Worldwide trends in the development of education and academic research, 15 – 18 June 2015

Distinctive Features of Mathematical Language Formation in the Context of Bilingual Education

Natalya Spiridonova, Anna Savvinova*

Institute of National Schools of the Sakha Republic (Yakutia), 22 Oktyabr’skaya St, Yakutsk 677027, Russia

Abstract

This article discusses distinctive features of formation and development of mathematical language among students of educational institutions with native (Yakut) language of instruction (case study of the Sakha Republic (Yakutia). The article showcases the results of a study investigating the organization of bilingual educational process in the field of mathematical education. It reveals the psychological and pedagogical conditions conducive to successful formation of mathematical language among school students. The authors have studied a project of a bilingual mathematical dictionary for fifth-sixth forms, developed within the project of the Institute of National Schools of the Sakha Republic (Yakutia) ‘Two Languages - Two Wings’. The dictionary will be an effective tool for implementation of the transitional model of bilingual education. The article also provides the results of experimental work to identify the level of formedness of mathematical language based on native (Yakut), and non-native (Russian) languages.

© 2015 The Authors. Published by Elsevier Ltd.

Keywords: bilingual instruction in mathematics; mathematical language; transitional model of bilingual education; interference; transposition; psychological and pedagogical conditions

1. Introduction

Bilingual education recently has become a subject of focused attention in the Russian Federation (hereinafter the RF), and bilingualism has become a standard for every person. According to the 2010 National Census, the RF, being one of the multinational countries, was home to 193 nationalities; and the Sakha Republic (Yakutia) (hereinafter the SR(Y)), being one of the largest multinational regions of the RF, was home to 128 nationalities. It is known that 277 languages and dialects are spoken in the RF; 89 languages are used in the public educational system,
30 of them - as the language of instruction, and 59 - as a subject of the curriculum. Along with the Russian language, which is the official language of the RF, in 1992 the SR(Y) legally enacted the second official language – the Yakut language (language of the Sakha people). In addition, such languages as Even, Evenki, Yukaghir, Dolgan, and Chukchi were also established as official languages in areas of compact settlement of these northern minorities.

In line with the concept of bilingual education researchers W. Mackey and M. Siguán (1990), since instruction is implemented in the two official languages, the educational system of the republic is bilingual. The official languages are taught as subjects within curriculum. According to the Ministry of Education of the Sakha Republic (Yakutia) (hereinafter the SR(Y) ME), there were 608 general education institutions in the republic in 2013-2014 academic year, out of which in 186 schools the Russian language was studied as native, in 308 schools a non-Russian language was studied as native, and in 114 schools Russian was studied as non-native language. The Sakha language was studied in 188 schools of the republic as a state language, and in 398 schools as native language. Bilingual education in the republic, as well as in other regions with two official languages, can be seen as a priority to ensure the preservation of languages and cultures, as well as the development of bilingual education.

With a view to the implementation of the Federal State Educational Standards (hereinafter FGOS) of general education, the general education institutions with native (non-Russian) instruction medium, which reflect the nation’s multicultural environment, should also have an opportunity to enjoy psychological and pedagogical conditions aimed not only at formation of a creative, socially active person, but also at formation of a balanced bilingualism among children. Psychological and pedagogical conditions should be understood as a set of psychological, organizational, pedagogical, and didactic conditions, providing certain educational interventions for the development of bilingual children, i.e. children who speak two languages.

2. Methodological framework of the study

At the present stage of Russian education development L.L. Salekhova tailored a didactic model of bilingual teaching of mathematics at the higher pedagogical school (Salekhova, 2004). A.I. Petrova was the first to consider in her work the issues of teaching mathematics in national schools via an interconnected use of native and Russian languages (Petrova, 2004). Based on the theoretical foundations of the assimilation of mathematical knowledge in conditions of bilingual education, surveyed in this study, we noted that bilingual teaching of mathematics suggests that the second language, as well as the native one, can be used as means of mastering the mathematical knowledge in the process of learning and cognitive activities of students. And along with these activities, the language of mathematics is not only a means of lexical and grammatical structuring and formulation of mathematical ideas, but also the aim of education. We shall assume that bilingual teaching of mathematics is a set of interrelated activities of teachers and students, aimed at studying mathematics by means of native and non-native languages, which results in the assimilation of subject content, the development of speech and thinking, and the development of ethnic-Russian and Russian-ethnic bilingualism.

In the regions of Russia with two official languages, the following models of bilingual education have evolved over the last century:

1) Linguistic Heritage programs – bilingualism with a focus on native (ethnic) language;
2) transitional model – a gradual transition from the native (ethnic) to the Russian language;
3) immersion training – bilingualism with a focus on the Russian language;
4) submersion in a foreign language environment (Submersion) – adaptation to life in the community of linguistic majority (Petrova, 2004).

Experience of educational institutions with native (non-Russian) language of instruction in the SR(Y) shows that in primary school learning occurs in the native language of learners (Yakut), i.e. Linguistic Heritage programs are functioning. Further in the middle school, a transition to bilingual education occurs. The transitional phase, when students gain experience in the use of the Russian language, embraces fifth-sixth forms, thus implementing transitional bilingual education model. Starting from the seventh form training mainly occurs with a focus on the Russian language, which means immersion training. By the time of admission to the high school, a transition to the language of professional training would have been completed, that is, Russian would be the only teaching medium.
Fig. 1 represents an approximate scheme of bilingual education in secondary schools with native (non-Russian) language medium (as exemplified by the system of mathematical education in the SR(Y)).

**MODEL OF BILINGUAL EDUCATION SYSTEM IN THE SR(Y)**

- **IMMERSION TRAINING**
  - Language of instruction: Russian
  - Language of instruction: Yakut
  - 7-9th forms

- **TRANSITIONAL MODEL**
  - Language of instruction: Russian
  - Language of instruction: Yakut
  - 5-6th forms

- **LINGUISTIC HERITAGE PROGRAMS**
  - Language of instruction – native (Yakut)
  - 1-4th forms

Fig. 1. Scheme of bilingual education system

Further let us define the types of bilingualism. Classification of bilingualism types suggested by E.M. Vereshchagin, which is based on the number of speech acts, is the most suitable for our study:

- Receptive (ability to navigate in bilingual texts regarding well-known subjects, but not more);
- Reproductive (ability to understand and reproduce foreign-language texts regarding known topics and problems);
- Productive (ability to understand and produce texts in two languages) (Vereshchagin, 1969).

In case of bilingual teaching of mathematics, when the second (Russian) language is used as the language of learning and self-education, Yakut school students should practice receptive bilingualism type. That is, in primary school, when expressing their thoughts, children should construct whole meaningful statements in their native language, and navigate in Russian text. In fifth-sixth forms children should develop reproductive type of bilingualism, in which case they would be able to understand and reproduce the learning material in both languages. And starting from the seventh form, students should learn to understand and produce their own statements in the native and Russian languages, that is, they should develop a productive type of bilingualism. On the way towards achieving this objective in the process of bilingual teaching of mathematics, special attention should be paid to how students express mathematical ideas, when their minds are concentrated on the content of the statements, regardless of the language used.

It is known that the mathematical activity occurs in a symbolic form, i.e. in the form of mathematical language. And, since speech is a process of communication by means of language, in order to develop speech when learning mathematics, the students need to master the mathematical language. Indeed, one of the main FGOS requirements for the results of training and understanding of the mathematics curriculum content is the ability to work with mathematical texts, accurately and competently express thoughts in spoken and written using mathematical terminology and symbols, that is, children should have a formed and developed mathematical language.

By definition of J. Ikramov, the term ‘mathematical language’ includes logical and mathematical symbols, graphics, diagrams, drawings, and scientific terms, along with elements of natural language. School mathematical language includes elements of natural language and terminology, and takes into account the features of the teaching media. Mastering the mathematical language implies a conscious assimilation of the content of mathematical concepts, relationships between them (theorems, axioms), and the ability to efficiently and competently express mathematical thoughts orally or in writing, using verbal or non-verbal means of mathematical language, as well as free manipulation with mathematical knowledge and skills in practice (Ikramov, 1989).

It is known that during linguistic communication, the work with information is closely linked to the activities of human mind, in which its content is carefully processed. According to A.I. Petrova, conscious assimilation and understanding of the meaning of a new concept (a term and its definition) is possible only when students find
relevant equivalents in their native language, when they consciously have a foothold on it, and compare some difficult cases of non-native (Russian) language with the corresponding phenomena of their native language (Petrova, 2004). The most important component and the source of lexical abundance of the school mathematical language is terminology. If a word (phrase) is given a certain meaning, which expresses the essence of the concept, it can be called a term. That is, every concept can be expressed by a word or a group of words that result from human mental activity (Ikramov, 1989). We believe that in order to make speech sensible, as well as for the thinking process, the speech activity needs such a semantic component as understanding, which manifests itself in mathematics in form of relation between mathematical terms and their meanings. Ultimately, the process of formation of mathematical language in bilingual environment is the development of students' ability to use mathematical terms and definitions simultaneously in two languages, in accordance with the situation, both in class, and during extracurricular activities.

As stated by J. Cummins, mastering a second language depends to a large extent on the level of development of the first language. If the first language is developed so much that a child is able to use it out of context, the development of a second language is relatively easy (Druzhinin, 1999). That is, subject to the transitional model of bilingual education, students use skills previously learned in their native language, thus implementing the transposition of knowledge and skills. For the Sakha children reliance on the native language in learning mathematics is a great opportunity to develop their mental possibilities, since it is their usual form of expressing and transmitting ideas. Upon assimilation of a mathematical concept, a linguistic shell of this concept appears in the minds of students in form of a term. To do this, students first mentally operate concepts, judgments, and conclusions, using logical methods of thinking, then verbally express them. That is, mathematical speech is a result of a complex cognitive process that occurs in two natural languages.

As noted by I.F. Teslenko, the activities of students of first-sixth forms are characterized mainly by transition from the sensual-concrete to the abstract, and are aimed at creating a common understanding of scientific concepts, while in seventh-eleventh forms there is rather a movement from the abstract to the concrete thinking and mainly scientific concepts are formed (Teslenko, 1979). Hence, during the transition to subject-focused teaching in primary school it is necessary to pay special attention to linguistic characteristics of learners, considering age peculiarities of students’ mental activity and knowing that it is in fifth-sixth forms when theoretical basis for further assimilation of mathematical concepts and laws is laid.

Another important point is to provide conditions that would ensure avoidance of interference, that is, an interaction of language systems, in which there is a violation of the norms of speech. Since 1950s linguists have been intensively discussing the problem of switching codes. In our region code switching is a common phenomenon and involves using words that belong to different language systems within one text. At lessons of mathematics in fifth-sixth forms, when students’ level of the Russian language is low and language of mathematics is a subject of learning, it is recommended to use their native language as the primary language of instruction and the language of code switching situations, so that a text in their own language would include words or phrases in Russian. It can also be used when doing exercises in written or oral form for deeper assimilation of educational content, as well as for studying the terminology. For example, the definition of a term and the description of its meaning are given in the mother tongue of students, and the term itself – in Russian, and vice versa.

In addition, we want to note that in conditions of bilingual education, competence in natural languages, on which mathematical language is based, also should be divided by types of speech activity (oral and written language, listening, and reading). Since it is this set of speech activities that underlie the process of verbal communication, including in the process of learning mathematics. The success of verbal communication and, ultimately, assimilation of the studied material, depend on the level of matureness of students' skills and abilities in said kinds of speech activity.

3. Stages of the study, results and discussion

In 2012-2014 based on mentioned works FGBNU ‘Institute of National Schools of the Sakha Republic (Yakutia)’ (hereinafter SR(Y) INS) conducted a study of distinctive features of organization of bilingual educational process in general education institutions in the domain of mathematical education.
The study incorporated three stages listed further. Stage 1 (2012): analysis of scientific-methodical and educational literature; identification of psychological and educational environment conducive to the formation of mathematical language in the context of bilingual education (monitoring of educational process in first-seventh forms, questioning students and teachers). Stage 2 (2013): development of a transitional model of bilingual teaching of mathematics for formation of mathematical language among students of first-sixth forms of Yakut schools; development of a conceptual and terminological bilingual dictionary for students of fifth-sixth forms; creation of a public professional community dealing with the issues of bilingual mathematical education in the SR(Y). Stage 3 (2014): experimental work to identify the level of formedness of mathematical language based on the native (Yakut), and non-native (Russian) languages. The data resulting from our study will be further used for tailoring a didactic model of bilingual mathematical education technology and testing thereof in educational process of urban and rural schools with native (Yakut) language of instruction.

At Stage 1 of the study in 2012-2013 academic year, in order to identify psychological and educational environment conducive to formation and development of mathematical language among students, we surveyed educational process in first-seventh forms. Summarizing the results of this observation it should be noted that in cases when learning language coincides with the mother tongue of students, it contributes to a better explanation of the meaning of mathematical concepts, in which case there is a better link between a term and corresponding mathematical object. That is, children better cope with the task, when mathematical concepts are provided in their native language. In first forms of Yakut schools the language of instruction is Yakut, which contributes to the development of cognitive activity and positively influences the intellectual abilities of children.

Since mathematics is learned by Russian textbooks starting from the fifth form in Yakut schools, students often find it difficult to understand the meaning of mathematical terms used. The experience of teachers and supervision of educational process indicates that in fifth-sixth forms teachers often resort to the help of the native language of students, and starting from the seventh form the need for further clarification of educational material gradually decreases. That is, children, relying on their native language, are able to better absorb the content of mathematical concepts. This approach does not produce large gaps in teaching of mathematics, doesn’t break the continuity of education. In high school students fully switch to the Russian language – the language of professional training, which, in turn, ensures the continuity of education at its current level.

This implies that when learning mathematics in fifth-sixth forms within transitional model of bilingual education, it is more effective to use translation of terms, denoting scientific concepts and their definitions, from one language to another, in form of an exercise or as a mnemonic trick, but mixing the languages should be avoided. That is, at first, acquisition of mathematical language occurs in the mother tongue, and then acquired knowledge is transferred into Russian (transposition), and vice versa. And in case of the immersion training, formation of concept and learning of mathematical language should be carried out both in the Yakut and Russian languages, which will indicate the students’ competence in productive type of bilingualism. Fig. 2 depicts features of acquiring mathematical language by schoolchildren with a native (non-Russian) learning language.

![Fig. 2. Flowchart of mathematical language formation in the system of bilingual mathematical education](image)

We conducted a survey among students and teachers. Out of 129 students of fifth-sixth forms, 30 elementary students studied their native language only as a subject. The results of the survey among students say that 36% of
respondents couldn’t identify specifically in what language they better absorbed knowledge at mathematics classes. 30% of children preferred the Russian language, 26% were bilingual, and only 7% were willing to study in their native language further. Survey among teachers showed that when teaching mathematics to fifth-sixth forms students, teachers often relied on the Yakut language, for example when studying new material, for generalization and systematization, control and correction of knowledge and skills of students. Teachers also explained that they resorted to the help of children’s native language at such phases of a lesson as preparation for introduction of new material, learning of new concepts and methods of action, initial verification of understanding of a topic, generalization and systematization of knowledge, correction of knowledge and methods of action, reflection and evaluation (summarizing). In addition, they pointed out that some of the children mainly used their native language in speech. But teachers also underlined that children came to the fifth form from different schools where the language of instruction could have been either Yakut or Russian. That is, in such cases the mechanism of implementing transitional models of bilingual teaching of mathematics doesn’t work. Thus, both children and teachers are experiencing some difficulties at transitional stage of training, which mainly occurs in the fifth-sixth forms.

At this stage of the study we found the following important psychological and pedagogical conditions of mastering mathematical language within transitional model of bilingual education: to perform propaedeutic work to develop general logical and logical-mathematical operations;  to account for psychological and linguistic characteristics of learners; to consider the interaction of verbal and symbolic components of mathematical language; to prevent interference; to develop speech activities, and provide appropriate scientific and methodological literature.

At Stage 2 of the research, in view of the identified psychological and pedagogical conditions for successful formation of mathematical language, we have developed a bilingual concept-terminology dictionary of mathematical terms to be used in fifth-sixth forms of Yakut schools in the framework of the ‘Two Languages - Two Wings’ project of the SR(Y) INS. This dictionary will be one of the tools for implementation of transitional model of bilingual teaching of mathematics. The dictionary is based on ‘Russian-Yakut Glossary of Mathematical Terms’ (I.G. Yegorov, P.P. Petrov, A.I. Petrova), approved for use in the secondary school by the SR(Y) ME in 1998. The structure of the bilingual dictionary was tailored taking into account psychological characteristics of students; it reflects not only the linguistic shell of mathematical concepts, but also such crucial structural components of mathematical language as semantics and syntax. It should be clarified that the semantic component of mathematical language characterizes relation of mathematical terms to proper concepts and to their content, while syntax - relations between words and symbols. The layout of the dictionary represents three columns, which reveal the essence of a mathematical concept through three languages (language of mathematics, Yakut, and Russian). The first and the third columns give a mathematical term and its definition in the native (Yakut) and Russian languages, to reveal the semantic component of mathematical language; the second column gives logical-mathematical symbols, graphics, diagrams, and drawings that determine the syntactic component. For ease and productivity of use by participants of the educational process, the dictionary will be a two-way one, combining the Yakut-Russian and Russian-Yakut versions.

![Fig. 3. Page spread of the bilingual dictionary of mathematical terms for students of 5-6 grades of Yakut schools](image-url)
Above described work was carried out with scientific advice of scholars: a doctor of pedagogical sciences and a
candidate of pedagogical and philological sciences. The reviewing also involved scientists: a doctor of physical and
mathematical sciences, and candidates of pedagogical and philological sciences.

To create a glossary for the dictionary we considered mathematical concepts and terminology, as well as set
phrases expressing those concepts, which were included in textbooks on mathematics for the fifth and the sixth
forms from the federal list of textbooks approved for the 2012-2013 academic year. Nine textbooks were considered
per each form. The number of terms for the fifth form was 731, for the sixth form - 757. After eliminating
duplication, a total of 1,325 terms were left for fifth-sixth forms. We identified 392 terms, which, in our opinion,
could have been classified as follows: non-translatable terms; terms used with phonetization, translatable terms;
controversial terms. At the moment, special attention should be paid to the controversial terms.

In addition, we examined textbooks on mathematics and numeracy in the Sakha language, published from 1936
to 2010, and analyzed textbooks on mathematics (first-fourth forms) from 12 primary school teaching kits.
At the initiative of the SR(Y) INS a public and professional community on issues of bilingual education was
formed. Among activities of the public and professional community, which embraces mathematics teachers,
linguists, mathematicians, and methodology experts, was a series of round tables to discuss the manuscript of the
dictionary. Currently, based on the comments of reviewers and members of the community, the dictionary has been
prepared for approbation in educational process at Yakut schools in 2015-2016 academic year.

At Stage 3 of the study in the 2014-2015 academic year in order to identify the level of matureness of
mathematical language among students of fifth-sixth forms on the basis of native (Yakut), and non-native (Russian)
languages, we carried out a screening work that embraced 121 students of the Yakut schools (of Yakutsk). We have
developed exercises in two languages, corresponding to the basic educational program. The level of assimilation of
mathematical language by students was assessed by the level of semantic skills (the ability to explain, to understand
the meaning of a term, or a symbol), and syntactic skills (the ability to restructure linguistic units), as well as
mathematical modeling skills.

Having compared performance in different groups of skills, we can say that students have better command of
semantic skills (the number of correctly executed tasks on semantic skills was 9.59% larger than on syntactic skills,
and 42.56% larger than ability to model). In semantic skills block understanding of terms was better than
understanding of symbols (by 7.66%). In the sixth form we observed an increase in level of semantization of terms
(by 5.42%) and decline in the level of semantization of symbols (by 36.01%). Levels of mastering syntactic skills in
fifth and sixth forms, respectively, were 65.66% and 19.6%, indicating a significant decrease in the level of syntactic
skills among children (by 46.06%). Ability to model was characterized by the following data. The rate of correctly
executed tasks in the fifth form made 16.92%, in the sixth form - 2.73%. Here the decrease of indicators in the sixth
form compared to the fifth form characterized a significant decrease in the level of skills.

Having compared the levels to which students had mastered skills that characterize levels of development of
mathematical speech in two languages, we found that students understand the meaning of terms in the Yakut and
Russian languages. The deviation from the average was not more than 9%. But with transition to the sixth form
children demonstrated a general decrease in the level of formation of semantic skills (terms). It is evident that it is
easier for them to explain the terms of relations in their native language. As for the semantics of symbols, here the
overall performance was also lower in the sixth form. In the fifth form students better assimilated the subject content
in Russian. In the sixth form children better coped with tasks related to the symbols of operations and relationships
based on their native language. Students of both the fifth and the sixth forms were able to write down verbal
expressions using mathematical symbols, perform transformations of symbolic expressions, construct expressions
using given symbols, and simulate verbal situation better in Russian, but their syntactic skills, in general, became
much weaker in the sixth form. While the ability to read symbolic records, on the contrary, improved over time,
particularly, in their native language. Consequently, students of fifth-sixth forms of Yakut schools assimilate subject
content, namely formation of semantic and syntactic skills inherent to mathematical language, in the Yakut and
Russian languages.
4. Conclusion

Due to the fact that there is no systematic approach to bilingual education, in particular, due to insufficiently developed theoretical base for formation of mathematical knowledge among students in conditions of two functioning official languages, introduction of transitional bilingual model of teaching mathematics occurs intuitively and depends on the level of a student’s development, as well as on the linguistic and special training of a teacher.

In course of the study, we saw once again that the development of language in children was closely linked with the development of their cognitive and mental abilities. Aside from assimilating the subject content, interference of instruction languages should be strictly avoided during the process of formation of mathematical language in bilingual educational environment. It is also needed to be taken into account that language of mathematics includes not only terminology, but also elements of natural language; hence it should be learned taking into account the peculiarities of instruction language. Transitional model of bilingual education, which involves a gradual transition from the native language to Russian, is the most optimal for further successful learning of mathematics, which is determined by degree of completeness to which the students assimilate educational material, by their ability to perform mental operations, and by development of their mathematical language and productive bilingualism.

We believe that the bilingual mathematical dictionary can become an effective tool for implementing the transitional model of bilingual teaching of mathematics, as well as the basis for translation and creation of textbooks and teaching aids. And we are confident that in case this model is implemented, educational objectives will be achieved at all levels, and the system of bilingual school education will become a stepping stone, preparing children for life in a multicultural environment.

References

Druzhinin, V. N. (1999). Sovremennaya psikhologia [Contemporary psychology]. Reference manual. Moscow: Infra-M.
Ikramov, J. (1989). Yazyk obuchenia matematike [Language of learning mathematics]. Tashkent: Ukituvchi.
Petrova, A.I. (2004). Stanovlenie i razvitie sistemy dvuyazychnogo obrazovaniya: istoriya, teoriya, opyt i perspektivy (na primere matematicheskogo obrazovaniya v Respublike Sakha (Yakutia)) [Formation and development of bilingual education system: history, theory, experience and prospects (as exemplified by mathematical education in the Sakha Republic (Yakutia))]. Moscow: MGOU.
Salekhova, L.L. (2004). Teoriya i praktika razvitiya shkol s bilingval'nym obucheniem [Theory and practice of development of schools with bilingual education]. Kazan: Izdatel’stvo Kazanskogo universiteta [Kazan University Publishing].
Siguán, M. & Mackey, W. (1990). Obrazovanie i dvuyazychie [Education and bilingualism]. Moscow: Pedagogika.
Teslenko, I.F. (1979). Formirovanie dialektiko-materialisticheskoj mirovozзreniya uchashchikhsya pri izuchenii matematiki [Formation of a dialectic-materialistic outlook in students at studying mathematics]. Moscow: Prosveshchienie.
Vereshchagin, M.E. (1969). Voprosy teori i metodiki prepodavaniya inostrannykh yazykov [Issues of theory and methodology of teaching foreign languages]. Moscow: Izdatel’stvo Moskovskogo universiteta [Moscow University Publishing].