Occupation of shell model orbitals extracted from knockout reactions

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Abstract. In recent years new facilities have been developed for quasifree knockout studies of unstable atomic nuclei. This has stimulated renewed interest in several unresolved issues from earlier work on stable targets. With selected examples the present paper provides a brief explanation to what extent results from the traditional investigations are reliable. As is known, current insight suggests a problem with the mean-field approximation to nuclear matter. To some extent this result is vulnerable to sensitivity to input ingredients of the theoretical model used to arrive at this conclusion. This review also implicitly provides some guidance in this respect.

1. Introduction

Early \((p,2p)\) nuclear reaction studies have provided direct and conclusive evidence of the validity of the concept of a shell model structure of atomic nuclei. Afterwards quasifree reactions were successfully interpreted in terms of a distorted-wave impulse approximation (DWIA), which benefited from a rapid evolution of essential refinements to the theoretical formulation \([1, 2]\).

The most recent review on the topic of quasifree reactions by Wakasa et al. \([3]\) updates the review paper of Kitching et al. \([4]\) of more than 30 years ago, as well as the two seminal ones of Jacob and Maris \([5, 6]\).

Proton-induced quasifree nucleon knockout enjoys renewed interest as a consequence of the development of modern accelerators, which facilitate investigations of unstable nuclei as projectiles in inverse kinematic arrangements on a hydrogen target. A consistent reduction to about 70\% of the prediction of the extreme shell model found in electron scattering knockout over a wide mass range of stable nuclei \([7]\) is a related problem of which the detailed origin also still needs to be investigated. Re-analysis \([3]\) of the large set of available \((p,2p)\) knockout data provides values which are in excellent quantitative agreement with the known degree of suppression of the extracted spectroscopic information found in \((e,e'p)\) experiments. Of course, even the earliest \((p,2p)\) work already suggested lower numbers than those predicted by a simple shell model formulation.

Another interesting development was that Gade et al. \([8]\) demonstrated a correlation of the reduction of spectroscopic strength as a function of the specific separation energy difference between protons and neutrons. These, and other issues \([9]\), are of particular interest for a current programme of Crespo et al. \([10, 11]\) to compare different reaction-model approaches to describe \((p,2p)\) observables.
The purpose of this paper is to discuss some details and results of existing \((p,2p)\) investigations that demonstrate salient features of the studies. The focus will be on sensitivities and limitations which are inherent to the DWIA when it is used to unravel the physics. Illustrative examples, mostly from my own work, will be presented. I am obviously very familiar with those, therefore it is by no means suggested that there are not other, or perhaps even better, examples available in the literature to demonstrate the same issues.

2. Theoretical Details

The DWIA has been one of the most widely applied theoretical formulations with which to study quasifree knockout reactions. The differential cross section for a knockout reaction expressed as \(A(a,cd)B\), in the notation of notation of Chant and Roos [2], is given by

\[
\frac{d^3 \sigma}{d\Omega_a d\Omega_d dE_c} = S_a F_K \sum_{\rho_c L \Lambda} \sum_{\rho_a \sigma_a \sigma_d \sigma_c} D^\rho_a (R_{ap}) \times D^\rho_c (R_{ac}) \\
\times T_{\sigma_a \sigma_c \rho_a \rho_c}^{\Lambda} (\sigma_c |t| \sigma_a) ,
\]

(1)

where \(S_a\) is a spectroscopic factor, \(F_K\) is a kinematic factor, the \(D\)'s are rotation matrices and \(\langle \sigma_c |t| \sigma_a \rangle\) denotes the matrix element of the two-body \(N-N\) transition operator. The target \(A\) is viewed as a \(b+B\) system. The quantity \(T_{\sigma_a \sigma_c \rho_a \rho_c}^{\Lambda}\) is expressed as

\[
T_{\sigma_a \sigma_c \rho_a \rho_c}^{\Lambda} = (2L + 1)^{-1/2} \int \chi_{\sigma_i \rho_c}^{(-)}(r) \chi_{\sigma_a}^{(-)}(r) \phi_{\Lambda \tau}(r) \chi_{\sigma_a \rho_a}^{(+)}(r) dr ,
\]

(2)

where \(\gamma = A/B\), \(\chi\)'s represent the distorted waves for the incoming and outgoing particles, \(\phi_{\Lambda \tau}\) is the bound state wave function of the nucleon \(b\) in the target nucleus. The detailed description of Eq. 1 is provided in Refs. [1, 2].

If spin-orbit terms are omitted in the distorting potentials for the protons (projectile and ejectile), the triple differential cross section reduces to the factorized form for the \((p,2p)\) reaction

\[
\frac{d^3 \sigma}{d\Omega_1 d\Omega_2 dE_1} = S_a F_K \left\{ \sum_{\Lambda} |T_{A \tau}^{\alpha \Lambda}|^2 \right\} \frac{d\sigma}{d\Omega} \bigg|_{p-p} ,
\]

(3)

where the two emitted protons are labeled 1 and 2, respectively, and \(\frac{d\sigma}{d\Omega} \bigg|_{p-p}\) is a half-shell two body cross section for \(N-N\) scattering.

3. Discussion of typical \((p,2p)\) quasifree knockout results

An interesting phenomenon in \((p,2p)\) reactions is that the distorted waves in the DWIA formulation have a very large influence on the observed cross section. This, in turn, would be expected to have a profound influence on the extraction of quantitative information on occupation of shell orbitals from experimental data. Clearly the sensitivity to optical model potentials is much more severe than experienced in \((e,e'p)\) reactions, which has only one emitted hadronic particle undergoing a final state interaction. Nevertheless, as will be discussed later, concerns that this could prevent determination of accurate spectroscopic information from \((p,2p)\) experiments appears to be groundless.

In Fig. 1 an energy-sharing cross section distribution, from Cowley et al. [12], for the \(^{208}\text{Pb}(p,2p)^{207}\text{Tl}\) reaction at an incident energy of 200 MeV, is shown. The coplanar-symmetric
Figure 1. Cross section coplanar symmetric quasifree energy-sharing distribution for the \(^{208}\text{Pb}(p,2p)^{207}\text{Tl}\) reaction at an incident energy of 200 MeV. Cross section yields include unresolved valence states of the residual nucleus. Results are from Cowley et al. [12]. For a prominent contrast of the difference between the cross section prediction for DWIA compared to PWIA, the same results are displayed in a linear scale in the left panel, and as a logarithmic display in the right panel.

angle pair allows zero recoil momentum at equal energy of the emitted protons. At the chosen kinematic conditions the DWIA is expected to offer a reasonably good theoretical description of the reaction mechanism. For comparison the prediction of a plane wave impulse approximation (PWIA) is also shown. The normalization of the PWIA cross section is relative to the DWIA value, in other words the same spectroscopic factor is introduced in both cases.

The very large (by about a factor of 25) overprediction of PWIA cross section demonstrates the extent of the drastic influence of the distorted waves. Not only is the absolute cross section corrected by the DWIA formulation, but the shape of the energy distribution is drastically modified. Of course, the exact cross section reproduction is not obvious from Fig. 1 because the theory is normalized based on agreement of the DWIA with the experimental data. However, as will be shown later, the spectroscopic value which is extracted by normalization is consistent with \((e,e'p)\) results.

Examples of the extent to which shape distortions in energy sharing distributions of \((p,2p)\) distributions are reproduced by the DWIA are shown in Figs. 2 and 3. These examples are at kinematic conditions not too far from the range where the DWIA is known to be reliable [13, 12]. However, even under drastically-varying shape distortions over a wide angular range, Cowley et
Figure 2. Cross section energy-sharing distribution for the $^{12}$C($p,2p$)$^{11}$B reaction at an incident energy of 200 MeV and coplanar scattering angles of $\theta_1 = 35^\circ$ and is $\theta_2 = -45^\circ$. Experimental data are from Ref. [13] projected towards the kinetic energy of the proton observed at $\theta_2$. The theoretical curve is a recent DWIA calculation of Mecca [15].

Figure 3. Cross section energy-sharing distribution for the $^{208}$Pb($p,2p$)$^{207}$Tl reaction at an incident energy of 200 MeV and coplanar scattering angles of $\theta_1 = 35^\circ$ and is $\theta_2 = -45^\circ$. Results [16] are projected towards the kinetic energy of the proton observed at $\theta_1$. The curve is a DWIA prediction.

Al. [14] find a similar extent of agreement for $^{16}$O($p,2p$)$^{15}$N at an incident energy of 150 MeV. In addition, in Ref. [14] an average spectroscopic factors of 1.3 is obtained for knockout to the ground state of $^{15}$N compared with 1.264 for $^{16}$O($e,e'p$)$^{15}$N [17]. This excellent quantitative agreement is also found for the first excited state of $^{15}$N; a value of 2.4 is extracted compared with 2.348 for ($e,e'p$).

The extreme sensitivity of cross section and shape distortion experienced for a target as heavy as $^{208}$Pb, implies that the details such as the energy- and target-mass dependence of optical model potentials employed to generate distorted waves need to be beyond reproach. Fortunately this criterion seems to be satisfied by global optical model sets available from elastic scattering in a wide range of mass and incident energy.

Walasa et al. [3] discuss extraction of spectroscopic information from ($p,2p$) reactions in a methodology which is as mutually-consistent with electron knockout as possible. Reassuringly, the extracted trend of values are in fairly good agreement, especially if one considers the more complicated interaction of a hadronic probe. This is shown in Fig. 4. The implied lowered occupation of valence shell orbitals relative to a mean-field approximation over the wide range of target masses is replicated in ($p,2p$).

As we have seen in the discussion of the results on $^{208}$Pb in Fig. 3, the predicted DWIA
Figure 4. Spectroscopic factors, expressed as a fraction of the extreme shell model limit, extracted from $(e,e'p)$ and $(p,2p)$ reactions for a series of target masses. Values from electron knockout (open circles) are from Lapikas [7], and those from proton-induced knockout (triangles) are from Wakasa et al. [3]. The spectroscopic limit of a mean-field approximation is indicated by the horizontal line at unity. The dashed line serves only to guide the eye. Note the logarithmic mass scale on the horizontal-axis.

Cross section at its peak for the $(p,2p)$ reaction at an incident energy of 200 MeV is only 4% of the PWIA value. In other words, distortions remove more than 95% of the yield from initial $N$-$N$ collisions which could potentially contribute to quasifree knockout. In spite of this drastic reduction the spectroscopic factor, which is included for this case also in Fig. 4, is still in remarkable agreement with the trend extracted from $(e,e'p)$. This inspires confidence in the reliability of the DWIA and its implementation.

It is reasonable to expect that the DWIA will break down as applicable kinematic limits are exceeded. This has been explored to its full extent in Refs. [18, 19, 20, 21, 22, 23]. The kinematic conditions under which one or both of the initial participants in a quasifree collision suffer further violent $N$-$N$ interactions are easily identified, and it is found that such events can be successfully associated with a mechanism related to a secondary [20, 23] pre-equilibrium reaction mechanism. Fortunately such secondary interactions are spatially removed from the region associated with a quasifree knockout mechanism.
4. Summary and Conclusions
A selection of results from quasifree \((p,2p)\) knockout reactions were reviewed. The DWIA formulation is shown to reproduce experimental energy-sharing distributions remarkably well, in spite of severe distortions encountered. This is especially true for a heavy target such as \(^{208}\)Pb. It appears that spectroscopic information, which is related to occupation of shell orbitals, is consistent with the underprediction observed in \((e,e'p)\) investigations. If one considers the theory and its ingredients used to extract information from \((p,2p)\) and \((e,e'p)\) studies, it appears that the low occupation could be related to the mean-field approximation, which the present analyses of both types of experiments share. Other features associated with the reaction mechanism are so drastically different between the reaction types that those are not likely to cause a common flaw.

It would be wise to explore the details of phenomena which are not understood yet further.

References
[1] Chant N S and Roos P G, 1977 Phys. Rev. C 15 57
[2] Chant N S and Roos P G, 1983 Phys. Rev. C 27 1060
[3] Wakasa T, Ogata K and Noro T, 2017 Prog. Part. Nucl. Phys. 96 32
[4] Kitching P, McDonald W J, Maris Th A J and Vasconcellos C A S, 1985 Advances in Nuclear Physics (Eds. Negele and Vogt, Plenum N.Y., 1985) Vol 15 p. 43
[5] Jacob G and Maris Th A J 1973 Rev. Mod. Phys. 45 6
[6] Jacob G and Maris Th A J 1966 Rev. Mod. Phys. 38 121
[7] Lapikas L, 1993 Nucl. Phys. A 553 297c
[8] Gade A et al., 2008 Phys. Rev. C 77 044306
[9] Gómez-Ramos M and Moro A M, 2018 Physics Letters B 785 511
[10] Crespo R et al. (in progress)
[11] Mecca A, Cravo E, Deltuva A, Crespo R, Cowley A A, Arriaga A, Wiringa R B and Noro T, 2019 Physics Letters B 798 134989
[12] Cowley A A, Arendse G J, Stander J A and Richter W A, 1995 Phys. Lett. B 359 300
[13] Cowley A A, Pilcher J V, Lawrie J J and Whittal D M, 1989 Phys. Rev. C 40 1950
[14] Cowley A A, Lawrie J J, Hillhouse G C, Whittal D M, Förtsch S V, Pilcher J V, Smit F D and Roos P G, 1991 Phys. Rev. C 44 329
[15] Mecca A, 2019 (private communication)
[16] Arendse G J, 1997 PhD thesis, Stellenbosch University
[17] Leuschner M, Calarco J R, Hersman W F, Jans E, Kramer G J, Lapikas L, van der Steenhoven G, de Witt Huberts P K A, Blok H P, Kalantar-Nayestanaki N and Friedrich J, 1994 Phys. Rev. C 49 955
[18] Cowley A A, Pilcher J V, Lawrie J J and Whittal D M, 1988 Phys. Lett. B 201 196
[19] Cowley A A, Chang C C, Holmgren H D, Silk J D, Hendrie D L, Koontz R W, Roos P G and Samanta C, 1980 Phys. Rev. Lett. 45 1930
[20] Ciangaru G, Chang C C, Holmgren H D, Nadasen A, Roos P G, Cowley A A, Mills S, Singh P P, Saber M K and Hall J R, 1983 Phys. Rev. C 27 1360
[21] Pilcher J V, Cowley A A, Whittal D M and Lawrie J W, 1989 Phys. Rev. C 40 1937
[22] Förtsch S V, Cowley A A, Lawrie J J , Pilcher J V, SmitF D and Whittal D M, 1993 Phys. Rev. C 48 743
[23] Cowley A A, Arendse G J, Visser R F, Steyn G F, Förtsch S V, Lawrie J J, Pilcher J V, Noro T, Baba T, Hatanaka K, Kawabata M, Matsuoka N, Mizuno Y, Nomachi M, Takahisa K, Tamura K, Yuasa Y, Sakaguchi H, Itoh T, Takeda H and Watanabe Y, 1998 Phys. Rev. C 57 3185