Abstract—A scenario in which a single source communicates with a single destination via a distributed MIMO transceiver is considered. The source operates each of the transmit antennas via finite-capacity links, and likewise the destination is connected to the receiving antennas through capacity-constrained channels. Targeting a nomadic communication scenario, in which the distributed MIMO transceiver is designed to serve different standards or services, transmitters and receivers are assumed to be oblivious to the encoding functions shared by source and destination. Adopting a Gaussian symmetric interference network as the channel model (as for regularly placed transmitters and receivers), achievable rates are investigated and compared with an upper bound. It is concluded that in certain asymptotic and non-asymptotic regimes obliviousness of transmitters and receivers does not cause any loss of optimality.

I. INTRODUCTION

MIMO systems implemented via distributed antennas, connected via a wireless or wired backbone, have been recently advocated as a viable solution to provide multiplexing, array and micro- or macro-diversity gains in infrastructure or mesh/ ad hoc networks (see, e.g., [1]-[3] and references therein). In such systems with non-collocated antennas, the main challenge in realizing the full gains of MIMO systems is the efficient use of the channel resources needed to coordinate the participating antennas at the transmit and/or receive sides into effective multi-antenna arrays. These channel resources can be either in-band, that is, in the same time and frequency band of the main end-to-end transmission [1] [3], or out-of-band, i.e., over orthogonal channels, possibly realized via wired connections or different wireless radio interfaces [2] [4]. Moreover, the feasibility of different transmission schemes depends on the amount of information that the available transceivers have regarding the encoding functions shared by the sources and destinations of the transmitted data. For example, decode-and-forward-type schemes require full knowledge of the codebooks used to encode the received data, while compress-and-forward or amplify-and-forward strategies do not have this requirement.

Basic $2 \times 2$ distributed MIMO systems with full codebook information and with in-band signalling between transmit antennas, at one end, and receive antennas, at the other, were considered in [1] and [5] for half and full-duplex transceivers, respectively. Reference [6] studies a general MIMO system with infinite-capacity out-of-band links connecting the transmit-side antennas and finite-capacity links at the receive side, where antennas are assumed to be either codebook-unaware (i.e., oblivious) or informed. Another line of work that is of interest here deals with distributed-antenna transmitters or receivers in a cellular scenario in the presence of non-ideal out-of-band links connecting the cooperating (transmit or receive) antennas (base stations), which can be either oblivious or not [4].

In this paper, we consider the scenario depicted in Fig. 1 in which a single source $S$ has data to communicate to a remote destination $D$. Communication takes place via a distributed $M \times M$ MIMO system constructed by connecting the source to the $M$ transmitting antennas through (equal) finite-capacity links and likewise the $M$ receiving antennas to the destination. The finite-capacity links are assumed to be orthogonal among themselves and out-of-band. This is the case when the source and destination are connected to the transmitters and receivers, respectively, via a wired backbone or via orthogonal wireless interfaces. Targeting a scenario where the infrastructure of transmitting and receiving antennas is meant to serve different communication standards, we assume that transmitters and receivers are oblivious to the encoding function shared by source $S$ and destination $D$ as in [4] [6]. Our interest is in obtaining analytical insights into the role of finite capacities.
To this end, we adopt a simplified channel model for the MIMO channel between transmitters and receivers that corresponds to a Gaussian interference network described by a single parameter $\alpha$, as shown in Fig. 1. Beside allowing analytical tractability, this channel is a variant of the Wyner model for infrastructure networks [8] that has been studied in a number of works (see [2] and [4] for a review). An upper bound and achievable rates are derived. It is shown that, in certain asymptotic and non-asymptotic regimes, no loss of optimality is incurred in designing the system for nomadic applications (i.e., assuming oblivious transmitters and receivers).

II. SYSTEM MODEL

A source $S$ is connected via finite-capacity (error-free) links of capacity $C$ [bit/ symbol] to $M$ distributed transmitters. Each transmitter has power constraint $P$. The source aims at communicating with a remote destination $D$, which is in turn connected to $M$ distributed receivers via links of capacity $C'$. Targeting nomadic applications, transmitters and receivers are assumed to be oblivious to the encoding function shared by source $S$ and destination $D$. More specifically: (i) Each transmitter is equipped with an independently generated standard complex Gaussian codebook of size $2^nC$ ($n$ is the length of the transmission block) with average power $P$, which is known to the source $S$. These $M$ codebooks can be obtained by $S$ via, e.g., a local public database. Through the finite-capacity link, the source selects which codeword in the codebook should be sent by each antenna in a given transmission block. In other words, no processing is carried out at the transmitters, except simple mapping of the index received by the source and the codebook; (ii) Each receiver is unaware of the processing carried out at the source and of the codebooks of the transmitters, and merely performs quantization of the received signals, which are then relayed to the destination; (iii) The destination $D$ is assumed to be aware of the quantization scheme used at the receivers. Moreover, we consider both cases in which the destination knows and does not know the codebooks of the transmitters. Finally, perfect block and symbol synchronization is assumed.

The complex Gaussian channel between transmitters and receivers is described by a Gaussian interference network as in Fig. 1, which is further described by an interference parameter $\alpha \in [0, 1]$. This channel corresponds to the circulant version of the Wyner model [8] for cellular networks considered in various works (see [2] and [4] for reviews). Accordingly, the received signal at any given time instant by the $m$th receiver is given by

$$Y_m = X_m + \alpha X_{m-1} + Z_m,$$

where $X_m$ is the complex symbol transmitted by the $m$th transmitter, $m-1$ represents the modulo-$M$ operation, and $Z_m$ is complex Gaussian noise with unit power ($Z_m \sim CN(0, 1)$). The per-transmitter input power constraint requires $E[|X_m|^2] = P$. Parameters $\alpha$, $C$ and $C'$ are assumed to be known by all the involved nodes. In order to obtain compact results, we will focus on the case $M \to \infty$. Results with finite $M$ can be easily inferred by using the circular structure of the channel model at hand and the corresponding circularity of the channel matrix, based on the standard arguments (see [2] for a review and a discussion on the validity of this asymptotic analysis). Finally, we normalize the achievable rate from $S$ and $D$ to the number of transmit and receive antennas $M$ and define it as $R$ [bit/ (symbol × antenna)]. Results will be stated here without formal proof. The reader is referred to [7] for proofs and further discussion.

III. PRELIMINARIES

In this section, we review some basic definitions and establish a reference result. At first, we recall that for $M \to \infty$ and perfectly cooperating receivers ($C' \to \infty$), the signal received by the destination of the network in Fig. 1 can be interpreted in the spatial domain as an inter-symbol-interference channel with impulse response $h_m = \delta_m + \alpha \delta_{m-1}$ ($\delta_m$ is the Kronecker delta function) and frequency response $\delta$ (see also [2]):

$$|H(f)|^2 = 1 + \alpha^2 + 2\alpha \cos(2\pi f).$$  \hspace{1cm} (2)

We then present two basic definitions and related results.

Definition 1: We define the waterfilling power spectral density with respect to the sum-power constraint $P$ and the SNR power spectral density $\rho(f)$ as $S_{WF}(f, P, \rho(f)) = \left(\mu - \rho(f)^{-1}\right)^+ \int_0^1 S_{WF}(f, P, \rho(f)) df = P$, and the corresponding rate as

$$R_{WF}(P, \rho(f)) = \int_0^1 \left(\log_2 \left(\mu \rho(f)\right)\right)^+ df.$$  \hspace{1cm} (3)

For short, we also define $R_{WF}(P, |H(f)|^2) = R_{WF}(P)$.

In [7], the result below is proved, following [9].

Lemma 1: If $\rho(f) = |H(f)|^2/N$, then we have:

$$R_{WF}\left(P, \frac{|H(f)|^2}{N}\right) \leq \log_2 \left(\frac{P}{N} + \frac{1}{1 - \alpha^2}\right),$$  \hspace{1cm} (4)

where equality holds for $\frac{P}{N} \geq \frac{2\alpha}{1 - \alpha^2}$. Having set the basic definitions above, we can now present an upper bound on the achievable rate between the source and destination for the network in Fig. 1. The bound also holds for the case where the transmitters and receivers are informed about the codebooks used by source and destination, and it is a straightforward consequence of cut-set arguments.

Proposition 1: The achievable rate $\bar{R}$ is upper bounded by

$$R_{UB} = \min\{C, C', R_{WF}(P)\} \leq \min\left\{C, C', \log_2 \left(\frac{P}{N} + \frac{1}{1 - \alpha^2}\right)\right\},$$  \hspace{1cm} (6)

with equality in (6) for $P \geq 2\alpha/(1 - \alpha)(1 - \alpha^2)$.

It should be noted that while the waterfilling solution (3) is based on the total power constraint, due to the symmetry of the channel at hand (see Fig. 1), it also satisfies the assumed per-transmitter power constraint for any $M$ (see also [2]). Moreover, the result (6) is a consequence of Lemma 1.
IV. FINITE-CAPACITY LINKS AT THE TRANSMITTER SIDE ONLY

In this section, we consider the case in which $C$ is finite and $C' \to \infty$, and derive achievable rates under the assumptions discussed above of oblivious antennas. It is noted that, due to the infinite-capacity links at the receiver side, the assumption of oblivious receivers has no impact on the results of this section. Two achievable rates are derived, one that assumes knowledge at the destination of the transmitters’ codebooks and one that does not require such assumption.

A. Independent messages

In this section, we consider a simple scheme that assumes that the destination is aware of the codebooks available at the transmitters. The source splits its message (of rate $MR$) into $M$ equal-rate messages, and delivers each to one transmitter via the finite capacity links. Each transmitter then maps the rate-$R$ message into a codeword, using a mapping which is known at both source and destination. The destination performs joint decoding. It is noted that here the codebooks available at the sources are used directly as channel codes. The following rate is achievable.

Proposition 2: Let $C' \to \infty$. Then, the following rate is achievable by transmitting independent messages (IM) from each transmitter

$$R_{IM} = \min\{C, R_{NC}\},$$

where $R_{NC}$ is the maximum rate achievable with no cooperation (NC) among the transmitters and $C' \to \infty$, which is given by

$$R_{NC} = \int_0^1 \log_2 \left( 1 + P |H(f)|^2 \right) df = \log_2 \left( 1 + (1 + \alpha^2)P + \sqrt{1 + 2(1 + \alpha^2)P + (1 - \alpha^2)^2 P^2} \right).$$

Remark 1: It is easy to see that this scheme is optimal (i.e., it achieves the upper bound $\mathcal{U}B$) if $C \leq R_{NC}$ (and thus in particular if $P \to \infty$). Instead, when $C > R_{NC}$, the rate achievable by this scheme does not achieve the upper bound $\mathcal{U}B$, suffering from the performance penalty caused by independent encoding as compared to the waterfilling solution $\mathcal{W}F$ [10].

B. Quantized waterfilling

Here we consider an alternative transmission scheme in which the transmitters’ codewords are assumed to be unknown to the destination, and thus are exploited by the source merely as quantization codebooks (of size $2^{nC'}$), as explained in the following. The source performs encoding for the $M$-antenna transmitter according to the waterfilling solution $\mathcal{W}F$, then it quantizes the obtained codewords using the codebooks available at the transmitters, and send the corresponding index to the given transmitter. Any transmitter simply transmits the codeword corresponding to the received indices, following our assumptions. The performance of this scheme can be proved to correspond to that of a fully cooperative MIMO system with additional (colored) noise due to quantization, as stated in the following proposition.

Proposition 3: Let $C' \to \infty$. Then, the following rate is achievable with quantized waterfilling (QW):

$$R_{QW} = R_{WF} \left( P \frac{(1 - 2^{-C})|H(f)|^2}{1 + P 2^{-C} |H(f)|^2} \right),$$

and we have:

$$R_{QW} \leq \log \left( P + \frac{1}{1 - \alpha^2} \right) - R_{NC}(P 2^{-C}),$$

with equality in the high-SNR regime where $P \geq \frac{1 - \alpha^2}{1 - \alpha(1 - 2^{-C})}$.}

Remark 2: The rate (10) reveals that for extremely large SNR ($P \to \infty$), the rate obtained with quantized waterfilling achieves the upper bound $\mathcal{U}B$. Moreover, for large capacity $C \to \infty$, it is easy to see from (9) that we have $R_{QW} \to R_{UB}$. This contrasts with the case of independent message transmission studied above, where the upper bound was not achievable for large $C$.

V. FINITE-CAPACITY LINKS AT THE RECEIVE SIDE ONLY

In this section, we focus on the scenario characterized by finite $C'$ and $C \to \infty$. It is noted that, dually to the scenario considered in Sec. IV here the assumption of oblivious transmitters has no impact on the results. We recall that we assume oblivious receivers in the sense specified in Sec. II. Following [6], we consider achievable rates with two quantization strategies carried out at the receivers, in order of complexity. The first is based on elementary compression, whereby correlation between the signals received by different antennas is not exploited for compression, and the second is based on distributed compression techniques. In both cases, we use Gaussian test channels for compression.

A. Elementary compression

With elementary compression, correlation among the received signals is not exploited in the design of the quantization functions.

Proposition 4. Let $C \to \infty$. Then, the following rate is achievable with elementary compression (EC):

$$R_{EC} = R_{WF} \left( \frac{P}{N_{EC}(P, C')} \right),$$

with

$$N_{EC}(P, C') = \frac{1 + (1 + \alpha^2)P 2^{-C'}}{1 - 2^{-C'}}.$$ 

Moreover, we have

$$R_{EC} \leq \log_2 \left( \frac{P}{N_{EC}(P, C')} + \frac{1}{1 - \alpha^2} \right),$$

with equality if conditions $P \geq \frac{2\alpha}{(1 + \alpha)(1 + \alpha^2)(1 - 2^{-C'} - 2\alpha)}$ and $C' > \log_2 \left( \frac{1 + \alpha^2}{1 - \alpha^2} \right)$ are satisfied.
Remark 3: From (13), it can be seen that for extremely large SNR ($P \to \infty$) (and if the condition on $C'$ stated above holds), the rate achieved with elementary compression is

$$R_{EC} \to \log_2 \left( \frac{2^{C'} - 1}{1 + \alpha^2} + \frac{1}{1 - \alpha^2} \right),$$

which is smaller than the upper bound (5) $R_{UB} \to C'$ for $P \to \infty$. This shows that there is a penalty to be paid for obliviousness at the receive side, at least if elementary compression is employed, even when $P \to \infty$. Moreover, for large capacity $C' \to \infty$, we clearly have optimal performance $R_{EC} \to R_{UB}$.

B. Distributed compression

The premise of the scheme discussed in this section is the observation that, since decoding of all quantization codewords takes place at the destination, the correlation of the signals observed at the receivers can be leveraged in order to decrease the equivalent quantization noise. Following [6], the quantization scheme employed here is based on the distributed compression approach used for the CEO problem.

Proposition 5: Let $C \to \infty$. Then, the following rate is achievable with distributed compression (DC):

$$R_{DC} = R_{WF} \left( P(1 - 2^{-r^*}) \right),$$

with $r^*$ satisfying the fixed-point equation

$$R_{WF} \left( P(1 - 2^{-r^*}) \right) = C' - r^*.$$  

Moreover,

$$R_{DC} \leq \log_2 \left( \frac{P + \frac{1}{1 - \alpha^2}}{1 + P2^{-C'}} \right),$$

with equality if conditions $P \geq \frac{2^{\alpha}}{(1 - \alpha)(1 - \alpha^2) - 2^{-c'}}$ and $C' > 2 \log_2 \left( \frac{1}{1 - \alpha} \right)$ are satisfied.

Remark 4: Equation (15) is easily solved numerically since $R_{WF} \left( P(1 - 2^{-r^*}) \right)$ is a monotonic function of $r^*$. Moreover, the expression (17) shows that for extremely large SNR ($P \to \infty$), the rate with oblivious transmitters (if the condition on $C'$ given above is satisfied, which requires sufficiently small $\alpha$ or large $C'$) achieves the upper bound (5), i.e., $R_{DC} \to C'$. This contrasts with the result discussed in Remark 3 for elementary compression, which was shown to be unable to achieve the upper bound. Finally, it can be seen that for large capacity $C' \to \infty$, we have $R_{DC} \to R_{UB}$.

VI. Finite-capacity links at the transmit and receive sides

In the two previous sections, we have considered the two limiting cases $C' \to \infty$ (Sec. IV) and $C \to \infty$ (Sec. V), and constructed basic transmission and reception strategies based on oblivious antennas, namely, transmission of independent messages (IM) versus quantized waterfilling (QW) at the transmit side, and elementary (EC) versus distributed compression (DC) at the receive side. These techniques can be combined giving rise to four transmission/reception strategies (IM-EC, IM-DC, QW-EC and QW-DC), as discussed below.

A. Independent messages and elementary compression

It is recalled that, when using transmission of independent messages, it is assumed that the destination is aware of the codebooks available at the transmitters.

Proposition 6: The following rate is achievable by transmitting independent messages (IM) and using elementary compression (EC) at the receive side:

$$R_{IM-EC} = \min \{ C', R' \},$$

with $R' = \log_2 \left( \frac{N_{EC}(P, C') + P2^{-C} \cdot |H(f)|^2}{2N_{EC}} \right)$ and $N_{EC}(P, C')$ as in (12) (we have dropped the dependence on $P, C'$ for the sake of legibility).

Remark 5: The result in Proposition 2 can be found as a special case of Proposition 6 for $C' \to \infty$. Moreover, this scheme is optimal whenever the second term in (18) is larger than $C$. For $P \to \infty$, as shown in [7], the scheme is not optimal and when $C, C' \to \infty$, the we have $R_{IM-EC} \to R_{NC} \leq R_{UB}$, thus suffering from the performance loss due to transmission of independent messages (see Remark 2).

B. Independent messages and distributed compression

Proposition 7: The following rate is achievable by transmitting independent messages (IM) and using distributed compression (DC) at the receive side:

$$R_{IM-DC} = \min \{ C, R' \}$$

with $R' = \log_2 \left( \frac{1 + 2\alpha^2 + 2\alpha2^{-C'} + (B^2 + 4\alpha^2 - 2^{-C'})P^2}{2(1 + 2^{-C'}P)(1 + \alpha^2 - 2^{-C'})P} \right)$.

Remark 6: Proposition 2 follows from Proposition 7 when $C' \to \infty$. Moreover, as for the previous scheme, optimality is guaranteed if the second term in (19) is larger than the capacity $C$. However, optimality is also attained with $P \to \infty$ (see also Remark 4), while for $C$ and $C' \to \infty$, we have $R_{IM-DC} \to R_{NC} \leq R_{UB}$.

C. Quantized waterfilling and elementary compression

We recall that, unlike the previous two subsections, the scheme considered here, based on quantized waterfilling, does not require the destination to be aware of the codebooks available at the transmitters.

Proposition 8: The following rate is achievable by using quantized waterfilling (QW) at the transmit side and elementary compression (EC) at the receive side:

$$R_{QW-EC} = R_{WF} \left( P, \frac{(1 - 2^{-C}) |H(f)|^2}{N_{EC}(P, C') + P2^{-C} |H(f)|^2} \right),$$

with $N_{EC}(P, C')$ as in (12). An upper bound and large-$P$ closed-form expression for (20) can be found in [7].
D. Quantized waterfilling and distributed compression

Proposition 9: The following rate is achievable by using quantized waterfilling (QW) at the transmit side and distributed compression (DC) at the receive side:

\[ R_{QW-DC} = R_{WF} \left( P, \frac{(1 - 2^{-r^*})(1 - 2^{-C})|H(f)|^2}{1 + P 2^{-C}|H(f)|^2} \right) \]

with \( r^* \) satisfying the fixed-point equation

\[ R_{WF} \left( P, \frac{(1 - 2^{-r^*})(1 - 2^{-C})|H(f)|^2}{1 + P 2^{-C}|H(f)|^2} \right) = C' - r^*. \]

An upper bound and a large-\( P \) closed-form expression can be found in [7].

Remark 7: While Proposition 8 subsumes Propositions 3 for \( C' \to \infty \) and 4 for \( C \to \infty \). Proposition 9 entails Proposition \( 3 \) for \( C' \to \infty \) and Proposition \( 5 \) for \( C \to \infty \). Moreover, equation (22) is easily solved numerically since the left-hand side is a monotonic function of \( r^* \). Finally, reference [7] shows that, for both QW-DC and QW-DC, for \( P \to \infty \) the upper bound is not attained, while when \( C \) and \( C' \to \infty \) the opposite is true.

VII. Numerical results

Fig. 2 shows the achievable rates of interest versus the SNR \( P \) for \( \alpha^2 = 0.6 \). Starting with the case \( C \to \infty \) of Sec. IV and \( C' = 4 \), it is noted that in this scenario, exploiting knowledge of the transmitters’ codebooks at the destination via independent encoding (\( R_{IM} \)) enables the upper bound \( R_{UB} \) to be closely approached and attained for sufficiently large SNR, here \( P \gtrsim 10dB \) (see Remark 1). The use of quantized waterfilling, instead, allows the upper bound to be achieved only for extreme SNR; here \( P \gtrsim 40dB \) (see Remark 2). For the case \( C' \to \infty \) of Sec. IV and \( C = 4 \), it is concluded that, while distributed compression is able to achieve the upper bound \( R_{UB} \) for \( P \to \infty \), the same is not true for elementary compression (see Remarks 3 and 4). Similar conclusions carry over to the case of finite \( C \) and \( C' = 4 \): for instance, with large power \( P \), the upper bound can be reached only if independent messages are transmitted with distributed compression (see Remark 6). Moreover, distributed compression significantly outperforms elementary compression, especially for high power \( P \).

VIII. Concluding remarks

A distributed MIMO scenario with transmit and receive antennas that are oblivious to the codebook of source and destination has been considered. Achievable rates have been derived based on several proposed techniques that exploit both channel and source coding principles. Referring the reader to [7] for a full discussion, here we point out that the analysis has shown that the considered design with oblivious antennas does not entail any loss of optimality in specific asymptotic and non-asymptotic regimes of SNR and link capacities. These results are in accord with the conclusions of recent work reviewed in [4] for uplink and downlink channels with finite-capacity backhaul.

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