Physically based bead topology model coupled with electro-mechanical power source model applied for wire and arc additive manufacturing

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Abstract. The present work is carried out in the framework of the WAS project \cite{1} which deals with WAAM process. The process relies on an automatized welding process in which a part is built by successively deposed metal bead. We propose a physically based bead topology model using the equilibrium between the hydrostatic pressure and the capillarity force, under two-dimensional hypothesis. This equilibrium can be described by the Laplace equation \cite{2}. The proposed model is used to estimate a bead topology which is deposed on an inclined support. Moreover, a deposed melted metal volume is necessary for the bead topology model. By modelling a gas metal arc welding (GMAW) power source system \cite{3}, the volume can be estimated and be used as a physical parameter for the bead topology model. Combining the topology and the power source models, the coupling model allows to simulate the topology of a weld bead through WAAM. In addition to the modelling, experimental profiles of the beads are used to validate the model.

1 INTRODUCTION

Metal additive manufacturing (MAM) has grown, in recent years, very strong interest in academic researches as well as industrial applications. Among MAM processes, wire and arc additive manufacturing (WAAM) became very popular due to its advantages in manufacturing of medium and large-scale components \cite{4}. Even though, manufacturability of a part relies on manufacturing topology because the part’s geometry defects such planarity defect may cause the collision between the welding torch and the part or the part’s geometry deviation from the targeted geometry. This defect is linked to the process parameters such as the synergy of power generator: wire speed vs current and voltage and the robot parameters: trajectory and torch speed. To avoid these defects, it is necessary to be able to simulate the topology of a weld bead. Firstly, the topology of a weld bead is depending on the deposed wire feed volume. One needs to be able to correctly estimate the deposed volume from the wire feed controlled by the power generator. Consequently, a dynamic model of drops detaching from a gas metal arc welding electrode has been proposed \cite{5}. The model allows to determine the kinematics of the drops as
well as the wire speed, the current and the voltage of the welding arc. The model is simplified in the study in order to only simulate the wire speed to estimate the deposited volume of a weld bead.

[6] has proposed a weld bead deposition using Laplace equation by the 2D prismatic assumption in order to simulate a 2D profile of a weld bead. However, to be able to simulate the profile, the model parameters (contact angle and curvature) need to be estimated from the experimental profiles according to the process parameters.

Physically, when the support is in an inclined position, the drop could lose its equilibrium and falls along the support. However, if the drop is in equilibrium, the hysteretic contact angle occurs. The advance and recession contact angles of the drop appears. [7] has proposed the equilibrium condition of a water drop with a given volume on an inclined plan with a hysteretic contact angle. The equilibrium condition is used to verify whether a drop is in equilibrium on an inclined plan. For a given inclined plan, the author has determined the critical volume at which the drop lose its equilibrium. On the other hand, in the study, we use the same equilibrium condition under 2D assumption coupling with Laplace equation to build a weld bead 2D profile on an inclined plan.

2 METHODOLOGIES

To simulate a weld bead topology according to the WAAM process, the volume of a deposited weld bead needs to be correctly estimated. The volume can be computed by the matter flow from the wire feed, hence the wire feed speed. The wire speed is a synergy parameter related directly to current and voltage of power source generator. The wire speed can be varied depending on the considered welding process. For example, in the Cold Metal Transfer (CMT Fronius) welding process, in order to stabilize some parameters, the wire speed may evolve during the weld bead deposition, on the other hand, in the pulsed welding process, the wire speed remains unchanged. The wire speed, space and plate power source model of the CMT welding process in order to simulate the wire speed. Then, the wire speed is used to estimate the deposited weld bead area section considering the shrinkage solidification. After that, the estimated area section is used to feed the topology model in order to simulate the weld bead profile on an inclined plan. The following subsections describes a power source model for CMT welding process, and a topology model for a weld bead deposition on an inclined plan.

2.1 Electro-mechanical power source model

Fig. 1 illustrates a system used to simulate and reproduce the CMT process behavior. The power source produces a constant voltage to feed the electrical circuit such that the wire is an electrode and the substrate an anode. The given synergy of wire feed speed and voltage of the power source are applied to the system. As shown in the fig. 1, Contact Tube Welding Distance (CTWD), is an external parameter which affects the current and arc voltage wave shapes. Therefore, it impacts also on the control of wire speed. This change in wire speed allows to stabilize the targeted arc-length.
The depicted system is described by a system of physical equations [5]. As previously described, we are only interested in wire speed in order to determine the molten pool volume for physical topology simulation. The system of equations can be simplified as follows:

\[
\begin{align*}
\frac{dI}{dt} &= V_0c - (R_a + R_s + \phi)I - V_0 - E_a(C_T - l_s) \\
\frac{dl_s}{dt} &= ws - C_1I + C_2\phi l_s I^2
\end{align*}
\]

Equation (1) enables to get the dynamics of current and wire speed. Required inputs for the system are power source voltage \(V_0c\), initial wire speed \(ws\), contact tube welding distance \(CT\), and wire radius \(r_w\). The fusion flow constant parameters \(C_1\) and \(C_2\) have direct impact on the molten flow. Hence, these constants are dependent directly on the synergy power generator. These parameters require to be identified from experimental measures. Other parameters are chosen according to steel properties as shown in table 1.

**Table 1**: Parameters of the power source model of the steel material.

| \(R_s (m\Omega)\) | \(L_s (\mu \text{H})\) | \(V_0 (V)\) | \(R_a (m\Omega)\) | \(E_a (V/m)\) | \(\phi (\Omega/m)\) |
|---|---|---|---|---|---|
| 75 | 20 | 15.7 | 4 | 1500 | 0.13 |
In order to identify the fusion flow constant parameters $C_1$ and $C_2$, we need to manufacture a weld bead for given synergy process parameters $V_{oc}$, $r_w$, $w_s$ and $CT$. For a given value of $CT$, the measured wire speed is extracted from the experiment. The mean value of experiment wire speed ($w_{meas}$) is then computed. From the equation (2), we can calculate the mean value of simulation wire speed in function of $C_1$ and $C_2$: $w_{sim}(C_1, C_2)$. We can find the optimal fusion flow constants as follows:

$$(C_1^*, C_2^*) = \arg\min |w_{sim}(C_1, C_2) - w_{meas}|$$

(2)

The calibration and validation of the model is carried out on an experiment of a manufactured weld bead with three different values of contact tube welding distance $CT = 10, 15$ and $20$ mm. The results are shown in the following discussion section.

2.2 Physical topology model

We focused, in this study, topology models of a weld bead deposed on an inclined plan. Weld bead topology can be described by Laplace equation of drop’s hydrostatic pressure and capillary force equilibrium. In the following sections, we describe the 2D Laplace weld bead profile and topology models for a weld bead deposed on an inclined plan.

2.2.1 Laplace weld bead profile

Molten pool under the electrical arc can be acted on by several kinds of forces such as Lorentz force, buoyance force, arc shear stress and surface tension force [6]. By considering these forces, the molten pool topology can have a very complicated shape. Under 2D hypothesis, it is stated that the approximated topology of the molten pool can be handled by a static liquid state which is the equilibrium between the hydrostatic pressure and the capillarity force. This equilibrium can be expressed as Laplace equation:

$$\Delta P = \gamma C$$

(3)

By assuming that the curvature radius is isotropic, we can write equation in 2D as following:

$$0.5\kappa^{-2}z^2 - C_0 z + 2\left(\frac{dx}{dz}\right)^2 = 0$$

(4)

or

$$\begin{align*}
\frac{dx}{dz} &= \frac{1}{\tan(\theta_0)} \\
0.5\kappa^{-2}z^2 - C_0 z &= 2(\cos(\theta_0) - 1)
\end{align*}$$

(5)

Where $\theta_0$ is Laplace contact angle illustrated in the fig. 2 which is formed by a tangent to
the contact point of the Laplace profile and the horizontal line;

\( C_0 \) is curvature at point \( P_0 \) (\( x = 0, \ z = 0 \)) the origin of the local reference frame \((Oxz)\);

\[ \kappa = \sqrt{\frac{\gamma}{\rho g}} \] is the capillary length;

\( \gamma \) is the surface tension force between liquid-vapor surface;

\( g \) is the gravity;

\( \rho \) is the metal density at the melting temperature;

In the case of steel, \( \gamma = 1.8N/m, \rho = 7150kg/m^3 \) and \( g = 9.8m/s^2 \). These values are used through this work.

As shown in the fig. 2, the topology of the drop at equilibrium can be built by the equation depending on the position of the local reference frame at \((x = 0, \ z = 0)\). There are two different positions of local reference frame origin, for an example, the origin \( P_0 \) or \( P'_0 \) which can be chosen depending on the normal direction of the support (up or down).

The above equation can be solved numerically by finite difference method or analytically by the elliptic function of first and second kind. As illustrated in fig. 3, the solution of the numerical method is in suitable agreement with the analytical one. However, the computational time of the numerical method is only taken 0.02 s which is 15 times faster than the analytical one which takes 0.3 s.

**Figure 2**: 2D weld bead profile scheme.
2.2.2 Inclined plan deposition

A liquid drop is equilibrated on an inclined plan due to the occurrence of the hysteretic contact angle \([2]\). This angle is defined by the difference between the advanced contact angle \((\theta_a)\) and the recession contact angle \((\theta_r)\) of a drop on an inclined plan of angle \(\alpha\) as shown in fig. 7. \([7]\) has demonstrated a method to determine the equilibrium condition of a known volume of water drop on an inclined plan.

The surface tension force \(f_p = l\gamma(\cos(\theta_r) - \cos(\theta_a))\) at the tri-phase line is balanced with gravity force \(f_g = \rho g\Omega \sin(\alpha)\) where \(l\) is the tri-phase line length, \(\rho\) is the density of the liquid, \(\Omega\) is the volume of the liquid, \(g\) is the gravity constant. The equilibrium condition is satisfied when \(f_p \geq f_g\).

In the case of a weld bead deposition, a molten pool is forming by deposition of many metal liquid drops. The volume of the molten pool is very difficult to estimate. According to 2D topology profile, an area section can be correctly computed. To simplify the volume estimation, we suppose that \(l \approx \Omega/A\) where \(A\) is the area section of the weld bead. We can express tension force per unit length as \(f_p^l = \gamma \Delta \theta \sin(\theta)\) where hysteretic contact angle \(\Delta \theta = \theta_a - \theta_r\) and mean contact angle \(\theta = \frac{\theta_a + \theta_r}{2}\) and gravity per unit length \(f_g^l = \rho g A \sin(\alpha)\). The equilibrium condition can be written as \(f_p^l \geq f_g^l\). To construct a complete profile of 2D topology weld bead on an inclined plan, we can solve the following system of equation:
\[
\begin{align*}
\frac{dx}{dz} &= \frac{1}{\tan(\theta_0)} \\
0.5\kappa^{-2}z^2 - C_0 z &= 2(\cos(\theta_0) - 1) \\
A(f)(C_0) &= A \\
\gamma \Delta \theta \sin(\theta) &= \rho g A \sin(\alpha)
\end{align*}
\]

(6)

Where \( f \) is Laplace profile of the weld bead.

We solve the system step by step as following:

- First, solving for the Laplace profile \( f \) for a given \( \theta_0 \) and \( C_0 \) by the first two equations of the equation system (6);
- Second, with the Laplace profile, solving for \( C_0 \) to satisfy the third equation of the equation system (6);
- Third, solving for \( \theta_0 \) such that the last equation of the equation system (6) is satisfied. The angle can be called the critical Laplace contact angle;

The validation of the proposed weld bead deposition on an inclined plan will be performed with the experimental specimen manufactured with different inclined angles. The results are illustrated in the discussion section.

3 RESULTS AND DISCUSSIONS

3.1 Calibration and validation of power source model

In this section, the fusion flow constant parameters \( C_1 \) and \( C_2 \) of the power source model are calibrated with the mean value of the measured wire speed of \( CT = 15\text{mm} \) (blue line as illustrated in fig. 5). Then, with calibrated values, the power source model is used to simulate the wire speed to validate with the mean values of the measured wire speed of \( CT = 10\text{mm} \) and \( 20\text{mm} \). Table 2 indicates the synergy parameters used in power source generator.
Table 2: Parameters for the power source generator.

| $V_{oc}$ (V) | $ws$ (m/s) | $r_w$ (mm) | $l_s$ (mm) |
|--------------|------------|------------|------------|
| 17.2         | 0.133      | 0.5        | 4          |

Using the calibration method as indicated in the section 2.1, table 3 shows the values of the fusion flow constant parameters identified for $CT = 15mm$.

Table 3: Calibrated value of the fusion flow constant parameters.

| $C_1 \times 10^{-11}$ | $C_2 \times 10^{-11}$ |
|------------------------|------------------------|
| 9.6                    | 87                     |

Table 4 displays the comparison between the experimental wire speed and the simulated one. The maximum error is under 5%. It shows a good agreement between the experimental and simulated wire speeds.

In the following section, the power source model is used with given process parameters in order to estimate the weld bead area section considering the solidification shrinkage.

Table 4: Comparison of the simulated and measured wire speed.

| $CT$ (mm) | $ws_{meas}$ (m/s) | $ws_{sim}$ (m/s) | $err.\,rela.$ (%) |
|-----------|-------------------|------------------|-------------------|
| 15        | 0.166             | 0.166            | 0                 |
| 10        | 0.151             | 0.155            | 2.9               |
| 20        | 0.169             | 0.178            | 4.8               |
3.2 Weld bead simulation for inclined plan

This section is dedicated to the validation of contact angles (advance and recession) of weld bead model on an inclined plan and experimental measurement. The inclined angles of 5.47°, 12.5°, 25.6°, 33.9°, and 44.2° are used to manufacture the weld beads.

Fig. 6 (a) and (b) illustrate the measured advance and recession contact angle of a weld bead on the inclined plan of 33.9°. Fig. 7 (a) depicts the evolution of surface tension and the gravity force in function of Laplace contact angle. At the intersection of the two forces, critical Laplace contact angle can be determined, then the advance and recession contact angles are estimated.

Fig. 7 (b) shows the simulated weld bead profile and the computed advance and recession contact angles. The simulated contact angles are close to the measurement one with the maximum error of 0.6°.

Table 5 displays the comparison between the advance and recession contact angles of different inclined angles. The maximum error is around 2°. When the inclined angle increases, the error between simulation and experiments decreases. Globally, the simulated contact angles are in good agreement with the experimental ones.

When the inclined angle increases, the gravity force in the direction of the inclined plan also increases. The balance of forces may not reach its equilibrium. The weld bead on inclined plan of angle 65° is simulated. Advance and recession contact angles are estimated around 99.5° and 79.5° respectively. Fig. 8 (a) shows the weld bead profile on the inclined plan. At 66°, the balance of forces cannot reach its equilibrium. This angle is considered as the inclined critical angle for a weld bead deposition with a given volume.

When the inclined angle decreases the gravity force along the inclined plan decreases. The capillary force also decreases because the hysteretic contact angle becomes smaller. The divergence of the resolution algorithm of equations (6) is observed due to the balance of forces. The optimal critical contact angle cannot be determined in this case. At the inclined angle of 1°, we can simulate the weld bead profile as shown in fig. 8 (b). The advance and recession contact angles are estimated around 60.6° and 61.1° respectively.
Figure 6: Measure of the advance (a) and recession (b) contact angle of a weld bead on the inclined plan of 33.9°.

Figure 7: (a) evolution of the gravity and capillary forces in function of contact angle and (b) simulated

Table 5: Comparison of the measure and simulated advance and recession contact angles of different inclined angles.

| α (°) | θ_a,meas (°) | θ_r,meas (°) | θ_a,sim (°) | θ_r,sim (°) |
|-------|--------------|--------------|--------------|--------------|
| 5.47  | 58.34        | 55.08        | 60.32        | 57.81        |
| 12.5  | 62.27        | 54.23        | 62.46        | 56.72        |
| 25.6  | 65.93        | 53.45        | 65.18        | 54.00        |
| 33.9  | 67.41        | 53.30        | 68.01        | 53.56        |
| 44.2  | 71.23        | 54.57        | 71.54        | 54.15        |
4 CONCLUSIONS

A methodology using simultaneously both a power source model and a physical topology model has been introduced in order to simulate the topology of a deposited weld bead on an inclined or a horizontal plan. The description of a simplified power source model has been detailed. The fusion molten flow constant parameters need to be calibrated with the measured wire speed for a given process parameter. The simulated wire speed is in good agreement with the measured one for different values of $CT$. The power source model is used to simulate the wire speed in order to estimate the area section from the conservation of the flow.

On an inclined plan, we have proposed a simplified physical model to simulate the 2D topology of a weld bead using the Laplace equation and the capillary-gravity force equilibrium. Step by step resolution method is proposed to solve the proposed system of physical equations.

Using this physical weld bead topology model coupling with a power source model, one can deposit a weld bead with different process parameters on an inclined plan. We need to extend the work to find a simplified model capable of simulating a weld bead profile deposited on a general support in order to assemble the weld bead to build the topology of a part manufactured by the WAAM process.

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