Revisiting Agegraphic Dark Energy in Brans-Dicke Cosmology

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We explore a spatially homogeneous and isotropic Friedmann-Robertson-Walker (FRW) universe which is filled with agegraphic dark energy (ADE) with mutual interaction with pressureless dark matter in the background of Brans-Dicke (BD) theory. We consider both original and new type of agegraphic dark energy (NADE) and further assume the sign of the interaction term can change during the history of the Universe. We obtain the equation of state parameter, the deceleration parameter and the evolutionary equation for the sign-changeable interacting ADE and NADE in BD theory. We find that, in both models, the equation of state parameter, \(w_D\), cannot cross the phantom line, although they can predict the Universe evolution from the early deceleration phase to the late time acceleration, compatible with observations. We also investigate the sound stability of these models and find out that both models cannot show a signal of stability for different model parameters.

I. INTRODUCTION

Cosmological probes such as type Ia Supernova \cite{1–3}, Weak Lensing \cite{4}, Cosmic Microwave Background (CMB) anisotropies \cite{5,6}, Large-Scale Structure (LSS) \cite{7–9}, Plank data \cite{10} and Baryon Acoustic Oscillations (BAO) \cite{11}, have given us cross-checked data to determine cosmological parameters with high precision. Combining the analysis of cosmological observations we realize that our observable Universe is nearly flat, homogeneous and isotropic at large scale and is currently experiencing a phase of accelerated expansion. Besides, a phase transition from deceleration to the acceleration was occurred in the redshift around \(0.45 \leq z \leq 0.9\) \cite{12,13}.

A great variety of scenarios have been proposed to explain this acceleration such as some attempts to investigate the nature of dark energy according to some principles of quantum gravity, although a complete theory of quantum gravity has not established yet. The ADE model is such an example, which is based on the uncertainty relation of quantum mechanics together with the gravitational effects in general relativity. In this model it is assumed that the observed dark energy comes from the quantum fluctuations of the space time \cite{14,15}. In Refs. \cite{16–18}, Karolyhazy and his collaborators showed that the distance \(t\) in Minkowski space time cannot be known to a better accuracy than \(\delta t = \beta t_p^{2/3} t^{1/3}\) where \(\beta\) is a dimensionless constant of order unity and \(t_p\) denotes reduced Plank time. Based on karolyhazy relation together with the time-energy uncertainty relation, Maziashvili \cite{19,20} and Sasakura \cite{21} have independently obtained the energy density of spacetime fluctuations as

\[
\rho_D \sim \frac{1}{t_p^{2/3} t^{2/3}} \sim \frac{m_p^2}{t^2},
\]

where \(m_p\) and \(t\) are the reduced Plank mass and proper time scale, respectively. In the following, Cai \cite{14} proposed the energy density of the original ADE has the form

\[
\rho_D = \frac{3n^2 m_p^2}{T^2} = \frac{3n^2}{8\pi G T^2},
\]

where \(T\) is the age of the universe and the numerical factor \(3n^2\) is introduced to parameterize some uncertainties, such as the species of quantum fields in the universe, the effects of curved spacetime, etc. Since the original ADE model suffers from the difficulty to describe the matter-dominated epoch, for avoiding these internal in consistencies, the NADE model was proposed by Wei and Cai \cite{15}, by replacing the cosmic age \(T\) with the cosmic conformal age \(\eta\) for the time scale. The ADE models have been studied extensively and constrained by various astronomical observations \cite{22}. On the other side, it is interesting to analyze both ADE and NADE models in the framework of BD gravity. The motivation for this study comes from the fact that in string theory, gravity becomes scalar-tensor in nature which its low energy limit leads to the Einstein gravity, coupled non-minimally to a scalar field \cite{23}. Besides, the ADE and NADE energy densities belong to a dynamical cosmological constant, thus we need a dynamical frame to

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accommodate they instead of Einstein gravity. The investigation on the ADE and NADE models in the framework of BD cosmology, have been carried out in [24].

On the other side, there are also several observations which indicate that the possibility of a mutual interaction between the DM and DE is not zero. It was argued that the mutual interaction may solve the coincidence problem [25]. On the other hand, the simplest form of this mutual interaction can be written as \( Q = 3b^2 H (\rho_m + \rho_D) \). Clearly, this interaction is always positive and hence cannot change itself sign. Considering the latest observational data, Cai and Su [12], discussed that the sign of the interaction between DM and DE can change in the redshift around 0.45 ≤ z ≤ 0.9. Motivated by [12], Wei proposed a sign-changeable interaction term as \( Q = q (\alpha \dot{\phi} + 3 \beta H \rho) \), where \( \alpha \) and \( \beta \) are dimensionless constant, \( q \) is the deceleration parameter and \( \rho \) is the energy density of DE, DM or the sum of them [26, 27]. Clearly, the sign of \( Q \) is changed when the expansion of our Universe changes from deceleration \( (q > 0) \) to acceleration \( (q < 0) \). DE models with sign-changeable interaction term between two dark sectors have been carried out in [28, 29].

In the present work, we would like to investigate the ADE and NADE models with sign-changeable interaction term in the background of BD theory. At first, we study the cosmological implications of these models and then we perform the stability analysis by calculating the squared of sound speed \( v_s^2 = dP/d\rho \) [40]. When \( v_s^2 > 0 \) we have the classical stability of a given perturbation. In the framework of Einstein gravity instability of DE models have been studied in [31]. While, stability of interacting HDE with GO cutoff in BD theory has been investigated in [32], sound instability of nonlinearly interacting ghost dark energy have been discussed in [33]. Recently, we have studied the stability of the HDE model with the sign-changeable interaction in BD theory with various IR cutoffs [34].

We organize the paper as follows. In section II, we give a brief review of the interacting ADE model in context of BD cosmology. In section III and IV we investigate ADE and NADE in the framework of BD theory by assuming a sign-changeable interaction term, respectively. In each cases, the cosmological implications of the model as well as the squared sound stability \( v_s^2 \) of the model are studied. Finally, the summary of the result is discussed in the last section.

II. INTERACTING ADE IN BD COSMOLOGY

We begin with the action of BD theory, with one scalar field \( \phi \) which in the canonical form can be written [33]

\[
S = \int d^4x \sqrt{g} \left( -\frac{1}{8\omega} \phi^2 R + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + L_M \right),
\]

where \( \omega \) represents a coupling between scalar field and gravity, \( g \) the determinate of metric tensor \( g_{\mu\nu} \), \( L_M \) the matter part of the lagrangian, \( R \) is the scalar curvature and \( \phi \) is the BD scalar field which replaces with the Einstein-Hilbert term \( R/G \) in such a way that \( G_{\text{eff}}^{-1} = 2\pi \phi^2/\omega \). \( G_{\text{eff}} \) is the effective gravitational constant as long as the dynamical scalar field \( \phi \) varies slowly. In order to study the evolution of the universe, we assume a homogeneous and isotropic FRW spacetime which is described by the line element

\[
ds^2 = dt^2 - a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right).
\]

where \( a(t) \) is the scale factor and \( k \) is the curvature parameter. Since a closed universe with a small positive curvature (\( \Omega_k \approx 0.01 \)) is compatible with observations [33], from three possible values \( k = -1, 0, 1 \), which represent to open, flat and closed geometry of the universe, we select case \( k = +1 \). The variation of the action (3) with respect to the metric (4) for universe filled with dust and ADE yields the following field equations

\[
\frac{3}{4\omega} \phi^2 \left( H^2 + \frac{k}{a^2} \right) - \frac{1}{2} \phi^2 + \frac{3}{2\omega} H \dot{\phi} \dot{\phi} = \rho_m + \rho_D,
\]

\[
\frac{1}{4\omega} \phi^2 \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) - \frac{1}{\omega} H \dot{\phi} - \frac{1}{2\omega} \dot{\phi} - \frac{1}{2} \left( 1 + \frac{1}{\omega} \right) \ddot{\phi} = p_D,
\]

\[
\dot{\phi} + 3H \phi - \frac{3}{2\omega} \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} \right) \phi = 0,
\]

where the dot is the derivative with respect to time and \( \rho_m \) and \( \rho_D \) denote the energy density of DM and DE, respectively, also \( H = \dot{a}/a \) is the Hubble parameter and \( p_D \) is the pressure of DE. Furthermore, we exclude baryonic matter and radiation due to their negligible contribution to the total energy budget in the late time evolution. Based
on the previous experiences in the BD theory, let us assume the relation between BD scalar field and scale factor as a power law of the scale factor, $\phi = \phi_0 a^\alpha(t)$. Thus, we have

$$\frac{\dot{\phi}}{\phi} = \alpha H, \quad \frac{\ddot{\phi}}{\phi} = \alpha^2 H^2 + \alpha \dot{H}, \quad \ddot{\phi} = \left(\alpha + \frac{\dot{H}}{H^2}\right) H. \tag{8}$$

Observational evidences provided by the galaxy cluster Abell A586 supports the interaction between DE and DM \[37\]. In the presence of interaction, the semi-conservation equations for DE and DM are given by

$$\dot{\rho}_D + 3H\rho_D(1 + w_D) = -Q, \tag{9}$$

$$\dot{\rho}_m + 3H\rho_m = Q, \tag{10}$$

where $w_D$ is the equation of state parameter of DE and $Q$ is the interaction term which we assume has the form $Q = 3b^2 q H (\rho_m + \rho_D)$ \[27, 37, 38, 39\], $b^2$ is a coupling constant and $q$ is the deceleration parameter,

$$q = -\frac{\ddot{a}}{a H^2} = -1 - \frac{\dot{H}}{H^2}. \tag{11}$$

The energy density of ADE in standard cosmology is given by Eq. \(2\), where the age of universe is defined as

$$T = \int_0^a dt = \int_0^a \frac{da}{Ha}. \tag{12}$$

In the framework of BD cosmology, we write down the energy density of ADE as

$$\rho_D = \frac{3n^2 \phi^2}{4\omega T^2}, \tag{13}$$

where for the NADE we should replace $T$ with $\eta$. The critical energy density $\rho_{cr}$ and the energy density of the curvature $\rho_k$ are introduced as

$$\rho_{cr} = \frac{3\phi^2 H^2}{4\omega}, \quad \rho_k = \frac{3k\phi^2}{4\omega a^2}. \tag{14}$$

Then the dimensionless density parameters can be written

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{4\omega \rho_m}{3\phi^2 H^2}, \tag{15}$$

$$\Omega_k = \frac{\rho_k}{\rho_{cr}} = \frac{k}{H^2 a^2}, \tag{16}$$

$$\Omega_D = \frac{\rho_D}{\rho_{cr}} = \frac{4\omega \rho_D}{3\phi^2 H^2}. \tag{17}$$

Based on these definitions, and using Eqs.\(8\) and \(14\), the first Friedmann equation \(5\) can be rewritten as

$$\rho_{cr} + \rho_k = \rho_m + \rho_D + \rho_\phi, \tag{18}$$

where we have defined

$$\rho_\phi \equiv \frac{1}{2} \alpha H^2 \phi^2 \left(\alpha - \frac{3}{\omega}\right). \tag{19}$$

Dividing Eq.\(18\) by $\rho_{cr}$, this equation can be rewritten as

$$\Omega_m + \Omega_D + \Omega_\phi = 1 + \Omega_k, \tag{20}$$

where

$$\Omega_\phi = \frac{\rho_\phi}{\rho_{cr}} = -2\alpha \left(1 - \frac{\alpha \omega}{3}\right). \tag{21}$$
We introduce the ratio of the energy densities as,
\[ r = \frac{\Omega_m}{\Omega_D} = -1 + \frac{1}{\Omega_D} \left[ 1 + \Omega_k + 2\alpha \left( 1 - \frac{\omega}{3} \right) \right]. \tag{22} \]

Next, we introduce our approach for investigating the stability of ADE and NADE in BD theory with sign-changeable interaction against perturbations. Assuming a small fluctuation in the background of the energy density, we would like to check whether the perturbation will grow with time or it propagates as a sound wave in the medium. In classical perturbation theory, if we consider \( \rho(t) \) as an unperturbed background energy density, then the perturbed energy density of the background in the linear perturbation factor, can be written as
\[ \rho(t, x) = \rho(t) + \delta \rho(t, x), \tag{23} \]
which its energy conservation equation, \( \nabla \mu T^\mu = 0 \) yields \[ \ddot{\delta \rho} = v_s^2 \nabla^2 \delta \rho(t, x), \tag{24} \]
where \( v_s^2 = \frac{dP}{d\rho} \) is the square of the sound speed. For case \( v_s^2 > 0 \), Eq.(24) becomes an ordinary wave equation which have a wave solution in the form \( \delta \rho = \delta \rho_0 e^{-i\omega t + ik \cdot x} \). Obviously it show a propagation mode for the density perturbations and system is stable. For case \( v_s^2 < 0 \), the frequency of the oscillations becomes pure imaginary and density perturbations will grow with time as \( \delta \rho = \delta \rho_0 e^{i\omega t + ik \cdot x} \) and system cannot be stable. The quantity \( v_s^2 \) for a nonflat FRW universe is obtained as
\[ v_s^2 = \frac{\dot{\rho}_D w_D + \rho_D \dot{w}_D}{\dot{\rho}_D (1 + r) + \rho_D \dot{r}}, \tag{25} \]
where \( P = P_D \) is the pressure of DE and \( \rho = \rho_m + \rho_D \) is the total energy density of DE and DM.

III. SIGN-CHANGEABLE ADE IN BD THEORY

We begin with the ADE in BD theory, whose energy density is given by Eq.(13). Differentiating the expression of the energy density of ADE given in Eq.(13) and using Eqs.(8) and (17) we obtain
\[ \dot{\rho}_D = 2H\rho_D \left( \alpha - \frac{\sqrt{\Omega_D}}{n} \right). \tag{26} \]
Inserting this equation in the semi-conservation law \( \hat{\rho} \) and using Eq.(22), we obtain the equation of state parameter for sign-changeable ADE in BD theory
\[ w_D = -1 - \frac{2\alpha}{3} + \frac{2}{3n} \sqrt{\Omega_D} - b^2 q(1 + r). \tag{27} \]
Dividing Eq. (6) by \( H^2 \) and using Eqs. (8), (11), (13), (16) and (17) we can obtain the following expression for the deceleration parameter
\[ q = \frac{1}{2\alpha+2} \left[ (2\alpha+1)^2 + 2\alpha(\alpha\omega - 1) + 3\Omega_D w_D \right]. \tag{28} \]
Substituting \( w_D \) from Eq. (27) in the above relation, we arrive at
\[ q = \frac{1 + 2\alpha(1 + \alpha(2 + \omega)) + \Omega_k - (3 + 2\alpha)\Omega_D + \frac{2\Omega_D^{3/2}}{n}}{2(1 + \alpha) + 3b^2 \Omega_D (1 + r)}. \tag{29} \]
The equation of motion for ADE may be obtained by substituting Eq. (13) in Eq.(17). We find
\[ \Omega_D = \frac{n^2}{H^2 T^2}. \tag{30} \]
Taking the time derivative of Eq. (30) and using Eq. (11) as well as the fact that \( \dot{\Omega}_D = \Omega_D' H \), we get
\[ \Omega_D' = 2\Omega_D \left( 1 + q - \frac{\sqrt{\Omega_D}}{n} \right), \tag{31} \]
FIG. 1: Evolution of $w_D$ versus redshift parameter $z$ for the sign-changeable interacting ADE in BD cosmology. Here, we have taken $\alpha = 0.003$, $\omega = 10^4$, $\Omega_k = 0.01$ and $b^2 = 0.1$ as the initial condition.

FIG. 2: Evolution of the deceleration parameter $q$ against redshift parameter $z$ for the sign-changeable interacting ADE in BD cosmology. Here, we have taken $\alpha = 0.003$, $\omega = 10^4$, $\Omega_k = 0.01$ and $b^2 = 0.1$ as the initial condition.

where the prime denotes derivative with respect to $x = \ln a$. Stability of this model can be studied by taking derivative of Eq. (27) and using Eqs. (26) and (25). Since the expression of $v_s^2$ is too long, for the economic reason we do not present it here, instead we focus on its behaviour via figures.

To describe the evolution of the universe, we plot the cosmological parameters for the sign-changeable interacting ADE in BD cosmology. From Fig. 1 we see that $w_D$ cannot cross the phantom line, while according to Fig. 2 we see that the deceleration parameter $q$ transits from deceleration ($q > 0$) in the early time to acceleration ($q < 0$) in the last time around $z \approx 0.6$. Again, by keeping the same initial condition, we plot the evolution of $\Omega_D$ against redshift parameter in Fig. 3 which show that at the late time where the DE is dominated we have $\Omega_D \to 1$, while $\Omega_D \to 0$ at the early time. Finally, we plot the squared sound speed for ADE model in BD theory in Figs. 4 and 5 by considering the different parameters $\alpha$, $b^2$, $n$ and $\omega$. In Fig. 4 we plot $v_s^2$ versus $z$ with different values of $n$ also $\alpha$, which show we cannot have the stable model. According to Fig. 5 we see that for different values of $b^2$ and $\omega$, the model does not show a signal of stability.

IV. SIGN-CHANGEABLE NADE IN BD THEORY

Since the original model of ADE model suffers the difficulty to describe the matter-dominated epoch, the NADE was proposed by Wei and Cai [15] to describe the late time acceleration. In the NADE model, the conformal time $\eta$ is chosen as the cutoff instead the age of the universe, which leads to the energy density in the form [15]

$$\rho_D = \frac{3n^2m_p^2}{\eta^2},$$  \hspace{1cm} (32)
FIG. 3: Evolution of $\Omega_D$ versus redshift parameter $z$ for the sign-changeable interacting ADE in BD cosmology. Here, we have taken $\alpha = 0.003$, $\omega = 10^4$, $\Omega_k = 0.01$ and $b^2 = 0.1$ as the initial condition.

FIG. 4: Evolution of the squared of sound speed $v_s^2$ against redshift parameter $z$ for the sign-changeable interacting ADE in BD cosmology. Here, we have taken $\alpha = 0.003$, $\omega = 10^4$, $\Omega_k = 0.01$ and $b^2 = 0.1$ in the left panel and $\Omega_k = 0.01$, $\omega = 10^4$, $n = 2.5$ and $b^2 = 0.1$ in the right panel, as the initial condition, respectively.

FIG. 5: Evolution of the squared of sound speed $v_s^2$ against redshift parameter $z$ for the sign-changeable interacting ADE in BD cosmology. Here, we have taken $\alpha = 0.003$, $\omega = 10^4$, $\Omega_k = 0.01$ and $n = 2.5$ in the left panel and $\alpha = 0.003$, $n = 2.5$, $\Omega_k = 0.01$ and $b^2 = 0.1$ in the right panel, as the initial condition, respectively.
where the conformal time is given by
\[ \eta = \int_0^a \frac{da}{Ha^2}. \]  
(33)

In the framework of BD cosmology, by using Eqs. (2) and (17) we can write the energy density of NADE as
\[ \rho_D = \frac{3n^2\phi^2}{4\omega\eta^2}. \]  
(34)

and
\[ \Omega_D = \frac{n^2}{H^2\eta^2}. \]  
(35)

Differentiating Eq.(34), we arrive at
\[ \dot{\rho}_D = 2H\rho_D \left( \alpha - \frac{\sqrt{\Omega_D}}{na} \right). \]  
(36)

Substituting this relation in Eq. (9), after using Eq.(22), we find
\[ w_D = -1 - \frac{2\alpha}{3} + \frac{2}{3na}\sqrt{\Omega_D} - b^2q(1 + r). \]  
(37)

When \( q = 1 \), Eq.(37) restores the equation of state of the NADE in the BD theory. Setting \( \alpha = 0 \) and \( q = 1 \), this equation recovers its respective expression for interacting NADE in Einstein gravity. We can also obtain the deceleration parameter \( q \) by substituting Eq.(37) in Eq.(28). The result is
\[ q = \frac{1 + 2\alpha(1 + \alpha(2 + \omega)) + \Omega_k - (3 + 2\alpha)\Omega_D + \frac{2\Omega_D^{\frac{3}{2}}}{na}}{2(1 + \alpha) + 3b^2\Omega_D(1 + r)}. \]  
(38)

On the other hand, the equation of motion for \( \Omega_D \) takes the form
\[ \Omega'_D = 2\Omega_D \left( 1 + q - \frac{\sqrt{\Omega_D}}{na} \right). \]  
(39)

Finally, we investigate stability of NADE by calculating \( v_s^2 \). For the economic reason, we do not bring the explicit expression for \( v_s^2 \), instead we study the evolution of \( v_s^2 \) via figures.

![Graph showing the evolution of \( w_D \) versus redshift parameter \( z \) for the sign-changeable interacting NADE in BD cosmology. Here, we have taken \( n = 2.5, \omega = 10^4, \Omega_k = 0.01 \) and \( b^2 = 0.01 \) as the initial condition.]

FIG. 6: Evolution of \( w_D \) versus redshift parameter \( z \) for the sign-changeable interacting NADE in BD cosmology. Here, we have taken \( n = 2.5, \omega = 10^4, \Omega_k = 0.01 \) and \( b^2 = 0.01 \) as the initial condition.

The behaviors of \( w_D, q \) and \( \Omega_D \) against redshift parameter \( z \) are plotted in Figs. 6-8. Our analysis of these figures show that \( w_D \) cannot cross phantom line. From Fig. 7 we see that a deceleration phase ends at past and transits to a phase of acceleration at late time. In Fig. 8 which we plot \( \Omega_D \) versus \( z \) for the sign-changeable interacting NADE in BD cosmology, we see \( \Omega_D \rightarrow 1 \) at late time for different values of \( n \). Finally, in Figs. 9 and 10 the stability of model is studied which show for different values of parameters which confirm that this system cannot show signal of stability in our universe.
FIG. 7: Evolution of the deceleration parameter $q$ against redshift parameter $z$ for the sign-changeable interacting NADE in BD cosmology. Here, we have taken $n = 2.5$, $\alpha = 0.003$, $\Omega_k = 0.01$ and $b^2 = 0.01$ as the initial condition.

FIG. 8: Evolution of $\Omega_D$ versus redshift parameter $z$ for the sign-changeable interacting NADE in BD cosmology. Here, we have taken $\omega = 10^4$, $\alpha = 0.003$, $\Omega_k = 0.01$ and $b^2 = 0.01$.

V. CLOSING REMARKS

In this paper, we have considered the ADE and NADE with sign-changeable interaction term in the framework of BD theory. We have discussed the physical behavior of the EoS parameter, the deceleration parameter, the evolution of density parameter $\Omega_D$ and the squared sound speed $v_s^2$ versus the redshift parameter $z$ for ADE and NADE in BD cosmology with sign-changeable interaction term. For ADE model, we found out that the EoS parameter $w_D$ cannot cross the phantom line, while the deceleration parameter $q$ transits from deceleration ($q > 0$) in the early time to acceleration ($q < 0$) at the last time around $z \approx 0.6$ which is compatible with recent observations. We also plotted the evolution of $v_s^2$ versus $z$ in and observed that this model cannot lead to stable DE dominated universe. For NADE model, we again see that $w_D$ cannot cross the phantom line, while at late time where the DE dominates we have $\Omega_D \rightarrow 1$. Finally, we observed that $v_s^2$ remains negative so we have a sign of instability for the NADE in BD theory.

In conclusion, our studies show that for the sign-changeable ADE and NADE models in the set up of BD cosmology we cannot have a stable DE dominated universe.
FIG. 9: Evolution of the squared of sound speed $v_s^2$ against redshift parameter $z$ for the sign-changeable interacting NADE in BD cosmology. Here, we have taken $n = 2.5$, $\omega = 10^4$, $\Omega_k = 0.01$ and $b^2 = 0.01$ in the left panel and $\alpha = 0.003$, $\omega = 10^4$, $\Omega_k = 0.01$ and $n = 2.5$ in the right panel, as the initial condition, respectively.

FIG. 10: Evolution of the squared of sound speed $v_s^2$ against redshift parameter $z$ for the sign-changeable interacting NADE in BD cosmology for $\alpha = 0.003$, $\Omega_k = 0.01$ and $b^2 = 0.01$. As the initial condition, we have taken in the left panel $\omega = 10^4$, and in the right panel $n = 2.5$.

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