UNCERTAINTIES IN THE DEPROJECTION OF THE OBSERVED BAR PROPERTIES

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1. INTRODUCTION

Bars are commonly seen in disk galaxies and play an important role in secular evolution (Friedli & Benz 1993; Sheth et al. 2005; Masters et al. 2011; Gadotti 2011; see Kormendy & Kennicutt 2004 for a review). Optical studies found that the bar fraction is typically around 50% (Marinova & Jogee 2004 for a review). Optical studies found that the bar fraction (Jogee et al. 2004; Barazza et al. 2008; Sheth et al. 2012). There are several methods to measure the bar strength in the literature, such as the ellipticity of the bar (Martin 1995) and the bar–interbar contrast (Elmegreen & Elmegreen 1985; Elmegreen et al. 1996). Buta & Block (2001) suggested that the maximum value (\(Q_b\)) of the ratio between the tangential force and the mean axisymmetric radial force in a barred disk galaxy can also be used to characterize the bar strength. This parameter gives us a direct impression of the actual force due to a bar. Observationally, stronger bars are usually longer with higher ellipticities (Martin 1995; Menéndez-Delmestre et al. 2007; Gadotti 2011). In this paper, we focus on two fundamental observed quantities characterizing the bar strength, i.e., bar length and ellipticity, which can be easily measured from the galaxy image. Besides the simple visual estimation of the bar properties (Kormendy 1979; Martin 1995), commonly used bar identification and measurement approaches include the maximum or minimum of the bar ellipticity, the radial variation of the isophotal position angle (PA), the radial profile of the phase angle and the relative amplitude of the Fourier \(m = 2\) mode, and the bar–interbar contrast (Laine et al. 2002; Aguerri et al. 2003; Sheth et al. 2003; Erwin 2005; Marinova & Jogee 2007; Menéndez-Delmestre et al. 2007; Li et al. 2011).

Since real galaxies are all inclined to a certain extent, the measured bar parameters should first be deprojected to their corresponding face-on values. The basic assumption of the
depresentation is that the outer part of a bar is infinitely thin (Gadotti et al. 2007). However, real bars are usually thickened via the buckling instability, showing a boxy/peanut-shaped bulge in the inner region (Sellwood 1981; Combes & Sanders 1981; Raha et al. 1991; Bureau & Freeman 1999; Bureau et al. 2006). As a good example, the Milky Way also harbors a buckled bar with the boxy bulge in the central region (Blitz & Spergel 1991; Bissantz & Gerhard 2002; Babusiaux & Gilmore 2005; Rattenbury et al. 2007; Cao et al. 2013). The vertically thick boxy bulge, as evident in COBE images (Weiland et al. 1994; Dwek et al. 1995), may simply be the Galactic bar viewed edge-on with the major axis tilted 20° away from the Sun–Galactic center line (e.g., Shen et al. 2010). It even presents a vertical X-shaped structure related to the buckling process (Nataf et al. 2010; McWilliam & Zoccali 2010; Li & Shen 2012). Thus, this basic assumption needs to be treated with caution when deprojecting bars.

In the one-dimensional (1D) approximation, a bar is treated as a straight line segment that can be easily deprojected (Martin 1995). This approximation is, of course, too simple for real bars. Assuming that the bar can be described by a planar ellipse, Gadotti et al. (2007) provided a more sophisticated method to analytically deproject the bar. They also tried to directly stretch the inclined galaxy image to the face-on one with the total flux conserved. After comparing different bar deprojection methods, they concluded that when the inclination angle is less than 50°, all methods agree very well with each other within a 20% difference. However, for the inclined galaxies in their sample, the face-on bar properties are actually unknown. Moreover, the estimated inclination angles introduce additional uncertainties. As a matter of fact, several popular deprojection methods are widely used in observations to characterize the bar structures (Martin 1995; Marinova & Jogee 2007; Gadotti et al. 2007; Li et al. 2011; Aguerri et al. 2000). The majority of these studies ignore the uncertainties in the deprojection process since it is impossible to know the true face-on values of the bar from observations. Consequently, it is hard to know to what extent these deprojection methods are accurate.

The uncertainties in various deprojection methods can be best assessed by analyzing a simulated disk galaxy. The true face-on bar properties of a simulated galaxy can always be measured readily. A simulated galaxy can also be “observed” from different viewing angles. The bar properties measured in inclined systems are then deprojected to the face-on values using different deprojection methods. Thus, the uncertainties of the deprojection methods can be tested by comparing the deprojected bar properties to the true face-on values. If inclination angles are treated as known quantities, the uncertainties are mainly from deprojection methods. By projecting the three-dimensional (3D) simulations onto a two-dimensional (2D) plane from different viewing angles, we create mock images and apply different bar measurement and deprojection methods. The creation of mock images and the bar measurements are described in Section 2. Section 3 presents different deprojection methods and the corresponding results, which are further discussed in Section 4 and, finally, concluded in Section 5.

2. IMAGE CREATION AND BAR MEASUREMENT

2.1. Mock Images

For the purpose of this work, we study two N-body disk simulations here (Model A and Model B) shown in Figure 1. Model A is taken from the simulation in Shen et al. (2010), which well matches the BRAVA stellar kinematics in the Milky Way bulge region. This simple N-body model simulates a disk galaxy with 10⁶ particles evolving in a rigid dark matter halo potential. Initially, the disk is dynamically cold (Toomre’s Q ∼ 1.2). A bar forms spontaneously and quickly buckles in the vertical direction. The snapshot at 1.8 Gyr is adopted to create the mock images. The scale length of the initial disk is R_d,0 = 1.9 kpc. In Model B, the density distribution of the dark matter halo is described by an adiabatically compressed King profile (Φ(0)/σ² = 3 and r_c = 10 R_d; see Sellwood & McGaugh 2005 for details of adiabatic compression). The halo consists of 2.5 million particles, and its total mass M_halo = 8 M_⊙. The snapshots at 2.4 Gyr is adopted for this model. Bars in both Model A and Model B have experienced buckling instabilities (Raha et al. 1991). The snapshots we choose have relatively strong spiral arms that are often seen in real observed galaxies. The bar ellipticity (∼0.5 in Model A, ∼0.4 in Model B) and
the ratio of the bar length to the disk size (∼0.5 in Model A, ∼0.65 in Model B) are also consistent with observations (Erwin 2005). We estimated the ratio between the boxy peanut and bar length of Model A (∼0.57) and Model B (∼0.44). This ratio is consistent with other simulations (Erwin & Debattista 2013). Therefore, the two models are reasonably representative and adequate for the present study. In addition, the bar in Model B is longer than that in Model A, which enables a consistency check of the different deprojection methods.

The mock images are created by projecting the 3D simulations onto a 2D plane (200 × 200 pixels) with different disk inclination (i) and different bar orientations (φbar).4 The inclination angle i varies from 0° to 75°, and φbar varies from 0° to 90°. In the end, we generate 42 mock galaxy images with equally sampled i and φbar for each simulation.

2.2. Bar Measurement

Bars leave distinct features in the isophotal geometric profiles. We fit isophotal ellipses of these mock images with IRAF task ELLIPSE. During the fitting, the center, PA, and ellipticity (e) of the isophotal ellipses are all set to be free parameters. In the literature, there are several methods commonly used to identify and measure a bar. Bars usually correspond to the position of the bar (Martin 1995). The deprojected bar length is $a_{\text{bar}}$ = $a_{\text{bar}}^{\text{obs}}(\cos^2 \alpha + \sin^2 \alpha \sec^2 i)^{1/2}$, where $a_{\text{bar}}^{\text{obs}}$ and $a_{\text{bar}}^{\text{dep}}$ are the observed and deprojected bar length, respectively. For galaxy images, i is the disk inclination angle, and α is the angle between the projected major axes of the bar and the inclined disk ($\alpha = \phi_{\text{bar}}$).

In Figure 3, we plot the ratios of the deprojected bar lengths ($a_{\text{dep}}^{\text{max}}, a_{\text{dep}}^{\text{min}}, a_{\text{dep}}^{i=10}$) to the intrinsic face-on values ($a_{\text{int}}^{\text{max}}, a_{\text{int}}^{\text{min}}, a_{\text{int}}^{i=10}$) as a function of i. The left column illustrates the deprojection results of Model A. Results of Model B are shown in the right column. Please note that in certain extreme cases, e.g., large i or $\phi_{\text{bar}}$, it is difficult to measure the bar properties due to the complex ellipticity radial profiles. Therefore, we did not measure bar parameters for some cases in Figure 3. As shown in panels (a) and (b), the deprojected $a_{\text{max}}$ tends to overestimate the corresponding face-on value, with larger i introducing higher uncertainties. The deprojection results at moderate inclinations ($i \leq 60^\circ$) are generally overestimated by ∼40%. However, in an extreme case ($i = 60^\circ, \phi_{\text{bar}} = 90^\circ$), the deprojected bar length can be overestimated by as much as 100%. When $i > 60^\circ$, the deprojected $a_{\text{max}}$ significantly overestimates the face-on values. In the case of $a_{\text{min}}$, the general uncertainty is ∼25% in moderately inclined disks ($i \leq 60^\circ$). Interestingly, at larger i, this uncertainty drops to ∼20%, which is much smaller than that of $a_{\text{max}}$. For both $a_{\text{max}}$ and $a_{\text{min}}$, a larger $\phi_{\text{bar}}$ usually results in a higher overestimation. The best case is $a_{\text{int}}$. As shown in panels (e) and (f), the overestimation is quite small at moderate inclinations (∼20%). The effect of $\phi_{\text{bar}}$ is minimal in this case. Similar to $a_{\text{min}}$, at large i (>60°), the deprojection uncertainty of $a_{\text{int}}$ also decreases. The 1D analytical deprojection results of Model B show similar trend and scatter as in Model A.

3.2. 2D Deprojections

3.2.1. 2D Analytical Deprojection

The 2D analytical deprojection has been discussed in detail in Gadotti et al. (2007). We briefly review this method here. The shape of the bar is assumed to be a planar ellipse, which is analytically deprojected to an ellipse in the face-on view. The semi-major and semi-minor axes after the deprojection are

$$S1 = \left\{ \frac{2(AF^2 + CD^2 + GB^2 - 2BDF - ACG)}{(B^2 - AC)((C - A)\sqrt{1 + \frac{4B^2}{(A-C)^2}} - (C + A))} \right\}^{1/2}$$

$$\theta_{\text{bar}}$$ is introduced during the mock image creation. Initially, the bar model is aligned with the X axis as shown in Figure 1. The model is rotated counterclockwise by $\theta_{\text{bar}}$. Then, we incline it with respect to the X axis (semi-major axis of the disk) by $i$ to create the mock image. After projection, the bar orientation ($\phi'_{\text{bar}}$) relative to the major axis of the inclined disk is slightly different from the initial $\theta_{\text{bar}}$ due to the projection effect. $\phi'_{\text{bar}}$ is measured directly from the mock images and used to deproject the bar properties in our analysis.
Figure 2. Examples of the best-fit isophotal ellipses and radial profiles of \( e \) and PA of each isophote for Model A (top row) and Model B (bottom row). From these radial profiles we can determine three different bar lengths, which are \( a_{\text{max}} \) (vertical solid line), \( a_{\text{min}} \) (vertical dotted line), and \( a_{10} \) (vertical dashed line). The position angle of the bar is marked with the horizontal dashed line.

\[
S_2 = \left\{ \frac{2(AF^2 + CD^2 + GB^2 - 2BDF - ACG)}{(B^2 - AC)(A - C)\sqrt{1 + \frac{4B}{(A - C)^2} - (C + A)}} \right\}^{1/2}, \tag{3}
\]

where \( i \) is the inclination angle and \( \alpha \) represents the projected bar orientation (\( \alpha = \phi_{\text{bar}}' \)). The semi-major and semi-minor axes lengths are \( \max(S1, S2) \) and \( \min(S1, S2) \), respectively. The deprojected ellipticity can then be calculated by \( e = \frac{1 - \min(S1, S2)}{\max(S1, S2)} \).

We perform the 2D analytical deprojection on all of the mock images with different \( i \) and \( \phi_{\text{bar}} \) and compare the deprojected values to the face-on ones. Results shown in Figure 4 are much better than those in the 1D deprojection shown in Figure 3 because an ellipse simply describes the shape of the bar better than a straight line segment. For all the three different bar measurements, the agreement is quite good (\( \sim 15\% \)) at small \( i \) (\( \leq 60^\circ \)). At large \( i \) (\( > 60^\circ \)), the deprojection on \( a_{\text{max}} \) can overestimate the intrinsic face-on values by as much as 100\%. For \( a_{\text{min}} \), depending on \( \phi_{\text{bar}} \), the deprojection can either overestimate or underestimate the bar length by \( \sim 10\% \) at moderate inclinations (\( i \leq 60^\circ \)). \( a_{10} \) tends to underestimate the bar length for Model A (\( \sim 10\% \)) but to overestimate the bar length for Model B.
length for Model B (≈10%). All the three deprojected results of the bar measurements depend weakly on $\phi_{\text{bar}}$. The results of Model B are similar to Model A with slightly smaller scatters.

As shown in Figure 5, the deprojected ellipticity is accurate when $i \leq 60^\circ$. At moderate inclinations, the general deviation is less than 10% for $e_{\text{max}}$, $e_{\text{min}}$, and $e_{10}$, but can be as large as 40% in some cases. This method behaves badly at large $i$. The trend depends on $\phi_{\text{bar}}$. Generally speaking, for $e_{\text{max}}$ and $e_{\text{min}}$, the method underestimates the face-on bar ellipticity for small $\phi_{\text{bar}} (<50^\circ)$, while it overestimates the ellipticity at large $\phi_{\text{bar}} (>50^\circ)$. On the other hand, $e_{10}$ has much better agreement than $e_{\text{max}}$ and $e_{\text{min}}$. However, for Model B, at $i \sim 45^\circ$, $e_{10}$ shows very large uncertainties. Since $e_{10}$ is the ellipticity at $a_{10}$, which is well beyond the visual bar length, the influence of the bar thickness to $e_{10}$ is much less than that to $e_{\text{max}}$ and $e_{\text{min}}$. Comparing the left column with the right column, we can see that the deprojected ellipticities of Model A and Model B are very similar regardless of the different bar lengths.

### 3.2.2. 2D Image Deprojection

The mock inclined images can first be deprojected to the face-on images with the IRAF task GEOTRAN, which basically stretches the inclined image along the minor axis of the disk. Then, the bar properties are extracted from the stretched images. The GEOTRAN routine enables us to correct the geometric distortion of galaxy images while keeping the total flux conserved. First, the major axis of the inclined galaxy is rotated to the direction of the X axis. Then, the size of the minor axis of the disk is linearly magnified to the original value according to the disk inclination. Hence, this method deprojects the whole galaxy image to face-on directly (see Figure 6 for an example). Another image deprojection method is the IRAF task IMLINTRAN (Gadotti et al. 2007). We have also tested this method and found that the deprojected images using these two routines are almost identical. The former task is adopted in this work. Comparing the deprojected bar properties extracted from these stretched images to the ones measured in the original face-on images, we can estimate the uncertainty of this method. However, once $i$ exceeds $60^\circ$, the images after stretching completely betray the real face-on ones. In such cases, it is almost impossible to identify the bar with the measured radial profiles of ellipticity and PA. Thus, the 2D image deprojection is confined to small $i$ only ($\leq 60^\circ$).

Figures 7 and 8 show the results of the deprojected bar length ($a_{\text{max}}$, $a_{\text{min}}$, and $a_{10}$) and ellipticity ($e_{\text{max}}$, $e_{\text{min}}$, and $e_{10}$), respectively. For Model A, the left column of Figure 7 shows that the deprojected $a_{\text{max}}$, $a_{\text{min}}$, and $a_{10}$ agree with the face-on values very well; the uncertainty is about 10% at moderate
inclinations. Systematic overestimation is found in $a_{\text{max}}$, while for $a_{\text{min}}$ and $a_{10}$, the situation is uncertain. For Model B, the right column indicates that the results have similar trends but different scatters compared to Model A. The deprojected results of $a_{\text{max}}$ and $a_{\text{min}}$ are very good ($\sim 5\%$), except that the deprojected $a_{10}$ has a relatively large uncertainty (up to $\sim 10\%$) at moderate inclinations. Generally speaking, the deprojection of Model B is more accurate than Model A. All of the panels show that the deprojected bar length depends weakly on $\phi_{\text{bar}}$.

For $e_{\text{max}}$ and $e_{\text{min}}$, the 2D image deprojection tends to underestimate the face-on values by $\sim 10\%$ at small $\phi_{\text{bar}} (< 50^\circ)$. This trend reverses at large $\phi_{\text{bar}} (> 50^\circ)$. The deprojected $e_{10}$ seems to underestimate the face-on value by $\sim 10\%$ regardless of $\phi_{\text{bar}}$. As shown in Figure 8, the deprojected ellipticity behaves similar to the 2D analytical deprojection in Figure 5. This figure suggests that the deprojected ellipticity is more accurate when $i$ is less than $60^\circ$. There is no difference in the deprojection of the ellipticity between Model B and Model A.

### 3.3. Fourier-based Deprojections

#### 3.3.1. Fourier Decomposition

In this work, we also test the Fourier decomposition method in recovering the bar length measured in the inclined image (Noordermeer & van der Hulst 2007; Li et al. 2011). We fit the galaxy images with the center, PA, and $e$ of each elliptical annulus fixed to the values measured at the outskirts of the disk. Then, we decompose the intensities within each elliptical annulus with

$$I(\theta) = I_0 + \sum_{m=1}^{\infty} I_m \cos (m\theta + \phi_m),$$

where $I$ is the intensity on the annulus in the direction of $\theta$. $I_0$ is the averaged intensity of each annulus. $I_m$ is the amplitude of the $m$th mode of the Fourier series. $\phi_m$ is the corresponding phase angle. Figure 9 shows an example of the Fourier decomposition. The upper left panel illustrates the image with the fixed ellipses overlaid. The upper right panel is the radial profile of the relative amplitude of the Fourier $m = 2$ mode. The bar corresponds to large $I_2/I_0$. The peak position of $I_2/I_0$ is within the bar, where the bar–interbar contrast is the strongest. Based on our empirical tests, we choose the position at $0.85(I_2/I_0)_{\text{max}}$ outside the peak position as the end of the bar. Assuming the disk is purely circular in its face-on view, the semi-major axes of the ellipses (bottom left panel) in fact equals the radii of the face-on circular annulus (bottom right panel). The bar length is marked by the semi-major axis of the particular ellipse that encloses the bar region. This value is actually the radius of the circle passing right through the bar ends in the face-on view. Thus, the bar length measured in the inclined image is the same as the length in the face-on image, which can be directly compared to the face-on values.

As shown in Figure 10, the bar length measured in the inclined image agrees quite well with that of the face-on image.
The typical difference at moderate inclination is $\sim 10\%$. However, when $i$ is larger than $60^\circ$, the measured bar length overestimates the face-on value by as much as 50%. Similar to the previous results, at lower $i$ ($\lesssim 60^\circ$), the influence of $\phi_{\text{bar}}$ is negligible. Most of the uncertainties in the deprojection come from the large inclination angles.

3.3.2. Bar–Interbar Contrast

In the Fourier decomposition method, the largest contribution to the bar comes from the Fourier $m = 2$ component. As demonstrated in Figure 11, the higher even-order components have weaker amplitudes. Ohta et al. (1990) argued that the
Figure 7. As in Figure 3, but for the 2D image deprojection (image stretching) of the bar length.
(A color version of this figure is available in the online journal.)

density distribution of the bar should be approximated by all the important even Fourier components. Based on these even Fourier modes, they further suggested the construction of radial profiles of the luminosity contrast between the bar and interbar regions. The bar intensity ($I_b$) is defined as $I_0 + I_2 + I_4 + I_6$, and the interbar intensity ($I_{ib}$) is $I_0 - I_2 + I_4 - I_6$. They defined the bar region as the zone where $I_b/I_{ib}$ is larger than 2. However, setting the value as 2 is not physically meaningful because it cannot account for all the morphological differences among galaxies. Aguerri et al. (2000) proposed a more reasonable method to find the bar region:

$$I_b/I_{ib} > \frac{(I_b/I_{ib})_{\text{max}} - (I_b/I_{ib})_{\text{min}}}{2} + (I_b/I_{ib})_{\text{min}},$$

which is equivalent to the FWHM of $I_b/I_{ib}$ profile.

The left panel of Figure 12 shows that, overall, this method tends to overestimate the deprojected bar length of Model A. The amount of the overestimation is comparable to that of the 2D analytical deprojection. When $i$ is larger than 60°, the deprojected bar length is typically overestimated by about 15%. This panel also shows different behavior compared to the previous results on $\phi_{\text{bar}}$. The overestimation actually decreases with increasing $\phi_{\text{bar}}$. However, it plays a minor role in determining the deprojected bar length. The right panel of Figure 12 indicates that the deprojected bar length of Model B is more accurate than that of Model A. At small $i$ (≤60°), the deprojected lengths agree very well with the face-on value. When $i > 60°$, similar to Model A, the deprojected lengths are overestimated by as much as 50%.

4. DISCUSSION

4.1. Deprojection Uncertainties

In this work, we investigate uncertainties in the deprojection of the bar properties of two models, i.e., Model A and Model B. Figure 13 summarizes the typical scatter ranges in different deprojection methods for the bar length and ellipticity at moderate inclinations ($i \leq 60°$). The scatter comes from measures corresponding to different values of $\phi_{\text{bar}}$ and $i$. For each method, the upper (lower) limit of the scatter is the first (third) quartile of the overall distribution of absolute values of scatters at $30° \leq i \leq 60°$. This figure shows that the 1D analytical deprojection always has the largest scatters in the two models, which is independent on the length of the bar. The 1D deprojection has the simplest assumption of the bar structure; the only measured property of the bar used in this deprojection process is the major axis (Equation (1)), which alone does not reflect the 3D structure of the bar at all. In addition, the 2D analytical deprojection and the 2D image deprojection produce consistent results for the two models. The scatters of these two methods are relatively small at $i \leq 60°$. However, the 2D image deprojection (image stretching) is almost impossible to deproject the bar properties at $i > 60°$. All the barred galaxies in our simulations show a vertical X-shaped structure in the inner
Figure 8. As in Figure 3, but for the 2D image deprojection (image stretching) of the bar ellipticity. (A color version of this figure is available in the online journal.)

region when $i > 60^\circ$. Thus, the stretched image is very different from the true face-on image. Figure 14 shows an example of the 2D image deprojection at $i = 75^\circ$. Comparing the top left panel to the bottom left panel, we can see that the deprojected image is completely different from the original face-on one. The bottom right panel also shows different features in the ellipticity profile, in which the bar is difficult to recognize.

The deprojection results shown in Section 3 suggest that scatters of the Fourier decomposition is $\sim 10\%$ at moderate inclinations ($i \leq 60^\circ$), which is generally consistent between the two models. Another Fourier-based method, namely, the bar–interbar contrast method, has relatively larger uncertainties than the Fourier decomposition method. On the other hand, uncertainties of the bar–interbar contrast method seems to be model dependent because this method considers the even ($m = 2, 4, 6$) modes to calculate the bar–interbar contrast, which in fact depends on the detailed stellar distribution of the bar.

For the deprojected ellipticity, the 2D analytical deprojection and the 2D image deprojection produce very similar results. Uncertainties increase at large $i$. As $\phi_{\text{bar}}$ increases from $0^\circ$ to $90^\circ$, the deprojected ellipticity gradually transitions from underestimation to overestimation. In addition, at certain $i$, uncertainties of the deprojected bar ellipticity are generally larger than the deprojected bar length because the vertical structure of the bar has more influence on the measured isophotal ellipticity than the length.

Uncertainties in the deprojected ellipticities of these two models are almost the same. This confirms the trend in the ellipticity deprojection as $\phi_{\text{bar}}$ increases from $0^\circ$ to $90^\circ$. This is not surprising because at large $\phi_{\text{bar}}$, the bar orientation aligns close to the disk minor axis. Therefore, the thickness of the bar increases the projected bar length when the galaxy is inclined, which makes the measured ellipticity larger than that expected from a planar bar. The results are reversed at small $\phi_{\text{bar}}$ since the thickness of the bar enlarges the bar minor axis length when the galaxy is inclined. Thus, the measured ellipticity becomes smaller.

In general, most of the results presented in this work suggest that the deprojection uncertainties of Model A and Model B have similar scatters and trends. Despite this consistency, scatters in galaxies with a long bar (Model B) are slightly smaller than that with a short bar (Model A), which could be due to the fact that the outer part of the bar in Model B is less affected by the inner thickened bulge than in Model A.

Theoretically, one of the deprojection uncertainties stems from the calculation of the inclination angle of the outer disk. The simplest way assumes that the disk is round and thin. Then, the inclination angle can be derived from a simple formula, i.e., $\cos(i) = 1 - e_{\text{disk}}$. This method is applied to late-type galaxies because their disk is relatively thin. The Hubble (1926) method utilizes the ellipticity of the disk outskirts to derive the inclination angle under the assumption of a certain intrinsic thickness and shape of the disk, which gives a more accurate
Figure 9. Example of the isophote measurement with the geometric parameters fixed to the outermost isophote (upper left panel) and the radial profile of the relative amplitude of Fourier $m = 2$ mode (upper right panel) for Model A ($i = 45^\circ$, $\phi_{\text{bar}} = 30^\circ$). The bar end marked by the solid line corresponds to $0.85I_2/I_0$ max. The bottom right panel shows the face-on image of Model A, overlaid with a circle enclosing the bar ends. The bottom left panel shows the projection of the image in the bottom right panel.

Figure 10. Results of the Fourier decomposition method. This figure shows the deprojected bar length ($a_{\text{dep}}$) to the intrinsic face-on value ($a_{\text{int}}$) as a function of the inclination angle ($i$). The left panel shows the result of Model A, while the right panel shows the result of Model B. Different colors represent different $\phi_{\text{bar}}$, which is given in panel (a). The black dashed line denotes unity.

(A color version of this figure is available in the online journal.)
estimation of the inclination angle for early-type galaxies. Since the intrinsic thickness and roundness of the disk are unknown, these methods inevitably introduce uncertainties to the bar deprojection. To test these uncertainties, the differences between the observed $i$ and the corresponding given values are investigated in this work. The first method mentioned above is adopted to measure the inclination angle. We find that the inclination uncertainty is very small ($\sim 5^\circ$) at intermediate inclinations ($i \leq 75^\circ$). However, at very small inclination angles, the difference is relatively large due to the simple assumption. In our simulation, the outer skirt of the galaxy is not perfectly featureless and circular ($\epsilon \sim 0.15$). A slight distortion of the isophotes in the outer part will result in a relatively large inclination angle ($\sim 20^\circ$). However, the influence of $i$ on the deprojection should be trivial. In our work, $10^\circ$ difference in inclinations does not affect the deprojected bar parameters too much. We assume that the inclination angle is exactly known. Even though the influence of the inclination is limited in our models, it is worth pointing out that the uncertainty of deprojection derived in this work is only a lower limit. The true uncertainty will be higher if the inclination error is considered. The error in $\phi'_{\text{bar}}$ measurement is also studied. The difference between the measured bar angle and the given value during the mock image creation is very small ($\sim 5^\circ$) at intermediate inclination ($i < 75^\circ$), but it becomes relatively large at very small $i$. When the disk is close to face-on, the line of nodes (LON) of disk is highly uncertain. A slight change in the outskirt shape could result in a huge difference between the bar angle and the disk LON. In this work, $\phi'_{\text{bar}}$ is measured from the mock images. The error in $\phi'_{\text{bar}}$ measurement will not influence our deprojection results.

Generally speaking, the measurement error in $i$ and $\phi'_{\text{bar}}$ could introduce uncertainties to the deprojection. However, such uncertainties are small compared to the one caused by the 3D structure of the bar itself. We will carefully investigate this with toy models in the next section.

### 4.2. Toy Models of the 3D Bar Structure

Simulations found that an evolved bar is thick in the inner part due to the vertical buckling instability (Combes & Sanders 1981; Raha et al. 1991; Debattista & Williams 2004; Athanassoula 2005; Debattista et al. 2006). Observations of intermediately inclined barred galaxies also found non-negligible thickness of the bar (Bureau & Freeman 1999; Bureau et al. 2006). We want to know to what extent this vertical structure influences the accuracy of the bar deprojection.

In the previous section, we conclude that all the deprojection methods behave badly at large $i$ ($> 60^\circ$). To better understand this result, we first look at the projected bar at different viewing angles ($i$ and $\phi'_{\text{bar}}$). In the edge-on image, a bar structure contains a boxy bulge in the inner region. To simplify the calculation, we use toy models here to show the projection process. Figure 15 shows an illustration of a toy model. There are two different parameter sets corresponding to Model A and Model B. The values are listed in Table 1. The structure of the bar is treated as a triaxial ellipsoidal shell with the axis ratio as $a:b:h$, where $a$ is the semi-major axis, $b$ is the semi-minor axis, and $h$ is the vertical thickness. The values in Table 1 are estimated from the measured bar properties in the face-on and edge-on mock images. Then, the triaxial ellipsoids with axis ratio of $a:b:h$ are constructed. Several simulations suggest that the outer part of the bar should be much thinner than the inner region (Athanassoula 2005). However, we found that one-ellipsoid models could represent...
The 3D structure of the bar reasonably well.\footnote{We also tested two ellipsoid in the toy model construction, i.e., an inner thick one and an extended thin one. The deprojection results are consistent with the single-ellipsoid toy model.} At large inclination angles, the thickness of the outer bar region plays a minor role in deprojection uncertainties; it is mainly the inner thick part that significantly changes the projected bar shape. At small inclinations, the effects of outer bar thickness in the deprojection uncertainties are also limited.

After projecting these 3D ellipsoids to a 2D plane from different $i$ and $\phi_{\text{bar}}$, the major axis and the minor axis of the projected 2D ellipses can be calculated. We deproject these 2D bars to the face-on properties using both the 1D and 2D analytical deprojection methods and compare them to the true face-on values. The 1D analytical deprojection results of the toy models are shown in Figure 16. The deprojection trend and scatter of the toy models are very similar to the results of our simulations shown in Figure 3. It is obvious that the true shape of a bar is not a simple straight line segment; it has some finite width and height, making the semi-major axis and orientation of the face-on bar differ from those of inclined bar.\footnote{At extremely large $i$ and $\phi_{\text{bar}}$, the projected major axis can be even smaller than the projected minor axis, which will make us mistakenly treat the projected minor axis as the intrinsic major axis. In our toy models, we deproject the bar length at $\phi_{\text{bar}} = 85^\circ$ rather than $\phi_{\text{bar}} = 90^\circ$.} In addition, the toy models do not consider the spatial density variations inside the bar and the projection effect from the surrounding disk, which could add additional errors to the bar measurement. These will be discussed in the next section.

The 2D analytical deprojection is also tested using the toy models. Comparing to Figure 16, we can see that the 2D analytical deprojection is indeed better than 1D as shown in Figure 17. This figure shows that the 2D analytical deprojection tends to give higher overestimation on the bar length at larger $\phi_{\text{bar}}$. At small $i$, the scatter of the deprojection is relatively small. However, this deprojection method is unable to recover the bar length at large $i$. Comparing to our previous simulations in the deprojected bar length, it is nice to see that Figure 17 generally matches Figure 4. On the other hand, Figure 18 shows the 2D analytical deprojection results of the ellipticity based on the toy models. Comparing to Figure 5, we also find good agreements between deprojected ellipticities of the toy models and our simulations.

Additionally, comparison among the results of $a_{\text{max}}$, $a_{\text{min}}$, and $a_{10}$ suggests that the turning point at which the deprojection uncertainties become large is different for these three kinds of bar length. The deprojected $a_{\text{max}}$, $a_{\text{min}}$, and $a_{10}$ have turning points at about $50^\circ$, $60^\circ$, and $70^\circ$ inclination angles, respectively. As shown in Figure 2, $a_{\text{max}}$ is the shortest (inside the visually identified bar), which can be easily affected by the thick part of the bar. $a_{10}$ is the longest (outside the visually identified bar). Hence, the influence by the thickness of the bar is the smallest. This suggests that the 2D analytical deprojection can be affected

Figure 13. Typical scatters in different deprojection methods for both bar length and ellipticity at $i \leq 60^\circ$. Black and red represent Model A and Model B, respectively. Error bars show the scatter range. The upper (lower) limit represents the first (third) quartile of the distribution of all the scatter values for a given method at $i \leq 60^\circ$. Note that the scatter of each method comes from measures corresponding to different values of $\phi_{\text{bar}}$ and $i$.

(A color version of this figure is available in the online journal.)

Table 1
Geometric Parameters of the Toy Models

|       | $a_{\text{max}}$ (kpc) | $b_{\text{max}}$ (kpc) | $a_{\text{min}}$ (kpc) | $b_{\text{min}}$ (kpc) | $a_{10}$ (kpc) | $b_{10}$ (kpc) | $h$ (kpc) |
|-------|-------------------------|-------------------------|-------------------------|-------------------------|----------------|----------------|-----------|
| Model A | 2.7                     | 1.2                     | 4.7                     | 2.4                     | 8.0            | 3.5            | 1.5       |
| Model B | 3.0                     | 1.4                     | 6.8                     | 4.3                     | 7.9            | 4.6            | 2.0       |

Notes. Columns 1 and 2: major and minor axis of the bar traced by maximum ellipticity. Columns 3 and 4: major and minor axis of the bar traced by minimum ellipticity. Columns 5 and 6: major and minor axis of the bar traced by 10$^\circ$ position angle variation. Column 7: vertical thickness of the bar.
by the identification of the bar. The results of Figure 4 show that
the deprojected $a_{\text{max}}$ tends to overestimate the true face-on $a_{\text{max}}$,
while the deprojected $a_{10}$ is prone to underestimate the true
face-on $a_{10}$. However, results of our toy models do not show
such trends. This is probably related to uncertainties in the bar
measurement. Briefly speaking, in the cases of $a_{\text{min}}$ and $a_{10}$,
the deprojected results underestimate the true face-on value at
low $\phi_{\text{bar}}$. That is because the bar lengths ($a_{\text{min}}$ or $a_{10}$) directly
measured in the inclined images shrink with respect to the true
face-on bar length after projection. Thus, it is reasonable that
the results of our toy models show some different features when
compared to the simulated galaxies. The uncertainties of bar
measurement will be discussed in the next section.

Since the distribution of stars within the bar is too complicated
for a toy model to represent, we did not directly test the
2D image deprojection and other Fourier-based deprojection
methods with our toy models. Fourier decomposition analyzes
the azimuthal variations in the light distribution and compares
the intensity between the bar and interbar regions. It gives us
more information about the material distribution in the bar
component (Fourier $m = 2$ mode). Near the ends of the bar,
Fourier $m = 2$ mode reaches its maximum value. Thus, in
principle, the influence of the bar thickness should be less
important on the Fourier-based methods at small $i$ ($\leq 60^\circ$).

### 4.3. Bar Measurement Uncertainties

Except the uncertainties mentioned above, there are some
uncertainties in the bar measurement which could also influence
the accuracy of deprojection. First, the methods used here
produce bar parameters with noticeable differences. $a_{\text{min}}$ seems
to represent the visual bar length well (Erwin 2005). $a_{\text{max}}$ is
located inside the bar region, which tends to underestimate
the visual bar length (Wozniak et al. 1995; Athanassoula &
Misiriotis 2002). $a_{10}$ is always found in the disk region outside
the bar, which tends to overestimate the visual bar length. Thus,
the accuracy of the deprojection also relies on the choice of the
bar identification method.

From our results of the 2D analytical deprojection, the
deprojected $a_{\text{max}}$ generally overpredicts the true face-on $a_{\text{max}}$.
$a_{\text{min}}$ and $a_{10}$ can either overestimate or underestimate the true
face-on values, depending on $\phi_{\text{bar}}$. After excluding the disk
Figure 15. Illustration of the toy model. The major axis, minor axis, and the height are along the \( Y \), \( X \), and \( Z \) axes, respectively. The dashed line shows the viewing direction (PP'), which passes through the center O. Segment OC is the projection of OP onto the \( X-Y \) plane.

Figure 16. As in Figure 3, but for the 1D analytical deprojection of the bar length in the toy models. (A color version of this figure is available in the online journal.)
particles outside the barred region in our model, we find that $a_{\text{max}}$ measured from particles in the bar is located in the boxy bulge region, where the bar is thickened in the vertical direction. Thus, the uncertainty of the deprojected $a_{\text{max}}$ is mainly affected by the 3D structure of the bar itself. The deprojection results of $a_{\text{max}}$ in the toy model gives the best agreement with our simulation. $a_{\text{min}}$ is quite close to the visually identified bar end. At small $\phi_{\text{bar}}$, the measured $a_{\text{min}}$ in the inclined images is affected by the interplay between the bar and the disk, which is usually smaller than the true face-on $a_{\text{min}}$ after projection. Therefore, the deprojected $a_{\text{min}}$ at small $\phi_{\text{bar}}$ could underestimate the true face-on value. Apparently, the errors in the measure of $a_{10}$ originate from the disk since $a_{10}$ is always larger than $a_{\text{min}}$. $a_{10}$ measured from the inclined images are also smaller than the directly deprojected true face-on $a_{10}$ after projection. That is the reason for the underestimation of deprojected $a_{10}$ at low $\phi_{\text{bar}}$.

For the Fourier decomposition, our results show that this method also produces consistent results. The main reason is due to the bar measurement method, where the bar length is determined by the relative amplitude of the Fourier $m = 2$ mode, i.e., $0.85(I_2/I_0)_{\text{max}}$. This position is closely related to the underlying elliptical annulus with the largest intensity difference between the bar and the interbar region, which usually varies little with $i$. However, the bar–interbar contrast, another method based on the Fourier analysis, has larger uncertainties compared to the Fourier decomposition. The most likely explanation is that this method takes FWHM of the bar–interbar contrast radial profile as the bar length, which actually changes significantly with inclination angle.

Another uncertainty in the bar measurement stems from the irregularity of the ellipticity and PA radial profiles measured from the deprojected images. In some mock images, the measured ellipticity profile is flat in the barred region without a clear peak (e.g., bottom panel in Figure 2), making it hard to identify $a_{\text{max}}$ or $a_{\text{min}}$. Thus, we take $a_{\text{max}}$ as the average value of two radii where the ellipticity decreases to 90% of the typical value in the flat region. $a_{10}$ may also have problems. Model B shows that the measured $e_{10}$ has a drastic change at different $i$ and $\phi_{\text{bar}}$, causing large uncertainties in the 2D analytical deprojection of $a_{10}$ and $e_{10}$. We try to use $a_{5}$ (5° PA deviation) instead of $a_{10}$, but it does not make a huge difference.

5. CONCLUSION

In this work, we use two simulated galaxies to investigate uncertainties of bar deprojection. The simulated barred galaxies are projected onto a 2D plane with different bar orientations and disk inclinations. The bar properties are measured with three different tracers, i.e., the maximum ellipticity ($a_{\text{max}}$, $e_{\text{max}}$), minimum ellipticity ($a_{\text{min}}$, $e_{\text{min}}$), and 10° PA variation ($a_{10}$, $e_{10}$). Comparing the deprojected parameters with the intrinsic face-on values, we find that the uncertainties increase with increasing $i$. When $i$ is larger than 60°, all deprojection methods fail badly.

Among all the deprojection methods tested here, the 1D analytical deprojection has the largest uncertainties (up to $\sim 100\%$).
Figure 18. As in Figure 3, but for the 2D analytical deprojection of the bar ellipticity in the toy models. (A color version of this figure is available in the online journal.)

This method assumes that the bar can be treated as a simple straight line segment, which obviously oversimplifies the structure of the bar. It is not surprising that it has relatively large errors because the projected major axis of the bar does not coincide with the real one in the face-on view. At relatively smaller $i$ ($\leq 60^\circ$), 2D deprojection methods (analytical and image stretching) and Fourier-based methods (Fourier decomposition and bar–interbar contrast) perform reasonably well with uncertainties $\sim 10\%$ in both the bar length and ellipticity. Different bar measurement methods also show systematic differences in the deprojection uncertainty. For $a_{\text{max}}$, both the 1D and 2D methods tend to overestimate the intrinsic bar length, whereas no clear trend can be found for $a_{\text{min}}$ and $a_{10}$. For the ellipticity, as the bar orientation increases from $0^\circ$ to $90^\circ$, the deprojected $e_{\text{max}}$ and $e_{\text{min}}$ from 2D methods transition from underestimation to overestimation, while the deprojected $e_{10}$ is generally underestimated. Bar ellipticity starts to have greater errors at lower inclinations as compared to bar length.

Theoretically, deprojection uncertainties stem from two factors. The uncertainties caused by the measurement of inclination angle and $\phi_{\text{bar}}$ are much smaller compared to the 3D structure of the bar itself. We construct two triaxial toy bar models that can reproduce the results of the 1D and 2D analytical deprojections fairly well; they confirm the vertical thickness of the bar as the main source of the uncertainties. By comparing the projected ellipse of a 3D triaxial bar with that of a planar ellipse, we find that the projected ellipticity difference is $\sim 0.1$ at $\sim 60^\circ$ inclination angle, which increases at larger $i$. Indeed, this difference is the fundamental reason for the deprojection uncertainty.

This is the first work performed on simulated disk galaxies to systematically investigate uncertainties of the deprojection methods, which can provide guidelines for the sample selection and error estimation of future statistical researches on barred galaxies. However, our models can be further improved. For example, including a classical bulge may create an even more realistic ellipticity profile. We will extend our work to more realistic models in the future.

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