The role of an $e^+e^-$ linear collider in the study of cosmic dark matter

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Abstract. The potential of a high energy, high luminosity $e^+e^-$ linear collider in the study of a weakly interacting massive new particle as a cosmic dark matter candidate is reviewed, with special emphasis on supersymmetric scenarios. Results of detailed simulation studies for supersymmetric neutralino dark matter indicate that the accuracy from linear collider data of sufficient energy may allow us to infer the dark matter relic density to accuracies comparable to those already obtained from the study of cosmic microwave background and other astrophysical data, thus providing a powerful test on the nature of dark matter by combining results from particle colliders with satellite and direct detection experiment data.

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1. Introduction

The study of the interactions of elementary particles at colliders has outlined the structure of the standard model (SM) of electroweak interactions. After the discovery of the $W^\pm$ and $Z^0$ bosons in proton–antiproton collisions at CERN $\bar{p}pS$, it has been through the concurrent operation of $e^+e^-$ and proton colliders that we have progressed so far in the understanding of the fundamental interactions of elementary particles to now be able to consider the connections between particle physics and cosmology through dark matter (DM). The data collected at particle colliders have confirmed the validity of the SM from the high energy frontier at LEP-2 and the Tevatron to the high precision frontier at LEP, the SLC and, more recently, the B-factories. Despite having successfully passed these tests, it is widely accepted the SM does not describe the full extent of particle physics phenomenology and that new physics (NP) must exist beyond it. Because strong bounds exist from data we have collected so far, this NP is best described by models where extra symmetries protect the phenomenology of the SM at low energies. These extra symmetries also prevent new particles to be produced and destroyed alone. As a result, models typically predict the existence of a new, heavy stable particle, which can be taken to be neutral. These models have been built and studied in parallel to the emergence of clear evidence for the existence of large amounts of DM in the universe. As none of the particles that make up the SM, as we know it today, has the right properties to be responsible for DM, it is at least highly suggestive to search for the origin of DM in relation to the new stable particles, which would appear in extensions of the SM. We can go even further along this line of reasoning and claim that the existence of DM, with the characteristics of a weakly interacting massive particle (WIMP), is possibly the strongest experimental evidence of NP we have at present. There are several models of NP, which address both (some of) the long-standing problems with the SM and also include a viable WIMP candidate. Among the best motivated, and most studied so far, are supersymmetric (SUSY) extensions of the SM, the models with extra dimensions (EDs) and little Higgs models of electro-weak symmetry breaking. Their structure in relation with DM is discussed in detail in [15]–[22]. The physics of the neutrino sector has also highlighted new challenges to the SM picture, but the contribution of the SM neutrinos cannot represent a significant fraction of the relic DM density [23].

The Tevatron proton–antiproton collider has provided us with the highest energy hadronic collisions so far, at 1.96 TeV, without yet uncovering any signals of physics beyond the SM. The imminent start of LHC operation is now widely anticipated as a turning point in our quest for NP. Colliding proton beams at energies up to 14 TeV, the LHC will be sensitive to new particles with masses up to the TeV scale, depending on their interaction and properties [24]. With the exception of some corners of the parameter space (which carry, however, a particular relevance for the DM study, as we shall discuss later) the LHC experiments should observe (at least some) signals of production of SUSY particles. If SUSY or one of the other models is indeed realized in nature and it is relevant to solve the SM problems, we expect signals of new particles associated with NP and possibly relevant to understand the nature of cosmic DM to be soon observed at CERN.

Beyond a first observation, it will become essential to reconstruct the profile of the DM candidate particle and precisely measure those properties that will be most important for determining its relic abundance in the early universe. A detailed understanding of DM will only be possible by relating microscopic data obtained from particle collider experiments to cosmology data and possible signals to be obtained in direct DM detection experiments.
the third generation of satellite CMB missions is now starting with the recent launch of the PLANCK surveyor and several direct DM search experiments are in operation, construction and development, this is likely to become one of the most fascinating scientific endeavours of the next several decades of fundamental physics. Similar to the case of understanding the structure of the SM, this program is best pursued with the data from an $e^+e^-$ collider of sufficient energy. Such data will bring the precision on the microscopic observables, which is needed to perform sensitive tests in conjunction with results from satellite and ground-based direct detection DM experiments.

2. An $e^+e^-$ linear collider

The importance of extending the high energy frontier in lepton collisions, beyond LEP-2, motivates an intense world-wide R&D effort towards an $e^+e^-$ linear collider, since several decades. Replacing the storage ring scheme, used until LEP-2, with a single-pass linear accelerator solves the problem of radiation loss from the high energy electrons, which represented a fundamental limitation to high energy electron collisions. The concept dates back to 1965 [1] while the possible structure and parameters were developed about a decade later. That study resulted in the SLC project, which has been a successful precursor of the $e^+e^-$ linear collider projects currently under consideration. Beam energy, collision luminosity and background conditions at the interaction region are the three parameters most important to experimentation. The luminosity, $\mathcal{L}$, defined as the proportionality factor between the number of collisions per unit of time and the process cross section $\sigma$, has requirements, which depend on the cross sections of the processes of interest. These are now identified as $\mathcal{L} > 10^{34}$ cm$^{-2}$ s$^{-1}$ at 0.5 TeV and increasing to $\sim 10^{35}$ cm$^{-2}$ s$^{-1}$ for multi-TeV collisions. Since in a linear machine beams collide only once, the collision frequency is small and high luminosity can be practically achieved only by decreasing the transverse beam size at the interaction region. A small beam size induces stronger beam–beam interactions, which result in an increase of beamstrahlung, the energy loss from particle radiation due to trajectory bending in the interactions with the incoming bunch [2, 3]. Beamstrahlung results in a larger energy spread of the colliding particles, a degraded luminosity spectrum and higher backgrounds and needs to be accounted in the overall parameter optimization together with the luminosity, at each collision energy. After two decades marked by important progress in the R&D on various RF technologies the International Linear Collider (ILC) design, based on the use of superconducting (s.c.) RF cavities, emerged as a viable solution for intermediate collision energies, with $\sqrt{s} = 0.5$ TeV, $\mathcal{L} = 2 \times 10^{34}$ cm$^{-2}$ s$^{-1}$ and a beamstrahlung parameter of 0.022, as baseline, later upgradeable to 1 TeV with $\mathcal{L} = 2.8 \times 10^{34}$ cm$^{-2}$ s$^{-1}$ [4, 5]. The linear collider physics program and its parameter optimization are reviewed in [6].

Beyond 1 TeV the extension of conventional RF technologies is more speculative. Since the gradient of s.c. RF cavities is limited below $\simeq 50$ MV m$^{-1}$, other technologies must be pursued to keep the total linac length within some acceptable constraints dictated by costs, site characteristics and alignment. The CLIC technology, which is based on the transfer of energy from an intense, low energy drive beam to a low charge, high energy main beam, has demonstrated gradients up to 150 MV m$^{-1}$ [7]. A linear collider based on the CLIC scheme aims at collisions from 0.5 to 3 TeV, with a total luminosity of $6 \times 10^{34}$ cm$^{-2}$ s$^{-1}$ and a beamstrahlung momentum spread of 0.29 at the highest energy [9]. CLIC offers a unique opportunity to match
and exceed the LHC energy reach with $e^+e^-$ collisions and presents new challenges to preserve that cleanliness, which has been a signature of $e^+e^-$ events, up to multi-TeV energies [7, 8].

In the longer term, laser-plasma accelerators, which use the plasma wave excited by an intense laser pulse in a plasma and have already demonstrated accelerating gradients up to 50 GV m$^{-1}$ [10, 11], may open an exciting path towards very high energy lepton collisions [12].

3. $e^+e^-$ collider measurements

There are three main features of an $e^+e^-$ linear collider that play a major role in providing precision data for understanding DM and justify a new large scale facility in accelerator particle physics to complement and extend the LHC. Firstly, virtually all the energy of the incoming electron and positron beams is transferred to the final state particles. This makes kinematics simpler and, more importantly, provides powerful constraints for studying invisible particles (such as a DM candidate) and for improving the resolution on the mass of observed resonances. Secondly, all $e^+e^-$ annihilation processes have roughly the same cross sections. This ensures that the production of new particles is not swamped by the underlying strong-interaction background and that all (or most) of their main decay channels are visible. Finally, in $e^+e^-$ collisions the energy, particle species and polarization of the beams can be adjusted. This enables detailed energy scans of the onset of new particle pair production and the choice of optimal energy, particle and polarization states, depending on the physics of interest, to maximize the analysing power of the collider data. All these features are instrumental for a linear collider to establish the connection between particle physics and cosmology through DM, as discussed below.

In this review, I consider primarily SUSY neutralino DM since it has been studied more thoroughly at both the linear collider and the LHC. The study of SUSY DM can be usefully started within the constrained minimal SUSY extension of the SM (cMSSM). In this model, the scalar masses, the gaugino masses and the trilinear parameters are assumed to be universal at a high scale with values $m_0$, $m_{1/2}$ and $A_0$, respectively. The only other parameters that remain free in the theory are the ratio of the vacuum expectation values of the two Higgs doublets, $\tan \beta$ and the sign of $\mu$. As this model is simple and it is fully described by a small number of parameters, it has been used most often for detailed phenomenology studies. In the cMSSM, there are four main regions that are compatible with current data on cosmic DM. These are shown in figure 1, which also gives the location of four specific benchmark points adopted in the study of [13], points which are referred.

3.1. Mass measurements

Since the DM candidate particle is stable and only weakly interacting, its mass cannot be directly measured in a collider detector. However, since it is the lightest particle with the (conserved) quantum number of the NP sector, it will appear at the end of each decay chain involving heavier new particles, independent of the specific model, and the kinematics of the associated SM particles can be used to determine its mass. More specifically, if we consider the case of SUSY with neutralino DM, the minimum and maximum energy of an SM particle produced together with the neutralino can be related to the mass difference between the original SUSY particle produced and the neutralino itself. This method, which was first developed in the framework of SUSY for scalar quarks [27], equally applies to scalar leptons, as well as to Kaluza–Klein (KK) excitations of SM particles, in ED models, where the neutralino is replaced, for example, by the
Figure 1. DM favoured regions in the universal gaugino and sfermion mass parameters of the cMSSM with the four benchmark points adopted for the study of [13] (figure modified from [14]).

lightest excitation of the photon, $\gamma^{(1)}$, as DM candidate. This measurement requires to precisely determine the endpoint energy of the detected particle, as well as the beam energy. Generally this is possible at an $e^+e^-$ collider, where the measurement is performed in pair production processes, such as $e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^-$, with $E_{\tilde{\mu}} = E_{\text{beam}}$, which is precisely known. The endpoint energies are measured to high accuracy in two body-decays to stable leptons, such as $\tilde{\mu} \rightarrow \mu \chi_1^0$ (see figure 2), taking into account that the anticipated momentum resolution of the tracking system of a linear collider detector is $\delta p/p^2 \approx 10^{-5}$ GeV$^{-1}$, i.e. about an order of magnitude better compared to that of LEP and LHC detectors. The main source of resolution smearing comes from the beam energy spread. The nominal beam energy spread is 0.15% for the ILC 0.25 TeV beam. For a 1.5 TeV CLIC beam at CLIC, because of beamstrahlung, one-third of the delivered luminosity falls within 1% of the nominal energy. The $W^+W^-$ background, which is significant in parts of the parameter space, can be suppressed by colliding polarized beams and profits from availability of both electron and positron polarization [39]. In addition, an $e^+e^-$ collider has a second, and even more precise, technique for independently measuring the mass of new particles. The option of precisely tuning the centre-of-mass energy, $\sqrt{s}$, makes it possible to scan the onset of the pair production process. The sparticle mass, and also its width, can then be extracted from a fit to the signal event yield as a function of $\sqrt{s}$. The anticipated analysis strategy is to determine the slepton mass using the threshold scan in a short dedicated run at and then determine the neutralino mass using kinematic endpoints, while running at the highest available $\sqrt{s}$ energy where the cross section is larger and the full physics program can be exploited [41]. Typical relative accuracies for masses are of the order of $\pm 5 \times 10^{-3} - 5 \times 10^{-4}$ from threshold scans and $\pm (1-5) \times 10^{-3}$ from momentum endpoint in the continuum [40]. These accuracies are typically one to two orders of magnitude better compared to those expected from the LHC.
data [65]. Estimated accuracies for sparticle masses in the four LCC benchmarks scenarios are summarized in table 1. Similar results apply to KK excitations of SM particles in Universal Extra Dimension (UED) models [66]. The assumption of operating the collider varying energy and beam polarization, when not even the beam particle species, may rise doubts on the feasibility to accomplish all of the required measurements within a pre-defined run period. Indeed, the issue has been studied in details for the case of two SUSY scenarios, which require operating at various energies and with different beam conditions [41]. While the construction of a run plan, i.e. the repartition of the run time between energies of operation, initial state electron polarizations and the use of special $e^+e^-$ runs, depends quite sensitively on the specifics of the assumed physics model, the result of the study indicates that an integrated luminosity of 1 ab$^{-1}$, i.e. 6–7 years of collider operation, guarantees high precision measurements of all of the SUSY states within kinematic reach [41].

While the neutralino mass plays a key role in defining the DM density, the masses of other particles, which may have interacted with the neutralinos in the early universe, also need to be precisely known to determine the neutralino annihilation cross section. The heavy Higgs boson $A^0$ plays a special role, in this respect. If $M_A \simeq 2M_{\chi_1^0}$, the annihilation process $\chi_1^0\chi_1^0 \rightarrow b\bar{b}$ is significantly enhanced through the $A^0$ pole, $\chi_1^0\chi_1^0 \rightarrow A^0 \rightarrow b\bar{b}$. This annihilation is so efficient that exactly on the pole, neutralino DM is almost completely wiped out. Since we do indeed have evidence of relic DM, acceptable solutions are restricted, in this scenario, to parameters giving just a tiny mismatch between $M_A$ and $2M_{\chi_1^0}$ and the amount of relic DM changes very rapidly when varying the $A^0$ mass and width. Therefore, it is essential for a precise study of DM at colliders to observe the heavy Higgs bosons and measure their properties. For the relic density calculation, these measurements remove a major source of uncertainty. For DM direct detection, they are even more important, since the dominant contribution to the scattering cross section comes most often from Higgs boson exchange diagrams. The study of the heavy $A^0$ boson exemplifies the complementarity between the LHC and a linear collider and the case for a staged approach to the linear collider energy. We do expect the LHC to observe the $H^0$ and $A^0$ heavy Higgs bosons in the region of parameters where the $A^0$ boson significantly contributes to the neutralino annihilation cross section (see figure 3) [42].
Since the mass relation between the two particles may be responsible for the point LCC-4 of figure 1, a linear collider can determine the mass and guide an energy upgrade. For masses almost up to the pair production kinematic limit, as study at a linear collider will likely require operation at 1 TeV, or above, which would motivate New Journal of Physics

The so-called co-annihilation region. Here is the small mass splitting between the neutralino and another SUSY particle to enhance the neutralino annihilation cross section. In particular, in the cMSSM, stau-neutralino co-annihilation is dominant along a narrow strip in the $m_0$--$m_{1/2}$ parameter plane. Typical mass differences are of the order of 1–10 GeV and extracting the soft signal events from underlying machine-induced background is an experimental challenge [49, 50]. The $\tilde{\tau} - \chi^0_1$ mass difference can be determined quite precisely through the use of an effective mass variable (see figure 5). The contribution from $\tilde{\tau}^+\tilde{\tau}^-$ and $\chi^0_1\chi^0_1$ that lead to the same $\tau^+\tau^-\chi^0_1\chi^0_1$ final state can be disentangled by varying the beam polarization. A co-annihilation region arises also in baryogenesis-motivated SUSY scenarios with very heavy scalars but a light scalar top particle, which is almost degenerate with the lightest neutralino [51, 52]. In

### Table 1. Summary of the accuracies (in GeV) on the main mass determinations by the ILC at 0.5 TeV for the four LCC benchmark points. Results in [ ] brackets also include data at 1 TeV (from [6]).

| Observable | LCC1  | LCC2  | LCC3  | LCC4  |
|------------|-------|-------|-------|-------|
| $\delta M(\tilde{\chi}^0_1)$ | $\pm 0.05$ | $\pm 1.0$ | $\pm 0.1$ | $[\pm 1.4]$ |
| $\delta M(\tilde{\tau}_R)$   | $\pm 0.05$ | – | $[\pm 1.0]$ | $[\pm 0.6]$ |
| $\delta M(\tilde{\tau}_1)$   | $\pm 0.3$ | – | $\pm 0.5$ | $\pm 0.9$ |
| $\delta M(\tilde{\tau}_2)$   | $\pm 1.1$ | – | – | – |
| $\delta (M(\mu_R) - M(\tilde{\chi}^0_1))$ | $\pm 0.2$ | – | $[\pm 0.2]$ | $\pm 0.6$ |
| $\delta (M(\tilde{\tau}_1) - M(\tilde{\chi}^0_1))$ | $0.3$ | – | $\pm 1.0$ | $\pm 1.0$ |
| $\delta (M(\tilde{\tau}_2) - M(\tilde{\chi}^0_1))$ | $\pm 1.1$ | – | $[\pm 3.0]$ | – |
| $\delta (M(\tilde{\chi}^0_2) - M(\tilde{\chi}^0_1))$ | $\pm 0.07$ | $\pm 0.3$ | $\pm 0.6$ | $[\pm 1.8]$ |
| $\delta (M(\tilde{\chi}^0_3) - M(\tilde{\chi}^0_1))$ | $\pm 4.0$ | $\pm 0.2$ | $[\pm 2.0]$ | $[\pm 2.0]$ |
| $\delta (M(A^0) - M(\tilde{\chi}^0_1))$ | $\pm 0.6$ | $\pm 0.25$ | $[\pm 0.7]$ | $\pm 2.0$ |
| $\delta (M(A^0))$ | $[\pm 3.0]$ | – | $[\pm 2.0]$ | $\pm 2.0$ |
| $\delta M(A^0)$ | $[\pm 1.5]$ | – | $[\pm 0.8]$ | $[\pm 0.8]$ |
| $\delta \Gamma(A^0)$ | $\pm 0$ | $\pm 1.2$ | $[\pm 1.2]$ | – |

$A^0$ mass accuracy crucially depends on the availability of the $A^0 \to \mu^+\mu^-$ channel [43]. At the lower end of the mass range of interest, $M_A$ can be measured very accurately: ATLAS reported a relative precision of $\mathcal{O}(0.1\%)$ for $M_A = 300$ GeV at $\tan \beta = 30$ with $300 \text{ fb}^{-1}$ of integrated luminosity [44]. However, at larger masses the main signal decay mode available is $A^0 \to \tau^+\tau^-$ and the mass determination is affected by the sizable energy scale uncertainty, which puts the LHC accuracy about an order of magnitude away from that required for the DM relic density computation to match the precision of the current CMB data [43]. After an early determination of the neutralino mass at a linear collider and with the prior knowledge of the LHC result on $M_A$, it will become apparent if the mass relation between the two particles may be responsible for setting the relic density. Since the $A^0$ boson is pair produced with the $H^0$ in $e^+e^-$ interactions, its study at a linear collider will likely require operation at 1 TeV, or above, which would motivate and guide an energy upgrade. For masses almost up to the pair production kinematic limit, as the point LCC-4 of figure 1, a linear collider can determine the $A^0$ mass and width with relative accuracies of $\sim 0.2\%$ and $\sim 15–20\%$, which exactly match the DM study requirements [47, 48] (see figure 4).

Another scenario where mass relations among SUSY particles set the relic density is in the so-called co-annihilation region. Here is the small mass splitting between the neutralino and another SUSY particle to enhance the neutralino annihilation cross section. In particular, in the cMSSM, stau-neutralino co-annihilation is dominant along a narrow strip in the $m_0$--$m_{1/2}$ parameter plane. Typical mass differences are of the order of 1–10 GeV and extracting the soft signal events from underlying machine-induced background is an experimental challenge [49, 50]. The $\tilde{\tau} - \chi^0_1$ mass difference can be determined quite precisely through the use of an effective mass variable (see figure 5). The contribution from $\tilde{\tau}^+\tilde{\tau}^-$ and $\chi^0_1\chi^0_1$ that lead to the same $\tau^+\tau^-\chi^0_1\chi^0_1$ final state can be disentangled by varying the beam polarization. A co-annihilation region arises also in baryogenesis-motivated SUSY scenarios with very heavy scalars but a light scalar top particle, which is almost degenerate with the lightest neutralino [51, 52]. In

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Figure 3. Colliders reach in Higgs boson searches in the $M_A$–$\tan \beta$ parameter plane of the MSSM. The LEP limit obtained from light $h^0$ boson searches is shown by the curve labelled LEP-2. The regions labelled by the names of the various Higgs bosons show which of them could be observed across the parameter space at the LHC with 300 fb$^{-1}$ of integrated luminosity by combining the ATLAS and CMS data (from [45]). The increased sensitivity from higher luminosity at the SLHC is also shown (from [45]). An $e^+e^-$ linear collider is able to cover most of this plane on the left of the kinematic limit $M_A \simeq \sqrt{s}/2$ using the dominant decays modes $b\bar{b}$ and $\tau^+\tau^-$. Specifically, the expected reach at 3 TeV is shown by the curve labelled CLIC (data from [46]). The dots at large $\tan \beta$ show the distribution of MSSM solutions in the $A_0$ annihilation funnel compatible with the WMAP $\Omega_{\chi h^2}$ result (from [43]).

These scenarios, a linear collider at 0.5 TeV can precisely determine the $\tilde{t}_1$ and $\chi^0_1$ masses and the stop mixing angles through its polarized production cross sections [51].

Finally, it is important to mention that in $e^+e^-$ collisions it is possible to search for a WIMP, $\chi$, in an (almost) model-independent way through the process $e^+e^- \rightarrow \chi \chi \gamma$, where only the initial state radiation photon is detected. The main background comes from the SM process $e^+e^- \rightarrow \nu \bar{\nu} \gamma$ [53]. A detailed study [54] has demonstrated how a WIMP with mass in the range 100–220 GeV (most relevant to DM) and pair annihilation to electrons of the order of 0.01 (0.1) can be observed with 500 fb$^{-1}$ of integrated luminosity at 0.5 TeV with (without) polarization (see figure 6). This region is of special interest in view of the anomalous positron abundance in the cosmic radiation reported by PAMELA [55]. The WIMP mass could be measured to a relative accuracy of 0.04–0.01 depending on the availability of beam polarization [54].
3.2. Identifying the DM nature

With many NP models, each offering possible WIMP candidates, having observed events with missing energy will not specifically point to any one of them. Rather, the analysing power of a lepton collider will be needed to pin down the model parameters and possibly even the nature of NP. The problem has been studied in detail with reference to the case of SUSY and UEDs [56], a particular class of ED models with a conserved quantum number, the KK parity. This case is particularly compelling since the two models are constructed with an almost exact symmetry. In SUSY, as well as in UED, for each SM particle there exists a partner particle in the new sector. The lightest of these particles, the lightest neutralino, $\tilde{\chi}^0_1$, in SUSY and the lightest KK excitation of the photon, $\gamma^{(1)}$, are stable and are the DM candidate. Similar decay chains arise from decays of heavy squarks (higher KK excitations of the quarks) and sleptons (higher KK excitations...
Figure 6. WIMP signal seen as an excess of events in the single photon energy distribution due to the $e^+e^- \rightarrow \chi\chi\gamma$ process for $M_\chi = 180$ GeV above the $e^+e^- \rightarrow \nu\bar{\nu}\gamma$ SM background. Both beams are assumed to be polarized (from [54]).

of the leptons) in SUSY (UED). What makes the two models fundamentally different, and thus distinguishable, is the spin quantum number of these new particles. In fact, the SUSY partners of quarks and leptons are bosons, while the KK excitations of fermions have the same spin as their SM counterparts. The superb analytical properties of $e^+e^-$ colliders ensure that the nature of NP can be identified in most cases. Unlike hadron colliders, the pair production of new particles in lepton collisions at a well-defined energy, allows to determine their spin from their production properties [62]. In fact, in the case of slepton production in SUSY

$$\left( \frac{d\sigma}{d \cos \theta} \right)_{\text{SUSY}} \sim 1 - \cos^2 \theta,$$

(1)

while for the KK excitations of leptons in UED

$$\left( \frac{d\sigma}{d \cos \theta} \right)_{\text{UED}} \sim 1 + \cos^2 \theta.$$  

(2)

The angular distributions for these two models are well distinguishable also when accounting for physics and machine-induced backgrounds and detector effects (see figure 7) [66]. Another technique to assess the spin quantum numbers of the new particle is to study the rate of increase of the pair production cross section near threshold, which is again available at an $e^+e^-$ collider, where the centre-of-mass energy can be precisely tuned [62]. Since the c.m. frame is not well defined, extracting the spin of new particles at the LHC appears challenging [63, 64].

Even within a specific scenario of NP such as the MSSM, determining its fundamental parameters from collider data will be essential to understand the mechanism(s) controlling DM annihilation in the early universe. Measurements of mass differences for new particles, which are expected to be the main deliverable of the LHC data for a scenario-like SUSY, do not provide enough constraints to fix the mixing angles and the nature of the neutralino. For example, in the case of the LCC-2 benchmark point of figure 1, the LHC data will not be able to assess whether the lightest neutralino, as DM candidate, is bino-, wino- or higgsino-like. To those three
Figure 7. SUSY (left) versus UED (right) at a linear collider: (left) muon momentum distribution and (right) polar angle for $e^+e^- \rightarrow \tilde{\mu}^+\tilde{\mu}^- \rightarrow \mu^+\chi^0\mu^-\chi^0$ and $e^+e^- \rightarrow \mu^{(1)}\mu^{(1)} \rightarrow \mu^+\gamma^{(1)}\mu^-\gamma^{(1)}$ (from [66]).

Figure 8. Disentangling parameters with linear collider measurements: MSSM models compatible with LCC-2 for the LHC data, presented as a scatter plot in the plane of gaugino production cross section $\sigma$ versus $\Omega_\chi h^2$. The polarized cross sections measurements at a linear collider with their associated uncertainties are shown by the horizontal line (from [13]).

solutions correspond different annihilation cross sections and thus different relic densities. The linear collider allows to solve these degeneracies by precise measurements of the polarized $\chi\chi$ cross sections, which fix the values of the mixing angles (see figure 8).
3.3. Collision energy requirements

While we follow the constant progress of the Tevatron experiments in improving not only our knowledge of the mass of the top quark \cite{28} but also the mass limits for a Higgs boson \cite{29} and have high expectations for the LHC data, we can already layout possible scenarios of NP based on the sizable set of precision electroweak, low energy and, in fact, cosmology data. Much has been written on this subject in recent years \cite{30}–\cite{35}. Even when restricting ourselves to highly constrained models, such as the cMSSM, current data do not indicate a clearly favoured mass scale for new particles. Rather, the most likely regions extend across the parameter space along contours largely determined by the cosmic DM density where, at increasing DM candidate masses, the annihilation cross section increases accordingly to restore agreement with the WMAP data (see figure 1) \cite{36}. It is interesting to observe that viable solutions can be obtained up to very large mass scales, depending on the mechanism of WIMP annihilation or on the nature of DM. In the so-called ‘focus point region’ of the MSSM, the DM candidate, the lightest neutralino, is a gaugino–Higgsino mixture. This makes its annihilation in $WW$ and $ZZ$ pairs, through $t$-channel diagrams, the dominant process determining the relic density \cite{37, 38, 57}. Here, the masses of fermion SUSY partners typically exceed 1 TeV and only the lighter neutralinos and charginos populate the mass spectrum at the $O$(100 GeV) scale. Such a scenario would be particularly interesting for an $e^+e^−$ collider of sufficient energy, since the LHC reach in studying, or even observing, SUSY would be limited along the narrow focus point line at high masses.

If the DM candidate is not the lightest neutralino but rather the gravitino, as the lightest SUSY particle, and the $\tilde{\tau}$ is the next to lightest particle new scenarios appear. In particular, the $\tilde{\tau}$ can create bound metastable states with several nuclei, which would affect Big Bang Nucleosynthesis (BBN). Now, it has been observed that there exists a small region of the parameter space where the $\tilde{\tau}$ effect improves the agreement between data and the model prediction on $^6$Li and $^7$Li concentration \cite{58}. This region corresponds to $\tilde{\tau}$ lifetimes shorter than 1000 s and very heavy particle masses, with thresholds for slepton pair production appearing above 2 TeV \cite{59}. The existence of such scenarios giving importance to the outer edges of the parameter phase space, where new particles are heavier, remind us the importance to develop a sustained accelerator R&D program towards multi-TeV $e^+e^−$ collisions, to be able to study these cases in full.

Depending on model parameters, we may expect the spectrum of new particles to extend up to masses of the order of 1 TeV or more, where not all of these particles can be accessed and precisely measured at a linear collider. However, it must be observed that particles of large mass in the NP sector quickly decouple from the DM candidate and do not significantly contribute to determine its relic density \cite{60}. In fact, the relic density depends on the WIMP annihilation cross section at temperatures of the order of $T/M_\chi \sim 1/25$, which correspond to non-relativistic velocities. In $t$-channel exchange, the contribution by heavy particles are suppressed by the factor $1/M^4$, where $M$ is their mass. The dominant diagrams for computing the annihilation cross section only involve the lightest particles in the model.

A benchmark point located in the focus point region brings an useful example. The LCC-2 point has sleptons and squarks too heavy to be pair-produced even at a 1 TeV linear collider. At 0.5 TeV only the three lightest neutralinos and the two lightest charginos are accessible. This scenario has been studied in detail in \cite{61}. The lightest neutralino mass...
can be reconstructed with a relative accuracy of 0.01 and the linear collider data provides a 
\[ \delta \Omega_1 h^2 / \Omega_1 h^2 = 0.076 \] [13].

This property is common to the vast majority of points in the SUSY parameter space: new particles with masses much larger than 2\(M_\chi\) decouple in the determination of the WIMP relic density. The study of DM thus gives special importance to precision measurements on the lightest states of the new particle spectrum, which will be accessible to a TeV-class linear collider.

### 3.4. Experimental implications

Providing us with well-defined, quantitative requirements in terms of physics measurement accuracy over a broad range of physics scenarios, DM-motivated NP plays an important role in benchmarking the physics potential of a linear collider and the response of its detectors [25]. In this respect, \(b\)-quark identification capabilities and very forward electron tagging are two examples of the implications of the phenomenology arising from DM-motivated NP on the linear collider detector design.

The importance of determining the properties of the \(A^0\) boson underscores the crucial role of fermion flavour tagging at the linear collider. The dominant decay mode of \(A^0\) and \(H^0\) bosons in the region of parameters where these heavy Higgs bosons are most relevant to DM is \(b\bar{b}\) and the signal-to-background ratio, before \(b\)-jet identification is \(2.5 \times 10^{-4}\). The typical production cross section for the process \(e^+e^- \rightarrow H^0A^0 \rightarrow b\bar{b}b\bar{b}\) is only of the order of 1 fb, yielding approximately \(10^3\) signals events per year of operation. If \(\epsilon_b\) is the efficiency for tagging a single \(b\)-jet, the total tagging efficiency for signal events is \(\simeq \epsilon_b^4\), making mandatory to achieve a tagging efficiency \(\epsilon_b > 0.75\).

In the so-called co-annihilation region of the cMSSM, the study of new particles with small mass splitting, puts special emphasis on efficient reconstruction of events with just a pair of soft particles and in particular on the detector coverage in the forward region to veto on primary electrons in background \(\gamma\gamma \rightarrow\) hadrons events, down to angles of a few mrad [26, 49]. This is an important requirement that touches upon the machine parameters and the complex design of the machine-detector interface.

### 4. Determinations of \(\Omega_1 h^2\) from linear collider data

Having identified the nature of the physics responsible for the new particles observed, an \(e^+e^-\) collider can measure mass and couplings of those particles which can be produced in pairs, i.e. with masses below \(\simeq \sqrt{s}/2\). These measurements can then be used to determine the properties most relevant to astrophysics. The quantity of most interest in this respect is the WIMP relic density \(\Omega_1 h^2\). The envisaged strategy is to infer \(\Omega_1 h^2\) from microscopic properties measured at colliders and compare it to the result of studies of cosmic microwave background and other astrophysical observables. If the two results agree, this would be a remarkable achievement towards understanding DM. The accuracy achievable at the LHC and at an \(e^+e^-\) linear collider has been studied in detail using results from both parametric simulation of detector response and full simulation and reconstruction in the context of the minimal SUSY extension of the SM [13, 43, 67].

In these analyses the anticipated accuracies on particle masses and other observables are used to define probability density functions for the WIMP relic density \(\Omega_1\) in optimized scans
Figure 9. Probability distributions of the microscopic prediction for $\Omega_\chi h^2$ for the four benchmark models LCC-1–LCC-4. In each plot, the three curves show the results for expected LHC measurements, ILC measurements up to 500 GeV and ILC measurements up to 1 TeV (from [13]).

of the parameter phase space of the MSSM. In particular, the study in [13] considers four points chosen to represent the variety of neutralino annihilation mechanisms (see figure 1) and for each of them scans the full 24-dimensional parameter space of the MSSM. The probability distribution functions for the neutralino relic density at these benchmark points are compared in figure 9 and the corresponding relative accuracies are summarized in table 2 for the LHC and a linear collider.

In each case, the estimated distribution that would be obtained in a model-independent analysis of SUSY neutralino DM from LHC data alone, from the LHC data plus the data from an $e^+e^-$ linear collider at 0.5 or 1 TeV are presented. Despite the greatly different nature and number of measurements required in the different regions of the MSSM parameter space, due to the different annihilation mechanisms, the study shows that the combination of data from the LHC and an $e^+e^-$ collider of sufficient energy can predict $\Omega_\chi h^2$ to a relative accuracy, which
Table 2. Relative accuracies on the derivation of $\Omega_\chi h^2$ using LHC and LC data at 0.5 and 1.0 TeV (from [13] and update in [48]).

| Benchmark point | LHC | +0.5 TeV | LHC | +1.0 TeV |
|-----------------|-----|---------|-----|---------|
| LCC-1           | 0.072 | 0.018  | 0.002 |
| LCC-2           | 0.82  | 0.14    | 0.076 |
| LCC-3           | 1.57  | 0.50    | 0.18  |
| LCC-4           | 4.05  | 0.85    | 0.080 |

is comparable to that resulting from the latest 5 years’ WMAP CMB data [36]. This is due to the combination of mass measurements to accuracies of the order of few per-mil and coupling measurements obtained through the determination of decay branching fractions and production cross sections, also exploiting the availability of polarized beams [39]. The anticipated accuracies of these measurements at an $e^+e^-$ linear collider remain virtually unchanged when performing the analysis with full simulation of the detector and detailed event reconstruction, compared to the use of a simpler parametrization of the detector response [48, 68]. This is a consequence of the relatively simple nature of $e^+e^-$ events and of the envisaged performance of the detectors.

What we expect to learn from an $e^+e^-$ collider is not limited to inferring the DM relic density. Rather, the precision measurement of the DM candidate properties and interactions will allow to fix the particle physics of DM and then attempt to move forward elucidating its role in cosmology. This addresses key question such as testing standard Big Bang cosmology back to $10^{-9}$ s, at which time the DM relic density was produced, and determining the local DM density profile in the Galaxy. In turn, these data are essential in the interpretation of direct DM detection experiments, which are pushing their sensitivity closer to the region where SUSY predicts the observation of a signal [69]–[72].

If DM is due to a weakly interacting, massive new particle, an $e^+e^-$ linear collider of sufficient energy is expected to measure its properties with the precision needed to perform crucial, model-independent tests in conjunction with satellite and DM direct detection experiments. Today we do not know with precision what the required collision energy is. The LHC will likely provide us with an answer, if NP, such as SUSY or EDs, is indeed responsible for cosmic DM. With the first LHC results in hand, proceeding with the construction and operation of an high-energy, high-luminosity linear collider will open up the path towards a new, fascinating era of fundamental science.

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