Vacuum-dressed cavity magnetotransport of a 2D electron gas

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We present a theory predicting how the linear magnetotransport of a two-dimensional electron gas is modified by the presence of a passive cavity resonator where no real photon is injected. For a cavity photon mode with in-plane polarization, the dc bulk magnetoresistivity of the 2D electron gas is anisotropic. In the regime of high filling factors of the Landau levels, the envelope of the Shubnikov-de Haas oscillations is profoundly modified. In the limit of low magnetic fields, the resistivity along the cavity-mode polarization direction is enhanced in the ultrastrong light-matter coupling regime.

The physics of strong light-matter coupling has been attracting the interest of a large community thanks to the manipulation of entangled quantum states in cavity [1, 2] and circuit QED [3, 4], as well as for the control of linear and nonlinear optical properties in polaritonic systems [5]. Recently, a few experiments have suggested that electron transport can be modified due to the strong light-matter coupling in disordered molecular films embedded in metallic resonators [6]. Interesting theoretical analysis of these complex systems has been based on simplified models describing a one-dimensional chain of two-level systems coupled to a cavity photon mode [7, 8]. Many questions remain open on the so-called vacuum-dressed systems coupled to a cavity photon mode [9], as demonstrated by several remarkable experiments [10–15]. The magnetotransport of a bare high-mobility 2DEG displays a rich phenomenology: for relatively-low magnetic fields, the bulk Drude-like longitudinal magnetoresistivity exhibits Shubnikov-de Haas oscillations [16]; for high magnetic fields, the bulk becomes an insulator and the transport is dominated by the edge states with the emergence of the quantum Hall effects [17].

Here, we present a theory predicting how a passive cavity resonator modifies the bulk linear magnetotransport of an embedded 2DEG in the dc regime where no real photon is injected. We determine the magnetoconductivity tensor via a linear response Kubo approach, consistently including the diamagnetic current contribution associated to the cavity mode, which is assumed to have an in-plane polarization. We show that, due to the cavity coupling, the magnetoresistivity tensor becomes anisotropic and the envelope of the Shubnikov-de Haas oscillations is drastically modified. Predictions for realistic parameters are presented and future perspectives are discussed.

Let us start by introducing the light-matter Hamiltonian describing a 2DEG coupled to a cavity mode in the presence of a perpendicular magnetic field $B$:

$$\hat{H}_{\text{lm}} = \hbar \omega_{\text{cav}} \hat{a}^\dagger \hat{a} + \sum_i \frac{1}{2m_*} \left( \hat{p}_i + e\hat{A}_i \right)^2$$

being $\hat{a}^\dagger$ the creation operator of a cavity photon in the considered mode, $e$ the electron charge and $m_*$ the effective electron mass. The sum runs over all the electrons, being $\hat{p}_i$ the momentum operator for one electron and $\hat{A}_i$ the electromagnetic vector potential operator. We choose a gauge for the electromagnetic field with zero scalar potential and vector potential

$$\hat{A}_i = \{0; B\hat{e}_z; 0\} + A_0 (\hat{\alpha} + \hat{\alpha}^\dagger) \hat{e}_{\text{cav}},$$

giving a static magnetic field $B$ perpendicular to the plane and an electric field operator $\hat{E}_{\text{cav}} = i \omega_{\text{cav}} A_0 (\hat{\alpha} - \hat{\alpha}^\dagger) \hat{e}_{\text{cav}}$ [18, 19] with $\hat{e}_{\text{cav}} = \{1; 0; 0\}$. Hence, we are considering a spatially uniform cavity mode polarized along $x$ in the region where the 2DEG is located. This is an excellent approximation for what experimentally achieved using metamaterial resonators [10, 11, 13].

In the following, we will consider a 2DEG living on a rectangular area $A = L_x L_y$ with periodic boundary conditions along the $y$ direction [20]. A sketch of the system is presented in Fig. 1(a). The one-electron eigenstates for $A_0 = 0$ correspond to the Landau levels, with eigenfunctions $\psi_{nk}(x, y) \propto H_n \left( \frac{x-x_c}{\ell_{\text{cyc}}} \right) \exp \left[ -\frac{(y-y_c)^2 + 2\pi x y}{2\ell_{\text{cyc}}^2} \right]$ and energies $E_{nk} = (n\hbar\omega_{\text{cyc}} + 1/2)$, where $\omega_{\text{cyc}} = eB/m_*$ is the cyclotron frequency, $\ell_{\text{cyc}} = \sqrt{\hbar/(eB)}$ the magnetic length and $x_c = -2\pi k \ell_{\text{cyc}}^2 / L_y$ the orbit center position. The quantum number $n$ is a non-negative integer, while $k$ is an integer such that $|k| < A/\langle 4\pi \ell_{\text{cyc}}^2 \rangle$. Each level has degeneracy $N_{\text{deg}} = A/(2\pi \ell_{\text{cyc}}^2)$. In the following, we will omit the spin degrees of freedom (we consider magnetic fields where the Zeeman splitting can be neglected).

In the framework of second quantization, we introduce the fermionic operator $\hat{c}_{nk}^\dagger (\hat{c}_{nk})$ which creates (annihilates) an electron in the single-particle state $|nk\rangle$. The
following expressions hold:

\[ \sum_i \hat{x}_i = \sum_{n\kappa} x_n \hat{c}_n^\dagger \hat{c}_n + \frac{\hbar}{\sqrt{2}} \sum_{n\kappa} n \left( \hat{c}_n^\dagger \hat{c}_{n-1} + \text{H.c.} \right), \]

\[ \sum_i \hat{p}_{x,i} = -i \frac{\hbar}{\sqrt{2} \ell_{\text{cyc}}} \sum_{n\kappa} \sqrt{n} \left( \hat{c}_n^\dagger \hat{c}_{n-1} + \text{H.c.} \right), \]

\[ \sum_i \hat{p}_{y,i} = -eB \sum_{n\kappa} x_n \hat{c}_n^\dagger \hat{c}_n. \]

(3)

Thanks to (3), we can recast the light-matter Hamiltonian (1) as

\[ \hat{H}_\text{lm} = \hbar \omega_\text{cav} \hat{a}^\dagger \hat{a} + \hat{H}_e + \hat{H}_I + \hat{H}_D, \]

where the bare electron contribution is \( \hat{H}_e = \hbar \omega_{\text{cyc}} \sum_{n\kappa} n \hat{c}_n^\dagger \hat{c}_n \). It is convenient to introduce the bright collective excitation operator

\[ \hat{b}^\dagger = \frac{1}{\sqrt{N_e}} \sum_{n\kappa} \sqrt{n} \hat{c}_n^\dagger \hat{c}_{n-1}, \]

so that the light-matter coupling \( \hat{H}_I \) can be written as

\[ \hat{H}_I = i \hbar \Omega (\hat{a} + \hat{a}^\dagger) (\hat{b} - \hat{b}^\dagger), \]

while the diamagnetic energy term \( \hat{H}_D \) is given by

\[ \hat{H}_D = \frac{\hbar \Omega^2}{\omega_{\text{cyc}}} (\hat{a} + \hat{a}^\dagger)^2. \]

Both \( \hat{H}_I \) and \( \hat{H}_D \) depend on \( \Omega \), the collective polariton Rabi frequency, defined as

\[ h \Omega = e A_0 \sqrt{N_e \frac{\hbar \omega_{\text{cyc}}}{2m_*}}, \]

(8)

where \( N_e \) is the total number of electrons.

For non-integer filling factor \( \nu = N_e/N_{\text{deg}} \), the bare many-body ground state of \( \hat{H}_e \) is degenerate. We call \( \bar{n} \) the Landau quantum number of the Landau band partially filled by \( \bar{N}_{\bar{n}} \) electrons, i.e., \( \nu = \bar{n} + \bar{N}_{\bar{n}}/N_{\text{deg}} \). The generic bare ground state of \( \hbar \omega_{\text{cav}} \hat{a}^\dagger \hat{a} + \hat{H}_e \) is

\[ |\text{FS}, \zeta \rangle = \prod_{n<\bar{n}} \prod_\kappa \hat{c}_n^\dagger \prod_{\kappa \in (\bar{n}+1)\zeta} \hat{c}_n^\dagger |\text{vac}\rangle, \]

(9)

where \( |\text{vac}\rangle \) is the electron and photon vacuum. The degeneracy is equal to the number of distinguishable permutations for the \( \bar{N}_{\bar{n}} \) electrons in the \( N_{\text{deg}} \) possible states, namely \( D_{\text{FS}} = \binom{N_{\text{deg}}}{\bar{N}_{\bar{n}}} \). The set \( \{k\}_\zeta \) in Eq. (9) corresponds to the \( \zeta \)-th permutation. For each \( \zeta \), we can identify a bright-excitation sector, spanned by the states \( a_m^* b_s^* |\text{FS}, \zeta \rangle \) \((m, s \in \mathbb{N})\). In the thermodynamic limit \( (N_e \gg 1) \), the bright excitation operator \( \hat{b}^\dagger \) behaves as a bosonic operator. Hence, within the considered bright-excitation sector, \( \hat{H}_e \) can be replaced by the effective bosonic Hamiltonian \( \hat{H}_e \rightarrow \hbar \omega_{\text{cav}} \hat{b}^\dagger \hat{b} \). We emphasize that the light-matter interaction does not couple bright sectors originating from different Fermi seas (i.e., having \( \zeta' \neq \zeta \)). Hence, \( \hat{H}_\text{lm} \) (4) can be block-diagonalized.

To determine the linear response of the 2DEG under the action of an electric bias of frequency \( \omega \), we follow a Kubo approach [21]. Knowing the eigenstates \( |\xi\rangle \) and energies \( E_\xi \) of the manybody Hamiltonian, the magnetoconductivity reads:

\[ \sigma_{ij}(\omega) = i \sum_{\xi \neq \xi'} e^{-\beta E_{\xi'}} - e^{-\beta E_\xi} \langle \xi | \hat{J}_j | \xi' \rangle \langle \xi' | \hat{J}_i | \xi \rangle \frac{\omega + i/\tau_{\xi \xi'}}{(-\omega + \omega_{\xi'} + \omega_{\xi})}, \]

(10)

where \( i, j \in \{x, y\} \). In Eq. (10), \( \beta = 1/(k_B T) \) is the inverse thermal energy, \( Z \) the partition function, and \( \tau_{\xi \xi'} \) is the transport scattering time [22]. For homogeneous bulk transport, the current operator entering the matrix elements of the Kubo formula (10) is

\[ \hat{J}_x = -\frac{e}{m_*} \sum_i (\hat{p}_i + eA_i) \] [21]. Using again Eq. (3) we obtain

\[ \hat{J}_x = -\frac{\hbar \omega_{\text{cav}} N_{\text{e}} e^2}{2m_*} \left[ i \hat{b}^\dagger + \frac{2\Omega}{\hbar \omega_{\text{cav}}} (\hat{a} + \hat{a}^\dagger) \right], \]

\[ \hat{J}_y = -\frac{\hbar \omega_{\text{cav}} N_{\text{e}} e^2}{2m_*} (\hat{b} + \hat{b}^\dagger). \]

(11)

Importantly, the current operators depend only on collective bright excitation and cavity mode operators. Therefore, to investigate the effects of the light-matter coupling on the magnetoco nductivity, we can restrict our treatment to the bright sector where the Hamiltonian \( \hat{H}_\text{lm} \) (4) can be exactly diagonalized through a Hopfield-Bogoliubov transformation [9, 23, 24], giving:

\[ \hat{H}_\text{lm} = E_{\text{GS}} + \hbar \omega_{\text{LP}} \hat{p}_{\text{LP}}^\dagger \hat{p}_{\text{LP}} + \hbar \omega_{\text{UP}} \hat{p}_{\text{UP}}^\dagger \hat{p}_{\text{UP}}, \]

(12)
where $E_{GS}$ is the ground-state energy, while $\omega_{LP} (\omega_{UP})$ is the frequency of the lower (upper) polariton excitation, whose bosonic creation operator is $\hat{p}_{LP}^\dagger (\hat{p}_{UP}^\dagger)$. Each ground state is now a polariton vacuum, such that $\hat{p}_{LP}|GS, \zeta\rangle = 0 = \hat{p}_{UP} |GS, \zeta\rangle$. The polariton operators are given by $\hat{p}_r = w_r \hat{a} + x_r \hat{b} + y_r \hat{a}^\dagger + z_r \hat{b}^\dagger$ with $r \in \{LP, UP\}$. The vector $\vec{r}_c = (w_r, x_r, y_r, z_r)^T$ satisfies the eigenvalue equation $M \vec{r}_c = \omega_c \vec{r}_c$, where

$$M = \begin{pmatrix}
\omega_c + 2D & -i\Omega & -2D & -i\Omega \\
-\Omega & -\omega_c & 0 & 2D \\
2D & -\Omega & -\omega_c & -2D \\
-\Omega & \omega_c & 0 & -2D
\end{pmatrix}, \quad (13)$$

being $D = \Omega^2/\omega_c$ due to the diamagnetic term (7). The Hopfield-Bogoliubov coefficients satisfy the normalization condition $|w_r|^2 + |x_r|^2 + |y_r|^2 + |z_r|^2 = 1$. The anomalous coefficients $y_r$ and $z_r$ are different from zero due to the anti-resonant (non-rotating-wave) terms of the light-matter interaction, which become significant in the ultrastrong coupling regime [24]. The electronic (photonic) weight of the polariton created by $\hat{p}_r$ is $W_{e,r} = |x_r|^2 - |y_r|^2$ and $W_{p,r} = |w_r|^2 - |y_r|^2$, given that $W_{e,r} + W_{p,r} = 1$ and it can be shown that $\sum_r W_{e,r} = \sum_r W_{p,r} = 1$. A typical polaritonic dispersion [25] is plotted in Fig. 1(b). Expressing $\hat{b}$ and $\hat{a}$ in terms of the polariton operators via the exact relations

$$\hat{a} = w_{LP}^* \hat{p}_{LP} + w_{UP} \hat{p}_{UP} - y_{LP} \hat{b}_{LP} - y_{UP} \hat{b}_{UP},$$

$$\hat{b} = x_{LP}^\dagger \hat{p}_{LP} + x_{UP} \hat{p}_{UP} - z_{LP} \hat{b}_{LP} - z_{UP} \hat{b}_{UP}, \quad (14)$$

and exploiting exact identities [26], the current operators (11) read:

$$\hat{J}_x = -\sqrt{\hbar \omega_c^2 N_e} \sum_r \frac{i}{2m^*} \frac{\omega_r}{\omega_c} [(x_r - z_r)^* \hat{p}_r - \text{H.c.}],$$

$$\hat{J}_y = -\sqrt{\hbar \omega_c^2 N_e} \sum_r [(x_r - z_r)^* \hat{p}_r + \text{H.c.}]. \quad (15)$$

In the low-temperature limit ($\beta \to +\infty$), the expression (10) for the magnetoconductivity can be simplified since only for $|\zeta| = |GS, \zeta\rangle$ or $|\zeta'| = |GS, \zeta\rangle$ the contribution to the sum is nonzero. Moreover, the only excited states coupled to $|GS, \zeta\rangle$ by the current operators (15) are the polariton states $|LP, \zeta\rangle = p_{LP}^\dagger |GS, \zeta\rangle$ and $|UP, \zeta\rangle = p_{UP}^\dagger |GS, \zeta\rangle$. Thus, we obtain:

$$\sigma_{ij} = \frac{1}{A} \frac{e^{-\beta E_{GS}}}{Z} \sum_{GS} \sum_{r \in \{LP, UP\}} \frac{1 - e^{-\hbar \omega_r / \beta}}{\hbar \omega_r} \left( \langle GS, \zeta | \hat{J}_i | r, \zeta \rangle \langle r, \zeta | \hat{J}_j | GS, \zeta \rangle / (\omega - \omega_r + 1/\tau_r) + \langle GS, \zeta | \hat{J}_j | r, \zeta \rangle \langle r, \zeta | \hat{J}_i | GS, \zeta \rangle / (\omega + \omega_r + 1/\tau_r) \right), \quad (16)$$

where $D_{GS} = D_{FS}$ is the ground-state degeneracy, which is not altered by the light-matter coupling. Due to the form of the $\hat{J}_i$ operators, each $\zeta$ gives the same contribution to $\sigma_{ij}$. Moreover, in the zero-temperature limit, the partition function becomes simply $Z \simeq D_{GS} e^{-\beta E_{GS}}$, allowing us to further simplify Eq. (16). In the dc limit ($\omega \to 0$), the components of the conductivity tensor become

$$\sigma_{ij}^{dc} = \frac{1}{A} \sum_{r \in \{LP, UP\}} \frac{2\tau_r}{\hbar \omega_r} \text{Re} \left[ \Theta_{ij}^{(r)} - \omega_r \tau_r \text{Im} \left[ \Theta_{ij}^{(r)} \right] / (\omega_r \tau_r)^2 \right], \quad (17)$$

where we have introduced

$$\Theta_{ij}^{(r)} = \langle GS, \zeta | \hat{J}_i | r, \zeta \rangle \langle r, \zeta | \hat{J}_j | GS, \zeta \rangle, \quad (18)$$

which does not depend on $\zeta$. Finally, inserting (15) in (18), we find the analytic result

$$\sigma_{ij}^{dc} = \frac{n_e e^2}{m^*} \sum_r \frac{|x_r - z_r|^2 \tau_r}{1 + (\omega_r \tau_r)^2} \left( \frac{\omega_r}{\omega_r + \omega_c} - \frac{\omega_c}{\omega_c + \omega_r} \right), \quad (19)$$

where $n_e = N_e/A$ is the density of electrons. The resistivity tensor $\rho^{dc}$ is obtained by inverting $\sigma_{ij}^{dc}$.

The formula (19) shows that the dc bulk magnetoconductivity tensor of the cavity-embedded 2DEG depends on the cavity-induced change of the ground state (polariton vacuum) and bright excited states (polaritons). Note that the diagonal components are different, an asymmetry due to the in-plane linear polarization of the cavity mode. Another crucial ingredient is the transport scattering time $\tau_r$ entering the Kubo conductivity. This can be written as the sum of two contributions, depending on the electronic and photonic weights as

$$\frac{1}{\tau_r} = \frac{W_{e,r}}{\tau_e} + \frac{W_{p,r}}{\tau_p}, \quad (20)$$

where $\tau_e$ is the electronic transport scattering time (typically due to disorder) and $\tau_p$ is a transport scattering time due to environmental fluctuations affecting the cavity mode (it can be much longer than the cavity photon lifetime). For no cavity coupling ($\Omega = 0$), we recover the standard Drude-like magnetoconductivity tensor [17]:

$$\sigma_{ij}^{dc, \Omega=0} = \frac{n_e e^2}{m^*} \frac{\tau_e}{1 + (\tau_e \omega_c)^2} \left( \frac{1}{\tau_e \omega_c} - \frac{\tau_e \omega_c}{1} \right), \quad (21)$$
In the regime of Shubnikov-de Haas (SdH) oscillations and in the low-temperature limit, the electron transport time can be modeled as \[ \tau_{\text{e}} = \frac{1}{\tau_{\text{c}}} \left[ 1 - 2 \exp \left( -\frac{\pi}{\tau_{\text{c}} \omega_{\text{cyc}}} \right) \cos (\pi \nu) \right], \tag{23} \]
where \( \tau_{\text{c}} \) is the Drude transport time at \( B = 0 \) and \( \tau_{\text{q}} \) is the so-called quantum lifetime.

In Fig. 2, we plot the predictions of our theory for the diagonal components of the resistivity tensor as a function of \( B \). Different curves correspond to different values of \( \Omega_{B=B_{\text{res}}} \), the collective vacuum Rabi frequency \( \Omega \) for \( B = B_{\text{res}} \) such that \( \omega_{\text{cyc}}(B_{\text{res}}) = \omega_{\text{cav}} \). Here we consider \( \tau_{\text{p}} \gg \tau_{\text{c}} \), i.e., the transport scattering time depends only on the electronic weight of the excitations. The dc longitudinal resistivity shows typical SdH oscillations, but the envelope is significantly modified by the coupling to the cavity mode. Panel 2(a) displays the results for the diagonal resistivity along the \( x \)-direction (parallel to the cavity-mode polarization). The main effect here is an overall increase of the resistivity for decreasing magnetic field. Indeed, in the limit of low magnetic field, we analytically derived the following result:

\[
\lim_{B \to 0} \frac{\rho_{xx}^{\text{dc}}}{\rho_{xx,\Omega=0}^{\text{dc}}} = 1 + 4 \left( \frac{\Omega_{B=B_{\text{res}}}}{\omega_{\text{cav}}} \right)^2. \tag{24}
\]

Such enhancement becomes quantitatively important in the ultrastrong light-matter coupling regime. For \( \Omega_{B=B_{\text{res}}} = 0.5 \omega_{\text{cav}} \), the enhancement is exactly a factor 2, in agreement with the numerical plots in Fig. 2(a). Note that such enhancement is already approached for relatively large magnetic fields \( B/B_{\text{res}} \sim 0.5 \). Panel 2(b) displays the results for the diagonal resistivity along the \( y \)-direction. Overall, the amplitude of the oscillations is reduced and around \( B = B_{\text{res}} \), the mean value of the resistivity is suppressed, an effect which is due to the cavity-induced change of the hybrid scattering times \( \tau_{\text{p}} \). However, for \( B \to 0 \) we retrieve the \( \Omega = 0 \) behavior (22).

In Fig. 3, we present our predictions for finite \( \tau_{\text{p}} \), taking \( \Omega_{B=B_{\text{res}}} = 0.5 \omega_{\text{cav}} \). When \( \tau_{\text{p}} > \tau_{0} \), the phenomenology is similar to Fig. 2 (\( \tau_{\text{p}} \gg \tau_{0} \)). When \( \tau_{\text{p}} = \tau_{0} \), the SdH
oscillations become symmetric with respect to their mean value. For $\tau_p < \tau_0$ and relatively large $B$ also the resistivity $\rho_{\nu\nu}^{dc}$ is increased. In the limit of low magnetic fields, however, the phenomenology is robust with respect to the ratio $\tau_p/\tau_0$ for both longitudinal components of $\rho^{dc}$.

In conclusion, we have derived an analytical theory showing how the bulk magnetotransport of a 2DEG can be strongly modified by the coupling to a cavity photon mode with in-plane polarization. The results are remarkable since strong modifications and anisotropy appear in the dc linear resistivity in a regime where no real photons are injected. An intriguing perspective is the study of the quantum Hall regime when the bulk is insulating and the transport is due to the edge states. The findings of the quantum Hall regime when the bulk is insulating and relatively large $B$ also the resistivity $\rho_{\nu\nu}^{dc}$ is increased. In the limit of low magnetic fields, however, the phenomenology is robust with respect to the ratio $\tau_p/\tau_0$ for both longitudinal components of $\rho^{dc}$.

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[25] When the geometric size $L_x$ (and/or $L_y$) is small enough, the Kohn’s theorem [29] for translationally-invariant systems cannot be applied and Coulomb magnetoplasmon corrections [20] of the cyclotron frequency can occur in the limit of low magnetic field.

[26] By definition, the eigenvector $\vec{v}_e = (w_x, x_r, y_r, z_r)^T$ satisfies $\vec{v}_e = \omega_c M^{-1} \vec{v}_e$. Inverting explicitly the matrix $M$, we found $\omega_{cy}(x_r + z_r) + 2i\Omega(w_r - y_r) = \omega_c(x_r - z_r)$.

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