New cosmic accelerating scenario without dark energy

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We propose an alternative, nonsingular, cosmic scenario based on gravitationally induced particle production. The model is an attempt to evade the coincidence and cosmological constant problems of the standard model (ΛCDM) and also to connect the early and late time accelerating stages of the Universe. Our space-time emerges from a pure initial de Sitter stage thereby providing a natural solution to the horizon problem. Subsequently, due to an instability provoked by the production of massless particles, the Universe evolves smoothly to the standard radiation dominated era thereby ending the production of radiation as required by the conformal invariance. Next, the radiation becomes subdominant with the Universe entering in the cold dark matter dominated era. Finally, the negative pressure associated with the creation of cold dark matter (CCDM model) particles accelerates the expansion and drives the Universe to a final de Sitter stage. The late time cosmic expansion history of the CCDM model is exactly like in the standard ΛCDM model, however, there is no dark energy. The model evolves between two limiting (early and late time) de Sitter regimes. All the stages are also discussed in terms of a scalar field description. This complete scenario is fully determined by two extreme energy densities, or equivalently, the associated de Sitter Hubble scales connected by \( \rho_I/\rho_f = (H_I/H_f)^2 \sim 10^{22} \), a result that has no correlation with the cosmological constant problem. We also study the linear growth of matter perturbations at the final accelerating stage. It is found that the CCDM growth index can be written as a function of the Λ growth index, \( \gamma_\Lambda \sim 6/11 \). In this framework, we also compare the observed growth rate of clustering with that predicted by the current CCDM model. Performing a \( \chi^2 \) statistical test we show that the CCDM model provides growth rates that match sufficiently well with the observed growth rate of structure.

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I. INTRODUCTION

The existence of a cosmological constant Λ which can be used in order to explain the recent cosmic acceleration, has brought the following major theoretical problem: within the framework of the quantum field theory the vacuum energy density is more than 120 orders of magnitude larger than the observed Λ value measured by the current cosmological data. This is the so called “old” cosmological constant problem (CCP) [1,2]. The “new” problem [3] asks why is the vacuum density so similar to the matter density just now? Many solutions to both theoretical problems have been proposed in the literature [4-6]. An easy way to overpass the above problems is to replace the constant vacuum energy with a dark energy (DE) that evolves with time. However the nature of DE is far from being understood. Indeed a main caveat of this methodology is the fact that the majority of the DE models appeared in the literature are plagued with no physical basis and/or many free parameters.

Nevertheless, there are other possibilities to explain the present accelerating stage. In particular, the inclusion of the backreaction in the Einstein Field Equations (EFE) via an effective pressure (which is negative for an expanding space-time) opened the way for cosmological applications. In these models, the gravitational production of radiation or cold dark matter provides a mechanism for cosmic acceleration as earlier discussed in [7,8]. As a consequence, several interesting features of cosmologies where the dark sector is reduced due to the creation of CDM matter have been discussed in the last decade [10,12,15].

In brief, the merits of the particle creation scenario with respect to the usual DE ideology are a) the former has a strong physical basis namely nonequilibrium thermodynamics, while the latter (DE) has not and b) the particle creation mechanism unifies the dark sector (dark energy and dark matter), since a single dark component (the dark matter) needs to be introduced into the cosmic fluid and thus it contains only one free parameter. Interestingly, from the viewpoint of a statistical Bayesian analysis models which include only one free parameter should be preferred along the hierarchy of cosmological models [16]. We would like to emphasize here that the only cosmological models (to our knowledge) which satisfy the above statistical condition are:

i) the concordance ΛCDM which however suffers from the coincidence and fine tuning problems [4,6].

ii) the braneworld cosmology of [17] which however does not fit the SNIa+BAO+CMB(shift-parameter) data (see [18]).

(iii) the current particle creation model which simultaneously fits the observational data and alleviates the coincidence and fine tuning problems [12,15].
In this paper, we are proposing a new cosmological scenario which is complete in the following sense: all the accelerating stages of the cosmic evolution are powered uniquely by the gravitational creation of massless (at the very early stage) and massive cold dark matter particles (at the late stages).

In our scenario, the Universe starts from an unstable de Sitter dominated phase \((a \propto e^{H_{\text{de}S}t})\) powered by the production of massless particles, and, as such, there is no horizon problem. Subsequently, it deflates and evolves to the standard radiation phase \((a \propto t^{1/2})\) thereby ending the creation of massless particles. Due to expansion, the radiation becomes subdominant with the Universe entering in the cold dark matter (CDM) dominated era, in which the linear growth of matter fluctuations is taking place in a natural way. Finally, the negative pressure associated with the creation of cold dark matter particles accelerates the expansion and drives the Universe to a final de Sitter stage. In addition, the transition from Einstein-de Sitter \((a \propto t^{2/3})\) to a de Sitter final stage \((a \propto e^{H_{\text{f}S}t})\) guarantees the consistency of the model with the supernovae type Ia data and complementary observations, including the growth rate of clustering. A transition redshift of the order of a few (exactly the same value predicted by \(\Lambda\)CDM) is also obtained.

The paper is structured as follows. In Sec. II, we discuss the basic ideas underlying the particle production in an expanding Universe and set up the basic equations whose solutions describe the complete evolution of our model. In Sec. III, we study the linear growth of perturbations, whereas in Sec. IV, we constrain the growth index through a statistical analysis involving the latest observational results. In Sec. V, a possible scalar field description for all stages is discussed, and, finally, in Sec. VI we summarize the basic results.

II. A COMPLETE COSMOLOGICAL SCENARIO WITH GRAVITATIONAL PARTICLE PRODUCTION

The microscopic description for gravitationally induced particle production in an expanding Universe began with Schrödinger’s seminal paper, which referred to it as an alarming phenomenon. In the late 1960s, this issue was rediscussed by Parker and others based on the Bogoliubov mode-mixing technique in the context of quantum field theory in a curved space-time described by the general relativity theory. Physically, one may think that the (classical) time varying gravitational field works like a ‘pump’ supplying energy to the quantum fields.

In order to understand the basic approach, let us consider a real minimally coupled massive scalar field \(\phi\) evolving in a flat expanding Friedman-Robertson-Walker (FRW) geometry. In units where \(\hbar = k_B = c = 1\), the field is described by the following action

\[
S = \frac{1}{2} \int \sqrt{-g} d^4x \left[ g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - m^2 \phi^2 \right]. \tag{1}
\]

In terms of the conformal time \(\eta\) \((dt = a(\eta)d\eta)\), the metric tensor \(g_{\mu\nu}\) is conformally equivalent to the Minkowski metric \(\eta_{\mu\nu}\), so that the line element is \(ds^2 = a^2(\eta) \eta_{\mu\nu} dx^\mu dx^\nu\), where \(a(\eta)\) is the cosmological scale factor. Writing the field \(\phi(\eta, x) = a(\eta)^{-1} \chi\), one obtains from the above action

\[
\chi'' - \nabla^2 \chi + \left( m^2 a^2 - \frac{a''}{a} \right) \chi = 0, \tag{2}
\]

where the prime here denotes derivatives with respect to \(\eta\). Notice that the field \(\chi\) obeys the same equation of motion as a massive scalar field in Minkowski space-time, but now with a time dependent effective mass,

\[
m_{\text{eff}}^2(\eta) \equiv m^2 a^2 - \frac{a''}{a}. \tag{3}
\]

This time varying mass accounts for the interaction between the scalar and the gravitational fields. The energy of the field \(\chi\) is not conserved (its action is explicitly time dependent), and, more important, its quantization leads to particle creation at the expense of the classical gravitational background.

On the other hand, in the framework of general relativity theory, the scale factor of a FRW type universe dominated by radiation \((a \propto t^{1/2})\) satisfies the following relation \([a \ddot{a} + \dot{a}^2 = 0]\), or, in the conformal time, \(a'' = 0\). Therefore, for massless fields \((m = 0)\), there is no particle production since Eq. \ref{eq:conformal} reduces to the same of a massless field in Minkowski space-time, and, as such, its quantization becomes trivial. This is the basis of Parker’s theorem concerning the absence of massless particle production in the early stages of the Universe. Note that Parker’s result does not forbid the production of massless particles in a very early de Sitter stage \((a'' \neq 0)\). Potentially, we also see that massive particles can always be produced by a time varying gravitational field. As we shall see, such features are incorporated in the scenario proposed here.

In principle, for applications in cosmology, the above semiclassical results have three basic difficulties, namely:

(i) The scalar field was treated as a test field, and, therefore, the FRW background is not modified by the newly produced particles.

(ii) The particle production is an irreversible process, and, as such, it should be constrained by the second law of thermodynamics.

(iii) There is no a clear prescription of how an irreversible mechanism of quantum origin can be incorporated in the EFE.

Later on, a possible macroscopic solution for these problems was put forward by Prigogine and coworkers using nonequilibrium thermodynamics for open systems, and by Calvão, Lima & Waga through a covariant
relativistic treatment for imperfect fluids (see also [9]). The novelty of such an approach is that particle production, at the expense of the gravitational field, is an irreversible process constrained by the usual requirements of nonequilibrium thermodynamics. This irreversible process is described by a negative pressure term in the stress tensor whose form is constrained by the second law of thermodynamics1. This macroscopic approach has also microscopically been justified by Zimdahl and collaborators through a relativistic kinetic theoretical formulation [24]. In comparison to the standard equilibrium equations, the irreversible creation process is described by two new ingredients: a balance equation for the particle number density and a negative pressure term in the stress tensor. Such quantities are related to each other in a very definite way by the second law of thermodynamics. Since the middle of the nineties, several interesting features of cosmologies with creation of cold dark matter and radiation have been investigated by many authors [10, 25, 26, 28, 30].

In what follows, as theoretically predicted by inflation and observationally indicated the angular power spectrum of the temperature fluctuations, we consider the EFE for a homogeneous, isotropic, spatially flat universe with gravitationally induced particle production:

\[ 8\pi G \rho = \frac{3}{2} \frac{\dot{a}^2}{a^2}, \] (4)

\[ 8\pi G (p + p_c) = -2 \frac{\ddot{a}}{a} - \frac{a^2}{a^2}, \] (5)

where an overdot means time derivative, \( \rho \) and \( p \) are the dominant energy density and pressure of the cosmic fluid, respectively, and \( p_c \) is a dynamic pressure which depends on the particle production rate. Special attention has been paid to the simpler process termed “adiabatic” particle production. It means that particles and entropy are produced in the space-time, but the specific entropy (per particle), \( \sigma = S/N \), remains constant [8]. In this case, the creation pressure reads \([7, 11]\)

\[ p_c = - \frac{(\rho + p)\Gamma}{3H}, \] (6)

where \( \Gamma \) with units of \((time)^{-1}\) is the particle production rate and \( H = \dot{a}/a \) is the Hubble parameter [13]. In principle, the quantity \( \Gamma \) should be determined from quantum field theory in curved spacetimes by taking into account that particle production is an irreversible process.

But how the evolution of \( a(t) \) affected by \( \Gamma \)? By assuming a dominant cosmic fluid satisfying the equation of state (EoS), \( p = \omega \rho \), where \( \omega \) is a constant, the EFE imply that

\[ \dot{H} + \frac{3}{2} (1 + \omega) H^2 \left( 1 - \frac{\Gamma}{3H} \right) = 0. \] (7)

The de Sitter solution \( (\dot{H} = 0, \Gamma = 3H = \text{constant}) \) is now possible regardless of the EoS defining the cosmic fluid. Since the Universe is evolving, such a solution is unstable, and, as long as \( \Gamma \ll 3H \), conventional solutions without particle production are recovered. From the above equation, one may conclude that the main effect of \( \Gamma \) is to provoke a dynamic instability in the space-time thereby allowing a transition from a de Sitter regime \( (\Gamma \sim 3H) \) to a conventional solution, and vice versa (see subsections A and B below).

A. From an early de Sitter stage to the standard radiation phase

Let us first discuss the transition from an initial de Sitter stage to the standard radiation phase. The main theoretical constraints are:

- **The model must not only solve the horizon problem but also provide a quasiclassical boundary condition to quantum cosmology (a hint on how to solve the initial singularity problem).**
- **Massless particles cannot be quantum-mechanically produced in the conventional radiation phase (Parker’s Theorem).**

To begin with, let us assume a radiation dominated Universe \( (\omega = 1/3, \Gamma \equiv \Gamma_r) \). The dynamics is determined by the ratio \( \Gamma_r/3H \) [see Eq. (7)]. The most natural choice would be a ratio which favors no epoch in the evolution of the Universe \( (\Gamma_r/3H = \text{constant}) \). However, the particle production must be strongly suppressed, \( \Gamma_r/3H \ll 1 \), when the Universe enters the radiation phase. The simplest formula satisfying such a criterion is linear, namely:

\[ \frac{\Gamma_r}{3H} = \frac{H}{H_I}, \] (8)

where \( H_I \) is the inflationary expansion rate associated to the initial de Sitter \( (H \leq H_I) \). It is worth to notice that such particle creation rates have been previously discussed by several authors (see [7, 9, 27, 28] and references therein). It is also worth noticing that for “adiabatic” photon creation the form of the blackbody spectrum is preserved in the course of the expansion [29].

Now, inserting Eq. (5) into Eq. (7) it becomes:

\[ \dot{H} + 2H^2 \left( 1 - \frac{H}{H_I} \right) = 0. \] (9)

The solution of the above equation can be written as

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1 The quantum approach is unable to provide the entropy burst accompanying the particle production since it is adiabatic and reversible.
where $D \geq 0$ is an integration constant. Note that $H = H_I$ is a special solution of Eq. (10) describing the exponentially expanding de Sitter space-time. This solution is unstable with respect to the critical value $D = 0$. For $D > 0$, the universe starts without a singularity and evolves continuously towards a radiation stage, $a \sim t^{1/2}$, when $Da^2 \gg 1$. By integrating Eq. (11), we obtain the scale factor:

$$H_I t = \ln \left( \frac{a}{a_*} \right) + \frac{\lambda^2}{2} \left( \frac{a}{a_*} \right)^2,$$

(11)

where $\lambda^2 = Da^2$ is an integration constant and $a_*$ defines the transition from the de Sitter stage to the beginning of the standard radiation epoch\footnote{This kind of evolution was first discussed by G. L. Murphy\textsuperscript{31} by studying possible effects of the second viscosity in the very early Universe. Later on, it was also investigated in a more general framework involving cosmic strings by J. D. Barrow\textsuperscript{32} who coined the expression “Deflationary Universes”. It has also been discussed in connection with decaying $\Lambda(t)$-models\textsuperscript{33}.}. At early times ($a \ll a_*), when the logarithmic term dominates, one finds $a \simeq a_* e^{H_I t}$, while at late times, $a \gg a_*, H \ll H_I$, Eq. (11) reduces to $a \simeq a_* \left( \frac{H_I t}{2} \right)^{1/2}$, and the standard radiation phase is reached.

It should be noticed that the time scale $H_I^{-1}$ provides the greatest value of the energy density, $\rho_I = 3H_I^2/8\pi G$, characterizing the initial de Sitter stage which is supported by the maximal radiation production rate, $\Gamma_r = 3H_I$. From Eqs. (11) and (10) we obtain the radiation energy density:

$$\rho_r = \rho_I \left[ 1 + \lambda^2 \left( \frac{a}{a_*} \right)^2 \right]^{-2}.$$  

(12)

As expected, we see again that the conventional radiation phase, $\rho_r \sim a^{-4}$, is attained when $a \gg a_*$. Now we pose the following question: how does the cosmic temperature evolve at the very early stages? For “adiabatic” production of relativistic particles the energy density scales as $\rho_r \sim T^4$\textsuperscript{3, 4}, and the above equation implies that

$$T = T_I \left[ 1 + \lambda^2 \left( \frac{a}{a_*} \right)^2 \right]^{-1/2},$$

(13)

where $T_I$ is the temperature of the initial de Sitter phase which must be uniquely determined by the scale $H_I$. We see that the expansion proceeds isothermally during the de Sitter phase ($a \ll a_*$) which means that the supercooling and subsequent reheating that is taking place in several inflationary variants are avoided\textsuperscript{3, 4, 33}. In other words, there is no ‘graceful exit’ problem.

After the de Sitter stage, the temperature decreases continuously in the course of the expansion. For $a \gg a_*$ ($H \ll H_I$), we obtain $T \sim a^{-1}$. Accordingly, the comoving number of photons becomes constant since $n \propto a^{-3}$, as expected for the standard radiation stage.\footnote{Since $n_r \propto T^3$, the average photon concentration reads $n_r = n_I \left[ 1 + \lambda^2 (a/a_*)^2 \right]^{-3/2}$.
}

In this context, we also need to answer to the following question: what about the initial temperature $T_I$? Since the model starts as a de Sitter space-time, the most natural choice is to define $T_I$ as the Gibbons-Hawking temperature\textsuperscript{30} of its event horizon, $T_I = H_I/2\pi$. Naively, one may expect $T_I$ of the same order or smaller than the Planck temperature because of the classical description. From EFE we have $\rho_I = 3m_{Pl}^2H_I^2/8\pi G$ (where $m_{Pl} \approx 1.22 \times 10^{19}$ GeV), and since the energy density is $\rho_I = N_s(T)T_I^4$, one finds $T_I \sim H_I \sim 10^{19}$ GeV (where $N_s(T) = \pi^2 g_*(T)/30$ depends on the number of effectively massless particles).

Naturally, due to the initial de Sitter phase, the model is free of particle horizons. A light pulse beginning at $t = -\infty$ will have traveled by the cosmic time $t$ a physical distance, $d_H(t) = a(t) \int_{-\infty}^{t} \frac{dt'}{a(t')}$, which diverges thereby implying the absence of particle horizons. The latter feature means that the local interactions may homogenize the whole Universe.

Since photons are not produced in the radiation phase, the Big-Bang Nucleosynthesis (BBN) may work in the conventional way\textsuperscript{37, 38}. Subsequently, the Universe enters the cold dark matter [Einstein-de Sitter, $a(t) \propto t^{2/3}$] dominated phase. Finally, we have also verified that a large class of $\Gamma_r$ is capable to overcome the “graceful exit” and fine tuning problem. Indeed we have found that the correct transition from an early de Sitter to the radiation phase is valid even for $\Gamma_r \propto H^n$ with $n \geq 2$ (work in progress). The details are of course different depending on the power $n$, but the qualitative fact of the transition is universal, and to our opinion this is very good news because it shows that it can be a clue for a general graceful exit mechanism.

### B. From Einstein-de Sitter to a late time de Sitter stage

Due to the conservation of baryon number the remaining question is the production rate of cold dark matter particles and the overall late time evolution. In other words, what is the form of $\Gamma_{\text{DM}}$? For simplicity, we consider here only the dominant CDM component.

$$H(a) = \frac{H_I}{1 + Da^2},$$

(10)
In principle, $\Gamma_{dm}$ should be determined from quantum field theory in curved spacetimes. In the absence of a rigorous treatment, we consider (phenomenologically) the following fact:  

$$\Gamma_{dm} \text{ should be determined from quantum field theory in curved spacetimes.}$$

By comparing Eqs. (14) and (15), we see that the same rate, $\Gamma$ does not depend on a given scale but rather it is considered homogeneous. The limiting value of the creation rate, $\Gamma_{dm} = 3H_f$, leads to a late time de Sitter phase ($\dot{H} = 0$, $H = H_f$) thereby showing that the de Sitter solution now becomes an attractor at late times. With this proviso, the solution of Eq. (15) reads:

$$H^2 = H_0^2E(z)^2 = H_0^2\left[\Omega_m(1+z)^3 + \Omega_\Lambda \right],$$

where $\Omega_\Lambda \equiv (H_f/H_0)^2 = 1 - \Omega_m$ is smaller than unity and $1+z = a^{-1}$. Such solution mimics the Hubble function $H(z)$ of the traditional flat $\Lambda$-cosmology, with $\Omega_\Lambda$ playing the dynamical role of $\Omega_\Lambda$ (dark energy appearing in the concordance model). The dark matter parameter ($\Omega_{dm} = 1$) is also replaced by an effective parameter, $\Omega_{eff} \equiv 1 - \Omega_\Lambda$, which quantifies the amount of matter that is clustering. This explains why this model is in agreement with the dynamical determinations related to the amount of the cold dark matter at the cluster scale, and, simultaneously, may also be compatible with the position of the first acoustic peak in the pattern of CMB anisotropies which requires $\Omega_{total} = 1$.

By integrating Eq. (16) we obtain

$$a(t) = \left(\frac{\Omega_m}{\Omega_\Lambda}\right)^{1/3} \sinh^2 \left(\frac{3H_0\sqrt{\Omega_\Lambda} t}{2}\right).$$

Note that the late time dynamics is determined by a single parameter, namely: $\Omega_\Lambda = 1 - \Omega_m$, and is identical to that predicted by the flat $\Lambda$CDM model. Using the current, a joint statistical analysis, involving the latest observational data (SN Ia [41], BAO [42] and CMB shift parameter [40]) is implemented. We find that the overall likelihood function peaks at $\Omega_m = 0.274 \pm 0.011$ with $\chi^2_{total}(\Omega_m) \approx 543.18$ for 557 degrees of freedom. Since the current statistical results are in excellent agreement with those provided by WMAP7 [40], for the rest of the paper we will restrict our present analysis to the choice ($\Omega_m, \sigma_{8,0} = (0.273, 0.811)$, where $\sigma_{8,0}$ is the rms mass fluctuations on scales of $8h^{-1}$ Mpc at redshift $z = 0$.

In Figure 1, we show the overall evolution (radiation, matter and dark energy dominated eras) of our complete cosmological scenario which coincides exactly with the one recently discussed in Refs. [12, 15] following a slightly different approach. Note also that by replacing the value of $\Gamma_{dm}$ into the definition of the creation pressure [see Eq. (10)] one obtains that it is negative and constant ($p_c = -3H_f^2/8\pi G = -3\Omega_\Lambda H_0^2/8\pi G$).

III. THE EVOLUTION OF THE LINEAR GROWTH FACTOR

In this section, we briefly discuss the basic equation which governs the behavior of the linear matter pertur-
bations $\delta_m = 0$ on subhorizon scales in the CDM model, assuming that the particle creation rate remains homogeneous and only the corresponding effective dark matter forms structures. The reason for introducing the growth analysis here is to give the reader the opportunity to appreciate also at the perturbative level, the relative strength and similarities of the CDM and ACDM models used to constrain the growth index. As discussed by Jesus et al. {43} based on the Neo-Newtonian approach 44, the evolution equation of the matter fluctuations of the linear growing mode (usually scaled to unity at the present time). The functions appearing in (18) are defined by:

\[
\frac{d^2 \delta_m}{d\eta^2} + F(\eta) \frac{d\delta_m}{d\eta} + \mathcal{G}(\eta) \delta_m = 0,
\]

whose solution is $\delta_m(\eta) \propto D(\eta)$, with $D(\eta)$ denoting the linear growing mode (usually scaled to unity at the present time). The functions appearing in (18) are defined by:

\[
F(\eta) = \frac{\Omega_m(1 + 6c_p) + 2\Omega_\Lambda e^{3\eta}(8 + 3c_p)}{2(\Omega_m + \Omega_\Lambda e^{3\eta})},
\]

\[
\mathcal{G}(\eta) = \frac{9\Omega_m^2}{2(\Omega_m + \Omega_\Lambda e^{3\eta})^2} + \frac{15\Omega_\Lambda e^{3\eta}(1 + c_p) - 3\Omega_m(2 + c_p)}{\Omega_m + \Omega_\Lambda e^{3\eta}},
\]

where $\eta = \ln a(t)$ and $\Omega_m = 1 - \Omega_r$. The quantity $c_p$ can be viewed as the “effective adiabatic” sound speed

\[
c_p \equiv c_{e f f, a d}^2 = \frac{\delta p_e}{\delta \rho_m},
\]

where $\delta p_e$ is the perturbation of creation pressure. Obviously using the above equation and Eq. (18) we find that the corresponding “effective adiabatic” sound speed must be negative $c_p = c_{e f f, a d} < 0$. Note that the latter restriction is valid also for the interacting dark energy models (see [43]). We would like to stress that for simplicity we are using a constant $c_p$ (for more discussions see [43]). In this context, the functions $F(\eta)$ and $\mathcal{G}(\eta)$ defined above can be written in terms of $\Omega_m(\eta)$ as

\[
F(\Omega_m) = \frac{1 + c_p + \Omega_m(1 - \Omega_m)}{2},
\]

and

\[
\mathcal{G}(\Omega_m) = \frac{9}{2} \Omega_m^2 + 15(1 + c_p)(1 - \Omega_m) - 3(2 + c_p)\Omega_m,
\]

where the definition

\[
\Omega_m(\eta) = 1 - \Omega_\Lambda(\eta) = \frac{\Omega_m}{\Omega_m + \Omega_\Lambda e^{3\eta}},
\]

has been adopted. At this point, we remind the reader that solving Eq. (18) for the ACDM cosmology, we derive the well-known perturbation growth factor (see [46]):

\[
D_\Lambda(z) = \frac{5\Omega_m E(z)}{2} \int_z^{\infty} \frac{(1 + \omega(\eta))}{E^3(\eta)} d\eta.
\]

Obviously, for $E(z) \simeq \Omega_m^{1/2}(1 + z)^{3/2}$ it gives the standard result $D(z) \simeq a = e^\eta = (1 + z)^{-1}$, which corresponds to the matter dominated epoch, as expected.

Now, for any type of DE, an efficient parametrization of the matter perturbations is based on the growth rate of clustering originally introduced by Peebles [46]. This is

\[
f(\eta) = \frac{d \ln \delta_m}{d\eta} \simeq \Omega_m^\gamma(\eta),
\]

which implies

\[
D(a) = \exp \left[ \int_1^{a} \frac{\Omega_m^\gamma(x)}{x} dx \right],
\]

where $\gamma$ is the so called growth index (see Refs. 47–52) which plays a key role in cosmological studies, especially in the light of recent large redshift surveys (like the WiggleZ and SDSS (DR9); see 53 and references therein). As an example, it was theoretically shown that for DE models which adhere to general relativity the growth index $\gamma$ is well approximated in terms of the equation of state parameter $\gamma_{DE} \simeq \frac{1}{6(1 - w - 1)}$ (see 47, 48, 51, 52), which boils down to $\gamma_\Lambda \simeq 6/11$ for the ACDM cosmology $w(z) = -1$. Notice, that in the case of the braneworld model of Dvali, Gabadadze & Porrati 50 we have $\gamma_{DE} \simeq 11/16$ (see also 51, 57), while for the $f(R)$ gravity models we have $\gamma_R \simeq 0.41 - 0.43$ at the present time.

Differentiating Eq. (26) with respect to $\eta$ we have

\[
\frac{df}{d\eta} + f^2 = \frac{1}{\delta_m} \frac{d^2 \delta_m}{d\eta^2}.
\]

Using Eqs. (18), (26) and (28), we find after some algebra

\[
\frac{df}{d\Omega_m} \frac{d\Omega_m}{d\eta} + f^2 + F(\eta)f + \mathcal{G}(\eta) = 0,
\]

6 For the usual ACDM cosmological model we have $\frac{df}{d\Omega_m} \frac{d\Omega_m}{d\eta} + f^2 + X(\eta)f + \mathcal{G}(\eta) = 0$ where $X(\eta) = \frac{1}{2} + \frac{3}{2}[1 - \Omega_m(\eta)]$ and $\mathcal{G}(\eta) = -\frac{1}{2} \Omega_m(\eta)$. 
of excellent approximation to the numerically obtained form using Eq. (31) in Eq. (27) the latter provides an exact between the two approaches is less than 0.1.

The growth data that we will use in this work based on 2dF, VVDS, SDSS and WiggleZ galaxy surveys, for which their combination parameter of the growth rate of structure, \( f(z) \), and the redshift-dependent rms fluctuations of the linear density field, \( \sigma_s(z) \), is available as a function of redshift, \( f(z) \sigma_s(z) \). The \( f \sigma_s \equiv A \) estimator is almost a model-independent way of expressing the observed growth history of the universe [51]. In particular the data used are based on

- The 2dF (Percival et al. [60]), SDSS-LRG

TABLE I: Summary of the observed growth rate and references.

| Index | \( z \)  | \( \text{growth rate (} A_{\text{obs}}\text{)} \) | Refs. | Symbols (in Fig. 3) |
|-------|-------|---------------------|------|---------------------|
| 1     | 0.17  | 0.510 ± 0.060       | [59, 60] | open circles       |
| 2     | 0.35  | 0.440 ± 0.050       | [59, 61] | open circles       |
| 3     | 0.77  | 0.490 ± 0.180       | [59, 62] | open circles       |
| 4     | 0.25  | 0.351 ± 0.058       | [53]  | open triangles      |
| 5     | 0.37  | 0.460 ± 0.038       | [53]  | open triangles      |
| 6     | 0.22  | 0.420 ± 0.070       | [54]  | solid circles       |
| 7     | 0.41  | 0.450 ± 0.040       | [54]  | solid circles       |
| 8     | 0.60  | 0.430 ± 0.040       | [54]  | solid circles       |
| 9     | 0.78  | 0.380 ± 0.040       | [54]  | solid circles       |
| 10    | 0.57  | 0.427 ± 0.066       | [55]  | solid square        |

where

\[
\frac{d\Omega_m}{d\eta} = -3\Omega_m(\eta)\left[1 - \Omega_m(\eta)\right]. \tag{30}
\]

Inserting the ansatz \( f \approx \Omega_m^{\gamma}(\Omega_m) \) into Eq. (29), using simultaneously Eqs. (22), (23), (24) and performing a first order Taylor expansion around \( \Omega_m = 1 \) (for a similar analysis see [51, 52]) we find that the asymptotic value of the growth index to the lowest order is

\[
\gamma \approx \frac{3(13 + 12c_p)}{11 + 6c_p} = \frac{\gamma_\Lambda(13 + 12c_p)}{2(1 + \gamma_\Lambda c_p)}. \tag{31}
\]

Inverting the above equation we have

\[
c_p \approx \frac{13\gamma_\Lambda - 2\gamma}{2\gamma_\Lambda(\gamma - 6)}. \tag{32}
\]

We have checked for various values of \( c_p \) and \( \tilde{\Omega}_m \), that using Eq. (31) in Eq. (27) the latter provides an excellent approximation to the numerically obtained form of \( D(\eta) \) appears in Eq. (13). Indeed the difference between the two approaches is less than 0.1–0.2%. Finally, from the above analysis it becomes clear that the possible difference between the CCDM and \( \Lambda \)CDM predictions is quantified only at the perturbative level, via the value of \( \gamma \), because the two cosmological models share the same Hubble parameter as well as the same number of free parameters, namely the dimensionless matter density at the present epoch \( \Omega_m \) and the growth index. In the case of \( \gamma \equiv \gamma_\Lambda \approx 6/11 \) we find \( c_p^* \approx -1.008 \). Hence expanding Eq. (31) around \( c_p^* \) we can write

\[
\gamma \approx \frac{6}{11} + \gamma_{c_p^*}(c_p - c_p^*), \tag{33}
\]

where

\[
\gamma_{c_p^*} = \frac{dc_p}{dc_p}(c_p^*) = \frac{162}{(11 + 6c_p^*)^2}. \tag{34}
\]

FIG. 2: Left Panel: The variance \( \Delta \chi^2 = \chi^2 - \chi^2_{\text{min}} \) around the best fit \( \gamma \) value for the CCDM cosmology. Note that the cross corresponds to \( (\gamma_\Lambda, \Delta \chi^2_{\text{obs}}) = (0.108, 1) \). Right Panel: The \( \Delta \chi^2 \) versus \( c_p \). The corresponding cross is \( (c_p, \Delta \chi^2_{\text{obs}}) = (-1.008, 1) \).

IV. FITTING THE CCDM GROWTH INDEX TO THE DATA

In the following we briefly present some details of the statistical method and on the observational sample that we adopt in order to constrain either the growth index or the “effective adiabatic” sound speed, presented in the previous section.

A. The growth data
as:  

The different growth data sets are represented by free parameters entering in the theoretical of the cosmological model. In particular, the essential points with the corresponding errors bars.

In Table I we list the precise numerical values of the data points with the corresponding errors bars.

B. Observational constraints

In order to constrain the CCDM growth index (or \(c_p\)) we perform a standard \(\chi^2\) minimization procedure between the growth data measurements (see previous section), \(A_{\text{obs}} = f_{\text{obs}}(z)\sigma_{\text{obs}}(z)\), with the growth values predicted by the CCDM model at the corresponding redshifts, \(A(p, z) = f(p, z)\sigma_s(p, z)\) with \(\sigma_s(p, z) = \sigma_{s, 0}D(p, z)\). The vector \(p\) contains the free parameters of the cosmological model. In particular, the essential free parameters entering in the theoretical

where \(\sigma_i\) is the observed growth rate uncertainty. Note that we sample \(\gamma \in [0.1, 1.3]\) in steps of 0.001.

In the left panel of Fig. 2 we show the variation of \(\Delta \chi^2 = \chi^2 - \chi^2_{\text{min}}\) around the best \(\gamma\) fit value for the CCDM cosmology. We find that the likelihood function peaks at \(\gamma = 0.60 \pm 0.072\) (or \(c_p = -1 \pm 0.011\); see the right panel of Fig. 2) with \(\chi^2_{\text{min}} \approx 7.75\) for 8 degrees of freedom. Notice, that for the physically acceptance range \(0.22 \leq \Omega_m \leq 0.32\), we obtain either \(0.50 \leq \gamma \leq 0.70\) or \(-1.016 \leq c_p \leq -0.983\) with \(\chi^2_{\text{min}} \in [0.90, 1.03]\). Hence the effective sound speed varies very little in function of \(\Omega_m\). Alternatively, considering the ΛCDM theoretical value of \(\gamma(\equiv 6/11)\) and minimizing with respect to \(\Omega_m\) we find \(\Omega_m = 0.243 \pm 0.034\) (see also [22]) with \(\chi^2_{\text{min}}/\text{dof} \approx 7.37/8\).

Our best-fit \(\gamma\) value is in agreement within 1σ (\(\chi^2/\text{dof} \approx 1\)) uncertainty with the theoretically predicted value of \(\gamma \alpha \approx 6/11\) (see the cross in Fig. 2). Finally, in Fig. 3, we plot the measured \(A_{\text{obs}}(z)\) with the estimated growth function, \(A(z) = f(z)\sigma_s(z)\) (see solid line).

We would like to finish this section with the following observation: if the particles are created proportional to the DM density (see, for instance [5, 27, 28]) then at each point the growth of matter density perturbations will speed up with respect to an homogeneous particle creation rate, changing the growth factor and thus it could potentially affect the observational constraints. However, in our case we do not face such a problem because the corresponding particle creation rate term \((\Gamma_{\text{dm}}/H)^2 = (H_f/H)^2\) in the matter dominated era becomes \(\Gamma_{\text{dm}}/3H \propto 1/p\). Notice that in order to obtain the latter relation we use Eq. (4).

V. SCALAR FIELD DESCRIPTION

Matter creation models constitute a possible way to explain the cosmic acceleration without the introduction of a dark energy component. However, it is sometimes desirable to represent the cosmic evolution in a field theoretical language, i.e., in terms of the dynamics of an ordinary scalar field \((\phi)\). In a point of fact, all the dynamical stages discussed here can be described through a simple scalar field model (for a similar analysis see [28]).

To begin with, let us replace \(\rho\) and \(p_{\text{tot}} = p + p_c\) in Eqs. (4) and (5) by corresponding scalar field expressions

\[
\rho \rightarrow \rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_{\text{tot}} \rightarrow p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi). \tag{36}
\]

Inserting the latter into the Friedmann’s equations we can separate the scalar field contributions and express them in terms of \(H\) and \(\dot{H}\), i.e.,

\[
\dot{\phi}^2 = -\frac{1}{4\pi G}\dot{H}, \tag{37}
\]

\[
V = \frac{3H^2}{8\pi G} \left(1 + \frac{\dot{H}}{3H^2}\right) = \frac{3H^2}{8\pi G} \left(1 + \frac{aH}{3H}\right), \tag{38}
\]
where \( \dot{H} = aHH' \) and prime here denotes derivative with respect to the scale factor. Now, considering that \( dt = da/aH \), Eq. (37) can be integrated to give

\[
\phi = \int \left( -\frac{\dot{H}}{4\pi G} \right)^{1/2} dt = \frac{1}{\sqrt{4\pi G}} \int \left( -\frac{H'}{aH} \right)^{1/2} da . \tag{39}
\]

### A. Early de Sitter - radiation: Deflationary stage

In this case the Hubble function is given by Eq. (10). Obviously we can integrate Eq. (39), in the interval \([0,a]\) to obtain

\[
\phi(a) = \frac{1}{\sqrt{2\pi G}} \sin^{-1} \left( \sqrt{Da} \right) , \\
= \frac{1}{\sqrt{2\pi G}} \ln \left( \sqrt{Da} + \sqrt{Da^2 + 1} \right) . \tag{40}
\]

Note that at the time of inflation \((a = a_*)\) the corresponding scalar field is

\[
\phi_* = \frac{1}{\sqrt{2\pi G}} \ln \left( \lambda + \sqrt{\lambda^2 + 1} \right) . \tag{41}
\]

Also after some simple algebra, the potential energy becomes

\[
V(a) = \frac{H_*^2}{8\pi G} \frac{3 + Da^2}{(1 + Da^2)^2} , \tag{42}
\]

or

\[
V(\phi) = \frac{H_*^2}{8\pi G} \frac{3 + \sinh^2(\sqrt{2\pi G} \phi)}{[1 + \sinh^2(\sqrt{2\pi G} \phi)]^2} . \tag{43}
\]

In the context of slow-roll approximation, one can prove that the density fluctuations are of the form \( \delta H \sim H^2/\phi^2 \sim 10^{-5} \) (see Peacock, *Cosmological Physics* 1999). Using Eqs. (10) and (37) the function \( H^2/\phi^2 \) becomes

\[
\frac{H^2}{\phi^2} = -4\pi G \frac{H}{H} = -4\pi G H \frac{a}{aH} = 2\pi G \frac{(1 + Da^2)}{Da^2} . \tag{44}
\]

At the epoch of inflation \((a = a_*)\) we get

\[
\frac{H_*^2}{\phi_*^2}(a_*) = 2\pi G \frac{(1 + \lambda^2)}{\lambda^2} . \tag{45}
\]

Inserting the latter into the density fluctuation constrain we obtain

\[
\frac{1}{\lambda^2} \sim \frac{\delta H}{2\pi G} - 1 \sim \frac{\delta H}{2\pi \rho_{pl}} , \tag{46}
\]

or

\[
\lambda \sim \sqrt{\frac{2\pi}{\delta H} l_{pl}} , \tag{47}
\]

where \( l_{pl} \simeq \sqrt{G} \) is the Planck length in units \( \hbar = c \equiv 1 \).

### B. Accelerating dark matter stage

In this case the late evolution of the Hubble function is given by Eq. (10). Now, inserting Eq. (16) as well as its derivative \((H')\) into Eq. (39) and integrating we obtain:

\[
\phi(a) = A \ln \left[ \frac{\sqrt{1 - \Omega_\Lambda + \Omega_\Lambda a^3} - \sqrt{1 - \Omega_\Lambda}}{\sqrt{1 - \Omega_\Lambda + \Omega_\Lambda a^3} + \sqrt{1 - \Omega_\Lambda}} \right] , \tag{48}
\]

where \( A = 1/\sqrt{24\pi G(1 - \Omega_\Lambda)} \). From Eqs. (16) and (38) it is possible to show that

\[
V(a) = \frac{H_*^2}{8\pi G} \left[ \tilde{\Omega}_\Lambda - \frac{1 + 2\Omega_\Lambda}{2} a^{-3} \right] . \tag{49}
\]

Finally, combining Eqs. (48) and (49) we find

\[
V(\phi) = B + C \cosh(\omega \phi) , \tag{50}
\]

where \( B = H_*^2(5\tilde{\Omega}_\Lambda + \tilde{\Omega}_\Lambda^2)/32\pi G(1 - \tilde{\Omega}_\Lambda) \), \( C = -H_*^2(1 + 2\tilde{\Omega}_\Lambda)\tilde{\Omega}_\Lambda/32\pi G(1 - \tilde{\Omega}_\Lambda) \) and \( \omega = \sqrt{24\pi G(1 - \Omega_\Lambda)} \).

### VI. FINAL REMARKS

A new cosmology based on the production of massless particles (in the early de Sitter phase) and CDM particles (in the transition to a late time de Sitter stage) has been discussed. The same mechanism avoids the initial singularity, particle horizon and the late time coincidence problem of the \( \Lambda \)CDM model has been phenomenologically eliminated \((\Lambda \equiv 0)\) because there is no dark energy in our accelerating scenario. In particular, this means that the dark energy component may be only a “mirage” (an effective description), since it can be mimicked (globally and locally) by the gravitationally induced particle production mechanism acting in the evolving Universe.

In this scenario, the standard cosmic phases - a radiation era followed by an Einstein-de Sitter evolution driven by nonrelativistic matter until redshifts of the order of a few - are not modified. However, the model has two extreme accelerating phases (very early and late time de Sitter phases) powered by the same mechanism (particle creation). Therefore, it sheds some light on a possible connection among the different accelerating stages of the universe. In particular our model, since \( H_T^2 = \Omega_\Lambda H_0^2 \), where \( \Omega_\Lambda \sim 0.7 \) and \( H_0 \simeq 1.5 \times 10^{-42} \text{GeV} \), it sets the ratio of the primeval and late time de Sitter scales to be \( \rho_j/\rho_{pl} = (H_j/H_J)^2 \sim 10^{122} \). This large number is ultimately obtained through a combination of thermodynamics and quantum field theory in curved spacetimes in virtue to the association of the Hawking-Gibbons temperature \((T = H/2\pi)\) to the early de Sitter phase. Due to the maximal radiation production rate, \( \Gamma_{r,\text{max}} = 3H_J \), the model deflates from an unstable de Sitter with initial
expansion rate ($H = H_1$). It is exactly this Gibbons-Hawking connection (and the present value of $H_0$) that lead for the given number under the proviso that the Universe starts with the Planck temperature. In principle, such a result in the present context has no correlation with the so-called cosmological constant problem (in this connection see [32]). In a forthcoming communication, it will be shown that the above ratio is preserved even if the initial de Sitter phase is powered by a production rate term proportional to $H^n, n \geq 2$.

As it appears, the cosmic history discussed here is semiclassically complete. However, there is no guarantee that the initial de Sitter configuration is not only the boundary condition of a true quantum gravitational effect. In other words, the very early de Sitter phase may be the result of a quantum fluctuation which is further semi-classically supported by the creation of massless particles (in this connection see [63] and Refs. therein).

Naturally, the existence of an early isothermal de Sitter phase suggests that thermal fluctuations (within the de Sitter event horizon) may be the causal origin of the primeval seeds that will form the galaxies. Such a possibility and its consequences for the structure formation problem deserves a closer investigation and is clearly out of the scope of the present paper.

We stress that our model provides a natural solution to the horizon problem and finally it connects smoothly the radiation and the matter dominated eras, respectively. At late times it also mimics perfectly the cosmic expansion history of the concordance $Λ$CDM model. In this context, we discuss the behavior of the linear matter perturbations $δ_m$ on subhorizon scales for the CCDM model and the main results are shown in Figs. 2 and 3.

For completeness, we have also represented the evolution of our model in terms of the dynamics of an ordinary scalar field ($φ$) and derived analytically the scalar field potential for two regimes: (i) when the Universe evolves from an early de Sitter to a Radiation phase and (ii) when it goes to the CDM phase to a late time de Sitter stage.

At present, we also know that a more complete version of the late time evolution must be filled with CDM ($\sim 96\%$) and baryons ($\sim 4\%$), and, unlike a cosmology, the baryon to dark matter ratio is a redshift function [12][13]. In particular, this means that studies involving the gas mass fraction may provide a crucial test of our scenario, potentially, modifying our present view of the dark sector. Some investigations along the above discussed lines are in progress and will be published elsewhere.

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