Chiral symmetry in electromagnetically induced resonant two hadron production

E. Oset$^{1,2}$, A. Ramos$^3$, E. Marco$^{1,2}$, J. C. Nacher$^{1,2}$, H. Toki$^1$

$^1$ Research Center for Nuclear Physics (RCNP), Osaka University, Ibaraki, Osaka 567-0047, Japan.
$^2$ Departamento de Física Teórica and IFIC, Centro Mixto Universidad de Valencia-CSIC 46100 Burjassot (Valencia), Spain.
$^3$ Departament d’Estructura i Constituents de la Materia, Universitat de Barcelona, Diagonal 647, 08028 Barcelona, Spain.

Abstract

We report on recent progress on the chiral unitary approach, analogous to the effective range expansion in Quantum Mechanics, which is shown to have a much larger convergence radius than ordinary chiral perturbation theory, allowing one to reproduce data for meson meson interaction up to 1.2 GeV. Applications to physical processes so far unsuited for a standard chiral perturbative approach are presented, in which two hadrons, either two mesons or a meson and a baryon, in a resonant state are produced in photoproduction processes.

1 Chiral Unitary Approach

Chiral perturbation theory ($\chi PT$) has proved to be a very suitable instrument to implement the basic dynamics and symmetries of the meson meson and meson baryon interaction [1] at low energies. The essence of the perturbative technique, however, precludes the possibility of tackling problems where resonances appear, hence limiting tremendously the realm of applicability. The method that we expose naturally leads to low lying resonances and allows one to face many problems so far intractable within $\chi PT$.

The method incorporates new elements: 1) Unitarity is implemented exactly; 2) It can deal with coupled channels allowed with pairs of particles from the octets of stable pseudoscalar mesons and $(\frac{1}{2}^+)$ baryons; 3) A chiral expansion in powers of the external four-momentum of the lightests pseudoscalars is done for $\text{Re } T^{-1}$, instead of the $T$ matrix itself which is the case in standard $\chi PT$. 
We sketch here the steps involved in this expansion for the meson meson interaction. One starts from a $K$ matrix approach in coupled channels where unitarity is automatically fulfilled and writes

$$T^{-1} = K^{-1} - i \sigma,$$  (1)

where $T$ is the scattering matrix, $K$ a real matrix in the physical region and $\sigma$ is a diagonal matrix which measures the phase-space available for the intermediate states

$$\sigma_{nn}(s) = -\frac{k_n}{8\pi \sqrt{s}} \theta \left( s - (m_1n + m_{2n})^2 \right),$$  (2)

where $k_n$ is the on shell CM momentum of the meson in the intermediate state $n$ and $m_1n, m_{2n}$ are the masses of the two mesons in the state $n$. The meson meson states considered here are $K\bar{K}, \pi\pi, \pi\eta, \eta\eta, \pi K, \bar{\pi} K, \eta K, \eta\bar{K}$. Since $K$ is real, from eq. (1) one sees that $K^{-1} = \text{Re } T^{-1}$. In non-relativistic Quantum Mechanics, in the scattering of a particle from a potential, it is possible to expand $K^{-1}$ in powers of the momentum of the particle at low energies as follows (in the s-wave for simplicity)

$$\text{Re } T^{-1} \equiv K^{-1} = \sigma \cdot \text{ctg} \delta \propto -\frac{1}{a} + \frac{1}{2} r_0 k^2,$$  (3)

with $k$ the particle momentum, $a$ the scattering length and $r_0$ the effective range.

The ordinary $\chi$PT expansion up to $O(p^4)$ is given by \[1\]

$$T = T_2 + T_4,$$  (4)

where $T_2$, which is $O(p^2)$, is obtained from the lowest order chiral Lagrangian, $L^{(2)}$, whereas $T_4$ contains one loop diagrams in the s, t, u channels, constructed from the lowest order Lagrangian, tadpoles and the finite contribution from the tree level diagrams of the $L^{(4)}$ Lagrangian. This last contribution, after a suitable renormalization, is just a polynomial, $T^{(p)}$. Our $T$ matrix, starting from eq. (1) is given by

$$T = [\text{Re } T^{-1} - i \sigma]^{-1} \equiv T_2 [T_2 \text{Re } T^{-1} T_2 - i T_2 \sigma T_2]^{-1} T_2,$$  (5)

where, in the last step, we have multiplied by $T_2 T_2^{-1}$ on the left and $T_2^{-1} T_2$ on the right for technical reasons. But using standard $\chi$PT we obtain the following expansion up to order $O(p^4)$,

$$T_2 \text{Re } T^{-1} T_2 = T_2 - \text{Re } T_4...$$  (6)

and hence, recalling that $\text{Im } T_4 = T_2 \sigma T_2$, one obtains

$$T = T_2 [T_2 - T_4]^{-1} T_2,$$  (7)

which is the coupled channel generalization of the inverse amplitude method of \[2\].
Once this point is reached one has several options to proceed:

a) A full calculation of $T_4$ within the same renormalization scheme as in $\chi PT$ can be done. The eight $L_i$ coefficients from $L^{(4)}$ are then fitted to the existing meson meson data on phase shifts and inelasticities up to 1.2 GeV, where 4 meson states are still unimportant. This procedure has been carried out in [2, 3]. The resulting $L_i$ parameters are compatible with those used in $\chi PT$. At low energies the $O(p^4)$ expansion for $T$ of eq. (7) is identical to that in $\chi PT$. However, at higher energies the nonperturbative structure of eq. (7), which implements unitarity exactly, allows one to extend the information contained in the chiral Lagrangians to much higher energy than in ordinary $\chi PT$, which is up to about $\sqrt{s} \simeq 400$ MeV. Indeed it reproduces the resonances present in the $L = 0, 1$ partial waves.

b) A technically simpler and equally successful additional approximation is generated by ignoring the crossed channel loops and tadpoles and reabsorbing them in the $\hat{L}_i$ coefficients given the weak structure of these terms in the physical region. The fit to the data with the new $\hat{L}_i$ coefficients reproduces the whole meson meson sector, with the position, widths and partial decay widths of the $f_0(980)$, $a_0(980)$, $\kappa(900)$, $\rho(770)$, $K^*(900)$ resonances in good agreement with experiment [4]. A cut off regularization is used in [4] for the loops in the s-channel. By taking the loop function with two intermediate mesons

$$G_{nn}(s) = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_{2n}^2 + i\epsilon} \frac{1}{(P - q)^2 - m_{2n}^2 + i\epsilon},$$

(8)

where $P$ is the total meson meson momentum, one immediately notices that

$$\text{Im} G_{nn}(s) = \sigma_{nn}.$$ (9)

Hence, we can write

$$\text{Re} T_4 = T_2 \text{Re} G T_2 + T_4^{(p)},$$

(10)

where $\text{Re} G$ depends on the cut off chosen for $|q|$. This means that the $\hat{L}_i$ coefficients of $T_4^{(p)}$ depend on the cut off choice, much as the $L_i$ coefficients in $\chi PT$ depend upon the regularization scale.

c) For the $L = 0$ sector (also in $L = 0$, $S = -1$ in the meson baryon interaction) a further technical simplification is possible. In these cases it is possible to choose the cut off such that, given the relation between $\text{Re} G$ and $T_4^{(p)}$, this latter term is very well approximated by $\text{Re} T_4 = T_2 \text{Re} G T_2$. This is impossible in those cases because of the predominant role played by the unitarization of the lowest order $\chi PT$ amplitude, which by itself leads to the low lying resonances, and because other genuine QCD resonances appear at higher energies.

In such a case eq. (5) becomes

$$T = T_2 [T_2 - T_2 G T_2]^{-1} T_2 = [1 - T_2 G]^{-1} T_2,$$

(11)
or, equivalently,

\[ T = T_2 + T_2 GT, \]  

which is a Bethe-Salpeter equation with \( T_2 \) and \( T \) factorized on shell outside the loop integral, with \( T_2 \) playing the role of the potential. This option has proved to be successful in the \( L = 0 \) meson meson sector in [3] and in the \( L = 0, S = -1 \) meson baryon sector in [6].

In the meson baryon sector with \( S = 0 \), given the disparity of the masses in the coupled channels \( \pi N, \eta N, K \Sigma, K \Lambda \), the simple “one cut off approach” is not possible. In [7] higher order Lagrangians are introduced while in [8] different subtraction constants (or equivalently different cut offs) in \( G \) are incorporated in each of the former channels leading in both cases to acceptable solutions when compared with the data.

### 2 Application to the photoproduction of meson baryon pairs in resonant states

As quoted above, a good description of the \( K^-p \) and coupled channel interaction is obtained in terms of the lowest order Lagrangians and the Bethe Salpeter equation with a single cut off. One of the interesting features of the approach is the dynamical generation of the \( \Lambda(1405) \) resonance just below the \( K^-p \) threshold. The threshold behavior of the \( K^-p \) amplitude is thus very much tied to the properties of this resonance. Modifications of these properties in a nuclear medium can substantially alter the \( K^-p \) and \( K^- \) nucleus interaction and experiments looking for these properties are most welcome. Some electromagnetic reactions appear well suited for these studies. Application of the chiral unitary approach to the \( K^-p \to \gamma \Lambda, \gamma \Sigma^0 \) reactions at threshold has been carried out in [9] and a fair agreement with experiment is found. In particular one sees that the coupled channels are essential to get a good description of the data, increasing the \( K^-p \to \gamma \Sigma^0 \) rate by about a factor 16 with respect to the Born approximation.

In a recent paper [10] we propose the \( \gamma p \to K^+ \Lambda(1405) \) reaction as a means to study the properties of the resonance, together with the \( \gamma A \to K^+ \Lambda(1405)A' \) reaction to see the modification of its properties in nuclei. The resonance \( \Lambda(1405) \) is seen in its decay products in the \( \pi \Sigma \) channel, but as shown in [10] the sum of the cross sections for \( \pi^0 \Sigma^0, \pi^+ \Sigma^-, \pi^- \Sigma^+ \) production has the shape of the resonance \( \Lambda(1405) \) in the \( I = 0 \) channel. Hence, the detection of the \( K^+ \) in the elementary reaction, looking at \( d\sigma/dM_I \) (\( M_I \) the invariant mass of the meson baryon system which can be induced from the \( K^+ \) momentum), is sufficient to get a clear \( \Lambda(1045) \) signal. In nuclear targets Fermi motion blurs this simple procedure (just detecting the \( K^+ \)), but the resonance properties can be reconstructed by observing the decay products in the \( \pi \Sigma \) channel. In fig. 1 we show the cross sections predicted for the \( \gamma p \to K^+ \Lambda(1405) \) reaction looking
at $K^+ \pi^0 \Sigma^0$, $K^+ \text{all}$ and $K^+ \Lambda(1405)$ (alone). All of them have approximately the same shape and strength given the fact that the $I = 1$ contribution is rather small.

![Graph showing cross section for $\gamma p \rightarrow K^+ X$ with $X = \text{all, } \pi^0 \Sigma^0, \Lambda(1405)$.](image)

Figure 1: Cross section for $\gamma p \rightarrow K^+ X$ with $X = \text{all, } \pi^0 \Sigma^0, \Lambda(1405)$.

The energy chosen for the photon is $E_\gamma = 1.7$ GeV which makes it suitable of experimentation at SPring8/RCNP, where the experiment is planned [11], and TJNAF.

One variant of this reaction is its time reversal $K^- p \rightarrow \Lambda(1405) \gamma$. This reaction, for a $K^-$ momentum in the 300 to 500 MeV/c range, shows clearly the $\Lambda(1405)$ resonant production [12] and has the advantage that the analogous reaction in nuclei still allows the observation of the $\Lambda(1405)$ resonance with the mere detection of the photon, the Fermi motion effects being far more moderate than in the case of the $\gamma A \rightarrow K^+ \Lambda(1405) X$ reaction which requires larger photon momenta and induces a broad distribution of $M_I$ for a given $K^+$ momentum.

One of the interesting developments around these lines is the interaction of the $K^-$ with the nuclei, with its relationship to problems like $K^-$ atoms or the possible condensation of $K^-$ in neutron stars. The problem has been looked at from the chiral perspective by evaluating Pauli blocking effects on the nucleons of the intermediate $\bar{K}N$ states [13, 14]. These effects lead to a $K^-$ self-energy in nuclei which is attractive already at very low densities, as a consequence of pushing the resonance at energies above $K^- p$ threshold. However, more recent investigations considering the $\bar{K}$ self-energy in a self-consistent way [15] lead to quite different results since the resonance barely changes its position.
Yet one still gets an attractive self-energy which is demanded by the $K^-$ atom data \cite{14}. A step forward in this direction is given in \cite{17}, where in addition to the $K^-$ self-energy in the medium, one also renormalizes the pions and takes into account the different binding of $N$, $\Sigma$ and $\Lambda$ in nuclei. Preliminary results from \cite{18} indicate that the $K^-$ self-energy obtained in \cite{17} can lead to a good microscopical description of present data on $K^-$ atoms, hence providing an accurate tool to study the properties of $K^-$ at higher densities and the eventual condensation in neutron stars.

3 Photoproduction of resonant two meson states

Another application which can be done using the same reaction is the photoproduction of resonant two meson states. Particularly the $f_0(980)$ and $a_0(980)$ resonances. These states appear in $L = 0$ in isospin zero and one respectively. The scalar sector of the meson is very controversial and the chiral unitary theory has allowed one to bring a new perspective on these states. Concretely the $a_0(980)$ is a resonant state of two mesons, mostly $KK$ coming from the interaction of the mesons, $\pi\eta$ and $KK$, in the coupled channel unitary approach, while the $f_0(980)$ state is still mostly a resonant state of $KK$ and $\pi\pi$, but it has also a small admixture of a genuine, $q\bar{q}$, state, a singlet of SU(3), which appears around 1 GeV in a study of the meson meson interaction in which the lowest order of $\chi PT$ is implemented together with the explicit exchange of preexisting resonances which provide the contribution to the higher order terms of the chiral Lagrangians \cite{19}. In the present case the reaction suggested is \cite{20}

$$\gamma p \to pM$$

where $M$ is either of the resonances $a_0(980)$ or $f(980)$. In practice the meson M will decay into two mesons, $\pi\pi$ or $KK$ in the case of the $f_0(980)$ or $KK$, $\pi\eta$ in the case of the $a_0(980)$.

In Fig. 2 we show the results for the 5 channels considered. We observe clear peaks for $\pi^+\pi^-$, $\pi^0\pi^0$ and $\pi^0\eta$ production around 980 MeV. The peaks in $\pi^+\pi^-$ and $\pi^0\pi^0$ clearly correspond to the formation of the $f_0(980)$ resonance, while the one in $\pi^0\eta$ corresponds to the formation of the $a_0(980)$ resonance. The $\pi^0\pi^0$ cross section is $\frac{1}{2}$ of the $\pi^+\pi^-$ one due to the symmetry factor. The $K^+K^-$ and $K^0\bar{K}^0$ production cross section appears at energies higher than that of the resonances and hence do not show the resonance structure. Yet, final state interaction is very important and increases appreciably the $K^+K^-$ production cross section for values close to threshold with respect to the Born approximation.

It is also interesting to see the shapes of the resonances which differ appreciably from a Breit-Wigner, due to the opening of the $KK$ channel just above the resonance \cite{21}.
Figure 2: Results for the photoproduction cross section on protons as a function of the invariant mass of the meson-meson system.

We would like to stress here that the invariant mass distributions for resonance excitation into the various pseudoscalar channels depicted in Fig. 2 are theoretical predictions of a chiral unitary model, in this case the one of [5], where only one parameter was fitted to reproduce all the data of the meson-meson interaction in the scalar sector.

A small variant of this reaction would be the $\gamma p \rightarrow nM\bar{M}$. In this case the $M\bar{M}$ system has charge +1 and hence $I = 0$ is excluded, hence, one isolates the $a_0$ production.

It is interesting to notice that the $f_0(980)$ resonance shows up as a peak in the reaction. This is in contrast to the cross section for $\pi\pi \rightarrow \pi\pi$ in $I = 0$ which exhibits a minimum at the $f_0$ energy because of the interference between the $f_0$ contribution and the $\sigma(500)$ broad resonance.

However, we should bear in mind that we have plotted there the contribution of the $f_0$ resonance alone. The tree level contact term and Bremsstrahlung diagrams, plus other contributions which would produce a background, are not considered there. We estimate the background from the experimental cross section for $\gamma p \rightarrow p\pi^+\pi^-$ of [22], which is around 45 $\mu$b at $E_\gamma = 1.7$ GeV. This provides a background of around 55 $\mu$b/GeV while the resonant peak has about 2.5 $\mu$b/GeV strength. This gives a ratio of 5% signal to background assuming that the background is mostly real versus an imaginary contribution from the resonance and hence there would be no interference. The situation with the $\pi^0\pi^0$ channel should be better because the $\gamma p \rightarrow \pi^0\pi^0 p$ cross section is about eight times smaller than the one for $\gamma p \rightarrow \pi^+\pi^- p$ [23, 24]. Considering that the resonant signal now is a factor two smaller than the $\gamma p \rightarrow \pi^+\pi^- p$ cross section, this would give a ratio of signal to background of 20%, which should
be more clearly visible in the experiment. The same or even better ratios than in the \( \pi^0 \pi^0 \) case are expected for \( \pi^0 \eta \) production in the \( a_0 \) channel, since estimates of the background along the lines of present models for \( \pi^0 \pi^0 \) production \[25, 26\] would provide a cross section smaller than for \( \pi^0 \pi^0 \) production.

However, there is a distinct feature about the \( f_0 \) resonance which makes its contribution, in principle, bigger than the estimates given above. Indeed, the \( f_0 \) is approximately a Breit-Wigner resonance with an extra phase of \( e^{i \pi/2} \). This means that the real part has a peak while the imaginary part changes sign around the resonance energy. This means that assuming the background basically real, there would be an interference with the \( f_0 \) resonance which would lead to an increase of about 50\% over the background, or a decrease by about 40\% (depending on the relative sign) for the \( \pi^+ \pi^- \) case and larger effects for the \( \pi^0 \pi^0 \) case. This is of course assuming weak dependence on momenta and spin of the background amplitudes. In any case, due to the particular feature of the \( f_0 \) resonance discussed above, it is quite reasonable to expect bigger signals than the estimates based on a pure incoherent sum of cross sections.

Certainly it is possible to obtain better ratios if one looks at angular correlations. If one looks in a frame where the two mesons are in their CM, the Bremsstrahlung pieces (both from the squared of the Bremsstrahlung term as well as from interference with s-wave terms) have a \( \sin^2 \theta \) dependence, with \( \theta \) the angle between the meson and the photon. Other terms from \[25, 26\] exhibit equally strong angular dependence, for what extraction of the angle independent part of the cross section would be an interesting exercise which would select the part of the cross section to which the resonant contribution obtained here belongs to.

4 Summary

We have reported on the unitary approach to meson meson and meson baryon interaction using chiral Lagrangians, which has proved to be an efficient method to extend the information contained in these Lagrangians to higher energies where \( \chi PT \) cannot be used. This new approach has opened the doors to the investigation of many problems so far intractable with \( \chi PT \) and a few examples have been reported here. We have applied these techniques to the problem of photoproduction of scalar mesons \( f_0(980) \), \( a_0(980) \) and the photoproduction of the \( \Lambda(1405) \), a resonant state of meson baryon in the \( S = -1 \) sector and have found signals which are well within measuring range in present facilities. The experimental implementation of these experiments contrasted with the theoretical predictions will contribute with new tests of these emerging pictures implementing chiral symmetry and unitarity, which for the moment represent the most practical approach to QCD at low energies.
Acknowledgments.

We are thankful to the COE Professorship program of Monbusho which enabled E. O. to stay at RCNP where part of the work reported here has been done. E. M. and J. C. N. would like to acknowledge the hospitality of the RCNP of the Osaka University and support from the Ministerio de Educacion y Cultura. This work is partly supported by DGICYT, contract number PB 96-0753.

References

[1] J. Gasser and H. Leutwyler, Ann. Phys. 158(1984)142; Nucl. Phys. B250(1985)465

[2] A. Dobado, M. J. Herrero and T. N. Truong, Phys. Lett. B235(1990)134; A. Dobado and J. R. Peláez, Phys. Rev. D47(1993)4883

[3] F. Guerrero and J. A. Oller, Nucl. Phys. B 537(1999)459

[4] J. A. Oller, E. Oset and J. R. Peláez, Phys. Rev. Lett. 80(1998)3452; Phys. Rev. D in print [hep-ph/9804209]

[5] J. A. Oller and E. Oset, Nucl. Phys. A620(1997)438; erratum Nucl. Phys. A624(1999)407

[6] E. Oset and A. Ramos, Nucl. Phys. A635(1998)99

[7] N. Kaiser, T. Waas and W. Weise, Nucl. Phys. A612 (1997) 297

[8] A. Parreño, A. Ramos and E. Oset, in preparation; A. Parreño, poster session in the Conference B321(1989)311

[9] T. S. H. Lee, J. A. Oller, E. Oset and A. Ramos, Nucl. Phys A643(1998)402

[10] J. C. Nacher, E. Oset, H. Toki and A. Ramos, Phys. Lett. B455(1999) 55

[11] T. Nakano, Talk at KEK Tanashi Symposium on Physics of Hadrons and Nuclei, Tokyo december 1998

[12] J. C. Nacher, E. Oset, H. Toki and A. Ramos, Phys. Lett. B in print, [nucl-th/9902071]

[13] V. Koch, Phys. Lett. B337(1994)7

[14] T. Waas, N. Kaiser and W. Weise, Phys. Lett. B365(1996)12; B379(1996)34; T. Waas and W. Weise, Nucl. Phys. A625(1997)287

[15] M. Lutz, Phys. Lett. B426(1998)12
[16] C. J. Batty, E. Friedman and A. Gal, Phys. Rep. 287(1997)385

[17] A. Ramos and E. Oset, nucl-th/9906016

[18] Y. Okumura, S. Hirenzaki, H. Toki, A. Ramos and E. Oset in preparation

[19] J. A. Oller and E. Oset, submitted to Phys. Rev. D in print hep-ph/9809337

[20] E. Marco, E. Oset and H. Toki, Phys. Rev. C in print, nucl-th/9905046

[21] S. M. Flatté, Phys. Lett. B63(1976) 224

[22] ABBHFM Collaboration, Phys. Rev. 175(1968)1669

[23] A. Zabrodin et al., Phys. Lett. B 401 (1997) 1617.

[24] F. Härter et al., Phys. Lett. B 401 (1997) 229.

[25] J.A. Gómez Tejedor and E. Oset, Nucl. Phys. A 571 (1994) 667; ibid A 600 (1996) 413.

[26] K. Ochi, M. Hirata and T. Takaki, Phys. Rev. C 56 (1997) 1472.