Influence of the material reverse flow process on the regularities of pressure formation along the length of the screw surface of extruders

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Abstract. In the present work, the problem of describing the law of increase in the internal screw pressure of a solid material along the length of the screw surface of the extruder is posed and analytically solved, depending not only on the distributed friction forces and concentrated resistance forces of the counter-rotation ribs, made on the inner surface of the extruder body, but also taking into account the process of “sluicing” - backflow of material, which significantly affects the formation of the pressure field. The obtained mathematical model makes it possible to more correctly substantiate the design and technological parameters of extruders and grinders. The complexity of the movement of material in screw aggregates during its transportation, extrusion and cutting is caused not only by the complex screw shape of the channel, but also by the influence of a large number of variable technological parameters, structural elements of the mechanism, as well as many factors of a physical, mechanical and rheodynamic nature. In particular, in this study, the processes in two characteristic zones of the screw space, which are fundamentally different in physical conditions, are structured and analytically described by differential calculus methods. Based on the obtained mathematical model for the law of pressure distribution along the length of the screw surface, the design and technological characteristics of extruders can be optimized to reduce the energy consumption of materials grinding processes.

1 Introduction
It is known that one of the main parameters determining the quality of materials grinding process, as well as the energy intensity of the extrusion and movement process in the top screw duct is the value of the pressure formed at the outlet in the cutting zone. The required value of this pressure is analytically determined in the classical work of A.I. Peleev [1] and refined in the study [2]. At the same time, the work [3] solved the problem of top clamp nut torque optimization, which eliminates material penetration into the joint of the blade-grid structural pair and ensuring the quality of cutting and shredding. Analytical problem solution of determining the law of pressure increase along the length of the screw duct at different degrees and levels of simplification is carried out in works [3, 4] and a few others. However, in these studies, the formation of the determining pressure at the outlet of the extruder is described only depending on the retarding action of friction forces on the material with the screw surface of the screw [5]. It is known that for such screw mechanisms as tops and grinders, the main factor determining the law of increasing pressure is the presence of contrarotation elements on the inner surface of the cylindrical body as shoulders or slots. Therefore, the work [6] solved the problem of mathematical description of the intra-screw pressure formation process considering the impact of the mechanical resistance forces of contrarotation elements on the extruded material. In the
study [7] the authors obtained a solution complicated by consideration of the conical shape of the extruder body. However, to date there is no theoretical description of the “sluicing” process effect, which is the occurrence of a reverse material flow through the screw gap between the internal screw surface of the contrarotation shoulder in the extruder case and the outer screw surface of the screw flight surface, to form the law of distribution of intra-screw pressure.

The purpose of the work is a mathematical description of the law of changing the intra-screw pressure considering the “sluicing” phenomenon in the second phase of the extrusion process.

2. Objects and methods of research
Objects of study are interaction processes of raw materials with structural elements of screw extruders, tops or meat grinders during movement, deformation and extrusion of solid material in back pressure conditions from the side of the contrarotation shoulders on the inner surface of the body, as well as considering the “sluicing” phenomenon in the zone of product's reverse current occurrence.

Mathematical modeling of the material intra-screw pressure formation process based on the constructed two-zone physical model and solution for each zone defining differential equations were chosen as research methods.

3. Discussion of results
3.1. Building a physical model of the extrusion process
For the mathematical description of intra-screw pressure formation process, we considered the physical product jam movement model in reverse motion on the screw duct surface in its two characteristic zones, considering the influence of such braking effect factors on the outer circuit of the first zone as friction force, mechanical resistance of contrarotation elements, and in the second screw space zone - additional appearance of the reverse product flow.

The structuring of the double-zone screw space and the structural parameters of the extruder elements are shown in Figure 1.

Figure 1. Structuring the zones of the screw space of the extruder

In the first screw space zone of the extruder (shredder, top, meat grinder), unidirectional translational material movement (velocity $V_1$) to a certain pressure growth values (from $P_o$ to $P_s$) is preserved. However, upon reaching the critical pressure $P_s$ (the "sluicing" start pressure - the reverse flow of the product), the compressive forces acting on the material become sufficient to carry out the product cut and its reverse movement (velocity $V_s$) through the screw gap, the clearance $\xi$, between
the inner screw surface of the extruder body contrarotation ribs and the outer screw surface of the flight screw as shown in Figures 1 and 2. Starting from this $P_s$ value, due to the redistribution of velocities $V_1$ and $V_2$, as well as the nature of the forces in action, the pressure in the second zone varies in accordance with a different law rather than in the first zone, increasing to the required material extrusion pressure $P_e$ through the holes of the output grid.

![Diagram](image)

**Figure 2.** Scheme and parameters of the annular gap in the cross section of the A-A extruder

The developed physical model and the meaningful description carried out allow to proceed to the statement of mathematical features of the pressure field formation in each of the selected zones.

3.2. **Construction of a general mathematical model of the extrusion process**

For the basis of the process general mathematical model, we take hydrodynamic motion equations in the Euler form for the product mass of volume unit [8], transformed into equilibrium equations of forces, acting on the elementary plate of the intra-screw space.

As an elementary plate, we will consider a prism formed by two cut planes normal to the screw helix line, with the distance $D_x$ between them as shown in figure 3.
Let us write down the equation of the extruded material prismatic plate movement in reverse motion in the form of the Euler equation, designing it on the longitudinal X axis of the reversed screw duct under action of the unknown pressure “P” force and the friction and resistance forces “R”, counteracting this displacement and applied to the mass of the volume unit:

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} + v_z \frac{\partial v_x}{\partial z} = R - \frac{1}{\rho} \frac{\partial P}{\partial x}.$$  

(1)

where:
- $v_x$ - projection of the material velocity vector on the axis OX, m/s;
- $v_y, v_z$ - corresponding projections of the velocity vector on the OY and OZ axis, m/s;
- $t$ - time, s;
- $\rho$ - density of extruded material, kg/m$^3$;
- $P$ - material pressure at the outlet of the screw mechanism, Pa;
- $R$ - projection of the main vector of external friction forces on the OX axis, as well as resistance acting on the mass of the prism volume unit, N/kg or m/s$^2$.

We are interested in the steady flow mode of extruded raw materials through the screw duct; therefore, neglecting the presence of transverse movement of food material along the axes OY and OZ we can put:

$$\frac{\partial v_x}{\partial t} = 0, \quad \frac{\partial v_x}{\partial y} v_y = \frac{\partial v_x}{\partial z} v_z = 0. \quad \text{(2)}$$

Considering the condition of inseparability of the mass flow, we believe that the inertial component is also zero:

$$\frac{\partial v_x}{\partial x} v_x = 0. \quad \text{(3)}$$

Thus, the basic equation for a two-zone problem considering assumptions (2) and (3), Euler equilibrium equation (1) for the mass of the material volume unit will take the form:

$$R - \frac{1}{\rho} \frac{\partial P}{\partial x} = 0.$$

For the mass of the prism of $M = \rho q$, we get:
\[ P \cdot Q \cdot R \cdot \frac{\partial \rho}{\partial x} = 0 \]  \hspace{1cm} (4)

where:
- \( M \) is the mass of the food material prism, kg;
- \( Q \) - the prism volume of the material being moved under the action of a pressure gradient, \( m^3 \);
- \( \rho \cdot Q \cdot R \) - projection on the OX axis of the main vector of all external forces impacting on the entire prism volume, N.

The resulting generalized equilibrium equation (4) should be revealed in the form of specific component equations for different forces in nature for the first and second zones of the screw space.

3.2.1. Construction of mathematical model of extrusion process in the first zone of the screw space

As follows from the calculation scheme presented in Figure 3, within the first zone of the screw space, the equilibrium equation does not contain the component \( F_{2av} \), so as the pressure is not enough for the cut and reverse current - the “sluicing” of the material, so we can write:

\[ S dP = dF_{fr} + dF_{cm} - dF_k \]  \hspace{1cm} (5)

We will open the content of the equation components (5), forming the pressure \( P \) on the elementary plate of the material area \( S \) and thickness \( D_x \) (Figure 3).

The differential of friction force \( dF_{fr} \) of the material against the screw surface forming the pressure field in the screw duct is determined by the obvious ratio

\[ dF_{fr} = \mu P v_1 - v P dX, \]  \hspace{1cm} (6)

where \( P \) is the pressure of the material in the screw duct, Pa;
- \( P_e \) - the perimeter of material friction on the screw surface, m.

According to the physical model of the extrusion process and the design scheme shown in Figure 3, the value of the material friction perimeter against the screw surface is determined by the obvious ratio:

\[ P_e = N - (D - D_m) \cdot \frac{\sin \beta}{\cos \beta}. \]

The pressure area \( S \) on the elementary plate is determined in accordance with Figure 3 by expression:

\[ S = \left[ H - \sin \beta \cdot \frac{D - D_m}{2} \right] \cdot \frac{D - D_m}{2}, \]

where \( D \) is the diameter size of the screw helix surface feathers formative cylinder, m;
- \( D_m \) - diameter of the screw helix surface cavities, m;
- \( \beta \) - half of the screw wedge angle (Figure 3), rad.

As shown in the work [9], the differential of the counteraction force from the contrarotation shoulders’ side is determined by their number, height \( \Delta \), geometric characteristics of the elementary plate and its angular coordination:

\[ dF_{cm} = kH \Delta \cos(\alpha - \gamma) dP, \]  \hspace{1cm} (7)

where \( k \) is the number of contrarotation shoulders;
- \( H \) - width of the screw duct along the feathers of the screw surface (Figure 3), m;
- \( \Delta \) - height of contrarotation shoulders, m;
- \( \gamma \) - the inclination angle of the shoulder generating line to the longitudinal screw axis (Figure 1), rad.

The width of the screw duct along the screw surface feathers is determined by the ratio

\[ H = \pi D \sin \alpha, \]

where \( D \) is the screw diameter on the screw surface feathers (Figure 3), m.

The differential of friction force \( dF_3 \) of material against the inner surface of the extruder's cylindrical body is determined according to Figures 1 and 3:
\[ dF_K = fP \frac{v}{1-v} Hsina dX. \] (8)

Substituting the obtained ratios (6) — (8) into equation (5), we get
\[ \text{SdP} = fP \frac{v}{1-v} PdX + kH\Delta \cos(\alpha - \gamma) dP - fP \frac{v}{1-v} Hsina dX. \] (9)

The solution of this differential equation for the boundary condition \( P = P_0 \), at \( X=0 \), is obtained as:
\[ P = P_0 e^{\left(\frac{fP}{1-v}\frac{v}{1-v}\left[S-Hk\Delta \cos(\alpha - \gamma)\right]\ln \frac{P}{P_0}\right)} X. \] (10)

We will show the condition of the reverse material flow process start through the ratio of equilibrium stresses of the material bunching on the contrarotation shoulder and the specific cutting force causing the sluicing process (figure 2), in the form of:
\[ P_s^* \xi (D/2) d\phi = P_{sp}(D/2) d\phi. \] (11)

Then we get the boundary condition for the “sluicing” pressure:
\[ P = P_s = P_{sp}/\xi. \] (11)

Solving the system of equations (10) and (11) relative to \( X \) together, we obtain the length of the screw line, after reaching which the desired pressure of material sluicing at the exit from the first extruder zone is ensured:
\[ X_s = \frac{(1-v)\left[S-Hk\Delta \cos(\alpha - \gamma)\right]}{fP} \ln \frac{P_{sp}}{\xi P_0}. \] (12)

Having determined the coordinate of the “sluicing” process start \( X_s \), we can proceed to the mathematical model development of the material extrusion process in the second zone.

3.2.2. Construction of mathematical model of extrusion process in the second zone of the screw space
As follows from the design scheme shown in Figure 3, within the second zone of the screw space the component \( F_{av}^2 \) is added to the equilibrium equation, since the material pressure exceeds the pressure sufficient for the cut and reverse current - “sluicing”, so we can write:
\[ \text{SdP} = dF_{fr} + dF_{cm} - dF_K - dF_{2av}^2. \] (13)

We will open the content of the last component characterizing the occurrence of the “sluicing” process starting with the \( X_s \) coordinate.

The differential \( dF_{2av}^2 \) the force causing the reverse material flow (the “sluicing” effect) is determined according to figure 1 similar to the ratio (7), but considering the lowering of the current pressure values by the value \( P_s = P_{sp}/\xi \).

Thus, we can write:
\[ dF_{2av}^2 = H\xi \cos(\alpha - \gamma) d\left(P - \frac{P_{sp}}{\xi}\right), \] (14)

where \( \xi \) is the gap size between the external formative cylinder of the screw surface feathers and the inner formative cylinder of the contrarotation shoulders feathers surface, as indicated in Figures 1 and 2, m.

Substituting the obtained ratio (14) into equation (13) and considering the equality of differentials \( dP \) and \( d\left(P - \frac{P_{sp}}{\xi}\right) \), we obtain:
\[ \text{SdP} = fP \frac{v}{1-v} PdX + kH\Delta \cos(\alpha - \gamma) dP - fP \frac{v}{1-v} Hsina dX - H\xi \cos(\alpha - \gamma) dP. \] (15)
The solution of this differential equation (15) for the second zone is obtained by the method of separating variables for boundary conditions, according to equations (11) and (12), at X=xs, P=Ps=Psp/ξ, in the form of:

\[ P = P_2 = \frac{Ps}{\xi} e^{\left(1-v\right)\left[S-Hk\alpha \cos(\alpha-\gamma)\right]} \]  

(16)

Given the ratio (12) for the coordinate of the "sluicing" process start, the resulting equation (16) of the pressure formation process in the second zone, after a series of algebraic transformations, will take the exponential law form:

\[ P = P_2 = \left\{ P_0 \frac{S-Hk\alpha - H\cos(\alpha-\gamma)}{S-Hk\alpha - H\cos(\alpha-\gamma)} \right\} e^{\left(1-v\right)\left[S-Hk\alpha - H\cos(\alpha-\gamma)\right]} \]  

(17)

The correctness of the obtained law for the second screw space zone is confirmed by the equality of equations (10) and (17) at ξ=0, that is, in the absence of the sluicing phenomenon.

3.3. Determination of the extrusion process optimality criterion, which excludes the phenomenon of material reverse flow ("sluicing") and ensures minimization of energy consumption

As noted earlier, in work [2] the theoretical substantiation of the amount of required extrusion pressure in the plane of the grinding grid is carried out considering the friction forces in the conditions of tight compression of food material

\[ Pe = \frac{4P_{sp}}{d-v(1-f)} \]  

(18)

From equation (18) it follows that the value of the required extrusion pressure Pe in the grid zone is determined not only by the diameter as a geometric parameter, and the specific cutting force Psp, characterizing the strength properties of raw materials, but also Poisson factor ν, characterizing material deformation properties, the friction coefficient of the raw material f, determining the nature of the tribological interaction of food material with the cylindrical surface of grinding grid of thickness δ [2, 10, 11].

After we receive the boundary condition for the "sluicing" pressure in the form of ratio (11), it becomes absolutely clear that during the movement of food raw material reverse flow through the ring gap between the outer screw surface and the inner forming surface of the contrarotation shoulder, the criterion of exclusion will be an obvious inequality Pe < P = Psp/ξ.

In this case, after solving the system of equations (11) and (18), the criterion for the absence of "gateway" becomes the following ratio, linking the design parameters of the output grinding grid (d, δ) and screw mechanism (Σ):

\[ \xi < \left(\frac{d}{4}\right) - \frac{f \nu \delta}{2\left[1-v(1-f)\right]} \]  

(19)

3.4. Graphic construction of pressure field along the screw helix surface

To build specific dependencies (10) and (17) in the first and second zones of the intra-screw space, we will accept the following real initial and calculated (P and S) data.

D=0.06 m; Dm=0.02 m; d=0.003 m; δ=0.005 m; H=0.133 m; β=0.2 rad.; f=0.10; ν=0.43; α=π/4; γ=0; Δ=0.002 m; ξ=0.001 m; k=4; S=0.0021 m²; P=0.162 m; Psp=400 N/m.

The coordinate of the "sluicing" process start is determined by the ratio (12) and for the accepted input data will equal to:

\[ Xs = \frac{1-v[S-Hk\alpha \cos(\alpha-\gamma)]}{\nu(P-H\sin(\alpha-\gamma))} \ln \frac{Ps}{P_0} = 0.570.050.021-0.133+0.002+0.707} \ln \frac{P_0}{0.43*0.1(0.162-0.133+0.707)} \ln \frac{400}{0.001+10^5} Xs = 0.2628*1.3863 = 0.364 m. \]

In this case, the equations (10), (11) and (16) will take the form:

\[ P_1 = P_0 e^{3.805*Xs} = 0.1e^{3.805*Xs} \]  

(20)

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In this case, the required extrusion pressure through the holes of the output grid is calculated by the ratio (18).

\[ P_e = \frac{4P_r}{d - \frac{2f\eta}{1 - v(1 - f)}} = \frac{4 + 400}{0.003} = 0.696 \text{ Mpa}. \]

Substituting the obtained value in the equation (22), we determine the required length of the screw surface, providing this value: \( X_t = 0.519 \text{ m} \).

As you can see, the "sluicing" process occurs at the length of \( X_s = 0.364 \text{ m} \) to \( X_t = 0.519 \text{ m} \).

Exclusion of the "sluicing" phenomenon requires the fulfillment of the condition (19):

\[ \xi < \left( \frac{d}{4} \right) - \frac{f\eta}{2[1 - v(1 - f)]} = (0.003/4) - 0.1*0.43*0.005/2[1-0.43(1-0.1)] = 0.0007 \text{ m}. \]

The construction of graphs of the obtained functions \( P_1(X) \) and \( P_2(X) \) is performed in Figure 4.

**Figure 4.** Functional dependence of pressures in the "no-sluice" zone and in the "sluice" zone on the length of the screw surface

Exclusion of the "sluicing" phenomenon requires the fulfillment of the condition (19):

\[ \xi < \left( \frac{d}{4} \right) - \frac{f\eta}{2[1 - v(1 - f)]} = (0.003/4) - 0.1*0.43*0.005/2[1-0.43(1-0.1)] = 0.0007 \text{ m}. \]

In this case, the required length of the screw surface is calculated according to the equation (20) and will be the value \( X_{t1} = 0.509 \text{ m} \), which allows designers to reduce dimensions and mass of the screw shredder by 2%. With the existing fleet of grinding and cutting equipment of different productivity in tens and hundreds of thousands of units, the value of savings is very significant.
4. Conclusions
In the paper it is shown that when a certain Pe pressure is exceeded in the screw space of the tops and shredders, there is a reverse material flow - the "sluicing" phenomenon, leading to an increased energy consumption and reduction of raw material quality due to its additional rumpling. Two characteristic zones (first - "no-sluice" and second - "sluice") are considered, for which differing analytical dependence of material pressure P on the screw surface length X are obtained. The criterion of the "sluicing" process start is determined: ratio of extrusion pressure and specific material cutting force. The obtained criterion is expressed through interconnected design parameters of the output grid (d, δ) and screw mechanism (Σ). At a known extrusion pressure, the Xs coordinate of the material reverse flow process start is determined. It is shown that by eliminating the "sluicing" process due to design improvement by reducing the gap ξ it is possible to reduce the size and material intensity of tops and meat grinders by 2%, that is a significant value with the existing equipment fleet of hundreds of thousands of units.

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