Scaling in the Integer Quantum Hall Effect: interactions and low magnetic fields

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Summary: Recent developments in the scaling theory of the integer quantum Hall effect are discussed. In particular, the influence of electron-electron interactions on the critical behavior are studied. It is further argued that recent experiments on the disappearance of the quantum Hall effect at low magnetic fields support rather than disprove the scaling theory, when interpreted properly.

While the remarkable accuracy of the quantization of the Hall conductivity is the most prominent feature of the integer quantum Hall effect (IQHE) \cite{1}, the nature of the transitions between different quantized plateaus is one of the most interesting aspects of the effect \cite{2}. The formation of the plateaus is due to the localization of the charge carriers by a residual disorder potential and the transition between the plateaus corresponds to a delocalization transition near the centers of the Landau bands. Despite two decades of research the correct field theoretical description of localization the IQHE is still controversial and to a large extent speculative \cite{3}. Even the essential ingredients of such a theory are not evident. In a real sample, the electrons are interacting, their wavefunction have a finite extent perpendicular to the plane of electron gas, and they move in a finite magnetic field. In contrast, the simplest theories assume strictly two-dimensional, non-interacting electrons moving in a magnetic field that is sufficiently strong to neglect Landau level mixing. In the present contribution we will discuss to what extent the Coulomb interactions between the electrons and strong Landau level mixing change the critical behavior associated with the plateau transitions.

The origin of the quantization of the Hall conductivity is an excitation gap in the clean system. In the IQHE this gap is the gap between different Landau levels, and hence of single particle origin, while in the fractional QHE electron correlations lead to a gap at magical filling factors \cite{4}. Since electron-electron interactions are unimportant for the origin of the quantization it is tempting to try to describe the IQHE by a theory of non-interacting electrons. Furthermore, since the quantization is most prominent at high magnetic fields where
the cyclotron energy is much larger than the disorder, a projection onto a single Landau level and the neglect of Landau level mixing seems reasonable. Unfortunately, even such a simplified model still resist analytic approaches. However, it is quite amenable to numerical simulations. The results compare quite favorably to experiments. In particular, they show evidence for a critical point near the center of each disorder broadened Landau band. Near the critical point single parameter scaling holds with a single relevant scaling index, the localization length exponent $\nu \approx 2.35 \pm 0.03$ [3, 8]. In fact, this value agrees with the experimental value of $\nu \approx 2.3 \pm 0.1$ [3]. Simulations show that the dissipative conductivity at the critical point in the lowest Landau band takes on a universal value of $\sigma_{xx} \approx 0.5e^2/h$ [7], in agreement with some experiments [3].

Despite these successes, a serious discrepancy exists between the theory for non-interacting electrons and experiments regarding the value of the dynamical critical exponent $z$. For non-interacting electrons $z$ equals to the space dimension $d = 2$ and the width of the transition region is then expected to scale as a power of an external frequency $\omega$ or temperature $T$ with an exponent $1/z \nu \approx 0.21$. In contrast, experiments show power law behavior, but with an exponent of about $0.42$ [9, 10], compatible with a dynamical critical exponent of $z \approx 1$. Apparently, the non-interacting theory is an inadequate description of the real world and in Section 1 we will discuss how the critical properties of the theory change upon inclusion of the effects of electron-electron interactions.

In addition to the effects of electron-electron interactions one has to consider the effects of a finite strength of the applied magnetic field and in particular the fate of the QHE as the magnetic field is switched off. Field-theoretic arguments suggest that the isolated critical points persist in the presence of Landau level coupling [11], though not necessarily anymore at the centers of the Landau levels. However, for vanishing magnetic field the scaling theory of Anderson localization tells us that all states below the Fermi energy are localized in two dimensions [12]. To reconcile this two results, Khmelnitskii and Laughlin have argued that the critical energies float upwards in energy as the magnetic field turns to zero and eventually move through the Fermi energy so that all states below are localized [13, 14]. In this levitation scenario the critical states move consecutively through the Fermi energy and the last transition to the insulator always happens from the plateau with Hall conductivity $ne^2/h$ with $n = 1$. Recently, this picture has been questioned by experiments that seem to show direct transitions from higher plateaus, $n = 2, \ldots, 6$, to the insulator [15, 16, 17, 18, 19]. As experiments are performed at finite temperatures and hence probe finite length scales and scaling theory deals with diverging length scales, it is important in this context to consider the implications of scaling for finite systems and temperatures. This will be done in Section 2 [20].
1 Electron-electron interactions

1.1 Relevance of interactions

We will use the language of the renormalization group for our discussion of the influence of interactions. In the absence of interactions, the plateau transitions correspond to quantum critical points that are characterized by a set of universal exponents, the localization length exponent $\nu$, the dynamical critical exponent $z$, the spectrum of generalized multifractal dimensions $D(q)$, where the exponent $D(2) = 2 - \eta$ enters the dynamical density correlator [21], as well as the critical conductivity $\sigma_c$ [7]. Numerical evidence suggests that these critical quantities are independent of microscopic details like the Landau level index or the nature of the disorder potential if corrections to scaling are properly taken into account [22].

The results are obtained for models of disordered non-interacting electrons in two dimensions. The finite extent of the electron wavefunction in the direction perpendicular to the two-dimensional plane does not change the scaling behavior as it only modifies the form factor of the disorder as long as only a single subband of the perpendicular motion is occupied. We will now consider the effect of an additional interaction potential

$$V(r) = Ar^{-\lambda}$$

between the electrons. The physical Coulomb interaction corresponds to $y = 1$ and $A = e^2/4\pi\epsilon$. In order to determine the relevance of this perturbation at the non-interacting fixed point of the renormalization group, we calculate the disorder average $\overline{V}$ of the interaction between two consecutive non-interacting eigenstates and compare this to the mean level spacing $\Delta$.

The average interaction $\overline{V}$ is just the disorder average of the Hartree-Fock operator,

$$\overline{V} = \int d^2r P_2(r, \omega \to 0)V(r),$$

$$P_2(r, \omega) = \sum_{i,j} \left( |\psi_i(0)|^2|\psi_j(r)|^2 - \psi_i^*(0)\psi_i(r)\psi_j^*(r)\psi_j(0) \right) \delta(E_i - E_j - \hbar\omega),$$

where $E_i < E_c$ and $E_j > E_c$, the sum runs over all eigenstates of the non-interacting system, and $E_c$ is the critical energy. At the critical point $P_2(r, \omega)$ shows power law scaling,

$$P_2(r, \omega) \propto \left( \frac{r}{L\omega} \right)^{-\tilde{\eta}},$$
with $L_\omega = (\rho_0 \hbar \omega)^{-1/2}$, the effective system size with level spacing $\hbar \omega$. The exponent $\tilde{\eta}$ reflects the multifractal character of the critical eigenstates. Fig. 1 shows numerical results for the scaling of the Fourier transform $P_2(qL_\omega)$ in the limit $q, \omega \to 0$. From the asymptotic power law $P_2(qL_\omega) \propto (qL_\omega)^{-2+\tilde{\eta}}$, we obtain $\tilde{\eta} = -0.5 \pm 0.1$.

The average interaction $\overline{V}$ scales then like

$$\overline{V} \propto A \left( L^{-\lambda} - L^{\tilde{\eta}-2} \right).$$

(1.5)

This energy scale has to be compared to the mean level spacing of the non-interacting system $\Delta = (\rho_0 L^2)^{-1}$. If $\overline{V}$ scales faster to zero than $\Delta$ with increasing system size then the non-interacting eigenstates are essentially unaffected by the interaction and the interaction is irrelevant, while in the opposite case increasingly more eigenstates get coupled with increasing system size and the interaction is relevant in the renormalization group sense. Defining the scaling index $x$ of the interaction by

$$\frac{\overline{V}}{\Delta} \propto L^x,$$

(1.6)

we see that for $\lambda > 2 - \tilde{\eta}$ the interactions are irrelevant with a scaling index $x = \tilde{\eta}$ (Fig. 2). For $2 < \lambda < 2 - \tilde{\eta}$ the interactions are still irrelevant but now with the range dependent scaling index $x = 2 - \lambda$. In both of these cases the fixed point structure is not changed and the critical exponents take on their non-interacting values. However, even these irrelevant interactions lead to a finite dissipative conductivity at finite temperatures in contrast to the non-interacting system [23]. For $\lambda < 2$ and hence for Coulomb interactions the interactions are
relevant with scaling index \( x = 2 - \lambda \) and the system is driven away from the non-interacting fixed point towards a new interacting fixed point [24].

### 1.2 Scaling for Coulomb interactions

Unfortunately, the linear stability analysis of the previous section does not tell us what this new fixed point looks like, only that infinitesimally weak interactions are sufficient to take us there. It is known that for sufficiently strong interactions the QH system develops new correlated ground states associated with the emergence of the fractional QHE [4]. At these fixed points the interaction strength is much larger than the disorder. These are not the fixed points that we are concerned with here. We want to know the ground state of the system for interaction strength much smaller than the disorder strength.

In order to study the nature of the new fixed point a numerical approach was chosen. Ideally, an exact method should be used but in the presence of both interactions and disorder exact diagonalization studies are limited to system sizes far too small to allow for any finite-size scaling analysis to extract critical exponents. We therefore treat the disorder exactly and incorporate the Coulomb interactions in a self-consistent Hartree-Fock approximation (HFA). The first calculation of this kind by Yang and MacDonald focused on the tunneling density of states (DoS) and found a linear Coulomb gap at all filling factors of the lowest Landau band, even at the critical energy [25]. In particular, it was found that the DoS \( \rho(E_F, L) \) at the Fermi energy in a system of linear dimension \( L \) scales as

\[
\rho(E_F, L) \propto \frac{1}{\gamma L},
\]

where \( \gamma = (e^2/4\pi\ell)/\Gamma \) is a measure of the interaction strength relative to the disorder strength \( \Gamma \). Note that this suppression of the DoS is basically of classical origin and not related to the quantum Hall critical point [21].
This system size dependence of the DoS is in contrast to the size independent DoS of the non-interacting system and offers an explanation for a change of the dynamical critical exponent $z$ from 2 to 1. The dynamical exponent relates energy and length scales, $E \propto L^{-z}$, and in a non-interacting system this relation is given by the DoS. The corresponding energy and length scales are the mean level spacing $\Delta$ and the system size $L$, respectively,

$$\Delta = \frac{1}{\rho L^d},$$

(1.8)

i.e., $z = d = 2$. With interactions, however, the DoS is no longer constant but scales like $1/L$ so that the dynamical exponent is reduced to $z = d - 1 = 1$. The suppression of the DoS is unrelated to the critical point and happens for all values of the Fermi energy, but when the Fermi energy coincides with the critical energy it changes the critical exponent $z$.

It remains to be checked whether this simple argument holds and whether other critical quantities are also changed by the interactions or not. Using the self-consistent Hartree-Fock eigenvalues and eigenfunctions, Yang, MacDonald, and Huckestein studied the scaling behavior of the participation ratio and compared the Thouless numbers of interacting and non-interacting systems [27]. The size and energy dependence of the participation ratio could be fitted by the same exponents $\nu$ and $D(2)$ that were obtained in the non-interacting system. The Thouless numbers, that are in a non-interacting system a measure of the conductance, also remained unchanged upon inclusion of the interactions, although the relation between the Thouless numbers and the conductance is unknown for an interacting system.

In order to directly obtain the conductivity and the dynamical scaling one has to go beyond the self-consistent HFA and include vertex corrections within the time-dependent Hartree-Fock approximation (TDHF). This is the corresponding conserving approximation and while the quantities calculated in the self-consistent HFA are single-particle properties, the TDHF allows to calculate true two-particle properties like the polarization and the conductivity. A finite-size scaling analysis shows that in the limit of vanishing frequency $\omega$ and wavevector $q$ the irreducible polarization $\Pi(q, \omega)$ scales like

$$\Pi(q, \omega) = \chi(q) \frac{\sigma^*(x)x}{\sigma^*(x)x - ie^2/\hbar},$$

(1.9)

with $x = q^2/\chi(q)\hbar\omega$. The static susceptibility $\chi(q)$ vanishes like $q$ for small $q$, even when the vertex corrections are included. Hence the scaling variable $x \propto q/\omega$ and the dynamical critical exponent $z$ is indeed unity. Despite the vanishing static susceptibility, the conductivity $\sigma^*(x) = e^2\chi(q)D(q, \omega)$ is finite even in the DC limit as the diffusion coefficient $D$ exhibits super-diffusive behavior and diverges in this limit. Fig. 3 shows the dynamical conductivity $\sigma^*$ as a function
Scaling in the IQHE

The DC conductivity obtained in the limit $x \to 0$ is independent of the strength of the interaction and is given by $\sigma_c = 0.5 \pm 0.1 e^2/h$. The scaling in the opposite limit $x \to \infty$, $\sigma^* \propto x^{1-\eta}$, is governed by multifractal correlation and allows to extract the anomalous diffusion exponent $\eta = 0.4 \pm 0.1$. Both of these values agree within the uncertainties with their non-interacting counterparts.

The numerical calculations are consistent with the picture that the interacting integer QH system is rather peculiar. The Coulomb interaction is relevant at the non-interacting fixed point but drives the system to a new fixed point, the Hartree-Fock fixed point, that differs from the old one only by the occurrence of the Coulomb gap and the associated reduction in the dynamical critical exponent $z$. Whether this picture is correct or due to the failure of the time-dependent Hartree-Fock approximation could be decided by numerically calculating the scaling dimension of the residual interactions at the HF fixed point, a rather daunting task that has not been attacked so far.

2 Fate of the QHE at low magnetic fields

The behavior of the critical energies, the conductivities, and the resistivities as expected from the scaling theory of the IQHE is sketched in Fig. 4. At high magnetic fields, the critical energies are located close to the Landau energies. At each of the critical points, the Hall conductivity changes by exactly 1 (from here on, we measure all conductivities in units of $e^2/h$). As no extended states exist
below the Fermi energy at zero magnetic field, the critical states can only move up in energy as the magnetic field goes to zero [13, 14]. At fixed Fermi energy or particle number, the sequence of Hall conductivities exhibited by the system as the magnetic field is lowered is thus 0–1–2–…–N–…–2–1–0. In particular, the transition from the last quantized plateau to the insulator at low fields can only happen from the $n = 1$ plateau.

It should be noted that the situation is fundamentally different in lattice models of the QHE. While for weak disorder and strong magnetic fields the low-lying magnetic subbands of a tight-binding band are faithful approximations of the Landau bands of the continuum, lattices models show completely different behavior at strong disorder/weak magnetic fields due to the presence of states with negative Hall conductivity that are absent in the continuum. These states can combine with states carrying positive Hall conductivity and alter the topology of the phase diagram in Fig. 4.

Since the dissipative conductivity is only finite at the plateau transitions, the Hall resistivity exhibits as non-monotonic series of plateaus, with the Hall resistivity increasing with decreasing magnetic field below the $1/N$ plateau. This behavior predicted by the scaling theory is in stark contrast to the experimental situation, where only monotonously increasing Hall resistivities are observed at low magnetic fields. Even more disturbingly, a series of experiments have observed a temperature independent point with insulating behavior for lower magnetic fields and emerging quantized plateau with $n > 1$ for higher mag-
Figure 5  Schematic flow diagram for the IQHE. The arrows indicate increasing length scale and the dashed curve represents the SCBA result.

netic fields [15, 16, 17, 18, 19]. Obviously, if this temperature independent point signals the presence of a quantum critical point at zero temperature, then the experiments are at odds with the phase diagram of Fig. 4.

However, the observed linear, classical behavior of the Hall resistivity at low fields gives us a hint how to properly interpret the experimental data. First, we have to realize that the results of scaling theory, such as the phase diagram of Fig. 4, deal with the behavior of the system at zero temperature on asymptotically large length scales. Experiments, on the other hand, are performed at finite temperatures and a finite temperature introduces a finite length scale $L_\Phi$, the phase coherence length. The question, whether or not an experiment performed at a certain temperature reflects the asymptotic scaling regime is thus a question of length scales.

How the conductivities change with length scale is determined by the renormalization group $\beta$-functions of the system. The exact form of these functions is not known for the QHE, only an approximation for large longitudinal conductivity $\sigma_{xx}$ is given in the literature [20]. The structure of the resulting flow diagram is given in Fig. 5 [31]. The flow starts at high temperatures, corresponding to short length scales, with the semiclassical values of the conductivity. The physics of the unstable QH fixed points at half-integer $\sigma_{xy}$ becomes apparent at length scales such that the longitudinal conductivity is of the order of $1/2$.

At high temperatures, the conductivities are given by the Drude expressions

$$\sigma_{xx}^0 = \frac{\sigma_0}{1 + (\omega_c \tau)^2},$$  
(2.10)

$$\sigma_{xy}^0 = \omega_c \tau \sigma_{xx}^0.$$  
(2.11)
Figure 6 Conductivities on short length scales: the Drude results (solid lines and dotted line) and the SCBA result for $\sigma_{xx}$ (dashed), appropriate for $\omega_c \tau \gg 1$.

with $\sigma_0 = e^2 n_c \tau / m^*$, $\omega_c = eB / m^*$, and $n_c$, $\tau = \ell / v_F$, and $\ell$ are the carrier density, transport time, and the elastic mean free path, respectively (Fig. 6). The resulting linear increase of the Hall resistivity is observed in all experiments. The change of the longitudinal conductivity $\sigma_{xx}$ due to quantum interference is given to lowest order in $1/\sigma_{xx}^0$ by the unitary $\beta$-function [32, 33]

\[
\sigma_{xx}(L) = \sigma_{xx}^0 - \frac{1}{\pi^2 \sigma_{xx}^0} \log \left( \frac{L}{\ell_c} \right).
\]  

(2.12)

While this expression is only valid for large $\sigma_{xx}$, it can provide a rough estimate of the length scale at which the weak localization corrections become comparable to the bare value $\sigma_{xx}^0$, at which point the influence of the QH fixed point becomes dominant. Integrating eq. (2.12), we get the crossover length scale

\[
\xi_0 = \ell_c \exp(\pi^2 \sigma_{xx}^0)^2).
\]  

(2.13)

We note the strong dependence of the crossover scale on the bare conductivity $\sigma_{xx}^0$. In order to get a handle on the possible values of $\sigma_{xx}^0$, we resort to Laughlin, who argued that the critical energies are given by the condition, that the Drude $\sigma_{xy}^0 = (n + 1/2)$ [4], corresponding to the Landau energies at high magnetic fields. In order to observe the $N$-th Hall plateau, the maximum of $\sigma_{xy}^0$ at $\omega_c \tau = 1$ needs to be larger than $N - 1/2$. Consequently, for magnetic fields up to $\omega_c \tau = 1$ the bare longitudinal conductivity $\sigma_{xx}^0$ exceeds $N - 1/2$. In the experiments that claim to observe direct transitions from higher plateaus, $N$ is at least 2. The corresponding crossover length scale $\xi_0$ is then at least several $10^9 \ell_c$, a macroscopic length, orders of magnitudes larger than any phase coherence length achievable experimentally. We thus conclude, that at low fields $\omega_c \tau < 1$ the attainable length scales are too short to observe the QHE, even if it is present in an infinite system. Instead, classical behavior with weak localization corrections is expected, in agreement with experiment.
So far, our discussion does not explain the experimentally observed strong change in the temperature dependence of the resistivities near $\omega_c \tau = 1$. Instead of the weakly insulating temperature dependence observed at low magnetic fields, at higher magnetic fields the onset of plateau quantization is seen. In order to understand this, we need to take into account the quantizing effect of the magnetic field. For $\omega_c \tau > 1$, the disorder broadening of the Landau levels becomes smaller than their separation, leading to minima in the density of states. These minima are accompanied by minima in the bare conductivity $\sigma_{xx}^0$. For $\omega_c \tau \gg 1$, the Drude result is no longer valid and the bare conductivity is given by the self-consistent Born approximation \[34\]. At integer filling factors the bare conductivity becomes small and the crossover length scale becomes microscopic, allowing the observation of QHE.

From these arguments, we expect an almost temperature independent point near $\omega_c \tau = 1$ separating a weakly temperature dependent, insulating regime at lower magnetic fields from a regime of stronger temperature dependence with emerging quantized Hall plateaus, in agreement with the experimental observations.

The present discussion leaves a lot of room for improvements. First, we use only approximate expressions for the bare conductivities. Next, the $\beta$-functions should be calculated all the way from large conductivities to the QH fixed points. While improvements in both of these areas are highly desirable, they are also hard to come by as the combined effects of finite magnetic field, disorder and interactions need to be taken into account. We should further distinguish between transport and scattering times, but since the experiments are performed on low mobility samples, there is not much difference. In spite of the limitations, we believe that our discussion captures the essential features of the physical situation. Finally, we want to point out that the whole discussion applies to samples that are actually insulating at zero magnetic field. It certainly does not apply to samples that are apparently metallic at zero field.

3 Conclusions

The scaling theory of the plateau transitions in the integer quantum Hall effect presents a picture that agrees with a wide range of experimental findings. However, when analyzing experimental or numerical data, it is imperative to consider the characteristic length scales in the system. This need becomes most prominent at low magnetic fields, where the emergence of an enormous crossover length scale prohibits the experimental observation of the QHE.

Coulomb interactions appear to play a peculiar role in the IQHE. Neglecting them, one obtains a theory that correctly reproduces the observed plateaus and
even most characteristic features of the transitions. However, the Coulomb interaction is a relevant perturbation at the non-interacting fixed point and the non-interacting theory is not the correct scaling theory. Incorporating the effects of the Coulomb interaction within a self-consistent time-dependent Hartree-Fock approximation, we find that only the dynamical critical exponent $z$ changes from 2 to 1, a change that can be traced to the occurrence of a Coulomb gap in the tunneling density of states.

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Bibliography

[1] K. von Klitzing, G. Dorda, M. Pepper, Phys. Rev. Lett. 45, 494 (1980).
[2] B. Huckestein, Rev. Mod. Phys. 67, 357 (1995).
[3] M. R. Zirnbauer, hep-th/9905054.
[4] R. Laughlin, Phys. Rev. Lett. 50, 1395 (1983).
[5] B. Huckestein, B. Kramer, Phys. Rev. Lett. 64, 1437 (1990).
[6] S. Koch, R. J. Haug, K. von Klitzing, K. Ploog, Phys. Rev. Lett. 67, 883 (1991).
[7] Y. Huo, R. E. Hetzel, R. N. Bhatt, Phys. Rev. Lett. 70, 481 (1993).
[8] D. Shahar et al., Phys. Rev. Lett. 74, 4511 (1995).
[9] H. P. Wei, D. C. Tsui, M. A. Paalanen, A. M. M. Pruisken, Phys. Rev. Lett. 61, 1294 (1988).
[10] L. W. Engel, D. Shahar, Ç. Kurdak, D. C. Tsui, Phys. Rev. Lett. 71, 2638 (1993).
[11] H. Levine, S. B. Libby, A. M. M. Pruisken, Phys. Rev. Lett. 51, 1915 (1983).
[12] E. Abrahams, P. W. Anderson, D. C. Liciardello, V. Ramakrishnan, Phys. Rev. Lett. 42, 673 (1979).
[13] D. Khmelnitskii, Phys. Lett. 106A, 182 (1984).
[14] R. Laughlin, Phys. Rev. Lett. 52, 2304 (1984).
[15] A. Shashkin, G. Kravchenko, V. Dolgopolov, Pis’ma Zh. Eksp. Teor. Fiz. 215 (1993), [JETP Lett. 58, 220 (1993)].
[16] S. Kravchenko, W. Mason, J. Furneaux, V. Pudalov, Phys. Rev. Lett. 75, 910 (1995).
[17] S.-H. Song et al., Phys. Rev. Lett. 78, 2200 (1997).
[18] C. Lee, Y. Chang, Y. Suen, H. Lin, Phys. Rev. B 58, 10629 (1998).
[19] M. Hilke et al., cond-mat/9906212.
[20] B. Huckestein, Phys. Rev. Lett. 84, (2000), cond-mat/9906450.
[21] J. T. Chalker, G. J. Daniell, Phys. Rev. Lett. 61, 593 (1988).
[22] B. Huckestein, Phys. Rev. Lett. 72, 1080 (1994).
[23] Z. Wang, M. P. Fisher, S. Girvin, J. Chalker, Phys. Rev. B 61, 8326 (2000).
[24] D.-H. Lee, Z. Wang, Phys. Rev. Lett. 76, 4014 (1996).
[25] S.-R. E. Yang, A. H. MacDonald, Phys. Rev. Lett. 70, 4110 (1993).
[26] A. Efros, B. Shklovskii, J. Phys. C 8, L49 (1975).
[27] S.-R. E. Yang, A. H. MacDonald, B. Huckestein, Phys. Rev. Lett. 74, 3229 (1995).
[28] B. Huckestein, M. Backhaus, Phys. Rev. Lett. 82, 5100 (1999).
[29] D. Polyakov, K. Samokhin, Phys. Rev. Lett. 80, 1509 (1998).
[30] A. M. M. Pruisken, in Field Theory, Scaling and the Localization Problem, Graduate Texts in Contemporary Physics, edited by R. E. Prange and S. M. Girvin (Springer, Berlin, 1987), Chap. 5, pp. 117–173.
[31] D. E. Khmel’nitskii, Pis’ma Zh. Eksp. Teor. Fiz. 38, 454 (1983), [JETP Lett. 38, 552 (1984)].
[32] S. Hikami, Phys. Rev. B 24, 2671 (1981).
[33] K. B. Efetov, Adv. Phys. 32, 53 (1983).
[34] T. Ando, Y. Uemura, J. Phys. Soc. Jpn. 36, 959 (1974).