Study of a method to reduce the harmonics of the output voltage of medium frequency power supply

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Abstract. Since the electrical equipment with nonlinear load began to be used, the harmonic suppression technology has become the focus of research. In this paper, a robust multi-resonance servo control method is proposed and applied to the medium frequency power supply. The method can not only perform the closed-loop system stability and asymptotic tracking, but also make the power supply output possess the advantage of rapid dynamic response, and low-order strong harmonic suppression. First, this paper presents a mathematical model, analyzes the time-domain robust control - linear matrix inequality and regional D stability and turns the analysis of the problem of robust control problem to a one of linear matrix inequality (LMI) convex optimization problem through the augmentation of system matrix dimension. Then, the YALMIP toolbox is used to solve for the status feedback gain matrix. Finally the whole system is simulated and tested. The results show that the designed controller makes the dynamic recovery time of the closed-loop system less than 5ms, and the output voltage THD is reduced to less than 4.5% with the nonlinear load.

1. Introduction

The basic control objectives of inverter are zero-error tracking of the reference signal, fast dynamic response, strong robust performance and low output voltage THD [1], around which a variety of control techniques have been applied to the inverter power supply design. The traditional double closed-loop PI control has been widely used, but it is known from the internal mode principle [2] that this control method has static error in AC power control [3]. The literature [4] proposed a dual closed-loop instantaneous value control strategy for voltage and current based on the PR controller, which can eliminate the system static error, but has a poor ability to carry non-linear loads. In recent years, improved LQR (linear quadratic optimum) control methods have started to be gradually applied in the field of inverter control. Literature [5] applied the LQR method to parallel inverter control and achieved steady-state current equalisation in the presence of a disturbance environment, and literature [6] discussed the method in more detail, but the method could not achieve closed-loop static error-free tracking; In [7], a time-domain robust controller based on the linear matrix inequality (LMI) method was applied to the excitation inverter control, which achieved relatively good steady-state and dynamic performance, but did not consider the case when the load was non-linear; In order to solve this problem, this paper proposes a low harmonic suppression technique based on the linear matrix inequality method, which achieves an optimal combination of robust control and servo control with strong low harmonic suppression capability. The article first analyses the circuit topology of the
medium power supply; secondly, it introduces the principle of the robust multi-resonant servo control method; finally, the proposed control method is simulated on the medium power supply platform, and the simulation results show the superiority and effectiveness of the method.

2. Main circuit topology and modelling analysis
The single-phase medium power supply discussed in this paper uses an H-bridge inverter structure with the circuit topology shown in Figure 1, where T1, T2, T3 and T4 are power devices with anti-parallel diodes IGBTs and the filters are LC-type filters.

![Figure 1. Topological structure of main circuit.](image)

The inverter part of the H-bridge can be approximated as a proportional link, $R_f$ and $L_f$ denote the internal resistance and inductance of the output filter inductor respectively, $C_f$ is the output filter capacitor, $I_L$ is the inductor current, $U_o$ is the output voltage, the output voltage of the H-bridge will vary due to the existence of load disturbance, etc., which is recorded as $U_o + \Delta U_o$. The load is unknown in the actual system, and the load variation is usually taken as the external system. The capacitor voltage and filter inductor current are used as state variables to obtain the following state space expressions.

$$
\dot{x} = Ax + Bu + Ed \\
y = Cx + Du
$$

$$
A = \begin{bmatrix}
-Y_0 & \frac{1}{C_f} \\
-\frac{1}{L_f} & \frac{1}{L_f}
\end{bmatrix},
A_{max} = \begin{bmatrix}
-Y_{max} & \frac{1}{C_f} \\
-\frac{1}{L_f} & \frac{1}{L_f}
\end{bmatrix},
A_{min} = \begin{bmatrix}
-Y_{min} & \frac{1}{C_f} \\
-\frac{1}{L_f} & \frac{1}{L_f}
\end{bmatrix},
B = \begin{bmatrix}
0 & 1 \\
1 & 0
\end{bmatrix},
C = [1 0],
D = [0 0],
u = U_T,
E = \begin{bmatrix}
\frac{1}{C_f} & 0 \\
0 & \frac{1}{L_f}
\end{bmatrix},
d = [\Delta U_T] x = [U_o I_L]
$$

Where: $x$ is the state variable, $y$ is the output variable and $u$ is the control input. The above state equations are discretised using zero-order hold:

$$
\dot{x}(k + 1) = A_dx(x(k)) + B_du(u(k)) + E_dd(k)
$$

$$
y(k) = C_dx(x(k)) + D_du(u(k))
$$

Where $A_d = e^{A_T}, A_d_{max} = e^{A_{max}T_s}, A_d_{min} = e^{A_{min}T_s}, B_d = \int_0^{T_s} e^{A(T_s-t)}B dt$

$$
C_d = C, \quad D_d = D, \quad E_d = \int_0^{T_s} e^{A(T_s-t)}E dt.
$$

Discrete robust multi-resonant servo and sliding mode controller design
The controller structure block diagram used in this paper is shown in Figure 2. The discrete sliding mode controller is the inner loop current loop, the outer loop voltage loop adopts a robust servo controller composed of the internal mode principle combined with the linear matrix inequality method, and the global optimal controller for multi-objective cooperative control is obtained by solving the set of linear matrix inequalities.
The discrete sliding-mode controller is chosen for the inner-loop current loop to avoid the chattering problem caused by the direct digitisation of the continuous sliding-mode control. When designing the inner-loop current loop, the external disturbance perturbations of the system and the uncertainty parameter $\Delta U_T$ present in the system are temporarily excluded, so the state space expression (2) becomes,

$$\dot{x}(k + 1) = A_d x(k) + B_d u(k) y_1(k) = C_{d1} x(k) + D_d u(k)$$  \hspace{1cm} (3)

$$C_{d1} = [0 \ 1], y_1 = I_L$$

When designing the current loop controller the output is inductive current so the matrix $C_{d1} = [0 \ 1]$.

The objective of the inner loop control is to track the reference current with zero error, choosing the sliding mode surface

$$s(k) = I_{ref}(k) - I_L(k),$$

where $I_{ref}(k)$ is the reference quantity for the voltage outer loop output which is also the current inner loop. From equation (3) it follows that

$$s(k + 1) = I_{ref}(k + 1) - I_L(k + 1) = I_{ref}(k + 1) - C_{d1} x(k + 1)$$  \hspace{1cm} (4)

In a control cycle $I_{ref}(k + 1)$ is approximately equal to $I_{ref}(k)$ and from equation (4) the inner loop equivalent control can be derived as

$$u_{eq}(k) = -(C_{d1} B_d)^{-1} [C_{d1} \cdot A_d x(k) - I_{ref}]$$  \hspace{1cm} (5)

The medium power supply designed in this paper has a rated output power of 20kVA and the circuit parameters are shown in Table 1.

| $T_S$ | $L_f$ | $C_f$ | $R_f$ | $Y_0$ | $U_1$ | $U_0$ | $f$ |
|------|------|------|------|------|------|------|------|
| 1/3200 | 0.8mH | 20\mu F | 0.1$\Omega$ | 1.492S | 600V | 115V | 400Hz |

The discrete sliding mode controller can be obtained as

$$u_{eq}(k) = -3.7977 \cdot [ [0.0297, 0.3423] x(k) - I_{ref}]$$  \hspace{1cm} (6)

3. Design of the outer-loop robust servo controller

When designing the voltage outer loop, considering the external disturbance perturbation of the system and the uncertainty parameter $\Delta U_T$ present in the system, the inner loop sliding mode controller and the medium power supply model are integrated together as the control object, then substituting the (5) formula into the (2) formula yield

$$x(k + 1) = A_m x(k) + B_m u_s(k) + E_d d(k)$$  \hspace{1cm} (7)

Definition: $u_s(k) = I_{ref}$.

$$x(k + 1) = A_d x(k) - B_d (C_{d1} B_d)^{-1} C_{d1} A_d x(k) + B_d (C_{d1} B_d)^{-1} u_s(k) + E_d d(k)$$

then:

$$x(k + 1) = A_m \cdot x(k) + B_m u_s(k) + E_d d(k)$$  \hspace{1cm} (8)

$$A_m = A_d - B_d (C_{d1} B_d)^{-1} C_{d1} A_d, B_m = B_d (C_{d1} B_d)^{-1}, u_s(k) = I_{ref}$$

The control object in this paper is a medium power supply, the reference input and the disturbance signal are both 400Hz sine waves, and the least common multiple of their characteristic
The polynomial is, which is known from the internal mode principle, which requires the inclusion of two poles in the controller, making a high control gain near the fundamental frequency, with the following expressions

\[ W_1 = \frac{K_3 s + K_4}{s^2 + \omega_1^2} \]  

where \( K_3, K_4 \) are the parameters to be found and the equation of state corresponding to equation (9) is

\[
\dot{y}(t) = A_{w3}y(t) + B_{w3}e \\
y = C_{w3}y(t)
\]  

\( A_{w1} = \begin{bmatrix} 0 & 1 \\ \omega_2^2 & 0 \end{bmatrix}, \ B_{w1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C_{w1} = [K_4 \ K_3]. \)

Similarly, The corresponding equations of state for the third and fifth harmonic servo compensators are:

\[
\dot{y}(t) = A_{w3}y(t) + B_{w3}e \\
y = C_{w3}y(t)
\]  

\( A_{w3} = \begin{bmatrix} 0 & 1 \\ \omega_3^2 & 0 \end{bmatrix}, \ B_{w3} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C_{w3} = [K_6 \ K_5], \ A_{w5} = \begin{bmatrix} 0 & 1 \\ \omega_5^2 & 0 \end{bmatrix}, \ B_{w5} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ C_{w5} = [K_8 \ K_7]. \)

Eqs. (10), (11) and (12) are discretized

\[
U_0 = 115 \sin \omega_1 t
\]

\( U_{ref} \) is the reference voltage, \( \eta_1(k), \ \eta_2(k), \ \eta_3(k) \) are the state variables of the single, third and fifth harmonic component servo controllers respectively; \( K_3, K_4, K_5, K_6, K_7, K_8 \) are the parameters to be solved, \( y_{ref} \), \( y_{1}(k) \), \( y_{2}(k) \), \( y_{3}(k) \) are the output variables of the fundamental, third and fifth harmonic servo controllers respectively.

The combination of equation (8) for the controlled object and equations (13), (14) and (15) for the discrete servo controller yields the extended system equation of state as

\[
x_p(k + 1) = A_p x_p(k) + B_p u_2(k) + E_d d(k) + F_d U_{ref}(k) \\
y_2(k) = C_p x_p(k) + D_p u_2(k)
\]

\[
A_p = \begin{bmatrix}
A_m & 0 & 0 & 0 \\
0 & A_{w1} & D_d & A_{w3} \\
0 & A_{w3} & D_d & 0 \\
0 & D_d & A_{w5} & D_d \\
\end{bmatrix}, \ B_p = [B_m - B_{w1} D_d - B_{w3} D_d - B_{w5} D_d].
\]

\[
C_p = [C_d C_{w1} C_{w3} C_{w5}], \ D_p = [0 0 0 0], \ F_d = \begin{bmatrix} 0 \\
B_{w1} \\
B_{w3} \\
B_{w5} \end{bmatrix}
\]

From the literature [9], when solving for the stability of the augmented system, one can set \( U_{ref} = 0 \). Thus, equation (16) transforms to

\[
x_p(k + 1) = A_p x_p(k) + B_p u_2(k) + E_d d(k) \\
y_2(k) = C_p x_p(k) + D_p u_2(k)
\]  

4. Robust D-stabilisation

The general LMI region is defined as follows: for a D region symmetric about the real axis in the complex plane, medium there exists a symmetry matrix L and a square matrix M such that holds, then D is said to be an LMI region
the closed-loop system poles are configured in the shaded region as shown in Figure 3, it is guaranteed that there is a minimum damping ratio $\varepsilon$ and a maximum natural frequency $\omega_n$, which further ensures that dynamic performance indicators such as overshoot, rise time and regulation time do not exceed the upper bound determined by $\varepsilon$ and $\omega_n$. The conditions for the closed-loop system poles to be configured in the shaded region are

$$L_1 \otimes P + M_1 \otimes (AP) + M_1^T \otimes (AP)^T < 0 \quad L_2 \otimes P + M_2 \otimes (AP) + M_2^T \otimes (AP)^T < 0 \quad L_3 \otimes P + M_3 \otimes (AP) + M_3^T \otimes (AP)^T < 0$$

(19)

where $\otimes$ represents the Kronecker product of the matrix,

$L_3 = 2\alpha, \quad M_1 = 1, \quad L_2 = \begin{bmatrix} -r & 0 \\ 0 & -r \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad M_3 = \begin{bmatrix} \sin \theta & \cos \theta \\ -\cos \theta & \sin \theta \end{bmatrix}$

From the literature [8], it is sufficient to ensure that this holds at the vertex throughout the variation of the load $Y_0 \in [Y_{min}, Y_{max}]$, i.e.

$$\begin{bmatrix} A_p(Y_{min})X + B_pW + (A_p(Y_{min})X + B_pW)^T & E & (C_pX + D_pW)^T \\ E^T & -\gamma I & 0 \\ C_pX + D_pW & 0 & -\gamma I \end{bmatrix} < 0$$

(20)

where $A_{min} = A(Y_{min})X + BW, \quad A_{max} = A(Y_{max})X + BW$. If the closed-loop system poles are configured in the LMI region as shown in Figure 3, they need to satisfy

$$L_1 \otimes P + M_1 \otimes (A_{min}) + M_1^T \otimes (A_{min})^T < 0 \quad L_2 \otimes P + M_2 \otimes (A_{min}) + M_2^T \otimes (A_{min})^T < 0 \quad L_3 \otimes P + M_3 \otimes (A_{min}) + M_3^T \otimes (A_{min})^T < 0 \quad L_4 \otimes P + M_4 \otimes (A_{max}) + M_4^T \otimes (A_{max})^T < 0 \quad L_5 \otimes P + M_5 \otimes (A_{max}) + M_5^T \otimes (A_{max})^T < 0 \quad L_6 \otimes P + M_6 \otimes (A_{max}) + M_6^T \otimes (A_{max})^T < 0$$

(21)

In summary, the objective function to be solved can be obtained as follows: the minimum $\gamma$ value is found and the constraints are inequalities (20) and (21). The final choice of parameters for the D domain is shown in Table 2, with $\theta = \pi/2$ to represents no requirement for the damping ratio of the closed-loop system.
Table. 2 parameters of D

| α   | θ  | r   |
|-----|----|-----|
| 100 | π/2| $1.5 \times 10^4$ |

Using the YALMIP toolbox the optimal feedback gain matrix is finally found as

$K_{opt} = [K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8] = [-1.3821, 0.6476, -4.0423 \times 10^5, 334.7280, 3.587 \times 10^6, 177.0978, -9.3552 \times 10^6, -150.3688].$

5. Simulation validation

According to a study by the Japanese Electrical Society, diode rectifiers account for nearly 70% of the classification of non-linear loads, with diode rectifiers-type non-linear load structures shown in Figure 4. Referring to the American UPS power supply IEC62040 standard can be obtained with uncontrolled diode rectifiers type non-linear load, the resistance and capacitance values are

$$R_L = \frac{0.66P}{1.22U_o^2}, \quad C_L = \frac{7.5}{f_0 \times R_L},$$

where $U_o$ is the output voltage RMS, $P$ is the output power and $f_0$ is the fundamental frequency. Based on the above formula, the resistance is calculated to be $R_L = 9.941 \Omega$ and the capacitance is $C_L = 1900 \mu\text{F}$ when carrying a 3kVA non-linear load.

The simulation results are shown in Figure 5 when the servo controller contains fundamental, 3rd harmonic and 5th harmonic controllers and the load is connected to a purely resistive load and a resistive inductive load respectively.

When the load is connected to a 3kVA non-linear load as shown in Figure 7 and the servo controller contains fundamental, fundamental + 3rd harmonic and fundamental + 3rd harmonic + 5th harmonic controllers respectively, the output voltage and current simulation waveforms and the output voltage THD when carrying the load are shown in Figure 6, Figure 7 and Figure 8 respectively.

![Fig. 4 Nonlinear load model](image)

![Fig. 5 Simulation results of the power supply under resistance load](image)

![Fig. 6 The nonlinear load simulation waveforms of the servo controller with fundamental wave](image)
When with non-linear loads, the THD of the output voltage is significantly reduced after the 3rd and 5th harmonic controllers are added to the servo controller, and the corresponding 3rd and 5th are also reduced to below 0.25%. After adding the 3rd harmonic servo controller, the total harmonics of the output voltage were reduced from 12.4% to 6.6%, of which the 3rd harmonic was reduced from 9.2% to 0.7%, and then after adding the 5th harmonic servo controller, the total harmonics of the output voltage were reduced to 4.4%, of which the 3rd harmonic was reduced from 9.2% to 0.4% and the 5th harmonic was reduced from 6.7% to 0.7%. The above results fully illustrate the designed controller.

6. Conclusion

The designed controller has good low harmonic suppression capability. After adding the 3rd and 5th harmonic servo control controller, the output voltage quality is significantly improved, the output voltage THD is only 4.4%, and the 3rd and 5th harmonics are reduced to less than 0.7%.

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