Note on de Sitter vacua from perturbative and non-perturbative dynamics in type IIB/F-theory compactifications

Vasileios Basiouris[1] George K. Leontaris[2]

Physics Department, University of Ioannina
45110, Ioannina, Greece

Abstract

The properties of the effective scalar potential are studied in the framework of type IIB string theory, taking into account perturbative and non-perturbative corrections. The former modify the Kähler potential and include $\alpha'$ and logarithmic corrections generated when intersecting $D7$ branes are part of the internal geometric configuration. The latter add exponentially suppressed Kähler moduli dependent terms to the fluxed superpotential. The possibility of partial elimination of such terms which may happen for particular choices of world fluxes is also taken into account. That being the case, a simple set up of three Kähler moduli is considered in the large volume regime, where only one of them is assumed to induce non-perturbative corrections. It is found that the shape of the F-term potential crucially depends on the parametric space associated with the perturbative sector and the volume modulus. De Sitter vacua can be obtained by implementing one of the standard mechanisms, i.e., either relying on D-terms related to $U(1)$ symmetries associated with the $D7$ branes, or introducing $D3$ branes. In general it is observed that the combined effects of non-perturbative dynamics and the recently introduced logarithmic corrections lead to an effective scalar potential displaying interesting cosmological and phenomenological properties.

[1] E-mail: v.basiouris@uoi.gr
[2] E-mail: leonta@uoi.gr
1 Introduction

Recent swampland conjectures \cite{1,2,3} have sparked an interesting discourse regarding the nature of string theory vacua \cite{4}. The “underlying essence” of these hypotheses is that the string landscape does not contain stable de Sitter (dS) vacua. However, the existence of the latter is essential in string motivated models and their cosmological predictions, providing in particular the groundwork for a successful cosmological inflationary scenario. Furthermore, a positive cosmological constant would be desirable since it is in agreement with the scenario of dark energy. The latter is considered as the most promising candidate to explain the observed accelerated expansion of the universe.

It was asserted by several authors, that dS minima are in principle possible in string theory, when the physics implications of perturbative and non-perturbative dynamics are taken into account. Indeed, quantum corrections in string theory play a key rôle in shaping the final form of the effective theory. In particular, they are indispensable for a non-vanishing scalar potential while at the same time they contribute decisively to the stabilisation of the Kähler moduli fields. In many cases, however, a straightforward approach to incorporate these effects ends up to an effective theory with anti de-Sitter (AdS) vacuum and thereby an appropriate mechanism is necessary to uplift it to a dS vacuum. Generally, the last two decades or so, in order to tackle the problems of moduli stabilisation and generate dS vacua, various sources of corrections were considered including non-perturbative objects such as D-branes, string loop effects etc. A great deal of endeavours to stabilise the Kähler moduli and construct stringy dS vacua has been focused on non-perturbative corrections \cite{6} and large volume compactifications \cite{7} generated by Euclidean D3 branes \cite{8}. The former use an uplift mechanism based on anti-D3 branes (D3 for short) to generate a dS vacuum. In the large volume compactification scenario (LVS), (see \cite{19} for a recent perspective) leading order $\alpha'$ perturbative corrections \cite{20} were also included in the Kähler potential and it has been argued that they are dominant at large as well as small volumes. The present work will be in the framework of type IIB string compactifications with D-branes and fluxes \cite{21} where such mechanisms are available.

Recently, it was shown that the issues of moduli stabilisation and de Sitter vacua can be successfully resolved with contributions arising only from perturbative string-loop corrections \cite{23}. These are related to higher derivative terms in the effective string action which generate localised Einstein-Hilbert terms and, in geometric configurations involving $D7$-branes, induce logarithmic corrections via string loop effects. It has also been shown that cosmological inflation is successfully implemented \cite{24}.

Quantum corrections of this type are standard in the presence of D-branes and were also studied in the past \cite{25,26} although in different contexts. Also, in \cite{27} it was shown that invariance of the effective classical action under $SL(2,R)$ transformations implies logarithmic corrections to the Kähler potential which depend on the untwisted Kähler moduli. Such contributions, break the no-scale structure of the Kähler potential and lead to an effective theory with all Kähler moduli stabilised. Furthermore, D-term contributions related to the abelian symmetries of the intersecting $D7$-branes, work as an “uplift” mechanism and ensure the existence of de Sitter vacua.

In the present work both perturbative logarithmic corrections in the Kähler potential as well as non-perturbative contributions to the superpotential are combined, and stabilisation of the moduli fields and the properties of the effective potential are investigated. With regard to the non-perturbative sector, an interesting and non-trivial case will be considered where only a subset of the Kähler moduli are involved in the superpotential leaving the remaining degrees of freedom to be stabilised by the quantum corrections breaking the no-scale structure

\footnote{For related reviews and further references see \cite{4} and \cite{5}.}

\footnote{For recent work on KKLT see \cite{8}-\cite{16}. For cases suggesting small $\mathcal{V}_0$ values see \cite{17}, \cite{18}.}

\footnote{For general review regarding four-dimensional compactifications with D-branes and fluxes see \cite{21}.}
of the Kähler potential. As described in [28] (see also [29]), such a situation can occur if the chosen world-volume fluxes lift certain fermionic zero-modes, whose presence would prevent the generation of non-perturbative superpotential terms. As a working example in the present analysis, a composition of three Kähler moduli is considered with only one of them involved in the non-perturbative part of the superpotential. Extensions to more than three moduli are straightforward, although the analysis becomes more involved and will not be considered in this letter.

Section 2 begins with a short description of quantum corrections where the main focus is on the logarithmic ones which prevail in a set up involving $D7$ branes. The analysis of the minimisation conditions on the F-term potential is presented in section 3 while in section 4 D-term contributions from abelian factors related to the $D7$ brane configuration are discussed. Then, the implementation of the uplift mechanism through D-terms after some of the Kähler moduli obtain their critical values is explained. A summary of the work and the main conclusions can be found in section 5.

2 Quantum corrections

The aim of this work is to propose a solution to the moduli stabilisation problem in II-B string theory, combining both perturbative and non-perturbative corrections. Furthermore, the necessary conditions for the existence of de Sitter minima will be investigated within the same string theory framework. The notation for various fields used in the subsequent analysis is as follows: The dilaton and Kalb-Ramond fields are denoted with $\phi$ and $B_2$ respectively while the various $p$-form potentials with $C_p$, $p = 0, 2, 4$. The $C_0$ potential and the dilaton field $\phi$ are combined in the usual axion-dilaton combination:

$$ S = C_0 + i e^{-\phi} \equiv C_0 + \frac{i}{g_s} . $$

Finally, $z_a$, $a = 1, 2, 3, \ldots$ stand for the complex structure (CS) moduli and $T_i$, $i = 1, 2, 3, \ldots$ for the Kähler fields. The fluxed induced superpotential, $W_0$, at the classical level is

$$ W_0 = \int G_3 \wedge \Omega(z_a) , $$

(1)

with $\Omega(z_a)$ being the holomorphic (3,0)-form and $G_3 := F_3 - S H_3$, where the field strengths are $F_p := d C_{p-1}$, $H_3 := d B_2$. The perturbative superpotential $W_0$ is a holomorphic function and depends on the axion-dilaton modulus $S$, and the CS moduli $z_a$. Thus, at the classical level, the supersymmetric conditions, $D_{z_a} W_0 = 0$ and $D_S W_0 = 0$ fix the moduli $z_a, S$, however, the Kähler moduli remain completely undetermined. At the same order, the Kähler potential depends logarithmically on the various fields, including the Kähler moduli

$$ K_0 = - \sum_{i=1}^{3} \ln(-i(T_i - \bar{T}_i)) - \ln(-i(S - \bar{S})) - \ln(-i \int \Omega \wedge \bar{\Omega}) . $$

(2)

Then, the effective potential is computed using the standard formula

$$ V_{\text{eff}} = e^K \left( \sum_{I,J} D_I W_0 K^{IJ} D_J W_0 - 3 |W_0|^2 \right) . $$

(3)

In the absence of any radiative corrections, the latter vanishes identically due to supersymmetric conditions and the no scale structure of the Kähler potential. Hence, it is readily inferred that in order to stabilise the Kähler moduli it is necessary to go beyond the classical level. In fact,
when quantum corrections are included they break the no-scale structure of the Kähler potential and presumably a non-vanishing contribution in the scalar potential, i.e. $V_{\text{eff}} \neq 0$, is feasible.

As already stated, in the quest for a stable dS minimum in effective string theories, the rôle of perturbative as well as non-perturbative corrections will be analysed. Furthermore it should be mentioned that this work takes place in the framework of type IIB string theory compactified on a 6-d Calabi-Yau (CY) manifold $X_6$, and the 10-d space is $M_4 \times X_6$ with the compact space factorised into three 2-tori $T_i^2$. In this framework a geometric configuration consisting of three intersecting $D7$ branes is considered, while the internal volume $V$ is expressed in terms of the imaginary parts $v_i$ (the two-cycle volumes) of the Kähler moduli

$$V = \frac{1}{6} k_{ijk} v^i v^j v^k, \quad v^i = -\text{Im}(T^i),$$

where $k_{ijk}$ are intersection numbers. The $v^i$ are related to 4-cycle volumes $\tau_i$ as follows:

$$\tau_i = \frac{1}{2} k_{ijk} v^j v^k.$$

In the present case it is simply assumed that $V = v^1 v^2 v^3$ or $V = \sqrt{\tau_1 \tau_2 \tau_3}.$

After these preliminaries, in the remaining of this section the various types of corrections will be presented.

Starting with non-perturbative corrections of the superpotential, in principle, all three Kähler moduli considered in this model may contribute. In this case the superpotential takes the form

$$W = W_0 + \sum_{i=1}^{3} A_i e^{-a_i \tau_i},$$

where $W_0 = \int G_3 \wedge \Omega$ is the tree-level superpotential. The second term in the right-hand side of (7) is the non-perturbative part. The constants $A_i$ in general depend on the complex structure moduli and the $a_i$ parameters are assumed to be small (for example in the case of gaugino condensation in an $SU(N)$, they are of the form $\frac{2\pi}{N}$). However, it maybe possible that only some of the Kähler moduli fields have non-vanishing non-perturbative contributions. This may happen, for example, when two $D7$ branes (or stacks of branes) wrap non-spin divisors and non-zero gauge fluxes are switched on for each one of them [28]. This would generate chiral matter that could eliminate the gaugino condensation on the given stack, and the corresponding non-perturbative terms of the superpotential [29]. In what follows, it is supposed that only the $\tau_1$ modulus induces a non-vanishing contribution in the non-perturbative part of the superpotential, thus

$$W = W_0 + Ae^{-a_1 \tau_1}.$$

Before proceeding to the next step, some comments are due with respect to (w.r.t.) the reliability of the instanton correction and the specific choices in the subsequent analysis. This type of corrections originates from the presence of Euclidean D3-branes wrapping four-cycles in the base of the compactification [31]. First of all, in order the supergravity approximation to be valid, the condition $\tau_1 \geq 1$ should be fulfilled. Two main reasons are in favor of this argument. First, shrinking one direction to small volume leads to highly curved Kähler cones or orbifolds where the effective approximation is at stake. Second, the logarithmic correction that has been added in the Kähler potential requires large transverse directions $\tau_i$. We come back to this issue in section 3.2.

Next, quantum corrections to the Kähler potential will be discussed, starting with the $\alpha'^3$ contributions, which, in the large volume limit imply a redefinition of the dilaton field $e^{-2\phi} = e^{-2\phi_{10}(V + \xi)} = e^{-\frac{1}{2} \phi_{10}(\hat{V} + \hat{\xi})}.$

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The last expression on the right-hand side of (9) holds in the Einstein frame and the volume is written in terms of the imaginary parts of the Kähler deformations $T^k$ as follows
\[ \mathcal{V} = \frac{1}{3!} \kappa_{ijk} v^i v^j v^k, \quad v^k = -\text{Im}(T^k) = \hat{v}^k e^{\frac{1}{2} \phi_{10}}. \] (10)

The modifications in the Kähler potential correspond to a shift of the volume by a constant $\xi$ which is determined in terms of the Euler characteristic $\xi = \frac{-\zeta(3)}{4(2\pi)^3} \chi$.

The origin of the second type of corrections comes from higher derivative terms which give rise to multigraviton scattering in string theory. The leading terms appearing in the 10-dimensional effective action are proportional to $R^4$, where $R$ is the Ricci scalar. After compactification to four dimensions $\mathcal{X}_{10} \rightarrow M_4 \times \mathcal{X}_6$, they induce a new Einstein-Hilbert (EH) term, multiplied by the Euler characteristic of the manifold. The one-loop amplitude of the on-shell scattering involving four gravitons has been worked out in [32–39] where it has been shown that the ten-dimensional action reduces to
\[ S_{\text{grav}} = \frac{1}{(2\pi)^7 \alpha'} \int_{M_4 \times \mathcal{X}_6} e^{-2\phi} R_{(10)} - \frac{\chi}{(2\pi)^3 \alpha'} \int_{M_4} \left( -2\zeta(3)e^{-2\phi} \pm 4\zeta(2) \right) R_{(4)}, \] (11)

where $R_{(4)}$ denotes the reduced Ricci scalar in four dimensions, the ± signs refer to the type IIA/B theory respectively, and the Euler characteristic is defined as
\[ \chi = \frac{3}{4\pi^3} \int_{\mathcal{X}_6} R \wedge R \wedge R. \] (12)

From (11), it is observed that a localised EH term is generated with a coefficient proportional to $\chi$ defined in (12). Consequently it is inferred that this term is possible only in four dimensions. In the geometry of the bulk space, the $R_{(4)}$ EH terms of (11) correspond to vertices at points where $\chi \neq 0$, and as such, they emit gravitons and Kaluza-Klein (KK) excitations in the six-dimensional space. Furthermore, in the presence of $D7$ branes which are an essential ingredient of the internal space configurations in type IIB and F-theory, new types of quantum contributions emerge. It is found thereby that the exchange of closed string modes between the EH-vertices and $D7$ branes and $O7$-planes give rise to logarithmic corrections. These take the form
\[ \frac{4\zeta(2)}{(2\pi)^2} \chi \int_{M_4} \left( 1 - \sum_k e^{2\phi T_k \ln(R_{ik}/\nu)} \right) R_{(4)}. \] (13)

In the above, $T_k$ is the tension of the $k^{th}$ 7-brane, $R_{ik}$ stands for the size of the two-dimensional space transverse to the brane, and $\nu$ is a ‘width’ related to an effective ultraviolet cutoff for the graviton KK modes propagating in the bulk [36].

### 3 The effective potential

Assembling the above ingredients the effective potential is readily constructed. To facilitate the presentation, firstly, the F-term part will be discussed and afterwards the contributions from D-terms will be included.

#### 3.1 The Kähler potential and the superpotential

Considering a geometric configuration of three intersecting $D7$ branes, the corrections (13) imply that the contributions involving the Kähler moduli are of the form
\[ \delta = \xi + \sum_{k=1}^{3} \eta_k \ln(\tau_k). \]
For simplicity it is assumed that all \( D7 \) branes have the same tension \( T = e^{-\phi}T_0 \), so the order one coefficients \( \eta_k \) and \( \xi \) are given by

\[
\eta_k \equiv \eta = -\frac{1}{2}g_s T_0 \ ; \ \xi = -\frac{\chi}{4} \times \begin{cases} \frac{\pi^2}{3} g_s^2 & \text{for orbifolds} \\ \zeta(3) & \text{for smooth CY} \end{cases}
\]

and \( \tau_k \) are defined in [5]. Thus, the Kähler potential takes the form

\[
K = -2 \ln \left( \sqrt{\tau_1 \tau_2 \tau_3} + \xi + \eta \ln (\tau_1 \tau_2 \tau_3) \right) \equiv -2 \ln (V + \xi + \eta \ln V) \cdot
\]

The covariant derivative of the superpotential w.r.t. the Kähler modulus is defined in the usual manner, i.e., \( D_{\tau_1} W = \partial_{\tau_1} W + W \partial_{\tau_1} K \). Working in the large volume limit, terms proportional to \( \xi \) and \( \eta \) coefficients compared to the volume \( V \) are ignored. Then it is readily found that

\[
D_{\tau_1} W \approx -e^{-\alpha \tau_1} \left( \alpha A + \frac{A + W_0 e^{\alpha \tau_1}}{2\tau_1} \right) \cdot
\]

The corresponding supersymmetric condition, \( D_{\tau_1} W = 0 \), fixes the value of the modulus \( \tau_1 \) in terms of the tree-level superpotential \( W_0 \) (determined by the choice of the fluxes) and the coefficients \( \alpha, A \) - related to non-perturbative contributions. Thus, the vanishing of the derivative (16) yields

\[
\tau_1 = -\frac{1 + 2w}{2\alpha} ,
\]

where \( w \) represents either of the two branches \( W_0, W_{-1} \), of the Lambert \( W \)-function

\[
w = W_{0/-1}(\frac{\gamma}{2\sqrt{e}}) .
\]

In (18), the convenient definition has been introduced

\[
\gamma = \frac{W_0}{A} .
\]

Real values of the solution are compatible with the bound \( \gamma \geq -2e^{-1/2} \approx -1.213 \) for both branches. For the “lower” branch \( W_0 \), equation (17) implies the constraint \( \alpha \tau_1 \leq 1/2 \). Requiring also \( \alpha \tau_1 > 0 \) it is found that the ratio \( \gamma = W_0/A \) is confined in the region:

\[
-1.213 \lesssim \gamma \leq -1 \cdot
\]

This solution is depicted with the blue curve in figure 1. The corresponding regions for the “higher” branch \( W_{-1} \), depicted with the orange curve in figure 1, are

\[
-1.213 \lesssim \gamma \leq 0 ,
\]

and \( \alpha \tau_1 \in [\frac{1}{2}, \infty) \).

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6We denote with calligraphic letters \( W_0, W \) the tree-level and corrected superpotential and reserve the symbols \( W, W_0, W_{-1} \) for the Lambert \( W \)-function.
Figure 1: Plot of solution (17) for $\alpha\tau_1$ as a function of the ratio $\gamma = \frac{W_0}{\tilde{A}}$. The orange (upper) and blue (lower) curves represent the $W_{-1}$ and $W_0$ branches, respectively. Acceptable values ($\alpha\tau_1 > 0$) for the blue curve are compatible only with its section satisfying $\gamma < -1$.

3.2 F-term potential with quantum corrections

The F-term scalar potential is computed by inserting (3) into (2). This yields a rather complicated formula which is not very illuminating, however, in the large volume limit it suffices to expand it w.r.t. the small parameters $\eta$ and $\xi/\sqrt{V}$ and obtain a simplified form. Thus, without loosing its essential features, in this approximation the potential is written as a sum of three parts, as follows:

$$V_F \approx V_{F1} + V_{F2} + V_{F3}.$$  

The various parts of the RHS in (22) are given by

$$V_{F1} = \frac{3}{2} \frac{W_0^2}{V^3} \frac{\xi - 2\eta(4 - \ln V)}{V^3} - 9W_0^2 \frac{\xi \eta \log(V)}{V^4},$$

$$V_{F2} = 4 \frac{\alpha\tau_1}{V^2} \tilde{A}(\tilde{A} + a\tau_1 \tilde{A} + W_0),$$

$$V_{F3} = \tilde{A}(\tilde{A} f + W_0 g).$$  

where $\tilde{A} = e^{-\alpha\tau_1} A$ and $O(\frac{1}{V^7})$ or higher terms in the expansion are ignored. Also

$$f = \frac{3\xi - 8\eta(2\alpha\tau_1(2\alpha\tau_1 + 3) + 3) - 4\xi \alpha\tau_1 (\alpha\tau_1 + 1) - 2\eta(2\alpha\tau_1 - 1)(2\alpha\tau_1 + 3) \log V}{2V^3},$$

$$g = \frac{(3 - 2\alpha\tau_1)(\xi + 2\eta \log(V)) - 24\eta(1 + \alpha\tau_1) - 6\eta \xi(3 - 2\alpha\tau_1) \log V + 2\alpha\tau_1}{V^4}.$$  

In the above all three Kähler moduli $\tau_i$ are expressed in terms of the volume $V$ with $\tau_1$ being considered at its critical value $\tau_1^{cr}$ given in (17), as it is stabilised from the supersymmetric conditions imposed on the superpotential. Therefore, only the two of them, namely $\tau_2$ and $\tau_3$ are left undetermined which appear only in the combination $\tau_2 \tau_3 = V/\tau_1^{cr}$.  

At this point, it is worth clarifying the origin of the components (23). The term $V_{F1}$ is derived solely from the perturbative string loop corrections (13) due to the localised EH terms, and enter in the Kähler potential (15). Indeed, switching off the non-perturbative corrections, i.e. setting $\gamma = W_0/\tilde{A}$, the potential gets dominated by $V_{F1} \approx 3W_0^2 \xi - 9W_0^2 \xi \eta \log(V)$ which is reproduced in (22) to leading order in $\eta$. From now on, we drop "$\tau_1^{cr}$" and write just $\tau_1$ for simplicity.
\[ w = W_0 \left( \frac{\alpha \tau}{2\sqrt{e}} \right) \]

\[ w = W_{-1} \left( \frac{\alpha \tau}{2\sqrt{e}} \right) \]

| Branch       | \( \alpha \tau \) | \( \gamma \) | \( w \) | \( \epsilon \) |
|--------------|-----------------|------------|--------|-------------|
| \( w = W_0 \left( \frac{\alpha \tau}{2\sqrt{e}} \right) \) | 0               | -1        | \(-\frac{1}{2}\) | 0             |
| \( w = W_{-1} \left( \frac{\alpha \tau}{2\sqrt{e}} \right) \) | \( \infty \) | 0         | \(-\infty\) | 2             |

Table 1: The range of the various parameters used in the analysis.

\( A = 0 \), the only term remaining in (23) is the \( V_{F_1} \) component which is identified with the one given in [10] where only perturbative corrections are studied. Setting \( \eta \) and \( \xi \) equal to zero, the only term that remains is the second component, \( V_{F_2} \). This contribution comes exclusively from the non-perturbative corrections which were included in the superpotential. Finally, the third component \( V_{F_3} \) is a mixing term and it is non-vanishing only when both perturbative and non-perturbative corrections are present.

As an additional check with regard to the non-perturbative part, the appropriate limit of (23) is taken to reproduce the already known results in the literature [6]. Indeed, for \( \eta = \xi = 0 \) the scalar potential becomes

\[ V_{F_2} = \frac{4e^{-2\alpha \tau}A}{\tau_2 \tau_3} (e^{\alpha \tau} W_0 + A + \alpha \tau_1 A) . \]  

Solving (16) w.r.t. the \( W_0 \), it is found that:

\[ W_0 = -Ae^{-\alpha \tau}(1 + 2\alpha \tau_1) . \]

Substituting in (24) while putting \( \tau_3 \to \tau, \tau_2 \to \tau, \tau_1 \to \tau \) the result is

\[ V_{\text{min}} = -\frac{4e^{-2\alpha \tau}A^2}{\tau} , \]

which (up to numerical factor related to the multiplicity of the Kähler moduli) coincides with the solution of [6].

To proceed with the minimisation of the scalar potential (23), a more convenient form will be worked out. To this end, the following parameter is introduced

\[ \epsilon = \frac{2w + 1}{w} . \]

Furthermore, for later convenience, the range of the various parameters defined up to this point for the two branches of the solution are shown in Table 1. As already noted, in the LVS regime it would be more suitable to have large directions given by the lower branch \( W_{-1} \). However, these solutions represent instanton corrections, and as it is obvious, the \( W_0 \) branch is a strongly coupled region, where higher order corrections should be taken into account. For the reasons discussed above and for the correctness of the effective approach, from now on only the \( W_{-1} \) branch will be considered as the solution for the \( \tau_1 \) modulus.

Using the above definitions, and the identities \( 2w = \gamma e^{\alpha \tau} = -(2\alpha \tau + 1) \) resulting from (17), (19) the F-term potential (22) can be cast in the following compact form

\[ V_F \approx (\epsilon W_0)^2 \left( \frac{2\xi - V + 4\eta (\log V - 1)}{4V^3} - \eta \xi \frac{3\log V - 1}{V^4} \right) + O \left( \frac{1}{V^5} \right) . \]

In the present approximation, valid in the large volume limit, it is observed that the parameters associated with the non-perturbative effects appear in the F-term potential as an overall positive-definite factor \( \epsilon^2 \) where \( \epsilon \) is defined in (27). Thus, the shape of \( V_F \) is controlled by the second

\[8\] The current understanding of the non-perturbative physics prevent a complete study of the other branch. A way of treating instanton corrections from \( D3 \)-branes is presented in [11].
factor which exhibits the volume dependence and involves the parameters $\xi$ and $\eta$ coming from the perturbative corrections in the Kähler potential. Indisputably, the properties of the potential depend decisively on the signs of $\xi, \eta$ given in (12) which convey topological and geometric information of the compactification manifold. For closed orientable smooth manifolds and the particular D7-branes set up [23] in the present study the choice $\chi < 0, \xi > 0$ will be adopted.

Then, dropping the subleading terms of order $\propto \frac{1}{V^4}$ and higher in the large volume, and requiring the vanishing of the first derivative, it is found that the volume at the minimum of the potential is given by

$$V_{\text{min}} = -6\eta W_0 \left(-\frac{1}{6\eta} e^{\frac{4}{3} - \frac{\xi}{2\eta}} \right),$$

(29)

where $W_0$ is the Lambert W-function. Substituting $V_{\text{min}}$ into the second derivative yields:

$$\frac{d^2 V_F}{dV^2} = (\xi W_0)^2 \frac{V - 6\eta}{2V^5}.$$  

(30)

Hence, a minimum exists as long as $V \geq 6\eta$ which is obviously true in the large volume regime. Nonetheless, this can be naturally uplifted to a dS minimum, when D-term contributions are taken into account.

4 De Sitter spacetime from D-terms

Various mechanisms have been proposed to uplift a supersymmetric AdS vacuum and obtain a metastable dS minimum and thus, a positive cosmological constant. Amongst others, these include $D^3$ branes, and nilpotent superfields [42]. In [40] it was proven that it is sufficient to consider the D-term contributions associated with $U(1)$ factors which arise in the presence of the intersecting $D7$ branes already included in the geometric configuration. Then, the generic form of the corresponding D-term potential can be approximated by [40]

$$V_D = \sum_{i=1}^{3} \frac{d_i}{\tau_i} \left(\frac{\partial K}{\partial \tau_i}\right)^2 \approx \sum_{i=1}^{3} \frac{d_i}{\tau_i^2} \equiv \frac{d_1}{\tau_1^2} + \frac{d_3}{\tau_3^2} + \frac{d_2}{\tau_1^2 \tau_3^2} V^6,$$

(31)

where, in the last expression the modulus $\tau_2$ has been traded with the internal volume modulus $V$, i.e., $\tau_2 = V^2/(\tau_1 \tau_3)$. Thus, the effective potential being the sum of the (28) and (31), $V_{\text{eff}} = V_F + V_D$ is given as a function of $\tau_1, \tau_3$ and $V$:

$$V_{\text{eff}} \approx -(\xi W_0)^2 \frac{V - 2\xi + 4\eta(1 - \log(V))}{4V^3} + \frac{d_1}{\tau_1^2} + \frac{d_3}{\tau_3^2} + \frac{d_2}{\tau_1^2 \tau_3^2} V^6.$$  

(32)

Firstly, let’s consider here that the $\tau_1$ modulus acquires its critical value already determined by the condition (17). Since $d_1 > 0$, the corresponding D-term automatically operates as a natural uplift mechanism of the potential, $V_{\text{up}} \equiv \frac{d_1}{\tau_1^2}$. Therefore, in addition to the volume there is only one more variable left, namely the modulus $\tau_3$. Then, the minimisation condition yields

$$\tau_3 = \left(\frac{d_3}{d_2}\right)^{\frac{1}{6}} \frac{V}{\tau_1^2}.$$  

(33)

Substituting this in (17) and minimising the resulting effective potential w.r.t. the volume, it is found that

$$V_{\text{eff}}|_{\tau_3} = -(\xi W_0)^2 \frac{V - 2\xi + 4\eta(1 - \log(V))}{4V^3} + \frac{2d_1}{\tau_1^2} V^{3/2} + \frac{d_1}{\tau_1^2}.$$  

(34)
The volume at the minimum is given now by

\[ V_{V'} = 0 = -6\eta W_0 \left( -\frac{1}{6\eta} e^{\frac{4}{3} - \frac{\zeta}{2\eta}} \right) , \quad (35) \]

where the following new parameters have been introduced

\[ \zeta = \xi + 4\delta, \quad \delta = \frac{d\tau_1^{3/2}}{(\epsilon W_0)^2}, \quad d = \sqrt{d_2 d_3} . \]

The value of the potential at the minimum is obtained by substituting (35) in (34). Requiring a dS solution, the following bound for the uplift parameter \( d_1 \) is derived

\[ d_1 > (\epsilon W_0)^2 \tau_1^3 V_{\text{min}} - 4\eta \frac{\tau_1^3}{12V_{\text{min}}^2} \approx (\epsilon W_0)^2 \frac{\tau_1^3}{12V_{\text{min}}^2} . \quad (36) \]

Clearly, in the large volume regime, this is easily satisfied even for small values of the coefficient \( d_1 \). In figures 2, 3 plots of \( V_{\text{eff}} \) vs the volume modulus \( V \) are shown for various values of the parameters obeying the appropriate constraints discussed in the analysis. Figure 4 shows the dS minimum of the potential vs \( \tau_3 \) and the volume modulus.

Figure 2: Plot of \( V_{\text{eff}} \times 10^6 \) potential vs the volume modulus for \( \epsilon W_0 = 1.9, \tau_1 = 40, \eta = -1/2, d = 2.35 \times 10^{-1}, d_1 = 0.2 \) and three values of the topological parameter \( \xi \).

Figure 3: Plot of \( V_{\text{eff}} \times 10^6 \) potential vs the volume modulus for \( \epsilon W_0 = 1.9, \tau_1 = 40, \eta = -1/2, \xi = 100, d_1 = 0.2 \) for three values of the D-term coefficient \( d \).
Figure 4: A 3D Plot of $V_{\text{eff}}$ potential vs the moduli $V, \tau_3$ where its dS minimum is displayed for the parameter values $cW_0 = 1.9, \xi = 50, \tau_1 = 40, d_2 = d_3 = 0.235, \eta = -0.5, d_1 = 0.2$.

At the level of the effective model, the properties of the scalar potential have significant phenomenological and cosmological implications. In this respect, the shape of $V_{\text{eff}}$ as shown in figure 3 looks promising in accommodating slow roll cosmological inflation and it is a theme worth contemplating in a future publication.

Next, an alternative approach is considered, where the potential is minimised w.r.t. both moduli $\tau_1, \tau_3$ as well as the volume $V$. Proceeding as in [40], it is found that the $\tau_{1,3}$ moduli as stabilised at

$$
\tau_k = \left(\frac{d_k}{d} V^2\right)^{\frac{1}{3}}, \ k = 1, 3, \text{ where } d = (d_1 d_2 d_3)^{\frac{1}{3}}
$$

(37)

It is worth noting that comparison of (37) and (17), for fixed $\gamma$ implies a relation among the D-term coefficient $d$ and the parameter $\alpha$.

The potential takes the form

$$
V_{\text{eff}}|_{\tau_{1,3}} = -(\epsilon W_0)^2 \frac{V - 2\xi + 4\eta(1 - \log V)}{4V^3} + \frac{3d}{V^2}
$$

(38)

At the minimum of the potential the volume modulus takes the value

$$
V_{V'=0} = \frac{6\eta(\epsilon W_0)^2}{12d - (\epsilon W_0)^2} W_0 \left(\frac{12d - (\epsilon W_0)^2}{6\eta(\epsilon W_0)^2} e^{\frac{4}{3} - \frac{\xi}{\eta}}\right)
$$

(39)

As in the previous case, the following two constraints are imposed: i) the argument of the $W_0$ function must be larger than $-1/e$ and ii) the potential at the minimum must be positive. Once these restrictions are implemented, the ratio $r = \frac{d}{(\epsilon W_0)^2}$ of the $F$- and $D$-term coefficients is found to be bounded in the region

$$
\frac{1}{12} - \frac{\eta}{3V_{\text{min}}^2} \leq r \leq \frac{1}{12} - \frac{\eta}{2} e^{\frac{4}{3} - \frac{\xi}{\eta}}, \quad r = \frac{d}{(\epsilon W_0)^2}
$$

(40)

For large volumes, the above bounds allow only a tiny region in the vicinity of 1/12. Given the ratio $r$, the inequalities (40) imply also an upper bound on $\xi$:

$$
-\frac{\eta}{3V_{\text{min}}^2} < -\frac{\eta}{2} e^{\frac{4}{3} - \frac{\xi}{\eta}} \Rightarrow \xi < 2|\eta| \left(\ln \frac{6|\eta|}{12r - 1} - \frac{7}{3}\right)
$$

(41)
In figure 5 the potential is plotted vs the volume for the set of parameters $\epsilon \mathcal{W}_0 = 1.9, \xi = 10, \eta = -1$ and three values of the D-term coefficient $d$. A dS minimum is obtained for a very short range of $d$.

![Figure 5: Plot of $V_{eff} \times 10^{10}$ potential vs the volume modulus for $\epsilon \mathcal{W}_0 = 1.9, \eta = -1, \xi = 10$ for three values of the D-term coefficient $d$.](image)

5 Conclusions

In this letter the issues of Kähler moduli stabilisation and the existence of de Sitter spacetime vacua in the string landscape have been revisited within the framework of type IIB string theory. The analysis has been conducted in the context of a geometric background of three intersecting seven-branes and three Kähler moduli $\tau_1, \tau_2, \tau_3$ associated with the large codimension-two volumes transverse to each one of them. More precisely the implications of the combined effects of the well-known non-perturbative contributions in the fluxed superpotential and the recently studied perturbative logarithmic corrections [23] in the Kähler potential were considered. Regarding the former corrections, only one modulus $\tau_1$ is taken into account in the present study. The origin of the latter come from string loop corrections when closed strings emitted from localised Einstein-Hilbert terms, $R(4)$, traverse the codimension-two volume towards the seven-brane probes. Such $R(4)$ terms come from the $R^4$ corrections of the effective ten-dimensional string action and, remarkably, appear only in four spacetime dimensions.

As in the previous studies with perturbative corrections [40], de Sitter vacua are possible only due to D-terms originating from intersecting $D7$-branes and their associated $U(1)$ factors. For finite values of the volume modulus the general shape of the effective potential $V_{eff}$ near the minimum is similar with those found in the study including only perturbative corrections. A substantial difference, however, arises in their asymptotic behavior. In the perturbative case the dS potential possesses a minimum at some finite $V_{\text{min}}$ as well as a (local) maximum at a finite value $V_{\text{max}} > V_{\text{min}}$ before it approaches zero at $V \to \infty$. On the contrary, for the case studied in this work, for values $V > V_{\text{min}}$ the potential approaches asymptotically this maximum at $V \to \infty$. This is due to the fact that the value of the Kähler modulus $\tau_1$ appearing in the non-perturbative superpotential is fixed by the supersymmetric conditions. Consequently, the corresponding D-term enacts as a constant uplift $V_{up}$ turning an AdS vacuum to a dS one, similar to the Fayet-Iliopoulos term considered in a supergravity context [43][44]. Furthermore, a separate study is conducted where the modulus $\tau_1$ is also treated dynamically in the minimisation of the scalar potential. In this case both $V_{\text{max}}$ and $V_{\text{min}}$ are finite and $V_{eff}$ exhibits a behaviour similar to that studied in [40].

As has been already emphasised, the above interesting features have been demonstrated for a small number of Kähler moduli while many CY manifolds predict large numbers of them.
Some recent attempts of the authors [19] have extended the analysis on arbitrary numbers, however, effects of logarithmic corrections considered here are not captured by their approach. It is envisaged that the present approach is a starting point in aspiring to more complicated cases where large numbers of moduli and various types of quantum corrections are involved in these computations. Furthermore, due to the variant features of the effective potential as compared to previous studies, novel cosmological implications are anticipated worth exploring in future work.

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