Multi-robot formation control based on high-order bilateral consensus

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Abstract
A high-order bilateral consensus robot formation control protocol for multi-agent systems is proposed in this paper. Considering the relationship between the state of the information exchange topology and derivatives, a third-order bilateral consistency protocol is presented and is extended it to a higher order bilateral consensus protocol. First, sufficient conditions for the third-order multi-agent system are given to achieve the bilateral consensus control protocol, and the system’s asymptotical stability is also achieved by adjusting the feedback system gain parameters. Then, by further studying the cohesive relationship between each state variable of the third-order protocol and the gauge transformation, the sufficient conditions of the higher order system are also provided. Finally, by applying the third-order control protocol to the control of multi-robot formation, the general control scheme of robot formation is given and the control of robot formation is successfully achieved.

Keywords
Bilateral consensus, high order, formation control, feedback system gain parameters

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Introduction
In the process of continuous research and exploration in the field of robotics, researchers have gradually found that it is difficult to use a single robot system to operate in more complex working environments. Therefore, the development of multi-robot systems has gradually become a focus of attention focus for scholars and experts. With the continuous development of the modern robot industry, multi-robot systems are increasingly exhibiting unique advantages over single robot structures. Multi-robots work together to accomplish complex mission objectives, that have been widely used in the field of robotics, such as coordinated cargo transportation and intelligent assembly, multi-robot automated welding, and the cooperative work of spacecraft in the space field.¹,²

In the field of multi-robot research, the realization of multi-robot motion control via robot formation control has become a key issue. Many experts and scholars have conducted relevant research in this field. Lee and Chong³ proposed a robotic cooperative control formation with an anonymous nature. Garrido et al.⁴ applied the Voronoi Fast Marching (VFM) method to the path planning of robot formations. Scheggi et al.⁵ achieved the control of robot formation via a visuo-haptic feedback mechanism. Aiming at the robotic avoidance obstacle problem, Nascimento et al.⁶ established a nonlinear multi-robot prediction model to achieve the control problem of robot formation. Xing et al.⁷ implemented distributed robot formation control by establishing a multi-robot model with nonholonomic constraints. Wang et al.⁸ mainly studied the control of robotic formations based on visual leader-followers in an unknown complex environment.

In the actual application process, asymptotical stability in the multi-robot system can be more easily achieved by combining the consensus protocol with the robot formation. Furthermore, the research and application of consensus issues have been of concern for many scholars. In the process of continuously exploring the formation control method of robots, many studies have shown that the consensus control theory of robot formation has strong stability and applicability compared with other theories. Xing et al.⁹ proposed
a distributed observation protocol for nonlinear dynamics, which transformed the formation tracking problem of multi-robot systems into a time-varying consensus problem. Sun et al.\textsuperscript{10} introduced a novel amplitude-saturated output feedback (OFB) control approach for underactuated crane systems that exhibited double-pendulum effects. Liu et al.\textsuperscript{11} concentrated on the problem of finite-time fault-tolerant control for a class of switched nonlinear systems in lower triangular form under arbitrary switching signals. Tian et al.\textsuperscript{12} investigated multi-agent systems with second-order dynamics and antagonistic interactions under both absolute and relative damping protocols. In addition, Nazari et al.\textsuperscript{13} converted the consensus problem into a local stability problem of error dynamics, evaluated its stability with the infinite dimension Floquet method, and applied it to the consensus control of space multi-spacecraft formation.

The high-order consensus problem has also been an important component of the research of many scholars. With the increasing requirements for multi-agent control in recent several years, the development and design of a reasonable and highly reliable control method has become necessary. High-order linear consensus agent systems with dynamics, high-order consensus protocols with fixed and switching topologies, and high-order consensus network control with continuous time for communication delays have also been studied by Wieland,\textsuperscript{14} Jiang\textsuperscript{15} and Lin.\textsuperscript{16} Khoi et al. and Chen et al., respectively, proposed the multi-surface sliding control and adaptive backstepping tracking control theories with semi-strict feedback multi-agent systems for higher order nonlinear agent systems.\textsuperscript{17,18} Furthermore, the relevant stationary average consensus control theory for high-order, multi-agent systems applied to spacecraft was proposed by Rezaei,\textsuperscript{19} and the self-triggered leader-following consensus protocol was also studied by You et al.\textsuperscript{20}

Although multi-robot formations and consensus issues have been recently investigated by numerous scholars, most research has focused on independent robot formations or consensus issues. This limits the application of the results of consensus research to a certain extent and also results in a lack of theoretical support for the research of robot formation. Some scholars have gradually discovered this problem throughout the course of research. They combined multi-robot formation with the second-order consensus problem and expanded the application of consensus research to multi-robot formation; the consensus research was limited to the exploration of two variables, such as the position and velocity of the robot. However, in the process of actual robotic movement, the control of multi-robot formation is by two variables that lack certain constraints and control accuracy. Furthermore, in the present study, to expand the control performance of the second-order system, an acceleration control variable is added, the second-order system is extended to a third-order system, and the control protocol of the high-order system is given, which further improves the control stability of the multi-robot formation. In addition, a large amount of simulation data and research results demonstrate that, via the interaction of positive and negative weights between agents that are both cooperative and antagonistic, the bilateral third-order consensus control protocol enables the robot formation to achieve consensus from two directions. This improves the control efficiency of the robot formation, facilitates the simplification of the algorithm, and expands the applicability of the protocol in multi-robot formation. Nevertheless, some disadvantages exist in high-order bilateral consensus; for example, the weight value cannot be too large, and the interference of communication delays may cause the divergence or oscillation of the network system.

This paper will combine these main results and related conclusions to study the control problems of robot formation for high-order, multi-agent systems under a bilateral consensus protocol. Considering the stability and reliability of the robot formation, a bilateral consensus protocol in the third-order state is proposed. Due to the existing relationship between cooperation and confrontation for robots, it is represented by the positive and negative weights of edges in the topological graph. To achieve the asymptotical consensus of the third-order system, by combining the concepts of graph theory and structural balance, the condition that the system has strong connectivity is given. The effectiveness and stability of the third-order system are proven via analysis of the Laplacian matrix eigenvalues and system gain parameters. For the more complex high-order systems, by simplifying the conditions for achieving the asymptotical consensus and by means of the properties and conclusions of the complex coefficient polynomials, the research in the complex domain is transformed into the real number domain, and the relevant evidence is presented. Furthermore, in this paper, the third-order protocol is applied to the control of robot formation to solve the problem of multi-robot cooperative operation consensus.

The remainder of this paper is structured as follows: Section “Precondition statement” provides a graphic description of the high-order consensus protocol and a statement of structural balance. Section “Main results” presents a bilateral consensus protocol in a high-order state and elaborates on the main results and the proof. Section “Robotic formation operation control” presents simulation analysis based on the protocol proposed in this paper. Section “Conclusion” summarizes the research results.

**Precondition statement**

**Graphic theory**

Let \( V = \{1, 2, \ldots, n\} \) represent the node set and \( \mu = \{(i, j)| \text{obtains information data from } j\} \) represent
the edge set. The signed diagraph is denoted by $G = (V, \mu, \Delta)$, $A = [a_{ij}] \in \mathbb{R}^{n \times n}$ is the adjacency matrix for $G$, and $a_{ij} \neq 0$ for $(i, j) \in \mu$. Moreover, provided that there exists cooperation in the relationship from $j$ to $i$, then $a_{ij} = 1$; if there exists antagonism in the relationship from $j$ to $i$, then $a_{ij} = -1$. In addition, $G(A_{0})$ refers to an undigraph obtained by graph $G$ for $A_{0} = (A + A^{T})/2$.

A directed path for $G$ is defined as

$$Q = \{(i_{2}, i_{1}), (i_{3}, i_{2}), \ldots, (i_{r}, i_{r-1})\} \subseteq \mu$$

Each node $i_{1}, i_{2}, \ldots, i_{r}$ represents different meanings. Provided that there exists more great or equal to one path from $i$ to $j$, and from $j$ to $i$, $\forall i, j \in V, i \neq j$, it can be said that the diagraph has strong connectivity.

**Definition of balanced structure**

Let $V_{1}, V_{2}$ be the set of the node. Provided that all the nodes of diagraph are divided into $V_{1}, V_{2}$, then $V_{1} \cup V_{2} = V$, $V_{1} \cap V_{2} = \emptyset$ and $a_{ij} \geq 0, \forall i, j \in V_{r}, \ (r \in \{1, 2\}, a_{ij} \leq 0, \forall i \in V_{r}, j \in V_{r}, \ (r, r \in \{1, 2\}, r \neq r)$, the diagraph is balanced structurally. Based on the previous statements, a set of more great or equal to $\Omega = \{D|D = \text{diag}\{d_{1}, \ldots, d_{n}\}, d_{i} = \pm 1, i = 1, 2, \ldots, n\}$ is used, and the Lemma can be obtained as described below.

**Lemma 1.** Provided the diagraph $G(A)$ has strong connectivity and the digon has signed symmetry, then it will strictly conform to any of the following conditions.

1. $G(A)$ is balanced in the structure;
2. If $G(A)$ has the cycles with the direction, then they are all positive;
3. $\exists \Omega \in \Omega$ such as existing all entries with non-negative property for $DAD$;
4. $L$ has a zero eigenvalue.

**Lemma 2.** Saber defined a weighted diagraph $G = (V, \mu, \Delta)$ for the Laplacian matrix $L$, and let each eigenvalue of $L$ be located in the disk as follows

$$D(G) = \{f \in C : |f - d_{\text{max}}(G)| \leq d_{\text{max}}(G)\}$$

Meanwhile, $f = d_{\text{max}}(G) + 0j$ is the center of the disk in complex plane and $d_{\text{max}}(G) = \max_{i,j}d_{ij}(i)$ represents the maximum node in-degree of the diagraph $G$.

**Lemma 3.** Saber supposed there exists strong connectivity in the diagraph $G$, the matrix $L$ within the $G$ conforms to $L\eta_{i} = 0, \eta_{i}^{T}L = 0, \text{ and } \eta_{i}^{T}\eta_{i} = 1$; then, $\lim_{i \to \infty} e^{-tL} = \eta_{i}^{T}$.

**Main results**

**Control protocol with third-order bilateral consensus**

In this paper, the position, velocity, and acceleration in the edge $(i, j)$ at time $t$ are expressed as $x$, $v$, and $a$, respectively. The third-order continuous time boundary dynamic model is as follows

$$\ddot{x}_{ij} = v_{ij}, \dot{v}_{ij} = a_{ij}, \ddot{a}_{ij} = u_{ij} \quad (1)$$

As for system (1), the control protocol is designed by

$$u_{ij} = -k_{ij}x_{ij} = \sum_{j \in N_{i}}\left[\frac{x_{ij}(t)|A_{ij}| - \text{sgn}(A_{ij})x_{ij}(t)A_{ij}}{-\sum_{j \in N_{i}}[v_{ij}(t)|A_{ij}||a_{ij} - a_{ij}\text{sgn}(A_{ij})v_{ij}(t)A_{ij}]} - \sum_{j \in N_{i}}[a_{ij}(t)|A_{ij}||a_{ij} - a_{ij}\text{sgn}(A_{ij})a_{ij}(t)A_{ij}]\right] \quad (2)$$

In addition, the feedback system gains are represented by $k, a_{1}, \text{ and } a_{2}$, and $u_{ij}$ denotes control input.

**Lemma 4.** Provided a following third-order complex coefficient polynomial

$$f_{3}(r) = r^{3} + (\delta_{2} + j\delta_{2})r^{2} + (\delta_{1} + j\delta_{1})r + \delta_{0} + j\delta_{0} \quad (3)$$

then, equation (3) is steady if and only if $\delta_{2} > 0$, $\delta_{2}\delta_{2}\delta_{1} + \delta_{2}\delta_{1} - \delta_{1} - \delta_{2} > 0$, and

$$\delta_{2} \cdot \text{det} \begin{pmatrix} \xi_{2} & -\xi_{1} & -\xi_{0} & 0 \\ \delta_{2} & \xi_{2} & -\xi_{0} & 0 \\ 0 & \delta_{2} & -\xi_{1} & -\delta_{0} \\ 0 & 0 & \xi_{1} & -\delta_{0} \end{pmatrix} + \text{det} \begin{pmatrix} \xi_{1} & -\delta_{1} & -\delta_{0} & 0 \\ -\delta_{1} & \xi_{1} & -\delta_{0} & 0 \\ 0 & \delta_{1} & \xi_{1} & -\delta_{0} \\ 0 & 0 & 0 & \xi_{1} \end{pmatrix} > 0$$

Let $x = (x_{ij}), \ v = (v_{ij}), \ a = (a_{ij}) \in \mathbb{R}^{n \times 1}, \ i = 1, 2, \ldots, n, j \in N_{i}; b = \sum_{i=1}^{n}d_{ii}(i)$, system (1) can be written as $[\dot{x} T \ \dot{v} T \ \dot{a} T]^{T} = B[x T \ \dot{v} T \ \dot{a} T]^{T}$, where

$$B = \begin{bmatrix} 0 & I_{b} & 0 \\ 0 & 0 & I_{b} \\ -L' & -kI_{b} - a_{1}L' - kI_{b} - a_{2}L' \end{bmatrix}$$

and matrix $L'$ can be described as above. Let $\tilde{x} = Dx, \ \tilde{v} = Dv, \ \tilde{a} = Da, \ D \in \Omega$, we have

$$[\dot{x} T \ \dot{v} T \ \dot{a} T]^{T} = \tilde{B}[\dot{x} T \ \dot{v} T \ \dot{a} T]^{T}$$

where

$$\tilde{B} = \begin{bmatrix} 0 & I_{b} & 0 \\ 0 & 0 & I_{b} \\ -L' & -kI_{b} - a_{1}L' - kI_{b} - a_{2}L' \end{bmatrix}$$

with $L' = DL'D$. Note that
Theorem 1. For multiple-agent coordination system (1) with dynamic continuous time, let the corresponding digraph $G$ have strong connectivity, and let the digon $G$ have sign symmetry and a balanced structure. When the Assumption holds, system (1) will asymptotically reach the bipartite consensus under protocol (2) for $\gamma_i \neq 0$, if the system conforms to the inequalities as follows

$$k - \alpha_2 \Re(\gamma_i) > 0$$

\(\alpha_1\alpha_2(\Im(\gamma_i))^2 + \Re(\gamma_i) + (k - \alpha_2 \Re(\gamma_i))(k - \alpha_1 \Re(\gamma_i))\)

\(\cdot (k - \alpha_2 \Re(\gamma_i)) - \alpha_1^2(\Im(\gamma_i))^2 > 0 \) \hspace{1cm} (5)

and

\[
\begin{vmatrix}
-\alpha_1 \Im(\gamma_i) & \Re(\gamma_i) & 0 \\
(k - \alpha_2 \Re(\gamma_i)) & 1 & 0 \\
1 & -\alpha_2 \Im(\gamma_i) & -\alpha_1 \Im(\gamma_i) \\
0 & 0 & k - \alpha_2 \Re(\gamma_i) \\
0 & 0 & 0 \\
0 & 0 & 0 \\
\end{vmatrix}
\]

\[
\begin{vmatrix}
\Im(\gamma_i) & 0 \\
\Re(\gamma_i) & 0 \\
(k - \alpha_1 \Re(\gamma_i)) & 0 \\
-\alpha_1 \Im(\gamma_i) & \Re(\gamma_i) \\
0 & \Re(\gamma_i) \\
-k - \alpha_2 \Re(\gamma_i) & -\alpha_1 \Im(\gamma_i) \\
\end{vmatrix}
\]

\(> 0 \) \hspace{1cm} (6)

Furthermore, provided $k \neq 0$, the consensus protocol values can be obtained as

\[
\lim_{t \to \infty} \bar{x}(t) = \eta_1^T \mathbf{D} \bar{x}(0) \mathbf{D} \eta_1 + \frac{1}{k} \eta_2^T \mathbf{D} \bar{a}(0) \mathbf{D} \eta_2
\]

\[
\lim_{t \to \infty} \bar{v}(t) = 0
\]

\[
\lim_{t \to \infty} \bar{a}(t) = 0
\]

where $\eta_1$ denotes the left eigenvector with the single zero eigenvalue of the matrix $L'$; similarly, $\eta_2$ represents the right eigenvector of the matrix $L'$ for a single zero eigenvalue, and $\eta_1^T \eta_2 = 1$.

Proof. Given $\gamma_i = 0$, if the Assumption holds, then the root values for polynomial $\lambda^2 + k$ are zero, and negative real parts exist in the complex numbers. Provided $\gamma_i \neq 0$, if preconditions (5), (6), or (7) can be met simultaneously, the root values of the polynomial are given negative real parts, which means that all eigenvalues except for zero of $\mathbf{B}$ are given the negative real parts.

When $k = 0$, Lemma 423 shows that the matrix $\mathbf{B}$ exists in just three zero eigenvalues; in addition, the other matrix eigenvalues also have in negative real parts. The inspiration for the steadiness theorem and the relevant results were obtained from Valcher and Misra; thus, the dynamic consensus protocol will be asymptotically achieved in system (1), and

\[
\lim_{t \to \infty} \bar{x}(t) = \eta_1^T \mathbf{D} \bar{x}(0) \mathbf{D} \eta_1 + \frac{1}{k} \eta_2^T \mathbf{D} \bar{a}(0) \mathbf{D} \eta_2
\]

\[
\lim_{t \to \infty} \bar{v}(t) = 0
\]

\[
\lim_{t \to \infty} \bar{a}(t) = 0
\]

where $\eta_1$ and $\eta_2$ are the left and right eigenvectors associated with $\gamma_i = 0$ of Laplacian matrix $L' = L' \mathbf{D} \mathbf{D} \mathbf{D}$, respectively, and $\eta_1^T \eta_2 = 1$. Thus, from $\mathbf{D} = \mathbf{D}^{-1}$, system (1) will asymptotically be achieved dynamic bipartite consensus in protocol (2), and

\[
\lim_{t \to \infty} \bar{x}(t) = \eta_1^T \mathbf{D} \bar{x}(0) \mathbf{D} \eta_1 + \frac{1}{k} \eta_2^T \mathbf{D} \bar{a}(0) \mathbf{D} \eta_2
\]

\[
\lim_{t \to \infty} \bar{v}(t) = 0
\]

\[
\lim_{t \to \infty} \bar{a}(t) = 0
\]
When the digraph $G$ does not achieve balance in structure, this denotes that all the eigenvalues of $-L'$ have negative real parts. Furthermore, system (1) will be asymptotically steady with protocol (2).

And we see $L'$ as Laplacian matrix of $L(G)$. Let $x^{(0)} = D x^{(0)}$, $c \in [0, s - 1]$, $D \in \Omega$, and $\hat{\rho} = [\hat{x}(t) \hat{x}(t)^T \ldots \hat{x}^{(s-1)}(t)^T]^T$. Then, we obtain

$$\dot{\hat{\rho}} = B \hat{\rho}$$

(11)

where

$$B = \begin{bmatrix}
0 & I_b & 0 & \ldots & 0 \\
0 & 0 & I_m & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & I_b \\
-L' & -kI_b - \alpha_1 L' & -kI_b - \alpha_2 L' & \ldots & -k_{s-1}I_b - \alpha_{s-1} L'
\end{bmatrix}$$

with $L' = DL'D$.

Note that

$$\det(\lambda L_b - B) = \det \left( \begin{bmatrix}
\lambda I_b & -I_b & 0 & \ldots & 0 \\
0 & \lambda I_b & -I_b & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & -I_b \\
L' & kI_b + \alpha_1 L' & kI_b + \alpha_2 L' & \ldots & (\lambda + k_{s-1}I_b + \alpha_{s-1} L')
\end{bmatrix} \right)$$

(12)

where $\gamma_i, i = 1, 2, \ldots, m$ are seen as the eigenvalues of matrix $-L'$ and there exists a special eigenvalue $\gamma_i = 0$. Moreover, matrix $B$ has more than one zero eigenvalue.

**Control protocol with high-order bipartite consensus**

The dynamic consensus model of the $s$ order with continuous time is given as follows

$$\dot{x}^{(0)}_t = x^{(1)}_t, \dot{x}^{(1)}_t = x^{(2)}_t, \ldots, \dot{x}^{(s-1)}_t = u_t$$

(8)

where $x^{(0)}_t, c \in [0, s - 1]$ are defined as the states at time variable $t$. For system (8), the following high-order control system protocol can be obtained

$$u_t = -\sum_{i=1}^{s-1} k_i x^{(i)}_t - \sum_{j \in N_t} \left[ x^{(0)}_t(t)A_{ij} - \text{sgn}(A_{ij})x^{(0)}_t(t)A_{ji} \right]$$

$$-\sum_{c=1}^{s} \sum_{j \in N_t} \left[ x^{(c)}_t(t)A_{ij} - \text{sgn}(A_{ij})x^{(c)}_t(t)A_{ji} \right]$$

(9)

where we see $k_1, k_2, \ldots, k_{s-1} \geq 0$ as feedback gains. Note that $x^{(c)}_t = (x^{(c)}_t(t)) \in \mathbb{R}^{b \times 1}, j \in N_t, b = \sum_{i=1}^{n} d_{in}(t)$, and $\rho = [x^{(0)}_t \ldots x^{(s-1)}_t]^T$. Then, equations (8) and (9) are rewritten as

$$\dot{\rho} = B \rho$$

(10)

where

$$B = \begin{bmatrix}
0 & I_b & 0 & \ldots & 0 \\
0 & 0 & I_m & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & I_b \\
-L' & -kI_b - \alpha_1 L' & -kI_b - \alpha_2 L' & \ldots & -k_{s-1}I_b - \alpha_{s-1} L'
\end{bmatrix}$$

In addition, $w(x)$ denotes a polynomial with complex coefficient and $l(x)$ also represents a polynomial of conjugate coefficients with the identical order

$$w(x) = (p_n - q_n i)x^n + (p_{n-1} - q_{n-1} i)x^{n-1} + \ldots + (p_1 + q_1 i)x + p_0 - q_0 i$$

The product is

$$l(x) = (p_n + q_n i)x^n + (p_{n-1} - q_{n-1} i)x^{n-1} + \ldots + (p_1 - q_1 i)x + p_0 - q_0 i$$

The product is
s(x) = \text{If}(x) \times w(x)

\begin{align*}
&= \sum_{c=1}^{n} \left( \sum_{j=0}^{2c-1} 2(p_j p_{j+c} + q_j q_{j+c}) \right) x^{2c-1} \\
&+ \sum_{c=0}^{n} \left( \sum_{i=j}^{2c} 2(p_i p_{i+c} + q_i q_{i+c}) + p_c^2 + q_c^2 \right) x^{2c}
\end{align*}

**Proof.** The preceding result can be proven by means of the multiplication algorithm for the polynomial with a complex coefficient; furthermore, each coefficient of \( s(x) \) will be greater than zero.

Moreover, \( \lambda^4 + (k_{s-1} - \alpha_{s-1} \gamma_i) \lambda^{s-1} + \ldots + (k - \alpha_2 \gamma_i) \lambda^2 + (k - \alpha_1 \gamma_i) \lambda - \gamma_i \) by \( P_s(\gamma_i) \), and notice that \( P_s(\gamma_i) \cdot P'_s(\gamma_i) = P_{2s}(\gamma_i) \). Furthermore, by means of Theorem 2, equation (10) can be obtained as follows

\[
det(\Lambda_{ps} - \Pi) = \prod_{i=1}^{c} P_s(\gamma_i) \cdot \prod_{i=c+1}^{c+k} P_{2s}(\gamma_i)
\]  

(13)

**Lemma 5.** Rogers and Owens\(^{25}\) provided a control system with feature polynomial

\[
F(l) = p_n l^n + p_{n-1} l^{n-1} + \ldots + p_1 l + p_0, p_i > 0, i \in [0, n]
\]

Moreover, the control system can reach a steady state if and only if its determinant is composed of characteristic polynomials

\[
\Delta_e = \begin{vmatrix}
  p_{n-1} & p_{n-3} & \cdots & 0 & 0 & 0 \\
  p_n & p_{n-2} & \cdots & 0 & 0 & 0 \\
  0 & p_{n-1} & \cdots & 0 & 0 & 0 \\
  0 & p_n & \cdots & 0 & 0 & 0 \\
  \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
  0 & 0 & \cdots & p_3 & p_1 & 0 \\
  0 & 0 & \cdots & p_2 & p_0 & 0 \\
\end{vmatrix}
\]

We see that the minimum value of its sequential master determinant \( \Delta_e(i = 1, 2, \ldots, n - 1) \) is greater than zero, namely

\[
\Delta_1 = p_{n-1} > 0, \Delta_e = \begin{vmatrix}
  p_{n-1} & p_{n-3} & \cdots & 0 & 0 & 0 \\
  p_n & p_{n-2} & \cdots & 0 & 0 & 0 \\
\end{vmatrix} > 0, \ldots, \Delta_e > 0
\]

**Theorem 2.** For multiple-agent coordination system (8) with dynamic continuous time, let the corresponding digraph \( G \) have strong connectivity, and let the digon have sign symmetry and a balanced structure. If the Assumption is satisfied, system (8) will asymptotically achieve dynamic bipartite consensus in protocol (9) when the determinants and sequential master determinants of \( P_s(\gamma_i) \) and \( P_{2s}(\gamma_i) \) in (13) are greater than zero for \( \gamma_i \neq 0 \), and \( \Delta_e > 0, j \in [1, s - c] \) of \( P_s(0) \) with \( k_e \neq 0, k_{e+1} \neq 0 \) for \( \gamma_i = 0 \).

**Proof.** In Lemma 5, by means of the results from Ren and Atkins,\(^{26}\) provided that the determinants and sequential master determinants of \( P_s(\gamma_i) \) and \( P_{2s}(\gamma_i) \) in (13) are greater than zero for \( \gamma_i \neq 0 \), the roots of the corresponding subpolynomial (12) all have negative real parts. Provided that \( \Delta_j > 0, j \in [1, s - c] \) with \( k_e = 0, k_{e+1} \neq 0 \) for \( \gamma_i = 0 \), the roots, except for zero of the corresponding subpolynomial (12), have negative real parts. Moreover, system (11) will achieve dynamic consensus.

When \( \mathbf{L}(\mathbf{G}) \) does not have balance in the structure, the eigenvalues of \( -\mathbf{L}' \) all have negative real parts. The eigenvalues of \( \mathbf{B} \) also have negative real parts, which means that equation (8) will be asymptotically steady under protocol (9) and \( \lim_{t \to \infty} x^{(0)} = 0, i \in [0, s - 1] \).

**Remark 1.** For the proposed high-order system, its pre-conditions are also valid for the three-order system. However, differences in the three-order system probably exist, such as the structure of the protocol, the matrix eigenvalues, and the relevant results. Furthermore, compared with the high-order system, the three-order system is advantageous in that it has a more simple and effective judgment method. More critically, the ultimate results derived by the three-order system can be well applied to the formation control of robots and also play a role in the further comprehension and deduction of meanings about high-order systems.

**Robotic formation operation control**

**Establishment of the robotic model**

The kinematic equations of the robot are given as Altafini\(^{21}\)

\[
\begin{cases}
  \dot{x} = v \cos \sigma \\
  \dot{y} = v \sin \sigma \\
  \dot{z} = \omega \\
  m \dot{v} = f \\
  J \dot{\omega} = T \\
  \dot{\sigma} = \omega
\end{cases}
\]

where \((x, y, z)\) represents the Cartesian position of robots; \(\sigma, \omega, v\) denote robotic orientation, angular velocity, and linear velocity; \(m\) and \(f\) denote mean robotic mass and force; and \(J\) and \(T\) are mass moment of inertia and torque acting on the robot, respectively. Meanwhile, the friction factors have been ignored.

In addition, we define another reference which is not robotic rotating center as follows

\[
\begin{cases}
  \dot{x}_i = x + \frac{1}{l} \cos \sigma \\
  \dot{y}_i = y + \frac{1}{l} \sin \sigma \\
  \dot{z}_i = \omega
\end{cases}
\]

where \((x_i, y_i, z_i)\) represents another coordinate and the distance is \(l\) between coordinate \((x, y, z)\) and \((x_i, y_i, z_i)\).
On the basis of above analysis, we can obtain the motional equation as below

\[
\begin{aligned}
\dot{x}_i &= v_{xi} = a_{xi} t_{si} = u_{xi} \\
\dot{y}_i &= v_{yi} = a_{yi} t_{yi} = u_{yi} \\
\dot{z}_i &= v_{zi} = a_{zi} t_{zi} = u_{zi}
\end{aligned}
\]

(15)

As follows, in our paper, \(u_{xi}, u_{yi}, \) and \(u_{zi}\) are input to the system; in addition, equation (14) represents the complete constraint state of the robot.

In this article, five robots are required to achieve the consensus in X, Y, and Z directions. Then, Figure 1 describes information exchange structure topology of the five robots, and the edge with directions \((i \rightarrow j \text{ robot})\) represents that \(x_i = x_{ig}, y_i = y_{ig}, \) and \(z_i = z_{ig}\) can be obtained by the means of the \(j\)th robot.

To make \((x_{ig}, y_{ig}, z_{ig})\) be an convergent point of ideal goal for the \(i\)th robot, a control law will be proposed among \(u_{xi}, u_{yi}, \) and \(u_{zi}\) as below

\[
\begin{aligned}
u_{xi} &= -k_{vxi} \\
&- \sum_{j \in N_i} \left( (x_i - x_{ig})(t - \varphi_j(t)) \right) |A_j| \cdot \text{sgn}(A_j)(x_j - x_{ig})(t - \varphi_j(t))A_j \\
&- \sum_{j \in N_i} \left( y_i(t - \varphi_j(t)) \right) |A_j| \cdot \text{sgn}(A_j)(y_j - y_{ig})(t - \varphi_j(t))A_j \\
&- \sum_{j \in N_i} \left( z_i(t - \varphi_j(t)) \right) |A_j| \cdot \text{sgn}(A_j)(z_j - z_{ig})(t - \varphi_j(t))A_j \\
u_{yi} &= -k_{vyi} \\
&- \sum_{j \in N_i} \left( (y_i - y_{ig})(t - \varphi_j(t)) \right) |A_j| \cdot \text{sgn}(A_j)(y_j - y_{ig})(t - \varphi_j(t))A_j \\
&- \sum_{j \in N_i} \left( v_{yi}(t - \varphi_j(t)) \right) |A_j| \cdot \text{sgn}(A_j)(v_{yi}(t - \varphi_j(t))A_j \\
&- \sum_{j \in N_i} \left( a_{yi}(t - \varphi_j(t)) \right) |A_j| \cdot \text{sgn}(A_j)(a_{yi}(t - \varphi_j(t))A_j \\
u_{zi} &= -k_{vzi} \\
&- \sum_{j \in N_i} \left( (z_i - z_{ig})(t - \varphi_j(t)) \right) |A_j| \cdot \text{sgn}(A_j)(z_j - z_{ig})(t - \varphi_j(t))A_j \\
&- \sum_{j \in N_i} \left( v_{zi}(t - \varphi_j(t)) \right) |A_j| \cdot \text{sgn}(A_j)(v_{zi}(t - \varphi_j(t))A_j \\
&- \sum_{j \in N_i} \left( a_{zi}(t - \varphi_j(t)) \right) |A_j| \cdot \text{sgn}(A_j)(a_{zi}(t - \varphi_j(t))A_j \
\end{aligned}
\]

(16)

Denote that equation (16) will be applied to ensure the desired formation of five robots.

**Simulation results**

The communication topology for a system composed of five robots is illustrated in Figure 1. Among them, the solid lines and the dotted lines represent the cooperation and competition relationship between the robots with weights of 1 and -1, respectively.

Let \(x = x_i - x_{ig}, y = y_i - y_{ig}, \) and \(z = z_i - z_{ig}\)

\[
\begin{aligned}
x_{p1}(0) &= [-2 1 1.5 -1 -3] \\
y_{p1}(0) &= [2.2 1.5 -1.5 -3] \\
z_{p2}(0) &= [4.3 2.5 -3.5 -3.5]
\end{aligned}
\]

Let \(D = \text{diag}(-1\ 1\ 1\ 1);\) thus, the following can be obtained

\[
L = DLD = \begin{bmatrix}
1 & 0 & 0 & -1 & 0 \\
-1 & 2 & -1 & 0 & 0 \\
0 & -1 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 \\
0 & -1 & 0 & 0 & 1
\end{bmatrix}
\]

Figure 1. Communication topology.

and 0 is the eigenvalue of the matrix \(L.\) The left and right eigenvectors \(\eta_i\) and \(\gamma_i\) can thus be obtained

\[
\eta_i = [-0.4472 -0.4472 -0.4472 -0.4472 -0.4472] \\
\gamma_i = [-0.4472 -0.4472 -0.4472 -0.4472 -0.4472]
\]

These eigenvectors conform to \(\eta_i^T \gamma_i = 1.\)

As presented in Figure 1, the solid line between vertexes indicates that there is a cooperative relationship between agents, and the corresponding weight is taken as 1. The dotted line between nodes indicates that there is a competitive relationship between agents, and the
Figure 2. X-direction trajectories of the five robots with $\phi_y = 0.1673$, $k = 2$, $\alpha_1 = 2$, $\alpha_2 = 1.5$.

Figure 3. Y-direction trajectories of the five robots with $\phi_y = 0.1673$, $k = 2$, $\alpha_1 = 2$, $\alpha_2 = 1.5$. 
Figure 4. Z-direction trajectories of the five robots with $\phi_y = 0.1673$, $k = 2$, $\alpha_1 = 2$, $\alpha_2 = 1.5$.

Figure 5. Position, velocity, and acceleration trajectories of the five robots with $\phi_y = 0.1673$, $k = 0$, $\alpha_1 = 2$, $\alpha_2 = 0.5$. 
corresponding weight is taken as $-1$. Let the upper bound of the time-varying delays $\phi_1 = 0.1673$; according to Figure 1, Figure 2, respectively, describes the trajectories of the position, velocity, and acceleration of each robot under the control rule (16); they conform to the law, and each robot can reach the desired consensus.

Figure 3 describes the Y-direction trajectories of each robot when $k = 2$, $\alpha_1 = 2$, and $\alpha_2 = 1.5$, which satisfy the consensus protocol and can reach bilateral consensus in the Y-direction. It is evident that the desired formation among seven robots is maintained, even though the information exchange topologies switch with time.

The position, velocity, and acceleration trajectories of each robot under control law (16) for Z-direction robotic trajectories of motion are illustrated in Figure 4, in which each robot of the same weight is able to reach its destination.

Figure 5, respectively, presents the position, velocity, and acceleration trajectories of the corresponding closed-loop system (1) under protocol (2) when $k = 0$, $\alpha_1 = 2$, and $\alpha_2 = 1.5$. The absolute values of the position, velocity, and acceleration of all the edges in the system converge to the specific value, which demonstrates that the multi-robot formation achieves consensus.

It is worth noting that the simulation results presented in the four figures are closely related to the choice of the parameters $k$, $\alpha_1$, and $\alpha_2$. The numerical simulation also indicates that the results will exhibit differences when designing different parameter values.

Conclusion

In this paper, a control protocol was proposed for a strongly connected, digon sign-symmetric structurally balanced graph with positive and negative weights under a high-order dynamics protocol. The relationship between competition and cooperation in robot formation was designed based on third-order dynamic protocols. The state values of all the edges for the third- and high-order multi-agent system were discussed in terms of the properties of feature polynomials and the sequential principal determinant. By combining the three-order bilateral consensus control protocol with the robot model, the control problem of multi-robot formation is well solved. The constraint problems of high-order systems, research on time-varying delays, and the precise control of robot formation still require further study in future work.

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