Experimental Study of Seismic Wave Velocity Tomography

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ABSTRACT

As a branch of seismic exploration methods, the seismic wave velocity tomography has importance in the geologic body velocity distribution inversion and the abnormal geologic body identification. In the beginning of this paper, the observation system, the coefficient matrix, projection and back projection transformations of the seismic wave velocity tomography, and specific applications of the iterative method are introduced through a simple numerical model. Comparing of slowness inversion results before and after the iteration shows: The iterative method improves the accuracy of tomography. Then, the observation system layout, coefficient settings of the data collecting system, the data processing method and explanations of inversion results are described combining with two cases of ground exploration in practice. The result shows: Specific locations of explored objectives can be determined through direct ray continual iterative inversion of seismic wave velocity tomography technology.

INTRODUCTION

The seismic tomography research is a complicated systematical program, has been focused on and developed incessantly due to its important guidance in the abnormal geologic body exploration research. In the narrow sense, the seismic tomography refers to the seismic wave transmission tomography which includes the wave velocity tomography, the attenuation tomography and the wave field tomography. The direct wave velocity tomography has unique advantages among seismic wave tomography technologies mainly because the continuity of geologic body mediums, easiness of measuring and extracting direct waves, speediness and efficiency of obtaining slowness distribution characteristics using appropriate path algorithms and consequently determining locations of abnormal bodies. In this paper, specific processes and actual application effects of the direct ray path

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A continuous iterative method is discussed in the direction of direct wave velocity tomography.

**BASIC THEORIES OF SEISMIC WAVE TRANSMISSION TRAVEL TIME TOMOGRAPHY**

Seismic tomography is a typical problem of geophysical inversion, most of which involve in following aspects: (1) model parameterization, (2) forward modelling (ray tracing), (3) inversion, (4) evaluation of solutions\(^1\). In the seismic tomography, the first step is to establish a model, arrange observation systems, simulate regional tiny anomalies by partition methods or node methods, turn a physical problem into a problem of finding solutions for a large-scale, sparse and usually sick linear system of equations, a matrix operation problem\(^2\). Grids should be well divided in order to obtain appropriate solutions. A division by space between source points or detection points is usually appropriate with improvement of tomography accuracy, and smaller divisions does not have a relationship with better effects\(^3\). Besides, rule out sick problems mathematically, and lead more prior information into the inversion process\(^4\). Start forward modeling after model parameterization, i.e. calculate theoretical travel time according to ray paths, and usually in two ways, one is direct ray simulation applying to homogeneous mediums, like the one proposed in passage\(^5\). Assume the path from the source point to the detection point is a straight line, thus to prepare for the inversion, units passed by every ray can be recorded and the number of rays passing through every grid unit can be counted. The other is curved ray simulation with several kinds, such as a shortest path algorithm based on Fermat Theory and Graph Theory which is proposed by Moser\(^6\), and the improved Dijkstra algorithm in paper\(^7\). The forward modeling provides theoretical arrival time of the direct wave, and prepares for the inversion. The inversion provides the value of slowness using the theoretical arrival time of the direct wave, actual arrival time and corresponding coefficient matrix. There are some inversion methods, such as the Back Projection Technology (BPT) which allots that passes through a grid unit to the whole length of the ray, and then verifies the slowness value of the grid unit with the sum of residual errors of travel time in the grid unit divided by the overall length of rays in the grid unit. It needs no iteration, and there is no divergent problem but the resolution is low\(^8\). Algebraic Reconstruction Technology (ART) is an iterative algorithm which distributes residual errors of travel time into every grid unit passed by ray according to the number of the ray, and then verifies grid unit slowness in turn without consideration of ray lengths in grid units. The problem of ART is that it has bad convergence. Unlike verifying slowness ray by ray in ART, Successive Iterative Reconstruction Technology (SIRT) verifies all slow values of grid units by substituting them with results from the previous round iteration\(^9\). In addition, SIRT has no limitations in tomographic inversion by direct waves between two wells or alleys due to well-matched relationship of coefficient matrix and the travel time residual error\(^10\). Influences of curved rays can be taken into consideration in the Successive Iterative Method, just insert a ray tracing process before calculating values of theoretical travel time\(^11\).
Establish a Simple Numerical Model

Establish a simple observation system. The observation area can be divided into four grids. X1, X2, X3 and X4 represent slowness values of the four grids respectively. There are six launching points and six receiving points around the observation area. One launching point corresponds to one receiving point. There are six rays in total, and there are six corresponding travel time which are t1~t6. The model is shown as in figure 1.

Figure 1. Model of seismic wave straight line transmission.

The following equations can be used to calculate the travel time of each ray.

\[
egin{align*}
1 \times X1 + 1 \times X2 + 0 \times X3 + 0 \times X4 &= t1 \\
0 \times X1 + 0 \times X2 + 1 \times X3 + 1 \times X4 &= t2 \\
1 \times X1 + 0 \times X2 + 1 \times X3 + 0 \times X4 &= t3 \\
0 \times X1 + 1 \times X2 + 0 \times X3 + 1 \times X4 &= t4 \\
1.4142 \times X1 + 0 \times X2 + 0 \times X3 + 1.4142 \times X4 &= t5 \\
0 \times X1 + 1.4142 \times X2 + 1.4142 \times X3 + 0 \times X4 &= t6
\end{align*}
\]

If written in the form of matrix

\[
A \times X = T
\]  

(1)

In which A is a coefficient matrix, X is a slowness matrix, T is a travel time matrix. In this case, A=

\[
\begin{bmatrix}
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 \\
1.4142 & 0 & 0 & 1.4142 \\
0 & 1.4142 & 1.4142 & 0
\end{bmatrix}
\]

X=[X1; X2; X3; X4]

Assume the slowness distribution is unknown, then assume from experience that each value of slowness in figure 1 is 100, i.e. X=[100; 100; 100; 100]

Denote the theoretical arrival time of the direct wave by T0, then

\[
T0 = A \times X = [200; 200; 200; 200; 282.84; 282.84]
\]
Disturb the slowness, and the slowness distribution is no longer homogeneous. For example, X=[100; 80; 60; 40]

The estimated theoretical travel time by the shortest travel time path tracing method is

\[ T = \begin{bmatrix} 180; 100; 160; 120; 198; 198 \end{bmatrix} \]

There is little difference between observed travel time and the estimated theoretical value. Then assume the direct wave travel time is \( T = [180, 110, 170, 130, 210, 200] \). Invert slowness X for first time according to A and T.

The result is \( X = \text{pinv}(A) * T = [102.38; 78.847; 63.847; 47.382] \)

In which \( \text{pinv}(A) \) is a function used to get the pseudo inverse of a matrix in MATLAB software.

Comparison and error analysis of slowness before and after first inversion is shown in Table 1.

| Slowness Module | True slowness value (ms/100m) | Inverted slowness value (ms/100m) | Error(%) |
|-----------------|-------------------------------|-----------------------------------|---------|
| X1              | 100                           | 102.38                            | 2.38    |
| X2              | 80                            | 78.847                            | -1.44   |
| X3              | 60                            | 63.847                            | 6.41    |
| X4              | 40                            | 47.382                            | 18.45   |

It can be seen from Table 1 that first inversion of the slowness distribution can be qualitatively evaluated by the above-mentioned coefficient matrix A, but with a big quantitative error. One of the reasons for errors is that successive iterative reconstruction technology is not used.

Now still use the coefficient matrix, do slowness inversion calculations by the successive iterative reconstruction technology (SIRT). Codes of slowness inversion by the SIRT method in MATLAB software are shown as below.

```
clc
clear
X0=[100 60 ;80 40];
A=[1 1 0 0;0 0 1 1;1 0 1 0;1 0 1:0 1:4142 1:4142 0;1:4142 0 0 1:4142];
T=[180 ;110 ;170 ;130 ;210 ;200];
n=100; N=4; err=1e-6;
figure(1); imshow(X0,
[min(X0(:)) max(X0(:))]); print(1, '-dbmp', 'InitialPic');
X=ones(N,1); Xerr=ones(N,1); Sclm=sum(A); k=n;
while k>0
    T1=A*X;
    for j=1:N
        AT=A(:,j).* (T-T1); B=sum(AT); C=0.0001+sum(Sclm,2);
        Xerr(j)=B/(C*Sclm(j)); X(j)=X(j)+Xerr(j);
    end
    er=sqrt(sum(Xerr.^2)/N);
    if er<err
        k=0;
    else
        k=k-1;
    end
end
```
X; Xerr; Xre=reshape(X,2,2);
figure(2); imshow(Xre,[min(X(:)) max(X(:))]); print(2, '-dbmp', ' XSirtPic');
The result after operation is $X=[98.5040; 82.2886; 67.4757; 44.1901]$

Comparison and error analysis of the slowness distribution before and after one hundred time iterations are shown in Table 2.

Table 2. Comparison of real value and 100th iteration value.

| Slowness module | True slowness value(ms/100m) | Slowness value after iterations(ms/100m) | Error(%) |
|----------------|-----------------------------|----------------------------------------|---------|
| X1             | 100                         | 98.504                                | -1.49   |
| X2             | 80                          | 82.289                                | 2.86    |
| X3             | 60                          | 67.476                                | 12.45   |
| X4             | 40                          | 44.19                                 | 10.47   |

Comparing Table 1 to Table 2, it can be seen that the maximum slowness error before iterations is 18.45%, the maximum slowness error after iterations is 12.45%, and the maximum error has a one third decrease after iterations. We can see from above that the SIRT method can decrease errors effectively, and increase the inversion accuracy if the coefficient matrix $A$ of the observation system is unchanged.

**APPLICATION EXAMPLE ONE**

Establish a Seismic Wave Observation System

There is a concrete road in the middle of the campus lawn. We arrange twelve vibration sources and twelve detectors on two sides of the road to establish a seismic waves observation system. The layout of the observation system on the experimental spot is shown in figure 2.

![Figure 2. Seismic waves observation system of road base.](image)

Model Parameterization

Establish a coordinate according to the observation system, divide slowness grid units (i.e. picture elements), and simulate paths of rays. The seismic wave CT software can calculate the length of rays that pass through each unit according to coordinates of shot points and detection points.
Seismic Wave Data Collection and Preprocessing

Artificially excited hammer sources are used for seismic wave data collecting, the signal collecting instrument is KDZ1114-3 type mine geologic detection instrument, and TZBS series 100 Hz sensors are used as wave detectors. Main sampling parameters are set in table 3.

| Range of Receiving Frequency | 0-4000Hz |
|-----------------------------|----------|
| sampling interval           | 120µs    |
| sampling number             | 1024     |
| fixed gain                  | 0dB      |
| advancing sampling number   | 0        |

In a method of 1 excitation and 12 detectors receive simultaneously, excite seismic wave point by point, and 144 seismic records can be obtained. Splice the records by the order of every shot (12 in total), the result is shown as in Figure 4.
Result of Seismic CT Inversion and Analysis

Collect the arrival time of direct waves in the seismic records and save, then invert slowness values in the method of SIRT in the seismic CT simulation software, and then get values of velocity of all grid units. Velocity distribution diagram is shown as in Figure 5.

![Velocity distribution inversion result of road base detection.](image)

Figure 5. Velocity distribution inversion result of road base detection.

By comparing Figure 5 to Figure 2, it can be seen that the road location got by velocity inversion and the actual location match well, which means direct ray inversion can be used in continuous mediums. After solving problems concerning data collection and arrival time pickup, the main reason of low image accuracy is that there are defects in the observation system, i.e., rays of the imperfect observation system have an inhomogeneous distribution, distinct blank slits in the middle can be seen in figure 3. If rays do not pass through a certain grid unit, or the ray density of this grid unit is low, values got after calculation are close to values in the background, and some defect in the middle of the road may appear.

APPLICATION EXAMPLE TWO

Establish a Model of Observation System

There is tree bed in the middle of the campus lawn which can be viewed as a high-speed abnormity area. Arrange 20 sources and 20 detectors on two sides of the tree bed according to locations designed to establish a seismic wave CT observation system. The layout of the observation system on the trial spot is shown as in figure 6.

![Observation system of tree bed.](image)

Figure 6. Observation system of tree bed.
Model Parameterization

Divide grid units of slowness and simulate ray paths. Ray paths of simulation are shown in Figure 7.

![Figure 7. Straight line modeling in tree bed detection.](image)

Seismic Wave Data Collection and Preprocessing

Artificially excited hammer sources are used for seismic wave data collecting, the signal collecting instrument is KDZ1114-3 type mine geologic detection instrument, and TZBS series 100 Hz sensors are used as wave detectors. Main sampling parameters are set as in table 4.

| Range of Receiving Frequency | 0-4000Hz  |
|-----------------------------|-----------|
| sampling interval           | 60µs      |
| sampling number             | 1024      |
| fixed gain                  | 72dB      |
| advancing sampling number   | 0         |

In a method of 1 excitation and 4 detectors receive simultaneously, excite seismic wave point by point, 400 seismic records can be obtained. Splice the records by the order of every shot (20 in total). The result is shown as in Figure 8.
Result of Seismic CT Inversion and Analysis

Pick out the arrival time of direct waves in the seismic records and save, then invert in the method of SIRT in the CT simulation software to get values of slowness, and then get values of velocity of all grid units. Velocity distribution is shown as in Figure 9.

By comparing Figure 9 to Figure 6, it can be seen that the tree bed location got by velocity inversion and the actual location match well, which further means direct ray slowness inversion can be used in continuous medium and can determine the location of the abnormal body. There are some false images in the figure as
well, typically high-speed abnormality band extended from the tree bed to four angles, this phenomenon is also presented in the paper[12], and it is probably a common feature of a round abnormal body in slowness inversion in the method of SIRT.

CONCLUSIONS

(1) Accurate locations of geologic abnormal bodies in continuous mediums can be determined in the method of direct ray path SIRT inversion.
(2) Parallel two sides observation system is imperfect, and leads to partial false images in the inversion figure.
(3) Explanations about geologic abnormal bodies need to be made with documents of geologic background, and with characters of numerical simulation imaging as well.

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