In this article we study certain $p$-adic master equations of some models of complex systems, which are connected with energy landscapes of the linear and logarithmic types. These equations were introduced by Avetisov et al, see [2], [3]. In a different way from [23] we will show that the fundamental solutions of these equations are transition density functions of strong Markov processes with state space $\mathbb{Q}_p^n$. We study some aspects of these processes, including the first passage time problem and the survival probability.

1. Introduction

Many problems in Biology, Chemistry and Material Science, as the dynamics of complex systems (such as glasses and proteins) are described by a random walk on a complex energy landscape, see e.g. [8]-[19], [22]-[29]. An energy landscape (or simply a landscape) is a continuous function $U : X \rightarrow \mathbb{R}$ that assigns to each physical state of a system its energy. In many cases we can take $X$ to be a subset of $\mathbb{R}^N$. The term complex landscape means that the function $U$ has many local minima; in which case, the method of interbasin kinetics is applied. In this approach the study of a random walk on a complex landscape is based on a description of the kinetics generated by transitions between groups of states (basins). Minimal basins correspond to local minima of energy, and large basins have hierarchical structure. The dynamics of the system is then encoded in a system of kinetic equations of the form:

$$\frac{\partial}{\partial t} u(i, t) = - \sum_j \{ T(i, j) u(i, t) - T(j, i) u(j, t) \} v(j),$$

where the indexes $i, j$ enumerate the states of the system (which correspond to local minima of energy), $T(i, j) \geq 0$ is the probability per unit of time of a transition from $i$ to $j$, and the $v(j) > 0$ are the basin volumes. For further details the reader may consult [13] and the references therein.

In [3]-[4] Avetisov et al. developed new class of models of interbasin kinetics using ultrametric diffusion generated by $p$-adic pseudodifferential operators. In these models, the time-evolution of the system is controlled by a master equation of the form

$$\frac{\partial u(x, t)}{\partial t} = \int_{\mathbb{Q}_p} \{ j(x \mid y) u(y, t) - j(y \mid x) u(x, t) \} dy, x \in \mathbb{Q}_p, t \in \mathbb{R}_+,$$

Key words and phrases. Markov processes, pseudodifferential operators, Feller semigroups, positive definite functions, non-Archimedean analysis.
where the function \( u(x,t) : Q_p \times R_+ \rightarrow R_+ \) is a probability density distribution, and the function \( j(x|y) : Q_p \times Q_p \rightarrow R_+ \) is the probability of transition from state \( y \) to the state \( x \) per unit of time. Master equation (1.2) is a continuous version of (1.1) obtained from it by passing to a ‘continuous limit’ in \( Q_p \) under the conditions \( v(j) = 1, T(i,j) = j(|i-j|_p) \), see e.g. [13]. The transition from a state \( y \) to a state \( x \) can be perceived as overcoming the energy barrier separating these states. In the energy landscapes studied in this article. In a different way in [23], will show that the fundamental solutions of these equations are degenerate energy landscape. In this case the master equation (1.2) takes the form

\[
\frac{\partial u(x,t)}{\partial t} = \int_{Q_p} j(|x-y|_p) \{ u(y,t) - u(x,t) \} dy,
\]

(1.3)

where \( j(|x-y|_p) = \frac{A(T)}{|x-y|_p} \exp \left\{ -\frac{U(|x-y|_p)}{kT} \right\} \). By choosing \( U \) conveniently, several energy landscapes can be obtained. Following [3], there are three basic landscapes: (i) (logarithmic) \( j(|x-y|_p) = \frac{1}{|x-y|_p \ln^{\alpha}(1+|x-y|_p)} \), \( \alpha > 1 \) (ii) (linear) \( j(|x-y|_p) = \frac{1}{|x-y|_p^\alpha} \), \( \alpha > 0 \), (iii) (exponential) \( j(|x-y|_p) = \frac{e^{-n|x-y|_p}}{|x-y|_p^\alpha} \), \( \alpha > 0 \).

The original models of Avetisov et al. were formulated in dimension one. In this way from [23] and [3], will show that the fundamental solutions of these equations are transition density functions of strong Markov processes with state space \( Q_p^n \), see Theorem [1].

By using similar techniques to [4] and [23], we study the problem of the first passage time for a random walk \( \mathcal{J}(t,\omega) \), see Theorem [2]. Moreover, in this article we study the survival probability \( S(t) \) (the probability that a path of \( \mathcal{J}(t,\omega) \) remains in \( \mathbb{Z}_p^n \) at the time \( t \), see Theorem [3].

The article is organized as follows: In Section [2] we will collect some basic results on the \( p \)-adic analysis and fix the notation that we will use through the article. In Section [4] we introduce some general results of nonlocal operators associated with any function \( J : Q_p^n \rightarrow R_+ \) that satisfies the Hypothesis \( A \) (see Definition [1]). We show that these operators are pseudodifferential and give some properties of their symbols. We also study the Cauchy problem naturally associated to these operators. In Section [4] we show the existence of a Feller semigroup on \( C_0(Q_p^n) \).
and strong Markov process \( \mathcal{X}(t, \omega) \) with state space \((\mathbb{Q}_p^n, \| \cdot \|_p)\) (whose paths are right continuous and have no discontinuities other than jumps) associated with the symbols of these operators previously treated. Moreover, we consider the problem of the first passage time for random walks on \( p \)-adic spaces in a similar way as in [23]. The results here are also given in a general way for any function \( J : \mathbb{Q}_p^n \rightarrow \mathbb{R}^+ \) that satisfies the Hypothesis A. Finally, in Section 5, we will study first passage time problem and survival probability for linear and logarithmic landscape.

2. Fourier Analysis on \( \mathbb{Q}_p^n \): Essential Ideas

2.1. The field of \( p \)-adic numbers. Along this article \( p \) will denote a prime number. The field of \( p \)-adic numbers \( \mathbb{Q}_p \) is defined as the completion of the field of rational numbers \( \mathbb{Q} \) with respect to the \( p \)-adic norm \( | \cdot |_p \), which is defined as

\[
|x|_p = \begin{cases} 
0, & \text{if } x = 0 \\
p^{-\gamma}, & \text{if } x = p^\gamma a
\end{cases}
\]

where \( a \) and \( b \) are integers coprime with \( p \). The integer \( \gamma := \text{ord}(x) \), with \( \text{ord}(0) := +\infty \), is called the \( p \)-adic order of \( x \).

Any \( p \)-adic number \( x \neq 0 \) has a unique expansion of the form

\[
x = p^{\text{ord}(x)} \sum_{j=0}^{\infty} x_j p^j,
\]

where \( x_j \in \{0, 1, 2, \ldots, p – 1\} \) and \( x_0 \neq 0 \). By using this expansion, we define the fractional part of \( x \in \mathbb{Q}_p \), denoted \( \{x\}_p \), as the rational number

\[
\{x\}_p = \begin{cases} 
0, & \text{if } x = 0 \text{ or } \text{ord}(x) \geq 0 \\
p^{\text{ord}(x)} \sum_{j=0}^{-\text{ord}(x) - 1} x_j p^j, & \text{if } \text{ord}(x) < 0.
\end{cases}
\]

We extend the \( p \)-adic norm to \( \mathbb{Q}_p^n \) by taking

\[
\|x\|_p := \max_{1 \leq i \leq n} |x_i|_p, \text{ for } x = (x_1, \ldots, x_n) \in \mathbb{Q}_p^n.
\]

For \( r \in \mathbb{Z} \), denote by \( B^n_r(a) = \{x \in \mathbb{Q}_p^n : \|x - a\|_p \leq p^r\} \) the ball of radius \( p^r \) with center at \( a = (a_1, \ldots, a_n) \in \mathbb{Q}_p^n \), and take \( B^n_r(0) =: B^n_r \). Note that \( B^n_r(a) = B_r(a_1) \times \cdots \times B_r(a_n) \), where \( B_r(a) := \{x \in \mathbb{Q}_p : \|x - a\|_p \leq p^r\} \) is the one-dimensional ball of radius \( p^r \) with center at \( a \in \mathbb{Q}_p \). The ball \( B^n_r \) equals the product of \( n \) copies of \( B_0 = \mathbb{Z}_p \), the ring of \( p \)-adic integers of \( \mathbb{Q}_p \). We also denote by \( S^n_r(a) = \{x \in \mathbb{Q}_p^n : \|x - a\|_p = p^r\} \) the sphere of radius \( p^r \) with center at \( a = (a_1, \ldots, a_n) \in \mathbb{Q}_p^n \), and take \( S^n_r(0) =: S^n_r \). The balls and spheres are both open and closed subsets in \( \mathbb{Q}_p^n \).

As a topological space \((\mathbb{Q}_p^n, \| \cdot \|_p)\) is totally disconnected, i.e. the only connected subsets of \( \mathbb{Q}_p^n \) are the empty set and the points. A subset of \( \mathbb{Q}_p^n \) is compact if and only if it is closed and bounded in \( \mathbb{Q}_p^n \), see e.g. [24, Section 1.3], or [1] Section 1.8]. The balls and spheres are compact subsets. Thus \((\mathbb{Q}_p^n, \| \cdot \|_p)\) is a locally compact topological space.

We will use \( \Omega(p^{-\gamma} ||x - a||_p) \) to denote the characteristic function of the ball \( B^n_r(a) \). We will use the notation \( 1_A \) for the characteristic function of a set \( A \). Along the article \( d^n x \) will denote a Haar measure on \( \mathbb{Q}_p^n \) normalized so that \( \int_{\mathbb{Q}_p^n} d^n x = 1 \).
2.2. Some function spaces. A complex-valued function \( \varphi \) defined on \( \mathbb{Q}_p^n \) is called \textit{locally constant} if for any \( x \in \mathbb{Q}_p^n \) there exist an integer \( l(x) \in \mathbb{Z} \) such that
\[
\varphi(x + x') = \varphi(x) \quad \text{for} \quad x' \in B_{l(x)}(x).
\]

A function \( \varphi : \mathbb{Q}_p^n \to \mathbb{C} \) is called a \textit{Bruhat-Schwartz function (or a test function)} if it is locally constant with compact support. The \( \mathbb{C} \)-vector space of Bruhat-Schwartz functions is denoted by \( \mathcal{D}(\mathbb{Q}_p^n) =: \mathcal{D} \). Let \( \mathcal{D}'(\mathbb{Q}_p^n) =: \mathcal{D}' \) denote the set of all continuous functional (distributions) on \( \mathcal{D} \). The natural pairing \( \mathcal{D}'(\mathbb{Q}_p^n) \times \mathcal{D}(\mathbb{Q}_p^n) \to \mathbb{C} \) is denoted as \((T, \varphi)\) for \( T \in \mathcal{D}'(\mathbb{Q}_p^n) \) and \( \varphi \in \mathcal{D}(\mathbb{Q}_p^n) \), see e.g. [1] Section 4.4.

Every \( f \in L^1_{\text{loc}} \) defines a distribution \( f \in \mathcal{D}'(\mathbb{Q}_p^n) \) by the formula
\[
(f, \varphi) = \int_{\mathbb{Q}_p^n} f(x) \varphi(x) \, d^n x.
\]

Such distributions are called \textit{regular distributions}.

Given \( \rho \in [0, \infty) \), we denote by \( L^\rho(\mathbb{Q}_p^n, d^n x) = L^\rho(\mathbb{Q}_p^n) := L^\rho \), the \( \mathbb{C} \)-vector space of all the complex valued functions \( g \) satisfying \( \int_{\mathbb{Q}_p^n} |g(x)|^\rho \, d^n x < \infty \), \( L^\infty := L^\infty(\mathbb{Q}_p^n) = L^\infty(\mathbb{Q}_p^n, d^n x) \) denotes the \( \mathbb{C} \)-vector space of all the complex valued functions \( g \) such that the essential supremum of \( |g| \) is bounded.

Let denote by \( C(\mathbb{Q}_p^n, \mathbb{C}) =: C_\mathbb{C} \) the \( \mathbb{C} \)-vector space of all the complex valued functions which are continuous, by \( C(\mathbb{Q}_p^n, \mathbb{R}) =: C_\mathbb{R} \) the \( \mathbb{R} \)-vector space of continuous functions. Set
\[
C_0(\mathbb{Q}_p^n, \mathbb{C}) := \left\{ f : \mathbb{Q}_p^n \to \mathbb{C} ; \ f \text{ is continuous and } \lim_{x \to \infty} f(x) = 0 \right\},
\]

where \( \lim_{x \to \infty} f(x) = 0 \) means that for every \( \epsilon > 0 \) there exists a compact subset \( B(\epsilon) \) such that \( |f(x)| < \epsilon \) for \( x \in \mathbb{Q}_p^n \setminus B(\epsilon) \). We recall that \( (C_0(\mathbb{Q}_p^n, \mathbb{C}), || \cdot ||_{L^\infty}) \) is a Banach space.

2.3. Fourier transform. Set \( \chi_p(y) = \exp(2\pi i \{y \} \rho) \) for \( y \in \mathbb{Q}_p \). The map \( \chi_p(\cdot) \) is an additive character on \( \mathbb{Q}_p \), i.e. a continuous map from \((\mathbb{Q}_p, +)\) into \( S \) (the unit circle considered as multiplicative group) satisfying \( \chi_p(x_0 + x_1) = \chi_p(x_0) \chi_p(x_1) \), \( x_0, x_1 \in \mathbb{Q}_p \). The additive characters of \( \mathbb{Q}_p \) form an Abelian group which is isomorphic to \((\mathbb{Q}_p, +)\), the isomorphism is given by \( \xi \mapsto \chi_p(\xi x) \), see e.g. [1] Section 2.3.

Given \( x = (x_1, \ldots, x_n) \), \( \xi = (\xi_1, \ldots, \xi_n) \in \mathbb{Q}_p^n \), we set \( x \cdot \xi := \sum_{j=1}^n x_j \xi_j \). If \( f \in L^1 \) its Fourier transform is defined by
\[
(\mathcal{F} f)(\xi) = \int_{\mathbb{Q}_p^n} \chi_p(\xi \cdot x) f(x) \, d^n x, \quad \text{for} \quad \xi \in \mathbb{Q}_p^n.
\]

We will also use the notation \( \mathcal{F}_{x \rightarrow \xi} f \) and \( \widehat{f} \) for the Fourier transform of \( f \). The Fourier transform is a linear isomorphism from \( \mathcal{D}(\mathbb{Q}_p^n) \) onto itself satisfying
\[
(\mathcal{F}(\mathcal{F} f))(\xi) = f(-\xi),
\]
for every \( f \in \mathcal{D}(\mathbb{Q}_p^n) \), see e.g. [1] Section 4.8]. If \( f \in L^2 \), its Fourier transform is defined as
\[
(\mathcal{F} f)(\xi) = \lim_{k \to \infty} \int_{||x|| \leq p^k} \chi_p(\xi \cdot x) f(x) \, d^n x, \quad \text{for} \quad \xi \in \mathbb{Q}_p^n,
\]
where the limit is taken in $L^2$. We recall that the Fourier transform is unitary on $L^2$, i.e. $|f|_{L^2} = |Ff|_{L^2}$ for $f \in L^2$ and that $2\Pi$ is also valid in $L^2$, see e.g. [20, Chapter III, Section 2].

3. A class of nonlocal p-adic Operators.

Definition 1. (Hypothesis A) We say that the function $J$ satisfies Hypothesis A, if $J: \mathbb{Q}_p^n \to \mathbb{R}_+$ is a radial (i.e. $J(x) = J(||x||_p)$) and continuous function (a.e.) with $\int_{\mathbb{Q}_p^n} J(||x||_p)d^n x = 1$.

Lemma 1. Assume that $J: \mathbb{Q}_p^n \to \mathbb{R}_+$ satisfies Hypothesis A. Then, the following assertions hold:

(i) $\widehat{\mathcal{J}}(\xi)$ is a real-valued, radial (i.e. $\widehat{\mathcal{J}}(\xi) = \widehat{\mathcal{J}}(||\xi||_p)$), and continuous function, satisfying $|\widehat{\mathcal{J}}(||\xi||_p)| \leq 1$ and $\widehat{\mathcal{J}}(0) = 1$;

(ii) For $\xi \in \mathbb{Q}_p^n \setminus \{0\}$,

$$1 - \widehat{\mathcal{J}}(||\xi||_p) = ||\xi||^{-n}_p J(p||\xi||_p^{-1}) + p^n ||\xi||^{-n}_p \sum_{l=0}^\infty p^n J(p^{1+l}||\xi||_p^{-1});$$

Proof. (i) The first two statements are obtained by applying the $n$-dimensional version in [24, Example 8, p. 43]. The continuity of $\widehat{\mathcal{J}}(||\xi||_p)$ follows from [20, Chapter II-1-Theorem 1.1-(b)]. The affirmations $|\widehat{\mathcal{J}}(||\xi||_p)| \leq 1$ and $\widehat{\mathcal{J}}(0) = 1$, are a direct consequence of the fact that $\int_{\mathbb{Q}_p^n} J(||x||_p)d^n x = 1$.

(ii) The proof is similar to the one given in [23, Lemma 1-(ii)].

Remark 1. By the previous lemma we have that $0 \leq 1 - \widehat{\mathcal{J}}(||\xi||_p) \leq 2$, for all $\xi \in \mathbb{Q}_p^n$.

For $f \in L^p(\mathbb{Q}_p^n)$ with $1 \leq \rho \leq \infty$, we define $\mathcal{A}f := J \ast f - f$. Then, for any $1 \leq \rho \leq \infty$, $\mathcal{A}: L^p \to L^p$ gives rise a well-defined linear bounded operator. Indeed, by the Young inequality

$$||\mathcal{A}f||_{L^p} \leq ||J \ast f||_{L^p} + ||f||_{L^p} \leq ||J||_{L^1} ||f||_{L^1} + ||f||_{L^p} \leq 2 ||f||_{L^p}.$$ 

Proposition 1. Consider $\mathcal{A} : L^2(\mathbb{Q}_p^n) \to L^2(\mathbb{Q}_p^n)$ given by

$$\mathcal{A}f(x) = -\mathcal{F}_{\widehat{\mathcal{J}}}^{-1} \left( (1 - \widehat{\mathcal{J}}(||\xi||_p)) \mathcal{F}_{x \to \xi} f \right),$$

and the Cauchy problem:

$$\begin{cases}
\frac{\partial u}{\partial t}(x,t) = \mathcal{A}u(x,t), & t \in [0, \infty), \ x \in \mathbb{Q}_p^n \\
u(x,0) = u_0(x) \in \mathcal{D}(\mathbb{Q}_p^n).
\end{cases}
$$

Then

$$u(x,t) = \int_{\mathbb{Q}_p^n} \chi_p (-\xi \cdot x) e^{-t(1-\widehat{\mathcal{J}}(||\xi||_p))} \widehat{u_0}(\xi) d^n \xi$$

is a classical solution of (3.1). In addition, $u(\cdot, t)$ is a continuous function for any $t \geq 0$.

Proof. The result follows from the following assertions.

Claim 1. $u(x, \cdot) \in C^1([0, \infty))$ and

$$\frac{\partial u}{\partial t}(x,t) = -\int_{\mathbb{Q}_p^n} \chi_p (-\xi \cdot x) (1 - \widehat{\mathcal{J}}(||\xi||_p)) e^{-t(1-\widehat{\mathcal{J}}(||\xi||_p))} \widehat{u_0}(\xi) d^n \xi$$
for \( t \geq 0, x \in \mathbb{Q}_p^n \).

The formula follows from the fact that
\[
\left| \chi_p (-\xi \cdot x) e^{-(1-\tilde{J}(||\xi||_p))\hat{u}_0(\xi)} \right| \leq |\hat{u}_0(\xi)| \in L^1
\]
and that
\[
\left| \chi_p (-\xi \cdot x) \left( \tilde{J}(||\xi||_p) - 1 \right) e^{\tilde{J}(||\xi||_p)-1,t}\hat{u}_0(\xi) \right| \leq 2 |\hat{u}_0(\xi)| \in L^1,
\]
cf. Lemma \[\mathbb{I}(i)\], by applying the Dominated Convergence Theorem.

**Claim 2.**

\[
Au(x,t) = -\int_{\mathbb{Q}_p^n} \chi_p (-\xi \cdot x) \left( 1 - \tilde{J}(||\xi||_p) \right) e^{-(1-\tilde{J}(||\xi||_p))t}\hat{u}_0(\xi) d^n \xi
\]
for \( t \in [0, \infty), x \in \mathbb{Q}_p^n \).

The formula follows from the fact that \( u(x,t) = \mathcal{F}_{\xi \to x}^{-1} \left( e^{-(1-\tilde{J}(||\xi||_p))t}\hat{u}_0(\xi) \right) \in L^2(\mathbb{Q}_p^n) \) for any \( t \geq 0 \) and that \( -(1-\tilde{J}(||\xi||_p))e^{-(1-\tilde{J}(||\xi||_p))t}\hat{u}_0(\xi) \in L^2(\mathbb{Q}_p^n) \) for any \( t \geq 0 \), cf. Lemma \([\mathbb{I}-i]\).

4. **Strong Markov Processes and the First Passage Time Problem**

**Remark 2.** (i) A function \( f : \mathbb{Q}_p^n \to \mathbb{C} \) is called positive definite, if
\[
\sum_{i,j=1}^m f(x_i - x_j) \lambda_i \lambda_j \geq 0
\]
for all \( m \in \mathbb{N} \setminus \{0\} \), \( x_1, \ldots, x_m \in \mathbb{Q}_p^n \) and \( \lambda_1, \ldots, \lambda_m \in \mathbb{C} \). By a direct calculation one verifies that \( \tilde{J}(||\xi||_p) \) is a positive definite function.

(ii) A function \( f : \mathbb{Q}_p^n \to \mathbb{C} \) is called negative definite, if
\[
\sum_{i,j=1}^m \left( f(x_i) + f(x_j) - f(x_i - x_j) \right) \lambda_i \lambda_j \geq 0
\]
for all \( m \in \mathbb{N} \setminus \{0\} \), \( x_1, \ldots, x_m \in \mathbb{Q}_p^n \), \( \lambda_1, \ldots, \lambda_m \in \mathbb{C} \). By \([\mathbb{I}]\) Corollary 7.7 we have that the function \( \tilde{J}(0) - \tilde{J}(||\xi||_p) = 1 - \tilde{J}(||\xi||_p) \) is negative definite.

**Definition 2.** A family \((\mu_t)_{t \geq 0}\) of positive bounded measures on \( \mathbb{Q}_p^n \) with the properties

(i) \( \mu_t(\mathbb{Q}_p^n) \leq 1 \) for \( t > 0 \),
(ii) \( \mu_t \ast \mu_s = \mu_{t+s} \) for \( t, s > 0 \),
(iii) \( \lim_{t \to 0^+} \mu_t = \delta_0 \) vaguely \( (\delta_0 \text{ denotes the Dirac measure at } 0 \in \mathbb{Q}_p^n) \),

is called a convolution semigroup on \( \mathbb{Q}_p^n \).

Condition (ii) of Definition \([\mathbb{I}]\) is equivalent to \( \tilde{\mu}_t \circ \tilde{\mu}_s = \tilde{\mu}_{t+s} \) for \( t, s > 0 \), because the Fourier transformation is injective, while condition (iii) of Definition \([\mathbb{I}]\) is equivalent to \( \lim_{t \to 0^+} \tilde{\mu}_t(\xi) = 1 \) for \( \xi \in \mathbb{Q}_p^n \), see \([\mathbb{K}]\) Chapter II-Section 10. Moreover, it is well known that there is a one-to-one correspondence between convolution semigroups \((\mu_t)_{t \geq 0}\) on \( \mathbb{Q}_p^n \) and continuous negative definite functions \( \psi \) on \( \mathbb{Q}_p^n \), where \( \tilde{\mu}_t(\gamma) = e^{-t\psi(\gamma)} \) for \( t > 0 \) and \( \gamma \in \mathbb{Q}_p^n \), see \([\mathbb{K}]\) Theorem 8.3. Therefore, the family \((\mu_t)_{t \geq 0}\), satisfying
\[
\tilde{\mu}_t(\xi) = e^{-t(1-\tilde{J}(||\xi||_p))}, \text{ for } t > 0 \text{ and } \xi \in \mathbb{Q}_p^n,
\]
determines a convolution semigroup on \( \mathbb{Q}_p^n \).
Theorem 1. (i) Let \( \{ T_t \}_{t \geq 0} \) the Feller semigroup previously obtained. There exists a transition function \( p_t(x, \cdot) \) on \( \mathbb{Q}_p^n \), satisfying condition (L), such that the formula

\[
T_t f(x) = \begin{cases} 
\int_{\mathbb{Q}_p^n} p_t(x, dy) f(y) & \text{if } t > 0 \\
f & \text{if } t = 0.
\end{cases}
\]  

holds. Moreover, \( p_t(x, \cdot) \) is a \( C_0 \)-function i.e. the space \( C_0(\mathbb{Q}_p^n) \) is an invariant subspace for the operators \( T_t, t \geq 0 \),

\[
f \in C_0(\mathbb{Q}_p^n) \rightarrow T_t f \in C_0(\mathbb{Q}_p^n)
\]

(ii) There exists a strong Markov process \( \mathcal{X}(t, \omega) \) with state space \( (\mathbb{Q}_p^n, \| \cdot \|_p) \) and transition function \( p_t(x, \cdot) \) whose paths are right continuous and have no discontinuities other than jumps.
Lemma 2. The probability density function $f(t)$ of the random variable $\tau_{Z_p^n}(\omega)$ satisfies the non-homogeneous Volterra equation of second kind

$$
(4.6) \quad g(t) = \int_0^\infty g(t - \tau)f(\tau)d\tau + f(t)
$$

where

$$
(4.7) \quad g(t) = \int_{Q_p^n \setminus Z_p^n} J(||y||_p)u(y, t)d^n y,
$$
is the probability density function for a path of $\mathcal{J}(t,\omega)$ to enter into $\mathbb{Z}_P^n$ at the instant of time $t$, with the condition that $\mathcal{J}(0,\omega)\in\mathbb{Z}_P^n$.

**Proof.** The result follow by using the arguments given in the proof of [2] Theorem 1] and [23, Lemma 8].

**Lemma 3.** The Laplace transform $G(s) \ (Re(s) > 0)$ of $g(t)$ is given by

\[
G(s) = (1 - p^{-n}) \sum_{i=1}^{\infty} J(p^i) \left[ (1 - p^{-n}) \sum_{j=i}^{\infty} \frac{p^{n(i-j)}}{s + 1 - \hat{f}(p^{-j})} - \frac{1}{s + 1 - \hat{f}(p^{1-i})} \right].
\]

**Proof.** The result follow by using the arguments given in the proof of [23, Lemma 9 and Theorem 3].

5. LINEAR AND LOGARITHMIC LANDSCAPES

Following [3], there are three basic landscapes: logarithmic, linear and exponential. In this section, we will study first passage time problem and survival probability for linear and logarithmic landscapes. First passage time problem for exponential landscapes was treated in [23].

Let $\mathcal{J}(t,\omega)$ the strong Markov process obtained in Section 4.

**Definition 7.** (i) We say that function $J: \mathbb{Q}_P^n \to \mathbb{R}_+$ is of linear type if it satisfies the Hypothesis A and there exist positive real constants $A, B, \alpha$, with $A \leq B$, such that

\[
\frac{A}{||x||_P^{\alpha+1}} \leq J(||x||_P) \leq \frac{B}{||x||_P^{\alpha+1}}, \text{ for any } x \in \mathbb{Q}_P^n.
\]

(ii) We say that function $J: \mathbb{Q}_P^n \to \mathbb{R}_+$ is of logarithmic type if it satisfies the Hypothesis A and there exist positive real constants $A, B, \alpha, \beta$, with $A \leq B$ such that

\[
\frac{A \ln^\alpha(1 + ||x||_P)}{||x||_P^{\beta}} \leq J(||x||_P) \leq \frac{B \ln^\alpha(1 + ||x||_P)}{||x||_P^{\beta}},
\]

for any $x \in \mathbb{Q}_P^n$.

**Remark 3.** (i) The function $J(||x||_P) = \frac{1}{|x|_P \ln^\alpha(1 + |x|_P)}$, $\alpha > 1$, which was used in [3] is not integrable, indeed,

\[
\int_{\mathbb{Q}_P^n} \frac{1}{|x|_P \ln^\alpha(1 + |x|_P)} dx = (1 - p^{-1}) \sum_{j=0}^{\infty} \frac{1}{\ln^\alpha(1 + p^{-j})} + (1 - p^{-1}) \sum_{j=1}^{\infty} \frac{1}{\ln^\alpha(1 + p^{j})}.
\]

Note that

\[
\ln^\alpha(1 + p^{-j}) \leq \frac{(1 - p^{-1})^\alpha}{(1 + p^{-j})^\alpha} \leq \frac{(1 - p^{-1})}{\ln^\alpha(1 + p^{-j})},
\]

moreover,

\[
(1 - p^{-1}) \lim_{j \to \infty} \frac{1}{(1 + p^{-j})^\alpha} = (1 - p^{-1}) \neq 0,
\]

so that by the series divergence criterion $\sum_{j=0}^{\infty} \frac{(1 - p^{-1})}{(1 + p^{-j})^\alpha} \to \infty$ and consequently by the series comparison criterion, $(1 - p^{-1}) \sum_{j=0}^{\infty} \frac{1}{\ln^\alpha(1 + p^{-j})} \to \infty$. In general, we
have that the function \( J(||x||_p) = \frac{B}{||x||_p \ln^{\alpha+1}(1+||x||_p)} \), \( \alpha > 1 \), \( B > 0 \), is not integrable in \( Q^n_p \). This situation causes serious mathematical problems to achieve the goals set in this paper. Hence the forms of the function \( J \) of the logarithmic type in Definition \( \ref{def:7} \).

(ii) By Lemma \ref{lem:7}-(ii) we have that

\[
1 - \tilde{J}(1) = J(p) + p^n \sum_{i=0}^{\infty} p^{ni} J(p^{i+1}) = J(p) + \sum_{k=1}^{\infty} p^{nk} J(p^k) = J(p) + \frac{1}{1 - p^{-n}} \int_{Q^n_p \setminus Z^n_p} J(||x||_p) d^n x.
\]

If \( 1 - \tilde{J}(1) = 0 \), then \( J(p) = 0 \) and \( J(||x||_p) \equiv 0 \) for \( x \in Q^n_p \setminus Z^n_p \), i.e. \( \text{supp} J(||x||_p) \subseteq Z^n_p \), which is not possible since \( J \) is of linear type or logarithmic type. Therefore, \( 1 - \tilde{J}(1) > 0 \).

Example 1. (i) Let \( \alpha > 0 \) for which there exists \( r \in \mathbb{Q} \setminus \{0\} \) such that \( \alpha + 1 = n + r \), and for this \( r \) there exists \( N := N(r) \in \mathbb{N} \) with the property that if \( j \in \mathbb{N} \) and \( j \geq N \) then \( j + \frac{\alpha}{\alpha+1} \), \( j - \frac{\alpha}{\alpha+1} \in \mathbb{N} \). Then the function \( J(||x||_p) := \frac{B}{||x||_p^{\alpha+1}} \), \( B > 0 \), is of the linear type where \( B \) is a constant so that \( \int_{Q^n_p} J(||x||_p) d^n x = 1 \). For example, the function \( J(||x||_p) = \frac{B}{||x||_p^{\alpha+1}} \) with \( \alpha + 1 = n + r \), where \( n = 3 \) and \( r = \frac{1}{2} \) is of the linear type.

(ii) The functions \( J(||x||_p) := \frac{B \ln^{\alpha+1}(1+||x||_p)}{||x||_p^{\alpha+1}} \) for any \( \alpha > 0 \) are of logarithmic type, here \( B \) is a positive constant so that \( \int_{Q^n_p} J(||x||_p) d^n x = 1 \).

Lemma 4. Suppose that the function \( J : Q^n_p \to \mathbb{R}_+ \) satisfies one of the following conditions:

(i) \( J \) is of linear type with \( \alpha + 1 - 2n \geq 0 \),

(ii) \( J \) is of logarithmic type with \( \beta - \alpha - 2n \geq 0 \),

Then

\[
\frac{\Omega(||\xi||_p)}{1 - J(||\xi||_p)} \notin L^1(Q^n_p, d^n \xi).
\]

Proof. (i) By Lemma \ref{lem:4}-(ii) we have for \( \xi \in Q^n_p \setminus \{0\} \),

\[
1 - \tilde{J}(||\xi||_p) \leq \frac{B}{p^{\alpha+1}} ||\xi||_p^{\alpha+1-n} + B p^{n-\alpha-1} ||\xi||_p^{\alpha+1-n} \sum_{i=0}^{\infty} p^i (n-\alpha-1).
\]

Note that the condition \( \alpha + 1 - 2n \geq 0 \) implies that \( \alpha + 1 > n \), so the series \( \sum_{i=0}^{\infty} p^i (n-\alpha-1) < \infty \). Then, there exists a positive constant \( B_1 \) such that

\[
1 - \tilde{J}(||\xi||_p) \leq B_1 ||\xi||_p^{\alpha+1-n}, \ \ \xi \in Q^n_p \setminus \{0\}.
\]

Therefore,

\[
\int_{Z^n_p} \frac{d^n \xi}{1 - \tilde{J}(||\xi||_p)} \geq \frac{1}{B_1} \int_{Z^n_p} \frac{d^n \xi}{||\xi||_p^{\alpha+1-n}} = \frac{1}{B_1} \sum_{j=0}^{\infty} \frac{1}{p^{-j (\alpha+1-n)}} \int_{||\xi||_p = p^{-j}} d^n \xi = \frac{(1 - p^{-n})}{B_1} \sum_{j=0}^{\infty} p^{j (\alpha+1-2n)} \to \infty.
\]
(ii) By Lemma \[\text{Lemma1}\](ii) and applying the inequality
\[\ln(1+p^{1+t}||\xi||_p^{-1}) \leq p^{1+t}||\xi||_p^{-1},\]
we have that \(1 - \tilde{J}(||\xi||_p), \xi \in \mathbb{Q}_p^n \setminus \{0\},\) is dominated by
\[
\frac{B}{p^{\alpha}}||\xi||_p^{\beta-n} \ln^a(1+p||\xi||_p^{-1}) + B p^{n-\beta}||\xi||_p^{\beta-n} \sum_{l=0}^{\infty} p^{l(n-\beta)} \ln^a(1+p^{1+l}||\xi||_p^{-1})
\]
\[
\leq B p^{n-\beta}||\xi||_p^{\beta-n-\alpha} + B p^{n-\beta+\alpha}||\xi||_p^{\beta-n-\alpha} \sum_{l=0}^{\infty} p^{l(n+\alpha-\beta)}.
\]

Note that the condition \(\beta - \alpha - 2n \geq 0\) implies that \(\beta > n + \alpha\), so the series
\[
\sum_{l=0}^{\infty} p^{l(n+\alpha-\beta)} < \infty.
\]
Then, there exists a positive constant \(B_2\) such that
\[
1 - \tilde{J}(||\xi||_p) \leq B_2 ||\xi||_p^{\beta-n-\alpha}, \xi \in \mathbb{Q}_p^n \setminus \{0\}.
\]
Therefore,
\[
\int_{\mathbb{Z}_p} \frac{d^n \xi}{1 - \tilde{J}(||\xi||_p)} \geq \frac{1}{B_2} \int_{\mathbb{Z}_p} \frac{d^n \xi}{||\xi||_p^{\beta-n-\alpha}} = \frac{1}{B_2} \sum_{j=0}^{\infty} \frac{1}{p^{-j(\beta-n-\alpha)}} \int_{||\xi||_p=p^{-j}} d^n \xi
\]
\[
= \frac{(1-p^{-n})}{B_2} \sum_{j=0}^{\infty} p^{j(\beta-\alpha-2n)} \to \infty.
\]

\[\square\]

**Remark 4.** By applying the Laplace transform to \(\tilde{J}(t, \omega)\), we have
\[
F(s) = \frac{G(s)}{1 + G(s)} = 1 - \frac{1}{1 + G(s)},
\]
where \(F(s)\) and \(G(s)\) are the Laplace transforms of \(f\) and \(g\), respectively. We understand \(F(0) = \lim_{s \to 0} F(s)\) and \(G(0) = \lim_{s \to 0} G(s)\). From \(F(0) = \int_0^\infty f(t)dt = \frac{G(0)}{1+G(0)}\), it follows that if \(G(0) = \infty\), then \(\tilde{J}(t, \omega)\) is recurrent.

The proof of the following theorem uses some arguments given in the proof of [23] Theorem 3 but are included for the sake of completeness.

**Theorem 2.** If \(J\) satisfies one of the hypothesis of Lemma 4, then \(\tilde{J}(t, \omega)\) is recurrent with respect to \(\mathbb{Z}_p^n\).

**Proof.** Since \(\lim_{j \to \infty} 1 - \tilde{J}(p^{-j}) = 0\), given any \(s > 0\), there exists \(j_0(s) \in \mathbb{N}\) such that \(1 - \tilde{J}(p^{-j}) < s\) for \(j > j_0(s)\). In addition, \(s \to 0^+\) implies that \(j_0(s) \to \infty\).

By using these observations from Lemma 3 we have
\[
G(s) \geq (1 - p^{-n})J(p) \left[ (1 - p^{-n}) \sum_{j=1}^{j_0(s)} \frac{p^{n(1-j)}}{s + 1 - \tilde{J}(p^{-j})} - \frac{1}{s + 1 - \tilde{J}(1)} \right]
\]
\[
\geq (1 - p^{-n})J(p) \left[ \frac{p^n (1 - p^{-n})}{2} \sum_{j=1}^{j_0(s)} \frac{p^{-nj}}{1 - \tilde{J}(p^{-j})} - \frac{1}{s + 1 - \tilde{J}(1)} \right],
\]
notice that by Remark 3(ii), $1 - \hat{J}(1) > 0$. Therefore,
\[
\lim_{s \to 0^+} G(s) \geq (1 - p^{-n})J(p) \left[ \frac{p^n (1 - p^{-n})}{2 \cdot 1 - \hat{J}(p^{-j})} - \frac{1 + p^n (1 - p^{-n})}{1 - \hat{J}(1)} \right] \\
= (1 - p^{-n})J(p) \left[ \frac{p^n}{2} \int_{\mathbb{Z}_p^n} d^n\xi - \frac{1 + p^n (1 - p^{-n})}{1 - \hat{J}(1)} \right] = \infty.
\]

\[\square\]

**Remark 5.** The survival probability $S(t)$ (the probability that a path of $\hat{J}(t, \omega)$ remains in $\mathbb{Z}_p^n$ at the time $t$) is given by
\[
S(t) := S_{Z_p^n}(t) = \int_{\mathbb{Z}_p^n} u(x,t)d^n x,
\]
where $u(x,t)$ is given by (4.3). Therefore,
\[
S(t) = \int_{\mathbb{Z}_p^n} \int_{\mathbb{Z}_p^n} \chi_p (\xi - x)e^{-t(1 - \hat{J}(\|\xi\|_p))}d^n\xi \\
= \int_{\mathbb{Z}_p^n} e^{-t(1 - \hat{J}(\|\xi\|_p))}d^n\xi = (1 - p^{-n}) \sum_{j=0}^{\infty} p^{-nj} e^{-t(1 - \hat{J}(p^{-j}))}.
\]

By using the technique of [2] Appendix A we prove the following results.

**Theorem 3.** (i) If $J$ is of linear type and satisfies the hypothesis of Proposition 4(i) then there exist positive real constants $B_3 = \frac{p^n - 1}{(\alpha + 1 - n)\ln p}$, $A_3 = \frac{p^n}{(\alpha + 1 - n)\ln p}$, such that
\[
\frac{B_3}{(tB_1)^{\alpha + 1 - n}} \gamma \left( \frac{n}{\alpha + 1 - n}, tB_1 \right) \leq S(t) \leq \frac{A_3}{(tA_1)^{\alpha + 1 - n}} \gamma \left( \frac{n}{\alpha + 1 - n}, tA_1 \right),
\]

where $\gamma(s, x) := \int_{0}^{x} z^{s-1} e^{-z}dz$ is the lower incomplete gamma function, $B_1$ is the constant given in (5.7) and $A_1 = \frac{A}{p^{\beta + 1}}$.

(ii) If $J$ is of logarithmic type and satisfies the hypothesis of Proposition 4(ii) then there exist positive real constants $B_4 = \frac{p^{n-1}}{(\beta - n - \alpha)\ln p}$, $A_4 = \frac{1}{(\beta - n - \alpha)\ln p}$, such that
\[
\frac{B_4}{(tB_2)^{\beta - n - \alpha}} \gamma \left( \frac{n}{\beta - n - \alpha}, tB_2 \right) \leq S(t) \leq \frac{A_4}{(tA_4)^{\beta - n - \alpha}} \gamma \left( \frac{n}{\beta - n - \alpha}, tA_4 \right),
\]

where $\gamma(s, x) := \int_{0}^{x} z^{s-1} e^{-z}dz$ is the lower incomplete gamma function, $B_2$ is the constant given in (5.2) and $A$ is the constant given in Definition 4(ii).

**Proof.** (i) From Lemma 1(ii), and the fact that $J$ is of linear type, we get that
\[
1 - \hat{J}(\|\xi\|_p) \geq A_1 \|\xi\|_p^{\alpha + 1 - n}, \quad \xi \in \mathbb{Q}_p^n \setminus \{0\}.
\]
So that by (5.4) we have that
\[
S(t) \leq \sum_{j=0}^{\infty} p^{-nj} e^{-tA_1 p^{-j(\alpha + 1 - n)}}.
\]

We know that $e^{-tA_1 p^{-j(\alpha + 1 - n)}}$ is an increasing function and $p^{-nx}$ is a decreasing function in the variable $x$. Therefore, we have on the interval $j \leq x \leq j + 1$ the
inequality 
\[
(5.7) \quad \frac{e^{-t A_1 p^{-x(1-n)}}}{p^{nx}} \leq \frac{e^{-t A_1 p^{-j(1-n)}}}{p^{nj}} \leq \frac{e^{-t A_1 p^{-x(1-n)}}}{p^{n(x-1)}}.
\]

Integrating \((5.7)\) in the variable \(x\) from \(j\) to \(j+1\), we get 
\[
\int_j^{j+1} \frac{e^{-t A_1 p^{-x(1-n)}}}{p^{nx}} \, dx \leq \int_j^{j+1} \frac{e^{-t A_1 p^{-j(1-n)}}}{p^{nj}} \, dx \leq \int_j^{j+1} \frac{e^{-t A_1 p^{-x(1-n)}}}{p^{n(x-1)}} \, dx.
\]

So that 
\[
\int_j^{j+1} \frac{e^{-t A_1 p^{(1-n)x}(1-n)}}{p^{nx}} \, dx \leq \int_j^{j+1} \frac{e^{-t A_1 p^{-j(1-n)}}}{p^{nj}} \, dx \leq \int_j^{j+1} \frac{e^{-t A_1 p^{-x(1-n)}}}{p^{n(x-1)}} \, dx.
\]

Now, summing with respect to \(j\) from 0 to \(\infty\), we have that 
\[
\int_0^{\infty} \frac{e^{-t A_1 p^{(1-n)x}(1-n)}}{p^{nx}} \, dx \leq \sum_{j=0}^{\infty} p^{-nj} e^{-t A_1 p^{-j(1-n)}} \leq p^n \int_0^{\infty} \frac{e^{-t A_1 p^{-x(1-n)}}}{p^{nx}} \, dx.
\]

By changing variables as \(z = t A_1 p^{-\gamma(1-n)}\) in the right side, we have 
\[
S(t) \leq \frac{p^n}{(\alpha + 1 - n)(t A_1)^{\frac{n}{\alpha + 1 - n}} \ln p} \left( n \left( \frac{n}{\alpha + 1 - n} , t A_1 \right) \right).
\]

On the other hand, by \((5.1)\) and \((5.4)\) we have that 
\[
S(t) \geq (1 - p^{-n}) \sum_{j=0}^{\infty} p^{-nj} e^{-t B_1 p^{-j(1-n)}}.
\]

We know that \(e^{-t B_1 p^{-x(1-n)}}\) is an increasing function and \(p^{-nx}\) is a decreasing function in the variable \(x\). Therefore, proceeding in a similar way to the case \((5.0)\) we have that 
\[
S(t) \geq \frac{p^n - 1}{(\alpha + 1 - n)(t B_1)^{\frac{n}{\alpha + 1 - n}} \ln p} \left( n \left( \frac{n}{\alpha + 1 - n} , t B_1 \right) \right).
\]

The result follows taking \(B_3 = \frac{p^n - 1}{(\alpha + 1 - n) \ln p}\) and \(A_3 = \frac{p^n}{(\alpha + 1 - n) \ln p}\).

(ii) From Lemma \((5.0)\) \((5.4)\), and the fact that \(J\) is of logarithmic type, we get that 
\[
1 - \hat{J}(||\xi||_p) \geq A p^{-\beta} ||\xi||_p^{\beta-n} \ln \left( 1 + p ||\xi||_p^{\beta-1} \right), \quad \xi \in \mathbb{Q}_p \setminus \{0\}.
\]

So that by \((5.4)\) we have that 
\[
S(t) \leq \sum_{j=0}^{\infty} p^{-nj} e^{-t A_1 p^{-j(\beta-n)}}.
\]

On the other hand, by \((5.2)\) and \((5.4)\) we have that 
\[
S(t) \geq (1 - p^{-n}) \sum_{j=0}^{\infty} p^{-nj} e^{-t B_2 p^{-j(\beta-n-n)}}.
\]

The result follows analogously as in \((i)\). \(\square\)
References

[1] Albeverio S., Khrennikov A. Yu., Shelkovich V. M., Theory of p-adic distributions: linear and nonlinear models. London Mathematical Society Lecture Note Series, 370. Cambridge University Press, Cambridge, 2010
[2] Avetisov V. A., Bikulov A. Kh., Zubarev, A. P., First passage time distribution and the number of returns for ultrametric random walks, J. Phys. A 42(8), 085003 (2009)
[3] Avetisov V. A., Bikulov A. Kh., Osipov V. A., p-adic description of characteristic relaxation in complex systems, J. Phys. A 36 (2003), no. 15, 4239–4246
[4] Avetisov V. A., Bikulov A. H., Kozyrev S. V., Osipov V. A., p-adic models of ultrametric diffusion constrained by hierarchical energy landscapes, J. Phys. A 35 (2002), no. 2, 177–189
[5] Berg Christian, Forst Gunnar, Potential theory on locally compact abelian groups. Springer-Verlag, New York-Heidelberg, 1975
[6] Chacón-Cortes L. F., Zúñiga-Galindo W. A., Nonlocal operators, parabolic-type equations, and ultrametric random walks. J. Math. Phys. 54, 113503 (2013); doi: 10.1063/1.4828857
[7] Edward N, “Feynman integrals and the Schrödinger equation”, J. Math. Phys. 5, pp 332-343 (1964)
[8] Frauenfelder H., Sligar S.G., Wolynes P.G., The energy landscape and motions of proteins, Science 254, pp. 1598–1603 (1991)
[9] Frauenfelder H., McMahon B. H., Fenimore P. W., Myoglobin: the hydrogen atom of biology and paradigm of complexity, PNAS 100 (15), pp. 8615–8617 (2003)
[10] Frauenfelder H., Chan S. S., Chan W. S. (eds), The Physics of Proteins. Springer, New York (2010)
[11] Khrennikov, A., Kozyrev, S., Zúñiga-Galindo, W. A.: Ultrametric Equations and Its Applications. Encyclopedia of Mathematics and Its Applications, vol. 168. Cambridge Univ. Press, Cambridge (2018)
[12] Kochubei, A.N.: Pseudo-differential Equations and Stochastics over Non-Archimedean Fields. Marcel Dekker, New York (2001)
[13] Kozyrev S. V., Methods and Applications of Ultrametric and p-Adic Analysis: From Wavelet Theory to Biophysics. In: Sovrem. Probl. Mat., vol. 12, pp. 3–168. Steklov Math. Inst., RAS, Moscow (2008)
[14] Mézard, M., Parisi, G., Virasoro, M.A.: Spin Glass Theory and Beyond. World Scientific, Singapore (1987)
[15] Ogieldski, A.T., Stein, D.L.: Dynamics on ultrametric spaces. Phys. Rev. Lett. 55(15), pp. 1634–1637 (1985)
[16] Rammal, R., Toulouse, G., Virasoro, M.A.: Ultrametricity for physicists. Rev. Mod. Phys. 58(3), pp. 765–788 (1986)
[17] Rodríguez-Vega, J.J., Zúñiga-Galindo, W.A.: Taibleson operators, p-adic parabolic equations and ultrametric diffusion. Pac. J. Math. 237(2), 327–347 (2008)
[18] Stillinger, F.H., Weber, T.A.: Hidden structure in liquids. Phys. Rev. A 25, 978–989 (1982)
[19] Stillinger, F.H., Weber, T.A.: Packing structures and transitions in liquids and solids. Science 225, 983–989 (1984)
[20] Taibleson M. H., Fourier analysis on local fields. Princeton University Press, 1975
[21] Taiara Kazuki, Boundary value problems and Markov processes. Second edition. Lecture Notes in Mathematics, 1499. Springer-Verlag, 2009
[22] Torresblanca-Badillo A., Zúñiga-Galindo W. A., Non-Archimedean Pseudodifferential Operators and Feller Semigroups, p-Adic Numbers, Ultrametric Analysis and Applications, pp. 57–73, Vol. 10, No. 1 (2018)
[23] Torresblanca-Badillo A., Zúñiga-Galindo W. A., Ultrametric Diffusion, exponential landscapes, and the first passage time problem. Acta Appl Math, pp 1-24 (2018)
[24] Vladimirov V. S., Volovich I. V., Zelenov E. I., p-adic analysis and mathematical physics. World Scientific, 1994.
[25] Wales D. J., Miller M. A., Walsh T. R., Archetypal Energy Landscapes, Nature 394, 758-760 (1998)
[26] Yoshino, H.: Hierarchical diffusion, aging and multifractality. J. Phys. A 30, 1143–1160 (1997)
[27] Zúñiga-Galindo, W.A.: Parabolic equations and Markov processes over p-adic fields. Potential Anal. 28(2), 185–200 (2008)
[28] Zúñiga-Galindo, W.A.: The non-Archimedean stochastic heat equation driven by Gaussian noise. J. Fourier Anal. Appl. 21(3), 600–627 (2015)

[29] Zúñiga-Galindo, W.A.: Pseudodifferential Equations over Non-Archimedean Spaces. Lectures Notes in Mathematics, vol. 2174. Springer, New York (2016)

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