We analyze final state strong interaction effects in $B \to D\rho$ and $B \to D^*\pi$ decays using the Regge model. We find that, due to the smallness of the contributions from the non-leading Regge trajectories ($\rho$, $f$, $\pi$ etc.), final state interaction phases are small if the Pomeron coupling to the charm quark is suppressed in comparison to lighter quarks. Our conclusion is that for $B$ decays into states containing charm, final state interaction effects should play a minor role.
The problem of final state strong interactions in non-leptonic heavy meson decays has recently received considerable theoretical interest [1, 2, 3, 4, 5]. The relevance of the problem is related to the present and future programs for studying CP-violation in heavy hadrons, in particular B meson, decays. As well known [6], CP-violation in such systems might be observed by measuring an interference effect between two different amplitudes, and the relevant physical observable, i.e the CP-odd asymmetry, turns out to depend crucially on the strong interaction phase difference between the two amplitudes.

Whereas the usual approach for charmed meson decays is the parametrization of the final state interaction effects by means of a resonant rescattering of the final particles, for B decays final state interaction effects are in general expected to be small. This seems rather plausible because the final decay products are moving away from each other with large momentum; due to the relativistic time dilatation, the formation time of the final particles is large and, when formed, they are far away from the color sources, which implies that strong phases induced by the color interactions should be small [7] (see also [8]). Some evidence for this comes for example from exclusive non-leptonic two-body B decays [9].

The expectation that the rescattering effects in the final state due to soft interactions become negligible in the \( m_B \to \infty \) limit has been challenged in [5]. Taking \( B \to \pi\pi \) and \( B \to K\pi \) as examples, these authors assume Pomeron dominance and Regge theory [10] at high energy (i.e. large \( \sqrt{s} = m_B \)) to estimate the size of the inelastic final state interaction and its effect on CP-violating asymmetries, such as \( \Delta \Gamma = \Gamma(B^- \to K^-\pi^0) - \Gamma(B^+ \to K^+\pi^0) \); their conclusion is that these inelastic effects, mainly induced by the Pomeron, are sizeable. Since a reliable way to compute these soft, non perturbative, effects is missing at the moment, the rather pessimistic conclusion reached in [5] is that the presence of final state interactions will limit the accuracy of the standard approaches to B decays that are based on the use of perturbative QCD supplemented by the factorization hypothesis, even though in the \( m_B \to \infty \) limit both methods are expected to be more and more reliable.

A rather different conclusion has been reached, on the other hand, by Zheng in [4]. Using basically the same approach (i.e. an approximate evaluation of the final state strong interaction S-matrix based on the Regge model) and considering \( B \to DK \) non leptonic decays, Zheng concludes that the approximation of neglecting final state strong phases is “accurate up to, roughly speaking, about 10%”. Because of these conflicting results, we believe worthwhile to investigate more accurately this problem. Therefore, in the present paper, we consider other decay channels: \( B \to D\rho, D^*\pi \) using the same model (Regge theory) already employed in [4] and [5]. Apart from
the choice of a different decay mode, our treatment differs from [4] and [5] because we explicitly compute some inelastic effects, i.e. the $D\rho \to D^*\pi$ rescattering. The two-body $D^*\pi$, $D\rho$ decays are presently investigated by several experiments (for a review see [11]) and present a noticeable theoretical and experimental interest, related to the validity of the factorization approximation and to the sign of the ratio of Wilson coefficients $a_2/a_1$.

For decay amplitudes, final state interactions are taken into account by means of the Watson’s theorem [8, 12]:

$$A = \sqrt{S}A_b$$

where $S$ is the $S$-matrix, $A_b$ are the bare amplitudes, i.e. decay amplitudes with no final state interactions, and $A$ are the full amplitudes. The $S$-matrix relates amplitudes with the same isospin $I$ and a given total angular momentum $J$. Since we consider the amplitudes $A_{1}^I = A(B \to D^*\pi)_I$ and $A_{2}^I = A(B \to D\rho)_I$, the final state has $J = 0$; moreover the vector mesons ($\rho$ or $D^*$) in the final state can only have longitudinal helicity ($\lambda = 0$), which means that, effectively, we are dealing with a situation analogous to a decay into scalar particles, which simplifies considerably the formalism.

Let us begin by writing down explicitly the isospin amplitudes ($I = 3/2$ and $I = 1/2$) in terms of the physical amplitudes

$$A(B \to D^*\pi)_{3/2} = -\sqrt{2/3}A(B^0 \to \bar{D}^*\pi^0) + \sqrt{1/3}A(B^0 \to D^*\pi^+)$$
$$A(B \to D^*\pi)_{1/2} = \sqrt{1/3}A(B^0 \to \bar{D}^*\pi^0) + \sqrt{2/3}A(B^0 \to D^*\pi^+)$$

and, similarly:

$$A(B \to D\rho)_{3/2} = -\sqrt{2/3}A(B^0 \to \bar{D}\rho^0) + \sqrt{1/3}A(B^0 \to D\rho^+)$$
$$A(B \to D\rho)_{1/2} = \sqrt{1/3}A(B^0 \to \bar{D}\rho^0) + \sqrt{2/3}A(B^0 \to D\rho^+)$$

The $J = 0$ $S$-matrix, for each given isospin channel, must satisfy the unitarity relation. In standard notation, the two-body $S$-matrix elements are given by ($i, j = 1, 2$)

$$S^{(0)I}_{ij} = \delta_{ij} + 2i\sqrt{\rho_i \rho_j}A^{(0)I}_{ij}$$

where the $J = 0$, isospin $I$ amplitude $A^{(0)I}_{ij} = A^{(0)I}_{ij}(s)$ is obtained by projecting the $J = 0$ angular momentum out of the amplitude $A_{ij}^I(s, t)$:

$$A^{(0)I}_{ij}(s) = \frac{1}{16\pi} \frac{s}{\sqrt{\lambda_i \lambda_j}} \int_{t-}^{t+} dt A_{ij}^I(s, t)$$
Working in the approximation \( m_D = m_{D^*}, m_\pi \simeq m_\rho \simeq 0 \), with \( \sqrt{s} = m_B \), we have \( \lambda_i = (s - m_D^2)^2, \rho_i = \sqrt{\lambda_i}/s \simeq (s - m_D^2)/s \), \( t_- = 0 \) and \( t_+ = -(s - m_D^2)^2/s \).

In order to compute Eq. (4) and Eq. (5), we need \( A_{ij}^I(s, t) \). As we stressed already, in order to evaluate these amplitudes, we will work in the Regge model, which should be a reasonable theoretical framework due to the rather large value of \( s = m_B^2 \). In terms of the Pomeron (\( P \)), which is the leading contribution, and the non-leading trajectories \( \rho, f(1270) \) and \( \pi \), neglecting Regge cuts \( [4] \) we have:

\[
\begin{align*}
A_{11}^{3/2} &= A(D^*\pi \rightarrow D^*\pi)_{3/2} = P + f + \rho \\
A_{12}^{3/2} &= A_{21}^{3/2} = A(D^*\pi \rightarrow D\rho)_{3/2} = \pi \\
A_{22}^{3/2} &= A(D\rho \rightarrow D\rho)_{3/2} = P' + f' + \rho' \\
A_{11}^{1/2} &= A(D^*\pi \rightarrow D^*\pi)_{1/2} = P + f - 2\rho \\
A_{12}^{1/2} &= A_{21}^{1/2} = A(D^*\pi \rightarrow D\rho)_{1/2} = -2\pi \\
A_{22}^{1/2} &= A(D\rho \rightarrow D\rho)_{1/2} = P' + f' - 2\rho' .
\end{align*}
\]

We observe that the primed \( (P', f', \rho') \) contributions may differ from the unprimed ones only for a numerical coefficient; we also observe that the leading Regge trajectories in the off-diagonal matrix elements should be \( \omega \) and \( A_2 \); however they only contribute to the helicity-flip amplitudes (with \( \lambda = \pm 1 \)) that are not of interest here, which is why we take into account the next-to-leading trajectory, i.e. the pion (\( \pi \)) Regge exchange.

Let us first consider the Pomeron contribution, that we parametrize as follows

\[
P = - \beta^P g(t) \left( \frac{s}{s_0} \right)^{\alpha_P(t)} e^{-i\frac{\pi}{2} \alpha_P(t)} ,
\]

with \( s_0 = 1 \text{ GeV}^2 \) and

\[
\alpha_P(t) = 1.08 + 0.25t \\
(\text{t in GeV}^2) ,
\]

as given by fits to hadron-hadron scattering total cross sections \([3] , [4] \). The product \( \beta^P \cdot g(t) = \beta^P(t) \) represents the Pomeron residue; for the \( t \)-dependence we assume

\[
g(t) = \frac{1}{(s - t/m_\rho^2)^2} \simeq e^{2st} ,
\]

which is motivated by the analogy with the electromagnetic form factor and by the smallness of \( t \), due to the exponential damping in \( (s/s_0)^{\alpha_P(t)} \). As for the residue at \( t = 0 \), i.e. \( \beta^P \), we assume, as usual, factorization:

\[
\beta^P = \beta_{D^*}^P \beta_{\pi}^P
\]
for the elastic $D^*\pi \to D^*\pi$ amplitude.

The residue at the vertex $P\pi\pi$ can be extracted from proton-proton and pion-proton high energy scattering; we find
\[ \beta_P^\pi \simeq \frac{2}{3} \beta_P^p = 5.1 \]
which is consistent with the hypothesis of the additive quark counting rule: $\beta_P^\pi = 2\beta_P^{(uu)}$, $\beta_P^p = 3\beta_P^{(uu)}$.

It is worthwhile to remark at this stage that we can obtain, from $\gamma p$ high energy scattering data, the $P\rho\rho$ residue $\beta_P^\rho$ by making the assumption of Vector Meson Dominance (VMD); in this way we find the approximate relation
\[ \beta_P^\rho \simeq \beta_P^\pi \]
which is numerically valid within 15%. Eq. (13) is also consistent with the additive quark counting rule. As for the coupling of the Pomeron to charm, assuming again the additive quark model, one has
\[ \beta_D^P = \beta_{D^*}^P = \beta_P^{(cu)} + \beta_P^{(uu)} \]
and for $\beta_P^{(cu)}$ one has to assume as an input some theoretical ansatz; for example in [4] it is assumed:
\[ \beta_P^{(cu)} = \frac{1}{10} \beta_P^{(uu)} \].

We shall assume Eq. (15) as well and will comment on this choice below. Let us observe that, by this assumption, we find
\[ P = P' \]
a result to be used in Eq. (6)-Eq. (7).

Let us now consider the non-leading Regge trajectories $R(= \rho, f, \pi)$. For these exchanges we write the general formula
\[ R = -\beta_R \frac{1 + (-)^s e^{-i\alpha_R(t)}}{2} \Gamma(l_R - \alpha_R(t))(\alpha')^{1-l_R} \alpha_R^{sR(t)} \]
as suggested in [13]. $\alpha_R(t)$ is the Regge trajectory given by
\[ \alpha_R(t) = \alpha_R(0) + \alpha't \]
with an universal slope $\alpha' = 0.93 \text{GeV}^{-2}$ and $\alpha_\rho(0) = \alpha_f(0) = 0.44$ and $\alpha_\pi(0) = -\alpha'm_\pi^2$. $l_R$ is the lowest spin occurring in the exchange degenerate trajectory ($l_\rho =$
$l_f = 1, l_\pi = 0$) and $s_R$ is the spin of the exchanged meson in the Regge amplitudes ($s_\rho = 1, s_f = 2, s_\pi = 0$). The choice Eq. (17) for the Reggeized amplitudes is suggested by the high energy limit of a Veneziano amplitude. Since $\alpha_R(t) = s_R + \alpha'(t - m_R^2)$, near $t = m_R^2$, Eq. (17) reduces to

$$R \approx \beta^R \frac{s^R}{(m_R^2 - t)} \quad (19)$$

which allows us to identify $\beta^R$ as the product of two on-shell coupling constants (as in [15], we neglect here the $t$-dependence of the Regge residues).

To compute the different residues, we assume factorization, exchange degeneracy and $SU(4)$ symmetry (which is of course largely violated, but should at least provide us with an order of magnitude estimate). Therefore we have $\beta^\rho = \beta^\rho_{\pi\pi}\beta^\rho_{D^*D^*}$, and also $\beta^\rho_{D^*D} = \beta^\rho_{D^*D^*}$ and $\beta^\rho_{\rho\rho} = \beta^\rho_{D^*D^*}$. Using as an input the $g_{\rho\pi\pi}$ coupling constant and Vector Meson Dominance to relate $\beta^\rho_{\rho\rho}$ to the electromagnetic coupling of the $\rho^+$ particle, we finally obtain $\beta^\rho = \beta^f \simeq 23$ and $\rho = \rho', f = f'$. As for the pion exchange, we obtain, by this method, $\beta^\pi_{\rho\rho} = 4.6$ GeV from $\rho \to \pi\pi$ decay, and $\beta^\pi_{D^*D^*} = 1.7$ GeV from a theoretical estimate of the $D^*D\pi$ coupling [16], with the result $\beta^\pi = 7.8$ GeV$^2$. Using these results, together with Eq. (14)-Eq. (17), we obtain the following results for the $J = 0$, isospin $I = 3/2$ and $1/2$ $2 \times 2$ $S$-matrices:

$$S^{(0)3/2} = \begin{pmatrix} 0.76 - 0.06i & -(0.17 + 1.6i) \times 10^{-2} \\ -(0.17 + 1.6i) \times 10^{-2} & 0.76 - 0.06i \end{pmatrix} \quad (20)$$

and

$$S^{(0)1/2} = \begin{pmatrix} 0.71 - 0.02i & +0.34 + 3.2i \times 10^{-2} \\ +0.34 + 3.2i \times 10^{-2} & 0.71 - 0.02i \end{pmatrix} \quad (21)$$

Let us now comment on these results. First of all, as one can see, $S^{(0)3/2}$ and $S^{(0)1/2}$ in Eq. (20) and Eq. (21) are not unitary matrices, the reason being that other inelastic effects, besides the $D\rho \to D^*\pi$ final state interaction, are present. We shall comment on this point later on; for the time being we observe that, because of the smallness of the off-diagonal matrix elements in Eq. (20) and Eq. (21), inelastic $D\rho \to D^*\pi$ scattering is not expected to play a major role in determining, through Watson’s theorem[15], the full $(B \to f)$ amplitude. For example, the bare amplitude $A_b(B^0 \to \bar{D}^0\rho^0)$ does contribute, through final state interactions, to the

\[\sqrt{S^{(0)3/2}}_{11} = \sqrt{S^{(0)3/2}}_{22} = 0.87 - 0.03i ; \quad \sqrt{S^{(0)3/2}}_{12} = \sqrt{S^{(0)3/2}}_{21} = -(0.07 + 0.9i) \times 10^{-2}\]

\[\sqrt{S^{(0)1/2}}_{11} = \sqrt{S^{(0)1/2}}_{22} = 0.84 - 0.02i ; \quad \sqrt{S^{(0)1/2}}_{12} = \sqrt{S^{(0)1/2}}_{21} = +(0.16 + 1.9i) \times 10^{-2}\]

\[\text{We have verified that this does not alter our numerical conclusions.}\]

\[\text{We find}\]

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decay amplitude $A(B^0 \rightarrow \bar{D}^* \pi^0)$, but its contribution to the width does not exceed a few percent. Moreover, the bare amplitudes $A_b(B^0 \rightarrow D^{*-} \pi^+)$ and $A_b(B^0 \rightarrow D^- \rho^+)$, that, in principle, also contribute to $A(B^0 \rightarrow \bar{D}^* \pi^0)$ via final state interactions and can destroy the simple predictions based on factorization\footnote{In the factorization approximation they are proportional to the Wilson coefficient $a_1$, while the bare amplitudes $A_b(B^0 \rightarrow \bar{D}^0 \rho^0)$ and $A_b(B^0 \rightarrow \bar{D}^0 \pi^0)$ depend on the much smaller coefficient $a_2$, see e.g. \cite{17}.}, give a negligibly small contribution (of the order 2%) to the width.

As mentioned earlier, the above results are obtained in the Regge model with exchange degeneracy. One could avoid assuming exchange degeneracy and take different residues and intercepts for the $\omega$ and $f$ as trajectories fitted by total cross sections and elastic scattering data \cite{18}. In this way, one would obtain: $\beta^\rho = 9$, $\beta^f = 21.5$, $\alpha^\rho(0) = 0.57$, $\alpha_f(0) = 0.43$, $\alpha' = 0.93 \text{GeV}^{-2}$. The difference in the residues ($\beta^\rho \neq \beta^f$) is largely compensated by the different intercept and the final results, as expressed by Eq. (20) and Eq. (21), would be unaffected (changes would be less than 3%).

These results should hold, at least qualitatively, also if we enlarge the basis of eigenstates to enforce the unitarity of the $S$-matrix. To show it explicitly implies a considerable amount of work, but we can be convinced of it by going to the approximation of neglecting the pion exchange contribution $\pi$ in Eq. (6) and Eq. (7) because of its smallness, and introducing, similarly to \cite{4} and \cite{5}, two effective states $|3\rangle$ and $|4\rangle$ to take into account the inelastic scatterings of $\bar{D}^* \pi$ and $\bar{D}^* \rho$ states respectively. In this way the unitary $4 \times 4$ $S$-matrix can be constructed

$$S^{(0)} = \begin{pmatrix}
\eta e^{2i\delta} & 0 & i\sqrt{1 - \eta^2} e^{i(\delta + \delta_1)} & 0 \\
0 & \eta e^{2i\delta} & 0 & i\sqrt{1 - \eta^2} e^{i(\delta + \delta_2)} \\
i\sqrt{1 - \eta^2} e^{i(\delta + \delta_1)} & 0 & \eta e^{2i\delta_1} & 0 \\
i\sqrt{1 - \eta^2} e^{i(\delta + \delta_2)} & 0 & 0 & \eta e^{2i\delta_2}
\end{pmatrix}$$

where, for $I = 3/2$, $\eta = 0.76$ and $\delta = -0.04$, while for $I = 1/2$, $\eta = 0.71$ and $\delta = -0.02$ ($\delta_1$ and $\delta_2$ are free parameters in this model). For reasonable values of $\delta_1$, $\delta_2$ ($\delta_1 \approx \delta_2$), we basically find the same result as in \cite{4}, i.e. that, within an uncertainty of $10 - 20\%$, the results that are obtained by including final state interactions do not differ from those obtained by the bare amplitudes with no final state interaction at all.

We can apply these results to the calculation of the width of some two-body $B$ decays into charmed states. With inelastic effects given mostly by the Pomeron contribution and neglecting the small strong phases $\delta$, the full decay amplitudes $A$
for $B \to D^*\pi$ and $B \to D\rho$, according to Eq. (1), are (see also [4]):

$$A = xA_b + \sqrt{1-x^2}A'_b$$  \hspace{1cm} (22)$$

where $A'_b$ is the $B$ decay amplitude into an inelastic final state and $x = \sqrt{(1+\eta)/2}$.

The decay rates with inelastic effects included can be obtained from

$$|A|^2 = |A_b|^2 + (1-x^2)(|A'_b|^2 - |A_b|^2).$$  \hspace{1cm} (23)$$

Now, if $A'_b$ is comparable to $A_b$, the decay rate thus obtained would be close to the rate obtained without final state interaction effects. This agrees well with experiment [8, 9, 11] for neutral $B$ decays into charged final states (e.g. $D^-\pi^+$, $D^-\rho^+$, $D^{*-}\pi^+$) where the factorization model works rather well for those amplitudes which do not receive contributions from the short-distance operator $O_2$ which depends on $a_2$. For those amplitudes which depend on both the short-distance operators $O_1$ and $O_2$, such as $B^+$ decays into $D^0\pi^+$, $\bar{D}^0\rho^+$, $D^{*0}\pi^+$, the rates obtained in the factorization model without final state interaction effects are significantly smaller than the experimental results using the value $a_2 \approx 0.11$ as predicted by QCD. This suggests that in $B$ decays there might be non-factorization contributions [19, 20].

Which conclusions can we draw from our analysis? As a preliminary remark, let us observe that, even though we have dealt here with specific decay channels, some aspects of our analysis are general enough to be applied also to other decay modes. For example we have found that the inelastic amplitude $D^*\pi \to D\rho$ is strongly suppressed as compared to the elastic ones. This result does not depend on the dominance, in this case, of the pion exchange which has a small intercept, i.e. $\alpha_\pi(0) \approx 0$, but would hold quite generally because of the suppression of the non-leading Regge trajectories as compared to the Pomeron contribution. Therefore the only surviving inelastic effects at $\sqrt{s} = m_B$ should consist of inelastic multiparticle production and should be dominated by Pomeron exchange. We have found, in agreement with [4], that also these inelastic effects should not destroy the predictions for exclusive two-body non-leptonic $B$ decays based on factorization and perturbative QCD. In [3], charmless final states were considered and the opposite conclusion was reached. The apparent contrast has its origin in the hypothesis contained in Eq. (15), that strongly reduces the Pomeron coupling to charm and therefore reduces the inelastic effects in all the channels with charmed mesons in the final state. This explains why [5] finds significant final strong interaction effects in the considered channels. Eq. (15) has been assumed in [4] on a pure theoretical ground, since there is no experimental information on $\beta^P(cu)$. However a dependence on the quark mass can be clearly seen in the $Kp$ total cross section which is asymptotically smaller than in $\pi p$ scattering; a fit to the
asymptotic cross section is obtained by taking a reduction of $2/3$ in the Pomeron-quark residue: $\beta_P^{su} \simeq \frac{2}{3} \beta_P^{uu}$; this shows a rather strong decrease with the quark mass. A powerlike dependence on the quark mass is certainly compatible with the result of Eq. (15), and, therefore, even if a clear numerical extrapolation is difficult, the assumption given by Eq. (15) is reasonable. Therefore we are led to the conclusion that final state interactions produce only moderate effects in the amplitudes provided that there are charmed (with or without open charm) particles among the decay products. On the other hand, for light charmless particles in the final state, the Pomeron contribution increases and rescattering effects may alter significantly the simple predictions based on perturbative QCD and heavy quark effective theory.

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