COMPREHENSIVE EXAMINATION OF THE THREE-DIMENSIONAL ROTATING FLOW OF A UCM NANOLIQUID OVER AN EXPONENTIALLY STRETCHABLE CONVECTIVE SURFACE UTILIZING THE OPTIMAL HOMOTOPY ANALYSIS METHOD

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Abstract
This article explores the three-dimensional (3D) rotating flow of Upper Convected Maxwell (UCM) nanoliquid over an exponentially stretching sheet with a convective boundary condition and zero mass flux for the nanoparticles concentration. The impacts of velocity slip and hall current are being considered. The suitable similarity transformations are employed to reduce the governing partial differential equations into ordinary ones. These systems of equations are highly non-linear, coupled and in turn solved by an efficient semi-analytical scheme known as optimal homotopy analysis method (OHAM). The effects of various physical constraints on velocity, temperature, and concentration fields are analyzed graphically and discussed in detail. The impact of hall current is reduced the temperature field whereas increase to the velocity and the concentration fields. The present results are compared with the available results in the literature to check the legitimacy of the present semi-analytical scheme and noted an excellent agreement for limiting cases.

Keywords: Rotating flow, Thermal diffusivity, Nanoparticles diffusivity, Hall effects, OHAM.

1. INTRODUCTION

Boundary layer flow of a viscous fluid due to an impulsive motion over an elastic surface is involved in several areas of science and technology such as “drawing, annealing, and timing of copper wires, rolling and manufacturing of plastic films, and artificial fibers,” etc. In these application processes, the end product primarily depends on the rate of stretching of the surface, which is very significant. The pioneering work of Sakiadis (1961) considering stationary ambient fluid over a moving plate has brought new dimensions to the boundary layer theory. Crane (1970) obtained the closed-form of the exact solution for the velocity distribution and examined the work of Sakiadis (1961) by considering the velocity of the stretching sheet which is proportional to the distance from the slit. Rajagopal et al. (1984) made a comparative analysis between viscoelastic fluid and viscous fluid and examined the rate of cooling of viscoelastic fluid flow over a stretching sheet. Grubka and Bobba (1985) and Lawrence and Rao (1992) extended the work of Rajagopal et al. (1984) for heat transfer characteristics. Further, several researchers have examined the stretching sheet geometry problems for two-dimensional flows (Ganj et al. (2014); Parand et al. (2017); Rahimi et al. (2017) and three-dimensional flows (Hayat et al. (2012); Nadeem et al. (2013); Weidman and Ishak (2015)).

All the above studies confined their examinations to two-dimensional/three-dimensional flows over linearly stretched sheets. On the other hand, the demands of the technological industries and previously mentioned applications are not only confined to a linearly stretched sheet but also to the non-linearly extruded sheet. The stretching of the sheet exponentially is one of the prominent methods to meet the nonlinear requirements of the industry. Given this, Magyari and Keller (1999) and Elbashbeshy (2001) analyzed the flow pattern due to an exponentially continuous stretching sheet.

and Hayat (2008) and Bidin and Nazar (2009) employed HAM/Keller-box method to obtain the analytical/numerical solution to examine the impact of thermal radiation on the flow using exponentially stretching sheet. Further, Swati Mukhopadhyay (2013) investigated that the slip effects over a magnetic flow field with suction/blowing are prominent. Of late, Fazle et al. (2017) and Srinivasacharya and Jagadeeshwar (2017) extended the work of Magyari and Keller (1999) and Swati Mukhopadhyay (2013) by considering radiation effects.

In recent advancement, the nanotechnology is an attractive area of discussion owing to its enriching characteristics of controlling the thermal conductivity. A blend (Solid-Liquid) of very small-sized nanoparticle (<100 nm) and base fluid is known as nanofluid. The colloids of the base fluids are usually made up of metal and oxides, which enhance both the conduction and convection coefficient and also improve significantly the heat transfer rates of the coolants. The nanofluids have emerged as special kinds of many applications in heat transfer such as nuclear reactor cooling, solar water heating, domestic refrigerators, drag reduction, and thermal energy storage, etc. Choi (1995) has initially experimented on the base fluid, which is the addition of the mixture of metal oxides to the base fluid and observed the enhancement of the thermal properties in the fluid. Buongiorno (2006) proposed a most conventional model to describe the convective transport based on the mechanism of Brownian motion with thermophoretic diffusion and remarked that the heat transfer performance has a vital role in the nanofluid. Further, Tiwari and Das (2007) examined two-sided lid-driven differentially heated square cavity filled with nanofluids model which different form conventional Buongiorno (2006) model and explained behavior of particle size, momentum/thermal diffusivity, and temperature. Some recent attempts to describe in this direction are Mustafa et al. (2016); Hayat et al. (2017); Animasaun et al. (2019); Prasad et al. (2018);
Vaidya et al. (2019a, 2019b); Majdi et al. (2019); Puroshotaman et al. (2019) and Amini et al. (2020).

Inspired by subsequent developments in the available literature, our main objective of the present investigation is to analyze the three-dimensional rotating flow of a UCM nanoliquid over an exponentially stretchable surface. Rotating flows usually involve in an anticlockwise flow circulation, geological stretching of tectonic plate beneath the rotating ocean, centrifugal filtration process, in rotor-stator systems, and cooling of skins of high-speed aircraft. The analysis is carried out in the presence of hall effect, velocity slip, convective boundary condition, and zero mass flux nanoparticle concentration. Here, the local similarity equations are derived and solved analytically for varying values of embedded parameters by the semi-analytical technique known as OHAM (see for details, Liao (2010); Marinca and Herisanu (2015); and Van Gorder (2019)). The impacts of different physical parameters on velocity, temperature, and concentration profiles are analyzed graphically. In addition to this, estimations of skin friction, local Nusselt number, and local Sherwood number are presented in the analysis, which is very important from the industrial application point of view.

2. MATHEMATICAL FORMULATION AND PHYSICAL DESCRIPTION OF THE STUDIED FLOW PROBLEM

Let’s consider a steady three-dimensional (3D) rotating flow of a viscous incompressible Upper Convected Maxwell (UCM) nanoliquid by an exponentially stretchable surface subjected to the slip velocity, convective boundary condition and zero mass flux concentration. The Cartesian coordinate system is adopted in such a way that the surface is aligned with x and y-axes and the fluid is taken in the space z ≥ 0 (see Fig. 1).

The fluid is rotating about z-axis with constant angular velocity \( \Omega \). The fluid is considered electrically conducting, and a transverse magnetic field \( B_0 \) is applied in the z-direction. Further, the hall current effect is taken into account. In general, the hall current and the electron pressure gradient are ignored. The generalized Ohm’s law with hall current is defined as

\[
\mathbf{J} = \sigma \left( \mathbf{E} + \mathbf{V} \times \mathbf{B} \right) + \frac{1}{en_c} \mathbf{J} \times \mathbf{B} + \frac{1}{en_c} \mathbf{J} \times \mathbf{B} + \frac{1}{en_c} \mathbf{J} \times \mathbf{B}
\]

where \( \mathbf{J} = \left( J_x, J_y, J_z \right) \) is the current density vector, \( \sigma \) is the electrical conductivity, \( \mathbf{E} \) is the induced electric field resulting from the charge separation, \( \mathbf{V} = \left( u, v, w \right) \) is the velocity vector, \( \mathbf{B} = \left( 0, 0, B_0 \right) \) is the magnetic induction vector, \( e \) is the electric charge, \( n_c \) is the electron number density and \( \rho_c \) is the electronic pressure. Furthermore, there is no applied or polarization voltage is imposed on the flow \( E = \left( 0, 0, 0 \right) \). For weakly ionized gases, ion slip effect and the electron pressure gradient are ignored. The generalized ohm’s law under the above-mentioned conditions for electrically non-conducting sheet \( J_z = 0 \). Hence the Eq. (1) becomes

\[
\begin{align*}
J_x &= \frac{\sigma B_0}{\left( 1 + m^2 \right)} (mu - v), \\
J_y &= \frac{\sigma B_0}{\left( 1 + m^2 \right)} (u + mv).
\end{align*}
\]

Here, the following assumptions are considered.

a) Joule heating is neglected.

b) The wall is impermeable (i.e., \( v_w = 0 \)).

c) The sheet is stretchable with a variable velocity and slip velocity is given by \( u_s(x) = U_0 e^{(x/l)} + k \frac{\partial u}{\partial z} \), where \( U_0 \) is the reference velocity, \( L \) is the characteristic length, \( k \) is the slip constant. The physical problem under consideration includes the connections of momentum, energy, and mass. These relations can be condensed as pursue

\[
\begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0, \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= -2\Omega v.
\end{align*}
\]

\[
\begin{align*}
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} &= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} + 2\nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2} \right), \\
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} &= \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} + 2\nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial x^2} \right),
\end{align*}
\]

with the following realistic boundary conditions (BCs)

\[
\begin{align*}
u &= u_s(x) = U_0 e^{(x/l)} + k \frac{\partial u}{\partial z}, \\
\nu &= v = 0, \\
w &= w = 0, \\
\frac{\partial C}{\partial z} &= \frac{\partial C}{\partial z} = \frac{\partial C}{\partial z} = 0 
\end{align*}
\]

Here, \( u, v, \) and \( w \) are the fluid velocity components along the \( x, y, \) and \( z \)-direction, respectively. Further, \( \nu \) is the kinematic viscosity, \( \lambda_e \) is the relaxation time, \( \rho_f \) is the density of the fluid, \( m \) is the hall effect parameter, \( T \) is the temperature, \( \alpha \) is the thermal diffusivity, \( \tau \) is the ratio of the effective heat capacity of the nanoparticle material and heat capacity of the fluid, \( D_{Th} \) is the Brownian diffusion coefficient, \( C \) is the concentration of nanoparticles, \( D_{Th} \) is the thermophoresis diffusion coefficient, \( T_f \) is the ambient fluid temperature, \( k \) is the thermal conductivity, \( h \) is the heat transport coefficient, \( T_f \) is the hot fluid temperature. The magnetic field \( B_0 \) is considered to be uniform.
2.1 Similarity variables and dimensionless governing equations

To simplify the mathematical analysis of the model by the following similarity variables (see details Hayat et al. (2017)) are evoked,

\[ u = U_e (\xi \zeta)^{1/4} f(\zeta), \quad v = U_e (\xi \zeta)^{1/4} g(\zeta), \quad \frac{w}{\sqrt{2L}} = \frac{U_e}{\sqrt{2L}} (f + \xi \zeta f') , \]

\[ T = T_e + (T_e - T_0) \theta(\zeta), \quad \text{where} \quad (T_e - T_0) = T_0 e^{1/2} \]

\[ C = C_e + (C_e - C) \phi(\xi \zeta), \quad \text{where} \quad (C_e - C) = C e^{1/2}, \quad \zeta = \frac{U_e}{\sqrt{2L}} e^{1/2} , \]

where the prime superscripts represent the differentiation concerning \( \zeta \).

Using the above transformations, the continuity equation given by Eq. (3) is automatically verified, while Eqs. (4) - (7) are reduced to

\[ -\beta (4 f'^2 + f^2 f'' - \xi f' f'' - 6 f' f^* ) = 0, \]

\[ - \frac{Mn}{1 + m^2} (f'^2 + \frac{2m}{3} g^2 + \frac{1}{2} \theta^2) = 0, \]

\[ g'' + \beta f(g' - \xi f' f'' - 6 f' f^*) = 0, \]

\[ - \frac{Mn}{1 + m^2} (f'^2 + \frac{2m}{3} g^2 + \frac{1}{2} \theta^2) = 0, \]

\[ \theta^* + \frac{Pr}{(1 - \beta)} (f \theta^* + \frac{Nt}{Nb} \phi^* + Nb \phi^* - \beta \theta^* ) = 0, \]

\[ \phi^* + \frac{Sc}{(B' - \beta)} (f \phi^* + \frac{Nt}{Nb} \theta^* ) = 0. \]

The local skin-friction coefficients are defined formally as

\[ f(\zeta) = \frac{1}{1 + K_1} (1 - e^{-\zeta}), \quad g(\zeta) = 0, \]

\[ \theta(\zeta) = \frac{Bi}{1 + Bi} e^{-\zeta}, \quad \phi(\zeta) = \frac{Bi}{1 + Bi} \frac{Nt}{Nb} e^{-\zeta}, \]

\[ L_f (\zeta) = f'' - f', \quad L_g (\zeta) = g'' - g, \]

\[ I_0 (\zeta) = \theta^* - \theta, \quad L_0 (\zeta) = \phi^* - \phi. \]

It is worth noting here that the auxiliary linear operators in Eq. (20) satisfy the properties

\[ L_f (A e^{C_f} + A e^{C_f}) = 0, \]

\[ L_g (A e^{C_g} + A e^{C_g}) = 0, \]

\[ L_0 (A e^{C_0} + A e^{C_0}) = 0, \]

where \( A \)'s \((i = 1-9)\) are arbitrary constants.

In the present procedure, we construct the following zeroth-order deformation equations are given by

\[ \frac{f'(0)}{f(0)} = 0, \quad \frac{f'(0)}{f(0)} = 1 + K_1 f'(0), \]

\[ \frac{\theta'(0)}{\theta(0)} = -\frac{Bi}{1-Nt} \theta(0), \quad \frac{\phi'(0)}{\phi(0)} = 0, \]

\[ f'(\infty) = 0, \quad g'(\infty) = 0, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0. \]

In the above expression, \( \lambda \) is the rotation parameter, \( \beta \) is the magnetic parameter, \( \alpha \) is the temperature exponent, \( Nb \) is the Brownian motion parameter, \( Nt \) is the thermophoresis motion parameter, \( Sc \) is the Schmidt number, \( B' \) is concentration exponent, \( K'_1 \) is velocity slip parameter, \( Bi \) is the Biots number and defined as follow

\[ \lambda = \frac{Q_{L}}{u_w}, \quad \beta = \frac{\alpha}{L}, \quad Mn = \frac{2 \sigma R_{f} L_{f}}{\mu}, \quad Pr = v/\alpha, \quad Nb = \frac{\tau_{f}(C_e - C)}{u_w}, \]

\[ Nt = \frac{\tau_{t}(T_e - T_0)}{v_{T_e}}, \quad Sc = \frac{v}{D_a}, \quad K'_1 = \frac{U_{e}}{2 \sqrt{2L}} e^{1/2}, \quad Bi = \frac{h}{k} \frac{2L}{u_w}. \]

2.2 Skin-friction, heat and mass transfer coefficients

The local skin-friction coefficients \((C_f, C_g)\) and the local Nusselt number \(Nu_f\) are defined formally as

\[ C_f = \frac{f'(0)}{u_w}, \quad C_g = \frac{g'(0)}{u_w}, \quad \text{and} \quad Nu_f = -\frac{x q}{(T_e - T_0)}, \]

where \( \tau_{\text{fr}} \) and \( \tau_{\text{w}} \) are the skin-friction (at wall) along \( x \) and \( y \) axis, and \( q \) is the heat flux from the plate are defined as

\[ \tau_{\text{fr}} = \left( \frac{\partial u}{\partial \xi} \right)_{\xi = 0}, \quad \tau_{\text{w}} = \left( \frac{\partial v}{\partial \xi} \right)_{\xi = 0} \quad \text{and} \quad q = \left( \frac{\partial T}{\partial \xi} \right)_{\xi = 0} \]

in terms of non-dimensional quantities are obtained as

\[ C_f = \frac{f'(0)}{u_w}, \quad C_g = \frac{g'(0)}{u_w}, \quad \text{and} \quad Nu_f = -\frac{x q}{(T_e - T_0)}. \]
\[ N_z = \dot{g}(\zeta, \eta) + \dot{j}(\zeta, \eta) \dot{g}(\zeta, \eta) - 2 \dot{j}(\zeta, \eta) \dot{g}(\zeta, \eta) + 4 \dot{j}(\zeta, \eta) + \beta \left[ \frac{\dot{j}(\zeta, \eta) - \dot{g}(\zeta, \eta)}{2} \right] + 2 \beta \left[ \frac{\dot{j}(\zeta, \eta) \dot{g}(\zeta, \eta) - \dot{j}(\zeta, \eta) \dot{g}(\zeta, \eta)}{2} \right] \]

\[ \frac{M_n}{1 + m^2} \left( \frac{\beta}{2} \dot{j}(\zeta, \eta) \dot{g}(\zeta, \eta) \right) \]

\[ N_\zeta = \dot{\phi}(\zeta, \eta) + \dot{\beta}(\zeta, \eta) \dot{\phi}(\zeta, \eta) + \frac{N_t}{N_b} \dot{\phi}(\zeta, \eta). \]

Methodologically, we choose the auxiliary function as
\[ H_\beta(\zeta, \eta) = H_\beta(\zeta) = H_\beta(\zeta) = e^{-\nu} \]

Hence, by setting \( q = 0 \) and \( q = 1 \), in Eq. (22) then we get the solutions as follows
\[ \dot{j}(\zeta, \eta) = \dot{g}(\zeta, \eta), \dot{g}(\zeta, \eta), \dot{g}(\zeta, \eta), \dot{g}(\zeta, \eta), \dot{g}(\zeta, \eta). \]

From Eq. (28), it is clear that \( \dot{j}(\zeta, \eta), \dot{g}(\zeta, \eta), \dot{g}(\zeta, \eta), \dot{g}(\zeta, \eta) \), and \( \dot{g}(\zeta, \eta) \) vary from the initial guesses \( f_\beta(\zeta, \eta), g_\beta(\zeta, \eta), \dot{g}_\beta(\zeta, \eta), \dot{g}_\beta(\zeta, \eta) \) to the final solutions \( f(\zeta, \eta), g(\zeta, \eta), \dot{g}(\zeta, \eta) \) and \( \dot{g}(\zeta, \eta) \) of Eqs. (10) - (13) with BCs (14), when the parameter \( q \) oscillates between 0 to 1. Moreover, the sought solutions \( \dot{j}(\zeta, \eta), \dot{g}(\zeta, \eta), \dot{g}(\zeta, \eta), \dot{g}(\zeta, \eta) \), and \( \dot{g}(\zeta, \eta) \) can be approximated accurately for the variable \( q \) by higher-order expansions with the help of Taylor’s series as follows
\[ \dot{g}(\zeta, \eta) = \frac{d^q f(\zeta, \eta)}{d \zeta^q} \]

\[ \dot{g}(\zeta, \eta) = \frac{d^q g(\zeta, \eta)}{d \zeta^q} \]

\[ \dot{g}(\zeta, \eta) = \frac{d^q \beta(\zeta, \eta)}{d \zeta^q} \]

\[ \dot{g}(\zeta, \eta) = \frac{d^q \phi(\zeta, \eta)}{d \zeta^q} \]

where the convergence of the above series strongly depends upon \( h, h_x, h_y, h_z \) and \( h \). Considering that the control parameter \( h, h_x, h_y, h_z \) are chosen in such a manner that Eq. (22) converges at \( q = 1 \). Then, we have
\[ (f, g, \beta, \phi) = f_0(\zeta, \eta) + \sum_{n=1}^{\infty} f_n(\zeta, \eta) + \sum_{n=1}^{\infty} g_n(\zeta, \eta) + \sum_{n=1}^{\infty} \beta_n(\zeta, \eta) + \sum_{n=1}^{\infty} \phi_n(\zeta, \eta) \]

The \( p \)-th order deformation equations and their corresponding BCs are
\[ f_\beta(\zeta, \eta) - \chi - \eta \dot{f}_\beta(\zeta, \eta) = h, R_\beta(\zeta, \eta), L_\beta(\zeta, \eta), f_\beta(\zeta, \eta) - \chi - \eta \dot{f}_\beta(\zeta, \eta) = h, R_\beta(\zeta, \eta) \]

\[ f_\beta(0) = 0, f'(0) = 0, f''(0) = 0, f'(\infty) = 0 \]

\[ g_\beta(0) = 0, g'(0) = 0, g''(0) = 0 \]

\[ \beta_\beta(0) = 0, \beta'(0) = 0, \beta''(0) = 0 \]

\[ \phi_\beta(0) = 0, \phi'(0) = 0, \phi''(0) = 0 \]

\[ \dot{g}(\zeta, \eta) = \dot{g}(\zeta, \eta) - 2 \dot{j}(\zeta, \eta) \dot{g}(\zeta, \eta) + 4 \dot{j}(\zeta, \eta) + \beta \left[ \frac{\dot{j}(\zeta, \eta) - \dot{g}(\zeta, \eta)}{2} \right] + 2 \beta \left[ \frac{\dot{j}(\zeta, \eta) \dot{g}(\zeta, \eta) - \dot{j}(\zeta, \eta) \dot{g}(\zeta, \eta)}{2} \right] \]

\[ \frac{M_n}{1 + m^2} \left( \frac{\beta}{2} \dot{j}(\zeta, \eta) \dot{g}(\zeta, \eta) \right) \]

\[ N_\zeta = \dot{\phi}(\zeta, \eta) + \dot{\beta}(\zeta, \eta) \dot{\phi}(\zeta, \eta) + \frac{N_t}{N_b} \dot{\phi}(\zeta, \eta). \]

The general solutions of the \( p \)-th order deformation equations are given by
\[ f_\beta(\zeta, \eta) = f_\beta(\zeta, \eta) + A_1 + A_2 e^\nu + A_3 e^{-\nu} \]

\[ g_\beta(\zeta, \eta) = g_\beta(\zeta, \eta) + A_1 + A_2 e^\nu + A_3 e^{-\nu} \]

\[ \beta_\beta(\zeta, \eta) = \beta_\beta(\zeta, \eta) + A_1 + A_2 e^\nu + A_3 e^{-\nu} \]

\[ \phi_\beta(\zeta, \eta) = \phi_\beta(\zeta, \eta) + A_1 + A_2 e^\nu + A_3 e^{-\nu} \]

in which \((f_\beta(\zeta, \eta), g_\beta(\zeta, \eta), \beta_\beta(\zeta, \eta))\) denote the special solutions.

The expressions of exact residual errors are written as follows
\[ \hat{E}_\nu(h) = \int_0^N \left[ f_\nu(h) \sum_{n=1}^{\infty} f_\nu(h) \right] d\zeta \]

\[ \hat{E}_\beta(h) = \int_0^N \left[ g_\nu(h) \sum_{n=1}^{\infty} g_\nu(h) \right] d\zeta \]

\[ \hat{E}_\phi(h) = \int_0^N \left[ \phi_\nu(h) \sum_{n=1}^{\infty} \phi_\nu(h) \right] d\zeta \]

In practice the evaluation of the residual errors are \((\hat{E}_\nu(h), \hat{E}_\beta(h), \hat{E}_\phi(h), \hat{E}_\phi(h))\) consumed a large amount of time. So, instead of computing the exact residual errors, it is feasible to handle the accuracy of the problem using the average residual errors defined by
corresponding to the functions and CPU time(s). Physical parameters are recorded as CPU time (in sec).

Table 2 provides the results obtained for the individual average squared residual error and total residual error at 12th-order approximation. It can be observed that the averaged squared residual error, and total residual errors are consistently reduced as increases the order of approximations. Further, the average squared residual error and total residual error of each governing equations are found in the diminishing function of the order of approximation, as shown in Fig. 2(a).

Table 1 The individual average squared residual error as a function of the number of iterations and CPU time is also listed. Physical parameters are \( Pr = 1.09, Sc = m = 1, Nb = Mn = Bi = 0.5, \)

\[ Pr = 1.09, Sc = m = 1, Nb = Mn = Bi = 0.5, \]

\( A = B = \beta = K_L = 0.2, \) fixed and CPU time is also listed. We obtain the optimal values of convergence control parameters are \( h_i = -1.18152, h_k = -1.29527, h_s = -1.30537, h_5 = -1.46363. \)

\[ E_i^p(h) = \frac{1}{P + 1} \sum_{k=0}^p \left( \sum_{n=0}^N f_k(\zeta) \hat{\phi}(\zeta) \right)^2 \zeta = n\Delta \]

\[ E^p(h) = \frac{1}{P + 1} \sum_{k=0}^p \left( \sum_{n=0}^N f_k(\zeta) \hat{\phi}(\zeta) \right)^2 \zeta = n\Delta \]

\[ E^p(h) = \frac{1}{P + 1} \sum_{k=0}^p \left( \sum_{n=0}^N f_k(\zeta) \hat{\phi}(\zeta) \right)^2 \zeta = n\Delta \]

\[ E^p(h) = \frac{1}{P + 1} \sum_{k=0}^p \left( \sum_{n=0}^N f_k(\zeta) \hat{\phi}(\zeta) \right)^2 \zeta = n\Delta \]

\[ E_i^p = E_i^p + E_k^p + E_s^p + E_5^p \]

where \( E_i^p \) represents the total squared residual error, \( \zeta = n\Delta = k/P, k = 0,1,\ldots,P. \)

Now we minimize the error function \( E_i^p(h), E_k^p(h), E_s^p(h) \) and \( E_5^p(h) \) and record the optimal values of \( (h_i, h_k, h_s, h_5) \). For the required order approximation, the optimal values of \( (h_i, h_k, h_s, h_5) \) corresponding to the functions \( (f, g, \phi, \theta) \) are obtained by utilizing the following mathematical restrictions

\[ \frac{dE_i^p(h)}{dh} = 0, \quad \frac{dE_k^p(h)}{dh} = 0, \quad \frac{dE_s^p(h)}{dh} = 0, \quad \frac{dE_5^p(h)}{dh} = 0 \]  

The convergent series solutions correspond to

\[ \lim_{p \to \infty} E_i^p(h) = 0, \quad \lim_{p \to \infty} E_k^p(h) = 0, \quad \lim_{p \to \infty} E_s^p(h) = 0, \quad \lim_{p \to \infty} E_5^p(h) = 0 \]

Table 1 and 2 provide the results obtained for the individual average squared residual error and total residual error by considering \( (h_i, h_k, h_s, h_5) = (1.8152, 2.9527, 3.0537, 1.4636) \) as the optimal values which have been analyzed by minimizing the averaged residual error and total residual error at 12th-order approximation. It can be observed that the averaged squared residual error, and total residual errors are consistently reduced as increases the order of approximations. Further, the average squared residual error and total residual error of each governing equations are found in the diminishing function of the order of approximation, as shown in Fig. 2(a).

**Table 1** The individual average squared residual error as a function of the number of iterations and CPU time is also listed. Physical parameters are \( Pr = 1.09, Sc = m = 1, Nb = Mn = Bi = 0.5, \theta = 0.1, A = B = \beta = K_L = 0.2, \) fixed and CPU time is also listed. We obtain the optimal values of convergence control parameters are \( h_i = -1.18152, h_s = -1.29527, h_5 = -1.30537, h_5 = -1.46363. \)

| \( P \) | \( E_i^p \) | \( E_k^p \) | \( E_s^p \) | \( E_5^p \) | CPU time |
|-----|-----|-----|-----|-----|-----|
| 2   | 6.09 \times 10^{-3} | 2.86 \times 10^{-3} | 2.57 \times 10^{-3} | 3.53 \times 10^{-6} | 3.51s |
| 4   | 3.59 \times 10^{-3} | 3.89 \times 10^{-4} | 8.28 \times 10^{-7} | 6.19 \times 10^{-7} | 34.9s |
| 6   | 4.96 \times 10^{-4} | 9.74 \times 10^{-4} | 4.40 \times 10^{-9} | 8.93 \times 10^{-8} | 263.18s |
| 8   | 1.17 \times 10^{-4} | 3.55 \times 10^{-5} | 1.31 \times 10^{-6} | 1.69 \times 10^{-5} | 1261.57s |
| 10  | 4.16 \times 10^{-5} | 1.67 \times 10^{-6} | 1.11 \times 10^{-9} | 5.22 \times 10^{-9} | 4898.92s |
| 12  | 1.97 \times 10^{-5} | 9.29 \times 10^{-6} | 3.21 \times 10^{-9} | 2.27 \times 10^{-8} | 24101.19s |

**Table 2** Total averaged squared residual error with no. of iteration and CPU time(s). Physical parameters are \( Pr = 1.09, Sc = m = 1, Nb = Mn = Bi = 0.5, \theta = 0.1, A = B = \beta = K_L = 0.2, \) fixed and recorded as CPU time (in sec).

| \( P \) | \( -\hat{R}_i^p \) | \( -\hat{R}_k^p \) | \( \hat{R}_s^p \) | \( \hat{R}_5^p \) | CPU time |
|-----|-----|-----|-----|-----|-----|
| 2   | 0.96748 1.17805 | 1.21602 1.39283 | 1.05 \times 10^{-2} | 1.81s |
| 4   | 1.05049 1.24464 | 1.20643 1.49248 | 9.36 \times 10^{-3} | 7.86s |
| 6   | 1.10042 1.26462 | 1.17431 1.52095 | 1.75 \times 10^{-4} | 15.46s |
| 8   | 1.13393 1.27572 | 0.95993 1.50951 | 3.38 \times 10^{-3} | 170.42s |
| 10  | 1.15996 1.28557 | 0.81760 1.48142 | 2.52 \times 10^{-3} | 318.52s |
| 12  | 1.18152 1.29527 | 0.71537 1.46363 | 1.39 \times 10^{-3} | 526.91s |

4. **MODEL VALIDATION**

Here, we present the exact solutions in certain special cases. These solutions have much importance due to they provide as a baseline for the comparison with the obtained results in the literature through the numerical solutions. In the absence of some non-dimensional parameters, \( (\beta, m, K_L, A, B) = (0,0,0,0,0,0) \), Eqs. (10) - (13) and BCs. (14) are reduced to those treated by Mustafa et al. (2016), which are

\[ f'' + f' - 2f'' + 4\lambda f = 0, \]

\[ g'' - fg' - 2f' + 4\lambda f = 0, \]

\[ \theta'' + Pr(f' + Nb\phi' + Nt\theta) = 0, \]

\[ \phi'' + Sc f\phi' + \frac{Nt}{Nb} \theta = 0, \]

with boundary conditions are

\[ f(0) = g(0) = 0, f'(0) = 1, g'(0) = 0, \theta(0) = 0, Nt\phi(0) = 0, \]

\[ f'(\infty) = 0, g(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0. \]

To authenticate and validate the exactness of the proposed OHAM procedure, the present outcomes are compared with those reported by
Hayat et al. (2017) for some special cases. The correctness found to be in superior agreement (see Table 3).

Table 3 Comparison results of local Nusselt number and local Sherwood number for various values of $\beta$ and $\lambda$ when $Mn = m = K_i = Bi = 0$.

| $\beta$ | $\lambda$ | Hayat et al. (2017) | Present Results By OHAM |
|---------|-----------|---------------------|--------------------------|
|         | $-\theta'(0)$ | $-\phi'(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
| 0.0     | 0.5323     | 0.5040 | 0.532478 | 0.504141 |
| 0.1     | 0.5239     | 0.4943 | 0.524068 | 0.494511 |
| 0.2     | 0.5172     | 0.4858 | 0.519813 | 0.490602 |
| 0.1     | 0.5239     | 0.4943 | 0.525566 | 0.498645 |
| 0.2     | 0.4988     | 0.4609 | 0.500721 | 0.465365 |

5. RESULTS AND DISCUSSION

The obtained numerical results by means of solving the transformed Eqs. (10) - (13) which are coupled and non-linear are treated with OHAM technique. Thus obtained are analyzed with the help of figures and tables. The results for the horizontal velocity profiles $f'(\zeta)$, transverse velocity profiles $g(\zeta)$, temperature profiles $\theta(\zeta)$ and concentration profiles $\phi(\zeta)$ are presented for various pertinent flow parameters such as rotation parameter ($\lambda$), Deborah number ($\beta$), magnetic parameter ($Mn$), hall current parameter ($m$), Prandtl number ($Pr$), temperature exponent ($A$), Brownian motion parameter ($Nb$), thermophoresis parameter ($Nt$), Schmidt number ($Sc$), concentration exponent ($B$), velocity slip parameter ($K_i$) and Biot’s number ($Bi$) in Figs. 3 - 7. The computed numerical values for the skin-friction coefficient $f^{*}(0)$ and $g^{*}(0)$, the local Nusselt number $\theta'(0)$, and the local Sherwood number $\phi'(0)$ are tabulated in Table 4.

Physically, this dual behavior of the fluid flow is due to the larger values of the rotation parameter $\lambda$ correspond to a higher rotation for the angular velocity $\Omega$, because the term $\lambda$ is defined as $\lambda = \Omega L / u_{c}$.
Further, the smaller Deborah number $\beta$ gives a viscous effect compared to the elastic effect, whereas the larger $\beta$ exhibit in the elastically solid material in nature. With reference to $\theta(\zeta)$, both $\lambda$ and $\beta$ increases the temperature field, which is recorded in Fig. 3(c). The concentration profile exhibits the decreasing trend for $\lambda$ and $\beta$ (see Fig. 3(d)). Figure 4(a) to 4(d) illustrates the impact of the magnetic parameter $Mn$ (presence and absence) and the hall current parameter $m$ on $f'(\zeta), g(\zeta), \theta(\zeta)$ and $\phi(\zeta)$. It is observed that the rising values of $m$ increases $f'(\zeta), g(\zeta)$ and $\phi(\zeta)$ and decreases $\theta(\zeta)$. This phenomenon is obtained due to the effective conductivity $[\sigma/(1+m^2)]$, which decreases with increasing values of $m$. Hence, it reduces the applied magnetic field, and consequently increases $f'(\zeta)$.

Further, for a larger value of hall current parameter $m$, the term $\left[1/(1+m^2)\right]$ becomes smaller and smaller, and the resistive force of the magnetic field is diminished. Besides this, the magnetic parameter $Mn$ reduces the fluid velocity, concentration profile in the boundary region and enhances the temperature field. The reason behind this development is the opposing force known as the Lorentz force, and this force tends to slow down the fluid flow.

**Fig. 4(a)** Horizontal velocity profiles for different values of $Mn$ and $m$ with $Pr=1.1, Sc=1, Nb=Bi=0.5, Nt=\lambda=0.1$, $A=B=K_n=\beta=0.2$.

**Fig. 4(b)** Transverse velocity profiles for different values of $Mn$ and $m$ with $Pr=1.1, Sc=1, Nb=Bi=0.5, Nt=\lambda=0.1$, $A=B=K_n=\beta=0.2$.

**Fig. 4(c)** Temperature profiles for different values of $Mn$ and $m$ with $Pr=1.1, Sc=1, Nb=Bi=0.5, Nt=\lambda=0.1$, $A=B=K_n=\beta=0.2$.

**Fig. 4(d)** Concentration profiles for different values of $Mn$ and $m$ with $Pr=1.1, Sc=1, Nb=Bi=0.5, Nt=\lambda=0.1$, $A=B=K_n=\beta=0.2$.

The influence of the magnetic parameter $Mn$ (in the presence /absence) and velocity slip $K_n$ on $f'(\zeta)$ is elucidated in Fig. 5. The velocity profile $f'(\zeta)$ decreases for varying values of $K_n$. Physically, the higher the value of $K_n$ reduces the kinematic viscosity $\nu$. This nature of the profile is attributed to the fact $K_n = k_n^\nu \sqrt{U_m/2\nu\epsilon^{(1/2)}}$.

**Fig. 5** Horizontal velocity profiles for different values of $Mn$ and $K_n$ with $Pr=1.1, Sc=m=1, Nb=Bi=0.5, Nt=\lambda=0.1$, $A=\beta=0.2$. 
Figure 6(a) demonstrates that the temperature profile \( T(\zeta) \) which reduces for increasing values of \( \text{Pr} \) and \( \text{A} \). Here, the increase in \( \text{Pr} = v / \alpha \) is responsible for lesser thermal diffusivity \( \alpha \) which results in the reduced thermal boundary layer. A similar trend may be observed in the case of \( \text{A} \). The impact of \( \text{Nt} \) and \( \text{Bi} \) on \( T(\zeta) \) is sketched in Fig. 6(b). The terms

\[ \text{Bi} = \frac{h}{k} \sqrt{2 \text{vL} / \text{U}_c} \]

predicts the enhancement in temperature as both \( \text{Nt} \) and \( \text{Bi} \) increase. It is noticed that the fluid temperature is zero when \( \text{Bi} = 0 \) and it is prescribed temperature at the wall when it tends to infinity.

Figure 7(a) explains the impact of \( \text{Nt} \) and \( \text{Nb} \) on \( \phi(\zeta) \). It is noted that the nanoparticle volume fraction increases with the increase in \( \text{Nt} \) (increase in thermophoresis force) and thus, an enhancement in the thickness of the concentration boundary layer is observed. In this case, the nanoparticles move away from the hot stretching sheet towards the cold ambient fluid under the influence of the temperature gradient. But in the case of \( \text{Nb} \) (smaller nanoparticles), the result is the opposite. However, \( \text{Nb} \) will stifle the diffusion of nanoparticles away from the surface, which results in a decrease in nanoparticle concentration values in the boundary layer. Finally, Figure 8(a-c) displays the 3D plot of velocities and these plots exhibit similar results as that of velocity profiles.

Table 4 is tabulated to exhibit the influence of embedding parameters on the skin-friction coefficient, the local Nusselt number, and the local Sherwood number. It is seen that the rising values of \( \text{Nt} \) and \( \text{Bi} \) decreases \( \tau_0(0), \gamma(0) \) and \( \phi(0) \). The effect of \( \text{m} \) shows a quite opposite trend as compared with \( \text{Nt} \) and \( \text{Bi} \).

Further, \( f(0), \gamma(0) \) and \( \phi(0) \) are the decreasing function of \( \text{Mn}, \text{A}, \text{Pr} \) and increasing function of \( K_i \) and \( \text{Sc} \).
Table 4 The values of skin-friction, local Nusselt number and local Sherwood number for various physical parameters with \( Nt = 0.1, Nb = 0.5 \).

| Pr  | A   | B   | Sc  | Bi  | \( \beta \) | Mn | \( \lambda \) | m  | \( K_i \) | \(- f''(0) \) | \(- h_i \) | \( \dot{E}_p' \) | \(- g'(0) \) | \(- h_s \) | \( \dot{E}_p^z \) | \(- \theta'(0) \) | \(- h_p \) | \( \dot{E}_p^x \) | \( \phi(0) \) | \(- h_y \) | \( \dot{E}_p^y \) | CPU time |
|-----|-----|-----|-----|-----|------|----|------|----|-------|----------------|--------|-------------|-----------|-------|-------------|-----------|--------|-------------|-----------|--------|-------------|----------|
| 1.09| 0.2 | 0.2 | 1   | 0.5 | 0.2  | 0.5 | 0.1  | 1   | 0.0   | 1.50171        | 1.09565 | 0.85932      | 1.10171    | 1.15027 | 1.18305      | 1.19386    | 1.04348 | 0.04897     | 1.21201    | 9.05305 | 423s        | 0.06652    |
| 1.09| 0.2 | 0.2 | 1   | 0.5 | 0.2  | 0.5 | 0.1  | 1   | 0.0   | 1.05204        | 1.13393 | 0.92313      | 1.05204    | 1.13933 | 1.17555      | 1.13933    | 1.05204 | 0.05266     | 1.05204    | 9.05305 | 423s        | 0.06652    |
| 1.09| 0.2 | 0.2 | 1   | 0.5 | 0.2  | 0.5 | 0.1  | 1   | 0.0   | 1.05204        | 1.13393 | 0.92313      | 1.05204    | 1.13933 | 1.17555      | 1.13933    | 1.05204 | 0.05266     | 1.05204    | 9.05305 | 423s        | 0.06652    |
| 1.09| 0.2 | 0.2 | 1   | 0.5 | 0.2  | 0.5 | 0.1  | 1   | 0.0   | 1.05204        | 1.13393 | 0.92313      | 1.05204    | 1.13933 | 1.17555      | 1.13933    | 1.05204 | 0.05266     | 1.05204    | 9.05305 | 423s        | 0.06652    |
| 1.09| 0.2 | 0.2 | 1   | 0.5 | 0.2  | 0.5 | 0.1  | 1   | 0.0   | 1.05204        | 1.13393 | 0.92313      | 1.05204    | 1.13933 | 1.17555      | 1.13933    | 1.05204 | 0.05266     | 1.05204    | 9.05305 | 423s        | 0.06652    |
6. CONCLUSIONS

Some of the interesting findings of the present work are summarized below.

- The rotation parameter decreases \( f'(\zeta) \), \( g(\zeta) \), \( \theta(\zeta) \) and \( \theta'(\zeta) \) whereas the hall current parameter exhibits reverse trend.
- A substantial variation in Deborah number reduces \( f'(\zeta) \) and \( \phi(\zeta) \) while \( g'(\zeta) \) and \( \theta'(\zeta) \) rises.
- The enhanced magnetic parameter and velocity slip parameter decreases \( f'(\zeta) \).
- Increased Prandtl number and temperature exponent and smaller magnetic parameter, reduces \( \theta(\zeta) \).
- An increase in \( \phi(\zeta) \) is due to the increase in the Schmidt number, temperature exponent, thermophoresis parameter and the Brownian motion parameter.

NOMENCLATURE

- \( A \): temperature exponent parameter
- \( B \): concentration exponent parameter
- \( B \): magnetic induction vector
- \( Bi \): Biots number
- \( B_h \): magnetic field strength [N m\(^{-1}\) A\(^{-1}\)]
- \( C \): nanoparticles concentration
- \( C_\mu, C_\beta \): skin friction co-efficient along x and y axis.
- \( C_w \): concentration at wall
- \( C_a \): ambient fluid concentration
- \( D_b \): Brownian diffusion coefficient [m\(^2\) s\(^{-1}\)]
- \( D_t \): thermophoretic diffusion coefficient [m\(^2\) s\(^{-1}\)]
- \( e \): electric charge
- \( E \): intensity vector of the electric field
- \( f, g \): dimensionless velocities
- \( h \): heat transfer coefficient [W m\(^{-2}\) K\(^{-1}\)]
- \( J \): current density vector
- \( K_s \): slip parameter
- \( k \): thermal conductivity of fluid [W m\(^{-1}\) K\(^{-1}\)]
- \( L \): characteristic length
- \( m \): Hall effect parameter
- \( M_n \): magnetic parameter
- \( n_e \): electron number density
- \( P_e \): electronic pressure
- \( N_b \): Brownian motion parameter
- \( N_t \): thermophoresis parameter
- \( N_u \): local Nusselt number
- \( Pr \): Prandtl number
- \( Re \): local Reynolds number
- \( Sc \): Schmidt number
- \( Sh \): local Sherwood number
- \( T \): fluid temperature [K]
- \( T_f \): hot fluid temperature
- \( T_e \): ambient fluid temperature [K]
- \( U_o \): reference velocity
- \( u, v, w \): velocity components in the \( x, y \) and \( z \) directions [m s\(^{-1}\)]
- \( u_n(x), v_n(y) \): stretching velocity in the \( x \) and \( y \) directions [m s\(^{-1}\)].
- \( V \): velocity vector
- \( x, y, z \): cartesian coordinate axes

Greek symbols:
- \( \alpha \): thermal diffusivity [m\(^2\) s\(^{-1}\)]
- \( \beta \): Deborah number
- \( \lambda \): rotation parameter
- \( \lambda_t \): relaxation time [s]
- \( \Omega \): constant angular velocity
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