We review various features of the $R$-parity breaking phenomenology, with particular attention to the low energy observables, and to the patterns of the $R$-parity breaking interactions that arise in Grand Unified models.

1 Introduction

The supersymmetrization of the Standard Model (SM) requires enlarging the spectrum of the theory. The quarks and leptons or the Higgs scalars become components of supersymmetry group representations, the chiral superfields. The notation is presented in table 1. According to the usual convention we denote the supersymmetric particles by a tilde.

| Chiral Superfield | Left Weyl fermion | Complex scalar | $SU(3)_c \times SU(2)_L \times U(1)_Y$ representation |
|------------------|-------------------|---------------|-------------------------------------------------|
| $L$              | $l$               | $\bar{l}$     | $(1, 2, -1/2)$                                  |
| $E^c$            | $e^c$             | $\bar{e}^c$   | $(1, 1, 1)$                                     |
| $Q$              | $q$               | $\bar{q}$     | $(3, 1, 1/6)$                                   |
| $U^c$            | $u^c$             | $\bar{u}^c$   | $(3^*, 1, -2/3)$                                |
| $D^c$            | $d^c$             | $\bar{d}^c$   | $(3^*, 1, 1/3)$                                 |
| $H_1$            | $h_1$             | $h_1$         | $(1, 2, -1/2)$                                  |
| $H_2$            | $h_2$             | $h_2$         | $(1, 2, 1/2)$                                   |

Table 1: Chiral superfields and corresponding component fields.

Notice that supersymmetry implies that two Higgs doublets, $h_1$ and $h_2$, are present.

Schemes of supersymmetry breaking allow us to generate superpartners mass patterns consistent with the present non-observation of superparticles. Theoretical arguments require these masses to be below the TeV range, making the supersymmetric extension of the Standard Model interesting for present and future searches.

Each interaction of the Standard Model can be generalized in a supersymmetric invariant form. For instance the down Yukawa interactions read:

$$\mathcal{L}_{\text{Yukawa}} = -Y_{D_{ij}} \left[ h_1 q_i \tilde{d}_j^c + \tilde{h}_1 \tilde{q}_i \tilde{d}_j^c + \bar{h}_1 \tilde{q}_i d_j^c \right] + \text{h.c.}$$

(YD are the down Yukawa couplings; \(i, j\) are family indices; $SU(2)_L$ doublets are contracted with $i\gamma_2$.) Due to supersymmetry, the SM interaction induces similar interactions between pairs of superpartners.

It is also easy to write interactions that have no SM analogue. This happens when superpartners behave as a dilepton, $l_i l_j\tilde{e}_k^c$; or as leptoquarks, $l_i \tilde{q}_j d_k^c$ and $l_i q_j \tilde{d}_k^c$; or finally as diquarks, $\bar{d}_i^c d_j^c \bar{u}_k^c$ and $d_i^c d_j^c \tilde{u}_k^c$ (the supersymmetric form can be easily inferred in analogy with eq. (1)). We conclude that SM gauge invariance does not assure lepton and/or baryon number conservation in supersymmetric context. Notice that these interactions are not necessarily linked to the supersymmetric breaking mechanism, or to the structure of the Higgs sector, about which we have not direct experimental informations yet, but just depend on the spectrum of the model and on SM gauge invariance.

Some or all the interactions above can be forbidden adding more symmetry to the model. Such a symmetry can be local or global, continuos or discrete. A widely used possibility is given by the $Z_2$ transformation upon which
only the superpartner changes sign: the $R$-parity. Since $R$-parity forbids the terms introduced above, this assumption amounts to baryon and lepton number conservation\textsuperscript{1}. One can even speculate about the origin of such a symmetry. But there are still no experimental keys to know which is the scheme chosen by Nature. Therefore, a phenomenological attitude toward the supersymmetric paradigm requires to study the consequences of relaxing the assumption of $R$-parity conservation.

The plan of the exposition is as follows: First, we define the $R$-parity breaking interactions, and study their possible manifestations and some experimental bounds. We pay particular attention to the rare and exotic low-energy processes. Then, in an effort to obtain finer control on these interactions, we consider them in the context of Grand Unification (GU). We discuss an interesting scenario, in which sizable $R$-parity breaking interactions can be reconciled with Grand Unification program.

\section{$R$-parity breaking}

In this section we define the $R$-parity breaking couplings, and study possible manifestations of their presence. We figure out important processes and give a feeling of the existing bounds on the couplings. The possibility-risk that $R$-parity breaking couplings make the ordinary matter unstable is analyzed.

\subsection{Definitions and fundamental facts.}

The superpotential $W$ gives us a compact formalism to describe the supersymmetric interactions of matter fields: By definition it is an analytic function of the chiral left superfields present in the theory. Let us decompose $W = W_R + W_{\tilde{R}}$. Considering the renormalizable interactions, the $R$-parity conserving part reads:

$$W_R = \frac{m_u}{v_1} H_1 L_i E_i^c + \frac{m_d}{v_1} H_1 Q_j V^c_{ji} D_i^c - \frac{m_u}{v_2} H_2 Q_i U_i^c + \mu H_1 H_2$$

whereas the $R$-parity violating part reads:

$$W_{\tilde{R}} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \lambda''_{ijk} D_i^c D_j^c U_k^c + \mu_i L_i H_2$$

The superpotential $W$ is written in terms of superfields with fermion mass eigenstates, so that the Cabibbo-Kobayashi-Maskawa matrix $V_{ij}$ appears in (2) and (3) explicitly as well as in $Q_i = (U_i, V_i D_i)$; $m_u$, $m_d$, $m_u$, are the fermion masses. Finally, $v_{1,2}$ are the vacuum expectation values of the scalar components of the superfields $H_1^0, H_2^0$.

Notice that with a proper redefinition, $\mu H_1 + \mu_i L_i \rightarrow \mu H_1$ the last term can be eliminated from the superpotential. Therefore we will assume in the following $\mu_i = 0$. In passing, we remark that the presence of all but the third term in (3) is due to the fact that the Higgs $H_1$ and the three lepton doublets $L_i$ are identical from the point of view of gauge symmetry.

$W_R$ is by definition the superpotential of the Minimal Supersymmetric extension of the SM (MSSM). Notice that in full analogy with the SM case \textsuperscript{2} conserves the four $U(1)$ numbers related to $B$ (baryon charge), $L_1$, $L_2$ and $L_3$ (lepton charges) where the definitions are done on the superfields\textsuperscript{2}. Let us therefore analyze the interactions in (3) from the point of view of the global symmetries. They violate either total lepton ($\lambda, \lambda', \mu_i$) or baryon number symmetries ($\lambda''$). One can further divide in two classes the lepton violating terms: The terms in the first class,

$$L_i H_2, \quad L_i Q_k D_i^c, \quad L_i L_j E^c_{ij}, \quad i \neq j$$

carry charges $L_i$, whereas those in the second class,

$$L_1 L_2 E^c_{3i}, \quad L_3 L_1 E^c_{2i}, \quad L_2 L_3 E^c_{1i}$$

carry charges $L_3 = L_1 + L_2 - L_3$, $L_2 = L_1 - L_2 + L_3$ and $L_1 = -L_1 + L_2 + L_3$ respectively. This classification has some importance for lepton violating phenomena. For instance, the neutrino mixing term $\nu_1 \nu_2$ cannot be generated by the operators (4) alone, since its charge is $1/2 (L_1 + L_2) + L_3$ (it would requires “half” vertices; similarly for the other mixings). For the same reason the terms (5) cannot be induced by those of (4) alone.

Few remarks, in order to give a perspective to the present study.

(1) It is of course possible to ascribe $B$- and $L$-violating phenomena to $R$-parity conserving theories, for example in the case of supersymmetric $SU(5)$ model; but, due to the different underlying mechanism, the resulting phenomenology is typically different.

\textsuperscript{1}Even if interactions which violate $R$-parity are forbidden, there is the interesting possibility of spontaneous breaking of the lepton number, due to the vacuum expectation value (VEV) of the sneutrino. Nonzero VEVs of other scalars would instead break color and/or electric charge.

\textsuperscript{2}This definition is forced by the gaugino interactions. Notice that the scalar masses can provide us with sources of violation of hadronic and leptonic flavours, if they are not diagonal in the same basis in which the fermion masses are diagonal.
2.2 Exotic interactions of ordinary matter

Let us consider the effective terms that SM inherits from the $R$-parity breaking interactions when the sleptons and the squarks fields are integrated away. The topologies of Feynman diagrams that is necessary to consider are listed in figure (b).

The operators of greatest interest are clearly those which violate SM conservation laws, the lepton and/or the baryon numbers, or give flavor-changing neutral currents. In the case of the two fermion operators there are the Majorana neutrino masses $\nu\nu$ (fig. (a)); for the six fermions operators, either those of the form $eudueu$, which give for instance $n-\bar{n}$ oscillations (fig. (a),(b)) or those of the form $uddudd$, which give for instance $n-\bar{n}$ oscillations (fig. (d)). We recall that the first two types of operators arise in pure lepton number violating framework, whereas the last just requires violation of baryon number; notice also that their flavor structure can be a priori generic.

Now let us focus the attention on the four fermions operators, arising by diagrams of the topology of fig. (b). They are listed in table 2, together with the couplings involved, the particle exchanged and a typical process triggered.

| Effective operator | Particle exchanged | Couplings involved | Example process |
|--------------------|--------------------|--------------------|-----------------|
| $ee\bar{e}$        | $\bar{\nu}$        | $\lambda^2$        | $\mu^- \rightarrow e^-e^-e^+$ |
| $ev\bar{e}$        | $\bar{\nu}, \bar{\nu}^c,e\bar{e}^c$ | $\lambda^2$        | $\mu^- \rightarrow e^-\nu_e\bar{\nu}_\mu$ |
| $\bar{d}\bar{d}\bar{d}$ | $\bar{\nu}, \bar{\nu}^c,d\bar{d}^c$ | $\lambda^2, \lambda^{2\prime}, \lambda^{2\prime\prime}$ | $K^0-\bar{K}^0$ oscill. |
| $\bar{u}\bar{d}\bar{d}$ | $\bar{\nu}, \bar{\nu}^c,e\bar{e}^c,d\bar{d}^c$ | $\lambda^2, \lambda^{2\prime}, \lambda^{2\prime\prime}$ | $B \rightarrow non\ charmed$ |
| $\bar{u}\bar{d}\bar{e}$ | $\bar{\nu}, \bar{\nu}^c,d\bar{d}^c$ | $\lambda^2, \lambda^{2\prime}, \lambda^{2\prime\prime}$ | $D^+ \rightarrow \pi^+\mu_e$ |
| $\bar{u}\bar{d}\bar{d}$ | $\bar{\nu}, \bar{\nu}^c$ | $\lambda\chi''$ | $B \rightarrow K\nu\bar{\nu}$ |
| $\bar{e}\bar{d}\bar{d}$ | $\bar{\nu}, \bar{\nu}^c$ | $\lambda\chi''$ | $B \rightarrow K\bar{\nu}$ |
| $\bar{d}\bar{d}\bar{d}$ | $\bar{\nu}, \bar{\nu}^c$ | $\lambda\chi''$ | $K_L \rightarrow \mu e$ |
| $\bar{e}\bar{u}\bar{d}$ | $\bar{\nu}, \bar{\nu}^c$ | $\lambda\chi''$ | $p \rightarrow \pi^0e^+$ |
| $\bar{u}\bar{u}\bar{d}$ | $\bar{\nu}, \bar{\nu}^c$ | $\lambda\chi''$ | $p \rightarrow K^+\nu$ |
| $\bar{d}\bar{d}\bar{d}$ | $\bar{\nu}, \bar{\nu}^c$ | $\lambda\chi''$ | $n \rightarrow K^+e^-$ |

Table 2: Four fermions operators resulting from $R$-parity breaking interactions. In first column $\nu$ denotes either the neutrino or the antineutrino field. The propagators like $ee^c$ in second column arise from the mixing of the scalar states $\tilde{e}$ and $\tilde{e}^c$ after $SU(2)_L$ breaking.

The most important operators are clearly those of last three rows of table 2, since they lead to instability of nucleons. As it is well known, they arise due to violations of both the baryon and the lepton number. The fact that there are no four-fermion operators which violates only the baryon number is a general consequence of $SU(3)_c \times U(1)_{e.m.}$ symmetry. Violations of the lepton number are possible, but only in the interactions involving neutrinos: As an example, the exchange of $\tau\tau^\pm$ induces the decay $\mu^- \rightarrow e^-\nu_e\bar{\nu}_\mu$ due to $\lambda_{123}$ and $\lambda_{231}$ couplings.

Table 2 illustrates the need to proceed carefully in introducing the $R$-parity violating couplings, since all kind of non-standard operators can be induced. According to previous observation, a safe possibility of introducing the

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3There is an important consequence of $R$-parity breaking interactions regarding the supersymmetric particles: the lightest supersymmetric particle becomes unstable. See [2] for searches at colliders.

4For further informations see references [1], [7] and [8].

5Unfortunately, existing limits on the single couplings render this process not experimentally interesting in the model under consideration—I thank M. Cooper for a clarification about this point.
$R$-parity violating terms is to forbid the $B$-violating terms, but to retain the lepton violating ones (or vice versa). A more daring possibility is to have both operators to a sufficiently suppressed level. We discuss these possibilities in the following, with particular attention to the manifestations in the low-energy physics.

### 2.3 Lepton-violating scenario

Suppose for a while that $B$-violating terms are absent. Consider the flavor structure of the $R$-parity breaking couplings. Are there couplings unconstrained by rare (or forbidden) processes? A partial answer is provided by table 3. It shows whether three “delicate” observables can be affected or not by the $\lambda'$-type couplings (the precise meaning of the table is: whether the coupling enters or not a tree level diagram relevant for the processes).

We deduce from table 3 that the couplings $\lambda'_{3jj}$ and $\lambda'_{j33}$ do not give contribution to the processes. This means that large values of these couplings are not incompatible with present experimental informations. As a common feature, these couplings do not violate hadronic flavours.

We can somewhat push the above argument. Let us suppose that one $\lambda'$ coupling, which is not in the class above, is large. Table 3 tell us that, in this case, some other $R$-parity couplings is constrained by present experimental bounds. To be quantitative, the observation of whichever coupling $\lambda'_{\text{obs}}$ at the level of $10^{-2}$ would imply a strong suppression ($\geq 10^{-4} \times \lambda'_{\text{obs}}$) of another $\lambda'$ coupling. In absence of a theoretical explanation, this scenario is questionable on the basis of naturalness.

| $K^0 - \bar{K}^0$ | $B^0 - \bar{B}^0$ | $K_L \to \mu e$ |
|-------------------|-------------------|-----------------|
| 111               | x                 | x               |
| 112               | x                 | x               |
| 121               | x                 | x               |
| 211               | x                 | x               |
| 122               | x                 | x               |
| 212               | x                 | x               |
| 221               | x                 | x               |
| 222               | x                 |                 |
| 113               | x                 |                 |
| 131               | x                 | x               |
| 311               | x                 |                 |
| 123               | x                 |                 |
| 132               | x                 | x               |
| 213               | x                 |                 |
| 231               | x                 | x               |
| 312               | x                 | x               |
| 321               | x                 | x               |
| 223               | x                 |                 |
| 232               | x                 | x               |
| 322               | x                 |                 |
| 133               | x                 |                 |
| 313               | x                 |                 |
| 331               | x                 |                 |
| 233               | x                 |                 |
| 323               | x                 |                 |
| 332               | x                 |                 |
| 333               |                   |                 |

Table 3: Rare processes in which the various $\lambda'_{ijk}$ couplings are involved. A sneutrino or an up squark is exchanged.
2.4 Lepton- and baryon-violating scenario

The simultaneous presence of the couplings $\lambda''$ ($B$-violating) and $\lambda'$ ($L$-violating) leads to the possibility of squark-mediated proton decay. This implies very strong bounds on the couplings which allow the decay at tree level:

$$|\lambda' \cdot \lambda''| \lesssim 10^{-24}$$

(6)

for squark masses around 1 TeV.

The bound does not affect certain couplings involving heavy generations. But, since the bounds are so stringent, it is important to check the one loop structure of the theory. It is possible to prove that, choosing whichever pair of couplings $\lambda'$ and $\lambda''$, there is always at least one diagram relevant for the decay at one loop level. This happens due to the flavor-changing interactions, which are present even in the absence of $R$-parity breaking, namely: the interactions of the quarks with the $W$ boson and the charged Higgs, and their supersymmetric counterparts. The less suppressed pair of couplings is still subject to a (conservative) bound on their product,

$$|\lambda' \cdot \lambda''| \lesssim 10^{-9},$$

(7)

according to \[10\]. The simultaneous presence of suitably chosen couplings $\lambda$ and $\lambda''$ seems instead to be less dangerous for proton decay \[11\]. Notice for instance that, due to the symmetry discussed above, the presence of operators of the class \[3\] requires that there are three different leptons in the final states, calling for dimension 9 effective operators for nucleon decay.

Coming to phenomenology, we remark that the squark-mediated nucleon decay may have a very neat experimental signature: the presence of the $(B + L)$-conserving channels\[12\]. These channels are related to effective operators at least of dimension 7 \[13\]. This calls for sources of $SU(2)_L$ breaking, which are provided by left-right squark mass mixing: for the top quark we have $m^2_{\tilde{t}e} \sim m_t \tilde{m}$ where $m_t$ is the top mass, that is not expected to be very different from the typical supersymmetric mass $\tilde{m}$. Regardless of the Lorentz structure, there is only one effective four-field operator at the quark level which mediate $(B + L)$ conserving nucleon decays: $d\bar{d}l\bar{l}$, where $l = e, \mu$. It gives rise to $n \rightarrow K^+l^-$ and $p \rightarrow K^+l^+\pi^+$ decay channels. The first decay, which proceeds with a faster rate, provokes the decay of the neutrons in the stable nuclei. This provide us with a quite clear signal in water Čerenkov detectors:

$$^{16}O \rightarrow ^{15}O + \gamma(6.2 \text{ MeV}) + \mu + l,$$

(8)

where $l$ is monochromatic, $\mu$ results from kaon decay and $\gamma$ from the transition of the excited nucleus to the ground state (a nonobservable neutrino from $K$ decay is also present).

A final remark. Even if it is allowed to speculate on the possibility of very small couplings, it would be much nicer to have a theoretical guideline to explain the size of the couplings. In the context of horizontal symmetry \[9\] \[14\], the smallness of the couplings can be related to suitably large horizontal charges. In our opinion however a defect of these approaches is that they still suffer of considerable latitude in the specification of the models.

3 Supersymmetric Grand Unification and $R$-parity breaking

In previous sections we assumed that:

(i) The Standard Model must be embedded into a supersymmetric theory;

(ii) all the interactions compatible with the gauge symmetry should be a priori present.

Unfortunately, at present, hypothesis (i) lacks of experimental support. This requires to convey special attentions to the theoretical motivations for supersymmetry. Among them, it is prominent the possibility to implement in the supersymmetric context the Grand Unification program (in its minimal form). Therefore we will further specify the theoretical context, and assume that:

(iii) The interactions of the supersymmetric Standard Model are the low energy manifestations of a $SU(5)$ invariant dynamics.

This hypothesis of course implies a specification of the $R$-parity breaking couplings.

In the $SU(5)$ model one can introduce the following $R$-parity violating interactions \[15\]:

$$\Lambda_{ijk} \tilde{5}_i \tilde{5}_j 10_k + \tilde{5}_i (M_i + h_t \Phi) H,$$

(9)

where $i, j, k = 1, 2, 3$ are generation indices, $\Lambda_{ijk}$ are the coupling constants and $\tilde{5}_i$, 10$ _i$ are the matter superfields which can be written (restating the gauge indices) as:

$$\tilde{5}^a = \left( \frac{D^{co}}{e_{AB} L^B} \right),$$  \hspace{1cm} 10_{ab} = \left( \begin{array}{cc} \epsilon_{\alpha\beta\gamma} U^{\alpha\beta} & -Q_B \epsilon_{AB} E^C \\ Q_A \epsilon_{AB} & \epsilon_{AB} E^C \end{array} \right).$$

(10)

\[A\] A different conclusion has been reached by \[1\].

\[B\] Another possible manifestation of these dynamics would be the presence of unexpected branching-ratios for the $(B - L)$-conserving channels of nucleon decay.
where $\epsilon^{12} = \epsilon_{21} = 1$. $M_i$ are mass parameters, $h_i$ are couplings, $\Phi$ and $H$ are the 24-plet and 5-plet of Higgs multiplets. Starting from (11), we will study in the following two possible scenarios for the $R$-parity breaking couplings.

### 3.1 A model with small $R$-parity breaking couplings

We first consider the effects of $\Lambda$ couplings, in a model in which the matter-Higgs mixing (the second term in (3)) is negligible.

It is convenient to define $\Lambda_{ijk}$ in the basis where $SU(2)_L$-singlets $u^c$ and $d^c$ coincide with mass eigenstates. This always can be done since $u^c$ and $d^c$ enter different $SU(5)$-multiplets. Note that due to the antisymmetry of 10-plets the interactions (9) are antisymmetric in generation indices: $\Lambda_{ijk} = -\Lambda_{jik}$.

Substituting the multiplets (10) in (3) and performing the redefinitions of the couplings which bring the $R$-parity conserving part of the superpotential with light fields in the form (3), we find the relations between original $\Lambda_{ijk}$ and $\Lambda_{ijk}$ couplings at the GU scale:

\begin{align}
\Lambda_{ij} & = -\Lambda_{ijk}^t U_{i\ell} U_{j\ell} V_{k\ell} \\
\Lambda_{ijk} & = 2 \Lambda_{ijk}^t U_{j\ell} W_{k\ell} \\
\Lambda_{ijk} & = \Lambda_{ijk}^t
\end{align}

where $U, W, V$ are unitary matrices. The appearance of these matrices can be explained considering that our choice of flavor basis does not fix the flavor structure of the superfield $L$ (respectively $E^c$ and $Q$) which appears together with $D^c$ ($U^c$) in the $SU(5)$ 5-plet (10-plet). They can be calculated fixing the mechanism of mass generation: which Higgs representation are present, which non-renormalizable operators, etc.. We will consider the case:

\begin{align}
U = W = 1, \\
V = V
\end{align}

which corresponds to the assumption that only Higgs 5-plets contribute to the fermion mass matrices.

As a consequence of quark and lepton unification in $SU(5)$, all types of $R$-parity violating couplings appear simultaneously. Moreover, different couplings $\lambda, \lambda'$ and $\lambda''$ are determined by unique GU coupling $\Lambda$. As follows from (11) and (12), these couplings basically coincide at GU scale:

\begin{align}
- \lambda_{ij} V_{ik}^{-1} = \frac{1}{2} \lambda_{jki}' = \lambda''_{ijk}.
\end{align}

Notice that Grand Unification implies that the $L$-violating couplings $\lambda'_{ijk}$ should be antisymmetric in the exchange of the first and third indices: $\lambda'_{ijk} = -\lambda'_{kji}$, similarly to other couplings; in the non-unified version (3) these couplings can have also a symmetric part.

The considerations above apply to the low energy theory up to minor modifications. A not completely negligible effect is the evolution of the couplings due to gauge renormalization. It leads to modification of GU relations (13) at the electroweak scale:

\begin{align}
\lambda_{ijk} & = -1.5 \Lambda_{ij} V_{ik} \\
\lambda'_{jki} & = 2 (3.4 \pm 0.3) \Lambda_{ijk} \\
\lambda''_{ijk} & = (4.4 \pm 0.4) \Lambda_{ijk}
\end{align}

(14)

(15)

The errors correspond to the uncertainty in strong coupling constant: $\alpha_s(M_Z) = 0.12 \pm 0.01$. The inclusion of other uncertainties related e.g. to threshold SUSY and GU corrections may require the doubling of the errors quoted. The renormalization effects due to third family Yukawa couplings do not drastically change the relations (14).

With previous remarks in mind, it is easy to understand that the couplings are subject to quite strong constraints from the proton decay bounds in the case under consideration. To be concrete, let us consider the bound on the coupling $\Lambda_{233}$ (which may be argued to be the dominating one). The proton decay, induced at the one loop level, implies (16):

\begin{align}
\Lambda \lesssim 3 \cdot 10^{-9}
\end{align}

This can be thought as a conservative bound in this kind of GU models for the $R$-parity breaking couplings. We conclude that, whereas present model easily encompasses nucleon instability phenomena (in particular decays which conserve $B + L$, or decays with exotic branching ratios), it cannot account for large $R$-parity breaking couplings.

### 3.2 A model with large $R$-parity breaking couplings

Let us consider a model where the matter-Higgs mixing is the only source of $R$-parity violation. Suggesting that third generation coupling dominates, we can write the appropriate terms of the superpotential in the following way

\begin{equation}
\bar{5}_3 \bar{m} H + \bar{H} M H + y_i \bar{5}_{i10} \hat{H},
\end{equation}

(16)
where $\delta_i$ and $10_i$ are defined in the diagonal basis for down quark Yukawa couplings $y_i$, $i = d, s, b$ so that $d^c_i$ and $d_i$ coincide, up to corrections $M_W/M_{\text{GU}}$, with mass eigenstates. The mass matrices of (14) can be written in the doublet-triplet form as:

$$\hat{m} = \text{diag}(m_{\text{tripl}}, m_{\text{doubl}}), \quad \hat{M} = \text{diag}(M_{\text{tripl}}, M_{\text{doubl}}),$$

where $M_{\text{tripl}} \sim M_{\text{GU}}$ and $M_{\text{doubl}}$, $m_{\text{doubl}}$ and $m_{\text{tripl}}$ are at the electroweak scale (large value of $m_{\text{tripl}}$ would result in the fast proton decay). The explanation of this mass pattern is clearly connected to the explanation of the doublet-triplet (DT) splitting.

The first term in (14) can be eliminated by rotations of the doublet and the triplet components of the 5-plets: $\bar{5}_3 = (B^c, L_3)$ and $H = (T, H_1)$. For triplet components we redefine:

$$c_{\text{tripl}} \bar{T} + s_{\text{tripl}} B^c \rightarrow \tilde{T},$$
$$c_{\text{tripl}} B^c - s_{\text{tripl}} \bar{T} \rightarrow B^c,$$

so that $B^c$ and $\tilde{T}$ are the mass states, $c_{\text{tripl}} \equiv \cos \theta_{\text{tripl}}$, $s_{\text{tripl}} \equiv \sin \theta_{\text{tripl}}$, and

$$s_{\text{tripl}}/c_{\text{tripl}} = m_{\text{tripl}}/M_{\text{tripl}}.$$

For doublet components:

$$c_{\text{doubl}} H_1 + s_{\text{doubl}} L_3 \rightarrow H_1,$$
$$c_{\text{doubl}} L_3 - s_{\text{doubl}} H_1 \rightarrow L_3,$$

and

$$s_{\text{doubl}}/c_{\text{doubl}} = m_{\text{doubl}}/M_{\text{doubl}}.$$

Since $m_{\text{doubl}}, m_{\text{tripl}}, M_{\text{tripl}} \sim M_W$ one gets from (21) and (13) that $s_{\text{tripl}}$ is strongly suppressed, $s_{\text{tripl}} \sim M_W/M_{\text{GU}} < 10^{-14}$, whereas $s_{\text{doubl}}$ can be of the order 1.

Substituting the expressions (13) and (21) into (14) we obtain the effective $R$-parity violating couplings (3). In particular the third generation Yukawa coupling gives

$$\chi_{333}^{\text{eff}} L_3 B^c Q'_3,$$

where

$$\chi_{333}^{\text{eff}} = s_{\text{doubl}} \cdot y_b,$$

and $Q'_3 \equiv V_{33} Q_i$. Baryon violating interactions as well as pure leptonic terms are absent due to the antisymmetry. The Yukawa coupling of the second generation leads to

$$y_s \left[ s_{\text{tripl}} B^c S^c U'_i + s_{\text{doubl}} L_3 S^c Q_i + s_{\text{doubl}} L_2 L_3 E^c_i \right]$$

(24)

(The first generation Yukawa coupling gives similar terms with the substitution $y_d V_{1s} \rightarrow y_d V_{1d}, S \rightarrow D, L_2 \rightarrow L_1$).

The leading contribution to the proton decay is induced by $L$-violating interaction (22) and $B$-violating interaction (24). The $b^c$ exchange dressed by $h^+, h^+$ results in the amplitude for proton decay

$$A \propto \chi_{333}^{\text{eff}} \cdot y_s \ s_{\text{tripl}} \cdot \xi = y_s y_b \ s_{\text{doubl}} s_{\text{tripl}} \ \xi,$$

where $\xi$ is the loop suppression factor. Substituting values of parameters, we find that even for large $\tan \beta$ ($y_b \sim 1$) this amplitude is small enough to allow for $s_{\text{doubl}}$, and consequently, $\chi_{333}^{\text{eff}}$ to be of the order 1. All other diagrams give smaller contributions. (Note that in the considered example all the $B$-violating interactions contain $b^c$ quark, so that even lowest family couplings need a loop “dressing”).

### 3.3 Neutrino masses and large $R$-parity breaking couplings

There is another consequence of the matter-Higgs mixing (16, 17, 18, 19): explicit $R$-parity violating terms in (16) induces in general VEV of sneutrino. Indeed, the relevant terms in the potential at the electroweak scale are:

$$V \geq (m^2_{L_3} + \delta m^2) |h_1|^2 + m^2_{L_3} |\tilde{l}_3|^2 - [B \cdot M_{\text{doubl}} \ h_1 h_2 + (B + \delta B) \cdot m_{\text{doubl}} \ \tilde{l}_3 h_2 + \text{h.c.}].$$

---

8We will not specify any underlying mechanism for DT splitting, but simply observe that it is technically possible to implement it in the present context, carefully choosing $M_i$, $h_i$ and $(\Phi)$ in (4).
To proceed in the discussion, we assume a definite scenario for supersymmetry breaking: the low-energy supergravity model. We suggest that soft breaking terms are universal at the scale $M_{\text{GU}}$ suggested by gauge coupling unification. Then the parameters $\delta m^2$ and $\delta B$ (20) describe the renormalization effect due to the bottom Yukawa coupling from $M_X$ to the electroweak scale. The corresponding renormalization group equations are:

$$
\frac{d}{dt} \delta B = 3 y_B^2 A_b, \\
\frac{d}{dt} \delta m^2 = 3 y_B^2 (m_Q^2 + m_{3/2}^2 + m_{H_1}^2 + A_b^2),
$$

where $t = 1/(4\pi)^2 \log(M_{\text{GU}}^2/Q^2)$. The rotation (20) which eliminates matter-Higgs mixing term in the superpotential generates mixing terms for the sleptons:

$$
V_L \approx \theta_{\text{doublet}} \times [\delta m^2 h_1^1 + \delta B \cdot \mu h_2] \tilde{t}_3 + \text{h.c.}
$$

(for small $\theta_{\text{doublet}}$). After electroweak symmetry breaking these mixing terms, together with soft symmetry breaking masses, induce a VEV of tau sneutrino of the order:

$$
\langle \tilde{\nu}_3 \rangle \sim v \theta_{\text{doublet}} \times \left( \frac{\delta m^2}{m_{L_3}^2} \cos \beta + \frac{\delta B \cdot \mu}{m_{3/2}^2} \sin \beta \right).
$$

The factor in brackets can be estimated as $y_B^6 (3 \cos \beta + 0.5 \mu/m_{L_3} \sin \beta)$, where the figures quoted arise from approximate integration of renormalization group equations (27). Consequently the tau sneutrino VEV is $\langle \tilde{\nu}_3 \rangle \sim v \theta_{\text{doublet}} y_B^2$. Due to this VEV the tau neutrino mixes with the zino, and consequently the mass of tau neutrino is generated via the see-saw mechanism:

$$
g_1^2 + g_2^2 \langle \tilde{\nu}_3 \rangle^2 \frac{1}{M_Z^2}
$$

(see [20, 21]). In the model under consideration this contribution to tau neutrino mass is typically larger than the one produced by the loop-diagram stipulated by the interaction (23).

We can derive from (30) the bound on $R$-parity violating couplings. Taking into account that $\chi_{333}^{\prime \prime \prime} \sim \theta_{\text{doublet}} y_B$, and $\langle \tilde{\nu}_3 \rangle \sim v \theta_{\text{doublet}} y_B^2$ we get the relation between $\chi_{333}^{\prime \prime \prime}$ and neutrino mass

$$
\chi_{333}^{\prime \prime \prime} \sim 0.06 \times \left( \frac{\theta_{\text{doublet}}}{0.1 \text{ rad.}} \right)^{1/2} \left( \frac{m_{\nu_\tau}}{10 \text{ MeV}} \right)^{1/4} \left( \frac{M_Z}{1 \text{ TeV}} \right)^{1/4}.
$$

Therefore it is possible to obtain large $R$-parity violating couplings with tau neutrino masses close to the present experimental limit. For $m_{\nu_\tau} = \mathcal{O}(30 \text{ eV})$, corresponding to the cosmological bound on stable $\nu_\tau$, the coupling $\chi_{333}^{\prime \prime \prime}$ becomes of the order 0.002. For such values of $\chi_{333}^{\prime \prime \prime}$ the detection of supersymmetric particle decays is still possible: the condition to be satisfied is in fact $\chi_{333}^{\prime \prime \prime} \gtrsim 2 \cdot 10^{-5} \sqrt{\gamma} (\bar{m}/1 \text{ TeV})^2 (150 \text{ GeV}/m_\chi)^{3/2}$, where $\gamma$ is the Lorentz boost factor [3].

4 Discussion and conclusions

The $R$-parity breaking couplings offer great possibilities for phenomenological speculations, but, up to now, no effect which should be related to them has been found. This may be due to the fact that $R$-parity breaking couplings are small; in this case one could expect physical effects in rare or forbidden processes. But, just on the basis of the observed phenomena, certain $R$-parity breaking couplings may be large. This unclear situation calls either for further theoretical or experimental informations. It is encouraging that rather clear patterns for $R$-parity breaking couplings emerge in the context of supersymmetric Grand Unification. Models in which both lepton and baryon-violating couplings are present in another kind of models, based on the doublet-triplet splitting. In the context of the low-energy supergravity models for supersymmetry breaking, we pointed to an interesting signature of this second scenario: the correlation between the size of the $R$-parity breaking coupling and the mass of the tau neutrino.

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Footnote: Technically it is possible to implement a cancellation between the two terms in (23) (see [a] for a phenomenological study of such a possibility). However we see no natural reason for this to happen in the supergravity context.
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Figure 1: Topologies of Feynman diagrams involving $R$-parity breaking couplings (represented by the blobs) which induce important interactions among the Standard Model particles. Fermions (bosons) are indicated by continuous (dashed) lines.