LIGHT-FRONT DYNAMICS OF CHERN-SIMONS SYSTEMS

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ABSTRACT

Chern-Simons theory coupled to complex scalars is quantized on the light-front in the local light-cone gauge by constructing the self-consistent hamiltonian theory. It is shown that no inconsistency arises on using two local gauge-fixing conditions in the Dirac procedure. The light-front Hamiltonian turns out to be simple and the framework may be useful to construct renormalized field theory of particles with fractional statistics (anyons). The theory is shown to be relativistic and the extra term in the transformation of the matter field under space rotations, interpreted in previous works as anomaly, is argued to be gauge artefact.

MIRAMARE- TRIESTE
October 1994

PACS number(s): 11.10.Ef, 11.30.Qc, 11.15.Tk

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1. Introduction

Chern-Simons (CS) gauge field theories [1,2] in three dimensional space time coupled to matter field have drawn much interest recently. They have been proposed to describe excitations with fractional statistics, anyons, which have been suggested to be relevant for an explanation [3] of the fractionally quantized Hall effect and possibly that of high-$T_c$ superconductivity [4] where the dynamics is effectively confined to a plane. There are, however, controversies related to the quantized field theoretical formulation. The lagrangian (path integral) formulation [5], for example, seems to give result which disagree with the canonical hamiltonian formulation [6-9]. It is claimed that the theory though shown relativistic has angular momentum anomaly [10] or shows anyonicity only in some nonlocal gauges [9,6]. Internal algebraic inconsistency [9] of using two local gauge-fixing conditions [11] in the context of the usual Coulomb gauge has also been stressed. The anomaly is also not found in some recent works [12,13] which avoid gauge-fixing and doubts have been raised about the anyonicity being gauge artefact [8].

We scrutinize these points in the paper by quantizing the CS theory coupled to the complex scalar field on the light-front [14] in the light-cone gauge. We show that there is no inconsistency in using two local gauge-fixing conditions in the theory. It becomes clear that a parallel discussion is valid also in the conventional Coulomb gauge and even when the fermionic fields are present. The hamiltonian theory on the light-front turns out to be simple and practical one and may serve to construct a renormalized theory in contrast to the complicated and difficult to handle hamiltonian obtained in the equal-time formulation in the local [6] or nonlocal [9] Coulomb gauge. The motivation for the new approach arises from the recent works [15,16] in the light-front quantization showing its potential for computing non-perturbative effects in QCD or the study of relativistic bound state problem. The light-front vacuum is known to be simpler than the conventional one and the anyonic excitations may here be studied transparently. In the context of the spontaneous symmetry breaking it was shown recently [17] that the light-front formulation leads to the same physical outcome as that from the equal-time one, though achieved through a different description. The conventional one requires to add external constraints in the theory based on physical considerations while the similar
constraints arise as self-consistency conditions in the light-front theory. The (anyonic) excitations obeying fractional statistics would also emerge in the simpler dynamics [14] on the light-front. We demonstrate also that the anomaly mentioned above should rather be interpreted as gauge artefact here and in the previous works as well. In the recently proposed gauge-independent theory [12], where extra terms are added to the canonical Hamiltonian in ad hoc fashion, it is not clear if it still describes the original lagrangian theory and the hamiltonian is as complicated as found in the earlier works.

In Sec. 2 theory with CS term coupled to complex scalar field is quantized on the light-front. The Dirac bracket is constructed and it is shown to lead to the well known light-front commutators for the independent scalar fields. The self-consistency [18] of the formulation is shown by recovering the lagrange eqs. The canonical Poincaré generators are constructed in Sec. 3 and the relativistic invariance of the theory checked. The commutators of the scalar field with Lorentz generators are found and it is argued that the so called rotational anomaly in the present context should rather be regarded as gauge artefact; with no bearing on the anyonicity.

2. Light-front Quantization of Chern-Simons Theory

The CS gauge theory we study is described by the following lagrangian density

$$\mathcal{L} = (D^\mu \phi)(\bar{D}_{\mu} \phi^*) + \frac{\kappa}{4\pi} \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$  \hspace{1cm} (1)

Here $\phi$ is a complex scalar filed, $A_\mu$ is the gauge field, $D_\mu = (\partial_\mu + ieA_\mu)$, $\bar{D}_\mu = (\partial_\mu - ieA_\mu)$, $\epsilon^{\mu\nu\rho}$ is the Levi-Civita tensor needed to construct the Chern-Simons kinetic term. For the coordinates $x^\mu$, and for all other vector or tensor quantities, we define the light-front $\pm$ components by $x^\pm = (x^0 \pm x^2)/\sqrt{2} = x_\mp$. We take $x^+ \equiv \tau$ to indicate the light-front time coordinate and $x^-$ the longitudinal space coordinate while $x^1$ is the transverse one. The metric tensor for the indices $\mu = (+, -, 1)$ is given by the nonvanishing elements $g^{+-} = g^{+ -} = -g^{11} = 1$ and $\epsilon^{+-1} = 1$. The conjugate momenta
\[ \pi = \frac{\partial L}{\partial (\partial_+ \phi)} = \tilde{D}_- \phi^*, \quad \pi^* = \frac{\partial L}{\partial (\partial_+ \phi^*)} = D_- \phi \] (2a)

\[ \pi^\mu = \frac{\partial L}{\partial (\partial_+ A^\mu)} = a \epsilon^{\mu\nu} A_\nu \] (2b)

where we set \( \kappa = 4\pi a \). They show that we are dealing with a constrained dynamical system (like in the equal-time case) and we will follow the well established Dirac procedure [18] to construct the hamiltonian theory. We observe that the conserved current \( j^\mu = ie(\phi^* D^\mu \phi - \phi \tilde{D}^\mu \phi^*) \) is gauge invariant and its contravariant vector property should remain intact if the theory constructed is relativistic.

We build the hamiltonian framework in the light-cone gauge, \( A_- \approx 0 \). We recall that it is mandatory for self-consistency [18] that the lagrange eqs. for the independent fields be recovered from the hamilton’s eqs. We therefore examine first the lagrange eqs. in our gauge

\[ \partial_+ \partial_- \phi = \frac{1}{2} D_1 D_1 \phi - ie A_+ \partial_- \phi - \frac{i}{2} e(\partial_- A_+) \phi \] (3a)

\[ 2a \partial_- A_1 = j^+ = ie(\phi^* \partial_- \phi - \phi \partial_- \phi^*) \] (3b)

\[ 2a (\partial_1 A_+ - \partial_+ A_1) = j^- = ie(\phi^* D_+ \phi - \phi \tilde{D}_+ \phi^*) \] (3c)

\[ -2a \partial_- A_+ = j^1 = -ie(\phi^* D_1 \phi - \phi \tilde{D}_1 \phi^*) \] (3d)

The gauge invariant field \( F_{-1} \) reduces in our gauge to \( \partial_- A_1 \) and it is proportional to the charge density \( j^+ \). The electric charge is given by \( Q = \int d^2 x j^+ = 2a \int dx^1 [A_1(x^- = \infty, x^1) - A_1(x^- = -\infty, x^1)] \). If the scalar field carries nonzero electric charge, it follows that \( A_1 \) may not be taken to satisfy the periodic or the vanishing boundary conditions along \( x^- \) at infinity. We assume for convenience the anti-periodic boundary conditions for the gauge fields at infinity along \( x^- \) and the vanishing ones along the \( x^1 \). These boundary conditions fix also the residual gauge invariance with respect to the \( x^1 \)-dependent gauge transformations. For the scalar fields we assume vanishing boundary conditions at infinity along the spatial coordinates.

The canonical Hamiltonian may then be written as
\[ H_c = \int d^2x \left[ (D_1 \phi)(\tilde{D}_1 \phi^*) - A_+ \Omega \right] \]  

where

\[ \Omega = ie(\pi \phi - \pi^* \phi^*) + a\epsilon^{+ij} \partial_i A_j + \partial_i \pi^i. \]  

The primary constraints following from (2) are

\[ \pi^+ \approx 0 \]  
\[ \top^i \equiv \pi^i - a\epsilon^{+ij} A_j \approx 0; \quad i = -, 1 \]  
\[ \top \equiv \pi - \tilde{D}_- \phi^* \approx 0 \]  
\[ \top^* \equiv \pi^* - D_- \phi \approx 0 \]  

and the preliminary Hamiltonian is

\[ H' = H_c + \int d^2x \left[ u\top + u^*\top^* + u_i\top^i + u_+\pi^+ \right] \]  

where \( u, u^*, u^i, u_+ \) are lagrange multiplier fields. We postulate initially the standard canonical \textit{equal-}\( \tau \) Poisson brackets with the nonvanishing ones given by

\[ \{\pi^\mu(x), A_\nu(y)\} = -\delta^\mu_\nu \delta^2(x-y), \quad \{\pi(x), \phi(y)\} = \{\pi^*(x), \phi^*(y)\} = -\delta^2(x-y), \]  

with \( x \) standing for \((x^-, x^1)\) and \( \tau \) suppressed for convenience. The following Poisson brackets among the constraints are easily derived

\[ \{\top^i, \top^j\} = -2a\epsilon^{+ij} \delta^2(x-y) \]  
\[ \{\top^i, \top\} = -i\delta^i_j \phi^* \delta^2(x-y) \]  
\[ \{\top^i, \top^*\} = i\delta^i_j \phi \delta^2(x-y) \]  
\[ \{\top, \top^*\} = -2\tilde{D}_x \delta^2(x-y) \]  
\[ \{\top, \top\} = \{\top^*, \top^*\} = 0 \]

while \( \pi^+ \) gives vanishing brackets with all of them. We make the \textit{convention} that the first variable in an equal-\( \tau \) bracket refers to the variable \( x \) while the second one to \( y \). The
\( \Omega \) generates gauge transformations of the canonical variables and gives \textit{weakly} vanishing brackets with the constraints (6) and \( H' \). The evolution in \( \tau \) of a dynamical variable is determined from \( df(x, \tau)/d\tau = \{ f(x, \tau), H'(\tau) \} + \partial f/\partial \tau \). Requiring the \textit{persistency in} \( \tau \) of the constraint \( \pi^+ \approx 0 \) and noting that \( \{ \pi^+(x, \tau), H'(\tau) \} \approx \Omega(x, \tau) \) we are led to a new \textit{secondary constraint} \( \Omega \approx 0 \). The persistency requirement for the others results in the consistency equations for determining the lagrange multiplier variables. We next go to the extended Hamiltonian \( H'' = H' + \int d^2x \, v \, \Omega \), where \( v \) is a lagrange multiplier and repeat the procedure. We find that no new secondary constraints are generated. The constraints \( \pi^+ \approx 0 \) and \( \Omega \approx 0 \) are first class while the remaining ones are second class. The \textit{light-cone gauge} on the phase space is defined by adding to the above set two \textit{gauge-fixing constraints} \( A_+ \approx 0 \) and \( A_- \approx 0 \), since we have two first class constraints. All the constraints in the set now become second class. The persistency of the gauge-fixing constraints is easily shown to be secured implying that the gauge is \textit{accessible} and thus there is no inconsistency in adopting the above two \textit{local} (weak) gauge-fixing conditions. Similar arguments clearly hold also in the conventional Coulomb gauge formulation.

We now construct the Dirac bracket to implement the above set of constraints in the theory. The following star bracket

\[
\{ f, g \}^* = \{ f, g \} - \int d^2 u \{ f, \pi^+(u) \} \{ A_+(u), g \} + \int d^2 u \{ f, A_+(u) \} \{ \pi^+(u), g \}
\]

has the property that it vanishes for arbitrary \( g \) (or \( f \)) when \( f \) (or \( g \)) is equal to one of the variables \( \pi^+ \) or \( A_+ \). We may then set \( \pi^+ = 0, A_+ = 0 \) as strong [18] equalities and they are \textit{eliminated} from the theory. The star brackets of the constraints are found to coincide with the corresponding Poisson brackets and the same holds true for the star brackets among the remaining canonical variables. We may thus effectively ignore \( A_+ \) and \( \pi^+ \) completely and continue using Poisson brackets. The modifications needed to take care of the remaining set of constraints, which we rename as \( \mathcal{T}_m, \ m = 1, 2..6: \mathcal{T}_1 \equiv \mathcal{T}^-, \mathcal{T}_2 \equiv \mathcal{T}^1, \mathcal{T}_3 \equiv \mathcal{T}, \mathcal{T}_4 \equiv \mathcal{T}^*, \mathcal{T}_5 \equiv \mathcal{A}_-, \mathcal{T}_6 \equiv \Omega \) is done by following the standard procedure.

Define the constraint matrix \( C(x, y) \)
The inverse matrix is defined by

\[ \int d^2 z C_{mk}(x, z) C^{-1}_{kn}(z, y) = \delta_{mn} \delta^2(x - y) \]  

(10)

We find \( C^{-1}(x, y) \) to be given by

\[
\begin{pmatrix}
0 & -4a \partial^x_+ & 0 & 0 & 0 \\
4a \partial^x_+ & \left[ \phi^*(x) \phi(y) + \phi(x) \phi^*(y) \right] & 2ai\phi(x) & -2ai\phi(x) & 0 & 0 \\
0 & 2ai\phi(y) & (2a)^2 & 0 & 0 \\
0 & -2ai\phi^*(y) & (2a)^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 2(2a)^2 \\
0 & -4a & 0 & 0 & 0 \\
\end{pmatrix}
\]

\[ \frac{K(x - y)}{(2a)^2} \]  

(11)

where \( K(x - y) = -(1/4) \epsilon(x^- - y^-) \delta(x^1 - y^1), \partial_-^x K(x, y) = (-1/2) \delta^2(x - y) \). Here \( \epsilon(x) = 1 \) for \( x > 0 \), \( -1 \) for \( x < 0 \) and \( \epsilon(0) = 0 \). The Dirac bracket which implements all the constraints \( \nabla_m \) is then constructed to be

\[ \{f, g\}_D = \{f, g\} - \int d^2u d^2v \{f, \nabla_m(u)\} C^{-1}_{mn}(u, v) \{\nabla_n(v), g\} \]  

(12)

It has the property \( \{f, \nabla_m\}_D = \{\nabla_m, f\}_D = 0 \) for arbitrary dynamical variable \( f \). We may then set \( \nabla_m = 0 \) as strong equalities and the hamilton’s eq. involves now the Dirac bracket in place of the Poisson one. The only independent variables left are \( \phi \) and \( \phi^* \) since \( \nabla_m = 0, A_+ = 0 \) lead to \( \pi = \partial_- \phi^* \), \( \pi^* = \partial_- \phi \), \( A_+ = \pi^1 = 0, \pi^- = aA_1 \) while (5) reduces to the Lagrange eq. (3c) which determines \( A_1 \) in terms of the charge density \( j^+ \).

From (12) we compute
\{\phi, \phi\}_D = 0, \ \{\phi^*, \phi^*\}_D = 0, \ \{\phi, \phi^*\}_D = \{\phi^*, \phi\}_D = K(x,y) \quad (13)

which are the well known light-front Dirac brackets. Some other useful ones are

\{\pi, A_1\}_D = -\frac{i}{4a} [-4\pi(x)K(x,y) + \phi^* \delta^2(x-y)] \quad (14a)

\{\pi^*, A_1\}_D = \frac{i}{4a} [-4\pi^*(x)K(x,y) + \phi^2(x-y)] \quad (14b)

\{\phi, A_1\}_D = \frac{i}{2a} [\phi(y) - 2\phi(x)] K(x,y) \quad (14c)

\{A_1, A_1\}_D = \frac{1}{(2a)^2} [\phi(x)\phi^*(y) + \phi^*(x)\phi(y)] K(x,y) \quad (14d)

The light-front Hamiltonian in the light-cone gauge, obtained by substituting all the constraints in $H''$, takes the simple form

$$ H(\tau) = \int d^2x \ (D_1\phi)(\tilde{D}_1\phi^*) $$

(15)

to be compared with the involved one obtained in the equal-time formulation [6-9]. There is still a $U(1)$ global gauge symmetry generated by $Q$. The scalar fields transform under this symmetry but they are left invariant under the local gauge transformations since, $\{\Omega, f\}_D = 0$.

It is mandatory to check the self-consistency. From the hamilton’s eq. for $\phi$ we derive (we set $e = 1$)

$$ \partial_- \partial_+ \phi(x, \tau) = \{\pi^*(x, \tau), H(\tau)\}_D $$

$$ = \frac{1}{2} D_1 D_1 \phi + \int d^2y \ j^1(y, \tau) \{\pi^*, A_1\}_D $$

$$ = \frac{1}{2} D_1 D_1 \phi + \frac{1}{4a} \left[ \phi (\phi^* D_1 \phi - \phi \tilde{D}_1 \phi^*) + \pi^*(x, \tau) \right. $$

$$ \left. \int d^2y (\phi^* D_1 \phi - \phi \tilde{D}_1 \phi^*)(y, \tau) \epsilon(x^- - y^-) \delta(x^1 - y^1) \right] \quad (16) $$

Comparing (16) with (3a) it is suggested to introduce a new variable in our formulation, indicated for convenience by (the above eliminated) $A_+$, and whose expression is that
obtained from solving (3d). Eq. (3b) is derived from $\Omega = 0$ and it is then straightforward to check (3c). The hamiltonian theory in the light-cone gauge constructed here is thus shown self-consistent. The variable $A_+$ has reappeared and we are effectively imposing $A_- = 0$ and not $A_\pm = 0$ which would in its turn imply setting the gauge invariant quantity $F_{+-}$ to be vanishing, leading in general to contradiction with (3d). Similar discussion can be made in the Coulomb gauge formulation as regards to $A^0$. Contrary to the suggestions made in [9] there arises no inconsistency on using the non-covariant local gauges. That only the nonlocal gauges may describe [9] the fractional statistics consistently in the present theory is not tenable. The conventional Coulomb gauge or nonlocal gauge-fixing conditions lead to quite complicated interactions and hamiltonian and renormalized theory seems difficult to construct. They do have the advantage of showing a dual description [6-9] in terms of free fields with multivalued operators and the manifest fractional statistics which arises from the graded equal-time commutation relations. In our case also it is possible to rewrite the Hamiltonian density in (15) as $\mathcal{H} = (\partial \hat{\phi})(\partial \hat{\phi}^*)$ if we note that $A_1 = \partial \Lambda$ where $8a\Lambda(x^- , x^1) = \int \epsilon(x^- - y^-)\epsilon(x^1 - y^1)j^+(y)$ and define $\hat{\phi} = e^{i\Lambda}\phi$, $\hat{\phi}^* = e^{-i\Lambda}\phi^*$. In view of (14c-d) the field $\hat{\phi}$ then does not have the vanishing Dirac bracket (or commutator) with itself. The theory is quantized via the correspondence of $i\{f , g\}_D$ with the commutator $[f , g]$ among the corresponding field theory operators. When there is ambiguity in the operator ordering we resort to the Weyl ordering.

3. Relativistic Covariance and Absence of Anomaly

In the non-covariant gauge like the one chosen here the manifest covariance is lost and even the scalar field may acquire some unconventional transformation properties. The relativistic invariance is shown by constructing the field theory space time symmetry generators and verifying that they give rise to Poincaré algebra. The canonical energy-momentum tensor derived from (1) is given by

$$\theta_c^{\mu\nu} = (D^\mu \phi^*)(\partial^\nu \phi) + (D^\mu \phi)(\partial^\nu \phi^*) + ae^{\sigma\mu\rho}A_\sigma \partial^\nu A_\rho - \eta^{\mu\nu}L$$

(17)
where $\partial_\mu \theta_c^{\mu\nu} = 0$ by construction. In the light-cone gauge they get simplified, for example,

\begin{align}
\theta_c^{++} &= 2\pi\pi^* \quad (18a) \\
\theta_c^{+1} &= -(\pi \partial_1 \phi + \pi^* \partial_1 \phi^*) \quad (18b) \\
\theta_c^{-+} &= (\partial_1 \phi)(\tilde{\partial}_1 \phi^*) = \mathcal{H} \quad (18c)
\end{align}

The momentum generators defined by $P^\mu = \int d^2x \theta_c^{+\mu}$ are conserved and shown to generate the translations, e.g., $\{\phi, P_\mu\}_D = \partial_\mu \phi$, $\{\phi^*, P_\mu\}_D = \partial_\mu \phi^*$ when we make use of the boundary conditions. The invariance of the classical lagrangian (1) under Lorentz transformation results [19] in the following conserved current

\[ J^{\mu\rho\sigma} = -J^{\mu\rho\sigma} = x^\rho \theta_c^{\mu\sigma} - x^\sigma \theta_c^{\mu\rho} - i \frac{\partial \mathcal{L}}{\partial \partial_\mu A_\alpha}(\Sigma^{\rho\sigma})^{\alpha\beta} A_\beta \]

where $\partial_\mu J^{\mu\rho\sigma} = 0$ on the mass shell and $(\Sigma^{\rho\sigma})^{\alpha\beta} = i(\eta_{\rho\alpha} \eta_{\sigma\beta} - \eta_{\rho\beta} \eta_{\sigma\alpha})$. The generators of the Lorentz transformation on the light-front may hence be defined by

\[ M^{\mu\nu} = -M^{\nu\mu} = \int d^2x J^{+\mu\nu} \]

\[ = \int d^2x \left[ x^\mu \theta_c^{+\nu} - x^\nu \theta_c^{+\mu} - (A_\mu A_\nu - A_\nu A_\mu) \right] \quad (20) \]

and in the light-cone gauge they simplify to

\begin{align}
M^{-1} &= \int d^2x \left[ x^{-} \theta_c^{+1} - x^{1} \theta_c^{+-} - aA_1^2 \right] \quad (21a) \\
M^{+1} &= x^+ P^1 - \int d^2x x^1 \theta_c^{++} \quad (21b) \\
M^{+-} &= x^+ P^- - \int d^2x x^- \theta_c^{++} \quad (21c)
\end{align}

The expressions of the generators as obtained on using the symmetric Belinfante tensor [20,19], $\theta_B^{\mu\nu} = [\theta_c^{\mu\nu} + a\epsilon^{\lambda\mu\beta} \partial_\lambda (A_\beta A^\nu)]$, or the symmetric gauge invariant one [6] differ
from (22) only by a surface term. It is to be stressed that the generators (21) and those for \( P^\mu \) above are \textit{perfectly legitimate set} \cite{19} to use in order to check the relativistic invariance of the theory under discussion. We showed already that the lagrange eqs. (3) are recovered in the hamiltonian framework. A direct verification of the closure of the Poincaré algebra on the mass shell, e.g., where we use (3), then becomes straightforward, though tedious. We note, for example,

\[
\{\theta_c^{++}, \theta_c^{+-}\}_D = -\pi(x)((\partial_1 \phi(y)) \partial^x - \delta^2(x - y) + \pi(y) \partial^x \delta^2(x - y)) - \pi^*(x)((\partial_1 \phi^*(y)) \partial^x - \delta^2(x - y) + \pi(y) \partial^x \delta^2(x - y)), \quad (22a)
\]

\[
\{\theta_c^{++}, \theta_c^{+-}\}_D = 2\pi(x)\{\pi^*, D_1 \phi \tilde{D}_1 \phi^*\}_D + 2\pi^*(x)\{\pi, D_1 \phi \tilde{D}_1 \phi^*\}_D
\]

\[
= 2a(\partial_- A_1)(\partial_- A_+^2) \delta^2(x - y) - \pi(x)(D_1 \phi(y)) \tilde{D}_1^y \delta^2(x - y)
\]

\[
- \pi^*(x)(\tilde{D}_1 \phi^*(y)) D^y_1 \delta^2(x - y). \quad (22b)
\]

\[
\{\theta_c^{++}, A_1^2\}_D = -2A_1(\partial_- A_1) \delta^2(x - y), \quad (22c)
\]

\[
(\theta_c^{1-} - \theta_c^{-1}) = a [\partial_1(A_+ A_1) - \partial_+ A_1^2 - \partial_- A_+^2], \quad (22d)
\]

The light-front hamiltonian formulation of (1) in the light-cone gauge is found to be Poincaré invariant.

The independent variables \( \phi, \phi^* \) satisfy the light-front Dirac brackets (13) and the expressions (19) and (21a) differ from those of the free field theory due to the contributions of the now dependent field \( A_1 \). Such extra terms have been called \cite{10,6} \textit{anomalous spin} induced on the scalar field due to the constrained dynamics generated by the Chern-Simons term. They may also not be removed by a redefinition of the generators which are required to satisfy the Poncaré algebra for relativistic invariance. We discuss this now in our context carefully by considering the Lorentz transformation of the scalar field. From (3), (13), (14), and (21) we derive after some algebra
\[ \{ \phi(x, \tau), M^{-1}(\tau) \}_D = [x^- \partial^1 - x^1 \partial^-] \phi(x, \tau) \]
\[ - \frac{i}{2} \phi(x, \tau) \int d^2 y \epsilon(x^- - y^-) \delta(x^1 - y^1) A_1(y, \tau) \] (23a)

\[ \{ \phi(x, \tau), M^{+1}(\tau) \}_D = [x^+ \partial^1 - x^1 \partial^+] \phi(x, \tau), \] (23b)

\[ \{ \phi(x, \tau), M^{+}(\tau) \}_D = [x^+ \partial^- - x^- \partial^+] \phi(x, \tau) \] (23c),

and on combining (23a) and (23b) we get

\[ \{ \phi(x, \tau), M^{21}(\tau) \}_D = [x^2 \partial^1 - x^1 \partial^2] \phi(x, \tau) \]
\[ + \frac{i}{2} \phi(x, \tau) \int d^2 y \epsilon(x^- - y^-) \delta(x^1 - y^1) A_1(y, \tau) \] (23d)

The extra second term in (23a) or (23d) has been called [10,6] in the the conventional treatment an *anomaly* arising from the anomalous spin term in the generator \( M^{-1} \) or \( M^{12} \). Its presence, however, does not lead to the breakdown of the closure of the space time generators to the Poincaré algebra. From our discussion, however, it is clear that we may as well interpret the anomalous transformation (23a) or (23d) here or those found in the previous works [10,6] as *gauge artefacts*. It is clear from the discussion here that, for example, the unusual transformations of \( A_- \) under space time rotations, viz, \( \{ M^{\mu\nu}, A_- \}_D = 0 \) or under translations, \( \{ P^\mu, A_- \}_D = 0 \), originate from the construction of the Dirac bracket (12). If \( A_- \) transformed normally we would be led in the light-cone gauge to \( A_1 = 0 \) as well. The unusual (anomalous) transformation above may not be thus considered as totally unexpected in the non-covariant gauge being used. This is reinforced by the verification of that the components of the *gauge invariant vector* \( j^\mu \), whether defined in terms of the scalar fields or in terms of the gauge fields according to (3), continue to possess the usual transformation properties of vector field and no anomalous terms are generated. To illustrate, we find
\begin{equation}
\{ A_1(x, \tau), M^{-1}(\tau) \}_D = (x^1 \partial^1 - x^1 \partial^-) A_1 - A_+ + \frac{1}{\partial^-} \partial_1 A_1. \tag{24}
\end{equation}

The last term on the right hand side is to be considered as gauge artefact rather than an anomalous term and it is correctly absent from

\begin{equation}
\{ \partial^- A_1(x, \tau), M^{-1}(\tau) \}_D = (x^1 \partial^1 - x^1 \partial^-)(\partial^- A_1) - (\partial^- A_+) \tag{25}
\end{equation}

since \( j^+ \sim \partial^- A_1 \) and \( j^1 \sim \partial^- A_+ \) and \( j^\mu \) is a contravariant vector. There is no genuine anomaly in the behavior of the scalar field (or the field \( A_1 \)) under the Lorentz transformations which agrees with the same result obtained in the recent gauge independent discussions \cite{12,13} in the equal-time formulation. The physical outcome, e.g., the emergence of (anyonic) excitations obeying fractional statistics should emerge in the dynamics of the relativistic theory here as described by (13) and (15), and not be considered as a consequence of the unconventional transformation law (gauge artefact) of the scalar field connected with the gauge-fixing conditions, which may as well be nonlocal and non-linear. A similar discussion may be given in the conventional local Coulomb gauge. Because of its simplicity the light-front quantized theory in the light-cone gauge promises to be a useful framework for discussing renormalization of the model (1).

\textbf{Acknowledgements}

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO for the hospitality at the International Centre for Theoretical Physics, Trieste. Acknowledgements are due to F. Caruso, R. Shellard, and B. Pimentel for comments.

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