Polarized inclusive lepton production, $\ell N \to hX$, and the hadron helicity density matrix, $\rho(h)$: possible measurements and predictions

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Abstract:
We discuss the production of hadrons in polarized lepton nucleon interactions and in the current jet fragmentation region; using the QCD hard scattering formalism we compute the helicity density matrix of the hadron and show how its elements, when measurable, can give information on the spin structure of the nucleon and the spin dependence of the quark fragmentation process. The cases of $\rho$ vector mesons and $\Lambda$ baryons are considered in more details and, within simplifying assumptions, some estimates are given.
1 - Introduction and general formalism

The full description of hard scattering processes involving hadrons always requires a knowledge of both the elementary interactions between the hadronic constituents and the constituent distribution or fragmentation properties; while the former interactions are computable in perturbative QCD or QED the latter properties, i.e. the amount of quarks and gluons inside hadrons and the amount of observed particles resulting from a quark or gluon fragmentation, are non perturbative and cannot be computed in QCD. However, their universality and the QCD knowledge of their $Q^2$ evolution allow, once some information is obtained from certain processes, to use it in other processes in order to make genuine predictions. It is then crucial to collect phenomenological information on these non perturbative quantities.

On the quark and gluon content of the nucleons we have by now gathered plenty of detailed information mainly from unpolarized Deep Inelastic lepton-nucleon Scattering; some information is also available on unpolarized quark fragmentation properties either from DIS or $e^-e^+$ annihilations. Much less we know about the inner structures of polarized hadrons and their dynamical properties. The proton and neutron spin structure functions have recently received much attention and their improved measurements have caused great surprise and enormous theoretical activity [1], but we still need a better knowledge; very little is known on polarized quark and gluon fragmentations. Several observed spin effects are not well understood and are certainly related to non perturbative hadronic properties.

We consider here the inclusive deep inelastic process

$$\ell N \to hX$$

in which an unpolarized or polarized lepton scatters off a polarized nucleon and one observes a final hadron $h$ whose spin state is studied through the measurement of its helicity density matrix $\rho(h)$. The incoming lepton interacts with a polarized quark inside the polarized nucleon and the quark then fragments into the hadron $h$ contributing to its spin; thus, we expect to learn something on the polarized quark distribution and fragmentation functions. We consider spin 1 and spin 1/2 final hadrons and different polarizations of the initial nucleon; we consider either unpolarized or longitudinally polarized leptons because, as we shall see, transversely polarized ones cannot add any further information.

According to the QCD hard scattering scheme and the factorization theorem [2]-[4], the helicity density matrix of the hadron $h$ inclusively produced in reaction (1) is given by

$$\rho^{(s,S)}(h) \dfrac{\sigma^{\ell,s,N,S\to h+X}}{d^3p_h} = \sum_{\lambda_h,\lambda_h'} \int \frac{dx}{\pi z} \dfrac{1}{16\pi x^2s^2} \rho_{\lambda_h,\lambda_h'}^{\ell,s} \rho_{\lambda_hq,\lambda_q}^{q/N,S} f_{q/N}(x) \hat{M}^{qq}_{\lambda_h,\lambda_q;\lambda_h',\lambda_q} D_{\lambda_h,\lambda_h'}(z)$$

where $\rho^{\ell,s}$ is the helicity density matrix of the initial lepton with spin $s$, $f_{q/N}(x)$ is the number density of unpolarized quarks $q$ with momentum fraction $x$ inside an
unpolarized nucleon and $\rho^{q/N,S}$ is the helicity density matrix of quark $q$ inside the polarized nucleon $N$ with spin $S$. The $M_{\lambda q, \lambda q; \lambda q}^q$’s are the helicity amplitudes for the elementary process $\ell q \rightarrow \ell q$. The final lepton spin is not observed and helicity conservation of perturbative QCD and QED has already been taken into account in the above equation: as a consequence only the diagonal elements of $\rho^{\ell,s}$ contribute to $\rho(h)$ and non diagonal elements, present in case of transversely polarized leptons, do not contribute. $D_{\lambda h, \lambda h}^{\lambda q, \lambda q}(z)$ is the product of fragmentation amplitudes

$$D_{\lambda h, \lambda h}^{\lambda q, \lambda q}(z) = \oint_{X, \lambda X} D_{\lambda X, \lambda h}^{\lambda q, \lambda q} D_{\lambda h, \lambda h}^{\lambda q, \lambda q} (z),$$

where the $\oint_{X, \lambda X}$ stands for a spin sum and phase space integration of the undetected particles, considered as a system $X$. The usual unpolarized fragmentation function $D_{h/q}(z)$, i.e. the density number of hadrons $h$ resulting from the fragmentation of an unpolarized quark $q$ and carrying a fraction $z$ of its momentum, is given by

$$D_{h/q}(z) = \frac{1}{2} \sum_{\lambda_q, \lambda_h} D_{\lambda q, \lambda h}^{\lambda q, \lambda q}(z) = \frac{1}{2} \sum_{\lambda_q, \lambda_h} D_{h/q}^{\lambda q, \lambda q}(z),$$

where $D_{\lambda h, \lambda q}^{\lambda q, \lambda q}(z) \equiv D_{h}^{\lambda q, \lambda q}(z)$ is a polarized fragmentation function, i.e. the density number of hadrons $h$ with helicity $\lambda_h$ resulting from the fragmentation of a quark $q$ with helicity $\lambda_q$. Notice that by definition and parity invariance the generalized fragmentation functions (3) obey the relationships

$$D_{\lambda h, \lambda q}^{\lambda q, \lambda q} = \left(D_{\lambda h, \lambda q}^{\lambda q, \lambda q}\right)^*$$

$$D_{-\lambda h, -\lambda q}^{\lambda q, -\lambda q} = -(-1)^{S_h} (-1)^{\lambda_q + \lambda_h + \lambda_q} D_{\lambda h, \lambda q}^{\lambda q, \lambda q},$$

where $S_h$ is the hadron spin; notice also that collinear configuration (intrinsic $k_\perp = 0$) together with angular momentum conservation in the forward fragmentation process imply

$$D_{\lambda h, \lambda h}^{\lambda q, \lambda q} = 0 \quad \text{when} \quad \lambda_q - \lambda_q' \neq \lambda_h - \lambda_h'.$$

Eq. (2) holds at leading twist, leading order in the coupling constants and large $Q^2$ values; the intrinsic $k_\perp$ of the partons have been integrated over and collinear configurations dominate both the distribution functions and the fragmentation processes. For simplicity of notations we have not indicated the $Q^2$ scale dependences in $f$ and $D$; the variable $z$ is related to $x$ by the usual imposition of energy momentum conservation in the elementary $2 \rightarrow 2$ process [1]:

$$z = -\frac{t + xu}{xs}$$

2
where \( s, t \) and \( u \) are the Mandelstam variables for the \( \ell N \to hX \) process, related to the Mandelstam variables \( \hat{s}, \hat{t} \) and \( \hat{u} \) for the subprocess \( \ell q \to \ell q \) by

\[
\hat{s} = xs \quad \hat{t} = xu/z = -Q^2 \quad \hat{u} = t/z.
\]  

Finally, the elementary amplitudes \( \hat{M}^q \) are normalized so that

\[
\frac{d\hat{\sigma}^q}{dt} = \frac{1}{16\pi\hat{s}^2} \frac{1}{4} \sum_{\lambda_q, \lambda_q} |\hat{M}^q_{\lambda_q, \lambda_q; \lambda_q, \lambda_q}|^2
\]

is the unpolarized elementary cross-section and the normalization factor in the left hand side of Eq. (2), such that \( \text{Tr} \rho(h) = 1 \), is the cross-section for the inclusive production of a hadron \( h \), summed over all possible hadron helicities, in the DIS scattering of a lepton with spin \( s \) off a nucleon with spin \( S \):

\[
\frac{E_h d^3 p_{\ell,s+N,S \to h+X}}{d^3 p_h} = \sum_{q; \lambda_q, \lambda_q} \int \frac{dx}{16\pi^2 x^2 z^2 s^2} \rho_{\lambda_q, \lambda_q} \rho_{\lambda_q, \lambda_q} f_{q/N}(x) |\hat{M}^q_{\lambda_q, \lambda_q; \lambda_q, \lambda_q}|^2 D_{h/q}(z)
\]

where we have made use of Eqs. (4), (6) and (7). In the sequel we shall adopt the short notation:

\[
\frac{\hat{M}^q_{++}}{4\sqrt{\pi} \hat{s}} \equiv \hat{M}^q_+ \quad \frac{\hat{M}^q_{-+}}{4\sqrt{\pi} \hat{s}} \equiv \hat{M}^q_-
\]

A measurement of \( \rho(h) \) allows, via Eq. (2) and the knowledge of the unpolarized distribution functions \( f_{q/N}(x) \) and of the elementary lepton-quark interaction, to obtain new information on the spin dependent quark distribution and fragmentation processes. The quark helicity density matrix \( \rho^{0/N,S} \) can be decomposed as

\[
\rho^{0/N,S} = P^0_{P(A)} \rho^{N,S} + P^0_{A} \rho^{N,-S}
\]

where \( P^0_{P(A)} \) (which, in general, depends on \( x \)) is the probability that the spin of the quark inside the polarized nucleon \( N \) is parallel (antiparallel) to the nucleon spin \( S \) and \( \rho^{N,S(-S)} \) is the helicity density matrix of the nucleon with spin \( S(-S) \). Notice that

\[
P^0_{q/N,S} = P^0_{P} - P^0_{A}
\]

is the component of the quark polarization vector along the parent nucleon spin direction.

In the next Sections we shall consider several particular cases of Eq. (2) and discuss what can be learned or expected from a measurement of \( \rho(h) \). For clarity and completeness of the discussion we also report or rederive known results. Towards completion of this work a most general analysis of polarized DIS lepto production has appeared in the literature [7], taking into account \( O(1/Q) \) corrections; our work puts more emphasis on possible measurements, gives numerical estimates and deals also with spin 1 final hadrons.
2 - Spin 1 final hadron; unpolarized leptons and polarized nucleons

We consider first the production of a spin 1 hadron \((h = V)\) with unpolarized leptons,

\[ \rho^{s,s}_{\lambda_L \lambda_{\ell}} = \frac{1}{2} \delta_{\lambda_L \lambda_{\ell}}, \quad (15) \]

and, for the moment, a generic nucleon spin \(S\). Eqs. (2), (7) and (10), together with parity invariance of the elementary QED process, then give

\[
\begin{align*}
\rho^{(s)}_{1,1}(V) d^3 \sigma &= \sum_q \int \frac{dx}{\pi z} f_{q/N} d\hat{\sigma}^q \left[ \rho^{q/N,S}_{++} D^{++}_{1,1} + \rho^{q/N,S}_{--} D^{-+}_{1,1} \right] \\
\rho^{(s)}_{0,0}(V) d^3 \sigma &= \sum_q \int \frac{dx}{\pi z} f_{q/N} d\hat{\sigma}^q D^{++}_{0,0} \\
\rho^{(s)}_{-1,-1}(V) d^3 \sigma &= \sum_q \int \frac{dx}{\pi z} f_{q/N} d\hat{\sigma}^q \left[ \rho^{q/N,S}_{++} D^{-+}_{1,-1} + \rho^{q/N,S}_{--} D^{-+}_{1,1} \right] \\
\rho^{(s)}_{1,0}(V) d^3 \sigma &= \sum_q \int \frac{dx}{\pi z} f_{q/N} \left[ \text{Re} \hat{M}^q \hat{M}^{\ast q} \right] \rho^{q/N,S}_{++} D^{++}_{1,0} \\
\rho^{(s)}_{-1,0}(V) d^3 \sigma &= \sum_q \int \frac{dx}{\pi z} f_{q/N} \left[ \text{Re} \hat{M}^q \hat{M}^{\ast q} \right] \rho^{q/N,S}_{--} D^{-+}_{1,0}
\end{align*}
\]

where the \(\pm\) indices stand respectively for \(\pm 1/2\) helicities; \(d\hat{\sigma}^q\) stands for \(d\hat{\sigma}^q / dt\), Eq. (10), and \(d^3 \sigma\) is a short notation for the cross-section of Eq. (11) with unpolarized leptons; it just equals the unpolarized cross-section

\[
\frac{E_h d^3 \sigma^{ELN \rightarrow hX}}{d^3 \mathbf{p}_h} = \sum_q \int \frac{dx}{\pi z} f_{q/N}(x) \frac{d\hat{\sigma}^q}{dt} D_{h/q}(z), \quad (21)
\]

as can be seen from Eq. (11) upon using Eqs. (10), (13) and the parity relation

\[ \sum_{\lambda_L} \hat{M}^q_{\lambda_L \lambda_{\ell}} \hat{M}^{\ast q}_{\lambda_L \lambda_{\ell}} = 32 \pi s^2 d\hat{\sigma}^q / dt. \]

By exploiting Eqs. (3) and (8) one can check that \(\rho_{1,1} + \rho_{0,0} + \rho_{-1,-1} = 1\) and \(\rho_{0,\pm 1} = (\rho_{\pm 1,0})^\ast\); notice also that, due to Eq. (4), \(\rho_{1,-1} = \rho_{-1,1} = 0\).

Eqs. (16)-(20) hold for any polarization \(S\) of the nucleon and in any reference frame; we shall now consider particular nucleon spin configurations in the lepton-nucleon centre of mass frame. Also the numerical results given in the last Section will be obtained in the c.m. frame. We choose \(xz\) as the hadron production plane with the lepton moving along the \(z\)-axis and the nucleon in the opposite direction; as usual we indicate by an index \(L\) the (longitudinal) nucleon spin orientation along the \(z\)-axis, by an index \(S\) the (sideway) orientation along the \(x\)-axis and by an index \(N\) the (normal) orientation along the \(y\)-axis.

a) Nucleon longitudinal polarization, \(S = S_L\)

In this case the helicity density matrix of the nucleon is given by (notice that \(S = \pm S_L\) means, for the nucleon moving opposite to the \(z\)-direction, \(\lambda_N = \mp 1/2\)
respectively)
\[ \rho^{N,S_L} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \quad \rho^{N,-S_L} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \]  
so that from Eq. (13) we have the quark helicity density matrix
\[ \rho^{q/N,S_L} = \begin{pmatrix} P^{q/N,S_L}_A & 0 \\ 0 & P^{q/N,S_L}_P \end{pmatrix}. \]  
(23)
By using Eqs. (4), (6) and (23) into Eqs. (16)-(20) we obtain the non-zero matrix elements:

\[ \rho^{(S_L)}_{1,1}(V) d^3\sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} d\hat{\sigma}^q \left[ P^{q/N,S_L}_A D_{V_1/q_+} + P^{q/N,S_L}_P D_{V_1/q_-} \right] \]  
(24)
\[ \rho^{(S_L)}_{0,0}(V) d^3\sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} d\hat{\sigma}^q D_{V_0/q_+} \]  
(25)
\[ \rho^{(S_L)}_{-1,-1}(V) d^3\sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} d\hat{\sigma}^q \left[ P^{q/N,S_L}_A D_{V_1/q_-} + P^{q/N,S_L}_P D_{V_1/q_+} \right] \]  
(26)
where the apex \((S_L)\) reminds of the nucleon spin configuration.

b) Nucleon transverse polarization, \(S = S_S\)

In this case we have
\[ \rho^{N,S_S} = \frac{1}{2} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \rho^{N,-S_S} = \frac{1}{2} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \]  
(27)
which, via Eqs. (13) and (14), imply
\[ \rho^{q/N,S_S} = \frac{1}{2} \begin{pmatrix} 1 & P^{q/N,S_S}_A \\ P^{q/N,S_S}_P & 1 \end{pmatrix}. \]  
(28)
Insertion of Eq. (28) into Eqs. (16)-(20) now obtains both diagonal and non-diagonal matrix elements; the diagonal ones are
\[ \rho^{(S_T)}_{1,1}(V) d^3\sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} d\hat{\sigma}^q \frac{1}{2} \left[ D_{V_1/q_+} + D_{V_1/q_-} \right] \]  
(29)
\[ \rho^{(S_T)}_{0,0}(V) d^3\sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} d\hat{\sigma}^q D_{V_0/q_+} \]  
(30)
\[ \rho^{(S_T)}_{-1,-1}(V) = \rho^{(S_T)}_{1,1}(V) = \frac{1 - \rho^{(S_T)}_{0,0}(V)}{2} \]  
(31)
where we have used an apex \(S_T\), rather than \(S_S\), because the same results, as we shall immediately see, hold also in the other transverse spin case, \(S = S_N\).
The non zero non diagonal matrix elements are

$$\rho_{1,0}^{(S_s)}(V) \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} \frac{P_{q/N,S_s}}{2} \left[ \text{Re} \hat{M}_q^q \hat{M}_q^{q*} \right] D_{1,0}^{+,-}$$

$$\rho_{-1,0}^{(S_s)}(V) = \rho_{1,0}^{(S_s)}(V)$$

which involve the non diagonal fragmentation functions (3).

c) Nucleon transverse polarization, \( S = S_N \)

We now have the nucleon helicity density matrices

$$\rho_{N,S_N} = \frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \quad \rho_{N,-S_N} = \frac{1}{2} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}$$

which lead to

$$\rho_{q/N,S_N} = \frac{1}{2} \begin{pmatrix} 1 & iP_{q/N,S_N} \\ -iP_{q/N,S_N} & 1 \end{pmatrix}.$$  

Insertion of Eq. (33) into Eqs. (16)-(20) gives the same results as those obtained with \( S = S_S \) for the diagonal matrix elements, Eqs. (24)-(26). The non zero non diagonal matrix elements are instead given by

$$\rho_{1,0}^{(S_N)}(V) \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} \frac{iP_{q/N,S_N}}{2} \left[ \text{Re} \hat{M}_q^q \hat{M}_q^{q*} \right] D_{1,0}^{+,-}$$

$$\rho_{-1,0}^{(S_N)}(V) = -\rho_{1,0}^{(S_N)}(V).$$

Notice that, by rotational invariance, \( P_{q/N,S_N} = P_{q/N,S_S} \), so that \( \rho_{1,0}^{(S_N)} = i \rho_{1,0}^{(S_S)} \).

3 - Spin 1/2 final hadron; unpolarized leptons and polarized nucleons

In case of final spin 1/2 hadrons (\( h = B \)), with unpolarized leptons and spin \( S \) nucleons, we have from Eqs. (2), (4) and (10) and in analogy to Eqs. (16)-(20):

$$\rho_{+,+}^{(S)}(B) \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} \, d\sigma^q \left[ \rho_{+,+}^{q/N,S} D_{+,+} + \rho_{-,+}^{q/N,S} D_{-,+} \right]$$

$$\rho_{-,-}^{(S)}(B) \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} \, d\sigma^q \left[ \rho_{+,+}^{q/N,S} D_{+,-} + \rho_{-,+}^{q/N,S} D_{-,+} \right]$$

$$\rho_{-,-}^{(S)}(B) \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} \left[ \text{Re} \hat{M}_q^q \hat{M}_q^{q*} \right] \rho_{+,+}^{q/N,S} D_{+,+}$$

with \( d^3 \sigma \) given by Eq. (21). Notice that \( \rho_{+,+} + \rho_{-,-} = 1 \), \( \rho_{-,+} = \rho_{+,+}^* \) and that, from Eqs. (3) and (2), \( D_{+,+} \) is real.

When considering particular nucleon spin configurations we obtain for \( S = S_L \), in analogy to Eqs. (24)-(26),

$$\rho_{+,+}^{(S_L)}(B) \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} \, d\sigma^q \left[ P_{A}^{q/N,S_L} D_{B,+} + P_{P}^{q/N,S_L} D_{B,+} \right]$$

(41)
\[
\rho^{(S_{L})}_{-,-}(B) \, d^{3}\sigma = \sum_{q} \int \frac{dx}{\pi z} f_{q/N} \, d\hat{\sigma}^{q} \left[ P_{A}^{q/N,S_{L}} D_{B_{+}/q_{-}} + P_{P}^{q/N,S_{L}} D_{B_{+}/q_{+}} \right] \tag{42}
\]

whereas, in the transverse spin cases, we have for the diagonal matrix elements

\[
\rho^{(S_{T})}_{+,-}(B) = \rho^{(S_{T})}_{-,+}(B) = \frac{1}{2} \tag{43}
\]

and for the off-diagonal ones, in analogy to Eqs. (32) and (36),

\[
\rho^{(S_{S})}_{+,-}(B) \, d^{3}\sigma = \sum_{q} \int \frac{dx}{\pi z} f_{q/N} \frac{P_{q/N,S_{S}}}{2} \left[ \text{Re} \hat{M}_{q}^{q_{+}} \hat{M}_{q_{-}}^{q_{+}} \right] D_{+,,-}^{+} \tag{44}
\]

\[
\rho^{(S_{N})}_{+,-}(B) \, d^{3}\sigma = \sum_{q} \int \frac{dx}{\pi z} f_{q/N} \frac{iP_{q/N,S_{N}}}{2} \left[ \text{Re} \hat{M}_{q}^{q_{+}} \hat{M}_{q_{-}}^{q_{+}} \right] D_{+,,-}^{+} \tag{45}
\]

The knowledge of the helicity density matrix \(\rho(B)\) allows to compute the expectation values of the components of the polarization vector \(P(B)\), in the helicity rest frame of \(B\) [3]:

\[
P_{i} = \text{Tr}(\sigma^{i} \rho), \tag{46}
\]

which yields

\[
P_{x}^{(S_{S})} \, d^{3}\sigma = \sum_{q} \int \frac{dx}{\pi z} f_{q/N} P_{q/N,S_{S}} \left[ \text{Re} \hat{M}_{q}^{q_{+}} \hat{M}_{q_{-}}^{q_{+}} \right] D_{+,,-}^{+} \tag{47}
\]

\[
P_{y}^{(S_{N})} = -P_{x}^{(S_{S})} \tag{48}
\]

\[
P_{z}^{(S_{L})} \, d^{3}\sigma = \sum_{q} \int \frac{dx}{\pi z} f_{q/N} P_{q/N,S_{L}} \, d\hat{\sigma}^{q} \left[ D_{B_{+}/q_{-}} - D_{B_{+}/q_{+}} \right] \tag{49}
\]

where we have used Eq. (14) and the fact that, by parity invariance, \(D_{+,,-}^{+} \) is real. All other components of the polarization vectors are zero. Let us stress once more that the \(x, y\) and \(z\) components in the above equations (47)-(49) refer to the coordinate axes in the helicity rest frame of hadron \(B\), whereas the apices \(S_{L}, S_{N}\) and \(S_{S}\) are related to the nucleon spin orientations in the reference frame where we compute the scattering [see comments after Eq. (21)].

The quantities in the right hand sides of Eqs. (47)-(49) might look more familiar if written in different notations or in different spin basis. In fact we have

\[
f_{q/N}(x) \left[ P_{A}^{q/N,S_{L}}(x) = f_{q_{+}(-)/N_{+}}(x) = f_{q_{-}(-)/N_{-}}(x) \right] \tag{50}
\]

where \(f_{q_{+}(-)/N_{+}}\) is the polarized distribution function, that is the density number of quarks with helicity \(+(--)\) inside a nucleon with helicity \(+\) and the last equality holds due to parity invariance. From Eq. (14) and Eq. (50) one has

\[
f_{q/N} P_{q/N,S_{L}} = f_{q_{+}/N_{+}} - f_{q_{-}/N_{+}} \equiv \Delta q \tag{51}
\]
and similarly for spin quantized along a transverse direction \( T = N, S \),

\[
f_{q/N} P_{q/N,S_T} = f_{q,S_T/N,S_T} - f_{q,-S_T/N,S_T} \equiv \Delta_T q. \tag{52}
\]

By switching from the helicity to the \( N \) spin quantization basis (notice that the \( N \) direction, i.e. the \( y \)-axis, is the same both for the initial nucleon and the final hadron helicity rest frame) one obtains [see Eq. (12)]:

\[
- \left[ \text{Re} \hat{M}_q^+ \hat{M}_q^- \right] = \frac{d \hat{\sigma}^{\ell+q,S_N \to \ell+q,S_N}}{d \hat{t}} - \frac{d \hat{\sigma}^{\ell+q,S_N \to \ell+q,-S_N}}{d \hat{t}} \equiv \Delta_N \hat{\sigma}^q \tag{53}
\]

and

\[
D_{+,\ell}^+ - D_{+,\ell}^- = D_{B,S_N/q,S_N} - D_{B,-S_N/q,S_N} \equiv \Delta_T D_{B/q}, \tag{54}
\]

which is a difference of transverse fragmentation functions and is the same for any transverse spin direction \( T = N, S \). Similarly, one defines

\[
D_{B,S_L/q,S_L} - D_{B,-S_L/q,S_L} = D_{B+/q,+} - D_{B-/+} \equiv \Delta D_{B/q}. \tag{55}
\]

Eqs. (47)-(49) then read

\[
P_{x}^{(S_S)} = -P_{y}^{(S_N)} \tag{56}
\]

\[
P_{z}^{(S_S)} d^3 \sigma = \sum_q \int \frac{dx}{\pi z} \Delta_T q \Delta_N \hat{\sigma}^q \Delta_T D_{B/q} \tag{57}
\]

\[
P_{z}^{(S_L)} d^3 \sigma = -\sum_q \int \frac{dx}{\pi z} \Delta q \Delta \hat{\sigma}^q \Delta D_{B/q}. \tag{58}
\]

4 - Longitudinally polarized leptons and polarized nucleons

We discuss now the case of polarized leptons; as we noticed after Eq. (2) only the diagonal elements of the lepton helicity density matrix \( \rho^{s,s} \) contribute to \( \rho(h) \), so that only longitudinal polarizations could affect the results. We consider then longitudinally polarized leptons, \( s = s_L \), which amounts to

\[
\rho^{s_L,s_L} = \delta_{\lambda_L \lambda_L} \delta_{\lambda_L \lambda_L}. \tag{59}
\]

Eq. (2) then reads

\[
\rho_{\lambda_h \lambda_h}^{s_L,S_S}(h) \frac{E_h d^3 \sigma_{\ell,s_L+N,S \to h+X}}{d^3 p_h} = \sum_{q,\lambda_q,\lambda_q'} \int \frac{dx}{\pi z 16 \pi x^2 s^2} \rho_{\lambda_q \lambda_q'}^{q/N,S} f_{q/N}(x) \hat{M}_q^{\ell+q,+} \hat{M}_q^{\ell+q,+} D_{\lambda_h \lambda_h}(z) \tag{60}
\]

which holds for any final hadron and any nucleon spin orientation. We briefly discuss the same cases treated in the previous two Sections; for convenience we use the notations defined in Eqs. (50)-(52).
i) $h = V, S = S_L$

Eqs. (60), (22) and (12) give [compare with Eqs. (24)-(26)]

$$
\rho_{1,1}^{(s_L,S_L)}(V)\,d^3\sigma_L = \sum_q \int \frac{dx}{\pi z} \left[ f_{q_+/N_+} |\hat{M}_+|^2 D_{V+/q+} + f_{q_+/N_+} |\hat{M}_+|^2 D_{V/-q-} \right] (61)
$$

$$
\rho_{0,0}^{(s_L,S_L)}(V)\,d^3\sigma_L = \sum_q \int \frac{dx}{\pi z} \left[ f_{q_+/N_+} |\hat{M}_+|^2 + f_{q_+/N_+} |\hat{M}_0|^2 \right] D_{V_0/q+} (62)
$$

$$
\rho_{-1,-1}^{(s_L,S_L)}(V)\,d^3\sigma_L = \sum_q \int \frac{dx}{\pi z} \left[ f_{q_+/N_+} |\hat{M}_+|^2 D_{V/-q-} + f_{q_+/N_+} |\hat{M}_0|^2 D_{V/+q+} \right] (63)
$$

where the apex $(s_L, S_L)$ reminds of the lepton and nucleon spin configurations and $d^3\sigma_L$ stands for

$$
\frac{E_V\,d^3\sigma^{L,S_L\rightarrow V+X}}{d^3p_V} = \sum_q \int \frac{dx}{\pi z} \left[ f_{q_+/N_+} |\hat{M}_+|^2 + f_{q_+/N_+} |\hat{M}_0|^2 \right] D_{V/q}. (64)
$$

ii) $h = V, S = S_T (T = S, N)$

Eqs. (29)-(31) now modify into

$$
\rho_{1,1}^{(s_L,S_T)}(V)\,d^3\sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} \frac{1}{2} \left[ |\hat{M}_+|^2 D_{V+/q+} + |\hat{M}_0|^2 D_{V/-q-} \right] (65)
$$

$$
\rho_{0,0}^{(s_L,S_T)}(V)\,d^3\sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} d\hat{\sigma}^0 D_{V_0/q+} (66)
$$

$$
\rho_{-1,-1}^{(s_L,S_T)}(V)\,d^3\sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} \frac{1}{2} \left[ |\hat{M}_+|^2 D_{V/-q-} + |\hat{M}_0|^2 D_{V/+q+} \right] (67)
$$

where $d^3\sigma$ is the unpolarized cross-section (21), and the non diagonal matrix elements are

$$
\rho_{1,0}^{(s_L,S_S)}(V)\,d^3\sigma = \sum_q \int \frac{dx}{\pi z} \frac{\Delta_Tq}{2} \hat{M}_0^+ \hat{M}_0^- D_{1,0}^{+,-} (68)
$$

$$
\rho_{-1,0}^{(s_L,S_S)}(V)\,d^3\sigma = \sum_q \int \frac{dx}{\pi z} \frac{\Delta_Tq}{2} \hat{M}_0^+ \hat{M}_0^- D_{1,0}^{+,-} (69)
$$

$$
\rho_{1,0}^{(s_L,S_N)}(V)\,d^3\sigma = \sum_q \int \frac{dx}{\pi z} \frac{i\Delta_Tq}{2} \hat{M}_0^+ \hat{M}_0^- D_{1,0}^{+,-} (70)
$$

$$
\rho_{-1,0}^{(s_L,S_N)}(V)\,d^3\sigma = -\sum_q \int \frac{dx}{\pi z} \frac{i\Delta_Tq}{2} \hat{M}_0^+ \hat{M}_0^- D_{1,0}^{+,-}. (71)
$$

The non diagonal elements $\rho_{\pm 1,0}$ might differ from those found with unpolarized leptons, Eqs. (32), (33), (36) and (37), only if the amplitude product $\hat{M}_0^+ \hat{M}_0^-$ is a complex quantity, which is certainly not the case at lowest perturbative order. Notice also that $\rho_{\pm 1,0}^{(s_L,S_S)} = \pm i\rho_{\pm 1,0}^{(s_L,S_S)}$. 


iii) \( h = B, S = S_L \)

Longitudinally polarized leptons and longitudinally polarized nucleons lead to final spin half hadrons with

\[
\rho_{+,-}^{(s_L, S_L)}(B) \, d^3 \sigma_L = \sum_q \int \frac{dx}{\pi z} \left[ f_{q^-/N+} \left| \hat{M}_+^q \right|^2 D_{B+/q+} + f_{q^+/N+} \left| \hat{M}_+^q \right|^2 D_{B+/q-} \right] \quad (72)
\]

\[
\rho_{-,-}^{(s_L, S_L)}(B) \, d^3 \sigma_L = \sum_q \int \frac{dx}{\pi z} \left[ f_{q^-/N+} \left| \hat{M}_-^q \right|^2 D_{B+/q+} + f_{q^+/N+} \left| \hat{M}_-^q \right|^2 D_{B+/q-} \right] \quad (73)
\]

whereas with transversely polarized nucleons one has

iv) \( h = B, S = S_T \)

\[
\rho_{+,-}^{(s_L, S_T)}(B) \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} \left[ \frac{1}{2} \left| \hat{M}_+^q \right|^2 D_{B+/q+} + \left| \hat{M}_+^q \right|^2 D_{B+/q-} \right] \quad (74)
\]

\[
\rho_{-,-}^{(s_L, S_T)}(B) \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} \left[ \frac{1}{2} \left| \hat{M}_-^q \right|^2 D_{B+/q+} + \left| \hat{M}_-^q \right|^2 D_{B+/q-} \right] \quad (75)
\]

\[
\rho_{+,-}^{(s_L, S_S)}(B) \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} \frac{i \Delta T q}{2} \hat{M}_+^q \hat{M}_-^{q*} \Delta_{++,-} + \Delta_{--,+} \quad (76)
\]

\[
\rho_{+,-}^{(s_L, S_N)}(B) \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} i \Delta T q \left| \hat{M}_+^q \hat{M}_-^{q*} \, \Delta_{++,-} \right| \quad (77)
\]

Again, in comparison to the case of unpolarized leptons [see Eqs. (14)-(18)], one finds different results for the diagonal matrix elements, but the same ones, provided \( \hat{M}_+^q \hat{M}_-^{q*} \) is real, for the non diagonal elements. Similarly to Eqs. (17)-(19) or (50)-(58), the above equations can also be written in terms of components of the polarization vector of hadron \( B \):

\[
P_x^{(s_L, S_S)} = -P_y^{(s_L, S_N)} = P_x^{(S_S)} = -P_y^{(S_N)} \quad (78)
\]

\[
P_z^{(s_L, S_L)} \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} \left[ f_{q^-/N+} \left| \hat{M}_+^q \right|^2 - f_{q^+/N+} \left| \hat{M}_-^q \right|^2 \right] \Delta D_{B/q} \quad (79)
\]

\[
P_z^{(s_L, S_T)} \, d^3 \sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} \left[ \frac{1}{2} \left| \hat{M}_-^q \right|^2 - \left| \hat{M}_+^q \right|^2 \right] \Delta D_{B/q} \quad (80)
\]

5 - Possible measurements

After the theoretical analysis of the previous Sections we should now address the question of which elements of the helicity density matrix of the produced hadrons can be measured; we discuss in details the cases of spin 1 \( \rho \) vector mesons and spin 1/2 \( \Lambda \) baryons as the most typical and simple ones, but the same procedure could be applied to other hadrons.
\(\rho\) particles decay into two pions and, in order to measure the helicity density matrix of the decaying \(\rho\), one has to look at the angular distribution of either one of the produced pions; such a decay is given by \[8\]

\[
W(\theta, \phi) = \frac{3}{4\pi} \left\{ \frac{1}{2} (1 - \rho_{0,0}) + \frac{1}{2} (3\rho_{0,0} - 1) \cos^2 \theta \right\} \nonumber
\]

\[
- \frac{1}{\sqrt{2}} \sin 2\theta \cos \phi \Re[\rho_{1,0} - \rho_{-1,0}^*] + \frac{1}{\sqrt{2}} \sin 2\theta \sin \phi \Im[\rho_{1,0} - \rho_{-1,0}^*] \nonumber
\]

\[
- \sin^2 \theta \cos 2\phi \Re[\rho_{1,-1}] + \sin^2 \theta \sin 2\phi \Im[\rho_{1,-1}] \right\} \quad (81)
\]

where \(\theta\) and \(\phi\) are respectively the polar and azimuthal angles of the pion in the helicity rest frame of the \(\rho\).

From our previous results we see that the above distribution simplifies, in that several matrix elements are zero, depending on the different spin configurations of the initial particles. Let us consider the case of unpolarized leptons with different nucleon spin orientations; we then expect angular distributions of the following types [see Eqs. (24)-(26), (29)-(33) and (36)-(37)]:

a) **Nucleon longitudinal polarization, \(S = S_L\)**

\[
W(\theta, \phi) = \frac{3}{4\pi} \left\{ \frac{1}{2} (1 - \rho_{0,0}^{(\text{SL})}) + \frac{1}{2} (3\rho_{0,0}^{(\text{SL})} - 1) \cos^2 \theta \right\} ; \quad (82)
\]

b) **Nucleon transverse polarization, \(S = S_S\)**

\[
W(\theta, \phi) = \frac{3}{4\pi} \left\{ \frac{1}{2} (1 - \rho_{0,0}^{(\text{ST})}) + \frac{1}{2} (3\rho_{0,0}^{(\text{ST})} - 1) \cos^2 \theta \right\} + \sqrt{2} \sin 2\theta \sin \phi \Im \rho_{1,0}^{(\text{SS})} \right\} ; \quad (83)
\]

c) **Nucleon transverse polarization, \(S = S_N\)**

\[
W(\theta, \phi) = \frac{3}{4\pi} \left\{ \frac{1}{2} (1 - \rho_{0,0}^{(\text{ST})}) + \frac{1}{2} (3\rho_{0,0}^{(\text{ST})} - 1) \cos^2 \theta \right\} - \sqrt{2} \sin 2\theta \cos \phi \Re \rho_{1,0}^{(\text{SN})} \right\} ; \quad (84)
\]

Notice that \(\rho_{0,0}^{(\text{SL})} = \rho_{0,0}^{(\text{ST})}\) [Eqs. (24), (30)] and that, from \(\rho_{1,0}^{(\text{SS})} = i \rho_{1,0}^{(\text{SN})}\) one has \(\Re \rho_{1,0}^{(\text{SN})} = - \Im \rho_{1,0}^{(\text{SS})}\); the observation of the angular distribution of the \(\rho \rightarrow \pi\pi\) decay supplies then only information on \(\rho_{0,0}^{(\text{SL})}(V)\) and \(\Re \rho_{1,0}^{(\text{SN})}(V)\), that is on the quantities:

\[
\rho_{0,0}^{(\text{SL})}(V) \ d^3\sigma = \sum_q \int \frac{dx}{\pi z} f_{q/N} d\hat{\sigma} q \ D_{0/\hat{q}+} \quad (85)
\]

\[
\Re \rho_{1,0}^{(\text{SN})}(V) \ d^3\sigma = - \sum_q \int \frac{dx}{\pi z} f_{q/N} \ P_{q/N,ST} \frac{P_{q/N,ST}}{2} \left[ \Re \hat{M}_+ \hat{M}^*_+ \right] \Im D_{1,0}^{+/-} \quad (86)
\]
with $d^3 \sigma$ given in Eq. (21).

Similar results hold in case of longitudinally polarized initial leptons, with the only difference that in such case $\rho_{0,0}^{(s_L,S_L)}$ differs from $\rho_{0,0}^{(s_L,S_T)}$ [Eqs. (62), (66)] and the two separate measurements might offer more information.

Let us now consider the production of a spin half baryon which, via its parity violating decay, allows a measurement of its polarization vector; the most typical example is the $\Lambda \rightarrow p\pi^-$ decay. The angular distribution of the proton as it emerges in the $\Lambda$ helicity rest frame is given by

$$W(\theta_p, \phi_p) = \frac{1}{4\pi} \left( 1 + \alpha \cos \theta_p (\cos 2\rho_{++} - 1) + 2\alpha \sin \theta_p \cos \phi_p \text{Re} \rho_{+-} - 2\alpha \sin \theta_p \sin \phi_p \text{Im} \rho_{+-} \right)$$

where $P$ is the $\Lambda$ polarization vector and $\hat{p}$ is the unit vector along the proton direction in the $\Lambda$ helicity rest frame. The decay parameter $\alpha$ is experimentally known and for the $\Lambda \rightarrow p\pi^-$ decay $\alpha = 0.642 \pm 0.013$.

From the results of Eqs. (56)-(58) we expect then, for unpolarized leptons and longitudinally polarized nucleon ($S = S_L$), the $\Lambda$ decay angular distribution

$$W(\theta_p, \phi_p) = \frac{1}{4\pi} \left( 1 + \alpha P_z^{(s_L)} \cos \theta_p \right); \quad (88)$$

for unpolarized leptons and $S = S_S$ we have

$$W(\theta_p, \phi_p) = \frac{1}{4\pi} \left( 1 + \alpha P_x^{(s_S)} \sin \theta_p \cos \phi_p \right); \quad (89)$$

and for $S = S_N$

$$W(\theta_p, \phi_p) = \frac{1}{4\pi} \left( 1 + \alpha P_y^{(s_N)} \sin \theta_p \sin \phi_p \right). \quad (90)$$

Recalling that $P_x^{(s_S)} = -P_y^{(s_N)}$ a measurement of $W(\theta_p, \phi_p)$ supplies information on the two quantities given in Eqs. (77), (88) or (77), (89).

Similar results are obtained when performing experiments with longitudinally polarized leptons, with the difference that one has a non zero $P_z$-component also for transversely polarized nucleons; one can then get further information on the polarized distribution and fragmentation functions via Eqs. (74) and (80).

### 6 - Some numerical estimates and conclusions

Which values could we expect for the measurable helicity density matrix elements? Let us consider first spin 1 vector mesons $\rho$ particles -- produced with unpolarized leptons and the matrix elements $\rho_{0,0}^{(s_L)}$ and $\text{Re} \rho_{1,0}^{(s_N)}$, Eqs. (53) and (80).
respectively; these are the only independent matrix elements which can be measured through the decay angular distributions (82) and (84).

It is not difficult to give an estimate of \( \rho_{0,0}^{(S_L)}(V) \) if one observes final mesons with large \( |x_F| \) and \( Q^2 \) values, so that one can safely argue that they contain the original fragmenting quark as a valence one; if one assumes that, for valence quarks, \( D_{V_0/q^+} = C D_{V/q^+} \), with \( C \) a flavour and \( z \)-independent constant, then from Eqs. (53) and (24) one obtains \( \rho_{0,0}^{(S_L)}(V) = C \). For \( \rho \) particles and according to \( SU(6) \) wave functions, one has \( C = 1/3 \), so that we expect

\[
\rho_{0,0}^{(S_L)}(\rho) = \frac{1}{3} \tag{91}
\]

for \( \rho \) mesons produced in a kinematical region dominated by valence quark hadronization. The same result holds in case of a production initiated by longitudinally polarized leptons, Eqs. (52) and (54). Notice that the above value of 1/3 leads to a constant angular decay distribution \( W(\theta_\pi, \phi_\pi) = 1/4\pi \), Eq. (82), which should be easily detected experimentally.

The evaluation of Re\(\rho_{1,0}^{(S_N)} \) via Eq. (56) requires the knowledge of non perturbative fragmentation properties, contained in Im\(D_{1,0}^{++} \); actually, we expect a measurement of Re\(\rho_{1,0}^{(S_N)} \) to give us information on such a quantity. However, we try here to obtain an idea of the possible maximum value of Re\(\rho_{1,0}^{(S_N)} \), by assuming, as naively suggested by Eqs. (3) and (4),

\[
\text{Im} D_{1,0}^{++} \simeq \left[ D_{V_1/q^+} D_{V_0/q^-} \right]^{1/2}. \tag{92}
\]

If, again, we consider only \( SU(6) \) valence quark contributions to \( \rho \) production, we have \( D_{\rho_0/q^+} = (1/3) D_{\rho/q} \) and \( D_{\rho_1/q^+} = (2/3) D_{\rho/q} \), so that we take

\[
\text{Im} D_{1,0}^{++} \simeq \frac{\sqrt{2}}{3} D_{\rho/q}. \tag{93}
\]

In order to evaluate Re\(\rho_{1,0}^{(S_N)} \) we also need to know the value of the transverse quark polarization inside the transversely polarized nucleon, \( P^{q/N,S_T} \). In general this quantity depends on \( x \); if, however, we consider large \( p_T \) final mesons originated by large \( x \) proton valence quarks which fragment into large \( z \) hadrons we can assume, according to \( SU(6) \) proton wave functions:

\[
P^{u/v,p,S_T} = \frac{2}{3}, \quad P^{d/v,p,S_T} = -\frac{1}{3} \tag{94}
\]

independent of \( x \). By interchanging \( u \) and \( d \) one obtains the analogous results for neutrons. Sea quarks are assumed not to be polarized.

The elementary interaction \( \ell q \rightarrow \ell q \), computed at lowest perturbative order, gives

\[
\hat{M}_q^{++} = 8\pi\alpha e_q \frac{\hat{s}}{t}, \quad \hat{M}_q^{+-} = 8\pi\alpha e_q \frac{\hat{u}}{t}. \tag{95}
\]
where $e_q$ is the quark charge in units of the proton charge; then [recall Eq. (12)]

$$Re \hat{M}^{q*} = 4\pi\alpha e_q^2 \frac{\hat{u}}{s t^2}. \quad \text{(96)}$$

By inserting Eqs. (62)-(64) into Eqs. (58) and (71), we obtain, for the process $\ell + p, S_N \rightarrow \rho^+ + X$ and within the above simplifying assumptions:

$$Re \rho^{(S_N)}_{1.0} (\rho^+) = \frac{2\sqrt{2} t}{9} \int dx \frac{t^2}{(t+ux)^2} f_{uv/p}(x) \frac{s}{u} D_{\rho^+/u}(z)$$

$$\text{Re} \rho^{(S_N)}_{1.0} (\rho^-) = \int dx \frac{t}{x} (t+ux)^2 f_{uv/p}(x) \frac{s}{u} D_{\rho^-/u}(z)$$

(97)

where we have taken $D_{\rho^+/u} = D_{\rho^+/d}$ and we have used Eqs. (6) and (3). Similar expressions can be derived for the production of $\rho^0$ and $\rho^-$ mesons.

The above result, Eq. (77), can only be considered as an upper estimate of the magnitude of $Re \rho^{(S_N)}_{1.0} (\rho^+)$; its sign is arbitrary, due to the phase uncertainty in Eq. (12). Notice, however, that, in case of production of a $\rho^-$, we find an opposite sign, due to a leading contribution from $d$ quarks and Eq. (94).

In Figures 1-4 we present numerical results obtained from Eq. (77) and the analogous ones for $\rho^-$ and $\rho^+$. We plot the values of $Re \rho^{(S_N)}_{1.0} (\rho^+)$, $Re \rho^{(S_N)}_{1.0} (\rho^0)$ and $Re \rho^{(S_N)}_{1.0} (\rho^-)$ for different choices of $\sqrt{s}$: in Figs. 1 and 2 we show results at $\sqrt{s} = 23$ GeV, respectively at fixed $x_F$ as functions of $p_T$ and at fixed $p_T$ as functions of $x_F$; analogous results are shown in Figs. 3 and 4 at $\sqrt{s} = 314$ GeV. $x_F$ is the $x$-Feynman variable in the $\ell - p$ c.m. frame and $p_T$ is the $\rho$ transverse momentum. We have used the $u$ and $\bar{d}$ distribution functions, including their $Q^2$ evolution, given in Ref. [3]; for the quark fragmentation functions we have taken $D_{\rho^+/u}(z) \sim z^{-1}(1-z)^{1.2}$ from a fit of the experimental data on $D_{\rho^+/u}(z)$ [10]. We have explicitly checked that choices of other available distribution and/or fragmentation functions leave the numerical results essentially unchanged. Figures 1-4 show that in all cases sizeable and hopefully detectable values of $|Re \rho^{(S_N)}_{1.0} (\rho)|$ are found; we notice once more that only the magnitudes and the relative signs of the results for $\rho^+$, $\rho^0$ and $\rho^-$ are meaningful.

Let us finally consider the production of a spin 1/2 baryon, say a $\Lambda$ particle. The decay angular distributions (58) and (70) allow a measurement of $P_y^{(S_N)}$ and $P_z^{(S_L)}$, given respectively in Eqs. (37), (48) and (13) [or (27) and (68)]. A much simplified version of Eq. (58) has been recently derived in Ref. [11].

According to $SU(6)$ wave function the entire $\Lambda$ polarization is due to the strange quark, so that the difference of polarized fragmentation functions in Eq. (13) is different from zero only for $s$ quarks, $D_{\Lambda^+/s} - D_{\Lambda^+/s} = -D_{\Lambda^+/s}$. Then Eq. (68) reads

$$P_z^{(s_L)} = -\frac{\int dx (\pi z)^{-1} \Delta s \tilde{d} \sigma_s D_{\Lambda/s}}{\sum_q \int dx (\pi z)^{-1} f_{q/N} \tilde{d} \sigma_q D_{\Lambda/q}} \quad \text{(98)}$$

Such a quantity is expected to be rather small; however, any non zero value would offer valuable information on the much debated issue of longitudinal strange quark

14
polarization, $\Delta s$, inside a longitudinally polarized nucleon $[11]$. A similar information on the transverse polarization can be obtained from a measurement of $P_y^{(SN)}$ and Eq. (57).

In conclusion, we have shown how a careful analysis of the spin of hadrons inclusively produced in the DIS scattering of leptons, either polarized or not, on polarized nucleons might yield further information on the quark distribution and on the quark fragmentation properties; we have performed our analysis in the framework of perturbative QCD and the factorization theorem, giving comprehensive and detailed expressions for measurable quantities in several different cases. Any measurement of spin observables would help in understanding subtle non perturbative spin properties of hadrons, which would otherwise be inaccessible; once enough information has been gathered, like in the unpolarized case, predictions for other processes or observables can reliably be made.

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Figure captions

**Fig. 1** - Plot of $\text{Re} \rho^{(S_N)}_{1,0}(\rho^+)$ (solid line), $\text{Re} \rho^{(S_N)}_{1,0}(\rho^0)$ (dashed line) and $\text{Re} \rho^{(S_N)}_{1,0}(\rho^-)$ (dot-dashed line) as functions of $p_T$ at $\sqrt{s} = 23$ GeV and $x_F = -0.3$. The results are obtained from Eq. (97) of the text and similar equations for $\rho^0$ and $\rho^-$; the minimum values of $x$ and $Q^2$ contributing are respectively $x_{\text{min}} \simeq 0.33$ and $Q^2_{\text{min}} \simeq 4$ (GeV/c)$^2$.

**Fig. 2** - Plot of $\text{Re} \rho^{(S_N)}_{1,0}(\rho^+)$ (solid line), $\text{Re} \rho^{(S_N)}_{1,0}(\rho^0)$ (dashed line) and $\text{Re} \rho^{(S_N)}_{1,0}(\rho^-)$ (dot-dashed line) as functions of $x_F$ at $\sqrt{s} = 23$ GeV and $p_T = 3$ GeV/c. The results are obtained from Eq. (97) of the text and similar equations for $\rho^0$ and $\rho^-$; the minimum values of $x$ and $Q^2$ contributing are respectively $x_{\text{min}} \simeq 0.28$ and $Q^2_{\text{min}} \simeq 10$ (GeV/c)$^2$.

**Fig. 3** - Same as Fig. 1 at $\sqrt{s} = 314$ GeV and $x_F = -0.3$; $x_{\text{min}} \simeq 0.30$ and $Q^2_{\text{min}} \simeq 16$ (GeV/c)$^2$.

**Fig. 4** - Same as Fig. 2 at $\sqrt{s} = 314$ GeV and $p_T = 6$ GeV/c; $x_{\text{min}} \simeq 0.20$ and $Q^2_{\text{min}} \simeq 36$ (GeV/c)$^2$. 


FIG. 1
\[ \text{Re}[\rho_{10}] \]

\[ x_F \]

**FIG. 4**