Device-independent quantum private comparison protocol without a third party

Guang Ping He
School of Physics, Sun Yat-sen University, Guangzhou 510275, China

Since unconditionally secure quantum two-party computations are known to be impossible, most existing quantum private comparison (QPC) protocols adopted a third party. Recently, we proposed a QPC protocol which involves two parties only, and showed that although it is not unconditionally secure, it only leaks an extremely small amount of information to the other party. Here we further propose the device-independent version of the protocol, so that it can be more convenient and dependable in practical applications.

PACS numbers: 03.67.Dd, 03.65.Ud, 03.67.Mn, 03.67.Ac, 03.65.Ta

I. INTRODUCTION

Device-independent (DI) quantum cryptography has caught great interests recently [1, 2]. It aims to replace the model of the physical devices used in cryptographic protocols with physically testable assumptions, e.g., the certification of nonlocality. Thus the devices can be treated as black boxes that produce outputs correlated with some inputs. This brings the advantage that the assumptions needed to guarantee the security of the protocol can be significantly reduced, so that the knowledge of the internal workings of the devices is not required. The protocol remains reliable even if the devices are provided by the adversary. Such a higher degree of security makes DI protocols more dependable in practical applications than traditional quantum cryptography.

Currently, most existing DI protocols focused on quantum key distribution (QKD) [3, 4], and related tasks [5], where all legitimate participants always collaborate honestly against the attack of external cheaters. On the other hand, as pointed out in Ref. [6], DI two-party cryptography remains a largely unexplored territory, as very few researches were dedicated to multi-party secure computation problems, where the legitimate participants do not trust each other since some of them may cheat. To our best knowledge, Refs. [7, 8] are the only references on this field so far, which studied quantum bit commitment, quantum coin tossing, weak string erasure, and position verification.

In this paper, we will take the first step towards the DI solution of another multi-party secure computation problem – quantum private comparison (QPC), a.k.a. the socialist millionaire problem [9]. The goal of QPC is to compare two secret numbers $a$ and $b$ of two parties Alice and Bob, respectively, so that they finally learn whether $a$ and $b$ are equal, without revealing any extra information on their values (other than what can be inferred from the comparison result) to the other party. As a typical example of multi-party secure computations, QPC plays essential roles in cryptography, with many applications in e-commerce and data mining.

As it is well-known, unconditionally secure quantum two-party secure computations are impossible [10–12], i.e., the dishonest party can always obtain a non-trivial amount of information on the secret data of the other party. QPC is also covered. To circumvent the problem, almost all existing QPC protocols ([23–30] and the references therein) added a third party to help Alice and Bob accomplish the comparison. The protocol in Ref. [30] is a rare exception, in which such a third party is absent. But it was later found to be completely insecure [31, 32], because the entire secret data of the honest party will always be exposed to the other party.

Very recently, we proposed a device-dependent (DD) QPC protocol which involves two parties only [33], and showed that the average amount of information leaked to the dishonest party can be as low as 14 bits for any length of the bit-string being compared. Therefore, despite that the protocol is not unconditionally secure, the performance is quite good, and this is achieved without relying on the third party. Here we will further upgrade the protocol to the device-independent (DI) version, so that it can enjoy even better security and versatility in practical applications.

The paper is organized as follows. In the next section, our previous DD protocol will be briefly reviewed. Then in section 3, it will be turned into the DI version, with its security analyzed in section 4.

II. THE DD PROTOCOL

Let $H(x)$ be a classical hash function which is a 1-to-1 mapping between the $n$-bit strings $x$ and $y = H(x)$ (i.e., $H: \{0, 1\}^n \rightarrow \{0, 1\}^n$). Denote the two orthogonal states of a qubit as $|0\rangle_0$ and $|1\rangle_0$, respectively, and define $|0\rangle_1 = (|0\rangle_0 + |1\rangle_0)/\sqrt{2}$, $|1\rangle_1 = (|0\rangle_0 - |1\rangle_0)/\sqrt{2}$. That is, the subscript $\sigma = 0, 1$ in $|\gamma\rangle_\sigma$ stands for two incompatible measurement bases, while $\gamma = 0, 1$ distinguishes the two states in the same basis. In Ref. [43], the following protocol was proposed.

The DD QPC Protocol (for comparing Alice’s $n$-bit string $a = a_1a_2...a_n$ and Bob’s $n$-bit string $b = b_1b_2...b_n$):

1. Alice and Bob each choose a random bit string $x = x_1x_2...x_n$ and compute $y = H(x)$.
2. Alice sends $y$ to Bob.
3. Bob sends $y$ to Alice.
4. Alice and Bob compare the bit strings $y$.

Guang Ping He
School of Physics, Sun Yat-sen University, Guangzhou 510275, China

---

*Electronic address: hegp@mail.sysu.edu.cn*
(1) Using the hash function $H(x)$, Alice calculates the $n$-bit string $h^A \equiv h^A_1 h^A_2 ... h^A_n = H(a)$, and Bob calculates the $n$-bit string $h^B \equiv h^B_1 h^B_2 ... h^B_n = H(b)$.

(2) From $i = 1$ to $n$, Alice and Bob compare $h^A$ and $h^B$ bit-by-bit as follows.

If $i$ is odd, then:

\[ (2.1A) \text{Alice randomly picks a bit } \gamma_i^A \in \{0, 1\} \text{ and sends Bob a qubit in the state } |\gamma_i^A\rangle_{h^A} \]

\[ (2.2A) \text{Bob measures it in the } h^B \text{ basis and obtains the result } |\gamma_i^B\rangle_{h^B}. \text{ He announces } \gamma_i^B \text{ while keeping } h^B_i \text{ secret.} \]

(2.3A) Alice announces $\gamma_i^A$.

If $i$ is even, then:

\[ (2.1B) \text{Bob randomly picks a bit } \gamma_i^B \in \{0, 1\} \text{ and sends Alice a qubit in the state } |\gamma_i^B\rangle_{h^B} \]

\[ (2.2B) \text{Alice measures it in the } h^A \text{ basis and obtains the result } |\gamma_i^A\rangle_{h^A}. \text{ She announces } \gamma_i^A \text{ while keeping } h^A_i \text{ secret.} \]

(2.3B) Bob announces $\gamma_i^B$.

(2.4) If $\gamma_i^A \neq \gamma_i^B$, then they conclude that $a \neq b$, and abort the protocol immediately without comparing the rest bits of $h^A$ and $h^B$. Otherwise they continue with the next $i$.

(3) If Alice and Bob find $\gamma_i^A = \gamma_i^B$ for all $i = 1, ..., n$ then they conclude that $a = b$.

Note that in the protocol, we compare the hash functions $h^A$ and $h^B$ instead of the secret strings $a$ and $b$ themselves. The purpose is to change the information leaked to the other party from direct information into mutual information on $a$ and $b$. It will not change the total amount of information leaked.

### III. THE DI PROTOCOL

In each round of step (2) of the above protocol, $h^A_i$ and $h^B_i$ are compared in a non-entangled way. That is, one party (e.g., Alice) prepares a qubit in the $h^A_i$ basis, then the other party (e.g., Bob) measures it in the $h^B_i$ basis. This can be replaced with the following entangled method. Alice prepares the Bell state $|\Phi^+\rangle = (|0\rangle_0 |0\rangle_1 + |1\rangle_0 |1\rangle_1)/\sqrt{2}$. She keeps the first qubit and sends Bob the second one. Then they measure their qubits in the $h^A_i$ or $h^B_i$ basis, respectively. Obviously, when $h^A_i = h^B_i$, their measurement results will always be equal, while if $h^A_i \neq h^B_i$, their results will be different with probability 1/2. Thus the resultant protocol is equivalent to our original DD protocol.

In DI cryptography, Bell states can further be replaced by pairs of devices called nonlocal boxes, which can be supplied by either Alice or Bob, or even other untrustworthy parties. These devices can be treated as DI black boxes that take inputs and produce outputs, without the need for the possibility to check how they work internally. Each of Alice’s (Bob’s) boxes has four inputs $S^{(0)}_A$, $S^{(1)}_A$, $S^{(2)}_A$, and $S^{(3)}_A$ ($S^{(0)}_B$, $S^{(1)}_B$, $S^{(2)}_B$, and $S^{(3)}_B$). For each input, Alice’s (Bob’s) box can produce either of the two outputs $\gamma^{(k)}_A = \pm 1$ ($\gamma^{(k)}_B = \pm 1$), $k = 0, 1, 2, 3$. When the boxes are manufactured honestly, they should act exactly like $|\Phi^+\rangle$. For example, if we treat $|0\rangle_0$ and $|1\rangle_0$ as the eigenstates of the Pauli operator $\sigma_x$, then an honest implementation of such a DI box pair can be realized using the Bell state $|\Phi^+\rangle$ itself, with the inputs of the box at Alice’s or Bob’s side being implemented with the measurements on her or his qubit, respectively, as 

\[ S^{(0)}_A = S^{(0)}_B = \sigma_z, \]

\[ S^{(1)}_A = S^{(1)}_B = \sigma_x, \]

\[ S^{(2)}_A = S^{(2)}_B = (\sigma_z + \sigma_x)/\sqrt{2}, \]

\[ S^{(3)}_A = S^{(3)}_B = (\sigma_z - \sigma_x)/\sqrt{2}. \]

Define the Clauser-Horne-Shimony-Holt (CHSH) polynomials $C_i$.

\[ C_1 = \langle \gamma^{(2)}_A \gamma^{(0)}_B \rangle + \langle \gamma^{(2)}_A \gamma^{(1)}_B \rangle + \langle \gamma^{(2)}_A \gamma^{(0)}_B \rangle - \langle \gamma^{(2)}_A \gamma^{(1)}_B \rangle \]

and

\[ C_2 = \langle \gamma^{(0)}_A \gamma^{(2)}_B \rangle + \langle \gamma^{(1)}_A \gamma^{(2)}_B \rangle + \langle \gamma^{(0)}_A \gamma^{(3)}_B \rangle - \langle \gamma^{(1)}_A \gamma^{(3)}_B \rangle, \]

where the correlator $\langle \gamma^{(k_1)}_A \gamma^{(k_2)}_B \rangle$ is defined as the probability $\Pr(\gamma^{(k_1)}_A = \gamma^{(k_2)}_B) - \Pr(\gamma^{(k_1)}_A \neq \gamma^{(k_2)}_B)$. If the boxes work as they were claimed, then the CHSH values should reach the point of maximal quantum violation of the CHSH Bell inequality, i.e., $C_1 = C_2 = 2\sqrt{2}$. Also, when Alice and Bob choose the same input $S^{(0)}$ (or $S^{(1)}$) for a pair of the boxes, they should always obtain maximally correlated outputs $\gamma^{(0)}_A = \gamma^{(0)}_B$ (or $\gamma^{(1)}_A = \gamma^{(1)}_B$). On the contrary, for local boxes which have fixed output values without displaying any nonlocal correlation, the CHSH values will satisfy the CHSH Bell inequality $C_1 \leq 2$ and $C_2 \leq 2$.

Also, like other DI cryptographic protocols (e.g., [2, 3, 5]), it is assumed that Alice’s and Bob’s locations are secure, in the sense that no unwanted information can leak out to the outside. Thus one party cannot know the other’s inputs and outputs to the DI boxes, unless the latter party announces them.

With such nonlocal boxes, our protocol can be turned into the following DI version.

**The DI QPC Protocol** (for comparing Alice’s $n$-bit string $a = a_1a_2...a_n$ and Bob’s $n$-bit string $b = b_1b_2...b_n$):

(A) The check mode: Alice and Bob share many pairs of DI boxes and check them as follows.

(i) Alice randomly chooses some of them, asking Bob to randomly pick inputs into his boxes and announce both his inputs and outputs, then she randomly picks inputs into her boxes and records the outputs.
(ii) Bob randomly chooses another portion of the box pairs, asking Alice to randomly pick inputs into her boxes and announce both her inputs and outputs; then he randomly picks inputs into his boxes and records the outputs.

In both (i) and (ii), each of them check whether they have correlated outputs when they picked the same inputs $S^{(0)}$ or $S^{(1)}$ for the boxes of the same pair, and use the outputs they obtained when picking different inputs to check whether the CHSH values $C_1$ and $C_2$ violate the CHSH Bell inequality.

(B) The compare mode: Alice and Bob randomly pick some of the rest unchecked pairs of boxes to continue with the following steps.

1. Using the hash function $H(x)$, Alice calculates the $n$-bit string $h^A = h_1^A h_2^A \ldots h_n^A = H(a)$, and Bob calculates the $n$-bit string $h^B = h_1^B h_2^B \ldots h_n^B = H(b)$.

2. From $i = 1$ to $n$, Alice and Bob compare $h^A$ and $h^B$ bit-by-bit as follows.

   If $i$ is odd, then:

   (2.1A) Alice randomly picks a DI box, inputs $S_A^{(h_i^A)}$ and records the output $\gamma_A^{(h_i^A)}$. Then she tells Bob which one she picked.

   (2.1B) Bob inputs $S_B^{(h_i^B)}$ into his corresponding box from the same pair and obtains the output $\gamma_B^{(h_i^B)}$. He announces $\gamma_B^{(h_i^B)}$ while keeping $S_B^{(h_i^B)}$ secret.

   (2.3A) Alice announces $\gamma_A^{(h_i^A)}$.

   If $i$ is even, then:

   (2.2A) Bob inputs $S_B^{(h_i^B)}$ into his corresponding box from the same pair and obtains the output $\gamma_B^{(h_i^B)}$. He announces $\gamma_B^{(h_i^B)}$ while keeping $S_B^{(h_i^B)}$ secret.

   (2.2B) Alice inputs $S_A^{(h_i^A)}$ into her corresponding box and obtains the output $\gamma_A^{(h_i^A)}$. She announces $\gamma_A^{(h_i^A)}$ while keeping $S_A^{(h_i^A)}$ secret.

(2.3B) Bob announces $\gamma_B^{(h_i^B)}$.

(2.4) If $\gamma_A^{(h_i^A)} \neq \gamma_B^{(h_i^B)}$, then they conclude that $a \neq b$ and abort the protocol immediately without comparing the rest bits of $h^A$ and $h^B$. Otherwise, they continue with the next $i$.

3. If Alice and Bob find $\gamma_A^{(h_i^A)} = \gamma_B^{(h_i^B)}$ for all $i = 1, \ldots, n$ then they conclude that $a = b$.

For clarity, in the above description we wrote the check mode and the compare mode separately. But for better security, the two modes should actually be mixed together. That is, Alice and Bob choose some DI boxes to run the check mode first, and they choose one of the unchecked DI box and shift to the compare mode to compare one bit of $h^A$ and $h^B$. Then they choose some other DI boxes and run the check mode again, followed by another round of the compare mode to compare the next bit of $h^A$ and $h^B$, and so on. The times for shifting modes are decided randomly by both parties in turns. Otherwise, if the compare mode is run only after the check mode is completed, the provider of the DI boxes may cheat by building a secret timer into each box, so that they all act honestly like the entangled state $|\Phi^+\rangle$ during the check mode, then switch to some kinds of cheating mode (e.g., giving fixed outputs regardless the input values) automatically at the time when the compare mode is expected to begin.

Also, it is important that in step (2.1A) (step (2.1B)), Alice (Bob) should finish input to her (his) box and record the output before telling the other party which box is picked. Otherwise, if she (he) announces which box is picked first, and postpone the input/output process to step (2.3) after the other party completed the input/output in step (2.2), then it could also bring security problems. That is, if the other party is dishonest and he provides the DI boxes, he may build a remote control into each box, so that they usually act like real nonlocal boxes that can certainly pass the CHSH inequality check. But when he knows which box is picked for the compare mode, he engages the remote control to turn the box into the cheating mode which gives a fixed output that known to himself beforehand, so that he can cheat with the method that we will describe below in the paragraph before Theorem 1.

IV. Security

Since unconditionally secure QPC is impossible when only two parties are involved, we do not attempt to make the information leaked in our protocol arbitrarily close to 0. Instead, our goal is merely to make it stay at a low level. That is, the cheating we are going to deal with is how the dishonest Alice/Bob tries to increase his/her information on the other’s secret data.

There is surely no security problem to consider when $a = b$, because both parties naturally know the secret data of each other from the comparison result. Now let us study the case when the protocol outputs $a \neq b$.

An important feature of our protocol is that, if it aborts after running $m$ ($1 \leq m \leq n$) rounds of step (2), the last $n - m$ bits of $h^A$ and $h^B$ will not be compared any more. They will not be input into the DI boxes, or enter the protocol in any other form at all. Thus it is obvious that these bits remain completely secret to the other party. As a consequence, the amount of mutual information leaked to each party is $m$ bits at the most.

Therefore, the goal of a dishonest party is to increase $m$. To do so, he/she has to make the protocol abort as late as possible. Without loss of generality, let us assume that Alice cheats, and suppose that she prepares and supplies all the DI boxes, which is a case that benefits her the most so that we can obtain a general upper bound of the successful cheating probability.

It is easy for Alice to cheat in each of the odd rounds, because in step (2.3A) she can always announce $\gamma_A^{(h_i^A)} = \gamma_B^{(h_i^B)}$ even if the actual result is $\gamma_A^{(h_i^A)} \neq \gamma_B^{(h_i^B)}$. This ensures that the protocol will never abort at these rounds.
But in each of the rest \( k = m/2 \) even rounds among the first \( m \) rounds of step (2), she is required to announce \( \gamma_A^{(h_B^k)} \) in step (2.2B) before Bob announces \( \gamma_B^{(h_B^k)} \) in step (2.3B). Since she wishes to announce a value that satisfies \( \gamma_A^{(h_B^k)} = \gamma_B^{(h_B^k)} \) so that the protocol will not abort, she needs to guess the \( \gamma_B^{(h_B^k)} \) value that Bob will obtain from his DI box. Now let us prove that the probability \( \text{pguess} \) for her to make a correct guess cannot equal exactly to 1.

**Theorem 1.** If all of Bob’s DI boxes always give the same output \( \gamma_B^{(0)} = \gamma_B^{(1)} \) no matter he inputs \( S_B^{(0)} \) or \( S_B^{(1)} \), then they cannot pass the CHSH inequality check.

**Proof.** Recall that step (ii) of the check mode requires Alice to announce both her inputs and outputs to a DI box when Bob has not input anything into his corresponding box yet. Therefore, for these boxes, Alice cannot monitor Bob’s announced values of his \( \gamma_B \)'s and \( \gamma_B^{(1)} \)'s, then first to fake her output values \( \gamma_A^{(2)} \)'s and \( \gamma_A^{(3)} \)'s to make them pass the CHSH inequality check. Consequently, if all boxes give \( \gamma_B^{(0)} = \gamma_B^{(1)} \), there will be \( \langle \gamma_A^{(2)} \gamma_B^{(0)} \rangle = \langle \gamma_A^{(2)} \gamma_B^{(1)} \rangle \) and \( \langle \gamma_A^{(3)} \gamma_B^{(0)} \rangle = \langle \gamma_A^{(3)} \gamma_B^{(1)} \rangle \). Substituting them into Eq. (2), we immediately obtain

\[
C_1 = 2 \langle \gamma_A^{(2)} \gamma_B^{(0)} \rangle \leq 2,
\]

which is far below the correct expected value \( 2\sqrt{2} \). Consequently, when Bob checks the \( C_1 \) value given by the boxes picked in step (ii), he will catch Alice cheating. This ends the proof.

Of course, dishonest Alice does not have to make all of Bob’s DI boxes always give the same output \( \gamma_B^{(0)} = \gamma_B^{(1)} \). She can mix a small number of such boxes with real nonlocal boxes (i.e., these act exactly like \( |\Phi^+\rangle \)). With this method, the corresponding CHSH value \( C_1 \) may merely deviate slightly from \( 2\sqrt{2} \), so that the cheating could be covered by statistical fluctuation. But then it is obvious that once Bob randomly picks a box in step (2.1B), this box does not necessarily be one of these which give fixed output \( \gamma_B^{(0)} = \gamma_B^{(1)} \) regardless Bob’s input. Once Bob picks a box whose output \( \gamma_B^{(h_B^k)} \) depends on his input \( S_B^{(h_B^k)} \), the no-signaling principle prevents Alice from knowing \( \gamma_B^{(h_B^k)} \) with certainty before Bob announces it. Thus Theorem 1 leads us to the following conclusion.

**Corollary.** In step (2.2B), the probability \( \text{pguess} \) for dishonest Alice to make a correct guess on the output value \( \gamma_B^{(h_B^k)} \) that Bob will announce in step (2.3B) cannot equal to 1.

In this case, the upper bound of the average amount of information on Bob’s secret data \( b \) that leaked to Alice is calculated as follows. Continue with the analysis before Theorem 1. As Alice has probability \( \text{pguess} \) to make a correct guess on Bob’s \( \gamma_B^{(h_B^k)} \), she can announce \( \gamma_A^{(h_B^k)} = \gamma_B^{(h_B^k)} \) in step (2.2B), so that in each of the first \( k \) even rounds there is probability \( \text{pguess} \) that the protocol will not abort. Consequently, the probability for the protocol to abort at the \( m \)th round (i.e., it happens to continue for the first \( k - 1 \) even rounds while aborts at the \( k \)-th even round) is

\[
p_{\text{abort}}^m = \text{pguess}^{k-1}(1 - \text{pguess}).
\]

That is, with probability \( p_{\text{abort}}^m \) the protocol will abort at the \( m \)th round, and dishonest Alice will know nothing about the rest \( n - m \) bits of Bob’s hash value, so that she learns \( m \) bits of information at the most. Also, with probability \( \text{pguess} \) the protocol does not aborts until all bits are compared (here \( [n/2] \) means the integer part of \( n/2 \)). In this case she learns all the \( n \) bits. Summing over all possible \( m \) values and recall that \( m = 2k \) (as the protocol will not abort at the odd rounds when Alice cheats), the average amount of mutual information leaked to dishonest Alice is bounded by

\[
I_A = \sum_{k=1}^{[n/2]} m \times p_{\text{abort}}^m + n \times \left[ \text{pguess} \right]^{[n/2]} = \sum_{k=1}^{[n/2]} 2k \times \text{pguess}^{k-1}(1 - \text{pguess}) + n \times \text{pguess}^{[n/2]},
\]

Note that this is merely a upper bound, as we have not considered the cases where dishonest Alice cannot even escape the detection in the check mode. That is, Alice may reach this bound only if her DI boxes can pass the check mode with the maximum probability 1.

When taking into consideration the probability for Alice to pass the check mode, the maximum value of \( \text{pguess} \) will also be bounded. But calculating the exact bound could be complicated, because the CHSH values are merely statistical results. In practice we cannot expect to find \( C_1 = C_2 = 2\sqrt{2} \) exactly. Some statistical fluctuation has to be allowed. Noise and manufacture imperfections of the experimental devices could also affected the actual outcome of the CHSH values. Thus it is hard to obtain a general result without knowing the specific performance of the experimental devices used for the implementation of the protocol. Here, take for example, let us follow Ref. [4] to take \( C_1, C_2 \geq 2.5 \) as acceptable values, and neglect the noise and device imperfections. In this case, suppose that Alice prepares 61% of the DI boxes as real nonlocal boxes (which have the theoretical expected values \( C_1 = C_2 = 2\sqrt{2} \)), while the rest 39% boxes are prepared as local boxes which will produce fixed output values known beforehand to herself regardless Bob’s input (which have \( C_1 = C_2 = 2 \)). Then the theoretical expected average CHSH values will be \( 61\% \times 2\sqrt{2} + 39\% \times 2 \approx 2.505 \), so that she stands a
nontrivial probability to pass the check mode. The local boxes enable Alice to guess Bob’s output $\gamma_B^{(h_B^m)}$ with probability 1. But for the real nonlocal boxes, as shown in Eq. (5) of Ref. [43], the maximum probability for her to guess Bob’s output is $p_{\text{max}} = \cos^2(\pi/8) \simeq 0.8536$. In this case we have

$$p_{\text{guess}} = 61\% \times 0.8536 + 39\% \times 1 \simeq 0.91.$$  (7)

In Fig. 1 we take $p_{\text{guess}} = 0.91$ and show the probability for the protocol to abort at exactly the $m$th round (i.e., $p_{\text{abort}}^m$ in Eq. 5) when Alice cheats. We can see that most of the time the protocol will abort very soon. Thus the corresponding amount of information leaked will be small. The cases that the protocol can last many rounds will occur with extremely small probabilities only. Therefore, there is very low chances that dishonest Alice can gain a large amount of information on Bob’s secret data.

Fig. 2 shows the average amount of information leaked to Alice (i.e., $I_A$ in Eq. 6) as a function of the length $n$ of the bit-strings $a$ and $b$ being compared. We can see that $I_A$ never exceeds 23 bits for any length $n$ of $a$ and $b$ when $p_{\text{guess}} = 0.91$. It is a little higher than that of the DD version, whose upper bound of the amount of information leaked to Alice is 14 bits, as proven in Ref. [43]. This is because in the DI protocol, 39% of the DI boxes can be local boxes which give fixed outputs, so that dishonest Alice’s $p_{\text{guess}}$ can go beyond the $p_{\text{max}}$ in the DD version. But we should note that both 23 bits (for the DI protocol) and 14 bits (for the DD protocol) are merely loose upper bounds. This is because, as elaborated in Ref. [43], when dishonest Alice saturates the maximum probability $p_{\text{max}}$ for guessing Bob’s output $\gamma_B^{(h_B^m)}$ in the DD protocol, there is no known method for her to learn Bob’s input $S_B^{(h_B^m)}$ with probability 1. Thus she cannot gain exactly $m$ bits of information on $h_B$ (and therefore Bob’s string $b$) when the protocol aborts in the $m$th round of step (2). The same thing is also true for the local boxes. That is, while they can enable Alice to guess Bob’s output $\gamma_B^{(h_B^m)}$ with probability 1, they cannot provide her the information on Bob’s input $S_B^{(h_B^m)}$ any more. It is worth studying what will be the tight upper bounds of the information leaked in both the DD and DI protocols.

For illustration purposes, in Fig. 2 we also plotted $I_A$ as a function of $n$ when $p_{\text{guess}} = 0.99$. It shows that $I_A$ never exceeds 200 bits for any length $n$ even for such a high $p_{\text{guess}}$ value. Considering that most data we use in real applications nowadays are generally at the size of megabytes to gigabytes, leaking only 200 bits is not serious at all. We would also like to emphasize that there is no known method to create DI boxes which can pass the CHSH test with a nontrivial probability while reaching $p_{\text{guess}} = 0.99$ simultaneously. But even if such “black magic” exists, our result shows that the security level of the DI QPC protocol is still quite acceptable.

From the symmetry of the protocol, it is trivial to show that the same conclusion also applies if Bob (instead of Alice) cheats.

V. SUMMARY

Thus we showed that although our DI QPC protocol is not unconditionally secure, the average of the amount of information leaked to the dishonest party is very low, and it has a fixed upper bound for any length of the secret strings being compared. Also, the elimination of the third party surely enhances the convenience for the practical applications of QPC.

This result also serves as yet another example that two-party cryptography can enjoy the advantage of the DI scenario too, so that its security can be even more reliable than their DD counterparts. It is also interesting to study in future works whether the method can be applied to quantum private query (QPQ) [44–46], which is another kind of multi-party secure computation protocols that became a hot topic recently due to its great practical significance.
1. A. Acín, S. Massar, and S. Pironio, *New J. Phys.*, 8, 126 (2006). Efficient quantum key distribution secure against no-signaling eavesdroppers
2. A. Acín, N. Brunner, N. Gisin, S. Massar, S. Pironio, and V. Scarani, *Phys. Rev. Lett.* 98, 230501 (2007). Device-independent security of quantum cryptography against collective attacks
3. S. Pironio, A. Acín, N. Brunner, N. Gisin, S. Massar, and V. Scarani, *New J. Phys.* 11, 045021 (2009). Device-independent quantum key distribution secure against collective attacks
4. M. McKague, *New J. Phys.* 11, 103037 (2009). Device independent quantum key distribution secure against coherent attacks with memoryless measurement devices
5. L. Masanes, S. Pironio, and A. Acín, *Nat. Commun.* 2, 238 (2011). Secure device-independent quantum key distribution with causally independent measurement devices
6. J. Barrett, R. Colbeck, and A. Kent, *Phys. Rev. Lett.* 110, 010503 (2013). Memory attacks on device-independent quantum cryptography
7. C. C. W. Lim, C. Portmann, M. Tomamichel, R. Renner, and N. Gisin, *Phys. Rev. X* 3, 031006 (2013). Device-independent quantum key distribution with local bell test
8. J. Ribeiro, G. Murta, and S. Wehner, *Phys. Rev. A* 97, 022307 (2018). Fully device-independent conference key agreement
9. J. Kaniewski and S. Wehner, *New J. Phys.* 18, 055004 (2016). Device-independent two-party cryptography secure against sequential attacks
10. J. Silman, A. Chailloux, N. Aharon, I. Kerenidis, S. Pironio, and S. Massar, *Phys. Rev. Lett.* 106, 220501 (2011). Fully distrustful quantum bit commitment and coin flipping
11. L. Y. Zhao, Z. Q. Yin, S. Wang, W. Chen, H. Chen, G. C. Guo, and Z. F. Han, *Phys. Rev. A* 92, 062327 (2015). Measurement-device-independent quantum coin tossing
12. E. Adlam and A. Kent, *Phys. Rev. A* 92, 022315 (2015). Device-independent relativistic quantum bit commitment
13. N. Aharon, S. Massar, S. Pironio, and J. Silman, *New J. Phys.* 18, 025014 (2016). Device-independent bit commitment based on the CHSH inequality
14. J. Ribeiro, G. Murta, and S. Wehner, arXiv:1609.08487. Fully general device-independence for two-party cryptography and position verification
15. J. Ribeiro, L. P. Thinh, J. Kaniewski, J. Helsen, and S. Wehner, *Phys. Rev. A* 97, 062307 (2018). Device independence for two-party cryptography and position verification with memoryless devices
16. M. Jakobsson and M. Yung, in *Advances in Cryptology: CRYPTO ’96, Lecture Notes in Computer Science, Vol. 1109* (Springer-Verlag, 1996), p. 186. Proving without knowing: On oblivious, agnostic and blindfolded provers
17. H. -K. Lo, *Phys. Rev. A* 56, 1154 (1997). Insecurity of quantum secure computations
18. T. Rudolph, quant-ph/0202143. The laws of physics and cryptographic security
19. R. Colbeck, *Phys. Rev. A* 76, 062308 (2007). Impossibility of secure two-party classical computation
20. L. Salvail, C. Schaffner, and M. Sotakova, in *ASIACRYPT 2009, Lecture Notes in Computer Science, Vol. 5912* (Springer-Verlag, 2009), p. 70. On the power of two-party quantum cryptography
21. L. Salvail and M. Sotakova, arXiv:0906.1671. Two-party quantum protocols do not compose securely against honest-but-curious adversaries
22. A. Chailloux, I. Kerenidis, and J. Sikora, *Quantum Inf. Comput.* 13, 158 (2013). Lower bounds for quantum oblivious transfer
23. Y. -G. Yang and Q. -Y. Wen, *J. Phys. A: Math. Theor.* 42, 055305 (2009). An efficient two-party quantum private comparison protocol with decoy photons and two-photon entanglement
24. Y. -G. Yang and Q. -Y. Wen, *J. Phys. A: Math. Theor.* 43, 209801 (2010). Corrigendum: An efficient two-party quantum private comparison protocol with decoy photons and two-photon entanglement
25. Z. W. Sun and D. Y. Long, arXiv:1204.4587. Cryptanalysis of the efficient two-party quantum private comparison protocol with decoy photons and two-photon entanglement
26. Y. -G. Yang, W. -F. Cao, and Q. -Y. Wen, *Phys. Scr.* 80, 065002 (2009). Secure quantum private comparison
27. X. -B. Chen, G. Xu, X. -X. Niu, Q. -Y. Wen, and Y. -X. Yang, *Opt. Commun.* 283, 1561 (2010). An efficient protocol for the private comparison of equal information based on the triplet entangled state and single-particle measurement
28. J. Lin, H. -Y. Tseng, and T. Hwang, *Opt. Commun.* 284, 2412 (2011). Intercept-resend attacks on Chen et al.’s quantum private comparison protocol and the improvements
29. H. -Y. Tseng, J. Lin, and T. Hwang, *Quantum Inf. Process.* 11, 373 (2012). New quantum private comparison protocol using EPR pairs
30. Y. -G. Yang, J. Xia, X. Jia, and H. Zhang, *Quantum Inf. Process.* 12, 877 (2013). Comment on quantum private comparison protocols with a semi-honest third party
31. W. Liu, Y. -B. Wang, and Z. -T. Jiang, *Opt. Commun.* 284, 3160 (2011). An efficient protocol for the quantum private comparison of equality with W state
32. W. Liu, Y. -B. Wang, Z. -T. Jiang, and Y. -Z. Cao, *Int. J. Theor. Phys.* 51, 69 (2012). A protocol for the quantum private comparison of equality with $\chi$-type state
33. W. Liu, Y. -B. Wang, Z. -T. Jiang, Y. -Z. Cao, and W. Cui, *Int. J. Theor. Phys.* 51, 1953 (2012). New quantum private comparison protocol using $\chi$-type state
34. Y. -G. Yang, J. Xia, X. Jia, L. Shi, and H. Zhang, *Int. J. Quantum Inf.* 10, 1250065 (2012). New quantum private comparison protocol without entanglement
35. B. Liu, F. Gao, H. -Y. Jia, W. Huang, W. -W. Zhang, and Q. -Y. Wen, *Quantum Inf. Process.* 12, 887 (2013). Efficient quantum private comparison employing single photons and collective detection
36. G. P. He, *Int. J. Quantum Inf.* 11, 1350025 (2013). Simple quantum protocols for the millionaire problem with a semi-honest third party
37. Y. B. Li, Q. Y. Wen, F. Gao, H. Y. Jia, and Y. Sun, *Eur. Phys. J. D* 66, 110 (2012). Information leak in Liu et al.’s quantum private comparison and a new protocol
38. W. J. Liu, C. Liu, H. B. Wang, and T. T. Jia, *IETE*
Quantum private comparison: A review

[39] V. Siddhu and Arvind, Quantum Inf. Process. 14, 3005 (2015). Quantum private comparison over noisy channels

[40] J. Lin, C.-W. Yang, T. Hwang, Quantum Inf. Process. 13, 239 (2014). Quantum private comparison of equality protocol without a third party

[41] G. P. He, Quantum Inf. Process. 14, 2301 (2015). Comment on “Quantum private comparison of equality protocol without a third party”

[42] B. Zhang, X. T. Liu, J. Wang, and C. J. Tang, Quantum Inf. Process. 14, 4593 (2015). Cryptanalysis and improvement of quantum private comparison of equality protocol without a third party

[43] G. P. He, Int. J. Quantum Inf. 15, 1750014 (2017). Quantum private comparison protocol without a third party

[44] V. Giovannetti, S. Lloyd, and L. Maccone, Phys. Rev. Lett. 100, 230502 (2008). Quantum private queries

[45] C.-Y. Wei, T.-Y. Wang, and F. Gao, Phys. Rev. A 93, 042318 (2016). Practical quantum private query with better performance in resisting joint-measurement attack

[46] C.-Y Wei, X. Q Cai, B. Liu, T.-Y. Wang, and F. Gao, IEEE T. Comput. 67, 2 (2018). A generic construction of quantum-oblivious-key-transfer-based private query with ideal database security and zero failure