Analysis and optimization of the two-dimensional model of the armature rotation on the electromagnetic railgun

Xiangrong Liu 1, Fuqiang Ma 1, Jiufu Wei 1 and Baoming Li 1,2

1 National Key Laboratory of Transient Physics, Nanjing University of Science and Technology, Nanjing 210094, Jiangsu, China
2 China Academy of Ordnance, 100089, Beijing, China

Email: 767812480@qq.com

Abstract: In order to generate enough electromagnetic torque on the armature during the electromagnetic launching process, an asymmetrical structure is adopted in this paper. And a two-dimensional armature model is established for analyzing this asymmetrical structure. Based on this model and the translational equation of the armature, the size parameters of the armature are optimized under the premise of the fixed device parameters of the pulse power supplies and the barrel. The effective length of the armature is between 5mm and 20mm, and the radius of the internal contour for the armature is between 0 and 4mm. The results show that when the above two parameters are 20mm and 4mm respectively, the armature can obtain the largest angular velocity. The maximum value is up to 1087.3rad/s. It is meaningful to guide the armature design of the electromagnetic railgun.

1. Introduction

It is well-known that the electromagnetic railgun is a hot topic in the research of the electromagnetic launch technology in recent decades. At present, there have been many researches on the mechanism of armature motion. A dynamic interior ballistic process considering the friction loss, nonlinear electrical contacts and aerodynamic drag is established to study the armature movement [1]. A normal pressure model caused by electromagnetic force is considered into the friction of the armature [2]. Besides, a ballistic model containing the movement and rotation of the armature is established to realize the ballistic control [3]. These methods have important influence on the research of the armature motion mechanism.

In this paper, first a movement equation of the armature is deduced for the augmentation railgun with four rails. Then in order to generate the electromagnetic torque on the armature, an asymmetrical rail layout is adopted. According to the Biot-Savart Law, a two-dimensional armature model is established to analyze this structure. At last, based on the movement model and the rotation model, the optimization for the size parameters of the armature are carried.
2. Model

2.1. Circuit model of the electromagnetic launch system

The circuit model of the electromagnetic launch system consists of two parts: the pulse power supply (PPS) circuit and the railgun circuit model. The circuit equation of the single pulse power supply is given by the following formula:

\[
\begin{align*}
-u_e + L_{\text{rad}} \frac{di}{dt} + iR_m + U_b &= 0 \\
i &= -C \frac{du_e}{dt} \\
L_{\text{rad}} \frac{di}{dt} + iR_m + U_b &= 0 
\end{align*}
\]

(1)

Where \( u_e \) is the voltage of the capacitor, \( L_{\text{rad}} \) is the inductance, \( i \) is the current of the module, \( R_m \) is the internal resistance, \( U_b \) is the breech voltage, and \( C \) is the capacitance.

For the augmentation railgun, the circuit model should be described as follows [4]:

\[U_b = \left[ (L_m' + 2M')x + L_n' \frac{df}{dt} + (L_m + 2M')v + I(R_m + R_n x + R_s l_{\text{rad}} + R_n) \right] \]

(2)

Where \( L_m' \) is the self-inductance gradient of the main rail, \( M' \) is the mutual inductance gradient between the additional rail and the mail rail, \( L_n' \) is the self-inductance gradient of the additional rail, \( l_{\text{rad}} \) is the length of the rail, \( L_s \) is the stray inductance, \( R_m \) is the resistance of the armature, \( R_n \) is the resistance gradient of the main rail, \( R_s \) is the resistance gradient of the additional rail and \( R_n \) is the stray resistance.

Considering that there are two modules of the PPS discharging simultaneously in this paper, \( i \) is equal to \( I/2 \). Thus, the equation (1) can be written as:

\[
\begin{align*}
\frac{du_e}{dt} &= -\frac{I}{2C} \\
\frac{df}{dt} &= u_e - \frac{I}{C} \left( R_m' x + R_m \frac{df}{dt} + R_n + R_n \frac{df}{dt} / 2 + (L_m' + 2M')v \right) \\
\frac{df}{dt} &= u_e \left( R_m' x + R_m \frac{df}{dt} + R_n + R_n \frac{df}{dt} / 2 + (L_m' + 2M')v \right) \\
\frac{df}{dt} &= u_e \left( R_m' x + R_m \frac{df}{dt} + R_n + R_n \frac{df}{dt} / 2 + (L_m' + 2M')v \right) 
\end{align*}
\]

(3)

Due to the displacement and speed of the armature in the above formula, it is necessary to supplement the movement equation of the armature. However, compared with the simple railgun, under the same current excitation, the magnetic field on the armature of the augmentation railgun increases obviously. Therefore, the armature can get more driving force. According to the reference [4], the movement equation of the armature in the augmentation railgun is derived.

Figure 1 shows the circuit diagram of the augmentation railgun. "1" refers to the position of armature at a certain time, and the armature reaches position "2" after \( dt \) time. The electrodynamic force is \( V \), the armature displacement is \( dx \), and the current flowing into the rail is \( I \) (assuming \( I \) is constant).
If the force acting on the armature is \( F \), then the mechanical work \( W_{\text{mech}} \) which the driving force \( F \) does is:

\[
W_{\text{mech}} = Fdx \tag{4}
\]

The increase of magnetic energy \( W_i \) is:

\[
W_i = \frac{1}{2} L_m 'dxI^2 + \frac{1}{2} M 'dxI^2 \times 2 = \frac{1}{2} (L_m ' + 2M ')dxI^2 \tag{5}
\]

The magnetic flux change in the whole circuit can be described as:

\[
\Phi = \frac{d(L_m I + 2MI)}{dt} = L_m 'Iv + 2M 'Iv = (L_m ' + 2M ')Iv \tag{6}
\]

Therefore, the total energy change \( W_g \) should be as follows:

\[
W_g = \Phi dt = (L_m ' + 2M ')I^2dx \tag{7}
\]

According to the law of conservation of energy, the total energy change \( W_g \) should be equal to the mechanical work \( W_{\text{mech}} \) plus the increase of magnetic energy \( W_i \). The driving force can be deduced as:

\[
F = \frac{1}{2} (L_m ' + 2M ')I^2 \tag{8}
\]

Thus, the movement equation of the armature is:

\[
m_a \frac{dv}{dt} = \frac{1}{2} (L_m ' + 2M ')I^2 \tag{9}
\]

Where \( m_a \) is the mass of the armature.

2.2. The rotation model of the armature

Figure 2 shows the diagram of the rail and the armature. As shown in figure 2, the origin of the reference system is fixed at the center of the armature tail end face. And the armature tail end face is considered as the \( xoxy \) plane.
The electromagnetic torque acting on the armature should be described as follows:

\[ M = \int_{\Omega} \bar{r} \times \bar{f} \, d\Omega \quad (10) \]

Where \( \Omega \) represents the space occupied by the armature, \( \bar{f} \) is the electromagnetic force density and \( \bar{r} \) is the corresponding radius vector, \( \bar{r} = (x, y, z)^T \).

According to the Lorentz force formula and the Ampere circuital theorem, the electromagnetic force density can be described as follows:

\[
\bar{f} = \frac{1}{\mu_0} (\nabla \times \bar{B}) \times \bar{B} = \frac{1}{\mu_0} \left[ (\bar{B} \cdot \nabla) \bar{B} - \nabla \left( \frac{\bar{B}^2}{2} \right) \right] \quad (11)
\]

Ignoring the torque in \( x \) and \( y \) direction, the rotation equation of the rotation is written as follows:

\[
J_{\text{inertia}} \frac{d\omega}{dt} = M_z + M_f \quad (12)
\]

Where \( \omega \) is the angular velocity of the armature, \( J_{\text{inertia}} \) is the inertia of the armature, and \( M_f \) is the resistance moment.

In order to solve the rotation equation conveniently, the following assumptions are made in this paper:

1. the armature should be considered as rigid body;
2. the magnetic field distribution on all cross-sections is consistent with that on the tail end face for the armature;
3. the cross-section of the armature is a ring shape;
4. the current flowing through the armature has little effect on the magnetic field of the armature;
5. the main rails and the additional rails are simplified to a linear-current-element model;
6. the resistance moment \( M_f \) should be ignored;
7. the rails within 4 times of the diameter from the tail end face of the armature have major contributions to the magnetic field.

In order to generate the electromagnetic torque on the armature, according to literature [5], the
method is to offset the additional rails by a certain distance in this paper. Figure 3 shows the specific rail layout containing the main rails and the additional rails.

Figure 3. The cross-section of the rails.

The parameter \( m \) represents the offset of the additional rails in \( x \) direction, and \( n \) represents the offset in \( y \) direction. The parameter \( r_1 \) is the radius of the outer contour of the armature and \( r_2 \) is the radius of the internal contour.

As shown in figure 3, taking No. 1 rail for example, if the field point is \((x, y, 0)\) and the source point is \((-m, n, z')\), the corresponding magnetic can be derived as follows according to the Biot-Savart Law:

\[
\bar{B} = \frac{\mu_0 I}{4\pi} \left[ (y-n)\bar{i} - (x+m)\bar{j} \right] \int \frac{dz'}{\sqrt{(x+m)^2 + (y-n)^2 + z'^2}}
\]  

(13)

Where \( \bar{i} \) represents the unit vector in \( x \) direction and \( \bar{j} \) represents the unit vector in \( y \) direction. After integral operation, the equation (13) is written as:

\[
\bar{B}_i = \frac{\mu_0 I}{4\pi} \frac{(y-n)\bar{i} - (x+m)\bar{j}}{(x+m)^2 + (y-n)^2 + 64r_i^2}
\]

(14)

Similarly, the magnetic field which the other rails generate can also be calculated as follows:
\[ \begin{align*}
\overrightarrow{B_2} & = \frac{\mu_0 I}{4\pi} \left( \frac{y - r_i}{x^2 + (y - r_i)^2} \right) \frac{8r_i}{\sqrt{x^2 + (y - r_i)^2 + 64r_i^2}} \\
& = \frac{\mu_0 I}{4\pi} \left( \frac{y + r_i}{x^2 + (y + r_i)^2} \right) \frac{8r_i}{\sqrt{x^2 + (y + r_i)^2 + 64r_i^2}} \\
\overrightarrow{B_3} & = \frac{\mu_0 I}{4\pi} \left( \frac{y + r_i}{x^2 + (y + r_i)^2} \right) \frac{8r_i}{\sqrt{x^2 + (y + r_i)^2 + 64r_i^2}} \\
& = \frac{\mu_0 I}{4\pi} \left( \frac{y + n}{x^2 + (y + n)^2} \right) \frac{16r_i}{\sqrt{x^2 + (y + n)^2 + 64r_i^2}}
\end{align*} \]

In summary, the magnetic field distribution should be described as follows:
\[ \overrightarrow{B} = \overrightarrow{B_1} + \overrightarrow{B_2} + \overrightarrow{B_3} + \overrightarrow{B_4} = \frac{\mu_0 I}{4\pi} \int g(x, y) \hat{i} + h(x, y) \hat{j} \]

Where \( g(x, y) \) and \( h(x, y) \) are coordinate functions which only related to \( x, y \).

Because there is no \( z \) component of the magnetic field in the equation (18), the operator \( \overrightarrow{B} \cdot \nabla \) in the equation (11) should be written as follows:
\[ \overrightarrow{B} \cdot \nabla = B_x \frac{\partial}{\partial x} + B_y \frac{\partial}{\partial y} \]

Therefore, the torque can be derived as follows:
\[ M_i = \int \left( x f_y - y f_x \right) \, d\Omega = \frac{1}{\mu_0} \int \left( x B_y + y B_x \right) \left( \frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \, d\Omega \]

In this paper, the following function is introduced. Then the torque will be written as follows:
\[ \Phi(x, y, z) = (x \cdot g + y \cdot h) \left( \frac{\partial h}{\partial x} - \frac{\partial g}{\partial y} \right) \]

\[ M_i = \mu_0 \left( \frac{I}{4\pi} \right)^2 \int_{\Omega} \Phi \, d\Omega \]

Introducing the cylindrical coordinate system, thus the torque can be described as follows:
\[ M_z = \mu_0 \left( \frac{I}{4\pi} \right)^2 \int_{\theta}^{2\pi} \int_{\rho}^{\rho_i} \Phi \rho \, d\rho \, d\theta \]

Where \( l_a \) is the effective length of the armature. It is not difficult to find that the triple integral in the above equation is a constant. In this case, the torque is directly proportional to the square of the rail current.
3. Results and Discussion

3.1. Numerical results

Table 1 lists the required parameters for the simulation. These parameters contain the device parameters of the PPS and the impedance characteristic parameters of the barrel. In addition, figure 4 shows the corresponding simulation results including the curves of the velocity, displacement, angular velocity, angle for the armature, the curve of the rail current and the curve of the capacitor voltage.

Table 1. The parameters of the PPS and the barrel.

| parameter        | value   | parameter        | value   |
|------------------|---------|------------------|---------|
| $L_{\text{ind}}$ ($\mu$H) | 16      | $R_s$ (m$\Omega$) | 2       |
| $C$ (mF)         | 7.8     | $l_{\text{rail}}$ (m) | 1.4     |
| $R_{\text{in}}$ (m$\Omega$) | 5       | $m_s$ (g)       | 7.5     |
| $L_{\text{a}}$' ($\mu$H/m) | 0.4     | $J_{\text{inertia}}$ (g·mm$^2$) | 169.86  |
| $M'$ ($\mu$H/m) | 0.1     | $m$ (mm)        | 6       |
| $L_{\text{a}}$' ($\mu$H/m) | 0.2     | $n$ (mm)       | 15      |
| $R_{\text{in}}$' (m$\Omega$/m) | 0.357   | $r_1$ (mm)     | 6       |
| $R_{\text{a}}$' (m$\Omega$/m) | 2.8     | $r_2$ (mm)     | 3       |
| $R_{\text{an}}$ (p$\Omega$) | 2.06    | $u_{\text{initial}}$ (kV) | 5       |
| $L_a$ ($\mu$H) | 5       | $I_a$ (mm)     | 10      |

![Figure 4. The simulation results under the above parameters.](image-url)
As shown in figure 4, the total current peak is 138.9kA, and the total current has dropped to 38.4kA when the armature comes out of the muzzle. The capacitor ends discharging at 0.79ms. The armature comes out of the muzzle at about 2.7ms. What’s more, the angular velocity $\omega$ reaches 962.4rad/s. In other words, the rotation rate of the projectile is about 153r/s. During this period, the armature has rotated about 1.59rad in the bore.

3.2. Optimization analysis

If the parameters of the PPS and the barrel are fixed, it is obvious that the main size parameters of the armature are $l_a$ and $r_2$ in this paper. In this case, in order to achieve the best muzzle angular velocity, taking $l_a$ and $r_2$ as independent variables, the corresponding optimization calculation is carried. Figure 5 shows the cloud chart of the muzzle angular velocity when $l_a$ is in the range of 5~20mm and $r_2$ is between 0 and 4mm.

![Figure 5. The cloud chart of the muzzle angular velocity.](image)

As shown in figure 5, when $l_a$ is less than 6mm, no matter how $r_2$ changes, the muzzle angular velocity is mostly less than 900rad/s. Furthermore, excluding the top right-hand corner, there is a little change for the muzzle angular velocity. Most of them are less than 1000rad/s. When $l_a$ is 20mm and $r_2$ is 4mm, the muzzle angular velocity reaches the maximum. The maximum value is 1087.3rad/s. However, in this case, although the muzzle angular velocity is the largest, the muzzle velocity is much smaller. The corresponding velocity value is 596.3m/s. It is equivalent to sacrificing the muzzle velocity in order to pursuit the larger angular velocity. Therefore, in practice, it is necessary to improve the muzzle angular velocity under the premise of meeting all requirements. The method in this paper has certain guiding significance.
4. Conclusion

In this paper, a translational equation of the armature is first established for the augmentation railgun with 4 rails. Then an asymmetrical rail layout is adopted to generate the electromagnetic torque. According to the Biot-Savart Law, a two-dimensional armature model is established to describe the rotation of the armature. Based on these above models, the optimization for the size parameters of the armature are carried. In the process of optimization, the device parameters of the PPS and the barrel are fixed. Taking the effective length of the armature and its radius of the internal contour as independent variables, these variables range are respectively 5~20mm and 0~4mm. The optimized results show that when the above two variables are 20mm and 4mm respectively, the armature can achieve the best angular velocity. The maximum angular velocity can reach 1087.3rad/s. It is meaningful to guide the research on the armature rotation of the electromagnetic railgun.

References

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