Let $F$ be an irreducible polynomial in 4 variables with complex coefficients with degree $\delta$, such that none of its first partial derivatives is identically zero. Let $A, B, C, D$ denote finite sets of complex numbers. Let $Z(F)$ denote the zero set of $F$. The paper proves that $|Z(F) \cap (A \times B \times C \times D)| = O(|A|^{2/3}|B|^{2/3}|C|^{2/3}|D|^{2/3} + |A||B| + |A||C| + |A||D| + |B||C| + |B||D| + |C||D|)$, where the constant in the $O(\cdot)$ term depends polynomially on $\delta$, or $Z(F)$ exhibits some very strong structure. This is the natural 4-variable analogue of the 3-variable result of the authors [Duke Math. J. 165, No. 18, 3517–3566 (2016; Zbl 1365.52023)]. Three applications of the result are shown.

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