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**Recommended Citation**

Ding, Lan; Liu, Jinsong; Wang, Kejia; and Wang, Dong: Coupling interaction of electromagnetic wave in a groove doublet configuration 2010, 21083-21089.  
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Coupling interaction of electromagnetic wave in a groove doublet configuration

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Abstract: Based on the waveguide mode (WGM) method, coupling interaction of electromagnetic wave in a groove doublet configuration is studied. The formulation obtained by WGM method for a single groove [Prog. Electromagn. Res. 18, 1–17 (1998)] is extended to two grooves. By exploring the total scattered field of the configuration, coupling interaction ratios are defined to describe the interaction between grooves quantitatively. Since each groove in this groove doublet configuration is regarded as the basic unit, the effects of coupling interaction on the scattered fields of each groove can be investigated respectively. Numerical results show that an oscillatory behavior of coupling interaction is damped with increasing groove spacing. The incident and scattering angle dependence of coupling interaction is symmetrical when the two grooves are the same. For the case of two subwavelength grooves, the coupling interaction is not sensitive to the incident angle and scattering angle. Although the case of two grooves is discussed for simplicity, the formulation developed in this article can be generalized to arbitrary number of grooves. Moreover, our study offers a simple alternative to investigate and design metallic gratings, compact directional antennas, couplers, and other devices especially in low frequency regime such as THz and microwave domain.

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OCIS codes: (290.5880) Scattering, rough surfaces; (050.0050) Diffraction and gratings; (050.2770) Gratings; (040.2235) Far infrared or terahertz.

References and links

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1. Introduction

The scattering of electromagnetic wave on a finite collection of grooves has recently sparked a wealth of experimental and theoretical works, which are aiming at understanding the underlying physics and at developing new applications of wave manipulation. In the past twenty years, a lot of methods, such as impedance boundary condition approach [1,2] and rigorous coupled-wave analysis [3], were established to study this scattering problem. Among these methods, the waveguide mode (WGM) method [4], which was formulated for the case...
of an isolated groove, required neither matrix inversions nor special function evaluations, thus it is efficient in computation. For a finite collection of grooves, treating the coupling interaction between the grooves as insignificant will make the formulation of WGM straightforward and easy to implement. However, although the importance of mutual coupling between grooves was demonstrated in [5], the formulation of WGM including the coupling interaction has not been developed for a finite collection of grooves.

In this paper, based on the WGM method, coupling interaction of electromagnetic wave in a groove doublet configuration is studied. The formulation obtained by WGM method for a single groove [4] is extended to two grooves. By exploring the total scattered field of the groove doublet configuration, coupling interaction ratios are defined to describe the interaction between grooves quantitatively. Since each of the two grooves is regarded as the basic unit here, one can even investigate the effects of coupling interaction on the scattered fields of each groove respectively. Numerical results show that an oscillatory behavior of coupling interaction is damped with increasing groove spacing. The incident and scattering angle dependence of coupling interaction is symmetrical when the two grooves are the same. For the case of two subwavelength grooves, the coupling interaction is not sensitive to the incident angle and scattering angle. It should be noted that the results obtained here can be generalized to a finite collection of grooves. Moreover, our study offers a simple alternative to investigate and design scattering devices, such as metallic gratings, compact directional antennas, and couplers, etc.

2. Problem formulation

In WGM method, the incident field is replaced by an impressed surface current in the aperture, and fields in and above the cavity are represented by the use of waveguide modes [4]. Based on this simplified approach, coupling interaction of electromagnetic wave in a groove doublet configuration is studied here. For simplicity, the configuration is assumed as two rectangular grooves in a perfect electrical conductor (PEC) plane, as shown in Fig. 1. A transverse magnetic (TM) plane wave \( E_{j} = E_{0j} \hat{x} + E_{0j} \hat{z}, \) and \( H_{j} = H_{0j} \hat{y} \) is obliquely incident at an arbitrary angle of incidence \( \theta_{i} \), which is between the incident wave vector \( k_{0} \) and the \( z \) direction, upon the configuration.

![Fig. 1. Schematic of the groove doublet configuration, including the geometrical parameters](image)

In this groove doublet configuration, each groove can be regarded as the basic unit. The compound incident field for each groove consists of the TM plane wave and the field scattered by the other groove. For example, the compound incident wave \( H_{\text{com}} \) for groove 2 can be represented as \( H_{2} + H_{12} \), where \( H_{12} \) is the scattered field of groove 1 obtained by ignoring the effect of groove 2. In other words, the compound scattered field of groove 2 can be expressed as \( H_{\text{com}} = H_{25} + H'_{25} \), in which \( H_{25} \) is the scattered field generated directly by \( H_{1} \).
and $\mathbf{H}'_{2S}$ is the additional scattered field generated by $\mathbf{H}_{1S}$, as illustrated in Fig. 1. Since the additional scattered field has been taken into account, the coupling interaction of electromagnetic wave between grooves is included here. In this section, we first explore the compound scattered field of each groove. Then, the total scattered field of the groove doublet configuration can be obtained as $\mathbf{H}_{s}^\text{total} = \mathbf{H}_{s}^\text{com} + \mathbf{H}_{s}^\text{2S}$.

We assume that the incident THz wave has amplitudes $|\mathbf{H}_1| = H_0$ and $|\mathbf{E}_2| = E_0 = \eta_0 H_0$, where $\eta_0 = 120\pi\Omega$. If the material in both regions are the same, where $\varepsilon_1 = \varepsilon_2 = \varepsilon_\text{r}$ and $\mu_1 = \mu_2 = \mu_\text{r}$, according to [4], the scattered field of each groove generated directly by $\mathbf{H}_i$ can be expressed as

$$
\mathbf{H}_j(\rho_j, \theta_j) = H_0 W_j \sqrt{\frac{ik_0}{2\pi \rho_j}} e^{-ik_0\rho_j} \sum_{n_{j}=0}^{N_j} c_{n_{j}} L_{n_{j}}(\theta_{j}) \hat{y},
$$

where $j = 1, 2$ represent groove 1 and groove 2, respectively, $k_0$ is the wave vector in free space, $\theta_j$ is the scattering angle, and $L_{n_j}(\theta_j)$ represents far-field Fourier integration of modal aperture field

$$
L_{n_{j}}(\theta_{j}) = \frac{-iu_{n_j}}{k_0 W_j \varepsilon_\text{r}} \int_{0}^{\rho_{j}} \cos(n_{j} \pi - x) e^{i k_0 (\sin \theta_{j}-x) / W_j} dx
$$

$$
= \frac{-iu_{n_j}}{k_0 \varepsilon_\text{r}} \left[ 1 + (-1)^{n_{j}} e^{i k_0 \sin \theta_{j}} \right] \frac{k_0 W_j \sin \theta_{j}}{(n_{j} \pi)^2 - (k_0 W_j \sin \theta_j)^2}.
$$

where $u_{n_j} = \sqrt{(n_{j} \pi / W_j)^2 - k_0^2 \varepsilon_\text{r} \mu_\text{r}}$. The field expansion coefficients $c_{n_{j}}$ in Eq. (1) is given by $c_{n_{j}} = \exp(-2u_{n_j} \rho_{j}) - 1 / a_{n_{j}}$, in which $a_{n_{j}}$ can be obtained as

$$
a_{n_{j}} = [1 - (-1)^{n_{j}} e^{-i k_0 \sin \theta_{j}} \sin \theta_{j} / (n_{j} \pi)^2 - (k_0 W_j \sin \theta_j)^2]^{-1} \int_{0}^{\rho_{j}} \cos(n_{j} \pi - x) e^{i k_0 (\sin \theta_{j}-x) / W_j} dx
$$

for $n_{j} \geq 0$.

where $\varepsilon_\text{r} = 1$ and $\varepsilon_\text{r} = 2$ for $n_{j} \geq 1$. Note that for lossless media in the groove, a series truncation of $N_j = 2W_j \sqrt{|\varepsilon_\text{r} \mu_\text{r}|} / k_0$ has provided excellent results [4].

Next, we will deduce the additional scattered field $\mathbf{H}'_{1S}$, which is generated by $\mathbf{H}_{1S}$. For simplicity, we only consider the scattered field propagating in the $+$ $x$ direction from groove 1. Substituting $\rho_1 = x$ and $\theta_1 = \pi / 2$ into Eq. (1), we can obtain

$$
\mathbf{H}_{1S}(x) = H_0 W_1 \left[ \sum_{n_{1}=0}^{N_1} c_{n_1} L_{n_1} \left( \frac{\pi}{2} \right) \right] \sqrt{\frac{ik_0}{2\pi x}} e^{-ik_0x} \hat{y},
$$

$$
= H'_1 e^{-ik_0x} \sqrt{x} \hat{y},
$$

in which $H'_1$ represents

$$
H'_1 = H_0 W_1 \left[ \sum_{n_{1}=0}^{N_1} c_{n_1} L_{n_1} \left( \frac{\pi}{2} \right) \right] \sqrt{\frac{ik_0}{2\pi x}}.
$$
Fourier cosine series are used to represent the additional impressed surface current $J'_s$ for the scattered field $H_{15}$ generated by groove 1. These series can be written as

$$J'_s = 2H'_e \frac{e^{-ik'_s \hat{x}}}{\sqrt{x}} \sum_{n=0}^{N} a'_{2n_0} \cos \left( \frac{n_0 \pi}{W_2} (x - \Lambda) \right) \hat{x},$$

where Fourier cosine series integration yield

$$a'_{2n_0} = \frac{2}{W_2} \int_{-W}^{W} e^{-ik'_s \hat{x}} \cos \left( \frac{n_0 \pi}{W_2} (x - \Lambda) \right) dx.$$  

The cavity modes for groove 2 are formed from pairs of $\pm z$ directed parallel plate waveguide modes [6] to satisfy $E'_s(x, -d_2) = 0$ on the groove bottom. Field components parallel to the aperture are given for $-d_2 \leq z \leq 0$ by

$$H'_s(x, z) \approx H'_e \sum_{n=0}^{N} b'_{2n_0} \cos \left( \frac{n_0 \pi}{W_2} (x - \Lambda) \right) \cosh \left[ \gamma_{2n_0} (z + d_2) \right],$$

$$E'_s(x, z) \approx \frac{-iE'_s}{k_0 E'_e} \sum_{n=0}^{N} b'_{2n_0} \gamma_{2n_0} \cos \left( \frac{n_0 \pi}{W_2} (x - \Lambda) \right) \sinh \left[ \gamma_{2n_0} (z + d_2) \right],$$

in which $\gamma_{2n_0} = \sqrt{(n_0 \pi/W_2)^2 - k_0^2 e_{\mu}}$.

Above groove 2, the fields are expanded for $+z$ directed waveguide modes as

$$H'_s(x, z) \approx H'_e \sum_{n=0}^{N} c_{2n_0} \cos \left( \frac{n_0 \pi}{W_2} (x - \Lambda) \right) \exp(-u_{2n_0} z),$$

$$E'_s(x, z) \approx \frac{iE'_s}{k_0 E'_e} \sum_{n=0}^{N} c_{2n_0} u_{2n_0} \cos \left( \frac{n_0 \pi}{W_2} (x - \Lambda) \right) \exp(-u_{2n_0} z),$$

where $u_{2n_0} = \gamma_{2n_0} = \sqrt{(n_0 \pi/W_2)^2 - k_0^2 e_{\mu}}$.

By enforcing continuity conditions at the aperture boundary, field expansion coefficients are found as

$$c'_{2n_0} = -\sinh(\gamma_{2n_0} d_2) b'_{2n_0} = \left[ \exp(-2u_{2n_0} d_2) - 1 \right] a'_{2n_0}.$$ (10)

The additional scattered field $H'_{2s}$ of groove 2 can now be evaluated using Green’s function integrations. We can obtain

$$H'_s(\rho_2, \theta_2) \approx H'_e W_2 \sqrt{\frac{ik_0}{2\pi\rho_2}} \sum_{n=0}^{N} c_{2n_0} L_{2n_0}(\theta_2) \hat{y},$$

where $\theta_2$ is the scattering angle, and $L_{2n_0}(\theta_2)$ represents far-field Fourier integrations of modal aperture fields, which can be computed by Eq. (2).

As mentioned above, the compound scattered field of groove 2 consists of the scattered field generated directly by the TM plane wave and the additional field generated by $H_{1s}$, where $H_{1s}$ is given by Eq. (1). Thus, the compound far-zone scattered field of groove 2 can be represented as
\[ H_{2s}^{\text{com}}(\rho_2, \theta_2) = H_{2s} + H_{2s}^{\prime} \]
\[ = H_0 W_2 \left( \frac{ik_0}{2\sigma \rho_2} \sum_{n=0}^{N} \left( c_{2n} + F_{n}^{1+2} \right) L_{2n}(\theta_2) \hat{y} \right). \]  
\( \text{(12)} \)

where \( F_{n}^{1+2} \) describes the electromagnetic mode effect of groove 1 on groove 2. Comparing Eq. (12) with Eq. (1) and (5), we can obtain

\[ F_{n}^{1+2} = W_1 \left( \frac{\pi}{2} \right) \left( \frac{ik_0}{2\sigma} \right) c_n. \]
\( \text{(13)} \)

By using the same method discussed above, the additional scattered field \( H_{1s}^{\prime} \) and the compound scattered field of groove 1 can also be obtained

\[ H_{1s}^{\text{com}}(\rho_1, \theta_1) = H_{1s} + H_{1s}^{\prime} \]
\[ = H_0 W_1 \left( \frac{ik_0}{2\sigma \rho_1} \sum_{n=0}^{N} \left( c_{1n} + F_{n}^{2+1} \right) L_{1n}(\theta_1) \hat{y} \right). \]  
\( \text{(14)} \)

in which \( F_{n}^{2+1} \) describes the effect of groove 2 on groove 1, and has the form

\[ F_{n}^{2+1} = W_1 \left( \frac{\pi}{2} \right) \left( \frac{ik_0}{2\sigma} \right) c_n. \]
\( \text{(15)} \)

In Eq. (15), the field expansion coefficients are decided by \( c_n = \exp(-2a_n d_1) - 1 \mid a_n' \),

where Fourier cosine series integration yield

\[ a_n' = \frac{2}{W_1} \int_0^\infty e^{-ik_0 \sqrt{\Lambda - x}} \cos(n \pi x) W_1 dx. \]
\( \text{(16)} \)

It should be noted that the polar coordinate origin of Eq. (12) is set at \((x = \Lambda, \ z = 0)\), while that of Eq. (14) is set at \((x = 0, \ z = 0)\). The compound scattered fields of groove 1 and groove 2 are given by Eq. (12) and Eq. (14), respectively. We can analyze the effect of coupling interaction on each of the grooves through the two equations. Then, by using of coordinate transformation and vector addition, the total scattered field of the groove doublet configuration can be expressed as

\[ H_{\text{total}}^{\text{com}}(\rho, \theta) = H_{1s}^{\text{com}}(\rho_1, \theta_1) + H_{2s}^{\text{com}}(\rho_2, \theta_2, \theta_1, \theta_1, \theta_1). \]
\( \text{(17)} \)

In Eq. (17), the process of coordinate transformation has the form

\[ \begin{align*}
\rho_1(\rho, \theta) &= \sqrt{\rho^2 + \Lambda^2} - 2\Lambda \rho \sin \theta \\
\theta_1(\rho, \theta) &= \arctan\left(\tan \theta - \frac{\Lambda}{\rho \cos \theta} \right).
\end{align*} \]
\( \text{(18)} \)

According to Eq. (12) to (18), coupling interaction has been included in the compound field of each groove and the total scattered field of the groove doublet configuration. Equation (17) is the main result of this paper.

Although the case of two grooves is discussed for simplicity in this article, the formulation developed for the groove doublet configuration can be generalized to arbitrary number of grooves. We take a configuration consisted of \( m \) grooves \((m>2)\) for an example. The total scattered field \( H_{\text{total}}^{\text{com}} \) can be represented as \( \sum_{l=1}^{m} H_{l}^{\text{com}}(\rho, \theta) \), in which \( (\rho, \theta) \) can be expressed by \( (\rho, \theta) \) through coordinate transformation. It means that Eq. (17) and Eq. (18) can be generalized to the case of \( m \) grooves. It should be noted that the compound scattered
field $\mathbf{H}_{m}^{\text{total}}$ for $1 < m < n$ contains two additional scattered fields generated by the two adjacent grooves respectively.

3. Numerical results

To investigate the effects of coupling interaction on the total scattered field, we pay our attention on the scattering width $\sigma$ (also referred to as radar cross section per unit length), which represents the scattering ability of a scatterer and can be evaluated using Green’s function integration [4]. Based on the concept of scattering width, we propose a coupling interaction ratio (CIR) to describe the interaction between grooves quantitatively, which can be defined as

$$CIR = \lim_{\lambda \to \infty} \frac{\sigma_{\text{total}} - \sigma_{\text{si}}}{\sigma_{\text{si}}} = \frac{2\pi \rho}{\lambda} \left( |\mathbf{H}_{s}^{\text{total}}|^2 - |\mathbf{H}_{s}^{\text{si}}|^2 \right) / |\mathbf{H}_{s}^{\text{si}}|^2,$$  \hspace{1cm} (19)

where $\mathbf{H}_{s}$ can be obtained by $\mathbf{H}_{s}^{1} + \mathbf{H}_{s}^{2}$. $\sigma_{\text{total}}$ is the scattering width corresponding to the total scattered field with coupling interaction, and $\sigma_{\text{si}}$ is the one ignoring the coupling interaction. It is clear that the coupling interaction gets weaker when $CIR$ approaches 0 while it gets stronger when $CIR$ departs from 0. In order to establish a concept of magnitude, we carried out the calculation of Eq. (19) for different $\Lambda$, $\theta_{1}$, and $\theta_{2}$ with given $\varepsilon_{r}$, $\mu_{r}$, $W_{1}$, $W_{2}$, $d_{1}$, and $d_{2}$. To make the numerical results clear, all size parameters of the grooves are compared with the incident wavelength $\lambda_{0}$, thus the two grooves are either subwavelength or non-subwavelength. The groove doublet configurations can be categorized into three types as the insets to Fig. 2, which are named as double-subwavelength (D-S), double-non-subwavelength (D-NS), and subwavelength-non-subwavelength (S-NS), respectively.

![Figure 2](image)

Figure 2 shows the groove spacing ($\Lambda$) dependence of $CIR$. It is clear that $CIR$ approaches 0 with increasing $\Lambda$, which means that coupling interaction between the two grooves decreases when the spacing between them is increasing. Meanwhile, total scattered field of the configuration has been strengthened/weakened by considering the coupling interaction when $CIR$ is more/less than zero. Moreover, a damped oscillatory behavior with period of $\lambda_{0}$, which is similar to slit-groove interaction [7], can be observed in Fig. 2. These oscillations come from interference between the compound scattered fields of groove 1 and 2.
The incident angle ($\theta_i$) and scattering angle ($\theta_1$) dependence of CIR for D-S, D-NS, and S-DS is displayed in Fig. 3. When the two grooves of the configuration are the same such as the cases of D-S and D-NS, the angle dependence of CIR is symmetrical, as illustrated in Fig. 3(a) and 3(b). Furthermore, one can see from Fig. 3 that the CIR surface changes smooth for D-S, and those for D-NS and S-DS change sharp. We have also calculated the angle dependence of CIR for different sizes of the three types of configurations (not displayed here), and the similar results have been obtained. It means that the coupling interaction is not sensitive to $\theta_i$ and $\theta_1$ when both of the two grooves are subwavelength.

These numerical examples indicate that the introducing of CIR is reasonable to study coupling interaction between grooves. Based on the results obtained above, the coupling interaction in the groove doublet configuration could be conveniently analyzed and designed by the formulation developed here.

Since our formulation is based on WGM, the numerical simulation is rapid, and has improved accuracy with increasing electrical size [4]. Moreover, as mentioned in part 2, since the compound scattered fields of groove 1 and 2 are deduced respectively, our approach would be more convenient when the two grooves are filled with different materials. However, the material of the groove doublet configuration is assumed as perfect electrical conductor (PEC), thus our results are applicable for metallic configurations in low frequency regime such as THz and microwave domain. Surface plasmon polaritons (SPPs) may be stimulated by optical waves on metallic groove configurations under some conditions. Then the coupling interaction between grooves will be different, thus the results obtained here will not be applicable any more.

4. Conclusion

In this article, coupling interaction of electromagnetic wave in a groove doublet configuration is studied based on WGM method. The formulation of total scattered field including coupling interaction between two grooves has been developed. To investigate the coupling interaction quantitatively, CIR is introduced and defined. Numerical results show that an oscillatory behavior of coupling interaction is damped with increasing groove spacing. The incident and scattering angle dependence of coupling interaction is symmetrical when the two grooves are the same. For the case of two subwavelength grooves, the coupling interaction is not sensitive to the incident angle and scattering angle. Although the case of two grooves is discussed for simplicity, the formulation developed in this article can be generalized to arbitrary number of grooves.

Moreover, our study offers a simple alternative to investigate and design metallic gratings, compact directional antennas, couplers, and other devices especially in low frequency regime such as THz and microwave domain.

Acknowledgments

The National Natural Science Foundation of China under grant No. 10974063, and the Research Foundation of Wuhan National Laboratory under Grant No. P080008 have supported this research.