Control of the Bose-Einstein condensate by dissipation. Nonlinear Zeno effect

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We show that controlled dissipation can be used as a tool for exploring fundamental phenomena and managing mesoscopic systems of cold atoms and Bose-Einstein condensates. Even the simplest boson-Josephson junction, that is, a Bose-Einstein condensate in a double-well trap, subjected to removal of atoms from one of the two potential minima allows one to observe such phenomena as the suppression of losses and the nonlinear Zeno effect. In such a system the controlled dissipation can be used to create desired macroscopic states and implement controlled switching among different quantum regimes.

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I. INTRODUCTION

Universal properties of condensed atomic gases as systems hosting spectacular nonlinear phenomena like instabilities and collapse, solitons and shock waves, localized modes, vortices and self-trapping and delocalizing transitions, nowadays are well known \cite{1}. These phenomena constitute the properties of Hamiltonian systems, where dissipation is considered as an undesirable destructing factor. This role of dissipation can be inverted, if a system possesses an intrinsic mechanism balancing the losses and giving origin to stable dissipative structures \cite{2}. Alternatively, the dissipation can lead to a constructive effect when its action is limited in time, allowing one to generate diverse nonlinear excitations \cite{3}, or when it has a nonlinear origin \cite{4} supporting localized patterns. These effects appear on the macroscopic scale. Study of the dissipative decay can reveal also the microscopic quantum properties of the atomic gases \cite{5} and even inhibit the losses of atoms, by inducing strong correlations due to a large imaginary scattering length \cite{6}.

The purpose of this article is to show that there are various dissipative regimes in systems of cold atoms and Bose-Einstein condensates (BECs) loaded in a multiwell trap and that removal of atoms can serve as a tool for exploring the fundamental quantum phenomena. In particular, we find that the dissipation can inhibit losses and allows one to manage the so-called Macroscopic Quantum Self-Trapping state (below simply the self-trapping state) \cite{7,8}. Moreover we observe the inhibition of losses due to atomic interactions which can be termed as the macroscopic nonlinear Zeno effect.

Specifically, we consider the case of a BEC in a double-well potential whose Hamiltonian dynamics is well understood theoretically and has been a subject of fundamental experiments and numerous potential applications. The basic model, the well-known boson-Josephson junction \cite{9,10}, is also analogous to a nonrigid quantum pendulum. In particular, it was already used for observation of the macroscopic quantum tunneling and self-trapping \cite{11}, whereas the future proposals include also the atomic Mach-Zehnder interferometer \cite{12}, the sensitive weak force detector \cite{13} and the atomic interferometer on a chip \cite{14}. It was previously used also to study thermal vs quantum decoherence \cite{15}.

In our setup, schematically depicted in Fig. 1 we introduce an additional element – the controlled removal of atoms from one of the two wells of the potential – and consider the effect of the dissipation, showing that the actual rate of atom losses has a dramatic dependence on the dissipation coefficient and on the atomic interactions strength.

We notice that in the earlier experiments \cite{16}, the Zeno effect was observed in a system of cold atoms loaded in

\begin{figure}
\centering
\includegraphics[width=\textwidth]{setup.png}
\caption{(Color online) A schematic representation of the setup. Atoms are removed from the left potential well through scattering process by an electron or laser beam.}
\end{figure}
an accelerating optical lattice with the magnitude of the acceleration varying in time. The macroscopic manifestation of Zeno and anti-Zeno effects in the Josephson junctions subjected to time-dependent perturbations was also studied [17]. Also, our statement of the problem, can be further developed to describe the experimental setup used in [19] for the observation of pulsed and continuous Zeno effects in an externally driven mixture of two hyperfine states of a $^{87}$Rb BEC, using the destructive measurement tool and can be described in the framework of the population of the states. However, this happens in a binary mixture with nonlinear interactions between the components, while separation of the two subsystems by a potential barrier in our setting induces only a linear coupling between them (i.e. the populations in the two wells) due to the quantum tunneling.

The organization of the article is as follows. In Sec. II, we deduce the master equation describing our system. An exact solution is found in Sec. III for noninteracting atoms. The nonlinear Zeno effect is described in Sec. IV. Sec. V is devoted to the description of several switching regimes which are induced and controlled by the dissipation. In the Conclusion (Sec. VI) we summarize our results and present an outlook.

II. THE MASTER EQUATION

We use the simplest boson-Josephson Hamiltonian [10, 11, 20] describing BEC in an asymmetric double-well potential $V(r)$, that is,

$$H = -J(a_1^\dagger a_2 + a_2^\dagger a_1) + \varepsilon n_1 + U_1 n_1^2 + U_2 n_2^2,$$  (1)

where $a_j$ and $a_j^\dagger$ ($j = 1, 2$) are boson operators for the wave functions $\varphi_{1,2}$ localized at the potential minima, $n_j = a_j^\dagger a_j$, $J$ is the single atom tunneling rate, $\varepsilon$ is the zero-point energy bias, $U_{1,2} = g/2 \int d^3r \varphi_{1,2}^4$, where $g = 4\hbar^2 a_s/m$, $a_s$ is the s-wave scattering length, and $m$ is the atomic mass (for a trap with the two quasi-degenerate lowest energy levels, $U_1 \approx U_2 = U$ with a good accuracy).

The controlled removal of atoms can be realized, e.g., by using the experimental technique based on the electron microscopy [21, 22]. A narrow electron beam is directed to one of the minima of the potential ionizing the atoms and the latter are removed from the condensate. Alternatively one can use a narrow laser beam. In both cases, the interaction with the beam serves as a continuous measurement tool and can be described in the framework of the standard Markovian approximation [23]. Introducing the probability $p \equiv p(k_1, \Delta t)$, where $\Delta t$ is the time interval and $k_m$ is a population of the $m$th well, the single-atom removal event can be cast as a quantum channel:

$$|k_1, k_2\rangle|0\rangle \rightarrow \sqrt{p}|k_1 - 1, k_2\rangle|1\rangle + \sqrt{1-p}|k_1, k_2\rangle|0\rangle,$$  (2)

where the atoms are removed from well 1, $|k_1, k_2\rangle = (a_1^\dagger)^{k_1} (a_2^\dagger)^{k_2}|0\rangle$ is the ket vector of the BEC state and $|j\rangle_R$ describes the atom counter. Introducing the atom removal rate $\Gamma$, we approximate $p(k_1, \Delta t) \approx \Gamma k_1 \Delta t$ for small $\Delta t$, much less than the tunneling time $\tau_T = \hbar/J$. In particular, in the experiments with the electron microscopy, the atom removal rate is computed to be $\Gamma \approx I\sigma_{en}/e$ [22], where $I$ is the current of the electron beam, $e$ is the electric charge of the electron, and $\sigma_{en}$ is the total ionization cross section.

In terms of the reduced density matrix $\rho$, describing the condensate alone, the quantum channel [2] is given by the Kraus superoperator representation $\rho \rightarrow M_0 \rho M_0^\dagger + M_1 \rho M_1^\dagger$, where for a small $\Delta t$ we have $M_0 = 1 - \Gamma n_1 \Delta t/2$ and $M_1 \approx \sqrt{\Gamma \Delta t} a_1$. This leads to the master equation in the Lindblad form

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \Gamma \left\{ a_1^\dagger a_1 \rho - \frac{n_1}{2} \rho - \frac{n_1}{2} \rho a_1 a_1^\dagger \right\}. \tag{3}$$

The Lindblad operator $\mathcal{D}(\cdot) = a(\cdot) a^\dagger - \{n_1, \cdot\}/2$ has the eigenvalues $\lambda = (-N, -N + 1/2, ..., -1/2, 0)$, where $N$ is the total number of atoms. The eigenstates are in the form $\rho = \rho^{(1)} \otimes \rho^{(2)}$, with $\rho^{(2)}$ corresponding to the $j$th well. Respectively, the right-hand side of Eq. (3), considered as a superoperator, has eigenvalues in the form $\lambda = i \Delta E - \mu$, where $\Delta E$ has the range of values of the difference between the energy levels of the Hamiltonian (1), while $0 \leq \mu \leq \Gamma N$. The only stationary state (i.e. attractor) is the zero eigenvalue eigenstate, which has $\rho_0^{(1)} = |0\rangle \langle 0|$, that is, no atoms in the left well. The dissipation part is also responsible, besides the removal of atoms, for the gradual loss of coherence between the wells of the double-well trap, counteracting the effect of the quantum tunneling. In its turn, the tunneling depends on the interactions between the atoms [2, 3, 10], thus leading to interesting dissipation regimes governed by the master equation (3).

III. NONINTERACTING-ATOMS CASE

Consider first the case of $U = 0$, which can be achieved by making $a_s$ negligible using the Feshbach resonance. Then Eq. (3) can be solved explicitly by using the adjoint equation (see, Ref. [22])

$$\frac{d\hat{A}}{dt} = \frac{i}{\hbar}[H, \hat{A}] + \Gamma \left\{ a_1^\dagger \hat{A} a_1 - \frac{n_1}{2} \hat{A} + \hat{A} \frac{n_1}{2} \right\} \tag{4}$$

for an observable $\hat{A}$ [we distinguish an operator solution of Eq. (4) with a hat]. First of all, for $\hat{a}_j(t)$ ($j = 1, 2$) Eq. (4) is solved by the ansatz $\hat{a}_j(t) = C_{1j}^t(t)a_1 + C_{2j}^t(t)a_2$ with the initial condition $\hat{a}_j(0) = a_j$. This gives

$$C_{11} = \frac{\lambda_+ e^{\lambda^+ t} - \lambda_- e^{\lambda^- t}}{2t\Omega}, \quad C_{22} = \frac{\lambda_+ e^{\lambda^- t} - \lambda_- e^{\lambda^+ t}}{2t\Omega},$$

$$C_{12} = C_{21} = \frac{J(e^{\lambda^+ t} - e^{\lambda^- t})}{2\hbar\Omega}.$$
where \( \lambda_\pm = -\frac{\Gamma}{4} - \frac{\Omega}{2\hbar} \pm i\Omega \) and \( \Omega = \sqrt{\frac{J^2}{\hbar^2} - \left[ \frac{\Gamma}{4} + \frac{\Omega}{2\hbar} \right]^2} \).

The time dependence of the operators \( \hat{a}_j(t) \) is given by the Hermitian conjugated expressions. Moreover, the time dependence of an arbitrary operator can be found in terms of the operators \( \hat{a}_j(t) \) and \( \hat{a}_j^\dagger(t) \). This is due to the conditional “Leibnitz rule” for the dissipation part of Eq. [1]: \( D^* [A\hat{B}] = D^* [\hat{A}\hat{B}] + AD^* [\hat{B}] \) valid when either \( [\hat{a}_1, \hat{A}] = 0 \) or \( [\hat{a}_1, \hat{B}] = 0 \). Hence, the coefficients \( C_{ij}(t) \) are sufficient to determine evolution of any observable.

Setting, for simplicity, \( \varepsilon = 0 \), and assuming that initially the condensate is in the ground state \( |\psi\rangle = (\hat{a}_1^\dagger + \hat{a}_2^\dagger)^{10} |0\rangle \) we obtain

\[
\langle N \rangle = e^{-\frac{\Gamma t}{4}} \left[ \frac{J^2}{(\hbar \Omega)^2} - \frac{\Gamma^2}{16\Omega^2} \cos(2\Omega t) \right] N_0, \tag{5}
\]

\[
\langle n_1 - n_2 \rangle = -e^{-\frac{\Gamma t}{4}} \frac{\Gamma}{4\Omega} \sin(2\Omega t) N_0. \tag{6}
\]

When \( \Gamma < 4J/\hbar \), Eqs. [4] and [5] describe the decaying Rabi oscillations with the decay rate \( \Gamma/2 \). Surprisingly, for \( \Gamma \gg 4J/\hbar \), the dynamics is characterized by two different loss rates: the initial stage with the rate \( \Gamma/2 \) and, for times exceeding \( 1/\Gamma \), a dramatically reduced dissipation rate \( \Gamma_{QT} \approx \frac{\Gamma_{QT}}{\Gamma} \). This can be explained as follows. Consider, for simplicity, the case \( \Gamma \gg J/\hbar \), that is, when \( \Gamma_{QT} \ll \Gamma \). We get for \( t \gg 1/\Gamma \):

\[
\langle n_1 \rangle \approx \frac{\Gamma_{QT} N_0}{\Gamma} e^{-\Gamma_{QT}t}, \quad \langle n_2 \rangle \approx \frac{N_0}{2} e^{-\Gamma_{QT}t}, \tag{7}
\]

Observe that initially the two potential minima are equally populated \( \langle n_i \rangle = N_0/2 \). Equation [7] shows that there few atoms \( \langle \Gamma_{QT}/\Gamma \ll 1 \rangle \) in well 1 after the time scale \( t \sim 1/\Gamma \), that is, the system state is close to the zero-eigenvalue eigenstate of the dissipation part of Eq. [3]. Further elimination of the atoms in this regime occurs via the quantum tunneling from well 2, thus giving origin to the dramatically reduced loss rate \( \Gamma_{QT} \).

The prevention of losses of atoms by a strong dissipation resembles the Zeno effect observed experimentally in a different setup [12]. Specifically, expressing our \( \Gamma_{QT} \) through the tunneling frequency \( \omega_R = 2J/\hbar \) we recover the decay rate \( \Gamma_{QT} = \omega_R^2/\Gamma \) which appears in the continuous Zeno effect of Ref. [13].

IV. THE CASE OF INTERACTING ATOMS

In the nonlinear case one cannot solve Eq. [4] exactly. The strategy now is to use the mean-field approximation, valid in the limit of a large number of atoms. To this end we consider the equations for the averaged quantities:

\[
\frac{d\langle n_1 \rangle}{dt} = iJ \left( \langle a_1^\dagger a_2 \rangle - \langle a_2^\dagger a_1 \rangle \right) - \Gamma \langle n_1 \rangle \tag{8a}
\]

\[
\frac{d\langle n_2 \rangle}{dt} = -iJ \left( \langle a_1^\dagger a_2 \rangle - \langle a_2^\dagger a_1 \rangle \right) \tag{8b}
\]

\[
\frac{d}{dt} \left( \langle a_1^\dagger a_2 \rangle - \langle a_2^\dagger a_1 \rangle \right) = 2iJ \left( \langle n_1 \rangle - \langle n_2 \rangle \right) + 2iU \left( \langle n_1 a_1^\dagger + a_2^\dagger \rangle a_2 - \langle a_2^\dagger a_1^\dagger \rangle \right) - \Gamma \left( \langle a_1^\dagger a_2 \rangle - \langle a_2^\dagger a_1 \rangle \right), \tag{8c}
\]

To obtain a closed system from Eqs. [8a]-[8c] in the nonlinear case one can decouple the fourth-order correlators as follows: \( \langle n_i a_j^\dagger a_j'' \rangle \approx \langle n_i \rangle \langle a_j^\dagger a_j'' \rangle \). This procedure corresponds to the mean-field approximation, that is, \( N \to \infty \), widely used for description of BEC and cold atoms for a large number of particles (we have checked its validity using the direct quantum Monte-Carlo simulations). The mean-field variables, \( z, \phi, \) and \( q \), correspond to the averaged quantities:

\[
\frac{d\langle n_1 \rangle}{dt} = \frac{\langle a_1^\dagger a_2 \rangle - \langle a_2^\dagger a_1 \rangle}{\langle n_1 \rangle + \langle n_2 \rangle}, \quad e^{i\phi} = \frac{\langle a_1^\dagger a_2 \rangle}{\langle n_1 \rangle + \langle n_2 \rangle}, \quad q = \frac{\langle n_1 \rangle + \langle n_2 \rangle}{N_0}. \tag{9}
\]

They satisfy the system

\[
\frac{dz}{d\tau} = -2\sqrt{1-z^2} \sin \phi - \frac{\gamma}{2} (1 - z^2), \tag{10a}
\]

\[
\frac{d\phi}{d\tau} = \frac{2}{\sqrt{1-z^2}} \cos \phi + \varepsilon + 2\Lambda qz, \tag{10b}
\]

\[
\frac{dq}{d\tau} = -\frac{\gamma}{2} q (1 + z). \tag{10c}
\]

Here we use the dimensionless time \( \tau = t/t_{QT} \) and the normalized atom removal rate \( \gamma = \Gamma_{QT} = \hbar \Gamma / J \). The parameter \( \Lambda = UN/J \) characterizes the atomic interactions in the condensate [3, 8].

We have checked, by comparing with the direct quantum Monte-Carlo simulations, that solutions of Eqs. [10a]-[10c] averaged in the classical phase space give an excellent agreement with Eq. [4] for \( N \sim 100 \) (whereas a good agreement is observed already for \( N \sim 10 \)). The essentially linear dynamics is observed for the interaction strength \( \Lambda \lesssim 1 \). However, for strong interactions \( \Lambda \gg 1 \) the self-trapping state [3, 8] features a dramatic reduction of the atom loss rate with increasing interaction strength (see Fig. 2) the self-trapping state is in well 2). The effect does not depend on the interaction type (attractive, \( \Lambda < 0 \), or repulsive, \( \Lambda > 0 \)) and appears for any value of \( \Gamma \).

This effect can be understood as follows. First, one has to distinguish between the two types of the self-trapping states in the Hamiltonian system: the running-phase states with \( \phi(t) \propto t \) and the fixed-phase states (see Refs. [8, 14]). There are the following fixed-phase self-trapping states: \( \cos \phi = -\text{sgn}(\Lambda) \) and \( z = \pm \sqrt{1 - \Lambda^2} \).

In the dissipative case, \( z = -1 \) defines an invariant subset of system [15] representing the mean-field description.
of the zero-eigenvalue eigenstate of the Lindblad term in Eq. (3), corresponding to the location of all atoms in well 2. Accordingly, the fixed-phase self-trapping state with \( z < 0 \) of the Hamiltonian system shows the new nonlinear dissipation rate, right after the self-trapping is significantly enhanced by the dissipation: \( z \) decreases to \(-1\), while the phase becomes equal to an odd (\( \Lambda > 0 \)) or even (\( \Lambda < 0 \)) multiple of \( \pi \) (the phase fluctuations, described by the expression \( \Delta e^{i\phi} = [1 - |e^{i\phi}|^2]^{1/2} \), decay to zero).

Further insight can be gained by visualizing the atomic interactions as an effective common potential experienced by the condensed atoms. Indeed, in the self-trapping state \( \langle n_1 \rangle \ll \langle n_2 \rangle \); hence, the nonlinear term in the averaged Hamiltonian (11) can be simplified

\[
U\left[\langle n_1^2 \rangle + \langle n_2^2 \rangle\right] \approx U\langle n_2^2 \rangle \approx U\langle N \rangle - 2U\langle N \rangle \langle n_1 \rangle.
\]

where we have used that the fluctuations of \( z \) are small (see Fig. 2) that is, \( \langle n_2^2 \rangle \approx \langle n_2 \rangle^2 \). The total number of atoms evolves adiabatically, since the dissipation rate is very small. Then, the last term in Eq. (11), which is proportional to \( \langle n_1 \rangle \), is but a simple renormalization of \( z: \varepsilon \rightarrow \varepsilon - 2U\langle N \rangle \) in Eq. (3) [respectively, (10)]. This allows one to derive the nonlinear dissipation rate of the self-trapping state. Considering the symmetric trap (\( \varepsilon = 0 \)) and setting \( \varepsilon_{NL} \equiv -2U\langle N \rangle \), provided that \( \hbar^2 \Gamma^2 + 4\varepsilon_{NL}^2 \gg J^2 \), we get for \( t \gtrsim 1/\Gamma \)

\[
\Gamma_{NL} \approx \frac{4J^2\Gamma}{\hbar^2 \Gamma^2 + 4\varepsilon_{NL}^2}.
\]

This estimate turns out to be in excellent agreement with the numerical results (at the initial stage of the decay of the self-trapping state), as it is shown in Fig. 2 (if one uses in Eq. (12) the actual numerical number of atoms \( \langle N \rangle \) remaining in the self-trapping state). In the regime with \( \varepsilon_{NL} \gg \hbar \Gamma \) the nonlinear dissipation rate (12) reduces to \( \Gamma_{NL} \approx J^2 \Gamma/\varepsilon_{NL}^2 \). The preceding inequality condition also means that \( \Gamma_{NL} \ll \Gamma \), that is, the actual dissipation rate is dramatically reduced. This inhibition of the losses for \( \Lambda \gg 1 \) can be viewed as a nonlinear Zeno effect which, in contrast to the usual Zeno effect (25), appears for arbitrary \( \Gamma \).

To conclude this section let us make two remarks on the nonlinear decay rate given by Eq. (12). First, we notice that while the analogy between the effect of two-body interactions and the bias \( \varepsilon \) in pure linear system worked well in the estimate for the decay rate in of the nonlinear Zeno effect, we emphasize that the role of these factors in the dynamics under consideration is very different. The linear bias is an external factor which defines the constant decay rate of the linear Zeno effect, that is, of a condensate of noninteracting atoms. The “induced bias” due to the two-body interactions, that is, \(-2U\langle N \rangle\), by itself is a dynamical quantity: It slowly changes with time, resulting, after all, in the change of the decay rate. In other words, after a sufficiently long time, sufficient for a significant loss of atoms, the nonlinear decay rate will coincide with the linear one. Second, the decay rate given in Eq. (12) has a broader application than just giving the decay of the self-trapping state (thus, it is a new nonlinear effect, different from the self-trapping itself). For instance, it gives a correction due to the nonlinearity to the decay rate of the linear Zeno effect also for a small \( \Lambda \). Indeed, there are two conditions of validity of Eq. (12): \( \hbar^2 \Gamma^2 + 4\varepsilon_{NL}^2 \gg J^2 \), satisfied also by taking large \( \Gamma \gg J/\hbar \) (and an arbitrary \( \Lambda \)), and that there few atoms in well 1, which is the stage of the Zeno effect.

V. QUANTUM SWITCHING INDUCED BY DISSIPATION

Quantum switching can be induced by simply removing atoms for a short time in a switch-on manner. For instance, application of the atom removal for a time interval on the order of \( 1/\Gamma \) draws the system close to the self-trapping state with \( z \approx -1 \). This is illustrated in Fig. 6 where about 50\% of initially loaded atoms remain in a self-trapping state, whereas until the action of dissipation, the system was in the Josephson oscillations regime.

By directing the dissipation tool to the well where the self-trapping state is located and by varying the time of application of the dissipation and the other parameters, for example, \( \Lambda \), one can obtain the switching between the self-trapping states in the two wells of the double-well trap, Fig. 3(a), or induce the switching to the macroscopic quantum tunneling regime, Fig. 3(b). The nonadiabatic variation of the dissipation induces large phase fluctuations, \( \Delta e^{i\phi} = [1 - |e^{i\phi}|^2]^{1/2} \rightarrow 1 \), see Fig. 3(b), in contrast to the continuous action of a constant dissipation, where the phase fluctuations rapidly decay.
or, alternatively, as an induced controlled dissipation of a mesoscopic system. Combining these, apparently different interpretations of the interaction between a given mesoscopic system and an external system considered as a reservoir, with the nonlinearity due to interatomic interactions, opens remarkable possibilities for observation of the fundamental quantum phenomena in open systems on one hand, and on the other hand, for preparation and controlled manipulation of mesoscopic systems, for example, a Bose-Einstein condensate. More specifically, we have shown that while the controlled dissipation attenuates losses of atoms (already known phenomenon, interpreted also as the celebrated Zeno effect), when combined with the strong atomic interactions it results in an essentially new regime, where the atomic decay rate is practically zero, due to emergence of a quasi-self-trapping state. Such a state still loses atoms (unlike the case of the standard self-trapping state at a fixed number of atoms) and in this sense, the nonlinear Zeno effect can be viewed as a signature of the self-trapping state. This new effect of drastic attenuation of the losses of atoms by the atomic interactions, which appears for any value of the dissipation parameter, can be viewed as the nonlinear Zeno effect. Moreover, application of the dissipation during a short interval of time opens new possibilities for control over the condensate, for example, inducing the switching between self-trapping states in the two wells of the double-well trap or between a self-trapping state and the macroscopic quantum tunneling regime.

We have considered only the simplest nontrivial trap, the double-well potential. In general, one should expect much richer behavior in the multiwell potentials and, moreover, in the case of BEC loaded in the optical lattices, manipulated by a local dissipation.

The macroscopic nature of the described phenomena suggests that they can be observed in any nonlinear physical system which is described by the nonlinear Schrödinger-like equation and has at least two different equilibria, one of which is subjected to dissipative losses, that is, in a fairly generic setup. For instance, there is a very close analogy with the nonlinear optics of Kerr media, in particular, the nonlinear optical fibers (say the standard Kerr fibers or hollow-core fibers filled with atomic gases), suggesting that the nonlinear Zeno effect can be observed also in the realm of nonlinear optics in the currently available experimental settings.

VI. CONCLUSION

Selective removal of atoms by a directed external beam of particles can be viewed as a continuous measurement or, alternatively, as an induced controlled dissipation of

VII. ACKNOWLEDGEMENTS

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[1] Emergent Nonlinear Phenomena in Bose-Einstein Condensates Theory and Experiment, edited by P. G. Kevrekidis, D. J. Frantzeskakis, and R. Carretero-Gonzlez (Springer, Berlin, 2008); Nonlinear Phenomena in Degenerate Quantum Gasses Physica D 38, edited by V. M. Pérez-García, et al. (2009).
[2] Dissipative Solitons, edited by N. Akhmediev and A. Ankiewicz (Springer-Verlag, Berlin, 2005).
[3] V. A. Brazhnyi, V. V. Konotop, V. M. Pérez-García, and H. Ott, Phys. Rev. Lett. 102, 144101 (2009).
[4] A. Alexandrescu and V. M. Pérez-García, Phys. Rev. A 73, 053610 (2006).
[5] E. A. Burt et al, Phy. Rev. Lett. 79, 337 (1997).
[6] N. Syassen at al, Science 320, 1329 (2008).
[7] A. Smerzi, S. Fantoni, S. Giovanazzi and S. R. Shenoy, Phys. Rev. Lett. 79, 4950 (1997).
[8] S. Raghavan, A. Smerzi, S. Fantoni, and S. R. Shenoy, Phys. Rev. A 59 620 (1999).
[9] A. J. Leggett, Rev. Mod. Phys. 73, 307 (2001).
[10] R. Gati and M. K. Oberthaler, J. Phys. B: At. Mol. Opt. Phys. 40, R61 (2007).
[11] M. Albiez et al, Phys. Rev. Lett. 95, 010402 (2005).
[12] Y. Shin et al, Phys. Rev. Lett. 92, 050405 (2004); C. Lee, Phys. Rev. Lett. 97 150402 (2006).
[13] M. Jääskeläinen and P. Meystre, Phys. Rev. A 73, 013602 (2006); B. V. Hall, S. Whitlock, R. Anderson, P. Hannaford, and A. I. Sidorov, Phys. Rev. Lett. 98, 030402 (2007).
[14] G.-B. Jo et al, Phys. Rev. Lett. 98, 030407 (2007).
[15] L. P. Pitaevskii and S. Stringari, Phys. Rev. Lett. 87, 180402 (2001).
[16] M. C. Fischer, B. Gutiérrez-Medina and M. G. Raizen, Phys. Rev. Lett. 87, 040402 (2001).
[17] A. Barone, G. Kurizki and A. G. Kofman, Phys. Rev. Lett. 92, 200403 (2004).
[18] E. W. Streed et al, Phys. Rev. Lett. 97, 260402 (2006).
[19] K. W. Mahmud, H. Perry and W. P. Reinhardt, Phys. Rev. A 71, 023615 (2005).
[20] D. Ananikian and T. Bergeman, Phys. Rev. A 73, 013604 (2006).
[21] T. Gericke, C. Utfeld, N. Hommerstad and H. Ott, Las. Phys. Lett. 3, 415 (2006).
[22] T. Gericke, P. Würtz, D. Reitz, T. Langen and H. Ott, Nat. Phys. 4, 949 (2008).
[23] H. P. Breuer and F. Petruccione, The Theory of Open Quantum Systems (Oxford University Press, Oxford, UK, 2002).
[24] V. S. Shchesnovich and M. Trippenbach, Phys. Rev. A 78, 023611 (2008).
[25] B. Misra and E. C. G. Sudarshan, J. Math. Phys. 18, 756 (1977).