Analytical calculation of matter effects in two mass-scale neutrino oscillations

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Abstract

We consider three active flavor neutrino oscillations where both the mass-square differences play a role in atmospheric neutrino problem. We calculate the matter effects arising due to propagation through earth. We demonstrate that these effects improve the fit to the electron data \textit{vis-a-vis} vacuum oscillations but make the fit to the muon data far worse, thus worsening the overall fit. The results of our analytical calculation verify the numerical investigations of this scheme presented earlier by Fogli \textit{et al.}

I. INTRODUCTION

There are three evidences for neutrino oscillations: 1) solar neutrino problem, 2) atmospheric neutrino problem and 3) the results of LSND experiment. Two flavor oscillation analysis of each individual problem gives three widely different values for neutrino mass-squared differences ($\Delta$’s). Oscillations between the three active flavor neutrinos have only two independent $\Delta$’s. Hence it was proposed that there should be at least one sterile neutrino if one wants to explain all three evidences in terms of neutrino oscillations [1]. However, various attempts were made to account for all three evidences in terms of three active flavor oscillations [2]. CHOOZ constraint on 13 mixing angle rules out most of these solutions [3].

Recently three active flavor oscillations as explanation for all three evidences was revived
with the following scenario [4,5]. The larger $\Delta$ is about 0.3 eV$^2$ which can explain LSND result. The smaller $\Delta$ is in the range $10^{-4} - 10^{-3}$ eV$^2$. The upper limit is set so that CHOOZ constraint on the relevant mixing angles will not apply to oscillations driven by this $\Delta$. The lower limit is set by demanding that the solar neutrino survival probability should be independent of energy. In these three flavor oscillation schemes, the atmospheric neutrino oscillations are driven by both the $\Delta$'s. In particular, for the oscillations driven by the smaller $\Delta$, the oscillation probability is almost zero for downward going neutrinos and is significant for upward going neutrinos. In particular these oscillations create the zenith angle dependence which is needed to explain the data.

This scheme reproduces the overall suppression seen in the atmospheric neutrino data and also the zenith dependence of the ratio of muon to electron events. But it does not give a flat dependence for the zenith distribution for electron events which is observed in the data. It also gives an incorrect zenith distribution for muon events. The large $\nu_e \leftrightarrow \nu_\mu$ transitions driven by the smaller $\Delta$, which are needed to explain solar neutrino problem, are responsible for this distortion of zenith angle distribution in atmospheric neutrino problem.

In ref. [8] this scheme was investigated numerically. Their results are summarized in figs (4) and (5) of [8]. There it was also pointed out that matter effects worsen the already bad fit to data. Three salient features emerge from these graphs.

- For both sub-GeV electron as well as muon events there is no appreciable difference in the event distributions with and without matter. This implies that matter and vacuum oscillation probabilities are the same.

- For multi-GeV electron events matter greatly suppresses the large $\nu_\mu \leftrightarrow \nu_e$ oscillations and thus drastically reduces the large excesses of electron events which vacuum oscillations generate especially at large distances of traversal, i.e for the upward going events.

- Both for multi-GeV muon events and upgoing muon events inclusion of matter effects results in essentially a flat profile as a function of the zenith angle.
In this paper we show that these features can be explained by a simple analytic calculation of matter effects in this scheme by using perturbation theory. Very recently two scale oscillations of atmospheric neutrinos were treated analytically \[\text{(9,10)}\] under a different set of neutrino masses and mixings. But these analyses can explain only LSND results and atmospheric neutrino data leaving solar neutrino problem untouched.

II. TWO SCALE OSCILLATIONS IN MATTER- A PERTURBATIVE ANALYSIS

We take Scheck-Barenboim (SB) scheme given in \[\text{(4)}\] as a representative of models in which all evidences for neutrino oscillations are explained in terms of three active flavors. A recent update using latest reactor and accelerator data is given in \[\text{(11)}\]. The masses of the vacuum mass eigenstates are taken to be $\mu_1$, $\mu_2$ and $\mu_3$. The vacuum oscillation here is controlled by both the mass-squared differences $\Delta_{21} = \mu_2^2 - \mu_1^2$ and $\Delta_{31} = \mu_3^2 - \mu_1^2$. $\Delta_{31}$ is chosen to be about 0.3 eV$^2$ to drive the oscillations seen at LSND and $\Delta_{21}$ is constrained to be a few times $10^{-4}$ eV$^2$. Both scales average out to give an energy independent suppression in the solar neutrino case. The (12) mixing angle $\omega$ should be close to $\pi/4$, so that the solar neutrino survival probability is 0.5. A recent analysis found that this energy independent solution of solar neutrino problem is allowed with good confidence level \[\text{(3,7)}\]. The (13) mixing angle $\phi$ is constrained to be quite small $\leq 10^\circ$ by reactor and LSND data. The (23) mixing angle $\psi$ is centered around 27 degrees to give a good fit to the ratio of muon to electron events in atmospheric neutrino data.

The three flavor eigenstates are related to the three mass eigenstates in vacuum through a unitary transformation,

$$
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= U^v
\begin{pmatrix}
\nu^v_1 \\
\nu^v_2 \\
\nu^v_3
\end{pmatrix},
$$

(1)

where the superscript $v$ on r.h.s. stands for vacuum. The 3 × 3 unitary matrix $U^v$ can be parametrized by three Euler angles ($\omega, \phi, \psi$) and a phase. The form of the unitary matrix
can therefore be written in general as, \( U^v = U_{23}(\psi) \times U_{\text{phase}} \times U_{13}(\phi) \times U_{12}(\omega) \), where \( U_{ij}(\theta_{ij}) \) is the mixing matrix between \( i \)th and \( j \)th mass eigenstates with the mixing angle \( \theta_{ij} \). The explicit form of \( U \) is

\[
U^v = \begin{pmatrix}
c_{\phi}c_{\omega} & c_{\phi}s_{\omega} & s_{\phi} \\
-c_{\phi}s_{\omega}e^{i\delta} - s_{\psi}s_{\phi}c_{\omega}e^{-i\delta} & c_{\psi}c_{\omega}e^{i\delta} - s_{\psi}s_{\phi}s_{\omega}e^{-i\delta} & s_{\psi}c_{\phi}e^{-i\delta} \\
s_{\psi}s_{\omega}e^{i\delta} - c_{\psi}s_{\phi}c_{\omega}e^{-i\delta} & -s_{\psi}c_{\omega}e^{i\delta} - c_{\psi}s_{\phi}s_{\omega}e^{-i\delta} & c_{\psi}c_{\phi}e^{-i\delta}
\end{pmatrix},
\]

(2)

where \( s_{\phi} = \sin \phi \) and \( c_{\phi} = \cos \phi \) etc. All the angles can take values between 0 and \( \pi/2 \). \( U^v \) can also be written as

\[
U^v = \begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} \\
U_{\mu1} & U_{\mu2} & U_{\mu3} \\
U_{\tau1} & U_{\tau2} & U_{\tau3}
\end{pmatrix},
\]

(3)

In the mass eigenbasis, the (mass)\(^2\) matrix is diagonal,

\[
M^2_0 = \begin{pmatrix}
0 & 0 & 0 \\
0 & \Delta_{21} & 0 \\
0 & 0 & \Delta_{31}
\end{pmatrix},
\]

(4)

In the flavour basis the (mass)\(^2\) matrix has the form

\[
M^2_v = U^v M^2_0 U^{v\dagger} = \Delta_{31} M_{31} + \Delta_{21} M_{21},
\]

(5)

where

\[
M_{31} = U^v \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix} U^{v\dagger}
\]

\[
M_{21} = U^v \begin{pmatrix}
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{pmatrix} U^{v\dagger}
\]

(6)
Matter effects can be included by adding $A(r)$, to the $e-e$ element of $M_v^2$ where

$$A = 2\sqrt{2} \left( \frac{G\nu_e}{m_n} \right) \rho E. \quad (7)$$

In the above equation $m_n$ is the mass of the nucleon, $Y_e$ the number of electrons per nucleon in the matter which is $\approx \frac{1}{2}$, and $\rho$ is the density of matter in gm/cc. $A$ can be written as

$$A = 0.76 \times 10^{-4} \rho \times E.$$  

$A$ is in $eV^2$, if $E$ is expressed in $GeV$. In the atmospheric neutrino case most of the trajectories are through the mantle of the earth. We take the mantle density to be a constant of value 5 gm/cc.

The matter corrected (mass)$^2$ matrix in the flavour basis is

$$M_m^2 = \Delta_{31} M_{31} + \Delta_{21} M_{21} + A M_A,$$  

where

$$M_A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (9)$$

Now $\Delta_{31} \gg \Delta_{21}$. Also for a typical neutrino energy in the multi-GeV range of about 3 GeV $\Delta_{21} \sim A$. Thus we work in an situation where $\Delta_{21}, A \ll \Delta_{31}$.

In this approximation, to the zeroth order, both the matter term and the term proportional to $\Delta_{21}$ can be neglected in eq. (8). Then $M_m^2 = \Delta_{31} M_{31}$, whose eigenvalues and eigenvectors are

$$0; |1\rangle = \begin{pmatrix} U_{e1} \\ U_{\mu1} \\ U_{\tau1} \end{pmatrix},$$

$$0; |2\rangle = \begin{pmatrix} U_{e2} \\ U_{\mu2} \\ U_{\tau2} \end{pmatrix},$$

$$0; |3\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$
$$\Delta_{31:3} = \begin{pmatrix} U_{e3} \\ U_{\mu3} \\ U_{\tau3} \end{pmatrix}. \quad (10)$$

We treat $AM_A + \Delta_{21}M_{21}$ as perturbation to the dominant term in $M^2_m$ and carry out degenerate perturbation theory. We first define two new states as follows

$$|1'\rangle = \alpha|1\rangle + \beta|2\rangle \quad (11)$$

$$|2'\rangle = -\beta|1\rangle + \alpha|2\rangle. \quad (12)$$

$\alpha$ and $\beta$ are determined by the conditions

$$\alpha^2 + \beta^2 = 1, \quad \text{and} \quad \langle 1'|H'|2'\rangle = 0, \quad (13)$$

where $H' = AM_A + \Delta_{21}M_{21}$. The condition eq. (13) leads to the following equation

$$\frac{\alpha \sqrt{1-\alpha^2}}{1-2\alpha^2} = \frac{U_{e1}U_{e2}}{(\Delta_{21}/A) + (U_{e2}^2 - U_{e1}^2)} \quad (14)$$

Using the explicit form of the vacuum mixing matrix given in eqn. (2) eq. (14) can be written in terms of mixing angles as

$$\frac{\alpha \sqrt{1-\alpha^2}}{1-2\alpha^2} = \frac{\frac{1}{2} \cos^2 \phi \sin 2\omega}{(\Delta_{21}/A) - (\cos^2 \phi \cos 2\omega)} \quad (15)$$

In the SB scheme $\omega$ is close to maximal mixing and $\phi$ is small so for this scheme we get

$$\frac{\alpha \sqrt{1-\alpha^2}}{1-2\alpha^2} \simeq \frac{A}{2\Delta_{21}} = k(say). \quad (16)$$

From the above equation we get

$$\alpha^2 = \frac{1}{2} \left[ 1 + \frac{1}{\sqrt{1+4k^2}} \right] \quad (17)$$

$$\beta^2 = \frac{1}{2} \left[ 1 - \frac{1}{\sqrt{1+4k^2}} \right] \quad (18)$$

For later use we also note that
\[ \begin{align*}
\alpha \beta &= \frac{k}{\sqrt{1 + 4k^2}} \\
\alpha^2 - \beta^2 &= \frac{1}{\sqrt{1 + 4k^2}}
\end{align*} \]

The matter dependent mass eigenvalues are now given by

\[ \begin{align*}
m_1^2 &= \beta^2 \Delta_{21} + A(\alpha^2 U_{e1}^2 + \beta^2 U_{e2}^2 + 2\alpha \beta U_{e1} U_{e2}), \\
m_2^2 &= \alpha^2 \Delta_{21} + A(\alpha^2 U_{e2}^2 + \beta^2 U_{e1}^2 - 2\alpha \beta U_{e1} U_{e2}), \\
m_3^2 &= \Delta_{31} + A U_{e3}^2 \simeq \Delta_{31}.
\end{align*} \tag{19} \]

Hence we get the mass squared differences in matter

\[ \begin{align*}
\Delta_{21}^m &= \frac{1}{\sqrt{1 + 4k^2}} [\Delta_{21} - A \cos^2 \phi], \\
\Delta_{31}^m &\simeq \Delta_{31}, \\
\Delta_{32}^m &\simeq \Delta_{31} \tag{20}
\end{align*} \]

By computing the first order correction to the old states one can derive the mixing angles in matter. It is straightforward to show that two of the mixing angles \( \phi \) and \( \psi \) are unaffected by matter (corrections of order \((A/\Delta_{31})\) and are negligible). This was also demonstrated in \cite{4}. The (12) mixing angle \( \omega \) is strongly modified by matter effects and is given by

\[ \sin \omega_m = -\beta \cos \omega + \alpha \sin \omega \tag{21} \]

Since \( \omega \) is constrained to be close to 45 degrees we get

\[ \sin \omega_m = \frac{1}{\sqrt{2}} (\alpha - \beta) = \frac{1}{\sqrt{2}} \left( 1 - \frac{A}{\Delta_{21}} \right)^{\frac{1}{2}} \tag{22} \]

Let us estimate the quantity \((A/\Delta_{21})\). Using the definition of \( A \) we get

\[ \frac{A}{\Delta_{21}} = \frac{0.76 \times 10^{-4} \times 5 \times E}{x \times 10^{-4}}, \tag{23} \]
where we set $\rho = 5 \text{ gm/cc}$, earth mantle density. The quantity $x$ in the SB scheme typically is between 2 to 5. So one sees that $(A/\Delta_{21}) \simeq E$ where $E$ is the neutrino energy in GeV. This implies

$$\sin \omega_m = \frac{1}{\sqrt{2}} \left( \left[ 1 - \frac{E}{\sqrt{1 + E^2}} \right] \right)^{\frac{1}{2}} \quad (24)$$

Now for typical multi-GeV events, $E \geq 2\text{GeV}$. This in turn implies

$$\frac{E}{\sqrt{1 + E^2}} \simeq 1 \quad (25)$$

From eq. (24) we see that $\sin \omega_m \to 0$. In the case of two flavor mixing, this effect is known previously [12]. If the vacuum mixing angle is maximal, then the matter dependent mixing angle goes to zero at high energies. Here we have demonstrated that the same effect occurs in three flavor oscillations also, in the limit of perturbation theory being valid. $\sin \omega_m = 0$ implies $U_{e2}^m = 0$ and hence the (12) scale in matter, $\Delta_{21}^m$, decouples from the oscillation probabilities involving electron neutrinos. Below we show that the same thing happens for electron anti-neutrinos also. The $\nu_\mu \leftrightarrow \nu_e$ oscillation probability in matter becomes

$$P_{\mu e}^m = 4U_{e1}^m U_{e3}^m U_{\mu 1}^m U_{\mu 3}^m \sin^2 \left( 1.27 \frac{d \Delta_{31}}{E} \right)$$

$$= \frac{1}{2} \sin^2 2\phi \sin^2 \psi \quad (26)$$

as the rapidly oscillating scale $\Delta_{31}$ will average out to half. This transition probability is at most a few percent because of the smallness of $\phi$. Let us now consider the case of antineutrinos. Since for antineutrinos $A \to -A$ we get

$$\sin \omega_m = \frac{1}{\sqrt{2}} \left( \left[ 1 + \frac{E}{\sqrt{1 + E^2}} \right] \right)^{\frac{1}{2}} \quad (27)$$

In this case for typical multi-GeV energies we see from eq. (27) that $\sin \omega_m \to \frac{\pi}{2}$. This implies $U_{e1}^m = 0$. The $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillation probability becomes

$$P_{\mu \bar{e}}^m = 4U_{e2}^m U_{e3}^m U_{\mu 2} U_{\mu 3} \sin^2 \left( 1.27 \frac{d \Delta_{31}}{E} \right)$$

$$= \frac{1}{2} \sin^2 2\phi \sin^2 \psi \quad (28)$$
which is again a few percent.

Let us now contrast eq. (26) and eq. (28) with the expression for the $\nu_\mu \leftrightarrow \nu_\mathrm{e}$ vacuum oscillation probability in the SB scheme. This is given by

$$
P_{\bar{\nu}_\mu \bar{\nu}_\mathrm{e}} = P_{\nu_\mu \mathrm{e}} = -4U_{e1}U_{\mu1}U_{\mu2}^2 \sin^2 \left( 1.27 \frac{d \Delta_{21}}{E} \right) - 2U_{e1}U_{\mu3} - 2U_{e2}U_{\mu3}U_{\mu2}U_{\mu3},
$$

where we have set the rapid oscillating term due to $\Delta_{31}$ equal to $1/2$. In the SB scheme, $\omega \simeq \pi/4$ and hence the magnitudes of $U_{e1}$, $U_{e2}$, $U_{\mu1}$ and $U_{\mu2}$ are 0.6 or more. Because of this, one gets large $\nu_\mu \leftrightarrow \nu_\mathrm{e}$ transitions driven by $\Delta_{21}$, for large distances of travel (i.e. for upward going neutrinos).

Let us now look at the survival probabilities for electron neutrinos and anti-neutrinos. By setting $\omega_m = 0$ for neutrinos and $\omega_m = \pi/2$ for anti-neutrinos, we obtain the matter dependent survival probabilities to be

$$
P_{ee}^m = P_{\bar{e}\bar{e}}^m = 1 - \frac{1}{2} \sin^2 2\phi \simeq 1.
$$

We should contrast these with the vacuum survival probabilities

$$
P_{ee}^v = P_{\bar{e}\bar{e}}^v = 1 - 4U_{e1}^2U_{e2}^2 \sin^2 \left( 1.27 \frac{d \Delta_{21}}{E} \right) - 2U_{e1}^2U_{e3}^2 - 2U_{e2}^2U_{e3}^2.
$$

The last two terms in the above equation are proportional $\sin^2 \phi$ and are negligible. However the term containing $\Delta_{21}$ has a large coefficient and can cause significant oscillations for upward going events.

Similarly, matter dependent survival probability for muon neutrinos (anti-neutrinos) are obtained by setting $\omega_m = 0(\pi/2)$. They turn out to be

$$
P_{\mu\mu}^m = 1 - 4 \sin^2 2\psi \sin^2 \phi \sin^2 \left( 1.27 \frac{d \Delta_{21}^m}{E} \right) - 2 \cos^2 \phi \sin^2 \psi (1 - \cos^2 \phi \sin^2 \psi)
$$

The anti-neutrino survival probability has a similar expression, except that $\Delta_{21}^m$ for anti-neutrinos is different from that of neutrinos. However, in both cases, the zenith angle
dependence is very weak because the oscillations driven by $\Delta_{21}^m$ have a coefficient proportional to $\sin^2 \phi$ and hence are very small. In case of vacuum survival probabilities, there is significant zenith angle dependence coming from the oscillations of $\Delta_{21}$ whose coefficient $4U_{\mu 1}^2 U_{\mu 2}^2$ is quite large.

Using the oscillation probabilities derived above, let us calculate the zenith angle distributions of electron and muon events in atmospheric neutrinos. They are given by

\[
N_{\mu} = \Phi_{\mu} P_{\mu \mu} + \Phi_{e} P_{e \mu} + \Phi_{\mu} P_{\mu \bar{\mu}} + \Phi_{\bar{\mu}} P_{\bar{\mu} \bar{\mu}}
\]

\[
N_{e} = \Phi_{\mu} P_{\mu e} + \Phi_{e} P_{e e} + \Phi_{\mu} P_{\mu \bar{e}} + \Phi_{\bar{e}} P_{\bar{e} \bar{e}},
\]

where the products are to be understood as convolutions over energy. To include matter effects, the matter dependent probabilities should be used in the above equations. The zenith angle dependence of vacuum probabilities, coming from the oscillations driven by $\Delta_{21}$, gives the required zenith angle dependence to upward going muon events for multi-GeV energies. For downward going muons, vacuum oscillations in SB scheme predict larger suppression compared to what is observed. In case of electron events, the vacuum oscillation prediction for downward going events is in agreement with data but for upward going events, a huge excess is predicted, which again occurs due to large $\nu_{\mu} \leftrightarrow \nu_{e}$ oscillations driven by $\Delta_{21}$. When matter effects are included, we have seen that $\Delta_{21}^m$ either decouples, as in $P_{ee}, P_{\bar{e} \bar{e}}, P_{\mu e}$ and $P_{\mu \bar{e}}$ or has a very small coefficient, as in $P_{\mu \mu}$ and $P_{\mu \bar{\mu}}$. Hence, with the inclusion of the matter effects, the zenith angle distribution for both electron and muon event distributions become very flat. This is desirable from the point of view of electron data but the flat distribution strongly disagrees with the muon data. Since the error bars in muon data are much smaller than those in electron data, getting a better fit to electron data at the expense of muon data, worsens the overall fit of the data. Thus we see that inclusion of matter effects in SB scheme worsen what was already not a good fit.

We see from fig. (3) of ref. [8] that the matter corrected oscillations of SB scheme predict a mild zenith angle distribution for the upward going multi-GeV electron events. Whereas the upward going muon events are predicted to have a flat distribution. These two features
can also be explained in terms of our calculation. For electron events, the visible energy is
the same as the neutrino energy (≥ 1.33 GeV). For energy range of 1.33 – 2 GeV, at the
beginning of multi-GeV events, \( \omega_m \neq 0 \) for neutrinos (and \( \omega_m \neq \pi/2 \) for anti-neutrinos).
Hence in this energy range, \( \Delta_{21}^m \) does not quite decouple. Since the flux is higher at lower
energy, these neutrinos lead to more events and hence we see some zenith angle dependence
of upgoing multi-GeV electron events. In case of muons, for a visible energy of 1.33 GeV, the
neutrino energy is greater than 1.5 GeV and hence the decoupling of \( \Delta_{21}^m \) is better because
of the higher neutrino energy threshold.

For the through going muon events the neutrino energy is much higher than typical
multi-GeV energies. So again

\[
\frac{E}{\sqrt{1 + E^2}} \simeq 1
\]

and the discussions of the previous section apply and we will get a flat distribution for the
muon events when matter effects are included. which is seen in the last panel of fig.(4) of

Lastly we discuss the sub-GeV events. For these events \( A < \Delta_{21} \). The parameter
\( k = (A/2\Delta_{21}) \) is about 15%. For most of the energy range, the matter effects are negligible
and one has essentially vacuum oscillations. This can also be seen in the first two panels of
fig. (4) of ref. [8].

III. SUMMARY AND DISCUSSION

We did an analytical calculation of matter effects in a scheme where all the three evidences
for neutrino oscillations are explained in terms of three active flavor oscillations. Our simple
calculation reproduced most of the features of the matter effects in this scheme, which
were numerically investigated earlier. Matter effects bring the predictions of this scheme for
electron events closer to the data and the fit to the electron data is improved. But they wipe
out the zenith angle dependence of multi-GeV muon events, which is in contradiction to the
data. Since the error bars of muon data are much smaller than those of electron data, the deviation away from the data makes the overall fit, with matter effects, to the data worse compared to the already bad fit of vacuum oscillation predictions of this scheme.

Finally we wish to make a rather interesting comment. If in future these kind of schemes become favored phenomenologically, the main motivation for building a muon storage ring based neutrino factories is lost. At such factories, the neutrino energies are in the range $10^{-50}$ GeV. We have seen that at such large energies, due to earth matter effects, $\omega_m \simeq 0$ for neutrinos and $\omega_m \simeq \pi/2$ for antineutrinos. In either case, the sub-dominant scale $\Delta_{21}^m$ decouples from oscillations and CP Violation in lepton sector will be unmeasurably small in a neutrino factory environment. In such a case, the best option to observe CP violation then will be a conventional superbeam with a short baseline (of about 100 km) [13].

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REFERENCES

[1] S. Goswami, Phys. Rev. D55, 2931 (1997).

[2] M. Narayan, G. Rajasekaran and S. Uma Sankar, Phys. Rev. D36, 437 (1997).

[3] CHOOZ Collaboration: M. Apollonio et al, Phys. Lett. 466B, 415 (1999).

[4] G. Barenboim and F. Scheck, Phys. Lett. 440B 332 (1998).

[5] R. P. Thun and S. McKee, Phys. Lett. 439B, 123 (1998).

[6] S. Choubey, S. Goswami, N. Gupta and D. P. Roy, Phys. Rev. D64, 053002 (2001).

[7] V. Berezinksy, M. C. Gonzalez-Garcia, C. Pena-Garay, Phys. Lett. 517B, 149 (2001).

[8] G. Fogli, E. Lisi, A. Marrone and G. Scioscia, hep-ph/9906450.

[9] Sandhya Choubey, Srubabati Goswami and Kamales Kar, hep-ph/0004100.

[10] G. Barenboim, A. Dighe and S. Skaudhaug, hep-ph/0106002.

[11] G. Barenboim, M. Narayan, and S. Uma Sankar, hep-ph/0009247.

[12] G. Fogli, E. Lisi, A. Marrone and G. Scioscia, Phys. Rev. D59, 033001 (1999).

[13] J. J. Gomez-Cadenas et al, hep-ph/0105297.