Rational Placement of a Macro Fibre Composite Actuator in Composite Rotating Beams

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Abstract. Abstract. In the presented research the dynamics of a thin rotating composite beam with surface bonded MFC actuator are considered. A parametric analysis aimed at finding the most efficient location of the actuator on the beam is presented. Gyroscopic effects resulting in the beam’s initial strain and therefore non-zero voltage in PZT are taken into account. Within the frame of the study maximising the system's response observed in vibration modes for uncoupled and coupled motions is examined. The results are compared to the case of a nonrotating beam and also to the maximum response of the beam with the actuator placed at different positions. To perform the analysis an ABAQUS finite element model of an electro-mechanical system under consideration is developed. The multi-layer composite beam structure is modelled by shell elements according to a layup-ply technique; the MFC actuator is modelled by 3D coupled field piezoelectric elements. Both modal analysis and frequency response spectra are performed to obtain the structural modal parameters and response amplitude, respectively. The analysis is repeated for three different orientations of the beam's cross-section with respect to the plane of rotation (i.e. arbitrary assumed pitch angles); in all cases the condition constant angular speed is preserved. This work is fundamental for continuing the research for control of dynamics of rotating composite beams with active elements.

1. Introduction

In recent years the idea of possible use of smart materials (e.g. piezoelectric elements, shape memory alloys or magnetorheological fluids) in modern structures has been the area of extensive interest. The significant research is related to the active control of flexible systems in order to improve their operating performance and to increase their service lifetime [1, 2]. In active vibration control, apart from the standard structural and controller design, the rational placement of an actuator for the most effective structural response plays a significant role.

Among different structures to be controlled lightweight rotating composite beams are of particular interest because of their rich dynamics. Numerous studies yield qualitatively and quantitatively their unique properties, like the rotating speed-dependent natural frequencies and mode shapes, the large deformations under centrifugal force and couplings between displacements/deformations along different directions [3]. These effects observed for isotropic materials are even more complex and more pronounced in case of lightweight composite beams [4], and also in multi-physics systems.
Therefore, the scope of this paper is a numerical parametric analysis of the rotating composite beam with an embedded MFC actuator. To perform the analysis an ABAQUS finite element electromechanical model of the beam with MFC patch is developed. The model is used for identification of a rational position of the actuator considering the influence of gyroscopic effects.

The study is aimed at finding the rational location of the transducer on the host structure to achieve the maximum system’s response in its uncoupled and coupled bending motions. Studies are performed for different rotating speeds $\omega$ and the results are compared to the case of a nonrotating beam. Moreover, different orientations $\theta$ of the beam’s cross-section with respect to the plane of rotation (i.e. pitch angles) are analysed. Effects resulting in the beam’s initial strain and therefore non-zero voltage in a $d_{33}$ type PZT patch are taken into account.

2. Rotating beam model

The model of a structure under consideration is presented in Figure 1 – the composite beam is clamped at the hub rotating with constant angular velocity $\omega$. Orientation of the beam’s local coordinate system $xyz$ with respect to the surface of revolution $XOY$ is defined by $\theta$ angle. In the current analysis a Macro Fibre Composite actuator of $d_{33}$ type is used (poling direction along the beam axis). This type of device, as opposed to traditional piezoceramic patch, is flexible and can therefore be applied to curved surfaces. In-plane poling is achieved by interdigitated electrode, which produces more induced actuating strain than monolithic piezoelectric ceramic patch and also more than $d_{31}$ type elements. It is assumed that the MFC element is perfectly bonded to the surface of the beam.

Within the frame of analysis three different possible positions of the actuator are taken into account – i.e. at hub, at the middle and at free end (see the figure below) in order to compare the system’s response to the harmonic excitation from the actuator.

![Figure 1. Rotating beam with single PZT element at free end](image)

3. Numerical studies

The composite beam is made of unidirectional glass fibre tape and epoxy Prime 20; average composite density $\rho = 1.85g/cm^3$, Young’s modulus along the fibres $E_1 = 36 250$ MPa, transversal Young’s modulus $E_2 = 6 662$ MPa, shear moduli $G_{12} = G_{13} = G_{23} = 2 446$ MPa and Poisson’s ratio $\nu_{12} = 0.285$. The layers of the composite are set in the following order: $0^\circ/90^\circ/+45^\circ/-45^\circ/+45^\circ/90^\circ/0^\circ$ (according to the Ox axis pointing side edgewise). A Rayleigh damping model is assumed for the composite material with coefficients $\alpha = 15.351$, $\beta = 2.172e-5$.

The MFC patch material data used in the numerical analysis corresponds to an M-8503-P1 element by Smart Material Corp. For the numerical calculations the following electric and elastic properties are used: isotropic ferroelectric material with constant permittivity $\xi = 8 \times 10^9$ F/m and mean value of piezoelectric coefficient $d_{33} = 400 \times 10^{-12}$ m/V; elastic orthotropic material having Young moduli $E_1 = E_2 = 15 857$ MPa, $E_3 = 30 336$ MPa, shear moduli $G_{12} = G_{13} = G_{23} = 5 515$ MPa and Poisson ratios $\nu_{12} = \nu_{13} = \nu_{23} = 0.31$; $\rho = 5.40g/cm^3$. Despite the modular structure of the
transducer under consideration the piezo-elements have not been modelled in micro scale – a substitute body made of orthotropic, homogenous piezoelectric material is proposed.

The numerical model of a beam is developed in ABAQUS ver. 6.10 software. Specimen is defined as a lamina type according to the layup technique and meshed by linear shell elements with reduced integration. The actuator domain is modelled with second order continuum piezo-elements C3D20E. The model of the piezo-element is 'glued' to the host structure by TIE constraints, which results in joining appropriate DOF of both bodies.

The boundary conditions of the electrical part used for modal analysis are at one pole zero electrostatic potential and the other pole is free. Three different pitch angles (θ = 0°, 45°, 90°) and at three rotating speeds (ω = 100 rpm, 6000 rpm, 12000 rpm) are studied. Mode shapes and frequencies of natural vibrations are recorded.

Following these results a steady-state response of the system to a continuous harmonic excitation by piezo actuator is studied. For the current analysis a direct steady-state method is used which is a perturbation procedure, where the perturbed solution is obtained by linearization about the current base state. For the calculation of the base state the system may exhibit material and geometrical nonlinear behaviour (centrifugal force). Moreover a direct calculation option is used because the modal based procedures do not adequately transform the charge loads into modal loads [5].

4. Results

Results of the modal analysis of the beam with the MFC patch are given in Table 1 and in Figure 1. In the case of a 45° pitch angle strong coupling of bending modes is observed – see Figure 1. Stressing of the structure caused by centrifugal loading is confirmed by increased natural frequencies at higher rotating speeds. Moreover a 'load stiffness' effect caused by the change in direction of centrifugal loading if the vibration causes motion in the plane normal to the axis of rotation is observed – 0° pitch angle vs 90°.

Table 1. First two natural frequencies [Hz] and modes of the rotating beam with an MFC patch

| Hub   | 0°         | 45°         | 90°         |
|-------|------------|-------------|-------------|
|       | 100 rpm    | 100 rpm     | 100 rpm     |
|       | 6000 rpm   | 6000 rpm    | 6000 rpm    |
|       | 12000 rpm  | 12000 rpm   | 12000 rpm   |
| 39.28 xz | 39.26 xz | 109.39 xz | 39.24 xz |
| 110.62 xz | 79.87 xz | 47.55 xz | 41.84 xz |
| 178.31 xy | 109.39 xz | 166.87 xy | 197.50 xy |
| 166.86 xy | 186.75 xy | 247.97 xy | 267.75 xy |
| 170.37 xy | 166.87 xy | 166.87 xy | 267.75 xy |
| 204.21 xz | 186.75 xy | 197.50 xy | 267.75 xy |

| Middle | 0°         | 45°         | 90°         |
|--------|------------|-------------|-------------|
|        | 100 rpm    | 100 rpm     | 100 rpm     |
|        | 6000 rpm   | 6000 rpm    | 6000 rpm    |
|        | 12000 rpm  | 12000 rpm   | 12000 rpm   |
| 35.99 xz | 35.98 xz | 104.41 xz | 35.96 xz |
| 107.68 xz | 75.65 xz | 39.91 xz | 30.44 xz |
| 171.72 xy | 104.41 xz | 165.74 xy | 195.21 xy |
| 165.74 xy | 184.37 xy | 243.82 xy | 263.38 xy |
| 167.75 xy | 184.37 xy | 165.74 xy | 263.38 xy |
| 202.31 xz | 184.37 xy | 165.74 xy | 263.38 xy |

| Free end | 0°         | 45°         | 90°         |
|----------|------------|-------------|-------------|
|          | 100 rpm    | 100 rpm     | 100 rpm     |
|          | 6000 rpm   | 6000 rpm    | 6000 rpm    |
|          | 12000 rpm  | 12000 rpm   | 12000 rpm   |
| 34.16 xz | 34.14 xz | 109.33 xz | 34.12 xz |
| 108.92 xz | 77.08 xz | 43.13 xz | 48.32 xz |
| 171.17 xy | 109.33 xz | 160.85 xy | 191.87 xy |
| 160.85 xy | 180.96 xy | 244.16 xy | 263.03 xy |
| 163.84 xy | 180.96 xy | 160.85 xy | 263.03 xy |
| 205.76 xz | 180.96 xy | 160.85 xy | 263.03 xy |

xz – bending in xz plane, xy – bending in xy plane, xz₁ – coupled xz-xy bendings xz plane dominant, xy² – coupled xz-xy bendings xy plane dominant.
Figure 2. Exemplary mode shapes of the beam with an MFC actuator; pitch angle 0° (cases a to c); pitch angle 45° (cases d to f) – coupled \(xz\) and \(xy\) bending motions are clearly visible.

At the next stage a series of tests to examine the steady-state response of the system to the continuous excitation by piezo actuator are performed. The system is excited by a harmonic voltage signal 
\[
U(t) = U_0 \cos(\Omega t), \quad U_0 = 500 \text{ V}
\]
applied to the PZT poles – see Figure 1.

Frequency response spectra are depicted for all the cases in Figure 3. In Figures 3a and 3b the amplitude of the beam tip displacement in case of 0° and 90° pitch angle considering only \(xz\) motion is presented. This motion is excited directly by the active element. Careful study of these two cases indicates that the active element is more efficient when it is placed at the hub, and as long as the rotation speed is increasing the effectiveness of the active element decreases. The frequency response spectra in case of a 45° pitch angle for \(xy\) and \(xz\) bending are shown in Figure 3c and 3d respectively.
Although the active element excites $xz$ bending motion directly – due to coupling – one can identify excitation also of $xy$ motion (Figure 3c) upon rotation.

Therefore the rational placement of an actuator is at the hub where the MFC patch is most effective, and as long as the rotation speed is increasing in the case of direct excited motion the effectiveness of the active element is decreasing. In case of $xy$ motion (Figure 3c) which is indirectly excited the phenomenon is opposite. This means the increase in rotation speed results in higher effectiveness of the active element due to stronger coupling in the equations of motion.

Figure 3. Steady state response of the system: a) pitch angle 0°, b) pitch angle 90°, c) pitch angle 45° – response in $xy$ plane, d) pitch angle 45° – response in $xz$ plane; notation: F – patch at the beam’s free end, M – patch in the middle, H – patch positioned at the hub.

6. Conclusions

In the presented paper properties of a light, thin rotating composite beam with an active element are examined. Parametric studies aimed at finding the rational location of this patch with respect to maximum structural response to actuator excitation are summarized. Different rotating speeds and different orientations of the beam’s cross-section with respect to the plane of rotation are considered.

The most important conclusions from the research undertaken are the following:

- In all the analysed cases locating the MFC actuator at the hub results in the highest system’s response in first bending mode.
• While the rotating speed is increased the response from the system gets smaller because of an increase in the stiffness due to centrifugal force. Moreover, the softening effect of ‘load stiffness’ caused by the change in direction of centrifugal loading, if the vibration causes motion in the plane normal to the axis of rotation, is confirmed.

• Setting the orientations of the beam’s cross-section with respect to the plane of rotation to 45° results in coupled bending motions – in-plane (xz) and out-of-plane (xy) directions.

• Since the actuator excites the beam in the xz plane the observed coupled response in the xy plane is significant only for the rotating system. Also, as opposed to directly excited motions the effectiveness of the active element in xy motion is increasing with an increase of the rotation.

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