Magnetically limited X-ray filaments in young SNR

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ABSTRACT

We discuss the damping of strong magnetic turbulence downstream of the forward shock of young supernova remnants (SNR). We find that strong magnetic fields, that have been produced by the streaming instability in the upstream region of the shock, or by other kinetic instabilities at the shock, will be efficiently reduced, so the region of enhanced magnetic field strength would typically have a thickness of the order $l_d = (10^{16} - 10^{17})$ cm. The non-thermal X-ray filaments observed in young SNR are thus likely limited by the magnetic field and not by the energy losses of the radiating electrons. Consequently the thickness of the filaments would not be a measure of the magnetic field strength and claims of efficient cosmic-ray acceleration on account of a run-away streaming instability appear premature.

Subject headings: acceleration of particles – supernova remnants – X-rays: ISM

1. Introduction

Supernova remnant shocks are prime candidates for the acceleration sites of galactic cosmic rays. A number of young shell-type SNR have been observed to emit nonthermal X-ray emission, which is commonly interpreted as being synchrotron emission of freshly accelerated electrons in the 10–100 TeV energy range. These electrons, and also newly accelerated cosmic-ray nucleons, should produce GeV–TeV scale $\gamma$-ray emission, which has been detected from a few objects (e.g. Aharonian et al. 2004).
Likely though the inference may appear, evidence for electron acceleration in SNR does not imply evidence for cosmic-ray nucleon acceleration. To solve the long-standing problem of the origin of galactic cosmic rays one is therefore interested in finding unambiguous signatures of a large flux of high-energy cosmic-ray nucleons in SNR, which obviously requires a good understanding of the nonthermal X-ray emission from SNR, i.e. the spatial distribution of magnetic field and the spatial and energy distribution of high-energy electrons, so the leptonic γ-ray emission can be accurately modeled.

High-resolution X-ray observations indicate that a large fraction of the nonthermal X-ray emission on the rims of young SNR is concentrated in narrow filaments (Bamba et al. 2005). In the literature these filaments are usually interpreted as reflecting the spatial distribution of the high-energy electrons around their acceleration site, presumably the forward shock (Yamazaki et al. 2004). By comparing the electron energy-loss time with the downstream flow velocity one then derives an estimate of the magnetic field strength that far exceeds the typical interstellar magnetic field strength modified according to the Rankine-Hugoniot conditions (Vink & Laming 2003; Bamba et al. 2004).

A number of authors have further advanced the argument by noting that strong near-equipartition magnetic fields can be produced by the streaming instability (Lucek & Bell 2000; Bell & Lucek 2001), thus apparently providing evidence for a very efficient acceleration of cosmic-ray nucleons at SNR forward shocks (Berezhko, Ksenofontov & Völk 2003; Berezhko, Pühlhofer & Völk 2003; Berezhko & Völk 2004).

It has been pointed out before that the observed X-ray filaments may actually be magnetic filaments, i.e. localized enhancements of the magnetic field, rather than limited by the distribution of high-energy electrons. Lyutikov & Pohl (2004) studied the pile-up of magnetic field at the contact discontinuity and concluded that it could explain the X-ray filaments, if a significant fraction of the electrons was accelerated close to the contact discontinuity.

In this Letter we investigate the damping of turbulent magnetic field near SNR shocks. In contrast to earlier studies (Völ, Zank & Zank 1988; Fedorenko 1992; Drury, Duffy & Kirk 1996; Ptuskin & Zirakashvili 2003, 2005) that concentrated on damping in the upstream region, where it is in competition with the streaming instability, we only consider the downstream region where the growth of turbulent magnetic field is negligible. We thus determine the spatial extent of amplified turbulent magnetic field that is produced at or upstream of the forward shock. Our analysis is independent of the amplification process, it applies to magnetic field produced by the streaming instability in the upstream region as well as to field generated by a kinetic plasma instability at the SNR shock (Akhiezer et al. 1975).
Substantial progress has been made in the understanding of strong magnetic turbulence, e.g. we have theoretical scalings that are supported by numerical simulations. Nevertheless, many questions remain unanswered, and in order to derive a generally valid picture, we use various models of the cascading and damping behaviour resulting from nonlinear wave interactions. While the models differ in their assumptions, they all predict a significant reduction of the turbulent magnetic energy density on a timescale shorter than that of energy losses of electrons that could synchrotron-radiate at a few keV X-ray energy. Given typical plasma streaming velocities downstream of a SNR shock, this would result in magnetic filaments of thickness $\lesssim 10^{17}$ cm. So even if a strong magnetic field is produced at or upstream of the forward shock by any mechanism, it is likely confined to a small volume around the shock and the observed X-ray filaments would just reflect the structure of the magnetic fields.

2. Wave damping in the downstream medium of SNR shocks

The energy density $W(k, x)$ of magnetic turbulence is related to the total turbulent magnetic field as

$$\frac{(\delta B)^2}{4\pi} = \int dk \, W(k, x)$$

In a plasma under steady-state conditions $W(k, x)$ obeys the continuity equation

$$U \frac{\partial W}{\partial x} = 2 (\Gamma_g - \Gamma_d) W$$

where $\Gamma_g$ is the growth rate and $\Gamma_d$ is the damping rate of turbulence. $U$ is the propagation velocity of the wave energy, i.e. the sum of the plasma velocity and the component of the group velocity of waves along the direction of the plasma flow. In the case under study we expect the excitation of turbulence by the streaming instability in the upstream region at $x \leq 0$, which is transported to the downstream region ($x > 0$) and eventually damped. Alternatively one could consider kinetic instabilities which are known to produce magnetic turbulence at the shock front ($x \approx 0$). The instabilities should not operate in the downstream region and hence the growth rate would vanish there.

Equation 2 must be separately solved on either side of the shock. As boundary condition we may assume absence of turbulence far ahead of the shock, $W(k, x = -\infty) = 0$, whereas the MHD jump conditions applied to the upstream solution at $x = 0$ provide the boundary condition for the downstream solution at $x \to 0$ (Vainio & Schlickeiser 1999).

Earlier studies found that the streaming instability during the early stages of SNR evolution can be so strong that the amplified turbulent magnetic field $\delta B \gtrsim 100$ $\mu$G far
exceed the undisturbed magnetic field $B \approx (3 - 10) \mu G$ upstream of the SNR blast wave (Lucek & Bell 2000; Bell & Lucek 2001). The cosmic-ray diffusion coefficient $\kappa$ should then be close to the Bohm limit, and consequently both the cosmic rays with energy $E$ and the amplified magnetic field would in the upstream region be confined to a shock precursor zone of thickness

$$l_{\text{prec}} \approx \frac{\kappa}{U_s}$$

$$\simeq (10^{15} \text{ cm})(\frac{U_s}{3000 \text{ km/s}})^{-1}(\frac{E}{\text{TeV}})(\frac{B}{100 \mu G})^{-1}$$

where $U_s$ is the SNR shock velocity.

Let us in the following assume that the streaming instability has efficiently produced strong magnetic fields in the precursor zone. The magnitude of the amplified magnetic field upstream of the shock depends on the efficiency of the various possible damping mechanisms. In any case a fraction of the magnetic turbulence will propagate through the shock to the downstream region, where it will convect away from the shock. We are interested in the spatial scale, on which the amplified magnetic energy is damped downstream of the shock. If this length scale is small compared with the dimensions of a supernova remnant, then the streaming instability essentially produces a magnetic filament at the location of the forward shock, that should be observable as a non-thermal X-ray filament on account of enhanced synchrotron emission of very high-energy electrons. If, on the other hand, the damping length scale were comparable to or larger than the typical SNR, then the entire interior of the remnant should be filled with strong magnetic field and the observed non-thermal X-ray filaments must be limited by electron energy losses.

Ion-neutral collisions will not efficiently damp magnetic turbulence in the downstream region on account of the high plasma temperature. Ordinary collisionless damping does occur, but is very inefficient for Alfvén and fast-mode perturbations that propagate along a large-scale magnetic field. Calculations of the damping rate of obliquely propagating fast modes for low-$\beta$ plasma (Ginzburg 1961) and high-$\beta$ systems (Foote & Kulsrud 1979) suggest that collisionless damping is also slow for small $k$ or large wavelengths. In the following we will discuss nonlinear wave-wave interactions and cascading as potential processes that may limit the thickness of the high magnetic field layer in the downstream region of the SNR shock.

The cascading of wave energy in astrophysical environments is not well understood to date. Much of the interstellar medium, and hence the upstream medium of SNR shocks, is thought to be turbulent. Goldreich & Sridhar (1995) have studied the cascading of wave energy and found a concentration of wave energy in transverse modes, which are noted to be an inefficient means of cosmic-ray scattering (Chandran 2000; Yan & Lazarian 2002).
In addition, wave damping may occur by interactions with background MHD turbulence. Wave packages are distorted in collisions with oppositely directed turbulent wave packets, resulting in a cascade of wave energy to smaller scalar where it is ultimately dissipated (Yan & Lazarian 2002). Analogous to MHD perturbations that can be decomposed into Alfvénic, slow and fast waves with well-defined dispersion relations, MHD perturbations that characterize turbulence can apparently be separated into distinct modes.

The separation into Alfvén and pseudo-Alfvén modes, which are the incompressible limit of slow modes, is an essential element of the Goldreich & Sridhar (1995) model. Even in a compressible medium, MHD turbulence is not an inseparable mess in spite of the fact that MHD turbulence is a highly non-linear phenomenon (Lithwick & Goldreich 2001; Cho & Lazarian 2002). The actual decomposition of MHD turbulence into Alfvén, slow and fast modes was addressed in Cho & Lazarian (2002, 2003), who used a statistical procedure of decomposition in the Fourier space, where the basis of the Alfvén, slow and fast perturbations was defined. For some particular cases, e.g. for a low \( \beta \) medium, the procedure was benchmarked successfully and therefore argued to be reliable.

Here we list the results of three previous studies on wave damping by cascading to very small scales, one using a very general picture of turbulence cascading, and the other two referring to the cascading of fast-mode and Alfvén waves in background MHD turbulence, respectively.

### 2.1. Kolmogorov-type energy cascade

For the case of a Kolmogorov-type nonlinearity Ptuskin & Zirakashvili (2003) have considered the simplified expression

\[
\Gamma_{nl} \approx \frac{V_A}{20} k A(> k), \quad A(> k) = \frac{\sqrt{4\pi k W(k, x)}}{B_0} \tag{4}
\]

where \( V_A \) is the Alfvén speed. This ansatz does not build on the disparity of the parallel and perpendicular scales, and therefore its applicability may be limited, but it has been used in part of the recent literature and thus merits our consideration.

In conjunction with a vanishing growth rate \( \Gamma_g = 0 \), inserting Eq. 4 in Eq. 2 yields the damping length scale

\[
l_k = 20 \frac{U \sqrt{\rho}}{\sqrt{W(k, 0)} k^{3/2}} \tag{5}
\]

with \( W(k, 0) \) being the energy density spectrum of turbulence that is transmitted through the shock to the downstream region.
Most of the magnetic energy density will reside at small \( k \) or large wavelengths \( \lambda \). Inserting numbers that are typical of the downstream region of young SNR blast waves and a highly amplified turbulent magnetic field the damping length scale is expected to be

\[
l_k \approx (1.5 \cdot 10^{16} \text{ cm}) \left( \frac{U}{1000 \text{ km/s}} \right) \times \left( \frac{n}{\text{atoms/cm}^3} \right)^{1/2} \left( \frac{\lambda}{10^{15} \text{ cm}} \right) \left( \frac{\delta B(\lambda)}{100 \mu G} \right)^{-1}
\]

Here \( \delta B \) is defined as

\[
\frac{[\delta B(\lambda)]^2}{4\pi} = \frac{[\delta B(k)]^2}{4\pi} = kW(k, x)
\]

The wavelength of the turbulent magnetic field, \( \lambda \), that is used in Eq. 6 should be related to the Larmor radius \( r_L \) of the high-energy particles, which are supposedly accelerated at the shock front. Noting that

\[
r_L = (3 \cdot 10^{13} \text{ cm}) \left( \frac{E}{\text{TeV}} \right) \left( \frac{B}{100 \mu G} \right)^{-1}
\]

we see that the value of \( \lambda \) used in Eq. 6 corresponds to about 30 TeV particle energy, in the case of electrons sufficient to account for X-ray synchrotron emission at a few keV.

For the smallest \( k \) or largest wavelength \( \lambda \) the magnetic fields components used in Eqs. 6 and 7 must be similar. Internal consistency should require that the damping length scale is larger than the largest wavelength of the waves, lest the waves at large wavelengths would have very little energy density in contradiction to our assumption. In addition, an efficient backscattering of cosmic rays to the upstream region requires that the high magnetic field layer in the downstream region has a thickness much larger than the scattering mean free path, which should be similar to the Larmor radius of the particles and hence to the wavelength of the waves. This leads to a limit for the cosmic-ray energy that is independent of the actual strength of the amplified magnetic field.

\[
l_k > \lambda \approx r_L \Rightarrow E \lesssim 500 \text{ TeV}
\]

2.2. Cascading of fast-mode turbulence

Yan & Lazarian (2004) have studied the cascading of fast-mode turbulence, that is randomly driven by turbulence on large scales \( L \) in a compressible medium. They find a cascading timescale

\[
\Gamma_{\text{fast}} \approx \sqrt{\frac{k}{L} \frac{V_t^2}{V_\phi}}
\]
where \( V_L \) is the turbulence velocity at the injection scale and \( V_\phi \) is the phase velocity of fast-mode wave and equal to the Alfvén and sound velocity for high- and low-\( \beta \) plasma, respectively. Typical values for the sound and Alfvén speed in the downstream region of the shock of a young SNR are

\[
C_s \approx 10^8 \text{ cm s}^{-1} \\
V_A \approx (2 \cdot 10^7 \text{ cm s}^{-1}) \left( \frac{B}{100 \mu \text{G}} \right) \left( \frac{n_d}{\text{atoms cm}^{-3}} \right)^{-1/2}
\]

so both characteristic velocities may be expected to be similar, corresponding to \( \beta \approx 1 \) in the downstream plasma. Inserting the cascading rate (10) as damping rate into the turbulence transport equation (2) we find that the turbulent magnetic field, if composed of fast-mode waves, should exponentially decay on a length scale

\[
l_f \approx (10^{16} \text{ cm}) \left( \frac{U}{1000 \text{ km/s}} \right) \left( \frac{V_\phi}{1000 \text{ km/s}} \right) \\
\times \left( \frac{V_L}{1000 \text{ km/s}} \right)^{-2} \left( \frac{L}{3 \text{ pc}} \right)^{1/2} \left( \frac{\lambda}{10^{15} \text{ cm}} \right)^{1/2}
\]

where for the turbulence driving scale and velocity we have used values that are characteristic of the SNR blast wave itself.

### 2.3. Cascading of Alfvén modes

Farmer & Goldreich (2004) have investigated the cascading of Alfvén modes by MHD turbulence that is driven on the outer scale \( L \). The damping rate is lowest, and the growth rate by cosmic-ray streaming is highest, for parallel-propagating Alfvén waves. All other waves with the same (parallel) wavelength should damp faster than the parallel propagating Alfvén waves, so a lower limit to the damping rate of Alfvén waves would be

\[
\Gamma_A \gtrsim \frac{V_A}{\sqrt{\lambda L}}
\]

Again, inserting the cascading rate (13) as damping rate into the turbulence transport equation (2) we find that the turbulent magnetic field, if composed of Alfvén waves, should exponentially decay on a length scale

\[
l_A \lesssim (10^{17} \text{ cm}) \left( \frac{U}{1000 \text{ km/s}} \right) \left( \frac{V_A}{500 \text{ km/s}} \right)^{-1} \\
\times \left( \frac{L}{3 \text{ pc}} \right)^{1/2} \left( \frac{\lambda}{10^{15} \text{ cm}} \right)^{1/2}
\]

which is very similar to the length scale obtained for the cascading of fast-mode waves in background MHD turbulence (Eq. 12).
3. Discussion

Limited and incomplete though they likely are, all models of strong turbulence cascading predict a damping length of magnetic turbulence $l_d = (10^{16} - 10^{17})$ cm that is generally small enough to produce a magnetic filament. The length scale of X-ray filaments, that are limited by electron energy losses, is larger than $10^{17}$ cm even for a strongly amplified magnetic field with $B \simeq 100 \, \mu$G.

To obtain a qualitative understanding of the X-ray properties of magnetic filaments we performed a simplified calculation for a spherically symmetric SNR with a forward shock located at the radius $r_s = 10$ pc. The magnetic is isotropic and its spatial distribution follows

$$B = (5 \, \mu G) + (45 \, \mu G) \exp \left( \frac{r - r_s}{0.1 \, \text{pc}} \right)$$

(15)

The differential electron density is at first assumed to follow

$$N(E) = N_0 \, E^{-2} \, \exp \left( -\frac{E}{100 \, \text{TeV}} \right) \Theta (r_s - r)$$

(16)

We have also studied an exponential decrease of the electron density on the lengthscale 0.5 pc, i.e. $\propto \exp((r - r_s)/0.5 \, \text{pc})$, as a rough approximation to the effects of a finite age of the high-energy electron population.

The resulting X-ray intensity distribution as a function of the projected distance from the SNR center, $r_p$, is an integral over the synchrotron emission coefficient, $j_\nu$,

$$I_\nu(r_p) = \int_{-\infty}^{\infty} dx \, j_\nu(r = \sqrt{x^2 + r_p^2})$$

(17)

and is shown in Fig.1 for three cases.

To be noted from the figure is a weak frequency dependence of the downstream width of the X-ray filaments, which is caused by the variation of the cut-off frequency of the synchrotron spectrum on account of the spatial variation of the magnetic field strength. Spectral modeling of the radio-to-X-ray spectra of young SNR suggests that the cut-off frequencies are generally found in the X-ray band (Reynolds & Keohane 1999), so we must expect some frequency dependence of the width of X-ray filaments, even if they are dominantly magnetic.

One should also note that any ageing or finite age of the electron population would enhance the contrast between the intensity of the filament and that of the plateau emission from the interior of the remnants. For young SNR electron energy losses would be generally unimportant outside of the magnetic filament, though.
Fig. 1.— The nonthermal X-ray intensity as a function of the projected distance from the SNR center. The acceleration site is assumed located at $r = 10$ pc. The solid line shows the intensity at 1 keV for a homogeneous electron density, and the dotted line displays the (scaled) intensity at 10 keV X-ray energy. The dashed line is derived for the same parameters as the solid line, except that the electron density is assumed to exponentially decay on a length scale of 0.5 pc.
Our results have serious consequences for the modeling of leptonic TeV-scale \( \gamma \)-ray emission, as that is produced anywhere, where high-energy electrons reside, and not only inside the filaments. A careful study of the acceleration history and thus of the electron distribution inside the remnant is required to obtain accurate estimates of the spectral energy distribution of leptonic emission from young SNR.

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