An analytical model for precision milling of screw rotor using a disk-like form-milling cutter with multiple inserts

A Arifin\textsuperscript{12} and Yu-Ren Wu\textsuperscript{1}

\textsuperscript{1}Department of Mechanical Engineering, National Central University, 300, Zhongda Rd., Zhongli District, Taoyuan City 32001, Taiwan (R.O.C.)
\textsuperscript{2}Department of Mechanical Engineering Educations, Universitas Negeri Yogyakarta, Indonesia

E-mail: yurenwu@ncu.edu.tw; Tel.: 886-3-4227151 ext. 34332; Fax: 886-3-4224501

Abstract. In screw rotors manufacturing, milling machining can be conducted by using a disk-like form-milling cutter. Design method of a disk-like form-milling cutter with multiple inserts applied on screw rotors machining for twin-screw compressors were presented in this study. An analytical calculation procedure of insert arrangement on the cutter body profile was constructed. Subsequently, the screw rotor cutting simulation was performed by considering the actual cutting conditions based on a horizontal CNC milling machine's established coordinate system. Finally, the simulated rotor profile could be obtained and the normal deviation was evaluated.

1. Introduction

The twin-screw compressors as a high-precision rotating machine, consisting of male and female screw rotors, are widely applied in many varieties of technological processes. There are numerous developments in geometry design, wear-resistance, and lifetime, also modern manufacturing techniques. \cite{1, 2}. Manufacturing for rotors profiles are commonly conducted in three machining process: roughing, hardening, and finishing. Notably, for screw rotors compressors, many manufacturers only consider the two roughing processes by milling and finishing by grinding \cite{3, 4}.

Many methods can be applied to generate a screw rotor profile. Zaytsev and Ferreira\cite{5} proposed a method for rotors profile generation of the twin-screw compressors from a prescribed meshing line. Cao et al. \cite{6} applied a B-spline curve as a composition curve to achieve the fast local adjustment of the rotors tooth profile and design a meshing line female, and male rotors profiles are derived in reverse.

Furthermore, designing a disk-like form-milling cutter with multiple inserts as a particular purpose to screw rotors machining can be acquired according to intermesh both of the rotors principles; however, in this case, the pair meshing is between the screw rotors and form milling cutter. The multiple inserts arrangement evenly to the form milling cutter body can be calculated using a mathematical method. Each insert's cutting-edge positions are defined by the local coordinate system and oriented on the cutter body using the cutter global coordinate system \cite{3, 7}.

The grinding process's cutting simulation can be adapted to screw rotors machining by using a disk-like form-milling cutter. Zhao et al. \cite{2} proposed a mathematical model of screw rotors precision grinding using a computer numerical control (CNC) grinding wheel dressing. Wu and Fan \cite{8} introduced a mathematical model for cutting simulation of screw rotors form grinding on a vertical multi-axis CNC machine. Kang Jia et al. \cite{9} presented a general mathematical model for two-parameter generating machining by adopting the discrete enveloping for machined profile calculation and instant contact.
analysis. Wu and Wei [10] developed a general mathematical model for the generating machining of screw rotors with a worm-shaped tool by applying two-parameter enveloping theory.

In this study, by using an analytical method, a simulated screw rotor profile could be generated according to the intermeshing condition among milling cutter and rotor on the screw rotor machining by using disk-like form-milling cutters with multiple inserts. Besides, the simulated rotor profile's normal deviation was evaluated to prove that the analytical model is practicable. Firstly, the stock and milling cutter profile curves were calculated according to the envelope theory of gearing and the shifted screw rotors’ profile. Next, the multiple inserts were arranged on the cutter body evenly distributed. Furthermore, a cutting simulation was performed using the designed disk-like form-milling cutter, considering the cutting conditions on the real machining. Finally, the simulated screw rotor profile could be obtained, and the normal deviation was evaluated.

2. Design method of a disk-like form-milling cutter

2.1. General coordinate system of rotor and cutter wheel relative motion

During the machining process on a horizontal multi-axis CNC milling, the screw rotor performs a helical motion on the workbench, and the cutter wheel rotates around its axis. Theoretically, when the cutter wheel is cutting the rotors tooth surface, an instant contact line will be formed between both, assuming that both of them must be in tangential and the normal vector of each contact point must get through the cutter wheel. The cutter wheel's profile curve can be obtained by using the contact line to sweep around the cutter wheel. The coordinate system of relative motion between the screw rotor and cutter wheel, as shown in Figure 1.

![Figure 1](image1.png)

**Figure 1**. General coordinate system of relative motion among screw rotor and cutter wheel

2.2. Screw rotors profile generation

The design quality of the rotor tooth profile is unique; it depends on the function. Assuming that the rotor tooth profile is given as a data point set according to its design quality, the Cubic-Spline is applied as a curve fitting method. To control the normal error of generated tooth profile on the screw rotor machining, a declination angle in the initial position of rotor tooth profile discrete point data needs to be set up [8], as shown in Figure 2(a). Then the shifted rotor tooth profile \( r_\alpha \) can be acquired by adding the normal vector magnitude to the rotated tooth profile by declination angle \( \mu \) as shown in Figure 2(b).

![Figure 2](image2.png)

**Figure 2** Female rotor: (a) rotor profile declination, (b) the shifted rotor transverse section profile
The screw rotor surfaces can be formed by the helical motion in the transverse tooth profile along the axial direction. Based on the transverse tooth profile of shifted rotor \( r_a [x_a(u), y_a(u)] \), the rotor tooth surfaces \( r_s \) and its unit normal vector, \( n_s \), are calculated as follows:

\[
\begin{align*}
    r_s(u, \theta) &= \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} = \begin{bmatrix} x_a(u) \cos \theta + y_a(u) \sin \theta \\ -x_a(u) \sin \theta + y_a(u) \cos \theta \\ sp \theta \end{bmatrix} \\
    n_s(u, \theta) &= \frac{\partial r_s^s \times \partial u r_s^s}{|\partial r_s^s \times \partial u r_s^s|}; \quad r_s^s = [x_s(u, \theta), y_s(u, \theta), sp \theta, 0]
\end{align*}
\]

(1)

(2)

where \( u \) and \( \theta \) are the surface variables of the rotors and parameter \( sp \) is the rotors unit lead.

2.3. Generation of stock and milling cutter wheel profile

The stock curve calculation can be conducted by meshing conditions among the shifted screw rotors and cutter wheel, and then the milling cutter curve was acquired by normal shifting method to the stock curve. Shifted screw rotors is assumed as the milling machining product result with a particular allowance dimension, which will be conducted on the finishing by grinding.

According to the general coordinate system as shown in Figure 1, further the defined equation of the rotor surfaces \( r_s \) and the unit normal vector \( n_s \) is transformed to the cutter wheel coordinate system \( S_2 \) from the rotor motion coordinate system and then the stock curve \( r_2 \) and the unit normal vectors \( n_2 \) can be defined as follows:

\[
\begin{align*}
    r_2(u, \theta, \varphi) &= M_{2a}(\varphi).r_s(u, \theta) \\
    n_2(u, \theta, \varphi) &= M_{2a}(\varphi).n_s(u, \theta)
\end{align*}
\]

(3)

(4)

where \( M_{2a} \) is the transformation matrix from the rotor motion coordinate system \( S_a \) to the milling cutter coordinate system \( S_2 \). \( \gamma \) is the setting angle of cutter wheel, \( \varphi \) is the rotor rotation angle, \( E_d \) is the shortest center distance among the rotor and cutter wheel, \( L_d \) is the cutter wheel translation along the rotor axis.

According to the theory of gearing, the meshing equation of the instant contact line can be derived as follows:

\[
f_2(u, \theta, \varphi) = n_2 . (k \times r_2) = 0
\]

(5)

where \((k \times r_2)\) means the tangent vector at any points on the cutter wheel profile, \( k = [0, 0, 1, 0] \) is the unit vector of the \( z_s \)-axis. The longitudinal parameter of the corresponding rotor tooth surfaces \( \theta \) can be obtained by substitute the parameter rotor tooth profile \( u \) to the value was obtained by using the Cubic-Spline before, and the rotor rotation angle \( \varphi = 0 \), then substitute all obtained parameter value to solve the Eq. (3) and (4). Furthermore, the instant contact position and unit normal vector of the rotor and stock curve will be obtained, and a single complete tooth can be obtained.

The axial-section profile of the stock curve at \( S_2 \) coordinate system can be constructed. The equation for the discrete points in the axial-section profile of the stock curve \( r_t \) can be defined in Eq. (6), where \( R_2 \) and \( z_2 \) are the coordinate values in the axial-section profile of the stock curve.

The milling cutter profile \( r_c \) can be acquired by finding the unit normal vector \( n_c \) of the stock curve profile \( r_t \) and then multiplied by cutter shifting coefficient \( s_1 \) that obtained from experimental work by Hanbell Company; it will be acquired normal shifting magnitude, \( n_{t1} \). Furthermore, to derive the milling cutter curve profile \( r_c \), adding the normal shifting magnitude \( n_{t1} \) to the stock curve profile \( r_t \) with the results that shifted rotors surface profile can be derived in Eq. (7-9).
\[ \mathbf{r}_t(u, \theta(u)) = \begin{bmatrix} x_t \\ y_t \\ z_t \\ 1 \end{bmatrix} = \begin{bmatrix} \sqrt{(x_2^*)^2 + (y_2^*)^2} \\ 0 \\ z_2^* \\ 1 \end{bmatrix} \]  

(6)

\[ n_t(u) = \frac{\partial r_t \times k}{|\partial r_t \times k|}, \quad k = [0, 0, 1, 0] \]  

(7)

\[ n_{t1}(u) = n_t(u) \times \sigma_1 \]

(8)

\[ r_c(u) = n_{t1}(u) + r_t(u) \]  

(9)

2.4. Analytical procedure of insert arrangement on the cutter body profile

The insert cutting edge geometry is defined in the local coordinate system of each insert, and the cutting edge locations are defined mathematically. The inserts position can be defined by projection the inserts cutting edge from the local coordinate system onto the milling cutter body profile on the axial plane. Each insert can be positioned on the cutter body mathematically by providing the coordinates of the insert center to the cutter body center [3, 7].

Figure 3 shows the local coordinate system of the insert’s cutting edge [3].

Figure 4. Coordinate system and motion relation of the horizontal CNC milling

Figure 3 shows the local coordinate system of the insert’s cutting edge and then can be described the spatial relation among the insert and the disk-like form-milling cutter body as follows: inclination \( \sigma \), which rotates around the \( x_5 \)-axis; rotational angle \( \zeta \), which rotates around the \( y_4 \)-axis; radial offset \( R_{bc} \); axial offset \( Z_{bc} \); and index angle \( \vartheta \), which rotates around the \( z_2 \)-axis. The inserts cutting edge can be expressed in coordinate system \( S_d \), where \( \tau \) is the insert curve parameter. The multiple inserts can be distributed on the disk-like form-milling cutter when all the parameters in Eq. (10) have solved, where \( M_{2d} \) is transformation matrix from insert coordinate \( S_d \) to the milling cutter coordinate system \( S_2 \).

\[ \mathbf{r}_2^{(d)} = \begin{bmatrix} x_2^{(d)} \\ y_2^{(d)} \\ z_2^{(d)} \\ 1 \end{bmatrix} = M_{2d} \cdot r_d, \quad r_d(\tau) = \begin{bmatrix} x_d \\ y_d \\ z_d \\ 1 \end{bmatrix} \]  

(10)
3. Analytical procedure of screw rotor machining on the horizontal CNC milling

This section explains the screw rotors cutting simulation using the designed disk-like form-milling cutter with multiple inserts before by adopting the grinding simulation model and a generating machining method that was applied on the multi-axis CNC machine [2, 8, 9].

3.1. Coordinate system and motion relation of the horizontal CNC milling

Based on the machine coordinate system in the horizontal CNC model as shown Figure 4 can be derived the transformation matrix for relation motion in cutting process among workpiece and disk-like form-milling cutter. The figure also presents the machine motion diagram including three sliding axes such as X, Y, and Z and three rotary axes such as A, B, and C. Where X is the radial sliding axis, Y is the tangential sliding axis, and Z is the axial sliding axis, all the axes with respect to the workpiece. A is the workpiece rotary axis, B is the milling cutter head rotary axis, and C is the milling cutter spindle.

The transformation matrix to present the relation motion in cutting process from disk-like form-milling cutter coordinate system transform to workpiece coordinate system can be acquired through the coordinate system in Figure 4 as follows:

$$M_{je} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

wherein, $a_{11} = \cos\phi_1 \cos\phi_3 + \cos\phi_2 \sin\phi_1 \sin\phi_3$, $a_{12} = -\cos\phi_3 \sin\phi_1 + \cos\phi_1 \cos\phi_2 \sin\phi_3$, $a_{13} = \sin\phi_2 \sin\phi_3$, $a_{14} = C_x \cos\phi_3 + C_y \sin\phi_3$, $a_{21} = \cos\phi_2 \cos\phi_3 \sin\phi_1 - \cos\phi_1 \sin\phi_3$, $a_{22} = \cos\phi_1 \cos\phi_2 \cos\phi_3 + \sin\phi_1 \sin\phi_3$, $a_{23} = \sin\phi_2 \cos\phi_3$, $a_{24} = C_y \cos\phi_3 - C_x \sin\phi_3$, $a_{31} = -\sin\phi_1 \sin\phi_2$, $a_{32} = -\cos\phi_1 \sin\phi_2$, $a_{33} = \cos\phi_2$, $a_{34} = C_z$.

The mechanical setup parameter of each axis on the horizontal multi-axis CNC machine can be expressed as a function of the motion parameter according to the workpiece rotation angle $\varphi$ as follows:

$$\phi_3(\varphi) = \varphi; \quad \phi_2 = \frac{\pi}{2} - \gamma; \quad C_x = E_d; \quad C_y = 0; \quad C_z(\varphi) = L_d(\varphi)$$

3.2. Generation of simulated rotor tooth profile by inserts path

The simulated screw rotor tooth profile can be acquired by referring to the inserts position on the disk-like form-milling cutter body and considered the machine coordinate system as shown Figure 4, the general equation of inserts path $r^{(d)}_3$ in the workpiece coordinate system $S_g$ can be obtained through the inserts position transformation equation $r_d$ as follows:

$$r^{(d)}_3(\varphi, \psi, \tau) = M_{je}(\varphi, \psi) \cdot M_{2d} \cdot r_d(\tau)$$

where $\tau$ is the insert curve parameter, $\varphi$ is the workpiece rotational angle, and $\psi$ is the cutter rotational angle.

To determine the multiple inserts path, it needs to consider the horizontal CNC machine's movements during the machining process, i.e., the workpiece rotational speed, the axial feeding speed of milling cutter, and the milling cutter rotational speed. According to the mechanical setup parameter, the workpiece rotational speed will control the axial feeding speed of the milling cutter on the workpiece's z axes with the corresponding parameters defined on Eq. (12). Whereas the workpiece rotational speed and the milling cutter rotational speed have no direct relationship. However, the multiple inserts path will be obtained from solving the Eq. (13) on the observed rotor length that considers the milling cutter movement on the axial feeding and rotational speed at a particular time. Finally, the simulated rotor
tooth profile $r_3^{(d)}$ can be observed at a single transverse plane of the workpiece surface on an arbitrary position of the rotor length.

3.3. Normal deviation calculation of rotor profile

The generated points on the simulated screw rotor tooth profile are calculated according to inserts path by using the proposed analytical method. In this case, the rotor tooth profile's normal deviation used to verify the proposed analytical method. The transverse tooth profile of shifted rotor $r_S$ defined as a datum curve, then compared with the simulated tooth profile that acquired by solving the Eq. (13). The normal deviation can be calculated by the equation as follow:

$$ (r_S + n_S \delta_n) - r_3^{(d)} = 0 $$  \hspace{1cm} (14)

where $r_s$ and $n_s$ are the position vector and the unit normal vector of the datum surface, respectively, and $\delta_n$ is the normal deviation.

4. Numerical Example

To demonstrate the proposed method, a pair of male and female screw rotors is applied as the example for generate the disk-like form-milling cutters with multiple inserts and cutting simulation. The rotor geometric parameter and the machine setup is given in Table 1.

| Item (unit)                      | Symbols | Male rotor | Female rotor |
|---------------------------------|---------|------------|--------------|
| Tooth number                    | $N$     | 5          | 6            |
| Declination angle (degree)      | $\mu$   | 5.0        | 12.0         |
| Pitch helix angle (degree)      | $\beta_p$ | 46.0       |              |
| Datum center distance between rotors | $C_d$ | 123.0       |              |
| Rotors screw length (mm)        | $L$     | 108.32     |              |
| Rotors normal shifting (mm)     | $\sigma_0$ | 0.15       |              |
| Rotors outer diameter (mm)      | $d_a$   | 173.84     | 138.99       |
| Helix direction                 | $\varepsilon$ | Right-hand | Left-hand    |
| Setting angle (degree)          | $\gamma$ | -44.0      | 44.0         |
| Wheel inner diameter (mm)       | $d_i$   | 95         |              |
| Stock normal shifting (mm)      | $\sigma_1$ | 0.3        |              |
| Datum center distance between wheel and rotors (mm) | $E_d$ | 181.92     | 164.495      |

The discrete point data of the rotor tooth profile was given from the Hanbell Company. By the Cubic-Spline and normal shifting method, a complete tooth profile curve can be fitted. Then by applying the Eq. (1) and considering the geometric parameter, the shifted rotor profile can be obtained. Furthermore, the stock curve profiles can be obtained by solving the Eq. (2-6), while the milling cutter wheel profile was acquired by solving the curve normal shifting equations on the Eq. (7-9). The stock and milling cutter wheel profiles of both male and female cutter, as shown in Figure 5.

For this instance, round type inserts were applied with the corresponding segment parameters refer to Sandvik Coromant RCHT09T300 PL Grade 1025 Carbide Milling (diameter 9.525 mm and thickness 3.97 mm). By considering the insert local coordinate system, as mentioned in Figure 3, and solved all inserts parameter on the Eq. (10), then the multiple inserts can be arranged on the cutter body evenly distributed, as shown in Figure 6.
Figure 5. Stock and milling cutter wheel profiles of both male and female cutter

Figure 6. Multiple inserts distribution on the cutter body profiles

Based on the input parameter in Table 2 and considering the Eq. (11-12), the screw cutting can be simulated using the stock curve profile $r_c$ as the cutter. The observed rotor length can be defined by giving the range $\varphi$ (min and max value) as the start and end length. Another corresponding parameter, $\psi$, is resolved by solving the Eq. (13), so that the cutter wheel path movement and the multiple insert path can be defined. Define a single transverse plane in between the observed rotor length as a projection layer on the rotor cross-section then the simulated rotor tooth profile can be acquired. Figure 7 shows the cutting simulation result of the screw-cutting of the female rotor. Similarly, the simulated male rotor profile can be obtained.

Table 2. Input parameters of the inserts trajectories calculation

| Parameter                      | Value  |
|--------------------------------|--------|
| Workpiece rotation speed ($n_1$) | 2 rpm  |
| Cutter rotation speed ($n_2$)   | 120 rpm|
| Machining time ($t$)            | 25 second|

Figure 7. The cutting simulation result of female rotor

According to both of the shifted and simulated rotor tooth profile, the normal deviation can be evaluated by applying the Eq. (14). The maximum value for the normal error along the tooth profile is approximately $-0.26$ mm as shown in Figure 8, so the proposed analytical method is verified, and the screw machining simulation by the method is entirely practicable. However, because the deviation is still quite broad, it is necessary to address improvement in the future study.
Figure 8. The normal deviation along the tooth profile curve

5. Conclusion

This paper proposes an analytical method to conduct the screw rotor milling using a disk-like form-milling cutter with multiple inserts. The shifted screw rotors' profile was applied to calculate the stock and milling cutter profile curves according to the enveloping theory of gearing. The inserts cutting edge were distributed along to the body profile of the disk-like form-milling cutter. The numerical example simulates a cutting process to obtain the simulated tooth profile by considering the multiple inserts path as on the real cutting conditions. The normal deviation of both original and simulated tooth profile was verified and shown that the proposed analytical method is entirely practicable.

6. References

[1] Patel, H.H. and V.J. Lakhera, A critical review of the experimental studies related to twin screw compressors. Proceedings of the Institution of Mechanical Engineers Part E-Journal of Process Mechanical Engineering, 2020. 234(1): p. 157-170.

[2] Zhao, Y.Q., et al., Precision grinding of screw rotors using CNC method. International Journal of Advanced Manufacturing Technology, 2017. 89(9-12): p. 2967-2979.

[3] Chiang, C.J. and Z.H. Fong, Design of form milling cutters with multiple inserts for screw rotors. Mechanism and Machine Theory, 2010. 45(11): p. 1613-1627.

[4] Stosic, N., A geometric approach to calculating tool wear in screw rotor machining. International Journal of Machine Tools & Manufacture, 2006. 46(15): p. 1961-1965.

[5] Zaytsev, D. and C.A.I. Ferreira, Profile generation method for twin screw compressor rotors based on the meshing line. International Journal of Refrigeration-Revue Internationale Du Froid, 2005. 28(5): p. 744-755.

[6] Cao, S., et al., Study on the reverse design of screw rotor profiles based on a B-spline curve. Advances in Mechanical Engineering, 2019. 11(10).

[7] Engin, S. and Y. Altintas, Mechanics and dynamics of general milling cutters. Part II: inserted cutters. International Journal of Machine Tools & Manufacture, 2001. 41(15): p. 2213-2231.

[8] Wu, Y.R. and C.W. Fan, Mathematical Modeling for Screw Rotor Form Grinding on Vertical Multi-Axis Computerized Numerical Control Form Grinder. Journal of Manufacturing Science and Engineering-Transactions of the Asme, 2013. 135(5).

[9] Jia, K., et al., A general mathematical model for two-parameter generating machining of involute cylindrical gears. Applied Mathematical Modelling, 2019. 75: p. 37-51.

[10] Wu, Y.R. and W.H. Hsu, A general mathematical model for continuous generating machining of screw rotors with worm-shaped tools. Applied Mathematical Modelling, 2014. 38(1): p. 28-37.