NEUTRINO MASSES, WHERE DO WE STAND? *

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ABSTRACT

I review the status of neutrino physics post-Neutrino 98, including the implications of solar and atmospheric neutrino data, which strongly indicate nonzero neutrino masses. LSND and the possible role of neutrinos as hot dark matter (HDM) are also mentioned. The simplest schemes proposed to reconcile these requirements invoke a light sterile neutrino in addition to the three active ones, two of them at the MSW scale and the other two maximally-mixed neutrinos at the HDM/LSND scale. In the simplest theory the latter scale arises at one-loop, while the solar and atmospheric parameters $\Delta m^2_{\odot}$ & $\Delta m^2_{\text{atm}}$ appear at the two-loop level. The lightness of the sterile neutrino, the nearly maximal atmospheric neutrino mixing, and the generation of $\Delta m^2_{\odot}$ & $\Delta m^2_{\text{atm}}$ follow naturally from the assumed lepton-number symmetry and its breaking. These two basic schemes can be distinguished at future solar & atmospheric neutrino experiments and have different cosmological implications.

1. Introduction

Neutrinos are the only fermions which the Standard Model (SM) predicts to be massless. This ansatz was justified due to the apparently masslessness of neutrinos in most experiments. However, the situation has changed due to the important impact of underground experiments, since the pioneer geochemical experiments of Davis and collaborators, to the more recent Gallex, Sage, Kamiokande and SuperKamiokande experiments. Altogether they provide solid evidence for the solar and the atmospheric neutrino problems, two milestones in the search for physics beyond the SM. Of particular importance has been the recent confirmation by the SuperKamiokande collaboration of the atmospheric neutrino zenith-angle-dependent deficit, which has marked a turning point in our understanding of neutrinos, providing a strong evidence for $\nu_\mu$ conversions. In addition to the neutrino data from underground experiments there is also some possible indication for neutrino oscillations from the LSND experiment. To this we may add the possible rôle of neutrinos in the dark matter problem and structure formation. If one boldly insists in including also the last two requirements, together with the data on solar and atmospheric neutrinos, then we have three mass scales involved in neutrino oscillations. The simplest way to reconcile these requirements invokes the existence of a light sterile neutrino. The pro-

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totype models proposed in\textsuperscript{10,11} enlarge the $SU(2) \otimes U(1)$ Higgs sector in such a way that neutrinos acquire mass radiatively, without unification nor seesaw. Out of the four neutrinos, two of them lie at the MSW scale and the other two maximally-mixed neutrinos are at the HDM/LSND scale. The latter scale arises at one-loop, while the solar and atmospheric scales come in at the two-loop level. The lightness of the sterile neutrino, the nearly maximal atmospheric neutrino mixing, and the generation of the solar and atmospheric neutrino scales all result naturally from the assumed lepton-number symmetry and its breaking. Either $\nu_e - \nu_\tau$ conversions explain the solar data with $\nu_\mu - \nu_\tau$ oscillations accounting for the atmospheric deficit\textsuperscript{10}, or else the roles of $\nu_\tau$ and $\nu_s$ are reversed\textsuperscript{11}. These two basic schemes have distinct implications at future solar & atmospheric neutrino experiments, as well as cosmology.

2. Theories of Neutrino Mass

One of the most unpleasant features of the SM is that the masslessness of neutrinos is not dictated by an underlying \textit{principle}, such as that of gauge invariance in the case of the photon: the SM simply postulates that neutrinos are massless by choosing a restricted multiplet content. \textit{Why are neutrinos so special when compared with the other fundamental fermions?} If massive, neutrinos would present another puzzle: \textit{Why are their masses so small compared to those of the charged fermions?} The fact that neutrinos are the only electrically neutral elementary fermions may hold the key to the answer, namely neutrinos could be Majorana fermions, the most fundamental kind of fermion. In this case the suppression of their mass could be associated to the breaking of lepton number symmetry at a very large energy scale within a \textit{unification approach}, which can be implemented in many extensions of the SM. Alternatively, neutrino masses could arise from garden-variety \textit{weak-scale physics} characterized by a scale $\langle \sigma \rangle = \mathcal{O}(m_Z)$ where $\langle \sigma \rangle$ denotes a $SU(2) \otimes U(1)$ singlet vacuum expectation value which owes its smallness to the symmetry enhancement which would result if $\langle \sigma \rangle$ and $m_\nu \to 0$.

One should realize however that, although the physics of neutrinos can be rather different in various gauge theories of neutrino mass, there is hardly any predictive power on masses and mixings, this is one of the aspects of the so-called flavour problem which is probably the toughest open problem in physics.

2.1. Unification Approach

An attractive possibility is to ascribe the origin of parity violation in the weak interaction to the spontaneous breaking of B-L symmetry in the context of left-right symmetric extensions such as the $SU(2)_L \otimes SU(2)_R \otimes U(1)$\textsuperscript{13}, $SU(4) \otimes SU(2) \otimes SU(2)$\textsuperscript{4} or $SO(10)$ gauge groups\textsuperscript{15}. In this case the masses of the light neutrinos are obtained
by diagonalizing the following mass matrix in the basis $\nu, \nu^c$

$$
\begin{bmatrix}
M_L & D \\
D^T & M_R
\end{bmatrix}
$$

(1)

where $D$ is the standard $SU(2) \otimes U(1)$ breaking Dirac mass term and $M_R = M_R^T$ is the isosinglet Majorana mass that may arise from a 126 vacuum expectation value (vev) in $SO(10)$. The magnitude of the $M_L \nu \nu$ term is also suppressed by the left-right breaking scale, $M_L \propto 1/M_R$.

In the seesaw approximation, one finds

$$M_{\nu \text{eff}} = M_L - D M_R^{-1} D^T.$$  

(2)

As a result one is able to explain naturally the relative smallness of neutrino masses since $m_\nu \propto 1/M_R$. Although $M_R$ is expected to be large, its magnitude heavily depends on the model and it may have different possible structures in flavour space (so-called textures). As a result it is hard to make firm predictions for the corresponding light neutrino masses and mixings that are generated through the seesaw mechanism. In fact this freedom has been exploited in model building in order to account for an almost degenerate seesaw-induced neutrino mass spectrum.

One virtue of the unification approach is that it may allow one to gain a deeper insight into the flavour problem. There have been interesting attempts at formulating supersymmetric unified schemes with flavour symmetries and texture zeros in the Yukawa couplings. In this context a challenge is to obtain the large lepton mixing now indicated by the atmospheric neutrino data.

2.2. Weak-Scale Approach

Although very attractive, the unification approach is by no means the only way to generate neutrino masses. There are many schemes which do not require any large mass scale. The extra particles employed to generate the neutrino masses have masses $\mathcal{O}(m_Z)$ accessible to present experiments. There is a variety of such mechanisms, in which neutrinos acquire mass either at the tree level or radiatively. Let us look at some.

2.2.1. Tree Level

For example, it is possible to extend the lepton sector of the $SU(2) \otimes U(1)$ theory by adding a set of two 2-component isosinglet neutral fermions, denoted $\nu^c_i$ and $S_i$, $i = e, \mu$ or $\tau$ in each generation. In this case one can consider the mass matrix (in the basis $\nu, \nu^c, S$)

$$
\begin{bmatrix}
0 & D & 0 \\
D^T & 0 & M \\
0 & M^T & \mu
\end{bmatrix}
$$

(3)
The Majorana masses for the neutrinos are determined from

\[ M_L = D M^{-1} \mu M^T \mu^{-1} D^T \]  

(4)

In the limit \( \mu \to 0 \) the exact lepton number symmetry is recovered and will keep neutrinos strictly massless to all orders in perturbation theory, as in the SM. The corresponding texture of the mass matrix has been suggested in various theoretical models \[20\], such as superstring inspired models \[21\]. In the latter the zeros arise due to the lack of Higgs fields to provide the usual Majorana mass terms. The smallness of neutrino mass then follows from the smallness of \( \mu \). The scale characterizing \( M \), unlike \( M_R \) in the seesaw scheme, can be low. As a result, in contrast to the heavy neutral leptons of the seesaw scheme, those of the present model can be light enough as to be produced at high energy colliders such as LEP \[22\] or at a future Linear Collider. The smallness of \( \mu \) is in turn natural, in t’Hooft’s sense, as the symmetry increases when \( \mu \to 0 \), i.e. total lepton number is restored. This scheme is a good alternative to the smallness of neutrino mass, as it bypasses the need for a large mass scale, present in the seesaw unification approach. One can show that, since the matrices \( D \) and \( M \) are not simultaneously diagonal, the leptonic charged current exhibits a non-trivial structure that cannot be rotated away, even if we set \( \mu \equiv 0 \). The phenomenological implication of this, otherwise innocuous twist on the SM, is that there is neutrino mixing despite the fact that light neutrinos are strictly massless. It follows that flavour and CP are violated in the leptonic currents, despite the masslessness of neutrinos. The loop-induced lepton flavour and CP non-conservation effects, such as \( \mu \to e + \gamma \) \[23\], or CP asymmetries in lepton-flavour-violating processes such as \( Z \to e\bar{\tau} \) or \( Z \to \tau\bar{e} \) \[24\] are precisely calculable. The resulting rates may be of experimental interest \[25\], since they are not constrained by the bounds on neutrino mass, only by those on universality, which are relatively poor. In short, this is a conceptually simple and phenomenologically rich scheme.

Another remarkable implication of this model is a new type of resonant neutrino conversion mechanism \[29\], which was the first resonant mechanism to be proposed after the MSW effect \[30\], in an unsuccessful attempt to bypass the need for neutrino mass in the resolution of the solar neutrino problem. According to the mechanism, massless neutrinos and anti-neutrinos may undergo resonant flavour conversion, under certain conditions. Though these do not occur in the Sun, they can be realized in the chemical environment of supernovae \[31\]. Recently it has been pointed out how they may provide an elegant approach for explaining the observed velocity of pulsars \[32\].

2.2.2. Radiative Level

There is also a large variety of radiative models, where the \( SU(2) \otimes U(1) \) multiplet content is extended in order to generate neutrino masses. The prototype one-loop scheme is the one proposed by Zee \[33\]. Supersymmetry with explicitly broken R-parity
also provides an alternative one-loop mechanism to generate neutrino mass. These arise, for example, from scalar quark or scalar lepton contributions, as shown in Fig. 1.

A two-loop scheme to induce neutrino mass was suggested by Babu. The relevant diagram is shown in Fig. 2.

In the above examples active neutrinos acquire radiative mass. One can also employ the radiative approach to construct models including sterile neutrinos, such as those in ref. [10,11]. In this case some new Feynman diagram topologies are encountered.

2.3. A Hybrid Approach

I now describe an interesting mechanism of neutrino mass generation that combines seesaw and radiative mechanisms. It invokes supersymmetry with broken R-parity, as the origin of neutrino mass and mixings [36]. The simplest model is a unified minimal supergravity model with universal soft breaking parameters (MSUGRA) and bilinear breaking of R-parity [36,37]. Contrary to a popular misconception, the bilinear violation of R-parity implied by the $\epsilon_3$ term in the superpotential is physical, and can not be rotated away [38]. It leads also by a minimization condition, to a non-zero

\[ a \]

Note here that I have used the slight variant of the Babu model suggested in ref. [35,36], which incorporates the idea of spontaneous, rather than explicit lepton number violation.
sneutrino vev, \( v_3 \). It is well-known that in such models of broken R–parity the tau neutrino \( \nu_\tau \) acquires a mass, due to the mixing between neutrinos and neutralinos. It comes from the matrix

\[
\begin{pmatrix}
M_1 & 0 & -\frac{1}{2} g' v_d & \frac{1}{2} g' v_u & -\frac{1}{2} g' v_3 \\
0 & M_2 & \frac{1}{2} g v_d & -\frac{1}{2} g v_u & \frac{1}{2} g v_3 \\
-\frac{1}{2} g' v_d & \frac{1}{2} g v_d & 0 & -\mu & 0 \\
\frac{1}{2} g' v_u & -\frac{1}{2} g v_u & -\mu & 0 & \epsilon_3 \\
-\frac{1}{2} g' v_3 & \frac{1}{2} g v_3 & 0 & \epsilon_3 & 0 \\
\end{pmatrix}
\] (5)

where the first two rows are gauginos, the next two Higgsinos, and the last one denotes the tau neutrino. The \( v_u \) and \( v_d \) are the standard vevs, \( g' \)s are gauge couplings and \( M_{1,2} \) are the gaugino mass parameters. Since the \( \epsilon_3 \) and the \( v_3 \) are related, the simplest (one-generation) version of this model contains only one extra free parameter in addition to those of the MSUGRA model. The universal soft supersymmetry-breaking parameters at the unification scale \( m_X \) are evolved via renormalization group equations down to the weak scale \( O(m_Z) \). This induces an effective non-universality of the soft terms at the weak scale which in turn implies a non-zero sneutrino vev \( v'_3 \) given as

\[
v'_3 \approx \frac{\epsilon_3 \mu}{m_Z} \left( v_d \Delta M^2 + \mu' v_u \Delta B \right)
\] (6)

where the primed quantities refer to a basis in which we eliminate the \( \epsilon_3 \) term from the superpotential (but reintroduce it, of course, in other sectors of the theory).

The scalar soft masses and bilinear mass parameters obey \( \Delta M^2 = 0 \) and \( \Delta B = 0 \) at \( m_X \). However at the weak scale they are calculable from radiative corrections as

\[
\Delta M^2 \approx \frac{3 h_b^2}{8 \pi^2} m_Z^2 \ln \frac{M_{\text{GUT}}}{m_Z}
\] (7)

Note that eq. (3) implies that the R–parity-violating effects induced by \( v'_3 \) are calculable in terms of the primordial R–parity-violating parameter \( \epsilon_3 \). It is clear that the universality of the soft terms plays a crucial rôle in the calculability of the \( v'_3 \) and hence of the resulting neutrino mass \( 36 \). Thus eq. (5) represents a new kind of see-saw scheme in which the \( M_R \) of eq. (1) is the neutralino mass, while the rôle of the Dirac entry \( D \) is played by the \( v'_3 \), which is induced radiatively as the parameters evolve from \( m_X \) to the weak scale. Thus we have a hybrid see-saw mechanism, with naturally suppressed Majorana \( \nu_\tau \) mass induced by the mixing between the weak eigenstate tau neutrino and the zino.

Let me now turn to estimate the expected \( \nu_\tau \) mass. For this purpose let me first determine the tau neutrino mass in the most general supersymmetric model with bilinear breaking of R–parity, without imposing soft universality. The \( \nu_\tau \) mass depends quadratically on an effective parameter \( \xi \) defined as \( \xi \equiv (\epsilon_3 v_d + \mu v_3)^2 \propto v_3^2 \) characterizing the violation of R–parity. The expected \( m_{\nu_\tau} \) values are illustrated in Fig. (3). The band shown in the figure is obtained through a scan over the
parameter space requiring that the supersymmetric particles are not too light. Let us now compare this with the cosmologically allowed values of the tau neutrino mass. The cosmological critical density bound $m_{\nu \tau} \lesssim 92 \Omega h^2$ eV only holds if neutrinos are stable. In the present model (with 3-generations) the $\nu_\tau$ can decay into 3 neutrinos, via the neutral current\cite{16,40}, or by slepton exchanges. This decay will reduce the relic $\nu_\tau$ abundance to the required level, as long as $\nu_\tau$ is heavier than about 100 KeV or so. On the other hand primordial Big-Bang nucleosynthesis implies that $\nu_\tau$ is lighter than about an MeV or so\cite{41}.

However, if one adopts a SUGRA scheme where universality of the soft supersymmetry breaking terms at $m_X$ is assumed, then the $\nu_\tau$ mass is theoretically predicted in terms of $h_b$ and can be small in this case due to a natural cancellation between the two terms in the parameter $\xi$, which follows from the assumed universality of the softs at $m_X$. One can verify that $m_{\nu_\tau}$ may easily lie in the electron-volt range, in which case $\nu_\tau$ could be a component of the hot dark matter of the Universe.

Notice that $\nu_e$ and $\nu_\mu$ remain massless in this approximation. They get masses either from scalar loop contributions in Fig. (1) or by mixing with singlets in models with spontaneous breaking of R-parity\cite{42}. It is important to notice that even when $m_{\nu_\tau}$ is small, many of the corresponding R-parity violating effects can be sizeable. An obvious example is the fact that the lightest neutralino decay will typically decay inside the detector, unlike standard R-parity-conserving supersymmetry. This leads to a vastly unexplored plethora of phenomenological possibilities in supersymmetric physics\cite{43}.

In conclusion I can say that, other than the seesaw scheme, none of the above models requires a large mass scale. As a result they lead to a potentially rich phenomenology, since the extra particles required have masses at scales that could be accessible to present experiments. In the simplest versions of these models the neutrino
mass arises from the explicit violation of lepton number. Their phenomenological potential gets richer if one generalizes the models so as to implement a spontaneous violation scheme. This brings me to the next section.

2.4. Weak-scale majoron

If lepton number (or B-L) is an ungauged symmetry and if it is arranged to break spontaneously, the generation of neutrino masses will be accompanied by the existence of a physical Goldstone boson that we generically call majoron. Except for the left-right symmetric unification approach, in which B-L is a gauge symmetry, in all of the above schemes one can implement the spontaneous violation of lepton number. One can also introduce it in an SU(2) \( \otimes \) U(1) seesaw framework\(^4\), as originally proposed, but I do not consider this case here, see ref.\(^45\) for a review. Here I will mainly concentrate on weak-scale physics. In all models I consider the lepton-number breaks at a scale given by a vacuum expectation value \( \langle \sigma \rangle \sim m_{\text{weak}} \). Such scale arises as the most natural one since in all of these models, as already mentioned, we have that the neutrino masses vanish as the lepton-breaking scale \( \langle \sigma \rangle \rightarrow 0 \)\(^46\).

It is also clear that in any acceptable model one must arrange for the majoron to be mainly an SU(2) \( \otimes \) U(1) singlet, ensuring that it does not affect the invisible Z decay width, well-measured at LEP. In models where the majoron has L=2 the neutrino mass is proportional to an insertion of \( \langle \sigma \rangle \), as indicated in Fig. (2). In the supersymmetric model with broken R-parity the majoron is mainly a singlet sneutrino, which has lepton number L=1, so that \( m_\nu \propto \langle \sigma \rangle^2 \), where \( \langle \sigma \rangle \equiv \langle \bar{\nu}^c \rangle \), with \( \bar{\nu}^c \) denoting the singlet sneutrino. The presence of the square, just as in the parameter \( \xi \) in Fig. (3), reflects the fact that the neutrino gets a Majorana mass which has lepton number L=2. The sneutrino gets a vev at the effective supersymmetry breaking scale \( m_{\text{susy}} = m_{\text{weak}} \).

The weak-scale majorons may have remarkable phenomenological implications, such as the possibility of invisibly decaying Higgs bosons\(^4\). Unfortunately I have no time to discuss it here (see, for instance\(^4\)).

If the majoron acquires a KeV mass (natural in weak-scale models) from gravitational effects at the Planck scale\(^47\) it may play a rôle in cosmology as dark matter\(^48\). In what follows I will just focus on two examples of how the underlying physics of weak-scale majoron models can affect neutrino cosmology in an important way.

2.4.1. Heavy neutrinos and the Universe Mass

Neutrinos of mass less than \( \mathcal{O} \) (100 KeV) or so, are cosmologically stable if they have only SM interactions. Their contribution to the present density of the universe implies\(^4\)

\[
\sum m_\nu \lesssim 92 \Omega_\nu h^2 \text{ eV} ,
\]  

(8)
where the sum is over all isodoublet neutrino species with mass less than $O(1\, \text{MeV})$. The parameter $\Omega_\nu h^2 \leq 1$, where $h^2$ measures the uncertainty in the present value of the Hubble parameter, $0.4 \lesssim h \lesssim 1$, while $\Omega_\nu = \rho_\nu/\rho_c$, measures the fraction of the critical density $\rho_c$ in neutrinos. For the $\nu_\mu$ and $\nu_\tau$ this bound is much more stringent than the laboratory limits.

In weak-scale majoron models the generation of neutrino mass is accompanied by the existence of a physical majoron, with potentially fast majoron-emitting decay channels such as

$$\nu' \to \nu + J \tag{9}$$

as well as new annihilations to majorons,

$$\nu' + \nu' \to J + J \tag{10}$$

These could eliminate relic neutrinos and therefore allow neutrinos of higher mass, as long as the rates are large enough to allow for an adequate red-shift of the heavy neutrino decay and/or annihilation products. While the annihilation involves a diagonal majoron-neutrino coupling $g$, the decays proceed only via the non-diagonal part of the coupling, in the physical mass basis. A careful diagonalization of both mass matrix and coupling matrix is essential in order to avoid wild over-estimates of the heavy neutrino decay rates, such as that in ref.\cite{44}. The point is that, once the neutrino mass matrix is diagonalized, there is a danger of simultaneously diagonalizing the majoron couplings to neutrinos. That would be analogous to the GIM mechanism present in the SM for the couplings of the Higgs to fermions. Models that avoid this GIM mechanism in the majoron-neutrino couplings have been proposed, e.g. in ref.\cite{50}. Many of them are weak-scale majoron models\cite{34,46,42}. A general method to determine the majoron couplings to neutrinos and hence the neutrino decay rates in any majoron model was first given in ref.\cite{40}. For an estimate in the model with spontaneously broken R-parity\cite{44} see ref.\cite{44}.

In short one may say that neutrino lifetimes can be shorter than required by the cosmological mass bound, for all values of the masses which are presently allowed by laboratory experiments.

2.4.2. Heavy neutrinos and Cosmological Nucleosynthesis

Similarly, the number of light neutrino species is also restricted by cosmological Big Bang Nucleosynthesis (BBN). Due to its large mass, an MeV stable (lifetime longer than $\sim 100$ sec) tau neutrino would be equivalent to several SM massless neutrino species and would therefore substantially increase the abundance of primordially produced elements, such as $^4\text{He}$ and deuterium\cite{34,46,44}. This can be converted into restrictions on the $\nu_\tau$ mass. If the bound on the effective number of massless neutrino species is taken as $N_\nu < 3.4 - 3.6$, one can rule out $\nu_\tau$ masses above 0.5 MeV\cite{44}. If we
Figure 4: The dashed line shows the effective number of massless SM neutrinos equivalent to the heavy $\nu_\tau$ ($g = 0$). Depending on the value of $g$ (in units of $10^{-5}$) one can lower $N_\nu$ below the canonical SM value $N_\nu = 3$ due to the effect of $\nu_\tau$ annihilations. From ref. 55

take $N_\nu < 4.5$ the $m_{\nu_\tau}$ limit loosens accordingly, as seen from Fig. (4), and allows a $\nu_\tau$ of about an MeV or so.

In the presence of $\nu_\tau$ annihilations the BBN $m_{\nu_\tau}$ bound is substantially weakened or eliminated. In Fig. (4) we also give the expected $N_\nu$ value for different values of the coupling $g$ between $\nu_\tau$'s and $J$'s, expressed in units of $10^{-5}$. Comparing with the SM $g = 0$ case one sees that for a fixed $N_\nu^{\text{max}}$, a wide range of tau neutrino masses is allowed for large enough values of $g$. No $\nu_\tau$ masses below the LEP limit can be ruled out, as long as $g$ exceeds a few times $10^{-4}$. One can also see from the figure that $N_\nu$ can also be lowered below the canonical SM value $N_\nu = 3$ due to the effect of the heavy $\nu_\tau$ annihilations to majorons. These results may be re-expressed in the $m_{\nu_\tau} - g$ plane, as shown in figure 4. We note that the required values of $g(m_{\nu_\tau})$ fit well with the theoretical expectations of many weak-scale majoron models.

The above discussion has been on the effect of $\nu_\tau$ annihilations to majorons in BBN. In some weak-scale majoron models decays in eq. (9) may lead to short enough $\nu_\tau$ lifetimes that they may also play an important rôle in BBN.

Before concluding the discussion on majorons, let me comment that the majoron may be realized even in the context of models where B-L is a gauge symmetry, such as left-right-symmetric models, by suitably implementing a spontaneously broken global $U(1)$ symmetry similar to lepton number. It plays an interesting rôle in such models as it allows the left-right scale to be relatively low.
3. Indications for Neutrino Mass

The most solid indications in favour of nonzero neutrino masses come from underground experiments on solar and atmospheric neutrinos. I will provide a theorist’s sketch of the present experimental situation.

3.1. Solar Neutrinos

The puzzle posed by the data collected by the Homestake, Kamiokande, and the radiochemical Gallex and Sage experiments still defy an explanation in terms of the Standard Model. The most recent data on rates are summarized as: $2.56 \pm 0.23$ SNU (chlorine), $72.2 \pm 5.6$ SNU (Gallex and Sage gallium experiments sensitive to the $pp$ neutrinos), and $(2.44 \pm 0.10) \times 10^6 \text{cm}^{-2}\text{s}^{-1}$ ($^8\text{B}$ flux from SuperKamiokande)\(^\[6\). This has been re-confirmed by the 504 days data sample now collected by the SuperKamiokande (SK) collaboration and reported at Neutrino 98\(^\[7\). In Fig. (6) one can see the predictions of various standard solar models in the plane defined by the $^7\text{Be}$ and $^8\text{B}$ neutrino fluxes, normalized to the predictions of the BP98 solar model\(^\[5\). Abbreviations such as BP95, identify different solar models, as given in ref.\(^\[6\). The rectangular error box gives the $3\sigma$ error range of the BP98 fluxes. The values of these fluxes indicated by present data on neutrino event rates are also shown by the contours in the figure. The best-fit $^7\text{Be}$ neutrino flux is negative! Possible non-standard astrophysical solutions are strongly constrained by helioseismology studies\(^\[8\],\(^\[9\). Within the standard solar model approach, the theoretical predictions clearly lie far from the best-fit solution, and even far from the $3\sigma$ contour, leading us to conclude that new
Figure 6: Recent SSM predictions, from ref. 58

Particle physics is the only way to account for the data. The most likely possibility is to assume the existence of neutrino conversions involving very small neutrino masses. The most attractive theoretical schemes are the MSW effect, vacuum neutrino oscillations or just-so solution and, possibly, the Spin-Flavour Precession mechanism proposed in ref. 63, aided by the Resonant enhancement due to matter effects in the Sun found in ref. 64. The resulting RSFP mechanism still provides a viable solution to the solar neutrino problem.

The recent SK data updates the 300 days situation we had before Neutrino 98 without major surprises, except that the SK collaboration has now given the first detailed report of the recoil energy spectrum produced by solar neutrino interactions. The measured spectrum they reported at Neutrino 98 shows more events at the highest bins than would have been expected from the most popular neutrino oscillation parameters discussed previously. At first sight this might seem bad news for the oscillation scenarios. However, Bahcall and Krastev have noted that if the low energy cross section for $^3\text{He} + p \rightarrow ^4\text{He} + e^+ + \nu_e$, the so-called hep reaction, is $\gtrsim 20$ times larger than the best (but uncertain) theoretical estimates, then this reaction could significantly influence the electron energy spectrum produced by solar neutrino interactions in the high recoil region. This would hardly have any effect at lower energies. They compare the predicted energy spectra for different assumed hep fluxes and different neutrino oscillation scenarios with the one measured at SuperKamiokande. Fig. 7 shows the ratio of the measured to the calculated number of events with electron recoil energy $E$. The crosses are the recent SK measurements, while the calculated curves are global fits to all of the data. The horizontal line at Ratio = 0.37 represents the ratio of the total event rate measured by SuperKamiokande to the predicted event rate with no oscillations and only $^8\text{B}$ neutrinos. One sees how the
spectra with enhanced hep contributions provide better fits to the SK data, suggesting that these neutrinos may be playing a rôle.

One can determine the required solar neutrino parameters $\Delta m^2$ and $\sin^2 2\theta$ through a $\chi^2$ fit of the experimental data. In Fig. (8) we show the allowed two-flavour regions obtained in an updated MSW global fit analysis of the solar neutrino data for the case of active neutrino conversions. The data include the chlorine, Gallex, Sage and SK total event rates, the SK energy spectrum, as well as the SK day-night asymmetry, which would be expected in the MSW scheme due to regeneration effects at the Earth. The data also includes the recent SK 504 days sample. The analysis uses the BP98 model but with an arbitrary hep neutrino flux. One notices from the analysis that rate-independent observables, such as the electron recoil energy spectrum and the day-night asymmetry (zenith angle distribution), play an important rôle in ruling out large regions of MSW parameters.

A theoretical issue which has raised some interest recently is the study of the possible effect of random fluctuations in the solar matter density. The possible existence of noise fluctuations at a few percent level is not excluded by present helioseismology studies. In Fig. (9) we show averaged solar neutrino survival probability as a function of $E/\Delta m^2$, for $\sin^2 2\theta = 0.01$. This figure was obtained via a numerical integration of the MSW evolution equation in the presence of noise, using the density profile in the Sun from BP95 in ref., and assuming that the correlation length $L_0$ (which corresponds to the scale of the fluctuation) is $L_0 = 0.1\lambda_m$, where $\lambda_m$ is the neutrino oscillation length in matter. An important assumption in the analysis is that $l_{\text{free}} \ll L_0 \ll \lambda_m$, where $l_{\text{free}} \sim 10 \text{ cm}$ is the mean free path of the electrons in the solar medium. The fluctuations may strongly affect the $^7\text{Be}$ neutrino component of the solar neutrino spectrum so that the Borexino experiment should provide an ideal
Figure 8: Presently allowed MSW solar neutrino parameters for 2-flavour active neutrino conversions with an enhanced hep flux, from ref. 66.

Figure 9: Solar neutrino survival probability in the presence of random density fluctuations, ref. 69.
test, if sufficiently small errors can be achieved. The potential of Borexino in probing the level of solar matter density fluctuations provides an additional motivation for the experiment. This is discussed in more detail in ref. 69.

The most popular alternative solution to the solar neutrino problem is the vacuum oscillation solution which clearly requires large neutrino mixing and just-so adjustment of the oscillation length so as to coincide roughly with the Earth-Sun distance. This solution fits with simplistic see-saw inspired-numerology and has attractive features, as recently advocated in ref. 72. Fig. 10 shows the regions of just-so oscillation parameters obtained in a recent global fit of the data including the 504 days SK data sample, both the rates and the recoil energy spectrum. Seasonal effects are expected in this scenario and could potentially be used to further constrain the parameters, as described in ref. 73, and also to help discriminating it from the MSW scenario.

3.2. Atmospheric Neutrinos

Showers initiated when primary cosmic rays hit the Earth’s atmosphere originate secondary mesons, mostly pions and kaons, which decay producing $\nu_e$ ’s, $\nu_\mu$ ’s as well as $\bar{\nu}_e$ ’s and $\bar{\nu}_\mu$ ’s. There has been a long-standing discrepancy between the predicted and measured $\nu_\mu / \nu_e$ ratio of the atmospheric neutrino fluxes. The anomaly was found both in water Cerenkov experiments, such as Kamiokande, SuperKamiokande and IMB, as well as in the iron calorimeter Soudan2 experiment. Negative experiments, such as Frejus and Nusex have much larger errors.

Although individual $\nu_\mu$ or $\nu_e$ fluxes are only known to within 30% accuracy, the $\nu_\mu / \nu_e$ ratio is known to 5%. The most important feature of the atmospheric neutrino 535-day data sample reported by the SK collaboration at Neutrino 98 is that it exhibits a zenith-angle-dependent deficit of muon neutrinos which is inconsistent
with expectations based on calculations of the atmospheric neutrino fluxes. For recent analyses see ref. [75, 76]. Experimental biases and uncertainties in the prediction of neutrino fluxes and cross sections are unable to explain the data.

In Fig. (11) I show the measured zenith angle distribution of electron-like and muon-like sub-GeV and multi-GeV events, as well as the one predicted in the absence of oscillation. I also give the expected distribution in various neutrino oscillation schemes. The thick-solid histogram is the theoretically expected distribution in the absence of oscillation, while the predictions for the best-fit points of the various oscillation channels is indicated as follows: for $\nu_\mu \to \nu_\tau$ (solid line), $\nu_\mu \to \nu_e$ (dashed line) and $\nu_\mu \to \nu_s$ (dotted line). The error displayed in the experimental points is only statistical.

In the theoretical analysis we have used the latest improved calculations of the atmospheric neutrino fluxes as a function of zenith angle, including the muon polarization effect and took into account a variable neutrino production point [78]. Clearly the data are not reproduced by the no-oscillation hypothesis, adding substantially to our confidence that the atmospheric neutrino anomaly is real.

In Fig. (12) I show the allowed neutrino oscillation parameters obtained in a recent global fit of the sub-GeV and multi-GeV (vertex-contained) atmospheric neutrino data [75, 76] including the recent data reported at Neutrino 98, as well as all other
Fig. 12. Allowed atmospheric oscillation parameters for all experiments including the SK data reported at Neutrino 98, combined at 90% (thick solid line) and 99% CL (thin solid line) for all possible oscillation channels, from ref. 75, 76. In each case the best-fit point is denoted by a star and always corresponds to maximal mixing, a feature which is well-reproduced by the theoretical predictions of the models proposed in ref. 10, 11. The sensitivity of the present accelerator and reactor experiments as well as the expectations of upcoming long-baseline experiments is also displayed.

experiments combined at 90% (thick solid line) and 99% CL (thin solid line) for each oscillation channel considered. The two lower panels (Fig. 12) differ in the sign of the $\Delta m^2$ which was assumed in the analysis of the matter effects in the Earth for the $\nu_\mu \rightarrow \nu_s$ oscillations. We found that $\nu_\mu \rightarrow \nu_\tau$ oscillations give a slightly better fit than $\nu_\mu \rightarrow \nu_s$ oscillations. At present the atmospheric neutrino data cannot distinguish between the $\nu_\mu$ to $\nu_\tau$ and $\nu_\mu$ to $\nu_s$ channels. It is well-known that the neutral-to-charged current ratios are important observables in neutrino oscillation phenomenology, which are especially sensitive to the existence of singlet neutrinos, light or heavy 16. The atmospheric neutrinos produce isolated neutral pions ($\pi^0$-events) mainly in neutral current interactions. One may therefore study the ratios of $\pi^0$-events and the events induced mainly by the charged currents, as recently advocated in ref. 79. This has the virtue of minimizing uncertainties related to the
original atmospheric neutrino fluxes. In fact the SK collaboration has already tried to do this by estimating the double ratio of $\pi^0$ over e-like events in their sample and found $R = 0.93 \pm 0.07 \pm 0.19$. This is consistent both with $\nu_\mu$ to $\nu_\tau$ or $\nu_\mu$ to $\nu_s$ channels, with a slight preference for the former. The situation should improve in the future.

We also display in Fig. (12) the sensitivity of present accelerator and reactor experiments, as well as that expected at future long-baseline (LBL) experiments in each channel. The first point to note is that the Chooz reactor data already excludes the region indicated for the $\nu_\mu \to \nu_e$ channel when all experiments are combined at 90% CL.

From the upper-left panel in Fig. (12) one sees that the regions of $\nu_\mu \to \nu_\tau$ oscillation parameters obtained from the atmospheric neutrino data analysis cannot be fully tested by the LBL experiments, as presently designed. One might expect that, due to the upward shift of the $\Delta m^2$ indicated by the fit for the sterile case (due to the effects of matter in the Earth) it would be possible to completely cover the corresponding region of oscillation parameters. Although this is the case for the MINOS disappearance test, in general most of the LBL experiments can not completely probe the region of oscillation parameters allowed by the $\nu_\mu \to \nu_s$ atmospheric neutrino analysis. This is so irrespective of the sign of $\Delta m^2$ assumed. For a discussion of the various potential tests that can be performed at the future LBL experiments in order to unravel the presence of oscillations into sterile channels see ref. [76].

3.3. LSND, Dark Matter & Pulsars

**LSND**

A search for $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations has been conducted at the Los Alamos Meson Physics Facility by using $\bar{\nu}_\mu$ from $\mu^+$ decay at rest. The $\bar{\nu}_e$'s are detected via the reaction $\bar{\nu}_e p \to e^+ n$, correlated with a $\gamma$ from $np \to d\gamma$ (2.2 MeV). The use of tight cuts to identify $e^+$ events with correlated $\gamma$ rays yields 22 events with $e^+$ energy between 36 and 60 MeV and only $4.6 \pm 0.6$ background events. A fit to the $e^+$ events between 20 and 60 MeV yields a total excess of $51.8^{+18.7}_{-16.9} \pm 8.0$ events. If attributed to $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations, this corresponds to an oscillation probability of $(0.31^{+0.11}_{-0.10} \pm 0.05)%$ and leads to the oscillation parameters shown in Fig. (13). The shaded regions are the favoured likelihood regions given in ref. [6]. The curves show the 90 % and 99 % likelihood allowed ranges from LSND, and compares them to limits from BNL776, KARMEN1, Bugey, CCFR, and NOMAD. A search for $\nu_\mu \to \nu_e$ oscillations has also been conducted by the LSND collaboration. Using $\nu_\mu$ from $\pi^+$ decay in flight, the $\nu_e$ appearance is detected via the charged-current reaction $C(\nu_e , e^-)X$. Two independent analyses are consistent with the above signature, after taking into account the events expected from the $\nu_e$ contamination in the beam and the beam-off background. If interpreted as an oscillation signal, the observed
oscillation probability of $2.6 \pm 1.0 \pm 0.5 \times 10^{-3}$ is consistent with the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation evidence described above. Fig. 14 compares the LSND region with the expected sensitivity from MiniBooNE, which was recently approved to run at Fermilab. A possible confirmation of the LSND anomaly would be a discovery of far-reaching implications.

**Dark Matter**

The research on the nature of the cosmological dark matter and the origin of galaxies and large scale structure in the Universe within the standard theoretical framework of gravitational collapse of fluctuations as the origin of structure in the expanding universe has undergone tremendous progress recently. Indeed the observations of cosmic background temperature anisotropies on large scales performed by the COBE satellite combined with cluster-cluster correlation data e.g. from IRAS can not be reconciled with the simplest cold dark matter (CDM) model. Barring a non-zero cosmological constant and high value of the Hubble parameter ($h \gtrsim 0.7$) the simplest model that have a chance to work is Cold + Hot Dark Matter (MDM, for mixed dark matter), if the Hubble parameter and age parameter allow for an $\Omega = 1$ cosmology, suggested by inflation. Electron-volt mass neutrinos are the most well-motivated HDM candidate. This mass scale is similar to that indicated by the hints reported by the LSND experiment.

However it is too early to be confident on the MDM scenario, and one should for the moment keep an open mind. For example, I note that an MeV range (unstable) tau neutrino is an interesting possibility to consider from the point of view of dark matter. If such neutrino decays before the matter dominance epoch, its decay products would add energy to the radiation, thereby delaying the time at which the matter and
radiation contributions to the energy density of the universe become equal. Such delay would allow one to reduce the density fluctuations on the smaller scales purely within the standard cold dark matter scenario.

Future sky maps of the cosmic microwave background radiation (CMBR) with high precision at the upcoming MAP and PLANCK missions should bring more light into the nature of the dark matter and the possible rôle of neutrinos.

**Pulsars**

One of the most challenging problems in modern astrophysics is to find a consistent explanation for the high velocity of pulsars. Observations show that these velocities range from zero up to 900 km/s with a mean value of $450 \pm 50$ km/s. An attractive possibility is that pulsar motion arises from an asymmetric neutrino emission during the supernova explosion. In fact, neutrinos carry more than 99% of the new-born proto-neutron star’s gravitational binding energy so that even a 1% asymmetry in the neutrino emission could generate the observed pulsar velocities. One possible explanation to this puzzle may reside in the interplay between the parity non-conservation present in weak interactions and the strong magnetic fields which are expected during a SN explosion. Possible realizations of this idea in the framework of the Standard Model (SM) have been proposed. However, it has recently been noted that no asymmetry in neutrino emission can be generated in thermal
equilibrium, even in the presence of parity violation. This suggests that alternative mechanism is at work. Several neutrino conversion mechanisms in matter have been invoked as a possible engine for powering pulsar motion. They all share in common the feature that neutrino propagation properties are affected by the polarization of the SN medium which is provided by the strong magnetic fields $10^{15}$ Gauss present during a SN explosion. This would give rise to some angular dependence of the matter-induced neutrino potentials leading to a deformation of the "neutrino-sphere" for, say, tau neutrinos and hence to an anisotropic neutrino emission. As a consequence, in the presence of non-vanishing $\nu_\tau$ mass and mixing the resonance sphere for the $\nu_e - \nu_\tau$ conversions is distorted. If the resonance surface lies between the $\nu_\tau$ and $\nu_e$ neutrino spheres, such a distortion would induce a temperature anisotropy in the flux of the escaping tau-neutrinos produced by the conversions, hence a recoil kick of the proto-neutron star. This mechanism was realized in ref. 89 invoking MSW conversions with $m_{\nu_\tau} \gtrsim 100 \text{ eV}$ or so, assuming a negligible $\nu_e$ mass. This is necessary in order for the resonance surface to be located between the two neutrino-spheres. It should be noted, however, that such requirement is at odds with cosmological bounds on neutrinos masses unless the $\tau$-neutrino is unstable. On the other hand in ref. 90 a realization was proposed in the resonant spin-flavour precession scheme (RSFP) 64. Here the magnetic field not only affects the medium properties, but also induces the spin-flavour precession through its coupling to the neutrino transition magnetic moment 63. Perhaps the simplest suggestion was proposed in ref. 32 where the required pulsar velocities would arise from anisotropic neutrino emission induced by resonant conversions of massless neutrinos (hence no magnetic moment) 29. This mechanism arises in the model described in eq. 30 and has been shown to be of potential relevance for SN physics 31.

Very recently, however, Raffelt and Janka 91 have claimed that the asymmetric neutrino emission effect was vastly overestimated, because the variation of the temperature over the deformed neutrino-sphere is not an adequate measure for the anisotropy of the neutrino emission. This would invalidate the oscillation mechanisms, leaving the pulsar velocity problem without any known viable solution. The only potential way out of their criticism would invoke conversions into sterile neutrinos, since the conversions would take place deeper in the star. However, it is too early to tell whether or not it works 92.

4. Reconciling the neutrino puzzles

It is easy to accommodate the solar and atmospheric neutrino data by themselves in a general gauge theory of neutrino mass, since it lacks predictivity. One could even have a situation where three-neutrino mixing could be bi-maximal, i.e. maximal in both the atmospheric as well as solar neutrino transitions, if the solution chosen by nature is just-so 72. The challenge to reconcile these two requirements arise mainly
if one wishes to do that in a predictive quark-lepton unification scheme that relates lepton and quark mixing angles. This especially so since the latter are small, in contrast to the lepton mixing indicated by the SK atmospheric data. The story gets more complicated if one wishes to account also for the LSND anomaly and for the hot dark matter. There has been a lot of effort to solve the bigger puzzle posed by the inclusion of any of these additional hints \[1, 2\]. As we have seen the atmospheric neutrino data requires $\Delta m^2_{\text{atm}}$ which is much larger than the scale $\Delta m^2_\odot$ which is indicated by the solar neutrino data, either in the context of the MSW mechanism or the just-so solution. These two experiments fix two different scales for neutrino mass differences, so that with just the three known neutrinos and without discarding any experimental data, there is no room to include the LSND scale indicated in Fig. 13, nor the HDM scale which is roughly similar \[b\].

Reconciling the neutrino puzzles may be attempted within the unification approach or the weak-scale approach to the theory of neutrino mass. I will concentrate mostly on the latter, because it is an interesting and simpler alternative to the former.

### 4.1. Almost Degenerate Neutrinos

The only possibility to fit solar, atmospheric and HDM scales in a world with just the three known neutrinos is if all of them have nearly the same mass \[12\], of about $\sim 1.5$ eV or so in order to provide the right amount of HDM \[8\] (all three active neutrinos contribute to HDM). There is no room in this case to accommodate the LSND anomaly. This can be arranged in the unification approach discussed in sec. 2 using the $M_L$ term present in general in seesaw models. With this in mind one can construct, e.g. unified $SO(10)$ seesaw models where all neutrinos lie at the above HDM mass scale ($\sim 1.5$ eV), due to a suitable horizontal symmetry, while the parameters $\Delta m^2_\odot$ & $\Delta m^2_{\text{atm}}$ appear as symmetry breaking effects. An interesting fact is that the ratio $\Delta m^2_\odot / \Delta m^2_{\text{atm}}$ appears as $m_e^2/m_\tau^2$ \[18\].

### 4.2. Four-Neutrino Models

The simplest way to open the possibility of incorporating the LSND scale is to invoke a sterile neutrino, i.e. one whose interaction with standard model particles (such as the $W$ and the $Z$) is much weaker than the SM weak interaction. It must come in as an $SU(2) \otimes U(1)$ singlet ensuring that it does not affect the invisible Z decay width, well-measured at LEP. The sterile neutrino $\nu_s$ must also be light enough in order to participate in the oscillations involving the three active neutrinos. The theoretical challenges we have are:

- to understand why the sterile neutrino is so light (it is clear that if a sterile

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\[b\] I will ignore the pulsar velocity problem since there is no clear working-model at the moment.
neutrino is introduced into the SM, the $SU(2) \otimes U(1)$ gauge symmetry allows it to have a bare mass, which could be large)

- to account for the maximal neutrino mixing indicated by the atmospheric data
- to account for the three scales $\Delta m^2_{\text{atm}}$, $\Delta m^2_\odot$ and $\Delta m^2_{\text{LSND}/\text{HDM}}$ from first principles

With this in mind we have formulated the simplest and first schemes which provide an answer to the above points. I will denote them, $(e\tau)(\mu s)$ and $(es)(\mu\tau)$, respectively. One should realize that a given phenomenological scheme (mainly determined by the structure of the leptonic charged current) may be realized in more than one theoretical model. For example, an alternative to the model in was suggested in ref. There have been many attempts to reproduce the above phenomenological scenarios from different theoretical assumptions, as has been discussed here.

These two basic schemes are characterized by a very symmetric mass spectrum in which there are two ultra-light neutrinos at the solar neutrino scale and two maximally mixed almost degenerate eV-mass neutrinos (LSND/HDM scale), split by the atmospheric neutrino scale. The HDM problem requires the heaviest neutrinos at about 2 eV mass. These scales are generated radiatively due to the additional Higgs bosons which are postulated, as follows: $\Delta m^2_{\text{LSND}/\text{HDM}}$ arises at one-loop, while $\Delta m^2_{\text{atm}}$ and $\Delta m^2_\odot$ are two-loop effects. Since this proposal pre-dated the LSND results, it naturally focussed on accounting for the HDM problem, rather than LSND. However, it has been realized that the LSND oscillation effects may be accounted for in its framework. These are the simplest theories based only on weak-scale physics, in which one explains the lightness of the sterile neutrino, the large lepton mixing required by the atmospheric neutrino data, as well as the generation of the mass splittings responsible for solar and atmospheric neutrino conversions. These follow naturally from the underlying lepton-number-like symmetry and its breaking.

These models are minimal in the sense that they add a single $SU(2) \otimes U(1)$ singlet lepton to the SM. Before breaking the symmetry the heaviest neutrinos are exactly degenerate, while the other two which will be responsible for the explanation of the solar neutrino problem are still massless. After the global $U(1)$ lepton symmetry breaks the massive ones split and the light ones get mass. The models differ according to whether the $\nu_s$ lies at the dark matter scale or at the solar neutrino scale. In the $(e\tau)(\mu s)$ scheme the $\nu_s$ lies at the LSND/HDM scale, as illustrated in Fig. while in the alternative $(es)(\mu\tau)$ model, $\nu_s$ is at the solar neutrino scale as shown in Fig. In the $(e\tau)(\mu s)$ case the atmospheric neutrino puzzle is explained by $\nu_\mu$ to $\nu_s$ oscillations, while in $(es)(\mu\tau)$ it is explained by $\nu_\mu$ to $\nu_\tau$ oscillations. Correspondingly, the deficit of solar neutrinos is explained in the first case by $\nu_e$ to $\nu_\tau$ conversions, while in the second the relevant channel is $\nu_e$ to $\nu_s$. The two models are therefore clearly inequivalent. In both cases it is possible to fit all present observations together.
Figure 15: $(e\tau)(\mu s)$ scheme: $\nu_e - \nu_\tau$ conversions explain the solar neutrino data and $\nu_\mu - \nu_s$ oscillations account for the atmospheric deficit, ref. 10.

I now turn to the consistency of the models with BBN. The presence of additional weakly interacting light particles, such as our light sterile neutrino $\nu_s$, is constrained by BBN since the $\nu_s$ would enter into equilibrium with the active neutrinos in the early Universe (and therefore would contribute to $N^{\text{max}}_{\nu}$) via neutrino oscillations unless $\Delta m^2 \sin^4 2\theta \lesssim 3 \times 10^{-6}$ eV$^2$ where $\Delta m^2$ denotes the mass-square difference of the active and sterile species and $\theta$ is the vacuum mixing angle. However, systematical uncertainties in the derivation of BBN bounds still caution us not to take them too literally. For example, it has been argued in 54 that present observations of primordial Helium and deuterium abundances can allow up to $N^{\text{max}}_{\nu} = 4.5$ neutrino species if the baryon to photon ratio is small. Adopting this as a limit, clearly both models described above are consistent. Should the BBN constraints get tighter, e.g. $N^{\text{max}}_{\nu} < 3.5$ they could rule out the $(e\tau)(\mu s)$ model, and leave out only the competing scheme as a viable alternative. For recent work on this see ref. 99.

The two models would be distinguishable both from the analysis of future solar as well as atmospheric neutrino data. For example they may be tested in the SNO experiment 100 once they measure the solar neutrino flux ($\Phi^\text{NC}_{\nu}$) in their neutral current data and compare it with the corresponding charged current value ($\Phi^\text{CC}_{\nu}$). If the solar neutrinos convert to active neutrinos, as in the $(e\tau)(\mu s)$ model, then one expects $\Phi^\text{CC}_{\nu}/\Phi^\text{NC}_{\nu} \approx .5$, whereas in the $(es)(\mu\tau)$ scheme ($\nu_e$ conversion to $\nu_s$), the

Figure 16: $(es)(\mu\tau)$ scheme: $\nu_e - \nu_s$ conversions explain the solar neutrino data and $\nu_\mu - \nu_\tau$ oscillations account for the atmospheric deficit, ref. 11.
above ratio would be nearly $\simeq 1$. Looking at pion production via the neutral current reaction $\nu_\tau + N \rightarrow \nu_\tau + \pi^0 + N$ in atmospheric data might also help in distinguishing between these two possibilities $\S$, since this reaction is absent in the case of sterile neutrinos, but would exist in the $(es)(\mu\tau)$ scheme.

If light sterile neutrinos indeed exist, as suggested by the current solar and atmospheric neutrino data, together with the LSND experiment, one can show that in some four-neutrino scenarios, neutrinos would contribute to a cosmic hot dark matter component and to an increased radiation content at the epoch of matter-radiation equality. These effects leave their imprint in sky maps of the cosmic microwave background radiation (CMBR) and may thus be detectable with the precision measurements of the upcoming MAP and PLANCK missions as noted recently in ref. $\S^2$.

4.3. MeV Tau Neutrino

In ref. $\S^{101}$ a model was presented where an unstable MeV Majorana tau neutrino naturally reconciles the cosmological observations of large and small-scale density fluctuations with the cold dark matter picture (CDM). The model assumes the spontaneous violation of a global lepton number symmetry at the weak scale. The breaking of this symmetry generates the cosmologically required decay of the $\nu_\tau$ with lifetime $\tau_{\nu_\tau} \sim 10^2 - 10^4$ sec, as well as the masses and oscillations of the three light neutrinos $\nu_e$, $\nu_\mu$, and $\nu_s$. One can also verify that the BBN constraints can be satisfied. The cosmological attractiveness of this scheme should encourage one to check whether one can indeed account for the present solar and atmospheric data through oscillations among the three light neutrinos, after taking into account the recent SK-data.

5. In conclusion

A major news has been the re-confirmation of an angle-dependent atmospheric neutrino deficit by the SK collaboration, providing a strong evidence for neutrino masses, similar to that offered by the solar neutrino data. Unfortunately future LBL experiments do not all probe the full region indicated by the atmospheric data. If the LSND result stands the test of time, this would be a puzzling indication for the existence of a light sterile neutrino. Who ordered it? The two most attractive schemes to reconcile these observations invoke either $\nu_e - \nu_\tau$ conversions to explain the solar data, with $\nu_\mu - \nu_s$ oscillations accounting for the atmospheric deficit, or the other way around. These two basic schemes have distinct implications at future solar & atmospheric neutrino experiments. SNO and SuperKamiokande have the potential to distinguish them due to their neutral current sensitivity.

How about heavy neutrinos? Although cosmological bounds are a fundamental tool to restrict neutrino masses, in many theories heavy neutrinos will either decay or
annihilate very fast, thereby loosening or evading the cosmological bounds. From this point of view, *neutrinos can have any mass presently allowed by laboratory experiments*, and it is therefore important to search for manifestations of heavy neutrinos at the laboratory.

Last but not least, though most of the recent excitement comes from underground experiments, one should note that models of neutrino mass may lead to a plethora of new signatures which may be accessible also at accelerators, thus illustrating the complementarity between the two approaches in unravelling the properties of neutrinos and probing for signals beyond the SM.

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