Preserving Local and Global Information for Network Embedding

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Abstract
Networks such as social networks, airplane networks, and citation networks are ubiquitous. The adjacency matrix is often adopted to represent a network, which is usually high dimensional and sparse. However, to apply advanced machine learning algorithms to network data, low-dimensional and continuous representations are desired. To achieve this goal, many network embedding methods have been proposed recently. The majority of existing methods facilitate the local information i.e. local connections between nodes, to learn the representations, while completely neglecting global information (or node status), which has been proven to boost numerous network mining tasks such as link prediction and social recommendation. Hence, it also has potential to advance network embedding. In this paper, we study the problem of preserving local and global information for network embedding. In particular, we introduce an approach to capture global information and propose a network embedding framework LOG, which can coherently model LOcal and Global information. Experimental results demonstrate the ability to preserve global information of the proposed framework. Further experiments are conducted to demonstrate the effectiveness of learned representations of the proposed framework.

1 Introduction
Networks, such as social networks, airplane networks, and citation networks, are ubiquitous and important in our daily life. Node representation learning/extraction is to learn or extract meaningful features for each node such that the representation can be facilitated in network analysis tasks such as link prediction, visualization, community detection and node classification. One naive way is to represent the nodes as an adjacency matrix, i.e., each node is represented as a sparse vector that contains link information. However, this representation usually suffers from the curse of dimensionality as the adjacency matrix is high dimensional and sparse because the networks are usually large with sparse links. Therefore, network embedding, which aims to learn low-dimensional and continuous node representations by preserving certain properties of the network, has attracted increasing attention in recent years [15, 21, 24, 3].

![Figure 1: Viewing a network from different perspectives.](image)

The majority of existing network embedding algorithms exploits local information to learn node representations. For example, DeepWalk [15] uses local information obtained from truncated random walks to learn latent representations; while LINE [21] preserves the first order proximity and second order proximity between nodes. By exploiting the local information, the learned vector representation of nodes captures certain local properties of the network, which has been demonstrated to advance many network analysis tasks such as link prediction [15, 24], node classification [20, 25] and visualization [19].

Despite the local information, global information (or node status) is another easily accessible but important information. Node status, which reflects where a node stands in the entire network, has different meanings under different “context”. For example, in the web networks, status can indicate a relevancy ranking of webpages [14]; in online social networks, it denotes reputations of users [9]; and in healthcare and biological networks, it can help determine the node representativeness [16, 22]. Most of the existing network embedding algorithms only facilitate local information by treating global status of each node equally as shown in Figure 1a. However, it is natural that nodes are of different global status as shown in Figure 1b. Many network based tasks such as web search [14], social network recommendation [15], active learning [5] and novel disease gene detection [7] have been proven to be advanced by
exploiting the global status. Therefore, global status can provide complementary information in addition to the local structure and incorporating global information has great potential to help learn better representation. However, the work on exploiting both local and global information is rather limited.

Therefore, in this paper, we study a new problem of investigating both local and global information for network representation learning. In essence, we need to solve two challenges: (1) How to mathematically capture global information; and (2) How to simultaneously model local and global information for network representation learning. In an attempt to solve these two challenges, we propose a novel framework LOG, which learns network representation that preserves both local and global information. The main contributions of the paper are summarized as follows:

- We propose a principled way to model global information for network representation learning;
- We propose a novel network embedding framework LOG, which integrates local and global information into a coherent model;
- We conduct experiments on real-world datasets to demonstrate the effectiveness of the proposed framework.

The remaining of the paper is organized as follows. In Section 2 we briefly review the related work. In Section 3 we formally define the problem and then, we introduce the proposed framework in Section 4. The experimental results and analysis are shown in Section 5. We conclude our paper with discussion and future work in Section 6.

### 2 Related Work

In this section, we briefly review some works related to our problem. Network embedding algorithms, which aim to learn low-dimensional node representation are attracting increasing attention recently. Inspired by word2vec [11, 10], DeepWalk [15] and LINE [21] are proposed. The general idea of word2vec is that a word can be explained by its surrounding neighbors. DeepWalk regards the nodes in a network as “words” of an “artificial” language and then use random walk to generate “sentences” for this language. Node representation can be learned following the procedure of word2vec. LINE tries to model the probability of the “context” of a node using the node representations and obtains the representations by minimizing the difference between the modeled probability and the empirical probability. node2vec[3] further extends DeepWalk by introducing parameters to allow biased random walk to explore the neighborhood of nodes. metapath2vec[11] extends the network embedding methods to heterogeneous network by introducing metapath based random walk. struc2vec[17] learns the node representation from a different perspective and it tries to preserve the structural identity between nodes. To achieve this goal, struc2vec first creates a new network based on the structural similarity between nodes and then follow the similar way of DeepWalk to learn node representation based on the created network. SDNE [23] uses an auto-encoder to compress indicate vector (the row corresponding to the node in the adjacency matrix) to obtain low-dimensional representations. LANE [6] tries to learn node representations on the attributed network with label information. In [24], a signed network embedding algorithm SiNE is proposed based on the notion that a user should be closer to their “friend” than their “enemy”. SiNE uses a deep neural network to optimize this objective when sampling triplets from the signed network. Two recent surveys [4, 2] give a comprehensive overview of network embedding algorithms. However, most of these existing methods cannot preserve the global information. In this paper, we propose a model which can preserve the local information as well as the global information.

### 3 Problem Statement

Before formally defining the problem we want to study, we first introduce the notations. Throughout the paper, matrices are written in boldface capital letters and vectors are denoted as boldface lowercase letters. Capital letters in calligraphic math font such as $\mathcal{P}$ are used to denote sets.

Let $\mathcal{G} = \{V, E\}$ be a network, where $V = \{v_1, \ldots, v_N\}$ denotes the set of $N$ nodes and $E = \{e_1, \ldots, e_M\}$ represents the set of $M$ edges between these nodes. Furthermore, let $t \in \mathbb{R}^{N \times 1}$ be the status scores to denote the global information for the $N$ nodes. The score can be calculated from various node status measures such as PageRank [13]. A larger score means a higher status.

With the aforementioned notations and definitions, the problem under study is formally stated as:

Given a network $\mathcal{G} = \{V, E\}$ and the global information $t$, we aim to learn the node representation matrix $U \in \mathbb{R}^{N \times d}$ by preserving the local structure information as well as the global information, where $d$ is the embedding dimension. Mathematically, the problem is written as:

$$Q(\mathcal{G}, t) \rightarrow U$$

where $Q$ is the learning algorithm we will investigate.
4 The Proposed Framework

In this section, we introduce the proposed model. Our model consists of two components – 1) the component to preserve the global information and 2) the component to preserve the local information. We first describe the component to preserve the global information, which leads to a new algorithm GINE. GINE can learn the embedding from only global information. We then introduce the component to preserve the local information and finally integrates both parts as the proposed framework LOG with both local and global information.

4.1 Global Information Preserved Network Embedding. In this subsection, we introduce a new embedding algorithm GINE which can preserve the global information for network embedding. Specifically, assume that we are given the status score vector \( t \) where \( t_i \) is the status score for node \( i \), which can be computed in various ways [14, 13, 12]. In this work, we calculate the status scores based on Pagerank [14]. The status rankings have been widely used in real-world applications instead of the status scores; hence, we first get the status rankings of nodes based on their status scores and then directly preserve the status rankings for network embedding.

To preserve the global ranking, we model this problem as a maximum likelihood problem. In other words, we need to maximize the probability that the ranking of the learned status vectors for the nodes \( \{v_1, \ldots, v_N\} \) follow the original Pagerank ranking \( \{r_1, \ldots, r_N\} \). For convenience, we use the ranking as the index for the nodes, that is, we use \( \{v_{(r_1)}, \ldots, v_{(r_N)}\} \) to represent the nodes. Intuitively, if we can preserve the relative ranking of all pairs of nodes, we can maintain the whole ranking for all nodes. Thus, we assume that the order between a pair of nodes is independent of the other pairs, we approximate this probability as:

\[
 p_{\text{global}} = \prod_{1 \leq i < j \leq N} p(v_{(i)}, v_{(j)});
\]

where \( p(v_{(i)}, v_{(j)}) \) is the probability that node \( v_{(i)} \) is ranked before node \( v_{(j)} \), which is defined as

\[
 p(v_{(i)}, v_{(j)}) = \sigma(f(u_i) - f(u_j)),
\]

where \( f(\cdot) \) is the mapping function that maps the node representation to the status and \( \sigma(\cdot) \) is the sigmoid function

\[
 \sigma(x) = \frac{1}{1 + e^{-x}}.
\]

Different mapping functions such as linear functions, non-linear functions and even neural networks can be chosen. In this paper, we adopt a linear function as

\[
 f(u) = w^T u.
\]

where \( w \) is the parameter of the function. We leave non-linear functions or neural networks as one future direction.

The representations and the mapping function \( f(\cdot) \) can be learned by minimizing the negative logarithm of (4.2) as

\[
 (4.6) \quad U, w = \arg \min_{U, w} \sum_{1 \leq i < j \leq N} \log(p(v_{(i)}, v_{(j)}))
 = \arg \min_{U, w} \sum_{1 \leq i < j \leq N} \log(\sigma(f(u_i) - f(u_j))).
\]

4.1.1 Reduce Computational Cost. It is computational expensive to optimize (4.6) w.r.t \( U \) and \( w \) as there are \( N(N - 1)/2 \) pairs in total. To accelerate the training speed, we relax the global ranking constraint.

Figure 2: The descending ordered pagerank score of nodes in BlogCatalog network.

Our relaxation comes from the observation from the distribution of status scores. Figure 2 plots the Pagerank value of the BlogCatalog network [26] where \( x \)-axis is the number of nodes and \( y \)-axis is the PageRank score. We make two observations – (1) many status scores are small; and (2) status scores in the long tail are very close to each other. Hence, the difference of the status scores of two nodes with different rankings in the long tail is very small. It is not useful to address the difference between these rankings. Therefore, we first transform the global status scores into status levels by splitting the scores into \( K \) equal size bins; and then we assume nodes in the same status level share the same status ranking. Instead of rankings 1 to \( N \), now we have \( K \) ranking levels. Then the global ranking constraint is relaxed to preserve status levels of nodes.

To maintain the order between the nodes from different levels, we randomly generate a set of lists \( \mathcal{L} = \{l_i\}_{i=1}^L \). Each list \( l = \{v_{[1]}, v_{[2]}, \ldots, v_{[K]}\} \) consists of \( K \) nodes, one from each level and the subscript of \( v_{[i]} \) represents the level and thus the ranking information of the node. If we preserve the ranking levels of all individual lists, we can preserve the status level globally. Therefore, we assume that all the lists are independent,
and the probability the ranking of all the lists hold can be modeled as:

\[
p_{\text{global}} = \prod_{l \in L} p(l);
\]

where \(p(l)\) is the probability that the order in list \(l\) is maintained, which is defined as

\[
p(l) = \prod_{1 \leq i < j \leq K} p(v_{[i]}, v_{[j]});
\]

With this relaxation, the objective function that we want to minimize can be formalized as follows

\[
O_1(U, w) = -\sum_{l \in L} \sum_{1 \leq i < j \leq K} \log(p(v_{[i]}, v_{[j]}))
= -\sum_{l \in L} \sum_{1 \leq i < j \leq K} \log(\sigma(w^T u_{[i]} - w^T u_{[j]}))
:= -\sum_{l \in L} O_l(U_l, w);
\]

where \(U_l\) is representations for the nodes in list \(l\).

To optimize (4.9), we adopt the Stochastic Gradient Descent method. The procedure of the optimization is described in Algorithm 1. The inputs of this algorithm are the size of the set of random list \(L\), the dimension of the representation \(d\) and the learning rate \(\eta\). Instead of generating the whole random list set beforehand, we generate a list \(l\) in each step as the procedure proceeds in Line 5. We then calculate the derivatives for \(w\) and \(U_l\) and update them by Gradient Descent in Line 6. The generation of the list costs \(O(K)\) time. For each list \(l\), there are \(K \cdot (K - 1)/2\) pairs, which takes \(O(d \cdot K^2)\) time to perform the update, thus, the whole procedure costs \(O(L \cdot d \cdot K^2)\).

**Algorithm 1 Optimization Procedure For GINE**

1: Input: \(L, d, \eta\)
2: Initialize \(d\)-dimension representations \(U\) and \(d\)-dimension parameter \(w\).
3: \(i \leftarrow 0\)
4: while \(i \leq L\) do
5: Generate a list \(l = \{v_{[1]}, v_{[2]}, \ldots, v_{[K]}\}\), which contains one node from each level.
6: Update the representations and parameters by Gradient Descent as follows
   \[
   U_l \leftarrow U_l - \eta \cdot \partial O_l(U_l, w) \over \partial U_l;
   w \leftarrow w - \eta \cdot \partial O_l(U_l, w) \over \partial w;
   \]
7: \(i \leftarrow i + 1\)
8: end while
9: return \(U, w\)

(4.10) using the representations and maximizing it with respect to the representations.

In a similar way, we can view the nodes in a network as the “words”. Then, for a node \(v\), we can regard all the nodes connected to \(v\) as the surrounding context “words”, which can be denoted as \(N(v)\).

Assuming the independence between observing different nodes given a center node \(v\), the probability that \(N(v)\) is the surrounding “context” of node \(v\) can be modeled as:

\[
p(N(v)|v) = \prod_{v_j \in N(v)} p(v_j|v);
\]

where \(p(v_j|v)\) can be modeled using a softmax function as suggested in [11].

\[
p(v_j|v) = \frac{\exp(u^T u_j)}{\sum_{v_i \in V} \exp(u^T u_i)};
\]

where \(V\) is the entire set of nodes in the network and \(u\) and \(u'\) are the source and target representations for node \(v\) as suggested in [11].

To learn the representations for all the nodes in the network, we model this problem as a maximum likelihood problem. In other words, we need to maximize the probability that \(N(v)\) is the “context” of node \(v\) for all the nodes \(v \in V\) with respect to the node representations:

\[
P_{\text{local}} = \prod_{v \in V} p(N(v)|v).
\]

The objective function to be minimized is the negative
logarithm of \((4.13)\)
\[
O_2(U) = - \sum_{v \in V} \sum_{v_j \in N(v)} \log p(v_j | v)
\]
\[
= - \sum_{v \in V} \sum_{v_j \in N(v)} \log \frac{\exp(u^T u'_{j})}{\sum_{v_i \in V} \exp(u^T u'_i)}
\]

4.2.2 The LOG Framework. To learn the representations preserving both the local and global information, the global information needs to be incorporated into the local information preserved embedding model. We combine the two objective \((4.14)\) and \((4.9)\) as:
\[
O(U, U', w, w') =
\]
\[
- \sum_{v \in V} \sum_{v_j \in N(v)} \log \frac{\exp(u^T u'_{j})}{\sum_{v_i \in V} \exp(u^T u'_i)};
\]
\[
- \lambda \sum_{l \in L} \sum_{1 \leq i < j \leq K} \log \sigma(f(u_{ij}, u'_{ij}) - f(u_{ij}, u'_{ij}))
\]
where \(\lambda \in [0, 1]\) is a hyperparameter which indicates the importance of the global information.

The node representations \(U, U'\) and the parameters \(w, w'\) can be obtained by minimizing the objective function \((4.15)\). Note that, the global part of this objective function can be used to extend some existing network embedding methods such as LINE to incorporate the global information.

Since for each node, we have two representations, hence, we need to slightly modify the mapping function \(f\) as
\[
f(u, u') = w^T u + w'^T u'.
\]
After we obtain the source and target representations \(U, U'\), we concatenate them to generate the final node representations.

4.2.3 An Optimization Method for LOG. To optimize \((4.15)\), we adopt Stochastic Gradient Descent method. We view the two terms as two optimization tasks. We regard the local information task as the main task which will always be performed and the global status task will be performed with probability \(\lambda\). The optimization of the global status task has been discussed in subsection 4.1.1, so, we only discuss the optimization of the local term and the joint optimization of the two tasks.

We observe that the minimization of the first term of \((4.15)\) is computational expensive due to the summation over the whole set of nodes \(V\) in \((4.12)\). Thus, we adopt the negative sampling approach proposed in \([11]\) to solve the computational issue in \((4.12)\). By using the negative sampling method, we replace each \(\log p(v_i | v)\) with
\[
g(v, v_i) = \log \sigma(u^T u'_{i}) + \sum_{n=1}^{N_\nu} \log \sigma(-u^T u'_{(n)})
\]
where \(\sigma(x)\) is the sigmoid function and \(N_\nu\) is the number of negative samples. The negative samples are sampled from the node set \(V\) according to the noise distribution \(P(v) \sim d_v^{3/4}\) as proposed in \([11]\), where, \(d_v\) is the in-degree of the node \(v\). The objective function becomes
\[
O(U, U', w, w') =
\]
\[
- \sum_{v \in V} \sum_{v_j \in N(v)} \log \sigma(u^T u'_{j}) + \sum_{n=1}^{N_\nu} \log \sigma(-u^T u'_{(n)})
\]
\[
- \lambda \sum_{l \in L} \sum_{1 \leq i < j \leq K} \log \sigma(f(u_{ij}, u'_{ij}) - f(u_{ij}, u'_{ij}))
\]
\[
:= - \sum_{v \in V} \sum_{v_j \in N(v)} g(v, v_i) - \lambda \sum_{l \in L} O_l(U_i, U'_j, w, w')
\]
The joint optimization procedure for LOG is described in Algorithm 2. The inputs of this algorithm are the representation dimension \(d\), the learning rates for the two tasks \(\eta_1\) and \(\eta_2\), the hyperparameter \(\lambda\) controlling the importance of the global term, the number of negative samples \(N_\nu\) and the number of running epochs \(E\). The representations \(U, U'\) and the parameters \(w, w'\) are initialized in line 2. The algorithm goes through the entire node set \(V\) \(E\) times. In the start of each epoch, the node set is shuffled to achieve better performance of SGD. Lines 7-10 perform the update of the representations corresponding to the local term of \((4.18)\). Lines 11-15 perform the update of the representations and parameters corresponding to the global term of \((4.18)\) with probability \(\lambda\). The update procedure of the local term in lines 8-9 takes \(O(d \cdot N_\nu)\) time, while the update procedure for the global term in lines 11-15 takes \(O(d \cdot K^2)\) time. As the for loop in line 6 goes through all the nodes and the for loop in line 7 goes through all the edges connected to the node, each epoch of procedure take \(O(M \cdot d \cdot N_\nu + N \cdot d \cdot K^2)\) time, where \(M\) is the number of edges and \(N\) is the number of nodes in the network. Therefore, the whole procedure costs \(O(E \cdot (M \cdot d \cdot N_\nu + N \cdot d \cdot K^2))\).

5 Experiments
In this section, we conduct experiments to verify the effectiveness of the proposed algorithms GINE and LOG. We first show that the embeddings learned by GINE and LOG can preserve the global information. Then, we perform link prediction to verify that including the global information can help learn better representations.

To evaluate the effectiveness of the proposed model, we conduct experiments on two network datasets, which have been widely used to evaluate network embedding algorithms \([13, 23]\).

- **BlogCatalog** \(26\) is a network of social relationships provided by blogger authors. This network
In our experiments, we empirically set the number of dimensions $d$ for the representations $U, U'$ and $d$-dimension parameter $w, w'$. The Optimization Procedure for LOG is as follows:

**Algorithm 2** The Optimization Procedure for LOG

1. **Input:** $d, \eta_1, \eta_2, \lambda, N_c, E$
2. Initialize $d$-dimension representations $U, U'$ and $d$-dimension parameter $w, w'$.
3. $i \leftarrow 0$
4. **while** $i \leq E$ **do**
5. Shuffle($\mathcal{V}$)
6. **for** $v \in \mathcal{V}$ **do**
7. **for** $v_j \in N(v)$ **do**
8. Generate $N_c$ negative samples.
9. Update the representations by Gradient Decent as follows
   $$Ψ \leftarrow Ψ - \eta_1 \cdot \frac{∂g(v, v_j)}{∂Ψ};$$
   where $Ψ = \{u, u', u'_n\}, n = 1, \ldots, N_c$ denotes all the involved representations in $g(v, v_j)$.
10. **end for**
11. Generate a random number $p \in [0, 1]$.
12. **if** $p \leq \lambda$ **then**
13. Generate a list $l = \{v_1, v_2, \ldots, v_K\}$, which contains one node from each status level.
14. Update the representations and parameters by Gradient Decent as follows
   $$Φ \leftarrow Φ - \eta_2 \cdot \frac{∂O_l(U, U', w, w')}{∂Φ};$$
   where $Φ = \{U_l, U'_l, w, w'\}$ denotes all the involved representations and parameters in $O_l(U_l, U'_l, w, w')$.
15. **end if**
16. **end for**
17. $i \leftarrow i + 1$
18. **end while**
19. **return** $U, U', w, w'$

consists of 10,312 nodes and 333,983 edges.

- **Flickr** [26] is a network of contacts between users from the photo sharing website Flickr. This network consists of 80,513 nodes and 5,899,882 edges.

In our experiments, we empirically set the number of levels $K$ to 60.

### 5.1 Validating Global Information Preserving

In this subsection, we show that the representations learned by our algorithms can preserve the global information. To visualize the global status of all the nodes in the network, we plot scatter plots for each method. For the BlogCatalog network, the scatter plot of the PageRank score has been shown in Figure 2. In this figure, the nodes are arranged in a descending order based on the PageRank score. For the plots of the other algorithms, we also arrange the nodes in this order. Figures 3a and 3b show the global status preserved by the proposed methods GINE and LOG respectively. Figure 3c and 3d show the global status preserved by DeepWalk and LINE respectively. DeepWalk and LINE do not learn a mapping function during the representation learning, therefore, we perform a linear regression using the learned representations and the original PageRank scores and then use this linear model to calculate the global status for each node. Note that since our models GINE and LOG use a linear mapping function from representations to the status scores, we also use a linear mapping function for DeepWalk and LINE for a fair comparison.

We only show the results in the BlogCatalog network in Figure 3 as the performance in the Flickr network is similar. As shown in Figure 3a, GINE can preserve the global status quite well, and the difference between nodes in different levels are clear. LOG can also roughly preserve the ranking information. Although it is noisier than GINE, the trend has been preserved, and the difference between levels can also be observed clearly, which achieves our goal of loosely preserving the global ranking. Figure 3c and Figure 3d do not show a specific trend, which means the representations learned by DeepWalk and LINE cannot preserve the global information.

### 5.2 Link Prediction

In this subsection, we perform link prediction task to further evaluate the effectiveness of the learned representations. The intuition is that a better embedding algorithm should learn better node representations, which will lead to better link prediction performance.

#### 5.2.1 Experiments Setting

In the link prediction task, a certain fraction of edges are removed from the network, we are supposed to use the remained network to predict whether the “missing” edges exist.

We perform the link prediction on the aforementioned two datasets. For each dataset, we set up three groups of experiments where 20%, 50% and 80% of edges are removed, while at least one edge for each node will remain in the network. After removing the edges, we use the remained network to learn the node representations. Then, to perform link prediction, we need to form representations for an edge or a pair of nodes. We use the element-wise addition of the two node representations as the representation of the edge.

To form the training set, we put all the edges remained in the network as positive samples and then...
sample an equal number of non-connected node pairs as negative samples. The testing set is formed in the same way. After forming the training set and testing set, we train a binary classifier using logistic regression on the training set and perform link prediction on the testing set. In this work, we use accuracy and AUC as the metrics to evaluate the link prediction performance.

5.2.2 Performance Comparison. To evaluate the performance of our algorithms, we compare them with the following representative baselines:

- **LINE** [21]: LINE is a network embedding algorithm which can preserve the first-order and second order proximity. We apply LINE to the networks and obtain the node representations.
- **DeepWalk** [15]: DeepWalk facilitates local information obtained from random walk to perform network embedding. We apply DeepWalk to the networks and obtain the node representations.
- **Handcrafted Feature Based (HFB)**: Given a pair of nodes, we calculate their number of common neighbors, Jaccard coefficient and preferential attachment value [8] as the representations for this node pair.
- **GINE**: This algorithm only uses the global information to learn the node representations.
- **LOG(0)**: This is a variant of our method LOG. We set $\lambda = 0$ so that only local information is used to learn the node representation.

For all the methods except for HFB, we set the dimension to 128. The experiment results are shown in Table 1. For LOG, we set $\lambda = 0.3$, so we denote it as LOG(0.3) in Table 1. The following observations can be made from the results:

- All embedding algorithms obtain better performance than HFB (Handcrafted Feature Based), which supports the advantage of the automated representation learning algorithms for networks.
- Although the performance of GINE is not as good as the other embedding based methods, it provides reasonable results. This shows the importance and effectiveness of the global information.
- The performance of LOG(0) is close to the performance of LINE, which is reasonable as LOG(0) is similar to LINE.
- LOG(0.3) consistently outperforms LOG(0), which further proves the importance of the Global information.
- Our method LOG(0.3) outperforms all the baselines in each setting of the two datasets, which shows that (1) local and global information are complementary; and (2) incorporating them can lead to better presentation learning.

5.2.3 Parameter Analysis. In this subsection, we analyze how the parameters in our model affect the performance of the link prediction task. There are mainly
two parameters in our model, the dimension of the representation $d$ and the hyperparameter $\lambda$ controlling the importance of the global information. We first analyze the stability of the performance of the link prediction task as the dimension $d$ changes. Then, we analyze how the performance changes as $\lambda$ varies.

### 5.2.4 Stability over dimensions

We analyze how the change of dimension affects the performance of the link prediction task. In particular, we set the dimension to $[8, 16, 32, 96, 128, 160, 192]$ with the setting of 50% edges removed and $\lambda = 0.1$. The results are shown in Figure 4, both the accuracy and AUC increase as the dimension gets larger at first and then when the dimension is larger than 96, the performance starts to be stable. The performance is not very sensitive to the change of the dimension after it reaches 96.

![Figure 4: Different dimensions](image)

#### 5.2.5 Stability over $\lambda$ values

We analyze how different values of $\lambda$ affect the link prediction performance. In particular, we set the $\lambda$ to $[0, 0.001, 0.01, 0.1, 0.3, 0.5, 0.7, 0.9]$ with the setting of 50% edges moved and dimension 128. The results are shown in Figure 5. Both the accuracy and the AUC score increase as $\lambda$ gets large, which indicates the effectiveness of including global information. The performance starts to be stable when $\lambda$ is large or equal to 0.3.

So the performance of link prediction is very sensitive to the change of $\lambda$ when it is small, which shows the effectiveness of the including global information. When the $\lambda$ is large, it is likely that enough global information has been absorbed in the representations and the performance cannot be improved by further increasing the value of $\lambda$.

![Figure 5: Stability over $\lambda$ values](image)

### 6 Discussion and Future Work

In this paper, we propose a principal way to model global information for network embedding. Based on this, we further propose a novel network embedding framework LOG, which can preserve both local and global information. The results of experiments show that LOG can well preserve the global information. The performance of link prediction further demonstrates that the inclusion of the global information can lead
to better representation learning.

In this paper, we use a linear function to model the global information. However, more complex functions can be adopted. The global status score we use in this paper is PageRank, while other global status scores could also be used. We will investigate these different possibilities in the future.

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