The gravitational wave emission from white dwarf interactions in globular clusters

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Abstract. In the dense central regions of globular clusters close encounters of two white dwarfs are relatively frequent. The estimated frequency is one or more strong encounters per star in the lifetime of the cluster. Such encounters should be then potential sources of gravitational wave radiation. Thus, it is foreseeable that these collisions could be either individually detected by LISA or they could contribute significantly to the background noise of the detector. We compute the pattern of gravitational wave emission from these encounters for a sufficiently broad range of system parameters, namely the masses, the relative velocities and the distances of the two white dwarfs involved in the encounter.

1. Introduction
Gravitational waves are a direct consequence of the General Theory of Relativity. Many efforts have been done so far to detect these elusive waves, but the intrinsic experimental difficulties have hampered their detection. However, with the advent of the current generation of terrestrial gravitational wave detectors, like LIGO \[1\] — which already has completed its fifth science run — VIRGO \[2\], GEO600 \[3\], or TAMA \[4\], and of space-borne interferometers like LISA \[5, 6\], gravitational wave astronomy is already a reality. It is therefore important to properly compute with the highest possible accuracy the gravitational wave pattern of all the sources of gravitational waves with potential astrophysical interest \[7\].

The Galactic population of field binary white dwarf is one of the most promising sources of gravitational waves, since it was shown long ago that LISA could detect them individually \[8\]. It is important to realize that field binary white dwarfs are the natural outcome of the evolution of many binary systems and it can be shown that due to tidal interactions their orbits are usually circular. Thus, the gravitational wave pattern of these systems is easy to compute since the two white dwarfs can be safely assumed to be point masses. Due to the ensuing emission of gravitational waves, the orbital separation of these systems decreases and the final fate of these systems is to coalesce. Consequently, the gravitational wave emission resulting form the merger of such systems has been the subject of some recent studies \[9, 10\]. These studies show that the resulting pattern of gravitational wave emission is rather simple. In particular, it has been
shown that these waveforms do not show a prominent peak and that the gravitational wave signal rapidly fades in a few orbital periods.

Globular clusters are old stellar systems which contain as many as $10^7$ stars. The typical stellar densities are $\sim 10^3$ pc$^{-3}$ and, due to their long lifetimes, the central regions of globular clusters contain many collapsed and degenerate objects, such as white dwarfs. In such a dense environment, where stars can pass very close one to another, every star is expected to suffer one or more strong encounters during its lifetime. In fact, it has been predicted [11] that dynamical interactions in globular clusters can form double white dwarf systems which could be eventually detectable by LISA. Moreover, in globular clusters, dynamical interactions can form binary white dwarf systems at an enhanced rate compared to the field [12]. Additionally, such systems are generally formed with non-zero eccentricities. This is an important issue because tidal interactions in eccentric systems substantially modify the gravitational wave emission pattern, thus providing us with a unique opportunity to use LISA to check our understanding of the interiors of white dwarfs [11]. Here we study the dynamical interaction of white dwarfs in globular clusters and we compute the associated gravitational wave radiation. Due to the complicated dynamics of the process, numerical simulations of such encounters are essential to obtain realistic sets of waveforms. In order to do so, we have performed a series of SPH simulations of strong encounters of white dwarfs in globular cluster, covering a broad enough range of system parameters, which include impact parameter and relative stellar velocities for two representative masses of the white dwarfs involved in the interaction.

2. Input physics and method of calculation

We follow the hydrodynamic evolution of the interacting white dwarfs using Smoothed Particle Hydrodynamics (SPH), a Lagrangian particle numerical code [13, 14]. The fact that the method is totally Lagrangian and does not require a grid makes it specially suitable for studying an intrinsically three-dimensional problem. We will not describe in detail the most basic equations of our numerical code, since this is a well-known technique. Instead, the reader is referred to Ref. [15] where the basic numerical scheme for solving the hydrodynamic equations can be found, and to Ref. [9] where all the relevant input physics for the problem at hand and of our specific implementation of this technique are explained in depth. We compute the gravitational wave emission in the slow-motion, weak-field quadrupole approximation [16]. The dimensionless wave strain, $h$, in the transverse-traceless gauge is given by:

$$h_{jk}^{TT}(t, x) = \frac{2G}{c^4 d} \frac{\partial^2 Q_{jk}^{TT}(t - R)}{\partial t^2}$$

where $t - R = t - d/c$ is the retarded time, $d$ is the distance to the observer and $Q_{jk}^{TT}(t - R)$ is the quadrupole moment of the mass distribution. For the case of a collection of SPH particles it can be shown (Lorén–Aguilar et al. 2005) that:

$$Q_{jk}^{TT}(t) \approx P_{ijkl}(N) \sum_{p=1}^{n} m(p)[2v^k(p)v^l(p) + x^k(p)a^l(p) + x^l(p)a^k(p)]$$

where $P_{ijkl}$ is the transverse-traceless projection operator onto the plane orthogonal to the outgoing wave direction, $N$, $m(p)$ is the mass of each SPH particle, and $x(p)$, $v(p)$ and $a(p)$ are, respectively, its position, velocity and acceleration.

In order to compute a large enough set of gravitational wave patterns we have relaxed two initial white dwarf models with masses $0.6M_\odot$ and $0.8M_\odot$, respectively. The mass of the SPH particles were the same in both cases in order to avoid numerical artifacts. Consequently, the $0.6M_\odot$ white dwarf was relaxed using $2.0 \times 10^4$ particles, whereas the $0.8M_\odot$ white dwarf needed $2.6 \times 10^4$ particles. For the interactions in which two equal-mass white dwarfs were involved...
Figure 1. Outcomes of the white dwarf close encounters for the $0.6 M_\odot + 0.8 M_\odot$ case as a function of the initial velocity $v_i$ and impact parameter $b$. In regions (a) and (b) the outcome of the interaction is a merger. In region (a) we obtain head-on impacts, whereas in region (b) a lateral collision occurs. Region (c) corresponds to the region in which an eccentric binary system is formed. The arrow points the direction of increasing eccentricity.

we have used the otherwise typical $0.6 M_\odot$ model, whereas for the other cases we have used the $0.6 M_\odot$ and the $0.8 M_\odot$ models. We have fixed the initial distance $x$ between the stars and their spin (fixed to typical values in field white dwarfs) allowing the impact parameter $b$ (distance between stars on the $y$ direction) and the initial velocity $\vec{v}_i = (v_i, 0, 0)$ to be our free parameters. We have performed 11 simulations with impact parameters ranging from $0.3 R_\odot$ to $0.9 R_\odot$ and initial velocities from 50 to 150 km/s. These values of the initial relative velocities and impact parameters are typical of globular clusters.

3. Results
We have obtained three different kinds of behavior, depending on the initial parameters adopted for the close encounter — see Fig. 1 where we show schematically the three different behaviors. When the initial velocity $v_i$ is relatively small and the impact parameters are not very large the gravitational forces bend the initial trajectories and the result of the interaction is a head-on collision. These close encounters correspond to initial velocities and impact parameters of region (a) in Fig. 1. For increasing initial velocities there is a transition region in which the initial trajectories are less affected by the gravitational interaction and the final outcome turns out to be a lateral collision. These interactions occur in the region labelled as (b) in Fig. 1. Finally, for sufficiently large initial velocities and above a threshold impact parameter which depends on the initial relative velocity of the interacting white dwarfs, the stars do not collide but, instead, form an eccentric binary system. This kind of interactions correspond to the region labelled as (c) in Fig. 1. The solid and dashed lines in Fig. 1 separate, respectively, the region of head-on collisions and lateral collisions and the region of lateral collisions and the region in which an eccentric binary system is formed. Finally, the arrow in figure 1 points the direction in which the eccentricity of the newly formed binary system increases.

In this work we will focus in those systems in which an eccentric binary system is formed.
Figure 2. Temporal evolution of the close encounters of the double white dwarf systems discussed here. The left panels show the evolution of the positions of the particles for the $0.6 + 0.8 \, M_\odot$ system. The right panels show the evolution of the $0.6 + 0.6 \, M_\odot$ system. In both cases the initial conditions were $v_i = 150 \, \text{km/s}$ and $b = 0.5 \, R_\odot$. The positions of the particles have been projected onto the $xy$ plane. The units of positions are $10^9 \, \text{cm}$. Times are shown in the right upper corner of each panel. These figures have been done using the visualization tool SPLASH [17].

That is, those systems which occupy the region of sufficiently large initial velocities and impact parameters in Fig. 1. The rest of the cases shown in Fig. 1 will be discussed at length elsewhere. Fig. 2 shows two typical examples of the temporal evolution of some of the close encounters studied here. The left panels show the case in which two white dwarfs of $0.6$ and $0.8 \, M_\odot$ are involved and the right panels the case in which two equal-mass white dwarfs interact. The trajectories of the centers of mass of each individual white dwarf are shown using dashed lines. These lines can be used as a visual aid to follow the trajectories of the intervening white dwarfs. For the sake of definiteness we have chosen to show two cases in which the initial conditions are the same, namely $v_i = 150 \, \text{km/s}$ and $b = 0.5 \, R_\odot$. As can be seen, the systems are well separated and the white dwarfs describe elliptical orbits around the center of mass. In both panels it can be appreciated that no mass transfer occurs between both components. This is so even at the minimum separation of the system. In all the cases studied here, at early times the system does not radiate gravitational waves because the accelerations are very small. Once the stars approach sufficiently each other, the gravitational wave signal rapidly grows. In the case of a head-on collision the signal first grows and then suddenly fades away. In the case of a lateral collision the gravitational wave emission is more irregular and does present several amplitude peaks. This occurs because in a lateral collision before the final merger, the two interacting white dwarfs describe a few orbits of decreasing separation, in which sizeable mass transfer occurs between the two components of the system. For the sake of conciseness we do not show here the corresponding waveforms and we refer the interested reader to a forthcoming publication. Finally, in those cases in which an eccentric binary forms the gravitational wave patterns present a regular, periodical behavior. In Fig. 3 we show the gravitational wave signal emitted by the systems shown in Fig. 2. In particular the dimensionless strains $h_+$ and $h_\times$ in
Figure 3. Gravitational wave emission for the cases shown in Fig. 2. The left panel corresponds to the case in which two white dwarfs of masses 0.6 and 0.8 $M_\odot$, respectively, interact to form an eccentric binary. The right panel corresponds to the case in which two equal-mass white dwarfs of 0.6 $M_\odot$ interact to form again an eccentric binary. The relative initial velocities and impact parameter were in both cases $v_i = 150$ km/s and $b = 0.5 R_\odot$, respectively. The dimensionless strains $h_+$ and $h_\times$ are measured in units of $10^{-22}$. The source is located at a distance of 10 kpc.

As can be seen the gravitational waveforms are characterized by a sharp peak, which occurs when the white dwarfs are at the minimum orbital separation. These systems will remain bound and after several periods the emission of gravitational waves will circularize and shrink their orbits. However, our numerical method does not allow to follow the evolution of these systems for long times and, thus, we only show in Fig. 3 the first period.

4. Conclusions
We have computed the emission of gravitational waves arising from the dynamical interaction of two white dwarfs in the dense central regions of a globular cluster. These interactions are thought to be quite frequent and thus it is important to characterize the resulting gravitational waveforms. We have done so for a sufficiently broad range of initial relative velocities and impact parameters and for two different component masses. In one set of calculations we have computed the gravitational wave pattern emitted in a close encounter of two equal-mass white dwarfs of 0.6 $M_\odot$, the typical white dwarf mass. The second set of calculations corresponds to close encounters in which an otherwise typical 0.6 $M_\odot$ white dwarf and a rather massive 0.8 $M_\odot$ companion are involved. We have found that there are three possible different outcomes, depending on the initial relative velocity and impact parameter of the system, which lead to distinctive patterns of gravitational wave radiation. In particular, we have characterized the range of parameters for which head-on mergers, lateral collisions and formation of an eccentric binary system occur. For the cases in which the outcome is a collision (either a head-on one or a lateral one) we find that signal is a strong burst of gravitational waves, followed in some cases by a series of smaller peaks. Such events will not be detectable by LISA. On the contrary if the initial conditions are such that the dynamical interaction leads to the formation of an eccentric binary system, the signal is periodical and will probably be detectable with LISA.
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