Delocalization induced by low-frequency driving in disordered superlattices

Dario F. Martinez and Rafael A. Molina
Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Str. 38, 01187 Dresden, Germany

We study the localization properties of disordered semiconductor superlattices driven by ac-fields. The localization length of the electrons in the superlattice increases when the frequency of the driving field is smaller than the miniband width. We show that there is an optimal value of the amplitude of the driving field for which the localization length of the system is maximal. This maximum localization length increases with the inverse of the driving frequency.

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Real materials always contain a certain degree of disorder since the atomic structure is never perfectly regular. In fact, many physical properties are either influenced or even mainly determined by this randomness. The understanding of the effects of disorder in the physical properties of a material is therefore of great practical importance and has played a central role in condensed matter physics in the last half-century[1].

One of the most simple disordered systems is the motion of a particle in a one-dimensional random potential. Realizations of this system can be studied experimentally, e.g. in semiconductor superlattices (SL) made by varying the alloy composition along one of the dimensions of a compound semiconductor such as AlxGa1-xAs[2]. With the technique of molecular-beam epitaxy the degree of control of the disorder in such systems can be very high. If the SL is long enough its eigenstates are exponentially localized due to disorder. Electrons in these localized states are spatially confined and their only contribution to transport is through thermally activated hopping. The main quantity of interest in this case is the localization length \( \lambda \) of the electron wave-functions, which is controlled by the ratio between the bandwidth \( \Delta \) and the strength of the disorder \( W \). A disordered system of length \( L > \lambda \) will behave as an insulator while a system with length \( L < \lambda \) will behave as a conductor[1,3].

The fascinating effects of radiation on the transport properties of SLs have been intensively studied both theoretically and experimentally in the last three decades[4]. One of the most interesting effects, first predicted by Dunlap and Kenkre, is dynamic localization (as defined in [3]), in which the electron wave-function can be strongly localized in the presence of an AC electric field. This effect was shown to be associated with miniband collapse[5] which occurs at the zeros of a Bessel function that depends on the field amplitude. At these points the width of the miniband goes to zero, the group velocity of a wave-packet vanishes and the electron becomes effectively localized. Experimentally, this effect was first observed as a suppression of current at some amplitudes of the AC electric field[6].

When a time-periodic driving is applied to a disordered SL, a new possibility emerges, i.e. controlling the localization of the system by changing the amplitude and/or frequency of the driving field. Holthaus et al. showed that for high frequencies, the Floquet states become localized (in contrast with dynamical localization) and their localization (participation ratio) depends on the ratio \( \Delta_{eff}/W \), where \( \Delta_{eff} = \Delta J_0(\epsilon V d/\hbar \omega) \). Here, \( \epsilon \) and \( \omega \) are the amplitude and frequency of the driving field and \( d \) is the spatial period of the lattice.

In this letter we calculate, for the first time, the localization length of a driven disordered SL and we focus on the previously unexplored low frequency regime. For this purpose we use a Floquet-Green function formalism which makes use of matrix continued fractions[8]. This formalism allows us to generalize the definition of \( \lambda \) for time-periodic systems and to calculate its value. We found that there are two distinct and clear-cut regimes in this system: High-frequency regime for \( \hbar \omega > (\Delta + W)/2 \) and low-frequency regime for \( \hbar \omega < (\Delta + W)/2 \). In the first one, we found that \( \lambda \) is a function of \( \Delta_{eff}/W \) in perfect agreement with previous works[8]. In this regime, the driving field always contributes to localize even more the wave-function (as compared to the non-driven case). In contrast, in the low frequency regime, we find that the driving can significantly delocalize the electrons. In addition to this new result, we provide an intuitive explanation of why this should be so: For low frequency, each additional Floquet channel created by the driving provides the electron with new paths which will differ in their degree of localization, with some having smaller and others having greater localization length as compared to the non-driven case. Since \( \lambda \) is determined only by the path with the greatest localization length, after the ensemble average has been performed, additional propagation channels should always contribute to increase \( \lambda \). The same situation does not occur in the high-frequency case because new paths introduced by the absorption or emission of one or more photons, always have localization lengths smaller than in the non-driven case.

We will model the one-dimensional disordered SL in the presence of an AC field by a single-band Anderson Hamiltonian with diagonal disorder[10] plus a time-periodic potential,

\[
H = -\frac{\Delta}{4} \sum_j (|j+1 \rangle \langle j| + |j \rangle \langle j+1|) + \sum_j \epsilon_j |j \rangle \langle j| + 2V \cos \omega t \sum_j |j \rangle \langle j| .
\] (1)
The on-site energies $\epsilon_j$ are distributed uniformly from $-W/2$ to $W/2$. The driving potential is due to an AC electric field of amplitude $2V$. (The factor of 2 is for convenience.) In a SL this term can represent THz radiation linearly polarized in the growth direction of the lattice. The quantity $\Delta$ is equal to the bandwidth of the non-driven system without disorder. We will use units in which $\hbar = 1$ and we set $\Delta = 4$.

For a system that obeys discrete time-translational symmetry with period $T$, there exists a complete set of solutions to the Schrödinger equation of the form $|\Psi^{\alpha,p}(t)\rangle = \exp(-i\epsilon_{\alpha,p}t/\hbar) |\phi^{\alpha,p}(t)\rangle$, where $|\phi^{\alpha,p}(t)\rangle = |\phi^{\alpha,p}(t+T)\rangle$. These are called the Floquet states of the system and the periodic functions $|\phi^{\alpha,p}(t)\rangle$ obey an eigenvalue equation similar to the static Schrödinger equation,

$$\left[H(t) - i\hbar \frac{d}{dt}\right] |\phi^{\alpha,p}(t)\rangle = \epsilon_{\alpha,p} |\phi^{\alpha,p}(t)\rangle,$$  \hspace{1cm} (2)

with $H(t)$ the time-periodic Hamiltonian of the system. The eigenvalues of this equation can be written as $\epsilon_{\alpha,p} = \epsilon_0 + p\hbar \omega$, for $0 \leq \epsilon_0 \leq \hbar \omega$ and $p$ an integer. A Floquet-Green operator corresponding to Eq. (2) can be defined and its Fourier components satisfy

$$G^{(k)}(E) \equiv \sum_{\alpha,p} \frac{|\phi^{\alpha,p}_{k+p}\rangle \langle \phi^{\alpha,p}_k|}{E - \epsilon_\alpha - p\hbar \omega},$$  \hspace{1cm} (3)

where $|\phi^{\alpha,p}_k\rangle$ are the Fourier components of the Floquet eigenfunctions $|\phi^{\alpha}_p\rangle = \sum_p e^{-ip\omega t} |\phi^{\alpha}_p\rangle$. The transport properties of driven systems have been formulated in terms of this Floquet-Green operator which plays a similar role to the Green operator in the Landauer formulism for conduction [11]. We generalize the usual definition of the localization length in terms of Green functions [3] and define the localization length of the $k$th Floquet component as

$$\frac{1}{\lambda^{(k)}(E)} = -\lim_{L \to \infty} \frac{1}{L} \left\langle \ln \left| G_{1L}^{(k)}(E) \right| \right\rangle.$$  \hspace{1cm} (4)

The different quantities $G_{1L}^{(k)}(E)$ are associated with the probability of a process where an electron starts with an energy $E$ at site 1 and ends at site $L$ with energy $E \pm k\hbar \omega$. Each one of these processes in principle can have a different localization length associated to it. However, assuming that the asymptotic behavior of the Floquet wave function is exponentially decreasing, it is easy to see that the dominant term in the sum over $p$ in Eq. (3) will always be the same, independently of the value of $k$. Therefore the functions $\lambda^{(k)}(E)$ are identical. From now on we will only compute $\lambda^{(0)}(E)$.

For the calculation of $G^{(0)}(E)$ we use a method developed by one of the authors (see [3] for details). We want to calculate the Floquet-Green operator for a periodic Hamiltonian of the form $H(t) = H_0 + 2\cos(\omega t)V$, where $H_0$ and $V$ are any time-independent operators in the Hilbert space of the system. The Floquet components of the Green operator for this Hamiltonian satisfy

$$(E + k\hbar \omega - H_0)G^{(k)} - V(G^{(k+1)} + G^{(k-1)}) = \delta_{k,0},$$  \hspace{1cm} (5)

These equations can be solved using matrix continued fractions. For the case $k = 0$, one gets

$$G^{(0)}(E) = (E - H_0 - V_{eff}(E))^{-1},$$  \hspace{1cm} (6)

where

$$V_{eff} = V^{+}_{eff}(E) + V^{-}_{eff}(E),$$  \hspace{1cm} (7)

with

$$V^{\pm}_{eff}(E) = V \frac{1}{E \pm \hbar \omega - H_0 - V \frac{1}{E \pm 2\hbar \omega - H_0 - V \frac{1}{E \pm 3\hbar \omega - H_0 - V \cdots}}}. \hspace{1cm} (8)$$

The convergence of equation (8) is system specific. For our Hamiltonian, Eq.(1), the number of bands necessary to ensure convergence increases linearly with $V L / \omega$. The numerical performance of our method is determined by the speed in the calculation of an $L \times L$-matrix inverse for each Floquet sideband. For our disordered system we know that the localization length in the middle of the band ($E = 0$) for $V = 0$ behaves as $\lambda_0 \approx 105 (\Delta/W)^2$ [12]. In this case it is also known that $\lambda$ as a function of the energy $E$ follows a parabolic law with a maximum at $E = 0$. For $|E| > (\Delta + W)/2$ the localization length decreases rapidly with $|E|$. In this work, we will always take $E = 0$.

In Fig.1 we show some examples of the results obtained for the ensemble average of $\ln G_{1L}(0)$ as a function of the
length of the system $L$. (We have dropped the superscript in the Floquet-Green operator.) As expected, a straight line fits the data very well. The wave-functions decay exponentially with the distance. The negative slope of these curves corresponds to the inverse of the localization length when $L \gg \lambda$. It is important to point out that the deviations from this exponential behavior are not due to a deficient ensemble average. For high-frequencies they show a regular pattern, a manifestation of dynamic localization which shuts off the tunnelling between the sites $L - 1$ and $L$ at the zeros of $J_0(2VL/\omega)$. In fact, for high-frequency and $L \gg \lambda$ one can show that the Floquet-Green function can be expressed analytically as $G_{1L}(0) = AJ_0(2VL/\omega) \exp(-L/\lambda)$. As shown in Fig. 1 for the second and last sets of data (from top to bottom), the thin (red online) continuous line representing this analytical result fits the numerical data very well. For low-frequencies, the Green function also decays exponentially. However, the deviations from this behavior cannot be expressed analytically and seem to occur equally on both sides of the straight-line fit.

We now show results for the high frequency regime, which has been discussed in the literature [8], although to the best of our knowledge, no calculation of $\lambda$ has been reported. The high frequency limit can be characterized as the regime in which the absorption or emission of any number of photons would leave the particle with an energy outside of the region where the eigenenergies of the non-driven system concentrate. This region is well known to have a width $\Delta + W$, and therefore, at the center of the band this condition is satisfied when $\omega > (\Delta + W)/2$. In the high-frequency regime, the results for $\lambda$ are obtainable from the function $\lambda_0(\Delta/W)$ but with a renormalized hopping term $\Delta \to \Delta J_0(2V/\omega)$. Fig. 2 shows, for $\omega = 20.0$ and for several different values of disorder, that the numerical data is in excellent agreement with Holthaus’ result. The minima of $\lambda$ correspond to the zeros of the Bessel function $J_0(2V/\omega)$.

In Fig. 3 we show the localization length $\lambda$ as a function of $V$ for low and high frequencies and for two values of disorder, $W = 10.0$ (top) and $W = 5.0$ (bottom). Here, one can see that there is a fundamental difference between those two regimes: For $\omega < (\Delta + W)/2$ and $V$ small, $\lambda$ increases with $V$, whereas for $\omega > (\Delta + W)/2$ it decreases. This delocalization of the wave function due to low frequency driving is the main result of this letter. For all the frequencies that we explored in the low frequency regime, as $V$ was increased $\lambda$ initially increased, then reached a maximum, and finally decreased (with some oscillations). This behavior can intuitively be understood if one assumes that the driving provides the electron with new propagating channels $n$, each one with a different localization length $\lambda_n$. According to this, one expects $\lambda = \max\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$. Given the linear dependence of the total number of Floquet sidebands with $V$, it is expected that $n \propto V$, and therefore $\lambda$ should initially increase with $V$. This is valid until we reach the maximum number of effective channels that can be supported in the region of the spectrum where the localization lengths are comparable (size $\approx \Delta + W$). Here, $\lambda$ reaches a maximum value $\lambda_{\text{max}}$. Beyond this point, $\lambda$ decreases with $V$ (with oscillations), as the weight of the Floquet eigenstates oscillates and shifts to energies outside the band.

In Fig. 4 we plot the data for $(\lambda_{\text{max}} - \lambda_0)/\lambda_0$ vs. $1/\omega$. 
If $\omega \tau < \frac{1}{\omega}$, the energy relaxation time $\tau = \frac{1}{\omega}$ seems to increase monotonically with $\omega$. However, in a real SL a limiting time-scale might appear, i.e., the energy relaxation time $\tau$. If $\omega \tau = 1$, the delocalization induced by the driving field could be limited. For a typical relaxation time $\tau = 10^{-12}$, the effective delocalizing a weakly disordered system than expected. The continuous curves correspond to a fit to a very simple mathematical model where the maximum number of propagating channels is inversely proportional to the frequency, $n_{\text{max}} = \frac{c}{\omega}$ and $\sigma$ is the standard deviation of each channel (with exponential probability function).

This figure suggests that low-frequency driving is more effective in delocalizing a weakly disordered system than expected. The continuous curves correspond to a fit to a very simple mathematical model where the maximum number of propagating channels is inversely proportional to the frequency, $n_{\text{max}} = \frac{c}{\omega}$ and $\sigma$. We also assume that each one of these channels has an exponential distribution function with standard deviation $\sigma$ (a Gaussian distribution gives similar results). We can see that this simple model fits the data very well. Despite the fact that some of the assumptions in this simple model are not formally justified, we believe that the agreement with many of the features in the numerical data supports our intuitive explanation for the delocalization of the wave-function at low frequencies. In our system, $\lambda_{\text{max}}$ seems to increase monotonically with the inverse of $\omega$. However, in a real SL a limiting timescale might appear, i.e., the energy relaxation time $\tau$. If $\omega \tau < 1$, the delocalization induced by the driving field could be limited. For a typical relaxation time $\tau = 10^{-12}$.

![Figure 4: Behavior of $\lambda_{\text{max}}$ as a function of $1/\omega$ for different values of disorder $W$. The numerical data has been fitted using a function $\lambda(n_{\text{max}}, \sigma)$ derived from a simple statistical model where the maximum number of effective channels is inversely proportional to the frequency, $n_{\text{max}} = \frac{c}{\omega}$ and $\sigma$ is the standard deviation of each channel (with exponential probability function).](image)

\[ \text{FIG. 4: Behavior of } \lambda_{\text{max}} \text{ as a function of } 1/\omega \text{ for different values of disorder } W. \text{ The numerical data has been fitted using a function } \lambda(n_{\text{max}}, \sigma) \text{ derived from a simple statistical model where the maximum number of effective channels is inversely proportional to the frequency, } n_{\text{max}} = \frac{c}{\omega} \text{ and } \sigma \text{ is the standard deviation of each channel (with exponential probability function).} \]

In this work we have calculated the localization length for one-dimensional SLs in a homogeneous AC electric field. We have found two very different regimes according to whether the frequency of the driving is smaller or bigger than the miniband width of the disordered SL. For low frequency driving, the localization length increases for moderate values of the driving amplitude, while for high frequency the localization length decreases due to the Bessel function renormalization of the miniband width. We have shown that low-frequency driving can increase the localization length by providing new propagation channels. In this sense, the effect of such a field can be compared to an increase in the dimensionality of the system, which suggests that, in disordered systems where a metal-insulator transition is expected when the dimension of the system increases, low-frequency driving could have very dramatic delocalizing effects.

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