.$$  

Users are assumed to have different willingness values to pay for different data rate levels. Therefore, the profit that is gained from an individual user $i$, $u_i$, can be modeled as

$$u_i = \begin{cases} 0, & r_i < R_1 \\ \phi_{ik}, & R_k \leq r_i < R_{k+1}, \forall i \in I, \forall k = 1, \ldots, s - 1, \\ \phi_{is}, & R_s \leq r_i \end{cases},$$

where $R_k$ shows the offered data rate level $k$ according to SLA. $\phi_{ik}$ represents the profit gained from a particular user $i$ if the actual data rate provided, $r_i$, is between the data rate level $k$, $R_k$, and $(k+1)$, $R_{k+1}$. Similarly, $\phi_{is}$ is the maximum profit gained from a user if the actual data rate provided is above the maximum data rate level offered, $R_s$.

The objective function that maximizes the total profit is

$$\max_{x_d, y_d, h_d, b_i, \{\tau_{ij}\}, \{z_i\}} \sum_{i \in I} \sum_{j \in J} u_i \tau_{ij}.$$  

Consequently, we have a MINLP formulation given by (Q) which is a generalized Gradual Maximal Covering Location Problem (GCLP) with a non-convex objective function and a
(Q) Maximize $\sum_{i \in I} \sum_{j \in J} u_i \tau_{ij}$
subject to:

Constraint (5), (8)-(11)

$$b_i \geq 0, \forall i \in I$$

$$\tau_{ij} \in \{0, 1\}, \forall i \in I, \forall j \in J$$

$$z_j \in \{0, 1\}, \forall j \in J$$

$$x_{min} \leq x_i \leq x_{max}$$

$$y_{min} \leq y_d \leq y_{max}$$

$$h_{min} \leq h_d \leq h_{max}$$

non-linear constraint set combined with binary and continuous variables [27].

Discrete version of the GCLP, where the alternatives to locate a facility is known, with a non-increasing coverage function can be solved by reducing the problem to the Uncapacitated Facility Location Problem (UFLP) [27]. Planar version of GCLP with a non-convex objective function and a non-increasing coverage function can be solved by the branch-and-bound algorithm with a predefined relative accuracy [28]. However, both the discrete and the planar GCLP becomes intractable with the increasing number of users [29]. Furthermore, there does not exist any solution approach for the problems with unimodal coverage functions and 3-D space. Hence, we propose a two-stage search algorithm that efficiently solves the problem in a reasonable time in the next section.

IV. Solution Approach

The capacity and the bandwidth constraints (5 and 8) are two important limiting constraints in (Q). The capacity constraint together with the user profit level constraint (11) are likely to extend the solution time of the problem (Q) since the total bandwidth is assumed to be known and all other constraints are auxiliary constraints to meet the SLA requirements of the users. Therefore, the solution complexity of (Q) can be reduced if the capacity is known.

Note that the backhaul capacity can be exactly calculated if the location of the UAV-BS is known. Then, (Q) can be reduced to the well-known Integer Knapsack Problem (IKP) [30] with two capacity constraints. This problem can be solved more efficiently by commercial optimization packages such as IBM CPLEX compared to MINLPs. The objective of the IKP is to maximize the total benefit by adding multiple items to a knapsack until its capacity is fulfilled. For our formulation, we can define items as user-level pairs. The capacity of the knapsack is determined through two dimensions: the backhaul capacity and the total available bandwidth. As a consequence, given the coordinates of the UAV-BS, (Q) can be reduced to ($Q_j'$), which can be solved separately for each MBS $j$ as an IKP.

($Q_j'$) has the same objective function with (Q) to maximize the total profit with a new binary variable, $t_{ik}$, which is equal to 1 if user $i$ is served at data rate level ($R_{ik}$), and 0 otherwise. The data rate function is concave with respect to $b_i$, see (6).

Although $b_i$ cannot be expressed in a closed form formulation, it can be estimated by a line search algorithm such as the bisection search, within a user-defined accuracy level, $\epsilon_{BS}$. Therefore, we introduce a new parameter $b_{ik}'$, that represents the amount of bandwidth required to serve user $i$ at data rate level $k$ within the accuracy of $\epsilon_{BS}$. Additionally, since the total amount of bandwidth is assumed to be known, the backhaul capacity of MBS $j$, $C_j$, can be calculated using (7). Eqs. (14) and (15) ensure that the total provided data rate and the total allocated bandwidth cannot exceed the backhaul capacity and total available bandwidth, respectively. Eq. (16) guarantees that a user can at most be served at one level.

($Q_j'$) Maximize $\sum_{i \in I} \sum_{k \in K} \phi_{ik} t_{ik}$
subject to:

$$\sum_{i \in I} \sum_{k \in K} R_{ik} t_{ik} \leq C_j$$

(14)

$$\sum_{i \in I} \sum_{k \in K} b_{ik}' t_{ik} \leq B_j$$

(15)

$$\sum_{k \in K} t_{ik} \leq 1, \forall i \in I$$

(16)

$$t_{ik} \in \{0, 1\}, \forall i \in I, \forall k \in K.$$  

(17)

By transforming the original problem to an IKP, we can solve it in multiple stages. Indeed, we propose a two-stage search algorithm, where the Golden Section Search (GSS) algorithm is used to search the altitude ($h_d$) of the UAV-BS in the first stage. For each fixed value of the altitude, in the second stage, a directional search algorithm (see Section VI-B for details) is used to search the 2D coordinates ($x_i$,$y_d$) of the UAV-BS. During this directional search, ($Q_j'$) is solved repeatedly to determine the bandwidth allocation to users.

A. First Stage: Golden Section Search

GSS is one of the efficient search algorithms to find the extremum of a unimodal function. In other words, if a function has one mode (minimum or maximum), the extremum of the function can be estimated by GSS within a predefined tolerance value, $\epsilon_{GSS}$. GSS searches the extremum by evaluating two different points in the domain of the function. Assume that function $f(x)$ is unimodal in the interval $[x_{min}, x_{max}]$ and it has one maximum value. Then, two interior points are selected by using the golden ratio, $g = \frac{\sqrt{5}-1}{2}$. The first point, $x_1$, is equal to $x_{min} + g(x_{max} - x_{min})$, and the second point, $x_2$, is equal to $x_{max} - g(x_{max} - x_{min})$. If $f(x_1) \geq f(x_2)$, then the extremum is between $x_2$ and $x_{max}$ and algorithm proceeds to the next iteration with assigning $x_{min}$ to $x_2$. Otherwise, it proceeds with assigning $x_{max}$ to $x_1$. It continues until $x_{max} - x_{min} \leq \epsilon_{GSS}$. At the end, the extremum is estimated as $\frac{x_{max} + x_{min}}{2}$.

Spectral efficiency is a measure of the ability of a communication system to utilize the available bandwidth efficiently and defined as the average number of bits per unit time that can be transmitted per unit bandwidth [31]. In our context, when all other variables are fixed, the data rate function of a user
Algorithm 1 GSS: Finds the location of the UAV-BS and the bandwidth allocation of the users

Input: User locations \((x_i, y_i)\), MBS locations \((x_j, y_j)\), data rate options \((R_k)\), willingness values of the users \((\phi_{ik})\), \(e_{GSS}\)

1: \(F_{max} \leftarrow \sum_{i \in I} \phi_{ik}\), \(F_{inc} \leftarrow 0\), \(x_{dinc} \leftarrow 0\), \(y_{dinc} \leftarrow 0\), \(h_{dinc} \leftarrow 0\), \(b_{inc} \leftarrow 0\), \(\forall i \in I\), \(h^l \leftarrow 0\), \(h^u \leftarrow h_{max}\)
2: while \(h^u - h^l > e_{GSS}\) do
3:     for \(a = 1\) to \(10\) do
4:         if \(a = 1\) then
5:             \(h_a \leftarrow h^l + \frac{\sqrt{a-1}}{a} (h^u - h^l)\)
6:         else
7:             \(h_a \leftarrow h^u - \frac{\sqrt{a-1}}{a} (h^u - h^l)\)
8:     end if
9:     Solve \(Q_j^a\) for each \(j \in J\) at \(h_a\), \(j^* \leftarrow \arg \max_j \{Q_j^a : Q_j^a \geq \Omega_j^a\}, \forall i \in J\). \(\Omega_j^a \leftarrow \Omega_j^a\), where \(\Omega_j^a\) is the objective value of \(Q_j^a\).
10: if \(\Omega_j^a = F_{max}\) then
11:     STOP. \(\Omega_j^a\) is the optimum. \(x_{dinc} \leftarrow x_{d, j^*}\), \(y_{dinc} \leftarrow y_{d, j^*}\), \(h_{dinc} \leftarrow h_a\), \(b_{inc} \leftarrow b_{inc}^*\), \(\forall i \in I\).
12: else
13:     Apply Algorithm 2\((SD_a)\) at \(h_a\), \(\Omega_j^a \leftarrow \Omega_j^{SD_a}\).
14: if \(\Omega_j^{SD} = F_{max}\) then
15:     STOP. \(\Omega_j^{SD}\) is the optimum. \(x_{dinc} \leftarrow x_{d, j^*}\), \(y_{dinc} \leftarrow y_{d, j^*}\), \(h_{dinc} \leftarrow h_a\), \(b_{inc} \leftarrow b_{inc}^*\), \(\forall i \in I\).
16: end if
17: end if

Algorithm 2 SD: Improve a given solution in terms of revenue

Input: User locations \((x_i, y_i)\), MBS locations \((x_j, y_j)\), data rate options \((R_k)\), willingness values of the users \((\phi_{ik})\), initial solution \((x_{init}, y_{init})\), \(h_{init}, b_{init}, \Omega_{init}\), \(\lambda_{SDx}, \lambda_{SDy}\)

1: \(x_{sd, \mbox{ init}} \leftarrow x_{d, \mbox{ init}}\), \(y_{sd, \mbox{ init}} \leftarrow y_{d, \mbox{ init}}\), \(\Omega_{SD} \leftarrow \Omega_{init}\), \(x_d \leftarrow x_{d, \mbox{ init}}\), \(y_d \leftarrow y_{d, \mbox{ init}}\), \(b_j^{SD} \leftarrow b_{init}, \forall i \in I\), \(\xi = 1\)
2: while \(\xi \leq \xi_{\mbox{ max}}\)
3:     \(\Delta t \leftarrow \frac{\sum_h \phi_{ih} \Delta t}{\|K\|}\)
4:     \(D_i \leftarrow \frac{\sum_{m \subseteq M} \sum_{j \in M} \phi_{jk} b_j^{SD} \times (x_i - x_{d, \mbox{ new}}, y_i - y_{d, \mbox{ new}})}{\sum_{m \subseteq M} \sum_{j \in M} \phi_{jk} b_j^{SD}}\)
5:     \(\Delta t \leftarrow \frac{\sum_{m \subseteq M} \sum_{j \in M} \phi_{jk} b_j^{SD} \times (x_i - x_{d, \mbox{ new}}, y_i - y_{d, \mbox{ new}})}{\sum_{m \subseteq M} \sum_{j \in M} \phi_{jk} b_j^{SD}}\)
6:     \(j^* \leftarrow \arg \max_j \{j : C_j < C_{m, j}, \forall j, m \in \{J\}\}\)
7:     Solve \(Q_j^a\) at \((x_{d, \mbox{ new}}, y_{d, \mbox{ new}}), h_{j, \mbox{ init}}\), \(\Omega^* \leftarrow Q_j^a\)
8: if \(\Omega^* > \Omega_{SD}\) then
9:     \(\Omega_{SD} \leftarrow \Omega^*\), \(x_d \leftarrow x_{d, \mbox{ new}}\), \(y_d \leftarrow y_{d, \mbox{ new}}\), \(b_j^{SD} \leftarrow b_j^{SD}, \forall i \in I\)
10: end if
11: \(\xi \leftarrow \xi + 1\)
12: end while
13: return \(x_d^{SD}, y_d^{SD}, \Omega^{SD}, b_j^{SD}, \forall i \in I\)

is unimodal with respect to the altitude of the UAV-BS \((h_d)\). It has a single maximum in the interval \([h_{min}, h_{max}]\) as can be seen in the sample spectral efficiency curves depicted in Fig. 2 for different environments. Spectral efficiency follows a similar pattern after a particular altitude of the UAV-BS for each environment, since the probability of achieving full LoS values are the same for higher altitudes, see [2]. However, the convergence rate of achieving full LoS, which means \(P_{LoS} = 1\), at lower altitudes changes depending on the environment. Achieving full LoS is highly likely at lower altitudes in suburban environment due to lack of barriers like tall buildings, while such an occurrence is far from certain in other environments such as highrise environment. Once full LoS is achieved, the only parameter that affects the path-loss is \(\mu_{LoS}\) and the altitude of the UAV-BS. Since \(\mu_{LoS}\) does not change dramatically for different environments, spectral efficiency decreases slowly for each environment as the effect of distance on the path-loss becomes more dominant than the effect of LoS.

If \(h_d\) is known, then the backhaul capacity, \(C_j\), is maximized when the UAV-BS is located just over one of the MBSs for a given altitude. The reason for such behavior is that \(\delta_j\), whose increasing values also increase \(C_j\), can be maximized when the distance between the UAV-BS and MBS \(j\) is minimized, see [7]. Note that the bandwidth allocated to user \(i\) can be obtained for a given data rate level when the coordinates of the UAV-BS is known. Therefore, we start every iteration by locating the UAV-BS over MBS \(j\) that maximizes \((Q^*_j)\) at a given altitude. Then, we apply the Steepest Descent (SD) Algorithm to
improve the solution by searching new horizontal coordinates that increase the objective at the same altitude. The algorithm terminates either whenever the maximum possible profit, $F_{\text{max}} = \sum_{i \in I} \phi_i s$, is achieved or the difference between the altitudes in consecutive iterations is smaller than $\epsilon_{\text{GSS}}$. The proposed GSS algorithm is summarized in Algorithm 1.

### B. Second Stage: Steepest Descent Algorithm

SD Algorithm is employed to improve a given solution by finding new horizontal coordinates for the UAV-BS with a direction that is expected to increase the objective function value. There should be a policy to find the direction in SD algorithms. At each iteration of the algorithm, the current solution is moved to a new candidate in the direction found by applying the policy and the unit step size, $\lambda_{\text{SD}}$. Whenever the new solution improves the current solution, it is saved as an incumbent solution. When the algorithm stops, the incumbent solution is returned as the output of the algorithm. The algorithm is terminated when the number of iterations reaches a predetermined maximum iteration value, $\xi_{\text{max}}$. The SD algorithm used in this paper is summarized in Algorithm 2.

### V. Computational Results

We test our algorithm in a suburban environment with instances formed according to the values given in Table I adopted from [8]. We simulate 432 instances with 10 replications, where the number of users set consists of 50, 100, 200, 300, 400, and 500, the number of MBSs set consists of 1, 2, 3, and 4, total bandwidth available at MBSs set consists of 1, 10, and 100 MHz, and the number of data rate levels ranges from 1 to 6. The data rate values for different number of levels are given in Table II. The minimum level of 0.1 Mbps is always offered to the users so that, at least, texting services can be utilized.

The users are randomly positioned to specific parts of the region on (\([0, x_{\text{max}}], [0, y_{\text{max}}]\)) cartesian plane with a random number of clustering points and clustering rate. For instance, if the number of clustering points is 5 and the clustering rate is 0.7 for a replication with 100 users, then 70 users are positioned in close proximities of uniformly selected 5 different points in the region while the remaining 30 users are located uniformly in the whole region. MBSs are located uniformly within sub-regions whose number is determined by the total number of MBSs in the instance. For example, if there exist 4 MBSs in an instance, the region is divided into 4 equal sub-regions and each MBS is located uniformly within those sub-regions. Willingness values of the users are calculated by using $\phi_{ik} = \phi_{i,k-1}(1 + (R_k - R_{k-1}) \times \text{Uniform}(0, 1))$, $k = 2, \ldots, 8 - 1$ and $\phi_{i1} = R_1 \times \text{Uniform}(0, 1)$. We coded our algorithm in Java SDK 1.7, ran all replications on an Intel i5-8250u CPU @1.80 GHz, 64-bit, and 8GB RAM computer, and used BARON v17.10.16 to solve the knapsack problems.

Fig. 3 illustrates two sample solutions of an instance with 300 users, 3 MBSs with 10 MHz available bandwidth, and 2 different data rate levels (0.1 and 1 Mbps) being offered to the users. Fig. 3a and 3b illustrate the provided data rate and the weighted average revenues of the users, respectively, where users are clustered around three clustering points with clustering rate of 0.96. Fig. 3c and 3d illustrate another replication of the same instance with a more uniformly distributed topology of the users where the clustering rate is 0.31 with four clustering points. The initial and final locations of the UAV-BS and the intermediate locations that improve the incumbent solution found out throughout the algorithm are shown by purple, green, and red diamonds in Fig. 3a and 3b, respectively. Users who are served at different data rate levels are demonstrated with different markers and the MBSs are shown by filled squares, while the MBSs with which the DBS has the backhaul link are shown by black squares. The weighted average revenue of the users are also illustrated in Figs. 3b and 3d where the profits are divided into four groups (e.g., Q4 band represents the highest 25% profit group).

Fig. 3 demonstrates that the algorithm starts from a higher altitude to search and moves the UAV-BS to better positions, iteratively, in terms of revenue. Furthermore, the UAV-BS is moved to a different point from where the SD algorithm starts for every altitude, although the backhaul capacity is maximized at the starting point of the algorithm. This behavior confirms that our algorithm changes the location of UAV-BS towards the users who have higher willingness values in order to increase the revenue. However, at the same time, it avoids moving too far away from the MBSs because of the backhaul capacity. Moreover, the UAV-BS is likely to be located at low altitudes when users are more clustered, while the altitude increases in more spread out topologies (e.g., 163 m vs. 201 m in Fig. 3). One reason for such behavior is to benefit from the LoS links when more people are clustered as much as possible to reduce the path-loss. Another reason is to provide services to the users who have higher willingness values but located far from the clustering points. By increasing its altitude, the UAV-BS can serve higher number of users, which directly increases the revenue. For instance, lowering the UAV-BS by 5 meters in the second replication would cause to lose three users who are located close to the edges of the network and served with 1 Mbps in return of serving five users who are closer to the UAV-BS. However, this would cause a 1.2% decrease in the revenue since the total willingness of the potential five users

### TABLE I: Simulation parameters for suburban environment.

| Parameter | Values |
|-----------|--------|
| $\eta, \alpha, \beta, \varphi_{\text{LoS}}, \varphi_{\text{NLoS}}, \sigma_{\omega}$ | 2.5, 4.88, 0.43, 0.1 dB, 21 dB, 24 dB |
| $f_c$ | 2 GHz |
| $x_{\text{max}}, y_{\text{max}}, h_{\text{max}}$ | 1500 m, 1500 m, 500 m |
| $G_{ij}, \forall j \in J$ | 46 dBm |
| $G_d$ | 30 dBm |
| $\epsilon_{\text{GSS}}$ | 10 m |
| $\epsilon_{\text{BS}}, \lambda_{\text{SD}}, \xi_{\text{max}}$ | $10^{-5}$ Hz, 100 m, 200 |

### TABLE II: Data rate sets for different service levels.

| Number of levels | Data rate values (Mbps) |
|------------------|------------------------|
| 1                | 0.1                    |
| 2                | 0.1, 1                 |
| 3                | 0.1, 1, 2              |
| 4                | 0.1, 1, 2, 4           |
| 5                | 0.1, 1, 2, 4, 6        |
| 6                | 0.1, 1, 2, 4, 6, 8     |
(a) The data rate provided to users with respect to the UAV-BS locations for Replication 1.

(b) The weighted average revenue gained from the users for Replication 1.

(c) The data rate provided to users with respect to the UAV-BS locations for Replication 2.

(d) The weighted average revenue gained from the users for Replication 2.

Fig. 3: Sample solutions showing data rate levels, revenue gained, and updates of the UAV-BS locations for different replications of the same instance. Qi’s denote the quarter rank (e.g., Q1 is the top one quarter and Q4 is the bottom one quarter).

is lower than the existing two users.

Fig. 4 presents the change in the spectral efficiency for different number of total users with respect to the clustering rate. Recall that the users are distributed to the region with respect to a random number of clustering points and a random clustering rate that defines the percentage of the users who are located in the close proximity of those points. Our algorithm achieves more efficient service provisioning when the users are in a more clustered formation. The reason for such behavior is due to the approach proposed in the SD Algorithm that determines the movements of the UAV-BS. Since the location of the UAV-BS is determined, primarily, according to the total impact on the users (i.e., the more the density in a specific area, the higher the possibility to move UAV-BS towards that area). The probability of having LoS with more users is higher when the UAV-BS is located right above a dense area which results in better path-loss links between the users and the UAV-BS.

Fig. 5 illustrates the change in the coverage and the normalized revenue with respect to different data rate options offered for different amounts of total bandwidth available. The normalized revenues for 1, 10, and 100 MHz are plotted by diamond, circled, and squared continuous lines, respectively, while the coverage values are shown by dashed lines with the same markers. What can be clearly seen in this figure is the gradual decrease in the coverage as the offered number data rates increases. Note that the coverage means the ratio of the number of users who are served with, at least, the minimum data rate to the total number of users in the instance. The highest decline in coverage, as the number of data rates increases from 1 to 6, is 59% in the instances with the available bandwidth of 10 MHz, whereas, the decline is 40%
and 30% for the instances with 1 and 100 MHz, respectively. On the other hand, the revenue sharply increases by 1074% in exchange for the decline in coverage for the instances with 100 MHz, whereas, the revenue increase for the instances with 10 and 1 MHz are 165% and 16%, respectively. Our algorithm improves the revenue as the offered data rates increase while reducing the fairness of the network in terms of the number of served users. Moreover, increasing the total bandwidth results in boosting both the coverage and the revenue.

Fig. 6 shows the change in the normalized revenue per Mbps and the spectral efficiency as functions of the number of offered data rate levels. Spectral efficiency increases as the number of offered data rates increases because UAV-BS moves towards the users with whom it can establish links with better spectral efficiency. In fact, the location of the UAV-BS is determined by evaluating the ratio between the revenue and the bandwidth required to achieve that revenue. Therefore, our algorithm exploits the advantage of having more efficient link between the users and the UAV-BS. Normalized revenue per Mbps for all bandwidth cases also increase as the number
of offered data rates increases up to a certain number of level. However, after those levels are exceeded, revenue begins to decrease (i.e., the increase is not sustainable) because in the instances with higher number data rate options, serving at higher data rate becomes too challenging to be realized. As the bandwidth is limited, allocating larger portion of the bandwidth to a small group of users who are to be served with high data rates sharply declines the normalized revenue per Mbps. For instance, in 100 and 10 MHz cases, approximately 65% and 60% of the total bandwidth are allocated to the users which are served with 8 Mbps although these users constitute only 8% and 4% of all the served users, respectively.

Fig. 7 illustrates how the variation of the total bandwidth affects the spectral efficiency, normalized revenue, and normalized coverage. As the total bandwidth increase, both revenue and coverage increase monotonically, yet, spectral efficiency decreases (e.g., spectral efficiency at 100 MHz is 67% less than the spectral efficiency at 1 MHz). The decrease in the spectral efficiency is due to the efficiency constraint in our model. In fact, allocating the bandwidth to higher number of users, thereby, creating more balanced allocations yield higher revenue, yet, it decreases spectral efficiency.

Fig. 8 presents the altitude of the UAV-BS and the normalized revenue with respect to the number of data rate levels offered. As the number of offered data rates increase, the UAV reduces its altitude to take advantage of the better channel conditions for a certain group of users because the spectral efficiency increases as the altitude decreases until a certain level as shown in Fig. 2. In other words, for a given total bandwidth value if only a low data rate can be provided to the users (e.g., 0.1 Mbps) then the coverage should be increased which is the only option to increase the revenue. However, if all data rate options are available then the best option to maximize the revenue is to serve a subset of the users with higher data rates. Indeed, as the number of data rate options increase from 1 to 6, the coverage decreases from 83% to 47%. Nevertheless, this result is closely related to the choice of the willingness function.

VI. CONCLUSION

UAV-BS location problems have attracted significant interest in both industry and academia due to their potential to bring unprecedented advantages like rapid deployment, dynamic coverage extension, and on-demand capacity increase. Most of the studies to date have focused on improving QoS while the QoE aspect is, mostly, overlooked. In this study, we address both the capacity aspect of QoS and price aspect of QoE. A novel mathematical model is constructed and an efficient two-phase solution procedure that combines GSS and SD is created for determining the location of a single UAV-BS to increase the revenue by jointly optimizing the location of the UAV-BS and the allocation of the bandwidth to the users. Proposed model considers the backhaul capacity and the price that users are willing to pay for different instantaneous data rate levels. The results of our analysis reveal that offering multiple data rate options to users improves revenue and spectral efficiency. The proposed procedure is shown to be capable of improving the QoE, significantly.

This study can be further extended by integrating the temporal dimension of the problem since users typically relocate in time and the willingness values of the users, possibly, change at different time intervals. Such dynamics can lead to changes in the location, the hovering time, and the trajectory of the UAV-BS. Another extension can be to combine energy-related performance metrics with the backhaul capacity. Covering all users with the minimum number of UAV-BSs by considering both QoS and QoE is also an interesting future research avenue.

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