Using a data sample of $448.1 \times 10^{6} \psi(3686)$ events collected with the BESIII detector operating at the BEPCII, we perform search for the hadronic transition $h_{c} \rightarrow \pi^{+} \pi^{-} J/\psi$ via $\psi(3686) \rightarrow \pi^{0} h_{c}$. No signals of the transition are observed, and the upper limit on the product branching fraction $B(\psi(3686) \rightarrow \pi^{0} h_{c})B(h_{c} \rightarrow \pi^{+} \pi^{-} J/\psi)$ at the 90% confidence level is determined to be $2.0 \times 10^{-6}$. This is the most stringent upper limit to date.

PACS numbers: 13.66.Bc, 14.40.Pq, 13.25.Gv

I. INTRODUCTION

Heavy quarkonium ($Q\bar{Q}$) presents an ideal environment for testing the interplay between perturbative and nonperturbative Quantum Chromodynamics (QCD) \footnote{I}. Hadronic transitions between the heavy $Q\bar{Q}$ states are particularly interesting. A
common approach for calculating these transitions is the QCD Multipole Expansion (QCDME) [2] for gluon emission. The calculation depends on experimental inputs and works well for transitions of heavy $Q\bar{Q}$ states below open flavor threshold [3]. But some puzzles remain to pose challenge to the theory. For example, the measured ratio $\frac{\Gamma(T(2S)-\eta\eta(1S))}{\Gamma(T(2S)-\eta/\gamma)}$ [4] is much smaller than the theoretical prediction. Hence, more experimental measurements for the transition of heavy $Q\bar{Q}$ are desirable to constrain and challenge the theory models. However to date, the only well-measured hadronic transitions in the charmonium sector are those for the $\psi(3686)$. 

For charmonium states below the $D\bar{D}$ threshold, the hadronic transitions of the spin-singlet P-wave state $h_c(1^1P_1)$ are one of the best places to test the spin-spin interaction between heavy quarks [5], but they remain the least accessible experimentally because the $h_c(1^1P_1)$ can not be produced resonantly in $e^+e^-$ annihilation or from electric-dipole radiative transitions of the $\psi(3686)$. Evidence for the $h_c$ state was reported in $p\bar{p} \to h_c \to \gamma\eta_c$ by E835 [6] at Fermilab. The first observation of the $h_c$ was reported by CLEO in a study of the cascade decay $\psi(3686) \to \pi^0h_c, h_c \to \gamma\eta_c$ [7]. With large statistics, CLEO measured the $h_c$ mass precisely [8], and presented evidence for multi-pion decay modes [9], which imply that the $h_c$ state has comparable rates for the decay to hadronic final states and the radiative transition to the $\eta_c$ state. Furthermore, for the first time the BESIII collaboration measured the branching fractions $B(\psi(3686) \to \pi^0h_c) = (8.4 \pm 1.3 \pm 1.0) \times 10^{-4}$ and $B(h_c \to \gamma\eta_c) = (54.3 \pm 6.7 \pm 5.2)\%$ [10], which were confirmed by CLEO [11]. 

The $h_c$ is also expected to decay to lower-mass charmonia state through hadronic transitions, but this has not been observed yet. In the framework of QCDME, the branching fraction of $h_c \to \pi\pi J/\psi$ (including charged and neutral modes) is predicted to be $2\%$ [12], while it is predicted to be $0.05\%$ when neglecting the nonlocality in time [13]. An experimental measurement is desirable to distinguish between these calculations. In this paper, we perform a search for the hadronic transition $h_c \to \pi^+\pi^- J/\psi$ using a data sample consisting of $(448.1 \pm 2.9)$ million $\psi(3686)$ events [14] collected at a center-of-mass energy of $3.686$ GeV, corresponding to the peak of $\psi(3686)$ resonance. Considering kinematic limitation and parity conservation, the angular momentum between the two-pion system (in a relative S-wave) and $J/\psi$ should be P-wave, and the transition rate of $h_c \to \pi^+\pi^- J/\psi$ is suppressed. Thus, statistical limitation and low detection efficiency for the soft pions are the two major challenges to study $h_c \to \pi^+\pi^- J/\psi$. Taking into account the theoretically predicted branching fraction for transition $h_c \to \pi^+\pi^- J/\psi$, the other related decay branching fractions from Particle Data Group (PDG) [15] and the total number of $\psi(3686)$ used in this analysis and without consideration of detection efficiency, the signal yield of $\psi(3686) \to \pi^0h_c, \eta \to \gamma\gamma, h_c \to \pi^+\pi^- J/\psi, J/\psi \to l^+l^- (l = e, \mu)$ is expected to be $600$ and $15$ for the predictions of Refs. [12] and [13], respectively.

This paper is structured as follows: In Section II the BESIII detector is described and details of the Monte Carlo (MC) samples are given. In Section III, the analysis strategy, event selection criteria and background analysis are introduced. Section IV presents the estimation of the upper limit, and Section V provides the systematic uncertainties of the measurement. Finally, a short summary and a discussion of the result are given in Section VI.

II. BESII DETECTOR AND MONTE CARLO SIMULATION

The BESIII detector is designed to facilitate physics research in the $\tau$-charm region in $e^+e^-$ annihilations with center-of-mass energies from $2$ to $4.6$ GeV at the Beijing Electron Positron Collider II (BEPCCI). The detector has a geometric acceptance of $93\%$ of the solid angle and mainly consists of five components: (1) a helium-gas-based main drift chamber (MDC) for tracking and particle identification using the specific energy loss d$E$/d$x$. The expected charged particle momentum resolution at 1 GeV and d$E$/d$x$ resolution are $0.5\%$ and $6\%$, respectively. (2) A plastic scintillator time-of-flight system with an intrinsic time resolution of $80\text{ps}$ in the barrel region and $110\text{ps}$ in the end-cap region. (3) A CsI(Tl) crystal calorimeter (EMC) with an energy resolution better than $2.5\%$ in the barrel region and $5\%$ in the end-cap region, and a position resolution better than $6\text{mm}$ for 1 GeV electrons and photons. (4) A superconductive solenoid magnet with a central field of 1.0 Tesla. (5) A muon chamber system composed of nine barrel layers and eight end-cap layers of resistive plate chambers with a spatial resolution better than $2\text{cm}$. More details on the construction and capabilities of BESIII detector may be found in Ref. [16].

The optimization of event selection criteria, study of backgrounds and determination of detection efficiency are based on samples of MC simulated events. A GEANT4-based [17] software is used to describe the geometry of the BESIII detector and simulate the detector response. A MC sample of 506 million generic $\psi(3686)$ decays (inclusive MC sample’) is generated to study the background processes. The $\psi(3686)$ resonance is generated by KKMC [18] with final state radiation (FSR) effects handled with PHOTOS [19]. The known decay modes are generated by EvtGen [20] with branching fractions set to the world average values according to the PDG [21]; the remaining unknown charmonium decays are generated with LundCharm [22]. The signal channel $\psi(3686) \to \pi^0h_c, \pi^+\pi^- J/\psi$ is excluded from the inclusive sample.

The signal MC sample of $\psi(3686) \to \pi^0h_c, h_c \to \pi^+\pi^- J/\psi$ is generated uniformly in phase space with the $\pi^0$ decaying to two photons and the $J/\psi$ decaying to $l^+l^-$ $(l = e, \mu)$. The MC sample of $\psi(3686) \to \eta J/\psi$ with $\eta$ decaying to $\pi^0\pi^+\pi^-$ and $J/\psi$ decaying to $l^+l^-$ is generated to study the background and determine the detection efficiency of this process. The angular distribution of the $\eta$ is modeled as $1 + \cos^2 \theta_{\eta}$, where $\theta_{\eta}$ is the angle between $\eta$ momentum and the positron beam in the rest frame of $\psi(3686)$. The decay $\eta \to \pi^0\pi^+\pi^-$ is generated by EvtGen [20] with the measured Dalitz plot amplitude [23], and $\pi^0 \to \gamma\gamma$ by a phase space distribution. The $J/\psi$ decays to $l^+l^-$ are generated with an angular distribution of $1 + \cos^2 \theta_l$, where $\theta_l$ is the angle be-
tween the $l^+$ momentum in the $J/\psi$ rest frame and the $J/\psi$ momentum in the $\psi(3686)$ rest frame.

### III. METHODOLOGY AND EVENT SELECTION

A relative measurement strategy is used to measure $h_c \rightarrow \pi^+\pi^- J/\psi$ according to

$$B(\psi(3686) \rightarrow \pi^0 h_c)B(h_c \rightarrow \pi^+\pi^- J/\psi) = \frac{N_{\text{obs}}^\text{sig} \epsilon_{\text{ref}}}{N_{\text{obs}}^\text{ref} \epsilon_{\text{sig}}}.$$

The decay $\psi(3686) \rightarrow \pi^0 h_c \rightarrow \pi^0 \pi^+ \pi^- J/\psi$ is the signal mode, and the decay $\psi(3686) \rightarrow \eta J/\psi \rightarrow \pi^0 \pi^+ \pi^- J/\psi$, which has the same final state as the signal, serves as the reference mode. These two processes will be selected simultaneously. Then the product $B(\psi(3686) \rightarrow \pi^0 h_c)B(h_c \rightarrow \pi^+\pi^- J/\psi)$ can be obtained by the ratio of the numbers of observed events $N_{\text{obs}}^\text{sig}/N_{\text{obs}}^\text{ref}$ and the ratio of detection efficiencies $\epsilon_{\text{ref}}/\epsilon_{\text{sig}}$ of these two processes. With this relative measurement method, most of the systematic uncertainties in the efficiencies and that of the total number of $\psi(3686)$ events cancel.

Charged tracks are reconstructed from hits in the MDC and are required to originate from the interaction point, i.e. passing within 10 cm to the interaction point in the beam direction and 1 cm in the plane perpendicular to the beam. In addition, the polar angle $\theta$ of each track is required to satisfy $|\cos \theta| < 0.93$. Electromagnetic showers are reconstructed from clusters in the EMC. A good photon candidate is an isolated shower that is required to have energy larger than 25 MeV in the barrel region of the EMC ($|\cos \theta| < 0.8$) or 50 MeV in the end-cap regions ($0.86 < |\cos \theta| < 0.92$). Showers in the transition region between the barrel and the end-cap are removed since they are not well reconstructed. In addition, timing information from the EMC ($0 \leq t \leq 700$ ns) is used to suppress electronic noise and energy deposits unrelated to the event.

For events of interest, including $\psi(3686) \rightarrow \pi^0 h_c, h_c \rightarrow \pi^+\pi^- J/\psi$ (signal mode), and $\psi(3686) \rightarrow \eta J/\psi, \eta \rightarrow \pi^0\pi^+\pi^-$ (reference mode), we require that there are four good charged tracks with zero net charge and at least two good photon candidates. The track momentum is used to separate leptons and pions since the momenta of leptons from $J/\psi$ decay are higher than 1 GeV/c. Charged tracks with momenta less than 1 GeV/c are assumed to be pions, while the remaining two tracks are taken as leptons. Electrons and muons are identified according to the ratio of energy ($E$) deposited in the EMC and momentum ($p$) measured in MDC. Tracks with $E/p > 0.7$ are taken as electrons, and those with $E/p < 0.3$ are identified as muons. A pair of pions with opposite charge and a pair of leptons with same flavor and opposite charge are required. Photon pairs with invariant mass in the region $120 < M(\gamma\gamma) < 145$ MeV/$c^2$ are combined into $\pi^0$ candidates. To avoid bias in choosing the best combination, all combinations due to multiple $\pi^0$ candidates are retained. Only 0.5% of all events contain more than one $\pi^0$ candidate, and this is modeled well in the simulation. The $\pi^+\pi^-$ invariant mass $M(\pi^+\pi^-)$ should be larger than 0.3 GeV/$c^2$ to reject backgrounds from $\pi^0\pi^0 J/\psi$ with $\gamma$ converting into an $e^+e^-$ pair in the beam pipe or inner wall of the MDC.

A five-constraint (5C) kinematic fit is performed for the $\pi^0\pi^+\pi^- l^+l^-$ combination enforcing energy and momentum conservation and constraining the invariant mass of the photon pair to the $\pi^0$ nominal mass $\left[1\right]$. Events with $\chi^2_{5C} < 60$ are accepted for further analysis. After imposing these criteria, clear $J/\psi$ peaks with low background levels are observed in both the $e^+e^-$ and $\mu^+\mu^-$ invariant mass distributions, as shown in Fig. [1]. For the selection of $J/\psi$ candidates, the invariant mass of lepton pairs $M(l^+l^-)$ is required to be in the $J/\psi$ mass region, i.e. $|M(l^+l^-) - M(J/\psi)| < 30$ MeV/$c^2$, where $M(J/\psi)$ is the nominal mass of the $J/\psi$ $\left[15\right]$. Based on studies of the inclusive MC sample, the dominant surviving event candidates are from $\psi(3686) \rightarrow \eta J/\psi, \eta \rightarrow \pi^0\pi^+\pi^-$, while background from events with different final states is negligible. A clear $\eta$ peak with a low level of background is observed in the $\pi^0\pi^+\pi^-$ invariant mass spectrum, $M(\pi^0\pi^+\pi^-)$, as shown in Fig. [2].

In order to validate the event selection criteria, we calcu-
late the branching fraction $\mathcal{B}(\psi(3686) \rightarrow \eta J/\psi)$ and compare it with a previous BESIII measurement [24], where $\eta$ is reconstructed via two photons and only the first set of the data sample of (107.0 ± 0.8) million $\psi(3686)$ taken in 2009 [14] was used. In our calculation, the yield of $\psi(3686) \rightarrow \eta J/\psi, \eta \rightarrow \pi^0\pi^+\pi^-$ is obtained by counting events in the $\eta$ signal region and subtracting the events in the $\eta$ sideband region. The $\eta$ signal region is defined as $|M(\pi^0\pi^+\pi^-) - M(\eta)| < 15$ MeV/c$^2$, where $M(\eta)$ is the $\eta$ nominal mass [15]. It covers about 99.2% of the $\eta J/\psi$ signal according to the MC simulation. The $\eta$ sideband region is defined as $30 < |M(\pi^0\pi^+\pi^-) - M(\eta)| < 45$ MeV/c$^2$. Using the same sample of 107 million $\psi(3686)$ events, we obtain $\mathcal{B}(\psi(3686) \rightarrow \eta J/\psi) = (33.89 \pm 0.27$ (stat.) $) \times 10^{-3}$, which is consistent with the previous measurement $33.75 \pm 0.17$(stat.) $\pm 0.86$(syst.) $) \times 10^{-3}$.

IV. UPPER LIMIT ON $\mathcal{B}(\psi(3686) \rightarrow \pi^0h_c)\mathcal{B}(h_c \rightarrow \pi^+\pi^- J/\psi)$

The two-dimensional distributions of $M(\pi^0\pi^+\pi^-)$ versus the $\pi^0$ recoil mass $RM(\pi^0)$ for the signal MC sample and data are shown in Fig. 3 and the distribution of $RM(\pi^0)$ is shown in Fig. 4. To improve the resolution, $RM(\pi^0)$ is calculated using the four-momenta after constraining the invariant mass of the photon pair to the $\pi^0$ nominal mass [15] (1C). The process $\psi(3686) \rightarrow \eta J/\psi$ is clearly dominant, but no obvious signal events from $\psi(3686) \rightarrow \pi^0h_c, h_c \rightarrow \pi^+\pi^- J/\psi$ are observed.

In order to obtain the yield of the decay of interest, we veto $\psi(3686) \rightarrow \eta J/\psi$ by imposing the further requirement $|M(\pi^0\pi^+\pi^-) - M(\eta)| > 32$ MeV/c$^2$. For $\psi(3686) \rightarrow \eta J/\psi$, events off the $\eta$ peak region are those with bad resolution and large $\chi^2_{5C}$. Thus, to further suppress the events from $\psi(3686) \rightarrow \eta J/\psi$ which are far from the $\eta$ signal region, a tighter requirement $\chi^2_{5C} < 15$ is imposed. With the above requirements, 99.99% of the $\psi(3686)$ into $\eta J/\psi$ backgrounds are removed according to MC simulation. No events in data survive in the full region of $RM(\pi^0)$. Based on a study of the inclusive MC sample, there are only two background events from $\psi(3686) \rightarrow 2(\pi^+\pi^-)\pi^0$ left. Neither event is in the signal region of $h_c$, which is defined as $3.51 < RM(\pi^0) < 3.534$ GeV/c$^2$. We therefore take the expected number of observed background events $N_{\text{bkg}}$ in the signal region as zero.

The upper limit on the number of observed signal events $N_{\text{obs}}$ at the 90% confidence level (C.L.) is 2.44, which is estimated by using the Feldman-Cousins frequentist approach [25] without considering the systematic uncertainties. All the numbers used to extract the upper limit of signal yield are summarized in Table 1.

TABLE I. Summary table. In order: upper limit on the number of observed signal events ($N_{\text{obs}}$)up, the number of observed background events $N_{\text{bkg}}$, signal efficiency ($\epsilon_{\text{sig}}$), the number of observed events of reference mode ($N_{\text{ref}}$), efficiency of reference mode ($\epsilon_{\text{ref}}$), statistical uncertainty ($\sigma_{\text{stat}}$) and total uncertainty ($\sigma_{\text{tot}}$)

| Quantity          | Value   |
|-------------------|---------|
| ($N_{\text{obs}}$)up | 2.44    |
| $N_{\text{bkg}}$   | 0       |
| $\epsilon_{\text{sig}}$ | 2.52%  |
| $N_{\text{ref}}$   | 31611 ± 178 |
| $\epsilon_{\text{ref}}$ | 8.25%  |
| $\sigma_{\text{stat}}$ | 0.57%  |
| $\sigma_{\text{tot}}$ | 15.4%  |

V. SYSTEMATIC UNCERTAINTY

In this analysis, the upper limit is obtained with a relative measurement strategy defined by Eq. (1). Since the signal mode and reference mode have same final states, and the uncertainty associated with the detection efficiency, i.e. trigger, photon detection, tracking and PID for charged tracks, $\pi^0$ reconstruction, and the SC kinematic fit cancel. The systematic uncertainty due to the $M(\pi^0\pi^+\pi^-)$ resolution is less than 0.1% and is negligible.

The $M(\pi^0\pi^+\pi^-)$ spectrum in the final state of $h_c \rightarrow \pi^+\pi^- J/\psi$ is unclear due to its unknown dynamics. In the nominal analysis, the signal MC sample is generated uniformly in the phase space without considering the angular distribu-
FIG. 3. Two-dimensional distributions of $M(\pi^0\pi^+\pi^-)$ versus $RM(\pi^0)$ for the signal MC sample (left) and data (right). The red box indicates the $h_c$ signal region.

FIG. 4. Distribution of $RM(\pi^0)$ after the 1C kinematic fit. Black dots with error bars show data. The red dashed histogram shows the MC simulated signal shape (with arbitrary normalization). The blue solid histogram is the MC distribution of the reference mode.

TABLE II. Summary of systematic uncertainties

| Sources | Systematic uncertainties (%) |
|---------|------------------------------|
| $B(\psi(3686) \rightarrow \eta J/\psi)$ | 1.5 |
| $B(\eta \rightarrow \pi^0\pi^+\pi^-)$ | 1.2 |
| MC model | 15.2 |
| Total | 15.4 |

tation. In order to estimate the related uncertainties of the MC model, an alternative signal MC sample is generated by assuming a pure P-wave production between the two-pion system ($S$-wave) and $J/\psi$, where the production amplitude is proportional to the third power of the momentum of the $\pi^+\pi^-$ system. The difference in detection efficiency between the two MC samples, 15.2%, is taken as the systematic uncertainty associated with the MC model.

The branching fractions of $\psi(3686) \rightarrow \eta J/\psi$ and $\eta \rightarrow \pi^0\pi^+\pi^-$ are taken from the PDG [15]. The uncertainties of the branching fractions, 1.5% and 1.2%, are considered as systematic uncertainties. The individual systematic uncertainties are summarized in Table II. Assuming that all sources of systematic uncertainties are independent, a total systematic uncertainty of 15.4% is obtained by taking the quadratic sum of the individual contributions.

VI. SUMMARY

In summary, a search for the hadronic transition $h_c \rightarrow \pi^+\pi^- J/\psi$ is carried out via $\psi(3686) \rightarrow \pi^0 h_c, h_c \rightarrow \pi^+\pi^- J/\psi$. No signal is observed. The upper limit of the product of branching fractions $B(\psi(3686) \rightarrow \pi^0 h_c)B(h_c \rightarrow \pi^+\pi^- J/\psi)$ at the 90% C.L. is determined to be $2.0 \times 10^{-6}$. Using the PDG value for the branching fraction of $\psi(3686) \rightarrow \pi^0 h_c$ of $(8.6 \pm 1.3) \times 10^{-4}$ [15], the upper limit on $B(h_c \rightarrow \pi^+\pi^- J/\psi)$ is determined to be $2.4 \times 10^{-3}$, which is the most stringent upper limit to date. Neglecting the small phase space difference between the charged and neutral $\pi\pi$ modes and assuming isospin symmetry, we obtain $B(h_c \rightarrow \pi\pi J/\psi) < 3.6 \times 10^{-3}$ (including charged and neutral modes) at the 90% C.L. It is noted that the measured branching fraction is smaller than the prediction in Ref. [12] by one order in magnitude, but does not contradict that in Ref. [13].

ACKNOWLEDGEMENT

The BESIII collaboration thanks the staff of BEPCII and the IHEP computing center for their strong support. This work is supported in part by National Key Basic Research Program of China under Contract No. 2015CB856700; National Natural Science Foundation of China (NSFC) under Contracts Nos. 11205117, 1122544, 11335008, 11425524, 11575133; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; the CAS Center for Excellence in Particle Physics (CCEPP); the
[1] H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47 B, 365 (1973).
[2] Y.-P. Kuang, Front. Phys. China 1, 19 (2006).
[3] E. Eichten, S. Godfrey, H. Mahlke, and J. L. Rosner, Rev. Mod. Phys. 80, 1161 (2008).
[4] N. Brambilla et al., Eur. Phys. J. C 71, 1534 (2011).
[5] S. Godfrey, J. Phys.: Conf. Ser. 9, 123 (2005).
[6] M. Andreotti et al. (E835 Collaboration), Phys. Rev. D 72, 032001 (2005).
[7] J. L. Rosner et al. (CLEO Collaboration), Phys. Rev. Lett. 95, 102003 (2005).
[8] S. Dobbs et al. (CLEO Collaboration), Phys. Rev. Lett. 101, 182003 (2008).
[9] G. S. Adams et al. (CLEO Collaboration), Phys. Rev. D 80, 051106 (2009).
[10] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. Lett. 104, 132002 (2010).
[11] J. Y. Ge et al. (CLEO Collaboration), Phys. Rev. D 84, 032008 (2011).
[12] Y.-P. Kuang, S.-F. Tuan, and T.-M. Yan, Phys. Rev. D 37, 1210 (1988).
[13] F. Ko, Phys. Rev. D 52, 1710 (1995).
[14] M. Ablikim et al. (BESIII Collaboration), arXiv:1709.03653
[15] C. Patrignani et al. (Particle Data Group), Chin. Phys. C 40, 100001 (2016).
[16] M. Ablikim et al. (BESIII Collaboration), Nucl. Instrum. Methods A 614, 345 (2010).
[17] S. Agostinelli et al. (GEANT Collaboration), Nucl. Instrum. Methods A 506, 250 (2003).
[18] S. Jadach, B. F. L. Ward, and Z. Was, Comput. Phys. Commun. 130, 260 (2000); Phys. Rev. D 63, 113009 (2001).
[19] E. Barberio and Z. Was, Comput. Phys. Commun. 79, 291 (1994).
[20] D. J. Lange, Nucl. Instrum. Methods A 462, 152 (2001); R.-G. Ping, Chin. Phys. C 32, 599 (2008).
[21] K. A. Olive et al. (Particle Data Group), Chin. Phys. C 38, 090001 (2014).
[22] J. C. Chen, G. S. Huang, X. R. Qi, D. H. Zhang, and Y. S. Zhu, Phys. Rev. D 62, 034003 (2000).
[23] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 92, 012014 (2015).
[24] M. Ablikim et al. (BESIII Collaboration), Phys. Rev. D 86, 092008 (2012).
[25] G. J. Feldman and R. D. Cousins, Phys. Rev. D 57, 3873 (1998).
[26] R. D. Cousins and V. L. Highland, Nucl. Instrum. Methods A 320, 331 (1992).