The Pion Photoproduction in the $\Delta(1232)$ Region

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Abstract

We investigate the pion photoproduction off the nucleon in the $\Delta$ region in the framework of an effective chiral Lagrangian including pions, nucleon, and $\Delta(1232)$. We work to third order in a small scale expansion with both $m_\pi$ and $M_\Delta - M_N$ treated as light scales. We note that in the $\Delta$ region, straightforward power counting breaks down as the amplitude becomes very large. To deal with this problem, we suggest that the appropriate way to compare the theoretical calculations with experimental data is via weighted integrals of the amplitudes through the $\Delta$ region.
1 Introduction

The $\Delta(1232)$ resonance enjoys a special status in the family of the nucleon resonances. It lies only about 300 MeV above the nucleon ground state, nicely isolated from the plethora of other densely populated resonances at higher energies, and only strongly couples to the $N\pi$ system. Therefore, the matrix of physical current-like $\langle N|J^{em}_\mu|\Delta \rangle$ is supposed to be easily extracted from experimental observables and expected to shed light on their structures.

For example, in simple constituent quark models, the interaction between quarks is believed to have a tensor part due to the color hyperfine interaction\[1\]. One consequence is the admixture of higher orbital angular momentum components in s-state quark wave functions of nucleon ground state and $\Delta$. The d-state mixture allows for an electric quadrupole E2 transition in $\gamma N \rightarrow \Delta(1232)$ excitation, which is otherwise a pure magnetic dipole M1 transition. The M1 and E2 transitions can be directly excited by photons and the subsequent pion decay can be observed. The amplitudes in $\pi N$ final states are usually denoted by $E_{l}^{I\pm}$ and $M_{l}^{I\pm}$ where E and M are respectively electric and magnetic multipoles, $l$ is the orbital angular momentum of the pion, the $\pm$ signs refer to the total angular momentum $j = l \pm 1/2$, and I is the isospin of the $\pi N$ system. Therefore, in this class of models, the ratio $R_{EM} \equiv E2/M1 = E_{1+}^{3/2}/M_{1+}^{3/2}$ is related to the tensor component in the interaction of quarks. The simplest noninteracting SU(6) quark model predicts $R_{EM}=0$; the nonrelativistic constituent quark model with gluon exchange predicts a rather small and negative result ranging between $-0.008\%$ to $-2\%$\[2,3\].

Many baryon models have the pion cloud playing an important role. Because of the derivative coupling pions are relatively efficient at generating strength for the E2. Cloudy bag models lead to $-2\% < R_{EM} < 0$\[4\]. Larger negative values are given by Skyrme and other hedgehog models: $-5.9\% < R_{EM} < -2.5\%$\[5,6\], and exchange current yields: $R_{EM} = -3.5\%$\[7\]. Thus $R_{EM}$ becomes a sensitive test for different models for baryon structure.

On the other hand, for several different reasons it is important to understand the $N \rightarrow \Delta$ EM transition and more generally, the nucleon and $\Delta$-isobars, without reference to any particular baryon model. For example, to study the photoproduction of the mesons off complex nuclei\[8\], it is necessary to use single nucleon information such as $\gamma \Delta N$ vertex strength as input. Such investigations help to clarify how the effective degrees of freedom, nucleons, mesons, and $\Delta$-isobars play their roles in nuclei. $\Delta$-isobars by themselves are particularly interesting objects in the large $N_c$ QCD because they will be degenerate with the nucleon if $N_c$ exactly goes to infinity. By the simple argument based on $N_c$ counting rules applied to the $\pi N$ scattering, it was shown that large $N_c$ QCD with only $I=J=1/2$ nucleon interacting with pions is inconsistent\[10\]; other states in the tower of $I=J=1/2,3/2$ ... must be included to satisfy the consistency conditions. Therefore, the study of $N \rightarrow \Delta$ transitions may help provide insights into the large $N_c$ QCD. Moreover, $R_{EM}$ was predicted to be unity in the domain of perturbative QCD\[9\]; thus the study of $N \rightarrow \Delta$ EM transition may provide a window into the breakdown of pQCD as momentum transfer drops.

It is difficult to determine the resonant $E_{1+}^{3/2}$ for two reasons. From the experimental
side, the precise measurement of this quadrupole amplitude is difficult due to its smallness compared to dominant magnetic dipole amplitude. Recently there has been substantial progress so that precise measurements of spin observables are now possible. At Mainz[11] a $p(\gamma p)\pi^0$ measurement was performed with tagged linearly polarized photons produced at 855 MeV Mainz Microtron MAMI; the differential cross section and photon asymmetry $\Sigma$ were measured simultaneously for the $p\pi^0$ and $n\pi^+$ channels. They took $R_{EM} = \text{Im}E_{3/2}/\text{Im}M_{1+}^{3/2}$ at the peak of the $\Delta(1232)$ resonance, and the reported value is $R_{EM} = -(2.5 \pm 0.2 \pm 0.2)\%$. There are also analyses of other groups based on their and BNL’s data[12]–[16].

However, there is a more serious problem on the theoretical side: The non-resonant contribution in $E_{1+}^{3/2}$ cannot be separated directly by the measurement[18]. Thus, the direct comparison between the experimental $R_{EM}$ with the results of the calculations based on the specific baryon models are meaningless unless one extends the model to include continuum states in a consistent way. Various methods have been proposed for the isolation of the resonant part from the measured multipoles results but their answers vary greatly. Different prescriptions result in different kinds of definitions of the resonant E2/M1 ratio[19].

For example, in the effective Lagrangian approach one main difficulty of this separation is due to the unitarization. The Born terms and the $\Delta$ excitation treated as the leading tree graphs, do not fulfill the requirement of unitarity[24]. The unitarity may be put in by hand but different unitarization prescriptions lead to different separations between the resonant part and background contributions[21]–[23].

On the other hand, models[24, 25], which treat pion photoproduction dynamically (i.e. solving the corresponding Lippman-Schwinger equation for a given $\pi N$ interaction) are automatically unitary since the rescattering process is included. Such models also provide a basis for the analysis of the role of final state interactions. However, to solve the dynamical equation, some phenomenological form factors must be included. They are needed to regularize the driving terms of the interaction. Since these form factors are put in by hand, they also make the separation between the resonant part and background contributions model dependent[26].

Recently, the “speed plot technique” was also applied to this problem[17]. The “speed” SP of the scattering amplitude $T$ is defined by:

$$SP(W) = \left| \frac{dT(W)}{dW} \right|.$$ 

Here $W$ is the total c.m energy. In the vicinity of the resonance pole the energy dependence of the full amplitude $T = T_R + T_B$ is determined by the resonance contribution:

$$T_R(W) = \frac{r \Gamma R e^{i \phi}}{M_R - W - i \Gamma_R/2},$$

while $T_B$ is a smooth function of energy. Application of this method to the amplitude of Tiator et al. derived by fixing the $t$ dispersion relation gives

$$R_\Delta \equiv \frac{r e^{i \phi_R}}{r e^{i \phi_M}} = -0.035 - 0.046i.$$
Th. Wilbois et al. [19] suggested a more sophisticated way to implement this and found $R_\Delta = -0.040 - 0.047i$. Note this $R_\Delta$ is a complex and energy-independent quantity; unfortunately it remains unclear how $R_\Delta$ could be compared to any microscopic baryon model, and it is difficult to determine what, if anything, about the chiral dynamics of $\pi\Delta N$ system.

Some of the ambiguities mentioned above will be ameliorated by using an approach based on chiral perturbation theory. Unitarity is guaranteed, at least perturbatively, because loop graphs are included. Since dimensional regularization can be implemented straightforwardly in this approach, one avoids the model dependence inherent in the introduction of phenomenological form factors. Therefore ChPT provides a model-independent picture of pion photoproduction—at least to the extent the expansion converges. Many previous ChPT calculations were limited to the threshold region because $\Delta$-isobars were not included explicitly. In such calculations, the $\Delta$ effects are supposed to be included in the form of local counterterms. The essential physical idea is that the delta propagator is treated as though it was shrunken to a point and the energy dependence of $\Delta$-isobars are reproduced by higher dimension operators which are suppressed by $1/(M_\Delta - M_N)$ [27], [28]. Of course, such a framework cannot be used for our problem since we wish to work in the $\Delta$ region.

It has been recognized, even in the threshold region, that the $\Delta$ is a low energy excitation and it is presumably sensible to include it dynamically, creating a more general effective field theory than simple ChPT. This is the spirit of the original work of Jenkins and Manohar [29], [30], and the applications by Butler, Savage and Springer to the SU(3)×SU(3) case [31], [32] although their works are never beyond the leading order.

Hermier et al. [33]–[35] have formalized such an approach. They have developed a consistent power expansion scheme, the so-called “small scale expansion”, which allows for nucleon and $\Delta$-isobar degrees of freedom to be treated simultaneously in an SU(2) effective chiral Lagrangian. Whereas in conventional HBChPT one expands in power of external momenta in analogy to the meson sector; a phenomenological expansion was set up in the small scale $\epsilon$. “Small scale” denotes the soft momentum, the pion mass or the mass difference $\Delta \equiv M_\Delta - M_N$. One natural reason to do this is: $\Delta$ now is treated as a new dimensionful parameter which stays finite in chiral limit but is nevertheless of comparable size to $m_\pi$ in the real world. To assert the accuracy of this novel approach one has to systematically calculate observables and compare the resulting predictions. The $\gamma N \to \Delta$ transition seems to be a promising case to implement such a scheme [50].

There is a potential problem, however. The perturbative power counting scheme fails in producing calculations for momenta transfers in the $\Delta$ region. This is unfortunate since this is precisely the region where we wish to work. The reason for this failure is quite obvious. The generic power counting has the $\Delta$ propagator behaving as $O(1/\epsilon)$, where $\epsilon$ is the small dimensionless parameter, $\epsilon \equiv m_\pi/L, \Delta/L$. For generic low momenta this is valid. As one approaches the $\Delta$ resonance, the propagator, treated at lowest order diverges, spoiling the power counting. One may hope to cure this by including the $\Delta$ self-energy in the propagator. The imaginary part of the self energy remains finite as there is a physical decay channel. There is a difficulty with this approach; namely, certain graphs are iterated to all orders—the $\Delta$ self energy insertions—while others are not. This makes it very difficult to assure that one has a systematic power counting scheme. However, even if the propagator
is anomalously large in the neighborhood of the resonance, we note that integrals of the propagator times smooth functions over regions of order $\epsilon \Lambda$ are order $1/\epsilon$—the same as the generic $\Delta$ propagator. Clearly, this means that so long as we are only sensitive to integrals over the propagator the power counting is safe. Loop diagrams with $\Delta$’s as intermediate states are of this type. However, there are also contributions coming from graphs such as the Born graphs, in which the four momentum flowing through the propagator is fixed by the external momenta. When those external momenta are such that four momentum in the $\Delta$ propagator are close to the pole the power counting has broken down.

We propose a possible way to avoid this problem. The power counting scheme used simply does not allow us to accurately calculate the amplitudes in the vicinity of the $\Delta$ pole. However, if we limit our ambitions to asking questions about integrals of the amplitudes through the $\Delta$ region the power counting scheme remains viable. Accordingly, our proposal is that one should not directly compare calculated amplitudes with experiment. Rather one should extract the amplitudes from experiment, estimate weighted integrals over the amplitudes and compare these integrals of the experimental amplitudes with theoretical ones. In this way, we can make predictions of quantities in the $\Delta$ region—albeit integrated quantities—for which the power counting scheme is viable. We note that this approach has some important limitations. The most obvious one, of course is that we cannot make a direct prediction of the experimental observables. There is also an important practical limitation. We do not make predictions for integrals of differential cross sections but for integrals of amplitudes. In order to do this, one must fix the amplitudes from the measurements. Unfortunately, the various spin-dependent differential cross sections each depend on several amplitudes. One needs to make several independent measurements to extract the amplitudes. To the best of our knowledge none of the amplitudes has been extracted from the experimental quantities to date. This means we can not presently use the methods discussed here to compare with experiment. However, future spin-dependent measurements could alter this situation.

This paper is organized as following: In Sec. 2 the formalism of HBChPT and the steps to include the $\Delta$ degree of freedom are briefly sketched. Sections 3 and 4 describe the loop and Born graphs. The general formalism on one analysis is given in Sec. 5, renormalization is discussed in section 6. Finally, in Sec. 7, the method for comparing the theoretical calculation with experimental data is reported; some related issues and further prospects are also discussed.

## 2 Effective Lagrangian

The starting point for our approach is the most general chiral invariant Lagrangian involving relativistic spin 1/2 and spin 3/2 fields:

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_\Delta + (\mathcal{L}_{\Delta N} + h.c.).$$

Although the expressions of the standard conventions of SU(2) HBChPT exists in literature as in Ref. [34], we believe that it is useful in establishing our notation to give relevant
expressions here:

\[ U = u^2 = \exp\left(\frac{i}{F_\pi} \vec{F} \cdot \vec{\pi}\right), \]  
(2)

\[ \nabla_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu), \]  
(3)

\[ u_\mu = iu^\dagger \nabla_\mu U u^\dagger, \]  
(4)

\[ w^i_\mu = \frac{1}{2} \text{Tr}(\tau^i u_\mu), \]  
(5)

\[ w^i_{\mu\nu} = \frac{1}{2} \text{Tr}(\tau^i [D_\mu, u_\nu]), \]  
(6)

\[ \Gamma_\mu = \frac{1}{2} [u^\dagger, \partial_\mu u] - \frac{i}{2} u^\dagger (v_\mu + a_\mu)u - \frac{i}{2} u(v_\mu - a_\mu)u^\dagger, \]  
(7)

\[ u^{ij} = \xi^{ik} u^j_k, \xi^{ij} = \delta^{ij} - \frac{1}{3} \tau^i \tau^j, \]  
(8)

\[ \Gamma^{ij}_\mu = \xi^{ik} \Gamma^{kj}_\mu, \]  
(9)

\[ D^{ij}_\mu \psi^\nu_j = (\partial_\mu \delta^{ij} + \Gamma^{ij}_\mu)\psi^\nu_j, \]  
(10)

\[ D_\mu \psi_N = \partial_\mu \psi_N + \Gamma_\mu \psi_N, \]  
(11)

\[ F^R_\mu = v_\mu + a_\mu, F^L_\mu = v_\mu - a_\mu, \]  
(12)

\[ F^{L,R}_{\mu\nu} = \partial_\mu F^L_{\nu} - \partial_\nu F^L_{\mu} - i[F^L_{\mu}, F^L_{\nu}], \]  
(13)

\[ f^{\pm}_{\mu\nu} = u^\dagger F^R_{\mu\nu} u \pm u F^L_{\mu\nu} u^\dagger, \]  
(14)

\[ \chi^\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u, \]  
(15)

When the only external fields are photons, \( f^{\pm}_{\mu\nu} \) simplifies to:

\[ f^{\pm}_{\mu\nu} = c(\partial_\mu A_\nu - \partial_\nu A_\mu)(uQu^\dagger \pm u^\dagger Qu), \]  
(16)

Q \equiv \text{diag}(1,0). \text{ } v_\mu \text{ denotes external vector field and } a_\mu \text{ denotes axial field. Here } \chi(x) = s(x) + ip(x) \text{ includes the explicit chiral symmetry breaking through the small current quark masses, } s(x) = B \text{ diag}(m_u, m_d), B = \frac{|\langle 0 | \bar{q}q | 0 \rangle|}{F_\pi}, \text{ with } F_\pi \text{ the weak pion decay constant. Here isospin invariance } (m_u = m_d) \text{ is assumed.}

The spin-3/2 field is represented via a Rarita-Schwinger spinor[33], i.e., a vector-spinor field \( \Psi_\mu(x) \) satisfies the equation of motion:

\[ (i\gamma_\nu \partial^\nu - M_\Delta) \Psi_\mu(x) = 0, \]  
(17)

with the subsidiary conditions:

\[ \gamma_\mu \Psi^\mu(x) = 0. \]  
(18)

This subsidiary condition ensures that the physical particle created by the field is spin-3/2 as opposed to 1/2. A consequence of the previous equations is the condition \( \partial_\mu \Psi^\mu(x) = 0 \). It’s important to note such a field always contains a spurious spin 1/2 degree of freedom, except on-shell where the subsidiary condition projects out the spin-1/2 piece. The \( \Delta(1232) \)
also has an isospin of 3/2. The four physical states $\Delta^{++}$, $\Delta^+$, $\Delta^0$, $\Delta^-$ can be described by $\psi^i_\mu$ as an isospin-doublet with a subsidiary condition:

$$\tau^i\psi^i_\mu = 0,$$

(19)

For the three isospin doublets we use the convenient representation:

$$\psi^1_\mu = \left( \frac{\Delta^{++}}{\sqrt{2}} - \frac{\Delta^0}{\sqrt{6}} \right)_\mu,$$

(20)

$$\psi^2_\mu = \left( \frac{i \Delta^{++}}{\sqrt{2}} + \frac{i \Delta^0}{\sqrt{6}} \right)_\mu,$$

(21)

$$\psi^3_\mu = \sqrt{\frac{2}{3}} \left( \frac{\Delta^+}{\Delta^0} \right)_\mu.$$

(22)

Following Hermert et al. the Lagrangian of $\Delta$-isobars is given by:

$$\mathcal{L}_\Delta = \bar{\psi}^i_\mu N^{ij}_\mu \psi^j_\nu,$$

(23)

$$N^{ij}_\mu = -[(i \not{D}) - M_\Delta \delta^{ij})g_{\mu\nu} + \frac{1}{4} \gamma^\mu \gamma^\nu (i \not{D}) - M_\Delta \delta^{ij})\gamma_\lambda \gamma_\nu$$

$$+ \frac{g_2}{2} g_{\mu\nu} \gamma^{ij} \gamma_5 + \frac{g_2}{2} (\gamma_\mu \gamma^{ij} + \gamma^{ij} \gamma_\mu) \gamma_5 + \frac{g_4}{2} \gamma_\mu \gamma^{ij} \gamma_5 \gamma_\nu].$$

(24)

The first two terms are the kinetic and mass terms of the free Rarita-Schwinger spinors, the remaining terms constitute the most general chiral invariant couplings to pions. Note that aside from the conventional $\pi\Delta\Delta$ coupling $g_1$, there are two additional pion-couplings characterized by $g_2$, $g_3$, which contribute only if at least one of the spin 3/2 fields is off-shell.

$\mathcal{L}_N$ is well known in HBChPT, to the third order it is given by:

$$\mathcal{L}_N = \mathcal{L}^{(1)}_N + \mathcal{L}^{(2)}_N + \mathcal{L}^{(3)}_N,$$

(25)

$$\mathcal{L}^{(1)}_N = \bar{\psi}_N (i \not{D} - M_N + \frac{g_A}{2} \gamma_5) \psi_N,$$

(26)

$$\mathcal{L}^{(2)}_N = c_1 \bar{\psi}_N \psi_N T(\chi^+ - \frac{c_2}{8m_N} \bar{\psi}_N T(u_\mu u_\nu) \{D^\mu, D^\nu\} \psi_N + h.c.)$$

$$+ \frac{g_4}{2} T(u^2) \bar{\psi}_N \psi_N + \frac{c_4}{4} \bar{\psi}_N \sigma^{\mu\nu}[u_\mu, u_\nu] \psi_N + c_5 (\chi^+ - \frac{1}{2} T(\chi^+)) \bar{\psi}_N \psi_N$$

$$+ \frac{c_6}{8m_N} \bar{\psi}_N \sigma^{\mu\nu} f^+_{\mu\nu} \psi_N + \frac{c_7}{8m_N} \bar{\psi}_N \sigma^{\mu\nu} T(f^+_{\mu\nu}) \psi_N,$$

(27)

$$\mathcal{L}^{(3)}_N = \sum_{i=1}^{23} \frac{b_i}{(4\pi F)^2} \bar{\psi}_N \mathcal{O}_i \psi_N,$$

(28)

The low energy constant $c_1$ is related to the pion-nucleon $\sigma$ term, $c_2$ and $c_3$ are related to axial polarizability $\alpha_A$, and isospin-even $\pi N$ S-wave scattering length $a^+$. The constant $c_4$ is determined from P-wave scattering volume, while $c_5$ is related to the strong (i.e. nonelectromagnetic) neutron-proton mass 1. It is useful to introduce $\tilde{c}_i = m_N c_i$, i=1,2...5. which are dimensionless. The $c_6$ and $c_7$ as defined are already dimensionless; they are related to the
anomalous magnetic momentums of nucleon. Only eight of the terms in \( \mathcal{L}^{(3)} \) are relevant in our problem:

\[
\mathcal{O}_9 = \frac{i}{2m_N} \epsilon^{\mu\nu\rho\sigma}Tr(u_\rho f_\mu^+)D_\sigma,
\]

\[
\mathcal{O}_{10} = \frac{i}{2m_N} \epsilon^{\mu\nu\rho\sigma}u_\rho Tr(f_\mu^+)D_\sigma,
\]

\[
\mathcal{O}_{17} = \frac{1}{2} \gamma_\mu \gamma_5 u^\mu Tr(\chi_+),
\]

\[
\mathcal{O}_{18} = \frac{1}{2} \gamma_\mu \gamma_5 Tr(u^\mu \chi_+),
\]

\[
\mathcal{O}_{19} = \frac{i}{2} \gamma_\mu \gamma_5 [D^\mu, \chi_-],
\]

\[
\mathcal{O}_{20} = \frac{i}{2} \gamma_\mu \gamma_5 [D^\mu, Tr(\chi_-)],
\]

\[
\mathcal{O}_{21} = \frac{i}{8m_N^2} \gamma_\mu \gamma_5 [u^\alpha, f_\mu^+]\{D^\nu, D^\alpha\},
\]

\[
\mathcal{O}_{22} = \frac{i}{2} \gamma_5 \gamma^\mu [u^\nu, f_\mu^+].
\]

As Fettes et al.\(^{[39]}\) pointed out that there is one counterterm too many in \( \mathcal{L}^{(3)} \) in \[38\], however the extra term \( \mathcal{O}_4 \) is irrelevant in the pion photoproduction process, so we still adopt the basis of \[38\].

The interaction of the \( \Delta \) field with photons, pions, and nucleons is given by the following effective Lagrangian\(^{[41]}\),\(^{[42]}\):

\[
\mathcal{L}_{\pi N} = \mathcal{L}_{\Delta N}^{(1)} + \mathcal{L}_{\Delta N}^{(2)} + \mathcal{L}_{\Delta N}^{(3)},
\]

\[
\mathcal{L}_{\Delta N}^{(1)} = g_{\pi N} \bar{\psi}_i \gamma^\mu \Theta_{\mu\nu}(z)u^\nu \psi_N + h.c,
\]

\[
\mathcal{L}_{\Delta N}^{(2)} = \frac{ig_{\pi N}}{2m_N} \bar{\psi}_i \gamma^\mu \Theta_{\mu\nu}(\tilde{z})u^\nu \gamma_\sigma \psi_N + h.c,
\]

\[
\mathcal{L}_{\gamma N}^{(2)} = \frac{iG_1}{2m_N} \bar{\psi}_i \gamma^\mu \Theta_{\mu\nu}(x)\gamma_\rho \gamma_5 \psi_N \frac{1}{2}Tr(\tau^\nu f_\mu^+) + h.c,
\]

\[
\mathcal{L}_{\Delta N}^{(3)} = \mathcal{L}_{\gamma N}^{(3,a)} + \mathcal{L}_{\gamma N}^{(3,b)},
\]

\[
\mathcal{L}_{\gamma N}^{(3,a)} = \frac{-G_2}{(2m_N)^2} \bar{\psi}_i \gamma^\mu \Theta_{\mu\nu}(y)\gamma_\rho \psi_N \frac{1}{2}Tr(\tau^\nu f_\mu^+) + h.c,
\]

\[
\mathcal{L}_{\gamma N}^{(3,b)} = \frac{-G_3}{(2m_N)^2} \bar{\psi}_i \gamma^\mu \Theta_{\mu\nu}(t)\gamma_\rho \psi_N \frac{1}{2}Tr(\tau^\nu \partial_\rho f_\mu^+) + h.c.
\]

The tensor \( \Theta_{\mu\nu}(z) = g_{\mu\nu} + z\gamma_\mu \gamma_\nu \) was introduced by Peccei\(^{[13]}\) as the most general form obeys the invariance under the contact transformation. The so-called “off-shell” parameters, \( z \) and \( x, y, \) etc. govern the couplings of the off-shell spin-1/2 fields to external fields. There has been considerable controversy in finding conditions to fix these parameters\(^{[13]}\)–\(^{[14]}\). We
find however, that at the end of our calculation that final result is completely insensitive to the off-shell parameters. This is quite reasonable. After all, as a general rule in a consistent scheme, off-shell effects are not observables; while they enter calculations in intermediate stages they should be canceled in the final answers. This is clear in the case of $A$ from the KOS theorem [37] and presumably applies for the other off-shell parameters as well. In this context it is useful to note that Tang and Ellis [46] explicitly showed such independence for a somewhat simpler system than the one considered here.

The next step in this approach consists of identifying the “light” and “heavy” degrees of freedom of spin-3/2 fields. The procedure is in analogy to the case of spin-1/2 fields. The situation becomes a little more complicated here because of the off-shell spin-1/2 components associated with the Rarita-Schwinger field. To separate them, it’s convenient to introduce a complete set of orthonormal spin projection operators with a fixed velocity $v$:

\[ P_{(33)\mu\nu} = g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{1}{3} (\not{\gamma} v_\mu v_\nu + v_\mu \gamma_\nu \not{\gamma}) ; \]

\[ P_{(11)\mu\nu}^{1/2} = \frac{1}{3} \gamma_\mu \gamma_\nu - v_\mu v_\nu + \frac{1}{3} (\not{\gamma} v_\mu v_\nu + v_\mu \gamma_\nu \not{\gamma}) ; \]

\[ P_{(22)\mu\nu}^{1/2} = v_\mu v_\nu ; \]

\[ P_{(12)\mu\nu}^{1/2} = \frac{1}{\sqrt{3}} (v_\mu v_\nu - \not{\gamma} v_\mu \gamma_\nu) ; \]

\[ P_{(21)\mu\nu}^{1/2} = \frac{1}{\sqrt{3}} (\not{\gamma} v_\mu \gamma_\nu - v_\mu v_\nu) . \]

The four-velocity $v_\mu$ is related to the four-momentum $p_\mu$ of the spin-3/2 particle by:

\[ p_\mu = M v_\mu + k_\mu , \]

where $M$ is the nucleon mass, and $k_\mu$ is taken to be the residual soft momentum. Note that $k_\mu$ includes both “off-shellness” and $M_\Delta - M_N$ effects. One can employ projection operators

\[ P^\pm_v = \frac{1}{2} (1 \pm \not{v}) , \]

and identify the “light” component as:

\[ T^i_\mu = P^+_v P_{(33)\mu\nu}^{3/2} \psi^i_\nu (x) \exp (i M_N v \cdot x) . \]

The remaining component:

\[ G^i_\mu = (g_{\mu\nu} - P^+_v P_{(33)\mu\nu}^{3/2} \psi^i_\nu (x) \exp (i M_N v \cdot x) \]

Note that $T^i_\mu (x)$ satisfies the constraints:

\[ v^\mu T^i_\mu (x) = \gamma^\mu T^i_\mu (x) = 0 . \]

In the heavy baryon formulation, in general, $\partial^\mu T^i_\mu \neq 0$—unlike the relativistic case.
The nucleon field is also split as “light” and “heavy” components as in the usual HBChPT:

\[ N(x) = P^+_v \psi_N \exp(iM_N v \cdot x), \]
\[ h(x) = P^-_v \psi_N \exp(iM_N v \cdot x). \]

Note that we have used the same mass \( M_N \) in the definition of heavy delta and nucleon fields; their difference is treated as part of the “light” physics.

The general Lagrangian now take the form:

\[
L_N = \bar{N} A_N N + (\bar{h} B_N N + h.c.) - \bar{h} C_N h, \\
L_{\Delta N} = \bar{T} A_{\Delta N} N + \bar{G} B_{\Delta N} N + \bar{h} D_{\Delta N} T + \bar{h} C_{\Delta N} G + h.c., \\
L_{\Delta} = \bar{T} A_{\Delta} T + (\bar{G} B_{\Delta} T + h.c.) - \bar{G} C_{\Delta} G.
\]

The matrix \( A_N, B_{\Delta N} \ldots \), admits a small energy scale expansion of the form:

\[ A_N = A_N^{(1)} + A_N^{(2)} + A_N^{(3)} + \ldots, \]

where \( A_N^{(n)} \) is of order \( \epsilon^n \). As emphasized in the introduction, we denote by \( \epsilon \) small quantities of order \( p \), like \( m_\pi \) or the soft momenta, as well as mass difference \( \Delta = M_\Delta - M_n \). This mass difference is distinct from the pion mass in the sense that it stays finite in the chiral limit but vanishes in the large \( N_c \) limit. However, in the physical world, \( \Delta \) and \( m_\pi \) are of the same scale—differing by only a factor of \( \sim 2 \). We therefore use a simultaneous expansion in both quantities. It is only through this small scale expansion that we may obtain a low energy expansion of \( \pi \Delta N \) system. Such an expansion is in the spirit of large \( N_c \) ChPT since \( \Delta \propto \frac{1}{N_c} \).

The standard procedure is to integrate the heavy components. This results in a relatively simple effective action:

\[
S_{eff} = \int d^4 x (\bar{T} \tilde{A}_\Delta T + \bar{N} \tilde{A}_N N + [\bar{T} \tilde{A}_{\Delta N} N + h.c.]),
\]

with

\[
\tilde{A}_\Delta = A_\Delta + \gamma_0 \tilde{D}_{\Delta N}^\dagger \gamma_0 \tilde{C}_{\Delta N}^{-1} \tilde{D}_{\Delta N} + \gamma_0 B_{\Delta N}^\dagger \gamma_0 C_{\Delta N}^{-1} B_{\Delta N}, \\
\tilde{A}_N = A_N + \gamma_0 \tilde{B}_N^\dagger \gamma_0 \tilde{C}_N^{-1} \tilde{B}_N + \gamma_0 B_{\Delta N}^\dagger \gamma_0 C_{\Delta N}^{-1} B_{\Delta N}, \\
\tilde{A}_{\Delta N} = A_{\Delta N} + \gamma_0 \tilde{D}_{\Delta N}^\dagger \gamma_0 \tilde{C}_{\Delta N}^{-1} \tilde{D}_{\Delta N} + \gamma_0 B_{\Delta N}^\dagger \gamma_0 C_{\Delta N}^{-1} B_{\Delta N}.
\]

Here

\[
\tilde{C}_N = C_N - C_{\Delta N} C_{\Delta N}^{-1} \gamma_0 C_{\Delta N}^\dagger \gamma_0, \\
\tilde{B}_N = B_N + C_{\Delta N} C_{\Delta N}^{-1} B_{\Delta N}, \\
\tilde{D}_{\Delta N} = D_{\Delta N} + C_{\Delta N} C_{\Delta N}^{-1} B_{\Delta}. \]

Note \( C_\Delta \) is a \( 5 \times 5 \) matrix [34]. The effect of heavy degrees of freedom shows up at order \( \epsilon^2 \). In order to calculate a given process to order \( \epsilon^n \), it is sufficient to construct matrix \( A \) to the same order, \( \epsilon^n \), \( B \) and \( D \) to order \( \epsilon^{n-1} \), and \( C \) to order \( \epsilon^{n-2} \).
3 Loop Graphs

To order $\epsilon^3$, only one-loop graphs with the lowest order of vertex need be considered. The vertices we need are from $A^{(1)}_N$, $A^{(1)}_\Delta$ and $A^{(1)}_{\Delta N}$:

$$A^{(1)}_N = iv \cdot D + g_A S \cdot u,$$

$$A^{(1)}_{\Delta N} = g_{\pi \Delta N} w^i,$$

$$A^{(1)}_\Delta = -[iv \cdot D^{ij} - \Delta \delta^{ij} + g_1 S \cdot u^{ij}] g_{\mu \nu}.$$

(65) (66) (67)

Here $S_\mu = \frac{i}{2} \gamma_5 \sigma_{\mu \nu} v^\nu$ denotes the Pauli-Lubanski spin vector. From $A^{(1)}_N$, $A^{(1)}_\Delta$ we determine the propagators in momentum space with residual soft momentum: $r_\mu = p_\mu - M_N v_\mu$:

$$S^{1/2}_\mu (v \cdot r) = \frac{i}{v \cdot r + i\epsilon},$$

$$S^{3/2}_{\mu \nu} (v \cdot r) = \frac{-iP^{3/2}_{\mu \nu}}{v \cdot r - \Delta + i\epsilon} \xi^{ij}_{i=3/2},$$

(68) (69)

with $P^{3/2}_{\mu \nu}$ denoting the spin 3/2 projector in d-dimensions:

$$P^{3/2}_{\mu \nu} = g_{\mu \nu} - v_\mu v_\nu + \frac{4}{d-4} S_\mu S_\nu,$$

(70)

and

$$\xi^{ij}_{i=3/2} = \delta^{ij} - \frac{1}{3} \tau^i \tau^j,$$

(71)

denotes an isospin 3/2 projector. At present, the coupling constant $g_{\pi \Delta N}$ has not been extracted from data within the context of a systematic implementation of the small scale expansion.

For simplicity, we work in the center of mass. We choose $v_\mu = (1, 0, 0, 0)$ which is equivalent to working in (or close to) the center of mass frame, and the entire calculation is done in Coulomb Gauge: $\epsilon \cdot v = 0$. This choice greatly reduces the number of graphs which contribute. The disadvantage is that the gauge invariance is no longer manifest. The properties of the “light” components of the delta:

$$\gamma_\mu T^{\mu i}(x) = v_\mu T^{\mu i}(x) = 0,$$

and

$$S_\mu T^{\mu i}(x) = 0,$$

also ensures that many graphs vanish identically. Note that $\pi \pi N$ vertex, unlike $\pi \pi NN$ vertex, starts from $O(\epsilon^3)$, and since $\Delta$-isobars must be the intermediate states, the $\pi \pi \Delta \Delta$ vertex and $\pi \pi \Delta N$ vertex do not enter our calculation. In Figs. 1, 2 all graphs are shown (1-a,b,c are grouped in gauge invariant classes); the amplitudes of those graphs which are not identically zero are listed in Appendix C.
4 Born Graphs

The Born graphs contributing to third order are shown in Figs. 3, 4 with the vertices given in Table 1:

| Vertex          | Lagrangian                                                                 | Remark                                          |
|-----------------|-----------------------------------------------------------------------------|-------------------------------------------------|
| $\mathcal{O}(\epsilon^0)\gamma NN$ | $A_N^{(1)}$                                                                | Vanishes at coulomb Gauge                        |
| $\mathcal{O}(\epsilon)\pi NN$      | $A_N^{(1)}$                                                                | Does not contribute in $\pi^0$ case             |
| $\mathcal{O}(\epsilon)\gamma NN$   | $\gamma_0 B_N^{(1)}\gamma_0 C_N^{0(0)-1} B_N^{(1)}, A_N^{(2)}$            |                                                  |
| $\mathcal{O}(\epsilon^2)\pi NN$    | $\gamma_0 B_N^{(1)}\gamma_0 C_N^{0(0)-1} B_N^{(1)}$                        |                                                  |
| $\mathcal{O}(\epsilon^2)\gamma NN$ | $\gamma_0 B_N^{(1)}\gamma_0 C_N^{0(0)-1} B_N^{(1)}$                        |                                                  |
| $\mathcal{O}(\epsilon^3)\gamma NN$ | $\gamma_0 B_N^{(1)}\gamma_0 C_N^{0(0)-1} B_N^{(1)}$                        |                                                  |
| $\mathcal{O}(\epsilon)\gamma N\Delta$ | $A_N^{(2)}$                                                                |                                                  |
| $\mathcal{O}(\epsilon)\pi N\Delta$ | $A_N^{(2)}$                                                                |                                                  |
| $\mathcal{O}(\epsilon^2)\gamma N\Delta$ | $\gamma_0 D_{N\Delta}^{(2)}\gamma_0 C_N^{0(0)-1} B_N^{(1)}, A_{N\Delta}^{(1)}$ |                                                  |
| $\mathcal{O}(\epsilon^2)\pi N\Delta$ | $\gamma_0 D_{N\Delta}^{(2)}\gamma_0 C_N^{0(0)-1} B_N^{(1)}$               |                                                  |

Table 1: Vertices for Born graphs in pion photoproduction.

The $\gamma N\Delta$ vertex is the focus of our study here. The leading order $\gamma N\Delta$ vertex is a pure M1 one, but through $1/M$ expansion, the coupling constants $G_1$ and $G_2$ still contribute to E2 transitions. Note also that the heavy and off-shell spin 1/2 component of $\Delta$-isobars modify the NN vertices, up to third order giving nonvanishing contributions. The off-shell parameters show up through the $1/M$ expansion. The combinations $h_1 = yG_2 - 2xG_1 - 2G_1$, and $h_2 = -2xG_1 + yG_2$ appear in the $\mathcal{O}(\epsilon^2)\gamma N\Delta$ vertex. However, the amplitudes depend only on $h_1 - h_2$, which is $2G_1$ and is independent of $x, y$. (Actually the vertex itself can be shown as independent of $x, y$ if the on-shell constraint: $S_\mu T^{\mu}_{\text{i}} = 0$ is implemented, then our vertex is identical with the one in [50].) The off-shell parameters are also encountered in the $\mathcal{O}(\epsilon^3)\gamma \pi NN$ vertex, but they cannot be distinguished from $b_9$; thus, in our calculation, the values of off-shell parameters are irrelevant. As discussed earlier, on very general grounds this is expected. Also the counterterm in $\mathcal{L}_{\pi\pi}^{(4)}$ appears in Fig. 3-C-7.
5 Invariant Amplitudes and Multipole Decomposition

We seek the S-matrix element for the process: \( N(p_1) + \gamma(k) \rightarrow N(p_2) + \pi^c(q) \),

\[
\langle p_2, q | p_1, k \rangle = \delta_{fi} - i(2\pi)^4 \delta(P_f - P_i) T(p_2, q | p_1, k),
\]

(72)

with \( P_{f,i} \) the total four-momentum in the final/initial state. The four momenta occurring in the reaction are denoted by \( k = (k^0, \vec{k}) \) for the photon; \( q = (q^0, \vec{q}) \) for the pion; \( p_{1,2} = (p_{1,2}^0, \vec{p}_{1,2}) \) for the nucleon in the initial and final states, respectively. The Mandelstam variables are:

\[
s = (k + p_1)^2 = (q + p_2)^2, t = (k - q)^2 = (p_1 - p_2)^2, u = (k - p_2)^2 = (q - p_1)^2.
\]

(73)

In the center-of-mass frame,

\[
\vec{k} + \vec{p}_1 = \vec{q} + \vec{p}_2 = 0,
\]

(74)

and \( \omega = v \cdot k = v \cdot q \) is valid up to \( \mathcal{O}(\epsilon^2) \). Furthermore the invariant four-momentum transfer squared \( t \) can be related to the scattering angle \( \Theta \) via the kinetic relation:

\[
t = M_N^2 - 2q^0 k^0 + 2|\vec{q}||\vec{k}|x, x = \cos \Theta,
\]

(75)

Note that for the residual momentum of the nucleon: \( r_\mu = p_{1\mu} - M_N v_\mu, \omega_r = v \cdot r = \sqrt{|\vec{k}|^2 + M^2_N} - M_N = \frac{\sqrt{s}}{2M_N} + \ldots \). In Appendix C for simplicity we ignore \( \omega_r \) because it is next order compared with \( \omega \), so the structure of the divergences up to \( \mathcal{O}(\epsilon^3) \) is the same. But in the numerical calculation of Sec. 7, we avoid any adjustment in the denominator including some series including all orders, therefore \( \omega_r \) was kept there. Physically it represents the recoil momentum effect.

For the multipole decomposition discussed below, one expresses the matrix elements in terms of Pauli matrices and a two-component spinor \( \chi \):

\[
T(p_2, q | p_1, k) = \frac{4\pi \sqrt{\frac{e}{m}}}{2} \chi^\dagger(2) \mathcal{T} \chi(1),
\]

(76)

\[
\mathcal{T} = i\vec{\sigma} \cdot \vec{\epsilon} \mathcal{T}_1 + \vec{\sigma} \cdot \vec{k} \vec{\epsilon} \cdot (\vec{q} \times \vec{\epsilon}) \mathcal{T}_2 + i\vec{\sigma} \cdot \vec{q} \vec{\epsilon} \cdot \mathcal{T}_3 + i\vec{\sigma} \cdot \hat{q} \vec{\epsilon} \cdot \hat{q} \mathcal{T}_4.
\]

(77)

The relation between the multipoles \( E_{l\pm}, M_{l\pm} \) and the amplitudes is given [18]:

\[
\begin{pmatrix}
E_{0+} \\
M_{1+} \\
M_{1-} \\
E_{1+}
\end{pmatrix} =
\begin{pmatrix}
P_0(x)/2 & P_1(x)/2 & 0 & (P_0(x) - P_2(x))/6 \\
P_1(x)/4 & -P_2(x)/4 & (P_2(x) - P_0(x))/12 & 0 \\
-P_1(x)/2 & P_0(x)/2 & (P_0(x) - P_2(x))/6 & 0 \\
P_1(x)/4 & -P_2(x)/4 & (P_0(x) - P_2(x))/12 & (P_1(x) - P_3(x))/10
\end{pmatrix}
\begin{pmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4
\end{pmatrix},
\]

(78)

Here \( P_l(x) \) is the Legendre polynomial.

The quantities like cross section and single-spin observables are the combinations of these multipole results. In addition, there are 12 double-spin observables classified into three types: beam-target, beam-recoil and target-recoil spin observables. These observables usually provide the bases of the comparison with theoretical results.
6 Renormalization

The one-loop diagrams contain divergences and have to be regularized and renormalized. Here we will use dimension regularization. The unrenormalized coefficients are then related to the renormalized (scale dependent) ones according to

\[ b_i = b'_i(\mu) + (4\pi)^2 \beta_i L , \]  

\[ L \equiv \frac{\mu^{d-4}}{(4\pi)^2} \left( \frac{1}{d-4} - \frac{1}{2} \ln(4\pi) + 1 + \Gamma'(1) \right) , \]

where \( b_i \) represents a generic coefficient and the \( \beta'_i \)'s are the associated dimensionless coefficients which govern the scale dependence of \( b'_i(\mu) \):

\[ b'_i(\mu) = b'_i(\mu_0) + \beta_i \ln \frac{\mu_0}{\mu} . \]

Ecker et al.\cite{38} have calculated all \( \beta_i \) in a theory without an explicit \( \Delta \) resonance and obtained

\[ \beta_9 = \beta_{10} = \beta_{18} = \beta_{19} = \beta_{20} = 0 , \]

\[ \beta_{17} = \frac{g_A^2}{2} + g_A^3 , \beta_{21} = g_A + g_A^3 , \beta_{22} = -g_A^3 . \]

(At first glance, the \( \beta \) functions of \cite{38} and \cite{39} are different but actually they are equivalent. The reason is some terms which are proportional to the nucleon EOM are kept in \cite{39}, but transformed away in \cite{38}. For convenience we adopt the \( \beta \) functions of \cite{38}.) Of course the \( \Delta-\pi \) loops will modify these functions.

However, before making such a modification, we would like to discuss an issue associated with diagrams, such as (2-A-1) (see Fig. 2). There is a complication because the divergences in such graphs are proportional to energy factors with an energy dependence of \( \omega/(\Delta \pm \omega) \). The difficulty is that tree diagrams with such an energy dependence could not absorb all divergences by themselves; clearly it is also impossible to absorb the divergences in any single coefficient \( b_i \). Instead we decompose the factor \( \omega/(\Delta \pm \omega) \) into the sum of \( \mp 1 \) and \( \pm \Delta/(\Delta \pm \omega) \). Each of these forms correspond to an energy dependence of a tree graph which can be renormalized. The divergences with the \( \pm \Delta/(\Delta \pm \omega) \) factors are absorbed in coupling constants:

\[ \kappa_v(\mu) = \dot{k}_v - \frac{m_N g_A^2}{4\pi F_\pi^2} + (4\pi)^2 \frac{\Delta M_N}{(4\pi F_\pi)^2} \frac{32 g_A^2}{9} (L + \frac{1}{16\pi^2} \ln \frac{m_\pi}{\mu} + \frac{\sqrt{\Delta^2 - m_\pi^2}}{16\pi^2 \Delta} \ln[R]) \]

\[ G_1(\mu) = \dot{G}_1 + (4\pi)^2 \frac{\Delta M_N}{(4\pi F_\pi)^2} \frac{2 g_A \Delta N}{3} \left( g_A + \frac{35}{9} g_1 \right) (L + \frac{1}{16\pi^2} \ln \frac{m_\pi}{\mu} + \frac{\sqrt{\Delta^2 - m_\pi^2}}{16\pi^2 \Delta} \ln[R]) \]

\[ R = \frac{\Delta}{m_\pi} + \sqrt{\left( \frac{\Delta}{m_\pi} \right)^2 - 1} , \]

\[ G_2(\mu) = \dot{G}_2 + (4\pi)^2 \frac{M_N^2}{(4\pi F_\pi)^2} \frac{8 g_\pi \Delta N}{3} \left( g_A - \frac{5}{9} g_1 \right) (L + \frac{1}{16\pi^2} \ln \frac{m_\pi}{\mu} ) , \]

\[ G_2(\mu) = \dot{G}_2 + (4\pi)^2 \frac{M_N^2}{(4\pi F_\pi)^2} \frac{8 g_\pi \Delta N}{3} \left( g_A - \frac{5}{9} g_1 \right) (L + \frac{1}{16\pi^2} \ln \frac{m_\pi}{\mu} ) , \]
where the $G_1$ and $G_2$ are the coefficients associated with the $\gamma \Delta N$ vertex and the $\cdot$ over the $G$’s indicates it is taken as lowest order value in the Lagrangian—in other words, taken in the limit: $m_\pi \to 0$, $\Delta \to 0$, and $\Delta m_\pi$ fixed. The remaining divergences are absorbed by altering the $\beta_i$. The $\pi^0$ photoproduction amplitude can be renormalized only if the $\beta$ functions are modified in this way:

$$\beta_9 = \frac{8 g_{\pi \Delta N}}{27} (g_A - \frac{5}{9} g_1), \beta_{10} = 0. \quad (87)$$

The $\pi^\pm$ photoproduction amplitude imposes an interesting constraint. To fit the forms of the counterterms in $\mathcal{L}_{\pi N N}^{(3)}$, $g_1$ must be $\frac{9}{5} g_A$, which in turn implies

$$\beta_9 = \beta_{10} = 0. \quad (88)$$

It is interesting to note that this constraint is required for self consistency. The coefficients $b_9$ and $b_{10}$ remain renormalization group invariant even after the $\Delta$ degree of freedom is introduced. Inserting the constraint on $g_1$ into the previous equations gives

$$G_1(\mu) = \hat{G}_1 + (4\pi^2) \frac{\Delta M_N}{(4\pi F)^2} \frac{16}{3} g_{\pi \Delta N} g_A (L + \frac{1}{16\pi^2} \ln(\frac{m_\pi}{\mu}) + \frac{\sqrt{\Delta^2 - m_\pi^2}}{16\pi^2 \Delta} \ln[R]), \quad (89)$$

$$G_2(\mu) = \hat{G}_2. \quad (90)$$

The charged pion amplitude also requires an alteration of $\beta_{21}$, $\beta_{22}$ and $\beta_{17}$:

$$\beta_{21} = g_A + g_A^3 - \frac{8}{9} g_A g_{\pi \Delta N}^2, \beta_{22} = -g_A^3 + \frac{8}{9} g_A g_{\pi \Delta N}^2, \quad (91)$$

$$\beta_{17} = \frac{g_A}{2} + g_A^3 - \frac{50}{81} g_1 g_{\pi \Delta N}^2 + \frac{2}{9} g_A g_{\pi \Delta N}^2. \quad (92)$$

The constraint on the value of $g_1$ at first sight seems surprising because $g_1$ was supposed to be an arbitrary number determined by fitting data. The consistency constraint we find suggests however that the only way we can do a simultaneous expansion in which $m_\pi$ and $\Delta$ are both small is if the coupling of pions to the $\Delta$’s is fixed to a special value. Note that is the large $N_c$ prediction for $g_1/g_A$ is $\frac{9}{5}$—precisely the value needed to satisfy the renormalization condition. We note in passing the same value for the ratio is given in the SU(6) quark model for $N_c = 3$. However, there is no systematic sense in which the quark model is the leading term in an expansion, while the $1/N_c$ expansion is well defined. Of course this constraint is only valid at leading order in our expansion. In general, the ratio is not precisely $9/5$. Formally, we may write

$$g_1 = \frac{9}{5} g_A + \sum_{n=1}^{\infty} \left(\frac{\Delta}{4\pi F_\pi}\right)^n \delta g_1^{(n)}. \quad (93)$$

Suppose $g_1$ is slightly different from $\frac{9}{5} g_A$; then some of the $\delta g_1^{(n)}$’s must be considered as a quantity of order $\epsilon^{-1}$. In other words, consistent power counting with the “small scale expansion” requires $g_1 = \frac{9}{5} g_A$ at leading order. On the other hand, amplitudes are still renormalizable even when $g_1$ is assigned another value if higher dimension operators suppressed by $\frac{\Delta}{4\pi F_\pi}$ are included.
As Kambor pointed out [35], one cannot identify the coupling constants of theory including delta degrees of freedom with those in HBChPT, even if they multiply the same structures in effective Lagrangians. The reason is the process of integrating out the additional degrees of freedom leads to a (general infinite) renormalization of the bare coupling constants of the underlying theory. So if we keep $\Delta$ finite, the bare coupling constants in our Lagrangians will differ with the usual one in HBChPT even in the chiral limit. From (83), (84) we have:

$$\kappa_v = \kappa^r_v - \frac{m^2 g^2 M N}{4\pi F^2} \frac{2m g^2 M_N}{9\pi^2 F^2} g^2 N(x \ln 2x - \sqrt{x^2 - 1} \ln [x + \sqrt{x^2 - 1}]),$$

$$\kappa^r_v = \kappa_v + \frac{2\pi M N}{9\pi^2 F^2} g^2 N(16\pi^2 L + \ln \frac{2\Delta}{\mu}),$$

$$G_1 = G^r_1 - \frac{m g^2 M_N}{3\pi^2 F^2} g A g^2 N(x \ln 2x - \sqrt{x^2 - 1} \ln [x + \sqrt{x^2 - 1}]),$$

$$G^r_1 = G_1 + \frac{\pi M N}{3\pi^2 F^2} g A g^2 N(16\pi^2 L + \ln \frac{2\Delta}{\mu}).$$

Here $x \equiv \frac{\Delta}{m^2}$. There are other examples; $m_N$ and $c_1$ are also infinitely renormalized:

$$m_N = m^r - 4c_1 M^2 = \frac{3g^2 M^3}{32\pi F^2} + \frac{g^2 N M N}{12\pi^2 F^2} R\left(\frac{\Delta}{M}\right),$$

where

$$R(x) = -4(x^2 - 1)^{3/2} \ln (x + \sqrt{x^2 - 1}) + 4x \ln 2x - x(2 + 6 \ln x),$$

$$m^r = m + \frac{g^2 N N}{3\pi^2 F^2} \Delta^3 (-16L + \frac{1}{\pi^2} \ln \frac{2\Delta}{\mu}),$$

$$c_1^r = c_1 + \frac{g^2 N N}{8\pi^2 F^2} \Delta (-16L + \frac{1}{\pi^2} \ln \frac{\Delta}{\mu}).$$

The renormalization of the axial coupling constant is similar but more complicated [48]:

$$g_A = g^r_A + \frac{4m^2}{(4\pi F^2)^2} \left\{ b^r_1 + S\left(\frac{\Delta}{M}\right) \right\},$$

$$g^r_A = g_A + \frac{4g A g^2 N}{27\pi^2 F^2} \Delta \left\{ 16\pi^2 L + \ln \frac{2\Delta}{\mu} + \frac{1}{2} \right\},$$

where

$$S(x) = a_1 + a_2 x^{-1} + \frac{3}{2} a_3 x^2 \ln 2x + a_4 x \sqrt{x^2 - 1} \ln [x + \sqrt{x^2 - 1}]$$

$$+ a_5 \left( \frac{x^2 - 1}{x} \right)^{3/2} \ln [x + \sqrt{x^2 - 1}],$$

$$a_1 = -\frac{3}{4} g^3 A - \frac{136}{27} g A g^2 N + \frac{200}{243} g_1 g^2 N,$$

$$a_2 = \frac{64\pi}{27} g A g^2 N,$$

$$a_3 = -\frac{304}{27} g A g^2 N + \frac{400}{81} g_1 g^2 N.$$
\[ a_4 = 16 g_A g_{\pi}^2 \Delta N - \frac{400}{81} g_1 g_{\pi}^2 \Delta N, \]
\[ a_5 = -\frac{128}{27} g_A g_{\pi}^2 \Delta N. \]

In general, the LED’s in the present expansion are different from HBChPT without an explicit \( \Delta \) degree of freedom. The reason for this is obvious: processes including the explicit \( \Delta \) contribute and serve to renormalize the parameters. However, in the chiral limit of \( m_{\pi} = 0 \), \( \kappa_v^r \), \( G_1^r \), \( m^r \), \( c_1^r \) and \( g_A^r \) can be identified with the analogous LECs in HBChPT without \( \Delta \)’s. In contrast, parameters of \( \dot{\kappa}_v \), \( \dot{G}_1 \), \( \dot{m}_N \) and \( \dot{c}_1 \) are defined in the limit: \( \Delta \to 0, m_{\pi} \to 0, \frac{\Delta}{m_{\pi}} \) fixed.

There is an interesting formal limit to consider apart from the chiral limit; namely the large \( N_c \) limit. Recall in this limit one naturally has \( \Delta \to 0 \) with \( m_{\pi} \) finite. Moreover, in the large \( N_c \) limit, \( g_A = \frac{8}{3} g_1 = \frac{2\sqrt{2}}{3} g_{\pi} \Delta N \). Inserting this into our expressions for the \( \beta \)'s we find:

\[ \beta_{21} = g_A, \beta_{22} = 0, \beta_{17} = \frac{g_A}{2}. \]

Actually such a result is required by the \( N_c \) counting rules. Note that \( g_A \) is \( \mathcal{O}(N_c) \) quantity, and \( F_{\pi} \sim \mathcal{O}(\sqrt{N_c}) \) and \( m_{\pi} \sim \mathcal{O}(1) \); therefore all terms with \( g_A^3 \) must vanish in the large \( N_c \) limit. Such a cancelation is possible only when the \( \Delta \)-isobar degrees of freedom are included. So we corroborate the idea \cite{10} that a chiral expansion has a sensible large \( N_c \) limit only if the entire tower of large \( N_c \) baryons is included. Furthermore we have:

\[ \kappa_v = \kappa_v^r - \frac{3 M_N m_{\pi} g_A^2}{8 \pi F_{\pi}^2}, \]
\[ G_1 = G_1^r - \frac{16 g_A g_{\pi}^2 \Delta N}{(4 \pi F_{\pi})^2} \frac{\pi}{3}, G_2 = G_2^r, \]
\[ g_A = g_A^r + \frac{4 m_{\pi}^2}{(4 \pi F_{\pi})^2} \left( b_{17} + \frac{g_A}{2} \ln \frac{m_{\pi}}{\mu} \right), \]
\[ m_N = \dot{m} - 4 \dot{c}_1 m_{\pi}^2 - \frac{9 g_A^2 m_{\pi}^3}{32 \pi F_{\pi}^2}. \]

It is interesting to compare the results in three distinct limits: \( N_c \to \infty, m_{\pi} \to 0, N_c m_{\pi} \) fixed, with ones in which the two limits are taken in different orders. That is, to consider the cases \( x \equiv \Delta/m_{\pi} = \) constant, \( x \to 0 \), and \( x \to \infty \). It is well known that for many baryon properties which are singular in the chiral limit, results are highly sensitive to the ordering of limits\cite{53, 52}. It is not hard to see that for \( g_A, m_N \) and \( c_1 \), the ordering of limits make no differences. However it becomes manifest that neither a pure \( 1/N_c \) expansion nor chiral expansion is suitable to describe various properties of baryons beyond the leading order. Because \( x = \frac{\Delta}{m_{\pi}} \approx 2.1 \), expanding around 0 or \( \infty \) is never a good way.

When higher-order calculations are performed, one must be very careful to use the appropriate parameters in each part of the calculation. For example, the \( \pi \) loop correction to
$G_1$ is an $O(\epsilon)$ quantity and $G_1$ contributes to the amplitudes from $O(\epsilon^2)$ tree graphs. Thus, in our third-order calculation the $G_1$ in $O(\epsilon^2)$ tree graphs must include the correction from the $\pi$ loop, but the $G_1$ in $O(\epsilon^3)$ tree graphs does not. The treatment of $\kappa_v$ is analogous. On the other hand, the $\pi$ loop correction to $g_A$ and $M_N$ are $O(\epsilon^2)$ quantities; thus, up to $O(\epsilon^3)$, only lowest-order quantities are used except in the $E_{+0}$ amplitude of $\pi^\pm$ photoproduction.

7 Comparison with Experiment

The values of unknown parameters such as $G_1, G_2...$ are expected to be extracted from the experimental data, and to be used to make predictions in other processes. However, as mentioned in the Introduction, there is an obstacle: The amplitudes at the $\Delta$ pole diverge, and amplitudes in the vicinity of the pole clearly cannot be taken seriously. At first glance, this appears to be a fatal problem for this approach since our interest is precisely in the $\Delta$ region. One natural way to cure this is to use a dressed delta propagator, i.e, put the self-energy part $\Sigma(\omega)$ in the delta propagator. With a self-energy included, amplitudes become smooth since the imaginary part in $\Sigma(\omega)$ shifts the pole from the real axis. However this approach breaks the power counting scheme in its purest form because it requires that part of the interaction (those associated with $\Delta$ decay into pion plus nucleon) be iterated to all orders while other parts are not. The overall expansion becomes questionable, and as we pointed out in the previous section, the power counting scheme is our only way to make predictions consistently.

At a technical level, the problem is that away from the $\Delta$ pole, the $\Delta$ propagator is $O(\epsilon^{-1})$, while in the immediate vicinity the pole is $O(\epsilon^{-2})$. As one moves up the resonance, the behavior changes, making a systematic treatment problematic. One purely phenomenological alternative is to simply insert the empirical decay width $\Gamma$ in the invariant amplitude as Adelseck et al. have done\cite{57}. Such a scheme is not systematic, however; and moreover, unitarity is violated\cite{45}.

Therefore we suggest that instead of directly comparing the theoretically calculated amplitudes with the experimentally extracted ones at all energies, we only compare weighted integrals of the amplitudes. For example,

$$\mathcal{M}^{(n)}_{l\pm} = \int_{m_{\pi}}^{\omega_{\max}} \mathcal{M}_{l\pm}(\omega)W_n(\omega)d\omega. \quad (115)$$

Here $\omega$ is the energy of the pion in the c.m frame, $W_n(\omega)$ is a smooth weight function, and $n$ is an integer index specifying the particular choice of weight function.

To justify such an approach, the weight functions need to be chosen with some care. First of all, the hierarchy introduced by power counting must be maintained after integration. In other words, the amplitude with factor $\frac{1}{\omega - \Delta + i\epsilon}$ should not be enhanced beyond what is permitted in our power counting. To satisfy this requirement we must integrate through the entire $\Delta$ region. Actually there’s another reason to do so: Recall that in the vicinity of the
pole, the theoretical resonant amplitude has a $\delta$ function in the strength through

$$\lim_{\epsilon \to 0} \frac{1}{\omega - \Delta + i\epsilon} = P\left(\frac{1}{\omega - \Delta}\right) - i\pi \delta(\omega - \Delta),$$

but the actual resonant amplitudes will be more like a Breit-Wigner form and one expects the imaginary part to spread the width $\Gamma$. Therefore, the weight functions are required to cover the whole $\Delta$ region to keep the full information of the imaginary part in an experimental resonant amplitude.

On the other hand, $\omega_{\text{max}}$ could not be put too high. The $O(\epsilon^4)$ contribution of amplitudes becomes more important when energy increases and our calculation loses its predictive power in the higher energy region. To satisfy both requirements, the best place for $\omega_{\text{max}}$ is at the upper end of the $\Delta$ region. Since we are interested in power counting it is sensible to look at a Taylor series for the $W_n(\omega)$:

$$W_n(\omega) = a_n^{(0)} + a_n^{(1)} \left(\frac{\omega}{m_p}\right) + a_n^{(2)} \left(\frac{\omega}{m_p}\right)^2 + a_n^{(3)} \left(\frac{\omega}{m_p}\right)^3 + a_n^{(4)} \left(\frac{\omega}{m_p}\right)^4 + ...$$

It is clear that the effect of higher order coefficients such as $a_n^{(4)}$ do not contain reliable information since they can not be distinguished from the effect of $O(\epsilon^4)$ amplitudes which are absent from our calculation. Accordingly we can choose

$$1, \left(\frac{\omega}{m_p}\right), \left(\frac{\omega}{m_p}\right)^2, \left(\frac{\omega}{m_p}\right)^3,$$

as our “basis functions”. Any $W_n(\omega)$ is equivalent to their linear combination once the higher order terms are thrown away. Thus we choose $W_n(\omega)$ as

$$W_n(\omega) = \left(\frac{\omega}{m_p}\right)^n.$$  

The preceding analysis also suggests another advantage of studying these integrated quantities apart from the problem of the $\Delta$ pole. Note there is a finite amount of information which can be extracted from a systematic expansion at a finite order. Direct predictions for cross sections as functions of energy formally have an infinite information content since it takes an infinite number of parameters to describe an arbitrary function. Clearly, much of the information content contained in a predicted functional dependence has considerable correlations. It is useful, therefore, to construct a number of discrete observables, such as our integrals which characterize the energy dependences. The scheme proposed here corresponds to picking the maximum number of independent predictions which we can make at this order.

We integrate from the threshold through the entire $\Delta$ region:

$$\mathcal{M}_{l\pm}^{(n)} = \frac{1}{m_p} \int_{m_{\pi}}^{\omega_{\text{max}}} \mathcal{M}_{l\pm}(\omega) \left(\frac{\omega}{m_p}\right)^n d\omega.$$  

(117)
The additional factor $1/m_p$ is only to ensure that $\bar{M}$ has the same dimension as $M$. The values of the unknown parameters can be fit from the amplitudes as follows:

\[
Re\tilde{P}_i^{\alpha P} = e g_A \zeta_i^A + e g_A \kappa_p \zeta_i^{B_{i;B}} + e g_A \kappa_p \zeta_i^{R_{i;R}} + e g_A (1 + \tilde{\kappa}_v) \tilde{c}_1 \zeta_i^C
\]
\[
+ e g_A^3 \zeta_i^D + e g_\pi \Delta_N \tilde{G}_{1i;B}^E + e g_\pi \Delta_N G_{1i;R}^E + e g_\pi \Delta_N G_{2i}^E
\]
\[
+ e g_\pi \Delta_N G_1 \xi_i^G + e g_\pi^2 \Delta_N g_A \xi_i^H + e g_\pi^2 \Delta_N g_1 \xi_i^K + e \tilde{b}_9 \xi_i^L, \\
\]
\[
i = 1, 2, 3.
\]

(118)

\[
Im\tilde{P}_i^{\alpha P} (q_0) = e g_3^3 \zeta_i^D + e g_\pi \Delta_N \tilde{G}_{1i;B}^E + e g_\pi \Delta_N G_{1i;R}^E + e g_\pi \Delta_N G_{2i}^E
\]
\[
+ e g_\pi \Delta_N G_1 \xi_i^G + e g_\pi^2 \Delta_N g_A \xi_i^H + e g_\pi^2 \Delta_N g_1 \xi_i^K, \\
\]
\[
i = 1, 2, 3.
\]

(119)

Here $\tilde{c}_1 = m_N c_1, \kappa_p = \frac{1}{2} (\kappa_v + \kappa_s), \tilde{b}_9 = b_9 - b_{10} - \frac{(4 \pi F_\pi)^2}{6 m_N^2} g_\pi \Delta_N G_1 (1 + 4 x + 4 z + 12 x z). \tilde{\kappa}_v$ means the parameter taken in the limit: $\Delta \to 0, m_\pi \to 0, \Delta/m_\pi$ fixed, and $\kappa_p$; the $\pi$ loop correction is included in this. The first four terms in (146) are from tree graphs without delta; the sixth to ninth terms are due to tree graphs with delta. Note that such tree graphs also contribute to the imaginary parts of amplitudes due to the delta function in $\frac{1}{\omega - \Delta + i \epsilon}$. The fifth term is from loop graphs without delta; the tenth and eleventh terms are $\Delta - \pi$ loop contributions; the last term, which only appears in $P_3$, is due to the counterterms in $\mathcal{L}_{\pi NN}^{(3)}$. Note that the quantities (as they must be), like $\xi_i^H, \xi_i^K$ are $\mu$-dependent, however final amplitudes are independent of $\mu$ because the $\kappa_v$ and $G_1$ are also $\mu$-dependent, and compensate the ones from the loop.

It is obvious that power counting is preserved in this scheme with this set of weight functions. The $O(\epsilon^2)$ amplitudes represented by $\zeta_i^{B_{i;B}}$ or $\zeta_i^{E_{i;R}}$, $i=1,2$ are significantly larger than the other terms, and $\zeta_i^{H_{i;R}}$ or $\zeta_i^{K_{i;R}}$ and $\zeta_i^F$, $\zeta_i^G$ are not particularly enhanced which shows that the wild behavior of $\frac{1}{\omega - \Delta + i \epsilon}$ is tamed by our weight functions.

We set $\Delta$=294 Mev, $F_\pi$=92.4 Mev, $M_N$=938 Mev, $\omega_{\text{max}}$=340 Mev, and $\mu$=500 Mev. The following results are given (in the unit of $10^{-4}/m_\pi$):

20
### Table 2: \(\xi_i^H, \xi_i^G\) and \(\xi_i^K\) represent the contributions due to the delta function of \(\frac{1}{\omega - \Delta + i\epsilon}\) in the loop diagrams like (2-A-1). The other \(\xi_i^i\)'s are from \(\pi\) loops.

| \(n\) | \(\xi_1^A\) | \(\xi_1^B\) | \(\xi_1^C\) | \(\xi_1^D\) | \(\xi_1^E\) | \(\xi_1^F\) | \(\xi_1^G\) | \(\xi_1^H\) | \(\xi_1^I\) | \(\xi_1^J\) | \(\xi_1^K\) | \(\xi_1^L\) |
|-------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 0     | 23.89       | 26.93       | -1.04       | 2.40        | 12.14       | -12.07      | -3.92       | 0.74        | -1.48       | -0.96       | -4.97       | 7.18        |
| 1     | 6.60        | 7.52        | -0.36       | 0.67        | 3.53        | -4.65       | 1.03        | 0.29        | -0.59       | -0.68       | -0.38       | 3.20        |
| 2     | 1.90        | 2.19        | -0.12       | 0.20        | 1.06        | -1.67       | -0.28       | 0.11        | 0.04        | -0.30       | -0.10       | 1.23        |
| 3     | 0.57        | 0.66        | -0.04       | 0.06        | 0.33        | -0.58       | -0.08       | 0.04        | 0.01        | -0.11       | -0.10       | 0.44        |

| \(\xi_i^D\) | \(\xi_i^E\) | \(\xi_i^F\) | \(\xi_i^G\) | \(\xi_i^H\) | \(\xi_i^K\) | \(\xi_i^L\) |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 0           | -10.91      | -6.86       | 0.95        | -0.95       | 9.95        | -3.89       |
| 1           | -3.06       | -1.90       | 0.26        | -0.26       | 3.89        | -1.07       |
| 2           | -0.89       | -0.52       | 0.07        | -0.07       | 1.42        | -0.30       |
| 3           | -0.27       | -0.14       | 0.02        | -0.02       | 0.49        | -0.08       |

| \(\xi_i^G\) | \(\xi_i^H\) | \(\xi_i^K\) | \(\xi_i^L\) |
|-------------|-------------|-------------|-------------|
| 0           | -41.53      | -24.82      | 1.66        |
| 1           | -11.41      | -6.91       | 0.54        |
| 2           | -3.27       | -2.00       | 0.18        |
| 3           | -0.97       | -0.60       | 0.06        |

| \(\xi_i^I\) | \(\xi_i^J\) | \(\xi_i^K\) | \(\xi_i^L\) |
|-------------|-------------|-------------|-------------|
| 0           | 2.52        | 0.86        | 1.89        |
| 1           | 0.73        | 1.89        | 0.52        |
| 2           | 0.22        | 0.52        | 0.14        |
| 3           | 0.06        | 0.14        | 0.04        |

| \(\xi_i^K\) | \(\xi_i^L\) |
|-------------|-------------|
| 0           | -0.36       |
| 1           | -0.16       |
| 2           | -0.06       |
| 3           | -0.02       |

Unfortunately, at present there is not enough data to test this theory. This scheme only predicts integrals of amplitudes through the Delta region. In order to compare with experiment it is essential that the amplitudes be separated out so that these integrals can be estimated. To do this, more precise and complete measures of spin observables are
required. The cross section plus three single-spin observables determine the magnitudes of the amplitudes, but double-spin observables determine their relative phase. By carefully selecting four of the double-spin observables, one can extract all of the requisite phase without discrete ambiguities \cite{11,54}--\ALL Once the experimental data of amplitudes are available, the values of these unknown parameters can be extracted and we can test our predictions.

A fundamental issue is the predictive power of our method of comparison. The scheme is designed to separate out the maximum number of independent quantities extractable from experiment at a given in the small scale expansion. To be predictive, there must be more observables than free parameters. In neutral pion photoproduction, the P-wave amplitudes are determined by ten unknown parameters. Of these ten, all but three can be fit in pion-nucleon scattering. With these three parameters we have 18 integrated observables to fit. With the S-wave there is just one new parameter: $b_{21} + b_{22}$. Toward the charge pion photoproduction, the P-wave and S-wave have respectively only one new parameters: $b_{22}$ and $b_{17} - \frac{b_{19}}{2}$. If we generalize to the case of electroproduction, there are only two additional parameters which need to be fit, and the number of observables increases since there are additional C2 amplitudes. Therefore up to $\mathcal{O}(\epsilon^3)$ the predictive power of our approach is clear. If one works at higher order, the number of basis functions increases, therefore a larger number of independent observables becomes available. However many new parameters will be involved. Whether one ultimately has increased the predictive power has not yet been settled.

In conclusion, the “small scale expansion” provides us a systematic way to calculate the processes of the $\pi \Delta N$ system, and with the power counting scheme we can fit the unknown parameters to make predictions. Our method is designed to isolate the maximum number of independent predictions which can be made at a given order.

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Appendix A: Loop Functions

Here we define many of the loop functions which occur frequently in our calculations and we will give some closed analytical forms. Divergent loop functions are regularized via dimensional regularization and expand around $d=4$ space-time dimensions. All propagators here are understood to have an infinitesimal negative imaginary part.

\[ \frac{1}{i} \int \frac{d^4l}{(2\pi)^d} \frac{1}{m^2_{\pi} - l^2} \equiv \Delta_{\pi}, \quad (120) \]

\[ \Delta_{\pi} = 2m^2_{\pi}(L + \frac{1}{16\pi^2} \ln \frac{m_{\pi}}{\mu}) + O(d - 4), \quad (121) \]

\[ L = \frac{\mu^{d-4}}{16\pi^2} \left[ \frac{1}{d-4} + \frac{1}{2}(\gamma_E - 1 - \ln 4\pi) \right], \quad (122) \]

\[ \frac{1}{i} \int \frac{d^4l}{(2\pi)^d} \frac{[1,l_{\mu},l_{\mu}l_{\nu}]}{(v \cdot l - A)(m^2_{\pi} - l^2)} \equiv \{ J_0(A), v_{\mu}J_1(A), g_{\mu\nu}J_2(A) + v_{\mu}v_{\nu}J_3(A) \}, \quad (123) \]

\[ J_0(A) = -4LA + \frac{A}{8\pi^2}(1 - 2 \ln \frac{m_{\pi}}{\mu}) - \frac{1}{4\pi^2}\sqrt{m^2_{\pi} - A^2} \cos^{-1}(\frac{A}{m_{\pi}}), \quad (124) \]

\[ J_1(A) = AJ_0(A) + \Delta_{\pi}, \quad (125) \]

\[ J_2(A) = \frac{1}{d - 1}[(M^2_{\pi} - A^2)J_0(A) - A\Delta_{\pi}], \quad (126) \]

\[ J_3(A) = AJ_1(A) - J_2(A), \quad (127) \]

The analytic continuation of these expressions above the branch point $A=m_{\pi}$ is obtained by following substitutions:

\[ \sqrt{m^2_{\pi} - A^2} \rightarrow -i\sqrt{A^2 - m^2_{\pi}}, \sin^{-1}(\frac{A}{m_{\pi}}) \rightarrow \frac{\pi}{2} + i \ln[A + \sqrt{A^2 - m^2_{\pi}}], \quad (128) \]

Similarly,

\[ \frac{1}{i} \int \frac{d^4l}{(2\pi)^d} \frac{[1,l_{\mu},l_{\mu}l_{\nu}]}{(v \cdot l + A)(m^2_{\pi} - (l+k)^2)} \equiv \{ \gamma^0_A, k_{\mu}\gamma^1_A + v_{\mu}\gamma^2_A, g_{\mu\nu}\gamma^3_A + k_{\mu}k_{\nu}\gamma^4_A + (k_{\mu}v_{\nu} + k_{\nu}v_{\mu})\gamma^5_A + v_{\mu}v_{\nu}\gamma^6_A \} \quad (129) \]

\[ \frac{1}{i} \int \frac{d^4l}{(2\pi)^d} \frac{l_{\mu}l_{\nu}l_{\sigma}}{(v \cdot l + A)(m^2_{\pi} - (l+k)^2)} \equiv (g_{\mu\nu}k_{\sigma} + \text{perm.})\gamma^7_A + (g_{\mu\nu}v_{\sigma} + \text{perm})\gamma^8_A + (k_{\mu}k_{\nu}v_{\sigma} + \text{perm})\gamma^9_A \]

\[ + (k_{\mu}v_{\nu}v_{\sigma} + \text{perm})\gamma^{10}_A + k_{\mu}k_{\nu}k_{\sigma}\gamma^{11}_A + v_{\mu}v_{\nu}v_{\sigma}\gamma^{12}_A. \quad (130) \]

Here $\omega = v \cdot k$, $k^2 = 0$ since we consider only real photons:

\[ \gamma^0_A = -\frac{\partial}{\partial m^2_{\pi}} \int_0^1 dx J_0(x\omega - A), \quad (131) \]

\[ \gamma^3_A = \frac{m^2_{\pi}}{2}\gamma^0_A - \frac{1}{4\omega}(\omega - A)J_0(\omega - A) + AJ_0(-A), \quad (132) \]
\[ \gamma_A^7 = -\frac{A m_\pi^2}{2 \omega} \gamma_A^0 - \frac{m_\pi^2}{4 \omega^2} [J_0(\omega - A) - J_0(-A)] + \frac{J_1(\omega - A)}{4 \omega} + \frac{J_2(\omega - A) - J_2(-A)}{4 \omega^2}, \tag{133} \]

\[ \gamma_A^8 = \frac{J_2(\omega - A) - J_2(-A)}{2 \omega}. \tag{134} \]

The divergences of \( \gamma_A^i \) are:

\[ [\gamma_A^3]^{\text{div}} = (\omega - 2A)L, [\gamma_A^7]^{\text{div}} = \left( -\frac{2}{3} \omega + A \right)L, [\gamma_A^8]^{\text{div}} = \left( -m_\pi^2 + \frac{2}{3} \omega^2 - 2\omega A + 2A^2 \right)L. \tag{135} \]

**Appendix B: Feynman Rules**

Here we collect the Feynman rules which are needed to calculate tree and loop diagrams. The following notations are used:

- \( l \): momentum of a pion or nucleon or delta propagator;
- \( k \): momentum of photon;
- \( q \): momentum of external pion;
- \( \epsilon \): Photon polarization vector;
- \( p \): momentum of a nucleon in heavy mass formulation;
- \( r \): momentum of a delta in heavy mass formulation.

Isospin indices of pion are \( a, b, c, d... \), the isospin indices of \( \Delta \) are \( i, j, k... \), and the spin indices of \( \Delta \) are \( \mu, \nu, \sigma \). \( v_\mu \) is the nucleon four-velocity and \( S_\mu \) is a covariant spin-vector. We also give the orientation of momenta at the vertices, i.e, which are “in”-going or “out”-going. \( Q \equiv \text{diag}(1,0) \). Here we only present the ones related to \( \Delta \) since others can be found in [28].

**Vertices from \( L^{(1)}_\Delta \):**

1. \( \Delta \) propagator:

   \[ -i \frac{P^{3/2}_{\mu \nu} \xi_{I=3/2}^{ij}}{v \cdot l - \Delta + i\epsilon}, \]

2. One pion (q out):

   \[ -\frac{g_1}{F_\pi} \epsilon_{I=3/2}^{ik} \xi_{I=3/2}^{c\epsilon kj} (S \cdot q) g_{\mu \nu} \]

3. One pion, one photon:

   \[ -i \frac{eg_1}{F_\pi} \epsilon_{I=3/2}^{c3b} \epsilon_{I=3/2}^{ik} \xi_{I=3/2}^{b\epsilon kj} (S \cdot \epsilon) g_{\mu \nu} \]

**Vertices from \( L^{(2)}_\Delta \):**

1. \( \Delta \) propagator:

   \[ -i \frac{P^{3/2}_{\mu \nu} \xi_{I=3/2}^{ij}}{(v \cdot l - \Delta + i\epsilon)^2} \frac{-1}{2m_N} \frac{(l^2 - (v \cdot l)^2)}{s} \]
Vertices from $\mathcal{L}_{\Delta N}^{(1)}$:

1. One pion (q, out):

\[ \frac{g_{\pi \Delta N}}{F_{\pi}} q_{\mu} \delta^{ci} \]

2. One photon (k, in):

\[ \frac{i e G_{1}}{m_{N}} \delta^{3i} \left[ (S \cdot k) \epsilon_{\mu} - (S \cdot \epsilon) k_{\mu} \right] \]

3. One pion, one photon:

\[ \frac{i e g_{\pi \Delta N}}{F_{\pi}} \epsilon_{\mu} \epsilon^{3i} \]

Vertices from $\mathcal{L}_{\Delta N}^{(2)}$:

1. One pion (p in; r, q out):

\[ -\frac{g_{\pi \Delta N}}{m_{N} F_{\pi}} r_{\mu} (v \cdot q) \delta^{ci} - \frac{\tilde{g}_{\pi \Delta N}}{2 m_{N} F_{\pi}} q_{\mu} (v \cdot q) \delta^{ci} \]

2. One photon (k, p in; r out):

\[ \frac{i e}{2m_{N}^{2}} (v \cdot k) \delta^{3i} \left[ -i h_{1} (S \cdot \epsilon) r_{\mu} + \frac{i h_{2}}{2} (S \cdot r) \epsilon_{\mu} + h_{2} (S \cdot r) \epsilon^{\mu \alpha \beta} \epsilon_{\rho} v_{\alpha} S_{\beta} + \frac{G_{2}}{2} \epsilon_{\mu} (S \cdot r - p) - G_{1} \epsilon_{\mu} (S \cdot r + p) \right] + \frac{i e}{2m_{N}^{2}} \left[ i h_{1} (S \cdot k) r_{\mu} - \frac{i h_{2}}{2} (S \cdot r) k_{\mu} + h_{2} (S \cdot r) \epsilon^{\mu \alpha \beta} k_{\rho} v_{\alpha} S_{\beta} + \frac{G_{2}}{2} k_{\mu} (S \cdot r - p) - \frac{G_{1}}{2} k_{\mu} (S \cdot r + p) \right] \]

\[ h_{1} = -2G_{1} - 2xG_{1} + yG_{2}; \quad h_{2} = yG_{2} - 2xG_{1} \]

3. One pion, one photon (p, k in; r, q out):

\[ \frac{e g_{\pi \Delta N}}{m_{N} F_{\pi}} (Q \delta^{3i} - i e^{ib} \tau^{b}) (v \cdot q) \epsilon_{\mu} \]

Appendix C: Diagrams

Here we collect all graphs in Fig. 1-4 except which is identical to zero in our calculation and the graph only contributes to the renormalization of nucleon wavefunctions and nucleon mass.

(1-A-1):

\[ -\frac{e g_{A}}{2F_{\pi}^{3}} \left( \delta^{3i} + \frac{1}{4} [\tau^{c}, \tau^{3}] \right) (\omega \gamma^{7} + \gamma^{8} + 2 \omega \gamma^{3}) (\bar{\sigma} \cdot \bar{\epsilon}). \]  \hfill (136)

(1-A-1-C):

\[ \frac{e g_{A}}{2F_{\pi}^{3}} \left( \delta^{3i} - \frac{1}{4} [\tau^{c}, \tau^{3}] \right) (\omega \gamma^{7} + \gamma^{8} - \omega \gamma^{3}) (\bar{\sigma} \cdot \bar{\epsilon}). \]  \hfill (137)
(1-A-3): \[-\frac{e g A}{4 F^3_\pi} (\delta^{c_3} + \frac{1}{4}[\tau^c, \tau^3])(J_1(\omega) + \omega J_0(\omega))(\vec{\sigma} \cdot \vec{e}).\] (138)

(1-A-3-C): \[\frac{e g A}{4 F^3_\pi} (\delta^{c_3} - \frac{1}{4}[\tau^c, \tau^3])(J_1(-\omega) - \omega J_0(-\omega))(\vec{\sigma} \cdot \vec{e}).\] (139)

(1-A-7): \[\frac{e g A}{4 F^3_\pi} [\tau^c, \tau^3](\vec{\sigma} \cdot \vec{e})(\omega \gamma^7 + \gamma^8).\] (140)

(1-A-8): \[\frac{e g A}{8 F^3_\pi} [\tau^c, \tau^3] \frac{1}{\omega^2 - k \cdot q} \{-(\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q}) + (\vec{\sigma} \cdot \vec{q})(\vec{e} \cdot \vec{q})\} \Delta_\pi.\] (141)

(1-A-15): \[-\frac{e g A}{8 F^3_\pi} [\tau^c, \tau^3] \frac{1}{\omega^2 - k \cdot q} \{(\vec{\sigma} \cdot \vec{q})(\vec{e} \cdot \vec{q})[-4 \omega \gamma_7 - 4 \gamma_8] + (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q})[4 \omega \gamma_7 + 4 \gamma_8]\}.\] (142)

(1-B-2): \[\frac{e g A}{2 F^3_\pi} (\delta^{c_3} + \frac{1}{2}[\tau^c, \tau^3]) \frac{\gamma^3}{\omega + i\epsilon} \{i(\vec{q} \times \vec{k} \cdot \vec{e}) + (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q}) - (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q})\}.\] (143)

(1-B-2-C): \[\frac{e g A}{2 F^3_\pi} (\delta^{c_3} - \frac{1}{2}[\tau^c, \tau^3]) \frac{\gamma^3}{\omega - i\epsilon} \{i(\vec{q} \times \vec{k} \cdot \vec{e}) - (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q}) + (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q})\}.\] (144)

(1-C-1): \[\frac{e g A}{2 F^3_\pi} [\tau^c, \tau^3] \frac{\gamma^3}{\omega} \{i(\vec{q} \times \vec{k} \cdot \vec{e})[\gamma^3_\omega - \gamma^3] + (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q})[2(\gamma^7 - \gamma^7_\omega) + (\gamma^3 - \gamma^3_\omega)\} + (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q})[2(\gamma^7 - \gamma^7_\omega) + (\gamma^3 - \gamma^3_\omega)]\}.\] (145)

(1-C-5): \[\left.\frac{e g A}{16 F^3_\pi} [\tau^c, \tau^3] \frac{\partial J_2(\omega)}{\partial \omega}\right|_{\omega=0}(\vec{\sigma} \cdot \vec{e}).\] (146)

(1-C-6): \[\frac{e g A}{9 F^3_\pi} [\tau^c, \tau^3] \frac{1}{\omega^2 - k \cdot q} \{(\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q}) - (\vec{\sigma} \cdot \vec{q})(\vec{e} \cdot \vec{q})\} \left.\frac{\partial J_2(\omega)}{\partial \omega}\right|_{\omega=0}.\] (147)

(2-A-1): \[\frac{2 e g A g^2_{\pi N A}}{9 F^3_\pi} (\delta^{c_3} + \frac{1}{2}[\tau^c, \tau^3]) \frac{\gamma^3}{\omega + i\epsilon} \{i(\vec{q} \times \vec{k} \cdot \vec{e}) + (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q}) - (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q})\}.\] (148)
\[
\frac{-2 e ga G^{2} \Delta N}{9 F^{3} \pi} \left(\delta^{3} - \frac{1}{2} \tau^{3} \right) \frac{1}{\omega - \Delta + i\epsilon} \left\{ i (\vec{q} \times \vec{k} \cdot \vec{e}) - (\vec{\sigma} \cdot \vec{e}) (\vec{k} \cdot \vec{q}) + (\vec{\sigma} \cdot \vec{k}) (\vec{e} \cdot \vec{q}) \right\}. \tag{149}
\]

\[
\frac{-2 e ga G^{2} \Delta N}{9 F^{3} \pi} \left(\delta^{3} - \frac{1}{4} \tau^{3} \right) \frac{1}{\omega - \Delta + i\epsilon} \left\{ i (\vec{q} \times \vec{k} \cdot \vec{e}) [-\gamma^{3}] + (\vec{\sigma} \cdot \vec{e}) (\vec{k} \cdot \vec{q}) [2\gamma^{3} + 3\gamma^{7}] + (\vec{\sigma} \cdot \vec{k}) (\vec{e} \cdot \vec{q}) [\gamma^{3} + 3\gamma^{7}] \right\}. \tag{150}
\]

\[
\frac{2 e ga G^{2} \Delta N}{9 F^{3} \pi} \left(\delta^{3} + \frac{1}{4} \tau^{3} \right) \frac{1}{\omega + \Delta - i\epsilon} \left\{ i (\vec{q} \times \vec{k} \cdot \vec{e}) [-\gamma^{3}] + (\vec{\sigma} \cdot \vec{e}) (\vec{k} \cdot \vec{q}) [3\gamma^{7} + \gamma^{3}] + (\vec{\sigma} \cdot \vec{k}) (\vec{e} \cdot \vec{q}) [2\gamma^{3} + 3\gamma^{7}] \right\}. \tag{151}
\]

\[
\frac{10 e ga G^{2} \Delta N}{81 F^{3} \pi} \left(\delta^{3} - \frac{1}{4} \tau^{3} \right) \frac{1}{\omega - \Delta + i\epsilon} \left\{ i (\vec{q} \times \vec{k} \cdot \vec{e}) [5\gamma^{3}] + (\vec{\sigma} \cdot \vec{e}) (\vec{k} \cdot \vec{q}) [3\gamma^{3} - \gamma^{3}_{A}] + (\vec{\sigma} \cdot \vec{k}) (\vec{e} \cdot \vec{q}) [4\gamma^{3}_{A} + 3\gamma^{7}_{A}] \right\}. \tag{152}
\]

\[
\frac{-10 e ga G^{2} \Delta N}{81 F^{3} \pi} \left(\delta^{3} + \frac{1}{4} \tau^{3} \right) \frac{1}{\omega + \Delta - i\epsilon} \left\{ i (\vec{q} \times \vec{k} \cdot \vec{e}) [5\gamma^{3}_{A+\omega}] + (\vec{\sigma} \cdot \vec{e}) (\vec{k} \cdot \vec{q}) [4\gamma^{3}_{A+\omega} + 3\gamma^{7}_{A+\omega}] + (\vec{\sigma} \cdot \vec{k}) (\vec{e} \cdot \vec{q}) [3\gamma^{7}_{A+\omega} - \gamma^{3}_{A+\omega}] \right\}. \tag{153}
\]

\[
\frac{2 e ga G^{2} \Delta N}{9 F^{3} \pi} \left(\delta^{3} - \frac{3}{4} \tau^{3} \right) \frac{1}{\omega - \Delta} \left\{ i (\vec{q} \times \vec{k} \cdot \vec{e}) [\gamma^{3}_{\omega} - \gamma^{3}_{A}] + (\vec{\sigma} \cdot \vec{e}) (\vec{k} \cdot \vec{q}) [(\gamma^{7}_{\omega} - \gamma^{3}_{A}) - (\gamma^{3}_{\omega} - \gamma^{3}_{A})] + (\vec{\sigma} \cdot \vec{k}) (\vec{e} \cdot \vec{q}) [(\gamma^{7}_{\omega} - \gamma^{3}_{A}) + 2(\gamma^{3}_{\omega} - \gamma^{3}_{A})] \right\}. \tag{154}
\]

\[
\frac{-2 e ga G^{2} \Delta N}{9 F^{3} \pi} \left(\delta^{3} + \frac{3}{4} \tau^{3} \right) \frac{1}{\omega + \Delta} \left\{ i (\vec{q} \times \vec{k} \cdot \vec{e}) [\gamma^{3}_{A+\omega} - \gamma^{3}_{A}] + (\vec{\sigma} \cdot \vec{e}) (\vec{k} \cdot \vec{q}) [2(\gamma^{3}_{A+\omega} - \gamma^{3}_{A}) + (\gamma^{7}_{A+\omega} - \gamma^{7}_{A})] + (\vec{\sigma} \cdot \vec{k}) (\vec{e} \cdot \vec{q}) [(\gamma^{7}_{A+\omega} - \gamma^{3}_{A}) - (\gamma^{3}_{A+\omega} - \gamma^{3}_{A})] \right\}. \tag{155}
\]

\[
\frac{25 e ga G^{2} \Delta N}{81 F^{3} \pi} \delta^{3} \frac{1}{\omega} \left\{ i (\vec{q} \times \vec{k} \cdot \vec{e}) [\gamma^{3}_{A+\omega} - \gamma^{3}_{A}] + (\vec{\sigma} \cdot \vec{e}) (\vec{k} \cdot \vec{q}) [2(\gamma^{3}_{A+\omega} - \gamma^{3}_{A}) + 2(\gamma^{7}_{A+\omega} - \gamma^{7}_{A})] + (\vec{\sigma} \cdot \vec{k}) (\vec{e} \cdot \vec{q}) [(\gamma^{3}_{A+\omega} - \gamma^{3}_{A}) + 2(\gamma^{7}_{A+\omega} - \gamma^{7}_{A})] \right\}. \tag{156}
\]

27
\[
\frac{4eg_A g_π^2 ΔN}{9F_π^3} \frac{1}{(ω^2 - \vec{k} \cdot \vec{q})} \left\{ (\vec{σ} \cdot \vec{k})(\vec{ε} \cdot \vec{q}) - (\vec{σ} \cdot \vec{q})(\vec{ε} \cdot \vec{k}) \right\} \frac{J_2(0) - J_2(-Δ)}{Δ}. \tag{157}
\]

\[
\frac{25g_1 g_π^2 ΔN}{81F_π^3} \frac{1}{(ω^2 - \vec{k} \cdot \vec{q})} \left\{ (\vec{σ} \cdot \vec{k})(\vec{ε} \cdot \vec{q}) - (\vec{σ} \cdot \vec{q})(\vec{ε} \cdot \vec{k}) \right\} \frac{∂J_2(ω)}{∂ω} \bigg|_{ω = -Δ}. \tag{158}
\]

\[
\frac{4eg_A g_π^2 ΔN}{9F_π^3} \frac{J_2(0) - J_2(-Δ)}{Δ}(\vec{σ} \cdot \vec{ε}). \tag{159}
\]

\[
\frac{25eg_1 g_π^2 ΔN}{81F_π^3} \frac{∂J_2(ω)}{∂ω} \bigg|_{ω = -Δ}(\vec{σ} \cdot \vec{ε}). \tag{160}
\]

\[-\frac{eg_A}{4F_π} [τ^c, τ^3] (\vec{σ} \cdot \vec{ε}). \tag{161}\]

\[
\frac{eg_A}{4F_π} [τ^c, τ^3] \frac{1}{ω^2 - \vec{k} \cdot \vec{q}} \left\{ (\vec{σ} \cdot \vec{q})(\vec{ε} \cdot \vec{q}) - (\vec{σ} \cdot \vec{k})(\vec{ε} \cdot \vec{k}) \right\}. \tag{162}\]

\[
\frac{eg_A}{4m_N F_π} (\vec{σ} \cdot \vec{ε})(v \cdot q)(τ^c + δ^{c3}). \tag{163}\]

\[-\frac{eg_A}{8m_N F_π} τ^c (1 + κ_3 + (1 + κ_3)τ^3) \frac{1}{ω + iε} \left\{ i(\vec{q} \times \vec{k} \cdot \vec{ε}) + (\vec{σ} \cdot \vec{ε})(\vec{k} \cdot \vec{q}) - (\vec{σ} \cdot \vec{k})(\vec{ε} \cdot \vec{q}) \right\}. \tag{164}\]

\[-\frac{eg_A}{8m_N F_π} (1 + κ_3 + (1 + κ_3)τ^3) τ^c \frac{1}{ω - iε} \left\{ i(\vec{q} \times \vec{k} \cdot \vec{ε}) - (\vec{σ} \cdot \vec{ε})(\vec{k} \cdot \vec{q}) + (\vec{σ} \cdot \vec{k})(\vec{ε} \cdot \vec{q}) \right\} \frac{1}{Q}. \tag{165}\]

\[
\frac{eg_A}{8m_N F_π} \frac{ω}{ω^2 - \vec{k} \cdot \vec{q}} [τ^c, τ^3] \left\{ (\vec{σ} \cdot \vec{k})(\vec{ε} \cdot \vec{q}) + (\vec{σ} \cdot \vec{q})(\vec{ε} \cdot \vec{k}) \right\}. \tag{166}\]
\[ (3\text{-C-1}): \]
\[
- \frac{e g_A N A_1}{3 m_N F^2} \delta^{\alpha_3} (1 + 4x + 4z + 12xz) i (\vec{q} \times \vec{k} \cdot \vec{e}) \\
+ \frac{e}{8 m^2 F^2} (b_9 \delta^{\alpha_3} - b_{10} \tau^c) i (\vec{q} \times \vec{k} \cdot \vec{e}) \\
- \frac{e}{16 m^2 F^2} (b_{17} - \frac{b_{19}}{2}) m^2 \{ \tau^c, \tau^3 \}(\vec{\sigma} \cdot \vec{e}) \\
+ \frac{e}{32 \pi^2 F^2} [\tau^c, \tau^3] \{- (b_{21} + b_{22}) (v \cdot k) (v \cdot q) (\vec{\sigma} \cdot \vec{e}) + b_{22} ((\vec{\sigma} \cdot \vec{e}) (\vec{k} \cdot \vec{q}) - (\vec{\sigma} \cdot \vec{k}) (\vec{e} \cdot \vec{q})) \} \\
+ \frac{e g_A}{8 m^2 F^2} Q \tau^c \{ i (\vec{q} \times \vec{k} \cdot \vec{e}) + (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q}) + (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q}) \} \\
+ \frac{e g_A}{16 m^2 F^2} \tau^c Q (\vec{\sigma} \cdot \vec{e})(\vec{q} \cdot \vec{q}) + \frac{e g_A}{16 m^2 F^2} [\tau^c, \tau^3] (v \cdot q) (v \cdot k)(\vec{\sigma} \cdot \vec{e}). \]  

\[ (3\text{-C-2}): \]
\[
- \frac{e g_A}{16 m^2 F^2} \tau^c (\kappa_s + \kappa_v \tau^3) \{ i \vec{q} \times \vec{k} \cdot \vec{e} + (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q}) - (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q}) \}. \]  

\[ (3\text{-C-2-C}): \]
\[
\frac{e g_A}{8 m^2 F^2} Q \tau^c \{ i (\vec{q} \times \vec{k} \cdot \vec{e}) - (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q}) + (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q}) \} \\
+ \frac{e g_A}{16 m^2 F^2} (\kappa_s + \kappa_v \tau^3) \tau^c \{ i (\vec{q} \times \vec{k} \cdot \vec{e}) - (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q}) + (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q}) \\
- 2(\vec{\sigma} \cdot \vec{e})(\vec{q} \cdot \vec{q}) + 2(\vec{\sigma} \cdot \vec{q})(\vec{e} \cdot \vec{q}) \}. \]  

\[ (3\text{-C-3}): \]
\[
- \frac{e g_A}{16 m^2 F^2} \tau^c (1 + \kappa_s + (1 + \kappa_v) \tau^3) \{ i \vec{q} \times \vec{k} \cdot \vec{e} + (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q}) - (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q}) \}. \]  

\[ (3\text{-C-3-C}): \]
\[
\frac{e g_A}{16 m^2 F^2} \{ (1 + \kappa_s + (1 + \kappa_v) \tau^3) \tau^c \{ i \vec{q} \times \vec{k} \cdot \vec{e} - (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q}) \\
+ (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q}) - 2(\vec{k} \cdot \vec{k})(\vec{\sigma} \cdot \vec{e}) \} + 2Q \tau^c (-2(\vec{\sigma} \cdot \vec{q})(\vec{e} \cdot \vec{q}) + 4(\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q})) \}. \]  

\[ (3\text{-C-4}): \]
\[
- \frac{e g_A}{m_N^2 F^2} \tau^c (1 + \kappa_s + (1 + \kappa_v) \tau^3) \frac{m_2^2 \vec{c}_1}{2(\omega + i \epsilon)^2} \{ i \vec{q} \times \vec{k} \cdot \vec{e} - (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q}) - (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q}) \}. \]  

\[ (3\text{-C-4-C}): \]
\[
- \frac{e g_A}{8 m_N^2 F^2} \frac{2 \omega^2 - m_2^2 + 2 \vec{k} \cdot \vec{q} + 8m_2 \vec{c}_1}{2(\omega - i \epsilon)^2} \{ (1 + \kappa_s + (1 + \kappa_v) \tau^3) \tau^c \{ i \vec{q} \times \vec{k} \cdot \vec{e} \\
- (\vec{\sigma} \cdot \vec{e})(\vec{k} \cdot \vec{q}) + (\vec{\sigma} \cdot \vec{k})(\vec{e} \cdot \vec{q}) \} - 4Q \tau^c (\vec{\sigma} \cdot \vec{q})(\vec{e} \cdot \vec{q}). \]
\[(3-C-6):\]
\[-\frac{e g_A}{32 m_N F_\pi} \left[ \tau^c, \tau^3 \right] \frac{m_\pi^2}{\omega^2 - k \cdot q} \{(\bar{\sigma} \cdot q)(\bar{\epsilon} \cdot \bar{q}) + (\bar{\sigma} \cdot \bar{q})(\bar{\epsilon} \cdot q)\}
- \frac{e g_A}{16 m_N^2 F_\pi^2} (b_{177} - \frac{2}{17}) \left[ \tau^c, \tau^3 \right] \frac{m_\pi^2}{\omega^2 - k \cdot q} (\bar{\sigma} \cdot q)(\bar{\epsilon} \cdot \bar{q}). \] (174)

\[(3-C-7):\]
\[\frac{e g_A}{2 F_\pi^3 \omega^2 - \bar{k} \cdot \bar{q}} \{(\bar{\sigma} \cdot \bar{q})(\bar{\epsilon} \cdot \bar{q}) - (\bar{\sigma} \cdot \bar{k})(\bar{\epsilon} \cdot \bar{q})\}. \] (175)

\[(4-A-1):\]
\[-\frac{e G_1 g_{\pi N}}{9 m_N F_\pi} (\delta^{c3} - \frac{1}{4} [\tau^c, \tau^3]) \frac{1}{\omega - \Delta + i \epsilon} \{-2i(\bar{q} \times \bar{k} \cdot \bar{\epsilon}) + (\bar{\sigma} \cdot \bar{\epsilon})(\bar{k} \cdot \bar{q}) - (\bar{\sigma} \cdot \bar{k})(\bar{\epsilon} \cdot \bar{q})\}. \] (176)

\[(4-A-1-C):\]
\[-\frac{e G_1 g_{\pi N}}{9 m_N F_\pi} (\delta^{c3} + \frac{1}{4} [\tau^c, \tau^3]) \frac{1}{\omega + \Delta - i \epsilon} \{2i(\bar{q} \times \bar{k} \cdot \bar{\epsilon}) + (\bar{\sigma} \cdot \bar{\epsilon})(\bar{k} \cdot \bar{q}) - (\bar{\sigma} \cdot \bar{k})(\bar{\epsilon} \cdot \bar{q})\}. \] (177)

\[(4-B-1):\]
\[\frac{e g_A \pi N}{3 m_N^2 F_\pi} (\delta^{c3} - \frac{1}{4} [\tau^c, \tau^3]) \frac{\omega}{\omega^2 - \Delta + i \epsilon} \{i(\bar{q} \times \bar{k} \cdot \bar{\epsilon})[G_2 + 2G_1] + (\bar{\sigma} \cdot \bar{\epsilon})(\bar{k} \cdot \bar{q})[G_2 + 2G_1] + (\bar{\sigma} \cdot \bar{k})(\bar{\epsilon} \cdot \bar{q})[4G_1 + 2G_2]\}. \] (178)

\[(4-B-1-C):\]
\[\frac{e g_A \pi N}{3 m_N^2 F_\pi} (\delta^{c3} + \frac{1}{4} [\tau^c, \tau^3]) \frac{\omega}{\omega^2 - \Delta - i \epsilon} \{-2i(\bar{q} \times \bar{k} \cdot \bar{\epsilon}) + (\bar{\sigma} \cdot \bar{\epsilon})(\bar{k} \cdot \bar{q}) - (\bar{\sigma} \cdot \bar{k})(\bar{\epsilon} \cdot \bar{q})\}. \] (179)

\[(4-B-2):\]
\[-\frac{e G_1 g_{\pi N}}{18 m_N^2 F_\pi^2} (\delta^{c3} - \frac{1}{4} [\tau^c, \tau^3]) \frac{\omega}{\omega^2 - \Delta + i \epsilon} \{-2i(\bar{q} \times \bar{k} \cdot \bar{\epsilon}) + (\bar{\sigma} \cdot \bar{\epsilon})(\bar{k} \cdot \bar{q}) - (\bar{\sigma} \cdot \bar{k})(\bar{\epsilon} \cdot \bar{q})\}. \] (180)

\[(4-B-2-C):\]
\[\frac{e G_1 g_{\pi N}}{6 m_N^2 F_\pi^2} (\delta^{c3} + \frac{1}{4} [\tau^c, \tau^3]) \frac{\omega}{\omega^2 - \Delta - i \epsilon} \{2i(\bar{q} \times \bar{k} \cdot \bar{\epsilon}) + (\bar{\sigma} \cdot \bar{\epsilon})(\bar{k} \cdot \bar{q}) - (\bar{\sigma} \cdot \bar{k})(\bar{\epsilon} \cdot \bar{q})\} \] \[+ \frac{e G_1 g_{\pi N}}{18 m_N^2 F_\pi^2} (\delta^{c3} + \frac{1}{4} [\tau^c, \tau^3]) \frac{\omega}{\omega^2 - \Delta - i \epsilon} \{2i(\bar{q} \times \bar{k} \cdot \bar{\epsilon}) + (\bar{\sigma} \cdot \bar{\epsilon})(\bar{k} \cdot \bar{q}) - (\bar{\sigma} \cdot \bar{k})(\bar{\epsilon} \cdot \bar{q})\}. \] (181)

\[(4-B-3-C):\]
\[\frac{e G_1 g_{\pi N}}{6 m_N^2 F_\pi^2} (\delta^{c3} + \frac{1}{4} [\tau^c, \tau^3]) \frac{2 \omega^2 - M_\pi^2 + 2(\bar{k} \cdot \bar{q})}{(\omega + \Delta - i \epsilon)^2} \{2i(\bar{q} \times \bar{k} \cdot \bar{\epsilon}) + (\bar{\sigma} \cdot \bar{\epsilon})(\bar{k} \cdot \bar{q}) - (\bar{\sigma} \cdot \bar{k})(\bar{\epsilon} \cdot \bar{q})\}. \] (182)
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Photon
Pion
Nucleon

Fig 1.B

Fig 1.C

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Photon
Pion
Nucleon
Fig. 2

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Nucleon

Pion

Delta

Photon
4.B.1

Fig4

4.B.2

4.B.3

2nd order delta propagator