Erratum: “Dynamics of pure spin current in high-frequency quantum regime” [Appl. Phys. Express 10, 053001 (2017)]

A few equations including Eq. (3) were incorrect and Fig. 3 should be corrected accordingly. These changes do not affect the overall discussion of this paper.

1) Some suffixes and notations of the noise spectrum were incorrect.

2) In Eq. (3), a few factors in the final brackets were incorrect. Thus, Eq. (3), Fig. 3, and its caption should be replaced as below. In Fig. 3, the label on the left axis should be replaced so that it is consistent with the corrected Eq. (3).

Corrected Eq. (3)

\[ C(t, \delta \mu, T) = \frac{4e^2 \sqrt{2(k_B T)^2}}{\hbar^2 \pi \sqrt{\pi}} \left( 1 - \tau \right) \cos \left( \frac{\delta \mu \tau}{\hbar} \right) + \tau \left[ \frac{\hbar}{2k_B T} \right]^2 \frac{\pi^2}{\exp \left( \frac{2\pi t}{\hbar k_B T} \right) + \exp \left( -\frac{2\pi t}{\hbar k_B T} \right) - 2} ] \]

Corrected Fig. 3

Corrected Fig. 3 caption:
Normalized autocorrelation of the spin current \( C(t, \delta \mu, T)/C(0, \delta \mu, T) \) with various \( \delta \mu/k_B T \) values ranging from 1 to 7. The dotted curves are the envelopes for each condition. We fix the transmission to \( \tau = 0.01 \). A clear oscillation with frequency \( \delta \mu / \hbar \) is observed.

(3) For consistency with the above correction, the online supplementary data should also be replaced (see the online supplementary data at http://stacks.iop.org/APEX/10/053001/mmedia).
Dynamics of pure spin current in high-frequency quantum regime

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Pure spin current is a powerful tool for manipulating spintronic devices, and its dynamical behavior is an important issue. By using the mesoscopic transport theory for electron tunneling induced by spin accumulation, we investigate the dynamics of the spin current in the high-frequency quantum regime, where the effect of frequency is much greater than those of temperature and bias voltage. Besides the thermal noise, frequency-dependent finite noise emerges, signaling the spin current across the tunneling barrier. We also find that the autocorrelation of the spin current exhibits sinusoidal oscillation with time as a consequence of the Pauli exclusion principle even without any net charge current.

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Spin current, a flow of spin angular momentum, is one of the central concepts in spintronics.¹⁻³) It plays an important role not only as a tool for controlling various spintronic devices but also as a probe for spin dynamics in solids.¹⁻⁹) In particular, pure spin current, a flow of spin without net charge current, is of considerable interest because it is thought to be dissipationless.¹⁰⁻¹²) Besides various studies on its utilization, there are several attempts to reveal its fundamental nature.¹³⁻¹⁵) Nevertheless, there remains much to be done to understand the spin current itself, and new measurement probes are required. As spin is a quantum object, it is of significance to address the quantum nature of the spin current by investigating its dynamics.¹⁵)

Shot noise, or nonequilibrium current fluctuation, can be a unique tool in tackling this problem. Conventionally, it appears when electrons tunnel through a potential barrier such that the current noise power spectral density $S_f$ is expressed as the Schottky formula $S_f = 2e|f|$ in the zero-frequency and zero-temperature limit ($I$: current and $e$: electric charge).¹⁶) Interestingly, shot noise is not trivial for spin current.¹⁷,¹⁸) Consider a pure spin current, where currents with spin-up ($I_↑$) and spin-down electrons ($I_↓$) move in opposite directions (say, $I_↑ = -I_↓$). While there is no net charge current, the fluctuation of each current adds up in accordance with the Schottky formula, yielding a finite shot noise of $S_f = 2e|f_↑| + |f_↓|$.

Recently, the relevance of this concept of “spin shot noise” has been experimentally demonstrated in the Ga(Mn)As-based all-semiconductor lateral spin-valve device.¹⁹) On the other hand, the above treatment is applied with the zero-frequency limit, and the shot noise at the finite frequency enables us to obtain new information on transport dynamics.

Recently, the shot noise in this regime has been experimentally addressed.¹⁹⁻²¹) Thibault et al.¹⁹) clarified an important aspect of the current fluctuation in the charge transport in a tunnel barrier. They measured noise spectra in the high-frequency quantum regime, namely, the frequency region much higher than both temperature and bias voltage, and derived the autocorrelation of the charge current. They observed the oscillation of autocorrelation with time as a direct consequence of the Pauli exclusion principle in the current. This is a clear manifestation of the quantum nature of the charge current. However, such an attempt has been lacking for the spin current.

In this Letter, expanding the concept of the spin shot noise to the finite frequency region, we show that the Pauli exclusion principle is also relevant for the spin current even when there is no net charge current. We evaluate the noise spectrum and autocorrelation of the spin current induced by the spin accumulation at the tunnel barrier. We also discuss the experimental feasibility of our results.

Figure 1(a) shows the setup we consider here, which is similar to that of the conventional lateral-spin valve system.²²,²³) The device consists of one ferromagnetic lead (FM) that is magnetized along the lead, and two nonmagnetic leads attached to it, namely, the middle lead (NM1) and the right lead (NM2). There is a tunnel barrier between NM1 and NM2. Note that we assume NM2 to be nonmagnetic just for simplicity, while the following result can be easily generalized for the ferromagnetic case by taking the spin polarization into account phenomenologically.

By injecting a spin-polarized current from FM to NM1, spin accumulation is created inside NM1 along the $x$-axis [Fig. 1(a)]. The chemical potentials of the spin-up and spin-down electrons become $x$-dependent, generating a spin current, as shown in Fig. 1(b). A part of this spin current flows down along the $z$-axis into NM2 through the tunnel barrier. The energy diagram in the vicinity of the barrier is presented in Fig. 1(c), where the chemical potentials of the spin-up and spin-down electrons steeply change at the barrier. The resulting potential difference at the barrier corresponds to the spin accumulation ($\delta\mu$). Here, we neglect the effect of spin diffusion along the $z$-axis across the barrier, as the signature of such an effect was never detected in the experiment described...
in Ref. 18. The spin diffusion does not play an important role as long as we consider devices similar to those described in Ref. 18. We apply \( V \) as the voltage difference between NM1 and NM2 [Fig. 1(c) shows the case of \( V = 0 \)]. We consider the noise generated at this barrier. Note that, as it occurs locally, the noise is irrelevant to the spin diffusion process in NM2. The spin flip during tunneling can be neglected, which was validated by a recent experiment.18

The calculation is performed using the mesoscopic transport theory based on the Landauer–Büttiker formalism (see the online supplementary data at http://stacks.iop.org/APEX/10/053001/mmedia).24 Tunnel the barrier is treated as a one-dimensional single-channel scatterer with an energy-independent transmission probability (\( \tau \)). Note that it is straightforward to extend our analysis to the multichannel case. We define a current operator \( \hat{I}_{\nu,\sigma} \) using the second quantization. Here, \( \gamma \) and \( \sigma \) denote leads (L: NM1 or R: NM2) and spin (\( \uparrow \) or \( \downarrow \)), respectively. By taking the quantum statistical average, the mean current is given by the well-known formula

\[
\langle \hat{I}_{\nu,\sigma}\rangle = \frac{e}{h} \int_{-\infty}^{\infty} dE [f_{\nu,\sigma}(E, \mu_{\nu,\sigma}, T) - f_{\nu,\sigma}(E, \mu_{\nu,\sigma}, T)],
\]

where \( h \) is the Planck constant, \( E \) is the electron energy, and \( f_{\nu,\sigma}(E) \) is the Fermi distribution function for electrons with the chemical potential \( \mu_{\nu,\sigma} \) and the temperature \( T \).

The spin current operator is defined as \( \hat{I}_{S} = \hat{I} - \hat{I}_{t} \). Here, the spin-up and spin-down channels are independent of each other as we can neglect the spin flip. By integrating the Fermi distribution function, we obtain \( \langle \hat{I}_{\nu,\sigma}\rangle = (e/h) \tau (\mu_{\nu,\sigma} - \mu_{\nu',\sigma})(1 - \delta_{\nu,\nu'}) \). Substituting the chemical potential of each lead \( \mu_{L/1} = \mu_{0} + (eV/2) \pm (\delta \mu/2) \) and \( \mu_{R/1} = \mu_{0} - (eV/2) \), we obtain \( \langle \hat{I}_{C}\rangle = (2e^{2}/h)eV \) for the charge current and \( \langle \hat{I}_{S}\rangle = (e/h)\tau (eV + \delta \mu) \) for the spin current. This is consistent with the previous results.18

Now, we discuss the noise spectrum. By defining the current noise operator \( \delta \hat{I}_{\nu,\sigma} = \hat{I}_{\nu,\sigma} - \langle \hat{I}_{\nu,\sigma}\rangle \), the noise spectrum is expressed as

\[
S_{\nu,\sigma}(\nu) = \int_{-\infty}^{\infty} \langle \delta \hat{I}_{\nu,\sigma}(t)\delta \hat{I}_{\nu,\sigma}(t + \tau) \rangle e^{2\pi i \nu \tau} d\tau,
\]

where \( t \) is time and \( \nu \) is frequency defined from \(-\infty \) to \(+\infty \). Following the work by Meair et al.17 in the framework of the Landauer–Büttiker formalism, the spin-dependent transmission channels can be treated as if they form a parallel circuit. Thus, the total noise spectrum of the spin current, \( S_{\text{total}} = \sum_{\nu,\sigma} S_{\nu,\sigma}(\nu) \), is analytically given as

\[
S_{\text{total}}(\nu, T, V, \delta \mu) = \frac{e^{2}}{h} \left[ \frac{4\pi^{2}h\nu}{1 - \exp\left(-\frac{h\nu}{k_{B}T}\right)} + e^{\nu} \tau(1 - \tau) \left( eV + \frac{\delta \mu}{2} + h\nu \right) \right] + e^{\nu} \tau(1 - \tau) \left( -eV - \frac{\delta \mu}{2} + h\nu \right) \]

\[
+ e\frac{\tau(1 - \tau)}{T} \left( eV - \frac{\delta \mu}{2} + h\nu \right) + e\frac{\tau(1 - \tau)}{T} \left( -eV + \frac{\delta \mu}{2} + h\nu \right) \right] \]

\[
+ e\frac{\tau(1 - \tau)}{T} \left( eV - \frac{\delta \mu}{2} + h\nu \right) \right]

By taking the zero-frequency and zero-temperature limit, we obtain an expression consistent with that given in Ref. 18. The quantum nature of the current appears in the finite-frequency component of the calculated noise.24 As it is generated via the process where there is a finite energy difference between the initial and final states, the system absorbs/emits a photon to conserve energy. Actually, Eq. (2) is understood in terms of the one-dimensional emission (\( \nu < 0 \)) and absorption (\( \nu > 0 \)) spectrum of a photon with energies of \( h\nu \pm eV + (\delta \mu/2) \) for the spin-up channel and \( h\nu \pm eV - (\delta \mu/2) \) for the spin-down channel. Thus, the quantum nature is naturally introduced when we consider the finite frequency noise. When the emission and absorption processes occur with the same probability, the noise spectrum can be symmetrized with regard to the positive and negative frequencies: \( S_{\text{sym}}(\nu, T, V, \delta \mu) = S(\nu, T, V, \delta \mu) + S(-\nu, T, V, \delta \mu) \) with \( \nu = [0, \infty) \).

In the rest of this paper, we focus only on the zero bias regime [\( V = 0 \), see Fig. 1(c)], where there is no net charge current across the barrier. For simplicity, we redefine \( S_{\text{sym}}(\nu, T, \delta \mu) \equiv S_{\text{sym}}(\nu, T, V = 0, \delta \mu) \). Two remarks are made for the zero-frequency limit. First, \( S_{\text{sym}}(\nu = 0, T, \delta \mu = 0) \equiv S_{0} \) gives the classical thermal (Johnson–Nyquist) noise.25 Second, \( S_{\text{sym}}(\nu = 0, T, \delta \mu) \) reproduces the previous result.18

Now, for the finite frequency, in Fig. 2, we show \( S_{\text{sym}}(\nu, T, \delta \mu)/S_{0} \) as a function of the normalized frequency \( (h\nu/k_{B}T) \) for the cases of \( \delta \mu = 0 \) (no spin accumulation) and \( \delta \mu/k_{B}T = 1, 3, \) and \( 5 \) (finite spin accumulation). While \( S_{\text{sym}}(\nu, T, \delta \mu = 0) \) again is equal to the well-known thermal noise spectra, we obtain an increase in the noise when the spin accumulation was finite. This indicates that the shot noise in the spin current is relevant to the spin diffusion process in NM2.
noise is generated by the spin current even without any net charge current.

What does the increase in the noise mean? To understand this, we investigate the dynamics of the system in real time. Applying the Wiener–Khinchin theorem to Eq. (2), we derive the autocorrelation of the spin current. Before this treatment, following Ref. 19, we need to redefine the current noise spectral density as \( S_{sym}(\nu, T, \delta \mu) \equiv S_{sym}(\nu, T, \delta \mu) - S_{sym}(\nu, T, 0, \delta \mu) \), because \( S_{sym} \) diverges such that \( S_{sym} \to 4e^{\nu \delta \mu} \) for \( \nu \to \infty \). This subtraction means that we ignore the contribution of the vacuum fluctuation in the noise spectrum (see the dotted line in Fig. 2), and focus only on the noise generated by electron tunneling.

We found that the autocorrelation of the spin current is given as

\[
C(t, \delta \mu, T) = 4\tau \left[ 1 - \tau \cos \left( \frac{\delta \mu}{\hbar} \right) \right] + \tau \left[ \frac{1}{2} \left( \frac{\hbar}{k_B T} \right) - 2 \exp \left( \frac{\pi}{2k_B T} \right) \right].
\]  

We plot Eq. (3) as a function of time in units of \( h/\delta \mu \) in Fig. 3. In the absence of the spin accumulation (\( \delta \mu = 0 \)), the thermal noise is the only origin of the noise, making the autocorrelation \( C(t, \delta \mu = 0, T) \) monotonically decrease with time. This means that the quantum coherence of electrons decays owing to thermal agitation with a characteristic time scale of \( h/k_B T \). In the presence of the spin accumulation (\( \delta \mu \neq 0 \)), the autocorrelation oscillates with the envelope \( C(t, \delta \mu = 0, T) \). Thibault et al. observed a similar oscillation when a charge current flowed across the voltage-biased barrier, which directly indicates that electrons can tunnel only when there is no net charge current. Here, spin-up and spin-down electrons sequentially come into the barrier. Owing to the Pauli exclusion principle, only one spin-up electron and one spin-down electron can tunnel at a certain time. This means that the quantum coherence of electrons always present regardless of temperature.

This shows that the oscillation with the frequency \( \delta \mu/\hbar \) is always present regardless of temperature.

The time dependence of the autocorrelation clearly shows the quantum nature of the spin current. With the same analogy as discussed in Ref. 19, the oscillation of the autocorrelation can be understood as follows. The tunneling of the spin current occurs as spin-up and spin-down electrons sequentially come into the barrier. Owing to the Pauli exclusion principle, only one spin-up electron and one spin-down electron can tunnel at a certain time. Here, spin-up and spin-down electrons have the same chemical potential difference with the same absolute value but opposite signs. In the quantum regime, because the energies before and after the tunneling are different, it takes a finite time to resolve the electron wave functions of the two states. This time is \( h/\delta \mu \) which corresponds to Heisenberg’s uncertainty principle.

Thus, the autocorrelation oscillates with the period of \( h/\delta \mu \), indicating that the Pauli exclusion principle acts on the spin current. Such a quantum nature can be destroyed owing to the decoherence processes. In this case, because of thermal agitation, the coherence vanishes as a function of time, and thus the oscillation of the autocorrelation decays, as shown in Fig. 3. Here, \( h/k_B T \) has the dimension of time and corresponds to the coherence time.

We now discuss the feasibility of the experiment. The measurement frequency must be higher than \( \delta \mu/\hbar \) to observe more than one period of the oscillation. At the same time, it must be higher than \( k_B T/\hbar \) to inhibit the thermal decay of electron coherence. For example, when \( T = 30 \) mK and \( \delta \mu = 5 \) µeV, the noise spectra need to be measured up to a frequency higher than \( 1 \) GHz. This can be realized using a dilution refrigerator at \( \lesssim 30 \) mK with a high-frequency-noise measurement setup. For example, the measurement sensitivity can reach up to \( 10^{-25} \) A²/Hz, while the shot noise of the sample with \( 50 \) Ω and \( 5 \) µV bias is of the order of \( 10^{-26} \) A²/Hz according to the Schottky formula.

Finally, we mention several points that we have ignored but may affect the behavior of the noise and the autocorrelation. First, if there exists a decoherence mechanism other than thermal agitation, it affects the decaying behavior. Investigating it experimentally is important as it gives critical information on decoherence sources for the spin current. Second, the transmission may be energy-dependent in the high-temperature regime, whereas we assumed the energy-independent transmission here. We believe that this assumption is valid on the energy scale of a few tens of mK and a few µeV. For example, in the case of Fe/MgO, according to Fig. 3 of Ref. 26, the typical energy scale governing the spin-dependent transport is at least of the order of a few meV. Thus, we believe that the energy dependence of the tunneling can be ignored if we consider such a very low energy scale.
However, this assumption is certainly invalid at high temperature (such as room temperature) and the energy dependence becomes very important. Third, the transmission may be channel-dependent with some materials whose unique band structure can yield the channel dependence. For example, the conduction band of Fe\textsubscript{3}MgO has a strong spin dependence. In our calculation, this can be included as spin-channel-dependent transmission, which is equivalent to including the spin polarization of the electrodes, as shown previously. We may experimentally detect the energy and channel dependences by checking the temperature dependence of Eq. (4) or changing the materials, the results of which would certainly be very interesting as they will give us new information.

In summary, according to the Landauer–Büttiker formalism, we have analytically derived the noise spectrum and the autocorrelation of the spin current at the tunnel junction. We show that the temperature-independent behavior of the autocorrelation is due to the Pauli exclusion principle for the spin current, which can be detected experimentally. Such an experiment would enable us to directly address several unexplored aspects of the spin current, for example, its quantum coherence and dissipationless property.

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1) S. Maekawa, H. Adachi, and K. Uchida, J. Phys. Soc. Jpn. 82, 102002 (2013).

2) A. Hirohata and K. Takamachi, J. Phys. D 47, 193001 (2014).
3) J. Žutić, J. Fabian, and S. D. Sarma, Rev. Mod. Phys. 76, 323 (2004).
4) Y. Niimi, D. Wei, and Y. Otani, J. Phys. Soc. Jpn. 86, 011004 (2017).
5) D. H. Wei, Y. Niimi, B. Gu, T. Ziman, S. Maekawa, and Y. Otani, Nat. Commun. 3, 1058 (2012).
6) J. Grollier, V. Cros, A. Hamzic, J. M. George, H. Jaffrès, A. Fert, G. Faini, J. Ben Youssef, and H. Legall, Appl. Phys. Lett. 78, 3663 (2001).
7) E. Saitoh, M. Ueda, H. Miyajima, and G. Tataru, Appl. Phys. Lett. 88, 182509 (2006).
8) T. Yang, T. Kimura, and Y. Otani, Nat. Phys. 4, 851 (2008).
9) A. M. Deac, A. Fukushima, H. Kubota, H. Maehara, Y. Suzuki, S. Yuasa, Y. Nagamine, K. Tsunekawa, D. D. Djayaprawira, and N. Watanabe, Nat. Phys. 4, 803 (2008).
10) S. Murakami, N. Nagaosa, and S.-C. Zhang, Science 301, 1348 (2003).
11) J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. 92, 126603 (2004).
12) E. I. Rashba, Phys. Rev. B 68, 241315 (2003).
13) Y. Tokunaga, A. Brataas, and G. E. W. Bauer, Phys. Rev. B 66, 224403 (2002).
14) T. Taniguchi and W. M. Saslow, Phys. Rev. B 90, 214407 (2014).
15) A. Kamra and W. Belzig, Phys. Rev. Lett. 116, 146601 (2016).
16) W. Schottky, Ann. Phys. 362, 541 (1918).
17) J. Mean, P. Stano, and P. Jacquod, Phys. Rev. B 84, 073302 (2011).
18) T. Arakawa, I. Shiogai, M. Ciorga, M. Utz, D. Schuh, M. Kohda, J. Nitta, D. Bougeard, D. Weiss, T. Ono, and K. Kobayashi, Phys. Rev. Lett. 114, 016601 (2015).
19) K. Thibault, J. Gabelli, C. Lupien, and B. Reulet, Phys. Rev. Lett. 114, 236604 (2015).
20) J. Basset, A. Yu. Kasumov, C. P. Moos, G. Zarand, P. Simon, H. Bouchiat, and R. Deblock, Phys. Rev. Lett. 108, 046802 (2012).
21) A. L. Grimsmo, F. Qassemi, B. Reulet, and A. Blais, Phys. Rev. Lett. 116, 043602 (2016).
22) S. Takahashi and S. Maekawa, Phys. Rev. B 67, 052409 (2003).
23) F. J. Jedema, A. T. Filip, and B. J. Wees, Nature 410, 345 (2001).
24) Y. M. Blanter and M. Büttiker, Phys. Rep. 336, 1 (2000).
25) H. Nyquist, Phys. Rev. 32, 110 (1928).
26) K. Liu, K. Xia, and G. E. W. Bauer, Phys. Rev. B 86, 020408 (2012).