A chiral symmetry breaking AdS / QCD model with scalar interactions

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January 20, 2013

Abstract

We consider chiral symmetry breaking AdS / QCD models with cubic and quartic potentials in the scalar sector and a $z$ dependent mass for scalars in the bulk. In these models it is possible to get good mesonic spectra using current quark masses.

January 20, 2013

1 Introduction

The gauge / gravity duality is currently applied to several areas in theoretical physics. In the QCD case it provides a new methodology to study strong interaction properties at low energy. Within the range of hadronic properties that can be studied using AdS / QCD models, we find models that consider effects of chiral symmetry breaking in the Lagrangian. At the beginning these models were of the hard wall type [1, 2], and their main problem is that they cannot describe correctly the hadronic spectra. This situation can be improved in the mesonic sector by introducing a dilaton field that depends on the holographical coordinate $z$ [3, 4], although this procedure does not work for baryons [5, 6]. Here it is necessary to look for complementary alternatives to the dilaton, such as using a warp factor in the AdS metric [7], and / or to consider a $z$ dependent mass for modes propagating in the bulk, which describe hadrons [6, 8, 9, 10]. Additionally, variable masses in holographical models are interesting to consider because in this way we can take into account effects of anomalous dimensions of operators described by AdS modes [11, 6].

The use of variable mass in AdS / QCD models with chiral symmetry breaking was considered in [12], in a AdS spacetime with quadratic dilaton, and without interactions in the scalar sector. The natural next step is to study models which include these interactions;
here we discuss the effect of a $z$ dependent mass in holographical models that consider cubic [13] and quartic [14] interactions in the scalar sector.

In [12], in order to get good mesonic spectra with current quark masses in the model, it was necessary to consider a vacuum expectation value (vev) with linear behavior when $z \to \infty$. As we will see below, the inclusion of cubic [13] and quartic [14] interactions plus $z$ dependent masses allows to get spectra with Regge behavior, with current quark mass values, and without the above restriction on the vev.

2 Model

We consider a soft wall AdS / QCD model as in [12], but we add cubic [13] or quartic [14] potentials in the bulk scalar potential. The 5d AdS background is defined by

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu + dz^2),$$

(1)

where $R$ is the AdS radius, the Minkowski metric is $\eta_{\mu\nu} = \text{diag}(-1,+1,+1,+1)$ and $z$ is a holographical coordinate defined in the range $0 \leq z < \infty$. In this paper we consider a usual quadratic dilaton $\phi(z) = \lambda^2 z^2$. To describe chiral symmetry breaking in the mesonic sector and in the 5d AdS side, the action contains $SU(2)_L \times SU(2)_R$ gauge fields and a scalar field $X$, and it is given by

$$S_5 = - \int d^5x \sqrt{-g} e^{-\phi(z)} Tr \left[ DX^\mu DX^\nu + m_X^2(z) |X|^2 - \kappa |X|^n \right]$$

$$+ \frac{1}{4g_5^2} (F_{L}^2 + F_{R}^2).$$

(2)

This action shows explicitly that the scalar modes masses are $z$ dependent, besides including cubic or quartic bulk scalar potentials for $n$ equal 3 or 4.

Here $g_5^2 = \frac{12\pi^2}{N_c}$, where $N_c$ is the number of colors, and the fields $F_{L,R}$ are defined by

$$F_{L,R}^{MN} = \partial^M A_{L,R}^N - \partial^N A_{L,R}^M - i [A_{L,R}^M , A_{L,R}^N],$$

here $A_{L,R}^{MN} = A_{L,R}^{MN} t^a, Tr[t^a t^b] = \frac{1}{2} \delta^{ab}$, and the covariant derivative is

$$D^M X = \partial^M X - i A_{L}^M X + i X A_{R}^M.$$
Starting from (2), the equations that describe the vev and the scalar, change if we consider $n$ equal 3 or 4. For $n = 3$ we have

$$- z^2 \partial_z^2 v(z) + z (3 + 2 \lambda^2 z^2) \partial_z v(z) + m_X^2(z) R^2 v(z)$$

$$- \frac{3}{4} R^2 \kappa v(z)^2 = 0,$$

(3)

$$- \partial_z^2 S_n(z) + \frac{3 + 2 \kappa^2 z^2}{z} \partial_z S_n(z) + \frac{m_X^2(z) R^2}{z^2} S_n(z)$$

$$- \frac{R^2 \kappa v(z)}{z^2} S_n(z) = M_S^2 S_n(z),$$

(4)

and when $n = 4$,

$$- z^2 \partial_z^2 v(z) + z (3 + 2 \lambda^2 z^2) \partial_z v(z) + m_X^2(z) R^2 v(z)$$

$$- \frac{1}{2} R^2 \kappa v(z)^3 = 0,$$

(5)

$$- \partial_z^2 S_n(z) + \frac{3 + 2 \kappa^2 z^2}{z} \partial_z S_n(z) + \frac{m_X^2(z) R^2}{z^2} S_n(z)$$

$$- \frac{3 R^2 \kappa v^2(z)}{2 z^2} S_n(z) = M_S^2 S_n(z),$$

(6)

On the other hand, vector and axial vector equations are the same in both cases

$$- \partial_z^2 V_n(z) + \left( \frac{1}{z} + 2 \lambda^2 z \right) \partial_z V_n(z) = M_V^2 V_n(z),$$

(7)

$$- \partial_z^2 A_n(z) + \left( \frac{1}{z} + 2 \lambda^2 z \right) \partial_z A_n(z) +$$

$$\frac{R^2 g_v^2 v^2(z)}{z^2} A_n(z) = M_A^2 A_n(z).$$

(8)

To discuss the phenomenology of this model, it is necessary to know the precise form of $m_X^2(z)$. This can be done as in Ref. [12]. The mass for scalar modes in the bulk, $m_X^2(z)$, is
obtained starting from (3) for the cubic case and from (5) in the quartic case, provided that the function \( v(z) \) is known.

The behavior of \( v(z) \) can be known in two limits. First we consider the usual limit \( z \to 0 \), according to which

\[
v(z \to 0) = \alpha z + \beta z^3,
\]

where the \( \alpha \) and \( \beta \) coefficients are associated with the quark mass and chiral condensate respectively.

The other limit for \( v(z) \) is when \( z \to \infty \). Here we consider the condition that we should get Regge spectra when \( z \) is high in equation (8). In fact, equation (8) does not change when \( \kappa = 0 \), which was the case analyzed in [12], so we can use the expressions considered obtained there, where it was shown that \( v(z \to \infty) \) can be constant, linear or quadratic in \( z \). For this reason we use the ansätze considered in [12] to reproduce both limits for each possible \( z \) behavior in the high \( z \) limit.

\[
v_I(z) = \frac{c_I}{R} \arctan(Az + Bz^3), \quad (\text{Model I})
\]

\[
v_{II}(z) = \frac{z}{R}(A + B \tanh(c_{II}z^2)), \quad (\text{Model II})
\]

\[
v_{III}(z) = \frac{Az + Bz^3}{R\sqrt{1 + c_{III}^2z^2}}. \quad (\text{Model III})
\]

In relation to the parameters of model, \( \lambda \) is fixed using data from the spectrum. We choose \( \lambda = 0.400 \text{GeV} \), which allows us to obtain correct mass values for vector mesons. The remaining parameters can be fixed using (10), (11) and (12). Comparing with the value established in the AdS / CFT dictionary, with the notation used in [14]

\[
v(z \to 0) = \frac{m_q\zeta}{R} z + \frac{\sigma}{R\zeta}z^3,
\]

The parameter \( \zeta \) was introduced in [11] to get the right normalization, and its value is \( \zeta = \sqrt{3}/(2\pi) \).

| Table 2: Parameters used in the model. |
|----------------------------------------|
| Cubic       | \( c_I = 0.4 \) | \( m_q = 4.3 \text{MeV} \) | \( R\kappa = -11 \) |
|            | \( c_{II} = 28 \) | \( m_q = 0.8 \text{MeV} \) | \( R\kappa = -1 \) |
|            | \( c_{III} = 3 \) | \( m_q = 0.8 \text{MeV} \) | \( R\kappa = -10 \) |
| Quartic    | \( c_I = 0.4 \) | \( m_q = 4.3 \text{MeV} \) | \( \kappa = -11 \) |
|            | \( c_{II} = 28 \) | \( m_q = 0.8 \text{MeV} \) | \( \kappa = -0.02 \) |
|            | \( c_{III} = 3 \) | \( m_q = 0.8 \text{MeV} \) | \( \kappa = -0.76 \) |
|            | \( \lambda = 0.4 \text{GeV} \) | | |
Table 3: Scalar meson spectra in cubic case. We give experimental values, masses calculated in each model considered here, and compare them with a couple of holographical models. All masses are in MeV. Notice that the mass of the lightest scalar meson in [13] is less than the mass of the pion, contradicting a well-established QCD theorem [16, 17].

| n  | $f_0(\text{Exp})$ | $f_0$ | $f_0$ | $f_0$ | $f_0(\text{Ref.}[12])$ | $f_0(\text{Ref.}[13])$ |
|----|------------------|-------|-------|-------|------------------------|------------------------|
|    | $m_q = 4.3$      | $m_q = 0.8$ | $m_q = 0.8$ | $m_q = 0.8$ | $c_{ij} = 3$ | $c_{ij} = 3$ |
| 0  | 550 $\pm$ 150   | 487   | 552   | 555   | 485        | 118        |
| 1  | 980 $\pm$ 10    | 1193  | 916   | 986   | 903        | 953        |
| 2  | 1350 $\pm$ 150  | 1452  | 1213  | 1275  | 1208       | 1335       |
| 3  | 1505 $\pm$ 6    | 1665  | 1454  | 1500  | 1451       | 1627       |
| 4  | 1724 $\pm$ 7    | 1851  | 1661  | 1710  | 1659       | 1873       |
| 5  | 1992 $\pm$ 16   | 2019  | 1845  | 1890  | 1841       | 2089       |
| 6  | 2103 $\pm$ 8    | 2172  | 2012  | 2054  | 2012       | 2285       |
| 7  | 2314 $\pm$ 25   | 2316  | 2166  | 2205  | 2166       | 2465       |

Table 4: As in Table III, but for scalar meson spectra in quartic case.

| n  | $f_0(\text{Exp})$ | $f_0$ | $f_0$ | $f_0$ | $f_0(\text{Ref.}[12])$ | $f_0(\text{Ref.}[13])$ |
|----|------------------|-------|-------|-------|------------------------|------------------------|
|    | $m_q = 4.3$      | $m_q = 0.8$ | $m_q = 0.8$ | $m_q = 0.8$ | $c_{ij} = 3$ | $c_{ij} = 3$ |
| 0  | 550 $\pm$ 150   | 548   | 595   | 553   | 485        | 118        |
| 1  | 980 $\pm$ 10    | 1232  | 1146  | 1251  | 903        | 953        |
| 2  | 1350 $\pm$ 150  | 1477  | 1478  | 1675  | 1208       | 1335       |
| 3  | 1505 $\pm$ 6    | 1683  | 1661  | 2010  | 1451       | 1627       |
| 4  | 1724 $\pm$ 7    | 1865  | 1831  | 2296  | 1659       | 1873       |
| 5  | 1992 $\pm$ 16   | 2030  | 1989  | 2549  | 1841       | 2089       |
| 6  | 2103 $\pm$ 8    | 2182  | 2137  | 2779  | 2012       | 2285       |
| 7  | 2314 $\pm$ 25   | 2324  | 2276  | 2991  | 2166       | 2465       |

In order to finish the model description, it is necessary to specify the values for $m_q$ and $\sigma$, which are related by the Gell-Mann-Oakes-Renner relation $m_\pi^2f_\pi^2 = 2m_q\sigma$, and therefore we need to fix only one of them. In this case we use $m_\pi = 140$ MeV and $f_\pi = 92$ MeV, and we fix the quark mass using

$$f_\pi^2 = -\frac{1}{g_5^2} \lim_{\epsilon \to 0} \frac{\partial_z A_0(0,z)}{z} \bigg|_{z=\epsilon},$$

where $A_0(0,z)$ is solution of (8), with $M_A^2 = 0$, and the boundary conditions used are $A_0(0,0) = 1$ and $\partial_z A_0(0,z \to \infty) = 0$.

As can be observed in (8), the $A_0(0,z)$ equation as a term that depends on $m_q$, so using (14) we get $f_\pi(m_q)$. In general we obtain two possible quark masses in each model, and one of them can be considered as a current mass.
Table 5: Axial vector mesons spectra. We give experimental values, masses calculated in each model considered here, and compare them with a couple of holographical models. All masses are in MeV.

| n | $a_1(Exp)$ | $a_1(m_q = 4.3, c_i = 0.4)$ | $a_1(m_q = 0.8, c_i = 28)$ | $a_1(Ref. [12])$ | $a_1(Ref. [13])$ | $a_1(Ref. [14])$ |
|---|---|---|---|---|---|---|
| 0 | 1230 ± 40 | 1236 | 804 | 1778 | 811 | 997 |
| 1 | 1647 ± 22 | 1412 | 1135 | 2526 | 1133 | 1541 |
| 2 | 1930 ± 20 | 1563 | 1388 | 3099 | 1384 | 1934 |
| 3 | 2096 ± 122 | 1731 | 1602 | 3582 | 1601 | 2258 |
| 4 | 2270 ± 55 | 1896 | 1791 | 4006 | 1789 | 2540 |

Table 6: Vector mesons spectra in MeV.

| n | $\rho(Exp)$ | $\rho(Model)$ | $\rho(Ref. [13])$ | $\rho(Ref. [14])$ |
|---|---|---|---|---|
| 0 | 775.5 ± 1 | 800 | 759 | 475 |
| 1 | 1282 ± 37 | 1131 | 1202 | 1129 |
| 2 | 1465 ± 25 | 1386 | 1519 | 1529 |
| 3 | 1720 ± 20 | 1600 | 1779 | 1674 |
| 4 | 1909 ± 30 | 1789 | 2005 | 1884 |
| 5 | 2149 ± 17 | 1960 | 2207 | 2072 |
| 6 | 2265 ± 40 | 2117 | 2393 | 2243 |

3 Mesonic spectrum

The model parameters are summarized in Table II, and are fixed as in [12]. We can then calculate meson masses, which correspond to eigenvalues in the equations (4) or (6), (7) and (8). In this set of equations, only (7) can be solved analytically. For this reason we prefer to change equations (4), (6) and (8) into Schrödinger like ones, and solve them numerically using a MATHEMATICA code [13], which was adapted to our potentials.

For scalars, depending on whether we consider cubic or quartic interactions, we have

$$V_{Sc}(z) = 2\lambda^2 + \lambda^4 z^2 + \frac{15}{4z^2} + \frac{m_X^2(z)R^2}{z^2} - \frac{R^2\kappa v(z)}{z^2}. \quad (15)$$

$$V_{Sq}(z) = 2\lambda^2 + \lambda^4 z^2 + \frac{15}{4z^2} + \frac{m_X^2(z)R^2}{z^2} - \frac{6R^2\kappa v(z)^2}{z^2}. \quad (16)$$

For vector mesons, in all cases we get the same potential

$$V_V(z) = \frac{3}{4z^2} + \lambda^4 z^2, \quad (17)$$

and in this case it is possible to get an exact spectrum,

$$M_V^2 = 4\lambda^2(n + 1). \quad (18)$$
Finally, for axial vector mesons the potential is
\[ V_A(z) = \frac{3}{4z^2} + \lambda^4 z^2 + \frac{R^2 g_5^2 v^2(z)}{z^2}. \] (19)

The mesonic mass spectra that we get are summarized in Tables III to VI, where we additionally include masses calculated in other holographical models.

4 Conclusions

This paper is an extension of ideas discussed in a previous work [12], taking into account cubic or quartic interactions in the scalar sector of the Lagrangian. Some specific equations are modified in relation to [12], such as the vev equation, which produces a different \( m_x(z) \). Changes in the \( S_n \) equation, considering both cubic or quartic additional interactions, provide better spectra, allowing us to get good mesonic masses with current quark mass values, for every \( v(z) \) used.

The model used is of the Bottom-Up type, and the mesonic spectra is satisfactory, with an easy hadron identification, in contrast to what happens in models such as Sakai-Sugimoto [18, 19], the best known Top-Down model, where there exist modes that cannot be identified with mesons in QCD.

Additionally we like to highlight that although the model was used in this paper to describe mesons, it is possible to apply it to study exotics [20, 6, 10] (this application is postponed to future work), since the mass of scalar modes that describe hadrons in the bulk depends for these modes on the conformal dimension \( \Delta \) according to \( m_x^2 R^2 = \Delta (\Delta - 4) \), and the dictionary tells us that the conformal dimension is related to operator dimensions, which looks like \( \Delta_0 + \delta \), where for states with angular momentum equal to zero \( \Delta_0 \) depends on the number and kind of hadrons constituents, and \( \delta \) is the anomalous dimension. This opens up the possibility to consider \( z \) dependent masses of some AdS modes in this kind of models [11, 12, 6]. Here we have only considered mesons, but the possibility to consider exotics implies the use of the right \( \Delta_0 \) as in [20, 6, 10], and in particular this could allow us to differentiate between the lightest meson and a tetraquark from a holographic perspective, as was studied in detail in [10] for an AdS / QCD model that considers chiral symmetry breaking.

The results obtained in this paper reinforce the idea that variable masses can be considered as a complementary alternative in holographic models.

Acknowledgments

Work supported by Fondecyt (Chile) under Grants No. 3100028 and 1100287.
References

[1] J. Erlich, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. Lett. 95, 261602 (2005) [arXiv:hep-ph/0501128].

[2] L. Da Rold and A. Pomarol, Nucl. Phys. B 721, 79 (2005) [arXiv:hep-ph/0501128].

[3] A. Karch, E. Katz, D. T. Son and M. A. Stephanov, Phys. Rev. D 74, 015005 (2006) [arXiv:hep-ph/0602229].

[4] P. Colangelo, F. De Fazio, F. Giannuzzi, F. Jugeau and S. Nicotri, Phys. Rev. D 78, 055009 (2008) [arXiv:0807.1054 [hep-ph]].

[5] I. Kirsch, JHEP 0609, 052 (2006) [arXiv:hep-th/0607205].

[6] A. Vega and I. Schmidt, Phys. Rev. D 79, 055003 (2009) [arXiv:0811.4638 [hep-ph]].

[7] H. Forkel, M. Beyer and T. Frederico, JHEP 0707, 077 (2007) [arXiv:0705.1857 [hep-ph]].

[8] Z. Abidin and C. E. Carlson, Phys. Rev. D 79, 115003 (2009) [arXiv:0903.4818 [hep-ph]].

[9] H. Forkel and E. Klempt, Phys. Lett. B 679, 77 (2009) [arXiv:0810.2959 [hep-ph]].

[10] H. Forkel, Phys. Lett. B 694, 252 (2010) [arXiv:1007.4341 [hep-ph]].

[11] A. Cherman, T. D. Cohen and E. S. Werbos, Phys. Rev. C 79, 045203 (2009) [arXiv:0804.1096 [hep-ph]].

[12] A. Vega and I. Schmidt, Phys. Rev. D 82, 115023 (2010) [arXiv:1005.3000 [hep-ph]].

[13] P. Zhang, JHEP 1005, 039 (2010) [arXiv:1003.0558 [hep-ph]].

[14] T. Gherghetta, J. I. Kapusta and T. M. Kelley, Phys. Rev. D 79, 076003 (2009) [arXiv:0902.1998 [hep-ph]].

[15] W. Lucha and F. F. Schöberl, Int. J. Mod. Phys. C 10, 607 (1999) [arXiv:hep-ph/9811453].

[16] D. Weingarten, Phys. Rev. Lett. 51, 1830 (1983).

[17] E. Witten, Phys. Rev. Lett. 51, 2351 (1983).

[18] T. Sakai and S. Sugimoto, Prog. Theor. Phys. 113, 843 (2005) [arXiv:hep-th/0412141].

[19] T. Sakai and S. Sugimoto, Prog. Theor. Phys. 114, 1083 (2005) [arXiv:hep-th/0507073].

[20] A. Vega and I. Schmidt, Phys. Rev. D 78, 017703 (2008) [arXiv:0806.2267 [hep-ph]].