Reconstruction of non-classical cavity field states with snapshots of their decoherence

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The state of a microscopic system encodes its complete quantum description, from which the probabilities of all measurement outcomes are inferred. Being a statistical concept, the state cannot be obtained from a single system realization. It can be reconstructed¹ from an ensemble of copies, by performing measurements on different realizations²-⁸. Reconstructing the state of a set of trapped particles shielded from their environment is an important step for the investigation of the quantum to classical boundary⁵. While trapped atom state reconstructions⁶-⁸ have been achieved, it is challenging to perform similar experiments with trapped photons which require cavities storing light for very long times. Here, we report the complete reconstruction and pictorial representation of a variety of radiation states trapped in a cavity in which several photons survive long enough to be repeatedly measured. Information is extracted from the field by atoms crossing the cavity one by one. We exhibit a gallery of pictures featuring coherent states⁴, Fock states with a definite photon number and Schrödinger cat states which are superpositions of coherent states with different phases⁶. These states are equivalently represented by their density matrices in the photon-number basis or by their Wigner functions, which are distributions of the field complex amplitude¹¹. Quasi-classical coherent states have a Gaussian-shaped Wigner function while Fock and Schrödinger cat Wigner functions show oscillations and negativities revealing quantum interferences. Cavity damping induces decoherence which quickly washes out the Wigner functions oscillations⁵. We observe this process and realize movies of decoherence by reconstructing snapshots of Schrödinger cat states at successive times. Our reconstruction procedure is a useful tool for further decoherence and quantum feedback studies of fields trapped in one or two cavities.

Engineering and reconstructing non-classical states of trapped light requires cavities preventing the escape of a single photon during the preparation and read-out procedures. We have built a cavity made of highly reflecting superconducting mirrors¹² whose long damping time, Tc=0.13 s, allows the trapped field to interact with thousands of atoms crossing it one by one. The interaction with atoms is used to turn an initial coherent field into a Fock or Schrödinger cat state and, subsequently, to reconstruct it. An ensemble of trapped photons becomes, like a collection of trapped atoms, an “object of investigation” to be manipulated and observed for fundamental tests and quantum information purposes.

Our set-up is sketched in Fig. 1a. The cavity C, resonant at 51 GHz, is cooled to a temperature of 0.8 K (mean number of residual blackbody photons n0 = 0.05). A coherent microwave field with a Poisson photon number distribution (mean n0, standard deviation Δn = √n0) is initially injected in C using a classical pulsed source S. Rubidium atoms from an atomic beam are prepared in box B into the circular Rydberg state with principal quantum number 50 (|g⟩). The cavity is detuned from the transition between |g⟩ and the adjacent circular state 51 (|e⟩) by an amount δ, precluding atom-field energy exchange. The pulsed atom preparation produces Rydberg atoms with a 250 m/s velocity. Auxiliary microwave cavities R1 and R2 sandwiching C are connected to a microwave source S’. They are used to apply resonant pulses to the atoms. The R1 pulse performs the |g⟩ → (|e⟩ + |g⟩)√2 transformation. The same pulse, differing by an adjustable phase-shift δ, is applied in R2. Atoms are counted by the detector D discriminating |e⟩ and |g⟩ (one atom on average every 0.5ms). For experimental details, see refs 13 and 14.

The R1-R2 combination forms a Ramsey interferometer¹⁴. It is sensitive to the atomic state superposition phase-shift induced by the atom’s interaction with the field in C which is characterized by the Rabi frequency Ω/2π=49 kHz. This phase-shift is described by an operator Φ(N,δ) depending upon δ and the photon number operator N=a†a (a and a†: photon annihilation and creation operators). To lower order, Φ(N,δ) is linear in N, but for the small δΩ values of our experiment, we take into account its exact non-linear expression¹⁵. The interferometer measures cos(Φ(N,δ)+φ) which is sensitive to the diagonal elements of the field density matrix in the Fock state basis, but tells nothing about the coherences between these states. To get this information, we measure the phase-shifts produced by the field after it has been translated in phase space, by mixing it with reference coherent fields of adjustable complex amplitudes α. These translations, described by the operators Ω(α) = exp(αa† - α∗a), are achieved by injecting a second field pulse in C.

Figure 1 | Reconstructing a coherent state. a, Sketch of the set-up showing the stream of atoms prepared in box B and crossing the R1-R2 interferometer in which the cavity C, made of two mirrors facing each other, is inserted. The source S coupled to a waveguide generates a coherent microwave pulse irradiating C on the side. By diffraction on the mirrors’ edges, it injects in C a small coherent field with controlled amplitude and phase. The outside field vanishes quasi-instantaneously after S is switched off. The source S is used to prepare an initial field in C and, later, to translate the field for state reconstruction. Another pulsed source S’ feeds the interferometer cavities R1 and R2. Information is extracted from the field by state selective atomic counting in D. b, Density matrix (absolute values of matrix elements) of a coherent state of amplitude β = 2.5, reconstructed in an 11-dimension Hilbert space. The reconstruction parameters are δ(2π) = 65 kHz and φ = −Φ(0,δ)+π. We sample 161 points in phase space and for each point detect ~7,000 atoms over 600 realizations. The fidelity F(||ρ0||) of the reconstructed state is 0.98. c, Wigner function (in units of 2/Δn) obtained from the density matrix shown in b.

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We call \( \rho \) the density matrix of the field to be reconstructed (matrix elements \( \rho_{nm} \)), \( \rho^{(1)} = D(\alpha)\rho D(-\alpha) \) the density matrix after field translation and \( P_+ (P_-) \) the probability for finding in \( |e \rangle \) \((|g \rangle) \) the first atom having crossed the interferometer (experimentally obtained by averaging over many field realizations). The difference \( P_+ - P_- = \text{Tr}(\rho^{(1)} \cos(\Phi(N,\delta) + \phi)\rho) \) is the expectation value in the translated state of the diagonal field operator \( \cos(\Phi(N,\delta) + \phi) \) in state \( \rho \). The measurement is non-demolition for the photon number 15, and the ensemble average of first crossing atoms does not change \( \rho^{(1)} \). Hence, the same \( P_+ - P_- \) expression holds for the second (or any subsequent) atom. We thus determine \( P_+ - P_- \) by averaging the detections of successive atoms along a single field realization together with those coming from different realizations. A measuring sequence on each realization lasts 4 ms, a time short compared to the state characteristic evolution time. We also correct the raw \( P_+ - P_- \) values by taking into account the known imperfections of the interferometer.

The \( P_+ - P_- \) difference is also the expectation value of \( G(\alpha,\phi,\delta) = D(-\alpha)\cos(\Phi(N,\delta) + \phi)D(\alpha) \) in state \( \rho \). By sampling \( \alpha \) values, we obtain the expectations \( G(\alpha,\phi,\delta) \) of an ensemble of non-commuting \( G(\alpha,\phi,\delta) \) operators satisfying:

\[
\text{Tr}[\rho G(\alpha,\phi,\delta)] = g(\alpha,\phi,\delta).
\]

Provided we sample a large enough number of \( \alpha \)-points in phase space, formula (1) allows us to reconstruct \( \rho \). To insure that the reconstructed state does not contain any information other than that extracted from the data, we also maximize the field entropy

\[
-\text{Tr}[\rho \log \rho]
\]

during the reconstruction procedure (principle of maximum entropy\(^{16}\)).

The Wigner function (WF) associated to state \( \rho \) is defined at point \( \alpha \) in phase space as\(^{14}\)

\[
W(\alpha) = 2\text{Tr}(D(-\alpha) \rho D(\alpha) e^{i\alpha N})/\pi
\]

and (to within a normalization) is the expectation value of the photon number parity operator \( \exp(\pi N) \) in the state translated by \(-\alpha \). The WF could be determined directly\(^{17}\) if the atoms underwent an exact phase shift of \( \pi \) per photon, realizing the measurement of \( \exp(\pi N) \) after field translation by different \( \alpha \)'s. This would be a special case of reconstruction corresponding to \( \Phi(N,\delta) = 0 \).

Figure 2 displays the obtained density matrices together with the corresponding WFs for \( n_0 = 0 \) (vacuum), 1, 2, 3, and 4. As expected, the density matrices mainly exhibit a single diagonal peak. Each WF shows circular rings around phase space origin, where it is positive for even \( n_0 \), negative for odd \( n_0 \). The number of rings and their size increases as expected with \( n_0 \). Photonic Fock states with small \( n_0 \) have already been reconstructed in free-space\(^{18-20}\) or in a cavity\(^{21}\), but this is to our knowledge the first Fock state reconstruction with \( n_0 > 2 \).

To generate a SC state\(^{22}\), we first inject in C a coherent field of amplitude \( \beta = \sqrt{n_0} \). We then prepare an atom in the state \( |e \rangle + |g \rangle \sqrt{2} \) using R\(_1\) and send it into C. The two atomic components shift the phase of the field in opposite directions. Neglecting atom-field phase shift non-linearity, the field is split into two coherent states of opposite phase, from which two different states \( \chi \) and \( -\chi \) are obtained. These states have the same field amplitudes, but differ by a relative phase \( \chi = (|e \rangle - |g \rangle \sqrt{2})/2 \). The two states \( \chi \) and \( -\chi \) are equal in phase space, but have opposite parity. The WF of the field state \( |\chi \rangle \) is given by

\[
W(\alpha) = 2\text{Tr}(D(-\alpha) \rho D(\alpha) e^{i\alpha N})/\pi
\]

and (to within a normalization) is the expectation value of the photon number parity operator \( \exp(\pi N) \) in the state translated by \(-\alpha \). The WF could be determined directly\(^{17}\) if the atoms underwent an exact phase shift of \( \pi \) per photon, realizing the measurement of \( \exp(\pi N) \) after field translation by different \( \alpha \)'s. This would be a special case of reconstruction corresponding to \( \Phi(N,\delta) = 0 \).

\[
W(\alpha) = 2\text{Tr}(D(-\alpha) \rho D(\alpha) e^{i\alpha N})/\pi
\]

The fidelities \( F = (|n_0\rangle|n_0\rangle) \) of the reconstructed states are 0.89, 0.98, 0.92, 0.82, 0.51 for \( n_0 = 0 \) to 4, respectively. The WF could be determined directly\(^{17}\) if the atoms underwent an exact phase shift of \( \pi \) per photon, realizing the measurement of \( \exp(\pi N) \) after field translation by different \( \alpha \)'s. This would be a special case of reconstruction corresponding to \( \Phi(N,\delta) = 0 \).
Figure 3 | Reconstructing Schrödinger cat states. The WFs of even (a) and odd (b) SC states (in units of 2π) with n₀=3.5 and χ=0.37π are reconstructed, following state preparation. The same detuning (δ/2π = 51 kHz) and interferometer phase (θ = -θ0(0.5)+π) are used for state preparation and reconstruction. The number of sampling points is 600, with ~2,000 atoms detected at each point, in 400 realizations. The dimension of the Hilbert space used for reconstruction is 11. The small insets present for comparison the theoretical WFs computed in the case of ideal preparation and detection of the atomic state superpositions. Decoherence during state preparation is taken into account. The maximum theoretical values of the classical components and interference fringes are close to 0.5 and 1, respectively. In the reconstructed states, the quantum interference is smaller, mainly due to imperfections of the Ramsey interferometer which affect the cat state preparation (and not its reconstruction). c, Reconstructed WF of the field prepared in C when the state of the preparation atom is not read-out (statistical mixture of two classical fields). In the inset: corresponding theoretical WF.

Figure 3a and b shows the WFs of the even and odd cat states obtained from the same coherent field (n₀ = 3.5 and χ = 0.37π). They exhibit two well-separated positive peaks associated to the classical components, whose slightly elongated shape is due to the phase-shift non-linearity neglected above. The “size” of each SC state, defined as the squared distance between peaks, is d²=4n₀ sin²χ = 11.8 photons. Between these peaks, oscillating features with alternating positive and negative values are the signatures of the SCs' quantum interference. The even and odd SCs' have nearly identical quantum interference. The even and odd SCs have nearly identical alternating positive and negative values are the signatures of the SCs.

Schrödinger cats are paradigmatic states for exploring decoherence, the phenomenon accounting for the transition between quantum and classical behaviours. Our reconstruction method allows us to study this process. Immediately after state preparation, we observe the WF at increasing times in the WF. Their interference terms are the quantum interference term.

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The measured quantum coherence of the even and odd cats is plotted versus time in Fig. 3c. The coherence of each SC component from their mutual quantum coherence, we consider the mathematically translated reconstructed density matrix ρ = ρ(0) exp[iχ(0)] whose classical components are close to the vacuum state, and to |iχ(0)|. This formal translation leaves unchanged the distance of the two classical components in the phase plane as well as their mutual coherence.

In Fig. 4b, we present the density matrix ρ(1)(t) of the SC state in Fig. 4a, reconstructed for the same times. In each frame, the diagonal elements present two maxima around n = 0 and n = 11. The off-diagonal elements are of two kinds. Those for which |n-n'| ≈ 11 describe the classical coherence of the non-vacuum component and remain nearly unchanged on the observed timescale. The off-diagonal terms in the first row and column of the matrix (respectively ρ^T_{0n} and ρ^T_{nn}) exhibit a bell-shaped variation with n, centred at n = 11. These terms correspond to the SC quantum coherence responsible for the oscillations observed in the WF. The fast decay is the signature of decoherence.

The measured quantum coherence of the even and odd cats is plotted versus time in Fig. 4c. A common exponential fit yields a decoherence time T_d = 17 ± 3 ms. A simple analytical model of decoherence predicts T_d = 2T_c/d² = 22 ms at T = 0 K, reduced to T_d = 2T_c/d²(1+2n₀+4n₀²) = 19.5 ms when including thermal background at T = 0.8 K, in good agreement with the measured value. A movie of a smaller SC (d² = 8) yields T_d = 28 ms, illustrating the dependence of the decoherence time on the cat size. Earlier experiments have studied the relaxation of photonic and atomic SCs by observing specific features of their states, but this experiment is the first to realize a movie of decoherence on a fully reconstructed SC.
We have shown that atoms interacting with a cavity field can be used to engineer and reconstruct a wide variety of photonic states and to study their evolution. Pushing one step further, we plan to use information provided by the atoms to implement feedback procedures and preserve the quantum coherence over longer time intervals. We will also extend these studies to fields stored in two cavities. Atoms will be used to entangle the cavity fields into non-local quantum states, reconstruct these states and protect them against decoherence by quantum feedback operations.

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