On the Aggregate Interference in Random CSMA/CA Networks

June Hwang, Riku Jäntti and Seong-Lyun Kim

Abstract—In this paper, we investigate the cumulative distribution function (CDF) of the aggregate interference in carrier sensing multiple access/collision avoidance (CSMA/CA) networks measured at an arbitrary time and position. We assume that nodes are deployed in an infinite two-dimensional plane by Poisson point process (PPP) and the channel model follows the singular path loss function and Rayleigh fading. To find the effective active node density we analyze the distributed coordinate function (DCF) dynamics in a common sensing area and obtain the steady-state power distribution within a spatial disk of radius $R/2$, where $R$ is the effective carrier sensing distance. The results of massive simulation using Network Simulator-2 (NS-2) show a high correlation with the derived CDF.

Index Terms—Aggregate interference, CSMA/CA, DCF, Poisson point process, NS-2

I. INTRODUCTION

A. Motivations

Due to the inherent scarcity of frequency spectrum and increasing wireless traffic demands nowadays, frequency reuse has become an essential key technological issue associated with contemporary wireless communication systems. Frequency reuse intrinsically causes interference between wireless links in not only homogeneous but also heterogeneous systems using the same frequency. Accordingly, the state of the aggregate interference at an arbitrary position in the random node topology has become of great importance.

In this paper, we are interested in the interference of carrier sense multiple access/collision avoidance (CSMA/CA) networks. In particular, we analyze the aggregate interference in randomly deployed IEEE 802.11 distributed coordination function (DCF) networks. From an understanding of the distribution of the aggregate interference at the protocol level, we can control this interference using the relationships discovered among the protocol parameters. To the best of our knowledge, there has been no massive test at the simulator level for the aggregate interference of CSMA/CA networks. Thus, we test as well as analyze the interference at the protocol level. Consequently, our goal in this study is to obtain the statistical inference of the aggregate interference and verify the results via simulations.

Most of the work previously done in this area focused on ALOHA-like systems in which the aggregate interference can be analyzed by assuming that transmitting nodes have independent locations and behaviors [1], [2]. Although broadband cellular systems such as LTE, LTE-A, WCDMA or its femto cell networks can also be modeled using this transmission-independent behavior, this is not a realistic assumption for CSMA/CA networks in which a certain interference level always needs to be maintained in a distributed manner. In a network of CSMA/CA nodes, every communication entity first senses the ongoing transmission in the channel and then determines when to start transmitting. The inappropriateness of the independent model in such a scenario was noted in [3], in which the authors proposed the alternative dependent point process to mimic real CSMA/CA networks. However, this proposed point process still cannot describe the DCF operation, in which collision and idle time, as well as successful transmission, can occur even in the exclusion area by carrier sense. Compared with these previous research efforts, our work is the first to investigate the exact distribution of aggregate interference in CSMA/CA networks and to validate it by means of massive simulations.

B. Summary of contributions and organization of this paper

Our paper has the following notable results:

- Owing to the possibility of concurrent transmission incurred by DCF operation occurring within an exclusion area, CSMA/CA random networks can only be modeled by the Poisson point process (PPP), not the dependent point process. Section III addresses the difference between the dependent and independent point processes, and explains how real CSMA/CA networks can be dealt with.

- We derive the effective node density reflecting CSMA/CA MAC layer operation. Section III explains this. This section is the core of our work since all the cross-layer parameters are coupled to model the network behavior. Further, this is used in PPP shot noise analysis, which is the result of the first item.

- Aggregate interference in PPP shot noise analysis using our derived effective node density is verified using the NS-2 network simulator and MATLAB simulations. Unlike other related theoretical analyses of stochastic geometry researches, we verify exactly how much our model reflects the NS-2 simulation results. We also compare our results with the MATLAB simulation on dependent point processes and thereby show that the PPP model with our new effective node density performs the best in modeling the aggregate interference. Related issues are outlined in Section IV.
The aggregate interference is either normal nor log-normal distributions, as contrary to the most common assumption that the aggregate interference follows normal (in dBm unit) or log-normal (in W unit) distributions.

II. POINT PROCESS FOR MODELING RANDOM CSMA/CA NETWORKS

In this section, we focus on determining which type of point process is suitable for modeling CSMA/CA networks. A generic wireless network consisting of multiple randomly deployed nodes can be described via a point process. In the point process, a mark (a scalar or a vector) can be assigned to each point independently, which is useful for modeling node-oriented properties such as transmission power, medium access delay. In particular, the case where the number of nodes in a network is Poisson-distributed and their positions at a given time instant are independent of each other, is adequately explained by means of the PPP. The method to derive the aggregate power emitted from points at an arbitrary position under independent marked PPP was previously studied as shot noise field, which was originally used to model the noise in electronic circuits in the time domain.

A. Inappropriateness of PPP for CSMA/CA modeling

However, the PPP approach as it currently exists may be insufficient to model the CSMA/CA. The reason is that the carrier sensing philosophy is not reflected in it. PPP is a typical independent point process in which the points are deployed independently of each other. On the other hand, in the carrier sensing operation, a sensing node always senses the shared medium and it delays its transmission once it senses that the medium is busy. The result is that active nodes are affected by each other, which means that the process is not independent.

B. Inappropriateness of the dependent point process

Let us now consider the dependent point process as a possible alternative. Here, the dependent point process means that some initially deployed points are discarded or selected by the metric related to the other points’ marks or locations. There are two dependent point processes most related to the modeling of CSMA/CA networks [1], [2]: the Matern hardcore (MHC) process and the simple sequential inhibition (SSI) point process. In [3], the authors are also motivated by the inappropriateness of PPP, i.e., the independence of points. In the paper, they compared the aggregate power distributions of PPP, MHC, and SSI with simulations, and concluded that SSI is the most appropriate one for modeling CSMA networks. However, their result is not fully acceptable because the operation they used in the simulation was not the real one but a modified version of a dependent point process based on IEEE 802.15.4 PHY parameters. More specifically, they neither considered any details of practical MAC layer parameters nor channel characteristics. Leaving this result aside, their MHC and SSI are fundamentally based on the hard exclusion area; nevertheless, they contain ambiguities in the determination of this area. To further illustrate the issue, let us look at our simulation results.

We simulated a realistic CSMA/CA network using NS-2 in order to observe the concurrent transmission behavior. Fig. 1 shows our simulation topology and the effect on the distribution of the number of concurrent transmitting nodes. In the grid topology, the black dot represents the transmitter and the corresponding receiver is located at 5 (m) right and 5 (m) up away from its transmitter; the receiver is omitted from the figure. In the simulation, a 500 B payload was given to each transmitter and the traffic was saturated. The distance between the two nearest black dots was 50 (m), and the carrier sensing (CS) threshold was tuned so that the resulting CS range was 70 (m), derived using Equation (6). All other parameters were the same as those in Table I. The large circle including set C nodes in the center denotes a CS area of the white dot (one of the transmitters), while the smaller circle including set B nodes has a radius that is one half of the CS range. We depict the relative time durations on the number of concurrent transmissions in each set in the bar plot.

In the graph, two things are of note: First, there is a time period in which two nodes are concurrently transmitting when all transmitters are even in each other’s CS area (see the Set B in the bar plot). Second, there is a time period in which the medium is idle in the full CS area (Set C). The first case occurs due to CS failure or collision in real situation. MHC and SSI are totally dependent thinning of PPP with the exclusion area, and they cannot model these events. The resulting effective node density of the dependent point process is likely to be lower than that of the real one. These approaches may work well in collision-less CSMA/CA networks where slot time is zero and backoff time is a continuous random variable (RV) rather than real situations. The second case occurs due to the idle time from the binary exponential backoff (BEB) and the DCF. This waste of time resource is the intrinsic cost of the distributed random access MAC. In the dependent point process, any point having none of the other points in its exclusion area always survives. The resulting effective node density of this process is likely to be higher than that of the real one. As a result, the real operation of a CSMA/CA network has both factors having higher and lower effective node density than that of the dependent point process. This difference is from the lack of MAC layer operation modeling in the dependent point process.
C. Revisit of PPP with a New Density

These collision events occur with a certain probability in real situations. This means that the concurrent transmission in an exclusion area occurs with some probability, not with deterministic patterns. This stochastic characteristic of real networks is appropriately modeled using the independent point process. Therefore, we believe that a possible way to model a CSMA/CA network is again to use the independent PPP, but with a new effective active node density reflecting MAC layer operations.

This is notable since recent research efforts such as [2] and literature therein point out that CSMA/CA can be modeled with MHC point process, which is different from our conclusion. At the very least, the aggregate power at an arbitrary position can be elaborated more when using PPP with a new effective node density rather than pure PPP or MHC/SSI.

For our analysis, we consider the infinite planar where the transmitting nodes are deployed randomly at positions specified by a Poisson distribution with its intensity $\lambda$. Each node transmits with a constant power $p$. The radio channel attenuates with the pass-loss exponent $\alpha = 4$ and Rayleigh fading. Then we have CDF and PDF of the aggregate interference $I$ at an arbitrary receiver as follows [5], [6]:

$$F_{I}(t) = \text{erfc} \left( \frac{\lambda \pi^2 \sqrt{p}}{4\sqrt{t}} \right),$$  \hspace{1cm} (1)

$$f_{I}(t) = \frac{\lambda^2 \pi^2 \gamma p}{4 \sqrt{t} \gamma \pi^3 \lambda^3} P^2,$$  \hspace{1cm} (2)

where \( \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} e^{-t^2} dt \) is a complementary error function.

Our idea is to use the above PDF and CDF again for calculating the aggregate interference of the CSMA/CA network, but with a new density $\lambda'$, called **effective active node density** reflecting all the CSMA parameters. Section III is devoted to describing how we obtain $\lambda'$, and its verification by massive NS-2 simulations is contained in Section IV. For the readers who are more interested in our results, please directly jump to Section IV.

III. Effective Active Node Density

A. Effective carrier sensing range and sharing area

Let us introduce a CS range $R$ so that a sensing node can sense any on-going transmission in this range. Then within the disk of radius $R/2$, every node senses each other. We set this disk as **sharing area**.

CS is based on the threshold $\gamma$, i.e., if the sensed power level at a sensing node is greater or lower than $\gamma$, a sensing node regards the channel is busy or idle, respectively. Assuming there is only one interferer near the sensing node, the CS probability versus the distance to this interferer is calculated as followings:

$$P[\text{Channel is busy}] = P[P_i + \nu \geq \gamma],$$  \hspace{1cm} (3)

where $P_i$ is a random variable (RV) representing the product of the fading effect and the constant transmission power from a typical node $i$, $r$ is the distance between the sensing node and the interferer, and $\nu$ is the receiver noise power. Considering Rayleigh fading, $P_i$ follows \( \text{Exp}(1/p) \) with a constant transmission power $p$.

Consequently, with the CS range $R$, we convert the stochastic CS to a deterministic one. First, the average sensing area is calculated by integrating the parts of the circumference, of which the radius and the center are $r$ and the sensing node, respectively. The CS probability of a point on this circumference is from (3):

$$\int_{0}^{\infty} 2\pi r \cdot P[P_i^2 + \nu \geq \gamma] dr = \int_{0}^{\infty} 2\pi r e^{-\frac{1}{r}(\gamma-\nu)r^4} dr = \frac{\pi^{3/2}}{2 \sqrt{\frac{2-\nu}{p}}}$$  \hspace{1cm} (4)

Assuming that the deterministic CS region should have the same average sensing area (sensing resolution) as the stochastic CS, the following equation is derived:

$$\frac{\pi^{3/2}}{2 \sqrt{\frac{2-\nu}{p}}} = \pi R^2$$  \hspace{1cm} (5)

Finally, we get the CS distance $R$ as follows:

$$R = \frac{1}{\sqrt{2} \left( \frac{\pi p}{\gamma - \nu} \right)^{1/4}}$$  \hspace{1cm} (6)

By this deterministic CS distance, which we will call **effective carrier sensing range**, the interference is regarded as boolean at a given distance rather than stochastic.

Let us consider an infinite plane with the nodes randomly deployed. Suppose an arbitrary disk having of radius $R/2$ in the plane (sharing area), where every nodes in this area can sense other nodes’ transmissions by the definition of CS range $R$. By PPP, the number of deployed nodes in the sharing area follows Poisson distribution with the parameter $\lambda \pi (R/2)^2$ as follows:

$$P[N = n] = \frac{(\lambda \pi (R/2)^2)^n}{n!} \exp(-\lambda \pi (R/2)^2)$$  \hspace{1cm} (7)

Once we know $P[N = n]$, we will derive the probability on the number of active nodes, $N_a$ in the sharing area $\mathbb{P}[N_a = a|N = n]$ and the power distribution at an arbitrary time instant in the sharing area. These will be explained in the following subsections. For the purpose, let us define the following probability:

**Definition 1.** $p_{on}$ is the probability that there are on-going transmissions in a given sharing area at a certain time.

B. Steady-state power distribution in a sharing area

1) DCF dynamics in a sharing area: Consider the IEEE 802.11 DCF protocol for CSMA/CA MAC. If all the transmitting nodes can sense each other in a sharing area and the given traffic to each node is a saturated one, we know the steady-state behavior in this area. As shown in [7] and
subsequent research efforts, the backoff stage of each node in the network is random at a certain time and this can be elaborated through a two-dimensional Markov chain. We have two main quantities for addressing this: $p_c$ is the probability that the collision happens conditioned on the transmission of each node, and $\tau$ is the transmission probability of a node at a randomly chosen time slot. These two quantities are derived by finding the steady-state solution of the discrete time Markov chain.

By following the notations of [6], we have the BEB dynamics with $m$ maximum backoff stage, $K (\geq m + 2)$ maximum retry limit and $W_0$ initial window size. The probability $\tau$ that a node transmits in a randomly chosen time slot is:

$$\tau = \left\{ \frac{(1 - p_c)W_0(1 - (2p_c)^m)}{2(1 - p_c^m)(1 - 2p_c)} + \frac{2^mW_0(1 - p_c^m)}{2(1 - p_c^m)} - \frac{1}{2} \right\}^{-1} \tag{9}$$

Again, $p_c$ is obtained through:

$$p_c = 1 - (1 - \tau)^N_a^{-1}, \tag{10}$$

where $N_a$ is the number of active (contending) nodes. We can solve the system dynamics by solving two independent Equations (8) and (9) and the existence of this solution is guaranteed by the fixed point theorem [7].

2) Power distribution in a sharing area: Since we know $\tau$, we can obtain the steady state power density. The probability that $i$ nodes transmit simultaneously at an arbitrary time slot, given that $N_a$ transmitting nodes are deployed in a sharing area is:

$$p_a(m) = \mathbb{P}[i = m|N_a = a] = \binom{a}{m} \tau^m(1 - \tau)^{a-m}, \quad m = 0, \ldots, a. \tag{10}$$

Each transmitting node’s operation in a sharing area is synchronized since the medium is sensed perfectly and every node has the inner clock. Thus, idle time is also segmented into multiple slot times ($\sigma$). Therefore, all events (idle time slot, successful and collision time slot) can be distinguished by their durations. At an arbitrary time, the sharing area medium is in one of three events and we call this random time slot as the virtual time slot. The virtual time slot has the random duration $T_v$. We assume that the payload size is the same as PAY for all nodes. In basic mode,

$$T_v = \begin{cases} \sigma, & \text{for the idle slot time}, \\ T_s^{BASE}(= PHY + \lfloor MAC + PAY \rfloor R_s) + di f s, & \text{for the successful slot time}, \\ T_c^{BASE}(= PHY + \lfloor MAC + PAY \rfloor R_s), & \text{for the collision slot time}, \end{cases}$$

where $PHY, si f s, ACK, di f s$ are the durations for PHY header, SIFS time, ACK packet, DIFS time respectively. And $MAC, R_s$ and $T_s$ are the MAC header size, symbol rate and symbol duration respectively. Once the transmission starts, irrespective of success or not, the packet of size $PHY + \lfloor MAC + PAY \rfloor R_s$ is transmitted first. And then the remaining parts ($si f s + ACK + di f s$ or $di f s$) are determined according to the existence of collision. Of course, in RTS-CTS mode, successful slot time and collision slot time will be changed $T_s^{RTS}(= RT S + CT S + PHY + \lfloor MAC + PAY \rfloor R_s)$ and $T_c^{RTS}(= RT S + di f s)$ respectively. $RT S$ and $CT S$ are the durations of RTS and CTS packet, respectively.

$T_v$ has the PMF induced from (10) such as $p_a(0), p_a(1)$, and $1 - p_a(0) - p_a(1)$ are for idle, successful transmission, and collision events, respectively. We derive the mean virtual time slot, $\mathbb{E}[T_v]$ using this PMF in each mode.

$$\mathbb{E}[T_v^{BASE}] = \sigma p_a(0) + T_s^{BASE} p_a(1) + T_c^{BASE} (1 - p_a(0) - p_a(1)),$$

$$\mathbb{E}[T_v^{RTS}] = \sigma p_a(0) + T_s^{RTS} p_a(1) + T_c^{RTS} (1 - p_a(0) - p_a(1)).$$

The distribution of the number of concurrent transmissions (which is also the power distribution) is based on this PMF. In each virtual slot, the number of concurrent transmissions is various from 0 to $N_a$. And even when the number of concurrent transmitting nodes are given, the virtual slot consists of nobody transmission portion and somebody transmission portion. In the basic mode, nobody transmits during $\sigma$. During $si f s$ and $di f s$ in the both of successful and collision slot time, nobody transmits, too. During packet transmission time ($PHY + \lfloor MAC + PAY \rfloor R_s$ for MAC in a successful slot, one node transmits, while multiple nodes transmit during $PHY + \lfloor MAC + PAY \rfloor R_s$ in a collision slot. In RTS-CTS mode, the power density is changed in the same manner. Therefore, the actual power distribution, $\mathbb{P}[j \text{ nodes transmit}, N_a = a]$ is obtained using this distribution in the basic mode (Equation (11)) and RTS mode (Equation (12)). The probability of busy channel in a sharing area $p_{on}$ is $\sum_{m=1}^{a} B_a^{BASE/CTS}(j) = 1 - B_a^{BASE/RTS}(0)$.

C. Number of active nodes in a sharing area

Consider a given sharing area $H$. Let sensing area be defined as the sensing node’s CS area excluding $H$. (See the asymmetric donut in Fig. 2 for the relation between the sensing and sharing areas.) The active node is defined here as the sensing node that has no on-going transmissions in its sensing area. The CS results for each sensing node in $H$ is random. Therefore, $N_a$ is a RV that varies within $[0, N]$.

For a sensing node in $H$ to be active, the CS result sensed from its sensing area must be idle and that sensed from $H$ must also be idle. Since the areas of overlapped regions among sensing areas are varied due to the different locations of the sensing nodes in $H$ and the activities of nodes in those overlapped regions affect multiple corresponding sensing nodes simultaneously, the CS result of a typical sensing node in $H$ cannot be modeled independently.

By accounting for the combinations of the activities of each quantized area and the possible sensing node locations, we can find the distribution of $N_a$ in $H$ as in (13) when the probability
We can get the value of $\pi_\tau$ in Definition 1, is given:

$$\mathbb{P}[N_a = a|N = n] = \sum_{\eta=0}^{8} P_{n,a,\eta} p_\eta,$$

where

$$P_{n,a,\eta} = \binom{n}{a} \left(\frac{\eta}{8}\right)^a \left(1 - \frac{\eta}{8}\right)^{n-a}.$$ 

and

$$p_\eta = \sum_{D=0}^{8} O_{\eta}^D p_{\text{on}}^D (1 - p_{\text{on}})^{8-D}. \tag{14}$$

Description on detailed derivation of (13) is necessary but due to the space limitation, we only report the results.

### D. Effective active node density

So far, we introduced a sharing area; using the spatial boundary of the boolean CS algorithm and derived the number of nodes distribution therein, which is $\mathbb{P}[N = n]$ as in (7), through Section III-A. The transmission probability of a node $\tau$ is derived in Section III-B1. Based on these derivations, the power distributions in the sharing area, (11) and (12) can be calculated in Section III-B2. We derived the number of active nodes in a sharing area, $\mathbb{P}[N_a = a|N = n]$ as in (13) in Section III-C. As seen from (14), these results are all based on the value of $p_{\text{on}}$, the probability that the sharing area is busy. We can get the value of $p_{\text{on}}$, by finding the intersection of the right hand side and left hand side of (13), which we will call $p_{\text{on}}^*$. Details on derivation of $p_{\text{on}}^*$ are given in Appendix.

If we obtain $p_{\text{on}}^*$, the distribution of the number of transmitting nodes in the sharing area can be derived as in (17), where the number of actual transmitting nodes (active and non-frozen) in the area is denoted by $Z$. The expected number of transmitting nodes is derived from this result:

$$\mathbb{E}[Z] = \sum_{z=0}^{\infty} z \cdot \mathbb{P}[Z = z]. \tag{15}$$

The effective active node density is defined as the average number of transmitting nodes per unit area. Thus, we finally obtain the effective active node density as follows:

$$\lambda' = \frac{\mathbb{E}[Z]}{\pi (\frac{R}{2})^2}. \tag{16}$$

This is used in the cumulative distribution function (CDF) (1) and the probability density function (PDF) (2) of the aggregate interference. We plot the resulting CDF and PDF for varying network parameters and compare these with the simulation results in Section IV.

### IV. Verification of the Analysis

In this section, we first, plot the analyzed $p_{\text{on}}$ ($p_{\text{on}}^*$) and $\lambda'$, and plot CDF (1) and PDF (2) of the aggregate interference using this $\lambda'$. Next, we compare these derived results with those of NS-2 simulations and two other dependent point process simulations (MHC and SSI). For all simulation scenarios, the exclusion radius $r$ in dependent processes are given to 70 (m). In the NS-2 simulation, MAC/PHY parameters and channel model are given so that the CS radius $R$ is determined to be between 50 and 100 (m) for investigation of $p_{\text{on}}$ and $\lambda'$. We later change these parameters for $R$ to be equal to 70 (m) for the comparison with the dependent point process.

#### A. Simulation setup

1) Monte Carlo simulation for MHC and SSI: We deployed the points using MHC and SSI processes, explained in [3], with MATLAB. The parameters used in the simulations are listed in Table II. For a given number of nodes, the aggregated power was measured at $O$ the center of ball $B$ with radius $r_M$ or $r_S$ for MHC or SSI respectively. The number of deployed nodes was generated using CDF of Poisson distribution with the parameter $\lambda |B|$ where $\lambda$ is the initial node density and $|B|$ is the area of ball $B$. We deployed these points uniformly and measured the aggregate power at $O$. We repeated this procedure for more than 100,000 iterations.

2) Monte Carlo simulation for SSI: We deployed the points using SSI processes, explained in [3], following the same procedure as in the MHC simulation. We changed the parameters of $R$ to be equal to 70 (m) for the comparison with the dependent point process.
\[ P[Z = z] = \sum_{n=2}^{\infty} \left( \frac{\lambda \pi (\frac{r}{2})^2}{n!} e^{-\lambda \pi (\frac{r}{2})^2} \sum_{a=2}^{n} P[N_a = a | N = n] \right) B_a(z), \text{ for } z \in \{0, 1, \ldots\} \] (17)

### TABLE I
**SIMULATION PARAMETERS FOR MHC AND SSI PROCESSES.**

| Varying Parameters | Fixed Parameters |
|--------------------|------------------|
| Number of Packets in the application queue | 3000 |
| Number of Packets in the application queue | 3000 |
| Channel Model | \( \frac{z}{z} \text{, where } X \sim \text{Exp}(1/p) \) |
| Node Density (\( \lambda \)) | \( \{1, 2, 3, 4, 5\} \times 10^{-4} \) |

### TABLE II
**SIMULATION PARAMETERS FOR NS-2.**

| Varying Parameters | Fixed Parameters |
|--------------------|------------------|
| Payload Size | 500 or 1000 (Bytes) |
| RTS Threshold | 0 or 1000 |
| Node Density (\( \lambda \)) | \( \{1, 2, 3, 4, 5\} \times 10^{-4} \) |
| Effective CS Range (\( \mathcal{R} \)) | 50, 70, 100(m) |

2) **NS-2 simulation for PPP:** To verify the analysis results, we conducted simulations using NS-2 [8]. Unlike the previous version, NS-2 version 2.34 (released June 2009), includes wireless PHY and MAC layer patches for the realistic IEEE 802.11 DCF. The simulation parameters, which are the default ones for IEEE 802.11a PHY and MAC, are listed in Table II. They are from the previous research [10].

The overall simulation procedure consisted of genuine NS-2 simulations, pre-processing of the scenario, and post-processing of the data. Generating the number of transmitting nodes using Poisson distribution, deploying them uniformly, attaching designated receiving nodes to each transmitting node, and generating traffic for each transmitting node were done in the pre-processing stage. For the saturated traffic situation, we obtained the time duration from \( \max_i \{ \text{the first transmission time of node } i \} \) to \( \min_i \{ \text{the last transmission time of node } i \} \). We call this the time window. Finding the time window, measuring the received power at the measuring node, recording the lasting time of each received power value, and accumulating all the measurements were done in the post-processing stage.

In attaching the receivers to transmitters, we fixed the relative location of each receiver at 5 (m) right and 5 (m) up from its transmitter. To measure the aggregate power, we put the measuring node in the center of the simulation grid. This node then reports on the received power level and we recorded the lasting time and power level of each received signal.

The simulation conducted in this paper is full-scaled, which takes a long time to collect meaningful results because of two reasons: First, each simulation per geometry scenario takes a long time. This time includes the simulation time in NS-2 and the post-processing time for handling received power instances. NS-2 traces all of the packet-level transactions with the received power recorded at every receiver. In post-processing stage, calculation of the received power from all of the on-going transmissions at a measuring node takes computation time. Moreover, the simulation time itself (not the computation time) has to be long enough to reflect the steady-state behavior, which theoretically requires infinite investigation time. We consider at least 30 seconds per scenario as the simulation time (this takes about 2 hours in real time using a Quad core i7 processor computer). Second, to get the sound statistical inference of PPP, we repeat per-geometry simulation many times. We do 50 repetitions, since all the resulting PDFs of aggregate interference obtained from the simulations show the convergence before 30 repetitions. The simulation time for all 50 scenarios takes approximately four days on average. We repeat this process for each combination of PHY and MAC layer parameters.

The saturated traffic was assigned to all transmitters so that there was no idle time by the traffic itself during the simulations. The background grid for all the simulation scenarios was always the same: a square 500 (m) by 500 (m) in size. The transmission times for RTS, CTS, PPDU (PHY+MAC+PAY) with 500 B (or 1000 B) of payload and ACK were 52, 44, 728 (or 1396), and 44 (\( \mu s \)), respectively (see Table II and [10]). We ignored the propagation delay, even though this exists in
the simulator, since the value was quite small compared to the other transmission times. A generated traffic of 5,000,000 packets per second (pps) was given to all the transmitters. This means a value of 0.2 (µs) for inter-arrival time from the application layer to the physical layer, which is less than the whole transmission time for one successful packet, i.e., $RTS + CTS + PHY + MAC + PAY + ACK + 3*\text{IFS} + \text{difs} = 948(\mu s)$, which is enough for the traffic to be saturated.

B. Discussions

![Fig. 3](image-url)  
**Fig. 3.** $P_{on}$ versus $\lambda$ for all combinations of MAC layer parameters.

![Fig. 4](image-url)  
**Fig. 4.** New node density $\lambda'$ for initial node density $\lambda$ and MAC layer parameter combinations. The thicker curves are using $\lambda''$ in (16).

1) Analysis results on $P_{on}$ and $\lambda'$: All $P_{on}$ in this subsection refer to $p_{on}^*$, the final solution of $p_{on}$ for simplicity of expression. In Fig. 3 $P_{on}$ are shown for various combinations of MAC parameters. The factor that affects $p_{on}$ the most is the effective CS distance $R$, followed by $\lambda$, the initial node density. As $\lambda$ increases, $p_{on}$ naturally increases due to the increased congestion level with semi-log style trend. If there is only the reduced idle time factor without increasing collision probability, as in [4]’s ideal CSMA, the rate of increase of $p_{on}$ must be a log function style. However, our model reflects the increased collision probability and reduced idle time simultaneously, as obtained in real situations. The resulting $p_{on}$ shows a mixture of linear and log functions. Within the same $R$, the combination of mode and payload size that has the lowest $p_{on}$ is the RTS mode and short payload. In other words, the order of $\{RTS-500B, Basic-500B, RTS-1000B, Basic-1000B\}$ is for lower $p_{on}$. In general, RTS-CTS mode has a lower congestion level than the basic mode since the system cost was only the RTS-CTS packet collision and the waiting time for retransmission compared to the basic mode. A large payload in the basic mode makes for a higher congestion level. However, in the cases of some $\lambda$ values, RTS-CTS packets were small enough to compensate for the payload size. Therefore Basic-500B might have higher $p_{on}$ than RTS-1000B. In this case, the effect of mode was weaker than that of the payload size.

This $p_{on}$ was used in the new density $\lambda'$ and we plotted this as shown in Fig. 4. This figure shows the smaller $R$ makes for a higher $\lambda'$, which is the opposite case to $p_{on}$. This is understandable since a smaller $R$ signifies more insensitivity to the interference around. Therefore, we expected the result of $R = 0$, the ALOHA system, to approach the line $\lambda' = \lambda$ in the figure. This line also represents wireless access systems that have no MAC, such as macro or femto cellular systems. The curve $\lambda'$ is a version of $\lambda$ filtered by the CSMA/CA and BEB mechanism. By showing the $\lambda' = \lambda$ line and the curves together, Fig. 4 also addresses how large the gap between these two node densities is and how effectively CSMA/CA MAC operates. The bold curves are from:

$$\lambda'' = \frac{1 - \exp(-\lambda\pi D^2)}{\pi D^2} \quad (18)$$

where $D$ is the exclusion distance with an ambiguity. For comparison, we put $R$ into $D$ in this figure. This expression is the approximated node density for modeling MHC adopted in [1] and [11]. As can be seen in the figure, the variation of $\lambda'$ is higher than $\lambda''$ for varying $\lambda$. Since $\lambda''$ is used for modeling dependent point processes, it cannot trace the real operation. As shown in the next section, our aggregate power distribution adopting this $\lambda'$ is the most elaborate among other point processes. Therefore, Fig. 4 shows the gap in aggregate interference between simplified MHC and the real situation. This is significant because this simplified MHC expression is widely used in academia [1], [2], [11], [12].

2) Comparison of the resulting aggregate interference with the simulation: Using this $\lambda'$ and shot noise analysis, we plotted the distribution of aggregate interference. The PDF and CDF of the analysis in each node density showed high correlations with those of the NS-2 simulations, as seen in Fig. 5 and 6. We depict all the PDFs in a log scale. Although at first glance they resemble a log-normal distribution, they are asymmetric based on the main lobe.

Therefore, they are definitely neither normal nor log-normal distributions. This is notable as some research efforts in the signal processing field assume that the aggregate interference follows normal (in dBm unit) or log-normal (in $W$ unit) distributions. For the other features, the higher the mean of the aggregate power, the lower the probability of that mean.
\[ p_{on} = \sum_{n,a} \sum_{j=1}^{a} B_a(j) = \sum_{n=0}^{\infty} \sum_{a=0}^{n} \sum_{j=1}^{a} \mathbb{P}[N = n] \mathbb{P}[N_a = a|N = n] \sum_{j=1}^{a} B_a(j) (a) \sum_{n=1}^{\infty} \sum_{a=1}^{n} \mathbb{P}[N = n] \sum_{j=1}^{a} B_a(j) (b) \sum_{n=1}^{\infty} \{\frac{\lambda \pi (R^2)}{n!}\}^n e^{-\lambda \pi (\frac{R^2}{4})^2} \sum_{a=1}^{\infty} \sum_{j=1}^{a} P_{n,a} \mathbb{P}[N_a = a|N = n] \sum_{j=1}^{a} B_a(j) (c) \]\n
\[ \sum_{n=1}^{\infty} \{\frac{\lambda \pi (R^2)}{n!}\}^n e^{-\lambda \pi (\frac{R^2}{4})^2} \sum_{a=1}^{\infty} \sum_{j=1}^{a} P_{n,a} \left( \sum_{D} O_{\eta} D \right) p_{on} (1 - p_{on})^{\lambda - D} \sum_{j=1}^{a} B_a(j) \] (19)

---

**Fig. 5.** Probability density of aggregate interference in the condition of RTS mode, 500B payload and \( R = 70(m) \). Other parameters are in Table III.

**Fig. 6.** Cumulative distribution of aggregate interference in the condition of RTS mode, 500B payload and \( R = 70(m) \).

value. Therefore, low-mean high-probability and high-mean low-probability patterns are shown in all the results. This is because the total sum of the probability is fixed to 1 and the x-axis is log-scaled and not linear.

Compared with dependent point processes, at any given \( \lambda \) value, our analysis is the closest one to the simulation results, as depicted in Fig. 7 and Fig. 8. The interference of MHC (Matern) is always less than that of SSI since MHC is the lower bound of SSI, which is also commented on in [3]. However, these two are not as sensitive as our model to variations in node density. Moreover, these two do not have sufficient MAC and PHY layer parameters to reflect the real situation, while our analysis can model any combination of the system parameters, as in Fig. 9.

As shown in the figures so far, our model of the interference has slightly lower values than that of the simulation in each case. This is mainly because the simulator allows the capture situation. In our analysis, the collision between transmitters is regarded as a failure of transmission and this increases each collided nodes’ backoff stages. In contrast, there might be a successful transmission even when multiple nodes in a CS area are transmitting at the same time. This is because if
the ratio of one incoming signal to the others is guaranteed to be higher than a certain threshold, this stronger incoming signal can always be decoded. In our analysis, we ignored that situation in order to simplify the analysis. Nevertheless, our result is competitive.

From these results, we learned the following lessons: If the network is required to maintain a lower interference than a certain level, there are multiple combinations of parameters that need to be controlled. Since the PDF of the aggregate interference is a function of \( \lambda' \), there are multiple combinations of parameters that can result in the same value of \( \lambda' \). Those controllable parameters are \( R \), transmission mode, payload size, etc. This can be used for the interference management in uncontrolled interference limited systems such as cognitive radio networks.

V. CONCLUSION

In this paper, we analyzed the aggregate interference from randomly deployed CSMA/CA nodes. CSMA/CA MAC operations on the stochastic geometry makes the understanding of a node’s behavior difficult. However, the homogeneity of PPP provides a clue to solving the problem. We derived the effective active node density by spatially quantizing the infinite space and analyzing the steady-state power distribution on this quantization unit. Verification is also nontrivial since much repetition is needed in order to get sound statistical inference from the random topology. Although the exact closed form expression of the interference distribution cannot be obtained, the analysis framework proposed in this paper shows high applicability to many related problems in modern wireless networks where the amount of interference in a random node geometry significantly affects system performance.

APPENDIX: DERIVATION OF \( p_{on} \)

The power distributions \( B_a(j) \) of (11) and (12) are conditioned on \( N_a \), while \( N_a \) is conditioned on \( N \). Therefore, the marginal PMF of \( p_{on} \) is obtained by summing all of the probabilities on conditioned variables, as in (19). The condition on \( N_a \) is eliminated using (13). The condition on \( N \) is eliminated by the homogeneity of PPP, i.e., every geometric subset satisfies Poisson distribution on the number of points in that subset, i.e., (7).

In (19), \( (a) \) is from the fact that the possibility of a sharing area being “on” is zero when there is no initial deployed nodes and active nodes. \( (b) \) is from (7) and (13). \( (c) \) is from (14). In this equation, \( p_{\eta} \) includes \( p_{on} \) term, which is also the marginal PMF from the unconditional event of channel busyness. Therefore, Equation (19) has the unknown variable \( p_{on} \) on both sides of the equation and the solution can be obtained by solving the weighted eighth order polynomial of \( p_{on} \), where the weight is the product form of Poisson and binomial probabilities. More specifically, \( \sum_{n=1}^{\infty} \sum_{a,n} p_{\eta}.a, n \{ \sum_{D} O_{D} p_{on} (1 - p_{on})^{D-1} \} \sum_{k=1}^{n} B_{a}(j) \) is the eighth-order polynomial of the unknown \( p_{on} \) and all other terms are known, and \( \frac{\lambda(n(\frac{1}{2})^{2})^n e^{-\lambda(\frac{1}{2})^2}}{n!} \) is a constant from Poisson distribution for the given \( n \). Therefore, summing the product of these two terms for varying \( n \) from \( n = 1 \) to \( n = \infty \) also results in an eighth-order polynomial. Since there is no general solution for polynomials with order higher than four and \( n \) goes to infinity, we cannot derive the closed form expression of \( p_{on}^{*} \), the solution of \( p_{on} \). However, we can still get a solution by finding the intersection of the right hand side and left hand side of (19), which we will call \( p_{on}^{*} \).

REFERENCES

[1] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, “An ALOHA protocol for multi-hop mobile wireless networks,” IEEE Trans. on Information Theory, Vol. 52, No. 2, pp. 421-436, 2006.
[2] J. Andrews, R. K. Ganti, M. Haenggi, N. Jindal, and S. Weber, “A primer on spatial modeling and analysis in wireless networks,” IEEE Comm. Magazine, Vol. 48, No. 11, pp. 156-163, 2010.
[3] A. Busson, G. Chelius, and J.-M. Gorce, “Interference modeling in CSMA multi-hop wireless networks,” Technical report INRIA, iria-00316029-ver3, pp. 1-21, 2009.
[4] L. Jiang, and J. Walrand, “A distributed CSMA algorithm for throughput and utility maximization in wireless networks,” IEEE/ACM Trans. on Networking, Vol. 18, No. 3, pp. 960-972, 2010.
[5] F. Baccelli, and B. Blaszczyszyn, “Stochastic geometry and wireless networks volume 1: theory,” Foundations and Trends in Networking, Vol. 3, No. 3-4, pp. 249-449, 2009.
[6] J. Hwang, and S.-L. Kim, “Cross-layer optimization and network coding in CSMA/CA-based wireless multihop networks,” IEEE/ACM Trans. on Networking, Vol. 19, No. 4, pp. 1028-1042, 2011.
[7] G. Bianchi, “Performance analysis of the IEEE 802.11 distributed coordination function,” IEEE Journal of Sel. Areas on Comm., Vol. 18, No. 3, pp. 535-547, 2000.
[8] Network Simulator-2, http://www.isi.edu/nsnam/ns/.
[9] Q. Chen, F. Schmidt-Eisenlohr, D. Jiang, M. Torrent-Moreno, L. Delgrossi, and H. Hartenstein, “Overhaul of IEEE 802.11 modeling and simulation in NS-2,” Proc. ACM MSWiM 07, 2007.
[10] D. Qiao, S. Choi, and K. G. Shin, “Goodput analysis and link adaptation for IEEE 802.11a wireless LANs,” IEEE Trans. on Mobile Computing, Vol. 1, No. 4, pp. 278-292, 2002.
[11] D. Stoyan, W. S. Kendall, and J. Mecke, “Stochastic geometry and its applications,” John Wiley and Sons, pp. 1-345, 1987.
[12] M. Haenggi, and R. K. Ganti, “Interference in large wireless networks,” Foundations and Trends in Networking, Vol. 3, No. 2, pp. 127-248, 2009.