Spin Conductivity and Spin-Charge Separation in the High \(T_c\) Cuprates

Qimiao Si

Department of Physics, University of Illinois, Urbana, IL 61801

We study both the spin and electrical conductivities in models of relevance to the high \(T_c\) cuprates. These models describe metallic states with or without spin-charge separation. We demonstrate that, given a linear in temperature dependence of the electrical resistivity, the spin resistivity should also be linear in temperature in the absence of spin-charge separation and under conditions appropriate at least for the optimally doped cuprates, but is in general not so in the presence of spin-charge separation. Based on these results, we propose to use the temperature dependence of the electron spin diffusion constant to diagnose spin-charge separation in the cuprates.

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One of the most striking behaviors in the normal state of the high $T_c$ cuprates is the linear in temperature dependence of the electrical resistivity $[1]$. The extended temperature range over which this $T-$linear resistivity occurs, along with other experimental signatures, have led to the conclusion that the electrical resistivity is dominated by electron-electron scattering instead of electron-phonon scattering. In a canonical Fermi liquid, however, the Pauli exclusion principle strongly suppresses the phase space available for quasiparticle-quasiparticle scattering, and the resistivity due to electron-electron scattering is expected to have a $T^2$ dependence. The observed $T-$linear resistivity, therefore, suggests that the normal state of the high $T_c$ cuprates deviates from a canonical Fermi liquid.

The precise nature of the normal state, and exactly how electron-electron scattering leads to this $T-$linear resistivity, remain subjects of debate. Starting from the work of Anderson $[2]$, spin-charge separation has been suggested to characterize the normal state. In a spin-charge-separated non-Fermi liquid, the elementary excitations are divided into two species, each carrying either spin or charge quantum numbers only. Such a decomposition alters the phase space available for electron-electron scattering and can, in some cases, result in a $T-$linear resistivity. Spin-charge separation occurs in the Luttinger liquid in one dimension $[3]$ and possibly in two dimensions as well $[2]$, in a phase of the two dimensional $t – J$ model with massless transverse gauge fields $[4]$, and in a mixed valence state of an extended Hubbard model in infinite dimensions with competing spin and charge fluctuations $[5]$. Alternatively, it has been proposed that some form of Fermi liquids with low energy scales can describe the normal state $[6–8]$. These states represent minimal modifications of the canonical Fermi liquid. The elementary excitations are quasiparticle-like, carrying both spin and charge degrees of freedom; these quasiparticles are coupled to some soft collective charge and/or spin fluctuations. While the resistivity is necessarily quadratic in $T$ below the soft energy scale, $T^*$, it can become linear in $T$ at $T \gg T^*$.

While the existence or absence of spin-charge separation remains to be established, it is worth noting that a difference does appear to exist between the spin dynamics and charge transport properties. This is most clearly seen in the optimally doped LaSrCuO in which
data are available over a wide range of temperatures. The Cu-site NMR relaxation rate, \( \frac{1}{T_1} \), crosses over from a low temperature \( T \)–linear dependence to a high temperature \( T \)–independent behavior while, over the corresponding temperature range, the electrical resistivity shows essentially no deviation from the \( T \)–linear behavior. This difference, however, can not be used as direct evidence for spin-charge separation as the correlation functions measured by these two probes can not be simply related.

In this communication we propose that a comparison between the temperature dependences of the electrical and spin conductivities can clarify whether or not spin-charge separation occurs in the high \( T_c \) cuprates. To be concrete, we consider several models that might be relevant to the physics of the metallic cuprates. We find that, given a \( T \)–linear electrical resistivity, and under conditions appropriate at least for the optimally doped cuprates, the spin resistivity is necessarily linear in temperature in the absence of spin-charge separation, but is in general not so in a spin-charge separated state. We note that, the spin conductivity \( (\sigma_s) \) describes the response of the spin current \( (j_s) \) to a gradient of magnetic field. The Einstein relation states that \( \sigma_s = \chi_s D_s \). Here, \( \chi_s \) is the uniform spin susceptibility. \( D_s \) is the spin diffusion constant, which can be measured using the technique of spin-injection. It should be noted that the measurement of spin conductivity is feasible in the cuprates as the effective interactions induced by the spin-orbit coupling are relatively small in the cuprates [of the order of a few meV].

**Without Spin-Charge Separation:** In a Fermi-liquid-like state, a \( T \)–linear resistivity can arise from quasiparticles being scattered from soft collective fluctuations. We can study the conductivities in these states within the following phenomenological action,

\[
S = S_{qp} + S_{collective} + S_{int}
\]

\[
S_{qp} = \int d\omega \sum_{k\sigma} c_{k\sigma}^\dagger (-\omega + \epsilon_k) c_{k\sigma}
\]

\[
S_{collective} = \int d\omega \sum_q [N_q \chi_c^{-1}(q, \omega) N_q + \vec{S}_q \chi_s^{-1}(q, \omega) \cdot \vec{S}_q]
\]

\[
S_{int} = \int d\omega \sum_{qk} [V_q (\sum_\sigma c_{k+q,\sigma} c_{k\sigma}) N_q + J_q (\sum_{\sigma\sigma'} c_{k+q,\sigma} \vec{s}_{\sigma\sigma'} c_{k\sigma'}) \cdot \vec{S}_q]
\] (1)
Here, $S_{qp}$ describes the single particle states, created by $c_{k\sigma}^\dagger$, with a dispersion $\epsilon_k$. $S_{collective}$ describes overdamped collective charge and spin degrees of freedom, $N_q$ and $\vec{S}_q$, with fluctuation spectra $\chi_{cf}(q, \omega)$ and $\chi_{sf}(q, \omega)$, respectively. Finally, $S_{int}$ describes the coupling of the single particle states to the collective fluctuations. $V_q$ and $J_q$ are the charge and spin coupling constants, respectively. Eq. (1) is quite general. It incorporates as special cases several proposed scenarios for the cuprates. For instance, the marginal Fermi liquid approach \cite{8} corresponds to choosing both $\text{Im}\chi_{sf}(q, \omega)$ and $\text{Im}\chi_{cf}(q, \omega)$ to be $\rho_o\text{sgn}\omega$ for $\omega > T$, and $\rho_o\omega/T$ for $\omega < T$ (where $\rho_o$ is the density of states). The nearly antiferromagnetic Fermi liquid approach \cite{7} corresponds to neglecting charge fluctuations, and assuming a mean field form for $\text{Im}\chi_{sf}(q, \omega)$. Other scenarios, as reviewed in Ref. \cite{6}, are also incorporated in Eq. (1) in a similar fashion.

We calculate the electrical and spin conductivities using the Kubo formula. The electrical and spin current operators associated with the quasiparticles are given by $\vec{j} = -e \sum_{k\sigma} \vec{v}_k c_{k\sigma}^\dagger c_{k\sigma}$ and $\vec{j}_s = (g\mu_B/2) \sum_{k\sigma} \vec{v}_k \sigma c_{k\sigma}^\dagger c_{k\sigma}$, respectively, where $\vec{v}_k = \partial\epsilon_k/\partial\vec{k}$. The current-current correlation functions are evaluated to the leading nonvanishing order within the memory function formalism \cite{15}. This is equivalent to the semi-classical approach through solving the linearized Boltzmann equation for the quasiparticle distribution function \cite{15}. Diagrammatically, it amounts to a resummation of the conductivity diagrams including the vertex corrections (ladder diagrams only) and self-energy corrections (to the leading non-vanishing order). The standard assumption made in this procedure is that there exists enough electron-electron Umklapp scatterings so that the scattering of the quasiparticles off of the collective fluctuations do contribute to the dissipation of the electrical current. This condition, while not satisfied for a jellium model, is expected to be well satisfied when an underlying lattice exists, and when the Fermi surface is large. The latter is well established at least for the optimally doped cuprates. The resulting expressions of the electrical and spin resistivities, $\rho$ and $\rho_{\text{spin}} = 1/\sigma_s$, can be written in terms of the transport scattering rate, $1/\tau_{tr}$, and the spin transport scattering rate, $1/\tau_{tr,s}$, respectively: $\rho = \frac{4\pi}{\omega_p^2} \frac{1}{\tau_{tr}}$, and $\rho_{\text{spin}} = \frac{4\pi}{\omega_{p,s}^2} \frac{1}{\tau_{tr,s}}$. 

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Here, $\omega_p^2/4\pi = e^2 A$ and $\omega_{p,s}^2/4\pi = (g\mu_B/2)^2 A$, where $A = N(\epsilon_F) << v_{ks} >> FS$, and $\tau_{tr}$ and $\tau_{tr,s}$ are given by $\tau_{tr} = (\tau_{tr})_{cf} + (\tau_{tr})_{sf}$ and $\tau_{tr,s} = (\tau_{tr,s})_{cf} + (\tau_{tr,s})_{sf}$. Here $(\tau_{tr})_{cf}, (\tau_{tr,s})_{cf}$, and $(\tau_{tr,s})_{sf}$ correspond to the contributions from electron scatterings off of charge fluctuations and spin fluctuations respectively.

Consider first the contribution from charge fluctuations alone. We find that $(\tau_{tr})_{cf}$ and $(\tau_{tr,s})_{cf}$ are equal and given by

$$\tau_{tr}^{cf} = \tau_{tr,s}^{cf} = \frac{1}{A} \sum_{k,q} \gamma_{kq}^2 B(k,q) V_q^2 Im\chi_{cf}(q,\epsilon_{k+q} - \epsilon_k) \tag{2}$$

where $\gamma_{kq} = v_{k+q} - v_k$ is the difference between the group velocities of the quasiparticles before and after a scattering event, and $B(k,q) = (-\partial n_b(\epsilon)/\partial \epsilon)_{\epsilon_{k+q}-\epsilon_k} [f(\epsilon_{k+q}) - f(\epsilon_k)]$ with $n_b(\epsilon)$ and $f(\epsilon)$ representing the boson and fermion distribution functions. The scattering rates become linear in $T$ when the integrated spectral weight, $\sum_q V_q^2 Im\chi_{cf}(q,\omega)$, is either independent of $\omega$ and $T$, or depends on them only through a combination $\omega/T$. The soft energy scale $T_{cf}^*$ is defined such that this condition is satisfied at $\omega, T \gg T_{cf}^*$. At $\omega, T \ll T_{cf}^*$, $Im\chi_{cf}(q,\omega)$ is linear in $\omega$. As a result, $(\tau_{tr})_{cf}$ and $(\tau_{tr,s})_{cf}$ are both quadratic in $T$ at $T \ll T_{cf}^*$.

We conclude that, if only charge fluctuations are important, the spin and electrical resistivities will always have the same temperature dependence.

The spin fluctuation contribution can be considered in a similar fashion. The corresponding contributions to the transport and spin transport scattering rates are given as follows,

$$\tau_{tr}^{sf} = \tau_{tr,s}^{sf} = \frac{3}{A} \sum_{k,q} \gamma_{kq}^2 B(k,q) J_q^2 Im\chi_{sf}(q,\epsilon_{k+q} - \epsilon_k)$$

$$\tau_{tr}^{sf} = \frac{1}{3} \tau_{tr}^{sf} + \frac{2}{A} \sum_{k,q} \gamma_{kq}^2 B(k,q) J_q^2 Im\chi_{sf}(q,\epsilon_{k+q} - \epsilon_k) \tag{3}$$

where $\gamma_{kq} = v_{k+q} + v_k$ is the sum of the group velocities of the quasiparticle states before and after a scattering event. Physically, the spin orientation of the quasiparticle is reversed after a spin-flip scattering, and so its contribution to the spin current. Two possibilities need to be considered. If the spin fluctuation spectrum, $Im\chi_{sf}(q,\omega)$, is only weakly $q$—dependent,
then the factors $\gamma_{kq}$ and $\tilde{\gamma}_{kq}$ will not lead to differences in the temperature dependences of
\[ \left( \frac{1}{\tau_{tr}} \right)_{sf} \text{ and } \left( \frac{1}{\tau_{tr,s}} \right)_{sf}. \]
Again, $\left( \frac{1}{\tau_{tr}} \right)_{sf}$ and $\left( \frac{1}{\tau_{tr,s}} \right)_{sf}$ are linear in $T$ at $T \gg T_{sf}^*$, and quadratic
in $T$ at $T \ll T_{sf}^*$, where $T_{sf}^*$ is the corresponding soft energy scale associated with the spin fluctuations; the prefactors are in general different, though of the same order of magnitude.

If $\operatorname{Im}\chi_{sf}(q, \omega)$ is sharply peaked at a particular wavevector $Q$, it is possible that $\tilde{\gamma}_{kq}$ and $\gamma_{kq}$ act as different form factors leading to different temperature dependences in $\left( \frac{1}{\tau_{tr}} \right)_{sf}$ and $\left( \frac{1}{\tau_{tr,s}} \right)_{sf}$. Such will be the case if, and only if, when we expand the $q$–dependence of $\tilde{\gamma}_{kq}$ and $\gamma_{kq}$ around $Q$, the leading terms for $\tilde{\gamma}_{kq}$ and $\gamma_{kq}$ have different powers in $\delta q = (q - Q)$.

It is easy to see that, this can occur only if $Q = 0$. As long as $Q \neq 0$, which can safely be assumed to be the case for the cuprates\[16\], the difference between $\tilde{\gamma}_{kq}$ and $\gamma_{kq}$ can lead to a difference between $\left( \frac{1}{\tau_{tr}} \right)_{sf}$ and $\left( \frac{1}{\tau_{tr,s}} \right)_{sf}$ only in the overall magnitude, not in the temperature dependence. Therefore, the spin and electrical resistivities will both be linear in $T$ at $T \gg T_{sf}^*$ and quadratic in $T$ at $T \ll T_{sf}^*$.

We now turn to the situation that the spin fluctuations and charge fluctuations are both important. If $T_{sf}^*$ and $T_{cf}^*$ are well separated, the spin and electrical resistivities will have different temperature dependences for temperatures from $\min(T_{sf}^*, T_{cf}^*)$ to $\max(T_{sf}^*, T_{cf}^*)$. However, the electrical resistivity is linear in $T$ only at $T \gg \max(T_{sf}^*, T_{cf}^*)$. In this same temperature range, the spin resistivity is also linear in $T$.

The discussion of a $T$–linear resistivity in these Fermi-liquid-like states\[6–8\] is usually restricted to the level of semi-classical description, which corresponds to the leading order terms we have considered so far. Should contributions beyond the leading order become important, it is not clear how a $T$–linear resistivity can arise from these Fermi-liquid-based schemes. In this regard, a particularly relevant possibility involves the transport of the collective modes themselves. This would occur if, at relevant length and energy scales, the collective fluctuations, $\chi_{sf}(q, \omega)$ and/or $\chi_{cf}(q, \omega)$, in fact describe some well-defined excitations. In this case, two additional contributions to the conductivities arise. One describes the spin conductivity from the spin-wave contribution to the spin-current. The other corresponds to the fluctuating conductivities coming from both $\chi_{sf}(q, \omega)$ and $\chi_{cf}(q, \omega)$, as described by
the Aslamasov-Larkin-type diagrams [17]. From a general analysis of the vector vertices, it
can be shown that their contributions to the electrical conductivity and spin conductivity
are in general different. While these additional contributions might be of relevance to the
underdoped cuprates [18], for the optimally doped case, given the simple behavior of the
observed thermodynamic properties (the essentially \( T \)-independent susceptibility [6] and
specific heat coefficient [19]) and an antiferromagnetic correlation length of the order of a
lattice spacing, it is expected that these collective transport should be negligible. We there-
fore conclude that, if the \( T \)-linear resistivity in the optimally doped cuprates originates
from quasiparticle scatterings off of soft collective modes, the spin resistivity should also be
\( T \)-linear.

**Spin-Charge Separated States: Luttinger Liquid** Different kinds of spin-charge sep-
eration may occur, and we will illustrate our idea by considering several examples. First,
the Luttinger liquid in 1D [3]. Here, the spin and charge excitations propagate with differ-
ent velocities, and a complete spin-charge separation is realized. Linearizing the dispersion
around the two Fermi points, and introducing a boson representation of the fermion fields,
the general interacting spin–\( \frac{1}{2} \) Fermion model in 1D can be written as,

\[
H_{lut} = H_\rho + H_\sigma + H_{g3} + H_{g1}
\]

where

\[
H_\nu = \frac{1}{2\pi} v_\nu \int dx \left[ K_\nu(\pi \Pi_\nu)^2 + \frac{1}{K_\nu(\partial_x \phi_\nu)^2} \right]
\]

describes the kinetic term for the free charge (\( \nu = \rho \)) and spin (\( \nu = \sigma \)) bosons, \( \phi_\rho \) and \( \phi_\sigma \).
Here, \( \Pi_\rho \) and \( \Pi_\sigma \) are the corresponding conjugate momenta. The charge and spin velocities,
\( v_\rho \) and \( v_\sigma \), as well as the charge and spin coupling constants, \( K_\rho \) and \( K_\sigma \), are determined by
the forward scattering interactions. The Umklapp interaction

\[
H_{g3} = \frac{g_3}{(2\pi a)^2} \int dx \cos(\sqrt{8}\phi_\rho + \delta x)
\]

describes two electrons with opposite spins being scattered from one Fermi point to an-
other. Here, \( a \) is a cutoff parameter, and \( \delta \) measures the deviation from half-filling. The
backscattering interaction

\[ H_{g_1} = \frac{g_1}{(2\pi a)^2} \int dx \cos(\sqrt{8\phi_\sigma}) \]

(7)
describes two electrons, from the opposite Fermi points and with opposite spins, inter-
changing branches. The electrical resistivity in this model has been studied extensively by
Giamarchi [22], whose notation we follow closely.

The dissipation of the electrical current, \( j = (-e)\frac{\sqrt{2}}{\pi} \partial_t \phi_\rho \), is due to the Umklapp term. Away from half-filling, there exists an energy scale [22], \( \Delta^* \sim \delta W \) (where \( W \sim v_F/a \) is the
characteristic bandwidth), below which all the Umklapp scatterings are frozen. At \( T \ll \Delta^* \),
the electrical resistivity goes to zero exponentially. At \( T \gg \Delta^* \), it has the algebraic form
with an interaction-dependent exponent,

\[ \rho \sim \frac{4\pi}{\omega_p^2} (\rho_0 g_3)^2 W \left( \frac{T}{W} \right)^{4K_\rho-3} \quad \text{for} \quad T \gg \Delta^* \]

(8)

In contrast, the dissipation of the spin current, \( j_s = (g\mu_B/2)\frac{\sqrt{2}}{\pi} \partial_t \phi_\sigma \), comes entirely from
the backscattering term and we find that,

\[ \rho_{\text{spin}} \sim \frac{4\pi}{\omega_{p,s}^2} (\rho_0 g_1)^2 W \left( \frac{T}{W} \right)^{4K_\sigma-3} \]

(9)

for all temperatures. \( K_\rho \) and \( K_\sigma \) are different for any non-zero interaction. Within the
repulsive Hubbard model, the exponents for \( \rho \) and \( \rho_{\text{spin}} \) can differ by as large as 2.

**Spin-Charge Separated States: Gauge Theory of the 2D \( t-J \) Model.** We now
consider the gauge theory of the \( t-J \) model in two dimensions [4]. This theory describes
a state with spinon-like and holon-like excitations coupled by a massless transverse gauge
field. The presence of this coupling to the gauge field leads to a situation in between that
of a complete spin-charge separation and that of no spin-charge separation. As we will see,
such an intermediate situation is also reflected in the relationship between \( \rho_{\text{spin}} \) and \( \rho \).

In terms of the slave-fields \( f_{i\sigma}^\dagger \) and \( b_i^\dagger \), which create singly occupied and empty configu-

\[ H_{tJ} = -t \sum_{<ij>} (f_{i\sigma}^\dagger b_i)(b_j^\dagger f_{j\sigma}) + J \sum_{<ij>} \sum_{\sigma\sigma'} (f_{i\sigma}^\dagger f_{i\sigma'})(f_{j\sigma'}^\dagger f_{j\sigma}) \]

(10)
with a no-double-occupancy constraint

$$\sum f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i = 1$$  \hspace{1cm} (11)$$

at every site \(i\). In Eq. (10), \(<ij>\) labels the nearest neighbors, \(t\) the hopping amplitude, and \(J\) the exchange interaction. The gauge theory description applies when there is a uniform nearest neighbor RVB order parameter, \(\sum_{\sigma} <f_i^{\dagger}\sigma f_j^{\dagger}\bar{\sigma}> = \Delta_o\), and when the boson field is not condensed. The latter ensures the existence of the massless transverse (unscreened) gauge field, which formally corresponds to the phase of \(\Delta_o\). This gauge field is coupled to both the bosons and fermions described by the \(b-\) and \(f-\) fields respectively. The electrical current operator is given by

$$j_x = (-ite) \sum_{\sigma} \left((f_{i\sigma}^\dagger b_i)(b_{i+x\bar{\sigma}}^\dagger f_{i+\bar{\sigma}}) - H.c.\right).$$

The current-current correlation function can be shown \([20]\) to be

$$\Pi_{jj} = \Pi_{jj,ff} - (\Pi_{jj,ff})^2[\Pi_{jj,ff} + \Pi_{jj,bb}]^{-1}$$  \hspace{1cm} (12)$$

where \(\Pi_{jj,bb}\) and \(\Pi_{jj,ff}\) are the current-current correlation functions for the boson and fermion currents, \(j_b = -e \sum_{\kappa\sigma} v^b_{k\sigma} b_{k\sigma}\) and \(j_f = -e \sum_{\kappa\sigma} v^f_{k\sigma} f_{k\sigma}\), where \(v^b_k\) and \(v^f_k\) are the fermion and boson velocities. The second term of Eq. (12) reflects the screening by the gauge field. Such a screening enforces the no-double occupancy constraint. Eq. (12) leads to an electrical resistivity

$$\rho = \rho_b + \rho_f$$  \hspace{1cm} (13)$$

where \(\rho_b\) and \(\rho_f\) are the boson and fermion resistivities, respectively, reflecting the scattering of the bosons and fermions by the gauge field. Using \(\rho_f \sim \frac{4\pi}{\omega_p^2 E_f(T/E_f)^{4/3}}\) \([21]\), and \(\rho_b \sim \frac{4\pi}{\omega_p^2 E_b(T/E_f)}\) \([4]\), where \(E_f \sim J, E_b \sim t\), and \(\omega_p^2/\omega_p^2 \sim \frac{1}{\delta}\) with \(\delta\) representing the doping concentration, the resistivity is approximately linear in temperature.

Using the spin current operator, \((j_s)_x = (g\mu_B) \sum_{\sigma\sigma'} \sigma [(it/2)(f_{i\sigma}^\dagger b_i)(b_{i+x\bar{\sigma}}^\dagger f_{i+\bar{\sigma}}) - (iJ/4)(f_{i\sigma f_{i\sigma'}})(f_{i+\bar{\sigma}}^\dagger f_{i+\bar{\sigma}}) - H.c.\], we find that the dominant contribution to the spin current-current correlation function is given as follows,

$$\Pi_{j_s j_s} = \Pi_{j_s j_s,ff} - (\sum_{\sigma\sigma'} \sigma \Pi_{j_s j_s,ff})^2[\Pi_{j_s j_s,ff} + \Pi_{j_s j_s,bb}]^{-1}$$  \hspace{1cm} (14)$$
where $\Pi_{j_{fs}j'_{fs}}$ corresponds to the current-current correlation function for the current operators $j_{fs} = (g\mu_B/2) \sum_k v_{kfs}^f f_k^\sigma f_k^\sigma$, and $\Pi_{j_{fs}j'_{fs}} = \sum_{\sigma\sigma'} \Pi_{j_{fs}j'_{fs}}^{\sigma\sigma'}$. The second term in Eq. (14) again comes from the screening of the gauge field. However, this term vanishes! Eq. (14) implies that,

$$\rho_{\text{spin}} \sim \frac{\omega_{p,f}^2}{\omega_{p,s,f}^2} \rho_f \sim \frac{4\pi}{\omega_{p,s,f}^2} E_f (T/E_f)^{4/3} \quad (15)$$

As a result of spin-charge separation, the spin resistivity is not linear in temperature despite of a $T$–linear electrical resistivity.

To summarize, we have demonstrated that it is possible to diagnose spin-charge separation in the optimally doped cuprates through a comparison of the spin and electrical conductivities. If the measured inverse spin diffusion constant turns out to be not linear in $T$ in, for example, the optimally doped YBCO or LSCO for which the electrical resistivity is known to be $T$–linear and the uniform static spin susceptibility essentially $T$–independent, it would provide a direct evidence for spin-charge separation in the cuprates. Finally, our analysis also suggests that measuring the spin-diffusion constant in the quasi-one-dimensional materials can help clarify the spin-charge separation theoretically expected in these systems.

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* On leave from Physics Department, Rice University, Houston, TX 77251-1892.

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