A non-tracking semi-circular trough solar concentrator

Wattana Ratismith | Dilawer Ali | John S. Briggs

1Energy Systems Research Institute, Prince of Songkla University, Songkhla, Thailand
2Program of Sustainable Energy Management, Faculty of Environmental Management, Prince of Songkla University, Songkhla, Thailand
3Institute of Physics, University of Freiburg, Freiburg im Breisgau, Germany

Correspondence
John S. Briggs, Institute of Physics, University of Freiburg, Freiburg im Breisgau, Germany.
Email: john.briggs@physik.uni-freiburg.de

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Summary
We report on the development of non-tracking solar concentrator troughs designed subject to the condition that they operate efficiently for diffuse irradiation from all angles during daylight hours. Troughs of the usual compound parabolic form were considered initially. Remarkably however, the most efficient shape for the above requirement turns out to be a trough of simple semi-circular cross-section incorporating a tube of height equal to the circle radius and with a planar absorber plate placed vertically. This trough has a limited concentration ratio of exactly 2.0 but an intercept factor for incident radiation of 100% for all angles of incidence. This desirable feature is explained from the properties of caustics arising from reflection of incident light at the semi-circular trough (SCT) surface. This very simplest of trough design has the clear advantages of small size and low-cost of manufacture. Experimental tests conducted under varying solar irradiation conditions demonstrate that the simple step of placing an evacuated tube in a SCT can lead to a power output gain of a factor of at least 1.7 on average.

KEYWORDS
100% intercept factor, compound parabolic concentrators, reflection caustics, semi-circular trough, solar thermal concentrator

1 | INTRODUCTION

A solar trough concentrator which can be moved to track the sun under clear-sky conditions usually has a pure parabolic shape and concentrates the direct sunlight at a focal point. Such a pure parabolic tracking concentrator is used in large solar facilities and captures almost 100% of normal incident radiation over the whole day. Of course the fraction of the energy utilisable is subject to losses for example, reflection losses, transmission losses to the absorber and from the absorber to the working fluid, which are specific to a particular facility. For small-scale low-cost solar concentration non-tracking troughs are used. Then arises the question of how much of the incident radiation over the daylight hours can be captured by a non-tracking trough of given shape and dimensions? The same considerations apply to the capture of diffuse radiation entering a concentrator at all angles. Over the past 50 years or so, many designs of trough concentrators for both solar thermal and photovoltaic application have been proposed.

1.1 | Compound parabolic concentrator (CPC) troughs

It is interesting to trace briefly the history of non-tracking concentrating troughs from their introduction. The first
suggestions were based on light-capture techniques of high-energy physics and the fundamental principles of cylindrical concentrators were laid down in 1975. Plainly, the proposed designs in early work were still strongly influenced by the focussing properties of the parabolic shape, for example, Reference 3. Hence, right up to today most troughs are composed of parabolic segments and go under the generic name of “compound parabolic concentrators” (CPCs). The reason for the employment of parabolic segments is that they still retain some of the focussing aspects of a complete parabola.

An absorber plate of planar, cylindrical or more complicated shape is placed in the concentrating trough. In units of the effective absorber width, clearly, the larger the width of the trough the more light enters. The basic idea is that through the use of multiple parabolae, rather than a single pure parabola, the trough width can be increased relative to its height and so increase the acceptance angle of a non-tracking collector. Furthermore the intercept factor, that is the fraction of incident light which is directed by reflection to the absorber or intercepts it directly, can be enhanced for large-angle incidence.

The most popular form is a double-parabolic trough (DPT) composed of two parabolae with the origins separated and the overlapping portions of the parabolae cut out. This gives a trough with a ridge running along the long axis where the base of the absorber tube is situated, see, for example References 6,7. This design increases the width compared to a single parabola, as is desirable for a non-tracking concentrator. Nevertheless, for a parabolic wall the height increases quadratically with the width. Then the CPC design leads to a limited angle of acceptance for light to enter the absorber which is placed normally near the bottom of the trough. A smaller angle of acceptance leads to a limited period of operation for a non-tracking device and an inability to capture much of diffusion irradiation.

Since the 1970s there have been hundreds of papers on the design, performance and application of very many concentrators based upon the CPC principle. To quote only a few examples, there are papers reporting the performance and efficiency of CPC troughs with various absorber shapes,8-11 applications to hot water heating12-17 to refrigeration,18-20 to chemical processes21 and for water de-salination.22 In a recent paper23 it is suggested to mount an assembly of standard double-wall CPC on the side of buildings for water heating applications. In addition there is a large body of work on the coupling of CPC concentrators to photovoltaic devices, see References 24, 25 and references therein. For each application different criteria dictate the design and each seeks to optimise various qualities, for example, output power, hours of operation, thermal efficiency and so on, to suit the application. In view of this it is not realistic to compare the capabilities of the very many individual CPC designed for quite different applications.

The present state of CPC development in all its myriad forms is documented fully in several recent reviews. A “comprehensive and up-to-date review” of the CPC design and applications up to 2018 has been given in Reference 26 under the title “A review on the recent research progress in the compound parabolic concentrator (CPC) for solar energy applications”. A review spanning the complete history of CPC development and applications is to be found in the paper27 “A 50 year review of basic and applied research in compound parabolic concentrating solar thermal collector for domestic and industrial applications”.

Most recently a “Review of the Compound Parabolic concentrator (CPC) with a Tubular Absorber” eponymously emphasising a cylindrical absorber is given in Reference 28. Some of the suggested designs of CPC and absorber shapes are quite sophisticated resulting in complicated manufacturing processes.

1.2 The semi-circular trough (SCT)

The CPC concept certainly has demonstrated its value in non-tracking concentrator design. However, here, in contrast to all previous works, we propose a radical departure from the CPC design in that parabolic segments are abandoned in favour of the much simpler SCT shape. In physical optics terms we utilise not the focussing property of the parabolic shape but rather the reflection into “caustic singularities” of the semi-circle. We suggest that such “tea-cup” caustic reflection from a semi-circular surface plays as fundamental a role in light concentration as does the focus for a parabolic surface.

The semi-circle has the advantage that the height increases only linearly with the width. As shown below, this results in a specific trough size where light entering the trough at all angles is directed onto a planar absorber plate. This feature ensures operation at all daylight hours and the optimum capture of diffuse radiation. Also manufacturing costs can be expected to be much reduced compared to sophisticated multi-parabolic collectors.

The paper is developed in the following way. In Section 2 we introduce numerical parameters with which to characterise trough solar collection performance and to define the gain attainable when a collector tube is incorporated in a trough compared to when it is not. Then in Section 3 we illustrate these criteria by discussing two different troughs composed of parabolic segments that is,
CPC form. In Section 4 we introduce a trough of semi-circular cross-section and discuss its unique light concentration properties. We analyse the advantages and the shortcomings of this simplest of concentrator trough and compare to other CPC troughs. In Section 5 we assess the properties of SCTs with a width increased compared to the size of the absorber plate, with a view to increasing the concentration gain.

Another main purpose of this paper is to compare the light-gathering properties of a commercial absorber tube placed in our SCT, with that of the same evacuated tube collector (ETC) without the trough. That is, we compare the properties of two identical tubes. One tube is within the trough (SCT) and one tube is without a trough (ETC). The results of experimental tests of such collector assemblies on typical clear and cloudy days in Hat Yai, Thailand are presented in Section 6. In Section 7 we discuss some conclusions gained from this study.

2 | TROUGH CHARACTERISTICS

To characterise a concentrator trough, two measures of the trough performance are particularly important.

The first factor is a measure of the concentration gain, we call this factor \( C \). It expresses the advantage that more light enters the aperture area of the concentrating trough than is intercepted by the absorber area presented by “naked” tubes exposed to the sun. For a given trough this enhancement is essentially a geometric quantity, that is, it is decided by the size of the trough and the shape, size and orientation of the absorber.

The collector plate inside the evacuated tube can be either of cylindrical or flat plate shape. Here we consider only the flat plate. For a naked collector tube the flat plate is placed horizontally to maximise the irradiance capture. In the case of a tube inside the trough, we have shown in Reference 11 that it is most efficacious to place the absorber plate vertically.

The concentration gain is a theoretical maximum not accounting for losses for example, due to reflection at the trough surface and transmission losses. In the collector of interest here it is defined as \( C = A/a \), where \( A \) is the width of the trough aperture and \( a \) the width of the absorber plate. Hence the relevant parameter is the trough width divided by the absorber plate width, see Reference 16 for details. For a given absorber width \( a \), the trough light reflection characteristics are decided by the value of \( C \). Therefore, in the following, we take the absorber width \( a \) as the unit of length and express all other lengths as multiples of it.

The second important quantity deciding trough performance is the intercept factor, which we denote by \( J(\theta) \). The angle \( \theta \) is the angle with respect to the vertical at which light is incident on the trough. The quantity \( J(\theta) \) expresses the percentage (or fraction) of the light entering the aperture which is incident upon the absorber plate. We emphasise that the intercept factor is an ideal geometric quantity which does not take account of reflection and other energy losses. For a tracking concentrator trough, \( J(\theta) \) is clearly close to 100%, independent of the angle \( \theta \). For a non-tracking trough, it can vary significantly and usually diminishes as the angle \( \theta \) increases to its maximum value of 90°.

The intercept factor describes the efficiency of the non-focussing optics in directing the incident radiation to the collector plate. However, in Reference 16 we introduced an important measure of trough performance which we call the power concentration factor (PCF). This factor is defined simply as \( P(\theta) = C \cdot J(\theta) \cdot L \) and is a measure of the power captured by the tube in the trough compared to the power captured by the tube without trough, where \( L \) is a factor (less than one) introduced to represent additional losses (reflectivity, heat loss) of the concentrating trough. Details of the derivation are to be found in Reference 16.

Note that losses due to absorption/radiation of the glass tube surrounding the collector plate will be very nearly the same in the two cases. These losses are discussed in more detail below and are of course device dependent. Hence, as we wish to concentrate on the light-capture characteristics, initially the factor \( L \) will be ignored. It will be discussed further when the actual performance of tubes with or without concentrating trough are compared, for example in Section 6. With this simplification, the PCF \( P(\theta) \) is a dimensionless ideal parameter dependent only upon trough size and geometry. The factor should, of course, at least exceed unity which is its value for the tube without trough.

Since the concentration factor \( C \) is fixed for each trough design, then \( P(\theta) \) can be viewed as a scaling of the intercept factor \( J(\theta) \). The strategy then is to optimise \( P(\theta) \) over as large an angular range as possible, subject to constraints of cost, size, output temperature and so on. These latter conditions seriously affect the optimum trough shape for particular applications. To optimise the PCF, clearly one should seek to increase both \( C \) and \( J(\theta) \). Unfortunately it is often the case that an increase of one of these factors is only to the detriment of the other. For example, since \( a \) is fixed, one can increase \( C \) by increasing the trough aperture area \( A \). However, this usually requires that the trough walls become also higher such that large-angle incident sunlight does not reflect so as to enter the absorber. Then \( J \) tends to zero at these angles. Precisely the fundamental problem of non-tracking concentrators is that increasing the concentration gain by
increasing the aperture width, leads to a trough which has high walls compared to the absorber size. In this respect, another trough parameter which is useful is the acceptance half-angle, defined as the maximum angle of incidence of a ray which reflects so as to intersect the absorber. This decides the hours before and after noon in which the absorber collects direct sunlight, or the angular range over which diffuse light is accepted. For most non-tracking trough designs the acceptance angle is considerably less than the maximum of 90°. Clearly, the lower the height of the trough, the larger is its acceptance angle.

To optimise the trough performance one seeks to lower the trough height in order to achieve the maximum acceptance angle and intercept factor but still maintain a significant concentration gain. As will be shown, this requires giving up the parabolic shape and somewhat noteworthy, replacing it by the simplicity of a semi-circle. We show that if one requires an acceptance angle of 90° and intercept factor \( I(\theta) \) of 100% for all angles, this limits the \( C \) factor of the SCT to exactly 2.0. That is, the semi-circle diameter \( A \) is exactly twice the width \( a \) of the absorber plate.

The advantage is that the semi-circular shape makes the trough cheap to build and, since light incident at all angles is accepted, the design is optimum for non-tracking operation in conditions of diffuse radiation. These are the conditions accounted most frequently in tropical or semi-tropical areas, where also the cost of the installation can be a big factor.

### 3 | THE TROUGH SHAPE AND THE PCF

The basic idea of the CPC is that through the use of multiple parabolae the trough width can be increased relative to its height and so enhance the acceptance angle of a non-tracking collector. Furthermore the intercept factor for large-angle incidence can be enhanced. As stated already, the most popular form is a double parabolic trough composed of two parabolae with the origins separated and the overlapping portions of the parabolae cut out.

The height of truncation of the parabolic-sided trough is arbitrary but increases quadratically with the aperture size \( A \). Hence, if one makes \( C \) large by increasing \( A \), then rapidly the large angle rays do not reflect to the absorber and \( I \) is zero for these angles. In the DPT discussed in Reference 11 the acceptance half-angle is only 40°. Then there is no absorption of directed sunlight by the absorber plate after around 2:30 PM in the afternoon or before 9:30 AM Correspondingly in diffuse irradiation a large part of the angular region does not enter the absorber.

This fundamental problem of the trade-off between concentration gain and acceptance half-angle in CPC of standard design is still a subject of discussion, for example, the question of truncating the height is analysed in the paper of Khalid et al.\(^{29}\)

A detailed and interesting discussion of this aspect of CPC design is given in the paper of Chen, Chen and Zhang\(^{6}\) where truncation of the parabolic wall height and other strategies are proposed whereby the concentration ratio and intercept factor can be optimised.

The lesson is that one must modify the walls of the parabola if it is desirable to have absorption at larger angles. In Reference 11, this was achieved by utilising the double parabolic shape but such that the trough base is flat. This trough achieves an acceptance half-angle of 63° and has a width which is 1.4 times larger than its height. Our aim is to increase this and extend the 100% intercept factor to even larger angles, indeed out to the maximum acceptance half-angle of \( \theta = 90° \). To achieve this, we experimented with troughs composed not of two but of four parabolic segments and a higher width to height ratio.

An indication of the construction of such a trough is shown in Figure 1. In this design, the slope of the sides is

![Figure 1](image-url)  
*Figure 1* The multiple-parabolic trough (MPT) showing (A) the four separate parabolic segments and (B) the trough with segments connected smoothly [Colour figure can be viewed at wileyonlinelibrary.com]
increased by adding two further parabolic segments as shown in Figure 1A. This is intended to reflect light incident at large angles into the absorber. The four segments are fused together smoothly as in Figure 1B. Again the trough height is truncated at the point at which the sides of the trough become vertical. Now this gives a $C$ of 2.0, that is, the width is double the height of the absorber, which is also the height of the trough. This trough we refer to as the “multiple-parabolic” trough (MPT).

The ray-tracing patterns for the MPT are shown in Figure 2 from $\theta = 0^\circ$ to $\theta = 75^\circ$. In fact the intercept factor is very close to 1.0 for all angles up to $90^\circ$. Hence we have almost achieved the aim of 100% absorption of all light entering the trough during daylight hours. The average of the PCF over the complete angular range is 2.0 for the MPT. Thus the MPT, due to its relatively low height is an effective compound parabolic trough in that it accepts almost all the light incident at all angles, clearly important for operation in diffuse irradiation conditions.

Referring again to Figure 2 for the ray-tracing for the MPT, one notes that for normal incidence a clear “tea-cup” caustic as observed in reflection from a spherical surface is evident. This, and the fact that the width is twice the height, prompted us to compare the MPT to a semi-circle. It turns out that the shape is quite close. This led us to abandon pre-occupation with complicated parabolic forms which are expensive to manufacture and to explore the concentrating properties of the simplest of all possible troughs, a semi-circle of radius equal to the width of the absorber plate ($C = 2.0$). This we call the SCT.

4 | THE SCT; THE TROUGH WITH SEMI-CIRCULAR CROSS-SECTION

The ray-tracing diagrams for a SCT with radius equal to the height of the absorber plate are shown in Figure 3. The close similarity to those of the MPT in Figure 2 is obvious, particularly at larger angles of incidence. For normal incidence ($0^\circ$ on Figure 3) the classic tea-cup caustic again is evident. Changing the angle of incidence gives a new normal passing through the centre of the trough circle. This centre point coincides with the upper tip of the absorber plate. Hence, changing the angle of incidence is equivalent to rotating the absorber counter-clockwise through the cusp structure. This is particularly evident at $\theta = 15^\circ$ and $\theta = 30^\circ$ on Figure 3.

The caustic terminating in a cusp located along the central ray is an invariant feature of reflection by a semi-circle. Mathematically it is a part of catastrophe theory as explained in Reference 30. The equation of the caustic line is given in Reference 31. From this equation one can show analytically that the cusp location along the central ($\theta = 0^\circ$) ray is exactly at a distance of one half the radius of the circle that is, it coincides with the centre of the absorber plate. Indeed mathematically the local curvature of the parabola is also that of a semi-circle which indicates that the cusp at the tip of the caustic is the vestige of the focal point of the parabola.

This is seen readily if one considers a Cartesian $(x, y)$ coordinate system with origin at the bottom of the trough, for example, in Figure 1B. For a parabolic
through the curve describing the trough surface is given by the equation \( y = bx^2 \), where \( b \) is a constant. This gives the position of the focal point as \( x = 0, y = 1/(4b) \).

For a SCT of radius \( R \) the centre of the circle is at the point \((0, R)\) and the trough surface obeys the equation of a circle,

\[
x^2 + (y-R)^2 = R^2 \tag{1}
\]

This can be written

\[
x^2 + y^2 - 2Ry = 0. \tag{2}
\]

For small deviations from the origin, the term \( y^2 \) can be neglected to give approximately,

\[
y \approx \frac{1}{2R} x^2 \tag{3}
\]

the equation of a parabola with \( b = 1/(2R) \) and so a focus at \( y = R/2 \). This result is well-known in Optics, see Reference 32. It implies that for near normal incidence rays falling on a spherical mirror with radius \( R \) will focus at a distance \( R/2 \). That is, locally a spherical mirror has the same effect as a parabolic reflector.

Hence, in the limit of very many short parabolic segments fused together one obtains a circular shape. This explains the similarity of the MPT of Figure 1, with four parabolic segments, to the SCT.

We suggest that the caustic plays a role as fundamental for non-focussing optics as the focus of the parabola does in focussing optics. Unlike the focus, however, due to the symmetry of the circle it is present at all angles. In Figure 4 one can see, that at \( \theta = 0^\circ \), the caustic, at which all reflected rays are tangential, essentially “traps” the light in the area below it and bounded by the absorber plate. This trapped light is forced to intersect the absorber giving an intercept factor of 1.0. However, the intercept factor is also precisely unity at all angles. This unique feature is peculiar to an SCT with radius equal to absorber height and can be explained from the structure of the caustic as a function of the angle of incidence. To illustrate this in Figure 4 we have plotted the reflected rays for various angles of incidence but only showing the rays incident in the left-hand half of the semi-circle.

For normal incidence \( \theta = 0^\circ \) the ray passing through the centre of the semi-circle is reflected back that is, at \( \theta = 180^\circ \), see Figure 4. All other rays, successively as one moves to the edge of the semi-circle, are reflected through angles less than \( \theta = 180^\circ \). Then they must cross the radius line from the bottom point of the trough to the centre. Of course this is just the line occupied by the absorber plate, so one has 100% interception or \( \mathcal{J} = 1.0 \). This pattern is repeated as \( \theta \) increases all the way to \( 90^\circ \). The extremal ray passing through the centre of the circle at angle \( \theta \) to the horizontal is reflected back along itself that is, through an angle \( 180 + \theta \), see the \( 60^\circ \) case on Figure 4 for example. All other rays incident out to the trough edge are reflected through smaller angles and therefore must intersect the absorber. This explains the achievement of \( \mathcal{J} = 1.0 \) at all angles of incidence.

In agreement with Equation (3), on Figure 3 at \( \theta = 0^\circ \) one sees the tip (cusp) of the caustic coincides with the point \( R/2 \) on the vertical axis (the absorber plate has the

**Figure 3** Ray tracing diagram for the SCT. The intercept factor is 100% at all angles of incidence [Colour figure can be viewed at wileyonlinelibrary.com]
length $R$ equal to the trough radius). This is the focussing of light rays incident very close to the central vertical axis occupied by the absorber plate. At large angles $\theta = 45^\circ$, $60^\circ$, and $75^\circ$ on Figure 3 one sees clearly a focus near to a distance $R/2$ from the trough wall. Again this focus is due to the corresponding circular wall segment behaving approximately as a parabola, according to Equation (3).

The simple semi-circular geometry represents a significant departure from the CPCs used to date, see Reference 26. The notable features, again referring to an ideal loss-free situation, are:

- The SCT is unique in that it transmits 100% of the radiation entering the aperture to the absorber plate at all angles of incidence from $0^\circ$ to $90^\circ$. That is $\mathcal{F} = 1.0$ independent of $\theta$.
- Since the diameter $A$ is twice the absorber width $a$, then the concentration gain $C = 2.0$.
- Clearly, the simplicity of the semi-circular shape has important implications for cost of manufacture, mechanical stability and so on.
- The width (diameter) to height (radius) ratio is 2.0 giving an acceptance half-angle equal to the maximum value of $90^\circ$.

5 INCREASING THE CONCENTRATION GAIN

The excellent capture characteristics of the SCT lead one to ask if they are retained when the concentration ratio $C$, that is the circle radius or trough aperture width, is increased. However, from the outset one can see that the 100% intercept factor $\mathcal{F} = 1.0$, will not be achieved for all angles of incidence since it depends precisely upon the trough width being double that of the absorber height.

To test the consequences of increasing trough width in order to increase the concentration ratio, in Figure 5, left panel, we show the absorption characteristics for $C = 4.0$. Let us call this trough SCT4. This corresponds to a semi-circle radius equal to two times the height of the absorber. Hence, for normal incidence, according to Equation (3) the top of the absorber plate now coincides exactly with the cusp of the caustic. The result is an intercept factor of 1.0 for normal incidence since all rays fall on the absorber. However, with increasing incident angle, some of the rays near to the cusp are not absorbed and the intercept factor has fallen to 0.42 at $45^\circ$.

The SCT4 shows the clear tea-cup cusp at zero degree incidence and $\mathcal{F} = 1.0$ to give a PCF $\mathcal{P} = C \cdot \mathcal{F} = 4.0$ compared to the constant 2.0 of the standard SCT. However, the decrease of the intercept factor at larger angles leads to a drop to $\mathcal{P} \approx 2$ already at $\theta = 35^\circ$ (see Figure 6). It is clear also that a $C$ value of 4 is perhaps the largest value to make full use of the cusp properties. Were the trough to be made larger, then at normal incidence the cusp would lie higher than the absorber tip and less radiation would be intercepted by the absorber that is, the intercept factor would be less than unity at all angles.

For comparison, in Figure 5, right panel B, we show also the ray tracing and intercept factor for the multiple parabolic trough of the same width 4.0 under the same conditions of irradiation. This we call MPT4. The intercept factor is significantly lower at normal incidence, although marginally greater at some larger angles of incidence. The large difference of SCT4 and MPT4 at $\theta = 0^\circ$ is interesting in that the patterns for SCT and MPT of width two (Figures 2 and 3) are virtually identical.

The PCF for the SCT4 and MPT4 are shown in Figure 6 in comparison with the constant value 2.0 of the SCT with width 2.0. The SCT4 gives an enhanced PCF of 4.0 for small angles but, as with MPT4, it drops rapidly to zero around $60^\circ$.

The average value of the PCF over the complete angular range clearly is 2.0 for the SCT and interestingly almost the same, 2.1, for the double-size SCT4. At an average of only 1.4 the double-size MPT4 is significantly lower than the MPT at average 2.0. Hence there appears no great advantage for these designs of scaling to larger aperture size.

Nevertheless the integrated power output from a single tube in the SCT4 is greater than that of the same tube in the smaller SCT. Hence, if trough size is no impediment, then the SCT4 would yield more power and higher output temperatures but only up to an angle of incidence of $40^\circ$.

It is clear that the parameters $C$ and $\mathcal{P}$, the PCF, are trough characteristics decided largely by the trough.
geometry. Whilst high values of PCF enhances initial power collection, the overall efficiency of a collector assembly depends also upon the tube properties, the heat transfer to the working fluid and overall heat losses. The efficiency of tube assemblies with and without concentrating SCT is considered in the next section.

6 | EXPERIMENTAL RESULTS

In the experiments we compare the output power of assemblies of ETC tubes without troughs and a flat absorber plate oriented horizontally, with the output power of identical tubes placed in an SCT such that
the absorber plate is oriented vertically. The measurements on the two assemblies are conducted simultaneously.

The absorber tubes are standard evacuated glass-walled tubes of length 2 m and outer glass diameter 100 mm. The flat absorber area per tube is 0.17 m² and the absorbance is given as 94%. The SCTs are manufactured from aluminium with a reflectivity of 89.9%.

The SCT has $\mathcal{J} = 1.0$ as a constant value and, from the trough cross-section circular geometry, one has also the fixed $C = 2.0$. This gives a PCF, neglecting losses, of $\mathcal{P} = 2.0$, independent of $\theta$, see Figure 6.

Since the ideal power gain of SCT compared to ETC is a factor of two, to test this we have made many observations of the output power of an assembly of eight tubes embedded in the SCT compared to an identical assembly of eight ETC without troughs. These experiments have been performed in a variety of experimental and climatic conditions.

The solar collector modules and test arrangement are illustrated in Figure 7. The setup and procedure for the efficiency testing follows closely the guidelines of ISO 9806:2017 for standard test conditions of solar collectors. The two test modules were configured in parallel connection, aligned in a north-south direction and tilted to the sun at an angle of approximately 20°. The inlet and outlet temperature differences of the flowing fluid (water) were measured by inserting temperature sensors across the test modules, see Figure 7. A pyranometer (Huksfluex SR20-T1) with a sensitivity of $13.58 \times 10^{-6} \text{ Vm}^2 \text{ W}^{-1}$ and a calibration uncertainty of $\pm 0.13 \times 10^{-6} \text{ Vm}^2 \text{ W}^{-1}$ was mounted with the same inclination as that of the solar collectors to measure the solar radiation. Two flow meters, attached to the SCT and ETC modules measured the flow rate of the working fluid. A cooler and heater were used to control the inlet fluid temperature. The whole testing process was executed by varying the inlet temperature from 40°C to 90°C, in constant steps of 10°C. The testing facility was monitored for at least 15 minutes to ensure the correct fluid measurement temperature at the inlet, followed by a steady-state measurement period of at least 15 minutes at each measured temperature. Data acquisition systems sampled data for inlet and outlet temperatures across the solar collectors, as well as the ambient temperature and solar irradiance after every minute.

Here we present typical results for a mostly clear day and a mostly cloudy day. In Figure 8, we show the results for a day of clear sky, except for a half-hour period around noon. The upper panel gives the solar irradiance

![Figure 6](image1.png)

**FIGURE 6** The PCF for the troughs SCT (dot-dash line), SCT4 (dashed line) and MPT4 (continuous line) [Colour figure can be viewed at wileyonlinelibrary.com]

![Figure 7](image2.png)

**FIGURE 7** Schematic diagram of the testing facility comprising eight SCT collectors (right) connected in parallel with eight ETC collectors (left). $T_i$ is the inlet temperature and $T_o$ the outlet temperature. The arrows indicate the direction of water flow [Colour figure can be viewed at wileyonlinelibrary.com]
from 10 AM to 3 PM on March 14, 2019. Apart from the short period the clear sky gives a near uniform irradiance of average around 900 W/m².

The middle panel shows the output power from the two assemblies. The output power $Q_e$ in Watts is calculated as

$$Q_e(t) = \dot{m} C_p (T_e - T_i).$$

In this equation, see Reference 16, the water flow rate is $\dot{m}$ and the inlet and outlet temperatures of the flowing water are $T_i$ and $T_e$ respectively. The specific heat of the water is $C_p$.

In Figure 8 one sees a strong similarity in the variation of the two output power curves, with the SCT uniformly roughly a factor of two higher. The two output powers track the variations in solar irradiance rather well, particularly the sudden fall around noon. This ratio of these two output powers of the middle panel is shown explicitly in the lower panel and varies from around 1.6 to a little over 2.0. The average over the time period shown is 1.72. This ratio reflects directly the advantage of placing the ETC in a concentrating SCT.

The deviation from the ideal value of 2.0 is probably due to the additional reflection and irradiation losses accompanying the embedding of the vacuum tubes in a trough. For the bare tubes the transmission through the evacuated glass cylinder is given as 91% and the absorption coefficient of the absorber as 94% by the manufacturer. This implies a loss factor of 14% of incident radiation for the bare tubes. For the tubes in the trough we assume the same loss but there is additional loss, principally due to reflection. We estimate a total loss of between 30% for normal irradiation (all light entering the absorber is reflected light) to around 20% for larger angles where a large proportion of the radiation intersects the collector tube without reflection (see Figure 3). This extra loss, between 16% and 6% due to embedding in the concentrator trough, is the prime factor in reducing the output power gain of the trough assembly from the theoretical ideal of 2.0. It gives a loss factor of $\mathcal{L} = 0.84$ for normal incidence rising to $\mathcal{L} = 0.94$ at large angles. This lowers the PCF of the SCT from the ideal value of 2.0 to between 1.68 and 1.88, in agreement with our measured average value of 1.72.

In Figure 9 we show results for February 6, 2019, beginning measurements at 9:30 AM. This was a cloudy morning with extreme solar irradiance fluctuations up to around 1 PM, followed by relatively uniform irradiance in the afternoon, as evident in the upper panel of the figure. The middle panel shows the separate output powers of the two assemblies. Again one notes the very close similarity of the fluctuations in output power of the two curves, testifying to the similar response behaviour of the two sets of eight tubes. The output power fluctuations tend to smooth out the more rapid insolation fluctuations. However, the assembly embedded in the SCT gives again a consistently higher power output than the assembly of ETC without troughs. This ratio is shown on the lower panel and again, as in Figure 8, has
an average value of 1.72. Hence, under very different conditions of solar irradiation, the simple SCT consistently gives a significant increase in power output compared to the ETC.

That the SCT module can achieve high temperature is demonstrated in Figure 10. Here, in the upper panel, we show the strongly varying insolation between the hours of 10 AM and 1 PM on a typical partly cloudy day.

For an assembly of 10 SCT's, the lower panel shows the temperature variation of 300 L of water in the storage vessel. Despite the fluctuations in irradiance, the water temperature rises steadily and monotonously, in this case to a maximum around 120°C. On a clear day temperatures up to around 200°C are readily achieved by this assembly.

6.1  |  The thermal efficiency

For applications, the thermal efficiency of the total collector assembly is an important parameter. This efficiency $\eta$ is defined simply as the ratio of output power to input power. Details of its calculation are given in Reference 16 and in the ISO standard for testing evacuated tube efficiency. Following the ISO standard the efficiency is plotted against the variable $x = (T_m - T_a)/G$ in units of m²K/W, where $T_m$ is the mean of the input and output temperatures and $T_a$ is the ambient temperature. $G$ is the solar insolation.

The overall efficiency depends upon the geometry of the assembly in deciding the effective aperture for
collection of solar insolation and the efficiency of concentration of this irradiation into the absorber plate (the PCF factor). In addition there are the heat losses, conductive, radiative and convective, from the assembly tubes and associated connectors and pipework.

The heat losses from the evacuated tubes themselves are quite small and are of course the same for tubes within the trough or without a trough. The vacuum of the tube eliminates convective loss and the emittance from the absorber plate is given as 5%. A measure of the heat losses is the rate at which the efficiency falls as the temperature difference ($T_m - T_a$) increases. This is discussed below.

There are two ways of expressing the efficiency, either with respect to the gross collecting area of a concentrator assembly or with respect to the smaller aperture area afforded by the troughs. The gross area of the assembly is more relevant to commercial applications. This is because for commercial application the area occupied by an assembly, for example, a roof area, may be limited. For laboratory experimental purposes, for example, in the ISO measurement, the smaller aperture area is used, so that these efficiencies are larger than those of the gross area.

In a sequence of experiments, we have measured the efficiency of the SCT in comparison with the tubes without trough. An assembly of eight SCT troughs was compared directly (ie, simultaneously) with an assembly of 16 tubes without trough. This is because then the two assemblies have almost the same gross area and therefore the same value of $Q_i$. The gross area of 16 tubes without troughs is (4.1 m$^2$) and that of eight tubes in the SCT is (3.8 m$^2$).

**FIGURE 11** (A) Red points: the measured efficiency $\eta$ with respect to the gross area of the assembly of 16 ETC tubes without troughs, the fit to the linear equation shown is the dashed line (red). Blue points: the measured efficiency of the SCT assembly with eight tubes, the fit to the linear equation shown is the continuous line (blue). (B) Same as in (A) but showing the efficiencies expressed in terms of the aperture areas [Colour figure can be viewed at wileyonlinelibrary.com]

**FIGURE 12** The pilot plant concentrator assembly operating at a food canning factory. The evacuated tubes embedded in the SCTs are roughly 2 m in length [Colour figure can be viewed at wileyonlinelibrary.com]
The results are shown in Figure 11, plotted as a function of the standard parameter \( x \) and for completeness we show efficiencies based on gross area and on aperture area. In the parameter region covered by our experiments the efficiencies are almost a linear function of \( x \) and the fits of the linear equation are indicated on the curves.

The efficiencies of the two assemblies are rather close and the range of \( \eta_0 \equiv \eta(x = 0) \) obtained by extrapolation back to \( x = 0 \) fall within the range 45%-50% for gross area and correspondingly higher at 56%-68% for aperture area. This is typical for collectors of non-tracking type, see References 18,33. The efficiency decreases, as is usual, with increasing \( x \), or an increase of mean temperature with respect to ambient temperature. This is due to heat loss processes but the fits indicate that this loss factor is somewhat lower for the SCT collector than for the tubes without trough.

A very useful application of solar collectors is to pre-heat water for commercial or industrial processes, as shown for example in Reference 35. To demonstrate such an application and to show that the simple SCT collector can be implemented successfully as a commercial solar concentrator, we have established a pilot plant of some ten, eight-tube modules at a food canning factory in Songkhla, Thailand (see Figure 12). This is currently in use to pre-heat water from the ambient temperature of around 30°C to a temperature of 60°C before passing into a boiler powered by fossil fuel. A similar pilot plant has been installed on the roof of a hotel in Hat Yai, Thailand.

7 | CONCLUSIONS

In the course of a research programme to develop CPCs with the ability to accept sunlight at all angles of incidence we arrived at a multiple parabolic trough (MPT) shape composed of four parabolic segments. This shape shows great similarity to a trough of simple semi-circular cross-section (SCT). Hence, here we have compared different CPC troughs with this SCT. In order to achieve an intercept factor \( \mathcal{I} = 1.0 \) for all angles, the radius of the semi-circle is restricted to equal the height of a vertical absorber plate. The key optical feature in reflection from a semi-circular surface is the cusp caustic. We have shown how geometric properties of the caustic ensures that \( \mathcal{I} = 1.0 \), that is, that all light entering the trough apertures intersects the absorber plate.

The PCF defined as \( \mathscr{P} = C \mathcal{I} \) (ignoring trough power losses) has a constant value of 2.0 for all angles of incidence making the collector particularly suitable for use in diffuse irradiation conditions. Although the concentration gain \( C = 2 \) is modest, this is offset by this ability to concentrate all light from any angle and the extreme simplicity of manufacture of the SCT, compared for example to a CPC.

We considered also an SCT4 with trough width four times the width of the absorber plate. Although of course more sunlight enters the trough, only in the hours around noon is the PCF greater than that of the SCT. After 3 PM (incident angle \( \theta \approx 45^\circ \)) the PCF of SCT4 falls rapidly to zero. For this reason we consider the SCT with \( C = 2 \) as the ideal size for the simple SCT optimised to capture diffuse light.

In a sequence of experiments under various conditions, from which two examples are presented, we have measured the ratio of the output power of a tube with semi-circular concentrating trough to that without trough. We found that the average gain in output power lies between a factor 1.5 and 2.0, with a typical value around 1.7.

That the proposed SCT design is commercially viable has been demonstrated by its implementation in a pilot plant at a canning factory to pre-heat process water. The trough size used, approximately 10 cm high and 20 cm wide, is dictated by the approximately 10 cm diameter of the cylindrical evacuated tube used. However, it is important to remark that the SCT scales in size according to the diameter. Recently, a design of solar water heater using capillary absorbing tubes of only 4 mm, embedded in CPCs of around 50 mm in height has been proposed.36 Hence, incorporating a capillary of 50 mm in an SCT of diameter 100 mm would achieve the same PCF of 2.0, see Figure 6, as the SCT tested in this paper. Clearly such a compact unit of many troughs is unobtrusive and therefore very suitable for use on the roofs of private residential buildings.

Hence, the SCT, due to its simplicity, compact size and low cost of manufacture is ideal for use in a multitude of energy-saving applications for heating and cooling in industrial buildings, hotels and residential complexes.

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NOMENCLATURE

\( Q_i \) input power to the solar collector
\( Q_e \) power output of solar collector
\( J(\theta) \) **intercept factor**  
\( \mathcal{F} \) **loss factor**  
\( C \) **concentration gain**  
\( \mathcal{P} \) **power concentration factor**  
\( A \) **trough width (m)**  
\( a \) **absorber plate width (m)**  
\( \eta \) **efficiency**  
\( C_p \) **water specific heat \( J/(kg \cdot ^{\circ}C) \)**  
\( T_a \) **ambient temperature (\(^{\circ}C\))**  
\( T_i \) **inlet water temperature (\(^{\circ}C\))**  
\( T_e \) **exit water temperature (\(^{\circ}C\))**  
\( T_m \) **mean of the inlet and exit water temperature (\(^{\circ}C\))**  
\( \dot{m} \) **water flow rate (kg/s)**  
\( G \) **solar irradiance (W/m\(^2\))**  
\( \text{ETC} \) **evacuated tube collector**  
\( \text{CPC} \) **compound parabolic concentrator**  
\( \text{SCT} \) **semi-circular trough**  
\( \text{MPT} \) **multiple-parabolic trough**  
\( \text{DPT} \) **double-parabolic trough**  
\( \text{SCT4} \) **semi-circular trough with concentration factor of 4**  
\( \text{PCF} \) **power concentration factor**

**ORCID**

*John S. Briggs* https://orcid.org/0000-0001-8794-2233

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