The relationship between the Sérsic law profiles measured along the major and minor axes of elliptical galaxies*

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ABSTRACT

In this paper we discuss the reason why the parameters of the Sérsic model best-fitting the major axis light profile of elliptical galaxies can differ significantly from those derived for the minor axis profile. We show that this discrepancy is a natural consequence of the fact that the isophote eccentricity varies with the radius of the isophote and present a mathematical transformation that allows the minor axis Sérsic model to be calculated from the major axis model, provided that the elliptical isophotes are aligned and concentric and that their eccentricity can be represented by a well behaved, though quite general, function of the radius. When there is no variation in eccentricity only the effective radius changes in the Sérsic model, while for radial-dependent eccentricity the transformation which allows the minor axis Sérsic model to be calculated from the major axis model is given by the Lerch \(\Phi\) transcendental function. The proposed transformation was tested using photometric data for 28 early-type galaxies.

Key words: Galaxies: fundamental parameters – galaxies: photometry – galaxies: structure.

1 INTRODUCTION

It is now recognized that the de Vaucouleurs (1948) \(R^{1/4}\) law does not fit the observed light distribution of elliptical galaxies (e.g. Schombert 1986). A much better representation of the light distribution in bright and dwarf elliptical galaxies and the bulges of spiral galaxies is provided by the Sérsic (1968) law:

\[
\log \left( \frac{I(R)}{I_n} \right) = -b_n \left[ \left( \frac{R}{R_n} \right)^{1/n} - 1 \right] \quad (1)
\]

where \(R_n\) is the radius encircling half the total galaxy luminosity and \(I_n\) is the intensity at \(R_n\). The coefficient \(b_n\) is a function of \(n\), which can be approximated by the relation \(b_n \simeq 2n - 0.327\) (Ciotti 1991).

The shape index \(n\), which parametrizes the curvature of the Sérsic model has been shown to correlate with the luminosity and size of the galaxy – brighter and larger galaxies having larger values of \(n\) (Caon, Capaccioli & D’Onofrio 1993; subsequently cited as CCD93) – and also, notably, with the central velocity dispersion \(\sigma_0\) and the mass of the central supermassive black hole (Graham, Trujillo & Caon 2001; Graham et al. 2001).

An important source of uncertainty affecting the determination of parameters of the Sérsic model that best describes the light distribution of a galaxy, is on which axis (major, minor or equivalent) the light profile should be fitted.

CCD93 extensively studied the light profiles of many Virgo cluster E and S0 galaxies by independently fitting Sérsic models to their major and minor axes, finding that in ~ 40% of the galaxies there were large discrepancies between the Sérsic parameters determined along the major and the minor axes. Such discrepancies were found not only among S0 galaxies which could be misclassified as E galaxies but also among genuine elliptical galaxies such as the E4 galaxy NGC 4621 and E3 galaxy NGC 4406.

Eccentricity gradients imply that both the major and minor axes cannot be, for example, described by the \(R^{1/4}\) model. The long observed ellipticity gradients in elliptical galaxies implies that the \(R^{1/4}\) model cannot be universal, but this obvious fact has been largely ignored in the literature.

In this paper we demonstrate that the discrepancy between major and minor axes Sérsic models in elliptical galaxies can be accounted for by radial variations of the eccentricity of the isophotes. We also present a mathematical formula that, coupled with the eccentricity profile, permits transformation of the major axis Sérsic model into the minor axis model, provided that the galaxy has well-behaved isophotes whose eccentricity varies with radius but which have the same center and position angle.

In section 2 we describe the proposed mathematical transformation, whose applicability and validity is tested by using a sample of galaxies selected from those studied by CCD93, as described in...
In section 3 we present the fitting method and in section 5 we analyze and discuss our results.

2 THE LINK BETWEEN MAJOR AND MINOR AXES SÉRISIC PROFILES

A simpler and more convenient representation of the Sérsic law is the form given in CCD93:

\[ \mu(R) = A + B \frac{R^n}{r}, \]

where, according to equation (1), A = \(-2.5 \) \( b_n + \log I_a \), B = \( 2.5 b_n / R_0^{1/n} \). R may represent the radial variable along the semi major axis a, the semi minor axis b, or the equivalent radius \( \sqrt{ab} \). The differential of the surface brightness profile can then be written as:

\[ d\mu(R) = \frac{B}{n} R^{n-1} dR. \]

Consider two nearby isophotes whose major and minor axes are respectively a and b for the inner isophote, and a’ and b’ for the outer one, as sketched in Fig. 1. The surface brightness gradient along the major axis may be written as:

\[ \frac{d\mu}{da} = \lim_{\Delta a \to 0} \frac{\mu(a') - \mu(a)}{\Delta a} \]

with a similar expression holding true for the minor axis (b).

From the definition of an isophote, we know that \( \mu(a) = \mu(b) \) and \( \mu(a') = \mu(b') \), so the numerators in the right hand side of expression (4) and in the equivalent expression for b are equal, while the denominators \( \Delta a \) and \( \Delta b \) will differ according to the radial behavior of the eccentricity \( \varepsilon \). In general we have:

\[ \frac{d\mu(b)}{db} = \frac{1}{F(a)} \frac{d\mu(a)}{da} \]

where \( F(a) \) will depend on the eccentricity function \( e(a) \). We discuss the case of constant and variable eccentricity functions in the following sections.

2.1 Constant eccentricity

The simplest case is that of concentric isophotes having constant eccentricity. If the eccentricity \( e \equiv b/a = e_c \) is constant, then we have \( b = e_c a \) and \( db = e_c da \), thus:

\[ \frac{d\mu(b)}{db} = \frac{1}{e_c} \frac{d\mu(a)}{da}. \]

1 For analytical simplicity, we use the eccentricity \( e \equiv b/a \) instead of ellipticity \( e \equiv 1 - e \).

By direct integration of equations (2) and (5), we see that in this case the Sérsic index \( n \) will be the same along the major (a) and the minor (b) axes, \( n_a = n_b \), and that the B coefficients on the major and minor axes are related by: \( B_a = B_a / e_c \). Equation (4) shows that the values of B obtained from the fits along the major and minor axes should not be considered as independent of each other, as was implicitly assumed by CCD93 (see Section 4). By analyzing the relationship between B and \( b_n \) in equations (1) and (2), it can be seen that the effect of \( e_c \) is to stretch out the relationship between \( B_a \) and \( B_b \) (Figure 2).

Theoretically, the integration constants should be equal, i.e. \( A_a = A_b \), since \( \mu(a = 0) = \mu(b = 0) \). However, in real cases (e.g. CCD93) this equality is broken by a variety of observational uncertainties and practical constraints (for instance, light profiles are fitted within a surface brightness interval whose limits in general differ on the major and minor axes). As a consequence, different values for \( A_a \) and \( A_b \) are obtained when the fitted profile is extrapolated to \( R = 0 \).

The Sérsic model along the minor axis is related to the Sérsic model along the major axis by the equation:

\[ \mu(b) = A_a + \frac{B_a}{e_c} b^{1/n_a} \]

where \( n_a \) is the major axis Sérsic index.

2.2 Variable eccentricity

In most galaxies, eccentricity is neither constant, nor is it a simple function of the radius. Indeed, no general rules seem to govern the radial variation of \( e \), and it is not clear what the physical significance of this variation is (Binney & Merrifield 1998). In cD galaxies, \( e \) generally decreases from the center outwards, while in other galaxies \( e(R) \) may increase, and sometimes it is found to vary non-monotonically with the radius.

Now, if the eccentricity is a differentiable function \( e = e(a) \), then \( db = e(a) da + a de \) or, equivalently,

\[ db = \left[ e(a) + a \frac{de}{da} \right] da \equiv F(a) da. \]

In this case, the minor axis profile may have a shape very different from that of the major axis, depending on the form of function \( e(a) \). We have integrated equation (5) for a general case in which \( e(a) \) can be expressed as a function of the form

\[ e(a) = e_0 + (e_1 - e_0) \left( \frac{a}{a_{3D}} \right)^l, \]

where \( a_{3D} \) is the scale length where the eccentricity equals \( e_1 \). Depending on \( l, e_0 \) and \( e_1 \), equation (8) may describe radial increasing (\( e_0 < e_1 \)) or decreasing (\( e_0 > e_1 \)) eccentricities, with different
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3 DATA SET USED

We applied the algorithm developed in the previous section to 28 galaxies selected from those studied by CCD93. Surface brightness and ellipticity profiles for these objects were published by Caon, Capaccioli & Rampazzo (1990), and Caon, Capaccioli & D’Onofrio (1994). The sample we used covered a wide interval of absolute magnitudes (−22.43 < $M_B$ < −17.29) and included at least one object for each morphological type (E0 to E7, dS0 and S0).

The correspondence between the Sérsic model index $n$ for the major (a) and minor (b) axis also varied: $n_a > n_b$ for 8 galaxies; $n_a < n_b$ for 17, and $n_a \approx n_b$ for 3. The eccentricity (Figures 1 and 2) increased with radius for 12 objects, decreased for another 12 and remained approximately constant for 4. The central parts of the light profiles, affected by seeing convolution, were excluded when fitting our eccentricity model (equation 9) to the observed profiles.

3.1 Errors

The photometric uncertainties on the CCD93 $B$-band surface brightness measurements were estimated by Caon et al. (1990), and are shown in Figure 3 of their paper. They can be approximated by the power-law function:

$$\delta \mu = \alpha \mu^\beta$$

where $\delta \mu$ is the error, $\mu$ the surface brightness in magnitudes, $\alpha \approx 3.25 \times 10^{-15}$ and $\beta \approx 0.7$.

The error in the eccentricity can be estimated by approximating the differentials in equation 4 by small variations, i.e., $d\mu \approx \delta \mu$ and $dR \approx \delta R$, thus obtaining $\delta \mu = (B/n) R^{1/2} \delta R$. Rearranging the terms with the help of equation 17 we can write the fractional error $\delta R/R$ as:

$$\frac{\delta R}{R} = \frac{n \alpha \mu^\beta}{B R^{1/2}} = \frac{n \alpha (A + B R^{1/2})^\beta}{B R^{1/2}}$$

Here $R$ may be the a or b variable and the coefficients $A, B, n$ may refer to the major or minor axis accordingly. Since the eccentricity is calculated as the quotient $b/a$, the fractional uncertainties add to give:

$$\frac{\delta e}{e} \approx \frac{\delta a}{a} + \frac{\delta b}{b}$$

For example, in the outer parts ($a = 296''$, $b = 180''$) of NGC 4473 we have $\delta \mu(a) \approx 0.31$ mag/arcsec$^2$, $\delta \mu(b) \approx 0.43$ mag/arcsec$^2$, $\delta a/a \approx 0.08$ and $\delta b/b \approx 0.15$, which yields $\delta e/e \approx 0.23$. For NGC 4406 ($a = 510''$, $b = 330''$, $\delta \mu(a) = 0.17$ mag/arcsec$^2$, $\delta \mu(b) = 0.28$ mag/arcsec$^2$, $\delta a/a \approx 0.06$ and $\delta b/b \approx 0.11$, thus $\delta e/e \approx 0.17$.

4 FITTING METHOD

For each of the 28 galaxies of the sample, a Levenberg-Marquardt algorithm was used to fit the minor axis surface brightness profile using the transformed major axis Sérsic law. The data for the major and minor axes light profiles are those analyzed by CCD93.

The fit was done for both the approximation of constant eccentricity, and for the more general case of variable eccentricity. We use the following notation:

$$\mu_c = A_c + \frac{B_c}{e_c^{1/n_a}}$$

and

$$\mu_L = A_L + \frac{B_L}{e_0 n_a l} a^{1/n_a} \Phi \left( \frac{1 - F(a)}{e_0} ; 1 ; \frac{1}{n_a} \right)$$

Equation 20 is for constant eccentricity and equation 21 is for variable eccentricity.

We decided to leave the parameters $A$ and $B$ completely free. The parameters $n_a$ is the major axis Sérsic index measured by CCD93, while the parameters $e_0$ and $l$ and the function $F(a)$ are...
Table 1. Galaxy name, type and the eccentricity profile parameters. $e_0$ is the eccentricity at $a = 0$, $e_1$ the eccentricity at $a = a_M$, $e_c$ is the value for the case of constant eccentricity, and $l$ is the exponent.

| Galaxy     | Type | $e_0$ | $e_1$ | $e_c$ | $l$  | $a_M$ |
|------------|------|-------|-------|-------|------|-------|
| NGC 4168   | E2   | 0.88  | 0.78  | 0.83  | 1.50 | 120   |
| NGC 4261   | E2   | 0.84  | 0.75  | 0.78  | 0.75 | 250   |
| NGC 4339   | S0(0) | 0.95 | 0.86  | 0.93  | 2.00 | 120   |
| NGC 4360   | E2   | 0.90  | 0.75  | 0.81  | 0.80 | 120   |
| NGC 4365   | E3   | 0.75  | 0.66  | 0.74  | 3.50 | 300   |
| NGC 4374   | E1   | 0.70  | 0.96  | 0.93  | 1.15 | 380   |
| NGC 4387   | E5   | 0.60  | 0.76  | 0.80  | 1.22 | 110   |
| NGC 4406   | E3   | 0.88  | 0.57  | 0.65  | 0.35 | 700   |
| NGC 4415   | dE1N | 0.90  | 0.86  | 0.89  | 1.00 | 80    |
| NGC 4431   | dS0N | 0.53  | 0.75  | 0.65  | 1.35 | 72    |
| NGC 4434   | E0   | 0.96  | 0.82  | 0.95  | 2.50 | 84    |
| NGC 4436   | dS0N | 0.47  | 0.60  | 0.70  | 2.50 | 110   |
| NGC 4458   | E1   | 0.84  | 0.98  | 0.90  | 0.51 | 90    |
| NGC 4472   | E2   | 1.00  | 0.75  | 0.80  | 0.16 | 715   |
| NGC 4473   | E5   | 0.45  | 0.69  | 0.60  | 0.55 | 330   |
| NGC 4476   | S0(5) | 0.58 | 0.91  | 0.85  | 0.75 | 154   |
| NGC 4478   | E2   | 0.82  | 0.97  | 0.88  | 3.00 | 77    |
| NGC 4486   | E0   | 1.00  | 0.60  | 0.85  | 0.65 | 550   |
| NGC 4550   | S0(7) | 0.39 | 0.22  | 0.30  | 0.78 | 154   |
| NGC 4551   | E2   | 0.68  | 0.82  | 0.75  | 1.00 | 85    |
| NGC 4552   | S0(0) | 1.00 | 0.81  | 0.88  | 0.43 | 300   |
| NGC 4564   | E6   | 0.44  | 0.61  | 0.60  | 1.00 | 190   |
| NGC 4600   | S0(6) | 0.62 | 0.85  | 0.80  | 1.00 | 77    |
| NGC 4621   | E4   | 0.65  | 0.95  | 0.90  | 1.00 | 360   |
| NGC 4623   | E7   | 0.90  | 0.22  | 0.41  | 0.15 | 110   |
| NGC 4636   | E1   | 1.00  | 0.62  | 0.72  | 0.39 | 400   |
| NGC 4649   | S0(2) | 0.77 | 0.83  | 0.82  | 0.60 | 640   |
| NGC 4660   | E3   | 0.55  | 0.86  | 0.82  | 0.70 | 130   |

5 THE RESULTS

The analysis of the results shown in Table 2 reveals an overall good agreement between the computed and the expected values.

For 14 of the galaxies, both $A_e$ (the zero point in the constant eccentricity model) and $A_l$ (the zero point in the variable eccentricity model) differ by less than 0.5 mag from the best fit $A_a$ values determined by CCD93. For further 8 galaxies the difference for both coefficients is less than 1 mag. The most discrepant galaxies are NGC 4406, NGC 4374 and NGC 4552 for which $|A_M - A_a| > 1.5$ mag.

As for scale lengths, the $B$ parameters in Table 2, 15 galaxies have $B_e$ and $B_l$, values which both differ by less than 20% from $B_a$, while for 8 galaxies the difference is less than 30%, the most discrepant object being NGC 4564 for which $|B_e - B_a| / B_a = 0.38$.

Figure 3 shows how the minor axis Sérsic parameters derived using our method correlates well with the major axis parameters, this new correlation being a remarkable improvement over that shown in Figure 1. The fact that the values of $A_e$, $A_l$, $B_e$, $B_l$ are close to their expected values ($A_a$ and $B_a$) indicates that our transformed major axis Sérsic models can fit the minor axis light profiles quite well.

These results support our proposal that the differences in the Sérsic model of the major and minor axes can be accounted for by radial variations of the isophotes eccentricity, indeed our model seems to be able to provide a valid mathematical description of the links between major and minor axes light profiles and the eccentricity profile.

There is increasing interest in using the $R^{1/4}$ law to address some issues related to the fundamental plane (FP) of elliptical galaxies (Ciotti, Lanzoni & Renzini 1996; Graham & Colless 1997; Ciotti & Lanzoni 1997), thus an extension of the work presented in our current paper would be to investigate how fitting the Sérsic model on different axes may affect the distribution of galaxies on the fundamental plane. This is because two galaxies with the same major axis light profile, but different eccentricity profiles, can give different values for the index $n$ when the Sérsic model is fitted to their equivalent axis profile. This is because $R_{eq} = \sqrt{ab} = a \sqrt{e(a)}$, which may account for some of the scatter observed in the fundamental plane. A full study of this topic is, however, outside the scope of the present paper.
### Table 2

| Galaxy     | $A_a$ | $A_b$ | $A_c$ | $A_l$ | $B_a$ | $B_b$ | $B_c$ | $B_l$ | RMS$_a$ | RMS$_b$ | RMS$_c$ | RMS$_l$ |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|---------|---------|---------|---------|
| NGC 4168  | 11.24 | 16.22 | 11.36 | 11.55 | 6.689 | 2.406 | 5.650 | 5.862 | 0.205   | 0.722   | 0.613   |
| NGC 4261  | 8.17  | 13.05 | 8.61  | 9.11  | 8.522 | 4.760 | 6.900 | 7.085 | 0.692   | 0.892   | 0.666   |
| NGC 4339  | 16.96 | 15.49 | 16.64 | 16.68 | 1.772 | 2.950 | 1.790 | 1.806 | 0.461   | 0.759   | 0.825   |
| NGC 4360  | 17.10 | 15.21 | 16.88 | 17.15 | 2.190 | 3.850 | 1.894 | 1.950 | 0.459   | 0.827   | 1.156   |
| NGC 4365  | 10.84 | 12.05 | 10.73 | 10.83 | 6.215 | 5.349 | 4.876 | 4.887 | 0.473   | 0.626   | 0.460   |
| NGC 4374  | 7.60  | 9.48  | 7.60  | 5.23  | 8.682 | 7.299 | 8.373 | 8.338 | 0.339   | 0.315   | 0.458   |
| NGC 4387  | 17.55 | 10.39 | 18.38 | 17.94 | 0.983 | 7.191 | 0.795 | 0.682 | 0.838   | 2.527   | 1.695   |
| NGC 4406  | 0.44  | 8.08  | −0.57 | 2.23  | 16.950| 9.159 | 11.477| 13.142| 0.394   | 0.805   | 0.334   |
| NGC 4415  | 19.55 | 17.69 | 19.68 | 19.72 | 0.640 | 1.921 | 0.576 | 0.568 | 1.407   | 0.669   | 0.754   |
| NGC 4431  | 20.14 | 19.79 | 20.32 | 20.02 | 0.418 | 0.894 | 0.345 | 0.337 | 0.459   | 0.923   | 0.399   |
| NGC 4434  | 14.00 | 16.46 | 13.83 | 14.00 | 4.053 | 2.111 | 4.011 | 3.945 | 0.996   | 0.639   | 0.455   |
| NGC 4436  | 19.48 | 15.13 | 19.30 | 19.28 | 0.643 | 4.595 | 0.732 | 0.494 | 0.633   | 0.554   | 0.530   |
| NGC 4458  | 17.28 | 18.17 | 16.86 | 16.64 | 1.635 | 1.168 | 1.614 | 1.661 | 3.313   | 1.399   | 1.130   |
| NGC 4472  | 12.07 | 9.75  | 11.99 | 13.15 | 4.680 | 6.814 | 3.905 | 3.833 | 0.258   | 0.435   | 0.909   |
| NGC 4473  | 15.73 | 5.27  | 15.53 | 14.52 | 1.897 | 11.238| 1.360 | 1.287 | 0.330   | 1.872   | 0.957   |
| NGC 4476  | 15.81 | 12.64 | 16.52 | 15.32 | 2.446 | 5.759 | 2.089 | 1.842 | 0.885   | 1.440   | 0.779   |
| NGC 4478  | 16.99 | 16.25 | 17.07 | 16.92 | 0.954 | 1.474 | 0.904 | 0.883 | 0.883   | 0.481   | 0.397   |
| NGC 4486  | 12.57 | 11.25 | 11.73 | 13.47 | 4.346 | 5.215 | 4.129 | 3.836 | 0.467   | 0.409   | 0.983   |
| NGC 4550  | 18.07 | 17.30 | 17.41 | 17.75 | 0.468 | 1.149 | 0.321 | 0.359 | 0.758   | 0.584   | 0.923   |
| NGC 4551  | 17.97 | 16.87 | 17.97 | 17.71 | 0.816 | 1.754 | 0.711 | 0.719 | 1.079   | 0.840   | 0.620   |
| NGC 4552  | −3.97 | −0.61 | −3.94 | −1.52 | 20.087| 16.850| 17.816| 17.982| 1.205   | 1.241   | 1.040   |
| NGC 4564  | 18.57 | 10.48 | 17.36 | 17.28 | 0.329 | 6.819 | 0.454 | 0.349 | 0.298   | 1.796   | 1.667   |
| NGC 4600  | 20.10 | 18.16 | 19.89 | 19.83 | 0.163 | 1.484 | 0.206 | 0.175 | 0.550   | 0.567   | 0.442   |
| NGC 4621  | 12.17 | 1.52  | 12.07 | 11.66 | 4.714 | 15.363| 4.641 | 3.556 | 0.590   | 0.590   | 0.293   |
| NGC 4623  | 19.98 | 16.89 | 19.68 | 20.15 | 0.130 | 2.302 | 0.130 | 0.096 | 0.104   | 2.588   | 3.749   |
| NGC 4636  | 15.69 | 16.13 | 14.80 | 15.75 | 2.608 | 2.069 | 2.246 | 2.407 | 1.054   | 1.062   | 1.203   |
| NGC 4649  | 12.70 | 10.34 | 12.58 | 12.31 | 4.038 | 6.122 | 3.499 | 3.416 | 0.797   | 0.725   | 0.595   |
| NGC 4660  | 14.98 | 6.55  | 14.76 | 14.20 | 2.140 | 10.251| 2.133 | 1.671 | 0.711   | 0.976   | 0.476   |

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Table 2. Best-fit Sérsic parameters (following the notation in equation (2)): zero point $A$ and scale length $B$. $A_a$, $A_b$, $A_c$ and $B_a$ are the parameters measured by CCD93 on major (subscript ‘a’) and minor (subscript ‘b’) axes. $A_c$, $A_L$, $B_c$ and $B_L$ are the parameters computed by us for constant (subscript ‘c’) and variable (subscript ‘L’) eccentricity. The root mean square (RMS) residuals of the fits are shown in the last three columns.

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**Figure A1.** Eccentricity profiles. The dotted line shows the observed eccentricity from CCD93 data; the solid line is the least-square fit of formula (2) to the data.

**Figure A2.** Eccentricity profiles. Continued.
Figure B1. Surface brightness profiles. Solid and dotted lines represent the CCD93 Sérsic fits to the galaxies major and minor axes profiles respectively; the short and long dashed lines represent our transformation of the major axis Sérsic law by constant and variable eccentricity, respectively. The bottom panel shows the residuals between the CCD93 data and the best-fit models, using the same line styles as described above.

Figure B3. Surface brightness profiles. Continued.

Figure B4. Surface brightness profiles. Continued.

APPENDIX C: LERCH $\Phi$ FUNCTION

The Lerch $\Phi$ function (named after Mathias Lerch, 1860-1922) is defined as an infinite series (Gradshteyn & Ryzhik 2000)

$$\Phi(z, \alpha, v) = \sum_{i=0}^{\infty} \frac{z^i}{(v + i)^{\alpha}}$$

(C1)
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where \(v + i \neq 0\). In the case studied in equation (13) we have

\[
\Phi \{ 1 - \frac{F(a)}{e_0}; 1; 1 \} = \sum_{i=0}^{\infty} \frac{nl}{1 + nl} \left( 1 + l \right) \left( 1 - \frac{c_1}{e_0} \right) \left( \frac{a}{a_M} \right)^{1+i}.
\]

In this case \((a = 1)\), one of the constraints for \(\Phi\) to be finite is that we must have \(|z| = |1 - F(a)/e_0| < 1\), which corresponds to a critical radius \(a_c\) beyond which \(\Phi\) is finite, given by

\[
a_c \equiv \frac{a_M}{(1 + l)(1 - e_0/e_1)^{1/2}}.
\]

We now may write eq. (C2) in terms of \(a_c\)

\[
\Phi \{ 1 - \frac{F(a)}{e_0}; 1; 1 \} = \sum_{i=0}^{\infty} \frac{nl}{1 + nl} \left( \frac{a}{a_c} \right)^{i+1}.
\]

The other constraint is that \(1 + inf \neq 0\) in equation (C4) above, thus \(nl \neq \ldots, -2, -1, 0\). When fitting the galaxy eccentricity profiles to equation (9) we must take these constraints into account.

The dependence of the Lerch \(\Phi\) function on the \(n\) and \(l\) parameters is shown in Figures C1 and C2. Figure C1 shows how \(\Phi_L\) changes for values of \(n = 1, 3, 5, 7, 9\), \(n\) raising in the direction indicated by the arrow. The solid curves have \(l = 0.3\) and the dotted curves have \(l = 0.7\). The same is true for Figure C2, for which we plot the values \(l = 1, 1/3, 1/5, 1/7, 1/9\), the solid curves having \(n = 3\) and the dotted curves having \(n = 9\). For all cases, \(e_0 = 0.9\) and \(e_1 = 0.1\). The critical radius \(a_c\) beyond which the function diverges should be noted. For example, in Figure C1 the solid line has \(a_c/a_M = 0.62\) and the dotted lines \(a_c/a_M = 0.55\), cf. equation (C4) and since \(a_c\) does not depend on \(n\) all the curves in Figure C1 have the same critical radius.

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Figure C1. The dependence of the Lerch $\Phi$ function on the Sérsic index $n$. The plotted values are $n = 1, 3, 5, 7, 9$ increasing as indicated by the arrow. The solid lines are for $l = 0.3$ and the dotted lines for $l = 0.7$. In both cases $e_0 = 0.9$ and $e_1 = 0.1$.

Figure C2. The dependence of the Lerch $\Phi$ function on the eccentricity parameter $l$. The plotted values are $l = 1, 1/3, 1/5, 1/7, 1/9$ increasing as indicated by the arrow. The solid lines are for $n = 3$ and the dotted lines for $n = 9$. In both cases $e_0 = 0.9$ and $e_1 = 0.1$.

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