A STATIONARY DRAKE EQUATION DISTRIBUTION
AS A BALANCE OF BIRTH-DEATH PROCESSES

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ABSTRACT

Previous critiques of the Drake Equation have highlighted its deterministic nature, implying that the number of civilizations is the same at all times. Here, I build upon earlier work and present a stochastic formulation. The birth of civilizations within the galaxy is modeled as following a uniform rate (Poisson) stochastic process, with a mean rate of $\lambda_C$. Each then experiences a constant hazard rate of collapse, which defines an exponential distribution with rate parameter $\lambda_L$. Thus, the galaxy is viewed as a frothing landscape of civilization birth and collapse. Under these assumptions, I show that $N$ in the Drake Equation must follow another Poisson distribution, with a mean rate $(\lambda_C/\lambda_L)$. This is used to demonstrate why the Copernican Principle does not allow one to infer $N$, as well evaluating the algebraic probability of being alone in the galaxy.

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A STOCHASTIC FORMALISM

The Drake Equation formulates the number of communicative civilizations in the galaxy, $N$, as product of 1) the star formation rate, 2) numerous conditional probabilities concerning life, and 3) the lifetime, $L$, of said civilizations. More succinctly, it’s the rate at which communicative civilizations emerge multiplied by their lifetime:

$$ N = \Gamma_C L. \quad (1) $$

The formulation wasn’t originally intended as a true calculator (Drake & Sobel 1991), rather as a pedagogical and organizational framework. A basic limitation is that for any specific choice for the inputs, $N$ is a fixed number - implying that at all times there is precisely the same number of civilizations (Cirković 2004). A better interpretation of the Drake Equation, then, is that it describes the mean number of civilizations, cast as the mean rate of emergence multiplied by the mean lifetime i.e. $E[N] = E[\Gamma_C]E[L]$.

Efforts to update the Drake Equation, from its original deterministic form to a probabilistic one, have been previously proposed (Forgan 2011). For example, Maccone (2010) highlight that given a lengthy series of multiplicative distributions, the Central Limit theorem dictates that $N$ is log-normally distributed - although following our earlier argument it’s more accurate to state that $E[\Gamma_C]$ is log-normally distributed. To make $N$ truly stochastic, rather than $E[N]$, Glade et al. (2012) (G12) proposed that a homogeneous Poisson process is suitable to describe the births of new civilizations.

A homogeneous Poisson process describes a stochastic process whose success rate is a time-independent quantity. If one chooses a recent epoch of $\lesssim 100$ Myr, the galaxy can be argued to hardly evolve and thus it’s reasonable to suggest that the mean rate of emerging civilizations is approximately constant during this time. Although the galaxy is made up of many different star-types, ages and environments, after averaging over the $\sim 10^{11}$ examples, the mean rate of emergence does not evolve.

In this note, I highlight that the formulation of G12 can be expanded to reveal a simple yet reasonable stochastic Drake Equation. Specifically, I adopt a stochastic model for the lifetime of civilizations following Kipping et al. (2020), who propose an exponential distribution. The basic assumption, similar to the Poisson process, is that the existential risk is the same in any given time interval (e.g. the probability of a fatal gamma ray burst (Piran 2014) doesn’t change in time).

Let us denote that the rate of civilization birth, $\Gamma_C$, follows a Poisson distribution characterized by the mean rate parameter $\lambda_C (\equiv E[\Gamma_C])$, such that the mean number of births in a time $t$ is $\lambda_C t$. Next, let us write that lifetime of civilizations, $L$, follows an exponential distribution characterized by a rate parameter $\lambda_L$, such that the mean lifetime of civilizations is $E[L] = 1/\lambda_L$. The Drake Equation is thus governed solely by $\Gamma_C$ and $L$, where $\Gamma_C \sim \text{Po}[\lambda_C]$ and $L \sim \text{exp}[\lambda_L]$, but what is the probability distribution for $N$ then?

Consider an infinitesimal time interval, $\delta t$. Let us write that the mean number of civilization births in this interval is $\delta n$. Via the theorem of linearity of expectation, the mean number of extant civilizations in this interval, $E[N]$, must equal $\delta n$ plus the mean number of survivors from earlier intervals. In the previous interval, the mean number of births must also be $\delta n$ (by definition of a Poisson process), but not all will survive. If one writes the risk of death as $\delta d$, then the mean number of survivors will be $\delta n(1-\delta d)$ from the previous cycle, $\delta n(1-\delta d)^2$ from the cycle before, etc. Summing over an infinite number of intervals one finds a convergent series where

$$ E[N] = \sum_{i=0}^{\infty} \delta n(1-\delta d)^i = \frac{\delta n}{\delta d}. \quad (2) $$

Let us now turn to $\delta d$. An exponential distribution is defined by a constant hazard function, such that the mean probability of failure in a time interval $\delta t$ equals $\lambda_L \delta t$. Using this, and taking the infinitesimal limit, one has

$$ E[N] = \frac{1}{\lambda_L} \frac{\partial n}{\partial t}. \quad (3) $$

Note that $\partial n/\partial t$ defines the mean number of civilizations birthed per unit time, which by definition equals $\lambda_C$, thus:
\( E[N] = \frac{\lambda_C}{\lambda_L} \) \hspace{1cm} (4)

Since the time interval \( \delta t \) is arbitrary, the above is true at all times and thus defines a Poisson process with a rate parameter \( \lambda_N = \frac{\lambda_C}{\lambda_L} \). Accordingly, one can write our new stochastic Drake Equation as \( N \sim \text{Po}[\lambda_C/\lambda_L](= \text{Po}[\lambda_N]) \). This remarkably simple form\(^1\) highlights how the number of civilizations present over various time intervals follows the classic Poisson distribution governed by just two terms - a balancing act of birth and death. This expression was verified through Monte Carlo experiments depicted in Figure 1. This formulation presents several key insights highlighted in what follows.

THE ABJECT FAILURE OF THE COPERNICAN PRINCIPLE

The Copernican Principle is often used to motivate the plurality of life (e.g. Westby & Conselice 2020). However, using our formulation, a complete failure of this principle is revealed, as a result of selection effects. The key is that any self-aware entity is living in a galaxy with at least one success, it cannot reside within the realization of \( N = 0 \). Consequently, \( N = 0 \) cases must be discounted, and so self-aware entities must be drawn from the truncated Poisson distribution

\[
\Pr(N | \text{selfaware}) = \begin{cases} 
\frac{1}{\exp(\lambda_N) - 1} \frac{\lambda_N^N}{N!} & \text{if } N \geq 1, \\
0 & \text{if } N = 0.
\end{cases} \hspace{1cm} (5)
\]

The Copernican Principle makes inferences about others based on itself as an example, and here the inference would be about \( \lambda_N \), which controls \( N \). Accordingly, one might now attempt to infer \( \lambda_N \) given one’s own existence - such that the “data” we are conditioning upon is that \( N \geq 1 \). Via Bayes’ theorem, one has

\[
\Pr(\lambda_N | N \geq 1, \text{selfaware}) \propto \Pr(N \geq 1 | \lambda_N, \text{selfaware}) \Pr(\lambda_N | \text{selfaware}), \hspace{1cm} (6)
\]

where the “selfaware” conditional is explicit throughout. The first-term on the right is known as the likelihood, and governs how the data informs our inference. In this case, the likelihood function can be solved analytically as

\[
\Pr(N \geq 1 | \lambda_N, \text{selfaware}) = \sum_{N' = 1}^{\infty} \Pr(N' | \text{selfaware}). \hspace{1cm} (7)
\]

Using Equation (5) and summing over all \( N' \) indices returns 1 - which is no surprise because the distribution is normalized over the interval \([1, \infty]\) by virtue of its truncation. Thus, the likelihood function is a constant and contains no information about \( \lambda_N \), and so inferences using the Copernican Principle are meaningless.

ARE WE ALONE?

Using our stochastic distribution, one can straight-forwardly calculate the probability that we are not alone in the galaxy. As before, since we know that we are self-aware, then probabilities using this work’s formulation should account for this and use Equation (5) - the truncated form. Accordingly, the probability that we are not alone (within some given volume governed by \( \Gamma_C \)) is

\[
\Pr(N \geq 2 | \text{selfaware}) = 1 - \frac{\lambda_C/\lambda_L}{e^{\lambda_C/\lambda_L} - 1} \hspace{1cm} (8)
\]

which is better than 0.5 for all \( (\lambda_C/\lambda_L) > 1.26 \) and approaches unity as \( (\lambda_C/\lambda_L) \gg 1 \). The \( \lambda_L \) rate is broadly unknown but, as an example, Simpson (2016) suggests \( \lambda_L = 0.002/\text{year} \) using the Doomsday Argument. In such a case, we are likely not alone if \( \lambda_C \gg 0.2/\text{century} \).

\(^1\) It is noted that an alternative derivation is possible exploiting the fact that for independent random variables the mean of the product equals the product of the means.
Figure 1. Panels A and B show a histogram (black) of the number of civilizations present in each snapshot of a Monte Carlo simulation with $10^6$ time steps with $\lambda_C = 5.5 \& \lambda_L = 1.5$ for A and $\lambda_C = 20.5 \& \lambda_L = 1.5$ for B. The colored points show the prediction of our stochastic distribution. Panel C shows the mean (black) and standard deviation (gray) of a broader set of examples, with the predictions shown as lines through each.

Finally, it’s highlighted that the mean number of civilizations who’ve ever arisen over a time interval $T$ is $\lambda_C T$, whereas the current number is $\lambda_C/\lambda_L$, and thus if $\lambda_C T \gg (\lambda_C/\lambda_L)$, or simply $T \gg E[L]$, then one expects there to be far more extinct civilizations that extant, in which case one might place greater weight on artifact SETI searches.

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