Effect of asymmetry in peak profiles on solar oscillation frequencies

Sarbani Basu
Institute for Advanced Study, Olden Lane, Princeton NJ 08540, U. S. A.

and

H. M. Antia
Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India

Received 2 Nov 1999; accepted 2 Nov 1999
Most helioseismic analyses are based on solar oscillations frequencies obtained by fitting symmetric peak profiles to the power spectra. However, it has now been demonstrated that the peaks are not symmetric. In this work we study the effects of asymmetry of the peak profiles on the solar oscillations frequencies of $p$-modes for low and intermediate degrees. We also investigate how the resulting shift in frequencies affects helioseismic inferences.

*Subject headings:* Sun: oscillations; Sun: interior
1. Introduction

Accurately measured frequencies of solar oscillations have been extensively used to infer the properties of solar interior. Most of the frequency tables available so far (e.g., Hill et al. 1996; Rhodes et al. 1997) have been obtained by fitting symmetric Lorentzian peak profiles to the observed power spectra. However, it has been demonstrated that in general, the peaks in solar oscillation power spectra are not symmetric (Duvall et al. 1993; Toutain 1993; Nigam & Kosovichev 1998; Toutain et al. 1998; Antia & Basu 1999) and the use of symmetric profiles may cause the fitted frequency to be shifted away from the true value. These frequency shifts may affect the helioseismic inferences obtained from existing frequency tables based on fits to symmetric peak profiles.

While the asymmetric nature of the peaks in the power spectra is well established, what is not known for certain is how much the frequency shifts caused by fitting the symmetric profiles to the peaks affect inferences about the solar interior. Toutain et al. (1998) have studied the effect of asymmetry on the frequencies of low degree modes and concluded that the inferred sound speed in the solar core can be significantly affected by the resulting frequency shifts. However, they did not include the effects of asymmetry on intermediate degree modes, which are also needed for inferring conditions in solar interior accurately. To get a proper idea of this effect it is necessary to include the effect of asymmetry on the intermediate degree modes also. Christensen-Dalsgaard et al. (1998) on the other hand concluded that the frequency shifts due to asymmetry in peak profiles should not cause any significant change in inversion results. This conclusion was reinforced by the inverse analyses carried out by Rabello-Soares et al. (1999). Their results, however, are based on artificial data, where they have assumed that the dimensionless asymmetry parameter characterizing the asymmetry of peak profiles is a function of frequency alone. Thus these results need to be checked against those obtained from real spectra for solar oscillations.
Apart from low degree modes, the frequency shifts due to asymmetry in peak profiles are also found to be significant in high degree modes obtained from ring diagram analysis (Antia & Basu 1999). Thus it would be interesting to study how the use of asymmetric profiles affects the frequencies of intermediate degree modes obtained from full-disk observations. In this work we use data from the Global Oscillations Network Group (GONG) to study the effects of peak-profile asymmetry on frequencies of p-modes with degree $0 \leq \ell \leq 200$ using the rotationally corrected, $m$-averaged power spectra, $m$ being the azimuthal order of the mode.

The rest of the paper is organized as follows: the basic technique used to determine solar oscillations frequencies using asymmetric peak profiles is described in § 2. The resulting frequency shifts are described in § 3, while the effects on helioseismic inferences are described in § 4. The conclusions from our study are summarized in § 5.

2. The technique

We use GONG power spectra to determine the frequencies of solar oscillations. The GONG project determines the frequencies of modes with different values of $n$ (radial order), $\ell$ (degree) and $m$ (azimuthal order) by fitting symmetric Lorentzian peak profiles to spectra for individual values of $\ell, m$ (Hill et al. 1996). The mean frequency of a multiplet for a given $(n, \ell)$ pair is then calculated by fitting the frequencies of all modes with same $n$ and $\ell$ but different values of $m$ to polynomials in $m$. Since the asymmetry in peak profiles is relatively small, it is difficult to distinguish between fits to symmetric and asymmetric profiles in spectra for individual $\ell, m$. To improve statistics we use the $m$-averaged spectra obtained by taking a sum over the azimuthal order $m$ for each $\ell$. Since in this work we are only interested in the mean frequency for each $n, \ell$ multiplet, we correct for the rotational splitting by shifting the spectrum for each $m$ by the approximately known rotational
splitting before summing. Such spectra are available from the GONG project (Pohl & Anderson 1998) for each of the GONG months 1–35. Each GONG ‘month’ covers a period of 36 days. In order to improve statistics still further, we have summed the spectra for different months. We can, in principle, sum all 35 spectra, but in view of the solar cycle variation in frequencies this may not be advisable. As a result we have taken sum over 16 months from month 7 to 22 (9 December 1995 to 6 July 1997), which is the period when the solar activity was close to minimum and there is little change in frequencies during this period. Most of our results about asymmetry in peak profiles have been obtained from this spectrum.

To determine the frequencies and other mode parameters from the power spectra, we fit a model of the form

\[
P(\ell, \nu) = \sum_i \left( \frac{\exp(A_i)(S^2 + (1 + Sx_i)^2)}{x_i^2 + 1} \right) + B_1 + B_2(\nu - \nu_c),
\]

where \(x_i = (\nu - \nu_i)/w_i\) and the summation is carried over all peaks in the fitting interval. If there are \(N\) peaks in the fitting interval then the \(3N + 3\) parameters \(A_i, \nu_i, w_i, S, B_1\) and \(B_2\) are determined by fitting a section of the spectra at constant \(\ell\) using a maximum likelihood approach (Anderson, Duvall & Jefferies 1990). In Eq. (1), \(\nu_c\) is the central value of \(\nu\) in the fitting interval and \(\exp(A_i)\) is the peak power in the mode, \(\nu_i\) is the mean frequency of the corresponding peak, \(w_i\) is the half-width. The terms involving \(B_1, B_2\) define the background power, which is assumed to be linear in \(\nu\). \(S\) is a parameter that controls the asymmetry, and the form of asymmetry is the same as that prescribed by Nigam & Kosovichev (1998). This parameter is positive for positive asymmetry, i.e., more power on the higher frequency side of the peak, and negative for negative asymmetry. By setting \(S = 0\) we can fit symmetric Lorentzian profiles. We have assumed that \(S\) has the same value for all peaks in the fitting interval. This may not be strictly true but the variation in \(S\) is not very large between neighboring peaks and for simplicity we neglect its variation.
between peaks in the fitting interval. This improves the convergence of fitting procedure. Even then, inclusion of asymmetry parameter in the fits reduces the number of modes that are successfully fitted. This could be due to some cross-correlation between $S$ and other parameters of the model, particularly, the background.

We fit each mode separately by using the portion of power spectrum extending halfway to the adjoining modes. Apart from the target mode there are other peaks in the spectra arising due to leaks from neighboring $\ell$ and $n$ values. We include all leaks from modes for which $\ell$ differs by at most 3 from those of the target mode, provided they occur within the fitting interval. Although all these peaks are fitted to obtain a good fit to the observed spectra, we ultimately use only the parameters obtained for the target peak and ignore the leaks.

The use of rotationally corrected, $m$-averaged spectra may introduce some systematic errors in the frequency due to incorrect even order splitting coefficients used in constructing the spectra. These coefficients are assumed to be zero while constructing the GONG $m$-averaged spectra. The non-zero values of these coefficients will introduce a small shift in the frequencies, but that is not relevant in the current work as we are only interested in the effect of asymmetry on the frequencies. The splitting coefficients will affect the fits to both symmetric and asymmetric spectra equally and its effect will cancel out when we take frequency differences between the two fits. Nevertheless, we can estimate this systematic error by taking the difference between frequencies fitted by us and those obtained by the GONG project using individual $\ell, m$ spectra.
3. The frequency shifts

We follow the procedure outlined in § 2 to fit the model given by Eq. (1) to suitable regions of the spectra obtained by summing over the 16 spectra for GONG months 7–22 in order to determine the mode parameters. Although other parameters may be of interest, in this work we only concentrate on the frequencies $\nu_i$ and the asymmetry parameter $S$. We fit both symmetric and asymmetric peak profiles to the spectra for studying the shift in frequency arising due to asymmetry in peak profiles. Fig. 1 shows a fit to the $\ell = 100$ spectrum using both symmetric and asymmetric profiles. It is clear from the figure that asymmetric profile gives a better fit to the observed spectra and it is probably desirable to use asymmetric profiles to determine frequencies. For comparison this figure also shows a fit to the $\ell = 0$ power spectrum over a similar frequency range. The $\ell = 0$ spectrum does not involve any sum over spectra for different $m$. It is clear that the spectrum in this case is too noisy to distinguish between the two fits. This shows why we have used the $m$-averaged spectra in this study — summing over spectra for all values of $m$ increases the signal to noise ratio.

Fig. 2 shows the asymmetry parameter $S$ for the modes. It is clear that this parameter is significant at frequencies around 2–2.5 mHz. This parameter is negative for all modes and hence there is more power on the low frequency side of the peak. The magnitude of $S$ is similar to what has been found by Toutain et al. (1998) for low degree modes and by Antia & Basu (1999) for high degree modes. The variation of $S$ with frequency is somewhat different from what was found at high degree using the ring diagram analysis. This probably implies that apart from frequency the asymmetry may also depend on $\ell$.

The shift in frequencies that result from using asymmetric peak profiles rather than the standard Lorentzian profiles are shown in Fig. 3. These frequency shifts are positive, i.e., frequencies tend to increase when asymmetric profiles are used. These frequency shifts
are clearly larger than the estimated errors and hence this effect must be included in helioseismic analysis. However, the frequency shift appears to be a function predominantly of frequency and is only weakly dependent on $\ell$. If this is true then the difference may be accounted for by the surface term in helioseismic inversions (Christensen-Dalsgaard et al. 1998). In order to check for any depth dependence we also show in Fig. 3 the frequency difference as a function of the lower turning point ($r_t$) for the mode. It is clear that there is a weak dependence of frequency difference with $r_t$, and there appears to be a change around the base of the convection zone, which is located at a radial distance of $0.713R_\odot$ (Christensen-Dalsgaard, Gough & Thompson 1991; Basu 1998). Thus we may expect some change in the inferred properties of solar interior when asymmetry in peak profiles is incorporated. In the next section we investigate the effect of these frequency shifts on various helioseismic inferences. Similar frequency shifts have been obtained for spectra from different time periods.

As mentioned in Section 2, there may be some systematic errors introduced by using the $m$-averaged spectra for determining the frequencies. In order to estimate this error we repeat the calculations for the summed spectra from the GONG months 4–14 (23 August, 1995 to 21 September, 1996) using symmetric peak profiles and compare the results with the mean frequencies determined from fitting the individual $n, \ell, m$ modes by the GONG project. The frequency difference is shown in Fig. 4. It is clear that the systematic errors are of order of 0.01$\mu$Hz, which is much smaller than the frequency shift due to asymmetry in peak profiles. Moreover, as mentioned earlier these systematic errors will cancel when we take the difference between frequencies from symmetric and asymmetric peak profiles.
4. Effect of asymmetry on structure inversion results

To investigate the effect of frequency shift due to asymmetric peak profiles on helioseismic inversions for solar structure, we first try the asymptotic inversion technique (Christensen-Dalsgaard, Gough & Thompson 1989). For this purpose the frequency difference is expressed as

\[ S(w) \frac{\delta \omega}{\omega} = H_1(w) + H_2(\omega) \]

(2)

where \( w = \frac{\omega}{(\ell + 1/2)} \) and

\[ S(w) = \int_{R_\odot}^{R} \left(1 - \frac{c^2}{w^2 r^2}\right)^{-1/2} \frac{dr}{c} \]

(3)

Here, the function \( H_1(w) \) contains information about the variation of sound speed with depth and can be inverted to obtain the sound speed, while \( H_2(\omega) \) represents the effect of differences in surface layers. This analysis can be applied to the frequency shifts shown in Fig. 3 to obtain the error introduced in inversion results due to asymmetry in peak profiles. The results are shown in Fig. 5. It may be noted that both \( H_1(w) \) and \( H_2(\omega) \) are comparable in magnitude and hence the frequency shift can not entirely be considered as surface effects and we would expect some change in inferred solar structure also. Fig. 6 shows the inferred relative difference in sound speed due to the frequency shifts shown in Fig. 3 and it is clear that the difference is fairly small, being comparable to the estimated errors in inversions. There is a very small hump near the base of the convection zone, which may give some difference in the estimated depth of the convection zone or the extent of overshoot below the convection zone. The small difference in the convection zone may account for some of the observed difference between the Sun (as inferred using fits to symmetric profiles) and standard solar models.

Instead of asymptotic inversion we can perform non-asymptotic inversions using the Regularized Least Squares (RLS) method (Antia 1996) or Subtractive Optimally Localized
Averages (SOLA) technique (Basu et al. 1996). These results are also shown in Fig. 6 and are similar to those obtained using asymptotic inversion technique. It is clear from all these results that the frequency shifts due to asymmetry in peak profiles do not affect the structure inversion results significantly. The difference $\delta c/c$ in the core is much less than what was found by Toutain et al. (1998) who found relative sound speed differences exceeding 0.002. Since the small change appears to manifest as hump around the base of the convection zone, in the next two subsections we investigate the effect of this frequency shifts on the inferred depth of the convection zone and the extent of overshoot below the convection zone. There is also a small dip near $r = 0.5R_\odot$, which can be seen in frequency difference shown in Fig. 3 too. This dip is comparable to error estimates in the individual modes, though after averaging over neighboring modes it may appear to be somewhat significant. The origin of this dip is not clear and it may be a numerical artifact arising from some correlations in spectra or between different parameters of model fitted. This dip is present in results obtained from most of the spectra that we have fitted. However, in averaged spectra from GONG months 24 to 35 and months 32 to 35 this dip can barely be seen.

4.1. Depth of the convection zone

Using solar p-mode frequencies it is possible to determine the depth of the convection zone quite precisely (Christensen-Dalsgaard, Gough & Thompson 1991; Basu & Antia 1997) and it would be interesting to check if this depth is affected by the frequency shifts resulting from asymmetry in peak profiles. We follow the approach used by Basu & Antia (1997) to determine the depth of the convection zone using the frequencies as obtained by fitting both symmetric and asymmetric profiles. We use the same set of reference models to determine the depth of convection zone using the two sets of frequencies.
The fits to symmetric profiles yield the position of the base of the convection zone at 
\((0.71336 \pm 0.00004)R_\odot\), while the use of frequencies obtained by fitting asymmetric profiles yield a value \((0.71344 \pm 0.00005)R_\odot\). Thus there is a marginal decrease in the inferred depth of the convection zone by \(0.00008R_\odot = 56\) km due to asymmetry in peak profiles, which is comparable to the error estimates. However, these error estimates do not include systematic errors as discussed by Basu & Antia (1997) and Basu (1998), which are an order of magnitude larger. Thus the difference arising due to asymmetry is essentially insignificant. Note that the error estimate is slightly larger for frequencies obtained from asymmetric profiles since in that case the number of modes successfully fitted is somewhat smaller.

### 4.2. Overshoot below the convection zone

Apart from the depth of the convection zone, it is also possible to estimate the extent of overshoot below the solar convection zone from the measured frequencies of solar oscillations (Gough 1990; Monteiro, Christensen-Dalsgaard & Thompson 1994; Basu, Antia & Narasimha 1994; Basu 1997). This measurement is obtained from a characteristic oscillatory component in frequencies of oscillations as a function of \(n\), which is introduced by steep changes in derivatives of the sound speed near the base of the convection zone, where the temperature gradient changes from adiabatic value inside the convection zone to the radiative gradient in the radiative interior. The amplitude of the oscillatory component is a measure of the extent of overshoot, while the ‘frequency’ of oscillatory component gives the acoustic depth \(\tau\) of the discontinuity in derivatives of sound speed. This oscillatory signal can be magnified by taking the fourth difference of the frequencies as a function of \(n\). We follow the approach used by Basu (1997) to determine the amplitude and ‘frequency’ of oscillatory component in frequencies.
The results obtained using the two sets of frequencies obtained from fits to symmetric and asymmetric profiles are shown in Fig. 7. These can be compared with earlier results obtained using the GONG data from individual $\ell, n, m$ modes, as well as those from the MDI data for the first 144 days of its operation (Rhodes et al. 1997). It is interesting to note that the results obtained from fits to asymmetric profiles are closer to those obtained from individual $\ell, n, m$ modes which were fitted to a symmetric profile. It is possible that this is a coincidence where systematic errors due to use of $m$-averaged spectra is cancelled by the frequency shift due to asymmetry. However, the results from MDI data which also employ symmetric profiles is close to what we find using symmetric profiles. It appears that the abnormally low amplitude obtained from MDI data, which is smaller than the amplitude in a model without overshoot, might be due to use of symmetric profiles in fitting the power spectra.

5. Conclusions

Using the rotationally corrected, $m$-averaged spectra of solar oscillations obtained by the GONG network we have determined the frequencies of solar oscillations for degree $0 \leq \ell \leq 200$. The use of asymmetric peak profiles improves the fit to observed spectra and the frequencies are increased as compared to those obtained when symmetric profiles are used. This frequency shift of about 0.2 $\mu\text{Hz}$ is larger than the estimated errors in fitted frequency and thus could affect results of helioseismic analyses. However, we find that this frequency shift is partly a function of frequency alone and its effect on helioseismic inferences is generally smaller than other systematic errors. We have confirmed this by inverting the frequency differences to estimate the error in sound speed caused by asymmetry in peak profiles.

We have also investigated how the frequency shifts affect results about the depth of
the convection zone and the extent of overshoot below the convection zone. The inferred depth of the convection zone is reduced by about 56 km, when the effect of asymmetry is included. Similarly, the amplitude of oscillatory component in frequencies increases when asymmetric profiles are used. However, this increase does not change existing limits on the extent of overshoot below the solar convection zone (Monteiro et al. 1994; Basu 1997) since the resulting amplitude is comparable to that obtained from a solar model without overshoot. In fact, the resulting amplitude using asymmetric peak profiles is similar to what is found from the GONG data from individual $\ell, n, m$ modes, which was used in obtaining earlier limits.

In this work we have investigated the effect of asymmetry in peak profile on mean frequencies only. In principle, the frequency splittings may also be affected by asymmetry. Basu & Antia (1999) have studied the effect of asymmetry on the ring diagram analysis of the large scale flows. They find that the asymmetry in peak profiles does not affect the inferred velocity field significantly. The changes in inferred flow velocities due to asymmetry of peaks are equivalent to changes in odd frequency splitting coefficients in global p-modes, thus it is possible that the splittings do not change significantly. This may be expected as to a first approximation asymmetry will shift the frequencies of all modes in a multiplet for given $n, \ell$ by the same amount and hence the splittings may not be affected. Christensen-Dalsgaard et al. (1998) have also argued that the effect of asymmetry in peak profile will not significantly affect the odd splitting coefficients which are useful in determining the rotation rate in solar interior. However, the even splitting coefficients which are determined by aspherical distortions may be affected by asymmetry in peak profiles. Clearly, more work is required to investigate the effect of asymmetry on splitting coefficients. With availability of better data and better understanding of asymmetry it may be possible to fit asymmetric profiles to find the splitting coefficients in addition to the mean frequencies studied in this work.
This work utilizes data obtained by the Global Oscillation Network Group (GONG) project, managed by the National Solar Observatory, a Division of the National Optical Astronomy Observatories, which is operated by AURA, Inc. under a cooperative agreement with the National Science Foundation. The data were acquired by instruments operated by the Big Bear Solar Observatory, High Altitude Observatory, Learmonth Solar Observatory, Udaipur Solar Observatory, Instituto de Astrofisico de Canarias, and Cerro Tololo Interamerican Observatory. This work also utilizes data from the Solar Oscillations Investigation / Michelson Doppler Imager (SOI/MDI) on the Solar and Heliospheric Observatory (SOHO). SOHO is a project of international cooperation between ESA and NASA.
REFERENCES

Anderson, E. R., Duvall, T. L., Jr., & Jefferies, S. M. 1990, ApJ, 364, 699

Antia, H. M. 1996, A&A, 307, 609

Antia, H. M., & Chitre, S. M. 1998, A&A, 339, 239

Antia, H. M., & Basu, S. 1999, ApJ, 519, 400

Basu, S. 1997, MNRAS, 288, 572

Basu, S. 1998, MNRAS, 298, 719

Basu, S., & Antia, H. M. 1997, MNRAS, 287, 189

Basu, S., & Antia, H. M. 1999, ApJ, (in press) astro-ph/9906252

Basu, S., Antia, H. M., & Narasimha, D. 1994, MNRAS, 267, 209

Basu, S., Christensen-Dalsgaard, J., Pérez Hernández, F., & Thompson, M. J. 1996, MNRAS, 280, 651

Christensen-Dalsgaard, J., Gough, D. O., & Thompson, M. J. 1989, MNRAS, 238, 481

Christensen-Dalsgaard, J., Gough, D. O., & Thompson, M. J. 1991, ApJ, 378, 413

Christensen-Dalsgaard, J., Rabello-Soares, M. C., Rosenthal, C. S., & Thompson, M. J. 1998, in Proc: SOHO6/GONG98 workshop, Structure and Dynamics of the Interior of the Sun and Sun-like Stars, eds. S. Korzennik, A. Wilson, ESA SP-418 (ESA: Noordwijk), p 147

Duvall, T. L., Jr., Jefferies, S. M., Harvey, J. W., Osaki, Y., & Pomerantz, M. A. 1993, ApJ, 410, 829
Gough, D. O., 1990, in Osaki, Y., Shibahashi, H., eds., Lecture Notes in Physics, 367, Springer, Berlin, p.283

Hill, F., Stark, P. B., Stebbins, R. T. et al. 1996, Science, 272, 1292

Monteiro, M. J. P. F. G., Christensen-Dalsgaard, J., & Thompson, M. J. 1994, A&A, 283, 247

Nigam, R., & Kosovichev, A. G. 1998, ApJ, 505, L51

Pohl, B., & Anderson, E. 1998, in Structure and Dynamics of the Interior of the Sun and Sun-like Stars, Eds., S. Korzennik, & A. Wilson, ESA SP418, p297

Rabello-Soares, M. C., Christensen-Dalsgaard, J., Rosenthal, C. S., & Thompson, M. J. 1999, A&A, 350, 672

Rhodes, E.J., Kosovichev, A.G., Schou, J., Scherrer, P.H., & Reiter, J. 1997, Solar Phys., 175, 287

Richard, O., Vauclair, S., Charbonnel, C., & Dziembowski, W. A. 1996, A&A, 312, 1000

Toutain, T. 1993, in Proc. Sixth IRIS Workshop, eds. D. O. Gough, & I. W. Roxburgh (Cambridge: Univ. Cambridge Press), 28

Toutain, T., Appourchaux, T., Fröhlich, C., Kosovichev, A. G., Nigam, R., & Scherrer, P. H. 1998, ApJ, 506, L147

This manuscript was prepared with the AAS \LaTeX\ macros v4.0.
Fig. 1.— Fits to power spectra for $\ell = 100$, $n = 3$ and $\ell = 0$, $n = 15$ mode obtained using symmetric and asymmetric peak profiles. The continuous line shows the observed power spectra, the dotted line shows the fit using symmetric profile and the dashed line shows the fit using asymmetric profile.
Fig. 2.— The asymmetry parameter $S$ for the fits to summed spectra for GONG months 7–22. For clarity error-bars are shown only for a few points.
Fig. 3.— The frequency shift due to asymmetry in peak profiles for fits to summed spectra for GONG months 7–22. For clarity error-bars are shown only for a few points.
Fig. 4.— The frequency difference between the fit to symmetric profiles for the summed $m$-averaged spectra and those determined by the GONG project using fits to individual $n, \ell, m$ modes for the GONG months 4–14.
Fig. 5.— The functions $H_1(w)$ and $H_2(\nu)$ resulting from the asymptotic fit to the frequency shifts shown in Fig. 3.
Fig. 6.— The relative sound speed difference inferred by various inversion techniques from the frequency shifts shown in Fig. 3. These represent the error introduced in helioseismic inversions due to use of symmetric profiles. The continuous line represents the results obtained using asymptotic inversion, while dashed line represents that obtained by RLS technique for nonasymptotic inversion, with the dotted lines giving the 1σ error estimates. The points with error bars represent the results obtained using OLA technique.
Fig. 7.— The amplitude of the oscillatory signal in the fourth difference of the frequencies plotted as a function of the ‘frequency’ of the signal. ASYM refers to the frequencies obtained by fitting asymmetric profiles to the peaks, while SYM is from frequencies obtained with symmetric profiles for the GONG months 7–22 summed spectra. MDI is the results for the MDI 144 day data and GONG is the result obtained from GONG months 4–10 data where the frequencies were obtained for each individual m-peak. RICH is for a model using the composition profiles from Model 5 of Richard et al. (1996) which includes rotational mixing of elements below the base of the convection zone, while INV is that for a model constructed with the composition profile obtained from inversions (Antia & Chitre 1998). Neither model has overshoot below the base of the convection zone.