Research Article

On Computation of Entropy of Hex-Derived Network

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A graph’s entropy is a functional one, based on both the graph itself and the distribution of probability on its vertex set. In the theory of information, graph entropy has its origins. Hex-derived networks have a variety of important applications in medication store, hardware, and system administration. In this article, we discuss hex-derived network of type 1 and 2, written as HDN1(n) and HDN2(n), respectively of order n. We also compute some degree-based entropies such as Randić, ABC, and GA entropy of HDN1(n) and HDN2(n).

1. Introduction and Preliminary Results

A graph G is a tuple G(V, E), where V is the set of vertices and E is the set of edges. A graph can be represented by a numerical quantity which is known as topological index. These indices have a vast number of applications in various fields, biology, computer science, information technology, and chemistry. Topological indices are used in QSAR/QSPR studies.

In order to understand the properties and information contained in the connectivity pattern of graphs, there are a number of numerical quantities, known as structure invariants, topological indices, or topological descriptors, which have been derived and studied over the past few decades. The topological indices have vast number of applications in the chemical graph theory which is the special branch of mathematical chemistry.

The combination of mathematics, information technology, and chemistry is a new division known as cheminformatics. It deals with QSAR and QSPR studies which predict the bio and physical chemical activities of compounds. The theory of topological indices was started by Wiener [1], when he was working on the boiling point of paraffins. The Wiener index is stated as

\[ W(G) = \sum_{(u,v) \in V(G)} d(u,v). \]  

(1)

A number of problems that occur in discrete mathematics, statistics, biology, computer science, chemistry, information theory, etc., investigate the entropy of structures to deal with them. The idea of entropy was given by Shannon in 1948 [2]. The entropy of a graph G is defined as follows.

Let G be a graph and V(G) = [1, 2, . . . , n] be the vertex set of G. Let \( P = (p_1, p_2, . . . , p_n) \) be the probability density of V(G) and VP(G) is the vertex packing polytope of G. Then, entropy of G with respect to P is

\[ H(G, P) = \min_{v \in V(P)} \sum_{i=1}^{n} p_i \log \left( \frac{1}{a_i} \right). \]  

(2)

Graph entropy has been utilized broadly to portray the structure of graph-based frameworks in numerical science [3]. Rashevsky gave the idea of graph entropy in 1955 [4]. He said that the graph entropy is dependent on order of vertices.

Hexagonal mesh was firstly proposed by Chen [5]. A hexagonal mesh consists of six triangles with dimension more than one. A 2-dimensional mesh \( HX(2) \) (Figure 1) is obtained from six triangles and \( HX(3) \) (Figure 1) is obtained from
$HX(2)$ by adding the layer of triangles around the boundary. Similarly, by adding the layer of triangles, we got $HX(n)$.

The number of arc-wise connected open sets obtained by the partition of plane by a graph are known as faces of $G$ [6]. In Figure 1, (3) shows the faces of $HX(2)$. By combining the faces of $HX(n)$, with the vertices, we obtain $\text{HDN}_1(n)$ hex-derived network of type 1. Figure 2 shows the $\text{HDN}_1(4)$ hex-derived network of type 1 with 4 vertices [7, 8].

The vertex and edge partition of $\text{HDN}_1(n)$ is shown in Tables 1 and 2, respectively.

As discussed before in the formation $\text{HDN}_1(n)$, if we join the vertices of $\text{HDN}_1(n)$ with each other, then the new figure formed by this is hex-derived network of type 2 $\text{HDN}_2(n)$. It is clear that $\text{HDN}_1(n)$ is a subgraph of $\text{HDN}_2(n)$. The hex-derived network of type 2 with 4 vertices is shown in Figure 3.

The vertex and edge partition of $\text{HDN}_2(n)$ is shown in Tables 3 and 4, respectively.

1.1. Degree-Based Topological Indices. The first degree-based topological index was presented by Milan Randić [9] and generalised by Bollobás and Erdos [10], and Amić et al. [11], in 1998.

$R_\alpha(G) = \sum_{e=uv \in E(G)} (d_u \times d_v)^\alpha,$ \hspace{1cm} (3)

where $\alpha = 1, -1, (1/2), - (1/2)$.

ABC index was introduced in 1998 by Estrada et al. [12]. It has the formulae:

$\text{ABC}(G) = \sum_{e=uv \in E(G)} \sqrt{d_u + d_v - 2 \over d_u \times d_v},$ \hspace{1cm} (4)

Vukičević was the person who studied this index for the first time [13]. It is written as $\text{GA}$ index and written as follows:

$\text{GA}(G) = \sum_{e=uv \in E(G)} 2 \sqrt{d_u \times d_v \over (d_u + d_v)},$ \hspace{1cm} (5)

Table 1: Vertex partition of $\text{HDN}_1(n)$.

| $d_u$, where $u \in V(\text{HDN}_1(n))$ | Cardinality of vertices |
|--------------------------------------|-------------------------|
| 3                                   | $6n^2 - 12n + 6$        |
| 5                                   | 6                       |
| 7                                   | $6n - 12$               |
| 12                                  | $3n^2 - 9n + 7$         |

1.2. Degree-Based Entropy of Graph. The entropy of a graph $G$ is defined as

$\text{ENT}_\psi(G) = -\sum_{u=1}^p d(u) \log \left[ d(u) \over \sum_{j=1}^p d(u_j) \right],$ \hspace{1cm} (6)

where $d_{ui}$ is the degree or vertex $u_i$. 

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Hexagonal meshes.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Hex-derived network $\text{HDN}_1(4)$.}
\end{figure}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
$u$ & $d_u$ & \text{Cardinality of vertices} \\
\hline
3 & $6n^2 - 12n + 6$ & \\
5 & 6 & \\
7 & $6n - 12$ & \\
12 & $3n^2 - 9n + 7$ & \\
\hline
\end{tabular}
\caption{Vertex partition of $\text{HDN}_1(n)$.}
\end{table}
Table 2: Edge partition of HDN 1 (n).

| (d(u), d(v)), where \( uv \in E(HDN 1(n)) \) | Cardinality of edges |
|-----------------------------------------------|----------------------|
| (3, 5)                                        | 12                   |
| (3, 7)                                        | 18n - 36             |
| (3, 12)                                       | 18n^2 - 54n + 42    |
| (5, 7)                                        | 12                   |
| (5, 12)                                       | 6                    |
| (7, 7)                                        | 6n - 18              |
| (7, 12)                                       | 12n - 24             |
| (12, 12)                                      | 9n^2 - 33n + 30     |

Figure 3: Hex-derived network HDN 2 (4).

Table 3: Vertex partition of HDN 2 (n).

| d(u), where \( u \in V(HDN 2(n)) \) | Cardinality of vertices |
|-------------------------------------|-------------------------|
| 5                                   | 6n                      |
| 6                                   | 6n^2 - 18n + 12         |
| 7                                   | 6n - 12                 |
| 12                                  | 3n^2 - 9n + 7           |

Table 4: Edge partition of HDN 2 (n).

| (d(u), d(v)), where \( uv \in E(HDN 2(n)) \) | Cardinality of edges |
|-----------------------------------------------|----------------------|
| (5, 5)                                        | 18                   |
| (5, 6)                                        | 12n - 24             |
| (5, 7)                                        | 12n - 12             |
| (5, 12)                                       | 6n                   |
| (6, 6)                                        | 9n^2 - 33n + 30      |
| (6, 7)                                        | 6n - 12              |
| (6, 12)                                       | 18n^2 - 60n + 48     |
| (7, 7)                                        | 6n - 18              |
| (7, 12)                                       | 12n - 24             |
| (12, 12)                                      | 9n^2 - 33n + 30      |

ENT_d(G) = - \( \sum_{i=1}^{p} \frac{d(u_i)}{\sum_{j=1}^{p} d(u_j)} \left[ \log d(u_i) - \log \sum_{j=1}^{p} d(u_j) \right] \),

\( \implies \) ENT_d(G) = \( \log \sum_{i=1}^{p} d(u_i)^d(u_i) \) - \( \frac{1}{\sum_{j=1}^{p} d(u_j)} \sum_{i=1}^{p} \left[ d(u_i) \log d(u_i) \right] \),

\( \implies \) ENT_d(G) = \( \log \sum_{i=1}^{p} d(u_i)^d(u_i) \) - \( \frac{1}{\sum_{j=1}^{p} d(u_j)} \sum_{i=1}^{p} \log d(u_i)^d(u_i) \).

By using the hand shaking lemma, we have \( \sum_{j=1}^{p} d(u_j) = 2q \). So,

ENT_d(G) = \( \log (2q) - \frac{1}{2q} \sum_{i=1}^{p} \log (d(u_i))^{d(u_i)} \).

1.3. Edge Partition-Based Entropy of Graph. The edge partition entropy of a graph G was introduced in 2014 by Chen et al. [14].

ENT_d(G) = \( - \sum_{u'v' \in E(G)} \sum_{uv \in E(G)} \frac{d(u')}{{d(u')} \cdot \log \left( \sum_{uv \in E(G)} \frac{d(u')}{d(u)} \right) \cdot \left( \frac{d(u')}{d(u)} \right)^a} \cdot \left( \frac{d(u)}{d(v)} \right)^a \).

1.4. Randic Entropy. Using equation (3), equation (9) is reduced as

ENT_{R_{a}}(G) = \log(R_a) - \frac{1}{R_a} \sum_{i=1}^{m} \left[ (d(u') \times d(v))^a \right] \cdot \left( d(u') \times d(v) \right)^a.

1.5. Atom Bond Connectivity Entropy. Using equation (4), equation (9) is reduced as

ENT_{ABC}(G) = \log(ABC) - \frac{1}{ABC} \sum_{i=1}^{m} \left[ \frac{d(u')+d(v)-2}{d(u) \times d(v)} \right] \cdot \left( d(u) \times d(v) \right)^{d(u)+d(v)-2}.

1.6. The Geometric Arithmetic Entropy. Using equation (5), equation (9) is reduced as


\[ \text{ENT}_{GA}(G) = \log(GA) - \frac{1}{GA} \sum_{i=1}^{m} \log \left[ \frac{2 \sqrt{d(u) \times d(v)}}{d(u) + d(v)} \right]^2 \]  

(12)

\[ \sum_{uv \in E(G)} \log \left[ \frac{2 \sqrt{d(u) \times d(v)}}{d(u) + d(v)} \right]^2 \] .

2. Main Results

Dacheng Xu et al. [15], Zehui et al. [16], and Imran et al. [8] computed the metric dimension and topological indices for hex-derived networks, respectively. Here, we discuss the first two types of hex-derived networks in this work and calculate the exact results for entropies based on degree and edges. These entropies and their variants are currently subjected to extensive research activity, see [7, 17–27]. For basic notations and definitions, see [28–31].

2.1. Results on Hex-Derived Network of Type 1. In this section, we calculate certain degree-based entropies of hex-derived network of type 1. We compute Randic entropy, ABC entropy, and GA entropy for hex-derived network HDN1(n).

2.2. Degree-Based Entropy of HDN1(n). If \( H_1 \equiv \text{HDN1}(n) \), then by using equation (8) and Table 1, we get

\[ \text{ENT}_d(H_1) = \log(2(27n^2 - 51n + 24)) - \frac{1}{2(27n^2 - 51n + 24)} \sum_{i=1}^{P} \left[ \log(d(r_i))^d(r_i) \right], \]

\[ \implies \text{ENT}_d(H_1) = \log(2(27n^2 - 51n + 24)) - \frac{1}{2(27n^2 - 51n + 24)} \left[ (6n^2 - 12n + 6) \times \log(3) + (6n^2 - 12n + 6) \times \log(12) \right], \]

\[ \implies \text{ENT}_d(H_1) = \log(54n^2 - 102n + 48) - \frac{1}{54n^2 - 102n + 48} \left[ (6n^2 - 12n + 6) \times \log(3) + (6n^2 - 12n + 6) \times \log(12) \right], \]

\[ \implies \text{ENT}_d(H_1) = \log(54n^2 - 102n + 48) - \frac{1}{54n^2 - 102n + 48} \left( 47.438707n^2 - 98.233822n + 49.220272 \right). \]

2.3. Randic Entropy. If \( H_1 \equiv \text{HDN1}(n) \), then by using Table 2 and equation (3), we have

\[ R_a(H_1) = 12 \times (15)^a + ((18n - 36) \times (21)^a) \]

\[ + ((18n^2 - 54n + 42) \times (36)^a) \]

\[ + ((12) \times (35)^a) + ((6) \times (60)^a) \]

\[ + ((6n - 18) \times (49)^a) \]

\[ + ((12n - 24) \times (84)^a) \]

\[ + ((9n^2 - 33n + 30) \times (144)^a), \]

and for \( a = 1 \), Randic index is

\[ R_1(H_1) = 1944n^2 - 5016n + 3138. \]

For \( a = -1 \),

\[ R_{-1}(H_1) = \frac{9}{16}n^2 - \frac{35}{48}n + \frac{173}{280}. \]

For \( a = (1/2) \),

\[ R_{(1/2)}(H_1) = 216n^2 + (42 \sqrt{21} - 720)n + 24 \sqrt{15} - 84 \sqrt{21} + 12 \sqrt{35} + 612. \]

For \( a = -(1/2) \),

\[ R_{-(1/2)}(H_1) = \frac{15}{4}n^2 + \left( \frac{24}{\sqrt{21}} \frac{141}{12} \right)n + \sqrt{15} - \frac{48}{\sqrt{21}} + \frac{12}{\sqrt{35}} + \frac{125}{14}. \]

Using equation (10) and Table 2, the Randic entropy is
\[
\text{ENT}_{R_a}(H_1) = \log(R_a) - \frac{1}{R_a} \left[ (12) \times \log \left( 15^a \right)^{15^a} + (18n - 36) \times \log \left( 21^a \right)^{21^a} \\
+ \left( 18n^2 - 54n + 42 \right) \times \log \left( 36^a \right)^{36^a} + 12 \times \log \left( 35^a \right)^{35^a} \\
+ 6 \times \log \left( 60^a \right)^{60^a} + (6n - 18) \times \log \left( 49^a \right)^{49^a} \\
+ (12n - 24) \times \log \left( 84^a \right)^{84^a} + (9n^2 - 33n + 30) \times \log \left( 144^a \right)^{144^a} \right].
\]

For \( \alpha = 1 \),

\[
\text{ENT}_{R_1}(H_1) = \log(R_1) - \frac{1}{R_1} \left[ (12) \times \log \left( 15 \right)^{15} + (18n - 36) \times \log \left( 21 \right)^{21} \\
+ \left( 18n^2 - 54n + 42 \right) \times \log \left( 36 \right)^{36} + 12 \times \log \left( 35 \right)^{35} \\
+ 6 \times \log \left( 60 \right)^{60} + (6n - 18) \times \log \left( 49 \right)^{49} \\
+ (12n - 24) \times \log \left( 84 \right)^{84} + (9n^2 - 33n + 30) \times \log \left( 144 \right)^{144} \right],
\]

\[ \implies \text{ENT}_{R_1}(H_1) = \log(R_1) - \frac{1}{R_1} \left( 3805.72181n^2 - 10345.600562n + 6807.897032 \right). \]

For \( \alpha = (1/2) \),

\[
\text{ENT}_{R_{(1/2)}}(H_1) = \log\left( R_{(1/2)} \right) - \frac{1}{R_{(1/2)}} \left[ (12) \times \log \left( \sqrt{15} \right)^{\sqrt{15}} + (18n - 36) \times \log \left( \sqrt{21} \right)^{\sqrt{21}} \\
+ \left( 18n^2 - 54n + 42 \right) \times \log \left( \sqrt{36} \right)^{\sqrt{36}} + 12 \times \log \left( \sqrt{35} \right)^{\sqrt{35}} \\
+ 6 \times \log \left( \sqrt{60} \right)^{\sqrt{60}} + (6n - 18) \times \log \left( \sqrt{49} \right)^{\sqrt{49}} \\
+ (12n - 24) \times \log \left( \sqrt{84} \right)^{\sqrt{84}} + (9n^2 - 33n + 30) \times \log \left( \sqrt{144} \right)^{\sqrt{144}} \right],
\]

\[ \implies \text{ENT}_{R_{(1/2)}}(H_1) = \log\left( R_{(1/2)} \right) - \frac{1}{R_{(1/2)}} \left( 200.59191n^2 - 483.632264n + 280.875589 \right). \]

For \( \alpha = -1 \),

\[
\text{ENT}_{R_{-1}}(H_1) = \log(R_{-1}) - \frac{1}{R_{-1}} \left[ (12) \times \log \left( \frac{1}{15} \right)^{1/15} + (18n - 36) \times \log \left( \frac{1}{21} \right)^{1/21} \\
+ \left( 18n^2 - 54n + 42 \right) \times \log \left( \frac{1}{36} \right)^{1/36} + 12 \times \log \left( \frac{1}{35} \right)^{1/35} \\
+ 6 \times \log \left( \frac{1}{60} \right)^{1/60} + (6n - 18) \times \log \left( \frac{1}{49} \right)^{1/49} \\
+ (12n - 24) \times \log \left( \frac{1}{84} \right)^{1/84} + (9n^2 - 33n + 30) \times \log \left( \frac{1}{144} \right)^{1/144} \right],
\]

\[ \implies \text{ENT}_{R_{-1}}(H_1) = \log(R_{-1}) - \frac{1}{R_{-1}} \left( -0.913049n^2 + 1.213888n - 0.476083 \right). \]
For $\alpha = -(1/2)$,

\[
\text{ENT}_{R_{-1/2}}(H_1) = \log\left( \frac{R_{-1/2}}{R_{1/2}} \right) - \frac{1}{R_{-1/2}} \left[ (12) \times \log \left( \frac{1}{\sqrt{15}} \right)^{1/\sqrt{15}} ight. \\
\left. + (18n - 36) \times \log \left( \frac{1}{\sqrt{21}} \right)^{1/\sqrt{21}} + 12 \times \log \left( \frac{1}{\sqrt{35}} \right)^{1/\sqrt{35}} \right. \\
\left. + \left( 18n^2 - 54n + 42 \right) \times \log \left( \frac{1}{\sqrt{36}} \right)^{1/\sqrt{36}} + 6 \times \log \left( \frac{1}{\sqrt{60}} \right)^{1/\sqrt{60}} \right. \\
\left. + (6n - 18) \times \log \left( \frac{1}{\sqrt{49}} \right)^{1/\sqrt{49}} + (12n - 24) \times \log \left( \frac{1}{\sqrt{84}} \right)^{1/\sqrt{84}} \right. \\
\left. + \left( 9n^2 - 33n + 30 \right) \times \log \left( \frac{1}{\sqrt{144}} \right)^{1/\sqrt{144}} \right], \\
\implies \text{ENT}_{R_{-1/2}}(H_1) = \log\left( \frac{R_{1/2}}{R_{-1/2}} \right) - \frac{1}{R_{-1/2}} \left( -3.14384n^2 + 5.390216n - 2.335494 \right). 
\]

where $R_n$ for $\alpha = 1, -1, (1/2), -(1/2)$ is written in equations (15)–(18), respectively.

\[\text{ABC}(H_1) = (12) \times \sqrt{\frac{(3 + 5 - 2)}{(3 \times 5)}} + (18n - 36) \times \sqrt{\frac{(3 + 7 - 2)}{(3 \times 7)}} \]
\[+ \left( 18n^2 - 54n + 42 \right) \times \sqrt{\frac{(3 + 12 - 2)}{(3 \times 12)}} + (12) \times \sqrt{\frac{(5 + 7 - 2)}{(5 \times 7)}} \]
\[+ (6) \times \sqrt{\frac{(5 + 12 - 2)}{(5 \times 12)}} + (6n - 18) \times \sqrt{\frac{(7 + 7 - 2)}{(7 \times 7)}} \]
\[+ (12n - 24) \times \sqrt{\frac{(7 + 12 - 2)}{(7 \times 12)}} + (9n^2 - 51n + 24) \times \sqrt{\frac{(12 + 12 - 2)}{(12 \times 12)}}. \]
\[\implies \text{ABC}(H_1) = \left( 3 \sqrt{\frac{22}{4}} + 3 \sqrt{13} \right) n^2 + \left( 12 \sqrt{\frac{22}{7}} - 9 \sqrt{13} - 2 \sqrt{\frac{357}{7}} - 33 \sqrt{\frac{22}{12}} \right) n \]
\[+ 12 \sqrt{\frac{10}{5}} - 24 \sqrt{\frac{42}{7}} + 7 \sqrt{13} + 12 \sqrt{\frac{144}{7}} - 4 \sqrt{\frac{357}{7}} + 5 \sqrt{\frac{22}{2}}. \]

Using equation (11) and Table 2, the ABC entropy is

\[\text{ABC}_{\text{ent}}(H_1) = \left( 3 \sqrt{\frac{22}{4}} + 3 \sqrt{13} \right) n^2 + \left( 12 \sqrt{\frac{22}{7}} - 9 \sqrt{13} - 2 \sqrt{\frac{357}{7}} - 33 \sqrt{\frac{22}{12}} \right) n \]
\[+ 12 \sqrt{\frac{10}{5}} - 24 \sqrt{\frac{42}{7}} + 7 \sqrt{13} + 12 \sqrt{\frac{144}{7}} - 4 \sqrt{\frac{357}{7}} + 5 \sqrt{\frac{22}{2}}. \]
\[
\begin{align*}
\text{ENT}_{ABC}(H_1) &= \log(ABC) - \frac{1}{ABC} \left[ 12 \times \log \left( \frac{\sqrt{\frac{3 + 5 - 2}{(3 \times 5)}}}{\sqrt{((3+5-2)/(3\times5))}} \right) \right. \\
&\quad + (18n - 36) \times \log \left( \sqrt{\frac{3 + 7 - 2}{(3 \times 7)}} \right) \cdot \sqrt{((3+7-2)/(3\times7))} \\
&\quad + \left(18n^2 - 54n + 42\right) \times \log \left( \frac{(3 + 12 - 2)}{(3 \times 12)} \right) \cdot \sqrt{((3+12-2)/(3\times12))} \\
&\quad + (12) \times \log \left( \sqrt{\frac{5 + 7 - 2}{(5 \times 7)}} \right) \cdot \sqrt{((5+7-2)/(5\times7))} \\
&\quad + (6) \times \log \left( \sqrt{\frac{5 + 12 - 2}{(5 \times 12)}} \right) \cdot \sqrt{((5+12-2)/(5\times12))} \\
&\quad + (6n - 18) \times \log \left( \frac{(7 + 7 - 2)}{(7 \times 7)} \right) \cdot \sqrt{((7+7-2)/(7\times7))} \\
&\quad + (12n - 24) \times \log \left( \frac{(7 + 12 - 2)}{(7 \times 12)} \right) \cdot \sqrt{((7+12-2)/(7\times12))} \\
&\quad + \left(9n^2 - 51n + 24\right) \times \log \left( \frac{(12 + 12 - 2)}{(12 \times 12)} \right) \cdot \sqrt{((12+12-2)/(12\times12))} \\
&\left. \right]\end{align*}
\]

\[
\begin{align*}
\Rightarrow \text{ENT}_{ABC}(H_1) &= \log(ABC) - \frac{1}{ABC} \left[ 12 \times \log \left( \frac{\sqrt{\frac{6}{15}}}{\sqrt{16/15}} \right) \right. \\
&\quad + (18n - 36) \times \log \left( \sqrt{\frac{8}{21}} \right) \cdot \sqrt{(8/21)} \\
&\quad + \left(18n^2 - 54n + 42\right) \times \log \left( \frac{\sqrt{13/36}}{\sqrt{13/36}} \right) \cdot \sqrt{((13/36)/13/36)} \\
&\quad + (12) \times \log \left( \frac{\sqrt{10/35}}{\sqrt{10/35}} \right) \cdot \sqrt{(10/35)/(10/35)} + (6) \times \log \left( \frac{\sqrt{15/60}}{\sqrt{15/60}} \right) \cdot \sqrt{(15/60)/(15/60)} \\
&\quad + (6n - 18) \times \log \left( \frac{\sqrt{12/49}}{\sqrt{12/49}} \right) \cdot \sqrt{(12/49)/(12/49)} + (12n - 24) \times \log \left( \frac{\sqrt{17/84}}{\sqrt{17/84}} \right) \cdot \sqrt{(17/84)/(17/84)} \\
&\quad + \left(9n^2 - 51n + 24\right) \times \log \left( \frac{\sqrt{22/144}}{\sqrt{22/144}} \right) \cdot \sqrt{(22/144)/(22/144)} \\
&\left. \right]\end{align*}
\]
where ABC index of HDN 1 \((n)\) is written in equation (24).

2.5. The Geometric Arithmetic Entropy of HDN 1 \((n)\). If \(H_1 \equiv \text{HDN} 1 \,(n)\), then by using equation (5) and Table 2, GA index is

\[
AG(H_1) = (12) \times \frac{2\sqrt{3 \times 5}}{3 + 5} \times (18n - 36) \times \frac{2\sqrt{3 \times 7}}{3 + 7} + (18n^2 - 54n + 42) \times \frac{2\sqrt{3 \times 12}}{3 + 12} \times (12) \times \frac{2\sqrt{5 \times 7}}{5 + 7} + (6) \times \frac{2\sqrt{5 \times 12}}{5 + 12} \times (6n - 18) \times \frac{2\sqrt{7 \times 7}}{7 + 7} + (12n - 24) \times \frac{2\sqrt{7 \times 12}}{7 + 12} + (9n^2 - 33n + 30) \times \frac{2\sqrt{12 \times 12}}{12 + 12}.
\]

\[
\Longrightarrow AG(H_1) = \frac{117}{5}n^3 + \left(\frac{582\sqrt{21}}{95} - \frac{381}{5}\right)n + 72\sqrt{\frac{35}{17}} - 1128\sqrt{\frac{21}{95}} + 2\sqrt{35} + \frac{318}{5}.
\]

Using equation (10) and Table 2, we have

\[
\begin{align*}
\text{ENT}_{GA}(H_1) &= \log(GA) - \frac{1}{GA} \left[(12) \times \log\left(\frac{2\sqrt{15}}{8}\right)^{(2\sqrt{3}/8)} + (18n - 36) \times \log\left(\frac{2\sqrt{21}}{10}\right)^{(2\sqrt{3}/10)} + (18n^2 - 54n + 42) \times \log\left(\frac{2\sqrt{36}}{15}\right)^{(2\sqrt{3}/15)} + (12) \times \log\left(\frac{2\sqrt{35}}{12}\right)^{(2\sqrt{3}/12)} + (6) \times \log\left(\frac{2\sqrt{60}}{17}\right)^{(2\sqrt{6}/17)} + (6n - 18) \times \log\left(\frac{2\sqrt{49}}{14}\right)^{(2\sqrt{7}/14)} + (12n - 24) \times \log\left(\frac{2\sqrt{84}}{19}\right)^{(2\sqrt{6}\sqrt{7}/19)} + (9n^2 - 33n + 30) \times \log\left(\frac{2\sqrt{144}}{24}\right)^{(2\sqrt{14}\sqrt{6}/24)} \right],
\end{align*}
\]

\[
\Longrightarrow \text{ENT}_{GA}(H_1) = \log(GA) - \frac{1}{GA} \left[-1.395504n^2 + 3.381504n - 2.101958\right],
\]

where GA index of HDN 1 \((n)\) is written in equation (26).

2.6. Results on Hex-Derived Network of Type 2. In this section, we calculate certain degree-based entropies of hex-derived network of type 2. We compute Randic entropy, ABC entropy, and GA entropy for hex-derived network HDN 2 \((n)\).

2.7. Degree-Based Entropy of HDN 2 \((n)\). If \(H_2 \equiv \text{HDN} 2 \,(n)\), then by using equation (8) and Table 3, we have

\[
\begin{align*}
\text{ENT}_d(H_2) &= \log\left(2\left(36n^2 - 72n + 36\right)\right) - \frac{1}{\left(2\left(36n^2 - 72n + 36\right)\right)} \left[\left(6n \times \log\left(\frac{5}{3}\right)\right) + \left(6n^2 - 18n + 12 \times \log(6)^a\right) + \left(6n - 12 \times \log\left(\frac{7}{5}\right)\right) + \left(3n^2 - 9n + 7 \times \log(12)^{12}\right)\right]
\end{align*}
\]

\[
\Longrightarrow \text{ENT}_d(H_2) = \log\left(72n^2 - 144n + 72\right) - \frac{1}{\left(72n^2 - 144n + 72\right)} \left(66.86397n^2 - 144.128692n + 75.689879\right).
\]

(28)

2.8. Randic Entropy of HDN 2 \((n)\). If \(H_2 \equiv \text{HDN} 2 \,(n)\), then by using equation (3) and Table 4, we have

\[
\begin{align*}
\text{R}_a(H_2) &= (18) \times \left(25\right)^a + (12n - 24) \times \left(30\right)^a + (6n - 12) \times \left(35\right)^a + (6n - 12) \times \left(60\right)^a + (9n^2 - 33n + 30) \times \left(36\right)^a + (6n - 12) \times \left(42\right)^a + (18n^2 - 60n + 48) \times \left(72\right)^a + (6n - 12) \times \left(49\right)^a + (12n - 24) \times \left(84\right)^a + (9n^2 - 33n + 30) \times \left(144\right)^a.
\end{align*}
\]

For \(\alpha = 1\),

\[
\begin{align*}
\text{R}_1(H_2) &= 6\left(486n^2 - 1261n + 794\right).
\end{align*}
\]

(30)
For $\alpha = -1$, 
\[
R_{-1}(H_2) = \frac{33075n^2 + 42815n - 20386}{58000}.
\] (31)

For $\alpha = 1/2$, 
\[
R_{1/2}(H_2) = 6(105 + 7(-3 + n) + 4\sqrt{21}(-2 + n) + 2\sqrt{30}(-2 + n) + \sqrt{42}(-2 + n)
+2\sqrt{35}(-1 + n) - 99n + 2\sqrt{15n} + 27n^2 + 6\sqrt{2}(8 - 10n + 3n^2)).
\] (32)

For $\alpha = -1/2$, 
\[
R_{-1/2}(H_2) = \frac{18}{5} + \frac{6}{7}(-3 + n) + 2\sqrt{\frac{5}{7}}(-2 + n) + \sqrt{\frac{6}{7}}(-2 + n) + 2\sqrt{\frac{6}{5}}(-2 + n)\alpha
+12\frac{(1 - n)}{\sqrt{35}} + \frac{5}{3}n + \frac{3}{4}(10 - 11n + 3n^2) + \frac{8 - 10n + 3n^2}{\sqrt{2}}.
\] (33)

Using equation (10) and Table 4, we have
\[
\text{ENT}_{R_{\alpha}}(H_2) = \log(R_\alpha) - \frac{1}{R_\alpha}[(18) \times \log ((25^\alpha)^{25}) + (12n - 24) \times \log (30^\alpha)^{30}] + (12n - 12) \times \log (35^\alpha)^{35} + (6n) \times \log (60^\alpha)^{60}
+(9n^2 - 33n + 30) \times \log (36^\alpha)^{36} + (6n - 12) \times \log (42^\alpha)^{42} + (18n^2 - 60n + 48) \times \log (72^\alpha)^{72} + (6n - 18) \times \log (49^\alpha)^{49}
+(12n - 24) \times \log (84^\alpha)^{84} + (9n^2 - 33n + 30) \times \log (144^\alpha)^{144}].
\] (34)

For $\alpha = 1$, 
\[
\text{ENT}_{R_{1}}(H_2) = \log(R_1) - \frac{1}{R_1}[(18) \times \log (25^{25}) + (12n - 24) \times \log (30^{30})]
+(12n - 12) \times \log (35^{35}) + (6n) \times \log (60^{60})
+(9n^2 - 33n + 30) \times \log (36^{36}) + (6n - 12) \times \log (42^{42}) + (18n^2 - 60n + 48) \times \log (72^{72}) + (6n - 18) \times \log (49^{49})
+(12n - 24) \times \log (84^{84}) + (9n^2 - 33n + 30) \times \log (144^{144})
\]
\[
\Rightarrow \text{ENT}_{R_{1}}(H_2) = \log(R_1) - \frac{1}{R_1}(5708.582715n^2 - 15463.045648n + 10152.693271).
\]
For $\alpha = (1/2)$,

$$\begin{align*}
\text{ENT}_{R_{(1/2)}}(H_2) &= \log(R_{(1/2)}) - \frac{1}{R_{(1/2)}} \left[ (18) \times \log(\sqrt[125]{25})^{\frac{1}{25}} + (12n - 24) \times \log(\sqrt[30]{30})^{\frac{1}{30}} \right. \\
&+ (12n - 12) \times \log(\sqrt[35]{35})^{\frac{1}{35}} + (6n) \times \log(\sqrt[60]{60})^{\frac{1}{60}} \\
&+ \left. (9n^2 - 33n + 30) \times \log(\sqrt[36]{36})^{\frac{1}{36}} + (6n - 12) \times \log(\sqrt[42]{42})^{\frac{1}{42}} \right. \\
&+ (18n^2 - 60n + 48) \times \log(\sqrt[72]{72})^{\frac{1}{72}} + (6n - 18) \times \log(\sqrt[49]{49})^{\frac{1}{49}} \\
&+ (12n - 24) \times \log(\sqrt[84]{84})^{\frac{1}{84}} + (9n^2 - 33n + 30) \times \log(\sqrt[144]{144})^{\frac{1}{144}} \right], \\
\implies \text{ENT}_{R_{(1/2)}}(H_2) &= \log(R_{(1/2)}) - \frac{1}{R_{(1/2)}} \left( 258.391474n^2 - 614.170818n + 359.638902 \right).
\end{align*}$$

For $\alpha = -1$,

$$\begin{align*}
\text{ENT}_{R_{(-1)}}(H_2) &= \log(R_{-1}) - \frac{1}{R_{-1}} \left[ (18) \times \log\left(\frac{1}{25}\right)^{(1/25)} + (12n - 24) \times \log\left(\frac{1}{30}\right)^{(1/30)} \right. \\
&+ (12n - 12) \times \log\left(\frac{1}{35}\right)^{(1/35)} + (6n) \times \log\left(\frac{1}{60}\right)^{(1/60)} \\
&+ (9n^2 - 33n + 30) \times \log\left(\frac{1}{36}\right)^{(1/36)} + (6n - 12) \times \log\left(\frac{1}{42}\right)^{(1/42)} \\
&+ (18n^2 - 60n + 48) \times \log\left(\frac{1}{72}\right)^{(1/72)} + (6n - 18) \times \log\left(\frac{1}{49}\right)^{(1/49)} \\
&+ (12n - 24) \times \log\left(\frac{1}{84}\right)^{(1/84)} + (9n^2 - 33n + 30) \times \log\left(\frac{1}{144}\right)^{(1/144}) \right], \\
\implies \text{ENT}_{R_{(-1)}}(H_2) &= \log(R_{-1}) - \frac{1}{R_{-1}} \left( -0.599231n^2 + 0.262484n + 0.187377 \right).
\end{align*}$$

For $\alpha = -(1/2)$,
Using equation (11) and Table 4, we have

\[
\text{ENT}_{R_{c1/2}}(H_2) = \log(R_{c1/2}) - \frac{1}{R_{c1/2}} \left[ (18) \times \log \left( \frac{1}{\sqrt{25}} \right)^{(1/\sqrt{25})} + (12n - 24) \times \log \left( \frac{1}{\sqrt{30}} \right)^{(1/\sqrt{30})} + (12n - 12) \times \log \left( \frac{1}{\sqrt{35}} \right)^{(1/\sqrt{35})} \right. \\
+ (6n) \times \log \left( \frac{1}{\sqrt{60}} \right)^{(1/\sqrt{60})} + (9n^2 - 33n + 30) \times \log \left( \frac{1}{\sqrt{36}} \right)^{(1/\sqrt{36})} \\
+ (6n - 12) \times \log \left( \frac{1}{\sqrt{42}} \right)^{(1/\sqrt{42})} + (18n^2 - 60n + 48) \times \log \left( \frac{1}{\sqrt{72}} \right)^{(1/\sqrt{72})} \\
+ (6n - 18) \times \log \left( \frac{1}{\sqrt{49}} \right)^{(1/\sqrt{49})} + (12n - 24) \times \log \left( \frac{1}{\sqrt{84}} \right)^{(1/\sqrt{84})} \\
+ (9n^2 - 33n + 30) \times \log \left( \frac{1}{\sqrt{144}} \right)^{(1/\sqrt{144})} \right],
\]

\[
\implies \text{ENT}_{R_{c1/2}}(H_2) = \log(R_{c1/2}) - \frac{1}{R_{c1/2}} \left[ -2.779385n^2 + 3.677553n - 0.972811 \right],
\]

where \( R_n \) for \( a = 1, -1, (1/2), -(1/2) \) is written in equations (30)–(33), respectively.

### 2.9. The Atom Bond Entropy of HDN \( 2(n) \)

If \( H_2 \equiv \text{HDN} \ 2(n) \), then by using equation (4) and Table 4, we have

\[
\text{ABC}(H_2) = (18) \times \sqrt{\frac{5 + 5 - 2}{5 \times 5}} + (12n - 24) \times \sqrt{\frac{5 + 6 - 2}{5 \times 6}} \\
+ (12n - 12) \times \sqrt{\frac{5 + 7 - 2}{5 \times 7}} + (6n) \times \sqrt{\frac{5 + 12 - 2}{5 \times 12}} \\
+ (9n^2 - 33n + 30) \times \sqrt{\frac{6 + 6 - 2}{6 \times 6}} + (6n - 12) \times \sqrt{\frac{6 + 7 - 2}{6 \times 7}} \\
+ (18n^2 - 60n + 48) \times \sqrt{\frac{6 + 12 - 2}{6 \times 12}} + (6n - 18) \times \sqrt{\frac{7 + 7 - 2}{7 \times 7}} \\
+ (12n - 24) \times \sqrt{\frac{7 + 12 - 2}{7 \times 12}} + (9n^2 - 33n + 30) \times \sqrt{\frac{12 + 12 - 2}{12 \times 12}},
\]

\[
\implies \text{ABC}(H_2) = \left( \frac{36\sqrt{2}}{5} \right) + \frac{12}{\sqrt{7}} \sqrt{3(-3 + n) + 6 \left( \frac{6}{\sqrt{5}}(-2 + n) + 2 \sqrt{\frac{51}{7}}(-2 + n) + \sqrt{\frac{66}{7}}(-2 + n) \right)} \\
+ 12 \sqrt{\frac{5}{7}}(-1 + n) + 3n + \frac{\sqrt{5}}{2}(10 - 11n + 3n^2) + \frac{1}{2} \sqrt{\frac{11}{2}(10 - 11n + 3n^2) + 2 \sqrt{2}(8 - 10n + 3n^2)}.
\]
\[ \text{ENT}_{ABC}(H_2) = \log(ABC) - \frac{1}{ABC} \left[ 18 \times \log \left( \frac{5 + 5 - 2}{5 \times 5} \right) \right. \]
\[ + (12n - 24) \times \log \left( \frac{5 + 6 - 2}{5 \times 6} \right) \left[ 5^{3+6-2/5} \right] \]
\[ + (12n - 12) \times \log \left( \frac{5 + 7 - 2}{5 \times 7} \right) \left[ 5^{3+7-2/5} \right] \]
\[ + (6n) \times \log \left( \frac{5 + 12 - 2}{5 \times 12} \right) \left[ 5^{3+12-2/5} \right] \]
\[ + \left( 9n^2 - 33n + 30 \right) \times \log \left( \frac{6 + 6 - 2}{6 \times 6} \right) \left[ 6^{3+6-2/6} \right] \]
\[ + (9n^2 - 33n + 30) \times \log \left( \frac{6 + 7 - 2}{6 \times 7} \right) \left[ 6^{3+7-2/6} \right] \]
\[ + (18n^2 - 60n + 48) \times \log \left( \frac{6 + 12 - 2}{6 \times 12} \right) \left[ 6^{3+12-2/6} \right] \]
\[ + (6n - 18) \times \log \left( \frac{7 + 7 - 2}{7 \times 7} \right) \left[ 7^{3+7-2/7} \right] \]
\[ + (12n - 24) \times \log \left( \frac{7 + 12 - 2}{7 \times 12} \right) \left[ 7^{3+12-2/7} \right] \]
\[ \left. + \left( 9n^2 - 33n + 30 \right) \times \log \left( \frac{12 + 12 - 2}{12 \times 12} \right) \right] \left[ 12^{3+12-2/12} \right] \]
\[ \text{ENT}_{ABC}(H_2) = \log(ABC) - \frac{1}{ABC} \left[ 18 \times \log \left( \frac{8}{25} \right) \right. \]
\[ + (12n - 24) \times \log \left( \frac{9}{30} \right) \left[ 9^{3/30} \right] \]
\[ + (12n - 24) \times \log \left( \frac{10}{35} \right) \left[ 10^{3/35} \right] \]
\[ + (6n) \times \log \left( \frac{15}{60} \right) \left[ 15^{3/60} \right] \]
\[ + (9n^2 - 33n + 30) \times \log \left( \frac{10}{36} \right) \left[ 10^{3/36} \right] \]
\[ + (18n^2 - 60n + 48) \times \log \left( \frac{11}{42} \right) \left[ 11^{3/42} \right] \]
\[ + (12n - 24) \times \log \left( \frac{16}{72} \right) \left[ 16^{3/72} \right] \]
\[ + (6n - 18) \times \log \left( \frac{12}{49} \right) \left[ 12^{3/49} \right] \]
\[ + (12n - 24) \times \log \left( \frac{17}{84} \right) \left[ 17^{3/84} \right] \]
\[ + (9n^2 - 33n + 30) \times \log \left( \frac{22}{144} \right) \left[ 22^{3/144} \right] \]
\[ \left. \right] \left. \right] \left. \right] \]
Table 5: Comparison table of entropies for HDN1 (n).

| n  | ENT_{R_1} | ENT_{R_{1/2}} | ENT_{R_{1/2}} | ENT_{ABC} | ENT_{GA} |
|----|-----------|---------------|---------------|-----------|---------|
| 2  | 1.426667  | 1.720856      | 1.34741       | 1.453922  | 1.668768|
| 3  | 1.952867  | 2.032098      | 1.389521      | 2.047508  | 1.672082|
| 4  | 2.295426  | 2.344017      | 2.352644      | 2.394541  | 2.205965|
| 5  | 2.541855  | 2.586246      | 2.604477      | 2.640122  | 2.51529 |
| 6  | 2.733943  | 2.779384      | 2.797527      | 2.830621  | 2.738371|
| 7  | 2.891273  | 2.938869      | 2.954339      | 2.986417  | 2.914042|
| 8  | 3.024485  | 3.074317      | 3.086541      | 3.118292  | 3.059337|
| 9  | 3.13985   | 3.191869      | 3.200907      | 3.232655  | 3.18381 |
| 10 | 3.241931  | 3.295628      | 3.301732      | 3.333636  | 3.291679|

Table 6: Comparison table of entropies for HDN2 (n).

| n  | ENT_{R_1} | ENT_{R_{1/2}} | ENT_{R_{1/2}} | ENT_{ABC} | ENT_{GA} |
|----|-----------|---------------|---------------|-----------|---------|
| 2  | 1.522379  | 1.548362      | 1.027711      | 1.549944  | 1.55593 |
| 3  | 2.097603  | 2.226359      | 1.479168      | 2.035114  | 2.147775|
| 4  | 2.451836  | 2.598216      | 1.785163      | 2.340108  | 2.496413|
| 5  | 2.703582  | 2.857086      | 2.016172      | 2.56459   | 2.744816|
| 6  | 2.89872   | 3.056051      | 2.202171      | 2.742565  | 2.937918|
| 7  | 3.058031  | 3.217744      | 2.355403      | 2.890104  | 3.095892|
| 8  | 3.192631  | 3.353968      | 2.487275      | 3.016135  | 3.22956 |
| 9  | 3.309161  | 3.471675      | 2.602399      | 3.126152  | 3.345406|
| 10 | 3.411899  | 3.575305      | 2.704483      | 3.223772  | 3.447626|

Figure 4: Randić entropy of HDN1 (n) for $\alpha = 1$.

Figure 5: Randić entropy of HDN1 (n) for $\alpha = (1/2)$. 
Figure 6: Randić entropy of HDN1 \((n)\) for \(\alpha = -1\).

Figure 7: Randić entropy of HDN1 \((n)\) for \(\alpha = -(1/2)\).

Figure 8: ABC entropy of HDN1 \((n)\).

Figure 9: GA entropy of HDN1 \((n)\).

Figure 10: Randić entropy of HDN2 \((n)\) for \(\alpha = 1\).

Figure 11: Randić entropy of HDN2 \((n)\) for \(\alpha = (1/2)\).
2.10. The Geometric Arithmetic Entropy of HDN 2\((n)\). If \(H_2 \equiv \text{HDN} 2\(n\)\), by using equation (5) and Table 4, we have
\[
\begin{align*}
\text{GA}(H_2) &= (18) \times \left( \frac{2\sqrt{5} \times 5}{5 + 5} \right) + (12n - 24) \times \frac{2\sqrt{5} \times 6}{5 + 6} \\
&\quad + (12n - 12) \times \frac{2\sqrt{5} \times 7}{5 + 7} + (6n) \times \frac{2\sqrt{5} \times 12}{5 + 12} \\
&\quad + (9n^2 - 33n + 30) \times \frac{2\sqrt{6} \times 6}{6 + 6} \\
&\quad + (6n - 12) \times \frac{2\sqrt{6} \times 7}{6 + 7} \\
&\quad + (6n - 18) \times \frac{2\sqrt{7} \times 7}{7 + 7} + (18n^2 - 60n + 48) \times \frac{2\sqrt{6} \times 12}{6 + 12} \\
&\quad + (12n - 24) \times \frac{2\sqrt{7} \times 12}{7 + 12} \\
&\quad + (9n^2 - 33n + 30) \times \frac{2\sqrt{12} \times 12}{12 + 12},
\end{align*}
\]
\[
\Rightarrow \text{GA}(H_2) = 60 + \frac{48}{19} \sqrt{21}(-2 + n) + \frac{24}{11} \sqrt{30}(-2 + n) \\
+ \frac{12}{13} \sqrt{42}(-2 + n) + 2\sqrt{35}(-1 + n) \\
- 60n + \frac{(24\sqrt{15}n)}{17} + 18n^2 + 4\sqrt{2}(8 - 10n + 3n^2).
\]
\[(41)\]
Using equation (12) and Table 4, we have

\[
\text{ENT}_{GA}(H_2) = \log(GA) - \frac{1}{GA} \left[ (18) \times \log \left( \frac{2\sqrt{5 \times 5}}{5 + 5} \right) \right]^{(2 \sqrt{5} / 5)^{5+5}}
\]

\[
+ (12n - 24) \times \log \left( \frac{2\sqrt{5 \times 6}}{5 + 6} \right) \]

\[
+ (12n - 12) \times \log \left( \frac{2\sqrt{5 \times 7}}{5 + 7} \right) + (6n) \times \log \left( \frac{2\sqrt{5 \times 12}}{5 + 12} \right)
\]

\[
+ (9n^2 - 33n + 30) \times \log \left( \frac{2\sqrt{5 \times 6 \times 6}}{6 + 6} \right) \]

\[
+ (6n - 12) \times \log \left( \frac{2\sqrt{5 \times 7 \times 7}}{6 + 7} \right) \]

\[
+ (18n^2 - 60n + 48) \times \log \left( \frac{2\sqrt{5 \times 7 \times 12}}{6 + 12} \right)
\]

\[
+ (6n - 18) \times \log \left( \frac{2\sqrt{7 \times 7 \times 7 \times 7}}{5 + 7} \right) \]

\[
+ (12n - 24) \times \log \left( \frac{2\sqrt{7 \times 7 \times 12}}{7 + 12} \right) \]

\[
+ (9n^2 - 33n + 30) \times \log \left( \frac{2\sqrt{12 \times 12 \times 12}}{12 + 12} \right)
\]

\[
\text{ENT}_{GA}(H_2) = \log(GA) - \frac{1}{GA} \left[ (18) \times \log \left( \frac{2\sqrt{5 \times 5 \times 10}}{10} \right) \right]^{(2 \sqrt{5} / 5)^{10}}
\]

\[
+ (12n - 24) \times \log \left( \frac{2\sqrt{30 \times 11}}{11} \right) + (12n - 12) \times \log \left( \frac{2\sqrt{35 \times 12}}{12} \right)
\]

\[
+ (6n) \times \log \left( \frac{2\sqrt{60 \times 17}}{12} \right) + (9n^2 - 33n + 30) \times \log \left( \frac{2\sqrt{36 \times 12}}{12} \right)
\]

\[
+ (6n - 12) \times \log \left( \frac{2\sqrt{4\times 13}}{13} \right) + (9n^2 - 33n + 30) \times \log \left( \frac{2\sqrt{5 \times 6 \times 12}}{12} \right)
\]

\[
+ (18n^2 - 60n + 48) \times \log \left( \frac{2\sqrt{72 \times 18}}{18} \right)
\]

\[
+ (6n - 18) \times \log \left( \frac{2\sqrt{48 \times 14}}{14} \right) + (12n - 24) \times \log \left( \frac{2\sqrt{84 \times 19}}{19} \right)
\]

\[
+ (9n^2 - 33n + 30) \times \log \left( \frac{2\sqrt{144 \times 24}}{24} \right)
\]

\[
\implies \text{ENT}_{GA}(H_2) = \log(GA) - \frac{1}{GA} \left( -0.434044n^2 + 0.944184n - 0.665748 \right),
\]
3. Conclusion

In this article, taking into account the definition of Shannon and Chen’s entropy, we studied two classifications of hex-derived network and also compute the entropies of them. We discuss the degree-based topological indices such as Randic, ABC, and GA index and find their closed formulae of entropy for hex-derived network. We in like manner enlisted the mathematical assessments of these entropies in Tables 5 and 6. We gave the relation of these entropies which help us to know the physiochemical activity of these networks.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

All authors contributed to this article equally.

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