The Moon turns out to be the perfect object to use the linear elasticity theory

V.P. Pavlov

\textsuperscript{a}Steklov Mathematical Institute of RAS, 8, Gubkina Str, Moscow, Russia.

Abstract

The applicability of the linear theory of elasticity to the Moon has been studied. As a criterion was taken the smallness of the strain tensor. The elastic moduli are obtained from the data on the longitudinal and transverse sound velocities in the Moon interior. The pressure was calculated in the framework of the model of a homogeneous solid sphere under the action of its own gravity. The strain tensor trace is of the order 0.02, which indicates the applicability.

The equilibrium condition in the body of the Moon is considered in the reference system rigidly connected to the rotating Moon. Except the elastic forces, all the mass forces in the body are potential. It means that acceleration of each of them (gravity, the Earth’s tidal and solar, centrifugal and inertia forces associated with the precession of the rotation axis) is minus gradient of the corresponding potential.

It turns out that there is a hierarchy among these potentials. If you take the order of gravity for 1, the relative order of the Earth’s tide is $\gamma \sim 10^{-5}$, the Sun’s one is 20 times smaller, and the rest - less than $10^{-8}$. This allows you to keep in equilibrium condition only the Earth’s tidal potential and apply the perturbation theory on $\gamma$.

The strain tensor in the body of the Moon is calculated in the first two (zero and first) order in $\gamma$ (i.e. taking into account the Earth’s tidal potential). Respectively the free energy density is calculated. Since the axis of the Moon rotation has its own non-zero declination to the ecliptic plane, the tidal potential variations take place during the rotation of the Moon around the Earth.
The estimation of the corresponding free energy density variations are made. Their dependence on the depth exhibit qualitative agreement with the data of depth dependence in the energy of deep moonquakes obtained in the project "Apollo."

Integral estimations of variations in the free energy for the year shows that it is many orders of magnitude greater than estimates of energy for the year of deep moonquakes energy. Thus offering an answer to the fundamental question: where is the source of energy released in the deep moonquakes.

Keywords: Linear elasticity, Moon, Tides, Strain tensor, Free energy density variations, Seismicity

1. Introduction.

Any statements on the planets seismicity are based on the accumulated information about their internal structure. To a large extent, this information is obtained by processing of data on seismic events. First of all, these are the data on the depth dependence of the longitudinal c_p and transverse c_s speed of sound in a solid body of a planet. Here one should note three circumstances. First, the speed values are reconstructed within linear elasticity theory. Second, the conventional relation between elastic moduli (bulk elastic modulus K and shear modulus µ) and speeds c_p and c_s and density ρ holds only when the coefficients in the wave equation describing the propagation of sound are constant, i.e. when all the above mentioned parameters of the medium are constant. Third, there is a natural applicability criterion of elasticity linear theory, the smallness of the strain tensor u_{ik}. In this theory, the trace of the strain tensor u_{ii} = p/K, where p is pressure. Let us take as a criterion of applicability of the smallness of the trace and see how it is met in geophysical models.

We note first of all that in those models (e.g., [Bullen and Haddon, 1967; Stacey and Davis, 2008; Garcia et al., 2011]) is used for the pressure the Bullen-Haddon hydrostatic approximation (µ = 0) in which the pressure at the surface is zero. In general, this is not so: body forces create internal stresses (including shear) even in the absence of external forces. In particular, the model of a homogeneous gravitating sphere (exercise 3 of §7 in
(Landau and Lifshitz, 2012)) gives

\[ p(r) = p(0) \left(1 - \left(1 + \frac{8\mu}{15K}\right)^{-1} \left(\frac{r}{R}\right)^2 \right) \]  \tag{2}  

Accordingly, for estimation of the criterion we use the model (2) for the pressure and the formulas (1) to the elastic moduli.

Conventional data (see, for example, (Stacey and Davis, 2008)) on the mechanical parameters of the Earth give \(\frac{8\mu}{15K} \sim 0.27\) for the whole mantle. For the pressure is reasonably to use normalization to the common value of the pressure at the lower boundary of the mantle. Then there are the following values \(u_{ii}\) at depths of 60, 800 and 2878 km: 0.19, 0.15 and 0.21. It means that with such precision linear elasticity, in framework of which the mechanical parameters and evaluated, is applicable. (Note that four significant figures for them are issued everywhere in conventional tables!)

For the Moon ((Garcia at al., 2011)) \(\frac{8\mu}{15K} \sim 0.3\), the value \(p(0)\) is calculated in (Pavlov, 2015):

\[ p(0) = \frac{2}{3} \pi \kappa g^2 R^2 \left(1 + \frac{4\mu}{3K}\right)^{-1} \left(1 + \frac{8\mu}{15K}\right) \sim 29 \times 10^9 \text{din cm}^{-2} \]  \tag{3}  

and estimates for the values of \(u_{ii}\) at the depth of 60, 270, 400 and 800 km are as follows: 0.008, 0.011, 0.012 and 0.017 respectively. Estimation errors are relates only to errors in the measurement of sound velocity. For \(c_p\) they are within 3% throughout the entire interior of the Moon, and for \(c_s\) they increase to 11% only on the boundary of the mantle and the liquid core. Moreover, with 7% accuracy density and elastic modules can be considered constant throughout the thickness of the mantle since the depths of 28 km to the border with the liquid core. Thus, unlike the Earth, the Moon is a perfect object to use linear elasticity.

2. The Moon model.

Let us apply the linear elasticity theory to describe the variations of the Moon interior stress state. We will work in a frame of reference rigidly attached to the rotating Moon. In this system, all the mechanical forces (except elastic) acting in the Moon interior (gravity, inertia forces and tidal) are potential: the acceleration of each of them is the negative gradient of the corresponding potential.
Since the polar flattening of the Moon is extremely small \((1.25 \times 10^{-3})\), for the gravitational potential \(\varphi_0\) it is enough to take the approximation of a homogeneous sphere

\[
\varphi_0(r) = \frac{2}{3} \pi \kappa \varrho R^2 \left( 3 - \left( \frac{r}{R} \right)^2 \right),
\]

where \(\kappa = 6.67 \times 10^{-8} cm^3 g^{-1} s^{-2}\) is the gravitational constant, \(\varrho = 3.34 g cm^{-3}\) the average density of the Moon, \(R = 1.737 \times 10^8 cm\) is its radius, \(r\) is the length of the radius vector \(\mathbf{r}\) of point inside the Moon. The order of \(\varphi_0\) is determined by the coefficients in front of the bracket (4) and is \(1.4 \times 10^{10} cm^2 s^{-2}\).

Motion of the Moon in the solar system is rather complicated. Working in our frame of reference, we must consider the potentials of all the inertial forces associated with the acceleration of this movement. We are interested in the dependence on \(r\) of the forces generated by these potentials.

Centrifugal potential associated with the proper rotation of the Moon, expressed by the formula

\[
\varphi_{cf}(\mathbf{r}) = \frac{1}{2} \left( \omega_j^2 r^2 - (\omega_j r_j)^2 \right),
\]

where \(\omega_j\) is angular velocity vector with the value of \(1.75 \times 10^{-7} sec^{-1}\) and an inclination relative to the ecliptic plane \(\epsilon = 1.54^\circ\). The order of this potential we estimate as its magnitude for \(r = R\) and obtain \(4.6 \times 10^2 cm^2 s^{-2}\).

Centrifugal potential \(\varphi_{cf(1)}\) associated with the rotation of the Moon around the Earth, expressed by the similar to (5) formula. It contains instead of \(\omega\) the angular velocity \(\omega_1\) of the Moon orbital motion around the Earth. The magnitude of \(\omega_1\) is the same as of \(\omega\), the average inclination \(i\) with respect to the ecliptic plane equals \(5.145^\circ\), and the order of the \(\varphi_{cf(1)}\) is same that of (5). A similar formula holds for the centrifugal potential \(\varphi_{cf(2)}\), associated with the rotation of the Earth-Moon pair around the Sun. The angular velocity \(\omega_2\) has the magnitude of \(1.5 \times 10^{-8} s^{-1}\), \(\omega_2\) is perpendicular to the plane of the ecliptic; the order of \(\varphi_{cf(2)}\) equals to \(3.4 cm^2 s^{-2}\).

The last of the force of inertia potential \(\varphi_{pr}\) is associated with the precession of the lunar orbit axis with a period of \(18.6 years = 7.8 \times 10^9 s\). Its angular velocity vector \(\varphi_{pr}\) has the magnitude \(0.8 \times 10^{-9} s^{-1}\) and deviates from the direction of \(\omega_1\) at an angle \(i\). \(\varphi_{pr}\) order is a hundred times less than that of \(\varphi_{cf(1)}\).
For the tidal potentials of the Earth and the Sun is sufficient to take the Laplace approximation

$$\varphi_1(r) = \frac{1}{2} \kappa \tilde{M} \tilde{R}^{-3} R^2 \left( \frac{r^2}{R^2} - \frac{3 (r \tilde{R})^2}{R^2 R^2} \right),$$

(6)

where $\tilde{M}$ and $\tilde{R}$ are the mass of the tide generating body and distance to it from the Moon center. Orders of these two potentials are defined as for (4), by the coefficients before of the bracket; for the Earth the order is $1.1 \times 10^5 cm^2 s^{-2}$, and for the Sun it is $6 \times 10^3 cm^2 s^{-2}$.

We see that there is a kind of hierarchy for the potentials: if we take the gravitational potential as 1, the Earth tidal potential has the order $0.8 \times 10^{-5}$, the Sun one has the order $4 \times 10^{-7}$, the centrifugal potentials $\varphi_{cf}$ and $\varphi_{cf1}$ have the order $3 \times 10^{-8}$, for $\varphi_{cf2}$ the order is $2.4 \times 10^{-10}$. We emphasize that the time dependence is only at the tidal potential with a characteristic period of 1 month: in our frame of reference only $\tilde{R}$ depend on time. Therefore, only the variations of the tidal potential can cause variations in the stress state in the the Moon interior. Since the own oscillations are damped in the Moon interior in a few hours, to estimate the tidal effects impact on the stress state of the Moon interior, we can use the condition of equilibrium instead of the equations of motion.

3. The linear elasticity equations.

In the linear theory of elasticity, Hooke’s law is valid:

$$\sigma_{ik} = K \delta_{ik} u_{jj} + 2\mu (u_{ik} - 3^{-1} \delta_{ik} u_{jj}) = -p \delta_{ik} + 2\mu u'_{ik},$$

(7)

where $u_{ik} = 2^{-1}(\partial_i u_k + \partial_k u_i)$ is the strain tensor, $u_i = u_i(r)$ is the displacement vector, $\partial_i = \partial/\partial r_i$, $p$ is pressure and $u'_{ik}$ defined by formula (7) is the deviator.

We write the equilibrium condition as a differential equation for the displacement vector:

$$\rho \partial_i \varphi = \partial_k \sigma_{ik} = ((K + 3^{-1} \mu) \partial_i \partial_k + \mu \delta_{ik} \Delta) u_k \equiv L_{ik} u_k,$$

(8)

where $\rho$ is density, $\Delta = \partial^2$ is Laplace operator and $\varphi = \Sigma u_i \varphi_i$ is the potentials sum of mass forces (gravity, centrifugal and tidal). $L_{ik}$ for our Moon model is the second order differential operator with the constant coefficients.
Everywhere in previous formulas and later the sum on repeated indices is implied: $a_k b_k \equiv \Sigma_k a_k b_k$.

Boundary condition for a differential equation (8) for $u_i$ is set on the Moon surface $\Sigma$ defined as $\varphi = const$:

$$\sigma_{ik} \partial_k \varphi|_{\Sigma} = 0. \quad (9)$$

The solution of equilibrium condition for $u_i$ is the sum of a particular solution of the inhomogeneous equation $L_{ik} u_k = \varrho \partial_i \varphi$, represented as a simple gradient, $u_i = \partial_i u(r)$, and a general solution of the homogeneous equation $L_{ik} \bar{u}_k = 0$, represented as the sum $\bar{u}_k = \partial_i \bar{u}(r) + w_i (r)$ of the gradient and the vector $w_i$ divergence of which is zero: $\partial_i w_i = 0$. The scalar functions $u$ and $\bar{u}$ naturally called deformation potentials.

The inhomogeneous equation takes the form of equation for $p = -K \Delta u$ — the contribution to the pressure from the inhomogeneous equation solution:

$$\partial_i p = -A \varrho \partial_i \varphi, \quad A = (1 + 4K/3\mu). \quad (10)$$

The solution of equation (10) can be obtained by the following argument ([Pavlov, 2015]). The variables $p, \varrho$ and $\varphi$ are the scalars relative to rotation group $O(3)$ and must depend on scalar combinations of their argument $r$ only. (Here we may temporarily consider $\varrho$ as changeable.) We may take the potentials $\varphi_0$ and two $\varphi_1$ for such scalar combinations denoting them as $\varphi_a, a = 0, 1, 2$. Then the equations (10) take the form

$$(\partial_a p = A \varrho) \partial_i \varphi_a, \partial_a = \partial/\partial \varphi_a \quad (11)$$

for each $a = 0, 1, 2$. But the gradients $\partial \varphi_a$ are linearly independent and therefore $\partial_a p - A \varrho = 0$. It means that $p$ depends on whole sum $\varphi$ only and it is valid the formula

$$p(r) = -A \int \varrho d\varphi = -A \varrho \varphi(r) + p_0. \quad (12)$$

(In (12) we come back to the constant $\varrho$ of our Moon model.) Note that formula (12) gives the generalization of the above relation (2) and allows for corrections by other than the gravitational potential. Integration constant $p_0$ is to be found from the boundary conditions (9).

The homogeneous equation has the form

$$\partial_i \ddot{p} - A \mu \Delta w_i = 0, \quad \ddot{p} = -K \Delta \bar{u}. \quad (13)$$
Take its divergence. We obtain the Laplace equation for $\tilde{p}$. The boundary condition for it can be taken zero as the pressures $p$ and $\tilde{p}$ are included in boundary condition (9) as summands. But harmonic function with zero values at the boundary is equal to 0 everywhere. Therefore $\tilde{u}$ and $w_i$ are harmonic functions.

To use the boundary condition, we need to find the deformation potentials $u$ and $\tilde{u}$ and harmonic vector-function $w_i$.

Having formula (12) for pressure and the relation $p = -K\Delta u$ between pressure and the trace of the strain tensor, we can recover the deformation potential $u$ and then the stress and strain tensors, as well as satisfy the boundary condition (9).

According to the arguments presented above, the scalar function $u$ depends only on $\varphi_0$ and $\varphi_1$. In what follows, as the scalar arguments of the function $u$ it is convenient to take the variable part of the gravity potential $x = \alpha_0 r^2/2$ and the dimensionless ratio $z = \varphi_1/x$ while distinguishing the small parameter $\gamma = \alpha_1/\alpha_0 \sim 10^{-5}$ in it: $z = \gamma \zeta$, $\zeta = -r^{-2} \tau_{ik} r_i r_k$. (Here we denote by $\alpha_0$ and $\alpha_1$ the coefficients before $r^2/2$ in expressions (4) and (6).)

In these terms, the equation for recovering $u$ has the form

$$\Delta u(x, z) = B(x(1 + \gamma \zeta) - c), \quad B = AK^{-1} \varrho, \quad c = \frac{3}{2} \alpha_0 R^2 + (A\varrho)^{-1} p_0. \quad (14)$$

It is shown in [Pavlov, 2015] that a solution to equation (14) is given by

$$u = \tilde{B} \left( x^2 \left( \frac{1}{5} - \frac{2}{7} \gamma \zeta \right) - \frac{2}{3} cx \right), \quad \tilde{B} = (2\alpha_0)^{-1} B. \quad (15)$$

It gives us a particular solution of the inhomogeneous equation $L_{ik} u_k = \varrho \partial_i \varphi$ for the deformation vector $u_k = \partial_k u$.

The general solutions for harmonic functions $\tilde{u}$ and $w_i$ are constructed within the framework of perturbation theory in the small parameter $\gamma$. The Laplace equation for $\tilde{u}(x, z)$ is given by

$$\Delta \tilde{u}(x, z) = 2\alpha_0 x^{-1} \left( \frac{3}{2} x u_x + x^2 u_{xx} - \frac{3}{2} \gamma \zeta u_z + \gamma^2 (\zeta^2 - \zeta + 2) u_{zz} \right) = 0. \quad (16)$$

In the first two orders in $\gamma$ the general solution for $\tilde{u}(x, z)$ with the natural condition $\tilde{u}(0, z) < \infty$ has the form

$$\tilde{u} = c_1 x z, \quad (17)$$
where \( c_1 \) is an arbitrary constant. It is shown in [Pavlov, 2015], that the vector function \( w_i \) vanishes in the first two orders in \( \gamma \).

Thus, in the first two orders in \( \gamma \), the general solution of the equilibrium conditions for the displacement vector \( u_i(r) \) is given by the sum of the gradient of the deformation potential \( u \) defined by formula (15) and the gradient of the deformation potential \( \tilde{u} \) defined by (17). It is shown in [Pavlov, 2015], that the terms of the zeroth order in \( \gamma \) of the boundary condition (9) fix the constant \( c \):

\[
c = \frac{\alpha_0}{2} R^2 \left( 1 + \frac{8\mu}{15K} \right),
\]

and the terms of the first order do it for the constant \( c_1 \):

\[
c_1 = -\frac{\alpha_0}{2} R^2 \frac{2}{35}.
\]

As a result, formula (15) and (17) – (19) allow to calculate the strain tensor and stress tensor up to first order in \( \gamma \). In particular, we get for the pressure

\[
p(r) = \frac{1}{2} \alpha_0 \varrho \left( 1 + \frac{4\mu}{3K} \right)^{-1} \left( \left( 1 + \frac{8\mu}{15K} \right) R^2 - r^2 \left( 1 - \gamma \tau_{ik} \frac{r_i r_k}{r^2} \right) \right). \tag{20}
\]

Obviously when \( \gamma = 0 \) we obtain (2) for uniform solid sphere, and when \( r = 0 \), the value \( p(0) \) of formula (3). It should be noted that the members of the first order in \( \gamma \) in (20) do not contribute to the value \( p(0) \).

4. Variations of the free energy density

In linear elasticity theory, the volume density \( f \) of the free energy of elastic stresses is expressed in terms of the stress tensor:

\[
f = \frac{K}{2} u_{jj}^2 + \mu (u'_{ik})^2. \tag{21}
\]

In Section 3, we have calculated the trace of the strain tensor \( u_{jj} = \Delta u \) (formula (14)). The results of the same section allows to calculate the deviator \( u'_{ik} = u_{ik} - 3^{-1} \delta_{ik} u_{jj} \). In both formulas, only the dimensionless function \( \zeta = 1 - 3 \cos \vartheta \) depends on time, where \( \vartheta \) is the angle between the radius vector \( r \) of a point in the Moon’s body and the radius vector \( \bar{R} \) of the Earth.
The function $\zeta$ depends periodically (with a period of $T = 1$ month) on time and experiences variations $\delta \zeta$. Define $\delta \zeta$ as the difference of absolute values of $\zeta$ for two extreme positions of $\tilde{R}$ in our coordinate system. In the spherical coordinates, $r = r(\cos \theta \cos \phi, \cos \theta \sin \phi, \sin \theta)$, where $\theta$ and $\phi$ are the latitude and longitude of the radius vector $r$ in the Moon’s body (the axis $z$ is directed along the self-rotation axis of the Moon). Since the self-rotation of the Moon and its motion along the orbit are synchronized and the direction of the angular velocity vector $\omega_j$ is constant, from the standpoint of the Moon the extreme positions of the Earth correspond to $\tilde{R} = R(\cos \varepsilon, 0, \pm \sin \varepsilon)$, where $\varepsilon$ is the angle of inclination of $\omega_j$ to the ecliptic plane. Corrections related to the ellipticity of the Moon’s orbit for the $r$-dependent part of $\zeta$ are on the order of 0.05, and we neglect them. As a result, we obtain the following formula for the Earth tides:

$$\delta \zeta = |\sin 2\varepsilon| \cdot |\sin 2\theta| \cdot |\cos \phi|. \quad (22)$$

The variation of the function $\zeta$ corresponds to the variation of the free energy density (21):

$$\delta f = K u_{0jj} \delta u_{1jj} + 2\mu u_{0ik} \delta u_{1ik}', \quad (23)$$

where $u_{0jj} = B(x - c)$ is the trace of the strain tensor in the zeroth order in $\gamma$, $\delta u_{1jj} = B \gamma x \delta \zeta$ is the variation of the trace in the first order in $\gamma$, and the convolution $u_{0ik}' \delta u_{1ik}'$ is calculated in paper (Pavlov, 2015):

$$u_{0ik}' \delta u_{1ik}' = B^2 \gamma \frac{22}{105} x \left(x + \frac{21}{22} c_1\right) \delta \zeta. \quad (24)$$

As a result, we obtain the following formula for the variations of the free energy density:

$$\delta f = B^2 \gamma \delta \zeta x \left(K(x - c) + \frac{44}{105} \mu \left(x + \frac{21}{22} c_1\right)\right). \quad (25)$$

Here it is useful to introduce a new variable $y = r/R$ and substitute the mean values of elastic moduli, $\mu/K \sim 0.57$. Then (25) reduces to

$$\delta f = -0.75\gamma \delta \zeta K^{-1} p^2(0)y^2(y^2 - 1.04^2). \quad (26)$$

The graph of the variation of the free energy density versus the dimensionless radius $y = r/R$ has a zero at the center, a maximum at $y = 0.52$, and one-third of the maximum value on the surface (in the hydrostatic approximation it would be zero).
5. Comparison with seismic data

The data on moonquakes obtained in 1969 – 1977 by the expeditions of the Apollo project ((Nakamura at al., 1973; Lammlein at al., 1974)) have been discussed in the literature for more than four decades. Today, there is a catalogue ((Nakamura, 2003, 2005)) of more than 12 500 seismic events. Most events (more than 7000) are identified as deep moonquakes whose sources are concentrated at about 300 "nests" at a depth of between 700 and 1200 km.

According to the modern views on the mechanics of strength and destructions (see, for example, (Volkov and Korotkikh, 2008)), periodic variations of pressure make the main contribution to the variations of the free energy density and lead (in addition to ordinary dissipation into heat) to the accumulation of defects in a solid medium (dislocations, cracks, etc.). Such a process is accompanied by the concentration of energy in these structures. When the concentration reaches a certain critical limit, the medium is destroyed with a release of accumulated energy.

We draw attention to the qualitative similarity between two patterns: depth distributions for deep focus moonquakes and variations of the free energy density due to tides. This similarity suggests that it is the energy concentrated at defects that is released in deep focus moonquakes.

This suggestion is corroborated by the integral (over the depths between 700 and 1200 km, which correspond to the interval $[0.3R, 0.7R]$) estimate for the variations of the free energy, which is possible due to formula (26):

$$\delta F = \int \delta f(r) r^2 dr \cos \vartheta d\varphi$$  \hspace{1cm} (27)

over a half-period $T/2$. For the Earth tides, this estimate yields $0.9 \times 10^{29}$ erg during a period of $T/2 = 1.8 \times 10^7$ s, or $2 \times 10^{30}$ erg per year. This is mainly the energy of oscillations. However, in a nonideal continuum, the dissipation of the energy of oscillations occurs both due to viscosity and due to the accumulation of defects.

Available estimates for the fraction of dissipating energy are very rough, depend on how the nonideality of the medium is modeled, and are on the order of $10^{-2}$. Apparently, the overwhelming part of this energy is converted into heat and is spent on heating the body of the Moon. If all the heat goes outside, then the power of its flux amounts to $1.4 \times 10^2 ergs^{-1} cm^{-2} = 14 \times 10^{-6} W cm^{-2}$. The available experimental estimates ((Langseth at al.,
yield a value of $2 \times 10^{-6} \text{W cm}^{-2}$. Hence, theoretically, all the heat is spent on heating the body of the Moon.

The fraction of dissipating energy that is spent on the accumulation of defects is also estimated roughly ((Volkov and Korotkikh, 2008)). The data of laboratory experiments are formulated in terms of the number of cycles of periodic loading that lead to the destruction of a sample. Translation of these data into the language of energy stored in defects depends on the model of the destruction process and is estimated as $10^{-5}$ of the oscillation energy. In any case, this estimate is many orders of magnitude greater than the estimate of energy release in deep focus moonquakes (see Goins at al. (1981)), which amounts to $8 \times 10^{13} \text{erg}$ per year. We conclude that the energy of tidal oscillations is more than enough to explain where the energy released in deep focus moonquakes comes from.

6. Acknowledgements

I am grateful to N.A. Slavnov and to the participants of the seminar led by A.G. Kulikovskii for the discussion of the results and valuable remarks.

This work is supported by the Russian Science Foundation under grant 14-50-00005.

References

Bullen and Haddon, 1967 K.E. Bullen, R.A.W. Haddon
Derivation of an Earth model from free oscillation data
Proc. Natl. Acad. Sci. USA, 58(1967), 3, pp. 846 – 852

Garcia at al., 2011 R.F. Garcia, J. Gagnepain-Beyneix, S. Chevrot, P. Lognomné
Very preliminary reference Moon model
Phys. Earth Planet. Inter. 188(2011), pp. 96 – 113

Goins at al., 1981 N.R. Goins, A.M. Dainty, M. N. Toksöz
Seismic energy release of the Moon
J. Geophys. Res. 86(1981), B1, pp. 378 – 388
Lammlein at al., 1974 R. Lammlein, G.V. Latham, J. Dorman, Y. Nakamura, M. Ewing  
**Lunar seismicity, structure, and tectonics**  
Rev. Geophys. Space Phys. 12(1974), 1, pp. 1 – 21.

Landau and Lifshitz, 2012 L.D. Landau, E.M. Lifshitz  
**Course of Theoretical Physics, Vol. 7: Theory of Elasticity**  
Elsevier, Oxford, UK (2012)

Langseth at al., 1976 M.G. Langseth, S.J. Keihm, K. Peters  
**Revised lunar heat-flow values**  
Geochim. Cosmochim. Acta, Suppl. 7(1976), pp. 3143 – 3171

Nakamura at al., 1973 Y. Nakamura, D. Lammlein, G. Latham, M. Ewing, J. Dorman, F. Press, N. Toksöz  
**New seismic data on the state of the deep lunar interior**  
Science 181(1973), pp. 49 – 51

Nakamura, 2003 Y. Nakamura  
**New identification of deep moonquakes in the Apollo lunar seismic data**  
Phys. Earth Planet. Inter. 139(2003), pp. 197 – 205

Nakamura, 2005 Y.Nakamura  
**Farside deep moonquakes and the deep interior of the Moon**  
J. Geophys. Res. 110(2005), E1, p E01001

Pavlov, 2015 V.P.Pavlov  
**Perturbation Theory for the Stress Tensor in the Moon’s Body with Tidal Effects Taken into Account**  
Proceedings of the Steklov Institute of Mathematics, Vol. 289(2015), pp. 183 – 193

Stacey and Davis, 2008 F.D.Stacey, P.M. Davis  
**Physics of the Earth**  
Cambridge Univ. Press, Cambridge, UK(2008)

Volkov and Korotkikh, 2008 I.A.Volkov, Korotkikh Yu.G.  
**Equations of State of Damaged Viscoelastoplastic Media**  
Fizmatlit, Moscow, RF(2008) (in Russian)