Present Status of Polarized Parton Distributions

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Abstract

A review of the present knowledge on polarized parton distributions is given. The effects of perturbative evolution on these distributions are discussed qualitatively and a comparison of various recent parametrizations is made.

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1 Introduction

A series of precision measurements \[1\] of the polarized structure function \(g_1(x, Q^2)\) off proton and deuteron targets has considerably improved our knowledge on the spin structure of the nucleon over the last two years. In combination with several older measurements \[2\], these experiments now cover an \(x\)-range of \(0.003 \leq x \leq 0.8\), although the \(Q^2\) range for fixed \(x\) is still rather restricted.

In the ‘naive’ parton model \(g_1\) can, like the unpolarized structure function \(F_1\), be expressed in terms of the probability distributions for finding quarks with spin parallel or antiparallel to the longitudinally polarized parent proton:

\[
F_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 [q(x) + \bar{q}(x)]
\]

\[
g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 [\Delta q(x) + \Delta \bar{q}(x)] ,
\]

where

\[
q = q_\uparrow + q_\downarrow , \quad \Delta q = q_\uparrow - q_\downarrow .
\]

Furthermore, the naive parton model predicts exact scaling behaviour for the above distributions, i.e. independence of the \(Q^2\) scale of the measurement. These distributions are intrinsic, nonperturbative features of the nucleon, and can therefore at present only be determined from a fit to the structure function data. Some insight can however be gained from thermodynamical models \[3\] or from the light-cone wavefunctions of partons in the nucleon \[4\].

Perturbative QCD yields corrections \[5\] to the simple parton model picture, which are manifest in a scale-dependence of the parton distributions. The quantitative features of these corrections will be discussed in Section 2.

Various groups have used the recent data on \(g_1(x, Q^2)\) to determine the polarized parton distributions, taking into account the leading-order QCD corrections\[6\]. The concepts of the various approaches and their results are compared in section 3. Finally, section 4 contains a brief summary.

\[\uparrow\]The next-to-leading order QCD corrections to the scale dependence of polarized parton distributions have only been calculated very recently \[6\]. So far, only one group has used these to produce a set of NLO polarized distributions \[7\].
2  Evolution of polarized parton distributions

In the QCD corrected parton model $g_1$ is expressed in terms of parton densities for the polarization of quarks and gluons,

$$
g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 \int_x^1 \frac{dy}{y} \left[ \Delta q(x/y, Q^2) + \Delta \bar{q}(x/y, Q^2) \right]
$$

$$
\times \left\{ \delta(1-y) + \frac{\alpha_s(Q^2)}{2\pi} \Delta C_q(y) + \ldots \right\}
$$

$$
+ \frac{1}{9} \int_x^1 \frac{dy}{y} \Delta G(x/y, Q^2) \left\{ n_f \frac{\alpha_s(Q^2)}{2\pi} \Delta C_G(y) + \ldots \right\}.
$$

(4)

The emission of collinear partons gives rise to an evolution of the parton densities \cite{5},

$$
\frac{\partial}{\partial \ln Q^2} \left( \frac{\Delta q}{\Delta G} \right)(x, Q^2)
$$

$$
= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} \left( \frac{\Delta P_{qq}}{2n_f \Delta P_{gg}} \right)(y) \left( \frac{\Delta q}{\Delta G} \right)(x/y, Q^2).
$$

(5)

This equation only determines how the distributions change with $Q^2$, not the distributions themselves. The boundary conditions for the solution enter as initial distributions $\Delta q(x, Q^2_0)$ and $\Delta G(x, Q^2_0)$.

Even if the precise form of the polarized parton distributions is still not yet known, several qualitative features of the $Q^2$ dependence can be determined from the splitting functions (Fig. 1). The evolution of the polarized valence quark density is identical to the unpolarized one: the distribution decreases in the large $x$ region and increases for smaller $x$. The polarized singlet distribution is changed by two splitting processes: $q \to q$ lowers the distribution at large $x$ and increases at small $x$ while $g \to q$ slightly increases it at large $x$ and lowers it at small $x$. The overall change of $\Delta \Sigma(x, Q^2)$ is therefore sensitive to the relative magnitude of the quark and gluon polarizations. The evolution of $\Delta G(x, Q^2)$ is dominated by the $g \to g$ splitting, which strongly increases the distribution at small and medium $x$. Only if the gluon polarization is initially smaller than the total quark polarization, will effects from $q \to g$ contribute visibly to the increase of $\Delta G(x, Q^2)$. 


Figure 1: Qualitative description of the evolution of polarized valence quark (a), polarized singlet quark (b) and polarized gluon (c) densities. Solid line: $x \times f(x)$ at $Q_0^2$, dashed line: $x \times f(x)$ at $Q^2 > Q_0^2$. 
3 Parametrizations of polarized parton distributions

Although the change of the polarized parton distributions with increasing $Q^2$ is determined by the above evolution equations, the distributions themselves are incalculable in perturbative QCD. The starting distributions at some low scale $Q_0^2$ reflect the nonperturbative spin-structure of the nucleon and can only be fitted to the experimental data. Various groups have performed these fits using the leading-order evolution equations. In the following we will restrict our discussion to the most recent parametrizations from Glück-Reya-Vogelsang (GRV), the La Plata group (LP) and to our own results (GS [10, 11]). These parametrizations differ in various aspects:

(i) Data selection:

| expt.    | $x$     | $Q^2$[GeV$^2$] | GRV | LP | GS94 | GS95 |
|----------|---------|---------------|-----|----|------|------|
| E130-p   | 0.180 – 0.7 | 3.5 – 10      | ●   | ●  | ●    | ●    |
| EMC-p    | 0.010 – 0.7 | 1.5 – 70      | ●   | ●  | ●    | ●    |
| SMC-p    | 0.003 - 0.7 | 1.0 – 60      | ●   | ●  | ●    | ●    |
| E143-p   | 0.029 – 0.8 | 1.3 – 10      | ●   | ●  | ●    | ●    |
| SMC-d93  | 0.006 - 0.6 | 1.0 – 30      | ●   | ●  | ●    | ●    |
| SMC-d95  | 0.003 - 0.7 | 1.0 – 60      |     |    | ●    |      |
| E143-d   | 0.029 – 0.8 | 1.0 – 30      | ●   |    | ●    | ●    |
| E142-n   | 0.030 – 0.6 | 1.0 – 10      | ●   | ●  | ●    | ●    |

(ii) Construction of $g_1(x, Q^2)$: The above experiments have measured the cross section asymmetry

$$A_1(x, Q^2) = \frac{g_1(x, Q^2)}{F_1(x, Q^2)}$$  \hspace{1cm} (6)

While this review was in preparation, the first set of next-to-leading order polarized parton distributions has appeared [7].
Therefore, one has to use $F_1(x, Q^2)$ for the extraction of $g_1(x, Q^2)$. The various parametrizations use either the unpolarized parton distributions of Refs. [12] (GRV) and [13] (LP) or the SLAC/NMC parametrizations [14] (GS).

(iii) **Positivity of the distributions:** The most fundamental constraint on the polarized parton distribution is the positivity of the individual helicity distributions. This forces the polarized distributions to be less in magnitude than the unpolarized ones,

$$|\Delta f| \leq f(x) \quad \text{with } f = q, G. \quad (7)$$

This constraint is either incorporated by defining polarizations (GRV, LP)

$$\Delta f(x, Q^2_0) = \chi_f(x) f(x, Q^2_0), \quad (8)$$

or by constraining combinations of parameters in the initial distributions (GS).

(iv) **Constraints from the Ellis-Jaffe sum rule:** The Ellis-Jaffe sum rule [15]

$$\Gamma_{1}^{p,n}(Q^2) = \pm \frac{1}{12} a_3 + \frac{1}{36} a_8 + \frac{1}{9} \Delta \Sigma^{(1)}(Q^2) \quad (9)$$

relates the first moment of $g_1(x, Q^2)$ to the expectation values

$$a_3 = \Delta u^{(1)}(Q^2) - \Delta d^{(1)}(Q^2) = F + D = 1.257$$
$$a_8 = \Delta u^{(1)}(Q^2) + \Delta d^{(1)}(Q^2) - 2 \Delta s^{(1)}(Q^2) = 3F - D = 0.579 \quad (10)$$

of the conserved nonsinglet axial vector currents. The nonconserved singlet axial vector current

$$\Delta \Sigma^{(1)}(Q^2) = \Delta u^{(1)}(Q^2) + \Delta d^{(1)}(Q^2) + \Delta s^{(1)}(Q^2) \quad (11)$$

is then matched to the experimental data. Note that the Ellis-Jaffe sum rule is independent of $Q^2$ in a leading-order model. Therefore, only an average value of all experimental measurements can be reproduced. Furthermore, the above procedure is not unique, as the different parametrizations use different assumptions for the flavour decomposition of the sea quark polarization.
(v) **Constraints from asymptotic estimates:** The experimental data on $g_1(x, Q^2)$ are presently insufficient for a global determination of all polarized parton distributions. Therefore some properties – such as the behaviour at small and large $x$ – have to be estimated from theoretical considerations. The estimates used differ from parametrization to parametrization, the most common being: color coherence and Regge arguments at small $x$, and counting rules at large $x$ (see Ref. [4] for a recent review).

As the data on $g_1^p$ and $g_1^d$ have almost the same statistical quality, the non-singlet distributions $\Delta u_{val}(x, Q_{0}^{2})$ and $\Delta d_{val}(x, Q_{0}^{2})$ can be determined from these measurements. Figure 2 illustrates the agreement between the parametrizations. The differences arise mainly from the data included in the fit and from different assumptions on the magnitude of the sea quark polarization entering the Ellis-Jaffe sum rule.

![Figure 2: Parametrizations of the polarized valence quark distributions.](image)

The sea quark and gluon distributions are much more sensitive to the various constraints used to determine the distributions. As these differ for all parametrizations, the corresponding distributions span a wide range of possibilities, as can be seen in Figs. 3 and 4.
Figure 3: Parametrizations of the polarized sea ($q = u = \bar{u} = d = \bar{d}$) and polarized strange ($s = \bar{s}$) quark distributions.

Figure 4: Parametrizations of the polarized gluon distribution.

Without experimental data on processes other than the structure function $g_1$, it will be very difficult to distinguish between these different possibilities. Future experiments at RHIC, CERN and SLAC, together with final state measurements at a polarized HERA collider [16] and at HERMES, could provide this information on $\Delta q(x)$ and $\Delta G(x)$ from various different processes.

4 Conclusions

In this talk, we have given a brief review of the quantitative features of the $Q^2$ evolution of polarized parton densities. Comparing various recent
parametrizations of these densities, we have found that the present data on \( g_1^{p,n}(x, Q^2) \) determine the valence quark polarization in the nucleon to some accuracy. In contrast, the polarization of the quark sea and the gluon are strongly dependent on additional theoretical constraints imposed on the distributions. This situation is not expected to improve significantly with more precise data on \( g_1 \), which clearly shows the need for complementary measurements on polarized nucleons.

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